Towards a $g$-factor determination of the electron bound in highly-charged calcium ions

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Abstract. Bound-state quantum electrodynamical calculations can be tested by high precision measurements of the magnetic moment of the electron bound in hydrogen-like and lithium-like ions. Measurements of hydrogen-like carbon and oxygen achieved relative experimental uncertainties as low as $2 \times 10^{-9}$. In the current experiment we plan to measure the $g$-factor of hydrogen-like and lithium-like calcium ions. The aim is to reach a relative uncertainty $\delta g / g$ in the order of $10^{-9}$. Here, we will give the motivation for the experiment, present the experimental techniques and the status of the experiment.

1. Introduction

The $g$-factor of an electron is a dimensionless proportionality constant between its magnetic moment $\mu$ and its angular momentum $\vec{J}$

$$\mu = -g e \frac{e}{2m_e} \vec{J},$$

where $e$ is the electron charge and $m_e$ its mass. The $g$-factor of a Dirac point-like free electron is predicted to be exactly 2 \cite{1}. However, relativistic \cite{2} and quantum electrodynamical \cite{3} effects slightly shift this value. Recently the $g$-factor of the free electron has been determined with an uncertainty as low as $8 \times 10^{-13}$ \cite{4} improving the previously best known value \cite{5} by a factor of six. If the electron is part of a bound system, the electric field created by the nucleus at the mean distance of the electron in the ground state ranges from $10^9$ V/cm for helium ($Z=2$) to $10^{15}$ V/cm for uranium ($Z=92$). Such huge fields result in much more complicated theoretical calculations, dealt with from a branch of quantum electrodynamics called bound-state quantum electrodynamics (BS-QED). Despite the difficulty of these calculations, substantial progress has been obtained in the field, and the numbers extracted for the $g$-factors of the electron bound to spinless, hydrogen-like nuclei are extremely accurate \cite{6}. $g$-factor experiments realized on hydrogen-like carbon and oxygen have also achieved very small uncertainties at the level of $10^{-9}$ \cite{7,8}, which, in combination with the values predicted theoretically, have led to the most precise determination of the mass of the electron in atomic mass units \cite{9,10}. For a more detailed review of the current status of the field see Ref. \cite{11}.

Since BS-QED effects become increasingly important with the nuclear charge $Z$, it is of interest to analyze heavier elements. For our current experiment \cite{12} we chose calcium for several reasons: it is considerably heavier than the species studied until now; it has two stable spinless isotopes (masses
40u and 48u), so the total angular momentum of the system is equal to the spin of the electron; and the binding energy of the 1s-electrons is \( \sim 5 \) keV, which is still affordable for our experimental setup. Besides the measurement of the g-factor on hydrogen-like calcium Ca\(^{19+}\), it is planned to perform it also on lithium-like calcium Ca\(^{17+}\) to test calculations and models of interelectronic interaction, which claim an accuracy of \( 9 \times 10^{-8} \) [13]. Finally, by comparing the measurements on \(^{40}\)Ca and \(^{48}\)Ca, mass effects and nuclear structure contributions can be studied for the first time.

2. g-factor measurement

The g-factor determination of a single stored ion is based on the “continuous Stern-Gerlach-effect” [14]. The Larmor frequency

\[
\omega_L = g \frac{e}{2m_e} B
\]

is the frequency with which the spin precesses in a magnetic field \( B \). The value of the magnetic field at the ion’s position can be determined from its cyclotron frequency, given by

\[
\omega_c = \frac{q}{m_{ion}} B,
\]

\( q \) being the charge of the ion and \( m_{ion} \) its mass. The g-factor is thus given by

\[
g = 2 \frac{\omega_L}{\omega_c} \frac{m_e}{e m_{ion}},
\]

where the masses of the electron and the ion are known from other publications [10, 15].

**Figure 1.** Schematic view of the cryogenic trap setup. At the right the double-Penning trap setup can be seen, which consists of the precision trap, where the magnetic field is extremely homogeneous for a proper determination of the eigen-frequencies of the ion, and the analysis trap, where the magnetic field is intentionally distorted to enable the detection of spin-flips. At the left, the EBIS is shown, including the electron gun and the creation trap, where the charge breeding takes place.

2.1. Triple-Penning trap setup

Figure 1 shows the experimental setup with which the g-factor is to be measured. It is based on a set of three cylindrical Penning traps [16]. A review of their applications was recently published in [17].

The confinement of the ions in a Penning trap is achieved with an electrostatic potential well (axial confinement) superposed to a constant magnetic field in the direction of the trap axis (radial confinement).
The motion of a particle in the trap can be seen as a composition of three independent oscillations, one in the axial direction and two in the radial plane, due to the non-vanishing cross term between the electric and magnetic fields. The axial frequency is given by \( \omega_z^2 = \frac{qU_0}{m_{\text{ion}}d^2} \), where \( U_0 \) is the electric potential depth and \( d \) indicates the dimensions of the trap. The reduced cyclotron frequency \( \omega_+ \) and the magnetron drift frequency \( \omega_- \) are \( \omega_{\pm} = \frac{\omega_z}{2} \pm \sqrt{\left(\frac{\omega_z}{2}\right)^2 - \frac{\omega_c^2}{2}} \).

### 2.2. Measurement of the axial frequency

In order to measure the axial frequency of an ion in a Penning trap, a tank circuit with a resonance frequency close to the frequency of the ion is attached to an electrode of the Penning trap [11]. The image charges that the ion induces in the electrode are picked up by the resonant circuit and, if the ion is in thermal equilibrium with the tank circuit, a dip will appear superimposed to the characteristic resonant noise curve of the tank circuit, exactly at the frequency with which the ion is oscillating.

### 2.3. Measurement of the free cyclotron frequency

Since the cyclotron motion in equation 4 is not a real motion in a Penning trap due to the influence of the electric field on the magnetic potential, it is not possible to measure it directly with the technique applied here. However, it can be derived from measuring all three real frequencies and using the Brown-Gabrielse invariance theorem [16]:

\[
\omega_c = \sqrt{\omega_+^2 + \omega_-^2 - \omega_z^2}. \tag{5}
\]

The reduced cyclotron frequency and the magnetron drift frequency will be determined by non-resonant coupling, based on the coupling of two independent trapping modes, e.g. the reduced cyclotron frequency and the axial frequency. This is done by irradiation of a suited electric radio-frequency (rf) field through which the corresponding motions are amplitude modulated [18]. The coupling frequency is slightly detuned by a frequency \( \delta \) relative to the sideband frequency, leading to a splitting of the resonance into two (avoided crossing). This technique is known as the “double-dip method”. In case of the reduced cyclotron frequency, the frequency components \( \omega_{r,l} \) are given by

\[
\omega_{r,l} = \omega_z - \frac{\delta}{2} \pm \sqrt{\frac{\delta^2}{4} + |A|^2} \tag{6}
\]

where \( |A| \) is measured in units of frequency and accounts for the strength of the coupling rf field. Thus, the reduced cyclotron frequency can be expressed as

\[
\omega_+ = (\omega_+ - \omega_z + \delta) + \omega_z + 2 \left( \frac{\omega_l + \omega_r}{2} - \omega_z \right) \tag{7}
\]

where \( \omega_+ - \omega_z + \delta \) is the known irradiated coupling frequency and \( \omega_z \) and \( \omega_{r,l} \) are observables. An analogous analysis can be derived for the magnetron motion.

### 2.4. Measurement of the Larmor frequency

Depending on the orientation of the spin with respect to the magnetic field (parallel or antiparallel), the coupling between the magnetic moment of the ion and the inhomogeneous magnetic field, which characterizes our analysis trap (figure 1), will superpose a small force in the axial direction, which will in turn increase or decrease the axial frequency that the ion would have in absence of the magnetic field. Thus, it is possible to determine the spin orientation through its influence on the axial frequency of the ion in the trap [19].

To determine the Larmor frequency, the only remaining quantity in Eq. (4), one can try to resonantly induce a spin-flip. The way to proceed experimentally is to irradiate the trapped ion with microwaves at a frequency close to the Larmor frequency and check the spin state before and after the irradiation. The Larmor frequency will be the microwave frequency for which the spin-flip probability is at maximum [18].
2.5. Ion creation

In order to conduct such high-accuracy measurements as intended, extreme vacuum and temperature conditions have to be fulfilled ($p \leq 10^{-16}$ mbar, $T \sim 4$ K). The charge breeding of the ions is therefore done in-trap with an electron beam ion source (see figure 1) located inside the sealed vacuum chamber [20]. By applying a voltage difference between field emission point array and an acceleration electrode, electrons tunnel out of the tips and are accelerated to an energy of up to 8 kV. The electron beam is guided axially by the strong magnetic field, so it flies through the creation trap and is reflected back by a high negative potential at the hyperbolical reflection electrode. As it flies back and forth, the electron beam expands because of Coulomb-repulsion and after several passes some electrons will hit the graphite target, with a sputtered layer of $^{40}$Ca and $^{48}$Ca of $\sim 10 \mu$m. The electrons will evaporate atoms from the target and the beam will ionize them, so they get trapped in the potential minimum of the creation trap where further charge breeding takes place by consecutive electron-impact ionization.

To be able to detect the signal of the image currents induced by the ions’ radial motion, a highly sensitive charge amplifier is attached to the split ring electrode of the trap. Since each charge state induces voltages at a specific frequency, it is possible to follow the evolution of up to all charge states simultaneously in real time, by means of a multi-channel FFT analyzer. From the observation and analysis of the charge breeding process, it is possible to obtain information about the ionization cross sections by electron impact [20].

3. Current status

The construction of the setup is completed and the detection electronics have been mounted and tested. Presently, the charge state breeding process is being investigated as required for the above mentioned possibility of measuring ionization cross sections. In the future, we plan to extend our $g$-factor measurements to heavy highly charged ions up to uranium $^{238}$U$^{91+}$ at the HITRAP facility [21].

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