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Principal component analysis to study the relations between the spread rates of COVID-19 in high risks countries

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Abstract In this research, the number of patients with Covid-19 and the number of deaths due to this disease in France, Germany, Iran, Italy, Spain, United Kingdom, and Unites States America are considered. First, the relations between the considered countries are studied using Pearson’s correlation. Then, based on the spread rate of Covid-19, these countries are categorized using principal component analysis.

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1. Background

In the last months of 2019, the scientist reported a new type of coronavirus, called 2019-nCoV (or Covid-19), in Wuhan [1]. It is a dangerous virus that causes severe damage to the body’s respiratory systems. There are many scientific researches to find the main source of the Covid-19. Since January until now (18 April 2020), this epidemic changed to pandemic in
the whole of the world and the number of patients with this disease and the number of deaths due to this disease is increasing day by day in most of countries, particularly in France [2–5], Germany [6], Iran [7], Italy [8–10], Spain [11], United Kingdom [12–14], and Unites States America [15]. The modeling of the different natural phenomena were studied using different data analysis techniques, such as statistical and mathematical modeling containing time series analysis, regression modeling, optimization and numerical analysis [16–40], and artificial intelligence modeling containing deep learning, machine learning and clustering [41–46]. Because of different social, economic and environmental effects of Covid-19, it is critical to study and compare the spread rate of this disease in different countries.

In this research, the number of patients with Covid-19 and the number of deaths due to this disease in France, Germany, Iran, Italy, Spain, United Kingdom, and Unites States America are considered. First, the relations between the considered countries are studied using Pearson’s correlation. Then, based on the spread rate of Covid-19, these countries are categorized using principal component analysis.

2. Material and method

In this section, the description about the dataset of the study and the introduction of principal component analysis are presented.

2.1. Dataset

The dataset of this research contains the number of patients with Covid-19 and the number of deaths due to this disease in France, Germany, Iran, Italy, Spain, United Kingdom, and Unites States America from 22 February 2020 until 18 April 2020. The descriptive statistics and the plot of the considered dataset are summarized in Table 1 and Fig. 1, respectively.

The results indicated that Unites States America and Iran have the maximum and the minimum of the daily mean of patients with Covid-19, respectively. Also, Unites States America and Germany have the maximum and the minimum of the daily mean of deaths, respectively.

Table 2 summarizes the values of the Pearson’s correlation to study the relations between the considered countries, based on the number of patients with Covid-19 and the number of deaths due to this disease. It be noticed that all of the values are significant in level of 0.001 (P-Value < 0.001).

Since all Pearson’s correlations are significant and more than 0.5, it can be concluded that there are extreme positive relations between the considered countries, based on the number of patients with Covid-19 and the number of deaths due to this disease.

2.2. Principal component analysis

Principal component analysis (PCA, in abbreviation) is a famous multivariate approach that converts several correlated variables into several linearly uncorrelated variables named principal components. In this conversion, the first principal components contain the most information about the dataset [47]. In applications, PCA is applied to transform a high-dimensional dataset to a lower-dimensional dataset, by using only the first few principal components so that the dimensionality of the transformed data is reduced. Based on the Kaiser Index, the number of the important principal components is equal to the number of the eigenvalues of the correlation matrix with values larger than 1.

PCA is the method of finding a new coordinate system so that the information of the data is mainly concentrated in several coordinates, the rest only carries a small amount of information. Simplify, PCA will find an orthonormal basis to be the new base.

Suppose the new orthonormal basis is $U$, each column of $U$ is a one-dimensional unit vector and we want to retain the $K$ coordinates of this system. Don’t lose generality assuming it’s the first $K$ component. The new base $U = [U_K, \bar{U}_K]$ is an orthonormal basis. Where, $U_K$ is a sub-matrix created by the first $K$ columns of $U$, the data matrix is written

$$X = U_KZ + \bar{U}_KY.$$ 

Then

$$[Z^T, Y^T] = [U_K^T, \bar{U}_K^T]X.$$ 

Its mean

$$Z = U_K^TX,$$

and

$$Y = \bar{U}_K^TX.$$ 

The purpose of PCA is to find the orthonormal matrix $U$ so that most of the information is retained, $\bar{U}_K^T$ omitted and replaced by a matrix regardless of the data point. Specifically, we will approximate $Y$ by a matrix with the same of all columns. Let this column is $b$ and it as bias, then we will approximate $Y \approx b1^T$ with $1^T \in R^{1 \times N}$ is a row vector that has all elements equal to 1. Suppose to find $U$, we need to find $b$ satisfied:

| Table 1 | Descriptive statistics about the number of patients with Covid-19 and the number of deaths. |
|---------|-------------------------------------------------|
|         | Country         | Patients (Mean ± Standard Deviation) | Deaths (Mean ± Standard Deviation) |
|         | France          | 1851.5 ± 1876.0 | 316.6 ± 447.3 |
|         | Germany         | 2329.2 ± 2245.2 | 69.7 ± 94.9 |
|         | Iran            | 1294.5 ± 921.5  | 80.7 ± 59.6  |
|         | Italy           | 2922.6 ± 2056.1 | 385.5 ± 309.5 |
|         | Spain           | 3187.6 ± 3016.8 | 330.1 ± 340.5 |
|         | United Kingdom  | 2922.6 ± 2056.1 | 247.1 ± 342.0 |
|         | Unites States America | 11900.8 ± 13327.5 | 628.0 ± 1013.4 |
\[ b = \arg \min_b \| Y - b^1 \|_F^2 = \arg \min_b \| \hat{U}_k^T X - b^1 \|_F^2. \]

Solving the derivative equation according to \( b \) of the objective function, that is 0:

\[
\left( b^1^T - \hat{U}_k^T X \right) 1 = 0 \Rightarrow Nb = \hat{U}_k^T X 1 \Rightarrow b = \hat{U}_k^T \bar{x},
\]

where, \( 1^T 1 = N \) and \( \bar{x} = 1^T X 1 \) is the means vector of all columns in \( X \).

We have, the initial data is approximated by

\[ X = U_k Z + \hat{U}_k Y \approx U_k Z + \hat{U}_k b^1^T \]

\[ = U_k Z + \hat{U}_k \hat{U}_k^T \bar{x} \bar{x}^T = X. \]

PCA is a problem to find orthonormal matrix \( U \) so that approximation is the best. Assume that the means vector \( \bar{x} = 0 \). Then,

\[ X \approx X = U_k Z. \]

The optimal problem of PCA will become

\[ U_k Z = \arg \min_{U_k Z} \| X - U_k Z \|_F \]

satisfied \( U_k^T U_k = I_k \) with \( I_k \in \mathbb{R}^{k \times k} \) is the unit matrix in \( K \)-dimensional space and this condition to ensure \( U_k \) is an orthonormal basis.

**Fig. 1** Frequency of patients with Covid-19 (Top and Left), frequency of deaths (Top and Right), cumulative frequency of patients with Covid-19 (Bottom and Left), and cumulative frequency of deaths (Bottom and Right).
The algorithm
The following is the algorithm of PCA:

Step 1. Calculate the means vector of all data
\[ \bar{x} = \frac{1}{N} \sum_{n=1}^{N} x_n \]

Step 2. Subtract the means vector from each data point of all data: \( \hat{x}_n = x_n - \bar{x} \).

Step 3. Let \( \hat{X} = [\hat{x}_1, \hat{x}_2, ..., \hat{x}_D] \) is an orthonormal data matrix. We have the covariance matrix
\[ S = \frac{1}{N} \hat{X} \hat{X}^T. \]

Step 4. Calculate eigenvalues and eigenvectors of the covariance (or correlation) matrix, arrange them in descending order of eigenvalues.

Step 5. Select \( K \) eigenvectors corresponding to the \( K \) largest eigenvalues to construct the \( U_K \) matrix with columns forming an orthogonal system. These \( K \) vectors, also known as the main components, form a subspace that close to the orthonormal data matrix.

Step 6. Make a projection the orthonormal data matrix to the subspace, that be found.

Step 7. The new data is the coordinates of the data points on the new space
\[ Z = U_k^T \hat{X}. \]

The original data can be approximated by the new data as follows:
\[ x \approx U_k Z + \bar{x}. \]

3. Results

3.1. PCA for number of patients

The results of PCA method for classification of countries, based on the number of patients, are provided in Fig. 2. Based on Kaiser Index and Fig. 2, there are significant differences between the number of patients in counties and these countries can be classified in following two groups:

First group: Italy, Iran, Germany, Spain and France.

Second group: Unites States America and United Kingdom.

3.2. PCA for number of deaths

The results of PCA method for classification of countries, based on the number of deaths, are provided in Fig. 3. Based on Kaiser Index and Fig. 3, there are significant differences between the number of deaths in counties and these countries can be classified in following two groups:

First group: Unites States America, United Kingdom, Germany and France.

### Table 2  Pearson’s correlation between different countries, based on the number of patients with Covid-19 and the number of deaths in different.

|                  | Unites States America | Spain | Italy | Germany | United Kingdom | France | Iran |
|-------------------|-----------------------|-------|-------|---------|----------------|-------|------|
| Patients          | 1                     | 0.666 | 0.567 | 0.942   | 0.654          | 0.673 | 0.718|
| Spain             | 0.666                 | 1     | 0.842 | 0.514   | 0.879          | 0.766 | 0.848|
| Italy             | 0.567                 | 0.842 | 1     | 0.514   | 0.879          | 0.766 | 0.848|
| Germany           | 0.942                 | 0.562 | 0.514 | 1       | 0.550          | 0.586 | 0.654|
| United Kingdom    | 0.654                 | 0.929 | 0.879 | 0.550   | 1              | 0.856 | 0.866|
| France            | 0.673                 | 0.851 | 0.766 | 0.586   | 0.856          | 1     | 0.850|
| Iran              | 0.718                 | 0.884 | 0.848 | 0.654   | 0.866          | 0.850 | 1    |

|                  | Unites States America | Spain | Italy | Germany | United Kingdom | France | Iran |
|-------------------|-----------------------|-------|-------|---------|----------------|-------|------|
| Deaths            | 1                     | 0.627 | 0.508 | 0.885   | 0.851          | 0.850 | 1    |
| Spain             | 0.627                 | 1     | 0.892 | 0.764   | 0.743          | 0.815 | 0.794|
| Italy             | 0.508                 | 0.892 | 1     | 0.629   | 0.626          | 0.677 | 0.934|
| Germany           | 0.885                 | 0.764 | 0.629 | 1       | 0.927          | 0.802 | 0.565|
| United Kingdom    | 0.889                 | 0.743 | 0.626 | 0.927   | 1              | 0.804 | 0.555|
| France            | 0.716                 | 0.815 | 0.677 | 0.802   | 0.804          | 1     | 0.621|
| Iran              | 0.449                 | 0.794 | 0.934 | 0.565   | 0.555          | 0.621 | 1    |

|                  | Unites States America | Spain | Italy | Germany | United Kingdom | France | Iran |
|-------------------|-----------------------|-------|-------|---------|----------------|-------|------|
| Cumulative Patients| 1                     | 0.966 | 0.947 | 0.999   | 0.963          | 0.971 | 0.959|
| Spain             | 0.966                 | 1     | 0.991 | 0.956   | 1.000          | 1.000 | 0.994|
| Italy             | 0.947                 | 0.991 | 1     | 0.935   | 0.994          | 0.991 | 0.998|
| Germany           | 0.999                 | 0.956 | 0.935 | 1       | 0.952          | 0.961 | 0.949|
| United Kingdom    | 0.963                 | 1.000 | 0.994 | 0.952   | 1              | 0.999 | 0.996|
| France            | 0.971                 | 1.000 | 0.991 | 0.961   | 0.999          | 1     | 0.995|
| Iran              | 0.959                 | 0.994 | 0.998 | 0.949   | 0.996          | 0.995 | 1    |

|                  | Unites States America | Spain | Italy | Germany | United Kingdom | France | Iran |
|-------------------|-----------------------|-------|-------|---------|----------------|-------|------|
| Cumulative Deaths | 1                     | 0.953 | 0.925 | 0.996   | 0.996          | 0.976 | 0.909|
| Spain             | 0.953                 | 1     | 0.992 | 0.974   | 0.976          | 0.993 | 0.983|
| Italy             | 0.925                 | 0.992 | 1     | 0.949   | 0.952          | 0.974 | 0.998|
| Germany           | 0.996                 | 0.974 | 0.949 | 1       | 1.000          | 0.991 | 0.934|
| United Kingdom    | 0.996                 | 0.976 | 0.952 | 1.000   | 1              | 0.992 | 0.937|
| France            | 0.976                 | 0.993 | 0.974 | 0.991   | 0.992          | 1     | 0.961|
| Iran              | 0.909                 | 0.983 | 0.998 | 0.934   | 0.937          | 0.961 | 1    |
Second group: Spain, Italy and Iran.

3.3. PCA for cumulative number of patients

The results of PCA method for classification of countries, based on the cumulative number of patients, are provided in Fig. 4. Based on Kaiser Index and Fig. 4, there are significant differences between the cumulative number of patients in counties and these countries can be classified in following two groups:

First group: United Kingdom and United States America.
Second group: France, Spain, Germany, Iran and Italy.

3.4. PCA for cumulative number of deaths

The results of PCA method for classification of countries, based on the cumulative number of deaths, are provided in Fig. 5. Based on Kaiser Index and Fig. 5, there are significant differences between the cumulative number of deaths in countries and these countries can be classified in following two groups:

First group: Unites States America, United Kingdom, Germany and France.
Second group: Spain, Italy and Iran.
4. Conclusion

Because of different social, economic and environmental effects of Covid-19, it is critical to study and compare the spread rate of this disease in different countries. In this research, the number of patients with Covid-19 and the number of deaths due to this disease in France, Germany, Iran, Italy, Spain, United Kingdom, and United States America were considered. First, the relations between the considered countries were studied using Pearson’s correlation. The results indicated that there were extreme positive relations between the considered countries, based on the number of patients and the number of deaths due to this disease. Then, based on the spread rate of Covid-19, these countries were categorized using principal component analysis. The results indicated that, for the number of patients, the distribution of spreading in United States America and United Kingdom was similar to each other and differed from other countries. Also, for the number of deaths, the distribution of spreading in Spain, Italy and Iran was similar to each other and differed from other countries. Moreover, the distributions of spreading in other countries were similar. The authors suggest the researchers to consider the more countries and to classify them based on PCA or other methods such as factor analysis.

CRediT authorship contribution statement

Mohammad Reza Mahmoudi: Validation, Methodology. Mohammad Hossein Heydari: Supervision, Software. Sultan
Noman Qasem: Visualization, Validation. Amirhossein Mosavi: Visualization, Validation. S. Band: Visualization, Validation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] R. Lu, X. Zhao, J. Li, P. Niu, B. Yang, H. Wu, W. Wang, H. Song, B. Huang, N. Zhu, Y. Bi, Genomic characterisation and epidemiology of 2019 novel coronavirus: implications for virus origins and receptor binding, Lancet 395 (10224) (2020) 565–574.
[2] D. Fanelli, F. Piazza, Analysis and forecast of COVID-19 spreading in China, Italy and France, Chaos Solitons Fractals 134 (2020) 109761.
[3] P. Gautret, J.C. Lagier, P. Parola, L. Meddeb, V. Giordano, V.E. Vieira, H.T. Dupont, S. Honére, Hydroxychloroquine and azithromycin as a treatment of COVID-19: results of an open-label non-randomized clinical trial, Int. J. Antimicrob. Agents 20 (2020) 105949.
[4] S.B. Stocklin, P. Rolland, Y. Silue, A. Mailles, C. Campese, A. Simondon, M. Mechin, L. Meurice, M. Nguyen, C. Bassi, E. Yamani, First cases of coronavirus disease 2019 (COVID-19) in France: surveillance, investigations and ambulatory management of suspected COVID-19 cases presenting at the infectious diseases referral hospital in Marseille, France, January 31st to March 1st, 2020: A respiratory virus snapshot, Travel Medicine and Infectious Disease 2020. 101632.
[5] A. Remuzzi, G. Remuzzi, COVID-19 and Italy: what next?, Lancet (2020).
[6] G. Onder, G. Rezza, S. Brusaferro, Case-fatality rate and characteristics of patients dying in relation to COVID-19 in Italy, JAMA (2020).
[7] M. Lazzerini, G. Putoto, COVID-19 in Italy: momentous decisions and many uncertainties, The Lancet, Global Health. (2020).
[8] H. Legido-Quigley, J.T. Mateos-Garcia, V.R. Campos, M. Gea-Sánchez, C. Munian, M. McKee, The resilience of the Spanish health system against the COVID-19 pandemic, The Lancet, Public Health. (2020).
[9] P.J. Lillie, A. Samson, A. Li, K. Adams, R. Capstick, G.D. Barlow, N. Easom, E. Hamilton, P.J. Moss, A. Evans, M. Ivan, Novel coronavirus disease (COVID-19): the first two patients in the UK with person to person transmission, J. Infect. (2020).
[10] M.S. Razai, K. Doehr, S. Ladhani, P. Oakeshott, Coronavirus disease 2019 (COVID-19): a guide for UK GPs, BMJ 6 (2020) 368.
[11] D.J. Hunter, Covid-19 and the stiff upper lip—the pandemic response in the united kingdom, N. Engl. J. Med. (2020).
[12] J.C. Rapid viral diagnosis and ambulatory management of suspected COVID-19 cases presenting at the infectious diseases referral hospital in Marseille, France, January 31st to March 1st, 2020: A respiratory virus snapshot, Travel Medicine and Infectious Disease 2020. 101632.
[13] R.M. Burke, C.M. Midgley, A. Dratch, et al, Active Monitoring of Persons Exposed to Patients with Confirmed COVID-19 — United States, January–February 2020, MMWR Morb. Mortal. Wkly. Rep. 69 (2020) 245–246, https://doi.org/10.15585/mmwr.mm6909e1.
[14] H. Haghbin, M.R. Mahmoudi, Z. Shishebor, Large Sample Inference on the Ratio of Two Independent Binomial Proportions, J. Math. Ext. 5 (1) (2011) 87–95.
[15] M.R. Mahmoudi, B. Behboodian, M. Maleki, Inference on the Ratio of Means in Two Independent Populations, J. Stat. Theory Appl. 16 (3) (2017) 366–374.
[16] M.R. Mahmoudi, M. Maleki, Inference on the Ratio of Variances of Two Independent Populations, J. Math. Ext. 7 (2) (2014) 83–91.
[17] M.H. Heydari, M. Mahmoudi, Inference on the Ratio of Correlations of Two Independent Populations, J. Math. Ext. 7 (4) (2014) 71–82.
[18] M.R. Mahmoudi, R. Nasirzadeh, M. Mohammadi, On the Ratio of Two Independent Skewnesses, Commun. Stat-Theor. Meth. 48 (7) (2019) 1721–1727.
[19] M.R. Mahmoudi, M. Maleki, A. Pak, Testing the Difference between Two Independent Time Series Models, Iran J. Sci. Technol. A. Sci. 41 (2017) 665–669.
[20] M.R. Mahmoudi, M.H. Heydari, R. Roohi, A new method to compare the spectral densities of two independent periodically correlated time series, Math. Comput. Simulat. 160 (2019) 103–110.
[21] M.R. Mahmoudi, M.H. Heydari, Z. Avazzadeh, Testing the difference between spectral densities of two independent periodically correlated (cyclostationary) time series models, Commun. Stat. Theory Meth. 48 (9) (2019) 2320–2328.
[22] M.R. Mahmoudi, M. Maleki, E. Nahavandi, Testing the Difference between Two Independent Regression Models, Commun. Stat. Theory Meth. 45 (21) (2016) 6284–6289.
[23] M.R. Mahmoudi, M. Maleki, A. Pak, Testing the Equality of Two Independent Regression Models, Commun. Stat. Theory Meth. 47 (12) (2018) 2919–2926.
[24] M.R. Mahmoudi, On Comparing Two Dependent Linear and Nonlinear Regression Models, J. Test Eval. 47 (1) (2018) 449–458.
[25] P. Ji-jun, M.R. Mahmoudi, D. Baleanu, M. Maleki, On Comparing and Classifying Several Independent Linear and Non-Linear Regression Models with Symmetric Errors, Symmetry 11 (6) (2019) 820.
[26] M. Bahrami, M.J. Amiri, M.R. Mahmoudi, S. Koochaki, Modeling caffeine adsorption by multi-walled carbon nanotubes using multiple polynomial regression with interaction effects, J. Water Health 15 (4) (2017) 526–535.
[27] A.R. Zarei, M.R. Mahmoudi, Evaluation of changes in RDIst index effected by different Potential Evapotranspiration calculation methods, Water Resour. Manag. 31 (15) (2017) 4981–4999.
[28] M.H. Heydari, Z. Avazzadeh, M.R. Mahmoudi, Chebyshev cardinal wavelets for nonlinear stochastic differential equations driven with variable-order fractional Brownian motion, Chaos, Solitons Fractals 124 (2019) 105–124.
[29] M.R. Mahmoudi, M. Maleki, A New Method to Detect Periodically Correlated Structure, Comput. Statistics 32 (4) (2017) 1569–1581.
[30] A.R. Nematollahi, A.R. Soltani, M.R. Mahmoudi, Periodically Correlated Modeling by Means of the Periodograms Asymptotic Distributions, J. Math. Ext. 8 (4) (2014) 1267–1278.
[31] M.R. Mahmoudi, M.H. Heydari, Z. Avazzadeh, On the Asymmetric Distribution for the Periodograms of Almost Periodically Correlated (Cyclostationary) Processes, Digital Signal Process. 81 (2018) 186–197.
[34] M.R. Mahmoudi, M.H. Heydari, Z. Avazzadeh, K.H. Pho, Goodness of fit test for almost cyclostationary processes, Digital Signal Process. 96 (2020) 102597.

[35] M.R. Mahmoudi, M. Maleki, K. Borodin, K.H. Pho, D. Baleanu, On comparing and clustering the spectral densities of several almost cyclostationary processes, Alexandria Eng. J. 59 (4) (2020) 2555–2565.

[36] Zhou, R., Mahmoudi, M. R., Mohammed, S. N. Q., & Pho, K. H., 2020. Testing the equality of the spectral densities of several uncorrelated almost cyclostationary processes. Alexandria Engineering Journal, 59 (2020) 3545–3550.

[37] M.R. Mahmoudi, D. Baleanu, B.A. Tuan, K.H. Pho, A novel method to detect almost cyclostationary structure, Alexandria Eng. J. 59 (4) (2020) 2339–2346.

[38] R. Roohi, M.H. Heydari, M. Aslami, M.R. Mahmoudi, A comprehensive numerical study of space-time fractional bioheat equation using fractional-order Legendre functions, The European Phys. J. Plus 133 (2018) 412.

[39] M.R. Mahmoudi, A.R. Nematollahi, A.R. Soltani, On the detection and estimation of the simple harmonizable processes, Iranian J. Sci. Technol. (Sci.) 39 (2) (2015) 239–242.

[40] M. Maleki, M.R. Mahmoudi, D. Wraith, K.H. Pho, Time series modelling to forecast the confirmed and recovered cases of COVID-19, Travel Med. Infect. Dis. 101742 (2020).

[41] S.M.J. Jalali, M. Karimi, A. Khosravi, S. Nahavandi, in: October). An efficient neuroevolution approach for heart disease detection, IEEE, 2019, pp. 3771–3776.

[42] Jalali, S. M. J., Khoasrvi, A., Alizadehsani, R., Jalali, S. M. J., Khoasrvi, A., Alizadehsani, R., Salaken, S. M., Kebria, P. M., Puri, R., & Nahavandi, S., 2019. Parsimonious Evolutionary-based Model Development for Detecting Artery Disease. In ICIT (pp. 800-805).

[43] S.M.J. Jalali, S. Ahmadian, A. Khoasrvi, S. Mirjalili, M.R. Mahmoudi, S. Nahavandi, Neuroevolution-based Autonomous Robot Navigation: A Comparative Study, Cognit. Syst. Res. (2020).

[44] S.J. Mousavirad, G. Schaefer, S.M.J. Jalali, I. Korovin, July). A benchmark of recent population-based metaheuristic algorithms for multi-layer neural network training, in: In Proceedings of the 2020 Genetic and Evolutionary Computation Conference Companion, 2020, pp. 1402–1408.

[45] S.M.J. Jalali, S. Ahmadian, P.M. Kebria, A. Khoasrvi, C.P. Lim, S. Nahavandi, in: December). Evolving artificial neural networks using butterfly optimization algorithm for data classification, Springer, Cham, 2019, pp. 596–607.

[46] A.R. Abbasi, M.R. Mahmoudi, Z. Avazzadeh, Diagnosis and clustering of power transformer winding fault types by cross-correlation and clustering analysis of FRA results, IET Gener. Transm. Distri. 12 (19) (2018) 4301–4309.

[47] R.A. Johnson, D. Wichern, Multivariate Analysis, John Wiley & Sons, Ltd, 2002.