New type of Pairing interactions in nuclear matter and finite nuclei

H Sagawa1, J Margueron2 and K Hagino3

1Center for Mathematics and Physics, University of Aizu, Aizu-Wakamatsu, 965-8580
Fukushima, Japan
2 Institut de Physique Nucléaire, Université Paris-Sud, IN2P3-CNRS, F-91406 Orsay Cedex,
France
3 Department of Physics, Tohoku University, Sendai, 980-8578, Japan
E-mail: sagawa@u-aizu.ac.jp

Abstract. We propose new types of density dependent contact pairing interaction which
reproduce pairing gaps in a wide range of nuclear mass table.

We discuss also the relation between the proposed pairing interactions and the pairing gaps
in symmetric and neutron matters obtained by a microscopic treatment based on a bare nucleon-
nucleon interaction. It is shown that the isovector type pairing interaction is necessary on top
of the isoscalar term to reproduce systematically nuclear empirical pairing gaps. The BCS-BEC
crossover of neutrons pairs in symmetric and asymmetric nuclear matters is studied by using
these contact interactions. It is shown that the bare and screened pairing interactions lead to
different features of the BCS-BEC crossover in symmetric nuclear matter.

1. Introduction

It has been known that the pairing correlations play an important role in finite and also
infinite nuclear systems. There are mainly two different approaches for a calculation of pairing
 correlations in finite nuclei. The first approach is based on phenomenological pairing interactions
whose parameters are determined using some selected data [1], while the second approach starts
from a bare nucleon-nucleon interaction and eventually includes the effect of phonon coupling [2].
The latter approach has shown that the medium polarization reduces the pairing gaps in neutron
matter while the neutron pairing gaps in symmetric matter are much enlarged at low density
compared to that of the bare calculation. This enhancement takes place especially for neutron
Fermi momenta $k_Fn < 0.7 \text{ fm}^{-1}$.

In this report, we propose effective density-dependent pairing interactions which reproduce
both the neutron-neutron ($nn$) scattering length at zero density and the neutron pairing gap in
uniform matter. In order to simultaneously describe the density dependence of the neutron
pairing gap for both symmetric and neutron matter, it is necessary to include an isospin
dependence in the effective pairing interaction. Depending on whether the medium polarization
effects on the pairing gap given in Ref. [3] are taken into account or not, we invent two different
density dependences in the pairing interaction. Then, we apply these interactions to study BCS-
BEC cross-over in infinite systems and also pairing gaps in semi-magic finite nuclei, such as Ca,
Ni, Sn and Pb isotopic chains.
Figure 1. (Color online) Top panels: Comparison between the rms radius $\xi_{\text{rms}}$ of the neutron pair and the average inter-neutron distance $d_n = \rho^{-1/3}$ (thin line) as a function of the neutron Fermi momentum $k_{F_n}$ in symmetric (left panel), asymmetric (central panel) and neutron matters (right panel). Bottom panels: The order parameter $\xi_{\text{rms}}/d_n$ is plotted as a function of $k_{F_n}$. The boundaries of the BCS-BEC crossover are represented by the two dashed lines, while the unitary limit is shown by the dotted line. The two pairing interactions defined in Eqs. (1) and (2) are used for the calculations.

2. Isospin dependent pairing interaction and BCS-BEC crossover in nuclear matter

The density–dependent pairing interaction can be read as

$$v_{\text{pair}}(1, 2) = \frac{1 - P_{\sigma}}{2} v_0 g[\rho, I] \delta(r_1 - r_2).$$

where $\rho$ is the nuclear density and $I$ is defined as $I = (\rho_n - \rho_p) / \rho \cdot \tau_z$ [4]. In Ref. [4], an isovector dependence in the density-dependent term $g$ is separated into two parts $g = g^1 + g^2$. The function $g^1$ is determined to mimic the bare pairing gaps in nuclear matter and the function $g^2$ takes care of the medium polarization effect. The functional form of $g^1$ is given by

$$g^1[\rho, I] = 1 - f_s(I) \eta_s \left( \frac{\rho}{\rho_0} \right)^{\alpha_s} - f_n(I) \eta_n \left( \frac{\rho}{\rho_0} \right)^{\alpha_n},$$

where $\rho_0 = 0.16$ fm$^{-3}$ is the saturation density of symmetric nuclear matter and the functions $f_s(I)$ and $f_n(I)$ are $f_s(I) = 1 - f_s(I)$ and $f_n(I) = I = (\rho_n(r) - \rho_p(r)) / \rho(r) \cdot \tau_z$. The values of parameters $\eta_s, \eta_n$ and powers of density dependence $\alpha_s, \alpha_n$ are given elsewhere [4].

We study the Fermi momentum dependence (equivalently density dependence) of BCS-BEC crossover by solving the BCS gap equation [6]

$$\Delta_n = -\frac{v_0 g}{2(2\pi)^3} \int d^3k \frac{\Delta_n}{E_n(k)} \theta(k, k),$$

2
for a given Fermi momentum. The Fourier transform of the Cooper pair wave function \( \Psi_{\text{pair}}(k) = C_{\text{ut}}(v_k) \) is used to evaluate the rms radius \( \xi_{\text{rms}} \) of the Cooper pair in nuclear matter. In Fig. 1, the rms radius \( \xi_{\text{rms}} \) is shown as a function of the neutron Fermi momentum \( k_{F_n} \) as well as the order parameter \( \xi_{\text{rms}}/d_n \) where \( d_n = \rho_n^{-1/3} \). The rms radius of the Cooper pair is less than 5 fm in the region \( k_{F_n} \sim (0.4-0.9) \) fm\(^{-1} \) \((\rho_n/\rho_0 \sim 0.01 - 0.15)\) in the three panels for the bare interaction. The screened interaction gives different effects in symmetric and asymmetric matters: it increases the rms radius for the neutron matter, while the rms radius stays small around 4 fm even at very low density at \( k_{F_n} \sim 0.15 \) fm\(^{-1} \) \((\rho_n/\rho_0 \sim 0.0007)\) in symmetric matter. In the lower panels is shown the ratio of the rms radius to the average distance between neutrons \( d_n \). For the bare interaction, the size of the Cooper pair becomes smaller than \( d_n \) for the Fermi momentum \( k_{F_n} < 0.8 \) fm\(^{-1} \) \((\rho_n/\rho_0 \sim 0.1)\) in general. There are substantial differences for symmetric and asymmetric matters in the case of the screened interaction. The boundaries of BCS and BEC phases are indicative because the phase transition is smooth at the boundaries being of the second order. The crossover region becomes smaller for the neutron matter, while the crossover region increases in the cases of asymmetric \((x_p = 0.3)\) and symmetric matters. Especially, the correlations become strong in symmetric matter and the Cooper pair reaches almost the BEC boundary at \( k_{F_n} \sim 0.2 \) fm\(^{-1} \) \((\rho_n/\rho_0 \sim 0.002)\). Even if the nuclear matter does not enter into the BEC regime, it is shown that the Cooper pair wave function is already very similar to the BEC one when it is close \([4]\).

In ref. \([7]\), the wave functions of di-neutrons participating to the halo nuclei \(^{11}\)Li and \(^{6}\)He have been analyzed to see the analogous behavior to the BCS-BEC crossover in terms of the correlation length. It has been shown that as the distance between the center of mass of the two-neutrons and the core increases, the wave function changes from the weak coupling BCS regime to the strongly correlated BEC regime. This is due to the fact that the pairing correlations are strongly density dependent and the distance between the two-neutrons and the core provides a measure of the pairing strength.

3. Isospin Dependence of Pairing Gaps in Finite Nuclei
We perform Hartree-Fock-Bogoliubov (HFB) calculations for semi-magic Calcium, Nickel, Tin and Lead isotopes using these density-dependent pairing interactions in Eq. (1) derived from a microscopic nucleon-nucleon interaction. Our calculations reproduce well the neutron number dependence of experimental data for binding energy, two-neutron separation energy, and odd-even mass staggering of these isotopes \([5]\). Especially the interaction IS+IV Bare without the medium polarization effect gives satisfactory results for all the isotopes as is seen in Fig. 2. It is clear in the comparison between IS bare and IS+IV bare that the isospin dependence of the pairing interaction plays an important role in the pairing gaps of neutron-rich nuclei. The isospin dependent pairing interaction is further applied to both even-even and even-odd nuclei by using EV8-odd program. The results successfully reproduce the empirical isotope and isotope dependences of the odd-even mass differences of several medium-heavy and heavy nuclei \([8]\).

4. Summary
A new type of density-dependent contact pairing interaction was obtained to reproduce the microscopic pairing gaps in symmetric and neutron matter \([3]\). We have applied these density-dependent pairing interactions to study the BCS-BEC crossover phenomenon in symmetric and asymmetric nuclear matters. We found that the spatial di-neutron correlation is strong in general in a wide range of low matter densities, up to \( k_{F_n} \sim 0.9 \) fm\(^{-1} \) \((\rho_n/\rho_0 \sim 0.15)\). This result is independent of the pairing interactions, either bare or screened one, as well as of the asymmetry of the uniform matter.

We performed also HFB calculations for semi-magic Calcium, Nickel, Tin and Lead isotopes using these density-dependent pairing interactions. Our calculations reproduce well the neutron
Figure 2. Comparison of the neutron HFB pairing gaps $\Delta_n$ with the odd-even mass staggering given by the three-point formula $\Delta^{(3)}$. The dotted line shows the results of the pairing interactions IS+IV Bare, while the short dashed, and long dashed lines are obtained with the pairing interactions IS+IV Screened and IS Bare, respectively. The difference $\delta(\Delta_n)$ is defined as $\delta(\Delta_n) = \Delta_n(\text{th.}) - \Delta_n(\text{exp.})$. All units are given in MeV.

number dependence of experimental data for binding energy, two-neutron separation energy, and odd-even mass staggering of these isotopes with the interaction IS+IV Bare. Recently, the isospin dependent interaction is extended to introduce the quadratic terms of isospin and applied to a wide region of nuclei in the mass table [9]. These results suggest that by introducing the isovector term in the pairing interaction, one can construct a global effective pairing interaction which is applicable to nuclei in a wide range of the nuclear chart.

Acknowledgments
This work was supported by the Japanese Ministry of Education, Culture, Sports, Science and Technology by Grant-in-Aid for Scientific Research under the program numbers (C) 22540262 and 20540277.

References
[1] Dobaczewski J, Nazarewicz W, and Reinhard P-G 2001 Nucl. Phys. A693, 361
[2] Barranco F et al. 1999 Phys. Rev. Lett. 83, 2147
[3] Cao L G, Lombardo U, and Schuck P 2006 Phys. Rev. C 74, 064301
[4] Margueron J, Sagawa H, and Hagino K 2007 Phys. Rev. C 76, 064316
[5] Margueron J, Sagawa H, and Hagino K 2008 Phys. Rev. C 77, 054309
[6] Engelbrecht J R, Randeria M, Sá de Melo C A R 1997 Phys. Rev B 55, 15153
[7] Hagino K, Sagawa H, Carbonell J and Schuck P 2007 Phys. Rev. Lett. 99, 025506
[8] Bertulani C A, Lu H F and Sagawa H 2009 Phys. Rev. C 80, 027303
[9] Yamagami M, Shimizu Y R and Nakatsukasa T 2009 Phys. Rev. C 80, 064301