The evolution of circular loops of a cosmic string with periodic tension

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Abstract

In this paper the equation of circular loops of cosmic string with periodic tension is investigated in the Minkowski spacetime and Robertson-Walker universe respectively. We find that the cosmic string loops possessing this kind of time-varying tension will evolve to oscillate instead of collapsing to form a black hole if their initial radii are not small enough.

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The topological defects such as domain walls, cosmic strings, monopoles etc. are expected to generate naturally in the early universe due to phase transitions followed by spontaneously broken symmetries. The cosmic strings, including their formation, evolution and observational effects, attracted more attentions of the physical community in the 1980s and most of 1990s [1, 2]. As one-dimensional defects at a symmetry breaking phase transition in the early universe, cosmic strings can arise generally at the end of an inflationary era within the framework of supersymmetric grand unified theories [3, 4], which provides us with a potential window on M theory [5-7]. Cosmic strings could not be the dominant source of the primordial fluctuations associated with the large scale structure formation in the universe because of limits on their tension $G\mu \leq 10^{-6}$, but they can still be a secondary source of fluctuation which can not be neglected [8-10]. The cosmic strings have also several potentially important astrophysical features such as gravitational lensing effects [10-13], gravitational wave background [14-16], early reionization [17, 18], etc.

A lot of effort has also been contributed to the cosmic string loops. At any epoch in the history of the universe once the cosmic strings formed, they are not static and would evolve under their own tension continuously instead. Within their evolution the cosmic strings collide and intersect to undergo reconnections although the strings stretch under the influence of the Hubble expansion or the environment and the strings lose energy as gravitational radiation when they oscillate. The results of reconnections of long strings and large loops is that more and more small loops will be generated copiously. It is clear that the cosmic strings exist as string network consisting of long strings and closed string loops. The cosmic string loops can oscillate with time rather randomly to become the complicated time-dependent gravitational sources. Schild et.al. observed and analyzed the anomalous brightness fluctuation in the multiple-image lens system Q0957+561A, B which has been investigated for many years [11, 19, 20]. They thought that the phenomena that the system consists of two quasar images separated by approximately 6 degree are known to be images of the same quasar not only because of the spectroscopic match, but also because the images fluctuate in brightness, and the time delay between fluctuations is always the same. This effect may be due to lensing by an oscillating loops of cosmic string between the lensing system and us because cosmic string loops can provide with a quantitative explanation of such synchronous variations in two images. In addition the cosmic string loops can also give rise to the distinct signatures [13, 14, 21]. The contribution of kinks on cosmic string loops to stochastic background of gravitational waves is estimated [15]. A large complicated loop of cosmic string fragment and the shape of small loops are investigated [22]. The number density distribution of cosmic string loops at any redshift soon after the time of string formation is derived analytically based on the Polchinski-Rocha model of loop formation from long strings [23]. Having considered the dynamics of cosmic strings and cosmic string loops in anisotropic backgrounds, the authors of Ref. [24] show the imprint on a cosmic string network in the process before the universe got to isotrope during inflation. In a word once the cosmic strings formed, they evolve to generate the cosmic string loops inevitably, so the evolution and fate of cosmic string loops have attracted more attention. More contributions have
been made to the research in various cases. In the Minkowski spacetime and the Robertson-Walker universe, the loops will collapse to form black holes or become a long cosmic string instead of remaining oscillating loops [25, 26]. In de Sitter worlds only loops with larger initial radii can survive [25, 27]. In the Kerr-de Sitter surrounding, around rotating gravitational sources with positive cosmological constant, a lot of cosmic string loops, including smaller ones, can evolve to survive when the gravitational source rotates faster [28]. The evolution of cosmic string loops is also discussed in the Gauss-Bonnet-de Sitter spacetimes [29]. The dynamics of cosmic string loops which carry current is investigated around the Schwarzschild black hole with a repulsive cosmological constant [30]. It should be pointed out that for all examples mentioned above the tension of cosmic string is chosen to be constant.

It is necessary to explore the evolution of cosmic string loops when the string tension is changeable. It should be emphasized that the tensions of cosmic strings are constant is just an assumption. In the cosmological situations a lot of cosmic strings have time-varying tension. The important issue that the tensions of cosmic strings can depend on the cosmic time was put forward by Yamaguchi [31]. Further the string tension $\mu$ can be denoted as $\mu \propto a^{-3}$, where $a$ is the universe scale factor which is proportional to $t^{\frac{3}{2}}$ in the radiation-dominated era and $t$ in the matter-dominated era respectively [31, 32]. There cosmic strings whose tension depends on the power of the cosmic time assumed as $\mu \propto t^q$ going into the scaling solution when $q < 1$ in the radiation domination and $q < \frac{2}{3}$ in the matter domination. The cosmological imprints from cosmic string with changeable tension and conventional cosmic strings with constant tension could be different. The cosmic strings with time-varying tension can also collide and intersect to undergo reconnections. During this process the loops of this kind of cosmic string emerge. We paid our efforts to the evolution of cosmic string loops with changeable tension in the Robertson-walker universe and de Sitter spacetimes respectively [33, 34]. We find that the cosmic string loops whose tension depends on some power of the cosmic time should not collapse to form a black hole if the power is lower than a critical value belonging to the power's region mentioned above in Ref. [31, 32], which means that a lot of cosmic string loops can survive in the universe. The evolution of circular loops of cosmic string with time-dependent tension is discussed in the BTZ black hole background [35]. We should point out that although the cosmic string loops with time-dependent tension could evolve to exist, the changeable tension as a decreasing function of time is just assumed to be related to the power of the cosmic time simply. When the power of time is negative and only when the negative number is lower than a critical value, these cosmic string loops will not contract to their Schwarzschild radii. The dependence of tension on time may be more complicated. The authors of Ref. [36] introduced $z$-dependent coupling in Abelian-Higgs vortex string to put forward that the string tension is periodic subject to the length of string, which means that the magnitude of tension oscillates along the string instead of getting bigger and bigger or smaller and smaller. As the loops evolve to change their size, the magnitude of tension oscillate and the periodic tension of string will modify the movement of string loops. According to our knowledge little contribution is made to investigate
In our paper we plan to derive the equation of circular loops of cosmic string with periodic tension in the Minkowski spacetime and Robertson-walker universe. We wonder how the periodic change of tension influences on the evolution and fate of the cosmic string loops. First of all, we search for the equations of circular loops of cosmic string in the static and flat spacetime and in the expanding world respectively by means of the Nambu-Goto action while the string tension is associated with the loop size. We substitute the string tension expression which shows its periodicity depending on the loop’s radius into the equation describing the motion of loop. We solve the equations numerically to study the evolution of loops and the periodicity of tension on the fate of cosmic string loops. At the end of paper, the conclusions and discussions are emphasized.

We research on the evolution of cosmic string loops whose tensions are functions of loop size in the expanding universe. The Robertson-Walker metric reads,

$$ds^2 = dt^2 - R^2(t) (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2)$$ (1)

where the scale factor is

$$R(t) = R_0 t^\alpha$$ (2)

Here we choose $\alpha = \frac{1}{2}$ for radiation-dominated era and $\alpha = \frac{2}{3}$ for matter-dominated era respectively. When the string tension changes periodically according to the loop evolution, the Nambu-Goto action for a cosmic string is given by,

$$S = -\int d^2 \mu(x) \left[ \left( \frac{\partial x}{\partial \sigma^0} \cdot \frac{\partial x}{\partial \sigma^1} \right)^2 - \left( \frac{\partial x}{\partial \sigma^0} \right)^2 \left( \frac{\partial x}{\partial \sigma^1} \right)^2 \right]^{\frac{1}{2}}$$ (3)

where $\mu$ is the string tension and the function of loop size. $\sigma^a = (t, \varphi), (a = 0, 1)$ are time-like and space-like string coordinates respectively. $x^\mu(t, \varphi)$ ($\mu, \nu = 0, 1, 2, 3$) are the coordinates of the string world sheet in the spacetime. For simplicity, let us assume that the string sheet we focus on lies in the hyper-surface $\theta = \frac{\pi}{2}$, then the spacetime coordinates of the world-sheet parametrized by $\sigma^0 = t, \sigma^1 = \varphi$ can be chosen as $x = (t, \varphi)$.

In the case of planar circular loops, the radius of loop is just associated with the cosmic time like $r = r(t)$, leading the size-dependent tension changes with the time. According to the metric (1) and the spacetime coordinates mentioned above, the Nambu-Goto action (3) is reduced to,

$$S = -\int dt d\varphi r(1 - R^2 r^2)^{\frac{3}{2}}$$ (4)

leading to the following equation of motion for loops,

$$r\dddot{r} + \frac{d\ln \mu}{dt} r\dot{r}^2 (1 - R^2 r^2) + \frac{\partial \ln \mu}{\partial r} R \frac{r}{R^2} (1 - R^2 r^2)^2 + \frac{1}{R^2} \dot{r}^2 - \frac{3R}{R} r\dddot{r} + 2R\dot{r}^3 = 0$$ (5)
where the dot denotes the differential with respect to time. Within the frame of Abelian-Higgs model, when the couplings as periodic functions of length of string were introduced, the string tension would be periodic in stead of being constant or decreasing [36]. The approximate expression for tension of this kind of string is,

$$\mu = \mu_0 [1 + \sin^2 \left(\frac{2\pi r}{\Delta}\right) + \sin^4 \left(\frac{2\pi r}{\Delta}\right)]$$  \hspace{1cm} (6)

where $\Delta$ is the period. $\mu_0$ is a constant. It is clear that the tension has something to do with the loop size shown as the perimeter or the radius equivalently because the circular loop perimeter is $2\pi r$. The magnitude of the string tension is within a region which can be denoted as $\mu \in [\mu_0, 3\mu_0]$ for Eq. (6). When the cosmic string loops expand or contract, their tensions will become stronger or weaker alternatively. Now we investigate the evolution of this kind of cosmic string loops. In the case of Minkowski spacetime, i.e. $R(t) = constant$, the equation of motion (5) is reduced to,

$$r\ddot{r} + \frac{2\pi}{1 + \sin^2 \left(\frac{2\pi r}{\Delta}\right) + \sin^4 \left(\frac{2\pi r}{\Delta}\right)} \left(1 + \frac{2\sin^2 \frac{2\pi r}{\Delta}}{1 + \sin^2 \frac{2\pi r}{\Delta} + \sin^4 \frac{2\pi r}{\Delta}}\right) r(1 - \dot{r}^2) - \dot{r}^2 + 1 = 0$$ \hspace{1cm} (7)

Here we set $R(t) = 1$. As the equation of circular loops of cosmic string goes for enough time, once the loop radii approach a constant meaning $\dot{r} = 0$, the equation of motion becomes,

$$\frac{2\pi (1 + 2 \sin^2 2\pi r) \sin 4\pi r}{1 + \sin^2 2\pi r + \sin^4 2\pi r} r + 1 = 0$$ \hspace{1cm} (8)

The solutions can be listed partly as $r = 0.294, 0.473, 0.767, 0.987, \ldots$. It should be pointed out that the constant radii of loops will lead the constant tension of cosmic string, so these loops shrink immediately in the Minkowski world [1, 25, 26]. The smaller and smaller size of loops will let the string tension to be periodic, and the loops start to oscillate. The equation can be solved numerically. We find that there must exist a critical value $r_f = 0.301$ while the period is chosen as $\Delta = 1$. When the initial radius of cosmic string loops $r(t_0) > r_f$ under $\dot{r}(t_0) = 0$, the loops will enlarge or contract alternatively or contrarily will collapse to form black holes. The evolution of radii of circular loops with periodic tension in the Minkowski spacetime is depicted in Fig. 1. It is shown that not all cosmic string loops will become black holes in the Minkowski background like in the case of constant tension of cosmic string. According to the scale factor (2) as the description of the Robertson-Walker universe and the periodic string tension (6), the equation of motion (5) becomes,

$$r\ddot{r} + \frac{2\pi}{1 + \sin^2 \left(\frac{2\pi r}{\Delta}\right) + \sin^4 \left(\frac{2\pi r}{\Delta}\right)} \left(\frac{1}{R_0^2 t^{2\alpha}} - \dot{r}^2\right) + \frac{3\alpha}{t} \ddot{r} - \dot{r}^2 - 2\alpha R_0^2 t^{\alpha-1} r^3 + \frac{1}{R_0^2 t^{2\alpha}} = 0$$ \hspace{1cm} (9)

At late times $t \rightarrow \infty$ or equivalently $R(t) \rightarrow \infty$, some of terms in Eq. (9) can be neglected and the equation introduce,

$$r\ddot{r} - \frac{2\pi (1 + 2 \sin^2 2\pi r) \sin 4\pi r}{1 + \sin^2 2\pi r + \sin^4 2\pi r} r^2 - \dot{r}^2 - 2\alpha R_0^2 t^{\alpha-1} r^3 = 0$$ \hspace{1cm} (10)
We find that there exists a constant solution to Eq. (10). The cosmic string loops with constant radii possess the constant tension and will collapse in the Robertson-Walker universe [1, 25, 26]. The decreasing radii of loops lead their tension to be periodic and the cosmic string loops are in the oscillating process. The circular loops of cosmic string are static just for a moment. In Eq. (10) describing the evolution of loops of cosmic string with periodic tension at late times the last term involves $t^{2\alpha-1}$ and $r^3$. The factor $t^{2\alpha-1}$ will be larger as the time passes. For Eq. (10) only the derivative of loop radius with respect to time like $\dot{r}$ should become weaker and weaker in order to mediate the increasing factor $t^{2\alpha-1}$. For the loops of cosmic string whose tension changes periodically, the complicated equation of motion (8) will also be solved numerically by means of burden calculation. We find that there also exist the special limits on the initial size of cosmic string loops in energy era, $r_R = 0.299$ for radiation-dominated era and $r_M = 0.295$ for matter-dominated era. In each era when the initial radii $r(t_0) > r_R$ or $r(t_0) > r_M$ under $\dot{r}(t_0) = 0$, respectively, all cosmic string loops will become larger and smaller by turns instead of expanding endlessly in the case that the string tension is a decreasing function of cosmic time. Contrarily all loops will collapse to form black holes at last. It is also shown that the larger cosmic string loops will not collapse to form black holes in the expanding universe if their tension is in the oscillating state. In the Robertson-Walker world the evolutions of radii of circular loops with periodic tension in the radiation-dominated era and matter-dominated era are plotted in Fig. 2 and Fig. 3 respectively. The evolution of radius of cosmic string loop becomes damped oscillation when the time is sufficiently long in the case of expanding universe. The results in the Robertson-Walker universe are similar to those in Minkowski spacetime. The smaller string loops will become black holes and the other loops will evolve to exist in the vibrant state of loops size.

There could exist a lot of cosmic string loops in our universe according to the phenomena [10, 19, 20]. We have investigated the fate of cosmic string loops with decreasing unceasingly tension in the Robertson-Walker universe except for constant tension ones [33, 34]. When the tension becomes weaker fast with time, the cosmic string loops with this kind of tension will expand without contracting. Here we discuss the evolution of planar circular loops of cosmic string with periodic tension in the Minkowski spacetime and the Robertson-Walker universe respectively. The so-called periodic tension means that its magnitude changes to be bigger or smaller by turns. It is found that more cosmic string loops will evolve instead of collapsing if their initial radii are not small enough. During their evolution the radii of circular loops vibrate about a configuration of stable equilibrium, meaning that the loops become larger and smaller in turn instead of expanding forever. Our findings indicate that there may exist a considerable number of loops of cosmic string but they are not too small. The states of the loops are oscillating.

In this work we find the circular loop equation for a cosmic string with periodic tension evolving in the hypersurface with $\theta = \frac{\pi}{2}$ in the Robertson-Walker universe. A remarkable solution to this equation is that a loop may never contract to one with a Schwarzschild radius if the tension of cosmic string is periodic and the loops are not too small. The evolution of this kind of cosmic
string loops is an oscillatory motion although the motion is not strictly periodic. Therefore more cosmic string loops whose tensions vary periodically within a region can evolve to survive in our universe except for the loops of cosmic string with weaker and weaker tension. Our recent research generalize completely and significantly the previous results belonging to the case that string tension is a decreasing function of cosmic time.

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Figure 1: The solid, dotted and dashed curves of $r(t)$, the radii of circular loops as functions of cosmic time with initial values $r(t_0) = 0.24, 0.31, 0.8$ respectively and $\dot{r}(t_0) = 0$ in the Minkowski spacetime.
Figure 2: The solid, dotted and dashed curves of $r(t)$, the radii of circular loops as functions of cosmic time with initial values $r(t_0) = 0.24, 0.3, 0.76$ respectively and $\dot{r}(t_0) = 0$ in the radiation-dominated era.
Figure 3: The solid, dotted and dashed curves of $r(t)$, the radii of circular loops as functions of cosmic time with initial values $r(t_0) = 0.24, 0.3, 0.76$ respectively and $\dot{r}(t_0) = 0$ in the matter-dominated era.