The following full text is a publisher's version.

For additional information about this publication click this link.
https://hdl.handle.net/2066/230750

Please be advised that this information was generated on 2021-07-16 and may be subject to change.
On the pumping of the CS($v=0$) masers in W51 e2e

D. J. van der Walt,1* A. Ginsburg2 and C. Goddi3

1Centre for Space Research, North West University, Potchefstroom, 2520, South Africa
2Department of Astronomy, University of Florida, P.O. Box 112055, Gainesville, FL 32611-2055, USA
3Department of Astrophysics/IMAPP, Radboud University Nijmegen, P.O. Box 9010, NL-6500 GL Nijmegen, the Netherlands

Accepted 2020 November 20. Received 2020 November 20; in original form 2020 August 22

ABSTRACT
We present the results of numerically solving the rate equations for the first 31 rotational states of CS in the ground vibrational state to determine the conditions under which the $J = 1 - 0$, $J = 2 - 1$, and $J = 3 - 2$ transitions are inverted to produce maser emission. The essence of our results is that the CS($v=0$) masers are collisionally pumped and that, depending on the spectral energy distribution, dust emission can suppress the masers. Apart from the $J = 1 - 0$ and $J = 2 - 1$ masers, the calculations also show that the $J = 3 - 2$ transition can be inverted to produce maser emission. It is found that beaming is necessary to explain the observed brightness temperatures of the recently discovered CS masers in W51 e2e. The model calculations suggest that a CS abundance of a few times $10^{21}$ and CS($v=0$) column densities of the order of $10^{16}$ cm$^{-2}$ are required for these masers. The rarity of the CS masers in high-mass star-forming regions might be the result of a required high CS abundance as well as due to attenuation of the maser emission inside as well as outside of the hot core.

Key words: masers – methods: numerical – stars: formation – stars: massive.

1 INTRODUCTION
Molecular maser emission has always been a fascinating phenomenon associated with various astrophysical environments such as evolved stars, star-forming regions, and supernova remnants. As for high-mass star-forming regions, this has been particularly emphasized over the last two decades through the discovery of periodic class II methanol, formaldehyde, and OH masers associated with high-mass star-forming regions (see eg. Godhart, Gaylard & van der Walt 2003, 2004; Araya et al. 2010; Goedhart et al. 2014, 2019; Szymczak, Wolak & Bartkiewicz 2015; Szymczak et al. 2016, 2018; Sugiyama et al. 2017; Olech et al. 2019). More recently water and methanol maser flaring events associated with high-mass star-forming regions attracted significant attention as such flaring events might be associated with episodic accretion outbursts (Brogan et al. 2018; Szymczak et al. 2018). The periodic masers and maser flaring events are just two examples which illustrate how masers can reveal changes in the star-forming environment which otherwise would not be detected with thermal line emission.

While widespread masers such as e.g. H$_2$O and CH$_3$OH are useful in studying some aspects of high-mass star-forming regions, rare masers, on the other hand, must reveal something special about the star-forming regions with which they are associated. Amongst those masers for which until recently only a single detection has been reported is that of CS. In the course of studying vibrationally excited emission from the carbon-rich giant IRC +10216, Highberger et al. (2000) noted that the CS($v=1$) $J = 3 - 2$ transition has two components with exceptionally small linewidths of, respectively, 1.1 and 2.5 km s$^{-1}$ compared to other lines they detected. In contrast, the linewidths for other molecules reflect the full expansion velocity of 14.5 km s$^{-1}$ of the gas in IRC+10216. In addition to the small linewidths, the brightness temperature of the CS($v=1$) $J = 3 - 2$ transition was found to be 526 K compared to significantly smaller values for the other transitions of CS. This led Highberger et al. (2000) to conclude that non-LTE conditions affect the CS ($v=1$) $J = 3 - 2$ transition and that it might be a weak maser.

Recently, Ginsburg & Goddi (2019) reported the first discovery of maser emission for the CS($v=0$) $J = 1 - 0$ and $J = 2 - 1$ transitions associated with a high-mass young stellar object, viz. W51 e2e. Although the observed linewidths for the two masers of, respectively, 7.3 and 5.3 km s$^{-1}$ were determined by the experimental velocity resolution, both transitions have brightness temperatures of about 6700 K which rules out local thermodynamic equilibrium (LTE) excitation. The two masers were found to be spatially separated by about 190 au and spectrally by about 30 km s$^{-1}$. Ginsburg & Goddi (2019) also imaged all three hot cores in the W51 A region in the CS($v=1$) $J = 1 - 0$ transition with a sensitivity of 1.1 mJy beam$^{-1}$ but did not detect it. This suggests that the physical environment of the CS($v=0$) $J = 1 - 0$ and $J = 2 - 1$ masers in W51 e2e is such that excitation to the $v=1$ state is either weak or completely absent.

This detection of CS maser emission associated with a very young high-mass star raises the questions of what the pumping mechanism is and why these masers are so rare. The aim of this paper is first to investigate the pumping mechanism and based on that consider why these masers seem to be rare. We restrict our investigation here to the CS($v=0$) masers in W51 e2e, leaving the case of the $v=1$ masers, as in IRC +10216, for later follow-up work.

* E-mail: johan.vanderwalt@nwu.ac.za

© 2020 The Author(s)
Published by Oxford University Press on behalf of Royal Astronomical Society
2 NUMERICAL METHOD AND MOLECULAR DATA

The level populations were found by solving the well-known set of rate equations in the escape probability approach given by

\[ \frac{dN_i}{dt} = \sum_{j} \left[ \left( -N_i + \left( \frac{g_i}{g_j} N_j - N_i \right) W N_{ij} \right) \beta_{ij} A_{ij} \right. \]

\[ + C_{ij} \left( N_i \frac{g_j}{g_i} e^{-E_{ij}/kT} - N_i \right) \]

\[ + \sum_{j} \left[ \left( N_j + \left( N_j - \frac{g_j}{g_i} N_i \right) W N_{ji} \right) \beta_{ji} A_{ji} \right. \]

\[ + C_{ji} \left( N_j - N_j \frac{g_i}{g_j} e^{-E_{ji}/kT} \right) \right], \tag{1} \]

where \( N_i \) is the number density in level \( i \), \( g_i \) the statistical weight of level \( i \), \( A_{ij} \) the Einstein-A coefficient for spontaneous emission between levels \( i \) and \( j \), \( W \) the geometric dilution factor for an external radiation field, and \( N_{ij} \) the photon occupation number for this field at frequency \( v_{ij} \). \( C_{ij} \) is the collision rate with \( N_{H_2} \), the H_2 number density and \( K_{ij} \) the collision rate coefficient. \( \beta_{ij} \) is the escape probability. The rate equations were supplemented with the particle number conservation requirement

\[ N_{\text{tot}} = X n_{H_2} = \sum_i N_i \tag{2} \]

with \( X \) the abundance relative to H_2 of the molecule under consideration.

The maser region is assumed to be embedded in or surrounded by a layer of warm dust at temperature \( T_d \). We assume that the dust radiation field is given by \( I_r = \tau_{d}(v) B_r(T_d) \) with \( B_r(T_d) \) the Planck function and \( \tau_{d}(v) = \tau_{d}(v_{0}) (v/v_{0})^{\beta_{d}} \) the dust optical depth which we have set \( \tau_{d}(v_{0}) = 10^{13} \) Hz = 1 (see e.g. Pavlakis & Kylafis 1996; Sobolev, Cragg & Godfrey 1997; Cragg, Sobolev & Godfrey 2002). We note that for dust emission Sobolev et al. (1997) and Cragg et al. (2002) used \( I_r \propto [1 - \exp(-\tau_{d})] B_r(T_d) \) which, in the optically thin case is equivalent to \( (v/v_{0})^{\beta_{d}} B_r(T_d) \).

We solve the rate equations using a fourth-order Runge–Kutta method, which is more accurate and stable than Heun’s method which was used in van der Walt (2014). The initial distribution of the level populations was a Boltzmann distribution with a temperature equal to the average of the dust and kinetic temperatures. For a given \( n_{H_2} \), the calculation started with a small CS specific column density such that the system is completely optically thin in all transitions. An equilibrium solution for this initial specific column density was then found by letting the system evolve in time-steps of 0.5 s till the convergence condition \( |N_i(t_{i+1}) - N_i(t_i)|/N_i(t_i) < 10^{-6} \) is reached for all levels \( i \). The specific column density is then increased by a small amount with the equilibrium solution for the previous value of the specific column density being used as the initial distribution. The process is repeated until the required specific column density is reached.

For the escape probability, we used the expression for the large velocity gradient approximation, i.e.

\[ \beta_{ij} = \frac{1 - e^{-\tau_{ij}}}{\tau_{ij}} \tag{3} \]

with the optical depth given by

\[ \tau_{ij} = \frac{A_{ij}}{8\pi} \left( \frac{c}{v} \right)^{3} \left( \frac{g_j}{g_i} x_j - x_i \right) \frac{N_{\text{col}}}{\Delta v}. \tag{4} \]

**Figure 1.** Comparison of the variation of the beaming factor for the four different dependencies of \( \Omega/4\pi \) on the gain: (1) \((2\tau + 1)^{-1}\), (2) \((2\tau + 1)^{-1.5}\), (3) \((2\tau + 1)^{-2}\), and (4) \(10^{-0.433\tau}\).

\( x_i \) and \( x_j \) are the fractional number densities of molecules in the lower level, \( i \), and upper level, \( j \), respectively. \( N_{\text{col}} \) the CS column density and \( \Delta v \) the linewidth. The ratio \( N_{\text{col}}/\Delta v \) is the specific column density. When a population inversion occurs, beaming becomes important in which case equation (3) becomes

\[ \beta_{ij} = \frac{\Omega_{ij} e^{\nu_{ij}} - 1}{4\pi \tau_{ij}} \tag{5} \]

(Elitzur 1992) with \( \Omega/4\pi \) the fractional beam solid angle. According to Elitzur (1992), \( \Omega/4\pi = (2\tau)^{-1} \). This dependence suggests that beaming only starts when \( \tau_{ij} = 0.5 \). On the other hand, Spans & van Langewede (1992) argued that \( \Omega_{ij}/4\pi \approx \tau_{ij}^{-2} \). We combined these two and assumed the beaming factor to be of the form \( \Omega_{ij}/4\pi = (2\tau_{ij} + 1)^{-\eta} \) to allow for beaming to start as soon as a population inversion occurs as well as to consider stronger beaming than that of Elitzur (1992).

We also considered the case when the beaming factor is that used by Sobolev et al. (1997). In their calculations, done within the framework of the large velocity gradient approach, beaming was included through a parameter \( \varepsilon = \tau_{ij}/\tau_0 = d(\ln \eta)/d(\ln r) \) with \( \tau_0 \) the optical depth in the direction of the beam and \( \tau_{ij} \) the optical depth perpendicular to the direction of the beam. The escape probability is then given by (Castor 1970)

\[ \beta(\tau, \varepsilon) = \int_{0}^{\frac{1}{1 + \varepsilon x^2}} \left( 1 - \exp \left( -\frac{\tau}{1 + \varepsilon x^2} \right) \right) dx. \tag{6} \]

\( \beta(\tau, \varepsilon) \) can be evaluated as a function of \( \tau \) for a given value of \( \varepsilon \) and be equated to the right-hand side of equation (5) for which the corresponding value of \( \Omega/4\pi \) can be found. Following this procedure and taking \( \varepsilon = 0.1 \) (Sobolev et al. 1997), it was found that \( \Omega/4\pi = 10^{-0.433\tau} \). In Fig. 1, we show the dependence of the beaming factor on the gain \( \tau \) for \( \eta = 1, 1.5, 2 \) and the ‘Sobolev’ beaming.

The Einstein-A and collisional rate coefficients of the first 31 levels of CS were taken from the Leiden Atomic and Molecular Database (Schöier et al. 2005). The collisional rate coefficients were extrapolated from 300 to 500 K using equation (13) of Schöier et al. (2005) to allow also for the possibility of higher gas kinetic temperatures than what is given on the Leiden Atomic and Molecular Database.

The parameters involved in the present calculations are e.g. \( H_2 \) density \( (n_{H_2}) \), gas kinetic temperature \( T_k \), dust temperature \( T_d \), the geometric dust dilution factor \( W \), the index \( p \) for the modified...
switched off. All calculations started at a specific column density of
and internal generated radiation only, i.e. the dust radiation field is
increased in steps of 0.01 dex when excitation is through collisions
of the variation of the optical depth as the specific column density is
small steps in specific column density. Fig. 2 shows some examples
density where the system is optically thin, and to evolve the system in
and CS specific column density is to start at a small specific column
is no beaming. For both transitions, the optical depth decreases
toward a minimum value (maximum gain) after which it increases to
eventually become positive. While both transitions can be inverted
at the same specific column density, the maximum gain for the
transition affects both the
rotational states.
A further interesting result of increasing the beaming is that the
transition is lifted. This behaviour is quite general
since the two masers share the
1 state in a single ladder of
transitions when there is no beaming. For both transitions, the optical depth decreases
toward a minimum value (maximum gain) after which it increases to
eventually become positive. While both transitions can be inverted
at the same specific column density, the maximum gain for the
J = 2 − 1 transition is seen to occur at a smaller specific column density
than for the J = 1 − 0 transition. As will be shown later, it is generally
the case that the maximum gain for the two masers does not occur at
the same specific column density.

Panels (b)–(d) show how the behaviour of the optical depth changes
when beaming is introduced. Comparison with panel (a) shows that beaming has a marked effect on how the optical depth changes
with specific column density. Panel (b) shows that the
specific column density where maximum gain occurs has shifted
to larger specific column densities. Also, the maximum gain has increased. Perhaps the most interesting and noteworthy difference with Panel (a) is that the lifting of the inversion for the J = 2 − 1 transition occurs over one step in specific column density. When this happens the gain of the J = 1 − 0 maser increases since population in the J = 1 state ‘suddenly’ increases. This behaviour is quite general
since the two masers share the J = 1 state in a single ladder of
rotational states.

A further interesting result of increasing the beaming is that the J = 3 − 2 transition also becomes inverted as is seen in panels (c) and
(d). Note how now three masers are linked through J = 1, and
J = 2 states. Thus, when the inversion of the J = 3 − 2 transition is lifted, it affects the J = 2 − 1 and the J = 1 − 0 transitions. The
inversion of the J = 3 − 2 transition occurs only when collisions dominate the excitation and does not occur at all combinations of
nH2 and Td. For most of what follows the focus will be on the J = 1 − 0 and J = 2 − 1 masers in the ground vibrational state.

The results presented in Fig. 2 show that inversion of the J = 1 − 0
collisionally pumped. An infrared radiation field, however, permeates
the star formation environment and it is necessary to examine what
the effect of the presence of such a radiation field is. In addition to
the temperature of the dust, it is also necessary to consider the effect
of geometric dilution. Instead of considering the maser gain, we will
henceforth consider the maser brightness temperature, calculated

\[
T_b = \frac{\hbar v_{ij} x_i (1 - e^{-\tau_{ij}})}{k (g_j x_j / g_i) - x_i} \tag{7}
\]

with \( i > j \) and \( x_i \) and \( x_j \) as in equation (4).

3 RESULTS

As explained in the previous section, the calculational procedure to
find the equilibrium solution for a given combination of \( n_{H_2} \), \( T_d \), \( T_b \),
and CS specific column density is to start at a small specific column
density where the system is optically thin, and to evolve the system in
small steps in specific column density. Fig. 2 shows some examples
of the variation of the optical depth as the specific column density is
increased in steps of 0.01 dex when excitation is through collisions
and internal generated radiation only, i.e. the dust radiation field is
switched off. All calculations started at a specific column density of
\( 10^6 \text{ cm}^{-3} \text{s} \) and ended at \( 2 \times 10^{12} \text{ cm}^{-3} \text{s} \).

Panel (a) shows the optical depth as a function of specific column
density for the J = 1 − 0 and J = 2 − 1 transitions when there is no beaming. For both transitions, the optical depth decreases
toward a minimum value (maximum gain) after which it increases to
eventually become positive. While both transitions can be inverted
at the same specific column density, the maximum gain for the J = 2 − 1 transition is seen to occur at a smaller specific column density
than for the J = 1 − 0 transition. As will be shown later, it is generally
the case that the maximum gain for the two masers does not occur at
the same specific column density.

Panels (b)–(d) show how the behaviour of the optical depth changes
when beaming is introduced. Comparison with panel (a) shows that beaming has a marked effect on how the optical depth changes
with specific column density. Panel (b) shows that the
specific column density where maximum gain occurs has shifted
to larger specific column densities. Also, the maximum gain has increased. Perhaps the most interesting and noteworthy difference with Panel (a) is that the lifting of the inversion for the J = 2 − 1 transition occurs over one step in specific column density. When this happens the gain of the J = 1 − 0 maser increases since population in the J = 1 state ‘suddenly’ increases. This behaviour is quite general
since the two masers share the J = 1 state in a single ladder of
rotational states.

A further interesting result of increasing the beaming is that the J = 3 − 2 transition also becomes inverted as is seen in panels (c) and
(d). Note how now three masers are linked through J = 1, and
J = 2 states. Thus, when the inversion of the J = 3 − 2 transition is lifted, it affects the J = 2 − 1 and the J = 1 − 0 transitions. The
inversion of the J = 3 − 2 transition occurs only when collisions dominate the excitation and does not occur at all combinations of
nH2 and Td. For most of what follows the focus will be on the J = 1 − 0 and J = 2 − 1 masers in the ground vibrational state.

The results presented in Fig. 2 show that inversion of the J = 1 − 0
collisionally pumped. An infrared radiation field, however, permeates
the star formation environment and it is necessary to examine what
the effect of the presence of such a radiation field is. In addition to
the temperature of the dust, it is also necessary to consider the effect
of geometric dilution. Instead of considering the maser gain, we will
henceforth consider the maser brightness temperature, calculated

\[
CS(v = 0) \text{ masers in W51 e2e} \quad 3873
\]

Figure 2. Figure to illustrate how the optical depth changes with specific column density as well as the effect of increased beaming. In all four panels, only collisional excitation was considered. Note in (b) how the J = 1 − 0 transition is affected when the inversion of the J = 2 − 1 transition is lifted. Similar behaviour can also be seen in (c) and (d) where it is shown that the J = 3 − 2 transition is also inverted. Lifting of the inversion in the J = 3 − 2 transition affects both the J = 2 − 1 and J = 1 − 0 transitions. Inversion of the J = 3 − 2 transition occurs only when collisional excitation is the dominant mechanism. Parameter values used: \( n_{H_2} = 3.7 \times 10^5 \text{ cm}^{-3} \), \( T_d = 300 \text{ K} \).
from equation (7), which is more relevant for comparison with observations. The effect of the dust temperature on the brightness temperature is shown in Fig. 3 and that of the geometric dilution in Fig. 4 when the dust spectral energy distribution is that of a blackbody. Increasing the dust temperature results in a decrease in the maser brightness temperature with a similar behaviour when the dilution factor is increased. Note that a dilution factor of zero implies collisions only and a value of one means that the dust emission originates inside the masing region. These results imply that both masers are affected negatively when the dust emission becomes stronger. The case when the dust emissivity is of the more general type, i.e. where the optical depth has its largest negative value. The brightness temperature was calculated using the corresponding level populations at maximum gain for the particular transition. The brightness temperature was set equal to zero when there was no inversion.

In Fig. 5, we show the variation of the brightness temperature when collisions are the only excitation mechanism (infrared radiation field switched off) and with no beaming. It is seen that an inversion can obtained with collisions as the only pumping mechanism over a significant part of the nH2−T* plane. Both panels suggest that inversions occur also outside the ranges used for nH2 and T*K, i.e. for densities greater than 10^6 cm^-3 and T*K > 500 K. For the J = 1 − 0 transition the maximum brightness temperature is 114 and 91 K for the J = 2 − 1 transition. The brightness temperatures for both transitions are significantly smaller than what has been observed by Ginsburg & Goddi (2019). Although the region over which there is an inversion in the J = 2 − 1 transition is completely overlapped by the region over which there is inversion of the J = 1 − 0 transition, it is again noted that the inversions do not occur at the same CS specific column densities as will be discussed later.

Although radiative excitation alone does not result in inversion for any of the two transitions, the presence of a dust radiation field does have an effect on the inverted level populations as was shown in Figs 3 and 4. Fig. 6 shows the variation of the brightness temperature when the radiation field is that of a 100 K blackbody modified according to the wavelength-dependent escape probability.
Comparison with Fig. 5 shows that in the presence of such a radiation field the brightness temperature at a specific \((n_{\text{H}_2}, T_k)\) point is reduced for both transitions. Also, for both transitions, the region in the \(n_{\text{H}_2} - T_k\) plane where inversion occurs is smaller and is shifted to higher \(H_2\) densities.

In Figs 7–9, we show the effect of beaming on the maser brightness temperature when \(\Omega_i/4\pi = (2\tau_{ij} + 1)^{-\eta}\). Comparison of Fig. 7 (\(\eta = 1\)) with Fig. 6 shows that not only is the region over which inversion occurs now larger but the brightness temperatures are also higher. However, the maximum brightness temperatures of 593 and 346 K for the \(J = 1 - 0\) and \(J = 2 - 1\) masers, respectively, are still significantly less than what has been observed for W51 e2e. Increasing the beaming further (Fig. 8, \(\eta = 1.5\)) results in significant changes in the shape of the region in the \(n_{\text{H}_2}-T_k\) plane where inversion occurs. In this case, the maximum brightness temperatures for the \(J = 1 - 0\) and \(J = 2 - 1\) masers are 2550 and 1400 K, respectively, which are still too low to explain the observed brightness temperatures of the masers in W51 e2e.

The result when \(\Omega_i/4\pi = (2\tau_{ij} + 1)^{-2}\) is shown in Fig. 9. The maximum brightness temperatures for the \(J = 1 - 0\) and \(J = 2 - 1\) masers are respectively 24700 K and 8580 K. The white hatched regions show where the brightness temperature for each maser lies between 5000 and 9000 K. For the \(J = 1 - 0\) transition, the region has an arch-like shape and lies more or less between kinetic temperatures 300 and 400 K and \(10^{4.5} \lesssim n_{\text{H}_2} \lesssim 10^5 \text{ cm}^{-3}\). For the \(J = 2 - 1\) transition, the corresponding values are \(T_k \gtrsim 350\) K and \(10^5 \lesssim n_{\text{H}_2} \lesssim 10^{5.5} \text{ cm}^{-3}\). Fig. 9 shows that with stronger beaming the regions in the \(n_{\text{H}_2}-T_k\) plane where the one maser is significantly brighter than the other become separated. Closer comparison of the upper and lower panels of Figs 6 and 7, with Fig. 9 shows that all the masers in W51 e2e are located in the region for which \(\Omega_i/4\pi = (2\tau_{ij} + 1)^{-2}\). This result is in agreement with the previous results that \(\Omega_i/4\pi = (2\tau_{ij} + 1)^{-1}\) results in a smaller region and that \(\Omega_i/4\pi = (2\tau_{ij} + 1)^{-2}\).
lower panels shows that the $J = 2 - 1$ transition is not inverted over most of the region where the $J = 1 - 0$ transition is inverted. The $J = 1 - 0$ transition, on the other hand, is inverted over most of the region where the $J = 2 - 1$ maser occur but has brightness temperature which is significantly less than that of the $J = 2 - 1$ maser. Only at the very high temperatures is there a small region where there is overlap. It is also seen that there are two regions where the $J = 1 - 0$ maser can occur, one at $\text{H}_2$ densities smaller and one at densities greater than where the $J = 2 - 1$ maser occur.

We also considered the case when the dust emissivity has the form $(\nu/\nu_0)B_\nu(T_d)$ with $\nu_0 = 10^{13} \text{ Hz}$. Fig. 10 shows the result for the same values of the other parameters as for Fig. 9. It is interesting to note that the region over which the $J = 1 - 0$ transition is inverted is significantly larger compared to the case when it is assumed that the dust emissivity behaves like a blackbody. The arch-like region where $5000 < T_B < 9000$ for the $J = 1 - 0$ transition in Fig. 9, now extends down to lower $\text{H}_2$ densities and higher kinetic temperatures. To understand this change note that the largest energy difference amongst the 31 levels, i.e. between $J = 30$ and 29, is equivalent to a frequency of about $1.5 \times 10^{12} \text{ Hz}$. Having a dust emissivity that varies as $(\nu/\nu_0)B_\nu(T_d)$ with $\nu_0 = 10^{13} \text{ Hz}$ means that the energy density in the radiation field at those frequencies that can affect the level populations, is reduced by more than 15 per cent compared to that of a blackbody emissivity. The effect of this change in the spectral energy distribution is that collisions and spontaneous emission are effectively the only mechanisms that affect the level populations. To confirm this, we also performed calculations when collisions are the only excitation mechanism and with the beaming factor given by $(2\tau_{ij} + 1)^{-2}$. The result is shown in Fig. 11. The similarity with Fig. 10 is obvious and implies that dust emissivities of the form $(\nu/\nu_0)B_\nu(T_d)$
with $1 \leq p \leq 2$, which is often used (e.g. Pavlakis & Kylafis 1996, 2000; Cragg et al. 2002), will result in the $J = 1 - 0$ and $J = 2 - 1$ masers having the same behaviour as that in Fig. 11.

A noticeable feature in Figs 9–11 is the stair-step behaviour which needs some explanation. Closer inspection of the model results shows that at a given kinetic temperature, the brightness temperatures of the $J = 1 - 0$ and $J = 2 - 1$ transitions increase significantly with an increase of 0.1 or 0.2 dex from the density where there first is an inversion. For example, in the case of Fig. 11, at $T_k = 200$ K the $J = 1 - 0$ transition already has a very weak inversion at H$_2$ densities of $10^4$ and $10^{4.1}$ cm$^{-3}$ with brightness temperatures of 0.5 and 1.0 K, respectively. Increasing the density to $10^{4.2}$ cm$^{-3}$ raises the brightness temperature to 2833 K. The same behaviour occurs at 180 K where the brightness temperature changes from 0.8 K at an H$_2$ density of $10^{4.1}$ cm$^{-3}$ to 2215 K at $10^{4.2}$ cm$^{-3}$. At 240 K the increase is from 1.3 to 3300 K. Although there is an increase of about 1000 K in the brightness temperature from $T_k = 180$ to 240 K at $n_{H_2} = 10^{4.2}$ cm$^{-3}$, such a change is not visible on the contour plots. A contour level of, say, 2000 K for the brightness temperature will therefore occur at the same H$_2$ density for kinetic temperatures between 180 and 240 K in this particular example. The same explanation applies to other kinetic temperature intervals, e.g. from 250 to 340 K, where the maser brightness temperature, as represented by the contours, seems to stay constant. The stair-step effect is therefore a consequence of the 0.1 dex step size in $n_{H_2}$ and would have been smaller if a smaller step size was used. We note that such a rapid increase in brightness temperature with a small change in $n_{H_2}$ is also seen in the results of Pavlakis & Kylafis (2000) on OH masers.

Considering the results presented in Panels (c) and (d) of Fig. 2 as well as that in Figs 10 and 11, we examined over what region in the $n_{H_2} - T_k$ plane the $J = 3 - 2$ transition is inverted. The result is shown in Fig. 12. Since the inversion occurs only at higher densities the calculations were done for H$_2$ densities greater than $10^5$ cm$^{-3}$ only and $T_k \geq 100$ K. In this particular case, the step size in density was 0.05 dex. Comparison of the two panels shows that the region where the $J = 3 - 2$ transition is inverted is basically the same as the higher density region (there is also a region at lower densities not shown in this Figure) where the $J = 1 - 0$ transition is inverted but not the $J = 2 - 1$ transition. However, as was shown in Fig. 2, the specific column density where the $J = 3 - 2$ transition has maximum gain is smaller than where the $J = 1 - 0$ transition has maximum gain. The second point to note is that the brightness temperature of the $J = 3 - 2$ maser is comparable with that of the $J = 1 - 0$ and $J = 2 - 1$ masers observed in W51 e2e, i.e. in excess of several thousand Kelvin. Thus, at least theoretically, these results suggest that, based on the brightness temperature, the $J = 3 - 2$ maser should be detectable in W51 e2e depending on the presence of gas with densities $\sim 10^{5.4}$ to $\sim 10^{5.8}$ cm$^{-3}$ and with kinetic temperatures greater than about 250 K.

We also considered the case when beaming is taken to be the same as that used by Sobolev et al. (1997). The result is shown in Fig. 13. While to a large extent the distribution in the $n_{H_2} - T_k$ plane of where inversion occurs is the same as for the other cases considered, it seen that the maximum brightness temperatures for the $J = 1 - 0$ and $J = 2 - 1$ transitions are much higher. In the case of the $J = 1 - 0$ transition, it exceeds $10^8$ K! For both the $J = 1 - 0$ and $J = 2 - 1$ transitions
transitions the regions where $5000 \leq T_B \leq 9000$ K are two narrow strips. It was already noted briefly with regard to Fig. 5, that although there is overlap in the $n_{H_2} - T_k$ plane where the $J = 1 - 0$ and $J = 2 - 1$ masers occur, these masers have their maximum gains at different specific column densities. This is illustrated in the upper panel of Fig. 14 where the specific column density where the $J = 2 - 1$ maser has maximum gain is plotted against the specific column density where the $J = 1 - 0$ maser has maximum gain. Upper panel: for collisions only and no beaming (see Fig. 5). Bottom panel: for collisions only and with the beaming factor given by $(2\tau_{ij} + 1)^{-2}$ (see Fig. 11).

Figure 14. Plot of the specific column density where the $J = 2 - 1$ maser has maximum gain against the specific column density where the $J = 1 - 0$ maser has maximum gain. Upper panel: for collisions only and no beaming (see Fig. 5). Bottom panel: for collisions only and with the beaming factor given by $(2\tau_{ij} + 1)^{-2}$ (see Fig. 11).

for group A the brightness temperature of the $J = 1 - 0$ maser is greater than 100 K while for the $J = 2 - 1$ maser it is only about 1 or 2 K. Group B is just the opposite of group A, viz. the $J = 2 - 1$ maser is bright and the $J = 1 - 0$ maser very weak. For group C both masers have brightness temperatures greater than 1000 K. The location of the three groups relative to the $y = x$ line shows that generally the $J = 1 - 0$ and $J = 2 - 1$ masers do not have their respective maximum gains at the same CS specific column density.

4 DISCUSSION

4.1 General comments

One of the first questions asked about a particular astronomical maser is how it is pumped. In the case of the CS masers the results of the numerical calculations presented above clearly suggest that these masers are pumped collisionally. It was shown that the $J = 1 - 0$, $J = 2 - 1$, and $J = 3 - 2$ transitions can be inverted in the absence of an external radiation field or when the dust emissivity is of the form $(\nu \tau_{ij})^p B_\nu(T_k)$ with $1 \leq p \leq 2$ and $\nu_0 = 10^{13}$ Hz, in which case the emission below $10^{13}$ Hz is strongly reduced relative to that of a blackbody. On the other hand, dust emission with a blackbody spectral energy distribution below about $10^{13}$ Hz negatively affects the population inversion of these three transitions. Considering all of these results it can be concluded that the CS masers are collisionally pumped.

We also note that there is some qualitative agreement with the observations of Ginsburg & Goddi (2019). These authors found that the $J = 1 - 0$ and $J = 2 - 1$ masers are clearly separated spatially and kinematically. Considering the cases where the brightness temperatures of the two masers fall between 5000 and 9000 K (Figs 9 and 10) as well as that the two masers do not have maximum gain at the same specific column densities (Fig. 14) very strongly suggests that the $J = 1 - 0$ and $J = 2 - 1$ masers will observationally not be found to be co-spatial.

As pointed out by Ginsburg & Goddi (2019), the newly discovered CS $J = 1 - 0$ and $J = 2 - 1$ masers associated with W51 e2e form part of a group of rare masers which raises the question of why these masers are so rare. These authors speculate that W51 e2e may have unique characteristics regarding the infrared radiation field, chemistry, geometry or any other unique aspect that favours CS masers. As for the radiation field, we already noted that brighter masers are produced in an environment with a dust emissivity of the form $(\nu \tau_{ij})^p B_\nu(T_k)$ rather than with a blackbody dust emissivity. However, such a dust emissivity is not unique and cannot be considered as a reason why these masers are rare. Another factor may be the geometry of the masing region which can affect the beam and therefore the brightness of the masers. It was found that beam which is stronger than that given by Elitzur (1992) but similar to that found by Spaans & van Langevelde (1992) is needed to explain the observed brightness temperatures of the masers. However, it should not be as strong as that used by Sobolev et al. (1997). On the other hand, W51 e2e also has associated class II methanol masers, which quite generally have very high brightness temperatures and which requires strong beaming. Why a different behaviour of the beaming factor is required for CS and CH$_3$OH masers is not clear.

4.2 Column densities

Two other factors that can play a role in explaining the uniqueness of W51 e2e for favouring CS masers are the CS abundance and column density. We first consider the column density (cm$^{-2}$) which is found...
from the CS specific column density where the inversion is maximum and the linewidth. The linewidths given by Ginsburg & Goddi (2019) are basically instrumental linewidths and it can therefore be expected that the real maser linewidths are smaller. Here, we use a linewidth of 1.0 km s\(^{-1}\) for both masers which can be considered as a lower limit. For illustrative purposes, CS column densities were calculated for both masers at two combinations of kinetic temperature and \(H_2\) density as indicated by the crosses in Fig. 10. For the \(J = 1 - 0\) maser at \(T_K = 270\) K and \(n_{H_2} = 10^{5.6}\) cm\(^{-3}\), the column density is found to be \(\log N_{CS} = 15.9\). For \(T_K = 400\) K and \(n_{H_2} = 10^{5.2}\) cm\(^{-3}\) the column density is \(\log N_{CS} = 16.1\) which, for practical purposes, is the same as for the lower temperature case. For the two combinations for the \(J = 2 - 1\) maser, the CS column density was found to be the same viz. \(\log N_{CS} = 15.6\).

These values can be compared with column densities derived by a number of authors using LVG model fitting of observations of high-mass star-forming regions. For 11 northern cores Zinchenko et al. (1994) found (\(\log N_{CS}\)) = 13.6. Juvela (1996) did a multitransition study toward 33 southern \(H_2O\) maser sources and found (\(\log N_{CS}\)) = 13.62. Plume et al. (1997) also did a multitransition CS study of 150 high-mass star-forming regions with associated \(H_2O\) masers. These authors found (\(\log N_{CS}\)) = 14.36 for 71 of these regions. Lariiov et al. (1999) did a survey of 158 bipolar outflow and methanol maser sources in the Northern sky and give CS column densities for 50 of these sources with (\(\log N_{CS}\)) = 14.47. Simply taken at face value, these column densities are at least an order of magnitude less than what is derived from the model calculations. However, it has to be noted that all four of these studies were using single-dish telescopes and that the derived column densities are averages over significantly large regions of the individual star-forming regions observed. For example, the beam size for the observations by Plume et al. (1997) range between 10 and 30 arcsec and for W51 included many unresolved cores for which the CS column densities certainly are higher than the beam averaged values.

4.3 Abundances

As in the case for the column densities, a quantitative comparison of abundances derived from observations with that from the model is not simple for two reasons. First, abundances determined from single-dish observations are, like column densities, averages along the line of sight as well as beam averaged and therefore do not take source structure into account (Wakelem et al. 2004). Secondly, abundances derived from the model results are calculated from the relation \(X_{CS} n_{H_2} \ell = N_{CS}\), where \(X_{CS}\) is the CS abundance relative to \(H_2\), \(\ell\) is the maser path-length and, \(N_{CS}\) is the CS column density calculated from the product of the specific column density where the maser gain is maximum and the maser linewidth (taken as 1 km s\(^{-1}\)). The maser path-length is unknown and the derived CS abundance will therefore depend on the assumed maser path-length. In spite of these limitations, it is nevertheless useful to compare the abundances inferred from the model calculations with some observationally derived values. We first consider CS abundances derived from observations and then maser path-lengths used by other authors.

Dickens et al. (2000) derived CS abundance ranges between \(7 \times 10^{-10}\) and \(7 \times 10^{-9}\) for the dark cloud L134N. Such low abundance is expected for dark clouds. For the Orion hot core, Chandler & Wood (1997) derived a CS abundance of \(1.2 \times 10^{-8}\), which is only slightly higher than the maximum CS abundance for L134N. Similar abundances were also found for W43MM1, IRAS18264-1152, IRAS05358+3543, IRAS18162-2048 (Herpin et al. 2009) and AFGL2591 (van der Tak et al. 2003). Bergin et al. (1997) derived CS abundances relative to CO of between \(2 \times 10^{-5}\) and \(1 \times 10^{-4}\) for M17 and Cepheus, respectively, which gives CS abundances relative to \(H_2\) of the order of \(10^{-8}\) to \(10^{-9}\) using [CO]/[H\(_2\)] = \(10^{-4}\) (Krugeel 2003).

More recently Barr et al. (2018) derived a CS fractional abundance relative to \(H_2\) of \(2 \times 10^{-6}\) for the hot molecular core associated with the massive protostar AFGL 2591 using mid-infrared spectroscopy. These observations probed the hot gas (\(< 700\) K) which strongly points a higher CS abundance closer to the protostar.

As for maser path-lengths, we note the following. Based on an angular diameter of 0.005 arcsec for OH masers in W3(OH), Moran et al. (1978) inferred a total maser path-length of about \(10^{15}\) cm for unsaturated OH masers. Reid et al. (1980) similarly concluded that gain lengths of the order of \(2 \times 10^{15}\) cm are required to explain very bright OH masers in W3(OH). Also considering OH masers, Gray, Field & Doel (1992) concluded that long maser path-lengths, i.e. greater than \(10^{14}\) cm may be composed of a series of regions along the line of sight having similar velocities. These authors also conclude that for OH masers, path-lengths as short as \(10^{15}\) cm may be adequate to explain observed maser brightness temperatures. Pavlakis & Kylafis (1996) and Pavlakis & Kylafis (2000) used a maser path-length of \(5 \times 10^{15}\) cm for OH masers in star-forming regions. On the other hand, Sobolev et al. (1997) and Cragg et al. (2002) assumed a path-length of \(10^{17}\) cm in their models for class II CH\(_3\)OH masers.

In spite of the fact that deriving a CS abundance from the model results depends on an assumed maser path-length, some constraints can be used to put limits on the maser path-length and therefore on the CS abundance. As already noted earlier, the \(J = 2 - 1\) maser is pumped at a higher \(H_2\) density compared to the \(J = 1 - 0\) maser which might suggest that the \(J = 2 - 1\) masing region is closer to the exciting star and that the kinetic temperature is also higher than in the masing region of the \(J = 1 - 0\) maser. To take such a scenario approximately into account, we calculated \((N_{CS}/n_{H_2})\) over selected regions of the \(n_{H_2}-T_K\) plane where the brightness temperature for the \(J = 1 - 0\) and \(J = 2 - 1\) masers lie between 5000 and 9000 K. For the \(J = 1 - 0\) maser, we considered \(H_2\) densities greater than \(10^{4.6}\) cm\(^{-3}\) and kinetic temperature between 220 and 290 K. For the \(J = 2 - 1\) maser, we assumed the kinetic temperature to be between 300 and 400 K. (see Figs 10 and 11). Maser path-lengths can then be calculated using abundances based on observations.

Thus, assuming a CS abundance of, say, \(10^{-8}\), path-lengths for the \(J = 1 - 0\) and \(J = 2 - 1\) masers of, respectively, \(1.5 \times 10^8\) and \(2.6 \times 10^9\) au are found which, obviously, are completely unrealistic. The implication is that the CS abundance associated with the two masing regions must be greater than \(10^{-8}\). Increasing the abundance to \(10^{-7}\) means that the corresponding maser path-lengths become \(1.5 \times 10^9\) and \(2.6 \times 10^10\) au, respectively. A path-length of \(1.5 \times 10^9\) au for the \(J = 1 - 0\) maser is also doubtful since it implies a maser path-length significantly larger than the outflows and the dust continuum emission associated with W51 e2e (see fig. 1 of Goddi et al. 2018). The \(J = 1 - 0\) and \(J = 2 - 1\) masers are projected at the base of the outflow and may be associated with a disc or other rotating structure. Given the size of a possible disc as shown by Goddi et al. (2018), it seems reasonable to argue that the longest path-length should be less than the diameter of the disc, i.e. \(< 720\) au. For example, a CS abundance of \(2.5 \times 10^{-5}\) results in path-lengths of \(\sim 670\) au for the \(J = 1 - 0\) and \(J = 2 - 1\) masers, respectively. The path-lengths may even be smaller due to radial-velocity gradients in a rotating structure which means that the CS abundance may also be higher. However, the CS abundance is expected to be less than the general CO abundance of \(< 10^{-4}\)
which requires the path-length for the \( J = 1 - 0 \) maser to be greater than about 150 au and greater than about 26 au for the \( J = 2 - 1 \) maser. Even though it is not possible to fix the maser path-length, it is rather clear that the model calculations require the CS abundance in W51 e2 to be at least two orders and maybe even three orders of magnitude higher than the CS abundances based on single-dish observations. The higher CS abundance suggested by the model calculations is at least qualitatively in agreement with the observational results of Barr et al. (2018).

### 4.4 Maser amplification or attenuation in the surrounding medium?

The results in Figs 9–11 show the regions in the \( H_2 - T_k \) plane where the brightness temperatures for \( J = 1 - 0 \) and \( J = 2 - 1 \) masers corresponds to that observed by Ginsburg & Goddi (2019). Considering the \( H_2 \) densities and kinetic temperatures where the masers occur it follows that the maser emission has to propagate through considerable molecular material before ‘emerging’ from the star-forming region. The question therefore arises whether there is additional amplification or attenuation of the masers in the surrounding molecular envelope.

Further amplification of the maser emission will take place if it propagates through regions where a population inversion also exists. Our calculations do not take such a scenario explicitly into account. Consider, however, for example the solutions at \( T_k = 270 \) K and \( n_{H_2} = 10^{4.5} \) cm\(^{-3}\) for the \( J = 1 - 0 \) maser and for the \( J = 2 - 1 \) maser at \( T_k = 320 \) K and \( n_{H_2} = 10^{4.2} \) cm\(^{-3}\) as indicated by the crosses in the upper and lower panels of Fig. 10. Quite generally it can be assumed that no beaming occurs outside of the masing region and that excitation is dominated by collisions, the situation as shown in Fig. 5 applies. Fig. 15 shows the variation of the amplification factor (\( e^{-\tau} \)) corresponding to the brightness temperature as shown in Fig. 5. It is seen that without any beaming further amplification outside the masing region is rather small for both the \( J = 1 - 0 \) and \( J = 2 - 1 \) masers and that it decreases toward lower densities and kinetic temperatures.

It is also seen from Fig. 15 that there is a significant part in the \( H_2 - T_k \) plane where, for both masers, there is no amplification which implies that the maser beam might be attenuated when propagating through the surrounding molecular medium. To estimate the attenuation, we calculated (assuming LTE) the optical depth in the \( J = 1 - 0 \) and \( J = 2 - 1 \) lines from

\[
\tau = \frac{1}{8\pi} \left( \frac{c}{v_{ad}} \right)^3 \frac{g_0}{g_1} A_0 \int_{r_{min}}^{r_{max}} [1 - \exp(-E_{ad}/kT(r))] n_l(r)dr
\]

with

\[
n_l(r) = \frac{1}{Z(r)} \frac{1}{\Delta \sigma(r)} X_{CS}(r)n_{H_2}(r)g_l \exp(-E_{l,0}/kT(r)).
\]

In equations (8) and (9), \( n_l(r) \) is the radial dependent CS number density, \( Z(r) \) the partition function, \( \Delta \sigma(r) \) the radial dependent velocity dispersion of the absorbing gas, \( X_{CS} \) the CS abundance, and \( E_{l,0} = E_l - E_0 \), with \( E_0 \) the ground-state energy.

Applying equations (8) and (9) to the possible attenuation of the masers requires the presence of hot gas as implied by the model results. The presence of hot gas up to \( \sim 600 \) K in W51 e2 was recently demonstrated by Ginsburg et al. (2017) from the analysis of a number of methanol lines for which \( 45 < E_u < 800 \) K which means that applying equations (8) and (9) to the case of W51 e2 is justified. For the temperature profile, we adopt that used by Gieser et al. (2019) for the AFGL 2591 VLA 3 hot core,

\[
T_k(r) = 255 K \left( \frac{r}{691 \text{AU}} \right)^{-0.41}
\]

for \( r < 4000 \text{ au} = 0.019 \text{ pc} \). For \( 0.019 < r < 0.1 \text{ pc} \), the kinetic temperature varies with a power law such that \( T_k = 30 \text{ K} \) at \( 0.1 \text{ pc} \) beyond which it stays constant at 30 K.

For the \( H_2 \) density we assumed

\[
n_{H_2}(r) = \left[ 3 \times 10^3 + n_{H_2}(r_0) \left( \frac{r}{r_0} \right)^{-1} \right] \text{ cm}^{-3},
\]

where \( r_0 \equiv 39 \text{ au} \) is the radial distance for the \( J = 2 - 1 \) maser calculated from equation (10) for \( T_k = 320 \text{ K} \). This behaviour gives \( n_{H_2} = 3.3 \times 10^3 \text{ cm}^{-3} \) at \( r = 1 \text{ pc} \). For the \( J = 1 - 0 \) maser with \( T_k = 270 \text{ K} \), the radial distance is found to be 601 au and \( n_{H_2} \sim 10^2 \text{ cm}^{-3} \). Inspection of Fig. 10 shows that such a density is only slightly larger than the maximum density for \( T_k = 270 \text{ K} \) for which the maser brightness temperature lies between 5000 and 9000 K. It is rather interesting to note that the radial separation of \( \sim 200 \text{ au} \) between the masers as implied by the temperature profile in equation (10) agrees remarkably with observed separation of \( 190 \pm 60 \text{ au} \) between the masers as found by Ginsburg & Goddi (2019).

The radial dependence of the velocity dispersion was related to the kinetic temperature according to \( \Delta \sigma(r) = 8 \text{ km s}^{-1} (T_k(r)/700 \text{ K})^{0.31} \) which gives \( \Delta \sigma \sim 3 \text{ km s}^{-1} \) at 0.1 pc.
and beyond which it stayed constant at 3.0 km s\(^{-1}\). The velocity dispersion of 8.0 km s\(^{-1}\) at 700 K follows from the work Barr et al. (2018). For the radial dependence of the CS fractional abundance, the result by Bruderer, Dasy & Benz (2009) (see their Fig. 6) was modified to account for a higher CS fractional abundance as was argued earlier. The fractional abundance was therefore set at 2 \(\times 10^{-9}\) at the position of the \(J = 2 - 1\) maser and to decrease as a power law to 1.0 \(\times 10^{-7}\) at 2 \(\times 10^{16}\) cm. For \(r > 2 \times 10^{16}\) cm, the fractional abundance was set at 5 \(\times 10^{-9}\).

Using these radial dependencies in equations (8) and (9) and integrating up to \(r_{\text{max}} = 1\) parsec gives \(\tau = 0.6\) for the \(J = 1 - 0\) maser and \(\tau = 1.4\) for the \(J = 2 - 1\) maser. The difference in optical depth between the two transitions is mainly due to the smaller radial position of the \(J = 2 - 1\) maser and therefore also the increased density. It is also worth noting that \(\sim 60\) per cent of the optical depth for the \(J = 2 - 1\) maser originates inside the hot core region while for the \(J = 1 - 0\) maser \(\sim 77\) per cent of the optical depth is due to the cold gas outside the hot core. Since the radial dependencies used the calculation may not reflect the real situation for W51 e2e and does not consider any clumpiness, these values should only be used as a rough estimate. In spite of the uncertainties, these results suggest that attenuation of the \(J = 1 - 0\) and \(J = 2 - 1\) masers might be non-negligible. The implication is that such attenuation may result in completely obscuring the masers and be a contributing factor to the rarity of these masers. For W51 e2e, the implication is that the detection of the masers might be due to a ‘hole’ in the surrounding molecular material as suggested by Ginsburg & Goddi (2019).

5 CONCLUSIONS

We solved the rate equations for the first 31 rotational states of CS in the ground vibrational state and considered the effect of collisions, radiation and beaming on the level populations to determine the conditions under which the \(J = 1 - 0, J = 2 - 1, \) and \(J = 3 - 2\) transitions are inverted. The main result from this study is that the three transitions can be inverted with collisions as the only excitation mechanism. To reproduce the brightness temperatures of the three transitions can be inverted with collisions as the only excitation and beyond which it stayed constant at 3.0 km s\(^{-1}\). The velocity dispersion of 8.0 km s\(^{-1}\) at 700 K follows from the work Barr et al. (2018). For the radial dependence of the CS fractional abundance, the result by Bruderer, Dasy & Benz (2009) (see their Fig. 6) was modified to account for a higher CS fractional abundance as was argued earlier. The fractional abundance was therefore set at 2 \(\times 10^{-9}\) at the position of the \(J = 2 - 1\) maser and to decrease as a power law to 1.0 \(\times 10^{-7}\) at 2 \(\times 10^{16}\) cm. For \(r > 2 \times 10^{16}\) cm, the fractional abundance was set at 5 \(\times 10^{-9}\).

Using these radial dependencies in equations (8) and (9) and integrating up to \(r_{\text{max}} = 1\) parsec gives \(\tau = 0.6\) for the \(J = 1 - 0\) maser and \(\tau = 1.4\) for the \(J = 2 - 1\) maser. The difference in optical depth between the two transitions is mainly due to the smaller radial position of the \(J = 2 - 1\) maser and therefore also the increased density. It is also worth noting that \(\sim 60\) per cent of the optical depth for the \(J = 2 - 1\) maser originates inside the hot core region while for the \(J = 1 - 0\) maser \(\sim 77\) per cent of the optical depth is due to the cold gas outside the hot core. Since the radial dependencies used the calculation may not reflect the real situation for W51 e2e and does not consider any clumpiness, these values should only be used as a rough estimate. In spite of the uncertainties, these results suggest that attenuation of the \(J = 1 - 0\) and \(J = 2 - 1\) masers might be non-negligible. The implication is that such attenuation may result in completely obscuring the masers and be a contributing factor to the rarity of these masers. For W51 e2e, the implication is that the detection of the masers might be due to a ‘hole’ in the surrounding molecular material as suggested by Ginsburg & Goddi (2019).

The detection of the CS(\(J = 1 - 0\)) and CS(\(J = 2 - 1\)) masers in W51 e2e raises other questions as well. For example, since the pumping of the 6.7-GHz methanol masers require the kinetic temperature to be less than the dust temperature (\(\sim 100\) K) (Sobolev et al. 1997; Cragg et al. 2002), what is the relation between the CS \(J = 1 - 0\) and the 6.7-GHz methanol masers that are in projection rather close to each other? Furthermore, if the 6.7-GHz methanol masers indeed require maser path-lengths of about \(10^{16}\) cm (6700 au), what physical structure are they associated with? A second question is that if a high CS abundance is not the reason for the rarity of the CS masers, what else can be the reason? If the rarity of the CS masers in high-mass star-forming regions is indeed due to a required high CS abundance, why is it that this requirement is met in W51 e2e and not in the other high-mass young stellar objects in W51?

DATA AVAILABILITY

No new data were generated or analysed in support of this research. The code used to obtain the numerical results will be shared on reasonable request to the corresponding author.

REFERENCES

Araya E. D., Hofner P., Goss W. M., Kurtz S., Richards A. M. S., Linz H., Olmi L., Sewilo M., 2010, ApJ, 717, L133
Barr A. G. et al., 2018, ApJ, 868, L2
Bergin E. A., Ungerechts H., Goldsmith P. F., Snell R. L., Irvine W. M., Schloerb F. P., 1997, ApJ, 482, 267
Brogan C. L. et al., 2018, ApJ, 866, 87
Bruderer S., Dasy S. D., Benz A. O., 2009, ApJS, 183, 179
Castor J. L., 1970, MNRAS, 149, 111
Chandler C. J., Wood D. D. S., 1997, MNRAS, 287, 445
Cragg D. M., Sobolev A. M., Godfrey P. D., 2002, MNRAS, 331, 521
Dickens J. E., Irvine W. M., Snell R. L., Bergin E. A., Schloerb F. P., Pratap P., Miralles M. P., 2000, ApJ, 542, 870
Elitzur M., 1992, Astronomical Masers. Astrophysics and Space Science Library Vol. 170. Kluwer, Dordrecht
Gieser C. et al., 2019, A&A, 631, A142
Ginsburg A. et al., 2017, ApJ, 842, 92
Ginsburg A., Goddi C., 2019, AJ, 158, 208
Goddi C., Ginsburg A., Maud L., Zhang Q., Zapata L., 2020, ApJ, 905, 25
Goedhart S., Gaylard M. J., van der Walt D. J., 2003, MNRAS, 339, L33
Goedhart S., Gaylard M. J., van der Walt D. J., 2004, MNRAS, 355, L53
Goedhart S., Maswanganyi J. P., Gaylard M. J., van der Walt D. J., 2014, MNRAS, 437, 1808
Goedhart S., van Roojen R., van der Walt D. J., Maswanganyi J. P., Sanna A., MacLeod G. C., van den Heever S. P., 2019, MNRAS, 485, 4676
Gray M. D., Field D., Doel R. C., 1992, A&A, 262, 555
Herpin F., Marseille M., Wakelam V., Bon temps S., Lis D. C., 2009, A&A, 504, 853
Highbarger J. L., Apponi A. J., Bieging J. H., Ziurys L. M., Mangum J. G., 2000, ApJ, 544, 881
Juvela M., 1996, A&AS, 118, 191
Kruegel E., 2003, in Kruegel E., ed, The Physics of Interstellar Dust, IOP Publishing, Bristol, 428p
Larionov G. M., Val’ts I. E., Winnberg A., Johansson L. E. B., Booth R. S., Golubev V. V., 1999, A&AS, 139, p. 257
Moran J. M., Reid M. J., Lada C. J., Jen L. J., Johnston K. J., Spencer J. H., 1978, ApJ, 224, L67
Olech M., Szymczak M., Wolak P., Sarniak R., Bartkiewicz A., 2019, MNRAS, 486, 1236
Pavlakis K. G., Kylafis N. D., 2000, ApJ, 546, 370
Pavlakis K. G., Kylafis N. D., 2000, ApJ, 534, 770
Plume R., Jaffe D. T., Evans Neal J. I., Martín-Pintado J., Gómez-González J., 1997, ApJ, 476, 730
Reid M. J., Haschick A. D., Burke B. F., Moran J. M., Johnston K. J., Swenson G. W. J., 1980, ApJ, 239, 89
Schöier F. L., van der Tak F. F. S., van Dishoeck E. F., Black J. H., 2005, A&A, 432, 369
Sobolev A. M., Cragg D. M., Godfrey P. D., 1997, A&A, 324, 211
Spaans M., van Langevelde H. J., 1992, MNRAS, 258, 159
Sugiyama K. et al., 2017, PASJ, 69, 59
Szymczak M., Wolak P., Bartkiewicz A., 2015, MNRAS, 448, 2284
Szymczak M., Olech M., Wolak P., Bartkiewicz A., Gawroński M., 2016, MNRAS, 459, L56
Szymczak M., Olech M., Wolak P., Gérard E., Bartkiewicz A., 2018, A&A, 617, A80
van der Tak F. F. S., Boonman A. M. S., Braakman R., van Dishoeck E. F., 2003, A&A, 412, 133
van der Walt D. J., 2014, A&A, 562, A68
Wakelam V., Caselli P., Ceccarelli C., Herbst E., Castets A., 2004, A&A, 422, 159
Zinchenko I., Forsstroem V., Lapinov A., Mattila K., 1994, A&A, 288, 601

This paper has been typeset from a TeX/LaTeX file prepared by the author.