NanoGrav 12.5-yr data and different stochastic Gravitational wave background sources

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Abstract

The NANOGrav Collaboration recently observed a strong evidence for a stochastic common-spectrum process in the pulsar timing data. In this work, we evaluate the possibility to interpret this process as stochastic gravitational wave backgrounds produced from first-order phase transitions, cosmic strings, domain walls, and large amplitude curvature perturbations.

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I. INTRODUCTION

The observation of gravitational waves (GWs) from the mergers of the binary black holes [1] and neutron stars [2] opens a fascinating new era to probe physics in the early Universe. Many new physics models beyond the Standard Model of the particle physics predict a first-order phase transition (FOPT) that can produce a stochastic gravitational wave background (SGWB). The spontaneous breaking of a (global or local) \(U(1)\) symmetry (a discrete symmetry) after the cosmic phase transition can produce a SGWB from the cosmic string decay (domain wall decay). The SGWB may also come from inflation, preheating, and the formation of primordial black holes (PBHs). The observation of the SGWBs produced by first-order phase transitions, cosmic strings and domain walls, would certainly provide crucial information on cosmology and high-energy physics, thus providing an amazing opportunity to explore new physics beyond the Standard Model of the particle physics [3-5]. The SGWB from the electroweak scale FOPT can be probed by the future space-based interferometers, such as LISA [6], TianQin [7], Taiji [8], BBO [9], and DECIGO [10]. The SGWB from the FOPT at a much higher scale can be probed by the ground-based interferometers, such as LIGO [11], CE [12], and ET [13, 14]. The pulsar timing array (PTA) experiments (such as EPTA [15], PPTA [16], and NANOGrav [17]) are supposed to detect or constrain the SGWBs from cosmic string, domain wall, and scalar perturbations.

Recently, with the 12.5-yr data set, the NANOGrav Collaboration reports a strong evidence of a stochastic common-spectrum process, although the evidence is not strong enough to claim a detection of GWs [18]. Motivated by the results, there are several attempts to interpret the result as GWs from super massive black holes [19-21], cosmic strings [22-24], dark phase transition [19, 25, 26], and PBH formation [19, 20]. In this work, we intend to perform a complete study on the implications of the possible SGWB detection by NANOGrav, and find out the model parameters space of different SGWB sources.

II. NANOGRAM 12.5-YR RESULTS VERSUS SGWB SOURCES

With the first five bins of the NANOGrav 12.5-yr data [18], roughly \(f \in (2.5 \times 10^{-9}, 1.2 \times 10^{-8})\) Hz, a power-law form of the characteristic strain is fitted by NANOGrav around a reference frequency \(f_{yr} = 1\text{yr}^{-1}\) [18]

\[
h_c(f) = A \left( \frac{f}{f_{yr}} \right)^{(3-\gamma)/2},
\]

(1)

with \((3-\gamma)/2\) being the index of the power-law. The corresponding GW energy spectrum is given by:

\[
\Omega_{GW}(f) = \frac{2\pi^2}{3H_0^2} f^2 h_c(f)^2 = \Omega_{yr} \left( \frac{f}{f_{yr}} \right)^n, \quad \text{where} \quad \Omega_{yr} = \frac{2\pi^2}{3H_0^2} A^2 f_{yr}^2.
\]

(2)

\(H_0\) is the Hubble constant and we adopt \(n = 5 - \gamma\). To estimate the prospective GW signal for different model parameters, we take a logarithmic derivative of the GW energy spectrum
from different SGWB sources with the following slope

\[ n = \left. \frac{d \log \Omega_{GW}(f)}{d \log f} \right|_{f=f_*} , \quad A = \sqrt{\frac{3H_0^2}{2\pi^2}} \frac{\Omega_{GW}(f_*) (f_*/f_*)^n}{f_{yr}^2} , \quad (3) \]

with the reference frequency \( f_* \approx 5.6 \times 10^{-9} \) Hz.

A. Various stochastic GW sources

As a preparation to investigate the possibility to explain the 12.5-yr NANOGrav result, we first review different SGWB sources and the corresponding calculation methods.

1. GWs from the FOPT

Two crucial parameters for the calculation of GWs from FOPT are the latent heat and the inverse duration of the phase transition: 1) the normalized by the radiative energy \( \frac{\Delta \rho}{\rho_R} \), with \( \Delta \rho \) being the released latent heat from the phase transition to the energy density of the plasma background; 2) the inverse time duration of the phase transition is parameterized by the parameter \( \frac{\beta}{H_*} = \frac{T}{d\left(S_2(T)/T\right)|_{T=T_n}} \). For the GW prospects we adopt the wall velocity \( v_b \approx 1 \). For the non run-away bubbles, the SGWB from FOPT is dominated by sound waves, and the GW energy spectrum reads \( \Omega_{sw} h^2(f) = 2.65 \times 10^{-6} \left( H_* \tau_{sw} \right) \left( \frac{\beta}{H_*} \right)^{-1} v_b \left( \frac{\kappa_\nu \alpha_{PT}}{1 + \alpha_{PT}} \right)^2 \left( \frac{g_*}{100} \right)^{-\frac{3}{2}} \left( \frac{f}{f_{sw}} \right) \left( \frac{7}{4 + 3 \left( f/f_{sw} \right)^2} \right)^{7/2} \) \( (4) \). Here, the factor \( \tau_{sw} = \min \left[ \frac{1}{H_0 T_f^0}, \frac{R_*}{U_f^0} \right] \) is adopted to account for the duration of the phase transition, where \( H_* R_* = v_b (8\pi)^{1/3} (\beta/H_*)^{-1} \) \( (51) \), and the root-mean-square (RMS) fluid velocity is \( \bar{U}_f^2 \approx \frac{3 \kappa_\nu \alpha}{4 + \alpha} \) \( (5) \). The factor \( \kappa_\nu \) is the fraction of the latent heat transferred into the kinetic energy of plasma, which is obtained by the hydrodynamic analysis \( (55) \). The peak frequency of sound waves locates at \( f_{sw} = 1.9 \times 10^{-5} \left( \frac{\beta}{H v_b} \right) \left( \frac{T_0}{100} \right) \left( \frac{g_*}{100} \right)^{1/6} \) Hz \( (6) \). For this study, we consider the plasma temperature being \( T_* \approx T_n \).

2. GWs from cosmic strings

The vastly adopted Nambu-Goto cosmic strings are characterized solely by the dimensionless parameter \( G\mu \), where \( G \) is Newton’s constant and \( \mu \) is the string tension. We calculate
the GW energy spectrum emitted from cosmic string networks following Ref. [61],

\[ \Omega_{\text{GW}}^a(f) = \sum_k \Omega_{\text{GW}}^{(k)}(f) , \]  

(7)

with each k-mode

\[ \Omega_{\text{GW}}^{(k)}(f) = \frac{2k}{\rho_c f} \mathcal{F}_\alpha \Gamma^{(k)} G \mu^2 \int_{t_F}^{t_0} \frac{d\tau}{a(\tau)} \left[ \frac{a(t)}{a(t_0)} \right]^5 \left[ \frac{a(t_k)}{a(t)} \right]^3 \Theta(t_k - t_F) , \]  

(8)

where \( \rho_c = 3H_0^2/8\pi G \) is the critical density, and we take \( \mathcal{F}_\alpha = 0.1 \) to characterize the fraction of the energy released by long strings and \( \alpha_{CS} = 0.1 \) to quantify the length of the string loops. The loop production efficiency is adopted as \( \Gamma \approx 50 \) [62]. Fourier modes of cusps [63] are [62, 64]:

\[ \Gamma^{(k)} = \Gamma_k - \frac{4}{3} \sum_{m=1}^{\infty} m^{-\frac{4}{3}} , \]  

(9)

here, \( \sum_{m=1}^{\infty} m^{-\frac{4}{3}} \approx 3.60 \) and \( \sum_k \Gamma^{(k)} \). The formation time of loops of the k-mode casts the form of

\[ t_i^{(k)}(\tilde{t}, f) = \frac{1}{\alpha_{CS} + \Gamma G \mu} \left[ \frac{2k}{f} a(t_0) \right] \left[ \frac{a(t)}{a(t_0)} \right] + \Gamma G \mu \tilde{t} , \]  

(10)

where \( \tilde{t} \) is the GW emission time. The cosmic string network reaches an attractor scaling solution after the formation time \( (t_F) \). When the small-scale structure of loops is dominated by cusps, the high mode relates to the low mode as:

\[ \Omega_{\text{GW}}^{(k)}(f) = \frac{\Gamma^{(k)}}{\Gamma^{(1)}} \Omega_{\text{GW}}^{(1)}(f/k) = k^{-4/3} \Omega_{\text{GW}}^{(1)}(f/k) . \]  

3. GWs from domain walls decay

For GWs from domain wall decay, the peak amplitude of GWs is estimated as [28, 29]

\[ \Omega_{\text{GW}}^{dw} h^2 (t_0)_{\text{peak}} \simeq 5.20 \times 10^{-20} \times \tilde{\epsilon}_{gw} A^4 \left( \frac{10.75}{g_*} \right)^{1/3} \left( \frac{10^{-2}}{\text{TeV}^3} \right)^4 \left( \frac{1\text{MeV}^4}{\Delta V} \right), \]  

(11)

at the present time \( t_0 \), with the peak frequency given by the Hubble parameter at the decay time \( t_{\text{dec}} \) [28]:

\[ f_{\text{dw}} (t_0)_{\text{peak}} = \frac{a(t_{\text{dec}})}{a(t_0)} H (t_{\text{dec}}) \simeq 3.99 \times 10^{-9} \text{Hz} A^{-1/2} \left( \frac{1\text{TeV}^3}{\sigma} \right)^{1/2} \left( \frac{\Delta V}{1\text{MeV}^4} \right)^{1/2} . \]  

(12)

The bias term \( \Delta V \) in Eqs. (12) [11] is introduced to explicitly break the discrete symmetry and lead to decay of domain walls. Domain walls should decay before the BBN, which yields the bound on the magnitude of the bias term

\[ \Delta V \gtrsim 6.6 \times 10^{-2} \text{MeV}^4 A \left( \frac{\sigma}{1\text{TeV}^3} \right) . \]  

(13)
We note that the magnitude of the bias term should be much less than the potential barrier \(\Delta V \ll V\), so that the discrete symmetry holds approximately and do not affect the phase transition dynamics. The surface energy density \(\sigma\) is a function of the discrete symmetry scale. In this study, we take the area parameter \(A = 1.2\) as in Ref. [29], the efficiency parameter \(\tilde{\epsilon}_{gw} = 0.7\) [28], and the effective relativistic degree of freedom at the domain wall decay time \(g_* = 10.75\) [29]. We use the slope of spectrum \(\Omega_{GW}^{dW} h^2 \propto f^3\) when \(f < f_{peak}\), and \(\Omega_{GW}^{dW} h^2 \propto f^{-1}\) when \(f \geq f_{peak}\) as suggested by the estimation of Ref. [28].

4. Scalar-induced GWs

Primordial curvature perturbations caused by quantum fluctuations during inflation can successfully seed large-scale structure and explain the cosmic microwave background (CMB) temperature anisotropies [92, 93]. The amplitude of the power spectrum of curvature perturbations on CMB scales is approximately \(2^{2} \times 10^{-9}\), however, the amplitude could be much larger on relatively small scales since the constraints are very loose [94–96]. Large amplitude curvature perturbations may cause a detectable scalar-induced SGWB after reentering the Hubble horizon due to the coupling between tensor and scalar metric perturbations at the second order. We use the methods derived in Ref. [97] to calculate the energy spectrum of the induced SGWB, which could explain the common-spectrum process detected by NANOGrav.

The GW production process happens almost around the horizon reentry of the corresponding modes, after that \(\Omega_{GW}\) soon reaches a constant. The energy spectrum \(\Omega_{GW}\) observed at \(t_0\) can be expressed in terms of the power spectrum of scalar perturbations \(P_R(k)\) as

\[
\Omega_{GW}^{si}(f) h^2 = \frac{1}{12} \Omega_{rad} h^2 \left(\frac{g_0}{g_*}\right)^{\frac{3}{2}} \times \int_0^\infty dv \int_{1-v}^{1+v} du \left(\frac{4v^2 - (1 + v^2 - u^2)^2}{4uv}\right)^2 P_R(2\pi fu)P_R(2\pi fv)I^2(u, v),
\]

(14)

where \(g_0\) and \(g_*\) are the effective relativistic degrees of freedom at \(t_0\) and at the time when the \(k\)-mode crosses the Hubble horizon, \(\Omega_{rad} h^2 = 4.2 \times 10^{-5}\) is the density fraction of radiation at \(t_0\), and

\[
I^2(u, v) = \frac{1}{2} \left(\frac{3}{4u^3v^3\pi}\right)^2 (u^2 + v^2 - 3)^2 \left\{ \left[ -4uv + (u^2 + v^2 - 3) \ln \left[ \frac{3 - (u + v)^2}{3 - (u - v)^2} \right] \right]^2 
+ \left[ \pi (u^2 + v^2 - 3) \Theta(u + v - \sqrt{3}) \right]^2 \right\}.
\]

(15)

Consider the case \(P_R(k)\) has a power-law form around \(k_* = \frac{2\pi}{1yr} = 20.6\) pc\(^{-1}\),

\[
P_R(k) = P_{R0} \left(\frac{k}{k_*}\right)^m \Theta(k - k_{min})\Theta(k_{max} - k),
\]

(16)
where $\Theta(x)$ is the Heaviside function and $k_\ast = \frac{2\pi}{1\,\text{yr}} = 20.6\,\text{pc}^{-1}$. To prevent overproduction of PBHs, we set cutoffs at $k_{\text{min}} = 0.03k_\ast$ and $k_{\text{max}} = 100k_\ast$ so that $P_R(k)$ obtains an upper bound.

From the quadratic $P_R$-dependence of $\Omega_{\text{GW}}(t_0, k)$ implied by Eq. (14), one can simply obtain $\Omega_{\text{GW}}(t_0, f = 1\,\text{yr}^{-1}) \propto P_R^2$ and $\Omega_{\text{GW}}(k) \propto k^{2m}$.

**B. Numerical results**

![Figure 1: Gravitational wave fits for FOPT.](image)

In the top-left (top-right) panel we take $T_n = 0.5\,\text{MeV}$ ($\beta/H_n = 40$) with $v_b = 1$, $\alpha_{\text{PT}} = 1$; in the bottom-left (bottom-right) panel we take $T_n = 1.5\,\text{MeV}$ ($\beta/H_n = 15$) with $v_b = 1$, $\alpha_{\text{PT}} = 0.3$.

In this section, we present the numerical results by fitting NANOGrav results at the 68% confidence level with GW energy spectrum from different sources. The top panel of the Fig. 1 shows that the FOPT can fit the NANOGrav result fairly well for $\beta/H \sim \mathcal{O}(10)$ at temperature $T_n \sim 0.1\,\text{MeV}$, when one consider a supersonic wall velocity $v_b = 1$ and a large latent heat $\alpha_{\text{PT}} = 1$. It was noted that for such low phase transition temperature, the phase transition should occur in the dark sector which interacts with the Standard Model pretty weakly [100]. The temperature difference between the dark sector and the visible sector should be taken into account to avoid the BBN constraints. Considering this effect, we also demonstrate the case for $v_b = 1$ and $\alpha_{\text{PT}} = 0.3$ in the bottom panels of Fig. 1. All these
plots indicate that a large magnitude of the GW energy spectrum and positive index $n$ can be obtained with large $\beta/H_n$ and $T_n$.

Figure 2: Gravitational wave fits for cosmic strings.

Figure 3: Gravitational wave fits for domain walls.

In Fig. 2, we show that when the SGWB is solely produced from the cosmic string network, the string tension should be $G\mu \sim \mathcal{O}(10^{-10})$ GeV ($G\mu \sim \mathcal{O}(10^{-8})$ GeV) for $\alpha = 0.1$ ($\alpha = 0.01$), which suggests the $U(1)$ symmetry breaking scale of the new physics is around $\eta \sim \mathcal{O}(10^{13})$ GeV ($\mathcal{O}(10^{14})$ GeV) for local strings (where $\mu \approx 2\pi \eta^2$ for the winding number $n = 1$). The magnitude of GWs from cosmic strings is characterized by the $\alpha_{CS}$, a large $\alpha_{CS}$ yields a higher magnitude of the GW spectrum. Fig. 3 shows that the surface energy density $\sigma \sim 10^{5-6}$ TeV$^3$ is required when the SGWB from domain walls dominates in the considered frequency region. As indicated by Eq. 14, a higher magnitude of GWs is obtained with a
large surface energy density $\sigma$. Utilizing $\sigma \sim 2\sqrt{2}\lambda\eta^3/3$ (here $\lambda$ and $\eta$ are the interaction couplings and breaking scale for the $Z_2$ discrete symmetry), one can estimate the breaking scale of the discrete symmetry being $\eta \sim 10^2$ TeV for $\lambda \sim \mathcal{O}(10^{-2})$.

Figure 4: Gravitational wave fits for scalar perturbations, where the power spectrum amplitude of scalar perturbations ($P_R$) is fixed, and $m$ denotes the spectral index of scalar perturbations.

In the case of scalar induced GWs, applying the power-law $P_R$ in Eq. (16), we find the scalar induced SGWB with $P_{R0} \sim 6 \times 10^{-3}$ and $m \sim -0.37$ can fit the NANOGrav data at the 68% confidence level, as shown in Fig. 4. It is well-known that large amplitude scalar perturbations are also responsible for the production of PBHs, which may constitute dark matter and provide merger events of black hole binaries [101–103]. The PBH abundance is less than $10^{-12}$, corresponding to the best fit $P_R$ of Gaussian curvature perturbations. However, if we consider non-Gaussian scalar perturbations reported in Ref. [104–106], the PBH abundance can be $10^{-3}$ to explain the merger rate observed by LIGO.

For comparison, we take several benchmarks (Table. I) from these SGWB sources that can fit the NANOGrav data at the 68% confidence level (Fig. 1, 2, 3, 4) and display the GW energy spectrum in Fig. 5 within the low frequency range of $[10^{-10}, 5 \times 10^{-8}]$ Hz. We found that the GW signal from all these SGWB sources that can fit the NANOGrav 12.5-yr results are in tension with the previous bound of PPTA [16] for some parameter spaces. Meanwhile, the measurements from future GW detectors are required to probe or constrain the GWs from phase transition with lower $\alpha_{PT}$ (weak phase transition strength), cosmic strings with small $\alpha_{CS}$, domain walls with small surface energy density $\sigma$, and scalar-induced GWs with small $P_{R0}$.

III. CONCLUSION AND DISCUSSION

In this paper, we study the possibility to interpret the NANOGrav result as different SGWB sources, including phase transition, cosmic string, domain wall, and primordial scalar
perturbations. Our study shows that all these SGWB sources can fit the low frequency five bins of the NANOGrav results fairly well, depending on parameter spaces of the GW models: 1) the GWs from phase transition in the dark sector with $T_n > 1$ MeV can successfully fit the NANOGrav data; 2) the symmetry breaking scale of new physics could be obtained from the fit of GWs from cosmic strings and domain walls; 3) the fit of the scalar induced GWs may hint the mass of PBHs and the possibility to constitute dark matter.

The fragmentation of the inflaton could produce GWs during preheating process [107, 108]. At the end of inflation, the inflaton start to oscillate around the minimum of the effective potential, and the perturbations of the inflaton are exponentially amplified due to parametric resonance. The peak value of $\Omega_{GW}$ comes within $10^{-9}$ to $10^{-11}$. However, since the peak frequency of the SGWB is proportional to the inflationary energy scale [109], to

| BM   | $\alpha_{CS}$ | $G\mu$ | $\alpha_{PT}$ | $\beta/H_n$ | $T_n$ (MeV) | $\sigma/\text{TeV}^3$ | $\Delta V/\text{MeV}^4$ | $P_{R0}$ | $m$   |
|------|---------------|--------|---------------|--------------|-------------|------------------|------------------|--------|------|
| $CS - BM_1$ | 0.1 | $10^{-9.5}$ | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| $CS - BM_2$ | 0.01 | $10^{-9}$ | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| $PT - BM_3$ | 0.0 | 0.0 | 0.3 | 15 | 2 | 6.50 | 2.81 | 0.0 | 0.0 | 0.0 | 0.0 |
| $PT - BM_4$ | 0.0 | 0.0 | 1 | 40 | 0.5 | 6.00 | 1.26 | 0.0 | 0.0 | 0.0 | 0.0 |
| $DW - BM_5$ | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| $DW - BM_6$ | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| $SI - BM_7$ | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| $SI - BM_8$ | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |

Table I: The eight benchmark points in Fig. 5.
generate a SGWB with peak frequency $10^{-9}$ Hz, the inflationary energy scale is below 100 MeV, which is too low to reheat the universe.

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