Gravitational clustering in N-body simulations

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Abstract. In this talk we discuss some of the main theoretical problems in the understanding of the statistical properties of gravity. By means of N-body simulations we approach the problem of understanding the rôle of gravity in the clustering of a finite set of N-interacting particles which samples a portion of an infinite system. Through the use of the conditional average density, we study the evolution of the clustering for the system putting in evidence some interesting and not yet understood features of the process.

INTRODUCTION

The study of the features of gravitational clustering is one of the most challenging problems of classical physics, and an interesting example of interplay between statistical mechanics and astrophysics.

From the point of view of statistical mechanics, it is very hard to study the properties of an infinite system of self-gravitating particles. This is mainly due to the long range nature of gravitational potential, which is not shielded by the balance of far away charges, as e.g. in a plasma. Therefore all scales contribute to the evaluation of the potential energy of a particle. As a consequence of such difficulties, a satisfying thermodinamic equilibrium treatment of such systems is still lacking.

Most studies on self gravitating systems have been focusing on clustering properties of well-defined physical systems: from interstellar cold molecular clouds to globular clusters, to clusters and superclusters of galaxies. The range of scales on which such processes take place is impressive (from $10^{-1} \text{pc}$ to $10^{5} \text{pc}$). This also implies interactions with physical processes of different types, depending on the scale (from e.g. turbulence in cold molecular clouds to cosmological expansion above galaxy scale).

In spite of the large amount of analytical and numerical studies on gravitational clustering, it is very difficult to retrieve a clear picture of the statistical properties
of self-gravitating systems, exactly because of the richness of physical processes which they take into account.

What has especially raised our interest in the subject was the results of a series of analyses of space correlations in galaxy catalogues by Pietronero et al. [2]. They revealed that galaxies form a fractal set with well defined properties at least up to a certain scale. The discovery of the fractal characteristics of large scale structures in the universe raises a number of new and extremely appealing questions, which have nevertheless been almost completely neglected. For example we can ask whether gravity is a self organizing process, or if the formation of fractal properties depend on the choice of a particular set of initial conditions, or even one has to invoke some other physical process.

In a more general perspective, we intend to study in detail the characteristics of the evolution of self gravitating systems, from the point of view of statistical mechanics. In particular we focused on the evolution of spatial correlations in self-gravitating systems, and on the possibility that they may develop some kind of self similar spatial or temporal feature.

N-body simulations represent in fact a valuable experimental setting for a careful investigation of the dynamics of clustering. Current astrophysical simulations have reached a high level of refinement, both in resolution and in the number of different physical processes which they take into account. Such characteristics allow them to study in great detail the single physical problem for which they are developed. On the other hand they don’t allow a clarification the common rôle and the peculiarities of gravitational interaction.

Our approach has been completely different: we start from the simplest possible model and study in detail what happens during the clustering. Even such a simple model gives raise to a rich phenomenology, which has not been studied in a systematic way up to now.

THEORETICAL ISSUES ON GRAVITATIONAL CLUSTERING

The peculiar form of the gravitational potential makes the statistical properties of self-gravitating systems a very difficult subject to study. Two classes of problems arise: those due to the short range (i.e. $r \to 0$) divergence and those due to the long range (i.e. $r \to +\infty$) behaviour.

The former is not uncommon, since it is the same problem which arises in electromagnetism. The divergence would cause, e.g., the Boltzmann factor to diverge in the limit $r \to 0$. A typical prescription is to put a small distance cut-off in the potential. The physical nature of this cut-off may be due to many effects, e.g. the dynamical emergence of angular momentum barriers.

The long range behaviour is of much more concern and is, in fact, the problem. It is an easy exercise to verify that the energy of a particle in an infinite self-gravitating system diverges. This causes the energy to be non-extensive. As a
consequence, a thermodynamical limit is not achieved, since as the number of particles goes to infinity while keeping the density constant, the energy per particle diverges. Strangely enough, such a problem has not been fully appreciated by many physicists in the field (see e.g. [1]), as they try to avoid the long range divergence by putting the system in a box “as it is usually done with ordinary gas”. In fact, the difference is that in ordinary gas, when confining the system in a box, the energy per particle is equal to a constant plus a surface term that goes to zero in the thermodynamical limit. In self-gravitating systems, due to non extensivity, the energy per particle is neither a constant, nor the surface term goes to zero (in fact, it is of the same order of magnitude as the potential energy due to particles belonging to the system).

Another very interesting consequence, which is often not appreciated, is that the thermodynamical definition of temperature, as the parameter which controls the equilibrium of the system, doesn’t hold for a self gravitating system, since one cannot divide a system into smaller subsystems with the same thermodynamic properties of the larger system.

So far we have discussed the problems one encounters when one tries to build a theory for thermodynamic equilibrium. However we are much more interested in what happens out of equilibrium, during the evolution of a system. For example it is highly probable that any fractal state could not survive when the system would reach the equilibrium. Problems due to long range interactions are present all the same in most approaches, but they are more subtle to be put in evidence.

**N-BODY SIMULATIONS**

N-body simulation is one of the main tools for the study of gravitational clustering. They provide in fact the possibility to test theoretical models and to study the evolution of selfgravitating systems, a feature that is very hard to obtain from observations. Several different algorithms are used (see [3] for a review).

A brute force method to evaluate forces acting on particles in a simulation requires $O(N^2)$ computations, a feature which forbids them to be useful if large number of particles are required. The tree algorithm (see [4], and [5] for a detailed explanation) is an approximation scheme which involves $O(N\log N)$ computations, and whose level of accuracy can be tuned to match the problem requirements.

It is based on the decomposition of the simulation volume into a tree-like structure. Particles are grouped with well-defined rules in a hierarchy of clusters. The contribution of distant clusters to the force on a particle is taken into account by a multipole expansion of the potential generated by them. This approximation greatly reduces the computational expense of a simulation.

The algorithm has to be elaborated for our purposes by adding periodic boundary conditions, which allow a statistically equivalent environment for all the particles, and also include the long range interactions.

In our simulation we started from the simplest possible model:
i) random (white noise) initial positions of particles;
ii) no cosmological expansion;
iii) zero initial velocities;
iv) equal mass particles.

The main tool we use to analyse spatial correlation of the system is to measure the conditional average density $\Gamma^*(r)$. It is defined as $\Gamma^*(r) = \langle N(r) \rangle / V(r)$ where $\langle N(r) \rangle$ is the average number of points (not considering the center) in a sphere of radius $r$ and volume $V(r)$ centered on each point of the system.

RESULTS

All the simulations we have performed have the same average number density, $\bar{n} = 1$ which sets the physical scale of the system.

Figure 1 shows the evolution of $\Gamma^*(r)$ with time for a simulation of $32^3$ particles. This picture shows some interesting features, and allow us to focus on the different aspects of gravitational clustering. We can identify the region left of the dash-crossed line as a regime of mainly two particles interactions. In fact for such small

**FIGURE 1.** Evolution of $\Gamma^*(r)$ in a simulation with $32^3$ particles. Measures are taken each 500s. $\Gamma^*(r)$ moves from left to right with time. Left of the dash-crossed line we have less than one particle on average in a sphere of radius $r$ centered on a particle (the particle at the centre is not included).
scales, we find on average less than one particle in each sphere centered on each particle of the system. This region shows a very nice power law behaviour which seems stable for quite a long time, Such behaviour is not reported in the literature, as far as we know. The growing front of the correlations shows beautiful features of “temporal self-similar behaviour”, in the sense that if we rescale length scales with a function of time the curves collapse. The scaling factor is exponential. It is interesting to notice that while the exponential time dependence can be naively retrieved (with the right coefficients) using the solutions for the linear theory of the growth of density perturbation in a self gravitating fluid, the behaviour of the correlations is not well described by such solutions. In fact they predict a selfsimilar (in time) growth of the initial correlations of density fluctuations, until $\delta n/n < 1$. On the other hand, an inspection of the fig.1 shows that such growth takes place at large number density contrasts (i.e. $\delta n/n > 1$), and does not correspond to a pure amplification of the initial conditions. Anyway, such theory is considered very well verified in cosmological simulations. There may be several explanations. One possibility is that some of the approximations used to derive such solutions break earlier without expansion. Another interesting remark could be that in our simulations we don’t suppress the granular features of particles with large smoothing length, as it is customary in many cosmological simulations (see e.g. [6]). Such an observation paves the way to a very interesting issue which we are currently investigating: whether the granularity is an essential feature to the understanding of gravitational clustering, or which is the limit of validity of the fluid description of a self gravitating medium. Another region of interest would be the one between the two we have just described. With present data, we cannot resolve the behaviour of correlations in such regime, due to small numbers of particles. Since simulations are independent of finite size effects until $\Gamma^*(r)$ starts differing from $\bar{n}$ on scales of the order of the system size, the more particles we have, the larger the system size, the farther in time we can observe the evolution of correlations. For this reason we are now working on a parallelisation of our code, which would allow us to simulate systems with more than a million of particles.

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