Relativistic plasma control for single attosecond pulse generation

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To describe the high harmonic generation at plasma surfaces in the relativistic regime, we introduce the concept of \textit{apparent reflection point} (ARP). It appears to an external observer that the radiation electric field is zero at the ARP. The relativistic dynamics of the ARP completely defines the generation of high harmonics and attosecond pulses. The ARP velocity is a smooth function of time. The corresponding \( \gamma \)-factor, however, has sharp spikes at the times when the tangential vector potential vanishes and the surface velocity becomes close to the speed of light. We show that managing the laser polarization, one can efficiently control the ARP dynamics, e.g., to gate a single (sub-)attosecond pulse out of the short pulse train generated by a multi-cycle driver. This relativistic control is demonstrated numerically by particle-in-cell simulations.

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High harmonics generated at plasma surfaces in the relativistic regime\textsuperscript{1,2,3} are a promising new source of short wavelength radiation and attosecond pulses, necessary for the study of ultra-fast processes in atoms, molecules and solids at intensities significantly higher than those obtained from strong field laser-atom interactions\textsuperscript{4,5}.

The plasma harmonics are generated in the relativistic regime, when the laser pulse intensity \( I \gg 10^{18} \text{ W/cm}^2 \). Laser pulses in this intensity range usually are several cycles long. As a consequence, the reflected radiation contains a comb of attosecond pulses. Yet, applications like molecular imaging\textsuperscript{5,6} or quantum control\textsuperscript{7} usually require a single short pulse to prevent undesirable effects, e.g., Coulomb explosion. The single attosecond pulse can be selected either using a phase-stabilized single cycle laser\textsuperscript{8} or controlling the atomic response by time-dependent laser polarization\textsuperscript{9}.

It was shown recently by particle-in-cell (PIC) simulations that in the \( \lambda^3 \)-regime, when a single-cycle laser pulse is focused down to a spot of a one wavelength size, a single attosecond pulse running in a particular direction can be isolated\textsuperscript{10}. However, in the case of a more realistic several-cycle laser pulse, such casual separation can be difficult.

In the present work we show for the first time that the managed time-dependent polarization (from elliptical to linear and back) of the incident laser pulse allows to gate a single (sub-)attosecond pulse from relativistically driven plasma surface in a well controlled way. Physically, the laser pulse polarization controls the relativistic \( \gamma \)-factor of the apparent reflection point (ARP) as seen by the observer. Although the driving laser pulse can be long and intense, the ARP velocity becomes highly relativistic only at the specific time, when the vector potential component tangential to the plasma surface vanishes. Exactly at this moment, the single attosecond pulse is emitted. The time-dependent polarization corresponds to two perpendicularly polarized laser pulses with slightly different frequencies and a well chosen phase shift, see Fig. 1. Experimentally, the time-dependent laser pulse ellipticity can be achieved by the femtosecond polarization pulse shaping techniques\textsuperscript{11}.

We consider the interaction of an ultra-intense short laser pulse with a slab of overdense plasma. We suppose that the plasma ions are immobile during the short interaction time and study the electron fluid oscillations only. It was shown in\textsuperscript{12} that to describe the high harmonic generation analytically, the boundary condition \( E_\tau = 0 \) must be used. Here \( E_\tau \) is the electric field component tangential to the plasma surface. The physical meaning of this boundary condition is quite clear. Let us consider the Poynting vector \( \mathbf{S} = \mathbf{E} \times \mathbf{B} / 4\pi \) that gives the electromagnetic energy flux. In the vacuum region in front of the plasma, it consists of the two parts: the incident flux \( \mathbf{S}^i = \mathbf{e} \mathbf{E}^i \times \mathbf{B}^i / 4\pi \) and the reflected one.

![FIG. 1: Geometry for relativistic plasma control of attosecond plasma surface dynamics. The high intensity driving pulse is polarization-managed through a low intensity controlling pulse. After proper filtering of the reflected radiation, a single attosecond pulse can be isolated. The interaction process can be described in terms of the apparent reflection point (ARP). It appears to an external observer that the radiation electric field and the Poynting vector \( \mathbf{S} \) are zero at the ARP.](image-url)
The electrons in the skin layer move with ultra-relativistic speed. If we also neglect the small absorption, then the observer sees the apparent reflection at the point \( x_{\text{ARP}}(t) \), where the normal component of the Poynting vector \( S_n = cE_x \times B_z / 4\pi = 0 \), implied by \( E_x(x_{\text{ARP}}) = 0 \). Thus the boundary condition ensures the energy conservation. The detailed microscopic derivation of the boundary condition \( E_x(x_{\text{ARP}}) = 0 \) and its relation to the electron dynamics can be found in [11].

The incident laser field in vacuum runs in the positive \( x \)-direction, \( E_i(x,t) = E_i(x - ct) \), while the reflected field is translated backwards: \( E_r(x,t) = E_i(x + ct) \). The tangential components of these fields interfere destructively at the ARP position \( x_{\text{ARP}}(t) \), so that the implicit equation for the apparent reflection point \( x_{\text{ARP}}(t) \) is:

\[
E_x^i(x_{\text{ARP}} - ct) + E_x^r(x_{\text{ARP}} + ct) = 0. \tag{1}
\]

We stress that Eq. (1) contains the electromagnetic fields in vacuum. That is why the reflection point \( x_{\text{ARP}} \) is apparent. The real interaction within the plasma skin layer can be very complex. Yet, an external observer, who has information about the radiation in vacuum only, sees that \( E_x = 0 \) at \( x_{\text{ARP}} \). Strictly speaking, one cannot ascribe \( x_{\text{ARP}} \) to any particular density level, e.g., the “critical density”. One can only state that the ARP is located within the skin layer at the electron fluid surface.

The ARP dynamics completely defines the high harmonic generation. In the ultra-relativistic regime, when the dimensionless vector potential \( a_0 = eA_0/mc^2 \) of the laser is large, \( a_0^2 \gg 1 \), we can apply ultra-relativistic similarity theory [12] to characterize this motion. The basic statement of this theory is that when we change the plasma density \( N_e \) and the laser amplitude \( a_0 \) simultaneously keeping the similarity parameter \( S = N_e/a_0 N_c \) constant the laser-plasma dynamics remains similar. Here \( N_c = \omega_0^2m/4\pi e^2 \) is the critical plasma density for the laser pulse with the frequency \( \omega_0 \). This means that for different interactions with the same similarity parameter \( S = \text{const} \), the plasma electrons move along the same trajectories, while their momenta \( p \) scale with the laser amplitude: \( p \propto a_0 \). Consequently, the electron momentum components tangential and normal to the plasma surface scale simultaneously with \( a_0 \): \( p_\tau \propto a_0 \) and \( p_n \propto a_0 \). Therefore, in general, the total electron momentum is not perpendicular to the surface. Moreover, the characteristic angle between their direction and the surface normal does not depend on \( a_0 \) provided that \( S \) is fixed.

This result is crucial for the plasma surface dynamics. Since we consider an ultra-relativistic laser pulse, \( a_0^2 \gg 1 \), the electrons in the skin layer move with ultra-relativistic velocities almost all the time:

\[
v = c\sqrt{\frac{p_n^2 + p_\tau^2}{m_e^2c^2 + p_n^2 + p_\tau^2}} = c(1 - O(a_0^{-2})). \tag{2}
\]

Yet the relativistic \( \gamma \)-factor of the plasma surface \( \gamma_s(t) \) and its velocity \( v_s(t) \) behave in a quite different way. Let us consider electrons at the very boundary of the plasma. The similarity theory states that the electron momenta can be represented as \( p_s(t) = a_0 p_n(S, \omega t) \) and \( p_\tau(t) = a_0 p_\tau(S, \omega t) \), where \( p_n \) and \( p_\tau \) are universal functions, which do not depend on \( a_0 \) explicitly, but rather on the similarity parameter \( S \) and on the pulse shape. Thus, the plasma surface velocity \( \beta_s(t) = v_s(t)/c \) and \( \gamma_s(t) \) are

\[
\beta_s(t) = \frac{p_n(t)}{\sqrt{m_e^2c^2 + p_n^2(t) + p_\tau^2(t)}} = \frac{P_n(t)}{\sqrt{P_n^2(t) + P_\tau^2(t) - O(a_0^{-2})}}, \tag{3}
\]

\[
\gamma_s(t) = \frac{1}{\sqrt{1 - \beta_s^2(t)}} = \frac{1 + P_n^2(t) + P_\tau^2(t)}{P_n^2(t) + P_\tau^2(t)} + O(a_0^{-2}). \tag{4}
\]

It follows from (3)-(4) that the relativistic \( \gamma \)-factor of the plasma boundary is of the order of unity for almost all times, except for those times \( t_0 \), when the tangential momentum component vanishes \( p_\tau(S, t_0) = 0 \). Exactly at these times, there are spikes of the \( \gamma \)-factor:

\[
\gamma_s = \frac{1}{\sqrt{1 - \beta_s^2}} = \frac{P_n^2 + m_e^2c^2}{m_e^2c^2} \propto a_0. \tag{5}
\]

On the contrary, the plasma surface velocity \( v_s \) as given by Eq. (3) is a smooth function. It approaches \( \pm \) when the electron momentum parallel to the surface vanishes, i.e., at the same times \( t_0 \) corresponding to the \( \gamma \)-spikes. The high harmonics are generated at the spikes, when the surface velocity is negative and close to \(-c\). Rigorous microscopic analysis [11] confirms that high harmonics are generated by bunches of fast electrons moving towards the laser pulse.

It is possible to estimate the width of a \( \gamma \)-spike. Since the surface velocity is a smooth function, we can expand it in Taylor series around the maximum as \( v_s(t) \approx v_{max} - \alpha_0(t - t_{max})^2 \), where the parameter \( \alpha \) depends only on \( S \). The width \( \Delta t = |t - t_{max}| \) of the \( \gamma_s \) spikes is then

\[
\Delta t \propto \sqrt{1 - v_{max}^2/c^2} / (\omega_0 \sqrt{\alpha}), \quad \Delta t = 1/(\omega_0 \sqrt{\alpha} \gamma_{max}). \tag{6}
\]

It follows from (3) that the \( \gamma_s \) spikes get higher and narrower when we increase \( a_0 \), keeping \( S = \text{const} \). Thus, the spikes behave as quasi-singularities. These ultra-relativistic spikes are the inherent cause for the high harmonic generation. The (sub-)attosecond pulses [3] are emitted exactly at the times of the spikes.

We study the motion of the plasma boundary and the specific behavior of \( v_{\text{LRP}} \) and \( \gamma_{\text{LRP}} \) numerically using the 1D PIC code VLPL [13]. The plasma slab is initially positioned between \( x_L = 1.5\lambda \) and \( x_R = 3.9\lambda \), where \( \lambda = 2\pi/\omega_0 \) is the laser wavelength. The laser pulse has the gaussian envelope: \( a(x,t = 0) = a_0 \exp(-x^2/\sigma^2) \cos(2\pi x/\lambda) \) with \( \sigma = 2\lambda \).
At every time step, the incident and the reflected fields are recorded at $x = 0$. Being solutions of the wave equation in vacuum, these fields can be easily chased to arbitrary $x$ and $t$. To find the ARP position $x_{\text{ARP}}$, we solve numerically the equation (1). The trajectory of $x_{\text{ARP}}(t)$ for the simulation parameters $a_0 = 20$ and $N_e/N_c = 90$ ($S = 4.5$) is presented in Fig. 2a. One can clearly see the oscillatory motion of the point $x_{\text{ARP}}(t)$. The equilibrium position is displaced from the initial plasma boundary position $x_L$ due to the mean laser light pressure.

Since only the ARP motion towards the laser pulse is of importance for the high harmonic generation, we cut out the positive ARP velocities $v_{\text{ARP}}(t) = dx_{\text{ARP}}(t)/dt$ and calculate only the negative ones, Fig. 2b). The corresponding $\gamma$-factor $\gamma_{\text{ARP}}(t) = 1/\sqrt{1 - v_{\text{ARP}}(t)^2/c^2}$ contains sharp spikes, which coincide with the velocity extrema. These spikes of the surface $\gamma$-factor are responsible for the high harmonic generation.

The numerically obtained spectrum of the high-harmonics is shown in Fig. 3a). Filtering out the lower harmonics and keeping only the harmonics with $\omega > 25\omega_0$, we obtain a train of short pulses in the reflected radiation, Fig. 3b).

For various applications, such as molecule imaging, it is of great importance to have a single short pulse, instead of a train of short pulses. We have shown above that the attosecond pulses are emitted when the tangential components of the surface electron momentum vanish. This property can be used to control the high harmonic generation and to gate a particular attosecond pulse out of the train.

In the 1D geometry, the transverse generalized momentum is conserved: $p_x = eA_x/c$, where $p_x$ and $A_x$ are the tangential components of the electron momentum $p$ and the vector potential $A$. Consequently, the attosecond pulses are emitted when the vector potential is zero. If the vector potential vanishes at several moments, there are several $\gamma$-spikes and correspondingly, after proper filtering, several short pulses are observed in the reflected radiation, see Fig. 3b).

To select a single attosecond pulse, we must ensure that the vector potential $A_x$ turns zero exactly once. Since $A_x$ has two components, how often it vanishes depends on its polarization. For linear polarization it vanishes twice per laser period, while for circular or elliptic polarization it never equals zero. A laser pulse with a time-dependent polarization can be prepared in such a way that its vector potential turns zero just once. The time-dependent polarization corresponds to two perpendicularly polarized
laser pulses with slightly different frequencies, see Fig. 1. Our PIC simulations suggest that a signal with a few percent of the driver intensity is sufficient to control the high harmonic generation, if the phase difference between the two laser pulses is chosen carefully.

To demonstrate the relativistic plasma control, we perform a PIC simulation where we add the z-polarized controlling pulse with amplitude $a_c = 6$ and frequency $\omega_c = 1.25$ and retain the same driver pulse with amplitude $a_0 = 20$, frequency $\omega_0 = 1$ and y-polarization. We keep the same plasma density $N_e/N_0 = 90$. The optimal phase difference between the two lasers is found empirically to be $\Delta \phi = \pi/8$. The simulation results are presented in Fig. 4 a) and b). Comparing the surface $\gamma$-factor dynamics in the regime of linear polarization, Fig. 2 c) and in the controlled regime, Fig. 4 a), we see that the central $\gamma$-spike is slightly larger while the both side spikes are significantly damped. This effect becomes much more pronounced when we compare the filtered radiation plots, Fig. 2 b) and Fig. 4 b). The control signal allows us to select the single attosecond pulse corresponding to the highest $\gamma$-spike in the surface motion.

Varying the control parameters $a_0/a_c, \omega_0/\omega_c$ and $\Delta \phi$ we are able to select different attosecond pulses one-by-one or in groups out of the original pulse train.

To recapitulate, we studied analytically and numerically the dynamics of the apparent reflection point at the overdense plasma surface. We have shown that the velocity of this point is a smooth function of time. However, the corresponding $\gamma$-factor has quasi-singularities or spikes when the surface velocity approaches the speed of light. These ultra-relativistic spikes are responsible for the high harmonic generation in the form of an attosecond pulse train. We show that the attosecond pulse emission can be efficiently controlled by managing the laser polarization. This is done by adding a low intensity control pulse with perpendicular polarization and frequency slightly different from that of the driving pulse. This relativistic plasma control allows to gate a single attosecond pulse or a prescribed group of attosecond pulses.

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