Quantum gate using qubit states separated by terahertz

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A two-qubit quantum gate is realized using electronic excited states in a single ion with an energy separation on the order of a terahertz times the Planck constant as a qubit. Two phase locked lasers are used to excite a stimulated Raman transition between two metastable states $D_{5/2}$ and $D_{3/2}$ separated by 1.82 THz in a single trapped $^{40}$Ca$^+$ ion to construct a qubit, which is used as the target bit for the Cirac-Zoller two-qubit controlled NOT gate. Quantum dynamics conditioned on a motional qubit is clearly observed as a fringe reversal in Ramsey interferometry.

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Atomic systems including trapped ions and neutral atoms are considered attractive for quantum information processing (QIP) since they can be made to be well isolated from the environment and hence enable construction of qubits with small decoherence/dephasing. Among experimental approaches toward QIP using different physical systems, some of the most advanced have been experiments using trapped ions,1,2 which are based on qubit levels with separation in the rf/microwave region and the optical region. Recent advances in optical comb generation and optical frequency metrology3–5 offers much flexibility in choosing qubit states, including atomic states with frequency separations that have not been explored before.

In view of recent progress in experiments of ultracold molecules transferred to the ground state of both internal and external degrees of freedom, molecular systems which have rich internal structures are also considered attractive for application to QIP. In the recent works6–8, by performing stimulated Raman adiabatic passage using two lasers with high relative coherence, weakly bound ultracold Feshbach molecules are transferred to their rovibronic ground state. In addition, there are proposals to encode qubits in molecular states with small dipole moments and transfer these to states with larger dipole moments, thereby realizing switchable interaction between molecular qubits9,10. The required transfer can be performed by applying two phase locked lasers through stimulated Raman process.

In this article, we present the result of a quantum gate experiment using phase-locked lasers to excite a stimulated Raman transition. Two metastable states $D_{3/2}$ and $D_{5/2}$ in $^{40}$Ca$^+$ separated by 1.82 THz are used as the target bit to perform the Cirac-Zoller gate14. This is the first attempt to use phase locked lasers to bridge an energy separation larger than a terahertz and realize a quantum gate, and is an important step toward obtaining a wider choice of qubit levels including internal levels of atoms and molecules.

Cirac and Zoller proposed in 1995 a realistic scheme for scalable quantum computation using a string of ions in a linear trap14. It was experimentally demonstrated in a simplified form using internal states and a motional degree of freedom in a single $^9$Be ion15. A full implementation of the scheme in a scalable manner using a $^{40}$Ca$^+$ ion string with the technique of individual addressing is reported in 200316.

It has been shown that all unitary operations on arbitrary many qubits can be decomposed into two-bit gates and one-bit gates17. One example of such decomposition of unitary operations uses controlled NOT (CNOT) gates and rotation operations on single qubits18,19. Analogously to a classical exclusive-OR (XOR) gate, a CNOT quantum gate realizes the following operation: $|\epsilon_1\rangle|\epsilon_2\rangle \rightarrow |\epsilon_1\rangle|\epsilon_1 \oplus \epsilon_2\rangle$ with $\epsilon_{1,2} = 1, 2$ and $\oplus$ representing an addition modulo 2.

In the Cirac-Zoller (CZ) proposal14, for implementing this CNOT operation, a red-sideband pulse, which is detuned to the lower side of the resonance of the qubit transition by the frequency of a collective motional mode, is applied between one basis state of the target qubit and an auxiliary state. When the collective motional state has one quantum, the red-sideband pulse applied for a duration corresponding to a $2\pi$ rotation causes a $\pi$ phase shift.

![FIG. 1: (Color online) Level scheme for $^{40}$Ca$^+$ and transitions relevant for implementing the CZ gate. Two sublevels each from $D_{3/2}$ and $D_{5/2}$ metastable states, $|\uparrow\rangle \equiv |D_{5/2}(m_j = 1/2)\rangle$ and $|\downarrow\rangle \equiv |D_{3/2}(m_j = 1/2)\rangle$, are used as the qubit states here.](image-url)
shift between two basis states of the target qubit states. On the other hand, when the collective motional state has no quantum, such rotation does not occur and no phase shift is given to the target qubit. This corresponds to a unitary operation conditioned on the motional quantum number, thereby implementing a controlled phase gate, and a CNOT gate is realized when this is combined with certain single qubit operations.

To realize the CZ gate using the metastable states in $^{40}$Ca$^+$ and its motional states, we adopt an excitation scheme using the stimulated Raman transition between $D_{3/2}$ and $D_{5/2}$ along with a quadrupole transition that connects the ground state $S_{1/2}$ with $D_{5/2}$ (see Fig. 1). The stimulated Raman transition is used for single qubit operation on the metastable states qubit, while the quadrupole transition is used for realizing conditional phase shift required for a CZ gate, as well as sideband cooling and state preparation. As the target qubit states $|\uparrow\rangle \equiv D_{5/2}(m_J = 1/2)$ and $|\downarrow\rangle \equiv D_{3/2}(m_J = 1/2)$ are chosen, while as the control qubit the low-lying two states of the axial motion initialized to the ground state are used: $|0\rangle$ ($|1\rangle$) $\equiv |n_z = 0\rangle$ ($|n_z = 1\rangle$), where $n_z$ denotes the axial motional quantum number. A conditional phase shift is implemented by applying a blue sideband (BSB) $2\pi$ pulse between $|\uparrow\rangle$ and $|g\rangle \equiv S_{1/2}(m_J = -1/2)$ state, which gives a $\pi$ phase shift to the $|1\rangle$ $|\uparrow\rangle$.

We use a single $^{40}$Ca$^+$ ion trapped in a vacuum pressure of $6 \times 10^{-9}$ Pa. The trap used here is a conventional linear trap with an operating frequency of 23 MHz and secular frequencies of $(\omega_x, \omega_y, \omega_z)/2\pi = (1.91, 1.68, 0.72)$ MHz. A magnetic field of $3.1 \times 10^{-4}$ T is applied to lift the degeneracy of Zeeman states and to define the quantization axis for optical pumping. To reduce the effects of the ambient ac magnetic field, the vacuum chamber is enclosed in a magnetic shield. Loading of single ions is performed by using two-step photoionization from the $4s^1S_0$ ground state of Ca via $4p^1P_1$ with corresponding wavelengths of 423 nm and 375 nm for the first and second step of the photoionization, respectively.

About the phase locked lasers used for excitation of the stimulated Raman transition, the setup has been modified from the one described in our previous article[20] in order to improve the noise in the difference frequency of the two lasers. Two Ti-sapphire lasers at 850 and 854 nm phase locked by using a passive-type optical comb[5] in combination with an acousto-optic modulator (AOM) and an electro-optic modulator[21] are used to excite the stimulated Raman transition. For excitation of the quadrupole transition, a titanium sapphire laser at 729 nm stabilized to a high-finesse low-thermal-expansion cavity having a linewidth of $< 400$ Hz and a root-mean-square intensity noise of 0.3% is used. Control of optical frequency/phase/amplitude is done by AOM and rf fields used for them are generated by direct-digital synthesis (DDS) boards which are controlled by a field-programmable gate array (FPGA).

See Fig. 2 for the details of the beam configuration. For realizing gate experiments, all the motional degrees of the ion are cooled to near the ground states using Doppler cooling (with 397 and 866 nm lasers) and sideband cooling (SBC). For SBC, the $S_{1/2}(m_J = -1/2)$ $D_{3/2}(m_J' = -5/2)$ transition at 729 nm is used, and an additional quenching laser resonant to $D_{5/2}$ $P_{3/2}$ at 854 nm is also applied. All the three dimensions are cooled for 2 ms each and then this is repeated for 20 times. Optical pumping is performed using 397 nm $\pi$-transitions in $S_{1/2}(m_J = +1/2)$ $P_{1/2}(m_J = -1/2)$ before, every 6 ms during, and after SBC, each with duration of $6 \mu$s. The final numbers of the states after SBC are $(n_{x, y, z}) \sim (1, 1, 0.02)$.

For preparation to the $|\uparrow\rangle$ state, a carrier/BSB $\pi$ pulse on $|g\rangle$ $|\uparrow\rangle$ is applied. This is a $|\Delta m_J = 1\rangle$ transition which requires a polarization different from that used for sideband cooling transition for which $|\Delta m_J = 2\rangle$. In our case the former is parallel with and the latter is perpendicular to the surface of the optical table on which the trap chamber is placed (see Fig. 2). In order to perform both in one configuration, the polarization of the 729nm light is chosen to be linear and rotated from the perpendicular direction by 45 degree.

The target qubit states are discriminated by shining the cooling lasers at 397 and 866 nm for 7 ms and observing fluorescence photons by a photomultiplier.

The coherence times of the Raman transition have been measured to be $\sim 5.1$ ms (1.6 ms) with (without) spin echo in a setup without a magnetic shield[22]. The coherence time for the quadrupole transition with a magnetic shield, which is deduced from decay of Rabi oscillation signals, is about 0.8 ms.

Figure 3 shows Rabi oscillation signals on the relevant transitions including the carrier/BSB on $|g\rangle$ $|\uparrow\rangle$ and the carrier on the stimulated Raman transition. Based on these results, we can expect nearly unit fidelity for carrier pulses on $|g\rangle$ $|\uparrow\rangle$ while less fidelity for BSB Rabi pulses and carrier pulses on the stimulated Raman transition. The figure also shows results of numerical simulation, the details of which are given later. By comparing the simulation with the experiment, we can quantitatively characterize the fidelity limiting factors, and this helps estimation of possible fidelity of Bell state generation as described later.
The first pulse (preparation pulse) is applied either on the carrier or BSB on |g⟩− |↑⟩ to prepare the motional state |0⟩ or |1⟩ respectively. Then the first stimulated-Raman π/2 pulse is applied, which is followed by a BSB 2π pulse on |g⟩− |↑⟩ and the second stimulated-Raman π/2 pulse. For |0⟩ preparation the BSB 2π pulse cause no effect since there is no motional state to reach in |g⟩, while for |1⟩ preparation the BSB 2π pulse cause 2π rotation and gives a π phase shift to the original state. This conditional phase flip (π rotation around the z axis in the Bloch sphere) is converted into a conditional bit flip (π rotation around a horizontal axis in the Bloch sphere) by the two π/2 pulses.

Figure 3(b) shows the result of the Cirac-Zoller gate experiment. Here the phase of the second pulse is rotated from 0 to 4π and the population in |↑⟩ is measured. Crosses represent the |0⟩ preparation case, and filled circles the |1⟩ preparation case. The interference fringes for the two cases clearly show a π phase difference to each other, which is an evidence of a conditional dynamics caused by the BSB 2π pulse. The contrasts of the fringes are limited to 0.4 ~ 0.6, and for the BSB preparation case there is a negative offset ~ 0.05, which is consistently observed in similar measurements. These imperfections are explained later.

Numerical simulation is performed to quantitatively analyze the CZ gate result and to estimate possible fidelity for Bell state generation. A Liouville equation with exponential decay is solved for three internal levels (|g⟩, |↑⟩ and |↓⟩) and 5 external levels representing the axial motional states truncated at n_z = 4. Coupling between the internal and external states is considered to the second order of the Lamb-Dicke factor η for carrier excitation and to the first order for sideband excitation.

Fidelity limiting factors except for that from axial motional state distribution, which include laser phase fluctuation and magnetic field fluctuation, are incorporated into the equation as exponential decay of off-diagonal density matrix elements for the internal degrees of freedom. For the axial motional state, the initial distribution is assumed to be a thermal distribution based on the experimental results of sideband cooling (0π~ 0.02). Heating during the gate operation is neglected, which is reasonable since our measured heating rate is ~0.005 quanta/ms for the axial motion and the typical gate sequences are shorter than 1 ms.

The parameters for the exponential dephasing are extracted from experimental results by manually fitting simulation results for simple one-pulse sequences to the experimental Rabi oscillation results. Dotted curves in Fig. 3 represent such manually fitted simulation results. Representing exponential decay of off-diagonal density matrix elements using a proportionality factor \(\exp[-(γ/2)t]\), the values of γ are extracted to be 2π × 400 Hz for |g⟩−|↑⟩ and 2π × 300 Hz for |↑⟩−|↓⟩.

Based on the above-mentioned assumptions, the CZ gate experiment is simulated with 4 pulses assumed, and the result is shown in the Fig. 3(b) as curves. It well reproduces the reduction of fringe contrasts and also the negative offset in the case of |1⟩ preparation without any fitting parameters. It is presumable that the negative offset is caused by the infidelity in the BSB excitation on...
Bell state $|\Psi_B\rangle = (1/\sqrt{2})(|0\rangle |\uparrow\rangle + |1\rangle |\downarrow\rangle)$ can be produced from an initial state $|g\rangle$ by first applying a $\pi/2$ carrier pulse and a $\pi$ BSB pulse on $|g\rangle – |\uparrow\rangle$ to prepare $(1/\sqrt{2})(|0\rangle + |1\rangle) |\uparrow\rangle$ and then performing a controlled NOT operation that flips the internal qubit conditioned on the motional state. Using exactly the same parameters as used for the simulation in Fig. 1(b), the time dependence of the density matrix in the process of the generation of the Bell state $|\Psi_B\rangle$ is simulated. The fidelity of the final state $F \equiv \langle \Psi_B | P | \Psi_B \rangle$ is obtained to be 0.74, which well exceed the value 0.5 expected for product states. 

Loss of fidelity in the Bell state generation process can be also estimated by simulation. Infidelity in excitation of the stimulated Raman transition $|\uparrow\rangle – |\downarrow\rangle$, which include phase noise between the lasers and magnetic field fluctuation, contributes 12-14%. Infidelity in excitation of $|g\rangle – |\uparrow\rangle$, which include 729-nm laser frequency noise and magnetic field fluctuation, contributes 5-7%. Axial quantum number distribution contributes 6-7%. Intensity fluctuation is estimated to contribute by as low as 0.1%, and the effect of spontaneous emission is $\sim$0.1%.

In conclusion, a quantum gate is demonstrated with an atomic qubit consisting of electronic excited states with a separation on the order of a terahertz. A conditional dynamics is clearly observed as a fringe reversal in Ramsey interferometry. Fidelity for Bell-state generation is estimated to be 0.74, and decoherence factors are analysed. The excitation scheme using stimulated Raman transitions with phase-locked lasers offers much flexibility, and is eventually used for atomic transitions which are not explored before as qubit transitions as well as for molecular transitions.

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