Meshing theory of axial arc tooth profile cylindrical worm drive

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Abstract
In this paper, the axial arc tooth profile cylindrical worm drive is proposed, whose worm is cut by turning tool. Due to the simple processing equipment, short manufacturing time and low cost, this kind of worm can replace the cylindrical worm ground by grinding wheel under some conditions. The meshing theory of the worm drive is founded comprehensively. Moreover, a movable orthogonal frame is established on the non-orthogonal parametric curves net helical surface. Based on the founded meshing theory, the simulating study on the meshing quality of the worm drive is performed systematically. The numerical outcome shows that the meshing quality of this worm drive is quite favorable, and the condition of forming lubricating oil film is excellent. The working orthogonal clearance of the turning tool is decreased with the increase of the worm thread number, which must be a positive value in the process of worm cutting. This explains that the number of the worm thread is \( \leq 4 \).

Keywords
Cylindrical worm drive, non-orthogonal parametric curves net, conjugate zone, sliding angle, working orthogonal clearance, worm thread number

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Introduction
In 1932, the arc cylindrical worm drive was invented by Merrit of David Brown Company.¹ This type of worm drive was used for replacing the Archimedes worm drive to overcome disadvantages, such as low transmission efficiency and short service life. The axial arc tooth profile cylindrical worm drive is a branch of the arc cylindrical worm drive, as shown in Figure 1.

The axial arc tooth profile cylindrical worm is cut by a lathe tool with an arc cutting edge. In the process of the worm cutting, the lathe tool makes a relative screw motion along the worm axis, and trajectory surface of the cutting edge forms the worm helical surface.²,³

Because of its simple processing equipment, short manufacturing time and low cost, it attracts scholar to research this type of the worm pair.⁴–⁶ Wu et al.¹ deduced equations of the worm tooth surface, and assessed the distribution of instantaneous contact lines of the worm pair by the meshing hinge lines. However, such assessment is rough. Wu also pointed out that the number of worm thread does not exceed four, but didn’t explicate specific reasons. By using trajectory surface method, Wang and Liu,⁷,⁸ deduced some meshing parameters of the worm drive. However, the process of deduction is trivial and troublesome. Yang⁹ researched the influence of modification coefficient and

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radius of the arc profile on the meshing characteristics. In 1984, Zhang\textsuperscript{10} put forward some suggestions on the selection of the worm turning tools. Zhang et al.\textsuperscript{11} carried out meshing analysis and computing some geometric parameters of the worm drive in 1987. In 1990, Yang et al.\textsuperscript{12} and Fu et al.\textsuperscript{13} discussed collocation of geometric parameters of the worm pair. Zhang et al.\textsuperscript{14} calculated the central film thickness and minimum film thickness of the worm drive at different worm rotation angles.

Simon\textsuperscript{15} presented a method for the determination of load sharing between the instantaneously engaged worm threads and gear teeth, for the calculation of load distribution along the teeth of cylindrical worm gears. He also put forward a method for computer aided loaded tooth contact analysis of cylindrical worm gears and performed the full loaded tooth contact analysis of such a gear pair by this method.\textsuperscript{16} Zhao and Zhang,\textsuperscript{17} proposed a novel method for curvature analysis and its application to the TA worm. Zhao\textsuperscript{18} carried out the meshing analysis for the TA worm drive and presented to improve the meshing performance by modifying the displacement of the constant contact line. In addition, Zhao et al.\textsuperscript{19} established the meshing geometry of a modified globoidal worm drive. These methods are also suitable for the worm drive in this paper. Zhao and Sun\textsuperscript{20} studied the meshing limit line of the toroidal enveloping cylindrical worm pair. Chen et al.\textsuperscript{21} proposed a calculation method of time-varying meshing stiffness of the worm gear.

In this study, the meshing theory is fully established for the axial arc tooth profile cylindrical worm drive. The deduction and operation for this type of worm drive are optimized adequately by using matrix rotation method. For geometrical analysis of the worm helical surface with non-orthogonal parametric curves net, a movable orthogonal frame is constructed. Base on established meshing theory, the numerical simulation is implemented. Distribution of contact lines in the conjugate zone is drawn in detail. According to working orthogonal clearance, a method for researching the number of the worm thread is presented.

Geometry of worm helicoidal surface

Coordinate systems of worm

As described in Section 1, the trajectory surface of the cutting edge of the turning tool forms the worm helical surface. During the worm cutting, a coordinate system $\sigma_1\{O_1; \vec{i}_1, \vec{j}_1, \vec{k}_1\}$ is connected to the worm roughcast, and unit vector $\vec{k}_1$ is along the worm axis. In Figure 2, origin $O_1$ is the middle point of the worm thread length, and point $Q$ is the intersection of the cutting edge and the worm.

The cutting edge is assumed to rotate though an angle $\theta$ about axis $\vec{k}_1$. The center of the cutting edge $O_g$ moves to $O_g'$ along the positive orientation of axis $\vec{k}_1'$, because the researched worm is right-hand. The length of the vector $\overrightarrow{O_gO_g'}$ along axis $\vec{k}_1'$ is $p\theta$, where $p$ is helix parameter of the worm.

In the $\sigma_1$ rotation process, unit vectors $\vec{i}_1$ and $\vec{j}_1$ become $\vec{e}_1(\theta)$ and $\vec{g}_1(\theta)$, respectively.\textsuperscript{22} The cutting edge rotates around the axis $\vec{g}_1(\theta)$ by angle $\phi$, and a vector from the center of the cutting edge to intersection $Q$ can be expressed as $\rho \vec{m}_1(\theta, \phi)$, where $\rho$ is the radius of the cutting edge.\textsuperscript{22}

Equation and its fundamental forms of worm helicoidal surface

From the geometric construction in Figure 2, the equation of the worm helical surface $\Sigma_1$ in $\sigma_1$ is

$$\overrightarrow{r}_1 = \overrightarrow{(O_1Q)}_1 = \overrightarrow{(O_1O_g')_1} + \overrightarrow{(O_gQ)_1}$$

(1)
where \( (O_1O_2)^t = -b\vec{e}_1(\theta) - (c - \rho \theta)\vec{k}_1 \), \( (O_2O)^t = \rho \vec{m}_1(\theta, \phi) = \rho \sin \phi \vec{e}(\theta) + \rho \cos \phi \vec{K}. \) Thus, the \((\vec{r}_1)\) can be represented by

\[
(\vec{r}_1) = (\rho \sin \phi - b)\vec{e}_1(\theta) + (\rho \cos \phi + \rho \theta - c)\vec{K}_1 = x_1\vec{i} + y_1\vec{j} + z_1\vec{k}_1
\]

where \( x_1 = (\rho \sin \phi - b) \cos \theta, \ y_1 = (\rho \sin \phi - b) \sin \theta, \ z_1 = \rho \cos \phi + \rho \theta - c. \) Symbols \( b \) and \( c \) are radial coordinate and axial coordinate of the cutting edge center, respectively.

Assuming that \( \theta = 0 \), it is easy to verify that \((x_1 + b)^2 + (z_1 + c)^2 = \rho^2. \) Thus, the tooth profile of the worm axial section is a segment of circular arc.

Partial derivatives of \((\vec{r}_1)\) are

\[
\frac{\partial(\vec{r}_1)}{\partial \theta} = (\rho \sin \phi - b)\vec{e}_1(\theta) + p\vec{k}_1 \\
\frac{\partial(\vec{r}_1)}{\partial b} = \rho \cos \phi \vec{e}_1(\theta) - \rho \sin \phi \vec{K}_1
\]

(3)

Based on differential geometry,\(^{23}\) the first fundamental quantities of \( \Sigma_1 \) is

\[
E = \frac{\partial(\vec{r}_1)}{\partial \theta} \cdot \frac{\partial(\vec{r}_1)}{\partial \theta} = (\rho \sin \phi - b)^2 + p^2 \\
F = \frac{\partial(\vec{r}_1)}{\partial \theta} \cdot \frac{\partial(\vec{r}_1)}{\partial b} = - \rho \rho \sin \phi \\
G = \frac{\partial(\vec{r}_1)}{\partial b} \cdot \frac{\partial(\vec{r}_1)}{\partial b} = \rho^2
\]

(4)

From equation (3), the unit normal vector of \( \Sigma_1 \) is

\[
(\vec{n}_1) = \frac{\partial(\vec{r}_1)}{\partial \theta} \times \frac{\partial(\vec{r}_1)}{\partial b} \left/ \left| \frac{\partial(\vec{r}_1)}{\partial \theta} \times \frac{\partial(\vec{r}_1)}{\partial b} \right| \right.
\]

\[
= \frac{1}{D} \left[ n_x\vec{e}_1(\theta) + n_y\vec{g}_1(\theta) + n_z\vec{k}_1 \right]
\]

(5)

where \( n_x = (b - \rho \sin \phi) \cos \phi, \ n_y = \rho \cos \phi, \ n_z = (b - \rho \sin \phi) \cos \phi, \) and \( D = \sqrt{(\rho \sin \phi - b)^2 + (\rho \cos \phi)^2}. \)

The second partial derivatives of \((\vec{r}_1)\) are

\[
\frac{\partial^2(\vec{r}_1)}{\partial \theta^2} = (b - \rho \sin \phi)\vec{e}_1(\theta), \\
\frac{\partial^2(\vec{r}_1)}{\partial \theta \partial \phi} = \rho \cos \phi \vec{g}_1(\theta), \\
\frac{\partial^2(\vec{r}_1)}{\partial \phi^2} = - \rho \sin \phi \vec{e}_1(\theta) - \rho \cos \phi \vec{k}_1
\]

(6)

From equations (5) and (6), the second fundamental quantities of \( \Sigma_1 \) is

\[
L = \vec{n}_1 \cdot \frac{\partial^2(\vec{r}_1)}{\partial \theta^2} = \frac{1}{D}(\rho \sin \phi - b)^2 \sin \phi \\
M = \vec{n}_1 \cdot \frac{\partial^2(\vec{r}_1)}{\partial \theta \partial \phi} = \frac{1}{D} \rho \cos^2 \phi \\
N = \vec{n}_1 \cdot \frac{\partial^2(\vec{r}_1)}{\partial \phi^2} = \frac{1}{D} \rho (\rho \sin \phi - b)
\]

(7)

Frame of worm helical surface and relative geometrical parameters

In equations (4) and (7), neither \( F \) nor \( M \) is constant 0. Therefore, orientations of parametric curves at an arbitrary point on \( \Sigma_1 \) are not orthogonal to each other.\(^{23}\) It is necessary to found a moving orthogonal frame for determining geometric parameters of \( \Sigma_1. \)

From the first equation in equation (3), unit tangent vector \((\vec{\xi}_1)\) of \( \Sigma_1 \) along the \( \theta \) line is

\[
(\vec{\xi}_1) = \frac{\partial(\vec{r}_1)}{\partial \theta} \left/ \left| \frac{\partial(\vec{r}_1)}{\partial \theta} \right| \right.
\]

\[
= \frac{1}{D} \left[ n_x\vec{e}_1(\theta) + n_y\vec{g}_1(\theta) + n_z\vec{k}_1 \right]
\]

\[
(8)
\]

At an arbitrary point \( M \) on \( \Sigma_1, \) a moving orthogonal frame \( \sigma_M = (\vec{\xi}_1, \vec{\eta}_1, \vec{\zeta}_1) \) can be established. By the cross product of \((\vec{n}_1)\) and \((\vec{\xi}_1)\), the unit tangent vector \((\vec{\eta}_1)\) can be acquired

\[
(\vec{\eta}_1) = (\vec{\xi}_1) \times (\vec{\xi}_1) = \vec{\eta}_1\vec{r}_1 + \vec{\eta}_1\vec{j}_1 + \vec{\eta}_1\vec{k}_1
\]

(9)

where \( \vec{\eta}_1 = \frac{\vec{\xi}_1}{D}, \vec{\eta}_1 = \frac{\vec{\xi}_1}{D}, \vec{\eta}_1 = \frac{\vec{\xi}_1}{D}. \)

The \( \delta \phi = 0 \) is substituted into normal curvature

\[
k_n = \frac{Ld\phi^2 + 2Mdd\delta\phi + Ndd\delta^2}{Ed\phi^2 + 2Fdd\delta\phi + Gdd\delta^2}, \]

so the normal curvature along \((\vec{\xi}_1)\) can be obtained as \( k_n(1) = L/E.\)

The mean curvature of \( \Sigma_1 \) is

\[
H = \frac{LG - 2MF + NE}{2(EG - F^2)} = \frac{\rho \sin \phi (p^2 + 2D^2) - bE}{2pD^3}
\]

(10)

The normal curvature along \((\vec{n}_1)\) can be expressed as

\[
k_n = 2H - k_n(1).\]

From equations (4) and (7), the Gaussian curvature of helical surface \( \Sigma_1 \) can be achieved as

\[
K = \frac{LN - M^2}{EG - F^2} = \frac{1}{p^3D^4} [\sin \phi (\rho \sin \phi - b)^3 - \rho \rho \cos^4 \phi].\]

The \( K \) is not constant 0, thus \( \Sigma_1 \) is undevelopable surface.
It can be derived \( \frac{d(\vec{r}_1)}{ds_1} = \frac{\partial(\vec{r}_1)}{\partial \theta} \frac{d\theta}{ds_1} + \frac{\partial(\vec{r}_1)}{\partial \phi} \frac{d\phi}{ds_1} \), where \( s_1 \) is arc length of \( \Sigma_1 \). From \( \frac{d(\vec{r}_1)}{ds_1} \), \( d\phi = 0 \) and the formula derived, it can be obtained

\[
\frac{\partial(\vec{r}_1)}{\partial \theta} \bigg/ \left| \frac{\partial(\vec{r}_1)}{\partial \theta} \right| = \frac{\partial(\vec{r}_1)}{\partial \phi} \frac{d\theta}{ds_1}
\]

(11)

It can be deduced that \( \frac{d\phi}{ds_1} = \frac{1}{E} \) from equation (11). By definition, the geodesic torsion \( \tau^{(1)}_\xi \) along the \( \theta \) line is

\[
\tau^{(1)}_\xi = \frac{1}{\rho D} \left| \begin{array}{c} E \frac{d\theta}{ds_1} + F \frac{d\phi}{ds_1} \frac{F \frac{d\theta}{ds_1} + G \frac{d\phi}{ds_1}}{L} + M \frac{d\phi}{ds_1} \end{array} \right| = \frac{1}{\rho D E (EM - FL)}
\]

Mesh of worm drive

Relative motion and equation of helicoid family

Because the worm drive studied in this paper is a theoretical line contact model, the mating worm gear researched is manufactured by a hob similar to the worm. In Figure 3, static coordinate systems \( \sigma_1 \{O_1; \vec{i}_1, \vec{j}_1, \vec{k}_1\} \) and \( \sigma_2 \{O_2; \vec{i}_2, \vec{j}_2, \vec{k}_2\} \) are connected to the worm and the worm gear, respectively. Unit vectors \( \vec{k}_1 \) and \( \vec{k}_2 \) coincide with \( \vec{k}_{\omega_1} \) and \( \vec{k}_{\omega_2} \), respectively. The worm revolves around \( \vec{k}_1 \) by angle \( \varphi_1 \), and the worm gear rotates through angle \( \varphi_2 \) about \( \vec{k}_2 \). The relation between \( \varphi_1 \) and \( \varphi_2 \) is \( \varphi_1 = \varphi_1/\omega_1 \), where \( \omega_1 \) is gear ratio. Angular velocities of the worm \( \vec{\omega}_1 \) and the gear \( \vec{\omega}_2 \) are toward orientations of axes \( \vec{k}_1 \) and \( \vec{k}_2 \), respectively.

When the worm revolves around \( \vec{k}_1 \), the helical surface can form a surface family \( \{\Sigma_1\} \) in \( \sigma_{o1} \), whose equation is

\[
(\vec{r}_1)_{o1} = R \left[ \vec{k}_{\omega_1}, \varphi_1 \right] (\vec{r}_1)_1 = x_{o1} \vec{i}_{o1} + y_{o1} \vec{j}_{o1} + z_{o1} \vec{k}_{o1}
\]

(13)

where \( x_{o1} = \cos (\theta + \varphi_1)(\rho \sin \phi - b) \), \( y_{o1} = \sin (\theta + \varphi_1)(\rho \sin \phi - b) \), \( z_{o1} = z_1 \). In equation (13), the \( R \left[ \vec{k}_{\omega_1}, \varphi_1 \right] \) denotes the rotation transformation matrix and it can be expressed as

\[
R \left[ \vec{k}_{\omega_1}, \varphi_1 \right] = \begin{bmatrix}
\cos \varphi_1 & -\sin \varphi_1 & 0 \\
\sin \varphi_1 & \cos \varphi_1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

By exploiting \( R \left[ \vec{k}_{\omega_1}, \varphi_1 \right] \), the unit normal vector of \( \{\Sigma_1\} \) can be determined in \( \sigma_{o1} \)

\[
(\vec{\nu}_{o1}) = R \left[ \vec{k}_{\omega_1}, \varphi_1 \right] (\vec{\nu}_1) = (n_{o1}^i \vec{i}_{o1} + n_{o1}^j \vec{j}_{o1} + n_{o1}^k \vec{k}_{o1})/D
\]

(14)

where

\[
n_{o1}^i = n_e \cos (\theta + \varphi_1) - n_g \sin (\theta + \varphi_1), \\
n_{o1}^j = n_e \sin (\theta + \varphi_1) + n_g \cos (\theta + \varphi_1), \\
n_{o1}^k = n_g
\]

Not losing the generality, the worm angular velocity can be assumed \( |\vec{\omega}_{\omega_1}| = 1 \text{ rad/s} \). Thus, it can be acquired \( |\vec{\omega}_{\omega_2}| = 1/\omega_2 \text{ rad/s} \). The relative angular velocity vector of the worm pair in \( \sigma_{o1} \) is

\[
(\vec{\omega}_{12})_{o1} = (\vec{\omega}_1)_{o1} - (\vec{\omega}_2)_{o1} = \vec{k}_{o1} + \vec{J}_{12}/\omega_1
\]

(15)

Due to \( (\vec{O}_2\vec{O}_1)_{o1} = -a \vec{t}_{o1} \), the partial derivative of \( (\vec{O}_2\vec{O}_1)_{o1} \) with respect to \( \varphi_1 = 0 \).

The relative velocity at an arbitrary meshing point \( P \) of the worm pair can be ascertained in \( \sigma_{o1} \)

\[
(\vec{V}_{12})_{o1} = (\vec{\omega}_{12})_{o1} \times (\vec{r}_1)_{o1} - (\vec{\omega}_2)_{o1} \times (\vec{O}_2\vec{O}_1)_{o1} \\
+ \frac{d(\vec{O}_2\vec{O}_1)_{o1}}{d\varphi_1}
\]

(16)

Meshing equation and equation of worm gear tooth face

By definition, the meshing equation of the worm pair is
\[
\Phi = (\tilde{V}_{12})_{o1} \cdot (\tilde{n})_{o1} = \frac{1}{i_{12}D}[ A \sin(\theta + \phi_1) + B \cos(\theta + \phi_1) + C ] = 0
\]

(17)

where \( A = -z_1n_x \), \( B = (b - \rho \sin \phi)(z_1 \sin \phi + n_x) \), 
\( C = n_2(a - p i_{12}) \).

The equation of the worm gear tooth surface \( \Sigma_2 \) can be determined in \( \sigma_2 \)

\[
\begin{bmatrix}
(x_{21}, y_{21}, z_{21}) \\
(\Phi(\theta, \phi, \psi) = 0)
\end{bmatrix}
= \left[ R \left[ k_2, -\phi_1/i_{12} \right] R \left[ i_2, -\frac{\pi}{2} \right] (\tilde{O}_2O_{1})_{o1} + (\tilde{r})_{o1} \right]
\]

where \( x_2 = (x_{o1} - a) \cos \frac{\phi_1}{i_{12}} + z_1 \sin \frac{\phi_1}{i_{12}}, y_2 = -(x_{o1} - a) \)
\( \sin \frac{\phi_1}{i_{12}} + z_1 \cos \frac{\phi_1}{i_{12}}, z_2 = -y_{o1}, \) and rotation matrixes
\( R \left[ k_2, -\phi_1/i_{12} \right] = \begin{bmatrix}
\cos(\phi_1/i_{12}) & \sin(\phi_1/i_{12}) & 0 \\
-\sin(\phi_1/i_{12}) & \cos(\phi_1/i_{12}) & 0 \\
0 & 0 & 1
\end{bmatrix} \)
\( R \left[ i_2, -\frac{\pi}{2} \right] = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix} \)

Characteristic parameters of worm drive

By definition,\(^{22}\) the meshing limit function can be ascertained

\[
\Phi_{\varphi_1} = \frac{\partial \Phi}{\partial \varphi_1} = \frac{1}{i_{12}D} [ A \cos(\theta + \phi_1) - B \sin(\theta + \phi_1) ]
\]

(19)

According to matrix rotation method, \( (\tilde{\xi})_1 \) and \( (\tilde{\eta})_1 \) can be represented in \( \sigma_{o1} \) by

\[
(\tilde{\xi})_{o1} = R \left[ k_{o1}, \phi_1 \right] (\tilde{\xi})_1 = \xi_x \tilde{i}_o + \xi_y \tilde{j}_o + \xi_z \tilde{k}_o
\]

(20)

\[
(\tilde{\eta})_{o1} = R \left[ k_{o1}, \phi_1 \right] (\tilde{\eta})_1 = \eta_x \tilde{i}_o + \eta_y \tilde{j}_o + \eta_z \tilde{k}_o
\]

(21)

where \( \xi_x = (b - \rho \sin \phi) \sin(\theta + \phi_1)/\sqrt{E}, \quad \xi_y = (\rho \sin \phi - b) \cos(\theta + \phi_1)/\sqrt{E} \), 
\( \xi_z = \sqrt{E} \cos \phi \cos(\theta + \phi_1)/D - pN \sin(\theta + \phi_1)/\sqrt{E} \).

\( \eta_x = \sqrt{E} \cos \phi \cos(\theta + \phi_1)/D - pN \sin(\theta + \phi_1)/\sqrt{E} \), 
\( \eta_y = \sqrt{E} \cos \phi \sin(\theta + \phi_1)/D + 
\( pN \sin(\theta + \phi_1)/\sqrt{E} \), 
\( \eta_z = -L/\sqrt{E} \).

The normal vector of instantaneous contact line can be determined in \( \sigma_{o1} \)
\[ (\tilde{N})_{o1} = N_x(\tilde{\xi})_{o1} + N_y(\tilde{\eta})_{o1} \]

(22)

Coefficients \( N_x \) and \( N_y \) can be figured out from \( k_{o1}^{(1)} \), 
\( k_{y}^{(1)}, \tau_{\xi}^{(1)}(\tilde{\omega})_{o1}, (\tilde{V})_{o1}, (\tilde{\xi})_{o1}, \) and \( (\tilde{\eta})_{o1} \)
\[ N_x = \tau_{\xi}^{(1)}(\tilde{\eta})_{o1} \cdot (\tilde{V}_{12})_{o1} + k_{y}^{(1)}(\tilde{\xi})_{o1} \cdot (\tilde{V}_{12})_{o1} + (\tilde{\eta})_{o1} \cdot (\tilde{\omega})_{o1} \]

(23)

\[ N_y = \tau_{\xi}^{(1)}(\tilde{\eta})_{o1} \cdot (\tilde{V}_{12})_{o1} + k_{y}^{(1)}(\tilde{\xi})_{o1} \cdot (\tilde{V}_{12})_{o1} - (\tilde{\xi})_{o1} \cdot (\tilde{\omega})_{o1} \]

(24)

where \( (\tilde{\eta})_{o1} \cdot (\tilde{V}_{12})_{o1} = \left[N_xz_1 + N_y(a - x_{o1})\right]/i_{12} + pN \sin \phi(\rho \sin \phi - b)/\sqrt{E} \), 
\( (\tilde{\xi})_{o1} \cdot (\tilde{V}_{12})_{o1} = \left[N_xz_1 + N_y(a - x_{o1})\right]/i_{12} + (\rho \sin \phi - b)^2 \sqrt{E} \).

\[
(\sqrt{E}, (\tilde{\eta})_{o1} \cdot (\tilde{\omega})_{o1}) = N_x \eta, \quad (\tilde{\xi})_{o1} \cdot (\tilde{\omega})_{o1} = -\xi_x - \xi_y/i_{12}.
\]

Based on \( (\tilde{V}_{12})_{o1}, \Phi_{\varphi_1}, \) and \( (\tilde{N})_{o1} \), the curvature interference limit function is

\[
\Psi = \Phi_{\varphi_1} + (\tilde{N})_{o1} \cdot (\tilde{V})_{o1}
\]

(25)

From \( N_x, N_y, \) and \( \Psi \), induced principal curvature has following form

\[
k_{N}^{(12)} = (N_x^2 + N_y^2)/\Psi
\]

(26)

Generally, if induced principal curvatures are not all positive or all negative, undercutting will occur during the worm pair meshing.

From \( (\tilde{V})_{o1}, \Phi_{\varphi_1}, (\tilde{N})_{o1}, \) and \( \Psi \), the acute angle between \( (\tilde{V})_{o1} \) and \( (\tilde{N})_{o1} \), called sliding angle, can be figured out as

\[
\theta_s = \arcsin \left| \frac{\Psi - \Phi_{\varphi_1}}{(\tilde{V})_{o1} \cdot (\tilde{N})_{o1}} \right|
\]

(27)

When the value of \( \theta_s \) is close to 90°, lubrication of the meshing point is excellent.

By means of matrix rotation method, deductions of the worm drive surface equations and characteristic parameters are simplified.

Geometric equations of worm pair section

In the worm axial section, a coordinate system \( O_1 - \tilde{x}_{R1} \tilde{y}_{R1} \) is established (Figure 4), and coordinate relations are

\[
x_{R1} = x_{o1}, \quad y_{R1} = \sqrt{x_{o1}^2 + y_{o1}^2}
\]

(28)

In coordinate system \( O_1 - \tilde{x}_{R1} \tilde{y}_{R1} \), equations of worm addendum and dedendum are

\[
\sqrt{x_{o1}^2 + y_{o1}^2} - r_{a1} = 0, \quad \sqrt{x_{o1}^2 + y_{o1}^2} - r_{f1} = 0
\]

(29)
where \( r_{a1} \) and \( r_{f1} \) are radii of top and root circles of worm, respectively.

In the axial section of the worm gear, a coordinate system \( O_2-x_2y_2z_2 \) is established (Figure 5), and coordinate relations are

\[
x_{R2} = -y_{o1}, y_{R2} = \sqrt{(x_{o1} - a)^2 + z_{1}^2}
\]  

(30)

In coordinate system \( O_2-x_2y_2z_2 \), equation of gear addendum arc is

\[
\sqrt{x_{R2}^2 + (y_{R2} - a)^2} - r_{g2} = 0
\]  

(31)

where \( r_{g2} \) is arc radius of worm gear addendum.

Equations of gear addendum and dedendum are

\[
y_{R2} - r_{e2} = 0, \sqrt{x_{R2}^2 + (y_{R2} - a)^2} + r_{f2} - a = 0
\]  

(32)

where \( r_{e2} \) and \( r_{f2} \) are radii of top and root circles of worm gear, respectively.

Equations of gear left and right chamfer are

\[
cot \left( \frac{\alpha}{2} \right) (x_{R2} - b_2) - y_{R2} - r_{e2} = 0,
\quad \cot \left( \frac{\alpha}{2} \right) (x_{R2} - b_2) + y_{R2} - r_{e2} = 0
\]  

(33)

where \( \alpha \) and \( b_2 \) are tooth width angle and tooth width of worm gear, respectively.

**Meshing stimulation studies**

In this section, the meshing simulation of worm pair is implemented to achieve parameters of meshing points. From achieved parameters, the position of meshing point can be determined. Then, the distribution of contact lines is precisely presented by use of smooth connection meshing points. Moreover, the meshing simulation is the foundation of meshing analysis of the worm drive.

**Main parameters of worm drive**

In this paper, the numerical simulation is carried out to investigate the meshing property of the worm drive. Main parameters of numerical simulation and their calculation methods are listed in Table 1.

**Basic parameters of turning tool**

In Figure 6, \( \phi_{\text{min}} \) and \( \phi_{\text{max}} \) can be fixed at addendum and dedendum of the worm, respectively.

Based on \( x_{o1} \) and \( y_{o1} \), the first formula of equation (29) can be rewritten as \( x_1^2 + y_1^2 = r_{a1}^2 \), which can be simplified as \( b - \rho \sin \phi = r_{a1} \). Thus, the \( \phi_{\text{min}} \) can be represented by

\[
\phi_{\text{min}} = \arcsin\left(\frac{b - r_{a1}}{\rho}\right)
\]  

(34)

It can also be obtained \( \phi_{\text{max}} = \arcsin\left(\frac{b - r_{f1}}{\rho}\right) \). The range of \( \sin \phi \) can be assumed \([0, 1]\). Thus, the range of \( b \) can be figured out as \([r_{a1}, r_{f1} + \rho]\). Generally, the \( \rho \) is chosen from 5\( m \) to 5.5\( m \). According to \( z_1 = 2 \) and \( m = 7 \) mm, the \( \rho \) can be acquired 35 mm. Therefore, parameter \( b \) can be chosen from 45 to 64.88 mm.

In terms of the worm pair processing principle, the coordinates of the cutting edge center can be computed as \( b = \rho \sin \alpha_s + r_1 \) and \( c = \rho \cos \alpha_s + s_4/2 \), respectively. Symbols \( \alpha_s \) and \( s_4 \) are axial profile angle and axial tooth thickness respectively, as shown in Table 1. Thus, the \( b \) is determined 51.67 mm probably, which conforms to the range of \( b \). The \( c \) is about 36.62 mm.
Computation of conjugate zone of the worm drive

Contact zone DACB of the worm gear surface is drawn in Figure 7. Line DAC is the boundary of the contact zone formed by the worm addendum. In order to reflect the contact zone of the worm, contact zone DACB is projected to the coordinate system $O_1$ as depicted in Figure 8.

Conjugate zone DACB is divided into DAB and BAC two zones by the meshing limit line AB in Figures 7 and 8. Therefore, the solution $\phi_1$ of equation (17) in zone DAB and zone BAC correspond to $\phi_1^{(1)}$ and $\phi_1^{(2)}$, respectively

$$\phi_1^{(1)} = \pi + \arcsin \left( C / \sqrt{A^2 + B^2} \right) - \varphi_0 - \theta,$$
$$\phi_1^{(2)} = - \arcsin \left( C / \sqrt{A^2 + B^2} \right) - \varphi_0 - \theta$$

where $\sin \varphi_0 = B / \sqrt{A^2 + B^2}$, $\cos \varphi_0 = A / \sqrt{A^2 + B^2}$.

By use of equation (35), calculation process of nonlinear equations can be reduced, and meshing points in different zones of the same instantaneous contact line can be determined easily. Moreover, parameters obtained can belong to the same meshing cycle.

### Table 1. Main parameters of the worm drive.

| Description                      | Symbol and formula | Cases and unit |
|----------------------------------|--------------------|----------------|
| Center distance                  | $a$                | 160 (mm)       |
| Modulus                          | $m$                | 7 (mm)         |
| Reference circle diameter of worm| $d_1$              | 76 (mm)        |
| Number of worm thread            | $Z_1$              | 2              |
| Number of worm gear tooth        | $Z_2$              | 33             |
| Gear ratio                       | $i_{12} = Z_2/Z_1$ | 16.5           |
| Modification coefficient         | $x_3$              | 0.93           |
| Addendum of the worm             | $h_a = m$          | 7 (mm)         |
| Dedendum of the worm             | $h_f$              | 8.12 (mm)      |
| Reference circle radius of worm  | $r_1 = d_1/2$      | 38 (mm)        |
| Addendum circle radius of the worm| $r_a = r_1 + h_a$ | 45 (mm)        |
| Dedendum circle radius of the worm| $r_f = r_1 - h_f$ | 29.88 (mm)     |
| Tooth width of worm              | $L_w \approx 2.5m\sqrt{Z_2 + 1}$ | 102.04 (mm)   |
| Helix parameter of the worm      | $p = mZ_1/2$       | 7              |
| Axial tooth profile angle        | $\alpha_s = 0.4\pi m$ | 23 (°)        |
| Axial tooth thickness            | $s_x = (1 + x_s)m$ | 8.80 (mm)      |
| Addendum of the worm gear        | $h_a = (1 + x_s)m$ | 13.50 (mm)     |
| Dedendum of the worm gear        | $h_f = (1.16 - x_s)m$ | 1.62 (mm)    |
| Reference circle radius of the worm gear | $r_2 = mZ_2/2$ | 115.5 (mm)     |
| Throat circle radius of the worm gear | $r_3 = r_2 + h_a$ | 129 (mm)       |
| Addendum circle radius of the worm gear | $r_4 = r_3 + 0.4m$ | 131.8 (mm)    |
| Dedendum circle radius of the worm gear | $r_5 = r_3 - h_f$ | 113.88 (mm)    |
| Throat generate circle radius of the worm gear | $r_6 = a - r_2$ | 31 (mm)        |
| Tooth width of worm gear         | $b_2 \approx 2m(0.5 + \sqrt{d_1/m + 1})$ | 55.21 (mm)    |
| Tooth width angle of worm gear   | $\alpha = 2 \arcsin (b_2/d_1)$ | 93.17 (°)     |
By means of setting on line AB is a function only about \( u \) fixed expressed as equations (17) and (19). Based on trigonometric function (36), \( f_{\text{AB}} \), thus \( A \) is the intersection of the worm addendum and line AB. It can be obtained from graphical method. From equation (36), the expression (37) can be simplified as a nonlinear function \( f_{\text{B}}(\phi) \) with respect to \( \phi \). The curve \( f_{\text{B}}(\phi) \) can be drawn in the interval \( [\phi_{\text{min}}, \phi_{\text{max}}] \) as shown in Figure 9.

The intersection of \( f_{\text{B}}(\phi) \) and the abscissa axis is near point \((0.82, 0)\). The \( \phi = 0.82 \) can be employed as the initial value to determine \( \phi \) of point B. Methods to acquire \( \theta \) and \( \varphi_1 \) of point B is similar to those of point A.

As feature point C is the intersection of the worm addendum and the worm gear right chamfer, \( \phi_C = \phi_{\text{min}} \). From \( \varphi_1 \), the second expression of equation (33) and its component expressions, the nonlinear function of point C is

\[
 f_{\text{C}}(\theta) = 
\sqrt{[\cos(\theta + \varphi_1) - b]^2 + [\rho \cos \phi_C + \rho \theta - c]^2} 
+ \cot \frac{\alpha}{2} \left[ -\sin(\theta + \varphi_1) - b \right] - r_{e2} 
\]

(38)

From graphical method, the \( \theta \) of point C can be received. Method to ascertain \( \varphi_1 \) of point C is similar to point A.

Feature point D is the intersection of the worm addendum and the worm gear addendum, therefore \( \phi_D = \phi_{\text{min}} \). By using of formula \( \varphi_1 \), \( x_{a1} \), \( y_{a1} \) and \( z_1 \), the first expression of equation (32) can be rewritten as

\[
 f_{\text{D}}(\phi, \theta) = \sqrt{\left( -\frac{A}{C} \right)^2 (\rho \sin -b)^2 + \left[ \left( -\frac{B}{C} \right)(\rho \sin -b - a) \right]^2 + z_1^2} - r_{g2} 
\]

(37)
Methods of determining point D parameters are similar to those of point C.

\[ f_D(\theta) = \sqrt{\left[ \cos(\theta + \varphi_1^{(1)}) - \rho \sin \phi_D - b - a \right]^2 + \left( \rho \cos \phi_D + p\theta - c \right)^2 - r_c^2} \]  

(39)

The related parameters of feature points are listed in Table 2.

**Determination of meshing points on instantaneous contact line**

Based on Table 2, the changing range of rotation angle \( \varphi_2 \) can be determined. In this range, values of \( \varphi_1 \) can be assumed. By means of assumed \( \varphi_1 \) and other conditions, parameters of meshing points can be acquired. Different categories of meshing points solving methods will be introduced as follows.

**Determining meshing points on the worm addendum.** In Figure 7, meshing points on line DAC are intersections of contact lines and the worm addendum. Taking point \( a_1 \) of zone DAB as an example, \( \phi \) of \( a_1 \) is the same as \( \phi_{\text{min}} \). From formula \( \varphi_1^{(1)} \), the meshing function of point \( a_1 \) can be represented by

\[ f_{a_1}(\theta) = \varphi_1^{(1)} + \theta + \varphi_0 - \arcsin(C/\sqrt{A^2 + B^2}) - \pi \]  

(40)

Since value of \( \varphi_1 \) has been assumed, the solution \( \theta \) of \( f_{a_1}(\theta) = 0 \) can be acquired. Parameters of meshing points in zone BAC can be achieved from formula \( \varphi_1^{(2)} \).

**Determining meshing points on worm gear addendum.** In Figure 7, meshing point, such as \( a_2 \) on line (3), is intersection of contact line and the worm gear addendum. With setting \( \varphi_1 = 395.5878^\circ \), point \( a_2 \) on line (1) is taken as an example. By means of formula \( \varphi_1^{(1)} \),

\[
\begin{align*}
\left\{ 
\begin{aligned}
 f_{a_2}^{(1)}(\phi, \theta) &= \varphi_1^{(1)} + \phi + \varphi_0 - \arcsin(C/\sqrt{A^2 + B^2}) - \pi \\
 f_{a_2}^{(2)}(\phi, \theta) &= \sqrt{\left[ \cos(\theta + \varphi_1^{(1)}) - \rho \sin \phi - b - a \right]^2 + \left( \rho \cos \phi - c \right)^2 - r_c^2}
\end{aligned}
\right.
\]  

(41)

Taking \([\phi_{\text{min}}, \phi_{\text{max}}]\) as horizontal axis range, curves of equation (41) are drawn Figure 10. The intersection of \( f_{a_2}^{(1)}(\phi, \theta) \) and \( f_{a_2}^{(2)}(\phi, \theta) \) is near the point (0.42, 6.3), so \( \phi = 0.42 \) and \( \theta = 6.3 \) can be regarded as initial solutions of nonlinear equations.

From formula \( \varphi_1^{(2)} \), parameters of meshing points of zone BAC can be acquired. When meshing points are on the worm gear addendum arc, equation (31) is chosen to replace the second formula of equation (41).

**Determining general meshing points.** It is necessary to ascertain general meshing points for estimating position of instantaneous contact line accurately. Horizontal coordinates of these points can be calculated by meshing points determined above. Based on the first formula of equation (30), an equation can be established as

\[ x_{a_2} = -y_{a_2} = \sin(\theta + \varphi_1)(b - \rho \sin \phi) \]  

(42)

Since parameter \( \varphi_1 \) has been assumed, parameter \( \theta \) can be expressed as a function with respect to \( \phi \) from equation (42). Then, equation (35) can be rewritten as

\[
\begin{align*}
 f^{(1)}(\phi) &= \varphi_0 + \arcsin(y_{a_2}/(\rho \sin \phi - b)) \\
 &\quad - \arcsin(C/\sqrt{A^2 + B^2}) - \pi
\end{align*}
\]  

(43)

![Figure 10. The image of curves \( f_{a_2}^{(1)}(\phi, \theta) \) and \( f_{a_2}^{(2)}(\phi, \theta) \).](image-url)
From equations (43) and (44), parameters of general meshing points in zone DAB and zone BAC can be obtained, respectively.

**Meshing quality of worm drive**

In zone BAC, intersections of line (1) and tooth top of the worm pair are not on the worm gear tooth surface. Intersections of line (1)–(4) and line AB are not on the worm gear tooth surface. Only meshing points on the tooth surface are of research significance. Thus, the part of instantaneous contact line on the tooth surface of the worm pair is drawn.

As shown in Figure 7, contact lines near the worm gear addendum fluctuate greatly. Contact lines near the worm gear dedendum fluctuate gently. In Figures 7 and 8, instantaneous contact lines pass through the meshing limit line and enter from one sub contact zone to another. Separated instantaneous contact lines in two sub zones can be called sub contact lines. Rotation angles of two sub contact lines are the same. In this sense, it can be considered that the worm pair is meshed along two sub contact lines at the same time.

Numerical simulation results of the worm drive meshing are listed in Table 3.

**Induced principal curvature of worm gear tooth surface in meshing process.** As already pointed out in Section 3, values of $k^{(12)}_n$ in Table 3 (a) and (b) are all negative, so curvature interference didn’t happen in the meshing simulation. The absolute value of $k^{(12)}_n$ is smaller in the middle part of instantaneous contact lines and larger on both sides. The absolute value of $k^{(12)}_n$ is smaller near the worm gear reference circle and larger near the worm gear dedendum. The maximum absolute value of $k^{(12)}_n$ takes place at the right side of the worm gear dedendum approximately, where may be possible to lose efficacy.

**Lubricating property of worm pair in meshing process.** In Table 3 (a) and (b), values of $\theta_s$ are between 2.0973° and 88.5801° approximately. Meshing points cover 50% of contact zone on the worm gear tooth surface, whose $\theta_s$ is between 40° and 88.58°. It shows that the worm drive has good lubrication performance. The result shows that the $\theta_s$ is larger on both sides of instantaneous contact lines and smaller in the middle. The $\theta_s$ is closer 90° near the worm gear addendum and closer 0° near the worm gear dedendum.

As described in Section 3, the lubricating oil film is easy to form on both sides of the worm gear tooth surface, but difficult to form in the middle. The lubrication of the worm gear addendum is better, and that of the worm gear dedendum is worse. The minimum value of sliding angle approximately appears on the middle of the worm gear dedendum (Black squares in Figure 7), and there may be hard to form lubricating oil film.

**Influence of worm thread number on worm drive**

According to previous literatures, the number of the worm thread $Z_1$ cannot exceed four, when the worm drive parameters are matched. Based on the principle of metal cutting, the geometric construction of turning tool and the worm is established as shown in Figure 11.

In Figure 11, tool reference plane $P_t$ is parallel to the bottom plane of turning tool, which passes through cutting point. Tool cutting edge plane $P_e$, passes through cutting point, is perpendicular to the $P_t$. The $\gamma_{oe}$ is working orthogonal rake, and the $\gamma_o$ is tool orthogonal rake. Symbols $\alpha_{oe}$ and $\alpha_o$ are working orthogonal clearance and tool orthogonal clearance, respectively. The assumed working plane of turning tool is $P_t$, which is the plane of the feed direction $\psi$. The tool cutting edge angle $\kappa_f$ is the angle between the plane $P_s$ and the plane $P_t$, which can be represented by $\kappa_f = \frac{\pi}{2} - \arcsin \frac{b - r}{\rho}$. The $r$ is radial distance of the point on the worm tooth profile.

Considering the feed motion, the working cutting edge plane $P_{oe}$ is tangent to the worm helical surface. The working reference plane $P_{oe}$ is perpendicular to the orientation of resultant cutting speed. Symbols $\gamma_{oe}$ and $\gamma_o$ are working side rake and tool side rake, respectively. The $\alpha_{fe}$ is working side clearance and the $\alpha_f$ is tool side clearance. The angle of resultant cutting speed is the lead angle $\gamma$ of the worm.

In the working plane, relations of turning tool angles are

$$\gamma_{fe} = \gamma + \gamma, \alpha_{fe} = \alpha_f - \gamma$$

where the lead angle $\gamma$ can be expressed as

$$\gamma = \arctan \frac{Z_1m}{2r}$$

Above relations can be expressed in working orthogonal plane

$$\gamma_{oe} = \gamma + \gamma_0 = \gamma_o + \arctan \left[\frac{Z_1m}{2r} \cos \left(\arcsin \frac{b - r}{\rho}\right)\right]$$

$$\alpha_{oe} = \alpha_o - \gamma_0 = \alpha_o - \arctan \left[\frac{Z_1m}{2r} \cos \left(\arcsin \frac{b - r}{\rho}\right)\right]$$

where $\tan \gamma_0 = \tan \gamma \sin \kappa_f$.25
In equations (46) and (47), $Z_1$ and $r$ are variable, and remaining parameters are consistent with those in Section 5.

Because the worm material is 20CrMnTi, the corresponding turning tool material is WC – TiC – Co cemented carbide, in which TiC content is 30%. Tool orthogonal clearance $\alpha_o$ and tool orthogonal rake $\gamma_o$ are chosen from $10^\circ$ to $15^\circ$ and $-5^\circ$ to $-15^\circ$, respectively. In this study, $\alpha_o = 15^\circ$ and $\gamma_o = -5^\circ$ are assumed.

Variations of working orthogonal angles of the worm with different $Z_1$ along the worm tooth profile are shown in Figure 12.

In Figure 12(a), the increasing $Z_1$ leads to the decrease of $\alpha_{oe}$. When $Z_1$ is 5, values of $\alpha_{oe}$ are mostly negative, and the maximum of $\alpha_{oe}$ is $0.1044^\circ$. Too small $\alpha_{oe}$ will increase friction between the flank face of turning tool and the tooth surface of the worm. Moreover, if the $\alpha_{oe}$ is negative, the cutting edge strength will be lessen, and the cutting tool wear will be

Table 3. Numerical results of meshing simulation.

(a) Induced principal curvature and sliding angle on line (1)–(5).

| Points | Meshing parameters | Instantaneous contact lines |
|--------|--------------------|-----------------------------|
|        | $k_{12}^{(1)}$ | (1) | (2) | (3) | (4) | (5) |
| $a_1$  | $k_{12}^{(1)}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ |
| $a_2$  | $k_{12}^{(1)}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ |
| $a_3$  | $k_{12}^{(1)}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ |
| $a_4$  | $k_{12}^{(1)}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ |
| $a_5$  | $k_{12}^{(1)}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ |
| $a_6$  | $k_{12}^{(1)}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ |
| $a_7$  | $k_{12}^{(1)}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ |
| $a_8$  | $k_{12}^{(1)}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ |

(b) Induced principal curvature and sliding angle on line (6)–(10).

| Points | Meshing parameters | Instantaneous contact lines |
|--------|--------------------|-----------------------------|
|        | $k_{12}^{(1)}$ | (6) | (7) | (8) | (9) | (10) |
| $a_1$  | $k_{12}^{(1)}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ |
| $a_2$  | $k_{12}^{(1)}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ |
| $a_3$  | $k_{12}^{(1)}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ |
| $a_4$  | $k_{12}^{(1)}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ |
| $a_5$  | $k_{12}^{(1)}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ |
| $a_6$  | $k_{12}^{(1)}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ |
| $a_7$  | $k_{12}^{(1)}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ |
| $a_8$  | $k_{12}^{(1)}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ | $a_{12}$ |
accelerated. In addition, the minimum value of $\alpha_{oe}$ appear at the tooth root of the worm. The lead angles of the worm dedendum are 13.19°, 25.11° and 30.36°, corresponding to $Z_1 = 2, 4$ and 5 respectively. In fact, undercutting is easy to occur in the worm drive meshing, when $\gamma > 30°$. Thus, the case of $Z_1 = 5$ is vetoed.

Because $Z_1 = 5$ is vetoed, two curves are drawn in Figure 12(b). The $\gamma_{oe}$ increases with the increase of $Z_1$. The working orthogonal wedge angle $\beta_{oe}$ is the angle between the rake face and the flank surface, which is an index to measure the turning tool. It can be expressed as $\beta_{oe} = 90° - \gamma_{oe} - \alpha_{oe}$. In the numerical simulation, $\beta_{oe}$ values of $Z_1 = 2$ and $Z_1 = 4$ are between 60° and 70°, which conform to standard.

Based on preceding results, the number of worm thread cannot be $>4$.

**Conclusions**

This paper minutely presented the meshing theory of the axial arc tooth profile arc cylindrical worm drive. For the worm helical surface with non-orthogonal parametric curve net, a movable orthogonal frame is established to acquire geometrical parameters. The matrix rotation method is fully utilized to optimize deduction and calculation of the worm drive meshing theory.

By means of elimination method, meshing points of different sub contact zones can be obtained in the same meshing cycle easily and the efficiency of solving non-linear equations can be improved. The distribution of instantaneous contact lines can be precisely determined from obtained meshing points.

The numerical simulation was carried out to study the worm drive. It is easy to form lubricating oil film near the worm gear addendum and both sides of worm gear tooth surface. However, the lubricating oil film near the surface middle of worm gear tooth root forms hardly.

A new method to research the number of the worm thread $Z_1$ based on working orthogonal angles is proposed. In simulation results, the working orthogonal clearance decreases with the increasing $Z_1$. If the $Z_1$ exceeds 4, the working orthogonal clearance will be negative, which leads wear increasing and service life reducing of the lathe tool. These need to be avoided during the worm machining. Therefore, the maximum of the worm thread is 4.

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**Figure 11.** Geometric relations of turning tool angles in worm cutting.

**Figure 12.** Change of working orthogonal angel along worm tooth profile: (a) working orthogonal clearance and (b) working orthogonal rake.
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References
1. Wu H, Zhang Y, Lin Q, et al. Design of worm drives. Beijing: China Machine Press, 1986.
2. Wang Sh. Arc cylindrical worm drive. Tianjin: Tianjin University Press, 1991.
3. Editorial Committee for Gear Handbook. Gear handbook, 2nd ed. Beijing: China Machine Press, 2004.
4. Zhu J, et al. Research on arc tooth worm and worm gear drive. J Taiyuan Inst Technol 1963; 1: 1–12.
5. Wu H. Gear meshing theory. Harbin: Harbin Institute of Technology Press, 1979.
6. Han M. Several problems in design and manufacture of arc tooth cylindrical worm drive. Mining Machine J 1981; 9: 24–29.
7. Wang Sh. Meshing theory of cylindrical worm with circular profile cut by lathe and worm gear. J Tianjin Univ 1980; 2: 15–39.
8. Wang Sh and Liu P. Meshing principle of cylindrical worm drive. Tianjin: Tianjin Science and Technology Press, 1982.
9. Yang L. Arc tooth cylinder worm drive. Taiyuan: Shanxi People’s Publishing House, 1984.
10. Zhang Y. Cylindrical worm and worm gear meshing and cutting tool. Chongqing: Chongqing Publishing House, 1984.
11. Zhang Y, Dong X, Songchun H, et al. Design of worm drives, vol. 2. Beijing: China Machine Press, 1987.
12. Lanheyn Y, Ziqin L, Zhan Z, et al. Handbook of worm drives. Shanghai: East China University of Science and Technology Press, 1990.
13. Fu Z, Zhao S, Yang L, et al. New types of worm drive. Xi’an: Shaanxi Science and Technology Press, 1990.
14. Zhang Y, Kai K, Liren Z, et al. Analysis of the elastohydrodynamic lubrication (EHL) of arc cylindrical worm drive. J Beijing Univ Chem Technol 2001; 28: 52–55.
15. Simon V. Load distribution in cylindrical worm gears. ASME J Mech Des 2003; 125: 356–364.
16. Simon V. Computer aided loaded tooth contact analysis in cylindrical worm gears. ASME J Mech Des 2005; 127: 973–981.
17. Zhao Y and Zhang Y. Novel methods for curvature analysis and their application to TA worm. Mech Mach Theory 2016; 97: 155–170.
18. Zhao Y. Meshing analysis for TA worm. Mech Mach Theory 2016; 43: 13–20.
19. Zhao Y, Huai Ch and Zhang Y. Compound modification of globoidal worm drive with variable parameters. Appl Math Modell J 2017; 50: 17–38.
20. Zhao Y and Sun X. On meshing limit line of ZC1 worm pair. In: European conference on mechanism science, Aachen, Germany, 2018, pp.292–298.
21. Wei C, Guomin W, Jiajie L, et al. Meshing stiffness calculation of ZC3 worm gear based on finite element method. J Mech Transm 2018; 1: 61–64.
22. Dong X. Foundation of meshing theory for gear drives. Beijing: China Machine Press, 1989.
23. Chen W. Preliminary differential geometry. Beijing: Peking University Press, 1990.
24. Wu D and Luo J. Gear meshing theory. Beijing: Science Press, 1985.
25. Zhou Z. Metal cutting principle. Shanghai: Shanghai Scientific & Technical Publishers, 1984.
26. Yuan Z and Liu H. Metal cutting tool design manual. Beijing: China Machine Press, 2009.

Appendix
Notation
\[ \begin{align*}
\theta & \text{ rotational angle of cutting edge around } \vec{k}_1 \\
\rho & \text{ helix parameter of the worm} \\
\phi & \text{ rotational angle of cutting edge around } \vec{j}_1 \\
\rho & \text{ radius of cutting edge arc} \\
b & \text{ radial coordinate of the center of cutting edge} \\
c & \text{ axial coordinate of the center of cutting edge} \\
\phi_1 & \text{ worm rotational angle of the worm drive meshing} \\
l_{12} & \text{ gear ratio} \\
a & \text{ center distance of the worm drive} \\
\lambda_2 & \text{ modification coefficient} \\
r_{a1} & \text{ addendum circle radius of the worm} \\
r_{d1} & \text{ dedendum circle radius of the worm} \\
r_{t1} & \text{ throat generate circle radius of the worm gear} \\
r_{a2} & \text{ addendum circle radius of the worm gear} \\
r_{d2} & \text{ dedendum circle radius of the worm gear} \\
r & \text{ tooth width angle of gear wheel} \\
\alpha & \text{ tooth width of worm gear} \\
\alpha_c & \text{ axial tooth profile angle} \\
r_1 & \text{ reference circle radius of the worm} \\
s & \text{ axial tooth thickness} \\
Z_1 & \text{ the number of the worm thread} \\
m & \text{ modulus} \\
r & \text{ radial distance of the point on the worm tooth profile} \\
\gamma_0 & \text{ tool orthogonal rake} \\
\alpha_o & \text{ tool orthogonal clearance}
\end{align*} \]