Loop Corrections to the Neutral Higgs Boson Sector of the MSSM with Explicit CP Violation

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Abstract

We compute one–loop corrections to the mass matrix of the neutral Higgs bosons of the Minimal Supersymmetric Standard Model with explicit CP violation. We use the effective potential method, allowing for arbitrary splitting between squark masses. We include terms $O(g^2h^2)$, where $g$ and $h$ stand for electroweak gauge and Yukawa couplings, respectively. Leading two–loop corrections are taken into account by means of appropriately defined running quark masses.

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1) Introduction

Supersymmetry is the currently best motivated extension of the Standard Model (SM) of particle physics, since it allows to stabilize the gauge hierarchy without getting into conflict with electroweak precision data. Among all possible supersymmetric theories, the Minimal Supersymmetric Standard Model (MSSM) occupies a special position. It is not only the simplest, i.e. most economical, potentially realistic supersymmetric field theory, it also has just the right particle content to allow for the unification of all gauge interactions \[1\]. Within the MSSM, the Higgs sector can be singled out: among all the as yet undetected new particles in the MSSM spectrum, the lightest neutral Higgs boson \(h_1\) is the only one for which a strict upper bound on the mass can be given \[2\], \(m_{h_1} \lesssim 130\) GeV. A good understanding of the Higgs sector is therefore of crucial importance for experimental tests of the MSSM.

It has recently been realized \[3\] that explicit CP violation in the mass matrices of third generation squarks can induce sizable CP violation in the MSSM Higgs sector through loop corrections. Note that CP violating phases for third generation sfermions can be quite large, since they contribute to the electric dipole moments of the electron and neutron only at the two–loop level \[4\]. Although a one–loop effect, the induced CP violation in the MSSM Higgs sector can be large enough to significantly affect Higgs phenomenology at present \[3, 5\] and future \[3, 6\] colliders. An accurate treatment of CP violating loop corrections to the MSSM Higgs sector is therefore of some importance.

The first calculations \[3\] used diagrammatic methods, and diagonalized the resulting mass matrix only approximately. More recently, the effective potential has been used \[5, 7\] to compute the \(3 \times 3\) mass matrix of the neutral Higgs bosons of the MSSM. However, the results of ref.\[5\] are not valid for large mass splitting between squark mass eigenstates, while ref.\[7\] does not include contributions from the bottom–sbottom sector, which can be important for large ratio of vacuum expectation values \(\tan \beta\). Here we present a calculation based on the full one–loop effective potential, valid for all values of the relevant soft breaking parameters. This extends older results \[8\] where CP was assumed to be conserved. Moreover, unlike refs.\[5, 7, 8\], we include terms \(O(g^2h^2)\), where \(g\) and \(h\) are a weak gauge and third generation Yukawa coupling, respectively. These new terms do not change the spectrum very much, but alter CP violating mixing angles by typically 20%. Finally, we absorb leading two–loop corrections into appropriately defined running quark masses \[2, 2\].

The remainder of this article is organized as follows. In Sec. 2 we present analytical results for the \(3 \times 3\) mass matrix of the neutral Higgs bosons of the MSSM. In the appropriate limit we find complete agreement with ref.\[5\], but there are some discrepancies between our results and those of ref.\[7\]. In Sec. 3 we show some numerical results, and compare them with results obtained using the formalism of ref.\[5\]. Finally, Sec. 4 is devoted to a brief summary and conclusions.

2) Analytical results

The MSSM contains two Higgs doublets \(H_1, H_2\), with hypercharges \(Y(H_1) = -Y(H_2) = -1/2\). Here we are only interested in the neutral components, which we write as

\[
H_1^0 = \frac{1}{\sqrt{2}} (\phi_1 + i a_1) ; \quad H_2^0 = e^{i \xi} \frac{1}{\sqrt{2}} (\phi_2 + i a_2), \quad (1)
\]
where $\phi_{1,2}$ and $a_{1,2}$ are real fields. The constant phase $\xi$ can be set to zero at tree level, but will in general become non-zero once loop corrections are included.

The mass matrix of the neutral Higgs bosons can be computed from the effective potential $\mathcal{V}_{\text{Higgs}}$:

$$V_{\text{Higgs}} = \frac{1}{2} m^2_1 (\phi_1^2 + a_1^2) + \frac{1}{2} m^2_2 (\phi_2^2 + a_2^2) - |m_{12}^2| (\phi_1 a_2 - a_1 \phi_2) \cos(\xi + \theta_{12})$$

where

$$\frac{g^2}{8} \mathcal{D}^2 + \frac{1}{64\pi^2} \text{Str} \left[ \mathcal{M}^4 \left( \log \frac{\mathcal{M}^2}{Q_0^2} - \frac{3}{2} \right) \right], \quad (2)$$

where we have allowed the soft breaking parameter $m_{12}^2 = |m_{12}^2| e^{i\theta_{12}}$ to be complex. We have introduced the quantities

$$\mathcal{D} = \phi_2^2 + a_2^2 - \phi_1^2 - a_1^2; \quad \tilde{g}^2 = \frac{g^2 + g'^2}{4}, \quad (3)$$

where the symbols $g$ and $g'$ stand for the $SU(2)$ and $U(1)_Y$ gauge couplings, respectively. $Q_0$ in eq. (2) is the renormalization scale; the parameters of the tree–level potential, in particular the mass parameters $m_1^2$, $m_2^2$ and $m_{12}^2$, are running parameters, taken at scale $Q_0$. The potential (2) is then independent of $Q_0$, up to two–loop corrections.

$\mathcal{M}$ is the field–dependent mass matrix of all modes that couple to the Higgs bosons. The by far dominant contributions come from third generation quarks and squarks. The (real) masses of the former are given by

$$m_b^2 = \frac{1}{2} |h_b|^2 \left( \phi_1^2 + a_1^2 \right); \quad m_t^2 = \frac{1}{2} |h_t|^2 \left( \phi_2^2 + a_2^2 \right), \quad (4)$$

where $h_b$ and $h_t$ are the bottom and top Yukawa couplings. The corresponding squark mass matrices can be written as

$$\mathcal{M}^2_t = \begin{pmatrix} m_Q^2 + m_t^2 - \frac{1}{8} \left( g^2 - \frac{g'^2}{3} \right) \mathcal{D} & -h_t^* \left[ A_t^* (H_0^q)^* + \mu H_0^t \right] \\ -h_t \left[ A_t^* (H_0^q)^* + \mu H_0^t \right] & m_U^2 + m_t^2 - \frac{g^2}{6} \mathcal{D} \end{pmatrix} \quad (5a)$$

$$\mathcal{M}^2_b = \begin{pmatrix} m_Q^2 + m_b^2 + \frac{1}{8} \left( g^2 + \frac{g'^2}{3} \right) \mathcal{D} & -h_b^* \left[ A_b^* (H_0^q)^* + \mu H_0^t \right] \\ -h_b \left[ A_b^* (H_0^q)^* + \mu H_0^t \right] & m_D^2 + m_b^2 + \frac{g^2}{12} \mathcal{D} \end{pmatrix} \quad (5b)$$

Here, $H_0^q$ and $H_0^t$ are given by eqs. (1) while $m_b^2$ and $m_t^2$ are as in eqs. (4) and $\mathcal{D}$ has been defined in eqs. (3). In eqs. (5) $m_Q^2$, $m_U^2$ and $m_D^2$ are real soft breaking parameters, $A_b$ and $A_t$ are complex soft breaking parameters, and $\mu$ is the complex supersymmetric Higgs(ino) mass parameter. The eigenvalues of the mass matrices (5) are:

$$m^2_{b,1,2} = \frac{1}{2} \left[ m_Q^2 + m_b^2 + |h_b|^2 \left( \phi_2^2 + a_2^2 \right) \right. \left. + \frac{\tilde{g}^2}{2} \mathcal{D} \right]$$

$$\pm \sqrt{\left[ \left( m_Q^2 - m_U^2 - \frac{3g^2 - 5g'^2}{24} \mathcal{D} \right)^2 + 2 |h_t|^2 \left| A_t e^{i\xi} (\phi_2 + i a_2) + \mu^* (\phi_1 - i a_1) \right|^2 \right]^{1/2}} \quad (6a)$$

$$m^2_{b,1,2} = \frac{1}{2} \left[ m_Q^2 + m_D^2 + |h_b|^2 \left( \phi_1^2 + a_1^2 \right) \right. \left. + \frac{\tilde{g}^2}{2} \mathcal{D} \right]$$

$$\pm \sqrt{\left[ \left( m_Q^2 - m_D^2 + \frac{3g^2 - g'^2}{24} \mathcal{D} \right)^2 + 2 |h_b|^2 \left| A_b (\phi_1 + i a_1) + \mu^* e^{-i\xi} (\phi_2 - i a_2) \right|^2 \right]^{1/2}} \quad (6b)$$
The calculation proceeds by plugging the field–dependent eigenvalues $\Pi_1$ and $\Pi_\beta$ into the potential $\Pi$; here the complex scalar squarks and Dirac fermion quarks enter with overall factors +2 and −4, respectively. The mass matrix of the Higgs bosons (at vanishing external momentum) is given by the matrix of second derivatives of this potential, computed at its minimum. In order to make sure that we are indeed in the minimum of the potential, we solve the stationarity relations, i.e. set the first derivatives of the potential to zero. This allows us to, e.g., express $m_1^2$, $m_2^2$ and $m_{12}^2 \sin(\xi + \theta_{12})$ as functions of the vacuum expectation values (vevs) and the remaining parameters appearing in the loop–corrected Higgs potential. Note that the equations $\partial V_{\text{Higgs}} / \partial a_1 = 0$ and $\partial V_{\text{Higgs}} / \partial a_2 = 0$ are linearly dependent, i.e. lead to only one constraint on parameters, if we demand that $\langle a_1 \rangle = \langle a_2 \rangle = 0$; the remaining vevs are defined through

$$\langle \phi_1 \rangle^2 + \langle \phi_2 \rangle^2 = \frac{M_\beta^2}{g^2} \simeq (246 \ \text{GeV})^2; \quad \frac{\langle \phi_2 \rangle}{\langle \phi_1 \rangle} = \tan \beta.$$  \hspace{1cm} (7)

This leads to the following expression for the re–phasing invariant sum $\xi + \theta_{12}$:

$$m_{12}^2 \sin(\xi + \theta_{12}) = - \frac{3}{32 \pi^2} \left\{ f(m_{11}^2) - f(m_{22}^2) \right\} |h_t|^2 \Delta_\xi + \left\{ f(m_{11}^2) - f(m_{22}^2) \right\} |h_b|^2 \Delta_\beta,$$  \hspace{1cm} (8)

where

$$f(m^2) = 2m^2 \left( \log \frac{m^2}{Q_0^2} - 1 \right).$$  \hspace{1cm} (9)

In eq.(8) we have introduced the quantities

$$\Delta_\xi = \frac{3 \text{Im}(A_t \mu e^{i \xi})}{m_{11}^2 - m_{22}^2}; \quad \Delta_\beta = \frac{3 \text{Im}(A_b \mu e^{i \xi})}{m_{b2}^2 - m_{b1}^2},$$  \hspace{1cm} (10)

which describe the amount of CP violation in the squark mass matrices. Note that $\Delta_\xi$ remains finite as $m_{11}^2 \rightarrow m_{22}^2$, since this implies $\text{Im}(A_t \mu e^{i \xi}) \rightarrow 0$. Most tree–level analyses use the convention $\xi = 0$; in this case we could set it to zero in the right–hand side of eq.(8), to one–loop order. We kept it in order to illustrate that only the phases of the re–phasing invariant quantities $A_t \mu e^{i \xi}$ and $A_b \mu e^{i \xi}$ have physical meaning.

The mass matrix of the neutral Higgs bosons can now be computed from the matrix of second derivatives of the potential $\Pi$, where (after taking the derivatives) $m_1^2$, $m_2^2$ and $m_{12}^2 \sin(\xi + \theta_{12})$ are determined by the stationarity conditions. We find that the state $G^0 = a_1 \cos \beta - a_2 \sin \beta$ is massless; it describes the would–be Goldstone mode that gets “eaten” by the longitudinal $Z$ boson. We are thus left with a squared mass matrix $M_H^2$ for the three states $a = a_1 \sin \beta + a_2 \cos \beta$, $\phi_1$ and $\phi_2$. This matrix is real and symmetric, i.e. it has 6 independent entries. The diagonal entry for $a$ reads:

$$M_{H}^2 \big|_{aa} = m_a^2 + \frac{3}{8 \pi^2} \left\{ |h_t|^2 m_{12}^2 g(m_{11}^2, m_{22}^2) \Delta_\xi^2 + |h_b|^2 m_{b2}^2 g(m_{b1}^2, m_{b2}^2) \Delta_\beta^2 \right\},$$  \hspace{1cm} (11)

where $\Delta_\xi$ and $\Delta_\beta$ are as in eq.(10), and we have introduced the function

$$g(m_1^2, m_2^2) = 2 - \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2} \log \frac{m_1^2}{m_2^2}.$$  \hspace{1cm} (12)

The quantity $m_A^2$ in eq.(11) is given by

$$m_A^2 = \frac{2m_{12}^2 \cos(\xi + \theta_{12})}{\sin(2\beta)} + \frac{2}{\sin(2\beta)} \left\{ |h_t|^2 \text{Re}(A_t \mu e^{i \xi}) F(m_{11}^2, m_{22}^2) + |h_b|^2 \text{Re}(A_b \mu e^{i \xi}) F(m_{b1}^2, m_{b2}^2) \right\},$$  \hspace{1cm} (13)
where

$$F(m_a^2, m_b^2) = \frac{3}{32\pi^2} \frac{f(m_a^2) - f(m_b^2)}{m_b^2 - m_a^2};$$

(14)

the function \(f\) has been defined in eq.(9).

If we consider \(m_A^2\), the values of the soft breaking parameters and \(\mu\) to be inputs, eqs. (8) and (13) can be combined to give an explicit expression for the induced phase \(\xi\) of the vev:

$$\sin \xi = - \frac{|h_t|^2 \Im(A_t \mu) F(m_{t_1}^2, m_{t_2}^2) + |h_b|^2 \Im(A_b \mu) F(m_{b_1}^2, m_{b_2}^2) + \Im(m_{12}^2)}{m_A^2 \sin \beta \cos \beta}.$$  

(15)

It should be emphasized that \(\xi\) and \(\theta_{12}\) are not separately physical quantities; only their sum is re-phasing invariant. Nevertheless eq.(15) can be useful. Note that the leading \(Q_0\) dependence in the numerator cancels between the explicit terms \(\propto \log Q_0\) contained in \(F(m_{t_1}^2, m_{t_2}^2)\) and \(F(m_{b_1}^2, m_{b_2}^2)\), and the RG–induced running of \(\Im(m_{12}^2)\). If one sets the phase of the Higgs field at some scale, eq.(15) shows that it will remain scale–independent, at least to 1–loop order. In particular, \(\xi\) can be set to zero, as pointed out by Pilaftsis [3]. Eq.(15) then determines the loop–induced phase of \(m_{12}^2\) (called a counter–term in ref.[3]). On the other hand, it is also possible to set \(\theta_{12} = 0\). The price one has to pay is that the phase of the Higgs fields will show a strong scale dependence even after explicit 1–loop corrections to the Higgs potential have been added.

We will see below that most of the \(Q_0\) dependence of the loop corrections to the Higgs mass matrix can be absorbed into the parameter \(m_A\). Within \(m_A^2\) itself, the \(Q_0\) dependence largely cancels between the running of \(m_{12}^2\) and the explicit \(\log Q_0\) dependence of \(f(m^2)\), eq.(14), just as in the CP conserving case [3]. Note that our \(m_A\) differs from \(M_a\) defined in ref. [3], since our \(m_A\) only includes corrections that are nonzero in the limit of exact CP invariance, whereas \(M_a^2\) corresponds to our \(M^2_H|_{aa}\) of eq.(11). In the remainder of this paper we will consider \(m_A^2\) as well as the re-phasing invariant quantities \(A_t \mu e^{i\xi}\) and \(A_b \mu e^{i\xi}\) to be input parameters. The results are then independent of the convention adopted for \(\theta_{12}\).

The CP violating entries of the mass matrix, which mix \(a\) with \(\phi_1\) and \(\phi_2\), are:

$$M_A^2|_{a\phi_1} = \frac{3}{16\pi^2} \left\{ \frac{m_A^2 \Delta_t}{\sin \beta} \left[ g(m_{t_1}^2, m_{t_2}^2) \left( X_t \cot \beta - 2 |h_t|^2 R_t \right) - \hat{g}^2 \cot \beta \log \frac{m_{t_2}^2}{m_{t_1}^2} \right] \right. + \left. \frac{m_A^2 \Delta_b}{\cos \beta} \left[ -g(m_{b_1}^2, m_{b_2}^2) \left( X_b + 2 |h_b|^2 R_b \right) + \left( \hat{g}^2 - 2 |h_b|^2 \right) \log \frac{m_{b_2}^2}{m_{b_1}^2} \right] \right\};$$  

(16a)

$$M_A^2|_{a\phi_2} = \frac{3}{16\pi^2} \left\{ \frac{m_A^2 \Delta_t}{\sin \beta} \left[ -g(m_{t_1}^2, m_{t_2}^2) \left( X_t + 2 |h_t|^2 R_t \right) + \left( \hat{g}^2 - 2 |h_t|^2 \right) \log \frac{m_{t_2}^2}{m_{t_1}^2} \right] + \frac{m_A^2 \Delta_b}{\cos \beta} \left[ g(m_{b_1}^2, m_{b_2}^2) \left( X_b \tan \beta - 2 |h_b|^2 R_b \right) - \hat{g}^2 \tan \beta \log \frac{m_{b_2}^2}{m_{b_1}^2} \right] \right\};$$  

(16b)

As noted earlier, the size of these entries is controlled by \(\Delta_t\) and \(\Delta_b\). In addition we have introduced the dimensionless quantities

$$X_t = \frac{5\hat{g}^2 - 3g^2}{12} \cdot \frac{m_{t_2}^2 - m_{t_1}^2}{m_{t_2}^2 - m_{t_1}^2}; \quad X_b = \frac{\hat{g}^2 - 3g^2}{12} \cdot \frac{m_{b_2}^2 - m_{b_1}^2}{m_{b_2}^2 - m_{b_1}^2};$$  

(17a)
As in eqs. (10), phases only appear in the re-phasing invariant combinations $A_t \mu e^{i\xi}$ and $A_b \mu e^{i\xi}$. The terms proportional to $\tilde{g}^2$ [defined in eqs. (3)], $X_t$ or $X_b$ are mixed gauge–Yukawa contributions, which were neglected in refs. [5, 7]; note that no corrections of this kind appear in eqs. (8) and (11). We do not include pure gauge, $O(g^4)$ corrections, since there are many additional corrections of this order from first and second generation sfermions as well as from loops involving gauge and Higgs bosons and their superpartners.

While eqs. (10) are pure loop corrections, the remaining entries of the Higgs boson mass matrix also receive tree-level contributions:

$$M_H^2|_{\phi_1 \phi_1} = M_Z^2 \cos^2 \beta + m_A^2 \sin^2 \beta$$

$$+ \frac{3m_t^2}{8\pi^2} \left( g(m^2_{t_1}, m^2_{t_2}) R_t \left( |h_t|^2 R_t - \cot \beta X_t \right) + \tilde{g}^2 \cot \beta R_t \log \frac{m^2_{t_2}}{m^2_{t_1}} \right)$$

$$+ \frac{3m_b^2}{8\pi^2} \left( |h_b|^2 \log \frac{m^2_{b_1} m^2_{b_2}}{m^4_b} - \tilde{g}^2 \log \frac{m^2_{b_1} m^2_{b_2}}{Q_0^2} \right)$$

$$+ g(m^2_{b_1}, m^2_{b_2}) R_b \left( |h_b|^2 R_b + X_b \right) + \log \frac{m^2_{b_1}}{m^2_{b_1}} \left[ X_b + \left( 2 |h_b|^2 - \tilde{g}^2 \right) R_b \right] \right); \quad (18a)$$

$$M_H^2|_{\phi_1 \phi_2} = - \left( M_Z^2 + m_A^2 \right) \sin \beta \cos \beta$$

$$+ \frac{3m_t^2}{8\pi^2} \left( g(m^2_{t_1}, m^2_{t_2}) \left[ |h_t|^2 R_t R'_t + \frac{X_t}{2} (R_t - R'_t \cot \beta) \right] + \tilde{g}^2 \cot \beta \log \frac{m^2_{t_1} m^2_{t_2}}{Q_0^2} \right)$$

$$+ \log \frac{m^2_{t_1}}{m^2_{t_1}} \left[ |h_t|^2 R_t - \frac{X_t}{2} \cot \beta + \tilde{g}^2 \left( R'_t \cot \beta - R_t \right) \right]\right)$$

$$+ \frac{3m_b^2}{8\pi^2} \left( g(m^2_{b_1}, m^2_{b_2}) \left[ |h_b|^2 R_b R'_b + \frac{X_b}{2} (R_b - R'_b \tan \beta) \right] + \tilde{g}^2 \tan \beta \log \frac{m^2_{b_1} m^2_{b_2}}{Q_0^2} \right)$$

$$+ \log \frac{m^2_{b_1}}{m^2_{b_1}} \left[ |h_b|^2 R_b - \frac{X_b}{2} \tan \beta + \tilde{g}^2 \left( R'_b \tan \beta - R_b \right) \right] \right); \quad (18b)$$

$$M_H^2|_{\phi_2 \phi_2} = M_Z^2 \sin^2 \beta + m_A^2 \cos^2 \beta$$

$$+ \frac{3m_t^2}{8\pi^2} \left( |h_t|^2 \log \frac{m^2_{t_1} m^2_{t_2}}{m^4_t} - \tilde{g}^2 \log \frac{m^2_{t_1} m^2_{t_2}}{Q_0^2} \right)$$

$$+ g(m^2_{t_1}, m^2_{t_2}) R'_t \left( |h_t|^2 R'_t + X_t \right) + \log \frac{m^2_{t_2}}{m^2_{t_1}} \left[ X_t + \left( 2 |h_t|^2 - \tilde{g}^2 \right) R_t \right]$$

$$+ \frac{3m_b^2}{8\pi^2} \left( g(m^2_{b_1}, m^2_{b_2}) R_b \left( |h_b|^2 R_b - \tan \beta X_b \right) + \tilde{g}^2 \tan \beta R_b \log \frac{m^2_{b_2}}{m^2_{b_1}} \right). \quad (18c)$$
As noted earlier, the explicit dependence on the renormalization scale \( Q_0 \) has mostly been absorbed into \( m_A^2 \). The only exceptions are terms proportional to the combination \( \hat{g}^2 \) of electroweak gauge couplings, eqs. (3). They come from the wave function renormalization of the Higgs fields, which leads to a logarithmic \( Q_0 \) dependence of the tree–level contributions \( \hat{g}^2 \langle \phi_1 \rangle^2 \) (in \( \mathcal{M}_\phi \mid_{\phi_1} \)), \( -\hat{g}^2 \langle \phi_1 \rangle \langle \phi_2 \rangle \) (in \( \mathcal{M}_\phi \mid_{\phi_1, \phi_2} \)) and \( \hat{g}^2 \langle \phi_2 \rangle^2 \) (in \( \mathcal{M}_\phi \mid_{\phi_2} \)), respectively. Following ref. [4] we define these vevs at scale \( Q_0 = m_t \), i.e. we use \( Q_0 = m_t \) in the explicitly \( Q_0 \) dependent terms in eqs. (18).

In order to compare our results with those of Demir [4], we have to set most of Pilaftsis and Wagner also include logarithmically enhanced two–loop corrections \( \mathcal{O}(g_s^2 h^4) \) and \( \mathcal{O}(h^6) \), where \( g_s \) is the strong gauge coupling. As shown in refs. [4, 2], leading QCD corrections can be included by interpreting the quark masses and Yukawa couplings appearing in eqs. (11), (13), (16) and (18) to be running parameters. For example,

\[
\overline{m}_t(Q) = \overline{m}_t(m_t) \cdot \left[ \frac{g_s^2(Q)}{g_s^2(m_t)} \right]^{12/21},
\]

where we have assumed \( Q \leq M_{\text{SUSY}} \), i.e. only standard QCD contributes; a pole mass \( m_t^{\text{pole}} = 173 \) GeV corresponds to \( \overline{m}_t(m_t) = 165 \) GeV. The corresponding expression for the running Yukawa coupling follows from eq. (4) and the observation that, to one–loop order, QCD corrections do not contribute to the running of \( \langle \phi_2 \rangle \). Corrections \( \mathcal{O}(h^6) \) to the Higgs mass matrix come from the one–loop running of the Yukawa couplings, as well as (for \( Q \leq M_{\text{SUSY}} \)) from the two–loop running of the quartic Higgs coupling(s) in the effective non–supersymmetric theory. Finally, as pointed out in ref. [3], one can absorb significant \( \mathcal{O}(g_s^2 h^4) \) corrections that are not enhanced by large logarithms by including gluino–stop loop corrections to the running top mass at scale \( Q \simeq M_{\text{SUSY}} \). In order to (approximately) include all these corrections, we introduce three different top masses that appear in various contributions to \( \mathcal{M}_\phi^2 \):

- The leading–log term \( \propto |h_t|^2 m_t^2 \log \frac{m_t^2}{m_t} \) should be computed with a running top mass
at the intermediate scale $Q_{\text{int}}^2 = m_t M_{\text{SUSY}}$:

$$m_{t,\text{int}} = \overline{m}_t(Q_{\text{int}}) \left[ 1 + \frac{3 |h_t|^2}{64 \pi^2} \log \frac{Q_{\text{int}}^2}{m_t^2} \right]. \quad (21)$$

- In terms proportional to powers of the CP–odd quantity $\Delta_1$, one should use the top mass

$$m_{t,\text{odd}} = m_{t,\text{high}} \left[ 1 + \frac{6 |h_t|^2}{64 \pi^2} \log \frac{M_{\text{SUSY}}^2}{m_t^2} \right]. \quad (22)$$

- Finally, in the remaining terms the top mass should be interpreted as

$$m_{t,\text{even}} = m_{t,\text{high}} \left[ 1 + \frac{3 |h_t|^2}{64 \pi^2} \log \frac{M_{\text{SUSY}}^2}{m_t^2} \right]. \quad (23)$$

The quantity $m_{t,\text{high}}$ appearing in eqs. (22, 23) is given by

$$m_{t,\text{high}} = \overline{m}_t(M_{\text{SUSY}}) - \frac{g^2}{12 \pi^2} \sin(2 \theta_i) \text{Re}(e^{-i \phi_i} m_{\tilde{g}}) \left[ B_0(m_t, m_{\tilde{t}_i}, |m_{\tilde{g}}|) - B_0(m_t, m_{\tilde{t}_z}, |m_{\tilde{g}}|) \right]. \quad (24)$$

Here $\theta_i$ and $\phi_i$ are the angles needed to diagonalize the stop mass matrix \cite{3}, i.e. $\tilde{t}_i = \cos \theta_i \tilde{t}_L + \sin \theta_i e^{-i \phi_i} \tilde{t}_R$. $m_{\tilde{g}}$ is the gluino mass, which can be complex, and $B_0$ is the Passarino–Veltman two–point function. Eq. (24) generalizes corresponding results of ref. \cite{2} to the case with CP–odd phases; note that the quantity $e^{-i \phi_i} m_{\tilde{g}}$ is re–phasing invariant. Eqs. (22)–(24) can be extended straightforwardly to the bottom–sbottom sector.

3) Numerical examples

We are now ready to present some numerical results. It is clear from eqs. (13) and (14) that loop–induced CP violation in the Higgs sector can only be large if both $|\mu|$ and $|A_t|$ (or $|A_b|$), if $\tan \beta > 1$) are sizable \cite{3}. We therefore choose $|A_t| = |A_b| = |\mu| = 2 m_{\overline{Q}}$. For definiteness we only present results for fixed $\tan \beta = 10$, and real and positive gluino mass. The value of $\tan \beta$ is not very important, unless it reaches $\sim m_t/m_b$, where $b-\overline{b}$ contributions can be important. Since for the given moderate value of $\tan \beta$ contributions from the (s)bottom sector are still quite small, our result are not sensitive to $m_{\overline{D}}$ and $A_b$; we therefore fix $m_{\overline{D}} = m_{\overline{G}}$ and also take equal phases for $A_t$ and $A_b$. However, we allow different values for the soft breaking masses of $SU(2)$ doublet and singlet squarks, $m_{\overline{Q}} \neq m_{\overline{G}}$. This allows us to study scenarios with large $\tilde{t}_1 - \tilde{t}_2$ mass difference even if $m_{\overline{Q}} \gg m_t$. Note that renormalization group effects do in fact produce significant differences between $m_{\overline{Q}}$ and $m_{\overline{G}}$ even if they are equal at some very large energy scale \cite{11}. Finally, we take $\overline{m}_t(m_t) = 165$ GeV and $\overline{m}_b(m_b) = 4.2$ GeV.

We first study a scenario with $m_{\overline{Q}} = m_{\overline{G}}$. Here we expect good agreement with results of ref. \cite{3} as long as $m_{\overline{Q}} \gg m_t$, if we “switch off” the new contributions $\propto g^2 h^2$ presented in Sec. 2 as well as the SUSY–QCD correction \cite{24}. This expectation is borne out by Fig. 1, which shows the mass of the lightest neutral Higgs boson (left panel) as well as the CP violating Higgs

\footnote{In eq. (24) we have assumed that $m_t < m_{\tilde{g}} + m_{\tilde{t}_z}$. If this is not the case, the loop functions will develop imaginary parts. To one–loop order, only the real part of the correction contributes.}
mixing angle $\alpha_2$ (right panel) as a function of $m_{\tilde{Q}}$, for $m_A = 1$ TeV and $\theta_{\text{eff}} \equiv \arg(A_t \mu e^{i\xi}) = \pi/2$. Defining the orthogonal Higgs mixing matrix $O^H$ through $h_i = O^H_{ij} \phi_j$, where $h_i$ denotes the physical Higgs bosons (mass eigenstates) and we have set $\phi_3 \equiv a$, the $\alpha_i$ are given by

$$\alpha_i = \min \left[ \frac{|O^H_{i3}|}{\sqrt{|O^H_{i1}|^2 + |O^H_{i2}|^2}}, \frac{\sqrt{|O^H_{i1}|^2 + |O^H_{i2}|^2}}{|O^H_{i3}|} \right].$$

(25)

If CP is conserved, $h_i$ is either purely CP–odd ($O^H_{i3} = 1$), or purely CP–even ($|O^H_{i1}|^2 + |O^H_{i2}|^2 = 1$), in which case $\alpha_i = 0$.

The dotted curves in Fig. 1 have been obtained using the expressions of ref.[3], while the dashed and solid curves show our results without and with $\mathcal{O}(g^2h^2)$ and 2–loop SUSY–QCD contributions switched off. For this comparison we

Figure 1: The mass of the lightest neutral Higgs boson (left) and the CP violating mixing angle of the second–lightest neutral Higgs boson (right) as a function of the soft breaking mass $m_{\tilde{Q}}$ of $SU(2)$ doublet squarks. The values of the other free parameters are: $m_{\tilde{U}} = m_{\tilde{D}} = m_{\tilde{Q}}, |A_t| = |A_b| = |\mu| = 2m_{\tilde{Q}}, \tan\beta = 10, m_A = 1$ TeV, $m_{\tilde{g}} = 1$ TeV and squark phase $\theta_{\text{eff}} = \pi/2$. The dotted lines have been obtained using the expressions of ref.[3], while the dashed and solid curves show our results without and with $\mathcal{O}(g^2h^2)$ and 2–loop SUSY–QCD corrections, respectively.
have chosen the parameter $M_a^2$ of ref. [4] to agree with the $(a,a)$ element (11) of the Higgs mass matrix. We then find that the expressions of ref. [5] reproduce the mass of the lightest neutral Higgs boson very well even for quite small values of $m_Q$. This is somewhat surprising, since for the smallest experimentally allowed value of $m_Q$, defined through the requirement $m_{t_1} \geq 90$ GeV [12], the stop mass splitting is very large, $m_{t_2} \simeq 4m_{t_1}$. On the other hand, the two predictions for $\alpha_2$ do start to deviate significantly for $m_Q \lesssim 500$ GeV, which corresponds to $m_{t_2} \gtrsim 2m_{t_1}$. Note that $\alpha_1$ is very small in this example, $\mathcal{O}(10^{-3})$, due to the large difference between $m_A$ and $m_{h_1}$.

The solid curves show our full results, including all contributions described in Sec. 2; we choose a gluino mass of 1 TeV. The new contributions to $m_{h_1}$ are not very large, although the reduction by 4 GeV for the smallest allowed value of $m_Q$ is not entirely negligible. In contrast, these new contributions reduce $\alpha_2$ by $\sim 20\%$ for all values of $m_Q$. Contributions of this magnitude should certainly be taken into account when translating observed (bounds on) CP violating effects into (bounds on the) values of the fundamental soft breaking parameters.

\[ m_{\tilde{u}} = m_{\tilde{d}} = 2m_{\tilde{Q}}, \quad m_A = 0.1 \text{ TeV} \]

![Graph showing mass of lightest neutral Higgs boson and CP violating mixing angle as a function of $m_Q$.](image)

**Figure 2:** The mass of the lightest neutral Higgs boson (left) and the CP violating mixing angle of this Higgs boson (right) as a function of $m_Q$. Notation and parameters are as in Fig. 1, except that we have taken $m_{\tilde{u}} = m_{\tilde{d}} = 2m_{\tilde{Q}}$ and $m_A = 100$ GeV.

In Fig. 2 we show results for a scenario with $m_{\tilde{u}} = 2m_{\tilde{Q}}$, so that $m_{t_2} > 2m_{t_1}$ for all values of $m_Q$. We nevertheless find good agreement between our calculation of $m_{h_1}$ and the results
of ref.[3]. The reason is that we now have chosen \( m_A = 100 \text{ GeV} \), in the range of interest to current LEP experiments. In the absence of CP violation we would then have \( m_{h_1} \simeq m_A \) [5], since \( \tan^2 \beta \gg 1 \) in our example. In the presence of CP violation in the squark sector the corrections \( (\underline{11}) \) to the \( (a,a) \) element of the Higgs mass matrix are negative; \( m_{h_1} \) is reduced further by mixing between CP–even and CP–odd Higgs bosons. However, these effects remain quite small as far as the Higgs spectrum is concerned, and therefore need not be computed with very high accuracy.

In contrast, the three predictions of \( \alpha_1 \) now differ significantly over the entire allowed range of \( m_{\tilde{Q}} \). In the given case we observe a partial cancellation between effects due to squark mass splitting and the new \( \mathcal{O}(g^2h^2) \) and 2–loop SUSY–QCD contributions. As a result the prediction from ref.[3] agrees more closely with our full result than with the approximation that only includes the interactions also included in ref.[3]. We see that the new mixed gauge–Yukawa and SUSY–QCD corrections again reduce CP violation in the Higgs sector by 10 to 20%. Note, however, that this loop–induced CP violation in the Higgs sector can still be very large, i.e. \( \alpha_1 \sim \mathcal{O}(1) \) is possible here.

Finally, in Fig. 3 we show that \( \alpha_1 \) can in fact reach its theoretical maximum of unity. In this figure we keep all dimensionful parameters fixed, and show \( m_{h_1} \) and \( \alpha_1 \) as a function of the phase \( \theta_{\text{eff}} \) appearing in the squark mass matrices. The dependence of \( m_{h_1} \) on this phase is mild. Note that the lightest Higgs mass now takes its maximal value for maximal CP violation in the squark sector, \( \theta_{\text{eff}} = \pi/2 \). In contrast, the CP violating Higgs mixing angle \( \alpha_1 \) becomes maximal for intermediate values, \( \theta_{\text{eff}} \approx 0.9 \) and \( \theta_{\text{eff}} \approx 2.2 \). In between these two values, \( h_1 \) is mostly CP–odd, i.e. \( |O_{13}^H| \geq 1/\sqrt{2} \); in the remaining region \( h_1 \) is dominated by its CP–even components.

Note that \( \alpha_1 \) is actually quite small, \( \sim 0.1 \), for “maximal” CP violation in the squark sector. The reason is that the \( (a,\phi_1) \) element \( (\underline{16a}) \) of the Higgs mass matrix is small here. The leading contribution to this element is proportional to the quantity \( R_t \) of eq.(17b), which is small if \( A_t \mu e^{i\xi} \) is purely imaginary, since the contribution \( \propto |\mu|^2 \) is suppressed for our choice \( \tan \beta = 10 \). As a result, this element of the Higgs mass matrix goes through zero for \( \theta_{\text{eff}} \approx 0.53\pi \). The quantity \( R'_t \) remains large in this region of parameter space, leading to a sizable \( (a,\phi_2) \) element of the Higgs mass matrix. However, its contribution to \( \alpha_1 \) is suppressed by the relatively large difference between the diagonal \( (a,a) \) and \( (\phi_2,\phi_2) \) elements.

4) Summary and conclusions

In this paper we calculated quark and squark loop corrections to the mass matrix of the neutral Higgs bosons of the MSSM using the effective potential method. We allowed for CP violating phases in the squark sector as well as arbitrary splitting between squark masses. We also for the first time included mixed weak gauge–Yukawa contributions, as well as leading 2–loop SUSY–QCD contributions, in the calculation of induced CP violation in the Higgs sector; the latter have been absorbed into properly defined running quark masses, following ref.[2]. Our formulas can be used to accurately calculate the masses and mixing angles of the MSSM Higgs bosons for all values of the (possibly complex) parameters describing the third generation squark mass matrices.

When comparing our results with earlier expressions [3] that are valid for relatively small squark mass splitting we found surprisingly good agreement for the Higgs spectra even in cases with very large squark mass splitting. However, we also found \( \sim 30\% \) discrepancies in the
predictions of CP violating Higgs mixing angles if the difference between squark masses is large. Similarly, we found that the mixed weak gauge–Yukawa and 2–loop SUSY–QCD contributions do not change the predictions for the Higgs spectrum very much, but can change the prediction for CP violating Higgs mixing angles by $\sim 20\%$.

Finally, we showed that maximal mixing between CP–even and CP–odd Higgs states remains possible for reasonable choices of the free parameters. Note that this usually does not happen when CP violation in the squark sector is maximal, i.e. CP violating Higgs mixing angles reach their maximum for intermediate values of the effective CP violating phase in the scalar top mass matrix (away from $\pi/2$). Since the loop–induced CP violation in the MSSM Higgs sector can be very large, an accurate treatment of this effect, as described in this paper, is required for the interpretation of searches for Higgs bosons at LEP and elsewhere.

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