Research on DOA Estimation Method of Sonar Radar Target Based on MUSIC Algorithm

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Abstract. In array signal processing, direction of arrival estimation (DOA estimation) is a common and extremely important method[1]. This paper is based on the existing DOA algorithm, and conducts deeper research. Firstly, compare three DOA estimation algorithms using simulation program. The experimental results prove that the MUSIC algorithm has the highest accuracy. Secondly, we analysis the basic theory of MUSIC algorithm, and then discusses the factors of affecting the estimation accuracy of MUSIC algorithm. Finally, in view of the shortcomings of the MUSIC algorithm for solving the problem of coherent sources, this paper improves the MUSIC algorithm, then proposes an improved algorithm, which is verified by simulation experiments.

1. Introduction
Array signal processing technology is a very important technology in the field of communication and rapidly developed in many fields, such as sonar, radar, exploration and so on[2]. The two main research directions of array signal processing are spatial filtering and spatial spectrum estimation. DOA (Direction of Arrival) estimation is an important part of spatial spectrum estimation[3]. Since the spatial parameters of the signal or the orientation of the source can be accurately measured, it is also called the spatial spectrum estimation as the DOA estimation. This is a significant task in many fields such as radar and sonar[4].

The Multiple Signal Classification (MUSIC) algorithm was proposed by Schmidt in 1979. This algorithm is the earliest super-resolution DOA estimation method. It has led to the rapid development of this type of algorithm research. In the classic DOA estimation, MUSIC algorithm is the most basic and efficient algorithm, and has a wide range of applications.
2. DOA Estimation Algorithm

2.1. Comparison of three algorithms
Delay-addition algorithm is a common beamforming method, also known as Bartleet beamforming. However, the low resolution of this method is aimed at this problem. Capon proposed a minimum variance method to solve this problem, namely the Capon algorithm.

Then used MATLAB to simulate and analyze the above three methods.

Simulation conditions: It is assumed that three independent narrow-band signals are incident on the uniform array of 8 elements at 20°, 40°, and 45° respectively. The spacing of the elements is half of the incident wavelength, the snapshots is 200, and the SNR is 20dB. The result is shown below.

![Figure 1. Comparison of the resolution of three algorithms](image)

It can be seen from the figure that for the three 20°, 40°, 60° signal incident angles with small phase difference, only the peak of the MUSIC algorithm is sharpest, which can accurately distinguish them. Therefore, the MUSIC algorithm has the highest resolution.

2.2. MUSIC algorithm
2.2.1. Principle of the MUSIC algorithm. Processing the covariance matrix of the array output matrix to obtain information about the source (azimuth, elevation, etc.).
2.2.2. Factors Affecting Estimation Accuracy of MUSIC Algorithm. The factors affecting the direction finding accuracy of the MUSIC algorithm are signal-to-noise ratio SNR, snapshot number N, array element spacing d, angle difference of incident signal $\Delta \theta$, etc. And the DOA estimation accuracy is positively correlated with the SNR, the snapshots N, and the angular difference of the incident signal $\Delta \theta$.

2.3. An improved MUSIC algorithm
Aiming at the problem that the MUSIC algorithm has insufficient estimation accuracy and cannot distinguish the coherent signal under some condition, I improved the traditional MUSIC algorithm and proposed an improved MUSIC algorithm.
2.3.1. Mathematical Model. For an uniform linear array, the assumptions are all isotropic elements and
array spacing is d. In the array, there are N narrow-band point sources at $\theta_k$ (k = 1, 2, ..., N) with reference to the normal of the line axis, incident at plane wave (wavelength is $\lambda$). Take the first array as a sample, the receives data as

$$X(t) = AS(t) + U(t)$$

(1)

$X(t)$ is $M \times 1$ data vector, $X(t) = [x_1(t), x_2(t), \ldots, x_M(t)]^T$. $U(t)$ is $M \times 1$ array element noise vector, $U(t) = [u_1(t), u_2(t), \ldots, u_M(t)]^T$, $u_i(t)(i = 1, 2, \ldots, M)$ is a white noise, not related to the signal source.

$$A = [a(\theta_1), a(\theta_2), \ldots, a(\theta_N)], a(\theta_i) = [1, e^{-j\omega_1}, e^{-j\omega_2}, \ldots, e^{-j(M-1)\omega}], S(t) = [s_1(t), s_2(t), \ldots, s_N(t)]^T, s_i(t)(i = 1, 2, \ldots, N). T$ is the transpose of the matrix, $\lambda$ is carrier wavelength, $d$ is array spacing.

The covariance matrix is

$$R_{XX} = E[XX^H] = AR_{SS}A^H + \delta^2 I_M$$

(2)

where $R_{ss} = E[S(t)S^H(t)]$ is the covariance matrix of the source, $I_M$ is an M-order unit matrix.

The traditional MUSIC algorithm uses $R_{XX}$ to perform matrix processing to obtain DOA estimation.

2.3.2. Theoretical Derivation of Improved Algorithm. Assume that

$$Y(n) = J_MX^*(n)$$

(3)

where $J_M$ is an M $\times$ M order exchange matrix, the elements on the negative diagonal are 1, and the others are all zero, defined as

$$J_M = \begin{bmatrix} 0 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 0 \end{bmatrix}_{M \times M}$$

(4)

obviously, $J_MJ_M^* = I_M$, the covariance matrix of $Y(n)$ is

$$R_{YY} = E[Y(n)Y^H(n)] = J_M A^*R_{SS}(A^*)^HJ_M + \delta^2 I_M = J_MR_{XX}^*J_M$$

(5)

Suppose there is a matrix

$$D = diag[e^{-j(M-1)\omega_1}, e^{-j(M-1)\omega_2}, \ldots, e^{-j(M-1)\omega_N}]$$

(6)

$$J_MA^* = AD^*$$

(7)
For non-correlated sources, the matrix $R_{SS}$ should be a real diagonal matrix, according to equations (6) and (7), we can obtain that

$$R_{YY} = AR_{SS}A^H + \delta^2 I$$

(8)

cross-covariance matrix of $X(n), Y(n)$ is

$$R_{XY} = E[XY^H]$$

(9)

Assume that

$$R_{11} = R_{XX} + J_M R_{XY}^* J_M^*, R_{22} = R_{YY} + J_M R_{XY}^* J_M^*, R_{33} = R_{XY} + J_M R_{XY}^* J_M^*$$

(10)

So, the total covariance matrix is

$$\tilde{R} = \frac{R_{11} + R_{22} + R_{33}}{3}$$

(11)

Then, do DOA estimation based on the matrix $\tilde{R}$.

2.3.3. Computer Simulation. Computer simulation for improved MUSIC algorithm.

Simulation 1. Set the signal source to 3, and both are 10 array elements. The array spacing $d = \lambda/2$, the angular position $\alpha_1 = 20^\circ, \alpha_2 = 40^\circ, \alpha_3 = 60^\circ$. Non-coherent signal source, the SNR = 20 dB, and the snapshots $N = 200$. The experimental results are shown in the figure.

![Figure 2. Comparison of two algorithms under non-coherent signals](image)

Simulation 2. Set the signal source to 2, and both are 10 array elements. The array spacing $d = \lambda/2$, the angular position $\alpha_1 = 20^\circ, \alpha_2 = 60^\circ$. Coherent signal source, the SNR = 20 dB, and the snapshots $N = 200$. The experimental results are shown in the figure.
3. Conclusion
It can be seen from the results that when the source is incoherent, the traditional MUSIC algorithm and the improved algorithm can well determine the direction of the signal source; when the source is coherent, the traditional MUSIC algorithm loses its effectiveness. The direction of arrival of the signal cannot be accurately estimated, and the false spectrum peak appears in the estimated spectrum, which interferes with the DOA estimation. So the resolution of the improved algorithm is higher.

This improved algorithm greatly improves the traditional MUSIC algorithm and greatly improves the decorrelation performance. Correspondingly, due to the complexity of the derivation formula, the problem of large computational complexity is generated, and it cannot be applied to other complex arrays. This is a problem that needs to be solved in the future.

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