On the treatment of confinement in three-quark calculations

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Abstract

The treatment of confining interactions in non-relativistic three-quark systems is revised. Usually in the Faddeev equations the Faddeev components are coupled by the total potential. In the new treatment the Faddeev components are coupled only by the non-confining short-range part of the potential, allowing thus its channel-by-channel investigation. The convergence in angular momentum channels is much faster.

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The Faddeev equations are the fundamental equations of three-body problems. Besides giving a unified formulation, they are superior to the Schrödinger equation in many respects: in incorporating boundary conditions, in treating symmetries, in handling correlations, etc., and only this formalism can handle all types of interactions. These unique advantages also appear in three-quark calculations [1–3]. It has been observed, however, that in the Faddeev scheme many angular momentum channels are needed. So, in practice, the apparent advantages of the Faddeev method are burdened by the need for many angular momentum channels.

A similar situation occurs in atomic physics. There are several bound-state atomic three-body calculations using Faddeev technique. They demonstrate the power of this scheme in all kinds of mass ratios and also for excited states, on the expense of many angular momentum channels, however. In a recent publication, Ref. [4], I have pointed out that this phenomenon is the consequence of the inadequate way of using the Faddeev technique. A new scheme has been proposed, which means splitting the Coulomb interactions into long-range and short-range terms, and applying the Faddeev decomposition only for the short-range terms. This way all the asymptotically important terms, like the kinetic energy and the long-range part of the interaction were treated on equal footing and in an asymptotically exact way. Numerical studies showed that in this scheme, to reach a good accuracy, much less angular momentum channels are needed.

The role of the Coulomb and confinement potentials in three-body atomic and in three-quark systems is similar: both kinds of potentials modify the character of the asymptotic motion, and thus, following the concept of scattering theory, both of them should be treated on equal footing with the kinetic energy. In the present work, taking over the concept of Ref. [4], I propose a new way of treating confinement potentials in three-quark calculations. I split the quark-quark interaction into a long-range confining and a short-range non-confining interactions, and apply the Faddeev decomposition only for the non-confining part. In the resulting Faddeev equations, in complete agreement with our physical expectation, the long-range confining parts play a similar role as the kinetic energy, thus the asymptotics of the Faddeev components are determined by them together. The power of the new method is demonstrated by numerical illustrations.

The Schrödinger equation of a three-quark system reads

\[ (H^0 + v_\alpha + v_\beta + v_\gamma) |\Psi\rangle = E |\Psi\rangle, \]

where \(H^0\) is the three-body kinetic energy operator and \(v_\alpha\) denotes the quark-quark interaction in subsystem \(\alpha\). In the conventional method [4], the Faddeev procedure is applied literally. The the wave function \(\Psi\) is written as a superposition of three Faddeev components

\[ |\Psi\rangle = |\psi_\alpha\rangle + |\psi_\beta\rangle + |\psi_\gamma\rangle, \]

and the components are required to satisfy the set of Faddeev equations

\[ (E - H^0 - v_\alpha) |\psi_\alpha\rangle = v_\alpha [ |\psi_\beta\rangle + |\psi_\gamma\rangle ], \]

with a cyclic permutation for \(\alpha, \beta, \gamma\). Of course, the sum of three Faddeev equations gives back the original Schrödinger equation (I).
Since $v_\alpha$ depends only on $\xi_\alpha$ Jacobi distance, it is natural to express the Faddeev components with the corresponding $(\xi_\alpha, \eta_\alpha)$ Jacobi coordinates. Each Faddeev component is expanded on an angular momentum basis, which should carry also the necessary spin and isospin indices. Usually, the energy reference corresponds to the case where all particles are infinitely separated. For confining particles this makes no sense. Since the asymptotics of $|\psi_\alpha\rangle$ in the $\eta_\alpha$ coordinate is determined by the kinetic energy component $h^0_{\eta_\alpha}$ only, this asymptotics is strongly dependent on the energy reference. The practical way out of this problem is choosing the energy reference in such a way that the relevant eigenvalues appear as very deeply bound states. This way the asymptotics of $|\psi_\alpha\rangle$ in the $\eta_\alpha$ coordinate becomes similar to a confinement asymptotics. In Ref. [1] the equations were solved in configuration space. In Ref. [2] a simplified version of the method of Ref. [3] was used. In fact, the Faddeev components were expanded in terms of Coulomb–Sturmian functions.

It should be noted that the splitting of the wave function $|\Psi\rangle$ into Faddeev components is not unique. If the potential $v_\alpha$ is split into a confining and non-confining terms,

$$v_\alpha = v^c_\alpha + v^{nc}_\alpha,$$

(4)

where $v^c_\alpha$ should contain all the asymptotically relevant terms, like constant and confinement terms, and $v^{nc}_\alpha$ is short-ranged compared to $v^c_\alpha$, the Faddeev procedure can also be applied only for the non-confining short-range part. The wave function $|\Psi\rangle$ can also be written as a superposition of the modified Faddeev components,

$$|\Psi\rangle = |\tilde{\psi}_\alpha\rangle + |\tilde{\psi}_\beta\rangle + |\tilde{\psi}_\gamma\rangle,$$

(5)

which are required to satisfy the set of modified Faddeev equations

$$(E - H^0 - v^c_\alpha - v^c_\beta - v^c_\gamma - v^{nc}_\alpha)|\tilde{\psi}_\alpha\rangle = v^{nc}_\alpha[|\tilde{\psi}_\beta\rangle + |\tilde{\psi}_\gamma\rangle],$$

(6)

with a cyclic permutation for $\alpha, \beta, \gamma$. Of course, the sum of three Faddeev equations also gives back the original Schrödinger equation (1). In this equation the asymptotics of $|\tilde{\psi}_\alpha\rangle$ along the coordinate $\eta_\alpha$ is determined not only by the kinetic energy, but also by the sum of confining interactions $v^c_\beta + v^c_\gamma$, and thus the asymptotics does not depend on the choice of energy reference.

The Faddeev procedure is not merely rewriting the wave function $|\Psi\rangle$ as a sum of three components: the decomposition should also act as an asymptotic filtering [3]. While the wave function $|\Psi\rangle$ describes all the three different two-cluster fragmentations each Faddeev component describes only one fragmentation. The necessary condition for that is that the term $v^c_\beta + v^c_\gamma$ on the left hand side of Eq. (1) should not generate bound states. Since they are infinite range confining interactions this is not possible. However, if the splitting (4) is performed in such a way that $v^c$ contains a repulsive core the bound states generated by $v^c_\beta + v^c_\gamma$ can be removed from the spectrum of physical interest. So, in the energy region of physical interest the decomposition (5) acts as an asymptotic filtering as well.

Eqs. (6) can be solved as before. For comparison, I present here the convergence of the energy (mass) of barions by increasing the number of angular momentum channels using both Eqs. (3) and Eqs. (6). In Refs. [2] and [3] the barions are described as three-quark systems interacting by two different kind of parametrization of a linear confinement plus Goldstone-boson-exchange potential,
\[ V_{qq}(\vec{r}) = V_0 + Cr + V_\chi. \]  

(7)

In the parametrization of Ref. [2] the strength of the confining force is relatively weak, \( C = 0.474 \text{ fm}^{-2} \), while in the other one, in the alternative parametrization of Ref. [3], it is rather strong, \( C = 0.77 \text{ fm}^{-2} \). The following kind of splitting of the total potential is adopted:

\[ v^c = V_0 + Cr + V_g \exp\left(-\left(\frac{r}{r_g}\right)^2\right) \]  

(8)

and

\[ v^{nc} = V_\chi - V_g \exp\left(-\left(\frac{r}{r_g}\right)^2\right), \]  

(9)

with \( r_g = 1.0 \text{ fm} \) and \( V_g = 3.0 \text{ fm}^{-1} \) in both cases. The role of the Gaussian term in \( v^c \) is to remove the spurious states from the spectrum, and, in the other hand, it should not destroy the convergence with respect to angular momentum channels. The results are given in Table I. The convergence in the new scheme, Eqs. (8), is much faster; in fact channels up to \( l = 2 \) are completely sufficient in both cases. The converged results from Eqs. (3) are in complete agreements with the fully converged results from Eqs. (3), which in Ref. [3] were cross-checked with the stochastic variational method [4].

Summarizing, in this paper I have proposed a new way of writing down the Faddeev equations for confining potentials. In this new scheme all the asymptotically important parts, like constant and confining parts, are treated on equal footing with the kinetic energy, and, while all the previous advantages of the Faddeev technique are preserved, the convergence with respect to angular momentum channels is much faster. The fact that the confining and non-confining parts are treated in different ways makes the channel-by-channel investigation of the physically interesting non-confining part possible. For example from Table I it is obvious, that in the model of Refs. [4] and [3] the higher partial waves of the non-confining interaction do not contribute to the mass of the nucleon and its excitations. This conclusion could have hardly been inferred from the results of Eqs. (3), and even less from other non-Faddeev calculations. I believe that for analyzing the quark-quark interaction in non-relativistic quark model by investigating the properties of barions the modified Faddeev equations (6) provide the best tools.

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TABLES

TABLE I. Convergence of the mass of the nucleon and “excited” nucleons with respect to angular momentum channels taken into account up to \( l = 0, l = 1, l = 2, l = 3 \) and \( l = 4 \). The energy values are given in MeV.

| Angular momentum channels | \( l = 0 \) | \( l = 1 \) | \( l = 2 \) | \( l = 3 \) | \( l = 4 \) |
|---------------------------|-----------|-----------|-----------|-----------|-----------|
| \( N \) with Eqs. (3)     | 950       | 945       | 939       | 939       | 939       |
| \( N \) with Eqs. (6)     | 939       | 939       | 938       | 938       | 938       |
| \( N^* \) with Eqs. (3)   | 1574      | 1565      | 1510      | 1502      | 1493      |
| \( N^* \) with Eqs. (6)   | 1495      | 1490      | 1490      | 1490      | 1490      |
| \( N^{**} \) with Eqs. (3) | 1859     | 1780      | 1724      | 1698      | 1690      |
| \( N^{**} \) with Eqs. (6) | 1704     | 1689      | 1681      | 1682      | 1681      |
| \( N^{**} \) with Eqs. (3) | 1900     | 1802      | 1780      | 1756      | 1748      |
| \( N^{**} \) with Eqs. (6) | 1780     | 1723      | 1714      | 1714      | 1714      |