Implementation of standard quantum error correction codes for solid-state qubits

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In quantum error-correcting code (QECC), many quantum operations and measurements are necessary to correct errors in logical qubits. In the stabilizer formalism, which is widely used in QECC, generators \( G_i (i = 1, 2, \ldots) \) consist of multiples of Pauli matrices and perform encoding, decoding and measurement. In order to maintain encoding states, the stabilizer Hamiltonian \( H_{\text{stab}} = - \sum_i G_i \) is suitable because its ground state corresponds to the code space. On the other hand, Hamiltonians of most solid-state qubits have two-body interactions and show their own dynamics. In addition solid-state qubits are fixed on substrate and qubit-qubit operation is restricted in their neighborhood. The main purpose of this paper is to show how to directly generate the stabilizer Hamiltonian \( H_{\text{stab}} \) from conventional two-body Hamiltonians with Ising interaction and XY interaction by applying a pulse control method such as an NMR technique. We show that generation times of \( H_{\text{stab}} \) for nine-qubit code, five-qubit code and Steane code are estimated to be less than 300 ns when typical experimental data of superconducting qubits are used, and sufficient pulse control is assumed. We also show how to prepare encoded states from an initial state \([0\ldots0]\). In addition, we discuss an appropriate arrangement of two- or three-dimensional arrayed qubits.

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I. INTRODUCTION

Similar to the digital computer, a rigid error-correcting system is required in the quantum computer. Various quantum error-correcting codes (QECC) have been developed such as the standard code \([0\ldots0]\), the subsystem codes \([10\,12]\), and the topological code \([13\,18]\). In QECC, it is necessary for many qubits to be coherently entangled for constructing logical qubits. For instance, nine qubits are required for a logical qubit of the nine-qubit code \([1]\), seven qubits are required for the seven-qubit Steane code, which is the smallest code of the general CSS code \([2]\), and so on \([3\,4]\). In any quantum codes, many operations and measurements are required for encoding, decoding and error-correcting processes. There are strict requirements concerning the maximum error rate for the success of QECC \([3\,5\,18]\). All manipulations of many qubits should be done sufficiently within the coherence time.

In general, it is difficult to produce desired encoded states consisting of many qubits. However, it is also difficult to maintain each entangled state during the time required in a flow of quantum computation \([3\,5]\). This problem arises when the encoded state is not the eigenstate of a system Hamiltonian. The encoded state changes following the dynamics of the system Hamiltonian. Assume that a computer system consists of many blocks. Each block must correlate with every other block to carry out a definite set of quantum computations. As a simple structure of a computing system, let us consider a system in which operations are synchronized to a system clock, which is the case with the present widely-used digital computers. Then, all operations are carried out step by step as the system clock ticks the system time. Entangled states produced by CNOT gates or other quantum gates appear only periodically when the entangled states are not the ground states of the system Hamiltonian. In such case, if each block of a system includes an individual entangled state, it will be difficult to control the synchronization of the total system because the period of the desired entangled states differs depending on the dynamics of each block. Thus, it will be desirable for encoded states to be the ground states of Hamiltonians of the blocks. Moreover, because each block of a system changes its role as system time passes, it is desirable that the Hamiltonian of each block changes depending on each calculation step.

In this paper, we show how to efficiently implement standard QECC in solid-state qubit systems with natural two-body interactions, focusing on the stabilizer formalism. Stabilizer operators \( \{ G_i | 1 \leq i \leq l \} \) are mutually commuting operators given by products of multiple Pauli matrices \([3\,4]\). Logical qubit states are encoded into a mutual eigenspace \( H_S \) of dimension \( 2^l \) of these operators through measurements. For \( l \) different stabilizers and \( n \) physical qubits, a maximum number of \( k = n - l \) logical qubits can be encoded into \( H_S \), whereas \( k < n - l \) in case of subsystem encoding \([10\,12]\). Although preparation of some “quantum memory” blocks to where logical qubit states can be transferred or teleported is one solution to preserve logical qubit states, we consider that it is better to change a system Hamiltonian into a stabilizer Hamiltonian defined by \( H_{\text{stab}} = - \sum_i G_i \), because transformation or teleportation of encoded states requires more complexity. We would also like to show how to generate encoded states without measurements. The encoded states are generated by using operators that are modified from stabilizer operators. Therefore, in this paper, we mainly describe the generation process of \( H_{\text{stab}} \).

In previous papers \([19\,20]\), we showed that we can construct \( G_i \) one by one based on the two-body Hamiltonian by using the appropriate pulse sequence. However,
it is much more efficient to directly produce \( H_{\text{stab}} \). In this paper, we show how to directly create \( H_{\text{stab}} \) starting from the two-body Hamiltonian. \( H_{\text{stab}} \) has a complicated form of multiplied Pauli matrices. We show that appropriate pulse sequences to generate \( H_{\text{stab}} \) can be found by inversely tracing a transformation from \( H_{\text{stab}} \) into single-qubit Hamiltonian. We show that the direct creation of \( H_{\text{stab}} \) greatly reduces the number of operations compared with our previous method in Ref. \[19, 20\]. This reduction is remarkable in the case of qubits with \( XY \) interaction. For example, the number of single-qubit rotations \( N_{\text{rot}} \) and that of qubit-qubit \( XY \) interaction \( N_{\text{int}} \) are reduced from \( N_{\text{rot}} = 44 \) to \( N_{\text{rot}} = 20 \), and from \( N_{\text{int}} = 288 \) to \( N_{\text{int}} = 132 \), respectively, for the Steane code. Similar results are obtained for the nine-qubit code and the five-qubit code. Accordingly, operation time can also be reduced. If we use a typical experimental parameter of superconducting qubits, we can reduce the time required to generate \( H_{\text{stab}} \) by 48.4 \% (194 ns), 59.1 \% (127.5 ns) and 54.4 \% (257 ns) for the nine-qubit code, the five-qubit code and the Steane code, respectively. The present method has the advantage that, as pulse control technology progresses, pulse error rate and speed are improved. Pulse errors can be corrected by using NMR techniques such as the composite-pulse method \[21, 22\], and the speed is increased by improving a control system operated by a digital computer.

We also investigate a possible architecture of standard codes for solid-state qubits on lattice sites. In general, interactions between solid-state qubits are restricted to their nearest-neighbor or next nearest-neighbor sites \[26–32\]. In order to prevent unexpected external noise, it is preferable for physical qubits in a logical qubit to be placed compactly in a small region. Moreover, for logical qubits to interact effectively with one another, it is desirable to place logical qubits side by side. Therefore, it is natural to construct logical qubit by one-dimensional (1D) qubit arrays and place them parallel as shown in Fig. \[1\]. In addition, frequent measurements in QECC require other qubit arrays for measurements. We will discuss possible setups of a qubit system.

As a general case, we consider always-on Hamiltonian in this paper. We think that we can separate logical qubits by effectively eliminating qubit-qubit interaction through the use of appropriate pulse sequences.

This paper is organized as follows: In Sec. \[III\] we establish the general procedure of generating the stabilizer Hamiltonian. In Sec. \[IV\] we show examples of generating the stabilizer Hamiltonian in the standard code, and in Sec. \[V\] we show how to generate the code state. Finally, in Sec. \[VII\] we consider possible qubit architecture realized by solid-state qubits. We close with a summary and conclusions in Sec. \[VII\].

II. STABILIZER HAMILTONIAN GENERATION METHOD

A. Stabilizer coding and stabilizer Hamiltonian

In the stabilizer code \[3, 4\], encoding, decoding and error-correction are carried out based on the stabilizers, which are mutually commutative and can be expressed by the Pauli matrices:

\[
G_i = \bigotimes_{j=1}^n (X_i)^{x_i(G_i)} (Z_i)^{z_i(G_i)} \tag{1}
\]

\((x_i(G_i), z_i(G_i) \in \{0, 1\})\), where Pauli matrices are given by

\[
X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2}
\]

The codeword \(|\Psi_m\rangle\) obeys the eigenvalue equation

\[
G_i |\Psi_m\rangle = |\Psi_m\rangle \tag{3}
\]

Conventionally, in order to construct encoding states, starting from an initial state \(\Pi_{i=1}^k |0\rangle_i\), measurements over stabilizer operators of the selected code are repeated. Depending on the measurement outcome, the common eigenstate is fixed to be the desirable encoded state. The correction procedure for the stabilizer code is carried out by measuring all relevant stabilizer operators. The stabilizer Hamiltonian \(H_{\text{stab}}\) is defined by

\[
H_{\text{stab}} = - \sum_{i=1}^l G_i, \tag{4}
\]

where the summation is taken over the constituent stabilizers of each code. Owing to the commutativity of the stabilizers \(G_i\), the ground state of Eq. (4) is a common eigenstate of the stabilizers, which is the encoded logical state. For the sake of simplicity, we consider the standard codes without considering subsystem code (\(k = n - l\)).

B. System Hamiltonian

The solid-state Hamiltonian controlled by pulse signals can be written as \[33, 34\]

\[
H(t) = \sum_i [\Omega_0 Z_i + 2\Omega_i \cos(\omega_i t + \delta_i) X_i] + \sum_{i<j} J_{ij} X_i X_j, \tag{5}
\]
where $\Omega_i$ and $\omega_i^{rf}$ are an amplitude and a frequency of a controlled signal applied to a qubit $i$. If we move to a frame rotating with the radio-frequency $\omega_i^{rf}$ about the z-axis, $H' = R^{-1}(t)H(t)R$, with $R = \exp[-i\sum\omega_i^{rf}t/2Z_i]$, then the transferred static Hamiltonian $H' = H'' - \sum_i(\omega_i^{rf}t/2)Z_i$ is approximately given by

$$H' = \sum_i \left[ \left( \Omega_i - \frac{\omega_i^{rf}}{2} \right) Z_i \right] + \Omega_i(\cos\delta X_i - \sin\delta Y_i) + \sum_{i<j} \frac{J_{ij}}{2} [X_iX_j + Y_iY_j].$$

(high-frequency components $2\omega_i^{rf}$ can be neglected). If Eq. (6) includes an interaction of $\sum_{i<j} J_{ij}Z_iZ_j$ instead of $\sum_{i<j} J_{ij}X_iX_j$, the final Hamiltonian Eq. (6) includes the Ising interaction. The x-pulse and y-pulse for qubit $i$ are realized when $\delta = 0$ and $\delta = -\pi/2$ signals are respectively applied to the qubit with $\omega_i^{rf} = 2\Omega_0i$. We assume that each pulse is sufficiently strong for interactions between qubits to be neglected during the pulse sequences $(\Omega_i, \Omega_0i > J_{ij})$.

Then the qubit Hamiltonian in the rotating frame of $\omega_i^{rf} = 2\Omega_0i$ is expressed by $H_q = H_0 + H_{\text{int}}$ where a single-qubit part $H_0$ is given by

$$H_0 = \sum_i H_{0i} = \sum_i \Omega_iX_i$$

The interacting part $H_{\text{int}} = \sum_{i<j} H_{\text{int}}^{ij}$ is expressed by

$$H_{XY} = \sum_{i<j} H_{XY}^{ij} = \sum_{i<j} J_{ij} [X_iX_j + Y_iY_j],$$

for XY interaction, and

$$H_{\text{Ising}} = \sum_{i<j} H_{\text{Ising}}^{ij} = \sum_{i<j} J_{ij}Z_iZ_j,$$

for Ising interaction.

\section{C. Dynamic generation of stabilizer Hamiltonian}

The generation of $H_{\text{stab}}$ from $H_q$ consists of two steps. The first step is to extract a single-qubit part or a pure two-body interaction part from a qubit Hamiltonian $H_q$. The second step is to construct $H_{\text{stab}}$ dynamically with pulse sequences by using a selected single qubit part $H_{\text{ini}}$ and qubit-qubit interactions $H_{\text{int}}^{ij}$. Because the second step is the core framework of this paper, we first describe the second step of dynamical transformation to $H_{\text{stab}}$. The extraction method is described in the next section \ref{III.D}.

The transformation from the two-body Hamiltonian $H_q$ to the many-body Hamiltonian $H_{\text{stab}}$ is carried out dynamically by using a time evolution of a system starting from a simple initial Hamiltonian $H_{\text{ini}} \propto X_i, Y_i, \text{ or } Z_i$ \cite{19, 20}. The time evolution of the generation process is illustrated with the schematic notation $\rho(0) \overset{1H}{\rightarrow} \rho(t)$, where $\rho(t) = \exp(-iHt)\rho(0)\exp(iHt)$ is the density matrix for a time-independent Hamiltonian $H$, or for an effective $H$ in the sense of the average Hamiltonian theory \cite{21}. After the application of mutually inverse, unitary operations according to

$$\rho(\tau_{\text{op}}) \overset{H_{\text{op}}}{\rightarrow} \rho(\tau_{\text{ini}}) \overset{H_{\text{ini}}}{\rightarrow} \rho(\tau_{\text{op}} + \tau_{\text{ini}}),$$

the system evolves as if propagated by the effective Hamiltonian $\exp(-i\tau_{\text{op}}H_{\text{op}})\rho(\tau_{\text{ini}})\exp(i\tau_{\text{op}}H_{\text{op}})$ for a time $\tau_{\text{ini}}$ \cite{18}.

To build $H_{\text{stab}}$ from $H_{\text{ini}}$, we need two elementary transformations: one that rotates arbitrary single-qubit terms through an angle of $\pi/2$ and another that increases the order of Pauli-matrix terms by one. Higher-order products of Pauli matrices can be generated using the following transformations \cite{19}:

$$e^{-i\theta[XY]_{12}}X_1 e^{i\theta[XY]_{12}} = \cos(2\theta)X_1 - \sin(2\theta)Y_2, \quad (11)$$

$$e^{-i\theta[XY]_{12}}Y_1 e^{i\theta[XY]_{12}} = \cos(2\theta)Y_1 + \sin(2\theta)Z_2, \quad (12)$$

$$e^{-i\theta[XY]_{12}}Z_1 e^{i\theta[XY]_{12}} = \cos^2(2\theta)Z_1 + \sin^2(2\theta)Z_2 + \frac{1}{2}\sin(4\theta)[X_1Y_2 - Y_1Z_2]. \quad (13)$$

for XY interaction. When $\theta = \pi/4$ we can change the number of Pauli matrices given by

$$X_1 \rightarrow -Z_1Y_2, \quad (14)$$

$$Y_1 \rightarrow Z_1X_2, \quad (15)$$

$$Z_1 \rightarrow Z_2. \quad (16)$$

For Ising interaction, we use the relations given by

$$e^{-i\theta[Z_1Z_2]X_1 e^{i\theta[Z_1Z_2]} = \cos(2\theta)X_1 + \sin(2\theta)Y_1Z_2, \quad (17)$$

$$e^{-i\theta[Z_1Z_2]Y_1 e^{i\theta[Z_1Z_2]} = \cos(2\theta)Y_1 - \sin(2\theta)X_1Z_2. \quad (18)$$

Then, for $\theta = \pi/4$, we can change the number of Pauli matrices given by

$$X_1 \rightarrow Y_1Z_2, \quad (19)$$

$$Y_1 \rightarrow -X_1Z_2, \quad (20)$$

$$Z_1 \rightarrow Z_1. \quad (21)$$

By combining these equations with single-qubit rotations, we can change $H_q$ to $H_{\text{stab}}$.

\section{D. Extracting $H_{\text{ini}}$ and $H_{\text{op}}$ from a qubit Hamiltonian}

In order to use the above-mentioned dynamic method, the important step is to extract a single-qubit part or a pure two-body interaction part from a qubit Hamiltonian $H_q$. This process is carried out using the Baker-Campbell-Hausdorff (BCH) formula \cite{21}. Here, we assume that qubits interact with their nearest-neighbor
qubits. Then, in order to define a logical qubit, we have to determine the locations of physical qubits in a logical qubit. In this section, after we explain the BCH formula, we would like to define a logical qubit arranged on lattice sites. Then, finally we will show how to extract a single-qubit part $H_{\text{int}}$ and a pure two-body interaction $H_{\text{op}}$ from the Hamiltonian of a qubit lattice.

1. Manipulation by using the Baker-Campbell-Hausdorff (BCH) formula

A desirable part of the original Hamiltonian $H_0$ is extracted by using appropriate pulse sequences $\pi$. The basic idea can be illustrated by using the standard NMR Hamiltonian $H_{\text{int}} = \sum_{i} \varepsilon_i Z_i + \sum_{i<j} J Z_i Z_j$. In this case, because of the property $[H_0, H_{\text{int}}] = 0$, $H_0$ and $H_{\text{int}}$ can be separately obtained by using a simple pulse sequence. The interaction part $H_{\text{int}}$ can be extracted by using two sandwiched $\pi$-pulses such as $\exp(i\tau H_{\text{int}}) = e^{-i(\pi/2) \sum_j \varepsilon_j Z_j e^{(\pi/2) H_{\text{int}} \sum_j \varepsilon_j Z_j}}$. For the general Hamiltonian (Eqs. (30)-(32)), because $[H_0, H_{\text{int}}] \neq 0$, we can approximately obtain a desirable part by repeatedly applying the Baker-Campbell-Hausdorff (BCH) formula. For $A = a + b$ (original Hamiltonian) and $B = a - b$ (transferred by applying a $\pi$ pulse) with $h = i\tau H_0$ and $h_0 = i\tau H_0$, we can extract $h_0 = i\tau H_0$ by using the relation given by

\[ (e^{A}e^{B})^n \approx \exp(i2t_0A + (t_0^2/n)[A, B]) \]  (22)

($t_0 = n\tau$). Thus, as long as $(t_0/n)||H_0|| \ll 1$ where $||A|| = |\text{Tr}(A^\dagger A)/d|^{1/2}$ is the standard operator norm in a Hilbert space of dimension $d$, we can neglect the second term. As the number $n$ of repetitions increases, this approximation improves.

In the following sections, we use an extended form of Eq. (22) described by

\[ (e^{A}e^{B}e^{A'})^n \approx \exp(2(nh + [h, h_0]) + n[h, h_0] - n[h_0, h_0] + 4n[h, h_0]) \]  (23)

where $A' = h_0 + h_0'$ and $B' = h_0 - h_0'$. $2(h + h_0' + h_0')$ is the target Hamiltonian. In the following two subsections, we show how to extract a desirable interaction term $H_{\text{int}}^{03}$ and a single-qubit part $H_0$ from $H_q$ by using Eq. (23).

2. Qubit lattice and logical qubit

We consider a qubit lattice in which physical qubits are arrayed on a lattice site interacting with their neighboring qubits. The simplest arrangement is a 1D array as shown in Fig. 1. Then we can interact logical qubits with their nearest-neighbor logical qubits by using interactions between physical qubits. The number of qubits in each 1D array depends on how many physical qubits are required to construct a single logical qubit. In Fig. 1 nine qubits constitute a logical qubit.

3. Selection of a single-qubit Hamiltonian

Here we show how to extract $H_0$ from $H_q$ for 2D qubit lattice, assuming always-on interactions between qubits. As an example, we consider logical qubits consisting of five qubits. In a 1D qubit array, $H_0$ is obtained by choosing $B = -h_1 + h_2 - h_3 + h_4 - h_5 - \sqrt{2}i\sum h_{ij}$ and $B' = h_1 - h_2 + h_3 - h_4 + h_5 - \sqrt{2}i\sum h_1 h_{ij}$ while $A$ and $A'$ are $H_q$ in Eq. (24) $[h_i = \tau H_0]$ and $h_{ij} = \tau H_{\text{int}}$. This procedure can be extended to the 2D lattice case by taking into account interactions between different logical qubits.

In this section, we treat Hamiltonians that include two types of Pauli matrices or fewer such as Eq. (20) or Eq. (20) with $\omega_{11}^f = 2\Omega_0$. For Eq. (20), “$\pi$-pulse” corresponds to $\pi$-pulse around $y$-axis. For Eq. (20) with $\omega_{11}^f = 2\Omega_0$, “$\pi$-pulse” corresponds to $\pi$-pulse around $z$-axis, which can also be produced by $\pi$-pulse around $y$-axis after that around $x$-axis. Extraction of $H_0$ and two-body interaction from the Hamiltonian Eq. (20) with $\omega_{11}^f \neq 2\Omega_0$ is described in Appendix A.

The 2D lattice Hamiltonian is given by

\[ H_{2D}^{Q} = \sum_k H_q^{(k)} \]  (25)

where

\[ H_q^{(k)} = H_0^{(k)} + H_{\text{int}}^{(k)} + H_{\text{int}}^{(k,k+1)} \]  (26)

$H_{\text{int}}^{(k,k+1)}$ shows an interaction term between $k$-th logical qubits and $k+1$-th qubits. In order to separate different logical qubits, $H_{\text{int}}^{(k,k+1)}$ should be erased. We apply $\pi$-pulses to (i) qubits $1,3,5$ of $\ldots,k-1$-th, $k+1$-th, $\ldots$ arrays for $A$, (ii) qubits $1,3,5$ of $\ldots,k$-th, $k+2$-th, $\ldots$ arrays for $B$, (iii) qubits $2,4,6$ of $\ldots,k-1$-th, $k+1$-th, $\ldots$ arrays for $B'$, and (iv) qubits $2,4$ of qubits of $\ldots,k$-th, $k+2$-th, $\ldots$ arrays for $A'$:

\[ A = \ldots \]

\[ - h_1^{(k-1)} + h_2^{(k-1)} - h_3^{(k-1)} + h_4^{(k-1)} - h_5^{(k-1)} - h_1^{(k-1)} \]

\[ - h_1^{(k-1,k)} + h_2^{(k-1,k)} - h_3^{(k-1,k)} + h_4^{(k-1,k)} - h_5^{(k-1,k)} \]

\[ + h_1^{(k)} \]

\[ - h_1^{(k,k+1)} + h_2^{(k,k+1)} - h_3^{(k,k+1)} + h_4^{(k,k+1)} - h_5^{(k,k+1)} \]

\[ - h_1^{(k+1)} + h_2^{(k+1)} - h_3^{(k+1)} + h_4^{(k+1)} - h_5^{(k+1)} - h_1^{(k+1)} \]

\[ \ldots \]

\[ B = \ldots + h_1^{(k-1)} \]

\[ - h_1^{(k-1,k)} + h_2^{(k-1,k)} - h_3^{(k-1,k)} + h_4^{(k-1,k)} - h_5^{(k-1,k)} \]

\[ - h_1^{(k)} + h_2^{(k)} - h_3^{(k)} + h_4^{(k)} - h_5^{(k)} \]

\[ - h_1^{(k,k+1)} + h_2^{(k,k+1)} - h_3^{(k,k+1)} + h_4^{(k,k+1)} - h_5^{(k,k+1)} + h_1^{(k+1)} \ldots \]  (27)
\[ B' = \ldots \]
\[ + h_1^{(k-1)} - h_2^{(k-1)} + h_3^{(k-1)} - h_4^{(k-1)} + h_5^{(k-1)} - h_{\text{int}}^{(k-1)} \]
\[ + h_{11}^{(k-1,k)} - h_{22}^{(k-1,k)} + h_{33}^{(k-1,k)} - h_{44}^{(k-1,k)} + h_{55}^{(k-1,k)} \]
\[ + h_q^{(k-1)} \]
\[ + h_{11}^{(k+1,k+1)} - h_{22}^{(k+1,k+1)} + h_{33}^{(k+1,k+1)} - h_{44}^{(k+1,k+1)} + h_{55}^{(k+1,k+1)} \]
\[ + h_1^{(k+1)} - h_2^{(k+1)} + h_3^{(k+1)} - h_4^{(k+1)} + h_5^{(k+1)} - h_{\text{int}}^{(k+1)} \]
\[ \ldots \]

(28)

\[ A' = \ldots + h_q^{(k-1)} \]
\[ + h_{11}^{(k-1,k)} - h_{22}^{(k-1,k)} + h_{33}^{(k-1,k)} - h_{44}^{(k-1,k)} + h_{55}^{(k-1,k)} \]
\[ + h_1^{(k+1)} - h_2^{(k+1)} + h_3^{(k+1)} - h_4^{(k+1)} + h_5^{(k+1)} - h_{\text{int}}^{(k+1)} \]
\[ + h_q^{(k-1)} \ldots \]

(29)

where \( h_q^{(k)} = \tau(H_0^{(k)} + H_{\text{int}}^{(k)}) \). By using Eq. (28), we obtain \( H_{\text{eff}} = 2 \sum_k H_0^{(k)} \).

4. Selection of two-body interaction

Next, we show how to extract the interaction term \( H_0^{(2)} \) between two qubits in order to use Eqs. (11)-(13) or Eqs. (17)-(18) for the 2D lattice qubits. As an example, we consider a case of extracting \( h_{23} = i\tau H_{23}^{(2)} \) in five-qubit array. The required transformation is given by extending the results of Ref. [20]. A in Eq. (23) is the original Hamiltonian such as \( A = \tau(H_0 + H_{\text{int}}) \). B in Eq. (23) is given by applying \( \pi \) pulse to qubits 2, 3, 5 of \( (k + 2n - 1) \)-th logical qubits and qubits 1, 4 of \( (k + 2n - 1) \)-th logical qubits \((n \text{ is an integer})\):

\[ B' = \ldots \]
\[ + h_1^{(k-1)} - h_2^{(k-1)} + h_3^{(k-1)} - h_4^{(k-1)} + h_5^{(k-1)} \]
\[ - h_1^{(k-1)} + h_2^{(k-1)} - h_3^{(k-1)} - h_4^{(k-1)} - h_5^{(k-1)} \]
\[ + h_1^{(k)} - h_2^{(k)} - h_3^{(k)} + h_4^{(k)} - h_5^{(k)} \]
\[ - h_1^{(k)} + h_2^{(k)} - h_3^{(k)} - h_4^{(k)} - h_5^{(k)} \]
\[ + h_1^{(k+1)} - h_2^{(k+1)} - h_3^{(k+1)} - h_4^{(k+1)} - h_5^{(k+1)} \]
\[ - h_1^{(k+1)} + h_2^{(k+1)} - h_3^{(k+1)} - h_4^{(k+1)} - h_5^{(k+1)} \]
\[ \ldots \]

(30)

where \( h_i^{(k)} = i\tau H_0^{(k)} \) and \( h_{ij}^{(k)} = i\tau H_{ij}^{(k)} \). \( B' \) is given by applying \( \pi \) pulse to qubits 2, 3, 5 of \( (k + 2n - 1) \)-th logical qubits and qubits 1, 4 of \( (k + 2n) \)-th logical qubits \((n \text{ is an integer})\):

\[ B' = \ldots \]
\[ + h_1^{(k-1)} - h_2^{(k-1)} + h_3^{(k-1)} - h_4^{(k-1)} - h_5^{(k-1)} \]
\[ - h_1^{(k-1)} + h_2^{(k-1)} - h_3^{(k-1)} - h_4^{(k-1)} - h_5^{(k-1)} \]
\[ - h_1^{(k)} - h_2^{(k)} - h_3^{(k)} - h_4^{(k)} - h_5^{(k)} \]
\[ + h_1^{(k)} - h_2^{(k)} - h_3^{(k)} - h_4^{(k)} - h_5^{(k)} \]
\[ + h_1^{(k+1)} - h_2^{(k+1)} - h_3^{(k+1)} - h_4^{(k+1)} - h_5^{(k+1)} \]
\[ - h_1^{(k+1)} + h_2^{(k+1)} - h_3^{(k+1)} - h_4^{(k+1)} - h_5^{(k+1)} \]
\[ \ldots \]

(31)

The \( A' \) is obtained by applying \( \pi \) pulse to all qubits given by

\[ A' = \tau(-H_0 + H_{\text{int}}) \].

(32)

By using Eq. (23), we can obtain \( \sum_k A h_{23}^{(k)} \).

The perturbation terms in Eq. (23) are described in Appendix [B]. For the selection of \( \sum_k A h_{23}^{(k)} \), the perturbation is estimated as \( ||H_{\text{pert}}|| \approx 10\tau N_{\text{qubit}} J_0 \), and for the case of \( H_0 \), we have \( ||H_{\text{pert}}|| \approx 20\tau N_{\text{qubit}} J_0 \), where \( N_{\text{qubit}} \) is the number of connected qubits. As long as \( N_{\text{qubit}} \) is not large, these perturbation terms can be neglected by repeating Eq. (23) with \( J_{ij} J_0 / n \ll 1 \). Hereafter, we consider the case of \( n = 1 \) for simplicity. Note that the procedure described in this section can be easily extended to three-dimensionally (3D) arrayed qubits.

E. Estimation of elapsed time

In order to estimate an operation time of pulse manipulations, we express the time for single-qubit rotation as \( \tau_{\text{rot}} \). For preparing a single Hamiltonian \( H_0 \), it takes an extra time of \( 5\tau_{\text{rot}} \), because, in Eq. (23), four Hamiltonians \( A, B, B', A' \) are transformed from \( H_0 \) by being sandwiched by \( \pi \)-pulses. It also takes extra times of \( 4\tau_{\text{rot}} \) and \( 5\tau_{\text{rot}} \) to obtain \( \exp(i\tau_{\text{op}} H_{\text{op}}) \) and \( \exp(-i\tau_{\text{op}} H_{\text{op}}) \), respectively, in Eq. (11). In the latter case, \( \tau_{\text{rot}} \) is required to reverse the sign of \( H_{\text{op}} \). Thus, for \( N_{\text{qubit}} \)-qubitubit operations, it takes a time of \( N_{\text{qubit}}[2\tau_{\text{rot}} + 9\tau_{\text{rot}}] \).

In the following, we would like to address the feasibility of our scheme in a typical superconducting qubit system. Note that our qubit lattice model can be applied not only to solid-state coupling qubits [30, 31], but also to circuit-QED qubits [40, 43]. For two superconducting qubits in a circuit-QED setup the effective inter-qubit interaction can be treated as \( XY \) type [44, 13]. For instance, for \( g / \Delta = 0.1, g / (2\pi) = 200 \text{ MHz}, \Delta / (2\pi) = 2 \text{ GHz} \), where \( g \) is the Jaynes-Cummings coupling constant and \( \Delta \) the detuning between the resonator frequency and the qubit splitting, we have \( J / (2\pi) = 20 \text{ MHz} \). Thus, \( \tau_{\text{op}} \approx 6.25 \text{ ns} \). We also take \( \tau_{\text{rot}} \approx 1 \text{ ns} \) [20]. The criterion is whether all pulse sequences can be done during the dephasing time \( T_2 \). We will show that all generation times are less than 300 ns. Thus, if we assume \( T_2 \approx 10 \text{ ns} \),
to 20 μs with well-controlled pulses, which was realized by Paik et al. [43], we will be able to use the standard QECC process and correct qubit errors, as long as the number of errors is small.

III. GENERATION OF STABILIZER CODE FROM CONVENTIONAL HAMILTONIAN

Here, we show concrete pulse sequences to produce the target stabilizer Hamiltonians of the three major codes: the nine-qubit code, the five-qubit code, and the Steane code. In general, it is difficult to find a pulse sequence of the transformation from the conventional two-body solid-state Hamiltonian to the target stabilizer Hamiltonian, because the target Hamiltonians have Pauli matrices whose form is complicated. The best way to look for an appropriate pulse sequence is to change the target stabilizer Hamiltonian into single-qubit Hamiltonian, because it is easier to reduce the number of multiplications of the Pauli matrices to single-qubit Hamiltonian. In the following, we show the transformation process of $H_{\text{stab}}$ of the three major codes to the initial single-qubit Hamiltonian. We also count the number of pulses and estimate generation time of the codes. We show that the direct generation of $H_{\text{stab}}$ is more effective than the previous method [20] in which $G_i$ is generated one by one. The comparison of the present results with those of the previous results is summarized in Tables I and II.

A. Nine-qubit code

We would like to start from Shor’s nine-qubit code that was the first advanced QECC to be invented [1]. This code can correct single-qubit error ($n = 9$, $k = 1$), and the number of stabilizers is $l = 8$. The stabilizers are given by

$G_1 = Z_1Z_2$, $G_2 = Z_2Z_3$, $G_3 = Z_3Z_5$, $G_4 = Z_5Z_6$ $G_5 = Z_7Z_8$, $G_6 = Z_8Z_9$ $G_7 = X_1X_2X_3X_4X_5X_6$, and $G_8 = X_4X_5X_6X_7X_8X_9$ [3, 4]. Then, the target stabilizer Hamiltonian is given by $H_{\text{code}} = \sum_{i=1}^{8} G_i$ in which $\Omega_i$ is omitted, and we treat $H_{\text{code}} = \sum_{i=1}^{8} G_i$ instead of $H_{\text{code}} = -\sum_{i=1}^{8} G_i$ for clarity. We will treat the stabilizer Hamiltonians of the five-qubit code and the Steane code similarly. We consider how this target Hamiltonian is transformed to a single-qubit Hamiltonian by using Eqs. (14)–(16) for the XY interaction or Eqs. (19)–(21) for the Ising interaction. Let us first consider a case of the XY interaction. $H_{\text{code}}$ is changed as follows:

$$H_{\text{code}} = Z_1Z_2 + Z_2Z_3 + Z_3Z_5 + Z_5Z_6 + Z_7Z_8 + Z_8Z_9 + X_1X_2X_3X_4X_5X_6 + X_4X_5X_6X_7X_8X_9, \quad (x \leftrightarrow z : 2, 4, 6, 8),$$

$$\rightarrow Z_1X_2 + X_2Z_3 + X_3Z_5 + Z_5X_6 + Z_7X_8 + X_8Z_9 - X_1Z_2X_3Z_4Z_5Z_6 - Z_4Z_5Z_6Z_7X_8X_9,$n

$$H_{\text{code}} = \sum_{i=1}^{8} G_i = H^{12}_{XY} + H^{34}_{XY} + H^{56}_{XY} + H^{78}_{XY},$$

$$(x \leftrightarrow z : 3, 4, 5, 9),$$

$$\rightarrow -Z_1 - Z_2Z_3 + X_3Z_5 - Z_5 - Z_7 - Z_8Z_9 + X_2Y_4Y_6 + Y_1Y_2X_4X_5X_6 + Z_5Z_6X_7X_9, \quad (y \leftrightarrow z : 3, 4, 5, 9),$$

$$\rightarrow -Z_1Z_2X_3 - Z_4X_5 - Z_6 - Z_7 - Y_2Y_3Z_4Z_5Z_6 + X_1Z_6Z_7X_9, \quad (y \leftrightarrow z : 3, 4, 5, 9),$$

$$\rightarrow -Z_1 + Z_2 + Y_3Y_4 - Z_6 - Z_7 - Z_8 - X_7 - X_3X_6 - X_6X_8, \quad (x \leftrightarrow z : 2, 4, 8),$$

$$\rightarrow -Z_2 - Z_3 + X_4Z_5 - Z_5 - Z_7 - Z_8 - X_3X_6 - X_6X_8, \quad (x \leftrightarrow z : 2, 4, 8),$$

$$\rightarrow -Z_1 - Z_2Z_3 - Z_3Z_5 - Z_5 - Z_7 - Z_8 - X_3X_6 - X_6X_8, \quad (x \leftrightarrow z : 1, 3),$$

$$\rightarrow -Z_2 - Z_1 + Z_4 - Z_5 - Z_9 - Z_7 - Z_3X_6 - X_6X_8, \quad : H^{34}_{XY} + H^{78}_{XY},$$

$$\rightarrow -Z_2 - Z_1 + Z_3 - Z_5 - Z_9 - Z_8 + Z_4X_5 - X_6X_7, \quad : H^{45}_{XY} + H^{67}_{XY},$$

$$\rightarrow -Z_2 - Z_1 + Z_3 - Z_4 - Z_9 - Z_8 + Z_5Z_6Y_7 + Y_7, \quad (x \leftrightarrow z : 5)(y \leftrightarrow z : 7),$$

$$\rightarrow -Z_2 - Z_1 + Z_3 - Z_4 - Z_9 - Z_8 + X_3Z_6Z_7 + Z_7, \quad : H^{56}_{XY},$$

$$\rightarrow -Z_2 - Z_1 + Z_3 - Z_4 - Z_9 - X_7 + X_7 + Z_6.$$

(A)
we can produce the stabilizer Hamiltonian $H^{\text{code}}$ by using the pulse sequence described by the reverse operations of Eq. (33). The initial Hamiltonian Eq. (35) is obtained by $e^{-i H_0 t}e^{i \Gamma_{X5}/2}e^{-i H_0 t}e^{-i \Gamma_{X5}/2}$ in which $e^{-i H_0 t}$ term is obtained from $H_0$ as shown in the previous section. For the Ising interaction, we obtain

$$H^{\text{code}} \rightarrow X_1 Z_2 + Z_2 Z_3 + X_4 Z_5 + Z_5 X_6 + Z_7 Z_8 + Z_8 X_9 - Z_1 X_2 Z_3 Z_4 X_5 Z_6 - Z_4 X_5 Z_6 X_7 X_8 Z_9,$$

$$t_{\text{Ising}}^{12} + t_{\text{Ising}}^{56} + t_{\text{Ising}}^{69}$$

$$\rightarrow Y_1 + Z_2 Z_3 + X_4 Z_5 + Y_6 + Z_7 Z_8 + Y_9 - Y_2 X_3 Z_4 Y_5 - Z_4 Y_5 X_7 Y_8,$$  

$$t_{\text{Ising}}^{56} + t_{\text{Ising}}^{69}$$

$$\rightarrow Y_1 - Y_2 Z_3 + X_4 Z_5 + Y_6 - Z_7 Y_8 + Y_9 - Z_2 X_3 Z_4 Y_5 - Z_4 Y_5 X_7 Z_8,$$

$$t_{\text{Ising}}^{23} + t_{\text{Ising}}^{45} + t_{\text{Ising}}^{78}$$

$$\rightarrow Y_1 + Z_2 X_3 + Y_4 + Y_6 + X_8 + Y_9 + Y_3 X_5 + X_7 Y_7,$$  

$$t_{\text{Ising}}^{34} + t_{\text{Ising}}^{67}$$

$$\rightarrow Y_1 + Z_2 - X_3 X_4 - X_6 Z_7 + X_8 + Y_9 - X_2 X_4 X_5 - X_5 Z_6 X_7,$$  

$$t_{\text{Ising}}^{56} + t_{\text{Ising}}^{67}$$

$$\rightarrow Y_1 + X_2 + X_3 Z_4 + Z_5 X_7 + X_8 + Y_9 - Z_3 X_4 Z_5 - Z_5 X_6 Z_7.$$  

$$t_{\text{Ising}}^{34} + t_{\text{Ising}}^{67}$$

$$\rightarrow Y_1 + X_2 + X_3 Z_4 + Y_7 + X_8 + Y_9 - Z_3 Y_4 - Z_5 Y_6.$$  

$$t_{\text{Ising}}^{34} + t_{\text{Ising}}^{56}$$

$$\rightarrow Y_1 + X_2 + Y_3 + Y_7 + X_8 + Y_9 + X_4 + X_6.$$  

(36)

After single-qubit rotations, we obtain an initial Hamiltonian:

$$H_{\text{ini}}^{\text{code}} = \Omega_1 X_1 + \Omega_2 X_2 + \Omega_3 X_3 + \Omega_4 X_4 + \Omega_5 X_5 + \Omega_7 X_7 + \Omega_8 X_8 + \Omega_9 X_9.$$  

(37)

This Hamiltonian is obtained by eliminating $X_5$ term in $H_0$ as in the case of the XY interaction.

Let us count the number of pulses necessary to obtain the nine-qubit code. Because the present method mainly relies on the control of many pulses, as the number of pulses increases, pulse errors become the principal origin of decoherence. Thus, the number of pulses is an indicator of decoherence in which it is desirable to have fewer pulses. Eq. (35) shows that eight qubit-qubit interaction processes and five single-qubit rotation processes are needed. Note that because $\tau_{\text{op}} > \tau_{\text{rot}}$, the sixth operations of Eq. (35) can be represented by $\tau_{\text{rot}}$. From the result of Sec.III, it takes a time of $N_{\text{op}}[2\tau_{\text{op}} + 9\tau_{\text{rot}}]$ for $N_{\text{op}}$ uses of $H_{\text{XY}}^{\text{op}}$. The initial state $H_{\text{ini}}^{\text{code}(\text{old})}$ is obtained by twice using the generating process of $H_0$, and thus it takes a time of $10\tau_{\text{rot}}$. Thus, we need a time of $\tau_{\text{XY}}^{\text{code}(\text{new})} = 8[2\tau_{\text{op}} + 9\tau_{\text{rot}}] + 12\tau_{\text{rot}} + 10\tau_{\text{rot}} = 16\tau_{\text{rot}} + 94\tau_{\text{rot}}$.  

(38)

In order to compare the present method with that of Ref. [20], let us consider constructing $H_{\text{stab}}$ of the XY interaction by summing up $G_i$ as in Ref. [20]. For $G_1 \sim G_6$, it takes a time of $[2\tau_{\text{op}} + 9\tau_{\text{rot}}] + (4 + 10)\tau_{\text{rot}} = 2\tau_{\text{op}} + 23\tau_{\text{rot}}$, because $G_1 \rightarrow Z_i X_{i+1} \rightarrow Y_i \rightarrow X_i$. For $G_7$, we have

$$G_7 = X_1 Z_2 X_3 Z_4 X_5 X_6,$$  

$$t_{\text{Ising}} = 10\tau_{\text{op}} + 63\tau_{\text{rot}} = 125.5\text{ns}.$$  

Here, we used the experimental value of $\tau_{\text{op}} \approx 6.25$ ns and $\tau_{\text{rot}} \approx 1$ ns (See Sec. B.1). $G_1 \sim G_6$ have the form of the two-body interaction, thus they are directly extracted from $H_0$ as shown in Sec. B.1. Thus it takes a time of $4\tau_{\text{rot}}$ for each process. $G_7$ and $G_8$ are reduced to $Z_5 Z_4$ and $Z_6 Z_7$ with a time of $4\tau_{\text{op}} + 28\tau_{\text{rot}}$, respectively. Therefore, we obtain $\tau_{\text{XY}}^{\text{code}(\text{old})} = 8\tau_{\text{op}} + 80\tau_{\text{rot}} = 130$ ns. For this case, 3.5 % reduction of time is achieved.

Thus, it takes a time of $3[2\tau_{\text{op}} + 9\tau_{\text{rot}}] + 8\tau_{\text{rot}} + 10\tau_{\text{rot}} = 6\tau_{\text{op}} + 45\tau_{\text{rot}}$ for obtaining $G_7$ and $G_8$. Thus, total generation time for the nine-qubit code by the method of Ref. [20] is given by $\tau_{\text{XY}}^{\text{code}(\text{old})} = 24\tau_{\text{op}} + 228\tau_{\text{rot}}$. Thus, the number of the qubit-qubit interaction of the present method is reduced to two-thirds of that of the previous method and the number of the single-qubit rotations is reduced to 41.2 % of that of the previous method. When we use the experimental values in Sec. B.1, $\tau_{\text{XY}}^{\text{code}(\text{new})} = 194$ ns and $\tau_{\text{XY}}^{\text{code}(\text{old})} = 376$ ns, thus 48.7 % reduction of the operation time is achieved.

B. Five-qubit code

Next, we consider $H_{\text{stab}}$ of the five-qubit code ($n = 5$ and $k = 1$). The stabilizers $G_i (i = 1,...,4)$ of this code are given by $G_1 = X_1 Z_2 Z_3 X_4, G_2 = X_2 Z_3 Z_4 X_5, G_3 = X_3 Z_4 Z_5 X_1, G_4 = X_4 Z_5 Z_1 X_2$. The process of constructing $H_{\text{code}}^{\text{ini}} \equiv \sum_{i=1}^{4} G_i$ is obtained by changing $H_{\text{code}}^{\text{ini}}$ reversely into a single-qubit Hamiltonian. For XY model, this process is obtained by
Thus, the initial Hamiltonian is given by

\[ H_{\text{ini}}^{\text{code}} = \Omega_1 X_1 + \Omega_2 X_2 + \Omega_3 X_3 + \Omega_4 X_4. \]

(42)

The time of constructing this code is given by

\[ \tau_{X_Y}^{\text{code(new)}} = 5[2\tau_{\text{op}} + 9\tau_{\text{rot}}] + 10\tau_{\text{rot}} + 10\tau_{\text{rot}} = 10\tau_{\text{op}} + 65\tau_{\text{rot}} = 127.5 \text{ ns}. \]

(43)

If we use the previous method in Ref. [20], we have \( \tau_{X_Y}^{\text{code(old)}} = 24\tau_{\text{op}} + 162\tau_{\text{rot}} = 312 \text{ ns}. \) This result is a little different from that in Ref. [20] in that here we start from 2D Hamiltonian. Thus, 59.1% reduction of time is expected with the present method. For the Ising interaction, we have

\[ H_{\text{ini}}^{\text{code}} \rightarrow Z_1 X_2 X_3 X_4 + Z_2 X_3 X_4 Z_5 + Z_1 Z_3 X_4 X_5 + Z_1 Z_2 Z_4 X_5, : H_{\text{Ising}}^{23} \]

\[ \rightarrow Z_1 X_2 X_3 X_4 + Y_3 X_4 Z_5 + Z_1 Z_3 X_4 X_5 + Z_1 Z_2 Z_4 X_5, : (x \leftrightarrow z : 2) (y \leftrightarrow z : 3) \]

\[ \rightarrow -X_1 Z_3 X_3 Z_4 + Z_2 X_3 X_4 - X_1 Y_3 X_4 X_5 - Z_1 X_2 Z_4 X_5, : H_{\text{Ising}}^{12} + H_{\text{Ising}}^{23} \]

\[ \rightarrow -Y_1 Y_3 + Z_4 Z_5 - Y_1 Z_3 Y_4 X_5 - Y_2 Z_4 X_5, : (y \leftrightarrow z : 4) (x \leftrightarrow z : 5) \]

\[ \rightarrow -Z_1 Y_3 + Z_4 X_4 - Z_1 Y_2 Y_4 X_4 - Z_2 Y_4 X_5, : H_{\text{Ising}}^{12} + H_{\text{Ising}}^{23} \]

\[ \rightarrow -Z_1 Y_3 + Y_5 + X_2 Y_3 Y_4 + Z_2 X_4, : (x \leftrightarrow z : 2) \]

\[ \rightarrow -Z_1 Y_3 + Y_5 - Z_2 Y_3 Y_4 + X_2 X_4, : H_{\text{Ising}}^{23} \]

\[ \rightarrow Z_1 Z_2 X_3 + Y_5 + X_3 Y_4 + Z_2 Z_3 X_4, : (y \leftrightarrow z : 2) (x \leftrightarrow z : 3, 4) \]

\[ \rightarrow Z_1 Y_2 Z_3 + Y_5 - Z_3 Y_4 - Z_2 X_3 X_4, : H_{\text{Ising}}^{23} \]

\[ \rightarrow -Z_1 X_2 + Y_5 - Z_2 Y_4 - Z_3 X_4, : H_{\text{Ising}}^{12} + H_{\text{Ising}}^{23} \]

\[ \rightarrow -Y_2 + Y_5 + X_4 + X_3, \quad (44) \]

Thus, the initial Hamiltonian from which \( H_{\text{stab}} \) is derived is given by

\[ H_{\text{ini}}^{\text{code}} = \Omega_2 X_2 + \Omega_3 X_3 + \Omega_4 X_4 + \Omega_5 X_5. \]

(45)

\[ G_3 \text{ is estimated from} \]

\[ G_3 = X_1 X_3 Z_4 Z_5, : (x \leftrightarrow z : 1, 4) \]

\[ \rightarrow -Z_1 X_3 X_4 Z_5, : H_{X_Y}^{23} + H_{X_Y}^{45} \]

\[ \rightarrow -Z_2 Y_2 Y_4, : (y \leftrightarrow z : 2, 4) \]

\[ \rightarrow Z_1 Y_2 Y_3 Z_4, : H_{X_Y}^{12} + H_{X_Y}^{45} \]

\[ \rightarrow X_2 X_3, : (x \leftrightarrow z : 2, 3) \]

\[ \rightarrow Z_2 Z_3. \]

(46)

Thus it takes \( 4\tau_{\text{op}} + 28\tau_{\text{rot}} \). Similarly, it takes \( 6\tau_{\text{op}} + 35\tau_{\text{rot}} \) for \( G_4 \). Therefore, in total, it takes \( 14\tau_{\text{op}} + 97\tau_{\text{rot}} = 184.5 \text{ ns} \) for summing up \( G_1 \sim G_4 \) in the Ising interaction. In this case the present method reduces the generation time by 18.2%.
C. Steane code

The stabilizers of the Steane code are described by $G_1 = X_1X_2X_3X_4$, $G_2 = X_1X_2X_3X_6$, $G_3 = X_1X_3X_5X_7$, $G_4 = Z_1Z_2Z_4Z_1$, $G_5 = Z_1Z_2Z_5Z_6$, and $G_6 = Z_1Z_3Z_5Z_7$. Because $G_4$, $G_5$, and $G_7$ are obtained from $G_1$, $G_2$, and $G_3$ by applying $\pi$-pulses, we first consider the generation process of $H_{X}^{\text{Steane}} \equiv G_1 + G_2 + G_3$. The process of the construction of $H_{X}^{\text{Steane}}$ is obtained by resolving it to a single-qubit Hamiltonian. For the case of $XY$ Hamiltonian, this process is given by,

\[
H_{X}^{\text{Steane}} = X_1X_2X_3X_4 + X_1X_2X_5X_6 + X_1X_3X_4X_7, \quad (x \leftrightarrow z : 2, 3, 5)
\]

\[
\rightarrow X_1Z_2Z_3X_4 + X_1Z_2Z_5X_6 + X_1Z_3Z_5X_7, \quad : H_{XY}^{12} + H_{XY}^{34} + H_{XY}^{56}
\]

\[
\rightarrow Y_2Y_3 + Y_2Y_5 - Z_1Y_2Z_4X_6X_7, \quad (y \leftrightarrow z : 2, 5)
\]

\[
\rightarrow Z_2Y_3 + Z_2Z_5 - Z_1Z_2Z_4Z_6X_7, \quad : H_{XY}^{23} + H_{XY}^{45} + H_{XY}^{67}
\]

\[
\rightarrow Z_2 + Z_3X_4 + Z_1Z_3Z_5Y_6, \quad : (x \leftrightarrow z : 2, 4)
\]

\[
\rightarrow -Z_2 + Z_3X_4 + Z_1Z_3Z_5Y_6, \quad : H_{XY}^{12} + H_{XY}^{34} + H_{XY}^{56}
\]

\[
\rightarrow -Z_1 - Y_3 + Z_2Z_4X_5, \quad : (y \leftrightarrow z : 3)
\]

\[
\rightarrow -Z_1 - Z_3 + Z_2Z_4X_5, \quad : H_{XY}^{23} + H_{XY}^{45}
\]

\[
\rightarrow -Z_1 - Z_2 - Z_3Y_4, \quad : H_{XY}^{34}
\]

\[
\rightarrow -Z_1 - Z_2 - X_3.
\]

(47)

Thus, we obtain the initial Hamiltonian:

\[
H_{X,\text{ini}}^{\text{Steane}} = \Omega_1X_1 + \Omega_2X_2 + \Omega_3X_3,
\]

(48)

The time of generation of $H_{X}^{\text{Steane}}$ is the same as Eq. 43. For the previous method, the time for obtaining $H_{X}^{\text{Steane}}$ is given by $22\tau_{\text{op}} + 143\tau_{\text{rot}}$. As mentioned above, because $H_{X}^{\text{Steane}} \equiv G_1 + G_5 + G_6$ is obtained by

\[
H_{Z}^{\text{Steane}} = e^{-i\pi(Y_1 + Y_2 + Y_3)/4}H_{X}^{\text{Steane}}e^{i\pi(Y_1 + Y_2 + Y_3)/4},
\]

(49)

the generation time of $H_{Z}^{\text{Steane}}$ is increased by $2\tau_{\text{rot}}$ compared with that of $H_{X}^{\text{Steane}}$. Then, the time of obtaining the Steane code by the present method is given by

\[
\tau_{XY}^{\text{Steane(new)}} = 20\tau_{\text{op}} + 132\tau_{\text{rot}} = 257\text{ns}.
\]

(50)

When we use the previous method in Ref. 20, the time for the code generation is given by $\tau_{XY}^{\text{Steane(old)}} = 44\tau_{\text{op}} + 288\tau_{\text{rot}} = 563\text{ ns}$. Thus, 54.4% reduction of time is expected.

For the Ising Hamiltonian, we have

\[
H_{X}^{\text{Steane}} = X_1X_2X_3X_4 + X_1X_2X_5X_6 + X_1X_3X_5X_7, \quad (z \leftrightarrow x : 1, 4, 6)
\]

\[
\rightarrow Z_1X_2X_3Z_4 + Z_1X_2X_5Z_6 - Z_1X_3X_5X_7, \quad : H_{xy}^{12} + H_{xy}^{34} + H_{xy}^{56}
\]

\[
\rightarrow Y_2Y_3 + Y_2Y_5 - Z_1Y_2Z_4X_6X_7, \quad (y \leftrightarrow z : 2)(x \leftrightarrow z : 6, 7)
\]

\[
\rightarrow Y_2Z_3 + Y_2Y_5 + Z_1Z_3Z_4Y_6X_7, \quad : H_{xy}^{23} + H_{xy}^{45} + H_{xy}^{67}
\]

\[
\rightarrow -X_2 + X_2Z_3X_4X_5 - Z_1Z_3X_5Y_6, \quad (z \leftrightarrow z : 2, 3, 4, 5)
\]

\[
\rightarrow Z_2 + Z_2X_3X_4X_5 + Z_1X_3X_5Y_6, \quad : H_{xy}^{34} + H_{xy}^{67}
\]

\[
\rightarrow Z_2 + Y_3Y_4 + Z_1Z_3Y_3Y_6, \quad : (y \leftrightarrow z : 2, 4, 5, 6)
\]

\[
\rightarrow -Y_2 + Y_3Z_4 + Z_1Y_2Y_5Z_6, \quad : H_{xy}^{12} + H_{xy}^{34} + H_{xy}^{56}
\]

\[
\rightarrow Z_1X_2 - X_3 - Z_2X_3Z_4X_5, \quad (x \leftrightarrow z : 2, 4, 5)
\]

\[
\rightarrow -Z_1Z_2 - Z_3 - Z_2X_3X_4Z_5, \quad : H_{xy}^{34} + H_{xy}^{56}
\]

\[
\rightarrow -Z_1Z_2 - Z_2Y_3 - Y_3Y_4.
\]

(51)

Thus, the initial Hamiltonian is given by

\[
H_{X,\text{ini}}^{\text{Steane}} = J_{12}Z_1Z_2 + J_{23}Z_2Z_3 + J_{34}Z_3Z_4.
\]

(52)

This Hamiltonian is obtained by erasing $H_{xy}^{34}$ from $H_{xy}$. $H_{xy}$ is obtained by applying $\pi$-pulses to all
qubits in $B$ of Eq. (22). Then, Eq. (22) is obtained by applying a $\pi$-pulse only to qubit 5 in $B$ of Eq. (22) for $H_{\text{Ising}}$. Then, the time of the preparation of Eq. (22) is estimated by $4\tau_{\text{rot}}$. Therefore, the total time of the generation of Eq. (51) is given by $5[2\tau_{\text{op}} + 9\tau_{\text{rot}}] + 12\tau_{\text{rot}} + 4\tau_{\text{rot}} = 123.5$ ns, and $\tau_{\text{Ising(new)}} = 20\tau_{\text{op}} + 124\tau_{\text{rot}} = 249$ ns. On the other hand, when we use the previous method, times for generating $G_1, G_2$ and $G_3$ are $4\tau_{\text{op}} + 26\tau_{\text{rot}}, 6\tau_{\text{rot}} + 35\tau_{\text{rot}}$, and $8\tau_{\text{op}} + 48\tau_{\text{rot}}$, respectively. Therefore, we obtain $\tau_{\text{Ising(0ld)}} = 36\tau_{\text{op}} + 220\tau_{\text{rot}} = 445$ ns, resulting in 44% reduction of time.

All the results of the above-mentioned three codes are summarized in Tables I and II for the $XY$ interaction and Ising interaction, respectively. From Tables I and II, we can see the large reduction of the generation time is achieved in the $XY$ interaction.

IV. CREATION OF THE STANDARD CODES

As briefly reviewed in Sec. IIA, encoded states are generated by repeating measurements of the stabilizers $G_i$ ($i = 1, \ldots, l$) for an initial state $\Pi_{i=1}^l|0\rangle_n$. Considering that measurements induce extra decoherence, the effectiveness of this conventional method is limited. In Ref. [20], we presented the more effective method of directly generating the operators obtained from $G_i$ by $\text{exp}[\frac{i\pi}{4}G_i]$ with the action of the projector $(1 + G_j)$ when qubit $a_j$ is in state $|0\rangle$. To fulfill Eq. (53) for all $1 \leq j \leq m$ simultaneously, all the $a_j$ have to be different and the modified stabilizers have to be generated in an order such that prior to $G_j^{a_j}$ none of the $G_k^{a_k}$ with $k < j$ have acted on qubit $a_j$ with an $X$ or $Y$ operator. The time for generating the encoded state is given by $\tau_{\text{stab}} + (\sum_i c_i)\tau_{\text{rot}}$.

Here, we extend this idea further and consider whether we can replace this equation by

$$|0\rangle = \exp\left(-i\frac{\pi}{4}H_{\text{stab}}\right)|0\rangle,$$

$$H_{\text{stab}} = \sum_i \hat{G}_i.$$  (55)

For the five-qubit code, we need $\hat{G}_1 = Y_1Z_2Z_3X_4$, $\hat{G}_2 = X_2Z_3Z_4Y_5$, $\hat{G}_3 = X_1Y_2Z_4Z_5$, $\hat{G}_4 = Z_1Y_2X_3Z_5$, and the multiplication is carried out in the following order: $\exp[i(\pi/4)\hat{G}_2] \exp[i(\pi/4)\hat{G}_1] \exp[i(\pi/4)\hat{G}_3] \exp[i(\pi/4)\hat{G}_4]$. However, only $\hat{G}_3$ and $\hat{G}_4$ commute; thus, we cannot replace Eq. (55) by Eq. (55).

For the Steane code, we need three generators:

$$\hat{G}_1 = X_1X_2X_3Y_4,$$  (56)
$$\hat{G}_2 = X_1X_2X_5Y_6,$$  (57)
$$\hat{G}_3 = X_1X_3X_5Y_7.$$  (58)

Because these three generators mutually commute, such as $[\hat{G}_i, \hat{G}_j] = 0$. Therefore we can apply Eq. (55) and reduce the generation time of the encoded state. Thus, it is observed that sparse distribution of the Pauli operators in a logical qubit is preferable for the code generation, because it results in simpler generation of encoded states. Next, we consider an encoding of unknown state $a|0\rangle + b|1\rangle$ to $a|0\rangle + b|1\rangle$ ($a$ and $b$ are arbitrary complex numbers). Because, in Eq. (53), $\hat{G}_j^{a_j}$ was introduced to hold $\exp[-i(\pi/4)\hat{G}_j^{a_j}]|0\rangle = (1 + G_j)|0\rangle$, we need different operations for obtaining $|1\rangle$. For simplicity, we consider $|1\rangle = \hat{X}|0\rangle$. Then, we can solve this problem if we can prepare a modified initial state for $|1\rangle$ defined by

$$|1\rangle' = M^{-1}X\bar{M}|0, \ldots, 0\rangle,$$  (59)

with $M = \prod_{i=1}^m \exp[-i(\pi/4)\hat{G}_i^{a_i}]$. This is because we can use the following relation:

$$\bar{M}(a|0\rangle + b|1\rangle') = a|0\rangle + b|1\rangle.$$  (60)

For the five-qubit code, $\hat{X}$ is given by $\hat{X} = X_1X_2X_3X_4X_5$, and the modified initial state $|1\rangle'$ is expressed by $|1\rangle' = -\hat{G}_3\hat{G}_2\hat{X}|00000\rangle = |00100\rangle$. This means that we can obtain an encoded unknown state $a|0\rangle + b|1\rangle$ when we encode an initial unknown state $a|0\rangle + b|1\rangle$ into the fourth qubit described by $|0\rangle_1|0\rangle_2|0\rangle_3|a|0\rangle_4 - b|1\rangle_4|0\rangle_5$ (the phase of $|1\rangle_4$ is changed). For the Steane code, $X$ is given by $X = X_2X_3X_5$, and the modified initial state $|1\rangle'$ is expressed by $|1\rangle' = X_2X_3X_5|00000\rangle = |01100\rangle$. Hence, we have to prepare $a|00000\rangle + b|01100\rangle$ to which $M$ is applied. This state is transformed from $|0\rangle_1(a|0\rangle_2 + b|1\rangle_2)|00000\rangle$ by applying CNOT gates in which qubits 3 and 5 are target qubits while qubit 2 is the control qubit.

The nine-qubit codes can be generated in a different way, because the nine-qubit code is expressed by the product of three parts given by $\text{III}$:

$$|0\rangle \equiv (|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle),$$  (61)
$$|1\rangle \equiv (|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle).$$  (62)

Each three-qubit block is a Greenberger-Horne-Zeilinger(GHZ) state. From $|000\rangle \pm |111\rangle = \exp[\pm i(\pi/4)X_1Y_2X_3]|000\rangle$, we have

$$|0\rangle = \exp\left(-i\frac{\pi}{4}H_0^{\text{code}}\right)|000\rangle,$$  (63)
$$|1\rangle = \exp\left(i\frac{\pi}{4}H_0^{\text{code}}\right)|000\rangle.$$  (64)
where the Hamiltonian $H_{0}^{\text{code}} = X_1Y_2X_3 + X_4Y_5X_6 + X_7Y_8X_9$ is obtained starting from $X_1 + X_4 + X_7$ by applying operations discussed in the previous sections. The concrete pulse sequence is given by (1) $H_{XY}^{12} + H_{XZ}^{45} + H_{XY}^{67}$, (2) $H_{XY}^{12} + H_{XY}^{45} + H_{XY}^{67}$, and (3) single-qubit rotations. Unknown state $a|0\rangle + b|1\rangle$ is encoded by applying $\exp(-i\frac{\pi}{4}H_{0}^{\text{code}})$ to a changed state $a|0\rangle + b|1\rangle$ which can be obtained by CNOT gate to $(a|0\rangle + b|1\rangle)|0\rangle$. 

V. QUBIT ARCHITECTURE

Let us consider possible encoded qubit architectures for solid-state qubits controlled by local gate electrodes. In general, solid-state qubits are fabricated on some substrate and, unlike optical qubits and ion trap qubits [47], they cannot be moved, being subject to the restriction that the interactions between qubits are limited to the nearest qubits. Thus, as discussed in Sec. [ID2] it is natural to set a logical qubit as a 1D array. In order to construct various stabilizer codes, every qubit should be accessed by an appropriate gate electrode. This means that a gate electrode layer should be placed along logical qubits. Because logical qubits interact with each other in a 2D plane, the gate electrode layer will be constructed on or under the logical qubit layer.

Next, let us consider a structure of measurements. For the fault-tolerant computation, additional measurement circuits are required as described in Ref. [4, 5]. Figure 2 shows the measurement circuit for a single-qubit measurement and the multi-qubit measurement. The multi-qubit measurement is used for stabilizer formalism (Fig. 2(b)). In Fig. 2(b), the number of qubits in the cat state $|0\cdots 0\rangle + |1\cdots 1\rangle$, depends on the number of the Pauli matrices of the stabilizer (Fig. 2(b) is the case of three-qubit stabilizer). This means that the number of ancilla qubits for the whole measurement circuit is of the same order as that of qubits in a logical qubit layer. Therefore, so as to avoid direct measurements and achieve the fault-tolerant computation, it is appropriate to set an independent qubit layer for measurements. Because we already have a logical qubit layer, it is natural that the additional measurement layer should be stacked as shown in Fig 3. Note that physical qubits and electrodes in Fig. 3 are described in a abstract form. Real qubits and electrodes are more complicated than a box. Thus, a stacked 3D qubit system will be straightforward architecture for an effective QECC system, as long as we assume that the interaction between physical qubits is restricted to their neighboring qubits. For generating the cat state of qubits in the measurement layer, our method shown in the previous section regarding the nine-qubit code generation is useful.

The stacked 3D qubit system can be applied to spin qubits and charge qubits. However, not all qubits can be stacked in the 3D system. Consider an example of standard superconducting flux qubits. If we stack flux qubits, the same flux penetrates stacked two qubits, resulting in confusion of signal between the stacked flux qubits. In

| System        | Previous generation time | New generation time | Improvement |
|---------------|--------------------------|---------------------|-------------|
| Nine-qubit code | $24\tau_{op} + 228\tau_{rot}$ | $16\tau_{op} + 92\tau_{rot}$ | $48.7\%$ |
| Five-qubit code | $24\tau_{op} + 162\tau_{rot}$ | $10\tau_{op} + 65\tau_{rot}$ | $59.1\%$ |
| Steane code   | $44\tau_{op} + 288\tau_{rot}$ | $20\tau_{op} + 132\tau_{rot}$ | $54.4\%$ |

TABLE I. The generation time of the stabilizer Hamiltonian from the Ising model. Parameters are the same as those in Table II.

| System        | Previous generation time | New generation time | Improvement |
|---------------|--------------------------|---------------------|-------------|
| Nine-qubit code | $8\tau_{op} + 80\tau_{rot}$ | $10\tau_{op} + 61\tau_{rot}$ | $3.5\%$ |
| Five-qubit code | $14\tau_{op} + 97\tau_{rot}$ | $12\tau_{op} + 76\tau_{rot}$ | $18.2\%$ |
| Steane code   | $36\tau_{op} + 220\tau_{rot}$ | $20\tau_{op} + 124\tau_{rot}$ | $44\%$ |

TABLE II. The generation time of the stabilizer Hamiltonian from the Ising model. Parameters are the same as those in Table I.
unavoidable pulse errors are likely to be the predominant cause of leakage, at low temperatures.

In the present method, each logical qubit is constructed by starting from a single-qubit Hamiltonian \( \sum_i \Omega_i Z_i \), and multiplying operators like \( X_1 \rightarrow X_1 X_2 \rightarrow X_1 \cdots X_N \). Hence, it is possible that this process makes operation errors transmit through each logical qubit. If we model the pulse errors by randomly distributed, unbiased, and uncorrelated deviations \( \delta \theta \) with \( \sigma_\theta = \sqrt{\langle \delta \theta^2 \rangle} \) from the ideal angle of \( \pi/2 \). The leakage from the twofold-degenerate ground-state manifold \( |0\rangle \) and \( |1\rangle \) can then be estimated by looking at the average of the ground state fidelity 
\[ F(t) \approx 1 - N_P \sigma_\theta^2 t / (8 T), \]
where \( N_P \) is the number of pulses in the sequence to generate \( H_{\text{stab}} \), and \( T \) its duration [20]. Thus, the reduction of the number of pulses \( N_P \) for generating stabilizer codes (Tables I and II) is very important.

For the QECC scheme to succeed, the error rate of each qubit operation should be less than \( 10^{-7} \sim 10^{-5} \) [4, 5]. Thus the accuracy of operation pulses is crucial. In this regard, we can also use one of many NMR techniques. If we construct each single pulse by composite pulses, the accuracy of the pulse increases dramatically [21]. The composite-pulse method generalizes the concept of spin echo, and has already been applied in the field of quantum computation to greatly improve both single-qubit rotations and CNOT operations [22, 23]. As the number of pulses \( N_P \) decreases and the dephasing time \( T_2 \) increases, more accurate composite pulses can be implemented, resulting in the success of QECC scheme.

VII. SUMMARY AND CONCLUSIONS

In summary, we showed how to produce stabilizer Hamiltonians starting from natural two-body Hamiltonians by using appropriate pulse sequences. We demonstrated our method by using typical codes: the nine-qubit code, the five-qubit code and the Steane code. The key method of finding the pulse sequence is to inversely trace the derivation process from the stabilizer Hamiltonian to the single-qubit Hamiltonian. We also showed how to generate encoded states without using measurements. Stabilizer Hamiltonians are important for preserving encoded states as ground states of the system. Effective preparation of stabilizers is considered to be critical to the succeed of QECC.

Many important experiments have been performed to enlarge coherence time in solid-state qubits [40]. The criteria for the realization of quantum computing is whether a sufficient number of quantum operations can be carried out during a given coherence time. Thus, manipulation speed of each quantum operation is one of the most important factors for practical quantum computing. Considering the fact that a quantum computer exceeds a digital computer only in several fields such as search algorithm, it will be natural to embed a quantum computer as a part of a digital computer system. Moreover, as in
the present experiments, a quantum circuit will be operated by a digital computer. Although the speed of a single processing unit of a commercial digital computer seems to become saturated, performance of digital computers will continue to increase by parallel processing. Accordingly, it is expected that the manipulation speed of a pulse sequence will also increase. Therefore, the approach presented in this paper enables faster quantum operations by using the cutting-edge technology of computer science. How to achieve an appropriate and smooth connection between a quantum computer and a digital computer will be a future problem.

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Appendix A: Extraction part of other type of Hamiltonian

Here, we show how to extract $H_0$ or interaction parts from a Hamiltonian that includes three Pauli matrices $X$, $Y$ and $Z$. This situation appears for Eq. (10) with $\omega_i^{[1]} \neq 2\Omega_0$ or Ref. [18] in which the Hamiltonian is given by

$$H = \sum_i \omega_i Z_i + \epsilon_i X_i + \sum_{i<j} \frac{J_{ij}}{2} [X_i X_j + Y_i Y_j].$$

For this Hamiltonian, one more step is required to obtain both $H_0$ and an interaction part. When the method of Sec. 11D.3 is applied, we obtain $H_{\text{eff}} = \sum_{k,i} [4\omega_i Z_i + 2\epsilon_i X_i]$ after using Eq. (22). When the method of Sect. 11D.4 is applied, we obtain $H_{\text{eff}} = \sum_k [\sum_i 4Z_i + \sum_k H^{23}_{XY}]$. In both cases, extra $\sum_i Z_i$ term remains. Therefore, we need one more step to delete $\sum_i Z_i$ term such as $e^{-iH_{\text{eff}}t} e^{-i\sum_i X_i/2} e^{-iH_{\text{eff}}t} e^{i\sum_i X_i/2}$, and $e^{-iH'_{\text{eff}}t} e^{-i\sum_i X_i/2} e^{-iH'_{\text{eff}}t} e^{i\sum_i X_i/2}$.

Appendix B: Perturbation terms in BCH formula

Here, we show the first-order perturbation terms that appear during the process of extracting $H^{23}_{XY}$ and a single-qubit Hamiltonian $H_0$ from the original Hamiltonian $H_0$ discussed in Sec. 11D.

The first-order perturbation terms from the ideal Hamiltonian $H^{23}_{XY}$ in five qubits are given by

$$H_{\text{pert}} \approx - \sum_k 2\tau \left( P^{(k)} + Q^{(k)} + R^{(k)} \right)$$

where

$$P^{(k)} = J_{12}^{(k)} \Omega_1^{(k-1)} Y_1^{(k-1)} Z_1^{(k-1)}$$

$$+ J_{34}^{(k)} \Omega_4^{(k-1)} Y_3^{(k-1)} Z_4^{(k-1)}$$

$$+ J_{45}^{(k)} \Omega_5^{(k-1)} Y_5^{(k-1)} Z_4^{(k-1)}$$

$$+ S_{\beta,4}^{(k)} + S_{\beta,4}^{(k)} + S_{\alpha,5}^{(k)} + S_{\alpha,3}^{(k)} + S_{\alpha,5}^{(k)},$$

$$Q^{(k)} = J_{23}^{(k)} \Omega_3^{(k)} Y_3^{(k)} Z_2^{(k)} + \Omega_3^{(k)} Y_3^{(k)} Z_3^{(k)},$$

$$R^{(k)} = J_{34}^{(k)} \Omega_4^{(k)} Y_4^{(k)} Z_4^{(k)} + J_{45}^{(k)} \Omega_5^{(k)} Y_5^{(k)} Z_5^{(k)}.$$
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