Fault Tolerance in Iterative-Convergent Machine Learning

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Abstract

Machine learning (ML) training algorithms often possess an inherent self-correcting behavior due to their iterative-convergent nature. Recent systems exploit this property to achieve adaptability and efficiency in unreliable computing environments by relaxing the consistency of execution and allowing calculation errors to be self-corrected during training. However, the behavior of such systems are only well understood for specific types of calculation errors, such as those caused by staleness, reduced precision, or asynchronicity, and for specific types of training algorithms, such as stochastic gradient descent. In this paper, we develop a general framework to quantify the effects of calculation errors on iterative-convergent algorithms and use this framework to design new strategies for checkpoint-based fault tolerance. Our framework yields a worst-case upper bound on the iteration cost of arbitrary perturbations to model parameters during training. Our system, SCAR, employs strategies which reduce the iteration cost upper bound due to perturbations incurred when recovering from checkpoints. We show that SCAR can reduce the iteration cost of partial failures by 78\%-95\% when compared with traditional checkpoint-based fault tolerance across a variety of ML models and training algorithms.

1 Introduction

Distributed model training for machine learning (ML) is a workload which is typically long-running and resource-intensive. Throughout a job’s lifetime, it is susceptible to hardware failures, performance fluctuations, and other uncertainties inherent to real-world cluster environments. For example, processes can be preempted by a cluster resource allocator (Vavilapalli et al., 2013; Hindman et al., 2011), parameter synchronization can be bottlenecked on a slow or congested network (Li et al., 2014b; Zhang et al., 2017b), and stragglers can severely impact overall job throughput (Cipar et al., 2013; Harlap et al., 2016). Thus, developing new fault-tolerance strategies for modern ML systems is an important area of research.

ML-agnostic distributed systems approaches for addressing such problems often adopt strong consistency semantics. They aim to provide strong execution guarantees at a per-operation level (such as linearizability or serializability), but may also incur higher performance overhead. On the other hand, ML training is often tolerant to small calculation errors and may not require such strong consistency guarantees. This observation has been exploited by recent ML systems to overcome cluster unreliability and resource limitation issues. For example, bounded staleness consistency allows stale model parameters to be used, reducing the cost of synchronization and mitigating the effects of stragglers and/or congested networks (Ho et al., 2013; Cipar et al., 2013; Cui et al., 2014). Training using quantized or low-precision floating point representations drastically reduces the overhead of computation and communication limitations (Courbariaux et al., 2014; Gupta et al., 2015; Hubara et al., 2017). Lock-free execution eliminates the cost of blocking synchronization primitives, achieving higher throughput parallel training (Niu et al., 2011; Dean et al., 2012). One notable exception to this trend is checkpoint-based fault tolerance, a common strategy in current ML systems for mitigating hardware failures (Abadi et al., 2016; Wei et al., 2015; Low et al., 2012) which continues to enforce strong consistency semantics at a high cost of re-computing lost work.
Figure 1: The self-correcting behavior of iterative-convergent algorithms. Even though a calculation error results in an undesirable perturbation of $\delta$ at iteration $T$, the subsequent iterations still brings the solution closer to the optimum value of $x^*$.

Figure 2: A framework for designing robust training systems by exploiting the self-correcting behavior of ML. First, through system design, resource instabilities and constraints in unreliable computing environments are allowed to manifest as perturbations in model parameters. Then, through the self-correcting behavior of ML, the perturbations are automatically corrected but incur a cost in the number of iterations to convergence.

This trend of relaxing consistency in ML systems relies on a fundamental trade-off, allowing the training algorithm to incur computation errors in order to gain more freedom to optimize execution and adapt to environmental faults. To preserve correctness, it relies on the self-correcting behavior of iterative-convergent ML training algorithms. During each step, the training algorithm calculates updates based on the current values of model parameters, and then applies the updates to obtain a “better” set of model parameters. By iteratively performing this computation, the model parameters eventually converge to a set of optimal values. Small computation errors made during this procedure are eventually washed out by the successive iterative improvements (see Fig. 1).

This self-correcting behavior of ML training suggests a general strategy for designing robust training systems for unreliable environments, as follows:

(A) The execution system allows certain environmental faults and/or resource limitations to manifest as calculation errors in model training. These errors can be conceptualized as perturbations to the model parameters.

(B) The perturbations are self-corrected by the model training algorithm, which incurs an extra number of iterations. We refer to this number of extra iterations as the iteration cost of the perturbations.

Unfortunately, this approach has only been used and analyzed individually for a few limited cases such as the ones listed above, while other opportunities still exist.

Motivated by this general strategy, in this paper we develop a framework for exploiting self-correction in ML systems in a way that is adaptive to generic perturbations whose cause or origin is unknown. It provides a theoretical foundation for understanding the self-correcting behavior of iterative-convergent model training as well as the tools needed by ML systems to take advantage of this behavior. Our main contributions are:

1. We quantify the impact of generic perturbations on iterative-convergent algorithms in terms of their iteration cost. Under reasonable convergence assumptions, we bound the iteration cost in terms of the sizes of these perturbations.

2. We propose new strategies for checkpoint-based fault tolerance in distributed model training. Partially
reverting from checkpoints, combined with prioritizing checkpoints in a way that reduces the size of
perturbations, can significantly reduce the iteration cost due to partial failures.

3. We design and implement SCAR, a parameter server system which employs our checkpoint strategies.
   We show that SCAR reduces the iteration cost of partial failures by 78%–95% across a variety of
   popular ML models and training algorithms when compared with traditional checkpoint recovery.

2 Modeling Faults in Iterative-Convergent Machine Learning

Most ML algorithms are iterative, i.e. model parameters are updated given a current estimate of the
model parameters \( x^{(k)} \) until convergence to some target parameter \( x^* \). Such algorithms are commonly
called iterative-convergent. Examples include classical optimization algorithms such as gradient descent
and stochastic gradient descent (SGD) as well as Monte Carlo methods such as Gibbs sampling and Metropolis-
Hastings. These algorithms are the basic building blocks of models such as deep neural networks, matrix
factorization, and latent Dirichlet allocation. In their most general form, such schemes can be written

\[
x^{(k+1)} = f(x^{(k)}), \quad x^{(k)} \in \mathbb{R}^d,
\]

where \( x^{(k)} \) represents the model parameter values at iteration \( k \) and \( f \) updates the current state \( x^{(k)} \) to
obtain the new state \( x^{(k+1)} \). In practice, \( f \) is a known function that depends on the data available at time
\( k \).

This model of iterative-convergent algorithms assumes that the current state \( x^{(k)} \) is stored persistently
and losslessly in memory. In practice, modern distributed ML systems are subject to faults such as hardware
failures, memory corruption, and performance fluctuations. Thus, it is unrealistic to assume that \( x^{(k)} \)
can always be retrieved with perfect fidelity. To model this uncertainty, let \( \delta_k \) be a random variable that
represents an unknown perturbation that corrupts the current state to produce a perturbed state \( x^{(k)} + \delta_k \).
We make no assumptions about the cause, size, or behavior of the perturbations \( \delta_k \). More specifically, we
assume the iterates obey the following scheme:

\[
\begin{align*}
y^{(0)} &= x^{(0)} \\
y^{(1)} &= f(y^{(0)} + \delta_0) \\
&\vdots \\
y^{(k+1)} &= f(y^{(k)} + \delta_k)
\end{align*}
\]

In the absence of errors, i.e. \( \delta_k = 0 \), we have \( y^{(k)} = x^{(k)} \), which reduces to the basic iterative scheme (1).
Moreover, since \( \delta_k \) is arbitrary, this model allows for any type of perturbation. In particular, perturbations
may occur in every iteration or periodically according to some random process.

This setup captures many of the ways that cluster environment faults can be manifested as perturbations
in distributed ML systems, and we give a few important examples below.

**Example 2.1 (Reduced Precision).** A simple practical example is using reduced precision floating/fixed
point representations for storing parameter values. Suppose \( \tilde{y}^{(k)} \) is the reduced precision version of the exact
parameter values \( y^{(k)} \), then the algorithm suffers perturbations of \( \delta_k = \tilde{y}^{(k)} - y^{(k)} \) at each iteration \( k \). If
the representation has a \( p \)-bit mantissa, then the size of \( \delta_k \) is bounded by \(|\delta_k| < 2^{-(p-1)}|y^{(k)}|\) (Higham, 2002).

**Example 2.2 (Bounded Staleness Consistency).** In SGD under the stale synchronous parallel (SSP) consistency
model (Ho et al., 2013), gradients are computed in a data-parallel fashion where each of \( M \) machines
may observe a stale version of the model parameters \( \tilde{x}_m^{(k)} \). Suppose \( \nabla(\tilde{x}_m^{(k)}, D_m) \) are the gradients computed
during iteration \( k \) using input data \( D_m \) at machine \( m \). If \( \nabla(x^{(k)}, D) \) is the true stochastic gradient at
iteration \( k \), then the algorithm suffers a perturbation at iteration \( k + 1 \) of:

\[
\delta_{k+1} = \frac{1}{M} \sum_{m=1}^{M} \nabla(\tilde{x}_m^{(k)}, D_m) - \nabla(x^{(k)}, D)
\]
In order to keep things simple, we assume that the unperturbed sequence satisfies

$$\delta_T = x^{(T)} - x^{(C)}.$$ 

Although from the system’s point of view the application is returned to an exact prior state, we can still view the act of checkpoint recovery as a perturbation to the model parameters.

**Remark 2.1.** Reduced precision (Example 2.1) and bounded staleness consistency (Example 2.2) have already been the focus of much attention in both the ML and systems communities (Zhang et al., 2017a; Jia et al., 2018; Wei et al., 2015; Dai et al., 2015). Although not typically studied within the explicit set-up of (2), these strategies generate perturbations which fit within our framework, and preserve the correctness of training by keeping the sizes of these perturbations small. This is accomplished via bounded floating-point/fixed-point rounding errors for reduced precision and via a maximum staleness limit for bounded staleness consistency. In Section 4, we apply the general set-up of (2) to devise new strategies for checkpoint-based fault tolerance (Example 2.3). Our system, SCAR, applies the same principle of reducing the sizes of perturbations in order to reduce the overall iteration cost of machine failures.

**Remark 2.2.** The iteration in (2) is closely related to perturbed gradient descent (PGD) (Ge et al., 2015; Jin et al., 2017; Du et al., 2017). Formally, PGD is a special case of (2). The main difference lies in the motivation: Jin et al. (2017) show that by choosing $\delta_k$ cleverly, it is possible to escape saddle points and guarantee that the iteration (2) converges to a second-order stationary point. The key idea in PGD is to design the perturbations $\delta_k$ to an advantage, which is in stark contrast to our set-up, in which we have no control over $\delta_k$. In the worst case, we allow $\delta_k$ to be chosen adversarially.

## 3 Analysis

Suppose that an ML system has experienced perturbations $\delta_1, \ldots, \delta_T$ up to the $T$th iteration. A (random) sequence $a_k$ is called $\varepsilon$-optimal if $E(\|a_k - x^*\| < \varepsilon$. The main question we seek to address in this section is the following: **Given $\varepsilon > 0$, what is the “cost” in number of iterations for $y^{(k)}$ to reach $\varepsilon$-optimality compared to the unperturbed sequence $x^{(k)}$?** We write “cost” in quotations to emphasize that this number can be negative—for example, $\delta_k$ could randomly move $y^{(k)}$ closer to $x^*$, or $\delta_k$ can be constructed in advance to improve convergence as in perturbed gradient descent (see Remark 2.2).

We call this quantity the **iteration cost** of the perturbed sequence $y^{(k)}$, introduced in Sec. 1. Our goal in the present section is to bound the iteration cost, which will be formally defined next.

### 3.1 Iteration cost

In order to keep things simple, we assume that the unperturbed sequence satisfies

$$\|f(x^{(k)}) - x^*\| \leq c \|x^{(k)} - x^*\|, \quad 0 < c < 1,$$

i.e. the iterates $x^{(k)}$ converge linearly. Although some algorithms (e.g. SGD) do not converge linearly, many of the most popular algorithms in practice do (e.g. gradient descent, proximal quasi-Newton, Metropolis-Hastings). This assumption is made purely for simplicity: We use (3) as a baseline for comparison, and the analysis can be easily extended to more general schemes such as SGD if desired (see Example 3.4).

Formally, the iteration cost is defined as follows: Let $\kappa(y^{(k)}, \varepsilon)$ be a lower bound such that $m > \kappa(y^{(k)}, \varepsilon)$ implies $E(\|y^{(m)} - x^*\| < \varepsilon$ (this may be $+\infty$ or negative). Under (3), it is straightforward to derive a similar lower bound for the unperturbed sequence $x^{(k)}$ as $\kappa(x^{(k)}, \varepsilon) = \log \left( \frac{1}{c} \|x^{(0)} - x^*\| / \log(1/c) \right)$. This will be used as a baseline for comparison: The iteration cost for the perturbations $\delta_k$ is defined to be

$$\iota(\delta_k, \varepsilon) := \kappa(y^{(k)}, \varepsilon) - \kappa(x^{(k)}, \varepsilon).$$
Figure 3: Illustrations of iteration costs using gradient descent on a simple 4-D quadratic program. Each plot consists of 1,000 trials with perturbation(s) randomly generated according to a normal distribution. The red line is the iteration cost bound according to Theorem 3.2. The value of $c$ is determined empirically, and the value of $\epsilon$ is set so that an unperturbed trial converges in roughly 1,000 iterations.

Using the unperturbed sequence $x^{(k)}$ as a benchmark, $\iota(\delta_k, \epsilon)$ bounds the additional number of iterations needed for the perturbed sequence $y^{(k)}$ to reach $\epsilon$-optimality (where this can be negative). Clearly, $\iota(\delta_k, \epsilon)$ depends on the sequence $\delta_k$, and should be smaller whenever the $\delta_k$ are smaller. We seek a bound on $\iota(\delta_k, \epsilon)$ that holds for arbitrary $\delta_k$.

Remark 3.1. We use the criterion $E\|y^{(k)} - x^*\| < \epsilon$ as an optimality criterion instead of directly bounding $P\|y^{(k)} - x^*\| < \epsilon$. This is commonly done (e.g. Bottou et al., 2016) since bounds on $E\|y^{(k)} - x^*\|$ imply bounds on the latter probability via standard concentration arguments (see e.g. Rakhlin et al., 2012). These bounds will of course depend on the tail behavior of $\delta_k$.

3.2 Bounding the iteration cost

To bound the iteration cost, we also require that the update $f$ satisfies a convergence rate similar to (3) for the perturbed data $\tilde{y}^{(k)} := y^{(k)} + \delta_k$:

$$E\|f(\tilde{y}^{(k)}) - x^*\| \leq c E\|\tilde{y}^{(k)} - x^*\|, \quad 0 < c < 1.$$  \hspace{1cm} (5)

This simply says that wherever the algorithm is, on average, a single step according to $f$ will not move the iterates further from $x^*$.

For example, consider gradient descent, which is arguably one of the simplest iterative schemes in ML. For gradient descent, we have $f(\tilde{y}^{(k)}) = \tilde{y}^{(k)} - \alpha \nabla \ell(\tilde{y}^{(k)})$, where $\ell$ is the objective function that is being minimized. Then we have the following:

Lemma 3.1 (Strongly convex objective). Suppose the objective function $\ell$ is strongly convex. Then the gradient descent algorithm satisfies (5).

Proof. See Appendix A.1.

Similar results hold for other optimization schemes such as proximal methods and Newton’s method. In fact, the assumption of strong convexity can be substantially relaxed to include various nonconvex problems (Xu and Yin, 2017; Attouch et al., 2010). See Example 3.2.

Under (3) and (5), we have the following general bound on the iteration cost:

Theorem 3.2. Assume $E\|\delta_k\| < \infty$ for $k \leq T$ and $\delta_k = 0$ for $k > T$. Under (3) and (5), we have for any $\epsilon > 0$,

$$\iota(\delta_k, \epsilon) \leq \frac{\log \left(1 + \frac{\Delta_T}{\|x^{(0)} - x^*\|}\right)}{\log(1/c)}$$  \hspace{1cm} (6)

where $\Delta_T := \sum_{t=0}^{T} c^{-t} E\|\delta_t\|$. 

5
Proof. See Appendix A.2.

In fact, the bound (6) is tight in the following sense: As long as (3) cannot be improved, there exists a deterministic sequence $\delta_1, \ldots, \delta_T$ such that (6) holds with equality. Theorem 3.2 is illustrated on a simple quadratic program (QP) in Figure 3, which provides empirical evidence of the tightness of the bound. Additional empirical experiments are illustrated in Figure 5.

The interesting part of the bound (6) is the ratio $\Delta_T/\|x(0) - x^*\|$, which is essentially a ratio between the aggregated cost of the perturbations and the “badness” of the initialization. For more intuition, re-write this ratio as

$$
\frac{\Delta_T}{\|x(0) - x^*\|} = \frac{\sum_{\ell=0}^{T} c^{k-\ell} E\|\delta_\ell\|}{c^k \|x(0) - x^*\|}.
$$

Up to constants, the denominator is just the error of the original sequence $x(k)$ after $k$ iterations. The numerator is more interesting: It represents a time-discounted aggregate of the overall cost of each perturbation. Each perturbation $\delta_\ell$ is weighted by a discount factor $c^{k-\ell}$, which is larger for more recent perturbations (e.g. $\delta_T$) and smaller for older perturbations (e.g. $\delta_0$). Thus, the dominant quantity in (6) is a ratio between the re-weighted perturbations and the expected error from the original sequence. As expected, if the original sequence converges very quickly and the perturbations are large, the iteration cost increases proportionally.

Theorem 3.2 also assumes that there are no perturbations after time $T$. The idea is that if there are no more perturbations, (6) bounds the cost of the perturbations incurred so far. Of course, in practice, the system may experience faults after time $T$, in which case (6) can be adjusted to include the most recent fault. The difficulty in directly accounting for future perturbations lies in our assumption that the $\delta_k$ can be arbitrary: If future iterations can experience any perturbation, it is clear that convergence cannot be guaranteed (e.g. consider $\delta_k = x - y(k)$ for some fixed $x \neq x^*$ and all $k > T$). Under some additional assumptions, something can be said about this case; see Example 3.3 in the next section.

3.3 Examples

In this section, we discuss some examples where the bound (6) is applicable, along with some generalizations.

Example 3.1 (Convex optimization). Lemma 3.1 implies that Theorem 3.2 applies to ML systems that are based on minimizing a strongly convex objective. This includes many classical problems such as linear and logistic regression.

Example 3.2 (Nonconvex optimization). If the loss function $\ell$ is nonconvex, then Theorem 3.2 still applies with some modifications. The assumptions (3) and (5) can be verified using known results on nonconvex optimization (Xu and Yin, 2017; Attouch et al., 2010) under the so-called Kurdyka-Lojasiewicz property, from which the bound (6) follows directly. Trouble arises, however, when $\ell$ has multiple basins of attraction: A perturbation $\delta_k$ could “push” the perturbed iterate $\tilde{y}^{(k)}$ into a different basin, resulting in a limit point that is different from $x^*$. Theorem 3.2 continues to hold as long as this can be avoided, i.e. the $\delta_k$ are not too large. We leave it to future work to study this case in more detail.

Example 3.3 (Infinite perturbations). An interesting case occurs when $\delta_k \neq 0$ for all $k$. In other words, there is a possibility of a fault in every iteration. For arbitrary $\delta_k$, it is clearly impossible to establish any kind of convergence result. In fact, suppose $\|\delta_k\| \leq \Delta$ for each $k$. Then there is an irreducible error of $(c/(1-c))\Delta$, meaning that we cannot hope to obtain an $\varepsilon$-optimal solution for any $\varepsilon < (c/(1-c))\Delta$. This helps to explain why we focus on the nontrivial case with $\delta_k = 0$ for $k > T$ in Theorem 3.2. One setting in which the analysis with infinite perturbations is nontrivial is when $\Delta$ is known to be small, e.g. when using reduced precision as in Example 2.1. This setting can be analyzed by setting $\Delta \geq 2^{-(p-1)}\|x^{(k)}\|$ for all $k$. For details, see Appendix B.1.

Example 3.4 (SGD). The assumption (5) does not hold for SGD, which has a sublinear convergence rate in general. Nonetheless, it is straightforward to extend our framework to sublinear algorithms, with the caveat that analogous bounds on the iteration cost become more complicated. In fact, it is not hard to see from our proof how to do this: Lemma A.1 in the Appendix establishes the following useful general inequality

$$
E\|y^{(k+1)} - x^*\| \leq c^{k+1} \left[\|x^{(0)} - x^*\| + \Delta_T\right].
$$
Evidently, the factor of $c$ governs how quickly $\Delta T$ (i.e. the cost incurred by perturbations) gets washed out as $k$ increases. For algorithms that converge sublinearly such as SGD, this effect will also be sublinear, but still tend to zero as long as the perturbations are not too large (see Appendix B.2 for a brief discussion). This is further corroborated by the empirical experiments in Section 5, where we show that the strategies for checkpoint-based fault tolerance proposed in the next section are successful on SGD as well as other optimization schemes such as alternating least squares.

4 Strategies for Checkpoint-Based Fault Tolerance

As an application of our iteration cost bounds, we study new strategies for checkpoint-based fault tolerance, by which stateful computations are made resilient to hardware failures by periodically checkpointing the entire program state. Whenever such a failure occurs, the most recent checkpoint is restored and computation is resumed (Example 2.3). The total running time $T$ of the system can be modeled as (Daly, 2006):

$$T = T_{\text{solve}} + T_{\text{dump}} + T_{\text{rework}} + T_{\text{restart}}$$

where $T_{\text{solve}}$ is the normal runtime of the program without any failures, $T_{\text{dump}}$ is the time taken saving a checkpoint, $T_{\text{rework}}$ is the time spent repeating lost work due to restoring to a previous checkpoint, and $T_{\text{restart}}$ is the time spent restoring a checkpoint. For iterative ML training, $T_{\text{rework}} = \iota(\delta_k, \varepsilon) \cdot T_{\text{iter}}$, where $\iota(\delta_k, \varepsilon)$ is the iteration cost of the failure and $T_{\text{iter}}$ is the time taken per iteration of the training algorithm. We focus our strategies primarily on reducing $T_{\text{rework}}$, since $T_{\text{solve}}$ and $T_{\text{iter}}$ are constant regardless of checkpoints and failures, and $T_{\text{dump}}$ and $T_{\text{restart}}$ are typically small fractions of $T_{\text{iter}}$ in our experiments.

Using the traditional checkpoint-based fault tolerance mechanism, the entire program state is saved during each checkpoint, restored after a failure, and all computation since the previous checkpoint repeated. Thus, $T_{\text{rework}}$ is the total amount of time between the previous checkpoint and the failure, spanning the computation which must be repeated. This process maximizes the consistency of recovery by restoring the system to an exact state it was in during the past, but can incur a high rework overhead if the checkpoint interval is long.

For iterative-convergent ML, however, we can exploit its self-correcting behavior to reduce $T_{\text{rework}}$. In particular, we can give up the consistency of checkpoint-recovery, and design a system which tries to reduce the size of the perturbation $\|\delta T\|$ incurred upon failure. By doing so, Theorem 3.2 shows that the iteration cost bound is also reduced, lowering the worst case iteration cost and thus reducing $T_{\text{rework}}$.

Our system, SCAR\footnote{SCAR stands for Self-Correcting Algorithm Recovery.}, implements two strategies which reduce $\|\delta T\|$ compared to traditional checkpoint recovery: (1) Partial recovery, and (2) Prioritized checkpoints. SCAR extends the popular parameter server (PS) architecture for distributed model training (Ho et al., 2013; Li et al., 2014b,a)—the model parameters are partitioned across a number of PS nodes, which are accessed by worker nodes. We assume that during a failure, any number of PS nodes can go down, causing the loss of their partitions of the model parameters. We present these strategies and the design of SCAR below, and show evaluation of SCAR in Section 5.

4.1 Partial Recovery

Our first strategy is to only recover (i.e. from a previous checkpoint) the part of the model parameters which are lost due to the failure. Since the model parameters are partitioned across several PS nodes, a partial failure of PS nodes should only cause a partial loss of model parameters. Mathematically, the partial recovery strategy should result in a smaller perturbation to the model parameters and, according to Theorem 3.2, incur a smaller iteration cost.

Suppose that a fully-consistent checkpoint is taken after iteration $C$, and a failure occurs during iteration $T > C$ which triggers checkpoint recovery.

**Theorem 4.1.** Let $\delta$ be the perturbation incurred by full checkpoint recovery, and $\delta'$ be the perturbation incurred by partial checkpoint recovery, then $\|\delta'\| < \|\delta\|$.

**Proof.** See Appendix A.3.
Furthermore, the size of the perturbation should also be related to the fraction of model parameters which are lost—losing fewer model parameters should generate a smaller perturbation. To establish this relationship, we will assume that parameters are partitioned uniformly at random across the PS nodes, and so a random subset of parameters will be lost. This assumption is reasonable as the partitioning scheme is typically within the control of the PS system, which can choose a random partitioning.

**Theorem 4.2.** Suppose that a failure causes the loss of a fraction $0 < p \leq 1$ of all model parameters chosen uniformly at random. Let $\delta$ be the perturbation incurred by full checkpoint recovery, and $\delta'$ be the perturbation incurred by partial checkpoint recovery, then $E||\delta'||^2 = p||\delta||^2$.

**Proof.** See Appendix A.4.

Thus, the expected size of perturbation incurred by partially restoring from a checkpoint decreases as the fraction of parameters lost decreases.

### 4.2 Priority Checkpoint

With the partial recovery strategy, we have shown that relaxing the consistency of checkpoint recovery can reduce the size of perturbations (i.e. $\delta_k$) experienced by the training algorithm due to a failure, and thus reduce the iteration cost. In this section, we further consider relaxing the consistency of saving checkpoints by taking more frequent, partial checkpoints.

Rather than saving all parameters every $C$ iterations, consider saving a fraction $r < 1$ of the parameters every $rC$ iterations. A running checkpoint is kept in persistent storage, which is initialized to the initial parameter values $x^{(0)}$ and updated each time a partial checkpoint is saved. At a given time, this checkpoint may consist of a mix of parameters saved during different iterations, and the choice of which subset of parameters to checkpoint can be controlled via system design. This strategy enables, e.g., prioritization of which parameters are saved during each checkpoint so as to prioritize saving parameters that will minimize the size of the perturbation caused by a failure. To do this, we consider a simple heuristic: Save the parameters which have changed the most since they were previously saved.

The checkpoint period $rC$ is chosen so that the number of parameters saved every $C$ iterations remains roughly constant across different values of $r$. As a result the prioritized checkpoint strategy writes the same amount of data per constant number of iterations to persistent storage as the full checkpoint strategy, while having more frequent opportunities to prioritize and save parameters to the running checkpoint. We evaluate the system overhead implications of this scheme in Section 5.5.

### 4.3 SCAR Architecture and Implementation

We implement our system, SCAR, using these two checkpoint-based fault tolerance strategies. SCAR is implemented as a PS architecture—the parameters of the ML model are randomly partitioned across PS nodes, while the input data is partitioned across worker nodes. During each iteration, the workers read values from the PS nodes, compute updates using their local input data, and send the updates to the PS nodes to be applied.

Figure 4 illustrates the architecture of SCAR. A fault tolerance controller runs as a separate service and consists of (1) a checkpoint coordinator responsible for coordinating periodic checkpoints at a fixed time interval, and (2) a recovery coordinator responsible for coordinating the failure recovery process whenever a failure is detected. The detection of failures is performed by a failure detector service, which can leverage heartbeating mechanisms in existing systems for distributed consensus such as ZooKeeper (Hunt et al., 2010). Checkpoints are saved to shared persistent storage, such as distributed filesystems like NFS (Sandberg et al., 1988), CephFS (Weil et al., 2006), or distributed databases like Cassandra (Lakshman and Malik, 2010). To speed up distance calculations between the current and previously saved parameters, each PS node keeps an in-memory cache of the current checkpoint, which is updated whenever a new partial checkpoint is saved.

When a checkpoint is triggered:

1. The checkpoint coordinator sends a message to each PS node, which computes the distance of each of its parameters from their previously saved values in the running checkpoint using its in-memory cache.

2. Each PS node sends its model parameter IDs and computed distances to the checkpoint coordinator.
3. Upon receipt of the computed distances from all PS nodes, the checkpoint coordinator selects the fraction $r$ of parameters with the largest distances, and sends their IDs back to their corresponding PS nodes.

4. Upon receipt of the parameter IDs, each PS node updates its in-memory cache, and saves those parameters to the shared persistent storage.

During step 4, the training algorithm can be resumed as soon as the in-memory caches have been updated, while output to the shared persistent storage happens asynchronously in the background. Thus, the checkpointing overhead $T_{\text{dump}}$ in SCAR is just the time needed for prioritizing parameters and updating the in-memory cache.

When a failure is detected:

1. The failure detector notifies the recovery coordinator, which determines how the parameters belonging to the failed PS nodes should be re-partitioned.

2. The recovery coordinator partitions and sends the failed parameter IDs to the remaining PS nodes, which re-load the parameters from the current running checkpoint in shared persistent storage.

SCAR is implemented using C++ and leverages an existing elastic ML framework (Qiao et al., 2018), which provides mechanisms for transparently re-routing requests from workers away from failed PS nodes, as well as for new PS nodes to join the active training job, replacing the old failed PS nodes.

5 Experiments

With our evaluation, we wish to (1) illustrate our iteration cost bounds for different types of perturbations using practical ML models, (2) empirically measure the iteration costs of a variety of models under the partial recovery and prioritized checkpoint strategies in SCAR, and (3) show that SCAR has low performance overhead.

5.1 Models and Datasets

We use several popular models as examples for our analysis and checkpoint strategies. We describe their training algorithms, datasets, and parameter partitioning schemes below, and refer to Appendix C for more
Multinomial Logistic Regression (MLR). We use the standard stochastic gradient descent approach to minimize the multi-logit loss. The model parameters are an $M \times N$ matrix of real numbers, where $M$ is the dimensionality of the data, and $N$ is the number of output classes. When distributed, the rows of the parameter matrix are randomly partitioned. We train MLR on the MNIST (Lecun et al., 1998) and CoverType (Dheeru and Karra Taniskidou, 2017) datasets.

Matrix Factorization (MF). We use the standard alternating least squares (ALS) approach to minimize the objective function. The model parameters are matrices $L \in \mathbb{R}^{m \times p}$ and $R \in \mathbb{R}^{p \times n}$. When distributed, the rows of $L$ and the columns of $R$ are randomly partitioned. We train MF on the MovieLens (Harper and Konstan, 2015) and Jester (Goldberg et al., 2001) datasets.

Latent Dirichlet Allocation (LDA). We use the standard collapsed Gibbs sampling (Liu, 1994) approach to learn the model parameters, which are the document-topic and word-topic distributions. We use a scaled total variation between document-topic distributions as the norm for computing distances between parameters. When distributed, the document-topic distributions are randomly partitioned across nodes. We do not consider loss of word-topic distributions because they can be re-generated from the latent token-topic assignments. More details on this setup are in Appendix C. We train LDA on the 20 Newsgroups (Lang, 1995) and Reuters (Lewis et al., 2004) datasets.

Convolutional Neural Network (CNN). We train a network consisting of 2 convolution layers with ReLU activations (Nair and Hinton, 2010) and max pooling followed by 3 fully-connected layers with ReLU activations using Adam (Kingma and Ba, 2014). Because of the structure in neural network models, we consider two different partitioning strategies: 1) In by-layer partitioning, we assume that the layers of the network are randomly partitioned across nodes; and 2) In by-shard partitioning, we further divide each layer’s parameters into shards, and all shards are randomly partitioned across nodes. We train this CNN on the MNIST (Lecun et al., 1998) dataset.

5.2 Iteration Cost Bounds

To illustrate the behavior of the iteration cost and to verify Theorem 3.2 for different types of models and perturbations, we train MLR and LDA and generate a perturbation according to one of three types: random, adversarial, and resets.
Figure 6: Perturbations are generated by resetting a random fraction of parameters back to their initial values, for both (a) MLR and (b) LDA. Other settings are the same as Figure 5.

For random perturbations (Figure 5(a)), the iteration cost bound is a loose upper bound on the actual iteration cost. This is in contrast to the simpler quadratic program (QP) experiments shown in Figure 3, in which the bound is relatively tight. On the other hand, we also do not observe any perturbations resulting in a negative iteration cost as for QP. This experiment shows that for MLR, a perturbation in a random direction is unlikely to greatly impact the total number of iterations to convergence.

We run a second experiment in which we generate “adversarial” perturbations opposite the direction of convergence (Figure 5(b)). In this case, we see that our bound is much closer to the actual iteration costs, indicating that it is still a tight worst-case upper bound on the iteration cost for MLR.

While Figure 5 shows the iteration costs for synthetically generated perturbations, Figure 6 generates more realistic perturbations for both MLR and LDA. We generate perturbations by resetting a random subset of model parameters back to their initial values. This scheme simulates the type of perturbations the training algorithm would observe in the partial recovery scenario described in Section 4.1. In this case, we see that the behavior of actual iteration costs is closer to the scenario with adversarial perturbations, although not quite as costly.

5.3 Partial Recovery

To empirically characterize the behavior of partial recovery from checkpoints, we simulate failures of varying fractions of model parameters for the MLR, MF, LDA, and CNN models. We compare the iteration costs incurred by full recovery with the iteration costs incurred by partial recovery. For each model, we sample the failure iteration from a geometric distribution, which causes the loss of a subset of model parameters chosen uniformly at random.

Fig. 7 shows the results. For all models and datasets, we see the average iteration cost incurred by partial recovery decreases as the failure fraction decreases. Meanwhile, the average iteration cost incurred by full recovery remains constant at its maximum value, since all parameters are loaded from the checkpoint regardless of which are actually lost.

Across all models and datasets tested, SCAR with partial recovery reduces the iteration cost by 12%–42% for 3/4 failures, 31%–62% for 1/2 failures, and 59%–89% for 1/4 failures.

5.4 Priority Checkpoint

In this section, we evaluate the effectiveness of our priority checkpoint strategy for the MLR, MF, LDA, and CNN models. We compare the iteration costs incurred by different fractions of partial checkpoints, while keeping constant the number of parameters saved per constant number of iterations, as described in Section 4.2. As before, we sample the failure iteration from a geometric distribution. In this experiment keep the fraction of lost parameters fixed at 1/2.

To gauge the effectiveness of prioritization, we compare between the following strategies:
Figure 7: Partial vs. full recovery for a variety of models and datasets, where the set of failed parameters are selected uniformly at random. The $x$-axis shows the fraction of failed parameters, and the $y$-axis shows the number of rework iterations. The error bars indicate 95% confidence intervals, calculated by repeating each trial 100 times.

Figure 8: Prioritized checkpoint experiments comparing between the random, round-robin, and priority strategies. The $x$-axis indicates checkpoint frequency relative to full checkpoints, where 1 indicates full checkpoints, 2 indicates $1/2$ checkpoints at $2\times$ frequency, etc., and the $y$-axis shows the number of rework iterations. The error bars indicate 95% confidence intervals, calculated by repeating each trial 100 times, and the dashed black line represents the rework cost of a full checkpoint.
Figure 9: SCAR vs traditional checkpoint-recovery for LDA on ClueWeb. Using SCAR, we save 1/4 of model parameters every iteration. Using the traditional method, we save all model parameters every 4 iterations. SCAR reaches the same likelihood value roughly 3 iterations sooner, saving ≈ 6 min.

1. **priority**: Parameters saved to checkpoint are selected based on the prioritization described in Section 4.2.

2. **round**: Parameters saved to checkpoint are selected in a round-robin manner.

3. **random**: Parameters saved to checkpoint are selected uniformly at random.

Fig. 8 shows the results. For all models and datasets, we see the priority strategy results in decreasing iteration costs when the fraction of each checkpoint decreases (and frequency of checkpoints increases). On the other hand, the round strategy either reduces or increases the iteration cost depending on the model and dataset, while the random strategy nearly always increases the iteration cost.

Across all models and datasets tested, combining partial recovery with prioritized 1/8th checkpoints at 8× frequency reduces the iteration cost of losing 1/2 of all model parameters by 78%–95% when compared with traditional checkpoint recovery.

5.5 System Overhead

Lastly, we evaluate the system overhead of SCAR by training LDA on a 12GB subset of the ClueWeb12 dataset (Gabrilovich et al., 2013). The dataset contains 480K documents and 2B tokens, and the number of topics is set to 1K. We use four AWS i3.2xlarge instances, each with 1.9TB NVMe SSDs. For persistent storage, we install a CephFS on these machines.

We trigger a failure of 1/2 of model parameters during the 7th iteration, and compare SCAR using 1/4th checkpoints with the traditional full checkpoint-recovery mechanism. Figure 9 shows the convergence plots. Using SCAR, the same likelihood value is reached roughly 3 iterations sooner than using traditional checkpoint-recovery. Each iteration takes ≈ 243 seconds, and SCAR spends an extra ≈ 13 seconds for checkpointing after each iteration. Overall, performance overhead of checkpointing in SCAR is small in comparison to the run-time of the training job, and results in a net reduction of $T_{\text{rework}} \approx 6$ min in rework time incurred due to the failure. In dynamic-resource shared computing environments, this extra time can be leveraged by a scheduler to make more fine-grained resource allocation decisions between competing jobs.

6 Discussion and Related Work

The model (2) is closely related to several models in the literature. As discussed in Remark 2.2, perturbed gradient descent (Jin et al., 2017) is (formally) a special case of (2), however, the motivations are quite
different. Mania et al. (2015) and El Gamal and Lai (2017) also consider a model similar to (2), however, perturbations are only added to the gradients. Neither of these works consider perturbations in both the gradients and the current state $x^{(k)}$, as we do.

A related body of work is distributed training under Byzantine faults (Blanchard et al., 2017; Chen et al., 2017; Damaskinos et al., 2018; Guerraoui et al., 2018), where a proportion of machines may act adversarially. Byzantine failures are one of the most general assumptions on failures, and thus a Byzantine fault-tolerant training system is naturally tolerant to many other types of faults and perturbations. However, perturbations to parameters during training are not always Byzantine, and can often be controlled via system implementations, such as bounded staleness consistency models, or partial recovery and prioritized checkpointing as in the present work.

In existing distributed ML systems, the fault tolerance problem is approached from an ML-agnostic perspective. For example, TensorFlow (Abadi et al., 2016) offers recovery from periodic checkpoints, while the parameter server of Li et al. (2014a) offers live replication of parameter values. Proteus (Harlap et al., 2017) proposes an approach for fault-tolerance on transient machines by using more reliable machines for active backup of program state. In comparison, our system takes advantage of the self-correcting nature of ML, offering lower iteration cost compared with traditional checkpoint-restart, and without the performance overhead of live replication or storing parameter state on designated reliable machines.

7 Conclusions and Future Work

The self-correcting behavior of ML forms the basis of system techniques that allow model training to achieve adaptability and efficiency in unreliable and resource-limited environments. In this paper, we outlined a general approach to design such systems by reducing the sizes of perturbations to model parameters. We derived an upper bound on the iteration cost of perturbations which can guide the design of new systems. We then proposed and implemented new strategies for checkpoint-based fault tolerance in our system SCAR. We showed that SCAR is able to reduce the iteration cost of failures by an order of magnitude or more when compared to traditional checkpoint-based fault tolerance.

As for future work, we have already observed that our main assumptions (3) and (5) can be relaxed, however, it remains to study these generalizations in more detail. In particular, it would be interesting to study the case of nonconvex $\ell$ (Example 3.2) more carefully in addition to sublinear schemes such as SGD (Example 3.4). Furthermore, we have avoided making assumptions on the perturbations $\delta_k$, however, by imposing additional assumptions on the frequency or size of these perturbations, one could derive tighter upper bounds on the iteration cost.

On the systems side, there exists opportunities for systems to more directly utilize Theorem 3.2. By approximating $c$ and $\|x^{(0)} - x^*\|$, we may obtain a predictive model which can be evaluated on-the-fly to inform decisions made by a system during run-time. Furthermore, it would be interesting to see how the strategies used in SCAR can be applied to non-PS architectures such as all-reduce, which has recently become popular for distributed training (Sergeev and Balso, 2018).

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A Proofs

A.1 Proof of Lemma 3.1

Lemma 3.1 follows from standard results on gradient descent, see e.g. the proof of Theorem 2.1.5 in Nesterov (2013).

A.2 Proof of Theorem 3.2

We start with the following useful lemma:

Lemma A.1. Assuming (5), we have for any k

\[ \mathbb{E} \| y^{(k+1)} - x^* \| \leq c^{k+1} \left[ \| x^{(0)} - x^* \| + k \sum_{\ell=0}^{k} c^{-\ell} \mathbb{E} \| \delta_\ell \| \right]. \]  

(7)

Proof. For any \( k > 0 \) we have

\[
\begin{align*}
\mathbb{E} \| y^{(k+1)} - x^* \| &= \mathbb{E} \| f(\tilde{y}^{(k)}) - x^* \| \\
&\leq c \mathbb{E} \| \tilde{y}^{(k)} - x^* \| \\
&= c \mathbb{E} \| y^{(k)} + \delta_k - x^* \| \\
&\leq c \left[ \mathbb{E} \| y^{(k)} - x^* \| + \mathbb{E} \| \delta_k \| \right],
\end{align*}
\]  

(8)
where we have invoked (5). Iterating this inequality, we obtain:

\[ c\left[ \mathbb{E}\|y^{(k)} - x^*\| + \mathbb{E}\|\delta_k\| \right] \]
\[ \leq c^2\mathbb{E}\|y^{(k-1)} - x^*\| + c^2\mathbb{E}\|\delta_{k-1}\| + c\mathbb{E}\|\delta_k\| \]
\[ \vdots \]
\[ \leq c^{k+1}\mathbb{E}\|y^{(0)} - x^*\| + \sum_{i=0}^{k} c^{i+1}\mathbb{E}\|\delta_{k-i}\| \]
\[ = c^{k+1}\|x^{(0)} - x^*\| + \sum_{\ell=0}^{k} c^{k-\ell+1}\mathbb{E}\|\delta_\ell\|. \tag{9} \]

In the last step we simply re-indexed the summation and use \( y^{(0)} = x^{(0)} \). Combining (8) and (10) yields the desired bound.

\[ \text{Proof of Theorem 3.2.} \] By Lemma A.1, we have for any \( k > T \),

\[ \mathbb{E}\|y^{(k)} - x^*\| \leq c^k \left[ \|x^{(0)} - x^*\| + \sum_{\ell=0}^{T} c^{-\ell}\mathbb{E}\|\delta_\ell\| \right] < \varepsilon \tag{11} \]
\[ \iff \frac{1}{\varepsilon} \left[ \|x^{(0)} - x^*\| + \Delta_T \right] < c^{-k} \tag{12} \]

Re-arranging, we deduce that \( \mathbb{E}\|y^{(k)} - x^*\| < \varepsilon \) if

\[ k > \frac{\log \left( \frac{1}{\varepsilon} \left[ \|x^{(0)} - x^*\| + \Delta_T \right] \right)}{\log(1/c)} \geq \kappa(y^{(k)}, \varepsilon). \]

It is easy to check (e.g. take \( \delta_k = 0 \) in the previous derivation) that \( \kappa(x^{(k)}, \varepsilon) = \log \left( \frac{1}{\varepsilon} \|x^{(0)} - x^*\| / \log(1/c) \right) \) is a bound on the number of iterations required for the unperturbed sequence \( x^{(k)} \) to reach \( \varepsilon \)-optimality. Thus, the iteration cost is given by

\[ \iota(\delta_k, \varepsilon) = \kappa(y^{(k)}, \varepsilon) - \kappa(x^{(k)}, \varepsilon) \]
\[ \leq \log \left( \frac{1}{\varepsilon} \left[ \|x^{(0)} - x^*\| + \Delta_T \right] \right) - \log \left( \frac{1}{\varepsilon} \|x^{(0)} - x^*\| \right) \]
\[ = \log \left( \frac{1 + \Delta_T}{\|x^{(0)} - x^*\|} \right) \frac{\log(1/c)}{\log(1/c)} \]

as claimed.

\[ \text{Proof of Theorem 4.1} \]

Let \( z = x^{(C)} \) be the checkpoint of the model parameters saved at iteration \( C \), and let \( S \) be the subset of model parameters lost during a failure at iteration \( T \). Then

\[ \|\delta\| = \|z - x^{(T)}\| \]

is the perturbation due to full recovery, and

\[ \|\delta'\| = \|z_S - x_S^{(T)}\| \]

as claimed.

\[ \text{A.3 Proof of Theorem 4.1} \]

Let \( z = x^{(C)} \) be the checkpoint of the model parameters saved at iteration \( C \), and let \( S \) be the subset of model parameters lost during a failure at iteration \( T \). Then

\[ \|\delta\| = \|z - x^{(T)}\| \]

is the perturbation due to full recovery, and

\[ \|\delta'\| = \|z_S - x_S^{(T)}\| \]
is the perturbation due to partial recovery, since $x_{S^c}^{(T)}$ does not change due to failure, where $S^c$ is the complement set of $S$. Then we have

$$||\delta'||^2 = ||z_S - x_S^{(T)}||^2$$

$$\leq ||z_S - x_S^{(T)}||^2 + ||z_{S^c} - x_{S^c}^{(T)}||^2$$

$$+ (z_S - x_S^{(T)}) \cdot (z_{S^c} - x_{S^c}^{(T)})$$

$$\leq ||z_S - x_S^{(T)}||^2 + (z_{S^c} - x_{S^c}^{(T)})||^2$$

$$= ||z - x(T)||^2 = ||\delta||^2$$

Thus $||\delta'|| \leq ||\delta||$, as claimed.

### A.4 Proof of Theorem 4.2

Let $z = x^{(C)}$ be the checkpoint of the model parameters saved at iteration $C$, and let $S$ be the subset (chosen uniformly at random) of model parameters lost during a failure at iteration $T$. Then

$$E||\delta'||^2 = E||z_S - x_S^{(T)}||^2$$

$$= E \left[ (z_S - x_S^{(T)}) \cdot (z_S - x_S^{(T)}) \right]$$

$$= \sum_i E \left[ (z_S - x_S^{(T)})_i^2 \right]$$

$$= \sum_i E \left[ i \in S \right] (z_i - x_i^{(T)})^2$$

$$= \sum_i P(i \in S) (z_i - x_i^{(T)})^2$$

$$= \sum_i p(z_i - x_i^{(T)})^2$$

$$= ||z - x(T)||^2 = p||\delta||^2$$

Thus $E||\delta'||^2 = p||\delta||^2$, as claimed.

### B Extensions

This appendix discusses extensions of our framework to (a) Infinite perturbations (Example 3.3) and (b) SGD (Example 3.4).

#### B.1 Analysis for $T = \infty$

Suppose $E||\delta_k|| \leq \Delta$ for each $k$. For intuition, note that Lemma A.1 implies that for any $k$,

$$E||y^{(k+1)} - x^*|| \leq c^{k+1} \left[ ||x^{(0)} - x^*|| + \sum_{\ell=0}^k c^{-\ell} \Delta \right]$$

$$= c^{k+1} \left[ ||x^{(0)} - x^*|| + \Delta \frac{1 - c^{-(k+1)}}{1 - c^{-1}} \right]$$

$$= c^{k+1} ||x^{(0)} - x^*|| + \Delta \frac{c - c^{k+2}}{1 - c}$$

$$\xrightarrow{k \to \infty} \frac{c}{1 - c} \Delta.$$

Evidently, there is an irreducible, positive error if we are subject to fault in every single iteration.
Thus, the best we can hope for is convergence to within some tolerance $\varepsilon > (c/(1-c))\Delta$. Re-arranging and solving for $k$ in (13) as in the proof of Theorem 3.2, we deduce that $\mathbb{E}\|y^{(k+1)} - x^*\| < \varepsilon$ as long as

$$k > \frac{\log \left( \frac{\|x^{(0)} - x^*\| - \frac{c\Delta}{1-c\Delta}}{\varepsilon - \frac{c\Delta}{1-c\Delta}} \right)}{\log(1/c)}.$$ 

The resulting iteration cost bound is (cf. (6)):

$$\iota(\delta_k, \varepsilon) \leq \frac{\log \left( \frac{1 - \frac{\|x^{(0)} - x^*\| - \frac{c\Delta}{1-c\Delta}}{1 - \frac{c\Delta}{1-c\Delta}}}{\log(1/c)} \right)}{\log(1/c)}.$$ 

This bound is only informative if $\|x^{(0)} - x^*\| > (c/(1-c))\Delta$ and $\varepsilon > (c/(1-c))\Delta$.

**B.2 Stochastic gradient descent**

Assume the objective function $\ell$ is strongly convex, as in Lemma 3.1. In order to derive upper bounds on the iteration cost for SGD, we start from following general recursion, which is standard from the literature (Nemirovski et al., 2009; Rakhlin et al., 2012):

$$\mathbb{E}\|x^{(k+1)} - x^*\|^2 \leq (1 - \alpha_k)\mathbb{E}\|x^{(k)} - x^*\|^2 + \alpha_k^2 G^2,$$ 

where $\alpha_k \to 0$ is a sequence that depends on $\ell$ and the step size, and $G$ is an upper bound on the expected norm of the stochastic gradients. Comparing (15) to (8), the only difference is that instead of a constant $c < 1$, we have a sequence $1 - \alpha_k \to 1$. Thus, instead of decaying at the geometric rate $c^k$, the iterates of SGD converge at a slower rate $(1 - \alpha_1) \cdots (1 - \alpha_k)$.

Define $a_k := (1 - \alpha_1) \cdots (1 - \alpha_k)$. Under the assumptions of Theorem 3.2, we have the following analogue of (11):

$$\mathbb{E}\|y^{(k)} - x^*\| \leq a_k \left[ \|x^{(0)} - x^*\| + T \sum_{\ell=0}^{T} a_\ell^{-1} (\mathbb{E}\|\delta_\ell\| + \alpha_\ell^2 G^2) \right] < \varepsilon.$$ 

This yields an implicit formula for $k$, which can be used to upper bound the iteration cost for SGD. For example, a popular choice of $\alpha_k$ is $\alpha_k \propto 1/k$, in which case $a_k \propto 1/k$ (this follows from an induction argument), and solving for $k$ yields the desired upper bound.

**C Details of Models and Datasets**

In this brief appendix, we collect some details of the different models and datasets used in the experiments in Section 5.1.

**Multinomial Logistic Regression (MLR).** For MNIST, we use a batch size of 10,000, a learning rate of $1 \times 10^{-5}$, and a convergence criteria of $2.5 \times 10^4$ in cross-entropy loss. For CoverType, we use a batch size of 1,000, a learning rate of $1 \times 10^{-7}$, and a convergence criteria of $6.7 \times 10^5$ in cross-entropy loss. For both datasets, the convergence criteria is reached in roughly 60 iterations.

**Matrix Factorization (MF).** For MovieLens, we use 20 factors and a convergence criteria of $9.2 \times 10^2$ in mean squared error loss. For Jester, we use 5 factors and a convergence criteria of $5.57 \times 10^3$ in mean squared error loss. The MovieLens dataset is the movielens-small version consisting of 671 users and 9,125 items. The Jester dataset is the Jester 2+ version, We further remove users with no ratings, and re-scale ratings from $[-10, 10]$ to $[0, 10]$. For both datasets, the factor matrices $L$ and $R$ are randomly initialized with each entry sampled uniformly at random from $[0, 1)$. And the convergence criteria is reached in roughly 60 iterations.
**Latent Dirichlet Allocation (LDA).** In LDA, each document in the input data consists of a series of tokens, where each token is assigned a categorical topic. Topic assignments are repeatedly randomly sampled during the lifetime of a job. From these token-topic assignments, a document-topic distribution is constructed for each document, and a word-topic distribution is constructed for each unique word. Both the document-topic and word-topic distributions can be re-generated given the token-topic assignments, so losing the distributions themselves is not a problem. However, when distributed, each document-topic distribution is typically co-located with the document it corresponds to. Thus, losing a document-topic distribution is typically associated with also losing the token-topic assignments of that document, which do require recovery from a saved checkpoint. Therefore, we only consider the loss of document-topic distributions, and assume that the word-topic distributions can be reconstructed at any time.

Since the parameters of LDA are distributions, a natural norm to use is the total variation norm. However, the total variation norm when applied to LDA puts the same weight onto every document-topic distribution. This means that re-sampling a token-topic assignment in a shorter document has a greater impact on the overall norm than re-sampling a token-topic assignment in a longer document, which biases checkpoint prioritization towards shorter documents. To address this, we scale the total variation norm of each document-topic distribution by the length of the document it corresponds to. The result is still a valid norm, since it is a positive linear combination (which is constant with respect to the input data) of total variation norms.

For 20 Newsgroups, we use a convergence criteria of $9.5 \times 10^6$ in negative log-likelihood. For Reuters, we use a convergence criteria of $8.5 \times 10^5$ in negative log-likelihood. For both datasets we train using 20 topics and hyperparameters $\alpha = \beta = 1$. The convergence criteria is reached in roughly 60 iterations.

**Convolutional Neural Network (CNN).** We use a batch size of 64, the recommended Adam settings of $\alpha = 0.001$, $\beta_1 = 0.9$, $\beta_2 = 0.999$, and $\epsilon = 10^{-8}$, and a convergence criteria of 0.08 in cross-entropy loss. In by-layer partitioning, the weight and bias parameters are independent and partitioned separately (so they can either be lost together, or not). In by-shard partitioning, each parameter tensor is evenly partitioning according to its first dimension.