Nonextensive distributions of rotation periods and diameters of asteroids

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ABSTRACT

Context. To investigate the distribution of rotation periods of asteroids from different regions of the Solar System and distribution of diameters of near-Earth asteroids (NEAs).

Aims. Verify if nonextensive statistics satisfactorily describes the data.

Methods. Light curve data was taken from Planetary Database System (PDS) with Rel ≥ 2. Taxonomic class and region of the Solar System was also considered. Data of NEA were taken from Minor Planet Center.

Results. The rotation periods of asteroids follow a q-Gaussian with q = 2.6 regardless of taxonomy, diameter or region of the Solar System of the object. The distribution of rotation periods is influenced by observational bias. The diameters of NEAs are described by a q-exponential with q = 1.3. According to this distribution, there are expected to be 994 ± 30 NEAs with diameters greater than 1 km.

Key words. asteroids – rotation periods – diameters – nonextensivity

1. Introduction

Asteroids and comets are primordial bodies of the Solar System (SS). The study of the physical properties of these objects may lead to a better understanding of the processes of formation of the SS, and, by inference, of the hundreds of exo-Solar systems already known. Distribution of rotation periods and diameters of asteroids are two parameters that may give pieces of information concerning the evolution of the SS. The first attempt to describe histograms of rotation periods of asteroids was made by Harris & Burns (1979). This work and also others that have followed have shown that rotation periods of big asteroids (D > 30–40 km) follow a Maxwellian distribution. Harris & Pravec (2000) have analyzed a sample with 984 objects and have confirmed that the distribution of rotation periods of asteroids with diameters D ≥ 40 km is Maxwellian, with 99% of confidence, though this hypothesis can be rejected at 95% of confidence. They suggest that objects within this diameter range are primordial, or originated from collisions of primordial bodies. It is known that for median sized (10 < D ≤ 40 km) and small (D < 10 km) asteroids, the distribution of rotation periods is not Maxwellian. The analysis of the data suggests the existence of a spin-barrier for the asteroids with diameters between hundreds of meters and 10 km and with more than 11 rotations per day (d⁻¹) (period of about 2.2 h). The absence of a substantial quantity of asteroids with period less than 2.2 h may be due to the low degree of internal cohesion of these objects. The majority of the sample may contain rubble pile asteroids (Davis et al. 1979; Harris 1996) that are composed by fragments of rocks kept together by self-gravitation. For objects below 0.2 km it is observed rotation periods smaller than the spin-barrier, suggesting that these objects have a high inter-nal cohesion, implying that they may be monolithic bodies.

The difficulty in the modelling of rotation periods of asteroids as a whole may be associated to the combined action of many mechanisms such as collisions (Paolicchi, Burns & Weidenschilling 2002), gravitational interactions with planets (Scheeres, Marzari & Rossi 2004), angular momentum exchange in binary or multiple asteroid systems (Scheeres 2002), or torques induced by solar radiation, known as YORP effect (from Yarkovsky–O’Keefe–Radzievskii–Paddack) (Rubincam 2000). Particularly, YORP effect is strongly dependent on the shape and size of the object and its distance to the Sun.

Near-Earth Asteroids (NEAs) is a subgroup of SS asteroids whose heliocentric orbits lead them close to the Earth’s orbit. More than 7000 NEAs are known up to 2011. The study of these objects is relevant once it may bring information regarding the birth and dynamic evolution of the SS. Also a special interest in these objects is related to the possibility of collision with the Earth with obvious catastrophic consequences (Alvarez et al. 1980). They also may be potential sources of raw material for future space projects.

The evaluation of the number of asteroids per year that may reach the Earth as a function of their diameters is essential for the determination of the potential risk of a collision. One of the first attempts to estimate this flux was done by Shoemaker et al. (1979).

The impact flux may be taken from the accumulated distribution of diameters of the NEAs. This distribution is indirectly obtained by the current asteroid surveys, according to the abso-
lute magnitude $H$. The distribution of absolute magnitude $H$ is described by Jedzick, Larsen & Spahr (2002).

$$\log N = a H + \beta,$$

(1)

where $N$ is the number of objects, $a$ is the “slope parameter” and $\beta$ is a constant. This relation asymptotically models the observed distribution of $H$. Departure from this power law is probably associated with the observational bias due to physical and dynamical properties of the asteroids (orbital elements, size, albedo), and instrumental limitations (CCD, detection software, among others). So Eq. (1) may be used to describe a given population of asteroids if a correction of the bias is made in the raw data.

The diameters of asteroids $D$ may be given in function of their absolute magnitudes and their albedos $p_i$ according to Bowell et al. (1989)

$$D = 1329 \left( \frac{10^{-H/5}}{p_i} \right)^{1/0.6}.$$  

(2)

The albedo is the rate of superficial reflection and its value is essential for the estimation of the diameters of the asteroids. The values of the albedos of asteroids varies according to the superficial mineralogical composition (taxonomic complex) and object’s shape. Typical values range from 0 to 0.6 for low albedo objects of C taxonomic complex up to 0.46 ± 0.06 for high albedo objects of V type (Warner, Harris & Pravec 2009).

Combination of equations (1) and (2) leads to a power law behavior:

$$N(> D) = k D^{-\beta}.$$  

(3)

The parameters are estimated by Stuart (2003), $b = 1.95$ ($a = b/5$, admitting the same albedo for all the sample) $k = 1090$, and $D$ is given in km. According to this expression and taking into account the uncertainties of the measures, Stuart & Binzel (2004) have estimated that there may exist 1090 ± 180 objects with diameters equal to or greater than 1 km ($H = 17.8$).

2. Nonextensive statistics

In order to model the accumulated distribution of periods and diameters of asteroids, we have applied results from Tsallis nonextensive statistics. This choice comes from observational evidences that astrophysical systems are somehow related to nonextensive behavior. It is known that system with long-range interactions (typically gravitational systems) are not properly described by Boltzmann-Gibbs statistical mechanics (Landsberg 1990). Along the last two decades it has been continuously developed the nonextensive statistical mechanics that is a generalization of Boltzmann-Gibbs statistical mechanics. Tsallis proposed in 1988 (Tsallis 1988) a generalization of the entropy, $S_q = k \sum_{i=1}^{W} p_i^q / q - 1$.

$$S_q = k \left[ 1 - \sum_{i=1}^{W} p_i^q \right] / q - 1,$$

(4)

where $p_i$ is the probability of the i-th microscopical state, $W$ is the number of states, $k$ is a constant (Boltzmann’s constant) and $q$ is the entropic index. As $q \to 1$ $S_q$ is reduced to Boltzmann-Gibbs entropy $S_1 = -k \sum_{i=1}^{W} p_i \ln p_i$. It was soon realized that the nonextensive statistical mechanics could be successfully applied to self-gravitating systems: Plastino & Plastino (1993) found a possible solution to the problem of the existence of a self-gravitating system with total mass, total energy and total entropy simultaneously finite, within a nonextensive framework.

Many examples of nonextensivity in astrophysical systems may be found. We list some instances. Nonextensivity was observed in the analysis of magnetic field at distant heliosphere associated to the solar wind observed by Voyager 1 and 2 (Burlaga & Viñas 2005 Burlaga & Ness 2009 Burlaga & Ness 2010). The distribution of stellar rotational velocities in the Pleiades open cluster was found to be satisfactorily modelled by a $q$-Maxwellian distribution (Soares et al. 2006). The problem of Jeans gravitational instability was considered according to nonextensive kinetic theory (Lima, Silva & Santos 2002). Nonextensive statistical mechanics was also used to describe galaxy clustering processes (Wuenschke et al. 2004 and temperature fluctuation of the cosmic background radiation (Bernui, Tsallis & Villela 2006 Bernui, Tsallis & Villela 2009). Fluxes of cosmic rays can be accurately described by distributions that emerge from nonextensive statistical mechanics (Tsallis, Anjos & Borges 2003). A list of more instances of applications of nonextensive statistical mechanics in astrophysical systems may be found in Tsallis (2009).

It is important to mention that the index $q$ has a physical interpretation — it expresses the degree of nonextensivity — and for some systems it can be determined $a$ priori (based on dynamical properties). In fact a nonextensive system is characterized by a $q$-triplet and not just by a single $q$ (additional information can be found in Tsallis (2009). Such triplet was already obtained in an astrophysical system (Burlaga & Viñas 2005 Burlaga & Ness 2009 Burlaga & Ness 2010).

Maximization of $S_q$ under proper constraints leads to distributions that are generalizations of those that appear within Boltzmann-Gibbs context. For instance, if it is required that the (generalized) energy of the system is constant (Curado & Tsallis 1991) then the probability distribution that emerges is a $q$-exponential,

$$p(x) \propto \exp_q(-\beta_q x),$$

(5)

$\beta_q$ is the Lagrange multiplier (not to confound with $\beta$ of Eq. (1)). The $q$-exponential function is defined as (Tsallis 1994)

$$\exp_q x = [1 + (1 - q)x]_+^{1/(1-q)}.$$  

(6)

The symbol $[a]_+$ means that $[a]_+ = a$ if $a > 0$ and $[a]_+ = 0$ if $a \leq 0$. The $q$-exponential is a generalization of the exponential function, that is recovered if $q \to 1$. If the constraint imposes that the (generalized) variance of the distribution is constant, then the distribution that maximizes $S_q$ is a $q$-Gaussian (Tsallis et al 1995 Prato & Tsallis 1999),

$$p(x) \propto \exp_q(-\beta_q x^2).$$  

(7)

The $q$-Gaussian recovers the usual Gaussian at $q = 1$, and particular values of the entropic index $q$ turn $p(x)$ into various known distributions, as Lorentz distribution, uniform distribution, Dirac’s delta (see Tsallis et al. 1995 Prato & Tsallis 1999 for details).

The Lagrange parameter $\beta_q$ also has a precise physical meaning. Within the statistical mechanics context, the Lagrange parameter $\beta_q$ in Eq. (5) is related to the inverse of the temperature ($\beta_q = 1/(k T)$ if $q = 1$), and in Eq. (7) it is related to the inverse of the variance ($\beta_q = 1/(2\sigma^2)$ in normal diffusion, $\sigma^2$ is the variance). Generally speaking, the inverse of the Lagrange parameters are associated to the finiteness of the first or second moments of the distribution.

Inverse cumulative distributions of the family of the $q$-exponentials are usually, and conveniently, graphically represented by means of log-log plots, and Fig. 1 shows a general
We are calling this region as “quasi-flat” to distinguish it from the asymptotical power law region. Figure 1 shows the intersection of two straight lines that represent the two regimes. The intersection is the transition point between the regimes (the crossover), and it is given by

\[ x^* = \frac{1}{(q - 1)\beta_q^\gamma}. \]

Figure 1 also displays an ordinary \((q = 1)\) exponential, for comparison. Exponentials of negative arguments decay vary fast, and when represented in log-log plots this feature becomes clear, once the exponential asymptotically presents slope \(-\infty\). Coherently, Eq. (8) gives \(x^* \to \infty\) for \(q = 1\), indicating that there is no crossover of an exponential to a power law regime.

We have found that the distribution of diameters of NEAs follows a \(q\)-exponential and the observed rotation periods of all asteroids, regardless of their diameters, mineralogical composition or region of the SS, are well approximated by a \(q\)-Gaussian. We have used two samples from databases of different years, in order to verify the effect of the observational bias.

3. Observational data

One important problem in the evaluation of distributions of rotation periods and diameters of asteroids (and of course the same applies for other observables) is that the data are possibly influenced by observational bias. In order to take into account this effect, we have considered samples from databases of two different years: 2005 and 2010 for rotation periods, and 2001 and 2010 for diameters of asteroids.

Two versions of the Lightcurve Derived Data, available at the Planetary Database System (PDS), were used: version 7 (V7), with 1971 periods, and version 11 (V11), with 4310 periods (Harris, Warner & Pravec 2005, 2010). The periods are classified according to a quality code of the reliability of the estimated period, defined by Harris & Young (1983). We have used periods with Rel \(\geq\ 2\) (Rel from reliability) that means they are accurate to \(\approx 20\%\) which resulted in 1621 entries for the V7 and 3567 asteroids for the V11. Cross-checking the V11 sample with a compilation of taxonomic classifications, also available at PDS, has revealed that about 40\% (1487) of these asteroids have approximately known mineralogical composition. The asteroids have been separated into three main classes: C, S and X complexes, following the SMASS II system of Bus & Binzel (2002), with respectively 503, 663 and 321 objects. The diameters of these sub-samples have been calculated with Eq. (9) with the absolute magnitude \(H\) available from MPCORB – Minor Planet Center Orbit Database (MPC 2010).

We have also used two versions of the compilation of absolute magnitudes \(H\), namely that of Oct., 2001, with 1649 NEA’s (similar to Stuart’s (2001) procedure) and Oct., 2010, with 7078 objects (MPC 2010). We have adopted \(p_c = 0.14 \pm 0.02\) for the NEAs population albedo. This value was estimated by Stuart & Binzel (2004) and it takes into account the great variety of taxonomic types that are found in the NEAs (Binzel et al. 2004).

In order to estimate the validity of the diameters estimated by Eq. (2), we have considered the diameters of 101 asteroids obtained from Spitzer Space Telescope data (Trilling et al. 2010). This resulted in about 20\% of error. We considered that this value, though not small, is reasonable for the purposes of our study.

4. Distribution of rotation periods

Fig. 2 shows the decreasing cumulative distribution of periods of V7 and V11, and superposed \(q\)-Gaussians \((N_S(p) = M \exp(-\beta_q p^\gamma))\) and it is seen that these functions quite satisfactorily describe almost all the data with \(q = 2.0 \pm 0.1\), \(\beta_q = 0.016 \pm 0.001\) \(h^{-2}\) and \(M = 1621\) (\(M\) is the number of objects) for V7, and \(q = 2.6 \pm 0.2\), \(\beta_q = 0.025 \pm 0.002\) \(h^{-2}\) and \(M = 3567\) for V11. Parameters were found by a nonlinear least square method. Fig. 2 also presents two ordinary Gaussians, and it is evident that these \(q = 1\) distributions are completely unable to represent the data. Confidence level for both fits is 95\%, according to \(\chi^2\) test. This suggests that the distribution does not depend on (i) the diameters; (ii) the mineralogical composition; (iii) the region of the SS in which the object is found. The latter is particularly important once the sample includes NEAs, trans-Netunian objects (TNO), asteroids from the main belt (CMB), Jupiter Trojans (JT) and dwarf planets like Ceres and Pluto. The values of the entropic indexes (\(q = 2.0\) for V7 and \(q = 2.6\) for V11) — rather distant from unit, that is, distant from the Maxwellian distribution — may indicate that long-range interactions play an essential role in the distribution of rotation periods. According to Eq. (8) (with \(x^* \equiv p^\gamma\), \(\gamma = 2\)), the transition point is \(p^* = 7.91 \pm 0.01\) \((f \sim 3\) d\(^{-1}\)) for the data of V7, and \(p^* = 5.00 \pm 0.02\) \((f \sim 5\) d\(^{-1}\)) for the data of V11. The transition points for both samples differ from the critical period of the spin barrier, and thus the transition is not a consequence of physical processes. Warner & Harris (2010) have demonstrated that the periods are more accurately determined for objects with
Fig. 2. Decreasing cumulative distribution of periods of V7 (green dots on line) and V11 (black dots on line) of PDS (NASA) with Rel ≥ 2, and superposed $q$-Gaussians ($N_q(p) = M \exp\left(-\beta_q p^q\right)$). V7: $q = 2.0, \beta_q = 0.0161$ h$^{-2}, M = 1621$; V11: $q = 2.6, \beta_q = 0.025$ h$^{-2}, M = 3567$. Fittings of the periods of V7 for $P > 50$ h are not good. This does not happen with V11, and it possibly indicates the increase in the accuracy of the data. Dashed (violet on line) and dot-dashed (magenta on line) lines are usual ($q = 1$) Gaussians, with $\beta_l = 0.0161$ h$^{-2}, M = 1621$, and $\beta_l = 0.025$ h$^{-2}, M = 3567$.

Fig. 3. Log-log plot of the decreasing cumulative distribution of periods of 3567 asteroids (dots) with Rel ≥ 2 taken from the PDS (NASA) and a $q$-Gaussian distribution ($N_q(p) = M \exp\left(-\beta_q p^q\right)$) (solid line), with $q = 2.6, \beta_q = 0.025$ h$^{-2}, M = 3567$. The other curves are 663 S-complex asteroids (diamonds, blue on-line), 503 C-complex asteroids (squares, green on-line), 321 X-complex asteroids (triangles, magenta on-line). Inset shows the 3567 asteroids and the $q$-Gaussian in a linear-linear plot.

Fig. 4. Decreasing cumulative distribution of diameters of known NEAs in 2001 (1649 objects, green dots) and in 2010 (7078 objects, black dots). Solid lines are best fits of $q$-exponentials ($N_q(D) = M \exp\left(-\beta_q D^q\right)$). Blue line (2001): $q = 1.3, \beta_q = 1.5$ km$^{-1}, M = 1649$, red line (2010): $q = 1.3, \beta_q = 3$ km$^{-1}, M = 7078$. Usual exponentials ($q = 1$) are displayed in the main panel for comparison (dashed violet, with $\beta_l = 1.5$ km$^{-1}, M = 1649$, and dot-dashed magenta, with $\beta_l = 3$ km$^{-1}, M = 7078$).

5. Distribution of diameters of near-Earth asteroids

We have found that the decreasing cumulative distribution of diameters of NEAs can be fitted by $q$-exponentials. The fitting of a $q$-exponential to the diameters of 7078 NEAs ($N_q(D) = M \exp\left(-\beta_q D^q\right)$), shown in Fig. 4B is quite good for the entire range of the data, with a confidence level of 95%: $q = 1.3 \pm 0.1$ and $\beta_q = 3.0 \pm 0.2$ km$^{-1}$ (found by nonlinear least squares method). This distribution, however, is influenced by observational bias. The $q$-exponential distribution can be used to determine the point in which the sample is supposed to be complete. Figure 4B compares $q$-exponentials that fit observed distribution of diameters of known NEAs in October of 2001 and October of 2010. The observed distribution of diameters of the 2011 database follows a $q$-exponential with $q = 1.3 \pm 0.1$ and $\beta_q = 1.5 \pm 0.1$ km$^{-1}$ and the same confidence level. The Figure also shows usual ($q = 1$) exponentials, and it can be promptly verified their inadequacy in the representation of the whole range of the data.

Once the value of $q$ is the same for both 2001 and 2010 samples, we may argue that this parameter is not influenced by the bias in this case, and it reflects real physical processes. Both curves are practically identical in the power-law region, and the point of transition to the quasi-flat region differ, as expressed by the different values of $\beta_q$. The value of $q = 1.3$ (different from one) is an indication that not only collisional processes are implied in the formation of these objects. Other mechanisms may also be present: the YORP effect may lead to the decrease of the period of rotation up to the point of rupture. This fragmentation process may yield the formation of binary or multiple systems. About (15 ± 4)% of NEAs with $D \geq 0.3$ km and rotation periods between 2 and 3 h possibly are binary systems (Pravec, Harris & Warner 2007). The transition points according to Eq. 8 (with $x^* \equiv D^*, \gamma = 1$) are $D^* = 2.22 \pm 0.05$ km (2001), and $D^* = 1.11 \pm 0.05$ km (2010).
The sample is complete up to the upper limit of these intervals, 2.22 + 0.05 = 2.27 km (H = 16) for 2001 basis and 1.11 + 0.05 = 1.16 km (H = 17.5) for 2010 basis. The number of NEA with D ≥ 2.27 km is virtually the same for the sample of 2001 and 2010 (166 ± 8 objects). This is a confirmation of the completeness of the sample up to H ~ 15 (Jedicke, Larsen & Spahr, 2002), and the extension of this limit up to H ~ 16 (Harris, 2008). Once there has been an increase in the efficiency of detection and in the number of surveys (Stokes & Larson, 2002; Larson, 2007), we conclude that the parameter β1 indicates the limit of completeness of the sample. For D ≥ 1.16 km, the 2010 data are best described by a power-law. We have found for Eq. (4), k = 994 ± 30 and b = 2.24 ± 0.01, with correlation coefficient R2 = 0.987. The value of b corresponds to α = 0.448 ± 0.002 in Eq. (1), that is a reasonable value if compared to the slope of 0.44 found by Zavodny et al. (2008). The value of q may be found from b using q = 1 + 1/b, thus q = 1.446 ± 0.001, that is within the interval found in the whole sample. According to the distribution we have found 994 ± 30 asteroids with D ≥ 1 km (H ≤ 17.7), in a close agreement with Mainzer et al. (2011). Analysis of distributions of diameters of MB and TNO according to lines similar to those used in this work are welcome.

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Methods. Data was taken from Planetary Database System (PDS) with Rel ≥ 2 (total of 3567 asteroids). Taxonomic class and region of the Solar System was also considered. Data for NEA was taken from Minor Planet Center (total of 7078 NEAs).
Results. The periods of rotation of asteroids follow a \( q \)-gaussian with \( q = 2.6 \) regardless of taxonomy, diameter or region of the Solar System of the object. The diameters of NEAs are described by a \( q \)-exponential with \( q = 1.3 \). According to this distribution, there are expected to be 1240 ± 180 NEAs with diameters greater than 1 km.

Key words. asteroids – rotation periods – diameters – nonextensivity

1. Introduction

Asteroids and comets are primordial bodies of the Solar System (SS). The study of the physical properties of these objects may lead to a better understanding of the processes of formation of the SS, and, by inference, of the hundreds of exo-Solar systems already known. Distribution of rotation periods and diameters of asteroids are two parameters that may give pieces of information concerning the collisional evolution of the SS. Pravec & Harris (2000) have analyzed a sample with 984 objects and have argued that the distribution of rotation periods of asteroids with diameters \( D \geq 40 \text{ km} \) is maxwellian, with 99% of confidence. They suggest that objects within this diameter range are primordial, or originated from collisions of primordial bodies. It is known that for small and very small asteroids \( (D < 10 \text{ km}) \), the distribution of rotation periods is not maxwellian. The analysis of the data suggests the existence of a spin-barrier for the asteroids with diameters between hundreds of meters and 10 km and with less than 11 rotations per day (period of about 2.2 h) (Pravec, Harris & Warner 2007). The absence of a substantial quantity of asteroids with period less than 2.2 h may be due to the low degree of internal cohesion of these objects. The majority of the sample may contain rubble pile asteroids (Davis et al. 1979, Harris 1990) that are composed by fragments of rocks kept together by self-gravitation. For objects below 0.2 km it is observed rotation periods smaller than the spin-barrier, suggesting that these objects have a high internal cohesion, implying that they may be monolithic bodies. The difficulty in the modelling of periods of rotation of asteroids as a whole may be associated to the combined action of many mechanisms such as collisions (Paolicchi, Burns & Weidenschilling 2002), gravitational interactions with planets (Scheeres, Marzari & Rossi 2004), angular momentum exchange in binary or multiple asteroid systems (Bottke et al. 2002), or torques induced by solar radiation, known as YORP effect (from Yarkovsky–O’Keefe–Radzievskii–Paddack) (Rubincam 2000). Particularly, YORP effect is strongly dependent on the shape and size of the object and its distance to the Sun.

Near-Earth Asteroids (NEAs) is a subgroup of SS asteroids whose heliocentric orbits lead them close to the Earth’s orbit. More than 7000 NEAs are known up to 2011. The study of these objects is relevant once it may bring information regarding the birth and dynamic evolution of the SS. Also a special interest in these objects is related to the possibility of collision with the Earth with obvious catastrophic consequences (Alvarez et al. 1980). They also may be potential sources of raw material for future space projects.

The evaluation of the number of asteroids per year that may reach the Earth as a function of their diameters is essential for the determination of the potential risk of a collision. One of the first attempts to estimate this flux was done by Shoemaker et al. (1979).

The impact flux may be taken from the accumulated distribution of diameters of the NEAs. This distribution is indirectly obtained by the current asteroid surveys, according to the absolute magnitude \( H \). The distribution of absolute magnitude \( H \) is described by Michelson (2004).

\[
\log N = \alpha H + \beta,
\]

where \( N \) is the number of objects and \( \alpha \) and \( \beta \) are constants. The diameters of asteroids \( D \) may be given in function of their...
absolute magnitudes and their albedos $p_i$ according to Bowell et al. (1989).

\[ D = 1329 \frac{10^{-W/5}}{\sqrt{p_i}}. \]  

(2)

The albedo is the rate of superficial reflection and its value is essential for the estimation of the diameters of the asteroids. The values of the albedos of asteroids varies according to the superficial mineralogical composition (taxonomic complex) and object’s shape. Typical values range from 0.06 ± 0.02 for low albedo objects of C taxonomic complex up to 0.46 ± 0.06 for high albedo objects of V type (Warner, Harris & Pravec 2009).

Combination of equations (1) and (2) leads to a power law behavior:

\[ N(> D) = kD^{-b}. \]  

(3)

The parameters are estimated by Stuart (2003), $b = 1.95$, $k = 1090$, and $D$ is given in km. According to this expression and taking into account the uncertainties of the measures, Stuart & Binzel (2004) estimated that there may exist 1090 ± 180 objects with diameters equal to or greater than 1 km.

2. Nonextensive statistics

In order to model the accumulated distribution of periods and diameters of asteroids, we have applied results from Tsallis nonextensive statistics. This choice comes from observational evidences that astrophysical systems are somehow related to nonextensive behavior. It is known that system with long-range interactions (typically gravitational systems) are not properly described by Boltzmann-Gibbs statistical mechanics (Landsberg 1990). Along the last two decades it has been continuously developed the nonextensive statistical mechanics that is a generalization of Boltzmann-Gibbs statistical mechanics. Tsallis proposed in 1988 (Tsallis 1988) a generalization of the entropy, $S_q = k \left[ 1 - \sum_i^W p_i^q \right]$, a particular values of the entropic index.

\[ p(x) \propto \exp_q(-\beta_q x), \]  

(5)

The $q$-exponential is defined as (Tsallis 1994)

\[ \exp_q x = [1 + (1 - q)x]^\frac{1}{q}. \]  

(6)

The $q$-gaussian recovers the usual gaussian at $q = 1$, and particular values of the entropic index $q$ turn $p(x)$ into various known distributions. The symbol $[a]_q$ means that $[a] = a$ if $a > 0$ and $[a] = 0$ if $a \leq 0$. The $q$-exponential is a generalization of the exponential function, that is recovered if $q \to 1$. In the constraint imposes that the (generalized) variance of the distribution is constant, then the distribution that maximizes $S_q$ is the $q$-gaussian (Tsallis et al. 1995, Prato & Tsallis 1999).

\[ p(x) \propto \exp_q(-\beta_q x^2). \]  

(7)

The $q$-gaussian recovers the usual gaussian at $q = 1$, and particular values of the entropic index $q$ turn $p(x)$ into various known distributions, as Lorentz distribution, uniform distribution, Dirac’s delta (Tsallis et al. 1995, Prato & Tsallis 1999 for details).

We have found that the distribution of diameters of NEAs follows a $q$-exponential and the observed rotation periods of all asteroids, regardless of their diameters, mineralogical composition or region of the SS, are well approximated by a $q$-gaussian.

3. Observational data

In order to obtain a distribution of asteroids rotation periods we used the compilation of 4127 periods available at the Planetary Database System (PDS) (Harris, Warner & Pravec 2010). In this sample the periods are classified according to a quality code of the reliability of the estimated period, defined by Harris & Young (1983). We have used periods with Rel $\geq 2$ (Rel from reliability) that means they are accurate to $\approx 20\%$ which resulted in 3567 asteroids taken into account. Cross-checking the samples with a compilation of taxonomic classifications, also available at PDS, has revealed that about 40% of these asteroids have approximately known mineralogical composition. The 1487 asteroids with taxonomic classification have been separated into three main classes: C, S and X complexes following the SMAX II system of Bus & Binzel (2002), with respectively 503, 663 and 321 objects. The diameters of these sub-samples have been calculated with Eq. (2) with the absolute magnitude $H$ available from MPCORB – Minor Planet Center Orbit Database (MPCORB). Absolute magnitudes of 7078 NEAs were obtained from MPCORB. We have adopted $p_i$ = 0.14±0.02 for the NEAs population albedo. This value was found by Stuart & Binzel (2004) and it takes into account the great variety of taxonomic groups that are found in the NEAs (Binzel et al. 2004). In order to estimate the validity of the diameters estimated by Eq. (4), we have considered the diameters of 101 asteroids obtained from Spitzer Space Telescope data (Trilling et al. 2010). This resulted in about 20% of error. We considered that this value, though not small, is reasonable for the purposes of our study.
Fig. 1. Log-log plot of the decreasing cumulative distribution of periods of 3567 asteroids (dots) with $Rel \geq 2$ taken from the PDS (NASA) and a $q$-gaussian distribution (solid line), Eq. (7), with $q = 2.6, \beta = 0.028 \text{ h}^{-2}$ and the normalizing constant $M = 3567$. Particular asteroids are indicated with blue dots (see Tables 1 and 2).

Table 1. Asteroids separated by the region in the Solar System. Data from PDS (Harris, Warner & Pravec 2010).

| Object       | Period (h) | Diameter (km) | Region |
|--------------|------------|----------------|--------|
| 1999 CU 3    | 3.7829     | 0.04           | NEA    |
| Hektor       | 6.294      | 0.04           | JT     |
| Ceres        | 9.074170   | 0.06           | MB (dwarf planet) |
| Sedna        | 10.273     | 0.08           | TNO    |
| Pluto        | 153.2935   |                | TNO (dwarf planet) |

Table 2. Small fast rotators NEAs at the approximately flat region of Fig. 1. Data from Binzel et al. (2002).

| Object           | $H$   | Diameter (km) | Period (h) |
|------------------|-------|---------------|------------|
| 1998-KY26        | 23.6  | 0.04          | 0.1784     |
| 2000 DO8         | 24.8  | 0.04          | 0.022      |
| 1999-SF10        | 24.0  | 0.06          | 0.0411     |
| 1999 TY2         | 23.1  | 0.08          | 0.1213     |

4. Distribution of periods

Fig. 1 shows the decreasing cumulative distribution of 3567 periods and a superposed $q$-gaussian $(N_q(h) = M \exp(-\beta q h^2))$ and it is seen that it quite satisfactorily describes almost all the data with $q = 2.6$ and $\beta = 0.028 \text{ h}^{-2}$ ($M = 3567$ is the number of objects). This suggests that the distribution does not depend on (i) the diameters; (ii) the mineralogical composition; (iii) the region of the SS in which the object is found. The latter is particularly important once the sample includes NEAs, trans-netunian objects (TNO), asteroids from the main belt (MB), Jupiter Trojans (JT) and dwarf planets like Ceres and Pluto. Some asteroids are identified in the Figure and in Tables 1 and 2. The value of the entropic index $q = 2.6$ — rather distant from unit, that is, distant from the Maxwellian distribution — indicates that long-range interactions play an essential role in the distribution of periods of rotation.

The spin-barrier period of 2.2 h is close to the intersection of the asymptotic power-law and the approximately flat region of Fig. 1. This result suggests that in the approximately flat region we may find monolithic fast-rotating asteroids (Whiteley, Hergenrother & Tholen 2002) with absolute magnitude $H > 23$ and $D < 0.2 \text{ km}$. Some of these asteroids are identified in Fig. 1 and in Table 2. Consequently in the asymptotic power-law we probably find gravitational bound objects.

The distributions for each complex taken separately (C, S, X) is shown in Fig. 2. They can be satisfactorily approximated by $q$-gaussians ($q = 2.7$ for S, $q = 2.0$ for C and $q = 2.0$ for X). For C and X complexes, the fittings can be improved if we admit that the number of objects are 10% higher than that found in the sample.

5. Distribution of Diameters of near-Earth asteroids

The decreasing cumulative distribution of diameters of NEAs is not described by Eq. (3) once it is only asymptotically a power law. We have tried to fit a $q$-exponential to the diameters of 7078 NEAs ($N_q(D) = M \exp(-\beta q D)$) and the result, shown in Fig. 3, is quite good for the entire range of the data.

The value of $q = 1.3$ (different from one) is an indication that not only collisional processes are implied in the formation of these objects. Other mechanisms may also be present: the YORP effect may lead to the decrease of the period of rotation up to the point of rupture. This fragmentation process may yield the formation of binary or multiple systems. About $(15 \pm 4)\%$ of NEAs with $D \geq 0.3 \text{ km}$ and periods of rotation between 2 and...
3 h possibly are binary systems (Pravec, Harris & Warner 2007). According to the distribution we have found there are 1240 ± 180 NEAs with $D ≥ 1$ km, that is superior than the 1090 asteroids found by Stuart & Binzel (2004). The analysis of the distributions of diameters of MBA and TNO are welcome.

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