U.S. Oil Reserves and Peak Oil

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Abstract
We use calculus methods to estimate the quantity of U.S. oil reserves. We consider a model that consists of an exponential function with four unknown constants. We fit real oil production data to determine the unknown constants. With the constants determined we use the function to find the year in which the U.S. oil production reached its peak. We also estimate the amount of petroleum produced until the end of 2006, and the undiscovered oil reserves to be produced in the future.

Keywords
Oil Reserves, Peak Oil Production, Conservation

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PROBLEM STATEMENT

In 1956, M. King Hubbert, an American geologist, postulated that US oil reserves are finite and he predicted that the oil productions would reach a maximum in the early 1970s. Therefore, the total petroleum produced is the area under the production curve in Figure 1.

Figure 1: U.S. oil production since 1859.

Making use of the mathematical model and the real data of U.S. oil production, the goal of this project is to estimate the quantity of undiscovered U.S. oil reserves to available to the oil industry in America in the future.

MOTIVATION

Recently, society has become concerned about the lack of energy resources available to supply the energy needs of large populations. Although renewable energy sources are more environmentally friendly and appropriated to the long-term progress, fossil fuels still play an essential role in most industries. Specifically in America, oil reserves are one of the most important natural resources for the development of the energy industry, transportation, and nearly all manufacturing industries. As petroleum reserves are a finite resource, the undiscovered
oil reserves in the U.S. are a top priority to the American government, economists, and industrial policy makers ((EIA), Crude Oil Production). With our calculations, we have determined the undiscovered oil reserves of America are about 38 billion barrels, roughly enough for the oil industry to continue at their current pace for ten more years.

**Mathematical Description and Solution Approach**

Since the production curve (Figure 1) has the shape of the graph of an exponential function we fit a model of the form:

\[ y(t) = \frac{Ae^{-b(t-c)}}{1+ae^{-b(t-c)}} \]  

where \( y(t) \) gives the annual production of oil (in thousands of barrels) in year \( t \), and \( A, a, b, c \) are constants.

First, we simultaneously graph the model of the real data and the projected data according to the model depending on the value of the four unknown constants. As each constant plays a significant role in model, we change each constant step by step to attempt to find a good fit for the data. Figure 2 shows the graph of the real data with the projected data we found to be the best fit.

Subsequently, we use SOLVER, an Excel Tool, to minimize the differences of real data by changing our estimated constants. The minimum deviation is calculated by the formula:

\[ \sigma = \min \left\{ \sum_{i=1}^{n} (y_{min}(t_i) - y_i)^2 \right\} \]  

The determination of the constants \( A, a, b \) and \( c \) is addressed later in the Discussion section.
To find the year in which the model predicts the peak production of oil, we differentiate (1) to obtain:

\[ y'(t) = \frac{e^{-b(t-c)}(A_{\alpha}b e^{-b(t-c)} - A_{\beta}b)}{(1+e^{-b(t-c)})^3} \]  

(3)

To find the extreme values of \( y(t) \) we set \( y'(t) = 0 \). From (3) we see \( y'(t) = 0 \) implies:

\[ A_{\alpha}b e^{-b(t-c)} - A_{\beta}b = 0 \]

(4)

and

\[ t = c + \frac{\ln(\alpha)}{b}. \]

(5)

With the values of our constants (given in the Nomenclature table) (5) gives:

\[ t_{max} = 1977 \text{ and } y_{max} = 3,284,390 \text{ (thousand barrels)}. \]  

(6)

Second, we integrate equation (1) to estimate the US petroleum that was once available. Mathematically, integrating the production curve from \(-\infty\) to \(\infty\) provides the total amount of petroleum, including amount already recovered, known reserves and unknown reserves.
We calculate:

\[
\int_{-\infty}^{\infty} y(t) dt = \lim_{m \to -\infty} \int_{m}^{0} y(t) dt + \lim_{m \to \infty} \int_{0}^{m} y(t) dt \tag{7}
\]

and define:

\[
Y(t) = \frac{A}{ab(1+ae^{-b(t-c)})}. \tag{8}
\]

Note that \(\frac{d}{dt} Y(t) = y(t)\) so (7) becomes:

\[
\lim_{m \to -\infty} [Y(0) - Y(m)] + \lim_{m \to \infty} [Y(m) - Y(0)] = \frac{A}{ab}. \tag{9}
\]

Therefore, we obtain the total US petroleum reserves consists of 234,645,178.6 thousand barrels.

Third, we integrate equation (1) to obtain the total US petroleum that was produced up to the end of 2006. Technically, we integrate the area from \(-\infty\) to 2006 to provide the total amount of recovered crude oil through 2006. This is given by:

\[
\int_{-\infty}^{2006} \frac{Ae^{-b(t-c)}}{(1+ae^{-b(t-c)})^2} dt = \lim_{m \to -\infty} [Y(2006) - Y(m)] = 196,904,262 \tag{10}
\]

So the total amount of recovered crude oil through 2006 is estimated to be 196,904,262 thousand barrels. However, the oil industry first began to extract crude oil in the year 1859 so for a more realistic estimate we integrate the production curve from 1859 to 2006:

\[
\int_{1859}^{2006} \frac{Ae^{-b(t-c)}}{(1+ae^{-b(t-c)})^2} dt = Y(2006) - Y(1859) = 196,579,070 \tag{11}
\]

Finally, we subtract our value of US oil production from the total reserves to predict the undiscovered US petroleum reserves:

\[
234,645,178.6 - 196,579,070 = 38,066,108 \text{ (thousand barrels)} \tag{12}
\]

At the end of 2006, the reported US oil reserves totaled 21 billion barrels ((EIA), U.S. Crude Oil, Natural Gas, and Natural Gas Liquids Reserves.); our model prediction is roughly 38 million
barrels, which is much higher than the reported number. We note that the undiscovered reserves are an unknown quantity so it is quite difficult to evaluate the correct number. For example, USGS predicted the mean conventional Oil Reserves by May, 2011 are 45 billion barrels (Figure 5, Appendix C). Additionally, the U.S. Energy Information Administration’s graph shows that the oil reserves are about 48 billion barrels until 2035 (Figure 3).

**Figure 3:** Past and projected U.S. crude oil production by location.

**DISCUSSION**

As we have presented the general formula, the detailed equation with real constants will be presented in Appendix B, and the value of the four constants is given in the Nomenclature table. Not only do we try to have the best fit model by minimizing the standard deviation (3); we claim our constants give the best fit for this formula because they yield the minimum summation of the partial derivative with respect to each of the determined constants. (Detailed calculations in Appendix B)
First we define the residual at each data point \( t_i \) (see table 2 in the appendix):

\[
E_i = y_i - \frac{A e^{-b(t_i-c)}}{(1 + a e^{-b(t_i-c)})^2}
\]  

(13)

Now the sum of the square of the residuals \( S_r \) is given by:

\[
S_r = \sum_{i=1}^{n} E_i^2 = \sum_{i=1}^{n} y_i - \frac{A e^{-b(t_i-c)}}{(1 + a e^{-b(t_i-c)})^2}
\]  

(14)

Differentiating with respect to \( A \) yields:

\[
\frac{\partial S_r}{\partial A} = \sum_{i=1}^{n} 2 \left[ y_i - \frac{A e^{-b(t_i-c)}}{(1 + a e^{-b(t_i-c)})^2} \right] \left[ -\frac{e^{-b(t_i-c)}}{(1 + a e^{-b(t_i-c)})^2} \right] = -1.14875 \times 10^{-36} \approx 0
\]  

(15)

Differentiating with respect to \( a \) yields:

\[
\frac{\partial S_r}{\partial a} = \sum_{i=1}^{n} 2 \left[ y_i - \frac{A e^{-b(t_i-c)}}{(1 + a e^{-b(t_i-c)})^2} \right] \left[ 2Ae^{-2b(t_i-c)} \right] \left[ \frac{2Ae^{-2b(t_i-c)}}{(1 + a e^{-b(t_i-c)})^3} \right] = 1.71127 \times 10^{-74} \approx 0
\]  

(16)

Differentiating with respect to \( b \) yields:

\[
\frac{\partial S_r}{\partial b} = \sum_{i=1}^{n} 2 \left[ y_i - \frac{A e^{-b(t_i-c)}}{(1 + a e^{-b(t_i-c)})^2} \right] \left[ -A(t_i - c)e^{-b(t_i-c)}(1 - a e^{-b(t_i-c)}) \right] \left[ -\frac{A(t_i - c)e^{-b(t_i-c)}}{(1 + a e^{-b(t_i-c)})^3} \right] = -8.4472 \times 10^{-27} \approx 0
\]  

(17)

Lastly differentiating with respect to \( c \) yields:

\[
\frac{\partial S_r}{\partial c} = \sum_{i=1}^{n} 2 \left[ y_i - \frac{A e^{-b(t_i-c)}}{(1 + a e^{-b(t_i-c)})^2} \right] \left[ -\frac{A e^{-b(t_i-c)}(1 - a e^{-b(t_i-c)})}{(1 + a e^{-b(t_i-c)})^3} \right] = 2.5359 \times 10^{-31} \approx 0
\]  

(18)

We see in (15)-(18) the partial derivative with respect to each of our determined constants is essentially zero, confirming the constants provide a good fit for the data.
CONCLUSION AND RECOMMENDATIONS

The entirety of U.S. oil reserves is small enough that the oil industry must be concerned about the most effective strategy to maintain the oil supply in the future. However, the total oil reserves on the planet are predicted to be large enough for the development and needs of high populations over the next several decades. As a result of the oil wells becoming dryer, everyone should be informed about the scarcity of finite resources, especially petroleum in order to save energy resources as much as possible. Saving the precious oil reserves not only aids the long-lasting development of modern society, but also helps to protect the environment.
NOMENCLATURE

| Symbol | Description       | Units                      |
|--------|-------------------|----------------------------|
| $y(t)$ | US oil production | Thousands of barrels/year  |
| $t$    | Time              | Years                      |
| $A = 3,942,039$ | Constant         | Thousands of barrels       |
| $a = 0.3$ | Constant     | -                          |
| $b = 0.056$ | Constant     | -                          |
| $c = 1998$ | Constant       | -                          |

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## APPENDICES

### APPENDIX A - TABLES

| Year | Barrels (Thousands) |
|------|---------------------|
| 1859 | 2                   |
| 1860 | 500                 |
| 1861 | 2,114               |
| 1862 | 3,057               |
| 1863 | 2,611               |
| 1864 | 2,116               |
| 1865 | 2,498               |
| 1866 | 3,598               |
| 1867 | 3,347               |
| 1868 | 3,646               |
| 1869 | 4,215               |
| 1870 | 5,261               |
| 1871 | 5,205               |
| 1872 | 6,293               |
| 1873 | 9,894               |
| 1874 | 10,927              |
| 1875 | 12,163              |
| 1876 | 91,33               |
| 1877 | 13,350              |
| 1878 | 15,397              |
| 1879 | 199,14              |
| 1880 | 26,286              |
| 1881 | 27,661              |
| 1882 | 30,350              |
| 1883 | 23,450              |
| 1884 | 24,218              |
| 1885 | 21,859              |
| 1886 | 28,065              |
| 1887 | 28,283              |
| 1888 | 27,612              |
| 1889 | 35,164              |
| 1890 | 45,824              |
| 1891 | 54,293              |
| 1892 | 50,515              |
| 1893 | 48,431              |
| 1894 | 49,344              |
| 1895 | 52,892              |
| 1896 | 60,960              |

### Table 1: Oil Production in the United States from 1859 to 2010.

https://digitalcommons.usf.edu/ujmm/vol4/iss2/1
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### Table 2: Comparision of the actual US oil production from 1859 to 2009 and the modeled oil production with optimal parameters $A$, $a$, $b$, and $c$.  

| Year ($t_i$) | US Oil Production ($y_i$) | Estimated Production ($y(t_i)$) | $\frac{\partial y}{\partial A}$ | $\frac{\partial y}{\partial a}$ | $\frac{\partial y}{\partial b}$ | $\frac{\partial y}{\partial c}$ |
|--------------|--------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 1859         | 2                        | 18,186                        | -1.14 E-47                    | 7.63 E-89                      | -8.95 E-38                    | 2.51 E-42                     |
| 1869         | 4,215                    | 31,771                        | -4.19 E-44                    | 4.93 E-85                      | -3.29 E-34                    | 9.25 E-39                     |
| 1879         | 19,914                   | 55,418                        | -3.47 E-43                    | 7.14 E-84                      | -2.70 E-33                    | 7.65 E-38                     |
| 1889         | 35,164                   | 96,405                        | -1.07 E-42                    | 3.86 E-83                      | -8.32 E-33                    | 2.37 E-37                     |
| 1899         | 57,071                   | 166,917                       | -3.05 E-42                    | 1.92 E-82                      | -2.35 E-32                    | 6.72 E-37                     |
| 1909         | 183,171                  | 286,652                       | -1.71 E-41                    | 1.89 E-81                      | -1.31 E-31                    | 3.78 E-36                     |
| 1919         | 378,367                  | 485,435                       | -6.19 E-41                    | 1.20 E-80                      | -4.73 E-31                    | 1.37 E-35                     |
| 1929         | 1,007,323                | 802,861                       | -2.88 E-40                    | 9.77 E-80                      | -2.19 E-30                    | 6.37 E-35                     |
| 1939         | 1,264,256                | 1,277,124                     | -6.34 E-40                    | 3.76 E-79                      | -4.79 E-30                    | 1.40 E-34                     |
| 1949         | 1,841,940                | 1,910,155                     | -1.62 E-39                    | 1.68 E-78                      | -1.22 E-29                    | 3.57 E-34                     |
| 1959         | 2,574,590                | 2,607,257                     | -3.96 E-39                    | 7.18 E-78                      | -2.96 E-29                    | 8.73 E-34                     |
| 1969         | 3,371,751                | 3,144,300                     | -9.07 E-39                    | 2.88 E-77                      | -6.75 E-29                    | 2.00 E-33                     |
| 1979         | 3,121,310                | 3,268,994                     | -1.47 E-38                    | 8.18 E-77                      | -1.09 E-28                    | 3.24 E-33                     |
| 1989         | 2,778,773                | 2,913,363                     | -2.29 E-38                    | 2.23 E-76                      | -1.69 E-28                    | 5.06 E-33                     |
| 1999         | 2,146,732                | 2,262,035                     | -3.10 E-38                    | 5.28 E-76                      | -2.27 E-28                    | 6.84 E-33                     |
| 2009         | 1,956,596                | 1,576,742                     | -4.94 E-38                    | 1.48 E-75                      | -3.60 E-28                    | 1.09 E-32                     |
| **Sum:**     |                          |                               | **-1.15 E-36**               | **1.71 E-74**                  | **-8.45 E-27**               | **2.54 E-31**                 |
APPENDIX B - CALCULATIONS

With $A = 3942039$, $a = 0.3$, $b = 0.056$, $c = 1998$, we have:

$$y(t) = \frac{3942039e^{-0.056(t-1998)}}{(1 + 0.3e^{-0.056(t-1998)})^2}$$

Thus the model’s derivative is:

$$\frac{dy}{dt} = \frac{-220754.184e^{-0.056(t-1998)} + 66226.2552e^{-2 \times 0.056(t-1998)}}{(1 + 0.3e^{-0.056(t-1998)})^3}$$

$$\frac{dy}{dt} = e^{-0.056(t-1998)} \left(-220754.184 + 66226.2552e^{-0.056(t-1998)}\right)$$

$$\Rightarrow \left(-220754.184 + 66226.2552e^{-0.056(t-1998)}\right) = 0$$

Thus:

$$e^{-0.056(t-1998)} = \frac{1}{0.3}$$
APPENDIX C – FIGURES

Figure 4: Mean conventional oil resources by location as of October 2007. (U.S. Geological Survey)