UNIVERSALITY OF FREQUENCY AND FIELD SCALING OF THE CONDUCTIVITY MEASURED BY AC-SUSCEPTIBILITY OF A YBCO-FILM

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I. INTRODUCTION

The widely observed scaling feature of the electrical conductivity in the presence of a transverse magnetic field \( B \) and the universality of the critical exponents obtained in such scaling analyses on thin \( \text{YBa}_2\text{Cu}_3\text{O}_7 \)-films serve as the best evidences for the existence of a continuous thermodynamic phase transition to a genuine superconducting phase occuring at some transition line \( T_g(B) \). Considering the interplay between the elastic properties of the vortex lattice and the disordering potential exerted on the vortices by density fluctuations of point-like pinning centers on a microscopic level, the possibility of a glassy vortex state with infinite barriers for vortex motion, \( U(j \to 0) \) and, hence, zero linear resistance, \( \rho(j \to 0) = 0 \), was argued previously for 3-dimensional lattices (see e.g. Ref. 12 and references therein). However, a theoretical proof for the hypothesized second order phase transition to this vortex glass (VG) which is characterized by both a singular correlation length and diverging relaxation time of the order parameter fluctuations,

\[
\xi_g(T, B) = \xi_g(B)|1 - T/T_g|^{-\nu} \quad (1a)
\]

\[
\tau_g(T, B) \simeq \tau_g(B)|1 - T/T_g|^{-\nu z} \quad (1b)
\]

is still lacking. Hence, also the analogy between the transition to the Meissner-phase at \( B = 0 \) (relaxational 3d-XY-model) and to the VG-phase at \( B \neq 0 \) defining the scaling property of the conductivity does not have a firm basis yet. Within this model, magnitude and phase angle are predicted to obey the following forms:

\[
|\sigma(\omega)| = \frac{\tau_g}{\xi_g} S_\pm(\omega \tau), \quad (2a)
\]

\[
\arctan \left( \frac{\sigma''}{\sigma'} \right) = P_\pm(\omega \tau), \quad (2b)
\]

where \( S_\pm(x) \) and \( P_\pm(x) \) represent homogeneous scaling functions above and below \( T_g \), which describe the change from pure ohmic behaviour (\( \sigma'' = 0 \)) for \( T \gg T_g \) to pure screening (\( \sigma' = 0 \)) at \( T \ll T_g \). While their limiting behaviours are known, the detailed shapes of these scaling functions have not yet been worked out.

This somewhat unsatisfactory theoretical situation contrasts to the ample evidence for scaling and universality obtained on thin YBCO-films. Most of these analyses were based on the nonlinear dc-conductivity \( \sigma(j) \) a recent one on the non-linear Hall resistance, and in two studies the linear dynamic conductivity at \( T_g \) has been investigated. Within the given error margins, almost all of the published critical exponents can be summarized by the r.m.s. averages \( \nu = 1.7(1) \) and \( z = 5.5(5) \). This represents a nice example for universality, since most films have been prepared under different conditions and do deviate from each other in microscopic composition and structure, normal resistivity, transition temperature and width. Moreover, the exponents appear to be insensitive against variation of the vortex density \( B/\Phi_0 \) and direction of the field with respect to the c-axis.

On the other hand, notable exceptions from this universality have been seen. Dekker et al. observed a crossover to two-dimensional-critical behaviour when the film thickness is reduced to about 16 \( \text{Å} \), i.e. to the thermal correlation length \( \xi_T \). I-V data from microbridges \((80 \times 10\mu\text{m}^2)\) patterned from epitaxial YBCO-films have been interpreted in terms of thermal activation across Josephson-barriers using the Ambegaokar-Halperin model. For YBCO-crystals distinctly different exponents have been reported (see e.g. Ref. 17) indicating the possibility of another universality class for the transition to true superconductivity due to the presence of a twin boundary network.

This paper reports a first detailed experimental study of the scaling behaviour of the linear dynamic conductivity \( \sigma(\omega) \) of a thin YBCO-film in a wide frequency range
range covering more than ten decades from 3 mHz to 50 MHz. This compares to the existing measurements of $\sigma(\omega)$ on films between 0.5 MHz and 500 MHz, which, moreover, focused only on the critical behaviour of $\sigma(\omega)$ at $T = T_g$. Using experimentally well defined scaling functions for $\sigma(\omega)$, we will also examine the not yet explained effect of the magnetic field on the vortex-glass scaling, at least on an empirical level. Recently, within the mean field approximation, the scaling behaviour of $\sigma(\omega)$ has been studied on the basis of the Edwards-Anderson model.

Our experimental method employs a contact-free access to the dynamic conductivity of a thin film. We measure the linear dynamic susceptibility $\chi(\omega)$ parallel to both the applied dc-field and the c-axis, which is normal to the film plane (see section II). Using an exact inversion scheme outlined in section III, we then evaluate from $\chi(\omega)$ the complex linear conductivity $\sigma(\omega)$ in the film plane. Section IV presents the scaling analysis of $\sigma(\omega)$ and determines the relevant results, i.e. the critical exponents, the glass line, and the field effect on the scaling functions. The results are discussed in section V.

II. EXPERIMENTAL

The 250 nm thick disk (R = 1.5 mm) under investigation has been cut from a film, epitaxially grown parallel to the (001)-substrate of a SrTiO$_3$-substrate by pulsed laser deposition being described in detail elsewhere. AFM images show a rather flat surface from which some isolated hills of 1 $\mu$m average diameter and up to 0.2 $\mu$m height grew up. The dc-resistance at 100 K is about 100 $\Omega$cm and the transition temperature is $T_c = 91.4$ K with a 10:90% width of less than 1 K. From the non-linear ac-susceptibility critical current densities of $2 \times 10^9$ A/cm$^2$ and $10^7$ A/cm$^2$ were obtained at 77 K and 20 K, respectively.

In order to measure the linear susceptibility, the sample, a thin circular disk (R = 1.5 mm) was inserted in well balanced mutual inductances, the excitation amplitude $\mu$ increasing up to 12 hours. In the entire frequency and temperature range of interest here, the complex output of the mutual inducances has been carefully calibrated against the known signals of para- and ferromagnetic samples (Pd, EuO, GdCl$_3$·7H$_2$O) of the appropriate volumes.

III. AC-CONDUCTIVITY

To determine the dynamic susceptibility $\chi(\omega) = (1/2V h_{ac}) \int r \times j(r, \omega) d^3r$ of a thin disk of thickness $d$ in a perpendicular ac field $h_{ac}e^{i\omega t}$ one has to solve an integral equation which describes nonlocal diffusion of the normalized in-plane sheet current density $J(r, z) = j(r, z)/h_{ac}$. In the following, complete penetration $|\delta| > d$ is assumed, equivalent to $\omega < 2/\mu_0 d^2|\sigma_{ac}|$ with the skin depth $\delta = (2/\mu_0\omega\sigma)^{1/2}$. This integral equation follows by inserting the nonlocal relation between $J(r, z)$ and the perpendicular field $h_{ac}(r, z)$ it generates (Ampère's law) into the induction law $\nabla \times e = -b$ using the appropriate material laws $j = \sigma_{ac}e$ and $b = \mu_0 h$. For a disk of radius $R (r \leq R)$ this integral equation reads

$$J(r, \omega) = w \left[ \pi r + \int_0^{1} Q(r, u) J(u, \omega) du \right],$$

where

$$w = \frac{i\omega R \delta h_0}{2\pi} \sigma_{ac}(\omega).$$

In the Ohmic case, $\sigma_{ac} = \sigma$ is real, thus $w$ is purely imaginary, whereas in the Meissner state or for rigid pinning, $\sigma_{ac}$ is purely imaginary and thus $w$ is real and frequency-independent. The integral kernel $Q$(4)

$$Q(r, u) = -\frac{1}{r} \int_0^{r/u} \left[ \frac{E(k)}{1-v} + \frac{K(k)}{1+v} \right] v \, dv.$$

Here $E(k)$ and $K(k)$ are complete elliptic integrals and $k^2 = 4v/(1+v)^2$. Expanding $J(r, \omega)$ in terms of the eigenfunctions $f_n(r)$ (Fig. 2) of the eigenvalue problem

$$f_n(r) = -A_n \int_0^{1} Q(r, u) f_n(u) du,$$

one gets for the planar current density $J(r, \omega) = \sum_n a_n(\omega) f_n(r)$ with

$$a_n(\omega) = \frac{\pi w b_n}{1+w/A_n}, \quad b_n = \int_0^{1} f_n(r) r^2 \, dr,$$

yielding the susceptibility

$$\chi(\omega) = \pi \sum_n a_n(\omega) b_n.$$
In order to invert the measured ac-susceptibilities
\( \chi(\omega) - i\chi''(\omega) \) into ac-conductivities \( \sigma'(\omega) + i\sigma''(\omega) \) by means of the analytical expression (9), we used a numerical inversion (searching) procedure. The choices \( N = 30 \) (Table I) and \( N = 50 \) yielded identical results within the accuracy of our data.

As an example, Fig. 3 displays the phase angle and the modulus of \( \sigma(\omega) \) evaluated from \( \chi(\omega) \) at 0.4T (see Fig.1(a)). Even slightly below \( T_c \) these data exhibit a frequency dependence which cannot be explained within the proposed extended models of thermally activated flux flow (TAFF) [2]. The most striking difference occurs in the phase \( P \), which according to TAFF continuously increases from zero to \( \pi/2 \) when passing \( \omega \approx \tau_0^{-1}\exp(-U/T) \) and then decreases in the same manner to zero above the much higher frequency \( \omega \approx \tau_0^{-1} \) due to losses originating from the vortex motion within their pinning wells. This is in stark contrast to the present behaviour (Fig. 3) where, with increasing temperature, the increase of the phase \( P \) (\( dP/d\omega > 0 \)) comes to a halt below \( P = \pi/2 \) and is followed by a region with \( dP/d\omega < 0 \) towards \( P = \pi/2 \). Hence our data indicate the onset of collective effects in the fluid vortex phase near \( T_c \), which are not included in the TAFF model.

IV. SCALING ANALYSES

When comparing Figs. 1 and 3, the interesting feature emerges that \( \chi(\omega) \) does not display any direct signal of a thermodynamic phase transition, whereas the phase \( \arctan(\sigma''/\sigma') \) of the ac-conductivity provides strong evidence for such a transition: its frequency derivative changes sign at some finite value \( P \) below \( \pi/2 \) demanded by the continuity of the scaling function \( P_\pm(x) \) (Eq. (2b)) at \( T_c \). The physical reason behind this characteristic feature can be associated with the coexistence of infinitely large clusters \( (\xi \rightarrow \infty) \) of superconducting glassy and a metallic (liquid) vortex phase at \( T_g \). With increasing temperature, the life time and size of the glassy fluctuations decrease so that within larger observation times \( 1/\omega \) they cannot contribute to the screening component \( \sigma'' \), whereas towards lower temperatures the reverse is true: within increasing \( 1/\omega \) the lossy contributions \( \sigma' \) arising from vortex loop excitations die out at the expense of the stable glassy phase.

Basing on this signature for the continuous phase transition at \( T_g \), the next step is to check the scaling property predicted for the phase angle (Eq. (2b)) in terms of the variable \( \omega T \sim \omega|1-T/T_g|^{-\nu z} \) by a proper adjustment of the exponent \( \nu z \). In fact, Fig. 4(a) shows that by taking \( \nu z = 9.35 \) the data scale very nicely over more than 14 decades in the scaled frequency \( \omega T \). Together with the value \( z = 5.5(5) \) following from the phase angle at the transition \( P(T_g) = (1 - 1/z)\pi/2 \) also the exponent \( \nu \) is determined, \( \nu = 1.7(1) \). The 4T-data also shown in Fig. 4(a) prove that both exponents are independent of
the magnetic field, at least between 0.4 T and 4 T studied here. Moreover, these results are identical with the averages for \( n \) and \( z \), defined by the numerous scaling analyses of the nonlinear conductivity of thin YBCO-films as outlined in the introduction. This constitutes an additional independent confirmation of the universality feature for YBCO films. Note that this transition from \textquoteleft metallic\textquoteright flux-flow behaviour (\( P = 0 \)) to true superconductivity (\( P = \pi/2 \)) occurs in rather narrow temperature intervals. For 0.4 T and 4 T, we find the \( \sigma(\omega)\)-data from the intervals \( \pm 0.9 \) K and \( \pm 3.0 \) K around \( T_g \) to collapse on the scaling functions. These intervals correspond to reduced temperatures \( |1 - T/T_g| \) smaller than \( 10^{-2} \) and \( 3 \cdot 10^{-2} \) as expected for a critical phenomenon. Interestingly, they are in almost quantitative agreement with the critical regions quoted by Reed et al.\(^{12}\) on a twinned crystal.

Additional support for the scaling property of \( \sigma(\omega) \) arises from the modulus of the conductivity shown in Fig. 4(b) for the same two vortex densities. Here the natural scaling variable is the dc-conductivity above \( T_g \), \( \sigma_g(0) \sim \tau_g/\xi_g \sim |1 - T/T_g|^{-\nu(z-1)} \). Due to the validity of the Kramers-Kronig relations for \( \sigma(\omega) \), no additional parameter is required to determine the scaling according to Eq. (2a). Using essentially the same glass temperatures and critical exponents, excellent scaling of all data from the indicated temperature intervals above and below \( T_g \) is found. Obviously, they collapse on two branches defining the scaling functions \( S_\pm(x) \). Again, their limiting cases \( S_+(0) \) and \( S_-(0) \) represent the pure lossy \(|\sigma| = \sigma'\) and pure screening \(|\sigma| \sim \sigma'' \sim \omega^{-1}\) responses, while at \( T_g \) one finds the algebraic behaviour \(|\sigma| \sim \omega^{-1+z} \) predicted for the dynamical criticality. Principally, these results for \( |\sigma| \) can be also considered as a successful check of the measuring and new inversion procedures presented here.

The inset to Fig. 5(b) shows the phase diagram defined by these scaling analyses. The transition temperatures define the glass line separating the normal conducting vortex fluid from the superconducting vortex glass, which can be described by the power law:

\[
B_g(T) = B_g(0) \left( 1 - \frac{T}{T_c} \right)^\beta,
\]

where \( \beta = 1.5(2) \) and \( B_g(0) = 85(5)\) T.

Looking at the scaling functions \( S_\pm \) and \( P_\pm \) exemplified in Fig. 4 for 0.4 T and 4 T, one notes that their shapes appear rather similar to each other. This strongly suggests to perform an additional scaling procedure in a phenomenological manner. We simply assign suitable powers of the applied field \( B \) to the scaled frequency and conductivity. In fact, Fig. 5 demonstrates nicely that for both, phase and modulus of \( \sigma(\omega) \), all experimental scaling functions fall on single curves over 15 decades of the scaled frequency if we assume that the lifetime \( \tau_g \) of the glassy fluctuations increases with \( B^{5.5(1.0)} \) and the dc-conductivity, \( \sigma(0) \sim \tau_g/\xi_g \) (Eq. (2a)), grows proportional to \( B^{4.5(0.5)} \). Interestingly, both results can be related to the same physical origin, namely to the field dependence of the vortex-glass correlation length. We obtain the field dependence of the principal scaling variable in the following form

\[
\xi_g(T, B) = \xi_g(B_0) \left( \frac{(B/B_0)^\gamma}{(T/T_g)^\nu} \right),
\]

where \( \gamma = 0.59(8) \) and \( B_0 \) denotes an arbitrary normalization field. This finding implies that the shape of the scaling functions does not depend explicitly on the magnetic field, but that the field effect on the scaling can be fully accounted for by the \( B \) dependence of the glass correlation length \( \xi_g(T, B) \). Moreover, if we follow Ref. 17 to express the field dependence of \( \xi_g \) in terms of the distance between \( T_g \) and \( T_{c2} \), i.e. \( |T_{c2} - T_g| \sim T_g B^{\beta'} \), with \( \beta' = 1.6(2) \) for the present glass line, we find within the given error margins, \( \xi_g^{1/\nu}(T, B) \sim (T_{c2} - T_g)/(T_{c2} - T_g) \).

This is fully consistent with the previous result\(^{22}\) obtained on a single crystal.

V. DISCUSSION

The main information obtained from scaling analyses are the critical exponents, which are determined by the rather general features of the phase transition, i.e. the spatial dimensionality, the symmetry of the order-parameter, and the type of disorder. The present values for the exponents of the correlation length and the relaxation time, \( \nu = 1.7(1) \) and \( z = 5.5(5) \), respectively, agree nicely with most of the existing values determined for YBCO-films\(^{22}\) from non-linear I-V characteristics. On the experimental side, this provides a strong support for the ac-susceptibility technique and the evaluation procedure applied here for the first time on a superconducting thin film. For bulk YBCO-crystals such an exact transformation of the complex susceptibility to the dynamic conductivity using conventional schemes has been performed recently\(^{22}\), while another recent scaling analysis was performed for the modulus of \( \chi(\omega) \) resting on the assumption \(|\chi(\omega)| \approx a/\lambda_{ac}(\omega)\). Of course, this assumption is only valid for \( |\lambda_{ac}| \) being large compared to the sample dimension and should be limited to small susceptibilities, i.e. to just the onset of screening.

The present values of the exponents \( \nu \) and \( z \) are significantly larger than the mean-field values\(^{22}\) for the vortex-glass (VG) \( \nu = 0.5 \) and \( z = 4 \), indicating the presence of strong critical fluctuations. For the case of a 3-dimensional isotropic VG one expects from analogy with spin-glasses\(^{22}\) larger numbers, e.g. \( z = 2(2 - \eta) \) with \( \eta < 1 \). However, exact values, are not known yet. Monte-Carlo simulations of the gauge-glass model revealed \( \nu = 1.3(4) \) and \( z = 4.7(7) \)\(^{22}\), but the applicability to the vortex state in real disordered materials like YBCO-films has not been proven to date.
On the other hand, the results $\nu = z = 3.1(3)$ from scaling analyses of $\sigma(\omega)$ for $\vec{B} \parallel \vec{c}$ of twinned YBCO-crystals indicate the existence of a distinctly different universality class for the transition to a superconducting vortex state. It has been suggested to associate this class with correlated pinning along the boundaries of twin colonies ($\parallel \vec{c}$), where single twin planes terminate. In this case of anisotropic pinning, one has to distinguish between two correlation lengths $\xi_1$ and $\xi_\perp$ of glassy fluctuations parallel and perpendicular to the preferred $c$-axis. Then the measured exponents can be explained by the assumption $\xi_1 \sim \xi_\perp^2$. These anisotropic fluctuations are considered by the so-called boson-glass model for a vortex-state with columnar pinning, for which by analogy with the dirty boson problem exists now good evidence for a phase-transition to genuine superconductivity at a finite temperature $T_{b0}(B)$.

Similar evidence is still lacking for the isotropic VG-model, where the vortices are pinned by fluctuations of pointlike defects of extension $r_0 \ll \xi_T$. Interestingly for the YBCO-crystal, the isotropic values of the exponents $\nu$ and $z$ have been recovered for $\vec{B} \perp \vec{c}$, i.e. for fields perpendicular to the columnar pinning centers, which means that these line defects are ineffective in this geometry. In contrast, when passing from $\vec{B} \perp \vec{c}$ to $\vec{B} \parallel \vec{c}$ for thin YBCO-films, no change of the isotropic exponents occurs $\xi_\perp$ which reveals an absence of columnar pinning. For our film such a conclusion for isotropic pinning is supported by the fact that twin colonies could not be detected down to a length-scale of 0.8 $\mu$m.

Another universal property of VG seems to be the effect of the magnetic field on the transition. The field dependence of the glass-line could frequently be explained by the power law, Eq. (7), with exponents similar to the one obtained here, $\beta = 1.5(2)$. Usually, $\beta = 2r_0$ is related to the exponent of the thermal correlation length of the homogeneous state, $\xi_T \sim (1 - T/T_g)^{-r_0}$. On the other hand, a detailed understanding of the non-universal amplitude $\Phi_0(B_0)$, i.e. its dependence on the pinning mechanisms has not yet been achieved.

Much less evidence exists for the field dependence of the dynamic scaling functions $S_k$ and $P_2$ themselves. Our analysis between 0.4T and 4T, shown in Fig. 5, suggests that their shape is not influenced by the vortex density, but that the entire effect on the dynamic conductivity near $T_g$ can be accounted for by that of the VG correlation length. Our result, Eq. (8), implies, that the amplitudes in (Eq. (1)), $\xi_\parallel(B)$ and $\gamma_\parallel(B)$, increase approximately linearly with $B$ (due to $\gamma_\parallel \approx 1.0(2)$) and with $B^2$, respectively. Because this finding agrees with that of our previous study on an YBCO single crystal for both orientations of the applied field, it appears to be another rather general property of the VG-transition. The most obvious consequence of $\xi_\parallel \sim B$ is that below $T_g$ the squared screening length $\lambda^2_s$ defined at low frequencies increases linearly with $B$:

$$\sigma(\omega \to 0, T < T_g, B) = \frac{i}{\mu_0 \omega \lambda_s^2},$$

$$\lambda^2_s(T, B) = \frac{\lambda^2_s(B_0)}{(1 - T/T_g)^\nu B_0},$$

with $\lambda_s(1T) = 0.7\mu$m.

Let us first note that this behaviour agrees with that of the elastic pinning length, $\lambda^2_e = B\Phi_0/C_p$, introduced by Campbell for a vortex lattice exposed to a pinning energy density $C_p$. In this classical model, the screening length $\lambda_s$ results from the elastic response of the vortex lattice and diverges near $T_{c2}(B)$, where the lattice melts. According to the present results, this melting occurs at $T_g(B)$ and the elastic response is determined by a field-independent pinning energy $C_p$.

When we associate the screening behaviour, Eq. (9), with the VG-fluctuations, as suggested by the scaling property (Fig. 5), then the screening depth $\lambda_s$ is related to the VG correlation length by the form

$$\xi_\parallel(T, B) \approx \frac{\lambda^2_s(T, B)}{\Lambda_T},$$

where $\Lambda_T = \Phi_0/4\pi \mu_0 k_BT$ represents the thermal length. Inserting the experimental $\lambda_s$ we obtain for the amplitude defined by Eq. (1a), $\xi_\parallel(B) \approx 2.5nm \times B/T(T)$. For this field, this value is in the order of the thermal coherence length at zero temperature $\xi_T(0)$ which is a natural lower bound to $\xi_\parallel$ at $T = 0$. This implies that, in the critical region, $0.002 \leq |1 - T/T_g| \leq 0.03$, where scaling is observed here, $\xi_\parallel$ varies from about 1$\mu$m to 100$\mu$m. This range is plausible, since it is located between the vortex-lattice constant, e.g. $a_0 \approx 50nm$ at 1T and the radius of our disk, $R = 1.5$mm, which both set natural bounds for the range of the fluctuations.

Fisher et al. argued that $\xi_\parallel(B)$ may be associated with the collective pinning length introduced by Larkin and Ovchinikov, which results from the competition of the elastic and the pinning energies, $l_p \approx l_i(c_{cl}/c_p)^2$, where $l_i$ is the mean distance between the pinning centers. Taking $\xi_\parallel \approx l_p$ for granted, our result implies $c_{cl}/c_p \approx B^{1/2}$ which is not among the predictions for $l_p$ in the various pinning regimes. Since the theories of collective pinning do not consider critical fluctuations but rather apply to $T \ll T_g$ this discrepancy should not be taken as too serious.

One interesting consequence of our finding $\xi_\parallel \sim B$ is that due to $\tau_\parallel \sim \xi_\parallel^2 \sim B^{5.5}$ the relaxation time of the fluctuations is growing dramatically with field. In particular, this leads to an increase of the dc-limit of the fluctuation conductivity, $\sigma(0) \sim \xi_\parallel^{-1} \sim B^{4.5}$ as evidenced by Fig. 5(b). This is in sharp contrast to the conventional flux-flow behaviour, $\sigma_{df} \sim B^{-1}$, and indicates that even slightly below $T_c$ the dc-limit of $\sigma(\omega)$ in the present YBCO-film is dominated by cooperative effects. Most likely, this is also true for other films with glass lines $T_g(B)$ sufficiently close to $T_c$. 

5
Let us finally recall the physical difference between the vortex dynamics according to the TAFF model and the highly collective VG-model. In the former, single vortices or bundles become increasingly depinned at elevated temperatures, and this is described by a single activation rate $\tau^{-1}(T)$. At fixed frequency, more and more vortex bundles depin within a time $\omega^{-1}$ and generate electric fields, i.e. ac-losses; this implies that the phase angle $P = \arctan(\sigma''/\sigma')$ decreases gradually from $\pi/2$ to zero with increasing $\omega \tau$. In the VG picture, on the other hand, a flow of vortex bundles of extremely long relaxation rates, $\tau_0 \sim \xi_0^2$, the phase angle $P$ assumes a nontrivial value $1 - 1/(\omega \tau)$, which lies below the superconducting limit $P = \pi/2$. This limit is reached for lower temperatures, where the size $\xi_0(T < T_g)$ of the vortex-liquid clusters, i.e. of the “metallic” regions shrinks. Since their lifetime is then strongly reduced, these clusters do not cause losses at low frequencies $\omega < \tau^{-1}$. It is this process which gives rise to the change of the slope of the phase $dP/d\omega$ at $T_g$ and is not contained in the TAFF model[2]. In the extended TAFF-mode[2], a negative slope of $P(\omega)$ appears at frequencies, where the (viscous) damping of the pinned vortices is reached. In contrast to the VG-transition, the crossover from the low-frequency TAFF-regime ($dP/d\omega > 0$) to the viscous damping one ($dP/d\omega < 0$) is of pure dynamic origin: at finite temperatures, there is no diverging time scale, and as long as the activation energy is large enough, the slope of the phase changes its sign always at $P = \pi/2$, and the high-frequency limit is $P = 0$ as compared to $(1 - z^{-1})\pi/2$ in the VG-model. This dynamic crossover to the London-screening, where $\lambda_s = \lambda_L$, will be a matter of future research in the VG-state.

VI. CONCLUSIONS

In conclusion, we have presented an extensive study of the linear ac-susceptibility measured perpendicular to a thin circular YBCO disk, which qualitatively shows the transition from full penetration to full screening of the ac-field with increasing frequency and decreasing temperature and field. In contrast to previous investigations, which discuss just the shift of the maximum of $\chi''$ in terms of the TAFF or other conventional pinning models, we have evaluated here the full information available from the data for $\chi(\omega, T, B) = \chi' - i\chi''$. This evaluation was achieved by a novel exact inversion routine, which allows to determine the linear dynamic conductivity of the bulk material, $\sigma(\omega)$. The frequency dependence of the phase, $\sigma''/\sigma'$, provides clear evidence for the existence of a continuous phase transition to a truly superconducting state at a well defined temperature $T_g(B)$. Using a step-by-step evaluation of the dynamical scaling proposed for this transition it was possible to extract the critical exponents $\nu$ and $z$, which agree with those obtained on a large number of YBCO-films[2] utilizing the nonlinear dc-conductivity. Since these films differ in their microscopic structure as well as in their transition temperatures, the present agreement obtained with a new experimental method, strongly supports the universality which has been conjectured for a transition to a vortex-glass[2].

The dynamical scaling functions for the linear conductivity determined here for the first time in some detail, perhaps may serve as a challenge to the theory to work out a realistic model of the vortex-glass state and its critical behaviour. As a hint for such an ansatz one may consider the observed field dependence of the coherence length $\xi_0(T, B)$ associated with this transition. Other interesting questions are the crossovers from the critical regime to the London-screening on the low-temperature side and to the flux-flow limit, at $T \gg T_g$.

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TABLE I. Positions of the poles $\Lambda_n$ and amplitudes $c_n$ entering the transverse susceptibility (6) of a circular disk. With these numbers inserted, the finite sum (6) approximates the complex function $\chi$ of the complex argument $w$ with high precision up to $|w| \approx 4000$. These numbers are for $N=30$.

| $n$ | $\Lambda_n$ | $c_n$ |
|----|-------------|--------|
| 1  | 0.8768551  | 0.6355414 |
| 2  | 1.8751352  | 0.2334135 |
| 3  | 2.8725246  | 0.1380313 |
| 4  | 3.8651025  | 0.0965691 |
| 5  | 4.8480195  | 0.0734911 |
| 6  | 5.8147181  | 0.0585680 |
| 7  | 6.7563510  | 0.0479884 |
| 8  | 7.6611613  | 0.0397497 |
| 9  | 8.5126922  | 0.0328696 |
| 10 | 9.2858252  | 0.0264066 |
| 11 | 9.9596476  | 0.0218315 |
| 12 | 10.652940  | 0.0246335 |
| 13 | 11.565066  | 0.0303418 |
| 14 | 12.125054  | 0.0000002 |
| 15 | 12.775746  | 0.0357576 |

FIG. 1. Temperature variation of the dispersion and absorption of the linear magnetic susceptibility measured at fixed frequencies in fields applied parallel to the c-axis of a YBCO-film: (a) $B = 0.4$ T and (b) $B = 4$ T.

FIG. 2. Eigenfunctions $f_n(r)$ of the eigenvalue problem, Eq. (4), determining the normalized current density $J(r, \omega)$ of a circular disk.

FIG. 3. Frequency dependence (a) of the phase angle $P$ and (b) of the modulus of the dynamic conductivity $\sigma(\omega)$ evaluated from the dynamic susceptibility $\chi(\omega)$ measured at 0.4 T. Full lines are drawn as guides to the eye. Note that the limit of thermally activated flux flow, $P = 0$ and $\sigma(\omega) = \text{const.}$, is reached only very close to $T_c$.

FIG. 4. Dynamical scaling of the linear conductivity. (a) Phase angle and (b) modulus of $\sigma$, both showing the crossover from pure ohmic to full screening behaviour at two different magnetic fields.

FIG. 5. Field-effect on the dynamical scaling functions for (a) the phase angle and (b) the modulus of $\sigma(\omega)$. The collapse of data is achieved by introducing appropriate powers of $B$ in the scaling variables. The inset shows the glass-line (Eq. (7)), relative to the mean-field transition $B_{c2}$. 