Multivariate Markov Chain Model for Sales Demand Estimation in a Company

Annisa Martina 1*

1Mathematics Department, Faculty of Science and Technology, UIN Sunan Gunung Djati Bandung
Jalan A.H. Nasution 105, Bandung, Indonesia

*annisamartina@uinsgd.ac.id

Abstract

Estimation of the number of demands for a product must be done correctly, so that the company can get maximum profit. Therefore, this study discusses how to estimate the amount of sales demand in a company correctly. The model that will be used to estimate sales demand is the Multivariate Markov Chain Model. This model can estimate the future state by observing the present state. The model requires parameter estimation values first, namely the transition probability matrix and the weighted Markov chain, where in previous studies an estimation of the transition probability matrix has been carried out, so that in this study we will continue to estimate the weighted Markov chain parameters. This model is compatible with 5 data sequences (product types) defined as product 1, product 2, product 3, product 4, and product 5, with 6 conditions (no sales volume, very slow-moving, slow-moving, standard, fast moving, and very fast moving). As the result, the state probability for product 1, product 2 and product 3 in company 1 are stationary at state 6 (very fast moving), product 4 and product 5 are stationary at state 2 (very slow moving).

Keywords: Multivariate Markov chain model, demand, estimation, the weighted Markov chain

I. INTRODUCTION

Markov chain was first created by a Russian professor named Andrei A. Markov (1856-1922), the following is Markov's statement which was later referred to as Markov property (Markovian Property): "The conditional probability of a future event, given that the future event is known, past and present events are independent of past events and dependent only on present events.” [1] The formulation of the problem in this research is how to model the optimization problem of sales demand by using the multivariate Markov chain model. In previous studies [2], the estimation of the transition probability matrix has been carried out, so that in this research it will be continued to estimate the weighted Markov chain parameters. This model will be solved by linear program optimization method, using linprog facility in Matlab software.

II. LITERATURE REVIEW

In this multivariate Markov chain model, [3] it is assumed that there are s categories of categorical data (product types), each of which has m states (for example: many, few, etc.).

\[ M = \{1, 2, ..., m\} \]
Let $x_{n}^{(j)}$ be the state probability vector of the $j$th sequence (product types) at time $n$. If at the $n$th time, the $j$th sequence (product types) is in state $l$, then it can be written [4]:

$$x_{n}^{(j)} = e_{l} = (0, \ldots, 0, 1, 0, \ldots, 0)^{T}.$$ 

In constructing the multivariate Markov chain model, the following equation is assumed [5]:

$$x_{n+1}^{(j)} = \sum_{k=1}^{s} \lambda_{jk} p_{jk}^{(k)} x_{n}^{(k)}, \text{for } j = 1, 2, \ldots, s$$

(1)

where $\lambda_{jk} \geq 0, 1 \leq j, k \leq s$, and $\sum_{k=1}^{s} \lambda_{jk} = 1$, for $j = 1, 2, \ldots, s$.

Thus, based on equation (1), the state probability distribution of the sequence (product type) $j$ at time $(n+1)$ depends on the states of the sequence (product type) $j$ and $k$ at time $n$ [6]. Here $\lambda_{jk}$ is the weighted Markov chain which includes the effect of the sequence state (product type) $k$ to $j$. As $p_{jk}^{(k)}$ is the probability of the sequence state (product type) $k$ to $j$,[7] and $x_{n}^{(k)}$ is the probability of the sequence state (product type) $k$ at time $n$. [8] The following is writing in matrix:

$$x_{n+1} = \begin{pmatrix} x_{n+1}^{(1)} \\ x_{n+1}^{(2)} \\ \vdots \\ x_{n+1}^{(s)} \end{pmatrix} = \begin{pmatrix} \lambda_{11} p_{11}^{(1)} & \lambda_{12} p_{12}^{(12)} & \cdots & \lambda_{1s} p_{1s}^{(1s)} \\ \lambda_{21} p_{21}^{(2)} & \lambda_{22} p_{22}^{(22)} & \cdots & \lambda_{2s} p_{2s}^{(2s)} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{s1} p_{s1}^{(s)} & \lambda_{s2} p_{s2}^{(s2)} & \cdots & \lambda_{ss} p_{ss}^{(ss)} \end{pmatrix} \begin{pmatrix} x_{n}^{(1)} \\ x_{n}^{(2)} \\ \vdots \\ x_{n}^{(s)} \end{pmatrix}$$

$$= Q x_{n}$$

(2)

Model (2) shows that there is only a positive correlation between product lines, so it is assumed $\lambda_{jk}$ to have a non-negative value [9]. This illustrates that increasing the probability of each sequence (product) at time $n$ can only increase the probability of the state at time $(n+1)$. While in actual conditions it is often found a decrease in circumstances, for example from a behavior state to not selling. Therefore, in order for this model to include a state derivation, [10] the state probability vector $x_{n}^{(j)}$ must be negatively correlated with the probability vector $x_{n+1}^{(j)}$ as follows:

$$z_{n+1}^{(j)} = \frac{1}{m-1} (1 - x_{n}^{(j)})$$

(3)

1 is a vector of one, and $\frac{1}{m-1}$ is a normalization constant with $m > 2$.

The next step is to build a new model for $s$ sequences (products) $x_{n}^{(1)}, x_{n}^{(2)}, \ldots, x_{n}^{(s)}$ by including the probability of deriving the state as in equation (3).

The following is a new multivariate Markov chain model that includes state derivation:

$$x_{n+1} = \begin{pmatrix} x_{n+1}^{(1)} \\ x_{n+1}^{(2)} \\ \vdots \\ x_{n+1}^{(s)} \end{pmatrix} = \Lambda^{+} \begin{pmatrix} x_{n}^{(1)} \\ x_{n}^{(2)} \\ \vdots \\ x_{n}^{(s)} \end{pmatrix} + \left( \frac{1}{m-1} \right) \Lambda^{-} \begin{pmatrix} 1 - x_{n}^{(1)} \\ 1 - x_{n}^{(2)} \\ \vdots \\ 1 - x_{n}^{(s)} \end{pmatrix}$$

positive correlation

(4)

negative correlation

where $\Lambda^{+} = \begin{pmatrix} \lambda_{11} p_{11}^{(11)} & \lambda_{12} p_{12}^{(12)} & \cdots & \lambda_{1s} p_{1s}^{(1s)} \\ \lambda_{21} p_{21}^{(21)} & \lambda_{22} p_{22}^{(22)} & \cdots & \lambda_{2s} p_{2s}^{(2s)} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{s1} p_{s1}^{(s1)} & \lambda_{s2} p_{s2}^{(s2)} & \cdots & \lambda_{ss} p_{ss}^{(ss)} \end{pmatrix}$
and \( \Lambda = \begin{pmatrix} \lambda_{1,-1} P^{(11)} & \lambda_{1,-2} P^{(12)} & \ldots & \lambda_{1,-s} P^{(1s)} \\ \lambda_{2,-1} P^{(21)} & \lambda_{2,-2} P^{(22)} & \ldots & \lambda_{2,-s} P^{(2s)} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{s,-1} P^{(s1)} & \lambda_{s,-2} P^{(s2)} & \ldots & \lambda_{s,-s} P^{(ss)} \end{pmatrix} \)

where \( \lambda_{j,k} \geq 0, j = 1,2,\ldots,s, k = 1,2,\ldots,s, \) and \( \sum_{k=-s}^{s} \lambda_{j,k} = 1 \)

Equation (4) is equivalent to,

\[
\begin{aligned}
x_{n+1}^{(1)} &= H_{1,1} x_n^{(1)} + H_{1,2} x_{n+1}^{(2)} + H_{1,s} x_{n+1}^{(s)} \\
x_{n+1}^{(2)} &= H_{2,1} x_n^{(1)} + H_{2,2} x_{n+1}^{(2)} + H_{2,s} x_{n+1}^{(s)} \\
&\vdots \\
x_{n+1}^{(s)} &= H_{s,1} x_n^{(1)} + H_{s,2} x_{n+1}^{(2)} + H_{s,s} x_{n+1}^{(s)} \\
\end{aligned}
\]

\[
\begin{pmatrix} x_{n+1}^{(1)} \\ x_{n+1}^{(2)} \\ \vdots \\ x_{n+1}^{(s)} \end{pmatrix} = M_s x_n + b
\]

where \( H_{j,k} = \left( \lambda_{j,k} - \frac{1}{m-1} \lambda_{j,-k} \right) P^{(jk)} \), and \( I_{j,-k} = \lambda_{j,-k} P^{(jk)} \)

for \( j = 1,2,\ldots,s \) and \( k = 1,2,\ldots,s \)

Therefore,

\[
x_{n+1} = M_s^{-1} x_{n+1}^{1} b
\]

\[
= M_s^{2} x_{n+1}^{1} (I + M_s^{-1}) b
\]

\[
= M_s^{3} x_{n+1}^{2} (I + M_s^{-1} + M_s^{-2}) b
\]

\[
\vdots
\]

\[
= M_s^{(n+1)} x_{n+1}^{(n)} (I + M_s^{-1} + \ldots + M_s^{-n}) b = M_s^{(n+1)} x_{n+1}^{(0)} + \left( \sum_{k=0}^{n} M_s^{k} \right) b
\]

where \( M_s^{0} = I \) dan \( M_s^{n} = M_s, M_s^{2}, M_s^{3}, \ldots, M_s^{n} \)

Multiplication of matrix \( M_s \) by \( n \) times

The model used has a stationary distribution for a certain norm matrix\( \| \cdot \| \) (in this case \( \| M_s \| \_\infty \)) where \( \| M_s \| < 1 \). For any matrix \( M \) size \( nxn \) choose:

\[
\| M \| \_\infty = \max \left( \sum_{j=1}^{n} | M_{ij} | \right)
\]

In this case it can be written:

\[
\lim_{n \to \infty} x_n = \lim_{n \to \infty} \sum_{k=0}^{n} M_s^{k} b = (I - M_s)^{-1} b
\]

Furthermore,

\[
\| M_s \| \_\infty \leq \max_{1 \leq k \leq \infty} \left( m \left| \lambda_{j,j} - \frac{1}{m-1} \lambda_{j,-j} \right| + \sum_{j \neq k} \left| \lambda_{j,k} - \frac{1}{m-1} \lambda_{j,-k} \right| \right)
\]

The smaller the \( \| M_s \| \_\infty \), the faster the convergence rate [9].
III. RESEARCH METHOD

The presentation of the experimental methods should be clear and complete in every detail facilitating reproducibility by other scientists. The value \( \lambda_{jk} \) can be estimated by minimizing the difference between the left and right sides of equation (4). This minimization process can be solved by solving a linear program optimization problem for a certain norm matrix, namely: \( \| \cdot \|_1 \) or \( \| \cdot \|_\infty \) [6]. The following is the meaning of the two matrix norms [11]:

Let matrix \( \mathbf{A} = \begin{bmatrix} a_{ij} \end{bmatrix} \in \mathbb{R}^{n \times n} \), then

1. \( \| \mathbf{A} \|_1 = \max_j \sum_{i=1}^{n} |a_{ij}| \) (the largest number of column modulus)
2. \( \| \mathbf{A} \|_\infty = \max_i \sum_{j=1}^{n} |a_{ij}| \) (the largest number of row modulus)

The writer chooses to use \( \| \cdot \|_\infty \) to estimate the value \( \lambda_{jk} \). The following is a linear program optimization problem for the norm matrix \( \| \cdot \|_\infty \):

\[
\begin{align*}
\min_{\lambda} & \quad \left\| \begin{bmatrix} \sum_{k=1}^{s} M_s (\lambda) \mathbf{x}_n^{(k)} + \mathbf{b} \end{bmatrix} - \mathbf{x}_n^{(j)} \right\|_1 \\
\text{subject to} & \quad \sum_{k=-s}^{s} \lambda_{jk} = 1, \forall j = 1,2,\ldots,s \\
& \quad \lambda_{jk} \geq 0, \forall k
\end{align*}
\]

(5)

The optimization problem (5) is equivalent to:

\[
\begin{align*}
\min_{\lambda} & \quad \left\| \mathbf{a}^{(j)} - \mathbf{x}_n^{(j)} \right\|_1 \\
\text{subject to} & \quad \mathbf{a}^{(j)} = \sum_{k=1}^{s} \left( \lambda_{jk} - \frac{1}{m-1} \lambda_{j,-k} \right) \mathbf{P}^{(jk)} \mathbf{x}_n^{(k)} + \frac{1}{m-1} \lambda_{j,-k} \mathbf{P}^{(jk)} \mathbf{1} \\
& \quad \sum_{k=-s}^{s} \lambda_{jk} = 1, \forall j = 1,2,\ldots,s \\
& \quad \lambda_{jk} \geq 0, \forall j = 1,2,\ldots,s, k = \pm 1, \pm 2,\ldots, \pm s
\end{align*}
\]

(6)

The Where 1 is a vector of size \( \mathbf{m} \times 1 \) where each entry contains the number 1.

The linear programming optimization problem (6) can be formulated as a linear programming problem as follows:

\[
\begin{align*}
\min & \quad \mathbf{w}_j \\
\text{subject to} & \quad \mathbf{w}_j \geq \mathbf{a}^{(j)} - \mathbf{x}_n^{(j)}, \\
& \quad \mathbf{w}_j \geq -\mathbf{a}^{(j)} + \mathbf{x}_n^{(j)}, \\
& \quad \mathbf{w}_j = \begin{bmatrix} \mathbf{w}_j \\ \vdots \\ \mathbf{w}_j \end{bmatrix}, \mathbf{w}_j \text{ as much as } \mathbf{m} \text{ (condition)}
\end{align*}
\]

(7)

\[
\begin{align*}
\mathbf{a}^{(j)} = \sum_{k=1}^{s} \left( \lambda_{jk} - \frac{1}{m-1} \lambda_{j,-k} \right) \mathbf{P}^{(jk)} \mathbf{x}^{(k)} + \frac{1}{m-1} \lambda_{j,-k} \mathbf{P}^{(jk)} \mathbf{1} \\
& \quad \mathbf{w}_j \geq 0, \\
& \quad \sum_{k=-s}^{s} \lambda_{jk} = 1, \lambda_{jk} \geq 0, \forall k
\end{align*}
\]

The following is the optimization problem (7) can be solved by using the linprog facility in Matlab.
IV. RESULTS AND DISCUSSION

A. The Current State Probability

The following is the current state probability value for five products in company 1 with each product having 6 states:

\[
\begin{align*}
\mathbf{x}_n^{(1)} &= 
\begin{pmatrix}
0.0818 \\ 0.4052 \\ 0.0483 \\ 0.0335 \\ 0.0037 \\ 0.4275
\end{pmatrix},
\mathbf{x}_n^{(2)} &= 
\begin{pmatrix}
0.3680 \\ 0.1970 \\ 0.0335 \\ 0.0037 \\ 0.3978 \\ 0.6283
\end{pmatrix},
\mathbf{x}_n^{(3)} &= 
\begin{pmatrix}
0.1450 \\ 0.2045 \\ 0.0186 \\ 0.0037 \\ 0.6283 \\ 0.2565
\end{pmatrix},
\mathbf{x}_n^{(4)} &= 
\begin{pmatrix}
0 \\ 0.3569 \\ 0.1338 \\ 0.0632 \\ 0.0632 \\ 0.2543
\end{pmatrix},
\mathbf{x}_n^{(5)} &= 
\begin{pmatrix}
0 \\ 0.3569 \\ 0.1227 \\ 0.0520 \\ 0.2416 \\ 0.0000
\end{pmatrix}
\end{align*}
\]

The greatest probability value for a product describes the state of the product. The current state probability in the five products form company 1: products 1, 2, and 3 are in the state of 6 (very fast moving), while products 4 and 5 are in state 2 (very slow moving).

B. The Weighted Markov Chain

After obtaining the transition probability matrix and the state probability vector in part A, the next step is to calculate the weighted Markov chain using the two results, according to the previous explanation in part III. The following are the weighted Markov chain for the five products in firm 3 with each product having 6 states:

\[
\begin{align*}
\lambda_{j,k} &= \begin{pmatrix}
\lambda_{1,1} & \lambda_{1,2} & \lambda_{1,3} & \lambda_{1,4} & \lambda_{1,5} \\
\lambda_{2,1} & \lambda_{2,2} & \lambda_{2,3} & \lambda_{2,4} & \lambda_{2,5} \\
\lambda_{3,1} & \lambda_{3,2} & \lambda_{3,3} & \lambda_{3,4} & \lambda_{3,5} \\
\lambda_{4,1} & \lambda_{4,2} & \lambda_{4,3} & \lambda_{4,4} & \lambda_{4,5} \\
\lambda_{5,1} & \lambda_{5,2} & \lambda_{5,3} & \lambda_{5,4} & \lambda_{5,5}
\end{pmatrix} = 
\begin{pmatrix}
0 & 0.9874 & 0 & 0 & 0 \\
0 & 0.9806 & 0 & 0 & 0 \\
0 & 0.9494 & 0 & 0 & 0.0289 \\
0 & 0 & 0 & 0.0067 & 0.9933 \\
0 & 0 & 0 & 1 & 0
\end{pmatrix}
\end{align*}
\]

\[
\begin{align*}
\lambda_{j,-k} &= \begin{pmatrix}
\lambda_{1,-1} & \lambda_{1,-2} & \lambda_{1,-3} & \lambda_{1,-4} & \lambda_{1,-5} \\
\lambda_{2,-1} & \lambda_{2,-2} & \lambda_{2,-3} & \lambda_{2,-4} & \lambda_{2,-5} \\
\lambda_{3,-1} & \lambda_{3,-2} & \lambda_{3,-3} & \lambda_{3,-4} & \lambda_{3,-5} \\
\lambda_{4,-1} & \lambda_{4,-2} & \lambda_{4,-3} & \lambda_{4,-4} & \lambda_{4,-5} \\
\lambda_{5,-1} & \lambda_{5,-2} & \lambda_{5,-3} & \lambda_{5,-4} & \lambda_{5,-5}
\end{pmatrix} = 
\begin{pmatrix}
0.0113 & 0 & 0.0113 & 0 & 0 \\
0.0194 & 0 & 0 & 0 & 0 \\
0.0199 & 0.0018 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{align*}
\]

where \(\sum_{k=1}^{5}(\lambda_{j,k} + \lambda_{j,-k}) = 1\) for each \(j\), where \(j = 1, 2, \ldots, 5\)

After getting the transition probability matrix \(P^{(jk)}\), the state probability vector \(\mathbf{x}_n^{(s)}\), the probability weight \(\lambda_{j,k}\) and \(\lambda_{j,-k}\), then the multivariate Markov chain model simulation can be performed to estimate the sales demand for five products in company 1 with each product having six conditions. This model simulation results in the form of a probability vector for the future, where the largest probability value in a row (state) is the state of the product.

C. The Future State Probability Vector

The following are the future state probability values for the five products in company 1, with each product having 6 states:

\[
\begin{align*}
\mathbf{x}_{n+1}^{(1)} &= 
\begin{pmatrix}
0.0819 \\ 0.4039 \\ 0.0490 \\ 0.0348 \\ 0.0039 \\ 0.4264
\end{pmatrix},
\mathbf{x}_{n+1}^{(2)} &= 
\begin{pmatrix}
0.3662 \\ 0.1989 \\ 0.0336 \\ 0.0037 \\ 0.3977 \\ 0.6283
\end{pmatrix},
\mathbf{x}_{n+1}^{(3)} &= 
\begin{pmatrix}
0.1449 \\ 0.2048 \\ 0.0184 \\ 0.0037 \\ 0.6283 \\ 0.2543
\end{pmatrix},
\mathbf{x}_{n+1}^{(4)} &= 
\begin{pmatrix}
0 \\ 0.3586 \\ 0.1342 \\ 0.0632 \\ 0.2543 \\ 0.0520
\end{pmatrix},
\mathbf{x}_{n+1}^{(5)} &= 
\begin{pmatrix}
0 \\ 0.3569 \\ 0.1227 \\ 0.0520 \\ 0.2416 \\ 0.0000
\end{pmatrix}
\end{align*}
\]
The greatest probability value for a product describes the state of the product. The future probability for the five products of company 1, namely: product 1, product 2, and product 3 are in state 6 (very fast moving), while product 4 and product 5 are in state 2 (very slow moving).

D. Analysis

The simulation results in sections A and C can be compared the results of the current state probability vector \( x_n \) with the future state probability vector \( x_{n+1} \). As a result, the state probability at company 1 has a state that is not much different from the current state. So, it can be concluded that the future state probability \( x_{n+1} \) is stationary. If it is continued until \( n+5 \) it will be obtained a table of probabilities as follows:

| TABLE I |
|---|
| TABLE OF THE STATE PROBABILITY IN COMPANY 1 |

| Product 1 | Product 2 | Product 3 | Product 4 | Product 5 |
|---|---|---|---|---|
| \( t \) | 1 | 2 | 3 | 4 | 5 | 6 | 1 | 2 | 3 | 4 | 5 | 6 | 1 | 2 | 3 | 4 | 5 | 6 |
| \( n \) | | | | | | | | | | | | | | | | | | | |
| \( n+1 \) | | | | | | | | | | | | | | | | | | | |
| \( n+2 \) | | | | | | | | | | | | | | | | | | | |
| \( n+3 \) | | | | | | | | | | | | | | | | | | | |
| \( n+4 \) | | | | | | | | | | | | | | | | | | | |
| \( n+5 \) | | | | | | | | | | | | | | | | | | | |

Based on Table 1, the state probability for product 1, product 2 and product 3 in company 1 are stationary at state 6 (very fast moving), product 4 and product 5 are stationary at state 2 (very slow moving). So that we get a new conclusion that from the state probability \( x_{n+1} \) until \( x_{n+5} \) are stationary (this strengthens the previous conclusion, namely the state probability \( x_{n+1} \) is stationary). Table 1 can only provide general information about the condition of a product. Therefore, to see in more detail the changes in the probability value of circumstances from time \( t=n \) to \( t=n+5 \), the author completes this study by presenting a bar graph of the probability value of each state in a company's product as follows:

**Fig. 1. The State Probability Graph of Each Product 1 in Company 1**
Based on Fig. 1 it can be concluded that each state changes from time $t=n$ to $n+5$, although the value of the change is very small. From the graph, it can also be seen that there is a trend, both uptrend and downtrend, such as state-1 (no sales volume) showing an uptrend, which means that within a certain period of time product 1 is likely to be in state 1. On the contrary, state-6 (very fast moving) showing a downward trend, which means that within a certain period of time product 1 may no longer be in state-6, but has moved to state-1, for example. This information can be used by the company to determine the amount of production of a product in the future. In addition, this information can also be used in preparing the company's marketing strategy for the product.
Based on Fig. 3 it can be seen that the uptrend is only found in state-6 (very fast moving) meaning that within a certain period of time product 3 will be in state-6 and will not move to another state.

Based on Fig. 4 it can be seen that there is a trend, both an uptrend and a downtrend, such as state-3 (slow-moving), state-4 (standard) and state-5 (fast moving) experiencing an uptrend which means that within a certain period of time product 4 there is a possibility is in state-3, state-4, or state-5. On the other hand, state-2 (very slow-moving) experiences a downward trend, which means that within a certain period of time product 4 may no longer be in state-6, but has moved to state-3, state-4 or state-5.
Based on fig. 5 it can be seen that there is a trend, both an uptrend and a downtrend, such as state-3 (slow-moving), state-4 (standard) and state-5 (fast moving) experiencing an uptrend which means that within a certain period of time product 4 there is a possibility is in state-3, state-4, or state-5. On the other hand, state-2 (very slow-moving) experiences a downward trend, which means that within a certain period of time product 4 may no longer be in state-6, but has moved to state-3, state-4 or state-5.

V. CONCLUSION

In completing the multivariate Markov chain model, it takes two parameters that must be estimated, namely: the transition probability matrix and the weighted Markov chain. The transition probability matrix has been calculated in previous studies. The weighted Markov chain is estimated by solving the linear program optimization problem using the linprog facility in Matlab. By substituting the transition probability matrix, the weighted Markov chain, and the state probability vector into the multivariate Markov chain model, the estimated value of future sales demand is obtained. The estimated value of future sales demand is a vector for the probability of the state of each product in the future. Where each row is the probability value for each state. If the probability value of a row has the greatest value, then the product in the future will be in that state. Based on the simulation results of the multivariate Markov chain model, the probability value of the future state \( x_{n+1} \) is stationary, that is \( x_n \) and \( x_{n+1} \) the value is not much different. If passed until \( x_{n+5} \), the value is not much different from \( x_n \). So, the probability value of the future state is not much different from the probability of the present state (stationary). When viewed in more detail using a bar graph, it can be seen that there is an uptrend or downtrend in the value of the state probability. This trend can describe the future state probability.

ACKNOWLEDGMENT

I would like to thank Dr. Agus Yodi Gunawan, S.Si., M.Si. for as the thesis supervisor, which is the idea of writing in this paper.

REFERENCES

[1] A. Martina, “Penggunaan Model Rantai Markov Multivariat Untuk Estimasi Permintaan Pada Suatu Perusahaan”, Thesis, Indonesia: Bandung Institute of Technology, 2015

[2] A. Martina, “Sales Demand Forecasting Using One of Multivariate Markov Chain Model Parameter,” International Journal of Information Communication Technology (IJoICT), 2020, doi: 10.21108/IJOICT.2020.00.533

[3] A. Martina, “Analysis the Increment of COVID-19 in Indonesia with One of MUltivariate Markov Chain Parameter”, in Indonesia: 1st International Conference on Mathematics and Mathematics Education (ICMMEd), 2020.

[4] W. Ching, Li, L., Li, T., Zhang, S., “A New Multivariate Markov Chain Model with Applications to Sales Demand Forecasting” in China: International Conference on Industrial Engineering and Systems Management, 2007.

[5] W. Ching and Ng. Michael K. “Markov Chains: Models, Algorithms and Applications”, United States of America: Springer+Business Media, Inc., 2006.

[6] L. Megasalindri, “Prediksi Permintaan Penjualan dengan Menggunakan Model Rantai Markov Multivariat” undergraduate final project, Indonesia: Bandung Institute of Technology, 2013.

[7] Cruz. Juan A.R. “Sensitivity of the Stationary Distributions of Denumerable Markov Chains,” Statistics & Probability Letters, 2020, doi: 10.1016/j.spl.2020.108866.
[8] Huang, W., An, Yuting, Pan, Yue, Li, Jinghua, Chen, Chun., “Predicting Transient Particle Transport in Periodic Ventilation Using Markov Chain Model with Pre-Stored Transition Probabilities”, Building and Environment, 2021, doi: 10.1016/j.buildenv.2021.108730.

[9] Wei, W., Xu, W., Liu, Jiankang, “Stochastic P-Bifurcation Analysis of a Class of Nonlinear Markov Jump Systems Under Combined Harmonic and Random Excitations,” Physica A: Statistical Mechanics and its Applications, 2021, doi: 10.1016/j.physa.2021.126246.

[10] Hu, Mengqiang, Liu, Wei, Xue, Kai, Liu, Lumeng, Liu, Huan, Liu, Meng, “Comparing calculation methods of state transfer matrix in Markov chain models for indoor contaminant transport,” Building and Environment, 2021, doi: 10.1016/j.buildenv.2021.108515.

[11] Muchlis, Ahmad. “Analisis Matriks,” Diktat Lectures, Indonesia: Bandung Institute of Technology. 2014.

[12] Cao, Yi, Yao, Yuan, “A Neural network enhanced hidden Markov model for tourism demand forecasting,” Applied Soft Computing, 2020, doi: 10.1016/j.asoc.2020.106465.

[13] Tian, Xin, Wang, Haoqing, E. Erjiang, “Forecasting Intermittent Demand for Inventory Management by Retailers: A New Approach,” Journal of Retailing and Consumer Services, 2021, doi: 10.1016/j.jretconser.2021.102662.

[14] Sperandio, M, Bernardon, D.P., Bordin, G., Oliveira, M.O., Bordignon, S, “Probabilistic Demand Forecasting to Minimize Overtaking the Transmission Contract,” Electric Power Systems Research, 2014, doi: 10.1016/j.epsr.2014.03.005.

[15] Pince. Cerag, Turrini, Laura, Meissner. Joern, “Intermittent Demand Forecasting for Spare Parts: A Critical Review,” Omega, 2021, doi: 10.1016/j.omega.2021.102513.

[16] Wilinski, Antoni, “Time Series Modeling and Forecasting Based on A Markov Chain with Changing Transition Matrices,” Expert Systems with Applications, 2019, doi: 10.1016/j.eswa.2019.04.067.

[17] Xie, Nai-Ming, Yuan, Chao-qing, Yang, Ying-Jie, “Forecasting China’s Energy Demand and Self-Sufficiency Rate by Grey Forecasting Model and Markov Model,” International Journal of Electrical Power & Energy Systems, 2014, doi: 10.1016/j.ijepes.2014.10.028.

[18] Erreygers. Alexander, Bock. Jasper De, “Bounding Inferences for Large-Scale Continuous-Time Markov Chains: A New Approach Based on Lumping and Imprecise Markov Chains,” International Journal of Approximate Reasoning, 2019, doi: 10.1016/g.rent.2019.09.003.

[19] Alvin, Lori, Kelly. James P., “Markov Set-Valued Functions and Their Inverse Limits,” Topology and its Applications, 2018, doi: 10.1016/j.topol.2018.03.035.

[20] Lian, Bosen, Zhang, Qingling, Li, Jinna, “Sliding Mode Control and Sampling Rate Strategy for Networked Control Systems with Packet Disordering Via Markov Chain Prediction,” ISA Transactions, 2018, doi: 10.1016/j.isatral.2018.08.009.