Relaxing Cosmological Constraints on Large Extra Dimensions

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Abstract

We reconsider cosmological constraints on extra dimension theories from the excess production of Kaluza-Klein gravitons. We point out that, if the normalcy temperature is above 1 GeV, then graviton states produced at this temperature will decay early enough that they do not affect the present day dark matter density, or the diffuse gamma ray background. We rederive the relevant cosmological constraints for this scenario.

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I. INTRODUCTION

Beyond the three spatial dimensions we observe may lie many others, with total volume small enough to have escaped detection through microphysical or cosmological measurements. While such a proposal is not new [1, 2], there exist a host of contemporary incarnations [3, 4, 5, 6, 7, 8, 9, 10, 11, 12] that allow the extra dimensions to have a significantly larger spatial extent than had previously been imagined. The central feature in these new constructions is the idea that standard model particles may be confined to a 3 + 1 dimensional submanifold - or brane - while gravitational degrees of freedom may propagate in the entire space - the bulk. Such an approach liberates the Kaluza-Klein (KK) idea from the strong constraints posed by precision laboratory and collider measurements of the electroweak theory and opens up new avenues for addressing long-standing particle physics and cosmological problems.

In place of traditional constraints, large extra dimension models face a set of new issues at the high energy frontier, both through collider experiments and cosmology. A particularly general class of cosmological constraints arise from the overproduction of Kaluza-Klein gravitons. In the four dimensional effective theory describing our universe for most of its history, the matter content consists of fields confined to the brane, the graviton zero mode (playing the role of our graviton) and a tower of massive graviton excitations having non-zero momenta in the extra dimensions. Cosmological constraints arise because standard model particles at high energy may create these KK gravitons. Since these particles are massive and long-lived (having only gravitational strength couplings), their overproduction can lead to them dominating the universe and coming into conflict with observations. This can occur indirectly, for example they may interfere with primordial nucleosynthesis, or directly, for example they may overclose the universe.

In this paper we reconsider cosmological constraints arising from the overproduction of KK gravitons. Previous studies [13, 14, 15, 16] concentrate on scenarios in which the temperature at which graviton production effectively starts (the so-called normalcy temperature $T_\star$) is in the MeV range. Then the produced gravitons are long-lived, and the most stringent cosmological limits came from constraints on the present day dark matter density and on the diffuse gamma ray background. We note that when the normalcy temperature is in the GeV range or above, the gravitons produced at these temperatures decay before
recombination time. We then find that the above constraints are significantly ameliorated by such decays. Note also that astrophysical constraints \[17, 18, 19, 20\] will be relaxed too, since in our scenario the fundamental scale of gravity is larger than in previous analyses (For another way to avoid these constraints see \[21\]). However, new constraints become significant, the most stringent one coming from the requirement that the graviton decay products do not destroy the predictions of the abundances of light elements created during Big-Bang Nucleosynthesis (BBN).

The structure of this paper is as follows. In the next section we briefly describe the main theoretical framework of the model. In section \[\text{III}\] we (re)compute the standard cosmological constraints on KK graviton production for the case when the graviton decays are negligible. Then, in \[\text{IV}\] we introduce the effects of decays and rederive the relevant constraints and in section \[\text{V}\] we briefly address the effect of early KK graviton production on the evolution of other cosmological parameters. In section \[\text{VI}\] we consider the possibility that the graviton-matter interaction at high energies might be modified, for example in soft or fat brane scenarios, and in section \[\text{VII}\] we offer a summary and our concluding comments.

II. THE MODEL

The general framework consists of a \(4 + d\) dimensional spacetime, with \(3 + 1\) dimensions corresponding to those we are familiar with, and the extra \(d\) spatial dimensions compactified. We follow the conventions that \(M, N, \ldots = 0, 1, \ldots, 4 + d, \mu, \nu, \ldots = 0, 1, 2, 3\) and \(a, b, \ldots = 5, 6, \ldots, 4 + d\). For simplicity, in this paper we shall assume compactification on a \(d\)-torus of common radius \(r/2\pi\), although other geometries may also be studied \[12, 22, 23, 24\]. Writing the bulk metric as \(G_{MN}\), we define the linearized metric \(H_{MN}\), describing the gravitational degrees of freedom, by \(G_{MN} = \eta_{MN} + H_{MN}\). Gravity propagates in the entire bulk and so it is convenient to expand the linearized metric as

\[
H_{MN}(x, y) = \sum_{\vec{n}} H_{MN}(x) \exp \left( i \frac{2\pi \vec{n} \cdot \vec{y}}{r} \right),
\]

where \(x^\mu\) are brane coordinates and \(y^a\) are those in the bulk.

This field may be decomposed into its scalar, vector and tensor components with respect
to the $3+1$ dimensional Poincaré group as

$$H_{MN} = \frac{1}{\sqrt{V_d}} \left( \begin{array}{c} h_{\mu\nu} + \eta_{\mu\nu} \phi \ A_{\mu a} \\ A_{\nu b} \\ 2\phi_{ab} \end{array} \right),$$

(2)

where $\phi \equiv \phi_a^a$ and $V_d \equiv r^d$ is the volume of the $d$-torus.

All other fields, and in particular those of the standard model, are restricted to propagate only on the brane, so that they do not have equivalent Kaluza-Klein excitations.

### III. DENSITY OF KK GRAVITONS

As mentioned above, we are interested in the possibility that KK gravitons may be produced through high-energy processes involving standard model particles on our $3$-brane. If such processes are abundant at high cosmic temperatures, then the cooling of the universe will be radically different from the usual effect of cosmic expansion.

The Kaluza-Klein gravitons are the tower of four dimensional excitations, described by (1), of the field $h_{\mu\nu}$, defined in (2) (the vector gravitons $A_{\mu a}$ do not couple with Standard Model matter, and we neglect production of scalar gravitons $\phi_{ab}$). The Feynman rules for the matter-graviton interaction have been derived in [25, 26]. Our goal is to compute the density of these particles produced at relevant epochs during the expansion of the universe.

To simplify our analysis, let us consider a particular graviton state of mass $m = 4\pi^2 n^2 / r^2$, and label the state by its mass, rather than by its KK vector. The number density of these gravitons evolves according to

$$\dot{n}_m + 3n_m H = P_m - \Gamma_m n_m ,$$

(3)

where $P_m$ is the production rate and $\Gamma_m$ is the decay rate. In order to compute the density of gravitons of mass $m$ produced in a given period of time, one must integrate Eq. (3). To achieve this it will be convenient, as usual, to transform the equation in two ways. First, we change the dependent variable from cosmic time to temperature, using the time-temperature relation which holds during the radiation dominated era

$$t = \frac{1.5}{\sqrt{g_*}} \frac{\bar{M}_p}{T^2} ,$$

(4)

(since generally only gravitons produced during this epoch are relevant). Here $\bar{M}_p \equiv (8\pi G)^{-1/2}$ is the reduced Planck mass. Second, we introduce the scaled number density
Denoting by a prime differentiation with respect to temperature, equation (3) now becomes

\[ Y'_m = \frac{3}{\sqrt{g^* T^3}} \left( \Gamma_m Y_m - \frac{P_m}{T^3} \right) . \]  

(5)

Now, there are two types of processes that contribute to the right hand side of this equation. The first is inverse decay, occurring when the graviton is generated, for example, via neutrino-antineutrino annihilation or through photon-photon interactions

\[ \nu\bar{\nu} \to G_m \quad \gamma\gamma \to G_m . \]  

(6)

The second type of interaction is graviton radiation, generated for example through

\[ e^+e^- \to \gamma G_m \quad e^-\gamma \to e^- G_m \]  

(7)

Note that these are just illustrations of the types of processes, not a full enumeration, and that the actual processes depend on the temperature \( T \) at which the production takes place.

Let us begin with inverse decays and, for simplicity, ignore the subsequent decays of the gravitons produced in this way. Recalling that the spin-summed amplitude squared for \( \nu\bar{\nu} \to G_m \) is \( s^2/4\bar{M}^2 \), the production rate for this process is given by

\[ P_m(\nu\bar{\nu} \to G_m) = \frac{m^5 T}{128\pi^3 \bar{M}^2} K_1 \left( \frac{m}{T} \right) . \]  

(8)

Here \( K_1 \) is the modified Bessel function of the second kind, with asymptotic behavior

\[ K_1(z) \sim \begin{cases} z^{-1} & \text{for } z \ll 1 \\ \sqrt{\frac{\pi}{2z}} e^{-z} & \text{for } z \gg 1 \end{cases} . \]  

(9)

As one would expect, the exponential suppression for \( m > T \) implies that it is not possible to produce gravitons whose mass is much larger than the temperature.

Denoting the temperature at which production commences by \( T_i \), the number density of gravitons at a lower temperature \( T_f \) is obtained by integrating (3), using (8), yielding

\[ Y_m(T_f) \simeq \frac{10^{-3}}{\sqrt{g^*}} \frac{m}{\bar{M}^2} \int_{m/T_i}^{m/T_f} dz\ z^3 K_1(z) . \]  

(10)

We may perform a similar calculation for graviton radiation processes, for which the graviton production rate is

\[ P_m(a\ b \to c\ G_n) = \langle \sigma v \rangle n_a n_b \]  

(11)
where \( n_a, n_b \) are the number densities of particles in the initial states at temperature \( T \) (\( n_\gamma \simeq 2.4 \ T^3/\pi^2, n_f \simeq 1.8 \ T^3/\pi^2 \) for relativistic bosons and fermions respectively). The thermally averaged cross-section is

\[
\langle \sigma v \rangle = \frac{T^6}{16\pi^4 n_a n_b} \int_{m/T}^\infty dz \ z^4 K_1(z) \sigma(z^2 T^2) ,
\]

(12)

with \( zT = \sqrt{s} \) the center of mass (CM) energy at which the collision takes place. Taking \( \langle \sigma v \rangle \simeq \alpha/\bar{M}_p^2 \) as a general approximation valid for these types of processes, it is then straightforward to integrate (3) to obtain

\[
Y_m(T_f) \simeq \frac{12\alpha}{\pi^4 \sqrt{g_*}} \frac{T_i - T_f}{\bar{M}_p} \simeq \frac{12\alpha}{\pi^4 \sqrt{g_*}} \frac{T_i}{\bar{M}_p} ,
\]

(13)

where the final step merely acknowledges that, generally, \( T_f \ll T_i \). It is worth commenting that if \( m > T_f \), then \( T_f \) should be replaced with \( m \) in the above expression, while if \( m \gg T_i \), the result will be close to zero, due to the exponential suppression in (12). Also note that if the mass of the graviton is of the same order of magnitude as \( T_i \), then the graviton densities (10), (13) generated by the two types of processes (inverse decay and radiation) are roughly of the same order magnitude, while if \( T_i \gg m \) the graviton radiation type will dominate.

In order to compute the total graviton density, it remains to sum over all the KK excitations of the graviton via

\[
\rho_G = \sum_{\eta} \frac{M_D}{M_0^2} \ f \int_0^{m_{\eta\to n}} dm \ m^{d-1} dm ,
\]

(14)

To accomplish this we replace the sum by an integral via

\[
\sum_{\eta} \to S_d \frac{M_D^2}{M_0^2} \int_0^{m_{\eta\to n}} dm \ m^{d-1} dm ,
\]

(15)

where \( M_D \) is the fundamental Planck scale in the full 4 + \( d \) dimensional theory, defined through \( M_D^2 = M_0^2/(d/2)^d \), and \( S_d = 2\pi^{d/2}/\Gamma(d/2) \) is the surface area of the unit sphere in \( d \) dimensions.

Since we wish to compute the present day graviton energy density, we take \( T_f = T_0 \sim 2K \) in (10), yielding

\[
\rho_G = S_d T_0^3 \frac{M_D^2}{M_0^2} \ f \int_0^{m_{\eta\to n}} \ dm \ m^{d+1} \int_{m/T_i}^\infty dz \ z^3 K_1(z) ,
\]

(16)

where \( f = 10^{-3}/\sqrt{g_*} \simeq 3 \times 10^{-4} \) (taking \( g_* \simeq 10 \), valid for temperatures smaller than 1MeV) and where, since \( m \gg T_0 \), we have approximated the upper limit of the integral in (10) by \( \infty \).
Usually one would also take $m_{\text{max}} \to \infty$, since the contribution from higher mass states will be suppressed by the exponential decay of the integral over $K_1$. However, because of this effective cutoff, we shall instead introduce an effective maximal mass $m_{\text{max}} \equiv rT_i$, with $r$ a phenomenological constant, and approximate the lower limit of the integral in (16) by zero so that the double integral in (16) becomes

$$\int_0^{rT_i} dm \, m^{d+1} \int_0^\infty dz \, z^3 K_1(z) = \frac{3\pi (rT_i)^{d+2}}{2(d+2)}. \tag{17}$$

It is easily checked that $r \sim 6$ yields a good fit to the exact results obtained by numerical integration (for $d = 6$ $r$ will be slightly higher than 6, while for $d = 2$ slightly lower). Note that the highest mass which can be produced effectively is somewhat larger than what one might naively assume, namely $2T_i$. This is to be expected, since there exists a large polynomial enhancement in the number of KK states available, which partially compensates for the Boltzmann suppression.

We then obtain

$$\rho_G \simeq \frac{3\pi}{2} f \frac{S_d}{d+2} \frac{M_p T_0^3}{M_D} \left( \frac{rT_\ast}{M_D} \right)^{d+2}, \tag{18}$$

where we have denoted the temperature at which effective graviton production starts by $T_\ast$ (also called normalcy temperature). Requiring that the fraction of the cosmological critical density in KK gravitons $\Omega_G$ be less than that in matter ($\Omega_G < 0.3$) then implies

$$5 \times 10^{-5} \frac{3\pi}{2} f \frac{S_d}{d+2} \frac{M_p}{T_0} \left( \frac{rT_\ast}{M_D} \right)^{d+2} < 0.3 \tag{19}$$

or

$$\left( \frac{rT_\ast}{M_D} \right)^{d+2} < 0.5 \times 10^7 \frac{T_0}{M_p} \sim 0.5 \times 10^{-24} \tag{20}$$

(here we have taken $S_d/(d+2) \sim 1$ for all $d$). Thus, in the case of $d = 2$, we obtain $rT_\ast/M_D < 10^{-6}$, so that if $T_\ast = 1$ MeV, then $M_D > 6$ TeV.

Note that in Eq. (19) we have taken into account the contribution of only one type of neutrino. It is necessary to multiply the left hand side of the equation by a factor $R_c$, which takes into account the number of channels through which this process can proceed and the relative strengths of the cross sections in these channels. For example, if the gravitons are produced at energies lower than 1 MeV, then $R_c = 7$, where a factor of 3 comes from the three neutrino families, and an additional factor of 4 arises because the $\gamma\gamma \to G_m$ annihilation cross-section is 4 times larger than the neutrino one.
Now let us move on to evaluate the graviton density created by radiation type processes. Using (13), we obtain
\[ \rho_{G} = S_d T_0^3 \frac{\bar{M}^2}{M_D^{2+d}} \frac{\tilde{f}}{M_p} \int_{0}^{m_{\text{max}}} dm T_* m^d, \]
where \( \tilde{f} \equiv 12\alpha / (\pi^4 \sqrt{g_*}) \simeq 4 \times 10^{-4} \). Choosing the same upper limit of integration, \( m_{\text{max}} = r T_* \), as for the previous case yields
\[ \rho_{G} \simeq \frac{\tilde{f}}{r} \frac{S_d}{d+1} \frac{M_p T_0^3}{M_D} \left( \frac{r T_*}{M_D} \right)^{d+2}, \]
which is an order of magnitude smaller than the contribution coming from \( \nu \bar{\nu} \) annihilation. However, this conclusion holds for gravitons produced at low temperatures. If \( T_* > 300 \) MeV, then gravitons can also be produced, for example, through \( q\bar{q} \rightarrow gG_m, qg \rightarrow qG_m \) and \( gg \rightarrow gG_m \) processes, for which the cross-section is proportional to the strong coupling constant rather than the electroweak one, and these contributions may become important.

**IV. THE EFFECTS OF DECAYs**

Thus far we have operated under the assumption that the gravitons produced are stable. We would now like to examine the validity of this approximation. The relevant decay process is that of gravitons into two Standard Model particles. To estimate this, consider the lifetime for decay into photons, given by [25]
\[ \tau_{\gamma\gamma} = 3 \times 10^9 \text{ yr} \left( \frac{100 \text{MeV}}{m} \right)^3. \]
Since the age of the Universe today is about \( 1.5 \times 10^{10} \) yr, we are clearly justified in neglecting decays of gravitons with mass lower than about 100 MeV.

However, higher mass gravitons will decay before the present epoch, and so cannot contribute to the present day dark matter density. If such gravitons decay after recombination, their decay products will contribute to the diffuse cosmic gamma ray background. It is therefore useful to calculate how high the mass of gravitons must be so that they decay before recombination at \( t_{\text{rec}} \simeq 5 \times 10^5 \) yr.

Since we are dealing with gravitons with masses in the GeV range, it is necessary to consider decays to gluon-gluon, lepton-lepton and quark-quark final states involving those particles with masses at or below this magnitude. These decays obey \( \Gamma_{gg} = \Gamma_{\gamma\gamma} \) and \( \Gamma_{ff} = \)
(1/2)Γ_{γγ} for individual gluons and fermions. Since there are one photon, 8 gluons, 5 quarks (each of 3 colors), 3 leptons and 3 neutrinos, the total decay width is
\[ \Gamma_t = \left\{ 1 + 8 + \frac{1}{2} [(3 \times 5) + 3 + 3] \right\} \Gamma_{γγ} = 19.5\Gamma_{γγ}, \] (24)
so that the relevant lifetime is
\[ \tau_G \simeq 1.5 \times 10^5 \text{ yr} \left( \frac{1\text{ GeV}}{m} \right)^3. \] (25)
Therefore, it is clear that most gravitons with mass greater than 1 GeV will decay before recombination.

We now revisit the results of the previous section in light of what we have just learned. To compute the “surviving” graviton density we clearly must take an upper limit \( m_{\text{max}} \simeq 1 \text{ GeV} \) in the integral (16). Our result (18) then becomes
\[ \rho_G(T_{\text{rec}}) \simeq \frac{3\pi}{2} f_p \frac{S_{d-1}}{d+2} \tilde{M}_p T_{\text{rec}}^3 \left( \frac{m_{\text{max}}}{\tilde{M}_D} \right)^{d+2}. \] (26)

We next need to identify those processes that contribute to the production of gravitons with mass around 1 GeV. At low temperatures, the possibilities were \( ν\bar{ν}, γγ \rightarrow G_m \). At higher temperatures, heavier particles can appear in the initial state. From Eq. (10) we see that, for these types of processes, most gravitons of mass \( m \) are produced at temperatures of order \( m \). Therefore, the initial state in this case will also contain \( e^+e^−, μ^+μ^−, gg \) and \( q\bar{q} \) pairs, where \( q \) stands for the three quarks with mass lower than 1 GeV (\( u, d, s \)). In this case the graviton density (26) should be multiplied by a factor \( R_c = 61 \) (remembering that gauge bosons contribute a factor of four times that of neutrinos, and leptons and quarks twice as much). Also, the effective number of degrees of freedom for graviton produced at \( T \sim 1 \text{ GeV} \) will be \( g_* = 61.75 \) (assuming Standard Model particle content).

The situation will be somewhat different for gravitons produced in radiation type processes. In this case, most of the gravitons with mass \( m \) will be produced at temperatures larger than \( m \) (assuming that the normalcy temperature \( T_* \gg m \)). The number density (22) then becomes
\[ \rho_G \simeq f_p \frac{s_{d-1}}{d+1} \tilde{M}_p T_{\text{rec}}^3 \left( \frac{m_{\text{max}}}{\tilde{M}_D} \right)^{d+1}. \] (27)

Strong interaction processes give the largest contribution to graviton production through radiation. These processes are \( q\bar{q} \rightarrow gG_m \), for which the cross-section has to be multiplied by
a factor $4N_f$ with respect to the $e\bar{e} \rightarrow \gamma G_m$ cross-section ($N_f$ is the number of quark flavors contributing and 4 is the color factor), $qg \rightarrow q G_m$ with a factor $8N_f$, and $gg \rightarrow g G_m$ with a color factor 24. If $T_* > 200$ GeV, all quark flavors contribute, and the total multiplicative factor for the right-hand side of Eq. (27) will be $R_c = 96$, while the number of effective degrees of freedom at production time will then be $g_* = 106.75$.

Since $T_* \gg m_{\text{max}} \simeq 1$ GeV, most gravitons with this mass will be produced through radiation type processes. Thus, requiring that $\rho_G/\rho_\gamma \ll 1$ at recombination implies

$$\left( \frac{m_{\text{max}}}{M_D} \right)^{d+1} \ll 5 \times 10^{-28} \times \frac{M_D}{T_*}.$$  \hspace{1cm} (28)

If we take $T_* \sim M_D$ then, for $d = 6$, this constraint is satisfied for $M_D \simeq 10$ TeV. In addition the present day graviton contribution to the dark matter density will be negligible.

A more complete calculation would entail evaluating the contribution of the photons resulting from late graviton decays to the diffuse gamma ray background. Also, the photons resulting from graviton decays close to recombination time ($\tau \gtrsim 10^{10}$ sec) might distort the CMB distribution from black body spectrum [27]. However, we will not address this here.

V. OTHER CONSTRAINTS

In the previous section we have seen that, as long as $M_D > 10$ TeV (for $d = 6$), KK gravitons may be produced at any temperature without impact on the present day dark matter density or on the diffuse gamma ray background. In this section we briefly examine other possible effects of graviton production at temperatures of the order of 100 GeV or greater on the cosmological parameters.

One first test is to look at the depletion of the cosmological photon (or other SM particle) density due to annihilation into gravitons. Such a depletion rate must be much smaller than the dilution due to the expansion of the universe

$$2 \sum_n P_m \ll 3 n_\gamma H.$$  \hspace{1cm} (29)

This can be written explicitly as

$$2 \times 10^{-3} S_{d-1} \frac{T}{M_D^{2+d}} \int_0^{m_{\text{max}}} dm \ m^{d+4} K_1 \left( \frac{m}{T} \right) \ll \frac{7.2}{\pi^2} T^3 \sqrt{\frac{\pi^2}{90 g_*}} \frac{T^2}{M_p},$$  \hspace{1cm} (30)
which yields
\[ \left( \frac{rT}{M_D} \right)^{d+2} \ll 10^{-16} \left( \frac{T}{1 \text{ GeV}} \right). \]  
(31)

For example this gives \( rT < M_D/100 \) for \( d = 6 \). Similar constraints are obtained if one considers the depletion of the gluon and quark densities.

The success of the theory of primordial nucleosynthesis also sets quite stringent constraints on the expansion rate of the universe at \( T = T_{BBN} \simeq 1 \text{ MeV} \). In particular, it is necessary that the universe be radiation dominated at that time. Therefore we require
\[ \frac{\rho_G}{\rho_\gamma}(T_{BBN}) \ll 1. \]  
(32)

Using (18) and (22) to evaluate \( \rho_G \) (and taking into account all contributing processes) this constraint becomes
\[ \left( \frac{rT^*}{M_D} \right)^{d+2} \ll 5 \times 10^{-21}, \]  
(33)
which is a significantly stronger constraint than the previous one, yielding, for example, \( T^* < 0.4 \times 10^{-3} M_D \) for \( d = 6 \).

It is likely, however, that the strongest constraint on such a scenario will come from requiring that the decay products of the massive gravitons do not destroy the light elements abundances predicted by BBN (see, for example [28]). An analysis using recent data indicates that the abundance of a generic unstable massive particle \( X \) decaying mainly to hadrons at a time between \( 10^4 - 10^{10} \) seconds has an upper limit \( Y_X \lesssim 10^{-14}/m_X(\text{GeV}) \) [29]. This constraint is about ten orders of magnitude more stringent than (33)
\[ \left( \frac{rT^*}{M_D} \right)^{d+2} \lesssim 10^{-31}. \]  
(34)

Constraints for the case when the heavy particle decays radiatively (to photons) are somewhat weaker [30] (For some applications to specific scenarios see [31, 32, 33]). However, neither of these numbers may be directly applicable to our case. Since the gravitons decay mostly to hadrons, but have a sizable decay branching ratio to electroweak gauge bosons and leptons, one should perform an analysis taking into account both types of decays. This may weaken somewhat the constraint (33), since the overproduction of D and \(^6\text{Li}\) through hadronic processes (which sets the strongest constraint on \( Y_X \) in the interesting region of parameter space) might potentially be compensated by a destruction of these elements due to energetic photons.
Another potential constraint one might consider arises from the entropy production from the decays of KK gravitons. Such production could unacceptably dilute a pre-existing baryon to entropy ratio. In the well-known example of gravitino decay, this dilution factor can be as high as $10^7$ and is of real concern for most baryogenesis mechanisms. A rough calculation in our case reveals a number that at most is of order $10^2$ (provided that (33) is satisfied). Given that a number of baryogenesis models are able to accommodate such a number, we shall not pursue this constraint further here.

Finally, we note that the constraints discussed in this section have the potential to make the observation of KK gravitons at colliders quite challenging. For example, if $T_*$ is in the 10 GeV range, then the weaker BBN constraint (33) will require that the fundamental gravity scale $M_D$ is in the 10 TeV range (for $d = 6$), which still allows for the observation of graviton effects at near-future colliders (see, for example [34] and references therein). However, if the stronger BBN constraint (34) is valid, then this will push $M_D$ to 100 TeV range or higher.

VI. BRANE SOFTENING AND OTHER NATURAL CUTOFFS

The validity of the constraints derived above assumes that the matter-graviton interaction stays unchanged up to energies of order $T_*$. However, at energies close to the fundamental scale of gravity new effects may appear.

One such effect would be the softening of the gravity-matter interaction due to brane fluctuations [35]. The interactions derived in [25] assume that the SM brane is rigid. However, this is not necessarily so; for example if the energy is high enough that brane oscillations can be excited, then the matter-graviton interaction vertex will acquire an effective form factor $F = e^{-\frac{1}{2} \frac{m^2}{\Delta^2}}$, with the ‘softening scale’ $\Delta$ related to the brane tension. As a result the cross-section for production of gravitons with mass greater than $\Delta$ will be exponentially suppressed.

The softening scale $\Delta$ therefore provides a natural origin for the cutoff on the magnitude of the masses of gravitons produced in the early universe. Assuming that the normalcy temperature is much larger than $\Delta$, most of the gravitons with mass of order $\Delta$ will be produced through radiation processes, and the graviton energy density [24] at temperature
\[ T \text{ becomes} \]
\[
\rho_G = S_d T^3 \frac{\bar{M}^2_p}{M_D^{2+d}} \frac{\tilde{f}}{M_p} \int_{0}^{m_{\text{max}}} dm \ T_* m^d \ e^{-\frac{m^2}{2T^2}}
\]
\[ \simeq \tilde{f} \frac{S_d}{d+1} M_p T^3 \left( \frac{\Delta}{M_D} \right)^{d+1}, \tag{35} \]

where we have taken \( T_* \simeq M_D \). As we saw in the previous section, the strongest constraints on graviton production in early universe comes from requiring the preservation of BBN predictions. (In this section we will use the expansion rate constraint (32), since we do not know the precise numbers for (34)). For \( \Delta > 1 \text{ GeV} \) this constraint now becomes
\[ \left( \frac{\Delta}{M_D} \right)^{d+1} \ll 3 \times 10^{-21}. \tag{36} \]

Note that there is still some hierarchy involved - the most natural scale for \( \Delta \) is close to \( M_D \), while the above constraint requires several orders of magnitude between the two quantities. However, the normalcy temperature \( T_* \) in this scenario can be as high as the fundamental gravity scale \( M_D \).

An alternate possibility, often referred to as the fat brane scenario \[36\] allows the SM particles to be localized around a brane, rather than strictly confined to one. This means that they may propagate in one or more of the extra dimensions in which gravity lives, but only for a reduced distance \( R \leq \mathcal{O}(\text{TeV}^{-1}) \) to ensure that the SM KK excitations satisfy collider bounds.

For simplicity assume that the confining potential is an infinite square well, so that the wave functions of the SM particles are unity on the brane \((0 < y_i < \pi R)\) and zero outside. The graviton-matter vertex function now acquires a form-factor \[36, 37\]
\[ \mathcal{F} = \frac{1}{(\pi R)^d} \int_0^{\pi R} \frac{d \vec{y}}{r} \exp \left( i \frac{2\pi \vec{n} \cdot \vec{y}}{r} \right) \tag{37} \]
and the production cross-section is correspondingly multiplied by
\[ |\mathcal{F}|^2 = \prod_{i} \left( \frac{M}{m_i \bar{\pi}} \right)^2 4 \sin^2 \left( \frac{\pi m_i}{2M} \right), \tag{38} \]
with \( M \equiv 1/R \) and \( m_i \equiv 2\pi n_i / r \). The energy density of gravitons produced in the early universe through inverse decay processes then becomes
\[ \rho_G(T) = T^3 \frac{\bar{M}^2_p}{M_D^{2+d}} \frac{f}{M_p} \int_{m < M_D} m^2 \prod_{i} dm_i \left( \frac{2M}{m_i \bar{\pi}} \right)^2 \sin^2 \left( \frac{\pi m_i}{2M} \right) \int_{m/T_i}^{\infty} dz \ z^3 K_1(z), \tag{39} \]
where \( m^2 = \sum_i m_i^2 \).

Note that the result of the integral over \( m \) increases linearly with the upper limit of integration (which we take to be \( M_D \)). This can be seen by integrating each part separately

\[
\sum_i \int_{-M_D}^{M_D} dm_i \sin^2 \left( \frac{\pi m_i}{2M} \right) \prod_{j \neq i} \int \frac{dm_j}{m_j^2} \sin^2 \left( \frac{\pi m_j}{2M} \right) \simeq M_D \left( \frac{\pi^2}{2M} \right)^{d-1} d .
\] (40)

Taking \( T_i \gg m \) in (39), we obtain

\[
\rho_G(T) \simeq 3 \frac{\pi}{2} f \bar{M}_p T^3 d \left( \frac{2M}{M_D} \right)^{d+1} .
\] (41)

We similarly evaluate the graviton energy density created in radiation processes to be (with \( T_\ast \simeq M_D \)):

\[
\rho_G(T) \simeq \bar{f} \bar{M}_p T^3 d \left( \frac{2M}{M_D} \right)^{d+1} \ln \left( \frac{M_D}{\pi M} \right) .
\] (42)

The BBN expansion rate constraint (32) then reads

\[
\left( \frac{2M}{M_D} \right)^{d+1} \ln \left( \frac{M_D}{\pi M} \right) \ll 5 \times 10^{-21} .
\] (43)

This again requires that the new scale \( M \) (related to the confining potential) should be several orders of magnitude smaller than the gravity scale \( M_D \), but one does not need a low normalcy temperature in this scenario either.

VII. CONCLUSIONS

The possibility of large extra dimensions has opened up an entirely new set of approaches to the problems of both particle physics and cosmology. The brane world construction is designed to obviate the traditional bounds on higher-dimensional versions of the electromagnetic and weak interactions by confining our standard model particles to the usual 3 + 1 dimensional submanifold. However, there are significant constraints on large extra dimension models from both particle physics and cosmology.

A generic constraint arises from the potentially problematic production of Kaluza-Klein graviton states during the evolution of the early universe. While these effects have been considered before, in this paper we have reconsidered them in the range of normalcy temperatures above 1 GeV. There are good reasons to consider this a particularly attractive region of parameter space. Perhaps the most compelling candidate for dark matter is a
weakly interacting massive particle (WIMP) - a heavy particle with electroweak scale interaction strength. The mass of this particle is expected to be in the 10 GeV - 10 TeV range (see, for example [38]) and in order to produce a sufficient relic density of this particle as the universe cools one generally then requires \( T^* = \mathcal{O}(\text{GeV}) \) or larger.

In this cosmologically interesting range of normalcy temperatures, we have demonstrated that one may not neglect the effects of KK graviton decay when computing constraints from KK graviton production. We have recomputed the relic density, taking into account the decay of these particles, and have rederived the corresponding constraints. The results cover a larger region of parameter space compared to previous calculations and open up a new window of viability for these models.

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