Mesoscopic conductance fluctuations in the insulating regime of small, disordered transistors were first observed by Pepper\cite{Pepper} in GaAs MESFETs and then studied in detail in Si MOSFETs by Fowler, Webb and coworkers\cite{Fowler} in the early 1980s. Extremely strong random fluctuations, spanning several orders of magnitude, were observed at low temperatures in the conductances of narrow-channel devices as the gate voltage was varied. The lognormal distribution for one-dimensional (1D) conductors was long ago analytically predicted by V.I. Mel’nikov\cite{Mel'nikov} and A.A.Abrikosov\cite{Abrikosov}. The explanation was provided by Lee\cite{Lee} who proposed a model in which electrons move by variable-range hopping (VRH) along a 1D chain. Serota, Kalia and Lee\cite{Serota} went on to simulate the ensemble distribution of the total chain resistance and its dependence on the temperature $T$ and the sample length $L$. In their ensemble, the random impurities are distributed uniformly in energy and position along the chain. In experiments a single device is generally used, so that the impurity configuration is fixed, and fluctuations are observed as a function of some variable external parameter such as the chemical potential. An ergodicity hypothesis is then invoked to the effect that the same ensemble is sampled in both cases, something that has been verified experimentally by Orlov et al.\cite{Orlov}. Using the natural logarithm of the resistance, the authors of Ref.\cite{Orlov} obtained for the mean and standard deviation:

$$
\langle \ln \rho \rangle \sim \left( \frac{T_0}{T} \right)^{1/2} \left[ \ln \left( \frac{2L}{\xi} \right) \right]^{1/2} \tag{1}
$$

$$
s \equiv \langle (\ln \rho - \langle \ln \rho \rangle)^2 \rangle \sim \left( \frac{T_0}{T} \right)^{1/2} \left[ \ln \left( \frac{2L}{\xi} \right) \right]^{-1/2} \tag{2}
$$

where $\xi$ is the localization radius and $T_0$ is the characteristic temperature for Mott VRH: $T_0 = 1/k_B \rho \xi$ ($\rho$ is the density of states at the Fermi energy). It can be seen that the size $s$ of the fluctuations decreases extremely slowly with length, a result characteristic of 1D which was first pointed out by Kurkijarvi\cite{Kurkijarvi}. The explanation is simply that exceptionally large resistance elements, even though they may be statistically rare, dominate the overall resistance since they cannot be bypassed in this geometry. The averaging assumed in the derivation of Mott’s hopping law for 1D does not occur and the total resistance takes on the activated form of the largest individual element. As $\xi$ is the localization radius and $T_0$ is the characteristic temperature for Mott VRH: $T_0 = 1/k_B \rho \xi$ ($\rho$ is the density of states at the Fermi energy).

The crossover from 1D to 2D were also studied\cite{Kurkijarvi, Serota}. The first theoretical description of hopping conductivity in narrow 2D strips was given by Serota\cite{Serota} and numerical simulations from narrow 2D to square 2D at a certain temperature were also done by X.C. Xie and S. Das Sarma\cite{DasSarma}. The conductance DF (on the metallic side) was first fully considered analytically in 2D and above by Altshuler, Kravtsov and I. V. Lerner\cite{Altshuler}, where they have predicted the Gaussian distribution with long lognormal tails. Due to the finite widths of channels, the 2D VRH is also affected by the width of system. To observe this effect fully, L. He et al\cite{He} did numerical simulations on all 2D cases, including the short 2D which has been studied experimentally by Hughes et al\cite{Hughes}. Although most of the results are in accordance with those of the previous work, the DF for short 2D is of a long tail to the low conductance opposed to the low resistance as expected. To check the results, we numerically simulate these samples by searching for the shortest path across system other than by percolating it. The shortest paths would be punctures which short out less conductive paths in the 2D geometry. The aim of the present work is to report this study, and first of all, it is also started by

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replacing the transport problem across a 2D mesoscopic system in Mott hopping regime with a random resistor network as did previously.

Imagine that a particle is about to transport a conductive network. In the principle of lowest energy, it prefers to the path of the smallest overall resistance among all possible paths. The path of lowest energy cost is equivalent to the shortest path of graph theory, which is just what the nature of puncture of the system means. It means that we could also approximate the resistance of the system by looking for the shortest path, as well as by percolating the system as usual. The shortest paths among percolation systems have been studied considerably recently\[14,15,16,17,18\] in terms of minimal path or optimal path between a pair of sites within the same cluster, and the study on the scaling form of the probability of the shortest path with regard to their Euclidean distance and the cluster mass ($M_D$)\[14\] has shown that the average conductance of the percolation backbone is strongly correlated with the shortest path, and it decreases with increasing minimal path. This means the shortest path determines the average conductance in nature.

A network is set up by resistors, $\rho_{ij}$, between sites $i$ and $j$

$$\ln \rho_{ij} = 2\alpha d + (|E_i - \mu| + |E_j - \mu| + |E_i - E_j|)/2kT, \quad (3)$$

Here $\alpha$ is the inverse localization length, $d$ is the distance of two localized sites, $E_i$ and $E_j$ are energies of site $i, j$, and $\mu$ is the chemical potential, $T$ is the temperature. Energy is chosen randomly from a uniform distribution in the range $-0.5 \sim +0.5$. Thus the mesoscopic system is reduced to a random resistor network (RRN). To percolate the network, the resistors joining electrodes are selected in ascending order until the first percolation path connects the reservoirs. The resistance of the percolation path is taken to be the resistance of the entire system.

To solve it by the shortest path, Dijkstra algorithm\[19\] is applied, which is efficient in finding shortest paths to all nodes from a single source in a fully connected graph. In views of graph, the RRN is partially connected, and there are more than one source nodes except for the 1D case. So, some modifications on this algorithm has been done in the simulation.

In calculation, positions of impurities are uniformly distributed over the system, their energies are distributed evenly between $-0.5 \sim +0.5$, and gate voltage, $\mu$, is randomly chosen. Thus we can consider the chemical potential distributions (for a fixed impurity configuration) and the ensemble distributions (for a fixed chemical potential ). For a 1D system of $L = 1000$, chemical potential $\mu = 0$ and temperature $T = 0.001$, we have got the profile of their individual resistances similar to that along percolating path (Fig.1 of Ref.\[12\]). It means that it is also the single largest hop along the shortest path that controls the overall conductance of the system. With temperature increasing, the sizes of fluctuation of individual resistors along the shortest path become close. As has been done previously\[13\], we also simulate the 1D systems firstly to validate the simulation. The results are shown in Fig.(1). The size of 1D system is 1000 in length, and 50 in localization length. The temperature is $T = 0.001$, which gives $\nu = 0.225$. The chemical potential range is $\mu = -0.1 \sim +0.1$. As found in Ref.\[12\], with the increase of temperature, the fluctuation of 1D resistance become small.

On the basis of the above validity, the 2D cases are numerically simulated by the shortest path, including cases of narrow 2D, square 2D and short 2D. In our simulation, localization length, temperature, etc., for a 2D system, $w \times L$, are the same as in the 1D case, while $w = 100$, $L = 1000$ for narrow 2D, $w = 1000$ and $L = 100$ for short 2D and $w = L = 1000$ for square 2D system. The results are shown in Fig.2. From this, it is seen clearly that the magnitude of conductance substantially increases with the increase of system width, which is in agreement with the prediction of both Serota\[2\] and Altshuler et al\[10\], and the numerical results of X.C. Xie et al\[1\] for narrow 2D and square 2D cases. The results also reveal that the situations of narrow 2D is close to the 1D case, and the normal 2D is close to Gaussian. As discovered previously\[12\], the DF of conductance across the short 2D system is still close to that of 1D case other than a mirror reflection of the latter as expected. Note that the peak of short 2D is shifted to the low end of resistance more than the other 2D cases. According to this, it seems to be more proper to explain the puncture nature of 2D systems by the position of the peak of DF while the long tail of DF shows the bias conductance fluctuations.

In conclusion, the paper has studied the DF of the conductance in mesoscopic systems by the shortest path which could reflect the puncture nature of VRH system more directly. The validity is done and the results are

FIG. 1: The conductance of 1D system for $\nu = 0.225$: a) ensemble distribution function; b) chemical potential distribution function. The histograms in both figures are the numerical result.
found to be quite close to the numerical results of the narrow and square 2D systems by P.A. Lee[5], Serota[6] and X.C. Xie et al.[11], i.e., the result for narrow 2D system is close to that of 1D case, and the result for square 2D is close to Gaussian[10]. But the unexpected results for short 2D system hints that the puncture nature of 2D systems could be explained more properly by its peak position of DFs and the long tail shows the preference of conductance fluctuation.

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