Self-interacting hidden sector dark matter and small scale galaxy structure anomalies

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The short distance behavior of dark matter (DM) at galaxy scales exhibits several features not explained by the typical WIMP DM including velocity dependence of DM cross-sections. We discuss a particle physics model with a hidden sector interacting feebly with the visible sector where a dark fermion self-interacts via a light dark photon as a mediator. We study coupled Boltzmann equations involving two temperatures, one for the visible sector and the other for the hidden sector. It is shown that a hidden sector which starts out very cold eventually thermalizes with the visible sector irrespective of the initial conditions. We fit the velocity dependence of the DM cross-section to the galaxy data consistent with relic density constraint.

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Introduction.—While the ΛCDM model works very well at large scales, several issues have arisen recently concerning WIMPS as CDM with regards to physics at galaxy scales. Some of these issues are described as the cusp-core problem, the missing satellite problem, the too-big-to-fail problem, and the diversity problem. A comprehensive review of these issues can be found in the paper by Tulin and Yu [1]. There are various suggestions on how to overcome some of these problems such as using complex dynamics and baryonic physics along with WIMP simulations [2], ultralight axions [3–5] as alternative to WIMPS and self-interacting dark matter. The last suggestion first made by Spergel and Steinhardt [6] has attracted considerable interest recently [7–20]. Galaxy data indicates that σ/m_D = O(1 cm^2/g), and further, that dark matter cross-sections have velocity dependence. Most of the analyses to fit the data use Yukawa interactions to model self-interactions. It is of interest to construct particle physics models which can explain the galaxy data along with satisfying the relic density constraint. Since the hidden sector and the visible sector in general will have different temperatures [21–23], a proper analysis of the coupled hidden and visible sectors requires study of Boltzmann equations involving temperatures of both the hidden and the visible sectors, which we carry out in the analysis below.

The model.—In this work we construct models where the dark matter particles have feeble interactions with the visible sector and are produced in the early universe by the freeze-in mechanism [24–27]. Specifically we consider an extended standard model with a hidden sector which has matter and gauge fields with a U(1)_X gauge invariance which has mixings with the visible sector U(1)_Y via gauge kinetic [28–30] and Stueckelberg mass mixings [31–35]. The relevant part of the Lagrangian of the extended model is

\[
\mathcal{L} = -\frac{1}{4} C^\mu\nu C_{\mu\nu} - g_X \bar{D} \gamma^\mu D C_{\mu} + m_D \bar{D} D - \frac{\delta}{2} C^\mu\nu B_{\mu\nu} - \frac{1}{2} (M_1 C_{\mu} + M_2 B_{\mu} + \partial_\mu \sigma)^2,
\]

where C_{\mu} is the gauge field of U(1)_X, B_{\mu} is the gauge field for the U(1)_Y, σ is an axion field which gives mass to C_{\mu}, and is absorbed in the unitary gauge, D is a Dirac fermion which is charged under U(1)_X, \delta is the kinetic mixing parameter, M_1 and M_2 are the mass parameters in the Stueckelberg mass mixing. The diagonalization of the gauge boson mass matrix along with the mass matrix arising from the spontaneous breaking of the Higgs boson in SU(2) \times U(1)_Y gives the following mass eigenstates: the photon (\gamma), the Z boson, and Z’(\gamma’). Because the mass of the third neutral boson would turn out to be in MeV region we will refer to it as a dark photon or \gamma’ which, however, is unstable and decays.

Two-temperature Boltzmann equations.—The Boltzmann equations for the number densities n_D and n_{\gamma'} depend on the two temperatures T and T_h. Thus the solution to the relic density involves three coupled equations for dn_D/dt, dn_{\gamma'}/dt and dn_{\eta}/dt, where \eta = T/T_h. In the analysis we will use the constraint that the total entropy S = sR^3 is conserved which gives ds/dt + 3H s = 0. Here s = s_v + s_h, where s_v depends on T and s_h on T_h so that

\[
s = \frac{2\pi^2}{45} (h^v_{\text{eff}} T_h^3 + h^v_{\text{eff}} T^3),
\]

where h^v_{\text{eff}} (h^h_{\text{eff}}) is the visible(hidden) effective entropy degrees of freedom. The Hubble parameter also depends on both T and T_h as can be seen from the Friedman equation

\[
H^2 = \frac{8\pi G_N}{3} (\rho_v(T) + \rho_h(T_h)),
\]
where \( \rho_v(T)(\rho_h(T_h)) \) is the energy density in the visible (hidden) sector at temperature \( T(T_h) \) and given by
\[
\rho_v = \frac{\pi^2}{30} g_{\text{eff}}^v T^4, \quad \rho_h = \frac{\pi^2}{30} g_{\text{eff}}^h T_h^4.
\]
(4)

\( g_{\text{eff}}^v, h_{\text{eff}}^v \) are functions of \( T \) and we use the fits given in [30] to parametrize them while \( g_{\text{eff}}^h, h_{\text{eff}}^h \) are functions of \( T_h \) and we use temperature dependent integrals given in [37] to parametrize them. The time evolution of \( \rho_h \) is given by
\[
d\rho_h \over dt + 3H(\rho_h + p_h) = j_h,
\]
(5)

where \( p_h \) is the pressure and \( j_h \) is the source in the hidden sector and is given in Eq. (11). We will use \( T_h \) as the reference temperature and replace \( t \) by \( T_h \) and analyze the evolution of \( n_D, n_{\gamma'} \) and \( \eta \) as a function of \( T_h \). For the computation of the relic densities it is more convenient to deal directly with yields defined by \( Y_a = n_a/s \) for a particle species \( a \) with number density \( n_a \). We assume that the dark particles \( D, \gamma' \) are feeble and there is no initial abundance and that they are initially produced only via freeze-in processes such as \( i \bar{i} \rightarrow DD, \ i \bar{i} \rightarrow \gamma' \), where \( i \) refers to standard model particles. However, \( D \) and \( \gamma' \) have interactions such as \( DD \rightarrow \gamma' \gamma' \) within the hidden sector which, in our case, are not feeble. The Boltzmann equations for the yields \( Y_D \) and \( Y_{\gamma'} \) and for \( \eta \) then take the form
\[
dY_D \over dT_h = -s \left( \frac{dp_h/dT_h}{4\rho_h - j_h/H} \right) \left[ (\sigma v)_{D \rightarrow i\bar{i}}(T)Y_D^{\text{eq}}(T)^2 \right.
\]
\[
\left. - (\sigma v)_{D \rightarrow \gamma'\gamma'}(T_h)Y_{\gamma'}^2 \right] + (\sigma v)_{i\bar{i} \rightarrow DD}(T_h)Y_D^2 \right],
\]
(6)

\[
dY_{\gamma'} \over dT_h = -s \left( \frac{dp_h/dT_h}{4\rho_h - j_h/H} \right) \left[ (\sigma v)_{D \rightarrow \gamma'\gamma'}(T_h)Y_{\gamma'}^2 \right.
\]
\[
\left. - (\sigma v)_{i\bar{i} \rightarrow \gamma'}(T_h)Y_{\gamma'} \right] + \left( \sigma v \right)_{i\bar{i} \rightarrow \gamma'}(T)Y_{\gamma'}^{\text{eq}}(T)^2 \right],
\]
(7)

\[
d\eta \over dT_h = -\frac{A_v}{B_v} + \frac{\rho_v + j_h/(4H)}{B_v} \frac{dp_h/dT_h}{B_v},
\]
(8)

where
\[
A_v = \frac{dg_{\text{eff}}^v}{dT} \eta^5 T_h^4 + 4g_{\text{eff}}^v \eta^4 T_h^3,
\]
(9)

\[
B_v = \frac{dg_{\text{eff}}^h}{dT} \eta^5 T_h^4 + 4g_{\text{eff}}^h \eta^3 T_h^4,
\]
(10)

and
\[
j_h = \sum_i \left[ 2Y_{\gamma'}^{\text{eq}}(T)^2 J(i \bar{i} \rightarrow DD)(T)
\right.
\]
\[
+ Y_{\gamma'}^{\text{eq}}(T)^2 J(i \bar{i} \rightarrow \gamma')(T) \right] s^2
\]
\[
+ Y_{\gamma'}(\gamma' \rightarrow e^+ e^-)(T_h)s,
\]
(11)

\[
Y_{\gamma'}^{\text{eq}} = \frac{n_{\gamma'}^{\text{eq}}}{s} = \frac{g_i}{2\pi^2} m_i^2 TK_2(m_i/T),
\]
(12)

where \( q_i \) is the number of degrees of freedom of particle \( i \) and mass \( m_i \). The functions \( J \) are discussed in the Appendix and \( K_2 \) is the modified Bessel function of the second kind and degree two. In Eq. (7) there are contributions one can add on the right hand side which involve processes \( i \bar{i} \rightarrow \gamma' \gamma', \gamma' Z, \gamma' \gamma' \). However, their contributions are relatively small compared to \( i \bar{i} \rightarrow \gamma' \). The relic density of \( D \) is related to \( Y_D \) by
\[
\Omega h^2 = \frac{m_D Y_D s_0 h^2}{\rho_c},
\]
(13)

where \( \rho_c \) is the critical density, \( s_0 \) is today’s entropy density and \( h = 0.678 \).

Table I: The benchmarks used in the analysis where we set \( M_2 = 0 \) and \( \delta \) is in units of \( 10^{-9} \).

| Model | \( m_D \) (GeV) | \( M_1 \) (MeV) | \( g_X \) | \( \delta \) |
|-------|----------------|----------------|--------|--------|
| (a)   | 1.50           | 1.20           | 0.016  | 25     |
| (b)   | 2.0            | 1.22           | 0.014  | 4.5    |
| (c)   | 2.16           | 1.13           | 0.015  | 5.5    |
| (d)   | 3.2            | 1.77           | 0.018  | 5.0    |
| (e)   | 3.26           | 1.99           | 0.018  | 5.0    |
| (f)   | 4.0            | 2.20           | 0.020  | 4.0    |

**Dark freeze-out, relic density, and fits to galaxy data.** —We now give a numerical analysis based on the above. In Table I we give a set of six benchmarks which satisfy the relic density constraint and where the dark photon decays before the BBN. The values of \( \sigma/m \) at low velocities for these model points lie in the range \( 1.2-3.7 \) \( \text{cm}^2/\text{g} \) which are needed to explain the short distance structure of dark matter at galaxy scales. In Fig. 1 we exhibit the dark freeze-out where the decoupling between the dark photon and the dark fermion, i.e. \( n_D(T_h)(\sigma v)_{D \rightarrow \gamma' \gamma'}(T_h) \sim H(T) \) occurs for values of \( x = T_h/m_D \sim \mathcal{O}(1-7) \times 10^{-3} \) exhibited by the knee in the lower part of the plot. In the top panel of Fig. 2 we exhibit the phenomenon of thermalization of the hidden sector for one model point. Here one finds that starting with different initial conditions on \( \xi \equiv \eta^{-1} \) at some high temperature, one ends up with \( \xi = 1, \ i.e., T_h = T \) at low temperatures. In the bottom panel of Fig. 2 we give a plot of the yields \( Y_D \) and \( Y_{\gamma'} \) as a function of \( T_h \) for the six parameter points of Table I. Here one finds that the dark photon yield has a precipitous fall at low temperature due to its decay and does not contribute to the relic density. In the top panel of Fig. 3 we give a plot of \( \langle \sigma v \rangle/m_D \) where
FIG. 1: A display of dark freeze-out showing a plot of $n_D\langle\sigma v\rangle_{D\bar{D}\to\gamma\gamma'}$ (solid line) and $H(T)$ (dashed line) versus $T_h$ for three benchmarks of Table I. $\sigma$ refers to self-interaction cross-section and $v$ is the Moller velocity. The theory curves are for six model points of Table I using THINGS and LSB galaxies and clusters’ data taken from [38], showing that the models can produce the observed velocity dependence of dark matter cross-sections. Finally, in the bottom panel of Fig. 3 we exhibit the spin-independent p-DM cross-section as a function of the dark matter mass $m_D$ where the current limits from CDMSlite R3 [39], DarkSide-50 [40] and PandaX-II [41] are also exhibited. One finds that the model points are consistent with the current limits including CMB constraints [42] and can be explored in future improved experiments.

Conclusion.—New analytic results of this work are the three coupled equations defined by Eqs. (6) – (12) which allow one to solve the Boltzmann equations for the relic density of dark matter where the evolution depends on two temperatures, one for the hidden and the other for the visible sector. It is then seen that one must simultaneously evolve the ratio $\eta = T/T_h$ consistently to solve for the relic density. The analysis shows that thermalization of the hidden sector eventually occurs, i.e., $T_h \rightarrow T$ independent of the initial value of $T/T_h$. The hidden sector model we consider consists of a dark fermion $D$ and a dark photon $\gamma'$ as mediator where the dark photon is unstable and decays before BBN. We present a set of model points which satisfy the relic density constraint and their self-interactions produce velocity dependence of dark matter cross-sections observed in galaxy data. The model points can be tested in future direct detection experiments via the spin-independent p-DM cross-sections.

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Appendix.—The $J$-functions that appear in Eq. (11) are defined as

\begin{align}
n_i^{eq}(T)^2 J(i \bar{i} \rightarrow D \bar{D})(T) & = \frac{T}{64\pi^4} \int_{s_0}^{\infty} ds \sigma_{D\bar{D}\rightarrow i\bar{i}}s(s-s_0)K_2(\sqrt{s}/T), \\
n_i^{eq}(T)^2 J(i \bar{i} \rightarrow \gamma)(T) & = \frac{T}{32\pi^4} \int_{s_0}^{\infty} ds \sigma_{i\bar{i}\rightarrow \gamma}s(s-s_0)K_2(\sqrt{s}/T),
\end{align}
\[ n_{\gamma'}J(\gamma' \rightarrow e^+e^-)(T_h) = n_{\gamma'}m_{\gamma'}\Gamma_{\gamma' \rightarrow e^+e^-}, \]

and

\[
n_{\gamma'}^{\text{eq}}(T)^2\langle \sigma v \rangle_{\bar{\gamma} \rightarrow \gamma'}(T) = \frac{T}{32\pi^4} \int_{s_0}^{\infty} ds \sigma(s)\sqrt{s} (s-s_0)K_1(\sqrt{s}/T), \]

where \( K_1 \) is the modified Bessel function of the second kind and degree one and \( s_0 \) is the minimum of the Mandelstam variable \( s \). The self-interaction cross-sections for \( D\bar{D} \rightarrow D\bar{D} \), \( DD \rightarrow DD \), and \( D\bar{D} \rightarrow D\bar{D} \) are given by

\[
\frac{d\sigma}{d\Omega} = 3 \frac{|M_1|^2}{64\pi^2 s},
\]

where for \( D\bar{D} \rightarrow D\bar{D} \)

\[
|M_1|^2 = 2g_X^4 \left\{ t^2 + u^2 + 8m_D^2s - 8m_D^4 \right\} \frac{s^2 + u^2 - 8m_D^2(s + u) + 24m_D^4}{(t - m_{\gamma'})^2}
+ \frac{t^2 + s^2 - 8m_D^2(s + t) + 24m_D^4}{(u - m_{\gamma'})^2}
+ \frac{2[m_{\gamma'}^4 - m_{\gamma'}^2(u + t) + ut + \Gamma_{\gamma'}^2m_{\gamma'}^2]{(m_{\gamma'}^4 - m_{\gamma'}^2)(u + t) + ut}}{(t - m_{\gamma'})^2}
\times (s^2 - 8m_D^2s + 12m_D^4),
\]

For \( DD \rightarrow DD \)

\[
|M_2|^2 = 2g_X^4 \left\{ s^2 + u^2 - 8m_D^2(s + u) + 24m_D^4 \right\} \frac{t^2 + s^2 - 8m_D^2(s + t) + 24m_D^4}{(t - m_{\gamma'})^2}
+ \frac{t^2 + s^2 - 8m_D^2(s + t) + 24m_D^4}{(u - m_{\gamma'})^2}
+ \frac{2[m_{\gamma'}^4 - m_{\gamma'}^2(u + t) + ut + \Gamma_{\gamma'}^2m_{\gamma'}^2]{(m_{\gamma'}^4 - m_{\gamma'}^2)(u + t) + ut}}{(t - m_{\gamma'})^2}
\times (s^2 - 8m_D^2s + 12m_D^4),
\]

where \( s, t, u \) are the Mandelstam variables. For \( D\bar{D} \rightarrow D\bar{D} \), \( |M_3|^2 = |M_2|^2 \). The cross-section for the process \( D\bar{D} \rightarrow \gamma'\gamma' \) is given by

\[
\sigma_{D\bar{D} \rightarrow \gamma'\gamma'}(s) = \frac{g_X^4}{8\pi s} \left\{ \frac{\mathcal{R}_{11} - s}\mathcal{R}_{21} \right\}^4
\times \left\{ \frac{\sqrt{(s - 4m_{\gamma'}^2)(s - 4m_D^2)}}{m_{\gamma'}^4 + m_D^4(s - 4m_{\gamma'}^2)}[2m_{\gamma'}^4 + m_D^4(s + 4m_{\gamma'}^2)]\right\}
+ \frac{\log A}{s - 2m_{\gamma'}^2}(s^2 + 4m_D^4s + 4m_{\gamma'}^4 - 8m_D^4 - 8m_{\gamma'}^2m_{\gamma'}^2),
\]

with

\[
A = \frac{s - 2m_{\gamma'}^2 + \sqrt{(s - 4m_D^2)(s - 4m_{\gamma'}^2)}}{s - 2m_{\gamma'}^2 - \sqrt{(s - 4m_D^2)(s - 4m_{\gamma'}^2)}}.
\]

Here \( \mathcal{R}_{11} \) and \( \mathcal{R}_{21} \) are matrix elements of \( \mathcal{R} \) which diagonalizes the mass and kinetic energy matrices as given in [33]. When kinematically allowed the process \( \gamma'\gamma' \rightarrow D\bar{D} \) is given by

\[
8(s - 4m_D^2)\sigma_{\gamma'\gamma' \rightarrow D\bar{D}}(s) = 9(s - m_{\gamma'}^2)\sigma_{D\bar{D} \rightarrow \gamma'\gamma'}(s).
\]

FIG. 3: Top panel: Fits to the galaxy data taken from [38] where \( \langle \sigma v \rangle \) is plotted vs \( \langle v \rangle \) in the halo using self-interacting dark matter cross-section for the six model points of Table I. Bottom panel: the spin-independent proton-DM scattering cross-sections for the six benchmarks of Table I calculated using micrOMEGAs 5.0 [44] with model files generated by SARAH [45, 46]. Also shown are the current exclusion limits from CDMSlite R3, DarkSide-50 and PandaX-II.
