Peculiar velocities in the local Universe: comparison of different models and the implications for $H_0$ and dark matter

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ABSTRACT

When measuring the value of the Hubble parameter, $H_0$, it is necessary to know the recession velocity free of the effects of peculiar velocities. In this work, we study different models of peculiar velocity in the local Universe. In particular, we compare models based on density reconstruction from galaxy redshift surveys and kernel smoothing of peculiar velocity data. The velocity field from the density reconstruction is obtained using the 2M++ galaxy redshift compilation, which is compared to two adaptive kernel-smoothed velocity fields: the first obtained from the 6dF Fundamental Plane sample and the other using a Tully–Fisher catalogue obtained by combining SFI++ and 2MTF. We highlight that smoothed velocity fields should be rescaled to obtain unbiased velocity estimates. Comparing the predictions of these models to the observations from a few test sets of peculiar velocity data, obtained from the Second Amendment Supernovae catalogue and the Tully–Fisher catalogues, we find that 2M++ reconstruction provides a better model of the peculiar velocity in the local Universe than the kernel-smoothed peculiar velocity models. We study the impact of peculiar velocities on the measurement of $H_0$ from gravitational waves and masers. In doing so, we introduce a probabilistic framework to marginalize over the peculiar velocity corrections along the line of sight. For the megamasers, we find $H_0 = 70.1 \pm 2.9 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$ using the 2M++ velocity field. We also study the peculiar velocity of the galaxy NGC 1052-DF2, concluding that a short $\sim 13 \, \text{Mpc}$ distance is not a likely explanation of the anomalously low dark matter fraction of that galaxy.

Key words: Galaxy: kinematics and dynamics – large-scale structure of Universe – cosmology: observations.

1 INTRODUCTION

Peculiar velocities, deviations from the regular Hubble flow, are sourced by the gravitational pull of large-scale structures, thus providing the only way to measure growth of structure on large scale in the low-redshift Universe (Huterer et al. 2017; Adams & Blake 2020; Boruah, Hudson & Lavaux 2020). Apart from their use as a probe of cosmological structure growth, one also has to account for the peculiar velocity of galaxies in other studies of cosmology and galaxy formation. For example, peculiar velocity corrections for nearby standard candles and standard sirens are an important step in trying to measure the expansion rate of the Universe. It was noted in Hui & Greene (2006) that correlated errors in the redshfits of supernovae, introduced due to peculiar velocities, are important to account for in cosmological analyses. Neill, Hudson & Conley (2007) applied peculiar velocity corrections while inferring the equation of state from supernovae, finding a systematic bias of $\Delta w \sim 0.04$ when not corrected for the peculiar velocities. In Riess et al. (2011), peculiar velocity corrections were applied for the first time in the measurement of the Hubble constant from supernovae.

Peculiar velocity corrections are especially important given the increasing discrepancy (Verde, Treu & Riess 2019) in the value of the Hubble constant, $H_0$, measured using the cosmic microwave background (CMB; Planck Collaboration VI 2018) and other low-$z$ measurements (Riess et al. 2019; Wong et al. 2020). The current tension between the measurements from CMB and the low-redshift supernovae has been estimated to be $\sim 4.4\sigma$. Other methods such as standard sirens and megamasers, which measure distances without any calibration to the distance ladder, are crucial in resolution of the $H_0$ tension. $H_0$ has already been measured from the first detection of gravitational waves from a binary neutron star merger, GW170817 (Abbott et al. 2017a) yielding a value of $H_0 = 70.8^{+12.5}_{-8.0} \, \text{km s}^{-1} \, \text{Mpc}^{-1}$ (Abbott et al. 2017b). The megamaser cosmology project (hereafter MCP, Pesce et al. 2020) has also measured distances to six megamasers giving a measurement, $H_0 = 73.9 \pm 3.0 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$.

The value of $H_0$ measured from local distance indicators depends on the peculiar velocity corrections. The observed redshift, $z_{\text{obs}}$, for standard candles and standard sirens gets a contribution from both the cosmological redshift due to the Hubble flow, $z_{\text{cos}} \equiv z_{\text{cos}}(H_0)$,
and the radial peculiar velocity of the object, \( v_r \),
\[
1 + z_{\text{obs}} = (1 + z_{\text{cos}}) \left[ 1 + \frac{v_r}{c} \right].
\] (1)

Therefore, in order to measure the value of \( H_0 \), the radial peculiar velocity needs to be subtracted from the observed redshift. The inferred value of \( H_0 \) can have significant difference depending on the model of velocity corrections (see e.g. Pesce et al. 2020). To measure the peculiar velocity, we need accurate measurements of the redshifts as well as distances. Accurate distances are also important for measuring, e.g. the masses of galaxies, which in turn has implications for understanding the nature of dark matter.

To measure the peculiar velocity of galaxies, one has to rely on some distance indicator. Commonly used distance tracers for the measurement of peculiar velocity include empirical galaxy scaling relationships such as the Tully–Fisher (TF; Tully & Fisher 1977) and the Fundamental Plane (FP; Djorgovski & Davis 1987; Dressler et al. 1987) relations as well as Type Ia supernovae. The distance measurements from TF and FP relations have \( \sim 15-25 \) per cent uncertainty (Springob et al. 2007; 2014), while Type Ia SNe gives a distance estimate that is accurate to \( 5-10 \) per cent (Foley et al. 2018). However, not all galaxies have a distance estimate obtained by one of these methods. Hence, we need some method to map out the peculiar velocity field of the nearby Universe.

In this work, we compare different peculiar velocity fields by comparing their predictions to independent peculiar velocity catalogues. Broadly, we compare the velocity field predicted using density reconstruction and the velocity field predicted using the adaptive kernel smoothing technique. Predicting peculiar velocities based on density reconstruction has a long history (see e.g. Kaiser et al. 1991; Hudson 1994b). In this approach, one uses the galaxy redshift surveys to ‘reconstruct’ the density field, which in turn is used to predict the peculiar velocity field in the local Universe. In contrast, the adaptive kernel smoothing method smooths the peculiar velocity data to map out the peculiar velocity field of the Universe. This has been used for cosmography with the 6dF (Springob et al. 2014) and the 2MTF (Springob et al. 2016) peculiar velocity surveys. The reconstructed velocity field used in this work is obtained using the 2M++ galaxy redshift compilation (Lavaux & Hudson 2011) and the adaptive kernel-smoothed velocity fields are obtained by smoothing the 6dF peculiar velocity catalogue and a combined TF catalogue from SFI++ and 2MTF. We use two different methods, a simple comparison of the MSE and a forward likelihood method, to compare the different velocity field predictions to the observations.

We also study the impact of different peculiar velocity models on cosmology and galaxy formation. First, we study its impact on the value of \( H_0 \) measured from gravitational wave standard sirens and megamaser galaxies. Neither of these techniques rely on intermediate distance calibrators, providing new, direct ways of measuring \( H_0 \). Finally, we study the peculiar velocity of NGC 1052-DF2, a galaxy that has been found to contain little or no dark matter and where it had been argued that a smaller distance to this galaxy can explain the anomaly.

This paper is structured as follows: in Section 2, we describe the peculiar velocity data, which we use for the adaptive kernel smoothing and to test peculiar velocity models. Section 3 describes the two methods for predicting the peculiar velocity fields. We highlight the importance of scaling the smoothed velocity fields by a constant factor to obtain unbiased velocity estimate of the galaxies in Section 4. In Section 5, the predictions of the peculiar velocity models are compared to the observed peculiar velocity from the test sets. We discuss the implications of the peculiar velocity fields for measurements of \( H_0 \) in Section 6. Finally, we discuss the case of NGC 1052-DF2 in Section 7 before summarizing our results in Section 8.

In Appendix A, we tested the predictions of the kernel smoothing method with an N-body simulation to determine the smoothing scale for unbiased velocity estimate. Appendix B presents the detailed results of the posterior ratios used for comparing the different peculiar velocity models.

## 2 Peculiar Velocity Data

We use a few different peculiar velocity catalogues in this work. These catalogues serve two purposes: first, as a ‘tracer sample’ to map the velocity field of the local Universe using an adaptive kernel smoothing technique, and secondly, as test sets to test the predictions of the peculiar velocity models. In this section, we describe the different catalogues we use in this work, their main features, and the corresponding data processing required.

### 2.1 6dF peculiar velocity catalogue

The 6dF peculiar velocity sample (Springob et al. 2014) consists of galaxies from the FP survey (Magoulas et al. 2012; Campbell et al. 2016) of the 6dF galaxy survey. It is presently the largest peculiar velocity survey with a total of 8885 galaxies. The distance (and hence the radial peculiar velocity) of these galaxies is estimated using the FP relation. The sample is restricted to the Southern hemisphere with a galactic cut-off \([|b| > 10^\circ\] and \(cz < 16000 \text{ km s}^{-1}\) in the CMB frame. The mean distance uncertainty of the sample was found to be \(\sim 26\) per cent. We plot the sky distribution of the 6dF peculiar velocity catalogue along with the tracers we use to compare the velocity field in the Southern hemisphere in the left-hand panel of Fig. 1. We use the 6dF peculiar velocity sample as a tracer sample to predict the peculiar velocity using the adaptive kernel smoothing method described in Section 3.2.

### 2.2 Tully–Fisher catalogues

In this work, we use two TF catalogues: SFI++ and 2MTF. The TF catalogues serve dual purposes in this work. First, we use the objects in the Southern hemisphere as a test set to test the predictions of the 2M++ reconstructed velocity field and the adaptive kernel-smoothed velocity obtained from 6dF. Secondly, we use a combined TF catalogue of SFI++ and 2MTF to predict the velocity field using the adaptive kernel smoothing method and compare it with the other velocity field models.

#### 2.2.1 SFI++

SFI++ (Masters et al. 2006; Springob et al. 2007) is an \( I \)-band TF survey with more than \( 4000 \) peculiar velocity measurements. As noted in Boruah et al. (2020), there is a deviation from the linear TF relation for objects in the faint and bright end. We therefore only use galaxies with \(-0.1 < \eta < 0.2\), where \( \eta \) is related to the velocity width, \( W \), as \( \eta = \log_{10} W - 2.5 \). To remove the outliers, we iteratively fit the TF relation using the redshift space distances and remove the 3.5\( r \) outliers. For this work, while comparing the peculiar velocities, we only consider the galaxies that are within \( cz < 10000 \text{ km s}^{-1}\). With these cuts, we have a total of 1607 field galaxies and 584 groups in our sample. Of these, 949 field galaxies and 204 groups are in the Southern hemisphere. While comparing the 6dF adaptive kernel-smoothed peculiar velocity field and the 2M++ reconstructed
velocity field, we use only the galaxies in the Southern hemisphere as a test set.

2.2.2 2MTF

The 2MTF survey (Masters, Springob & Huchra 2008) is a TF survey in the \( J \), \( H \), and \( K \) bands. The final catalogue (Hong et al. 2019) consists of 2062 galaxies within \( c_z < 10 \,000 \, \text{km s}^{-1} \). We remove the duplicates from 2MTF that are already contained in the SFI++ catalogue. Similar to the SFI++ catalogue, we only use galaxies with \(-0.1 < \eta < 0.2\) and reject outliers by iteratively fitting the TF relation. We have a total of 1248 galaxies after these cuts. Of these, 567 galaxies are in the Southern hemisphere and we use these galaxies to compare with the 6dF adaptive smoothed peculiar velocity field and the 2M++ reconstructed velocity field.

2.2.3 SuperTF

We combine the SFI+++ and the 2MTF catalogue into a ‘super TF’ catalogue which we then use to produce an adaptive kernel-smoothed peculiar velocity map. Unlike 6dF, we can use this catalogue to map out the velocity field in both hemispheres using the kernel smoothing method. The \( F \)-band TF relation, used in the SFI+++ catalogue, has a smaller intrinsic scatter compared to the TF relation in the infrared bands, which is used by the 2MTF survey (see e.g. Boruah et al. 2020). Therefore, when there are duplicates in the SFI+++ and 2MTF data sets, we use the SFI+++ objects. The final data set consists of 584 SFI+++ groups, 1607 SFI+++ field galaxies and 1248 2MTF galaxies. We show the sky distribution of the objects in this combined catalogue in Fig. 1. We also compare the redshift distribution of the objects in the SuperTF catalogue with the 6dF peculiar velocity catalogue in Fig. 2. Note that the SuperTF catalogue has a higher density of objects compared to 6dF at lower redshifts (\( z \leq 0.015 \)).

much smaller distance error than TF or FP galaxies. We presented the Second Amendment (A2) sample of supernovae in Boruah et al. (2020). It consists of low-redshift (low-\( z \)) supernovae from the CfA supernovae sample (Hicken et al. 2009), Carnegie Supernovae project (Folatelli et al. 2010; Krisciunas et al. 2017), the Lick Observatory Supernova Survey (Ganeshalingam, Li & Filippenko 2013) and the Foundation supernovae sample (Foley et al. 2018; Jones et al. 2019). The final sample consists of 465 low-\( z \) supernovae, resulting in the largest peculiar velocity catalogue based on supernovae. While comparing to the 6dF adaptive smoothed peculiar velocity field, we only use the SNe in the Southern hemisphere and \(|b| < 10^\circ\). We call this data set consisting of 150 SNe, ‘A2-South’.

3 PECULIAR VELOCITY MODELS

In this work, we compare two different types of peculiar velocity models – (i) the velocity field reconstructed from the 2M+++ galaxy redshift compilation and (ii) velocity field mapped out using an adaptive kernel smoothing technique. In this section, we briefly describe both these methods.
of galaxy redshifts (Lavaux & Hudson 2011). The 2M++ catalogue consists of a total of 69160 galaxies. The catalogue was found to be highly complete up to a distance of 200h⁻¹Mpc (or K < 12.5) for the region covered by the 6dF and SDSS and up to 125h⁻¹Mpc (or K < 11.5) for the region that is not covered by these surveys.

We use the luminosity-weighted density field from Carrick et al. (2015) in this work. The velocity field is predicted from the density field using linear perturbation theory. The predicted velocity field from the luminosity-weighted density field is scaled by a factor of β = f/θb and an external velocity, Vext is added to this. To fit for the value of β and the external velocity Vext, we compared the predicted velocity to the observed velocities from a peculiar velocity survey. In order to avoid using the same survey twice, we do not fit the flow parameters to the survey being compared. For example, when we want to compare the peculiar velocity model to the SFI++ flow fields, we only use the A2 data set to fit for the flow parameters. The values of β and Vext can be found in Carrick et al. (2015) and Boruah et al. (2020). More details on the reconstruction procedure can be found in Carrick et al. (2015).

3.2 Adaptive kernel smoothing method

An adaptive kernel smoothing technique to map the peculiar velocity field using measured peculiar velocities was presented in Springob et al. (2014, 2016). We use this method to map the velocity field in the local Universe using the 6dF and the SuperTF catalogues.

In this scheme, a Gaussian kernel is used to smooth the peculiar velocity measurements from the catalogues. The measured radial velocities, \(v_i(r_i)\), in peculiar velocity catalogue at locations, \(r_i\), is used to predict the peculiar velocity at \(r\) as,

\[
v(r) = \frac{\sum_{i=1}^{N_{gal}} v_i(r_i) \cos(\delta_i) e^{-\Delta r_i^2/2\sigma_i^2} \sigma_i^{-3}}{\sum_{i=1}^{N_{gal}} e^{-\Delta r_i^2/2\sigma_i^2} \sigma_i^{-3}},
\]

where, \(\Delta r_i = |r - r_i|\) and \(\cos(\delta_i) = \hat{r} \cdot \hat{r}_i\). Note that in this method, the location, \(r\), is given in the redshift space. The kernel width for each galaxy is adaptively computed using the prescription presented in Springob et al. (2014, 2016). Assuming a fiducial smoothing length, \(\sigma\), the adaptive smoothing length for each object is defined as

\[
\delta_i = \frac{(3)}{\exp \left( \frac{\sum_{j=1}^{N} \ln(\delta_j)/N}{\delta_i} \right) \right]^{1/2},
\]

where \(\delta_i\) is computed as

\[
\delta_i = \sum_{j=1}^{N} \exp \left( - \frac{\sigma r_j - r_i}{2\sigma^2} \right).
\]

The sum in equation (4) is over the \(N\) objects that are within distance, \(3\sigma\), of the \(i\)th object. The quantity, \(\delta_i\), roughly calculates the density of peculiar velocity tracers near the \(i\)th object. Equation (3) then calculates the kernel width adaptively based on the density of the peculiar velocity tracers. The calculated kernel size in regions with larger density is thus smaller.

We use the adaptive kernel smoothing method to predict the velocity field using the 6dF FP and the SuperTF catalogue. In this work, we use two different fiducial smoothing lengths, 8 and 16h⁻¹Mpc to predict the kernel-smoothed velocity field. If we use a fiducial smoothing length of \(\sigma = 8\ h^{-1}\) Mpc, the mean smoothing length, \(\langle \sigma \rangle\), for the 6dF and the SuperTF catalogues are 8.61 and 8.83 h⁻¹ Mpc, respectively. The spread of the same smoothing lengths as measured using the standard deviation of the distribution are, 3.60 and 4.39 h⁻¹ Mpc for the 6dF and Super TF catalogues, respectively. The distribution of the adaptively calculated smoothing lengths with fiducial smoothing length of 8h⁻¹ Mpc for the 6dF and the SuperTF catalogues is shown in Fig. 4.

A comparison of the peculiar velocity fields in the supagalactic plane derived using the 2M++ reconstruction, and the adaptive kernel smoothing on the 6dF and the SuperTF catalogue is shown in Fig. 3.

4 SCALING THE SMOOTHED VELOCITY

In the previous section, we presented two ways to predict the velocities of galaxies. Both these methods rely on smoothing the peculiar velocity field in some way. However, one must be careful while using a smoothed field to predict velocities since certain smoothing scales may lead to biased estimates of the peculiar velocity.

Figure 3. The radial velocity in the supagalactic plane for the three different peculiar velocity models: Left: 2M++ reconstruction, Centre: Adaptive kernel smoothing with the 6dF peculiar velocity catalogue, Right: Adaptive kernel smoothing with the SuperTF catalogue. The coordinates for the adaptive smoothed fields are in real space, while that for 2M++ reconstruction is in the real space. We also show the location of a few prominent superclusters and NGC 4993.

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To compare the models of peculiar velocity of the local Universe, we use independent peculiar velocity data sets to compare the predictions of the models to observations. We propose two ways to test this – the first method is based on a simple comparison of the MSE between predicted and observed peculiar velocity of the tracer peculiar velocity data set. In this method, we use the velocity estimate from the peculiar velocity models at the estimated distance of the peculiar velocity tracer and compare it with the observed value of the velocity. However, this method is known to be affected by inhomogeneous Malmquist bias. The second method is the forward likelihood method that can correct for the inhomogeneous Malmquist bias. We use Bayesian model comparison with this method to compare the peculiar velocity models presented in Section 3.

5 COMPARING PECULIAR VELOCITY MODELS

The kernel smoothing method also predicts the velocity by smoothing the peculiar velocity data in the neighbouring region. Given the large error bars and the sparse sample of peculiar velocity data, smoothing over a larger region may be necessary to reduce the uncertainties on the predictions to an acceptable level. However, smoothing over larger regions also biases low the peculiar velocity estimates. Therefore, if we smooth the peculiar velocity field using a smoothing scale \( \sigma_0 \gtrsim 4h^{-1}\) Mpc, we need to correct for this bias. To do so, we use a simple scaling of the smoothed peculiar velocity. More precisely, the peculiar velocity estimate, \( v_p(R) \), obtained by using a smoothing scale, \( R \) is scaled by factor of \( A(R) \) such that

\[
v_p(R) \rightarrow A(R)v_p(R). \tag{5}
\]

The scaling factor, \( A(R) \) is determined by comparing the kernel-smoothed velocity of an N-body simulation to the true velocity of the haloes. The details of the study on the simulation is presented in Appendix A. With the help of these simulations, we determine the scaling factor for a smoothing length of 8 and 16 \( h^{-1}\) Mpc to be 1.07 and 1.16, respectively. We compare both the scaled and the unscaled versions of the adaptive kernel-smoothed peculiar velocity fields in the next section.

5.1 Comparing the mean-squared error

In the first approach, we calculate the MSE between the predicted radial velocity from the peculiar velocity models and the measured radial peculiar velocity of the test set. The model with the better predictions for the peculiar velocity should have a lower value of MSE. For the adaptive kernel smoothing approach, we use the velocity estimate of the test objects using the approach of Section 3.2. In this approach, we use the redshift space position of the test object to estimate the velocity. For the reconstructed velocity field, we predict the velocity at the reported mean position of the object. We plot the predicted velocity from 6dF adaptive kernel smoothing technique and from the 2M++ reconstruction against the observed velocity of the 2MTF, SFI++, and the A2 supernovae in Fig. 5. Note that to calibrate the flow parameters (\( \beta \) and \( V_{\text{esc}} \)), we do not use the survey for which we are predicting the peculiar velocity. That is when predicting the 2MTF velocity field, we use the flow parameters fitted on A2 and SFI++ only.

We then compute the estimated velocity with the measured radial velocity. The MSE between the observed velocity and the predicted velocity is defined as

\[
\text{MSE} = \frac{1}{N_{\text{tracers}}} \sum_{i=0}^{N_{\text{tracers}}} \frac{(V_{\text{pred}} - V_{\text{obs}})^2}{\Delta V_{\text{obs}}^2}. \tag{6}
\]

The value of MSE obtained for the different models is presented in Table 1. Since the SuperTF catalogue consists of SFI++ and 2MTF galaxies, we do not use these data sets to compare to the velocity field obtained by using the adaptive kernel smoothing on the SuperTF catalogue. We notice that for all the test sets, the 2M++ reconstructed velocity field gives a lower value of the MSE than the adaptive kernel-smoothed velocity fields. Hence, it hints that 2M++ reconstructed peculiar velocity is a better model for the peculiar velocity of the local Universe than the one obtained by using adaptive kernel smoothing. However, given the inherent drawbacks of the \( \chi^2 \) minimization technique, we need further work to confirm this. This is done by using Bayesian model comparison in Section 5.3 using the forward likelihood method.

5.2 Forward likelihood

The approach used in Section 5.1 is simplistic and is known to be affected by inhomogeneous Malmquist bias. Hudson (1994a, b) introduced an approach for peculiar velocity analysis that can deal with inhomogeneous Malmquist bias by using an improved radial distribution for the peculiar velocity tracer (see also Pike & Hudson 2005). This approach is called forward likelihood. In this section, we use this approach to determine which peculiar velocity field fits the data well.

In the forward likelihood approach, the difference in the observed and predicted redshifts are minimized along the line of sight and the radial distribution is marginalized. We take into account the

Figure 4. Distribution of the adaptively calculated kernel smoothing length computed using a fiducial smoothing length, \( \sigma_0 = 8h^{-1}\) Mpc, for the two peculiar velocity catalogues, 6dF and SuperTF. The mean smoothing lengths for the 6dF and the SuperTF catalogues are 8.61 and 8.83 \( h^{-1}\) Mpc, respectively.
Figure 5. Plot of the predicted velocity, $V_{\text{pred}}$, predicted using adaptive kernel smoothing on 6dF (left) and 2M++ reconstruction (right) versus the observed velocity, $V_{\text{obs}}$ for the different test sets. The black markers denote the peculiar velocity estimate for each object in the test set. The blue curve shows a binned version where we plot the weighted average of the observed peculiar velocity objects in each $50 \, \text{km} \, \text{s}^{-1}$ bins of predicted peculiar velocity. The red line represents, $V_{\text{pred}} = V_{\text{obs}}$.

Table 1. The value of MSE measured for different test data sets.

| Test               | Velocity field model | 2M++ | 6dF | SuperTF |
|--------------------|----------------------|------|-----|---------|
| A2 (South)         |                      | 0.742| 1.856| 1.970   |
| A2 (low-z)        |                      | 0.899| –   | 2.214   |
| SFI++ groups       |                      | 0.814| 0.949| –       |
| SFI++ field galaxies|                    | 0.703| 0.763| –       |
| 2MTF               |                      | 0.875| 1.072| –       |

inhomogeneities along the line of sight to correct for inhomogeneous Malmquist bias. This is done by assuming the following radial distribution:

$$P(r) \propto \frac{r^2}{\sqrt{2\pi}\sigma_d^2} \exp \left( -\frac{r^2}{2\sigma_d^2} \right) (1 + \delta_d(r)), \quad (7)$$

where $d$ is the distance reported in the peculiar velocity survey converted to comoving distance, $\sigma_d$ is the associated uncertainty, and $\delta_d$ is the overdensity in the galaxy field. In this work, we use the luminosity-weighted density field from the 2M++ reconstruction for the inhomogeneous Malmquist bias correction.

To account for the errors that arise because of the triple-valued regions and inhomogeneities along the line of sight, the likelihood is marginalized over the above radial distribution. The likelihood for observing the redshift given a peculiar velocity model, $v$, can be written as

$$P(z_{\text{obs}}|v) = \int_{0}^{\infty} \text{d}r P(z_{\text{obs}}|r, v) P(r), \quad (8)$$

where $P(z_{\text{obs}}|r, v)$ is modelled as a Gaussian with standard deviation, $\sigma_r$,

$$P(z_{\text{obs}}|r, v) = \frac{1}{\sqrt{2\pi}\sigma_r^2} \exp \left( -\frac{(z_{\text{obs}} - cz_{\text{pred}}(r, v))^2}{2\sigma_r^2} \right). \quad (9)$$

and $P(r)$ is given by equation (7). $z_{\text{pred}} \equiv z_{\text{pred}}(r, v)$ as given as

$$1 + z_{\text{pred}} = \left( 1 + \frac{v_r}{c} \right), \quad (10)$$

where $v_r$ is predicted using the velocity model, $v$, at the position, $r$. In the case of adaptive kernel smoothing, the radial velocity is
calculated in the redshift space. Therefore, we introduce the redshift space coordinate, \( s \), to facilitate the calculation for this case:

\[
\mathbf{s} = H_0 r + v_t(r) \hat{r}.
\]

Note that, in the above equation distance coordinates, \( s \) and \( r \) has the same units as that for the velocity. That is, we convert the distance units into velocity units (km s\(^{-1}\)). We use this change of coordinates to calculate equation (8) in the redshift space:

\[
\mathcal{P}(z_{\text{obs}}|v) = \int_0^\infty \mathcal{P}(z_{\text{obs}}|r(s), \mathbf{v}(s))\mathcal{P}(r(s)),
\]

\[
= \int_0^\infty \frac{ds}{H_0} \left( 1 - \frac{\partial v_t}{\partial s} \right) \mathcal{P}(z_{\text{obs}}|r(s), \mathbf{v}(s))\mathcal{P}(r(s)).
\]

5.3 Bayesian model comparison with forward likelihood

We use Bayesian model comparison to compare the two peculiar velocity models described in Section 3. Given two models, \( \mathcal{M}_1, \mathcal{M}_2 \) describing the same data, \( \mathcal{D} \), Bayesian model comparison gives a way to compare the two models. The plausibility of the two models can be compared by calculating the posterior probability ratio (Mackay 2003):

\[
\mathcal{P}(\mathcal{M}_1|D) = \frac{\mathcal{P}(\mathcal{M}_1) \mathcal{P}(D|\mathcal{M}_1)}{\mathcal{P}(\mathcal{M}_2) \mathcal{P}(D|\mathcal{M}_2)},
\]

\( \mathcal{P}(\mathcal{M}) \) denotes the prior belief in the model, \( \mathcal{M} \).

Hence, Bayesian model comparison is well suited to compare the different peculiar velocity models. We do so by using the forward likelihood to calculate the likelihood, \( \mathcal{P}(D|\mathbf{v}) \). If we assign equal prior probability to two different peculiar velocity models, \( \mathbf{v}_1 \) and \( \mathbf{v}_2 \), i.e. \( \mathcal{P}(\mathbf{v}_1) = \mathcal{P}(\mathbf{v}_2) \), the posterior probability ratio can be written as

\[
\mathcal{P}(\mathbf{v}_1|D) = \frac{\mathcal{P}(D|\mathbf{v}_1)}{\mathcal{P}(D|\mathbf{v}_2)}.
\]

6 IMPLICATIONS FOR \( H_0 \)

As we saw in equation (1), the redshift of a galaxy gets a contribution from both the peculiar velocity and the Hubble recession. Therefore, one needs to correct for the contribution of the peculiar velocity in order to correctly infer \( H_0 \). In this section, we consider the peculiar velocity corrections for two data sets: the distance measurement from gravitational wave for NGC 4993 and the peculiar velocity corrections for megamasers from the MCP. We begin with the Bayesian model we use for the treatment of peculiar velocity for these data sets.

6.1 \( H_0 \) likelihood

When correcting the redshifts for the peculiar velocities, traditionally one uses a point estimate for the peculiar velocity at a given redshift. However, the distance tracers used to measure \( H_0 \) usually have a large distance uncertainty. Given this uncertainty on the distances, we need
assuming $\sigma_v$ by expanding it in terms of the intermediate distance variable, a fixed peculiar velocity correction leads to a small, but significant bias in the inferred value of $H$.

In order to go further, we simplify the likelihood, $P$($H_0$|$z_{\text{obs}}$, $x_{\text{dist}}$), by expanding it in terms of the intermediate distance variable, $d$, and the velocity field, $v$, used to correct the observed redshifts.

$$P(z_{\text{obs}}^i, x_{\text{dist}}^i|H_0) = \int dd P(d) P(z_{\text{obs}}^i, H_0, v, d) P(x_{\text{dist}}^i|d).$$

From this, we have to model the likelihood for the redshift, $P(z_{\text{obs}}^i|H_0, v, d)$:

$$P(z_{\text{obs}}^i|H_0, v, d) = \frac{1}{\sqrt{2\pi \sigma_v}} \exp \left( -\frac{(z_{\text{obs}}^i - cz_{\text{pred}})}{2\sigma_v^2} \right).$$

where $cz_{\text{pred}} = cz_{\text{pred}}(H_0, d, v)$ and $\sigma_v$ is the typical uncertainty in the predicted peculiar velocity.

We test this method of inferring $H_0$ using simulation in Appendix C, where we show that marginalized peculiar velocity correction leads to an unbiased inference of $H_0$. On the other hand, using a fixed peculiar velocity correction leads to a small, but significant bias in the inferred value of $H_0$.

### 6.2 Peculiar velocity of NGC 4993

The exact peculiar velocity of NGC 4993 has been the discussion of many recent works (e.g. Howlett & Davis 2020; Nicolaou et al. 2020; Mukherjee et al. 2021). NGC 4993 was the host galaxy for the binary neutron star event, GW170817 (Abbott et al. 2017a) discovered by LIGO. The distance measurement from the gravitational wave event can be used to put constraint on the value of $H_0$. Because it is relatively nearby, the contribution to the redshift from the peculiar velocity is substantial ($\gtrsim 10$ per cent) and needs to be corrected.

The original estimate of $H_0$ from GW170817 (Abbott et al. 2017b), used adaptive kernel smoothing on the 6dF peculiar velocity sample to predict the velocity of NGC 4993. It was noted that reconstruction based method with 2M++ also gives a similar velocity estimate. Howlett & Davis (2020) tested the dependence of the peculiar velocity predictions on different assumptions, such as group assignment and different peculiar velocity catalogues for kernel smoothing. It was demonstrated that different assumptions leads to different estimates for the peculiar velocity, thus leading to a larger uncertainty in the measured value of $H_0$. Mukherjee et al. (2021) used the forward-modelled reconstruction framework, BORG (Jasche & Wandelt 2013; Jasche & Lavaux 2019), to predict the velocity field with the 2M++ catalogue, finding a velocity estimate of $330 \pm 130$ km s$^{-1}$ for NGC 4993.

In this section, we check the predictions for the peculiar velocity of NGC 4993 using different choices of the peculiar velocity models considered in this work. Nicolaou et al. (2020) noted that the estimates of the peculiar velocity for NGC 4993 using the kernel smoothing technique depends strongly on the choice of kernel width and used a Bayesian model to account for the uncertainty due to the choice of the kernel width. As we already point in Section 4, predictions from a smoothed velocity field need to be scaled up in order to obtain unbiased estimates for the velocity. We show the dependence of the predicted peculiar velocity of NGC 4993 on the smoothing length in right-hand panel of Fig. 6. As can be seen, after scaling the velocity by the required factor, there is no longer a strong dependence on the smoothing scale. The predictions from 2M++ reconstruction and adaptive kernel smoothing give consistent results. Since our methods of estimating the peculiar velocity smooths the velocity field at a scale much larger than that of individual galaxies, it is useful to predict the velocities of groups of galaxies. Averaging the redshifts of galaxies in a group suppresses the non-linear velocity contributions. In order to correct for the grouping, we identify the groups from the 2M++ catalogue. For NGC 4993, the average group
redshift is \( c_z = 3339 \, \text{km} \, \text{s}^{-1} \), while that of the galaxy is \( c_z = 3216 \, \text{km} \, \text{s}^{-1} \). Using a fiducial smoothing length of 8 \( h^{-1} \, \text{Mpc} \), the adaptive kernel smoothing technique predicts a velocity of \( v_t = 357 \pm 219 \, \text{km} \, \text{s}^{-1} \) for the 6dF catalogue and \( v_t = 388 \pm 162 \, \text{km} \, \text{s}^{-1} \) for the SuperTF catalogue. The uncertainty for the two fields are obtained by adding the measurement error from the kernel smoothing added in quadrature to \( \sigma_v = 150 \, \text{km} \, \text{s}^{-1} \). The justification for using this uncertainty is as follows. The uncertainty in the peculiar velocity estimated using the kernel smoothing has two contributions: (i) The measurement uncertainty for the peculiar velocity data, and, (ii) the scatter of the peculiar velocity of the galaxies from which the kernel-smoothed velocity is estimated. The first part can be estimated from the measurement errors on the peculiar velocity estimates. For the second part, we assume a value, \( \sigma_v = 150 \, \text{km} \, \text{s}^{-1} \). Since the 2M++ reconstruction gives the velocity in the real space coordinates, we use an iterative method to estimate the velocity along the line of sight of the galaxy. For NGC 4993, we find the 2M++ predictions to be, \( v_t = 456 \pm 150 \, \text{km} \, \text{s}^{-1} \). Therefore, we find consistent predictions from the 2M++ reconstruction and the kernel smoothing methods.

We also used the likelihood described in Section 6.1 to derive the constraint on \( H_0 \) from the gravitational wave distance. For NGC 4993, the luminosity distance can be measured from the gravitational wave event GW170817.\(^2\) A volumetric prior, \( P_{\text{prior}}(d) \propto d^2 \), was used inferring the distance posterior. Therefore, we also used a volumetric prior for inferring \( H_0 \) with the GW data. We compare the results from the likelihood for measuring \( H_0 \) that was used in Abbott et al. (2017b), where a fixed velocity estimate was used. The inferred value of \( H_0 \) for the different peculiar velocity fields and the two likelihoods are given in Table 2. The \( H_0 \) posterior using the GW distance is shown in Fig. 7. We show the effect of using the different peculiar velocity models considered in this work. The table and the figure shows that marginalizing the peculiar velocity correction along the line-of-sight inflates the error bar by \( \sim 20 \) per cent. In Fig. 8, we show the peculiar velocity along the line-of-sight of NGC 4993 as predicted by the different peculiar velocity models.

### 6.3 Peculiar velocity correction for megamasers

Water megamasers in the active galactic nuclei of galaxies provide a completely geometric method to measure distances without the need for intermediate calibration. In Pesce et al. (2020), the MCP measured the value of \( H_0 \) from the distance measurement to 6 such megamasers. It was found that the inferred value of \( H_0 \) depended strongly on the treatment of peculiar velocities.

\(^2\)The posterior samples are publicly available at [https://dcc.ligo.org/LIGO-P1800061/public](https://dcc.ligo.org/LIGO-P1800061/public).
(iii) We use the group-corrected redshift instead of the individual redshift for each galaxy.

(iv) We use two different values of \( \sigma_v \) to check the robustness of the result to the choice of this parameter.

We discuss the effect of each of these modelling assumptions on the inferred value of \( H_0 \) in the following. We use the peculiar velocity fields described in the previous sections to correct for peculiar velocities. Since all six megamasers are located in the northern sky, we cannot use the 6dF velocity field for our purpose. We used both the 2M++ and the SuperTF velocity fields for our treatment of the peculiar velocity.

**Marginalizing the line-of-sight peculiar velocity:** We use the probability model described in Section 6.1, where we marginalize over the line-of-sight peculiar velocity, as opposed to a point estimate that is usually used. We compared the effect of not marginalizing the line-of-sight peculiar velocity. When using the simple point estimate of the peculiar velocity, we find that the \( H_0 \) posterior shifts by \( \sim 1.5 \text{ km s}^{-1} \text{ Mpc}^{-1} \). This is a non-negligible effect on the inferred value of \( H_0 \). Such a shift is not present when using the SuperTF velocities, suggesting that the effect of marginalizing over the line-of-sight peculiar velocity may be non-trivial. In addition to using a fixed peculiar velocity, if we use a uniform prior on the distances, we get \( H_0 = 71.5 \pm 2.7 \text{ km s}^{-1} \text{ Mpc}^{-1} \), which is consistent with the values obtained by Pesce et al. (2020) for the 2M++ velocity field.

**Distance priors:** It has been noted that assuming a wrong distribution biases the distance measurement, an effect usually called Malmquist (1920) bias, although the first derivation of this effect is in Eddington (1914). In Pesce et al. (2020), a uniform distance prior was used to derive the distances to the megamasers, which may bias the distance measurements to a lower value, leading to an inferred value of \( H_0 \).

For the forward method used in this work, in general, the magnitude of the Malmquist bias is independent of the selection effect (Strauss & Willick 1995). However, as pointed out in Gould (1993), this assumption breaks down if the selection is on a variable that has a covariance with the distance measurements. In this case, if the selection variable has a correlation coefficient, \( \rho \) with the distance measurement, the magnitude of the fractional Malmquist bias is given as

\[
\Delta_{MB} = 3 \rho \Delta^2,
\]

where \( \Delta \) is the fractional distance uncertainty. In the case where the selection variable is the same as the distance measurement variable (i.e., \( \rho = 1 \)), this gives a fractional bias of \( 3 \Delta^2 \) as pointed in Lynden-Bell et al. (1988). However, for the case of megamasers, the selection function has complicated dependence on multiple observation features of the megamaser. Therefore, quantifying the bias due to the ‘Gould effect’ is complicated and beyond the scope of this paper. If the selection is completely independent on the distance measurement (\( \rho = 0 \)), using a uniform prior induces no bias. For the case of megamasers, we use a uniform prior as the fiducial analysis choice. However, there can be a residual bias due to covariance of the selection variable with the distance measurement. Therefore, we also include the result by assuming a volumetric prior. The difference in the result between the two priors is \( \sim 1 \text{ km s}^{-1} \text{ Mpc}^{-1} (~0.4\sigma) \). This should be interpreted as the maximal bias (if \( \rho = 1 \)) possible due to Malmquist bias. The true Malmquist bias will be an intermediate value between 0 and this maximal value. In the future, with a large sample of megamasers, it may be possible to measure the correlation coefficient between the selection variable and the distance measurements directly from the observations.

### Table 3. Inferred value of \( H_0 \) from megamasers for different treatment of peculiar velocities. Results are reported as the median with 1\( \sigma \) confidence interval (16–84th percentile).

| Peculiar velocity | Model assumption | \( H_0 \) (km s\(^{-1}\) Mpc\(^{-1}\)) |
|------------------|------------------|----------------------------------|
| 2M++             | Fiducial         | 70.1 ± 2.9                      |
|                  | Volumetric distance prior | 69.0 ± 2.8                  |
|                  | No group redshift correction | 69.6 ± 2.9             |
|                  | \( \sigma_v = 200 \text{ km s}^{-1} \) | 70.5 ± 3.1                  |
|                  | Fixed \( v_{pec} \) | 71.5 ± 2.7                      |
|                  | Pesce et al. (2020) 2M++ fit | 71.8 ± 2.7                  |
| SuperTF          | Fiducial         | 74.4 ± 7.3                      |
|                  | Volumetric prior  | 72.6 ± 6.3                      |
|                  | No group redshift correction | 73.8 ± 6.2             |
|                  | Fixed \( v_{pec} \) | 73.7 ± 6.7                      |

**Group redshift corrections:** As in the previous section, we use the group mean redshifts for the megamaser galaxies to suppress the non-linear velocity contributions. Except for NGC 4258, we identify the groups from the 2M++ catalogue. In Kourkchi & Tully (2017), it was noted that the NGC 4258 group is located directly in the foreground of the NGC 4217 group. This can lead to mistaken identification of the two groups into the same group. In the group catalogue of 2M++, the two groups are identified into a single group. Therefore, we use the group information for NGC 4258 from Kourkchi & Tully (2017). We note that for the MCP megamaser sample, using the group redshift has a small effect, leading to a value \( H_0 \) that is 0.4 km s\(^{-1}\) Mpc\(^{-1}\) higher than when using the galaxy redshifts.

**Value of \( \sigma_v \):** We also checked the effect of using a different value of \( \sigma_v \) for the peculiar velocity error. The \( \sigma_v = 150 \text{ km s}^{-1} \) uncertainty was obtained by calibrating our reconstruction with simulations. If for some reason, the peculiar velocity error is underestimated, the value of \( \sigma_v \) may be higher. We therefore test the effect of adopting a larger uncertainty, \( \sigma_v = 200 \text{ km s}^{-1} \) for our measurements. The effect of different values of \( \sigma_v \) in non-trivial. As expected, the uncertainty increases for the higher \( \sigma_v \) value. The mean inferred value \( H_0 \) is also 0.4 km s\(^{-1}\) Mpc\(^{-1}\) higher than for the fiducial value of \( \sigma_v \).

We present our results in Table 3, where the inferred value of \( H_0 \) for different model assumptions are reported for the 2M++ and the SuperTF peculiar velocity fields. For the SuperTF velocity field, we add in quadrature to \( \sigma_v = 150 \text{ km s}^{-1} \), the measurement uncertainty in the kernel-smoothed velocity field. As can be seen in Fig. 2, the distribution of SuperTF galaxies at high redshift is sparse. Furthermore, as discussed in Section 5.4, at higher redshift, the peculiar velocity uncertainties are also larger. Hence, the velocity corrections for the megamasers obtained using the SuperTF field is a very noisy, leading to the much larger uncertainty in \( H_0 \). In the ‘fiducial model’, we marginalize over the line-of-sight peculiar velocity, assume a uniform prior for the distance, and the group corrected redshifts are used. For 2M++, we assume \( \sigma_v = 150 \text{ km s}^{-1} \) for the fiducial model. The posteriors derived for the different model assumptions are shown in Fig. 9. As can be seen from the figure, we find that using the SuperTF velocity field yields a much larger uncertainty in \( H_0 \) compared to the 2M++ velocity field. Of the different assumptions that we checked, we highlight the importance of marginalizing the line-of-sight peculiar velocity and possible bias due to wrong assumption on the distance prior. For the six megamasers, a combination of these two effects shifts the \( H_0 \) posterior by \( \sim 1\sigma \). In Fig. 10, we show the \( H_0 \) posterior
The peculiar velocity field can be expressed in terms of the density
peculiar velocity do not have a substantial dependence on the various
the cosmological parameter, \( \Omega_m \).

Given that we use the peculiar velocity corrections to measure
6.4 Cosmology dependence of the peculiar velocity field
from each of the 6 individual megamasers. As can be seen from
from the 2M++ velocity field, while the blue curves are obtained using the SuperTF velocity field.
Figure 9. \( H_0 \) posteriors from the megamaser distances for different peculiar velocity treatments. The red curves are obtained using the 2M++ velocity field, while the blue curves are obtained using the SuperTF velocity field. Different line styles correspond to different model assumptions as indicated in the plot. The Planck and SH0ES confidence intervals are shown with the green and orange vertical bands.

Figure 10. The \( H_0 \) posterior for each of the individual megamasers for our fiducial model assumptions with the 2M++ velocity field. NGC 5764b provides the tightest constraint on \( H_0 \) among the six megamasers.

First, note that the derived peculiar velocity field does not explicitly
depends on \( H_0 \). Since the distances to the peculiar velocity tracers are derived without an absolute calibration, the measured distances can be expressed in units of \( h^{-1} \) Mpc. Once written in these units, \( H_0 \) dependence drops out from equation (19).

The velocity field depends on \( \Omega_m \) in two ways: (i) Through the dependence of \( \Omega_m \) on the cosmological redshift or \( E(z) \) and (ii) through the dependence of \( \beta \beta \) on \( \Omega_m \).

Figure 11. \( H_0 \) posteriors from the megamaser distances for different peculiar velocity treatments. The red curves are obtained using the 2M++ velocity field, while the blue curves are obtained using the SuperTF velocity field. Different line styles correspond to different model assumptions as indicated in the plot. The Planck and SH0ES confidence intervals are shown with the green and orange vertical bands.

We tested the impact of these effects on the inferred value of \( H_0 \) using simulations in Appendix C. For the first of these effects, we run our inference pipeline using two values of \( \Omega_m = (0.25, 0.35) \). In that case, the difference in the inferred value of \( H_0 \) is \( \sim 0.2 \) per cent. The impact of cosmological models on the inference of \( H_0 \) has also been tested for a wide variety of cosmological models in Dhawan et al. (2020), who found that the inferred value of \( H_0 \) does not depend significantly on the cosmological model. We similarly test for the effect of miscalibration of \( \beta \) on the inference of \( H_0 \) with the same simulations. We observe that a 10 per cent \((\sim 2\sigma)\) uncertainty on \( \beta \) leads to a 0.39 per cent shift on the inferred value of \( H_0 \). For the applications in this paper, both of these effects are small compared to the statistical uncertainty \((\sim 4\) per cent). However, this may not be a negligible effect for a statistical uncertainty of 1 per cent which is the target for a number of direct probe measurements of \( H_0 \). Finally, if we use a value of \( \beta = 0 \), which is equivalent to having no peculiar velocity corrections, we find that the inferred value of \( H_0 \) is shifted by 3.43 \( \pm \) 0.68 per cent, which is similar in magnitude of the statistical uncertainty in the \( H_0 \) measured from megamasers.

6.4 Cosmology dependence of the peculiar velocity field

Given that we use the peculiar velocity corrections to measure
the cosmological parameter, \( H_0 \), we need to test that the derived peculiar velocity do not have a substantial dependence on the various cosmological parameter used in deriving the peculiar velocity field.

The peculiar velocity field can be expressed in terms of the density field as

\[
\psi(r) = \frac{f H_0}{4\pi} \int d^3r' \delta(r) \frac{r - r'}{|r - r'|^3}. \tag{19}
\]

First, note that the derived peculiar velocity field does not explicitly depends on \( H_0 \). Since the distances to the peculiar velocity tracers are derived without an absolute calibration, the measured distances can be expressed in units of \( h^{-1} \) Mpc. Once written in these units, \( H_0 \) dependence drops out from equation (19).

The velocity field depends on \( \Omega_m \) in two ways: (i) Through the dependence of \( \Omega_m \) on the cosmological redshift or \( E(z) \) and (ii) through the dependence of \( \beta \beta \) on \( \Omega_m \). We tested the impact of these effects on the inferred value of \( H_0 \) using simulations in Appendix C. For the first of these effects, we run our inference pipeline using two values of \( \Omega_m = (0.25, 0.35) \). In that case, the difference in the inferred value of \( H_0 \) is \( \sim 0.2 \) per cent. The impact of cosmological models on the inference of \( H_0 \) has also been tested for a wide variety of cosmological models in Dhawan et al. (2020), who found that the inferred value of \( H_0 \) does not depend significantly on the cosmological model. We similarly test for the effect of miscalibration of \( \beta \) on the inference of \( H_0 \) with the same simulations. We observe that a 10 per cent \((\sim 2\sigma)\) uncertainty on \( \beta \) leads to a 0.39 per cent shift on the inferred value of \( H_0 \). For the applications in this paper, both of these effects are small compared to the statistical uncertainty \((\sim 4\) per cent). However, this may not be a negligible effect for a statistical uncertainty of 1 per cent which is the target for a number of direct probe measurements of \( H_0 \). Finally, if we use a value of \( \beta = 0 \), which is equivalent to having no peculiar velocity corrections, we find that the inferred value of \( H_0 \) is shifted by 3.43 \( \pm \) 0.68 per cent, which is similar in magnitude of the statistical uncertainty in the \( H_0 \) measured from megamasers.

7 DISTANCE AND PECULIAR VELOCITY OF NGC 1052-DF2

In an interesting result, van Dokkum et al. (2018) discovered that a galaxy, NGC 1052-DF2, contains little or no dark matter. This challenges the conventional wisdom about galaxy formation and shows that at least some galaxies may have baryonic component without any dark matter in it. However, their result was contested by Trujillo et al. (2019), where using tip of red giant branch and other distance measurements, the authors calculated a distance of \( \sim 13 \) Mpc to the galaxy as opposed to the earlier estimates of \( \sim 19 \) Mpc as derived from surface brightness fluctuations in van Dokkum et al. (2018). An analysis adopting the shorter distance to NGC 1052-DF2 results in total-mass-to-stellar mass ratio, \( M_\text{halo}/M_\star > 20 \) as opposed to the value of order unity derived in van Dokkum et al. (2018).

However, the shorter (\( 13 \) Mpc) distance implies a radial peculiar velocity of \( 640 \pm 25 \) km s\(^{-1}\) in the CMB frame for NGC 1052-DF2. While Trujillo et al. (2019) claim that the spread of peculiar velocities in this region is high, we argue that most of this spread is due to uncertainty in the distance indicator and does not reflect the real velocity noise on the underlying flow field. For example, for the TF and FP methods, the scatter in the distance indicator is \( \sim 20 \) per cent, which translates to a scatter of \( \sim 350 \) km s\(^{-1}\) in the peculiar velocity from galaxy to galaxy at the distance of NGC 1052. In this section, we infer the peculiar velocity of the galaxy by the two methods previously described. The result of this analysis is shown in Fig. 12.

The mean redshift of the nine members of the NGC 1052 group (in which NGC 1052-DF2 is assumed to reside) as identified in the 2M++ catalogue is \( z_{\text{CMB}} = 1256 \) km s\(^{-1}\). This is in agreement with Kourkchi & Tully (2017), who find a mean CMB redshift of \( 1256 \) km s\(^{-1}\) from 16 group members. Note that this is considerably lower than the CMB redshift of NGC 1052-DF2 itself (1587 km s\(^{-1}\)). Using the 2M++ reconstruction, the peculiar velocity for NGC 1052 group is \( v_r = -162 \pm 150 \) km s\(^{-1}\). Assuming \( H_0 = 72 \) km s\(^{-1}\) Mpc\(^{-1}\), this
peculiar velocity implies a distance of $19.7 \pm 2.1$ Mpc. The distance estimates are consistent with the FP distance estimate for NGC 1052, $d = 19.4 \pm 2.4$ Mpc (Tonry et al. 2001) and the SBF distances derived for NGC 1052-DF2 itself, $19 \pm 1.7$ and $20.4 \pm 2.0$ Mpc by van Dokkum et al. (2018) and Blakeslee & Cantiello (2018), respectively. We find consistent results from the kernel smoothing approaches: with a fiducial smoothing radius of $8 \ h^{-1}$ Mpc, with the 6dF velocity data there are a total of nine 6dF velocities within 1 kernel length of NGC 1052-DF2 and these yield a kernel-smoothed mean velocity of $v_r = -124 \pm 165 \ km \ s^{-1}$. With the SuperTF data, the kernel-smoothed mean velocity is $v_r = -191 \pm 155 \ km \ s^{-1}$ from 32 SuperTF velocities within 1 kernel length of NGC 1052-DF2.

We also consider the possibility that NGC 1052-DF2 may be in the foreground of the NGC 1052 group with a distance of $\sim 13$ Mpc. In this case, if it is isolated, then the peculiar velocity needed to explain its redshift is $\sim 600 \ km \ s^{-1}$. In Fig. 13, we plot the predicted redshift along the line of sight of NGC 1052-DF2 for the different peculiar velocity models. As can be seen from the Figure, there are no locations in the foreground of the NGC 1052 group with such high outward radial peculiar velocity.

More recently, Monelli & Trujillo (2019) have suggested that NGC 1052-DF2 is a member of a foreground group dominated by the spirals NGC 1042 and NGC 1035 at a distance of $\sim 13.5$ Mpc. The mean CMB velocity of these two galaxies is $1184 \ km \ s^{-1}$, which would mean that NGC 1052-DF2 would have a peculiar velocity of $403 \ km \ s^{-1}$ with respect to this poor group: unlikely, but perhaps not impossible if NGC 1052-DF2 is close to the bottom of the group’s potential well and is falling in directly along the line of sight. However the group itself, if at a distance of $13.5$ Mpc, would have a CMB frame peculiar velocity (assuming $H_0 = 72 \ km \ s^{-1} \ Mpc^{-1}$).
Turning the problem around, to be consistent with the 2M++ different methods for a few specific galaxies. First, we investigated velocity test data sets across all range of redshifts. We find that the 2M++ using a simple comparison of the MSE and a forward likelihood method. We need to rescale the predictions by a smoothed velocity field, we need to rescale the predictions by a smoothing the peculiar velocity data. In this work, we compared the performance of different peculiar velocity models of the local Universe. The first model we studied is the reconstructed velocity of the density field in the local Universe or by kernel unbiased estimates of peculiar velocity are essential for different applications in cosmology and galaxy formation. There are different methods of estimating the peculiar velocities of galaxies, e.g. using reconstruction of the density field in the local Universe or by kernel smoothing the peculiar velocity data. In this work, we compared the performance of different peculiar velocity models of the local Universe. The first model we studied is the reconstructed velocity field from the 2M++ redshift compilation. The others are based on an adaptive kernel smoothing technique, which we apply on the 6dF peculiar velocity data and SuperTF, a compilation of the TF peculiar velocity data from SFI++ and 2MTF. We highlight that, when using a smoothed velocity field, we need to rescale the predictions by a scaling factor to get unbiased estimate of the peculiar velocity. We compared the peculiar velocity predictions to a few test data sets using a simple comparison of the MSE and a forward likelihood method. We find that the 2M++ reconstruction performs better than both the kernel-smoothed peculiar velocity data for all the peculiar velocity test data sets across all range of redshifts.

We also compared the peculiar velocity estimates from these different methods for a few specific galaxies. First, we investigated the implications of our peculiar velocity fields for the measurement of $H_0$ from standard sirens and megamasers. In doing so, we introduced a probabilistic framework where we marginalize over the line-of-sight peculiar velocity to accurately capture the effect of peculiar velocity corrections. By testing on mock simulations, we showed that such a line-of-sight marginalization method gives an unbiased estimate of $H_0$, while a fixed peculiar velocity correction can lead to a small, but significant bias in the estimate of $H_0$. NGC 4993 was the host galaxy for the first binary neutron star event detected by LIGO. Because of it nearby location, accurate peculiar velocity is required to correct the redshift of the galaxy in order to obtain the measurement of the Hubble constant. The different models considered in this work give remarkably consistent peculiar velocity estimates for the galaxy. For NGC 4993, we notice that marginalizing over the line-of-sight peculiar velocity inflates the uncertainty on $H_0$ by a factor of $\sim 1.5$. Another distance indicator that does not rely on intermediate distance calibrator is megamaser. We also checked the effects of different assumptions about the peculiar velocity correction on the inferred value of $H_0$ from the megamasers. We highlight two key factors that can significantly bias the inferred value of $H_0$ from the megamasers: (i) using the wrong prior for the distances and (ii) not marginalizing over the line-of-sight peculiar velocity. With our fiducial model assumptions with the 2M++ velocity field, we find $H_0 = 70.1 \pm 2.9 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$, which is $\sim 1.2\sigma$ lower than the value obtained by the SH0ES collaboration from Type Ia supernovae. Finally, we also investigate the peculiar velocity of NGC 1052-DF2, which is an ultradiffuse galaxy that has been claimed to be almost free of dark matter. This result has been contested with the claim that a shorter distance to the galaxy solves the anomalous stellar mass fraction. However, we find that this claim is not supported by the models of peculiar velocity that we use in our study.

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**DATA AVAILABILITY STATEMENT**

The SFI++, 2MTF, and the 6dF catalogues are publicly available with their respective publications as cited in Section 2. The Second Amendment supernovae compilation is available with the supplementary data of \(\text{https://doi.org/10.1093/mnras/staa2485}\). The 2M++ reconstruction used in this work is publicly available at \(\text{https://cosmicflows.iap.fr/}\). Finally, the distance data from MCP were provided by Dom Pesce and Jim Braatz.

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\(^3\)https://www.aquila-consortium.org/
In Appendix A, we discuss the testing of simulations. In Section 4, we used a value of $R_{\text{unbiased}}$, which denotes the smoothing scale at which the predictions from kernel smoothing of peculiar velocities give an unbiased result. In this section, we determine the value of this scale using an N-body simulation. We used an N-body simulation from the VELMASS simulation suite. The VELMASS simulation was used to perform a cubic box of size $2h^{-1}$ Gpc with a total of 2048 $^3$ particles with mass $9.387 \times 10^{10} h^{-1} M_\odot$. The cosmological parameters for this simulation are as follows: $\Omega_m = 0.315$, $\Omega_b = 0.049$, $H_0 = 68 \text{ km s}^{-1} \text{ h}^{-1} \text{ Mpc}^{-1}$, $\sigma_8 = 0.81$, $n_s = 0.97$, and $Y_{\text{He}} = 0.248$. We identified the haloes in the simulation with the ROCKSTAR halo finding software (Behroozi, Wechsler & Wu 2013), using only the haloes of mass larger than $2 \times 10^{12} h^{-1} M_\odot$. We used the haloes within a $(250 h^{-1} \text{ Mpc})^3$ sub-box of the full simulation. We do not consider the sub-haloes. There were a total of 32 784 haloes in this region. We then split these haloes into two sets – a ‘tracer’ sample which is used to obtain the kernel-smoothed velocity and a ‘test’ sample, for which the predicted velocity is compared to the true velocity.

We use three different number densities for the tracer population to validate the prediction. In the fiducial configuration, we used 12 000 randomly selected tracer velocities to predict the velocity. This roughly corresponds to the tracer density of the 2MTF survey. In addition to this fiducial tracer density, we also used a high (low) tracer density configuration with 24 000 (6 000) randomly selected tracers. For each of these tracer densities, we compared the predicted velocity of the test haloes to its true velocity, thus fitting for the scaling factor, $A$ and the velocity error, $\sigma_v$. The scaling factor scales the predicted velocity such that

$$V_{\text{true}} = AV_{\text{pred}}. \quad (A1)$$

Note that, for the simulations, we are smoothing the full 3D velocity and not just the radial velocity. The value of $A$ and $\sigma_v$ is fitted for different smoothing scales. The results of these fits are shown in Fig. A1. As expected, the velocity error, $\sigma_v$ is larger for the sample with low tracer density. For each of the three tracer densities, the unbiased smoothing length is around $3-5 h^{-1} \text{ Mpc}$. There is, nevertheless, a dependence of this scale on the tracer density – for a higher tracer density, the unbiased smoothing scale is lower. A dependence of this scale on the tracer density is shown in Fig. A1. As expected, the velocity error, $\sigma_v$ is larger for the sample with low tracer density. For each of the three tracer densities, the unbiased smoothing length is around $3-5 h^{-1} \text{ Mpc}$. There is, nevertheless, a dependence of this scale on the tracer density – for a higher tracer density, the unbiased smoothing scale is lower. A similar trend is also obtained for the scale where the velocity error is minimum (See the right-hand panel of Fig. A1).
Peculiar velocity corrections

Figure A1. The dependence of the (left) scaling factor, and (right) the velocity error, on the smoothing scale in our simulations. The blue, green, and the red curves show the results for the fiducial, high and low tracer density samples. For the scaling factor, we also plot the results from the linear theory. As can be seen from the figure, the velocity error is lower for a high tracer density. Also note that the scale for unbiased velocity prediction depends on the tracer density of the sample. For the sample with high tracer density, the smoothing scale of unbiased velocity predictions is lower.

APPENDIX B: RESULTS FOR THE POSTERIOR RATIO

In Section 5.2, we used Bayesian model comparison to compare the reconstructed peculiar velocity field from 2M++ with the adaptive kernel-smoothed velocity fields. In doing so, we compared the posterior ratios using the forward likelihood presented in the same section. We present the values of the posterior ratios in Tables B1 and B2, where we compared the adaptive kernel-smoothed velocity fields smoothed with a fiducial smoothing length of 8 and 16 $h^{-1}$ Mpc, respectively. The reported quantity is the ratio of likelihood obtained for the 2M++ field and the adaptive kernel-smoothed field. We make various cuts in redshifts as indicated in the tables. We compare the likelihoods both with (denoted as a superscript) and without the scaling factor introduced in Section 4. In the following tables, a positive value for the posterior ratio implies that the 2M++ velocity field is preferred.

Table B1. Ratio of $\log(P)$ calculated using the 2M++ reconstructed velocity field and the adaptive kernel smoothing technique for 6dF and SuperTF. For this table, we used a fixed value of $\sigma_v = 150$ km s$^{-1}$ and the fiducial smoothing length is 8 $h^{-1}$ Mpc. For each test set, we make cuts in the redshift as indicated. The prefix ‘scaled’ denotes that the kernel-smoothed velocity is scaled up by a scaling factor. The scaling factor for 8 $h^{-1}$ Mpc smoothing is 1.07.

| Test set | Redshift selection | $\ln\left(\frac{P_{2M++}}{P_{6dF}}\right)$ | $\ln\left(\frac{P_{2M++}}{P_{SuperTF}}\right)$ | $\ln\left(\frac{P_{2M++}}{P_{SuperTF}}\right)$ | $\ln\left(\frac{P_{2M++}}{P_{SuperTF}}\right)$ | $N_{tracers}$ |
|----------|---------------------|-----------------|-----------------|-----------------|-----------------|-------------|
| A2-South | cz $< 3000$ km s$^{-1}$ | 2.21 | 2.70 | 2.41 | 2.88 | 16 |
|          | cz $< 4500$ km s$^{-1}$ | 2.33 | 3.86 | 5.07 | 5.89 | 32 |
|          | cz $< 6000$ km s$^{-1}$ | 5.21 | 6.05 | 8.85 | 10.21 | 53 |
|          | cz $< 9000$ km s$^{-1}$ | 9.19 | 10.58 | 15.56 | 17.80 | 79 |
| A2-low-z | cz $< 3000$ km s$^{-1}$ | – | – | 11.38 | 11.95 | 49 |
|          | cz $< 4500$ km s$^{-1}$ | – | – | 20.76 | 21.89 | 92 |
|          | cz $< 6000$ km s$^{-1}$ | – | – | 49.82 | 52.86 | 168 |
|          | cz $< 9000$ km s$^{-1}$ | – | – | 85.92 | 92.90 | 318 |
| 2MTF     | cz $< 3000$ km s$^{-1}$ | 24.6 | 24.4 | – | – | 108 |
|          | cz $< 4500$ km s$^{-1}$ | 39.00 | 36.28 | – | – | 247 |
|          | cz $< 6000$ km s$^{-1}$ | 55.65 | 53.06 | – | – | 379 |
|          | cz $< 9000$ km s$^{-1}$ | 69.49 | 69.01 | – | – | 483 |
| SFI++ Groups | cz $< 3000$ km s$^{-1}$ | 12.08 | 11.88 | – | – | 61 |
|          | cz $< 4500$ km s$^{-1}$ | 9.35 | 9.29 | – | – | 100 |
|          | cz $< 6000$ km s$^{-1}$ | 18.89 | 17.87 | – | – | 165 |
|          | cz $< 9000$ km s$^{-1}$ | 18.78 | 17.78 | – | – | 170 |
| SFI++ Field | cz $< 3000$ km s$^{-1}$ | 9.94 | 9.01 | – | – | 63 |
|          | cz $< 4500$ km s$^{-1}$ | 5.72 | 5.24 | – | – | 153 |
|          | cz $< 6000$ km s$^{-1}$ | 13.52 | 11.70 | – | – | 388 |
|          | cz $< 9000$ km s$^{-1}$ | 53.66 | 44.29 | – | – | 736 |
Appendix C: validation test of the method

In Section 6.1, we introduced a new method to account for the peculiar velocity corrections while inferring \( H_0 \). While marginalization over the line of sight seems like the principal way to account for peculiar velocity correction, this method needs to be validated on simulation. In order to validate our method, we test our method on the same VELMASS simulation used in Section A. We created 100 magnitude-limited mock surveys from the simulation assuming a fiducial value of \( H_0 = 68 \text{ km s}^{-1} \text{ Mpc}^{-1} \). Each mock survey in our simulation contains 50 distance tracers. On each mock survey, we run out inference pipeline using both our line-of-sight marginalization method and assuming a fixed peculiar velocity correction as is usually used. The velocity field used for correction is the linear velocity field calculated from the true density field of the simulation. In order to test for any bias, we compare a shift in the inferred \( H_0 \) value defined as

\[
\text{shift} = \frac{H_0^{\text{mean}} - H_0^{\text{true}}}{\sigma_{H_0}}.
\]

The distribution of this shift for both the methods is plotted in Fig. C1. For an unbiased inference of \( H_0 \), we expect the shift to be centred around 0. With the 100 mock surveys, we find that the mean of the shift for using as fixed peculiar velocity survey is \(-0.353 \pm 0.089\), while the mean for using the line-of-sight marginalization is \(-0.084 \pm 0.096\). Thus, when using a fixed peculiar velocity correction, we observe a bias in the inference of \( H_0 \) that is small compared to the scatter in \( H_0 \) \((\sim 0.35 \sigma_{H_0})\), but is statistically significant at \( \sim 4 \sigma \), where \( \sigma \) is the error in the mean. But when marginalizing over the line-of-sight peculiar velocity, we do not get any significant bias. Furthermore, the standard deviation of the shift in \( H_0 \) is 0.956 for using the line-of-sight marginalization method and 0.890 for using a fixed peculiar velocity correction. We note that this conclusion may be different depending on the exact details of the survey and needs further study.

We also compared the shift in \( H_0 \) using the two methods for each simulation to test for systematic differences in the inferred value of \( H_0 \) using the same simulations. In Fig. C2, we plot the shift using a fixed peculiar velocity correction versus the shift using the LOS marginalization method for each of the 100 simulation. We see that in general the LOS marginalization gives higher value of \( H_0 \). More precisely, for 75 mock surveys, the value of \( H_0 \) inferred using LOS marginalization is higher than using a fixed peculiar velocity correction.

\[
\text{Figure C1.} \text{ Distribution of the } H_0 \text{ shift in the mock surveys using the line-of-sight marginalization method and the fixed peculiar velocity correction. A normal distribution is shown for comparison. We see a small but significant (0.35\sigma \pm 0.09\sigma) bias in the inferred value of } H_0 \text{ when using using a fixed peculiar velocity correction, while we see no significant bias when we marginalize over the line-of-sight peculiar velocities.}
\]

Table C2. Same as Table B1, but with a fiducial smoothing length of 16 h^{-1} Mpc. The scaling factor for 16 h^{-1} Mpc smoothing is 1.16.

| Test set       | Redshift selection | \( \ln \left( \frac{P_{\text{mean}}}{P_{\text{true}}} \right) \) | \( \ln \left( \frac{P_{\text{mean}}}{P_{\text{SuperTF}}} \right) \) | \( \ln \left( \frac{P_{\text{mean}}}{P_{\text{2M++}}} \right) \) | \( \ln \left( \frac{P_{\text{mean}}}{P_{\text{SuperTF}}} \right) \) | \( N_{\text{tracers}} \) |
|----------------|-------------------|------------------|------------------|------------------|------------------|------------------|
| A2-South       | \( c_2 < 3000 \text{ km s}^{-1} \) | 2.39             | 2.61             | 3.01             | 3.11             | 23               |
|                | \( c_2 < 4500 \text{ km s}^{-1} \) | 4.37             | 4.48             | 6.19             | 6.47             | 42               |
|                | \( c_2 < 6000 \text{ km s}^{-1} \) | 6.18             | 6.33             | 9.70             | 10.08            | 66               |
|                | \( c_2 < 9000 \text{ km s}^{-1} \) | 8.01             | 8.05             | 12.83            | 12.83            | 94               |
| A2-low-\( c \) | \( c_2 < 3000 \text{ km s}^{-1} \) | –                | –                | 9.96             | 9.56             | 49               |
|                | \( c_2 < 4500 \text{ km s}^{-1} \) | –                | –                | 17.50            | 16.80            | 92               |
|                | \( c_2 < 6000 \text{ km s}^{-1} \) | –                | –                | 42.78            | 42.42            | 168              |
|                | \( c_2 < 9000 \text{ km s}^{-1} \) | –                | –                | 69.66            | 68.81            | 310              |
| 2MTF           | \( c_2 < 3000 \text{ km s}^{-1} \) | 20.81            | 18.51            | –                | –                | 118              |
|                | \( c_2 < 4500 \text{ km s}^{-1} \) | 45.63            | 39.00            | –                | –                | 282              |
|                | \( c_2 < 6000 \text{ km s}^{-1} \) | 59.87            | 51.25            | –                | –                | 443              |
|                | \( c_2 < 9000 \text{ km s}^{-1} \) | 68.19            | 59.85            | –                | –                | 563              |
| SFI++ Groups   | \( c_2 < 3000 \text{ km s}^{-1} \) | 8.36             | 7.18             | –                | –                | 70               |
|                | \( c_2 < 4500 \text{ km s}^{-1} \) | 7.22             | 5.17             | –                | –                | 119              |
|                | \( c_2 < 6000 \text{ km s}^{-1} \) | 10.19            | 7.56             | –                | –                | 198              |
|                | \( c_2 < 9000 \text{ km s}^{-1} \) | 9.99             | 7.33             | –                | –                | 203              |
| SFI++ Field    | \( c_2 < 3000 \text{ km s}^{-1} \) | 8.93             | 9.32             | –                | –                | 75               |
|                | \( c_2 < 4500 \text{ km s}^{-1} \) | 5.16             | 4.58             | –                | –                | 180              |
|                | \( c_2 < 6000 \text{ km s}^{-1} \) | 9.40             | 7.14             | –                | –                | 450              |
|                | \( c_2 < 9000 \text{ km s}^{-1} \) | 10.87            | 10.86            | –                | –                | 863              |
Figure C2. Shift in $H_0$ for each simulation using a fixed peculiar velocity correction versus the shift in $H_0$ using the LOS marginalization method.

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