Gravimagnetic shock waves and gravitational-wave experiments

Yu.G. Ignat’ev
Kazan State Pedagogical University
1 Mezhlauk Str., Kazan 420021, Russia

Abstract
Causes of the unsatisfactory condition of the gravitational-wave experiments are discussed and a new outlook at the detection of gravitational waves of astrophysical origin is proposed. It is shown that there are strong grounds for identifying the so-called giant pulses in the pulsar NP 0532 radiation with gravimagnetic shock waves (GMSW) excited in the neutron star magnetosphere by sporadic gravitational radiation of this pulsar.

1 Introduction
In the history of physics of the 20th century, I suppose, there is no such a grave experimental problem (except the controlled thermonuclear fusion problem) that, being solved for over thirty years by different research groups as the gravitational wave (GW) detection problem. Although much means are used to solve it, no sufficiently convincing positive results have been obtained. What are the reasons for this situation? An error in the gravitational theory? The experimentalists’ incapability? Are there realizable opportunities to detect gravitational radiation in the visible future? Is the GW detection problem worth studying? We will try to answer these questions in the present paper. As any other radiation detection problem, this one also splits into two independent problems: (1) “GW sources” and (2) “GW detectors”. It is important not to forget to join both branches by solving a concrete experimental problem.

2 Sources of gravitational radiation
2.1 Estimates of gravitational radiation power
The average power of gravitational radiation from a source is calculated by the formula [9]

\[ L_{GW} = \frac{G}{5c^5} \langle \dddot{t}_{ik} \dddot{t}_{ik} \rangle, \]  

(1)

where \( G \) is the gravitational constant, \( c \) is the velocity of light,

\[ t_{ik} = \int \rho(x_{i}x_{k} - \frac{1}{3} \delta_{ik} r^2)dV \]  

(2)

is the reduced quadrupole moment of the source; dots mean time derivatives. There are two parameters of interest in the GW detection problems: the GW magnitude (i.e. the deviation from the flat metric \( h_{ik} = g_{ik} - \eta_{ik} \)) and the GW frequency, \( \omega \). The GW energy flow is expressed in terms of these parameters by the formula [9]:

\[ ct^{14} = \mathcal{P} = \frac{c^3}{16\pi G} \left[ h_{23}^2 + \frac{1}{4} (\dot{h}_{22} - \dot{h}_{33})^2 \right]. \]  

(3)
Throughout the paper we use the metric signature \((-1, -1, -1, +1)\). From (1) - (3) follow estimating formulae for the gravitational radiation power and magnitude:

\[ L_{GW} = \frac{G E_q \omega^2}{5c^5}, \]  
\[ L_{GW} = \frac{c^3 \omega^2 R^2}{4G} h^2, \]

where \(E_q\) is the energy of quadrupole oscillations of the source of the characteristic frequency \(\omega\) \((L_0 = \omega E_q\) is the quadrupole oscillation power), \(R\) is the distance from the source to an observer.

### 2.2 Restrictions on GW magnitude

In particular, a useful formula follows from (4) – (5):

\[ \frac{h}{h_0} = \frac{E_q}{NC^2}, \]

where \(h_0\) is the gravitational potential of the source of the total mass \(M\):

\[ h_0 = \frac{GM}{c^2 R}. \]

According to (5) and (7), the ratio of GW magnitude to the Newtonian gravitational potential of the source is of the order of the ratio of the quadrupole energy of the source oscillations to its complete energy at rest, \(E_0 = Mc^2\). It is obvious that always \(E_q < E_0\), and \(E_q/E_0 \ll 1\) in a generic situations, therefore Eq. (7) gives an upper limit of the GW magnitude of a source, which is a good sobering factor by itself. Let us list some values for reference. For the Solar mass \((M_\odot = 2 \cdot 10^{33} \text{g})\) and a distance of 1 pc \((3.26 \text{ light years} = 3 \cdot 10^{18} \text{ cm})\) 1, from Eq. (7) we obtain:

\[ h_0 \cdot (\text{pc}/M_\odot) = 4.8 \cdot 10^{-14}. \]

For a mass of 1 kg at a distance of 1 m:

\[ h_0 \cdot (\text{m kg}^{-1}) = 7.4 \cdot 10^{-18}. \]

Therefore for an A-bomb explosion with \((\Delta M/M \sim 10^{-3})\), under the condition that the whole explosion energy turns into the quadrupole oscillations energy, at a distance of 1 m (!) from the epicentre, we get the GW magnitude \(h \sim 10^{-21}\). If one makes all atoms of a compact graser oscillate in the optic range (the radiation energy is about \((\hbar \nu \sim 1 \text{ ev, } l \sim 1 \text{ m})\), we get the following maximum estimate for the GW magnitude on the graser end-wall: \(h \sim 10^{-28}\).

GW sources can be divided into two classes: (1) stable (quasistable) sources, which cannot be destroyed during the GW radiation process; (2) catastrophic sources, being destroyed in the GW radiation process. A graser represents a source of the first type, an A-bomb a source of the second type. For first type sources the quadrupole oscillation energy cannot exceed the binding energy of the source as a whole, unlike second-type ones, which are sources for one occasion. For example, close binaries and quadrupole oscillations of neutron stars are first-type astrophysical sources, and Supernovae are second-type ones.

---

1 The nearest stars’ distance is about 1.3 pc.
As mentioned above, stable radiation sources are subject to the condition

$$E_{\text{kin}} < E_b,$$  \hspace{1cm} (8)

where $E_{\text{kin}}$ is the inner kinetic energy of separate parts of the source, $E_b$ is their binding energy. Since it is always $E_q \leq E_{\text{kin}}$, the condition (8) takes the form

$$E_q \leq E_b.$$  \hspace{1cm} (9)

Hence the upper limit of GW magnitude from such sources can be obtained from the formula

$$h < h_0 \frac{E_b}{M c^2}.$$  \hspace{1cm} (10)

For astrophysical sources the binding energy is essentially that of gravitational attraction. Let $\Delta M$ be the part of the mass of an astrophysical object performing quadrupole oscillations. Its gravitational binding energy is

$$E_b < G \frac{\Delta M \cdot M}{l},$$  \hspace{1cm} (11)

where $l$ is the characteristic size of the system. Thus for the upper limit of a GW magnitude from such a source Eqs. (10) and (11) give:

$$h \leq h_0 \frac{r_g \Delta M}{2l},$$  \hspace{1cm} (12)

where $r_g = 2GM/c^2$ is the gravitational radius of the radiating system.

### 2.3 Radiation frequency

Consider first a source of total mass $M$, which consists of two parts, so that the second part $\Delta M$ performs a free motion in the gravitational field of the system (rotation or free fall). Let $\omega$ be a characteristic frequency of this process.\(^2\) Equating the centrifugal and free fall accelerations, we obtain the well-known relation

$$GM = \omega^2 l^3,$$  \hspace{1cm} (13)

which connects the characteristic size of the system with its characteristic frequency.

Now let the gravitational attraction in the system be held by the forces of pressure (for stellar quadrupole oscillations). Equating these forces, we get the hydrostatic balance condition

$$|\nabla P| = \rho \frac{GM}{l^2},$$  \hspace{1cm} (14)

where $P$ is the pressure and $\rho$ is the density. Using the known relation $dP = v_f^2 d\rho$, where $v_f$ is the velocity of sound, from Eq. (14) we obtain:

$$lv_f^2 \approx MG.$$  \hspace{1cm} (15)

\(^2\)Evidently the order of magnitude of this quantity coincides with the frequency of gravitational radiation from the system.
But $v_f/l \approx \omega$ is the system proper oscillation frequency. Therefore for systems supported by the forces of pressure we return to the estimate of Eq. (13).

Thus for stable astrophysical GW sources Eq. (13) has a universal nature if omega is understood as a characteristic frequency of the system oscillations.

From the law (13) we can estimate the radiation characteristics of the collapsing objects, colliding stars and the like. It follows from this law that a maximum radiation frequency can be achieved for objects close to the gravitational collapse condition. In this case, by (12), the maximum magnitude of radiated GW is achieved (see [1]). For objects of masses of the order of the Solar mass ($r_g = 2.96$ km) the maximum radiation frequency is

$$\omega_{\text{max}} \sim c/r_g \approx 10^5 \text{sec}^{-1}.$$

### 2.4 Close binary stellar systems

For intense astrophysical sources of gravitational radiation, this radiation is the basic mechanism of quadrupole oscillation energy loss. Therefore, more rigorously, such sources should be called quasistable. The gravitational radiation power of a system of two orbiting gravitating masses $m_1$ and $m_2$ is calculated from the known formula [10]

$$L_g = -\frac{dE}{dt} = \frac{32G^4m_1^2m_2^2(m_1 + m_2)}{5c^3r^5},$$

(16)

where $r$ is the separation of the centres of mass. The energy balance leads to the mass approaching law [10]

$$\dot{r} = \frac{64G^3m_1m_2(m_1 + m_2)}{5c^3r^3},$$

(17)

Its integration yields a formula for the time $t$ needed for the mass centres to approach to a distance of $r$ from $r_0$:

$$t = \frac{5c^5}{192G^3m_1m_2(m_1 + m_2)}(r^4 - r_0^4).$$

(18)

Further for simplicity we will study a pair of equal stars, setting $m_1 = m_2 = M$, $r_0 = 2R_0$ where $R_0$ is the stellar radius, i.e. we will calculate the time until the catastrophic stellar collision, $\tau$ (the lifetime). Then from (18) we get:

$$\tau = \frac{5c^5(l^4 - 16R_0^4)}{384G^3M^3}$$

and for $l \gg R_0$

$$\tau \approx \frac{5}{384} \left( \frac{l}{r_g} \right)^3 \frac{l}{c}.$$  

(19)

The gravitational radiation frequency of a binary system increases with time; the ratio of the frequency shift per period $\Delta \omega$ to the radiation frequency $\omega$ is, by order of magnitude,

$$\frac{\Delta \omega}{\omega} \sim \left( \frac{r_g}{l} \right)^{\frac{3}{2}}.$$  

(20)

Figs. 1 and 2 show the dependence of the gravitational radiation power of a binary system and the radiated GW magnitude on the distance between the stars. Fig. 3 shows the binary lifetime versus their separation for stars with the masses $m_1 = m_2 = M_\odot$.  


Let us estimate the probability of GW detection from a close binary system in the Galaxy at given rotation period, assuming that the Galaxy age is of the order of $1 \cdot 10^{10}$ years. Further, we assume that the average stellar number density in the Galaxy is of the order of $0,120$ stars/pc$^3$ [15], the Galactic volume is $300$ kpc$^3$ [16], then the number of stars in the Galaxy is about $0,35 \cdot 10^{11}$. Besides, we take into account that approximately half of the stars are in binary systems [17]. Then the possibility of existence of a binary with a prescribed lifetime $\tau$ is proportional to the ratio $\tau/t$, where $t$ is the age of the Galaxy. In the columns of Table 1 corresponding to system lifetimes smaller than 1 year, the detection probability of such systems in experiments lasting 1 year is shown. Evidently for such systems the probability of detection in a year-lasting experiment coincides with that for a binary having a lifetime of 1 year. The number of such systems in the Galaxy is estimated to be of the order of one.
The presented data show that at the instant preceding the catastrophic collision, the gravitational radiation power from the binary is of the order of Supernova luminosity. Thus, a stellar collision in a close binary is an event whose scale is of the order of a Supernova explosion. As mentioned above, in the Galaxy the probability of GW detection from a binary with a lifetime of the order of 1 year is close to one. This means that catastrophic phenomena with energy release of the order of $1 \times 10^{54}$ erg/s should happen once a year. However, in reality such phenomena happen once in 40 to 80 years in the Galaxy \[19\]. A reason for such a discrepancy is that in a close binary with a lifetime of the order of 1 year the stars’ separation is about 12000 km.

Therefore for such a system to exist it is necessary that both stars be at least white dwarfs. But in this case even in a stellar collision the energy released is 4 orders of magnitude smaller than that of a Supernova explosion. Thus for a catastrophic collision of this scale it is at least necessary that one of the components be a neutron star, while the second one is a white dwarf. The existence probability of such systems in the Galaxy is much smaller. Note that it is difficult to understand the experimental programmes intended for registration of GW from the binaries with periods of the order of a few seconds. According to Table 1, their lifetime does not exceed 5 years, and in this case it would be more reasonable to wait these 5 years and to detect the gravitational radiation from a catastrophic collision: its power is higher by at least 13 orders and the GW magnitude is greater by 3 orders (!), as follows from Table 1. However, at least in the last 10 years nobody detected catastrophic events on such a scale at distances smaller than 15 kpc.

Since the pulsars are identified with Supernovae remnants, the average frequency of Supernova bursts may be estimated from the data on pulsars spreading in the Solar neighbourhood. Thus, at distances within about 1 kpc, on the whole, about 20 pulsars are observed. Table 2 shows the data on the pulsars nearest to the Solar system\[3\]. As follows from this Table, almost all the pulsars are younger than 108 years. Therefore, it may be stated that the observed pulsars are remnants of Supernovae which exploded in the last hundred million years. It gives 1 flash per 50 years, coinciding with the estimate of Ref. \[19\]. It seems likely to be also close to the average frequency of catastrophic collisions in close binaries.
Table 1: Characteristics of gravitational radiation from a close binary

|  | 5(4) | 2(4) | 1(4) | 7(3) | 4(3) | 1(3) | 320 | 100 | 40 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\tau$ | 8.2(10) | 2.1(9) | 1.3(8) | 3.15(7) | 3.3(6) | 1.3(4) | 129 | 130 | 0.031 | 0 |
| $\omega$ | 0.046 | 0.18 | 0.52 | 0.88 | 2.05 | 16.4 | 92.6 | 518 | 2047 | 5.96 |
| $T$ | 136 | 34.3 | 12.1 | 7.10 | 3.07 | 0.38 | 0.068 | 0.012 | 0.003 | 0.001 |
| $h$ | 7.5(-21) | 1.9(-20) | 3.7(-20) | 5.4(-20) | 9.4(-20) | 3.7(-19) | 1.2(-18) | 3.7(-18) | 9.4(-18) | 1.9(-17) |
| $P$ | 9.7(-14) | 9.5(-12) | 3.0(-10) | 1.8(-9) | 3.0(-8) | 3.0(-5) | 9.8(-3) | 3.0 | 296 | 9463 |
| $N$ | 2343 | 60 | 3.7 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $h_{\oplus}$ | 1.5(-20) | 4.4(-21) | 1.7(-21) | 3.7(-21) | 6.5(-21) | 2.6(-20) | 8.3(-20) | 2.6(-19) | 6.5(-19) | 1.3(-18) |
| $P_{\oplus}$ | 3.8(-13) | 3.2(-12) | 1.6(-11) | 9.0(-12) | 1.5(-10) | 1.5(-7) | 4.7(-5) | 1.5(-2) | 1.4 | 46 |

*) Here and henceforth, the figures in parentheses indicate the order of magnitude \(a(b) = a \cdot 10^b\); the quantities $h$ and $P$ (the gravitational radiation flow density) are calculated at a distance of 1 kpc from the binary; $T = 2\pi/\omega$ is the GW period; $N$ is the expected number of binaries in the Galaxy; $R = < R >$ is the expected distance to the binary in kpc; $h_{\oplus}$ is the expected GW magnitude on the Earth; $P_{\oplus}$ is the expected gravitational radiation flow density on the Earth. The quantities $T$ and $\tau$ are given in seconds, $\omega$ in $\text{sec}^{-1}$, $l$ in km, $L_{\text{g}}$ in erg/sec, $P$ and $P_{\oplus}$ in W/cm$^2$.

Table 2: Data on pulsars located at distances smaller or of the order of 1 kpc from the Sun

| No | Pulsar (name) | Distance (kpc) | $P$ (sec) | $\dot{P}/P$ (years) |
|---|---|---|---|---|
| 1 | MP 0031 | 0.21 | 0.94 | 7.1(7) |
| 2 | MP 0450 | 0.33 | 0.55 | |
| 3 | NP 0532 | 2.0 | 0.033 | 2.5(3) |
| 4 | MP 0628-28 | 0.170 | 1.24 | 1.6(7) |
| 5 | CP 0809 | 0.19 | 1.29 | 2.5(8) |
| 6 | AP 0823+26 | 0.38 | 0.53 | 1.0(7) |
| 7 | PSR 0833-45 | 0.5 | 0.089 | 2.3(4) |
| 8 | CP 0834 | 0.43 | 1.27 | 5.9(6) |
| 9 | PP 0943 | 0.30 | 1.098 | |
| 10 | CP 0950 | 0.10 | 0.253 | 3.5(7) |
| 11 | CP 1133 | 0.16 | 1.188 | 1.0(7) |
| 12 | AP 1237+25 | 0.20 | 1.382 | 4.6(7) |
| 13 | PSR 1451-68 | 0.40 | 0.263 | 2.8(6) |
| 14 | HP 1508 | 0.26 | 0.740 | 4.7(6) |
| 15 | CP 1919 | 0.42 | 1.337 | 3.2(7) |
| 16 | PSR 1929+10 | 0.27 | 0.227 | 6.2(6) |
| 17 | JP 1933+16 | 3 | 0.359 | 1.9(6) |
| 18 | AP 2016+28 | 0.47 | 0.558 | 1.2(8) |
| 19 | PSR 2045 | 0.38 | 1.962 | 5.7(6) |
Thus we have to deal at best with system linear sizes of the order of $110000 \div 20000\text{km}$. It gives:
\[ T \sim 10 \div 40\text{sec}, \quad h_\oplus \sim (1 \div 5) \cdot 10^{-21}, \quad P_\oplus \sim 10^{-11} \div 3 \cdot 10^{-12}\text{W/cm}^2. \] GW with such parameters can hardly be detected in the coming decades. In this situation it only remains to hope for a case, rare and simultaneously dangerous for the Earth, of a Supernova burst or a catastrophic end of a close binary.

### 2.5 Neutron star oscillations

There is, however, one more class of stable astrophysical GW sources — quadrupole oscillations of neutron stars. Table 3 shows the calculated parameters of gravitational radiation from neutron stars \( \left[9\right] \) (columns 1 \div 7). According to this table, the following characteristics of radiation are to be expected from these sources:
\[ \sim (0, 3 \div 1) \cdot 10^4\text{s}^{-1}; \quad h_\oplus \sim 10^{-25}, \quad P_\oplus \sim 3 \cdot 10^{-12}\text{W/cm}^2. \] by the energy of stellar quadrupole oscillations of the order of $10^{38} \div 10^{39}\text{erg}$ and the whole gravitational luminosity of $L_g \sim 4 \cdot 10639\text{erg/sec}$. Note that due to the smallness of the GW magnitude from this source and the absence of a mechanism able to support the excitation of a necessary quadrupole moment during a sufficiently long time, quadrupole oscillations of neutron stars have not been considered as a competitive GW source.

### 3 GMSW and GW detection

The cause of the unsatisfactory condition in the GW detection problem is, in the author's opinion, the originally chosen erroneous way of its solution — the programme of creating GW detectors. Direct GW detection can be realized either due to their tidal effect on a nonrelativistic (solid-state) detector, or due to their relativistic effect on a detector having a relativistic component (a laser ray). In both cases the GW effect on a detector (test-body displacement or laser ray deviation) is proportional to the GW magnitude. And the expected GW magnitudes from astrophysical sources are extremely small (see, for example, \( \left[9\right] \)).

The existing GW detection programmes are generally meant for astrophysics sources of two types: 1. Supernovae; 2 close binaries. In the first case one may expect GW magnitudes about $10^{-17} \div 10^{-18}$ with the radiation in a wide frequency range with the characteristic frequency of the order of $10^3\text{sec}^{-1}$, in the second case magnitudes about $10^{-20} \div 10^{-21}$ with a fixed frequency in the range of $0, 1 \div 10\text{sec}^{-1}$. Due to the very small expected GW magnitudes on the Earth, the experimental programmes intended for direct GW detection inevitably come across the problem of noise of external thermal and quantum nature. This struggle is already in its third decade and requires the creation of high-precision deeply cooled detectors.

On the other hand, it is well-known that even such weak-magnitude GW carry rather a high energy: in the above examples, this energy is of the order of $1\text{W/cm}^2$ in the first case and about $10^{-13} \div 10^{-11}\text{W/cm}^2$ in the second case. Electromagnetic signal detection on such a power level has no problems. Therefore, the GW detection problem should be solved in a different way: by looking for specific electromagnetic signals from GW effect upon matter in those regions of the Galaxy where the gravitational radiation intensity is high. Setting the problem in such a way, we should above all study the GW effect on plasma-like media. The corresponding studies, carried in the eighties mainly in the Kazan school of gravitation, revealed a number of specific electromagnetic reactions of plasma to GW. In Refs. \( \left[2\right] - \left[5\right] \) the effect of plane GW (PGW) on plasma-like media was investigated by...
the methods of relativistic kinetic theory in the approximation of negligible back reaction of matter on the PGW:

$$(8\pi G/c^2)v \ll \omega^2.$$  \hfill (21)

where $\omega$ is the GW characteristic frequency, $v$ is the matter energy density. These papers have revealed a number of phenomena of interest, consisting in induction of longitudinal electric oscillations in the plasma by PGW. In spite of the strictness of the results obtained in [2]–[5], the effects discovered have very little to do with the real problem of GW detection. Moreover, the above calculations show a lack of any prospects for GW detectors based on dynamical excitation of electric oscillations by gravitational radiation. There are two reasons for that: the smallness of the ratio $m^2G/e^2 = 10^{-43}$ and the small relativistic factor $\langle v^2 \rangle/c^2$ of standard plasmalike systems. The GW energy conversion coefficient to plasma oscillations is directly proportional to a product of these factors.

However, the situation may change radically if strong electric or magnetic fields are present in the plasma. In Ref. [6], where the induction of surface currents on a metal-vacuum interface by a PGW was studied, it was shown that the values of currents thus induced can be of experimental interest. In [7], on the basis of relativistic kinetic equations, a set of magneto-hydrodynamics (MHD) equations was obtained, which described the motion of collisionless magnetoactive plasma against the background of a PGW of arbitrary magnitude in drift approximation and it was shown that, provided the propagation of the PGW is transversal, there arises a plasma drift in the PGW propagation direction.

In Ref. [1] an exact solution of the relativistic MHD equations in the PGW background of arbitrary magnitude was obtained and, on its basis, a fundamentally new class of sufficiently nonlinear threshold effects was discovered, named GMSW (“jimmysway”) - gravimagnetic shock waves.

3.1 GMSW

The PGW metric of the polarisation $e^\pm$ is described by the expression [9]:

$$ds^2 = 2dudv - L^2[e^{2\beta}(dx^2)^2 + e^{-2\beta}(dx^3)^2],$$  \hfill (22)

where $\beta(u)$ is an arbitrary function (the PGW magnitude), while $L(u)$ (the PGW background factor) obeys the ordinary second-order differential equation

$$L'' + L\beta^2 = 0;$$  \hfill (23)

$u = 1/\sqrt{2}(t - x^1)$ is the retarded time and $v = 1/\sqrt{2}(t + x^1)$ is the advanced time. Let there be no PGW at ($u \leq 0$):

$$\beta(u)|_{u \leq 0} = 0; \quad L(u)|_{u \leq 0} = 1,$$  \hfill (24)

while the plasma be homogeneous and at rest:

$$v^\nu(u)|_{u \leq 0} = v^\nu(u)|_{u \leq 0} = 1/\sqrt{2}; \quad v^2|_{u \leq 0} = v^3|_{u \leq 0} = 0;$$

$$\varepsilon(u)|_{u \leq 0}; \quad p(u)|_{u \leq 0} = p_0$$  \hfill (25)

9
\( p = p(\varepsilon) \) is the plasma pressure, \( v^k \) is its dynamic velocity vector) and a homogeneous magnetic field is directed in the \((x^1, x^2)\) plane:

\[
\begin{align*}
H_1(u)|_{u \leq 0} &= H_0 \cos \Omega; \quad H_2(u)|_{u \leq 0} = H_0 \sin \Omega; \\
H_3(u)|_{u \leq 0} &= 0; \quad H_\alpha(u)|_{u \leq 0} = 0,
\end{align*}
\]  

(26)

where \( \Omega \) is the angle between the axis \( Ox^1 \) (the PGW propagation direction) and the magnetic field \( \mathbf{H} \) direction. The conditions (26) correspond to the vector potential:

\[
\begin{align*}
A_v &= A_u = A_2 = 0; \\
A_3 &= H_0 (x^1 \sin \Omega - x^2 \cos \Omega); \quad (u \leq 0).
\end{align*}
\]  

(27)

The exact solution of the relativistic MHD equations against the metrics background (22). The exact solution of the relativistic MHD equations against the metrics background (22) obtained in [1] satisfies the initial conditions (24) - (26) and is determined by the governing function \( \Delta(u) = 1 - \alpha^2 (e^{2\beta} - 1) \), obtained in [1] satisfies the initial conditions (25) - (27) and is determined by the governing function:

\[
\Delta(u) \equiv 1 - \alpha^2 (e^{2\beta} - 1),
\]  

(28)

where \( \alpha \) is a dimensionless parameter:

\[
\alpha^2 = \frac{H_0^2 \sin^2 \Omega}{4\pi(\varepsilon_0 + p_0)}.
\]  

(29)

This solution contains a physical singularity on the hypersurface \( \Sigma : u = u_* \):

\[
\Delta(u_*) = 1 - \alpha^2 (u_*) (e^{2\beta(u_*)} - 1) = 0,
\]  

(30)

where the plasma and magnetic field energy densities tend to infinity and the dynamic velocity of the plasma as a whole tends to the velocity of light in the PGW propagation direction. In this case the ratio of the magnetic field energy density and the plasma energy tends to infinity. This singularity is a gravimagnetic shock wave (GMSW, [1]), spreading in the PGW propagation direction at a subluminal velocity. According to Eq. (30), necessary conditions for the occurrence of the singularity are

\[
\begin{align*}
\beta(u) &> 0; \\
\alpha^2 &> 1.
\end{align*}
\]  

(31)

An extremely important fact is that a singular state is even possible in a weak PGW (\(|\beta| \ll 1\)) under the condition that the plasma is highly magnetized (\(\alpha^2 \gg 1\)); in this case the singularity condition arises according to (30) on the hypersurfaces \( u = u_* \):

\[
\beta(u_*) = 1/(2\alpha^2).
\]  

(33)

In particular, for a barotropic equation of state \( p = k\varepsilon, \ 0 \leq k < 1 \)

\[
\varepsilon = \varepsilon_0 \Lambda^{-1+\nu},
\]  

(34)

\[
v_v = \frac{1}{\sqrt{2}} L^{\nu} \Delta^{1+\frac{\nu}{2}};
\]  

(35)
\[ \frac{v_u}{v_v} = \Delta^{-2} \left[ \Lambda^{-\nu} + (\Delta - 1)^2 L^{-2} e^{-2\beta} \cot^2 \Omega \right]; \quad (36) \]

\[ H^2 = \frac{H_0^2}{\Lambda^2} = \left( \cos^2 \Omega + L^2 \Lambda^{-\nu} e^{2\beta} \sin^2 \Omega \right), \quad (37) \]

where

\[ \Lambda = L^2(u)\Delta(u), \quad \nu = \frac{2k}{1-k} > 0, \]

and

\[ H^2 = \frac{1}{2} F_{ik} F^{ik} \]

is the electromagnetic field invariant, (squared magnetic field strength in the frame of reference comoving with the plasma).

It follows from (34) - (37) that if \( \beta > 0 \), the plasma moves in the GW propagation direction \( (v^1 = 1/\sqrt{2}(v_u - v_v) > 0) \) and if \( \beta < 0 \), in the opposite direction. The effect is maximum in the PGW propagation direction, that is, perpendicular to that of the original magnetic field strength, and vanishes in the direction parallel to the magnetic field strength.

In the case of strictly transversal PGW propagation \( (\Omega = \pi/2) \), in the direction \( Ox^2 \) the plasma drift vanishes, and the component of the plasma physical 3-velocity in the \( Ox^1 \) direction \( v^1 \) is

\[ v^1 = c \frac{v_u - v_v}{v_u + v_v} = c \frac{1 - \Delta^2 \Lambda^\nu}{1 + \Delta^2 \Lambda^\nu}. \quad (38) \]

The components of the total (including the magnetic field) EMT of the magnetoactive plasma, \( T^\alpha_k \) \( (\alpha = 1, 4) \) have a hydromagnetic structure:

\[ T^\alpha_k = (E + P)v^\alpha v_k - P \delta^\alpha_k, \quad (39) \]

where

\[ \varepsilon_H = P_H = \frac{H^2}{8\pi}; \quad E = \varepsilon + \varepsilon_H; \quad P = p + P_H, \quad (40) \]

where \( P \) and \( E \) are the total pressure and energy density of the magnetoactive plasma. There arises an energy flow in the plasma in the direction \( Ox^1 \):

\[ T^{14} = \frac{\varepsilon_0 + p_0}{4L^4} (\Delta^{-4} \Lambda^{2\nu} - 1)(\Delta \Lambda^{2\nu} + \alpha^2 e^{2\beta}). \quad (41) \]

The parameter \( \nu \) takes in these formulae the following values in the two extreme cases:

\[ \nu = \begin{cases} 0; & k = 0; \\ 1; & k = 1/3. \end{cases} \quad (42) \]

For a weak GW

\[ |\beta(u)| \ll 1; \quad L^2(u) = 1 + O(\beta^2) \approx 1, \quad (43) \]

the expressions (37), (41) and (38) take the form

\[ \frac{v^1}{c} = \frac{1 - \Delta^m}{1 + \Delta^m}; \]
where the coefficients \( m \) and \( n \) take integer values for nonrelativistic \((k = 0)\) and ultrarelativistic \((k = 1/3)\) plasma:

\[
\begin{align*}
    k = 0; & \quad m = 2; \quad n = 4; \\
    k = 1/3; & \quad m = 3; \quad n = 2.
\end{align*}
\]

\[\text{(46)}\]

### 4 GW energy transmission to plasma: a half-self-consistent solution

#### 4.1 Total momentum conservation

Since on the singular hypersurface \((30)\) \(\Delta(u) = 0\), the energy densities of the plasma and the magnetic field tend to infinity, and the velocity of the plasma as a whole tends to the speed of light, the total energy of the magnetohydrodynamic shock wave and its flow in the GW propagation direction tend to infinity. The singular state emerging in the plasma due to the PGW violates the basic assumption \((21)\) of the weakness of GW interaction with the plasma. In a more complete self-consistent problem including gravitation, the back reaction of the shock wave upon the PGW should lead to PGW energy loss and its magnitude damping up to the values

\[
\max|\beta| < 1/2\alpha^2.
\]

\[\text{(47)}\]

Thus a GMSW is an effective mechanism of a gravitational wave energy pumping over into plasma \[1\]. A rigorous solution of the problem of PGW energy transformation into the shock wave energy is only possible by studying the self-consistent set of the Einstein equations and the MHD equations.

Ref. \[1\] suggested a semiquantitative solution of this problem on the basis of a simple model of energo–ballance. Due to its extreme importance, we do not restrict ourselves to \[1\] and return to a more complete study of the problem of energy transmission from a GW to magnetoactive plasma. However, instead of solving the Einstein equations, we make use of their consequence, the conservation law of the total momentum of the system “plasma + gravitational waves”. Clearly, this model is only approximate and cannot replace a rigorous solution to the Einstein equations. According to \[10\], an arbitrary gravitational field provides the conservation of the system’s total momentum

\[
P^i = \frac{1}{c} \int (-g)(T^{i4} + t^{ik})dV
\]

\[\text{(48)}\]

where \(t^{ik}\) is the energy-momentum pseudotensor of the gravitational field and the integration covers the whole 3-dimensional space. Let us take into account that the above solution is plane-symmetric and only depends on the retarded “time” \(u\). Consequently the integration over the “plane” \((x^2, x^3)\) in \[18\] reduces to simply multiplying by an infinite 2-dimensional area. Dividing both sides of \[18\]
by this area and bearing in mind that with Ω = π/2 among the 3-dimensional flows only $P^1$ is nonzero, we obtain the conservation law of the surface density of the momentum $P^1_\Sigma$:

$$P^1_\Sigma = \frac{1}{c} \int_{-\infty}^{+\infty} (-g)(T^{14} + t^{i4})x = \text{Const.} \quad (49)$$

Let the right semispace $x > 0$ be filled with magnetoactive plasma and the left one $x < 0$ with matter which does not interact with a weak GW. Let further the whole gravitational momentum be concentrated in the interval $u \in [0, u_f]$, where $t_f = \sqrt{2}u_f$ is the gravitational pulse duration. Since the integral in Eq. (49) is conserved all the time, let us consider it at $t_0 < 0$, when the GW has not yet reached the magnetoactive plasma, and $t_f > t > 0$, when the GW has reached the plasma. Taking into account that the vacuum solution depends only on the retarded time, we get for the integral in Eq. (49):

$$u_f \int_{t_0}^{t_f} = \int_{t_0}^{t_f} (T^{14} + t^{14})du + \int_{t/\sqrt{2}}^{u_f} t^{14}, \quad (50)$$

where $t_f^{14} = t^{14}(\beta_0(u))$; $t^{14} = t^{14}(\beta(u))$, $\beta_0(u)$ is the vacuum magnitude of the PGW, $\beta(u)$ is the PGW magnitude with allowance for interaction with the plasma. Transferring one of the integrals to the left-hand side of Eq. (50), we arrive at the relation

$$\int_{t_0}^{u} = \int_{t_0}^{u} (T^{14} + t^{14})du, \quad (51)$$

where the variable $u = t/\sqrt{2} > 0$ can now take any positive values.

A similar law may be written for the plasma total energy; in this case instead of Eq. (51) we obtain:

$$\int_{t_0}^{u} = \int_{t_0}^{u} (T^{14} - E_0 + t^{14})du,$$

where $E_0$ is the total energy density of the unperturbed plasma.

### 4.2 Local analysis of the conservation law

Since the relation (51) must be valid at any values of the variable $u$, the corresponding local relation should hold:

$$T^{41}(\beta) + t^{41}(\beta) = t^{41}(\beta_0), \quad (52)$$

i.e. a local conservation law of the energy flow density should hold, as was assumed in Ref. [1]. It should be pointed out that the local conservation law (52) is a direct consequence of the solution stationarity, i.e. the solution dependence on the retarded time $u = (ct - x)/\sqrt{2}$. There are two factors preventing the solution in a rigorous model from being stationary: (1) PGW interaction with the
plasma; (2) the boundary conditions on the surface \( x = 0 \). In accordance with the approximation (21), we introduce a small dimensionless parameter \( \chi \): \[ \chi^2 = \frac{\pi G(\varepsilon_0 + p_0)(1 + \alpha^2)}{e^2 \omega^2} \sim \frac{\omega^2_g}{\omega^2}. \]

where \( \omega \) is the characteristic GW frequency, \[ \omega^2_g = \frac{8\pi G\varepsilon_0}{e^2}. \]

The approximation (21) is equivalent to the condition \[ \chi^2 \ll 1. \] (54)

Under the condition (54) the GW velocity tends to that of light, thus providing the required solution stationarity even in inhomogeneous plasma [24]. Let \( \beta_0 = \text{Const} > 0 \) be a maximum value of the PGW vacuum magnitude, \( \beta_* \). Let us introduce one more dimensionless parameter, the first GMSW parameter \( \xi^2 \): \[ \xi^2 = \frac{\chi^2}{\beta_0^2} \sim E/E_{GW} \] (55)

where \[ E_{GW} = \frac{\beta_0^2 \omega^2 c^2}{(4\pi G)}. \]

Thus, the parameter \( \xi^2 \) is of the order of the ratio of the total magnetoactive plasma energy to the vacuum GW energy.

Making use of the solution of the MHD equations in the case of strict transversal PGW propagation \( (\Omega = \pi i/2) \) as well as the expression of the total plasma EMT (44) and that for the energy flow of a weak PGW (3), we reduce Eq. (52) to the form \[ \dot{\beta}^2 + \nabla(\beta) = \beta_*^2, \] (56)

where \[ \nabla(\beta) = \chi^2[\Delta^{-n}(\beta) - 1] \quad (\nu = 1) \] (57)

is a function of \( \beta \); \( \dot{\beta} \) is now a derivative in the dimensionless “time” \( s = \sqrt{\omega u}/c. \)

Introduce the relative PGW magnitude: \[ q = \beta/\beta_0; \quad q_* = \beta_*/\beta_0. \] (58)

Then Eq. (56) may be rewritten in the form \[ \dot{q}^2 + V(q) = q_*^2, \] (59)

where \[ V(q) = \xi^2[(1 - \Upsilon q)^{-n} - 1] \] (60)

and a new dimensionless parameter has been introduced: \[ \Upsilon = 2\alpha^2 \beta_0^2 \] (61)
(the second GMSW parameter). The total energy conservation law leads roughly to the same result. Eq. (59) may be treated as an equation with respect to the variable \( q \). On the other hand, (59) completely coincides in its form with the energy conservation law of a 1-dimensional mechanical system described by the canonical variables \( \{ q(s), \dot{q}(s) \} \) [23], where \( V(q) \) is the potential, \( \dot{q}^2 \) is its kinetic energy and \( \dot{q}_{e}^2 = E_0 \) is its total energy. Fig. 4. shows the qualitative form of the potential \( V(q) \).

![Figure: 4. Potential \( V(q) \) of Eq.(59)](image)

Two points on the potential curve (A and B) correspond to any positive value of \( E_0 \). These points are the system trajectory turning points. No real system states exist under the potential curve \( V(q) \). At the point

\[
q = q_c = Y^{-1}; \quad (\beta = 1/2\alpha^2)\]

\[
V(q_c) \to \infty.
\]

To analyze the system behaviour, let us suppose that the moment \( s = 0 \) corresponds to the front edge of the GW, while

\[
\beta_* \approx \beta_0 \sin s \Rightarrow q_* \approx \sin(s).
\]

Thus, provided the initial conditions (24) are satisfied, the system always starts from the point \( S_0 \) along the line (AB) towards A (for \( \beta > 0 \)). Since this is a turning point, the maximum accessible value of the variable \( q \) in the system is \( q(A) \). This is the smallest root \( q_\rightarrow = q(\chi, Y, E_0) \) of the algebraic equation

\[
V(q) - E_0 = 0.
\]

The maximum attainable PGW magnitude in the system, \( \beta_{\text{max}} \), is

\[
\beta_{\text{max}} = q_* - \beta_0.
\]

Thus \( q_\rightarrow \) coincides in its sense with the “PGW magnitude damping factor” \( \gamma \) introduced Ref. [1]. Solving Eq. (65), we obtain the required root \( q_\rightarrow \):

\[
q_\rightarrow = \frac{1}{Y} \left[ 1 - \left( 1 + \frac{q_e^2}{\xi^2} \right)^{-1/n} \right].
\]
From (65) it follows that always
\[ q_- \leq \Upsilon^{-1}, \]
and also, as \( E_0 \to 0 \)
\[ q_- \approx \frac{q_-^2}{4\Upsilon^2} \to 0. \]

With increasing \( E_0 \) this magnitude grows and for \( E_0 \to \infty \) it reaches the value \( (q = q_c) \):
\[ \beta_{max} \to \beta_0 \Upsilon. \]

After the turning point the GW magnitude diminishes, reaching negative values. For \( s \to +\infty \)
\[ s \to +\infty \quad \beta' \to \beta'_\infty = \text{Const} < 0; \quad \beta \sim \beta'_\infty u \to -\infty; \]
the metric (22) degenerates \((g_{22} \to 0, g_{33} \to -\infty)\); the only nonzero components of the curvature tensor take the following form due to the Einstein equations (see (23)):
\[ R_{u2u2} = (L^2/\beta'_\infty \exp(2\beta'_\infty u) \to -0; \]
\[ R_{u3u3} = (L^2/\beta'_\infty \exp(2\beta'_\infty u) \to +\infty. \]

Thus, as \( s \to +\infty \), a true singularity is formed in the system. It is easily verified that in this case \( H^2 \to 0, \varepsilon \to 0, V^1 \to -c \). The plasma in the final state moves to meet the original GW direction. This reverse of the plasma needs a more detailed self-consistent analysis.

4.3 Numerical analysis of GMSW

Let us pass to a more detailed study on the selfconsistent motion of the system. From Eq. (59) we obtain the differential equation
\[ \frac{dq}{ds} = \pm \sqrt{q_2^2 - V(q)} \]
where the plus sign is chosen before and the minus sign after the turning point \( q_- \). It is helpful to solve and analyze Eq. (71) using the new dimensionless variables: \( \Delta(\beta) \) and
\[ S = \Upsilon s \equiv \sqrt{2} \Upsilon \omega u \]
Substituting into (71), for example, \( q_s(s) = \sin s \), we reduce it to the form
\[ d\Delta/dS = \mp \sqrt{\cos^2 S - V(\Delta)}, \]
moreover, the initial condition is to be fulfilled:
\[ \Delta(0) = 1. \]

Figs. 5–11 show some results of numerical integration of Eq. (73) with the initial condition (74). An analysis of the formulae describing GMSW and numerical calculations make it possible to discover a number of general laws of the GMSW excitation process in homogeneous and isotropic plasma under the condition that the PGW propagation is strictly transversal:
1. A GMSW is completely described by three nonnegative dimensionless parameters: the parameter \( k \) in the plasma equation of state, the first (\( \xi^2 \)) and the second (\( \Upsilon \)) GMSW parameters.

2. Necessary conditions for GMSW excitation are (31) and (32):
\[
\Upsilon \geq 1. \tag{75}
\]

3. The only criterion of strong GW absorption is, according to (69), a large value of the second GMSW parameter (\( \Upsilon \)):
\[
\Upsilon \gg 1. \tag{76}
\]

4. Under these conditions a maximum response of the plasma to GW is achieved when the values of the first GMSW parameter are small:
\[
\xi^2 \ll 1. \tag{77}
\]

5. The plasma response to GW is a single pulse, and the shock wave stage is always replaced by a reverse stage, when the plasma turns back. Simultaneously its density, pressure and magnetic field strength fall off.

6. The ultrarelativistic (\( k = 1/3 \)) plasma response is much greater (by approximately 2 orders) than that of a plasma with the nonrelativistic equation of state (\( k = 0 \)), and in ultrarelativistic plasma the pulse duration is also slightly greater.

7. The profiles of the plasma response at sufficiently large values of the second GMSW parameter (\( \Upsilon \geq 5 \)) actually coincide on the \( S \) scale. This means that in the conventional time scale \( t \) the pulse duration is inversly proportional to the second parameter, or more precisely
\[
\Delta \tau \approx \frac{\pi}{2\omega \Upsilon} = \frac{T}{4\pi \Upsilon}, \quad (\Upsilon \geq 5), \tag{78}
\]
where \( T \) is the GW period.

8. With \( \Upsilon < 5 \) the response magnitude rapidly decreases and at \( \Upsilon \sim 1 \) becomes smaller by an order of magnitude. In this case a maximum pulse duration is achieved:
\[
\tau \leq T/4 = \pi/(2\omega). \tag{79}
\]

9. A decrease in the first GMSW parameter causes a rapid increases in the response (roughly proportional to \( 1/\xi^2 \)); simultaneously increases the pulse duration approximately by a factor of 2.

10. A maximum response is achieved at the instant \( S \approx 1 \).

---

4Here and further on, speaking of a maximum response of the plasma, we mean its energy characteristics: the plasma energy flow density and the magnetic field energy density.
11. Under the optimal GMSW conditions (76) and (77), the total surface density of the magnetic field energy, transported in the pulse, $E_\Sigma$, is for ultrarelativistic plasma of the order of

$$E_\Sigma \sim cH_0^2 / (\omega \xi^2 \Upsilon).$$

(80)

12. The shock wave energy is taken from the GW energy, so under the conditions (76) and (77) the GMSW is an effective mechanism of gravitational waves energy transformation into other forms of energy.

Figure: 5. Relative GW magnitude in ultrarelativistic plasma, $q(S)$; everywhere $\Upsilon = 10$. 1 — $\xi^2 = 1$; 2 — $\xi^2 = 0,1$; 3 — $\xi^2 = 0,01$.

Figure: 6. Physical velocity of plasma in GW field, $v^1(S)/c$: 1 — nonrelativistic, 2 — ultrarelativistic equation of state; $\Upsilon = 10$; $\xi^2 = 0,01$. 
Figure: 7. Physical velocity of plasma in GW field, $v^1(S)/c$: 1 - $\Upsilon = 10$; 2 - $\Upsilon = 100$. Everywhere $\xi^2 = 0, 1$. The lines practically coincide.

Figure: 8. Dimensionless density of ultrarelativistic plasma energy flow, $T^{14}(S) = 4\pi GT^{14}/\rho^2_0 \omega^2 c^2$: 1 - $\Upsilon = 10$; 2 - $\Upsilon = 100$. Everywhere $\xi^2 = 0, 1$. The lines practically coincide.
Figure: 9. Magnetic field strength $\log H^2(S)/H_0^2$ in GW field, 1 — $k = 0$, 2 — $k = 1/3$; $\Upsilon = 10$; $\xi^2 = 0, 01$.

Figure: 10. Magnetic field strength in GW field for ultrarelativistic plasma, $H^2(S)/H_0^2$: 1 — $\Upsilon = 10$; 2 — $\Upsilon = 100$. Everywhere $\xi^2 = 0, 1$. The lines practically coincide.
5 GMSW in neutron star magnetospheres

5.1 GSMW parameters in neutron star magnetospheres

In [1] it was shown that in the magnetospheres of neutron stars performing quadrupole oscillations, large values of the second GMSW parameter are realized. That is, the necessary condition for GMSW excitation (76) is fulfilled. Let us study this problem in more detail. The electron number density in a pulsar magnetosphere, \( n_e(r) \), which is necessary for calculating the parameter \( \Upsilon \), may be obtained by dimensional estimation from the Maxwell equations [12]:

\[
    n_e(r) \sim H(r)/(4\pi er).
\]  

(81)
Further, as is known from (see [15]), the pulsar period slowing-down rate $t_0$ is connected with the pulsar parameters as follows:

$$t_0 \approx \frac{3c^2M P^2}{8\pi^2 H^2 R^4},$$

where $R$ is the neutron star radius, $M$ is its mass, $H$ is the magnetic field strength at the stellar surface, $P$ is the rotation period. This formula gives for the pulsar NP 0532 $H \approx 5 \cdot 10^{12}$ G (see Table 3) according to the known slowing-down rate of this pulsar. Actually, as pointed out in Ref. [11], the magnetic field strength at the surface of NP 0532 is somewhat smaller than the value obtained on the basis of (82), and is of the order of $10^{12}$ G. Further on we use this value. In the figures shown below (unless specially indicated) the following values of the parameters are adopted: $R = 1.2 \cdot 10^6$ cm, $\beta_0(R) = 10^{-8}$ and the magnetic field in the magnetosphere is assumed to be dipole: $H(r) \sim (R/r)^3$.

### Table 3. GMSW in a neutron star magnetosphere

| $r_g/R$ | $\delta M$ | $T_n$ | $\tau_n$ | $E_m/\Delta R$ | $L_g/\Delta R$ | $R$ | $H(R)$ | $\sqrt{\Delta R}$ | $\alpha^2$ | $\Upsilon$ | $E_m$  |
|---------|-----------|------|---------|----------------|----------------|----|--------|----------------|---------|---------|-------|
| 0.057   | 0.405     | 1.197| 13.0    | 7.8(50)        | 1.2(50)        | 21 | 7.7(11)| 1.0(-4)         | 5.2(11) | 1.1(5)  | 8.4(42)|
| 0.159   | 0.677     | 0.699| 1.7     | 5.7(52)        | 7.0(52)        | 13 | 2.7(12)| 4.2(-6)         | 4.7(11) | 2.4(4)  | 1.1(42)|
| 0.240   | 0.682     | 0.311| 0.2     | 2.8(52)        | 2.9(53)        | 13 | 6.1(12)| 3.3(-6)         | 7.5(11) | 4.4(4)  | 3.0(41)|
| 0.580   | 1.954     | 0.378| 0.2     | 1.7(54)        | 1.6(55)        | 10 | 7.4(12)| 4.2(-7)         | 8.0(11) | 4.2(4)  | 3.1(41)|
| 0.434   | 1.670     | 0.349| 0.2     | 5.0(53)        | 5.0(54)        | 12 | 5.2(12)| 9.5(-7)         | 6.3(11) | 4.1(4)  | 4.7(41)|

* Comments to Table 3. The data placed in columns 1 - 7, 9 and 12 are taken from the book [9]. $\delta M = M/M_\odot$ is the neutron star mass related to the Solar mass; $R$ is the star radius in km; $T_n$ is the neutron star eigen-oscillation period in the basic quadrupole mode (in milliseconds); $\tau_n$ is the oscillation damping time (in seconds); $\Delta R = \langle (\delta R/R)^2 \rangle$ is the root-mean-square relative magnitude of the neutron star oscillations; $E_m$ is the oscillations kinetic energy in erg; $L_g$ is the the star’s gravitational luminosity in erg/sec. The GW magnitude value at the neutron star surface, $\beta_0(R)$, is assumed to be equal to $10^{-9}$. $H(R)$ is the magnetic field strength in gauss (G). The data placed in columns 8÷11 are calculated using Eqs. (3), (82) and (81) for the observed Crab pulsar (NP 0532) parameters; $P = 0.033$ s, $t_0 = 2500$ years. The data placed in the last line of the table (columns 2÷7 and 9) are obtained by extrapolation of the values from the book [9]. The values of the parameters $\alpha^2$ and $\Upsilon$ are given for the magnetosphere near the stellar surface.

If the magnetic field of a neutron star is described as that of a dipole, then the geographic angle $\Theta$ (counted from the magnetic equator) will be connected with the above angle $\Omega$ by the relation $\Omega = \pi/2 - \Theta$. Therefore the GMSW excitation condition depends on the angle $\Theta$:

$$\sin^2 \Theta < 1 - \frac{1}{2\alpha_0^2/|\beta|} \sim 1 - \Upsilon^{-1}.$$

Thus, in the magnetosphere of a neutron star (or a Supernova) a GMSW can be excited in the vicinity of the magnetic equator, similarly to pulsars, with a knife radiation pattern. In this region, as was demonstrated by the above examples, the gravitational radiation can be absorbed almost completely by shock wave excitation. Fig. 13 shows the radial dependence of the GMSW parameters.
magnetic equator plane ($\Theta = 0$) of a neutron star magnetosphere with the above parameters $R$, $H(R)$ and $\beta_0(R)$. According to Table 3, in the case of NP 0532 such values of the parameters $\beta_0(R)$ and $R$ correspond to the gravitational radiation power $L_g \approx 4.5 \cdot 10^{42}$ erg/sec.

As is seen from Fig. 13, the region favourable for the GMSW formation lies in the range $6R \div 16R$, i.e., where the local magnetic field strength is $3\Delta 10^8 \div 4\Delta 10^9$ G. When the GMSW pulse passes, these local values increase by a factor of 10 to 30. Thus a neutron star in whose magnetosphere a GMSW zone is formed, is able to radiate GW only from its magnetic poles, like pulsars with a pencil radiation pattern. In this case the probability of direct GW detection from such sources is drastically decreased. However, GMSW open another way for GW observation. A formed GMSW carries above all strong magnetic fields. They move from the neutron star in the magnetic equator plane, and therefore should lead to an increased pulsar magnetic bremsstrahlung intensity at the moment when the GMSW front passes. Thus anomalous electromagnetic radiation flashes in the pulsar radiation should be observed at moments when quadrupole oscillations are excited.

The total magnetic bremsstrahlung intensity of a relativistic electron is proportional to squared magnetic field strength [10]:

$$I = 2e^4 H^2 \vec{p}^2 / (3m^4 c^5),$$

where

$$\vec{p} = m\vec{v} \sqrt{1 - v^2 / c^2}$$

is the electron momentum.

Therefore the curves $H^2(S)$ shown in Figs. 9–12, actually describe the time dependence of the magnetosphere magnetic bremsstrahlung intensity, i.e. the local electromagnetic response to the gravitational radiation of the neutron star. Such a response might be detected by an observer at rest placed in the magnetosphere and screened from the electromagnetic radiation coming from other regions. The situation is more difficult with a total response of the magnetosphere to the GW, detected by a distant observer. We will later return to this problem.

\footnote{In all further figures the magnetosphere is considered in the magnetic equator plane.}
As was mentioned above, the response of a homogeneous magnetoactive plasma even to strictly periodic gravitational radiation has the form of a single pulse. But even if it were not the case, the response of a neutron star magnetosphere to a GW would still have the same form. Indeed, a shock wave (GMSW), emerging after the excitation of quadrupole oscillations of a neutron star, should throw the equatorial sector of the magnetosphere away into the interstellar space. For the next pulse to be formed, the magnetosphere should restore. The necessary time for its restoration is of the order of $\Delta t \sim l/v_s$ where $l$ is the characteristic size of the magnetosphere and $v_s$ is the velocity of sound. For a typical neutron star magnetosphere $\Delta t \sim 1$ sec. And typical quadrupole oscillation damping times comprise tenths of a second, according to Table 3.

### 5.2 Effect of magnetosphere inhomogeneity upon GMSW

According to Table 3, neutron star eigen-oscillation periods vary depending on the stellar mass in the range of 0.3 to 1.2 ms. Therefore the local duration of GMSW pulses should, by (79), satisfy the condition

$$\Delta \tau < 7 \cdot 10^{-5} \div 3 \cdot 10^{-4} s$$

i.e., be shorter than 70 to 300 microseconds. Fig. 14 shows the dependence of the GMSW pulse local duration on the radial coordinate $r$, $\Delta \tau(r)$, calculated according to Eq. (78).

![Figure: 14. The dependence of local pulse duration $\Delta \tau$ (in microseconds) on the distance form the star centre $r/c$ (in microseconds). The duration was calculated by Eq. (78). The black circles mark the boundaries of a region where the GMSW effect is sufficiently well-developed. Outside this region the result is of a formal nature. On the left boundary of the region the pulse duration should quickly grow up to the values (84).](image)

As is seen from Fig. 14, the actual local pulse duration in the region where the GMSW mechanism is fairly effective, ranges from 1 to 10 $\mu$s. Thus in this region $\Delta \tau \sim 10^{-2} r$ the following condition is fulfilled with a large spare:

$$\Delta \tau c \ll r,$$

(85)
justifying the use of the GMSW formulae for describing an inhomogeneous magnetosphere.

An observer out of the neutron star magnetosphere would detect the magnetic bremsstrahlung from the magnetospheric electrons during the whole time while the local pulse passes through the magnetospheric region \( r_- < r < r_+ \) where favourable conditions for GMSW development are realized, namely, (76), (77). With a certain caution these conditions may be specified: \( \xi^2 < 0.5 \) (the lower bound of the GMSW range, \( r_- \)) and \( \Upsilon > 5 \) (its upper bound, \( r_+ \)). So the size of the GMSW zone is

\[
\Delta r = r_+ - r_-
\]

If \( \Delta r < 0 \), a GMSW zone does not appear in the neutron star magnetosphere at all. Since, as we have seen, the GMSW pulse spreading velocity is very close to \( c \), the whole magnetic bremsstrahlung detected by remote observer will be concentrated in the time “window” of duration \( \Delta T \)

\[
\Delta T = \Delta r / c = t_+ - t_-
\]

(86)

where \( t_\pm = r_\pm / c \) are the instants when the GW leading front reaches the upper and lower boundaries of the GMSW zone. Near its boundaries \( r_- \) and \( r_+ \) the GMSW is poorly developed (in the first case the first GMSW parameter is too large, in the second case the second parameter is too small). Therefore the intensity of the electromagnetic signal is small near the boundaries of the window, while in its medium domain a radiation maximum (large \( \Upsilon \) and small \( \xi^2 \)) is achieved. The form of the signal itself is yet to be calculated. Fig. 15 shows the dependence of the window width on the magnetic field strength and the GW magnitude.

![Figure 15. Dependence of the GMSW existence range \( \Delta T \) (in \( \mu s \)) in the magnetosphere of a neutron star of radius \( R = 1, 2 \cdot 10^6 \) cm on the magnetic field strength \( H(R) \), (related to \( 10^{12} \) G) and the GW magnitude \( \beta_0(R) \). The thin line corresponds to \( \beta_0(R) = 5 \cdot 10^{-9} \), the points to \( \beta_0(R) = 7, 5 \cdot 10^{-9} \), the thick line to \( \beta_0(R) = 10^{-8} \).](image)

5.3 Magnetic bremsstrahlung intensity

Note that the local density of the bremsstrahlung intensity \( W(t, r) \) is determined by the local values of the squared magnetic field strength, \( H^2(t, r) \), and the local electron number density in the
magnetosphere, \( n_e(t, r) \). For ultrarelativistic electrons by [10] it is

\[
W = \frac{2e^4H^2}{3m^2c^3} \left( \frac{E}{mc^2} \right)^2 n_e,
\]

where \( E \) is the electron energy. Further we will assume that the size of a local pulse is much smaller than both the characteristic scale of the magnetospheric inhomogeneity \( r \) and the window width \( \Delta r \). Thus the retarded time \( u \) is a quick variable and the radial coordinate \( r \) is a slow variable.

Then a GMSW may be described by the formulae for homogeneous plasma, where it is necessary to use the local values of the GMSW parameters, \( \xi^2(r) \) and \( \Upsilon(r) \). Meanwhile in the exact stationary solutions there arises a weak dependence on the radial coordinate \( r \), i.e. the solution will be weakly nonstationary and the nonstationarity will show itself in the form of a functional dependence of the GMSW solutions on the local values of the parameters, e.g.,

\[
\Delta(u; r) = 1 - \Upsilon(r)q(u; r),
\]

etc. Thus, from the particle number conservation law and the solution stationarity it follows:

\[
L^2 n_e(r, t) v_v(u) = \text{Const} \approx \frac{1}{\sqrt{2}} n_e^0(r),
\]  

(87)

where \( n_e^0(r) \) is the unperturbed electron number density in the magnetosphere. Taking into account that \( E n_e = \varepsilon/2 \) (half energy of a relativistic magnetosphere belongs to the electrons) and using the solutions (34), (35) and (45) for a weak GW, we obtain:

\[
n_e(r, t) = n_e^0(r) \Delta^{-3/2}(u);
\]  

(88)

\[
E(r, t) = E_0(r) \Delta^{-1/2}(u);
\]  

(89)

where \( E_0(r) \) is the unperturbed energy of magnetospheric electrons and

\[
H^2(u) = H^2_0(r) \Delta^{-3}(u).
\]  

(90)

Thus we get the following relation for the magnetic bremsstrahlung of the ultrarelativistic magnetosphere:

\[
W(r, t) = W_0(r) \Delta^{-11/2}(u),
\]  

(91)

where \( W_0(r) \) is the magnetic bremsstrahlung intensity density for a nonperturbed magnetosphere. Eq. (91) needs a relativistic correction taking into account the plasma motion: the radiation density should be multiplied by the relativistic factor \( (1 - v^2/c^2)^{-1/2} \). The net result is

\[
W(r, t) = W_0(r) \Delta^{-11/2} \frac{1}{2} (\Delta^{3/2} + \Delta^{-3/2}).
\]  

(92)

Thus \( W(r, t) \sim W_0(r) \Delta^{-7} \). Note a large value of the exponent of the governing function \( \Delta(u) \), which leads to a large steepness of the local magnetic bremsstrahlung pulse. Integrating Eq. (92) over the GMSW zone near the magnetic equator, we obtain a formula for the variation of the complete magnetospheric magnetic bremsstrahlung resulting from a GW pass:

\[
\Delta J(t) = 2\pi \Theta_0 \int_{r_-}^{r_+} \Phi(\Delta) W_0(r) r^2 dr,
\]  

(93)
where
\[ \Phi(\Delta) = \frac{1}{2}\Delta^{-11/2}(\Delta^{3/2} + \Delta^{-3/2}) - 1; \] (94)

\( \Theta_0 \) is the angle of the knife radiation pattern. This formula completely describes the shape of the signal to be detected by a remote observer. The expression in the square brackets in (93) is notably nonzero only in the domain of the local GMSW pulse, i.e. in the domain
\[ 0 < S \leq \pi \iff ct - \pi/(\omega \Upsilon(r)) \leq r < ct. \] (95)

Therefore the integral (93) tends to zero for \( t < r_-/c \) and \( t > r_+/c + \pi/(\omega \Upsilon(r)) \). The observed pulse duration is formally determined by these limits. However, since, as noted above, near the zone boundaries the GMSW is poorly developed, the actually observed pulse duration (more precisely, its half-width) can turn out to be smaller than this value. Without solving the problem of the observed pulse shape, let us estimate its magnitude in its medium domain
\[ r_-/c < t < r_+/c, \] (96)

when the pulse local duration is much smaller than the window width:
\[ \Delta \tau \ll \Delta T. \] (97)

Under these conditions the integrand in Eq. (93) is \( \delta \)-like, therefore with a good precision the following estimate is valid:
\[ \Delta J(t) \approx \frac{2\pi}{\omega \Upsilon} \left< W_0(r) r^2 > \right> c \int_{S_-}^{S_+} \Phi(\Delta(S))dS, \] (98)

where \( S_\pm = \omega \Upsilon(t - r_+/c), < W_0(r) r^2 > \) and the value of \( r_* \) is determined from the equation
\[ r_* = ct - 1/((\omega \Upsilon(r_*)). \] (99)

Thus
\[ \Delta J(t) \approx \frac{2\pi \Theta_0}{\omega \Upsilon(r_*)} W_0(r_*) r_*^2 c \Delta T \langle \Delta^{-7} \rangle, \] (100)

where
\[ \langle \Delta^{-7} \rangle = \frac{1}{\Delta S} \int_{S_-}^{S_+} \Delta^{-7}(S) dS; \] (101)

\( \Delta S = \Delta T \omega \Upsilon. \) Thus, by order of magnitude, the intensity change of the magnetic bremsstrahlung (in its maximum) as a result of the GMSW excitation is equal to a product of the unperturbed magnetospheric radiation intensity in the GMSW zone by the dimensionless factor \( \langle \Delta^{-7} \rangle \) (which can reach \( 10^{-7} \)). We will return to a calculation of the observed pulse shape in our next paper. Here we restrict ourselves to estimates of the complete energy of a GMSW pulse (Fig. 16).
Figure: 16. The radial coordinate \(r\) dependence of the pulse energy \(E_{\text{pulse}} = 4\pi r^2 \mathcal{E}_\Sigma\) (in erg) by Eq. (80). The GMSW effect is sufficiently well-developed only in the range \(0.8 < \log(r/R) < 1.2\) (\(7.6 \cdot 10^6\text{ cm} < r < 1.9 \cdot 10^7\text{ cm}\)). Outside this region the results are of clearly formal nature. The pulse energy should rapidly fall near the boundaries of this range.

5.4 Source of quadrupole oscillations

There naturally arises the question of a source of pulsars' quadrupole oscillations. A possible energy source for such oscillations might be represented by explosive nuclear reactions with heavy hyperons like \(n + n \leftrightarrow p + \Sigma^+\) taking place in neutron stars cores at densities over \(10^{15}\text{ g/cm}^3\) [13]. The presence of strong magnetic fields should lead to an asymmetry of the explosions, i.e. to the quadrupole moment excitation. For such processes to occur in a neutron star, it must be sufficiently young. Numerical simulations of the process of a neutron star cooling shows [14] that after a Supernova explosion the neutron star temperature falls approximately by an order of magnitude in \(10^4\) years. Consequently, GMSW should be sought in radiation from sufficiently young pulsars formed no earlier than \(10^4\) years ago.

6 The Crab pulsar NP 0532 emits gravitational waves

A pulsar with the required parameters does exist: it is the famous pulsar in the Crab nebula, NP 0532, life-time is less than 1000 years (the 1054 Supernova). This pulsar is the youngest of all known ones (and consequently the hottest), it has the shortest period (at least among the closest pulsars, enumerated in Table 2): \(T = 0.033\) sec. What is suprising is that the radio emission of this pulsar contains anomalies which can be with a large degree of confidence identified with the GMSW. Namely: there are single irregular, the so-called giant pulses (on the average a pulse in every 5 to 10 minutes) [11]. The radiation intensity in the giant pulses is a few tens of times higher (by roughly a factor of 60) than in common pulses. But the most interesting is that the duration of the giant pulses is no more than \(9 \cdot 10^{-5}\) s, i.e., almost by 2(!) orders shorter than that of the common pulses from NP 0532 (\(\tau \sim 6 \cdot 10^{-3}\) s). The common pulse duration, as is easily seen, is about \(1/7\) of the NP 0532 rotation period, so that the common pulses are clearly explained geometrically by pulsar rotation.
The giant pulse duration is 300 times shorter than the pulsar rotation period, and consequently the existence of the giant pulses has not yet found any satisfactory theoretical model.

However, the giant pulses are easily explained by the GMSW, and their duration is not related to the pulsar rotation period, or an angle of the knife radiation pattern, but to its eigen-oscillation period, $T_0$. A comparison of the NP 0532 giant pulse duration with that of a GMSW pulse (84) shows a striking coincidence. Indeed, for the NP 0532 pulsar, as known from the annihilation line shift in the $\gamma$ radiation spectrum (400 keV instead of 511 keV), the gravitational redshift is known [21, 22]:

$$\Delta E/E = MG/Rc^2 = 0.217.$$ 

Then from Table 3 we find the pulsar mass: $M = 1.67 M_\odot$ and the corresponding neutron star radius: $R = 12$ km. According to Eq. (79) and Table 3, the GMSW pulse duration for the NP 0532 pulsar should be about 87 microseconds, while the observed NP 0532 giant pulse duration is approximately 90 microseconds (1).

For the Crab pulsar $t_0 = 2.5 \cdot 10^3$ years (2); then, setting $\delta M = 1.67$, $R = 12$ km, we find from (82): $H \sim 5.2 \cdot 10^{12}$ G. The angle $\Theta_0$ of the knife radiation pattern is connected with the observed pulse duration $\tau$ by the relation $\Theta_0 = 2\pi\tau/P$. For the pulsar NP 0532 this angle is $0.48 \approx 28^\circ$.

Assuming the complete pulsar luminosity in the continuous spectrum to be about $5 \cdot 10^{36}$ erg/s, we find the giant pulse intensity recalculated for the whole neutron star surface:

$$L_{giant} \approx 4 \cdot 10^{39} \text{erg/s}.$$ 

As has been pointed out above, the real magnetic field strength on the pulsar NP 0532 surface is $10^{12}$ G [11]. To explain the observed giant pulse emission power, one needs GW magnitude values on the stellar surface of the order of $10^{-8}$. Note that, according to Fig. 15, the window width $\Delta T \approx 180 \mu s$ corresponds observed GMSW pulse should be about 90 $\mu$s, which again precisely coincides with the observed giant pulse duration!

The indicated magnitude $\beta_0(R)$ corresponds to the gravitational radiation power of the order of $4 \cdot 10^{42}$ erg/s and the neutron star oscillations energy about $E_m \approx 4 \cdot 10^{41}$ erg. In this case the neutron star surface oscillation magnitude is about 1 cm. Taking into account that in the whole lifetime of NP 0532 (1000 years) approximately $7 \cdot 10^7$ giant pulses have been emitted, we get an estimate of the energy carried away from the neutron star by GW for the whole time of its existence: $E = 2.8 \cdot 10^{49}$ erg. It is $10^{-5}$ of the rest energy of this neutron star, which completely agrees with the assumption of a permanent rebuilding of its core. Thus, to a high degree of confidence we can state that the giant pulses observed in the pulsar NP 0532 radiiation are optical manifestations of gravimagnetic shock waves (GMSW) excited by the gravitational radiation of the neutron star corresponding to the pulsar NP 0532 [20].

7 Conclusion

Besides NP 0532, among all known pulsars only PSR 0833 seems to be able to emit (but more seldom) giant pulses. Other pulsars are too old for it. Therefore it is necessary to concentrate the main effort on observations of these two pulsars. It should be stressed that there is no other mechanism able to accelerate a shock wave to subluminal velocities. Therefore an investigation of the giant pulse spectrum in the X-ray range, aimed at discovering a violet shift in the radiation spectrum, is of utmost importance. A comprehensive study of the giant pulses (their shapes and instantaneous
spectrum) will allow one not only to verify the existence of gravitational radiation, but also to get additional information on the neutron stars structure and the processes in their interior. In turn it is necessary to study the GMSW pulse formation in detail theoretically.

Acknowledgement
The author is thankful to S.V. Sushkov for help in carrying out the calculations and to N.A. Zvereva for translating the article into English.

References
[1] Yu.G. Ignat’ev, Gravitation & Cosmology 1, 287 (1995).
[2] Yu.G. Ignat’ev, ZhETF 81, 3 (1981).
[3] Yu.G. Ignat’ev and A.B. Balakin, Izvestia VUZov, Fizika 24, 7, 20(1981) (in Russian).
[4] Yu.G. Ignat’ev, Izvestia VUZov, Fizika 27, 12, 70 (1984) (in Russian).
[5] Yu.G. Ignat’ev, Izvestia VUZov, Fizika 28, 1, 74 (1985) (in Russian).
[6] A.B. Balakin and Yu.G. Ignat’ev, Phys. Lett. 96A, 10 (1983).
[7] A.B. Balakin and Yu.G. Ignat’ev, in: “Problems of Gravitation Theory and Particle Theory”, 14th issue, Energoatomizdat, Moscow, 1984.
[8] Yu.G. Ignat’ev, Ukr. Fiz. Zh. 29, 1025 (1984) (in Russian).
[9] C.W. Misner, K.S. Torn and J.A. Wheeler, “Gravitation”, W.H. Freeman and Company, San Francisco, 1973
[10] L.D. Landau and E.M. Lifshitz, “Field Theory”, Nauka, Moscow, 1975 (in Russian).
[11] F.G. Smith, “Pulsars”, Cambridge University Press, Cambridge, 1977.
[12] F. Pachini, Nature 219, 145 (1968).
[13] W.D. Langer and A.G.W. Cameron, Ap. & Space Sci. 5, 213 (1969).
[14] S. Tsuruta and A.G.W. Cameron, Can. J. Phys. 43, 2056 (1965).
[15] K.R. Lang, “Astrophysical Formulae”, Springer - Verlag, Berlin - Heidelberg - New-York, 1974.
[16] Ya.B. Zeldovich and I.D. Novikov, “Theory of Gravity and Stellar Evolution”, Nauka, Moscow, 1971 (in Russian).
[17] T.A. Agekyan, “Stars, Galaxy, Metagalaxy”, Nauka, Moscow, 1966 (in Russian).
[18] U.H. Kopvillem and V.N. Nagibarov, Pisma v ZhETF 2, 529 (1965) (in Russian).
[19] T.A. Lozinskaya, Itogi nauki i tehniki., Astronomia 22, 33 (1988) (in Russian).
[20] Yu.G. Ignat’ev, to appear in Phys. Lett. A.
[21] C.M. Varma, Nature 267, 686 (1977).
[22] Yu.G. Ignat’ev, Ukr. Fiz. Zh., 24, 742 (1979).
[23] L.D. Landau and E.M. Lifshitz, “Mechanics”, Nauka, Moscow, 1975 (in Russian).
[24] Yu.G. Ignat’ev, Izvestia VUZov, Fizika 17, 12, 136 (1974)