IMPLICATIONS OF A HEAVY TOP IN SUPERSYMMETRIC THEORIES

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ABSTRACT

In the context of the radiative electroweak symmetry breaking scenario, we investigate the implications of a heavy top quark mass, close to its infrared fixed point, on the low energy parameters of the minimal supersymmetric standard model. We use analytic expressions to calculate the Higgs masses as well as the supersymmetric masses of the third generation. We further assume bottom-tau unification at the GUT scale and examine the constraints put by this condition on the parameter space \((\tan \beta, \alpha_3)\), using the renormalization group procedure at the two-loop level. We find only a small fraction of the parameter space where the above conditions can be satisfied, namely \(1 \leq \tan \beta \leq 2\), while \(0.111 \leq \alpha_3(M_Z) \leq 0.118\). We further analyse the case where all three Yukawa couplings reach the perturbative limit just after the unification scale. In this latter case, the situation turns out to be very strict demanding \(\tan \beta \sim 63\).
Introduction

The last years there has been a revived interest in the supergravity unified models and their low energy effective theories, mainly due to the fact that LEP measurements are in good agreement with a gauge coupling constant unification scenario with supersymmetric $\beta$-function coefficients down to the scale of $\sim 1\text{TeV}$. However, the existence of supersymmetry will only be confirmed when new particles – the superpartners of the standard model spectrum – will be observed in (near) future experiments. Thus, the study of supersymmetric grand unification is very important and should be seen in conjunction with the predictions for the new particles which may be observed soon. So far, the constraints put by the unification of the three gauge couplings require a superpartner mass spectrum in the range of $(0.1 - 1)\text{TeV}$ which can be accessible in the near future.

There is another experimental fact the last few years which seems to be related with the fate of the electroweak symmetry breaking in an effective supersymmetric low energy theory. The non-observation of the top quark gives a lower bound on its mass $m_t \gtrsim 100\text{GeV}$. Although this result is disappointing from the experimental point of view, on the other hand, it fits perfectly with the idea of radiative symmetry breaking scenario suggested several years ago \cite{1,2}. Indeed the renormalization group improved SUSY Higgs potential breaks the $[SU(2) \times U(1)]_{\text{EW}}$ symmetry when the top Yukawa coupling is large enough to drive one of the soft supersymmetry breaking parameters (namely $m_{H_2}^2$) negative.

Grand unification based on the most popular groups, with the minimal number of fermion and Higgs content, implies additional relations in the initial values of the parameters of the theory. Thus, for example in the Yukawa sector, one such well known constraint requires the bottom and tau lepton Yukawa couplings $h_b$ and $h_\tau$, to be equal at the unification scale $E_G$

$$h_b(E_G) = h_\tau(E_G)$$  \hspace{1cm} (1)

In certain cases, and particularly in string derived unified models, additional constraints on the Yukawa sector of the theory are often obtained, i.e.

$$h_b(E_G) \approx h_r(E_G) \approx h_t(E_G) \sim g_{\text{String}}$$  \hspace{1cm} (2)

where $h_t$ is the top Yukawa coupling and $g_{\text{String}}$ is the value of the unified gauge coupling at the string scale $E_{\text{String}} \geq E_G \sim 10^{16}\text{GeV}$. In particular, a large top Yukawa coupling which is implied by the last equality in Eq.\hspace{.1cm}(2), motivates again the study of the fixed point solutions proposed several years ago in the context of non supersymmetric theories \cite{3}.

Motivated by the experimental fact that the top quark mass is rather high as well as from the aforementioned theoretical speculations, in the present
work we wish to study the implications of the above considerations on the low energy theory. In order to minimize the arbitrary parameters and to avoid complications with flavour changing neutral currents, we assume universality of the scalar mass parameters at the GUT scale. Using renormalization group techniques, we derive the mass formulae of the scalar masses (in particular those affected by a large top quark mass) and examine their properties close to the infrared fixed point of the top mass. Furthermore we investigate the regions of the parameter $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$ which are compatible with the above constraints and the minimization conditions put by the renormalization group improved Higgs potential.

**Radiative Symmetry Breaking in the Presence of a Heavy Top Quark**

One of the most appealing features of supergravity theories is the radiative symmetry breaking mechanism which may occur in the presence of a heavy top quark mass. Indeed, the renormalization group improved Higgs potential breaks the electroweak symmetry if the top Yukawa coupling is large enough to drive the $m_{H_2}^2$ mass parameter negative below a certain scale $Q_0$.

At the tree level the supersymmetric Higgs potential can be written as follows

$$V_0(Q) = m_1^2 |H_1|^2 + m_2^3 |H_2|^2 + m_3^3 (\epsilon_{ij} H_1^i H_2^j + \text{h.c.}) + \frac{1}{8} (g^2 + g'^2) \left( |H_1|^2 - |H_2|^2 \right)^2 + \frac{1}{2} g^2 |H_1^i H_2^j|^2,$$  \hspace{1cm} (3)

where $H_1 = (H_1^0, H_1^-)$ and $H_2 = (H_2^+, H_2^0)$ are the standard Higgs superfields and $\epsilon_{ij}$ is the antisymmetric tensor in two dimensions. We have also introduced the two Higgs mass parameters

$$m_1^2 = m_{H_1}^2 + \mu^2, \hspace{1cm} (4)$$
$$m_2^2 = m_{H_2}^2 + \mu^2. \hspace{1cm} (5)$$

Finally $m_{H_1,2}$ and $m_3$ are the soft SUSY breaking mass terms and $\mu$ is the Higgs mixing mass parameter.

The above tree-level potential $V_0(Q)$ depends strongly on the energy scale $Q$. It has been shown however, that a correct minimization procedure can be achieved (making the potential relatively stable), if one includes the one-loop corrections $\Delta V_1(Q)$

$$\Delta V_1(Q) = \frac{1}{64 \pi^2} \text{Str} \left[ M^4 \left( \ln \frac{M^2}{Q^2} - \frac{3}{2} \right) \right], \hspace{1cm} (6)$$

where $M$ is the field dependent tree level mass matrix squared. Thus finally

$$V_H(Q) = V_0(Q) + \Delta V_1(Q) \hspace{1cm} (7)$$
The symbol \( \text{Str} \) stands for the supertrace which is defined as follows

\[
\text{Str} f(M^2) = \sum_i q_i (-1)^{2s_i} (2s_i + 1) f(m_i^2)
\]

with \( q_i \) being the color degrees of freedom while \( m_i \) and \( s_i \) are the mass and the spin of the corresponding particle.

Now, electroweak symmetry breaking occurs if the following two conditions are met:

- The supersymmetric Higgs potential should develop an asymmetric minimum below some scale \( Q \leq Q_0 \). This requirement is expressed by the condition

\[
m_1^2(Q)m_2^2(Q) - m_3^4(Q) \leq 0.
\]

- The Higgs potential should be bounded from below. This requirement sets the second condition, which reads

\[
m_1^2(Q) + m_2^2(Q) \geq 2|m_3(Q)|^2
\]

The minimization conditions \( \frac{\partial V_H}{\partial v_i} = 0 \), where \( v_i \equiv < H_i > \), result the well known equations

\[
\frac{1}{2} M_Z^2 = \frac{\mu_1^2 - \mu_2^2 \tan^2 \beta}{\tan^2 \beta - 1}
\]

\[
\frac{1}{2} \sin 2\beta = -\frac{m_3^2}{\mu_1^2 + \mu_2^2}
\]

where we have introduced the new mass parameters \( \mu_i^2 = m_H^2 + \mu^2 + \sigma_i^2 \), which take into account the corrections to the Higgs potential from the one-loop contributions \( \sigma_i^2 \)

\[
\sigma_i^2 \equiv \frac{\partial \Delta V_1}{\partial v_i}
\]

From the above equations one can conclude that the one-loop corrections to the Higgs potential appear in the minimization conditions through shifts of the Higgs mass parameters \( m_i^2 \to m_i^2 + \sigma_i^2 \). It has been shown \[5\] that although 31 particles contribute to \( \sigma_i^2 \) corrections, there are finally large cancellations which reduce significantly their effect to the electroweak symmetry breaking. Moreover, the one-loop contribution of the t-squark-quark sector to the masses of the neutralinos, Higgsinos and gauginos seems to be well below the 10% \[6\] (except in the unfavorable case of a very light tree-level mass).
The most important contributions arise from the squarks of the third generation and the top quark mass. Therefore, it is obvious that the Higgs mass parameters $m_{H_1}$ and the $t$-squarks play an important role in the minimization of the Higgs potential.

The scale dependence of $m_{H_1}$ and $t$-squarks is given by the renormalization group equations which can be integrated to give the following results. The $m_{H_1}$ Higgs mass parameter is given by

$$m_{H_1}^2 = m_0^2 + C_{H_1}(t)m_{1/2}^2$$

where $t = \ln Q$, $m_0$ and $m_{1/2}$ are the universal scalar and gaugino mass parameters at $E_G$, and $C_{H_1} \sim 0.57$ for $t \sim \ln M_Z$. For the rest of the scalar masses, denoting for convenience $\tilde{m}_t \equiv \tilde{m}_1$, $\tilde{m}_t \equiv \tilde{m}_2$ and $m_{H_2} \equiv \tilde{m}_3$, we can write the general analytic form

$$\tilde{m}_n^2 = \alpha_n m_0^2 + C_n(t)m_{1/2}^2 - n\delta_m^2(t) - n\delta_A^2(t)$$

where $\alpha_n$ depends on the K"aller manifold and hereafter we assume that $\alpha_n = 1$. The quantities $\delta_{m,A}^2(t)$ are given by

$$\delta_m^2(t) = \left(\frac{m_t(t)}{2\pi v \gamma_Q(t) \sin \beta}\right)^2 \times (3m_0^2 I(t) + m_{1/2}^2 J(t))$$

and

$$\delta_A^2(t) = \Delta_A^2(t) - \frac{3}{2} \left(\frac{m_t(t)}{2\pi v \gamma_Q(t) \sin \beta}\right)^2 E_A^2(t)$$

where $v = 246\text{GeV}$ and $I$, $J$, $\Delta_A^2$ and $E_A^2$, are integrals containing functions of gauge couplings, i.e.

$$I = \int_t^{t_0} dt' \gamma_Q^2(t')$$

$$J = \int_t^{t_0} dt' \gamma_Q^2(t') C(t')$$

$$\Delta_A^2 = \int_t^{t_0} \frac{h_2^2(t')}{8\pi^2} A^2(t') dt'$$

$$E_A^2 = \int_t^{t_0} dt' \gamma_Q^2(t') \Delta_A^2(t')$$

with $t_0 = \ln E_G$, $C(t) \equiv \sum_{n=1}^3 C_n(t)$, while $\gamma_Q(t) = \prod_{j=1}^3 (\alpha_{j,0}/\alpha_j)^{C_j/2b_j}$. Clearly, a large top mass implies also a large value of the top-Yukawa coupling, and therefore the negative contributions $\delta^2$ will become also significant. It is possible then to have $\tilde{m}_3^2 \equiv m_{H_2}^2$ negative, and the radiative symmetry breaking scenario will take place.
A very interesting possibly arises in the case where the top mass is close to its infrared quasi-fixed point. The evolution of the top quark coupling, assuming all the way down supersymmetry, is given by

\[ h_t = \frac{h_t(t_0)\gamma_Q(t)}{(1 + \frac{3}{4\pi h_t(t_0)I(t)})^{1/2}} \]  

(22)

In this case, i.e. for \( \frac{h_t(t_0)}{4\pi} \sim 1 \), since \( I(t \sim \ln m_t) \gg 1 \), we can approximate the above

\[ h_t(fixed) \approx \frac{2\pi}{\sqrt{3I(t)}}\gamma_Q(t) \]  

(23)

i.e. independent of the initial value \( h_t(t_0) \). Thus \( m_t(fixed) = m_t^0 \sin \beta \sim (190 - 200)\sin \beta \) GeV, depending on the precise values of \( \alpha_3, E_G \) etc.

The scalar masses of \( m_{\tilde{t}_L}, m_{\tilde{t}_R} \) and the Higgs which couples to the up-quarks, take a very simple \( m_t \)-independent form in this case. For \( h_t = h_t(fixed) \) Eq.(15) simplifies to

\[ \tilde{m}_n^2 = (1 - \frac{n}{2})m_0^2 + [C_n(t) - \frac{nJ}{6I}]m_{1/2}^2 \]  

(24)

As far as one assumes \( A(E_G) \leq 3|m_0| \), corrections due to \( A \)-contributions to the above formula have been found to be very small and thus they have been totally ignored. Calculation of the various \( t \)-dependent quantities at \( t \approx \ln m_t \) gives

\[ C_1(t) \approx 5.30, \quad C_2(t) \approx 4.90, \quad C_3 \approx 5.7, \quad I \approx 113, \quad J \approx 590 \]  

(25)

There are some worth-noting properties of the above mass formulae. Indeed, first note that \( m_{\tilde{t}_R} \) depends only on \( m_{1/2}^2 \) up to \( A \) corrections which have been found negligible. A second property is that the dependence of the sum \( m_{\tilde{t}_L}^2 + m_{H^\pm}^2 \) on \( m_0^2 \), vanishes at the limit \( m_t \rightarrow m_t(fixed) \). The above properties have also been noticed in Ref.[8]. It is interesting to see the implications of the above simplified formulae in the case of the minimization conditions of the Higgs potential. We start first with the Higgs mixing parameter \( \mu \) involved in the minimization conditions Eqs.(11, 12). Ignoring one-loop effects for simplicity, the \( \mu \) parameter can be given in terms of the known parameters \( I, J \), the unkown Higgs-vev ratio and the initial values \( (m_0, m_{1/2}) \), by the following equation

\[ |\mu| = \frac{1}{\sqrt{2}}\left\{ \frac{k^2 + 2}{k^2 - 1}m_0^2 + \left( \frac{k^2}{k^2 - 1} - \frac{J}{I} \right)m_{1/2}^2 - M_Z^2 \right\}^{1/2} \]  

(26)

with \( k = \tan \beta \). In Fig.(1),we plot the \( |\mu| \)-values in the parameter space \( (m_0, m_{1/2}) \), for \( \tan \beta = 1.1 \) and \( \tan \beta = 5 \). For the most of the parameter
space, $|\mu| \leq 1.5\text{TeV}$. Of course, as $\tan \beta \rightarrow 1$, $\mu$ grows larger, and a fine tuning problem may arise in Eq.(14), in order to obtain the correct experimental value of $M_Z$. Thus, to avoid fine tuning, we may put the condition on $\tan \beta \geq 1.1$, which finally translates to the bound $M_t \equiv m_t(pole) \geq (150 - 155)\text{GeV}$.

The $\mu$-parameter plays also important role in the squark mass matrices. In particular, the t-squark mass matrix is

$$M^2_Q = \begin{pmatrix} M^2_{LL} & M^2_{LR} \\ M^2_{LR} & M^2_{RR} \end{pmatrix}$$

(27)

with eigenvalues given by

$$m^2_{t_{1,2}} = \frac{M^2_{LL} + M^2_{RR} \pm \sqrt{(M^2_{LL} - M^2_{RR})^2 + 4M^4_{LR}}}{2}$$

(28)

where

$$M^2_{LL} + M^2_{RR} = \frac{1}{2}m^2_0 + (C_1 + C_2 - \frac{J}{2})m^2_{1/2} + 2m^2_t + \frac{1}{2}m^2_Z \cos 2\beta$$

$$M^2_{LL} - M^2_{RR} = \frac{1}{2}m^2_0 + (C_1 - C_2 + \frac{J}{6})m^2_{1/2} + \frac{4}{3}M^2_W - \frac{5}{6}M^2_Z \cos 2\beta$$

$$M^2_{LR} = m^0_t (A \sin \beta + \mu \cos \beta)$$

In Fig.(2) (assuming $\mu > 0$), we plot contours of the above eigenmasses in the parameter space $(m_0, m_{1/2})$ for the choice $A(E_G) = -\sqrt{3}m_0$, and two representative values of $\tan \beta$ in the low range $(1.1 - 10)$, namely $\tan \beta = 1.1$ and $\tan \beta = 5$. In most of the parameter space the light eigenstate preserves the independence of $m_0$ mass parameter. For reasonable initial values of the parameters $m_0$ and $m_{1/2}$, the squark masses are well below the 1TeV, and therefore accessible to future experiments.

Notice finally that the one-loop contributions to the effective potential will also result to a shift in the $|\mu|$ parameter. Making use of the fact that in the limit $m_{1/2} \gg m_0$ we can approximate

$$\ln \frac{m^2_{t_1}}{M^2_Z} \sim \ln \frac{m^2_{t_2}}{M^2_Z} \sim \ln \frac{<m^2_t>}{M^2_Z}$$

we may obtain an analytic form for the one-loop corrected $|\mu|$ parameter, when $m_t$ and $m_{t_{1,2}}$ corrections are taken into account

$$|\mu| = \sqrt{(\mu^2_0 + \eta^2)/(1 - \Omega^2)}$$

(29)

where

$$\eta^2 = \frac{\alpha_2}{8\pi \cos^2 \theta_W} \left\{ \left( M^2_{LL} + M^2_{RR} \right) \left( \frac{1}{4} - \rho^2 \right) \right\}$$

6
\[
\begin{align*}
&+ \left( M_{LL}^2 - M_{RR}^2 \right) \left( \frac{1}{4} - \frac{2}{3} \sin^2 \theta_W \right) - \rho^2 A^2 \right] \left( \ln \rho^2 - 1 \right) \\
&- 2m_i^2 \left( \ln \rho^2 - 1 \right) \left( \frac{\rho^2}{k^2} \right) \left( \frac{k^2 + 1}{k^2 - 1} \right) \\
\Omega^2 &= \frac{\alpha_2}{8\pi \cos^2 \theta_W} \left[ \rho^2 \left( k^2 + 1 \right) \right] \left( \ln \tilde{\rho}^2 - 1 \right)
\end{align*}
\]  

with \( \rho = m_t/M_Z \), \( \tilde{\rho} = < m_i > /M_Z \) and \( \mu_0 \) the tree level parameter define in (26). For moderate values of \( m_{1/2} \) however, these corrections are not going to alter substantially our previous results.

**Bottom–Tau Yukawa Unification and the IR Fixed Point**

One of the great successes of the most popular GUTs is the equality of the bottom and tau Yukawa couplings at the GUT scale which lead to the correct prediction of the experimentally determined relation \( m_b \approx 3 m_\tau \) at low energies. Several groups have examined the effects of \( h_b, h_\tau \) relations implied by various unified theories, assuming minimal supersymmetry with grand unification at an energy scale close to \( 10^{16} \)GeV. It has been claimed that the GUT relation \( h_b = h_\tau \) implies a heavy top quark with a value of the Yukawa coupling close to its infrared fixed point. In this section, we wish to present a detailed numerical analysis in the context of the GUT constraints mentioned above. We will mainly discuss the constraints on the parameter space \((\tan \beta, \alpha_3)\) when bottom-tau Yukawa unification is assumed and examine the connection of this constraint in relation with the top-mass. We will further examine the case where the three Yukawa couplings reach the perturbative limit just after the unification scale. Our analysis will be done at the two-loop level, taking into account the contribution of the Yukawa couplings, and in particular that of the \( h_t \) into the running of the gauge coupling constants. Our results largely agree with previous analyses, however the allowed region in the parameter space \((\tan \beta, \alpha_3)\) is more constrained. In particular, we find that \( h_b = h_\tau \) can be satisfied only in a small region of \( 1 \leq \tan \beta \leq 2 \) and \( .111 \leq \alpha_3 \leq .118 \). The case where all three Yukawa couplings are equal at \( E_G \) occurs theoretically in (minimal) \( SO(10) \) and in \( SU(4) \times SU(2)_L \times SU(2)_R \). The allowed region of \( \tan \beta \) shortens around the value 63 for that case while \( \alpha_3(M_Z) \) stays on the lower edge of the experimentally allowed region \((\sim .11)\).

We shall present now a detailed description of the procedure we are following. We adopt the so called bottom-up approach starting from \( M_Z \). Which are the inputs at this energy level?

- the experimentally known values of \( \alpha, \sin^2 \theta_W \) and \( \alpha_3 \), or equivalently of the three gauge couplings \( \alpha_i, i = 1, 2, 3 \). The relatively small exper-
imental errors on $\alpha$ and $\sin^2 \theta_W$ permit us to talk about the “bands” of $\alpha_1$ and $\alpha_2$ in the running of those couplings while we treat $\alpha_3$ as a “free” parameter, inside its experimental limits of course.

- the value of $\tan \beta$, starting its rôle when we reach the energy $E_S$ where SUSY is valid.

- the value of the $h_t$ Yukawa coupling of the top quark. Essentially it is a free parameter as long as it gives the mass of the top quark in the allowed experimental region $(110–190)\text{GeV}$. We use the 1 loop QCD corrections to define the pole mass $M_t$ of the top quark

$$M_t = \frac{h_t(M_t)v/\sqrt{2}}{1 + \frac{4}{3\pi \alpha_3(M_t)}}$$

- the values of $h_b$ and $h_\tau$, taken from the relations

$$m_b(m_b) = \frac{h_b(M_Z)v/\sqrt{2}}{\eta_b}, \quad m_\tau = h_\tau(M_Z)v/\sqrt{2}$$

We take the mass of the bottom quark $m_b(m_b) = (4.15–4.35)\text{GeV}$ while that of the $\tau$ lepton $m_\tau(m_\tau) = 1.7841\text{GeV}$. The factor $\eta_b$, appearing in the mass of the bottom quark, includes the 1 loop QCD corrections from $m_b$ to $M_Z$.

Between $M_Z$ and $E_S$, which we take to be 1TeV, we run the couplings with the $\beta$-functions of the S.M. At $E_S$ we apply the following boundary conditions for the Yukawa couplings

$$h_t^S = \frac{h_t}{\sin \beta}, \quad h_b^S = \frac{h_b}{\cos \beta} \quad \text{and} \quad h_\tau^S = \frac{h_\tau}{\cos \beta}$$

Then onwards we run the couplings using the MSSM $\beta$-functions. At an energy $E_G$, around $10^{16}\text{GeV}$, the “bands” of the couplings $\alpha_1$ and $\alpha_2$ meet and determine what we call “the unification band”. The strong coupling $\alpha_3(M_Z)$ should be chosen so that it passes through this unification band in order to achieve gauge coupling unification. A short comment is in order at this point. Since we are using 2 loop $\beta$-functions the differential equations for all the couplings are coupled (this fact shows its presence even harder when we demand one or all the Yukawa couplings to grow large at $E_G$). Therefore the unification band is not uniquely determined but depends, though not strongly, on the particular choice for $\alpha_3$ as well as on the Yukawa couplings at $M_Z$. We try to find the values of $\tan \beta$ that permit the growth of $h_t$ to the perturbative limit ($h_t \sim 3.5$) at the energy scale $E_G$ or later, checking always that $\alpha_3$ passes through the unification band. At the same time we try
to unify, again at $E_G$, the other two Yukawa couplings: $h_b(E_G) = h_\tau(E_G)$. This could be achieved by varying the mass of the bottom quark inside its experimentally allowed region. Finally we try to arrange the possibility that all three Yukawa couplings grow to the perturbative limit at $E_G$. This last step could be achieved by using large values of $\tan \beta$.

We approach, step by step, the above three points, constraining in each step the allowed region of the parameter space of our inputs. In Fig.3 we plot, for several values of the mass of the top quark $M_t$, $\tan \beta$ versus $\alpha_3(M_Z)$ demanding gauge coupling unification and $h_t(E_G) \lesssim 3.5$. Let us explain the features of the graph. The lower limit on $\alpha_3(M_Z)$ appears because the lower the gauge couplings the larger the slope $dh_t/dt$ (recall that gauge and Yukawa couplings have opposite contributions to the $\beta$-functions). This fact permits $h_t$ to grow very fast and reach the perturbative limit before gauge coupling unification is achieved. The same line of thought explains the slope of the “lines” in Fig.3. Choosing a higher $\alpha_3(M_Z)$ we need a higher initial point $h_t(E_S)$ to reach the perturbative limit, therefore we need a smaller $\tan \beta$. The turning edges of each line is more intriguing. At the right end the value of $\alpha_3(M_Z)$ is so high that, although $h_t$ permits gauge coupling unification, $\alpha_3(E_G)$ passes above the unification band of $(\alpha_1, \alpha_2)$. Choosing a higher value of $\tan \beta$ (therefore smaller $h_t$) $d\alpha_i/dt$ grows to larger values. The coupling $\alpha_2$ receives the biggest contribution, the unification band shifts to higher values and allows $\alpha_3$ to pass through it. Of course, in that case $h_t(E_G) < 3.5$. For each line in Fig.3, the region where $h_t(E_G) < 3.5$ (dashed lines) grows bigger as $M_t$ grows. Similar arguments explain the left end of the lines. Now $\alpha_3(M_Z)$ is so small that very easily drops below the unification band. Choosing a somewhat higher $\alpha_3(M_Z)$ permits a higher $\tan \beta$. Again, in this case, $h_t(E_G) < 3.5$. Therefore the allowed region for each $M_t$ is inside the envelope–like shape.

Our next step is to demand $h_b(E_G) = h_\tau(E_G)$. For each $M_t$, we plot in Fig.3 a band (shaded region) corresponding to $m_b(m_b) = 4.15\text{GeV}$ (lower line of the band) and to $m_b(m_b) = 4.35\text{GeV}$ (upper line of the band). We notice that $b-\tau$ unification requires $M_t$ to be near its fixed point, being closer for $M_t \sim (150 - 160)\text{GeV}$.

The last step is to require all three Yukawa couplings to reach the perturbative limit near $E_G$. To achieve that point we need a large value for $\tan \beta$ (for $h_b$ and $h_\tau$) and a large $M_t$ (for $h_t$) The situation is very strict. For example, using as inputs

\[
\tan \beta = 63.4, \quad \alpha_3(M_Z) = 0.112, \quad M_t = 190\text{GeV}, \quad \text{and} \quad m_b(m_b) = 4.2\text{GeV}
\]

we get gauge coupling unification at $E_G = (10^{16.0} - 10^{16.1})\text{GeV}$, while the three Yukawa couplings at the scale $10^{16.2}\text{GeV}$ reach values in the range
3.1 – 3.5. Trying to achieve those large values of $h_t$, $h_b$ and $h_\tau$ with a lower value of $M_t$, one needs to choose either a lower value of $\alpha_3(M_Z)$ or a smaller $\tan \beta$. The latter does not help since, at such large values, $\sin \beta$ does not change much while $\cos \beta$ does, preventing $h_b$ and $h_\tau$ to reach the perturbative limit. On the other hand, the change of $\alpha_3(M_Z)$ does not affect $h_\tau$ in contrast with $h_t$ and $h_b$. The situation is greatly complicated since all Yukawa couplings are large.

In conclusion, in this paper we examined the implications of a heavy top quark, and bottom tau unification at the GUT scale, implied by popular unified models, in the minimal supersymmetric standard model. We have assumed a top Yukawa coupling close to its infrared fixed point and we have given analytic forms of the t-squark masses and the Higgs mass parameter responsible for the radiative electroweak symmetry breaking scenario. We have found that $m_{\tilde{t}_L}$ does not depend on the $m_0$ mass parameter, while all masses under consideration are very weakly dependent on the trilinear scalar parameter $A$. The bottom-tau unification turned out to be very restrictive. We have found only small ranges in the $(\tan \beta, \alpha_3)$-plane where this condition can be satisfied. Moreover, this condition demands a heavy top with a mass close to its infrared fixed point, as was previously assumed. In the case of the large $\tan \beta$ scenario, the above requirements can be satisfied only in a tiny region with $\tan \beta \approx 63$.

One of us (NDT) would like to thank G. Bathas, K. Farakos, G. Koutsoumbas and S.D.P. Vlassopulos for useful discussions. The work of G.K.L. is partially supported by a C.E.C. Science Program SCI-0221-C(TT), while of N.D.T. by C.E.C. Science Program SCI-CT91-0729.
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Figure Captions

**Fig.1.** Surfaces of constant $|\mu|$ in the parameter space $(m_0, m_{1/2})$. The upper surface corresponds to $\tan \beta = 1.1$ while the lower one corresponds to $\tan \beta = 5$.

**Fig.2.** Contours of constant $m_{t_1, t_2}$ in the parameter space $(m_0, m_{1/2})$, for two values of $\tan \beta = 1.1$ and 5. (a) and (c) corresponds to the lighter eigenstate while (b) and (d) to the heavier one.

**Fig.3.** Allowed regions in the space of $(\tan \beta, \alpha_3(M_Z))$, in order to achieve gauge coupling unification, for several values of the top mass $M_t$. Below the solid part of the contours, $h_t$ reaches the perturbative limit before gauge coupling unification, while below the dashed part $\alpha_3$ passes above the unification band of $\alpha_1$ and $\alpha_2$. Demanding $h_b = h_\tau$ at $E_G$, the allowed regions, for each $M_t$ value, shrink to the corresponding shaded bands.