Laboratory Investigation on the Stress-Dependent Anisotropic Shear Wave Velocity (Vs) and Coefficient of Lateral Earth Pressure at Rest ($K_0$) of Granular Materials

Guangbo Du $^{1,2}$, Nina Liu $^{1,2}$, Zhao Xia $^3$ and Xin Kang $^3$

$^1$Key Laboratory of Western China’s Mineral Resource and Geological Engineering, Ministry of Education, Chang’an University, Xi’an 710054, China
$^2$School of Geology Engineering and Geomatics, Chang’an University, Xi’an 710054, China
$^3$College of Civil Engineering, Hunan University, Changsha 410082, China

Correspondence should be addressed to Xin Kang; kangxin@hnu.edu.cn

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The stress-dependent $K_0$, $V_s$, and $V_h$ anisotropy and their correlations with sand for 1D consolidation stress were tested with a custom-designed floating-wall consolidometer-type Bender Element (BE) testing apparatus. $K_0$ of a soil sample was calculated using stress measurements through soil pressure transducers installed at the midsection of the consolidometer. The $V_s$ and $V_h$ anisotropy were measured by the bender elements installed in three orthogonal directions in the consolidometer, i.e., $v_h$, $h_v$, and $h_h$. Granular soils with different sizes and shapes were tested. The effects of the stress level, overconsolidation ratio (OCR), particle size and shape on the $V_s$ anisotropy, and $K_0$ of the granular soils during one-dimensional consolidation were investigated. The laboratory investigations suggested (1) the $K_0$ showed a constant value during loading, while it increased as the OCR increased during unloading; (2) soils with smaller particle sizes, rough surfaces, and angular geometry tended to have a lower value of $K_0$, and vice versa; (3) both the anisotropic stress state and the anisotropic fabric (geometry) could lead to the $V_s$ anisotropy, but the $V_s$ anisotropy was manifested due to the horizontal stress-lock during unloading stage; and (4) the published correlation between $V_s$ and $K_0$ was modified by introducing the influence of the OCR, which could effectively reduce the variation and improve the prediction accuracy. Therefore, the modified correlation could be used as a robust approach to estimate $K_0$ for both normally consolidated and highly overly consolidated granular soils.

1. Introduction

$K_0$ (equation (1)) is an intrinsic soil property that is related to soil characteristics, stress state, and stress history [1–5]. Therefore, it is used frequently in engineering practices such as retaining walls, embankment construction, mining and tunneling, and deep excavation:

$$K_0 = \frac{\sigma_h}{\sigma_v}$$

Based on the stress distribution analysis within wedge-shaped sand dunes, Jaky [6] reported that the value of $K_0$ was related to the internal friction angle of the sand. For normally consolidated soil, i.e., clay or loose sand, Jaky [6] derived a theoretical formula to calculate $K_0$:

$$K_0 = 1 - \sin \phi'$$

where $\phi'$ is the effective critical-state friction angle of the granular soil. There are theoretical flaws with predicting the lateral Earth pressure at rest in this equation because the equation indicates that $K_0$ is related to the frictional force of the soil, while the stress state at rest is stress path dependent. Michalowski [4] revisited the original derivation of the equation of Jaky [6] and concluded that Jaky’s derivation was stress path dependent. It was a 1D strain condition that was associated with the $K_0$ state. This finding indicates that $K_0$ at
the symmetry axis of a sand pile covers the entire range from the active stress state to the passive stress state. However, the stress state (the distribution of the active and passive stresses) in the 1D strain process may not be the same as that for a wedge-shaped sand pile [4]. In addition, $K_0$ was also found to be related to the stress history (i.e., $OCR$) and physical properties of geomaterials [7]. For instance, pre-loading can induce the residual horizontal stresses to be locked, which will eventually lead to an increased $K_0$. Therefore, in order to predict $K_0$ during unloading and reloading, the overconsolidation ratio ($OCR$) should be taken into consideration. For this reason, Mayne and Kulhawy [8] made modifications to equation (2) that accounted for the effect of overconsolidation. However, this equation is still empirical in nature:

$$K_0 = (1 - \sin \phi') (OCR)^\alpha,$$  

where $\alpha$ is the exponential parameter that defines the effect of the $OCR$ on $K_0$ and $OCR$ is the overconsolidation ratio of soil, which is usually approximated by using $\sin \phi'$. This modified correlation is good for overconsolidated soils during expansion but not for recompression. Therefore, for soils during unloading and reloading cycles, Mayne and Kulhawy [8] proposed another empirical equation:

$$K_0 = K_{0, NC} \left\{ \frac{OCR}{OCR_{\max}} (1 - \sin \phi') + 3 \left( 1 - \frac{OCR}{OCR_{\max}} \right) \right\},$$

where $K_{0, NC}$ can be interpreted with Jaky’s equation and $OCR_{\max}$ is the maximum $OCR$ of the soil in its stress history. Equations (1)–(3) indicated that $K_0$ of a soil would stay quite constant during loading [8]. During the unloading process, $K_0$ increases with the $OCR$ [1]. Although the proposed estimations are widely used to estimate $K_0$, there are still limitations associated with these previous published classical theoretical equations. For Jaky’s equation, the special stress state in the wedge-shaped sand pile is different from that in the one-dimensional consolidation, thus inhibiting its application and producing the inaccurate estimation of $K_0$. Equations (2) and (3) could offer reasonable estimation of $K_0$, however, they are empirical in nature. Therefore, the prediction of $K_0$ still needs further investigation.

Except for the theoretical equations, $K_0$ can be measured in a laboratory, i.e., using specially equipped oedometers [1, 3, 6, 9–17], a triaxial apparatus [18–22], a “soft” thin wall oedometer [2], and a plane-strain apparatus [23]. However, all these $K_0$ test apparatuses have their own limitations, i.e., high soil-wall resistance and the so-called “zero” radial strain for $K_0$ consolidation. $K_0$ can also be obtained from an in situ test. However, in situ tests will inevitably cause soil displacement issues during penetration, and they may significantly influence the actual $K_0$ state of stress. Kang et al. [24] manufactured a floating-wall oedometer bender element testing apparatus that overcame the limitations of the previous $K_0$ devices. The testing system [24], which was integrated with stress sensors and the measurements of the shear wave velocity, claimed a truly nonintrusive approach. In addition, the use of $V_s$ to predict the in situ horizontal stress, and $K_0$ has also been reported by previous studies [5, 25]. Ku and Mayne [5] used different types of shear wave velocities to estimate $K_0$. Once the $V_s$ profiles (either $V_{s-hh}$ and $V_{s-hv}$ or $V_{s-hh}$ and $V_{s-vh}$) are obtained, $K_0$ may be calculated with either of the following equations:

$$K_0 = \left( \frac{V_{s-hh}}{V_{p-hv}} \right)^{(2/n)}$$  

or

$$K_0 = \left( \frac{V_{s-hh}}{V_{s-vh}} \right)^{(2/n)}.$$

Through a literature review and field measurements, Ku and Mayne [5] suggested that the “hh-hv” mode yielded good prediction of $K_0$. The proposed fitting constants $a$ and $b$ were given as 0.78 and 2.55, respectively:

$$K_0 = a \left( \frac{V_{s-hh}}{V_{s-vh}} \right)^b.$$

Soils often have strong anisotropy properties [26–33]. Therefore, $V_s$ depends on the structure of the soil fabric and the measuring directions, such as longitudinal or transverse [24, 34–37]. By comparing $V_s$ in three orthogonal directions, i.e., $V_{s-vh}$, $V_{s-hv}$, and $V_{s-hh}$, the $V_s$ anisotropy of soils can be evaluated, where the first subscript letter represents the direction of the shear wave propagation direction and the second subscript letter represents the direction of the soil particle vibration ($v = \text{vertical}$ and $h = \text{horizontal}$). The $V_{s-vh}$, $V_{s-hv}$, and $V_{s-hh}$ are compared during the loading and unloading stages. The stiffness anisotropy is solely related to $V_s$ because $G = \rho V_s^2$. Since there were three pairs of BE sets installed in three different directions inside a sample that could measure $V_s$ in three directions, the term “$V_s$ anisotropy” is considered the same as the term “small-strain stiffness anisotropy” in this paper. The measurement of $V_s$ also offers a different way to investigate the anisotropy of geomaterials, which is related to the applied stress, the inherent soil fabric, and the physicochemical conditions [24]. Therefore, $K_0$ of granular materials was experimentally investigated in this research, which was focused on the correlations with the shear wave velocity, stress levels, and different particle sizes and shapes. The main aims of this study were to (1) study the $K_0$ and $V_s$ characteristics of granular materials during one-dimensional consolidation, (2) evaluate the current correlations between $K_0$ and the shear wave velocity of granular materials, and (3) characterize the $V_s$ anisotropy of granular materials.

1.1. Test Setup. A custom-designed floating consolidometer bender element testing system (Figure 1(a)) was used in this study. The testing system consisted of a water chamber, a floating-wall oedometer, two soil pressure sensors (one sensor installed at the base, and the other sensor located at the center of the cell), three BEs, and the wave generation and processing apparatus. A piece of bridge-shaped steel was used to cover the benders and the soil pressure sensors as well as to apply the vertical load. The floating wall worked as a constrained ring so that the $K_0$ condition was guaranteed. The horizontal stress was measured through a horizontally installed pressure sensor. A Teflon coating was put in the inner wall to reduce the soil-wall interface friction [38, 39]. A
fluid chamber was used in which the floating wall was submerged in water during the test operation. This chamber maintained the saturation of the sample and controlled the physicochemical conditions of the bulk fluid, including the pH, temperature, and salt concentration. An Agilent signal generator 33210A was used to generate the source waves. A Krohn-Hite filter/signal conditioner 3364 was used to filter the received signals. All the received signals were read from the oscilloscope (54622A, Agilent). A single-cycle sine wave with a frequency ranging from 2 kHz to 10 kHz and an amplitude of 10 V was adopted. A BED-A soil pressure sensor was purchased from Civil Engineering and

**Figure 1:** (a) Floating-wall consolidometer-type bender element testing system. (b) The detailed schematic setup of the floating-wall consolidometer cell [24].
Construction Instruments, Kyowa Electronic Instruments Co., Ltd. The sensor had an outer diameter of 20 mm, and the sensing surface had a diameter of 27 mm. Its maximum capacity was 1 MPa. The BED-A sensors were able to work at a temperature range from −10 to 60 °C and had a light weight, a good waterproof shield, and very good test sensitivity. All the soil pressure sensors were calibrated against a known load cell, and the test data was automatically read from the data logger. The floating-wall consolidometer design had the advantage of reducing the wall friction and protecting the side bender elements from detrimental bending. The weight of the oedometer could be counterbalanced through the built-in frictionless pulley system [24]. The bender element (Piezo Systems, Inc.) used in this study was cut into 12.7 mm × 8.0 mm × 0.6 mm (length × width × thickness) plates to fit the floating consolidometer bender element testing system. After trimming, polyurethane and silver powder were painted on the bender element surface to produce the physical and electrical shield. The first arrival time (t) was manually picked at the first zero-crossing of the first major peak by the cursor function in MathCAD (PTC, Needham, MA), and the tip-to-tip distance (l) was considered to be the shear wave propagation distance, so the shear wave velocity (V_s) could be calculated with V_s = l/t.

The schematic view of the floating-wall cell, soil pressure sensors, and BE sets is shown in Figure 1(a), and the detailed schematic setup of the floating-wall consolidometer cell is shown in Figure 1(b). The cell had a diameter of 11.4 cm, and its thickness was 1.3 cm. The height of the wall was 12.7 cm. Shear waves in different vibration directions could be generated in a soil specimen by the paired BE sets: V_{i-j}, where “i” is the propagation direction and “j” is the polarization direction. One BE was installed in the center of the top platen vertically (represents vh direction), and two BEs were installed at the base platen with an alignment perpendicular to each other. Four horizontal BEs were installed in the side wall (Figure 1). One soil pressure transducer was installed in the center of the consolidometer cell with a screw plate fixed to the outside of the wall. Another soil sensor was located at the center of the base plate, right between the two bottom bender elements.

2. Materials and Methods

Glass beads (GB) with diameters of 0.5 mm, 3 mm, and 6 mm, Ottawa sands (20–30, 50–70, and 200–325), and silica sand 30–50 were used in this study to investigate the K_0 and V_s anisotropy. The grain size distribution curves of all these materials (ASTM D 422-63) are shown in Figure 2. e_{max} and e_{min} are shown in Table 1 based on ASTM D 4253 and ASTM D 4254. The sand and the GB are classified as poorly graded sands, SP (ASTM D 2487). Sample preparation was performed using an air purification method [40, 41]. Dry specimens with a diameter of 114.3 mm (4.5 in) and a height of approximately 50.8 mm (2 in) were prepared in the consolidometer. A fixed falling distance of 80 cm from the funnel to the cell was predetermined to achieve approximately the same initial void ratio (relative density) for all the samples. A target mass of the soil was weighed and uniformly poured into the funnel. During each lift, a rubber hammer was used to tap the side of the cell gently in order to avoid any segregation and to internally build voids during falling. After setting the sample in the consolidometer, a steel tamper was used to carefully compact the top of the sand specimen and flatten the soil surface. Before putting on the top cap, measurements of the sample height were taken, and the average height was used to calculate the initial void ratio. A seating load of 16 kPa was then applied to push the top bender element into the soil and to maintain good contact between the soil and the sensors [42].

3. Results

One-dimensional consolidation tests were performed in this research. At the end of each loading stage, the base platen soil pressure and lateral Earth pressure were measured. At the end of consolidation, BE tests were carried out, during which the shear wave velocities at three orthogonal directions were measured and used to correlate with the K_0. Each test material was subjected to triaxial compression tests to obtain the critical-state friction angles. For the specimens in the oedometer tests and CD triaxial compression tests, identical void ratios were maintained in order to avoid any differences that might have been induced from the initial density. The soil-wall interface friction coefficient, e_{max}, e_{min}, and e and the critical-state friction angle for each type of soil are shown in Table 1.

3.1. K_0. In this study, the method proposed by Kang et al. [24] was applied to compute the effective normal stress, σ_v', of the specimens. By using the measured horizontal stress (σ_h'), we calculated K_0. K_0 values of different materials are shown in Figure 3. All of the specimens were consolidated to...
a maximum vertical stress of 800 kPa and then unloaded to different stress levels. Loading (Figure 3(a)): in general, $K_0$ was found to be constant during the loading stage. $K_0$ of the glass beads was higher than $K_0$ of the sand during loading (Figure 3(a)). For the glass beads, $K_0$ decreased as the diameter decreased. As the consolidation loading increased, $K_0$ increased slightly, as also observed by Mesri and Hayat [14] and Vardhanabhuti and Mesri [43]. However, the trend of the increase was not very large, which was similar to the trends reported by Wang and Gao [7]. The smallest sand (Ottawa sand 200–325) showed the highest value of $K_0$. However, the Ottawa sand 20–30 (larger grains) exhibited the lowest $K_0$. Unloading (Figure 3(b)): for all the test materials, $K_0$ was found to increase as the vertical load was removed. In other words, $K_0$ increased nonlinearly as the OCR increased. When the vertical stress decreased below 200 kPa (OCR greater than 3.0), $K_0$ increased dramatically. Compared with the glass beads, the sand particles had a smaller $K_0$ given the same vertical load. With the decreasing particle size, $K_0$ of the glass beads decreased slightly. Unlike Ottawa sand 200–325, silica sand 30–50, Ottawa sand 50–70, and Ottawa sand 20–30 showed almost the same values of $K_0$ during the entire loading range (Figure 3(b)).

### Table 1: Physical properties of the tested materials.

| Soil          | $D_{50}$ (mm) | $\varepsilon_{\text{max}}$ | $\varepsilon_{\text{min}}$ | Void ratio | Density index | CS friction angle |
|---------------|---------------|-----------------------------|-----------------------------|------------|---------------|-------------------|
| GB 6.0 mm     | 6             | 0.95                        | 0.61                        | 0.622      | 0.9647        | 20                |
| GB 3.0 mm     | 3             | 0.95                        | 0.61                        | 0.688      | 0.7705        | 21                |
| Ottawa 20–30  | 0.73          | 0.95                        | 0.56                        | 0.610      | 0.7058        | 33                |
| GB 0.5 mm     | 0.5           | 0.95                        | 0.61                        | 0.714      | 0.6941        | 24                |
| Silica 30–50  | 0.45          | 0.77                        | 0.54                        | 0.652      | 0.5095        | 37                |
| Ottawa 50–70  | 0.26          | 0.85                        | 0.65                        | 0.754      | 0.4775        | 35                |
| Ottawa 200–325| 0.06          | 1.42                        | 0.82                        | 1.228      | 0.3184        | 30                |

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**Figure 3:** $K_0$ of different materials during (a) loading and (b) unloading.

3.2. $V_s$ and $V_s$ Anisotropy. Santamarina et al. [44] developed the following equation to uncover the stress-dependent characteristics of $V_s$:

$$V_s = a \left( \frac{\sigma_m'}{1 \text{kPa}} \right)^\beta,$$

where $\sigma_m'$ is the mean normal stress, $a$ factor (m/s) is the velocity of a medium subject to 1 kPa confinement, and the exponent $\beta$ represents the amount of stress-dependent effect. For the $vh$ and $hv$ directions, $\sigma_m' = (1 + K_0) / 2 \sigma_v'$. In the $hh$ direction, $\sigma_m' = K_0 \sigma_v'$. Figure 4 shows the shear wave velocities of the GB 3 mm, GB 0.5 mm, Ottawa sand 20–30, and Ottawa sand 50–70 samples during the loading and unloading stages in the $vh$, $hv$ and $hh$ directions. The curves with solid symbols represent the unloading behavior and the curves with open symbols represent the loading behavior. In general, three typical features were identified from figures.
The $V_s$ values during unloading were slightly higher than the $V_s$ values during the loading stage. For the glass beads samples, $V_s$-hh was the smallest during loading. However, $V_s$-hh became the largest among the three orthogonal directions during the unloading stage (Figures 4(a) and 4(b)). In contrast to the glass beads, the Ottawa sand displayed distinct behaviors during both loading and unloading (Figures 4(c) and 4(d)). $V_s$-hh was always the largest among the three directions in both the loading and unloading stages. The hierarchy of the $V_s$ magnitude followed $V_s$-hh > $V_s$-hv > $V_s$-vh.

The $V_s$ anisotropy of a soil is generally quantified by the ratio of $V_s$-hh/$V_s$-vh or $V_s$-hv/$V_s$-vh. The higher the ratio is, the higher the degree of anisotropy is. In order to compare the effect of the anisotropic stress induced $V_s$ anisotropy and the inherent fabric induced $V_s$ anisotropy, the ratios of $V_s$-hh/$V_s$-vh and $V_s$-hv/$V_s$-vh of the glass beads samples and the Ottawa sand samples were plotted, as shown in Figures 5(a) and 5(b). As observed in the figures, the stress-induced $V_s$ anisotropy gradually increased when the vertical stress was gradually removed. The stress-induced $V_s$ anisotropies of the glass beads and the Ottawa sand could reach up to 1.28 and 1.32, respectively. For the glass beads samples, the ratios of $V_s$-hh/$V_s$-vh and $V_s$-hv/$V_s$-vh were almost the same during unloading, which indicated that the anisotropy was largely due to the horizontal stress lock. For the Ottawa sand,
however, the \( hh \) direction displaced the highest \( V_s \) anisotropy during unloading, indicating that fabric anisotropy governed the \( V_s \) anisotropy.

4. Discussion

4.1. Stress Dependency of \( V_s \). The magnitude of \( V_s \) depends on the particle stiffness, effective stress conditions, and soil structure, as well as aging and other factors. The stress dependence of the shear wave velocity can be represented by equation (7), where the \( \alpha \) term relates to the structural soil anisotropy. The value of the exponent \( \beta \) is affected by soil particle contact effects \[44\]. The fitted \( \alpha \) and \( \beta \) were plotted as shown in Figure 6. Based on the observations, we found that most of the \( \alpha \) and \( \beta \) values were plotted underneath the \( \beta = 1/4 \) area that represented the cone-to-plane contacts, and some of the data were plotted within the \( \beta = 1/6 \) area (Hertzian contact). As shown by the dotted line, a similar trend was observed for the fitted \( \alpha \) and \( \beta \) values \((\beta = 0.36 - \alpha/700)\) and the hyperbolic relationship (equation (9)): \[44, 45\]

\[
\beta = -0.0011\alpha + 0.3099, \tag{8}
\]

\[
\beta = \frac{1217.93}{(\alpha + 117.21)^{10/7}}. \tag{9}
\]

The coefficient of determination was \( R^2 \), and the values for the linear and hyperbola fittings were 0.75504 and 0.872071, respectively. In both the loading and unloading stages, equations (5) and (6) showed good fitting for the \( \alpha \) values that were larger than 25 m/s. Equations (8) and (9) were no longer valid when the \( \alpha \) value was smaller than 25 m/s because the governed contacts were not Hertzian elastic contacts.

4.2. Correlations between \( V_s \) and \( K_0 \). The shear wave velocities from both the \( hh-\text{vh} \) and \( hh-\text{hv} \) modes and the \( K_0 \) data during the unloading stages were fitted with equation (6). The results are presented in Figures 7 and 8. The 0.5 mm GB, silica sand 30–50, and Ottawa sand 50–70 were

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Figure 5: Anisotropic stress-induced anisotropy during unloading: (a) glass beads samples; (b) Ottawa sand samples.

Figure 6: Fitted values of \( \alpha \) and \( \beta \) coefficients for granular materials in this study. L: loading, in open symbol. U: unloading, in solid symbol.
taken as examples to compare the fitted $K_0$ and the direct measurements of the $K_0$ from original test data. As indicated in Figure 8, the fitted $K_0$ (lines with solid symbols) were very far off from the test data (lines with open symbols), which indicated that the fitting equation (6) might not have been applicable for the unloading behaviors. If the soil was highly overconsolidated, the estimated $K_0$ might have been very far off from the real $K_0$. The fitting parameters of “a” and “b” for all the test materials are presented in Figure 8. As indicated in Figure 8, the fitting parameter “a” was generally smaller than 1 and very constant (with a very small variation). However, the fitting parameter “b” showed very large scatter, which ranged from 0.5 to 6. The large variability of the fitting parameters and the “un-match” between the fitted $K_0$ and the tested $K_0$ indicated that the proposed fitting equation (6) might not be applicable for estimating $K_0$ of granular soils during unloading conditions, especially for highly overly consolidated soils, because many other influencing factors were not accounted for in equation (6), e.g., the OCR.

Since $K_0$ was found to be dependent on the OCR, the unsuccessful prediction of $K_0$ with equation (6) might have been due to the omission of the OCR effects. The authors speculated that a good fitting between $K_0$ and $V_s$ might be achieved if the OCR was incorporated into the fitting equation. Therefore, the correlation (Equation (6)) that was developed by Ku and Mayne [5] was slightly modified so that the effect of the OCR was included, and the equation was rearranged:

$$K_0 = a \left( \frac{V_{sHH}}{V_{sHV}} \right) ^b \cdot$$

(10)

By using the modified correlation (equation (10)), the entirety of the test data was replotted and compared with the tested $K_0$ (see Figure 9). Similarly, the curves with solid symbols in the figure are the fitted $K_0$, and the lines with open symbols are the tested $K_0$. In contrast to the previous fitting, after incorporating the OCR into equation (6), all the fitted curves matched fairly well with the test data because all the estimated data points overlapped with the test data points (see Figure 9). The fitting parameters of “a” and “b” that were obtained from the modified fitting correlation (equation (9)) were plotted as shown in Figure 10. For the same vertical scale as that in Figure 8, the values of “a” and “b” from the modified correlation were very constant compared with the previous correlation (equation (6)). The average values of “a” and “b” for the modified correlation were 0.7 and 0.2, respectively. The good match between the fitted $K_0$ and the tested $K_0$ from the modified correlation and the smooth trends of the fitting parameters of “a” and “b” (see Figure 10) largely supported the concept that incorporating of the OCR could effectively reduce the variation and improve the correlation between $V_s$ and $K_0$ of highly overconsolidated granular soils.

4.3. Particle Shape and Size Effects on the $K_0$ and $V_s$

Anisotropy. In order to have a view of the microscopic surface texture of the test materials, scanning electron microscope (SEM) tests were conducted. The results are shown in Figure 11. The GBs were identical for the SEM, and all the GB particles were round in shape with smooth surfaces (Figure 11(a)). The relatively rough surfaces could be seen in the SEM photos of all the sands. The Ottawa sand particles were almost rounded to subrounded (Figures 11(b) and 11(d)). The silica sand 30–50 and Ottawa sand 200–325 particles were mostly subangular to angular (Figures 11(c) and 11(e)).

The glass beads samples tested in this study had almost the same initial density and packing. $K_0$ was decreased slightly during unloading as the particle size decreased (Figure 3). The decrease of $K_0$ implied that the degree of
horizontal stress lock was decreased with the decreasing mean particle size. The force chain development among the granular materials has been evaluated in the literature [46, 47]. The smaller the particle was, the more uniform the stress distribution was inside the sample, and there was less chance of a “stress arch” occurring. Therefore, $K_0$ might decrease slightly. In addition to the particle size, the angularity and the particle shape were found to influence $K_0$ of the sand. The Ottawa sand 200–325 and the silica sand 30–50 both had high degrees of angularity, of which $K_0$ were slightly higher than those of the other two sand samples under the same condition. The Ottawa sand 200–325 showed the highest $K_0$ among all the sand specimens, which could be attributed to the relatively loose initial density and small critical-state friction angle. Mesri and Vardhanabhuti [3] reported that the looser samples normally have higher values of $K_0$, which implied that $K_0$ varied and was dependent on the initial density condition of the soils. By comparing the density index in Table 1, Ottawa sand 200–325 was found to have a relatively loose condition compared with the other sand samples. Thus, it exhibited a higher $K_0$.

Stress anisotropy could cause the $V_s$ anisotropy of the geomaterials composed of nonangular particles, such as round sand/spherical fly ash/glass beads [36], because the $V_s$ anisotropy of coarse grained soils is majorly dependent on the stress components in the direction of shear wave propagation and polarization but relatively independent of the out-of-plane stress component. Under high overburden
pressure, platy-clay particles often possessed preferential alignment which is the originality of the fabric anisotropy [48]. Therefore, $V_{s-hh}$ is always higher than $V_{s-hv}$ and $V_{s-vh}$ [24]. The anisotropic initial state of stress, interparticle contact orientation, stress history of the medium, micro-cracks inside the materials, inclusions, and laminated media could also contribute to the fabric anisotropy of the clays, thus leading to the $V_s$ anisotropy, i.e., the $K_0$ consolidation induces the preferential alignment of clay fabric in a direction that is normal to the applied loading [24, 49].

During the one-dimensional consolidation, the anisotropic stress state could induce $V_s$ anisotropy; e.g., $V_{s-hh}$ was smaller than $V_{s-vh}$ and $V_{s-vh}$ (see Figures 4(a) and 4(b)) during loading. This has been proven by many researchers.

**Figure 11:** Scanning electron microscopy images of the GB, Ottawa sand, and Silica sand.
for the anisotropic loading condition [24, 34–36, 50]. In general, two types of $V_s$ anisotropy could be observed, namely, the anisotropic stress state-induced $V_s$ anisotropy and the inherent fabric anisotropy-induced $V_s$ anisotropy. Since the glass beads were perfectly round, as indicated by the SEM images (Figure 11), the anisotropic stress state-induced $V_s$ anisotropy was as shown in Figures 4(a) and 4(b) during the loading stage. However, during the unloading stage, the $V_s$ anisotropic behavior was dramatically changed. $V_s$-hh gradually became the largest among the three orthogonal directions. This was related to the increased $K_0$ during unloading, for which relatively high horizontal stresses were locked inside the consolidometer. The locked high horizontal stress could cause principal stress rotation inside the specimen when the horizontal direction became the major principal stress, and the vertical direction became the minor principal stress. The rotated principal stress states thus caused $V_s$-hh to evolve gradually to the highest $V_s$ among the three orthogonal directions during unloading (the curves with solid symbols in Figures 4(a) and 4(b)).

The Ottawa sands had geometric anisotropy (indicated by the SEM images, where the particles had a subrounded shape with a longitudinal direction longer than the vertical direction). Due to the vertical stress, the sand grains might rotate and align more in the horizontal direction during loading, which was called the “preferred orientation.” Thus, the fabric might be anisotropic when the $V_s$ anisotropy was manifested due to the fabric anisotropy that was induced by the anisotropic stress state. The anisotropic fabric of the Ottawa sand-induced $V_s$ anisotropy during the loading stage when $V_s$-hh in general was higher than $V_s$-vh and $V_s$-hv (see Figures 4(c) and 5(b)). During unloading, the $V_s$ anisotropy was manifested by the “stress rotation” when the horizontal stress became the major principal stress. $V_s$-hh remained the largest among the three orthogonal directions during the entire unloading stage, as indicated by the curves with solid symbols in Figures 4(c) and 4(d). The ratios of $V_s$-hh/$V_s$-vh and $V_s$-hv/$V_s$-vh could reach up to 1.32 and 1.10 during unloading.

5. Conclusions

The $K_0$, $V_o$, and $V_s$ anisotropies and their correlations with granular soils during one-dimensional consolidation loading were evaluated with a custom-designed floating-wall oedometer BE testing system. $K_0$ stayed almost constant during loading. However, it increased dramatically when the consolidation loading was removed. The higher the OCR was, the greater the $K_0$ was. $K_0$ decreased slightly as the particle size decreased. The decrease of $K_0$ could be attributed to the decrease of the “horizontal stress lock” due to the mean particle size decrease. The anisotropy and the particle shape were also found to influence $K_0$ of the sand. Specimens with smaller particle sizes, rough surfaces, and angular geometry tended to have a lower value of $K_0$. In addition, $K_0$ also varied and it depended on the initial density condition of the soils. The Ottawa 200–325 sand was found to have a relatively loose condition compared with the other sand samples, thus exhibiting a higher value of $K_0$. The stress dependence of $V_s$ was evaluated, and $V_s$ was found to increase with the increase of the norm stress. Both the anisotropic stress state and the fabric/geometry anisotropy of the sand had induced $V_s$ anisotropy. During unloading, the $V_s$ anisotropy was manifested by the “stress rotation and stress lock.” Furthermore, the directional $V_s$ was validated to be a good tool for estimating the field stress state and predicting $K_0$. The incorporation of OCR into $V_s$ and $K_0$ correlation effectively improved the prediction accuracy, which offers a robust way to predict $K_0$ of both normally consolidated and highly overconsolidated granular soils in geotechnical practice.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest.

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References

[1] J. A. Yamamuro, P. A. Bopp, and P. V. Lade, “One-dimensional compression of sands at high pressures,” Journal of Geotechnical Engineering, vol. 122, no. 2, pp. 147–154, 1996.
[2] H. Shin and J. C. Santamarina, “Mineral dissolution and the evolution of $k_0$,” Journal of Geotechnical and Geoenvironmental Engineering, vol. 135, no. 8, pp. 1114–1147, 2009.
[3] G. Mesri and B. Vardhanabhuti, “Compression of granular materials,” Canadian Geotechnical Journal, vol. 46, no. 4, pp. 369–392, 2009.
[4] R. L. Michalowski, “Coefficient of earth pressure at rest,” Journal of Geotechnical and Geoenvironmental Engineering, vol. 131, no. 11, pp. 1429–1433, 2005.
[5] T. Ku and P. W. Mayne, “Evaluating the in situ lateral stress coefficient ($K_0$) of soils via paired shear wave velocity modes,” Journal of Geotechnical and Geoenvironmental Engineering, vol. 139, no. 5, pp. 775–787, 2013.
[6] J. Jaky, “A nyugalmi nyomas tenyezoje—the coefficient of earth pressure at rest,” Magyar Mernok Es Espitesz-Egylet Kozlonye, pp. 355–358, 1944.
[7] Y.-H. Wang and Y. Gao, “Examining the behavior and mechanisms of structuration in sand under the $S$/$S_0$ [K_0]$S_0$ condition,” Granular Matter, vol. 16, no. 1, pp. 55–68, 2014.
[8] P. W. Mayne and F. H. Kulhawy, “$K_0$—OCR relationships in soil,” Journal of the Geotechnical Engineering Division, vol. 108, no. 6, pp. 851–872, 1982.
[9] T. G. Thomann and R. D. Hryciv, “Laboratory measurement of small strain shear modulus under $K_0$ conditions,” Geotechnical Testing Journal, ASTM, vol. 13, no. 2, pp. 97–105, 1990.
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[10] G. Mesri and T. M. Hayat, "The coefficient of earth pressure at rest," *Canadian Geotechnical Journal*, vol. 36, no. 4, pp. 647–666, 1993.

[11] A. Komornik and J. G. Zeitlen, "An apparatus for measuring lateral soil swelling pressure in the laboratory," in *Proceedings of the 6th International Conference Soil Mechanics and Foundations Engineering*, pp. 278–281, Toronto University Press, Montreal, Canada, September 1965.

[12] E. W. Brooker and H. O. Ireland, "Earth pressures at rest related to stress history," *Canadian Geotechnical Journal*, vol. 2, no. 1, pp. 1–15, 1965.

[13] S. Lirer, A. Flora, and M. V. Nicotera, "Some remarks on the coefficient of earth pressure at rest in compacted sandy gravel," *Acta Geotechnica*, vol. 6, no. 1, pp. 1–12, 2011.

[14] G. Mesri, T. W. Feng, and J. M. Benak, "Post densification penetration resistance of clean sands," *Journal of Geotechnical & Geoenvironmental Engineering*, vol. 116, no. 1095, pp. 1095–1115, 1990.

[15] Z. Ofer, "Laboratory instrument for measuring lateral soil pressure and swelling pressure," *Geotechnical Testing Journal*, vol. 4, no. 4, pp. 177–182, 1981.

[16] H. Singh, D. J. Henkel, and D. A. Sangrey, "Shear and K₀ swelling of overconsolidated clay," *Proceedings of the Eighth International Conference on Soil Mechanics and Foundation Engineering*, vol. 1, no. 2, pp. 367–376, 1973.

[17] SK. Saxena, J. Hadberg, and C. C. Ladd, "Geotechnical properties of Hackensack valley varved clay of N," *Geotechnical Testing Journal*, vol. 1, no. 3, pp. 239–242, 1978.

[18] T. Tsuchida and Y. Kikuchi, "O-coefficient of sand in triaxial apparatus," *Canadian Geotechnical Journal*, vol. 44, no. 2, pp. 519–524, 2007.

[19] Y. Watabe, M. Tanaka, H. Tanaka, and T. Tsuchida, "Triaxial strength properties of gravelly sand under different stress paths," *Advances in Civil Engineering*, vol. 21, Article ID 8898814, 21 pages, 2021.

[20] X. Zhang, M. Ren, Z. Meng, B. Zhang, and J. Li, "Experimental study on the mechanical behavior of yunnan limestone in natural and saturated states," *Advances in Civil Engineering*, vol. 2021, Article ID 6614412, 16 pages, 2021.

[21] Y. Zhang, Y. Zhao, J. Liu, T.-Y. Meng, S.-J. Shao, and F.-T. She, "An experimental study on the deformation and strength characteristics of Q3 loess under a plain strain anisotropic consolidation condition," *Advances in Civil Engineering*, vol. 2021, Article ID 8813707, 13 pages, 2021.

[22] J. Feda, "Measurement and correlations of K₀ and vs anisotropy of granular soils," *Geotechnique*, vol. 44, no. 10, pp. 1242–1263, 2007.

[23] B. Vardhanabhuti and G. Mesri, "Coefficient of earth pressure at rest for sands subjected to vibration," *Canadian Geotechnical Journal*, vol. 44, no. 10, pp. 1242–1263, 2007.

[24] J. C. Santamarina, K. A. Klein, and M. A. Fam, *Soils and Waves—Particulate Materials Behavior, Characterization and Process Monitoring*, John Wiley & Sons, New York, NY, USA, 2001.

[25] T. Ku, P. W. Mayne, and B. J. Gutierrez, "Hierarchy of Vs modes and stress-dependency in geomaterials," in *Proceedings of the Fifth International Symposium on Deformation
Characteristics of Geomaterials, pp. 533–540, Seoul, South Korea, September 2011.

[46] S. Ostojic, E. Somfai, and B. Nienhuis, “Scale invariance and universality of force networks in static granular matter,” *Nature*, vol. 439, no. 7078, pp. 828–830, 2006.

[47] J. F. Peters, M. Muthuswamy, J. Wibowo, and A. Tordesillas, “Characterization of force chains in granular material,” *Physical Review E*, vol. 72, no. 4, Article ID 041307, 2005.

[48] R. Bellotti, M. Jamiolkowski, D. C. F. L. Presti, and D. A. O’Neill, “Anisotropy of small strain stiffness in ticino sand,” *Géotechnique*, vol. 46, no. 1, pp. 115–131, 1996.

[49] J. C. Santamarina and G. Cascante, “Stress anisotropy and wave propagation: a micromechanical view,” *Canadian Geotechnical Journal*, vol. 33, no. 5, pp. 770–782, 1996.

[50] B. O. Hardin and G. E. Blandford, “Elasticity of particulate materials,” *Journal of Geotechnical Engineering*, vol. 115, no. 6, pp. 788–805, 1989.