Bayesian acoustic impedance inversion with gamma distribution

Hao Wu¹², Shu Li¹², Yingpin Chen¹², Zhenming Peng¹²*

¹School of Information and Communication Engineering, University of Electronic Science and Technology 5 of China, Chengdu 610054, China
²Center for Information Geoscience, University of Electronic Science and Technology of China, Chengdu 611731, China
zmpeng@uestc.edu.cn

Abstract. Bayesian model is popular in seismic acoustic impedance inversion because of the statistical information from Bayesian model. However, there are many distributions to simulate impedance distribution. Different distribution has different result. In this paper, the Gamma distribution is used as the prior distribution. A Bayesian impedance inversion method is proposed, under the assumption that the prior distribution is a Gamma distribution. Through the test from the field and model data, this method is feasible and this model based on Gamma distribution has a result with tall resolution.

1. Introduction
Seismic acoustic impedance (AI) inversion is an optimization process of estimating the AI from the observed seismic data[1]. However, the observed seismic data has noise that lead to that multiple impedance model can match the observed seismic. So AI inverse problem is an ill-posed optimization problem. Nevertheless, because of the close relationship with lithology and porosity, acoustic impedance is an important geophysical property[2]. In the past decades, many methods of AI inversion have been proposed, such as the band-limited inversion[3], the linear inversion[4, 5], the sparse inversion[6-8]. However, the resolution of the inverse result is low.

With the popularization of Bayesian model, it provides a theoretical basis for improving the resolution in seismic impedance inversion because of the statistical information from Bayesian.[9] Because of the convenience on how to solve the inversion problem, Gaussian model is used to simulate distribution[10, 11]. However, Gaussian model is not the best choice to simulate distribution. In the past decades, there are many distributions used to simulate distribution. They can be divided into two categories. The first categories are the distributions based on Gaussian model, such as, the alpha-stable distribution[12-14], the Gaussian mixture distribution[15-17]. The others are the distributions that are not related to Gaussian model, such as, Cauchy distribution[18].

In this paper, with the research of impedance distribution, the Gamma distribution can simulate impedance distribution well. Then, a Bayesian AI inversion is proposed under the assumption that the prior distribution is a Gamma distribution. Finally, through the test from the field and model data, this method is feasible and this model based on Gamma distribution has a result with tall resolution.
2. Method

2.1. Acoustic impedance forward model based on Bayesian

In order to get accurate impedance, a maximum likelihood estimation model based on Bayesian is used to establish the forward model.

$$\text{argmax}(p(\text{AI} | S))$$

where $\text{AI}$ is the acoustic impedance, and $S$ is the seismic. The likelihood estimation model can not be solved directly. The full probability formula is applied to solve this problem.

$$p(\text{AI} | S) \propto p(S | \text{AI}) p(\text{AI})$$

where $p(S | \text{AI})$ is posterior probability density function. $p(\text{AI})$ is prior probability density function. In order to compute $p(S | \text{AI})$, the relation between $S$ and $\text{AI}$ is applied.

$$S = \omega \otimes r + N$$

where $\omega$ is wavelet and $r$ is reflectivity, and $N$ is noise. What’s more, the reflectivity can be applied by impedance.

$$r_i \approx \frac{1}{2} (\ln \text{AI}_{i+1} - \ln \text{AI}_i)$$

where $r_i$ is the element of $r$, $\text{AI}_i$ is the element of $\text{AI}$.

For ease of calculation, combining (3) and (4) by matrix.

$$S = \frac{1}{2} WDL + N$$

where $W$ is the matrix of $\omega$, $D$ is a matrix to compute the error of $L$. Suppose $n$ is the length of $\text{AI}$, $m$ is the length of $\omega$.

$$W = \begin{pmatrix} \omega_1 & 0 & \cdots & 0 \\ \omega_2 & \omega_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \omega_m & \cdots & \cdots & \omega_1 \\ 0 & \omega_m & \cdots & \omega_2 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \omega_m \end{pmatrix}^{(n + m - 1) \times (n - 1)}$$

$$D = \begin{pmatrix} 1 & -1 & 0 & \cdots & 0 \\ 0 & 1 & -1 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 1 & -1 \end{pmatrix}^{(n - 1) \times n}$$

where $L$ is exponent of $\text{AI}$. So $\text{AI}$ and $L$ are corresponding one by one. $L$ replaces $\text{AI}$ in the following article.

From (5), the distribution of $p(S | \text{AI})$ is decided by noise. Gaussian model is usually applied to model noise. So Gaussian model is applied to model $p(S | L)$.

$$p(S | L) = \frac{1}{(\sqrt{2\pi\delta_n})^{n-1}} \exp(-\frac{1}{2\delta_n^2} \left\| S - \frac{1}{2} WBL \right\|^2_2)$$

2.2. The prior probability density function based on Gamma distribution

By analyzing the distribution of the acoustic impedance, the Gamma distribution can model the $p(L)$ well (Fig. 1). The blue histogram is the distribution of $L$ from well. The red line is the Gamma distribution.
Fig. 1 Match between Gamma distribution and the histogram of L from well.

So the Gamma distribution is used to model $p(L)$ in this article.

$$p(L) = \left( \frac{\lambda^\alpha L^{\alpha-1}}{\Gamma(\alpha)} \right) \exp(-\lambda L)$$  \hspace{1cm} (7)

where $\alpha, \lambda$ are constants.

Combining (6) and (7), $p(L|S)$ can be given by

$$p(L|S) = \frac{\lambda^\alpha}{\Gamma(\alpha)} \frac{1}{(2\pi \delta_n)^{n-1}} L^{\alpha-1} \exp \left[ -\left( \frac{1}{2} \frac{S-1/2}{WBL} + \lambda L \right) / 2\delta^2 \right]$$  \hspace{1cm} (8)

The Monte Carlo Markov Chains method is used to compute the likelihood estimation model $p(L|S)$. The iteration cycle is displayed in Algorithm 1.

---

**Algorithm 1: The Monte Carlo Markov Chains method**

1. **Input**: delta, tol, S, trace, $\alpha, \lambda$
2. **Initialize**: $t=1, i=1$
3. While $i < \text{trace}$
   4. $s = S(i)$
   5. While $\|L^{t+1} - L^t\|_2 > \text{tol}$
   6. $p(L'|S) = \lambda^\alpha L^{\alpha-1} \exp \left[ -\left( \frac{1}{2} \frac{S-1/2}{WBL} + \lambda L' \right) / 2\delta^2 \right] / \Gamma(\alpha)(2\pi \delta_n)^{n-1}$
   7. $L^{\text{new}} = L' + \delta \cdot \Delta L, \Delta L \sim U(0,1)$
   8. $p(L^{\text{new}}|S) = \lambda^\alpha L^{\alpha-1} \exp \left[ -\left( \frac{1}{2} \frac{S-1/2}{WBL^{\text{new}}} + \lambda L^{\text{new}} \right) / 2\delta^2 \right] / \Gamma(\alpha)(2\pi \delta_n)^{n-1}$
   9. $a(L', L^{\text{new}}) = \min \left\{ \frac{p(L^{\text{new}}|S)}{p(L'|S)}, 1 \right\}$
   10. $u \sim U(0,1)$
   11. $L^{t+1} = \begin{cases} L' & \text{if } u > a(L', L^{\text{new}}) \\ L^{\text{new}} & \text{if } u \leq a(L', L^{\text{new}}) \end{cases}$
12. End
13. $AI(i) = \exp(L^{t+1})$
14. End

---
3. Examples

3.1. Marmous2 model

To test the method, Marmous2 model is applied. Fig. 2 (a) is a part of true impedance model from Marmous2 model that combines 2951 trace, and each trace has 301 points. Fig. 2 (b) is seismic synthetic seismogram based on true impedance model. It is computed by the impedance model and a Ricker wavelet of dominant frequency 40 Hz. Fig. 2 (c) is initial impedance model that is obtained by filtering the impedance model using a Gaussian low-pass filter of size 51 with standard deviation $1 \times 10^3$. Fig. 2 (d) is inversion result of impedance. Comparing Fig. 2 (c) and Fig. 2 (d), the method in this article can get a clear horizon information.

![Fig. 2 The Marmous2 model](image)

In order to observe result clearly, the result of trace 1000 (Fig. 3 (a)), and the result of depth 100 (Fig. 3 (b)) are applied. The red line, blue line, and green line represents the inversion result, the true impedance model, the initial impedance model. From these two figures, the horizon from result is closer to the horizon from true impedance model. However, the inversion result is not smooth. This problem is due to the random of MCMC method and the smooth model. A field data is applied to test the method.
3.2. Field data
The field data comes from the northeastern Sichuan Province, China that consists 512 trace. The sampling interval is 1ms (Fig. 4 (a)). Fig. 4 (b) is the initial model which resulting from Kriging interpolation of well log under the guidance of horizons, and followed by Gaussian low-pass filtering. The wavelet is extracted by using the Roy-White wavelet extraction method. Through method in this article, the inversion result (Fig. 4 (c)) and the seismic synthetic seismogram based on inversion result (Fig. 4 (d)) are given. From Fig. 4 (a) and Fig. 4 (d), there are more details in the inversion result.
4. Conclusions
In this paper, the Gamma distribution is used as the prior distribution. Then, the Bayesian AI inversion method under the assumptions that the prior distribution is a Gamma distribution is proposed. Finally, through the test from the field and model data, this method is feasible and this model based on Gamma distribution has a result with tall resolution.

Acknowledgment
This work is supported by the National Natural Science Foundation of China [grant numbers 61775030, 61571096, 41274127]

Bibliography
[1] Li Shu, Peng Zhenming. Seismic acoustic impedance inversion with multi-parameter regularization [J]. Journal of Geophysics & Engineering, 2017, 14(3): 520-532.
[2] Russell, Brian, Domenico. Introduction to Seismic Inversion Methods [M]. Society of Exploration Geophysicists, 1988.
[3] Bickel. Resolution performance of Wiener filters [J]. Geophysics, 1983, 48(7): 887-899.
[4] Berkhout. Least-squares inverse filtering and wavelet deconvolution [J]. Geophysics, 1977, 42(7): 1369-1383.
[5] Yuan Sanyi, Wang Shangxu, Luo Chunmei, He Yanxiao. Simultaneous mutitrace impedance inversion with transform-domain sparsity promotion [J]. Geophysics, 2015, 80(2): 71-80.
[6] Kong Dehui, Peng Zhenming. Seismic random noise attenuation using shearlet and total generalized variation [J]. Journal of Geophysics & Engineering, 2015, 12(6): 1024-1035.
[7] Li Shu, He Yanmin, Chen Yingpin, Liu Wei, Yang Xi, Peng Zhenming. Fast multi-trace impedance inversion using anisotropic total p-variation regularization in the frequency domain [J]. Journal of Geophysics and Engineering, 2018, 15(5): 2171-2182.
[8] Li Shu, Chen Yingpin, Wu Hao, Peng Zhenming, Wu Ru-Shan. Seismic acoustic impedance inversion using total variation with overlapping group sparsity [M]. SEG Technical Program Expanded Abstracts 2018. Society of Exploration Geophysicists. 2018: 411-415.
[9] Bosch Miguel, Mukerji Tapan, Gonzalez Ezequiel F. Seismic inversion for reservoir properties combining statistical rock physics and geostatistics: A review [J]. Geophysics, 2010, 75(5): 165-176.
[10] Buland, Omre. Bayesian linearized AVO inversion [J]. Geophysics, 2003, 68(1): 185-198.
[11] Grana Dario. Bayesian linearized rock-physics inversion [J]. Geophysics, 2016, 81(6): 625-641.
[12] Yue Bibo, Peng Zhenming, Zhang Fanchang. Seismic inversion method with alpha-stable distribution [J]. Chinese Journal of Geophysics-Chinese Edition, 2012, 55(4): 1307-1317.
[13] Yue Bibo, Peng Zhenming. A validation study of α - stable distribution characteristic for seismic
[14] Yue Bibo, Peng Zhenming. Acoustic impedance inversion with covariation approach [J]. Journal of Inverse and Ill-posed Problems, 2014, 22(5): 609-623.

[15] Grana Dario, Fjeldstad Torstein, Omre Henning. Bayesian Gaussian mixture linear inversion for geophysical inverse problems [J]. Mathematical Geosciences, 2017, 49(4): 493-515.

[16] de Figueiredo Leandro Passos, Grana Dario, Santos Marcio, Figueiredo Wagner, Roisenberg Mauro, Neto Guenther Schwedersky. Bayesian seismic inversion based on rock-physics prior modeling for the joint estimation of acoustic impedance, porosity and lithofacies [J]. Journal of Computational Physics, 2017, 336(128-142).

[17] Vestergaard Peter D., Mosegaard Klaus. Inversion of post-stack seismic data using simulated annealing [J]. Geophysical Prospecting, 2010, 39(5): 613-624.

[18] Liu Chengming, Wang Deli, Wang Tong, Feng Fei, Wang Yonggang. Multichannel sparse deconvolution of seismic data with shearlet-Cauchy constrained inversion [J]. Journal of Geophysics & Engineering, 2017, 14(5): 1275.