The basic parameters of \( \gamma \)-ray-loud blazars

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Abstract. We determined the basic parameters, such as the central black hole mass \( (M) \), the boosting factor (or Doppler factor) \( (\delta) \), the propagation angle \( (\Phi) \) and the distance along the axis to the site of \( \gamma \)-ray production \( (d) \) for 23 \( \gamma \)-ray-loud blazars using their available variability timescales. In this method, the absorption effect depends on the \( \gamma \)-ray energy, emission size and property of the accretion disk. Using the intrinsic \( \gamma \)-ray luminosity as a fraction \( \lambda \) of the Eddington luminosity, \( L_{\gamma}^{in} = \lambda L_{\text{Edd}} \) and the optical depth equal to unity, we can determine the upper limit of the central black hole masses. We found that the black hole masses range between \( 10^7 \) \( M_\odot \) and \( 10^9 \) \( M_\odot \) when \( \lambda = 0.1 \) and 1.0 are adopted. Since this method is based on gamma-ray emissions and the short time-scale of the sources, it can also be used for central black hole mass determination of high redshift gamma-ray sources. In the case of the upper limit of black hole mass there is no clear difference between BLs and FSRQs, which suggests that the central black hole masses do not play an important role in the evolutionary sequence of blazars.

Key words. Galaxies:quasars-galaxies:BL Lacertae objects-galaxies:jet- galaxies:\( \gamma \)-rayst

1. Introduction

The EGRET instrument at CGRO has detected many blazars (i.e., flat spectrum radio quasars (FSRQs) and BL Lacertae objects (BLs)). Blazars emit most of their bolometric luminosity in \( \gamma \)-rays (\( E > 100 \) MeV) (Hartman et al. 1999). Many \( \gamma \)-ray emitters are also superluminal radio sources (von Montigny et al. 1995). These objects share some common properties, such as luminous \( \gamma \)-ray emission and strong variability in the \( \gamma \)-ray and other bands on timescales from hours to days (see below). These facts suggest that \( \gamma \)-ray emission in blazars is likely arise from a jet. To explain its observational properties, a beaming (black hole + accretion disk + jet) model has been proposed. In the beaming model, a supermassive black hole is surrounded by an accretion disk. Many authors have tried to estimate the masses using different methods, (1) the reverberation mapping technique (e.g. Wandel, Peterson et al. 1999; Kaspi et al. 2000), (2) the gas and stellar dynamics technique (see Genzel et al. 1997; Magorrian et al. 1998; Kormendy & Gebhardt 2001), (3) the variability time-scale technique (Fan et al. 1999; Cheng et al. 1999), (4) the broad-line width technique (Dibai 1984; Wandel & Yahil 1985; Padovani & Rafanelli 1988; Laor 1998; Melure & Dunlop 2001; Vestergaard 2000 based on the assumption that the clouds in the broad-line region (BLR) are gravitationally bound and orbiting with Keplerian velocities). The central black hole mass is also found to be correlated with bulge luminosities (Kormendy & Richstone 1995), the bulge mass (Magorrian et al. 1998), the bulge velocity dispersion (Ferrarese & Merritt 2000; Gebhardt et al. 2000; Ferrarese et al. 2001) and the radio power (Franceschini et al. 1998). However, Ho (2002) found that the radio continuum power either integrated for the whole galaxy or isolated for the core is poorly correlated with the central black hole mass. The tight \( M_{\text{BH}} - \sigma \) correlation can be used for black hole mass determination. This relation is also used by Barth et al. (2002) and Wu et al. (2002) for black hole mass determination for Mrk 501 and other AGNs. Cao (2002) estimated the black hole mass for a sample of BL Lacertae objects based on the assumption that broad emission lines are emitted from clouds ionized by the radiation of the accretion disk surrounding the black hole.

Since there is a large number of soft photons around the central black hole, it is generally believed that the escape of high energy \( \gamma \)-rays from the AGN depends on the \( \gamma - \gamma \) pair production process. Therefore, the opacity of \( \gamma - \gamma \) pair production in \( \gamma \)-ray-loud blazars can be used to constrain the basic parameters. Becker & Kafatos
focused in a solid angle, $\Omega = 2\pi$, scale may carry the information about the blazars are discussed. In addition, the variability time "focusing" is related to the more general issue of $\gamma$-pair production cross section. The phenomenon of the disk, due to the strong angular dependence of the radiation, which sets a minimum distance between the central black hole and the site of $\gamma$-ray production (Bednarek 1993, Dermer & Schlickeiser 1994, Becker & Kafatos 1995, Zhang & Cheng 1997). So, the $\gamma$-rays are observed $\gamma$-ray energy flux and luminosity distance respectively. The observed $\gamma$-rays from an AGN require that the jet almost points towards us and that the optical depth $\tau$ is not greater than unity. The $\gamma$-rays are from a solid angle, $\Omega$, instead of being isotropic. In this sense, the non-isotropic radiation, absorption and beaming (boosting) effects should be considered when the properties of a $\gamma$-ray-loud blazar are discussed. In addition, the variability time scale may carry the information about the $\gamma$-ray emission region. These considerations require a new method to estimate the central black hole mass and other basic parameters of a $\gamma$-ray-loud blazar, which is the focus of the present paper.

In section 2, we introduce the method used to estimate the black hole mass and three other parameters (the Doppler factor, the propagation angle of the $\gamma$-rays and their emission distance at the symmetric axis above the accretion disk). In section 3, we present the discussion and a brief summary of the paper.

$H_0 = 75$ km s$^{-1}$ Mpc$^{-1}$, and $q_0 = 0.5$ are adopted throughout the paper.

2. Mass estimation method and result

2.1. Method

Here we describe our method of estimating the basic parameters, namely, the central black hole mass ($M$), the boosting factor (or Doppler factor) ($\delta$), the propagation angle ($\Phi$) and the distance along the axis to the site of the $\gamma$-ray production ($d$) for $\gamma$-ray-loud blazars with short timescale variabilities (see Cheng et al. 1999 for detail).

To do so, we consider a two-temperature disk (see Fig. 1). The $\gamma$-ray observations suggest that the $\gamma$-rays are strongly boosted. From the high energy $\gamma$-ray emission we know that the optical depth of $\gamma-\gamma$ pair production should not be larger than unity. In addition, the observed short-time scale gives some information about the size of emitting region. This can be used to constrain the basic parameters of a $\gamma$-ray-loud blazar as in the following.

**Optical depth** Based on the paper by Becker & Kafatos (1995), we can obtain an approximate empirical formula for the optical depth for a two-temperature disk case at an arbitrary angle, $\Phi$ (see Cheng et al. 1999),

$$\tau_{\gamma\gamma}(M_7, \Phi, d) = 9 \times \Phi^{2.5} \left(\frac{d}{R_g}\right)^{-2a_X+3} + kM_7^{-1} \left(\frac{d}{R_g}\right)^{-2a_X-3},$$

where $k$ is

$$k = 4.61 \times 10^9 \frac{\Psi(a_X)(1+z)^{(2a_X+3)}X_{\alpha}^0(1+z+\sqrt{1+z})^2}{(2a_X+1)(2a_X+3)} \times$$

![Schematic diagram of $\gamma$-ray propagation above a two-temperature disk surrounding a supermassive black hole. $\gamma$-rays interact with the soft X-ray photons produced at all points on the disk. The interaction angle between the $\gamma$-ray and X-ray photons is $\Theta$, the angle between the $\gamma$-ray trajectory and the z-axis is $\Phi$. $\eta$ is the distance between the photon-photon interaction point and the soft photon emission point in the disk.](https://example.com/image.png)
the observed luminosity is correlated with the intrinsic luminosity
$L_{\text{obs}}$.

In a relativistic beaming model, the observed luminosity is correlated with the intrinsic one in the frame comoving with the relativistic jet by

$$L_{\gamma\text{obs}} = \frac{\delta \sigma_{\gamma} + 4}{(1 + z)^{\sigma_{\gamma} - 1}} L_{\gamma}^{\text{in}},$$

where $\sigma_{\gamma}$ is the $\gamma$-ray spectral index. As mentioned above, the observed $\gamma$-ray flux, $F_{\gamma\text{obs}}(>100\text{MeV})$, which is in units of ergs $\text{cm}^{-2} \text{s}^{-1}$, can be expressed as a function of the intrinsic luminosity $L_{\gamma}^{\text{in}}$, the Doppler factor $\delta$, the luminosity distance $D$, and the solid angle $\Omega$ (or propagation angle $\Phi$)

$$F_{\gamma\text{obs}}(>100\text{MeV}) = (1 + z)^{-\sigma_{\gamma}} \delta^{\sigma_{\gamma} + 4} L_{\gamma}^{\text{in}}/\Omega D^2.$$  

If we define an isotropic luminosity as $L_{\text{iso}} = 4\pi D^2 F_{\gamma\text{obs}}(>100\text{MeV})$, we have

$$L_{\text{iso}}^{45} = \frac{\lambda L_{\text{Edd}}}{(1 - \cos\Phi)(1 + z)^{\sigma_{\gamma} - 1}} M_7.$$

where $L_{\text{in}}^{\text{in}} = \lambda L_{\text{Edd}} = 1.26 \times 10^{15} M_7$ is adopted, $\lambda$ is a parameter depending on specific $\gamma$-ray emission models, $L_{\text{iso}}^{45}$ is the isotropic luminosity in units of $10^{45}$ ergs $\text{s}^{-1}$.

From equation (4), we can get the Doppler factor

$$\delta = \left( \frac{L_{\text{iso}}^{45}(1 - \cos\Phi)(1 + z)^{\sigma_{\gamma} - 1}}{\lambda L_{\text{Edd}}} \right)^{\frac{1}{\sigma_{\gamma} - 1}}.$$

Substituting Eqs. (4) and (5) into Eq. (1), we obtain a relation for $\tau_{\gamma\gamma}(\Phi, M, L_{\text{iso}})$,

$$\tau_{\gamma\gamma}(\Phi, M_7, L_{\text{iso}}) = [9 \times \Phi^2.5 (1 - \cos\Phi)]^{-\frac{22}{25}} A^{-\frac{22}{25}} k M_7^{-1} k M_7^{-1} (1 - \cos\Phi) \frac{20 \times 4 + 3}{20 \times 4 + 8} \sin\Phi \times \sin\Phi.$$

(7)

From the high energy $\gamma$-ray emission, we know that the optical depth of $\gamma$-$\gamma$ pair production should not be larger than unity. So, we can assume $\tau_{\gamma\gamma}(\Phi, M_7, L_{\text{iso}}) = 1.0$ for our purposes. However, there are two variables in the equation of $\tau_{\gamma\gamma}(\Phi, M_7, L_{\text{iso}}) = 1.0$, so one should have one more equation to determine the basic parameters. Fortunately, the $\tau_{\gamma\gamma}(\Phi, M, L_{\text{iso}})$ shows a minimum for a certain mass $M_7$ and angle $\Phi$. For a given mass, $M_7$, the dependance of $\tau_{\gamma\gamma}(\Phi, M, L_{\text{iso}})$ on $\Phi$ is illustrated in Figure 2, in which we show the case of $0208-512$. For the source, the relevant values are $\alpha_X = 1.04$, $\alpha_{\gamma} = 0.69$, $k = 6.41$, $L_{\text{iso}} = 2.0 \times 10^{46} \text{ergs}^{-1}$, $z = 1.003$, $\lambda = 0.1$, and $\Delta T = 134.4$ hours respectively. In this sense, if we assume that the minimum value of $\tau_{\gamma\gamma}(\Phi, M, L_{\text{iso}})$ is equal to 1.0, then we will have the relation $\frac{\delta}{\partial \Phi}|_{M} = 0$.

Equation (4) gives

$$\frac{\partial \tau_{\gamma\gamma}}{\partial \Phi}|_{M} = \frac{[22.5 \Phi^2.5 (1 - \cos\Phi) - 9 \times \frac{22}{25} A^{-\frac{22}{25}} k M_7^{-1} k M_7^{-1} \Phi^2.5 \sin\Phi]}{\frac{22}{25} \Phi^2.5 \sin\Phi} \times \sin\Phi \times \frac{22}{25} \Phi^2.5 \sin\Phi \times \sin\Phi = 0 \quad (9)$$

Then, $\frac{\partial \tau_{\gamma\gamma}}{\partial \Phi}|_{M} = 0$ suggests that

$$22.5 \Phi^2.5 (1 - \cos\Phi) - 9 \times \frac{22}{25} A^{-\frac{22}{25}} k M_7^{-1} k M_7^{-1} \Phi^2.5 \sin\Phi \times \sin\Phi = 0 \quad (9)$$

Under this consideration, we can finally get four relations,

$$\frac{d}{R_g} = 1.73 \times 10^3 \frac{\Delta T_D}{1 + z} \delta M_7^{-1},$$

$$L_{\text{iso}}^{45} = \frac{\lambda L_{\text{Edd}}}{(1 - \cos\Phi)(1 + z)^{\sigma_{\gamma} - 1}} M_7,$$

$$9 \times \Phi^2.5 \frac{d}{R_g} k M_7^{-1} k M_7^{-1} \Phi^2.5 \sin\Phi = 1 \quad (10)$$

in which there are four basic parameters. So, for a source with available data in the X-ray and $\gamma$-ray bands, the masses of the central black holes, $M_7$, the Doppler factor, $\delta$, the distance along the axis to the site of the $\gamma$-ray production, $d$, and the propagation angle with respect to the axis of the accretion disk, $\Phi$, can be derived from Eq. (10), where $R_{ms} = 6 R_g$, $R_0 = 30 R_g$, and $E_{\gamma} = 1 \text{GeV}$ are adopted.
different sources and/or different observational periods, we use the doubling timescale, $\Delta T_D = (F_{\text{minimum}}/\Delta F)\Delta T$, as the variability timescale, where $\Delta F = F_{\text{maximum}} - F_{\text{minimum}}$ is the variation of the flux over the time $\Delta T$. There are few simultaneous X-ray and $\gamma$-ray band observations, so the data considered here are not simultaneous. The $\gamma$-ray data by Hartman et al. (1999) are used to calculate the $\gamma$-ray luminosity. The X-ray data are taken from recent publications (see col. 5 in Table 1). Except for the two sources 1226+023 (Courvoisier et al. 1988) and 2230+114 (Pica et al. 1988), the doubling time is taken from Dondi & Ghisellini (1995).

The intrinsic $\gamma$-ray luminosity is unknown, so we assume it to be close to the Eddington luminosity, say $\lambda L_{\text{Edd}}$. In the present work, $\lambda = 0.1$ and 1.0 are adopted for the calculations. From the available X-ray and $\gamma$-ray data, we can estimate the central black hole mass, $M_\gamma$ (see Table 1) and three other parameters ($\Phi$, $\delta$, $d$, these values are not listed in Table 1). Since short-term time scales and $\gamma$-ray emissions are included in our consideration, only objects with those values can be involved in the present paper. However, at present, short term timescales are available only for 23 $\gamma$-ray loud blazars as listed in Table 1, in which Col. 1 gives the name, Col. 2 the redshift, Col. 3 the identification where Q stands for a flat spectral radio quasar and B for a BL Lacertae object, Col. 4 the 1 KeV X-ray flux density in units of $\mu$ Jy, Col. 5 the reference for Col. 4, Col. 6 the X-ray spectral index $\alpha_X$ (the averaged value of $<\alpha_X>$ = 0.67 (Comastri et al. 1997) is adopted for FSRQs and $\alpha_{OX} = 1.31$ is used for $\alpha_X$ for BLs if their X-ray spectral indices are unknown, as done by Ghisellini et al. (1998)), Col. 7 the reference for Col. 6, Col. 8 the flux $F(>100\text{MeV})$ in units of $10^{-6}$ photon cm$^{-2}$ s$^{-1}$, Col. 9 the $\gamma$-ray spectral index $\alpha_\gamma$ = 1.0 is adopted for 0537-441 (see Fan et al. 1998); The data in col no. (7) and (8) are mainly from by Hartman et al. (1999) except for Mkn 501, which showed a flux of $F(>100\text{MeV}) = (0.32 \pm 0.13) \times 10^{-6}$ photons cm$^{-2}$ s$^{-1}$ with a photon index of 1.6$\pm$0.5 during a 1996 multiwavelength campaign (see Kataoka et al. 1999), Col. 10 the doubling time scale in units of hours, Col. 11 reference for Col. 10, Col. 12 the observed isotropic luminosity in units of $10^{48}$ erg s$^{-1}$, Col. 13, the central black hole mass in units of $10^8 M_\odot$ ($\lambda = 1$), Col. 14, the central black hole mass in units of $10^7 M_\odot$ ($\lambda = 0.1$). The references in Table 1 (B97: Bloom et al. 1997; C97: Comastri et al. 1997; Ch99: Chiappetti et al. 1999; DG: Dondi & Ghisellini 1995; F98: Fan et al. 1998; Fo98: Fossati et al. 1998; H96: Hartman 1996; H96b: Hartman et al. 1996; H: Hartman et al. 1999; K93: Kniffen et al. 1993; L98: Lawson & McHardy 1998; M93: Mattax et al. 1993; M96: Madejski et al. 1996; M97: Mattox et al. 1997; P96: Perlman et al. 1996; P88: Pica et al. 1988; Q96: Quinn et al. 1996; S96: Stacy et al. 1996; U97: Urry et al. 1997; W98: Wehrle et al. 1998).

**Fig. 2.** Plot of the optical depth against the angle $\Phi$. For illustration, we used the data of 0208-512, the solid curve stands for the $M_\gamma = 95$ case, while the dash-dotted curve for $M_\gamma = 60$ and the dashed curve for $M_\gamma = 130$. Here the relevant values are $\alpha_X = 1.04$, $\alpha_\gamma = 0.69$, $k = 6.41$, $L_{\text{iso}} = 2.0 \times 10^{48}$ erg s$^{-1}$, $z = 1.003$, $\lambda = 0.1$, and $\Delta T = 134.4$ hours respectively.

### 2.2. Results

Since we are interested in the variability timescale, we present here only the $\gamma$-ray-loud blazars with detected short time scale variability. Since the variability timescale corresponds to the different amplitude of variation for different sources and/or different observational periods, we use the doubling timescale, $\Delta T_D = (F_{\text{minimum}}/\Delta F)\Delta T$, as the variability timescale, where $\Delta F = F_{\text{maximum}} - F_{\text{minimum}}$ is the variation of the flux over the time $\Delta T$. There are few simultaneous X-ray and $\gamma$-ray band observations, so the data considered here are not simultaneous. The $\gamma$-ray data by Hartman et al. (1999) are used to calculate the $\gamma$-ray luminosity. The X-ray data are taken from recent publications (see col. 5 in Table 1). Except for the two sources 1226+023 (Courvoisier et al. 1988) and 2230+114 (Pica et al. 1988), the doubling time is taken from Dondi & Ghisellini (1995).

The intrinsic $\gamma$-ray luminosity is unknown, so we assume it to be close to the Eddington luminosity, say $\lambda L_{\text{Edd}}$. In the present work, $\lambda = 0.1$ and 1.0 are adopted for the calculations. From the available X-ray and $\gamma$-ray data, we can estimate the central black hole mass, $M_\gamma$ (see Table 1) and three other parameters ($\Phi$, $\delta$, $d$, these values are not listed in Table 1). Since short-term time scales and $\gamma$-ray emissions are included in our consideration, only objects with those values can be involved in the present paper. However, at present, short term timescales are available only for 23 $\gamma$-ray loud blazars as listed in Table 1, in which Col. 1 gives the name, Col. 2 the redshift, Col. 3 the identification where Q stands for a flat spectral radio quasar and B for a BL Lacertae object, Col. 4 the 1 KeV X-ray flux density in units of $\mu$ Jy, Col. 5 the reference for Col. 4, Col. 6 the X-ray spectral index $\alpha_X$ (the averaged value of $<\alpha_X>$ = 0.67 (Comastri et al. 1997) is adopted for FSRQs and $\alpha_{OX} = 1.31$ is used for $\alpha_X$ for BLs if their X-ray spectral indices are unknown, as done by Ghisellini et al. (1998)), Col. 7 the reference for Col. 6, Col. 8 the flux $F(>100\text{MeV})$ in units of $10^{-6}$ photon cm$^{-2}$ s$^{-1}$, Col. 9 the $\gamma$-ray spectral index $\alpha_\gamma$ = 1.0 is adopted for 0537-441 (see Fan et al. 1998); The data in col no. (7) and (8) are mainly from by Hartman et al. (1999) except for Mkn 501, which showed a flux of $F(>100\text{MeV}) = (0.32 \pm 0.13) \times 10^{-6}$ photons cm$^{-2}$ s$^{-1}$ with a photon index of 1.6$\pm$0.5 during a 1996 multiwavelength campaign (see Kataoka et al. 1999), Col. 10 the doubling time scale in units of hours, Col. 11 reference for Col. 10, Col. 12 the observed isotropic luminosity in units of $10^{48}$ erg s$^{-1}$, Col. 13, the central black hole mass in units of $10^8 M_\odot$ ($\lambda = 1$), Col. 14, the central black hole mass in units of $10^7 M_\odot$ ($\lambda = 0.1$). The references in Table 1 (B97: Bloom et al. 1997; C97: Comastri et al. 1997; Ch99: Chiappetti et al. 1999; DG: Dondi & Ghisellini 1995; F98: Fan et al. 1998; Fo98: Fossati et al. 1998; H96: Hartman 1996; H96b: Hartman et al. 1996; H: Hartman et al. 1999; K93: Kniffen et al. 1993; L98: Lawson & McHardy 1998; M93: Mattax et al. 1993; M96: Madejski et al. 1996; M97: Mattox et al. 1997; P96: Perlman et al. 1996; P88: Pica et al. 1988; Q96: Quinn et al. 1996; S96: Stacy et al. 1996; U97: Urry et al. 1997; W98: Wehrle et al. 1998).
3. Discussion

The central black hole plays an important role in the observational properties of AGNs and has drawn much attention. It may also shed some light on the evolution (Wang et al. 2001; Barth et al. 2002; Cao 2002). There are several methods for black hole mass determinations although consensus has not been reached. In the present work, we proposed a new method to estimate the central black hole mass. It is constrained by the optical depth of the $\gamma - \gamma$ pair production and can be used if X-ray and $\gamma$-ray emissions and short time-scale are known. This method can be used to determine the central black hole mass of high redshift gamma-ray sources. It is an approximate empirical method, which is obtained from the data/figures of Becker & Kafatos (1995). The mass determined in the present paper corresponds to an optical depth of unity and therefore the results correspond to the upper limit of the central black hole mass. The main difference between our consideration and others is that we assumed that the $\gamma$-rays originate from a cone while others think that the $\gamma$-rays are isotropic. However, our results are consistent with those obtained by others as discussed in the following.

In our consideration, the estimated mass upper limits for a sample presented here are in the range of $10^7 M_\odot$ to $10^9 M_\odot$, $(0.57 \sim 60) \times 10^7 M_\odot$ for $\lambda = 1.0$ and $(0.87 \sim 95.0) \times 10^7 M_\odot$ for $\lambda = 0.1$. The real value of $\lambda$ will cause an uncertainty in the mass, but the uncertainty caused by $\lambda$ will be negligible. When $\lambda$ decreases by a factor of 10, the mass increases by a factor of $\sim 1.5$. The results obtained with the present method are independent of the $\gamma$-ray emission mechanism although it will depend the X-ray emission mechanism.

For illustration, we will compare the mass estimation for two sources, 3C 279 and Mkn 501. Several groups of authors have estimated their masses.

3C 279 This quasar displayed two outbursts, one in 1991 and another in 1996. The upper limit of the central black hole masses obtained from these outbursts are similar. For $\lambda = 1.0$ the masses were $M = 6.57 \times 10^7 M_\odot$ and $M = 5.25 \times 10^7 M_\odot$ whereas for $\lambda = 0.1$, $M = 10.2 \times 10^7 M_\odot$ and $M = 8.12 \times 10^7 M_\odot$ for the 1991 and 1996 outbursts respectively. To fit the 3C 279 multiwavelength energy spectrum corresponding to the 1991 $\gamma$-ray flare, Hartman et al. (1996) used an accreting black hole of $10^9 M_\odot$; our result of $(6.57 - 10.2) \times 10^7 M_\odot$ is consistent with their.

Mkn 501 TeV and X-ray emission result shows that there is a possible period of 23 days in the light curves of Mkn 501 (Hayashida et al. 1998; Kranich et al. 1999), which may suggest binary black holes at center (Rieger & Mannheim 2000, 2003; Villata & Raiteri 1999). The black hole mass has been determined by many authors ($2 - 3.4) \times 10^9 M_\odot$ (Barth et al. 2002; Kormendy & Gebhardt 2001; Merritt & Ferrarese 2001; Rieger & Mannheim 2003). Our present result shows that the central black hole mass upper limit is $(4.01 - 6.27) \times 10^7 M_\odot$. This result is lower than those claimed by the authors mentioned above, but this difference is probably from the facts that (1) the methods used to estimate the mass by different authors are based on different assumptions, (2) there is a binary black hole system at the center of the BL Lacertae objects, our result corresponds to the less massive black hole. A binary black hole system is successfully used in the explanation of the periodic variability of the BL Lacertae object OJ 287 by Sillanpaa et al. (1988), who claimed that the ratio of the secondary black hole mass to the primary black hole mass is 0.004. Recently, Komossa et al. (2003) reported that there is a binary black hole system in NGC 6240. If there is a binary black hole system at the center of Mkn 501, and the more massive black hole of $\sim 10^9 M_\odot$ corresponds to the primary black hole and the less massive black hole of $10^7 M_\odot$ corresponds to the secondary, then the ratio is 0.01, which is consistent with the result for OJ 287 by Sillanpaa et al. (1988). In addition, our result is consistent with that of De Jager et al. (1999), $(1 - 6) \times 10^7 M_\odot$.

BLs and FSRQs are two subclasses of blazars. From the observational point of view, except for the emission line properties (the emission line strength in FSRQs is strong while that in BL is weak or invisible), other observational properties are very similar between them (Fan 2002). Their relationship has drawn much attention (e.g. Sambruna et al. 1996; Scarpa & Falomo, 1997; Ghisellini et al. 1998; D’Elia & Cavaliere 2000; Bottcher & Dermer 2002; Fan 2002; Ciaramella et al. 2004). Bottcher & Dermer (2002) proposed that the accretion rate rather than the central black hole masses play an important role in the evolutionary sequence (from FSRQs evolving to BLs) of blazars. The accretion rate in FSRQs is much larger than in BLs, there is not much gas in BLs to fuel the central black hole. In the present paper, if we consider BLs and FSRQs separately, the distribution of the mass upper limits is not very much different (see Fig. 3), their average masses are $\log M = 8.06 \pm 0.54$ for FSRQs, and $\log M = 8.13 \pm 0.46$ for BLs. There is no difference in black hole mass between BLs and FSRQs. Recently, Wu et al. (2002) also found that there is no mass difference between different subclasses of AGNs. This result suggests that the central black hole mass play a less important role in the evolutionary sequence as pointed out by Bottcher & Dermer (2002). To investigate the evolutionary process further, one should take into account the black-hole spin and the accretion processes since the former effect is important for non-thermal radio emission while the latter will change the properties of the outflow.

The high luminosity, rapid variability and superluminal motion observed in some $\gamma$-ray-loud blazars suggest that the $\gamma$-ray emission is strongly beamed, therefore, one can assume that the beamed emissions arise from a certain solid angle. In the present paper, our calculation show that the propagation angle $\Phi$ and Doppler factor $\delta$ are in the range of $9.68$ to $61.14$ and $0.12$ to $3.31$ for $\lambda = 1.0$. For $\lambda = 0.1$ the value of $\Phi$ ranges from $8.91$ to $56.49$ and $\delta$ 0.16 to 4.6. In the isotropic emission case, the $\gamma$-rays can be detected at any angle, but in the case of non-isotropic
emission, the emission is produced in the cone of a solid angle of $\Omega$, then $\gamma$-rays will not be detected at any angle. From the observational properties such as high luminosity, rapid $\gamma$-ray variability and superluminal motion, boosting effects should be present in the $\gamma$-ray sources. So, the optical depth should depend on the angle $\Phi$, the boosting factor $\delta$, the central black hole masses and the distance along the axis to the site of $\gamma$-ray production, $d$, which can be expressed as $d(\Phi, M, L_{iso}) = AR_g(1 - \cos \Phi)^{\frac{1}{3}}$. If the angle $\Phi$ is too small, then $d$ is small, therefore, the site of the $\gamma$-ray production is near the center where the X-ray photon density will be high. So, the optical depth is large. In this sense, the smaller the angle $\Phi$, the smaller the $d$, and the higher the X-ray photon density, which results in a higher optical depth. On the other hand, when the angle $\Phi$ is too large, the boosting factor $\delta$ is very small, so that the optical depth is also large. Therefore, there are angles that correspond to a small optical depth. From Fig. 2, one can see that the minimum value depends on the central black hole mass, so one can choose a mass so that the minimum value of the optical depth is 1.0. The mass estimated in this way is an upper limit. From our calculation, we found that Dopper factor $\delta < 1$ in some cases; the lower than unity Dopper factors do not conflict with the beaming argument since we proposed that the emission is not isotropic in the present work. Other authors assumed that the emission is isotropic. The different assumptions will result in a $\frac{1}{3}(1 - \cos \Phi)$ times difference in the Doppler factor. For the $\gamma$-ray emission regions, the obtained results indicate that they are in the range of $17.2R_g$ to $713R_g (\lambda = 1.0)$ or $18.8R_g$ to $640R_g (\lambda = 0.1)$.

In this paper, the optical depth of a $\gamma$-ray travelling in the field of a two-temperature disk and beaming effects have been used to determine the central mass, $M$, for 23 $\gamma$-ray-loud blazars with available short time-scales. The masses obtained in the present paper are in the range of $10^7M_\odot$ to $10^9M_\odot$ for the whole sample. In the case of black hole mass, there is no clear difference between BLs and FSRQs, which suggests that the central black hole masses do not play an important role in the evolutionary sequence of blazars.

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Table 1. Black hole mass for 23 $\gamma$-ray-loud blazars

| Name     | $z$ | $ID$ | $f_{K\alpha}$ | Ref | $\alpha_X$ | Ref | $\alpha_\gamma$ | $\Delta T_D$ | $L_{iso}^\gamma$ | $M_1$ | $M_{0.1}$ |
|----------|-----|------|---------------|-----|------------|-----|-----------------|--------------|-----------------|-------|-----------|
| 0208-512 | 1.003 | Q 0.61 | C97 1.04 | C97 9.1 | 0.69 | 134.4 | S96 2.0 | 60.9 | 95.0 |
| 0219+428 | 0.444 | B 1.56 | Fe98 1.6 | Fe98 0.25 | 1.01 | 30.0 | DG 0.018 | 19.73 | 29.80 |
| 0235+164 | 0.94 | B 2.5 | M96 1.01 | M96 0.65 | 1.85 | 72 | M96 2.0 | 36.75 | 53.69 |
| 0420-014 | 0.915 | Q 1.08 | Fe98 0.67 | C97 0.64 | 1.44 | 33.6 | DG 0.175 | 10.98 | 16.43 |
| 0458-020 | 2.286 | Q 0.1 | DG 0.67 | C97 0.68 | 1.45 | 144.0 | DG 1.424 | 31.48 | 47.2 |
| 0521-365 | 0.055 | B 1.78 | DG 0.68 | DG 0.32 | 1.63 | 72 | DG 0.0002 | 31.1 | 46.44 |
| 0528+134 | 2.07 | Q 0.65 | C97 0.54 | C97 3.08 | 1.21 | 24 | DG 18.4 | 4.52 | 6.97 |
| 0537-441 | 0.894 | Q 0.81 | C97 1.16 | C97 2.0 | 1.0 | 16. | H96 3.01 | 10.46 | 15.96 |
| 0716+714 | 0.3 | B 1.35 | Fe98 1.77 | Fe98 0.46 | 1.19 | 1.92 | DG 0.013 | 2.22 | 3.28 |
| 0735+178 | 0.424 | B 0.248 | Fe98 1.34 | Fe98 0.30 | 1.6 | 28.8 | DG 0.013 | 20.03 | 29.37 |
| 0829+046 | 0.18 | Q 1.07 | DG 0.67 | C97 0.34 | 1.47 | 24. | DG 0.0013 | 12.14 | 18.2 |
| 0836+710 | 2.17 | Q 0.819 | Fe98 0.42 | Fe98 0.33 | 1.62 | 24. | DG 0.548 | 1.67 | 2.53 |
| 1101+384 | 0.031 | B 37.33 | Fo98 2.10 | Fe98 0.27 | 0.57 | 1.92 | DG 0.0001 | 8.9 | 13.27 |
| 1226+023 | 0.158 | Q 12.07 | Fe98 0.81 | Fe98 0.09 | 1.58 | 24 | C88 0.003 | 8.9 | 13.27 |
| 1253-055 | 0.537 | B 2.43 | H96b 0.68 | H96b 2.8 | 1.02 | 12. | K93 1.34 | 6.57 | 10.2 |
| 1253-055 | 0.538 | B 2.0 | L98 0.78 | L98 1.1 | 0.97 | 6. | W98 5.75 | 5.25 | 8.12 |
| 1510-089 | 0.361 | Q 0.718 | Fe98 0.90 | Fe98 0.49 | 1.47 | 57.6 | DG 0.018 | 28.03 | 41.9 |
| 1622-297 | 0.815 | Q 0.08 | M97 0.67 | C97 1.7 | 0.87 | 4.85 | M97 26.9 | 5.44 | 8.51 |
| 1633+382 | 1.814 | Q 0.42 | C97 0.53 | C97 0.96 | 0.86 | 16. | M93 9.72 | 3.43 | 5.42 |
| 1652+399 | 0.033 | B 10.1 | C97 1.60 | C97 0.32 | 0.68 | 6. | Q96 0.0003 | 4.01 | 6.27 |
| 2155-304 | 0.117 | B 0.058 | U97 1.25 | U97 0.34 | 0.56 | 3.3 | C99 0.003 | 3.26 | 5.11 |
| 2200+420 | 0.07 | B 1.84 | P96 1.31 | P96 1.71 | 0.68 | 3.2 | B97 0.019 | 11.0 | 15.95 |
| 2230+114 | 1.04 | Q 0.486 | Fe98 0.67 | C97 0.51 | 1.45 | 48 | P88 0.184 | 15.33 | 22.95 |
| 2251+158 | 0.859 | Q 1.08 | Fe98 0.62 | Fe98 1.16 | 1.21 | 1.92 | DG 0.32 | 0.57 | 0.87 |
Fig. 3. Histogram of black hole mass for BLs (dotted lines) and FSRQs (solid lines) for the case of $\lambda = 0.1$. 

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