Multi-pion production in the $dd \rightarrow \alpha X$ reaction

Anders Gårdestig

Nuclear Physics Division, Uppsala University, Box 535, S-751 21 Uppsala, Sweden

Abstract: A simple model, based on two parallel and independent $NN \rightarrow d\pi$ processes, has recently been proposed for two-pion production in the $dd \rightarrow \alpha X$ reaction. It reproduces all observed features, including the sharp peak structure in momentum distributions (the ABC effect) and the strong oscillations in the deuteron vector and tensor analyzing powers. This model is now extended to describe also four-pion production with the same basic mechanism, but with two $np \rightarrow d\pi\pi$ processes as input. The calculations of the high missing mass spectra are within about 30% of the experimental data for beam energies in the range $1.9 < T_d < 2.4$ GeV.

1Electronic address: grdstg@tsl.uu.se
It has long been known that the momentum distributions of the \( np \rightarrow dX \) [1, 2, 3], \( pd \rightarrow {^3}HeX \) [4, 5], and \( dd \rightarrow \alpha X \) [6, 7] reactions at intermediate energies share some common and spectacular features. Characteristic are the sharp peaks, located slightly above the two-pion threshold, in association with a broader bump around the maximal missing mass. This behaviour, known as the ABC effect, is observed only in isospin \( I_X = 0 \) channels. The phenomenon is believed to originate from a largely kinematic enhancement in the production of two pions, but all previous attempts to give a quantitative description have so far had only limited success. In particular, the theoretical models for the \( np \rightarrow dX \) [8, 9, 10, 11] and \( pd \rightarrow {^3}HeX \) [12, 10] reactions have problems in reproducing the angular dependences [3, 12]. A recently published model for \( dd \rightarrow \alpha X \) [13] has, however, had remarkable success in reproducing data. It is based on two almost free \( NN \rightarrow d\pi \) processes happening in parallel, where the two final deuterons merge to form the \( \alpha \) particle. Since the deuteron is loosely bound, these processes are almost free and their c.m. systems coincide with the overall c.m. frame. Because of the strong forward-backward peaking in the two \( d\pi \) systems, the small missing mass (parallel pions) and maximal missing mass (back-to-back pions) configurations are both enhanced. In the latter case, the large relative momentum between the deuterons in the \( \alpha \) particle does cause a suppression relative to the low \( \pi\pi \) mass case. The ABC peaks and the central bump are hence simultaneously explained. The model, depicted by the Feynman diagram in Fig. 1, is able to reproduce quantitatively all the observed features of \( dd \rightarrow \alpha X \) in the energy range \( 0.8 < T_d < 1.9 \) GeV, including angular and energy distributions [13] and deuteron vector and tensor analyzing powers [7, 13]. The purpose of this work is to use the same type of mechanism to estimate four-pion production with two parallel \( np \rightarrow d\pi\pi \) processes as input.

At higher energies (\( T_d > 1.9 \) GeV), where production of heavier mesons and multiple (> 2) pions begins to be important, the central bump in \( 2\pi \)-production at large \( \pi\pi \) effective masses is strongly suppressed. The large relative momentum between the two pions requires the deuterons to be far out in the tail of the \( \alpha \)-particle wave function in momentum space. The angular dependences of the subprocesses are here of minor importance — it is the dynamics of the \( dd : \alpha \) vertex that dominates the behavior. The situation will be very similar when heavier mesons are produced by the same mechanism, the effect being more pronounced the larger the maximal missing mass is compared to the meson mass. At high enough energy the formation of the \( \alpha \) particle will force the two mesons to be close together in momentum space and suppress other configurations. For the production of pairs of mesons with definite masses we then expect sharp peaks close to the two-meson threshold and a rapid decrease towards maximal missing mass. On the other hand, when the meson has a broad width, like the controversial \( \sigma \) meson, the peaks corresponding to different masses should be added together with appropriate weights. In the present analysis we will use the \( \sigma \) meson as a convenient tool to describe and handle the \( I_X = 0 \) pion pair produced in \( np \rightarrow dX \). By considering two parallel \( \sigma \) productions, the four-pion contribution could be estimated in the same framework as the two-pion one. The missing mass distributions of \( np \rightarrow dX \) is then the weight functions needed as input to this \( dd \rightarrow \alpha\sigma\sigma \) model.

Since the mathematical details of the model have been published [13], only the main...
features are recapitulated here. The dashed lines in the Feynman diagram (Fig. 1) are now considered to be $\sigma$ mesons with the (possibly unequal) masses $m_1$ and $m_2$. Instead of the complicated and many $NN \rightarrow d\pi$ amplitudes used in our previous work, we here assume that the $np \rightarrow \sigma d$ reaction can be parametrized by one single $s$-wave amplitude

$$M_{np \rightarrow \sigma d} = A \eta_{d}^\dagger \left[ -\frac{1}{\sqrt{2}} \sigma \cdot \epsilon_d^\dagger \right] \eta_p,$$

where $\eta_x$ is a nonrelativistic two-component spinor, $\eta^c = -i\sigma_2 \eta^*$ its charge conjugate, $\epsilon_d$ the deuteron polarization vector, and $A = A(m_\sigma)$ the $\sigma$ meson production amplitude. After summing over internal spins the spin kernel $K$ \cite{13} is, with this normalization,

$$K = \frac{\epsilon_1 \cdot \epsilon_2}{2\sqrt{3}} A^2,$$

where $\epsilon_i$ are the polarization vectors of the initial deuterons. The summed square is then $\sum |K|^2 = (1/4)|A|^4$, but an extra factor of four is needed because there are two ways of distributing the four nucleons between the two $\sigma d$ vertices.

The mass dependence of the amplitude $A$ was obtained from experimental data for $np \rightarrow dX$ \cite{1,3} by transforming the laboratory momentum distribution to a missing mass distribution in the c.m. frame, where it should be symmetric. The data were taken with a spectrum of neutron momenta and their quality suggests that one should use the forward ABC peak (low $k_{lab}^2\alpha$) at the highest energy \cite{3}, while the backward ABC is the better choice at 991 MeV \cite{1}. By integrating this over a small mass range $\Delta m_x$ the differential cross section for $\sigma d$ could be estimated:

$$\frac{d\sigma}{d\Omega} (np \rightarrow \sigma d) \bigg|_{cm} \approx \frac{d^2\sigma}{d\Omega dm_x} \Delta m_x.$$

In the normalization we are using, the relation between the cross section and the amplitude, and in particular its mass dependence, is given by

$$\frac{d\sigma}{d\Omega} (np \rightarrow \sigma d) \bigg|_{cm} \approx \frac{3}{16(2\pi)^2 s_{\sigma d} p} \frac{m^2 k}{dm_x} |A|^2 \Delta m_x,$$

where $m$ is the nucleon mass and $p$ and $k$ are the nucleon and deuteron c.m. momenta. The squared amplitude $d|A|^2/dm_x$, extracted from Eqs. (3) and (4), was fitted to a third degree polynomial in $m_x$ for each beam energy. At $T_d = 1163$ MeV an angular distribution has been measured \cite{3} and the transformation was done for each of the angles. The variation of the amplitude with the $\sigma$ mass is roughly independent of the laboratory angle, which supports our $s$-wave assumption, and the polynomial was fitted to all of the 1163 MeV data simultaneously. Since none of the energies where the amplitude functions were extracted coincides with the energies required for our $dd$ estimate, a linear interpolation in $E_i = \sqrt{s_{\sigma d}}$ was applied between the fitted curves. The masses used in the fitted polynomials were estimated by letting them be in the same proportion to the mass range at the fitted
energies as the actual mass is to its allowed mass range. The amplitudes were then finally evaluated for each individual \( m_x \) by multiplying the fitted and interpolated values by the increment \( \Delta m_x \).

The matrix element \( \mathcal{M}_{\alpha\sigma\sigma} \) is written as the product \( \mathcal{M} = -i(m_\alpha/v_d)\mathcal{KW} \), where \( \mathcal{W} \) is the dimensionless form factor defined in Ref. [13]. In this factorization approximation the amplitudes are calculated without considering the Fermi momenta. Nevertheless it is possible to get a crude estimate of their influence by assuming that the distribution of energy between the two subprocesses follows a simple Gaussian, \( \exp[-(\Delta E_i^2)/(2\sigma_E^2)] \). Here \( \Delta E_i = E_i - \sqrt{s_{\alpha\sigma\sigma}}/2 \) is the deviation in energy from equal sharing and \( \sigma_E \approx 25 \text{ MeV} \) is calculated from a Gaussian deuteron wave function. The calculations are very stable against variations of this distribution. Only for small assumed widths \( (\sigma_E < 5 \text{ MeV}) \) could significant changes be observed.

The calculation of the form factor \( \mathcal{W} \) follows the lines of [13], apart from obvious mass and kinematical changes. The integration over meson angles was performed directly since there are no angular or momentum dependences in the amplitudes. For each choice of total missing mass \( M_X \), the momentum distributions were calculated for all possible combination of the two \( \sigma \) masses. The results were then accumulated to give the full four-pion spectrum. With this procedure, all the allowed four-body pion phase space was completely accounted for.

Our predictions of the four-pion missing mass distributions in the c.m. frame can be seen in Fig. 2 together with data points taken from [6]. Note that the scale factors applied to the calculations are not directly comparable to those in [13]: the latter values should be multiplied by the relativistic factor \( 1/\gamma^2 = (m_d/E_d)^2 \), which was overlooked in the programs (but not in the formulae) of that reference.

A plausible reason for the underestimate of the cross section is the neglect of contributions from \( pp \to pp\pi\pi \). The cross section for this process is at \( \approx 1.2 \text{ GeV} \) about half that for \( np \to d\pi\pi \) [14] but, lacking information about angular distributions and the \( nnpp: \alpha \) wave function, it is impossible to describe its influence. Another effect will occur at low missing masses \( (M_X \approx 4m_\pi) \), where the statistics of 4 equal bosons should increase the counting rate. In the semi-classical model we are employing — and with no knowledge of the \( np \to d\pi\pi \) amplitudes — this effect cannot be estimated. Distortion due to rescattering of the pions might decrease the cross section.

Superoposed upon all three of the \( 4\pi \) spectra there is an additional structure which, since it peaks at \( M_X \approx 780 \text{ MeV}/c^2 \) independent of beam energy, might be attributed to the \( \omega \) meson. This interpretation is however not unambiguous since the widths of the peaks are much larger than the stated experimental resolution. The experimentalists noted this difference and remarked that the resolution was calculated from known properties of the beam and spectrometer but that they were never able to test it directly because of poor statistics for two-body reactions in \( dd \to \alpha X \) [15]. The upper curves in Fig. 2 result from single Gaussians fitted to these points which, after integration give the c.m. cross sections \( d\sigma/d\Omega(\omega') = 9, 6, \) and \( 2 \text{ nb}/\text{sr} \) for 1.9, 2.2, and 2.4 GeV. Because of the threshold \( 4\pi \) probable enhancement mentioned in the previous paragraph, these values are probably overestimates.
The assumption that the four pions are produced via two $\sigma$’s has the consequence that the relative weights for charged and neutral pions should be $N(+---):N(00+-):N(0000) = 4:4:1$, which would be a test of the model, since other possible production channels have different weights. In the case of $\rho \rho$ production, where $4\pi^0$ is forbidden, $N(+---):N(00+-) = 2:1$. This contribution can be estimated from $pp \rightarrow d\pi^0\pi^+$. The cross section for this process is smaller than that for $np \rightarrow d\pi\pi$ by roughly a factor of 10\(^{14}\) and, in addition, there is only one diagram in this case, giving an extra factor $1/4$. In total this will give a contribution of less than 1\% of the $\sigma\sigma$ cross section.

Despite the crudeness and simplistic nature of the assumptions and approximations made in this model, the calculations are in pleasing agreement with data. Taken together with the results of\(^{13}\), it seems likely that most of the multi-meson production in the $dd \rightarrow \alpha X$ reaction in the $0.8 < T_d < 2.4$ GeV range could be attributed to the same general double-interaction mechanism. Further support for this suggestion could be obtained from the estimate of $\gamma\gamma$ production via two $np \rightarrow d\gamma$ processes. This is shown to be of considerable importance as background to the charge-symmetry-breaking $dd \rightarrow \alpha\pi^0$ reaction\(^{16}\).

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Figure 1: The Feynman diagram for the $dd \rightarrow \alpha\pi\pi$ and $dd \rightarrow \alpha\sigma\sigma$ reactions.

Figure 2: The missing mass distributions for $dd \rightarrow \alpha X$ at $\theta^\alpha_{cm} \approx 0^\circ$. The data [6] are compared with the $\sigma\sigma$ (lower solid line) model, the latter curves multiplied by scale factors 1.2, 1.0, and 1.3 for 1.9, 2.2, and 2.4 GeV. The upper solid line is the fit to the ‘$\omega$’ peak.