Cosmological solutions from models with unified dark energy and dark matter and with inflaton field

Denitsa Staicova and Michail Stoilov

Abstract Recently, few cosmological models with additional non-Riemannian volume form(s) have been proposed. In this article we use Supernovae type Ia experimental data to test one of these models which provides a unified description of both dark energy via dynamically generated cosmological constant and dark matter as a "dust" fluid due to a hidden nonlinear Noether symmetry. It turns out that the model allows various scenarios of the future Universe evolution and in the same time perfectly fits contemporary observational data. Further, we investigate the influence of an additional inflaton field with a step like potential. With its help we can reproduce the Universe inflation epoch, matter dominated epoch and present accelerating expansion in a seamless way. Interesting feature is that inflaton undergoes a finite change during its evolution. It can be speculated that the inflaton asymptotic value is connected to the vacuum expectation value of the Higgs field.

1 The two-measures model

The application of the two-measures model [1, 2, 3, 4] to cosmology has been pioneered in series of articles by Guendelman, Nissimov and Pacheva ([5, 6, 7, 8, 9, 10, 11, 12]). In those articles, it has been described a model which is able to describe both dark matter and dark energy in the Universe and also
early inflation. This is achieved by the introduction of two scalar fields – a
darkon and an inflaton – in a scalar Lagrangian coupled both to the standard
Riemannian volume-form (the square root of the metric determinant) and to
another non-Riemannian volume form (given in terms of auxiliary maximal-
rank antisymmetric tensor gauge field). The effect of the additional measure
in the theory is felt only through the ratio of the two measures – a constraint
determined by an algebraic equation. The equations of motion of such a
theory generate dynamically a cosmological constant and a dark matter dust
fluid term and also inflation-inducing terms.

In this article, we will discuss our recent numerical investigations of this
model in the case of both darkon-only universe and of darkon-inflaton uni-
verse. In the first case, we were able to successfully fit the model with the
Supernova Type 1 data and to limit its parameter space to observationally
acceptable values. We also showed that in this case it is possible for the
Universe to undergo a phase transition. In the second case, we were able to
reproduce the stages of the Universe expansion – early inflation, matter dom-
ination and late inflation under certain choices for the parameters. We also
observed some novel features, like the matter-dominated early epoch and a
non-zero scalar field in the late Universe. In both cases, we have numerically
confirmed that the two-measures model can be a viable cosmological model.

2 The darkon model in FLRW metric

The action of the two-measures darkon model in the f(R) gravity (Guendel-
man, Nissimov and Pacheva ([6],[9])) has the following form:

\[ S_{\text{darkon}} = \int d^4x \sqrt{-g} \left( R(g, \Gamma) - \alpha R^2(g, \Gamma) \right) + \int d^4x \left( \sqrt{-g} + \Phi(C) \right) L(u, X) \]

where \( \Phi(C) = \frac{1}{3} \epsilon^{\mu\nu\kappa\lambda} \partial_\mu C_{\nu\kappa\lambda} \) is the non-Riemannian measure and \( L(u, X) = -\frac{1}{2} g^{\mu\nu} \partial_\mu u \partial_\nu u - V(u) \) is the matter Lagrangian of the darkon scalar field \( u \).

If one applies the equations of motion obtained from this action to the
Friedman–Lemaître–Robertson–Walker metric with \( k = 0 \):

\[ ds^2 = -dt^2 + a(t) \left[ dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right] \]

one obtains from the Friedman equations \((G_{00} = T_{00})\) the following relations
for the energy density:

\[ \rho = \frac{1}{8\alpha} \dot{u}^2 + \frac{3}{4} \frac{p_u}{a(t)^3} \dot{u} - \frac{1}{4\alpha} \]

\[ p_u = a(t)^3 \left[ -\frac{1}{2\alpha} \dot{u} + \left( \frac{1}{4\alpha} - 2M_0 \right) \dot{u}^3 \right] \]
where \( p_u = \text{const.} \)

Following our work in [13], we rewrite the last cubic equation for \( \dot{u} \) (Eq. (3)), as \( y^3 + 3ay + 2b = 0 \) with \( a = -\frac{2}{3-24\alpha M_0} \) and \( b = -\frac{2\alpha p_u}{a(t)^{2(1-8\alpha M_0)}} \), \( y = \dot{u} \).

The solutions are:

\[
\begin{align*}
y_1 &= -\frac{a}{(-b + \sqrt{a^2 + b^2})^{1/3}} + (-b + \sqrt{a^3 + b^2})^{1/3} \\
y_2 &= \frac{a}{(b - \sqrt{a^2 + b^2})^{1/3}} - (b - \sqrt{a^3 + b^2})^{1/3} \\
y_3 &= \frac{y_2 - i\sqrt{3}y_1}{2}
\end{align*}
\]

Since no real smooth solution exists in the whole \([a, b]\) plane, we define the following piecewise functions, \textbf{real} in the whole plane \([a, b]\):

\[
y_b = \begin{cases} 
y_1 & \text{for } (a, b) \in \{a \geq 0\} \cup \{a < 0 \cap b < 0\} \\
y_2 & \text{for } (a, b) \in \{a < 0 \cap b > 0\}
\end{cases} \quad y_s = \begin{cases} 
y_1 & \text{for } b > 0 \\
y_2 & \text{for } b < 0.
\end{cases}
\]

We obtain the final form of the Friedman equation after rescaling time by \( 2|\alpha|/3 = 1 \) and absorbing \( \alpha \) into Hubble constant \( \bar{\rho} = \frac{4|\alpha|}{\rho} \):

\[
\left( \frac{\dot{a}(t)}{a(t)} \right)^2 = \bar{\rho} = \left( \frac{1}{2} y^2 + \frac{b}{a} y - 1 \right) \quad (4)
\]

The asymptotics, corresponding to the dark energy term in the late universe, is:

\[
\bar{\rho} \xrightarrow{a(t) \to \infty} \begin{cases} 1 & \text{for } a > 0 \\
\frac{3}{2}a - 1 & \text{for } a < 0
\end{cases}
\]

We use as independent real solutions \( y_b \) (our basic solution) and \( y_s \) and integrate numerically Eq. (4) to find the evolution of the universe.

**Phase transition:** From the numerical integration we have seen that it is possible to obtain both Universes with or without phase transition. The possibility for such transition comes from the fact that in the \([a, b]\) plane exist sectors where two of our solutions have positive energy density \( \rho \).

Explicitly, let’s denote \( \bar{\rho} \) the density corresponding to solution \( y_s \) (\( \bar{\rho} \) corresponds to \( y_b \)). At the moment \( t_1 = 0 \), it will be negative, i.e. \( \bar{\rho}(t_1) < 0 \). For certain moment \( t_p \), however, it will change sign: \( \bar{\rho}(a(t_p)) = 0 \). Therefore, for any moment \( t > t_p \) we have two ”states” of the Universe \( \bar{\rho} \) and \( \bar{\rho} \) such as:

\[
0 \leq \bar{\rho} < \bar{\rho} \quad \text{for } t \geq t_p.
\]

This opens the possibility the Universe to undergo ”phase transition” or ”quenching” to the lower state.

The moment of the phase transition is crucial for the further evolution, since if \( t = t_p \) the evolution stops (\( \bar{\rho} = 0 \)), if \( t = t_p + \delta t \), we observe phase
transition of the first kind. An illustration of this process can be seen on Fig. 1, where the transition happens between lines \( b, b_1 \) and \( b_2 \).

![Fig. 1: Graphics of the \( a(t) \) evolution for: \( a = -5.987, b = \frac{-2.932}{a^3} \) (a) \( a = -1, b = \frac{-2}{a^3} \) (b), \( t_p = 1.5074, a_s(t_p) = 2.0825 \) (b_2), \( a = -0.5, b = \frac{-0.5}{a^3} \) (c)](image)

**The Supernova Fit:** Using the freely available data of Supernovae Type 1a [14], we were able to fit the distance modulus \( d_m \) as a function of \( z \) using energy density from the two-measures model:

\[
d_m = 5 \log_{10} \left( (1 + z) \int_0^z dx \frac{a(x)}{\dot{a}(x)} \right) = \text{const} + 5 \log_{10} \left( (1 + z) \int_0^z dx \frac{1}{\sqrt{\rho(x)}} \right) \tag{6}
\]

Using a symplectic fit, we were able to prove that our model is able to reproduce the observational data. The details on the fit can be seen in [13], here we will emphasize only that the precision of the fit (\( \chi^2 \sim 562 \) for \( a < -2/3, \chi^2 \sim 578 \) for \( a > 1 \)) is similar to the one of the standard model (\( \chi^2 = 562 \)).

The best fit of SN data using the proposed model is not unique. We find one parametric family of solutions producing the same \( d_m(z) \) function. An approximate formula for the dependency \( b(a) \) can be obtained using the LeastSquares algorithm in Maple and it gives:

\[
b_{\pm} = \sum_{0}^{4} \pm c_i a^i + O(a^5), \text{ with coefficients}
\]

\[
c_i = [0.337906, 0.376679, -0.0251697, 0.00148545, 0.11272710^{-3}]
\]

\(^1 \) \( d_m = 5 \log_{10} \left( \frac{d}{10} \right) \), where \( d \) is the distance in parsecs.
On Fig. 1, curve (d) represents one such evolution with parameters $a$ and $b$ corresponding to the observational data.

### 3 The two-measures theory – including the inflaton

In order to produce inflation in the model, one needs to include a new scalar field – the inflaton $\phi$. Following Guendelman, Nissimov and Pacheva [12, 6] (where in $S_{darkon} \alpha = 0$), the action, featuring two non-Riemannian measures $\Phi_1(A)$ and $\Phi_2(B)$, becomes:

$$S = S_{darkon} + \int d^4x \Phi_1(A)(R + L^{(1)}) + \int d^4x \Phi_2(B) \left( L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}} \right)$$

where:

$$L^{(1)} = -\frac{1}{2}g^{\mu\nu}\partial_\mu \phi \partial_\nu \phi - V(\phi), \quad V(\phi) = f_1 e^{-\alpha \phi}$$

(7)

$$L^{(2)} = -\frac{b}{2} e^{-\alpha \phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + U(\phi), \quad U(\phi) = f_2 e^{-2\alpha \phi}$$

(8)

From the equations of motion we have:

$$p = -2M_0 = const, \quad \frac{\Phi_2(B)}{\sqrt{-g}} = \chi_2 = const$$

$$R + L^{(1)} = -M_1 = const, \quad L^{(2)} + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = const$$

$$U_{eff}(\phi) = \frac{(V_1(\phi) + M_1)^2}{4\chi_2(U(\phi) + M_2)}$$

An important condition following from the requirement that the vacuum energy density of the early Universe $U_-$ should be much higher than that of the late Universe $U_+$ gives:

$$\frac{f_1^2}{f_2} \gg \frac{M_1^2}{M_2}$$

(10)

This ensures that the effective potential has the form of two infinite plateaus connected with a steep slope.

Additionally, one can postulate:

$$|M_1| \sim M_{EW}^4, \quad M_2 \sim M_{Pl}^4, \quad f_1 \sim f_2 \sim 10^{-8} M_{Pl}^4,$$

so that one can connect the theory with the electroweak and the Planck scales.

The system of equations that need to be solved numerically in order to obtain the evolution of the Universe is the following:
In order to work in the sector III, where both solutions have chosen the parameters in such a way that during the late (current) expansion trajectory for one set of parameters (dots). It starts at which the Universe will describe in its evolution won't be straight lines like during matter domination period ($w < 0$. One can see the different epochs by plotting the equation of state $w = p/\rho$.

\[ v^3 + 3av + 2b = 0 \]  
\[ a = \frac{-1}{3} V(\phi) + M_1 - \frac{1}{2} \chi_2 b e^{-\alpha \phi} \dot{\phi}^2, \quad b = \frac{-p_u}{2a(t)^3(\chi_2(U(\phi) + M_2) - 2M_0)} \]  
\[ \dot{a}(t) = \sqrt{\frac{\rho}{6}}(t), \quad \rho = \frac{1}{2} \dot{\phi}^2(1 + \frac{3}{4} \chi_2 b e^{-\alpha \phi} v^2) + \frac{v^2}{4} (V + M_1) + \frac{3p_u v}{4a(t)^3} \]  
\[ \ddot{a}(t) = -\frac{1}{12} (\rho + 3p) a(t), \quad p = \frac{1}{2} \dot{\phi}^2(1 + \frac{1}{2} \chi_2 b e^{-\alpha \phi} v^2) - \frac{v^2}{4} (V + M_1) + \frac{p_u v}{4a(t)^3} \]  
\[ \frac{d}{dt}\left( a(t)^3 \phi(1 + \frac{\chi_2 b e^{-\alpha \phi} v^2}{2}) + a(t)^3 \frac{\dot{\phi}^2}{4} \chi_2 b e^{-\alpha \phi} v^2 + \frac{1}{2} V_\phi v^2 - \chi_2 U_\phi v^4 \right) = 0 \]

Here Eq. (14) is optional and it offers an independent way to evaluate $\ddot{a}(t)$. This differential system is of first order with respect to $a(t)$ and of second order with respect to $\phi(t)$. Once again, we first solve the cubic equation by choosing a base solution and then we use it, to integrate the differential system with the implemented in Maple Fehlberg fourth-fifth order Runge-Kutta method with degree four interpolant.

**Study the $[a, b]$ plane** Unlike the previous case, where $a$ was a constant, here it depends on $\phi, \dot{\phi}$. Because of this, the trajectories in the $[a, b]$ plane which the Universe will describe in its evolution won’t be straight lines like in the darkon case, but curves. For example, on Fig. 2 we have plotted the trajectory for one set of parameters (dots). It starts at $b \rightarrow -\infty$ and ends at $b \rightarrow 0$. In its evolution, it crosses the $a^3 + b^2 = 0$ line (solid line). On the plot, one can see also the trajectories for the darkon case plotted with dashed lines. In order to work in the sector III, where both solutions $y_b$ and $y_s$ are valid we have chosen the parameters in such a way that $b = \frac{-p_u}{2a(t)^3(\chi_2(U(\phi) + M_2) - 2M_0)} < 0$.

**Numerical integration and equation of state** For the numerical integration, we impose as initial conditions:

\[ a(0) = 10^{-12}, \phi(0) = \phi_0, \dot{\phi}(0) = 0. \]

Additionally, we impose the gauge condition: $a(1) = 1$.

This problem has 12 parameters of the system. Our numerical experiments show that the system is extremely sensitive to them and small changes can lead to either eternally exponentially expanding Universe or collapsing without inflation Universe. In order for an evolution to reproduce the known past of the Universe the second derivative of the scale factor $a''(t)$ has to change sign at least 2 times: to be positive during early inflation $a''(t) > 0$, to be negative during matter domination period ($a''_{MD}(t) < 0$, and to be positive during the late (current) expansion $a''_{LE}(t) > 0$.

Our numerical investigations show that such “physical” cases are indeed possible, for example Fig. 3, but require careful fine-tuning of the parameters $^2$. One can see the different epochs by plotting the equation of state $w = p/\rho$.

$^2$ The plots are for parameters $M_0 = -0.01, M_1 = 0.1, M_2 = 4, a = .7, b_0 = 1 \times 10^{-6}, p_u = .15, \chi_2 = 3.3 \times 10^{-4}, f_1 = 3 \times 10^{-3}, f_2 = 1 \times 10^{-8}$, integrated for $t = 0.4$.
Fig. 2: The evolution of the solutions in the \([a, b]\) plane. The dashed line correspond to the darkon case, the dash-dot line – to the inflation case, the solid lines – the lines of validity of the solutions.

A notable result from our work is that it is not numerically possible to start from the left plateau and to obtain a "physical" evolution. Instead, the evolution explodes to eternal inflation. It is not possible also to finish on the right plateau, because the evolution of the scalar field stops before reaching it (there is a friction term). This illustrated on Fig. 3 (a), where one can see the effective potential in this case.

A very important feature of the model is that it starts with a pre-inflation mater domination epoch with exponentially high energy density, which quickly cools to enter in the early inflation stage (see Fig. 3 b). Another important feature is the fact that the scalar field does not reach zero in the late Universe as expected by the theory. This is also due to the friction term which stops its evolution \(\dot{\phi} = 0\) before it can reach zero (Fig. 4).

In our numerical simulations we have discovered some interesting features of the model, which is not in accordance with the asymptotic found in [6]. One needs to keep in mind, however, that due to the numerical complexity of the problem and its big parameter-space, we have not yet reached the theoretically predicted values of some of the parameters discussed in there. While the main requirement of the model Eq. (10) is satisfied, the values of the other parameters, which can be found in the table 3, should be extended to a more physically realistic domain.
Fig. 3: a) The effective potential, where the crosses signify the moments: $t = 0$, $t = 1$ and $t = 4$. b) Plot of $\ddot{a}(t)$ and the equation of state $w = p/\rho$. One can see the different epochs – ultra-relativistic matter domination (UM), the early inflation (EI), the matter domination (MD) and the late inflation (LI).

Fig. 4: a) The inflaton scalar field $\phi(t)$ b) Its derivative $\dot{\phi}(t)$
Another way to define “physicality” of the problem, is the problem of the time scales. The observationally expected values for the time at which $\ddot{a}(t)$ changes its sign comes from the time when different epochs start. Theoretically, matter domination is considered to start at $a_{MD}(t) \sim 3 \times 10^{-4}$ and the accelerated expansion – at $a_{AE}(t) \gtrsim 0.6$. Our current best result is $a_{MD} = 0.2, a_{AE} = 1.2$. It is yet to be seen whether the observational values can be reached trough fine-tuning of the parameters. Because of the complexity of the problem, this fine-tuning needs to be done step by step and cannot be automatized for the moment.

Finally, due to the extreme predicted ratio $U^+ / U^- \sim 10^{120}$, reaching the theoretically predicted values of the parameters may be computationally impossible, due to possible increase in the required precision for the numerical integration of the system. A fuller investigation of the parameter space of the problem will be presented in future works.

### 4 Conclusions

In our numerical work on the application of the two-measures model to cosmology we have confirmed that this model can be considered as an alternative of the standard model of dark matter and dark energy. Through numerical integration of the Friedman equations in the K-essence theory in the darkon and the inflaton case, along with detailed study of the plane $[a, b]$, we have obtained interesting numerical results.

It was shown that in the darkon model we can obtain both a Universe with and without phase transition and those models were fit to the data of Supernovae Type 1a. In the case of inflaton model, we have performed first steps in the study of the the parameter space of the model and we have found solutions for which one can obtain the two inflationary epochs and one matter dominated epoch. It was shown that the inflation experiences friction, due to which inflation stops before reaching the $U^+$ part of the potential. This was unexpected result which is to be further investigated, because it also means that there should be a non-zero scalar field surviving to the modern epoch.
Acknowledgments

It is a pleasure to thank E. Nissimov and S. Pacheva for the discussions.

The work is supported by BAS contract DFNP – 49/21.04.2016, by Bulgarian NSF grant DN-18/1/10.12.2017 and by Bulgarian NSF Grant 08-17.

References

1. E.I. Guendelman, Mod. Phys. Lett. A 14 1043-1052 (1999), arXiv:gr-qc/9901017;
2. E. Guendelman and A. Kaganovich, Phys. Rev. D 60 065004 (1999), arXiv:gr-gc/9905029;
3. E. Guendelman and O. Katz, Class. Quantum Grav. 20 1715-1728 (2003), arXiv:gr-qc/0211095
4. E.I. Guendelman and P. Labrana, Int. J. Mod. Phys. D22 (2013) 1330018, arxiv:1303.7267 [astro-ph.CO]; E.I. Guendelman, D. Singleton and N. Yongram, JCAP 1211 (2012) 044, arxiv:1205.1056 [gr-qc]; E.I. Guendelman, H.Nishino and S. Rajpoot, Phys. Lett. 732B (2014) 156, arXiv:1403.4199 [hep-th].
5. E.I. Guendelman, E. Nissimov and S. Pacheva, Unification of Inflation and Dark Energy from Spontaneous Breaking of Scale Invariance, "Eight Mathematical Physics Meeting", pp.93-103, B. Dragovic and I. Salom (eds.), Belgrade Inst. Phys. Press (2015) 10, arXiv:1407.6281 [hep-th]
6. E. Guendelman, R. Herrera, P. Labrana, E. Nissimov, S. Pacheva, Emergent Cosmology, Inflation and Dark Energy, General Relativity and Gravitation 47 (2015) art.10, arXiv:1409.5344 [gr-qc]
7. E. Guendelman, E. Nissimov, S. Pacheva, Metric-Independent Volume-Forms in Gravity and Cosmology, Invited talk at the Memorial "Matey Mateev Symposium", April 2015, Bulgarian Journal of Physics, 42 (2015), arXiv:1505.07680 [gr-qc];
8. E. Guendelman, R. Herrera, P. Labrana, E. Nissimov, S. Pacheva, Stable Emergent Universe – A Creation without Big-Bang, Astronomische Nachrichten 336 (2015) 810-814, arXiv:1507.08878 [hep-th]
9. E. Guendelman, E. Nissimov, S. Pacheva, Dark Energy and Dark Matter From Hidden Symmetry of Gravity Model with a Non-Riemannian Volume Form, European Physics Journal C 75 (2015) 472-479, arXiv:1508.02008 [gr-qc];
10. E. Guendelman, E. Nissimov and S. Pacheva, Unified dark energy and dust dark matter dual to quadratic purely kinetic k-essence, Eur.Phys.J. C 76:90 (2016), arXiv:1511.07071[gr-qc]
11. E. Guendelman, E. Nissimov, S. Pacheva, Gravity-Assisted Emergent Higgs Mechanism in the Post-Inflationary Epoch, International Journal of Modern Physics D 25 (2016) 1644008, arXiv:1603.06231 [hep-th]
12. Eduardo Guendelman, Emil Nissimov, Svetlana Pacheva, Quintessential Inflation, Unified Dark Energy and Dark Matter, and Higgs Mechanism, Bulgarian Journal of Physics 44 (2017) 15-30, arXiv:1609.06915[gr-qc]
13. D. Staicova, M. Stoilov, Cosmological Aspects Of A Unified Dark Energy And Dust Dark Matter Model, Mod. Phys. Lett. A, Vol. 32, No. 1 (2017) 1750006, arXiv:1610.08368
14. Suzuki et al. (The Supernova Cosmology Project), The Hubble Space Telescope Cluster Supernova Survey: V. Improving the Dark Energy Constraints Above z=1 and Building an Early-Type-Hosted Supernova Sample, ApJ 746, 85 (2012)