Analysis of amplitude-frequency characteristics of spiral vibrating feeder system

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Abstract. Spiral Vibrating Feeder is a kind of common equipment in Machining Automation. Traditional theoretical analysis of spiral vibrating feeder is based on the dynamic model of vertical vibration, which can not really reflect the working condition of the system. On the basis of explaining the working principle of the spiral vibration feeder system, a three-degree-of-freedom vibration mechanics model of the system is established. The theoretical formulas of the amplitude-frequency characteristics of the system are deduced and analyzed, and Taking PEF120A feeding system as an example, the amplitude-frequency characteristics of the system are analyzed comprehensively. The experiments prove that the re-established system model is closer to the actual situation, which will lay a good theoretical foundation for the design of vibrating feeder for conveying precision components.

1. Introduction
Spiral vibration feeder is a common equipment in processing automation. It can arrange all kinds of products in an orderly way and complete the processing of workpieces with automatic processing machinery [1]. According to different driving modes, spiral vibration feeder can be divided into electromagnetic and piezoelectric modes. Electromagnetic driven feeder has been widely used in industry because of its high feeding efficiency and strong driving ability [2-3]. In recent years, with the rapid development of microelectronics, precision machinery and other industries, the requirement for the precision and rapid alignment and transportation of light and small workpieces is also increasing. The traditional electromagnetic feeder can not meet the production requirements in these fields because of its shortcomings such as insufficient sorting ability, high energy consumption and high noise. The feeder driven by piezoelectric ceramic micro-amplitude came into being [4].

After decades of development and application, the relevant theoretical research of spiral vibration feeder is relatively mature, but these theories only consider the vertical vibration of the system, ignoring the torsional vibration of the system when establishing the vibration mechanics model of the feeder [9-13]. Therefore, the current theoretical research on the spiral vibration feeder system can not fully and truly reflect the system work. In this paper, based on PEF120A piezoelectric vibration feeder, the vibration mechanics model of the system including vertical and torsional vibration is established, and the amplitude-frequency characteristics of the system are analyzed in detail by using MATLAB software.
Through theoretical deduction and physical verification, the dynamic model established in this paper can more truly reflect the working condition of spiral vibration feeder system, which will lay a good theoretical foundation for the design of feeder for conveying precision components. This makes the hopper which is closely connected with the top plate vibrate with high frequency and small amplitude along a small space spiral trajectory besides its central axis. By adjusting the frequency of the excitation source, when the excitation frequency is equal to the natural frequency of the feeder system, the whole system reaches the resonance state. At this time, the hopper obtains a larger vibration amplitude and intensity, thus forming the ability to transport materials. By adjusting the excitation voltage, the feeding speed can be controlled.

2. Structure and Principle
The structure of spiral vibration feeder is shown in Figure 1, which is mainly composed of hopper, top plate, support spring, base, rubber sole and excitation source. The spiral vibration feeder has the same system structure and feeding principle except that the excitation mode and the excitation structure are different. The working principle is that the vertical or torsional excitation force generated by the excitation source is transmitted to the top plate, which is restrained by four sets of oblique supporting springs connected with the top plate and the base, resulting in vertical vibration of the top plate and torsional vibration around its central axis. This makes the hopper, which is closely connected with the top plate, vibrate with high frequency and small amplitude along a small space spiral trajectory.

3. Dynamic Model of Feeder
According to the working principle of the feeder mentioned above, the vertical vibration and torsional vibration around the central axis of the feeder system are simultaneously carried out when the feeder system works. These two modes of vibration are synchronized and coupled with each other. According to the vibration theory, the dynamic model of the system is established as shown in Fig. 2. Among them, m1 and J1 are equivalent mass and moment of inertia of hopper and top plate, m2 and J2 are equivalent mass and moment of inertia of the base, x1 and x2 are vertical vibration displacement of hopper.
And base, φ1 and φ2 are torsional vibration displacements of hopper and base, k1 and k1φ are vertical equivalent stiffness and torsional equivalent stiffness of support spring group, k1 and k1φ are vertical equivalent stiffness and torsional equivalent stiffness of rubber foot, β are installation inclination angle of support spring group, R is torsional radius of support spring group, C2 and C2φ are equivalent damping coefficients of rubber foot, Fsinωt is the exciting force for the excitation source and ω is the excitation frequency.

According to the vibration of the system, the force and deformation of the supporting spring are shown in Figure 3. In the picture, \( \Delta \delta = x_1 - x_2, \Delta \xi = (\varphi_1 - \varphi_2)R \).

Since the deformation of the supporting spring is relatively small compared with its size, the linearization of the deformation of the supporting spring has the following relationship.

\[
\varphi_1 = \varphi_2 + K(x_1 - x_2)
\]  

(1)

Where

\[
K = \frac{\tan \beta}{R}
\]
Using the second kind of Lagrange equation, the vibration equation of the system can be obtained as follows.

\[
\begin{aligned}
(m_1 + K^2J_1)\dddot{x}_1 - K^2J_1\dddot{x}_2 + KJ_1\dddot{\varphi}_2 + k_e x_1 - k_e x_2 &= -F\sin\omega t \\
-K^2J_1\dddot{x}_1 + (m_2' + K^2J_1)\dddot{x}_2 - KJ_1\dddot{\varphi}_2 + C_2\dot{x}_2 - k_e x_1 + k_e x_2 &= F\sin\omega t \\
KJ_1\dddot{x}_1 - KJ_1\dddot{x}_2 + (J_1 + J_2')\dddot{\varphi}_2 + C_2\varphi\dddot{\varphi}_2 &= 0
\end{aligned}
\]  
(2)

Where

\[k_e = k_1 + K^2k_1\varphi; \ m_2' = m_2 - \frac{k_2}{\omega^2}; \ J_2' = J_2 - \frac{k_2\varphi}{\omega^2}\]

According to (2), it follows that

\[
\begin{aligned}
x_1 &= \text{Im}\left[\lambda e^{i\omega t}\right] \\
x_2 &= \text{Im}\left[A\lambda e^{i\omega t}\right] \\
\varphi_2 &= \text{Im}\left[B\lambda e^{i\omega t}\right]
\end{aligned}
\]  
(3)

\[
\omega_n = \sqrt{\frac{k_e}{m}}
\]  
(4)

Where \(\omega_n\) is natural frequency of the feeder system,

\[
\lambda = \frac{-F}{-\omega^2(m_1 + K^2J_1 - K^2J_1A + KJ_1B) + k_e(1 - A)}
\]

\[
A = \frac{\omega^2m_1}{-\omega^2m_2 + k_2 + i\omega C_2}
\]

\[
B = \frac{KJ_1\omega^2[m_1^2 + m_2^2] - k_2 - i\omega C_2)\}(-\omega^2m_2 + k_2 + i\omega C_2)}{\omega^2(J_1 + J_2) - k_2\varphi - i\omega C_2\varphi}
\]

\[
m = \frac{m_1m_2'}{m_1 + m_2'} + K^2\frac{J_1J_2'}{J_1 + J_2'}
\]

According to the theory of mechanical vibration, when the whole vibration feeding system reaches resonance, the amplitude of the system reaches the maximum, and the corresponding feeding effect of the system reaches the best. Formula 4 shows that the natural frequency of the system is a function of the excitation frequency. In order to clarify the relationship between them, the structural parameters of PEF120A feeder (Table 1) are brought into formula (4), and the curve shown in Figure 4 is drawn by MATLAB software.
Tab 1. Theoretical equivalent parameters of the system

| Equivalent parameter | Value      | Equivalent parameter | Value      |
|----------------------|------------|----------------------|------------|
| \( m_1 \)           | 1.35kg     | \( m_2 \)           | 3.5kg      |
| \( J_1 \)           | 2.5\times 10^3 kg.m^2 | \( J_2 \) | 8.3\times 10^3 kg.m^2 |
| \( k_1 \)           | 1.15\times 10^6 N/m | \( k_2 \) | 17314N/m          |
| \( k_{1\phi} \)      | 2875Nm/rad | \( k_{2\phi} \)     | 3004Nm/rad |
| \( \beta \)         | 75°        | \( R \)              | 0.05m      |
| \( K \)             | 74.64      | \( k_e \)            | 1.72\times 10^3 N/m |
| \( C_2 \)           | 2.5m\omega | \( C_{2\phi} \)      | 2.5m\omega |

Fig 4. The natural frequency varying against excitation frequency

From the red curve in Fig. 4, we can see that the natural frequency \( \omega_n \) of the system varies with the excitation frequency \( \omega \). There are three points of \( \omega_n = \omega \) in the whole curve, that is, \( \omega_n1 = 510.8 \text{ rad/s} \) (81.3Hz), \( \omega_n2 = 595.6 \text{ rad/s} \) (94.8Hz) and \( \omega_n3 = 1251 \text{ rad/s} \) (199.2Hz). Based on these three frequency points, the corresponding values of \( A, B \) and \( \lambda \) are calculated by formula (5) - (7), as shown in Table 2.

Tab 2. System Amplitude and Corresponding Coefficient

| \( \omega_n = 510.8 \) rad/s | \( \omega_n = 595.6 \) rad/s | \( \omega_n = 1251 \) rad/s |
|-----------------------------|-----------------------------|-----------------------------|
| \( m \)             | 63.71                       | -0.034                      | 10.98                      |
| \( C_2 \)           | 8.14\times 10^4             | —                           | 3.43\times 10^4            |
| \( C_{2\phi} \)      | —                           | —                           | —                          |
| \( A \)             | -0.0002 - 0.0085i            | —                           | -0.0062 - 0.0485i          |
| \( B \)             | 0.00000 - 0.0012i            | —                           | 0.0003 - 0.0068i           |
| \( \lambda \)        | -F(7.57\times 10^{-8})      | —                           | -F(-1.48\times 10^{-7}+4.86\times 10^{-9}i) |

As shown in Table 2, when \( \omega_n2 = 595.6 \text{ rad/s} \), the equivalent mass of the system is negative, which does not accord with the actual situation. Therefore, the frequency point has no practical significance and no further study is needed. When \( \omega_n1 = 510.8 \text{ rad/s} \) and \( \omega_n3 = 1251 \text{ rad/s} \), the coefficients of Table 3 are substituted into formulas (1) - (3) respectively, \( x_1, x_2, \phi_1 \) and \( \phi_2 \) are computed as:

When \( \omega_n1 = 510.8 \text{ rad/s} \),

\[ x_1 = \frac{-F(7.57 \times 10^{-8}) \sin(510.8t - 8.73 \times 10^{-3})}{(5)} \]
corresponding natural frequency of the system increases rapidly along the curve, resulting in the rate of the curve near \( n_1 \) point is relatively large. When the excitation frequency compensating the instability of the system. From the sub the rated load of the system or the fatigue attenuation of the feeder system occurs in the course of use, resonance state, i.e. the working frequency is \( n_1 = 510.8 \text{rad/s} \).

Thus, when \( \omega = 510.8 \text{rad/s} \), the excitation source at the two resonance points of the system. It can be considered that the two vibrations are in the same phase, and the corresponding amplitude ratios are as follows:

When \( \omega = 510.8 \text{rad/s} \),

\[
x_1 = F(1.48 \times 10^{-7}) \sin(1251t - 3.28 \times 10^{-2})
\]

\[
x_2 = -F(7.25 \times 10^{-9}) \sin(1251t + 1.41)
\]

\[
\varphi_1 = F(1.12 \times 10^{-5}) \sin(1251t + 1.53 \times 10^{-2})
\]

\[
\varphi_2 = F(1.01 \times 10^{-9}) \sin(1251t - 1.56)
\]

The hopper is a part that contacts directly with the material, so the vibration state of the hopper directly determines the material delivery effect, and the base plays the main role of supporting and counterweight in the system, so the vibration state of the base is not the focus of research. From the expressions (5), (7), (9) and (11), it can be seen that the phase difference between the response of mass 1 and the excitation source is very small at the two resonance points of the system. It can be considered that the two vibrations are in the same phase, and the corresponding amplitude ratios are as follows:

When \( \omega = 510.8 \text{rad/s} \),

\[
\frac{X_1}{\delta_{st}} = \frac{F(7.57 \times 10^{-8})}{F(5.81 \times 10^{-8})} = 1.30
\]

\[
\frac{\Theta_1}{\delta_{st}} = \frac{F(5.65 \times 10^{-6})}{F(5.81 \times 10^{-8})} = 97.2
\]

When \( \omega = 1251 \text{rad/s} \),

\[
\frac{X_1}{\delta_{st}} = \frac{F(1.48 \times 10^{-7})}{F(5.81 \times 10^{-8})} = 2.55
\]

\[
\frac{\Theta_1}{\delta_{st}} = \frac{F(1.12 \times 10^{-5})}{F(5.81 \times 10^{-8})} = 192.77
\]

Where, \( X_1 \) and \( \Theta_1 \) are the amplitude of hopper and \( \delta_{st} = F/k_e \) are the static displacement of the system. Thus, when \( \omega = 1251 \text{rad/s} \), the amplitude amplification factor of hopper is almost twice as much as that of \( \omega = 510.8 \text{rad/s} \). That is to say, when the hopper reaches the same vibration amplitude, the output force of the excitation source at \( \omega = 510.8 \text{rad/s} \) is 1/2 of that at \( \omega = 510.8 \text{rad/s} \), which can greatly improve the service life of the excitation source. Reference [14] shows that the vibration feeder generally works in the sub-resonance state, i.e. the working frequency is (0.85–0.95)\( n_{on} \). When the weight of the material exceeds the rated load of the system or the fatigue attenuation of the feeder system occurs in the course of use, the resonance frequency of the system will be reduced, which makes the system change from the original sub-resonance state to the resonance state, thus reinforcing the vibration amplitude and automatically compensating the instability of the system. From the curve in Figure 4, it can be seen that the change rate of the curve near \( n_1 \) point is relatively large. When the excitation frequency \( \omega = (0.85–0.95) \text{rad/s} \), the corresponding natural frequency of the system increases rapidly along the curve, resulting in the
deviation between the excitation frequency and the natural frequency of the system is too large to reach the sub-resonance state, so n1 point does not have self-compensation function. Similarly, the curve change rate near ωn3 point is almost zero. When the excitation frequency $\omega=(0.85-0.95)\omega_n$, the deviation between $\omega_n$ and $\omega_n3$ is very small. At this time, the system can achieve sub-resonance state, so the system at $\omega_n3$ has self-compensation function.

Considering the service life of the excitation source and the stability of the feeder system, it is more appropriate to select $\omega_n3$ as the resonance point of the system. However, due to the influence of damping and excitation, the point value should be smaller than 1251 rad/s (199.2 Hz) in the actual system.

4. Experiment
When the output voltage of the driving power supply is 220 V, the driving frequency is adjusted to 35-245 Hz with an interval of 0.2 Hz. The vertical amplitude A of the hopper is measured with a laser micrometer at each frequency point, and the relationship between the driving frequency and the amplitude is plotted, as shown in Figure 6.

![Figure 5](image)

**Fig 5.** The Driving frequency varying against amplitude

Fig. 5 shows that in the range of 40-250Hz, the vertical amplitude of hopper has two peaks with the increase of driving frequency. The driving frequencies corresponding to these two peaks are the natural frequencies of the system, which are 78.4Hz and 195Hz, respectively. Although they are smaller than the theoretical values, they are within the range of error acceptance. The values of the two peaks are 15μm and 32μm microns respectively, and the latter is almost twice as much as the former, which is basically in accordance with the theoretical calculation. The experiment verifies the correctness of the theoretical analysis.

5. Conclusion
On the basis of previous research, the system dynamics model based on vertical and torsional vibration is established, and the theoretical formula of system response is derived. The amplitude-frequency characteristics of PEF120A feeding system are analyzed comprehensively. The following conclusions can be drawn. The experiments prove the correctness of the dynamic model and theoretical analysis and calculation in this paper.

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