Super-resolution enhancement of UAV images based on fractional calculus and POCS

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ABSTRACT
A super-resolution enhancement algorithm was proposed based on the combination of fractional calculus and Projection onto Convex Sets (POCS) for unmanned aerial vehicles (UAVs) images. The representative problems of UAV images including motion blur, fisheye effect distortion, overexposed, and so on can be improved by the proposed algorithm. The fractional calculus operator is used to enhance the high-resolution and low-resolution reference frames for POCS. The affine transformation parameters between low-resolution images and reference frame are calculated by Scale Invariant Feature Transform (SIFT) for matching. The point spread function of POCS is simulated by a fractional integral filter instead of Gaussian filter for more clarity of texture and detail. The objective indices and subjective effect are compared between the proposed and other methods. The experimental results indicate that the proposed method outperforms other algorithms in most cases, especially in the structure and detail clarity of the reconstructed images.

1. Introduction
In recent years, the use of unmanned aerial vehicles (UAVs) has increased in various fields. The UAVs are gaining outstanding success because of their versatility, flexibility, low-cost and minimized operational risk. Nonetheless, the quality of image from UAVs is sensitive to various internal factors (sensor performance, motion modes, exposure times, etc.) and to complex external conditions (fog, rain, turbulence, bad illumination of the scene, etc.). Thus, the resulting image usually contains deterioration problems, such as dynamic blur, fisheye effect, downsampling. In addition, UAV is with the flight-time limitations. Taking aerial images of a large field will consume a large amount of time, but maximizing the height, which increases the viewing perspective, will reduce the optical detail of image. These problems limit the extensive application of UAVs, especially for obtaining the higher resolution and precision images of target objects. Therefore, a super-resolution enhancement technique is necessary.

Image super-resolution refers to a class of algorithms that synthesize high-resolution images by estimating the values of unknown pixels from original single or multiple low-resolution images. It can provide better visual effect, and have a broad application prospect in remote sensing, military, medical imaging, public security, and other fields (Zhang et al. 2015). Image super-resolution methods are mainly divided into three categories: the interpolation-based methods (Bätz et al. 2015), the reconstruction-based methods (Bengtsson et al. 2012; Xiao et al. 2016), and the learning-based methods (Freedman and Fattal 2011; Lian and Zhang 2012). The interpolation-based methods have simple calculation, but often cannot obtain the ideal super-resolution results. The reconstruction-based methods allow different prior conditions in the reconstruction process to achieve better reconstruction results without cumbersome computational complexity. But it may lose the texture detail information of images because of limited prior knowledge. The learning-based methods rely on external training libraries to restore the ideal high-resolution images. Accordingly, its memory and time consumption are large, and the computational complexity is high. This paper focuses on the reconstruction-based methods combined with Projection onto Convex Sets (POCS) and fractional calculus.

POCS algorithm solves the problem of super-resolution reconstruction based on sets theory. First, the conditions that the solution must meet should be listed according to the prior knowledge, such as positive definite, energy bounded, data reliable, smooth. Then, these conditions are abstracted into several constrained convex sets mathematically. The combination of prior knowledge and solution consequently restricts the solution to the
has been widely introduced in the fields of cybernetics, the concept of calculus was proposed. In recent years, it has been studied in the field of mathematics since calculus in the real number domain, fractional calculus, which has good effect in both image denoising and enhancement. At the same time, the matching accuracy is improved by SIFT matching in the process of POCS. The fractional integral kernel is used to construct and improve the point spread function in POCS. Experimental data demonstrate that, compared with the traditional algorithm, the proposed algorithm can effectively improve the effect of convex sets reconstruction and enhance both the subjective and objective effect of reconstruction results, especially for blurred images.

2. Basic principle of POCS

The main idea of POCS is as follows. The unknown sequence $S$ is restricted in a closed convex set $C$ by every constraint. So if there are $n$ known constraints, there are $n$ corresponding closed convex sets. The reconstructed image is a point within the intersection of all constraint sets. As long as the intersection between the constraints exists, and $C_i (i \in N)$ is known as well as the corresponding operators $P_i$ of each set, the sequence $S$ will gradually converge to a point in the set through multiple iterations. In super-resolution reconstruction, we consider that there is a transformational model between high-resolution images and low-resolution images. In this model prior constraints can be such that a high-resolution image by POCS iterative convergence calculation in model space. From the degradation model (Stark and Oskoui 1989) of the image, we can know the relationship between the images before and after the degradation in image reconstruction as:

$$g(x_1, y_2) = \sum_{(x,y) \in \mathcal{S}} f(x_0, y_0)h(x_0, y_0; x_1', y_1') + m(x_1, y_1)$$

(1)

where $g(x_1, y_2)$ is a degraded image, $h(x_0, y_0; x_1', y_1')$ is the point spread function at point $(x_0, y_0)$ in the degenerate model, $f(x_0, y_0)$ is the original high-resolution image, and $m(x_1, y_1)$ is the possible additive noise.

We assume that $ref(x, y)$ is the result of the interpolation from one of the low-resolution image sequence frames, and $m(x, y)$ is the estimated value of the current high-resolution image through the imaging system. The high-resolution image is obtained according to the degradation model. If the point spread function is normalized to obtain the function $h(x, y)$, then the estimated value of $m(x, y)$ is:

$$m(x, y) = \sum_{(x_0, y_0)} ref(x, y)h(x_0, y_0)$$

(2)

After the estimated value is obtained from the degenerate model of the image, the residual value $r$ between the pixel value $g(x, y)$ of the degraded image and the estimated value $m(x, y)$ is:

$$r = m(x, y) - g(x, y)$$

(3)

The acquisition of the high-resolution image is based on the convex set theory. So, we firstly obtain the residual between the gray value at the observed image pixel and the gray value corrected with the estimated value. The absolute value of the residual is limited to the pre-set boundaries after iteration. Here, it is represented by $\delta_0$.

The correction process is as follows:

$$ref_{ref}(x, y) = \begin{cases} 
ref(x, y) + (r - \delta_0)h(x, y) & r < -\delta_0 \\
ref(x, y) - \delta_0 & -\delta_0 \leq r \leq \delta_0 \\
ref(x, y) + (r + \delta_0)h(x, y) & r > \delta_0
\end{cases}$$

(4)
where \( i = 0, 1, \ldots, L \). If \( f(x, y) \) is the ideal high-resolution image, according to Equation (1), the residual should be consistent with the additive noise \( n(x, y) \). Accordingly, the residual \( r \) should depend on the statistical properties of the noise. Therefore, the POCS algorithm has good robustness to noise. The resulting high-resolution image will converge to a certain set with a certain value under the random noise.

It can be found that to a large extent, the accuracy of the low-resolution image estimation depends on the matching degree relative to the original high-resolution image. The better the matching, the better the accuracy will be. Therefore, in order to improve the effect of super-resolution reconstruction, image registration is also introduced as a constraint to the POCS algorithm. It can obtain the motion parameters between the low-resolution image and the high-resolution image. Thereby the constraint effect of the point spread function is improved.

The resolution improving can be understood as the scale change and fuzzy matching of image. As SIFT feature is invariant to image scaling, it can be used to calculate the motion parameters. SIFT feature is the local feature of the image, which has been widely used to calculate the motion parameters, the accuracy can be improved. Therefore, in order to improve the effect of super-resolution reconstruction, image registration is also introduced as a constraint to the POCS algorithm. It can obtain the motion parameters between the low-resolution image and the high-resolution image. Thereby the constraint effect of the point spread function is improved.

The fractional calculus is proposed by Leibniz at the same time as the calculus theory (Huang, Xu, and Pu 2012). The order of calculus is extended from integer domain to the real domain and even imaginary domain. Out of habit, we still call it fractional calculus. In the application, the fractional calculus has the following several general definitions: the Grünewald–Letnikov (G–L) definition which transforms the fractional transformations into convolution operations, the Riemann–Liouville (R–L) definition which is convenient to calculate and analyze the solution, the Capotu definition which is applicable to fractional differential equations and so on. G–L definition is mainly used herein.

Starting with the classic definition of differential, it extends the order from an integer to a non-integer, which can be expressed as

\[
\begin{align*}
C_{a}^{v}D_{t}^{v}f(t) &= \lim_{h \to 0} h^{-v} \sum_{n=0}^{[t-1/h]} (-1)^{b} \frac{\Gamma(v+1)}{b! \Gamma(v+1-b)} f(t-bh) \\
&= \lim_{h \to 0} h^{-v} \left[ \sum_{n=0}^{[t-1/h]} (-1)^{b} \frac{\Gamma(v+1)}{b! \Gamma(v+1-b)} f(t-bh) \right] \\
&= \left[ \sum_{n=0}^{[t-1/h]} (-1)^{b} \frac{\Gamma(v+1)}{b! \Gamma(v+1-b)} f(t-bh) \right] \\
&= \lim_{h \to 0} h^{-v} \sum_{n=0}^{[t-1/h]} (-1)^{b} \frac{\Gamma(v+1)}{b! \Gamma(v+1-b)} f(t-bh) \\
&= \lim_{h \to 0} h^{-v} \sum_{n=0}^{[t-1/h]} (-1)^{b} \frac{\Gamma(v+1)}{b! \Gamma(v+1-b)} f(t-bh) \\
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&= \lim_{h \to 0} h^{-v} \sum_{n=0}^{[t-1/h]} (-1)^{b} \frac{\Gamma(v+1)}{b! \Gamma(v+1-b)} f(t-bh)
\end{align*}
\]

where the Gamma function is

\[
\Gamma(n) = \int_{0}^{\infty} e^{-t} t^{n-1} dt
\]

when \( n \) is an integer, \( \Gamma(n) = (n-1)! \). \( \Gamma \) denotes the Gamma function, \( a \) and \( t \) are the upper and lower bounds of the calculated differential values; \( v \) is the order of the fractional calculus. The positive and negative of \( v \) in \( C_{a}^{v}D_{t}^{v} \) represent the fractional differential and integral, respectively.

When the fractional theory is applied in the image signal, the equal interval of the image signal is at least \( 1 \) pixel. Let \( h = 1 \), then we can get \( [(t-a)/h] = [t-a] \). Consequently, the definition can be changed to:

\[
\begin{align*}
\frac{d_{f}(t)}{d_{t}} &\approx f(t) + (-v) f(t - 1) + \frac{(-v)(-v+1)}{2} f(t - 2) \\
&+ \cdots + (-1)^{n-1} \frac{\Gamma(v+1)}{n! \Gamma(v-n+1)} f(t - n)
\end{align*}
\]

The coefficients \( n \), \( -v \), \( \frac{(-v)(-v+1)}{2} \), \( \cdots \) are as the weight. The result of fractional calculus processing can be obtained by convolving the pixel. In practical applications, it is also important to take the errors into account in the different processing directions of the two-dimensional image signal, such as the direction of vertical or parallel to the edge. Handling of different directions can result in significant differences in treatment outcomes.

### 3. Image super-resolution enhancement algorithm based on fractional calculus and POCS

The traditional POCS reconstruction algorithm cannot achieve good results for the blurred image. The fractional calculus operator which can effectively enhance the high-frequency information of image is introduced into the reconstruction process. The fractional differential window is used to enhance the original interpolated reference image and increase the texture details of the image. Then, the noise or offset introduced in the enhancement is modified by the POCS reconstruction based on the fractional integral kernel function.

#### 3.1. Fractional differential for image enhancement analysis

Considering the directionality of the two-dimensional image, we can establish the isotropic filters of the image in eight directions: 0°, 45°, 90°, 135°, 180°, 225°, 270°, and 315° according to Equation (6). Respectively, the mask window \( A \) with size of \( 3 \times 3 \) and the mask window \( B \) with size of \( 5 \times 5 \) are obtained as follows:
In order to evaluate the effect for different order of fractional calculus in image processing, the objective evaluation index of information entropy and clarity (Xiao et al. 2017) was selected. The image “koala” is processed by fractional operators with different orders. The results are shown in Figure 2.

Information entropy and clarity are used to score the differential processing results. The size of the mask window is 3 × 3. In Figure 2, we can find that the information entropy and clarity of image change with the fractional differential order. The evaluation value is approximately linear with the order when the order is less than 0.7. It is no longer linear when the order is greater than 0.7. In Figure 2, the edge of the image will be over-sharpened in the place where the evaluation target is mutated. It can be considered that the image will not be over-sharpened when the information entropy and clarity of image vary linearly with the order, meanwhile, there will be better effect of image progressing. In the practical algorithm, we use more explicit clarity of linear effect as the index, and make the order as large as possible in ensuring the clarity index in the linear range. From the data in Figure 2, we can know the ideal order of fractional calculus is 0.7.

3.2. The influence of fractional calculus on the effect of POCS

In the traditional POCS algorithm, the initial high-resolution reference image used in the projection iteration is generally obtained by interpolation. In this paper, we

![Figure 1. Effects of fractional calculus.](image)

![Figure 2. Objective evaluation index with different order of fractional calculus.](image)
introduce the fractional differential window, and the initial high-resolution reference image is convoluted by fractional mask to enhance the detail texture of the initial high-resolution image. We compare the two results of the traditional POCS algorithm and the proposed algorithm, as shown in Figure 3.

From the contours of the leaves, the branches of the shrubs, the texture of the bark and the hair of the koala in Figure 3, we can conclude that the fractional differential operator effectively enhances the texture of the picture, and improves the overall high-frequency information of reconstruction image.

### 3.3. The proposed algorithm

Based on the analysis of the previous two sections, more high-frequency information in the acquisition of the initial reference frame is obtained in POCS processing. Then the image is subjected to POCS iterations. SIFT operator is used to obtain the matching point of the pixel on the reference frame on the low resolution image. Compared to the image block matching, SIFT cannot only obtain higher accuracy of motion estimation, but also reduce the computational complexity. Using the pixel values at that point on the low-resolution image, the reference frame is corrected by the point spread function processing based on fractional integral. After several iterations, the convergent results of super-resolution reconstruction can be obtained.

According to the above process, we design the following algorithm flow chart as shown in Figure 4.

The input low-resolution image sequence is denoted as \( X_i \), and the output high-resolution image is denoted as \( Y \). Several steps of the algorithm are described as follows in detail.

**Step 1.** Take a frame in the low-resolution image sequence. The interpolation of it can be the initial high-resolution reference image \( \text{Ref} \). Let \( k = 0 \).

**Step 2.** With the clarity as the reference value, the fractional differential of the low-resolution images \( X_i \) and \( \text{Ref} \) are \( L_i \) and \( \text{Ref}_k \), respectively. \( B_i \) denotes fractional differential operator.

**Step 3.** Calculate each point \((x, y)\) in \( \text{Ref}_k \) by the transformation matrix \( T_i \), and obtain the corresponding point \((x', y')\) in \( T_i \).

\[
\begin{align*}
L_i &= B_i \times X_i \\
\text{Ref}_k &= B_i \times \text{Ref} \\
\end{align*}
\]

(8)

At the same time, the SIFT operator is used to calculate the transformation matrix \( T_i \) between \( L_i \) and \( \text{Ref}_k \).

\[
T_i = \text{SIFT}(L_i, \text{Ref}_k), \quad i = 0, 1, 2, 3
\]

(9)

**Step 3.** Calculate each point \((x, y)\) in \( \text{Ref}_k \) by the transformation matrix \( T_i \), and obtain the corresponding point \((x', y')\) in \( T_i \).

\[
\begin{align*}
(x_i, y_i) &= T_i \cdot (x, y) \\
A &= G(x, y) \times \text{Ref}_k(x, y)
\end{align*}
\]

(10)

where the point spread function \( G(x, y) \) denotes the fractional integral kernel function centered on point \((x, y)\) with the order of \(-0.7\) and the window of \(5 \times 5\).

Then calculate the corresponding residuals:

\[
r_i = L_i(x_i, y_i) - A, \quad i = 0, 1, 2, 3
\]

(11)

Finally, the pixel value of each point \((x, y)\) is corrected in four low-resolution images. The correction threshold \( \delta_0 \) of the residual is set for the empirical value of 0.05.

\[
\text{Ref}_{k+1}(x, y) = \begin{cases} 
\text{Ref}_k(x, y) + (r_i + \delta_0)G(x, y), & r < -\delta_0 \\
\text{Ref}_k(x, y), & -\delta_0 \leq r \leq \delta_0 \\
\text{Ref}_k(x, y) + (r_i - \delta_0)G(x, y), & r > \delta_0 
\end{cases}
\]

(12)
Step 4. After all points have been corrected, turn to Step 5, if \( k \geq 10 \) or there is no correction process (that is, \( r \) always satisfies the inequality \( -\delta_0 \leq r \leq \delta_0 \)). Otherwise, let \( k = k + 1 \), and turn to Step 3.

Step 5. Output \( ref_{k+1} \), which is the reconstructed high-resolution image \( Y \).

4. Experiment results

In order to evaluate the performance of the proposed algorithm, this paper lists other reconstruction algorithms such as BICUBIC interpolation, POCS reconstruction combined with the kernel registration (Stark and Oskoui 1989), SIFT combined with POCS (Wang, Chen, and Zhou 2014), and Laplace operator initialization reference (Liu, Quan, and Wu 2016) as contrasts. The results of super-resolution reconstruction are based on subjective visual perception and five objective evaluation indicators (Xiao et al. 2016).

The software platform was set as Windows 7 × 64, Visual Studio 2010, and OpenCV 2.4.4; the hardware configuration as Intel Core i3-2120 3.30 GHz; GPU as NVIDIA GeForce GTX 550 Ti. As shown in Figure 5, the images by which we carry out the super-resolution reconstruction include a standard image Lena and the UAV images taken from the image data-set.

4.1. Objective evaluation indexes

The purpose of image super-resolution is to get a better image signal. In order to measure the performance of the algorithm objectively, we conduct a certain evaluation of the reconstructed image. The evaluation of image quality plays a key role in image processing. The following is a brief introduction to the common objective image evaluation method in the field of image super-resolution.

1. Peak Signal to Noise Ratio (PSNR). PSNR can effectively evaluate the image and meet the subjective feelings of the human eye (Xiao et al. 2016). The larger PSNR value means that the target image is closer to the reference image.

2. Structural Similarity (SSIM). The SSIM between images is an index of the degree of similarity between two images, which is between 0 and 1 (Xiao et al. 2016). If the SSIM value is larger, the structure of the original image is more similar to the reconstructed image, and the result of image reconstruction is better.

3. Edge Preservation Index (EPI). EPI is an indicator of the degree to keep edges before and after image reconstruction. If the EPI value is closer to 1, the effect of image reconstruction is better.

4. Entropy. The entropy evaluates the image by the probability of the gray value (Xiao et al. 2016). The larger entropy means that an image contains more information.

5. Clarity. Image clarity is also called the image mean gradient, which is obtained by calculating the gradient around the pixel (Xiao et al. 2016). It describes the tiny details of the image. If the Clarity value of an image is larger, the image contains richer textures in general.
the $2 \times 2$ region are placed in different low-resolution sequences, respectively. In this case, when the image is reconstructed, the reconstructed results can be compared directly to the original image. The generation process of

4.2. Test low-resolution images

In this paper, we use the simulation method to obtain the low-resolution image sequence. The main method is the original image sub-sampling. The four pixels in

Figure 7. Image reconstruction results of the Lena picture.

Figure 8. Image reconstruction results of the Normal picture.

Figure 9. Image reconstruction results of the Contrast picture.
low-resolution image sequence frame is described above. The four low-resolution images produced by the test image Lena are shown in Figure 6, which are, respectively, based on left-top pixels, left-bottom pixels, right-top pixels, and right-bottom pixels. We can see that four images are similar to each other and have slight different.

**Figure 10.** Image reconstruction results of the Motion-blur Picture 1.

**Figure 11.** Image reconstruction results of the Motion-blur Picture 2.

**Figure 12.** Image reconstruction results of the Fisheye-effect Picture 1.
4.3. Test results

The reconstruction results of different images are shown in Figures 7–13.

Figures 7–13 show the reconstruction results of several different images under different methods. The edge of the image reconstructed by BICUBIC is relatively obscure; the image reconstructed by Stark and Oskoui (1989) shows a serious pixel shift due to registration error; the edge of the image reconstructed by Liu, Quan, and Wu (2016) has been exposed; the subjective effect of Wang, Chen, and Zhou (2014) is better than that of the previous three algorithms, but the overall contour is blurred; the result of the proposed algorithm has the best visual observation effect.

We carried out experiments on the remaining images in the aerial image library. The objective evaluation results are shown in Table 1.

![Fisheye-effect Picture 2](image1)
![BICUBIC](image2)
![Stark and Oskoui (1989)](image3)
![Liu, Quan, and Wu (2016)](image4)
![Wang, Chen, and Zhou (2014)](image5)
![Proposed method](image6)

**Figure 13.** Image reconstruction results of the fisheye-effect picture 2.

| Image | Objective index | BICUBIC | Stark and Oskoui (1989) | Wang, Chen, and Zhou (2014) | Liu, Quan, and Wu (2016) | Proposed method |
|-------|----------------|---------|------------------------|----------------------------|-------------------------|----------------|
| 5-a   | PSNR           | 20.4734 | 17.6757                | 21.6333                    | 23.8028                 | 28.7037        |
|       | SSIM           | 0.8611  | 0.4762                 | 0.7874                     | 0.8135                  | 0.9316         |
|       | EPI            | 0.9672  | 1.3619                 | 1.0816                     | 0.9659                  | 1.0189         |
|       | Entropy        | 7.3844  | 7.4670                 | 7.5050                     | 7.4685                  | 7.5256         |
|       | Clarity        | 6.6139  | 8.7920                 | 7.1703                     | 6.3441                  | 6.7459         |
| 5-b   | PSNR           | 28.5828 | 21.3599                | 29.0265                    | 28.3656                 | 26.4906        |
|       | SSIM           | 0.7791  | 0.4471                 | 0.8020                     | 0.8416                  | 0.8110         |
|       | EPI            | 0.4420  | 1.5091                 | 0.4864                     | 0.6853                  | 1.0874         |
|       | Entropy        | 6.9072  | 7.0162                 | 6.9276                     | 7.0236                  | 7.2346         |
|       | Clarity        | 5.2157  | 17.3780                | 5.7293                     | 8.1037                  | 12.6885        |
| 5-c   | PSNR           | 33.7031 | 24.7998                | 34.3155                    | 33.8253                 | 29.0918        |
|       | SSIM           | 0.8903  | 0.6080                 | 0.9043                     | 0.9200                  | 0.8649         |
|       | EPI            | 0.5818  | 1.5586                 | 0.6245                     | 0.8187                  | 1.3202         |
|       | Entropy        | 6.7818  | 6.8593                 | 6.7954                     | 6.8560                  | 7.0293         |
|       | Clarity        | 4.0157  | 10.5003                | 4.3038                     | 5.6613                  | 9.1688         |
| 5-d   | PSNR           | 26.4575 | 20.0503                | 26.9148                    | 26.7901                 | 24.5698        |
|       | SSIM           | 0.8025  | 0.5283                 | 0.8239                     | 0.8601                  | 0.8282         |
|       | EPI            | 0.4793  | 1.2831                 | 0.5235                     | 0.7208                  | 1.0907         |
|       | Entropy        | 7.2268  | 7.2669                 | 7.2424                     | 7.3397                  | 7.5205         |
|       | Clarity        | 6.7908  | 17.1433                | 7.3979                     | 10.2413                 | 15.5899        |
| 5-e   | PSNR           | 28.7442 | 21.5713                | 29.2497                    | 29.0897                 | 26.6343        |
|       | SSIM           | 0.7987  | 0.4560                 | 0.8222                     | 0.8577                  | 0.8244         |
|       | EPI            | 0.4598  | 1.5244                 | 0.5069                     | 0.7080                  | 1.1222         |
|       | Entropy        | 6.9308  | 7.0483                 | 6.9518                     | 7.0464                  | 7.2588         |
|       | Clarity        | 5.2041  | 16.8582                | 5.7124                     | 8.0174                  | 12.6690        |
| 5-f   | PSNR           | 28.8234 | 21.3475                | 29.3487                    | 29.1007                 | 26.3387        |
|       | SSIM           | 0.8141  | 0.4555                 | 0.8369                     | 0.8706                  | 0.8323         |
|       | EPI            | 0.4997  | 1.5971                 | 0.5479                     | 0.7545                  | 1.1860         |
|       | Entropy        | 6.8326  | 6.9585                 | 6.8603                     | 6.9688                  | 7.2379         |
|       | Clarity        | 5.8324  | 18.2451                | 6.3678                     | 8.8238                  | 13.7080        |
| 5-g   | PSNR           | 30.1912 | 26.0488                | 30.9352                    | 22.4229                 | 31.6486        |
|       | SSIM           | 0.8149  | 0.6204                 | 0.8179                     | 0.5392                  | 0.8448         |
|       | EPI            | 0.7234  | 1.3632                 | 0.6481                     | 1.4322                  | 0.8740         |
|       | Entropy        | 7.1604  | 7.1777                 | 7.1538                     | 7.2657                  | 7.1791         |
|       | Clarity        | 4.6329  | 6.7681                 | 4.1275                     | 9.0123                  | 5.5916         |

Note: In this table, the bold indicates the best result of the objective index in these algorithms.
In Table 1, it can be seen that the evaluation results of the BICUBIC and Stark and Oskoui (1989) are poor when compared with the other three algorithms. The Clarity value of Stark and Oskoui (1989) is too large to result in overexposing for image reconstruction. The Entropy and SSIM value of Liu et al’s method are sometimes higher than those of the proposed algorithm. This is because the details of the high-frequency part of the image are increased after the Laplace operation. In some pictures these two indicators of Liu, Quan, and Wu (2016) are dominant, but the fractional differential shows the better effect on the middle- and low-frequency information than Laplace operator. Compared with Wang, Chen, and Zhou (2014), we can find that the proposed algorithm is more effective on EPI and Entropy. Considering the existence of the degradation of the original image, the algorithm is not such good in the PSNR index for the fractional operator enhances the high-frequency information. It can be seen from the subjective visual perception that the algorithm results are obviously better than the rest algorithms. With the five objective evaluation indicators, proposed algorithm is the maximum in many cases in the table. It is more effective than other methods.

5. Conclusions

In this paper, a more valuable solution is derived from the initial reference frame in POCS, especially for UAV images. Fractional differential is proposed to convolute with the reference frame to enhance the image details. Then the fractional integral is used as the point spread function to reduce the error caused by fractional differential. The results of this algorithm are compared with some other super-resolution reconstruction methods. The objective index is higher and better in most cases, and the subjective visual effect also shows that the proposed algorithm improves the contrast of the image, while the edge of the image is well preserved. When the initial image degradation occurs, the algorithm is more accommodative.

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References

Bätz, M., A. Eichenseer, J. Seiler, M. Jonscher, and A. Kaup. 2015. “Hybrid Super-resolution Combining Example-Based Single-Image and Interpolation-based Multi-image Reconstruction Approaches.” Paper presented at the Proceedings of the IEEE International Conference on Image Processing, Quebec City, Canada, September 27–30, 58–62.

Bengtsson, T., Y. H. Gu, M. Viberg, and K. Lindström. 2012. “Regularized Optimization for Joint Super-Resolution and High Dynamic Range Image Reconstruction in a Perceptually Uniform Domain.” Paper presented at the Proceedings of the IEEE International Conference on Acoustics, Speech and Signal Processing, Kyoto, Japan, March 25–30, 1097–1100.

Cao, Y., X. Liu, W. Wang, and Z. Xing. 2009. “Super-resolution Image Reconstruction Algorithm Based on Projection onto Convex Sets and Wavelet Fusion.” Journal of Biomedical Engineering 26 (5): 947–952.

Freedman, G., and R. Fattal. 2011. “Image and Video Upscaling from Local Self-examples.” ACM Transactions on Graphics 30 (2): 12.
Xiao, J., G. Pang, Y. Zhang, Y. Kuang, Y. Yan, and Y. Wang. 2016. "Adaptive Shock Filter for Image Super-resolution and Enhancement." Journal of Visual Communication and Image Representation 40 (Part A): 168–177.

Xiao, J., T. Liu, Y. Zhang, B. Zou, J. Lei, and Q. Li. 2016. "Monitoring Image Fusion Based on Depth Extraction with Inhomogeneous Diffusion Equation." Signal Processing 125: 171–186.

Xiao, J., E. Liu, L. Zhao, Y. Wang, and W. Jiang. 2017. "Detail Enhancement of Image Super-resolution Based on Detail Synthesis." Signal Processing: Image Communication 50: 21–33.

Xu, M., and W. Tan. 2006. "Intermediate Processes and Critical Phenomena: Theory, Method and Progress of Fractional Operators and Their Applications to Modern Mechanics." Science in China (Series G): Physics, Mechanics & Astronomy 49 (3): 257–272.

Yang, Z., J. Zhou, X. Yan, and M. Huang. 2008. "Image Enhancement Based on Fractional Differentials." Journal of Computer - Aided Design & Computer Graphics 20 (3): 343–348.

Yao, K., W. Su, and S. Zhou. 2004. "On the Fractional Calculus Functions of a Type of Weierstrass Function." Chinese Annals of Mathematics, Series a 6 (6): 711–716.

Zhang, Y., K. Xu, and Y. Li. 2010. "Remote Sensing Image Super-resolution Based on POCS and out-of-Core." Journal of Tsinghua University (Science and Technology) 50 (10): 1743–1746.

Zhang, K., D. Tao, X. Gao, X. Li, and Z. Xiong. 2015. "Learning Multiple Linear Mappings for Efficient Single Image Super-resolution." IEEE Transactions on Image Processing 24 (3): 846–861.