Detecting baryon acoustic oscillations by 3d weak lensing

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ABSTRACT

We investigate the possibility of detecting baryon acoustic oscillation features in the cosmic matter distribution by 3d weak lensing. Constraints on baryon oscillations coming from tomography are expected to be rather weak, due to wide line-of-sight weighting functions and therefore a specialized approach via 3D shear estimates is required. We quantify the uncertainty of estimating the matter spectrum amplitude at the baryon oscillations wavevectors by a Fisher matrix approach with a fixed cosmology and show in this way that future weak lensing surveys such as Euclid and dark energy survey are able to pick up the first four and three wiggles, respectively, with Euclid overall giving a better precision in the measurement. We also provide a detailed investigation of the correlation existing between errors and of their scaling behaviour with respect to survey parameters such as median redshift, error on redshift, error on the galaxy shape measurement, sky coverage and finally with respect to the number of wiggles one is trying to determine.

Key words: methods: analytical.

1 INTRODUCTION

Baryon acoustic oscillation (BAO) features are modulations in the cosmic matter distribution on very large spatial scales of roughly \(100 \text{ Mpc} h^{-1}\) (for a review, see Bassett & Hlozek 2010). These BAOs are the imprint of oscillations of the photon–baryon fluid in the early universe on the matter density field driven by gravity and the equation of state providing a restoring force, and they are observable in two primary channels: through the observation of anisotropies in the cosmic microwave background (CMB) and through galaxy surveys. The most important features such as their spatial scales, their signature in the CMB, their statistical properties and their dependence on cosmological parameters is very well understood analytically (Hu & Sugiyama 1996; Seljak & Zaldarriaga 1996; Montanari & Durrer 2011; Sutherland 2012).

Concerning the determination of cosmological parameters, it is a fortunate situation that they are observable at high redshifts through the primary CMB and at much lower redshifts in the galaxy distribution. Due to the fact that BAOs provide a standard yardstick at two different cosmological epochs, it is possible to constrain the density parameters of cosmic fluids and the possible time evolution of their equation-of-state parameters in a geometric way, breaking degeneracies that may arise if the equations of state of cosmological fluids are allowed to change with time.

BAO observations carried out by the Cosmic Background Explorer (Bennett et al. 1994; Wright et al. 1996) first revealed anisotropies in the CMB, but only the Wilkinson Microwave Anisotropy Probe (Hinshaw et al. 2003, 2007; Nolta et al. 2009; Larson et al. 2011) had sufficient angular resolution such that the BAO scale of \(100 \text{ Mpc} h^{-1}\) could be resolved at a comoving distance of \(10 \text{ Gpc} h^{-1}\), revealing temperature modulations of the CMB of the order of \(\Delta T/T_{\text{CMB}} \approx 10^{-5}\) at an angular scale of roughly 2°, with subsequent higher harmonics. Likewise, galaxy surveys have now reached sufficient depth and solid angle that BAO features could be detected as modulations of the galaxy density of the order of 10 per cent in both radial and transverse directions. With the assumption of a galaxy biasing model, the longest wavelength BAO modes survive non-linear structure formation to the present epoch (Meiksin, White & Peacock 1999) and will be targeted by future surveys for the precision determination of cosmological parameters (Dolney, Jain & Takada 2006; Angulo et al. 2008; Labatie, Starck & Lachièze-Rey 2012), in particular dark energy (Seo & Eisenstein 2003; Eisenstein et al. 2007). Both avenues have contributed significantly to the estimation of cosmological parameters and to the selection of most plausible cosmological models.

Specifically, there are quite a number of detection reports with ongoing surveys, for instance with the Sloan Digital Sky Survey (Eisenstein et al. 2005; Padmanabhan et al. 2007, 2012; Percival et al. 2007, 2010; Kazin et al. 2010a; Mehta et al. 2012), the 2-Degree Field Galaxy Redshift Survey (Percival et al. 2007; Beutler et al. 2011), the WiggleZ Survey (Parkinson et al. 2012) and Lyman \(\alpha\) data (Busca et al. 2012) with subsequent determination of cosmological parameters which confirm spatial flatness and the low matter density found by CMB observations, if flatness is assumed prior to the analysis. Recent studies (Parejko et al. 2012; Zhao et al. 2013) were able to constrain neutrino masses. BAO modulations have been found as longitudinal as well as transverse modes in the galaxy density (Gaztañaga et al. 2009a; Gaztañaga, Cabré &
Hui 2009b; Kazin et al. 2010b) and their issues of model selection and parameter estimation have been addressed thoroughly (Cabrè & Gaztañaga 2011).

The motivation for this paper is the fact that the detection of BAOs as a modulation feature in the galaxy field depends on the assumption of a biasing mechanism (Gaztañaga et al. 2009a; Desjacques et al. 2010) which relates the galaxy number density to the ambient density of dark matter as well as a control of redshift-space distortions effects (Nishimichi et al. 2007; Taruya et al. 2009), and it would be desirable to measure the dark matter density directly. Weak lensing would be a prime candidate for such a measurement, but the wide line-of-sight weighting functions cause the weak lensing signal to depend rather on the integral of the spectrum of cold dark matter (CDM) than on individual, localized features, even in the case of tomographic lensing surveys (Hu 1999). An interesting analysis of the possibility of using angular-dependent tomography for measuring BAOs was made by Simpson (2006), showing that although this method cannot compete with CMB or galaxy surveys constraints, it can constitute a consistency check. In this paper, we choose to go beyond tomography and investigate the sensitivity of 3d weak lensing (3dWL; Heavens 2003) for constraining the dark matter spectrum on BAO scales: 3dWL provides a direct estimate of the three-dimensional matter distribution and gives Gaussian errors on the amplitude of the CDM spectrum in wavelength bands from sparsely sampled data (Leonard, Dupé & Stark 2012). In this way, we aim to quantify the statistical precision at which 3dWL constrains the CDM spectrum at the BAO wavelengths, and the statistical significance for inferring the presence of one or more wiggles from 3dWL data relative to the null-hypothesis of absent wiggles.

After a short compilation of basic results concerning distances, structure growth, structure statistics and conventional weak lensing in Section 2, we recapitulate the main results of 3dWL in Section 3 and motivate its usage in constraining BAO wiggles. Our statistical approach and the estimation of statistical errors on the BAO measurement is given in Section 4 followed by a discussion of our main results in Section 5.

The reference cosmological model used is a spatially flat wCDM cosmology with Gaussian adiabatic initial perturbations in the matter distribution. The specific parameter choices are Ω_m = 0.25, n_s = 1, σ_8 = 0.8 and H_0 = 100 h km s^{-1} Mpc^{-1}, with h = 0.72. The dark energy equation of state is set to w = -0.9, and we assume the dark energy to be smooth. The baryon density Ω_b = 0.04 is used for correcting the CDM shape parameter and for predicting BAO-wiggle amplitudes and wavevectors.

### 2 COSMOLOGY AND STRUCTURE FORMATION

#### 2.1 Dark energy cosmologies

In spatially flat dark energy cosmologies with the present matter density Ω_m, the Hubble function aH(a) = da/dt is given by

$$\frac{H^2(a)}{H_0^2} = \Omega_m \frac{a^3}{a^3} + (1 - \Omega_m) \exp \left( 3 \int_a^1 \frac{a}{(1 + w(a))} \right),$$

with the dark energy equation of state w(a). A constant value w \equiv -1 corresponds to the cosmological constant. The relation between comoving distance \( \chi \) and scale factor \( a \) is given by

$$\chi = c \int_a^1 \frac{da}{a^2 H(a)},$$

with the Hubble distance \( \chi_H = c/H_0 \) as the cosmological distance scale. Redshift \( z \) and comoving distance are related by \( dz/d\chi = H(z)/c \).

#### 2.2 CDM power spectrum

The linear CDM-density power spectrum \( P(k) \) describes the fluctuation amplitude of the Gaussian homogeneous density field \( \delta \),

$$\langle \delta(k)\delta(k')^* \rangle = (2\pi)^3 \delta_D(k - k')P(k) \propto k^n T^2(k),$$

with the spectral index \( n \) and the transfer function \( T(k) \). The restoring force provided by the baryon–photon fluid in the early Universe generates a set of wiggles in the spectrum \( P(k) \) and an overall suppression due to diffusion. Both effects are discussed in detail by Eisenstein & Hu (1998, 1999) who also provide a fitting formula for \( T(k) \) in terms of the density parameters \( \Omega_m, \Omega_b \), and the Hubble parameter \( h \).

The spectrum \( P(k) \) is normalized in such a way that it exhibits the variance \( \sigma_8^2 \) on the scale \( R = 8 \text{ Mpc} h^{-1} \),

$$\sigma_8^2 = \frac{k^3 dk}{2\pi^2} P(k) W^2(k R)$$

with a Fourier-transformed spherical top hat filter function, \( W(x) = 3 j_1(x)/x \), where \( j_1(x) \) is the spherical Bessel function of the first kind of order \( \ell \) (Abramowitz & Stegun 1972).

#### 2.3 Structure growth

The growth of density fluctuations in the cosmic matter distribution can be described as a self-gravitating hydrodynamical phenomenon, in the limit of Newtonian gravity. Homogeneous growth of the density field

$$\delta(x, a) = D_+(a) \delta(x, a = 1)$$

in the linear regime \( |\delta| \ll 1 \) is described by the growth function \( D_+(a) \), which is the solution to the growth equation (Turner & White 1997; Wang & Steinhardt 1998; Linder & Jenkins 2003),

$$\frac{d^2}{da^2} D_+(a) + \frac{1}{a} \left( 3 + 4 \frac{\ln H}{\ln a} \right) \frac{d}{da} D_+(a) = \frac{3}{2a^2} \Omega_m(a) D_+(a) + \frac{3}{2a^2} \Omega_b(a) D_+(a).$$

Non-linear structure formation leads to a strongly enhanced structure growth on small scales, generates non-Gaussian features and, most importantly, wipes out BAO wiggles as features in the initial matter distribution. This can be understood in an intuitive way as corrections to the CDM spectrum in perturbation theory to order \( n \) assume the shape of integrals over polyspectra up to order \( 2n \) (which separate into a product of \( n \) spectra by application of the Wick theorem, see the review by Bernardeau et al. 2002) and are therefore becoming insensitive to localized features that are not strongly influencing the normalization of \( P(k) \) (Springel et al. 2005; Jeong & Komatsu 2006; Crocce & Scoccimarro 2008; Matarrese & Pietroni 2008; Pietroni 2008; Jeong & Komatsu 2009; Nishimichi et al. 2009; Anselmi & Pietroni 2012; Jürgens & Bartelmann 2013).

Since non-linear structure formation affects small scales first, we will target BAO wiggles with 3dWL beginning at the largest wavelength before proceeding to successively shorter wavelengths.

#### 2.4 Weak gravitational lensing

The weak lensing convergence \( \kappa \) provides a weighted line-of-sight average of the matter density \( \delta \) (for reviews, see Bartelmann
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\[ \kappa = \int_0^\chi d\chi W_\kappa(\chi) \delta, \tag{7} \]

with the weak lensing efficiency \( W_\kappa(\chi) \) as the weighting function,

\[ W_\kappa(\chi) = \frac{3\Omega_m D_+}{2\chi_H a} G(\chi) \frac{\delta}{\chi}, \tag{8} \]

and the lensing-efficiency-weighted galaxy redshift distribution, rewritten in terms of comoving distance,

\[ G(\chi) = \int_0^\chi d\chi' n(\chi') \left( 1 - \frac{\chi'}{\chi} \right). \tag{9} \]

Here, \( n(\chi) \) denotes a common parametrization of the redshift distribution of the lensed background galaxy sample,

\[ n(\chi) = n_0 \left( \frac{\chi}{z_0} \right)^2 \exp \left( - \left( \frac{\chi}{z_0} \right)^{\beta} \right) d\chi \text{ with } \frac{1}{n_0} = \frac{z_0}{\beta} \Gamma \left( \frac{3}{\beta} \right), \tag{10} \]

which can be rewritten in terms of a distribution in comoving distance with the relation \( n(\chi) d\chi = n(\chi) d\chi \) using \( d\chi/d\chi = c/H(a) \).

These expressions allow to carry out a Limber projection (Limber 1954) of the weak lensing convergence, which yields the angular convergence spectrum \( C_\kappa(\ell) \),

\[ C_\kappa(\ell) = \int_0^\chi d\chi \frac{\delta}{\chi^2} W_\kappa^2(\chi) P(k = \ell/\chi). \tag{11} \]

We will formulate our derivations in terms of the lensing convergence \( \kappa \) instead of the observable shear \( \gamma \) because it is a scalar quantity and possesses identical statistical properties. Equation (11) illustrates why line-of-sight-averaged weak lensing spectra are ineffective in picking up BAO wiggles (and is almost a repetition of the previous argument why non-linear structure formation destroys BAO features). They provide only an integrated measure of the CDM spectrum \( P(k) \) weighted with wide weighting functions \( W_\kappa(\chi) \) that is very insensitive to local features of the spectrum such as BAO wiggles. This argument holds even for advanced tomographic surveys (Hu 1999; Takada & Jain 2004) and motivates the need of a three-dimensional mapping of the cosmic matter distribution. With reference to Gaztanaga et al. (2009b) and Kazin et al. (2010b), we would like to emphasize that weak lensing, due to its sensitivity to gravitational shear components perpendicular to the line of sight, will provide measurements of BAO wiggles in the transverse direction.

### 3D Weak Lensing

The method of 3dWL was introduced by Heavens (2003), who proposed to include distances of lensed galaxies estimated from their photometric redshifts to infer the three-dimensional unprojected tidal shear, i.e. the second derivatives of the gravitational potential perpendicular to the line of sight from distortions in the galaxies’ ellipticity. Therefore, this approach differs from estimations of the angular line-of-sight-averaged spectrum \( C_\kappa(\ell) \) or corresponding tomographic spectra \( C_\kappa^{\chi}(\ell) \) in the important respect that the statistics of the full three-dimensional matter distribution is inferred without any averaging of shears with the line-of-sight galaxy distribution, which has been performed in equation (9). As such, 3dWL is particularly suited for the problem at hand, namely to provide a precise estimate of the amplitude of the dark matter power spectrum at the BAO wavelengths. Additionally, Heavens (2003) showed that if 3dWL is used for constraining \( P(k) \) at a fixed cosmology, the smallest errors are expected in the BAO regime of the CDM spectrum.

In this section, we recapitulate the main results of 3dWL in terms of the weak lensing convergence in the Fourier convention we prefer to work with; please also refer to Castro, Heavens & Kitching (2005), Massey et al. (2007), Heavens, Kitching & Taylor (2006) and Kitching et al. (2008a) for a detailed description of the theory, to Munshi, Heavens & Coles (2011) for higher order statistics through 3dWL and to Ayaita, Schäfer & Weber (2012) for details of our numerical implementation. We assume spatial flatness and lensing in linearly evolving structures, which can be, in principle, relaxed from the 3dWL point of view (Pratten & Munshi 2013). The impact of systematic errors is nicely investigated by Kitching, Taylor & Heavens (2008b), and for an application to observational data we refer the reader to Kitching et al. (2007).

The most natural choice for carrying out a Fourier transform in spherical coordinates is a combination of spherical harmonics for the angular and spherical Bessel functions for the radial dependence. We can therefore write the transformation for the convergence \( \kappa \) as

\[ \kappa_{\ell m}(k) = \sqrt{\frac{2}{\pi}} \int d\chi \frac{\delta}{\chi} j_\ell(k \chi) Y_{\ell m}^*(\theta) \tag{12} \]

(see Ballinger, Heavens & Taylor 1995; Heavens & Taylor 1995), where \( j_\ell \) and \( Y_{\ell m} \) are, respectively, a spherical Bessel function of the first kind and a spherical harmonic, and \( \theta \equiv (\theta, \phi) \). There exist algorithms for fast computation of \( \kappa_{\ell m}(k) \) (Percival et al. 2004; Lusanne, Rassat & Starck 2012; Leistedt et al. 2012; Rassat & Refregier 2012). Such a transformation is particularly convenient as the combination of \( j_\ell \) and \( Y_{\ell m} \) is an eigenfunction of the Laplacian in spherical coordinates, leading to a quite simple relationship between the coefficients of the density field \( \delta_{\ell m}(k) \) and the lensing convergence \( \kappa_{\ell m}(k) \) as the observable:

\[ \kappa_{\ell m}(k) = \frac{3\Omega_m (\ell + 1) \eta_{\ell m}(k,k)}{2X_H^2} \delta_{\ell m}(k), \tag{13} \]

with the lensing-induced mode coupling \( \eta_{\ell m}(k,k') \)

\[ \eta_{\ell m}(k,k') = \frac{4}{\pi} \int_0^\infty d\chi \frac{\delta}{\chi} j_\ell(k \chi) j_\ell(k' \chi), \tag{14} \]

(14) with implicit assumption of the Einstein summation convention

\[ X(k,k') Y(k',k'') = \int_0^\infty k^2 dk' X(k,k') Y(k',k''). \tag{15} \]

It is then possible to construct an estimator for \( \kappa_{\ell m}(k) \) by including the uncertainty of the galaxy distance estimates coming from errors in the measurements of redshift. If we denote by \( \chi \) the true radial coordinate of a galaxy and by \( \chi' \) the one inferred by its observed redshift \( \chi' = z(\chi') \), then they will be related by the probability \( p(\chi'|\chi) \), which we assume to be Gaussian for simplicity:

\[ p(\chi'|\chi) d\chi' = \frac{1}{\sqrt{2\pi\sigma_\chi}} \exp \left( -\frac{d(\chi,\chi')^2}{2\sigma_\chi^2} \right) d\chi', \tag{16} \]

where \( \sigma_\chi \) is the width of the distribution and is assumed to be constant throughout the entire galaxy sample. Furthermore, galaxies receive a statistical weight according to their distribution in distance \( n(\chi) d\chi \). Following the derivation in Heavens (2003), we define the two additional matrices

\[ Z_{\ell m}(k,k') = \frac{2}{\pi} \int d\chi' p(\chi'|\chi) j_\ell(k \chi') j_\ell(k' \chi'). \tag{17} \]
\begin{align}
M_i(k, k') = \frac{2}{\pi} \int \chi^2 d\chi \ n(\chi) \ j_i(k\chi) j_i(k'\chi),
\end{align}

where \( n(\chi) \) is the number density of galaxies, as defined in equation (10). These matrices describe the correlations in spherical Fourier modes generated by the measurement process: while \( \eta_i(k, k') \) describes mode couplings due to weak lensing, \( Z_i(k, k') \) and \( M_i(k, k') \) define, respectively, the contributions in the mode couplings coming from redshift errors and from the galaxy distribution along the radial coordinate \( \chi \).

We restrict ourselves to observations of the entire sky. In this case, the expression for the estimator \( \bar{k}_{im} \) of the convergence is then expected to be

\begin{align}
\bar{k}_{im}(k) = \frac{3 \Omega_m}{2 \sqrt{H}} \frac{\ell(\ell + 1)}{2} B_i(k, k') \frac{1}{(k')^2} \delta_{im}(k''),
\end{align}

where the mode-coupling matrix \( B_i(k, k') \) describes two integrations over \( k_1 \) and \( k_2 \):

\begin{align}
B_i(k, k') = Z_i(k, k_1) M_i(k_1, k_2) \eta_i(k_2, k'').
\end{align}

Since the average values of a field like \( \kappa_{im}(k) \) are zero for all-sky surveys, we can only infer information about any parameter the field may depend on by means of its covariance,

\begin{align}
\langle \kappa_{im}(k) \kappa_{im}(k') \rangle = S_{\kappa_{im}}(k, k') + N_{\kappa_{im}}(k, k') = C_{\kappa_{im}}(k, k')
\end{align}

which consists of a signal term \( S_{\kappa_{im}}(k, k') \) and a noise term \( N_{\kappa_{im}}(k, k') \). The signal term \( S_{\kappa_{im}} \) can be calculated directly from equation (19):

\begin{align}
S_{\kappa_{im}}(k, k') = \left( \frac{3 \Omega_m}{2 \sqrt{H}} \right)^2 \frac{\ell(\ell + 1)}{2} B_i(k, k') B_i(k', k'') \frac{1}{(k')^2} \delta_{im} P_i(k''),
\end{align}

with the abbreviations

\begin{align}
B_i(k, k') = Z_i(k, k_1) M_i(k_1, k_2) \eta_i(k_2, k''),
\end{align}

\begin{align}
B_i(k', k'') = Z_i(k', k_3) M_i(k_3, k_4) \eta_i(k_4, k''),
\end{align}

with implicit integration over \( k_1, k_2 \) and \( k_3, k_4 \). The corresponding noise part \( N_{\kappa_{im}} \) is given by

\begin{align}
N_{\kappa_{im}}(k, k') = \sigma_i^2 M_i(k, k'),
\end{align}

which is proportional to the shape noise \( \sigma_i^2 \), namely the variance of the galaxy ellipticity distribution. It is important to notice that \( N_{\kappa_{im}} \) is independent of cosmology or variations in the CDM spectrum \( P(k) \). Intrinsic ellipticity correlations were neglected, which would greatly complicate the 3dWL description.

4 DETECTING BAO WIGGLES

4.1 Construction of the Fisher matrix

We choose a Fisher matrix approach to determine how precisely 3dWL can constrain BAOs in the matter power spectrum \( P(k) \). The Fisher matrix is a square matrix whose elements are defined as the expectation values of the second derivative of the logarithmic likelihood with respect to the fiducial parameters \( \theta_\alpha \) and \( \theta_\beta \):

\begin{align}
F_{\alpha\beta} = - \left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial \theta_\alpha \partial \theta_\beta} \right\rangle.
\end{align}

As a general statement, if the likelihood \( \mathcal{L} \) can be expressed as an \( N \)-dimensional Gaussian

\begin{align}
\mathcal{L} = \frac{1}{\sqrt{(2\pi)^N \det(C)}} \exp \left( -\frac{1}{2} x^T C^{-1} x \right),
\end{align}

where \( x \) is a generic data vector and \( C \) is the corresponding covariance, we can then write

\begin{align}
F_{\alpha\beta} = \frac{1}{2} \left[ \left( \partial_\alpha C \right) \left( \partial_\beta C \right) - \left( \partial_\beta C \right) \left( \partial_\alpha C \right) \right],
\end{align}

or, equivalently,

\begin{align}
F_{\alpha\beta} = \frac{1}{2} \left( \partial_\alpha \ln C \times \partial_\beta \ln C \right).
\end{align}

where \( \partial_\alpha \) and \( \partial_\beta \) stand for the derivatives with respect to the parameters \( \theta_\alpha \) and \( \theta_\beta \). Given a particular experimental framework, the Fisher matrix specifies what are the best errors to expect for the inferred parameters \( \theta_\alpha \) via the Cramér–Rao relation.

It can be proved that, since \( \ell \)-measurements are independent in the case of full-sky coverage, we can reformulate equation (28): we consider our estimator to be \( \bar{k}_{im}(k) \) and its covariance as defined in equation (21), and find

\begin{align}
F_{\alpha\beta} = \frac{f_{\text{sky}}}{2} \sum_{\ell=\ell_{\text{min}}}^{\ell_{\text{max}}} (2\ell + 1) \left[ \left( C_{\alpha,\ell}^{-1} \partial_\alpha C_{\ell,\ell} \right) \left( C_{\beta,\ell}^{-1} \partial_\beta C_{\ell,\ell} \right) \right].
\end{align}

In our specific case, we consider the power spectrum as parametrized not by usual cosmological parameters, such as \( \Omega_m \) or \( \sigma_8 \), but rather by its own wiggles amplitudes, namely by the values assumed by \( P(k) \) in a range of \( k \) where the wiggles \( w_\ell \) are located. In Fig. 1, we plot the wiggle-only power spectrum, i.e. the ratio between the power spectrum with BAOs \( P(k) \) and an equivalent, smoothed out spectrum that has the same shape as \( P(k) \) but shows no oscillating feature, \( P_s(k) \). We highlight the wiggles that have been used to parametrize \( P(k) \), \( w_\alpha \). By calculating the derivative \( \partial_\alpha C \) of the covariance with respect to a variation of the amplitude of a maximum number of wiggles \( n_{\alpha} \), we can build up (equation 28) a Fisher matrix \( F_{\alpha\beta} \), where \( \alpha, \beta = 1, \ldots, 6 \).

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Ratio between the power spectrum with BAOs \( P(k) \) and the smooth power spectrum \( P_s(k) \). The largest wiggles in amplitude have been highlighted and labelled as \( w_\ell \), \( \alpha = 1, \ldots, 6 \). These are the wiggles used to parametrize the power spectrum in our Fisher matrix approach.
n_w. Such a matrix carries information about the best errors to expect on the detection of each wiggle w_α, α = 1, ..., n_w, and the cross-correlations between inferred wiggle amplitudes. Given our aim, what we are actually performing in the calculation of ∂_σ C is a functional derivative, also known as Fréchet derivative. In fact, we can imagine the power spectrum as depending on features, i.e. the wiggles in Fig. 1. Each one of them can be approximated to a sin-like function defined in a range of k as wide as λ/2, where λ is the wavelength of the function itself. The covariance derivative is numerically estimated for one wiggle at a time as a finite difference:

$$\partial_\sigma C_{\ell,\ell'} = \frac{C_{\ell,\ell'}^{+} - C_{\ell,\ell'}^{-}}{2\epsilon},$$

where C_{\ell,\ell'}^{±} are the covariance matrices calculated using the power spectra P_{α}(k) and ε is an arbitrarily small number. The spectra P_{α}(k) are equivalent to the original P(k) for all k of the domain, exception made for the wavenumbers belonging to the interval I_{k} that corresponds to wiggle w_α. In this interval, P_{α}(k) is then

$$P_{α}(k) = P(k) \pm \epsilon P(k).$$

We would like to point out that, since what we are actually performing by means of the spectrum variation in equation (32) is in a way a logarithmic derivative of C_{ℓ}, the denominator in equation (31) lacks a factor P(k) and is therefore just two times the fraction of the spectrum used in the variation.

In Fig. 2, we show a representation of an example of the variation performed in the calculation of the derivative of the covariance matrix, ∂_σ C; in this case, the second wiggle, w_2, has been considered.

It is worth noticing that the Fisher matrix approach for inferring the error σ_α on the dark matter spectrum P(k_α) (where k_α are simply the k ∈ I_{k}, for brevity) as a Gaussian standard deviation is perfectly justified because of the linearity of the lensing observable and the linearity of the random field, so we do not need to use Monte Carlo sampling for evaluating the likelihood L(P(k_α)) and to measure its widths σ_α from Monte Carlo samples of the likelihood.

As noise sources for the inference of P(k_α), we consider a Gaussian shape measurement error σ_z for the galaxy ellipticities, which are assumed to be intrinsically uncorrelated, and a Gaussian error σ_α for the redshift determination uncertainty. Likewise, we work in the approximation of neglecting all geodesic effects (Seitz & Schneider 1994; Seitz, Schneider & Ehlers 1994), like deviations from the Born approximation, lens–lens couplings (Shapiro & Cooray 2006; Krause & Hirata 2010), source clustering (Schneider, van Waerbeke & Mellier 2002), source–lens correlations (Hamana et al. 2002) and deviations from Newtonian gravity (Acquaviva, Baccigalupi & Perrotta 2004). While performing the necessary variations for computing the Fisher matrix, we keep all other cosmological parameters fixed and calculate everything using an ℓ-range between ℓ_{min} = 2 and ℓ_{max} = 100 (please see the next section for a justification of this choice). As surveys, we consider the cases of Euclid, dark energy survey (DES) and a hypothetical deep-reaching survey labelled DEEP.

### Table 1. Basic survey characteristics used for the Fisher analysis: median redshift z_{med} of the galaxy sample, galaxy density per squared arcminute h, sky coverage fraction f_{sky}, redshift error σ_z and shape measurement error σ_α of the surveys Euclid, dark energy survey (DES) and a hypothetical deep-reaching survey labelled DEEP.

| Survey | z_{med} | h | f_{sky} | σ_z | σ_α |
|--------|---------|---|---------|-----|-----|
| Euclid | 0.9     | 30| 0.5     | 0.1 | 0.3 |
| DES    | 0.7     | 10| 0.1     | 0.12| 0.3 |
| DEEP   | 1.5     | 40| 0.1     | 0.05| 0.3 |

4.2 Statistical errors

The error σ_α for inferring the amplitude of the CDM spectrum P(k_α) at wiggle positions k_α is given by the Cramér–Rao relation,

$$\sigma_α^2 = (F^{-1})_{αα},$$

and

$$\sigma_z^2 = 1/F_{zz},$$

for marginalized and conditional likelihoods, respectively. Before carrying out our analysis for surveys like Euclid, DES and DEEP, we implement some tests in order to determine the optimal value for the maximum number of modes to be used in the calculation of the Fisher matrix, ℓ_{max}. Besides, we tried to find out how and by how much are the errors sensitive to some of the usual survey parameters, such as

(i) the shape noise σ_z,
(ii) the error σ_z in the measurement of redshift,
(iii) the median redshift z_{med},
(iv) the fraction of sky coverage f_{sky}.

Throughout these tests, when not stated otherwise, we make use of a default set of survey parameters such that σ_z = 0.02, σ_α = 0.3, z_{med} = 0.9, f_{sky} = 0.4 and h = 20. Additionally, we assumed that we want to constrain simultaneously the first four wiggles (see Fig. 1).

We start our investigation by determining the errors σ_α, α = 1, 2, 3, 4, as the maximum number of modes ℓ_{max} in the summation in equation (30) increases. Please refer to Fig. 3 for a plot of the behaviour of σ_α normalized to the oscillation amplitude A_α, where A_α is defined as the maximum value of |P(k) − P_α(k)| for each
In particular, we considered \( \ell_{\text{max}} = 10, 30, 100, 300 \), and observe for instance that, between \( \ell_{\text{max}} = 100 \) and \( \ell_{\text{max}} = 300 \), the gain in the precision with which the wiggles would be constrained in a 3dWL approach is not significantly large and it is of the order of 0.01, 0.1, 0.6 and 2 per cent of the relative error for, respectively, the first, second, third and fourth wiggle. Therefore, we decide to stick to a maximum number of modes of 100 for all the subsequent calculations. This is a fair approximation not only because of its numerical convenience, but also from a theoretical point of view: in fact, extending too much the \( \ell \) interval for the \( F_{\alpha \beta} \) summation could make us fall out of the linear regime; in addition, the assumption of a Gaussian shape for the likelihood \( L \) could not to be anymore reasonable in such a multipole range (Heavens 2003). In Fig. 4, we show what happens as soon as we keep all the survey parameters fixed and vary \( \sigma_z \). As one can expect, larger values of \( \sigma_z \) produce larger errors on all the wiggles under investigation, although here the relation is somewhat slower, as long as \( \sigma_z \lesssim 0.1 \); the relation also appears to be slightly dependent on the wiggle, becoming steeper as higher order oscillations are taken. In fact, by incrementing the error on redshift from 0.01 to 0.1, we get an error larger only by a factor of \( \sim 2 \) on the first wiggle and by a factor of \( \sim 8 \) on the second. Additionally, the correspondence between \( \sigma_\alpha \) and the median redshift of the survey (Fig. 5) turns out to be quite peculiar (Fig. 5). While it seems to behave like a power law for \( z_{\text{med}} \gtrsim 0.4 \), it slows considerably down for smaller values, until it starts to raise again (\( z_{\text{med}} \lesssim 0.1 \)). Qualitatively, this trend makes sense in light of the fact that, as we increase the median redshift, we keep fixed all other survey parameters, such as the galaxy density per squared arcminute \( \bar{n} \). By doing so, we consider surveys where a number \( \bar{n} \) of galaxies is distributed over a deeper cone, meaning that we are actually sampling the 3D convergence field in a more diluted way, and therefore inheriting a larger noise. This behaviour is also quantitatively expected, as going from \( z = 0.4 \) to \( z = 1 \) corresponds to an increase of a factor of 10 in comoving volume, which is in good agreement with the results shown in Fig. 5. The trend observed for \( z_{\text{med}} \lesssim 0.4 \), on the other hand, is probably due to a combination of factors: at lower redshifts cosmic variance starts to play a role, the lensing signal is considerably weaker, slowing down the increase in the precision of the wiggles measurement for relatively small values of \( z_{\text{med}} \). If we vary \( \bar{n} \), instead, and consider a higher density of galaxies, we get an increase in the sensitivity, as we expect (Fig. 6), that nonetheless hits a plateau and stops after \( \bar{n} = 1000 \). We assume that this feature comes from the fact that we start to be limited not only by the 3dWL noise but also by the cosmic variance. Naturally, varying the sky coverage propagates to the errors \( \sigma_\alpha \propto 1/\sqrt{\bar{f}_{\text{sky}}} \), whereas the precision in the wiggle measurements scales as \( \sigma^2_z \) (see, respectively, equation 30 and equation 25).
Figure 6. Relative marginalized errors on the first four wiggles when the average number of galaxies per squared arcminute $\bar{n}$ of the survey increases. Of course a larger $\bar{n}$ will lead to higher precision in the BAO measurement, as the survey volume is sampled via a higher number of galaxies.

4.3 Detectability of BAO wiggles

In this section, we would like to present the results obtained when estimating the best errors to expect on BAO wiggles for the surveys Euclid, DES and DEEP (please see Table 1 for specifications).

We started our analysis by calculating both marginalized ($\sigma_\alpha = \sqrt{(F^{-1})_{\alpha\alpha}}$) and conditional errors ($\sigma_\alpha = 1/\sqrt{\text{F}_{\alpha\alpha}}$) relative to the wiggle amplitude. These errors were computed for the three types of surveys, considering the first four oscillations, as shown in Fig. 7. As we could expect, marginalized errors are always larger than the correspondent conditional ones, namely the $\sigma_\alpha$ on each wiggle when we assume to know precisely all the other wiggle amplitudes.

We continue the investigation considering the confidence ellipses calculated from the corresponding Fisher matrices obtained for the three surveys. We assume that we are aiming to jointly constrain the first four wiggles and plot the results in Figs 8, 9 and 10 for, respectively, Euclid, DES and DEEP. The sizes of the ellipses, whose contours stand for $1-2-3\sigma$, already tell us that, among the ones evaluated, Euclid will probably be the survey with largest constraining power on the BAO wiggles. It is of particular interest noticing the orientation of the ellipses, or their correlation coefficients (upper-right corner in every panel), that tell us something about the interdependence between different wiggles: in fact, neighboring wiggles are anti-correlated, i.e. increasing the amplitude of $\mathcal{P}(k)$ at the position of wiggle 2, we must then have the amplitude at $w_3$ decreased in order to remain in the confidence region. The x- and y-axes show the variation of the wiggle in terms of percentage of its amplitude $A_\alpha$, the three contours areas correspond to $1-2-3\sigma$ and every panel shows the correlation coefficient in the upper-right corner.

**Figure 7.** Conditional (magenta lines) and marginalized relative errors (black lines) for the three surveys under investigations Euclid (solid line), DES (dashed line) and DEEP (dash-dotted line) as a function of the wiggles, when the first four oscillations are simultaneously constrained.

**Figure 8.** Confidence ellipses for the first four wiggles in a Euclid-like survey, showing that the wiggles are indeed highly correlated. In fact, taking, for example, two consecutive wiggles, such as $w_2$ and $w_3$, we see that by increasing the amplitude of $\mathcal{P}(k)$ at the position of wiggle 2, we must then have the amplitude at $w_3$ decreased in order to remain in the confidence region. The x- and y-axes show the variation of the wiggle in terms of percentage of its amplitude $A_\alpha$, the three contours areas correspond to $1-2-3\sigma$ and every panel shows the correlation coefficient in the upper-right corner.

**Figure 9.** Fisher confidence ellipses for DES, when one tries to simultaneously constrain the first four wiggles. Again, contours areas correspond to $1-2-3\sigma$ and the number in every panel is the correlation coefficient; the axes represent variation of wiggles in terms of their amplitude fraction. Also, in this case, we can observe correlation between the amplitudes of $\mathcal{P}(k)$ at different wiggle positions.

In order to better understand whether the constraining power of the three surveys will allow us to detect any oscillatory feature in the evaluated, Euclid will probably be the survey with largest constraining power on the BAO wiggles. It is of particular interest noticing the orientation of the ellipses, or their correlation coefficients (upper-right corner in every panel), that tell us something about the interdependence between different wiggles: in fact, neighbouring wiggles are anti-correlated, i.e. increasing the amplitude of the power spectrum in correspondence to one oscillation would cause the $\mathcal{P}(k)$ at the position of the adjacent wiggle to take smaller values in order to stay among the confidence region, and vice versa, whereas the opposite holds for alternated wiggles.
BAOs with lensing

Figure 10. Confidence ellipses for the DEEP survey, with the variation of the oscillations in terms of the wiggle amplitude. Again, we assumed we wanted to constrain jointly the first four wiggles, contour areas stand for $1-2-3\sigma$ and the correlation coefficient can be read in the panels.

Figure 11. The bars show the marginalized errors in the detection of the wiggles, normalized with respect to $P_s(k)$, when the first two (upper-left panel), three (upper-right panel), four (bottom-left panel) five (bottom-right panel) wiggles are used to parametrize the power spectrum. Here, we considered a Euclid-like survey with $\sigma_z = 0.1, \sigma_\epsilon = 0.3, z_{\text{med}} = 0.9, f_{\text{sky}} = 0.5$ and $\bar{n} = 30$.

Figure 12. Marginalized errors on the detection of BAO wiggles when one tries to detect the first two (upper-left panel), three (upper-right panel), four (bottom-left panel), five (bottom-right panel) wiggles at the same time, for a DES-like survey ($\sigma_z = 0.12, \sigma_\epsilon = 0.3, z_{\text{med}} = 1.5, f_{\text{sky}} = 0.12$ and $\bar{n} = 10$). Again, we considered the relative marginalized errors, dividing by $P_s(k)$.

Figure 13. The marginalized errors on the detection of the wiggles in the power spectrum, relative to $P_s(k)$, for the hypothetical survey DEEP, characterized by the parameters $\sigma_z = 0.02, \sigma_\epsilon = 0.3, z_{\text{med}} = 1.5, f_{\text{sky}} = 0.1$ and $\bar{n} = 40$.

CDM power spectrum, we plot the $\sigma_\alpha$ obtained from the Cramér–Rao relation in equation (33) as error bars in the usual wiggle-only power spectrum for Euclid (Fig. 11), DES (Fig. 12) and DEEP (Fig. 13). Since what is shown is a ratio between $P(k)$ and a smooth spectrum, the $\sigma_\alpha$ have of course also been normalized with respect to $P_s(k)$. The four different panels show how the errors change when we try to jointly constrain the first two, three, four, or five wiggles with a 3dWL approach.

What these and the following plots show, first of all, is an expected feature: as we increment the number of wiggles we expect to simultaneously examine, the precision with which the amplitudes $P(k_\alpha)$ would be measured gets poorer and poorer for all the oscillations. Our purpose would then be to evaluate how many BAO wiggles one is allowed to constrain before the errors on them become too large. It can be seen that all three surveys would allow for quite good constraints on the first two wiggles. The hypothetical survey DEEP already shows error bars of the order of the wiggle amplitude $A_\alpha$ when the first three wiggles are considered, and the errors become much larger than $A_\alpha$ ($\alpha > 1$) as soon as one tries to detect four or more wiggles (Fig. 13). On the other hand, DES and Euclid give a better performance, allowing for, respectively, the first three and four wiggles to be simultaneously constrained, with Euclid giving smaller errors overall (Fig. 11).

A better comparison between the three surveys can be carried out analysing Figs 14 and 15, where we plotted relative errors $\sigma_\alpha/A_\alpha$ as functions of the maximum number of wiggles we want to jointly constrain, $n_\alpha$, and we collate results coming from, respectively, DEEP and Euclid, DES, and Euclid. It becomes straightforward that a DEEP-like survey cannot compete against Euclid: the relative errors coming from DEEP are always larger than the latter's,
Subject of this paper has been a statistical investigation on whether future weak lensing surveys are able to detect BAOs in the cosmic matter distribution by application of the 3dWL method. For a fixed $\omega_{CDM}$ cosmology, we have estimated the statistical precision $\sigma_\alpha$ on the amplitude of the CDM spectrum $P(k)$ at the BAO wiggle positions in a Fisher matrix approach. Throughout, we worked under the assumption of Gaussian statistics, independent Fourier modes and in the limit of weak lensing. Noise sources were idealized and consisted in independent Gaussian-distributed shape–noise measurements for the lensed background galaxy sample, as well as a Gaussian error for the redshift determination. As surveys, we considered the cases of Euclid, DES and a hypothetical deep-reaching survey DEEP.

(i) We have constructed the Fisher matrix considering our model as parametrized by the amplitudes of the CDM power spectrum at the BAOs anticipated positions. In particular, we started taking the two BAO wiggles with largest amplitude and progressively increased the number of oscillations considered. Keeping the cosmology fixed to a standard $\omega_{CDM}$ parameter choice, we carried out variations of $P(k)$ that preserved its wiggle-shape in those wavenumber intervals; we then estimated the Fisher matrix accordingly, in order to quantify whether the statistical power of future weak lensing surveys suffices to place bounds on the amplitudes of the considered harmonics. By means of the Cramér–Rao relation, we calculated the best errors $\sigma_\alpha$ to expect for the amplitudes of $P(k)$ at wiggle positions.

(ii) The sensitivity of $\sigma_\alpha$ with respect to some typical survey/parameters was tested. In particular, we considered the shape noise $\sigma_z$, the redshift error $\sigma_z$, the median redshift $z_{\text{med}}$, the average number of galaxies per squared arcminute $\bar{n}$ and the sky coverage $f_{\text{sky}}$. We found that, as expected, increasing the uncertainty in the estimate of either the redshift or the galaxy shapes brings a larger error in the inference of the presence of wiggles, and that the sensitivity of these errors on $\sigma_z$ is less pronounced for small values of $\sigma_z$, although it grows as soon as we consider higher order wiggles or large $\sigma_z$. An increase of $z_{\text{med}}$ leads as well to larger errors on the wiggles amplitudes, as one would expect from considering less and less populated surveys, whereas a larger $\bar{n}$ brings smaller errors, because of the same mechanism; similarly, a wider sky coverage, i.e. larger $f_{\text{sky}}$, yields to higher precision in constraining the wiggle amplitudes $\sigma_\alpha \propto 1/\sqrt{f_{\text{sky}}}$. Overall, we may conclude that the volume of a survey and how densely it is populated seem to be overcoming the importance of a high precision in the redshift measurement of galaxies, at least for $\sigma_z < 0.1–0.2$.

(iii) Finally, we evaluated the $\sigma_\alpha$ for the surveys under investigation and found that, among them, Euclid gave the best results, potentially allowing for the detection of up to the first four BAO wiggles with a good statistical confidence. Given our tests on the sensitivity of the errors on $P(k_\alpha)$ to certain survey parameters, we may conclude that Euclid’s good performance is probably due to the volume of the survey in terms of total galaxy number and sky coverage, which seem to prevail over the negative effects brought by the error on redshift measurements, $\sigma_z$, larger than the ones predicted for the other two surveys.

5 SUMMARY AND CONCLUSIONS

Given these results, we conclude that measurements of BAO wiggles based on future weak lensing data are entirely possible, and avoid issues related to galaxy biasing and redshift-space distortions. We forecast a detection of the first four or three wiggles with Euclid.
and DES by applying 3dWL techniques. Future developments from our side include estimates of the precision that can be reached on inferring dark energy density and equation of state by including the estimate of the BAO scale at low redshifts probed by lensing to the estimates at intermediate redshift provided by galaxy surveys and those at high redshifts, such as the CMB. Additionally, we are investigating the impact of systematical errors on the estimation process from 3dWL data and biases in the estimation of BAO wiggle amplitudes.

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