Optimization of Reinforced Concrete Cantilever Retaining Wall using Particle Swarm Optimization

Shubham Srivastava*, Saurabh Pandey¹, Rajesh Kumar²

*Research Scholar, Department of Civil Engineering, IIT BHU Varanasi, India,
shubhamsrivastava.rs.civ19@iitbhu.ac.in
¹M.Tech, Department of Civil Engineering, IIT BHU Varanasi, India,
jazzysaurabh@gmail.com
²Professor, Department of Civil Engineering, IIT BHU Varanasi, India,
rkumar.civ@iitbhu.ac.in

ABSTRACT The design of structures depends on designer’s experience and generally, the designer proceeds with trial and error until he arrives at a design which satisfies prescribed limit states. This is especially true for reinforced concrete structures in which the structural configuration is decided first and then reinforcement requirements are determined, resulting in higher cost. Retaining walls involve a large number of variables and therefore have been far from optimization. However, since retaining walls comprise of 20-30 percent of the cost of highways in hilly regions, their optimization is critical to economy of the project. Particle Swarm Optimization (PSO) does not require the objective function to be linear or differentiable and hence is ideally suited for optimization of retaining walls. This study uses PSO as a tool for the optimizing a Cantilever RC Retaining Wall. The objective function consists of two parts - cost function and penalty term. A program and GUI was developed to implement PSO. A reduction in cost was achieved from 8% - 17% depending on the height of retaining wall, while the optimization in weight varied from 9% - 34%.

Keywords: Particle Swarm Optimization, Retaining Wall, MATLAB, Cantilever, Optimization

1. INTRODUCTION

Retaining walls are structures used to support soils or other such material behind them. Earth retaining walls have several application such as in bridge abutments and for getting the desired carriageway in case of roads in hilly regions. According to an estimate the cost of retaining wall comprises of 20-30% of the cost of a highway project in a hilly region. The design of structures is largely dependent on the experience of the designer and rule of thumb. The designer proceeds with the practice of trial and error and arrives at a design which satisfies the limit states prescribed in the design codes. This is especially true for concrete structures in which the structure configuration is decided by the designer who then proceeds to find out the reinforcement requirements. This leads to a design, the cost of which is highly dependent on the experience and expertise of the design engineer.

Particle swarm optimization (PSO) is an optimization method developed by Eberhart and Kennedy [1]. They explored its efficacy on several benchmark functions and training of neural networks. They also described the relationships between particle swarm optimization and artificial life and genetic algorithms. The particle swarm optimization tested by them achieved 92% correct training of Neural Networks as compared to the conventional back-propagation method which achieved 89%. However, the origin of particle swarm optimization may be traced to the work of Reeves (1983), who introduced particle systems to simulate dynamic objects which could not easily be represented by particles like smoke, fire, clouds etc. in the field of computer graphics. Later, particle systems were used to simulate the behaviour of a flock of birds. PSO caught the attention of several researchers which lead to many
improvements and variants [2, 3]. It has also been used with other evolutionary and non-evolutionary algorithms like Genetic Algorithms, Ant-Colony Optimization, and Simulated Annealing etc. to derive what are known as hybrid swarm algorithms. Recently, Plasma Generation Optimization technique was used to optimize retaining structures based on ACI 318-19 [4]. Studies were carried out to access the performance of other optimization methods example differential evolution (DE), evolutionary strategy (ES) and biogeography based optimization algorithm (BBO), for nonlinear constrained optimum design of cantilever retaining wall [5]. The application of optimization under earthquake loads was also studied [6,7].

A modified version of Particle Swarm Optimization known as passive congregation for cost-effective design of Retaining Wall [8]. PSO was also applied for solving the problem of Beam-Slab layout design [9]. Genetic Algorithms was used to the optimization of cantilever reinforced concrete retaining wall [10] and to optimize reinforced concrete retaining walls giving heuristic rules for the proportioning of retaining walls [11]. The hybrid metaheuristic algorithm has also been applied for optimization for retaining walls [12]. Genetic Algorithm (GA), Simulated Annealing (SA) and Particle Swarm Optimization (PSO) - the three heuristic optimization algorithms for the optimization of retaining walls were compared for efficiency and PSO was recommended to be used as it was found to be superior in terms of effectiveness and efficiency [13]. Further, the Optimtool in MATLAB was used for optimized design of cantilever wall [14]. The application of evolutionary algorithms applied to the optimization of structures was studied the structural optimization was identified to be of three types: topology, shape and size [15]. Most recently, flower pollination algorithm [16] and reliability based design optimization [17] was also utilised for optimum design of reinforced concrete retaining walls.

This study aims at testing PSO as a tool for the optimization of structures, particularly reinforced concrete cantilevered retaining walls. The study uses inbuilt MATLAB PSO toolbox to implement the PSO algorithm. A program was written to calculated constraints, bounds and to implement the PSO. A GUI was also made to make the changing of design and Particle Swarm Parameters easier. Observations were taken at height of 3-12 m for both non-cohesive and cohesive soil to find the reduction in cost and weight. The effect of swarm size and number of iterations was also observed on the cost of computations.

2. PARTICLE SWARM OPTIMIZATION (PSO)

PSO is primarily inspired by swarms of living organisms like swarm of bees, flocks of birds etc. It is a stochastic computational method that optimizes a problem over many iterations. It was originally introduced to simulate the social behaviour of living organisms. PSO is quite similar to other evolutionary methods like Genetic Algorithms (GA) in the sense that it is also a population based optimization method and the optimization takes place due to the updating of generations. It however differs from GA as it does not employ mutation, cross-over or any other such evolutionary operators. The objective solution is represented as food source (as shown in Figure 1). The particles try to find the path to the food source using their own intelligence and the collective intelligence of the swarm.
Figure 1. Illustration of PSO

The PSO algorithm starts by randomly allocating initial velocities to the initial particles created. Objective function at each location is estimated and the best (lowest) function value and the best location are determined. Based on current velocity, particles' individual best locations and the best locations of their neighbours the new velocities were selected. The particle locations, velocities, and neighbours were iteratively updated. The iterations continued until the algorithm reached a threshold criterion.

Figure 2. Convergence of swarm particles in PSO

The reiteration leads to convergence of swarm particles in PSO as shown in Figure 2. The cost function represents the actual function to be optimized while the penalty term is a means for the implementation of the constraints on the design. The algorithm flowchart for PSO is shown in Figure 3.
Figure 3. PSO Algorithm Flowchart

3. DESIGN OF RETAINING WALL

For design of retaining wall, the various forces acting on the retaining wall as shown in Figure 4 are needed to be determined. The coefficients of active ($k_a$) and passive ($k_p$) earth pressure were calculated using Rankine’s Earth Pressure theory, where $i$ is the angle of inclination of backfill and $\phi$ is the friction angle of the soil. For the stability of the retaining wall, the resultant of the forces must lie in the middle one third of the foundation base width. For safety against bearing failure, the maximum base pressure must not exceed the safe bearing capacity (SBC) of soil. The horizontal component of the earth pressure tends to overturn the retaining wall which is resisted by the combination of weight of the retaining wall and the earth over the heel portion. Further, it tends to slide the retaining wall which is resisted by the cohesive forces (in case of cohesive soils), the frictional forces between the base of the retaining wall and the soil and the shear key (if provided).

The design of the retaining wall consists of determining the parameters from (i) to (vi).
(i) Estimation of earth pressure:
\[ k_a = \cos \theta \cdot \frac{\cos^2 \theta - \cos \phi^2}{\cos^2 \theta + \cos \phi^2} \]
\[ k_p = \cos \theta \cdot \frac{\cos^2 \theta - \cos \phi^2}{\cos^2 \theta - \cos \phi^2} \]

(ii) Eccentricity: \( e \geq B/6 \)

(iii) Bearing Pressure:
\[ q_{\text{min}} = \frac{R}{L} \left( 1 - \frac{6e}{L} \right) \geq 0 \]
\[ q_{\text{max}} = \frac{R}{L} \left( 1 + \frac{6e}{L} \right) \leq SBC \]

(iv) Moments and Shear Forces at Toe and heel sections

(v) Stability against overturning:
\[ FOS_{\text{overturning}} = \frac{0.9 \times M_{\text{correcting}}}{M_{\text{overturning}}} \geq 1.4 \]

(vi) Stability against sliding:
\[ FOS_{\text{sliding}} = \frac{0.9 \times (F_{\text{friction}} + F_{\text{cohesion}} + F_{\text{key}})}{F_{\text{sliding}}} \geq 1.4 \]

The minimum and maximum tension reinforcement areas were taken as 0.12 and 4 percent respectively according to the directives of IS: 456 – 2000.

4. FORMULATION OF DESIGN OPTIMIZATION PROBLEM

The Optimization of both weight and cost was undertaken and the results compared with conventional design results for heights 3 to 12 metres based on the guidelines provided in IS 456:2000. Also, the variation of cost with grade of steel and concrete was studied.

4.1 Design Variables

As shown in the Figure 5 and Table 1, a total of nine design variables were considered for optimization. They consisted of six geometric variables and three variables giving areas of steel in various sections, namely: the heel, the toe and the stem.
Table 1. Design Variables

| Sr. No. | Design Variable | Description           |
|---------|-----------------|-----------------------|
| 1.      | Lh              | Length of the heel    |
| 2.      | Lt              | Length of the toe     |
| 3.      | wb              | Width of the base slab|
| 4.      | Ass             | Area of steel in the stem |
| 5.      | Ash             | Area of steel in the heel |
| 6.      | Ast             | Area of steel in the toe |
| 7.      | Lb              | Width of the stem slab at the base |
| 8.      | ws              | Width of the shear key |
| 9.      | Ds              | Depth of the shear key |

Figure 5. Design variables

4.2 Design Parameters

Design parameters (as listed in Table 2 and illustrated in Figure 6) were used to define site conditions and were taken as input from the Graphic User interface (GUI) in MATLAB. The various parameters were soil properties, loadings, material properties and material cost rates.

Table 2. Design Parameters

| Design Parameter | Description                  | Input values            |
|------------------|------------------------------|-------------------------|
| H                | Height Above GL              |                         |
| hf               | Depth of Foundation          |                         |
| i                | Inclination of backfill      | 10°                     |
| q                | Surcharge                    | 20kN/m³                 |
| γₖ               | Unit weight of the backfill  | 16 kN/m³                |
| φₖ               | Friction Angle of backfill   | 25° (cohesive) 35° (Non-cohesive) |
| cₖ               | Cohesion of backfill         | 20 kN/m²                |
| γ                | Unit weight of the soil      | 18 kN/m³                |
| φ                | Friction Angle of Soil       | 30°                     |
| c                | Cohesion of Soil             | 10 kN/m²                |
| SBC              | Safe Bearing capacity        | 250 kN/m²               |
| μ                | Friction Coefficient         | 0.4                     |
| fck              | Concrete Grade              | 25 MPa                  |
| fy               | Steel Grade                 | 500 MPa                 |
| γc               | Concrete Density            | 25 kN/m³                |
| γs               | Steel Density               | 78.5 kN/m³              |
| r₉₉              | Concrete Price Rate          | Rs. 7000/m³             |
| rsteel           | Steel Price Rate             | 68.1 per kg             |

Figure 6. Design parameters

4.3 Design Variables bounds

The design variable bounds as suggested by Saribus and Erabatur [18] and based on IS: 456-2000, were taken. The design variable bounds (as listed in Table 3) are necessary to reduce the computation time and prevent impractical results. The variable bounds are related to the total height of the retaining wall.
### Table 3. Design variable bounds

| S.No. | Design Variable | Description                  | Lower Bound       | Upper Bound       |
|-------|-----------------|------------------------------|-------------------|-------------------|
| 1.    | Lh              | Length of the heel           | 2.3*h/11          | 0.7*h             |
| 2.    | Lt              | Length of the toe            | (0.4*12/11)*h/3   | 0.7*h             |
| 3.    | wb              | Width of the base slab       | h/15              | max{lh, lt}       |
| 4.    | Ass             | Area of steel in the stem    | 0.0012*0.2        | 0.04*(0.1*h+0.2)  |
| 5.    | Ash             | Area of steel in the heel    | 0.0012*h/15       | 0.04*max{lh, lt}  |
| 6.    | Ast             | Area of steel in the toe     | 0.0012*h/15       | 0.04*max{lh, lt}  |
| 7.    | Lb              | Width of the stem slab at the base | 0.2 | 0.1*h+0.2 |
| 8.    | ws              | Width of the shear key       | 0.3               | 0.1*h             |
| 9.    | ds              | Depth of the shear key       | 0.3               | 0.1*h             |

#### 4.4 Design Constraints

Design constraints (as given Table 4) were utilized to implement various constraints on design such factors of safety in stability and limit states of strength. Additional design constraints were also utilized for implementing minimum and maximum steel areas and preventing the design of over-reinforced sections. Constraints were converted into dimensionless quantities for their weights to be considered equally while calculating penalty term.

**Table 4. Design Constraints**

| Sr. No. | Design Constraint | Description                  |
|---------|-------------------|------------------------------|
| 1.      | g₁(x)             | Stability against overturning |
| 2.      | g₂(x)             | Stability against sliding    |
| 3.      | g₃(x)             | Eccentricity                 |
| 4.      | g₄(x)             | Maximum base pressure        |
| 5.      | g₅(x)             | No tension condition at base |
| 6.      | g₆(x)             | Shear capacity at toe slab   |
| 7.      | g₇(x)             | Moment capacity at toe slab   |
| 8.      | g₈(x)             | Shear capacity at heel slab   |
| 9.      | g₉(x)             | Moment capacity at heel slab  |
| 10.     | g₁₀(x)            | Shear capacity of the stem   |
| 11.     | g₁₁(x)            | Moment capacity of the stem  |
| 12.     | g₁₂(x)            | Shear capacity of the shear key |

#### 4.5 Objective Function

The objective function consisted of two parts namely the cost function and the penalty term. The cost function represents the actual function to be optimized while the penalty term is a means for the implementation of the constraints on the design. As the iteration proceeds the penalty term approaches zero and the objective function equals the cost function. The cost function is the total amount of concrete and steel used multiplied by their respective rates. Other costs like shuttering and formwork, temperature and nominal steels were neglected to make the computations simpler. The objective function used for cost optimization was:

\[ f(x) = R_{\text{conc}} \cdot V_{\text{conc}} + R_{\text{steel}} \cdot W_{\text{steel}} + h(k) \cdot H(x) \]  

Where the first two terms represent the cost of the structure and the third term is the penalty factor based on the violation of the constraints. The combination of continuous assignment function along with linear penalty parameter function gave the best results hence was adapted for the problem. Similarly the objective function used for the optimization of weight was:

\[ f(x) = d_{\text{con}} \cdot V_{\text{conc}} + d_{\text{steel}} \cdot V_{\text{steel}} + h(k) \cdot H(x) \]  

(2)
5. ANALYSIS AND RESULT

Reinforced retaining walls of height 3 to 12 m were analysed for the savings in cost and reduction in weight. The optimization was done for both the cases of cohesive and non-cohesive backfill. Also, the effect of swarm size and the iterations was studied. The effect of grade of concrete and the friction angle on optimal cost was also studied.

5.1 Non-cohesive backfill

The comparison of optimized cost and weight with conventional design was carried out for cohesionless backfill. Figure 7 shows graph obtained height optimization, Figure 8 shows graph obtained for weight optimization and Figure 9 shows the graph for various dimensions versus height for cost optimization.

5.2 Cohesive backfill

The comparison of optimized cost and weight with conventional design was carried out for cohesive backfill. Figure 10 shows graph obtained height optimization, Figure 11 shows graph obtained for weight optimization for cohesive backfill soil.
5.3 Variation of cost with friction angle of the soil for 6m wall

Figure 12 shows the variation of the optimal cost with the friction angle of soil for 6m wall.

5.4 Variation of optimal cost with the grade of concrete

Figure 13 shows the variation of the optimal cost with grade of concrete.
The program was able to obtain an 8-17% reduction in the cost depending on the height of the retaining wall. Also, a reduction of about 27% was observed on an average in the case of weight optimization. The difference in conventional and optimized weight was however, lower for higher retaining wall.

5.5 Effect of Number of Iterations and Swarm Size

Figure 14. Cost vs Iterations

Figure 14 shows the variation of cost with iterations for the retaining wall of height 6m. It may be noted that the programs started with a high initial cost and slowly converges to the final cost. The cost shown in the plot is of the best particle in the swarm. It is also evident from the plot that the improvement in the solution after about 100 iterations is negligible. Therefore, the number of iterations for a run can be limited to about 150 without affecting the accuracy by a significant amount. Figure 15 shows the relationship between the swarm size and the time taken for computation.

Figure 15. Computation time vs size of the swarm

It was found that the computation time increase drastically with the increase in the number of particles in the swarm, but there is not much significance in the quality of the solution. Therefore, a compromise has to be made between the computation time and the optimization. It is found that a population size of 25 is adequate for a satisfactory solution but the chances of the algorithm getting stuck in a local minimum is high. Therefore, a swarm size of 75-100 is suggested. Figure 16 shows the variation of the shape of the retaining wall as the iterations progress. The initial candidate solution in represented in a lighter shade of grey and the darker lines progressively show later iterations. It can be seen that the program starts with a comparatively heavier section which is slowly modified to a more efficient section.
6. CONCLUSION
The present study proposed to optimize reinforced concrete cantilever retaining walls using PSO (Particle Swarm Optimization). The optimization both in terms of cost and weight was performed using a program developed in MATLAB. The program featured the capability to vary various site and material parameters.

- The reduction in cost achieved was from 8 to 17 percent depending on the height of the retaining wall, while the optimization in weight varied from 9 to 34 percent.
- The percentage saving for cost showed a slight increasing trend with the height of the retaining wall while the percentage saving in terms of weight showed a decrease with the height of the retaining wall.
- The design solution in each case satisfied all the factors of safety as prescribed in the cost.
- The time taken for the optimized design was 50-70s on an average for a swarm of 100 particles over 150 iterations.
- The program was successful in optimizing the cost and weight and also reducing the design times dramatically.
- The effect on computation times due to the number of iterations and the number of particles in the swarm was also observed and it was concluded that a swarm size of 75-100 with the number of iterations at 150 provided satisfactory balance between computation times and quality of solutions.

Thus the study was successful in establishing PSO as a robust method for the optimization of RC cantilevered retaining walls.

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