Theoretical studies on multiple ionisation and electron capture processes in heavy ion induced M-shell ionisation

Soumya Chatterjee\textsuperscript{1}, Sumana Ghosh\textsuperscript{1} and D. Mitra\textsuperscript{1}

\textsuperscript{1}Department of Physics, University of Kalyani, Kalyani, West Bengal-741235, India.

Multiple ionisation and electron capture are found to be vital mechanisms for K and L x-ray emissions along with the direct coulomb ionisation in heavy ion-atom collisions. Naturally, these two mechanisms may also be significant for M x-ray emissions also. However, these mechanisms associating with the M-shell are highly complex and not yet studied convincingly. We, in this work, have discussed about theoretical techniques to study these mechanisms in more general way, which in turn, almost solved the wide gap between the theories and experiments regarding M-shell prevalent in literature up to a good extent. We have justified this technique with a few representative cases: for example, the silicon and sulphur projectile ions colliding with the gold and bismuth targets at beam energies ranging from 5 to 10 MeV. Here, we see that the capture contribution is much higher than that coming from the multiple ionisation effect. The combined effects of direct coulomb ionisation associated with simultaneous multiple ionisation and electron capture give a better agreement with the measurements up to a good extent.

I. INTRODUCTION

The emission of x-rays from targets has resulted in major advances in radiation physics \cite{1}, plasma physics \cite{2}, atomic and nuclear physics \cite{3}, and the particle-induced x-ray emission (PIXE) technique \cite{4,5}. Mostly the PIXE method has used light ions such as protons or alphas \cite{6-10}. However, there is an increasing interest to employ heavy ions since their cross sections are larger and have, thereby, better sensitivity \cite{11}. Nevertheless, this potentiality is discouraged by discrepancies observed between the theories and experiments. It is well-known that for asymmetric collisions, \( z_1/z_2 < 1 \), the direct coulomb ionisation (DCI) is dominant, whereas for symmetric collisions \( z_1/z_2 \approx 1 \), the multiple ionisation (MI) and electron capture (EC) processes become increasingly important. \( z_1 \) and \( z_2 \) are the projectile and target atomic number, respectively. Chatterjee et al. \cite{15,16} showcased few such cases for the K- and L-shell ionisation, and resolved the existing discrepancies between experiment and theory. They demonstrated that besides the MI process, K-K and L-K electron capture processes must be taken into account in the theory to justify such discrepancies. More importantly, the charge state of the heavy projectile ions inside the target must be considered in evaluating the contribution of MI and EC as the inner shell ionisation takes place mostly inside the target. They reported a theoretical methodology to predicting the charge state distributions inside (CSD-I) the target. The same approach has also been applied to study the L-subshell vacancy production by considering the DCI, MI, and EC by performing a detailed comparison between experiment and theory \cite{16}. Contrary to the agreement found in K-shell case, the contribution from the MI and EC does not account for all the discrepancies found between the measurements and the theoretical descriptions for L-shell scenario. It was found that the presently available atomic parameters for L-shell \cite{16} (the fluorescence and Coster-Kronig yields) are unable to achieve agreement between the experiment and the theory. So, they have optimised these values iteratively until a good agreement was achieved.

In the present study, we intend to validate the trend seen in K- and L-shell ionisation through M-shell ionisation also. Though the motivation is quite natural, applying MI and EC in M-shell is more complex than L-shell due to the involvement of five subshells (\( 3s_{1/2}, 3p_{1/2}, 3p_{3/2}, 3d_{3/2}, 3d_{5/2} \)) due to the spin-orbit coupling. Three different types of wave functions are required for describing the system theoretically. Previous studies have not accomplished it to a satisfactory limit. For example, Mitra et al. \cite{17}, in their work, interpreted their measured data considering the fully stripped projectile ion instead of the actual charge state of it inside the target to get the maximum limit of MI. There they took MI under consideration by using the Lapicki’s method \cite{18}. It is based on the fact that each electron in a manifold of the outer subshells is ionised with a probability \( P \), which is used to evaluate the modified fluorescence and Coster-Kronig yields due to multiple ionisation in the outer shells, as follows

\[
P = \frac{q_m^2}{2\beta v_p^2} \left(1 - \frac{\beta}{4v^2}\right),
\]

where \( \beta = 0.9 \), \( v \) is the velocity of the projectile ion and \( q_m \) is the equilibrium charge state of the projectile ion inside the target. EC has also not yet been applied to heavy-ion induced M-shell ionisation in their work. Here, we have extended the domain of the theory of EC from K- and L-shell to M-shell ionisation also and take MI in a realistic way by considering the projectile charge state inside the target.

II. MULTIPLE IONISATION

Phenomenon of multiple ionisation or multiple vacancy production in heavy ion-atom collisions takes place due
to strong perturbations. Not only a vacancy is created in M shell, but also during ion-atom collisions one or more spectator vacancies are created in higher shells. More than one target electron is expelled, stimulated, or caught in the majority of individual collisions - usually from distinct shells. This causes plenty of issues on the Auger, Coster-Kronig, and fluorescence yields, as well as on transition energies. Thus, estimation of the values of the different atomic parameters are quite complicated in this situation. Theoretical description of multiple ionisation turns out to be very complex due to the strong perturbations in the collision. Various theories have been employed to explain the phenomenon time to time, for example, the ionisation theories like semi classical approximation (SCA) and modified binary encounter approximation (BEA) in seventies \cite{19, 20} were employed for zero impact parameter ion-atom collisions only. In next decade, Lapicki et al. \cite{18} proposed a heuristic formula to account for multiple ionisation in terms of a classical probability. Sulik et al. \cite{21} generalized and extended the simple geometrical model developed for ionisation probabilities on the basis of a BEA model to take into account magnetic sub-states and non-zero impact parameter during ion-atom collisions. The latter was successful in comparing the results with a wide range of experimental data and no contradictions were found, whereas the formulation prescribed by Lapicki et al. \cite{22} was used in a few cases. Furthermore, the Sulik model \cite{21} works in wider energy range than the Lapicki model \cite{18}. Therefore, we have chosen the Sulik model in our present work.

According to the BEA model \cite{21}, the multiple ionisation probability can be written as

\[ P(X_n) = \frac{X_n^2}{4.2624 + X_n^2[1 + 0.5 \exp(-X_n^2/16)]} \]  

(2)

Where, \( X_n (= W/n) \) is the universal scaling parameter, that involves the principal quantum number \( (n) \) of the given shell and a universal scaling variable \( W \) defined as

\[ W = 4 \frac{1}{v} V[G(V)]^{1/2} \]  

(3)

It denotes the measurement of the perturbation strength that characterizes the collision. \( V (= v/v_2) \) is the scaled projectile velocity, \( v_2 (= \omega z - S_a)/v_2 \) is the target atomic electron velocity. \( z_2 \) is the target atomic number and \( S_a \) is the screening constant for ground state atom prescribed by Slater\cite{23}. \( G(V) \) is the BEA scaling function, which is derived from a large number of experiments and tabulated over a large range of \( V \). In McGuire and Richard \cite{20}, the values of \( G(V) \) were determined in two ways: in one Gerjuoy-Vriens-Garcia approach \cite{24} is considered and in other Gryzinski’s model \cite{25} is used. According to McGuire and Richard \cite{20}, Gerjuoy-Vriens-Garcia \( G(V) \) is more accurate than Gryzinski’s \( G(V) \) and thus, we have taken Gerjuoy-Vriens-Garcia \( G(V) \) to estimate the multiple ionisation probability.

Due to the simultaneous multiple ionisations, besides active shell, vacancy can also be created in higher shells. Non-availability of electrons in higher shell results the decreases of non-radiative transition (Auger transition as well as CK transition) rate, which in turn increases the fluorescence yield (\( \omega \)) also. We have to consider these modified values of atomic parameters in order to incorporate the MI effect in calculation of DCI. We have taken the \( \omega_i^0 \) and \( f_{ij}^0 \) values for single vacancy from Chauhan and Puri \cite{26} and then modified them for MI effect according to the prescription of Lapicki et al. \cite{22} as follows,

\[ \omega_i = \omega_i^0 [1 - P(1 - \omega_i^0)^{-1}] \]  

(4)

while the \( f_{ij} \) values for multiple ionisation are given by

\[ f_{ij} = f_{ij}^0 [1 - P]^2. \]  

(5)

This \( P \) is nothing but \( P(X_n) \) as given in Eqns. 2

The DCI is evaluated using the ISICS code \cite{27} using the \( \omega_i^0 \) and \( f_{ij}^0 \). Next to get DCI associated with MI, same code is used just by using the \( \omega_i \) and \( f_{ij} \) as obtained from Eqns. 4 and 5.
III. THEORY OF ELECTRON CAPTURE PHENOMENON

Long ago V. S. Nikolaev [30] developed an expression for evaluating electron capture cross sections in the framework of the Oppenheimer-Brinkman-Kramers (OBK) approach, where the capture taking place from target atoms to projectile ions during ion-atom collisions. Lapicki and McDaniel [31] modified the cross section formulation for electron capture from the K-shells of the target atoms to the K-, L-, M- shell of the stripped projectile ions only. In this work, we have made a general approach so that we can estimate the electron capture cross section from any target shells to any projectile shells as follows,

\[ \sigma_{\text{OBK}} = \frac{2^8 \pi N}{5} \frac{n_1^2}{v^2} \frac{v_1}{v_2} \frac{\xi^5}{(\xi + 1)^3} \phi_4(\zeta) \]  

(6)

Here, \( v \) is the projectile ion velocity and \( N \) is the total number of electrons in the target atomic shell. The projectile and target shells are characterized by the quantum numbers \( n_1 \) and \( n_2 \), respectively. Orbital electron velocity of the projectile ion (\( v_1 \)) cannot be calculated by simply scaling for higher shell electrons as done by Lapicki for bare projectile ions [32]. Rather, we have to consider the screening effect of all the existing electrons of the ions during the binding energy calculation for the projectile vacant shells where the target electrons are captured, from which we can get projectile electron velocity \( v_1 \). To calculate the binding energy, we have utilised the ionisation potential of projectile ions in its ground state prescribed by Agmon [33] as follows,

\[ E = \frac{\hbar c R_H}{n_1^2} (z_1 - S_z)^2 \]  

(7)

\( S_z \) is the screening constant and \( n_1^2 \) is the effective principal quantum number of projectile ions as listed in the reference [33]. \( \hbar c R_H \) is ionisation potential of hydrogen atom. \( z_1 \) be the projectile atomic number. In electron volt units it attains the value 13.6. So, the velocity of the captured electron in projectile orbit is written as

\[ v_1 = \sqrt{\frac{E}{13.6}} \]  

(8)

\( v_1 \) is expressed in atomic units. Orbital velocities of the target electron is \( v_2 = (z_2 - S_a)/n_2 \). \( z_2 \) is the target atomic number and \( S_a \) is the screening constant for ground state atom prescribed by Slater [23]. “\( S_a \)” for \( M_i(i = 1, 2, 3) \) sub-shells is 11.25, whereas for \( M_i(i = 4, 5) \) sub-shells it is 21.15. The reduced binding energy \( (\theta) \) of the target electron, which is being captured by the projectile ion [34] is defined as

\[ \theta = \frac{\text{Observed Binding Energy}}{v_2^2 \times 13.6} \]  

(9)

The parameter \( \xi(\theta) \) in Eqn. 6 is defined by Lapicki and McDaniel [31]

\[ \xi(\theta) = \frac{v_2}{\sqrt{v_4^2 + q^2(\theta)}} \]  

(10)

where \( q(\theta) \) measures the momentum transfer in the capture process

\[ q(\theta) = \frac{v}{2} + \frac{v_2^2 \theta - v_2^2}{2v} \]  

(11)

In Eqn. 6 the function \( \phi_4(X) \) takes the form as

\[ \phi_4(X) = [1 - \left( \frac{4}{X} \right)](1 + \frac{1}{X})^{-3/2}(1 + X)^{-2} - \frac{1}{3} \frac{1}{2} (1 + \frac{1}{X})^2 \]  

(12)

And the parameter \( \zeta \) in Eqn. 6 is written as

\[ \zeta = \left( 1 - \theta \right) \xi^2 \]  

(13)

Projectile velocity dependent electron capture cross section can be written according to Lapicki and McDaniel [31]. For low-velocity ions \((v \ll v_2)\) the cross section \((\sigma_{(\langle \rangle)}^{OBK})\) is given by

\[ \sigma_{(\langle \rangle)}^{OBK} = C \sigma_{\text{OBK}}(\lambda \theta, \lambda \theta) \]  

(14)

Where \( \lambda \) is the binding energy correction term for target M-shell and C is the Coulomb deflection factor, which are caused by the united atom formation in low velocity. The \( \lambda \) is described by Chen and Crasemann [35] as follows

\[ \lambda = (1 + \frac{z_1}{z_2 - S_a})^2 \]  

(15)

and the factor C is given by Lapicki and McDaniel [31]

\[ C = \exp[-\pi D q(\lambda \theta)] \]  

(16)

The half-distance \((D = z_1 z_2/M v^2)\) of closest approach in a head-on collision is approximated by the reduced mass \((M^{-1} = M_1^{-1} + M_2^{-1})\) of the scattering system. Where \( M_1 \) and \( M_2 \) are the atomic mass of projectile and target respectively. The binding effect reduces the ionisation cross section by effectively increasing \( \theta \) to \( \lambda \theta \).

Whereas, the capture cross-sections \((\sigma_{(\rangle)}^{OBK})\) for high velocity ions \((v \gg v_2)\) takes the form [31] as

\[ \sigma_{(\rangle)}^{OBK} = \frac{1}{3} \sigma_{\text{OBK}}[\xi(\lambda \theta), \lambda \theta] \]  

(17)

and for intermediate velocity range \((= v \approx v_2)\), the capture cross-section \((\sigma_{(\langle \rangle)}^{OBK})\) takes the following form [31]

\[ \sigma_{(\langle \rangle)}^{OBK} = \frac{\sigma_{(\rangle)}^{OBK} \sigma_{(\langle \rangle)}^{OBK}}{\sigma_{(\langle \rangle)}^{OBK} + 2 \sigma_{(\langle \rangle)}^{OBK}} \]  

(18)
FIG. 2: Charge state distributions inside different targets are calculated using the Voigt distribution with distribution width \([36]\) at two selected energies.

In the present study, velocity \((v)\) of the projectile ions lies in the range 3.25 to 4.61 a.u. Whereas electron orbital velocity in the M-shell of the targets Au is about 19.28 to 22.58 a.u. and 20.62 to 23.92 a.u. for Bi target, that leads to the value of \(\xi(\theta)\) to be \(0.25 \leq \xi(\theta) \leq 1\). So for obtaining the electron capture (EC) cross section, we have used the formulation of equation \([18]\) applicable for the intermediate velocity regime \((v \approx v_2)\) \([37]\).

To compare the theoretical predictions of total M-shell ionisation cross sections with the experimentally obtained value of the total M-shell production cross sections of Mitra \textit{et al.} \([17]\), we need to convert the theoretically calculated total ionisation cross sections to the total production cross sections. This conversion is a routine practice. One can do it by using ISICS code \([27]\). DCI along with MI can also be done in the ISICS code \([27]\) by including the modified atomic parameters for MI as given in equations \([4]\) and \([5]\). However, converting the ionisation contributions due to the electron capture to the M x-ray production cross sections are to be done and it is out of scope for the ISICS code. We have made use of a simple method where average florescence yield \(\omega_{avg}\) (ratio of total M-shell productions to ionisation cross sections as obtained from the ISICS code \([27]\)) is used to convert the total theoretical ionisation cross sections due to the electron capture \((\sigma_M^{IC}(tot))\) to the production cross sections due to the electron capture \((\sigma_M^{XC}(tot))\) as follows

\[
\sigma_M^{XC}(tot) = \omega_{avg}\sigma_M^{IC}(tot) \tag{19}
\]

In the present case, \(\sigma_M^{IC}(tot) = \sigma_O^{DK}\), see Eqn. \([18]\). Hence, the total M x-ray production cross section \(\sigma_M^{X}(tot)\) is given by

\[
\sigma_M^{X}(tot) = \sigma_M^{X}(tot) \text{ from ISICS code} + \sigma_M^{XC}(tot) \tag{20}
\]
experimental results of Mitra et al. 17. They used the silicon (\(14 \text{Si}^{3+},4^+\)) and sulphur (\(16 \text{S}^{4+},4^+\)) projectile ions on gold (200 \(\mu g/cm^2\)) and bismuth (80 \(\mu g/cm^2\)) targets in the energy range of 5-10 MeV. We have intended to compare the total M x-ray production cross sections at different stages of the theoretical predictions, viz., the direct coulomb ionisation (DCI), DCI associated with MI effects, and after taking consideration of EC. For DCI, we used ECPSSR theory as it gives the best representation for the K-and L-shell ionisation as shown in our recent studies 18,28. Next, to incorporate the effect of MI and EC in M x-ray production cross sections, we need to have a clear idea of the projectile charge states inside the target and these facts are clearly displayed in Figs.1 and 2. We have made use of the above mentioned methods to calculate the production cross section considering the multiple ionisation effect during DCI and the contribution from EC. All stages of calculations are depicted in Fig. 3 and 4. Furthermore, when united atom is considered within the ECPSSR theory, it is called ECUSAR theory. We checked in the present case, both ECPSSR and ECUSAR results the same. It means united atom formation is negligible for the systems used here. It can be seen from Fig. 3 and 4 that ECPSSR underestimates the M x-ray production cross sections very much. If we include the MI contribution using the method described above along with the DCI, we see certain improvement. In the next step, we need to add up the EC contribution along with the DCI associated with MI using the methodology given above. However, to calculate it we need to know the type of EC which is responsible to create the M-shell vacancies in the target atoms for the present case. In other words, we must know the vacancies present in the projectile ions while it is traversing the bulk of the target. Accordingly, we will know whether M-K or M-L capture is taking place. To account such aspects we followed the procedure described below.

Though a monochromatic ion beam is passed through the target, ion atom interaction leads to a certain charge exchange inside the target. It leads to a mean charge state \(q_m\) inside the target. To obtain the \(q_m\) inside the target, we employed the empirical formula based on the Schwietz Model 29. This model has developed the formula empirically by constructing least square-fit with a large number of data points (800). The target dependency of mean-charge-state is incorporated in the model by including target atomic number \(Z\), projectile atomic number \(Z_p\), projectile velocity \(v_p\), and reduced velocity \(v_r\) as

\[
q_m = \frac{8.29x + x^4}{(0.06/x) + 4 + 7.4x + x^4}
\]

where, 
\[x = c_1\left(\frac{v_r}{1.54r_B}\right)^{1+1.83/z_1}\] and the correction factors to be \(c_1 = 1 - 0.26e^{-\frac{r_B}{2}}e^{-\frac{(2z_2-Z_1)^2}{2}}\) and \(c_2 = 1 + 0.03v_rln(z_2)\) and reduced velocity \(v_r = \frac{Z_1}{1-0.543}v_p\), where \(v_p\) is projectile ion velocity and \(v_r\) is Bohr velocity. Where \(z_1\) is the projectile atomic number.

In second step, the \(q_m\) values of the projectile ions inside the target are substituted in the voigt charge state distribution \(F(q)\) as follows

\[
F(q) = \frac{1}{\pi} \frac{\Gamma}{(q-q_m)^2 + \left(\frac{\Gamma}{2}\right)^2} \quad \text{and} \quad \sum_q F(q) = 1 \quad \text{(22)}
\]

where distribution width \(\Gamma\) is taken from Novikov and Teplova 36 as follows

\[
\Gamma(x) = C[1 - exp(-x^\alpha)](1 - exp(-(1-x)^\beta)] \quad \text{(23)}
\]

where \(x = q_m/z_1\), \(\alpha = 0.23\), \(\beta = 0.32\) and \(C = 2.669 - 0.0098.52 + 0.058.21 + 0.00048.21.22\). The \(F(q)\) values so obtained are shown in Fig. 5. This CSD lets us know the charge state fraction \(F(q)\) of every charge state, which

![FIG. 4: Same as the Fig.3 but for the Bi target](image-url)
will dictate the type of EC, i.e., M-K or M-L capture as well as the contribution of each charge state in electron capture cross-section. From Fig. 2 it is clear that, inside the target, for both S and Si projectiles both M-K and M-L captures have to be taken under consideration.

The CSD inside the target for the four systems (two beams on two target elements) is shown in Fig. 3 for two selected beam energies. These charge state fractions are used to estimate the concerned capture contributions.

Once we include the electron capture cross sections ($\sigma_{EC}$) along with the DCI with the effect of MI, the total M-shell production cross section $\sigma^*_M(tot)$ as obtained from direct ionisation theories leads to better agreement with the measured $\sigma^*_M(tot)$ [17] as plotted in Fig. 3 and 4. Maybe the deviations that appeared in Fig. 3 and 4 from experimental predictions implies that the atomic parameters [20] used to undertake multiple ionisation effects as well as to convert theoretical ionisation cross sections to production one, needs improvement. For L-shell ionisation case, we have optimized the atomic parameters from proton induced L-ionisation data [16], but applying the same approach for M-shell ionisation case is too complicated to use. Hence, we leave the atomic parameters relevant to the M-shell ionisation to the theoretical evaluations only.

V. CONCLUSION

Multiple ionisation and electron capture are found to be vital mechanisms for K and L x-ray emissions along with the direct coulomb ionisation in heavy ion-atom collisions. Naturally, these two mechanisms may also be significant for M x-ray emissions also. However, these mechanisms are highly complex and not studied yet. Here in this work, we have extended the theoretical methods to study the multiple ionisation and electron capture processes for M-shell ionisation in heavy target atom by heavy projectile ion impact. The multiple ionisation theory is based on binary encounter approximation and the electron capture theory is founded on the basis of OBK approach. The incorporation of MI in DCI to calculate the production cross-section and the addition of the EC give us better agreement by resolving the wide gap between the theories and experiments prevalent in literature to a great extent. We have verified the present theory with the case of silicon and sulphur ions colliding with gold and bismuth targets at the energies ranging from 5 to 10 MeV. We see that capture contribution is much higher than that coming from the multiple ionisation effect. Combined effects of direct coulomb ionisation with MI effects and electron capture, together give a better agreement with the measurements. Hence, we reached a good level of understanding the multiple ionisation and electron capture in heavy atoms by heavy-ion impact. The discrepancies may be attributed to the shortcomings of the theoretical atomic parameters such as fluorescence and Coster-Kronig yields. Hence, the present theories developed for the multiple ionisation and electron capture are good enough to scrutinize well the atomic parameters.

VI. ACKNOWLEDGEMENTS

One of the authors, SC, acknowledges the University of Kalyani for providing him the senior research fellowship towards obtaining PhD.

[1] T. Satoh, International Journal of PIXE 25, 147 (2015).
[2] P. Sharma and T. Nandi, Physics of Plasmas 23, 083102 (2016).
[3] N. A. Dyson and N. A. Dyson, X-rays in Atomic and Nuclear Physics (Cambridge University Press, 2005).
[4] M. Antoszewska-Moneta, R. Brzozowski, and M. Moneta, The European Physical Journal D 69, 1 (2015).
[5] A. W. Gillespie, C. L. Phillips, J. J. Dynes, D. Chevrier, T. Z. Regier, and D. Peak, Advances in agronomy 133, 1 (2015).
[6] T. B. Johansson, R. Akselsson, and S. A. Johansson, Nuclear Instruments and Methods 84, 141 (1970).
[7] A. Bertol, R. Hinrichs, and M. Vasconcellos, Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms 363, 28 (2015).
[8] J. Garcia, Physical Review A 1, 280 (1970).
[9] D. Joseph, S. N. Rao, and S. Kailas, Mapana Journal of Sciences 12, 1 (2013).
[10] X. Zhou, Y. Zhao, R. Cheng, Y. Wang, Y. Lei, X. Wang, and Y. Sun, Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms 299, 61 (2013).
[11] J. Miranda, G. Murillo, B. Méndez, J. López-Monroy, J. Aspiazu, P. Villaseñor, J. Pineda, and J. Reyes-Herrera, Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms 316, 113 (2013).
[12] J. Miranda and G. Lapicki, Atomic data and nuclear data tables 100, 651 (2014).
[13] H. Mohan, A. K. Jain, M. Kaur, P. S. Singh, and S. Sharma, Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms 332, 103 (2014).
[14] R. Siegele, D. D. Cohen, and N. Dytlewski, Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms 158, 31 (1999).
[15] S. Chatterjee, P. Sharma, S. Singh, M. Oswal, S. Kumar, C. Montanari, D. Mitra, and T. Nandi, Physical Review A 104, 022810 (2021).
[16] S. Chatterjee, S. Kumar, S. Kumar, M. Oswal, B. Mohanty, D. Mehta, D. Mitra, A. Mendez, D. Mitnik, C. Montanari, et al., Physica Scripta 97, 045405 (2022).
[17] D. Mitra, M. Sarkar, D. Bhattacharya, S. Santra, A. Mandal, and G. Lapicki, Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms 268, 450 (2010).
[18] G. Lapicki, R. Mehta, J. L. Duggan, P. Kocur, J. Price, and F. D. McDaniel, Physical Review A 34, 3813 (1986).
[19] J. Hansen, Physical Review A 8, 822 (1973).
[20] J. H. McGuire and P. Richard, Physical Review A 8, 1374 (1973).
[21] B. Sulik, I. Kádár, S. Ricz, D. Varga, J. Végh, G. Hock, and D. Berényi, Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms 28, 509 (1987).
[22] G. Lapicki, G. R. Murty, G. N. Raju, B. S. Reddy, S. B. Reddy, and V. Vijayan, Physical Review A 70, 062718 (2004).
[23] J. C. Slater, Physical Review 36, 57 (1930).
[24] E. Gerjuoy, Physical Review 148, 54 (1966).
[25] M. Gryziński, Physical Review 138, A336 (1965).
[26] Y. Chauhan and S. Puri, Atomic Data and Nuclear Data Tables 94, 38 (2008).
[27] Z. Liu and S. J. Cipolla, Computer Physics Communications 97, 315 (1996).
[28] S. Puri, Atomic Data and Nuclear Data Tables 93, 730 (2007).
[29] G. Schiwietz, K. Czerski, M. Roth, F. Staufenbiel, and P. Grande, Nuclear Instruments and Methods in Physics Research Section B: Beam Interactions with Materials and Atoms 225, 4 (2004).
[30] Z. E. T. F. V. S. Nikolaev, Sov. Phys.—JETP 24, 847 (1967).
[31] G. Lapicki and F. D. McDaniel, Physical Review A 22, 1896 (1980).
[32] G. Lapicki and W. Losonsky, Physical Review A 15, 896 (1977).
[33] N. Agmon, Journal of Chemical Education 65, 42 (1988).
[34] W. Brandt and G. Lapicki, Physical Review A 10, 474 (1974).
[35] M. H. Chen and B. Crasemann, Atomic data and nuclear data tables 41, 257 (1989).
[36] N. Novikov and Y. A. Teplova, Physics Letters A 378, 1286 (2014).
[37] W. Brandt and G. Lapicki, Physical Review A 20, 465 (1979).
[38] S. Chatterjee, P. Sharma, D. Mitra, and T. Nandi, arXiv preprint arXiv:2103.08299v4 (2021).