Refraactive Thinking Profile In Solving Mathematical Problem Reviewed from Students Math Capability

M Maslukha1,*, A Lukito1 and R Ekawati1

1 Departement of Mathematic, Universitas Negeri Surabaya
*Corresponding Author: maslukhamaslukha@mhs.unesa.ac.id

Abstract. Refraction is a mental activity experienced by a person to make a decision through reflective thinking and critical thinking. Differences in mathematical capability have an influence on the difference of student’s refractive thinking processes in solving math problems. This descriptive research aims to generate a picture of refractive thinking of students in solving mathematical problems in terms of students’ math skill. Subjects in this study consisted of three students, namely students with high, medium, and low math skills based on mathematics capability test. Data collection methods used are test-based methods and interviews. After collected data is analyzed through three stages that are, condensing and displaying data, data display, and drawing and verifying conclusion. Results showed refractive thinking profiles of three subjects is different. This difference occurs at the planning and execution stage of the problem. This difference is influenced by mathematical capability and experience of each subject.

1. Introduction

Refraction is an event of change or distortion of direction of the wave as it passes through the boundary plane of medium having different density[1]. Using light metaphor the refraction process is illustrated from reflective thinking toward critical thinking then refractive thinking[2]. Illustrations of refractive thinking processes such as figure 1.

![Figure 1. Refraction illustration](image)

Based on illustration of figure 1, occurrence of refraction through reflection and critical thinking. Reflection is the first step in the knowledge development cycle. Reflection is the process of constructing experience in dealing with problems to acquire knowledge to solve it[3]. Reflection can be understood
as a general term for the intellectual activities in which individuals are engaged to explore their experiences in order to gain a new understanding[4]. Concerning reflective thinking, reflective thinking is a situation that occurs when a person is perplexed and investigates to find a solution to the problem. Perplexity is the uncertainty and difficulty in solving the problem[5].

After reflective thinking, the next process towards a more active mental is called critical thinking. Critical thinking demonstrates capability to evaluate relevant information obtained when performing systematic, cautious, and efficient reflections in solving the problem[3]. The most important thing in critical thinking is to consider and evaluate some of the information obtained so that it is possible to make a decision[6]. While critical thinking is thinking that involves the ability to test, connect and evaluate[7]. In this level of thinking also includes collecting, organizing, remembering and analyzing information. Reflective thinking is part of a critical thinking process that specifically refers to the process of analyzing and making judgments about what happens[5][8][9][10]. Reflection is the key of critical thinking, where reflection indicates that a person starts to think at a high level in three taxonomies that involves examining both process knowledge and results and producing new conclusions.

The third step in the knowledge development cycle is refractive thinking[3]. Refractive thinking is a learning transformation in which a person clarifies by providing new facts and evidence. The essence of refraction is the ability to see and identify experiences through different lenses[11]. The term refraction as an instrumental learning that is the process of learning that is done by changing, controlling and experimenting on the environment encountered and studying the resulting reactions[12]. Meanwhile, new knowledge generated from reflection and critical thinking[13]. From these ideas can be concluded that refractive thinking is the thinking process that generates decisions through reflective thinking and critical thinking.

Research related to refractive thinking has been done by Prayitno[3]. This research describes the refractive thinking process of students in solving math problems about decision-making. However in this research, subjects taken only one student and did not include differences in mathematical capability. In fact, differences in mathematical capability have an influence on the refractive thinking process of students in solving mathematical problems.

The purpose of this research was to get a description of refractive thinking profile of high, medium, and low mathematical capability in solving mathematical problems. The troubleshooting steps used in this study consisted of understanding the problem, making completion plan, executing settlement plan, and re-examining. The results of this research are expected to give information about the profile of refractive thinking of students in solving mathematical problems, so that information can be used as consideration for teachers in designing learning activities in accordance with refractive thinking profile with different levels of students' mathematical capability.

2. Method
This research is a qualitative descriptive research. Subjects in this study consisted of three students from the 8th grade of Islamic Junior High School. Three students are high, medium, and low mathematical capability based on mathematical capability tests. Mathematical ability test is taken from the national exam that is relevant to the 8th grade of Junior High School. To avoid having students answer truth by accident, mathematical capability test is made in essay form consisting of 10 questions with 90 minutes time. Description of the refractive thinking profile of students derived from problem-solving tests and semi-structured interviews. The problem-solving tests assigned to the subject are a matter of non-routine mathematical questions related to mathematical material. Interviews are used to clarify written data and explore information that may not be in the written data about students' refractive thinking profiles in solving mathematical problems. Triangulation technique used in this research is triangulation time. The distance between the first and second problem-solving tests is done within a period of about one week. And from the consistency of data collected then the data is analyzed through three stages namely, data condensation, data display, and drawing and verifying conclusion.

3. Results And Discussion
The problem was given to a student is:

Note the following parallelogram ADFE!

\[
\begin{array}{c}
\text{A} \\
\text{B} \\
\text{C} \\
\text{D} \\
\text{E} \\
\end{array}
\]

\[
\begin{array}{c}
\angle ABC = 134^\circ \\
\angle BCD = 360^\circ - 134^\circ = 196^\circ \\
\angle BDC = 35^\circ \\
\end{array}
\]

\[
m \angle BCD.
\]

**Figure 2. Problem was given to Students**

Based on data analysis results obtained refractive thinking process students with high mathematical capability in solving mathematical problems begins from the stage of understanding the problem. At this stage, students begin to reflect by gathering information and linking prior knowledge in interpreting the problems it receives. Once students are able to interpret the problem furthermore students connect the information they have with information on the matter, which indicates that the student is critical thinking. Furthermore from the information relationships obtained, from the refraction of students have a picture of the idea of settlement that will be used to answer the problem. Pagano states that refraction occurs because of a reflection signaled by light passing through a medium that triggers the occurrence of critical thinking, so that light coming out of the medium is not the same as reflection[3]. This shows that the components of refractive thinking are reflection and critical thinking. Therefore, the refractive thinking process that students do with high math skills in understanding the problem is in accordance with the process proposed by Pagano.

In making the completion plan, students with high math skills propose more than one settlement strategy. It shows students doing refraction. In accordance with the opinion [3] which states the core of refraction is the ability to see and identify issues and experiences through different view.

Furthermore, the student uses the experience and knowledge possessed beforehand that is the parallelogram and rectangular character to solve the problem given in accordance with the strategy that he previously set. In addition to these methods, students use the concept of angular relationships such as opposite angles, angular positions and number of angles in triangles. This means that at the stage of carrying out the settlement plan, the student does the refraction process because he is able to solve the problem more than one way.

**Figure 3. Results of the first completion by student with high mathematical capability**

**Figure 4. Results of the second completion by student with high mathematical capability**
After the student has found the answer, the student checks by comparing the first strategy obtained with another strategy which has the same final result. So he felt very confident that the answer he got was correct. This shows at the re-examining stage, student does refraction.

Refractive thinking profile of medium mathematical capability subjects At the stage of understanding the problem, student begins to reflect by gathering information and connecting it with the knowledge of interpreting the problem. Through critical thinking, students are able to connect the information they have with information on the matter. Then from the relationships obtained, the results of the refraction is that student is able to have an idea of the idea that he will use to solve the problem. Stages of refractive thinking through which students are capable of mathematics are already in line with the phases of refractive thinking put forward by Pagano. Reflection is the first step in the knowledge development cycle [3]. Reflection is a process by which we perceive experience, frame it and gain meaning from it. Critical thinking is the second step in the knowledge development cycle. Critical thinking shows the ability to evaluate pertinent information and opinions gathered in the reflection phase systematically, directionally and efficiently to develop problem-solving skills. Furthermore, refraction is the third step in the knowledge development cycle.

After understanding the problem on the question, students begin to strategize based on the experience and knowledge they have acquired. Students had thought to use quadrilateral to solve the problem. However, he could not continue the idea until the end of the settlement. He had already eliminated the idea and started thinking of other strategies that can be used to solve the problem. Based on his previous experience and knowledge of the nature of triangles and angular relationships, students chose a trial and error to make help in order to find the angle they are asking for. Students tried again and again until finally he got the best solution. The refractive thinking process shown by medium mathematical capability students is in submitting a plan in accordance with [3] stating that refraction occurs when in the face of a problem can not immediately get a solution, allowing one to reflect. Reflection in this situation means connecting the problem it faces with the possible solution of the settlement obtained from the knowledge it possesses. Next, through critical thinking, one will identify, analyze, synthesize, and evaluate some alternative solutions obtained at the reflection stage. On refractive thinking someone offers an proper solution in solving the problem. In the next stage students use the nature of triangles and angular relationships to execute the strategies that he set before in accordance with the knowledge he has received. Furthermore, students assess every step of the settlement done with a logical consideration. However, at this stage the student is only able to carry out the settlement plan in no more than one way.

![Figure 5. Results of Student with Medium Math Ability](image)

After the student found the answer, the next step was checking the answer. While doing the examination, he found no calculation errors then he fixed it. But at this stage, the student did not doing refraction process because he does not check by comparing the method or first idea obtained by other method or ideas.

Refractive thinking profile of subject has low mathematical capability. At the stage of understanding the problem, students with low mathematical capability need to read the problem until repeatedly to understand the problem on the matter. Then, the student collected the information and connected it to
the knowledge or experience it has gained to interpret the information on the problem. Unfortunately, students with low math skills do not have an idea of completion to solve the problem. So, at this stage students have not done the refraction process. He just did the reflection course.

Furthermore, in making the settlement plan, based on his experience, students had thought using the nature of the triangle to solve the problem. Unfortunately, students are unable to continue with this idea until the end of the settlement. So he eliminated the plan. Next he tried to make another strategic plan that is using trapezoidal properties to solve the problem. Because he only proposed one idea of completion, so that the refractive ability of the students did not appear. Students only do the reflection process.

After the student had successfully settled the plan of completion, the student also executed the plan. During the completion of the completion plan, student did not assess every step taken with logical consideration. So that students could not execute a strategy of completion well. So the last answer obtained by students is wrong. This means that students do not refract.

In the last stage, student did repetitive examination of the answers. But he was not careful so he did not realize a mistake he made. This shows that student is doing not critical thinking. So at this stage, student with low mathematical capability does not arrive at the refraction stage.

At each stage of problem-solving showed that student with low mathematical capability in solving mathematical problems did not arrive at the refractive phase of thinking. Because there is a critical stage of thinking that has not been passed. Because, process to do refractive thinking is reflection and critical thinking.

In general, the refractive thinking process of the three subjects is different. This difference occurs at the stage of planning and executing problem solving, this difference is influenced by the different levels of subject mathematics ability. Some research found that students’ mathematical ability had an effect on the solution[14]. It also explains the need of mathematical skills when doing mathematical manipulations including understanding of concepts and procedural knowledge[15].

4. Conclusion
The results of this research show the different of refractive thinking process of students with high, medium, and low mathematical capability. At the stage of planning and executing problem solving, students with high mathematical capability are able to propose the idea of completion in more than one way and he is able to carry out all the plans he previously set well. While a student with medium mathematical capability is able to propose two ideas of completion. However, only a single idea of completion is carried out well. For student with low mathematical capability, he is only able to pursue one idea of completion and can not carry out the idea properly until the end of the settlement. From this research result, teachers are expected to deliver problem that potentially have many solutions to students with medium and low mathematical capability.
References
[1] Prayitno A, Sutawidjaja A, Subanji, and Muksar M 2015 Karakterisasi Berpikir Refraktif Mahasiswa Menyelesaikan Matematika tentang Data Prosiding Seminar Nasional Matematika dan Pendidikan Matematika
[2] Downey G 2005 How to Guide and facilitate Self Reflective Practice in Re-Entry Programs (Miami: Presented at CIEE Conference)
[3] Pagano M and Roselle L 2009 Beyond: Refraction and International Experiential Education Frontiers: The Interdisciplinary Journal of Study Abroad 18 pp 221
[4] Boud D Keogh R and Walker 1985 Reflection: Turning Experience into Learning (London: Kogan Page)
[5] Dewey J 1933 How We Think A Restatement of the Relation of Reflective Thinking of the Education Process (Boston: D C health and Company)
[6] Ennis R H 2011 Critical Thinking (NJ: Prentice-Hall)
[7] Krulick S and Rudnick J A 1995 The New Sourcebook for Teaching and Problem solving in Elementary School (Needam Heights: Allyn and Bacon)
[8] Taylor L 1992 Mathematics Attitude Development from A Vygotskian Perspective Mathematics Education Research Journal 4 pp 8-23
[9] Park J Y and Son J B 2011 Expression and Connection: The Integration of the Reflective Learning Process and the Public Writing Process into Social Network Sites MERLOT Journal of Online Learning and Teaching 7 pp 170-178
[10] Colley B M, Billics A R, and Lerch C M 2012 Reflection: A Key Component of Thinking Critically The Canadian Journal for the Scholarship of Teaching and Learning 3 pp 1-19
[11] Mezirow J 1990 Fostering Critical Reflection in Adulthood: A Guide to Transformative and Emancipatory Learning (San Francisco: Jossey-Bass)
[12] Sherlock J and Nathan 2004 Producing Actionable Knowledge: Appllying Mezirow is Theory to the Managerial Learning Context, Academy of Management Best Conference Paper
[13] Medeni T D 2012 Reflection and Refraction for Knowledge Management systems International Journal of Ebusiness and Egovernment studies 4
[14] Ermawati 2015 Proses Berpikir Reflektif Siswa SMA dalam Memecahkan Masalah Matematika Ditinjau dari Kemampuan Matematika Siswa (UNESA: Tesis yang tidak dipublikasikan)
[15] Syaban M 2010 Menumbuh-kembangkan Daya Matematis Siswa EDUCARE: Jurnal Pendidikan dan Budaya