Tidal radii of main sequence stars - II. Simulation methodology and the character of full tidal disruptions

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ABSTRACT

This is the second in a series of papers presenting the results of fully general relativistic simulations of stellar tidal disruptions in which the stars’ initial states are realistic main-sequence models. We consider eight different stellar masses, from 0.15 M⊙ to 10 M⊙. In the first paper (Ryu et al. 2019a), we gave an overview of this program and discussed the principal observational implications of our work. Here we describe our calculational method and provide details about the outcomes of full disruptions. We find that, relative to the traditional order-of-magnitude estimate \( r_t \), the physical tidal radius of low-mass stars is larger by tens of percent, while for high-mass stars (\( M_\star \gtrsim 1 M_\odot \)) it is smaller by a factor 2–2.5. The traditional estimate of the range of energies found in the debris is approximately accurate for low-mass stars, but is a factor \(~2\) too small for high-mass stars; in addition, the energy distribution for high-mass stars has significant wings. For all stars undergoing tidal encounters, we find that mass-loss continues for a long time because the instantaneous tidal radius, the distance out to which the black hole’s tidal gravity competes with the instantaneous stellar gravity at the star’s surface, stays comparable to the distance to the black hole until the star has reached \( O(10) r_t \). These findings indicate significant failings in the popular “frozen-in” approximation.

Keywords: black hole physics – gravitation – hydrodynamics – galaxies:nuclei – stars: stellar dynamics

1. INTRODUCTION

Observations suggest that almost every massive galaxy hosts at least one supermassive black hole (SMBH) in its center (Kormendy & Ho 2013). As stars in a galaxy’s core interact gravitationally, some stars’ orbits can be perturbed in a way that places them on nearly radial orbits. If they approach the central BH sufficiently closely, these stars are tidally disrupted and lose some fraction of their mass. Roughly half the stellar debris is bound and returns back to the BH, while the other half is expelled outward at \( \sim 5000–10,000 \) km/s, producing a a luminous flare. A few dozen candidate tidal disruption events (TDEs) have been identified (Komossa 2015; van Velzen 2018), and the number is expected to grow with detections by the ongoing optical time-domain survey (e.g., ZTF1: Graham et al. 2019) as well as future surveys (e.g., eROSITA2 All-Sky Survey: Merloni et al. 2012, and LSST3: LSST Science Collaboration et al. 2009).

This paper is the second in a series of closely-related papers. Our central goal is to determine quantitatively the chief parameters governing these events: the physical tidal radius \( R_t \), i.e., the pericenter within which all encounters end in complete disruption; the relation between remnant mass and pericenter when the star is only partially disrupted; and the distribution functions of de-
bris mass with respect to energy and angular momentum in all cases. To trace the mass-dependence of these quantities, we simulate encounters with a $10^6 M_\odot$ black hole of realistic main-sequence (MS) stars for a wide range of mass ($0.15 M_\odot \leq M_\star \leq 10 M_\odot$), in each case examining parabolic orbits with a range of pericenters. In Ryu et al. (2019a) (Paper 1 hereafter), we presented $R_t$ and the characteristic energy width of stellar debris $\Delta E$ as functions of stellar mass; for some stars, $R_t$ is as much as $1.4 r_\star$, the usual order-of-magnitude estimate for the tidal radius (e.g., Shiokawa et al. 2015), onto binary black holes (e.g., Noble et al. 2009, 2010; Schnittman et al. 2013), accretion flow from a stellar core (e.g., Saumon et al. 1995). However, the resulting effective adiabatic index is $\gamma = 5/3$ although it is possible for the effective adiabatic index to be different. In fact, for treating the hydrostatic star, MESA employs equations of state tables constructed on the basis of quantum statistical calculations by Rogers & Nayfonov (2002) and Saumon et al. (1995). However, the resulting effective adiabatic index wherever $T \geq 10^5$ K, i.e., in the bulk of the stellar mass, is $\approx 5/3$. In the course of the TDE, both the density and temperature of the stellar material decrease. The only physical effect in the debris that might alter the adiabatic index is ionization state change, particularly where the temperature is low enough for H to recombine. Because, for the great majority of the stellar mass, H recombination takes place outside our simulation domain, $\gamma = 5/3$ is a well-justified approximation.

In a code adopting a conservative integration scheme like HARM3D, the transformation between the conserved quantities and the so-called primitive variables is performed at each time step more than once to update the fluid elements. In general, in a conservative GRMHD code, the transformation between the two sets of variables is not straightforward since simple analytic relations between the two sets do not exist. In our study, we numerically recover the primitive variables from the conserved variables assuming the conservation of momentum (time-component of the conservation law of the

2.1. Numerical Method

We use the fully conservative general relativistic magneto-hydrodynamics (GRMHD) code HARM3D (Noble et al. 2009). The code is an extended version of the 2D GRMHD HARM (Gammie et al. 2003), adopting the Lax-Friedrichs numerical flux formula and a parabolic interpolation method (Colella & Woodward 1984) with a monotonized central-differenced slope limiter. This code has been used for studying many problems in BH physics, including energy production in accretion onto Kerr black holes (Noble et al. 2009, 2010; Schnittman et al. 2013), accretion flow from a stellar tidal disruption (e.g., Shiokawa et al. 2015), onto binary black holes (e.g., Noble et al. 2012; dAscoli et al. 2018) and the X-ray spectra of stellar-mass black holes (Kinch et al. 2019).

The equations solved in our application of HARM3D are $\nabla_\mu T^\mu_\nu = 0$ and $\nabla_\mu \rho u^\mu = 0$, where the stress-energy tensor $T^\mu_\nu = \rho h u^\mu u_\nu - pg^\mu_\nu$, $\rho$ is the proper rest-mass density, $h$ is the enthalpy, and $u^\mu$ is the fluid 4-velocity.

We further assume an adiabatic equation of state with an adiabatic index $\gamma = 5/3$ although it is possible for the effective adiabatic index to be different. In fact, for treating the hydrostatic star, MESA employs equations of state tables constructed on the basis of quantum statistical calculations by Rogers & Nayfonov (2002) and Saumon et al. (1995). However, the resulting effective adiabatic index wherever $T \geq 10^5$ K, i.e., in the bulk of the stellar mass, is $\approx 5/3$. In the course of the TDE, both the density and temperature of the stellar material decrease. The only physical effect in the debris that might alter the adiabatic index is ionization state change, particularly where the temperature is low enough for H to recombine. Because, for the great majority of the stellar mass, H recombination takes place outside our simulation domain, $\gamma = 5/3$ is a well-justified approximation.

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In this paper, we provide a detailed description of our numerical simulations in Section 2, including discussions of: the code we use (Section 2.1); computational domain setup (Section 2.2); spacetime geometry, tidal force, and self-gravity (Section 2.3); our stellar models (Section 2.4); and the initial conditions (Section 3.1). In Section 4, we present the results for full disruptions, which occur when the pericenter distance $r_p < R_t$. In particular, we discuss the duration of tidal disruption (Section 4.2), the debris energy and angular momentum distribution, and the fallback rate of the debris (Section 4.3). In Section 5, we compare our results for $R_t$ with other simulation studies (Section 5.1) and discuss the validity of the “frozen-in” approximation (Section 5.2). Lastly, we summarize our results in Section 6.

In the remaining two papers of this series, we analyze the results of partial disruptions (Ryu et al. 2019b, Paper 3) and study the black hole mass-dependence of TDEs, especially the changing magnitude of relativistic corrections to the tidal stress (Ryu et al. 2019c, Paper 4).

Throughout this paper, symbols with the subscript $\star$, such as $r_\star$ (stellar vibration time, defined in Section 2.4), $R_\star$ (stellar radius) and $M_\star$ (stellar mass), always indicate the characteristic scales for the initial model star. All masses are measured in units of $M_\odot$ and all stellar radii in units of $R_\odot$.

2. SIMULATION SETUP

In this section we present our numerical models. We describe our numerical scheme for fluid calculations, pre-
stress-energy tensor, or Equation 27 in Noble et al. 2006) and entropy (Equation 19 in Noble et al. 2009).

2.2. Computational domain

Our computational domain is a rectangular box of fixed orientation that moves with the star. Midway through the simulation, we change the spatial size and shape of the box to accommodate the changing shape of the star and the debris. We use a cubic box until the star’s Boyer-Lindquist radial coordinate \( r \) reaches \( 2-4 \, r_t \), from the BH as it moves away from pericenter passage. At this point we replace it with an elongated rectangular box, larger in every dimension. We do so to ensure that the gas velocity is consistently supersonic outward at the box boundary. In a small number of cases for which \( r_p \) is well outside \( R_t \), the tidal effects are so weak that replacing the cubic box is unnecessary.

1. Cubic domain: Onset \( (r \simeq 10 \, r_t \text{ before pericenter passage}) \) to \( r \simeq 2 - 4 \, r_t \text{ after pericenter passage} \)

The sides of the cubic box are \( L_x = L_y = L_z = 5R_* \). The resolution of the cubic box is \( \approx 25 \text{ cells per } R_* \). The number of cells on each axis is 128.

2. Extended elongated domain: \( r \gtrsim 2 - 4 \, r_t \text{ after pericenter passage} \)

As the star is stretched due to the tidal forces of the BH, the star becomes elongated primarily in one dimension. When the size in that direction becomes longer than the width of the cubic box, we increase the box in all dimensions, but more in a dimension along the axis where the debris is extended. The size of the larger box is \( (L_x, L_y, L_z) = (17, 9, 10) \times R_* \). For the larger rectangular domain, we coarsen the grid by a factor of 2 in all dimensions.

We map the last snapshot of the simulation with the cubic box onto the central region of the same volume in the elongated domain such that the total mass, momentum, and internal energy are conserved. The rest of the extended domain is filled with gas at the fluid floor density.

Figure 1 schematically depicts how a star evolves in the comoving computational domain as it travels along an orbit, and how we change the computational box accordingly.

Because the debris stream is long and narrow, it is desirable to align the long axis of the rectangular box with the stream as well as possible. Computationally, this is easiest to do if this axis is parallel to one of the axes of the cubic box. For this reason, as shown in Figure 1, we start with a box rotated by an angle with respect to the semimajor axis of the orbit so that the debris is extended along the \( x \)-axis of the grid when the box has traveled out to \( r > 10 \, r_t \).

We also ran several simulations with a cubic box \( 2 \times \) larger than the standard in all dimensions and a rectangular domain \( 1.5 \times \) larger than the standard size. In addition we examined the effect of rotating the rectangular box, but without creating a large angle with the stream. We find no significant differences between runs with the different box sizes or orientations in terms of mass contained in the same volume around the domain origin and distinguishing between full and partial disruptions. We have also performed convergence tests with \( 1.5 \times \) finer resolution and find no significant differences between those simulations and runs with our standard resolution.

We give all primitive variables zero gradient at boundaries. However, to ensure outflow, we set the normal component of the primitive fluid velocities in the ghost cells to be zero if the fluid motion is found to be inward. The time-step is determined using a Courant number of 0.3.

2.3. Spacetime geometry of the comoving frame - tidal force and self-gravity

2.3.1. Global spacetime: tidal gravity in the box frame

All our simulations are carried out in a global Schwarzschild spacetime, but modified to include the star’s self-gravity within the computational box. To accomplish this, we proceed in a series of steps. These begin by describing the Schwarzschild spacetime in terms of Cartesian coordinates with an origin at the black hole (Godoi 2019) and oriented so that the \( x \)-axis is parallel to the orbital major axis. We then transform this metric to the moving frame of the box by a coordinate transformation in which the time coordinate does not change. The last step of this transformation is to rotate the spatial coordinate axes to align with the box sides. We call the resulting coordinate system the “comoving frame”. This procedure guarantees that the relativistic tidal gravity of the black hole is expressed exactly in the moving frame of the computational box. Note that because we fix the time coordinate, this is not a Lorentz transformation.

2.3.2. The self-gravity component \( h_{\mu\nu}^{\text{sg}} \)

The easiest way to combine stellar self-gravity with the background metric is to use a post-Newtonian approximation. In this approximation, the total metric is

\[
g_{\mu\nu} \simeq \tilde{g}_{\mu\nu} + h_{\mu\nu}^{\text{sg}}, \tag{1}\]

where \( \tilde{g}_{\mu\nu} \) is the background metric and \( h_{\mu\nu}^{\text{sg}} \) is the self-gravity contribution.
Figure 1. Schematic diagram showing successive moments in a TDE. The red line indicates the star’s orbit around the black hole (black circle). Each inset figure presents a snapshot of the density distribution in the orbital plane within our simulation box. The white circle in each snapshot shows the initial stellar radius. Partway through the event, we replace the cubic box with a rectangular box; we draw a red square in the rectangular boxes to show the position and size of the original cubic box. Note that the rectangular boxes are not drawn to the same scale as the cubic boxes, and the dotted curves marking $r_{\text{t}}$, $10r_{\text{t}}$ and $20r_{\text{t}}$ are likewise not drawn to scale.

where $\tilde{g}_{\mu\nu}$ is the global Schwarzschild metric as it is represented in the box frame, and

$$h_{00}^{\text{sg}} = -2\Phi_{\text{sg}},$$
$$h_{0i}^{\text{sg}} = h_{i0}^{\text{sg}} = 0,$$
$$h_{ij}^{\text{sg}} = 0,$$

where $\Phi_{\text{sg}}$ satisfies the Poisson equation, $\nabla^2 \Phi_{\text{sg}} = 4\pi \rho$.

In order for this approximation to be valid, two requirements must be met: $|h_{\mu\nu}^{\text{sg}}| \ll 1$. and $|\tilde{g}_{\mu\nu} - \eta_{\mu\nu}| \ll 1$ for all elements; here $\eta_{\mu\nu}$ is the Minkowski metric. The first condition is easily satisfied because $|\Phi_{\text{sg}}| \simeq (GM_*/R_*)c^{-2} \simeq 10^{-6}$.

Further steps must be taken to satisfy the second requirement: the departures from the Minkowski metric in the fluid frame are $\sim O(0.1)$ at $\sim 20r_g$ from the black hole. Here, $r_g$ is the gravitational radius of the BH. For the purpose of combining stellar self-gravity with the global spacetime, we therefore create a new frame, one defined by an orthonormal tetrad formalism. The metric in the tetrad system is, by construction, exactly Minkowski at the origin. Elsewhere in the box, it remains very close to $\eta_{\mu\nu}$: in the $\tilde{g}_{00}$ element, the greatest departure is $\sim 10^{-2}$, while in most of the volume the departure is $\sim 10^{-3}$.

We construct the tetrad system at the star’s starting location in the usual way. We choose the time-like unit vector $e_\mu^{(0)}$ to be the 4-velocity $u^\mu$. In the comoving frame, $e_\mu^{(0)} = (1/\sqrt{-\tilde{g}_{00}}, 0, 0, 0)$. The remaining components $e_\mu^{(i)}$ are found by a Gram-Schmidt method. This procedure could be performed at each point along the orbit. We find it more efficient, however, to perform it only once, at the starting point of the star. Once that first system has been calculated, we parallel-transport the tetrad basis along the star’s geodesic by integrating the equation of motion

$$\frac{d^2 e_\mu^{(a)}}{d\tau^2} + \Gamma^\mu_{\alpha\beta} e^{(a)}_{\alpha} e^{(b)}_{\beta} = 0,$$

where $\Gamma^\mu_{\alpha\beta}$ refers to the metric’s affine connection evaluated in the comoving frame.

Both $\tilde{g}_{\mu\nu}$ and the tetrad basis are functions of the orbital variables $X(t)$ (the star’s center-of-mass position in the black hole frame) and $dX(t)/dt$ (the star’s coordinate velocity in the black hole frame). Because the orbit is independent of fluid updates, we integrate the orbit of the star and the parallel-transport equation beforehand using a 4-th order Runge-Kutta integrator with adaptive time steps and make a lookup table with the orbital variables. At each time step, the code finds $X(t)$...
and \(d\mathbf{X}(t)/dt\) from the lookup table by linearly interpolating between the two sets of data at the two most adjacent times, and then calculates \(\tilde{g}_{\mu\nu}\). We ensure that time differences between lines of the lookup table are sufficiently small compared to the time steps for fluid updates.

The self-gravitational potential \(\Phi_{\text{sg}}\) of the star is computed at each step of the fluid simulation using a (discrete sine) Fourier transform method. Following Cheng & Evans (2013), we introduce an image mass on the box boundary so that \(\Phi_{\text{sg}}\) asymptotes to zero at infinity, not on the domain boundary. Its magnitude depends on the multipole moments of the mass inside the box; we carry out the sum up to \(l_{\text{max}} = 4\). We stress that this image mass is used only when calculating \(\Phi_{\text{sg}}\), and not when updating the fluid elements.

Once \(\Phi_{\text{sg}}\) has been calculated, we add it to \(\tilde{g}_{00}\) in the tetrad frame as in Equation 1. We then perform the inverse of the original tetrad coordinate transformation in order to find \(g_{\mu\nu}\)—now including both the star’s and the black hole’s gravity—in the comoving frame. This form of the metric governs the fluid simulation.

### 2.4. The stellar model

To provide the initial data for our simulations, we evolve stars using the stellar evolution code MESA (Paxton et al. 2011), assuming solar metallicity, until they reach half the MS life time for their mass. Since the life times of stars with \(M_\star < 1\) are longer than a Hubble time, we assume all low-mass stars have an age \(\sim 13 - 14\) Gyr.

For our suite of simulations, we consider eight MS stellar models, with masses, \(M_\star = 0.15, 0.3, 0.4, 0.5, 0.7, 1.0, 3.0\) and 10. The models represent a range of different interior structures: fully convective stars (0.15 - 0.3 M\(_\odot\)), stars with a shallow convective envelope and a large radiative inner region (0.4 - 0.7 M\(_\odot\)), fully-radiative stars (1 M\(_\odot\)), and stars with a radiative envelope with a convective core (3 M\(_\odot\) and 10 M\(_\odot\)) (Kippenhahn & Weigert 1994). Throughout this paper we will use the term “low-mass” for all stars with \(M_\star < 0.5\), and “high-mass” for stars with \(M_\star \geq 1\). Within both the low-mass and high-mass groups, the internal stellar structures are similar to one another; the \(M_\star = 0.7\) structure has an intermediate character. The density profiles of these stellar models are shown in Figure 2, together with a few polytropic stellar models. The \(M_\star = 0.15, 0.3\) and 0.4 stellar models are in good agreement with a polytropic model with \(\gamma = 5/3\) for the given mass and radius. The \(M_\star = 1\) star is closely matched by a polytrope with \(\gamma = 4/3\) at intermediate radii, but not near the core or the surface. The other stars are not well-described by a polytropic model. Stars with \(M_\star \geq 1\) tend to have a more concentrated inner region than low-mass stars or polytropic stars with \(\gamma = 4/3\). We summarize the model parameters of the MS stars in Table 1.

We find that the relation between \(M_\star\) and \(R_\star\) for \(0.15 \leq M_\star \leq 3\) is well-described by the formula

\[
R_\star = 0.93 M_\star^{0.88}.
\]  

(4)

The fractional differences between \(R_\star\) estimated using Equation 4 and \(R_\star\) taken from the MESA models are all less than 0.1. For \(M_\star = 10\), the fractional difference is 0.27. This relation is consistent with that of Kippenhahn & Weigert (1994) even though they found \(d\ln R_\star/d\ln M_\star \approx 0.8\) for low-mass stars and \(d\ln R_\star/d\ln M_\star \approx 0.6\) because those slopes did not apply to \(M_\star \approx 1\), where the slope was rather higher.

### 3. RUNNING THE SIMULATIONS

#### 3.1. Initial stellar structure

At the start of each simulation, a MS star in hydrostatic equilibrium is placed so that its center lies at the coordinate origin of the box. The density and pressure profiles of the star are determined by a linear interpolation between two adjacent data points in the MESA model whose positions are closest to each cell center of our HARM3D grid. After doing so, the profiles on our grid agree with the MESA profiles to within less than 0.1% out to a radius at which the enclosed mass \(\approx 99\% M_\star\).

To avoid creating too sharp a discontinuity between the stellar density and the external “vacuum”, we extrapolate the logarithmic density gradient at the 99% mass radius to larger radii, but not permitting the density to fall below the vacuum density. The pressure in the extrapolation region is determined by the hydrostatic equilibrium condition with a temperature comparable to the stellar surface temperature. We set the vacuum density low enough to ensure that the total mass of the domain, minus \(M_\star\), is < \(10^{-3}M_\star\). The simulation’s absolute density floor is \((10^{-1} - 10^{-2})\times\) the vacuum density.

These stellar models stay in hydrostatic equilibrium for much longer than the time it takes for the stars to pass the pericenter, i.e., > 25 \(\tau_\star\). Here \(\tau_\star\) is the stellar vibration time, which we define as

\[
\tau_\star = 1.0/\sqrt{GM_\star/(4\pi R_\star^2/3)}.
\]

#### 3.2. Stellar trajectories

For each stellar model, we select a number of parabolic Schwarzschild geodesics with different pericenter distances in order to explore the transition from partial...
Figure 2. The radial density profile of our MS MESA models. The thick red solid lines indicate the profiles from the MESA data. In each case, we show the profile only out to the radius at which we supersede the MESA data in order to create a smoother connection to the external atmosphere. The plots for the 3 M\(_{\odot}\) and 10 M\(_{\odot}\) stars have their own density scales in order to show the large range of density found in these stars. For a comparison, we overplot for each mass the density profiles predicted by polytropic models with \(\gamma = 4/3\) (dotted), 5/3 (dot-dashed) and 2.0 (dashed).
M /M ≃ mass (see Table 1). In every case, the initial distance of exception, decisions made on the basis of these criteria tion of three criteria at the end of a simulation. Without reaches \( r \) to follow the event until the center-of-mass of the star passes through pericenter at \( t \). Without \( \psi \equiv r_p / r_t \) considered in our TDE experiments.

\[
\begin{array}{cccccc}
M_* & R_*^6 & \tau_* & r_t / r_g & \psi = r_p / r_t \\
0.15 & 0.17 & 0.6 & 15 & 1.0, 1.4, 1.5, 1.6, 1.8, 2.0 \\
0.30 & 0.30 & 1.0 & 21 & 1.0, 1.2, 1.3, 1.4, 1.5, 1.8 \\
0.40 & 0.37 & 1.2 & 24 & 1.0, 1.2, 1.3, 1.4, 1.5, 1.8 \\
0.50 & 0.46 & 1.5 & 27 & 0.80, 1.0, 1.1, 1.2, 1.5, 1.8 \\
0.70 & 0.69 & 2.2 & 36 & 0.60, 0.65, 0.70, 0.80, 0.90, 1.5 \\
1.0 & 1.0 & 3.3 & 47 & 0.40, 0.45, 0.50, 0.55, 0.65, 1.00 \\
3.0 & 2.4 & 7.2 & 80 & 0.35, 0.40, 0.45, 0.50, 0.60, 0.85 \\
10 & 5.6 & 14 & 120 & 0.35, 0.40, 0.45, 0.50, 0.60, 0.85 \\
\end{array}
\]

Units: \(^a\) M_⊙; \(^b\) R_⊙; \(^c\) 10^3 s.

to full disruption. They are parameterized in terms of \( \psi \equiv r_p / r_t \). For stars with \( M_* < 0.7 \), we consider \( \psi \) in the range 0.8–2.0; for higher-mass stars, \( 0.35 \leq \psi \leq 1.5 \), with a small shift toward smaller \( \psi \) for stars with larger mass (see Table 1). In every case, the initial distance of the star from the BH is \( \simeq 10 \) \( r_t \); with this choice, the star passes through pericenter at \( t \simeq 8 \) \( \tau_* \). We continue to follow the event until the center-of-mass of the star reaches \( r \simeq 20 – 30 \) \( r_t \).

3.3. Distinguishing partial from full disruptions

We define complete disruption of a star as the satisfaction of three criteria at the end of a simulation. Without exception, decisions made on the basis of these criteria are consistent.

1. Lack of any approximately spherical bound structure.

2. Monotonic (as a function of time) decrease in the maximum pressure of the stellar debris.

3. Monotonic decrease in the mass within the computational box. This criterion is illustrated in Figure 3. The mass remaining in the box for complete disruption a falls with increasing distance from the BH \( \propto r^{-\alpha} \) with \( \alpha \simeq 1.5 – 2.0 \), whereas for partial disruptions the remaining mass eventually becomes constant, which signifies a persistent self-gravitating object.

4. RESULTS

4.1. The physical tidal radius

The first product of our simulations is the distinction between those pericenters yielding partial disruptions and those yielding full disruptions. Not surprisingly, the classic tidal radius estimator \( r_t \) is good at the order-of-magnitude level, but does not indicate the physical tidal radius (the divide between partial and full disruptions) to better than a factor of 2. As shown in Figure 4, the ratio \( \Psi = R_t / r_t \) rises to \( \simeq 1.4 \) for extremely low mass (\( M_* = 0.15 \)), drops gradually as the mass increases to \( M_* \simeq 0.5 \), and then drops rapidly to \( \simeq 0.4–0.45 \) for \( M_* > 1 \). Remarkably, as discussed at greater length in Paper 1, \( R_t / r_t \simeq 27 \) for \( M_{BH} = 10^6 \) nearly independent of \( M_* \) from \( M_* = 0.15 \) to \( M_* \simeq 3 \). As also reported in Paper 1, \( \Psi(M_*) \) can be very well represented by an analytic form involving a pair of exponentials,

\[
\Psi(M_*) = \frac{1.47 + \exp[(M_* - 0.669)/0.137]}{1 + 2.34 \exp[(M_* - 0.669)/0.137]}. \tag{5}
\]

4.2. Duration of tidal disruption

The classic order-of-magnitude estimate of the tidal radius is the statement that the tidal gravity of the black hole matches the self-gravity at the surface of the star. At the qualitative level, this comparison divides the realm of strong and weak tidal forces. However, because stars lose mass during a tidal encounter while also changing their distance from the black hole, the sense of this comparison can be a function of time. To study
how it evolves through an event, we introduce an instantaneous tidal radius:

$$\lambda_t(t) \equiv \left( \frac{M_{BH}}{\bar{\rho}(r)} \right)^{1/3},$$

where $\bar{\rho}$ is the average density of the cells containing 99% of the total mass in the domain when summed outward from the center.

Figure 5 shows how the distance of a star from the black hole in units of $\lambda_t$ changes as a function of its distance from the black hole in units of $r_t$. Although the example we show is for a 0.3 $M_{\odot}$ star, the same diagram for other masses is qualitatively very similar. The lines are initially straight because the incoming stars stay intact, i.e., both $\bar{\rho}$ and $r/\lambda_t \propto r$ remain constant. However, there is a noticeable contrast between the behavior of full and partial disruptions. When the encounter ends in the complete dissolution of the star, after the star passes pericenter, $r/\lambda_t$ increases quite slowly, approximately $\propto r^{1/3}$, and it remains near unity out to $r \gtrsim 20 r_t$. On the other hand, when the ultimate result is a partial disruption, after pericenter passage $r/\lambda_t$ is also $\propto r^{1/3}$, much like the full disruption tracks, but with a larger coefficient. However, this slope ends earlier, steepening sharply when $r \gtrsim 10 r_t$ (Steinberg et al. 2019 find a similar result for full disruptions in which $r_p \ll r_t$, but the outgoing track is slightly steeper: $r/\lambda_t \propto r^{1/2}$).

The same curves also show the pace of mass-loss. Both full and partial disruptions exhibit mass-loss during the entire period when $r/\lambda_t \sim 1$. In partial disruptions, mass-loss continues until the star has reached $\sim 10 r_t$, while mass loss continues until $r$ is at least $\sim 20 r_t$ in full disruptions. In other words, mass is lost for as long as $r \sim \lambda_t$, and this state can endure for as long as the time required for the star to swing from $r_p$ to 10–20 $r_t$.

### 4.3. Distribution of specific energy and angular momentum and fallback rate

The distribution of mass with energy and angular momentum determines both the orbits of tidal debris and the rate at which mass returns to the vicinity of the black hole. We first present in Figure 6 the map of $E - L$ for stars with $M_* = 0.3$ (top), 1 (middle) and 10 (bottom); in each case, we show data from the smallest $r_p$ we simulated. We normalize $E$ to $\Delta \epsilon$, the fiducial energy scale (Lacy et al. 1982; Rees 1988)

$$\Delta \epsilon = \frac{G M_{BH} R_*}{r_t^2},$$
The distribution of the specific energy $E$ and specific angular momentum $L$ for $M_* = 0.3$ (top), 1.0 (middle) and 10 (bottom), as representative cases of low- and high-mass stars at $r \simeq 22 r_t$. We consider the strongest encounter (smallest $\psi$) for each star. The color scale indicates the mass fraction $\Delta M/M_*$ in a logarithmic scale. We normalize $E$ by the fiducial energy spread $\Delta \epsilon$ (Equation 7) and $L_0$ refers to the initial specific angular momentum.

Figure 6.

The distribution of the specific energy $E$ and specific angular momentum $L$ for $M_* = 0.3$ (top), 1.0 (middle) and 10 (bottom), as representative cases of low- and high-mass stars at $r \simeq 22 r_t$. We consider the strongest encounter (smallest $\psi$) for each star. The color scale indicates the mass fraction $\Delta M/M_*$ in a logarithmic scale. We normalize $E$ by the fiducial energy spread $\Delta \epsilon$ (Equation 7) and $L_0$ refers to the initial specific angular momentum.

Figure 7. $\Delta E/\Delta \epsilon$ for all full disruption events ($r_p < R_t$). When we have data for two values of $r_p < R_t$ (see Table 1), the red circles indicate the smaller $r_p$, while the blue triangles indicate the larger. The black dotted line represents the fitting formula for $\Delta E/\Delta \epsilon$ (Equation 8).

and the $y$–axis indicates the relative difference of the specific angular momentum $L$ with respect to the initial angular momentum $L_0$ for a given orbit. In geometrized units in terms of $M_{BH}$, $L_0 \simeq 6.85$ for $M_* = 0.3$, $\simeq 6.51$ for $M_* = 1$ and $\simeq 9.49$ for $M_* = 10$.

The distributions are very nearly symmetric around the origin in all cases, but the ranges of $E$ and $L$ are different. To characterize the width of these distributions, we define $\Delta E$ and $\Delta L$ such that $90\%$ of the total mass is contained within $-\Delta E < E < +\Delta E$ and $-\Delta L < L - L_0 < +\Delta L$. The range of pink–red color in the figure is a good estimator of both $\Delta E/\Delta \epsilon$ and $\Delta L/L_0$.

Much as we found for $R_t$, there are strong contrasts between low-mass and high-mass stars for both $\Delta E/\Delta \epsilon$ and $\Delta L/L_0$. As $M_*$ increases, $\Delta E/\Delta \epsilon$ jumps from 0.8 to 1.5 around $M_* \simeq 1$ (see also Figure 7) while $\Delta L/L_0$ does not change significantly. As $M_*$ increases above 1, however, $\Delta E/\Delta \epsilon$ remains $\simeq 1.5$–2.0 while $\Delta L/L_0$ more than doubles. The mass-dependence of $\Delta E/\Delta \epsilon$ is well-described by a fitting formula introduced in Paper 1 (where this ratio is called $\Xi$),

$$\frac{\Delta E(M_*)}{\Delta \epsilon} = \frac{0.620 + \exp \left( (M_* - 0.674)/0.212 \right)}{1 + 0.553 \exp \left( (M_* - 0.674)/0.212 \right)}. \tag{8}$$

As demonstrated by the almost exact coincidence between the $\Delta E/\Delta \epsilon$ values for runs with the same mass but different $r_p < R_t$ (Figure 7), the energy spread for complete disruption of a given mass varies hardly at all over the $10$–$20\%$ contrast in $r_p$ we have explored.
Figure 8. $dM/dE$ for the model stars undergoing the strongest encounters. These distributions are normalized so that the area under the curve is integrated to unity. The diagonal dotted line depicts the case for $dM/dE \propto e^{-k|E/\Delta E|}$ with $k = 3.0$.

Figure 8 depicts $dM/dE$ for all of our model stars. As already mentioned, the energy spread for high-mass stars is close to a factor of 2 broader than for low-mass ones. No significant difference of the $E$ distribution is found for different $\psi$ values over the limited range we have simulated. The weak dependence on $\psi$ is consistent with the one found by Guillochon & Ramirez-Ruiz (2013).

Although $dM/dE$ does not vary by large factors within its central region, neither is it strictly flat, as is often assumed. For both low-mass and high-mass stars, the distribution has “shoulders”, larger $dM/dE$ for $|E|/\Delta E \lesssim 1$ than for $E/\Delta E \simeq 0$. The value of $dM/dE$ at the peaks of the shoulders is typically $\approx 1.5 \times dM/dE$ at the local minimum near $E = 0$. The distribution has fairly sharp outer boundaries for the low-mass stars, but a more gradual fall for the high-mass stars. Where $|E| > \Delta E$, $dM/dE$ is very well described by an exponential $\exp[-k|E/\Delta E|]$. For $M_* < 0.7$, $k \gtrsim 7$, but $k$ falls to $\approx 2.5-3.0$ for $M_* \gtrsim 1$.

The spikes at $E \approx 0$ represent the last remaining gas in the simulation box. As the remnant moves farther out, both the width of this spike and the integral under it decrease. These features are also reported in other studies (e.g. Lodato et al. 2009; Coughlin et al. 2016), but the peaks shown in Figure 8 are sharper and the wings are more noticeable, especially for our low-mass stars.

Using the energy distribution data from our simulations (Figure 8) and the expression for the fallback rate (Rees 1988; Phinney 1989),

$$\dot{M}_{\text{fb}} = \left( \frac{M_*}{3P_\Delta} \right) \left( \frac{dM/\dot{M}_*}{\Delta E/2\Delta_\epsilon} \right) \left( \frac{t}{P_\Delta} \right)^{-5/3},$$

we determine the fallback rate (see Figure 9). Here $P_\Delta = (\pi/\sqrt{2})GM_{BH}\Delta E^{-3/2}$ is the orbital period for orbital energy $-\Delta \epsilon$.

For full disruptions, the shapes of the fallback rate curves divide neatly into two classes, as expected from the distinctive shapes of the energy distributions. For low-mass stars, a steep rise that reaches a maximum fallback rate $\dot{M}_{\text{max}} \simeq 0.5\dot{M}_0$ at $t \simeq (1.5-2)P_\Delta$ is followed by a quick transition to a $t^{-5/3}$ decay. Here $\dot{M}_0 = M_*/(3P_\Delta)$. On the other hand, because the energy spread $\Delta E$ for the most-bound debris from high-mass stars is $\approx 2\Delta_\epsilon$, the fallback rate peaks earlier, at $t \simeq 0.5P_\Delta$, and at a higher rate, $\dot{M}_{\text{max}} \simeq (0.8-1.3)\dot{M}_0$. The return rate of the stellar debris from 0.7 $M_\odot$ stars lies between that of low-mass and high-mass stars. This contrast in fallback history is likely to translate to observational contrasts, which may help constrain the mass of the disrupted star.

5. DISCUSSION

5.1. Comparison with previous studies

5.1.1. Physical tidal radius

Figure 4 compares our results for $R_*/r_1(= \Psi)$ with other simulations and with the correction factor introduced by Phinney (1989). We also tabulate the results in Table 2.
The dramatic change in \( \Psi \) from \( M_\star = 0.4 \) to \( M_\star = 1 \) is due to change in the internal structure of the stars. This trend was predicted by Phinney (1989), who suggested adjusting \( r_t \) by the factor \((k/f)^{1/6}\), in which \( k \) is the apsidal motion constant, reflecting the degree of central concentration, and \( f \) is the non-dimensional binding energy. Low-mass stars, which are convective except possibly near their core, tend to be rather less centrally concentrated than high-mass stars, which are convective only near their cores (see Figure 2). This leads to a prediction that \( \Psi_{k/f} = 0.82 \) for fully-convective stars (e.g., 0.15–0.4 \( M_\odot \)) and \( \Psi_{k/f} = 0.52 \) for fully-radiative stars (e.g., 1 \( M_\odot \)). The qualitative sense of this prediction is consistent with our results (\( \Psi = 1.25 – 1.45 \) for \( M_\star \leq 0.3 \) and \( \Psi = 0.425 \) for \( M_\star \geq 3 \)), but it is quantitatively discrepant, particularly for the low-mass stars. Overall, the \((k/f)^{1/6}\) correction factor underestimates the contrast in \( \Psi \) from low-mass to high-mass on both ends.

Earlier numerical simulations of TDEs (Guillochon & Ramirez-Ruiz 2013; Mainetti et al. 2017) adopted polytropic spheres to model MS stars. Guillochon & Ramirez-Ruiz (2013) focused on the mass fallback rate, using the adaptive-mesh refinement (AMR) grid-based hydrodynamics code FLASH. They considered solar mass polytropic stars with \( \gamma = 4/3 \) and 5/3 and assumed that a star is completely disrupted when the logarithmic time derivative of the self-bound stellar mass remains \(~ O(1)\) for all times after the time of pericenter passage. With this definition, they found that \( \Psi \simeq 0.54 \) for \( \gamma = 4/3 \) and \( \simeq 1.1 \) for \( \gamma = 5/3 \). Mainetti et al. (2017) measured \( \Psi \) using three numerical techniques: mesh-free finite mass, smoothed particle, and AMR grid-based hydrodynamics simulations; they then checked that the different techniques gave consistent results. Likewise considering polytropic stars with the same values of \( \gamma \) and a similar disruption criterion, they found results very close to those of Guillochon & Ramirez-Ruiz (2013): \( \Psi \simeq 0.5 \) for \( \gamma = 4/3 \) and \( \simeq 1.08 \) for \( \gamma = 5/3 \). For our fully-convective stars, those with \( M_\star = 0.15 – 0.4 \), we find a value larger by 15–30%, \( \Psi = 1.25 – 1.45 \). It is possible that this discrepancy is due to relativistic effects in the tidal stress (See Paper 4). For \( M_\star = 1 \), a polytrope with index corresponding to \( \gamma = 4/3 \) coincidentally gives a fairly good approximation to the actual density profile (see Figure 2); at this mass, we find \( \Psi = 0.475 \), 14% less than the value found from the Newtonian polytropic assumption, and slightly closer to the \((k/f)^{1/6}\) prediction.

At higher masses, the \( \gamma = 4/3 \) Newtonian polytrope approximation becomes poorer, overestimating \( \Psi \) by 27%.

Recently, Goicovic et al. (2019) performed hydrodynamics simulations for TDEs using the moving-mesh code AREPO. Just as we did, they used MESA to create the initial stellar model for a \( M_\star = 1 \) star, but their definition of full disruption was that of Guillochon & Ramirez-Ruiz (2013). They found \( \Psi = 0.5 \), still closer to our value.

Thus, where our results pertain to the same stellar model, they agree qualitatively with previous work, but with an interesting discrepancy for low-mass stars: full tidal disruptions can occur for rather larger pericenters than previously thought. As we will analyze more carefully in Ryu et al. (2019c), this discrepancy can probably be attributed to relativistic effects that only we have included. Where we treat different stellar models, most notably for \( 0.5 \leq M_\star < 1 \) and for \( M_\star \geq 3 \), there has been no directly comparable previous work. In these mass ranges, no polytropic approximation fares well. Further ramifications of relativistic effects will appear in Ryu et al. (2019c).

### 5.1.2. Debris energy distribution

Both Guillochon & Ramirez-Ruiz (2013) and Goicovic et al. (2019) presented results on \( dM/dE \). For the \( \gamma = 5/3 \) polytrope, \( \Delta E \) as found by Guillochon & Ramirez-Ruiz (2013) appears to be somewhat smaller than \( \Delta E \) for a star with \( M_\star = 1 \), the case they modeled.
For their $\gamma = 4/3$, a better description of the $M_* = 1$ case, their $\Delta E$ appears to be somewhat larger. Their result may be consistent with that found by Goicovic et al. (2019), which appears to be $\simeq 1.5\Delta \epsilon$ for $M_* = 1$, in reasonable agreement with our result, given the imprecision of reading off the figures. Goicovic et al. (2019) attribute the fact that $\Delta E \gtrsim \Delta \epsilon$ to continuing forces acting on the stars’ material after it passes inside $r = r_1$. The energy distribution figure displayed by Goicovic et al. (2019) also shows exponential wings like our $dM/dE$, and with an approximately similar slope.

5.2. The “Frozen-in” approximation

The so-called “frozen-in” approximation is frequently discussed in previous papers, but is somewhat elastically defined. When first introduced by Evans & Kochanek (1989), it referred to the fact that when the disruption finishes, the debris then travel on ballistic orbits with conserved energy and angular momentum. Lodato et al. (2009) redefined it as an “impulse approximation”, in which $dM/dE$ can be identified with the orbital energies of the individual fluid elements within the star at the moment of pericenter (interestingly, Evans & Kochanek 1989 observed that their SPH simulation of a $\gamma = 5/3$ polytrope produced $\Delta E \approx 1.8\Delta \epsilon$, whereas the impulse approximation automatically yields $\Delta E \leq \Delta \epsilon$). Stone et al. (2013) argued on the basis of an analytic model for fluid element trajectories that the impulse approximation was valid, but should be applied to the moment when the star passes inward through $r = r_1$, not to the moment of pericenter passage. The weak dependence of the mass fallback rate on $r_p$ was seen by Guillochon & Ramirez-Ruiz (2013) as evidence in favor of focusing on the moment at which $r = r_1$, but they also pointed out that in their simulation the energy of a fluid element at the end of the encounter was in general quite different from its value when the star passed inside the tidal radius.

Our results have several new things to say about the various forms of this approximation. The facts that mass-loss continues for as long as $r \sim \lambda_t$, and this condition lasts until $r \gtrsim 20 \lambda_t$, strongly point to significant dynamics continuing for far longer than the time to pass through the pericenter region or even the sphere defined by $r = r_1$ (Section 4.2). Both stellar self-gravity and pressure gradients can be comparable to black hole tidal gravity when $r \sim \lambda_t$. This finding strengthens and deepens cognate remarks made by Guillochon & Ramirez-Ruiz (2013) regarding non-impulsive dynamics and by Steinberg et al. (2019) on the significance of $\lambda_t$.

In some versions of the “frozen-in” approximation (e.g Lodato et al. 2009; Stone et al. 2013), it is used as an explanation for the characteristic energy $\Delta \epsilon$ introduced by Lacy et al. (1982). Although we find that this is a good rough estimate of the debris energy scale, it is subject to corrections at the factor of 2 level, corrections that depend on the internal structure of the star.

On the other hand, we also find little dependence of $\Delta E$ upon the pericenter of the encounter, and that it is roughly as well estimated by use of $\lambda_t$ in the expression for $\Delta \epsilon$ as by use of $\mathcal{R}_t$ (Ryu et al. 2019a). These facts point toward a special role for the scale defined by $\lambda_t$, but, given the duration of the event, it is clear that this role cannot be due to an “impulse approximation”. We suggest that the special place of $\lambda_t$ in defining the debris energy scale is due instead to its function as the radial scale on which tidal gravity can compete with stellar self-gravity (before the star is significantly distended) and the star spends the greatest amount of time.

Lastly, we point out two basic respects in which the “frozen-in” approximation (except in the narrow sense of Evans & Kochanek 1989) cannot possibly be adequate. The first is that high-mass stars can pass well within $r_1$ while still suffering only a partial disruption—for these stars, $R_1$ is only $\simeq 0.4 \lambda_t$. Even though the star is well inside $r_1$, the fluid elements destined to stay in the remnant are never well-described as following ballistic orbits determined by the black hole’s gravity. In the same fashion, for low-mass stars the physical tidal radius $\mathcal{R}_t$ is greater than $\lambda_t$. If stellar disruption happens because of an “impulse” when the star passes through $r = r_1$, they would not be disrupted at all, much less completely torn apart.

6. SUMMARY

This is the second installment in a series of papers reporting on our program of tidal disruption simulations in which the stars are given realistic main-sequence internal structures, and the gravitational dynamics are treated in full general relativity.

In our first paper (Ryu et al. 2019a), we presented an overview and highlighted our results with the greatest observational implications. Here we described the details of our calculations and our findings regarding events in which the stars are completely disrupted.

Our calculations are noteworthy in several respects: their fully relativistic treatment of dynamics due to the black hole’s gravity; their employment of MESA to determine the initial conditions, so that they begin with density profiles of realistic stars; and the large range of stellar masses explored and the relatively dense coverage of that mass-range, properties that enable us to clearly determine how mass-dependence modifies the order of magnitude picture. Although in this work we present
results for a SMBH of $10^6 \, M_\odot$, in Ryu et al. (2019c) we also explore the mass-dependence of these correction factors.

Previous work employing Newtonian dynamics had noted that the physical tidal radius for polytropes with $\gamma = 5/3$, a good model for fully-convective stars, is actually slightly greater than the widely-used order-of-magnitude estimate $r_t \equiv R_\star (M_{BH}/M_\star)^{1/3}$, while the physical tidal radius for a polytrope with $\gamma = 4/3$, a coincidentally good match to stars of mass $M_\star = 1$, but not to any others, is $\geq 0.5 \, r_t$. We have shown that when the black hole mass is $10^6$, the actual physical tidal radius is several tens of percent greater than the Newtonian prediction ($\approx 1.4 \, r_t$ rather than $\approx 1.1 \, r_t$), and that for $M_\star \geq 3$, $R_\star \approx 0.4 \, r_t$. There is a sharp (but continuous) transition between these two limits across the range of masses $M_\star = 0.5-1$. For $M_{BH} = 10^6$, the physical tidal radius of all stars with $0.15 \leq M_\star \leq 3$ is $\approx 27 \, r_g$ to within $\pm 20\%$ (Ryu et al. 2019a).

We have further demonstrated that although the characteristic debris energy scale suggested by Lacy et al. (1982) is a reasonable estimator of the actual width of the debris energy distribution, it requires factor $\sim 2$ corrections dependent upon the stellar mass. Like the ratio between physical tidal radius and nominal tidal radius, these corrections are roughly constant as a function of stellar mass at both the high and low ends of the range, but these constants are different. In addition, although the distribution of mass with energy has been widely assumed to be flat between sharp edges ever since the work of Rees (1988) and Evans & Kochanek (1989), we have found that for all stars the distribution has “shoulders” near $E \approx \Delta E$ at which $dM/dE$ is $\approx 50\%$ greater than $dM/dE$ at $E = 0$, where there is a local minimum. Moreover, although the edges of the distribution for fully-convective stars are, indeed, quite sharp, the energy distribution for debris from stars with $M_\star \geq 1$ generically has wings containing a small, but possibly significant amount of mass with energy $2-3\Delta E$.

These results strengthen the critical questions raised by the popular “frozen-in” approximation. In its most ambitious form (Lodato et al. 2009; Stone et al. 2013), it has been used to predict the ultimate energy distribution of the debris based entirely on the matter’s potential energy within the undisturbed star at radii close to the black hole (sometimes $r_\gamma$, sometimes $r_p$). In particular, we have shown that mass-loss begins only shortly after pericenter passage, and continues (in complete disruptions) until the star has reached a distance from the black hole $\approx 20 \, r_t$, which can be $\approx 50 \, R_\star$. Throughout this entire time, the instantaneous tidal radius $\lambda_t \sim r_t$. Thus, the specifics of the energy distribution are determined by continued interaction between the black hole’s gravity, the star’s self-gravity, and internal fluid forces.

Our estimates of the physical tidal radius affect, among other things, the rate of full TDEs, as well as the relative rates for stars of different masses. Low-mass stars undergo complete tidal disruptions at a higher rate than predicted by the fiducial $r_t$, while high-mass stars undergo fewer (Ryu et al. 2019a). Our alterations to the expected energy distribution lead immediately to implications regarding the rate and time-delay at which matter falls back to the star. These changes are especially noteworthy for the more massive stars, as they predict a time of peak fallback several times earlier than the traditional prediction, and a maximum rate correspondingly larger. A spread of energy larger by a factor of 2 also implies that the fastest unbound debris has a speed at infinity $\sqrt{2} \times$ greater than previously thought; further factors of 2 related to matter in the wings of the distribution can augment that ratio. These factors of 2 can be important in any attempt to relate observed light curves to the fallback rate, and from the constraints obtained determine the system’s parameters.

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REFERENCES

Cheng, R. M., & Evans, C. R. 2013, prd, 87, 104010
Colella, P., & Woodward, P. R. 1984, Journal of Computational Physics, 54, 174
Coughlin, E. R., Nixon, C., Begelman, M. C., Armitage, P. J., & Price, D. J. 2016, M.N.R.A.S., 455, 3612
d'Ascoli, S., Noble, S. C., Bowen, D. B., et al. 2018, ApJ, 865, 140
Evans, C. R., & Kochanek, C. S. 1989, ApJL, 346, L13
Gammie, C. F., McKinney, J. C., & Tóth, G. 2003, ApJ, 589, 444
Godoi, V. M. d. S. 2019, viXra e-prints, viXra:1502.0089
Goicovic, F. G., Springel, V., Ohlmann, S. T., & Pakmor, R. 2019, arXiv e-prints, arXiv:1902.08202
Graham, M. J., Kulkarni, S. R., Bellm, E. C., et al. 2019, PASP, 131, 078001
Guillochon, J., & Ramirez-Ruiz, E. 2013, ApJ, 767, 25
Hunter, J. D. 2007, Computing In Science & Engineering, 9, 90
Kinch, B. E., Schnittman, J. D., Kallman, T. R., & Krolik, J. H. 2019, ApJ, 873, 71
Kippenhahn, R., & Weigert, A. 1994, Stellar Structure and Evolution
Komossa, S. 2015, Journal of High Energy Astrophysics, 7, 148
Kormendy, J., & Ho, L. C. 2013, Ann. Rev. A&A, 51, 511
Lacy, J. H., Townes, C. H., & Hollenbach, D. J. 1982, ApJ, 262, 120
Lodato, G., King, A. R., & Pringle, J. E. 2009, M.N.R.A.S., 392, 332
LSST Science Collaboration, Abell, P. A., Allison, J., et al. 2009, arXiv e-prints, arXiv:0912.0201
Mainetti, D., Lupi, A., Campana, S., et al. 2017, A&A, 600, A124
Merloni, A., Predehl, P., Becker, W., et al. 2012, arXiv e-prints, arXiv:1209.3114
Noble, S. C., Gammie, C. F., McKinney, J. C., & Del Zanna, L. 2006, ApJ, 641, 626
Noble, S. C., Krolik, J. H., & Hawley, J. F. 2009, ApJ, 692, 411
—. 2010, ApJ, 711, 959
Noble, S. C., Mundim, B. C., Nakano, H., et al. 2012, ApJ, 755, 51
Paxton, B., Bildsten, L., Dotter, A., et al. 2011, ApJ Supp., 192, 3
Phinney, E. S. 1989, in IAU Symposium, Vol. 136, The Center of the Galaxy, ed. M. Morris, 543
Rees, M. J. 1988, Nat., 333, 523
Rogers, F. J., & Nayfonov, A. 2002, ApJ, 576, 1064
Ryu, T., Krolik, J., & Piran, T. 2019a, arXiv e-prints, arXiv:1907.08205
Ryu, T., Krolik, J. H., & Piran, T. 2019b
—. 2019c
Saumon, D., Chabrier, G., & van Horn, H. M. 1995, ApJ Supp., 99, 713
Schnittman, J. D., Krolik, J. H., & Noble, S. C. 2013, ApJ, 769, 156
Shiokawa, H., Krolik, J. H., Cheng, R. M., Piran, T., & Noble, S. C. 2015, ApJ, 804, 85
Steinberg, E., Coughlin, E. R., Stone, N. C., & Metzger, B. D. 2019, arXiv e-prints, arXiv:1903.03898
Stone, N., Sari, R., & Loeb, A. 2013, M.N.R.A.S., 435, 1809
van Velzen, S. 2018, ApJ, 852, 72