Single transverse spin asymmetries in inclusive hadron production

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A consistent phenomenological approach to the computation of transverse single spin asymmetries in inclusive hadron production is presented, based on the assumed generalization of the QCD factorization theorem to the case in which quark intrinsic motion is taken into account. New $k_{\perp}$ and spin dependent quark distribution and fragmentation functions are considered; some of them are fixed by fitting data on $p^+p \rightarrow \pi X$ and predictions are given for single spin asymmetries in $\ell p^+ \rightarrow \pi X$ and $\gamma^* p^+ \rightarrow \pi X$ processes.

1. General Formalism

It is well known that perturbative QCD and the factorization theorem at leading twist \cite{2} can be used to describe the large $p_T$ production of a hadron $C$ resulting from the interaction of two polarized hadrons $A$ and $B$

\begin{equation}
\frac{E_C d^3 \sigma_{A,S_A+B,S_B\rightarrow C+X}}{d^3 p_C} = \sum_{a,b,c,d;\{\lambda\}} \int \frac{dx_a dx_b}{16\pi^2 z_2^2} \rho_{\lambda_a,\lambda'_b}^{a/B,S_B} f_a(x_a) \rho_{\lambda'_a,\lambda_b}^{b/B,S_B} f_b(x_b)
\end{equation}

$\hat{M}_{\lambda_c,\lambda_d;\lambda_a,\lambda_b} M_{\lambda_c,\lambda_d;\lambda_a,\lambda_b}' D_{\lambda_c,\lambda_c}'(z)$, and that the above equation leads to vanishing single spin asymmetries

\begin{equation}
A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{2 d\sigma^{\text{amp}}},
\end{equation}

due to helicity conservation in the elementary interactions and parton collinear configurations in distribution and fragmentation functions (the notations used above should be self-explanatory, further details can be found, e.g., in Ref. \cite{2}; $\uparrow$ and $\downarrow$ are directions perpendicular to the scattering plane).

Possible origins of single spin effects can be introduced by considering $k_{\perp}$ dependences in the quark distribution functions $\hat{f}_{q/p}(x, k_{\perp})$ \cite{3} or in the quark fragmentation functions $\hat{D}_{h/q}(z, k_{\perp})$ \cite{3}, \cite{4}; a consistent phenomenological approach has been developed taking these possibilities into account \cite{2}, \cite{3}, \cite{4} and assuming that the factorization theorem of Eq. (1) holds also when intrinsic parton motion is included. At leading order in $k_{\perp}$ one has, for the $p^+ p \rightarrow \pi X$ process:

\begin{equation}
\frac{E_{\pi} d^3 \sigma^{\uparrow}}{d^3 p_{\pi}} - \frac{E_{\pi} d^3 \sigma^{\downarrow}}{d^3 p_{\pi}} = \sum_{a,b,c,d} \int \frac{dx_a dx_b}{\pi z} \times
\end{equation}

\begin{equation*}
\left\{ \int d^2 k_{\perp} \Delta N f_{a/p^\uparrow}(k_{\perp}) f_{b/p} \frac{d\sigma}{dt}(k_{\perp}) D_{\pi/c}(k_{\perp}) + \int d^2 k'_{\perp} h_{1}^a/p \Delta N f_{b/p} \Delta N D_{\pi/c}(k'_{\perp}) + \int d^2 k''_{\perp} h_{1}^a/p \Delta N f_{b'/p} \Delta N \Delta N D_{\pi/c}(k''_{\perp}) \right\},
\end{equation*}

where the first line in brackets corresponds to the so-called Sivers effect \cite{3}, the second to Collins effect \cite{2} and the third one to a recently proposed mechanism \cite{4}.

The new $k_{\perp}$ and spin dependent functions appearing in Eq. (3) have a partonic interpretation
in terms of polarized quark distribution and fragmentation functions as:

\[ \Delta^N f_{q/p}^{\perp} = \hat{f}_{q/p}^{\perp}(x, k_\perp) - \hat{f}_{q/p}^{\perp}(x, k_\perp), \] (4)

\[ \Delta^N D_{h/q} = \hat{D}_{h/q}^{\perp}(z, k_\perp) - \hat{D}_{h/q}^{\perp}(z, k_\perp), \] (5)

\[ \Delta^N f_{q/p}^{\perp} = \hat{f}_{q/p}^{\perp}(x, k_\perp) - \hat{f}_{q/p}^{\perp}(x, k_\perp). \] (6)

The other quantities appearing in Eq. (6), apart from the unpolarized quark distribution and fragmentation functions, \( f \) and \( D \), are the transverse spin content of the proton:

\[ h_1^{q/p} = f_{q/p}(x) - f_{q/p}(x) \] (7)

and the elementary double spin asymmetries, computable in pQCD:

\[ \Delta^N N \hat{\sigma} = \frac{d\hat{a}^\perp b \rightarrow c^d}{dt} - \frac{d\hat{a}^\perp b \rightarrow c^d}{dt}, \] (8)

\[ \Delta' N \hat{\sigma} = \frac{d\hat{a}^\perp b^\perp \rightarrow c^d}{dt} - \frac{d\hat{a}^\perp b^\perp \rightarrow c^d}{dt}. \] (9)

Eq. (9) can be used to obtain information on the new quantities \( \Delta^N N \hat{\sigma} \) by fitting existing data on \( p^+ p \rightarrow \pi X \) [2]; once this has been done predictions for other processes can be given. This program has been partially performed in Refs. [2], [3], [4Leptonic] and [8].

2. Sivers and Collins effects alone

In Ref. [4], Sivers effect has been assumed to be the only origin of single spin asymmetries and the corresponding expression of Eq. (3) (i.e. its first two lines) has been used to fit the data on \( p^+ p \rightarrow \pi X \), leading to an explicit expression for \( \Delta^N f_{q/p}^{\perp}(x, \langle k_\perp \rangle) \) at an average intrinsic transverse \( k_\perp \) value; the data can be nicely fitted and the deduced expression of \( \Delta^N f_{q/p}^{\perp}(x, \langle k_\perp \rangle) \) leads to predictions for single spin asymmetries in \( p^+ p \rightarrow \pi X \) in agreement with data [2]. The resulting values of \( \Delta^N f_{q/p}^{\perp}/2f_{q/p} \) also turn out to be reasonable and plausible.

A similar program, taking only Collins effect into account, has been performed in Ref. [2], where the first and third line of Eq. (9) have been used to explain the existing data and to derive an explicit expression of \( \Delta^N D_{\pi/q}(z, \langle k_\perp \rangle) \): again, data on \( p^+ p \rightarrow \pi X \) can be fitted, with some difficulties at large \( z \) values, and values of \( A_N \) in \( p^+ p \rightarrow \pi X \) processes are computed in agreement with experiment. In this case, however, the resulting values of \( \Delta^N D_{\pi/q}/2D_{\pi/q} \) turn out to be at the border of acceptability, in that this ratio has to reach 1 at large \( z \) values in order to fit the data.

It is clear that both Sivers and Collins effects might be at work at the same time and contribute in different proportions to the observed values of \( A_N \). Not only: also the contribution suggested in Ref. [3] might be at work. Its consequences alone have not been studied yet, although it might be possible that the effects of \( \Delta^N f_{q/p}^{\perp} \) are evident in different \( x_F \) regions from Sivers and Collins effects.

However, both the new, time-reversal odd functions \( \Delta^N f_{q/p}^{\perp} \) and \( \Delta^N f_{q/p}^{\perp} \) require some initial state interactions [8]; these initial state interactions are expected in hadron-hadron processes where many soft gluons should be exchanged between the colliding particles. This is not so in lepton-hadron interactions where any extra exchange of photons gives only negligible corrections. Thus, we expect that only Collins effect might originate single spin asymmetries in lepto-production processes. We consider the most favourable situation, by assuming that this is true also in \( pp \) processes, and take the expression of \( \Delta^N D_{\pi/q} \) as determined in Ref. [4]; under such an assumption – only Collins effect at work to originate single spin asymmetries in any inclusive process – we are able to give predictions for \( A_N \) in semi-inclusive lepton initiated processes.

3. Predictions for DIS processes

We consider, in complete analogy to \( p^+ p \rightarrow \pi X \), the process \( \ell p \rightarrow \pi X \): the former process has an observed large \( A_N \) and, if Collins mechanism is responsible for it, we also expect a large asymmetry for the latter:

\[ \frac{E_\pi d^3\bar{\sigma}^{\perp}}{d^3p_\pi} - \frac{E_\pi d^3\bar{\sigma}^{\perp}}{d^3p_\pi} = \sum_q \int \frac{dx}{\pi z} \int d^2k_\perp \times (10) \]
\[ h_1^{q/p}(x) \Delta_{NN} \hat{\sigma}^q(x, k_\perp) \Delta^N D_{\pi/q}(z, k_\perp) \]

where

\[ \Delta_{NN} \hat{\sigma}^q = \frac{d\hat{\sigma}^{q\uparrow \rightarrow q\uparrow}}{dt} - \frac{d\hat{\sigma}^{q\uparrow \rightarrow q\downarrow}}{dt}. \] (11)

From the expressions of Ref. [2] we can compute Eq. (2) and (10) and we obtain indeed large values of \( A_N \); in Fig. 1 we show results at \( s = 52.5 \text{ GeV}^2 \) and \( p_T = 1.2 \text{ GeV/c} \) (typical HERMES values). Similar results hold at different energies [8].

A measurement of \( A_N \) in \( \ell p^{\uparrow} \rightarrow \pi X \) requires transversely polarized nucleons; however, single transverse spin asymmetries may be measurable also in the case of longitudinally polarized nucleons provided one looks at the double inclusive process, \( \ell p^{\uparrow} \rightarrow \ell \pi X \) from which one can reconstruct the \( \gamma^* p^{\uparrow} \rightarrow \pi X \) reaction, which, in general, occurs in a plane different from the \( \ell - \ell' \) plane where the longitudinal nucleon spin lies; in this case one has (see Ref. [8] for details):

\[ \frac{d\sigma_{\gamma^* p^{\uparrow} \rightarrow \pi X}}{dx \, dQ^2 \, dz \, d^2p_T} - \frac{d\sigma_{\gamma^* p^{\uparrow} \rightarrow \pi X}}{dx \, dQ^2 \, dz \, d^2p_T} = \sum_q \times \] (12)

\[ h_1^{q/p} \left[ \frac{d\hat{\sigma}^{q\uparrow \rightarrow q\uparrow}}{dQ^2} - \frac{d\hat{\sigma}^{q\uparrow \rightarrow q\downarrow}}{dQ^2} \right] \Delta^N D_{\pi/q}(p_T) \] (12)

In Fig. 2 we show \( A_N \) at the same energy values of Fig. 1, as a function of \( z \): it is large also in this case, although only at very large \( z \) values which might be difficult to reach experimentally.

![Figure 1](image1.png)

**Figure 1.** \( A_N(\ell p^{\uparrow} \rightarrow \pi X) \) for typical HERMES kinematics.

![Figure 2](image2.png)

**Figure 2.** \( A_N(\gamma^* p^{\uparrow} \rightarrow \pi X): \) \( s = 52.5 \text{ GeV}^2, \) \( Q^2 = 8 \text{ GeV}^2 \); solid, dashed and dotted line refer respectively to \( \pi^+, \pi^0 \) and \( \pi^- \).

Single transverse spin asymmetries might be large in semi-inclusive DIS: their measurement is important and is feasible at several laboratories either with the existing configurations or with transversely polarized future nucleon beams.

**REFERENCES**

1. J.C. Collins, Nucl. Phys. B396 (1993) 161.
2. M. Anselmino, M. Boglione and F. Murgia, e-Print Archive: [hep-ph/9901442](http://arxiv.org/abs/hep-ph/9901442) to appear in Phys. Rev. D (1999).
3. D. Sivers, Phys. Rev. D41 (1990) 83; D41 (1991) 261.
4. M. Anselmino, M. Boglione and F. Murgia, Phys. Lett. B362 (1995) 164.
5. D. Boer, e-Print Archive: [hep-ph/9902255](http://arxiv.org/abs/hep-ph/9902255).
6. M. Anselmino and F. Murgia, Phys. Lett. B442 (1998) 470.
7. D.L. Adams et al., Phys. Lett. B264 (1991) 462; Phys. Rev. Lett. 77 (1996) 2626.
8. M. Anselmino, M. Boglione, J. Hansson and F. Murgia, in preparation.