Strong electroweak phase transition in a model with extended scalar sector

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Abstract

In this paper we consider an extension of the Standard Model (SM) with additional gauge singlets which exhibits a strong first order phase transition. Due to this first order phase transition in the early universe gravitational waves are produced. We estimate the contributions such as the sound wave, the bubble wall collision and the plasma turbulence to the stochastic gravitational wave background, and we find that the strength at the peak frequency is large enough to be detected at future gravitational interferometers such as eLISA. Deviations in the various Higgs boson self couplings are also evaluated.

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1 Introduction

The discovery of a narrow resonance, with a mass near 125 GeV, at the Large Hadron Collider (LHC) with properties similar to those of the Higgs boson predicted by the Standard Model (SM) [1, 2], sparked a lot of excitement among high energy physicists. But in spite of this important discovery the (SM) is considered to be incomplete. For instance, in the Standard Model of particle physics a strong first order phase transition (SFPT) does not occur [3]. However SFPT is needed to justify the baryon asymmetry of our universe [4], moreover, the SM does not have a candidate for dark matter (DM).

Therefore, some new models are required to address these issues. A popular model is to couple a singlet scalar to Higgs boson. In Ref. [5] it is shown that, it is possible to modify the standard theory by adding a scalar which possesses a discrete $Z_2$ symmetry and to address the issue of

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the dark matter of the universe, within the framework of singlet extended SM, the issue of dark matter has been studied in [6–13], while electroweak phase transition was studied in Refs. [14 – 23] and in Refs. [24 – 25] the authors attempt to explain electroweak phase transition EWPT and dark matter by singlet extended SM.

Another class of models are the multi-singlet extensions of the SM model [26 – 36]. These models have a larger parameter space in comparison to the singlet extended models, hence they can address several issues, in Refs. [26, 27] cosmological implications of such models with classical conformal invariance is presented. Electroweak phase transitions in two-Higgs doublet model is analyzed in [37, 38] and within supersymmetric models in [39 – 45]. A comprehensive review of EWPT within various models has been given in [46].

In order to investigate the dynamics of the electroweak phase transition EWPT one has to utilize techniques from the domain of thermal field theory [47 – 50]. The occurrence of a first order phase transition, requires that the electroweak breaking and preserving minima to be degenerate, an event which happens at a critical temperature $T_c$. Moreover, to prevent the washout of any baryon asymmetry by electroweak sphalerons, the electroweak phase transition must be strongly first order. Namely the ratio of vacuum expectation value of the Higgs field to the critical temperature needs to be greater than unity. As we describe in the next section if a first order phase transition occurs in the early universe, the dynamics of bubble collision and subsequent turbulence of the plasma are expected to generate gravitational wave (GW). If we detect these GW then we can obtain information about symmetry breaking in early universe. GW signals from phase transitions has been discussed in [51 – 59].

In [60] the authors study EWPT within several exotic models and in [61] an analysis of the EWPT of a large number of minimal extensions of the SM and their classically conformal limits is presented. Complex conformal singlet extension of SM with emphasis on the issue of dark matter and Higgs phenomenology is studied in [62]. Recently baryogenesis within a $\varphi^6$ model is addressed in [63]. An investigation of electroweak phase transitions in a singlet extended model in the 100 (TeV) range is given in [64]. The authors of [65] study strong first order EWPT in a singlet scalar extension of the SM where the singlet scalar is coupled non-minimally to gravity. In this scheme the singlet field first derives inflation and at a later time causes a strong EWPT, in a new study a first order EWPT in the SM is obtained by varying Yukawas during phase transition [66].

In this work we propose a new model and we investigate the strength of EWPT within this model. In this model which is a generalization of [19], N real gauge singlet are coupled to Higgs boson via trilinear interactions. Previous studies of multi-scalar singlet extension of SM impose separate $Z_2$ symmetries on the singlets [26, 28, 29, 31, 33, 35, 36], however in our model we do not require such symmetry. In spite of it’s simple form, this model has a very rich phenomenology. The main feature of the singlet extended SM model (without $Z_2$ symmetry) is that the potential barrier between the true and the false vacua necessary for first order EWPT can be formed mainly by tree-level interactions. But in the singlet extended SM (with $Z_2$ symmetry) non-decoupling loop effects are needed for the occurrence of a strong first order EWPT, however, these models have DM candidate. In Ref. [67] a non-minimal composite model based on the coset $\frac{SO(7)}{SO(6)}$ has been considered. At low energy the scalar sector of their model is composed of two scalars, one with an unbroken $Z_2$ symmetry and another scalar with
a broken $Z_2$ symmetry.

The plan of this paper is as follows:

In section two we summarize the basic notions of $EWPT$. We describe the finite temperature effective potential at one-loop. Then by emphasizing the underlying physical mechanisms, we describe the basic quantities of interest such as the strength of a phase transition, the rate of variation of bubble nucleation rate per volume and the ratio of the latent heat released at the phase transition to the radiation energy density. In section three we consider a simple extension of the $SM$ by the addition of $N$ real scalar gauge singlets with trilinear coupling to Higgs. We present the phenomenology of the model for $N = 2$ and due to lack of protective symmetry these gauge singlets are not candidates for $DM$ and we discuss the issue of observability of gravitational wave of our model. And finally in section four we present our conclusions. Technical details are explained in the appendix.

2 Electroweak Phase Transition

In this section we summarize basic notions and definitions of the electroweak phase transitions. The observable universe consists predominantly of matter. The asymmetry between the matter and anti-matter content of the universe is expressed by the baryon to photon ratio

$$\rho = \frac{n_b - n_b^\bar{}}{n_\gamma} \sim 10^{-9}, \quad (1)$$

where $n_b$, $n_b^\bar{}$ and $n_\gamma$ are the number densities of baryons, antibaryons and photons. In a symmetric universe one expects $\rho = 0$ but experiments reveal that $\rho$ has a tiny but non-zero value. This paradox can be resolved by requiring baryon number violation, $C$ and CP violation and departure from thermal equilibrium [4].

Baryogenesis is the physical process which is responsible for this observed baryon asymmetry of universe (BAU). In electroweak baryogenesis one assumes that the physical mechanism is the occurrence of a strong first order electroweak phase transition (EWPT) namely a smooth or a weak phase transition can not explain BAU [68,69].

A convenient tool for investigation of EWPT is effective potential. For any quantum field theory if we replace the quantum field by it’s vacuum expectation value in the presence of a source the result for the potential energy to lowest order in perturbation theory is called the effective potential. The physical meaning of the effective potential is that it represent an energy density. In general effective potential contains other terms (loop corrections) [70,71]. By using path integrals it is possible to find effective potential. In an Euclidean space time at one loop order the result is

$$V^{\text{eff}}(\varphi_c) = V_0(\varphi_c) + \frac{i}{2} \int \frac{d^4p}{(2\pi)^4} \log\left[\frac{p^2 + m^2(\varphi_c)}{p^2}\right] + O(h^2) + ..., \quad (2)$$

which is known as the Coleman-Weinberg potential. The first term in eq.(2) is the classical tree-level potential.
While considering physical processes in a hot environment such as early universe a proper method is finite temperature field theory \([47 - 50]\). The expression for the one loop effective potential at finite temperature is \([72]\)

\[
V_{1}^{\text{eff}}(\varphi_{c}, T) = \sum_{i} n_{i} T^{4} J_{\pm}^{} \left( \frac{m_{i}(\varphi_{c})}{T} \right), \tag{3}
\]

where \(n_{i}\) is the number of degrees of freedom of the particle, \(m_{i}\) is the field dependent particle mass, and

\[
J_{\pm} = \pm \int_{0}^{\infty} dy \frac{y^{2}}{2\pi} \log[1 \mp \exp(-\sqrt{x^{2} + y^{2}})], \tag{4}
\]

and \(J_{-}(x), J_{+}(x)\) denotes the contribution from bosons, fermions. In addition there is another contribution to thermal effective potential from the daisy subtraction \([73, 74]\).

The best way for the evaluation of the finite temperature effective potential is to use special packages (codes) developed for a specific model. But when the temperature is much greater than the masses of the various particles of the system under considerations it is possible to expand the \(J_{\pm}\) in a power series of \(m_{i} T\). In this work we use this high temperature approximation. Recently, a new scheme for computation and resummation of thermal masses beyond the high-temperature approximation in general beyond SM scenarios has been proposed \([75]\).

Let us consider the shape of the effective potential. For a generic model, as the universe cooled down at temperature above a critical temperature \(T_{c}\) the effective potential had an absolute minima which was located at the origin. At \(T_{c}\) there were two degenerate minima, which were separated by an energy barrier. At temperature below the critical temperature the second minimum became the global one and presently there is no energy barrier. Moreover the rate of expansion of the universe slowed, as the Hubble parameter \(H\) which characterizes the rate of expansion of the universe depends quadratically on the temperature.

In 1976 \(t\) Hooft discovered that baryon number is violated in the Standard Model and it is due to transition between two topologically distinct \(SU(2)_{L}\) ground states \([76, 77]\). While at zero temperature, the probability for barrier penetration is vanishingly small, at non-zero temperature the transition between two ground state differing by a unit of topological charge can be achieved by a classical motion over the barrier. Unstable static solutions of the field equations with energy equal to the height of the barrier separating two topologically distinct \(SU(2)_{L}\) ground states (sphalerons) have been reported in \([78]\). Hence in the electroweak baryogenesis, the baryon asymmetry of universe is generated through the sphaleron process in the symmetric phase during the \(EWPT\). At the symmetric phase the rate of baryon violating process which we denote by \(\tilde{\Gamma}^{\text{sph}}\) is a quartic function of the temperature, hence in this phase \(\tilde{\Gamma}^{\text{sph}} \gg H\).

But the third conditions to generate \(\text{BAU}\) is the departure from thermal equilibrium. Hence, the baryon number changing sphaleron interaction must quickly decouple in the broken phase, that is \(\tilde{\Gamma}^{\text{sph}} < H\). But the rate of sphaleron induced baryon violating process is suppressed by a Boltzmann factor

\[
\tilde{\Gamma}^{\text{sph}} \propto \exp\left[ -\frac{E^{\text{sph}}(T)}{T} \right], \tag{5}
\]
but at phase transition $E_{sph}(T) \propto \varphi_c$

\[
\frac{\varphi_c}{T_c} \geq 1,
\]

where $\varphi_c$ is the broken phase minimum at the critical temperature $T_c$.

A first order phase transition proceeds by nucleation of bubbles of the broken symmetry phase within the symmetric phase. The underlying mechanisms for bubble nucleation are quantum tunneling and thermal fluctuations. These bubbles then expand, merge and collide. Gravitational waves are produced due to collision of bubble walls and turbulence in the plasma after the collisions. In addition while these bubbles pass through the plasma sound waves are created. These sound waves can provide additional sources of gravitational waves.

A crucial parameter for the calculation of the gravitational wave spectrum is the rate of variation of the bubble nucleation rate per volume, called $\beta$. It is common to use a normalized dimensionless parameter which is defined as

\[
\tilde{\beta} = \frac{\beta}{H_*},
\]

where, $H_*$ denotes the Hubble parameter at the time of phase transition. The density of latent heat released into the plasma is

\[
\epsilon_* = [-V_{min}^{eff}(T) + T \frac{d}{dT} V_{min}^{eff}(T)]_{T=T_*},
\]

where $V_{min}^{eff}(T)$ is the temperature-dependent true minimum of the effective potential of the scalar fields which causes the phase transition moreover, it’s value must be set to zero by adding a constant at each time. Another dimensionless parameter for characterizing the spectrum of gravitational wave is

\[
\alpha = \frac{\epsilon_*}{\rho_{rad}},
\]

where the radiation energy density of the plasma $\rho_{rad} = \frac{\pi^2}{30}g_*T^4$. And the parameter $g_*$ is the effective degrees of freedom in the thermal bath at the phase transition. In this work we assume $g_* = 106.75 + N_S$ where $N_S$ denotes the number of singlet scalars that facilitates the electroweak phase transition.

3. The Model

In Ref. [19] the effects of a light scalar on the electroweak phase transition has been considered. In their model the scalar sector has been extended by an addition of a singlet, which has a trilinear interaction with the Higgs boson. Recently an extension of the SM with addition of $N$ isospin-singlet, which has a quartic interaction with the Higgs boson has been considered [57]. Here we consider a generalization of model of [19]. At zero temperature the effective potential
of the scalar sector of our model is

\[ V_0 = -DT_0^2 \varphi^2 + \frac{\lambda}{4} \varphi^4 + \frac{1}{2} \sum_{i=1}^{N} m_i^2 s_i^2 + \varphi^2 \sum_{i=1}^{N} \kappa_i s_i. \]  

(10)

Since we have not included quartic self coupling for the extra scalars, in order to have a stable potential, we assume that the squared of the mass parameters of the extra scalars \( m_i^2 \) are positive definite (See appendix for detail).

At high temperature we have

\[ V_T = D(T^2 - T_0^2) \varphi^2 - ET \varphi^3 + \frac{\lambda_T}{4} \varphi^4 + \frac{1}{2} \sum_{i=1}^{N} m_i^2 s_i^2 + \varphi^2 \sum_{i=1}^{N} \kappa_i s_i + \frac{T^2}{12} \sum_{i=1}^{N} \kappa_i s_i. \]  

(11)

The parameters of eq.(11) are given by

\[ D = \frac{1}{8v^2}(2m_W^2 + m_Z^2 + 2m_t^2 + 2\lambda v^2) \]

\[ E = \frac{1}{8\pi v^3}(4m_W^3 + 2m_Z^3) \]

\[ \lambda_T = \lambda - \frac{1}{16\pi^2 v^4}(6m_W^4 \ln \frac{m_w}{a_B T^2} + 3m_Z^4 \ln \frac{m_Z}{a_B T^2} - 12m_t^4 \ln \frac{m_t}{a_F T^2}) \]

\[ \ln a_B = 3.91, \quad \ln a_F = 1.14. \]  

(12)

The critical temperature of model is given by

\[ T_c = \frac{T_0}{\sqrt{1 - \frac{E^2}{D(\lambda_T - 2\zeta)}} - \frac{\zeta}{2D}}, \quad \text{where} \quad \zeta = \sum_{i=1}^{N} \frac{\kappa_i^2}{m_i^2}. \]  

(13)

The strength of the phase transition is denoted by \( \xi \) and it is given by

\[ \xi = \frac{\varphi_c}{T_c} = \frac{2E}{\lambda_T - 2\zeta}. \]  

(14)

Moreover, \( s_{ic} \) is the vev of the scalar field \( s_i \) at the second minimum of the effective potential at \( T_c \) is

\[ s_{ic} = -\frac{\kappa_i}{m_i^2}(\varphi_c^2 + \frac{T_c^2}{12}). \]  

(15)

3.1 Phenomenology of the models with N trilinear interactions

The structure of the scalar mass matrix is

\[
\begin{pmatrix}
2\lambda v^2 & 2\kappa_1 v & 2\kappa_2 v & 2\kappa_3 v & \ldots & \ldots \\
2\kappa_1 v & m_1^2 & 0 & 0 & 0 & 0 \\
2\kappa_2 v & 0 & m_2^2 & 0 & 0 & 0 \\
2\kappa_3 v & 0 & m_3^2 & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\end{pmatrix}
\]
The physical mass squared of the scalar sector of our model are the eigenvalues of this matrix. The characteristic equation is

\[
\det \begin{pmatrix}
\omega - 2\lambda v^2 & -2\kappa_1 v & -2\kappa_2 v & -2\kappa_3 v & \ldots & \ldots \\
-2\kappa_1 v & \omega - m_1^2 & 0 & 0 & 0 & 0 \\
-2\kappa_2 v & 0 & \omega - m_2^2 & 0 & 0 & 0 \\
-2\kappa_3 v & 0 & \omega - m_3^2 & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\end{pmatrix} = 0.
\]

Hence in general one should solve a polynomial of degree \((N+1)\) of \(\omega\). From the above equation we see that the coefficient of \(\omega^{N+1}\) is unity. If we designate the coefficient of \(\omega^N\) by \(\alpha\), then

\[-\alpha = \sum_{i=1}^{N} \chi_i^2 - m_i^2 \]

where the physical mass of the \(i\)th scalar is denoted by \(\chi_i\). From this relation we find an important relation about the Higgs self coupling, namely

\[
\lambda - \lambda_{SM} = \frac{1}{2v^2} \sum_{i=1}^{N} (\chi_i^2 - m_i^2),
\]

where \(\lambda_{SM}\) is the Higgs quartic self coupling of the standard model. We see that if the mass parameters of the scalars as well as the physical masses of the scalars are much smaller than \(v\) then the the deviation of the parameter \(\lambda\) from the Higgs self-coupling of the standard model will be very small.

### 3.2 The special case \(N=2\)

Here we consider the case \(N = 2\). The scalar mass matrix of the model at zero temperature for this case is.

\[
M^2 = \begin{pmatrix}
2\lambda v^2 & 2\kappa_1 v & 2\kappa_2 v \\
2\kappa_1 v & m_1^2 & 0 \\
2\kappa_2 v & 0 & m_2^2
\end{pmatrix}.
\]

The physical mass squared of the model can be obtained from

\[
\omega^3 + A\omega^2 + B\omega + C = 0,
\]

where

\[
A = -(2\lambda v^2 + m_1^2 + m_2^2), \quad B = 2v^2[\lambda(m_1^2 + m_2^2) - 2(\kappa_1^2 + \kappa_2^2)] + m_1^2 m_2^2 \\
C = 4v^2(\kappa_1^2 m_2^2 + \kappa_2^2 m_1^2) - 2\lambda^2 v^2 m_1^2 m_2^2.
\]

Now one of the eigenvalues is equal to \(m_H^2\), hence we obtain

\[
m_H^6 + Am_H^4 + Bm_H^2 + C = 0,
\]
Moreover, by minimizing the potential with respect to variables $s_1, s_2$ we obtain

$$s_i = -\frac{\kappa_i v^2}{m_i^2}, \quad i = 1, 2. \quad (21)$$

By minimizing the potential with respect to variable $\varphi$ we obtain

$$-2 DT_0^2 + \lambda v^2 - 2 \sum_{i=1}^{2} \frac{\kappa_i^2 v^2}{m_i^2} = 0. \quad (22)$$

The parameters of the model must satisfy eqs.(20, 22). Hence the parameter space of the model in this case contains four independent parameters $(\kappa_1, \kappa_2, m_1, m_2)$.

If we subtract eq.(20) from eq.(18) we get

$$\chi^4 + (A + m_H^2) \chi^2 + m_H^4 + m_H^2 A + B = 0, \quad (23)$$

Therefore, the physical mass squared of the singlets are determined. But models with extended Higgs sectors predicting strongly first order phase transition simultaneously predict a significant deviation in the triple Higgs boson coupling as well [79]. This deviation at the tree level in [17] and at loop level in [33, 79] has been studied, with

$$\Delta_{hhh} = \frac{\lambda_{hhh}^{MSM} - \lambda_{hhh}^{SM}}{\lambda_{hhh}^{SM}}, \quad (24)$$

where $\lambda_{hhh}^{SM}$ is the Higgs triple coupling of the $SM$ and $\lambda_{hhh}^{MSM}$ is the Higgs triple coupling of the multi-singlet extension of the $SM$. Collider experiments could measure the Higgs triple coupling. Here we want to explore the region of intermediate mass of the singlets. But there are bounds on Higgs-Portal models from the LHC Higgs data [80, 81, 82]. For instance the Higgs doublet is mixed with the extra singlet scalars and the mixing element $\cos(\varphi)$ between the CP-even component of the doublet ($\varphi$) and the physical Higgs (whose $m_H=125.09$ GeV) is not arbitrary and it is subject of a constraint coming from the Higgs coupling to the W gauge bosons. Current data [81] suggests $\cos(\varphi) > 0.86$.

The results are presented in Table 1 for the onset of a strong $EWPT$, namely $\xi = 1$. For each configuration in the table we present our results for the deviation of Higgs triple coupling as well. Hence by adding one scalar to the model we can have singlets in the intermediate mass region.
\[ \kappa_1, \kappa_2, m_1(\text{GeV}), m_2(\text{GeV}), \chi_1(\text{GeV}), \chi_2(\text{GeV}), \cos(\varphi), \Delta \lambda_{hh}, \Delta \lambda_{hhhh} \]

| Set | \kappa_1 | \kappa_2 | m_1(\text{GeV}) | m_2(\text{GeV}) | \chi_1(\text{GeV}) | \chi_2(\text{GeV}) | \cos(\varphi) | \Delta \lambda_{hh} | \Delta \lambda_{hhhh} |
|-----|-----------|-----------|-----------------|-----------------|-----------------|-----------------|---------------|----------------|------------------|
| I   | 0.2       | 3.7       | 69.2            | 16.0            | 6.2             | 69.2            | 0.993         | 13.9 \%        | 4.1 \%           |
| II  | 0.5       | 3.1       | 18.7            | 13.5            | 5.2             | 18.6            | 0.995         | 14.5 \%        | 3.0 \%           |
| III | 0.8       | 2.3       | 11.2            | 10.4            | 4.0             | 11.1            | 0.997         | 15.1 \%        | 1.8 \%           |
| IV  | 1.2       | 0.7       | 18.1            | 3.2             | 17.5            | 1.3             | 0.999         | 5.7 \%         | 0.6 \%           |

Table 1: Different configurations associated with the onset of a strong first order phase transition ($\xi = 1$).

For completeness the predictions of the model for the deviation of Higgs boson quartic coupling $\Delta \lambda_{hhhh}$ are given in Table 1. Moreover, the existence of the extra singlet scalars could affect the total Higgs decay if they are light enough, which becomes

\[ \Gamma_{\text{total}}(h) = \cos^2(\varphi)\Gamma_{\text{total}}(h_{\text{SM}}), + \sum \Gamma(h \to s_i + s_k) \] (25)

where $s$ denote all the scalars. The deviation from the $SM$ value $\Delta \Gamma_{\text{total}} < 1.4(\text{MeV})$. In the $SM$, a total Higgs decay width around $4 \text{ MeV}$ is predicted. In this work we assume the decay $h \to s_i + s_k$ is an invisible decay. However, current analysis [82] suggests that the Higgs invisible decay branching ratio should be less than 17%. In Table 2 we present Higgs invisible decay branching ratio in various decay modes, as well the total Higgs invisible branching ratio for all of Higgs invisible decay modes. The invisible Higgs width, total Higgs width and the deviation of total Higgs width of our model from that of the $SM$ are also shown. The unit for the various width in this Table is (MeV), moreover in our calculation we have assumed $\Gamma_{\text{total}}(h_{\text{SM}}) = 4(\text{MeV})$.

It turns out that the critical temperature for this model $T_c < 100(\text{GeV})$. Hence it is a good approximation to use $\lambda$ instead of $\lambda_T$ and in Ref. [19] this approximation is used to study $SFPT$ for the case $N = 1$ but the mass of the light scalar is up to 12 GeV, in Ref. [20] a one loop study of the same model has been presented but the mass of the light scalar to catalyze a $SFPT$ is up to 20 GeV. But for the model presented in this work and using this approximation for the case $N = 2$ the mass of the scalar to catalyze a $SFPT$ is 69 GeV. By adding more scalars we expect to have heavy singlets.
Table 2: Values of Higgs invisible branching ratios for various decay modes, total value of Higgs invisible branching ratio, Higgs invisible width, Higgs total width and deviation of the Higgs width from the SM value are shown for various configurations. The unit for width in this table is $\text{MeV}$. All four configurations are consistent with current experimental data.

3.3 Detection of gravitational waves

The direct detection of gravitational waves by LIGO [83] generated a lot of interest among researchers in cosmology, astrophysics and particle physics. Major sources of gravitational waves are inflation, compact binary systems or cosmological phase transitions.

In this section under phenomenological description we aim to make an estimate for the observability of the gravitational waves produced by the model presented in this work. In all of previous multi-singlet models a $Z_2$ symmetry (either broken or unbroken) is imposed on the fields. However, neither of the two fields of our model has this symmetry. This is the main difference between our model and previous studies.

The main parameters that are of value for obtaining the spectrum of gravitational waves are the parameter $\alpha$ and $\beta$, which we discussed in section two. And they are obtained from the thermal effective potential.

For the set $I$ of Table 1 and from eqs. (8,9) we obtain $\alpha = 0.16$. But the velocity of the bubble wall and the efficiency factor, the fraction of the latent heat which is converted to the kinetic energy of the plasma are determined from

$$v_b = \frac{1}{\sqrt{3}} + \sqrt{\frac{\alpha^2 + \frac{2\alpha}{3}}{1 + \alpha}}, \quad \kappa = \frac{1}{1 + 0.715\alpha}(0.715\alpha + \frac{4}{27}\sqrt{\frac{3\alpha}{2}}),$$

Thus for this configuration $v_b = 0.81$ and $\kappa = 0.17$. The peak frequency for bubble collision contribution is given by [84]

$$f_{col} = 16.5 \times 10^{-6} \frac{0.62}{v_b^2 - 0.1v_b + 1.8} \frac{\beta}{H_* \text{100}} \frac{T_*}{H_* \text{100}} \left( \frac{g_*}{100} \right)^{\frac{1}{2}}, \quad Hz$$

And the energy density at this frequency is given by

$$\Omega h^2_{col} = 1.67 \times 10^{-5} \left( \frac{\beta}{H_*} \right)^2 \frac{0.11v_b^3}{0.42 + v_b^2} \left( \frac{\kappa\alpha}{1 + \alpha} \right)^2 \left( \frac{g_*}{100} \right)^{\frac{3}{2}}$$
where \( h \) is the reduced Hubble constant at present.

Another source of gravitational waves is the compression waves in the plasma (sound waves) and the peak frequency is

\[
f_{sw} = 1.9 \times 10^{-5} \frac{\beta}{H_*} v_b^{-1} T_* \left( \frac{g_*}{100} \right)^{\frac{1}{6}}, \quad Hz
\]  

(29)

the energy density at this peak frequency is obtained from

\[
\Omega h^2_{sw} = 2.65 \times 10^{-6} \left( \frac{\beta}{H_*} \right)^{-1} \left( \frac{\kappa \alpha}{1 + \alpha} \right)^2 \left( \frac{g_*}{100} \right)^{\frac{1}{6}} v_b.
\]  

(30)

And finally the peak frequency for gravitational waves which are caused by the turbulence of the plasma is

\[
f_{turb} = 2.7 \times 10^{-5} \frac{\beta}{H_*} v_b^{-1} T_* \left( \frac{g_*}{100} \right)^{\frac{1}{6}}, \quad Hz,
\]  

(31)

And the peak energy density of this part is given by

\[
\Omega h^2_{turb} = 3.35 \times 10^{-4} \left( \frac{\beta}{H_*} \right)^{-1} \left( \frac{\varepsilon \kappa \alpha}{1 + \alpha} \right)^{\frac{5}{3}} \left( \frac{g_*}{100} \right)^{\frac{1}{6}} v_b \frac{1}{2^{\frac{5}{3}} \left( 1 + 8\pi \frac{f_{turb}}{H_*} \right)}
\]  

(32)

where \( \varepsilon \) denotes the fraction of latent heat that is transformed into turbulent motion of the plasma. We choose \( \varepsilon = 0.05 \).

Hence by knowing the values of the parameters \( \alpha, \tilde{\beta} \) and \( T_* \) we can obtain the spectra of the GW. Even though the nucleation temperature \( T_* \) is lower than the critical temperature in this work we assume \( T_* \approx T_c \).

The standard method of calculation of the parameter \( \tilde{\beta} \) from the effective potential is to compute the Euclidean action of the model and it is explained in [36, 85], but in an approximate scheme it is found that [59],

\[
\tilde{\beta} \approx 170 - 4 \ln \left( \frac{T_*}{1 GeV} \right) - 2 \ln g_*,
\]  

(33)

and in our case when the phase transition happens at weak scale, \( \tilde{\beta} \approx 144 \).

Hence, in order to assess the implications of the model on the spectra of GW we present results by varying this parameter in the interval \( 50 \leq \tilde{\beta} \leq 250 \). In Table 3 we present our results. The unit for various frequencies is \( mHz \).

| GW Spectra | \( f_{sw} \) | \( f_{col} \) | \( f_{turb} \) | \( \Omega h^2_{sw} \) | \( \Omega h^2_{col} \) | \( \Omega h^2_{turb} \) | \( \Omega h^2_{total} \) |
|------------|----------|----------|----------|----------------|----------------|----------------|----------------|
| \( \tilde{\beta} = 50 \) | 0.65     | 0.12     | 1.85     | 2.34 \times 10^{-11} | 1.98 \times 10^{-13} | 1.67 \times 10^{-15} | 2.36 \times 10^{-11} |
| \( \tilde{\beta} = 100 \) | 1.30     | 0.24     | 3.69     | 1.17 \times 10^{-11} | 9.9 \times 10^{-14} | 8.35 \times 10^{-16} | 1.18 \times 10^{-11} |
| \( \tilde{\beta} = 250 \) | 3.25     | 0.60     | 9.23     | 4.67 \times 10^{-12} | 3.96 \times 10^{-14} | 3.34 \times 10^{-16} | 4.71 \times 10^{-12} |

Table 3: The spectra of gravitational wave as predicted by our model. The emitted GW are within the reach of eLISA C1.
The results of Table 3 shows that in our model the contribution from the turbulent motion of always a few orders of magnitude smaller than the previous two. Moreover, the contribution from the sound waves is the dominant source of the total GW spectrum. By studying the frequency dependent spectra [84], we find that the peak of energy density of the sound wave contribution and the peak of energy density due to collision are well separated. A desirable feature while detecting these waves. These waves are within the reach of future gravitational wave interferometers (eLISA C1).

4 Conclusions

In this work we have presented a new extension of the SM. In this model we amend the SM by \( N \) gauge singlets. In order to avoid proliferation of the parameters we considered the most economical model. For each scalar we allowed a mass term with a positive squared mass parameter to insure vacuum stability in the direction of that scalar and a triple coupling with the Higgs field to facilitate a strong electroweak phase transition, and for the special case \( N = 2 \) we find SFPT with gauge singlets in the intermediate mass range. We also investigated the deviations of the Higgs coupling constants from the SM values. And we find that the deviation of the triple Higgs boson coupling can be as large 15%.

Finally, we have obtained the gravitational wave spectrum from the electroweak phase transition. We have shown that the gravitational wave signal can be detected by eLISA. The present model has a large parameter space and as a result a richer phenomenology in comparison to [19]. It would be of interest to amend the model by inclusion of quartic self-coupling of the scalars as well as the mixing term between scalars (see appendix for detail).

It would be of interest to consider higher values of \( N \) for the model proposed in this work. These and other related issues are presently under considerations.

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Appendix : Vacuum stability conditions

For the special case \( N = 2 \) of the model the weak condition for vacuum stability is that if the value of the fields tend to infinity then the tree-level potential is not unbounded-from-below directions. This leads to \( \lambda > 0 \). But if only the field \( s_1 \) tend to infinity, then the condition vacuum stability will be \( m_1^2 > 0 \) and by similar argument for the field \( s_2 \) the restriction will be \( m_2^2 > 0 \).

However, the strong condition for vacuum stability as stated in [35, 86, 87] is that the masses of the particles of a particular model be positive.
The necessary conditions for a symmetric matrix $A$ of order 3 to have real eigenvalues are:

\[
\begin{align*}
    a_{11} &> 0, a_{22} > 0, a_{33} > 0, \\
    \bar{a}_{12} &= a_{12} + \sqrt{a_{11}a_{22}} > 0, \\
    \bar{a}_{13} &= a_{13} + \sqrt{a_{11}a_{33}} > 0, \\
    \bar{a}_{23} &= a_{23} + \sqrt{a_{22}a_{33}} > 0,
\end{align*}
\]

and

\[
\sqrt{a_{11}a_{22}a_{33}} + a_{12}\sqrt{a_{33}} + a_{13}\sqrt{a_{22}} + a_{23}\sqrt{a_{11}} + \sqrt{2\bar{a}_{12}\bar{a}_{13}\bar{a}_{23}} > 0.
\]

The constraints stated in eq. (34) applies to quantities with dimension of squared mass. From our mass matrix of subsection 3, the first constraint is $2\lambda v^2 > 0$, which leads to $\lambda > 0$. Hence, for the model presented in this work, the criteria which provide the necessary and sufficient vacuum stability conditions are given by,

\[
\begin{align*}
    \lambda &> 0, \quad m_1^2 > 0, \quad m_2^2 > 0, \\
    \kappa_1 &> -\sqrt{\frac{\lambda m_1^2}{2}}, \quad \kappa_2 > -\sqrt{\frac{\lambda m_2^2}{2}},
\end{align*}
\]

and

\[
\sqrt{\lambda m_1^2 m_2^2} + \kappa_1 \sqrt{2m_2^2} + \kappa_2 \sqrt{2m_1^2} + \sqrt{(2\kappa_1 + 2\lambda m_1^2)(2\kappa_2 + 2\lambda m_2^2)}\sqrt{m_1^2 m_2^2} > 0,
\]

where the conditions stated in the first line of eq.(36) has been obtained by using weak conditions for vacuum stability.

In our model we have not included quartic terms for the gauge singles such as

\[
V_{\text{quartic}} = \sum_{i=1}^{2} \frac{\lambda_i s_i^4}{4} + \delta s_1^2 s_2^2
\]

in the effective potential, where $\lambda_1$, $\lambda_2$ and $\delta$ are dimensionless coupling parameters, in this case the vacuum stability in the direction of the extra scalars is maintained if

\[
\lambda_1 > 0, \quad \lambda_2 > 0, \quad \sqrt{\lambda_1\lambda_2} > -2\delta,
\]

and the squared of the mass parameters $m_1^2$ and $m_2^2$ in principle can assume any value (positive, zero or negative), for instance in the multi-scalar model described in [26] all of the extra scalars do not have a mass term, while in the absence of quartic terms the squared mass parameters of the model presented in section three must be positive as required by vacuum stability.
References

[1] ATLAS Collaboration, G. Aad et al., Phys. Lett. B 716 (2012) 1.
[2] CMS Collaboration, S. Chatrchyan et al., Phys. Lett. B 716 (2012) 30.
[3] G. W. Anderson and L. J. Hall, Phys. Rev. D 45 (1992) 2685.
[4] A. Sakharov, Pisma Zh. Eksp. Teor. Fiz. 5 (1967) 32.
[5] V. Silveira and A. Zee, Phys. Lett. B 161 (1985) 136.
[6] J. McDonald, Phys. Rev. D 50 (1994) 3637.
[7] C. P. Burgess, M. Pospelov and T. ter Veldhuis, Nucl. Phys. B 619 (2001) 709.
[8] J. J. van der Bij, Phys. Lett. B 636 (2006) 56.
[9] V. Barger, P. Langacker, M. McCaskey, M. J. Ramsey-Musolf and G. Shaughnessy, Phys. Rev. D 77 (2008) 035005.
[10] W. -L. Guo, Y. -L. Wu, JHEP 1010 (2010) 083.
[11] C. E. Yaguna, JHEP 1108 (2011) 060.
[12] R. Coimbra, M. O. P. Sampaio and R. Santos, Eur. Phys. J. C 73 (2013) 2428.
[13] L. Feng, A. S. Profumo and B. L. Ubaldi, JHEP 03 (2015) 045.
[14] J. R. Espinosa and M. Quiros, Phys. Lett. B 305 (1993) 98.
[15] J. Choi and R. R. Volkas, Phys. Lett. B 317 (1993) 385.
[16] S. Profumo, M. J. Ramsey-Musolf, G. Shaughnessy, JHEP 0708 (2007) 010.
[17] A. Noble and M. Perelstein, Phys. Rev. D 78 (2008) 063518.
[18] A. Ashoorioon and T. Konstandin, JHEP 0907 (2009) 086.
[19] S. Das, P. J. Fox, A. Kumar and N. Weiner, JHEP 1011 (2010) 108.
[20] J. R. Espinosa, T. Konstandin, and F. Riva, Nucl. Phys. B 854 (2012) 592.
[21] T. Alanne, K. Tuominen, and V. Vaskonen, Nucl. Phys. B 889 (2014) 692.
[22] D. Curtin, P. Meade and C. T. Yu, JHEP 1411 (2014) 127.
[23] S. Profumo, M J. Ramsey-Musolf, C. L. Wainwright and P. Winslow, Phys. Rev. D 91 (2015) 035018.
[24] J. M. Cline, K. Kainulainen, P. Scott and C. Weniger, Phys. Rev. D 88 (2013) 055025.
[25] J. M. Cline, K. Kainulainen, JCAP 1301 (2013) 012.
[26] J. R. Espinosa and M. Quiros, Phys. Rev. D 76 (2007) 07600.
[27] J. R. Espinosa, T. Konstandin, J. M. No and M. Quiros, Phys. Rev. D 78 (2008) 123528.
[28] A. Drozd, B. Grzadkowski and J. Wudka, Acta Phys. Polon. B 42 (2011) 2255.
[29] A. Abada, S. Nasri and D. Ghaffor, Phys. Rev. D 83 (2011) 095021.
[30] A. Abada and S. Nasri, Phys. Rev. D 85 (2012) 075009.
[31] A. Drozd, B. Grzadkowski and J. Wudka, JHEP 1204 (2012) 006.
[32] A. Ahriche and S. Nasri, Phys. Rev. D 85 (2012) 093007.
[33] A. Ahriche, A. Arhrib and S. Nasri, JHEP 02 (2014) 042.
[34] A. Tofighi, O. N. Ghodsi and M. Saeedhoseini, Phys. Lett. B 748 (2015) 208.
[35] K. P. Modak, D. Majumdar and S. Rakshit, JCAP 1503 (2015) 011.
[36] R. Jinno, K. Nakayama and M. Takimoto, Phys. Rev. D 93 (2016) 045024.
[37] J. M. Cline and P.-A. Lemieux, Phys. Rev. D 55 (1997) 3873.
[38] L. Fromme, S. J. Huber, and M. Seniuch, JHEP 0611 (2006) 038.
[39] S. J. Huber and M. G. Schmidt, Eur. Phys. J. C 10 (1999) 473.
[40] K. Funakubo and E. Senaha, Phys. Rev. D 79 (2009) 115024.
[41] D. J. H. Chung and A. J. Long, Phys. Rev. D 81 (2010) 123531.
[42] J. Kozaczuk, S. Profumo, L. S. Haskins and C. L. Weinwright, JHEP 1501 (2015) 144.
[43] C. Balazs, A. Mazumdar, E. Pukartas and G. White, JHEP 1401 (2014) 073.
[44] W. Huang, Z. Kang, J. Shu and J. M. Yang, Phys. Rev. D 91 (2015) 025006.
[45] C. -W. Chiang and E. Senaha, JHEP 1006 (2010) 030.
[46] D. J. Chung, A. J. Long, and L.-T. Wang, Phys. Rev. D 87 (2013) 023509.
[47] L. Dolan and R. Jackiw, Phys. Rev. D 9 (1974) 3320.
[48] E. W. Kolb and M. S. Turner, The Early Universe, Front. Phys. 69 (1990) 1547.
[49] J. I. Kapusta, Finite temperature field theory, Cambridge University Press, (1989).
[50] M. Le-Bellac, Thermal field theory, Cambridge University Press, (2000).
[51] A. Kosowsky, A. Mack and T. Kahniashvili, Phys. Rev. D 66 (2002) 024030.
[52] A. Kosowsky, M. S. Turner and R. Watkins, Phys. Rev. D 45 (1992) 4514.
[53] M. Kamionkowski, A. Kosowsky and M. S. Turner, Phys. Rev. D 49 (1994) 2837.
[54] C. Caprini and R. Durrer, Phys. Rev. D 74 (2006) 063521.
[55] L. Leitao and A. Megevand, arxiv:1512.08962.
[56] M. Artymowski, M. Lewicki and J. D. Wells, arxiv:1609.07143.
[57] M. Kakizaki, S. Kanemura and T. Matsui, Phys. Rev. D 92 (2015) 115007.
[58] P. Huang, A. J. Long, L.-T. Wang, Phys. Rev. D 95 (2016) 075008.
[59] C. Balazs, A. Fowlie, A. Mazumdar and G. White, Phys. Rev. D 95 (2017) 043505.
[60] A. Katz and M. Perelstein, JHEP 1407 (2014) 108.
[61] F. Sannino and J. Virkaja¨rvi, Phys. Rev. D 92 (2015) 045015.
[62] Z. -W. Wang, T. G. Steele, T. Hanif and R. B. Mann, JHEP 08(2016) 065.
[63] M. Lewicki, T. Rindler-Daller and J. D. Wells, JHEP 06 (2016) 055.
[64] A. V. Kotwal, M. J. Ramsey-Musolf, J. M. No and P. Winslow, Phys. Rev. D 94 (2016) 035022.
[65] I. Baldes, T. Konstandin and J. Servant, arxiv:1604.04526.
[66] T. Tenkanen, K. Tuominen and V. Vaskonen, arxiv:1606.06063.
[67] M. Chala, G. Nardinib and I. Sobolevc, Phys. Rev. D 94 (2016) 055006.
[68] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B 155 (1985) 36.
[69] D. E. Morrissey and M. J. Ramsey-Musolf, New J. Phys. 14 (2012) 125003.
[70] S. Coleman, Aspects of symmetry: Selected Erice lectures, Cambridge University Press, Cambridge, (1988).
[71] A. Zee, Quantum field theory in a nutshell, Princeton University Press, Princeton, (2010).
[72] M. Carena, A. Megevand, M. Quiros and C. E. M. Wagner, Nucl. Phys. B 716 (2005) 319.
[73] P. B. Arnold and O. Espinosa, Phys. Rev. D 47 (1993) 3546.
[74] Z. Fodor and A. Hebecker, Nucl. Phys. B 432 (1994) 127.
[75] D. Curtin, P. Meade and H. Ramani, arxiv:1612.00466.
[76] G. ’t Hooft, Phys. Rev. Lett. 37 (1976) 8.

[77] G. ’t Hooft, Phys. Rev. D 14 (1976) 3432.

[78] N. S. Manton, Phys. Rev. D 28 (1983) 2019.

[79] S. Kanemura, M. Kikuchia and K. Yagyub, Nucl. Phys. B 917 (2017) 154.

[80] T. Robens and T. Stefaniak, Eur. Phys. J. C 76 (2016) 268.

[81] K. Cheung, P. Ko, J. S. Lee and P. Y. Tseng, JHEP 10 (2015) 057.

[82] P. Bechtle, S. Heinemeyer, O. Stål, T. Stefaniak, and G. Weiglein, JHEP 11 (2014) 039.

[83] Virgo, LIGO Scientific collaboration, B. P. Abbott et al., Phys. Rev. Lett. 116 (2016) 061102.

[84] C. Caprini, et al., JCAP 1604 (2016) 001.

[85] V. Vaskonen, arxiv:1611.02073.

[86] K. Kannike, Eur. Phys. J. C 72, 2093 (2012) 2093.

[87] J. Chakrabortty, P. Konar and T. Mondal, arXiv:1311.5666.