Abstract—In this paper, we devise the optimal caching placement to maximize the offloading probability for a two-tier wireless caching system, where the helpers and a part of users have caching ability. The offloading comes from the user local cache, other users’ D2D sharing and the helpers’ transmission. In particular, to maximize the offloading probability we reformulated the caching placement problem for users and helpers into a difference of convex (DC) problem which can be effectively solved by DC programming. Moreover, we analyze the two extreme cases where there is only help-tier caching network and only user-tier. Specifically, the placement problem for the helper-tier caching network is reduced to a convex problem, and can be effectively solved by the classical water-filling method. We notice that users and helpers prefer to cache the popular contents under low density and prefer to cache different contents evenly under a high density. Simulation results indicates a great performance gain of the proposed caching placement over existing approaches.

I. INTRODUCTION

The rapid proliferation of mobile devices has led to the unprecedented growth in wireless traffic demands. According to Cisco’s most recent report [1], the mobile multimedia data is forecasted to grow at a compound annual growth rate of more than 60%. On the other hand, user demand for multimedia contents is highly redundant, i.e., a small number of contents account for a majority of all requests [2]. Therefore, caching popular contents at various nodes in the network is a promising approach to alleviate the network bottleneck [3].

For the wireless caching systems where helpers (WiFi, femtocells) have high storage capacity, the performance depends heavily on the adopted caching replacement. The cache placement for helpers is firstly investigated in [4] to minimize the downloading time, where both uncoded and coded cases are considered. It is shown that the optimization problem for the uncoded case is NP-hard. In addition, [5] considers the channel fading and develops the cache placement to minimize the average bit error rate, where the optimal caching placement is to balance between the channel diversity gain and the caching diversity gain. Moreover, the problem of optimal MDS-encoded cache placement at the wireless edge is investigated in [6] to minimize the backhaul rate in heterogeneous networks. However, all above analyses [4]–[6] are based on the fixed topology between users and helpers. In [7], more realistic network models are adopted to characterize the stochastic natures of geographic location and the corresponding optimal caching placements are derived according to different criteria.

On the other hand, the potential cache capacity at user side can also be exploited, e.g., local cache offloading or D2D sharing. Various works have been done on the caching placement at user side. In [8], the D2D outage-throughput tradeoff problem is investigated and the optimal scaling laws are characterized. [9] analyzes the scaling behavior of the throughput with the number of devices per cell under Zipf distributed content request probability with exponent γr, and conclude that the optimal cache distribution is also a Zipf distribution with a different value γc. By modeling the mobile devices as a homogeneous Poisson Point Processes (PPP), [10] derives the optimal cache distribution such that the total probability of content delivery is maximized. However, the local offloading probability is not considered in their analysis.

In addition, coded caching is also an effective approach to exploit the content diversity [11]. By caching contents partially at user side according to the developed caching distribution during the first phase, a coded multicasting opportunity can be created even for different content requests in the second phase. Moreover, [12] further proposes the hierarchical coded caching to address the joint cache placement problem at both users and helpers. However, these analyses [11]–[13] are based on the fixed topology which is not suitable for the user mobility scenario.

Despite the aforementioned studies, to the best of our knowledge, the optimal caching placements for both helpers and users under realistic network models remain unsolved to date. Thus in this paper, we consider a two-tier caching system, where the helpers and users are spatially distributed according to two mutually independent homogeneous Poisson Point Processes (PPPs) with different intensities [14]. In order to alleviate the traffic load in the cellular network, we aim to develop an optimal caching placement scheme to maximize the offloading probability, where the offloading includes self-offloading, D2D-offloading and helper-offloading. More details along with the main contributions are as follows:

- We consider a D2D assisted two-tier wireless caching network consisting of users and helpers where the offloading comes from self-offloading, D2D-offloading and helper-offloading. Different from [10], we take self-offloading events into consideration. Moreover, the practical assumption that only a part of users has caching ability is considered.
We formulate the total offloading probability of caching placement in the two-tier wireless networks and adopt the DC programming to solve the non-convex maximization problem. In addition, we notice that, users and helpers ought to cache the popular contents while the density is low and ought to cache different contents while density is high. And our proposed caching placement can achieve a balance between them.

The two special cases of one-tier caching systems are considered. In absence of user caching ability, we model the caching placement for helper-tier as a convex problem, and can be effectively solved by the classical water-filling method; In absence of helper caching ability, the caching placement for users is also formulated into a convex problem. We combine the solution of the two cases as a non-joint optimal caching placement and compare it with the proposed placement.

II. SYSTEM MODEL AND CONTENT ACCESS PROTOCOL

In this section, we first introduce the two-tiered caching system as illustrated in Fig. 1, where the helpers and users are spatially distributed based on two mutually independent homogeneous Poisson Point Processes (PPPs) with density $\lambda_H$ and $\lambda_{UE}$, respectively. Then the content access protocol is provided.

A. System Model

1) Content module: The content library consists of $N$ contents. The popularity distribution vector of the contents is denoted by $q = \{q_1, \ldots, q_N\}$, where $q_i$ is the access probability for the $i$-th content. In this paper, we characterize the popularity distribution as a Zipf distribution with parameter $\gamma$. If we arrange contents in descending order of popularity, the popularity of the $i$-th ranked content is [13]

$$q_i = \frac{1/i^\gamma}{\sum_{j=1}^{M} 1/j^\gamma},$$

where $\gamma$ governs the skewness of the popularity. The popularity is uniform over contents for $\gamma = 0$, and becomes more skewed as $\gamma$ grows. For simplicity, we assume all the $N$ contents are of equal size $L$.

2) Network module: In addition to the macro base stations (BSs), the network module also consists of the helpers with caching ability, where helpers could successfully send the contents in its local cache to requesting users within radius $R_H$ at relatively low cost. For simplicity, we assume the caching ability for all helpers are the same, denoted by $M_H L$, where $M_H < N$. Therefore, the helper can cache up to $M_H$ different contents entirely. Also we assume a content can only be cached entirely rather than partially. Denote the cache placement at the helpers for each content as $P_H = [p^H_1, p^H_2, \ldots, p^H_N]$, where $p^H_i$ is the percent of helpers caching the $i$-th content and $0 \leq p^H_i \leq 1$ for $i = 1, 2, \ldots, N$. The cache storage constraint at the helpers can then be written as $\sum_{i=1}^{N} p^H_i \leq M_H$. The helpers caching the $i$-th content also follow a PPP with intensity $\lambda_H p^H_i$.

3) User module: We assume part of the users having caching ability. Let $\alpha$ denote the percent of cache-enabled users, where $0 \leq \alpha \leq 1$. The cache-enabled users also follow a thinning homogeneous PPP with intensity $\alpha \lambda_{UE}$. For simplicity, we assume the caching ability for the cache-enabled users are the same, denoted by $M_{UE} L$. Therefore, cache-enabled users can cache up to $M_{UE}$ different contents entirely in its local cache. Moreover, a device to device (D2D) communication can be established if the requesting user and the user caching the desired content are within distance $R_{UE}$, where $R_{UE} < R_H$ due to the transmitting power. Let $P_{UE} = [p^UE_1, p^UE_2, \ldots, p^UE_N]$ denote the cache placement at the users with caching ability for each content, where $p^UE_i$ is the percent of helpers caching the $i$-th content, and $0 \leq p^UE_i \leq 1$ for $i = 1, 2, \ldots, N$. The cache storage constraint at the cache-enabled users can then be written as $\sum_{i=1}^{N} p^UE_i \leq M_{UE}$.

Therefore, the users caching the $i$-th content also follows a PPP with intensity $\alpha \lambda_{UE} p^UE_i$.

B. Content Access Protocol

The content access protocol is as follows:

(a) Self-offloading: When a content request occurs, the user first checks its local storage whether the desired content has been stored in it. The request will be satisfied and offloaded immediately if the user has cached the desired content in its local storage space. We term it as “Self-offloading”.

(b) D2D-offloading: If the exact content has not been cached in the local storage or the user does not have cache ability, the user will turn to search for nearby devices with the desired content. If there is at least one user has stored the desired content within the radius $R_{UE}$, the request would be met and offloaded by establishing a D2D communication, termed as “D2D-offloading”.

(c) Helper-offloading: In addition to “D2D-offloading”, if there is at least one helper have stored the desired content within $R_H$, the request would be satisfied and offloaded by the helper transmission, termed as “Helper-offloading”.

Fig. 1. System model of the D2D assisted wireless caching system, where (a), (b), (c) and (d) stand for Self-offload, D2D-offload, Helper-offload and Cellular-response, respectively.
(d) **Cellular-response:** If the request could not be offloaded via local cache, D2D or the helpers then it need to be forwarded to the cellular base station and the cellular network transmit the requested content in response.

### III. Offloading Probability and Problem Formulation

In this paper, in order to alleviate the traffic load from the cellular network, our goal is to find the optimal caching placement to maximize the offloading probability. Therefore, we first analyze the offloading probability for the D2D assisted wireless caching network. Then, the optimal caching placement problem is formulated.

#### A. Offloading probability analysis

For a PPP distribution with density \( \lambda \), the probability that there are \( n \) devices in the area with a radius \( r \) is:

\[
F(n, r, \lambda) = \frac{(\pi r^2 \lambda)^n}{n!} e^{-\pi r^2 \lambda}
\]  

(2)

Therefore, for a reference user located at the origin, the probability that there are at least another user caching the \( i \)-th content within the transmission range is:

\[
P_{i,\text{off}}^{\text{D2D}} = 1 - F(0, R_{\text{UE}}, \lambda_{\text{UE}}) = 1 - e^{-\pi \alpha \lambda_{\text{UE}} R_{\text{UE}}^2}.
\]  

(3)

Similarly, the probability that at least one helper caching the \( i \)-th content within the radius \( R_H \) is:

\[
P_{i,\text{off}}^{\text{H}} = 1 - F(0, R_H, \lambda_H) = 1 - e^{-\pi \alpha \lambda_{\text{H}} R_H^2}.
\]  

(4)

The offloading probability for the \( i \)-th content of cache-enabled users, i.e. the probability at least one helper or one user caching the \( i \)-th content is:

\[
P_{i,\text{NC}} = 1 - (1 - P_{i,\text{off}}^{\text{D2D}})(1 - P_{i,\text{off}}^{\text{H}}) = 1 - e^{-(\pi \alpha \lambda_{\text{UE}} R_{\text{UE}}^2 + \pi \alpha \lambda_{\text{H}} R_H^2)}.
\]  

(5)

The corresponding offloading probability of the cache-enabled users for the \( i \)-th content is:

\[
P_{i,\text{C}} = p_{i,\text{UE}}^{\text{C}} + (1 - p_{i,\text{UE}}^{\text{C}}) P_{i,\text{NC}}.
\]  

(6)

Therefore, the offloading probability for the \( i \)-th content becomes:

\[
P_{i,\text{off}} = \alpha P_{i,\text{C}} + (1 - \alpha) P_{i,\text{NC}} = 1 - (1 - \alpha p_{i,\text{UE}}^{\text{C}}) e^{-(\pi \alpha \lambda_{\text{UE}} R_{\text{UE}}^2 + \pi \alpha \lambda_{\text{H}} R_H^2)}.\]

(7)

The total offloading probability for the D2D assisted wireless caching system becomes:

\[
P_{\text{off}} = \sum_{i=1}^{N} q_i P_{i,\text{off}},
\]

while more data offloaded by the wireless caching network, less data needs to be sent via the cellular network, alleviating the traffic load for the cellular network.

#### B. Problem Formulation

Let \( P = [P_H \quad P_{\text{UE}}] \) denote the caching placement at helper and user sides. The optimal caching placement that maximize the offloading probability for the wireless caching network can be formulated as:

\[
\begin{align*}
\max_{P} \quad & \sum_{i=1}^{N} q_i P_{i,\text{off}} \\
\text{s.t.} \quad & \sum_{i=1}^{N} p_{i,\text{UE}}^{\text{H}} \leq M_{\text{UE}} \\
& \sum_{i=1}^{N} p_{i,\text{H}}^{\text{H}} \leq M_{\text{H}} \\
& 0 \leq p_{i,\text{UE}}^{\text{H}} \leq 1, i \in \{1, \ldots, N\} \\
& 0 \leq p_{i,\text{H}}^{\text{H}} \leq 1, i \in \{1, \ldots, N\}
\end{align*}
\]

(9)

In this section, we adopt the difference of convex (DC) program to solve the above problem. The maximizing problem is equivalent to the following minimization problem:

\[
\begin{align*}
\min_{P} \quad & -\sum_{i=1}^{N} q_i P_{i,\text{off}} \\
\text{s.t.} \quad & \sum_{i=1}^{N} p_{i,\text{UE}}^{\text{H}} \leq M_{\text{UE}} \\
& \sum_{i=1}^{N} p_{i,\text{H}}^{\text{H}} \leq M_{\text{H}} \\
& 0 \leq p_{i,\text{UE}}^{\text{H}} \leq 1, i \in \{1, \ldots, N\} \\
& 0 \leq p_{i,\text{H}}^{\text{H}} \leq 1, i \in \{1, \ldots, N\}
\end{align*}
\]

(11)

Let \( F(P) = -\sum_{i=1}^{N} q_i P_{i,\text{off}} \) denote the objective function in problem (11) and it can be easily verified that the hessian matrix of \( F(P) \) is not positive definite and hence the optimal probability of (11) is non-convex.

Let \( H(P) = \sum_{i=1}^{N} h_i \), where \( h_i = q_i \alpha \lambda_H R_H^2 (p_{i,\text{UE}}^2 + p_{i,\text{H}}^2) \). Denote \( G(P) = F(P) + H(P) \), we then have the following proposition.

**Proposition 1:** \( H(P) \) and \( G(P) \) are both convex of \( P \).

**Proof:** Let \( A_i \) denote the hessian matrix of \( h_i \):

\[
A_i = \begin{bmatrix}
\frac{\partial^2 h_i}{\partial p_{i,\text{UE}}^2} & \frac{\partial^2 h_i}{\partial p_{i,\text{UE}} \partial p_{i,\text{H}}} \\
\frac{\partial^2 h_i}{\partial p_{i,\text{H}} \partial p_{i,\text{UE}}} & \frac{\partial^2 h_i}{\partial p_{i,\text{H}}^2}
\end{bmatrix} = \begin{bmatrix}
2\alpha \lambda_H R_H^2 & 0 \\
0 & 2\alpha \lambda_H R_H^2
\end{bmatrix}.
\]

Hence the matrix \( A_i \) is positive definite and \( h_i \) is convex. Since the linear combination of the convex functions is also convex, \( H(P) \) is convex. Similarly, we have \( G(P) \) convex of \( P \).

Hence, \( F(P) \) can be written as a difference of the following two convex functions:

\[
F(P) = G(P) - H(P).
\]

(13)

Therefore, we adopt the DC programming to solve this problem. DC programming is a quick convergence programming which can obtain a partial optimal solution and sometimes the global optimal solution of a non-convex function. Since \( \frac{\partial H(P)}{\partial P} \) is continuous, the DC programming can be simply described in Algorithm 1.
Algorithm 1: DC programming for caching placement

1: initial value: \( P^0_{0:} = \frac{M_H}{N_i}, P^0_i = \frac{M_H}{N_i}, i = 1, 2, \ldots N \)
2: solve the convex optimization problem:
   \[
   \min G(P) - H(P_k) - (P - P_k) \frac{H(P_k)}{P_k}
   \]
3: the solution of step 2 is \( P_{k+1} \);
4: if \( \|F(P_k) - F(P_{k+1})\| \leq \varepsilon \) or \( \|P_k - P_{k+1}\| \leq \varepsilon \), \( P_k \) is the optimal solution of \( F(P) \); otherwise, return to step 2;
5: RETURN: the result is: \( F(P_k), \) the solution is: \( P_k \).

V. EXTREME CASE ANALYSES

In this section, we consider the caching problem under extreme cases where only one tier of the caching system is considered and the optimal caching placement can be calculated. We analyze the caching placement and combine the result of the two extreme cases as a baseline and illustrated the performance in section [VII].

A. \( \alpha = 0 \): helper-tier caching network

1) problem formulation: In this case, all users have no caching ability and we only need to optimize the caching placement \( P_H \) at helper side. The offloading probability for the \( i \)-th content is reduced to

\[
P_{i,\text{off}} = P_i^H = 1 - e^{-\pi \lambda_H p_i^H R_H^2} \tag{14}
\]

Problem (11) can be written as

\[
\min_{P_H} - \sum_{i=1}^{N} q_i (1 - e^{-\pi \lambda_H p_i^H R_H^2}) \tag{15}
\]

s.t. \[
\begin{align*}
\sum_{i=1}^{N} p_i^H &\leq M_H \\
0 &\leq p_i^H, i \in \{1, \ldots, N\}
\end{align*} \tag{16}
\]

**Lemma 1:** (Water-filling method) The optimal caching placement of the helpers is

\[
p_i^H = \min \left( \beta + \frac{\ln q_i}{\pi \lambda_H R_H^2}, 1 \right) \tag{17}
\]

for \( i = 1, 2, \ldots, N \), where \( x^+ = \max (x, 0) \) and \( \beta \) is effectively solved by the bisection search with \( \sum_{i=1}^{N} p_i^H = M_H \).

**Proof:** The second derivative of \( P_{i,\text{off}} \) is

\[
\frac{\partial^2 P_{i,\text{off}}}{\partial p_i^H} = -\pi^2 \lambda_H^2 p_i^H e^{-\pi \lambda_H p_i^H R_H^2} < 0, \tag{18}
\]

thus \( -P_{i,\text{off}} \) is convex in \( p_i^H \) and the objective function \( -\sum_{i=1}^{N} q_i P_{i,\text{off}} \) is also convex. Therefore, the caching placement optimization problem is a convex problem. Consider the following Lagrangian

\[
\mathcal{L} = -\sum_{i=1}^{N} q_i (1 - e^{-\pi \lambda_H p_i^H R_H^2}) + \mu \left( \sum_{i=1}^{N} p_i^H - M_H \right) \tag{19}
\]

where \( \mu \) is the Lagrange multiplier. The KKT condition for the optimality of a caching placement is

\[
\frac{\partial \mathcal{L}}{\partial p_i^H} = -\pi \lambda_H R_H^2 q_i e^{-\pi \lambda_H p_i^H R_H^2} + \mu \begin{cases} 
0 & \text{if } p_i^H < 1 \\
\geq 0 & \text{if } p_i^H = 0 \\
\leq 0 & \text{if } p_i^H = 1
\end{cases} \tag{20}
\]

Let \( \beta = \frac{\ln (\pi \lambda_H R_H^2) - \ln \mu}{\pi \lambda_H R_H^2} \) and \( x^+ = \max (x, 0) \), we then have

\[
p_i^H = \min \left( \beta + \frac{\ln q_i}{\pi \lambda_H R_H^2}, 1 \right), \tag{21}
\]

where \( \beta \) can be solved by the bisection search method under the cache storage constraint.

As illustrated in Fig. 2, the water-filling method allocate more cache probability to contents with larger popularity. For instance, the contents with larger popularities under the water level, i.e., the first content and the second contents have been cached in all helpers. While the contents with smaller popularity above the water level, i.e., the 7-th content to the last content, have not been cached in any helper.

**Remark 1:** According to **Lemma 1**, it is straightforward that the most popular contents are cached in helper storage under relatively low helper density i.e., \( p_i^H = \ldots = p_{N_H}^H = 1 \) and \( p_{N_H+1}^H = \ldots = p_N^H = 0 \). While under relatively high density, contents are evenly cached at helper side, i.e, \( p_i^H = \ldots = p_N^H = \frac{M_H}{\lambda H} \).

B. \( \lambda_H M_H = 0 \): user-tier caching network

In this case, no helpers with caching ability participate in offloading the user requests, the optimization problem is reduced to the optimal caching placement of \( P_i^{\text{UE}} \). We thus rewritten the function of offloading probability as:

\[
P_{i,\text{off}} = 1 - (1 - \alpha p_i^{\text{UE}}) e^{-\pi \lambda H p_i^{\text{UE}} R_H^2}. \tag{22}
\]
Then problem (11) becomes

$$\min_{P_{i}^{\text{UE}}} \sum_{i=1}^{N} q_i (1 - (1 - \alpha P_{i}^{\text{UE}}) e^{-\pi \alpha \lambda_{\text{UE}} P_{i}^{\text{UE}} R_{\text{UE}}^2})$$

(23)

s.t. \begin{align*}
0 \leq P_{i}^{\text{UE}} \leq 1, i \in \{1, \ldots, N\} \\
0 \leq P_{i}^{\text{UE}} \leq M_{\text{UE}}
\end{align*}

(24)

**Proposition 2:** The above problem is also a convex problem.

*Proof:* The second derivative of the objective function becomes

$$\frac{\partial^2 P_{i,\text{off}}}{\partial P_{i}^{\text{UE}}^2} = -2b(1 - \alpha P_{i}^{\text{UE}}) e^{-b P_{i}^{\text{UE}}} < 0,$$

(25)

where $b = \pi \alpha \lambda_{\text{UE}} R_{\text{UE}}^2$. Therefore, $-P_{i,\text{off}}$ is convex in $P_{i}^{\text{UE}}$ and the objective function $-\sum_{i=1}^{N} q_i P_{i,\text{off}}$ is also convex.

Therefore, the caching placement optimization problem is convex.

As a result, we can adopt an inter-point method to achieve the optimal solution (10).

**VI. SIMULATIONS**

In this section, we provide some numerical results to verify our analysis and compare the performance of the proposed caching placement with other baselines. Parameter setting and the three baselines are described in Table I and Table II. In particular, we combine the optimal solutions of the two extreme cases as a baseline and named it non-joint optimal caching placement.

Fig. 3 shows that the offloading probability increases with $\lambda_{\text{H}}$. The performance of the proposed caching placement is better than other three baselines no matter how $\lambda_{\text{H}}$ changes. When $\lambda_{\text{H}} = 0$, the performance of the proposed caching placement is equal to the non-joint one, because in this situation there are no helpers joining to offload traffic data.

With the increasing of $\lambda_{\text{H}}$, the performance of the proposed caching placement becomes better than the non-joint one. While $\lambda_{\text{H}}$ is considerable large, non-joint caching placement approaches to the proposed case again. That is because the caching placement of non-joint case is also a optimal solution when there is only helper-tier, and the offloading is mostly consisted of helper-tier in this situation. From Fig. 4 we can draw a similar conclusion about $\lambda_{\text{UE}}$.

Furthermore, from Fig. 3 and Fig. 4 we can see that when both of $\lambda_{\text{H}}$ and $\lambda_{\text{UE}}$ are small, the performance of even-store placement is the worst one. As one of $\lambda$ increase, the performance of even-store placement becomes better. When $\lambda_{\text{H}} = 0.8 \times 10^{-4}$ and $\lambda_{\text{UE}} = 1.2 \times 10^{-2}$, it exceed over the popular-store placement. That is because while there are few devices participating in the caching network, users and helpers need to cache popular contents to cope with the corresponding high request probability thus the popular-store placement performs well. When the resource of the caching network is rich, the offloading probabilities for the most popular contents are easily satisfied, and the surplus storage can be used to cache other relatively less popular contents. So the even-store placement

**TABLE I**

| Parameters                        | values   |
|-----------------------------------|----------|
| D2D communication radius: $R_{\text{UE}}$ | 15 (m)   |
| helpers transmit radius $R_{\text{H}}$: | 100 (m)  |
| the proportion of cache-enabled users $\alpha$: | 0.5      |
| the intensity of users $\lambda_{\text{UE}}$: | 5000/\pi 500^2 |
| the intensity of helpers $\lambda_{\text{H}}$: | 50/\pi 500^2 |
| the cache size of users and helpers | $M_{\text{UE}} = 2$, $M_{\text{H}} = 8$ |
| the skewness of the popularity $\gamma$: | 30       |
| the size of content library $N$: | 1        |

**TABLE II**

| Schemes     | caching schemes of users | caching schemes of helpers |
|-------------|--------------------------|---------------------------|
| popular     | $P_{i}^{\text{UE}} = 1, i \in [1, M_{\text{UE}}]$ | $P_{i}^{\text{H}} = 1, i \in [1, M_{\text{H}}]$ |
| even        | $P_{i}^{\text{UE}} = 0, j \in [M_{\text{UE}} + 1, N]$ | $P_{j}^{\text{H}} = 0, j \in [M_{\text{H}} + 1, N]$ |
| Non-joint   | $P_{i}^{\text{UE}} = M_{\text{UE}}/N$ | $P_{i}^{\text{H}} = M_{\text{H}}/N$ |

**Proposition 2:** The above problem is also a convex problem.

*Proof:* The second derivative of the objective function becomes

$$\frac{\partial^2 P_{i,\text{off}}}{\partial P_{i}^{\text{UE}}^2} = -2b(1 - \alpha P_{i}^{\text{UE}}) e^{-b P_{i}^{\text{UE}}} < 0,$$

(25)

where $b = \pi \alpha \lambda_{\text{UE}} R_{\text{UE}}^2$. Therefore, $-P_{i,\text{off}}$ is convex in $P_{i}^{\text{UE}}$ and the objective function $-\sum_{i=1}^{N} q_i P_{i,\text{off}}$ is also convex. Therefore, the caching placement optimization problem is convex.

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Furthermore, from Fig. 3 and Fig. 4 we can see that when both of $\lambda_{\text{H}}$ and $\lambda_{\text{UE}}$ are small, the performance of even-store placement is the worst one. As one of $\lambda$ increase, the performance of even-store placement becomes better. When $\lambda_{\text{H}} = 0.8 \times 10^{-4}$ and $\lambda_{\text{UE}} = 1.2 \times 10^{-2}$, it exceed over the popular-store placement. That is because while there are few devices participating in the caching network, users and helpers need to cache popular contents to cope with the corresponding high request probability thus the popular-store placement performs well. When the resource of the caching network is rich, the offloading probabilities for the most popular contents are easily satisfied, and the surplus storage can be used to cache other relatively less popular contents. So the even-store placement
performs better and the offloading probability of popular-store placement no longer increases. When \( \lambda_{\text{UE}} \) is considerable large, the performance of even-store placement approaches to the optimal caching placement. In Fig. 5 we demonstrate the proposed caching placement which is calculated by DC programming when there are five contents. As \( \lambda_{\text{UE}} \) increases, we can see that the optimal caching placement changes from a popular-store placement to a even-store placement which is consistent with our analysis.

Fig. 5. the caching placement

Fig. 6. The impact of \( \alpha \) on the offloading probability

Fig. 7. The impact of \( \gamma \) on the offloading probability

Fig. 8. The impact of \( N \) on the offloading probability

In this paper, \( \gamma \) is denoted as the skewness of content popularity. While \( \gamma \) is large, the user requests focus on the popular contents and the caching system may have large probabilities to cache the "right" contents. Therefore the offloading probability usually increases with \( \gamma \) and we show it in Fig. 7. The performance of popular-store placement grows rapidly with increasing \( \gamma \) while the performance of even-store placement is not affected by \( \gamma \), because it stores every content with a same probability.

Fig. 8 illustrates that the offloading probability decreases with \( N \). To improve the size of content library \( N \), in a sense, is similar to decrease the storage capacity \( M \), thus the offloading probability will experience a decline accordingly. However we can notice that the performance of our proposed caching placement still performs well. It demonstrates that when the system is applied into a multi-contents situation, the proposed caching placement can finely adjust the caching proportion of each content by a joint optimization and keep a good performance.

VII. CONCLUSION

In this paper, the optimal caching placement are proposed to maximize the total offloading probability for the D2D assisted wireless caching network. Specifically, the caching placement problem for the two-tier caching network is formulated as a DC problem and be solved by the DC programming. In
addition, the extreme analysis are provided for helper-tier (or user-tier) caching case in absence of the other tier. The caching placements for both cases are proved to be convex. Moreover, the classical water-filling method was adopted to solve the helper-tier caching case. Simulation results indicate the most popular contents are ought to be cached under a relatively low node density, while contents are cached evenly for a relatively high node density. And our proposed caching placement can always make a balance of that.

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