Sudakov Form Factors with an Effective Theory of Particle Worldlines

N. G. Stefanis

Institut für Theoretische Physik II,
Ruhr-Universität Bochum,
D-44780 Bochum, Germany
Email: stefanis@hadron.tp2.ruhr-uni-bochum.de
(13 July, 1998)

Abstract

Sudakov-type form factors for quark vertex functions in QCD are discussed using a worldline approach.

*Invited talk presented at the XI International Conference Problems of Quantum Field Theory, in memory of D. I. Blokhintsev, July 13-17, 1998, JINR, Dubna, Russia. To be published in the Proceedings.
1. Introductory remarks

A very convenient context for investigating the low-energy regime of gauge field theories is provided by particle worldlines [1]. I begin by recalling the basics of this approach, discussed in more detail in a series of publications [2-5] on fermionic Green and vertex functions in the infrared (IR) domain of QED and QCD. Then I shall focus on the electromagnetic quark vertex function in QCD and present recent results for Sudakov-type form factors. A main point will be to show how one can obtain such results avoiding complicated diagrammatic techniques [6].

2. Basics of the worldline approach

The worldline approach is simply a framework for trading field degrees of freedom for those of particles, i.e., symbolically,

\[ \int [d\bar{\psi}] [d\psi] e^{S_{\text{field}}[\bar{\psi},\psi]} \rightarrow \int [dx(\tau)] [dp(\tau)] e^{S_{\text{particle}}[x(\tau), p(\tau)]} . \]  

The treatment of the bosonic sector is to be adapted to the particular dynamical situation and is encoded in the expectation value of the Wilson line operator. Once a soft sector of the theory in question has been isolated, i.e., a factorization scheme has been established, the infrared behavior of Green and vertex functions is determined in terms of universal anomalous dimensions [7], pertaining to the specific geometry of the dominant (particle) worldlines. In this way, physical conditions, e.g., interactions, off-mass-shellness, etc. translate into worldline obstructions, like cusps, end-points, etc. [8].

3. Three-point function

The fermion (quark) form factor within the worldline approach is defined as the functional derivative of the generating functional:

\[ G^{ij}_{\mu}(x, y; z) = -\frac{\delta}{\delta J_\mu(z)} \frac{\delta^2}{\delta \bar{\eta}_i(x) \delta \eta_j(y)} \ln Z[\bar{\eta}, \eta, J_\mu]_{\bar{\eta}, \eta, J_\mu=0} , \]  

where \( Z[\bar{\eta}, \eta, J_\mu] \) is the well-known expression for a non-Abelian gauge field theory. After conversion into particle variables, the form factor reads

\[ G^{ij}_{\mu}(x, y; z) = \int_0^\infty dT \int [dx(\tau)] [dp(\tau)] \mathcal{P} \exp \left\{ -\int_0^s d\tau \left[ ip(\tau) \cdot \gamma + m \right] \right\} \Gamma_{\mu} \]

\[ \times \mathcal{P} \exp \left\{ -\int_s^T d\tau \left[ ip(\tau) \cdot \gamma + m \right] \right\} \exp \left[ i \int_0^T d\tau p(\tau) \cdot \dot{x}(\tau) \right] \]

\[ \times \left\langle \mathcal{P} \exp \left[ ig \int_0^T d\tau \dot{x}(\tau) \cdot A(x(\tau)) \right] \right\rangle_A^{ij} , \]

which involves two fermion lines coupled to a color-singlet current of the form \( \bar{\psi}(z) \Gamma_\mu \psi(z) \), and where \([dx(\tau)]\) is to be evaluated under the constraints: \( x(0) = x, x(T) = y, x(s) = z. \)
Here \(< >_A\) denotes functional averaging of the bracketed quantity in the non-Abelian gauge field sector, while \(P\) stands for path ordering of the exponential in connection with \(\gamma\)-matrices and/or non-Abelian vector potentials. Our basic computational task is the evaluation of the expectation value of a Wilson line operator over paths from an initial point \(x\) to a final point \(y\) obliged to pass through the point of interaction \(z\) with the external current which injects the large momentum \(Q^2\). Note that in our approach such lines may have a finite length, corresponding to off-mass-shell particles. A perturbative expansion yields

\[
\left\langle P e^{ig \int_0^T d\tau \, A} \right\rangle_A = 1 + (ig)^2 \left\{ \int_0^s d\tau_1 \int_0^s d\tau_2 \theta(\tau_2 - \tau_1) + \int_s^T d\tau_1 \int_s^T d\tau_2 \theta(\tau_2 - \tau_1) \right.
\]

\[
+ \int_0^s d\tau_1 \int_s^T d\tau_2 \langle A_{\mu}(\tau_1) A_{\nu}(\tau_2) \rangle \left( A_{\mu}(x(\tau_1)) A_{\nu}(x(\tau_2)) \right)_A + \ldots
\]  

(4)

In the Feynman gauge and for a dimensionally regularized casting, the gauge field correlator reads \(\langle A_{\mu}(x) A_{\nu}(x')\rangle_{\text{reg}} = \delta_{\mu\nu} C_F \left( \mu^{4-D}/4\pi^{D/2} \right) \Gamma(D/2 - 1) \ |x - x'|^{2-D} \).

4. Factorization of a soft sector

The next major step is to isolate the soft contribution to Eq. (4). As mentioned, this is done for a particular worldline geometry. To this end, we adopt a straight-line path going from \(x\) to \(z\) (for which \(\dot{x}(\tau) = u_1\) with \(0 < \tau < s\)) and a second such path from \(z\) to \(y\) (for which \(\dot{x}(\tau) = u_2\) with \(s < \tau < T\)). The no-recoil situation entailed by this restriction, except at the cusp point \(z\), suggests that the active gauge-field degrees of freedom entering our computation are bound by an upper momentum scale which serves to separate “soft” from “hard” physics in our factorization scheme. The singularities at the cusp and the endpoints of the path will give rise to renormalization factors with corresponding anomalous dimensions. The latter will determine the renormalization-group evolution of the form factor and eventually produce Sudakov-type form factors.

Refraining from entering into technical details (see [5]), I present only results. Choosing a frame for which \(|u_1| = |u_2| = |u|\) and using the boson field correlator given above to reexpress Eq. (4) solely in terms of particle degrees of freedom, we obtain in the limit \(D \rightarrow 4\_\)

\[
\left\langle P e^{ig \int_0^T d\tau \, A} \right\rangle_A = 1 - \frac{g^2}{4\pi^2} C_F \left\{ \frac{1}{D - 4} \left[ \varphi(w) - 2 \right] + \frac{D - 4}{2} \ln \left( \pi e^{2\gamma_E} \right) \right.
\]

\[
+ \ln(\mu |u| s) \left[ F_4 \left( \frac{s}{T - s}, w \right) - 1 \right]
\]

\[
+ \ln(\mu |u| (T - s)) \left[ F_4 \left( \frac{T - s}{s}, w \right) - 1 \right]
\]

\[
+ \frac{1}{D - 4} \left[ F_D \left( \frac{s}{T - s}, w \right) + F_D \left( \frac{T - s}{s}, w \right) - \varphi(w) \right] \right\}, \tag{5}
\]

where \(\gamma_E\) is the Euler-Mascheroni constant and

\[
F_4(x, w) = \frac{w}{\sqrt{1 - w^2}} \arctan \frac{\sqrt{1 - w^2}}{w} - \frac{w}{\sqrt{1 - w^2}} \arctan \frac{\sqrt{1 - w^2}}{x + w}, \tag{6}
\]
\[ \varphi(w) \equiv F_4(x, w) + F_4\left(\frac{1}{x}, w\right) = \frac{w}{\sqrt{1-w^2}} \arctan \frac{\sqrt{1-w^2}}{w}, \]  
with the relative velocity \( w \) given by \( w \equiv u_1 \cdot u_2/|u_1||u_2| = p_1 \cdot p_2/|p_1||p_2|. \)

5. Effective low-energy theory

We have isolated a sector of the full theory in which the matter particles (fermions) are “dressed” to a scale that makes them appear extremely heavy to the active, in this low-energy sector, gauge field degrees of freedom. This “heaviness” prevents their derailment from the straight-line propagation – in close analogy to a Bloch-Nordsieck situation \[2\] – and inhibits the creation of fermion-antifermion pairs (this corresponds to integrating out of the theory hard gluons). This is nothing but eikonal behavior \[6\]. Hence the factorization scale of the full theory becomes now the UV scale of the high-energy domain of this “soft” sector and UV divergences give rise to anomalous dimensions, much like in HQETh. The advantage is clear: IR divergences (of the full theory) can now be treated by usual renormalization group techniques. The anomalous dimensions are then just the coefficients of the leading UV divergences in dimensional regularization and are due to the obstructions of the worldline: the cusp (interaction point with the external electromagnetic current) and the endpoints:

\[ \Gamma_{\text{cusp}} = \frac{\alpha_s}{\pi} C_F \left[ \frac{w}{\sqrt{w^2-1}} \tanh^{-1} \left( \frac{\sqrt{1-w^2}}{w} \right) - 1 \right], \quad \Gamma_{\text{end}} = -\frac{\alpha_s}{2\pi} C_F, \]  
with \( w = 1 + Q^2/2|p|^2 \) and is actually finite. Now, the reference mass scale \( \mu \) can range from a minimum value \( \mu_{\text{min}} \) all the way up to \( \mu_{\text{max}} \sim |Q| \). Below \( \mu_{\text{min}} \), we demand that

6. Off-mass-shell fermions

Now we return to the three-point function (Eq. \[3\]), take its Fourier transform, and removing the pole in \( D - 4 \), we get the renormalized expression:

\[ G_\mu(p_1, p_2) = G_\mu^{(0)}(p_1, p_2) \left[ 1 - \ln (\mu|u|/\lambda) \gamma(\alpha_s, w) - f(\alpha_s, w) \right] + O(g^4), \]

where \( G_\mu^{(0)}(p_1, p_2) \) is the product of two free propagators along the two branches of the cusped worldline. Above, \( \lambda = |m^2 - p^2|/m \) is an off-mass-shellness scale serving as an IR regulator, and \( Q^2 = -(p_1 - p_2)^2 = -2p^2 + 2p_1 \cdot p_2 \), or, equivalently, \( p_1 \cdot p_2/|p|^2 = w = 1 + Q^2/2|p|^2 \), where \( p_1 \) and \( p_2 \) are, respectively, the four-momenta of the initial and final quark with mass \( m \). The last term in on the rhs of Eq. \[4\] is given by \( f(\alpha_s, w) = \lim_{D \to 4} [2F_D(1, w) - \varphi(w)] \frac{\alpha_s}{D-4} \) and is actually finite. Now, the reference mass scale \( \mu \) can range from a minimum value \( \mu_{\text{min}} \) all the way up to \( \mu_{\text{max}} \sim |Q| \). Below \( \mu_{\text{min}} \), we demand that
the three-point function describes free propagation along the two branches of the worldline. We are thus led to the equation
\[ \ln (\mu_{\text{min}} |u|/\lambda) \gamma(\alpha_s, w) + f(\alpha_s, w) = 0 , \] (11)
which can be solved for \( \mu_{\text{min}} \), when \( Q^2/p^2 \to \infty \). Then
\[ \gamma(\alpha_s, w) \simeq \frac{\alpha_s}{\pi} C_F \ln \left( \frac{Q^2}{p^2} \right) , \quad f(\alpha_s, w) \simeq \frac{\alpha_s}{4\pi} C_F \ln^2 \left( \frac{Q^2}{p^2} \right) , \] (12)
whereupon we determine (see also [7])
\[ \mu_{\text{min}} = \frac{|m^2 - p^2|}{(Q^2/p^2)^{1/4}} . \] (13)
A similar result holds also for \( G_\mu(p_1, p_2) \) on-mass-shell, the only difference being that the contribution to the anomalous dimension due to the end-points is absent [3].

7. Renormalization group evolution

To simplify the discussion, I consider the on-mass-shell case and refer for the off-mass-shell case to [5]. Defining the evolution operator by \( \mathcal{D} \equiv \mu \partial / \partial \mu + \beta(g) \partial / \partial g + \Gamma_{\text{cusp}}(w, g) \) we have
\[ \mathcal{D} F_{\text{soft}} \left( w, \frac{\mu^2}{\lambda^2} \right) = 0 . \] (14)
In the limit \( w \to \infty \) we find – in accordance with [7]
\[ \Gamma_{\text{cusp}}(w, g) \simeq \ln \left( \frac{Q^2}{p^2} \right) \Gamma_{\text{cusp}}(g) , \quad \Gamma_{\text{cusp}}(g) = \frac{\alpha_s}{\pi} C_F + \mathcal{O}(g^4) . \] (15)
Imposing \( F_{\text{soft}}(w, 1) = 1 \), we obtain in LL approximation (\(|p| = |p_i|, i = 1, 2\))
\[ F_{\text{soft}} \left( \frac{Q^2}{p^2}, \frac{\mu^2}{\lambda^2} \right) = \exp \left[ - \ln \left( \frac{Q^2}{p^2} \right) \int_{\lambda^2}^{\mu^2} \frac{dt}{2t} \Gamma_{\text{cusp}}(g(t)) \right] \]
\[ = \exp \left[ - \frac{C_F}{2\pi} \frac{4\pi}{\beta_0} \ln \left( \frac{Q^2}{p^2} \right) \ln \left( \frac{\ln \frac{\mu^2}{\lambda^2}}{\ln \frac{\lambda^2}{\Lambda^2}} \right) \right] , \] (16)
where \( \mu \) now denotes the separation point between the soft and the hard sectors of the theory. Then the total form factor can be factorized according to
\[ F \left( \frac{Q^2}{\lambda^2}, \alpha_s(Q^2) \right) = F_{\text{hard}} \left( \frac{Q^2}{\mu^2}, \alpha_s(Q^2) \right) \otimes F_{\text{soft}} \left( \frac{Q^2}{\mu^2}, \frac{\mu^2}{\lambda^2}, \alpha_s(Q^2) \right) , \] (17)
and taking into account that \( \frac{2}{\beta_0 \ln \frac{Q^2}{\lambda^2}} \ln F_S = - \int_{\lambda^2}^{\mu^2} \frac{dt}{t} \gamma_{\text{cusp}}(g(t)) \), we find for \( Q^2/\lambda^2 \to \infty \) the final result for the form factor:
\[ F \left( \frac{Q^2}{\lambda^2} \right) = \exp \left\{ - \frac{C_F}{2\pi} \frac{4\pi}{\beta_0} \ln \left( \frac{Q^2}{\Lambda^2} \right) \ln \left[ \ln \left( \frac{Q^2/\Lambda^2}{\ln (\lambda^2/\Lambda^2)} \right) \right] + \frac{C_F}{2\pi} \frac{4\pi}{\beta_0} \ln \left( \frac{Q^2}{\lambda^2} \right) \right\} . \] (18)
8. Summary

I have shown that it is possible to derive useful information about the quark electromagnetic vertex in QCD without relying upon complicated Feynman diagram techniques, and pointed out the main features of the worldline techniques employed. Applications to hadronic form factors using the presented formalism just started being explored [3].

Acknowledgments

I would like to thank my long-term collaborators A. Karanikas and C. Ktorides for their valuable contributions, and the organizers of the conference for their generous hospitality and support.
REFERENCES

[1] A.M. Polyakov, Nucl. Phys. B 164 (1979) 171.
[2] A. Kernemann and N.G. Stefanis, Phys. Rev. D 40 (1989) 2103; R. Jakob and N.G. Stefanis, Ann. Phys. (N.Y.) 210 (1991) 112.
[3] A.I. Karanikas, C.N. Ktorides, and N.G. Stefanis, Phys. Lett. B 289 (1992) 176; B 301 (1993) 397; Phys. Rev. D 52 (1995) 5898.
[4] A.I. Karanikas and C.N. Ktorides, Phys. Rev. D 52 (1995) 5883.
[5] G. Gellas, A.I. Karanikas, C.N. Ktorides, and N.G. Stefanis, Phys. Lett. B 412 (1997) 95.
[6] J.C. Collins and D.E. Soper, Nucl. Phys. B 193 (1981) 381; J.C. Collins, in Perturbative Quantum Chromodynamics, edited by A.H. Mueller (World Scientific, Singapore, 1989), Vol. 5, p. 573.
[7] G.P. Korchemsky and A.V. Radyushkin, Nucl. Phys. B 283 (1987) 342; Phys. Lett. B 279 (1992) 359; G.P. Korchemsky, Phys. Lett. B 217 (1989) 330; B 220 (1989) 62.
[8] N.G. Stefanis, in 10th International Conference on Problems of Quantum Field Theory, Alushta, 13-18 May, 1996, Crimea, Ukraine, edited by D.V. Shirkov et al. (JINR, Dubna, Russia, 1996) p. 199-206 [hep-th/9607063].
[9] N.G. Stefanis, W. Schroers, and H.-Ch. Kim, [hep-ph/9807298].