Loop quantum black hole in a gravitational collapse model

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The gravitational collapse plays an important role in the formation of black holes as well as for our understanding of the spacetime structure. In this paper, we propose the exterior effective spacetime that are well matched to the interior effective model of loop quantum cosmology for the Datt-Oppenheimer-Snyder gravitational collapse model. The analysis shows that, as the collapse goes on, the quantum-corrected black hole can form with the occurrence of horizon. The quantum gravitational effects will stop the collapse of the dust matter when the energy density reaches the Planck scale and bounce it to an expanding phase, resulting in the resolution of the singularity of the classical black hole. Moreover, the quantum gravitational corrections can affect the black hole shadows by their sizes. The stability of the quantum-corrected black hole under perturbations is also studied by calculating the quasinormal modes. It turns out that the quantum-corrected black hole is stable against the scalar and vector perturbations.

I. INTRODUCTION

The singularity theorems in general relativity (GR) predicts the formation of black holes (BHs) and spacetime singularities [1, 2]. The appearance of singularities implies the breakdown of GR when spacetime curvature increases unboundedly. Therefore, it is nature to expect that some quantum theory of gravity becomes domain in these regions and can cure the singularities. Up to now, some competitive quantum theories of gravity have been proposed. Among them, loop quantum gravity (LQG), a background-independent and non-perturbative approach to quantum gravity, has received considerable attention and has been widely investigated (see, e.g., [3, 4] for books, and [5–11] for articles). After thirty years of theoretical research, the canonical (Hamiltonian) and the covariant (Lagrangian) formulations of LQG have achieved individual and remarkable successes. Besides the predictions of discretized geometries and the interpretation of BH entropy in the canonical formulation [12–19], well-defined dynamical descriptions in both formulations were proposed, and the consistency between them was checked in certain senses [20–31]. Gravity coupled to matters were studied and revisited in LQG, leading to the resolution of some long-standing problems [21, 32, 33]. Furthermore, loop quantization program has been successfully extended to high-dimensional theories of gravity as well as alternative gravitational theories [34–46].

Aiming to test the ideas and techniques of full LQG and to quantize the symmetry-reduced phase space of the theory, loop quantum cosmology (LQC) has made more substantial progress in solving the cosmological big-bang singularity [47, 48]. At the early stage, the big-bang singularity is resolved in the sense that quantum dynamical equation for the spatially flat Friedmann-Robertson-Walker (FRW) model is well defined at the big-bang singularity though which it can be evolved smoothly [47, 48]. As the numerical techniques improve, the quantum dynamical equation is solved numerically, and it shows that the big-bang singularity is replaced by a quantum bounce [49–51]. These achievements are based on the successful implementation of quantum dynamics in LQC. To determine quantum dynamics, one needs to quantize the Hamiltonian constraint for LQC in a suitable way. The gravitational Hamiltonian constraint consists of two terms, the Euclidean and Lorentzian terms. They are proportional to each other due to the homogeneity and flatness of spatial surface. For simplicity, one can quantize only the combined Euclidean term in LQC. Alternatively, inspired by the implementation of the gravitational Hamiltonian constraint in full LQG, these two terms can also be treated independently and then be quantized in a different way in LQC [52]. It turns out that the former leads to a symmetric quantum bounce, while the latter results in an asymmetrical bounce for gravity coupled to a massless scalar field [51–55]. It is worth noting that in these works the Hamiltonian constraint was quantized by employing the Thiemann’s trick in LQG. It has been shown that asymmetrical bounces hold for different quantization schemes [56–59]. Although achievements have made in LQC, a systematical derivation of LQG from LQC is still absent up to now. Nevertheless, some progress has been made on the relation between LQG and LQC by calculating the expectation value of the Hamiltonian in LQG under certain coherent state peaked states [52]. In Ref. [52] takes the same form as the effective one obtained by a suitable semiclassical analysis of Thiemann’s Hamiltonian in full LQG [65].

Taking into account the isometry between the interior of Schwarzschild BH with the Kantowski-Sachs spacetime, the techniques developed in LQC is employed to quantize the interior of Schwarzschild BH, leading to resolution of Schwarzschild BH singularity in certain senses [66–74]. In addition, the techniques of LQG were employed directly to quantize the spherically symmetric gravitational collapse plays an important role in understanding the formation of BH and its...
singularity in GR. Classically, the first gravitational collapse model was independently constructed by Datt [80] and by Oppenheimer and Snyder [81] (OSD model). In this model, the interior sourced matter is assumed to be a spherically symmetric and homogeneous pressureless dust, and thus it can be modeled by a FRW model. Due to the simplicity, this model is exactly solvable, providing us with a new widow to understand more complicated and realistic dynamical processes of gravitational collapse. There is no doubt that to have a competitiveness of LQG on the gravitational collapse models are being studied [82–90]. In particular, an effective Hamiltonian and an effective metric for the vacuum exterior solution for the OSD collapse model were derived in the effective context of spherically symmetric spacetime [84]. By matching the effective exterior solution, it turns out that the effective interior spacetime can be described by the LQC model in [49]. In this way, the interior collapse of dust matter field stops when its energy density reaches the Planck scale by the quantum gravity effects. The interior region $\mathcal{M}^-$ on shadows and quasinormal modes of BHs. Our results are summarized and discussed in Sec. IV.

II. THE STRATEGY TO GENERATE EXTERIOR SOLUTIONS

In this section, we consider the OSD model and set up our notations. In this model, the spacetime manifold $\mathcal{M}$ is divided into two regions, the interior region $\mathcal{M}^-$ and the exterior region $\mathcal{M}^+$, by its timelike boundary 3-surface $\Sigma = \mathcal{M}^- \cap \mathcal{M}^+$. The former is assumed to be a FRW spacetime region sourced with a spherical dust ball (cloud) with homogenous density, while the latter is described by a static and spherically symmetric spacetime region. To match these two regions at $\Sigma$, suitable boundary conditions need to be imposed. In the classical theory, the conditions have been studied by Darmois and Israel [91, 92], and hence are called as the Darmois-Israel junction conditions. In what follows, we will match the exterior to the interior of dust collapsing star via the Darmois-Israel junction conditions following Refs. [93–96], so that the exterior solutions could be generated dynamically by the interior solutions.

A. Matching the interior and exterior metrics

The interior region $\mathcal{M}^-$ consists of a dust ball of uniform perfect fluid with zero pressure, which can be described by a FRW cosmological model with the following line element

$$ds^2 = -d\tau^2 + a(\tau)^2 \left[ d\chi^2 + h(\chi)^2 d\Omega^2 \right], \quad (2.1)$$

where $\tau$ denotes the proper time of the comoving observer, $a(\tau)$ is the scale factor, $d\Omega^2 \equiv d\theta^2 + \sin^2 \theta d\phi^2$, and the function $h(\chi)$ is given by

$$h(\chi) = \begin{cases} \sin \chi, & \text{closed FRW model} \\ \chi, & \text{spatially flat FRW model} \\ \sinh \chi, & \text{open FRW model} \end{cases} \quad (2.2)$$

The boundary surface $\Sigma$ of $\mathcal{M}^-$ is located at a constant $\chi = \chi_0$ in the comoving coordinates, and can be parametrized by $(\tau, \theta, \phi)$. Then the induced line element on $\Sigma$ from the line element (2.1) of $\mathcal{M}^-$ reads

$$ds^2|_{\Sigma} = -d\tau^2 + a(\tau)^2 h(\chi_0)^2 d\Omega^2. \quad (2.3)$$

In the exterior region $\mathcal{M}^+$, we assume that the metric is still stationary and can be expressed in Schwarzschild coordinates $(\eta, r, \theta, \phi)$ as

$$ds^2 = -F(r) d\eta^2 + L(r)^{-1} dr^2 + r^2 d\Omega^2. \quad (2.4)$$

As seen from the outside, the surface $\Sigma$ can be described by the parametric equations $r = r(\tau)$ and $t = t(\tau)$. Thus, the induced line element on $\Sigma$ from the exterior line element (2.4) is given by

$$ds^2|_{\Sigma} = -\left( F' \tau^2 - L^{-1} \dot{r}^2 \right) d\tau^2 + r(\tau)^2 d\Omega^2, \quad (2.5)$$

where a dot over a letter denotes its derivative with respect to $\tau$.

To match the exterior region $\mathcal{M}^+$ with the interior region $\mathcal{M}^-$ along the boundary surface $\Sigma$ so that $\Sigma$ forms a unique surface in the entire spacetime $\mathcal{M}$, one needs to impose suitable boundary conditions (junction conditions). In the classical theory, the Darmois-Israel junction conditions [91–93] require that the first and the second fundamental forms on the two sides of the boundary surface $\Sigma$ equal to each other, respectively. In our case, the match of the first fundamental form yields

$$1 = F \dot{F} - L^{-1} \dot{r}^2 \quad \Leftrightarrow \quad F \dot{r} = \sqrt{F L^{-1} + \dot{F}} \equiv \beta(r, \dot{r}), \quad (2.6)$$

$$a(\tau) h(\chi_0) = r(\tau). \quad (2.7)$$

To calculate the extrinsic curvature (the second fundamental form) of $\Sigma$, we chose the direction of its normal towards $\mathcal{M}^+$. 
Then the components of extrinsic curvature as seen from $M^-$ read
\[ K^\tau_\tau = 0, \quad K^\theta_\theta = \frac{K^{-\phi}_\phi}{\sin^2 \theta} = a(\tau) h(\chi_0) h'(\chi_0), \] (2.8)
where $h'(\chi_0) \equiv \frac{\partial h(\chi)}{\partial \chi} |_{\chi=\chi_0}$. By using Eq. (2.6), the components of extrinsic curvature as seen from $M^+$ can be calculated as
\[ K^\tau_\tau^+ = -\frac{\beta}{r} \sqrt{F^{-1}L}, \quad K^\theta_\theta^+ = \frac{K^\phi_\phi^+}{\sin^2 \theta} = r \beta \sqrt{F^{-1}L}. \] (2.9)
Hence matching the extrinsic curvatures on the two sides of $\Sigma$ leads to
\[ 0 = -\frac{\beta}{r} \sqrt{F^{-1}L}, \quad a(\tau) h(\chi_0) h'(\chi_0) = r \beta \sqrt{F^{-1}L}. \] (2.10, 2.11)
Up to now, we have obtained the junction conditions determined completely by Eqs. (2.6), (2.7), (2.10), and (2.11). To satisfy Eq. (2.10), it is sufficient to require that the integral curve of $\partial/\partial \tau$ is geodesic in $M^+$. Since $\partial/\partial \tau$ is a Killing vector field in $M^+$, $\beta$ is just the conserved energy along the geodesic tangent to $\partial/\partial \tau$, i.e.,
\[ E = g_{ab} \left( \frac{\partial}{\partial \tau} \right)^a \left( \frac{\partial}{\partial \tau} \right)^b = F i = \beta, \] (2.12)
which leads to $\beta = 0$. Then, combing Eq. (2.11) with Eq. (2.7), we get
\[ \beta = h'(\chi_0) \sqrt{FL^{-1}}. \] (2.13)
Combing Eq. (2.6) with Eq. (2.13) and annihilating $\beta$, we have
\[ r^2 = \left[ h'(\chi_0) \right]^2 - L. \] (2.14)
Taking the derivative of Eq. (2.7) with respect to $\tau$ and using Eq. (2.14), we arrive at
\[ L = \left[ h'(\chi_0) \right]^2 - H^2 r^2, \] (2.15)
where $H(\tau) \equiv \frac{1}{a(\tau)} \frac{da(\tau)}{d\tau}$ is the Hubble parameter. Thus, by matching $M^+$ to $M^-$ along $\Sigma$ generated by geodesics, the junction conditions lead to the three equations (2.7), (2.13), and (2.15). Equation (2.7) gives a relation between the two coordinates on $\Sigma$, while Eqs. (2.13) and (2.15) determine a dynamical relation between the two regions.

B. The exterior solutions generated dynamically by the interior solutions

Now we impose the dynamical equation for the interior region $M^-$. In the classical theory, the dynamics of the interior spacetime satisfies the Friedmann equation
\[ H^2 = \frac{8 \pi G}{3} \rho - \frac{k}{a^2}, \] (2.16)
where $G$ is the Newtonian gravitational constant, $\rho(\tau) = \frac{M}{4 \pi r^3}$ is the energy density in $M^-$, and $k = +1, 0, -1$ for the closed, flat, and open FRW models, respectively. Inserting Eq. (2.16) into Eq. (2.15) and using Eq. (2.7), we find
\[ L = \left[ h'(\chi_0) \right]^2 + k \rho G r^2 = 1 - \frac{8 \pi G}{3} \rho r^2 = 1 - \frac{R_s}{r}, \] (2.17)
where $R_s \equiv 2GM$ denotes Schwarzschild radius. Combining Eq. (2.17) with (2.13) yields
\[ F = \left[ \frac{\beta}{h'(\chi_0)} \right]^2 \left( 1 - \frac{R_s}{r} \right). \] (2.18)
Hence, by matching $M^+$ to $M^-$ and using the interior dynamical equation (2.16), the exterior metric (2.4) can be generated as
\[ ds^2 = - \left[ \frac{\beta}{h'(\chi_0)} \right]^2 \left( 1 - \frac{R_s}{r} \right) dr^2 + \left( 1 - \frac{R_s}{r} \right)^{-1} dr^2 + r^2 d\Omega^2. \] (2.19)
Absorbing $\beta/h'(\chi_0)$ into $r$ in Eq. (2.19) yields
\[ ds^2 = - \left( 1 - \frac{R_s}{r} \right) dr^2 + \left( 1 - \frac{R_s}{r} \right)^{-1} dr^2 + r^2 d\Omega^2. \] (2.20)
This is nothing but Schwarzschild metric as expected.

We assume that the Darmois-Israel junction conditions be still valid for the effective theory, which is widely adopted in the literature [82, 87, 97]. In addition, to achieve the aim that the quantum effects in the interior can be carried into the exterior, one needs also to specify the interior effective dynamical equation. We focus on the spatially flat FRW model with $k = 0$. There are alternative approaches to derive the effective Friedmann equation of spatially flat FRW model in LQC. In particular, the two quantization schemes in [51–53] have been studied deeply in the literature. Note that the resulting effective Friedmann equations obtained in these two schemes are suitable for arbitrary matter sources though they are derived with a massless scalar field. The reason behind this is that the quantum corrections to Friedmann equation arise completely from the quantum modification in the gravitational part. In the scheme in [51] the backward evolution of the flat FRW universe will reach the critical energy density of matter, and then be symmetrically bounced into another universe, resolving the classical big-bang singularity. The effective Friedmann equation reads [51]
\[ H^2_{\text{eff}} = \frac{8 \pi G}{3} \rho \left( 1 - \frac{\rho}{\rho_c} \right), \] (2.21)
where $\rho_c = \frac{3}{8 \pi G \Delta}$ is the critical energy density. The parameter $\Delta = 4 \sqrt{3} y l_p^2$ is the area gap, where $l_p^2 = G\hbar$ and $y$ denotes the Immirzi parameter whose value has been fixed as 0.2375 by calculating BH entropy [98, 99]. We now consider the dynamics of the interior spacetime determined by the effective
equation (2.21). Therefore, repeating the matching procedure, the quantum-corrected exterior metric can be obtained as
\[
\text{d}s^2_{\text{eff}} = -f(r)\text{d}t^2 + f(r)^{-1}\text{d}r^2 + r^2\text{d}\Omega^2,
\] (2.22)
where
\[
f(r) = 1 - \frac{R_s}{r} + \gamma^2\Delta \frac{R_s^2}{r^4}.
\] (2.23)

It is remarkable that the effective exterior metric in Eq. (2.22) is just the one obtained directly by loop quantizing the spherically symmetric exterior spacetime in [79]. By the interior-exterior matching procedure presented above, we obtain the effective exterior metric (2.22). It is easy to see that the effective metric (2.22) tends to the classical metric (2.20) as \(\Delta \to 0\). This behavior also holds for large scale with \(r \gg \sqrt{\Lambda}\). Hence the effective exterior metric (2.22) go to the classical Schwarzschild metric as expected.

The energy density \(\rho_e\) in \(M^+\) measured by the static observer can be obtained as
\[
\rho_e(r) = -\frac{1}{8\pi G} \frac{G_{tt}}{g_{tt}} = \frac{1}{8\pi G} [3H(r) + 2rH'(r)],
\] (2.24)
where \(G_{tt}\) and \(g_{tt}\) denote the \(tr\)-components corresponding to the Einstein tensor \(G_{ab}\) and the metric tensor \(g_{ab}\), respectively. Here the Hubble parameter \(H\) should be understood as a function of \(\rho = \frac{M}{4\pi r^3}\), and thus as a function of \(r\), by using the Friedmann equation. In the classical case, the vanishing energy density determined by Eq. (2.24) indicates the the vacuum exterior. A straightforward calculation shows that the effective energy density reads
\[
\rho_e^{\text{eff}} = \frac{3}{8\pi G} \left(\frac{R_s}{r^3}\right)^2 \gamma^2 \Delta.
\] (2.25)

C. Effective descriptions of collapse model

As collapse goes on, a quantum-corrected BH will form with the occurrence of horizon for a large \(M\). Note that the exterior metric (2.22) is static. Hence the vector field \(\xi^a = (\frac{\partial}{\partial t})^a\) is a Killing vector field. By definition, the Killing horizon \(\mathcal{K}\) is a null hypersurface determined by \(\xi^a\xi_{a|\mathcal{K}} = 0\). Therefore, the Killing horizon corresponds to the surface with radius \(r\) satisfying \(f(r) = 0\). The functions \(f(r)\) of the quantum-corrected BH is compared to that of Schwarzschild BH in Fig. 1. Interestingly, it is easy to see from Fig. 1 that the quantum-corrected BH has two Killing horizons at \(f(r) = 0\), while there is only one horizon in the BH of classical theory. It turns out that the three horizons, the event, Killing and apparent horizons, are the same for the spacetime with metric (2.22).

In the late stage of the evolution in the effective LQC model, the collapse of the dust cloud will stop when \(v(r) = 0\) is satisfied, which leads to \(f(r) = 1\) by using Eq. (2.14). At this point, \(r\) takes the minimum value
\[
r_{\text{min}} = \left(\gamma^2 R_s \Delta\right)^{1/3}.
\] (2.26)

Thus the exterior effective metric is well defined for \(r \geq r_{\text{min}}\). In the limit \(r \to r_{\text{min}}\), the effective metric reduces to the Minkowski one, and the interior energy density \(\rho\) of the dust cloud reaches its maximum value \(\rho_c\). The functions \(v(r)\) are plotted for the Schwarzschild and quantum-corrected cases in Fig. 2. It is shown that, during the collapse with decreasing \(r\), the collapsing velocity of the boundary surface increases to its maximum and then decreases to 0 in the effective theory, while it increases to infinity in the classical case. After that, the collapsing phase will be bounced to an expanding phase in the effective theory, resulting in a transformation from a BH to something similar to a white hole. Therefore, the quantum gravitational effects resolve the classical BH singularity and replace it by a quantum bounce at small scale, and the effective metric agrees quite well with the Schwarzschild metric at large scale. The global causal structure of the maximally extended BH spacetime in the effective theory has been studied in [100].
III. THE QUANTUM GRAVITATIONAL EFFECTS

In this section, we will study the quantum gravity effects on observables such as the BH shadow, the quasinormal modes and the ringdown waveform, and compare them to those of Schwarzschild case.

A. Shadows, rings and lensing rings

BH shadow was initially studied in [101, 102]. Due to its importance for exploring BHs in astrophysics, many works have been focused on it. Recently, the Event Horizon Telescope Collaboration released the observational shadows of the supermassive BH M87* at the center of the galaxy Messier 87 as well as the one in the center of the Milky Way [103, 104], promoting strongly the observational investigation of BHs. In this subsection, we study the BH shadows in the effective theories following [105, 106], offering future observations of quantum gravity effects on BH shadows.

Considering the spherically symmetric BHs, without loss of generality, we concentrate on the trajectory of light lying on the equatorial pane with \( \theta = \frac{\pi}{2} \). Taking into account the conserved energy \( \bar{E} \) and angular momentum \( \bar{J} \), the orbital equation of light ray approaching to the BHs described by Eq. (2.22) can be obtained as an ordinary differential equation of the radius \( r \) in terms of the azimuthal angle \( \phi \) as

\[
\left( \frac{dr}{d\phi} \right)^2 = r^4 \left( \frac{1}{b^2} - \frac{f(r)}{r^2} \right) = \bar{V}(r),
\]

(3.1)

where \( b = J/\bar{E} \) is the impact parameter associated to the light ray. Hence the trajectory of light ray is completely determined by its impact parameter \( b \). The radius \( r_{\text{ph}} \) of the photon sphere formed by a bounded orbit of light is determined by

\[
\bar{V}(r)_{r=r_{\text{ph}}} = 0,
\]

(3.2)

\[
\frac{d\bar{V}(r)}{dr} \bigg|_{r=r_{\text{ph}}} = 0.
\]

(3.3)

Due to the spherical symmetry, Eq. (3.3) can be reduced to

\[
\frac{d}{dr} \left( \frac{f(r)}{r^2} \right) \bigg|_{r=r_{\text{ph}}} = 0.
\]

(3.4)

Inserting the solution \( r = r_{\text{ph}} \) of Eq. (3.4) into Eq. (3.2), one obtains the critical compact parameter \( b_c \) corresponding to the photon sphere as

\[
b_c = \frac{r_{\text{ph}}}{\sqrt{f(r_{\text{ph}})}}.
\]

(3.5)

According to the value of \( b \), the light trajectory near a BH can be classified into the following three situations: (i) \( b = b_c \), the light ray will surround BHs in the circular orbit; (ii) \( b < b_c \), the light ray will approach and fall into the BH; (iii) \( b > b_c \), the light ray will be binded by the BH and then escapes to spatial infinity. For the Schwarzschild BH, the orbital radius of photon sphere and the corresponding impact parameter read respectively as

\[
r_{\text{ph}}^{\text{Sch}} = \frac{3}{2} R_s, \quad b_c^{\text{Sch}} = 3 \sqrt{\frac{3}{2}} R_s.
\]

(3.6)

For the quantum-corrected BH, these physical quantities depend on \( \Delta \) as well as \( \gamma \). The parameters \( r_{\text{ph}} \) and \( b_c \) for different values of parameters \( \gamma \) and \( \Delta \) are plotted in Fig. 3 for the quantum-corrected BH and compared to those of the Schwarzschild BH. In the left and middle panels of Fig. 3, the Immirzi parameter is chosen as \( \gamma = 0.2375 \) and \( \gamma = 1 \), respectively. It is clear that as \( \Delta \) increases, both \( r_{\text{ph}} \) and \( b_c \) monotonically decrease for the quantum-corrected BH. The right panel of Fig. 3 shows that both \( r_{\text{ph}} \) and \( b_c \) have a similar behavior with fixed \( \Delta = 0.1 \) and varying \( \gamma \). It indicates that for small \( \Delta \), both \( r_{\text{ph}} \) and \( b_c \) for the quantum-corrected BH are always smaller than those of Schwarzschild BH with the same mass. It should be noted that by choosing \( \Delta = 0.1 \), its effect on the shadows of BHs are obviously different for different theories, although \( \Delta \) should be a very small number. Also, we will fix \( \gamma = 1 \) for simplicity in the following.

To study the light bending near a BH, it is convenient to introduce a variable \( u = 1/r \). Then the orbital equation (3.1) can be expressed in terms of \( u \) as

\[
\left( \frac{du}{d\phi} \right)^2 = \frac{1}{b^2} - \frac{f \left( \frac{1}{u} \right)}{u^2}.
\]

(3.7)

Hence the total change in azimuthal angle outside the horizon of the trajectory for the situation (ii) with \( b < b_c \) is given by

\[
\phi = \int_0^{u_{\text{ph}}} \frac{1}{\sqrt{\frac{1}{b^2} - f \left( \frac{1}{u} \right) u^2}} \, du,
\]

(3.8)

where \( u_{\text{ph}} := 1/r_b \) with \( r_b \) being the radius of the (outermost) horizon. In the situation (iii) with \( b > b_c \), the total change in angle reads

\[
\phi = 2 \int_0^{u_{\text{min}}} \frac{1}{\sqrt{\frac{1}{b^2} - f \left( \frac{1}{u} \right) u^2}} \, du,
\]

(3.9)

where \( u_{\text{min}} := 1/r_{\text{min}} \) with \( r_{\text{min}} \) being the light ray’s radial minimal distance from its trajectory to the BH. Let \( n = \phi/(2\pi) \) be the total number of orbits, which is a function of \( b \), satisfying [105, 106]

\[
n(b) = \frac{2m - 1}{4}, \quad m = 1, 2, 3, \cdots.
\]

(3.10)

For each given \( m \), there will be two solutions for Eq. (3.10) [105, 106], denoted by \( b_m^+ \) with \( b_m^- \) and \( b_m^+ \) being the minimum and the maximum solutions, respectively. Then the rays can be classified as follows:

1. Direct: \( n < 3/4 \) \( \iff \) \( b \in (0, b_2^-) \cup (b_2^+, \infty) \);
2. Lensed: \( 3/4 < n < 5/4 \) \( \iff \) \( b \in (b_2^-, b_3^-) \cup (b_3^+, b_4^+) \);
3. Shadowed: \( n = 3/4 \) \( \iff \) \( b = b_3^- \).
(3) Photon ring: \( n > 5/4 \) \( \iff \) \( b \in (b_1^+, b_2^+) \).

Note that the orbit equation of the time-like geodesic reads,

\[
\left( \frac{dr}{d\phi} \right)^2 = r^4 \left( \frac{1}{b^2} - \frac{f(r)}{r^2} - \frac{f'(r)}{f(r)} \right) = \tilde{V}(r). \tag{3.11}
\]

The radius \( r_{isco} \) of the innermost stable circular orbit (isco) is determined by

\[
\tilde{V}(r) = 0, \quad \left. \frac{d\tilde{V}(r)}{dr} \right|_{r=r_{isco}} = 0, \quad \left. \frac{d^2\tilde{V}(r)}{dr^2} \right|_{r=r_{isco}} = 0. \tag{3.12}
\]

In Table I, the involved physical quantities are shown for the two types of BHs. It indicates that for the fixed parameters \( R_s = 2, \gamma = 1, \) and \( \Delta = 0.1, \) the quantum effect always shrinks the corresponding quantities. It is worth noting that the universal conjecture \([107, 108]\)

\[
\frac{3}{2} r_h \leq r_{ph} \leq \frac{b_c}{\sqrt{3}} \leq 3GM \tag{3.13}
\]

is satisfied for Schwarzschild BHs as well as quantum-corrected BH. The behaviors of photons in the effective spacetime of the quantum-corrected BH are plotted in Fig. 4. The left panel of Fig. 4 depicts the total number of orbits, and the right panel shows the trajectories of light rays surrounding the quantum-corrected BH.

Now we study the shadows of the two kinds of BHs surrounded by an optically and geometrically thin accretion disk on the equatorial panel of BHs, with an observer located at the north pole. Let us consider the simple case, where the emission originates from the accretion disk near BHs, and the emission intensity \( \nu' \) depends only on the radial coordinate \( r. \) Here \( \nu \) denotes the emission frequency in a static frame. It turns out that the observed intensity is related to the emission intensity by \([105]\)

\[
I_{obs}(b) = \sum_m f(r)^4 I_{em}(r) \bigg|_{r=r_{m}(b)}, \tag{3.14}
\]

where \( I_{em}(r) := \int f(r)^4 \nu'^4 \, d\nu' \) is the integrated intensity, and \( r_{m}(b) (m = 1, 2, 3, \cdots) \) is the so-called transfer function describing the radial position of the \( m \)th intersection of the light ray and the accretion disk outside the horizon at \( \phi = \frac{2m-1}{2}. \) Here the absorption and reflection of light by the accretion disk are neglected for simplicity. For the case of \( m > 3, \) it turns out that the contributions from the corresponding photon rings to the total luminosity can be ignored. The first three transfer functions \( r_{m}(b) (m = 1, 2, 3) \) can be expressed as

\[
\begin{align*}
r_1(b) &= \frac{1}{u\left(\frac{\pi}{2}, b\right)}, \quad b \in (b_1^+, \infty), \quad \tag{3.15} \\
r_2(b) &= \frac{1}{u\left(\frac{\pi}{2}, b\right)}, \quad b \in (b_2^+, b_2^+), \quad \tag{3.16} \\
r_3(b) &= \frac{1}{u\left(\frac{\pi}{2}, b\right)}, \quad b \in (b_3^-, b_3^+), \quad \tag{3.17}
\end{align*}
\]

where \( u(\phi, b) \) denotes the solution to the orbit equation (3.7).

The first three transfer functions for Schwarzschild spacetime and the effective spacetime are plotted in Fig. 5.

To study the observational appearance of emission, one needs to specify the intensity of emission \( I_{em}. \) Now let us consider the following three specific intensities of emission,

\[
I_{em}(r) := \begin{cases} 
\int_0^1 \left( \frac{1}{r_{isco}} - 1 \right)^2, & r > r_{isco}, \\
0, & r \leq r_{isco},
\end{cases} \tag{3.18}
\]

\[
I_{em}(r) := \begin{cases} 
\int_0^{1/(r_{ph} - 1)} \left( \frac{1}{r_{ph} - 1} \right)^3, & r > r_{ph}, \\
0, & r \leq r_{ph},
\end{cases} \tag{3.19}
\]

\[
I_{em}(r) := \begin{cases} 
\int_0^{\frac{\pi}{2}-\arctan\left( r/(r_{isco} - 1) \right)} \left( \frac{1}{r_{isco} - 1} \right)^2, & r > r_{isco}, \\
\int_0^{\frac{\pi}{2}-\arctan\left( r_{ph} - r_{isco} \right)} \left( \frac{1}{r_{ph} - r_{isco}} \right)^2, & r > r_{ph}, \\
0, & r \leq r_{ph},
\end{cases} \tag{3.20}
\]

where the emission intensities are peaked at \( r_{isco}, r_{ph}, \) and \( r_h, \) respectively, and decay sharply for the first two cases and decay slowly for the last case. Here \( I_0 \) denote the maximum value of the emitted intensities. It is easy to see from Table I that the difference of the three values of \( r_{isco}, r_{ph}, \) and \( r_h \)
TABLE I. Various involved physical quantities for Schwarzschild spacetime (Sch) and the effective spacetime (Eff), with \( R_s = 2, \gamma = 1 \), and \( \Delta = 0.1 \).

| BHs | \( r_h \) | \( r_{ph} \) | \( r_{isco} \) | \( b_e \) | \( b_j \) | \( b_\gamma \) | \( b_j \) | \( b_\gamma \) |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Sch | 2   | 3   | 6   | 5.19615 | 2.84770 | 5.01514 | 6.16757 | 5.18781 | 5.22794 |
| Eff | 1.94570 | 2.95342 | 5.93746 | 5.15630 | 2.77984 | 4.95885 | 6.15099 | 5.14644 | 5.19107 |

FIG. 4. Behavior of photons in the effective spacetime as a function of \( b \). On the left panel, the total number of orbits, \( n = \phi/(2\pi) \), is shown. The black, gold and red lines correspond to the direct, lensed and photon rings, respectively. On the right panel, a selection of associated trajectories in the Euclidean polar coordinates \((r, \phi)\) is plotted. The impact parameter spacing is 0.1, 0.01 and 0.001 in the direct (black), lensed (gold) and photon rings (red), respectively. The BH is represented by a black disk, while the circular orbit of light is shown as a dashed black circular. The parameters are \( R_s = 2, \gamma = 1 \) and \( \Delta = 0.1 \).

between the Schwarzschild and effective spacetimes are very tiny, resulting in almost the same observational appearance of emission originating near BHs. The observational appearances of the thin disk near the BHs with these three different profiles are shown in Fig. 6. For comparison, the left and middle panels in each row of Fig. 6 depict the emission intensities and observational intensities for the Schwarzschild BH and its quantum correction. We observe that for the fixed parameters \( R_s = 2, \gamma = 1 \), and \( \Delta = 0.1 \), the quantum correction always shrinks the radius of shadows.

**B. Quasinormal modes**

Quasinormal modes (QNMs) are characterized by some complex frequencies of the linear perturbations around BH solutions (see [109, 110] for review). They describe the response of a BH to external perturbations. It is widely believed that the study of QNMs plays important roles not only in analyzing the stability of BHs, but also in understanding gravitational wave signals. The modes consist of the oscillation frequency (the real part) and the decay width (the imaginary part). In 1957, Regge and Wheeler studied the perturbation of Schwarzschild BH for the first time [111]. Since then, the QNMs of various BHs have been studied [112–126]. Recently the perturbations of some loop quantum corrected BHs have also been studied [127–138]. In this section, we will calculate the QNMs of the quantum-corrected BH described in Sec. II under certain perturbations.

Considering the spherical symmetry of the spacetime, the perturbation field \( \Psi \) can be expressed as

\[
\Psi(t,r,\theta,\phi) = Y_l(\theta,\phi) \frac{\psi(t,r)}{r},
\]

where \( Y_l(\theta,\phi) \) denotes the spherical harmonics with \( l \) being the multipole quantum number. It turns out that the equation of motion for the the perturbation field \( \Psi \) can be uniformly written in the following Schrödinger-like wave equation [110]:

\[
\frac{\partial^2 \psi(t,r_s)}{\partial t^2} - \frac{\partial^2 \psi(t,r_s)}{\partial r_s^2} + V(r_s)\psi(t,r_s) = 0,
\]

where \( r_s \) is the tortoise coordinate as the solution of

\[
dr_s = \frac{dr}{f(r)},
\]
FIG. 5. The first three transfer functions for a face-on thin disk in Schwarzschild spacetime (left) and the effective spacetime (right), representing the radial coordinate of the first (black), second (gold), and third (red) intersections with a face-on thin disk outside BH. The parameters are $R_s = 2, \gamma = 1$ and $\Delta = 0.1$.

FIG. 6. The observational appearances of the thin disk near the BHs with the three different profiles. In each row, the first two panels show the emission intensity $I_{em}/I_0$ and observational intensity $I_{obs}/I_0$, normalized to the maximum value $I_0$, of a thin disk near the quantum-corrected BH (blue) compared to those of the Schwarzschild BH (red), and the third panel depicts the density plot of $I_{obs}/I_0$ of a thin disk near the quantum-corrected BH. The parameters are $R_s = 2, \gamma = 1$ and $\Delta = 0.1$.

which maps the region $(r_h, \infty)$ into the region $(-\infty, +\infty)$, and $V(r_\ast)$ denotes the effective potential with the form

$$V(r_\ast) \equiv V(r(r_\ast)) = f(r) \left[ \frac{\ell(\ell + 1)}{r^2} + \frac{1 - s^2}{r} \frac{df(r)}{dr} \right], \quad (3.24)$$

here $s$ denotes the spin of the perturbation field with values 0, 1 and 2 for the scalar perturbation, vector perturbation, and axial perturbation, respectively.

Assuming that $\psi(t, r_\ast)$ can be split into

$$\psi(t, r_\ast) = e^{-i\omega t} \varphi(r_\ast), \quad (3.25)$$
Eq. (3.22) reduces to the time-independent wave equation
\[ \frac{d^2 \psi(r_s)}{dr_s^2} + \left[ \omega^2 - V(r_s) \right] \psi(r_s) = 0. \] (3.26)

The complex frequency \( \omega \) can be obtained by solving Eq. (3.26) under appropriate boundary conditions for the wave at event horizon \( (r_s = -\infty) \), and infinity \( (r_s = +\infty) \). It is convenient to impose the condition such that the wave is purely incoming at the horizon, while it is purely outgoing at spatial infinity, i.e.,
\[ \psi(r_s) \sim \begin{cases} e^{-i\omega r_s}, & r_s \to +\infty \\ e^{+i\omega r_s}, & r_s \to -\infty \end{cases}. \] (3.27)

Moreover, negative imaginary part of \( \omega \) indicates that \( \psi \) is damped and thus is stable, while positive imaginary part means an instability.

We now calculate the QNM frequencies \( \omega \) under the scalar and vector perturbations in the case of \( l = 2 \), starting from the time-independent wave equation (3.26) and the time evolution wave equation (3.22), respectively. Notice that both Eq. (3.26) and Eq. (3.22) are completely determined by the effective potential \( V(r_s) \). To obtain the precise values of \( V \) at \( r_s \), we integrate numerically Eq. (3.23) under suitable initial data, e.g., \( r(r_s = 0) = 10R \), and inserting the result into the right-hand side of Eq. (3.24). The effective potentials of the scalar and vector perturbations outside the two kinds of BHs are plotted in Fig. 7. As it can be seen, for both perturbations, the effective potential for the effective theory always takes the maximum at each point \( r_s \), and the effective potentials in the effective theory have the form of a barrier and take constant values at the event horizon and spatial infinity.

To solve \( \omega \) from Eq. (3.26), we adopt the WKB method, by treating it as the problem of scattering near the peak of the barrier potential in quantum mechanics. This method is initially utilized in [115], and is improved in [139, 140]. The resulting values for the fundamental QNM frequencies up to the 6th-order approximation are shown in Table II as well as in Fig. 8. Also, one can employ the time domain integration method to calculate the evolution of \( \psi(t, r_s) \) at a fixed point \( r_s \) and obtain the time-domain profile, from which the frequency \( \omega \) can be extracted by using the Prony method (See, e.g., [141, 142] for reference). To do this, we implement the time domain integration by employing the finite difference method. Introducing the light-like coordinates
\[ u := t - r_s, \quad w := t + r_s, \] (3.28)
then Eq. (3.22) can be expressed in terms of \( u \) and \( w \) as
\[ -4 \frac{\partial^2 \psi(u, w)}{\partial u \partial w} = V \frac{w - u}{2} \psi(u, w). \] (3.29)

To study numerically the differential equation (3.29), one can discretize it on the \( u \)-\( w \) null grid as [114]
\[ \psi_{i,j} = \psi_{i,j-1} + \psi_{i-1,j} - \psi_{i-1,j-1} - \frac{\delta^2}{8} V_{i-1,j-1} \left( \psi_{i,j-1} + \psi_{i-1,j} \right) + O(\delta^4), \] (3.30)
where \( \delta \) denotes the overall grid scale factor, \( \psi_{i,j} := \psi(i\delta, j\delta) \) and \( V_{i,j} := V \left( \frac{i\delta + j\delta}{2} \right) \). To implement the discretized evolution (3.30), one needs to specify certain initial data. It is shown that the QNMs depend neither on the initial data nor on the small \( \delta \). Following Ref. [114], on the null boundary \( u = 0, \psi \) is specified as a Gaussian wavepacket
\[ \psi_{0,0} := e^{-\frac{(w-0)^2}{2\sigma_0^2}}. \] (3.31)
where \( w_0 \) and \( \sigma \) are the median and width of the wavepacket, and on the null boundary \( w = 0, \psi \) is specified as a constant determined by \( \psi_{0,0} \). Then one can calculate the values of \( \psi_{i,j} \) on the whole grid from Eq. (3.30). In this way, one can generate the ringdown waveform, namely, the time domain profile of perturbations by extracting the values of \( \psi \) at constant \( r_s \). To determine the quasinormal frequency corresponding to the profile, we use the Prony method to fit the profile data, and set \( w_0 = 0, \sigma = 1, \delta = 0.5 \). The ringdown waveforms \( |\psi| \) of all the two kinds of BHs under the scalar and vector perturbations with \( l = 2 \) are plotted in Fig. 9. It shows that the quantum-corrected BH and the Schwarzschild BH have almost identical waveforms. All the waveforms have the same power-law tail due to the fact that the effective potentials of the two kinds of BHs have the same asymptotic behavior. We observe that the oscillation frequency for the quantum-corrected BH is the higher in each of the perturbations. Moreover, the damping rate for the Schwarzschild BH is the higher in each of the perturbations. These results are in agreement with those provided in Table II. To further check the consistency between the generated ringdown waveforms in Fig. 9 and the QNMs obtained by the WKB method and presented in Table II, the frequencies \( \omega \) are also extracted by the Prony method from the ringdown waveforms in Fig. 9 and shown in Table III. By comparison, we find that the data presented in Table II agree quite well with those shown in Table III. The fact that the imaginary frequencies always take negative values indicates that these oscillations will die off with time evolution, and thus the two kinds of BHs are all stable against the scalar and vector perturbations.

IV. SUMMARY

In previous sections, the gravitational collapse sourced by a spherically symmetric and homogeneous dust field in the effective theory of loop quantum BH has been studied. By adopting the Darmois-Israel junction conditions, we constructed the exterior effective metric (2.22) which is well matched to its interior effective FRW universe for the collapsing model. It has been shown in Figs. 1 and 2 that, as the collapse goes on, the quantum-corrected BH forms with the occurrence of horizon, and then the collapse of the dust matter stops when the energy density reaches the Planck scale, resulting in the resolution of the classical BH singularity. After bounce, the dust cloud will enter a expanding phase from the collapsing phase, leading to a procedure from a BH to something similar to a white hole.
and the quantum-corrected BH (QC), with ω profiles are obtained from Eq. (3.24) by inserting the solution to Eq. (3.23) with the initial data \( r(\tau = 0) = 10R_c \).

TABLE II. The QNMs \( \omega \) of the scalar and vector perturbations obtained by the WKB method up to 6th order for the Schwarzschild BH (Sch) and the quantum-corrected BH (QC), with \( R_c = 2, \gamma = 1, \) and \( \Delta = 0.1 \).

| BHs   | 1st-order | 2nd-order | 3rd-order | 4th-order | 5th-order | 6th-order |
|-------|-----------|-----------|-----------|-----------|-----------|-----------|
| Sch   | 0.506317  | 0.483977  | 0.483211  | 0.483647  | 0.483656  | 0.483642  |
| QC    | 0.509387  | 0.48769   | 0.486942  | 0.487488  | 0.487509  | 0.487474  |

| BHs   | 1st-order | 2nd-order | 3rd-order | 4th-order | 5th-order | 6th-order |
|-------|-----------|-----------|-----------|-----------|-----------|-----------|
| Sch   | 0.480754  | 0.45796   | 0.457131  | 0.457596  | 0.457605  | 0.457593  |
| QC    | 0.484045  | 0.461956  | 0.461135  | 0.461723  | 0.461746  | 0.46171   |

TABLE III. The QNMs \( \omega \) of the scalar and vector perturbations of \( l = 2 \) extracted by the Prony method from the profile shown in Fig. 9 with \( t \in (70, 150) \) for the Schwarzschild BH (Sch) and the quantum-corrected BH (QC). The parameters are \( R_c = 2, \gamma = 1, \) and \( \Delta = 0.1 \).

| BHs   | \( \omega \) (scalar perturbation) | \( \omega \) (vector perturbation) |
|-------|----------------------------------|----------------------------------|
| Sch   | 0.484111 – 0.0959327i | 0.458259 – 0.0942558i |
| QC    | 0.487941 – 0.0942513i | 0.462383 – 0.0925878i |

To understand how quantum corrections affect the observables of BHs, the BH shadows and QNMs of BHs have also been studied. We considered the shadows and appearances of BHs surrounded by the optically and geometrically thin accretion disk on the equatorial panel of BHs with the specified intensities of emission \( I_{em} \) by Eqs. (3.18)-(3.20). On one hand, we found that the quantum correction always shrinks the radius of shadows. On the other hand, we calculated the QNMs of the quantum-corrected BH under the scalar and vector perturbations by two methods, the WKB approximation method and the time domain integration method. The results are compared with the Schwarzschild case and presented in Tables II and III as well as Figs. 8 and 9. The data presented in Table II agree quite well with those shown in Table III. Moreover, we found that the quantum correction increases the real part and decreases the absolute value of the imaginary part. In particularity, the imaginary frequencies of QNMs always take negative values, and thus the quantum-corrected BH is stable against the scalar and vector perturbations.

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FIG. 8. The real (top) and imaginary (bottom) parts of the frequencies $\omega$ as a function of the order of the WKB method up to 6th order for the scalar (left panel) and vector (right panel) perturbations of $l = 2$, corresponding to the Schwarzschild BH (red) and the quantum-corrected BH (blue). The parameters are $R_{+} = 2$, $\gamma = 1$ and $\Delta = 0.1$.

FIG. 9. The time domain profiles of the scalar (left panel) and vector (right panel) perturbations, corresponding to the Schwarzschild BH (red) and the quantum-corrected BH (blue). The time domain profiles are observed at $r_s = 0$ (or $r = 10r_s$) under the initial Gaussian wavepacket (3.31) with $w_0 = 0$, $\sigma = 1$, and the grid spacing $\delta = 0.5$. The parameters are $R_{+} = 2$, $\gamma = 1$ and $\Delta = 0.1$.

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