Explanations of the Tentative New Physics Anomalies and Dark Matter in the Simple Extension of the Standard Model (SESM)

Tianjun Li,1,2,∗ Junle Pei,1,2,† Xiangwei Yin,1,2,‡ and Bin Zhu3,§

1CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China
2School of Physical Sciences, University of Chinese Academy of Sciences, No. 19A Yuquan Road, Beijing 100049, China
3Department of Physics, Yantai University, Yantai 264005, China

Abstract

We revisit the Simple Extension of the Standard Model (SESM) which can account for various tentative new physics anomalies and dark matter (DM). We consider a complete scalar potential which is needed to address the $W$ boson anomaly. Interestingly, the SESM can simultaneously explain the B physics anomaly, muon anomalous magnetic moment, $W$ mass anomaly, and dark matter, etc. Also, we study the unitarity constraint in this model. We perform the systematic study, and find the viable parameter spaces which can explain these anomalies and evade all the current experimental constraints. To be complete, we briefly comment on the neutrino masses and mixings, baryon asymmetry, and inflation.

∗tli@mail.itp.ac.cn
†peijunle@mail.itp.ac.cn
‡yinxiangwei@mail.itp.ac.cn
§zhubin@mail.nankai.edu.cn
I. INTRODUCTION

After the discovery of Higgs boson at the LHC in 2012 [1, 2], the Standard Model (SM) has been confirmed to be a correct effective theory at the low energy scale. However, we do have a few evidences of new physics Beyond the SM (BSM), for instance, dark energy, dark matter (DM), neutrino masses and mixings, baryon asymmetry, and inflation, etc [3]. Also, there exist some fine-tuning problems in the SM, for example, gauge hierarch problem, and strong CP problem, etc. Thus, the SM is not complete, and we need to explore the new physics.

In recent years, the LHCb Collaboration has declared the results in rare decays of B mesons. The persistent discrepancies between the Standard Model and the experimental measurements imply that there might be new physics. Such B physics anomalies can be observed in the angular distribution of $B \rightarrow K \mu^+\mu^-$ and Lepton Flavor Universality (LFU) ratios $R_K = BR(B \rightarrow \mu\mu)/BR(B \rightarrow ee)$ [4–8]. Also, there exists a 4.2 $\sigma$ discrepancy for the muon anomalous magnetic moment (muon $g - 2$) $a_\mu = \frac{(g_\mu - 2)}{2}$ between the experimental results and theoretical predictions [9]. Although the hadronic contribution might induce the discrepancy, muon $g - 2$ is still a promising hint for new physics beyond the SM [12–14], and has been studied extensively [15–19]. Recently, the CDF Collaboration announced a state-of-the-art measurement of the $W$ boson mass, which shows 7 $\sigma$ deviation from the prediction of the SM [20]. For recent studies, see Refs. [21–60]. Moreover, the DM is a crucial problem in both particle physics and astronomy, and the observations from astrophysics and cosmology provide the overwhelming evidence. The Weakly Interacting Massive Particle (WIMP) provide an excellent DM candidate to account for the relic density observed by the experiment of Cosmic Microwave Background (CMB). This is the so called “WIMP miracle”.

To address the above anomalies and dark matter, we revisit the Simple Extension of the Standard Model (SESM) [61]. We consider a complete scalar potential which is needed to account for the $W$ boson anomaly. Interestingly, the SESM can simultaneously explain the B physics anomaly, muon anomalous magnetic moment, $W$ boson mass anomaly, and dark matter, etc. Also, we study the unitarity constraint in this model. We perform the systematic study, and find the viable parameter spaces which can explain these anomalies and evade all the current experimental constraints. To be complete, similar to the New
Minimal SM (NMSM) [3], we can introduce two right-handed neutrinos to explain the neutrino masses and mixing as well as baryon asymmetry, and introduce a real scalar to address inflation. Because of the strong constraint on the tensor-to-scalar ratio, we need to choose the proper inflaton potential, for example, the inflaton potentials in Section 2 of Ref. [62]. The XENON1T excess [63] will be studied elsewhere as well.

This paper is organized as follows. In Section II, we briefly review the SESM and present the complete scalar potential. In Section III, the constraints of unitarity on the parameters in the Yukawa sector are studied. In Section IV, we explain the above anomalies and dark matter. In Section V, the $W$ boson mass anomaly is investigated. We conclude in Section VI.

II. THE SIMPLE EXTENSION OF THE STANDARD MODEL

Following Ref. [61] we introduce a complex singlet scalar $\Phi_S$, a doublet scalar $\Phi_D$, and two vectorlike pairs of Weyl fermions (that combine into two Dirac fermions) with the same quantum numbers as the SM quark and lepton doublets $Q'$ and $L'$. Under a discrete $Z_2$ symmetry, these extra fields are all odd while the SM fields are even. The quantum numbers of extotic fields under SM gauge group are

| Field | spin | SU(3)$_c$ | SU(2)$_L$ | U(1)$_Y$ |
|-------|------|-----------|-----------|-----------|
| $Q'$  | 1/2  | 3         | 2         | 1/6       |
| $L'$  | 1/2  | 1         | 2         | -1/2      |
| $\Phi_S$ | 0   | 1         | 1         | 0         |
| $\Phi_D$ | 0   | 1         | 2         | -1/2      |

The new vectorlike fermions and scalars can be written as

$$Q' = \begin{pmatrix} U' \\ D' \end{pmatrix}, \quad L' = \begin{pmatrix} L^0 \\ L'^- \end{pmatrix}, \quad \Phi_S \equiv S^0_s, \quad \Phi_D = \begin{pmatrix} S^0_d \\ S^- \end{pmatrix}.\tag{2}$$

The Lagrangian involving the new fields is given by

$$\mathcal{L} \supset \left( \lambda^Q_i \overline{Q}' Q_i \Phi_S + \lambda^U_i \overline{U}' U_i \Phi_D + \lambda^L_i \overline{L}' L_i \Phi_S + \lambda^D_i \overline{D}' D_i \Phi_D + a' H^\dagger \bar{\Phi}_D \Phi_S + \frac{\lambda_S}{2} \left( \bar{\Phi}_D H \right)^2 + \text{h.c.} \right) - M_{Q'Q'} Q'^\dagger Q' - M_{L'L'} L'^\dagger L' - M_S^2 \Phi_S^\dagger \Phi_S - M_D^2 \Phi_D^\dagger \Phi_D$$

$$+ \frac{\lambda_S}{2} \left( \Phi_S^\dagger \Phi_S \right)^2 + \frac{\lambda_D}{2} \left( \Phi_D^\dagger \Phi_D \right)^2 + \lambda_{SD} |\Phi_S|^2 |\Phi_D|^2 + \lambda_{SH} |\Phi_S|^2 |H|^2 + \lambda_{DH} |\Phi_D|^2 |H|^2$$

$$+ \lambda'_1 H^\dagger \bar{\Phi}_D \left( \Phi_D^\dagger H \right),\tag{3}$$
where we have systematically considered the scalar potential relative to the phenomenology that we are interested in. We denote the left-handed exotic quarks, right-handed exotic quarks, left-handed exotic leptons, right-handed exotic leptons, left-handed quark doublets, right-handed up-type quarks, right-handed down-type quarks, left-handed lepton doublets, and right-handed down-type leptons as $Q'_L, Q'_R, L'_L, L'_R, Q_i, U_i, D_i, L_i,$ and $E_i (i=1,2,3)$, respectively.

After the breaking of the electroweak symmetry, the terms involved $\lambda_{SH}$ and $\lambda_{DH}$ will contribute to masses of $\Phi_S$ and $\Phi_D$, respectively, while the term involved $a_H$ will induce mixing of $S^0_s$ and $S^0_d$. For simplicity, we choose $a'_H, \lambda'_1$ and $\lambda'_2$ to be zero. Therefore, the modified mass matrix of the BSM neutral scalars can be written as

$$U^\dagger \begin{pmatrix} M^2_S + \frac{\lambda_{SH} v^2}{2} & a'_H v/\sqrt{2} \\ a_H v/\sqrt{2} & M^2_D + \frac{\lambda_{DH} v^2}{2} \end{pmatrix} U = \begin{pmatrix} M^2_{S_1} \\ M^2_{S_2} \end{pmatrix},$$

(4)

where $v \simeq 246$ GeV. The corresponding mass eigenstates $S_1$ and $S_2$ have physical masses of

$$M^2_{S_{1,2}} = \left( M^2_S + \frac{\lambda_{SH} v^2}{2} + M^2_D + \frac{\lambda_{DH} v^2}{2} \mp \Delta M^2 \right)/2$$

(5)

where

$$\Delta M^2 = \sqrt{\left( M^2_D + \frac{\lambda_{DH} v^2}{2} - M^2_S - \frac{\lambda_{SH} v^2}{2} \right)^2 + 2a^2_H v^2}.$$  

(6)

The mixing of $S_1$ and $S_2$ is given by

$$U = \begin{pmatrix} \frac{\sqrt{2}a_H v}{\sqrt{M^2_D + \lambda_{DH} v^2 - M^2_S - \frac{\lambda_{SH} v^2}{2} - \Delta M^2} + 2a'_H v^2} & \frac{-M^2_D + \lambda_{DH} v^2 - M^2_S - \frac{\lambda_{SH} v^2}{2} - \Delta M^2}{\sqrt{2}a_H v} \\ \frac{-M^2_D + \lambda_{DH} v^2 - M^2_S - \frac{\lambda_{SH} v^2}{2} - \Delta M^2}{\sqrt{2}a_H v} & \frac{\sqrt{M^2_D + \lambda_{DH} v^2 - M^2_S - \frac{\lambda_{SH} v^2}{2} - \Delta M^2} + 2a'_H v^2}{\sqrt{2}a_H v} \end{pmatrix}.$$  

(7)

The couplings between up and down type quarks have a relative misalignment, and we choose

$$\lambda_{i}^{Q_u} = \lambda_{i}^{Q}, \quad \lambda_{i}^{Q_d} = \sum_{k} \lambda_{k}^{Q} V_{ki},$$

(8)

where $V$ is the CKM matrix.

In our model, we impose that the lightest BSM particle is $S_1$, which is the DM candidate. But the mass hierarchies between $M_{Q'}$, $M_{L'}$, and $M_{S_2}$ are not mandatory.
III. THE UNITARITY CONSTRAINT

The partial wave amplitude of any $2\rightarrow 2$ scattering process in the high-energy massless limit is [64],

$$a^J_{fi} = \frac{1}{32\pi} \int_{-1}^{1} d \cos \theta d^J_{\mu_i \mu_f}(\theta) T_{fi}(\sqrt{s}, \cos \theta) ,$$

where $\theta$ is the scattering angle in the center of mass frame, $d^J_{\mu_i \mu_f}(\theta)$ is Wigner $d$-function, and $\mu_{i/f} = \lambda_{i_1} - \lambda_{i_2}$ with $\lambda_{i_1/2}$ and $\lambda_{f_1/2}$ standing for the helicities of initial and final particles, respectively.

The unitarity of $S$ matrix respects $SS^\dagger = 1$. By considering elastic scattering and imposing that the intermediate states are two-particle states, the condition required by the unitarity of $S$ matrix can be obtained. For general unitarity bound, one has

$$\text{Re}^2 \left[ a^J_{ii} \right] + \left( \text{Im} \left[ a^J_{ii} \right] - \frac{1}{2} \right)^2 \leq \frac{1}{4} .$$

At tree level, the above unitarity bound becomes

$$\left| \text{Re} \left( a^J_{ii, \text{tree}} \right) \right| \leq \frac{1}{2} .$$

To compute the unitarity bounds in the Yukawa sector, the helicity amplitude approach demonstrated in [64] is employed, and the general form is given in TABLE I. $T$ is the non-trivial part of $S$ matrix. The absence of $T$ in $C^{(t)}$, $E^{(t)}$, and $F^{(t)}$ is due to the angular momentum conversation. In the massless limit, the entities in $A^{(t)}$ and $B^{(t)}$ vanish exactly since $T$ is proportional to the mass. Besides, the entity in D is present when considering the scattering of two scalars.

One can decompose the $T$ matrix into the following structure

$$T^{\lambda_{f_1} \lambda_{f_2} \lambda_{i_1} \lambda_{i_2}}_{f_1 f_2 i_1 i_2}(\sqrt{s}, \theta) = \bigoplus_r \sum_{m=s,t,u} T^{\lambda_{f_1} \lambda_{f_2} \lambda_{i_1} \lambda_{i_2}}_m(\sqrt{s}, \theta) F_{f_1 f_2 i_1 i_2}^m(N) 1_{d_r} ,$$

where $T^{\lambda_{f_1} \lambda_{f_2} \lambda_{i_1} \lambda_{i_2}}_m(\sqrt{s}, \theta)$ is the Lorentz part and $F_{f_1 f_2 i_1 i_2}^m(N)$ is the group factor. To obtain the partial wave amplitude, we should consider a concrete elastic scattering process which can occur in different channels and representations. Our strategy is

1) Considering which partial wave we are interested in;

2) Calculating the group factors of the corresponding elastic scattering process;

3) Combining Eqs. 9, 11, and 12 to obtain the constraint of unitarity on this elastic scattering process.
TABLE I: Scattering matrix $\mathcal{T}$.

A. The unitarity constraint of $\lambda^Q_2$

According to the classification in Ref. [64], the term $\lambda^Q_2 \bar{Q} Q \Phi_S$ in Eq. 3 corresponds to the first model of dirac type theory with respect to $SU(3)_C$ and $SU(2)_L$. Since $Q_i$ denotes a left-handed field, $\bar{Q}$ should be a right-handed field. The term for the second generation is $\lambda^Q_2 \bar{Q}_R Q^2 \Phi_S$. In the massless limit, (anti-)fermions described by the left-handed and right-handed spinors have helicities of $-\frac{1}{2} (\frac{1}{2})$ and $\frac{1}{2} (-\frac{1}{2})$, respectively.

1. $J = 0$ partial wave

TABLE. I shows that only $\mathcal{T}^{+++-}$ and its conjugation contribute to the $J = 0$ partial wave. Since $Q'_R$ and $\bar{Q}_2$ are in the fundamental and anti-fundamental representations of $SU(2)_L$ and $SU(3)_C$, the tensor decomposition of $Q'_R \bar{Q}_2$ gives

$$++ + + \left\{ Q'_R \bar{Q}_2 \sim \left( \mathbf{1} \oplus \text{Adj} \right)_{SU(2)_L} \otimes \left( \mathbf{1} \oplus \text{Adj} \right)_{SU(3)_C} \right. \right. \right.$$  \hspace{1cm} (13)

The elastic scattering process is mediated by the singlet complex scalar $\Phi_S$. Thus, only the channel of the $\mathbf{1}$ representation of $SU(2)_L \times SU(3)_C$ is present. As for the group factor of this process, we have

$$++ + + \left\{ \mathcal{F}^n_{Q'_R \bar{Q}_2 Q'_R \bar{Q}_2} = - - - - \right\} = N_2 \times N_3 = 2 \times 3 \right.$$  \hspace{1cm} (14)

where $N$ comes from that both fermions involved are in the (anti-)fundamental representation of the $SU(N)$ group (there are more details in Appendix A) which means that $N_2 = 2$
and $N_3 = 3$ in our case. In the basis of $(Q'_R \overline{Q}_2, \overline{Q}'_R Q_2)$, based on Eqs. 9 and 12, we get

$$d_{SU(3) = 1, SU(2) = 1}^j = \frac{\lambda_2^Q}{32\pi} \int_{-1}^{+1} d \cos \theta d_0^j(\theta) \begin{pmatrix} N_2 N_3 T_s^{+++} & 0 \\ 0 & N_2 N_3 T_s^{---} \end{pmatrix}$$

$$= \frac{\lambda_2^Q}{32\pi} \int_{-1}^{+1} d \cos \theta \begin{pmatrix} 2 \times 3 \times (-1) & 0 \\ 0 & 2 \times 3 \times (-1) \end{pmatrix}$$

$$= -\frac{3\lambda_2^Q}{8\pi} ,$$

(15)

where the last step means that we find the largest eigenvalue after the diagonalization of the matrix. Besides, we have used $d_0^j(\theta) = 1$ and $T_s^{+++} = T_s^{---} = -1$ according to Table 2 in [64]. Then, the perturbation unitarity condition of Eq. 11 leads to the bound

$$|\lambda_2^Q| \leq 2.05 .$$

(16)

**B. A brief summary of the unitarity constraints in Yukawa sector**

The elaborate constraints on the parameters of Yukawa sector in our model are given in TABLE II, and the details can be found in Appendix A.

| $J = 0$ | $|\lambda_2^Q| \leq 2.05$ | $|\lambda_3^Q| \leq 2.05$ | $|\lambda_2^L| \leq 3.54$ | $|\lambda_2^E| \leq 5.01$ |
|--------|----------------|----------------|----------------|----------------|
| Base I | $(Q'_R \overline{Q}_2, \overline{Q}'_R Q_2)$ $(Q'_R \overline{Q}_3, \overline{Q}'_R Q_3)$ $(L'_R \overline{T}_2, \overline{T}'_R L_2)$ $(\overline{T}'_L E, L'_L \overline{E})$ | $(\overline{Q}_2 \Phi_3^s, Q_2 T_s^*)$ $(\overline{Q}_3 \Phi_3^s, Q_3 \overline{T}_s^*)$ $(L'_R \Phi_3^s, \overline{T}'_R \Phi_3^s)$ $(E \overline{\Phi}_D, \overline{E} \Phi_3^*)$ |
| $J = \frac{1}{2}$ | $|\lambda_2^Q| \leq 5.01$ | $|\lambda_3^Q| \leq 5.01$ | $|\lambda_2^L| \leq 5.01$ | $|\lambda_2^E| \leq 7.09$ |
| Base II | $(Q'_R \Phi_3^s, \overline{Q}'_R \Phi_3^s)$ $(Q'_R \Phi_3^s, \overline{Q}'_R \Phi_3^s)$ $(L_2 \Phi_3^s, L_2 \Phi_3^s)$ $(\overline{T}'_L \overline{\Phi}_D, L'_L \Phi_3^*)$ | $(\overline{Q}_2 \Phi_3^s, \overline{Q}_2 T_s^*)$ $(\overline{Q}_3 \Phi_3^s, \overline{Q}_3 \overline{T}_s^*)$ $(L'_R \Phi_3^s, \overline{T}'_R \Phi_3^s)$ $(E \overline{\Phi}_D, \overline{E} \Phi_3^*)$ |
| $J = \frac{1}{2}$ | $|\lambda_2^Q| \leq 5.01$ | $|\lambda_3^Q| \leq 5.01$ | $|\lambda_2^L| \leq 5.01$ | $|\lambda_2^E| \leq 5.01$ |

TABLE II: Unitarity constraint in Yukawa sector.

For $\lambda_2^Q$, $\lambda_3^Q$ and $\lambda_2^L$, the strict constraints come from $J = 0$ partial wave, which are $|\lambda_2^Q| \leq 2.05$, $|\lambda_3^Q| \leq 2.05$ and $|\lambda_2^L| \leq 3.54$. While the $J = \frac{1}{2}$ partial wave leads to a relative weaker bound. For $\lambda_2^E$, the $J = 0$ partial wave gives the strict bound $|\lambda_2^E| \leq 5.01$ as well, however, $J = \frac{1}{2}$ partial wave in an alternative base $(\overline{T}'_L \overline{\Phi}_D, L'_L \Phi_3^*)$ leads to the same bound.
The unitarity constraints on Yukawa couplings eventually lead the following bounds

\[ |\lambda_Q^2| \leq 2.05 , \]
\[ |\lambda_Q^3| \leq 2.05 , \]
\[ |\lambda_L^2| \leq 3.54 , \]
\[ |\lambda_E^2| \leq 5.01 . \]

(17)

Of course, we need to consider the perturbative bounds on the Yukawa couplings, i.e., all the Yukawa couplings are smaller than \( \sqrt{4\pi} \).

IV. THE TENTATIVE NEW PHYSICS ANOAMLIES AND DARK MATTER

In this Section, we will address the \( R_K \) and \( B_s \) mixing, muon \( g - 2 \), and dark matter relic density simultaneously. Random scan of the model parameters is employed, and the benchmark points satisfying all the current constraints are selected.

A. \( R_K \) and \( B_s \) mixing

According to [61], the contribution to \( R_K \) can be described by the following effective Lagrangian,

\[ \mathcal{H}_{\text{eff}}^{bs\mu} \supset -\mathcal{N} \left[ C_{9}^{bs\mu} (\bar{s} \gamma_\mu P_L b) (\bar{\mu} \gamma_\mu \mu) + C_{10}^{bs\mu} (\bar{s} \gamma_\mu P_L b) (\bar{\mu} \gamma_\mu \gamma_5 \mu) + \text{h.c.} \right] , \]

(18)

where the normalization is given by

\[ \mathcal{N} \equiv \frac{4G_F}{\sqrt{2}} \frac{e^2}{16\pi^2} V_{tb} V_{ts}^* . \]

(19)

Contributions to \( C_{9,10}^{bs\mu} \) from the BSM particles are

\[ \Delta C_{9}^{bs\mu} = -\frac{\lambda_Q^d \lambda_Q^d*}{128\pi^2 N} \sum_{\alpha=1,2} |U_{1\alpha}|^4 \left| \frac{\lambda_L^2}{M_{S_\alpha}^2} \right|^2 + |U_{1\alpha}|^2 |U_{2\alpha}|^2 \left| \frac{\lambda_E^2}{M_{S_\alpha}^2} \right|^2 F_2 \left( \frac{M_{Q_s}^2}{M_{S_\alpha}^2}, \frac{M_{L}^2}{M_{S_\alpha}^2} \right) , \]

(20)

\[ \Delta C_{10}^{bs\mu} = \frac{\lambda_Q^d \lambda_Q^d*}{128\pi^2 N} \sum_{\alpha=1,2} |U_{1\alpha}|^4 \left| \frac{\lambda_L^2}{M_{S_\alpha}^2} \right|^2 - |U_{1\alpha}|^2 |U_{2\alpha}|^2 \left| \frac{\lambda_E^2}{M_{S_\alpha}^2} \right|^2 F_2 \left( \frac{M_{Q_s}^2}{M_{S_\alpha}^2}, \frac{M_{L}^2}{M_{S_\alpha}^2} \right) , \]

(21)

with loop function

\[ F_2(x, y) \equiv \frac{1}{(x - 1)(y - 1)} + \frac{x^2 \log x}{(x - 1)^2 (x - y)} + \frac{y^2 \log y}{(y - 1)^2 (y - x)} . \]

(22)
According to the up-to-date fitting [4] (2σ), the contribution to LFU violation in \( b \to s\mu\mu \) from \( \Delta C_9^{bs\mu\mu} \) and \( \Delta C_{10}^{bs\mu\mu} \) needs to satisfy
\[
6.74 + 9.04(\Delta C_9^{bs\mu\mu})^2 + \Delta C_9^{bs\mu\mu}(14.96 - 10.68\Delta C_{10}^{bs\mu\mu}) + \Delta C_{10}^{bs\mu\mu}(-13.22 + 11.90\Delta C_{10}^{bs\mu\mu}) \leq 1.
\]
(23)

The contribution to \( B_s - \bar{B}_s \) oscillation can be induced from the following operators
\[
\mathcal{H}_{\text{eff}} \supset C_1^{bd_i} (\bar{d}_i\gamma_\mu P_L b) (\bar{d}_i\gamma_\mu P_L b) + \text{h.c.} \quad d_i = d, s.
\]
(24)
The \( Q' - \Phi_S \) box diagram gives Wilson coefficients
\[
\Delta C_1^{bd_i} = \left(\frac{\lambda_3^{Q_d} \lambda_i^{Q_s^*}}{128\pi^2}\right)^2 \sum_{\alpha=1,2} |U_{1\alpha}|^4 \frac{M_Q^2}{M_{S\alpha}^2} F\left(\frac{M_{S\alpha}^2}{M_{S\alpha}^2}\right),
\]
(25)
where the loop function is
\[
F(x) \equiv \frac{x^2 - 1 - 2x \log x}{(x-1)^3}.
\]
(26)
The bound given in [10, 11] is
\[
\Delta C_1^{bs} < 2.1 \times 10^{-5}\text{TeV}^{-2}.
\]
(27)

**B. Muon g-2**

The contribution of BSM particles in this model to \( a_\mu \) is
\[
\Delta a_\mu \approx -\frac{m_\mu M_L}{8\pi^2} \sum_{\alpha=1,2} \text{Re} \left(\frac{\lambda_2^L \lambda_2^{E^*} U_{1\alpha} U_{2\alpha}^*}{M_{S\alpha}^2}\right) f_{LR}\left(\frac{M_L^2}{M_{S\alpha}^2}\right),
\]
(28)
where the loop function is
\[
f_{LR}(x) \equiv \frac{3 - 4x + x^2 + 2 \log x}{2(x-1)^3}.
\]
(29)
The latest (1σ) discrepancy is given by [9]
\[
a_\mu^{\text{EXP}} - a_\mu^{\text{SM}} = (2.51 \pm 0.59) \times 10^{-9}.
\]
(30)

**C. Dark matter phenomenology**

We consider the lightest neutral scalar \( S_1 \) as the DM candidate, and the typical freeze-out mechanism controls the DM production. In addition, the annihilation and co-annihilation processes can deplete the DM relic density.
FIG. 1: DM annihilation mediated by Higgs resonance. The black points satisfy the constraints of $R_K$ (2\(\sigma\)) and $B_s - \bar{B}_s$, the blue points satisfy the constraints of $R_K$ (2\(\sigma\)), $B_s - \bar{B}_s$ and muon g-2, the red points satisfy the constraints of $R_K$ (2\(\sigma\)), $B_s - \bar{B}_s$, muon g-2 and DM relic density, and the green points satisfy the constraints of $R_K$ (2\(\sigma\)), $B_s - \bar{B}_s$, muon g-2, DM relic density and XENON1T direct detection.

In our random scan, the ranges of parameters have to satisfy the bounds of unitarity given in TABLE II. In addition, we set the couplings to the right-handed quarks ($\lambda^U_i$ and $\lambda^D_i$) and the left-handed quarks of the first generation ($\lambda^Q_1$) to be zero, and only consider the second family couplings in the lepton sector ($\lambda^L_2$ and $\lambda^E_2$). The elaborate scan ranges of parameters are given in TABLE III.

| Parameters | $\lambda^Q_2$ | $\lambda^Q_3$ | $\lambda^L_2$ | $\lambda^E_2$ | $a_H$/TeV | $\lambda_S/2$ | $\lambda_D/2$ |
|------------|---------------|---------------|---------------|---------------|------------|--------------|--------------|
| Range      | [-2,2]        | [-2,2]        | [-3,3]        | [-3,3]        | [-1,1]     | [-3,3]       | [-3,3]       |

| Parameters | $M_S$/TeV | $M_D$/TeV | $M_L$/TeV | $M_Q$/TeV | $\lambda_{SH}$ | $\lambda_{DH}$ | $\lambda_{SD}$ |
|------------|-----------|-----------|-----------|-----------|---------------|---------------|---------------|
| Range      | [0,2]     | [0,5]     | [0.11,5]  | [1.2,6]   | [-1,1]        | [-1,1]        | [-3,3]        |

TABLE III: The scan ranges of parameters employed in the random scan. New parameters of this model which are not shown in this table are set to be zero.

FIG. 1 shows the DM relic density versus the mass of $S_1$. The conditions from $R_K$ (2\(\sigma\)) and $B_s - \bar{B}_s$ (black dot), the muon g-2 (1\(\sigma\)) constraint (blue dot), the constraint of (rescaled) DM relic density (red dot), and the constraint from XENON1T experiment (green dot) are applied in order. It can be seen from FIG. 1 that the Higgs mediated DM annihilation
FIG. 2: The DM (co-)annihilation processes.

channel efficiently depletes the DM, leading to the minimum value of relic density.

The DM relic density is controlled by the (co-)annihilation processes shown in FIG. 2.

1) Annihilation of two $S_1$ will always happen no matter what the mass differences between $S_1$ and other particles are, which are demonstrated in the first column of FIG. 2. The annihilation process mediated by the Higgs boson shown in the last diagram of the first column of FIG. 2 can efficiently deplete the DM relic density.

2) Co-annihilations of $S_1$ and other particles ($S_2$, $L'$, and $Q'$) will occur when their mass difference are small, which are shown in the second column of FIG. 2.

Ten benchmark points are given in TABLE IV, and all these points can evade the experimental constraints from $R_K$ and $B_s$ mixing, and muon g-2. Points 1 and 2 correspond to
annihilation of two $S_1$. The Higgs mediated DM annihilation is demonstrated by Points 3 and 4. Point 5 (6), Point 7 (8), and Point 9 (10) are characterized by the co-annihilation between $S_1$ and $S_2$, $L'$, and $Q'$, respectively. In addition, the dark matter candidate $S_1$ can explain some of the observed relic density and evade XENON1T experiment as well.

| Relic density | $\lambda^Q_2$ | $\lambda^Q_3$ | $\lambda^L_2$ | $\lambda^F_{S_1}$ | $\lambda^F_{S_2}$ | $\lambda^L_{S_1}/2$ | $\lambda^L_{S_2}/2$ | $\lambda^S_{SH}$ | $\lambda^D_{S_1}/2$ | $\lambda^D_{S_2}/2$ | $\lambda^S_{SH}/2$ | $\lambda_{SH}$ | $\lambda_{SD}$ |
|---------------|---------------|---------------|---------------|-----------------|-----------------|------------------|------------------|-----------------|------------------|------------------|------------------|---------------|-------------|
| 0.011522      | 0.10101       | -1.1951       | -2.6267       | 0.58956         | 0.093819        | 0.000076         | -0.12509         | 0.5996          | 0.011026         | 0.01447          | 0.017495         | 0.014744       | -1.0286      |
| 0.000046      | 1.1424        | 1.7261        | 1.9734        | 0.18295         | 0.017041        | 0.000000         | 0.000000         | 0.000000        | 1.3606           | 1.3606           | 1.3606           | 1.3606         | 1.3606       |
| 0.011447      | 1.3606        | -1.0692       | 2.9536        | 1.2603          | -1.9362         | -1.2747          | -1.226           | -1.226          | -1.226           | -1.226           | -1.226           | -1.226         | -1.226      |
| 0.017495      | 0.54168       | -0.48519      | 2.9733        | 1.3269          | -1.9258         | -1.5627          | -1.5627          | -1.5627         | -1.5627          | -1.5627          | -1.5627          | -1.5627       | -1.5627    |
| 0.016008      | 0.55338       | 1.3764        | -2.492        | -2.4106         | -2.7338         | -2.7466          | -2.7466          | -2.7466         | -2.7466          | -2.7466          | -2.7466          | -2.7466       | -2.7466   |
| 0.04798       | 0.22722       | 1.1424        | -2.492        | -2.4106         | -2.7338         | -2.7466          | -2.7466          | -2.7466         | -2.7466          | -2.7466          | -2.7466          | -2.7466       | -2.7466   |
| 0.048167      | 0.26312       | 1.3606        | -1.226        | -1.226           | -1.226           | -1.226           | -1.226           | -1.226          | -1.226           | -1.226           | -1.226           | -1.226        | -1.226    |

TABLE IV: Benchmark points in DM annihilation and co-annihilation.

In addition, we investigate the large mixing and relative small coupling case. We can see from Eqs. 20 and 21, $\Delta C_9$ and $\Delta C_{10}$ are quadratically depended in $\lambda^L_2$. To reduce the $\lambda^L_2$, one needs small $M_Q$ and $M_L$, which are already near the lower edge of experimental search. After a somehow simple analysis, one cannot have a small $\lambda^L_2$ to account for all the aforementioned anomalies. In the left panel of FIG. 3, we can see that only Higgs resonance can explain the $R_K$, $B_s$ mixing, muon $g - 2$, and the saturated DM relic density. The right panel of FIG. 3 shows the $S_1$ and $L'$ co-annihilation process with large mixing ($a_H = 500$ GeV). In this case, the $R_K$, $B_s$ mixing, and muon $g - 2$ can be explained simultaneously and the DM relic density is undersaturated. In FIG. 4 we display viable parameter space with different $a_H$. In top left panel, the flavor observables and DM relic density can be explained as well. As $a_H$ increases, the DM relic density is undersaturated while the constraints of flavour observables are satisfied.

We intend to give some comments on the parameter space on the edge of unitarity.
FIG. 3: The combined constraints on the $M_S$ versus $M_D$ plane and $M_L$ versus $M_S$ plane. The couplings are $\lambda_2^Q = 0.49$, $\lambda_3^Q = -0.49$, $\lambda_2^L = 1.4$, $\lambda_2^E = -0.4$, $\lambda_{SH} = \lambda_{DH} = 0.001$ (left), $\lambda_2^Q = 0.49$, $\lambda_3^Q = -0.49$, $\lambda_2^L = 1.5$, $\lambda_2^E = -0.06$, $\lambda_{SH} = \lambda_{DH} = 0.001$ (right).

In FIG. 5 we employ the conservative parameters which are consistent with the unitarity bounds in Eq. 17 and avoid large quantum corrections. Roughly speaking, we find the upper bounds on the masses of exotic particles, $M_{Q'} = 17.5$ TeV and $M_{L'} = 2.4$ TeV which can account for the $R_K$, $B_s$ mixing, muon $g - 2$, and the saturated DM relic density.

V. W Boson Mass

The oblique corrections contributing to $W$ boson mass are given by [65, 66]

$$m_W^2 = m_W^2(SM) + \frac{\alpha c^2}{c^2 - s^2} m_Z^2 \left[ -\frac{1}{2} \Delta S + c^2 \Delta T + \frac{c^2 - s^2}{4 s^2} \Delta U \right],$$

(31)

where $\alpha$ is the fine structure constant, $c = \cos \theta_W$, $s = \sin \theta_W$. The expressions of $\Delta S$ and $\Delta T$ are

$$\Delta S = \frac{1}{2\pi} \left[ \frac{1}{6} \log \left( \frac{m_R^2}{m_{S \pm}^2} \right) - \frac{5}{36} + \frac{m_R^2 m_A^2}{3 (m_A^2 - m_R^2)^2} + \frac{m_A^4 (m_A^2 - 3m_R^2)}{6 (m_A^2 - m_R^2)^3} \log \left( \frac{m_A^2}{m_R^2} \right) \right],$$

(32)

$$\Delta T = \frac{1}{32\pi^2 \alpha v^2} \left[ F \left( m_{S \pm}^2, m_A^2 \right) + F \left( m_{S \pm}^2, m_R^2 \right) - F \left( m_A^2, m_R^2 \right) \right],$$

(33)

with the loop function

$$F(x, y) = \begin{cases} \frac{x+y}{2} - \frac{xy}{x-y} \log \left( \frac{x}{y} \right), & x \neq y \\ 0, & x = y \end{cases},$$

(34)
FIG. 4: The combined constraints on $M_S$ and $M_D$ plane with different $a_H$. The couplings are setting as $\lambda_L^2 = 1.5, \lambda_Q^2 = 0.49, \lambda_Q^3 = 0.49, \lambda_{SH} = \lambda_{DH} = 0.001$ and $\lambda_E^2$ are -0.1, -0.06, -0.04, -0.04 respectively where we ignore $\Delta U$ and rewrite the scalar doublet as

$$\Phi_D = \begin{pmatrix} S_d^0 = R + iA \\ S^- \end{pmatrix}$$  \hspace{1cm} (35)$$

For convenience, we consider the small mixing between scalar singlet and doublet. To interpret the W boson mass anomaly, the scalar potential is essential to give an appropriate mass splitting between neutral and charge part of the doublet. We find that W boson mass cannot be explained if we consider $\lambda'_2$ only. However, if the $\lambda_{DH}, \lambda'_2, \lambda'_1$ are involved simultaneously, the W mass anomaly can be account for in a straightforward way. We can fit the up-to-date measurement of W boson mass ($1 \sigma$) by using Eqs. 31, 32 and 33. FIG.6 shows the corresponding oblique parameters between $\Delta S$ and $\Delta T$, and the mass splitting between neutral and charge part of the scalar doublet.
FIG. 5: The parameter space consistent with unitarity bounds. The couplings are setting as $\lambda_L^2 = 2.9$, $\lambda_E^2 = -2.9$, $\lambda_Q^2 = 1.4$, $\lambda_3^Q = -2.0$, $\lambda_{SH} = \lambda_{DH} = 0.001$

FIG. 6: The points that explain the up-to-date $W$ boson mass ($1\sigma$) in $\Delta S$ versus $\Delta T$ plane (left) and the mass splitting between neutral and charged part of the scalar doublet plane (right).

VI. CONCLUSION

We have revisited the SESM which can address various tentative new physics anomalies and DM. The mass splitting between the charged and neutral parts of scalar doublet via scalar potential can account for the $W$ boson mass anomaly. Moreover, we considered the unitarity constraints in the Yukawa sector. Also, we employed the random scan approach,
and obtained the viable parameter spaces which can explain the B physics anomaly, muon anomalous magnetic moment, W mass anomaly, and dark matter relic density simultaneously. To be concrete, we select some benchmark points. The various DM (co)-annihilation and resonance processes are investigated, and the benchmark points whose DM relic density are around or smaller than the observed DM relic density are demonstrated. In particular, the Higgs pole, $S_1$ and $S_1$ annihilation, $S_1$ and $S_2$, $L'$, $Q'$ co-annihilation are demonstrated as well, and all these points can evade the current flavor and XENON1T direct detection constraints.

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Appendix A: The Unitarity Constraint on $\lambda_2^Q$

1. The Unitarity Constraint on $\lambda_2^Q$

a. $J = 0$ Partial Wave

Two particle system is defined as

$$|\psi \bar{\psi}\rangle_1 = \frac{1}{\sqrt{N}} \delta^b_a |\psi_a \bar{\psi}^b\rangle,$$

$$|\psi \bar{\psi}\rangle_{A\text{d}j} = \sqrt{2} (T^A)^b_a |\psi_a \bar{\psi}^b\rangle,$$

$$|\psi \bar{\psi}\rangle_S = (T^A_S)^a_b |\psi_a \bar{\psi}_b\rangle.$$  \hfill (A1)

The group factor can be caculate as follows

$$\langle \psi \bar{\psi} | \psi \bar{\psi}\rangle_1 = \langle \psi_c \bar{\psi}^d | \frac{1}{\sqrt{N}} \delta^d_c \frac{1}{\sqrt{N}} \delta^b_a | \psi_a \bar{\psi}^b\rangle$$

$$= \frac{N}{\sqrt{N} \sqrt{N}} \langle \psi_1 \bar{\psi}_1 | \psi_1 \bar{\psi}_1 \rangle$$

$$= N \langle \psi_1 \bar{\psi}_1 | \psi_1 \bar{\psi}_1 \rangle \ , \hfill (A2)$$
where $\langle \psi \bar{\psi} | \psi \bar{\psi} \rangle_1$ denotes the elastic scattering process in singlet channel, and $\psi_1$ stands for one component of $\psi$ in the fundamental representation.

**b. $J = \frac{1}{2}$ Partial Wave**

$\mathcal{T}^{+0^*+0^*}$ and $\mathcal{T}^{-0^*0^*}$ contribute to $J = \frac{1}{2}$ partial wave as follows

$$+ 0^* \right\{ \bar{Q}_2 \Phi_S^* \sim \Box \right. .$$  \hspace{1cm} (A3)

The one fermion and one scalar state is

$$|\Psi S\rangle = 1|\Psi S\rangle ,$$  \hspace{1cm} (A4)

with group factor

$$\langle \Psi S | \Psi S \rangle = 1 \langle \Psi S | \Psi S \rangle .$$  \hspace{1cm} (A5)

We have $+0^* + 0^*$ or $-0^* - 0^*$ \left( \frac{\mathcal{F}^{u,\Box}}{\bar{Q}_2 \Phi_S^* \bar{Q}_2 \Phi_S} = \mathcal{F}^{u,\Box} = 1 \right)$. In $(\bar{Q}_2 \Phi_S^*, Q_2 \bar{\Phi}_S^*)$ basis, we have (Note that the base $(Q_R^* \Phi_s^*, Q_R \bar{\Phi}_s^*)$ leads to the same result.)

$$e^{J=1/2}_{SU(3) = \Box \Box, SU(2) = \Box \Box} = \frac{\lambda_2 Q^2}{32\pi} \int_{-1}^{1} d\cos \theta d\frac{1}{2} \frac{1}{2} \begin{pmatrix} T_u^{+0^*+0^*} & 0 \\ 0 & T_u^{-0^*0^*} \end{pmatrix}$$

$$= \frac{\lambda_2 Q^2}{32\pi} \int_{-1}^{1} d\cos \theta \cos \frac{\theta}{2} \begin{pmatrix} -\frac{1}{\cos \frac{\theta}{2}} & 0 \\ 0 & -\frac{1}{\cos \frac{\theta}{2}} \end{pmatrix}$$

$$= -\frac{\lambda_2 Q^2}{16\pi} .$$  \hspace{1cm} (A6)

This corresponds to the bound

$$|\lambda_2^Q| \leq 5.01 .$$  \hspace{1cm} (A7)

**2. The Unitarity Constraint on $\lambda_3^Q$**

The $\lambda_3^Q$ in Eq. 3 is the first model of dirac type theory in both $SU(3)_C$ and $SU(2)_L$ as well, and thus the constraint on $\lambda_3^Q$ is exact the same as $\lambda_2^Q$. We have the strict bound

$$|\lambda_3^Q| \leq 2.05$$ from $J = 0$ partial wave.
3. The Unitarity Constraint on $\lambda_2^L$

As we have claimed in the quark sector before, the Lagrangian becomes $\lambda_2^L \bar{L} L \Phi_S \rightarrow \lambda_2^L \bar{L} R L \Phi_S$. This kind of interaction can be classified by the first type of Dirac theory in $SU(2)_L$.

a. $J = 0$ Partial Wave

One can consider the elastic scattering processes $L'_R L_2 \rightarrow L'_R L_2$ or $\bar{L}'_R L_2 \rightarrow \bar{L}'_R L_2$. Group factor is $++$ or $- - - -$\{ $F_{s,1}^{s,1} = F_{s,1}^{s,1} = N$ \}.

In $(L'_R L_2, \bar{L}'_R L_2)$ basis, we have

$$a_{SU(2)=1}^{J=0} = \frac{\lambda_2^L}{32\pi} \int_{-1}^{+1} d\cos \theta \delta^{0}_0(\theta) \begin{pmatrix} N_2 T_{s}^{++} & 0 \\ 0 & N_2 T_{s}^{--} \end{pmatrix}$$

$$= \frac{\lambda_2^L}{32\pi} \int_{-1}^{+1} d\cos \theta \begin{pmatrix} 2 \times (-1) & 0 \\ 0 & 2 \times (-1) \end{pmatrix}$$

$$= -\frac{\lambda_2^L}{8\pi},$$

which gives the bound

$$|\lambda_2^L| < 3.54.$$ (A9)

b. $J = \frac{1}{2}$ Partial Wave

Consider the elastic scattering process $L'_R \Phi_S \rightarrow L'_R \Phi_S^*$ or $\bar{L}'_R \Phi_S \rightarrow \bar{L}'_R \Phi_S$, group factor is $+0^* + 0^* - 0 - 0$\{ $F_{s,1}^{s,1} = F_{s,1}^{s,1} = 1$ \}. In $(L'_R \Phi_S^*, \bar{L}'_R \Phi_S)$ basis, we get (the basis $(\bar{L}_2 \Phi_S^*, L_2 \Phi_S)$ gives the same result.)

$$a_{SU(2)=0\text{ or } 0}^{J=1/2} = \frac{\lambda_2^L}{32\pi} \int_{-1}^{+1} d\cos \theta \delta^{1/2}_{1/2}(\theta) \begin{pmatrix} T_{u}^{+0^* + 0^*} & 0 \\ 0 & T_{u}^{-0 - 0} \end{pmatrix}$$

$$= \frac{\lambda_2^L}{32\pi} \int_{-1}^{+1} d\cos \theta \cos \frac{\theta}{2} \begin{pmatrix} -\frac{1}{\cos \frac{\theta}{2}} & 0 \\ 0 & -\frac{1}{\cos \frac{\theta}{2}} \end{pmatrix}$$

$$= -\frac{\lambda_2^L}{16\pi},$$

with the relatively weak bound

$$|\lambda_2^L| < 5.01.$$ (A11)
4. The Unitarity Constraint on $\lambda_E^2$

The Lagrangian is given by $\lambda_E^2 T_1 E \Phi_D \to \lambda_E^2 T_2 E \Phi_D$. Since $E$ is the $SU(2)_L$ singlet and $\Phi_D$ is $SU(2)_L$ doublet, this kind of interaction can be classified into the second type of dirac theory.

a. $J = 0$ Partial Wave

Two particle state is defined ($E$ is $SU(2)_L$ singlet fermion) as

$$|\bar{L}_L E\rangle_a^a = |\bar{L}_L E\rangle.$$  \hspace{1cm} (A12)

The group factor is

$$\langle \bar{L}_L^a E | \bar{L}_L^a E \rangle = 1 \langle \bar{L}_L^a E | \bar{L}_L^a E \rangle.$$ \hspace{1cm} (A13)

We have $+++ \text{ or } --- \text{ or } +-- \text{ or } --+$ $\{ F_{s, \square}^{a, \square} \}_{LL_L E} = F_{L_L E}^{a, \square} = 1.$

In ($\bar{L}_L E, L'_L E$) basis, we have

$$a^J_{SU(2)=\square or \Box} = \frac{\lambda_E^2}{32\pi} \int_{-1}^{+1} d\cos \theta \Theta_0^0(\theta) \begin{pmatrix} T_{s,++} & 0 \\ 0 & T_{s,--} \end{pmatrix}$$

$$= \frac{\lambda_E^2}{32\pi} \int_{-1}^{+1} d\cos \theta \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= -\frac{\lambda_E^2}{16\pi},$$ \hspace{1cm} (A14)

with unitarity bound

$$|\lambda_E^2| < 5.01.$$ \hspace{1cm} (A15)
b. $J = \frac{1}{2}$ Partial Wave

Group factor is $+0 + 0$ or $-0^* - 0^* \left\{ \mathcal{F}_{E \Phi_D E \Phi_D}^s = \mathcal{F}_{E \Phi_D E \Phi_D}^{s*} = 1 \right\}$. In $(E \tilde{\Phi}_D, E \tilde{\Phi}_D^*)$ base, we have (We should claim that $\tilde{\Phi}_D$ is in the fundamental representation.)

$$a_{J=1/2}^{SU(2)\square \square} = \frac{\lambda_E^2}{32\pi} \int_{-1}^{+1} d\cos\theta d\frac{1}{2} \left( \begin{array}{cc} T_s^{+0+0} & 0 \\ 0 & T_s^{-0^*-0^*} \end{array} \right)$$

$$= \frac{\lambda_E^2}{32\pi} \int_{-1}^{+1} d\cos\theta \cos\frac{\theta}{2} \left( -\cos\frac{\theta}{2} & 0 \\ 0 & -\cos\frac{\theta}{2} \right)$$

(A16)

with unitarity bound

$$|\lambda_E^2| < 7.09 \quad .$$

(A17)

However, an alternative basis $(L'_L \tilde{\Phi}_D, L'_L \tilde{\Phi}_D^*)$ gives stronger constraint since the group factor is $+0 + 0$ or $-0^* - 0^*$ \left\{ \mathcal{F}_{L'_L \tilde{\Phi}_D L'_L \tilde{\Phi}_D}^{s,1} = \mathcal{F}_{L'_L \tilde{\Phi}_D L'_L \tilde{\Phi}_D}^{s,1*} = N \right\}$. In this case, we get

$$a_{J=-1/2}^{SU(2)=1} = \frac{\lambda_E^2}{32\pi} \int_{-1}^{+1} d\cos\theta d\frac{1}{2} \left( \begin{array}{cc} N_2 T_s^{+0+0} & 0 \\ 0 & N_2 T_s^{-0^*-0^*} \end{array} \right)$$

$$= \frac{\lambda_E^2}{32\pi} \int_{-1}^{+1} d\cos\theta \cos\frac{\theta}{2} \left( 2 \times (-\cos\frac{\theta}{2}) & 0 \\ 0 & 2 \times (-\cos\frac{\theta}{2}) \right)$$

(A18)

$$= -\frac{\lambda_E^2}{16\pi} ,$$

with the group factor enhancement, the unitarity bound becomes

$$|\lambda_E^2| < 5.01 \quad .$$

(A19)

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