Unified description of dark matter at the center and in the halo of the Galaxy

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Abstract

We consider a self-gravitating ideal fermion gas at nonzero temperature as a model for the Galactic halo. The Galactic halo of mass \( \sim 2 \times 10^{12} M_\odot \) enclosed within a radius of \( \sim 200 \) kpc is consistent with the existence of a supermassive compact dark object at the Galactic center that is in hydrostatic and quasi-thermal equilibrium with the halo. The central object has a maximal mass of \( \sim 2.3 \times 10^6 M_\odot \) within a minimal radius of \( \sim 18 \) mpc for fermion masses \( \sim 15 \) keV.

In the past, self-gravitating neutrino matter was suggested as a model for quasars, with neutrino masses in the range \( 0.2 \) keV \( \lesssim m \lesssim 0.5 \) MeV \[1\]. More recently, supermassive compact objects consisting of nearly non-interacting degenerate fermionic matter, with fermion masses in the range \( 10 \lesssim m/\)keV \( \lesssim 20 \), have been proposed \[2, 3, 4, 5, 6\] as an alternative to the supermassive black holes that are believed to reside at the centers of many galaxies.

So far the masses of \( \sim 20 \) supermassive compact dark objects at the galactic centers have been measured \[7\]. The most massive compact dark

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object ever observed is located at the center of M87 in the Virgo cluster, and it has a mass of $\sim 3 \times 10^9 M_\odot$. If we identify this object of maximal mass with a degenerate fermion star at the Oppenheimer-Volkoff (OV) limit, i.e., $M_{OV} = 0.54 M_{Pl}^3 m^{-2} g^{-1/2} \simeq 3 \times 10^9 M_\odot$, where $M_{Pl} = \sqrt{\hbar c/G}$, this allows us to fix the fermion mass to $m \simeq 15$ keV for a spin and particle-antiparticle degeneracy factor $g = 2$. Such a relativistic object would have a radius $R_{OV} = 4.45 R_S \simeq 1.5$ light-days, where $R_S$ is the Schwarzschild radius of the mass $M_{OV}$. It would thus be virtually indistinguishable from a black hole of the same mass, as the closest stable orbit around a black hole has a radius of $3 R_S$ anyway.

Near the lower end of the observed mass range is the compact dark object located at the Galactic center with a mass $M_c \simeq 2.6 \times 10^6 M_\odot$. Interpreting this object as a degenerate fermion star consisting of $m \simeq 15$ keV and $g = 2$ fermions, the radius is $R_c \simeq 21$ light-days $\simeq 7 \times 10^4 R_S$, $R_S$ being the Schwarzschild radius of the mass $M_c$. Such a nonrelativistic object is far from being a black hole. The observed motion of stars within a projected distance of $\sim 6$ to $\sim 50$ light-days from Sgr A* yields, apart from the mass, an upper limit for the radius of the fermion star $R_c \lesssim 22$ light-days.

The required nearly non-interacting fermion of $\sim 15$ keV mass cannot be an active neutrino, as it would overclose the Universe by orders of magnitude. However, the $\sim 15$ keV fermion could very well be a sterile neutrino mixed to active neutrinos with a mixing angle $\sin^2 2 \theta \sim 10^{-11}$. Indeed, as has been shown for an initial lepton asymmetry of $\sim 10^{-2}$, a sterile neutrino of mass $\sim 10$ keV may be resonantly or non-resonantly produced in the early Universe with near closure density, i.e., $\Omega_4 \sim 0.3$. As an alternative possibility, the required $\sim 15$ keV fermion could be the axino or the gravitino in soft supersymmetry breaking scenarios.

In the recent past, galactic halos were successfully modeled as a self-gravitating isothermal gas of particles of arbitrary mass, the density of which scales asymptotically as $r^{-2}$, yielding flat rotation curves. As the supermassive compact dark objects at the galactic centers are well described by a gas of fermions of mass $m \sim 15$ keV at $T = 0$, it is tempting to explore the possibility that one could describe both the supermassive compact dark objects and their galactic halos in a unified way in terms of a fermion gas at finite temperature. We show in this letter that this is indeed the case, and that the observed dark matter distribution in the Galactic halo is consistent
with the existence of a supermassive compact dark object at the center of the Galaxy which has about the right mass and size.

Degenerate fermion stars are well understood in terms of the Thomas-Fermi theory applied to self-gravitating fermionic matter at $T = 0$. Extending this theory to nonzero temperature, it has been shown that at some critical temperature $T = T_c$, a self-gravitating ideal fermion gas, having a mass below the OV limit enclosed in a sphere of radius $R$, may undergo a first-order gravitational phase transition from a diffuse state to a condensed state. However, this first-order phase transition can take place only if the Fermi gas is able to get rid of the large latent heat. As short-range interactions of the fermions are negligible, the gas cannot release its latent heat; it will thus be trapped for temperatures $T < T_c$ in a thermodynamic quasistable supercooled state close to the point of gravothermal collapse.

The formation of a supercooled state close to the point of gravothermal collapse may be due to violent relaxation. Through the gravitational collapse of an overdense fluctuation, part of gravitational energy transforms into the kinetic energy of random motion of small-scale density fluctuations. The resulting virialized cloud will thus be well approximated by a gravitationally stable quasi-thermalized halo. In order to estimate the particle mass-temperature ratio, we assume that a cold overdense cloud of the mass of the Galaxy $M$, stops expanding at the time $t_m$, reaching its maximal radius $R_m$ and minimal average density $\rho_m = 3M/(4\pi R_m^3)$. The total energy per particle is just the gravitational energy

$$E = -\frac{3GM}{5R_m}.$$  \hspace{1cm} (1)

Assuming spherical collapse, one arrives at

$$\rho_m = \frac{9\pi^2}{16} \bar{\rho}(t_m) = \frac{9\pi^2}{16} \Omega_d \rho_0 (1 + z_m)^3,$$  \hspace{1cm} (2)

where $\bar{\rho}(t_m)$ is the background density at the time $t_m$ or the cosmological redshift $z_m$, and $\rho_0 \equiv 3H_0^2/(8\pi G)$ is the present critical density. We now approximate the virialized cloud by a singular isothermal sphere of mass $M$ and radius $R$, characterized by a constant circular velocity $\Theta = (2T/m)^{1/2}$ and the density profile $\rho(r) = \Theta^2/4\pi Gr^2$. Its total energy per particle is the
sum of gravitational and thermal energies, i.e.,

\[ E = -\frac{1}{4} \frac{GM}{R} = -\frac{1}{4} \Theta^2. \]  

(3)

Combining Eqs. (1), (2), and (3), we find

\[ \Theta^2 = \frac{6\pi}{5} G(6\Omega_d\rho_0 M^2)^{1/3}(1 + z_m). \]  

(4)

Taking \( \Omega_d = 0.3 \), \( M = 2 \times 10^{12} M_\odot \), \( z_m = 4 \), and \( H_0 = 65 \text{ km s}^{-1} \text{Mpc}^{-1} \), we find \( \Theta \simeq 220 \text{ km s}^{-1} \), which corresponds to the mass-temperature ratio \( m/T \simeq 4 \times 10^6 \).

We now briefly discuss the Thomas-Fermi theory \([17, 18]\) for a self-gravitating gas of \( N \) fermions with mass \( m \) at the temperature \( T \) enclosed in a sphere of radius \( R \). We restrict our attention to the Newtonian theory since the general relativistic effects are not relevant to the physics we discuss in this paper. The general relativistic treatment will be reported elsewhere. For large \( N \), we can assume that fermions move in a spherically symmetric mean-field potential \( \varphi(r) \) which satisfies Poisson’s equation

\[ \frac{d\varphi}{dr} = G \frac{M}{r^2}; \quad \frac{dM}{dr} = 4\pi r^2 mn, \]  

(5)

\( M \) being the enclosed mass. The number density of fermions (including antifermions) \( n \) can be expressed in terms of the Fermi-Dirac distribution (in units \( \hbar = c = k = 1 \))

\[ n = \frac{\rho}{m} = g \int \frac{d^3 q}{(2\pi)^3} \left( 1 + \exp\left( \frac{q^2}{2mT} + \frac{m}{T} \varphi - \frac{\mu}{T} \right) \right)^{-1}. \]  

(6)

Here \( g \) denotes the combined spin-degeneracy factor of the neutral fermions and antifermions, i.e., \( g \) is 2 or 4 for Majorana or Dirac fermions, respectively. For each solution \( \varphi(r) \) of (5), the chemical potential \( \mu \) is adjusted so that the constraint

\[ \int_0^R dr 4\pi r^2 n(r) = N, \]  

(7)

is satisfied. Equations (5) with (6) should be integrated using the boundary conditions at the origin, i.e.,

\[ \varphi(0) = \varphi_0; \quad M(0) = 0. \]  

(8)
It is useful to introduce the degeneracy parameter

$$\eta = \frac{\mu}{T} - \frac{m}{T} \varphi .$$

(9)

As $\varphi$ is monotonously increasing with increasing $r$, the strongest degeneracy is obtained at the center with $\eta_0 = (\mu - m\varphi_0)/T$. The parameter $\eta_0$, uniquely related to the central density, will eventually be fixed by the constraint (9) or equivalently by the condition $M(R) = mN$ at the outer boundary. In this way, the explicit dependence on the chemical potential $\mu$ is absorbed in the degeneracy parameter $\eta_0$. For $r \geq R$, the function $\varphi$ yields the usual empty-space Newtonian potential

$$\varphi(r) = -\frac{mN}{r}.$$  

(10)

The set of self-consistency equations (5)-(7), with the boundary conditions (8), defines the gravitational Thomas-Fermi equation.

The numerical procedure is now straightforward. For a fixed, arbitrarily chosen ratio $m/T$, we first integrate Eqs. (5) numerically on the interval $[0, R]$ to find the solutions for various central values $\eta_0$. This yields $M(R)$ as a function of $\eta_0$. We then select the value of $\eta_0$ for which $M(R) = mN$.

The quantities $N, T,$ and $R$ are free parameters in our model and their values are dictated by physics. In the following, $N$ is required to be of the order $2 \times 10^{12} M_\odot/m$, so that for any $m$, the total mass is close to the estimated mass of the halo $^{25}$. As we have demonstrated, the expected temperature of the halo is given by $m/T = 4 \times 10^6$. Our choice $R = 200$ kpc is based on the estimated size of the Galactic halo. The only remaining free parameter is the fermion mass, which we fix at $m = 15$ keV, and justify its choice a posteriori.

For fixed $N$, there is a range of $T$ where the Thomas-Fermi equation has multiple solutions. For example, for $N = 2 \times 10^{12}$ and $m/T = 4 \times 10^6$, we find six solutions, which we denote by (a), (b), (c), (c'), (b'), and (a') corresponding to the values $\eta_0 = -30.53, -25.35, -22.39, 29.28, 33.38,$ and $40.48$, respectively. In Fig. 1 we plot the mass density profiles of the halo. For the negative central value $\eta_0$, for which the degeneracy parameter is negative everywhere, the system behaves basically as a Maxwell-Boltzmann isothermal sphere. Positive values of the central degeneracy parameter $\eta_0$ are characterized by a pronounced central core of mass of about $2.5 \times 10^6 M_\odot$.
within a radius of about 20 mpc. The presence of the core is obviously due to the degeneracy pressure. The core represents material which, having been cooled by expansion, experiences little entropy increase during the ensuing collapse. Thus the dynamics of its formation should be well approximated by a dynamical Thomas-Fermi theory based on the equation of state of a degenerate Fermi gas \[26\]. Conversely, the halo is formed from phase-mixed matter and estimates similar to those leading to \[1\] give an average entropy per particle increasing from few \(\times 10^0\) to few \(\times 10^1\).

A similar structure was obtained in collisionless stellar systems modeled as a nonrelativistic Fermi gas \[27\]. Note that while violent relaxation leads to a Fermi-Dirac distribution in either case, for stars the onset of degeneracy signals the breakdown of the assumption that collisions are unimportant, resulting in a Maxwell-Boltzmann distribution \[21\]. No such breakdown occurs for elementary fermions \[22\].

Fig. 1 shows two important features. First, a galactic halo at a given temperature may or may not have a central core depending on whether the central degeneracy parameter \(\eta_0\) is positive or negative. Second, the closer to zero \(\eta_0\) is, the smaller the radius at which the \(r^{-2}\) asymptotic behavior of density begins. The flattening of the Galactic rotation curve begins in the range \(1 \lesssim r/\text{kpc} \lesssim 10\), hence the solution (c') most likely describes the Galactic halo. This may be verified by calculating the rotation curves in our model. We know already from our estimate \[1\] that our model yields the correct asymptotic circular velocity of 220 km/s. In order to make a more realistic comparison with the observed Galactic rotation curve, we must include two additional matter components: the bulge and the disk. The bulge is modeled as a spherically symmetric matter distribution of the form \[28\]

\[
\rho_b(s) = \frac{e^{-hs}}{2s^3} \int_0^\infty du \frac{e^{-hus}}{[(u + 1)^s - 1]^{1/2}},
\]

where \(s = (r/r_0)^{1/4}\), \(r_0\) is the effective radius of the bulge and \(h\) is a parameter. We adopt \(r_0 = 2.67\) kpc and \(h\) yielding a bulge mass \(M_b = 1.5 \times 10^{10} M_\odot\) \[23\]. In Fig. 2 the mass of the halo and bulge enclosed within a given radius is plotted for various \(\eta_0\). The data points, indicated by squares, are the mass \(M_c = 2.6 \times 10^6 M_\odot\) within 18 mpc, estimated from the motion of the stars near Sgr A* \[10\], and the mass \(M_{50} = 5.4^{+0.2}_{-0.6} \times 10^{11}\) within 50 kpc, estimated from the motion of satellite galaxies and globular clusters \[24\]. Variation of
Figure 1: The mass density profile of the halo for $\eta_0 = 0$ (dotted line) and for the six $\eta_0$-values discussed in the text. Configurations with negative $\eta_0$ ((a), (b), (c)) are depicted by the dashed and those with positive $\eta_0$ ((a'), (b'), (c')) by the solid line.
Figure 2: Enclosed mass of the halo plus bulge versus radius for $\eta_0 = 24$ (dashed), 28 (solid), and 32 (dot-dashed line).
the central degeneracy parameter $\eta_0$ between 24 and 32 does not change the essential halo features.

In Fig. 3 we plot the circular velocity components: the halo, the bulge, and the disk. The contribution of the disk is modeled as

$$\Theta_d(r)^2 = \Theta_d(r_o)^2 \frac{1.97(r/r_o)^{1.22}}{[(r/r_o)^2 + 0.782^{1.43}, (12)]},$$

where we take $r_o = 13.5$ kpc and $\Theta_d = 100$ km/s. Here we have assumed for simplicity that the disc does not influence the mass distribution of the bulge and the halo. Choosing the central degeneracy $\eta_0 = 28$ for the halo, the data by Merrifield and Olling [31] are reasonably well fitted.

We now turn to the discussion of our choice of the fermion mass $m = 15$ keV for the degeneracy factor $g = 2$. To that end, we investigate how the mass of the central object, i.e., the mass $M_c$ within 18 mpc, depends on $m$ in the interval 5 to 25 keV, for various $\eta_0$. We find that $m \simeq 15$ keV always gives the maximal value of $M_c$ ranging between 1.7 and $2.3 \times 10^6 M_\odot$ for $\eta_0$ between 20 and 28. Hence, with $m \simeq 15$ keV we get the value closest to the mass of the central object $M_c$ estimated from the motion of the stars near Sgr A* [10].

The radius of our central object of about 18 mpc is much larger than the size of the radio source Sgr A*. In fact, very large array interferometric observations of Sgr A* at millimeter wavelength show that the radiowave emitting region is $\leq 1-3$ AU [32]. However, it has not yet been shown conclusively that Sgr A* is indeed the object that has a mass $\sim 3 \times 10^6 M_\odot$. There are arguments, based on the nonmotion of Sgr A* and equipartition of energy in the central star cluster, indicating that Sgr A* could have a mass $\geq 10^3 M_\odot$ [33]. This argument is only conclusive if equipartition of energy actually takes place in a reasonable time frame. It is, therefore, still possible that the compact radiosource Sgr A*, with a radius of a few AU, and the moderately compact supermassive dark object that has been detected gravitationally, and possibly also in X-rays in the quiescent state, with a radius of $\sim 20$ mpc [34], could be two distinct objects.

In summary, using the Thomas-Fermi theory, we have shown that a weakly interacting self-gravitating fermionic gas at finite temperature yields a mass distribution that successfully describes both the center and the halo of the Galaxy. For a fermion mass $m \simeq 15$ keV, a reasonable fit to the
Figure 3: Fit to the Galactic rotation curve. The data points are by Olling and Merrifield [31], for $R_0 = 8.5$ kpc and $\Theta_0 = 220$ km/s.
rotation curve is achieved with the temperature $T = 3.75$ meV and the degeneracy parameter at the center $\eta_0 = 28$. With the same parameters, the masses enclosed within 50 and 200 kpc are $M_{50} = 5.04 \times 10^{11} M_\odot$ and $M_{200} = 2.04 \times 10^{12} M_\odot$, respectively. These values agree quite well with the mass estimates based on the motion of satellite galaxies and globular clusters [25]. Moreover, the mass $M_c \simeq 2.27 \times 10^6 M_\odot$, enclosed within 18 mpc, agrees reasonably well with the observations of the compact dark object at the center of the Galaxy. We thus conclude that both the Galactic halo and the center could be made of the same fermions.

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