Validating continuous gravitational-wave candidates based on Doppler modulation

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Following up large numbers of candidates in continuous gravitational wave searches presents a challenge, particularly in regard to computational power and the time required to manually scrutinize each of the candidates. It is important to design and test good follow-up procedures that are safe (i.e., minimize false dismissals) and computationally efficient across many search configurations. We investigate two existing follow-up procedures, or “vetoes”, both of which exploit the Doppler modulation predicted in astrophysical signals. We take advantage of a well-established semicoherent search algorithm based on a hidden Markov model to study a wide variety of search configurations and to generalize the veto criteria by considering the overall veto performance in terms of efficiency and safety. The results can serve as a guideline for follow-up studies in future continuous gravitational wave searches using a hidden Markov model algorithm. The results also apply qualitatively to other semicoherent search algorithms.

I. INTRODUCTION

Gravitational waves (GWs), perturbations in spacetime which propagate at the speed of light, were first directly observed in 2015 when the Hanford and Livingston detectors of the Advanced Laser Interferometer Gravitational-Wave Observatory (Advanced LIGO) detected a merging binary black hole system (GW150914) [1, 2]. In the years since the first detection, the addition of Advanced Virgo [3] and KAGRA [4] to the network of observatories, together with improved sensitivity, has produced increasingly frequent detections of compact binary coalescences (CBCs) [5–7]. Other types of GW sources that are predicted to radiate at frequencies within the observational band of ground-based detectors remain undetected, e.g., the continuous gravitational waves (CWs) produced by spinning neutron stars. CWs, once detected, will provide invaluable information regarding the structure of neutron stars as well as the nuclear equation of state [8]. A great deal of work has been carried out to develop methods to search for CWs [9–11].

Since the expected strain amplitudes of CWs are orders of magnitude smaller than those produced by CBCs, the computational cost of searching large template banks (including parameters such as the signal frequency and time derivatives thereof) coherently over a long period of time, e.g., ~ 1 yr, is high [9]. In addition, intrinsic, stochastic wandering of the frequency, sometimes called “timing noise” in the context of radio pulsars, could degrade the sensitivity of a CW search [12, 13]. In this paper we focus on a computationally efficient, semicoherent search strategy based on a hidden Markov model (HMM) which is equipped to track a signal frequency that wanders stochastically and spins down secularly [14, 15]. The tracking scheme has its origins in engineering applications, and has recently been used in many directed CW searches (e.g., [16–25]).

The HMM-based searches are made up of two main procedures: (1) dividing the total observing time into subintervals and coherently calculating the signal power within each consecutive time interval (e.g., with length of ~ 1 d) using a frequency domain matched filter (e.g., the $F$-statistic) [26], and (2) using the Viterbi algorithm (an HMM tracking scheme) to find the most probable signal evolution over the total observing time (e.g., ~ 1 yr) [14, 16]. This optimal signal evolution, referred to as the Viterbi path, consists of a frequency estimated at each discrete time step. The search over the full frequency band is usually parallelized and carried out in narrow sub-bands (with width of ~ 1 Hz). The Viterbi path obtained in each sub-band, corresponding to the most likely CW candidate in that sub-band, is assigned a Viterbi score. This score evaluates the candidate significance, such that a higher score signifies a greater probability that the candidate is inconsistent with random noise fluctuations [14, 16]. For the mathematical details behind these procedures, see Secs. II A–D in Ref. [21]. Some searches rely on an unnormalized log-likelihood instead of the Viterbi score to evaluate the significance of a candidate [19, 22–25].
In this study, we focus on directed CW searches where the HMM-based methods have been most widely used. A directed search targets an astrophysical source at a known sky position with unknown ephemeres and hence is conducted over a wide frequency band (e.g., \( \sim 20-1000 \) Hz). A typical directed search will produce on the order of \( 10^3 \) CW candidates, many of which will require the use of multiple verification techniques to be identified as noise. As such, a computationally efficient follow-up procedure is needed in order to comb through the results and distinguish the candidates caused by noise artifacts from any real astrophysical signal.

Many of these candidates, which have a Viterbi score above the threshold \( S_{th} \) (e.g., corresponding to a 1\% false alarm probability), can be easily eliminated using already well-established procedures. Candidates are initially passed through a known-line veto and a single-interferometer veto, which are defined in Refs. [16, 21–23]. The known-line veto involves eliminating every candidate whose frequency evolution overlaps any known instrumental line present in any of the detectors [27, 28]. The single interferometer veto involves eliminating unidentified instrumental artifacts by checking if a candidate is significantly louder (i.e., if it has a larger Viterbi score) in one detector than in the other. After this, most searches use two additional well-defined vetoes to further identify noise artifacts, also described in Refs. [16, 21–23]. The total observing time can be split into multiple sub-intervals, and any candidates that are significantly louder in one particular sub-interval are eliminated. Then, if the estimated frequency evolution rate is sufficiently low, the coherent time, \( T_{coh} \), over which the data are integrated coherently can be increased and any candidate whose Viterbi score decreases when \( T_{coh} \) increases can be vetoed.

Although most candidates will have been eliminated after applying the aforementioned vetoes, some closely resemble astrophysical signals and require more careful inspection. Two additional veto strategies, both based on Doppler modulation (DM), have been developed for the candidate follow-up procedure [19, 29]. They prove to be both useful and complementary to each other, in that one often vetoes the candidates that the other does not. However, these procedures are quite dependent on the search configuration used in a particular study. For the first of these two strategies, called the DM-off veto, the signal significance is calculated with the DM correction for the Doppler shift due to Earth’s motion switched on and off. The candidates that are louder when the DM correction is switched off are not likely to be of astrophysical origin. For the second strategy, called the off-target veto, the search is shifted to a sky position away from the true position of the target, i.e., a shifted DM correction is applied. If the candidates become louder, they are vetoed. Despite the wide applications of these two vetoes in existing searches [21–24], the general safety (i.e., no astrophysical signal is falsely eliminated) of these veto procedures is still being studied for various search configurations, e.g., for HMM-based searches.

In this paper, we carry out an in-depth study of how the DM changes across the sky, investigate the impact on synthetic signals and noise outliers through simulations, and derive criteria for when and how the two DM-based vetoes can be used to discriminate between astrophysical signals and noise artifacts. Although the simulations are designed for signals from isolated sources, the results can be applied to signals from CW sources in binary systems, assuming the Doppler shift caused by the orbital motion is fully accounted for. The impact from the imperfect removal of the binary orbital modulation is out of the scope of this paper. While this study is carried out using an HMM-based method, in principle the resulting guidelines broadly apply to stack-slide-based semicoherent algorithms [30] (but rigorous studies using other algorithms is out of the scope of this paper). Moreover, our results can be generalized to follow up candidates in an all-sky search.

The organization of this paper is as follows. In Section II, an analytic investigation of the Doppler modulation is presented. Section III outlines the search methods used in this study. Section IV presents the results of the investigation of the DM-off veto. Section V details the results of the off-target veto study. A discussion of the results and concluding remarks are presented in Section VI.

II. DOPPLER SIGNATURE

In this section, we briefly review the CW signal model and calculate analytically how the DM affects CW signals. This provides a foundation for the empirical studies described in Sections IV and V.

The phase of a CW signal can be modeled as follows [26]:

\[
\Psi(t) = \Phi_0 + 2\pi \sum_{k=0}^{s} f_0^{(k)} \frac{k+1}{(k+1)!} + \frac{2\pi}{c} \mathbf{n}_0 \cdot \mathbf{r}_d(t) \sum_{k=0}^{s} \frac{f_0^{(k)} k^k}{k!},
\]

(1)

Here \( f_0 \) is the signal frequency at reference time \( t = 0 \) with respect to the solar system barycenter (SSB), \( f_0^{(k)} \) is the \( k \)th time derivative of the instantaneous frequency evaluated at \( t = 0 \) at the SSB, \( \mathbf{n}_0 \) is the constant unit vector in the source direction within the SSB reference frame, and \( \mathbf{r}_d(t) \) is the position vector of the detector relative to the SSB origin. For the coordinate system with the SSB reference frame, we take the \( x \)-axis parallel to the \( x \)-axis of the celestial sphere coordinate system and the \( z \)-axis perpendicular to the ecliptic \( z \)-axis. Then, the unit vector \( \mathbf{n}_0 \) has the components [26]

\[
\mathbf{n}_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \epsilon & \sin \epsilon \\ 0 & -\sin \epsilon & \cos \epsilon \end{pmatrix} \begin{pmatrix} \cos \alpha \cos \delta \\ \sin \alpha \sin \delta \\ \sin \delta \end{pmatrix},
\]

(2)

where \( \epsilon \) is the angle between the ecliptic plane and Earth’s equator (i.e., \( \epsilon = 23.5^\circ \)), \( \alpha \) is the right ascension (RA) of
the source, and \( \delta \) is the declination (Dec). Meanwhile, the detector’s position vector \( \mathbf{r}_d \) has the components \([26]\)

\[
\mathbf{r}_d = R_{ES} \begin{pmatrix} \cos(\phi_0 + \Omega_0 t) \\ \sin(\phi_0 + \Omega_0 t) \\ 0 \end{pmatrix} + R_E \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \epsilon & \sin \epsilon \\ 0 & -\sin \epsilon & \cos \epsilon \end{pmatrix} \begin{pmatrix} \cos \lambda \cos(\phi_r + \Omega_r t) \\ \cos \lambda \sin(\phi_r + \Omega_r t) \\ \sin \lambda \end{pmatrix}
\]

where \( R_{ES} = 1 \) AU is the mean distance between Earth’s center and the SSB, \( R_E \) is the mean radius of Earth, \( \Omega_0 \) is Earth’s mean orbital angular velocity, \( \Omega_r \) is Earth’s rotational angular velocity, \( \lambda \) is the detector latitude, and \( \phi_0 \) and \( \phi_r \) are phases specifying the exact location of Earth in its orbital and diurnal motion, respectively, at \( t = 0 \).

Substituting Equations (2) and (3) into (1) yields the following expression \([26]\):

\[
\Psi(t) = \Phi_0 + 2\pi \sum_{k=0}^{s} \frac{f_0^{(k)} t^{k+1}}{(k+1)!} + \frac{2\pi}{c} \{ R_{ES} \cos \alpha \cos \delta \cos(\phi_0 + \Omega_0 t) + (\cos \epsilon \sin \alpha \cos \delta + \sin \epsilon \sin \delta) \sin(\phi_0 + \Omega_0 t) \}
\]

We calculate the time derivative of \( \Psi(t) \) to obtain the signal frequency:

\[
f(t) \approx (f_0 + f_0^{(1)} t) \left( 1 + \frac{\mathbf{v}_d \cdot \mathbf{n}_0}{c} \right)
\]

\[
\approx (f_0 + f_0^{(1)} t) \left[ 1 + \frac{R_{ES} \Omega_0}{c} \left[ -\cos \alpha \cos \delta \sin(\phi_0 + \Omega_0 t) + (\cos \epsilon \sin \alpha \cos \delta + \sin \epsilon \sin \delta) \cos(\phi_0 + \Omega_0 t) \right] \right],
\]

where \( f_0^{(1)} \) is the first time derivative of the signal frequency at reference time \( t = 0 \) and \( \mathbf{v}_d \) is the velocity vector of the detector. In Equations (5)–(6), we omit the term corresponding to the DM effect due to Earth’s rotation, which is negligible compared to the DM effect caused by Earth’s orbital motion. We also omit the higher order time derivatives and other negligible terms in order to simplify calculations. (See Appendix A for the full derivation.) For convenience, we define the frequency shift induced by the DM as

\[
\Delta \equiv \frac{\mathbf{v}_d \cdot \mathbf{n}_0}{c} \approx \frac{R_{ES} \Omega_0 \kappa(\alpha, \delta, t)}{c}
\]

with

\[
\kappa = -\cos \alpha \cos \delta \sin(\phi_0 + \Omega_0 t) + (\cos \epsilon \sin \alpha \cos \delta + \sin \epsilon \sin \delta) \cos(\phi_0 + \Omega_0 t).
\]

In panel (a) of Figure 1, \( \kappa \) is plotted with \( \phi_0 = 0 \), \( \Omega_0 = 2\pi/31557600 \text{ rad s}^{-1} \), and \( t \) set to GPS time 1167545066, for a grid of evenly spaced RA and Dec values spanning the entire sky. Similar contour plots are computed for an equatorial coordinate system in Ref. [31] and an ecliptic coordinate system in Ref. [32]. Panels (b) and (c) show two sets of curves where \( \Delta \) is plotted as a function of time for an entire year (starting from GPS time 1167545066) and where each curve corresponds to a different sky position. These sky positions are marked on the contour map in panel (a) with white markers, each with a different shape that can be matched to its corresponding \( \Delta \)-curve (as a function of time) in panels (b) and (c).

We examine the DM of the signal frequency closely here in preparation for connecting the DM patterns plotted in Figure 1 to the empirical results discussed in later sections of this paper. In particular, for any series of markers that lie along the same \( \kappa \) contour in panel (a), the \( \Delta \)-curves that correspond to these markers will all intersect at the particular time used to calculate \( \kappa \) in (a). This is because the DM for sources at sky positions along a single contour are the same for this particular time. For instance, the \( \Delta \)-curves for four locations along the contour where \( \kappa = 0 \) are shown in panel (b), marked by red dots, light pink stars, green diamonds, and purple plus signs. The \( \Delta \) value evolves over time in different ways depending on the sky position, so although the \( \Delta \)-curves all intersect at the start time, they no longer overlap as time shifts forward. While the dot and star markers are near the equator and experience strong DM effects, the diamond and plus sign markers are near the poles and are only weakly impacted by the DM. Examples of \( \Delta \)-curves for locations with a range of other \( \kappa \) values are displayed in panel (c).

For a source with a known sky position, Figure 1 helps us to better understand how targeting a sky position shifted away from the true position of the source impacts the search results. That is, for all sky positions along a \( \kappa \) contour, despite being off-target from the true position, the DM effect is the same as that seen at the true position for a given instant time. As the integration time is increased, most of these similarities tend to disappear as the Doppler correction at each sky position evolves differently. However, the DM effects integrated over time for sky positions that lie along certain directions could still mimic each other, especially for sky positions that have similar \( \kappa \) values over the integration time. Therefore, when correcting the DM with a slightly incorrect sky position along some particular direction(s), which is the
basis for the off-target veto, the recovered signal at the offset position could mimic the signal from the true sky position and produce similar detection statistics in a CW search. Indeed, if one searches a grid of locations around the true position of the source, the detection statistic for an elliptical area around the true sky position is higher than other sky regions. This behavior is discussed in detail in Section V.

III. GENERATING CANDIDATES

In order to study the CW validation techniques, we first generate candidates using the HMM-based semicoherent search method. The two fundamental procedures that make up the searches are as follows. First, we divide the observation time into subintervals and coherently calculate the signal power within each time segment using the maximum likelihood matched filter, $\mathcal{F}$-statistic. Second, we use an HMM tracking scheme to discover the most

FIG. 1. Value of $\kappa$ as a function of RA and Dec (both in degrees) for the GPS time 1167545066 (a). The $\Delta$-values as a function of time over a year (starting from 1167545066) for the sky positions highlighted in (a) are plotted in (b) and (c). Each curve, generated using one of these sky positions, is drawn with the same symbol as the white marker at this specific sky position in panel (a). All curves in panel (b) originate from the same contour in panel (a) with $\kappa = 0$. See more detailed discussions in Sec. V; sky positions are given in Table II.
probable frequency evolution over the full observing time. These two procedures are briefly reviewed in Sections III A and III B, respectively. The detection statistic used in this study, known as the Viterbi score, is outlined in Section III C. More details can be found in Refs. [14, 16].

A. \( \mathcal{F} \)-statistic

We can write the time-domain data collected in the detector as

\[
x(t) = h(t) + n(t) = \mathcal{A}^\mu h_\mu(t) + n(t),
\]

where \( h(t) = \mathcal{A}^\mu h_\mu(t) \) is the signal, and \( n(t) \) is stationary, additive noise [26]. The amplitudes \( \mathcal{A}^\mu \), depending on the characteristic GW strain amplitude \( h_0 \), source orientation, and initial phase, are associated with the four linearly independent components \( h_\mu(t) \) that depend on the phase in Eq. (1) and the detector antenna patterns. (For more details, see Refs. [14, 26].)

The \( \mathcal{F} \)-statistic is a frequency-domain matched filter that estimates the likelihood that a signal, parameterized by its frequency and the frequency time derivatives, is present in the data [26]. We first define a scalar product \( (\cdot, \cdot) \) as a sum over single-detector inner products:

\[
(x|y) = \sum_X (x^X|y^X) = \sum_X 4\Re \int_0^\infty df \frac{x^X(f)y^{X*}(f)}{S_h^X(f)}.
\]

Here \( X \) indexes the detector, \( x \) is the data in detector \( X \), \( S_h^X(f) \) is the single-sided power spectral density (PSD) of detector \( X \), the tilde denotes a Fourier transform, and \( \Re \) is the real part of a complex number [33]. Using this definition, the \( \mathcal{F} \)-statistic can be expressed as

\[
\mathcal{F} = \frac{1}{2} x_\mu M^{\mu \nu} x_\nu,
\]

where we define \( x_\mu = (x|h_\mu) \), and \( M^{\mu \nu} \) is the matrix inverse of \( M_{\mu \nu} = (h_\mu|h_\nu) \) [34].

If we assume that the noise is Gaussian and that the single-sided PSD is the same in all detectors, the probability of having a signal in the data depends on the signal-to-noise ratio (SNR) when analyzing the data coherently for a time \( T_{\text{coh}} \), given by (cf. Sec. IIIC in Ref. [26])

\[
\rho_0^2 = \frac{K h_0^2 T_{\text{coh}}}{S_h(f)},
\]

where the constant \( K \) depends on the sky position, orientation of the source, and number of detectors.

B. HMM tracking and Viterbi algorithm

A general description of the HMM method can be found in Refs. [14, 15]. We briefly summarize the method as follows.

A Markov chain is a stochastic process transitioning from one discrete state to another at discrete times \( \{t_1, \cdots, t_{N_T}\} \), where \( N_T \) is the total number of time steps. An HMM is made up of two variables, the unobservable, hidden state variable \( q(t) \in \{q_1, \cdots, q_{N_Q}\} \) and the observable, measurement state variable \( o(t) \in \{o_1, \cdots, o_{N_O}\} \), where \( N_Q \) and \( N_O \) are the total number of hidden and measurement states, respectively. The hidden state at time \( t_{n+1} \) is solely dependent on the state at time \( t_n \) (this is the Markovian assumption) and has a transition probability of

\[
A_{q_i,q_j} = P(q(t_{n+1}) = q_j|q(t_n) = q_i).
\]

\[\text{(15)}\]

At time \( t_n \), the likelihood that the hidden state \( q_i \) is observed in state \( o_j \) is described by the emission probability, defined as

\[
L_{o_j,q_i} = P(o(t_n) = o_j|q(t_n) = q_i).
\]

\[\text{(16)}\]

The prior can be written as

\[
\Pi_{q_i} = P(q(t_1) = q_i).
\]

\[\text{(17)}\]

Then, the probability that an observed sequence \( O = o(t_1), \cdots, o(t_{N_T}) \) results from a hidden state path \( Q = q(t_1), \cdots, q(t_{N_T}) \) via a Markov chain can be expressed as

\[
P(Q|O) \propto L_{q(t_{N_T}),q(t_{N_T-1})} \cdots L_{o(t_2),q(t_2)} \times A_{q(t_2),q(t_1)} \Pi_{q(t_1)}.
\]

\[\text{(18)}\]

The most probable path, obtained by maximizing \( P(Q|O) \), is simply [14]

\[
Q^*(O) = \arg \max P(Q|O),
\]

\[\text{(19)}\]

where \( \arg \max (\cdots) \) returns the argument that maximizes (\cdots).

In this study, we track \( q(t) = f(t) \), where \( f(t) \) is the signal frequency at time \( t \). We then map the discrete hidden states one-to-one with the frequency bins in the output of \( \mathcal{F} \)-statistic calculated over a span of length \( T_{\text{coh}} \) (see Section III A), with each frequency bin size being \( \Delta f = 1/(2T_{\text{coh}}) \). Thus we choose \( T_{\text{coh}} \) to satisfy

\[
\int_t^{t+T_{\text{coh}}} dt' \dot{f}(t') \leq \Delta f,
\]

\[\text{(20)}\]

where \( \dot{f}(t) \) is the first time derivative of the signal frequency. This relationship must remain valid throughout the total observing time \( T_{\text{obs}} \).

The choice of \( A_{q,f} \) does not greatly impact the sensitivity of an HMM as long as it captures the general behavior of the signal, and as such, the particular transition matrix that is chosen should not impact the guidelines presented in this paper [15, 35]. Without loss of generality, we assume that, in the CW signal we are searching for, the effect of timing noise on the frequency evolution is orders of magnitude smaller than that of the secular spin down
of the star, and that $|f(t)|$ is uniformly distributed in the range from zero to the maximum estimated frequency derivative $|\dot{f}|_{\text{max}}$. By substituting $|\dot{f}|_{\text{max}}$ into (20), we are able to simplify (15) to

$$A_{q_i-1, q_i} = A_{q_i, q_i} = \frac{1}{2}, \quad (21)$$

with all other $A_{q_i, q_i}$ entries vanishing. (In searches where the signal frequency is assumed to walk randomly, such as in Ref. [15], we would instead have $A_{q_i+1, q_i} = A_{q_i, q_i} = A_{q_i, q_i} = 1/3$.) Using the definition of the $F$-statistic, we express the emission probability as

$$L_0(t) = P[\omega(t)|f(t) = f(t') \leq f_i + \Delta f] \propto \exp[F(f_i)], \quad (22)$$

$$\ln S = \ln f(t) - \ln f(t') \leq f_i + \Delta f, \quad (23)$$

for $t \leq t' \leq t+T_{\text{coh}}$, where $f_i$ denotes the central frequency in the $i$th bin. A uniform prior of $\Pi_{q_i} = N_{q_i}^{-1}$ is chosen because there is no independent knowledge of the signal frequency at $f_1$ [14].

We use the classic Viterbi algorithm [36] to efficiently solve the HMM, which outputs the most likely frequency evolution path $Q^*(O)$ over the entire $T_{\text{obs}}$, i.e., the Viterbi path.

### C. Detection statistic

We use the Viterbi score, denoted by $S$, to evaluate the significance of a candidate in this study [14, 16]. In each sub-band searched (with width of 1 Hz in this paper), $S$ is defined such that the log likelihood of the optimal Viterbi path is equal to the mean log likelihood of all paths ending in different bins of the sub-band plus $S$ standard deviations at final step $N_T$. This is written as

$$S = \frac{\ln \delta_q(T_{N_T}) - \mu_{\ln \delta(T_{N_T})}}{\sigma_{\ln \delta(T_{N_T})}}, \quad (24)$$

with mean

$$\mu_{\ln \delta(T_{N_T})} = N_{q_i}^{-1} \sum_{i=1}^{N_Q} \ln \delta_q_i(T_{N_T}), \quad (25)$$

and variance

$$\sigma_{\ln \delta(T_{N_T})}^2 = N_{q_i}^{-1} \sum_{i=1}^{N_Q} [\ln \delta_q_i(T_{N_T}) - \mu_{\ln \delta(T_{N_T})}]^2. \quad (26)$$

Here, $\delta_q(T_{N_T})$ is the likelihood of the path with the maximum probability ending in state $q_i$ ($1 \leq i \leq N_Q$) at step $N_T$ and $\delta_q^*(T_{N_T})$ is the likelihood of the optimal Viterbi path. In some applications of the HMM algorithm, e.g., [19, 22–25], the total log-likelihood along the Viterbi path, $\ln \delta_q^*(T_{N_T})$, is directly used as the detection statistic. Transforming to the Viterbi score in Eq. (24) does not impact the results in this paper.

To conclude this section, it is worth noting that the $F$-statistic is well-equipped to deal with the DM correction; it calculates the expected Doppler shift for a given signal and thus corrects for the Doppler modulation when it computes the signal likelihood. The Viterbi score, which is calculated using the resulting likelihoods, is then also impacted by the DM correction. Then, the DM effect acts as an interesting basis for vetoes; it provides a useful way to study the difference between signals, whose intrinsic frequencies can only be recovered with the DM correction, and noise artifacts, whose frequencies are not impacted by the DM (since they originate on Earth).

### IV. SWITCHING OFF THE DM CORRECTION

In this section, we define a veto based on switching on and off the DM correction (i.e., DM-off veto) in a Viterbi search. We demonstrate that the veto is safe using Monte Carlo simulations.

The DM-off veto, as first presented in Ref. [29], is a technique in which the DM correction, which accounts for the Doppler shift due to Earth’s orbital and rotational motion, is switched off and the detection statistic is reevaluated and compared to the one obtained with the DM correction applied. In a Viterbi search, the DM correction is switched off within the $F$-statistic calculation. When the DM correction is switched off, the Viterbi score of a signal of astrophysical origin should drop below the threshold $S_0$ and a different Viterbi path should be returned. A candidate resulting from a noise artifact, on the other hand, should yield a higher score (except in the rare case that a noise line wanders in a way that mimics the DM) and a Viterbi path that overlaps with the original. As such, a candidate is vetoed only if the Viterbi score increases when the DM correction is switched off and an overlapping Viterbi path is returned. In Ref. [29], the DM-off veto is described for a coherent $F$-statistic search.

#### A. Choose a $T_{\text{coh}}$

Recalling Eq. (20), we can select a $T_{\text{coh}}$ value that satisfies

$$T_{\text{coh}} \leq (2|\dot{f}|_{\text{max}})^{-1/2}, \quad (27)$$

in a directed search, depending on a rough estimate of $|\dot{f}|_{\text{max}}$ for the source. Because $T_{\text{coh}}$ directly depends on the rate of frequency evolution, it varies for searches targeting different sources. For example, for younger sources, a shorter $T_{\text{coh}}$ is required [14].

#### B. Search configurations

In order to safely use the DM-off veto in a semicoherent Viterbi search, we need to ensure that a single set of criteria can be applied to various search configurations. Thus, we test a wide range of CW search configurations.
in this study. We choose two $T_{\text{coh}}$ values, a longer $T_{\text{coh}}$ of 5 d and a shorter one of 12 hr, and we use a total observation time of 180 d for the analysis (starting at an arbitrarily chosen GPS time 1167545066). The detection threshold $S_{\text{th}}$ is estimated for each $T_{\text{coh}}$ by running a series of 600 Monte-Carlo simulations in pure Gaussian noise, with the detector’s amplitude spectral density (ASD) set to $S_{h}^{1/2} = 4 \times 10^{-24}$ Hz$^{-1/2}$ here and throughout the rest of this study, in the sub-bands 200–201 Hz and 500–501 Hz. The resulting Viterbi scores are sorted and the score at the 99th percentile is set as the threshold (corresponding to a 1% false alarm probability per 1 Hz sub-band). For $T_{\text{coh}} = 12$ hr and 5 d, we find $S_{\text{th}} = 5.56$ and 7.14, respectively. (These values are cross-checked with previous studies using similar search configurations, e.g., Refs. [21, 37].)

C. Test synthetic signals

To establish criteria for the DM-off veto, we run pairs of simulations in which we inject synthetic signals into Gaussian noise and first search with the DM correction applied (DM-on), then with DM-off. We compare the Viterbi scores and paths between the pairs of simulations to see if the candidate present in the DM-on case disappears in the DM-off case, as would be expected for a real astrophysical signal. Figures 2 and 3 show the DM-off score ($S_{\text{DM-off}}$) plotted against the DM-on score ($S_{\text{DM-on}}$) for $T_{\text{coh}} = 12$ hr and $T_{\text{coh}} = 5$ d, respectively, for randomly chosen sky positions distributed uniformly across the sky. A variety of different signal strengths are tested, ranging from $h_{0} = 3.0 \times 10^{-26}$ to $h_{0} = 2.4 \times 10^{-25}$, as shown in the legends. For each injected signal, we randomly draw $\cos \iota$, where $\iota$ is the source inclination angle, from a uniform distribution over the range $[-1, 1]$. When $\cos \iota$ is equal to 1 or $-1$, the signal is circularly polarized and its strength is maximized. In each figure, the horizontal and vertical black lines indicate the same detection threshold $S_{\text{th}}$ for that particular $T_{\text{coh}}$.

The expected behavior of a synthetic signal is as follows: although $S_{\text{DM-on}} > S_{\text{th}}$ could lie anywhere along the horizontal axis to the right of the threshold depending on the SNR, $S_{\text{DM-off}}$ should fall below the dashed diagonal line.
(i.e., \(S_{\text{DM-off}} < S_{\text{DM-on}}\)). This behavior is confirmed in all of the injections. In fact, almost all of the markers fall below the horizontal black line marking \(S_{\text{th}}\), with several exceptional cases. Among these exceptional candidates with \(S_{\text{DM-on}} > S_{\text{DM-off}} > S_{\text{th}}\), a black triangle marks each candidate whose Viterbi path recovered by the DM-off search does not overlap with the path returned by the DM-on search. When we describe the two paths as “overlapping” we mean that the DM-off path, widened by \(\pm 1 \times 10^{-4} f_0\) Hz to account for the maximum Doppler shift, intersects the DM-on path at some point along its frequency evolution.

We now further discuss the two scenarios that result in the candidates produced in this study falling above the threshold in the DM-off search. The first is for those without triangle markers, all found at \(S_{\text{DM-on}} \gtrsim 27\) for \(T_{\text{coh}} = 12\ \text{hr} \left(f_0 \geq 1.0 \times 10^{-25}\right)\) and \(S_{\text{DM-on}} \gtrsim 200\) for \(T_{\text{coh}} = 5\ \text{d} \left(f_0 \geq 2.0 \times 10^{-25}\right)\), shown within the shaded purple region which marks where the signals are so strong that we do not miss them even without the DM correction applied [37]. The signal is so loud in this case that it bleeds into nearby frequency bins within the \(F\)-statistic, so the likelihood values for the bins around the true frequency bin are all high. When this occurs, even if we do not apply the DM correction in the \(F\)-statistic, the total log likelihood is still significant enough to produce a score above the threshold (but lower than the score with the DM correction switched on), and the Viterbi path still roughly tracks the true signal frequency. Nevertheless, we set the criteria to only veto candidates with \(S_{\text{DM-off}} \gtrsim S_{\text{th}}\) increased, which we do not see here, so the veto is still reliable for such loud signals.

The second scenario is for those weaker signals to the left of the shaded region but with \(S_{\text{DM-off}} > S_{\text{th}}\). They are all marked by triangles, meaning that the original candidate is not found with DM-off. These candidates appear above \(S_{\text{th}}\) in the DM-off run because they are false alarms. We note that in both Figures 2 and 3, one false alarm in the DM-off run (with \(S_{\text{DM-off}} \approx 6.5\) and 7.0, respectively) does have an increased score relative to \(S_{\text{DM-on}}\) that is above threshold (with a different Viterbi path), but in fact the injected signal is too weak to be identified as a candidate in the first place (i.e., \(S_{\text{DM-on}} < S_{\text{th}}\)). According to these simulation results, one can even
safely veto a candidate if the score remains above \( S_{\text{th}} \)
and the same Viterbi path is returned in the DM-off search, as long as the candidate is not in the high SNR shaded region. (In practice, with the current detector sensitivity, candidates found in a real search with Viterbi scores \( S > 27 \) (\( T_{\text{coh}} = 12 \) hr) and \( S > 200 \) (\( T_{\text{coh}} = 5 \) d) are caused by noise artifacts and would most likely be eliminated by another CW veto.) However, comparing \( S_{\text{DM-off}} \) to \( S_{\text{DM-on}} \) rather than the threshold is generally a more conservative option, i.e., to be cautious, we keep the candidates with \( S_{\text{DM-off}} > S_{\text{DM-off}} > S_{\text{th}} \) for further scrutiny. In particular, setting veto criteria by comparing \( S_{\text{DM-off}} \) to \( S_{\text{DM-on}} \) rather than \( S_{\text{th}} \) ensures the veto safety for sources with sky positions close to the ecliptic poles, since the DM correction at these positions would be minimal and so switching off DM correction would not have much impact on the significance of the Viterbi score. Still, even for these sky positions, we would not expect the Viterbi score to increase when switching the DM correction off. Thus, based on the criteria we have defined, these candidates would not be vetoed and the DM-off veto remains safe.

D. Compare with noise

To provide a baseline for comparison, a series of synthetic monochromatic noise lines at fixed frequency (no DM is added in the simulation code) with different strain amplitudes \( h_{\text{noise}} \) are injected into the Gaussian noise background in the 200–201 Hz sub-band, and the results of the search are plotted in a similar fashion to the synthetic signals, shown in Figure 4. Noise lines, unlike astrophysical signals, should increase in significance when the DM correction is switched off and lie above the diagonal (dashed line) in the figure. Indeed, all outliers caused by synthetic noise lines show this behavior other than the weakest few with \( h_{\text{noise}} < 5.0 \times 10^{-26} \) that are below threshold for both the \( S_{\text{DM-on}} \) and \( S_{\text{DM-off}} \) searches (these exceptional ones are not identified as candidates in the first place).

The behavior caused by noise lines is also confirmed by studying the real noise lines from Advanced LIGO O2 (selected from the candidates identified as noise artifacts in Ref. [21]). The search results are shown in Figure 5. Although quite a few candidates caused by noise lines lie below the dashed line, many do not have overlapping frequency paths between the DM-on and DM-off runs.
FIG. 5. Comparison of the Viterbi scores $S_{\text{DM-on}}$ and $S_{\text{DM-off}}$ for real noise lines with $T_{\text{coh}} = 5$ d. The solid black lines mark $S_{\text{th}}$, and the dashed black line marks the diagonal $S_{\text{DM-off}} = S_{\text{DM-on}}$, above which we would expect the outliers caused by noise lines to lie. Candidates marked by black triangles are found at different frequencies in the DM-on and DM-off searches and thus cannot be regarded as a pair of results with and without the DM correction applied. Candidates marked with red circles have overlapping frequency paths with decreased $S_{\text{DM-off}}$ and are thus considered exceptional cases (see details in text).

(signified with black triangles), meaning that the original candidates are not recovered by the DM-off runs, so they cannot be evaluated in the DM-off veto procedure. There are nine candidates caused by noise lines that fall below the dashed line that do have overlapping frequency paths, marked with red circles. These artifacts are inspected individually. Two possibilities are as follows: (i) the noise lines wander in a way that mimics the Doppler modulation in an astrophysical signal; or (ii) when we integrate data from both detectors and switch off the DM correction, the noise artifact at the candidate frequency remains the most significant, but the likelihoods in other frequency bins happen to increase, decreasing the relative significance of the original artifact. In a real search, these candidates would not be eliminated by the DM-off veto because they do not satisfy the veto criteria, so they would be followed up using other methods. Thus, despite the small number of false positives that are unable to be vetoed, the DM-off veto does not incorrectly eliminate any synthetic astrophysical signals and is therefore safe.

V. SEARCHING OFF TARGET

When an incorrect sky position is used to perform the DM correction on an astrophysical signal, the detection statistic decreases. The off-target veto makes use of this knowledge and of the fact that noise artifacts originate on Earth and are not impacted by DM due to Earth’s motion—that is, performing a DM correction on a noise artifact at a sky position that is slightly off target from the true sky position of the source should not cause the Viterbi score to drop significantly.

In existing searches, the off-target veto is usually done as follows: the Viterbi score is computed for one or more offsets along one spatial direction from the source’s true position. For an astrophysical source, one would expect the Viterbi score to drop and remain below $S_{\text{th}}$ once a certain offset from the true sky position is reached. On the other hand, the Viterbi scores of candidates arising from noise artifacts should remain consistently above $S_{\text{th}}$ regardless of the offset [19]. The offsets typically tested in a CW search are either along RA, holding Dec fixed, or along Dec, holding RA fixed (e.g., see Ref. [19, 23]).
Occasionally, a search is conducted at offsets along both RA and Dec (e.g., Ref. [21, 38]). In a recent search for CWs from accreting millisecond X-ray pulsars, for each candidate that remained after the initial veto procedures, the log-likelihood is computed for a grid of off-target positions around the source’s true position [24]. Every candidate whose log likelihood contours do not match simulations in Gaussian noise gets eliminated. Despite all of these studies, more robust criteria is needed so that the off-target veto can be applied to a wide range of search configurations. Thus, in this section we discuss an in-depth empirical study of the off-target veto and define veto criteria based on this investigation for two different scenarios depending on the number of candidates one is processing.

### A. $T_{coh}$

In the first stage of the investigation, we test three different coherent lengths in order to help generalize the study to other stack-slide-based searches on various timescales as well as to allow for a broader application to other stack-slide-based semicoherent CW searches. We carry out the tests in both low and high SNR scenarios. As we increase $T_{coh}$, the recovered candidate is more significant at the position where the signal was injected, and the score drops below threshold more steeply as we move off target. This is exactly what we would expect to see; moreover, it holds true in both low and high SNR scenarios. The methods and results are detailed as follows.

We run a series of simulations in which a synthetic signal is injected into Gaussian noise in the 1 Hz sub-band starting at 200 Hz and the Viterbi score is computed for a grid of offsets around the center (i.e., the sky position of the injection). Two signal strengths are studied: a weak signal with $h_0 = 4.0 \times 10^{-26}$ and a loud one with $h_0 = 8.0 \times 10^{-26}$. The detector ASD is set to $S_{coh}^{1/2} = 4 \times 10^{-24} \text{Hz}^{-1/2}$. We assume the signals are circularly polarized with cos $\kappa$ fixed at unity such that $h_0^{eff}/h_0$ is held fixed, where $h_0^{eff}$ is defined in Eq. (33) in Ref. [14]. The simulation is run over 180 d, starting from the GPS time 1167545066 (arbitrarily chosen; same as the one used in the DM-off studies and in Figure 1 (a)). Three search configurations are tested in order to get a more complete picture of how a signal behaves around its center position in the sky: $T_{coh} = 12$ hr ($N_T = 360$), $T_{coh} = 5$ d ($N_T = 36$), and $T_{coh} = 30$ d ($N_T = 6$). The spacing and total span of the offset grid vary depending on the search configuration used; the offset grid ranges from $31 \times 21$ (finest) to $21 \times 15$ (coarsest) data points, the latter used to save on computational time.

Figure 6 shows the Viterbi scores (plotted as the color) of the recovered signal at its injected sky position (RA = 00 h 00 m 00 s, Dec = 00° 00′ 00″) and for a grid of offsets around that position. The left and right columns correspond to the low and high SNR scenarios, respectively. The three rows, from top to bottom, correspond to $T_{coh} = 12$ hr, 5 d, and 30 d. Table I lists the Viterbi scores at the source’s true position, denoted as $S_{target}$, for each panel.

| Label | SNR | $T_{coh}$ | $S_{target}$ |
|-------|-----|-----------|--------------|
| a     | low | 12 hr     | 11.28        |
| b     | high| 12 hr     | 27.39        |
| c     | low | 5 d       | 62.70        |
| d     | high| 5 d       | 176.59       |
| e     | how | 30 d      | 116.06       |
| f     | high| 30 d      | 478.04       |

**TABLE I.** Viterbi score obtained at the injection site for each panel shown in Fig. 6.

The bright ellipse (with the brightest spot in the center) shown in these plots, which we refer to as the effective point spread function (EPSF), is exactly what one would expect for an astrophysical signal in any stack-slide-based semicoherent search algorithm. Moreover, such patterns are consistent with those in existing literature: e.g., see Fig. 1 in Ref. [39], Fig. 1 in Ref. [33], and Fig. 14 in Ref. [40]. The ellipse in part comes about because of the different dependencies in RA versus Dec, outlined in Ref. [33]. The faint periodic features seen in some plots but not in others may be related to the periodic functions within the $\kappa$ term in Eq. (8) [32]. However, we rely only on the broader elliptical patterns in this study, so the minor features do not impact the results. A more detailed investigation of these minor features lies outside the scope of this paper. It should be noted that the ellipses shown in panels (a) and (b), both with $T_{coh} = 12$ hr, appear to be slightly off-center such that the brightest point is not exactly at the injection site. This is most likely due to the relatively poor sky resolution when a short $T_{coh} < 1$ d is used. Indeed, the peak is recovered almost exactly at the injection location for $T_{coh} = 5$ d and 30 d.

In comparing the images shown in Fig. 6, it is clear that as $T_{coh}$ increases, the EPSF becomes brighter in the center and narrower. In fact, it has already been established that the offset at which the detection statistic drops below threshold is related to $T_{coh}$: as $T_{coh}$ increases, this offset decreases [39]. This relationship is shown in Figure 7; the offset in Dec (holding RA fixed at the injection coordinate) at which the Viterbi score drops below 0.5$S_{target}$ is tracked as a function of $T_{coh}$. (To quantify this relation with better resolution, we conduct additional searches with two more coherent lengths $T_{coh} = 1$ d and 10 d in Dec.) Indeed, for both SNRs tested, the shortest coherent time, 12 hr, has the greatest Dec offset. As we increase $T_{coh}$, this offset decreases approximately linearly in log-log scale. We choose to take our offsets only in Dec because the behavior of the detection statistics is more dependent on the source position when moving only in RA, so taking offsets in Dec allows us to set more generally applicable guidelines for the veto that are independent of sky position. (More details are discussed in Section V B.)
FIG. 6. Contour of Viterbi scores as a function of the offset in RA and Dec for an injection at RA = 00 h 00 m 00 s and Dec = 00° 00′ 00″ with signal strength $h_0 = 4.0 \times 10^{-26}$ (left) and $h_0 = 8.0 \times 10^{-26}$ (right). From top to bottom, coherent lengths $T_{coh} = 12$ hr, 5 d, and 30 d are used with a total observing time of 180 d. (Other simulation parameters: $S_{h1/2} = 4 \times 10^{-24}$ Hz$^{-1/2}$, $\cos \iota = 1$.) See Table 1 for the Viterbi score obtained at the injection location ($S_{\text{target}}$) in each panel.

decreases as the Dec offset increases for $T_{coh} = 12$ hr, 5 d, and 30 d, from left to right. In general, in the first few steps away from the center, the scores drop steeply, especially for longer coherent times, then level out such that further increasing the offset leads to only small decreases in the score. Once a large enough offset is reached, the score drops below threshold and the signal is no longer detectable. This offset at which the signal becomes undetectable heavily depends on $T_{coh}$ and SNR. For extremely loud signals, the detection statistic may never drop below threshold, regardless of the sky position searched. However, current detectors do not operate in the high-SNR regime for CW sources—in fact, such a loud signal would be astrophysically implausible because the source would likely fall outside the range of extreme ellipticities and would be spinning down so rapidly that its $\dot{f}$-value would not lie within the ranges used in this study. Thus we do not take into consideration the extremely high-SNR scenario.
FIG. 7. Dec offset at which the Viterbi score drops below 0.5$S_{\text{target}}$ as a function of $T_{\text{coh}}$. The coherent lengths tested are $T_{\text{coh}} = 12$ hr, 1 d, 5 d, 10 d, and 30 d. The lines that join the sample points give a rough idea of where this offset would occur for other choices of $T_{\text{coh}}$ within this range. For the low (green) and high (purple) SNRs, the signal strength of the injection is $h_0 = 4.0 \times 10^{-26}$ and $h_0 = 8.0 \times 10^{-26}$, respectively. (Other simulation parameters: $S_{\text{th}}^{1/2} = 4 \times 10^{-24}$ Hz$^{-1/2}$, $\cos \iota = 1$).

FIG. 8. Viterbi score (as a fraction of $S_{\text{target}}$) obtained at positions away from the center along Dec (from left to right: $T_{\text{coh}} = 12$ hr, 5 d, 30 d). In each panel, orange dots and blue circles correspond to the low SNR case with $h_0 = 4.0 \times 10^{-26}$ and high SNR case with $h_0 = 8.0 \times 10^{-26}$, respectively. The solid orange and dashed blue lines mark the threshold, each one displayed as a fraction of $S_{\text{target}}$ for its corresponding SNR. (Although $S_{\text{th}}$ remains the same for a particular $T_{\text{coh}}$ regardless of SNR, $S_{\text{target}}$ obtained is larger for a higher SNR, and thus the orange and blue lines do not overlap.)

**B. Sky position**

The elliptical EPSF shown in Figure 6 is further explored in this section by varying the location of the injection. A series of simulations are run with different combinations of RA and Dec. We find that sky position is the dominant factor in determining the shape (i.e., the ratio of the major and minor axes) and orientation of
the EPSF. The EPSF can act as a precise marker of an astrophysical signal, so we use it in the off-target veto criteria. The details are discussed below.

In this set of simulations, we fix the injection SNR using the signal strength \( h_0 = 4.0 \times 10^{-26} \) and conduct the search with \( T_{\text{coh}} = 5 \) d. The other simulation parameters are the same as those in Section V A. Figure 9 shows the resulting contour plots, where the Viterbi score (color) is plotted as a function of RA and Dec for a grid of offsets centered on the injection site; see Table II for the ten injection locations and their corresponding scores. Random variations aside, the shape and orientation of the EPSF change systematically as functions of RA and Dec. Matching up each panel in Figure 9 to its position in Figure 1 (a), we observe that the inclination of the ellipse at a particular sky position roughly follows the slope of the \( \kappa \) contour in Figure 1 (a) passing through that same position.

| Label | RA   | Dec   | \( S_{\text{target}} \) |
|-------|------|-------|--------------------------|
| a     | 00:00:00 | 00'00'00'' | 62.70 |
| b     | 06:00:00 | −66'30'00'' | 73.20 |
| c     | 12:00:00 | 00'00'00'' | 60.35 |
| d     | 18:00:00 | 66'30'00''  | 73.71 |
| e     | 01:00:00 | −60'00'00'' | 70.93 |
| f     | 01:00:00 | −20'00'00'' | 55.82 |
| g     | 01:00:00 | 45'00'00''  | 74.35 |
| h     | 01:00:00 | 88'00'00''  | 69.42 |
| i     | 06:00:00 | 00'00'00''  | 68.89 |
| j     | 18:00:00 | −23'30'00'' | 61.64 |

TABLE II. Viterbi scores at the ten injection sites shown in Fig. 9.

Two notable examples are discussed below. The first is presented in Figure 9 (h); this image is centered on RA = 01 h 00 m 00 s and Dec = 88° 00' 00", and it shows an ellipse that spans more than two hours in RA—a significant fraction of the sky. Referring to where this position is found within the contour map in Figure 1 (a)—the triangle marker pointing towards the left—we immediately notice that the contour passing through this point moves along RA at a roughly fixed Dec. The same holds true for a source found at, for example, Dec = −88° 00' 00". Indeed, as the position approaches either pole, the EPSF extends broadly along RA because the DM effect is weak near the poles. This fact could lead to astrophysical signals being falsely vetoed as noise if the off-target veto is applied to sources near the poles using an offset only in RA. Certainly, choosing our criteria so that the off-target veto can be safely applied to candidates at the poles is important since it is unlikely that the DM-off veto will eliminate such candidates.

The second example worth discussing is shown in Figure 9 (j). This image is centered on RA = 18 h 00 m 00 s and Dec = −23° 30' 00" (the latter corresponding to the tilt of Earth’s axis) and shows an ellipse spanning roughly 15° in Dec. Once again, referring to the corresponding sky position in Figure 1 (a), we can see that the injection is located in the center of the dark region—the “dark spot” where \( \kappa = −1 \) (the cross marker). Based on the simulation results obtained at various positions and on how the EPSFs tend to vary with injection location, we find that this sky position at the dark spot—along with the “bright spot” with \( \kappa = 1 \)—in general produces the most extended EPSF in Dec. We use this to set veto guidelines in Sec. V D.

Since the extension of the ellipse also largely depends on \( T_{\text{coh}} \) (for a given SNR), we further conduct a set of tests for injections at the dark spot, using another two choices of coherent length: \( T_{\text{coh}} = 12 \) hr and 30 d. In Figure 10, the top row shows the EPSFs at the dark spot for \( T_{\text{coh}} = 12 \) hr (left), 5 d (middle), and 30 d (right). We inject a louder signal, \( h_0 = 5.0 \times 10^{-26} \), for the search which uses \( T_{\text{coh}} = 12 \) hr (as opposed to \( h_0 = 4.0 \times 10^{-26} \) for the other two coherent times) so that the EPSF is better resolved. Of all the search configurations tested throughout this study, the ellipse at this sky position using the shortest coherent length \( T_{\text{coh}} = 12 \) hr spans the largest angular distance in Dec—approximately 30°. The bottom row of Fig. 10 shows how the Viterbi score decreases as the Dec offset increases along the dashed cyan line plotted in the top panels, for \( T_{\text{coh}} = 12 \) hr, 5 d, and 30 d, from left to right (similar to Figure 8).

C. Noise artifact

To provide a baseline for comparison, the Viterbi scores for a grid of sky positions, centered at RA = 22 h 57 m 39.1 s and Dec = −29° 37’ 20.0", are calculated in the 462–463 Hz sub-band in LIGO O2 data, where an unidentified noise artifact originating in the Hanford detector is located (identified in Sec. IV of Ref. [21]). For consistency, a GPS start time of 1167545066 is used with a total observing time of 180 d. We repeat this procedure in Gaussian noise with a synthetic signal \( (h_0 = 2.0 \times 10^{-26}, \cos \iota = 1) \) injected at RA = 22 h 57 m 39.1 s and Dec = −29° 37’ 20.0". The results from LIGO O2 data and the simulated data containing a synthetic signal are plotted in the left and right panels in Figure 11, respectively. The left panel shows a spread of above-threshold Viterbi scores which fluctuate randomly across the sky. There is no trace of the bright peak in the center seen in the right panel resulting from a synthetic signal. This is the expected behavior of a candidate caused by a noise artifact, which would not be impacted by DM because it originates on Earth.

D. Veto guidelines

In practice, determining the exact shape and orientation of the EPSF of a CW signal is more complicated than simply following the direction of a Doppler pattern contour, e.g., in Figure 1 (a). In fact, the shape and
FIG. 9. Contour of Viterbi scores as a function of the offset in RA and Dec for injections at a variety of different sky positions with signal strength $h_0 = 4.0 \times 10^{-26}$ (Gaussian noise). A coherent length $T_{\text{coh}} = 5$ d is used with a total observing time of 180 d. (Other simulation parameters: $S_{h}^{1/2} = 4 \times 10^{-24}$ Hz$^{-1/2}$, $\cos \iota = 1$.) See Table II for the Viterbi score obtained at each injection location ($S_{\text{target}}$).
To conclude this discussion of the off-target veto, we outline concrete guidelines for how to best apply this veto in two different scenarios: (1) when following up a large number of candidates from various sky positions, and (2) when following up a handful of candidates from a limited number of positions. First, in the follow-up procedure of either a directed search which is targeting many sources at different sky positions (e.g., [23, 24]) or an all-sky CW search, if the earlier-stage vetoes are not effective enough, we need to use the off-target veto to process a large number of CW candidates from many sky positions. Thus, both safety and efficiency are important. Creating and inspecting detailed images of the EPSF for every candidate would be computationally expensive. Rather, this study allows us to use the detection statistic at a single Dec offset (as offsets in Dec have been shown in Sec. V B to be safer than offsets in RA) to decide whether or not to veto a candidate. Moreover, we choose a conservative offset such that it is valid for all sky positions; however, the choice of offset does depend on the T\text{coh} and T\text{obs} used in a particular search.

1. Veto a large number of candidates

We propose the following broad criteria for applying the off-target veto to many candidates at once: a candidate can be safely vetoed only if the score at a certain conservative Dec offset remains above 0.5S\text{target} or S\text{th} (whichever is larger), with the added condition that the recovered Viterbi path at this offset position must overlap

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**FIG. 10.** Top row: Contour of Viterbi scores as a function of the offset in RA and Dec for an injection at RA = 18 h 00 m 00 s and Dec = −23° 30′ 00″ with signal strength h₀ = 5.0 × 10⁻²⁷ (left plot) and h₀ = 4.0 × 10⁻²⁶ (middle and right plots) (Gaussian noise), using three different choices of T\text{coh}. From left to right, we have T\text{coh} = 12 hr, 5 d, and 30 d, with S\text{target} = 22.55, 54.44, and 131.16, respectively. The total observing time is 180 d. (Other simulation parameters: S\text{th}/S\text{target} = 4 × 10⁻²⁴ Hz⁻¹/², cos θ = 1.) Bottom row: Viterbi score S (as a fraction of S\text{target}) obtained at positions away from the center along Dec. In each panel, the search configuration is the same as that used in the contour image above it (from left to right: T\text{coh} = 12 hr, 5 d, 30 d). The solid lines mark the threshold for each search configuration displayed as a fraction of S\text{target}. The dashed cyan line in each contour image in the top row marks the direction along which the series of offsets shown in the bottom row are taken.
the original path. In practice, one can choose a different fraction other than 0.5 as needed, based on empirical studies. The reason we stipulate a fraction of the original score rather than a particular value of the score is so that, for a relatively loud signal with original score $S > 2S_{\text{th}}$, the dependence on the SNR is generally removed (as demonstrated in Figure 7). If such an astrophysical signal exists, the score should drop at least 50\% by the chosen Dec offset. We keep any candidate whose score is below $S_{\text{th}}$ at this offset position because a weak signal becomes undetectable at very small offsets from the true sky position. One could even consider setting the offset based on a more sophisticated parameter-space-based distance, as presented in Ref. [41], but such a detailed study is beyond the scope of this paper.

Figure 10 shows an example of how one might find this optimal offset position for the three $T_{\text{coh}}$ choices used most frequently in this paper. When processing many candidates at once, a plot like those along the bottom row of Fig. 10 can be created using a synthetic signal injected into Gaussian noise. To obtain a safe offset that can be applied to candidates from various sky positions, we carry out the simulations at the sky position where the detection statistic varies the slowest in Dec (i.e., the position at which the EPSF will be the most extended in Dec). The offset for applying the off-target veto can then be determined by looking at where the score drops below 0.5$S_{\text{target}}$ for a relatively loud signal such that 0.5$S_{\text{target}} > S_{\text{th}}$. For our particular search configuration, using the data presented in Fig. 10, we find this offset for $T_{\text{coh}} = 12$ hr, 5 d, and 30 d (for a total observing time of 180 d) to be 7°, 3°, and 1°, respectively. It should be noted for $T_{\text{coh}} = 12$ hr that, despite the EPSF being significantly off-center, as often occurs for short coherent times which have poorer resolution, the veto can still be safely applied as long as a conservative offset is chosen using the results from simulations in Gaussian noise.

2. Veto a handful of candidates

When we inspect a handful of candidates individually, a more comprehensive procedure is preferred, similar to what was done in Ref. [24]. That is, an image should be produced showing the Viterbi scores for a grid of sky positions centered at the candidate position. A second image also showing the EPSF, this time from a synthetic signal injected into Gaussian noise at the candidate sky position, should also be produced. The search configuration should remain unchanged when creating both images (i.e., observation start time, $T_{\text{coh}}$, and total observing time should all be the same), and the signal strength of the injection should be chosen such that the recovered signal at the center has roughly the same $S_{\text{target}}$ as the CW candidate. Then, these two images can be compared. Although we would not expect the EPSF of a real CW signal to perfectly match the simulations, the elliptical pattern should, in general, be centered around the same position and have roughly the same shape and orientation. If it does not, this candidate may be safely vetoed.

Indeed, in Ref. [24], for the single candidate which remained after passing all the candidates through a hierarchy of vetoes, the candidate’s EPSF in real data was not centered on the true sky position of the target and
did not match the EPSF produced in Gaussian noise, so the candidate was vetoed. We verify this procedure in Figure 11, where the Viterbi scores for a grid of sky positions centered at a candidate position in LIGO O2 data (vetoed as noise in Ref. [21]) and the EPSF for a synthetic signal injected into Gaussian noise at the same sky position are compared. No trace of the EPSF shown in the right panel can be seen in the real data in the left panel. Thus, as expected, we would veto this candidate.

It should be noted that although data gaps are usually present (due to maintenance, upgrading, etc.), the EPSFs are generally not impacted. For a more rigorous comparison, one can also include data gaps in the Gaussian noise simulations.

VI. CONCLUSION

In this study, we investigate the DM-off veto and the off-target veto in order to establish veto criteria for a semicoherent CW search. We draw conclusions about the safety of these two vetoes through Monte Carlo simulations in which synthetic signals are injected into Gaussian noise. We use a combination of the coherent $F$-statistic and an HMM scheme to track the signal, and we take the Viterbi score as our primary detection statistic.

We find that the DM-off veto is safe in all configurations tested (i.e., for all sky positions and for both a short coherent length of 12 hr and a longer coherent length of 5 d). Although the veto is not able to eliminate every noise line, the majority are successfully eliminated. More importantly, no synthetic signals are falsely eliminated in any configuration. Furthermore, we show that the off-target veto can be used in two different ways to follow up CW candidates depending on how many remain to be investigated. If many candidates from various sky positions need to be processed at once, a single Dec offset can be investigated. If many candidates from various sky positions centered at a candidate position in LIGO O2 data (vetoed as noise in Ref. [21]) and the EPSF for a synthetic signal injected into Gaussian noise at the same sky position are compared. No trace of the EPSF shown in the right panel can be seen in the real data in the left panel. Thus, as expected, we would veto this candidate.

It should be noted that although data gaps are usually present (due to maintenance, upgrading, etc.), the EPSFs are generally not impacted. For a more rigorous comparison, one can also include data gaps in the Gaussian noise simulations.

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Appendix A: Signal frequency

We simplify the CW phase shown in Eq. (4) by omitting the term that accounts for the DM effect due to Earth’s rotation, which is two orders of magnitude smaller than the effect of Earth’s orbital motion (i.e., $\Omega_0 R \, R_{ES}/c \sim 1.0 \times 10^{-4}$, compared to $\Omega_R R_E/c \sim 1.5 \times 10^{-6}$). The approximate phase becomes

$$
\Psi(t) \approx \Phi_0 + 2\pi \sum_{k=0}^{s} f^{(k)}_0 \frac{1}{(k+1)!} \left( R_{ES} \frac{1}{c} \cos \alpha \cos \delta \cos(\phi_0 + \Omega_0 t) + (cos \epsilon \sin \alpha \cos \delta + \sin \epsilon \sin \delta) \sin(\phi_0 + \Omega_0 t) \right) \sum_{k=0}^{s} f^{(k)}_1 \frac{1}{k!}.
$$

(A1)
We can then write the signal frequency as

\[ f(t) \approx (f_0 + f_0^{(1)} t) + \frac{R_{ES} \Omega_0}{c} \left[ - \cos \alpha \cos \delta \sin(\phi_0 + \Omega_0 t) + (\cos \epsilon \sin \alpha \cos \delta + \sin \epsilon \sin \delta) \cos(\phi_0 + \Omega_0 t) \right] (f_0 + f_0^{(1)} t) \]

\[ + \frac{R_{ES}}{c} \left[ \cos \alpha \cos \delta \cos(\phi_0 + \Omega_0 t) + (\cos \epsilon \sin \alpha \cos \delta + \sin \epsilon \sin \delta) \sin(\phi_0 + \Omega_0 t) \right] f_0^{(1)}, \]  

(A2)

where we have omitted higher order frequency derivative terms \( f_0^{(k)} \) with \( k \geq 2 \). Considering the fact that \( \Omega_0 f_0 \gg f_0^{(1)} \) in the parameter space searched, the last term in Eq. (A2) can be omitted, and we obtain

\[ f(t) \approx (f_0 + f_0^{(1)} t) \left[ 1 + \frac{R_{ES} \Omega_0}{c} \left( - \cos \alpha \cos \delta \sin(\phi_0 + \Omega_0 t) + (\cos \epsilon \sin \alpha \cos \delta + \sin \epsilon \sin \delta) \cos(\phi_0 + \Omega_0 t) \right) \right], \]  

(A3)

i.e., Eq. (5) in Section II.

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