ARTICLE TYPE

Fusion dynamics of $^{12}\text{C}+^{12}\text{C}$ reaction: An astrophysical interest within the relativistic mean-field approach

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Funding Information
DAE-BRNS Project Sanction No, 58/14/12/2019-BRNS, FOSTECT Project Code: FOSTECT.2019B.04, and FAPESP Project Nos. 2017/05660-0,

INTRODUCTION

Nuclear fusion reactions provide the source for stellar energy. In the process of stellar Helium burning, the main products are $^{12}\text{C}$ and $^{16}\text{O}$. For massive stars, with mass greater than $8M_{\odot}$, the Carbon and Oxygen burning reactions, and principally the $^{12}\text{C}+^{12}\text{C}$, plays a significant role in later phases of stellar evolution and explosions (Aguilera et al. (2006); Patterson, Winkle & Zaidins (1969); Tan et al. (2020)). The crucial temperature for such reactions to occur in the stars lies in the range of 0.8 to 1.2 GK, which corresponds to the center-of-mass energies ($E_{c.m.}$) of 1-3 MeV. The experimental measurement of the fusion cross-section at such low energies of astrophysical interest is tedious due to the suppression of cross-section by the Coulomb barrier (Assunção & Descouvemont (2013); Patterson et al. (1969)). In addition to this, there are resonant structures observed even at a very low energy region. As a result, large uncertainties persist in reaction rate while extrapolating the data at an astrophysically significant energy range (Beck, Mukhamedzhanov, & Tang (2020); Tan et al. (2020)). So the experimental measurement of the fusion cross-section for $^{12}\text{C}+^{12}\text{C}$ fusion reaction have been limited to the energies above $E_{c.m.} = 2.1$ MeV (Beck et al. (2020); Zhang et al. (2020)).

To extrapolate the data in the lower energy regions of astrophysical interest, the theoretical modeling of heavy-ion $^{12}\text{C}+^{12}\text{C}$ fusion reaction is necessary. Various phenomenological and microscopic models have been developed to explain the fusion dynamics of these heavy-ion reactions (Beck et al. (2020); Zhang et al. (2020)). Further, to remove the effects arising due to the Coulomb barrier, the fusion cross-section for astrophysical reactions is defined in terms of astrophysical

**KEYWORDS:** Relativistic Mean Field, Nucleon-Nucleon Potential, Nucleus-Nucleus Potential, $\ell$- summed Wong Model, Fusion Cross-section, S-factor

*Abbreviations: RMF, Relativistic mean field; NN, Nucleon-Nucleon
S-factor. This S-factor contains all the intrinsic nuclear factors which influence the reaction cross-section and is observed to follow a rising trend towards the lower energies \cite{Aguilera2006, Patterson1969, Zhang2020}. The barrier penetration model using the proximity adiabatic & Krappe-Nix-Sierk potentials \cite{Aguilera2006}, coupled channel calculations \cite{Assuncao2013, Ebensen2011}, density-constrained time-dependent Hartree-Fock method \cite{Umar2013, Esbensen2011}, density-constrained coupled channel calculations \cite{Assuncao2006, Descouvemont2006, Patterson1969, Zhang2020}. The barrier penetration model using the proximity adiabatic approach \cite{Satchler1979} to obtain the nuclear potential. The resultant of Coulomb and nuclear potentials gives the fusion barrier. We have adopted the double folding approach \cite{Satchler1979} to obtain the nuclear interaction potential \( V_n(R) \) and is given as,

\[
V_n(R) = \int \rho_p(\vec{r}_p)\rho_t(\vec{r}_t) V_{\text{eff}} \left| \vec{r}_p - \vec{r}_t + \vec{R} \right| d^3r_p d^3r_t.
\]

Here, \( \rho_p \) and \( \rho_t \) are the nuclear density distributions of interacting projectile and target nuclei respectively. \( V_{\text{eff}} \) is the effective nucleon-nucleon \( (\text{NN}) \) interaction potential.

The densities of the projectile and target nuclei in Eq. \( \text{(2)} \) are obtained using relativistic mean field (RMF) formalism. More details of RMF approach can be found in the Refs. \cite{Bhuyan2018, Bhuyan2020, Lalazissis2009, Ring1996} and references therein. Here we have used two kinds of nucleon-nucleon potentials, namely, (1) the most popular M3Y potential given in terms of three Yukawa terms \cite{Bhuyan2018, Bhuyan2020, Satchler1979}; and (2) the recently developed relativistic R3Y potential by solving the RMF equations of motion for mesons in limit of one-meson exchange \cite{Bhuyan2018, Bhuyan2020, Sahu2014, Singh2012}. For the present study we have used recently developed non-linear NL3* parameter set \cite{Lalazissis2009}, which is the refitted version of the NL3 parameter set. It is worth mentioning that the relativistic R3Y NN potential is analogous to the M3Y potential and can be used for various nuclear studies, such as proton and cluster radioactivity, nuclear decay, nuclear fusion and so on \cite{Bhuyan2018, Bhuyan2020, Sahu2014, Singh2012}. The barrier characteristics i.e. barrier height, position and frequency are extracted from the total interaction potential and are used to estimate the fusion cross-section for \(^{12}\text{C} + ^{12}\text{C} \) system.

Wong gave a formula that use s-wave barrier characteristics \cite{Wong1973} to obtain the cross-section for fusion. It excludes the angular momentum dependence of potential, which was later included by \cite{Kumar2009}. The extended formula is named as \( \ell \)-summed Wong model \cite{Bhuyan2018, Bhuyan2020, Kumar2009}. In this model, the fusion cross-section in terms of the partial wave is given as,

\[
\sigma(E_{c.m.}) = \frac{\pi}{k^2} \sum_{\ell=0}^{\ell_{\text{max}}} (2\ell + 1) P_{\ell}(E_{c.m.}).
\]

Here, \( E_{c.m.} \) is the center-of-mass energy of two colliding nuclei and \( P_{\ell} \) is known as the transmission coefficient for \( \ell \)-th partial wave. It is generated using Hill-Wheeler approximation \cite{Wong1973} and references therein). In terms of barrier height \( V_B^{\ell}(E_{c.m.}) \) and curvature \( h_{\ell}^{\ell}(E_{c.m.}) \), \( P_{\ell} \) is written as,

\[
P_{\ell} = \left[ 1 + \exp \left( \frac{2\pi V_{B}^{\ell}(E_{c.m.}) - E_{c.m.}}{h_{\ell}^{\ell}(E_{c.m.})} \right) \right].
\]

\( P_{\ell} \) describes the penetration of barrier given by Eq. \( \text{(1)} \). The \( \ell_{\text{max}} \)-values are obtained from the sharp cut-off model \cite{Beckerman1981} and extrapolated for below barrier energies. It is to be noted that this model can be used to calculate the fusion cross-section around the Coulomb barrier. At energies far below the barrier, the Coulomb force dominates. So to remove most of the Coulomb barrier penetration effect, the astrophysical S-factor was introduced. It depends upon the intrinsic effects of nuclear forces and for \(^{12}\text{C} + ^{12}\text{C} \) system it is given by \cite{Aguilera2006, Patterson1969},

\[
S = \sigma E_{c.m.}\exp(87.21E^{-1/2} + 0.46E).
\]
The fusion cross-section as a function of center-of-mass energy \( E_{c.m.} \) is presented in Fig. 2 for both M3Y (blue line) and R3Y (black line) NN interactions. The calculations of the fusion cross-section are started from \( E_{c.m.} = 0.5 \text{ MeV} \) onward. This energy range is far below the observed Coulomb barrier and lies well within the astrophysically significant energy range (1-3 MeV). The \( \ell_{\text{max}} \) values at above barrier regions are calculated from the sharp cut off model (Beckerman et al. (1981)) and extrapolated for below barrier energies. For comparison the experimental (black spheres) and the extrapolated (black circles) data from Ref. (Patterson et al. (1969)) and experimental data (solid red squares) from Ref. (Aguilera et al. (2006)) is also plotted in the Fig. 2. It can be observed here that R3Y NN interaction gives a higher cross-section as compared with the M3Y interaction potential at all energy regions. On comparing the calculated results with the experimental data it is observed that R3Y NN interaction gives a better fit to the experimental data than the M3Y interaction at lower energies.
below barrier as well as above barrier region. If we observe the far below barrier region ($E_{c.m.} < 4.0$ MeV) then it is found that both M3Y and R3Y NN interaction potentials give higher fusion cross-section as compared to the extrapolated ones (Patterson et al. [1969]). As no experimental data is available for this energy range. It is to be noted here that the $\ell_{\text{max}}$ can take only integer values else a relatively better fit to the data could also be achieved with R3Y interaction. So overall, the calculated fusion cross-section with R3Y NN interaction gives a nice fit to the measured experimental data and overestimates the extrapolated data of (Patterson et al. [1969]). The astrophysical S-factor calculated using Eq. (5) for R3Y (black line) and M3Y (blue line) NN-interactions is compared with the experimental (solid black dots from Ref. (Patterson et al. [1969]) and red squares from Ref. (Aguilera et al. [2006]) and extrapolated data (hollow circles from Ref. (Patterson et al. [1969]) as a function of $E_{c.m.}$ for $^{12}\text{C} + ^{12}\text{C}$ reaction in Fig. 3. It is observed that the astrophysical S-factor increases sharply at far below barrier energy. As evident from the fusion cross-section as well, the S-factor corresponding to R3Y NN interaction potential follows a similar trend of the experimental data. The calculated S-factor at far below barrier energies is higher for both the cases of M3Y and R3Y potentials as compared to the extrapolated fusion cross-section (Patterson et al. [1969]). This shows that either R3Y and M3Y potentials give relatively attractive nuclear interaction potential and/or modification of Eq. (5) by implementing proper structural input.

4 | SUMMARY AND CONCLUSIONS

We have calculated the fusion cross-section and S-factor for $^{12}\text{C} + ^{12}\text{C}$ system which holds a great astrophysical significance. The nuclear interaction potential is obtained from the double folding model furnished with relativistic mean-field (RMF) density distributions along with phenomenological M3Y and microscopic R3Y NN interaction. The $\ell'$-summed Wong formula is employed to calculate the fusion cross-section. The fusion cross-section and S-factor obtained using microscopic R3Y NN interaction derived from RMF theory is found to be more close to the experimental data as compared to phenomenological M3Y NN interaction both at around the barrier as well as above energy regions. At far below barrier center of mass energies ($E_{c.m.} < 4.0$ MeV), both M3Y and R3Y NN interaction potentials are observed to give higher values of fusion cross-section as well the S-factor as compared to the available extrapolated ones. Thus the present work has great motive to test the relativistic mean-field approach for giving a reasonable fit to the fusion cross-section of $^{12}\text{C} + ^{12}\text{C}$, and then use the same approach to predict the cross-section at below barrier energies of astrophysical interest as no experimental data is available. It will be of future interest to investigate the fusion of other Carbon and Oxygen burning reactions such as $^{12}\text{C} + ^{16}\text{O}$ and $^{16}\text{O} + ^{16}\text{O}$ etc. within this microscopic approach, which can lead to a better understanding of fusion cross-section at lower energies of astrophysical interest and hence the later phases of stellar evolution.

ACKNOWLEDGMENTS

This work was supported by DAE-BRNS Project Sanction No. 58/14/12/2019-BRNS, FOSTECT Project Code: FOSTECT.2019B.04, and FAPESP Project Nos. 2017/05660-0.

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How to cite this article: Shilpa Rana, Raj Kumar, and M. Bhuyan (2020), Fusion dynamics of $^{12}\text{C} + ^{12}\text{C}$ reaction: An astrophysical interest within the relativistic mean-field approach, Astronomische Nachrichten, XXXX; YY:xxxx.

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