HEAVISIDE TRANSFORM WITH RESPECT TO THE MASS IN QCD

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Abstract

We propose the use of Heaviside transform with respect to the quark mass to investigate dynamical aspects of QCD. We show that at large momentum transfer the transformed propagator of massive quarks behaves softly and thus the dominant effect of explicit chiral symmetry breaking disappears through Heaviside transform. This suggests that the massless approximation would be more convenient to do in the transformed quantity than in the original one. As an example of explicit approximation, we estimate the massless value of the quark condensate.
1 Introduction

In this paper we report the result of the application of Heaviside transform with respect to the mass to QCD. Let $m$ be the masses of up and down quarks and the massless value of some quantity, $\Omega(m)$, be of interest. Ordinary approach is to consider the limit that $\lim_{m \to 0} \Omega(m)$. However, in the framework of perturbation expansion, the limit leads to the trivial results and if $m$ is kept non-zero for the approximation of the massless value, the effect of explicit chiral symmetry breaking remains and gives, for example, the hard high energy behavior of the effective quark mass. In the present paper, we demonstrate that the use of Heaviside transform with respect to $m$ solves the above dilemma and provides us a new way of approximating the massless dynamics.

2 Heaviside transform

Since our approach is based on Heaviside transformation of perturbative series, let us state some basic features of the transform. Heaviside transform of $\Omega(m)$ is given by the Bromwich integral,

$$\hat{\Omega}(\hat{m}) = \int_{s-i\infty}^{s+i\infty} \frac{dm \exp(m/\hat{m})}{2\pi i} \Omega(m),$$

where the vertical straight contour should lie in the right of all the poles and the cut of $\Omega(m)/m$. Since the $\Omega(m)/m$ is analytic in the domain, $\Re(m) > s$, $\hat{\Omega}(\hat{m})$ is zero when $\hat{m} < 0$. The Heaviside transform is the inverse of the (second kind of ) Laplace transform given by

$$\Omega(m) = m \int_{-\infty}^{\infty} dx \exp(-mx)\hat{\Omega}(1/x), \quad x = \frac{1}{\hat{m}}.$$  

Since $\hat{\Omega}(\hat{m}) = 0$ for $\hat{m} < 0$, the integral (2) reduces to the familiar one in which the integration region is $[0, \infty)$. However the mathematical manipulation is straightforward for the form (2) because $\hat{\Omega}(\hat{m})$ involves Dirac $\delta$ function in usual cases.

From (2) we find that

$$\lim_{m \to +0} \Omega(m) = \lim_{\hat{m} \to +0} \hat{\Omega}(\hat{m}),$$

if the both limits exist. Eq.(3) states that the massless value is an invariant of the Heaviside transform. Hence the transformed function can be directly used to approximate $\Omega(0)$ without integration over $\hat{m}$. Here suppose that $\Omega(m)$ denotes the condensate for the quark with its explicit mass $m$. We note then that, since the massless limit in the both
functions cannot be taken at finite orders of perturbation expansion due to the bad infra-
red behavior of perturbative QCD, we must choose some non-zero $m$ or $\hat{m}$ to approximate
$\Omega(0)$ or $\hat{\Omega}(0)$, respectively. Then, some difference of the status of approximation may occur
between the two functions. We will show that, for the approximate calculation of quark
propagator and the quark condensation, the transformed functions are more convenient
than the original ones. Though $\hat{\Omega}$ is not a physical quantity, there is no objection to use
it in the approximation of $\Omega(0)$.

For the sake of later argument let us show transformations of typical functions. Using
(1) it is easy to find that
\[
m^{-\rho} \rightarrow \frac{\hat{m}^{-\rho}}{\Gamma(1 + \rho)} \theta(1/\hat{m}) \quad (\rho > -1),
\]
where $\theta(x)$ is the step function,
\[
\theta(x) = \begin{cases} 1 & (x > 0) \\ 0 & (x < 0) \end{cases}
\]
and $\mathcal{H}$ represents the Heaviside transformation. By expanding both sides of (4) in powers
of $\rho$, we have
\[
1 \mathcal{H} \rightarrow \theta(x),
\]
\[
\log m \mathcal{H} \rightarrow - (\gamma + \log x) \theta(x),
\]
and so on where $x = 1/\hat{m}$. From these results we have the transform of typical functions
appearing in perturbative expansions. To obtain the formulae, the following result is useful,
\[
m\Omega(m) \mathcal{H} \rightarrow \frac{\partial \hat{\Omega}(1/x)}{\partial x}.
\]
For example, using (6) and (8) we find
\[
m^k \mathcal{H} \rightarrow \delta^{(k-1)}(x), \quad (k = 1, 2, 3, \cdots).
\]
From (7) and (8) we also find
\[
m \log m \mathcal{H} \rightarrow - \frac{1}{x} \theta(x) - (\gamma + \log x) \delta(x),
\]
\[
m^2 \log m \mathcal{H} \rightarrow \frac{1}{x^2} \theta(x) - \frac{2}{x} \delta(x) - (\gamma + \log x) \delta'(x),
\]
\[
m^3 \log m \mathcal{H} \rightarrow - \frac{2}{x^3} \theta(x) + \frac{3}{x^2} \delta(x) - \frac{3}{x} \delta'(x) - (\gamma + \log x) \delta''(x).
\]
The terms containing $\delta$ functions play the role of "counter terms" since they cancel out the divergences coming from the Laplace integration of first terms.

In this paper we confine ourselves with studying the approximate calculation of QCD quantities at $m = 0$. From now on, we therefore omit the $\delta$ function terms of the transformed functions since the Laplace integration is un-necessary in our scheme that $\Omega(0)$ will be approximated by $\hat{\Omega}(\hat{m})$ at some fixed non-zero $\hat{m}$.

The result (9) shows that Heaviside transform forces the polynomial of $m$ vanish. This is desirable because the polynomial of $m$ should not have nothing to do with the massless physics. On the other hand the contributions of the mass-logarithms remains. Since the mass-log involves $\mu$, the renormalization parameter, we find that Heaviside transform keeps the effect of ultraviolet singularity. This is in accord with the point of view that the ultraviolet divergence plays the central role in the non-perturbative effects. Strictly speaking, renormalization-group invariant mass, $\Lambda_m$, corresponding to $m$ should be used in place of $m$ in (1). However for notational simplicity we use $m$ as long as $m$ is proportional to $\Lambda_m$. We may point out that $\hat{m}$ obeys the renormalization group equation same as that for $m$. This is because the argument of the exponential in (1) and (2) must be renormalization group invariant (This is obvious if one writes transformations (1) and (2) in terms of $\Lambda_m$ and the corresponding conjugate which is necessarily renormalization group invariant).

In what follows we first study the high energy behavior of the Heaviside function of the effective mass and find that it behaves softly for non-zero $\hat{m}$ at large momentum transfer. This fact suggests an advantage of dealing with the Heaviside function when one carry out the approximation of the massless case. Next, as an example, we perform a rough estimation of the massless value of the quark condensate. Throughout this paper we use dimensional regularization$^2$ and Landau gauge.

3 High energy behavior of the transformed function of the effective quark mass

The inverse of the quark propagator is written as,

$$S_F^{-1} = p - m - \Sigma(m, p) = A(p, m)p - B(p, m) = A(p, m)(p - M(p, m)),$$  \hspace{1cm} (13)

and the function $M(p, m)$ defines the effective quark mass. Heaviside transform of the
inverse propagator is given as

\[ S^{-1}_F = \hat{A}(p, \hat{m}) p - \hat{B}(p, \hat{m}), \]  

(14)

where

\[ S^{-1}_F(p, m) \xrightarrow{\mathcal{H}} \hat{S}^{-1}_F(p, \hat{m}), \quad A(p, m) \xrightarrow{\mathcal{H}} \hat{A}(p, \hat{m}), \quad B(p, m) \xrightarrow{\mathcal{H}} \hat{B}(p, \hat{m}). \]  

(15)

The transformed function of the effective mass is defined by

\[ \hat{M}(p, \hat{m}) = \hat{A}(p, \hat{m}) \hat{B}(p, \hat{m}). \]  

(16)

It is easy to show that

\[ M(p, 0) = \hat{M}(p, 0). \]  

(17)

To clarify the difference between \( M(p, m) \) and \( \hat{M}(p, \hat{m}) \), let us discuss the high energy behavior of \( \hat{M}(p, \hat{m}) \) at the one-loop level.

At the one-loop level, the self energy of quarks is given by

\[ -i (g \mu^e)^2 C_F m (D - 1) \frac{\Gamma(\epsilon)}{(4\pi)^{D/2}} \int_0^1 dx [m^2 x - p^2 x (1 - x)]^{-\epsilon}, \]  

(18)

where \( C_F = (N_c^2 - 1)/2 N_c \) and \( D = 4 - 2\epsilon \). Then at large \(-p^2\) the inverse of quark propagator behaves as

\[ S^{-1}_F = p - M(p, m), \]  

\[ M(p, m) = m \left( 1 - \frac{3C_F \alpha}{4\pi} \left( \log \frac{-p^2}{\mu^2} - \frac{4}{3} \right) \right) + \frac{m^3 3C_F \alpha}{p^2 4\pi} \left( \log \frac{-p^2}{m^2} + 1 \right) + O\left( \frac{m^4}{p^4} \right), \]  

(19)

where we have subtracted the ultraviolet divergence according to \( \overline{MS} \) scheme\(^3 \) which we use throughout this paper. Heaviside transform of (19) will be performed by using the following results coming from (6), (9) and (12),

\[ 1 \xrightarrow{\mathcal{H}} 1, \quad m \xrightarrow{\mathcal{H}} 0, \quad m^3 (\log m) \xrightarrow{\mathcal{H}} -2\hat{m}^3. \]  

(20)

We note from (20) that the so-called hard piece of order \( m \) in (19) disappears and the soft term of order \( m^3/p^2 \) survives after the transform. The reason that the soft term remains is that it involves \( \log(m) \). The fact that the mass-log exists at the order \( m^3/p^2 \) can be found by differentiating (18) with respect to \( m \). After the differentiation up to three times, one finds the infra-red singularity when \( m \to 0 \).
From (19) and (20) we have
\[
\hat{S}_F^{-1} = p - \hat{M}(p, \hat{m}),
\]
\[
\hat{M}(p, \hat{m}) = \frac{3C_F \alpha \hat{m}^3}{\pi p^2} + O(1/p^4),
\]
and find that \(\hat{M}\) behaves softly \(^\dagger\). We note that, for non zero \(\hat{m}\), \(\hat{M}(p, \hat{m})\) simulates the soft behavior of \(M(p, 0)\) with spontaneous symmetry breaking\(^5\). Thus, the dominant effect of explicit chiral symmetry breaking has been washed out through Heaviside transform. From the second equation in (20) the hard piece vanishes to all orders if it is simply proportional to \(m\). Actually it is pointed out in Ref.6 that, in pole subtraction schemes, the coefficient functions of operator product expansion\(^7\) are analytic in all masses involved. Then, since the hard piece belongs to the unit operator and the unit operator cannot involve any mass-log, the whole part of the order \(m\) is just proportional to \(m\). Thus, we conclude that the leading effect of the explicit mass disappears in the Heaviside function of the effective mass to all orders. This reveals an advantage of \(\hat{M}(p, \hat{m})\) in approximating \(M(p, 0)\), since keeping \(\hat{m}\) non-zero is compatible with the chiral symmetry. On the other hand, for \(M(p, m)\), we must set \(m = 0\) to force the hard piece vanish but this makes the perturbative \(M\) trivial.

To carry out the massless approximation, we must take higher order contributions into consideration and approximate \(\hat{M}(p, 0)\) by setting \(\hat{m}\) as small as possible in \(\hat{M}(p, \hat{m})\) \(^\ddagger\). A typical example is given in the next section.

4 Calculation of the quark condensate

To prove non-perturbative effects, the perturbative series would not contain enough information. However, it is not clear whether it is useless in the approximate calculation or not. Actually we have found that the perturbative series is effective in the approximate calculation of the dynamical mass in the Gross-Neveu model\(^8\). Since the Heaviside transform removes the dominant effect of the explicit mass which is irrelevant to massless

\(^\dagger\)The soft behavior was also found by the direct calculation in the generalized Hartree-Fock approach\(^4\). Also, we point out that, though all sub-leading terms in \(1/p^2\) are lost at the one-loop level, higher order contributions would involve \(\log m\) and give non-trivial results under \(H\) transformation.

\(^\ddagger\)Higher order contributions would produce powers of \(\log \hat{m}\) at the \(\hat{m}^3/p^2\) order term in \(\hat{M}\) and the resulting series of \(\hat{m}^3\) times the series of \(\log \hat{m}/\mu\) corresponds to the transformed quark condensate.
dynamics, we expect also in QCD that the transformed quantity is more convenient in the approximate calculation than the original one.

In this section we try to estimate the massless value of the condensate. For the purpose we directly calculate the condensate for the massive case to two-loops and transform it to evaluate the massless value, assuming the existence of the non-zero condensate.

As is well known, the naive product, \( \sum_{A,\alpha} \bar{q}^{A,\alpha}(0)q^{A\alpha}(0) = \bar{q}q \), is singular (\( A \) and \( \alpha \) denote color and Lorentz indices, respectively) and we need to regularize it. In this paper, we define the regular product according to \( \overline{MS} \) scheme\(^9\). We denote thus regularized product as \( \bar{q}q \). In this definition the two-loop calculation of the condensate is given by\(^10\)

\[
\langle \bar{q}q \rangle = \frac{N_c m^3 \mu^{-2\epsilon}}{4\pi^2} \left\{ 1 - \log \frac{m^2}{\mu^2} + \frac{3C_F g^2}{8\pi^2} \left( \log^2 \frac{m^2}{\mu^2} - \frac{5}{3} \log \frac{m^2}{\mu^2} + \text{constant} \right) \right\},
\]

(22)

where

\[
\bar{q}q = Z \bar{q}q - \frac{N_c m^3 \mu^{-2\epsilon}}{4\pi^2} \left( \frac{\hat{1}}{\epsilon} + \frac{C_F g^2}{8\pi^2} \left( 3 \frac{\hat{1}^2}{\epsilon} - \frac{\hat{1}}{\epsilon} \right) \right),
\]

(23)

and

\[
Z = 1 - \frac{3C_F g^2(\mu)}{(4\pi)^2} \frac{\hat{1}}{\epsilon}, \quad \frac{\hat{1}}{\epsilon} = \frac{1}{\epsilon} - \gamma + \log(4\pi).
\]

(24)

Note that we need non-multiplicative renormalization to make the condensate finite (see (23)). As a result, the transformation property of the product under the renormalization group changes from that of \( Z\bar{q}q \) and \( \langle \bar{q}q \rangle \) no longer satisfies \( \mu d\langle \bar{q}q \rangle/d\mu = -\gamma_m\langle \bar{q}q \rangle \) where \( \gamma_m = \mu \partial m/\partial \mu = \left( -\frac{3C_F}{2\epsilon} \alpha + O(\alpha^2) \right) m. \)

Our task is first making Heaviside transform of (22) and then improving the result by renormalization group. First, we point out that, since the non-multiplicative pole terms are the cube of \( m \), they disappear after Heaviside transform to give,

\[
\hat{\langle \bar{q}q \rangle} = \langle \hat{Z}\bar{q}q \rangle,
\]

(25)

where the hat denotes the Heaviside transform and we used (see (9))

\[
m^k \overset{\mathcal{H}}{\to} 0 \quad (k = \text{positive integer}).
\]

(26)

Since in the pole subtraction scheme the non-multiplicative piece is proportional to \( m^3 \) and free from the mass-log, eq.(25) holds to all orders. Thus, as in the exact massless case, we could use \( Z\bar{q}q \) as the local product. As well as the recovery of the soft behavior.
in the transformed effective mass, this result shows that the effect of the explicit mass is reduced and it would be better to use the transformed quantity to simulate the massless case. Now, from (9), (12) and
\[
m^3 \log^2(m) \xrightarrow{R} -2(m/n)^3 \left(3 + 2(\log(m/n) - \gamma)\right),
\]
we find
\[
\langle [\bar{q}q]\rangle \xrightarrow{R} \langle [\bar{q}q]\rangle = \frac{N_c \hat{m}^3}{\pi^2} \left[1 - \frac{3C_F \alpha(\mu)}{2\pi} \left(\log \frac{\hat{m}^2}{\mu^2} - 2\gamma + \frac{13}{6}\right)\right], \quad \alpha = \frac{g^2}{4\pi}.
\]
We find by the direct operation of \(\mu(d/d\mu)\) that the condensate satisfies the renormalization group equation,
\[
\mu \frac{d}{d\mu} \langle [\bar{q}q]\rangle = -\gamma_m \langle [\bar{q}q]\rangle, \quad \gamma_m = -\frac{3C_F}{2\pi} \alpha + O(\alpha^2),
\]
up to the lowest order. Thus, as implied by (25), the transformed condensate shows the correct property under the renormalization group. The solution of (29) is given by
\[
\langle [\bar{q}q]\rangle = \chi(\alpha, \hat{m}, \{M\}, \mu)|_{\mu = \mu_0} \alpha^{-A}, \quad \chi = \langle [\bar{q}q]\rangle_0 \alpha^A, \quad A = \frac{9C_F}{11C_G - 2n_F}
\]
where the subscript 0 means the value at \(\mu = \mu_0\) and \(\{M\}\) denotes the set of explicit masses of other quarks.

Let us improve the large \(\hat{m}_0\) behavior of \(\chi\) which can be settled by perturbation expansion. The behavior is improved by adjusting \(\mu_0\) in accordance with \(\hat{m}_0\). We impose \(\mu_0\) to satisfy
\[
\log \frac{\hat{m}_0}{\mu_0} - \gamma + \frac{13}{12} = t,
\]
where \(t\) is a fixed constant which value is yet un-specified. This condition fixes \(\mu_0\) as the function of \(\hat{\Lambda}_m\) (the renormalization group invariant mass corresponding to \(\hat{m}\)), \(\Lambda\) (finite QCD scale in \(\bar{MS}\) scheme), and \(t\). Further the \(\hat{\Lambda}_m\) dependence of \(\hat{m}_0\) changes from the simple proportional one. Actually, from the renormalization group equation and (31), we find the implicit equation for \(\hat{m}_0\),
\[
\hat{m}_0 = \hat{\Lambda}_m \alpha(\mu_0)^A = \hat{\Lambda}_m \left(\beta_0 \log \frac{\hat{m}_0 e^{-\gamma + 13/12 - t}}{\Lambda}\right)^{-A},
\]
at this order of expansion. Since \(\mu_0 > \Lambda\) to keep the coupling \(\alpha_0\) positive, \(\hat{m}_0\) must be larger than \(\Lambda e^{\gamma - 13/12 + t} (= \hat{m}^*\) from (31). When \(\hat{\Lambda}_m\) goes to zero, \(\hat{m}_0 \to \hat{m}^*\) and \(\alpha \to +\infty\)
from (31) and (32). Thus, $\hat{m}^*$ represents the limitation of the obtained perturbative result. Although the correspondence between $\hat{\Lambda}_m$ and $\hat{m}_0$ has been involved, $\hat{m}_0$ is a monotonic function of $\hat{\Lambda}_m$ and most manipulation can be carried out in terms of $\hat{m}_0$. We note from (31) that $\alpha_0$ depends on $\hat{m}_0$ as governed by $\hat{m}_0 \partial \alpha_0 / \partial \hat{m}_0 = \beta(\alpha_0) = -\beta_0 \alpha_0^2 + O(\alpha_0^3)$ where $\beta_0 = (11C_G - 2n_F)/6\pi$. The asymptotic freedom in QCD enters into the condensate through $\alpha_0(\hat{m}_0, t)$. Taking (31) into account, we thus have

$$\chi = \frac{N_c \hat{m}_0^3}{\pi^2} \left[ 1 - \frac{6C_F \alpha_0(\hat{m}_0, t)}{\pi} t \right] \alpha_0(\hat{m}_0, t)^A,$$

and arrive at the improved result,

$$\langle [\bar{q}q] \rangle = \chi \alpha(\mu)^{-A} = \frac{N_c \hat{m}_0^3}{\pi^2} \left[ 1 - \frac{6C_F \alpha_0(\hat{m}_0, t)}{\pi} t \right] \alpha_0(\hat{m}_0, t)^A \cdot \alpha(\mu)^{-A}. \quad (34)$$

Let us turn to the rough estimation of the condensate, $\langle [\bar{q}q] \rangle |_{m=0}$. Recall that one cannot take the $\hat{\Lambda}_m \to 0$ limit as we stated in the previous paragraph. Then what we can do is to fix $\hat{\Lambda}_m$ or equally $\hat{m}_0$ in terms of $\Lambda$ under the guiding principle that one should minimize the effect of the explicit mass as possible as one can. Here we note that $\lim_{m \to 0} m^j \langle [\bar{q}q] \rangle = 0$ for any positive integer $j$ and therefore from (3) that $\lim_{\hat{m} \to 0} H\{m^j \langle [\bar{q}q] \rangle\} = 0$ ($H$ denotes the operation of Heaviside transformation). Let us impose the case of $j = 1$. Further, since we have another parameter $t$ which we can choose freely, we demand the condition of $j = 2$. Now from (8), the conditions of $\hat{m}_0$ minimizing the effect of the explicit mass are

$$\frac{\partial \langle [\bar{q}q] \rangle}{\partial (1/\hat{m}_0)} = 0, \quad \frac{\partial^2 \langle [\bar{q}q] \rangle}{\partial (1/\hat{m}_0)^2} = 0,$$

which lead to the same conditions for $\chi$. Eq.(35) gives two non-trivial solutions. However, one of those corresponds to the large coupling, $\alpha \sim 8$, and we discard this since it is too large to rely upon within the two-loop level. The other solution gives $\alpha \sim 0.8$ for various flavors and is valuable to be discussed further. That value of $\alpha$ indicates that the renormalization scale $\mu_0$ is a few of $\Lambda$. Assuming the decoupling of heavy quarks, we therefore work with the choice that $n_F = 3$. Then we have $\alpha \sim 0.79$ and $t \sim 0.91$. These values give $\hat{m}_0 \sim 3.6\Lambda_3$, $\chi \sim -10.8\Lambda_3^3$ and $\hat{\Lambda}_m \sim 4\Lambda_3$ where (31) and the one-loop relation, $\Lambda_3 = \mu \exp(-1/\beta_0 \alpha)$, was used ($\Lambda_3$ denotes the QCD scale effective at three flavors). Thus we arrive at

$$\langle [\bar{q}q] \rangle \sim -10.8\Lambda_3^3 \alpha^{-A}(\mu). \quad (36)$$
Further numerical computation of the condensate can be done as follows: From the experimental data, \( \alpha(m_Z) \sim 0.117 \pm 0.005 \), and the definition of \( \Lambda \) at the lowest order, we have \( \Lambda_5 \sim 83 + 27 - 23 \text{MeV} \) for \( n_F = 5 \). With the help of the relation, \( \Lambda_3 \sim \Lambda_5(m_c m_b/\Lambda_5^2)^{2/27} \) \( (m_b = 4.3\text{GeV}, m_c = 1.3\text{GeV}) \), it is converted to the scale value for \( n_F = 3 \), giving \( \Lambda_3 \sim 135 + 28 - 31 \text{MeV} \). By setting \( \mu = 1\text{GeV} \) and \( n_F = 3 \) in (36), we thus have

\[
\langle [\bar{q}q] \rangle |_{\mu=1\text{GeV}} \sim -(351 + 88 - 77 \text{MeV})^3.
\]

5 Discussion

The phenomenological value, \( \langle \bar{u}u \rangle \sim \langle \bar{d}d \rangle \sim -(250\text{MeV})^3 \) is slightly below the range shown in (37). We think that the result is rather good, taking into account that the result (37) is just the lowest order one which comes from a rough ansatz, (34). Of course the higher order calculation is needed for the serious study of our approach. In particular the three loop calculation is important since at this order the non-linear gluon vertex and the loops of other quarks becomes explicitly active.

Acknowledgements

We wish to thank Dr. O. Morimatsu for the discussion and helpful comments in the early stage of this work. We also thanks Dr. H. Suzuki for the interest and fruitful discussion on the subjects treated in the paper. We acknowledge the warm hospitalities at Institute for Nuclear Study, University of Tokyo and Ochanomizu University. This work is financially supported by Iwanami Fujukai.
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