Aeolian transport layer
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We investigate the airborne transport of particles on a granular surface by the saltation mechanism through numerical simulation of particle motion coupled with turbulent flow. We determine the saturated flux $q_s$ and show that its behavior is consistent with a classical empirical relation obtained from wind tunnel measurements. Our results also allow to propose a new relation valid for small fluxes, namely, $q_s = a(u_\ast - u_\tau)^\alpha$, where $u_\ast$ and $u_\tau$ are the shear and threshold velocities of the wind, respectively, and the scaling exponent is $\alpha \approx 2$. We obtain an expression for the velocity profile of the wind distorted by the particle motion and present a dynamical scaling relation. We also find a novel expression for the dependence of the height of the saltation layer as function of the wind velocity.

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The transport of sand by wind is among others responsible for sand encroachment, dune motion and the formation of coastal and desert landscapes. The dominating transport mechanism is saltation as first described by Bagnold [1] which consists of grains being ejected upwards, accelerated by the wind and finally impacting onto the ground producing a splash of new ejected particles. Reviews are given in Refs. [2, 3]. Quantitatively this process is however far from being understood.

Due to Newton’s second law the wind loses more momentum with increasing number of airborne particles until a saturation is reached. The maximum number of grains a wind of given strength can carry through a unit area per unit time defines the saturated flux of sand $q_s$. This quantity has been measured by many authors in wind tunnel experiments and on the field, and numerous empirical expressions for its dependence on the strength of the wind have been proposed [4, 5, 6, 7]. In previous studies theoretical forms have also been derived using approximations for the drag in turbulent flow [8, 9]. All these relations are expressed as polynomials in the wind shear velocity $u_\ast$, which are of third order, under the assumption that the grain hopping length scales with $u_\ast$ [1, 4, 8, 9, 10] and otherwise can be more complex [5]. The velocity profile in a particle laden layer has also been the object of measurement [11, 12] and modelization [13]. Surprisingly however very few measurements of the height of the saltation layers as function of $u_\ast$ have been reported [14] and no systematic data close to the threshold are available. The complete analytical treatment of this problem remains out of reach not only because of the turbulent character of the wind, but also due to the underlying moving boundary conditions in the equations of motion. Despite much research in the past there remain many uncertainties about the trajectories of the particles and their feedback with the velocity field of the wind. It is this challenge which motivates the present work.

We will present the first numerical study of saltation which solves the turbulent wind field and its feedback with the dragged particles. As compared to real data, our values have no experimental fluctuations neither in the wind field nor in the particle sizes. As a consequence, we can determine all quantities with higher precision than ever before, and therefore with a better resolution close to the critical velocity at which the saltation process starts.

In order to get quantitative understanding of the layer of airborne particle transport above a granular surface, we simulate the situation inside a two-dimensional channel with a mobile top wall as schematically shown in Fig. 1. We impose a pressure gradient between the left and the right side. Gravity points down, i.e., in negative $y$-direction. The $y$-dependence of the pressure drop is adjusted in such a way as to insure a logarithmic velocity profile along the entire channel in the case without particles, as it is expected in fully developed turbulence [15].

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More precisely, this profile follows the classical form

\[ u_x(y) = \left( u_* / \kappa \right) \ln \left( y / y_0 \right) , \tag{1} \]

where \( u_x \) is the component of the wind velocity in the \( x \)-direction, \( u_* \) is the shear velocity, \( \kappa = 0.4 \) is the von Karman constant and \( y_0 \) is the roughness length which is typically between \( 10^{-4} \) and \( 10^{-2} \) m. The upper wall of the channel is moved with a velocity equal to the velocity of the wind at that height in order to insure a non-slip boundary condition.

The fluid mechanics inside the channel is based on the assumptions that we have an incompressible and Newtonian fluid flowing under steady-state and homogeneous turbulent conditions. The fluid is air with viscosity \( \mu = 1.7894 \times 10^{-5} \) kg m\(^{-1}\) s\(^{-1}\) and density \( \rho = 1.225 \) kg m\(^{-3}\). The Reynolds-averaged Navier-Stokes equations with the standard \( k - \epsilon \) model are used to describe turbulence. The numerical solution for the velocity and pressure fields is obtained through discretization by means of the control volume finite-difference technique \cite{18,19}. The integral version of the governing equations is considered at each cell of the numerical grid to generate a set of non-linear algebraic equations which are pseudo-linearized and solved. The criteria for convergence used in the simulations are defined in terms of residuals, i.e., a measure of the degree to which the conservation equations are satisfied throughout the flow field. In our simulations convergence is achieved only when each of the normalized residuals falls below \( 10^{-6} \).

After having produced a steady-state turbulent flow, we proceed with the simulation of the particle transport along the channel. Assuming that drag and gravity are the only relevant forces acting on the particles, their trajectory can be obtained by integrating the following equation of motion:

\[
\frac{du_p}{dt} = F_D(u - u_p) + g(\rho_p - \rho) / \rho_p , \tag{2}
\]

where \( u_p \) is the particle velocity, \( g \) is gravity and \( \rho_p = 2650 \) kg m\(^{-3}\) is a typical value for the density of sand particles. The term \( F_D(u - u_p) \) represents the drag force per unit particle mass where

\[
F_D = \frac{18\mu}{\rho_p d_p^2} C_D \text{Re} \frac{D}{24} , \tag{3}
\]

\( d_p = 2.5 \times 10^{-4} \) m is a typical particle diameter, \( \text{Re} \equiv \rho d_p |u_p - u| / \mu \) is the particle Reynolds number, and the drag coefficient \( C_D \) is taken from empirical relations \cite{19}. Each particle in our calculation represents in fact a stream of real grains. It is necessary to take into account the feedback on the local fluid velocity due to the momentum transfer to and from the particles. Specifically, this coupling effect is considered here by alternately solving the discrete and continuous phase equations until the solutions in both phases agree. The momentum transfer from one phase to another is computed by adding the momentum change of every particle as it passes through a control volume \cite{18}.

\[
F = \sum_{\text{particles}} F_D(u - u_p) \dot{m}_p \Delta t \tag{4}
\]

where \( \dot{m}_p \) is the mass flow rate of the particles and \( \Delta t \) the time step. The exchange term Eq. (4) appears as a sink in the continuous phase momentum balance.

In Fig. 1 we see the trajectory of one particle stream and the velocity vectors along the \( y \)-direction. Each time a particle hits the ground it loses a fraction \( r \) of its energy and a new stream of particles is ejected at that position with an angle \( \theta \). The parameters \( r = 0.84 \) and \( \theta = 36^\circ \) are chosen from experimental measurements \cite{20,21}. We also studied other values for \( r \) and \( \theta \) and even considered a continuous distribution of ejection angles. As expected, the choice of unrealistic values produces unphysical results. More details will be given in Ref. \cite{22}.

If \( u_* \) is below a threshold value \( u_t \) the energy loss at each impact prevails over the energy gain during the acceleration through drag and particle transport comes to a halt as illustrated in Fig. 2. Only for \( u_* > u_t \) steady sand motion is achieved. The resulting flux is given by

\[
q = m_p \dot{n}_p / u_p \tag{5}
\]

where \( m_p \) and \( \dot{n}_p \) are the mass and the average velocity of the particles, respectively, and \( n_p \) is the number of particle streams released. The first added particle streams are strongly accelerated in the channel and their jumping amplitude increases after each ejection until a maximum is reached as seen in Fig. 2. The more particles are injected the smaller is this final amplitude. Beyond a certain number \( n_s \) of particle streams, the trajectories however start to lose energy and the overall flux is reduced. This critical value \( n_s \) characterizes the saturated flux \( q_s \) through Eq. (5).

In Fig. 3 we see the plot of \( q_s \) as function of the wind velocity \( u_* \). Clearly, there exists a critical wind velocity threshold \( u_t \) below which no sand transport occurs at all. This agrees well with experimental observations \cite{4,22}. Also shown in Fig. 3 is the best fit to the simulation data using the classical expression proposed by Lettau and Lettau \cite{4}.

\[
q_s = C_L \frac{D}{g} u_*^2 (u_* - u_t) , \tag{6}
\]
where $C_L$ is an adjustable parameter. We find very good agreement using fit parameters of the same order as those of the original work and a threshold value of $u_t = 0.35 \pm 0.02$. This is in fact, to our knowledge, the first time a parabolic expression of the form

$$q_s = a(u_s - u_t)^2$$

fits the data at least as well as Eq. (6), as can be seen in the inset of Fig. 3. The threshold obtained in this case, namely, $u_t = 0.33 \pm 0.01$, is slightly lower than the one obtained for the classical cubic expression Eq. (9). We believe that our parabolic expression describes correctly the critical behavior very close the transition right above $u_t$, since at this point the classical assumption of proportionality between the saltation jump length and $u_s$ cannot hold.

Experimentally much more difficult to access is the velocity profile of the wind within the layer of grain transport. This profile clearly deviates from the undisturbed logarithmic form of Eq. (1) because of the momentum the fluid must locally exchange with the particles. In Fig. 4 we show the loss of velocity with respect to the logarithmic profile without particles of Eq. (1) for different values of $q$ as function of the height $y$. As clearly seen in Fig. 4, the loss of velocity is maximal at the same height $y_{max}$ regardless of the value of flux $q$. Except for large values of the flux, dividing the velocity axis by $q$ one can collapse all the profiles quite well on top of each other as can be verified in the inset of Fig. 4. The position $y_{max}$ of the height of maximum loss depends essentially linearly on $u_s$ as shown in Fig. 5. This is consistent with the observation that the saltation jump length is proportional to $u_s$. The proportionality constant obtained from the best linear fit to the data is $0.35 \, \text{m/s}$. Quantitatively the data in Fig. 5 also fit very well into the experiment data plots of Ref. [14]. By extrapolation to $y_{max} = 0$, we obtain an alternative estimate for
the threshold velocity, $u_t = 0.35 \text{ m/s}$, that is consistent with the values calculated before from the fits to the data using Eqs. 4 and 7.

Whoever has been in the desert or on a beach during a very windy day knows that the saltation process in nature looks like a sheet of particles floating above ground at a certain height $y_s$ which strongly depends on the wind velocity. This height corresponds to the position of the largest likelihood to find a particle as obtained from the maximum of the density profile of particles as function of height $y$. Fig. 5 implies that the profile of velocity difference of the wind has a minimum at a similar height, which is consistent with the maximal loss of momentum. Within the error bars our results in fact yield that $y_s$ coincides with the values of $y_{\text{max}}$ in Fig. 5. More details will be presented in Ref. [22]. It is important to note that both heights, $y_{\text{max}}$ and $y_s$, also have the same linear dependence on $u^*$. We have shown in this letter results of simulations giving insight about the layer of granular transport under conditions of turbulent flow. The lack of experimental noise allows for a precise study close to the critical threshold velocity $u_t$ that lead us to a parabolic dependence of the saturated flux. In addition, we show that the velocity profile disturbed by the presence of grains scales linearly with the flux of grains, except close to saturation. Notably a characteristic height appears at which the momentum loss in the fluid and the grain density are maximized. Moreover, this height increases linearly with the wind velocity $u_*$. It would be very interesting to verify experimentally these novel predictions. The present model can be extended in many ways including the study of the dependence of the aeolian transport layer on the grain diameter, the gas viscosity, and the solid or fluid densities. This would allow to calculate, for instance, the granular transport on Mars and compare with the expression presented in Ref. [10]. Work in this direction is under way [22].

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