Numerical study of supersonic flow over axisymmetric bodies with combined supply of energy into incoming undisturbed flow

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Abstract. The development of supersonic aircrafts should rely on advanced methods of flow control in order to improve the efficiency and reliability of the aircrafts. This article presents numerical simulation of energy supply to the incoming flow and simulation of the flow over bodies with hemispherical and conical nose parts. The numerical study was based on the model of an inviscid perfect gas described by the Euler equations which were solved by using the explicit unsteady Godunov’s numerical scheme with the second-order accuracy.

1. Introduction

The development of supersonic aircrafts requires new schemes and improvement of existing techniques of flow control near the aircraft surface. The goal is to increase the efficiency and reliability of the flow control technique.

A promising way of improvement of aerodynamic characteristics of aircrafts in a wide range of speeds, especially in the case of restrictions on the shapes of elements, is a control of flow, i.e. a purposeful impact on the air flow with the aim of achieving a required change in aerodynamic characteristics. An attractive method of impact on the air flow is the power (thermal) method, i.e. a supply of energy of different natures (chemical, electromagnetic, energy of laser or microwave radiations, kinetic) into the incoming flow [1] with its subsequent transformation into the thermal one.

Because of high power and thermal loads on the aeroballistic objects flying with high speeds in the atmosphere, of high importance is the task of stabilization of their flight aimed at increasing the static stability and decreasing the drag force.

2. Mathematical model

This article presents some results of our numerical research of flow over bodies with a hemispherical and conical nose parts. To organize a power supply ($Q$), a small body of a spherical shape (a sphere with a diameter $d = 0.1D$, where $D$ is the diameter of the hemispherical or conical part of the body) was located coaxially in an undisturbed flow in front of a streamlined body at distance $l$ (Figure 1).
Figure 1. Types of axisymmetric bodies. (a) spherical nose part, (b) conical nose part

The model of nonviscous perfect gas (heat capacity ratio $k = 1.4$) described by Euler's equations in the axisymmetric form (1-5) was used for the numerical simulation [4-7]:

$$\frac{\partial \sigma}{\partial t} + \frac{\partial a}{\partial x} + \frac{\partial b}{\partial r} = - \frac{1}{y} f_0 + f_1,$$

(1)

where

$$\sigma = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{bmatrix}, \quad a = \begin{bmatrix} \rho u \\ p + \rho u^2 \\ \rho uv \\ (e + p)u \end{bmatrix}, \quad b = \begin{bmatrix} \rho v \\ p + \rho v^2 \\ \rho uv \\ (e + p)v \end{bmatrix},$$

(2)

$$f_0 = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 \\ (e + p)v \end{bmatrix}, \quad f_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ X \end{bmatrix},$$

(3)

$$e = \rho \left( \varepsilon + \frac{u^2 + v^2}{2} \right),$$

(4)

$$X = Q$$

(5)

Here $\rho$ is the density, $p$ is the pressure, $\varepsilon$ is the specific internal energy, $u, v$ are the components of the velocity vector, and $Q$ is the power source near the sphere surface (power supply).

The obvious non-stationary second-order Godunov's numerical scheme was used to solve this system.

The calculations were carried out on an oblique-angled grid having refinements in the areas with high gradients of gasdynamic parameters (Figure 2) [10]. In view of an essential non-uniformity of the mesh and severe restrictions on the time step, in order to accelerate the process of solution, the time step individual for each cell was used in the calculation. $D, l$ and $Q$ were the varied parameters in the calculations. The supply of energy was carried out in the cells adjacent to the small sphere surface. At
the same time, the power input depended on the gas density in the particular cell. The power input approximately corresponded to the energy release due to hydrogen combustion.

Figure 2. Mesh in computational domain. (a) spherical body, (b) conical body

The calculations of the flow over the body with a hemispherical nose part were performed with the following values of the parameters: \( l = D \), \( d = 0.1D \) and \( Q = 0.04q \) (where \( q \) is the dynamic pressure in the nondisturbed flow).

The calculations of the flow over the conical body with length \( L/D = 1.5 \) was investigated in a wide range of parameters: \( l/D = 0.5 – 1.5; \) \( Q = 0.03q \) and \( 0.06q \); \( d = (0.05 \) and \( 0.1)D \).

In the next section the analysis of the calculation results for the flow over these bodies at the incoming Mach number \( M_{∞} = 5 \) and flow conditions corresponding to the height \( H_{∞} = 50 \text{ km} \) are given.

3. Results of numerical simulations

**Body with a hemispherical nose part**

The drag coefficients and relative decrease in the drag force of the body with a hemispherical nose part for the cases (i) when there was only a hemispherical body in the flow, (ii) when there were a hemispherical body and a small sphere in front of it in the incoming flow, and (iii) when there were a hemispherical body, a small sphere in front of it and an energy supply on the small sphere surface (Figure 3) are given in Table 1.

| Case                          | \( c_d \) | \( \Delta c_d / c_{d0} \) % |
|-------------------------------|----------|-----------------------------|
| Hemisphere                    | 0.9      | 0                           |
| Hemisphere + sphere           | 0.3487   | 60                          |
| Hemisphere + sphere + power supply | 0.2891 | 68                           |

It can be seen from Table 1 that the presence of the sphere in the undisturbed incoming flow in front of the streamlined body reduces the drag coefficient by 60%. This decrease in the drag is caused by the same factors, as for the case of flow over bodies with spikes and a power supply in the undisturbed incoming flow investigated by the authors earlier. These factors, which have become already classical, are the reorganization of shock-wave structure and formation of a circulating zone in front of a body. The mechanism of such shock-wave structure reorganization is well known and has been described in many publications in the last decades [2, 3].
Figure 3. Distribution of pressure near the body with a hemispherical nose part.
(a) without sphere in incoming flow and power supply, (b) with sphere in incoming flow, (c) with sphere in incoming flow and power supply from its surface.

Figure 3 presents the distribution of pressure near the body with a hemispherical nose part with and without the small sphere in the incoming flow. It can be seen from figure 3 that the shock-wave structure, typical of the cases listed above, is formed. However, it should be noted that there is a bigger decrease in drag as compared with the flow over a hemisphere with a spike [8, 9]. This can be explained by the fact that the small sphere is not connected with the hemisphere and, hence, the drags of the sphere and the hemisphere are not summed up in contrast to the hemisphere with spike which is an integral whole.

The energy supply to the flow from the small sphere surface does not lead to significant changes in the structure of the flow, which is evident from Figure 3(c). However, the hemisphere drag coefficient in this case is 17% less than for a case of absence of a power supply. It can be explained by changes in the shape and intensity of the head shock-wave and an increase in the extent of the circulating zone. The change in the shape of the head shock-wave (an increase in the angles of its inclination causing a bigger turn of the flow behind it) leads to an essential reduction in the intensity of bow shock-wave and, as a result, reduction of pressure in the front part of the hemisphere. Nevertheless, this pressure decrease does not lead to a reduction in the back-flow and circulating zone (which is observed in the cases of bodies with spikes) as the power supply on a surface of a sphere causes increase in the cross sizes of the trace after the small sphere with power supply on its surface.

Body with a conical nose part

The study of the flow over the body with a conical nose part has shown that the placement of a sphere in the undisturbed incoming flow and a power supply from its surface, like in the case considered above, leads to a decrease in the drag coefficient. The results are presented in Table 2.

| l/D | Drag coefficient of the body, cd | Drag coefficient of the body and small sphere, cd_s | Drag coefficient of the body, small sphere and power supply, cd_s+ps | Power supply (Ps) |
|-----|---------------------------------|---------------------------------|-------------------------------------------------------------|-----------------|
| 0.5 | 0.225                           | 0.2080                          | 0.1860                                                      | O = 0.03q       |
| 0.7 | 0.225                           | 0.2070                          | 0.1859                                                      | O = 0.03q       |
| 1.0 | 0.225                           | 0.2068                          | 0.1850                                                      | O = 0.03q       |
| 1.5 | 0.225                           | 0.2098                          | 0.1880                                                      | O = 0.03q       |
| 1.5 | 0.225                           | 0.2098                          | 0.1738                                                      | O = 0.06q       |
It is apparent from Table 2 that the placement of a small sphere in the undisturbed incoming flow leads to a decrease in the drag force of the cone by 8%. The change in the distance between the sphere and the cone tip in the $l=0.5–1.0$ range affects only slightly the drag coefficient. The power supply from the surface of the sphere gives a 17% decrease in the drag. At $l > 1$ the drag increases as compared with the case $l < 1$. The increase in the power supply to $0.06q$ in the latter case provides a decrease in the drag coefficient of the cone by 22%.

![Figure 4](image)

**Figure 4.** Distribution of pressure and density near the body with a conical nose part. (a) and (b) – small sphere, (c) and (d) – small sphere and a power supply.

The flow over the cone with the sphere located in front of it is characterized by the absence of a forward circulating zone, which is visible in Figure 4. The power supply from the surface of the sphere, like in case of the flow over the hemispherical body, leads to changes in the shape and intensity of the head shock-wave, and an increase in the cross section of the zone with a lower density (Figure 4 (b) and (d)) and also this leads to the shift of attached shock which is formed on a cone. The latter leads to a pressure decrease in the leading part of the cone adjoining its tip. A reduction in the diameter of the sphere leads to a reduction in the effects of the sphere impact on the cone drag. So for the $d = 0.05D$ case, the decrease in the drag coefficient of the cone was insignificant.

### 4. Conclusion

The method of change on undisturbed flow before the body by the placing power source in it as described above can be promising way for the solution of the problem of decreasing the body drag with a good aerodynamic characteristics. However, more extensive experimental and theoretical studies are needed to confirm this conclusion.

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