An analysis of olympiad students’ critical thinking in solving National Science Olympiad (OSN) problem on number theory material

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Abstract. This research aimed at describing the process of the mathematical Olympiad students in solving National Science Olympiad (OSN) problem on number theory material. The subjects of the research were 4 Olympiad students at Pythagoras Development Center of Mathematics and Natural Sciences Jember. The data collection methods used were test and interview. In this case, every student had strengths and weaknesses in solving the problem. All of the students were able to write and explain what was known and asked but they were still lack to model the problem into mathematical form. The students were able to formulate properly but were lack to construct it into various possible answers. All of the students were able to write systematic steps and correct conclusion. All of the students checked all the results, but there were still some students who were not careful in checking, so there were inappropriate answers.

1. Introduction
Education is a very important thing for all individuals to achieve an ideal. Mathematics was one of subjects which occupied an important role in the world of education, because mathematics was one of basic sciences in various science disciplines which developed the human thinking power [1]. The success and reformation of mathematics education are achieved when it can change the students’ thinking habit into thinking critically in a constructive way [2]. This statement is in line with the education system of Indonesia Nation. One of the content standards for primary and secondary education levels has mentioned that mathematics subject is one of compulsory subjects that must be followed by students in each level of education to provide students with the abilities to think logically, analytically, systematically, critically, innovatively and creatively. In general, the students learned in the following aspects: 1) do (work in group/individually), 2) speak up, 3) ask/question/discussion and 4) listen up [3].

Every student can be claimed as a problem solver, but the way of student in solving the problem needs a further investigation. The students who used to join Mathematics National Science Olympiad were familiar with the process of complex problem solving so that they were also familiar developing their ability of critical, creative, and innovative thinking [4]. Basically, mathematical Olympiad for elementary, junior high and high school levels similarly tested the students’ abilities in solving mathematical problems which were not routine [5]. Non-routine problem is a problem which contains many concepts and procedures taught and contains many usages and procedures of mathematics to solve the problem given unclearly [6]. The critical thinking ability of junior high school students in
Indonesia belonged to low [7]. One third of Indonesian students (33.1%) were only able to do the problem if all of the information needed were given correctly. Only 0.1% who were able to do mathematical modeling which required the critical thinking ability [8]. The critical thinking ability included knowledge to create a series of related critical questions at the correct time [9].

The success of the students in solving these non-routine problems depends on their creativity and innovation in using their mathematical communication abilities. This mathematical communication ability which the students often experience difficulties in practice, because mathematical Olympiad problems belong to high-level problems. In general, thinking ability consisted of four stages covered recall thinking, basic thinking, critical thinking and creative thinking [10]. The stages of critical thinking according to Krulick and Rudnick.

**Table 1.** Stages of critical thinking according to krulick and rudnick (1995).

| No | Problem Solving Stage | Critical Thinking Ability | Indicator |
|----|------------------------|--------------------------|-----------|
| 1  | Read and Think         | Investigating the context and spectrum of problems | Able to produce various assumptions/examples, categories, and able to write important relevant information used in problem solving, and develop various problem models. |
| 2  | Explore and Plan       | Formulating mathematical problem | Able to formulate meaningful mathematical questions/problems that provide direction for solving the various possible answers. |
| 3  | Select a Strategy      | Developing a concept of answer and arguments that make sense | Able to arrange various concepts of answers, formulate reasonable arguments that connect the concepts with the problems faced and able to develop a plausible solution model. |
| 4  | Find an Answer         | Evaluating               | Able to make assessments of the context of the problem, problem formulation or concept of meaningful answer and can find other alternative solutions. |
| 5  | Reflect and Extend     |                          |           |

Critical thinking student are students who are able to identify, evaluate, and construct arguments and are able to solve problems [11]. Critical thinking is a type of thinking that does not directly lead to conclusions, or accept some evidence, demands or decisions just like that, without really thinking about it and critical thinking clearly requires interpretation and evaluation of observations, communication and other sources of information [12]. There are four characteristics of a good problem solver, a person who has the following characteristics [13]:

- Curious, problem solvers are interested in understanding why things work that way.
- Persistent, because the problem is challenging, then to find a solution requires the problem solver to look for alternative strategies, approaches that can solve the problem better than before.
- Taking risks, often solving problems means making a few mistakes before finding a solution, problem solvers dare to take those risks.
- Reflective, effective problem solvers reflect what they learn from experience of solving problems. They take advantage of this reflection while solving other problems.
Olympiad students who can solve OSN problems properly can be analyzed to find out the level of critical thinking so that it can be used as an input or evaluation to be a better problem solver. Moreover, it also can be used as a reference for other students to be good problem solvers too. The mathematical concepts appeared in mathematical Olympiad questions were numbers, algebra, geometry, statistics and combinatorics [14]. Therefore, this research aimed to analyze the Olympiad students’ critical thinking processes in solving OSN questions on number theory material. This research described the Olympiad students’ critical thinking process which was held at the Pythagoras MIPA Development Center (PPM). The Pythagoras MIPA Development Center (PPM) was chosen because it was one of the Olympiad coaching groups for elementary and junior high schools in Jember.

2. Method
This research was a qualitative descriptive research. The data obtained were in the form of oral or written word from an object that had been observed. The subjects of this research were 4 junior high school students who were Olympiad students at the Pythagoras MIPA Development Center (PPM).

The research instruments used were test questions and interview guidelines. The instrument that was made was then validated by the validators, who were two lecturers of the Mathematics Education Study Program of UNEJ. The instrument was declared valid because $2.5 \leq V_d < 3$. The value of the observation sheet validation results obtained was $V_a = 2.75$, while for the interview guide value obtained was $V_a = 2.75$. Analysis of the data used in this research was the analysis of students’ work and interviews. Data obtained from test results were analyzed to describe the students’ critical thinking skills. The results of the interview were also analyzed to obtain the descriptive data as a complement to the test result data. The next was an assessment of the relationship between test results and interviews. This connection was used to draw conclusions about students’ critical thinking processes. The data obtained from the results of the test were analyzed to describe the students’ creative thinking skill. The results of the interview were also analyzed to get the data descriptively as the supporting data of the result of the test. Then, a study about the relationship between the result of the test and interview was conducted. This relationship was used to draw conclusion about the students’ critical thinking.

3. Result and discussion
The results of the analysis showed that the results of the test and interview from 4 Olympiad students at Pythagoras MIPA Development Center overall were able to answer the questions although there was still some mistakes. Each student had different way to solve the problem. Some students wrote down and explained the problem on the questions with what was “known” and “asked” as the process of questions understanding, however, other students were not able to write down the mathematics model of the questions. The next students were able to answer the questions from what was “known” and “asked” from the questions. This showed that the students were able to relate between what was “known” and “asked” in the questions so they were able to decide the way to answer the questions.

Question no 1 was: If $a$, $b$, $c$, and $d$ are positive integers divided by 19 respectively remains 12, 9, 7, and 5, then, determine the rest of $3a + 4b - 3c + 2d$ divided by 19. The four students S1, S2, S3, S4 were able to write down what was known and asked and were able to model the problem well, so they could formulate the problem well to get logical answer and argumentation. The four students were able to explain orally. The following was one of the students work:
Developing reasonable answer and argumentation concepts

Investigation of context and spectrum

Formulating mathematical problem

Figure 1. The result of students' work number 1.

Possible evaluation could be done only in the interview. S1, S2, S3, and S4 conducted evaluation through rechecking, however, they were not able to get the other alternative solutions. The result of the students' work showed that the students had fulfilled 3 critical thinking abilities but not the fourth critical thinking ability because they could not find the alternative solution to the problem. When the students were interviewed, they said that they had rechecked their result but they were not able to find the other alternative solution.

Question number 2 was: It is known that $1+3k$ is divided out by 5, $1+5k$ is divided out by 7, and $1+8k$ is divided out by 19. If $k$ is positive integers, then determine the smallest value of $k$. The four subjects could investigate the problem context and spectrum well so they could formulate the problem as well. The following was the result of the students' work.

Figure 2. The student’s work result of number 2.

Translate version

Know: $a,b,c$ and $d$ are positive integers, if divided by 19 respectively remains $12, 9, 7$, and $5$.

Asked: the rest of $3a + 4b - 3c + 2d$ divided by 19 = ... ?

Answer:
Math.model: $a = 19p + 12; b = 19q + 9; c = 19m + 7; d = 19n + 5$

exp: $p = 1, q = 2, m = 3, n = 4$

$a = 19 \cdot 1 + 12 = 31; b = 19 \cdot 2 + 9 = 47; c = 19 \cdot 3 + 7 = 64; d = 19 \cdot 4 + 5 = 81$

$3a + 4b - 3c + 2d = \frac{251}{19} = 13 \text{ rest } 4$

Conclusion: so, the rest of $3a + 4b - 3c + 2d$ divided by 19 = 4
When developing logical answer and argumentation, the students solved the problem well, however, some students directly wrote down the result obtained. The results of the students’ work showed that the students were less clear in writing down the steps of the solution or in other words the students wrote down the answer unsystematically. However, when the students were interviewed, they were able to explain the unclear steps well. The four subjects were less fulfilled the fourth critical thinking ability because they could not find the other alternative solution to the problem.

A 4 natural number $abcd$ with the same unit number and thousands number. If the 4 digit numbers are supposed as $x$, then $x$ is a quadratic number. Determine all possible $x$ numbers. From these problems $S1$, $S2$, $S3$, $S4$, write what was known and asked; but $S3$ and $S4$ could not model the problem properly. The following is one of the students’ work results.

**Figure 3.** The result of student’s work number 3.

S3 and S4 also were not able to formulate the problem properly. However, four students solved the problem by experimenting possible grades. The students were lack of developing the concept of answers and arguments that made sense to them. However, the students explained these problems well orally. Based on the results of interview with the students, it showed that students explained reasonably but were not careful in solving problems. So, S4 made a mistake, which was the answer to a question that should had 5 possibilities but S4 only got 4 possible answers.

Based on the result of analysis on Olympiad students’ critical thinking in solving Olympiad problem in number theory material above, it showed that, generally, the students were able to solve the problem by using the easiest way for them and not what their teacher or coach taught. It depended on the way they think and used their logic. In the process of understanding the problem, each student was able to understand the question well and knew what was known and asked from the question, yet in modeling the question into mathematical form, each student faced difficulty. However, it did not hinder them to solve the problem although they used their own way.

Generally, it can be said that the subjects were able to investigate the context and spectrum of the problem, formulate mathematics problem, and develop the concept of reasonable answer and argument, as well as conduct evaluation. Each subject had their own strength and weakness in solving the problem. All subjects were able to investigate the context and spectrum of the problem by writing down and explaining the information of what was known and asked. However, the subjects were less able to model the problem into mathematical form. The subjects were less able to formulate mathematics problem which was reflected when they did it which results in their tendency to dabble in
solving the problem. Furthermore, the subjects were able to develop the concept of reasonable answer and argument. This was seen when the subjects solved the problem by writing down the steps systematically as well as drawing the conclusion correctly. All subjects did not recheck their works so that they were less thorough in doing the evaluation.

4. Conclusion

The conclusion obtained from the results of the analysis and discussion of critical thinking was each student had advantages and disadvantages in solving problems. All students investigated the context and spectrum of problems well, but to formulate problems in mathematical form, they were not capable in doing it. Each student had his own way of developing concepts of answers and arguments that made sense in order to solve the problem. Students answered briefly or in a long way it depended on how critical the student was. But the weakness of students was in finding other alternatives to solve these problems.

Suggestions for teachers and Olympiad instructor from the results of this study are that students must be accustomed to solving material theory number problems not only in the form of answers, but by using ways that can develop students' critical thinking skills. This method is also possible to facilitate students in practicing critical thinking. In addition this research can be used as an additional reference for similar research.

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References

[1] Trapilasiwi D and Setiawani S 2016 Journal of Edukasi 3 21-26
[2] Schoenfeld A H 2016 Mathematical Thinking and Problem Solving (New York: Routledge)
[3] Rahmati S A, Hobri and Oktavianingtyas E 2018 Journal of Education 9 1–10
[4] Trisnowali A 2015 Journal of EST 1 47- 57
[5] Tohir M, Susanto and Dafik 2018 Advanced Science Letters 24 8361-8364
[6] Afghani, D J 2011 Materi Pokok Analisis Kurikulum Matematika (Jakarta: Universitas Terbuka)
[7] Purwati R, Hobri and Fatahillah A 2016 Kadikma 7 84-93
[8] Fatmawati H, Mardiyyana and Triyanto 2014 Electronic Journal of Mathematics Learning 2 899-910
[9] Damayanti E, Sunardi and Oktavianingtyas E 2017 Journal of Edukasi 8 1-10
[10] Yunita N W, Hobri and Oktavianingtyas E 2018 Kadikma 9 1-10
[11] Kusumawardani L, Hobri and Fatahillah A 2015 Student Scientific Articles 1 1-6
[12] Amir M F 2015 Journal of Math Educator Nusantara 1 159-170
[13] Kallicht B and Ross B 1997 How to Assess Problem-Solving Skills in Math (New York: Scholastic Professional Books)
[14] Latifah I W, Susanto and Sugarti T 2018 Kadikma 9 145-154