Review of QCD, Quark-Gluon Plasma, Heavy Quark Hybrids, and Heavy Quark State production in p-p and A-A collisions

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Abstract

This is a review of the Quantum Chromodynamics Cosmological Phase Transition, the Quark-Gluon Plasma, and the detection of the Quark-Gluon Plasma via RHIC production of heavy quark states using the mixed hybrid theory for the Ψ(2S) and Υ(3S) states.

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1 Outline of QCD Review

QCD Theory of the Strong Interaction
The QCD Phase Transition
Heavy Quark Mixed Hybrid States
Proton-Proton Collisions and Production of Heavy Quark States
RHIC and Production of Heavy Quark States
Production of Charmonium and Bottomonium States via Fragmentation
Brief Overview

2 Brief Review of Quantum Chromodynamics (QCD)

In the theory of strong interactions quarks, fermions, interact via coupling to gluons, vector (quantum spin 1) bosons, the quanta of the strong interaction fields, color replaces the electric charge in QED, which is why it is called Quantum Chromodynamics or QCD. See Refs[1],[2],[3], and Cheng-Li’s book on gauge theories[4].
The QCD Lagrangian is
\[ L_{\text{QCD}} = -\frac{1}{2} \text{tr}[G_{\mu\nu}G^{\mu\nu}] + \sum_k \bar{q}_k(i\gamma^\mu(\partial_\mu - igA_\mu) - m_k)q_k \]
\[ G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu A_\nu - A_\nu A_\mu] \]
\[ A_\mu = \sum_1^8 A_\mu^a \lambda^a / 2 , \]

where \( q_k \) is a quark field with flavor \( k \) and \( A_\mu^a \) is the strong interaction field, called the gluon field, with the quanta called gluons, \( \gamma^\mu \) are the Dirac matrices, \( a \) is color, and \( g \) is the strong interaction coupling constant. The quark flavors are \( q_k : u, d, s, c, b, t = \text{up, down, strange, charm, bottom, and top quarks} \); and \( m_k \) are the quark masses. The quarks which we shall call heavy quarks are charm (c) and bottom (b) quarks. Although quark masses are not well defined, as one cannot make a beam of particles with color, the heavy quark masses are \( m_c \simeq 1.5 \text{ GeV} \) and \( m_b \simeq 5.0 \text{ GeV} \).

The \( \lambda^a \) are the SU(3) color matrices, with
\[ \lambda^a \lambda^b - \lambda^b \lambda^a = i2 \sum_{c=1}^8 f^{abc} \lambda^c , \]
with \( f^{abc} \) the SU(3) structure constants. The nonvanishing \( f^{abc} \) are:
\[ f^{123} = 1 , \quad f^{458} = f^{678} = \sqrt{3}/2 , \]
\[ f^{147} = f^{165} = f^{246} = f^{257} = f^{345} = f^{376} = 1/2 . \]

The most important states with which we consider are mesons, which in the standard model consist of a quark and antiquark. For example, the \(|J/\psi(1S)>\propto |c\bar{c}(1S)>\), a charm-anticharm state, with a mass of about 3.1 GeV, approximately the mass of two charm quarks. Other states very important for this review are the Upsilon states \(|\Upsilon(mS)>\), which in the standard model are \(|\bar{b}b(mS)>\), with \( m=1,2,3 \).

The quarks have a strong interaction by coupling to gluons. They also have an electric charge and experience an electromagnetic force. This is a much more familiar force than QCD. The quantum field theory, QED, is similar to QCD, with a Lagrangian
\[ L_{\text{QED}} = i\bar{\psi}(i\gamma^\mu(\partial_\mu - ieA_\mu^{EM}) - m)\psi , \]
where \( \psi \) is a quantum field with electric charge \( e \) and \( A_\mu^{EM} \) is the electromagnetic quantum field. The quantum of \( A_\mu^{EM} \) is the photon, which is much more familiar than the gluon.

As shown in the figures below, the electromagnetic interaction with \( e^2 \simeq 1/137 \) is weak enough so the lowest order Feynman diagram illustrated in Fig.1 gives almost the entire electric force, while \( g^2 \simeq 100 \times e^2 \) is so large that Feynman diagrams are not useful. Non-perturbative theories, such as QCD sum rules discussed below, must be used.

The lowest order Feynman diagrams for two quarks interacting via the electromagnetic interaction and strong interaction are illustrated in Fig.1 and Fig. 2 below.
QED (Quantum Electrodynamics): electric force via photon exchange

Electric Force: \( F = \frac{q^2}{d^2} \), \( d \) = distance between quarks with \( q_e \) = electric charge

\( q_e = 2e/3 \) for \( u \)-quark and \( -e/3 \) for \( d \)-quark

Note \( e^2 \approx 1/137 \). Therefore higher order diagrams are small.

**Figure 1: Two quarks interacting via the exchange of a photon**

QCD (Quantum Chromodynamics): quark force via gluon exchange

**STRONG FORCE**

\( g^2 \approx 100 \times e^2 \)

Nonperturbative. Feynman diagrams do not converge—no good

**Figure 2: Two quarks interacting via the exchange of a gluon**
3 QCD Phase Transition

A phase transition is the transformation of a system with a well defined temperature from one phase of matter to another. The two basic types of phase transitions are CLASSICAL, when one phase transforms to another, and QUANTUM, when a state transforms to a different state.

The three most common classical phases are solid, liquid, and gaseous; and under special conditions there is a plasma phase. For early universe phase transitions the plasma phase is very important as the matter in the universe before the QCD phase transition was the Quark-Gluon Plasma, the main topic in this review. These classical phase transitions are illustrated in the figure below.

In the figure above, the “Recombination” transition is from a plasma to a gas. For the QCD Cosmological phases from a transition, discussed later in this section, as the Temperature of the universe dropped the matter went from a Quark-Gluon Plasma to our present universe of protons and neutrons, which is a gas (neither solid nor liquid), and later formed atomic nuclei during the first 10-100s (see figure on Evolution of the Universe below).

As we discuss in later sections, a major project of high energy nuclear physics is to form the Quark-Gluon Plasma via the collisions of atomic nuclei such as Copper (CU), lead (Pb), and gold (Au), and to detect it by studying the production of heavy quark states.

Next we briefly describe the evolution of the universe.
The universe has evolved for about 13.7 billion years. It has gone from a very dense universe with very high temperature to our present universe, with a number of important cosmological events, as illustrated in the figure below.

Figure 4: Evolution of the Universe

Inflation and Dark energy, which we do not discuss, occurred at about $10^{-34}$ seconds. The Electroweak Phase Transition (EWPT) occurred at a time about $10^{-11}$ seconds after the big bang when the temperature (a form of energy, so we use energy units) was $T \approx 125$ GeV, the mass of the Higgs particle (discussed below). During the EWPT it is believed that all particles except the photon got their mass. The QCD Phase Transition (QCDPT), the main topic in this review, occurred at $t \approx 10^{-5}$ s, with $T \approx 150$ MeV.

The main event that we discuss in this review is the QCDPT. Over three decades ago QCD and possible phase transitions at high $T$ and density were discussed[5]. Inflation, the EWPT, CMBR (Cosmological Microwave Background Radiation (from which the amount of Standard and Dark Mass and Dark Energy have been measured) and events that occurred after the QCDPT are discussed in detail in a recently published book[6].
3.1 Classical Phase Transitions and Latent Heat

During a first order phase transition, with a critical temperature $T_c$, as one adds heat the temperature stays at $T = T_c$ until all the matter has changes to the new phase. The heat energy that is added is called latent heat. This is illustrated in the figure below.

In contrast to a first order phase transition, a crossover transition is a transition form one phase to another over a range of temperatures, with no critical temperature or latent heat.

![First order vs higher order phase transitions](image)

Figure 5: First order and crossover phase transitions
For application to cosmology we are mainly interested in first order phase transitions. These phase transitions occur at a critical temperature, $T_c$, and the temperature stays the same until all matter in the system changes to the new phase. For example if one heats water (a liquid) at standard atmospheric pressure it starts to boil, with bubbles of steam (a gas), and the temperature stays at $100 \ C^\circ$. The heat energy that turns water to steam is called LATENT HEAT. This illustrated in the figure above.

A familiar example of first order phase transitions is ice, a solid, melting to form water, a liquid; and water boiling to form steam, a gas. The figure below shows these two first order phase transitions for one gallon of water.

Figure 6: Latent heat for ice to water and water to steam
3.2 Quantum Phase Transitions

3.2.1 Brief Review of Quantum Theory

In quantum theory one does not deal with physical matter, but with states and operators. A quantum phase transition is the transition from one state to a different state. For the study of Cosmological Phase Transitions a state is the state of the universe at a particular time and temperature.

We now review some basic aspects of quantum mechanics needed for quantum phase transitions. A quantum state represents the system, and a quantum operator operates on a state. For instance, a system is in state $|1\rangle$ and there is an operator $A$.

$$|1\rangle \equiv \text{state}[1]$$

$$A \equiv \text{operator } A.$$ (5)

An operator operating on a quantum state produces another quantum state. For example, operator $A$ operates on state $|1\rangle$

$$A|1\rangle = |2\rangle,$$ (6)

where state $|2\rangle = |2\rangle$ is a quantum state.

State $|2\rangle$ might also be the same as state $|1\rangle$, with $|1\rangle = |2\rangle = |A\rangle$, except for normalization,

$$A|A\rangle = a|A\rangle,$$ (7)

where $a$ is called the eigenvalue of the operator $A$ in state $|A\rangle$. It is the exact value of $A$. If a state is not an eigenstate of an operator, the operator does not have an exact value.

In general, if a system is in a quantum state, the value of an operator is given by the EXPECTATION VALUE. For example, consider state $|1\rangle$ and operator $A$.

$$<1|\equiv \text{adjoint of state}[1]$$

$$<1|A|1\rangle \equiv \text{EXPECTATION VALUE OF } A.$$ (8)

For example, classically an electron has momentum $\vec{p}$. In quantum theory the system is in a state $|e,\vec{p}\rangle$. The momentum operator when operating on $|e,\vec{p}\rangle$:

$$\vec{p}_{op}|e,\vec{p}\rangle = \vec{p}|e,\vec{p}\rangle,$$ (9)

since $|e,\vec{p}\rangle$ is an eigenstate of the operator $\vec{p}$.

In quantum theory both position $\vec{r}$ and momentum $\vec{p}$ are operators, with $p_x = (\hbar/i)(d/dx)$, where $\hbar = h/(2\pi)$ with $h$ Planks constant. Since $p_x \neq xp_x$, a state cannot be an eigenstate of both position and momentum. If the uncertainties in $x, p_x$ is $\Delta x, \Delta p_x$ satisfy

$$\Delta p_x \Delta x \geq \hbar/2,$$ (10)

which is the Heisenberg Uncertainty Principle.
3.2.2 Cosmological Phase Transitions

Calling \( |0, T > \) the state of the universe at time \( t \) when it has temperature \( T \), an operator \( A \) has the expectation value \( \langle 0, T | A | 0, T > \), as discussed above. If there is a cosmological first order phase transition, then there is a critical temperature \( T_c \) and

\[
\langle 0, T | A | 0, T >_{T<T_c} - \langle 0, T | A | 0, T >_{T>T_c} = \Delta A,
\]

with \( \Delta A \) the latent heat of the cosmological phase transitions. The two very important cosmological phase transitions are the Electroweak and QCD.

The Electroweak Phase Transition (EWPT) took place at a time \( t \simeq 10^{-11} \) seconds after the Big Bang, when the critical temperature was \( kT_c \simeq 125 GeV \). The operator \( A \) in Eq(11) is the Higgs field \( \Phi \). \( \langle 0, T | \Phi | 0, T >_{T>T_c} = 0 \), so the latent heat for the EWPT is

\[
\langle 0, T | \Phi | 0, T >_{T<T_c} \propto 125 GeV \simeq M_H,
\]

with the Higgs particle recently detected at the LHC at CERN, with the mass \( M_H \simeq 125 \) GeV measured by the CMS[7] and ATLAS[8] collaborations. During the EWPT all standard model particles got their masses. With an additional scalar field in the standard model, usually called the Stop, the EWPT is first order, with baryogenesis-more quarks than antiquarks.

The QCD Phase Transition (QCDPT), which is the main topic in this review, took place at \( t \simeq 10^{-5} \) seconds after the Big Bang, when the critical temperature was \( kT_{cQCDPT} \simeq 150 MeV \). It is a first order phase transition and bubbles of our present universe with protons, neutrons, etc (hadrons) nucleated within the universe with a dense plasma of quarks and gluons, the Quark-Gluon Plasma (QGP) that existed when the temperature of the universe was greater than \( T_{cQCDPT} \). This is illustrated in the figure below. We shall discuss the possible detection of the QGP via heavy ion collisions.

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Figure 7: Hadron phase forming within the QGP during the QCDPT
3.3 The QCDPT and Quark Condensate

As reviewed above, the QCD fermion fields and particles are quarks. The Latent Heat for the QCD Phase Transition (QCDPT) is the Quark Condensate, which we now define.

\[ q(x) = \text{quark field} \]
\[ \bar{q}(x) = \text{antiquark field} \]
\[ |0, T> = \text{vacuum state temperature} = T \]
\[ <0, T | \bar{q}(x)q(x) | 0, T> = \text{quark condensate} \]
\[ = \text{vacuum expectation value of} \bar{q}(x)q(x) \]
\[ <0, T | \bar{q}(x)q(x) | 0, T> \approx 0 \text{ in quark gluon plasma phase} T > T_{c}^{QCDPT} \]
\[ \approx (0.23 \text{ GeV})^3 \text{ in hadron phase} T < T_{c}^{QCDPT} \]

The QCDPT is first order, with a discontinuity on the quark condensate at critical temperature. In the figure below the results of a recent lattice gauge calculation for \(< \bar{q}q >\), the quark condensate is shown.

\[ T \approx 150 \text{ MeV} \]
\[ t = 10^{-5} \text{ s} \]
\[ t = 10^{-4} \text{ s} \]

Figure 8: The quark condensate as a function of T=temperature

As one can see from the figure, the quark condensate \(< \bar{q}q >\) goes from 0 to \((0.23)^3 \text{ GeV}^3\) at the critical temperature of about 150 MeV, and is therefore a first order phase transition.

Although we do not discuss Dark Energy in this review, note that Dark Energy is cosmological vacuum energy, as is the quark condensate. It has been shown that Dark Energy at the present time might have been created during the QCDPT via the quark condensate[9].

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4 Review of mixed hybrid heavy quark meson states

The Charmonium and Upsilon (nS) states which are important for this review are shown in the figure below.

![Figure 9: Lowest energy Charmonium and Upsilon states](image)

4.1 Heavy quark meson decay puzzles

Note that the standard model of the $\psi'(2S)$ and $\Upsilon(3S)$ as $c\bar{c}$ and $b\bar{b}$ mesons is not consistent with the following puzzles:

1) The ratio of branching rarios for $c\bar{c}$ decays into hadrons (h) given by the ratios (the wave functions at the origin canceling)

$$R = \frac{B(\Psi'(c\bar{c}) \rightarrow h)}{B(J/\Psi(c\bar{c}) \rightarrow h)} = \frac{B(\Psi'(c\bar{c}) \rightarrow e^+e^-)}{B(J/\Psi(c\bar{c}) \rightarrow e^+e^-)} \simeq 0.12,$$

the famous 12% RULE.

The $\rho - \pi$ puzzle: The $\Psi'(2S)$ to $J/\Psi$ ratios for $\rho - \pi$ and other h decays are more than an order of magnitude too small. Many theorists have tried and failed to explain this puzzle.
2) The Sigma Decays of Upsilon States puzzle: The $\sigma$ is a broad 600 MeV $\pi - \pi$ resonance.

$\Upsilon(2S) \to \Upsilon(1S) + 2\pi$ large branching ratio. No $\sigma$

$\Upsilon(3S) \to \Upsilon(1S) + 2\pi$ large branching ratio to $\sigma$

We call this the Vogel $\Delta n = 2$ Rule[10]. Neither of these puzzles can be solved using standard QCD models. They were solved using the mixed heavy hybrid theory.

### 4.2 Hybrid, mixed heavy quark hybrid mesons, and the puzzles

The method of QCD Sum Rules[11] was used to study the heavy quark Charmonium and Upsilon states, and show that two of them are mixed hybrid meson states[12], which we now review.

#### 4.2.1 Method of QCD Sum Rules

The starting point of the method of QCD sum rules[11] for finding the mass of a state $A$ is the correlator,

$$\Pi^A(x) = \langle |T[J_A(x) J_A(0)]| \rangle ,$$

with $|\rangle$ the vacuum state and the current $J_A(x)$ creating the states with quantum numbers $A$:

$$J_A(x)\rangle = c_A|A\rangle + \sum_n c_n|n; A\rangle ,$$

where $|A\rangle$ is the lowest energy state with quantum numbers $A$, and the states $|n; A\rangle$ are higher energy states with the $A$ quantum numbers, which we refer to as the continuum.

The QCD sum rule is obtained by evaluating $\Pi^A$ in two ways. First, after a Fourier transform to momentum space, a dispersion relation gives the left-hand side (lhs) of the sum rule:

$$\Pi(q)^A_{\text{lhs}} = \frac{\text{Im}\Pi^A(M_A)}{\pi(M_A^2 - q^2)} + \int_{s_o}^{\infty} ds \frac{\text{Im}\Pi^A(s)}{\pi(s - q^2)} ,$$

where $M_A$ is the mass of the state $A$ (assuming zero width) and $s_o$ is the start of the continuum—a parameter to be determined. The imaginary part of $\Pi^A(s)$, with the term for the state we are seeking shown as a pole (corresponding to a $\delta(s - M_A^2)$ term in Im$\Pi$), and the higher-lying states produced by $J_A$ known as the continuum. Next $\Pi^A(q)$ is evaluated by an operator product expansion (O.P.E.), giving the right-hand side (rhs) of the sum rule

$$\Pi(q)^A_{\text{rhs}} = \sum_k c_k(q)\langle 0 | O_k | 0 \rangle ,$$
where \( c_k(q) \) are the Wilson coefficients and \( \langle 0|\mathcal{O}_k|0 \rangle \) are gauge invariant operators constructed from quark and gluon fields, with increasing \( k \) corresponding to increasing dimension of \( \mathcal{O}_k \).

After a Borel transform, \( \mathcal{B} \), in which the \( q \) variable is replaced by the Borel mass, \( M_B \) (see Ref[11]), the final QCD sum rule, \( \mathcal{B}\Pi_A(q)(LHS) = \mathcal{B}\Pi_A(q)(RHS) \), has the form

\[
\frac{1}{\pi} e^{-M_A^2/M_B^2} + \mathcal{B} \int_{s_o}^{\infty} \frac{Im[\Pi_A(s)]}{\pi(s-q^2)} ds = \mathcal{B} \sum_k c_k^A(q) < 0|\mathcal{O}_k|0 > .
\]

This sum rule and tricks are used to find \( M_A \), which should vary little with \( M_B \). A gap between \( M_A^2 \) and \( s_o \) is needed for accuracy. If the gap is too large, the solution is unphysical.

### 4.3 Mixed charmonium-Hybrid charmonium States

Recognizing that there is strong mixing between a heavy quark meson and a hybrid heavy quark meson with the same quantum numbers (defined below), the following mixed vector \((J^{PC} = 1^{--})\) charmonium, hybrid charmonium current was used in QCD Sum Rules

\[
J^\mu = bJ_H^\mu + \sqrt{1-b^2}J_{HH}^\mu
\]

with

\[
J_H^\mu = \bar{q}_c^a \gamma^\mu q^a_c \quad J_{HH}^\mu = \bar{\Psi} \Gamma_\nu G^{\mu\nu} \Psi
\]

where \( \Psi \) is the heavy quark field, \( \Gamma_\nu = C\gamma_\nu \), \( \gamma_\nu \) is the usual Dirac matrix, \( C \) is the charge conjugation operator, and the gluon color field is

\[
G^{\mu\nu} = \sum_{a=1}^{8} \frac{\lambda_a}{2} G_a^{\mu\nu}
\]

with \( \lambda_a \) the SU(3) generator \((Tr[\lambda_a \lambda_b] = 2\delta_{ab})\), discussed above.

Therefore the correlator for the mixed state:

\[
\Pi_{H-HH}^{\mu\nu}(x) = <0|T[J^\mu(x)J^{\nu}(0)]|0>
\]

is

\[
\Pi_{H-HH}^{\mu\nu}(x) = b^2 \Pi_H^{\mu\nu}(x) + (1-b^2) \Pi_{HH}^{\mu\nu}(x) + 2b\sqrt{1-b^2} \Pi_{HHH}^{\mu\nu}(x)
\]

\[
\Pi_H^{\mu\nu}(x) = <0|T[J_H^\mu(x)J^{\nu}_H(0)]|0>
\]

\[
\Pi_{HH}^{\mu\nu}(x) = <0|T[J_{HH}^\mu(x)J^{\nu}_{HH}(0)]|0>
\]

\[
\Pi_{HHH}^{\mu\nu}(x) = <0|T[J_{HH}^\mu(x)J^{\nu}_{HH}(0)]|0>
\]

It was necessary to carry out many QCD sum rule calculations to determine the value of the parameter \( b \), which gives the relative probability of a normal to a hybrid meson.
The leading diagrams for the meson and meson-hybrid meson diagrams are shown in the figure below.

![Diagram](image)

Figure 10: (a) lowest order diagram for a heavy meson (b) lowest order diagram for a meson-hybrid meson

After a Fourier transform to find the correlator in momentum space, $\Pi^{\mu\nu}_{H-HH}(p)$, the standard procedure for QCD sum rules was carried out.

Finally the Borel transform of $\Pi^{\mu\nu}_{H-HH}(p)$ was found, from which the square of the mixed meson-hybrid meson mass as function of the Borel mass, $M_{H-HH}^2$ was found. The result is $M_{C-HC}^2 \approx 3.69$ GeV=energy of the $\Psi'(2S)$ state. A similar QCD sum rule calculation bottom heavy quarks found that the mixed upsilon-hybrid upsilon mass is $M_{\Upsilon-H\Upsilon}^2 \approx 10.4$ GeV= energy of the $\Upsilon(3S)$ state.

From this we conclude that the $\Psi'(2S)$ and $\Upsilon(3S)$ states are mixed meson-hybrid meson states. This is very important for the study heavy quark state production via proton-proton collisions and RHIC for the detection of the Quark-Gluon Plasma, since a hybrid mesons have a valence gluons, as does the QGP.
For the mixed Charmonium-hybrid charmonium mass, \( M_{C-HC}^2 \), the result of the QCD sum rule analysis is shown in the figure below for \( b^2 = 0.5 \).

![Figure 11: Mixed Charmonium-hybrid charmonium mass \( \simeq 3.69 \text{ GeV} \)](image)

From this figure one sees that the minimum in \( M_{C-HC}^2(M_B^2) \) corresponds to the \( \Psi'(2S) \) state being 50% normal and 50% hybrid. The analysis for upsilon states was similar, with the \( \Upsilon(3S) \) being 50% normal and 50% hybrid.

### 5 Heavy Quark State Production In p-p Collisions

There has been a great deal of interest in the production and polarization of heavy quark states in proton-proton collisions. In addition to the puzzles discussed above, the \( J/\Psi, \Psi' \) production anomaly\[13\], in which the charmonium production rate was larger than predicted for \( J/\Psi \), and much larger for \( \Psi' \) than theoretical predictions in proton-proton (p-p) collisions has motivated p-p heavy quark state production experiment. In addition to being an important study of QCD, these experiments also could provide the basis for testing the production of Quark-Gluon Plasma (QGP) via a Relativistic Heavy Ion Collider (RHIC).

At the proton-proton (p-p) energies of the Fermilab, BNL-RHIC, or the Large Hadron Collider (LHC) the color octet dominates the color singlet model, which we now review.

#### 5.1 Color Octet vs Color Singlet Heavy Quark State Production

The color octet model was shown to dominate the color singlet model\[14, 15, 16\]. We now discuss the Cho/Leibovich study\[14, 17\] which compared color octet to color singlet production. For the color singlet production they used the standard results of Ref[18] and others (with \( \alpha_s = g^2/(4\pi) \), where \( g \) is the strong coupling constant \( M = 2M_Q \) and \( q^0 = \vec{q}^2/M \)), with \( \vec{q} \) the colliding particles momentum:

\[
\sigma(gg \rightarrow Q\bar{Q}[^1S_0^{(1)}]) = \frac{\alpha_s^2 M}{384\pi^2 q^0 s} \delta(1 - M^2/s),
\]  
(23)
The two color octet diagrams are shown in the figure below.

\( \begin{align*}
\text{(a)} & \\
\text{(b)} & 
\end{align*} \)

Figure 12: Color octet diagrams for (a) \( q\bar{q} \rightarrow \Psi_Q(8) \) and (b) \( gg \rightarrow \Psi_Q(8) \)

The results for the \( p - \bar{p} \rightarrow J/\Psi \) theoretical transverse momentum differential cross section for the singlet and octet theories and CDF data\cite{17} are shown in the figure below. Solid curve is color octet and dashed curve is color singlet production.

From this figure and references given above one sees that the color octet theory dominates. As we shall see when discussing the theory of production cross sections, there are a number of parameters that must be determined, and the diagrams shown in the figure above are not simple Feynman diagrams from which one derives the matrix elements needed to predict the cross sections.

This rather complicated theory which we discuss in the next subsection is used for p-p production of heavy quark states, which we discuss in the next subsection. It is also used in RHIC, AA production of heavy quark states, as is discussed in the following section.
Transverse momentum differential cross section for $p - \bar{p} \rightarrow J/\Psi$:

5.2 Proton-Proton Collisions and Production of $\Psi$ and $\Upsilon$ States

In this subsection we review the publication of Ref[19] on heavy quark state production in p-p collisions. We only consider unpolarized p-p collisions. The production cross sections are obtained from

$$\sigma_{pp \rightarrow \Phi(\lambda)} = \int_0^1 \frac{dx}{x} f_q(x, 2m) f_{\bar{q}}(a/x, 2m) \sigma_{q\bar{q} \rightarrow \Phi(\lambda)} + f_g(x, 2m) f_g(a/x, 2m) \sigma_{gg \rightarrow \Phi(\lambda)}, \quad (24)$$

where $a = 4m^2/s$, with $m = 1.5$ GeV for charmonium, and 5 GeV for bottomonium. $f_g(x, 2m), f_q(x, 2m)$ are the gluonic and quark distribution functions evaluated at $Q = 2m$.

For the quark and gluon cross sections, $\sigma_{q\bar{q} \rightarrow \Phi(\lambda)}$ and $\sigma_{gg \rightarrow \Phi(\lambda)}$ one needs the octet matrix elements derived from the diagrams shown in Fig.10 by Braaten and Chen[15]. The procedure of Nyyak and Smith[20] was followed in Ref[19]. The three octet matrix elements needed are $< O_8^\Phi(1S_0) >$, $< O_8^\Phi(3S_1) >$, and $< O_8^\Phi(3P_0) >$, with $\Phi$ either $J/\Psi$, $\Psi(2S)$, or $\Upsilon(nS)$. Since these matrix elements are not well known, Nyyak and Smith[20] use three scenarios:

1) $< O_8^\Phi(1S_0) > = < O_8^\Phi(3P_0) > /m^2 = .0087,$
2) $< O_8^\Phi(1S_0) > = .039$ and $< O_8^\Phi(3P_0) > = 0,$
3) $< O_8^\Phi(1S_0) > = 0,$

and $< O_8^\Phi(3P_0) > /m^2 = .01125$, with $< O_8^\Phi(3S_1) >= .0112$ in all scenarios. All matrix elements have units GeV$^3$. Note that these matrix elements are not used to obtain the wave functions of the heavy quark meson states.
Using scenario 2 the production cross sections for $\Phi$ for helicity $\lambda = 0$ and 1 are

\[
\sigma_{pp \to \Phi(\lambda=0)} = A_\Phi \int_a^1 \frac{dx}{x} f_g(x, 2m) f_g(a/x, 2m)
\]

\[
\sigma_{pp \to \Phi(\lambda=1)} = A_\Phi \int_a^1 \frac{dx}{x} [f_g(x, 2m) f_g(a/x, 2m) + 0.613((f_d(x, 2m) f_d(a/x, 2m) + f_u(x, 2m) f_u(a/x, 2m))],
\]

with $A_\Phi = \frac{5\pi^2a^2}{288m^4} < O(1S_0^{1/2})$.

The main purpose of this work was to explore the effects of matrix elements for $\Psi'(2S)$ and $\Upsilon(3S)$, comparing results with the hybrid model to the standard model. In the standard model the states are $(nS)$ quark anti-quark states, and the ratios of the matrix elements for $n$ greater than 1 is given by the squares of the wave functions. Note that the basis for the octet model being used is the nonrelativistic QCD model[14, 15, 16], with a model potential for the quark anti-quark interaction giving bound states. A harmonic oscillator potential can be used to approximately give the energies of the first few states, which is what is needed in the present work. For the octet matrix elements the results of Refs.[14, 15, 16, 20] were used, as discussed above.

To approximate the ratios of matrix elements in a nonrelativistic quark model for these heavy quark meson states harmonic oscillator wave functions were used[21], with $\Phi(1S) = 2Exp[-r/a_0]/a_0^{3/2}$, $\Phi(2S) = \Phi(1S)(1-r/a_0)/2^{3/2}$, $\Phi(3S) = \Phi(1S)(1-2r/3a_0+2r^2/27a_0^2)/3^{3/2}$. Defining $N1= \int |\Phi(2S)|^2$ divided by $\int |\Phi(1S)|^2$ for the 2S to 1S probability, and similarly $N2$ for the 3S to 1S probability, we find $N1=0.039$, $N2=0.0064$, $N3=N2/N1=1.16$. This is a very rough estimate. Therefore, we use $A_{\Psi'(2S)} = 0.039A_{\Psi'(1S)}$, $A_{\Upsilon(2S)} = 0.039A_{\Upsilon'(1S)}$, and $A_{\Upsilon(3S)} = 0.0064A_{\Upsilon'(1S)}$ in the standard model.

On the other hand in the mixed hybrid study both $\Psi'(2S)$ and $\Upsilon(3S)$ were found to be approximately 50% hybrids. In Ref[12] it was shown, using the external field method, that the octet to singlet matrix element was enhanced by a factor of $\pi^2$ compared to the standard model, as illustrated in the figure below. For mixed hybrids an enhancement factor of 3.0 was used.

\[
\begin{align*}
\Psi'(2S) & \quad \Upsilon(3S) \\
\Xi & \quad \pi^2 \times
\end{align*}
\]

Figure 13: External field method for $\Psi'(2S)$ and $\Upsilon(3S)$ states
For differential cross sections the rapidity variable, $y$, is used,

$$y(x) = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right); \text{ with } E = \sqrt{M^2 + p_z^2}$$

$$p_z = \frac{\sqrt{s}}{2} \left( x - \frac{a}{x} \right), \quad (28)$$

or

$$x(y) = 0.5 \left[ \frac{m_s}{s} (\exp y - \exp (-y)) + \sqrt{\left( \frac{m_s}{s} (\exp y - \exp (-y)) \right)^2 + 4a} \right] \quad (29)$$

For the unpolarized proton collisions we use a polynomial fit to the parton distributions of Ref.[22]. Because of the wide range of values, in order to obtain a good polynomial fit to the parton distributions we limit the range of rapidity to $-1 < y < 1$.

For $Q=3$ GeV, with $m=$ Charmonium mass = 1.5 GeV, from Eq(29), $x$ has a range about 0.028 to 0.032, and $a/x$ 0.008 to 0.015. In Ref[19] the following expressions were derived for the gluon ($g$), $u$ and $d$ quark, and anti-quark distribution functions using QTEQ6 for $Q=3$ GeV, fitting the range $x=0.008$ to .004, which is needed for $\sqrt{s} \simeq 200$ to 500 GeV

$$f_g(x) \simeq 1334.21 - 67056.5x + 887962.0x^2$$

$$f_d(x) \simeq 72.956 - 3281.1x + 42247.6x^2$$

$$f_u(x) \simeq 82.33 - 3582.36x + 45867.3x^2$$

$$f_{\bar{d}}(x) \simeq 55.98 - 2722.04x + 35641.2x^2$$

$$f_{\bar{u}}(x) \simeq 57.44 - 2757.05x + 36030.5x^2. \quad (30)$$

For $Q=10$ GeV, $m=$ Bottomonium mass=5 GeV, from Eq(29), $x$ has a range about 0.05 to 0.08, and $a/x$ 0.03 to 0.05. We have derived the following expressions for the gluon ($g$), $u$ and $d$ quark, and antiquark distribution functions using QTEQ6 for $Q=10$ GeV, fitting the range $x=0.03$ to .08, which is needed for $\sqrt{s}=38.8$ GeV and 2.76 TeV.

$$f_g(x) \simeq 275.14 - 6167.6x + 36871.3x^2$$

$$f_d(x) \simeq 26.96 - 527.14x + 3119.13x^2$$

$$f_u(x) \simeq 32.92 - 604.38x + 3530.1x^2$$

$$f_{\bar{u}}(x) \simeq 16.64 - 377.53x + 2336.86x^2$$

$$f_{\bar{d}}(x) \simeq 17.81 - 390.64x + 2392.46x^2. \quad (31)$$

The differential rapidity distribution for $\lambda = 0$ is given by

$$\frac{d\sigma_{pp \to \Phi(\lambda=0)}}{dy} = A_\Phi \frac{1}{x(y)} f_g(x(y), 2m) f_g(a/x(y), 2m) \frac{dx}{dy}, \quad (32)$$

while for $\lambda=1$

$$\frac{d\sigma_{pp \to \Phi(\lambda=1)}}{dy} = A_\Phi \frac{1}{x(y)} \left[ f_g(x(y), 2m) f_g(a/x(y), 2m) + 0.613(f_d(x(y), 2m) f_d(a/x(y), 2m) \right. \left. + f_u(x(y), 2m) f_u(a/x(y), 2m) \right] \frac{dx}{dy}. \quad (33)$$
5.2.1 Charmonium Production Via Unpolarized p-p Collisions at $\sqrt{s}=200$ Gev at BNL-RHIC

Unpolarized p-p collisions for $\sqrt{s}=200$GeV corresponding to BNL energy, using scenario, with the nonperturbative matrix elements given above, $A_\Phi = \frac{5\pi^3\alpha^2}{25889s} < O_8^\Phi (1S_0) > =7.9 \times 10^{-4}$nb for $\Phi=J/\Psi$ and $2.13 \times 10^{-5}$nb for $\Upsilon(1S)$ heavy quark states.

For $\sqrt{s}=200$GeV

$$x(y) = 0.5 \left[ \frac{m}{200}(\exp y - \exp (-y)) + \sqrt{\left(\frac{m}{200}(\exp y - \exp (-y))^2 + 4\alpha\right)} \right]$$

$$\frac{dx(y)}{dy} = \frac{M}{400}(\exp y + \exp (-y)) \left[ 1 + \frac{\frac{M}{200}(\exp y - \exp (-y))}{\sqrt{\left(\frac{M}{200}(\exp y - \exp (-y))^2 + 4\alpha\right)}} \right].$$

Note that there was a typo in Ref[19], with $\frac{M}{200}(\exp y + \exp (-y))$ instead of $\frac{M}{200}(\exp y - \exp (-y))$ in the numerator of Eq(34). Using Eqs(32,33,34), with the parton distribution functions given in Eq(30), we find $d\sigma/dy$ for $Q=3$ GeV, $\lambda = 0$ and $\lambda = 1$ the results for $J/\Psi$ shown in Figure 14.

![Figure 14: $d\sigma/dy$ for $Q=3$ GeV, $E=200$ GeV unpolarized p-p collisions producing $J/\Psi$ with $\lambda = 0$, $\lambda = 1$](image)

Note that the shape of $d\sigma/dy$ is consistent with the BNL-RHIC-PHENIX detector rapidity distribution[23].
For $\Psi'(2S)$ the results are shown in Figure 15.

![Figure 15: $d\sigma/dy$ for $Q=3$ GeV, $E=200$ GeV unpolarized p-p collisions producing $\Psi'(2S)$ with $\lambda=1, \lambda=0$](image)

The results for $d\sigma/dy$ shown in Figure 15 labeled $\Psi'(2S)(a)$ are obtained by using for the standard nonperturbative matrix element $=0.039$ times the matrix elements for $J/\Psi$ production; while the results labeled $\Psi'(2S)(b)$ are obtained by using the matrix element derived using the result that the $\Psi'(2S)$ is approximately $50\%$ a hybrid with the enhancement is at least a factor of $\pi$, as discussed above.

### 5.2.2 Upsilon Production Via Unpolarized p-p Collisions at $E=\sqrt{s}=38.8$ GeV at Fermilab

In this subsection the cross sections calculated for for $\Upsilon(nS)$ production, with $n=1, 2, 3$ at $38.8$, which has been measured at Fermilab[24, 25], are reviewed.

For $Q=10$ GeV, using the parton distributions given in Eq(31) and Eqs(32,33) for helicity $\lambda=0, \lambda=1$, with $A_\Upsilon =5.66 \times 10^{-4}$nb and $a = 6.64 \times 10^{-2}$, one obtains $d\sigma/dy$ for $\Upsilon(nS)$ production.
The results for $\Upsilon(1S)$, $\Upsilon(2S)$ are shown in Figure 16, and for $\Upsilon(3S)$ in Figure 17.

Figure 16: $d\sigma/dy$ for $Q = 10$ GeV, $E=38.8$ GeV unpolarized $p$-$p$ collisions producing $\Upsilon(1S)$, $\Upsilon(2S)$ with $\lambda = 0$, $\lambda = 1$

Figure 17: $d\sigma/dy$ for $Q = 10$ GeV, $E=38.8$ GeV unpolarized $p$-$p$ collisions producing $\Upsilon(3S)$ with $\lambda = 0$, $\lambda = 1$. Results labeled a,b are for the standard, mixed hybrid theories.
It should be noted that the ratios of $d\sigma/dy$ for $\Psi'(2S)/J/\Psi$, and $\Upsilon(3S)/(\Upsilon(1S)+\Upsilon(2S))$ for the hybrid theory vs. the standard are our most significant results, as there are uncertainties in the absolute magnitudes and shapes of $d\sigma/dy$ on the scenarios, as well as the magnitudes of the matrix elements.

5.2.3 Polarized p-p collisions for $E=200$ GeV at BNL-RHIC

For polarized p-p collisions the equations for $d\sigma_{pp\rightarrow\Psi(\lambda=0)}/dy$ and $d\sigma_{pp\rightarrow\Psi(\lambda=1)}/dy$ are the same as Eqs(32,33) with the parton distribution functions $f_g$ and $f_q$ given in Eqs(30,31) replaced by $\Delta f_g$ and $\Delta f_q$, the parton distribution functions for longitudinally polarized p-p collisions. A fit to the parton distribution functions for polarized p-p collisions for $Q=3$ GeV obtained from CTEQ6[22] in the x range needed for $\sqrt{s}=200$ GeV is

$$
\Delta f_g(x) \simeq 15.99 - 700.34x + 13885.4x^2 - 97888.x^3
$$

$$
\Delta f_d(x) \simeq -5.378 + 205.60x - 4032.77x^2 + 28371.x^3
$$

$$
\Delta f_u(x) \simeq 8.44 - 292.19x + 5675.16x^2 - 39722.x^3
$$

$$
\Delta f_{\bar{u}}(x) \simeq -1.447 + 64.67x - 1268.24x^2 + 8878.32x^3
$$

(35)

$$
\Delta f_d(x) = \Delta f_{\bar{u}}(x),
$$

and for $Q=10$ GeV, which we do not use in the present work, as the $\Upsilon(nS)$ are not resolved at BNL-RHIC,

$$
\Delta f_g^{10}(x) \simeq 28.98 - 1435.47x + 29533.5x^2 - 211440.x^3
$$

$$
\Delta f_d^{10}(x) \simeq -6.074 + 241.57x - 4762.04x^2 + 33604.4x^3
$$

$$
\Delta f_u^{10}(x) \simeq 9.88 - 348.632x + 6729.49x^2 - 47058.x^3
$$

$$
\Delta f_{\bar{u}}^{10}(x) \simeq -1.552 + 75.731x - 1531.97x^2 + 10896.6x^3
$$

(36)

$$
\Delta f_d^{10}(x) = \Delta f_{\bar{u}}^{10}(x).
$$

The differential rapidity distribution for polarized p-p collisions are

$$
\frac{d\Delta\sigma_{pp\rightarrow\Psi(\lambda=0)}}{dy} = -A_{\Phi} \frac{1}{x(y)} \Delta f_g(x(y),2m)\Delta f_g(a/x(y),2m) \frac{dx}{dy},
$$

(37)

$$
\frac{d\Delta\sigma_{pp\rightarrow\Psi(\lambda=1)}}{dy} = -A_{\Phi} \frac{1}{x} [\Delta f_g(x(y),2m)\Delta f_g(a/x(y),2m) - 0.613(\Delta f_d(x(y),2m)] \frac{dx}{dy}.
$$

(38)

For polarized p-p collisions, $Q=3$ GeV, the results for $d\Delta\sigma/dy$ for $J/\Psi$ production using the standard model are shown in Figure 18; while for $\Psi'(2S)$ the results are shown in Figure 19. As above, the curves labelled $\Psi'(2S)a$ and are the standard model results, while that labelled $\Psi'(2S)b$ are the results for a mixed hybrid. The enhancement from active glue is once more quite evident. Since $\Upsilon(nS)$ states have not been resolved at BNL-RHIC, where polarized p-p collisions were measured, we do not calculate $d\Delta\sigma/dy$ for $\Upsilon(nS)$ states.
Once again, it is the ratios of $d\Delta \sigma/dy$ that are most significant, as there is uncertainty both the absolute magnitudes and shapes.
5.2.4 Ratios of Upsilon Production Cross Sections Via Unpolarized p-p Collisions at 2.76 TeV at LHC-CMS

The cross sections for $\Upsilon(nS)$ state production in p-p collisions have been measured at 2.76 TeV at the LHC-CMS[26] and at 38.8 GeV at Fermilab[24].

In this subsection the cross sections for $\Upsilon(nS)$ production, with $n=1, 2, 3$ are calculated, and then the theory that $\Upsilon(3S)$ is a hybrid is used to estimate the ratios of cross section. Since with scenario 2 with $\langle O_8^8(3P_0) \rangle = 0$, the $\lambda = 0$ helicity dominates the cross section[20], the $\lambda = 1$ terms were dropped. From Eq(27), for $\lambda = 0$, the cross section is determined from

$$\sigma_{pp \rightarrow \Phi(\lambda=0)} = A_\Phi \int_a^1 \frac{dx}{x} f g(x, 2m) f g(a/x, 2m).$$

From this it follows that

$$\sigma_{pp \rightarrow \Upsilon(1S)} \simeq 0.85 \text{ nb}$$  \hspace{1cm} (39)

What is significant for the present work are the ratios of the (1S), (2S), (3S) state production. In the present subsubsection experimental results from the Fermilab experiment[24] are used for the ratio of $\sigma(\Upsilon(2S))/\sigma(\Upsilon(1S))$, since the 2S and 1S states are given by the standard model, and $\sigma(\Upsilon(3S))/\sigma(\Upsilon(1S)) = N3 \times \sigma(\Upsilon(2S))/\sigma(\Upsilon(1S))$, as discussed above. Therefore, the estimate of the standard model is

$$\sigma(\Upsilon(2S))/\sigma(\Upsilon(1S)) \simeq 0.27 \text{ standard}$$

$$\sigma(\Upsilon(3S))/\sigma(\Upsilon(1S)) \simeq 0.04 \text{ standard, giving}$$

$$\frac{\sigma(\Upsilon(2S)) + \sigma(\Upsilon(3S))}{\sigma(\Upsilon(1S))} \simeq 0.31 \text{ standard}$$  \hspace{1cm} (40)

On the other hand, in the mixed hybrid theory with the $\Upsilon(3S)$ about 50% hybrid[12], one expects a factor of $\pi^2/4$ in the matrix element, and therefore a factor of about 2.45 for the $\Upsilon(3S)$ cross section compared to the standard model.

This results in the estimate

$$\frac{\sigma(\Upsilon(2S)) + \sigma(\Upsilon(3S))}{\sigma(\Upsilon(1S))} = 0.52.$$  \hspace{1cm} (41)

Compared to the LHC-CMS result[26] that this ratio is $0.78_{-0.14}^{+0.16} \pm 0.02$, while in the standard model it would be about 0.31 Therefore, the experimental results are consistent with the mixed heavy quark hybrid theory and not with the standard model.
5.2.5 Ratios of Upsilon Production Via Unpolarized p-p Collisions at 38.8 GeV at Fermilab

The study of the 38.8 GeV Upsilon production at Fermilab is similar to the preceding one for the LHC-CMS 2.76 TeV experiments. The result for the \( \frac{\sigma(\Upsilon(3S))}{\sigma(\Upsilon(1S))} \) expected at 38.8 GeV in the standard model, see Eq(40), compared to the mixed hybrid theory:

\[
\frac{\sigma(\Upsilon(3S))}{\sigma(\Upsilon(1S))} \approx 0.04 \text{ standard}
\]

\[
\frac{\sigma(\Upsilon(3S))}{\sigma(\Upsilon(1S))} \approx 0.147 - 0.22 \text{ hybrid (42)}
\]

compared to the experimental result\[24\] of about 0.12 to 0.16.

5.2.6 Conclusions of Ref[19]

The mixed hybrid theory for heavy quark states was used to predict that the cross sections for production of the charmonium \( \Psi'(2S) \) state in 200 GeV p-p collisions and bottomonium \( \Upsilon(3S) \) states in 38.8 GeV p-p collisions are much larger than the standard model. Also the estimated ratio of cross sections for 2.76 TeV and 38.8 GeV experiments, and the prediction for the \( \Upsilon(3S) \) production cross section is larger than the standard model, and closer to the experimental values.

Because of the importance of gluonic production in processes in a Quark Gluon Plasma, this could lead to a test of the creation of QGP in RHIC.

5.2.7 Upsilon Production In p-p Collisions For Forward Rapidities At LHC

In the work on p-p collisions producing heavy quark states reviewed above the rapidity was \( y = -1 \) to 1, while the present study is for \( y = 2.5 \) to 4.0 at the LHC\[28\]. The differential rapidity distribution for Upsilon production with \( \lambda = 0 \) (dominant for \( \Upsilon(nS) \) production), as is given by

\[
\frac{d\sigma_{pp \rightarrow \Phi(\lambda=0)}}{dy} = A_\Upsilon \frac{1}{x(y)} f_g(x(y), 2m) f_g(a/x(y), 2m) \frac{dx}{dy},
\]

with \( x(y), \frac{dx(y)}{dy} \) defined in Eq(34) and \( A_\Upsilon = 1.12 \times 10^{-7}, 1.73 \times 10^{-8} \text{ nb} \), for \( \sqrt{s} = 2.76, 7.0 \text{ TeV} \). \( f_g \) is the gluonic distribution function given in Eq(31) for the energies at the LHC.

Using Eqs(43,31) and parameters given in Ref[19] we obtain the results for \( \Upsilon(1S) \) and \( \Upsilon(3S) \) production shown in Fig. 20 and Fig. 21 at 2.76 TeV and 7.0 TeV in p-p collisions for \( 2.5 \leq y \leq 4.0 \). Although the units in Figs. 20, 21 are in pb, the actual magnitude is uncertain due to the normalization of the state. The overall magnitude and rapidity dependence of the differential rapidity distribution, however, provides satisfactory estimates at forward rapidities for LHC experiments.
Figure 20: $d\sigma/dy$ for pp collisions at $\sqrt{s} = 2.76$ and 7.0 TeV producing $\Upsilon(1S)$.

Figure 21: $d\sigma/dy$ for pp collisions at $\sqrt{s} = 2.76$ and 7.0 TeV producing $\Upsilon(3S)$. 
5.2.8 Ratios of $\Upsilon(2S)$ and $\Upsilon(3S)$ to $\Upsilon(1S)$

The ratios of $\Upsilon(2S)$ and $\Upsilon(3S)$ for the standard model and for the mixed hybrid model [12], upon which the present work [28] is based, are given by the premise that the $\Upsilon(3S)$ is 50% standard and 50% hybrid. One finds that

$$\left[ \frac{\sigma(\Upsilon(2S))}{\sigma(\Upsilon(1S))} \right]_{\text{mixed}} = \left[ \frac{\sigma(\Upsilon(2S))}{\sigma(\Upsilon(1S))} \right]_{\text{standard}} \simeq 0.27$$

$$\left[ \frac{\sigma(\Upsilon(3S))}{\sigma(\Upsilon(1S))} \right]_{\text{mixed}} \simeq 2.5 \times \left[ \frac{\sigma(\Upsilon(3S))}{\sigma(\Upsilon(1S))} \right]_{\text{standard}} \simeq 0.1.$$ (44)

These are the same as those given in Ref [27], and are consistent with CMS measurements.

Note that recent measurements by the LHCb experiment of the $\Upsilon$ production in p-p collisions at $\sqrt{s} = 7$ TeV [29] in the rapidity range $2.0 < y < 4.5$ found the cross sections times the branching fractions to $\mu^+\mu^-$ are $\Upsilon(2S)/\Upsilon(1S) \simeq 0.25$, in agreement with the standard and mixed hybrid theories, while $\Upsilon(3S)/\Upsilon(1S) \simeq 0.12$, in agreement with the mixed hybrid theory, but in disagreement with the standard model.

5.2.9 $\Psi$ and $\Upsilon$ Production In pp Collisions at 7.0 TeV

This is an extension of recent studies for $\Upsilon(nS)$ and $\Psi(1S, 2S)$ production at the LHC in p-p collisions with $E=7.0$ GeV and the ALICE detector [30]. The differential rapidity cross section is the same as Eq (43) with $A_{\Upsilon} = 1.73 \times 10^{-8}$ nb for $E=7.0$ TeV, and $A_{\Upsilon \to \Psi} = 6.46 \times 10^{-7}$. The gluonic distribution $f_g$ is the same as in Eq (31). The calculation of the production of $\Upsilon(3S)$ and $\Psi(2S)$ states is done with the mixed heavy hybrid theory [12].

![Figure 22: d\sigma/dy for pp collisions at \sqrt{s} = 7.0 TeV producing \Upsilon(1S).](image-url)
The differential rapidity cross sections for $\Upsilon(2S)$ with the standard model and for $\Upsilon(3S)$ with the standard model and mixed hybrid theory are shown in the figures below.

Figure 23: $d\sigma/dy$ for pp collisions at $\sqrt{s} = 7.0$ TeV producing $\Upsilon(2S)$.

Figure 24: $d\sigma/dy$ for pp collisions at $\sqrt{s} = 7.0$ TeV producing $\Upsilon(3S)$ for usual and hybrid theories.
The differential rapidity cross sections for $J/\Psi(1S)$ and $\Psi(2S)$ are shown in the figures below.

Figure 25: $d\sigma/dy$ for pp collisions at $\sqrt{s} = 7.0$ TeV producing $J/\Psi(1S)$.

Figure 26: $d\sigma/dy$ for pp collisions at $\sqrt{s} = 7.0$ TeV producing $\Psi(2S)$ for usual and hybrid theories.

For $\Upsilon(3S)$ and $\Psi(2S)$ the standard $q\bar{q}$ prediction is shown by dashed curves, while the prediction using the mixed hybrid theory[12] is shown with solid curves, with the difference explained in Ref[19].
5.2.10  Ψ and Υ Production In p-p Collisions at 8.0 TeV

This is an extension of the preceeding subsubsection for Υ(nS), n = 1, 2, 3, and J/Ψ(1S), Ψ(2S) production in p-p collisions with the ALICE detector at 7.0 TeV, with new predictions for p-p collisions at the LHC-ALICE with E=8.0 TeV[31]. The differential rapidity cross section is the same as Eq(43) with $A_\Upsilon = 1.33 \times 10^{-8}$ and $A_\Upsilon \rightarrow A_\Psi = 4.95 \times 10^{-7}$ for $E= 8.0$ TeV. The gluonic distribution $f_g(x(y), 2m)$ for the range of $x$ needed for $E= 8.0$ TeV is the same as Eq(31).

The calculation of the production of Υ(3S) and Ψ(2S) states is done with the usual quark-antiquark model and the mixed heavy quark hybrid theory, as in the previous subsections. The differential rapidity cross sections for J/Ψ(1S) and Ψ(2S) production for the standard model and the mixed hybrid theory are shown in Figure 27.

Figure 27: $d\sigma/dy$ for p-p collisions at $\sqrt{s} = 8.0$ TeV producing $J/\Psi(1S)$; and $\Psi(2S)$ for the standard model (dashed curve) and the mixed hybrid theory.
The differential rapidity cross sections for Υ(1S), Υ(2S), and Υ(3S) are shown in Figure 28.

Figure 28: dσ/dy for p-p collisions at √s = 8.0 TeV for producing Υ(1S), Υ(2S), Υ(3S) (dashed curve) using the standard model; and Υ(3S) with the mixed hybrid theory.
6 Heavy-quark state production in A-A collisions at \( \sqrt{s_{pp}} = 200 \) GeV

This section is a review of Ref[32]. The differential rapidity cross section for the production of a heavy quark state \( \Phi \) with helicirkyt \( \lambda = 0 \) in the color octet model via A-A collisions is given by

\[
\frac{d\sigma_{AA \rightarrow \Phi(\lambda=0)}}{dy} = R_{AA} N_{bin}^{AA} \frac{d\sigma_{pp \rightarrow \Phi(\lambda=0)}}{dy},
\]

where \( R_{AA} \) is the nuclear modification factor, defined in Ref[33], which includes the dissociation factor after the state \( \Phi \) is formed[34]. See Refs.[35],[36] for a discussion of “cold nuclear matter effects” and references to earlier experimental and theoretical publications. \( N_{bin}^{AA} \) is the number of binary collisions in the A-A collision, and \( \langle d\sigma_{pp \rightarrow \Phi(\lambda=0)} \rangle \) is the differential rapidity cross section for \( \Phi \) production via nucleon-nucleon collisions in the nuclear medium. Note that \( R_{AA}^{E} \), which we take as a constant, can be functions of rapidity. See Refs[37, 36] for a review and references to many publications.

Experimental studies show that for \( \sqrt{s_{pp}} = 200 \) GeV \( R_{AA} \simeq 0.5 \) both for Cu-Cu[38, 39] and Au-Au[40, 41, 52]. The number of binary collisions are \( N_{bin}^{AA} = 51.5 \) for Cu-Cu[53] and 258 for Au-Au. The differential rapidity cross section for p-p collisions in terms of \( f_{g}[22, 19] \), the gluon distribution function \(-0.8 \leq y \leq 0.8 \) for \( \sqrt{s_{pp}} = 200 \) GeV with \( f_{g} \) from Ref[19]), is

\[
\langle d\sigma_{pp \rightarrow \Phi(\lambda=0)} \rangle = A_{\Phi} \frac{1}{\bar{x}(y)} f_{g}(\bar{x}(y), 2m) f_{g}(a/\bar{x}(y), 2m) \frac{dx}{dy},
\]

where, as is discussed above, \( a = 4m^{2}/s \); with \( m = 1.5 \) GeV for charmonium, and \( 5 \) GeV for bottomonium, and \( A_{\Phi} = \frac{5\bar{x}^{3}a^{2}}{288m^{3}s} < O_{8}^{\Phi}(1S_{0}) > [19] \). For \( \sqrt{s_{pp}} = 200 \) GeV \( A_{\Phi} = 7.9 \times 10^{-4} \)nb for \( \Phi = J/\Psi \) and \( 2.13 \times 10^{-5} \)nb for \( \Upsilon(1S) \); \( a = 2.25 \times 10^{-4} \) for Charmonium and \( 2.5 \times 10^{-3} \) for Bottomonium.

The function \( \bar{x} \), the effective parton \( x \) in a nucleus (A), is given in Refs[42, 43]:

\[
\bar{x}(y) = x(y)(1 + \frac{\xi_{g}^{2}(A^{1/3} - 1)}{Q^{2}})
\]

\[
x(y) = 0.5 \left[ \frac{m}{\sqrt{s_{pp}}}(\exp y - \exp (-y)) + \sqrt{\frac{m}{\sqrt{s_{pp}}} \exp y \cdot \exp (-y)} \right]^{2} + 4a, \quad (47)
\]

with[44] \( \xi_{g}^{2} = .12 GeV^{2} \). For \( J/\Psi \) \( Q^{2} = 10 GeV^{2} \), so \( \bar{x} = 1.058x \) for Au and \( \bar{x} = 1.036x \) for Cu, while for \( \Upsilon(1S) \) \( Q^{2} = 100 GeV^{2} \), so \( \bar{x} = 1.006x \) for Au and \( \bar{x} = 1.004x \) for Cu.
From this we find the differential rapidity cross sections as shown in the following figures for $J/\Psi, \Psi(2S)$ and $\Upsilon(1S), \Upsilon(2S), \Upsilon(3S)$ production via Cu-Cu and Au-Au collisions at RHIC (E=200 GeV), with $\Psi(2S), \Upsilon(3S)$ enhanced by $\pi^2/4$ as discussed above. The absolute magnitudes are uncertain, and the shapes and relative magnitudes are our main prediction.

Figure 29: $d\sigma/dy$ for $2m=3$ GeV, $E=200$ GeV Cu-Cu collisions producing $J/\Psi$ with $\lambda = 0$

Figure 30: $d\sigma/dy$ for $2m=3$ GeV, $E=200$ GeV Au-Au collisions producing $J/\Psi$ with $\lambda = 0$
Figure 31: $d\sigma/dy$ for $2m=3$ GeV, $E=200$ GeV Cu-Cu collisions producing $\Psi(2S)$ with $\lambda = 0$. The dashed curve is for the standard $c\bar{c}$ model.

Figure 32: $d\sigma/dy$ for $2m=3$ GeV, $E=200$ GeV Au-Au collisions producing $\Psi(2S)$ with $\lambda = 0$. The dashed curve is for the standard $c\bar{c}$ model.
Figure 33: $d\sigma/dy$ for $2m=10$ GeV, $E=200$ GeV Cu-Cu collisions producing $\Upsilon(1S)$ with $\lambda = 0$

Figure 34: $d\sigma/dy$ for $2m=10$ GeV, $E=200$ GeV Au-Au collisions producing $\Upsilon(1S)$ with $\lambda = 0$
Figure 35: $d\sigma/dy$ for $2m=10$ GeV, $E=200$ GeV Cu-Cu collisions producing $\Upsilon(2S), \Upsilon(3S)$ with $\lambda = 0$. For $\Upsilon(3S)$ the dashed curve is for the standard $b\bar{b}$ model.

Figure 36: $d\sigma/dy$ for $2m=10$ GeV, $E=200$ GeV Au-Au collisions producing $\Upsilon(2S), \Upsilon(3S)$ with $\lambda = 0$. For $\Upsilon(3S)$ the dashed curve is for the standard $b\bar{b}$ model.
6.1 Ratios of $\Psi'(2S)$ to $J/\Psi$ cross sections

As discussed above, for the standard (st), hybrid model (hy) one finds for p-p production of $\Psi'(2S)$ and $J/\Psi$

$$\sigma(\Psi'(2S))/\sigma(J/\Psi(1S))|_{st} \simeq 0.27$$
$$\sigma(\Psi'(2S))/\sigma(J/\Psi(1S))|_{hy} \simeq 0.67 \pm 0.07,$$

(48)

while the PHENIX experimental result for the ratio[45] $\simeq 0.59$. Therefore, as in our earlier work the hybrid model is consistent with experiment, while the standard model ratio is too small.

The recent CMS/LHC result comparing Pb-Pb to p-p Upsilon product ion[26] found

$$\frac{[\Upsilon(2S) + \Upsilon(3S)]_{Pb-Pb}}{[\Upsilon(2S) + \Upsilon(3S)]_{p-p}} \simeq 0.31^{+0.19}_{-0.15}(syst),$$

(49)

while in the work discussed previously on $p-p$ collisions the ratio $\sigma(\Upsilon(3S))/\sigma(\Upsilon(1S))|_{p-p}$ of the standard $|b\bar{b}>$ model was $4/\pi^2 \simeq 0.4$ of the hybrid model. This suggests a suppression factor for $\sigma(\Upsilon(3S))/\sigma(\Upsilon(1S))$, or $\sigma(c\bar{c}/2S)/\sigma(c\bar{c}/1S)$ of $0.31/4$ as these components travel through the QGP; or an additional factor of 0.78 for $\Psi'(2S)$ to $J/\Psi$ production for $A-A$ vs $p-p$ collisions. Therefore from Eq(48) one obtains the estimate using the mixed hybrid theory for this ratio

$$\sigma(\Psi'(2S))/\sigma(J/\Psi(1S))|_{A-A \text{ collisions}} \simeq 0.52 \pm 0.05$$

(50)

6.2 Ratios of $\Upsilon(2S)$ and $\Upsilon(3S)$ to $\Upsilon(1S)$ cross sections

In work[19] discussed previously the ratios of $\Upsilon(2S)$ and $\Upsilon(3S)$ to $\Upsilon(1S)$ cross sections were estimated, in comparison with an experiment published in 1991[24]. The result for p-p collisions, with uncertainty due to separating $\Upsilon(2S)$ from $\Upsilon(3S)$, was

$$\Upsilon(3S)/\Upsilon(1S)|_{p-p} \simeq 0.14 - 0.22,$$

(51)

for the mixed hybrid theory, while the standard model would give $\Upsilon(3S)/\Upsilon(1S) \simeq 0.06$. A more recent CMS result[46], with a correction factor for acceptance and efficiency of the $\Upsilon(3S)$ to the $\Upsilon(1S)$ state, which was estimated to be approximately 0.29, was found to be

$$\Upsilon(3S)/\Upsilon(1S)|_{p-p} \simeq 0.12,$$

(52)

with the mixed hybrid theory in agreement within errors, while the standard model differs by a factor of two.

The new CMS experiment’s main objective[46] is to test for $\Upsilon$ suppression in PbPb collisions, with estimates of the following quantities:

$$\frac{[\Upsilon(2S)/\Upsilon(1S)]_{PbPb}}{[\Upsilon(2S)/\Upsilon(1S)]_{pp}},$$
$$\frac{[\Upsilon(3S)/\Upsilon(1S)]_{PbPb}}{[\Upsilon(3S)/\Upsilon(1S)]_{pp}}.$$

(53)

The studies of A-A collisions for Bottomonium states, which cannot be carried out at RHIC but are an important part of the LHC CMS program, is expected to be carried out in future research.
6.3 Creation of the QGP via A-A collisions

A main goal of the study of heavy quark state production in A-A collisions is the detection of the Quark Gluon Plasma. The energy of the atomic nuclei must be large enough so just after the nuclei collide the temperature is that of the universe about $10^{-5}$ seconds after the Big Bang, when the universe was too hot for protons or neutrons and consisted of quarks and gluons (the constituents of proton and nucleons)-the QGP. As the figure below illustrates, the emission of mixed hybrid mesons, the $\Psi(2S)$ and $\Upsilon(3S)$ as discussed above, with active gluons, could be a signal of the formation of the QGP.

Figure 37: Au-Au collisions producing $\Psi(2S)$ and $\Upsilon(3S)$ from the QGP.

6.4 Conclusions for Heavy-quark state production in A-A collisions at $\sqrt{s_{_{pp}}}=200$ GeV

The differential rapidity cross sections for $J/\Psi, \Psi(2S)$ and $\Upsilon(nS)(n = 1, 2, 3)$ production via Cu-Cu and Au-Au collisions at RHIC (E=200 GeV) were calculated using $R_{AA}$, the nuclear modification factor, $N_{bin}^{AA}$ the binary collision number, and the gluon distribution functions. This should give some guidance for future RHIC experiments, although at the present time the $\Upsilon(nS)$ states cannot be resolved.

The ratio of the production of $\sigma(\Psi'(2S))$, which in the mixed hybrid theory is 50% $c\bar{c}(2S)$ and 50% $c\bar{c}g(2S)$ with a 10% uncertainty, to $J/\Psi(1S)$, which is the standard $c\bar{c}(1S)$, could be an important test of the production of the Quark-Gluon Plasma. Using the hybrid model and suppression factors from previous theoretical estimates and experiments on $\Upsilon(mS)$ state production at the LHC, the ratio of $\Psi'(2S)$ to $J/\Psi(1S)$ production at RHIC via A-A collisions is estimated to be about $0.52 \pm 0.05$. In future studies at BNL and the LHC-CERN the study of RHIC producing $\Psi'(2S)$ and $\Upsilon(3S)$ mixed hybrid meson could be a method for determining the creation of the QGP.
6.5  $J/\Psi$ state production in Pb-Pb collisions at $\sqrt{s_{NN}}=2.76$ TeV

There have also been a number of experiments by the ALICE Collaboration on the production of $J/\Psi$ via Pb-Pb collisions at 2.76 TeV[54, 55, 56] which has measured $R_{AA}$ and other aspects of A-A collisions needed to establish the detection of the QGP. Since the present review is mainly focused on experimental tests of the mixed hybrid theory, we do not discuss these experimental publications.

7  Production of Charmonium and Upsilon States via Fragmentation

In the previous sections we reviewed the production of $c\bar{c}$ and $b\bar{b}$ states via p-p and A-A collisions. In this section we review the production of $|c\bar{q}>$ and $|b\bar{q}>$, with $q$ a light quark. Therefore the dominant octet processes illustrated in Figure 12, which produce $QQ$ states are not sufficient. To produce a $Qq$ state, with $Q = c$ or $b$ and $q$ a light quark, one needs the quark fragmentation processes, which was introduced for the study of $Z^0$ (a weak gauge boson) decay[47]. This is illustrated in Figure 38.

![Figure 38: Quark fragmentation for $Z^0 \rightarrow \Psi + c\bar{c}$](image)

The fragmentation probability which is used in the production of D-mesons via p-p collisions discussed in the following subsection was calculated by Bratten et.al.[48]

Gluon fragmentation into heavy quarkonium calculated in Ref[49] is illustrated in Figure 39. Although it is important for some charmonium or bottomium state production, we do not use it in the present review.
7.1 D Production In p-p Collisions

In this subsection the production of $D^+(c\bar{d}), D^0(c\bar{u})$ Charm mesons via unpolarized p-p collisions at 200 GeV is discussed. The main new aspect of the present work is that while a gluon can produce a $c\bar{c}$ or $b\bar{b}$ state, it cannot directly produce a $c\bar{d}$. A fragmentation process converts a $c\bar{c}$ into a $c\bar{d} - d\bar{c}$, for example. We use the fragmentation probability, $D_{c\rightarrow c\bar{q}}$ of Bratten et. al.[48], illustrated in Figure 40.

Using what in Ref[20] is called scenario 2, the production cross section with gluon dominance for $D_X$ is

$$\sigma_{pp\rightarrow D_X} = \int_a^1 \frac{dx}{x} f g(x, 2m) f g(a/x, 2m) \sigma_{gg\rightarrow D_X}, \quad (54)$$

with[48]

$$\sigma_{gg\rightarrow D_X} = 2\sigma_{gg\rightarrow c\bar{c}}D_{c\rightarrow c\bar{q}}, \quad (55)$$
where $\sigma_{gg\rightarrow c\bar{c}}$ is similar to the charmonium production cross section in Ref[19] and $D_{c\rightarrow c\bar{q}}$ is the total fragmentation probability.

For $E = \sqrt{s} = 200$ GeV the gluon distribution function is

$$f_{g}(y) = 1334.21 - 67056.5x(y) + 887962.0x(y)^2$$  \hspace{1cm} (56)

From Ref[48], using the light quark mass=(up quark mass+down quark mass)/2=3.5 MeV.

$$D_{c\rightarrow c\bar{q}} = 9.21 \times 10^5 \alpha_s|R(0)|^2/\pi,$$  \hspace{1cm} (57)

in units of $(1/GeV^3)$, with $\alpha_s = 0.26$. For a 1S state $|R(0)|^2 = 4/(a_0)^3$. For a $c\bar{q}$ state, $(1/a_0) = m_q \simeq 3.5$ MeV. Therefore,

$$|R(0)|^2 \simeq 1.71 \times 10^{-7} \text{ (GeV)}^3$$

$$D_{c\rightarrow c\bar{q}} \simeq 3.39 \times 10^{-3}.$$  \hspace{1cm} (58)

The calculation is similar to that in Ref[19].

$$\frac{d\sigma_{gg\rightarrow DX}}{dy} = Acc \ast f_{g}(x(y), 2m)f_{g}(a/x(y), 2m) \frac{dx(y)}{dy} \frac{1}{x(y)} D_{c\rightarrow c\bar{q}},$$  \hspace{1cm} (59)

with $Acc$ the matrix element for charmonium production[19] modified by an effective mass $ms[48]$

$$Acc = 7.9 \times 10^{-4}(1.5/ms)^3 \text{ nb}.$$  \hspace{1cm} (60)

The calculation of $\frac{d\sigma_{gg\rightarrow DX}}{dy}$ is a future project.

A number of experiments have measured $\sigma_{c\bar{c}}$ cross sections at $\sqrt{s_{pp}}=200$ GeV[50, 51, 41, 57]. Theoretical estimates of heavy quark state production via p-p collisions at RHIC and LHC energies were made almost two decades ago[58]. More recently estimates of $D$ production were made from data on d-Au collisions at $\sqrt{s_{NN}}= 200$ GeV[59]. Experimental measurements of $D^+, D^-, D^0$ production via p-p collisions are expected in the future.

8 Brief Overview

The theoretical basis for production of heavy quark states via p-p collisions, using the standard model for $J/\Psi, \Upsilon(1S), \Upsilon(2S)$ states and a mixed hybrid theory for $\Psi(2S), \Upsilon(3S)$ using QCD and QCD Sum Rules has been established by comparison with many experiments. For detection of the Quark-Gluon plasma, a main objective of RHIC and an important objective for the LHC, production of heavy quark states via A-A collisions is required. This is much more complicated, but there has been a great deal of progress in both experiment and theory. Also, the theory of production of open charm and bottom meson via p-p and A-A collisions is now greatly improved.

The detection of the Quark-Gluon Plasma via A-A collisions is closer to realization with this improved theory.

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