PHENOMENOLOGY OF “ONIUM” PRODUCTION

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Abstract

The phenomenology of heavy quarkonia production in hadron collisions is reviewed. The theoretical predictions are compared to data. Commonly used production models are shown to fail in explaining all the experimental findings. The shortcomings of these models are analysed and possible improvements are discussed.

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1 Introduction

The production of heavy quarkonium states in high energy processes probes the very border between perturbative and non perturbative domains. It provides therefore quite stringent tests of our understanding of Quantum Chromodynamics (QCD) and has hence recently attracted much of the theoretical and experimental interest.

A large amount of data is now available, both from fixed target \[1\] and from collider \[2, 3\] experiments. In this talk I’ll restrict myself to the inelastic hadroproduction case, and in particular I’ll focus on the collider results. This because that’s where the most serious discrepancies between theory and experiments have shown up, demanding for a careful reanalysis of the quarkonium production mechanisms.

The outline of the talk is as follows. I will first describe the general framework which is believed to encompass quarkonium production, and the way this framework has in the past been formalized into production models. The approximations which had been made will be pointed out, and the results of theoretical predictions based on these models will be compared to experimental data and found to be sometimes in disagreement. Possible reasons for the discrepancies will then be considered, and a new model which may help reconciling theory with experiments will be briefly presented and discussed.

2 Production models

When considering any bound state object production two key questions have to be answered:

1. What is it made of?

2. How are its components produced?

As for the first question, there is evidence that heavy quarkonia are (mainly) composed of a heavy quark-antiquark pair, i.e. $c\bar{c}$ for charmonium (the $J/\psi$ family) and $b\bar{b}$ for the $\Upsilon$ family.

As for the second questions, there seems to be general agreement that QCD is the theory of strong interactions. It is therefore within its context that the production of the heavy quark and antiquark has to be described.

These two questions and the answers given above constitute the general framework to quarkonium phenomenology. At this points the different models we can think of do diverge and provide different descriptions and hence different results. I’ll now describe two models which have been put forward some years ago and widely used till today: Fritzsch’s Color Evaporation Model [4] (CEM) and Berger and Jones’ Color Singlet Model [5] (CSM).

2.1 The Color Evaporation Model

This model makes use of the parton-hadron duality hypothesis to relate the charmonium cross section to the quark-antiquark production cross section. In particular, it assumes that all the non perturbative effects that lead to the bound state formation cancel when considering inclusive final states, and therefore writes the production cross section for the quarkonium state $H$ as:

$$\sigma[H] = P_H \int_{4m_Q^2}^{4m_H^2} ds \sigma[Q\bar{Q}]$$  \hspace{0.5cm} (1)
On the right hand side we have the QCD cross section for producing a $Q\bar{Q}$ pair, $\sigma[Q\bar{Q}]$, integrated over the invariant mass range up to the mass $m_M$ of the lowest lying heavy-light meson (for instance the $D$ meson in the charm case). The factor $P_H$ has to be determined phenomenologically and provides the only differentiation among the various quarkonium states. While being therefore quantitatively not very predictive, this model has its central feature in that the differential distributions (like energy, $x_F$, or $p_T$ dependencies) should be equal for all the quarkonium states considered. It is thus mainly on this ground that its degree of validity will have to be assessed. For a review of this model and its achievements see for instance ref. [6].

2.2 The Color Singlet Model

This model tackles the problem of quarkonium production from an opposite point of view with respect to the Color Evaporation Model. It tries to reach the highest possible predictivity at the price of making stronger approximations.

Its central assumption is that the $Q\bar{Q}$ pair that will form the bound state is produced by the hard (short distance) QCD interaction with the correct spin and angular quantum numbers for the quarkonium considered, in a color singlet (and therefore observable) configuration and with zero or small relative momentum:

$$\sigma[n^{2S+1}L_J] = P_{nL} \sigma[Q\bar{Q}(n^{2S+1}L_J, \perp)]$$

(2)

The non perturbative effects are assumed to factorize into a single parameter $P_{nL}$, which can be related to the wave function of the bound state (or its derivative, for $P$ states) evaluated at the origin. It can be calculated within potential models (see for example ref.[7]) or extracted from experimental data of quarkonium decay widths, which can be calculated within the same model and yield expressions analogous to (2).

The difference with the previous model can easily be appreciated: the structure of the quarkonium (“What is it made of?”) is now defined much more precisely, and this allows detailed quantitative calculations for each different state $H = n^{2S+1}L_J$. On the other hand, to assume that a quarkonium is well described by a $Q\bar{Q}$ pair in a color singlet configuration may be too a crude approximation.

3 Phenomenology

When we make use of the CSM to obtain phenomenological predictions for quarkonia production we have to live with a double approximation.

The first one is intrinsic in the definition of a quarkonium as a color singlet $Q\bar{Q}$ pair with zero relative momentum: this amounts to neglect relativistic corrections, i.e. to work at lowest order in $v^2$, $v$ being the heavy quark speed in the quarkonium center of mass frame.

The second is the perturbative order at which the cross section for producing the heavy quark-antiquark pair is evaluated. The lowest order in $\alpha_s$ was usually considered to represent the dominant term in the expansion.

When calculating large $p_T$ production rates for quarkonia at the Tevatron, i.e. in $p\bar{p}$ collisions at $\sqrt{s} = 1800$ GeV, within the above approximations we immediately find huge discrepancies between theory and experiment. The $J/\psi$ and $\psi'$ rates are found to be underestimated by one or two orders of magnitudes.
We may at this point ask ourselves if we are operating within a wrong model or if we are just neglecting important contributions when making the aforementioned approximations. I’ll pursue the second point of view and try to present the CSM as being the lowest order approximation of a wider model. It must therefore be supplemented by higher order terms, both in $\alpha_s$ and in $v^2$, in order to produce reliable predictions.

Following the historical development I start considering higher order terms in $\alpha_s$. In 1993 Braaten and Yuan [8] pointed out that the process

\[ gg \rightarrow gg \rightarrow J/\psi gg \] (3)

while being of higher order ($\alpha_5^s$ vs. $\alpha_3^s$) with respect to the lowest order one,

\[ gg \rightarrow J/\psi g \] (4)

becomes however dominant at large $p_T$ (say $p_T > 5$–6 GeV). The technical reason for this is that the process (3) is not suppressed by a form factor $(m/p_T)^2$, $m$ being the heavy quark mass, which is instead present in the cross section of (4). The physical reason for the existence of this form factor is that it is difficult to produce the heavy quark and antiquark in a small spatial region, of size $1/p_T$, and still keep their relative momentum close to zero so that they can form a bound state. This shortcoming is absent in the process (4), as the gluon that fragments to the $J/\psi$ can have invariant mass of order $m \ll p_T$ and thus make the heavy quark-antiquark formation a longer distance process. Moreover, they could also show that in the limit of the gluon energy being much larger than its invariant mass the cross section could be approximated by the convolution of a kernel cross section for producing on-shell gluons with a fragmentation function for the gluon going to the $J/\psi$.

The fragmentation functions for a gluon or a heavy quark to go into any quarkonium state can be calculated [8, 9] in perturbative QCD (with the exception of the usual non-perturbative parameters related to the bound state formation) and the cross section is given by

\[
\frac{d\sigma[H(p_T)]}{dp_T} = \int \frac{d\hat{\sigma}[i(p_T/z), \mu]}{dp_T} D_i^H(z, \mu) \tag{5}
\]

In this equation $\hat{\sigma}$ is the kernel cross section for producing the parton $i$, and $D_i^H$ is the fragmentation function of $i$ to the quarkonium $H$. $\mu$ is the factorization scale, to be taken of the order of $p_T$.

Phenomenological predictions based on this formula [10] have promptly shown that the CDF data [2] on $J/\psi$ production could now be explained within a factor of two to five, depending on the input parameters, (see fig. 1a) while predictions for $\psi'$ productions where still falling a factor of 30 below the data (see fig. 1b).

The solution to this discrepancy, which has been named the “CDF Anomaly”, may have been found within the context I’m going to describe in the next Section.
4 The Factorization Approach

A rigorous frame for treating quarkonium production and decays has been recently developed by Bodwin, Braaten and Lepage [11]. Their so-called "factorization model" expresses the cross section for quarkonium production as a sum of terms each of which contains a short-distance perturbative factor and a long-distance non-perturbative matrix element:

\[ \sigma[H] = \sum_n \frac{F_n(\Lambda)}{m_{n-4}^d} \langle 0|O_n^H(\Lambda)|0 \rangle \]  

(6)

\( F_n \) are short-distance coefficients which can be calculated in perturbative QCD by expanding in powers of \( \alpha_s \). \( \Lambda \) is a scale which separates short and long distance effects. The cross section is however independent of \( \Lambda \) as its effect is compensated by the \( \Lambda \)-dependence of the non-perturbative matrix elements. \( \delta_n \) are related to the dimension of the operator \( O_n^H \). Finally, the matrix elements \( \langle 0|O_n^H(\Lambda)|0 \rangle \) can be defined rigorously in Non Relativistic QCD. They absorb the non-perturbative features of the process and can either be extracted from data or calculated on a lattice.

The main feature of this model, and the main difference with respect to the CSM, is that - pretty much like the Color Evaporation Model but in a much more sophisticated way - it takes into account the full Fock space structure of the quarkonium state. The latter is no more assumed to be represented by a color singlet \( Q\bar{Q} \) pair with the correct quantum numbers, but rather by an infinite series of terms:

\[
|H = n^{2S+1}L_J\rangle = O(1)|Q\bar{Q}(n^{2S+1}L_J), \downarrow\rangle \\
+ O(v)|Q\bar{Q}(n^{2S+1}(L \pm 1), J', S)g\rangle \\
+ O(v^2)|Q\bar{Q}(n^{2S+1}L_J, S)gg\rangle + ... \\
+ ...
\]

(7)

The CSM can be recovered by taking the lowest order term in eq. (6).

Higher order components are suppressed by powers of \( v \), but can become important if their associated short-distance coefficient \( F_n \) in eq. (6) is large. Braaten and Fleming [12] have shown that \( \psi \) production via gluon fragmentation through a color octet \( ^3S_1 \) pair, being its short distance coefficient of order \( \alpha_s \) only, can easily overwhelm ordinary gluon fragmentation to a color singlet pair (of order \( \alpha_s^3 \)) and may thus explain the \( \psi' \) abundance that CDF observes.

5 Conclusions

In this talk I have presented evidence that lowest order Color Singlet Model results are incapable of explaining CDF data of quarkonium production. Higher order terms both in \( \alpha_s \) and in \( v^2 \) are important and must be included to ensure a consistent and satisfactory description of quarkonia production.

The interplay between higher orders in \( \alpha_s \) and in \( v^2 \) is far from being trivial, and quarkonia production has to be regarded as a two-parameter problem. A consistent framework for treating the double expansion can be found in the factorization model developed by Bodwin, Braaten and Lepage.
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