SUPERSYMMETRIC HOLST ACTION WITH MATTER COUPLING AND PARITY VIOLATION

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A general construction of the Holst action is discussed. Based on this, the $N = 1, 2, 4, 8$ supergravities and $N = 1$ supergravities with matter coupling are presented. It is shown that in all these cases the Immirzi parameter does not influence the field equations. The construction ensures that the theory is invariant under supersymmetry as well as gauge transformations, but the Holst extension breaks parity.

Keywords: SUGRA; Holst action; matter coupling; parity violation.

1. Introduction

The Holst action is an action used as a starting point to quantize gravity in theories employing real $SU(2)$ connection. In fact, Holst term is much older then Holst action formulation or even the Loop Quantum Gravity (LQG). It was first presented in [1] as a possible extension of the Hilbert-Einstein action invariant under gauge symmetries (e.g. Lorentz and diffeomorphisms symmetries) but breaking parity whenever the torsion is present. It turns out that the extension of the Lagrangian density $R$ by the term $\epsilon^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}$ which is proposed in this article is the only possible extension linear in $R$ in the Einstein-Cartan framework. It was suggested at the end of [1] that it would be interesting to extend the results to the supersymmetric case. Such extension is presented at this work.

The minimal/non-minimal coupling of the matter fields with spin-1/2 was already studied in the Holst action framework by several authors [2,3]. It was shown that the term with Immirzi parameter becomes dynamical i.e. it appears in the equation of motion (e.o.m.) and the parity is violated as expected. However, in [4] it has been shown that there is a coupling of fermions to the Holst action which does not affect the equations of motion\footnote{In the Ref. [4] it was shown that the Holst action can be rewritten as $\int d^4x (NY + \partial_\mu J^\mu)$, where $NY$ is the Nieh-Yan topological invariant and $J^\mu$ is a current of matter field. This property is more general and can be extended for derivations contained at this work. This turns out to be equivalent to the argument that if we take into account the terms quadratic in contorsion in the dynamical part of the action as well as in the Holst term (see Appendix A), the Immirzi parameter is not dynamical.}. In spite of the interest in coupling fermions to the theory, the supersymmetric Holst action was studied only in [5] as an exten-
sion of the Holst bosonic Lagrangian or in the $[6]$ where the supersymmetric Holst action was obtained as a result of super–BF theory with constrains. In both cases it turned out that the Holst term does not influence the e.o.m. As this work shows, these properties are general features of supersymmetric theories.

To clarify some misunderstanding in the role of the Holst term and Immirzi parameter, this paper will first recall the way of constructing Holst term, then will shortly present the role of the Immirzi parameter. The last part of this section is devoted to rudiments of general theory concerns the construction of the general parity violation term, based on the torsion equation. In the second part of this work the construction for higher supersymmetry will be presented and the super-Holst action $N = 1$ will be coupled, using the Noether method, to scalar-spinor-$1/2$, Maxwell and Yang–Mills multiplet.

2. Appearance of Immirzi parameter in General Relativity

The Immirzi parameter was introduced by Barbero $[7]$ and Immirzi $[8]$ as a parameter which allows to introduce real $SU(2)$ connection $\Gamma^i_{\mu}$ instead of complex $SL(2, C)$ Ashtekar connection $[9]$ in canonical quantization of general relativity.

The construction follows from noticing that if one partially fixes the $SO(3,1)$ gauge freedom by "time gauge", what is left is the invariance under local $SO(3)$, with frame space–like field $e^{\mu}(\mu, \nu, \ldots$ are space indexes, and $i, j, \ldots = 1, 2, 3$ internal indices), and 3–metric constructed from frame field $q^{\mu\nu} = e^{\mu}e^{\nu}$. Then one can construct a densitized triad $E^i_{\mu} = \frac{1}{2}q^{\mu\nu}(\epsilon_{ijk}e^j_{\mu}e^k_{\nu})^{\frac{1}{2}}$. There are two connection present in this construction, a $SO(3)$ Levi–Civita connection $\Gamma^i_{\mu} = \frac{1}{2}q^{\mu\nu}(\epsilon_{ijk}\omega^j_{\nu})$ and the connection $K^i_{\mu} = q^{\mu\nu}(\omega^j_{\nu}) = e^{\nu}K_{\mu}(\omega^j_{\nu})$ (the $\omega^j_{\nu}$ is a spin connection where $I, J, \cdots = 0, 1, 2, 3$) conjugated to $E^\mu_i$ i.e.

$$\{E^\mu_i(x), K^\nu_j(y)\} = \delta^{ij}\delta^{\mu\nu}\delta^3(x, y).$$

Barbero and Immirzi pointed out $[7,8]$, that there exist a canonical transformation of variables

$$E^\mu_i \rightarrow E^\mu_i$$
$$A^i_{\mu} \rightarrow \Gamma^i_{\mu} + \gamma K^i_{\mu}$$

which results in a single non vanishing Poisson brackets

$$\{E^\mu_i(x), A^\nu_j(y)\} = \gamma \delta^{ij}\delta^\mu\nu\delta^3(x, y).$$

The parameter $\gamma$ is called Immirzi parameter and its presence ensures that the theory has a Lorentzian signature as long as $\gamma \neq 1$ (which is Euclidean case). If one sets $\gamma = \pm i$ then one reconstructs the original Ashtekar self/anti-self dual variables $[9]$. For $\gamma \in \mathbb{R} – \{0\}$ the variables are real.

$^b$super–BF theory of the MacDowell–Mansouri type

$^c$Immirzi parameter is also named as Barbero–Immirzi parameter to honor both authors.

$^d$The group $SU(2)$ is double covering of the group $SO(3,1)$. 
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The Immirzi parameter also appears in a more general context in [10], where it was shown that by using the most general inner product on Lie algebra $so(3,1)$

$$<X,Y> = tr(X(c_0 + \ast c_1)Y),$$

and identifying $(c_0, c_1) = (1, \frac{1}{\gamma})$, one gets a theory, that includes the Immirzi parameter.$^{[3]}$

### 3. Holst action

An action which leads to Barbero formulation in Hamiltonian framework$^7$ is the action proposed in$^11$ $f$

$$S = \frac{1}{\alpha G} \int e e^\mu_I e^\nu_J (R^I_J - \beta \ast R^I_J).$$

This action was proposed earlier$^1$ in the form

$$S = \int d^4 x \frac{1}{16\pi G} \sqrt{-g} R + \frac{1}{16\pi G_P} \epsilon^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma},$$

where $G_P \sim \gamma$ governs the parity violating interaction.

Observe that the action (3.1) is based on the Barbero connection. One can take a variation of action

$$\delta_\omega S = \frac{1}{\alpha G} \int e e^\mu_I e^\nu_J \delta_\omega (R^I_J - \beta \ast R^I_J) = \frac{2}{\alpha G} \int \delta_\omega B^I_J \omega^\mu \epsilon^\mu I e^\nu J$$

where

$$\delta_\omega B^I_J = \delta_\omega \omega^I_J - \beta \ast \delta_\omega \omega^I_J.$$  

In the case where $\beta = \pm i$ one can recognizes the Ashtekar connection. If one uses the space-time decomposition of connection, i.e.

$$\omega_{\mu ij} = -\epsilon_{ijk} \Gamma^k_\mu = -\epsilon_{ijk}(-\frac{1}{2} \epsilon^{klm} e_{rl} \nabla_{\mu} e^r_k)$$

$$\omega_{\mu i0} = e_i^\nu \nabla_{\mu} e_{r0} = K_{\mu i}$$

and inserts it to the action, then it is easy to see that action depends exactly on the Barbero connection

$$A_{\mu i} = \Gamma_{\mu i} + \frac{1}{\beta} K_{\mu i},$$

with the identification $\frac{1}{\beta} = \gamma$.  

$^e$More precisely the Holst action (which is presented in section$^3$ with boundary terms.  

$^f$The $\alpha$ was introduced for simplification as it is a numerical factor.
4. General construction of the Holst term

The construction of connection (3.4) suggests how the Holst term is constructed. It is built from terms belonging to the torsion equation i.e. from e.o.m. calculated under variation $\delta \omega$, multiplied by $\frac{1}{\gamma} \star$. The torsion equation has the following general structure

$$T_{\mu \nu}^\lambda = D_{[\mu} e_{\nu]}^\lambda = K_{[\mu}^{\ IJ} e_{\nu]}^\lambda = -2K_{[\mu \nu]}^\ I. \quad (4.1)$$

The tensor $K_{\mu}^{\ IJ}$ is called contorsion and is defined as a difference between torsion and torsion-free connection

$$\Gamma_{\mu \nu}^\lambda = \tilde{\Gamma}_{\mu \nu}^\lambda - K_{\mu \nu}^\ lambda. \quad (4.2)$$

In terms of spin connection it can be rewritten as

$$\omega_{\mu \nu}^{IJ} = \omega_{\mu \nu}^{IJ} + K_{\mu \nu}^{IJ}. \quad (4.3)$$

The contorsion $K_{\mu \nu}^\lambda$ can be obtained from

$$e_\rho^\mu T_{\mu \nu}^I (\epsilon, \omega(e) + K) = -2K_{[\mu \nu]}^\rho \quad (4.4)$$

and can be expressed in term of torsion as

$$K_{\mu \nu}^\rho = \frac{1}{2} (T_{\mu \nu}^\rho - T_{\nu \mu}^\rho + T_{\nu \mu}^\rho). \quad (4.5)$$

The Holst term can be constructed from torsion and contorsion tensor by the following contractions

$$S_{\text{Holst}} = \frac{1}{G} \int \frac{1}{\gamma} \epsilon_{\mu \nu \rho \sigma} (T_{\mu \nu}^I + K_{\mu \nu}^I) (T_{\rho \sigma} + K_{\rho \sigma})$$

$$= \frac{1}{G} \int \frac{2}{\gamma} (T_{\mu \nu}^I + K_{\mu \nu}^I) \star (T_{\rho \sigma}^I + K_{\rho \sigma}^I). \quad (4.6)$$

It arises from the $\text{[3.4]}$ that a variation with respect to $\omega$ is

$$\delta_{\omega} S = \frac{4}{\alpha G} \int \epsilon_{\mu \nu \rho \sigma} \epsilon_{ijkl} \delta \omega_{\mu \nu} T_{ijkl} \left( (T_{\rho \mu}^L + K_{\rho \mu}^L) \frac{1}{\gamma} + (T_{\rho \mu}^L + K_{\rho \mu}^L) \right). \quad (4.7)$$

If one restricts oneself to a theory linear in $R$ and without any second covariant derivatives, the most general object of 3 indices (2 space-time and 1 internal) which can be contracted via $\epsilon_{\mu \nu \rho \sigma}$ and gives the second part of $\text{[4.7]}$ can be expressed in $\text{[4.6]}$ as

$$- \frac{4}{\alpha G} \int \epsilon_{\mu \nu \rho \sigma} (\delta \omega_{\mu \nu} T_{ijkl}^I + K_{\mu \nu}^I) \quad (4.8)$$

is exactly $\text{[4.6]}$.

There are some comments on this construction. First, if matter fields are not included to Lagrangian, there is no torsion $\text{[4]}$. In the other words the torsion

$$T_{\mu \nu}^\lambda = \Gamma_{[\mu \nu]}^\lambda. \quad (4.9)$$
is antisymmetric in lower indices and there is no contorsion term $K^{\alpha\beta\gamma} = 0$. To get dynamical torsion, due to the fact that torsion couples to spin and gravity field has spin-2, one can express contorsion field in a term of scalar field

$$K_{\alpha\beta\gamma} = \phi_{,\beta}g_{\alpha\gamma} - \phi_{,\gamma}g_{\alpha\beta}$$

(4.10)

and ensure that this equation will be satisfied by adding it to the action via Lagrange multiplier. However this procedure does not break parity symmetry \[1\]. In the case of matter field the term \[4.16\] bieng a pseudoscalar breaks parity. Second comment is that there exist much more terms quadratic in contorsion which can be added to the action (see \[11,12\]), even the terms added in nondynamical manner, which results in that there exist much more terms quadratic in contorsion which can be added to the action (see \[11,12\]), even the terms added in nondynamical manner, which results in the dynamical torsion i.e. terms with second derivatives of $K$ or equivalently, terms quadratic on the derivatives of $K$ \[11,13\].

5. Supersymmetric $N = 1, 2, 4, 8$ Holst action

Based on the construction \[4,0\], the supersymmetric extension \[2\] of the Holst action can be easily constructed and classified \[4\]. In the case of supergravity (SUGRA) expression in \[4,1\] is called ”super torsion” \[1\].

| $N$ | Super torsion | Holst term |
|-----|---------------|------------|
| 1   | $T_{\nu\rho} = \bar{\psi}_{\nu}\gamma^\lambda\psi_\rho$ | $e^\mu\nu\rho\sigma\bar{\psi}_\mu e_\nu^a e_\rho^b e_\sigma^c$ |
| 2   | $T_\mu^\nu = (\bar{\psi}_\mu\gamma^a\psi_\nu + \bar{\psi}_\nu\gamma^a\psi_\mu)$ | $e^\mu\nu\rho\sigma\bar{\psi}_\mu e_\nu^a e_\rho^b e_\sigma^c$ |
| 4   | $T_\mu^\nu = \bar{\psi}_\mu^I\gamma^a\psi_\nu^I + \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\lambda_I^a\lambda_I^b\lambda_I^c$ | $e^\mu\nu\rho\sigma\bar{\psi}_\mu e_\nu^a e_\rho^b e_\sigma^c$ |
| 8   | $T_\mu^\nu = \bar{\psi}_\mu^I\gamma^a\psi_\nu^I + \frac{1}{12}\epsilon_{\mu\nu\alpha\beta}\chi^{\alpha\beta\gamma\delta}_{\mu\nu\alpha\beta}$ | $e^\mu\nu\rho\sigma\bar{\psi}_\mu e_\nu^a e_\rho^b e_\sigma^c$ |

\[The action and torsion equation for case $N = 8$ can be found in \[14\].

\[The terms agree with published in \[6\] for SUGRA $N = 1$ and in \[5\] for SUGRA $N = 1, 2, 4$.

\[From this section $i = 0, \ldots, 3$ are internal indices, $\mu, \nu = 0, \ldots, 3$ are space-time indices and $I, J, K$ numbers the different supersymmetric charges.

Table 1. Super torsion equation and Holst term for different SUGRA theories.
6 Coupling matter to SUGRA $N = 1$

Matter can be coupled in the supersymmetric manner i.e. in the way which ensure that the Lagrangian will be invariant under supersymmetric, Lorentz and diffeomorphisms transformations. The method of coupling is called the Noether method and details can be found in [15,16,17].

The brief review of the method [17,16] for the scalar-spin-1/2 multiplet based on consideration of Wess-Zumino Lagrangian which is scalar-spinor-1/2 theory with global supersymmetric invariance

$$L = -\frac{1}{2} g^{\mu\nu} (\partial_\mu A \partial_\nu A + \partial_\mu B \partial_\nu B) - \frac{1}{2} \bar{\chi} \gamma^\mu D_\mu \chi,$$  

where $A$ is scalar, $B$ is pseudoscalar and $\chi$ is spin-1/2 spinor. If one replaces a global supersymmetric invariance with local, the coupling term, accurate to second order in coupling constant $\kappa$, can be expressed as

$$L + \kappa \frac{1}{2} e \bar{\psi} \mu (\partial / A + i \gamma^5 \partial / B) \gamma^\mu \chi - \frac{1}{16} e \kappa^2 (\bar{\chi} \chi)^2$$

and the supersymmetric transformation, accurate to first order in $\kappa$, are

$$\delta \chi = \partial (A + i \gamma_5 B) \epsilon - \frac{1}{2} \kappa (\bar{\psi}_\rho \gamma_5 \chi) \gamma^\rho \epsilon$$

$$\delta \psi_\mu = \frac{2}{\kappa} D_\mu \epsilon + \frac{ik}{2} \gamma_5 \epsilon (A \leftrightarrow \mu B) + \frac{\kappa}{4} (\chi \gamma^5 \gamma^\rho \epsilon) (\sigma_\mu \gamma_5 \epsilon)$$

$$\delta A = \bar{\epsilon} \chi \quad \delta B = i \bar{\epsilon} \gamma_5 \chi \quad \delta e_\mu = i k \bar{\epsilon} \gamma^i \psi_\mu$$

$$\delta \omega^{ij} = -\kappa \bar{\epsilon} \gamma^{ij} \psi_\mu.$$  

The total Lagrangian of supersymmetric gravity coupled to scalar multiplet is

$$L = L^{SUGRA} - \frac{1}{2} g^{\mu\nu} (\partial_\mu A \partial_\nu A + \partial_\mu B \partial_\nu B) - \frac{1}{2} \bar{\chi} \gamma^\mu D_\mu \chi$$

$$+ \frac{1}{2} e \kappa \bar{\psi}_\mu (\partial A + i \gamma_5 \partial B) \gamma^\mu \chi - \frac{1}{16} e \kappa^2 (\bar{\chi} \chi)^2$$

$$+ \frac{1}{32} (\bar{\chi} \gamma^5 \gamma^\tau \chi) [e^{\alpha \beta \mu \nu} \bar{\psi}_\alpha \gamma_5 \psi_\beta - 2 e (\bar{\psi}_\alpha \gamma^5 \gamma^\tau \psi_\beta) ]$$

$$+ \frac{1}{8} i \kappa^2 (A \leftrightarrow \mu B) [e \bar{\chi} \gamma^5 \gamma^\beta \chi - \epsilon^{\alpha \beta \mu \tau} \bar{\psi}_\alpha \gamma_\tau \psi_\mu].$$

The equation of torsion can be calculated from (6.3) as

$$T_{\nu\rho\mu} = D^{\nu}_{[\rho} e_{\mu]i} = \bar{\psi}_\nu \gamma_i \psi_\rho + \frac{1}{2} e_{[\nu \rho}^{\mu} \epsilon_{\mu]_{\nu \rho \sigma}} \bar{\chi} \gamma^\sigma \chi.$$
Using the construction (4.6) the Holst term reads
\[
\mathcal{L}^{\text{Holst}} = \frac{1}{\gamma} e^{\mu \nu \rho \sigma} \left( 4D_{\nu} e_{\mu} D_{\sigma} e_{\rho} e_{\tau} - \bar{\psi}_{\mu} \gamma_{\nu} \psi_{\rho} D_{\sigma} e_{\tau} - \frac{1}{e} e^{\alpha}_{\tau} e_{\mu \alpha \beta} \bar{\chi}_{\gamma}^{\beta} \chi_{\sigma}^{\gamma} e_{\rho e_{\tau}} + \frac{1}{4} \bar{\psi}_{\mu} \gamma_{\nu} \psi_{\rho} \gamma_{\tau} \psi_{\sigma} \\
+ \frac{1}{8e} e^{\alpha}_{\tau} e_{\mu \alpha \beta} \bar{\chi}_{\gamma}^{\beta} \chi_{\sigma}^{\gamma} e_{\rho} + \frac{1}{4e^2} e^{\alpha}_{\tau} e_{\mu \alpha \beta} \bar{\chi}_{\gamma}^{\beta} \chi_{\sigma}^{\gamma} e_{\rho} e_{\xi} e_{\delta} \chi \right). \tag{6.11}
\]

One can couple the Maxwell field using exactly the same procedure [17]. The action for coupled abelian field in supersymmetric way is
\[
\mathcal{L} = \mathcal{L}^{\text{SUGRA}} + \frac{1}{4} e^{\mu \nu \rho} g^{\alpha \beta} F_{\mu \nu} F_{\rho \sigma} - \frac{1}{2} e \bar{\chi}_{\gamma}^{\alpha} D_{\mu} \lambda + \frac{1}{4} e_{\kappa e} \bar{\psi}_{\mu} \gamma_{\alpha \beta} \lambda F_{\alpha \beta} \\
+ \frac{1}{8} e^2 \left( (\bar{\psi} \cdot \psi)(\bar{\chi} \lambda) + (\bar{\psi}_{\alpha} \gamma^5 \psi_{\alpha})(\bar{\chi} \gamma^5 \lambda) \\
+ \frac{1}{4} (\bar{\psi} \cdot \gamma \gamma \cdot \psi)(\bar{\chi} \lambda) + \frac{1}{4} (\bar{\psi} \cdot \gamma \gamma^5 \gamma \cdot \psi)(\bar{\chi} \gamma^5 \lambda) \\
- \frac{1}{2} (\bar{\psi} \cdot \gamma \gamma^5 \cdot \psi_{\alpha})(\bar{\chi} \gamma^5 \lambda) + \frac{1}{4} (\bar{\psi}_{\alpha} \cdot \gamma 5 \gamma_{\alpha} \cdot \psi_{\alpha})(\bar{\chi} \gamma^5 \lambda) \\
+ \frac{3}{2} (\bar{\chi} \lambda)(\bar{\chi} \lambda) \right). \tag{6.12}
\]

One can recognize the two connections, a spin connection and an abelian connection, but the impact to super torsion equation is given only by a spin connection and it looks exactly the same as (6.10).

If one considers a Yang–Mills multiplied given by the Lagrangian
\[
\mathcal{L} = \mathcal{L}^{\text{SUGRA}} + \frac{1}{4} e^{\mu \nu \rho} g^{\alpha \beta} F_{\mu \nu} F_{\rho \sigma} - \frac{1}{2} e \bar{\chi}_{\gamma}^{\alpha} D_{\mu} \lambda + \frac{1}{4} e_{\kappa e} \bar{\psi}_{\mu} \gamma_{\alpha \beta} \lambda F_{\alpha \beta} \\
+ \frac{1}{8} e^2 \left( (\bar{\psi} \cdot \psi)(\bar{\chi} \lambda) + (\bar{\psi}_{\alpha} \gamma^5 \psi_{\alpha})(\bar{\chi} \gamma^5 \lambda) \\
+ \frac{1}{4} (\bar{\psi} \cdot \gamma \gamma \cdot \psi)(\bar{\chi} \lambda) + \frac{1}{4} (\bar{\psi} \cdot \gamma \gamma^5 \gamma \cdot \psi)(\bar{\chi} \gamma^5 \lambda) \\
- \frac{1}{2} (\bar{\psi} \cdot \gamma \gamma^5 \cdot \psi_{\alpha})(\bar{\chi} \gamma^5 \lambda) + \frac{1}{4} (\bar{\psi}_{\alpha} \cdot \gamma 5 \gamma_{\alpha} \cdot \psi_{\alpha})(\bar{\chi} \gamma^5 \lambda) \\
+ \frac{3}{2} (\bar{\chi} \lambda)(\bar{\chi} \lambda) \right) + \frac{3}{64} e^2 \left( Tr(\bar{\chi} \gamma 5 \gamma \lambda) \right)^2, \tag{6.13}
\]

than immediately will see, that using non-abelian matter fields instead of abelian do not influence to torsion equation (6.10). Therefore, it is a general observation that Holst term in the scalar-spin-1/2, Maxwell and Yang–Mills theory has the same form of (6.11).

7. General Theory for SUGRA

It is possible to formulate general theory, which is valid for any supersymmetric matter couple to SUGRA N=1 and for any higher SUGRA (N=1,2,4,8).

**Theorem 1.** The Holst term constructed in SUGRA N = 1, 2, 4, 8 and SUGRA N = 1 coupled to matter is invariant under supersymmetric, Lorentz and diffeomorphisms transformation and does not influence the field equations.
Proof. The torsion equation has a general form

$$T^i_{\mu\nu} + K^i_{1\mu\nu} + \cdots + K^i_{N\mu\nu} = 0,$$

where $N$ is a number of different matter sources present in a theory. Using the construction (4.6) the Holst term reads

$$\mathcal{L} = \epsilon^{\mu\nu\rho\sigma}(T^i_{\mu\nu}T_{i\rho\sigma} + T^i_{\mu\nu}K^i_{1\rho\sigma} + \cdots + T^i_{\mu\nu}K^i_{N\rho\sigma}
+ \frac{1}{2}K^i_{1\mu\nu}K^i_{1\rho\sigma} + \cdots + K^i_{1\mu\nu}K^i_{N\rho\sigma}
+ \cdots + \frac{1}{2}K^i_{N\mu\nu}K^i_{N\rho\sigma}) \cdot$$

(7.2)

One can notices that any arbitrary infinitesimal variation of this Lagrangian

$$\delta\mathcal{L} = \epsilon^{\mu\nu\rho\sigma}(\delta T^i_{1\mu\nu}T_{i\rho\sigma} + 2\delta T^i_{1\mu\nu}T_{2\rho\sigma} + \cdots + 2\delta T^i_{1\mu\nu}T_{N\rho\sigma}
+ \delta T^i_{2\mu\nu}T_{2\rho\sigma} + \cdots + 2\delta T^i_{2\mu\nu}T_{N\rho\sigma}
+ \delta T^i_{N\mu\nu}T_{N\rho\sigma}) \cdot$$

(7.3)

vanishes according to torsion equation (7.1).

8. Conclusion

One can realize that in the supersymmetry theories the terms depending on Immirzi parameter are not dynamical. This result is much different from many works on coupling spinor fields to gravity [2,3]. Therefore, the understanding of the difference is very instructive. Usually, the strategy of coupling matter field to Holst action is based on adding the matter Lagrangian with some coupling constants. The significant is that the matter field does not contribute the Holst action. If one calculates the torsion equation then immediately realizes that it has some impact to torsion–free connection i.e. contorsion, and contorsion depend on the Immirzi parameter. Solution of this equation gives the new connection (4.3), which contribute to action by some additional terms (see (A.3) in the Appendix) e.g. $K^i_{\mu\nu\rho}K^i_{\mu\nu\rho}$, which appear in e.o.m. Strategy presented in this work assumes that the Holst term is constructed according to construction (4.6), which ensures that Holst term will be supersymmetric (using Theorem [1]), as the theory without Holst term has to be supersymmetric by itself. Then if super torsion equation is calculated, contorsion tensor will not depend on Immirzi parameter anymore. Therefore, the action with additional terms (A.2, A.3) also does not depend on Immirzi parameter. Thus one can see that the assumption of supersymmetry does not allow appearance of the Immirzi parameter in e.o.m.

However, considering the additional term [112,113] in the super–Holst action is very interesting according to the interpretation of Immirzi parameter as a regulator of the quantum fluctuations of the vanishing torsion condition. Author leaves this consideration for the future work.
Acknowledgments

Author would like to thank Wolfgang M. Wieland for reference [1] and helpful discussion, Simone Mercuri for discussion and indication of the many interesting problems, Jerzy Kowalski-Glikman for inspiration, discussion and comments on the manuscript, and Tomasz Bialach for some remarks. This work was partially supported by National Science Center grant No. 2011/01/N/ST2/00415.

Appendix A. The expanding tensor $R_{\mu\nu\rho\sigma}$ under full connection

The construction (4.6) can be done alternatively [12] by calculating curvature using
\[ e_{\mu K} e_{\nu L} R^{KL}_{\rho\sigma} (\omega(e) + K) = R_{\mu\nu\rho\sigma} (e) + 2 \nabla_{[\rho} K_{\sigma]\mu\nu} + K_{\rho\mu\lambda} K_{\lambda\nu} - K_{\sigma\mu\lambda} K_{\rho\nu} \]
(A.1)
and finding the dimension-two invariant of (A.1) under two possible contractions
\[ e^{\mu\nu\rho\sigma} e_{\mu K} e_{\nu L} R^{KL}_{\rho\sigma} = (d^4 x) K_{\mu\nu} K_{\rho\sigma} + (\text{boundary term}) \]  (A.2)
\[ e^{\mu\nu\rho\sigma} e^{IJKL} e_{\mu I} e_{\nu J} R_{\rho\sigma KL} = (d^4 x) e [ R(e) + K_{\mu
u\rho} K^{\mu\rho} - K_{\mu\rho\lambda} K_{\nu\rho\lambda} ] + (\text{boundary term}) \]  (A.3)

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