WORMHOLES AND COSMIC STRINGS

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ABSTRACT

A gravitational scenario is proposed where the euclidean action is invariant under the isotropic and homogeneous version of the euclidean $U(1)$ group of local transformations of the scale factor and scalar matter field, interpreting the trace of the second fundamental form as the gauge field. The model allows spontaneous breakdown of some involved symmetries, including possibly diffeomorphism invariance, and leads to the formation of topological defects. In particular, we consider here the case of the wormhole-induced formation of cosmic strings.

1. Introduction

This report proposes a possible framework where canonical theory of gravity coupled to matter can be interpreted as a gauge theory that may undergo spontaneous symmetry breaking. More precisely, we show that in a Robertson-Walker minisuperspace with scale factor $a$ and conformal time $\eta = \int \frac{d\tau}{a}$, the euclidean action for Hilbert-Einstein gravity conformally coupled to a scalar field $\Phi$ with generally nonzero mass $m$ satisfies an "axionic" symmetry that allows the action to become also invariant under the isotropic and homogeneous version of the euclidean $U(1)$ group of local transformations, provided we Wick rotate anti-clockwise and interpret the trace of the second fundamental form as the gauge field. The final lagrangian can be seen to have the form of a typical Higgs model, and spontaneous symmetry breaking provides the resulting baby universes with the maximal Higgs masses. We also show that in the resulting gauge theory the nucleation of baby universes can be regarded as a topological process leading to the formation of well known topological defeccts, such as cosmic strings. One could be tempted to suggest the generalization that there is a strong link between the topological changes leading to nucleation of baby universes in quantum gravity, and the emergence of all kinds
of topological defects that are allowed in some gauge theories with spontaneous symmetry breaking.

2. A Wormhole-Higgs Model

Using ordinary (matter-field) euclidean rotation, $t \to -i\tau$, the euclidean Hilbert-Einstein action for a conformally coupled scalar field in Robertson-Walker metric becomes

$$I = \frac{1}{2} \int d\eta N\left(-\frac{a'^2}{N^2} + \frac{\chi'^2}{N^2} - a^2 + \chi^2 + m^2\chi^2a^2\right),$$

where $N$ is the lapse function, $\chi = (\frac{4\pi G}{3})^{1/2}a\Phi$, and a prime denotes differentiation with respect to $\eta$. The equations of motion for $\chi$ and $a$ derived from (1), $\chi'' = \chi + m^2\chi^2a$ and $a'' = a - m^2\chi^2a$, should correspond to classical solutions that allow the effective gravitational constant $G_{eff} = G(1 - \frac{4\pi G}{3}\Phi^2)$ to change sign along the allowed phase-space region, and this would ultimately lead to negative energies and instabilities for perturbations about the classical solutions. This problem points to inadequacy of the naive insistence of a single general Wick rotation both for gravity and matter fields, and suggests thereby rotation in the opposite direction, $t \to +i\tau$, as the "correct" euclidean continuation when the coupled field is constant or pure gravity is dealt with.

Although (1) is invariant under the global transformation (rotation)

$$\Phi \to \Phi e^{i\alpha_0},$$
$$a \to ae^{-i\alpha_0},$$

with $\alpha_0$ a constant, it is not under the Robertson-Walker version of the corresponding local transformation where $\alpha_0 \to \alpha \equiv \alpha(\eta)$, unless asymptotically. Thus, applying transformation $a \to ae^{-i\alpha(\eta)}$, $\chi \to \chi$ to action (1), this is amounted with a new term $\triangle I(\alpha) = \frac{1}{2} \int d\eta Na^2\alpha'^2$, and from the equation of motion for $\alpha$ we obtain $\alpha' = \frac{B_0}{a}$, where $B_0$ is a constant. It follows $\triangle I(\alpha) = \frac{1}{2} \int d\eta NB_0^2/a^2$ which vanishes as $a \to \infty$. Note nevertheless that the set of the above two equations of motion for $\chi$ and $a$, as well as the hamiltonian constraint, $\delta I/\delta N$, and the action itself, remain all unchanged under the symmetry $\chi = ia$. It turns out that such a symmetry implies constant imaginary field $\Phi = i(\frac{4\pi G}{3})^{1/2}a\Phi$, i.e. a constant axionic field, and this validates rotation of time in the opposite direction, and should require introducing an additional boundary term, $I_B = Const.N \int_0^\infty d\eta$ in the euclidean action for transitions with constant $\Phi$. Consistency of the equations of motion and hamiltonian constraint with $\chi = ia$ leads then to $Const. = R_0^2$, with $R_0$ the integration constant for the first-integrated equations of motion which, after symmetry $\chi = ia$, can be written for the scale factor (in the gauge $N = 1$)

$$a'^2 = a'^2 + \frac{1}{2}m^2a^4 - R_0^2,$$

whose classical solution in terms of the Robertson-Walker time $\tau$ is

$$a(\tau) = m^{-1}\left[(1 + 2m^2R_0^2) \cosh(2^{1/2}m\tau) - 1\right]^{1/2}; \chi = ia,$$
which represents a nonsingular wormhole spacetime. The lorentzian continuation of (3) describes a baby universe with maximum size of the order (for very small $m$) $R_0$, with a singularity in the past at lorentzian time $t \simeq R_0$. Of course, for $m = 0$ we recover the usual Tolman-Hawking wormhole $a = R_0 \cosh \eta$, though still satisfying $\chi = ia$.

It is worth pointing out that if symmetry $\chi = ia$ holds and euclidean rotation is performed anti-clockwise, then remarkably the resulting euclidean action becomes positive-definite and hence free from the fatal conformal divergences which have many times been considered as one of the biggest problems in euclidean gravity.

Re-expressing (1) in terms of $\chi$ alone by using $\chi = ia$, after Wick rotating in the opposite direction, one can obtain the lagrangian density

$$L = \frac{1}{2} \frac{\dot{a}}{a}^2 \Phi^2 + \frac{1}{2} a \dot{\Phi} \dot{\Phi} + \frac{1}{2} \Phi^2,$$

$$-\frac{1}{2} \frac{\dot{a}}{a}^2 - \frac{1}{4} (\frac{m}{m_p})^2 \Phi^4 + \frac{m_p^2 R_0^2}{2 a^4},$$

(4)

where the overhead dot means differentiation with respect to $\tau$, $m_p$ denotes Planck mass and, if all symmetries are preserved, $\Phi = 0$.

It is only when the lagrangian density is expressed in the $\chi$-saturated form (4) that it becomes invariant under the above local transformations. Denoting $A = \frac{TrK}{3Q}$, where $TrK$ is the trace of the second fundamental form, $TrK = \frac{3a}{a}$, and $Q$ is the charge of the scalar field, such transformations can immediately be written in the more familiar form

$$\Phi \rightarrow \Phi e^{ia(\tau)}$$

(5)

$$A \rightarrow A - i\dot{a}(\tau),$$

(6)

which exactly are the transformations of the isotropic and homogeneous euclidean $(t \rightarrow +i\tau)$ version of the Abelian group $U(1)$ if $A$ is interpreted as the gauge field. Thus, if $\Phi$ is shifted by some real scalar $\varphi$, $\Phi \rightarrow \phi = \Phi + \varphi$, then upon substitution of $\Phi$ for $\phi$, (4) becomes a typical Higgs model for an arbitrary charge-$Q$ field $\phi$, a massless gauge field $A \propto K$, and variable "tachyonic" mass $\frac{1}{a}$. Without loss of generality one may regard that after spontaneous symmetry breaking it is only the originally zero real component of the scalar field that acquires some constant classical part. Actually, a full euclidean version of the usual isotropic and homogeneous Higgs mechanism for spontaneous symmetry breaking in the $U(1)$ group can be readily obtained. Therefore, after symmetry breaking in the unitary gauge, instead of the massless gauge scalar field $A$ and the axion field $\Phi$ one deals with the massive scalar field with mass $m_A = \frac{m_\varphi}{m_\varphi}$ and with the real scalar field $\varphi$ with the mass $m_\varphi = (\frac{2}{a^2})^{\frac{1}{2}}$. On the asymptotic region, $\eta \rightarrow \infty$, $A = \frac{1}{3} (\frac{m^2}{2a^2})^{\frac{1}{2}}$ and $m_A = m_\varphi = 0$, so the symmetry is restored on that region. The most stable broken phase condenses just at the wormhole neck where $A$ vanishes and the field masses become maximum, $m_A = \frac{Qm_\varphi}{m_\varphi R_0}$, $m_\varphi = (\frac{2}{R_0^2})^{\frac{1}{2}}$, so one should expect the branched baby universes to carry such maximum masses generated by the Higgs mechanism.
The masses generated in the above Higgs mechanism would provide the four-momentum constraints with nonzero r.h.s. terms, and it is in this sense that such a mechanism can be thought to break diffeomorphism invariance. In fact, Goldstone bosons appearing in the process could be regarded as gravitons arising from spontaneous breakdown of diffeomorphism invariance, and the trace of the second fundamental form - i.e. the gauge field - would play the role of the cosmological time needed to endow the Wheeler DeWitt equation with a r.h.s. energy operator. The gravitationally-induced Higgs mechanism discussed in this report suggests the possibility that baby universes carrying all the gauge mass of the broken phase be nucleated. For finite gauge coupling, masses $m_A$ and $m_\varphi$ will take on nonzero arbitrarily large values inside the baby universes, but become exactly zero on the asymptotic region where the symmetry is restored.

3. Gravitational Cosmic Strings

The gauge theory we have considered so far allows for spontaneous symmetry breaking, and the quantum-gravitational topological changes that lead to the nucleation of baby universes can be viewed as phase transitions where topological defects are produced. Such defects would be expected to manifest as e.g. euclidean cosmic strings, with static string metric:

$$ds^2 = e^{2\nu}d\tau^2 + e^{2\psi}d\phi^2 + e^{2\lambda}(d\rho^2 + dz^2), \quad (7)$$

where we have chosen the string to lie along the $z$ axis, and $\nu$, $\psi$ and $\lambda$ will now depend not just on the radial internal coordinate $\rho$, but also on the scale factor $a$, and $\phi = 0$ and $\phi = 2\pi$ are identified. Taking for the interior of the string a uniform energy density $\epsilon(a) > 0$, out to some cylindrical radius $\rho_0$, guarantees avoidance of any singularity if the transverse dimensions of the strings are comparable to the Planck length, and we assume $R_0 >> m_\gamma^{-1}$, $mR_0 << 1$. One can then use the thin-string approximation. Now, since we must choose

$$\epsilon = \frac{m_p}{2\pi^2 ma^4} \rho < \rho_0,$$

which is independent of $\rho$, and all components of the stress-tensor equal to zero, unless $T^{\tau}_{\tau} = T^z_z = -\epsilon$, from the Einstein equations for metric (7), we obtain an interior metric

$$ds^2 = d\tau^2 + d\rho^2 + dz^2 + m^*a^4 \sin^2\left(\frac{\rho}{m^*a^2}\right)d\phi^2, \quad (8)$$

where $m^* = \frac{\pi m}{4Gm_p}$, and $a$ is given by (3).

Metric (8) differs from the Hiscock internal metric by its dependence on the scale factor $a$; they only become the same at the wormhole neck, where $m^*a^2 = \frac{8\pi G\epsilon_0}{4}$, $\epsilon_0 = \frac{m_p}{2\pi^2 m^2 R_0^4}$. On the asymptotic region, $a \to \infty$, metric (8) reduces to an euclidean cylinder of radius $\rho$, so the interior of the string disappears. Following the same procedure as in Ref. 10, we also obtain an euclidean static, cylindrically
symmetric exterior metric

\[ ds^2 = d\tau^2 + dz^2 + dr^2 + (1 - 4\mu(a))r^2 d\phi^2, \]

where the mass per unit length will now depend on the wormhole scale factor

\[ \mu(a) = \frac{1}{4} [1 - \cos(\frac{\rho_0}{m^*a^2})]. \]

At the wormhole neck, (9) reduces to the euclidean version of the Vilenkin conical metric\(^{11}\). Asymptotically, \( \mu \to 0 \) and the string closes upon itself; i.e. on the asymptotic region there is neither exterior nor interior of strings: topological defects simply disappear. Rotating to the lorentzian region, \( \tau \to -it \), metrics (8) and (9), it can be seen that they become those of a usual cosmic string\(^{10,11}\) only when baby universes reach their maximum radius \( R_0 \).

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