Multiple Scattering Between Adjacent Sound Sources in Head-Related Transfer Function Measurement System

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Abstract: To accelerate head-related transfer functions (HRTFs) measurement, two or more independent sound sources are usually employed in the measurement system. However, the multiple scattering between adjacent sound sources may influence the accuracy of measurement. On the other hand, the directivity of sound source could induce measurement error. Therefore, a model consisting of two spherical sound sources with approximate omni-directivity and a rigid-spherical head is proposed to evaluate the errors in HRTF measurement caused by multiple scattering between sources. An example of analysis using multipole re-expansion indicates that the error of ipsilateral HRTFs are within the bound of ±1.0 dB below a frequency of 20 kHz, provided that the sound source radius does not exceed 0.025 m, the source distance relative to head center is not less than 0.5 m, and the angular interval between two adjacent sources is not less than 20 degrees. Similar conclusions under different conditions can also be analyzed and discussed by using this calculation method. Furthermore, the results are verified by measurements of HRTFs for a rigid sphere and a KEMAR artificial head.

Keywords: Head-related transfer function; near field; measurement error; multiple scattering

1 Introduction

Head-related transfer functions (HRTFs) are acoustical transfer functions from a point sound source to two ears in the free field and play an important role in binaural hearing researches and virtual auditory display (VAD). Generally, HRTFs vary with frequency and source position (including azimuth, elevation and distance), and also depend on individual anatomical parameters [1]. When the sound source distance relative to the head center is larger than 1.0 m, HRTFs are approximately independent of source distance and called far-field HRTFs. At a source distance less than 1.0 m, however, HRTFs are relevant to the source distance and called near-field HRTFs [2,3].

Experimental measurement is a common way to acquire individual HRTFs. A conventional measurement system is time-consuming, which usually contains one sound source and the relative position between the sound source and subject is changeable. Alternatively, some systems with two or more sound sources have been employed to accelerate the HRTF measurement [4,5]. However, practical sound sources are not strict point sources, which causes the multiple scattering between the sources or among the sources and head, and thus may result in errors in the measured HRTFs.

In our previous work, the multiple scattering between a single sound source and head in near-field HRTF measurement was evaluated by using a spherical pulsating sound source and a spherical head model [6]. It was suggested that a spherical dodecahedron sound source with radius not more than 0.035 m can be used in the near-field HRTF measurement [7]. In this study, the multiple scattering among two
sound sources and head in HRTF measurement is further analyzed by using the method of multipole reexpansion [8].

2 Calculation Model

In a head-centered coordinate system, the position of a point sound source is specified by vector \( \mathbf{r}_S \) or spherical coordinate \((r_S, \alpha_S, \beta_S)\), where \( r_S \) is the sound source distance relative to head center, \( 0^\circ \leq \alpha_S \leq 180^\circ \) and \( 0^\circ \leq \beta_S < 360^\circ \) are the elevation and azimuth, respectively, with \( \alpha_S = 0^\circ, 90^\circ, 180^\circ \) being the above, horizontal and down directions, respectively. In the horizontal plane, \( \beta_S = 0^\circ, 90^\circ, 180^\circ \) are front, right and back directions, respectively. Similarly, the position of arbitrary field point is specified by vector \( \mathbf{r} \) or spherical coordinate \((r, \alpha, \beta)\). Then, HRTFs are defined as the ratio between the pressure \( P(\mathbf{r}_{\text{ear}}, \mathbf{r}_S, f) \) at the left or right ear and the free-field pressure \( P_{\text{free}}(\mathbf{r}_S, f) \) at the position of head center with the head absent.

\[
H(\mathbf{r}_{\text{ear}}, \mathbf{r}_S, f) = \frac{P(\mathbf{r}_{\text{ear}}, \mathbf{r}_S, f)}{P_{\text{free}}(\mathbf{r}_S, f)},
\]

where \( f \) denotes the frequency and \( \mathbf{r}_{\text{ear}} \) denotes the ear position \( \mathbf{r}_L \) or \( \mathbf{r}_R \). Due to the symmetry of the head, only the right-ear HRTF \( H(\mathbf{r}_R, \mathbf{r}_S, f) \) is analyzed in this paper.

Fig. 1 shows the physical model of HRTF measurement system with two independent sound sources. The model consists of a rigid-spherical head (its center locates at point \( O_{\text{H}} \), and its radius is denoted as \( a_{\text{H}} \)) and two spherical sound sources (their centers locate at points \( O_S \) and \( O_T \), and their radii are denoted as \( a_S \) and \( a_T \), respectively). In practice, the sound sources are driven one by one in the multi-sources measurement system. As a representative case, only the sphere whose center locates at point \( O_S \) is an active source which is approximated as a spherical pulsating sound source, and the sphere whose center locates at point \( O_T \) serves as a scattering object. Accordingly, in the head-centered coordinate system, the position of \( O_S \) is specified by a vector \( \mathbf{r}_S \) or a spherical coordinate of \((r_S, \alpha_S, \beta_S)\), and the position of \( O_T \) is specified by a vector \( \mathbf{r}_T \) or a spherical coordinate of \((r_T, \alpha_T, \beta_T)\).

Figure 1: The calculation model and the spatial coordinate system

For the above model, pressure \( P(\mathbf{r}, \mathbf{r}_S, \mathbf{r}_T, f) \) satisfies the Helmholtz equation:

\[
\nabla^2 P(\mathbf{r}, \mathbf{r}_S, \mathbf{r}_T, f) + k^2 P(\mathbf{r}, \mathbf{r}_S, \mathbf{r}_T, f) = 0,
\]
where \( k \) is the wave number and \( \nabla^2 \) is Laplacian operation with respect to the field point \( r \). In the infinite distance, the Sommerfeld radiation condition requires,

\[
\lim_{r \to \infty} \left[ \frac{\partial P(r, r_S, r_T, f)}{\partial r} + jkP(r, r_S, r_T, f) \right] = 0 ,
\]

where \( j \) is the imaginary unit. Assume that the surface \( \Omega_H \) of spherical head and the surface \( \Omega_T \) of scattering object are rigid, and the normal velocity on the surface \( \Omega_S \) of the pulsating sound source is \( u_S \). Then the pressure satisfies the following boundary conditions,

\[
\frac{\partial P(r, r_S, r_T, f)}{\partial n} \bigg|_{\Omega_H} = \frac{\partial P(r, r_S, r_T, f)}{\partial n} \bigg|_{\Omega_T} = 0 ; \quad \frac{\partial P(r, r_S, r_T, f)}{\partial n} \bigg|_{\Omega_S} = -2\pi j \rho u_S ,
\]

where the symbol \( \partial/\partial n \) expresses the derivative with respect to outward normal direction, and the symbol \( \rho \) denotes the density of the air.

Generally, the overall sound pressure \( P(r, r_S, r_T, f) \) consists of four parts, the free-field pressure \( P_{rad} \) radiated by the pulsating sound source, the scattered pressures \( P_H, P_S \) and \( P_T \) caused by the spherical head, sound source and scattering object, respectively. This leads to

\[
P(r, r_S, r_T, f) = P_{rad} + P_H + P_S + P_T .
\]

The pressure \( P_{rad} \) can be written as,

\[
P_{rad} = j \frac{k \rho \chi Q_0}{4\pi |r - r_S|} e^{-jkr_S} ,
\]

where \( Q_0 \) is the source intensity. Within the region of \( r < r_S \), Eq. (6) can be further expanded as

\[
P_{rad} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} C_{lm}^{II}(k) j_l(kr_S) Y_{lm}(\alpha, \beta) ,
\]

where \( j_l(kr) \) and \( h_l(kr) \) are the spherical Bessel function and the spherical Hankel function of the second kind, respectively; \( Y_{lm}(\alpha, \beta) \) are the complex-value spherical harmonics functions; superscript "*" denotes complex-conjugate.

The scattering pressure caused by the spherical head can also be expanded as,

\[
P_H = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} A_{lm}^{II}(k) h_l(kr_S) Y_{lm}(\alpha, \beta) ,
\]

where \( A_{lm}^{II}(k) \) are a set of coefficients to be determined.

To calculate the scattered pressures \( P_S \) and \( P_T \), two new coordinate systems are specified. One is centered at the sound source \( \Omega_S \), and the position of arbitrary field point is denoted by \((r', \alpha', \beta')\), the other is centered at the scattering object \( \Omega_T \), and the position of arbitrary field point is denoted by \((r'', \alpha'', \beta'')\). The axes of two new coordinate systems are parallel to the corresponding axes in the head-centered coordinate. In these new coordinate systems, the scattered pressures \( P_S \) and \( P_T \) can be expanded as

\[
P_S = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} A_{lm}^{II}(k) h_l(kr') Y_{lm}(\alpha', \beta') ,
\]

\[
P_T = \sum_{l'=0}^{\infty} \sum_{m'=-l'}^{l'} A_{lm'}^{II}(k) h_{l'}(kr'') Y_{lm'}(\alpha'', \beta'') ,
\]

respectively, where \( A_{lm}^{II}(k) \) and \( A_{lm'}^{II}(k) \) are two set of coefficients to be determined.

To apply the boundary condition on the surface \( \Omega_H \) of the spherical head, the pressures \( P_S \) and \( P_T \) should be represented in terms of head-centered coordinate by using following transforming equations
The error in contralateral HRTF magnitude at high frequency is tolerable. In contrast, the errors in the other azimuths are small and prominent errors mainly appear at the source azimuths which are contralateral to the concerned ear (within 225° ≤ βS < 315° for the right ear). The SD reaches up to about 10.0 dB at high frequencies above about 7 kHz. However, because the high-frequency HRTF magnitude for contralateral source directions is naturally small due to the shadow of head, its contribution to auditory perception is relatively minor. The error in contralateral HRTF magnitude at high frequency is tolerable. In contrast, the errors in the other azimuths are small and SD varies within the bound of about ±1.0 dB in the whole audible frequency band, which satisfies the requirement of HRTF measurement [1].
Accordingly the error at an arbitrary ipsilateral azimuth, for instance the horizontal azimuth of $\beta_S = 90^\circ$, can be seen as the representative one. Therefore, in the following discussions, only the HRTF errors at azimuth $\beta_S = 90^\circ$ are calculated and analyzed. Under the conditions of ($\beta_S = 90^\circ$ and $a_H = 0.0875$ m), the errors depend on the spherical source radius ($a_S = a_T$), the source distance ($r_S = r_T$), and the angle interval $\Delta \alpha$ between the two sources.

At the source distance of 0.5 m, the HRTF errors as defined in Eq. (13) with source elevation intervals of $15^\circ$, $20^\circ$ and $25^\circ$ are shown in Fig 3. Results show that the errors increase as the source elevation interval decreases. For the elevation interval of $15^\circ$, the errors reach up to about $\pm 1.5$ dB at several frequencies. In the multi-sources HRTF measurement system, if the acceptable limit of error is $\pm 1.0$ dB, the elevation interval should not less than $20^\circ$ at the distance of 0.5 m.

In the case of the elevation interval of $20^\circ$, the HRTF errors at source distances of 0.2 m, 0.5 m, and 1.0 m are shown in Fig. 4. Results indicate that the errors increase as the source distance decreases and reach up to $\pm 2.5$ dB at source distance 0.2 m. On the other hand, the errors at 1.0 m are only within the bound of $\pm 0.5$ dB. Therefore, for the sources in far field (source distance not less than 1.0 m), the permitted elevation interval could be smaller and the allowable sizes of sound sources could be larger.

As shown in Fig. 5, at source distance of 1.0 m and under the error criterion of $\pm 1.0$ dB, the elevation interval can be decreased to $10^\circ$ and the source radius can be enlarged to 0.035 m. However, the errors further increase as the spherical source radius increase. For the source radius 0.05 m and the elevation interval $10^\circ$, the HRTF errors reach up to about $\pm 2.0$ dB, which is unacceptable in HRTF measurement. Above results yield the reasonable conditions of multi-sources measurement system for near-field and far-field HRTFs.

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**Figure 2:** (a) The conditions of a calculation example; (b) the errors of the calculated right ear HRTFs at various azimuths
Figure 3: The HRTF errors for the elevation intervals of 15°, 20° and 25°, the source distance of 0.5 m, the spherical source radius of 0.025 m, and the sound source azimuth of $\beta_s = 90°$.

Figure 4: For the spherical source radius of 0.025 m, the elevation interval of 20°, and the sound source azimuth of $\beta_s = 90°$, the errors at source distances ($r_s = r_t$) of 0.2 m, 0.5 m and 1.0 m.

4 Validation via Measurement

In order to validate the results, we design two spherical dodecahedron sound sources with radius of about 0.025 m and a woody spherical head model with radius of about 0.0875 m, and then the HRTFs of the head model are measured by using two sound sources and denoted by $\hat{H}(r_{\text{ear}}, r_s, r_t, f)$. Fig. 6(a) shows the configuration of head model and sound sources, which is identical that shown in Fig. 2(a). Removing the spherical source $O_T$, the measured HRTFs can be deemed as the accurate ones and denoted by $H(r_{\text{ear}}, r_s, f)$. Fig. 6(b) shows the errors of the measured HRTFs defined by Eq. (13).

In comparison with the calculated results in Fig. 2(b), some discrepancies are observed in Fig. 6(b), but only at high frequencies and the contralateral azimuths. It is interesting to observe that some prominent errors predicted by calculation disappear in the measured results. Actually, the interferences of sound wave from various paths around a perfect rigid-spherical head make the contralateral HRTFs very sensitive to the small disturbance. A slight multiple scattering from sound sources may cause a large change in the contralateral HRTF magnitudes. The woody spherical-head model is not as perfect as the theoretical one, which weakens the influence of interferences of sound wave at the contralateral ear and makes the corresponding HRTFs less sensitive to the small disturbance. For the human subjects, the head...
shape is more irregular than a spherical head, which could further decrease the contralateral errors in HRTF measurement. In other words, the error in practical HRTF measurement at contralateral directions may be overestimated by the calculation model shown in Fig. 2(a). This is beneficial to practical design of measurement system.

Figure 5: The HRTF errors at the azimuth $\beta_S = 90^\circ$ and the source distance 1.0 m, the spherical source radii are 0.025 m, 0.35 m and 0.05 m, and the elevation intervals include 10° and 20°, respectively

Figure 6: (a) The conditions of HRTF measurement; (b) the errors of the measured HRTFs at various azimuths

To verify the above-mentioned hypothesis, paying attention to the magnitude error less than 2.5 dB at distance 0.2 m in Fig. 4, 4 sound sources are set at distance 0.2 m with elevations ($\alpha = -30^\circ$, 0°, 30°, 60°, respectively) in the near-field HRTF measurement for a KEMAR artificial head. The results are shown in Fig. 7. Errors of most azimuths fall within about 1.0 dB when multiple sound sources are deployed. The errors reach up to or beyond 2.0 dB only when the sound sources are located at the contralateral side ($\beta = 270^\circ$). This is because, as mentioned previously in this paper, the multiple scattering sound waves among sources have greater influence on contralateral HRTF with small amplitude. With the increase of source distance, the spatial interval among sources get larger and the scattering errors will decrease further. This indicated that the above-mentioned spherical dodecahedron sound source is suitable for multiple-sound-source HRTF measurement system.
Figure 7: Influence of multiple-sound-source scattering to near-field HRTF measurement using a KEMAR artificial head. (a) the right-ear HRTF measured with deployment of multiple sound sources; (b) the right-ear HRTF measured with deployment of a single sound source; (c) magnitude spectral difference (SD in dB)

5 Conclusions

The model consisting of a rigid spherical head and two spherical sound sources has been used to analyse the errors in HRTF measurements caused by multiple scattering of the adjacent sound sources. The errors increase with the increasement of the source radius and the decrement of the source distance and elevation interval. Especially, the spectral distortion in ipsilateral HRTF measurement are within ±1.0 dB for the frequencies below 20 kHz, source radius less than 0.025 m, source distance not less than 0.5 m, and the elevation interval of the adjacent sources not less than 20°. Using similar method, the cases of various sound source distances and intervals can be analyzed. The measurement results of the multiple scattering of the rigid spherical head model and a KEMAR manikin further verified the calculation results. In practice, some sound absorption treatments on the source surface can further reduce the error caused by multiple scattering, and thus further improve the accuracy of measurement.

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