Study on detection method of distorted voltage sag in distribution network under the background of incorporating new energy

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Abstract. An advanced voltage sag detection method and compensation technique can improve the power quality of the distribution network to a entirely new level. A high performance voltage sag detection method demands fast response and good accuracy, even detecting distorted grid voltage sag caused by new energy accessing to distribution network. The voltage sag detection method, based on sliding window iterative discrete fourier transfer (DFT) algorithm, has excellent filtering ability but long detection time. On the other hand, the voltage sag detection method, based on nonrecursive DFT algorithm, has short detection time but poor filtering ability. Aiming to meet the requirements of fast response and good accuracy, this paper proposed a voltage sag detection method based on improved nonrecursive DFT algorithm. By means of disturbance filter, which is combined with several notch filters and a low-pass filter, the filter characteristic of nonrecursive DFT algorithm has been improved, and the detection delay time is reduced. The proposed algorithm can guarantee shorter detection time and higher detection accuracy.

1. Introduction
Voltage sag has become the main interference to the safe and stable operation of distribution network under the background of incorporating new energy [1]. When the distributed photovoltaic (PV) suddenly loses power, voltage sag will appear in the distribution network, and the shunt capacitor compensation device should be connected as fast as possible, to keep the bus voltage stable [2]. In addition, voltage sag will lead to some sensitive equipment malfunction or failure, such as computer restart [3]. A large amount of grid connection converter will cause distorted grid voltage, under the situation of new energy accessing to distribution network. The distorted grid voltage will be a challenge to voltage sag detection method. Therefore, rapid and accurate detection of voltage sag is worthy of study.

The d-q transform sag detection methods, by means of transformation of the three-phase voltages to a rotating reference, are widely used [4,5]. However, single-phase voltage sag is more common, which means that the detection method needs to be suitable for single-phase voltage sag [6]. To solve this problem, several single-phase voltage sag detection methods based on rms calculation and discrete fourier transfer (DFT) have been proposed [7-11]. Rms calculation algorithm is a most common way for its simplicity, but the algorithm will be inaccurate when the voltage is distorted or the sag contains
phase-angle jump [7,8]. The DFT algorithm has excellent performance of suppressing harmonic disturbance, but poor convergence speed [9]. The speed of voltage sag detection based on DFT algorithm depends on the length of window [10]. The rms voltage will be refreshed faster when the window is sufficiently short, but at the same time the harmonic components will result in inaccuracy of the rms value. In [11], the sliding-window rms calculation method based on Fourier transform simultaneously caters for the speed and accuracy. However, if the voltage sag contains phase-angle jump, the effect of proposed method will be limited.

Voltage sag detection method proposed in this paper is based on non-recursive DFT algorithm, and disturbance filter is combined to suppress harmonic interference. Through the rational design of filter parameters, the proposed method can achieve shorter detection time, but also to ensure higher detection accuracy.

2. Voltage sag detection method based on DFT algorithm

The DFT algorithm can accurately extract the fundamental component of distorted grid voltage, but the convergence of the algorithm is relatively slow, which cannot meet the requirement of fast detection. Therefore, there are some literatures on this basis to make improvements.

2.1. Voltage sag detection method based on non-recursive DFT algorithm

The fundamental component of the sampled signal f(t) is defined as

\[ f^*(t) = X_a \sin(\omega t) + X_b \cos(\omega t) \]  

Where, \( \omega \) represents fundamental angular velocity. Sampled waveform f(t) may be approximated by \( f^*(t) \) to minimize the mean square error. By means of discretizing f(t), the total square error \( E^2 \) is written as

\[ E^2 = \sum_{n=0}^{N-1} (f_n - X_a \sin(n\omega T_s) - X_b \cos(n\omega T_s))^2 \]  

Where, \( f_n \) represents discrete sampled signal, \( N \) represents length of window which is adjustable, and \( T_s \) is sampled time.

The partial derivative with respect to \( X_a \) and \( X_b \) should be equal to zero for the reason that \( E^2 \) should be as small as possible [10].

\[ \frac{\partial E^2}{\partial X_a} = -2 \sum_{n=0}^{N-1} \sin(n\omega T_s) (f_n - X_a \sin(n\omega T_s) - X_b \cos(n\omega T_s)) = 0 \]  

\[ \frac{\partial E^2}{\partial X_b} = -2 \sum_{n=0}^{N-1} \cos(n\omega T_s) (f_n - X_a \sin(n\omega T_s) - X_b \cos(n\omega T_s)) = 0 \]  

Equations (3) and (4) can be transformed to (5) and (6),

\[ X_a \sum_{n=0}^{N-1} \sin(n\omega T_s) \cos(n\omega T_s) + X_b \sum_{n=0}^{N-1} \sin^2(n\omega T_s) = \sum_{n=0}^{N-1} f_n \sin(n\omega T_s) \]  

\[ X_a \sum_{n=0}^{N-1} \cos^2(n\omega T_s) + X_b \sum_{n=0}^{N-1} \sin(n\omega T_s) \cos(n\omega T_s) = \sum_{n=0}^{N-1} f_n \cos(n\omega T_s) \]  

Substituting

\[ A = \sum_{n=0}^{N-1} \sin^2(n\omega T_s) \]  

\[ B = \sum_{n=0}^{N-1} \sin(n\omega T_s) \cos(n\omega T_s) \]  

\[ C = \sum_{n=0}^{N-1} \cos^2(n\omega T_s) \]
The following equations can be given as

\[ X_a = \frac{1}{AC - B^2} \sum_{n=0}^{N-1} f_n[C \sin(n\omega T_s) - B \cos(n\omega T_s)] \] (10)

\[ X_b = \frac{1}{AC - B^2} \sum_{n=0}^{N-1} f_n[A \cos(n\omega T_s) - B \sin(n\omega T_s)] \] (11)

Substituting

\[ K_{an} = \frac{1}{AC - B^2} [C \sin(n\omega T_s) - B \cos(n\omega T_s)] \] (12)

\[ K_{bn} = \frac{1}{AC - B^2} [A \cos(n\omega T_s) - B \sin(n\omega T_s)] \] (13)

\[ X_a \text{ and } X_b \text{ can be calculated as} \]

\[ X_a = \sum_{n=0}^{N-1} K_{an} f_n \] (14)

\[ X_b = \sum_{n=0}^{N-1} K_{bn} f_n \] (15)

The fundamental rms \( X \) is derived as

\[ X = \sqrt{X_a^2 + X_b^2} \] (16)

As shown in figure 1, there is no harmonics, and voltage sag occurs in 0.047s. The depth is 0.2 p.u., and the phase jump angle is 10°. The width of window is set to 1/8 of the fundamental period, and the convergence of the fundamental rms is shown in figure 1. According to IEEE 1159 standard, when the fundamental rms is less than 90% of its rated value, which is considered to voltage sag. Therefore, the voltage sag detection time is 2.2 ms, meeting the requirements of fast detection.

As shown in figure 2, there are 5% 3rd harmonic and 3.5% 5th harmonic components in the grid voltage, the other conditions are consistent with figure 1. It can be found that the fundamental rms calculated by non-recursive DFT algorithm has poor accuracy when the grid voltage is distorted.

In order to improve the detection accuracy of distorted grid, the common approach is to combine with a low-pass filter to suppress harmonic disturbances. Due to the lowest harmonic frequency of fundamental rms is 100 Hz, the cut-off frequency of low-pass filter is set as 50 Hz, the convergence of
the fundamental rms is shown in figure 3. Although the steady error is reduced to less than 2%, but the detection time is longer, about 7.7 ms.

![Image](image_url)

**Figure 3.** Nonrecursive DFT algorithm based on low-pass filter (Distorted voltage sag).

**Figure 4.** Improved sliding-window iterative DFT algorithm (Distorted voltage sag without phase-angle jump).

### 2.2. Improved sliding-window iterative DFT algorithm

The sliding-window iterative algorithm can ensure the detection accuracy even if the grid is distorted, but the detection rate is slow [9]. The test conditions in figure 2 are almost consistent with figure 4, except that there is no phase jump. As shown in figure 4, the steady error of fundamental rms is very small, but the detection time is 10 ms.

In order to ensure high detection accuracy and short detection time, some researcher proposed an improved sliding window iterative algorithm, which modified the sag detection criteria [11]. Assuming the critical sag occurs in T2, and the depth is 0.1 p.u. The convergence of the critical fundamental rms is shown in figure 4. $U_{T_2}$ is the fundamental rms before sag occurs. Suppose the desired detection time is $\Delta T$, then the fundamental rms in T3 ($T_3 = T_2 + \Delta T$) is $\hat{U}_{T_3}$, which is defined as

$$
\hat{U}_{T_3} = \sqrt[4]{T_1 \left[ T_1 \left[ \sqrt{U_{T_2}} \sin(\omega t) \right]^2 \right] dt} + T_1 \left[ 0.9 \sqrt{U_{T_2}} \sin(\omega t) \right] dt
$$

(17)

The new sag criterion is written as

$$
\Delta \hat{U} = U_{T_3} - \hat{U}_{T_3}
$$

(18)

When the voltage sag occurs, the fundamental rms in T3 can be obtained by sliding-window iterative DFT algorithm. The fundamental rms difference between T2 and T3 is expressed as

$$
\Delta U = U_{T_3} - U_{T_2}
$$

(19)

If $\Delta U > \Delta \hat{U}$, it is determined that the sag occurs; If $\Delta U \leq \Delta \hat{U}$, it is determined that the sag not occurs. According to the new sag criterion, the detection time is 2 ms.

Although the improved sliding-window DFT algorithm greatly reduced the detection time, but it did not take into account the influence of phase jump. Besides the voltage drop during a sag, there is also a phase-angle jump associated with it. Thus, the formula (17) may be rendered inaccurate, except that phase jump angle can be obtained in advance to modify the formula, which is hard to achieve.

As shown in figure 5, the test conditions are almost consistent with figure 4, except that the phase jump angle is $-10^\circ$. It can be found that the convergence of actual fundamental rms is not always lower than critical one. It means that $\Delta T$ should be longer than 5 ms to accurately determine the voltage sag. The phase jump angle will vary with different types of voltage sag, which means $\Delta T$ is difficult is set.
Therefore, the method proposed in literature [11] is not suitable for voltage sag with phase-angle jump.

![Figure 5](image.png)

**Figure 5.** Improved sliding-window iterative DFT algorithm (Distorted voltage sag with phase-angle jump).

2.3. *Comparison of different voltage sag detection methods*

Table 1 shows the comparison of different detection methods for distorted grid voltage sag. It can be seen that, the current voltage sag detection methods based on DFT algorithm cannot meet the requirements of dynamic voltage restorer (DVR).

|       | Detection time | Detection accuracy | Phase-angle jump |
|-------|----------------|--------------------|------------------|
| Nonrecursive DFT | short | low | suitable |
| Nonrecursive DFT + low-pass filter | long | high | suitable |
| Sliding-window iterative DFT | long | high | suitable |
| Improved sliding-window iterative DFT | short | high | unsuitable |

3. *The application of disturbance filter in voltage sag detection method based on nonrecursive DFT algorithm*

3.1. *Analysis of disturbance filter*

![Figure 6](image.png)

**Figure 6.** Schematic diagram of nonrecursive DFT algorithm based on disturbance filter.

The disturbance filter contains several second order notch filter and a second order low-pass filter. The
notch filter is used to suppress specific low frequency harmonic disturbance, and the low-pass filter is used to suppress high frequency harmonic disturbance. Compared to traditional low-pass filter, the disturbance filter can improve the response speed, but also to guarantee small steady error. The control block diagram is shown in figure 6.

The transfer function of second order notch filter is

$$G_{nf}(s) = \frac{s^2 + \omega_{nf}^2}{s^2 + \frac{\omega_{nf}^2}{Q_{nf}} s + \omega_{nf}^2}$$

(20)

$\omega_{nf}$ represents resonant frequency, $Q_{nf}$ represents quality factor. The transfer function of second order low-pass filter is

$$G_{lf}(s) = \frac{\omega_{lf}^2}{s^2 + \frac{\omega_{lf}^2}{Q_{lf}} s + \omega_{lf}^2}$$

(21)

$\omega_{lf}$ represents resonant frequency, $Q_{lf}$ represents quality factor.

3.2. Design of disturbance filter

The fundamental rms calculated by nonrecursive DFT algorithm may contain 2nd, 4th and 6th harmonic components, and so on. The more notch filters cause to better low-frequency harmonic disturbance rejection, but slower response speed. Therefore, two notch filters are chosen and the respective resonant frequency is set as 2nd and 4th harmonic frequency. The respective quality factor of notch filter is both set as 1. The cut-off frequency of low-pass filter is set as 4th harmonic frequency, and the quality factor is set as 0.5.

In order to be applied in digital control, the transfer function of disturbance filter need to be discrete. Traditional discretization method, such as zero-order hold, bilinear transform, cannot be adapted to grid frequency fluctuation. The performance of disturbance filter can be influenced by frequency fluctuation, especially notch filter. To solve this problem, some scholars made a structure conversion of the second-order generalized integrator, and then discretized the pure integrator [12]. The resonant frequency can be adjusted with a phase-locked loop to ensure a high performance. According to this thought, the block diagram of disturbance filter is shown in figure 7.

**Figure 7.** The block diagram of disturbance filter. (a) Second-order notch filter and (b) Second-order low-pass filter.

As shown in figure 8, the test conditions are consistent with figure 2, it can be found that the steady state error of fundamental rms is less than 2% and the detection time is 5.2 ms. The harmonic disturbance is well suppressed to improve the reliability of the detection, and the detection time just delays a little.
Figure 8. Non-recursive DFT algorithm based on disturbance filter (Distorted voltage sag).

4. Experimental results

4.1. Non-distorted voltage sag detection
In order to verify the effectiveness of the non-recursive DFT algorithm based on disturbance filter presented in this paper, four different depths of voltage sag are tested. The total harmonic distortion (THD) of grid voltage is 4.89%, as shown in figure 9.

Figure 9. Harmonic analysis of undistorted grid voltage.

As shown in figure 10, the depth of voltage sag in figure 10(a) is 0.2 p.u., the depth of voltage in figure 10(b) is 0.5 p.u., the depth of voltage in figure 10(c) is 0.7 p.u., the depth of voltage in figure 10(d) is 1 p.u. Channel 1 shows the voltage sag, and channel 2 shows the change of detection signal. When the detection signal is high, it means that the voltage is stable. When the detection signal turns low, it means that the voltage sag begins.

As shown in figure 10, the detection time are 4.8 ms, 3.9 ms, 3.2 ms and 2.1 ms, respectively. Deeper sag means faster detection because of higher rms declining slope. Therefore, the proposed algorithm is suitable for non-distorted voltage sag detection.
Figure 10. Waveforms of different voltage sags. (a) The depth of voltage sag: 0.2 p.u., (b) The depth of voltage sag: 0.5 p.u., (c) The depth of voltage sag: 0.7 p.u. and (d) The depth of voltage sag: 1 p.u.

Figure 11. Harmonic analysis of distorted grid voltage.
4.2. Distorted voltage sag detection
To further verify the distorted voltage sag detection capability of the proposed algorithm, some non-linear loads are connected to the grid, which will increase the harmonic components in the grid voltage. The THD of grid voltage is 9.09%, as shown in figure 11.

Figure 12(a) shows the distorted voltage sag detection based on simple non-recursive DFT algorithm without disturbance filter. When the grid voltage is distorted, the detection signal is always low, even if the grid voltage is stable. Due to the poor harmonic suppression ability of non-recursive DFT algorithm, there will be false judgement when the grid voltage is distorted.

Figure 12(b) shows the distorted voltage sag detection based on non-recursive DFT algorithm with disturbance filter. The voltage begins in 0.06 s, and the depth of voltage sag is 0.3 p.u., and the detection time is 4.2 ms. The addition of disturbance filter compensates for the defect of non-recursive DFT algorithm, therefore, the proposed algorithm is also suitable for distorted voltage sag detection.

![Waveforms of distorted voltage sag](image)

(a) Distorted voltage sag detection (without disturbance filter) and (b) Distorted voltage sag detection (with disturbance filter).

5. Conclusion
Due to existing voltage sag detection methods based on DFT algorithm cannot simultaneously meet the requirements of detection time, detection accuracy and adaptability of phase-angle jump, this paper presents an improved short-window DFT algorithm based on disturbance filter. Two second-order notch filters are designed to suppress 2nd and 4th harmonic disturbances, and a second-order low-pass filter is designed to suppress high frequency harmonic disturbance. Although the disturbance filter slightly delays detection time, but in exchange for higher detection accuracy. Experiments show that, in general situation, the proposed voltage sag detection method has high performance whether the grid voltage is distorted or non-distorted.

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