Nonparametric Method for Aircraft State Prediction

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Abstract. This paper is a part of a series of articles on nonparametric methods in dynamic systems theory. Here the authors propose a new method for direct dynamic problem solution based only on some past measurements of system input-output signals. The described nonparametric state prediction method needs no any priori information on system model parameters and does not require identification, training or statistical calculations. The methodological examples of nonparametric aircraft state prediction are presented.

1. Introduction
The aircraft state prediction is one of the core design problems of many functions of onboard and ground systems such as [1–4]: automatic flight envelope protection, aircraft takeoff and landing, terrain reference navigation, terrain awareness and warning, traffic alert and collision avoidance, 4D trajectory prediction, moving object guidance, and the others. The efficiency of the methods for aircraft state prediction depend mainly on available online information.

Traditional methods, based on aircraft mathematical models, work very well for own aircraft state prediction, when accurate model parameters are available, all input and output signals are measurable. But when enough information is not available the state prediction problem is dramatically more complex. The problems of nonstationarity and nonlinearity of aircraft complex dynamic models can partly be solved by inflight parameters identification methods [5–7]. But the problems of aircraft model unidentifiability in normal closed-loop operation conditions requires the additional controls exiting besides priori structural model information [8].

Moreover, traditional parametric methods can hardly be used for state prediction of other moving controlled objects because their mathematical models are very difficult to establish only by outer measurements. Using aircraft performance models with structures based on kinetic assumptions, also belonging to physics-based approaches, is also limited because most of these models based on ideal assumptions, rarely considered the real constraints, human behavior factors, and the intersection of trajectories [9, 10].

Among the comparatively new nonparametric prediction methods, that need no priori information on models parameters, the most widely used are statistical and artificial intelligence methods [11]. Compared with those parametric methods they are constructed with weak assumptions or even without assumptions. But these methods have high computational costs, need much sample size or learning time.

The authors propose a new method for nonparametric aircraft state prediction, that needs no any information on model parameters and does not require identification, training or statistical calculations. It significantly expands the applicability of nonparametric dynamic systems theory [12–15]. In this paper the simple own aircraft state prediction problem is solved, showing the principle of
the method. Further researches assume predicting of states of any observable moving objects with unexpected or expected command and control strategies given by pilots or flight management systems (known as aircraft intent).

2. Nonparametric aircraft state prediction problem formulation and solution

Let’s an aircraft dynamics is described by a discrete state space model in the form the following parametric vector

\[ x_{i+1} = Ax_i + Bu_i \]  

or matrix

\[
\begin{bmatrix}
  x_{i-h} & \ldots & x_i & x_{i+1}
\end{bmatrix} = A\begin{bmatrix}
  x_{i-h-1} & \ldots & x_{i-1}
\end{bmatrix} + B\begin{bmatrix}
  u_{i-h-1} & \ldots & u_{i-1}
\end{bmatrix},
\]

expressions, where \( A, B \) – eigen-dynamics and control efficiency matrices, \( x \) – state vector, \( u \) – control vector, \( h \) – number of observations, \( i \) – discrete time.

The aircraft state prediction problem is to find the its future state \( x_{i+1} \), knowing only some past values of states and controls, without any priori information on the aircraft model parameters \( A, B \).

To solve this problem, let’s write the equation (1) in a block-matrix form

\[
\begin{bmatrix}
  X_i & x_{i+1} \end{bmatrix} = A\begin{bmatrix}
  X_{i-1} & x_i
\end{bmatrix} + B\begin{bmatrix}
  U_{i-1} & u_i
\end{bmatrix},
\]

where \( X_i = \begin{bmatrix} x_{i-h} & \ldots & x_i \end{bmatrix} \), \( X_{i-1} = \begin{bmatrix} x_{i-h-1} & \ldots & x_{i-1} \end{bmatrix} \), \( U_{i-1} = \begin{bmatrix} u_{i-h-1} & \ldots & u_{i-1} \end{bmatrix} \), and convert it to the following linear matrix equation

\[
\begin{bmatrix}
  A & B
\end{bmatrix}\begin{bmatrix}
  X_{i-1} & x_i \\
  U_{i-1} & u_i
\end{bmatrix} = \begin{bmatrix}
  X_i & x_{i+1}
\end{bmatrix},
\]

representing aircraft eigen-dynamics and control efficiency parameters identification problem.

Equation (3) has the form of a right-hand side matrix equation

\[ ZC = D, \]

which is solvable if and only if

\[ DC^R = 0, \]

where \( C^R \) – the right-hand full rank zero divisor of the matrix \( C \), such that \( C^R = 0 \), which formally describes all the linear dependent columns of matrix \( C \) [8, 12–15].

According to (4), we always can find such \( h \), that the necessary and sufficient solvability condition of (3) has the form of equality

\[
\begin{bmatrix}
  X_i & x_{i+1} \\
  U_{i-1} & u_i
\end{bmatrix}^{\text{R}} \begin{bmatrix}
  X_{i-1} & x_i \\
  U_{i-1} & u_i
\end{bmatrix}\begin{bmatrix}
  r_i \\
  1
\end{bmatrix} = 0,
\]

where

\[
\begin{bmatrix}
  r_i \\
  1
\end{bmatrix} = \begin{bmatrix}
  X_{i-1} & x_i \\
  U_{i-1} & u_i
\end{bmatrix}^{\text{R}}
\]

is the single-column normalized right-hand zero divisor, such that

\[
\begin{bmatrix}
  X_{i-1} & x_i \\
  U_{i-1} & u_i
\end{bmatrix}\begin{bmatrix}
  r_i \\
  1
\end{bmatrix} = 0.
\]

So, from (5) we can find a simple nonparametric aircraft state prediction algorithm

\[ x_{i+1} = -X_i r_i, \]

allowing obtaining the future aircraft state \( x_{i+1} \) based only on some \( h \) past measurements without a direct solution of identification equation (3) and any information on the model parameters \( A \) and \( B \).

Note that the condition (5) is met for all discrete times, so to predict the next aircraft states we need iterating (7) and (8) for all future steps.
3. Nonparametric aircraft state prediction examples
Consider a simple aircraft roll motion model
\[ \begin{align*}
\dot{\omega}(t) &= k_\omega \omega(t) + k_\delta \delta(t), \\
\dot{\gamma}(t) &= \omega(t),
\end{align*} \tag{9} \]
where \( t \) – continuous time, \( \omega \) – angular rate of roll, \( \gamma \) – roll, \( \delta \) – deflection angles of ailerons, \( k_\omega \) – natural damping coefficient, \( k_\delta \) – efficiency of ailerons.

With the zero initial conditions \( \omega(0) = 0, \ \dot{\omega}(0) = 0, \ \gamma(0) = 0, \ \dot{\gamma}(0) = 0 \), constant control \( \delta(t) = 1 \), and model parameters \( k_\omega = -2, k_\delta = 2 \) we can find the exact solutions of the equations (9)
\[ \begin{align*}
\omega(t) &= 1 - e^{-2t}, \\
\gamma(t) &= t + \frac{e^{-2t} - 1}{2}.
\end{align*} \tag{10} \]
According to (10) the first five states with \( \Delta t = 0.1 \ sec \) discretization interval are presented in table 1.

| \( i \) | \( 0 \) | \( 1 \) | \( 2 \) | \( 3 \) | \( 4 \) |
|---|---|---|---|---|---|
| \( t \) | 0 | 0.1 | 0.2 | 0.3 | 0.4 |
| \( \omega(t) \) | 0 | 0.1813 | 0.3297 | 0.4512 | 0.5507 |
| \( \gamma(t) \) | 0 | 0.0094 | 0.0352 | 0.0744 | 0.1247 |
| \( \delta(t) \) | 1 | 1 | 1 | 1 | 1 |

Let we know only the first four columns in table 1 and we need to predict the fifth one using the proposed method. Then, according to (2) we have the following block matrix of states and controls
\[ \begin{bmatrix} X_{i-1} & x_i \\ U_{i-1} & u_i \end{bmatrix} = \begin{bmatrix} 0 & 0.1813 & 0.3297 & 0.4512 \\ 0 & 0.0094 & 0.0352 & 0.0744 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \tag{11} \]
It’s easy to check that for the following right-hand single-column zero divisor of matrix (11)
\[ \begin{bmatrix} X_{i-1} & x_i \\ U_{i-1} & u_i \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.8170 \\ 2.6340 \\ -2.8166 \end{bmatrix} \]
the null space equity (5) is always fulfilled:
\[ \begin{bmatrix} X_{i-1} & x_i \\ U_{i-1} & u_i \end{bmatrix} - \begin{bmatrix} 0 & 0.1813 & 0.3297 & 0.4512 \\ 0 & 0.0094 & 0.0352 & 0.0744 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -0.8170 \\ 2.6340 \\ -2.8166 \end{bmatrix} = 0. \]
Then, using the expression (8) we can find the predicted aircraft state
\[ \hat{x}_{i+1} = \begin{bmatrix} \hat{\omega}_{i+1} \\ \hat{\gamma}_{i+1} \end{bmatrix} = -X r_i = \begin{bmatrix} 0.1813 & 0.3297 & 0.4512 \\ 0.0094 & 0.0352 & 0.0744 \end{bmatrix} \begin{bmatrix} -0.8170 \\ 2.6340 \\ -2.8166 \end{bmatrix} = 0.5507 \\ 0.1245 \]
that entirely congruent with the exact values in last column of table 1.

Consider now the approximate numerical aircraft model (9) solutions using first-order Euler method for solving ordinary differential equations. Then (9) with the same parameters and initial conditions has the following discrete time-invariant form
\[
\begin{aligned}
\frac{\omega_{i+1} - \omega_i}{\Delta t} &= k_\omega \omega_i + k_\delta \delta_i, \\
\frac{\gamma_{i+1} - \gamma_i}{\Delta t} &= \omega_i.
\end{aligned}
\]

(12)

Rewriting (12)
\[
\begin{aligned}
\omega_{i+1} &= (1 + k_\omega \Delta t) \omega_i + (k_\delta \Delta t) \delta_i, \\
\gamma_{i+1} &= (\Delta t) \omega_i + \gamma_i,
\end{aligned}
\]

(13)

brings it to the matrix form (1), where
\[
X = \begin{bmatrix} \omega \\ \gamma \end{bmatrix}, \
U = \delta, \
A = \begin{bmatrix} 1 + k_\omega \Delta t & 0 \\ \Delta t & 1 \end{bmatrix} = \begin{bmatrix} 0.8 & 0 \\ 0.1 & 1 \end{bmatrix}, \
B = \begin{bmatrix} k_\delta \Delta t \\ 0 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}.
\]

Let the ailerons control strategy be a little bit more complex than in the first example as shown in table 2.

**Table 2. Aircraft control values.**

| $i$ | 0  | 1   | 2  | 3  | 4  | 5  | 6  | 7...20 |
|-----|----|-----|----|----|----|----|----|--------|
| $t_i$ | 0  | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7...2 |
| $u_i$ | 1  | 1   | 0   | 0   | 0.5 | 0.5 | 0   | 0...0  |

Simultaneous to simulation (13) we also iterate our prediction algorithm (7)–(8) with sample width $h = 4$. The first some simulated $\omega_i$, $\gamma_i$ and predicted $\hat{\omega}_i$, $\hat{\gamma}_i$ states are shown in table 3, the rest can be seen from figure 1.

**Table 3. Aircraft simulated and predicted states values**

| $i$ | 0  | 1   | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|-----|----|-----|----|----|----|----|----|----|----|----|
| $t_i$ | 0  | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| $\delta_i$ | 1  | 1   | 0   | 0   | 0.5 | 0.5 | 0   | 0   | 0   | 0   |
| $\omega_i$ | 0.00 | 0.20 | 0.36 | 0.29 | 0.23 | 0.28 | 0.33 | 0.26 | 0.21 | 0.17 |
| $\gamma_i$ | 0.00 | 0.00 | 0.02 | 0.06 | 0.08 | 0.11 | 0.14 | 0.17 | 0.20 | 0.22 |
| $\hat{\omega}_i$ | -  | -   | -   | -   | -   | 0.23 | 0.28 | 0.33 | 0.26 | 0.21 |
| $\hat{\gamma}_i$ | -  | -   | -   | -   | -   | 0.08 | 0.11 | 0.14 | 0.17 | 0.20 |

Figure 1. Aircraft control, simulated and predicted states values.
It can be seen, that proposed nonparametric prediction algorithm needs only four simulations steps or 0.4 seconds for tuning in this example. As soon as it collect enough data for equation (7) nontrivial solution existence it start to predict future aircraft states without using information of $A$ and $B$ matrices, just based only on $h$ past measurements and formula (8).

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