Light Higgsino from Axion Dark Radiation

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Abstract

The recent observations imply that there is an extra relativistic degree of freedom coined dark radiation. We argue that the QCD axion is a plausible candidate for the dark radiation, not only because of its extremely small mass, but also because in the supersymmetric extension of the Peccei-Quinn mechanism the saxion tends to dominate the Universe and decays into axions with a sizable branching fraction. We show that the Higgsino mixing parameter $\mu$ is bounded from above when the axions produced at the saxion decays constitute the dark radiation: $\mu \lesssim 300$ GeV for a saxion lighter than $2m_W$, and $\mu$ less than the saxion mass otherwise. Interestingly, the Higgsino can be light enough to be within the reach of LHC and/or ILC even when the other superparticles are heavy with mass about 1 TeV or higher. We also estimate the abundance of axino produced by the decays of Higgsino and saxion.

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I. INTRODUCTION

The present Universe is dominated by the dark sector, i.e., dark matter and dark energy, although it is not yet known what they are made of. Therefore, it may not be so surprising if there is another dark component, which behaves like radiation.

The presence of additional relativistic particles increases the expansion rate of the Universe, which affects the cosmic microwave background (CMB) as well as the big bang nucleosynthesis (BBN) yield of light elements, especially $^4\text{He}$. The amount of the relativistic particles is expressed in terms of the effective number of light fermion species, $N_{\text{eff}}$, and it is given by $N_{\text{eff}} \approx 3.046$ for the standard model. Therefore, if $N_{\text{eff}} > 3$ is confirmed by observation, it would immediately call for new physics.

Interestingly, there is accumulating evidence for the existence of additional relativistic degrees of freedom. The latest analysis using the CMB data (WMAP7 [1] and SPT [2]) has given $N_{\text{eff}} = 3.86 \pm 0.42$ (1$\sigma$ C.L.) [3]. Other recent analysis can be found in Refs. [1, 2, 4–8]. The $^4\text{He}$ mass fraction $Y_p$ is sensitive to the expansion rate of the Universe during the BBN epoch\(^1\), although it has somewhat checkered history since it is very difficult to estimate systematic errors for deriving the primordial abundance from $^4\text{He}$ observations [16]. Nevertheless, it is interesting that an excess of $Y_p$ at the 2$\sigma$ level, $Y_p = 0.2565 \pm 0.0010 \text{ (stat)} \pm 0.0050 \text{ (syst)}$, was reported in Ref. [17], which can be understood in terms of the effective number of neutrinos, $N_{\text{eff}} = 3.68^{+0.80}_{-0.70}$ (2$\sigma$).\(^2\) Interestingly, it was recently pointed out that the observed deuterium abundance $\text{D/H}$ also favors the presence of extra radiation [19, 20]: $N_{\text{eff}} = 3.90 \pm 0.44$ (1$\sigma$) was derived from the CMB and $\text{D/H}$ data [20]. It is intriguing that the CMB data as well as the Helium and Deuterium abundance favor additional relativistic species, $\Delta N_{\text{eff}} \sim 1$, while they are sensitive to the expansion rate of the Universe at vastly different times.

The extra radiation may be dark radiation composed of unknown particles. Then it is a puzzle why it is relativistic at the recombination epoch, why the abundance is given by $\Delta N_{\text{eff}} \sim 1$, and why it has very weak interactions with the standard-model particles.

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\(^1\) $Y_p$ is also sensitive to large lepton asymmetry, especially of the electron type, if any [9, 15].

\(^2\) The authors of Ref. [18] estimated the primordial Helium abundance with an unrestricted Monte Carlo taking account of all systematic corrections and obtained $Y_p = 0.2561 \pm 0.0108 \text{ (68\%CL)}$, which is in broad agreement with the WMAP result.
In fact, there is a well-motivated particle with the desired properties, namely, the QCD axion. The axion appears in the Peccei-Quinn (PQ) mechanism \cite{21}, one of the solutions to the strong CP problem \cite{22}, in association with the spontaneous breakdown of the PQ symmetry. The axion remains extremely light: its mass is originated from the QCD anomaly and is in the range of $10^{-5}$ eV $\lesssim m_a \lesssim 10^{-3}$ eV for the PQ breaking scale in the cosmological window, $F_a = 10^{10}$ GeV $\sim 10^{12}$ GeV. Furthermore, in a supersymmetric (SUSY) framework, the saxion tends to dominate the energy density of the Universe, and it decays mainly into a pair of axions. Such non-thermally produced axions naturally remain relativistic until present. Thus, the axion is a plausible candidate for the dark radiation.$^3$

In this paper we consider the QCD axion as a candidate for the dark radiation, and show that the branching fraction of the saxion into axions naturally falls in the right range, if the axion multiplet is coupled to the Higgs superfields as in the Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) axion model \cite{31,32}. Most important, we find that the $\mu$-parameter is bounded from above, namely $\mu \lesssim 300$ GeV for the saxion lighter than $2m_H$, and $\mu$ less than the saxion mass otherwise, independent of the PQ scale $F_a$. As we shall see later, if the radiative corrections play an important role in the stabilization of the saxion, its mass can be naturally smaller than the soft masses for the SUSY standard model (SSM) particles. This implies that, even if the other SUSY particles are so heavy that they are above the reach of the Large Hadron Collider (LHC), the Higgsino should remain within the reach of the LHC and/or future collider experiments such as International Linear Collider (ILC). This will be of great importance especially if the mass of the SM-like Higgs boson is indeed around $124 - 126$ GeV as suggested by the recent data from ATLAS and CMS at the LHC \cite{33}.

We also estimate the axino production from the decay of both Higgsino and saxion. In our scenario, the dark matter is made of the axion in the form of non-relativistic coherent field oscillations and the axino lightest SUSY particle (LSP), while the axion produced by the saxion decay accounts for the dark radiation. Thus, the axion and its superpartners play an important role to account for the dark matter and dark radiation.

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$^3$ The abundance of relativistic axions produced by flaton decays was studied in detail in Ref. \cite{23}. The late-time increase of $N_{\text{eff}}$ by decaying particles (e.g. saxion to two axions, and gravitino to axion and axino) was studied in Ref. \cite{24}. The light gravitinos produced by inflaton decay can account for dark radiation in analogous to Ref. \cite{25}. See also recent Refs. \cite{26-29}. The possibility that the $X$ particle was in thermal equilibrium was studied in Ref. \cite{30}.
II. PQ EXTENSION OF THE MSSM

A. Saxion Properties

Let us begin by examining the properties of the saxion in a simple PQ extension of the minimal SSM (MSSM) where the PQ sector consists of the axion super field \( S \) and \( N_\Psi \) pairs of PQ messengers \( \Psi + \bar{\Psi} \) forming \( 5 + \bar{5} \) representation of SU(5):

\[
W_{PQ} = y_\Psi S \Psi \bar{\Psi}.
\]  

Since the saxion is a flat direction in the supersymmetric limit, its properties are determined by how SUSY breaking is transmitted to the PQ sector. It is natural to expect that the PQ messengers, which are charged under the SM gauge groups, feel SUSY breaking in the same way as the MSSM superfields do.

The PQ messengers radiatively generate a potential for the saxion after SUSY breaking \([34]\), as well as an effective coupling of \( S \) to the gluon supermultiplet that implements the PQ mechanism. To see this, one can integrate out \( \Psi + \bar{\Psi} \) under a large background value of \( S \), which results in

\[
\mathcal{L}_{\text{eff}} = \int d^4 \theta Z_S(Q = y_\Psi |S|) |S|^2.
\]  

There then arises a potential, \( V_{\text{rad}} = m_S^2(Q = y_\Psi |S|) |S|^2 \). Here \( m_S^2 \) is the soft scalar mass squared of \( S \), and \( Q \) is the renormalization scale. Hence, if \( m_S^2 \) is positive at a high scale and the messenger Yukawa coupling is large enough to drive it negative through radiative correction, \( V_{\text{rad}} \) develops a minimum along \( |S| \) around the scale where \( m_S^2 \) changes its sign.

At the minimum, the Kähler potential \([2]\) gives masses to the saxion and axino

\[
m_\sigma^2 = \frac{1}{8\pi^2} \sum_\Psi y_\Psi^2 (m_\Psi^2 + m_{\bar{\Psi}}^2 + A_\Psi^2) \sim \frac{5N_\Psi y_\Psi^2}{8\pi^2} m_{\text{soft}}^2,
\]

\[
m_{\tilde{a}} = \frac{1}{8\pi^2} \sum_\Psi y_\Psi^2 A_\Psi \sim \frac{5N_\Psi y_\Psi^2}{8\pi^2} A_\Psi,
\]  

where \( m_\sigma^2 \) is a soft scalar mass squared, and \( A_\Psi \) is the trilinear soft parameter associated with \( y_\Psi \). The saxion acquires a mass suppressed by a factor \( \epsilon \sim \sqrt{5N_\Psi/8\pi^2} y_\Psi \) relative to other superparticle masses \( m_{\text{soft}} \), and the axino has a small mass \( m_{\tilde{a}} \sim \epsilon^2 A_\Psi \). From the effective Kähler potential, one can also find the saxion couplings

\[
\frac{\sigma}{\sqrt{2c_\alpha F_\alpha}} \left( (\partial^\mu a)\partial_\mu a + \frac{1}{2}(\lambda_{\tilde{a}} m_{\tilde{a}} a + \text{h.c.}) \right),
\]  

where
where \( F_a = c_a^{-1} \langle |S| \rangle \) is the axion decay constant. The model-dependent parameter \( c_a \) is generally of order unity, and the saxion coupling to the axino

\[
\lambda_a = \left. \frac{d \ln(y^2 \Psi A)}{d \ln Q} \right|_{Q = c_a F_a}
\]

generically has a value in the range between \( 10^{-1} \) and \( 10^{-2} \).

On the other hand, depending on the mediation mechanism of SUSY breaking, \( V_{\text{rad}} \) alone may not be able to stabilize \( S \). This happens, for instance, in gauge mediation, where only those carrying SM gauge charges acquire soft SUSY breaking masses at the messenger scale \( M_{\text{mess}} \). An interesting way to give a soft mass to the gauge singlet \( S \) at \( M_{\text{mess}} \) is to consider mixing between \( \Psi \) (or \( \bar{\Psi} \)) and the messenger fields transferring SUSY breaking \[^{[35]}\]. Then, there is an additional contribution to soft masses for the PQ sector fields, while the MSSM sector feels SUSY breaking only through the gauge mediation. The additional contribution makes \( V_{\text{rad}} \) develop a minimum in the same way as discussed above. Turning off the mixing, the saxion potential runs away to infinity, but can be lifted by supergravity effects \( \Delta V = \xi m_{3/2}^2 |S|^2 \) where \( m_{3/2} \) is the gravitino mass. For a positive \( \xi \) of order unity and \( m_{3/2} \) not so small compared to \( m_{\text{soft}} \), \( S \) is stabilized below \( M_{\text{mess}} \) by \( V_{\text{rad}} + \Delta V \). The situation is different when \( m_{\text{soft}} \gg m_{3/2} \), for which the supergravity effects become important at \( |S| \) larger than \( M_{\text{mess}} \).\(^4\)

We close this subsection by stressing that the potential generated by PQ messenger loops is naturally expected to play an important role in stabilizing the saxion in models where the PQ symmetry is spontaneously broken mainly by a single PQ field. Then, the saxion and axino are generally lighter than the MSSM superparticles.

**B. Saxion Cosmology**

Let us move to the cosmology of the saxion. The saxion has a very flat potential lifted only by the SUSY breaking effect, and so, the saxion potential may be significantly modified during inflation, because SUSY is largely broken by the inflaton potential energy. In

\(^4\) In this case, a minimum appears at a scale above \( M_{\text{mess}} \), where \( \Psi + \bar{\Psi} \) generate a potential for the saxion at the three-loop level \[^{[36]}\]. This results in that the saxion has a mass of the order of \( m_{3/2} \), and the axino acquires a tiny mass \( m_{\tilde{a}} = (\epsilon_1 + \epsilon_2) m_{3/2} \ll m_{3/2} \), where \( \epsilon_1 \sim m_{3/2} / (8\pi^2 m_{\text{soft}}) \) comes from the PQ messenger loops while \( \epsilon_2 \sim F_a^2 / M_{\text{Pl}}^2 \) is due to the supergravity contribution. The coupling \( \lambda_a \) has a value about \( (\epsilon_1 / 8\pi^2 + \epsilon_2) / (\epsilon_1 + \epsilon_2) \), and \( F_a \sim M_{\text{mess}} m_{\text{soft}} / (\sqrt{8\pi^2} m_{3/2}) \).
particular, the so-called Hubble-induced mass term generically deviates the saxion from the low-energy potential minimum.\footnote{This is not the case if there is a certain (approximate) symmetry among the PQ fields.}

For instance, the saxion may be stabilized at a large field value close to the Planck scale in the presence of the negative Hubble-induced mass term. In this case, the saxion starts to oscillate with a large initial amplitude when the Hubble parameter becomes comparable to the saxion mass $m_\sigma$. Then the saxion would dominate the energy density of the Universe, if the reheating temperature of the inflaton is higher than the saxion decay temperature.

Alternatively, the PQ symmetry may be restored during and/or after inflation. In particular, since the PQ messengers generate a thermal potential for the saxion, $V_{\text{thermal}} \sim y_q^2 T^2 |S|^2$ at $|S| \ll T$, the saxion could be thermally trapped at the origin if it sits around the origin after inflation. In this case thermal inflation\footnote{This is not the case if there is a certain (approximate) symmetry among the PQ fields.} takes place when the saxion potential energy at the origin ($\sim m_\sigma^2 F_a^2$) dominates the Universe.

Thus, it is plausible that the saxion dominates the energy density of the Universe. In the rest of the paper, we assume that this is indeed the case, and discuss how the saxion decay proceeds.

C. PQ Solution to the $\mu$ Problem

If $S$ has no other interactions with the MSSM sector than the loop-induced coupling to the gluon supermultiplet, the saxion dominantly decays into axions with a branching ratio $B_a \simeq 1$:

$$ B_a = \frac{\Gamma_{\sigma \rightarrow aa}}{\Gamma_\sigma} = \frac{1}{\Gamma_\sigma} \frac{1}{64\pi} \frac{m_\sigma^3}{(c_a F_a)^2}, $$

where $\Gamma_\sigma$ is the total decay width of the saxion. This is obviously problematic in a scenario where the Universe experiences a saxion-dominated epoch.

A natural way to suppress $B_a$ is to introduce a coupling of $S$ to the Higgs doublets so that a Higgs $\mu$ term is dynamically generated after PQ symmetry breaking. Among various types of couplings, we take the Kim-Nilles superpotential term\footnote{This is not the case if there is a certain (approximate) symmetry among the PQ fields.}

$$ W = \lambda \frac{S^2}{M_{Pl}} H_u H_d, $$
which generates an effective $\mu$ term

$$\mu = \lambda \frac{(c_a F_a)^2}{M_{Pl}},$$

(8)

around the weak scale for $\lambda \lesssim 1$ and $F_a = 10^{10-12}$ GeV, and thus naturally explains the smallness of $\mu$.

The above superpotential term also induces couplings of the saxion to the MSSM particles. These couplings are proportional to $\mu/F_a$, and open up the possibility to obtain $\Delta N_{\text{eff}} \sim 1$ from the axions non-thermally produced by saxion decays.

To evaluate $B_a$, one needs to know interactions of the saxion with the MSSM particles. For simplicity, we consider the decoupling limit where the effective Higgs sector below $m_{\text{soft}}$ consists only of a SM-like Higgs doublet and other heavy Higgs bosons decouple from the theory. Then, the saxion has the interactions

$$\frac{C_\sigma |\mu|^2}{\sqrt{2} c_a F_a} \left(1 - \frac{|B|^2}{m_A^2}\right) \sigma hh + \left(\frac{C_\sigma}{\sqrt{2} c_a F_a} \frac{\mu}{\sqrt{2}} \sigma \tilde{H}_u \tilde{H}_d + \text{h.c.}\right),$$

(9)

both of which are directly induced from (7). Here $h$ is the CP-even neutral Higgs boson, $m_A$ is the mass of the CP-odd neutral Higgs boson, and $B$ is the Higgs mixing soft parameter. The coefficient $C_\sigma = \partial \ln |\mu|/\partial \ln |S| = 2$ reflects the form of the coupling between $S$ and the Higgs doublets. In addition, the saxion interacts with other MSSM particles as well because there arises mixing between the saxion (axino) and neutral Higgs (Higgsinos) after electroweak symmetry breaking. These couplings can be read off from the MSSM Lagrangian by taking the substitution

$$h \rightarrow \frac{2 C_\sigma v}{c_a F_a} \left(1 - \frac{|B|^2}{m_A^2}\right) \frac{|\mu|^2}{m_\sigma^2 - m_\tilde{h}^2} \sigma,$$

(10)

where $\langle |H_u^0| \rangle = v \sin \beta$. It is straightforward to derive the partial decay widths of the saxion into SM particles and Higgsinos, which are presented in the appendix. The important decay channels are those into $hh$, $WW$, $ZZ$, $b\bar{b}$, $t\bar{t}$, and into a Higgsino pair, depending on $m_\sigma$. We note that $B_a$ is determined essentially by $\mu$ and $m_\sigma$, but insensitive to $F_a$. The saxion interacts with the SM particles more strongly for a larger $\mu$, making $B_a$ smaller.

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6 To solve the $\mu$ problem, one may instead consider the Giudice-Masiero mechanism \[39\] implemented by a Kähler potential term $S^* H_u H_d / S$. In this case, however, the couplings of the saxion to the MSSM sector arise at the loop level.
III. AXION DARK RADIATION

In this section, we examine the condition for the axion dark radiation produced at saxion decays to yield $\Delta N_{\text{eff}} \sim 1$ in the presence of the superpotential term \( \langle 7 \rangle \) that is responsible for generating a $\mu$ term. Then, we also examine if the axino can constitute the dark matter of the Universe with the correct amount of a relic density.

Our analysis does not assume any specific mechanism of the saxion stabilization, but the radiative potential induced by the PQ messengers may play an important role in fixing the vacuum expectation value of $S$. Keeping this in mind, we shall consider the case where the saxion and axino have masses lighter than $m_{\text{soft}}$. It should be noted that the Higgsino mixing parameter $\mu$, on which $B_a$ crucially depends, does not need to be related to the scale $m_{\text{soft}}$ because it is a supersymmetric coupling.

A. Axion Dark Radiation from Saxion Decays

The axions produced at the saxion decays have never been in thermal equilibrium for the saxion decay temperature much lower than $F^2_a/M_{\text{Pl}}$ \[40\], which is the case we shall deal with. The energy density of non-thermalized axions and thermalized radiation evolves as

\[
\rho_a(t) \propto a^{-4}(t), \\
\rho_{\text{SM}}(t) \propto g_*^{-1/3}(t)a^{-4}(t),
\]

where $a(t)$ is the scale factor, and $\rho_a : \rho_{\text{SM}} = B_a : (1 - B_a)$ at the saxion decay time $t = t_\sigma$. Because non-thermally produced axions contribute to $N_{\text{eff}}$ with

\[
\Delta N_{\text{eff}} = \frac{\rho_a}{\rho_\nu} \bigg|_{\nu \text{ decouple}} = \frac{\rho_{\text{SM}}(t)}{\rho_\nu(t)} \bigg|_{\nu \text{ decouple}} \times \frac{\rho_a(t)}{\rho_{\text{SM}}(t)} \bigg|_{\nu \text{ decouple}}
\]

\[
= 43 \frac{B_a}{7} \frac{1}{1 - B_a} \left( \frac{43/4}{g_*(T_\sigma)} \right)^{1/3},
\]

$\Delta N_{\text{eff}} = 1$ is achieved if the saxion couplings to the MSSM sector arising from the superpotential \( \langle 7 \rangle \) are strong enough to give $B_a$ between about 0.2 and 0.3. Here $\rho_\nu$ is the energy density of a single species of relativistic neutrino, and the effective number of relativistic degrees of freedom varies from $g_* \simeq 60$ to $g_* = 113.75$ for $0.2 \text{ GeV} < T < m_{\text{soft}}$, through which a mild dependence on $T_\sigma$ comes in. The saxion decay temperature $T_\sigma$ is defined at the time $t = t_\sigma$ when the energy density of radiation $\rho_a + \rho_{\text{SM}}$ becomes equal to that of the
saxion, which implies $\rho_{\text{SM}}(t) = (1 - B_a)\rho_a(t)$, where we have used that the ratio between energy densities of axion and thermalized radiation is $\rho_a/\rho_{\text{SM}} = B_a/(1 - B_a)$ at $t = t_\sigma$. One can then estimate,

$$T_\sigma \simeq \left( \frac{45(1 - B_a)}{4\pi^2 g_*(T_\sigma)} \right)^{1/4} \sqrt{T_\sigma M_{Pl}}$$

$$\simeq 3.5\text{GeV} \left( \frac{B_a^2/(1 - B_a)}{0.08} \right)^{-1/4} \left( \frac{g_*(T_\sigma)}{100} \right)^{-1/4} \left( \frac{m_\sigma}{10^3\text{GeV}} \right)^{3/2} \left( \frac{c_a F_a}{10^{11}\text{GeV}} \right)^{-1},$$

with $t_\sigma \simeq 1/\Gamma_\sigma$, by taking the approximation that the scale factor $a(t)$ of the Universe is determined mainly by the saxion energy density at $t < t_\sigma$ and by the energy density of the radiation at $t > t_\sigma$.

In fig. 1 we show constant contours of $\Delta N_{\text{eff}}$ in the $(m_\sigma, \mu)$ plane for $|B|/m_A = 0.6$ and $m_h = 125$ GeV. Here we take $g_* = 80$, which depends on $T_\sigma$ but modifies $\Delta N_{\text{eff}}$ only slightly for $0.2\text{GeV} < T_\sigma < m_{\text{soft}}$. It is interesting to observe that $\Delta N_{\text{eff}} \sim 1$ is obtained at $\mu$ of a few hundred GeV for $m_\sigma$ less than 1 TeV. A large $\mu$ renders the saxion couplings to the SM particles strong, suppressing $B_a$. We find that non-thermally produced axions yield

FIG. 1: The constant contours of $\Delta N_{\text{eff}}$ for $|B|/m_A = 0.6$ and $m_h = 125$ GeV in the $(m_\sigma, \mu)$ plane. The black lines represent the contours of $\Delta N_{\text{eff}} = 0.5, 1, 1.5$ from the above, respectively. In the shadowed region, $0.4 \leq \Delta N_{\text{eff}} \leq 2$. We also plot constant contours of the quantity, $(c_a F_a/10^{11}\text{GeV}) \times T_\sigma = 0.1, 0.5, 1, 3, 6, 10$ GeV from the below, respectively, in red lines.
\( \Delta N_{\text{eff}} = 1 \) at \( \mu \) similar to the saxion mass:

\[
\mu \sim 150 \text{GeV} \times \left| 1 - \frac{|B|^2}{m_{\sigma}^2} \right|^{1/2} \left( \frac{m_{\sigma}}{300 \text{GeV}} \right),
\]

(14)

for \( 2m_W < m_{\sigma} \lesssim m_{\text{soft}} \), in which case the saxion decays mainly through \( \sigma \rightarrow WW \) and through \( \sigma \rightarrow ZZ, hh, tt \) if kinematically allowed. Here \( m_W \) is the \( W \) boson mass. On the other hand, for \( m_{\sigma} < 2m_W \), where the dominant decay channel is \( \sigma \rightarrow b\bar{b} \), a \( \mu \) less than about 300 GeV is necessary to have \( \Delta N_{\text{eff}} = 1 \). In the figure, we also draw constant contours of the quantity

\[
\left( \frac{c_a F_a}{10^{11} \text{GeV}} \right) T_{\sigma},
\]

(15)

from which the value of \( T_{\sigma} \) can be read off for a given value of \( F_a \). As we will see soon, the dark matter abundance puts a stringent constraint on \( T_{\sigma} \) and also on the properties of the axino.

The saxion generically acquires a mass smaller than \( m_{\text{soft}} \) by one order of magnitude from the potential radiatively generated by the PQ messengers. Meanwhile, the recent data from the LHC Higgs search suggests that a SM-like Higgs boson may have mass around 125 GeV \cite{33}. To explain this within the MSSM, we need to push the stop mass to about 10 TeV or higher, or to invoke large stop mixing. For \( m_{\sigma} \lesssim 0.1 m_{\text{soft}} \) and \( 1 \text{TeV} \lesssim m_{\text{soft}} \lesssim 10 \text{TeV} \), the Higgsino should have a mass of a few hundred GeV when the axion dark radiation gives \( \Delta N_{\text{eff}} \sim 1 \). Thus, there is a chance to detect SUSY at multi-TeV hadron colliders even when the other MSSM superparticles are heavier than 1 TeV.

B. Axino Dark Matter

Under the \( R \)-parity conservation, the axino is a natural candidate for the dark matter in PQ extensions of the MSSM. The main processes of axino production for \( \mu < m_{\text{soft}} \) are (i) the decay of Higgsinos in thermal bath, and (ii) the decay of the saxion, which crucially depends on the properties of the axino.

Let us first examine the thermal process (i), which is mediated by the interactions

\[
C_5 \frac{\mu}{\sqrt{2} c_a F_a} \left( \tilde{H}_u^0 \cos \beta + \tilde{H}_d^0 \sin \beta \right) \tilde{a} + C_5 \frac{m_Z}{\sqrt{2} c_a F_a} Z_\mu \left( \tilde{H}_u^0 \sin \beta - \tilde{H}_d^0 \cos \beta \right) \sigma^\mu \tilde{a},
\]

(16)

with \( C_5 = \partial \ln \mu / \partial \ln S = 2 \), where the first coupling comes from the superpotential term \( S^2 H_u H_d \), while the other is a consequence of the axino-Higgsino mixing. The produced
axino number density is highly sensitive to $T_{\sigma}$,\footnote{As was noticed in Ref. \[41\], in the case that the Universe does not experience the saxion domination, thermally produced axinos by Higgsino decays would overclose the Universe unless the reheating temperature is much lower than the weak scale or the axino has a tiny mass less than $\mathcal{O}(100)$ keV for $F_a = 10^{10-12}$ GeV. See also Refs. \[42\,44\] for thermal production of the axino.} and is numerically approximated by $n_{\tilde{a}}/s \propto e^{-0.63 \mu/T_{\sigma}}$ for $T_f \lesssim T_{\sigma} \lesssim 0.2 \mu$ where $T_f \simeq \mu/20$ is the freeze-out temperature of the Higgsino.

The derivation can be found in the appendix. We find that $n_{\tilde{a}}/s \leq 10^{-10}$ requires $T_{\sigma} \lesssim T_f$ for $\mu \sim m_\sigma$ and $B_a \sim 0.1$. For $T_{\sigma} \leq T_f$, the axino energy density is approximated by

$$\frac{\rho_{\tilde{a}}}{s}_{\text{thermal}} = \frac{m_{\tilde{a}}}{s} \sum_{i=1}^{2} \left( \frac{1}{a^3} \int_0^{t_f} dt a^3(t) \Gamma_{\tilde{h}_i^0 n_{\tilde{h}_i^0}}(t) + n_{\tilde{h}_i^0}(t_f) \right) \approx 3.6 \times 10^{-10} \text{GeV} \left( \frac{m_{\tilde{a}}}{1 \text{GeV}} \right) \left( \frac{g_*(T_{\sigma})}{100} \right)^{-1} \left( \frac{B_a}{0.3} \right) \left( \frac{\mu/m_\sigma}{0.5} \right)^3 \left( \frac{T_{\sigma}/\mu}{1/34} \right)^9,$$  \hspace{1cm} (17)

where $\Gamma_{\tilde{h}_i^0}$ is the decay rate for $\tilde{H}_i^0 \rightarrow h(Z) + \tilde{a}$, $n_{\tilde{h}_i^0}$ is the number density, and $t_f$ is the time when the Higgsino freezes out of thermal equilibrium. We have taken into account that there are two neutral Higgsinos $\tilde{H}_1^0, \tilde{H}_2^0$, which are almost degenerate in mass, and neglected mixing between Higgsinos and gauginos. Meanwhile, because $m_\sigma < 2\mu$ is required for $\Delta N_{\text{eff}} \leq 2$ unless $B$ is much smaller than $m_{\text{soft}}$, the saxion decay into a Higgsino pair is kinematically forbidden. Even if this mode is open, the annihilation among the Higgsinos produced at the saxion decays would occur effectively \[45\], and consequently the axino relic abundance produced from the Higgsino decays can be smaller than the observed dark matter abundance for $m_{\tilde{a}}$ less than $T_{\sigma}$.

On the other hand, the non-thermal process (ii) crucially depends on the properties of the axino. This process gives rise to

$$\frac{\rho_{\tilde{a}}}{s}_{\text{non-th}} = \frac{2 \Gamma_{\sigma \rightarrow \tilde{a} \tilde{a}} m_{\tilde{a}}}{a^3 s} \frac{\sigma_\text{eff}}{m_\sigma} \int_0^\infty dt a^3(t) \rho_\sigma(t) \approx 3 e^{\Gamma_{\sigma t_\sigma}} \frac{2 T_{\sigma}}{1 - B_a m_\sigma} \frac{m_{\tilde{a}}}{m_\sigma} \Gamma_{\sigma \rightarrow \tilde{a} \tilde{a}} \approx 2.4 \times 10^{-10} \text{GeV} \left( \frac{\lambda_{\tilde{a}}}{0.01} \right)^2 \left( \frac{B_a/(1 - B_a)}{0.3} \right) \left( \frac{m_{\tilde{a}}/m_\sigma}{0.01} \right)^3 \left( \frac{T_{\sigma}/1 \text{GeV}}{1/34} \right)^9.$$  \hspace{1cm} (18)

Thus, in order not to overclose the Universe, the axino should have a small coupling to the saxion and/or small mass compared to $m_\sigma$. The non-thermally produced axinos can yield $\Omega_{\tilde{a}} h^2 \simeq 0.1$, for instance, if $m_{\tilde{a}}$ is less than a few GeV and $\lambda_{\tilde{a}} \lesssim 10^{-2}$ for $F_a = 10^{10-12}$ GeV. Such axino properties are indeed expected when $S$ is stabilized by the potential generated from the PQ messenger loops.
The axino abundance, (17) and (18), can be made consistent with the observed value of the dark matter abundance by taking appropriate values of \( m_{\tilde{a}}, \lambda_{\tilde{a}} \) and \( F_a \) for \( \mu \) and \( m_\sigma \) leading the axion dark radiation to give \( \Delta N_{\text{eff}} \sim 1 \). The energy density of the dark matter receives contribution also from the axion due to the vacuum misalignment

\[
\Omega_a h^2 \sim 0.4 \theta_a^2 \left( \frac{F_a}{10^{12} \text{GeV}} \right)^{1.18},
\]

where \( |\theta_a| < \pi \) is the initial misalignment angle. If the axino relic abundance is too small, which would be the case for \( F_a \) around \( 10^{12} \text{ GeV} \) or higher, the dark matter of the Universe can be explained by the axion from the misalignment.

One might consider other cases where the axino is heavier than the Higgsino or some other MSSM superparticle. Then, for \( \mu \) of a few hundred GeV as required to have \( \Delta N_{\text{eff}} \sim 1 \), one needs either \( m_\sigma < 2 m_{\tilde{a}} \) or an extremely small \( \lambda_{\tilde{a}} \) in order to avoid overproduction of the dark matter. In the case where the gravitino is the lightest superparticle, which is possible in gauge mediation, a small gravitino mass \( m_{3/2} \ll m_{\tilde{a}} \) or a tiny \( \lambda_{\tilde{a}} \) would be necessary since gravitinos produced at the axino decays behave like a hot dark matter with a free-streaming length much larger than 10 Mpc.

Finally, we mention the detection potential of SUSY at collider experiments. The charged Higgsino generally obtains a mass slightly heavier than the mass of \( \tilde{H}_1^0 \) when bino and wino masses have the same phase [46, 47]. Assuming that it is the lightest one of the MSSM superparticles, \( \tilde{H}_1^0 \) decays into \( h(Z) + \tilde{a} \) with

\[
\Gamma_{\tilde{H}_1^0} \approx \frac{C_{\tilde{a}}^2}{16\pi} \frac{\mu^3}{(c_a F_a)^2} \simeq \frac{1}{314 \text{cm}} \left( \frac{\mu}{200 \text{GeV}} \right)^3 \left( \frac{c_a F_a}{10^{11} \text{GeV}} \right)^{-2},
\]

for \( \mu > m_h + m_{\tilde{a}} \), as is the case in most of the parameter region giving \( \Delta N_{\text{eff}} \leq 2 \) for \( m_h \lesssim 130 \text{ GeV} \). Here we have neglected the masses of the final states and mixing with neutral gauginos. Thus, depending on \( \mu \) and \( F_a \), \( \tilde{H}_1^0 \) can decay inside the detector while leaving displaced vertices. Measuring its decay length would give us information about the axion decay constant.

IV. CONCLUSIONS

In this paper we have studied the possibility that dark radiation recently suggested by observations can be explained by the QCD axion non-thermally produced by the saxion
decay. In order to account for $\Delta N_{\text{eff}} \sim 1$, the axion superfield must have a sizable coupling to the Higgs sector. We have found that the Higgsino mixing parameter $\mu$ is bounded above and should be in the range of a few hundred GeV, for the saxion mass lighter than 1 TeV. Considering that the saxion mass could be naturally one order of magnitude smaller than the soft masses for the MSSM superparticles, the Higgsino can be within the reach of the LHC and/or ILC, even if the other SUSY particles are much heavier than $O(1)$ TeV. This will be of great importance especially if the SM-like Higgs boson mass is confirmed to be around $124 - 126$ GeV [33].

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Saxion decay rates and axino production

In this appendix, we present the partial decay widths of the saxion, and the derivation of the axino abundance thermally produced by Higgsino decays. In the decoupling limit $m_A \gg m_W$, the saxion decay occurs with

$$
\Gamma_{\sigma \rightarrow hh} = \left( 1 - \frac{4m_h^2}{m_\sigma^2} \right)^{1/2} \Lambda_\sigma,
$$

$$
\Gamma_{\sigma \rightarrow VV} = k_V \frac{m_\sigma^4}{(m_\sigma^2 - m_h^2)^2} \left( 1 - \frac{4m_V^2}{m_\sigma^2} \right)^{1/2} \left( 1 - \frac{4m_V^2}{m_\sigma^2} + 12 \frac{m_V^4}{m_\sigma^4} \right) \Lambda_\sigma,
$$

$$
\Gamma_{\sigma \rightarrow f\bar{f}} = 4N_f \frac{m_f^2 m_\sigma^2}{(m_\sigma^2 - m_f^2)^2} \left( 1 - \frac{4m_f^2}{m_\sigma^2} \right)^{3/2} \Lambda_\sigma,
$$

(21)

if the corresponding process is kinematically accessible. Here $k_V = 2(1)$ for $V = W(Z)$, and $N_f = 3(1)$ for quarks (leptons). The overall factor $\Lambda_\sigma$ is defined by

$$
\Lambda_\sigma = 4C_\sigma^2 B_\sigma \Gamma_\sigma \left( 1 - \frac{|B|^2}{m_A^2} \right)^2 \frac{|\mu|^4}{m_\sigma^4},
$$

(22)
where we have used the relation \( m_A^2 = 2 |B\mu|/\sin 2\beta \). On the other hand, if \( m_\sigma > 2\mu \), the saxion decays into a pair of Higgsinos with

\[
\Gamma_{\sigma \to H \bar{H}} = 8C_s^2 B_\sigma \frac{|\mu|^2}{m_\sigma^2} \left( 1 - 4 \frac{|\mu|^2}{m_\sigma^2} \right)^{3/2},
\]

(23)

ignoring mixing between the Higgsinos and gauginos. The above decay rate will be reduced by the mixing.

Let us move to the axino abundance. The energy densities of the saxion and thermalized radiation are given by

\[
\rho_\sigma(t) = \rho_0 \left( \frac{a_0}{a(t)} \right)^3 e^{-\Gamma_\sigma(t-t_0)},
\]

\[
\rho_{SM}(t) = (1-B_\sigma)\rho_0 \frac{a_0}{a(t)} \int_{t_0}^t dt \frac{a(t')}{a_0} e^{-\Gamma_\sigma(t'-t_0)},
\]

(24)

where \( a_0 = a(t_\sigma) \), \( \rho_0 = \rho_\sigma(t_\sigma) \). Thermal production of axinos is dominated by the Higgsino decays:

\[
\Gamma_{\tilde{H}_0^0 \to h \tilde{a}} \simeq (\cos \beta \pm \sin \beta)^2 \frac{C_s^2}{32\pi} \frac{|\mu|^3}{(c_a F_a)^2} \left( 1 - \frac{m_a^2}{|\mu|^2} \right)^2,
\]

\[
\Gamma_{\tilde{H}_1^0 \to Z \tilde{a}} \simeq (\cos \beta \mp \sin \beta)^2 \frac{C_s^2}{32\pi} \frac{|\mu|^3}{(c_a F_a)^2} \left( 1 - \frac{m_Z^2}{|\mu|^2} \right)^2 \left( 1 + 2 \frac{m_Z^2}{|\mu|^2} \right),
\]

(25)

for \( \mu > m_h, Z \) and \( \mu \gg m_\tilde{a} \). The number density of thermally produced axinos is thus estimated by

\[
\frac{n_{\tilde{a}}}{s} = \sum_i \left( \frac{\Gamma_{\tilde{H}_0^0 \to h \tilde{a}} + \Gamma_{\tilde{H}_1^0 \to Z \tilde{a}}}{a^3 s} \int_{t_0}^{t_f} dt a^3(t)n_{\tilde{H}_0^0}(t) + \frac{n_{\tilde{H}_1^0}(t_f)}{s} \right)
\]

\[
\simeq \frac{45}{\pi^4 g_*(T_\sigma)} \left( \sum_i \frac{\Gamma_{\tilde{H}_0^0 \to h \tilde{a}, Z \tilde{a}}}{\Gamma_\sigma} (P_{t<t_\sigma}(z_\sigma) + P_{t>t_\sigma}(z_\sigma)) + z_f^{-3} \int_0^\infty dk \frac{k^2}{e^{k^2/z_f} + 1} \times \text{Min}[(z_\sigma/z_f)^5, 1] \right),
\]

(26)

where \( z_f = T_f/\mu \) and \( z_\sigma = T_\sigma/\mu \). The axino production functions \( P_i(x) \) before and after \( t = t_\sigma \) are approximated by

\[
P_{t<t_\sigma}(x) \simeq 2x^9 \int_x^\infty dz \int_0^\infty dk \frac{z^{-13}k^2}{e^{k^2/z} + 1} \approx \left( \frac{x}{x_0} \right)^9 e^{-0.63/x_0},
\]

\[
P_{t>t_\sigma}(x) \simeq \theta(x-z_f) x^2 \int_{z_f}^x dz \int_0^\infty dk \frac{z^{-6}k^2}{e^{k^2/z} + 1} \approx \theta(x-z_f) x^{-2} e^{-1.26/x},
\]

(27)
for $x_0 = \text{Max}[x, z_f]$, and $\theta(x)$ being the step function. Here we have taken the approximation, $a(t)/a_0 \sim (t/t_\sigma)^{2/3} \sim (T_\sigma/T)^{8/3}$ at $t < t_\sigma$. For $z_\sigma < 0.2$, $P_{t<t_\sigma}(z_\sigma)$ is always larger than $P_{t>t_\sigma}(z_\sigma)$. We also find that the axino abundance computed by the approximated formulae is slightly smaller by a factor 2 or 3 than the one obtained by numerically solving the Boltzmann equation. However, the above approximation is enough to illustrate how strongly the axino abundance depends on $T_\sigma$.

[1] E. Komatsu et al. [WMAP Collaboration], “Seven-Year Wilkinson Microwave Anisotropy Probe (Wmap) Observations: Cosmological Interpretation,” Astrophys. J. Suppl. 192 (2011) 18 [arXiv:1001.4538 [astro-ph.CO]].

[2] J. Dunkley, R. Hlozek, J. Sievers, V. Acquaviva, P. A. R. Ade, P. Aguirre, M. Amiri, J. W. Appel et al., “The Atacama Cosmology Telescope: Cosmological Parameters from the 2008 Power Spectra,” Astrophys. J. 739, 52 (2011). [arXiv:1009.0866 [astro-ph.CO]].

[3] R. Keisler et al., “A Measurement of the Damping Tail of the Cosmic Microwave Background Power Spectrum with the South Pole Telescope,” Astrophys. J. 743 (2011) 28 [arXiv:1105.3182 [astro-ph.CO]].

[4] Z. Hou, R. Keisler, L. Knox, M. Millea and C. Reichardt, “How Additional Massless Neutrinos Affect the Cosmic Microwave Background Damping Tail,” [arXiv:1104.2333 [astro-ph.CO]].

[5] A. X. Gonzalez-Morales, R. Poltis, B. D. Sherwin and L. Verde, “Are Priors Responsible for Cosmology Favoring Additional Neutrino Species?,” [arXiv:1106.5052 [astro-ph.CO]].

[6] J. Hamann, S. Hannestad, G. G. Raffelt and Y. Y. Y. Wong, “Sterile Neutrinos with eV Masses in Cosmology – How Disfavoured Exactly?,” JCAP 1109 (2011) 034 [arXiv:1108.4136 [astro-ph.CO]].

[7] M. Archidiacono, E. Calabrese and A. Melchiorri, “The Case for Dark Radiation,” [arXiv:1109.2767 [astro-ph.CO]].

[8] J. Hamann, “Evidence for Extra Radiation? Profile Likelihood Versus Bayesian Posterior,” [arXiv:1110.4271 [astro-ph.CO]].

[9] K. Enqvist, K. Kainulainen and J. Maalampi, “Refraction and Oscillations of Neutrinos in the Early Universe,” Nucl. Phys. B 349 (1991) 754.

[10] R. Foot, M. J. Thomson and R. R. Volkas, “Large Neutrino Asymmetries from Neutrino
[11] X. D. Shi, “Chaotic Amplification of Neutrino Chemical Potentials by Neutrino Oscillations in Big Bang Nucleosynthesis,” Phys. Rev. D 54 (1996) 2753 [arXiv:astro-ph/9602135].

[12] J. March-Russell, H. Murayama and A. Riotto, “The Small Observed Baryon Asymmetry from a Large Lepton Asymmetry,” JHEP 9911 (1999) 015 [arXiv:hep-ph/9908396].

[13] M. Kawasaki, F. Takahashi and M. Yamaguchi, “Large Lepton Asymmetry from Q-Balls,” Phys. Rev. D 66 (2002) 043516 [arXiv:hep-ph/0205101].

[14] M. Yamaguchi, “Generation of cosmological large lepton asymmetry from a rolling scalar field,” Phys. Rev. D68, 063507 (2003). [hep-ph/0211163].

[15] K. Ichikawa, M. Kawasaki and F. Takahashi, “Solving the Discrepancy among the Light Elements Abundances and WMAP,” Phys. Lett. B 597 (2004) 1 [arXiv:astro-ph/0402522].

[16] K. A. Olive and E. D. Skillman, “A Realistic Determination of the Error on the Primordial Helium Abundance: Steps Toward Non-Parametric Nebular Helium Abundances,” Astrophys. J. 617 (2004) 29 [arXiv:astro-ph/0405588].

[17] Y. I. Izotov, T. X. Thuan, “The primordial abundance of 4He: evidence for non-standard big bang nucleosynthesis,” Astrophys. J. 710, L67-L71 (2010). [arXiv:1001.4440 [astro-ph.CO]].

[18] E. Aver, K. A. Olive, E. D. Skillman, “A New Approach to Systematic Uncertainties and Self-Consistency in Helium Abundance Determinations,” JCAP 1005, 003 (2010). [arXiv:1001.5218 [astro-ph.CO]].

[19] J. Hamann, S. Hannestad, G. G. Raffelt, I. Tamborra and Y. Y. Y. Wong, “Cosmology Seeking Friendship with Sterile Neutrinos,” Phys. Rev. Lett. 105 (2010) 181301 [arXiv:1006.5276 [hep-ph]].

[20] K. M. Nollett and G. P. Holder, “An analysis of constraints on relativistic species from primordial nucleosynthesis and the cosmic microwave background,” [arXiv:1112.2683 [astro-ph.CO]].

[21] R. D. Peccei and H. R. Quinn, “CP Conservation in the Presence of Instantons,” Phys. Rev. Lett. 38, 1440 (1977); “Constraints Imposed by CP Conservation in the Presence of Instantons,” Phys. Rev. D 16, 1791 (1977).

[22] For a review, see J. E. Kim, “Light Pseudoscalars, Particle Physics and Cosmology,” Phys. Rept. 150, 1 (1987); H. Y. Cheng, “The Strong CP Problem Revisited,” Phys. Rept. 158, 1 (1988); J. E. Kim and G. Carosi, “Axions and the Strong CP Problem,” Rev. Mod. Phys. 82, 557 (2010) [arXiv:0807.3125 [hep-ph]].
[23] E. J. Chun, D. Comelli and D. H. Lyth, “The Abundance of relativistic axions in a flaton model of Peccei-Quinn symmetry,” Phys. Rev. D 62, 095013 (2000) [hep-ph/0008133].

[24] K. Ichikawa, M. Kawasaki, K. Nakayama, M. Senami and F. Takahashi, “Increasing Effective Number of Neutrinos by Decaying Particles,” JCAP 0705 (2007) 008 [arXiv:hep-ph/0703034].

[25] F. Takahashi, “Gravitino dark matter from inflaton decay,” Phys. Lett. B 660, 100 (2008) [arXiv:0705.0579 [hep-ph]].

[26] J. Hasenkamp, “Dark radiation from the axino solution of the gravitino problem,” Phys. Lett. B 707, 121 (2012) [arXiv:1107.4319 [hep-ph]].

[27] J. L. Menestrina and R. J. Scherrer, “Dark Radiation from Particle Decays during Big Bang Nucleosynthesis,” arXiv:1111.0605 [astro-ph.CO].

[28] T. Kobayashi, F. Takahashi, T. Takahashi and M. Yamaguchi, “Dark Radiation from Modulated Reheating,” arXiv:1111.1336 [astro-ph.CO].

[29] D. Hooper, F. S. Queiroz and N. Y. Gnedin, “Non-Thermal Dark Matter Mimicking an Additional Neutrino Species in the Early Universe,” arXiv:1111.6599 [astro-ph.CO].

[30] K. Nakayama, F. Takahashi and T. T. Yanagida, “A Theory of Extra Radiation in the Universe,” Phys. Lett. B 697 (2011) 275 [arXiv:1010.5693 [hep-ph]].

[31] M. Dine, W. Fischler and M. Srednicki, “A Simple Solution To The Strong CP Problem With A Harmless Axion,” Phys. Lett. B 104, 199 (1981).

[32] A. R. Zhitnitsky, “On Possible Suppression Of The Axion Hadron Interactions. (In Russian),” Sov. J. Nucl. Phys. 31, 260 (1980) [Yad. Fiz. 31, 497 (1980)].

[33] The ATLAS and CMS collaborations, ATLAS-CONF-2011-163 and CMS-PAS-HIG-11-032 (December, 2011).

[34] S. R. Coleman and E. J. Weinberg, “Radiative Corrections As The Origin Of Spontaneous Symmetry Breaking,” Phys. Rev. D 7, 1888 (1973).

[35] K. S. Jeong and M. Yamaguchi, “Axion model in gauge-mediated supersymmetry breaking and a solution to the $\mu/B\mu$ problem,” JHEP 1107, 124 (2011) [arXiv:1102.3301 [hep-ph]].

[36] T. Asaka and M. Yamaguchi, “Hadronic axion model in gauge mediated supersymmetry breaking,” Phys. Lett. B 437, 51 (1998) [hep-ph/9805449]; “Hadronic axion model in gauge mediated supersymmetry breaking and cosmology of saxion,” Phys. Rev. D 59, 125003 (1999) [hep-ph/9811451].

[37] D. H. Lyth and E. D. Stewart, “Cosmology with a TeV mass GUT Higgs,” Phys. Rev. Lett.
75, 201 (1995) hep-ph/9502417; “Thermal inflation and the moduli problem,” Phys. Rev. D 53, 1784 (1996) hep-ph/9510204.

[38] J. E. Kim and H. P. Nilles, “The Mu Problem And The Strong CP Problem,” Phys. Lett. B 138, 150 (1984).

[39] G. F. Giudice and A. Masiero, “A Natural Solution to the mu Problem in Supergravity Theories,” Phys. Lett. B 206, 480 (1988).

[40] K. Choi, E. J. Chun and J. E. Kim, “Cosmological implications of radiatively generated axion scale,” Phys. Lett. B 403, 209 (1997) hep-ph/9608222.

[41] E. J. Chun, “Dark Matter in the Kim-Nilles Mechanism,” Phys. Rev. D 84 (2011) 043509 arXiv:1104.2219 [hep-ph].

[42] K. J. Bae, K. Choi and S. H. Im, “Effective interactions of axion supermultiplet and thermal production of axino dark matter,” JHEP 1108, 065 (2011) arXiv:1106.2452 [hep-ph].

[43] K. -Y. Choi, L. Covi, J. E. Kim and L. Roszkowski, “Axino Cold Dark Matter Revisited,” arXiv:1108.2282 [hep-ph].

[44] K. J. Bae, E. J. Chun and S. H. Im, “Cosmology of the DFSZ axino,” arXiv:1111.5962 [hep-ph].

[45] J. Hisano, K. Kohri and M. M. Nojiri, “Neutralino warm dark matter,” Phys. Lett. B 505, 169 (2001) hep-ph/0011216.

[46] H. E. Haber and G. L. Kane, “The Search For Supersymmetry: Probing Physics Beyond The Standard Model,” Phys. Rept. 117, 75 (1985).

[47] D. M. Pierce, J. A. Bagger, K. T. Matchev and R. j. Zhang, “Precision corrections in the minimal supersymmetric standard model,” Nucl. Phys. B 491, 3 (1997) arXiv:hep-ph/9606211.