Using Operations Research to Determine the Optimal Three Mashes for Three Stages of Broiler Feeding

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Abstract. In this research, one of the methods of operations research, a linear programming method, was used to build a mathematical model to minimize the costs of feeding chickens for the production of white meat. The feeding system adopted for the first three periods from the age of one day to the age of 22 days. The second period starts from age 23 days to age 42 days and the third period starts from the age of 43 days until the stage of direct marketing by taking advantage of feed materials available in local markets or imported from outside Iraq to choose three economic diets, and each diet is suitable for feeding the chickens for the production of white meat. These three stages of nutrition should ensure the presence of all the essential elements to be provided in chicken diets such as proteins, energy represented, fats, fiber, vitamins, mineral salts, amino acids and other elements required to be available in the poultry food at the same rates determined by the dietitian and for each three phase of feeding. The use of these feeds extracted through the application of one of the methods of operations research will lead to minimizing the cost of poultry feeding as well as reducing the time required for the arrival of chickens to the stage of marketing and profit taking.

Keywords: linear programming, broiler feeding, broiler diet, feed formulation

1. Introduction

Linear programming method, are effective methods that have achieved wide success in various areas of life. I will mention some of the production planning, transportation problems, distribution of energy sources, financial planning, health care planning, hospital management and other areas. One of the methods of operations research in the field of poultry nutrition through the identification of optimal feed mixes for the stages of breeding (the beginning stage - the growth stage - the end stage) and to take advantage of the feed materials available in the local markets for the production of chicken feed rations that meet the needs basic and necessary for the required chicken feed at the lowest possible financial costs, and this will increase the productivity of the profitability of these fields.

Increasing the profitability of poultry farms is achieved primarily through minimizing the cost of feeding poultry during the three feeding periods. Feeding chickens has become one of the most important challenges facing the owners of poultry farms. Feeding chickens to a minimum while providing all the basic needs required for feeding poultry without increasing or decreasing to obtain the highest productivity performance in the production of white meat, in addition to reducing the other expenses required by the process of raising chickens dedicated to produce meat whenever possible. Poultry farming in Iraq is facing the problem of the lack of use of modern scientific methods in
feeding poultry and reliance on the personal experiences of some owners of poultry fields, which led to high production costs and low productivity profitability of these fields, which led many owners of poultry fields to close their fields for lack of profits.

Therefore, some research needed to reduce the cost of feeding, which form a large percentage of the cost of breeding chickens by applying one of the methods of operations research.

2. Materials And Methods

N R C (1994) is the main source of the data in the tables 2, 3, 4

Table 1. Upper limits and the prices of the broiler diet ingredient

| N. | The ingredient         | Upper limit of ingredient in 100 kgm of diet | Price of 1 kgm | The ingredient         | Upper limit of ingredient in 100 kgm of diet | Price of 1 kgm |
|----|------------------------|---------------------------------------------|---------------|------------------------|---------------------------------------------|---------------|
| 1  | The wheat              | 70                                         | 510           | 14 Fish meal           | 5                                          | 1750          |
| 2  | The barley             | 20                                         | 430           | 15 Bone meal           | 2.5                                        | 850           |
| 3  | The maize              | 70                                         | 490           | 16 The broad bean      | 10                                         | 510           |
| 4  | White corn             | 10                                         | 410           | 17 The white Beans     | 10                                         | 1210          |
| 5  | The Rice               | 70                                         | 290           | 18 The Chickpeas       | 10                                         | 710           |
| 6  | The coarse wheat bran  | 5                                          | 400           | 19 The Lentils         | 10                                         | 710           |
| 7  | The fine wheat bran    | 5                                          | 400           | 20 Dehydrated alfalfa 17% protein | 7                                          | 610           |
| 8  | maize gluten 42% protein | 5                                    | 958           | 21 Dehydrated alfalfa leaves | 7                                          | 610           |
| 9  | maize gluten 60% protein | 5                                    | 1450          | 22 Limestone           | 0.5                                        | 110           |
| 10 | Soybean meal 44% protein | 40                                 | 838           | 23 Vegetable oils      | 4                                          | 1810          |
| 11 | Soybean meal 49% protein | 30                                 | 910           | 24 Crude vegetable oils | 4                                          | 1500          |
| 12 | Sunflower meal         | 40                                         | 460           | 25 Vegetable protein concentrates | 10                                        | 1930          |
| 13 | The Sesame meal        | 3                                          | 3000          | 26 Animal protein concentrates | 10                                        | 1800          |
|    |                        |                                             |               |                        | 27 Sodium chloride                          | 0.25          | 110           |
| N. | The Ingredient              | Energy | Lysine% | Methionine% | Phosphor% | Calcium% | Crude Protein% | Sodium% |
|----|----------------------------|--------|---------|-------------|-----------|----------|----------------|---------|
| 1  | the wheat                 | 3250   | 0.39    | 0.37        | 0.12      | 0.05     | 13             | 0.07    |
| 2  | the barely                | 2750   | 0.39    | 0.37        | 0.16      | 0.08     | 9              | 0.02    |
| 3  | the maize                 | 3370   | 0.24    | 0.4         | 0.11      | 0.07     | 8.72           | 0.01    |
| 4  | white corn                | 3210   | 0.25    | 0.35        | 0.13      | 0.04     | 11.4           | 0.01    |
| 5  | The rice                  | 3150   | 0.3     | 0.3         | 0.04      | 0.09     | 8.18           | 0.11    |
| 6  | The coarse wheat bran     | 1250   | 0.53    | 0.42        | 0.34      | 0.16     | 14.1           | 0.3     |
| 7  | The fine wheat bran       | 1650   | 0.59    | 0.47        | 0.37      | 0.15     | 15.5           | 0.07    |
| 8  | maiz gluten 42% protein   | 3335   | 0.73    | 1.91        | 0.15      | 0.45     | 42             | 0.1     |
| 9  | maiz gluten 60% protein   | 3750   | 1.29    | 2.79        | 0.19      | 0        | 60             | 0.01    |
| 10 | soybean meal 44% protein  | 2300   | 2.91    | 1.33        | 0.29      | 0.32     | 45             | 0.24    |
| 11 | soybean meal 49% protein  | 2500   | 3.17    | 1.47        | 0.19      | 0.26     | 50             | 0.34    |
| 12 | sunflower meal            | 1960   | 1.73    | 2.22        | 0.16      | 0.38     | 46.9           |         |
| 13 | The sesame meal           | 2415   | 1.09    | 1.86        | 0.42      | 2.02     | 44.5           | 0.3     |
| 14 | The fish meal             | 2963   | 4.83    | 2.32        | 2.95      | 5.02     | 64.7           |         |
| 15 | bone meal                 | 1398   | 0.87    | 0.29        | 0.14      | 0.3      | 12.1           | 0.46    |
| 16 | The broad bean            | 2520   | 1.55    | 0.47        | 0.12      | 0.17     | 26.1           | 0.08    |
| 17 | The white beans           | 2330   | 1.1     | 0.48        | 0.15      | 0.13     | 24             |         |
| 18 | The chickpeas             | 2756   | 1.34    | 0.59        | 0.18      | 0.2      | 20.8           |         |
| 19 | The lentils               | 2647   | 1.73    | 0.41        | 0.11      | 0.52     | 23.5           |         |
| 20 | Dehydrated alfalfa 17% protein | 1453 | 0.82    | 0.51        | 0.24      | 1.3      | 17.3           | 0.18    |
| 21 | Dehydrated alfalfa leaves 20% protein | 1580 | 0.9     | 0.55        | 0.27      | 1.5      | 21.1           | 0.19    |
| 22 | The lime stone            |        |         |             |           |          | 30             | 38      |
| 23 | Crude vegetable oils      | 7010   |         |             |           |          |                |         |
| 24 | Vegetable oils            | 8900   |         |             |           |          |                |         |
| N. | The Ingredient                        | Thiamin mg/kgm | Niacin mg/kgm | Pantothenic mg/kgm | Riboflavin mg/kgm | Crude Fiber % | Lipids % | Ash % | Humidity % |
|----|--------------------------------------|----------------|---------------|--------------------|------------------|---------------|----------|-------|------------|
| 1  | the wheat                            | 4.4            | 56.6          | 12.1               | 1.2              | 2.4           | 1.9      | 1.6   | 11         |
| 2  | the barley                           | 5.1            | 57.4          | 6.5                | 2                | 5             | 1.9      | 2.4   | 11         |
| 3  | the maize                            | 4.0            | 22.9          | 5.0                | 1.1              | 2             | 3.9      | 1.8   | 12         |
| 4  | white corn                           | 3.9            | 42.7          | 11.4               | 1.2              | 2             | 2.8      | 1.7   | 11         |
| 5  | The rice                             | 0.6            | 14.1          | 3.3                | 0.6              | 9             | 1.9      | 4.5   | 11         |
| 6  | The coarse wheat bran                | 7.9            | 209           | 29.0               | 3.1              | 10            | 4.1      | 6.1   | 11         |
| 7  | The fine wheat bran                  | 2.6            | 20.0          | 13.0               | 0.9              | 3             | 2.9      | 2.1   | 11         |
| 8  | maize gluten 42% protein              | 0.2            | 49.9          | 10.3               | 1.5              | 4             | 2.3      | 2.4   | 9          |
| 9  | maize gluten 60% protein              | 0.2            | 5.5           | 2.9                | 2.2              | 1.3           | 1.7      | 2     | 9          |
| 10 | soybean meal 44% protein              | 4.0            | 30.4          | 15.2               | 2.6              | 6             | 4.7      | 5.7   | 10.4       |
| 11 | soybean meal 49% protein              | 6.5            | 26.8          | 14.5               | 3.3              | 3             | 0.8      | 5.6   | 10         |
| 12 | sunflower meal                       | 220            | 10.1          | 3.3                | 5                | 5.1           | 9.3      | 7     |            |
| 13 | The sesame meal                      | 2.86           | 30.8          | 6.38               | 3.74             | 14            | 2.8      | 7.1   | 10         |
| 14 | The fish meal                        | 0.2            | 47.1          | 3.3                | 4.6              | 1             | 4.4      | 21.7  | 8          |
| 15 | bone meal                            | 3.6            | 10.6          | 33.9               | 18.9             | 2             | 3.2      | 71.8  | 5          |
| 16 | The broad bean                       |                |               |                    |                  | 5.7           | 1.5      | 3.6   | 13.4       |
| 17 | The white beans                      |                |               |                    |                  |               |          |       |            |
| 18 | The chickpeas                        |                |               |                    |                  |               |          |       |            |
| 19 | The lentil                           | 500            | 82.9          | 44.4               |                 |               |          |       |            |
| 20 | Dehydrated alfalfa 17% protein       | 4.0            | 54.6          | 32.8               | 15.4             | 24.8          | 2.6      | 9     | 7          |

Source NRC (1994)

Table 3. Percentage of humidity, fiber, lipids and vitamins amounts in broiler diet ingredient
| N.  | The Ingredient                  | Initial Mash | Grower Mash | Finisher mash |
|-----|---------------------------------|--------------|-------------|---------------|
| 1   | Crude protein %                 | 24           | 22          | 20            |
| 2   | Energy kilocalorie/kgm          | 2800         | 3000        | 3300          |
| 3   | Crude Fiber %                   | 3.5          | 4           | 4             |
| 4   | Crude Fat %                     | 5            | 6           | 7             |
| 5   | Energy/Protein                  | 117          | 136         | 165           |
| 6   | Ash %                           | 0.50%        | 0.50%       | 0.50%         |
| 7   | Calcium %                       | 1            | 0.9         | 0.8           |
| 8   | Sodium %                        | 0.2          | 0.15        | 0.12          |
| 9   | Chloride %                      | 0.2          | 0.15        | 0.12          |
| 10  | Phosphor %                      | 0.45         | 0.35        | 0.3           |
| 11  | Lysine %                        | 1.1          | 1           | 0.85          |
| 12  | Methionine %                    | 0.5          | 0.38        | 0.32          |
| 13  | Methionine + Systine            | 0.9          | 0.72        | 0.61          |
| 14  | Riboflavin (mg)                 | 3.6          | 3.5         | 3             |
| 15  | Niacin (mg)                     | 35           | 30          | 25            |
| 16  | Pantothenic (mg)                | 10           | 10          | 10            |
| 17  | Niacin (mg)                     | 35           | 30          | 25            |
| 18  | Folic Acid (mg)                 | 0.55         | 0.55        | 0.5           |
| 19  | Thiamin (mg)                    | 1.8          | 1.7         | 1.6           |

*Source NRC (1994)*

Table 4. Requirement of the broiler diet in the three intervals of nutrition

3. The Mathematical Model
3.1. Decision Variables

Table 5. Decision variables for the three mashses

| N. | Ingredient name                  | Symbol of the decision variable in the Initial mash | Symbol of the decision variable in the grower mash | Symbol of the decision variable in the finisher mash |
|----|----------------------------------|-----------------------------------------------------|-----------------------------------------------------|-----------------------------------------------------|
| 1  | The wheat 1                        | A1                                                  | B1                                                  | C1                                                  |
| 2  | The barley 2                        | A2                                                  | B2                                                  | C2                                                  |
| 3  | The maize 3                         | A3                                                  | B3                                                  | C3                                                  |
| 4  | The white corn 4                    | A4                                                  | B4                                                  | C4                                                  |
| 5  | The Rice 5                          | A5                                                  | B5                                                  | C5                                                  |
| 6  | The coarse wheat bran 6             | A6                                                  | B6                                                  | C6                                                  |
| 7  | The fine wheat bran 7               | A7                                                  | B7                                                  | C7                                                  |
| 8  | Maize gluten 42% protein 8          | A8                                                  | B8                                                  | C8                                                  |
| 9  | Maize gluten 60% protein 9          | A9                                                  | B9                                                  | C9                                                  |
| 10 | Soybean meal 44% protein 10         | A10                                                 | B10                                                 | C10                                                 |
| 11 | Soybean meal 49% protein 11         | A11                                                 | B11                                                 | C11                                                 |
| 12 | Sunflower meal 12                   | A12                                                 | B12                                                 | C12                                                 |
| 13 | The Sesame meal 13                  | A13                                                 | B13                                                 | C13                                                 |
| 14 | Fish meal 14                        | A14                                                 | B14                                                 | C14                                                 |
| 15 | Bone meal 15                        | A15                                                 | B15                                                 | C15                                                 |
| 16 | The bean 16                         | A16                                                 | B16                                                 | C16                                                 |
| 17 | The white Beans 17                  | A17                                                 | B17                                                 | C17                                                 |
| 18 | The Chickpeas 18                    | A18                                                 | B18                                                 | C18                                                 |
| 19 | The Lentil 19                       | A19                                                 | B19                                                 | C19                                                 |
| 20 | Dehydrated alfalfa 17% protein 20   | A20                                                 | B20                                                 | C20                                                 |
| 21 | Dehydrated alfalfa leaves 20% protein 21 | A21 | B21 | C21 |
| 22 | Limestone 22                        | A22                                                 | B22                                                 | C22                                                 |
| 23 | Vegetable oils 23                   | A23                                                 | B23                                                 | C23                                                 |
| 24 | Crude vegetable oils 24             | A24                                                 | B24                                                 | C24                                                 |
| 25 | Vegetable protein concentrates 25   | A25                                                 | B25                                                 | C25                                                 |
| 26 | Animal protein concentrates 26      | A26                                                 | B26                                                 | C26                                                 |
| 27 | Sodium chloride 27                  | A27                                                 | B27                                                 | C27                                                 |
3.2. The Initial Mash

3.3. The objective function

Minimize the summation of the cost of 100 kgm. of the initial mash

\[ \text{Min } z = 510A_1 + 430A_2 + 490A_3 + 410A_4 + 290A_5 + 400A_6 + 400A_7 + 958A_8 + 1450A_9 + 838A_{10} + 910A_{11} + 460A_{12} + 3100A_{13} + 1750A_{14} + 850A_{15} + 1210A_{16} + 710A_{17} + 710A_{19} + 610A_{20} + 610A_{21} + 110A_{22} + 110A_{23} + 1810A_{24} + 1930A_{25} + 1750A_{26} + 110A_{27} \ldots \ldots \ldots \ldots (1). \]

3.4. The Constraints

- Constraint of the wheat in the initial mash: \( A_1 \leq 70 \) … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … (2).
- Constraint of the barely in the initial mash: \( A_2 \leq 20 \) … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … (3).
- Constraint of the maize in the initial mash: \( A_3 \leq 70 \) … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … (4).
- Constraint of the white corn in the initial mash: \( A_4 \leq 10 \) … … … … … … … … … … … … … … … … … … … … … … … … … … … (5).
- Constraint of the rice in the initial mash: \( A_5 \leq 70 \) … … … … … … … … … … … … … … … … … … … … … … … … … … … (6).
- Constraint of the coarse wheat bran in the initial mash: \( A_6 \leq 5 \) … … … … … … … … … … … … … … … … … … … … (7).
- Constraint of the fine wheat bran in the initial mash: \( A_7 \leq 5 \) … … … … … … … … … … … … … … … … … … … … (8).
- Constraint of the maize gluten 42% protein in the initial mash: \( A_8 \leq 5 \) … … … … … … … … … … … … … … … … … … … (9).
- Constraint of the maize gluten 60% protein in the initial mash: \( A_9 \leq 5 \) … … … … … … … … … … … … … … … … … … … (10).
- Constraint of the soybean meal 44% protein in the initial mash: \( A_{10} \leq 40 \) … … … … … … … … … … … … … … … … … … (11).
- Constraint of the soybean meal 49% protein in the initial mash: \( A_{11} \leq 30 \) … … … … … … … … … … … … … … … … … … (12).
- Constraint of the sunflower meal in the initial mash: \( A_{12} \leq 40 \) … … … … … … … … … … … … … … … … … … … (13).
- Constraint of the sesame meal in the initial mash: \( A_{13} \leq 3 \) … … … … … … … … … … … … … … … … … … … (14).
- Constraint of the fish meal in the initial mash: \( A_{14} \leq 5 \) … … … … … … … … … … … … … … … … … … … (15).
- Constraint of the bone meal in the initial mash: \( A_{15} \leq 2.5 \) … … … … … … … … … … … … … … … … … … … (16).
- Constraint of the bean in the initial mash: \( A_{16} \leq 10 \) … … … … … … … … … … … … … … … … … … … (17).
- Constraint of the white beans in the initial mash: \( A_{17} \leq 10 \) … … … … … … … … … … … … … … … … … … (18).
- Constraint of the chick peas in the initial mash: \( A_{18} \leq 10 \) … … … … … … … … … … … … … … … … … … (19).
- Constraint of the lentils in the initial mash: \( A_{19} \leq 10 \) … … … … … … … … … … … … … … … … … … (20).
- Constraint of the dehydrated alfalfa 17% protein in the initial mash: \( A_{20} \leq 7 \) … … … … … … (21).
- Constraint of the dehydrated alfalfa leaves 20% protein in the initial mash: \( A_{21} \leq 7 \) … … … … … (22).
- Constraint of the Lime stone in the initial mash: \( A_{22} \leq 1 \) … … … … … … … … … … … … … … … … … … (23).
- Constraint of the vegetable oils in the initial mash: \( A_{23} \leq 4 \) … … … … … … … … … … … … … … … (24).
- Constraint of the crude vegetable oils in the initial mash: \( A_{24} \leq 4 \) … … … … … … … … … … … … … … (25).
- Constraint of the vegetable protein concentrates & vitamins in the initial mash: \( A_{25} \leq 10 \) … … (26).
- Constraint of the animal protein concentrates & vitamins in the initial mash: \( A_{26} \leq 1 \) … … (27).
- Constraint of the sodium chloride in the initial mash: \( A_{27} \leq 0.25 \) … … … … … … … … … … … … (28).

Constraint of all ingredients in the initial mash:

\[ A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8 + A_9 + A_{10} + A_{11} + A_{12} + A_{13} + A_{14} + A_{15} + A_{16} + A_{17} + A_{18} + A_{19} + A_{20} + A_{21} + A_{22} + A_{23} + A_{24} + A_{25} + A_{26} + A_{27} = 100 \] … … … … … … … … … … … … (29)
Constraint of the lower limit of the legumes in the initial mash: $A_{16}+A_{17}+A_{18}+A_{19} \geq 3$ ... 

Constraint of the upper limit of the legumes in the initial mash: $A_{16}+A_{17}+A_{18}+A_{19} \leq 10$ (31)

Constraint of the meals in the initial mash: $A_8+A_9+A_{10}+A_{11}+A_{12}+A_{13} \leq 40$ ... ... ... ... (32)

Constraint of the grains and another energy sources in the initial mash: $A_1+A_2+A_3+A_4+A_5+A_6+A_7 \leq 50$ ... ... ... ... ... ... ... ... ... ... (33)

Constraint of the oils in the initial mash: $A_{23}+A_{24} \leq 4$ ... ... ... ... ... ... ... ... ... ... (34)

Constraint of the concentrated proteins in the initial mash: $A_{25}+A_{26} \leq 10$ ... ... ... ... ... ... ... ... ... ... (35)

Constraint of the percentage in the initial mash:

$13A_1+9A_2+8.72A_3+11.4A_4+8.18A_5+14.1A_6+15.5A_7+42A_8+60A_9+45A_{10}+50A_{11}+46.9A_{12}+44.5A_{13}+64.7A_{14}+12.1A_{15}+26.1A_{16}+24A_{17}+20.8A_{18}+23.5A_{19}+17.3A_{20}+21.1A_{21}+50A_{25}+50A_{26}=2400$ ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... (36)

Constraint of the metabolizable energy in the initial mash:

$3250A_1+2750A_2+3370A_3+3210A_4+3150A_5+1650A_6+1650A_7+3335A_8+3760A_9+2300A_{10}+2500A_{11}+1960A_{12}+2416A_{13}+2963A_{14}+1398A_{15}+2520A_{16}+2330A_{17}+2756A_{18}+2647A_{19}+1453A_{20}+1580A_{21}+7010A_{23}+8900A_{24}=280000$ ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... (37)

Constraint the upper limit of the lipid percentage in the initial mash:

$1.9A_1+1.9A_2+3.9A_3+2.8A_4+1.9A_5+4.1A_6+2.9A_7+2.3A_8+1.7A_9+4.7A_{10}+0.8A_{11}+5.1A_{12}+2.8A_{13}+4.4A_{14}+3.2A_{15}+1.5A_{16}+2.6A_{20}+3.6A_{21}+90A_{23}+90A_{24} \leq 700$ ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... (38)

Constraint the lower limit of the lipid percentage in the initial mash:

$1.9A_1+1.9A_2+3.9A_3+2.8A_4+1.9A_5+4.1A_6+2.9A_7+2.3A_8+1.7A_9+4.7A_{10}+0.8A_{11}+5.1A_{12}+2.8A_{13}+4.4A_{14}+3.2A_{15}+1.5A_{16}+2.6A_{20}+3.6A_{21}+90A_{23}+90A_{24} \geq 400$ ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... (39)

Constraint of the Lower limit of the humidity percentage in the initial mash:

$11A_1+11A_2+12A_3+11A_4+11A_5+11A_6+11A_7+9.48+9.49+10.4A_{10}+10A_{11}+7A_{12}+10A_{13}+8A_{14}+5A_{15}+13.4A_{16}+7A_{20}+10.3A_{21} \geq 500$ ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... (40)

Constraint of the upper limit of the humidity percentage in the initial mash:

$11A_1+11A_2+12A_3+11A_4+11A_5+11A_6+11A_7+9.48+9.49+10.4A_{10}+10A_{11}+7A_{12}+10A_{13}+8A_{14}+5A_{15}+13.4A_{16}+7A_{20}+10.3A_{21} \leq 1000$ ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... (41)

Constraint of the lower limit of the crude fiber percentage in the initial mash:

$2.4A_1+5A_{2}+2A_{3}+2A_{4}+9A_{5}+10A_{6}+3A_{7}+4A_{8}+1.3A_{9}+6A_{10}+3A_{11}+5A_{12}+14A_{13}+A_{14}+2A_{15}+5.7A_{16}+24.8A_{20}+20.1A_{21} \geq 300$ ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... (42)

Constraint of the upper limit of the crude fiber percentage in the initial mash:
\[ 2.4A_1 + 5A_2 + 2A_3 + 2A_4 + 9A_5 + 10A_6 + 3A_7 + 4A_8 + 1.3A_9 + 6A_{10} + 3A_{11} + 14A_{12} + A_{14} + 2A_{15} + 5.7A_{16} + 24.8A_{20} + 20.1A_{21} \leq 700 \quad \text{(43)} \]

**Constraint of the lower limit of the ash percentage in the initial mash:**

\[ 1.6A_1 + 2.4A_2 + 1.8A_3 + 1.7A_4 + 4.5A_5 + 6.1A_6 + 2.1A_7 + 2.4A_8 + 2A_9 + 5.7A_{10} + 5.6A_{11} + 9.3A_{12} + 7.1A_{13} + 21.7A_{14} + 71.8A_{15} + 3.6A_{16} + 9A_{20} + 10.3A_{21} \geq 200 \quad \text{(44)} \]

**Constraint of the upper limit of the ash percentage in the initial mash:**

\[ 1.6A_1 + 2.4A_2 + 1.8A_3 + 1.7A_4 + 4.5A_5 + 6.1A_6 + 2.1A_7 + 2.4A_8 + 2A_9 + 5.7A_{10} + 5.6A_{11} + 9.3A_{12} + 7.1A_{13} + 21.7A_{14} + 71.8A_{15} + 3.6A_{16} + 9A_{20} + 10.3A_{21} \leq 500 \quad \text{(45)} \]

**Constraint of the lower limit of phosphor in the initial mash:**

\[ 0.12A_1 + 0.16A_2 + 0.11A_3 + 0.13A_4 + 0.04A_5 + 0.34A_6 + 0.37A_7 + 0.15A_8 + 0.19A_9 + 0.29A_{10} + 0.19A_{11} + 0.16A_{12} + 0.42A_{13} + 2.95A_{14} + 14A_{15} + 0.12A_{16} + 0.15A_{17} + 0.18A_{18} + 0.11A_{19} + 0.24A_{20} + 0.27A_{21} \geq 39 \quad \text{(46)} \]

**Constraint of the upper limit of phosphor in the initial mash:**

\[ 0.12A_1 + 0.16A_2 + 0.11A_3 + 0.13A_4 + 0.04A_5 + 0.34A_6 + 0.37A_7 + 0.15A_8 + 0.19A_9 + 0.29A_{10} + 0.19A_{11} + 0.16A_{12} + 0.42A_{13} + 2.95A_{14} + 14A_{15} + 0.12A_{16} + 0.15A_{17} + 0.18A_{18} + 0.11A_{19} + 0.24A_{20} + 0.27A_{21} \leq 50 \quad \text{(47)} \]

**Constraint of the lower limit of calcium percentage in the initial mash:**

\[ 0.05A_1 + 0.08A_2 + 0.07A_3 + 0.04A_4 + 0.09A_5 + 0.16A_6 + 0.15A_7 + 0.45A_8 + 0.32A_{10} + 0.26A_{11} + 0.38A_{12} + 2.02A_{13} + 5.02A_{14} + 30A_{15} + 0.17A_{16} + 0.13A_{17} + 0.2A_{18} + 0.52A_{19} + 1.3A_{20} + 1.5A_{21} + 38A_{22} \geq 95 \quad \text{(48)} \]

**Constraint of the upper limit of calcium percentage in the initial mash:**

\[ 0.05A_1 + 0.08A_2 + 0.07A_3 + 0.04A_4 + 0.09A_5 + 0.16A_6 + 0.15A_7 + 0.45A_8 + 0.32A_{10} + 0.26A_{11} + 0.38A_{12} + 2.02A_{13} + 5.02A_{14} + 30A_{15} + 0.17A_{16} + 0.13A_{17} + 0.2A_{18} + 0.52A_{19} + 1.3A_{20} + 1.5A_{21} + 38A_{22} \leq 110 \quad \text{(49)} \]

**Constraint of the lower limit of sodium percentage in the initial mash:**

\[ 0.07A_1 + 0.02A_2 + 0.01A_3 + 0.01A_4 + 0.11A_5 + 0.3A_6 + 0.07A_7 + 0.14A_8 + 0.03A_{9} + 0.24A_{10} + 0.34A_{11} + 0.3A_{13} + 0.46A_{15} + 0.08A_{16} + 0.18A_{20} + 0.19A_{21} + 50A_{27} \geq 20 \quad \text{(50)} \]

**Constraint of the upper limit of sodium percentage in the initial mash**
\[ 0.07A_1 + 0.02A_2 + 0.01A_3 + 0.01A_4 + 0.11A_5 + 0.3A_6 + 0.07A_7 + 0.1A_8 + 0.03A_9 + 0.24A_{10} + 0.34A_{11} + 0.3A_{13} + 0.46A_{15} + 0.0816 + 0.18A_{20} + 0.19A_{21} + 50A_{27} \leq 25 \] \[ \text{(51)} \]

Constraint of the lower limit of lysine amino acid percentage in the initial mash:

\[ 0.39A_1 + 0.39A_2 + 0.24A_3 + 0.25A_4 + 0.3A_5 + 0.53A_6 + 0.59A_7 + 0.3A_8 + 1.29A_9 + 2.91A_{10} + 3.17A_{11} + 1.73A_{12} + 1.09A_{13} + 4.83A_{14} + 0.87A_{15} + 1.55A_{16} + 1.3A_{17} + 1.34A_{18} + 1.73A_{19} + 0.82A_{20} + 0.9A_{21} \geq 100 \] \[ \text{(52)} \]

Constraint of the upper limit of lysine amino acid percentage in the initial mash:

\[ 0.39A_1 + 0.39A_2 + 0.24A_3 + 0.25A_4 + 0.3A_5 + 0.53A_6 + 0.59A_7 + 0.3A_8 + 1.29A_9 + 2.91A_{10} + 3.17A_{11} + 1.73A_{12} + 1.09A_{13} + 4.83A_{14} + 0.87A_{15} + 1.55A_{16} + 1.3A_{17} + 1.34A_{18} + 1.73A_{19} + 0.82A_{20} + 0.9A_{21} \leq 123 \] \[ \text{(53)} \]

Constraint of the lower limit of methionine amino acid percentage in the initial mash:

\[ 0.37A_1 + 0.37A_2 + 0.3A_3 + 0.3A_4 + 0.3A_5 + 0.42A_6 + 0.47A_7 + 1.91A_8 + 2.79A_9 + 1.33A_{10} + 1.47A_{11} + 2.22A_{12} + 1.86A_{13} + 2.32A_{14} + 0.29A_{15} + 0.47A_{16} + 0.48A_{17} + 0.59A_{18} + 0.4A_{19} + 0.51A_{20} + 0.55A_{21} \geq 70 \] \[ \text{(54)} \]

Constraint of the lower limit of methionine amino acid percentage in the initial mash:

\[ 0.37A_1 + 0.37A_2 + 0.3A_3 + 0.3A_4 + 0.3A_5 + 0.42A_6 + 0.47A_7 + 1.91A_8 + 2.79A_9 + 1.33A_{10} + 1.47A_{11} + 2.22A_{12} + 1.86A_{13} + 2.32A_{14} + 0.29A_{15} + 0.47A_{16} + 0.48A_{17} + 0.59A_{18} + 0.4A_{19} + 0.51A_{20} + 0.55A_{21} \leq 90 \] \[ \text{(55)} \]

Constraint of the lower limit of the thiamin amount in the initial mash:

\[ 4.4A_1 + 5.1A_2 + 4A_3 + 3.94A_4 + 0.6A_5 + 7.9A_6 + 2.6A_7 + 0.2A_8 + 0.2A_9 + 4A_{10} + 6.6A_{11} + 2.86A_{13} + 0.2A_{14} + 3.6A_{15} + 4A_{20} + 0.9A_{21} \geq 180 \] \[ \text{(56)} \]

Constraint of the upper limit of the thiamin amount in the initial mash:

\[ 4.4A_1 + 5.1A_2 + 4A_3 + 3.94A_4 + 0.6A_5 + 7.9A_6 + 2.6A_7 + 0.2A_8 + 0.2A_9 + 4A_{10} + 6.6A_{11} + 2.86A_{13} + 0.2A_{14} + 3.6A_{15} + 4A_{20} + 0.9A_{21} \leq 220 \] \[ \text{(57)} \]

Constraint of the lower limit of the riboflavin amount in the initial mash:

\[ 1.2A_1 + 2A_2 + 1.1A_3 + 1.2A_4 + 0.6A_5 + 3.1A_6 + 0.94A_7 + 1.54A_8 + 2.24A_9 + 2.6A_{10} + 3.3A_{11} + 3.3A_{12} + 3.74A_{13} + 4.6A_{14} + 18.9A_{15} + 44.4A_{19} + 15.4A_{20} + 2.4A_{21} \geq 220 \] \[ \text{(58)} \]

Constraint of the upper limit of the riboflavin amount in the initial mash:
\[ 1.2A1 + 2A2 + 1.1A3 + 1.2A4 + 0.6A5 + 3.1A6 + 0.9A7 + 1.5A8 + 2.2A9 + 2.6A10 + 3.3A11 + 3.3A12 + 3.7A13 + 4.6A14 + 18.9A15 + 44.4A19 + 15.4A20 + 2.4A21 \leq 360 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (59) \]

Constraint of the lower limit of the pantothenic amount in the initial mash:

\[ 12.1A1 + 6.5A2 + 5A3 + 11.4A4 + 3.3A5 + 29A6 + 13A7 + 10.3A8 + 2.9A9 + 15.2A10 + 14.5A11 + 10.1A12 + 6.3A13 + 3.3A14 + 33.9A15 + 82.9A19 + 32.8A20 + 39A21 \geq 950 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (60) \]

Constraint of the upper limit of the pantothenic amount in the initial mash:

\[ 12.1A1 + 6.5A2 + 5A3 + 11.4A4 + 3.3A5 + 29A6 + 13A7 + 10.3A8 + 2.9A9 + 15.2A10 + 14.5A11 + 10.1A12 + 6.3A13 + 3.3A14 + 33.9A15 + 82.9A19 + 32.8A20 + 39A21 \leq 1000 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (61) \]

Constraint of the lower limit of the niacin amount in the initial mash:

\[ 56.6A1 + 57.4A2 + 22.9A3 + 42.7A4 + 14.1A5 + 209A6 + 20A7 + 49.9A8 + 5.5A9 + 30.4A10 + 26.8A11 + 220A12 + 30.8A13 + 47.1A14 + 10.6A15 + 500A19 + 54.6A20 + 46A21 \geq 3400 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (62) \]

Constraint of the upper limit of the niacin amount in the initial mash:

\[ 56.6A1 + 57.4A2 + 22.9A3 + 42.7A4 + 14.1A5 + 209A6 + 20A7 + 49.9A8 + 5.5A9 + 30.4A10 + 26.8A11 + 220A12 + 30.8A13 + 47.1A14 + 10.6A15 + 500A19 + 54.6A20 + 46A21 \leq 3600 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (63) \]

Constraint of the upper limit for the ratio of (energy/protein) in the initial mash:

\[ 250A1 + 305.56A2 + 386.47A3 + 281.58A4 + 385A5 + 88.65A6 + 106.45A7 + 79.4A8 + 62.66A9 + 51.11A10 + 50.41A11 + 41.79A12 + 54.29A13 + 45.8A14 + 115.53A15 + 96.55A16 + 97.08A17 + 132.5A18 + 112.64A19 + 83.98A20 + 74.88A21 \leq 220 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (64) \]

Constraint of vegetable protein sources in the initial mash:

\[ A8 + A9 + A10 + A12 + A13 + A16 + A17 + A18 + A19 + A20 + A21 \leq 35 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (65) \]

3.5. The mathematical model for the grower mash

The objective function is to minimize the summation of the cost of 100 kgm of the grower mash

\[ \min \quad z = 510B1 + 430B2 + 490B3 + 410B4 + 290B5 + 400B6 + 400B7 + 958B8 + 1450B9 + 838B10 + 910B11 + 460B12 + 310B13 + 1750B14 + 850B15 + 510B16 + 1210B17 + 710B18 + 710B19 + 610B20 + 610B21 + 110B22 + 110B23 + 1810B24 + 1930B25 + 1750B26 + 110B27 \quad \ldots \quad \ldots \quad (66) \]

The objective function subject to the following constraints:

- Constraint of the wheat in the grower mash: \( B1 \leq 70 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (67) \)
- Constraint of the barely in the grower mash: \( B2 \leq 20 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (68) \)
• Constraint of the maize in the grower mash: B3<=70
• Constraint of the white corn in the grower mash: B4<=10
• Constraint of the rice in the grower mash: B5<=30
• Constraint of the coarse wheat bran in the grower mash: B6<=5
• Constraint of the fine wheat bran in the grower mash: B7<=5
• Constraint of the maize gluten 42% protein in the grower mash: B8<=50
• Constraint of the maize gluten 60% protein in the grower mash: B9<=5
• Constraint of the soybean meal 44% protein in the grower mash: B10<=300
• Constraint of the soybean meal 49% protein in the grower mash: B11<=20
• Constraint of the sunflower meal in the grower mash: B12<=40
• Constraint of the sesame meal in the grower mash: B13<=30
• Constraint of the fish meal in the grower mash: B14<=5
• Constraint of the bone meal in the grower mash: B15<=2.5
• Constraint of the bean in the grower mash: B16<=10
• Constraint of the white beans in the grower mash: B17<=10
• Constraint of the lentils in the grower mash: B19<=10
• Constraint of the dehydrated alfalfa 17% protein in the grower mash: B20<=7
• Constraint of the dehydrated alfalfa leaves 20% protein in the grower mash: B21<=7
• Constraint of the limestone in the grower mash: B22<=1
• Constraints of the vegetable oils in the grower mash: B23<=4
• Constraints of the crude vegetable oils in the grower mash: B24<=4
• Constraints of the oils in the grower mash: B23+B24<=4
• Constraint of the upper limit of the concentrated vegetable protein in the grower mash: B25+B26<=10

Constraint of all ingredients in grower mash:

- B1+ B2+ B3+ B4+ B5+ B6+ B7+ B8+ B9+ B10+ B11+ B12+ B13+ B15+ B16+ B17+ B18+ B19+ B20+ B21+ B22+ B23+ B24+ B25+ B26+ B27= 100
- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500
- B1+ B2+ B3+ B4+ B5+ B6+ B7+ B8+ B9+ B10+ B11+ B12+ B13+ B15+ B16+ B17+ B18+ B19+ B20+ B21+ B22+ B23+ B24+ B25+ B26+ B27= 100
- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500
- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500
- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500
- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500
- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500
- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500
- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500
- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500

Constraint of the grain and another energy sources in the grower mash:

- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500
- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500
- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500
- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500
- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500
- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500
- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500
- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500
- B1+ B2+ B3+ B4+ B5+ B6+ B7<= 500

Constraint of the upper limit of the concentrated vegetable protein in the grower mash:

- B25+B26<=10
Constraint of protein percentage in the grower mash:

\[ 13B1 + 9B2 + 8.72B3 + 11.4B4 + 8.18B5 + 14.1B6 + 15.5B7 + 42B8 + 60B9 + 45B10 + 50B11 + 46.9B12 + 44.5B13 + 64.7B14 + 12.1B15 + 26.1B16 + 24B17 + 20.8B18 + 23.5B19 + 17.3B20 + 21.1B21 = 2200 \] (102)

Constraint of the metabolizable energy in the grower mash:

\[ 3250B1 + 2750B2 + 3370B3 + 3210B4 + 3150B5 + 1250B6 + 1650B7 + 3355B8 + 3760B9 + 2300B10 + 2500B11 + 1960B12 + 2416B13 + 2963B14 + 1398B15 + 2520B16 + 2330B17 + 2756B18 + 2647B19 + 1453B20 + 1580B21 + 7010B23 + 8900B24 = 300000 \] (103)

Constraint of the upper limit of the lipids percentage in the grower mash:

\[ 1.9B1 + 1.9B2 + 3.9B3 + 2.8B4 + 1.9B5 + 4.1B6 + 2.94B7 + 2.3B8 + 1.7B9 + 4.7B10 + 0.8B11 + 5.1B12 + 2.8B13 + 4.4B14 + 3.2B15 + 1.5B16 + 2.6B20 + 3.6B21 + 90B23 + 90B24 \leq 700 \] (104)

Constraint of the lower limit of the lipid percentage in the grower mash:

\[ 1.9B1 + 1.9B2 + 3.9B3 + 2.8B4 + 1.9B5 + 4.1B6 + 2.94B7 + 2.3B8 + 1.7B9 + 4.7B10 + 0.8B11 + 5.1B12 + 2.8B13 + 4.4B14 + 3.2B15 + 1.5B16 + 2.6B20 + 3.6B21 + 90B23 + 90B24 \geq 390 \] (105)

Constraint of the upper limit of the humidity percentage in the grower mash:

\[ 11B1 + 11B2 + 12B3 + 11B4 + 11B5 + 11B6 + 11B7 + 9B8 + 9B9 + 10.4B10 + 10B11 + 7B12 + 10B13 + 8B14 + 5B15 + 13.4B16 + 7B20 + 10.3B21 \leq 1000 \] (106)

Constraint of the lower limit of the humidity percentage in the grower mash:

\[ 11B1 + 11B2 + 12B3 + 11B4 + 11B5 + 11B6 + 11B7 + 9B8 + 9B9 + 10.4B10 + 10B11 + 7B12 + 10B13 + 8B14 + 5B15 + 13.4B16 + 7B20 + 10.3B21 \geq 500 \] (107)

Constraint of upper limit of the crude fiber percentage in the grower mash:

\[ 2.4B1 + 5B2 + 2B3 + 2B4 + 9B5 + 10B6 + 3B7 + 4B8 + 1.3B9 + 6B10 + 3B11 + 5B12 + 14B13 + B14 + 2B15 + 5.7B16 + 24.8B20 + 20.1B21 \leq 700 \] (108)

Constraint of lower limit of the crude fiber percentage in the grower mash:

\[ 2.4B1 + 5B2 + 2B3 + 2B4 + 9B5 + 10B6 + 3B7 + 4B8 + 1.3B9 + 6B10 + 3B11 + 5B12 + 14B13 + B14 + 2B15 + 5.7B16 + 24.8B20 + 20.1B21 \geq 300 \] (109)
Constraint of the lower limit of the ash in the grower mash:

\[ 1.6B1 + 2.4B2 + 1.8B3 + 1.7B4 + 4.5B5 + 6.1B6 + 2.1B7 + 2.4B8 + 2B9 + 5.7B10 + 5.6B11 + 9.3B12 + 7.1B13 + 21.7B14 + 71.8B15 + 3.6B16 + 9B20 + 10.3B21 \geq 200 \] (110)

Constraint of the upper limit of the ash in the grower mash:

\[ 1.6B1 + 2.4B2 + 1.8B3 + 1.7B4 + 4.5B5 + 6.1B6 + 2.1B7 + 2.4B8 + 2B9 + 5.7B10 + 5.6B11 + 9.3B12 + 7.1B13 + 21.7B14 + 71.8B15 + 3.6B16 + 9B20 + 10.3B21 \leq 500 \] (111)

Constraint of the lower limit of the phosphor percentage in the grower mash:

\[ 0.12B1 + 0.16B2 + 0.11B3 + 0.13B4 + 0.04B5 + 0.34B6 + 0.37B7 + 0.15B8 + 0.19B9 + 0.29B10 + 0.19B11 + 0.16B12 + 0.42B13 + 2.95B14 + 14B15 + 0.12B16 + 0.15B17 + 0.18B18 + 0.11B19 + 0.24B20 + 0.28B21 \geq 35 \] (112)

Constraint of the upper limit of the phosphor percentage in the grower mash:

\[ 0.12B1 + 0.16B2 + 0.11B3 + 0.13B4 + 0.04B5 + 0.34B6 + 0.37B7 + 0.15B8 + 0.19B9 + 0.29B10 + 0.19B11 + 0.16B12 + 0.42B13 + 2.95B14 + 14B15 + 0.12B16 + 0.15B17 + 0.18B18 + 0.11B19 + 0.24B20 + 0.28B21 \leq 40 \] (113)

Constraint of the lower limit of the calcium percentage in the grower mash:

\[ 0.05B1 + 0.08B2 + 0.07B3 + 0.04B4 + 0.09B5 + 0.16B6 + 0.15B7 + 0.45B8 + 0.32B10 + 0.26B11 + 0.38B12 + 2.02B13 + 5.02B14 + 30B15 + 0.17B16 + 0.13B17 + 0.2B18 + 0.52B19 + 1.3B20 + 1.5B21 + 38B22 \geq 87 \] (114)

Constraint of the upper limit of the calcium percentage in the grower mash:

\[ 0.05B1 + 0.08B2 + 0.07B3 + 0.04B4 + 0.09B5 + 0.16B6 + 0.15B7 + 0.45B8 + 0.32B10 + 0.26B11 + 0.38B12 + 2.02B13 + 5.02B14 + 30B15 + 0.17B16 + 0.13B17 + 0.2B18 + 0.52B19 + 1.3B20 + 1.5B21 + 38B22 \leq 92 \] (115)

Constraint of the upper limit of the sodium percentage in the grower mash:

\[ 0.07B1 + 0.02B2 + 0.01B3 + 0.01B4 + 0.11B5 + 0.3B6 + 0.07B7 + 0.1B8 + 0.03B9 + 0.24B10 + 0.34B11 + 0.3B13 + 0.4B15 + 0.08B16 + 0.18B20 + 0.19B21 \leq 22 \] (116)

Constraint of the lower limit of the sodium percentage in the grower mash:

\[ 0.07B1 + 0.02B2 + 0.01B3 + 0.01B4 + 0.11B5 + 0.3B6 + 0.07B7 + 0.1B8 + 0.03B9 + 0.24B10 + 0.34B11 + 0.3B13 + 0.4B15 + 0.08B16 + 0.18B20 + 0.19B21 + 50B27 \geq 14 \] (117)
Constraint of the lower limit of the lysine amino acid percentage in the grower mash:

\[ 0.39B_1 + 0.39B_2 + 0.24B_3 + 0.25B_4 + 0.3B_5 + 0.53B_6 + 0.59B_7 + 0.73B_8 + 1.29B_9 + 2.91B_{10} + 3.17B_{11} + 1.73B_{12} + 1.09B_{13} + 4.83B_{14} + 0.87B_{15} + 1.55B_{16} + 1.3B_{17} + 1.34B_{18} + 1.73B_{19} + 0.82B_{20} + 0.9B_{21} \geq 76 \] \[ (118) \]

Constraint of the upper limit of the lysine amino acid percentage in the grower mash:

\[ 0.39B_1 + 0.39B_2 + 0.24B_3 + 0.25B_4 + 0.3B_5 + 0.53B_6 + 0.59B_7 + 0.73B_8 + 1.29B_9 + 2.91B_{10} + 3.17B_{11} + 1.73B_{12} + 1.09B_{13} + 4.83B_{14} + 0.87B_{15} + 1.55B_{16} + 1.3B_{17} + 1.34B_{18} + 1.73B_{19} + 0.82B_{20} + 0.9B_{21} \leq 114 \] \[ (119) \]

Constraint of the upper limit of the methionine amino acid percentage in the grower mash:

\[ 0.37B_1 + 0.37B_2 + 0.4B_3 + 0.35B_4 + 0.3B_5 + 0.42B_6 + 0.47B_7 + 1.91B_8 + 2.79B_9 + 1.33B_{10} + 1.47B_{11} + 2.22B_{12} + 1.86B_{13} + 2.32B_{14} + 0.29B_{15} + 0.47B_{16} + 0.48B_{17} + 0.59B_{18} + 0.41B_{19} + 0.51B_{20} + 0.55B_{21} \leq 76 \] \[ (120) \]

Constraint of the lower limit of the methionine amino acid percentage in the grower mash:

\[ 0.37B_1 + 0.37B_2 + 0.4B_3 + 0.35B_4 + 0.3B_5 + 0.42B_6 + 0.47B_7 + 1.91B_8 + 2.79B_9 + 1.33B_{10} + 1.47B_{11} + 2.22B_{12} + 1.86B_{13} + 2.32B_{14} + 0.29B_{15} + 0.47B_{16} + 0.48B_{17} + 0.59B_{18} + 0.41B_{19} + 0.51B_{20} + 0.55B_{21} \geq 32 \] \[ (121) \]

Constraint of the lower limit of the thiamin amount in the grower mash:

\[ 4.4B_1 + 5.1B_2 + 4B_3 + 3.9B_4 + 0.6B_5 + 7.9B_6 + 2.6B_7 + 0.2B_8 + 0.2B_9 + 4B_{10} + 6.6B_{11} + 2.86B_{13} + 0.2B_{14} + 3.6B_{15} + 4B_{20} + 0.9B_{21} \geq 160 \] \[ (122) \]

Constraint of the lower limit of the thiamin amount in the grower mash:

\[ 4.4B_1 + 5.1B_2 + 4B_3 + 3.9B_4 + 0.6B_5 + 7.9B_6 + 2.6B_7 + 0.2B_8 + 0.2B_9 + 4B_{10} + 6.6B_{11} + 2.86B_{13} + 0.2B_{14} + 3.6B_{15} + 4B_{20} + 0.9B_{21} \leq 322 \] \[ (123) \]

Constraint of the lower limit of the riboflavin amount in the grower mash:

\[ 1.2B_1 + 2B_2 + 1.1B_3 + 1.2B_4 + 0.6B_5 + 3.1B_6 + 0.9B_7 + 1.5B_8 + 2.2B_9 + 2.6B_{10} + 3.3B_{11} + 3.3B_{12} + 3.74B_{13} + 4.6B_{14} + 18.9B_{15} + 44.4B_{19} + 15.4B_{20} + 2.4B_{21} \geq 220 \] \[ (124) \]

Constraint of the upper limit of the riboflavin amount in the grower mash:

\[ 1.2B_1 + 2B_2 + 1.1B_3 + 1.2B_4 + 0.6B_5 + 3.1B_6 + 0.9B_7 + 1.5B_8 + 2.2B_9 + 2.6B_{10} + 3.3B_{11} + 3.3B_{12} + 3.74B_{13} + 4.6B_{14} + 18.9B_{15} + 44.4B_{19} + 15.4B_{20} + 2.4B_{21} \leq 350 \] \[ (125) \]
Constraint of the lower limit of the pantothinc amount in the grower mash:

\[ 12.1B_1 + 6.5B_2 + 5B_3 + 11.4B_4 + 3.3B_5 + 29B_6 + 13B_7 + 10.3B_8 + 2.9B_9 + 15.2B_{10} + 14.5B_{11} + 10.1B_{12} + 6.38B_{13} + 3.3B_{14} + 33.9B_{15} + 82.9B_{19} + 32.8B_{20} + 39B_{21} \geq 1000 \] (126)

Constraint of the upper limit of the pantothinc amount in the grower mash:

\[ 12.1B_1 + 6.5B_2 + 5B_3 + 11.4B_4 + 3.3B_5 + 29B_6 + 13B_7 + 10.3B_8 + 2.9B_9 + 15.2B_{10} + 14.5B_{11} + 10.1B_{12} + 6.38B_{13} + 3.3B_{14} + 33.9B_{15} + 82.9B_{19} + 32.8B_{20} + 39B_{21} \leq 1100 \] (127)

Constraint of the lower limit of the niacin amount in the grower mash:

\[ 56.6B_1 + 57.4B_2 + 22.9B_3 + 42.7B_4 + 14.1B_5 + 209B_6 + 20B_7 + 49.9B_8 + 5.5B_9 + 30.4B_{10} + 26.8B_{11} + 220B_{12} + 30.8B_{13} + 47.1B_{14} + 10.6B_{15} + 500B_{19} + 54.6B_{20} + 46B_{21} \geq 350 \] (128)

Constraint of the upper limit of the niacin amount in the grower mash:

\[ 56.6B_1 + 57.4B_2 + 22.9B_3 + 42.7B_4 + 14.1B_5 + 209B_6 + 20B_7 + 49.9B_8 + 5.5B_9 + 30.4B_{10} + 26.8B_{11} + 220B_{12} + 30.8B_{13} + 47.1B_{14} + 10.6B_{15} + 500B_{19} + 54.6B_{20} + 46B_{21} \leq 3750 \] (129)

Constraint of vegetable protein sources in the grower mash:

\[ B_8 + B_9 + B_{10} + B_{12} + B_{13} + B_{16} + B_{17} + B_{18} + B_{19} + B_{20} + B_{21} \leq 35 \] (130)

3.6. The mathematical model for the finisher mash

The objective function is to minimize the summation of the cost of 100 kg of the finisher mash

\[ \text{Min } z = 510C_1 + 430C_2 + 490C_3 + 410C_4 + 290C_5 + 400C_6 + 4000C_7 + 958C_8 + 1450C_9 + 838C_{10} + 910C_{11} + 460C_{12} + 3100C_{13} + 1750C_{14} + 850C_{15} + 510C_{16} + 1210C_{17} + 710C_{18} + 710C_{19} + 610C_{20} + 610C_{21} + 110C_{22} + 110C_{23} + 1810C_{24} + 1930C_{25} + 1750C_{26} + 110C_{27} \leq 100 \] (131)

The objective function subject to the following constraints

- \( \text{constraint of the wheat in the finisher mash: } C_1 \leq 70 \)
- \( \text{constraint of the barely in the finisher mash: } C_2 \leq 20 \)
- \( \text{constraint of the maiz in the finisher mash: } C_3 \leq 70 \)
- \( \text{constraint of the white corn in the finisher mash: } C_4 \leq 10 \)
- \( \text{constraint of the rice in the finisher mash: } C_5 \leq 30 \)
- \( \text{constraint of the coarse wheat bran in the finisher mash: } C_6 \leq 5 \)
- \( \text{constraint of the fine wheat bran in the finisher mash: } C_7 \leq 5 \)
- \( \text{constraint of the maiz gluten 42% protein in the finisher mash: } C_8 \leq 50 \)
- \( \text{constraint of the maiz gluten 60% protein in the finisher mash: } C_9 \leq 5 \)
- \( \text{constraint of the soybean meal 44% protein in the finisher mash: } C_{10} \leq 30 \)
- \( \text{constraint of the soybean meal 49% protein in the finisher mash: } C_{11} \leq 20 \)
- \( \text{constraint of the sunflower meal in the finisher mash: } C_{12} \leq 40 \)
- \( \text{constraint of the sesame meal in the finisher mash: } C_{13} \leq 30 \)
• Constraint of the fish meal in the finisher mash: \( C_{14} \leq 5 \) … … … … … … … … … … (145)
• Constraint of the bone meal in the finisher mash: \( C_{15} \leq 2.5 \) … … … … … … … … … … (146)
• Constraint of the bean in the finisher mash: \( C_{16} < 10 \) … … … … … … … … … … … … (147)
• Constraint of the white beans in the finisher mash: \( C_{17} \leq 10 \) … … … … … … … … … … (148)
• Constraint of the chickpeas in the finisher mash: \( C_{18} \leq 10 \) … … … … … … … … … … … … (149)
• Constraint of the lentis in the finisher mash: \( C_{19} \leq 10 \) … … … … … … … … … … … … (150)
• Constraint of dehydrated alfalfa 17% protein in the finisher mash: \( C_{20} \leq 7 \) … … … … (151)
• Constraint of the dehydrated alfalfa leaves 20% protein in the grower mash: \( C_{21} \leq 7 \) … (152)
• Constraint of the limestone in the finisher mash: \( C_{22} \leq 1 \) … … … … … … … … … … (153)
• Constraints of the vegetable oils in the finisher mash: \( C_{23} \leq 4 \) … … … … … … … … … (154)
• Constraints of the crude vegetable oils in the finisher mash: \( C_{24} \leq 4 \) … … … … … … … (155)
• Constraint of the vegetable protein concentrates & vitamins in the finisher mash: \( C_{25} < 10 \) … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … (156)
• Constraint of the animal protein concentrates & vitamins in the finisher mash: \( C_{26} \leq 10 \) … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … (157)
• Constraint of the sodium chloride in the finisher mash: \( C_{27} \leq 0.25 \) … … … … … … … … (158)

Constraint of all ingredients in finisher mash:

\[ C_{1} + C_{2} + C_{3} + C_{4} + C_{5} + C_{6} + C_{7} + C_{8} + C_{9} + C_{10} + C_{11} + C_{12} + C_{13} + C_{14} + C_{15} + C_{16} + C_{17} + C_{18} + C_{19} + C_{20} + C_{21} + C_{22} + C_{23} + C_{24} + C_{25} + C_{26} + C_{27} = 100 \] … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … (159)

Constraint of the lower limit of the legumes in the finisher mash:

\[ C_{16} + C_{17} + C_{18} = 3 \] … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … (160)

Constraint of the lower limit of the legumes in the finisher mash:

\[ C_{16} + C_{17} + C_{18} \leq 10 \] … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … (159)

Constraint of the meals in the finisher mash: \( C_{8} + C_{9} + C_{10} + C_{11} + C_{12} + C_{13} \leq 40 \) … … … … (161)

Constraint of the grain and another energy sources in the finisher mash:

\[ C_{1} + C_{2} + C_{3} + C_{4} + C_{5} + C_{6} + C_{7} \leq 500 \] … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … (162)

Constraint of the oils in the finisher mash: \( C_{23} + C_{24} \leq 4 \) … … … … … … … … … … … … … … … (163)

Constraint of the upper limit of the concentrated vegetable protein in the finisher mash:

\[ C_{25} + C_{26} \leq 10 \] … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … (164)

Constraint of protein percentage in the finisher mash

\[ 13C_{1} + 9C_{2} + 8.72C_{3} + 11.4C_{4} + 8.18C_{5} + 14.1C_{6} + 15.5C_{7} + 42C_{8} + 60C_{9} + 45C_{10} + 50C_{11} + 46.9C_{12} + 44.5C_{13} + 64.7C_{14} + 12.1C_{15} + 26.1C_{16} + 24C_{17} + 20.8C_{18} + 23.5C_{19} + 17.3C_{20} + 21.1C_{21} = 2000 \] … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … … (165)
Constraint of the metabolizable energy in the finisher mash:
\[3250C1 + 2750C2 + 3370C3 + 3210C4 + 3150C5 + 1250C6 + 1650C7 + 3335C8 + 3760C9 + 2300C10 + 2500C11 + 1960C12 + 2416C13 + 2963C14 + 1398C15 + 2520C16 + 2330C17 + 2756C18 + 2647C19 + 1453C20 + 1580C21 + 7010C23 + 8900C24 = 320000\ldots\ldots\ldots\ldots\ldots(166)\]

Constraint of the upper limit of the lipid percentage in the finisher mash:
\[1.9C1 + 1.9C2 + 3.9C3 + 2.8C4 + 1.9C5 + 4.1C6 + 2.94C7 + 2.3C8 + 1.7C9 + 4.7C10 + 0.8C11 + 5.1C12 + 2.8C13 + 4.4C14 + 3.2C15 + 1.5C16 + 2.6C20 + 3.6C21 + 90C23 + 90C24 = 700\ldots\ldots\ldots(167)\]

Constraint of the lower limit of the lipid percentage in the finisher mash:
\[1.9C1 + 1.9C2 + 3.9C3 + 2.8C4 + 1.9C5 + 4.1C6 + 2.94C7 + 2.3C8 + 1.7C9 + 4.7C10 + 0.8C11 + 5.1C12 + 2.8C13 + 4.4C14 + 3.2C15 + 1.5C16 + 2.6C20 + 3.6C21 + 90C23 + 90C24 = 310\ldots\ldots\ldots(168)\]

Constraint of the upper limit of the humidity percentage in the finisher mash:
\[11C1 + 11C2 + 12C3 + 11C4 + 11C5 + 11C6 + 11C7 + 9C8 + 9C9 + 10.4C10 + 10C11 + 10C13 + 8C14 + 5C15 + 13.4C16 + 7C20 + 10.3C21 = 1000\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(169)\]

Constraint of the lower limit of the humidity percentage in the finisher mash:
\[11C1 + 11C2 + 12C3 + 11C4 + 11C5 + 11C6 + 11C7 + 9C8 + 9C9 + 10.4C10 + 10C11 + 10C13 + 8C14 + 5C15 + 13.4C16 + 7C20 + 10.3C21 = 500\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(170)\]

Constraint of the upper limit of the crude fiber percentage in the finisher mash:
\[2.4C1 + 5C2 + 2C3 + 2C4 + 9C5 + 10C6 + 3C7 + 4C8 + 1.3C9 + 6C10 + 3C11 + 14C13 + C14 + 2C15 + 5.7C16 + 24.8C20 + 20.1C21 = 600\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(171)\]

Constraint of the lower limit of the crude fiber percentage in the finisher mash:
\[2.4C1 + 5C2 + 2C3 + 2C4 + 9C5 + 10C6 + 3C7 + 4C8 + 1.3C9 + 6C10 + 3C11 + 14C13 + C14 + 2C15 + 5.7C16 + 24.8C20 + 20.1C21 = 300\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(172)\]

Constraint of the lower limit of the ash in the finisher mash:
\[1.6C1 + 2.4C2 + 1.8C3 + 1.7C4 + 4.5C5 + 6.1C6 + 2.1C7 + 2.4C8 + 2C9 + 5.7C10 + 5.6C11 + 9.3C12 + 7.1C13 + 21.7C14 + 71.8C15 + 3.6C16 + 9C20 + 10.3C21 = 100\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(173)\]

Constraint of the upper limit of the ash in the finisher mash:
\[1.6C1 + 2.4C2 + 1.8C3 + 1.7C4 + 4.5C5 + 6.1C6 + 2.1C7 + 2.4C8 + 2C9 + 5.7C10 + 5.6C11 + 9.3C12 + 7.1C13 + 21.7C14 + 71.8C15 + 3.6C16 + 9C20 + 10.3C21 = 500\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(174)\]
Constraint of the upper limit of the phosphor percentage in the finisher mash

\[ 0.12C_1 + 0.16C_2 + 0.11C_3 + 0.13C_4 + 0.04C_5 + 0.34C_6 + 0.37C_7 + 0.15C_8 + 0.19C_9 + 0.29C_{10} + 0.19C_{11} + 0.16C_{12} + 0.42C_{13} + 2.95C_{14} + 14C_{15} + 0.12C_{16} + 0.15C_{17} + 0.18C_{18} + 0.11C_{19} + 0.24C_{20} + 0.27C_{21} \leq 35 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (175) \]

Constraint of the lower limit of the phosphor percentage in the finisher mash

\[ 0.12C_1 + 0.16C_2 + 0.11C_3 + 0.13C_4 + 0.04C_5 + 0.34C_6 + 0.37C_7 + 0.15C_8 + 0.19C_9 + 0.29C_{10} + 0.19C_{11} + 0.16C_{12} + 0.42C_{13} + 2.95C_{14} + 14C_{15} + 0.12C_{16} + 0.15C_{17} + 0.18C_{18} + 0.11C_{19} + 0.24C_{20} + 0.27C_{21} \geq 19 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (176) \]

Constraint of the lower limit of the calcium percentage in the finisher mash

\[ 0.05C_1 + 0.08C_2 + 0.07C_3 + 0.04C_4 + 0.09C_5 + 0.16C_6 + 0.15C_7 + 0.45C_8 + 0.32C_{10} + 0.26C_{11} + 0.38C_{12} + 2.02C_{13} = 5.02C_{14} + 30C_{15} + 0.17C_{16} + 0.13C_{17} + 0.2C_{18} + 0.52C_{19} + 1.3C_{20} + 1.5C_{21} + 38C_{22} \geq 33 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (177) \]

Constraint of the upper limit of the calcium percentage in the finisher mash

\[ 0.05C_1 + 0.08C_2 + 0.07C_3 + 0.04C_4 + 0.09C_5 + 0.16C_6 + 0.15C_7 + 0.45C_8 + 0.32C_{10} + 0.26C_{11} + 0.38C_{12} + 2.02C_{13} = 5.02C_{14} + 30C_{15} + 0.17C_{16} + 0.13C_{17} + 0.2C_{18} + 0.52C_{19} + 1.3C_{20} + 1.5C_{21} + 38C_{22} \leq 90 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (178) \]

Constraint of the upper limit of the sodium percentage in the finisher mash

\[ 0.07C_1 + 0.02C_2 + 0.01C_3 + 0.01C_4 + 0.11C_5 + 0.3C_6 + 0.3C_7 + 0.07C_8 + 0.1C_9 + 0.03C_9 + 0.24C_{10} + 0.34C_{11} + 0.3C_{13} + 0.46C_{15} + 0.08C_{20} + 0.19C_{21} \leq 50C_{27} = 19 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (179) \]

Constraint of the lower limit of the sodium percentage in the finisher mash

\[ 0.07C_1 + 0.02C_2 + 0.01C_3 + 0.01C_4 + 0.11C_5 + 0.3C_6 + 0.3C_7 + 0.07C_8 + 0.1C_9 + 0.03C_9 + 0.24C_{10} + 0.34C_{11} + 0.3C_{13} + 0.46C_{15} + 0.08C_{20} + 0.19C_{21} + 50C_{27} \geq 12 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (180) \]

Constraint of the lower limit of the lysine amino acid percentage in the finisher mash

\[ 0.39C_1 + 0.39C_2 + 0.24C_3 + 0.25C_4 + 0.3C_5 + 0.53C_6 + 0.59C_7 + 0.73C_8 + 1.29C_9 + 2.91C_{10} + 3.17C_{11} + 1.73C_{12} + 1.89C_{13} + 4.83C_{14} + 0.87C_{15} + 1.55C_{16} + 1.3C_{17} + 1.3C_{18} + 1.73C_{19} + 0.82C_{20} + 0.9C_{21} \leq 85 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (181) \]

Constraint of the upper limit of the lysine amino acid percentage in the finisher mash

\[ 0.39C_1 + 0.39C_2 + 0.24C_3 + 0.25C_4 + 0.3C_5 + 0.53C_6 + 0.59C_7 + 0.73C_8 + 1.29C_9 + 2.91C_{10} + 3.17C_{11} + 1.73C_{12} + 1.89C_{13} + 4.83C_{14} + 0.87C_{15} + 1.55C_{16} + 1.3C_{17} + 1.3C_{18} + 1.73C_{19} + 0.82C_{20} + 0.9C_{21} \geq 95 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (182) \]
Constraint of the lower limit of the methionine amino acid percentage in the finisher mash

\[ 0.37C_1 + 0.37C_2 + 0.4C_3 + 0.35C_4 + 0.3C_5 + 0.42C_6 + 0.47C_7 + 1.91C_8 + 2.79C_9 + 1.33C_{10} + 1.47C_{11} + 2.22C_{12} + 1.86C_{13} + 2.32C_{14} + 0.29C_{15} + 0.47C_{16} + 0.48C_{17} + 0.59C_{18} + 0.41C_{19} + 0.51C_{20} + 0.55C_{21} \geq 32 \] \hspace{1em}(183)

Constraint of the lower limit of the methionine amino acid percentage in the finisher mash

\[ 0.37C_1 + 0.37C_2 + 0.4C_3 + 0.35C_4 + 0.3C_5 + 0.42C_6 + 0.47C_7 + 1.91C_8 + 2.79C_9 + 1.33C_{10} + 1.47C_{11} + 2.22C_{12} + 1.86C_{13} + 2.32C_{14} + 0.29C_{15} + 0.47C_{16} + 0.48C_{17} + 0.59C_{18} + 0.41C_{19} + 0.51C_{20} + 0.55C_{21} \leq 75 \] \hspace{1em}(184)

Constraint of the lower limit of the thiamin amount in the finisher mash

\[ 4.4C_1 + 5.1C_2 + 4C_3 + 3.9C_4 + 0.6C_5 + 7.9C_6 + 2.6C_7 + 0.2C_8 + 0.2C_9 + 4C_{10} + 6.6C_{11} + 2.86C_{13} + 0.2C_{14} + 3.6C_{15} + 4C_{20} + 0.9C_{21} \geq 160 \] \hspace{1em}(185)

Constraint of the lower limit of the thiamin amount in the finisher mash

\[ 4.4C_1 + 5.1C_2 + 4C_3 + 3.9C_4 + 0.6C_5 + 7.9C_6 + 2.6C_7 + 0.2C_8 + 0.2C_9 + 4C_{10} + 6.6C_{11} + 2.86C_{13} + 0.2C_{14} + 3.6C_{15} + 4C_{20} + 0.9C_{21} \leq 270 \] \hspace{1em}(186)

Constraint of the upper limit of the riboflavin amount in the finisher mash

\[ 1.2C_1 + 2C_2 + 1.1C_3 + 1.2C_4 + 0.6C_5 + 3.1C_6 + 0.9C_7 + 1.5C_8 + 2.2C_9 + 2.6C_{10} + 3.3C_{11} + 3.3C_{12} + 3.74C_{13} + 4.6C_{14} + 18.9C_{15} + 44.4C_{19} + 15.4C_{20} + 2.4C_{21} \geq 360 \] \hspace{1em}(187)

Constraint of the upper limit of the riboflavin amount in the finisher mash

\[ 1.2C_1 + 2C_2 + 1.1C_3 + 1.2C_4 + 0.6C_5 + 3.1C_6 + 0.9C_7 + 1.5C_8 + 2.2C_9 + 2.6C_{10} + 3.3C_{11} + 3.3C_{12} + 3.74C_{13} + 4.6C_{14} + 18.9C_{15} + 44.4C_{19} + 15.4C_{20} + 2.4C_{21} \leq 370 \] \hspace{1em}(188)

Constraint of the lower limit of the pantothenic amount in the finisher mash

\[ 12.1C_1 + 6.5C_2 + 5C_3 + 11.4C_4 + 3.3C_5 + 29C_6 + 13C_7 + 10.3C_8 + 2.9C_9 + 15.2C_{10} + 14.5C_{11} + 10.1C_{12} + 6.38C_{13} + 3.3C_{14} + 33.9C_{15} + 82.9C_{19} + 32.8C_{20} + 39C_{21} \geq 860 \] \hspace{1em}(189)

Constraint of the upper limit of the pantothenic amount in the finisher mash

\[ 12.1C_1 + 6.5C_2 + 5C_3 + 11.4C_4 + 3.3C_5 + 29C_6 + 13C_7 + 10.3C_8 + 2.9C_9 + 15.2C_{10} + 14.5C_{11} + 10.1C_{12} + 6.38C_{13} + 3.3C_{14} + 33.9C_{15} + 82.9C_{19} + 32.8C_{20} + 39C_{21} \leq 1000 \] \hspace{1em}(190)

Constraint of the lower limit of the niacin amount in the finisher mash
\[ 56.6C1 + 57.4C2 + 22.9C3 + 42.7C4 + 14.1C5 + 209C6 + 20C7 + 49.9C8 + 5.5C9 + 30.4C10 + 26.8C11 + 220C12 + 30.8C13 + 47.1C14 + 10.6C15 + 500C19 + 54.6C20 + 46C21 \geq 2270 \ldots (191) \]

**Constraint of the upper limit of the niacin amount in the finisher mash**

\[ 56.6C1 + 57.4C2 + 22.9C3 + 42.7C4 + 14.1C5 + 209C6 + 20C7 + 49.9C8 + 5.5C9 + 30.4C10 + 26.8C11 + 220C12 + 30.8C13 + 47.1C14 + 10.6C15 + 500C19 + 54.6C20 + 46C21 \leq 2800 \ldots (192) \]

**Constraint of vegetable protein sources in the finisher mash**

\[ C8 + C9 + C10 + C12 + C13 + C16 + C17 + C18 + C19 + C20 + C21 \leq 35 \ldots \ldots (193) \]

4. **The solution of the three mathematical models**

The software winqsb was used to find the optimal solution for the three mathematical models.

**Table 6.** The optimal values of the decision variables for the three mashes

| Decision variable | Optimal value | Decision variable | Optimal value | Decision variable | Optimal value |
|-------------------|---------------|-------------------|---------------|-------------------|---------------|
| 1                 | A1            | 0                 | B1            | 0                 | C1            | 10.3174       |
| 2                 | A2            | 14.4707           | B2            | 0                 | C2            | 0             |
| 3                 | A3            | 0                 | B3            | 11.8519           | C3            | 38.7385       |
| 4                 | A4            | 10                | B4            | 10                | C4            | 0             |
| 5                 | A5            | 19.7253           | B5            | 23.0473           | C5            | 0             |
| 6                 | A6            | 0.1427            | B6            | 0                 | C6            | 0             |
| 7                 | A7            | 5                 | B7            | 5                 | C7            | 0.3822        |
| 8                 | A8            | 5                 | B8            | 5                 | C8            | 4.5824        |
| 9                 | A9            | 1.0967            | B9            | 5                 | C9            | 5             |
| 10                | A10           | 29.5640           | B10           | 17.6989           | C10           | 0.9694        |
| 11                | A11           | 0                 | B11           | 0                 | C11           | 5.8324        |
| 12                | A12           | 3.7959            | B12           | 0                 | C12           | 0             |
| 13                | A13           | 0                 | B13           | 0                 | C13           | 0             |
| 14                | A14           | 0                 | B14           | 0                 | C14           | 0.7385        |
| 15                | A15           | 1.6319            | B15           | 1.4845            | C15           | 0.2043        |
| 16                | A16           | 3.5992            | B16           | 10                | C16           | 10            |
| 17                | A17           | 0                 | B17           | 0                 | C17           | 0             |
| 18                | A18           | 0                 | B18           | 2.1753            | C18           | 10            |
| 19                | A19           | 0.7237            | B19           | 3.5841            | C19           | 0             |
| 20                | A20           | 0                 | B20           | 0                 | C20           | 2.8169        |
| 21                | A21           | 0                 | B21           | 0.0824            | C21           | 5.9933        |
Table 7. The characteristic of the three mashers

|  | Initial mash | Grower mash | Finisher mash |
|---|---|---|---|
| 1 | Protein% | 24 | 22 | 20 |
| 2 | Energy | 2800 | 3000 | 3200 |
| 3 | Lipids% | 6.5 | 6.15 | 6.07 |
| 4 | Humidity% | 9.9 | 9.7 | 9.59 |
| 5 | Crude fibers% | 5.27 | 4.6 | 4 |
| 6 | The ash% | 5 | 4.19 | 3 |
| 7 | The phosphor(mg)/kgm | 0.4 | 0.35 | 0.19 |
| 8 | The calcium(mg)/kgm | 1.05 | 0.916 | 0.38 |
| 9 | The sodium(mg)/kgm | 0.241 | 0.22 | 0.19 |
| 10 | Lysine(mg)/kgm | 1.23 | 1.02 | 0.834 |
| 11 | Methionine(mg)/kgm | 0.8 | 0.724 | 0.69 |
| 12 | Thiamin(mg)/kgm | 2.64 | 1.92 | 2.63 |
| 13 | Riboflavin(mg)/kgm | 2.2 | 2.95 | 1.6 |
| 14 | Pantothenic(mg)/kgm | 10 | 10 | 8.2 |
| 15 | Niacin(mg)/kgm | 40 | 37.5 | 24 |
| 16 | Energy/protein ratio | 116.6 | 136.6 | 160 |
| 17 | Calcium/phosphor ratio | 2.5 | 2.57 | 2 |
| 18 | Cost of 100kgm of diet (dinar) | 57210.2 | 56519.53 | 68159.22 |

Table 8. Sensitivity analysis of the objective function coefficients for the initial mash

|  | Decision variable | Solution value | Reduced cost | Unit cost or profit C(j) | Allowable Min. C(j) | Allowable Max. C(j) |
|---|---|---|---|---|---|---|
| 1 | A1 | 0 | 48.6237 | 510 | 461.3763 | M |
| 2 | A2 | 14.4707 | 0 | 430 | 419.6268 | 432.2101 |
| 3 | A3 | 0 | 55.4077 | 490 | 434.5923 | M |
| 4 | A4 | 0 | 0 | 0 | 410 | -M |
| 5 | A5 | 19.7253 | 0 | 290 | 282.0684 | 300.1610 |
| 6 | A6 | 0.1427 | 0 | 0 | 400 | 395.8073 | 411.8904 |
Table 9. Sensitivity analysis of the objective function coefficients for the grower mash

| Decision variable | Solution value | Reduced cost | Unit cost or profit C(j) | Allowable Min. C(j) | Allowable Max. C(j) |
|-------------------|----------------|--------------|--------------------------|---------------------|---------------------|
| B1                | 0              | 35.0649      | 510                      | 474.9351            | M                   |
| B2                | 0              | 16.9957      | 430                      | 413.0043            | M                   |
| B3                | 11.8519        | 0            | 490                      | 481.7990            | 497.8255            |
| B4                | 10             | 0            | 410                      | -M                  | 562.0387            |
| B5                | 23.0473        | 0            | 290                      | 276.4873            | 300.2552            |
| B6                | 0              | 465.7074     | 400                      | -65.7074            | M                   |
| B7                | 5              | 0            | 400                      | -M                  | 531.8535            |
| B8                | 5              | 0            | 958                      | -M                  | 986.4819            |
| B9                | 5              | 0            | 1450                     | -M                  | 1483.937            |
| B10               | 17.6989        | 0            | 838                      | 821.0748            | 862.1385            |
| B11               | 0              | 146.2615     | 910                      | 763.7385            | M                   |
| B12               | 0              | 32.0064      | 960                      | 927.9937            | M                   |
| B13               | 0              | 2342.755     | 3000                     | 657.2448            | M                   |
| B14               | 0              | 41.6486      | 1750                     | 1708.3510           | M                   |
| B15               | 1.4845         | 0            | 850                      | -84.9726            | 1085.859            |
| Decision variable | Solution value | Reduced cost | Unit cost or profit C(j) | Allowable Min. C(j) | Allowable Max. C(j) |
|-------------------|----------------|--------------|--------------------------|---------------------|---------------------|
| 1 C1               | 10.3174        | 0            | 510                      | 500.6506            | 524.6102            |
| 2 C2               | 0              | 23.9956      | 430                      | -M                  | M                   |
| 3 C3               | 38.7385        | 0            | 490                      | 475.5099            | 503.5728            |
| 4 C4               | 0              | -158.3258    | 410                      | -M                  | M                   |
| 5 C5               | 0              | -18.2852     | 290                      | -M                  | M                   |
| 6 C6               | 0              | 112.6969     | 400                      | -M                  | M                   |
| 7 C7               | 0.3822         | 0            | 400                      | -57.5590            | 412.0724            |
| 8 C8               | 4.5824         | 0            | 958                      | 822.6099            | 1537.564            |
| 9 C9               | 5              | 0            | 1450                     | -M                  | M                   |
| 10 C10             | 0.9694         | 0            | 838                      | 819.5104            | 908.7286            |
| 11 C11             | 5.8324         | 0            | 910                      | 789.6345            | 933.5669            |
| 12 C12             | 0              | -484.8367    | 960                      | -M                  | M                   |
| 13 C13             | 0              | 2374.8970    | 3000                     | -M                  | M                   |
| 14 C14             | 0.7385         | 0            | 1750                     | 1402.0960           | 1884.619            |
| 15 C15             | 0.2043         | 0            | 850                      | 102.3680            | 2717.005            |
| 16 C16             | 10             | 0            | 510                      | -M                  | 545.3395            |
| 17 C17             | 0              | 0            | 1210                     | -M                  | M                   |
| 18 C18             | 10             | 0            | 710                      | -M                  | M                   |
| 19 C19             | 0              | -1857.332    | 710                      | -M                  | M                   |
| 20 C20             | 2.8169         | 0            | 510                      | 305.7042            | 766.9684            |
| 21 C21             | 5.9933         | 0            | 610                      | -111.7691           | 825.5251            |
| 22 C22             | 0.1747         | 0            | 110                      | 80.4122             | 33254.94            |
| 23 C23             | 0              | 0            | 110                      | -M                  | M                   |
| 24 C24             | 4              | 0            | 1800                     | -M                  | M                   |
5. Conclusions

1. The three mashes mentioned their specifications in table 7 correspond to the specifications determined by the nutritionists and they are guaranteed of quality while the ready-made-feed that is purchased from the local markets is not guaranteed of quality.

2. The cost of 100 kgm of the initial mash is extracted from the application of the linear programming method 57210.2 iraqi dinar, while the cost of buying 100 kgm from the local markets is 70000 iraqi dinars, meaning that the price of 100 kgm of the developing diet resulting from the application of the linear programming method is reduced by 12789.78 iraqi dinars from the prices in the local markets.

3. The cost of 100 kgm of the grower mash is extracted from the application of the linear programming method 56519.53 iraqi dinar, while the cost of buying 100 kgm from the local markets is 70000 iraqi dinars, meaning that the price of 100 kgm of the developing diet resulting from the application of the linear programming method is reduced by 13480.47 iraqi dinars from the prices in the local markets.

4. The cost of 100 kgm of the finisher mash is extracted from the application of the linear programming method 68159.22 iraqi dinar, while the cost of buying 100 kgm from the local markets is 70000 iraqi dinars, meaning that the price of 100 kgm of the developing diet resulting from the application of the linear programming method is reduced by 1840.78 iraqi dinars from the prices in the local markets.

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