SU(3) flavor symmetry breaking and semileptonic decays of heavy mesons

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ABSTRACT

Assuming the $\bar{D}^0$, $D^-$, $D_s^-$ and $B^+$, $B^0$, $B^0_s$ mesons belong to triplets of SU(3) flavor symmetry, we analyse the form factors in the semileptonic decays of these mesons. Both quark and meson mass differences are taken into account. We find a number of relations, in agreement with the present data as well as with previous analyses, and predict certain ratios of form factors, not yet measured, most notably the $D$ meson decay constant $f_D = 209 \pm 39$ MeV.

PACS numbers(s): 13.20.-v, 13.20.Fc, 13.20.He

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The B meson semileptonic decays are of special importance in our understanding of the Standard Model (see e.g. [1, 2]). Many theoretical approaches have been developed to describe these decays: heavy quark effective theory (HQET) [3]-[10], lattice calculations; e.g., [11, 12], QCD sum rules; e.g., [13]-[15], the quark-hadron duality model [16], and other quark models [17]-[22].

The heavy quark symmetry [23], [24] relates the dynamical variables of D meson decays with the decay variables of B mesons. Using the heavy quark symmetry for the c and b quarks, the ratio of decay constants \( \frac{f_{B_s}}{f_{B_d}} \) and \( \frac{f_{D_s}}{f_{D_d}} \) in B and D meson decays has been estimated [4]. This approach is limited by the fact that charm and bottom quark masses, \( m_c \) and \( m_b \), are, indeed, finite and there are corrections to the results of HQET that are of the order \( m_s/m_c \) [4, 25]. Moreover, the predictions of heavy quark effective theory are expected to be reliable only in the region of small recoil momentum \( q^2 \approx q^2_{\text{max}} \).

On the experimental side, there are many new results in D leptonic and semileptonic decays [26]. The CLEO experiment [27] has measured the ratio \(|f_+^{B\pi}(0)/f_+^{BK}(0)| = 1.29 \pm 0.21 \pm 0.11\), assuming pole-dominance behavior of the form factors. The decay width for \( D_s \to \mu\nu\mu \) has now also been measured [28]. The accumulating experimental data [26] indicate that there is a timely need to better understand the behavior of semileptonic decay form factors at \( q^2 \simeq 0 \).

Recently [25], the ratios \( f_{B_s}/f_B \) and \( f_{D_s}/f_D \), as well as the double ratio \( (f_{B_s}/f_B)/(f_{D_s}/f_D) \) have been analysed, following the usual assumption that chiral symmetry is broken by quark mass terms in the energy density, and
assuming the heavy mesons $\bar{D}_0, D^-, D_{s}^-$ and $B^+, B^0, B_{s}^0$ belong to triplets of the $SU(3)$ flavor group; i.e., assuming the state vectors of these mesons in their rest frame are not appreciable mixed with other representations of $SU(3)$. Following the analysis of ref. [25], and relying on the current experimental data, we analyse the effects of the $SU(3)$ symmetry breaking, due to the substantial quark and meson mass splittings, in semileptonic decays of heavy pseudoscalar mesons into light pseudoscalar mesons.

We use the usual definitions

$$< 0 | A_\mu(x) | H(p) >= i f_H p_\mu e^{-i p\cdot x},$$

and

$$< P(p') | V_\mu(x) | H(p) >= [f_+(q^2)(p_\mu + p'_{\mu}) + f_-(q^2)(p_\mu - p'_{\mu})] e^{-i(q-p')\cdot x},$$

where $H(p)$ is generic for one of the heavy mesons $\bar{D}_0, D^-, D_{s}^-$ or $B^+, B^0, B_{s}^0$ and $P(p')$ denotes one of the light pseudoscalar states $\pi, K, \eta$ or $\eta'$, and $q = p-p'$. The vector and axial-vector current operators are $V_\mu = \bar{Q}(x) \gamma_\mu q(x)$ and $A_\mu = \bar{Q}(x) \gamma_\mu \gamma_5 q(x)$ with $q(x)$ and $Q(x)$ being the light and heavy quark field operators, respectively. We assume, as usual, that chiral symmetry is broken only by quark mass terms. The energy density is then of the form $H_0(x) + H_1(x)$, where $H_0$ is chiral symmetric and $H_1$ breaks this symmetry. In terms of (current) quark masses and fields $H_1$ is given by

$$H_1(x) = \sum_i m_i \bar{q}_i(x) q_i(x),$$
where the sum extends over all quarks, both light and heavy. The divergences of the axial vector and vector currents can readily be calculated from (3) using the local relations

\[ \partial^\mu A_\mu(x) = -i [Q_5, H_1(x)], \quad (4) \]

and

\[ \partial^\mu V_\mu(x) = -i [Q, H_1(x)], \quad (5) \]

where \( Q_5 \) and \( Q \) are the axial and the vector charges. One easily finds the matrix elements

\[ < 0 | \partial^\mu A_\mu(0) | H(p) > = i (m_Q + m_q) < 0 | \bar{Q}(0) \gamma_5 q(0) | H(p) > , \quad (6) \]

and

\[ < P(p') | \partial^\mu V_\mu(0) | H(p) > = i (m_Q - m_q) < P(p') | \bar{Q}(0) q(0) | H(p) > , \quad (7) \]

which together with (4) and (5) give the exact relations

\[ f_H m_H^2 = i (m_Q + m_q) < 0 | \bar{Q} \gamma_5 q | H(p) > , \quad (8) \]

and

\[ f_+(q^2)(m_H^2 - m_P^2) + f_-(q^2)q^2 = - (m_Q - m_q) < P(p') | \bar{Q} q | H(p) > . \quad (9) \]

Proceeding to analyse the constraints on the form factors coming from (4) and (5), we first notice that the density operators in the matrix elements,
by definition, belong to the same $SU(3)$ triplet representation. Next, we will assume the vacuum state is a singlet under $SU(3)$, while the light pseudoscalar mesons belong to the octet representation. While neither of these assumptions can be exact, since $SU(3)$ is broken, neglecting the mixing of other representations in these state vectors has proven to be an excellent approximation in numerous applications over many years [28]. It has been remarkably well established, empirically, that the breaking of light quark $SU(3)$ symmetry by only the current quark mass terms in $H_1$, while producing quite large mass splittings, does not appreciably mix the state vectors. We will further assume that the heavy meson state vectors transform as triplets under $SU(3)$. (Actually, it is only the short distance parts of the state vectors that need belong to pure $SU(3)$ representations, since we will be computing the matrix elements only of local operators.) The Wigner-Eckart theorem can then be applied to estimate the ratio of the matrix elements using the Clebsch-Gordan coefficients for the $3 \times \bar{3} = 8 + 1$ representations of the $SU(3)$ flavor group. Moreover, one can expect that this application of the Wigner-Eckart theorem to be even better for the heavy mesons since they are more compact systems than the light quark systems and $SU(3)$ flavor symmetry becomes exact in the short distance limit, where the light quark masses can be ignored altogether.

Using the definition of the scalar form factor $f_0(q^2)$

$$f_-(q^2) = [-f_+(q^2) + f_0(q^2)](m_H^2 - m_P^2)/q^2,$$

one also finds
\[(m_H^2 - m_p^2) f_0(q^2) = -(m_Q - m_q) < P(p') | \bar{Q} q | H(p) > . \quad (11)\]

Recall that in the heavy quark limit \( f_+ = -f_- \) and \( f_0 = 0 \) \[3, 4, 8\]. The effects of \( SU(3) \) flavor symmetry breaking on semileptonic decays in HQET were discussed in refs. \[3, 4, 9\]. However, these approaches are expected to be valid only in the region of small recoil momentum; that is, at \( q_{max}^2 \), which is not a limitation of the present approach. We emphasize that (8) - (11) are exact relations for all \( q^2 \). Consequently, we can analyse the form factors in the region \( q^2 \approx 0 \), which is more accessible, experimentally.

In the following we concentrate on the semileptonic decays of \( D^0, D^+, D_s^+ \) mesons. Putting \( q^2 = 0 \) in (9) and taking the ratio of the form factors for \( D \to Kl\nu_l \) and \( D \to \pi l\nu_l \) one obtains

\[
\frac{f_{DK}(0)(m_D^2 - m_K^2)}{f_{D\pi}(0)(m_D^2 - m_\pi^2)} = \frac{m_c - m_s}{m_c - m_u} \left. < K(p') | \bar{s} c | D(p) > \right|_{q^2=0} \left. < \pi(p') | \bar{u} c | D(p) > \right|_{q^2=0} . \quad (12)
\]

Applying the Wigner-Eckart theorem, as discussed above, one expects that

\[
\left. < K(p') | \bar{s} \Gamma c | D(p) > \right|_{q^2=0} \left. < \pi(p') | \bar{u} \Gamma c | D(p) > \right|_{q^2=0} \approx 1 \quad (13)
\]

\((\Gamma = 1, \gamma_5)\) is a very good approximation. From (8), as in ref. \[25\], we also have

\[
\frac{f_{D_k} m_{D_k}^2}{f_D m_D^2} = 1 + \frac{m_s}{m_c} . \quad (14)
\]

where \( m_u/m_c \) has been neglected. From (12) - (14) the dependence on quark masses can be eliminated, yielding the following relation among ratios of D meson decay form factors and meson masses:
\[ \frac{f^{DK}(0)}{f^{D\pi}(0)} = \left[ 2 - \frac{f_{D\pi}}{f_D} \frac{m_{D\pi}^2}{m_D^2} \right] \frac{m_D^2 - m_{\pi}^2}{m_K^2} . \] (15)

In (15) only \( f_D \) is not yet experimentally measured and therefore it can be used to calculate \( f_D \):

\[ f_D = f_{D\pi} \frac{m_{D\pi}^2}{m_D^2} \left[ 2 - \frac{f^{DK}(0)}{f^{D\pi}(0)} \frac{m_D^2 - m_{\pi}^2}{m_K^2} \right]^{-1} . \] (16)

The \( D_s \to \mu \nu \) decay constant has been measured to be \( f_{D_s} = 241 \pm 21 \pm 30 \) MeV \([29]\). From (16) we then predict

\[ f_D = 209 \pm 39 \text{ MeV}, \] (17)

where we have also used the experimentally measured value \(|\frac{f^{D\pi}(0)}{f^{DK}(0)}| = 1.29 \pm 0.21 \pm 0.11 \] \([27]\), assuming the positive sign for this ratio. This prediction for \( f_D \), (17), is in a remarkable agreement with the lattice calculations \([1]\), lending support for our approach. (For \( \frac{f^{D\pi}(0)}{f^{DK}(0)} = -1.29 \) we obtain the rather unlikely value \( f_D = 98 \text{ MeV} \).) The error in (17) reflects only the experimental errors on \( f^{D\pi}(0)/f^{DK}(0) \) and \( f_{D_s} \). One can also determine the ratio of the current quark masses, \( m_s/m_c = 0.27 \pm 0.13 \), from (14) and (15), which also agrees with the data \([26]\), although the uncertainty is large.

We can empirically check the validity of our approach using independent data as follows: The decay rates for \( D_s \to \eta(\eta')\mu\nu_\mu \) have been measured \([20]\). If we assume that the \( \eta \) and \( \eta' \) states mix as usual ( \( \eta = \eta_8 \cos \theta - \eta_0 \sin \theta \) and \( \eta' = \eta_8 \sin \theta + \eta_0 \cos \theta \)), and use (9), we have

\[ f^{D_s\eta}(0) = -\sqrt{\frac{2}{3}} (\cos \theta + \frac{1}{\sqrt{2}} \sin \theta) f^{DK}(0) , \] (18)
and
\[ f_+^{D_s\eta}(0) = -\sqrt{\frac{2}{3}}(\sin\theta - \frac{1}{\sqrt{2}}\cos\theta) f_+^{DK}(0). \] (19)

From the \( D \to Kl\nu_l \) decay data the form factor \( f_+^{DK}(0) = 0.74 \pm 0.03 \) \[26\]. This value was extrapolated from the branching ratio assuming pole-dominance. Assuming that the form factors \( f_+^{D_s\eta(\eta')} \) are pole-dominated, too, (see, e.g. \[30\]) we then also have
\[ f_+^{D_s\eta(\eta')}(q^2) = \frac{f_+^{D_s\eta(\eta')(0)}}{1 - q^2/m_{D_s}^2}. \] (20)

Calculating the decay rates for \( D_s \to \eta(\eta')\mu\nu_\mu \) as in ref. \[31\] one finds
\[ \Gamma(H \to Pl\nu_l) = \frac{G_F m_H^2}{24\pi^3} \int_0^{y_m} dy \left| V_{qq'} f_+(0) \right|^2 \left( \frac{(m_H^2(1 - y) + m_P^2)^2}{4m_H^2} - m_P^2 \right)^{3/2}, \] (21)

where \( V_{qq'} \) denotes the CKM matrix element, and \( m_H^* \) is the mass of the heavy vector meson causing the pole in the form factors. In Table I we present the branching ratios for the \( D_s \to \eta(\eta')\mu\nu_\mu \) decays, assuming the typical values of the mixing angle \( \theta = -10^0, -20^0 \), and \(-23^0\), as given in \[26\]. Table I indicates that \( \theta = -10^0 \) is preferred, although the experimental data are not very precise, which is consistent with the Gell-Mann-Okubo mass relation \[32\], which lends further phenomenological support for our approach.

Finally, we note that from (11) it follows that \( f_0(0) = f_+(0) \), and therefore (15), (18) and (19) are then also valid for the scalar form factor \( f_0(0) \).

The semileptonic decays of B mesons can be divided into two categories. First are the usual weak decays \( B \to \pi l\nu_l \), where knowledge of the form factor
f_+ enables one to estimate the CKM matrix element $|V_{ub}|^{33}$. The second type is the flavor-changing neutral decays $B \rightarrow K\mu^+\mu^-$. The calculation of the rate for these decays not only requires knowing the form factors $f^{BK}_+$ but also the additional form factor $h^{BK}$, defined by the matrix element $<K(p')|\bar{b}\sigma_{\mu\nu} s|B(p)> = -2ih^{BK}(p'_\mu p_\nu - p'_\nu p_\mu)$. These flavor changing neutral decays are beyond the scope of the present analysis, but are discussed in §3, 4, 6 using HQET.

To analyse the charged semileptonic decays, we make the replacement $c \rightarrow b$ in (14), finding

$$\frac{f_{Bs}}{f_B} \frac{m_{B_s}^2}{m_B^2} = 1 + \frac{m_s}{m_b}.$$  

(22)

Because there are fewer experimental data on B meson decays, and since $m_s$ is poorly known, we will use the estimate $m_s/m_c = 0.27$, derived from (14) and (15) above, together with the better established values of the current quark masses $m_c = 1.5$ GeV and $m_b = 5$ GeV [26]. Then, with $m_s/m_b = 0.081$, we obtain

$$\frac{f_{Bs}}{f_B} = 1.04,$$

(23)

which is very close to the values obtained previously [1, 4, 25]. Using (13) with the replacement $c \rightarrow b$, we also find

$$\frac{f^{BK}_+}{f^{B\pi}_+(0)} = [2 - \frac{f_{Bs} m_{B_s}^2}{f_B m_B^2}] \frac{m_B^2 - m_{\pi}^2}{m_B^2 - m_K^2},$$

(24)

from which one obtains

8
\[
\frac{f^{B\pi}_+(0)}{f^{BK}_+(0)} = 1.08. \tag{25}
\]
Combining this result with the experimental value of \([f^{D\pi}_+(0)]/[f^{DK}_+(0)]\), the resulting double ratio is

\[
\left| \frac{f^{BK}_+(0)}{f^{B\pi}_+(0)} \right| = 1.19. \tag{26}
\]
We note that the heavy quark effective theory prediction for this ratio is 1 in the regime of \(q^2_{max} \). While the majority of the present experimental data comes from the decay \(\bar{B}^0 \to \pi^+l\bar{\nu}_l\), within the antitriplet \(B^-\), \(\bar{B}^0\), and \(\bar{B}^0_s\) there are several other semileptonic decays to light mesons which are also of interest. These other decays also offer a possible means to determine the CKM matrix element \(|V_{ub}|\). Following our analysis above, with the replacement \(c \to b\), we have

\[
\frac{f^{B^+\pi^0}_+(0)}{f^{B^0\pi^0}_+(0)} = \frac{1}{\sqrt{2}}, \tag{27}
\]

\[
\frac{f^{B^+\eta}_+(0)}{f^{B^{0}\pi^-}_+(0)} = \frac{1}{\sqrt{6}}(\cos \theta - \sqrt{2}\sin \theta), \tag{28}
\]

\[
\frac{f^{B^+\eta'}_+(0)}{f^{B^{0}\pi^-}_+(0)} = \frac{1}{\sqrt{6}}(\sin \theta + \sqrt{2}\cos \theta), \tag{29}
\]
and

\[
\frac{f^{B^{0}K^-}_+(0)}{f^{B^0\pi^-}_+(0)} = \frac{m^2_\pi - m^2_\pi}{m^2_{B^-} - m^2_{K}}. \tag{30}
\]

Recent CLEO measurements give the branching ratio \(BR(\bar{B}^0 \to \pi^+l\bar{\nu}_l) = (1.8 \pm 0.4 \pm 0.3 \pm 0.2) \times 10^{-4}\), and \(|V_{ub}/V_{cb}| = 0.080 \pm 0.015, \) assuming \(V_{cb} = \).
Using these data we can make predictions for the other decay widths relative to \( B^0 \to \pi^- \nu_l \). In Table II we present the predictions for the ratios \( \Gamma(B \to Pl\nu_l)/\Gamma(B^0 \to \pi^- \nu_l) \), where \( B \) is either a \( B^+ \) or \( B^0 \) meson and \( P \) is a light pseudoscalar meson, for all three values of the \( \eta - \eta' \) mixing angle \( \theta \). Note that we also have assumed the vector meson pole dominance as in (20).

We analysed the effects of \( SU(3) \) flavor symmetry breaking on the semileptonic decays of heavy to light pseudoscalar mesons. The ratios of the matrix elements of local quark operators, which transform as triplets, by definition under flavor \( SU(3) \), were estimated using the Wigner-Eckart theorem for the state vectors, but including the effects of the substantial mass splittings. Using experimental data for \( f^{DK+}_{+}(0) \), \( |f^{D\pi}_{+}(0)/f^{PK+}_{+}(0)| \), and \( f_D \), we predict \( f_D = 209 \pm 39 \) MeV, in good agreement with the lattice calculations.

We also determined the ratios \( f^{D+\eta'(\eta)}_{+}(0)/f^{PK+}_{+}(0) \) assuming that the form factors \( f_+(q^2) \) are pole-dominated. We tested our approach using the \( D_s \to \eta(\eta')l\nu_l \) decay rates and verified its consistency with previous analyses. Within the flavor triplet of \( B^+, B^0, \) and \( B^0_s \) we calculated the ratios \( f_{B_s}/f_B = 1.04, \) and \( f^{B\pi}_{+}(0)/f^{BK+}_{+}(0) = 1.08, \) which agree well with previous estimates. Finally, the ratio of decay widths \( \Gamma(B \to Pl\nu_l)/\Gamma(B^0 \to \pi^- \nu_l) \) were predicted.

We emphasize that assuming only that the meson state vectors are not appreciably affected by flavor \( SU(3) \) breaking allows the effects of the substantial mass splittings due to flavor \( SU(3) \) breaking to be taken into account very straightforwardly, leading to several relations among D and B meson decay form factors, which are in good agreement with the data.
This work was supported in part by the Ministry of Science and Technology of the Republic of Slovenia (B.B. and S.F.), by the British Royal Society (B.B.) and by the U.S. Department of Energy, Division of High Energy Physics, under grant No. DE-FG02-91-ER4086 (R.J.O.). S.F. thanks the Department of Physics and Astronomy at Northwestern University, for very kind hospitality. She also thanks D. Bećirević for many very useful discussions.
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\[ \theta \rightarrow \eta \mu \nu_{\mu} \rightarrow \eta' \mu \nu_{\mu} \]

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| \( \theta \) | \( D_s \rightarrow \eta \mu \nu_{\mu} \) | \( D_s \rightarrow \eta' \mu \nu_{\mu} \) |
|-------------|----------------|----------------|
| \(-10^0\)   | \(2.1 \cdot 10^{-2}\) | \(5.8 \cdot 10^{-3}\) |
| \(-20^0\)   | \(1.4 \cdot 10^{-2}\) | \(7.7 \cdot 10^{-3}\) |
| \(-23^0\)   | \(1.1 \cdot 10^{-2}\) | \(8.2 \cdot 10^{-3}\) |
| \textit{exp.} | \((3.3 \pm 1.0)\%\) | \((8.7 \pm 3.4) \cdot 10^{-3}\) |

Table 1: The branching ratios for \( D_s \rightarrow \eta \mu \nu_{\mu} \) and \( D_s \rightarrow \eta' \mu \nu_{\mu} \) decays calculated for typical values of the \( \eta - \eta' \) mixing angle \( \theta \).

| \( B \rightarrow P \) | \( \Gamma(B \rightarrow P \nu_{\ell})/\Gamma(B^0 \rightarrow \pi^- \nu_{\ell}) \) |
|-------------------|---------------------------------|
| \( B^+ \rightarrow \pi^0 \) | 0.5 |
| \( B^+ \rightarrow \eta(\theta = -10^0) \) | 0.21 |
| \( B^+ \rightarrow \eta(\theta = -20^0) \) | 0.29 |
| \( B^+ \rightarrow \eta(\theta = -23^0) \) | 0.31 |
| \( B^+ \rightarrow \eta'(\theta = -10^0) \) | 0.16 |
| \( B^+ \rightarrow \eta'(\theta = -20^0) \) | 0.11 |
| \( B^+ \rightarrow \eta'(\theta = -23^0) \) | 0.09 |
| \( B_s^0 \rightarrow K^- \) | 0.92 |

Table 2: The ratios \( \Gamma(B \rightarrow P \nu_{\ell})/\Gamma(B^0 \rightarrow \pi^- \nu_{\ell}) \).