The Probability Distribution of the Double Neutron Star Coalescence Rate and Predictions for More Detections

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Abstract. We present an analysis method that allows us to estimate the Galactic formation of radio pulsar populations based on their observed properties and our understanding of survey selection effects. More importantly, this method allows us to assign a statistical significance to such rate estimates and calculate the allowed ranges of values at various confidence levels. Here, we apply the method to the question of the double neutron star (NS–NS) coalescence rate using the current observed sample, and we find calculate the most likely value for the total Galactic coalescence rate to lie in the range $3^{-22}_{+1}$ Myr$^{-1}$, for different pulsar population models. The corresponding range of expected detection rates of NS–NS inspiral are $(1-9)\times10^{-3}$ yr$^{-1}$ for the initial LIGO, and $6-50$ yr$^{-1}$ for the advanced LIGO. Based on this newly developed statistical method, we also calculate the probability distribution for the expected number of pulsars that could be observed by the Parkes Multibeam survey, when acceleration searches will alleviate the effects of Doppler smearing due to orbital motions. We suggest that the Parkes survey will probably detect $1-2$ new binary pulsars like PSRs B1913+16 and/or B1534+12.

1. Introduction

The detection of the double neutron star (NS–NS) prototype PSR B1913+16 as a binary pulsar (Hulse & Taylor 1975) and its orbital decay due to emission of gravitational waves have inspired a number of quantitative estimates of the coalescence rate, $\mathcal{R}$, of NS–NS binaries (Clark et al. 1979; Narayan et al. 1991; Phinney 1991; Curran & Lorimer 1995). Significant interest derives from their importance as gravitational-wave sources for the upcoming ground-based laser interferometers (such as LIGO).

We present a newly developed statistical analysis that allows the calculation of statistical confidence levels associated with rate estimates. The method can be applied to any radio pulsar population. Here, we consider PSR B1913+16 (Hulse & Taylor 1975) and PSR B1534+12 (Wolszczan 1991). For different assumed distributions of pulsar properties (luminosities, Galactic positions), we
derive the probability distribution function of the total Galactic coalescence rate weighted by the two observed binary systems. The method involves the simulation of selection effects inherent in all relevant radio pulsar surveys and a Bayesian statistical analysis for the probability distribution of $R$. The small-number bias and the effect of the faint-end of the luminosity function, previously identified as the main sources of uncertainty in rate estimates (Kalogera et al. 2001) are implicitly included in this analysis. We extrapolate the Galactic rate to cover the detection volume of LIGO and estimate the most likely detection rates of NS–NS inspiral events for the initial and advanced LIGO. Details of this work are given in Kim et al. (2003; hereafter KKL).

In the second part of this paper, we modify our statistical method in a way that allows us to calculate the probability distribution for the number of pulsars that could be detected by the Parkes Multibeam survey (hereafter PMB survey; Lyne et al. 2000; Manchester et al. 2001) PMB, when the effects of Doppler smearing due to orbital motions are corrected with acceleration searches.

2. Pulsar Survey Selection Effects

For a model pulsar population with a given spatial and luminosity distribution, we determine the fraction of the total population which are actually detectable by current large-scale pulsar surveys. In order to do this, we calculate the effective signal-to-noise ratio for each model pulsar in each survey, and compare this with the corresponding detection threshold. Only those pulsars which are nominally above the threshold count as being detectable. After performing this process on the entire model pulsar population of size $N_{\text{tot}}$, we are left with a sample of $N_{\text{obs}}$ pulsars that are nominally detectable by the surveys. By repeating this process many times, we can determine the probability distribution of $N_{\text{obs}}$, which we then use to constrain the population and coalescence rate of NS–NS binaries. More details are given in §2 of Lorimer et al. (1993) and in KKL.

Here we discuss in some detail the the Doppler smearing effect which is significant for recent surveys with relatively long exposures. For binary pulsars, we need to take account of the reduction in signal-to-noise ratio due to the Doppler shift in period during an observation. For observations of NS–NS binaries, where the orbital periods are of the order of 10 hours or less, the apparent pulse period can change significantly during a search observation causing the received power to be spread over a number of frequency bins in the Fourier domain. As all the surveys considered in this analysis search for periodicities in the amplitude spectrum of the Fourier transform of the time series, a signal spread over several bins can result in a loss of signal-to-noise ratio $\sigma$. To take account of this effect in our survey simulations, we need to multiply the apparent flux density of each model pulsar by a “degradation factor”, $F = \sigma_{\text{binary}}/\sigma_{\text{control}}$. Significant degradation occurs, when $F \ll 1$. Using an analysis method described in Camilo et al. (2000), we calculate the degradation factor for the two pulsars we consider in this work. As expected, we find that surveys with the longest integration times are most affected by Doppler smearing. For the PMB survey, which has an integration time of 35 min, mean values of $F$ are 0.7 and 0.3 for PSR B1913+16 and PSR B1534+12 respectively. The greater degradation for PSR B1534+12 is due to its mildly eccentric orbit ($e \sim 0.3$ versus 0.6 for PSR B1913+16) which results
in a much more persistent change in apparent pulse period when averaged over the entire orbit. In order to improve on the sensitivity to binary pulsars, the PMB survey data are now being reprocessed using various algorithms designed to account for binary motion during the integration time (Faulkner et al.; these proceedings). For the Jodrell Bank and Swinburne surveys (Nicastro et al. 1995; Edwards et al. 2001), which both have integration times of order 5 min, we find $F \sim 0.9$ for both systems. For all other surveys, which have significantly shorter integration times, no significant degradation is seen, and we take $F = 1$.

3. Probability Distribution of Double Neutron Star Coalescence Rates

3.1. Statistical Method

As already mentioned, we generate large numbers of "observed" pulsar samples by modeling the survey selection effects and applying them to model populations of PSR B1913+16–like and PSR B1534+12–like pulsars, separately. For a fixed value of $N_{\text{tot}}$, we use these “observed” samples to calculate their distribution, which we find to be very well described by a Poisson distribution:

$$P(N_{\text{obs}}; \lambda) = \frac{\lambda^{N_{\text{obs}}} e^{-\lambda}}{N_{\text{obs}}!},$$

(1)

where $\lambda \equiv < N_{\text{obs}} >$. With our Monte Carlo simulations we calculate $\lambda$ and find it to linearly correlate with $N_{\text{tot}}$ (for values in the range $10 - 10^4$):

$$\lambda = \alpha N_{\text{tot}},$$

(2)

where $\alpha$ is a constant that depends on the properties (space and luminosity distributions and pulse period and width) of the Galactic pulsar population.

For a given $N_{\text{tot}}$, we calculate the rate using estimates of the associated pulsar beaming correction factor $f_b$ and lifetime $\tau_{\text{life}}$: $R = \frac{N_{\text{tot}}}{\tau_{\text{life}} f_b}$. We adopt values discussed in Kalogera et al. (2001): 5.72 and $3.65 \times 10^8$ yr for PSR B1913+16, and 6.45 and $2.9 \times 10^9$ yr for PSR B1534+12.

Using Bayes' theorem and the best-fit Poisson distributions, we can calculate the probability distribution of the total number $N_{\text{tot}}$ of pulsars in the Galaxy. Further, using estimates of the associated pulsar beaming correction factor $f_b$ and lifetime $\tau_{\text{life}}$, we can calculate the distribution function of pulsar rates:

$$P(R) = \left(\frac{\alpha \tau_{\text{life}}}{f_b}\right)^2 R e^{-\left(\frac{\alpha \tau_{\text{life}}}{f_b}\right) R},$$

(3)

We use appropriate variable transformations to then calculate the total rate probability distribution:

$$P(R_{\text{tot}}) = \left(\frac{AB}{B - A}\right)^2 \left[ R_{\text{tot}} (e^{-A R_{\text{tot}}} + e^{-B R_{\text{tot}}}) - \left(\frac{2}{B - A}\right) (e^{-A R_{\text{tot}}} - e^{-B R_{\text{tot}}}) \right],$$

(4)
where $A$ and $B$ are defined as follows:

$$A \equiv \left( \frac{\alpha \tau_{\text{life}}}{f_b} \right)_{1913} \quad \text{and} \quad B \equiv \left( \frac{\alpha \tau_{\text{life}}}{f_b} \right)_{1534}.$$  \hspace{1cm} (5)

Having calculated the probability distribution of the Galactic coalescence rate, we can take one step further and also calculate ranges of values for the rate $R_{\text{tot}}$ at various confidence levels (CL). The lower ($R_a$) and upper ($R_b$) limits to these ranges are determined by the following conditions:

$$\int_{R_a}^{R_b} P(R_{\text{tot}}) dR_{\text{tot}} = \text{CL} \quad \text{and} \quad P(R_a) = P(R_b).$$  \hspace{1cm} (6)

Finally, we can calculate the detection rate for LIGO, $R_{\text{det}}$, defined by

$$R_{\text{det}} = \epsilon R_{\text{tot}} V_{\text{det}},$$  \hspace{1cm} (7)

where $\epsilon$ is the scaling factor (based on the blue luminosity density of the nearby universe) derived to be $\sim 10^{-2}\text{Mpc}^{-3}$ (for details see Kalogera et al. 2001). $V_{\text{det}}$ is the detection volume defined as a sphere with a radius equal to the maximum detection distance $D_{\text{max}}$ for the initial ($\sim 20\text{Mpc}$) and advanced LIGO ($\sim 350\text{Mpc}$; Finn 2001).

### 3.2. Results

We have chosen one of our pulsar population models to be our reference model based on the results presented by Cordes & Chernoff (1997). For this model, we find the most likely value of $N_{\text{tot}}$ to be $\sim 390$ pulsars for the “PSR B1913+16-like” population, and $\sim 350$ pulsars for the “PSR B1534+12-like” population. Using eq. (4–5), we evaluate the total Galactic coalescence rate of NS–NS binaries for this reference case. The most likely value of the coalescence rate is $R_{\text{peak}} \sim 8\text{ Myr}^{-1}$ and the ranges at different statistical confidence levels are: $\sim 3 - 20\text{ Myr}^{-1}$ at 68%, $\sim 1 - 30\text{ Myr}^{-1}$ at 95%, and $\sim 0.7 - 40\text{ Myr}^{-1}$ at 99%. Also, the most likely values of detection rates, which correspond to $R_{\text{peak}}$ are $\sim 3 \times 10^{-3}\text{yr}^{-1}$ and $\sim 18\text{yr}^{-1}$, for the initial and advanced LIGO.

In Fig. 1, $P(R_{\text{tot}})$ along with $P(R_{1913})$ and $P(R_{1534})$ are shown for the reference model. It is evident that the total rate distribution is dominated by that of PSR B1913+16. This is due to the fact that we calculate the two rate contributions having relaxed the constraint that pulsars have luminosities equal to that of the observed pulsar, and instead allowing for the full range in luminosity. In this case any differences in the two separate rate contributions depend only on differences in pulse periods, and widths. Given that the latter are rather small, it makes sense that, for example, the most likely values of $N_{\text{tot}}$ for the two pulsars come out to be very similar (e.g. $\sim 390$ and $\sim 350$, for PSR B1913+16 and PSR B1534+12, respectively, in the reference model). Consequently any difference in the rate contributions from the two populations is due to the difference in lifetimes (about a factor of 10) for the two observed pulsars (note that the two do not only have similar $N_{\text{tot}}$ estimates, but also similar beaming correction factors). Since the lifetime estimate for PSR B1913+16 is much smaller, the total rate distribution is dominated by its contribution.
We have found that there are strong correlations between the peak value of the total Galactic coalescence rate, $R_{\text{peak}}$, and the cut-off luminosity, $L_{\text{min}}$, and its power index, $p$. As seen in the Fig. 2, $R_{\text{peak}}$ increases rapidly with decreasing $L_{\text{min}}$ (left panel) or with increasing $p$ (right panel). The peak rate does not show any strong dependence on scale lengths of the spatial distribution (either $R_0$ or $Z_0$), except for rather extreme cases (e.g. $R_0 \leq 3$ kpc). Hence, the most important model parameter seems to be the slope and low-end cut-off of the luminosity function. We find that the most likely values for the rates are in the range $\simeq 3-22$ Myr$^{-1}$ for all models with luminosity-function parameters consistent with pulsar observations at 68% confidence level (Cordes & Chernoff 1997).

4. Probability Distribution of $N_{\text{obs}}$ for Parkes Multibeam Survey

4.1. Statistical Method

As mentioned in §2.1, because of the long integration time, the signal-to-noise ratio for the PMB survey is severely reduced by Doppler smearing due to the pulsars’ orbital motion. Acceleration searches in the reanalysis by Faulkner et al. (these proceedings) promise to alleviate this reduction in the near future. Here we calculate the probability distribution of the number of pulsars $N_{\text{obs}}$ that could be detected with the PMB survey, assuming that the reduction in flux due
Figure 2. Left panel: The correlation between $R_{\text{peak}}$ and the cut-off luminosity $L_{\text{min}}$ for different power indices $p$ of the luminosity distribution function. Right panel: The correlation between $R_{\text{peak}}$ and the power index of the luminosity distribution function $p$.

To Doppler smearing is corrected perfectly. Using this distribution $P(N_{\text{obs}})$, we calculate the average value of $N_{\text{obs}}$, $<N_{\text{obs}}>_{\text{PMB}}$, for the PMB survey.

We first calculate $N_{\text{obs}}$ (detected by PMB survey) for a range of total number of pulsars in our model galaxy and determine the slope, $\alpha_{\text{PMB}}$ using eq. (2)(§3.1). We then use $P(N_{\text{tot}})$ (in practice $P(\lambda)$) derived in §3, to calculate $P(N_{\text{obs}})$, for each type of pulsar:

$$P(N_{\text{obs}}) = \int P(N_{\text{obs}}; \lambda_{\text{PMB}})P(\lambda_{\text{PMB}}) d\lambda_{\text{PMB}},$$

where $P(N_{\text{obs}}; \lambda_{\text{PMB}})$ is given in eq. (1). Defining $\beta = \frac{\alpha}{\alpha_{\text{PMB}}}$, $P(\lambda_{\text{PMB}})$ can be derived from $P(N_{\text{tot}})$:

$$P(\lambda_{\text{PMB}}) = \beta^2 \lambda_{\text{PMB}} e^{-\beta \lambda_{\text{PMB}}}.$$

(9)

Since $\alpha$'s are different for each type of pulsar, $\beta$ is determined for each pulsar, separately. The normalized probability distribution of $N_{\text{obs}}$ for each type of pulsar, $P(N_{\text{obs}})$, is then calculated as:

$$P(N_{\text{obs}}) = \frac{\beta^2}{(1 + \beta)^2} \frac{(N_{\text{obs}} + 1)}{(1 + \beta)^{N_{\text{obs}}}}.$$

(10)

Once we have $P(N_{\text{obs}})_{1913}$ and $P(N_{\text{obs}})_{1534}$, we can calculate the combined $P(N_{\text{obs}}_{1913} + N_{\text{obs}}_{1534})$. We define $N_+ = N_{\text{obs}}_{1913} + N_{\text{obs}}_{1534}$ and $N_- = N_{\text{obs}}_{1913} - N_{\text{obs}}_{1534}$, where all variables are integers. The joint probability distribution function for observing either PSR B1913+16-like or PSR B1534+12-like pulsar, $P(N_+, N_-)$ is given by:

$$P(N_+, N_-) = \frac{\beta_1 \beta_2}{2(1 + \beta_1)(1 + \beta_2)} \frac{(N_+ + N_- + 2)(N_+ - N_- + 2)}{(1 + \beta_1)^{(N_+ + N_-)/2}(1 + \beta_2)^{(N_+ - N_-)/2}}.$$

(11)
Finally, we calculate $P(N_+)$ by the summation of $P(N_+, N_-)$ over $N_-$. 

$$P(N_+) = \sum_{N_-} P(N_+, N_-), \quad (12)$$

where $N_-$ lies in the range $[-N_+, N_+]$ with an increment of 2.

4.2. Results

In Fig. 3, we show the distribution function $P(N_+)$ that could be detected by the PMB survey for our reference pulsar population model. We have calculated the distributions and average predicted values, for other models with different cut-off luminosity $L_{\text{min}}$ and the power index of the luminosity function, $p$ (KKL). For 7 different models with luminosity-function parameters within a 68\% confidence level (Cordes & Chernoff 1997), we find values of $< N_+ >$ in the range 1.35–1.5 (1.4 for our reference model). We conclude that, if orbital motion does not affect the signal-to-noise ratio, then the PMB survey could be expected to detect 1-2 new binary pulsars with pulse profile and orbital properties similar to either PSR B1913+16 or PSR B1534+12 in the PMB survey.

5. Discussion

We have recently developed a new method for estimating the total number of pulsars in our Galaxy and have applied it to the calculation of the coalescence rate of double neutron star systems in the Galactic field (for more details see KKL). Here, we extend this method to obtain a prediction for the average number of observed pulsars that the PMB survey could detect when acceleration
Kalogera, Kim & Lorimer searches are used to correct for the Doppler smearing due to orbital motions. The modeling of pulsar survey selection effects is formulated in a “forward” way, by populating the Galaxy with model pulsar populations and calculating the likelihood of the real observed sample. This is in contrast to the “inverse” way of the calculation of scale factors used in previous studies.

We note that this method could be further extended to account for distributions of pulsar populations in pulse periods, widths, and orbital periods. It is important to note that both our rate estimates and the predictions for detections from the PMB survey do not apply to binary pulsars that are significantly different from with such properties that are significantly different PSRs B1913+16 and B1534+12 in terms of pulse shapes and orbital properties.

Most importantly the method can be applied to any type of pulsar population with appropriate modifications of the modeling of survey selection effects. Currently we are working on assessing the contribution of double neutron stars formed in globular clusters as well as the formation rate of binary pulsars with white dwarf companions that are important for gravitational-wave detection by LISA, the space-based interferometer planned by NASA and ESA for the end of this decade.

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