Meron excitations in the $\nu = 1$ quantum Hall bilayer and the plasma analogy

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We study meron quasiparticle excitations in the $\nu = 1$ quantum Hall bilayer. Considering the well known single meron state, we introduce its effective form, valid in the longdistance limit. That enables us to propose two (and more) meron states in the same limit. Further, establishing a plasma analogy of the (111) ground state, we find the impurities that play the role of merons and derive meron charge distributions. Using the introduced meron constructions in generalized (mixed) ground states and corresponding plasmas for arbitrary distance between the layers, we calculate the interaction between the construction implied impurities. We also find a correspondence between the impurity interactions and meron interactions. This suggests a possible explanation of the deconfinement of the merons recently observed in the experiments.
\[
\rho_\uparrow(q) = \frac{V(q) n_\uparrow V(q)}{1 - nV(q)}, \tag{4}
\]

for \(\uparrow\) and \(\downarrow\) charge respectively. We immediately see that the total charge is screened,

\[
\rho_c(q) \sim \rho_\uparrow + \rho_\downarrow = \frac{V(q)}{1 - nV(q)}, \tag{5}
\]

and \(\lim_{q \to 0} \rho_c(q) = \text{Const}\), like in the usual Laughlin quasihole case, but the pseudospin charge \(\rho_s(q) \sim \rho_\uparrow - \rho_\downarrow = V(q)\) is unscreened growing as \(\ln(r)\) (if \(w = 0\)) with distance \(r\). Therefore the capacitive energy defined as

\[
E_c = \int d^2r (\rho_\uparrow - \rho_\downarrow)^2, \tag{6}
\]

which is in the first approximation proportional to the energy to excite the quasihole, is proportional to (up to logarithmic factors) the area of the system. This is the conclusion of the numerical study in [3]. Therefore the plasma analogy is able to reproduce the main result of the detailed investigation [4] (which helps us to eliminate from further consideration the constructions of the form in Eq. (1) as relevant excitations for the bilayer). The agreement does not come as a surprise if we analyze more closely the diagrams in Fig. 1. In them we are justifiably using the screening properties of the charge channel, which behaves as a plasma. Because of this, from now on, we will refer to the statistical model based on the (111) state as plasma.

In the second quantization formalism the meron excitation of the pseudospin theory [3] that parallels the construction in Eq. (1) is

\[
|\Psi_m(w = 0) > = \prod_{m=0}^{N-1} (c_m^{\dagger,\uparrow} + c_m^{\dagger,\downarrow}) |0 > \tag{7}
\]

\[
\int_0^{\infty} dr \ r |z|^2 |\Phi_m(z)|^2 \equiv 2(m+1) \int_0^{\infty} dr \ r |\Phi_m(z)|^2 + 2(m+1) C \int_0^{\infty} dr \ r |\Phi_m(z)|^2. \tag{10}
\]

The condition and implied expansion are appropriate when we look for charge density distributions of the state in Eq. (10). In the limit \(m \to \infty\) we get \(C = 0.8\) as can be seen in Fig. 2. In Eq. (10) the searched for correction is for the orbital \(\Phi_m\) right at the longdistance cut-off \(R = \sqrt{2(m+1)} = \sqrt{2N}\) (i.e., the radius of the system). Therefore, in this approximation, in the approach with fixed up and down number of particles, the \(\uparrow\) charge density of the excitation in Eq. (7) can be extracted from the following integral, with \(|z_{\uparrow}| \equiv r\),

\[
\rho_{w=0}(r) \sim \int d^2z_{\uparrow} \int d^2z_{\downarrow} \exp\{\sum_i \frac{C}{|z_{\uparrow}|} |\Psi_{111}(z_{\uparrow}, z_{\downarrow})|^2\}, \tag{11}
\]

and analogously for the \(\downarrow\) charge density. In this way, in the (111) plasma, we consider a new type of impurity which connects via the interaction \(C/|z_{\uparrow}|\) to the \(\uparrow\) particles of the plasma. In this sense we can propose the following longdistance form of the meron excitation
This construction can be easily generalized to the case where more than one meron (of both vorticities) are present. Namely, we change the way impurity connects to the plasma by switching from \( V \) to \( V_m \). In this way \( \rho_\uparrow(q) \approx \frac{1}{q^2} V_m(q) \) and \( \rho_\downarrow(q) \approx -\frac{1}{q^2} V_m(q) \) in the \( q \to 0 \) limit and for \( n_\uparrow = n_\downarrow \), resulting in \( E_o \sim \ln R \), where \( R \) is the radius of the system, for the energy to excite a meron, in agreement with the XY model considerations and pseudospin theory [3].

By considering the new impurities in the (111) plasma and applying the plasma techniques, we can prove the usual XY model logarithmic interactions between them, which is a result without an obvious connection with the physics and XY model of the bilayer. By considering also a pair of the old impurities (that follow from the construction in Eq. (1)), with same charge and opposite vorticity, we can find that their interaction energy in the plasma grows quadratically as a function of distance. It was found in [4], in numerics, that their real (capacitive) interaction energy behaves in the same way. This all again shows that whenever we have an underlying bosonic analogy and corresponding quasiparticles, like for the Laughlin states [12], or transparent, like in the bilayer case [13], the corresponding plasma has non-linear interaction energy laws, up to the value of couplings, to the interactions among quasiparticles in the quantum Hall systems.

The mixed states proposed as the ground states at finite (not small) \( d \) as mixtures of composite bosons of the (111) state and composite fermions of the nearby phase of two decoupled Fermi-liquid-like states can be expressed as

\[
\Psi_o = \mathcal{P} \mathcal{A} \left( \prod_{i<j} (z_i \uparrow - z_j \uparrow) \prod_{k<l} (z_k \downarrow - z_l \downarrow) \prod_{p,q} (z_{p \uparrow} \downarrow - z_{q \uparrow} \downarrow) \right) \Phi_i^\uparrow(w_1 \uparrow, w_2 \uparrow) \Phi_i^\downarrow(w_1 \downarrow, w_2 \downarrow) \prod_{k,l} (z_k \downarrow - w_l \downarrow) \prod_{p,q} (z_{p \uparrow} \downarrow - w_{q \uparrow} \downarrow) \Psi_{111}(z_\uparrow, z_\downarrow) .
\]

\[
(12)
\]

\[
(13)
\]

\[
(14)
\]

We will take that the number of \( \uparrow \) and \( \downarrow \) bosons is the same and neglect the antisymmetrizer in \( \Psi_o \). Then, the charge distribution of \( \uparrow \) charge can be found, first considering bosonic part, which has only a single connection and contribution \( \sim V_m(q) \) because of the alternating sign in Eq. (13) and fermionic part that we symbolically depicted in Fig. 3. In Fig. 3 the doubly wriggly line denotes \( V_m(q) \), and the crossed circle, twice the static

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\[
(10)
\]
structure, \( s_{\uparrow}(q) \), of the Fermi gas. Though simple, the final contributions are results of massive cancellations that follow from different interaction signs in the way quasiparticles connect to the plasma. Examining the contributions, we find that despite the presence of the composite fermions and their screening, the \( \uparrow (\downarrow) \)

\[
V_{\text{int}}(q) = \frac{V_{m}(q)V(q)(n_{\uparrow}n_{\downarrow} + n_{\uparrow}s_{\uparrow}(q) + n_{\downarrow}s_{\downarrow}(q))}{1 - V(q)(n + 2s(q))} + \frac{V_{m}(q)s^{2}(q)}{1 - V^{2}(q)(2s(q))^{2}} \{ nV^{2}(q) + \frac{n^{2} + n(2s_{\uparrow}(q) + 2s_{\downarrow}(q))}{1 - V^{2}(q)(2s(q))^{2}} V^{3}(q) \},
\]

where \( s_{\uparrow}(q) = s_{\downarrow}(q) = s(q) \) and \( n_{\uparrow}, n_{\downarrow} \), and \( n = n_{\uparrow} + n_{\downarrow} \) denote bosonic densities. This can be obtained straightforwardly with the help of diagrams. In the \( q \to 0 \) limit we have,

\[
V_{\text{int}}(q) \to (-1)V_{m}^{2}\left(\frac{n_{\uparrow}n_{\downarrow}}{n} + \frac{2n_{\uparrow}n_{\downarrow}}{n^{2}} - \frac{1}{2}\right)V_{m}^{2}(q)s(q),
\]

i.e. the leading is the attractive ln\( r \) interaction, and the correction is a \( \frac{1}{2} \) interaction due to the screening by composite fermions that vanishes in the \( n_{\uparrow} = n_{\downarrow} \) case \[14\].

This is a result in the formal setting of plasma analogy, but very likely, due to the mentioned correspondence, also a relevant conclusion for the interaction between two merons in the quantum Hall system. Then please note again that \( n_{\uparrow}, n_{\downarrow} \), and \( n \) are not overall densities but reduced, due to the presence of fermions, bosonic densities. Therefore, though the type of interaction (ln\( r \)) stays the same, the coupling strength is weaker due to its proportionality to the density of bosons.

Certainly, it is appropriate to check the amounts of the screening charges of a single meron construction in a mixed state that is the generalization of the construction in Eq. \[12\]. Again with the help of diagrams, they can be easily found, and we will just state their limiting, \( q \to 0 \), behavior:

\[
\rho_{\uparrow}(q) \to \frac{V_{m}(q)}{2} + \frac{n_{\uparrow} - n_{\downarrow}}{n}V_{m}(q),
\]

and,

\[
\rho_{\downarrow}(q) \to \frac{V_{m}(q)}{2} - \frac{2n_{\downarrow}}{n}V_{m}(q).
\]

Therefore, in the \( n_{\uparrow} = n_{\downarrow} \) case, the limits do not differ from the case without composite fermions.

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[14] Very likely this correction is not a final result because of the known difficulties of the weakly-screening plasma approach [10] in generating nonanalytic ($\sim |q|^n, n$ - odd) correction to a leading behavior.