The placement of the head that minimizes online memory: a complex systems approach

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Abstract

It is well known that the length of a syntactic dependency determines its online memory cost. Thus, the problem of the placement of a head and its dependents (complements or modifiers) that minimizes online memory is equivalent to the problem of the minimum linear arrangement of a star tree. However, how that length is translated into a cognitive cost is not known. Here it is shown that the online memory cost is minimized when the head is placed at the center, regardless of the function that transforms length into cost, provided only that this function is strictly monotonically increasing. Online memory defines a quasi-convex adaptive landscape with a single central minimum if the number of elements is odd and two central minima if that number is even. We discuss various aspects of the dynamics of word order of subject (S), verb (V) and object (O) from a complex systems perspective. We suggest that word orders tend to evolve by swapping adjacent constituents from and initial or early SOV configuration that is attracted towards a central word order by online memory minimization. We also suggest that the stability of SVO is to due at least two factors, the quasi-convex shape of the adaptive landscape in the online memory dimension and online memory adaptations to avoid regression to SOV. Although OVS is also optimal for placing the verb at the center,
its low frequency is explained by its long distance to the seminal SOV in the permutation space.

Keywords: word order, head placement, adaptive landscape, neutrality, language dynamics, language evolution

1 Introduction

Word order is a complex phenomenon with different forces pulling in different directions (Langus & Nespor, 2010; Hawkins, 2004). One of these forces is the minimization of the length of a syntactic dependency between a head and its dependents (modifiers or complements). The constraints on word order imposed by this force are able to explain the interpretation of ambiguous sentences (Gibson & Pearlmutter, 1998), sentence comprehension difficulties (Gibson & Pearlmutter, 1998), sentence acceptability (Morrill, 2000), word order preferences (Hawkins, 1994), the exponential decay of the probability of syntactic dependency lengths (Ferrer-i-Cancho, 2004), Greenbergian universals (Ferrer-i-Cancho, 2008b), the low frequency of crossings between syntactic dependencies (Ferrer-i-Cancho, 2006) and the tendency of dependencies to not cover the root of a syntactic dependency structure (Ferrer-i-Cancho, 2008b). Various statistical properties of syntactic dependencies, such as the sublinear scaling of the mean dependency as a function of sentence length (Ferrer-i-Cancho, 2004; Ferrer-i-Cancho & Liu, 2013) and above chance global metrics of dependency lengths (Liu, 2010; Gildea & Temperley, 2010) are consistent with a principle of dependency length minimization. The aim of the present article is to illuminate the complex phenomenon of word order from the perspective of this force, hoping that progress in one dimension helps to solve the whole puzzle.

Various sources suggest that there is a tendency for the verb (V) to be placed between the subject (S) and the object (O). Table 1 indicates that among all the possible orderings of S, V and O, those with V at the center (SVO and OVS) represent 42% of the languages showing a dominant word order. Although the languages with an ordering in which V is put last are slightly more numerous, i.e. 48%,

- the number of languages with V at the center is still significantly large with regard to the null hypothesis that V can go anywhere (Ferrer-i-
Cancho, 2008a). Under this null hypothesis, the verb has a probability 1/3 of being placed first, second or third;

- the most frequent word order by far is SVO (Bentz & Christiansen, 2010) if frequency is measured in number of speakers and not in number of languages as in Table 1.

These statistical facts suggest that central verb placement is an attractor of the dynamics of word order evolution. However, the high frequency of SVO could be simply an accident of history, e.g., be the result of the higher diffusion of a certain culture through imperialism. A stronger support for the hypothesis of central verb placement as an attractor is provided by the direction of word order change which, when it occurs, has been for the most part from SOV to SVO and, beyond that, from SVO to VSO/VOS with a subsequent reversion to SVO occurring occasionally (interestingly, reversion to SOV occurs only via diffusion) (Gell-Mann & Ruhlen, 2011). The idea that SOV evolves towards SVO but evolution in the opposite direction is by far less common is not new (Newmeyer, 2000; Givón, 1979).

Here we aim to shed light on the origins of the attraction of V towards the center as a particular case of the problem of arranging sequentially a head (e.g. a verb) and its complement (a subject or an object). Imagine that there is a head and n dependents (modifiers or complements). For instance, the English phrase “a black cat” is a case of a head, the noun “cat” and two modifiers, i.e. the determiner “a” and the adjective “black”. The head or the modifiers/complements will be referred to as elements (there are thus n + 1 elements). Imagine that the elements are produced sequentially. It will be shown that placing the head at the center minimizes the online memory cost and that dependency length minimization is a consequence of online memory minimization. In particular, this implies that placing the verb between the subject and the object is optimal. By doing so, the claim that evidence of the selective advantage of SVO is missing is challenged (Gell-Mann & Ruhlen, 2011). The goal of this article is not to solve the puzzle of the ordering of S, V and O entirely but rather to shed light on a concrete question of word order dynamics: what force could be responsible for an SOV language becoming SVO? Similarly, why is the current number of SOV languages historically decreasing (Newmeyer, 2000)? Addressing the issue of why SOV is the most frequent word order in languages at present and apparently even more so in ancient times is beyond the scope of this article.
2 The placement of heads that minimizes the online memory expenditure

Imagine that the position of a head and its $n$ dependents in a sequence are specified using natural numbers from 1 to $n + 1$ and that the position of the head is $l$ (thus $l \in [1, n + 1]$). Following these conventions, the phrase “a black cat” has ’a’ at position 1, “black” at position 2 and “cat” at position 3 and $n = 3$. The distance between two elements is defined as the absolute value of the difference between element positions, i.e. the number of intermediate elements plus one (Ferrer-i-Cancho, 2004). For instance, in the example above, ”black” and ”cat” are at distance one, whereas ”a” and ”cat” are at distance two. Similarly, the length of a dependency is defined as the distance between the head and the dependent. There is a long tradition in linguistics and closely related fields of studying the dependency between cognitive cost and the distance between syntactically related items (Hawkins, 1994; Gibson & Pearlmutter, 1998; Morrill, 2000; Grodner & Gibson, 2005; Liu, 2010; Temperley, 2008; Ferrer-i-Cancho, 2008b). The distance between a head and its dependent can be seen as the time that is needed to keep an open or unresolved head-dependent dependency in online memory (Morrill, 2000). Accordingly, $g(d)$ is defined as the online memory cost of a dependency of length $d$ (length is measured in elements). For simplicity, we assume that the online memory cost of a dependency depends only on its length, and thus the only parameter of the online memory cost function is $d$. For instance, this implies that we assume that the cost of a dependency does not depend on whether the head precedes or follows its dependent. We assume that $g(d)$ is a strictly monotonically increasing function of $d \in [1, n]$. For instance, the identity function ($g(d) = d$) has been considered (Ferrer-i-Cancho, 2004; Liu, 2010). We will not work directly on distances but on their implied online memory cost. The total online memory cost of the dependencies between a head placed at position $l$ ($1 \leq l \leq n + 1$) and its $n$ dependents may be defined as the sum of the cost of dependencies with dependents to the left of the head and the sum of the cost of dependencies with dependents to its
right, i.e.

\[ D_l = \sum_{i=1}^{l-1} g(|i - l|) + \sum_{i=l+1}^{n+1} g(|i - l|) \]

\[ = \sum_{i=1}^{l-1} g(l - i) + \sum_{i=l+1}^{n+1} g(i - l), \quad (1) \]

where \(|...|\) is the absolute value operator. Eq. 1 can be rewritten as

\[ D_l = \sum_{d=1}^{l-1} g(d) + \sum_{d=1}^{n+1-l} g(d). \quad (2) \]

Although \(g(0) = 0\) is a reasonable assumption, notice that the definition of \(D_l\) in Eq. 2 does not need it because \(g(d)\) is always invoked satisfying \(1 \leq d \leq n\) there. Assuming \(g(0) = 0\) is neither necessary for the arguments below. When \(g(d) = d\), Eq. 2 yields a polynomial of the second degree, i.e.

\[ D_l = \sum_{d=1}^{l-1} d + \sum_{d=1}^{n+1-l} d \]
\[ = l(l - 1)/2 + (n + 2 - l)(n + 1 - l)/2 \]
\[ = l^2 - (n + 2)l + \frac{1}{2}(n + 1)(n + 2), \quad (3) \]

after some algebra. For instance, when \(n = 3\), the head has four possible placements, cf. Fig. 1. When \(n = 3\) and \(g\) is the identity function, \(D_l\) is maximum when \(D_l\) is placed in the extremes and minimum when it is placed in one of the two middle positions (Eq. 3 gives \(D_l = 6\) for \(l = 1\) and \(l = 4\) and \(D_l = 4\) otherwise).

From a theoretical perspective, the problem of the placement of the head that minimizes \(D_l\) is a particular case the minimum linear arrangement problem for a tree with \(g(d) = d\) (Chung, 1984). In our case, our tree is a star tree of \(N = n + 1\) vertices, being the head the hub of the star tree and the edges the syntactic dependencies between the governor and its dependents. Fig. 1 shows various linear arrangements of star trees. Star trees have maximum degree variance and their linear arrangement cannot have crossings (Ferrer-i-Cancho, 2013). Assuming that \(g(d) = d\), it has been shown that (Ferrer-i-Cancho, 2013)
• Star trees reach the maximum sum of dependency lengths ($D_l$) that a non-crossing tree can reach when the hub is placed first ($l = 1$) or last ($l = N$), i.e.

$$D_l = \frac{N(N - 1)}{2}.$$  \hfill (4)

• In a star tree, $D_l$ is minimized then the hub is placed at the center. If $N$ is odd, then $D_l$ is minimized by $l = (N + 1)/2$ with

$$D_l = \frac{N + 1}{4}.$$  \hfill (5)

If $N$ is even, then $D_l$ is minimized by either $l = N/2$ and $l = N/2 + 1$ with

$$D_l = \frac{N^2}{4(N - 1)}.$$  \hfill (6)

Here we aim to go beyond the common assumption of $g(d) = d$ of research on the minimum linear arrangement problem in computer science (Chung, 1984; Díaz, Petit, & Serna, 2002) and linguistics from a mathematical (Ferrer-i-Cancho, 2004, 2006, 2008b, 2013) or statistical perspective (Ferrer-i-Cancho, 2004; Liu, 2010; Temperley, 2008; Gildea & Temperley, 2010). In her milestone article on optimal linear arrangement of trees, F. Chung proposed to consider $g(d) = d^\delta$ with $\delta$ being a fixed power. Here we consider the case that $g(d)$ is a strictly monotonically increasing function of $d$, which covers the case $\delta > 0$ on star trees.

A sequence of $n + 1$ elements (with $n \geq 1$) has a single central position at $l^* = \lceil n + 1/2 \rceil$ if $n$ is even and two central positions, one at $l^*$ and another at $l^* + 1$ if $n$ is odd. Thus is $l^*$ is a central position of the sequence: the only central position if $n$ is odd and the first central position if $n$ is even. If $n$ is even $l^* = n/2 + 1$ while if $n$ is odd, $l^* = (n + 1)/2$. Hereafter it is assumed that $n \geq 2$ as there are no dependents when $n = 0$ and the placement of the head does not matter when $n = 1$ because $D_1 = D_2$. The following theorem states that the online memory cost is maximum when the head is placed at the extremes of the sequence and minimum at the center for any strictly monotonically increasing online memory cost function (some intuition about the result presented below can be obtained assuming $g(d) = d$ (Ferrer-i-Cancho, 2008b, 2013)).
Theorem 2.1 (Online memory cost of the dependencies) Let \( l^* = \lceil (n+1)/2 \rceil \) be a central position of a sequence of \( n+1 \) elements. If it is assumed that there is a head, \( n \geq 2 \) dependents, and \( g(d) \) is a strictly monotonically increasing function of \( d \) in \([1,n]\), then \( D_l \), the total cost of the dependencies between the head placed at position \( l \in [1,n+1] \subset \mathbb{N} \) and its \( n \) dependents, has

- a minimum for \( l = l^* \) if \( n \) is even;
- two minima for \( l = l^* \) and \( l = l^* + 1 \) if \( n \) is odd;
- two identical maxima for \( l = 1 \) and \( l = n + 1 \).

Besides, \( D_l \) is

- strictly monotonically decreasing for \( l \in [1,l^*] \) and strictly monotonically increasing for \( l \in [l^*,n+1] \) when \( n \) is even;
- strictly monotonically decreasing for \( l \in [1,l^*] \) and strictly monotonically increasing for \( l \in [l^* + 1,n+1] \) when \( n \) is odd.

**Proof** Applying the definition of \( D_l \) in Eq. 2 to \( \Delta_l = D_l - D_{l+1} \), yields

\[
\Delta_l = g(n+1-l) - g(l)
\]

for \( l \in [1,n] \) (and thus notice that \( g(d) \) in Eq. 7 is only applied for values of \( d \) within \([1,n]\) although one could have \( n+1-l = 0 \) or \( l = n+1 \) a priori). Knowing that \( g(d) \) is a strictly monotonically increasing function of \( d \in [1,n] \), Eq. 7 finally gives

a) \( \Delta_l > 0 \) iff \( 1 \leq l < l^* \).

b) \( \Delta_l < 0 \) iff

- \( l^* < l \leq n \) if \( n \) is odd
- \( l^* \leq l \leq n \) if \( n \) is even

c) \( \Delta_l = 0 \) iff \( n \) odd and \( l = l^* \).
First, assume that \( n \) is even. Then \( D_l \) is strictly monotonically decreasing for \( l \in [1, l^*] \) and strictly monotonically increasing for \( l \in [l^*, n + 1] \). Therefore, \( D_l \) has a single minimum at \( l = l^* \). Second, assume that \( n \) is odd. Then, \( D_l \) is strictly monotonically decreasing for \( l \in [1, l^*] \) and strictly monotonically increasing for \( l \in [l^* + 1, n + 1] \). Therefore, recalling c), i.e. \( \Delta_l = 0 \), it follows that \( D_l \) has two minima at \( l = l^* \) and \( l = l^* + 1 \).

Concerning the maxima, notice that \( D_l \) is a symmetric function of \( l \), i.e. \( D_l = D_{n+2-l} \), by its definition in Eq. 2. The shrinking and growth behaviour of \( D_l \) described above \( D_l \) has a maximum for \( l = 1 \) and another for \( l = n + 1 \). These two maxima are identical due to the symmetry of \( D_l \).

Theorem 2.1 indicates that \(-D_l\) is a unimodal function if \( n \) is even (Avriel, Diewert, Schaible, & Zhang, 1988, pp.63) and bimodal but almost unimodal if \( n \) is odd as the two modes are adjacent in that case. It will be shown next that \( D_l \) is a quasi-convex function. Quasi-convexity is a generalization of convexity (Avriel et al., 1988; H. J. Greenberg & Pierskalla, 1977). Quasi-convex functions can be optimized within a reasonable computation cost (Kiwiel, 2001).

**Corollary 2.2 (Quasi-convexity of the online memory cost)** If there are \( n \geq 2 \) dependents and \( g(d) \) is a strictly monotonically increasing function of \( d \), then \( D_l \) is quasi-convex within \([1, n + 1]\), i.e. for any \( l_1, l_2, \) and \( l_3 \) such that \( 1 \leq l_1 \leq l_2 \leq l_3 \leq n + 1 \) one has that

\[
D_{l_2} \leq \max(D_{l_1}, D_{l_3}).
\]  

**Proof** The condition defined in Eq. 8 is equivalent to (a) \( D_{l_2} \leq D_{l_1} \) or (b) \( D_{l_2} \leq D_{l_3} \). If \( l_2 \leq l^* \), Theorem 2.1 gives (a) while if \( l_2 \geq l^* \), Theorem 2.1 gives (b).

## 3 Discussion

It has been shown that placing a head at the center minimizes the online memory expenditure. If the number of elements is odd then there is a single minimum while if the number of elements is odd there are indeed two central minima. There are two novelties in our analysis. First, our approach which abstracts away from the particular form of the function that translates the distance between a head and its complements into a cognitive cost. The
point is subtle: even if one considers that online memory cost originates from the time that is needed to keep an open or unresolved dependency between a head an a dependent in online memory (Morrill, 2000), it is still not known how this time translates into an energy cost for the brain and if this final translation depends on the language. Second, the form of the adaptive landscape (Wright, 1932) defined by online memory cost has been unraveled. The landscape is quasi-convex as illustrated by the example of Figure 2. In that figure, it has been assumed that the length of a dependency and its cost are the same. Interestingly, Theorem 2.1 indicates that Figure 2 is representative of the form of the landscape in spite of being for a particular online memory cost function, because the landscape remains quasi-convex for any strictly monotonically increasing positive function that maps length onto cost. Therefore the landscape depicted in Fig. 2 would be the kind of landscape in which word order dynamics would operate on the online-memory dimension. The dynamics of the ordering of S, V and O would have at least three fundamental ingredients:

1. SOV as an initial or early word order configuration (Goldin-Meadow, So, Özyürek, & Mylander, 2008; Sandler, Meir, Padden, & Aronoff, 2005; Newmeyer, 2000),

2. An online memory landscape that would drag the verb from the final position found in SOV towards the center eventually leading to the SVO order.

3. Adaptations to increase the stability of a given order (e.g., adaptations in SVO to prevent regression to SOV).

Theorem 2.1 sheds new light on the transition from SOV to SVO (Newmeyer, 2000; Givón, 1979), which is a transition from maximum to minimum online memory cost. Future work should consider the inclusion of more dimensions, such as other sources of cognitive cost beyond dependency length. An important question for further research is determining whether verb last or central verb are stable or unstable attractors of word order evolution and what role factors other than dependency length have in determining such stability or instability.

The return to SVO witnessed occasionally during word order evolution and the difficulty of reversion to SOV except via diffusion (Gell-Mann & Ruhlen, 2011) suggests some degree of stability for central verbs. The shape
of the adaptive landscape of online memory adds new theoretical support for the stability of central verbs: a language tending to place the verb at the center will receive an increasing penalty as the verb is displaced to the beginning or end due to the quasi-convexity of that shape.

If there is a force that explains why SOV is initially selected, why SVO languages do not return to SOV easily because of that force, despite the attraction for the verb at the center due to online memory minimization? A possible explanation is that evolutionary successful SVO orders may have incorporated adaptations to hinder regression to SOV (Appendix). Indeed, further theoretical support for the stability of SVO comes from mathematical word order theory, which addresses the issue of the optimal placement of dependents relative to their heads within S, V and O according to online memory minimization (Ferrer-i-Cancho, 2008b). For a language that tends to put the verb last, e.g., a SOV language, the optimal solution in terms of online memory minimization for top level dependencies is placing dependents before the nominal head whereas by symmetry, for a language that tends to put the verb first, the optimal solution is placing the dependents after the nominal head (Appendix). Interestingly, if a language tends to put the verb at the center, whether dependents are placed to the left or to the right, is almost irrelevant in terms of online memory minimization of top level dependencies (Appendix). From an evolutionary perspective, a change in the relative placement of dependents of nominal heads is practically neutral for SVO but not for SOV. From the perspective that orders are competing and adapting to survive (Ferrer-i-Cancho, 2008b; Gong, Minnet, & Wang, 2009), it is not surprising that stable SVO languages put adjectives after the head (Ferrer-i-Cancho, 2008b), as SOV prefers the opposite, namely that dependents are located before the nominal head. Besides word order, there might be other kind of factors difficultéing a return to SOV. One possibility is case marking: case marking facilitates the learning of SOV structures (Lupyan & Christiansen, 2002). Thus, regression to SOV would be harder from SVO languages lacking case marking.

We hypothesize that online memory minimization is a universal principle of language. It is important not to be side-tracked by seemingly contradictory evidence. The fact that a certain language does not show SVO as dominant word order does not mean that language does not suffer pressure for online memory minimization:

- It is well known that SVO is an alternative word order in languages
where SVO is not the dominant word order (J. H. Greenberg, 1963). This has a simple explanation. A priori, placing the verb at the center is optimal in terms of online memory minimization; placing it somewhere else is not (Theorem 2.1). Therefore, a language that does not have a dominant word order with the verb at the center can compensate the cost of that non-optimal verb placement by adopting an alternative word order that is optimal, e.g., SVO.

The abundance of verb last orders does not contradict the principle of online memory cost minimization. First, the diversity of orders of S, V and O (Dryer, 2011) could result from various principles acting simultaneously (Langus & Nespor, 2010; Hawkins, 2004). Second, the relative position of adjectives and verbal auxiliaries in verb last orders can be explained in terms of online memory cost minimization (Ferrer-i-Cancho, 2008b). Thus, the fact that a language has SOV as dominant does not mean that online memory cost minimization is inactive.

It is tempting to think that if the hypothesis of an attraction towards the verb at the center is correct, then OVS, the other ordering with V at the center, should have high frequency and the transition from SOV to OVS should be as frequent as the transition from SOV to SVO. However, the failure of these predictions on OVS can be understood easily with the help of the permutation space, which was originally introduced to explain the low frequency of OVS with regard to SVO (Ferrer-i-Cancho, 2008b). If one assumes that word order evolves by swapping consecutive elements, the evolution from an initial or early word order SOV to SVO requires only one step: exchanging the positions of O and V. In contrast, the transition from SOV to OVS needs two steps (1) exchanging the position of S and O to obtain OSV and from it, (2) exchanging S and V to obtain finally OVS (Fig. 3 (a)). Therefore, the word order with V at the center that can be reached sooner from the initial or early SOV is SVO. SVO is the easiest way of minimizing online memory from SOV by swapping adjacent constituents. While it has been argued that the disproportion between the abundance of SVO (488 languages) and OVS (11) languages could be due to an arbitrary break of the symmetry between orders placing the verb at the center (Ferrer-i-Cancho, 2008b), we argue here that this is more likely to be caused by the proximity of SOV, an attractor of word order evolution at early stages. For simplicity, we have assumed that word order evolution proceeds by exchanging consecutive elements but it could be argued the exchange distant elements has an important role in
word order evolution. However, that is expected to be cognitively more expensive and therefore less likely (consider all the arguments supporting online memory minimization reviewed above). The path of the evolution of word order, i.e. SOV, SVO and VSO/VOS (Gell-Mann & Ruhlen, 2011) is consistent with a traversal of the permutation space essentially by swapping adjacent constituents. It could be argued that once SVO is reached, the emergence of its reverse, OVS should be easier. However, notice that OVS is the farthest order in the permutation ring depicted in Fig. 3 (a): at least three permutations of adjacent constituents are needed to reach OVS from SVO. Interestingly, the abundance of languages showing a certain dominant word order X is perfectly correlated with the number of swaps needed to reach X from SOV (Fig. 3 (b)): the number of languages always decreases as the number of those swaps increases in a clockwise sense in the permutation ring of Fig. 3 (a). Thus the Spearman rank correlation (Conover, 1999) is $\rho = -1$ and the p-value of a two-sided test is $\frac{2}{6!} \approx 0.003$ as all the abundances of dominant orders and the number of swaps do not have repeated values (Ferrer-i-Cancho & Hernández-Fernández, 2013)). Besides the shape of the permutation space and the clockwise sense, other factors could explain why OVS is rarely reached, such as a preference for S first that is suggested by the very high frequency of SOV and SVO together ($S$ first is found in 88% of languages showing a dominant word order (Dryer, 2011)).

Here we have considered the problem of the optimal linear arrangement of dependents with regard to their head but have focused on the particular case of the verb, subject and object. As our argument is abstract, one expects that it is also valid for the other heads and their dependents. For instance, a challenge is that SVO languages tend to have the adjective before the noun (Ferrer-i-Cancho, 2008b). The fact that no language consistently splits its noun phrases about a central, nominal pivot, with half of the modifiers to the left and half to the right might be seen as evidence that online memory minimization has a very restricted scope. However, online memory minimization is still a fundamental principle even for nominal heads. The point is that the optimal placement of modifiers around a noun in terms of online memory minimization is not independent from (a) the dominant placement of the verb governing the noun in a word order a $X$ as we have discussed above (b) is not necessarily independent from the dominant word order of a competitor $Y$. Consider that $X$ is SVO. Then the permutation ring in Fig. 3 (a) indicates that $Y$ could be SOV as it is easy to move from SVO to SOV. From the perspective of online memory minimization, SVO may put half of
the modifiers to the left and half of the modifiers to the right of the noun but as the placement of modifiers before the nominal head is optimal in SOV, we have seen that SVO languages can maximize the costs for SOV by putting modifiers in the opposite side (Ferrer-i-Cancho, 2008b). A non-reductionistic approach to word order needs at least that the interaction between the placement of heads of different levels and the interaction between competing word orders at the same level is taken into account.

A basin of attraction consists of an attractor and all the trajectories leading to this attractor (Wuensche, 2000). Fig. 2 defines the trajectories on their impact on online memory cost when moving the head one position forward or backward and thus defines the basin of attraction of word order evolution on the single dimension of online memory cost. Fig. 3 (a) defines the trajectories that can be followed a priori by swapping consecutive constituents till SVO is reached. Looking at the neighbours of SOV in the permutation ring, one notices that SVO (488 languages) is much more frequent than OSV (4 languages). This suggests that the bulk of the trajectories that dominant word order evolution describes might be more accurately described by a directed graph version of Fig. 3 (a) where the link direction is clockwise. Fig. 3 (b) shows (from left to right) the linear directed tree that results from a traversal of the ring of Fig. 3 (b) starting from SOV, moving clockwise and stopping when all word orders have been visited. The low frequency of OSV languages and the correlation between word order abundance and distance to SOV provides further support for the idea that word order evolution is essentially unidirectional (Gell-Mann & Ruhlen, 2011), a feature that is reminiscent of the unidirectionality of grammaticalization processes (Traugott & Heine, 1991). However, that directed graph with a ring backbone is still an incomplete draft of the possible trajectories under the influence (at least) of online memory minimization. Contingency (initial or early preference for SOV) would favour SVO over OVS. SOV could also be an attractor of the dynamics whose origin, from a dynamical and mathematical point of view, is still not well-understood.

In order to develop a deep understanding of language and word order in particular, it is important not to mistake a principle with an explanation. The fact that online memory minimization does not explain the abundance of SOV order does mean that it is not involved in explaining why and how it appears or why and how it is maintained. If a language leaves SOV, it is well known that SVO is a dominant direction of change and although SVO can be abandoned towards VSO/VOS, reversion to SVO occurs occasionally.
while reversion to SOV is not expected unless via diffusion, which is a secondary process in word order evolution (Gell-Mann & Ruhlen, 2011). It is perhaps here that the power of our mathematical results is revealed. Two languages could even have a different function for transforming the length of a dependency into a cognitive cost, but provided that the transformation is monotonically increasing, both will at least be attracted towards central head placement. Furthermore, the form of the adaptive landscape would reinforce the attraction towards a central placement of the verb (Fig. 2). Here online memory minimization is not regarded as the only principle nor the most important one. Language design is a multiconstraint engineering problem (Evans & Levinson, 2009) and therefore that principle would be one of the constraints to meet during word order evolution. Here our analysis has simple focused on one dimension of the problem.

A look through the eye of physics can help clarify our notion of principle. The force of gravity explains why objects fall, but when a rocket flies in the opposite direction of the force, one does not say that its movement constitutes an exception to the force. The force is still acting and is involved in explaining, for instance, the amount of fuel that is needed to fly in the opposite direction of the force. While the falling of an object is a manifestation of the force of gravity, central head placements or the movement of a head towards the center (perhaps not reaching the optimal center) are a manifestation of a principle of online memory minimization. As the force of gravity is still acting on a rocket flying in the opposite direction of that force, one should not conclude prematurely that online memory minimization is not acting upon a language when the head is not placed in the middle, i.e. the optimal position according to online memory minimization. Online memory minimization still determines the relative placement of adjectives and verbal auxiliaries in SOV languages (Ferrer-i-Cancho, 2008b). Indeed, although the Earth and Venus are very different planets, the force of gravity is valid in both (indeed universal) and physicists only care about the variation in its magnitude. Similarly, all languages can have online memory minimization in common, no matter how large the genetic, historical, typological or other differences are among the languages or among their speakers. We believe that a more physical point of view is needed for progress in our understanding of word order evolution. It is not true that the placing the verb at the center does not confer any selective advantage to a language that adopts it (Gell-Mann & Ruhlen, 2011). Online memory minimization could underlie the transition from SOV to SVO (Newmeyer, 2000; Givón, 1979). However,
understanding the tendency of subjects to precede objects (Cysouw, 2008) and how other principles can determine another verb placement such as the verb last placement that is found in the very frequent SOV are challenges for future research.

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Appendix: the optimal placement of dependents of nominal heads

Assuming that \( g(d) = d \), a mathematical theory for the ordering of three top level constituents: subject (S), object (O) and verb (V) was developed (Ferrer-i-Cancho, 2008b). \( |x| \) is defined as the length in words of constituent \( x \) (\( x \in \{S, V, 0\} \)). We assume that the constituents are not empty (\( |O|, |S|, |V| \geq 1 \)) and their lengths are constant (word order variations do not alter \( |O|, |S| \) or \( |V| \)). \( \delta_{x \sim x'}^{y} \) is defined as the length of the dependency between the head of constituents \( x \) and \( x' \) in word order \( y \) (\( y \in \{SOV, SVO, \ldots\} \)).

\[
\delta^{y} = \delta_{V \sim S}^{y} + \delta_{V \sim O}^{y}
\]

is thus the sum of the length of the dependencies (in words) between the head of the verb and subordinated heads of the subject and the object for a word order \( y \). The total cost of a sentence consisting of \( S, V \) and \( O \) following and order \( y \) is

\[
\Omega^{y} = \omega_{S}^{y} + \omega_{V}^{y} + \omega_{O}^{y} + \delta^{y},
\]

where \( \omega_{x}^{y} \) is the sum of the sum of internal dependency lengths of constituent \( x \) in word order \( y \).

\( L_{x}^{y} \) and \( R_{x}^{y} \) are defined, respectively, as the number of words to the left or to the right of the head word of constituent \( x \) (e.g., \( x \in \{S, V, 0\} \)) in word order \( y \) and thus \( |x| = L_{x}^{y} + 1 + R_{x}^{y} \). Assuming continuity, one has that (Ferrer-i-Cancho, 2008b)

\[
\delta^{SOV} = 2L_{V}^{SOV} + 2R_{O}^{SOV} + L_{O}^{SOV} + R_{S}^{SOV} + 3
\]

and

\[
\delta^{SVO} = R_{S}^{SOV} + |V| + L_{O}^{SOV} + 1.
\]

For simplicity, the original mathematical theory (Ferrer-i-Cancho, 2008b) focused on the problem of the optimal placement of dependents of nominal heads to minimize the sum of lengths of dependencies defined by the top level constituents (external dependencies), i.e. \( \delta^{y} \), thus neglecting the cost of dependencies formed by words within a given constituent, i.e. \( \omega_{S}^{y}, \omega_{V}^{y} \) and \( \omega_{O}^{y} \). As it will be shown next, this is equivalent to assuming conservation
for the lengths implied by other dependencies (internal dependencies) when varying the relative ordering of those top level constituents.

\( \delta_{y,left} \) and \( \delta_{y,right} \) are defined as the value of \( \delta_y \) implied by placing all the dependents of the nominal head before or after the noun, respectively. \( \omega_{x,left} \) and \( \omega_{x,right} \) are defined as the total sum of the internal dependencies of constituent \( x \) in word order \( y \) when dependents of nominal heads precede or follow the head, respectively. \( \Omega_{y,left} \) and \( \Omega_{y,right} \) are defined similarly for the total sum. Thus we have that

\[
\Omega_{y,left} = \omega_{y,left}^S + \omega_{y,left}^V + \omega_{y,left}^O + \delta_{y,left},
\]

\[
\Omega_{y,right} = \omega_{y,right}^S + \omega_{y,right}^V + \omega_{y,right}^O + \delta_{y,right},
\]

(12)

(13)

The conservation of the total sum of internal dependency lengths when varying the relative ordering of the dependents of nominal heads means that

\[
\omega_{y,left}^S + \omega_{y,left}^V + \omega_{y,left}^O = \omega_{y,right}^S + \omega_{y,right}^V + \omega_{y,right}^O
\]

(14)

or equivalently, that

\[
\Omega_{y,left} - \Omega_{y,right} = \delta_{y,left} - \delta_{y,right},
\]

(15)

i.e. conservation means that the difference in total cost between different relative placements depends only on the sum of dependency lengths of top level constituents. Investigating the extend to which this conservation is valid for human language and the consequences of violations of such conservation for our theoretical arguments is left for future research.

Next the sum of dependency lengths as a function of the relative placement of dependents of nominal heads for a given word order will be investigated. Notice that the condition \( \Omega_{y,left} < \Omega_{y,right} \) is equivalent to \( \delta_{y,left} < \delta_{y,right} \) under conservation (Eq. 15). Let us consider SOV first. For placements of dependents before their nominal head, one has \( R_{O,SOV,left} = 0 \), \( L_{O,SOV,left} = |O| - 1 \) and \( R_{S,SOV,left} = 0 \), which transforms Eq. 10 into

\[
\delta_{SOV,left} = 2L_{V,SOV,left} + |O| + 2.
\]

For placements after their nominal head, one has \( R_{O,SOV,right} = |O| - 1 \), \( L_{O,SOV,right} = 0 \) and \( R_{S,SOV,right} = |S| - 1 \), which transforms Eq. 10 into

\[
\delta_{SOV,right} = 2L_{V,SOV,right} + 2|O| + |S|.
\]
Assuming $L_{V}^{SOV,left} = L_{V}^{SOV,right}$ (the relative ordering of dependents of nominal heads should not alter the relative ordering of dependents of verbal heads), Eqs. 16 and 17 transform the condition $\delta^{SOV,left} < \delta^{SOV,right}$ into

$$|S| + |O| \geq 2$$

(18)

with equality if and only if $|S| = |O| = 1$ as $|S|, |O| \geq 1$. Therefore, placing the dependents before their nominal head is always advantageous in terms of top level dependency lengths for SOV (the case $|S| = |O| = 1$ does not bother in the last statement as it means that the nominal heads do not have dependents).

Let us consider SVO now. For placements of dependents before their nominal head, one has $R_{S}^{SVO,left} = 0$ and $L_{O}^{SVO,left} = |O| - 1$, which transforms Eq. 11 into

$$\delta^{SVO,left} = |V| + |O|.$$  

(19)

For placements after the nominal head, one has $R_{S}^{SVO,right} = |S| - 1$ and $L_{O}^{SVO,right} = 0$, which transforms Eq. 11 into

$$\delta^{SVO,right} = |V| + |S|.$$  

(20)

Eqs. 19 and 20 transform the condition $\delta^{SVO,left} < \delta^{SVO,right}$ into

$$|O| < |S|.$$  

(21)

This result indicates that the placement of modifiers with regard to the nominal head is practically irrelevant for SVO with regard to SOV. If one assumes that the object and the subject have the same length ($|O| = |S|$) then total cost of SVO does not depend on the relative ordering of dependents of nominal heads.

Next a mathematical argument on the cost of regression from SVO to SOV depending on the relative placement of dependents of nominal heads will be developed. By definition, the cost of an SVO order that places those dependents before their nominal head is

$$\Omega^{SVO} = \omega_{S}^{SVO,left} + \omega_{V}^{SVO,left} + \omega_{O}^{SVO,left} + \delta^{SVO,left}$$  

(22)

while that of an SVO order placing them before is

$$\Omega^{SVO} = \omega_{S}^{SVO,right} + \omega_{V}^{SVO,right} + \omega_{O}^{SVO,right} + \delta^{SVO,right}.$$  

(23)
Imagine the that one of those SVO orders is transformed into SOV simply by reordering the top level constituents (keeping their internal organization constant so that $\omega_{S}^{SVO, left}$, $\omega_{V}^{SVO, left}$, $\omega_{O}^{SVO, left}$ are not altered; this could be simply due to least effort). Eq. 22 implies that the cost of the SOV order placing those dependents before their head is

$$\Omega'_{SOV} = \omega_{S}^{SVO, left} + \omega_{V}^{SVO, left} + \omega_{O}^{SVO, left} + \delta^{SOV, left}$$

while that of the SOV order placing them before is

$$\Omega''_{SOV} = \omega_{S}^{SVO, right} + \omega_{V}^{SVO, right} + \omega_{O}^{SVO, right} + \delta^{SOV, right}$$

Thus, the condition $\Omega'_{SOV} < \Omega''_{SOV}$ reduces to $\delta^{SOV, left} < \delta^{SOV, right}$, which has been proven above to require only that $|S| \neq |O|$ to be true (recall Eq. 18 and that that $S$ and $O$ cannot be empty by definition). Thus, regression to SOV from SVO is more expensive from the perspective of SOV when dependents of nominal heads are placed last. One could argue that we are oversimplifying the problem when not considering the case of a reordering of constituents that includes an internal reordering of the constituents but this internal reordering increases the cost of regression to SOV (speakers and hearers may find the larger number of word order changes harder to produce or understand) and thus it is less likely. Generalizing the analysis above to include the case $g(d) \neq d$ is left for future research.
Table 1: The frequency of the placement of the verb in the ordering of the subject (S), verb (V) and object (O) in world languages showing a dominant word order. There are three possible placements: 1 for verb initial orderings (VSO and VOS), 2 for central verb placements (SVO and its reverse) and 3 for verb final orderings (SOV and OSV). Absolute frequencies are borrowed from Dryer (2011).

| Verb placement | Frequency (in languages) | Percentage |
|----------------|-------------------------|------------|
| 1              | 120                     | 10.1       |
| 2              | 499                     | 42.0       |
| 3              | 569                     | 47.9       |
| Total          | 1188                    |            |

Figure 1: All the four possible placements of a head with \( n = 3 \) dependents. Black filled circles are elements and edges indicate syntactic dependencies. \( l \) indicates the position of the head in the sequence of elements. It can be seen that the linear arrangements of the top row are symmetric to those of the bottom row.
Figure 2: $D_l$, the total online memory cost of placing a head and $n = 10$ dependents as a function of $l$, the position of the head. Eq. 3 is used to compute $D_l$. The placement of the head first or last yields maximum cost while at the center is minimum.
Figure 3: (a) Network of the possible trajectories between the six possible orderings of S, V and O. Two orderings are connected if one can lead to the other by swapping a pair of adjacent constituents (adapted from Ferrer-i-Cancho (2008b)). The network shows a ring structure. (b) The linear network of trajectories from an initial SOV order to other orders that results by swapping adjacent constituents following the permutation ring in a clockwise sense. Numbers indicate the number of languages having a certain word order as dominant (numbers borrowed from Dryer (2011)).