Small Dirac Neutrino Masses and $R$-parity from Anomalous $U(1)$ Symmetry

Ilia Gogoladze $^{a,b,*}$ and Abdel Pérez-Lorenzana$^{a,c,†}$

$^a$ The Abdus Salam International Centre for Theoretical Physics, I-34100, Trieste, Italy
$^b$ Department of Physics, Oklahoma State University, Stillwater, OK 74078, USA
$^c$ Departamento de Física, Centro de Investigación y de Estudios Avanzados del I.P.N.
Apdo. Post. 14-740, 07000, México, D.F., México
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Abstract

We suggest that many of the free parameters in the supersymmetric extensions of the Standard Model can be all linked together to the existence of a non universal $U(1)$ gauge symmetry, which has been spontaneously broken at very high scale. Such a symmetry can easily generate, via non-renormalizable operators, appropriate tree level fermion mass textures as well as the $\mu$-term of the Higgs potential. We give a general parametrization of those terms. As an output, $R$ parity breaking terms only appear at a non-renormalizable level and linked to the Yukawa couplings, giving rise to the possibility of having an effective (exact) $R$ parity conservation. As an interesting application of this idea we explore the case where neutrinos are Dirac particles. The scenario can be embedded in an extended $SU(5)$ unification theory where the extra right handed neutrinos are introduced as singlets. Such theory has an exact $R$ parity conservation and sneutrino as LSP.

I. INTRODUCTION

Supersymmetry (SUSY) [1] has been considered for many years as the preferred solution to the hierarchy problem that pollutes the Higgs sector in the Standard Model (SM) of particle physics. The natural cancellation of quadratic divergences in the SUSY theory, which takes place among particles and sparticles belonging to same representation, makes it a very appealing extension of the SM. However, as it is well known, SUSY is not an exact symmetry in nature but rather a broken one, and the effective SUSY SM has more than one
hundred free parameters. About twenty of these parameters correspond to those originally contained in the SM, among which one has masses and mixings of fermions, three coupling constants, a CP phase and the $\mu$ coupling on the Higgs potential. Many other of these parameters are the so called soft breaking terms, which encode our ignorance of the way SUSY is being broken at the high scale. In the SUSY limit, however, new renormalizable gauge invariant terms, which explicitly violate lepton and baryon number conservation, can also be written down. Usually such terms are dangerous since they may give rise to a fast proton decay, or to flavour changing neutral processes at large rates. The usually advocated solution of this problem is the addition of an extra discrete symmetry, called R parity, which distinguishes particles from their superpartners, and which also forbids such dangerous operators. On the contrary, in the absence of such a symmetry one needs to introduce small couplings which can be sources of new phenomena. An interesting use of R-parity breaking terms could be, for instance, the generation of neutrino masses [2]. Although in the SM, the neutrino is massless, recent experimental data points towards massive neutrinos (with masses smaller than few eV) and to one or possibly two large mixing angles in the lepton sector [3,4]. The situation is then opposite to that in quark sector where mixing angles are small.

With massive neutrinos there also comes the challenging problem of understanding whether neutrinos are Majorana or Dirac particles. So far, no evidence that may resolve this question exists. A positive signal in future neutrinoless double beta decay experiments would decide for a Majorana neutrino. However, a negative result would not shed any light up on this problem. From the theoretical point of view, the generation of neutrino masses usually needs extending the SM particle content by adding right handed neutrinos $N_R^i$ (where $i = 1, 2, 3$ is the family index), which can be either SM singlets or triplets under $SU(2)_L$ gauge symmetry. Notice, however, that (Majorana) neutrino masses can still be generated even without introducing extra fields (see for instance Ref. [2]). With a right handed singlet we can write the following Yukawa couplings:

$$h_{ij} L_i H_2 N_{R}^j,$$

where $L_i$ is left handed doublet, $H_2$ is SM Higgs field and $h$ are the Yukawa couplings. Once Higgs field gets a VEV, Eq. (1) induces Dirac masses for the neutrinos, $M_D L_i N_{R}^j$. On the other hand, a priori one can not forbid Majorana mass terms for right handed neutrinos, $M_R \bar{N}_{R}^c N_{R}$. Thus, the question of whether the neutrino is a Majorana or a Dirac field clearly depends on the $M_R$ scale. So, we define a neutrino to be a:

1. Majorana particle if $M_R \gg M_D$;
2. Pseudo-Dirac particle if $M_R \ll M_D$;
3. Dirac particle if $M_R = 0$.

Last two cases are difficult to accommodate theoretically since they may required a large fine tuning on the Yukawa couplings to get eV masses. In fact one needs $h \sim O(10^{-11})$ or less. Some models that attempt to explain the smallness of this coupling can be found in Refs. [2–4]. In contrast, the lightness of neutrino as a Majorana particle is easier to explain through the seesaw mechanism [8]. For this reason neutrinos as Majorana particles are usually considered as the most appealing possibility.
In any case, the extended theory that includes an explanation for neutrino anomalies will contain at least six more parameters: three neutrino masses and their three mixing angles. The sole existence in the theory of all those (fundamental) parameters has made us to believe that neither the SM, nor its minimal SUSY version (MSSM), can be the final theory of particle physics. Understanding their origin has motivated the exploration of many new ideas and models for particle physics at higher energies. Examples of it are the extended gauge models as the Left-Right models and the Grand Unified Theories (GUT). It is expected that some of these ideas may find a realization in String Theories. The usual trend in the study of this problem, however, is to analyze each sector, or set of parameters, in an almost independent way. The pattern of neutrino masses is usually studied separately from all other sectors and in fact new scales, and so new physics, are usually introduced [8]. GUT explains the values of the coupling constants but it can say very little about the masses and mixings among fermion families without the help of extra (flavour) symmetries, though some links between lepton and quark mixing and masses can still be obtained [9]. Indeed, using Abelian flavor symmetries [10,11] to understand the mass hierarchies is very popular.

In this paper we are re-addressing the question of whether all these ingredients of the theory may all be linked together through the existence of some Abelian $U(1)_A$ symmetry. The overall idea is as follows. We assume that there exists an extra symmetry that can distinguish among the different MSSM particle representations. Thus, except for gauge and soft SUSY breaking couplings, all other couplings (Yukawa; $\mu$ and R-parity breaking terms) might not be allowed at the renormalizable level by $U(1)_A$ gauge invariance. In fact, $U(1)_A$ charges might be such that only certain couplings, among all, would be acceptable. As there are not too many degrees of freedom (total number is equal to number of field representations), once we fix them by asking, for instance, for specific profiles (textures) of the lepton and quark mass matrices, all other allowed couplings of the theory would be predicted. Moreover, if our $U(1)_A$ is broken at some large energy scale by a SM singlet, then, those operators that were not present in the unbroken theory can now be generated by the use of non-renormalizable operators. Such operators come with a suppression respect to the renormalizable ones, thus they can be responsible for generating the observed hierarchy on the masses, as well as suppressing (forbidding) R-parity breaking terms. However, interestingly enough, this mechanism can only generate a very specific class of mass matrices for the minimal matter content. Although in our mechanism one can equally use global as well as local symmetries, we will rather prefer to use local ones. An interesting example of such a theory is the case of anomalous gauge symmetries [10,11]. Such anomalous $U(1)_A$ symmetries may be related to String Theory. They are usually broken close to string scale and the anomaly canceled by Green-Schwarz [12] mechanism. Note that the freedom that an Abelian symmetry has on charge assignments may be reduced if one considers non-Abelian symmetries instead. Alternative models of these sort can be found on Ref. [13].

We organize the present discussion as follows. First, to motivate the ideas, we briefly review the Green-Schwarz [12] mechanism used to cancel the $U(1)_A$ anomaly. Then, we shall discuss the general procedure used to determine the mass matrices induced by the $U(1)_A$ symmetry. As we will show, the mass textures generated by this mechanism can be parameterized by a small number of parameters, which encode the actual charge assignments on matter fields. As a direct application of our whole idea, we study the possibility of using an Abelian $U(1)_A$ symmetry for getting an effective theory that accommodates Dirac neutrinos.
We also show that R-parity breaking terms are linked to the effective Yukawa couplings. In fact in a theory with Dirac neutrinos only baryon number violating interactions would be present. We further explore the possibility that such a theory can be embedded in a GUT environment, particularly in an extended $SU(5)$ theory. There, one gets a theory that predicts exact R-parity conservation. Thus, LSP appears as a dark matter candidate, which in our example could correspond to the right handed sneutrino. Some explicit examples of tree level mass matrices are given at the end of the paper.

II. FRAMEWORK

Let us start by discussing the general mechanism we want to use for protecting mass textures and constraining R-parity breaking terms. In this paper we will always work in the context of SUSY. In order to allow for (Dirac) neutrino masses we extend the MSSM by introducing three right handed neutrino superfields, $N_i$. In addition, we introduce a $U(1)_A$ flavor symmetry, which distinguishes the different matter representations of the MSSM through the prescription of $U(1)_A$ charges. We will also assume that this symmetry has been broken spontaneously, at some high scale, by a heavy scalar superfield $S$. Given this, our working philosophy will be to consider, at tree level, all those renormalizable and non-renormalizable operators that are allowed for the theory. Among them, we will pay most of our attention to those which, upon symmetry breaking, reproduce Yukawa couplings, $\mu$ and R-parity breaking terms. Therefore, we will take the approach that try to understand the hierarchies as a consequence of large suppressions coming on high dimension operators. Our final goal will be parameterizing the most general form of all those couplings in the effective low energy theory. It is worth mentioning that the power of this effective field theory approach is that it gives a well defined framework for knowing where the small numbers appear on the theory. However, we should clearly state that we will not provide a dynamical model that would yield exactly the set of higher dimensional operators that we will discuss along the paper.

In the absence of any other heavy particles, it turns out that the $U(1)_A$ symmetry is anomalous. It is known that such anomalous $U(1)_A$ factors can arise in string theories. Cancellation of such anomaly occurs through the Green-Schwarz mechanism [12]. Though, we will assume so for the moment, this is an additional ingredient which may not be at all required for our mechanism, since heavy exotic matter may also take care of balancing the theory, for instance. Due to the Abelian $U(1)_A$ symmetry, a Fayet-Illiopoulos D-term $\xi f d^4 \theta V_A$ is always generated, where in string theory $\xi$ is given by [14]

$$\xi = \frac{g_A^2 M_P^2}{192 \pi^2} \text{Tr} \ Q .$$

(2)

The $D_A$-term will have the form:

$$\frac{g_A^2}{8} D_A^2 = \frac{g_A^2}{8} \left( \Sigma Q_a |\varphi_a|^2 + \xi \right)^2 ,$$

(3)

where $Q_a$ is the ‘anomalous’ charge of $\varphi_a$ superfields. In order to break down $U(1)$ we introduce a singlet superfield $S$ with $U(1)$ charge $Q_S$. Assuming $\xi > 0$ $[\text{Tr} Q > 0$ in (3)], and taking $Q_S = -1$ , the cancellation of $D_A$ fixes the VEV of the scalar component of $S$ field:
\[ \langle S \rangle = \sqrt{\xi}. \] 

Furthermore, we will take
\[ \frac{\langle S \rangle}{M_P} \equiv \epsilon \simeq 0.2. \] 

As we shall see below, \( \epsilon \) turns out to be an important expansion parameter. Indeed due to the coupling of \( S \) with all other MSSM fields, \( \epsilon \) parameter governs all mass textures in the theory, as we will now discuss. From this point of view, fermion masses and mixings would be the residual witnesses of the existence of the \( U(1)_A \) symmetry.

### III. QUARK TEXTURES FROM A \( U(1)_A \) SYMMETRY

Let's consider the following general assignment of \( U(1)_A \) charges to the MSSM and right handed neutrino field representations,

\[
\begin{align*}
L_i(1,2,-1/2) & : \alpha_i ; \\
N_i(1,1,0) & : \beta_i ; \\
E_i(1,1,1) & : \delta_i ; \\
Q_i(3,2,1/6) & : \theta_i ; \\
U^c_i(3,1,-2/3) & : \omega_i ; \\
D^c_i(3,1,1/3) & : \rho_i ; \\
H_u(1,2,1/2) & : \gamma ; \\
H_d(1,2,-1/2) & : \sigma ,
\end{align*}
\]

where in the brackets are given the \( SU(3)_c \times SU(2)_L \times U(1)_Y \) quantum numbers of the particle. Also, The index \( i \) stands for family replication. With these choices, Yukawa couplings are not trivially allowed by gauge invariance. Instead, the superpotential contains a class of operators that involve couplings to \( S \) field.

In the up quark sector, for instance, the most general couplings that are trilinear in the MSSM fields have the form
\[ W_u = \sum_{ij} h^u_{ij} Q_i H_u U^c_j \left( \frac{S}{M_P} \right)^{n_{ij}}, \] 

where \( n_{ij} \) is a set of integer numbers. Accordingly to our working philosophy, the dimensionless coupling constants, \( h_u \), would be taken hereon to be of order one. Notice that this apparent extra degrees of freedom would not actually play any important role on our understanding of small numbers on the theory. Indeed, such order one parameters are not really sizeable, nor arbitrarily adjustable, as to account for the features of the mass spectrum, and so we will omit them along our further discussion, though their presence should be understood. After the scalar part of the \( S \) superfield has developed its VEV, the above superpotential will give rise to the effective Yukawa couplings
\[ Y^{ij}_u = \epsilon^{n_{ij}}, \] 

which generate the up quark mass matrix \( M_u \sim Y_u v \), with \( v \) the Higgs VEV. Therefore, we can easily get a hierarchical mass pattern defined in terms of powers of a single parameter, \( \epsilon \), just as desiderated in many cases. We shall stress, however, that not just any arbitrary texture can be accommodated through this mechanism. Indeed, gauge invariance constrains the exponents by relating them to the charges of the MSSM fields in Eq. (4), such that
\[ n_{ij} = \theta_i + \omega_j + \gamma . \] (9)

Let us mention for completeness that a negative (or non integer) \( n_{ij} \) would actually indicate a forbidden coupling. It is then straightforward to show that the above formula translates into a symmetry relationship among the \( n_{ij} \) parameters. Indeed, for any four given family indexes one gets

\[ n_{ij} + n_{kl} = n_{kj} + n_{il} . \] (10)

Eq. (10) reduces the number of linearly independent \( n_{ij} \) parameters to five in a three family basis. An easy way to understand this point is as follows. First notice that Eq. (10) relates the exponents of the four elements of any two by two submatrix of \( Y_u \). Therefore, only three of those elements are actually independent. One actually gets

\[ \left( \begin{array}{ccc} \epsilon^{n_{ij}} & \epsilon^{n_{il}} \\ \epsilon^{n_{kj}} & \epsilon^{n_{il}+n_{il}-n_{ij}} \end{array} \right) . \] (11)

Now we proceed by adding a row (or a column) to the above matrix. Furthermore, Eq. (10) will constrain one of the two elements of such a row (column), and so, our degrees of freedom get increased only by one. By induction we are then led to the conclusion that in a model with \( N \) flavours the Yukawa couplings of Eq. (8) are defined by \( 2N - 1 \) degrees of freedom. Thus, for \( N = 3 \) one has only five.

A similar conclusion will follow from considering the down quark couplings

\[ W_d = \sum_{ij} Q_i H_d D_j \left( \frac{S}{M_p} \right)^{k_{ij}} , \] (12)

where now \( k_{ij} \) represents the power on \( \epsilon \) suppression that appears in the effective Yukawa couplings \( Y_d^{ij} = \epsilon^{k_{ij}} \). We shall notice that besides the already known constraint among the \( k_{ij} \) parameters, given by exchanging \( n \rightarrow k \) in Eq. (11), one also gets a new constraint that interrelates both, up and down, sectors, due to the fact that both Eqs. (7) and (12) share a common field, \( Q \). By combining the relation \( k_{ij} = \theta_i + \rho_j + \sigma \) with that in Eq. (9) one easily gets

\[ n_{ij} + k_{kl} = n_{kj} + k_{il} , \] (13)

for any arbitrary set of family indexes \( i, j, k, l \).

Once we have fixed all five free parameters that determine the up mass texture, above constraints will leave only three more independent parameters to define the down mass matrix. Therefore, the most general Yukawa couplings that one can build from our mechanism can be written as

\[ Y_u \sim \left( \begin{array}{ccc} \epsilon^{n_1+n_5} & \epsilon^{n_2+n_5} & \epsilon^{n_5} \\ \epsilon^{n_3+n_4} & \epsilon^{n_2+n_4} & \epsilon^{n_4} \\ \epsilon^{n_3} & \epsilon^{n_2} & 1 \end{array} \right) \epsilon^{n_1} ; \quad \text{and} \quad Y_d \sim \left( \begin{array}{ccc} \epsilon^{k_1+n_5} & \epsilon^{k_2+n_5} & \epsilon^{n_5} \\ \epsilon^{k_3+n_4} & \epsilon^{k_2+n_4} & \epsilon^{n_4} \\ \epsilon^{k_3} & \epsilon^{k_2} & 1 \end{array} \right) \epsilon^{k_1} ; \] (14)

where \( n_1, ..., 5 \) and \( k_{1,2,3} \) represent the eight arbitrary integer numbers that parametrize our (model dependent) degrees of freedom. Notice that we have factorized the overall suppressions, \( \epsilon^{n_1} \) and \( \epsilon^{k_1} \), which characterize the mass scales of quarks in third family.
IV. LEPTON TEXTURES.

We now turn our attention to the lepton sector. We then consider the following interactions

$$W_\ell = \sum_{ij} L_i H_u N_j \left( \frac{S}{M_P} \right)^{p_{ij}} + L_i H_d E_j^c \left( \frac{S}{M_P} \right)^{q_{ij}}$$

which give rise to the effective Yukawa couplings, $Y_{\nu}^{ij} = e^{p_{ij}}$ and $Y_e^{ij} = e^{q_{ij}}$, that are responsible for the generation of masses for Dirac neutrinos and charged leptons, respectively. Once again, the analysis follows same lines as in the quark sector above. $p_{ij}$ and $q_{ij}$ are related to $U(1)_A$ charges by $p_{ij} = \alpha_i + \beta_j + \gamma$ and $q_{ij} = \alpha_i + \delta_j + \sigma$. By combining this expressions we will find, again, that only eight free parameters govern the profiles in the mass matrices, $M_{e,\nu} = Y_{e,\nu} v$, also called the mass textures. We then choose those parameters in order to write the most general lepton textures in the useful form

$$Y_\nu \sim \begin{pmatrix} 1 & e^{p_2} & e^{p_3} \\ e^{p_4} & e^{p_{2+4}} & e^{p_{3+4}} \\ e^{p_5} & e^{p_{2+5}} & e^{p_{3+5}} \end{pmatrix} e^{p_1} \quad \text{and} \quad Y_e \sim \begin{pmatrix} e^{q_{3-p_5}} & e^{q_{2-p_5}} & e^{-p_5} \\ e^{q_{3+p_4-p_5}} & e^{q_{2+p_4-p_5}} & e^{p_{4-p_5}} \\ e^{q_3} & e^{q_2} & 1 \end{pmatrix} e^{q_1},$$

In this expressions we have used five $p$ integer numbers to parametrize the Dirac neutrino couplings, and three more, $q$, for $Y_e$. Notice that the matrices in Eq. (16) are just a reparametrization of those already given for the quark sector in Eq. (14). As before, the factors $e^{p_1}$ and $e^{q_1}$ give overall suppression scales. However, the larger scale in each sector may be different, since it actually depends on our specific choice for the numerical values of the $p$ and $q$ parameters.

There will also be some operators, which are bilinear in the low energy fields, that induce Majorana mass terms for the right handed neutrinos. They are written as

$$W_R = \sum_{ij} M_R N_i N_j \left( \frac{S}{M_P} \right)^{r_{ij}}$$

with $h_R$ of the order of one, and $r_{ij} = \beta_i + \beta_j$. Therefore, the induced Majorana masses will have the form $(M_R)_{ij} = Y_{R}^{ij} M_P$ where $Y_{R}^{ij} = e^{r_{ij}}$. A priori, and due to the symmetry in $r_{ij}$, the right handed mass terms should be defined in terms of three different $r$ parameters. However, once we have fixed the Dirac textures in Eq. (16), there would be only one extra degree of freedom that one can use to do the parametrization. One can easily understand this fact by noting that there is a constraint that relates the $p_{ij}$ parameters on the $Y_\nu$ texture with the $r_{ij}$ ones of $Y_R$, which is similar to that of Eq. (13). Indeed, from gauge invariance one implies $p_{ij} + r_{lk} = p_{ik} + r_{ij}$. Thus, we can parametrize the right handed masses as

$$M_R \sim \begin{pmatrix} 1 & e^{p_2} & e^{p_3} \\ e^{p_2} & e^{p_{2+p_3}} & e^{p_{2+p_3}} \\ e^{p_3} & e^{p_{2+p_3}} & e^{2p_3} \end{pmatrix} e^r M_P .$$

By looking at $M_R$ we notice that since we have chosen $p_{1,2,3}$ to be integer numbers, there is a possibility of forbidding all Majorana masses just by taking $r$ to be either a fractional
or a large negative number. This is an interesting possibility that we want to explore in the last part of the paper. On the other hand, a large and positive $r$ would add for a large suppression on $M_R$, thus leading to pseudo-Dirac neutrinos.

Among all other non-renormalizable operators that low energy theory would have, we will find those of the form

\[ \epsilon^{s_{ij}} \frac{L_i H_u L_j H_u}{M_P}; \]  

(19)

and

\[ \epsilon^{t_{ij}} \frac{N_i N_j H_u H_d}{M_P}. \]

(20)

In principle, they may induce tiny Majorana masses for left and right handed neutrinos. In the case where $M_R$ is large, we can safely neglect these two last contributions. However, in case $M_R$ terms were forbidding, it is not obvious that same holds for these two operators. If they were allowed they would split the Dirac neutrino components into a pseudo-Dirac pair. The induced mass squared difference could be too small as to contribute to neutrino oscillations. In this case it is easy to show that the exponents in the class of terms of Eq. (19) are totally fixed by the parameters already used for the other lepton textures. In fact, after some simple algebra one gets $s_{ij} = p_{i1} + p_{j1} - r$, which also means that

\[ \epsilon^{s_{ij}} \sim Y_\nu^i Y_\nu^j \epsilon^{-r}. \]

(21)

Clearly, if $r$ is chosen to be a non integer number (or a large enough negative one), then such terms would be forbidden. This is it due to our choice on the $p$ parameters as integer numbers. We shall postpone the discussion of the remaining terms for the next section, since they are closely related to the $\mu$-term that we will now discuss.

V. $\mu$-TERM

It is worth stressing that above textures have been calculated yet with out knowing the actual values of $U(1)_A$ charges in Eq. (6). In fact, our whole procedure has been rather to express such charges in terms of the above given parameters. Thus, all our low energy physics could be written in terms of a set of integers numbers with a clear physical meaning, that of the hierarchies. One should notice, however, that so far we have involved up to twenty different charges, corresponding to equal number of superfield representations, which were used in writing down the superpotential terms $W_u$, $W_d$, $W_\ell$ and $W_R$. Nevertheless, only seventeen of them can be fixed by any given choice of Yukawa textures. Thus, at least three of all will remain undefined at this point.

One extra constraint comes from the $\mu$-term, which is again non trivially allowed at the renormalizable level (unless of course $\gamma = -\sigma$ which we do not assume). Indeed, the most general coupling that induces a $\mu$-term on the effective low energy theory is given, up to an order one coupling constant, as

\[ M_P H_u H_d \left( \frac{S}{M_P} \right)^{a}, \]

(22)
for an integer number that satisfies $a = \gamma + \sigma$. Hence, by setting in the VEV of the $S$ field, one gets the induced $\mu H_uH_d$ term, where

$$\mu = e^a M_P.$$  \hfill (23)

The large hierarchy between Planck and electroweak scale fixes $a = \ln(\mu/M_P)/\ln \epsilon \sim 16$. We then choose $a$ as a parameter of the theory.

Notice that the coupling (22) also enters in the operator mentioned in Eq. (20), which generates tiny right handed neutrino masses. The possible texture of those terms would be completely fixed by that in $Y_R$. Indeed, one gets

$$\epsilon_{ij} \sim \left(\frac{\mu}{M_P^2}\right) (M_R)_{ij}.$$  \hfill (24)

Therefore, the couplings in Eq. (20) appear just as a correction to any already existing right handed mass term. Hence, forbidding $M_R$ will also forbid these extra terms. We shall notice, however, that in the special case where one forbids $M_R$ couplings by taking $r$ to be negative, the mass terms in Eq. (20) could still be generated due to possible conspiring cancellations among the parameters, which may render $a + r_{ij} > 0$. In such a case the theory would generate pseudo-Dirac neutrinos.

VI. R-PARITY BREAKING INTERACTIONS

Last condition imposed by the $\mu$-term reduces our ignorance on the charge assignments to only two undefined degrees of freedom. In the absence of new (model dependent) constraints, such degrees of freedom would be fixed by the experimental bounds on the induced R-parity breaking terms. As we have already used nine independent parameters to write the lepton textures, the remaining two degrees of freedom can only affect the baryon number violating interactions,

$$\lambda''_{ijk} U_i^c D_j^c D_k^c .$$  \hfill (25)

In the above expression the effective coupling constant $\lambda''_{ijk} \sim \epsilon_{ijk}$, where the exponent $t_{ijk} = \omega_i + \rho_j + \rho_k$. Now, we proceed to determine the number of linearly independent parameters that give $t_{ijk}$. First, we notice that $t_{ijk}$ contains only nine non zero entries due to the discrete symmetry under $j \leftrightarrow k$ exchange, and to the color symmetry of quarks. There are also some extra constraints that relate the different $t_{ijk}$ among themselves, $t_{ikl} + t_{jkm} = t_{jkl} + t_{ikm}$; and with the exponents in $Y_u$ and $Y_d$ textures through the symmetry relationships: $t_{ikl} - t_{jkl} = n_{jm} - n_{jm}$ and $t_{ijk} - t_{ijkl} = k_{ik} - k_{il}$, all for arbitrary family indexes. Putting all these constraints together, it is easy to realize that only one extra independent parameter is needed to completely parametrize the $\lambda''$ couplings. We then get

$$\lambda'' \equiv \left( \begin{array}{ccc} \lambda''_{123} & \lambda''_{131} & \lambda''_{112} \\ \lambda''_{223} & \lambda''_{231} & \lambda''_{212} \\ \lambda''_{323} & \lambda''_{331} & \lambda''_{312} \end{array} \right) \sim \left( \begin{array}{ccc} \epsilon^{n_{5} - k_{3}} & \epsilon^{n_{5} - k_{2}} & \epsilon^{n_{5}} \\ \epsilon^{n_{4} - k_{3}} & \epsilon^{n_{4} - k_{2}} & \epsilon^{n_{4}} \\ \epsilon^{- k_{3}} & \epsilon^{- k_{2}} & 1 \end{array} \right) \epsilon^t .$$  \hfill (26)

Lepton and R-parity violating interactions appear in the MSSM due to the field equivalence on lepton doublets, $L_i$, and the chiral Higgs superfield, $H_d$. In fact, in the MSSM they
carry the same quantum numbers associated to the SM gauge group. Thus, SUSY makes no distinction among them, and one can write in the superpotential the interactions
\[ m_i L_i H_u + \lambda_{ijk} L_i E_j^c L_k + \lambda'_{ijk} L_i Q_j D_k^c . \] (27)
However, those fields are actually different under the \( U(1)_A \) symmetry. At the level of the low energy effective theory, these terms will also be generated by non renormalizable operators. Moreover, the suppression on the different couplings appearing in Eq. (27) is governed by the lepton and quark textures. A simple calculation allows to show that the bilinear mass coupling goes as
\[ m^i \sim Y^{i1}_\nu \epsilon^{-r/2} M_P . \] (28)
The fractional exponent \( r/2 \) means that such terms are only accepted when \( r \) is an even number. For any other choice of this parameter this bilinear couplings will be forbidden. Notice, however, that when \( r \) is a large negative number this coupling is also zero (since then \( Y_\nu = 0 \)). A similar conclusion comes out in the case of the other two couplings, which are given as
\[ \lambda_{ijk} \sim Y^{i1}_\nu Y^{kj}_e \epsilon^{-a-r/2} ; \quad \text{and} \quad \lambda'_{ijk} \sim Y^{i1}_\nu Y^{jk}_d \epsilon^{-a-r/2} . \] (29)
This shows an interesting connection between the nature of neutrino and R-parity violating interactions in the theory. Actually, it reflects the well known conservation of lepton number that appears when neutrinos are Dirac particles. Interestingly enough, our mechanism is able to generate very light Dirac neutrino masses, and at the same time to protect them from all those potentially dangerous operators that could induce Majorana masses. Some clarification is needed at this point. The fact that relates R-parity violating terms in Eq. (27) with the Majorana mass parameter \( r \), that otherwise would not be expected since \( N \) does not appear at all on such couplings, is twofold. First, the couplings in Eq. (27) play no role in fixing the \( U(1)_A \) charges which at this point have been already defined by all other couplings. Second, our assumption made on the numbers involved in the parametrization of fermion masses (\( k, n, p \) and \( q's \)), which we take as integers, makes \( r \) the only possible non integer piece in the involved combination of charges.

VII. A \( SU(5) \) GUT EMBEDDING

There are few remarks of interest that one has to keep in mind while considering the mechanism that we have analyzed in the previous sections. Even though we have started with a theory that has twenty degrees of freedom, represented by the assignment of \( U(1)_A \) charges in Eq. (8), only nineteen independent parameters (\( n_{1, \ldots, 5}; k_{1,2,3}; p_{1,\ldots,5}; q_{1,2,3}; r; a \) and \( t \)) are actually needed to parametrize the effective low energy theory. Therefore, our present approach seems more economical. It allows to describe the physics of the low energy theory without bothering about the actual charge distribution. Moreover, as we have one parameter less, there is in principle a whole class of theories that may give same low energy physics. We will use these extra degree of freedom as a motivation to explore the embedding of our mechanism in the context of a \( SU(5) \) unification theory.
Let us consider a supersymmetric $SU(5)$ model where the MSSM matter content is accommodated in the 5 and 10 representations as usual.

$$5_i : (D^c, L)_i ; \quad 10_i : (U^c, Q, E^c)_i ; \quad 5_d : H_d \quad \text{and} \quad 5_u : H_u . \quad (30)$$

The right handed neutrinos, as well as the $S$ superfield are not originally in the theory, thus we will add them in the singlet representations: $1_{N_i}$ and $1_s$, respectively. As a comment, let us mention that the high energy fields needed to break the $SU(5)$ symmetry, and to solve the doublet triplet splitting may be chargeless under $U(1)_A$, such that their self couplings would not be affected. So that pieces of the theory would remain unchanged.

Since 5 and 10 contain both leptons and quarks, the $U(1)_A$ charges [Eq. (8)] will be constrained. This substantially reduces the number of degrees of freedom in the theory to eleven, same that we will use to rewrite our parametrization of mass matrices. First we write the Dirac mass terms, $\bar{5}_u 1_{N_i}$, and notice that no extra condition is imposed on the parameters from these couplings, so $Y_{\nu}$ is given as in Eq. (16). On the other hand, as the up quark masses now come from the couplings $10 \ 10 \ 5_u$, the corresponding mass matrix would be symmetric, and thus, its parametrization has only three degrees of freedom. Hence, one has to take $n_1 = n_2$, and $n_5 = n_3$ in Eq. (8). In contrast, now both charge lepton and down quark textures come from the same generic coupling, $10 \ 5 \ 5_d$, which implies that $Y_d = Y_e^T$. Moreover, now, the most general form of the $Y_e$ texture will only have a single degree of freedom, that enters as an overall scale factor. Actually, one now should use $q_2 = n_2$ and $q_3 = n_3$ in Eq. (19). Finally, the right handed mass terms, $M_{\nu} \ 1_N \ 1_N$, will remain as they were, and so our conclusions regarding all other Majorana mass terms.

Therefore, in the $SU(5)$ context one gets the general parametrization

$$Y_{\nu} \sim \left( \begin{array}{ccc} 1 & \epsilon^{p_2} & \epsilon^{p_3} \\ \epsilon^{p_4} & \epsilon^{p_2+p_4} & \epsilon^{p_3+p_4} \\ \epsilon^{p_5} & \epsilon^{p_2+p_5} & \epsilon^{p_3+p_5} \end{array} \right) \epsilon^p ; \quad Y_R \sim \left( \begin{array}{ccc} 1 & \epsilon^{p_2} & \epsilon^{p_3} \\ \epsilon^{p_4} & \epsilon^{p_2+p_4} & \epsilon^{p_3+p_4} \\ \epsilon^{p_5} & \epsilon^{p_2+p_5} & \epsilon^{p_3+p_5} \end{array} \right) \epsilon^r ;$$

$$Y_u \sim \left( \begin{array}{ccc} \epsilon^{n_3} & \epsilon^{n_2+n_3} & \epsilon^{n_3} \\ \epsilon^{n_2+n_3} & \epsilon^{n_2} & \epsilon^{n_2} \\ \epsilon^{n_3} & \epsilon^{n_2} & 1 \end{array} \right) \epsilon^{n_1} ; \quad Y_d^T ; Y_e \sim \left( \begin{array}{ccc} \epsilon^{n_3-p_5} & \epsilon^{n_2-p_5} & \epsilon^{-p_5} \\ \epsilon^{n_3-p_4-p_5} & \epsilon^{n_2-p_4-p_5} & \epsilon^{-p_4-p_5} \\ \epsilon^{n_3} & \epsilon^{n_2} & 1 \end{array} \right) \epsilon^q . \quad (31)$$

Ten independent parameters have been involved in the above expressions: $n_{1,2,3}$, $p_{1,...,5}$, $q$ and $r$. As before, the $\mu$-term, $5_d 5_u$, will fix one more, $a$. Therefore, once the above parametrization are given, the theory has no additional degrees of freedom. Any additional coupling on the theory will be a prediction that can be written in terms of the fermion mass textures. Such is the case of the R-parity breaking terms,

$$\lambda_{ijk} \bar{5}_i 10_j \bar{5}_k ; \quad (32)$$

where the coupling constants are given as in Eq. (29): $\lambda_{ijk} \sim Y_{\nu}^{i1} Y_d^{jk} \epsilon^{-a-r/2}$. It is worth noticing that the baryon number violating interactions in Eq. (20) will now follow the same fate of all other R-parity breaking terms. Interestingly, they all would be forbidden in case neutrinos were Dirac particles, which happens if one takes $r$ to be a fractional number. Thus, in the $SU(5)$ context, the link between the nature of the neutrino and the R-parity breaking terms becomes stronger.
Let's make a final remark regarding the observation that \( Y_e = Y_d \) from above analysis. Last might be considered as a negative observation. This is, however, a well known problem of minimal \( SU(5) \) GUT, for which solutions are also known. In order to break the degeneracy among down quarks and charged leptons on the second and third family, one can extend the theory by adding an extra field, \( \Sigma \), in the 24 representation, which carries no \( U(1)_A \) charge and develops a VEV along the direction \( \langle \Sigma \rangle = V \text{Diag}(2, 2, 2, -3, -3) \). This field has the coupling \( 10 \bar{5}_d \Sigma \) that corrects the \( Y_d \) and \( Y_T \) in a different way due to the different sign on the VEV. Some small fine tuning in this case has to be introduced to get the proper order of correction. Nevertheless, this will not affect our other conclusions since the main hierarchy on the Yukawa couplings would be given by similar expressions as before.

VIII. SOME MODELS WITH DIRAC NEUTRINOS AND R-PARITY

There is, in fact, a large number of possible tree level mass matrices that one can generate from an additional \( U(1)_A \) symmetry. Let us consider some specific examples for illustration proposes. The case of our interest would be that where the theory has Dirac neutrinos, so, we fix \( r = \frac{1}{2} \). To simplify, we set \( n_1 = 0; n_2 = n_4 = 1 \) and \( n_3 = n_5 = 2 \) for \( Y_u \) in Eq. (31). Also we take \( p_1 = p + \ell \) and \( p_2, \ldots, 5 = -\ell \) for \( Y_\nu \), with \( \ell > p \). That gives, up to order one factors, the quite appealing tree level mass matrices

\[
M_u \sim m_t \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}; \quad M_e^T; M_d \sim m \begin{pmatrix} \epsilon^{\ell+2} & \epsilon^2 & \epsilon^2 \\ \epsilon^{\ell+1} & \epsilon & \epsilon \\ \epsilon^\ell & 1 & 1 \end{pmatrix}; \quad M_\nu \sim m_0 \begin{pmatrix} \epsilon^\ell & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}.
\]

(33)

In above expressions \( M_e \) and \( M_d \) are given up to corrections that need to be added to break the \( m_\mu/m_s = 1 \) condition, as we have already mentioned. Notice that we have also reproduced the neutrino texture associated to models with the well known \( L_e - L_\mu - L_\tau \) symmetry [15,16]. As we expect neutrinos to be very light, in fact here \( m_0 = e^p \nu \) has to be \( \sim \sqrt{\Delta m^2_{atm}} \), we are forced to take \( p \sim 12 \). That makes \( \ell > 12 \). Thus, if no extra corrections are considered, one gets for solar splitting \( \Delta m^2_\odot \sim 4\epsilon^\ell \sim 10^{-9} \text{eV}^2 \), for \( \ell = 13 \), which may be good for oscillations in vacuum. Alternative explanations can be obtained if the model admits a soft hierarchy in the \( h \) couplings, since small departures from the texture would be introduced this way.

Another example. Let us instead take \( p_1 = p + 2\ell \) and the same former values of other parameters. With this choice we get the same mass matrices as in Eq. (33) but with the Dirac neutrino texture

\[
M_\nu \sim m_0 \begin{pmatrix} \epsilon^{2\ell} & \epsilon^\ell & \epsilon^\ell \\ \epsilon^\ell & 1 & 1 \\ \epsilon^\ell & 1 & 1 \end{pmatrix}.
\]

(34)

This generates a hierarchical pattern [15]. Here \( \ell \) can take any value. Many other textures are possible.

If the setup is the extended MSSM, only baryon number violating interactions will appear. In the context of \( SU(5) \) theory, R-parity will be conserved. Assuming universality of soft SUSY breaking terms at GUT scale, in the case \( SU(5) \) SUSY GUT, or at \( M_P \) scale in
Supposing \( m_{1/2} \geq 3m_0 \) [17], where \( m_{1/2} \) and \( m_0 \) are common gaugino and squark–slepton mass at GUT or at \( M_P \) scale. Then, the possible candidate for a LSP could be the right handed sneutrino, which can also play the role of a dark matter candidate. The phenomenology of this scenario has been studied for instance in Ref. [3,17].

**IX. CONCLUSIONS AND REMARKS**

We have studied the generation of masses and mixings in models with an anomalous \( U(1)_A \) symmetry, which is supposed to be spontaneously broken close to the string scale. In such models the low energy interactions that are responsible for the generation of masses and mixings and R-parity violation, in the context of an effective MSSM, appear only at the renormalizable level due to the non trivial assignments of \( U(1)_A \) charges. By extending the MSSM matter content via adding three generations of right handed neutrinos, which are singlets under the SM interactions, we have uncovered an intriguing link between R-parity breaking interactions and Dirac neutrinos.

We have shown that in the context of the so extended MSSM, or \( SU(5) \) unification theory, the fermion mass textures can be easily parameterized in terms of a small set of numerical parameters. The parametrization has been done in a way that does not need the specification of \( U(1)_A \) charge distribution among low energy fields. The couplings are actually given in terms of powers of a single parameter \( \epsilon \). Thus, our phenomenological parameters are represented by a set of exponents of \( \epsilon \) that appear on the effective couplings at low energy. This phenomenological approach has the advantage not only of predicting the most general form of the effective Yukawa interactions, but also of relating the R-parity breaking couplings with the Majorana mass textures and the hierarchy between weak and Planck scales. From here, we showed that models with Dirac neutrinos may only have baryon number violating interactions, which are governed by an overall scale. In \( SU(5) \) extension of the theory, R-parity appears as an exact symmetry provided that neutrinos are Dirac particles. Some explicit examples of the tree level textures produced for this class of models have been also given. The mechanism may as well generate Majorana or pseudo-Dirac neutrinos.

We would like also to mention that our parameterizations are equally good for theories where the textures are protected by a global \( U(1) \) symmetry, which may be softly broken at some intermediate scale \( \Lambda \) by a scalar field \( S \). In such a case, it is obvious that in all our analysis one should take the suppression in the non renormalizable operators to be given by the scale of decoupling of the \( U(1) \) theory, \( \Lambda \), instead of \( M_P \). Hence, the expansion parameter \( \epsilon \) would be replaced by the ratio

\[
\epsilon \to \frac{\langle S \rangle}{\Lambda}.
\]

The only apparent changes will be suffered by the overall scale of \( M_R \) and the \( \mu \)-term that explicitly depend on the choice of \( \Lambda \). No other important changes would arise.

Finally, we should mention that all the analysis in the present paper has been done in the tree level approximation where one takes all dimensionless couplings, \( h_{u,d,e,\nu} \), to be one, according to the philosophy of effective field theories. Small deviations on our effective
Yukawa couplings could be expected, though they will not affect the textures provided that not large hierarchies are introduced through $h_{u,d,e,\nu}$.

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