The dependency of dark stars properties on their compactness using vector meson exchange model

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Abstract. We study compactness of dark stars as the function of rotating period, tidal deformation, and moment of inertia of dark stars using vector meson exchange model. These stars are compact object formed of bosonic dark matter. In this work, bosonic dark particle mass is set to be 300 MeV and 400 MeV. We use total cross-section elastic scattering in low-density limit to constraint the effective interaction mass of vector meson. We obtain that compactness exceeds threshold compactness of dark stars using self-interaction model predicted in paper Maselli et al. We also compare the properties of the corresponding dark stars with the ones of the white dwarf and black holes.

1. Introduction
Complex massive scalar or vector field can form compact object which we call “Boson Stars”. Boson stars will collapse into the black hole if these stars among of them occur high energy collisions. If boson stars do not collapse, these stars accrete dark matter. Accretion of dark matter occurs when dark matter collision between oscillaton. Oscillaton is the real massive scalar or vector field minimally coupled to gravity. When accretion of dark matter occurs, boson stars are going to change into dark stars [1].

Dark matter annihilation has an important role to create dark stars. Dark matter annihilation provided as a heat source at high gas core density. Dark stars can shine at low temperatures. Dark stars are stable for a long time period. James Webb Telescope could find them at $z \sim 10$ [2].

In this work, we use bosonic dark matter as dark matter candidates. In addition, we use a vector meson exchange model as dark matter interaction [3-4]. We also set bosonic dark particle masses are 300 MeV and 400 MeV [5].

2. Methods
2.1. Formalism
In this section, we will divide into two subsections i.e., the model and dark stars properties.

2.1.1. The Model
This model refers to references [3-4]. Lagrangian density for this model is

$$L = \mathcal{D}_{\mu} \phi \mathcal{D}^{\mu} \phi - m^2 \phi^2 - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} m_{\pi} V_{\mu} V^{\mu}$$ (2.1)
which \( m \) and \( m_v \) are dark particle mass and vector meson mass. In addition, \( D_\mu \) and \( V_\mu \) are dark matter field coupled to the vector field with coupling constant and vector field respectively. \( D_\mu \) and \( V_\mu \), can be described by the below equation

\[
D_\mu = \partial_\mu + igV_\mu, \\
V_\mu = \partial_\mu V_r - \partial_r V_\mu. 
\]

(2.2) 

(2.3)

which \( g \) is coupling constant. We set stationary scalar field ansatz

\[
\phi(r, t) = \phi(r)e^{-i\omega t}. 
\]

(2.4)

From Euler-Lagrange equations of motion, we obtain

\[
\phi'' + \frac{2}{r}\phi' + [(\omega - gV)^2 - m^2]\phi = 0, \\
V'' + \frac{2}{r}V' + 2g(\omega - gV)|\phi|^2 - m_v^2V = 0. 
\]

(2.5) 

(2.6)

In mean field approximation, equations (2.5) and (2.6) reduces to

\[
\omega = m + gV_0, \\
m_v^2V_0 = 2g(\omega - gV_0)|\phi|^2. 
\]

(2.7) 

(2.8)

Insert equation (2.7) to (2.8), we obtain

\[
m_v^2V_0 = 2gm|\phi|^2. 
\]

(2.9)

Next, we find number density of the dark particle. The number density of dark particle can be found from conserved current [6]

\[
n = -i \left[ \phi \frac{\partial}{\partial t} - \frac{\partial\phi}{\partial t} \right], \\
n = (\omega - gV_0)|\phi|^2. 
\]

(2.10) 

(2.11)

Insert equation (2.7) to (2.11), we obtain

\[
n = 2m|\phi|^2. 
\]

(2.12)

Next, we find energy density from energy-momentum tensor [7]

\[
T^{\mu\nu} = \frac{\partial}{\partial x^\mu} \phi^{\nu} - \eta^{\mu\nu}L, 
\]

(2.13)

which \( \eta^{\mu\nu} \) is Minkowski metric. We obtain energy density from component-00 and also use mean-field approximation

\[
\epsilon = 2m^2|\phi|^2 + \frac{1}{2}m_v^2V_0^2. 
\]

(2.14)

Insert equations (2.9) and (2.12) to (2.14), we obtain

\[
\epsilon = mn + \frac{g}{2m_v^2}n^2. 
\]

(2.15)

Next, we find pressure from thermodynamically relation [8]

\[
P = n^2 \frac{\partial(\epsilon/n)}{\partial n}. 
\]

(2.16)

We obtain pressure

\[
P = \frac{g^2}{2m_v^2}n^2. 
\]

(2.17)

Insert equation (2.17) to (2.15), we obtain an EoS for this model

\[
\epsilon = m\sqrt{\frac{m}{\sigma}}(\sqrt{P} + P). 
\]

(2.18)

We set \( m_I = \sqrt{\frac{2m_v}{\sigma}} \) which \( m_I \) is effective interaction mass of vector meson. Thus, equation (2.18) turns into

\[
\epsilon = mm_I(\sqrt{P} + P). 
\]

(2.19)

The value of \( m_I \) obtained from the following equation [9-10]

\[
0.1 \leq \frac{m}{m} \leq 1 \frac{m}{m}, 
\]

(2.20)

which \( \sigma \) and \( m \) are the cross-section and dark particle mass, respectively. The cross-section on the non-relativistic limits for this model is
2.1.2. Dark Stars Properties

We compute the mass and radius of the dark stars using TOV equations
\[
\frac{dP}{dr} = \frac{-GM\epsilon}{r^2} \left( 1 + \frac{\epsilon}{\mu} \right) \left( 1 + \frac{4\pi\sigma^2 r^3}{M} \right)^{-1},
\]
\[
\frac{dM}{dr} = \frac{4\pi r^2 \epsilon}.
\]
Then, we obtain compactness of these stars
\[
C = \frac{GM}{R^3}.
\]

We compute rotating period, dimensionless tidal deformation, and moment of inertia of dark stars as function of compactness, respectively. Rotating period of dark stars described as following equation
\[
T = \frac{2\pi R}{c^2 \sqrt{C}}.
\]
Dimensionless tidal deformation of dark stars described as the following equation [11]
\[
\Lambda = \frac{2k_2}{3c^4}.
\]
Before we compute moment of inertia, we solving second ordinary differential equation first [12]
\[
\frac{1}{r^4} \frac{d}{dr} \left[ r^4 e^{-\nu} \left( 1 - \frac{2GM}{r} \omega \right) \right] + \frac{3}{r^4} \frac{d}{dr} \left[ r^4 e^{-\nu} \left( 1 - \frac{2GM}{r} \omega \right) \right] \omega = 0,
\]
which \(\omega(r) \equiv \Omega - \omega(r)\). \(\omega(r)\) is frame-dragging effect due to rotation from line element
\[
dS^2 = ds_0^2 - 2\omega(r)r^2 (\sin\theta)^2 dt d\phi,
\]
\[
dS_0^2 = -e^{2\nu} dt^2 + e^{2\nu} dr^2 + d\Omega^2.
\]
Moment of inertia of dark stars described as the following equation
\[
\frac{d\omega}{dr} = \frac{6GM}{r^4}.
\]

3. Result and Discussion

![Figure 1. Compactness and mass relation of dark stars](image)

In Figure 1, the maximum value of compactness of dark stars does not depend on dark particle mass and effective interaction mass of vector meson. However, the role of vector meson can give the maximum value of compactness of dark stars exceed the threshold of self-interaction model [5]. Furthermore, these stars different from black holes \((C \rightarrow 0.5)\) [13] and white dwarfs [5].
Figure 2. Rotating period (left), dimensionless tidal deformation (middle), and moment of inertia (right) as a function of compactness of dark stars

Impact compactness on these stars shown in Figure 2. The impact of compactness on the nature of the rotation period is that the rotation period is slower with the lighter dark particles and vector mesons as well as vice versa. WIMP is hypothetical particles as candidate of dark particle which are weakly interacting with standard model particles. If dark particles are WIMP, then these dark stars are very fast in its rotation period. PSR J1909-3744 belongs to the mass of dark particles are 300 MeV \[14\]. PSR is abbreviation of pulsars and J1909-3744 is identification number of pulsars. The impact of compactness on the properties of the tidal deformation is tidal deformation dependent on dark particles and vector mesons. GW170817 could detect these dark stars if dark particles are light \[15\]. The impact of compactness on the moment of inertia is the same as the impact of compactness on the rotation period.

4. Conclusion
Through compactness, vector meson interaction model and self-interaction model can be distinguished in dark stars. With this model, dark stars differ from white dwarfs and black holes. Mass of dark particles and vector mesons affect the nature of the rotation period and moment of inertia. Mass of dark particles and vector mesons does not affect the nature of tidal deformations.

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