Stochastic MPC with Dual Control for Autonomous Driving with Multi-Modal Interaction-Aware Predictions

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We propose a Stochastic MPC (SMPC) approach for autonomous driving which incorporates multi-modal, interaction-aware predictions of surrounding vehicles. For each mode, vehicle motion predictions are obtained by a control model described using a basis of fixed features with unknown weights. The proposed SMPC formulation finds optimal controls which serves two purposes: 1) reducing conservatism of the SMPC by optimizing over parameterized control laws and 2) prediction and estimation of feature weights used in interaction-aware modeling using Kalman filtering. The proposed approach is demonstrated on a longitudinal control example, with uncertainties in predictions of the autonomous and surrounding vehicles.

Autonomous Driving Systems, Advanced Driver Assistance Systems, Identification and Estimation

1. INTRODUCTION

Autonomous driving technologies have seen a surge in popularity over the last decade, with the potential to improve flow of traffic, safety and fuel efficiency [1]. While existing technology is gradually being introduced into traffic, the absence of V2V communication makes safe motion planning for autonomous agents a challenge. The difficulty arises because of the variability of possible behaviors of the surrounding agents. To address this, works like [2, 3] build prediction models for the surrounding agents with multiple modes (discrete decisions for distinct maneuvers), which may also incorporate interactions with the autonomous agent.

The focus of this work is to use these multi-modal, interaction-aware predictions of the surrounding agents (called Target Vehicles (TVs)) in a Model Predictive Control (MPC) [4] framework for the autonomous agent (called Ego Vehicle (EV)). We use Stochastic MPC (SMPC) for addressing uncertainties in EV and TVs predictions by using probabilistic collision avoidance, and state and actuation constraints. The TVs’ predictions for each mode are obtained using a driver/control model with a basis of known features¹ multiplied by unknown, time-varying weights (e.g., such a model can be obtained by regression [5]).

2. RELATED WORK

There is a large body of work focusing on SMPC for autonomous driving applications, e.g., autonomous lane change [6], cruise control [7], and platooning [8]. A typical SMPC algorithm involves solving a chance-constrained finite horizon optimal control problem in a receding horizon fashion [9]. The prevailing approaches ([10]) for autonomous driving solve the SMPC optimization problem to find a single sequence of control values to satisfy the collision avoidance constraints for TVs’ trajectory predictions of all the modes. This can be conservative and feasible solutions for such SMPC may not exist, which is undesirable in practice if a backup planner is unavailable.

We build on [11] to propose an SMPC scheme which optimizes over a sequence of control laws. This allows for different sequences of control values corresponding to different realizations of the EV’s and TVs’ trajectories, thus enhancing feasibility of the SMPC optimization problem. The SMPC also uses ideas from dual control [12, 13] for prediction and estimation of the unknown, feature weights (specific to TV drivers) corresponding to each mode using Kalman filtering. We also maintain a probability distribution over modes using Bayesian updates, which allows prioritising performance for the more probable modes.

The paper is organized as follows. Section 3 describes the control problem and modelling assumptions. In section 4 we detail our SMPC design for addressing the control problem and demonstrate our approach in simulation, along with ablation studies in section 5.

3. PROBLEM FORMULATION

In this section, we formalize our modeling assumptions and present our SMPC design.

3.1. EV Prediction Model

The EV is modeled using a linear time-varying (LTV) system with state $x_t$, input $u_t$ and noise $w_t$ at time $t$, given by eq. (1). The noise is normally distributed with

\[ x_{t+1} = \Phi x_t + \Gamma u_t + \Theta w_t \]

¹E.g., features for a longitudinal control driver model may consist of safety distance from its lead car and/or the distance to a stop sign.
3.2. TV Prediction Model

The TVs are jointly modeled in eq. (2) with all TVs’ states given by $\alpha_t$, i.i.d. noise $v_t$, and interaction-aware driver/controls models given by a feature matrix $\Phi^\sigma(x_t, \alpha_t)$ for a particular mode $\sigma \in \{1, \ldots, M\}$, multiplied by weights $\gamma_t$

$$
\alpha_{t+1} = \tilde{A}_t \alpha_t + \tilde{B}_t \Phi^\sigma(x_t, \alpha_t) \gamma_t + v_t \quad \sigma = j
$$

The mode $\sigma$ and weights $\gamma_t$ are unknown, although the feature matrices $\Phi^\sigma(x_t, \alpha_t)$ are known $\forall j \in \{1, \ldots, M\}$.

3.3. Constraints

The traffic rules, speed, actuation constraints are given by chance constraints $P[(x_{t+1}, u_t) \not\in \mathcal{U}_t] \leq \epsilon$, and the collision avoidance chance constraints are given by $P[(x_t, \alpha_t) \not\in \mathcal{C}_t] \leq \epsilon$ for some given risk level $\epsilon > 0$, where

$$
\mathcal{U}_t = \{(x, u) : f_{t+1}^x \cdot x + g_{t+1}^u \cdot u \leq f_t + g_t \}, \forall t = 1, \ldots, n_x \\
\mathcal{C}_t = \{(x, o) : g_{t}^x \cdot x + g_{t}^o \cdot o \leq g_t \}, \forall t = 1, \ldots, n_o
$$

3.4. SMPC Algorithm

The SMPC optimization problem is given as follows

$$
\min_{\theta_{t+1}} \sum_{j=1}^{M} \sum_{k=1}^{N-1} c_j(x_{k+1|t}, u_{k|t}) \\
\text{s.t.} \quad x_{k+1|t} = A_k x_{k|t} + B_k u_{k|t} + w_{k|t}, \\
\alpha_{k+1|t} = A_k \alpha_{k|t} + B_k \Phi^\sigma(x_{k|t}, \alpha_{k|t}) \gamma_{k|t} + v_{k|t}, \\
\gamma_{k+1|t} = \gamma_{k|t} + n_{k|t}, \\
u_{k|t} = \pi_{k|t} (x_{k|t}, \ldots, u_{k|t-1}, \alpha_{k|t}, \ldots, \alpha_{0|t}), \\
\mathbb{P}(x_{k+1|t}, u_{k+1|t}) \not\in \mathcal{U}_t \leq \epsilon, \\
\mathbb{P}(x_{k+1|t}, \alpha_{k+1|t}) \not\in \mathcal{C}_{k+1} \leq \epsilon, \\
\gamma_{k|t} \sim \mathcal{N}(\gamma^0_{k|t}, \Sigma_{k|t})
$$

where $c_j(\cdot)$ is the stage cost, notation $\gamma^0_{k|t}$ denotes the prediction of quantity $q$ for time $k$ and mode $j$. at time $t$. Problem (5a) is solved to find the optimal decision variables $\theta^*_{t} = \{\theta^*_{t+1|t}, \ldots, \theta^*_{N-1|t}\}$ parameterizing the control laws in (5c). The controller at time $t$ is $u_t = \pi_{t|t} (x_t, \alpha_t)$ and (5b) is solved again at time $t+1$. Since the mode $\sigma$ in (2b) is unobservable, the predictions for all modes (in (5b), (5c) are initialized using the observed $x_t, \alpha_t$ (via (5b)), and the control laws $\pi_{t|t} (\cdot)$ are mode-agnostic. Problem (5b) also requires initial distributions of the weights in (5b) which are obtained using the Kalman Filter (KF) for (3), with $\hat{o}_t$ being the observation model.

The goal is to design $\pi_{t|t} (\cdot)$, $\forall k = t, \ldots, t+N-1$, such that solving (5b) is computationally tractable. The control laws in (5c) offer greater flexibility for satisfying (5b) than using a single sequence of controls $\pi_{t|t} (\cdot) \equiv \theta_{t|t}$ (which are independent of different EV, TV trajectory realizations). To further reduce conservatism, the control laws must also serve a dual role of predicting and estimating the distributions of $\gamma^0_{k|t}$ using EV, TV trajectory realizations $x_{t+1|t}^{i}, \ldots, x_{t+1|t}^{n}, o_{t+1|t}^{i}, \ldots, o_{t+1|t}^{n}$ along the prediction horizon.

4. SMPC FOR MULTI-MODAL, INTERACTION-AWARE PREDICTIONS

The details of our SMPC design are as follows.

4.1. Linearized Predictions

Linearization of (5c): The nonlinear term $\Phi^\sigma(x_{k+1|t}, o_{k+1|t}) \gamma^0_{k|t}$ in (5c) is linearized about $(x_{k+1|t-1}, o_{k+1|t-1}, \gamma^0_{k|t})$ using the previous solution of (5), given as

$$
\hat{x}_{k+1|t-1} = A_k \hat{x}_{k|t-1} + B_k \hat{o}_{k|t-1} \\
\hat{o}_{k+1|t-1} = A_k \hat{o}_{k|t-1} + B_k \Phi^\sigma(\hat{x}_{k|t-1}, \hat{o}_{k|t-1}) \gamma^0_{k|t} \\
\hat{u}_{k|t} = \pi_{k|t} (x_{k|t}, \ldots, \hat{o}_{k|t-1}) \\
\hat{x}_{k|t} = x_t, \hat{\theta}_{k|t} = \theta_{t|t}
$$

The linearized term is denoted as $B_k \Phi^\sigma(x_{k|t}, \hat{o}_{k|t}) \gamma^0_{k|t} \approx G^0_{k|t} \hat{\gamma}_{k|t} + P^0_{k|t} \hat{x}_{k+1|t-1} + Q^0_{k|t} \hat{o}_{k|t} + l^0_{k|t}$, where the coefficient matrices are given by

$$
G^0_{k|t} = B_k \Phi^\sigma(\hat{x}_{k+1|t-1}, \hat{o}_{k+1|t-1}) \\
P^0_{k|t} \hat{Q}^0_{k|t} = B_k \sum_{i=1}^{n_x} \nabla \Phi^\sigma(\hat{x}_{k+1|t-1}, \hat{o}_{k+1|t-1}) \gamma_{k|t}^{i} \\
l^0_{k|t} = -B_k \hat{P}^0_{k|t} \hat{x}_{k+1|t-1} + Q^0_{k|t} \hat{o}_{k|t} + l^0_{k|t}
$$

using notation $\Phi^\sigma(x, o) = [\Phi^\sigma_1(x, o), \ldots, \Phi^\sigma_{n_o}(x, o)]$.

Prediction and Estimation of Weights: Let $\hat{x}_{k|t}$ denote the random variable $\gamma^0_{k|t}$ conditioned on $x_{k|t}, \ldots, x_{t-1|t}, o_{k|t}, \ldots, o_{t-1|t}$, with $\gamma_{k|t} = \gamma_{k|t}^0$. Given a prior distribution $\gamma^0_{k|t-1} \sim \mathcal{N}(\gamma^0_{k|t-1}, \Sigma^0_{k|t-1})$, we compute the posterior distribution using the measurements $(x_{k-1|t}, o_{k-1|t}, \hat{o}_{k|t})$ and (5c) to construct a measurement model for $\gamma_{k|t}$ as

$$
y_{k|t} = \hat{o}_{k|t} - (\hat{A}_k + Q_{k|t} \hat{Q}^0_{k|t}) \gamma_{k|t-1} - P^0_{k|t} \hat{x}_{k-1|t} + l_{k|t}^0 \\
y_{k|t}^{\pi} = G^0_{k-1|t} \gamma_{k-1|t} + \theta_{t|t}^o
$$

where $x_t, o_t$ are given by $x_t, o_t$.
where we have used the TVs’ state measurements and linearised dynamics to define the output in the first equation. The distribution of \( \gamma_{k|t} \) is obtained from the “update” step of the KF (with \( \Sigma_k \)) as the dynamics model and \( M_{k|k-1} \) as the measurement to give

\[
\gamma_{k|t} = (I - K_{k|t}(G_{k-1})^T)\gamma_{k-1|t} + K_{k|t} y_{k|t} + n_{k|t}
\]

(7)

where \( K_{k|t} = \Sigma_{k-1} G_{k-1} G_{k-1}^T \Sigma_{k-1} + \Sigma_{k|t-1} \) is the Kalman gain. The mean and covariance are given by

\[
\Sigma_{k|t} = (I - K_{k|t} G_{k-1})\Sigma_{k-1|t} + \Sigma_{n|k|t}
\]

(8)

In terms of the initial distribution in \( \Sigma_k \), \( \gamma_{1|t} \) can be alternatively expressed as

\[
\gamma_{1|t} = \sum_{i=1}^{k} W_i j_{i|t} + n_{1|t}
\]

(9)

where \( n_{1|t} \sim N(0, \Sigma_{n|1}) \) and \( W_i j_{i|t} = I - K_{1|t} G_{1} \). The initial distribution \( \Sigma_k \) is also similarly obtained by KF from the TVs’ state measurements \( o_t, o_{t-1} \).

**Consolidated TV Model:** Denote \( \bar{\gamma}_{k+1|t} \) as the random variable \( o_{k+1|t} \) conditioned on \( x_{t|t-1}, x_{t+1|t}, x_{t+2|t}, \ldots, x_{t+k-1|t} \), which is given by

\[
\bar{\gamma}_{k+1|t} = (\bar{A}_k + Q_k) \bar{o}_{k|t} + P_k \bar{x}_{t+k|t} + \bar{G}_k \bar{z}_{k|t} + \bar{t}_{k|t} + \bar{n}_{k|t}
\]

Using (10) and defining a new random variable \( z_{k|t} \) as

\[
z_{k|t} = v_{k|t} + G_k^T ( \sum_{i=1}^{k} W_i j_{i|t} + \sum_{i=t+1}^{k-1} W_i j_{i|t} )
\]

(10)

we can replace \( \Sigma_k \) and \( \Sigma_{n|1} \) by the following consolidated model

\[
\bar{\gamma}_{k+1|t} = (\bar{A}_k + Q_k) \bar{o}_{k|t} + P_k \bar{x}_{t+k|t} + \bar{G}_k \bar{z}_{k|t} + \bar{t}_{k|t}
\]

(11)

Note that \( \bar{z}_{k|t} \) serves as the “effective” process noise for TV prediction model, which can be measured using measurements of \( o_t, o_{t+1}, o_{t+2}, \ldots, o_{t+k-1} \). In contrast, see that \( u_{k|t} \) can’t be measured using \( \Sigma_k \) because \( \gamma_{k|t} \) is unknown. This observation will be important for designing the control laws \( \pi_{\theta_k} (\cdot) \), \( \forall k = t, \ldots, t + N - 1 \).

**Stacked Predictions:** Let \( x_{t|t} = [x_{t|t-1}, \ldots, x_{t+|t}] \) (similarly for \( \bar{x}_{t|t} \)), \( \bar{o}_{t|t} = [\bar{o}_{t|t-1}, \ldots, \bar{o}_{t+|t}] \), \( u_{t|t} = [u_{t|t-1}, \ldots, u_{t+|t}] \) (similarly for \( w_t, n_T, \bar{n}_T, x_t, \bar{x}_t \)). The stacked state predictions for the EV can be expressed as a function of the current state \( x_t \) and stacked inputs \( u_t \) as

\[
x_{t|t} = A_t x_t + B_t u_t + E_t w_t.
\]

(12)

For the TVs, the stacked, conditioned predictions \( \bar{o}_{t|t} \) are given by

\[
\bar{o}_{t|t} = \bar{A}_t \bar{o}_{t|t} + \bar{P}_t \bar{x}_{t|t} + \bar{G}_t \bar{n}_{t|t} + \bar{z}_{t|t} + \bar{t}_{t|t}.
\]

(13)

Using \( \gamma_t = G \gamma_{t-1} + \Gamma_t \) from (5a) and \( \gamma_t = \Gamma_t A \gamma_{t-1} + \Gamma_t B \gamma_{t-1}^x \) from (6) where \( n_t = [n_t, \bar{n}_t] \), the stacked, conditioned predictions \( \bar{o}_{t|t} \) are explicitly given as

\[
\bar{o}_{t|t} = \bar{A}_t \bar{o}_{t|t} + \bar{A}_t \bar{x}_{t|t} + \bar{B}_t \bar{u}_t + \bar{G}_t \bar{n}_{t|t} + \bar{z}_{t|t} + \bar{t}_{t|t}.
\]

(14)

We defer the various matrix definitions to the appendix.

**4.2. Control Law Parameterization**

We use the affine disturbance feedback parameterization for our control law,

\[
\pi_{\theta_k} (x_1, \ldots, x_t, o_t, o_{t+1}) = h_{k|t} + \sum_{i=t}^{k-1} M_{n|k|t} w_{i|t} + M_{x|k|t} x_{i|t}
\]

(15)

which is a function of the past process noise realizations \( w_1, \ldots, w_{k-1}, z_{k-1|t} \), with mode-agnostic parameters \( \theta_k = (h_{k|t}, \{ M_{n|k|t}, M_{x|k|t} \}) \) (see (15) for equivalence of disturbance feedback and state feedback). For an intuitive explanation of the equivalence, notice that each \( w_{i|t} \) can be measured from consecutive EV state measurements \( x_{i+1|t} \) using (5b) and each \( z_{i|t} \) can be measured from \( o_1, o_{t+1}, o_{t+2}, \ldots, o_{t+k-1} \) using (11). Conversely, given the process noise sequence \( w_1, \ldots, w_{k-1}, z_{k-1|t} \) the EV and TV state trajectories \( o_1, o_{t+1}, \ldots, o_{t+k-1} \) are completely determined. Thus, there exists an invertible transformation \( \Gamma (\cdot) \) such that \( o_1, \ldots, o_{k-1}, z_{k-1|t} = \Gamma (w_1, \ldots, w_{k-1}, z_{k-1|t}) \).

The stacked control inputs are given by

\[
u_{t} = h_t + M_{x|t} w_{t} + M_{x|t} x_{t}
\]

(16)

where the stacked control parameter matrices \( h_t, M_{x|t}, M_{x|t} \) are defined in the appendix. Note that these parameters will be the decision variables for our SMPC, and thus optimized online.

**4.3. Chance Constraint Reformulation**

The chance constraints (5a) and (5b) are reformulated using the following result from (16)

\[
\text{Pr}(a^T x > b) \leq \epsilon, x \sim N(\mu, \Sigma)
\]

\[
\Rightarrow a^T \mu + \Omega (1 - \epsilon) \sqrt{\Sigma} \leq b
\]

where \( \Omega (\cdot) \) is the quantile function of \( N (0, 1) \). The distributions of the stacked predictions \( x_{t|t} \) (12), (11) and control laws \( u_{t} \) (15) are determined by the random variables \( w_t, v_t, n_t \). Note that \( z_{t|t} = v_{t} + \Gamma_t n_t \), where \( \Gamma_t \) is defined in the appendix. Define constant matrices \( S_{x}^e, S_{x}^o, S_{o}^x \) such that \( S_{x}^e x_{t|t} = x_{t|t}, S_{x}^o o_t = o_t, S_{o}^x u_t = u_t \), to recover predictions at the \( k \)-th time step from the stacked predictions (12), (13), (15). Then the state-input chance constraints (5a) and the collision avoidance chance constraints (5b) can be rewritten as

\[
P ( (f_{k+1}^e x_{t|t} + f_{k+1}^o o_t) \notin X(k) ) \leq \epsilon
\]

\[
P ( (f_{k+1}^e x_{t|t} + f_{k+1}^o o_t) \notin X(k) ) \leq \epsilon
\]

\[
P ( \sum_{i=1}^{k} \sqrt{f_{i}^e (f_{i}^e + f_{i}^o)} + \sum_{i=1}^{k} \sqrt{f_{i}^o (f_{i}^e + f_{i}^o)} ) \leq \Omega (1 - \epsilon) \times
\]

\[
[ (f_{k}^e S_{x}^e (B_t M_{x|t}^e + E_t) + f_{k}^o S_{x}^o (M_{x|t}^o)^T ) \]
4.4. Cost Definition

Estimation of Mode Probabilities: The trajectory cost for each mode is weighted by the mode probability, \( p_i \), to prioritize minimizing the trajectory cost for more likely modes. For estimating \( p_i \) using (4), the likelihood functions are computed using the pdf which is obtained using the consolidated TV model [11] as

\[
P[\{\hat{z}_{t}, \hat{o}_{t}\}|(x_{t-1}, u_{t-1}, \alpha_{t-1}), \sigma = i] = P[\hat{z}_{t}^{s}_{t-1}|(\hat{z}_{t-1}^{s}_{t-1}) = \hat{z}_{t-1}^{s}_{t-1} - \hat{B}_{t} \hat{\Phi}(x_{t-1}, \alpha_{t-1})^{s}_{t-1]|]^{\hat{z}_{t-1}^{s}_{t-1}}
\]

where \( \hat{z}_{t-1}^{s}_{t-1} \approx N(0, \Sigma_{v} + G_{t-1}^{s}_{t-1} \Sigma_{\hat{z}}^{s}_{t-1} G_{t-1}^{s}_{t-1}) \)

Stage Cost: We choose a convex, quadratic cost to penalise deviations from a given reference, \( c(x, u) = (x - x_{ref}^{st})^{T} C_{x} (x - x_{ref}^{st}) + (u - u_{ref}^{st})^{T} C_{u} (u - u_{ref}^{st}) \) for positive definite \( C_{x}, C_{u} \).

Expected Trajectory Cost per Mode: The expected trajectory cost for mode \( j \) in (5a) can be calculated as

\[
\mathbb{E}\left[ \sum_{k=1}^{\epsilon} c(x_{t+k}^{st}, u_{t+k}^{st}) \right] = \mathbb{E}\left[ (x_{t}^{st} - x_{t}^{ref})^{T} C_{x} (x_{t}^{st} - x_{t}^{ref}) + (u_{t}^{st} - u_{t}^{ref})^{T} C_{u} (u_{t}^{st} - u_{t}^{ref}) \right] = h_{t}^{T} (B_{t}^{T} C_{x} B_{t} + C_{u} h_{t}) + \text{tr}(B_{t}^{T} C_{x} B_{t} + C_{u}) M_{t}^{x} M_{t}^{x} \Sigma_{v}^{s} M_{t}^{x} M_{t}^{x} \Sigma_{v}^{s} + (A_{t} x_{t} - x_{t}^{ref})^{T} C_{x} (A_{t} x_{t} - x_{t}^{ref} + 2 B_{t} h_{t}) - 2u_{t}^{ref T} C_{u} B_{t} h_{t} + \text{tr}(C_{u} E_{t} \Sigma_{v}^{s} E_{t}^{T}) \]

where \( C_{x} = I_{N} \otimes C_{x}, C_{u} = I_{N} \otimes C_{u}, x_{t}^{ref} = [x_{t+1}^{ref}, ..., x_{T}^{ref}], u_{t}^{ref} = [u_{t}^{ref}, ..., u_{T-1}^{ref}] \). Due to the positive definiteness of \( C_{x}, C_{u}, \) the cost is convex and quadratic in the control laws’ parameters \( \theta_{t} = (h_{t}, M_{t}^{x}, M_{t}^{u}) \).

4.5. SMPC Optimization Problem

The SMPC optimization problem is given in batch form, by explicitly substituting for the predictions (5b)-(11) and parameterised control laws (14), to yield

\[
\min_{n_{i}, M_{t}^{x}, M_{t}^{u}} \sum_{j=1}^{M} p_{j}^{T} \cdot \text{ (Cost per mode, eq. (18))}
\]

s.t. State-input constraints, eq. (16a)

Collision avoidance constraints, eq. (16b)

\[
\forall j = 1, ..., M, \forall k = t, t+1, ..., N - 1
\]

The optimization problem (19) is a convex, second-order cone programming problem that can be solved online for the control input \( u_{t} = h_{t}^{*}. \)

5. SIMULATIONS

5.1. Problem Formulation for Longitudinal Control:

Consider the longitudinal control problem as depicted in fig. 1 described by point-mass models with longitudinal position, speed describing the states, acceleration as the control input, for both the EV and TV respectively denoted as \( x_{t} = [s_{t}, v_{t}], u_{t} = a_{t}, \alpha_{t} = [s_{t}^{0}, v_{t}^{0}] \). The system matrices are time-invariant and obtained from Euler-discretization as

\[
A_{t} = \hat{A}_{t} = \begin{bmatrix} 1 & dt \\ 0 & 1 \end{bmatrix}, B_{t} = \hat{B}_{t} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

Fig. 1: EV must stop for the pedestrian while keeping safe distance from TV.

The TV control is assumed to have \( M = 2 \) modes with \( n_{s} = 1 \) feature each, given by

\[
\begin{align*}
\Phi_{1}^{t}(x_{t}, \alpha_{t}) &= k_{d1}^{t}(s_{t} - d_{safe} - s_{t}^{0}) + k_{d1}^{t}(0 - v_{t}^{0}) \\
\Phi_{2}^{t}(x_{t}, \alpha_{t}) &= k_{d2}^{t}(s_{t} - d_{safe} - s_{t}^{0}) + k_{d2}^{t}(v_{t} - v_{t}^{0})
\end{align*}
\]

where mode 1 is an EV-agnostic PD control to stop at \( s_{t} - d_{safe} \), and mode 2 is a PD control to follow behind the EV by \( d_{safe} \). The car-following control for mode 2 is adapted from [18], with constant headway \( d_{safe} \). Thus, the TV’s control law is given by

\[
u^{ref}(x_{t}, \alpha_{t}, \gamma_{t}, \sigma) = \begin{cases} 
\gamma_{t} \Phi_{1}(x_{t}, \alpha_{t}), & \text{if } \sigma = 1 \\
\gamma_{t} \Phi_{2}(x_{t}, \alpha_{t}), & \text{if } \sigma = 2
\end{cases}
\]

where the mode \( \sigma \), and the weight \( \gamma_{t} \) are unknown to the EV, and thus need to be inferred online. The challenge is that the EV must stop for the pedestrian, and reach the stop fast enough to avoid the TV if \( \sigma = 1 \), but must slow down smoothly to guide the TV to stop if \( \sigma = 2 \), to prevent colliding with the TV.

The sets describing the state-input constraints and collision avoidance constraints are given as

\[
\begin{align*}
\mathcal{A}_{t} &= \{ s_{t+1} \leq s_{t}, 0 \leq v_{t+1} \leq v_{max}, a_{min} \leq a_{t} \leq a_{max} \} \\
\mathcal{C}_{t} &= \{ s_{t} - s_{t}^{0} \leq d_{safe} \}
\end{align*}
\]

The cost is chosen to penalise deviations from the setpoint \( x_{ref} = [s_{f}, 0] \), \( u_{ref} = 0 \) and the EV acceleration \( a_{t} \) is obtained by solving (19).
5.2. Simulation Results:
The various parameters used in our simulation are given in Table 1. We demonstrate the performance of the proposed SMPC design in closed-loop, for both $\sigma = 1, \sigma = 2$ scenarios. For each scenario, we initialise the simulation with EV and TV states $x_0 = [0, 11], \sigma_0 = [-9, 15]$, and weight estimates $\hat{\sigma}_0 = 0$, $\Sigma_{0} = \Sigma_{0}^2 = 1$.

| Parameter | Value |
|-----------|-------|
| $dt$      | 0.1s  |
| $v_{\text{max}}$ | 14 m/s$^{-1}$ |
| $d_{\text{safe}}$ | 7m |
| $t_{\text{min}}$ | 0 m |
| $s_f$ | 50 m |
| $\epsilon$ | 0.1 |
| $C_s = \text{diag}(50, 20)$ |
| $\Sigma_{w} = \text{diag}(10^{-2}, 10^{-2})$ |
| $\Sigma_{v} = \text{diag}(10^{-2}, 10^{-1})$ |
| $\Sigma_{m} = 0.5$ |
| $k_1^s = 1$ |
| $k_1^v = 6$ |

The matrices for defining the stacked predictions are considered the matrix functions:

$M_{\lambda}(\{A_k\}_{k=1}^{N}) = \left[ I \quad A_1^T \quad \ldots \quad \prod_{k=1}^{N} A_k^T \right]^T,$

$M_{B}(\{A_k, B_k\}_{k=1}^{N}) = \begin{bmatrix} O & \cdots & O \\ B_1 & \cdots & O \\ \vdots & \ddots & \vdots \\ \prod_{k=1}^{N-1} A_k B_1 & \cdots & B_N \end{bmatrix},$

$M_{D}(\{A_k\}_{k=1}^{N}) = \text{blkdiag}(\{A_k\}_{k=1}^{N}).$

The matrices for defining the stacked predictions are obtained using these matrix functions as follows

$A_k = M_{\lambda}(\{A_k\}_{k=1}^{1+N-1}), B_k = M_{B}(\{A_k, B_k\}_{k=1}^{1+N-1}).$

We see that removal of feedback in the predictions results in deteriorated performance in terms of feasibility of the SMPC optimization problem and task success, especially in the scenario when interaction is required ($\sigma = 2$ in our example).

6. CONCLUSION

We have proposed a Stochastic MPC framework with interaction-aware, multi-modal predictions of TVs given by basis of known features multiplied by unknown, time-varying weights. The proposed approach finds an optimal sequence of EV and TV trajectory-dependent control laws given by the affine disturbance feedback parameterization, to 1) reduce conservatism in satisfaction of chance-constraints and 2) use dual control for prediction, and estimation of feature weights.

Appendix: Matrix Definitions for Stacked Predictions

Consider the following matrix functions:

$M_{\lambda}(\{A_k\}_{k=1}^{N}) = \left[ I \quad A_1^T \quad \ldots \quad \prod_{k=1}^{N} A_k^T \right]^T,$

$M_{B}(\{A_k, B_k\}_{k=1}^{N}) = \begin{bmatrix} O & \cdots & O \\ B_1 & \cdots & O \\ \vdots & \ddots & \vdots \\ \prod_{k=1}^{N-1} A_k B_1 & \cdots & B_N \end{bmatrix},$

$M_{D}(\{A_k\}_{k=1}^{N}) = \text{blkdiag}(\{A_k\}_{k=1}^{N}).$

We demonstrate the benefits of the including feedback in predictions of the SMPC design (19) via ablation studies. The SMPC (19) is compared against two ablations:

1. A modified SMPC formulation (denoted by (19)\text{KF}) that just uses (5d) instead of KF for $\gamma_t$ predictions.
2. A modified SMPC formulation (denoted by (19)\text{KF}) that just looks for an optimal sequence of control values instead of control laws, i.e., $u_t = b_i$. For each approach, both $\sigma = 1, \sigma = 2$ scenarios are simulated from 16 initial conditions chosen from the set: $x_0 \in \{[0 \pm 1] \times [11 \pm 1], \sigma_0 \in \{[-9 \pm 1] \times [14 \pm 1]\}$. We deem a control task as successful if the EV is able to stop at $s_f$, with the TV stopping $d_{\text{safe}}$ behind. We record the percentage of task success as (S%). We also record the percentage of feasibility of the SMPC optimization problems as (F%). When infeasibility is encountered during the simulation, the previous input is applied. Table 2 presents our results for the SMPC approaches for each scenario.

| Mode | Metric | (19) | (19)\text{KF} | (19)\text{KF} |
|------|--------|------|---------------|---------------|
| $\sigma = 1$ | S% | 100. | 93.75 | 100. |
| F% | 98.71 | 98.21 | 76.10 |
| $\sigma = 2$ | S% | 87.5 | 56.25 | 0 |
| F% | 95.30 | 93.38 | 8.33 |

Discussion:
We see that removal of feedback in the predictions results in deteriorated performance in terms of feasibility of the SMPC optimization problem and task success, especially in the scenario when interaction is required ($\sigma = 2$ in our example).
\[ E_t = M_B \left( \{A_k, I\}^{t+N-1}_{k=t} \right) \]
\[ A_t^* = M_A \left( \{A_k + Q_k^j\}^{t+N-1}_{k=t} \right), P_t^* = M_B \left( \{A_k + Q_k^j\}^{t+N-1}_{k=t} \right) \]
\[ G_t^* = M_B \left( \{A_k + Q_k^j\}^{t+N-1}_{k=t} \right), F_t^* = M_B \left( \{A_k + Q_k^j\}^{t+N-1}_{k=t} \right) \]
\[ A_t^\perp = M_D \left( \{A_k + Q_k^j\}^{t+N-1}_{k=t} \right), P_t^\perp = M_D \left( \{P_j^k\}^{t+N-1}_{k=t} \right) \]
\[ G_t^\perp = M_D \left( \{G_j^k\}^{t+N-1}_{k=t} \right), \Gamma = M_A \left( I^{t+N-1}_{k=t} \right) \]

\[ \Gamma^l = \text{Unit lower triangular matrix} \]

The matrices describing the predictions \( \hat{A}_t \) explicitly are given as

\[ \hat{A}_t = (A_t^* + G_t^\perp \Gamma^l) \hat{A}_t + G_t^\perp \Gamma^l A_t \]
\[ \hat{B}_t = (P_t^* + \hat{A}_t P_t^* + G_t^\perp \Gamma^l \Gamma^l P_t^* + B_t \Gamma^l) \]
\[ \hat{G}_t = G_t^\perp \Gamma^l + (G_t^\perp \Gamma^l)^2 \]
\[ \hat{P}_t^\perp = (P_t^* + \hat{A}_t P_t^* + G_t^\perp \Gamma^l \Gamma^l P_t^* + \hat{P}_t^* \Gamma^l) \]
\[ \hat{L}_t = (I + G_t^\perp \Gamma^l + \hat{A}_t) \Gamma^l \]

For \( u_t \), we have \( h_t = [h_{t+1}, ..., h_{t+N-1}]^T \) and \( M_w^\perp \) (similarly for \( M_w \)) given by

\[ M_w^\perp = \begin{bmatrix} O & \cdots & \cdots & O \\ \vdots & \ddots & \ddots & \vdots \\ \end{bmatrix} \]

The matrix \( \Gamma^l \) for defining \( z_t \) is given by

\[ \Gamma^l = M_D \left( \{G_{j,k}\}^{t+N-1}_{k=t} \right) \times \begin{bmatrix} I & \cdots & O \\ \vdots & \ddots & \vdots \\ W_t^l & \cdots & O \\ \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ \end{bmatrix} \]

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