Nonlinear wave in granular systems based on elastoplastic dashpot model

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Abstract
The dynamic dashpot models are widely used in EDEM commercial software. However, most dashpot models suffer from a serious numerical issue in calculating the granular chain because the denominator of damping force includes the initial impact velocity. Moreover, the existing dynamic dashpot models extended from the original Hertz contact law overestimated the contact stiffness in the elastoplastic contact phase. These two reasons above result in most dynamic dashpot models confronting some issues in calculating the multiple collision of the granular chain. Therefore, this investigation aims to propose a new composite dynamic dashpot model for the dynamic simulation of granular matters. First, the entire contact process is divided into three different phases: elastic, elastoplastic, and full plastic phases. The Hertz contact stiffness is still used in the elastic contact phase when the contact comes into the elastoplastic or full plastic phase. Hertz contact stiffness in the dynamic dashpot model is replaced by linearizing the contact stiffness from the Ma-Liu (ML) model in each time step. Second, the whole contact behavior is treated as a linear mass-spring-damper model, and the damping factor is obtained by solving the single-degree-freedom underdamped vibration equation. The new dynamic dashpot model is proposed by combining the contact stiffnesses in different contact phases and corresponding damping factors, which not only removes the initial impact velocity from the denominator of damping force but also updates the contact stiffness based on the constitutive relation of the contact body when the contact comes into the elastoplastic or full plastic phase. Finally, a granular chain is treated as numerical examples to check the reasonability and effectiveness of the new dynamic dashpot model by comparing it to the experimental data. The simulation shows that the solitary waves obtained using the new dashpot model are more accurate than the dashpot model used in EDEM software.

KEYWORDS
dynamic dashpot model, granular chain, numerical singularity, solitary wave
The dynamics of one-dimensional granular chains exhibit a rich landscape of unforeseen behavior phenomena by simple physical laws, which supports the formation of highly nonlinear, strongly localized solitary waves. This feature makes the granular chains a perfect candidate for various potential applications such as shock absorbers, impulse protectors, impact mitigation, acoustic switch, waveguide, and vibration filter, and so on. Although the granular chain has many known applications, it is hard to understand its intrinsic dynamic properties caused by the strong nonlinearity of contact force between the particles. The nonlinearity of the granular interaction is closely related to the local properties of the contact body and energy dissipation during impact.

In earlier works, the mechanics characteristic between the particles is described by the Hertz contact law. To consider the energy dissipation during contact, the equation of motion of granular chain is improved by introducing the dissipative term related to the relative contact velocity between the particles or the coefficient of restitution to represent the energy dissipation. In this process, the energy dissipation during contact is not represented by the damping factor from the continuous dashpot models. Namely, the contact force between the particles is still calculated based on the original Hertz contact law. This is one of the reasons why the existing dashpot models are seldom applied to the simulation of the granular chain. More importantly, almost all of the existing dashpot models derived based on the energy conservation before and after impact suffered from the serious numerical singular problem because the denominator of damping force in the dashpot models includes the initial impact velocity. In this scenario, when the initial impact velocity of the particle in the granular chain is equal to zero, the damping force in the dashpot model is infinite, which violates the mechanics characteristic between the particles, and triggers the numerical issues. To sidestep the numerical singular issue, some damping factors are assumed as constant according to the empirical evidence. However, this approach is not a universal method regarding the different impact scenarios. The alternative method is to develop the damping factor based on the spring-damper model, which removes the initial impact velocity from the denominator of the damping factor in the dashpot model. Therefore, the dashpot model without the initial impact velocity in the denominator of damping force is used in the commercial EDEM software. Nevertheless, most existing dashpot models are difficult to accurately describe the contact behavior involving elastoplastic or plastic deformation. That is mainly because the Hertz contact stiffness overestimates the contact stiffness in the elastoplastic or plastic contact phase. Accordingly, to overcome this problem, some scholars developed a series of quasi-static elastoplastic contact models to describe the elastoplastic contact events using the piecewise function, including the elastic, elastoplastic, and full plastic phase. Since these elastoplastic contact models depend entirely on the constitutive relation of contact bodies, they can accurately reflect the relationship between the force and displacement in the elastoplastic phase.

Whereupon, these quasi-static elastoplastic models had been used in calculating the solitary wave propagation of elastoplastic granular chain in past decades. However, since the contact behavior in the elastoplastic phase follows the different constitutive relation between the compression and recovery phase, the maximum contact deformation and residual deformation must be identified and saved at each impact for the sake of preparing for the following contact behavior. The situation makes the simulation process of the contact event complicated compared with the dynamic dashpot models. That is why the quasi-static elastoplastic contact models were not widely used in the EDEM commercial software.

In light of the research background above, two kinds of models, including quasi-static elastoplastic contact models and dynamic dashpot models, without the numerical singular issue, are applied to simulate the solitary wave propagation in the granular chain. However, both of them confront with some issues. As for the dynamic dashpot models, they can represent the energy dissipation by the damping factors and do not need to distinguish the compression and recovery phase in calculating the impact behavior. This approach extremely simplifies the simulation strategy of collision events and benefits for programming. Nevertheless, they cannot reflect the elastoplastic contact characteristic because of the Hertz contact stiffness. On the contrary, although the quasi-static elastoplastic contact models can accurately describe the relationship between the loading and displacement in the elastoplastic contact phase, they made the calculation strategy of elastoplastic contact behavior in the granular chain more complicated. It needs to be emphasized that the elastoplastic collision in the granular chain depends on the constitutive relation of contact bodies, and the convenient and straightforward calculation strategy is determined by the damping factor. Therefore, this investigation not only replaces the Hertz contact stiffness of existing dashpot models in the elastoplastic phase by means of the quasi-static elastoplastic contact model (Ma-Liu (ML) model), but also proposes a new damping factor without initial impact velocity by solving a single-degree-freedom underdamped vibration equation. The main contributions of this investigation can be highlighted as follows: (i) the new dashpot contact model with a linear elastoplastic contact stiffness and a new hysteresis damping factor is proposed; (ii) the new dashpot model does not suffer from a seriously numerical issue when the initial impact velocity is very small or equal to zero; (iii) compared with the dashpot model used in EDEM software, the new dashpot model can accurately capture the elastoplastic contact behavior in the granular chain. The structure of this investigation can be organized as follows: In Section 2, the ML model is introduced simply. A new dashpot model is proposed in Section 3. The horizontal granular chain is simulated based on the new dashpot model in Section 4. The main conclusions are summarized in Section 5.
## 2 ML STATIC CONTACT FORCE MODEL

The loading phase of the elastoplastic contact model is governed as

$$F(\delta) = \begin{cases} \frac{4}{3}E\sigma_0^2\delta^3 & \delta < \delta_e, \\ \delta\left(c_1 + c_2\ln\frac{\delta}{\delta_0} + c_3\delta - \delta_e\right) & \delta_e \leq \delta < \delta_p, \\ F_p + k_1(\delta - \delta_p) & \delta \geq \delta_p, \end{cases}$$

(1)

where $F$ is the contact force; $\delta$ is the contact deformation; the effective elastic modulus and radius are expressed as $1/E = (1 - \nu^2)/E_1 + (1 - \nu_2^2)/E_2$, $R = R_1R_2/(R_1 \pm R_2)$, $R_1$ and $R_2$ are the radii of curvature of the contact bodies, $E_1$ and $E_2$ are the Young’s modulus of the contact bodies, $\nu_1$ and $\nu_2$ are the Poisson’s ratios of the contact material, $\delta_e = \pi^2R_0^2/4E_1^2$ is the critical elastic deformation, $\sigma_0 = 1.61\sigma_y$ is the critical value of yielding, $\sigma_y$ is the yield stress. $\delta_p = \delta_0^2/2$ is the critical plastic deformation. $\epsilon$ is the dimensionless parameter that corresponds to a geometric relationship when the pressure on the contact surface approximately approaches uniformity. Its value is within the scope from 13 to 20 when the contact spheres are made of the same materials.43

Moreover, the specified form of the coefficient in this contact model is expressed as

$$k_1 = 2nR_0\psi\sigma_y, \\ F_p = \delta_0(c_1 + c_2\ln(\epsilon^2/2) + c_3, \\ c_1 = \frac{p_0(1 + \ln(\epsilon^2/2) - 2\psi\sigma_y)}{\ln(\epsilon^2/2)}\pi R, \\ c_2 = \frac{2\psi\sigma_y - p_0}{\ln(\epsilon^2/2)}\pi R, \\ c_3 = F_p - c_1\delta_e, \ F_p = n^2R_0^2p_0^2/6E_2, \ (2)$$

where $\psi$ is the dimensionless parameter that corresponds to a ratio between the Brinell hardness and the yielding strength of the material, which is in the range from 2.6 to 3.0.43

During the unloading phase, the constitutive relation between the contact force and displacement is expressed as

$$F(\delta) = \begin{cases} \frac{4}{3}E\sigma_0^2\delta^3 & \delta_{\text{max}} < \delta_e, \\ \frac{4}{3}E\left(R_0^2\delta \pm \delta_e\right)^3 & \delta_e \leq \delta_{\text{max}} < \delta_p, \\ \frac{4}{3}E\left(R_0^2\delta \pm \delta_p\right)^3 & \delta_{\text{max}} \geq \delta_p, \end{cases}$$

(3)

where $\delta_{\text{max}}$ is the maximum contact deformation and $\delta_e$ is the residual contact deformation.

Since both elastoplastic and plastic deformation happened before the restitution phase, the curvature radius is larger than before. Therefore, when the contact behavior aborts at the elastoplastic compression phase, the radius of curvature of the contact body can be written as

$$R_{\text{ep}} = \frac{F_{\text{ep}}}{F_{\text{max}}}, \ F_{\text{ep}} = \frac{4}{3}E\sigma_0^2\delta_{\text{ep}}^3, \ (4)$$

where $R_{\text{ep}}$ is the radius of curvature after impact in the elastoplastic phase and $F_{\text{max}}$ is the maximum contact force.

When the contact behavior ends at the plastic compression phase, the radius of curvature of the contact body can be written as

$$R_{\text{p}} = \frac{F_{\text{p}}}{F_{p_{\text{max}}}}, \ F_{p_{\text{max}}} = \frac{4}{3}E\sigma_0^2\delta_{p_{\text{max}}}^3, \ (5)$$

where $R_{\text{p}}$ is the radius of curvature after impact in the plastic phase. $F_{\text{ep}}$ and $\delta_{\text{ep}}$ are the load and critical plastic contact deformation at the inception of the plastic contact phase.

The quasi-static ML contact model describes the relationship between the loading, unloading, and displacement and calculates the dynamic elastoplastic collision behavior. When the initial impact velocity is assumed as $0.2 \text{ m/s}$, the simulation parameters of the contact bodies can be seen in Table 1. The dimensionless parameters are the same as the literature $\psi = 3.0$, $\epsilon = 13$. As for the ML model, the loading path is the same as the unloading path in the elastic phase, governed by the Hertz contact law. When the contact phase comes into the elastoplastic or full plastic phase, the loading path is significantly different from the unloading path. The elastic contact phase happens in a short time at the beginning stage, which has a slight effect on the entire contact behavior. Moreover, the critical elastic deformation equals $4.0542E-8 \text{ m}$, which means the elastoplastic deformation is prone to be activated despite the small initial impact velocity, especially in the contact materials with larger Young’s modulus and smaller yield strength. It is noteworthy that the relationship between the contact force and deformation in the compression phase is approximately linear when the contact behavior comes into the elastoplastic or plastic phase (Figure 1).

## 3 A NEW COMPOSITE DASHPOT MODEL APPLIED TO THE GRANULAR CHAIN

At present, there are around 15 kinds of dynamic dashpot models.33 Most existing dynamic dashpot models are only suited for simulating the single impact with initial contact velocity between contact bodies. When calculating the multiple impacts in the granular chain where some particles are at rest initially, such as granular chain, Hopkinson bar, and so on. Namely, most existing dynamic dashpot

| Element | Young’s modulus (Pa) | Poisson ratio | Radius (m) | Yield strength (Pa) | Density (kg/m³) |
|---------|----------------------|---------------|------------|---------------------|-----------------|
| Body 1  | 2.07E11              | 0.30          | 2.05E-2    | 1.00E7              | 2700            |
| Body 2  | 2.07E11              | 0.30          | 2.00E-2    | 1.03E9              | 7800            |
models suffer from a seriously numerical problem when the initial impact velocity is very small or equal to zero. The damping force in the existing dynamic dashpot models is infinite when the initial impact velocity equals zero, which is impossible and violates the realistic physical meaning. That is mainly because the denominator of damping force in most models includes the initial impact velocity. The other dashpot models removed the initial impact velocity from the denominator of damping force based on the nonlinear spring damper model, which is used in EDEM commercial software. However, the Hertz contact stiffness in this dashpot model cannot represent the contact stiffness in the elastoplastic contact phase, which induces the simulation error when the elastoplastic deformation is activated in the granular chain. To overcome the deficiency above, the derivation process of the damping factor is equivalent to the solution procedure of a single-degree-freedom underdamped vibration system. The elastoplastic contact behavior is treated as a linear spring dashpot model in Figure 2 because the elastoplastic contact stiffness can be linearized according to the constitutive relation of contact bodies in Figure 1.

The equation of motion during the contact phase can be written as

\[ M \ddot{\delta} + D \dot{\delta} + K \delta = 0, \quad (6) \]

where \( M \) is the mass of the contact body, \( D \) is the damping coefficient, and \( K \) is the stiffness coefficient of spring.

The solution of the underdamped system is expressed as

\[ \delta(t) = Ae^{-\xi\omega t} \sin(\omega_d t + \phi), \quad (7) \]

where \( A \) is the amplitude, \( \phi \) is the phase angle, \( \omega \) is the frequency, \( \omega = \sqrt{K/M} \), \( \omega_d \) is the damped natural frequency, \( \omega_d = \omega \sqrt{1 - \xi^2} \), \( \xi \) is the damping, and \( \xi = D/2\omega M \).

The amplitude and phase angle can be determined from the initial condition

\[ \begin{align*}
    t = 0, \; \delta = 0 & \Rightarrow 0 = \sin(\phi) \\
    t = 0, \; \dot{\delta} = \dot{\delta}^{(-)} & \Rightarrow \dot{\delta}^{(-)} = A\omega_d \cos(\phi) \\
    0 = \phi, \; A = \frac{\dot{\delta}^{(-)}}{\omega_d}
\end{align*} \quad (8) \]

Equation (7) can be rewritten as

\[ \delta(t) = \frac{\dot{\delta}^{(-)}}{\omega_d} e^{-\xi\omega t} \sin(\omega_d t), \quad \omega_d = \omega \sqrt{1 - \xi^2} = \frac{\sqrt{4MK - D^2}}{2M}. \quad (9) \]

The duration time of the entire contact process is expressed as

\[ t_i = \frac{\pi}{\omega_d} = \pi \left( \frac{K}{M} - \left( \frac{D}{2M} \right)^2 \right)^{-\frac{1}{2}}. \quad (10) \]

When an entire contact process is over, the deformation in Equation (9) is equal to zero, the deformation velocity is written as according to Equation (9)

\[ \delta(t_i) = \frac{\dot{\delta}^{(-)}}{\omega_d} e^{-\xi\omega t_i} \sin(\omega_d t_i) + \omega_d \cos(\omega_d t_i) \Rightarrow \delta(t_i) = \dot{\delta}^{(-)} e^{-\xi\omega t_i}. \quad (11) \]

Based on the definition of Newton’s coefficient of restitution (COR), one can obtain the following equation

\[ \begin{align*}
    \delta(t) &= Ae^{-\xi\omega t} \sin(\omega_d t + \phi) \\
    c_r &= \frac{\delta(t)}{\dot{\delta}^{(-)}} = \frac{\delta^{(-)} e^{-\xi\omega t}}{\omega_d} \Rightarrow c_r = e^{-\xi\omega t} \Rightarrow \ln c_r = -\xi\omega t_i \\
    &\Rightarrow \ln c_r = -\frac{D}{2\omega_d M} \pi = -\frac{\pi D}{2\omega_d M} D = 2 \ln c_r \sqrt{\frac{KM}{\pi^2 + \ln^2(c_r)}}.
\end{align*} \quad (12) \]

where \( c_r \) is the COR and \( D \) is the damping coefficient.
Therefore, according to Equation (6), the other new dynamic dashpot model in the elastoplastic or plastic phase can be written as

\[
F_{ep} = K\delta + D\delta = K_p\delta + 2|\ln c_r| \sqrt{\frac{K_pM}{\pi^2 + \ln^2(c_r)}} \delta.
\]  

(13)

where \(K_p\) is the linearized contact stiffness in the elastoplastic or plastic phase based on the ML model in Figure 1.

When the contact behavior is in the elastic phase, the contact stiffness \(K_c\) is assumed as \(K = K_p\delta^3\), which is still linear with the contact deformation. Namely, the dynamic dashpot model can also be obtained according to Equation (6):

\[
F_{ne} = K\delta + D\delta = K_c\delta + D\delta = K_c\delta^3 + 2|\ln c_r| \sqrt{\frac{K_cM}{\pi^2 + \ln^2(c_r)}} \delta\frac{1}{2}, K_c = \frac{4}{3}E\rho R^2.
\]  

(14)

It is noteworthy that the new dynamic dashpot model is almost the same as the dashpot model used in the EDEM software, which is expressed as:

\[
F_{dem} = K_c\delta^3 + \frac{\sqrt{3}}{2}\kappa D\delta^3\delta.
\]  

(15)

Equations (13) and (14) can evaluate the dynamic collision behavior and accurately calculate the dynamic elastoplastic collision event. Obviously, the dashpot model in Equation (15) used in the EDEM software cannot accurately capture the elastoplastic or plastic collision behavior because the Hertz contact stiffness fails to describe the relationship between the contact force and deformation in the elastoplastic or plastic phase.

### 4 | HORIZONTAL GRANULAR CHAIN

To exhibit the merit of the new dashpot model in calculating dynamic elastoplastic contact behavior, Figure 3 shows the schematic diagram of the experimental setup that is established by Daraio et al.\textsuperscript{18} for a monodisperse granular chain of 70 stainless steel particles assembled horizontally, the particle is made of stainless steel 316. The particle size and material properties in the chain are identical to each other, as shown in Table 2. The striker impacts along the ramp activated the solitary waves. The contact forces are measured by the calibrated piezo-sensor inside the selected particles. The other information about this experimental setup in detail can refer to the literature.\textsuperscript{17,18} The initial impact velocity of the striker is assumed as 1.77 m/s. The dimensionless parameters are e = 13. The critical elastic deformation is equal to 5.4822E−7 m, the critical plastic deformation is equal to 4.6324E−5 m. The COR is assumed as 0.7.

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**TABLE 2 Simulation parameters**

| Parameters         | Value       |
|--------------------|-------------|
| Radius             | 2.38 mm     |
| Poisson ratio      | 0.3         |
| Young’s modulus    | 193 GPa     |
| Yield stress       | 940 MPa     |
| Mass               | 0.45 kg     |
| Position of the sensor | 9, 16, 24, 31, 40, 50, 56, 63 |

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**FIGURE 3** The experimental setup of the one-dimension granular chain. Reproduced (adapted) with permission.\textsuperscript{18} Copyright 2009, American Physical Society
4.1 Comparison between new dashpot model and ML model

Since the ML model ultimately depends on the constitutive relation of particles, the solitary wave in the granular chain obtained using the ML model replicates the results of the experimental data well. However, the complicated calculation process is not beneficial for programming, especially in a considerable amount of granular matter. On the contrary, the new dashpot model only needs to identify which contact phase is happening, and then, the contact force between the particles can be calculated using Equation (14) in the elastic phase and Equation (13) in the elastoplastic or plastic phase. Therefore, the new dashpot model simplifies the calculation process of elastoplastic collision behavior because it ignores the residual deformation and maximum contact deformation in each contact, and does not need to differ the compression or recovery phase as well.

Figure 4 shows the comparison between our numerical findings and the existing experimental results from Figure 1B in the Ref. 18. As for the new dynamic dashpot model, when the contact deformation between the particles is smaller than the critical elastic deformation, the contact forces between the particles are calculated by Equation (14). When the contact deformation exceeds the critical elastic deformation, the contact force between the particles is evaluated based on Equation (13) by linearizing the elastoplastic or plastic contact stiffness in each time step based on the ML model. The maximum contact force estimated using the new dashpot model is larger than the loading force obtained using the ML model in the first five solitary waves, which is mainly because the new dashpot model has a redundant damping term compared with the ML model in the elastoplastic contact phase. In addition to this, the red error bar between the experimental data and the new dashpot model in Figure 4A is smaller than the blue error bar between the experimental data and the ML model in Figure 4B. This result means the maximum contact force obtained using the new dashpot model is more closed to the experimental data compared with the ML model in the elastoplastic phases in Figure 4C.

Significantly, in Figure 5B, the loading paths of the different particles are the same, but the unloading paths are different from each other. Namely, the ML model can obtain the residual deformation according to the unloading path. However, in Figure 5A, the hysteresis loop depends on the relative deformation velocity direction during impact. Hence, the compression and recovery paths are discrepant for each particle. Meanwhile, the energy dissipation obtained by the ML model is represented by the difference between the loading and unloading path in Figure 5B, which is homogenized to the hysteresis loop in the new dashpot model in Figure 5A. Although these two contact models, including the ML model and the new dashpot model, used different approaches to calculate the elastoplastic contact behavior, the maximum contact force and the maximum contact deformation can keep harmonizing with each other. Therefore, the relative impact velocity between the particles calculated using the ML model is closed to the results obtained using the new dynamic dashpot model in Figure 6.

As the solitary wave propagates forward ceaselessly, the kinetic energy is gradually dissipated, the contact behavior comes into the elastic

![Figures](A) Contact force obtained from the new dynamic dashpot model. (B) Contact force obtained from the ML model. (C) Contact force from the different contact models. Reproduced (adapted) with permission. Copyright 2009, American Physical Society
phase in the last three solitary waves, ML model considered there is no kinetic energy to be dissipated in the elastic phase. However, the new dynamic dashpot model in Equation (14) employs the damping factor in Equation (12) to calculate the energy dissipation during the elastic contact phase. Therefore, the peak of the contact force obtained using the new dynamic dashpot model is smaller than those obtained by the ML model. It is noteworthy that the solitary wave tail obtained using the new dashpot model arises from the coupling relationship between the particles in multiple-collision of the granular chain.

### 4.2 Comparison between new dashpot model and dashpot model used in EDEM

Since the dashpot model used in EDEM software Equation (15) is extended based on the Hertz contact law, the contact force between the particles is estimated using Equation (15) no matter what in the elastic or plastic phase. However, the new composite dashpot model distinguishes the entire contact process into three phases: the elastic in Equation (14), elastoplastic, and full plastic phase in Equation (13). This section
implements the comparison between the new dashpot model and the dashpot model used in EDEM software.

In Figure 7C, the solitary wave propagation obtained using the new dashpot model in Figure 4A is almost identical to the results using the dashpot model used in EDEM software in Figure 7A, which can keep consistent with the experimental data. However, in calculating the maximum contact force between the particles, the blue error bar between the new dynamic dashpot model and experimental data is smaller than the red one between the dashpot model used in the EDEM software and experimental data in Figure 7B. The absolute error percentage of the peak value of the solitary wave can be seen in Table 3. In each solitary wave, the results

**FIGURE 7** Comparison between the dynamic dashpot model used in the EDEM and the new dynamic dashpot model regarding these particles with the sensor in a one-dimensional granular chain. (A) Contact force obtained from the dashpot model used in EDEM software. (B) Error analysis. (C) Contact force from the different contact models. Reproduced (adapted) with permission18 Copyright 2009, American Physical Society

**TABLE 3** Error percentage of maximum contact force between the dashpot model and experimental data

| Number | 9   | 16  | 24  | 31  | 40  | 50  | 56  | 63  |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|
| New dashpot model | 4.23% | 6.68% | 19.44% | 1.44% | 9.12% | 0.5% | 12.33% | 25.39% |
| Dashpot model used in EDEM | 5.36% | 7.39% | 22.44% | 10.03% | 10.19% | 16.38% | 26.54% | 36.79% |

**FIGURE 8** Comparison between the dynamic dashpot model used in the EDEM and the new dynamic dashpot model regarding these particles with the sensor in a one-dimensional granular chain
from the new dashpot model are closed to the experimental data compared with the ML model. The reason lies in that the new dynamic dashpot model accurately captures the contact stiffness in the elastoplastic or plastic contact stiffness to obtain the real elastoplastic or plastic force in the first term $K_\rho \delta$ in Equation (13) because both damping forces from them are almost the same with each other in Figure 8. This conclusion illuminates that the new dynamic dashpot model possesses higher fidelity than the dashpot model used in the EDEM software. It can accurately describe the relationship between the contact force and deformation in three different contact phases, i.e., elastic, elastoplastic, and plastic phases.

5 | CONCLUSION

The motivation of this investigation lies in that most existing dynamic dashpot models suffer from a serious numerical issue when the initial impact velocity is very small or equal to zero. Moreover, since the dynamic simulation process of the elastoplastic collision behavior is complicated when using the static elastoplastic contact model, this investigation proposes a new composite dashpot model including the elastic, elastoplastic, and full plastic phases for the dynamic simulation of granular matter by means of the constitutive relation of the particles. In this process, the collision between the particles is treated as a linear mass-spring-damper model because the relationship between the force and deformation is approximately linear in the elastoplastic or plastic phase; based on this assumption, the damping factor is derived by solving a single-degree-freedom underdamped vibration equation. A new dashpot model is proposed by the linear elastoplastic contact stiffness and a new hysteresis damping factor.

The propagation law of solitary wave in the granular chain is calculated using the new dashpot model closed to the ML model, which is also validated by the experimental data. The simulation in Figure 4 illuminates the effectiveness of simplification using the damping factor from the new dashpot model in the calculation process of elastoplastic collision compared with the ML model. The other merit of the new dashpot model lies in that it depends on both the Hertz contact stiffness in the elastic phase and the elastoplastic contact stiffness in the elastoplastic phase. By contrast, the dashpot model used in EDEM software only depends on the Hertz contact stiffness in the entire contact process. The simulation in Figure 7 shows that the error from the dashpot model used in EDEM software is larger than the new dashpot model because the dashpot model used in EDEM software fails to capture the mechanics features in the elastoplastic phase. Therefore, the new dashpot model not only sidesteps the numerical singular issue, and accurately calculates the contact force between the particles in the elastoplastic or plastic phases compared with most existing dashpot models.

Nevertheless, the proposed dashpot model is valid when ignoring the strain rate or plastic flow of the contact body during impact. Furthermore, the proposed dashpot model can accurately evaluate the maximum contact force that is an index to measure whether or not the collision damages the mechanical structure. The wear phenomenon between the contact bodies caused by the elastoplastic contact events can be predicted based on the Archard’s wear model and the new dashpot model because it can accurately calculate the contact force compared with the most existing dashpot model. More importantly, it can be used to describe the coupling relationship between the local plastic deformation and the system’s elastic deformation in the flexible multibody system.

| NOMENCLATURE |
|---------------|
| $A$ | amplitude |
| $c_r$ | Newton’s coefficient of restitution (COR) |
| $D$ | damping coefficient |
| $E_1$ and $E_2$ | Young’s modulus |
| $F$ | loading force or contact force |
| $K$ | stiffness coefficient of spring |
| $K_p$ | linearized contact stiffness in the elastoplastic or plastic phase |
| $K_c$ | Hertz contact stiffness |
| $M$ | mass of the contact body |
| $p_y$ | critical value of yielding |
| $R_1$ and $R_2$ | radius of curvature |
| $R_{cp}$ | radius of curvature after impact in the elastoplastic phase |
| $R_p$ | radius of curvature after impact in the plastic phase |
| $t_1$ | duration time of the entire contact process |
| $\nu_1$ and $\nu_2$ | Poisson ratio |
| $\sigma_y$ | yield stress |
| $\delta$ | contact deformation |
| $\delta_c$ | critical elastic deformation |
| $\delta_p$ | critical plastic deformation |
| $\varepsilon$ and $\psi$ | dimensionless parameters |
| $\phi$ | phase angle |
| $\omega$ | frequency |
| $\omega_d$ | damped natural frequency |
| $\xi$ | damping |

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CONFLICT OF INTEREST
The authors declare that there are no conflict of interest.

DATA AVAILABILITY STATEMENT
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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