Is Holographic Entropy and Gravity the result of Quantum Mechanics?

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ABSTRACT: In this paper we suggest a connection between quantum mechanics and Verlinde’s recently proposed entropic force theory for the laws of Newton. We propose an entropy based on the quantum mechanical probability density distribution. With the assumption that the holographic principle holds we propose that our suggested quantum entropy generalizes the Bekenstein entropy used by Verlinde in his approach. Based on this assumption we suggest that Verlinde’s entropic theory of gravity has a quantum mechanical origin. We establish a reformulation of the Newtonian potential for gravity based on this quantum mechanical entropy. We also discuss the notion of observation and the correspondence to classical physics. Finally we give a discussion, a number of open problems and some concluding remarks.

KEYWORDS: Holographic Principle, Quantum Mechanics, Thermodynamics.
1. Introduction

In a remarkable paper Verlinde recently proposed a framework for gravity as an entropic force [20]. This theory, while related to Jacobsson’s approach [9] and subsequent work by Padmanabhan [13, 14, 15], showed that Newtonian gravity easily could be obtained by using entropic and holographic arguments. The assumption was that space is emergent and that the holographic principle holds [20]. Bekenstein entropy was also a key component in his approach. He thus reversed the line of research, assuming that the holographic principle was underlying Newtonian physics [14]. The change of entropy was linked to the change of the Newtonian potential, this led to the conclusion that inertia might be equivalent to the lack of entropy gradients [20]. As Verlinde states, the holographic principle has not been easy to extract from the laws of Newton and Einstein because it is deeply hidden within them [20]. His paper attracted quite some attention and several papers from various fields of theoretical physics, including Loop Quantum Gravity and
quantum mechanics, have been published relating to its topic \([7, 16, 19, 21]\). A shortcoming of the theory was the unknown origin of the coupling constant \(\hbar\) \([1, 20]\). This coupling constant was added by Bekenstein \([1]\) in the 1970s mainly for dimensional reasons and has since remained a mystery. We will suggest an origin of this constant in this paper.

2. Entropy and the holographic principle

In Verlinde’s view space is mainly a storage place of information, which is associated with positions, movements and mass of matter \([2, 13, 14, 17, 18, 20]\). This information is displayed to us on a surface, a holographic screen. The information is stored in discrete bits on the screen and since the number of bits is limited we get holographic effects. This means that if there is more information on the inside than the amount of information accessible on the screen then information will be hidden from us as we observe the dynamics. This is the holographic principle. Thus the dynamics on the screen is governed by some unknown rules which only can utilize the information on the screen. Since information is stored on a screen this means that space is emergent in the normal direction of the screen. The microstates may be thought of having all sorts of physical attributes such as energy, temperature etc. This is then related, via entropy, to the information associated with the system \([20]\).

2.1 Entropy as a force

Bekenstein related the area of a black hole to the entropy of it by assuming that all information lost down a black hole must still be conserved and therefore contained in some measure \([1, 8]\). This laid the foundation for an emergent holographic view of physics \([20]\).

The connection between entropy and information is that the change of information \(I\) is the negative change of entropy \(S\):

\[
\Delta I = -\Delta S. \tag{2.1}
\]

Considering a small piece of a holographic screen and a particle with mass \(m\) approaching it from the side at which time has already emerged, then Verlinde concluded (unitilizing Bekensteins arguments) that the entropy associated with this process should be Bekenstein entropy with an extra factor of \(2\pi\) \([20]\):

\[
\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x. \tag{2.2}
\]

Here \(k_B\) is Boltzmanns constant and the factor \(2\pi\) was added for reasons to be clear in connection with the gravitational force. Furthermore, an entropic force is a macroscopic force that originates in a system with many degrees of freedom by the universe’s statistical tendency to maximize its entropy. An entropic force \(F\) is defined as \([20]\):

\[
F \Delta x = T \Delta S \tag{2.3}
\]

where \(T\) is temperature. In order to relate the entropy to the screen the maximum number of bits \(N\) that can be associated with a screen is then assumed to be:

\[
N = \frac{A c^3}{G \hbar} = \frac{4\pi R^2 c^3}{G \hbar}, \tag{2.4}
\]
where \( A = 4\pi R^2 \) is the area of the screen. The temperature can be determined from the equipartition rule (2.3) [13, 14]:
\[
E = \frac{1}{2} N k_B T, \tag{2.5}
\]
which is the average energy per bit. We shall also assume the mass-energy relation:
\[
E = Mc^2. \tag{2.6}
\]
In a straightforward way these equations yield the gravitational force:
\[
F = G \frac{Mm}{R^2}. \tag{2.7}
\]
This is a surprising result considering it practically comes from first principles. In addition to this Verlinde also discusses the nature of inertia and via the equipartition rule for single a particle he gets:
\[
mc^2 = \frac{1}{2} nk_B T. \tag{2.8}
\]
Here \( n \) is the number of bits associated with a particle. He associated this with the Unruh effect:
\[
k_B T = \frac{1}{2\pi} \frac{\hbar a}{c} \tag{2.9}
\]
where \( a \) is the acceleration, which can be set equal to the gradient of the Newtonian potential:
\[
a \equiv \nabla \Phi. \tag{2.10}
\]
From this Verlinde derives the following relation [20]:
\[
\frac{\Delta S}{n} = -k_B a \frac{\Delta x}{2c^2} = -k_B \frac{\Delta \phi}{2c^2}. \tag{2.11}
\]
A general conclusion here is that the change of the Newtonian potential \( \phi \) is related to the change of entropy \( S \). Verlinde continued and generalized these concepts to a relativistic version of this entropic gravitational theory which for strong fields turns out to be equivalent to Einstein field equations, see [3, 20] for more information. His general conclusion is that inertia is due to the lack of entropy gradients, and conversely that gravity is due to the presence of them [20].

3. Quantum mechanics and entropy

Quantum mechanics has historically a number of different but equivalent approaches [13]. The perhaps most canonical is the path-integral formulation made by Feynman [6, 12]. A Quantum mechanical wave function \( \psi \) is linked to the classical action \( A \) via the relation:
\[
\psi = Re^{i\frac{A}{\hbar}}, \tag{3.1}
\]
where \( R^2 = \psi^* \psi = |\psi|^2 \) is the probability density distribution (\( \psi^\dagger \) is the complex conjugate of \( \psi \)) [3]. Feynman’s insight was that any quantum mechanical system is the sum of all
complex amplitudes relating to a particle's paths from one point to another [6]. This meant in practice:

$$\psi = Re^{i \frac{A_n}{\hbar}} = \sum_n e^{i \frac{A_n}{\hbar}},$$  \hspace{1cm} (3.2)

where the sum goes over all actions of the possible paths of a particle from one point to another. It has many similarities with the partition function of statistical mechanics [12]. We have the definition of the action as the time-integral over the lagrangian:

$$A_n = \int L_n dt.$$  \hspace{1cm} (3.3)

The probability density for a particle is defined as:

$$\rho = \psi \psi^\dagger = |\psi|^2,$$  \hspace{1cm} (3.4)

where $$\psi^\dagger$$ is the complex conjugate of $$\psi$$. When integrating (3.4) one gets the probability of a state in a particular domain of space. We shall here assume that the integration be over the volume $$V_S$$ inside a given holographic screen gives unity:

$$\int_{V_S} |\psi|^2 dV = 1.$$  \hspace{1cm} (3.5)

This excludes the possibility of the particle being outside the screen. The application of this assumption will be apparent when we discuss the nature of observation and correspondence to classical physics in section 5. Via the Feynman approach (3.2) to quantum mechanics we can conclude that $$|\psi|^2$$ is related to a sum of states of a quantum system. In the single particle situation it contains information regarding the probability of position of the particle. In light of this we suggest that the probability density $$|\psi|^2$$ is in fact related to a partition function $$Z$$ for different possible states:

$$Z \equiv |\psi|^{-2}.$$  \hspace{1cm} (3.6)

Although the particular sum of the partition function is not known it is no stretch of imagination to assume that at least one exists for every physical system, especially considering the structure of the Feynman approach to quantum mechanics (3.2). If we assume that the partition function (3.6) holds we can construct an entropy:

$$S = k_B \frac{\partial (T \ln(Z))}{\partial T} = k_B \frac{\partial (T \ln(|\psi|^{-2}))}{\partial T},$$  \hspace{1cm} (3.7)

which shall be referred to as the quantum entropy associated with a physical system. In those cases where $$\ln(Z)$$ are independent of $$T$$ in some way we have a simplified entropy:

$$S = k_B \ln(Z) = -2k_B \ln|\psi|.$$  \hspace{1cm} (3.8)

This relation between $$|\psi|$$ and $$S$$ will be used as a generalization of the Bekenstein entropy throughout the rest of this paper. Note that the entropy here is space dependent.
3.1 Single stationary particle entropy

Let's take the particular case of a single particle at rest to see how the entropy works. We have the Klein-Gordon equation from relativistic quantum mechanics as follows [3]:

\[
\left( \nabla^2 - \frac{1}{c^2} \frac{\partial}{\partial t} \right) \psi = \frac{m^2 c^2}{\hbar^2} \psi,
\]

which for a stationary particle becomes:

\[
\nabla^2 \psi = \frac{m^2 c^2}{\hbar^2} \psi.
\]

This equation has the solution (that is square integrable):

\[
\psi(x) = Ae^{-mc \frac{\hbar}{c} x},
\]

where \( A \) is a normalization constant and we consider \( x \) the radius outwards from the classical position of the particle. Note that in the case where \( x \) is considered the radius a very small potential \( \hbar c/r \) will be apparent in both the Schrödinger and Klein-Gordon equations, it is a potential which in most situations can completely be omitted. If we insert (3.11) into (3.7) we get:

\[
S = k_B \partial T \ln |\psi|^2 = -2k_B \partial T \left( -\frac{mc}{\hbar} x + \ln(A) \right),
\]

here we shall have to use the temperature from the equipartition law (to be used and derived in section 4):

\[
T = \frac{\hbar GM}{2kBc^2x^2}
\]

for an external mass \( M \). If we evaluate (3.12) we get:

\[
S = -2k_B \left( -\frac{mc}{\hbar} x + \ln(A) \right).
\]

Note that when using the particular temperature (3.13), which is canonical here, and a \( |\psi| \) that is exponentially decaying, most of the time (like in this case) the special entropy solution (3.8) will work. We shall therefore use the special entropy solution from here on. Now if we return to (3.14) and look at the difference in entropy \( \Delta S = S_1 - S_2 \) from the difference in \( \Delta x = x_1 - x_2 \) we get:

\[
S_1 - S_2 = \Delta S = 2k_B \frac{mc}{\hbar} (x_1 - x_2) - \ln(A) - \ln(A)
\]

The \( \ln(A) \)-terms disappear and the negative sign vanishes for the right choice of direction on the difference \( \Delta x \) making (3.15):

\[
\Delta S = 2k_B \frac{mc}{\hbar} \Delta x
\]

which is equivalent to the entropy used in Verlinde’s approach (2.2) up to a factor of \( \pi \). Verlinde added the \( 2\pi \)-factor for the convenience of cancelling it in the pursuit of the
gravitational force \[20\]. In order to remedy this problem we redefine the proportionality constant between the number of bits on the screen and the entropy by substituting $\hbar \rightarrow \pi \hbar$:

$$N \equiv \frac{A c^3}{\pi \hbar G} \quad (3.17)$$

This move ought to be considered legal since the proportionality \[2.4\] was estimated \[1, 20\]. Thus we have concluded that in this framework we get the expression for Bekenstein’s entropy in the situation of a stationary particle. In order to make sense of what entropy is in this approach we shall apply it to Verlinde’s approach to gravity in section 4.

4. Newtonian gravity from quantum mechanics via entropy

In light of his discovery that entropy might be the source of gravity, Ted Jacobsson stated that a quantization of general relativity is physically as absurd as the quantization of for example the wave equation for sound in air \[9\]. In a similar fashion we shall not quantize gravity here, but rather construct gravity based on the quantum mechanical entropy which is analogous to the case of the sound waves in air where the underlying microstates are quantum mechanical and the macroscopical wave is derived from them. Let’s take the equation for entropic force:

$$F \Delta x = T \Delta S \quad (4.1)$$

and for infinitesimal displacements we have the integral form:

$$U = \int F dx = \int T dS. \quad (4.2)$$

The potential energy is the result of the integral of the temperature over entropy in the emergent direction (normal to the screen). We assume that the particle subject to force creating the potential energy $U$ has mass $m$ and that this potential energy can be defined as the product of a potential $\phi$ and the mass $m$:

$$U = m \phi \quad (4.3)$$

The temperature $T$ on the screen in \(4.1\) is defined via the equipartition rule of all energy inside a holographic screen:

$$E = Mc^2 = \frac{1}{2} k_B n T, \quad (4.4)$$

and if we utilize the area to entropy relation \(3.17\) and rearrange for $T$ we get:

$$T = \frac{\hbar GM}{2 k_B c r^2}, \quad (4.5)$$

where $r$ is the radius of the screen. This is the temperature of the screen at radius $r$. With the aid of our entropy defined from quantum mechanics \(3.7\):

$$S = -2 k_B ln|\psi|, \quad (4.6)$$
we may re-express the entropy expression (3.16) on a differential formulation:

$$dS = -2k_B d(ln|\psi|) = -2k_B |\psi|^{-1} d|\psi|.$$  \hspace{1cm} (4.7)

If we insert (4.7) in (4.2) we arrive at a general potential energy $U$ for a particle:

$$U = m\phi = - \int \frac{hGM}{cr^2} d(ln|\psi|) = - \int \frac{hGM}{cr^2 |\psi|} d|\psi| = \frac{h}{c} \int \frac{\nabla \phi_N}{|\psi|} d|\psi|,$$  \hspace{1cm} (4.8)

where $\phi_N$ is the Newtonian potential. A generalized potential energy emerges as $U$. Note here that this for the single particle can be regarded as a generalization of the potential:

$$\phi = \frac{h}{mc} \int \frac{\nabla \phi_N}{|\psi|} d|\psi|.$$  \hspace{1cm} (4.9)

If we take the special case of the single particle solution (3.11) and insert it into (4.9) terms cancel and we get:

$$m\phi = m\phi_N = G\frac{Mm}{r}$$  \hspace{1cm} (4.10)

which is the Newtonian potential energy, just as in Verlinde’s approach [20]. Also, the gradient of the force is naturally identified with the gradient of the Newtonian potential in this special case:

$$\nabla \phi = \nabla \phi_N = -\frac{GM}{r^2}$$  \hspace{1cm} (4.11)

for which we have Poisson’s equation:

$$\nabla^2 \phi = 4\pi G \rho.$$  \hspace{1cm} (4.12)

Thus we have derived Poisson’s equation for gravity via the stationary solution of a single particle (3.11). The expression (4.12) can also be reformulated as Gauss’s law:

$$M = \frac{1}{4\pi G} \int_S \nabla \phi \cdot dA.$$  \hspace{1cm} (4.13)

We have the general potential (which is a generalization of the Newtonian potential) (4.9):

$$\phi = \frac{h}{mc} \int \frac{\nabla \phi_N}{|\psi|} d|\psi|.$$  \hspace{1cm} (4.14)

It should be stressed here that this potential is the potential that the single particle with mass $m$ experiences, so in reality the potential is directly coupled to the particular force of the particle. This reformulation of the Newtonian potential could be used in any quantum theory as an additional potential in order to incorporate gravity up to some generalized Newtonian limit. The Hamiltonian $\mathcal{H}$ should be supplied with the extra term containing the potential in order to contain gravity:

$$\mathcal{H} \rightarrow \mathcal{H} + \int TdS = \mathcal{H} + \frac{h}{c} \int \frac{\nabla \phi_N}{|\psi|} d|\psi|.$$  \hspace{1cm} (4.15)
If we assume that we have a particle $m$ in a potential with a massive mass $M$ as above in (4.15) then we can insert it in for example the Schrödinger equation \[3\] which gives:

\[
\frac{i\hbar}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + \int TdS = -\frac{\hbar^2}{2m} \nabla^2 \psi + \frac{\hbar}{c} \left( \int \frac{\nabla \phi_N}{|\psi|} d|\psi| \right) \psi.
\] (4.16)

Although we used the relativistic approach with the Klein-Gordon equation in order to construct the single stationary particle solution this equation should give interesting results within some reasonable limits. Note that the gravitational potential that arises here is only due to the entropic force acting on a particle, so gravity is not added, only the entropic force. We shall leave the relativistic approach to this quantum gravity for future research.

5. Observation and the correspondence principle

Our quantum approach to entropy suggests that information in a physical system is directly associated with its quantum mechanical entropy defined by the equality of the partition function with the inverse probability density distribution:

\[ Z \equiv |\psi|^{-2}. \] (5.1)

How does this relate to physical observation? If we condense the notion of observation in physics we can conclude that:

"Observing a physical system is obtaining information from it"

This asserts that an observer observes a system. In the holographic scenario we might imagine the observer being on the outside of a screen observing it. When observing a physical system it means that the entropy of the system will change because information is obtained on it. This information needs to originate from inside the screen and travel via some mediating particle, a photon for example, and transmit information to a detector outside the screen. In practice this means that the screen for the particle, when the particle is observed, has a very small radius. By the normalization condition:

\[
\int_{V_S} |\psi|^2 dV = 1,
\] (5.2)

where $V_S$ is the volume of the screen, we can see that as $V_S \to 0$ we have that:

\[ |\psi|^2(\vec{x}) \to \delta(\vec{x} - \vec{x}_0). \] (5.3)

Here $\delta(\vec{x} - \vec{x}_0)$ is the Dirac delta function where $\vec{x}_0$ is the point where the particle is detected. One should keep in mind that the entropy is related to the screen should be seen as a difference:

\[ \Delta S = S(r) - S(0). \] (5.4)
This means that entropy approaches zero for the particle as the radius of the sphere goes to zero (when observation takes place):

\[ V_S \to 0 \Rightarrow |\psi| \to \delta \Rightarrow \Delta S \to 0 \]  

(5.5)

Note here that even if \( \Delta S \to 0 \) it does not mean that \( \partial_r S \to 0 \) since \( \partial_r S \) is constant in for example the free single stationary solution (3.11). This entire situation is the equivalent to a wave function collapse.

### 5.1 Correspondence principle

The correspondence to classical physics is when all screens either become transparent (displaying all information) or collapse down to single points. This happens for example when \( h \to 0 \):

\[ \lim_{h \to 0} N = \lim_{h \to 0} \frac{A e^3}{\pi h G} = \infty. \]  

(5.6)

As all holographic effects vanish the probability becomes binary for each event for every observer. This means that the wave function \( |\psi| \) approaches the delta function (5.3) and all entropy vanishes. Generally one may conclude that in the classical scenario all entropy vanishes. A particular effect of this is that if all entropy vanishes, so does all quantum- and gravitational effects. That all quantum effects vanishes in the classical situation is not surprising, but that all gravitational effects vanish is quite remarkable.

### 6. Discussion

#### 6.1 Quantum mechanics, information and entropy

##### 6.1.1 The validity of quantum entropy

The interpretation of the quantum mechanical uncertainty as a form of entropy is perhaps not that strange considering that entropy in fact is practically equivalent to lack of information regarding an object. A main uncertainty in the approach proposed in this paper is the connection between the proposed quantum entropy arising in quantum mechanics and the thermodynamical entropy. In defence of this assumption, the entropy in quantum mechanics must in some way, for sure, be accounted for in the complete thermodynamical-entropy theory of physics. Especially if we consider the close information-to-physics connection arising from the holographic principle and black hole thermodynamics. Also, the relation between entropy and information hints that entropy should be additive in the final correct setup of physical entropy:

\[ \Delta S = \sum_i \Delta S_i = - \sum_i \Delta I_i \]  

(6.1)

which coupled with the indestructibility of information suggests that at least one term is quantum mechanical in origin, this provides a non-zero lowest estimate on the entropy contribution from quantum mechanics. In fact other types of contributions from quantum mechanics have been proposed recently [11].
6.1.2 The single particle solution

Our particular single particle solution (3.2) might be considered to be a special case given that it derives primarily from the Klein-Gordon equation, which does not hold for all types of particles. As far as free particle solutions goes the most reasonable probability density solutions that can be normalized will be those of exponential monotonic decreasing form. Whether it be linear or non-linear, the dynamics will be similar to the one proposed here, at least quantitatively. However there might be particular qualitative difference for certain situations, these are situations which are reasonably typical quantum-dominated situations such as particles in a box etc.

6.1.3 Prospects

An interesting aspect is that the principle law of the universe to minimize entropy gradients via the second law of thermodynamics will surely be important for the continued development of the relation between quantum mechanics and Verlinde’s theory. The junction of thermodynamics, quantum mechanics and relativity has not yet been fully understood, but it appears to have promising prospects for future research via Verlinde’s reversal of physics through the holographic foundation for Newtonian mechanics.

6.2 Conclusions

The great connections between matter and information made by Bekenstein, Hawking and others in the 1970s has turned out to have very interesting consequences. Verlinde’s framework for the origin of the laws of Newton including gravitation based on entropy is perhaps one of the greatest consequences of this. In what way space is emergent and how the holographic principle holds is starting to fall in to place. In this paper we have proposed an entropy arising from quantum mechanics and we have investigated its relation to Verlinde’s theory. This was then applied to generalize the Newtonian potential arising in Verlinde’s theory. There are many open problems remaining as this is a theory in progress. The study of multiple particle situations in the quantum mechanical approach should be interesting. A relativistic approach to quantum entropic gravity also needs to be established and investigated. The construction of various quantum field theories in curved spacetimes based on this approach should be of particular interest. Generally the connection to quantized gravity theories, if there are valid such [3], is left for further investigation. The nature of space and time as derived concepts, as spoken of by Verlinde [20], is not addressed in this paper and are features in need of investigation. The relation to AdS/CFT correspondence is also left open for further investigation. In conclusion, this paper, guided by a pure speculation, suggests that the gravitational attraction perhaps could be the result of a particular type of entropy arising in quantum mechanics.

6.3 Final comments

Verlinde proposed a theory of gravity where gravity no longer was a fundamental force, but rather an effect of entropy. His theory, as a reversal of physics research, is a remarkable framework that has many open questions and interesting consequences still to be
uncovered. This is why the greatest achievement of this paper is that it provides Verlinde’s remarkable theory with a possible physical explanation for the factor $\hbar$, which had previously been added by Bekenstein mainly for dimensional reasons.

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