THE PRIMORDIAL HELIUM ABUNDANCE

V. LURIDIANA
Instituto de Astrofísica de Andalucía (CSIC), Camino Bajo de Huétor 24, 18008 Granada, Spain

The primordial abundance of $^4\text{He}$, $Y_P$, is one of the hottest themes in present-day astronomy, mostly due to its cosmological relevance. The disagreement between different determinations has been currently reduced to the 1-2% level, but these differences are still large enough to have deep implications for Big-Bang nucleosynthesis. It is therefore crucial to estimate precisely the uncertainties involved in the measurement of $Y_P$. Here, I review the methods used in the determination of $Y_P$ and the related uncertainties. I also discuss some recent results, and emphasize the assumptions underlying the differences among them.

1 Introduction

Research on the abundance of primordial helium ($Y_P$) has entered its golden age. The exact value of $Y_P$ is one of the few missing pieces in the puzzle of the Big-Bang scenario, fueling a lively debate among astronomers, and motivating a huge amount of literature on the subject. The other side of the coin is that this literature is stuffed with numerically subtle argumentations, devoted to the quantitative determination of very specific aspects of the issue. Having established the basic principles, all the effort is now committed to the fine tuning of these details. $Y_P$ measurements are increasingly precise, but still not accurate enough to be used as strong cosmological constraints. The debate concerns now the third digit of $Y_P$: a scale small indeed, but where differences still matter.

My scope here is to present an overview of the subject and its implications. The most important part of this task is trying to help the reader understand, without getting lost in numbers, where the differences come from. Unfortunately, I find it impossible to be simultaneously clear and exhaustive in just a few pages. My own choice is to be as clear as I can, though it implies that many important studies on this topic will not be mentioned. I apologize for that to their authors, hoping that this review will have, at least, the effect to make readers grasp the essence of the debate, and, hopefully, want to know more.
2 Cosmological implications

Primordial nucleosynthesis is one of the three pillars supporting the Big-Bang model for the origin of the Universe, the other two being the cosmic microwave background and the Hubble expansion (Schramm, Dolgov). Since the Hubble expansion is also predicted by alternative cosmological models, the Big-Bang nucleosynthesis (BBN) has a fundamental role as a decisive proof of the Big Bang. In the scenario of standard BBN, the primordial abundances of four light isotopes (D, $^3$He, $^4$He, and $^7$Li) depend only on the baryon-to-photon ratio $\eta$; the corresponding relations for three of them are shown in Fig. 1. $\eta$ is a key parameter in cosmology, since it allows determination of $\Omega_b$, the baryon fraction of the closure density. $\eta$ is overdetermined by the four isotopes, therefore their agreement provides an extremely robust test of BBN, and the four are actively investigated. Each isotope tells a different history, and represents a unique technical and theoretical challenge (see, e.g., the review by S. Burles in these proceedings). Unfortunately, the data obtained in this field are still contradictory and uncertain. As for $^4$He, it is the easiest to observe among the four, but also the less sensitive to $\eta$, so that its measurements must be highly precise to be cosmologically relevant. In the following, I will describe how these measurements are carried out, highlighting the uncertainties involved in the analysis, and the causes of disagreement among different determinations.

3 $Y_P$ determinations: the method

The history of $Y_P$ determinations began in 1966 with Peebles, who estimated $0.26 < Y_P < 0.28$, based on a simple cosmological model. Many $Y_P$ determinations have followed since; some of them are plotted in the upper right panel of Fig. 1 as a function of the publication date. The plot shows that the data are progressively converging toward a common value, but also that a significant scatter remains, even among the most recent results. In particular, the data published in the last five years tend to cluster around two preferred values ($Y_P \sim 0.238$ and $Y_P \sim 0.245$), which are, with their error bars, mutually exclusive. Intermediate results are also found.

Before analyzing the causes underlying this disagreement, I will give a general description of the method used to determine $Y_P$. The basic strategy has been originally proposed by Peimbert
& Torres-Peimbert\textsuperscript{24}, since the Universe was born with zero metallicity \((Z)\), and both \(Z\) and the helium abundance \((Y)\) increase with time, \(Y_P\) can be found by extrapolating back to \(Z=0\) the \((Y, Z)\) relation for a sample of objects (Fig. 1, lower right panel). Variations of this basic strategy also exist: e.g., \(Y_P\) (or, more precisely, an upper limit to it) can also be found by averaging \(Y\) in a sample of extremely metal-poor objects (Searle & Sargent\textsuperscript{32}, Steigman, Viegas, & Gruenwald\textsuperscript{33}).

The application of this method relies on precise measurements of \(Y\) and \(Z\) in individual objects. This is done by means of nebular abundance diagnostics, which are relations linking the observed emission line intensities to the corresponding ionic abundances in the gas. Many types of objects have been used in this analysis: planetary nebulae (D’Odorico, Peimbert, & Sabbadin\textsuperscript{31}), H II regions, either galactic (Peimbert & Torres-Peimbert\textsuperscript{44, 45}, Magellanic (Peimbert & Torres-Peimbert\textsuperscript{44, 45}, Peimbert, Peimbert, & Luridiana\textsuperscript{28}), or extragalactic (Skillman\textsuperscript{46}, Torres-Peimbert, Peimbert, & Fierro\textsuperscript{44}), dwarf irregulars (dIrrs) and blue compact galaxies (BCGs) (Lequeux et al.\textsuperscript{24}, Kuntz\textsuperscript{11}, Pagel, Terlevich, & Melnick\textsuperscript{23}, Skillman & Kennicutt\textsuperscript{67}, Izotov, Thuan, & Lipovetsky\textsuperscript{19, 20}, Izotov & Thuan\textsuperscript{18, 17}, Fields & Olive\textsuperscript{14}, Peimbert et al.\textsuperscript{85}).

Since \(Z\) cannot, in practice, be directly measured, individual metals are used as metallicity tracers; the expected behavior of the element with respect to \(Y\) determines the type of fit to the data. Occasionally, carbon has been used for this scope (see, e.g., Steigman\textsuperscript{32}, and Fields & Olive\textsuperscript{43}) but it does not yield, for various reasons, a good determination of \(Y_P\). Nitrogen is more frequently used; the relation between \(Y\) and \(N\) is sometimes assumed to be linear, but this assumption is an oversimplification, roughly valid only at low \(Z\) (see, e.g., Fields & Olive\textsuperscript{43}; van Zee, Salzer, & Haynes\textsuperscript{53}). On the other hand, the linear behavior is a much better assumption in the case of oxygen. Oxygen, the most abundant heavy element, is the best metallicity tracer.

The slope in the \((Y, Z)\) relation can be determined either observationally (i.e., from the fit to the data; Peimbert & Torres-Peimbert\textsuperscript{44}, D’Odorico et al.\textsuperscript{6}, Melnick et al.\textsuperscript{22}, Pagel et al.\textsuperscript{83}, Izotov et al.\textsuperscript{18, 24}, Izotov & Thuan\textsuperscript{18}, see also Pagel & Portinari\textsuperscript{12} and Høg et al.\textsuperscript{14} for a different approach), or by means of chemical evolution models (Lequeux et al.\textsuperscript{24}, Carigi et al.\textsuperscript{6}). Most derived values are in the range \(~ 2 - 6\), with large uncertainties. Obviously, the impact on \(Y_P\) of the uncertainty in \(dY/dZ\) is minimized by the inclusion of BCGs and dIrrs in the sample, since these are the most metal-poor galaxies known. On the other hand, \(dY/dZ\) is better determined with a wide baseline in metallicity.

Helium abundance determinations are most often made by means of optical observations, although data from other wavelength ranges have also been used, e.g. infrared (Rubin et al.\textsuperscript{64}) and radio (Churchwell, Mezger, & Huchtmeier\textsuperscript{1}; Shaver et al.\textsuperscript{6}; Peimbert et al.\textsuperscript{47, 48}). Helium in photoionized regions can exist in all its three ionization stages. Neutral helium cannot be observed, and will be dealt with in the next section. Double-ionized helium, if present, gives rise to the He II recombination spectrum, which is straightforward to interpret in terms of abundance. None of these two ions is abundant in H II regions: most of the helium is always singly ionized, and shows up in the spectrum as prominent He i recombination lines. He i has two separate level systems, the singlets and the triplets, and the transitions between them are forbidden by electric dipole selection rules. While singlet lines are relatively easy to interpret, the triplet spectrum is complicated by the metastability of the lowest triplet level, the \(2^3S\), where electrons tend to accumulate (Osterbrock\textsuperscript{23}), with two important consequences.

First, photons emitted in transitions ending on \(2^3S\) can be reabsorbed, and, eventually, reemitted in different transitions. This self-absorption process alters the pure recombination line intensities, increasing or decreasing them according to the line considered.

Second, collisions with free electrons may remove electrons from the \(2^3S\) level and populate other levels, enhancing the intensities with respect to the pure recombination value. The most affected lines are triplets, but singlets are also enhanced.

Self-absorption effects depend strongly on density, and are generally more important in planetary nebulae than in H II regions (Robbins\textsuperscript{44}, Peimbert\textsuperscript{40}; Peimbert, Luridiana, & Torres-
Collisional rates depend both on density and temperature. Expressions for the collisional contribution to each line can be found in Kingdon & Ferland and Benjamin et al. Work on this topic has been made by, e.g., Ferland, Peimbert & Torres-Peimbert, Clegg, Sawey & Berrington, Benjamin, Skillman, and Smits. Both effects must be subtracted out of the total intensities before deriving $Y$ from the observed line intensities.

4 $Y_P$ determinations: sources of uncertainty

Atomic parameters Benjamin et al. identified three error sources affecting the analysis of emission lines: a) the use of a fitting function to represent the emissivity, introducing an uncertainty $\sigma_{\text{fit}}$; b) the uncertainty in the atomic data, $\sigma_{\text{atomic}}$; c) the uncertainty in the input density and temperatures used in the analysis, $\sigma_n$ and $\sigma_T$. These four $\sigma$s should be added in quadrature, and the result of these estimation should be further added to the observational uncertainty. These authors believe that $\sigma_{\text{atomic}}$ alone can be as high as 0.015.

Underlying absorption The nebular diagnostics used in abundance determinations work under the assumption that the spectrum observed is produced exclusively in the gas. In most cases, however, it includes also a stellar contribution; if this superposed stellar spectrum has absorption features, the corresponding emission lines will appear weaker than their true nebular value, introducing a bias in the analysis if no correction is applied.

Ionization structure The total helium abundance in mass, $Y$, can be computed from the number ratio He/H, which is, as a first approximation, equivalent to the (inferred from observations) ionic ratio He$^+$/H$^+$ (or [He$^+$ + He$^{++}$])/H$^+$ in high-excitation objects). This is equivalent to assume that the Strömgren spheres of He and H are coincident. When highly precise measurements are required, however, the observed ionic ratios must be corrected to account for either neutral helium inside the H$^+$ sphere, or neutral hydrogen in the He$^+$ sphere. This is generally done by multiplying the observed He$^+$/H$^+$ by an appropriate “ionization correction factor” ($icf$), defined by the expression $He/H = icf \times He^+/H^+$ (alternative definitions of the ionization correction factor also exist, e.g.: $He/H = [1 + icf] \times He^+/H^+$, or the capitalized ICF by Gruenwald et al.).

Collisional excitation of Balmer lines Balmer hydrogen lines can be enhanced through collisional excitation of H$^0$. Since this mechanism depends strongly on temperature, it plays a role only in hot, low-metallicity regions, where it can enhance the strongest Balmer lines by a few percent. Davidson & Kinman drew the attention to the fact that this mechanism could introduce a bias in the measurement of $Y$: if the Balmer flux is interpreted in terms of pure recombination, the inferred relative abundance of hydrogen is biased toward high values, and the He/H ratio is biased toward low values. A further problem is that, because collisions affect more H$\alpha$ than H$\beta$, they mimic the effect of reddening.

Temperature structure The concept of temperature fluctuations was first introduced by Peimbert, who developed a formalism to describe the departures from spatially constant temperature in nebulae, estimated their impact on abundance determinations, and provided tools to detect their observational signatures. Since the emissivity of each line has a unique dependence on temperature, each line weighs differently those parts of the nebula with different temperatures. As an example, the emissivity of a collisional line such as [O III] $\lambda5007$ is, in the typical range of nebular temperatures, a strongly increasing function of the temperature, therefore the observed intensity of $\lambda5007$ is dominated by the hottest parts of the nebula, and defines implicitly a typical O$^{++}$ temperature $T_e(\text{O III})$. Analogously, the recombination emissivities of
hydrogen or helium lines are mildly decreasing functions of the temperature, and these lines sample preferentially the coldest zones: their observed intensity define typical recombination temperatures, e.g. $T_e$(He II). (Collisional contribution to these lines slightly complicates this basic picture, because it increases with temperature; for the line as a whole, then, the way it weighs the nebula depends on the particular regime of the object. For example, Balmer lines in typical nebular conditions are always dominated by recombination.) From the explanation above, it is clear that $T_e$(O III) and $T_e$(He II) need not take the same value, and often indeed do not. When we step back from intensities to abundances, the temperature appropriate to each ion must be used to evaluate the average emissivities, or a bias will be introduced. This bias usually yields spuriously low $Z$ values, while the effect on $Y$ is more complex to predict since it is the combination of the opposing effects on the recombination and the collisional contributions.

5 $Y_P$ determinations: results

In this section, I will describe and compare a few recent $Y_P$ determinations. I will use the series of works published by Izotov’s group both as a starting point, and a reference in the comparison. This choice is motivated by two reasons. One is practical: their very large sample of objects has been re-analyzed by several other groups, so the comparison is straightforward in these cases. The other is methodological: they generally provide an extremely detailed report of their assumptions and computations, down to a very basic level (with the exception, perhaps, of the uncertainties in their line intensities, which are quoted to be extremely low and would therefore call for an explicit discussion), so that their results are highly reproducible. This is a very valuable aspect of their work, especially considering the tiny quantitative differences among results from different authors.

Izotov and his collaborators analyzed in a series of papers a large sample of metal-poor BCGs (Izotov et al. 1994, 2001, Izotov & Thuan 1997, hereinafter ITL94,97 and IT98). These works discuss critically the potential bias introduced by several physical effects, of which a few will be mentioned here. The amount of stellar absorption is determined by ITL94 simultaneously with the reddening coefficient, by fitting iteratively the dereddened intensity of several hydrogen lines to their recombination values. They find that stellar absorption is generally negligible; however, their procedure fails, for no evident reason, for two of the regions. On the contrary, stellar absorption is found by ITL97 to be extremely important in the case of I Zw 18 (a crucial object in $Y_P$ determinations since it is the most metal-poor galaxy known), which is therefore excluded from the analysis. As for the ionization structure, ITL94 compute the $icfs$ by means of a simple recipe by Pagel et al., linking the $icf$ to $\eta_{sof}$ (Vilchez & Pagel), and corroborate the result with a fit to the photoionization models by Stasinska. They find $icfs > 1$ for the objects in the sample, but in IT98 the question is re-analyzed and the $icfs$ are set to 1. As for the temperature structure, ITL94 adopt $T_e$(He II) = $T_e$(O III), based on a fit to the models by Stasinska. ITL97 maintain this assumption and exclude, based on several indirect pieces of evidence, that temperature fluctuations might play a role in the objects considered. On the other hand, ITL94 claim that a proper estimation of collisions in helium lines should rely on a self-consistent density value, $N_e$(He II), rather than the arbitrary assumption of a density obtained by other diagnostics. They calculate $N_e$(He II) by means of a self-consistent procedure, which constrains the three He I line ratios $\lambda 5876/\lambda 4471$, $\lambda 6678/\lambda 4471$, and $\lambda 7065/\lambda 4471$ to recover their recombination value after correcting for collisional enhancement. The importance of including the density-sensitive $\lambda 7065$ is stressed as a means to obtain a self-consistent result. Self-absorption effects are considered by ITL94 to be negligible, on the argument that the most sensitive line, He I $\lambda 3889$, has roughly its recombination value. However, the analysis performed by IT98 on a larger sample leads to the conclusion that self-absorption effects are

*I added the subindex soft to the customary symbol to avoid confusion with the cosmological $\eta$. 
indeed important, and λ3889 is explicitly added to their self-consistent procedure to detect self absorption. **Collisional effects on hydrogen lines** are evaluated by ITL97, but because the inclusion of such effects actually worsens the fit, the authors infer that they are probably overestimated, and choose not to include them in the analysis. The last paper of this series analyzes a sample of 45 BCGs, yielding a primordial helium value of $Y_P = 0.244 \pm 0.002$.

Let’s see now how these effects have been treated by other authors. Several of them have centered their analysis on the **ionization correction factor**: a) Olive & Steigman agree with ITL94 in that $icf \sim 1$. b) Based on photoionization models, Vegas et al. argue that $icf < 1$ in regions ionized by young, metal-poor stars, so that helium abundances derived in previous analyses should be corrected downwards; this effect is amplified by density inhomogeneities. Re-analyzing the sample of IT98, they find $Y_P = 0.2489 \pm 0.0030$. c) Ballantyne et al. find, by means of photoionization models, that at high stellar temperatures the $icfs$ can be significantly different from 1, with both negative or positive values possible according to the particular stellar atmosphere and temperature considered. They propose to use the metallicity-independent line ratio $\lambda 5007/\lambda 6300$ to discriminate the regions for which $icf \neq 1$. Applying this exclusion criterion to the IT98 sample, they find $Y_P = 0.2489 \pm 0.0030$. d) Sauer & Jedamzik consider the ionization structure a major source of uncertainty in the determination of $Y_P$, and, by means of photoionization models, develop a method to determine the $icf$ based on $\eta_{\text{soft}}$. They find characteristic $icf$s values smaller than 1 for the sample by IT98 and, though they don’t give any definite numbers, conclude that $Y_P$ was overestimated by those authors. e) Gruenwald et al. investigate the evolution of the $icf$ as the H II region evolves, and find that in the range of ages in which H II regions are observed, the $icf$ oscillates twice back and forth from negative to positive values. They argue that the criterion proposed by Ballantyne et al. is not sensitive to the shape of the ionizing spectrum, but rather to its intensity. They also argue that partially density-bounded regions may have high $\lambda 5007/\lambda 6300$ ratios, mimicking a high-excitation zone and biasing the application of the criterion. Re-analyzing the data by Izotov & Thuan, they find that $Y_P$ should be lowered to $Y_P = 0.238 \pm 0.003$.

Other authors have focused their attention on the treatment of **self-absorption**: for example, Olive, Steigman & Skillman argue against the use of λ7065 by ITL94 and ITL97. These authors believe that, because λ7065 is very sensitive to self-absorption effects, for which no correction has been done, it may introduce large uncertainties in the results.

The treatment of **underlying stellar absorption** has been carried out differently by different authors. a) Olive & Skillman believe that this effect might play a role, and propose to include He i λ4026 in the self-consistent analysis of helium lines: this line could serve as a diagnostic of underlying absorption, since it is weak and not much affected by collisions and self-absorption. b) Peimbert et al. correct the weakest helium lines for underlying absorption according to the synthetic spectra by González Delgado, Leitherer, & Heckman. A few authors have discussed the question of the **temperature structure**: a) Steigman et al. argue that temperature fluctuations bias differently hot, low-Z than cold, high-Z regions. The net effect of taking temperature fluctuations into account would be to tilt the $Y$ vs. $O$ relation, in the sense of making it flatter. b) The temperature structure is the main theme in the work of A. Peimbert, M. Peimbert and collaborators. These authors argue, based on several lines of evidence from observations and photoionization modeling, that in low-metallicity regions $T(\text{He} \, \text{ii})$ is systematically smaller than $T(\text{O} \, \text{iii})$. They analyze a small sample of metal-poor objects, and determine $T_e$(He ii) and $N_e$(He ii) self-consistently by means of a $\chi^2$ minimization procedure applied to the intensity of up to nine helium lines, obtaining on average $T_e$(O iii) – $T_e$(He ii) = 1300 K. The primordial helium abundance they determine is $Y_P = 0.2384 \pm 0.0025$.

Peimbert et al. also evaluate the **collisional enhancement of Balmer lines**, which acts in the sense of increasing their computed value of $Y_P$ by about 0.003 (this increase is already included in the $Y_P$ value quoted above). This effect has also been studied by Stasińska &
Izotov, who estimate that the correction for individual objects can be as high as 5%, making it one of the most important sources of systematic errors in the determination of $Y_P$.

6 Conclusions

From the discussion above, it is apparent that the central problem in the determination of $Y_P$ is that several physical mechanisms acting in H II regions are still not completely understood. Furthermore, although I described them separately for exposing convenience, these mechanisms interact with each other in complex ways. A huge collective effort is presently aiming to pinpoint the relevance of these effects, in part with direct observations, more often with numerical simulations. In the meanwhile, whether they actually play a role or not remains mostly a question of personal judgement, based on pieces of evidence that are more or less compelling, but rarely conclusive. Because personal judgement is so important, it is a natural question whether unconscious individual prejudices might be playing a role in obtaining one result or another. It is well known that, to some extent, this kind of bias is always present in any analysis, and that it may be particularly insidious. I therefore conclude with a personal remark: it would be extremely instructive for all of us if the relevant scientists in the field build up a kind of double-bind experiment, with both real and “placebo” data, to evaluate the impact of the human factor on $Y_P$ determinations.

Acknowledgments

This research has been supported by a Marie Curie Fellowship of the European Community programme “Improving Human Research Potential and the Socio-economic Knowledge Base” under contract number HPMF-CT-2000-00949.

References

1. D. R. Ballantyne, G. J. Ferland, & P. G. Martin, ApJ 536, 773 (2000)
2. R. A. Benjamin, E. D. Skillman, & D. P. Smits, ApJ 514, 307 (1999)
3. L. Carigi, P. Colín, M. Peimbert, & A. Sarmiento, ApJ 445, 98 (1995)
4. E. Churchwell, P. G. Mezger, & W. Huchtmeier, A&A 32, 283 (1974)
5. R. E. S. Clegg, MNRAS 229, 31P (1987)
6. K. Davidson & T. D. Kinman, ApJS 58, 321 (1985)
7. S. D’Odorico, M. Peimbert, & F. Sabbadin, A& A 47, 341 (1976)
8. A. D. Dolgov, NPPS 110, 137 (2002)
9. G. J. Ferland, ApJL 310, L67 (1986)
10. B. D. Fields & K. A. Olve, ApJ 506, 177 (1998)
11. G. Fiorentini et al., PhRD 58, f3506 (1998)
12. H. B. French, ApJ 240, 41 (1980)
13. R. M. González Delgado, C. Leitherer, & T. M. Heckman, ApJS 125, 489 (1999)
14. R. Gruenwald, G. Steigman, & S. M. Viegas, ApJ 567, 931 (2002)
15. C. J. Hogan, K. A. Olve, & S. T. Scully, ApJL 489, L119 (1997)
16. E. Høg et al., SSR 84, 115 (1998)
17. Y. I. Izotov et al., ApJ 527, 757 (1999)
18. Y. I. Izotov & T. X. Thuan, ApJ 500, 188 (1998)
19. Y. I. Izotov, T. X. Thuan, & V. A. Lipovetsky, ApJ 435, 647 (1994)
20. Y. I. Izotov, T. X. Thuan, & V. A. Lipovetsky, ApJS 108, 1 (1997)
21. J. Kingdon & G. J. Ferland, ApJ 442, 714 (1995)
22. D. Kunth, PASP 98, 984 (1986)
23. D. Kunth & W. L. W. Sargent, *ApJ* **273**, 81 (1983)
24. J. Lequeux et al., *A&A* **80**, 155 (1979)
25. G. J. Mathews, R. N. Boyd, & G. M. Fuller, *ApJ* **403**, 65 (1993)
26. J. Melnick, M. Heydari-Malayeri, & P. Leisy, *A&A* **253**, 16 (1992)
27. K. A. Olive & E. D. Skillman, *New Ast.* **6**, 119 (2001)
28. K. A. Olive & G. Steigman, *ApJS* **97**, 49 (1995)
29. K. A. Olive, G. Steigman, & E. D. Skillman, *ApJ* **483**, 788 (1997)
30. J. M. O'Meara et al., *ApJ* **552**, 718 (2001)
31. D. E. Osterbrock, in *Astrophysics of Gaseous Nebulae and Active Galactic Nuclei* (University Science Books, California) (1989)
32. B. E. J. Pagel & L. Portinari, *MNRAS* **298**, 747 (1998)
33. B. E. J. Pagel et al., *MNRAS* **255**, 325 (1992)
34. B. E. J. Pagel & E. A. Simonson, *RMAA* **18**, 153 (1989)
35. B. E. J. Pagel, R. J. Terlevich, & J. Melnick, *PASP* **98**, 1005 (1986)
36. P. J. E. Peebles, *ApJ* **146**, 542 (1966)
37. A. Peimbert, M. Peimbert, & V. Luridiana, *RMAAC* **10**, 148 (2001)
38. A. Peimbert, M. Peimbert, & V. Luridiana, *ApJ* **565**, 668 (2002)
39. M. Peimbert, *ApJ* **150**, 825 (1967)
40. M. Peimbert, in *The Analysis of Emission Lines*, eds. R. Williams & M. Livio, Cambridge University Press, **165** (1995)
41. M. Peimbert, V. Luridiana, & S. Torres-Peimbert, *RMAA* **31**, 147 (1995)
42. M. Peimbert, A. Peimbert, & M. T. Ruiz, *ApJ* **541**, 688 (2000)
43. M. Peimbert et al., *ApJ* **395**, 484 (1992)
44. M. Peimbert & S. Torres-Peimbert, *ApJ* **193**, 327 (1974)
45. M. Peimbert & S. Torres-Peimbert, *ApJ* **203**, 581 (1976)
46. M. Peimbert & S. Torres-Peimbert, *RMAA* **15**, 117 (1987)
47. M. Peimbert et al., *PASJ* **40**, 581 (1988)
48. M. Pettini & D. V. Bowen, *ApJ* **560**, 41 (2001)
49. R. R. Robbins, *ApJ* **151**, 511 (1968)
50. R. H. Rubin et al., *ApJL* **501**, L209 (1998)
51. D. Sauer & K. Jedamzik, *A&A* **381**, 361 (2002)
52. P. M. J. Sawey & K. A. Berrington, *ADNDT* **55**, 81 (1993)
53. D. N. Schramm, *SSR* **84**, 3 (1998)
54. L. Searle, & W. L. W. Sargent, *ApJ* **173**, 25 (1972)
55. P. A. Shaver et al., *MNRAS* **204**, 53 (1983)
56. E. D. Skillman, *ApJ* **347**, 883 (1989)
57. E. D. Skillman & R. C. Kennicutt, *ApJ* **411**, 655 (1993)
58. E. D. Skillman et al., *ApJ* **431**, 172 (1994)
59. A. Songaila, L. L. Cowie, C. J. Hogan, & M. Rugers, *Nature* **368**, 599 (1994)
60. G. Stačiunas, *A&A* **83**, 501 (1990)
61. G. Stačiunas & Y. I. Izotov, *A&A* **378**, 817 (2001)
62. G. Steigman, *RMAA* **14**, 71 (1987)
63. G. Steigman, S. M. Viegas, & R. Gruenwald, *ApJ* **490**, 187 (1997)
64. T. K. Suzuki, Y. Yoshii, & T. C. Beers, *ApJ* **540**, 99 (2000)
65. S. Théado & S. Vaclavir, *A&A* **375**, 70 (2001)
66. S. Torres-Peimbert, M. Peimbert, & J. Fierro, *ApJ* **345**, 186 (1989)
67. L. van Zee, J. J. Salzer, & M. P. Haynes, *ApJL* **497**, L1 (1998)
68. S. M. Viegas, R. Gruenwald, & G. Steigman, *ApJ* **531**, 813 (2000)
69. J. M. Vílchez & B. E. J. Pagel, *MNRAS* **231**, 257 (1988)