Mixing of superconducting $d_{x^2-y^2}$ state with $s$-wave states for different filling and temperature

Angsula Ghosh\textsuperscript{1}\textsuperscript{*} and Sadhan K Adhikari\textsuperscript{2}

\textsuperscript{1}Instituto de Física, Universidade de São Paulo, Caixa Postal 66318, 05315-970 São Paulo, Brazil
\textsuperscript{2}Instituto de Física Teórica, Universidade Estadual Paulista, 01.405-900 São Paulo, São Paulo, Brazil

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Abstract

We study the order parameter for mixed-symmetry states involving a major $d_{x^2-y^2}$ state and various minor $s$-wave states ($s$, $s_{xy}$, and $s_{x^2+y^2}$) for different filling and temperature for mixing angles 0 and $\pi/2$. We employ a two-dimensional tight-binding model incorporating second-neighbor hopping for tetragonal and orthorhombic lattice. There is mixing for the symmetric $s$ state both on tetragonal and orthorhombic lattice. The $s_{xy}$ state mixes with the $d_{x^2-y^2}$ state only on orthorhombic lattice. The $s_{x^2+y^2}$ state never mixes with the $d_{x^2-y^2}$ state. The temperature dependence of the order parameters is also studied.

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\textsuperscript{*}Corresponding author. e-mail: angsula@if.usp.br, Fax +55 11 3177 9080. Address: Instituto de Física, Universidade de São Paulo, Caixa Postal 66318, 05315-970 São Paulo, Brazil
I. INTRODUCTION

After many theoretical and experimental investigations on the high-$T_c$ cuprates [1], with a high critical temperature $T_c$, the symmetry of their order parameter is not yet completely known. It is now accepted [2–4] that the cuprates are quasi-two-dimensional superconductors and at higher temperatures the symmetry of the order parameter is of the $d_{x^2-y^2}$ type. However, many experiments [5–12] and related theoretical studies [13–22] suggest that at lower temperatures the order parameter of the cuprates has a mixed symmetry of the $d_{x^2-y^2} + \exp(i\theta)\chi$ type, where $\chi$ represents a minor component with a distinct symmetry superposed on the major component $d_{x^2-y^2}$. From theoretical analysis the mixing angle $\theta$ can have the values 0, $\pi/2$, $\pi$, and $3\pi/2$. For mixing angles $\pi/2$ and $3\pi/2$ the time-reversal symmetry is broken. Four possible candidates for the minor symmetry state $\chi$ are the $d_{xy}$, $s$, $s_{x^2+y^2}$ and $s_{xy}$ states. From a group theoretical point of view these states belong to the same irreducible representation of the orthorhombic point group. However, there is still controversy about the nature of the minor component and the value of the mixing angle in different high-$T_c$ materials [13–20].

The possibility of a mixed ($s-d$) wave symmetry in superconductors was suggested sometime ago by Ruckenstein et al. [21] and Koltiar [22]. Several phase-sensitive measurements on the order parameter indicate a significant mixing of minor s-wave component with a major $d_{x^2-y^2}$ state at lower temperatures in YBa$_2$Cu$_3$O$_7$ (YBCO). In several experimental analysis on the Josephson supercurrent for tunneling between a conventional s-wave superconductor (Pb) and twinned or untwinned single crystals of YBCO, the possibility of mixed states like $d_{x^2-y^2} \pm s$ or $d_{x^2-y^2} \pm is$ has been conjectured at lower temperatures [5–8]. More recently, Kouznetsov et al. [9] performed $c$-axis Josephson tunneling experiments by depositing a conventional superconductor (Pb) across the single twin boundary of a YBCO crystal and from a study of the critical current under an applied magnetic field they also conjecture a similar mixed state in YBCO. From the measurement of the microwave complex conductivity of high quality YBa$_2$Cu$_3$O$_{7-\delta}$ single crystals at 10 GHz using a high-Q Nb cavity Sridhar et al. [10] also suggested the possibility of $d_{x^2-y^2} \pm s$ or $d_{x^2-y^2} \pm is$ states. In view of this we perform a theoretical study of this problem using a tight-binding model including orthorhombic distortion and second-nearest-neighbor hopping.

There is some consensus about ($d-s$) mixing in YBCO, but in other cuprates one could have a mixing between $d_{x^2-y^2}$ and $d_{xy}$ components. However, we shall not study this mixing in this work. It is pertinent to refer to the studies which led to this conclusion in case of Bi$_2$Sr$_2$CaCu$_2$O$_8$ [11,12].

Although, there is no suitable microscopic theory for high-$T_c$ cuprates, the existence of Cooper pair as the charge carrier is usually accepted for these materials. However, there continues controversy about a proper description of the normal state and the pairing mecha-
anism for such materials. More recently, for underdoped systems it has not been possible
to accommodate the experimentally observed pseudogap \[23,24\] above the superconducting
critical temperature in a microscopic theory. However, with the increase of doping the su-
perconducting critical temperature increases and the pseudo gap is reduced and eventually
it disappears at optimal doping.

It is generally accepted that in high-$T_c$ cuprates, superconductivity resides mainly in the
CuO$_2$ planes with nearly tetragonal symmetry. Consequently, the critical temperature and
superconducting gap of high-$T_c$ materials are expected to be very sensitive to the level of
doping which determines the number of conduction electrons in the two-dimensional CuO$_2$
plane of the cuprates and it is interesting to study the effect of doping or filling on the
superconductivity in cuprates. The present single-band model can accommodate a maximum
filling of two electrons (spin up and down) per unit cell. In addition, the experimental
observation of a large anisotropy in the penetration depth between the $a$ and $b$ directions \[25\]
in YBCO suggests an orthorhombic distortion in YBCO and the present study is extended
to include such a distortion.

In the absence of a microscopic theory, we use a phenomenological two-dimensional
tight-binding model with appropriate lattice symmetry for studying some of the general
features of the mixed-symmetry states involving $d_{x^2-y^2}$ and different $s$ states. This model
has been used successfully in describing many properties of high-$T_c$ materials \[13–19\]. Both
orthorhombicity and filling are expected to play a crucial role in the evolution of these
mixed-symmetry states and we study in this paper the properties of these states for different
temperature, filling, orthorhombicity, and second-neighbor hopping.

In Sec. II we describe the formalism. In Sec. III we present our numerical results and
finally in Sec. IV we give a brief summary.

### II. THEORETICAL FORMULATION

The present tight-binding model is sufficiently general for considering mixed angular
momentum states on tetragonal and orthorhombic lattice, employing nearest and second-
nearest-neighbor hopping integrals. Here we take the effective interaction $V_{kq}$ for transition
from a momentum $q$ to $k$ to be separable, and is expanded in terms of some general basis
functions $\eta_{i}$, labeled by index $i$, so that

$$V_{kq} = -V_1 \eta_{1k} \eta_{1q} - V_2 \eta_{2k} \eta_{2q} \quad (1)$$

The separable nature of the interaction facilitates the solution of the gap equation. The
orthogonal functions $\eta_{ik}$ are associated with a one-dimensional irreducible representation
of the point group of square lattice $C_{4v}$ and are appropriate generalizations of the circular
harmonics incorporating the proper lattice symmetry. Here $V_i$ is the coupling of effective
interaction in the specific angular momentum state. In the present investigation we consider predominant singlet Cooper pairing and subsequent condensation in the $d$ and $s$ states, denoted by indices 1 and 2, respectively, and the mixed-symmetry state formed by these two.

The first function $\eta_{1q}$ corresponding to the $d_{x^2−y^2}$ state is given by

$$\eta_{1q} = \cos q_x - \beta \cos q_y,$$

$d_{x^2−y^2}$-wave, (2)

whereas the second function could be one of the following $s$ states

$$\eta_{2q} = 1,$$  
$s$-wave, (3)

$$\eta_{2q} = 2 \cos q_x \cos q_y,$$  
s$_{xy}$-wave, (4)

$$\eta_{2q} = \cos q_x + \beta \cos q_y,$$  
s$_{x^2+y^2}$-wave, (5)

etc. Here $\beta = 1$ corresponds to square lattice and $\beta \neq 1$ represents orthorhombic distortion. The orthogonality property of functions $\eta$’s is taken to be

$$\sum_q \eta_{1q}\eta_{2q} = 0, \ldots, i \neq j.$$  
(6)

Property (6) is approximate for choice (5) for $\beta \neq 1$.

We consider a single tight-binding two-dimensional band with an electron dispersion relation including second-nearest-neighbor hopping. In this case the quasiparticle dispersion relation relating the electronic energy $\epsilon_k$ and momentum $k$ is taken as

$$\epsilon_k = -2t(\cos k_x + \beta \cos k_y - 2\gamma \cos k_x \cos k_y) - \mu,$$

where $t$ and $\beta t$ are the nearest-neighbor hopping integrals along the in-plane $a$ and $b$ axes, respectively, and $\gamma t$ is the second-nearest-neighbor hopping integral. In Eq. (4) $\mu$ is the chemical potential measured with respect to the Fermi energy and is determined once the filling $n$ is specified. The nearest-neighbor hopping parameter $t$ is typically taken to be $\sim 0.1$ eV. The parameter $\beta$ destroys the symmetry between the $a$ and $b$ directions in the CuO$_2$ planes in this simple model. The potential $V_{kq}$ above also possesses such a symmetry-breaking term. The energy $\epsilon_k$ is measured with respect to the Fermi surface. Such a one-band model with different first-neighbor-hopping parameters in the $a$ and $b$ directions is the simplest approximate way of including in the theoretical description the effect of orthorhombicity.

At a finite temperature $T$, one has the following gap equation

$$\Delta_k = -\sum_q V_{kq} \Delta_q \tanh \frac{E_q}{2kBT},$$

(8)

with $E_q = [(\epsilon_q - \mu)^2 + |\Delta_q|^2]^{1/2}$, and $k_B$ the Boltzmann constant.
It has been observed that the critical temperature $T_c$ is sensitive to the level of doping which determines the number of available conduction electrons in the CuO$_2$ plane. In this model the chemical potential $\mu$ and the the filling $n$ are determined by the number equation

$$n = 1 - \sum_q \frac{\epsilon_q - \mu}{E_q} \tanh \frac{E_q}{2k_BT}. \quad (9)$$

The filling $n$ can be related to the the experimental doping $\delta$ in the three dimensional Brillouin zone by $n = 1 - \delta$. In this work we study the variation of $n$ from 0 to 1 (half filling).

The order parameter $\Delta_q$ has the following anisotropic form:

$$\Delta_q = \Delta_1 \eta_1 q + C \Delta_2 \eta_2 q, \quad (10)$$

where $C \equiv \exp(i\theta)$ is a complex number of unit modulus $|C|^2 = \cos^2 \theta + \sin^2 \theta = 1$. We substitute Eqs. (1) and (10) into the gap equation (8) and using the orthogonality property (6) obtain the two following coupled equations for $\Delta_1$ and $\Delta_2$:

$$\Delta_1 = \sum_q V_1 \eta_1 q \frac{\Delta_1 \eta_1 q + C \Delta_2 \eta_2 q}{2E_q} \tanh \frac{E_q}{2k_BT}, \quad (11)$$

$$\Delta_2 = \sum_q V_2 \eta_2 q \frac{\Delta_1 \eta_1 q + C \Delta_2 \eta_2 q}{2E_q} \tanh \frac{E_q}{2k_BT}. \quad (12)$$

Equations (11) and (12) can be substantially simplified for a purely imaginary $C$, e.g., for $C = \pm i$ ($\theta = \pm \pi/2$). In this case for real $\Delta_1$ and $\Delta_2$, the real and imaginary parts of these equations become, respectively,

$$\Delta_1 = \sum_q V_1 \eta_1 q \frac{\Delta_1 \eta_1 q}{2E_q} \tanh \frac{E_q}{2k_BT}, \quad (13)$$

$$\Delta_2 = \sum_q V_2 \eta_2 q \frac{\Delta_2 \eta_2 q}{2E_q} \tanh \frac{E_q}{2k_BT}, \quad (14)$$

since in this case

$$E_q = [(\epsilon_q - \mu)^2 + \Delta_1^2 \eta_1^2 q + \Delta_2^2 \eta_2^2 q]^{1/2}, \quad (15)$$

$$\sum_q \eta_1 q \eta_2 q \frac{E_q}{2E_q} \tanh \frac{E_q}{2k_BT} = 0, \quad (16)$$

which follows from the definitions of $\eta_1 q$ and $\eta_2 q$ and Eq. (15). However, Eq. (15), which is responsible for the simplification for $C = \pm i$ ($\theta = \pi/2$ and $3\pi/2$), does not hold for $C = \pm 1$ or for a general complex $C$.

For $C = \pm 1$ ($\theta = 0, \pi$) no further simplification of the coupled Eqs. (11) and (12) is possible and one has
\begin{align*}
\Delta_1 &= \sum_q V_1 \eta_{1q} \frac{\Delta_1 \eta_{1q} \pm \Delta_2 \eta_{2q}}{2E_q} \tanh \frac{E_q}{2k_B T}, \\
\pm \Delta_2 &= \sum_q V_2 \eta_{2q} \frac{\Delta_1 \eta_{1q} \pm \Delta_2 \eta_{2q}}{2E_q} \tanh \frac{E_q}{2k_B T},
\end{align*}

with

\[ E_q = [(\epsilon_q - \mu)^2 + (\Delta_1 \eta_{1q} \pm \Delta_2 \eta_{2q})^2]^{1/2}, \]

Finally, for a general complex \( C \) one can separate Eqs. (11) and (12) into their real and imaginary parts. In this case Eq. (16) is not valid, and the above procedure results in four equations for the two unknowns \( \Delta_1 \) and \( \Delta_2 \). These four equations are consistent only if \( \Delta_1 = 0 \) or \( \Delta_2 = 0 \), which means that there could not be mixing between the two components. So mixed-symmetry states are allowed only for mixing angles \( \theta = 0, \pi/2, \pi, \) and \( 3\pi/2 \), or for \( C = \pm 1 \), and \( \pm i \) and we shall consider only these cases in the following.

The ultraviolet momentum-space divergence of the Bardeen-Cooper-Schrieffer equation was originally neutralized by a physically-motivated Debye cutoff [26]. This procedure had the advantage of reproducing the experimentally observed isotope effect. It can also be handled by using the technique of renormalization [27,28]. Here we introduce a cut off in the momentum sums of the gap equation. As there is no pronounced isotope effect in the high-\( T_c \) cuprates, the present cut off is merely a mathematical one without any reference to the phonon-induced Debye cut off. In Eqs. (11) and (12) both the interactions \( V_1 \) and \( V_2 \) are assumed to be energy-independent constants for \( |\epsilon_q - \mu| < k_B T_D \) and zero for \( |\epsilon_q - \mu| > k_B T_D \), where \( k_B T_D \) is the present cut off.

\section*{III. NUMERICAL RESULT}

We solve the coupled set of equations (13) and (14) or (17) and (18) in conjunction with the number equation (9) numerically and calculate the gaps \( \Delta_1 \) and \( \Delta_2 \) at various filling and temperature. This gives us the opportunity to study the mixed-symmetry states for different filling and temperature on both tetragonal and orthorhombic lattice. Throughout the present study we consider the cut off \( k_B T_D = 0.1t \) with the parameter \( t = 0.2586 \) eV. This corresponds to a cut-off of \( T_D = 300 \) K. The order parameters \( \Delta_{x^2-y^2}, \Delta_{xy} \) and \( \Delta \) presented in this work are all in units of \( t \).

We study the mixture of the \( d_{x^2-y^2} \) and the symmetric \( s \) state (= 1) on tetragonal and orthorhombic lattice with second-nearest-neighbor hopping contribution. The results of our study have interesting variation as the second nearest hopping parameter is varied and this is studied in detail in the following for mixing angles 0 and \( \pi/2 \). First we consider the coupled \( d_{x^2-y^2} + is \) wave at \( T = 0 \) on a tetragonal lattice. In this case Eqs. (13) and (14)
are applicable. The parameters for this model on tetragonal lattice are the following: $\beta = 1$, $V_1 = 0.73t$, and $V_2 = 1.8t$. In Fig. 1(a) we plot $\Delta_1 \equiv \Delta_{x^2-y^2}$ and $\Delta_2 \equiv \Delta_s$ for different filling $n$ and for $\gamma = 0, 0.05, 0.1, \text{ and } 0.2$. The mixing between $d_{x^2-y^2}$ and $s$ states takes place for values of $n$ close to half filling. For $\gamma = 0.2$ the $d$ wave is completely suppressed and we have pure $s$ wave order parameter for all $n$. For orthorhombic distortion the parameters of the model are $\beta = 0.95$, $V_1 = 0.97t$, and $V_2 = 2.1t$. In Fig. 1(b) we plot $\Delta_{x^2-y^2}$ and $\Delta_s$ on orthorhombic lattice for different filling $n$ and for $\gamma = 0, 0.05, 0.1, \text{ and } 0.2$. The qualitative nature of the order parameters in Figs. 1 (a) and (b) are the same although there are quantitative differences. In both cases the mixing is limited to large values of $n$. However, in Fig. 1(b) there is mixing for $\gamma$, whereas there is none in Fig. 1(a). In Fig. 1(a) we observe a gradual decrease in $d$-wave order parameter at half filling with increase in $\gamma$. The mixing depends very much on doping and second nearest neighbour hopping.

We study superconductivity in coupled $d_{x^2-y^2} + s$ wave at $T = 0$, governed by Eqs. (17) and (18), which corresponds to the mixing angle 0. On a tetragonal lattice there is no mixing but meaningful mixing is possible on orthorhombic lattice and we study this case in detail. In Fig. 2 we plot $\Delta_{x^2-y^2}$ and $\Delta_s$ for different filling $n$ and $\gamma = 0, 0.05, 0.1, \text{ and } 0.2$ calculated with $\beta = 0.95$, $V_1 = 0.97t$, and $V_2 = 2.1t$. In this case the $d$ and the $s$ waves can coexist and the mixing occurs for large $n$ values. However, with increasing $\gamma$ the $d$-wave component is reduced in magnitude.

Next we consider the mixing of the $d_{x^2-y^2}$ wave with the $s_{xy}$ wave for both mixing angles 0 and $\pi/2$. In both cases there is no mixing on tetragonal lattice. However, there is mixing on a orthorhombic lattice and we discuss the detail below. First we consider the $d_{x^2-y^2} + is_{xy}$ case, where we solve Eqs. (13) and (14) with $\beta = 0.95$, $V_1 = 0.95t$, and $V_2 = 1.17t$, for $\gamma = 0, 0.05, 0.1, \text{ and } 0.2$. In Fig. 3(a) we plot $\Delta_{x^2-y^2}$ and $\Delta_{s_{xy}}$ for different filling $n$. The interesting region of mixing occurs for very large values of $n$. Next we consider the $d_{x^2-y^2} + s_{xy}$ case, where we solve Eqs. (17) and (18) with $\beta = 0.95$, $V_1 = 0.95t$, and $V_2 = 1.17t$, for $\gamma = 0, 0.05, 0.1, \text{ and } 0.2$. In Fig. 3(b) we plot $\Delta_{x^2-y^2}$ and $\Delta_{s_{xy}}$ for different filling $n$. No mixing is found between $\Delta_{x^2-y^2}$ and $s_{x^2+y^2}$ waves on tetragonal and orthorhombic lattice and we shall not discuss this case further.

Now we investigate the temperature dependence of the order parameters in different cases. We studied several cases for different values of $n$, $\gamma$ and $\beta(= 1, 0.95)$. The qualitative nature of the temperature dependence in different cases are different and we discuss them separately. Figure 4(a) illustrates the temperature dependencies of the order parameters for the $d_{x^2-y^2} + is$ case on square lattice calculated with the parameters of Fig. 1(a) for $\gamma = 0.05$ and $n = 0.95$ (critical temperature 69 K) and for $\gamma = 0.1$ and $n = 0.9$ (critical temperature 70 K). The nature of the order parameters of Fig. 4(a) does not change in the presence of orthorhombic distortion. The order parameters in this case are similar to those in the uncoupled case. The only difference is that at lower temperatures in the presence of
the \( s \)-wave component the \( d \)-wave order parameter gets a bit suppressed. In this case the \( s \)-wave order parameter goes to zero at a temperature lower than \( T_c \).

In Fig. 4(b) we show the temperature dependencies of the order parameters for the \( d_{x^2-y^2} + s \) case on orthorhombic lattice calculated with the parameters of Fig. 2 for \( \gamma = 0.05 \) and \( n = 0.9 \) (critical temperature 70 K), and for \( \gamma = 0 \) and \( n = 0.95 \) (critical temperature 90 K). There is a qualitative difference between the order parameters of Figs. 4(a) and (b). In Fig. 4(b) both the components become zero at the critical temperature and a mixed-wave order parameter is present for all temperatures below \( T_c \), whereas in Fig. 4(a) there is a temperature region where only the \( d \)-wave order parameter exists.

Next we study the order parameters in case of mixture with the \( s_{xy} \) state. First we consider the \( d_{x^2-y^2} + s_{xy} \) type mixture on orthorhombic lattice corresponding to Fig. 3(b) with \( \gamma = 0.1, \ n = 0.95, \ T_c = 98 \) K, and with \( \gamma = 0.2, \ n = 0.95, \ T_c = 64 \) K. The order parameters for different temperatures are plotted in Fig. 5(a). The temperature dependence in this case behaves as in Fig. 4(b). Both components are nonzero immediately below the critical temperature. Next we consider the \( d_{x^2-y^2} + i s_{xy} \) type mixture on orthorhombic lattice corresponding to Fig. 3(a) with \( \gamma = 0.1, \ n = 0.9, \ T_c = 68 \) K, and with \( \gamma = 0.0, \ n = 0.98, \ T_c = 73 \) K. The corresponding order parameters at different temperatures are plotted in Fig. 5(b). The nature of the order parameters in Fig. 5(a) is quite different from those in Fig. 5(b). In all cases we observe very different temperature dependencies of order parameter compared to the standard BCS-model results for uncoupled wave.

**IV. SUMMARY**

In this work we have studied the mixed-symmetry superconducting states comprising of \( d_{x^2-y^2} \) and different \( s \) waves appropriate for two-dimensional cuprates using a tight-binding model on tetragonal and orthorhombic lattice. We studied the variation of order parameters with filling \( n \) for tetragonal and orthorhombic lattices for different second-neighbor hopping for pure and mixed-symmetry states. The mixing of \( d_{x^2-y^2} \) and \( s \) waves varies considerably with doping and second-neighbor hopping. We observe mixing to take place at large values of filling. We have also studied the temperature dependence of the order parameter under different situations. The temperature dependence for \( d_{x^2-y^2} + i s \) is similar to our previous studies \([16,18]\) for the same mixing at \( n = 1 \). However, this dependence is very different for the time reversal symmetry cases \( d_{x^2-y^2} + s \) and \( d_{x^2-y^2} + s_{xy} \).

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**Figure Captions:**

1. The order parameters $\Delta_{x^2-y^2}$ (dashed line) and $\Delta_s$ (full line) for mixed wave $d_{x^2-y^2} + is$ on (a) tetragonal and (b) orthorhombic lattice for different $n$ and $\gamma$. The parameters for the model on tetragonal lattice are $\beta = 1, V_1 = 0.73t, V_2 = 1.8t$ and on orthorhombic lattice are $\beta = 0.95, V_1 = 0.97t, V_2 = 2.1t$.

2. The order parameters $\Delta_{x^2-y^2}$ (dashed line) and $\Delta_s$ (full line) for mixed wave $d_{x^2-y^2} + s$ on orthorhombic lattice for different $n$ and $\gamma$. The parameters for the model are $\beta = 0.95, V_1 = 0.97t, V_2 = 2.1t$.

3. The order parameters $\Delta_{x^2-y^2}$ (dashed line) and $\Delta_{sxy}$ (full line) for mixed wave (a) $d_{x^2-y^2} + is_{xy}$ and (b) $d_{x^2-y^2} + s_{xy}$ on orthorhombic lattice for different $n$ and $\gamma$. The parameters for both models (a) and (b) are $\beta = 0.95, V_1 = 0.95t, V_2 = 1.17t$.

4. Temperature dependence of the order parameters $\Delta_{x^2-y^2}$ and $\Delta_s$ for mixed-symmetry (a) $d_{x^2-y^2} + is$ state on tetragonal lattice for $\gamma = 0.05$ and $n = 0.95$ (dashed line) and for $\gamma = 0.1$ and $n = 0.9$ (full line) and for mixed symmetry (b) $d_{x^2-y^2} + s$ state on orthorhombic lattice for $\gamma = 0.05$ and $n = 0.9$ (dashed line) and for $\gamma = 0$ and $n = 0.95$ (full line).

5. Temperature dependence of the order parameters $\Delta_{x^2-y^2}$ and $\Delta_s$ for mixed-symmetry (a) $d_{x^2-y^2} + s_{xy}$ state on orthorhombic lattice for $\gamma = 0.1$ and $n = 0.95$ (dashed line) and for $\gamma = 0.2$ and $n = 0.95$ (full line) and for mixed symmetry (b) $d_{x^2-y^2} + is_{xy}$ state on orthorhombic lattice for $\gamma = 0.1$ and $n = 0.9$ (dashed line) and for $\gamma = 0$ and $n = 0.98$ (full line).
Fig. 1(a)

\[ \Delta x^2, \Delta y^2, \Delta \]

\[ n \]

\( \gamma = 0.2 \)
\( \gamma = 0.1 \)
\( \gamma = 0.05 \)
\( \gamma = 0 \)
\[ \Delta^2 x - y^2, \Delta \]

\[ \gamma = 0.2 \]

\[ \gamma = 0.1 \]

\[ \gamma = 0.05 \]

\[ \gamma = 0 \]
Fig. 2

\[ \Delta x^2 - \gamma^2, \Delta \]

\( \gamma = 0.2 \)

\( \gamma = 0.1 \)

\( \gamma = 0.05 \)

\( \gamma = 0 \)
Fig. 3(a)

\[ \Delta x^2 - y^2, \Delta \chi y \]

\( \gamma = 0.2 \)

\( \gamma = 0.1 \)

\( \gamma = 0.05 \)

\( \gamma = 0 \)
Fig. 3(b)

\[ \Delta x^2 - y^2, \Delta xy \]

\( \gamma = 0.2 \)
\( \gamma = 0.1 \)
\( \gamma = 0.05 \)
\( \gamma = 0 \)
Fig 4(a)

\[ \Delta^2 \Delta_{x-y} \]

\[ T (K) \]

\[ \Delta \]

\[ \Delta \]

\[ d \]

\[ s \]
Fig 4(b)
Fig 5(a)
Fig 5(b)