Optimization of Water-Food Nexus System under Dual Uncertainties

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Abstract. In this study, a fuzzy chance-constrained programming (FCCP) method is developed to synergetic plan water-food nexus (WFN) system under dual uncertainties. The developed method can tackle uncertainties expressed as probabilistic distribution and flexible parameter. Then, a FCCP-WFN model is formulated for the city of Jinan (China), in which 72 scenarios are designed with the consideration of different food demand levels, constraint-violation risk levels, and satisfactory degrees. Results indicate that (i) surface water would be the main water source for Jinan (accounting for 62.7% of water supply), and agriculture would be the largest water consumer (accounting for 55.5% of water allocation), therefore, rational management of surface water and reduction of agricultural water allocation are essential to alleviate the water shortage problem in Jinan; (ii) the annual arable land area is 628.4×10³ ha to 683.4×10³ ha, of which grain crops account for 62.1%, and the sufficient grain planting area can ensure food security in Jinan; (iii) uncertainties have significant influence on water allocation schemes, thus, managers should consider the impact of uncertainties during decision-making process, and make different management schemes according to different attitudes to system risks under undetermined conditions.

1. Introduction
The demands for water and food face mounting pressures with the growing population and accelerated urbanization. According to WHO and UNICEF (2017) [1], 29% of global population cannot obtain the safety drinking water. The global water scarcity will account for 40% by 2030 [2]. As the largest user of water resources, agriculture accounts for nearly 68% of global freshwater withdrawals. Thus, the water shortage problem would further lead to food insecurity. The inextricable interrelationship between water and food forms the water-food nexus (WFN) system. In order to alleviate water shortage and ensure food security, managers should make synergetic management of WFN system. However, the WFN system is often plagued by complicated uncertainties. For example, with the impact of natural process, the water availability would have highly random uncertainty, which can be presented as probability distribution. These uncertainties pose challenges to stakeholders in the synergetic planning of WFN system. Therefore, more robust planning method should be developed to cope with above problems.

Chance-constrained programming (CCP) can effectively tackle system’s random uncertainty. For example, Li et al. (2011) [3] developed a CCP-based optimization method to plan economic and environmental system under uncertainties, where CCP was applied to tackle random wastewater
treatment capacity. Khatakvark and Mays (2017) [4] formulated an optimization model based on CCP to plan pumping system for water allocation, in which the system cost was minimized. Stuhlmacher and Mathieu (2020) [5] developed a multiperiod optimization model to plan water distribution network, where the random resource demand was set based on Gaussian probability distribution. However, due to inaccurate information and human subjectivity, there would be some ambiguous information in the WFN system, and CCP has difficulty in tackling this vague uncertainty. Flexible fuzzy programming (FFP) method can be introduced to solve above-mentioned problem through setting soft constraint [6].

Therefore, this study aims to develop a fuzzy chance-constrained programming (FCCP) to plan WFN system under uncertainties. FCCP integrates CCP and FFP into a general framework. The novelty and contribution include: (i) FCCP is developed to synergetic management for WFN system, where different uncertainties expressed as probabilistic distribution and fuzzy parameter can be tackled; (ii) an FCCP-WFN model is formulated for the city of Jinan (China), where different food demand levels are considered to explore the interrelationship between water and food; (iii) the obtained results can provide decision makers with quantitative information support in alleviating water scarcity and ensuring food security.

2. Methodology
Chance-constrained programming (CCP) is effective in tackling random uncertainty which expressed as probability and analysing the risk of constraint violation. However, in real-world problems, due to the subjective judgment of the decision maker, some vague information would exist in the constraints. Therefore, fuzzy chance-constrained programming (FCCP) can to be developed to solve above-mentioned problems with the cooperation of flexible fuzzy programming (FFP) and CCP. The FCCP can be developed as [7]:

\[
\text{Max} f = \sum_{j=1}^{n} c_j x_j \tag{1}
\]

subject to:

\[
\Pr \left[ \{ t | a_j(t) x_j \leq b_j(t) \} \right] \geq 1 - p_j, \quad \forall j \tag{2}
\]

\[
g_j x_j \leq h_j, \quad \forall j \tag{3}
\]

\[
x_j \geq 0, \quad \forall j \tag{4}
\]

where \( x_j \) is decision variables, \( c_j, a_j \) and \( g_j \) are coefficients for the objective and constraint, \( b_j(t) \) is the random parameter presented based on probability space \( T, t \in T \), \( p_j \in [0, 1] \) means the constraint (1b) would be satisfied with a probability of \( 1 - p_j \), \( \xi \) is fuzzy inequality which means that the right-hand parameter is essentially larger than the left-hand one. Through introducing the fuzzy set theory, the ambiguity and vagueness in WFN system can be tackled. Then, a fuzzy number \( \tilde{\xi} \) (triangular fuzzy number) can be employed to tackle constraint (2c). where \( \tilde{\xi} \) is presented as three prominent points \( \tilde{\xi} = [\xi_{1(1)}, \xi_{1(2)}, \xi_{1(3)}] \). Based on fuzzy ranking method, \( \partial_{\xi} \) (i.e. \( \partial_{\xi} = \xi_{1(3)} - \xi_{1(1)} \)) and \( \partial'_{\xi} \) (i.e. \( \partial'_{\xi} = \xi_{1(2)} - \xi_{1(1)} \)) can be employed to describe the lateral margins of \( \tilde{\xi} \) [8]. Then, model (2) can be converted as follows:

\[
\text{Max} f = \sum_{j=1}^{n} c_j x_j \tag{5}
\]

subject to:

\[
\Pr \left[ \{ t | a_j(t) x_j \leq b_j(t) \} \right] \geq 1 - p_j, \quad \forall j \tag{6}
\]
3. Case study

3.1. Statement of problem

Jinan (China) faces serious water shortage problem. According to the Jinan Water Resources Bulletin (2019) [9], the total allocation of water resources for Jinan in 2019 was $1959.7 \times 10^6$ m$^3$ (the surface water accounted for 59.4%, the groundwater accounted for 32.9%, and other source accounted for 7.7%). The city’s per capita water availability was only 298.8 m$^3$, which was far below the limit of world’s water-shortage threshold (i.e., 1000 m$^3$ per capita). Agriculture is a major consumer of water resources. With the growth of population, water scarcity problem would further lead to food crisis. The total grain output of Jinan in 2019 was $2.5 \times 10^6$ kg, and the per capita grain occupation was 383.3kg. Although the city’s grain production is relatively excessive, food security issue still needs to be paid attention in the context of the new coronavirus pandemic. Therefore, it is vital to manage water-food nexus system synergetic for Jinan to alleviate water shortage and ensure food security.

3.2. FCCP-WFN modeling formulation

A FCCP-based WFN management (FCCP-WFN) model is formulated for the city of Jinan. Its objective is to maximize system benefit over the planning horizon. The model contains two water sources (i.e., surface water and groundwater), twelve water users (i.e., grain, oil-bearing crop, cotton, vegetable, melon, orchard, other farm crop, electricity, other industry, domestic, municipal, and ecology) and five planning periods (2021-2025). Besides, in order to tackle uncertainties which presented as random and fuzzy in WFN system, four $p$ levels (i.e., $p = 0.01, 0.05, 0.10, \text{ and } 0.15$) and six $\alpha$ levels (i.e., $\alpha = 0, 0.2, 0.4, 0.6, 0.8, \text{ and } 1$) are considered. To explore the impact of food demand on WFN system, three food demand levels (i.e., low, medium, and high) are also involved in the model, which leading to 72 scenarios. In detail, the FCCP-WFN model is formulated as:

$$
\text{Max} f = \sum_{i=1}^{7} \sum_{j=1}^{5} \xi_i \times (SW_a + GW_a) \times WL_i \times PL_i \times NB_i + \sum_{i=8}^{12} \sum_{j=1}^{5} \gamma_i \times (SW_a + GW_a) \times NB_i
$$

subject to:

(1) Water resources demand constraint

$$\xi_i \times (SW_a + GW_a) \geq WD_i, \quad i = 1, 2, \ldots, 7; \quad \forall t$$

$$\gamma_i \times (SW_a + GW_a) \geq WD_i, \quad i = 8, 9, \ldots, 12; \quad \forall t$$

(2) Water resources availability constraint

$$\text{Pr} \left[ \sum_{i=1}^{12} SW_a \leq TSW_i \right] \geq 1 - p, \quad \forall t$$

$$g_j x_j \leq h_j + \left[ \frac{\xi_j - \gamma_j}{3} \right] (1 - \alpha), \quad \forall j$$

$$x_j \geq 0, \quad \forall j$$

$$0 \leq \alpha \leq 1$$
Pr \left[ \sum_{i=1}^{12} GW_i \leq TGW_t \right] \geq 1 - P, \quad \forall t \quad (14)

(3) Arable area constraint
\begin{align*}
\sum_{i=1}^{7} \xi_i \times (SW_i + GW_i) \times WL_i \leq A_i^{\text{max}} + \left[ \frac{\xi_i}{\xi_{(i)}} + \frac{\theta_i}{3} \right] (1 - \alpha), \quad \forall t \quad (15) \\
\sum_{i=1}^{7} \xi_i \times (SW_i + GW_i) \times WL_i \geq A_i^{\text{min}} + \left[ \frac{\xi_i}{\xi_{(i)}} + \frac{\theta_i}{3} \right] (1 - \alpha), \quad \forall t \quad (16)
\end{align*}

(4) Constraint of electricity consumption during water resources allocation
\begin{align*}
\sum_{i=1}^{12} SW_i \times (EPS_i + ET_a + EC_a) \times CE_a + \sum_{i=1}^{12} GW_i \times (EPG_a + ET_a + EC_a) \times CE_a \leq EWM, \quad \forall t \quad (17)
\end{align*}

(5) Food security constraint
\begin{align*}
\sum_{i=1}^{12} \xi_i \times (SW_i + GW_i) \times WL_i \times PL_i \geq FD, \quad \forall t \quad (18)
\end{align*}

(6) Non-negative constraint
\begin{align*}
SW_i \geq 0, \quad \forall i, t \\
GW_i \geq 0, \quad \forall i, t \quad (19) \quad (20)
\end{align*}

4. Result and discussion

Figure 1 presents water allocation schemes under different scenarios. The results show that the total amount of water allocation for Jinan would be $12.2 \times 10^9$ m$^3$ to $12.6 \times 10^9$ m$^3$ over the planning horizon, with surface water accounting for nearly 62.7% and groundwater accounting for nearly 37.3%. During the planning periods, the total allocation of water resources in Jinan has an upward trend (i.e., with an average annual increase of about 11.7%), which is mainly owning to the increased water demand for economic development. In view of the current shortage of water resources in Jinan, reasonable planning of water resources in the future is critical for decision makers. Besides, the random uncertainty of water availability would also have impact on water allocation. As p level
increases from 0.01 to 0.15, the total amount of water allocation would increase 3.4%. This is because higher \( p \) level means higher risk of constraint violation, thus more water can be obtained to allocate for users. Therefore, decision makers should strike a balance between the risk of constraint violation and the increased availability of water resources.

Figure 2 shows the proportion of water allocation for different users. The results indicate that agriculture is the largest water consumer in Jinan, which accounts for 44.1% to 56.9% over the planning horizon. Besides, during the planning periods, the proportion of agricultural water allocation would decrease by 9.5% averagely, while the proportion of industrial water allocation would increase by nearly 15.6%. This is due to the high net benefit for industry. Therefore, on the premise of ensuring food security, reducing agricultural water allocation and increasing industrial water allocation can effectively optimize water allocation structure and increase system benefit.

Besides, with increase of \( \alpha \) level (i.e., from 0 to 1), the mean proportion of agriculture water allocation would decrease 4.6%. A higher \( \alpha \) level corresponds with lower constraint-violation degree and stronger desire of stakeholder to maximize system benefit, which leads to more arable land can be obtained for agriculture. With the increase of \( p \) level (i.e., from 0.01 to 0.15), the proportion of industrial water allocation would increase 0.9%, this is because more water can be obtained under high \( p \) level. However, the risk of constraint violation would also increase with the rising \( p \) level. Therefore, uncertainties have significant impact on water allocation schemes, decision makers should make different management plans according to different attitudes to system risks under undetermined conditions.

Figure 3 presents the cultivated area of different crops for Jinan. The results show that the annual cultivated area would be \( 628.4 \times 10^3 \) ha to \( 683.4 \times 10^3 \) ha over the planning horizon. Grain crop has the largest area, accounting for nearly 62.1% of the total cultivation. With the increase of \( \alpha \) level (i.e., from 0 to 1), the total cultivated area for Jinan would reduce 5.4% to 7.2%. In particular, when \( \alpha = 0 \) (\( p = 0.15 \)), Jinan has the largest cultivated area (i.e., \( 683.4 \times 10^3 \) ha). This is because lower \( \alpha \) level corresponds with higher degree of violating arable land constraint, thus more arable land can be obtained. Results imply that decision makers should adopt varied cultivation schemes based on local agricultural policies and different risk aversion attitudes.
Figure 3. Cultivated area of different crops.

Figure 4 shows the system benefit under different scenarios, which indicates that the system benefit would range from $324.8 \times 10^9$ yuan to $394.6 \times 10^9$ yuan. As $\alpha$ level increases (i.e., from 0 to 1), the system benefit would decrease by 9.8% to 12.7%. Besides, with the rise of $p$ level (i.e., from 0.01 to 0.15), the system benefit would increase by nearly 7.2%. Therefore, lower $\alpha$ and higher $p$ level can result in higher system benefit, while the system reliability would decrease. Different food demand levels would also have impact on system benefit. Compared with low food demand level, the system benefit under high demand level would decrease 2.1‰, which is owing to the relative low water benefit for grain crop. Therefore, to ensure the food security of the city, the system needs to sacrifice part of the benefit. Therefore, decision makers should make a trade-off between food security and economic development.

Figure 4. System benefit under all scenarios.
5. Conclusions
In this study, a fuzzy chance-constrained programming (FCCP) method has been developed to synergetic plan water-food nexus (WFN) system. FCCP has advantages in tackling uncertainties expressed as probabilistic distribution and flexible parameter. Then, a FCCP-WFN model has been formulated for the city of Jinan (China). Four $p$ levels, six $\alpha$ levels and three food demand levels are considered, which leading to 72 scenarios. The obtained results can provide numerical information to help decision makers alleviate water shortage and ensure food security.

Some major findings are: (i) during the planning periods, the surface water would still be the main water source for Jinan (accounting for 62.7%). Besides, the amount of water allocation would show an increase trend with an annual growth rate of 11.7%. Therefore, rational management of surface water is essential to alleviate the water shortage problem in Jinan. (ii) Jinan has sufficient grain production, with an annual cultivated area range from $628.4 \times 10^3$ ha to $683.4 \times 10^3$ ha. (iii) the system uncertainties have significant influence on water allocation schemes. With the growth of $p$ level (i.e., from 0.01 to 0.15), the total amount of water allocation would increase 3.4%, the proportion of industrial water allocation would increase 0.9%, and the system benefit would increase by nearly 7.2%; with the increase of $\alpha$ level (i.e., from 0 to 1), the mean proportion of agriculture water allocation would decrease 4.6%, the total cultivated area for Jinan would reduce 5.4% to 7.2%, and the system benefit would decrease by 9.8% to 12.7%. Therefore, decision makers should consider the impact of uncertainties when comprehensively managing the WFN system, and make different management schemes according to different attitudes to system risks under undetermined conditions.

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