An Effective Operators Analysis of CP Violation: The Semileptonic Case

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Abstract: Aiming at a model-independent analysis of possible new physics effects in semileptonic processes at various energy scales, we list and study a complete set of $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant 4-Fermi operators which consist of a pair of quarks and a pair of leptons above the electroweak symmetry breaking. We give a full 1-loop renormalization group treatment of the evolution of the Wilson coefficients associated with these 4-Fermi operators between low energy ($\sim$ meson masses) and the cutoff scale $\Lambda$, $\sim (1 - 10)$ TeV, where we assume new degree of freedom beyond standard model will begin to appear and an ultra-violet completion of our effective theory will take place.

Motivated by the existing phenomenological bounds, we argue that the new CP violation can only stem from the scalar and tensor types of 4-Fermi interaction. Some interesting constraints are obtained by studying the universality of kaon and pion leptonic decays, CP violating polarization of $K^\pm_{\mu3}$, charged lepton anomalous magnetic moments, and $(\mu \rightarrow e\gamma)$ like rare decays. In particular, we can use the limit of electron dipole moment to constrain the size of the CP violating triplet correlation in the $e^+e^- \rightarrow t\bar{t}$ process.

KEYWORDS: CP violation, Beyond Standard Model, Renormalization Group.
1. Introduction

Intense experimental efforts in the search for new physics have resulted in establishing the Standard Model (SM) as a very good effective theory at the weak scale given by the Higgs boson vacuum expectation value of $v \simeq 250 \text{ GeV}$ and below. Yet there are strong arguments that it is one with a cutoff scale $\Lambda$ much lower than the Planck scale; perhaps as low as a few TeV. More importantly the observation of neutrino mixings have given the first hint new physics beyond $v$. The exact form of physics beyond the SM is unknown. One can proceed by making a guess at this new physics and engage in constructing consistent models. The other approach is to make use of the effective field theory (EFT). The main assumption here is that at energies below $\Lambda$ physical observables are largely insensitive to the unknown new physics. The effective Lagrangian is then a sum of the SM term and non-renormalizable ones which are the results of integrating out the unknown degrees of freedom. This bottom up approach clearly has its drawbacks. One of them being that there can be many such non-renormalizable terms. This is not as hopeless as it seems. This operators can be classified by their dimensions and the higher ones are suppressed by higher powers of $1/\Lambda$. Strictly speaking the cutoff scale for each set of operators need not be the same. Hence, we need only focus on the lowest dimension operators.

The EFT approach is built upon two crucial ingredients. The first one to correctly identify the symmetry operative below the scale $\Lambda$. The second one is to know the degrees of freedom. The phenomenal success of the SM in confronting experiments suggest strongly that we take the gauge symmetry of the SM to be operative between $\Lambda$ and $v$. Below the
electroweak scale the gauge symmetry is $SU(3) \times U(1)$. To this we may also incorporate baryon number so as to ensure proton stability. This is not mandatory as is well known that operators leading to proton decay can be suppressed by mass scale of order $10^{16}$ GeV. While taking the SM as the gauge symmetry below $\Lambda$ is largely not controversial the same cannot be said about what states to include in constructing an EFT below $\Lambda$. The most conservative paradigm is to take only the SM fields of 45 chiral fermions plus the gauge bosons and one Higgs doublet. The effective Lagrangian will then take the form

$$L^{\text{eff}} = L_{\text{SM}} + \frac{1}{\Lambda'} L_5 + \frac{1}{\Lambda^2} L_6 + \cdots.$$  (1.1)

Here $L_5$ is the dimension 5 operator constructed from the neutrino and Higgs fields which is responsible for generating Majorana neutrino masses for the active neutrinos. The seesaw scale is denoted by $\Lambda'$ which may or may not be the same as $\Lambda$ depending on the origin of neutrino masses. If this is non-vanishing then lepton number is not conserved. On the other hand neutrinos may be Dirac particle in which case $L_5$ vanished\(^\dagger\). The details of how neutrinos get their masses will not be important and we only note that active neutrinos are massive and they mix. $L_6$ is a sum of dimension 6 operators composed of chiral fermions, gauge bosons, and the Higgs field. The dots are higher dimension operators suppressed by higher powers of $1/\Lambda$. The number of operators in $L_6$ is over 20\(^\dagger\) even after the use of equations of motion and not counting family dependence. Clearly a purely phenomenological analysis will be unwieldy. It will be a more tractable problem if we select a subset of these that are closed under renormalization to 1-loop and analyze their physical effects.

We begin with the dimension 6 four-Fermi operators that are made up of a pair of leptons and a pair of quarks. We are especially interested in the ones give CP violating (CPV) effects\(^*\). There are 10 such operators which we will divide into two groups: the vector type, and the scalar and tensor type. The vector terms are explicitly,

\begin{align*}
O_{V1}^{ij,kl} &= (\bar{Q}^i \gamma^\mu Q^j)(\bar{L}^k \gamma^\mu L^l) , \\
O_{V2}^{ij,kl} &= (\bar{Q}^i \gamma^\mu Q^j)(\bar{L}_b^k \gamma^\mu L_a^l) , \\
O_{V3}^{ij,kl} &= (\bar{Q}^i \gamma^\mu Q^j)(\bar{e}_k \gamma^\mu e^l) , \\
O_{V4}^{ij,kl} &= (\bar{d}^i \gamma^\mu d^j)(\bar{L}^k \gamma^\mu L^l) , \\
O_{V5}^{ij,kl} &= (\bar{u}^i \gamma^\mu u^j)(\bar{L}^k \gamma^\mu L^l) , \\
O_{V6}^{ij,kl} &= (\bar{d}^i \gamma^\mu d^j)(\bar{e}_k \gamma^\mu e^l) , \\
O_{V7}^{ij,kl} &= (\bar{u}^i \gamma^\mu u^j)(\bar{e}_k \gamma^\mu e^l) ,
\end{align*}

and the remaining scalar and tensor terms are

\begin{align*}
O_{S1}^{ij,kl} &= (\bar{Q}^i d^j)(\bar{e}^k L^l) ,
\end{align*}

\(^\dagger\)It is sufficient to have a $U(1)_{B-L}$ symmetry to ensure proton stability and no Majorana neutrino mass. However, we are mindful that Planck scale physics is likely to break such global symmetry.

\(^*\)Since we are considering only semileptonic decays and lepton dipole moments the 4-quark operators will not be included here.
\[ O_{S2}^{ij,kl} = (\bar{Q}^i_a u^j)(\bar{L}^k_b e^l)\epsilon^{ab}, \quad (1.10) \]
\[ O_{T}^{ij,kl} = (\bar{Q}^i_a \sigma^{\mu\nu} u^j)(\bar{L}^k_b \sigma_{\mu\nu} e^l)\epsilon^{ab}, \quad (1.11) \]

where \( Q, L, e, d, u \) are respectively the left-handed quark doublets, left-handed lepton doublets, right-handed charged leptons, down and up-type quarks of the SM. \( a, b \) are \( SU(2) \) indices and \( i, j, k, l \) are family indices. We have also suppressed the chiral projection operators. We use the convention that repeated indices are summed over and our metric \( g^{\mu\nu} \) is \((+ - - -)\) with \( \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \). The purely leptonic 4-Fermi operators are recently studied in detail in [2] and we shall follow the notations given there. Then in the weak basis we write

\[ -\mathcal{L}_6 = \sum_{A=1}^{7} C_{V,A}^{ij,kl} O_{V,A}^{ij,kl} + \sum_{A=1}^{2} C_{S,A}^{ij,kl} O_{S,A}^{ij,kl} + C_{T}^{ij,kl} O_{T}^{ij,kl} + \text{h.c.} \quad (1.12) \]

In general the Wilson coefficients denoted by \( C \)'s depend on the family index. Also hermiticity requires the 7 vector Wilson coefficients to satisfy

\[ C_{V}^{ijk} = (C_{V}^{ijkl})^*. \quad (1.13) \]

However, for the scalar and tensor types there is no such requirement. Clearly the most general flavor structure is too unwieldy for a fruitful analysis and we shall make phenomenologically motivated simplification that hopefully reflects some dynamics of the yet unknown new physics.

Even after going over to the mass basis of the fermions there will be flavor changing neutral currents (FCNC) generated at 1-loop. These are known to be highly suppressed. Following the discussions in [2] we make the ansatz

\[ C_{V,A}^{ij,kl} = C_{V,A}^{ii,kk} \delta^{ij} \delta^{kl}, \quad (1.14) \]

where \( C_{V,A}^{ii,kk} \) are constants that depend on the quark and lepton families. The operator \( O_{V,2} \) contains charged weak currents. Universality tests of tau and Pion decays suggest that their Wilson coefficients should be independent of lepton flavors. Thus it is reasonable to assume

\[ C_{V,2}^{ii,kk} = C_{V,2}^{ii}. \quad (1.15) \]

We shall see in the next section that renormalization will mix \( O_{V,1} \) and \( O_{V,2} \) and hence to satisfy the above mentioned experimental constraints we take \( C_{V,1}^{ii,kk} = C_{V,1}^{ii} \). Thus, we are emboldened to extend it to all vector Wilson coefficients and drop the dependence on the families indices. In fact most four dimensional unified models have this feature. Some notable exceptions are provided by extra dimension models and split fermion models [3].

With that in mind we rewrite the effective Lagrangian in the mass eigenbasis as

\[ -\mathcal{L}_6 = \sum_{A=1}^{7} C_{V,A}^{ii,kk} O_{V,A}^{ii,kk} + \sum_{A=1}^{2} C_{S,A}^{ij,kl} O_{S,A}^{ij,kl} + C_{T}^{ij,kl} O_{T}^{ij,kl} + \text{h.c.} \quad (1.16) \]

From now on all our operators expressed will be in the mass eigenbasis. Notice that the flavor insensitivity of the vector operators are not extended to the scalar and tensor
operators. Certainly the $C_S,T$'s in Eq. (1.16) differ from those in Eq. (1.12) by products of rotational matrices that diagonalize the fermion masses. For simplicity we have kept the same notation.

There are eight dimension-6 dipole operators with two fermions. They are listed as

$$D_{ij}^1 = H\bar{L}^i\sigma^{\mu\nu}e^jB_{\mu\nu},$$

(1.17)

$$D_{ij}^2 = H\bar{L}^i\sigma^{\mu\nu}e^jW_{\mu\nu},$$

(1.18)

$$D_{ij}^3 = \tilde{H}\bar{Q}^i\sigma^{\mu\nu}u^jB_{\mu\nu},$$

(1.19)

$$D_{ij}^4 = \tilde{H}\bar{Q}^i\sigma^{\mu\nu}u^jW_{\mu\nu},$$

(1.20)

$$D_{ij}^5 = H\bar{Q}^i\sigma^{\mu\nu}d^jB_{\mu\nu},$$

(1.21)

$$D_{ij}^6 = H\bar{Q}^i\sigma^{\mu\nu}d^jW_{\mu\nu},$$

(1.22)

$$D_{ij}^7 = H\bar{Q}^i\sigma^{\mu\nu}d^jG_{\mu\nu},$$

(1.23)

$$D_{ij}^8 = \tilde{H}\bar{Q}^i\sigma^{\mu\nu}d^jG_{\mu\nu},$$

(1.24)

where $\tilde{H} = i\sigma^2H^*$, and $B, W, G$ are the field strength of $U(1)_Y, SU(2)_L$ and $SU(3)$ respectively.

Notice that the operators are in general not flavor diagonal. After electroweak symmetry breaking (EWSB) we can substitute $H \to (v + h^0)/\sqrt{2}$. The terms from Eq.(1.17,1.18) become

$$-\mathcal{L}_6^D = \frac{vC_{ij}^{ij}}{\sqrt{2}}(\bar{e}^i\sigma^{\mu\nu}\tilde{e}^jF_{\mu\nu}) + h.c.$$  

(1.25)

They will induce flavor violating or conserving radiative transitions that result in lepton electric dipole moments (EDM's), $(g - 2)_\mu$, $\mu \to e\gamma$, and $b \to s\gamma$ at the level of the Born term. All these are strongly constrained by null experimental observation beyond SM \[2\]. For example, the muon $g - 2$ constrains $Re\, C_{ij}^{ij}$ and electron EDM limits $Im\, C_{ij}^{ij}$ and they have to be $< 10^{-6}$. From this observation we shall adopt a phenomenologically motivated assumption that any reasonable new physics beyond SM will only induce negligible tree-level dipole operators.

Proceeding further one recognizes that the dipole operators can be induced at 1-loop by the 4-Fermi operators and SM interactions. In Fig.1 we show the possible 1-loop diagrams that will mix the 4-Fermi operators and the dipole operators. The first three terms are vanishing from the above discussion. Since only the t-quark enjoys $O(1)$ Yukawa coupling and thus the mixing involving other light quarks are severely suppressed by their small Yukawa couplings and can be neglected. Fig.1(d) shows that we only need to consider the dipole term induced from the tensor operator involving the third generation quarks, $O_{ij}^{33,kl}$. Thus, we only need to consider the first two dipole operators involving 2 leptons. At low energies for EDM studies the relevant operator involves the photon thus we define

$$O_{ij}^{D\gamma} = \bar{L}^i\sigma^{\mu\nu}\tilde{e}^jF_{\mu\nu}H + h.c.$$  

(1.26)

and the Wilson coefficient is $C_{ij}^{D\gamma} = c_{d1}^{ij}\cos \theta_w + c_{d2}^{ij}\sin \theta_w$ in standard notations. The
The complete effective Lagrangian at the weak scale is now

$$- \mathcal{L}_6 = \sum_{A=1}^{7} C_{VA} O_{VA}^{ij,kk} + \sum_{A=1}^{2} C_{SA}^{ij,kl} O_{SA}^{ij,kl} + C_{T}^{ij,kl} O_{T}^{ij,kl} + \sum_{A=1}^{2} C_{DA}^{ij} O_{DA}^{ij} + \text{h.c.} \quad (1.27)$$

The physics is clearer if we expand $\mathcal{L}_6$ in terms of their components. We split it into the charged current (CC) terms and the neutral current (NC) terms. After some algebra the NC terms are given by, $(\hat{L} = \frac{1-\gamma_5}{2}, \hat{R} = \frac{1+\gamma_5}{2})$,

$$- \mathcal{L}_6^{NC} = \tau^{\mu} \gamma^{\mu} \left[ (C_{V1} + C_{V2}) \hat{L} + +C_{V5} \hat{R} \right] u^i \times (\vec{\tau}^k \gamma_\mu \hat{L} \nu^k)$$
$$+ \vec{d}^{\mu} \gamma^{\mu} \left[ C_{V1} \hat{L} + C_{V4} \hat{R} \right] d^i \times (\vec{\tau}^k \gamma_\mu \hat{L} \nu^k)$$
$$+ \vec{e}^{\mu} \gamma^{\mu} \left[ C_{V1} \hat{L} + C_{V5} \hat{R} \right] u^i \times (\vec{\tau}^k \gamma_\mu \hat{L} \nu^k)$$
$$+ \vec{d}^{\mu} \gamma^{\mu} \left[ (C_{V1} + C_{V2}) \hat{L} + C_{V4} \hat{R} \right] d^i \times (\vec{\tau}^k \gamma_\mu \hat{L} \nu^k)$$
$$+ \vec{e}^{\mu} \gamma^{\mu} \left[ C_{V3} \hat{L} + C_{V7} \hat{R} \right] u^i \times (\vec{\tau}^k \gamma_\mu \hat{R} e^k)$$
$$+ \vec{d}^{\mu} \gamma^{\mu} \left[ C_{V3} \hat{L} + C_{V6} \hat{R} \right] d^i \times (\vec{\tau}^k \gamma_\mu \hat{R} e^k)$$
$$+ C_{S1}^{ij,kl} \left( \vec{d} \sigma^{\mu \nu} \hat{R} u^l \right) (\vec{\tau}^k \hat{L} e^l) + C_{S2}^{ij,kl} \left( \vec{e} \sigma^{\mu \nu} \hat{R} d^l \right) (\vec{\tau}^k \hat{L} e^l)$$
$$+ C_{T}^{ij,kl} \left( \vec{d} \sigma^{\mu \nu} \hat{R} u^l \right) (\vec{e} \sigma_\mu \hat{R} e^l) + \text{h.c.} \quad (1.28)$$

If one probes the effective neutral current couplings using neutrinos versus electrons one would find the corrections to the SM are very different. Interestingly the NC effects of the electron can contain terms with different Lorentz structure then that of the SM. On the other hand the new NC couplings of the neutrinos are only of the vector and axial vector.
types. This follows from the assumption that there are no right-handed neutrinos below \( \Lambda \) and the gauge symmetry of the SM. The structure of Eq.\((1.28)\) also points to polarized electrons being powerful and versatile probes of new physics.

The CC terms are explicitly:

\[
-L_6^{CC} = C_{V2}' \left( \overline{u}^i \gamma_\mu \hat{L} d^i \right) \left( \overline{E}^j \gamma_\mu \hat{L} \nu^j \right) + C_{ij,kl} \left( \overline{u}^i \sigma^{\mu\nu} \hat{R} d^j \right) \left( \overline{\nu}^k \sigma^{\mu\nu} \hat{R} \nu^l \right) - C_{ij,kl} \left( \overline{d} \sigma^{\mu\nu} \hat{R} u^j \right) \left( \overline{\nu}^k \sigma^{\mu\nu} \hat{R} \nu^l \right) - C_{ij,kl} \left( \overline{d} \hat{R} u^j \right) \left( \overline{\nu}^k \hat{R} \nu^l \right) + \text{h.c.} \tag{1.29}
\]

\( C_{V2}' \) differs from its NC counterpart by quark and lepton rotational matrices. In principle if the parameters involved are all measured in the future the relation between \( C_{V2} \) and \( C_{V2}' \) will be a test the above framework. Currently we only have limits on these parameters. The Eq.\((1.29)\) renormalizes the canonical 4-Fermi interaction; thus \( C_{V2}' \) can be revealed by precision measurements comparing tau and muon leptonic decays with neutron beta decays as done in classic universality tests. The scalar and tensor terms adds incoherently and hence are not important. Nonetheless, these latter terms are very interesting since their discovery will be strong indication of new physics. They can be probed by \( K \) and \( B \) meson decays; for example by measuring the energy spectral of the charged leptons in semileptonic decays. This is analogous to what we have studied previously in \( \mu \) decays [2].

Furthermore, the Wilson coefficients are in general complex and thus can give rise to novel CP violation effects via interference with a SM amplitude. A prime example will be muon polarization measurements in \( K^+ \rightarrow \pi^0 \mu^+ \nu \) which are unique low energy probes of these terms [4]. Details will be given later.

Now we return to more theoretical issues. The Wilson coefficients are defined at the scale \( \Lambda \). In a top down approach one would be able to calculate them in terms of the parameters of the new physics theory. The bottom up path taken here does not afford such luxury. We can only treat them as parameters to be determined or constrained by experiments. To date all relevant CPV experiments are done at energies much lower than \( \Lambda \). We must then evolve the Wilson coefficients from \( \Lambda \) to \( v \) and then to the scale of \( K, B \) mesons or charged lepton masses. This is done by solving the renormalization group (RG) equations for the Wilson coefficients and matching the boundary conditions as one crosses each mass scale. The results here can be used to relate CP violation quantities measured at high energies at colliders to those at lower energies and vice versa. As an example some leptonic processes cases are studied in [4].

In this paper we are more interested in processes below the weak scale. In the next section we give a discussion of the evolution of the Wilson coefficients from \( \Lambda \) to the weak scale and calculated the anomalous dimension of operators. Below the weak scale we integrate out the t-quark and the heavy SM gauge bosons. The RG running then will involve only \( U(1)_{em} \) and \( SU(3)_c \) and will be discussed in detail. We choose the low energy boundary condition at \( m_p = 1 \) GeV. In section III we use the effective Hamiltonian and fold in the appropriate form factors to estimate the strength of signatures for CPV experiments in \( K \) meson and pion as well as semileptonic decays, and the electric dipole moments of lepton and neutron. They give stringent limits on the scalar and tensor coefficients. As an
example of the synergy between precision low energy measurements and high energy collider experiments we calculated the CPV triple momentum-spin-spin correlation in $e^+e^- \rightarrow t + \bar{t}$ at the linear collider. Our results are summarized in the concluding Sec.IV.

\[ \Lambda \]

\[ \mathcal{L}_{SM} + \sum \frac{\Lambda^2}{\Lambda^2} O \] Probed by Colliders

\[ SU(3)_c \times SU(2)_L \times U(1)_Y \]

\[ \mathcal{L}_{QED} + \mathcal{L}_{QCD} + \sum \frac{\Lambda^2}{\Lambda^2} O' \]

RGE

\[ U(1)_QED \times SU(3)_c \]

\[ \mu = m_p \]

EDM, Meson Decays

**Figure 2:** The connection between high and low energy interactions in effective field theory. The symmetry at each mass scale is given. All notations are given in the text.

A schematic representation of this approach is given in Fig. (2). Although we have made the very conservative assumption of no new states below \( \Lambda \) the effective operative approach can be easily generalized. For example if there were sterile neutrinos below \( \Lambda \) the list of additional lepton operators is given in [2]. If the minimal supersymmetric standard model turns out to be the correct description of physics just beyond the standard model then \( \Lambda \) can be identified with the supersymmetry breaking scale. Moreover, the operator list will have to be extended to include supersymmetric state or states and care has to be taken to respect the \( R \)-parity.

### 2. Effective Hamiltonian for Leptonic and Semileptonic Meson Decays

We begin by calculating the quark level effective Hamiltonian for $d_i \rightarrow u_j l \nu$. This given by the low energy SM with the W boson, Higgs boson, and heavy quarks integrated out plus the relevant terms in Eq.(1.29) with the Wilson coefficients taken at some energy \( \mu \) below the weak scale. The term \( C'_V \) is expected to make a undetectable correction to the Fermi coupling constant \( G_F \) since we are taking \( \Lambda \) to be TeV or higher. Thus, to leading order in weak interactions and the new physics scale \( \Lambda \) the effective Hamiltonian is given by

\[
\mathcal{H}_{\text{eff}}^{d \rightarrow \ell}(\mu) = 4 \frac{G_F}{\sqrt{2}} A(\mu) (V_{ij} P_h \tilde{u}_i \gamma_\mu \tilde{L}_d_j)(\tilde{L}_n \gamma_\mu \tilde{L}_n) \\
+ \frac{C_{ij,kl}^{ij,kl}(\mu)}{\Lambda^2} (\tilde{u}_i \tilde{R}d_j)(\tilde{L}_n \tilde{L}_n) - \frac{C_{ij,kl}^{ij,kl}(\mu)}{\Lambda^2} (\tilde{d}_i \tilde{R}u_j)(\tilde{\nu}_k \tilde{R}l_k) \\
- \frac{C_{ij,kl}^{ij,kl}(\mu)}{\Lambda^2} (\tilde{d}_i \sigma^{\mu\nu} \tilde{R}u_j)(\tilde{\nu}_k \sigma_{\mu\nu} \tilde{R}l_k) \\
+ h.c.
\]

(2.1)
where $V_{ij}$ is an element of the CKM matrix and $P_{lk}$ is an element of the neutrino mixing matrix. The Wilson coefficient $A$ is unity in the free quark limit and are modified by SM gauge interactions as ones goes from $\Lambda$ to the electroweak scale and then further down. The QED and QCD running of $A(\mu)$ is well known whereas similar behaviors of scalar and tensor coefficients, to the best of our knowledge, have not been presented before. Hence we shall briefly discuss below how one can obtain the leading logarithmic (LL) contributions by renormalization group methods. This also serves to establish our notations.

The operators $O_{S,V,T,D}$ which we shall generically denote by $O_6^A$ are bare operators. Upon renormalization they will mix via the equation

$$O_6^A = \sum_{B=1}^{10} Z_{AB}^{-1} (Z_L)^{n/2} (Z_e)^{m/2} (Z_Q)^{r/2} (Z_q)^{s/2} O_6^B$$

where $Z_L$, $Z_e$, $Z_Q$ and $Z_q$ $(q = u, d)$ are the wave function renormalization constants for the various fermion fields. $n, m, r, s$ are the number of such fields in each of the $O_A$ operators. Thus, $n, m, r, s = 0, 1, 2$. The sum here runs over the 10 terms of $O_S, O_V$ and $O_T$ and the prime denotes renormalized quantity.

The renormalized operator $O_6^B$ will depend on the t’Hooft renormalization scale $\mu$ whereas the bare operators do not. Correspondingly the $\mu$-dependence of the Wilson coefficients will be such as to render the renormalized effective Lagrangian $\mathcal{L}_6^\mu$ independent of $\mu$. This leads to the renormalization group equation (RGE) for the coefficients in Eqs.(1.16):

$$\mu \frac{d}{d\mu} C_A + \sum_B \gamma_{AB} C_B = 0$$

where $\gamma_{AB}$ is the anomalous dimension matrix which is non-diagonal. The values of $C_A$ at any scale $m$ are obtained by solving the above equation plus boundary conditions which are the values given at another scale say the weak scale $M_W$. At present there are only limits on a few of these coefficients from LEPII measurements. We now return to discuss $\gamma_{AB}$ calculation.

Below the weak scale the renormalization of the operators are given by photon and gluon exchanges. We will ignore all Yukawa couplings except that of the t-quark. The calculation becomes the evaluation of the high energy parts of the depicted diagrams in Fig. 3. The wave function renormalization graphs also must be calculated but not shown.

![Figure 3](image)

**Figure 3**: 1-loop corrections to the 4-Fermi effective operators by SM gauge bosons represented by the wavy lines.
\[
\gamma_{4F} = \frac{1}{4\pi} \begin{pmatrix}
G_1 & 2\alpha_1 & 0 & 0 & 0 & 0 \\
0 & -2\alpha_1 & 0 & 0 & 0 & 0 \\
0 & 0 & 4\alpha_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 4\alpha_1 & 0 & 0 \\
0 & 0 & 0 & 0 & -8\alpha_1 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{8}{3}\alpha_1 - 8\alpha_3
\end{pmatrix}
\]

in the basis of \(\{c_{V1, V7}, c_{S1}, c_{S2}, c_T\}\). And \(G_{1,2}\) are the two by two matrices:

\[
G_1 = \begin{pmatrix}
-\alpha_1 - 3\alpha_2 & 8\alpha_2 \\
6\alpha_2 & -\alpha_1 - 7\alpha_2
\end{pmatrix},
\]

\[
G_2 = \begin{pmatrix}
-\frac{11}{5}\alpha_1 - 8\alpha_3 & 30\alpha_1 - 6\alpha_2 \\
\frac{3}{5}\alpha_1 + \frac{2}{3}\alpha_2 & \frac{2}{5}\alpha_1 + \frac{8}{3}\alpha_3
\end{pmatrix}.
\]

After diagonalization and putting this into Eq. (2.3) and solving it gives the running of the scalar and tensor Wilson terms between \(\Lambda\) and \(M_W\).

To get a qualitative feeling of the RG running above the electroweak scale we first take the running of SU(2) and U(1) to be much smaller than from SU(3) when the cutoff is below 10 TeV. This is in accordance to usual expectation. Considering the SU(3) running alone leads to following simple solution. Suppressing indices we define the enhancing factor

\[
\mathcal{G}(M_W, \Lambda) \equiv \frac{C(\Lambda)}{C(M_W)}.
\]

Then we have

\[
\mathcal{G}_{V1, V7}(M_W, \Lambda) \sim 1.0 \pm \mathcal{O}(0.01)
\]

since only U(1) and SU(2) running play a role here. On the other hand QCD running affects the other coefficients and we have

\[
\mathcal{G}_{S1}(M_W, \Lambda) \sim \mathcal{G}_{S2}(M_W, \Lambda) \sim \left( \frac{\alpha_3(M_W)}{\alpha_3(M_t)} \right)^{-\frac{12}{23}} \left( \frac{\alpha_3(M_t)}{\alpha_3(\Lambda)} \right)^{-\frac{1}{23}},
\]

\[
\mathcal{G}_T(M_W, \Lambda) \sim \left( \frac{\alpha_3(M_W)}{\alpha_3(M_t)} \right)^{\frac{1}{23}} \left( \frac{\alpha_3(M_t)}{\alpha_3(\Lambda)} \right)^{\frac{11}{23}}.
\]

At \(\Lambda = 1\) TeV, we find that \(\mathcal{G}_{S1,2} = 0.865\) and \(\mathcal{G}_T = 1.049\). We can also give approximate expressions for the above quantities:

\[
\mathcal{G}_{S1,2}(M_W, \Lambda) \sim 0.9456 \left( \frac{m_t}{\Lambda} \right)^{0.036}, \quad \mathcal{G}_T(M_W, \Lambda) \sim 1.0188 \left( \frac{\Lambda}{m_t} \right)^{0.012}
\]

where the number factors in front of the parentheses are the result of RG running from \(M_W\) to \(m_t = 174\) GeV.

Below the EWSB scale the operating gauge symmetry is \(U(1)_{em} \times SU(3)_c\). It is well known that there are no large LL contribution from QCD for semileptonic decays in the
SM. The perturbative QCD corrections have been calculated and is found to be small [6]; henceforth, we shall neglect it in $A(\mu)$. However, QED gives rise to LL corrections in $A(\mu)$ [7] and the beta function is obtained from a subset of the diagrams listed in Fig.3.

Explicitly we have

$$A(m_i, m_W) = \left[ \frac{\alpha(m_r)}{\alpha(m_i)} \right]^{\alpha^{-1}(m_W)} \left[ \frac{\alpha(m_b)}{\alpha(m_r)} \right]^{\alpha^{-1}(m_b)} \left[ \frac{\alpha(M_W)}{\alpha(m_b)} \right]^{\alpha^{-1}(M_W)}$$

(2.12)

where $i = d, s$. In the above we have integrated out simultaneously the t-quark and the gauge bosons and then the light fermions in succession. $m_i$ is either the strange quark mass or the proton mass and we have normalized $A(m_i, m_i)$ to unity. Using the values

$$\alpha^{-1}(M_W) = 128.0, \quad \alpha^{-1}(m_b) = 132.14, \quad \alpha^{-1}(m_r) = 133.33, \quad \alpha^{-1}(m_c) = 133.69, \quad \alpha^{-1}(m_p) = 133.91,$$

(2.13)

where we choose $m_b = 4.3$ GeV, $m_c = 1.3$ GeV, and $m_p \sim 1$ GeV. We find

$$A(m_p, M_W) = 1.0107,$$

(2.14)

which is an important correction for precision measurements.

For $C_{S1}$, we find

$$\gamma_{S1} = \frac{1}{4\pi} \left( \frac{4}{3} - 8\alpha_3 \right),$$

(2.15)

and have the solution:

$$C_{S1}(m_p) = \left[ \frac{\alpha(m_c)}{\alpha(m_p)} \right]^{\alpha_3(m_c)} \left[ \frac{\alpha(m_r)}{\alpha(m_c)} \right]^{\alpha_3(m_r)} \left[ \frac{\alpha(m_b)}{\alpha(m_r)} \right]^{\alpha_3(m_b)} \left[ \frac{\alpha(M_W)}{\alpha(m_b)} \right]^{\alpha_3(M_W)} \times \left[ \frac{\alpha_3(m_c)}{\alpha_3(m_p)} \right]^{\frac{\alpha_3(m_b)}{\alpha_3(m_c)}} C_{S1}(M_W).$$

(2.16)

Using the boundary condition $\alpha_s(M_W) = 0.120$ and the standard 1-loop QCD beta function, we get

$$\alpha_3(m_b) = 0.211, \quad \alpha_3(m_c) = 0.316, \quad \alpha_3(m_p) = 0.359,$$

(2.17)

and

$$C_{S1}(m_p) = (0.9965) \times (1.7232) \times C_{S1}(M_W),$$

(2.18)

where the first and second brackets are the QED and QCD RG running effect respectively. We see that the RG effects enhance the coefficient almost by factor 2.
The running of $C_{S2}$ and $C_T$ are more complicated since they are coupled:

$$\gamma_{ST} = \frac{\alpha}{4\pi} \begin{pmatrix} 4/3 & 8 \\ 1/6 & -40/9 \end{pmatrix} + \frac{\alpha_3}{4\pi} \begin{pmatrix} -8 & 0 \\ 0 & 8/3 \end{pmatrix}$$ \tag{2.19}

in the basis of $\{C_{S2}, C_T\}^T$.

An analytic solution for Eq. (2.19) is unavailable. For a solution numerical methods are required. However, since the QED effect is expected to be much smaller than QCD; for simplicity, we can just keep the QCD part to have a rough idea how the Wilson coefficient evolves. Using the same parameter setting as in Eq. (2.18), we get

$$C_{S2}(m_p) \sim 1.72 C_{S2}(M_W), \quad C_T(m_p) \sim 0.83 C_T(M_W),$$ \tag{2.20}

which should be sufficient for order of magnitude estimates of new physics effects. It is interesting to note that the scalar coefficients increases as the energy decreases whereas the tensor one does the opposite.

### 3. Phenomenology

#### 3.1 Leptonic Decays of K and $\pi$ Mesons

We can now apply the above formalism to leptonic decays of pseudoscalar mesons. In particular the branching ratios $R_\pi = B(\pi \to e\nu/\pi \to \mu\nu)$ and $R_K = B(K \to e\nu/K \to \mu\nu)$ which are precisely measured. Theoretically they have been precisely calculated in the SM with the uncertainties below the experimental limits; thus making these decays very sensitive tests of new physics and in particular scalar interactions.

To calculate the amplitude a pseudoscalar meson $M (\pi^+, K^+, B^+)$ decays into a lepton pair we need to introduce the following form factors

$$\langle 0 | \bar{u}_i \gamma^\mu \gamma_5 d_j | M(p) \rangle = i f_M p^\mu,$$
$$\langle 0 | \bar{u}_i \sigma^{\mu\nu} d_j | M(p) \rangle = i f_M \left( M^2 / (m_i + m_j) \right),$$ \tag{3.1}

where $M$ is the meson mass. The pseudoscalar form factors take the values $f_\pi = 130.7(4)$ MeV and $f_K = 160(2)$ MeV although we do not need these values. On the other hand $f_B$ is not known but expected to be about 200 MeV.

Since the pseudoscalar meson carries only one momentum, $p$. It can’t have an antisymmetric form factor due to Lorentz invariance,

$$\langle 0 | \bar{u}_i \gamma_5 d_j | M(p) \rangle = 0,$$
$$\langle 0 | \bar{u}_i \sigma^{\mu\nu} \gamma_5 d_j | M(p) \rangle = 0.$$ \tag{3.2}

However, one can have nonzero form factors for tensor interaction if there is an extra particle in the final state such in the $\pi^- \to e\nu\gamma$ case to which we shall return at the end of the section.
Substituting Eq.(3.1) into Eq.(2.1) the amplitude for the decay $d_j \to u_i l \nu_k$ for a specific charged lepton $l$ but sum over all three active neutrinos is then given by

\[
iM = \frac{2G_F}{\sqrt{2}} J_M \left[ V_{ij} A(m_i) \left( P_{l k} \bar{p}^{\mu} \gamma^\mu \bar{L} \nu_k \right) - \frac{\sqrt{2} M^2}{4 G_F N^2 (m_i + m_j)} \left(C_{S1}^{ij,lk} - C_{S2}^{ij,kl} \right) \left( \bar{L} \nu_k \right) \right], \tag{3.3}
\]

and notice that only a sum over $k$ is taken. From this we obtain

\[
R_\pi = R^{SM}_\pi \times \left( 1 + \frac{K^e_\pi - K^\mu_\pi}{G_F \Lambda^2} \right) \tag{3.4}
\]

where

\[
R^{SM}_\pi = \frac{m^2_e (M^2_\pi - m^2_\mu)}{m^2_\mu (M^2_\pi - m^2_\mu)} \left( 1 - \frac{16.1 \alpha}{\pi} \right) = 1.2354(2) \times 10^{-4} \tag{3.5}
\]

is the SM prediction [8], and

\[
K^l_\pi = \frac{1}{\sqrt{2} A(m_i, M_W) V_{ud}} \frac{M^2_\pi}{m^2_l (m_u + m_d)} \sum_k \Re \left[ P^*_{kl} \left( C_{S2}^{11,kl*} - C_{S1}^{11,lk} \right) \right]. \tag{3.6}
\]

Because we have factored out the SM helicity suppression factor in Eq.(3.4) the correction factor given by Eq.(3.6) has a $1/m_l$. This is a reflection of the helicity flipping nature of scalar interactions.

Compared with the data $R^{exp}_\pi = 1.230(4) \times 10^{-4}$ [9] which is consistent with the SM we have the limit:

\[
\left| \frac{K^e_\pi - K^\mu_\pi}{G_F \Lambda^2} \right| \leq 0.007 \quad (68\% \text{ C.L.}) \tag{3.7}
\]

Besides the overall scale factor $\Lambda$ for the new physics, Eq.(3.7) gives valuable information of these scalar coefficients. The first observation is that the phases of $C_{S1}^{ij,kl}$ and $C_{S2}^{ij,kl}$ must not accidentally cancel for it to be useful. Next we define

\[
C^\pi,kl \equiv C^{11,kl*} - C^{11,lk} \quad (l = e, \mu) \tag{3.8}
\]

and consider the following general possibilities which cover most of the relevant models for this problem:

A. There is no hierarchy (NH) between the electron and muon modes.

An an example we take

\[
C^\pi,ke \sim C^\pi,k\mu. \tag{3.9}
\]

Then $K^e_\pi$ dominates in Eq.(3.7) and taking $M_\pi/(m_u + m_d) = 15$, we have:

\[
\left| \sum_k \Re \left[ P^*_{ke} \left( C_{S2}^{11,ke*} - C_{S1}^{11,ek} \right) \right] \right|_{NH} \leq 2.8 \times 10^{-5} \left( \frac{\Lambda}{\text{TeV}} \right)^2 \quad (68\% \text{ C.L.}) \tag{3.10}
\]
B. An exact hierarchy exists between the electron and muon modes.

As an illustration we take the coefficients to be proportional to the electron and muon Yukawa couplings, \( y_e \sim 3 \times 10^{-6} \) and \( y_\mu \sim 6 \times 10^{-4} \) respectively. Such is the case for two Higgs doublet model (2HMD). Explicitly we have

\[
C_\pi^{\sigma,ke} = y_e C, \\
C_\pi^{\sigma,k\mu} = y_\mu C, 
\]

(3.11)

where \( C \) is an undetermined constant which for simplicity we take to have no dependence on the neutrino indices. In this case the constraint from Eq.(3.7) becomes

\[
| C \sum_k \Re [P_{ke} - P_{k\mu}] | \leq 9.3 \left( \frac{\Lambda}{\text{TeV}} \right)^2. 
\]

(3.12)

Clearly the constraint from lepton universality tests is not as stringent as the previous case; nevertheless is still an interesting one. Furthermore, it also demonstrates the importance of taking into account neutrino mixings.

The above analysis can be repeated for K decays. We have

\[
R_K = R_{K}^{SM} \times \left( 1 + \frac{K^e_K - K^K_K}{G_F \Lambda^2} \right) 
\]

(3.13)

where \( R_{K}^{SM} \) is the SM prediction and

\[
R^\ell_K = \frac{1}{\sqrt{2} A(m_i, M_W)} V_{us} m_l(m_u + m_s) \sum_k \Re \left[ P_{kl}^* \left( C_{S2}^{21,kl*} - C_{S1}^{12,kl*} \right) \right]. 
\]

(3.14)

The most recent data from NA48/2 [11] is

\[
R_K^{exp} = (2.416 \pm 0.043_{\text{stat}} \pm 0.024_{\text{syst}}) \times 10^{-5} 
\]

(3.15)

which improves upon the PDG value [3]: \( R_K^{exp} = 2.44(11) \times 10^{-5} \). This is to be compared with the SM value of \( R_{K}^{SM} = 2.472(1) \times 10^{-5} \) [8]. Using \( M_K/(m_u + m_s) = 3 \) and \( V_{us} = 0.224 \), we have the limit:

\[
\left| \sum_k \Re \left[ P_{ke}^* \left( C_{S2}^{21,ke*} - C_{S1}^{12,ke*} \right) \right] \right|_{\text{NH}} \leq 6.4 \times 10^{-5} \left( \frac{\Lambda}{\text{TeV}} \right)^2 (68\% \text{ C.L.}) 
\]

(3.16)

For the exact hierarchy case we get

\[
\left| C' \sum_k \Re [P_{ke} - P_{k\mu}] \right| \leq 21.3 \left( \frac{\Lambda}{\text{TeV}} \right)^2
\]

(3.17)

where \( C' \) replaces \( C \) of Eq.(3.11).

As remarked earlier we have to use the radiative decays to get constraint on the tensor coefficients. However, as has been pointed out by [12], due to QED correction the tensor
operator will induce scalar operator. And it’s been widely believed that the limit on $C_T$ from $\pi^- \to e\nu$ is two orders better than the direct constraint from $\pi^- \to e\nu\gamma$.

Since we have a RG improved result given in Eq.(2.19), it’s interesting to see how this affects the above conclusion. To that end we carried out a full numerical analysis for the coupled equations Eq.(2.19) and the result is displayed in Fig.4 where the figure caption explains our notations.

**Figure 4:** The full RG running for the coupled scalar and tensor Wilson coefficients from $M_W$ to $m_p \sim 1$ GeV. The x-axis is the ratio $r = C_{S2}/C_T$ at $M_W$, and the left-handed y-axis is the ratio $r$ at $m_p$. The dash line (blue) is the amplitude at low energy, we set $A \equiv \sqrt{|C_{S2}|^2 + |C_T|^2} = 1$ at $M_W$, refer to the axis at the right-handed side.

Assuming there is no hierarchy between the Wilson coefficients $C_{S2}$ and $C_T$ at EW scale, for instance the two are within one order of magnitude. Basically, we find the RG evolution can be simply summarized by Eq.(2.20).

Let us consider the extreme examples that $(C_{S2}, C_T) = (0, 1)$ and $(1, 0)$ at EW scale. The first corresponds to only tensor interactions are induced by new physics whereas the second has only scalar being generated. Our numerical solution gives $(-0.03, 0.84)$ and $(1.717, -5 \times 10^{-4})$ at $m_p$ respectively. In other words, if the new physics beyond SM gives only tensor interaction at the weak scale; then at meson mass scale the RG evolution gives

$$\frac{C_{S2}(m_p)}{C_T(m_p)} = -0.036. \quad (3.18)$$

This is to be compared with the estimation given in [12]

$$\frac{C_{S2}(m_p)}{C_T(m_p)} = -\frac{\alpha}{\pi} \ln \frac{M_W^2}{m_p^2} = -0.020. \quad (3.19)$$
We see the full RG running gives about 80% correction. Using Eq.\((3.10)\) and Eq.\((3.18)\) we get the upper limit

\[
|\text{Re}C_T(m_p)| < 7.7 \times 10^{-4} \left(\frac{\Lambda}{\text{TeV}}\right)^2 ,
\]

which is a factor of \(\sim 250\) better than direct measurement of \(\pi \to e\nu\gamma\).

### 3.2 Polarization in \(K^+ \to \pi^0\mu^+\nu\)

It is well known that the transverse muon polarization in semileptonic pseudoscalar mesons decays are sensitive CP violation tests of effective scalar interactions \([13, 14]\). We revisit these studies by adding the RG running of the effective operators to the previous studies which were conducted mostly in the context of specific models \([15, 16]\).

The effective Hamiltonian at the quark level is the same as in Eq.\((2.1)\). The form factors for \(K^+(p_K) \to \pi^0(p_\pi)\mu^+(p_\mu)\nu(p_\nu)\) decay that we need are

\[
\langle \pi^0|\bar{s}\gamma^\mu u|K^+\rangle = f_+ p_\mu^0 + f_- p_\mu^- ,
\]

\[
\langle \pi^0|\bar{s}u|K^+\rangle \simeq f_+ \frac{M_K^2 - M_\pi^2}{m_s} ,
\]

\[
\langle \pi^0|\bar{s}\sigma^{\mu\nu} u|K^+\rangle \simeq i \frac{f_T}{M_K}\left(p_\mu^\nu p_\pi^\mu - p_\mu^\nu p_\pi^\mu\right) ,
\]

where \(p_\pm = (p_K \pm p_\pi)\) and we have neglected the u-quark mass. Again, the form factors of \((\bar{s}\gamma^\mu\gamma_5 u)\), \((\bar{s}\gamma_5 u)\), and \((\bar{s}\sigma^{\mu\nu}\gamma_5 u)\) vanish due to parity. The form factors are functions of \(p_\pi^-\) and is normalized to \(f_+(0) = 1\). Furthermore, one usually defines \(\xi \equiv \frac{f_T}{f_+}\). In the second of Eq.\((3.21)\) we have omitted a term proportional to \(\xi\) by taking advantage of its small value; i.e. \(\xi\) = 0.124 \(\text{[4]}\). In the third of Eq.\((3.21)\), we have dropped a term \(\delta f_T \times (\epsilon^{\alpha\beta\mu\nu} p_{K\alpha} p_{\pi\beta})\) which can be taken into account by redefining \(f_T \to (f_T - \delta f_T)\) when contract with the lepton current part. The decay amplitude is

\[
-\mathcal{M} = \sqrt{2} G_F f_+ \left[V_{us}^* P_{2k} A(m_s)(p_\mu^0 + \xi p_\nu^0)\bar{\nu}_\alpha \hat{L}_\mu\right. \\
+ \frac{(M_K^2 - M_\pi^2)}{2\sqrt{2} G_F \Lambda^2 m_s} \left(C_{S1}^{12,2k}(m_s) - C_{S2}^{21,2k}(m_s)\right)\bar{\nu}_k \hat{R}_\mu \\
- \left.i \frac{f_T}{f_+ \sqrt{2} G_F \Lambda^2 M_K}\right] C_{T}^{21,2k}(m_s) p_{K\mu} p_{\pi\nu} \bar{\nu}_k \sigma_{\alpha\beta} \hat{R}_\mu .
\]

(3.22)

The above can be easily compared with the standard notation \([11]\). We further define two variables

\[
\xi_S \equiv \frac{1}{2 \sqrt{2} V_{us} A(m_s)} \frac{M_K^2 - m_\pi^2}{M_K m_s} \frac{C_K^S}{\Lambda^2 G_F} ,
\]

\[
\xi_T \equiv - \frac{f_T/f_+}{\sqrt{2} V_{us} A(m_s)} \frac{C_K^T}{\Lambda^2 G_F} ,
\]

(3.23)

(3.24)

where

\[
C_K^S = \sum_k P_{k\mu}^* \left(C_{S1}^{12,2k}(m_s) - C_{S2}^{21,2k}(m_s)\right) ,
\]

(3.25)
and

\[ C^K_T = \sum_k P^*_{k\mu} C^{21,k2}_T(m_\pi). \]  

(3.26)

The CP violating transverse muon polarization is found to be:

\[ P_\perp \sim \left[ 3m_\pi s + \frac{p_K \cdot (p_\nu - p_\mu) + m_\mu^2/2}{M_K^2} \right] \frac{\vec{p}_\mu \times \vec{p}_\nu}{\Phi} \]  

(3.27)

where \( \vec{p} \)'s are leptons' 3-vector momentum in the kaon rest frame and we have ignored the correction of higher power in \( \xi_T \) and

\[ \Phi = (2E_\mu M_K - m_\mu^2) \left( 1 - \frac{E_\pi + E_\mu}{M_K} \right) \]

\[ - \frac{1}{2} \left( M_K^2 + m_\pi^2 - m_\mu^2 - 2E_\pi M_K \right) \left( 1 - \frac{m_\mu^2}{4M_K^2} \right). \]  

(3.28)

Plugging in the numbers, we have

\[ P_\perp \sim \left[ 0.38 \Im C^K_S - 0.27 \frac{p_K \cdot (p_\nu - p_\mu) + m_\mu^2/2}{M_K^2(f+/f_T)} \right] \left( \frac{\text{TeV}}{\Lambda} \right)^2 \frac{\vec{p}_\mu \times \vec{p}_\nu}{\Phi}. \]  

(3.29)

It is interesting to note the imaginary part of both tensor and scalar coefficients are sensitive to where measurements are made in the Dalitz plot distribution \( [17] \). \( C_T \) has maximum sensitivity when \( E_\mu - E_\nu \) is largest in the kaon rest frame. From the current limit |\( P_T \) - 0.0050 \( [18] \) and assuming that \( f_T \simeq f_+ \), we have

\[ |\Im C^K_S| \text{ and } |\Im C^K_T| \leq 2 \times 10^{-3} \left( \frac{\Lambda}{\text{TeV}} \right)^2. \]  

(3.30)

Notice here that the results from \( K_{l2} \) decays limits the real part of the scalar coefficients.

### 3.3 Constraints from Electron and Neutron EDM's

It is well known that the EDM of the electron is a very sensitive test of new physics phases. This is due to the fact that the SM gives an undetectably small value |\( d_e \) - 10^{-38} e-cm. In addition there are several ongoing experimental efforts to reach the level of 10^{-30}. If discovered it will be a clear sign of new physics. Similarly the neutron EDM, \( d_n \), is also very sensitive to new physics phases and are usually connected with electron EDM in model dependent ways. Hence, they are very complementary tests of new physics. Although the SM prediction for \( d_n \) is not as robust as \( d_e \) but is generally taken to be < 10^{-31} e-cm. This is still very much below the experimental bound of \( [19] \)

\[ |d_n^{\text{exp}}| < 6.3 \times 10^{-26} \text{ e-cm}. \]  

(3.31)

Among the terms in the effective Lagrangian new phases reside in the scalar and tensor Wilson coefficients in \( L^{NC}_6 \) and \( L^{CC}_6 \). Some NC terms will contribute to \( d_l \) at the 1-loop level and the CC term will induce it at the 2-loop level. This is explored in detail below.
Figure 5: 1-loop contribution to the EDM of a charged fermion $f$ from semileptonic 4-Fermi operators denoted by the box and $f$ represents $l$ or a $u$-quark and $f'$ a $u$-quark or $l$ respectively.

3.3.1 EDM at One Loop
The generic one loop diagram for $d_f$ is given in Fig.(5). It has been shown [20] that the RG running of $d_f$ below the EW scale is not significant and we shall ignore it. For the quark EDM, QCD correction is the biggest contribution. We found its QCD anomalous dimension is same as the tensor operator which we have already found its solution in Eq.(2.20), i.e.

$$d_u(m_p) \sim 0.83d_u(M_W). \quad (3.32)$$

Hence, for our purpose we can neglect the RG running in both cases.

It has been shown [2] that scalar operators will not contribute at this level. The vector terms in Eq.(1.28) have real coefficients and hence will play no role. This leaves the terms $C_{ii,ee}$ and only the $u$-type quarks come into play. A straightforward calculation leads to

$$d_e = \frac{eN_c}{3\pi^2\Lambda^2} \sum_{i=u,c,t} m_i \text{Im} C_{T_{ii,ee}}^{i} \ln \frac{\Lambda^2}{m_i^2} \quad (3.33)$$

at the EW scale which is where we set the boundary condition for the RGE of EDM operator and $N_c$ is the number of colors. For simplicity, we ignore the top threshold effects and the RG evolution between $m_t$ and $M_W$. Similarly, we have

$$d_u = -\frac{e}{2\pi^2\Lambda^2} \sum_{l=e,\mu,\tau} m_l \text{Im} C_{T_{11,ll}}^{ii} \ln \frac{\Lambda^2}{m_l^2}. \quad (3.34)$$

Notice that there is no $d_d$ at this level due to the $SU(2)_L$ symmetry.

Clearly the most important term is due to the $t$-quark. This has the unusual behavior of depending on the internal fermion mass and independent of $m_e$. The chirality changing nature of tensor interactions is responsible. Thus, one expects the same formula to hold for $d_\mu$ and $d_\tau$ with obvious change to the lepton indices. From the experimental limit of $|d_e| \leq 1.7 \times 10^{-27}$ e-cm [19] we deduce that

$$\left| \text{Im} C_{T_{33,ee}}^{ii} \right| \leq 1.4 \times 10^{-9} \left( \frac{\Lambda}{\text{TeV}} \right)^2 \quad (3.35)$$

which is much more stringent than one obtains from meson decays; see Eq.(3.30), albeit for different flavor indices.
Assuming that the leading contribution to $d_n$ comes from $d_u$ and using the quark model relation $d_n \sim (4d_d - d_u)/3$ for estimation, we obtain from Eq. (3.31) a less stringent bound

$$\left| \Im C_T^{11,\tau\tau} \right| \leq 8.4 \times 10^{-6} \left( \frac{\Lambda}{\text{TeV}} \right)^2.$$  

(3.36)

### 3.3.2 EDM at Two Loops

Unlike $\mathcal{L}_6^{NC}$ the complex coefficients in $\mathcal{L}_6^{CC}$ can only induce EDM at the 2-loop level. Similarly the scalar coefficients of $\mathcal{L}_6^{NC}$ come into play now. The generic diagrams for this are displayed in Fig. 6. Now both the scalar terms $C_{S1,2}$ contribute. From the above discussions we can ignore the tensor terms since they are constrained to be small (see also the next section).

An examination of $\mathcal{L}_6^{NC}$ and the structure of the Feynman diagrams shows that only the flavor diagonal terms come into play. Since the lepton masses are small they can be neglected in the calculation and we only need to keep the quark masses. Also the leading contribution comes from internal photon exchange. Then the NC contribution to the EDM $d_l$ of a charged lepton $l = e, \mu$ or $\tau$ is given by

$$d_l(\text{NC}) = \frac{e\alpha}{48\pi^3 \Lambda^2} \left[ 4 \sum_{i=u,c,t} m_i \Im C^{\nu\nu,LL}_{S_2} \mathcal{F}_u \left( \frac{\Lambda^2}{m_i^2} \right) + \sum_{j=b,s,t} m_j \Im C^{\nu\nu,LL}_{S_1} \mathcal{F}_d \left( \frac{\Lambda^2}{m_j^2} \right) \right],$$  

(3.37)

and the functions are

$$\mathcal{F}_u(z) = \int_0^z dx \int_0^1 dy \frac{1 - y(1 - y)}{1 + y(1 - y)x},$$  

(3.38)

$$\mathcal{F}_d(z) = \int_0^z dx \int_0^1 dy \frac{y(1 - y)}{1 + y(1 - y)x}.$$  

(3.39)

These results extend our previous expression in \[3\] where only partial contribution was considered. We have $\mathcal{F}_t = \{10.872, 59.552, 150.780\}$ and $\mathcal{F}_b = \{8.596, 13.200, 17.808\}$ for $\Lambda = \{1, 10, 100\}$ TeV. When $z \gg 1$, they can be approximated by

$$\mathcal{F}_t(z) \simeq (2 - \ln z + \ln^2 z),$$  

$$\mathcal{F}_b(z) \simeq -(2 - \ln z).$$  

(3.40)
There are two helicity flips in the above result. The first one involves the scalar 4-Fermi interactions $C_{S1,2}$. The second one is the helicity flip from the quarks in the loop due to the chiral nature of the weak interactions and is explicitly displayed in Eq. (3.37). As expected, in the limit of massless fermions there is no EDM. It is also clear that only the t and b-quarks are important.

For the contributions from $L^{CC}_D$ the calculation is similar. Now the internal gauge boson exchange comes from the W boson. The result is

$$d_l(CC) \sim \frac{e\alpha}{16\pi^3 \sin^2 \theta_w \Lambda^2} \left[ m_t \text{Im} C_{S2}^{\text{33,ll}} G_t \left( \frac{\Lambda^2}{M_W^2} \right) + m_b \text{Im} C_{S1}^{\text{33,ll}} G_b \left( \frac{\Lambda^2}{M_W^2} \right) \right] ,$$

(3.41)

$$G_t(z) = \frac{1}{8} \int_0^z dx \int_0^1 dy \frac{(2 - 3y)(2 - y)x}{(m_t^2/M_W^2 + yx)(1 + x)} ,$$

(3.42)

$$G_b(z) = -\frac{1}{8} \int_0^z dx \int_0^1 dy \frac{y(2 - 3y)x}{(m_b^2/M_W^2 + yx)(1 + x)} ,$$

(3.43)

where we have kept only the t quark mass in the quark propagators. The analytic expression for $G$ is rather complicated and not illuminating. From numerical study we have $G_t = \{1.708,11.450,31.670\}$ and $G_b = \{-0.0824,-0.347,-0.6343\}$ for $\Lambda = \{1,10,100\}$ TeV. For our purpose it is good enough to use the following approximation when $z \gg 1$:

$$G_t(z) \approx \frac{1}{4} \left( 13.7 - 6.31 \ln z + \ln^2 z \right) ,$$

$$G_b(z) \approx \frac{1}{4} \left( 1 - \frac{1}{4} \ln z \right) .$$

(3.44)

Combining the above two contributions, we arrive at the estimation:

$$d_l \sim \frac{e\alpha}{16\pi^3 \Lambda^2} \left\{ m_t \text{Im} C_{S2}^{\text{33,ll}} \left[ \frac{4}{3} F_t + \frac{G_t}{\sin^2 \theta_w} \right] + m_b \text{Im} C_{S1}^{\text{33,ll}} \left[ \frac{1}{3} F_b + \frac{G_b}{\sin^2 \theta_w} \right] \right\} .$$

(3.45)

The physics is now clear. For $C_{S1,S2}$, due to the helicity, it can only pick up the bottom/top quark mass in the loop. Barring accidental cancellation between the two scalar coefficients terms the upper limit of the imaginary parts can be deduced from electron EDM experiment:

$$\text{Im} C_{S2}^{\text{33,ee}} < 1.5 \times 10^{-6} \left( \frac{\Lambda}{\text{TeV}} \right)^2 ,$$

$$\text{Im} C_{S1}^{\text{33,ee}} < 4.7 \times 10^{-4} \left( \frac{\Lambda}{\text{TeV}} \right)^2 .$$

(3.46)

The results from our one and 2-loop study show that EDM’s are sensitive to the imaginary parts of the flavor diagonal scalar coefficients. On the other hand the charged lepton polarization experiments directly probe the flavor off diagonal terms although at a less sensitive level.
3.4 Muon Anomalous Magnetic Moment, $a_\mu$ and Rare Decays.

The exact same 1-loop calculation leads to the following effective Lagrangian:

$$\Delta L \sim -\frac{e m_i C_{33,ij}^{33}}{2\pi^2 \Lambda^2} \ln \frac{\Lambda^2}{m_t^2} \left( \epsilon_i \sigma^{\mu\nu} \hat{R} e_j \right) F_{\mu\nu} + h.c. + \frac{e m_i C_{33,ij}^{33}}{4\pi^2 \Lambda^2} \ln \frac{\Lambda^2}{m_t^2} \left( \tilde{u}_i \sigma^{\mu\nu} \hat{R} u_j \right) F_{\mu\nu} + h.c. \quad (3.47)$$

The flavor diagonal entries in the first term also contribute to $(g - 2)_\mu$ and off diagonal terms lead to the rare decay $(\mu \rightarrow e + \gamma)$. Thus, the branching ratio of the radiative decay of a charged lepton $L$ normalized to the decay width of $(L \rightarrow l + \nu_L + \bar{\nu}_l)$ is given by [3]:

$$B(L \rightarrow l + \gamma) = 48 \frac{e m_k^2}{\pi M_L^2} \left( \frac{\Lambda^2}{m_L^2} \right)^2 \left( |C_{33,li}^{33}|^2 + |C_{33,li}^{33}|^2 \right). \quad (3.48)$$

From the limit $B(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11} \text{, } B(\tau \rightarrow \mu\gamma) < 6.5 \times 10^{-6} \text{ and } B(\tau \rightarrow e\gamma) < 1.6 \times 10^{-5}$ [3], we get

$$|C_{33,\mu e}^{33}|^2 < 4.0 \times 10^{-16} \left( \frac{\Lambda}{\text{TeV}} \right)^4, \quad (3.49)$$

We see that the lepton flavor changing tensor coefficients are constrained to be very small unless $\Lambda > 100 \text{ TeV}$.

By the same token we obtain the following modification to the charged lepton anomalous magnetic moment

$$\Delta a_l = -\frac{m_l m_t}{\pi^2 \Lambda^2} \ln \frac{\Lambda^2}{m_t^2} Re C_{33,li}^{33} \quad (3.50)$$

From the current limits $|\Delta a_e| < 3.5 \times 10^{-11}$ and $|\Delta a_\mu| < 254 \times 10^{-11}$ [3] we deduce that

$$|Re C_{33,ee}^{33}| < 1.1 \times 10^{-3} \left( \frac{\Lambda}{\text{TeV}} \right)^2, \quad (3.51)$$

which are less stringent than Eq.(3.49) but still very strong.

3.5 Triple Spin-Momenta Correlations in $e^+e^- \rightarrow t\bar{t}$

We give as an example a physical quantity measurable in the process at high energy which can be related to low energy constraints by RGE’s. Consider at the TeV linear collider (LC) the process $e^+(p_+)e^-(p_-, s_e) \rightarrow t(k_+, s_t)\bar{t}(k_-)$, where the momenta and spins are specified. One can construct a T-odd quantity involving two independent spins and a momentum:
e.g. $\hat{p}_- \cdot (\hat{s}_t \times \hat{s}_e)$ where $\hat{p}_-$ is the unit momentum 3-vectors, and $\hat{s}_e$ is the unit 3-spin vector in the electron rest frame. We can relate this to $d_e$.

Since the tensor coefficient $C_{T}^{33,ee}$ has been determined to be around or smaller than $10^{-9}$ from the electron EDM, Eq.(3.35), we need not consider it. Only the effective scalar operators will be important. The triple correlation is a result of the interference between the SM amplitudes of photon and $Z$ exchange, and the scalar 4-Fermi interaction. To match the chirality, either the mass or the spin of electron has to be involved. Clearly, a LC with polarized electron beam offers higher possibility of probing CPV part of the scalar interaction beyond SM.

A straightforward calculation yields the CPV amplitude, normalized to the photon exchanging amplitude,

$$|\mathcal{M}|_{TO}^2 = \frac{\Im C_{S2}^{33,ee}(\Lambda)}{\Im C_{S2}^{33,ee}(m_p)} \left[ \left( 1 + \frac{m_t^2}{s} \right) (\hat{p}_- - \hat{p}_+) \cdot (\hat{s}_t \times \hat{s}_e) + \left( 1 + \frac{2m_t^2}{s} \right) (\hat{k}_+ - \hat{k}_-) \cdot (\hat{s}_t \times \hat{s}_e) \right]$$

where $s = (p_+ + p_-)^2$ is the Mandelstam variable, and $\hat{s}_t$ is the unit spin vector of the t-quark in its rest frame.

In arriving at the above expression, we have assumed that $\sqrt{s} \gg M_Z$ and $\sin^2 \theta_W \simeq 0.25$ for simplicity. Putting in all the RG running we have derived, Eqs.(2.11,2.20), we get the following relation

$$\frac{\Im C_{S2}^{33,ee}(\Lambda)}{\Im C_{S2}^{33,ee}(m_p)} = (1.72)^{-1} \times (0.9456) \times \left( \frac{m_t}{\Lambda} \right)^{0.036} \left( \frac{\Lambda}{\text{TeV}} \right)^2 = 0.516 \times \left( \frac{\Lambda}{\text{TeV}} \right)^{1.964}.$$ (3.53)

With the electron EDM constraint, Eq.(3.46), we conclude the relative size of CPV amplitude has to be

$$< 2.7 \times 10^{-7} \times \left( \frac{\sqrt{s}}{\text{TeV}} \right)^2 \times \left( \frac{\text{TeV}}{\Lambda} \right)^{0.036}.$$ (3.54)

It is very challenging if not impossible to achieve this kind of precision at the ILC.

4. Conclusions

We begin with the assumption that the SM gauge symmetry is valid from the some unknown new physics scale $\Lambda$ to the EW scale and the matter content of the SM. Using the effective field theory approach we studied all the dimension six operators which contain a pair each of lepton and quark fields. A previously derived result [3] allows us to cast them in the mass eigenstates. By a general assumption that the vector Wilson coefficients are constants we can eliminate zero order FCNC. This leaves the most the scalar and tensor operators as the most interesting ones that can have FCNC at the level of the Born term. They also provide new sources of CP violation in both charge and neutral current channels. Furthermore,
in the NC channels there can be flavor changing as well as flavor conserving modes. For notational simplicity we generically call the corresponding coefficients $C_S$ and $C_T$. We also argue that stringent experimental limits make the Wilson coefficients of dimension six operator of the dipole type negligible and permit us to concentrate on 4-Fermi operators.

An important aspect of field theory is that these Wilson coefficients evolved with energy and this can be calculated by the RG method. Once a coefficient is measured at one scale, say the EW scale, its value at higher energy is determined by the anomalous dimensions. This robust prediction depends only on the assumed gauge symmetry and the known states between the two energies. We have calculated the anomalous dimensions of the complete set of semileptonic 4-Fermi operators to 1-loop. The RG running of the associate Wilson coefficients are solved between the EW scale and $\Lambda$.

Below the EW scale the conserving gauge symmetries are $U(1)_{em}$ and color $SU(3)_c$. We found that the effect due to $U(1)_{em}$ is not large in agreement with previous studies; however the QCD corrections to $C_S$ and $C_T$ are large. The combine effect increases $C_S$ by almost a factor of two from the scale of $M_W$ to 1 GeV; whereas $C_T$ is decreased by approximately seventeen percent.

As is well known that pure leptonic meson decays are the most sensitive tests of $C_S$, we refined the previous analysis on the constraints by including RG effects. We also found that the large logarithms of the RG method change the result on $C_T$ by almost a factor of two. It turns out that the most stringent constraint on all the different $C_T$’s arise from 1-loop effects they induce in EDM, anomalous magnetic moments, and rare decays of leptons.

The CPV effects from $\Im C_S$ do not give rise to EDM’s at 1-loop. At 2-loops they constraint the flavor diagonal operators; especially those involving the t-quark. Since these are suppressed by an extra loop factor the bounds are not as tight as those for $C_T$. The flavor off-diagonal $\Im C_S$ are most directly constrained by polarization measurements in semileptonic meson decays such as in $K^+\mu\bar{\nu}_\mu$. Hence, these are very complementary tests in this very general framework of effective operator approach to new physics. Furthermore, given the limits obtained at low energies and the RG effects are not large we do not expect any of the $C_S$ coefficients to be measurable at the LHC. On the other hand high energy colliders may be more sensitive to the vector terms in $\mathcal{L}_6^{NC}$. This warrants a detail study which we shall reserve for future investigations.

Theoretically the bounds we obtained on $C_S$ and $C_T$ have profound implications for model building. They imply that if the new physics scale characterized by $\Lambda$ were to be below 10 TeV any tree-level origin of these effective operators must have small couplings. Although it is not impossible this implies some degree of fine tuning is at work. Well known examples are the multiple Higgs models\[13\] and R-parity violating supersymmetric models. An alternative and perhaps more natural one will be that they are dynamically suppressed. For example they are generated at 1-loop or higher level in the new physics model. Many supersymmetric models have this feature. Another possibility will be that the suppression is a result of some symmetry at work above $\Lambda$. Family symmetry is an example that comes to mind which will suppress off diagonal terms and leave the diagonal ones relatively free.

We are looking forward to more precision measurements and hope that some of these
operators will be measured. Our analysis will be useful to unraveling the new physics at work in a general way.

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