Decay of proton into Planck neutrino in the theory of gravity

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Abstract

It is considered gravitational interaction within the framework of the Newton theory and the quantum field theory. It is introduced the Planck neutrino $\nu_{Pl}$. Gravitational interaction of the fields $\psi\psi$ includes short-range interaction $\psi\nu_{Pl}$ and long-range interaction $\nu_{Pl}\nu_{Pl}$. Gravitational radiation can be identified with the Planck neutrino. The theory predicts the decay of proton into Planck neutrino. It is assumed that the Planck mass built from three fundamental constants $\hbar$, $c$, and $G$ is fixed in all the inertial frames. This leads to that the lifetime of proton relative to the decay into Planck neutrino decreases with the Lorentz factor as $\sim \gamma^{-5}$. Such a dependence of the lifetime of proton on the Lorentz factor yields a cut-off in the EHECRs spectrum. It is shown that the first ”knee” in the EHECRs spectrum $E \sim 3 \times 10^{15}$ eV corresponds to the lifetime of proton equal to the lifetime of the universe, the second ”knee” $E \sim 10^{17} - 10^{18}$ eV corresponds to the lifetime of proton equal to the thickness of our galactic disc. The EHECRs with the energies $E > 3 \times 10^{18}$ eV can be identified with the Planck neutrinos.

1 Introduction

The principle of equivalence of inertial and gravitational masses underlines the theory of gravity. In the Einstein theory of gravity [1], this leads to that the free gravitational field is nonlocalized. Under the presence of the matter, the gravity is described by the Einstein equations

$$G_{ik} = T_{ik} \quad (1)$$

where $G_{ik}$ is the Einstein tensor, $T_{ik}$ is the tensor of momentum-energy of the matter. Free gravitational field defined by the absence of the matter $T_{ik} = 0$ is described by the equations

$$R_{ik} = 0 \quad (2)$$

where $R_{ik}$ is the Ricci tensor. The localized field must be described by the tensor of momentum-energy. Einstein characterized the momentum-energy of the gravitational field by the pseudo-tensor defined as

$$t^{ik} = H^{ikm}_{,lm} - G^{ik} \quad (3)$$

where $H^{ikm}_{,lm}$ is the linearized part of $G_{ik}$. Thus in the Einstein theory gravitational field is nonlocalized.
The natural way proposed by Lorentz and Levi-Civita \cite{2} is to take \( G_{ik} \) as the momentum-energy of the gravitational field. However in this case \( G_{ik} \) is equal to zero for the free gravitational field \( G_{ik} = R_{ik} = 0 \). Such a situation may be interpreted as that the gravitational interaction occurs without gravitational field. Then the problem arises as to how to introduce gravitational radiation. The possible resolution of the problem is to introduce some material field as a radiation.

2 Theory

Consider gravitational interaction within the framework of the Newton theory and the quantum field theory. The Lagrangian of the Newton gravity is given by

\[
L = G \frac{m^2}{r},
\]

with the mass \( m \) being the gravitational charge. While expressing the Newton constant \( G \) via the charge \( g = (\hbar c)^{1/2} \) and via the Planck mass \( m_{Pl} = (\hbar c/G)^{1/2} \), the Lagrangian (4) can be rewritten in the form

\[
L = G \frac{m^2}{r} = \frac{g^2}{m_{Pl}^2} \frac{m^2}{r}.
\]

The Lagrangian of the Newton gravity in the form (5) describes gravitational interaction by means of the charge \( g \). In this way gravity may be implemented into the quantum field theory.

Rewrite the Lagrangian (5) in the form of the effective Lagrangian of interaction of the spinor fields \cite{3}

\[
L = \frac{g^2}{m_{Pl}^2} J^\mu(x) J^\mu(x).
\]

The term \( 1/m_{Pl}^2 \) in the Lagrangian (6) reads that gravitational interaction takes place at the Planck scale. At the same time gravity is characterized by the infinite radius of interaction. To resolve the problem consider the scheme of gravitational interaction which includes both the short-range interaction and the long-range interaction

\[
L = L_{short} + L_{long}.
\]

Let us introduce the Planck neutrino \( \nu_{Pl} \). Let the Planck neutrino is the massless particle of the spin 1/2. Suppose that the Planck neutrino interacts with the other fields at the Planck scale

\[
\psi \rightarrow \nu_{Pl}
\]

where \( \psi \) denotes all the fields of the spin 1/2. This interaction is of short-range and is governed by the Lagrangian (5)

\[
L_{short} = \frac{g^2}{m_{Pl}^2} J^\mu(x) J^\mu(x)
\]
where the current $J_\mu$ transforms the field $\psi$ into the field $\nu_{Pl}$. Let the interaction of the Planck neutrinos $\nu_{Pl}\nu_{Pl}$ be of long-range and is governed by the Lagrangian identically equal to zero

$$L_{long} \equiv 0. \quad (10)$$

The considered scheme allows one to describe both the classical gravity and the decay of the field $\psi$ into the Planck neutrino. In this scheme gravitational radiation can be identified with the Planck neutrino.

Within the framework of the standard quantum field theory, the above scheme of gravitational interaction should include two intermediate fields $\psi\nu_{Pl}$ and $\nu_{Pl}\nu_{Pl}$. Since the Lagrangian of the interaction $\nu_{Pl}\nu_{Pl}$ is identically equal to zero, the energy of the field $\nu_{Pl}\nu_{Pl}$ is identically equal to zero. The field $\psi\nu_{Pl}$ is defined by the Planck mass. In the theory of gravity there is the limit of ability to measure the length equal to the Planck length $\Delta l \geq 2(\hbar G/c^3)^{1/2} = 2l_{Pl}$. From this it follows that there is no possibility to measure the field $\psi\nu_{Pl}$ in the physical experiment. Thus both intermediate fields $\psi\nu_{Pl}$ and $\nu_{Pl}\nu_{Pl}$ cannot be measured. This means that the intermediate fields $\psi\nu_{Pl}$ and $\nu_{Pl}\nu_{Pl}$ do not exist. We arrive at the conclusion that gravitational interaction occurs without intermediate fields.

3 The lifetime of proton relative to the decay into Planck neutrino

In view of eq. (8), the decay of proton into Planck neutrino occurs at the Planck scale

$$p \rightarrow \nu_{Pl}. \quad (11)$$

The lifetime of proton relative to the decay into Planck neutrino is defined by the Lagrangian (6)

$$t_p = t_{Pl} \left( \frac{m_{Pl}}{2m_p} \right)^5 \quad (12)$$

where the factor 2 takes into account the transition from the massive particle to the massless one. This lifetime corresponds to the rest frame. Consider the lifetime of proton in the moving frame with the Lorentz factor

$$\gamma = \left( 1 - \frac{v^2}{c^2} \right)^{-1/2}. \quad (13)$$

In the moving frame the rest mass and time are multiplied by the Lorentz factor

$$m' = \gamma m \quad (14)$$

$$t' = \gamma t. \quad (15)$$

The Planck mass $m_{Pl} = (\hbar c/G)^{1/2}$ and the Planck time $t_{Pl} = (\hbar G/c^5)^{1/2}$ are built from three fundamental constants $\hbar$, $c$ and $G$. According to the special relativity [2], the speed of light...
is fixed in all the inertial frames. Extend the special relativity principle and suppose that
the three constants $\hbar$, $c$ and $G$ are fixed in all the inertial frames

$$\hbar' = \hbar, \quad c' = c, \quad G' = G.$$  \hspace{1cm} (16)

Hence the Planck mass and time are fixed in all the inertial frames. Then the lifetime of
proton in the moving frame is given by

$$t_p' = t_{Pl}\left(\frac{m_{Pl}}{2\gamma m_p}\right)^5.$$ \hspace{1cm} (17)

For comparison consider the decay of muon which is governed by the Lagrangian of
electroweak interaction $L$

$$L = \frac{g^2}{m_W^2}J_\mu(x)J^{\mu}(x)$$ \hspace{1cm} (18)

where $m_W$ is the mass of W-boson. In the rest frame the lifetime of muon is given by

$$t_\mu = t_W\left(\frac{m_W}{m_\mu}\right)^5.$$ \hspace{1cm} (19)

In the moving frame the lifetime of muon is given by

$$t_\mu' = \gamma t_W\left(\frac{m_W}{m_\mu}\right)^5.$$ \hspace{1cm} (20)

Thus unlike the usual situation when the lifetime of the particle, e. g. muon, grows with
the Lorentz factor as $\sim \gamma$, the lifetime of proton relative to the decay into Planck neutrino
decreases with the Lorentz factor as $\sim \gamma^{-5}$. State once again that such a behaviour is due
to that the Planck mass built from three fundamental constants $\hbar$, $c$ and $G$ is fixed in all
the inertial frames.

4 Extra high energy cosmic rays spectrum in view of
the decay of proton into Planck neutrino

In view of eq. (17), the lifetime of proton relative to the decay into Planck neutrino decreases
with the increase of the kinetic energy of proton. Then the decay of proton can be observed
for the extra high energy protons. In particular the decay of proton can be observed as a
cut-off in the energy spectrum of extra high energy cosmic rays (EHECRs).

The EHECRs spectrum above $10^{16}$ eV can be divided into three regions: two ”knees”
and one ”ankle" [5]. The first ”knee” appears around $3 \times 10^{15}$ eV where the spectral power
law index changes from $-2.7$ to $-3.0$. The second ”knee” is somewhere between $10^{17}$ eV
and $10^{18}$ eV where the spectral slope steepens from $-3.0$ to around $-3.3$. The ”ankle” is
seen in the region of $3 \times 10^{18}$ eV above which the spectral slope flattens out to about $-2.7$.  \hspace{1cm} 4
Consider the EHECRs spectrum in view of the decay of proton into Planck neutrino. Let the earth be the rest frame. For protons arrived at the earth, the travel time meets the condition

\[ t \leq t_p. \]  

(21)

From this the time required for proton travel from the source to the earth defines the limiting energy of proton

\[ E_{\text{lim}} = \frac{m_p}{2} \left( \frac{t_p}{t} \right)^{1/5}. \]  

(22)

Within the time \( t \), protons with the energies \( E > E_{\text{lim}} \) decay and do not give contribution in the EHECRs spectrum. Thus the energy \( E_{\text{lim}} \) defines a cut-off in the EHE proton spectrum. Planck neutrinos appeared due to the decay of the EHE protons may give a contribution in the EHECRs spectrum. If the contribution of Planck neutrinos in the EHECRs spectrum is less compared with the contribution of protons one can observe the cut-off at the energy \( E_{\text{lim}} \) in the EHECRs spectrum.

Determine the range of the limiting energies of proton depending on the range of distances to the EHECRs sources. Take the maximum and minimum distances to the source as the size of the universe and the thickness of our galactic disc respectively. For the lifetime of the universe \( \tau_0 = 14 \pm 2 \) Gyr \[3\], the limiting energy is equal to \( E_1 = 3.9 \times 10^{15} \) eV. This corresponds to the first ”knee” in the EHECRs spectrum. For the thickness of our galactic disc \( \simeq 300 \) pc, the limiting energy is equal to \( E_2 = 5.5 \times 10^{17} \) eV. This corresponds to the second ”knee” in the EHECRs spectrum. Thus the range of the limiting energies of proton due to the decay of proton into Planck neutrino lies between the first ”knee” \( E \sim 3 \times 10^{15} \) eV and the second ”knee” \( E \sim 10^{17} - 10^{18} \) eV.

From the above consideration it follows that the decrease of the spectral power law index from \(-2.7\) to \(-3.0\) at the first ”knee” \( E \sim 3 \times 10^{15} \) eV and from \(-3.0\) to around \(-3.3\) at the second ”knee” \( E \sim 10^{17} - 10^{18} \) eV can be explained as a result of the decay of proton into Planck neutrino. From this it seems natural that, below the ”ankle” \( E < 3 \times 10^{18} \) eV, the EHECRs events are mainly caused by the protons. Above the ”ankle” \( E > 3 \times 10^{18} \) eV, the EHECRs events are caused by the particles other than protons.

If Planck neutrinos take part in the strong interactions, they must give some contribution in the EHECRs events. To explain the observed EHECRs spectrum it is necessary to assume that the contribution of Planck neutrinos in the EHECRs spectrum is less compared with the contribution of protons. Suppose that proton decays into 5 Planck neutrinos. Then the energy of the Planck neutrino is \( 1/5 \) of the energy of the decayed proton. For the spectral power law index equal to \(-2.7\), the ratio of the proton flux to the Planck neutrino flux is given by \( J_p/J_\nu = 5^{1.7} = 15.4 \).

From the above consideration it is natural to identify EHE particles with the energies \( E > 3 \times 10^{18} \) eV with the Planck neutrinos. Continue the curve with the spectral power law index \(-2.7\) from the ”ankle” \( E \sim 3 \times 10^{18} \) eV to the first ”knee” \( E \sim 3 \times 10^{15} \) eV and compare the continued curve with the observational curve. Comparison gives the ratio of the proton flux to the Planck neutrino flux \( J_p/J_\nu \approx 15 \).
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