Spin-orbit-driven coherent oscillations in a few-electron quantum dot

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(Dated: March 23, 2022)

We propose an experiment to observe coherent oscillations in a single quantum dot with the oscillations driven by spin–orbit interaction. This is achieved without spin-polarised leads, and relies on changing the strength of the spin–orbit coupling via an applied gate pulse. We derive an effective model of this system which is formally equivalent to the Jaynes–Cummings model of quantum optics. For parameters relevant to a InGaAs dot, we calculate a Rabi frequency of 2 GHz.

PACS numbers: 73.63.Kv,71.70.Ej,03.65.-w

Motivated by the desire for a closer understanding of quantum coherence and by the drive to develop novel quantum computing architecture, a number of break-through solid-state experiments have focused on coherent oscillations — the back and forth flopping of that most fundamental of quantum objects, the two-level system [1,2,3,4]. The pioneering work of Nakamura et al. with the coherent superposition of charge states of a Cooper-pair box [1] first demonstrated the possibility of observing such oscillations in a wholly solid-state device; thus sparking the remarkable progress in qubit development in super-conducting systems [2,3].

The important double quantum dot experiment of Hayashi and co–workers [4] showed that coherent oscillations could also be observed in normal semiconductor systems. It is the purpose of this paper to propose an experiment in which coherent oscillations are observed in a single quantum dot (QD), with these oscillations being driven by the spin–orbit (SO) interaction.

The SO interaction in semiconductor heterostructures has its origin in the breaking of inversion symmetry, and is increasingly coming to be seen as a tool with which to manipulate electronic states, see e.g. [5]. The grandfather of these ideas is the spin–transistor of Datta and Das [6], in which the SO interaction causes electron spins to precess as they move through a two-dimensional electron gas (2DEG). In materials where the structural inversion asymmetry dominates, e.g. InGaAs, the SO interaction causes electron spins to precess as they move through a two-dimensional electron gas (2DEG). In materials where the structural inversion asymmetry dominates, e.g. InGaAs, the SO interaction can be described by the Rashba Hamiltonian [7]

\[ H_{SO} = -\frac{\alpha}{\hbar} \left[ (p + eA_c) \times \sigma \right]_z. \]  

In this letter we consider the effects of \( H_{SO} \) on electrons in a small, few-electron lateral quantum dot. Although such dots are yet to be realised in materials with strong SO coupling, there is currently a considerable effort to develop nanostructures in such materials [8]. Our interest here is not in open or chaotic QDs [9,10], but rather in small dots in the Coulomb blockade regime.

Such dots have been studied by a number of authors [11,12,13], but our analysis differs in a crucial respect: by making an analogy with quantum optics, we are able to derive an approximate Hamiltonian that captures the essential physics of the dot. This model is formally identical to the Jaynes–Cummings (JC) model [14], first derived in the context of the atom–light interaction. Here, the roles of the atomic pseudo-spin and light field are played by the spin and orbital angular momentum of the electron respectively. The system then naturally decomposes into a set of two-level systems (TLS), any of which may be considered as the qubit degree of freedom within which coherent oscillations can occur. These oscillations are genuine Rabi oscillations [15], with orbital and spin degrees of freedom exchanging excitation. This “spin–orbit pendulum” behaviour has been noted in three-dimensional models in nuclear physics [16].

Having elucidated the origin and properties of the TLS, we then describe an experimental scheme through which the coherent oscillations can be investigated. The key problem here is that of injecting into, and reading out from, states which are not eigenstates of the SO coupled system. In the Hayashi experiment [4], this was achieved through the spatial separation of the two dots, which makes the leads couple to the localised left and right states, rather than to the bonding and anti-bonding eigenstates. In our single dot system, the direct analogy of this would be the injection of spin–polarised electrons. Given the difficulty of interfacing ferromagnetic leads with semiconductors [3], we avoid their use by exploiting the fact that the strength of the SO interaction can be controlled by external gates [17,18,19].

Our starting point is the Fock-Darwin theory of a single electron in a 2DEG with parabolic confinement of energy \( h\omega_0 \) [20],

\[ H_0 = \frac{(p + eA)^2}{2m} + \frac{m}{2} \omega_0^2(x^2 + y^2), \]  

where \( m \) is the effective mass of the electron. Applying a perpendicular magnetic field in the symmetric gauge, in
second quantised notation we have

\[ H_0 = \hbar \omega (a_+^\dagger a_x + a_y^\dagger a_y + 1) + \hbar \omega_c (a_y a_x^\dagger - a_x a_y^\dagger), \]

with \( \omega_c \equiv eB/mc \) and \( \bar{\omega}^2 \equiv \omega_0^2 + \omega_c^2/4 \). Introduction of \( a_\pm = 2^{-1/2}(a_x \mp ia_y) \) decouples the system into eigenmodes of frequency \( \omega_\pm = \bar{\omega} \pm \omega_c/2 \).

We now include the Rashba interaction of Eq. (4), for which the coupling strength \( \alpha \) is related to the spin-rotation length \( l_{SO} \equiv h^2/2ma \). With magnetic length \( l_B \equiv \sqrt{\hbar/m\omega_c} \), we have

\[ H_{SO} = \frac{\alpha}{l} \left[ \gamma_+(a_+ \sigma_+ + a_x^\dagger \sigma_-) - \gamma_-(a_- \sigma_- + a_x^\dagger \sigma_+) \right], \]

with coefficients \( \gamma_\pm \equiv 1 \pm \frac{1}{2} \left( \frac{l}{l_B} \right)^2 \) and \( l \equiv \sqrt{\hbar/m\bar{\omega}} \).

Adding the Zeeman term, in which we take \( g \) to be negative as in InGaAs, performing a unitary rotation of the spin such that \( \sigma_+ \rightarrow -\sigma_- \) and \( \sigma_- \rightarrow -\sigma_+ \), and rescaling energies by \( \hbar \omega_0 \) we arrive at the Hamiltonian

\[ H = \omega_+ a_+^\dagger a_+ + \omega_- a_-^\dagger a_- + \frac{1}{2} E_2 \sigma_z 
+ \frac{l_B^2}{2l_{SO}^2} \left[ \gamma_-(a_- \sigma_- + a_x^\dagger \sigma_+) - \gamma_+(a_+ \sigma_+ + a_x^\dagger \sigma_-) \right]. \]

where \( l_0 = \sqrt{\hbar/m\omega_0} \) is the confinement length of the dot and \( E_2 = |g|m/(2mc)(l_B/l_0)^2 \) is the Zeeman energy with \( m_e \) the bare mass of the electron.

This single–particle picture is motivated by the good agreement between Fock-Darwin theory and experiment in the non-SO case [20], and by studies which have shown that many-body effects in QDs play only a small role at the magnetic fields we consider here [11, 12, 21].

We now derive an approximate form of this Hamiltonian by borrowing the observation from quantum optics that the terms preceded by \( \gamma_+ \) in Eq. (4) are counter-rotating, and thus negligible under the rotating–wave approximation [13] when the SO coupling is small compared to the confinement. This decouples the \( \omega_+ \) mode from the rest of the system, giving \( H = \omega_0 n_+ + H_{JC} \) where

\[ H_{JC}(\alpha) = \omega_- a_-^\dagger a_- + \frac{1}{2} E_2 \sigma_z + \lambda(a_- \sigma_- + a_x^\dagger \sigma_-), \]

with \( \lambda = l_B^2 \gamma_-/2l_{SO}^2 \). This is the well-known Jaynes-Cummings model (JCM) of quantum optics. It is completely integrable, and has ground state \( \{|0, \downarrow\} \) with energy \( E_G = -E_2/2 \) independent of coupling. The rest of the JCM Hilbert space decomposes into two-dimensional subspaces \( \{|n, \uparrow\}, \{|n+1, \downarrow\} : n = 0, 1, \ldots \). Diagonalisation in each subspace gives the energies \( E_n(\gamma) = (n + 1/2) \omega_0 + \Delta_n/2 \) with detuning \( \delta \equiv \omega_0 - E_2 \) and \( \Delta_n \equiv \sqrt{\delta^2 + 4\lambda^2(n+1)} \). The eigenstates are

\[ |\psi_n^{(\alpha, \pm)}\rangle = \cos \theta_n^{(\alpha, \pm)} |n, \uparrow\rangle + \sin \theta_n^{(\alpha, \pm)} |n+1, \downarrow\rangle, \]

with \( \tan \theta_n^{(\alpha, \pm)} = (\delta \pm \Delta_n)/2\lambda \sqrt{n+1} + 1 \).

Figure 1 shows a portion of the excitation spectrum obtained by exact numerical diagonalisation for a typical dot in InGaAs. The approximate \( H_{JC} \) describes the energy levels of the system to within 10% of the typical anticrossing width and 1% of \( \omega_0 \). This small discrepancy is visible in Fig. 1b. In the following, we are only concerned with the lowest–lying energy states in the dots. Without SO interaction, these states are described by \( n_+ = 0 \) – indicating that the states converge to the lowest Landau level in the high–field limit, and by \( n_- \) corresponding to the quantum number of angular momentum. The SO interaction thus couples two states of adjacent angular momentum and opposite spin. The detuning \( \delta \) uniquely identifies \( \omega_c \) for fixed material parameters and dot size.

Under the assumptions of the constant interaction model [20], the most important prediction of this model for linear transport is that, with the dot on resonance, the addition–energy spectrum for the first few electrons (up to 18 here) is described by a sequence of well-separated anticrossings, the width of which increases as \( \alpha \sqrt{n+1} \). This behaviour is shown in Fig. 1c, and its observation would be confirmation of our JC model, and would permit a determination of \( \alpha \) in quantum dots.

We now describe the procedure for observing spin-orbit driven Rabi oscillations. Our proposal is somewhat similar to that of Nakamura [1] with a voltage pulse driving.
the system, but with the crucial difference that the oscillations here are induced, not by a change in the detuning, but by a change in the SO coupling strength. We operate in the non-linear transport regime and address a single two-level system by being near resonance and by tuning the chemical potentials of the leads close to the $n$-th anticrossing. The SO coupling is set to $\alpha_1$ and the states taking part in the oscillation are eigenstates of $H_{JC}(\alpha_1)$, namely $\psi_{\alpha_1}^{\pm}$, which are situated symmetrically around the chemical potential of the right lead $\mu_R$, see Fig. 2a. The temperature is taken smaller than the detuning $k_BT < \delta$ to avoid the effects of thermal broadening. Assuming Coulomb blockade and considering first- or sequential tunnelling only, electrons can either tunnel from the left lead into the dot via state $\psi_{\alpha_1}^{+}$ and subsequently leave to the right or, alternatively, tunnel to state $\psi_{\alpha_1}^{-}$ blocking the dot, see Fig. 2a. Assuming tunnelling through the left/right barrier at a constant rate $\Gamma_{L/R}$, we set $\Gamma_L > \Gamma_R$ to assure that the dot is preferentially filled from the left; thus maximising the current. On average then, the dot will be initialised in state $\psi_{\alpha_1}$ for times $t_i > \Gamma_R^{-1}$.

Having trapped an electron in this state, we apply a voltage pulse to the gate. This has two effects. Firstly, this change in voltage alters the SO coupling to a new value $\alpha_2$. Since this change is performed non-adiabatically, the electron remains in the initial eigenstate $\psi_{\alpha_1}^{+}$ until Rabi oscillations begin between this state and $\psi_{\alpha_1}^{-}$ under the influence of the new Hamiltonian $H_{JC}(\alpha_2)$. Secondly, the TLS is drawn below both chemical potentials, assuring that oscillations can occur without tunnelling out of the dot, see Fig. 2c.

After a time $t_p$, the gate voltage is returned to its initial value, and the TLS resumes both to its original position and coupling $\alpha_1$, as in Fig. 2d. Tunnelling out of the dot can now occur, provided that the electron is found in the upper state, which happens with a probability given by the overlap of the oscillating wave function at time $t_p$ with the upper level,

$$P(t_p) = |\langle \psi_{\alpha_1}^+ | \Psi(t_p) \rangle|^2 = |\langle \psi_{\alpha_1}^+ | e^{-iH_{JC}t_p} | \psi_{\alpha_1}^- \rangle|^2. \quad (8)$$

This process is operated as a cycle and the current is measured. From probability arguments we see that $I \approx e\Gamma_R P(t_p)$, where we have used the simplification that $\Gamma_{R}^{-1} < t_p, \Gamma_{L}^{-1}$. Thus, by sweeping $t_p$ we are able to image the time evolution of Rabi oscillations, just as in the previous experiments of Nakamura and Hayashi.

The singular case of a non-adiabatic change in $\alpha$ from zero to a finite value produces oscillations with the maximum possible amplitude, $P_{\text{max}} = 1$. However, in realistic systems only changes between finite values of $\alpha$ are feasible. This leads to a reduction in the amplitude, and achieving a significant oscillation signal requires a suitably large change in $\alpha$. In experiments with 2DEGs, changes in $\alpha$ of a factor of 2 are reported, and in a recent Letter by Koga et al., $\alpha$ was shown to vary in the range $\approx (0.3 - 1.5) \times 10^{-12} \text{eV m}$ (a factor of 5) in one InGaAs sample. Grundler has shown that the large back-gate voltages usually used to change $\alpha$ can be drastically reduced by placing the gates closer to the 2DEG. Thus, it is conceivable that changes in $\alpha$ of a factor between 2 and 5 could be produced with voltages small enough to be pulsed with rise times substantially shorter than a typical coherent oscillation period.

In Fig. 3, we plot time–traces of the transition probability $P(t_p)$ calculated for the first anticrossing as a function of magnetic field. We have used the values $\alpha_1 = 1.5 \times 10^{-12}$ and $\alpha_2 = 0.3 \times 10^{-12} \text{eV m}$ from the Koga experiment . The amplitude of the oscillations $P_{\text{max}}$ for different ratios of $\alpha_2/\alpha_1$ is presented in Fig. 3b, which shows a node at $B = 0$ (\(\delta = 0\)). This is because, for $\delta = 0$, the eigenstates of JCM are $2^{-1/2} |\uparrow\rangle \pm |n+1, \downarrow\rangle$ for all $\alpha \neq 0$. Therefore, a finite detuning is required to obtain the maximum amplitude, which concurs with $\delta > kBT, \Gamma_R$ to overcome broadening effects. Both the amplitude $P_{\text{max}}$ and frequency $\Omega$ show non-trivial dependencies on $\alpha_1$ and $\alpha_2$ as well as on the magnetic field. This latter behaviour stems from the parametric dependence on $B$ of all three parameters in $H_{JC}$.

For our model parameters with $\alpha_2/\alpha_1 = 1/5$ and with the detuning set such that the amplitude is maximised, we have $P_{\text{max}} \approx 0.45$ with a Rabi frequency of $\Omega = 2 \text{GHz}$, which corresponds to a period of about 3 ns. This is within accessible range of state–of–the–art exper-
FIG. 3: Characteristics of the Rabi oscillation. (a) Probability $P(t_p)$ of finding electron in upper level after time $t_p$ following the non-adiabatic change $\alpha_1 = 1.5 \rightarrow \alpha_2 = 0.3 \times 10^{-12}$ eVm as function of magnetic field. (b) Amplitude of oscillation as function of $B/B_0$ for changing from $\alpha_1 = 1.5, 0.8, 0.6$ to $\alpha_2 = 0.3 \times 10^{-12}$ eVm (top to bottom). (c) Phonon-induced relaxation rate for InAs parameters $\alpha = 1.5 \times 10^{-12}$ eVm, $P = 3.0 \times 10^{-21} J^2/m^2$, $\rho = 5.7 \times 10^{3} kg/m^3$, $c = 3.8 \times 10^3 m/s$. Close to $B_0$ the rate is suppressed to $\Gamma_{cp} < 10^{-7} \omega_0$.

Experimental technique. Note that the period can be extended by using weaker confinement and SO coupling.

For both the observation of coherent oscillations, and the operation as a qubit, it is essential that the lifetime of state $|c\rangle$ is long. This is the case for a pure electronic spin in a QD, and we now show that the hybridisation of the spin with the orbitals, and the ensuing interaction phonons, does not affect this. We assume a piezo-electric coupling to acoustic phonons via the potential $V_{cp} = \lambda_{q} e^{\alpha q \tau}(b_{q} + b_{-q}^{\dagger})$, with phonon operators $b_{q}$ and $|\alpha q|^2 = h\omega/2\rho c q \nu$, with coupling $P$, mass density $\rho$, speed of sound $c$, and volume $\nu$. For $n=0$, a Golden Rule calculation yields the rate

$$\Gamma_{cp}/\omega_s = \frac{mP}{8\pi(\hbar\omega_s)^2\rho l_0^3} \frac{\sqrt{2}l_0}{L} \sin^2 \theta_+ \sin^2 \theta_-(\xi^5 I(\xi)), \quad (9)$$

with $\omega_s = c/l_0$, $\xi = 2^{-1/2}(l/l_0)(\Delta/\hbar\omega_s)$, and $I(\xi) \leq 8/15$. Close to $B_0$, $\xi \ll 1$, and thus the rate is extremely small $\Gamma_{cp} \approx 10^{-4} s^{-1}$ (Fig. 3). Therefore, the robustness of spin qubits is not significantly weakened by the SO hybridisation.

In general, residual relaxation affects our measurement scheme in two ways. During the oscillation (Fig. 3), the system may relax to the eigenstate $|\psi_{+}\rangle$. This damps the oscillation by a factor $\exp(-\Gamma t_p)$ to the constant value $I = eR P_{max}/2$. Relaxation during the read-out phase (Fig. 2B) simply reduces the overall amplitude of the signal by a factor $\exp(-\Gamma_2/\Gamma_R)$. Clearly then, to observe oscillations, we require $\Gamma_1 < \Omega$ and $\Gamma_2 < \Gamma_R$.

In summary, we have outlined a proposal for the observation of spin-orbit driven coherent oscillations in a single quantum dot. We have derived an approximate model, inspired by quantum optics, that shows the oscillating degree of freedom to represent a novel, composite spin-angular momentum qubit.

This work was supported by the EU via TMR/RTN projects, and the German and Dutch Science Foundations DFG, NWO/FOM. We are grateful to T. Brandes, C.W.J. Beenakker and D. Grundler for discussions, and to B. Kramer for guidance and hospitality in Hamburg.

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