Nudged Elastic Band in Analysis of Range Image High-contrast Patches

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Abstract. Range images are more interested in lately for several reasons, such as, range images could be used in object recognition; the scene geometry of the 3D world could be comprehended more effectively by using range images. As there is a distance between the laser scanner and the nearest object for each range image pixel, hence, every range image may be thought as a vector in a high dimensional space \( W \). It is very difficult to study a set of range images \( X \subseteq W \), because \( X \) has very high dimension and it is very sparse in \( W \). An efficient way for analysing range images is to study the space of small range image patches. The nudged elastic band method is a main tool for searching minimum energy paths in computational chemistry. In this paper, Morse functions are created by the sampled data from range image high contrast small patches, and then one-dimensional cell complexes are built from the Morse functions by using the nudged elastic band method, topological features of range image data are detected by a sequence of cell complexes. Particularly, we experimentally show that there exist subspaces of high-contrast 3×3, 4×4, 6×6 and 7×7 range image patches whose homology is that of a circle.

1. Introduction

The authors of the papers [1, 2, 3] analysed range image high-contrast small patches by using lazy witness complexes to find the topological properties of sets of high-dimensional range image data. These lazy witness complexes in general consist of thousands (even more) of simplexes, they are too big to calculate sometimes. Adams etc. ([4]) applied the nudged elastic band to build cell complexes by density functions, they created more thrifty models for some nonlinear data sets with a few of cell simplexes, and discovered efficient representation of the homology of these data sets, they shown how the method works on data of 5×5 range image high-contrast patches, and they got the primary circle model for the data set.

Now we utilize the methods of the paper [4] to range image small patches, and study the topological properties of spaces of 3×3, 4×4, 6×6 and 7×7 patches of range images, we find out a circle model for these sizes of patches, and the three circle model for 7×7 patches, which extend the results of the paper [4].

2. Preliminaries

2.1. NEB.
The nudged elastic band (NEB) method is a valid technique for searching a minimum energy path between fixed first and last states [5]. An elastic band with $N+1$ images, the first and last images (states) are given, the other $N-1$ centre images are altered by an optimal algorithm [6].

2.2. The three circles model
The Klein bottle is the quotient space represented by pasting a square shown in figure 1. In three circles are informed in the process of affixing a square, one is the primary circle (corresponding to $S_{\text{lin}}$) constructed by horizontal lines, the other two circles (corresponding to $S_1$ and $S_\theta$) are constructed from the vertical lines separately, which is named the three circle model (figure 2), denoted as $C_3$. The Betti numbers of $C_3$ are $\beta_0=1$, $\beta_1=5$ and $\beta_2=0$ ([7]).

2.3. Sublevel sets
For a function $g : M \rightarrow R$, the set $M_b = g^{-1}((-\infty, b])$ is called the sublevel set of $b \in R$, similarly set $M_c = g^{-1}([c, \infty))$ is a superlevel set for $c \in R$.

3. The spaces of range image small patches
We choose data sets of high contrast $3 \times 3, 4 \times 4, 6 \times 6$ and $7 \times 7$ patches from the Brown database by Huang and Lee ([8]). One picture is shown in Figure 3.

Here we apply the similar set marks as in the papers [2, 3], $M_n(k, p)$ is a core subsets of space $M_n$, $CC_3(n, 300)$ is picked subsets from $M_n$, and $M_n(10000)$ are random subsets of $M_n$ with 10000 points ($n = 3, 4, 6, 7$). We do the discrete cosine transform for these sets.

4. Outlines of computing
The technique used here come from the paper [4], the outlines of the technique are as following:

Step 1. A density function is constructed from the sampled data to approximate the unknowing density.

Step 2. Zero-cells are found by searching local maxima of the estimated density function.
Step 3. One-cells are discovered by finding the convergent bands from initial bands. Thus, we could detect the topological characterizations of the sampled data $X$ by making cell complexes $Z^\beta$. Please refer to [4, 9, 10] for more details.

5. Main experimental results

The authors of the papers [2, 3] use persistent homology to study the topological constructions of spaces $M_n$ of $n \times n$ range image patches ($n = 3, 4, 5, 6, 7$), and they show that the topology of different subsets in $M_n$ varies from a circle to a three circle model. In particular, there exist core subsets $M_3(600, 30)$ in $M_3$, $M_4(400, 30)$ in $M_4$, and $M_5(300, 30)$ in $M_5$ ($n = 5, 6, 7$), whose homology is that of a circle. There exists a subset $CC_3(n, 300)$ of $M_n$ ($n = 3, 4, 5, 6, 7$) possessing the topology of the three circle space $C_3$ ([8]).

For our analysis we select different subsets of $M_n(n = 3, 4, 6, 7)$: (1) random subsets $M_n(10000)$ of $M_n$ with 10000 points; (2) core subsets $M_n(k, 30)$ in $M_n$; (3) picked subset $CC_3(n, 300)$ in $M_n$.

5.1. $3 \times 3$ patches.

We apply the computing method to the data sets in Table 1. For $M_3(10000)$, put standard deviation $\sigma = 0.35$, we succeed in finding four zero-cells and four one-cells, their densities vary from 0.0975, 0.1023, 0.1056, 0.2133 to 0.08619, 0.08723, 0.09482, 0.09911, these 0-cells and 1-cells constitute a circle. Thus for $\beta = 0.08619$, the $Z^\beta$ is a circle (Figure 4).

Figure 4. $M_3(10000)$ and a circle $Z^{0.08619}$ thrown to a plane

For $M_3(600, 30)$, $\sigma = 0.39$, five 0-cells and five 1-cells are discovered, their densities are in [0.07832, 0.2693] and [0.07035, 0.08769] respectively, all the cells make up a loop (Figure 5).

Figure 5. $M_3(600, 30)$ and a loop $Z^{0.07035}$ thrown to a plane

For $CC_3(3, 300)$, $\sigma = 0.40$, there are four 0-cells and four 1-cells, whose densities fall in [0.1093, 0.1379] and [0.08846, 0.09973], all cells comprise a circle (Figure 6). When $\sigma$ taken as other values (for example, 0.20, 0.25, 0.30, 0.35), no an explicit conclusion are found. As shown in [2], $CC_3(3, 300)$ has same topology as the three circle model $C_3$, but $CC_3(3, 300)$ has the topology of $C_3$ is not found by using the current method.
Table 1. Information of data sets

|                  | $M_3(10000)$ | $M_3(600,30)$ | $CC_3(3,300)$ |
|------------------|--------------|---------------|---------------|
| points of data set | 10000        | 15000         | 900           |
| dimension $n$    | 8            | 8             | 8             |
| standard deviation | 0.35        | 0.39          | 0.40          |

Table 2. Information of data sets

|                  | $M_4(10000)$ | $M_4(400,30)$ | $CC_4(4,300)$ |
|------------------|--------------|---------------|---------------|
| points of data set | 10000        | 15000         | 900           |
| dimension $n$    | 15           | 15            | 15            |
| standard deviation | 0.35        | 0.40          | 0.45          |

5.2 4×4 patches

In this section we study the three kinds of data sets in Table 2. For $M_4(10000)$, $\sigma=0.35$, we obtain four 0-cells with densities in $[0.1927, 0.4127]$, and four 1-cells with densities in $[0.1688, 0.2034]$, which make up a loop (Figure 7).

For $M_4(400,30)$, let $\sigma=0.40$, we have four zero-cells with densities in $[0.0635, 0.1836]$ and four one-cells possessing densities in $[0.04934, 0.07593]$, a loop is formed by these cells (Figure 8).

For $CC_4(4,300)$, $\sigma=0.45$ we search four 0-dimensional cells having densities in $[0.02490, 0.02546]$ and four 1-dimensional cells with densities in $[0.001992, 0.02044]$, that make up a loop (Figure 9). We get similar results for $\sigma=0.35$, 0.40, 0.50 and 0.55 as for $\sigma=0.45$. When we adopt other values of standard deviation and make tests for them, we cannot discover that $CC_3(4,300)$ has the topology of $C_3$ applying the current method.

5.3 6×6 patches

Now we consider subsets of 6×6 patches (see Table 3), for $M_6(10000)$, $\sigma=0.35$, we detect 0-dimensional and 1-dimensional cells, we find that these cells compose a loop (Figure 10).

Table 3. Information of data sets

|                  | $M_6(10000)$ | $M_6(300,30)$ | $CC_6(6,300)$ |
|------------------|--------------|--------------|--------------|
| points of data set | 10000        | 15000        | 900          |
| dimension $n$    | 35           | 35           | 35           |
| standard deviation | 0.35        | 0.30         | 0.35         |
For $M_{6} (300, 30)$, $\sigma = 0.30$, four 0-dimensional cells and four 1-dimensional cells are discovered (Figure 11), their densities are in [711.4, 1948] and [282.3, 490.7] respectively.

For $CC_{3} (6, 300)$, supposed $\sigma = 0.35, 0.40, 0.45$, for each case a circle is formed (Figure 12) by found 0-cells and 1-cells. We cannot investigate that $CC_{3} (6, 300)$ possesses the topology of $C_{3}$ by utilizing the current method.

**Figure 10.** $M_{6} (10000)$ and a loop $Z^{2.03}$ thrown to plane

**Figure 11.** $M_{6} (600, 30)$ and a loop $Z^{282.3}$ thrown to a plane

**Figure 12.** $CC_{3} (6, 300)$ and a circle $Z^{4.818}$ thrown to a plane

### 5.4 $7 \times 7$ patches

Lastly we research $7 \times 7$ patches, and we use data sets as in Table 4.

| Table 4. Information of data sets |
|-----------------------------------|
| points of data set | $M_{7} (10000)$ | $M_{7} (300, 30)$ | $CC_{3} (7, 300)$ |
| dimension $n$ | 10000 | 15000 | 900 |
| standard deviation | 0.35 | 0.35 | 0.35 |

There exist 0-cells and four 1-cells forming a loop for $M_{7} (10000)$ (Figure 13), when $\sigma = 0.35$.

Four 0-dimensional cells with densities in [26.33, 58.2] and four 1-cells with densities in [16.32, 25.89] are found (Figure 14) for $M_{7} (300, 30)$ when $\sigma = 0.35$.

For $CC_{3} (7, 300)$, let $\sigma = 0.35$, we obtain nine 0-cells having densities in [30.83, 38.19] and thirteen 1-cells with densities in [23.66, 30.50], all the cells compose the three circle model (Figure 15).

**Figure 13.** $M_{7} (10000)$ and a loop $Z^{2.03}$ thrown to plane

**Figure 14.** $M_{7} (300, 30)$ and a loop $Z^{282.3}$ thrown to a plane

**Figure 15.** $CC_{3} (7, 300)$ and a circle $Z^{4.818}$ thrown to a plane
Remark. For $\sigma=0.35$, although we sometimes discover eight 0-cells and twelve 1-cells, all the cells form the three circle model (Figure 16). There are four lines being very closely in Figure 15, one need to amplify the figure to see them clearly. In order to see more clearly Figure 15, we make translations for some bands (indicated by dash dot line) as shown in figure 17, comparing it with Figure 2, we conclude that the Figure 17 (or Figure 15) constitutes the three circle model.

6. Conclusions

We experimentally prove that the spaces of $3 \times 3$, $4 \times 4$, $6 \times 6$, and $7 \times 7$-patches have some subsets modelled as a circle. From section 5, the main parameter of NEB is the standard deviation, one should adjust its value in a proper range to get a stable result. By comparing the outcomes of the article with that of the papers [2, 3], we may conclude that the advantage of NEB is its simplicity. Such as, to model $M_n(300,30)$ as a circle, only 8 cells are needed, when modelling $M_n(300,30)$ as a circle by lazy witness complexes, tens of thousands witness complexes are needed. It is very significant to detect that for how big of $n$, the $n \times n$ range image patches do not possess a circle feature. With increasing of $n$, the calculations of $n \times n$ patches change into more difficult. We will discuss $n \times n$ patches for large $n$ in future.

The disadvantage of NEB is that to produce higher dimensional cells is more difficult [4], it can only discover coarser topology of a dataset.

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References

[1] H. Adams and G. Carlsson, On the nonlinear statistics of range image patches. *SIAM J. Image Sci.*, 2(2009), 110–117.
[2] S. Xia, On the local behaviour of spaces of range image patches, to appear.
[3] Q. Yin and W. Wang, An analysis of spaces of range image small patches, *The Open Cybernetics & Systemics Journal*, 9 (2015), 275-279.
[4] H. Adams, A. Atanasov, and G. Carlsson, Nudged elastic band in topological data analysis, *Topological Methods in Nonlinear Analysis*, 45(2015), 247-272.
[5] D. Sheppard, R. Terrell, and G. Henkelman, Optimization methods for finding minimum energy paths, *Journal Chemical Physics*, 128 (2008), 134106–1-10.
[6] G. Henkelman, B. Uberuaga, H. Jónsson, A climbing image nudged elastic band method for finding saddle points and minimum energy paths. *Journal Chemical Physics*, 112 (2000), 9901–9904.
[7] G. Carlsson, T. Ishkhanov, V. de Silva, A. Zomorodian, On the local behaviour of spaces of natural images, *International Journal Computer Vision*, 76(2008), 1-12.
[8] J. Huang, A. Lee, and D. Mumford, Statistics of range images, In Proc. of IEEE Conf. on Computer Vision and Pattern Recognition, Hilton Head Island, SC, June 15-15, (2000), 324-331.

[9] G. Henkelmana and H. Jónsson, Improved tangent estimate in the nudged elastic band method for finding minimum energy paths and saddle points. *Journal Chemical Physics*, 113(2000), 9978–9985.

[10] Y. Cheng, Mean shift, mode seeking, and clustering, *IEEE Trans. Pattern Anal.*, 17(1995), 790-799.