Comparative Analysis on Multimode Sensitivity Kernels of Rayleigh Wave Based on Adjoint Method

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Abstract. Different modes of surface waves possess different phase velocities for each wavelength, which reflects the layer information of different depths. Penetrating depth of surface wave data is significant to determine stratigraphic models and invert their velocity structures. Instead of ray theory, we use the sensitivity kernels to investigate the relationship between multimode wavelengths and penetrating depth for a layered model based on the adjoint wave equation. We use linear Radon transform to perform Rayleigh-wave multimode separation and reconstruction. The results confirm that longer wavelength components are sensitive to deeper layers. Different modes of surface waves are sensitive to different depths. The kernels in HVL (high-velocity-layer model) and LVL (low-velocity-layer model) suggest that the velocity anomaly layer has a shielding effect on sensitivity penetrating. Rayleigh waves are insensitive to the layers beneath a velocity anomaly layer and the HVL itself.

1. Introduction
Rayleigh wave is a type of surface wave that travels along a free surface, such as the earth–air interface, as a result of interfering P and SV waves. Surface waves are dispersive and guided in a vertically heterogeneous earth model, which can be characterized by relatively low velocity, low frequency, and high amplitude [1]. Rayleigh wave is widely used for near surface geophysical problems [2]. Multichannel Analysis of Surface Waves (MASW) is an efficient tool to obtain the vertical shear (S)-wave velocity profile. More than one phase velocity, the fundamental mode, and higher modes can be associated with a given frequency of surface waves [3]. A particular mode of surface waves will possess a unique phase velocity for each wavelength. Because different Rayleigh-wave modes normally interfere with each other in the time and space domain, it is necessary to perform mode separation and reconstruction to increase the accuracy of phase velocity determination from a dispersion image [4].

Sensitivity analysis is the basis of surface-wave inversion. Instead of ray theory, the sensitivity kernels based on wave equation can be applied in further waveform tomography that is more efficient for lateral heterogeneity detection. Using the adjoint wave equation, the corresponding illumination of sensitivity kernel can be straightforwardly obtained in a time retrieval manner. This method involves the interaction between the conventional forward wavefield and the adjoint wavefield generated by the time reverse signal on the receiver [5]. This method has been popular in different seismic measurements in waveform tomography. In this paper, the spectral element method developed in recent decades with the geometric division flexibility of the finite element method with the high-precision convergence of high-order Lagrangian polynomials is applied to calculate the multimode Rayleigh wave sensitivity kernel. The mode separation and reconstruction are performed by linear Radon transform.
2. Theory

In seismology, $\mathbf{u}$ represents the displacement wave field corresponding to the wave equation. Wave equation can be written as,

\[ L(\mathbf{u}, \mathbf{m}) = f \]  

(1)

$f$ represents the external force source, $L$ represents a functional relationship, and the model parameter $\mathbf{m}$ represents the earth medium density, elastic modulus, etc.

The misfit function can be used to quantify the difference between the seismic records observed at $\mathbf{x} = \mathbf{x}_r$ and those calculated by the numerical method.

\[ \chi(\mathbf{m}) = \frac{1}{2} \int_T \int_G [\mathbf{u}(\mathbf{m}; \mathbf{x}, t) - \mathbf{u}^0(\mathbf{x}, t)]^2 \delta(\mathbf{x} - \mathbf{x}_r) dt d^3 \mathbf{x} \]  

(2)

The sensitive kernel $K_m$ reveals that the misfit function $\chi(\mathbf{m})$ is affected by the change of model parameters $\mathbf{m}$ at the position $\mathbf{x}$ of the earth.

\[ K_m := \frac{d}{dt} \nabla m \chi = \int_T \mathbf{u}^\dagger \cdot \nabla m L dt \]  

(3)

Based on the adjoint method, we use the adjoint wave field $\mathbf{u}^\dagger$ generated by the time reverse signal on the receiver

\[ \delta \chi = \int_{\Omega} (\delta p K_p + \delta \mathbf{c} \cdot \mathbf{K}_c) d^3 \mathbf{x} + \int_0^T \int_{\Omega} \mathbf{u}^\dagger \cdot \delta \mathbf{f} d^3 \mathbf{x} dt \]  

(4)

We can use (5), (6) to calculate the sensitive kernel

\[ K_p(\mathbf{x}) = -\int_0^T \mathbf{u}^\dagger(\mathbf{x}, T-t) \cdot \partial_t^2 \mathbf{u}(\mathbf{x}, t) dt \]  

(5)

\[ K_c(\mathbf{x}) = -\int_0^T \nabla \mathbf{u}^\dagger(\mathbf{x}, T-t) \cdot \nabla \mathbf{u}(\mathbf{x}, t) dt \]  

(6)

We only consider elastic medium, $\mathbf{c}$ represents elastic tensor (only shear modulus $\mu$ and bulk modulus $\kappa$ are involved in this paper), $\rho$ represents density, $[0, T]$ represents a time interval.

3. Comparative analysis of multimode sensitivity kernels

In this section, we use the three models (Models in Tables 1, 2, and 3) to compare the different penetrating depths of multimode sensitivity kernels. We use a unidirectional point force source with a Ricker wavelet time function. We put the source on the ground. With an offset of 15 m, one station array with 1 m interval is laid, as shown in Figure 1.

| Layer | $v_s$ (m/s) | $v_p$ (m/s) | $\rho$ (kg/m$^3$) | $h$(m) | $\kappa$ | $\mu$ |
|-------|-------------|-------------|-----------------|--------|---------|--------|
| 1     | 500         | 800         | 1800            | 5      | 10      | 10     |
| 2     | 700         | 1600        | 1900            | 5      | 10      | 10     |
| 3     | 900         | 2100        | 2000            | 5      | 10      | 10     |

| Layer | $v_s$ (m/s) | $v_p$ (m/s) | $\rho$ (kg/m$^3$) | $h$(m) | $\kappa$ | $\mu$ |
|-------|-------------|-------------|-----------------|--------|---------|--------|
| 1     | 500         | 800         | 1800            | 5      | 10      | 10     |
| 2     | 250         | 1600        | 1900            | 5      | 10      | 10     |
| 3     | 900         | 2100        | 2000            | 5      | 10      | 10     |

| Layer | $v_s$ (m/s) | $v_p$ (m/s) | $\rho$ (kg/m$^3$) | $h$(m) | $\kappa$ | $\mu$ |
|-------|-------------|-------------|-----------------|--------|---------|--------|
| 1     | 500         | 800         | 1800            | 5      | 10      | 10     |
| 2     | 1200        | 1600        | 1900            | 5      | 10      | 10     |
| 3     | 900         | 2100        | 2000            | 5      | 10      | 10     |
For 50 Hz peak frequency, Figures 2 - 4 show the synthetic vertical-component displacement. Using linear Radon transform, Figures 5 and 6 show the fundamental mode record in NVL and the 1st higher mode record in HVL, respectively.
Figure 6. 1st higher mode in HVL

Figure 7. 100 Hz source in NVL

Figure 8. 50 Hz source in NVL

Figure 9. 25 Hz source in NVL

Figure 10. 10 Hz source in NVL

Figure 11. 50 Hz source in HVL (mode separation)

Figure 12. 50 Hz source in NVL (mode separation)

In the NVL model, density kernel of 100, 50, 25, and 10 Hz sources are shown in Figures 7 -10. In the HVL and NVL models, shear modulus kernels of the fundamental mode and 1st-higher mode Rayleigh wave are shown in Figures 11 and 12, respectively. In the HVL model, we calculate the shear modulus kernel penetrating depth at x=30 m. When the depth is shallow, the absolute value of the
fundamental mode shear modulus kernel is greater than the 1st-higher mode. The value of the shear modulus kernel decreases sharply at depth=5 m, the stratification interface. First-higher mode shear modulus kernel absolute value is greater than the fundamental mode after passing through the interface.

![Figure 13](image.png)

**Figure 13.** Shear modulus kernels for 50 Hz source at x=30m in HVL (dotted lines represent the location of layered interfaces)

### 4. Conclusions
We use sensitivity kernel to analyze the penetrating depth of multimode Rayleigh wave based on the spectral element method and adjoint method. The results show that different modes of surface waves are sensitive to different depths. The First-higher mode wave has a deeper sensitivity than the fundamental mode wave does. Longer wavelength components are sensitive to deeper layers. Across the up interface, the value of shear modulus kernel decreases sharply. The interface hinders the sensitivity kernels penetrating more deeply.

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