Identifying the graphene $d$-wave superconducting symmetry by an anomalous splitting zero-bias conductance peak

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Abstract

Not until recently, was a gate-tunable, high-temperature superconducting proximity effect in graphene demonstrated experimentally. And usually in $d$-wave superconductor (SC) hybrid structure, ferromagnetism and spin-triplet states could result in a splitting zero-bias conductance peak (ZBCP). Herein, we theoretically present an anomalous splitting ZBCP in a graphene-based ferromagnet/Rashba spin–orbit coupling (RSOC)/insulator/$d$-wave SC hybrid structure. With increasing the exchange field from $h/E_F = 0$, the ZBCP starts to turn into a splitting one with a zero-bias conductance dip (ZBCD) sandwiched in between two subpeaks, while from $h/E_F = 1$, the two subpeaks and ZBCD begin to gradually shrink till the ZBCP reappears. The anomalous splitting ZBCP can be modulated by the RSOC strength, magnitude of Fermi wave vector mismatch as well as insulator barrier strength. These peculiar features are ascribed to the novel spin–triplet Andreev reflection in the context of the RSOC, characteristic by the anisotropic $d$-wave pair symmetry combined with the relativistic nodal fermions, which in turn can be experimentally used to directly identify not only the proximity–induced ferromagnetism and RSOC but $d$-wave pair symmetry in graphene. These results pave the way to a new class of tunable, high-temperature superconducting spintronic devices based on large-scale graphene.

1. Introduction

For an unconventional $d$-wave superconductor (SC), its anisotropic pair potential could lead to such singular features as midgap surface states and a zero-bias conductance peak (ZBCP)—an experimental signature of anisotropic superconductivity observed by using scanning tunneling spectroscopy [1–5]. However, in the ferromagnet (FM)/$d$-wave SC hybrid structure, as the exchange field increases, the height of the ZBCP is suppressed with the increase of spin polarization in the FM, and only for fully polarized limit, the ZBCP evolves to a zero-bias conductance dip (ZBCD) [6–8]. The ZBCD has been experimentally observed in a La$_2/3$Ba$_1/3$MnO$_3$/DyBa$_2$Cu$_3$O$_7$ junction whose amplitude is shown as an attenuation function of temperature and/or magnetic field [9]. In the FM/$d$-wave SC junction with broken time reversal symmetry (BTRS) states near the surface of the SC, the ZBCP splits into two peaks at zero field, and the splitting becomes weak and disappears gradually with the interface roughness increased [10]. Moreover, in the clean FM/FM/$d$-wave SC double tunnel junctions, a novel AR exhibits due to noncollinear magnetizations, resulting in spin-triplet pairing states near the FM/SC interface. The resultant spin-triplet superconducting correlation gives rise to a conversion from a ZBCP to a splitting ZBCP under the situation of the highly polarized FM [11].

Graphene, a single layer of carbon atoms, has been attracting considerable interests for the past decade, which is characterized by not only the linear dispersion with zero density of states at the Fermi level $E_F = \hbar^2 |k|^2$ but also the two nonequivalent valleys (K and K’ point) in the two-dimensional (2D) hexagonal Brillouin zone [12]. Linder and Sudbø theoretically investigated the influence of an anisotropic order parameter induced in graphene on the tunneling conductance in graphene-based normal metal (NM)/insulator (I)/$d$-wave SC...
The proposed graphene-based FM/1/d-wave SC junctions with or without a BTRS in SC region have also been investigated theoretically, which reveals that the exchange filed $h$ only decreases the height of the ZBCP and the split is absent [14–17]. This is very different from that for the conventional FM/1/d-wave SC junctions [6, 10, 11]. It has been widely accepted that such high-$T_c$ SCs as YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO) have a d-wave pairing symmetry [18]. However, until recently, the fabrication of graphene/YBCO contacts and their superconducting proximity effect have been reported, where both the temperature dependence and the current bias dependence of resistance for the contacts are investigated [19], showing the indication of the tunneling of Cooper pairs from YBCO to the graphene. The ZBCP and so-called Klein-like tunneling of Andreev electron-hole pairs that carry superconducting correlations from the high-$T_c$ SC YBCO into graphene have also been experimentally observed very recently [20], as in the usual Klein tunneling with the reflectionless transmission of electrons across a high energy barrier. Moreover, the latest experimental findings show the realization of the intrinsic unconventional superconductivity in a ‘magic angle’ twisted bilayer which is created by stacking two sheets of graphene that are twisted relative to each other by a small angle [21].

Similarly owing to the proximity effect, not only the ferromagnetism but also Rashba spin–orbit coupling (RSOC) in graphene was realized in experiments [22–27]. Itinerant electrons in graphene have weak intrinsic spin–orbit coupling, because they are formed primarily from the $p_z$ orbital with zero orbital momentum. The decoration of the graphene surface with heavy metal atoms such as gold (Au), indium (In) or thallium (Tl) has been proposed to increase the RSOC. Recently, it was experimentally demonstrated that at the artificial interface between monolayer graphene and few-layer semiconducting tungsten disulphide (WS$_2$), the graphene acquires spin–orbit coupling up to 17 meV, three orders of magnitude higher than its intrinsic value [26]. The RSOC in monolayer graphene removes the spin degeneracy and creates a spin-splitting $2\lambda_{\parallel}$ at the $K$ and $K'$ points, but the energy splitting does not break the time-reversal symmetry, unlike the exchange splitting in FM. However, the RSOC may affect the tunneling conductance in RSOC/SC junctions since it mixes spin–up and –down states [28, 29]. The RSOC-induced spin-triplet AR occurs in the graphene-based FM/RSOC/s-wave SC junction, where the incident electrons and the reflected holes come from the same spin sub-band. This novel equal-spin AR results in nonvanishing superconducting triplet equal-spin pairings in the ferromagnetic region near the FM/RSOC interface [30]. In most previous theoretical works on the spin-triplet pairing in graphene-based FM/SC structures, the SC layer was assumed to be of s-wave [30–32]. However, the spin-triplet AR in the graphene-based d-wave SC junctions has not been studied so far. It has been demonstrated that, the anisotropic d-wave pair symmetry could be merged with the relativistic nodal fermions of graphene and the anomaly of the ZBCP as an experimental signature of the high-$T_c$ superconducting proximity effect in graphene was also exhibited [20, 13]. Since tunneling conductance spectra of the graphene-based FM/1/d-wave SC junctions have been investigated [14–17], it is highly desirable to study the ZBCP of graphene-based FM/RSOC/1/d-wave SC multilayered structure so as to find whether or not the triplet states induced by the interplay between RSOC and FM possess some new features on the ZBCP compared with the one without RSOC? And thus, the features may be an experimental signature for demonstrating the existence of proximity-induced RSOC and d-wave pair symmetry combined with the spin-triplet pairing states in graphene layers.

To this end, in this paper, we extend the Blonder–Tinkham–Klapwijk (BTK) theory [33] to study an graphene-based FM/RSOC/1/d-wave SC hybrid structure. It is shown that with the exchange energy $h$ increased, the ZBCP splits into two subpeaks from the beginning, accompanied by a ZBCD, and then from $h/E_g = 1$, the splitting ZBCP starts to gradually shrink till the non-splitting ZBCP reoccurs, which is thoroughly different from that for s-wave case or without the RSOC. The triplet equal-spin AR characteristic by the anisotropic d-wave pair symmetry combined with the relativistic nodal fermions, is the origin of the anomalous ZBCP splitting. And with the enhancement of RSOC strength, the subpeaks are gradually narrowed but heightened. We also find that the split of the ZBCP can be controllable by the magnitude of Fermi wave vector mismatch (FWVM) as well as insulator barrier strength $\chi$, particularly, the periodic oscillating modulation of $\chi$ is exhibited whereas the curve at one period is not symmetric. These results could provide direct evidence for the existence of not only the RSOC and ferromagnetism but also the d-wave pairing symmetry, which are all combined with the nodal relativistic low-energy Dirac fermions.

2. Formalism and theory

The proposed graphene-based FM/RSOC/1/d-wave SC junction in the $x - y$ plane is schematically shown in figure 1. Here, the FM ($x \leq -L$) and SC ($x > d$) regions could be manufactured by contacting graphene with a ferromagnetic electrode and a superconducting electrode, respectively. And the RSOC region ($-L \leq x \leq 0$) can be created by depositing the graphene on a WS$_2$ substrate, while the I region ($0 \leq x \leq d$), modeled by a barrier potential $V_0$, can be implemented by using either a gate voltage or local chemical doping.
The electron and hole excitations in the present hybrid structure can be described by the Dirac–Bogoliubov–de Gennes (DBDG) equation in the context of RSOC and exchange filed $h$ as follows [34]

$$
\begin{pmatrix}
H(x) - E_F & \Delta(x) \\
\Delta'(x) & E_F - H(x)
\end{pmatrix}
\begin{pmatrix}
u \\
v'
\end{pmatrix}
= E
\begin{pmatrix}
u \\
v'
\end{pmatrix}
$$

with $E$ being the quasiparticle's energy relative to the Fermi energy $E_F$ and the spinor basis in the Nambu space $(\nu, v)^T = [(\psi_{A, \tau}^0, \psi_{B, \tau}^0, \psi_{A, \tau}^1, \psi_{B, \tau}^1, -\psi_{A, \tau}^2, \psi_{B, \tau}^2, -\psi_{A, \tau}^3, \psi_{B, \tau}^3)]^T$, where the index $\tau = +(-)$ with $\tau = -\tau$ indicates the two so-called valleys of $K(K')$, the arrow index $(\uparrow, \downarrow)$ corresponds to real spin, $A$ and $B$ denote the two trigonal sublattices [29, 35], and the single particle Hamiltonian is given by

$$
H(x) = H_0 + \begin{cases}
H_{FM} = -(s_z \otimes \sigma_0)h, & x \leq -L, \\
H_{RSOC} = \lambda (s_y \otimes \sigma_y - \tau s_x \otimes \sigma_y), & -L \leq x \leq 0, \\
H_r = V_{0s0} \otimes \sigma_0, & 0 \leq x \leq d, \\
H_{SC} = -U_0s0 \otimes \sigma_0, & x \geq d.
\end{cases}
$$

In equation (2), $H_0 = \hbar v_F s_0 \otimes (s_z k_x + \tau s_y k_y)$ is a two dimensional Dirac Hamiltonian with a linear dispersion at low energies, where $v_F$ is the Fermi velocity in graphene, $k_x$ and $k_y$ are components of wave vector in the $x$ and $y$ directions, and $\sigma_i$ and $s_i$ represent the Pauli matrices acting in sublattice space ($\sigma_0$ and $s_0$ are the $2 \times 2$ unit matrices). The ferromagnetic region is assumed to be additionally described by the Stoner band model with the magnetic exchange field $h$. $\lambda$ is the energy scale of spin–orbit coupling and $U_0$ of the last term refers to the electrostatic potential. Here, the decay of the order parameter in the vicinity of the interface is assumed to be neglected, and the orientational dependence of superconducting gap can be written as $\Delta(\theta_i) = \Delta_0 \cos(2\theta_i - 2\alpha)e^{i\phi_0} \Theta(x - L)$, where $\theta_i$ is the angle between the RSOC/SC interface normal and the momentum of the quasiparticle, $\alpha$ is the angle between the $a$ axis of the crystal and the interface normal, and $\phi_0$ is the macroscopic phase of SC. The temperature dependence of superconduction pairing potential $\Delta$ is given by $\Delta(T) = \Delta_0 \tanh(1.74\sqrt{T_c/T - 1})$ with $T_c$ the critical temperature of SC and $T = 0.01T_c$ throughout our calculations. The energies and lengths are respectively normalized by the superconducting gap at zero temperature $\Delta_0$ and the superconducting coherent length $\xi_s = \hbar v_F/\Delta_0$. To satisfy the mean field approximation in the SC region, the fermi wave vector in the SC can be expressed by $k_F^F = E_F + U_0$, where $U_0$ is used to tune the FWVM between the FM and SC regions. We can solve the DBDG equation to obtain the eigenfunctions of each region, which are given in the appendix.

For an incident electron with spin-up ($\uparrow$) at $x = 0$ from the FM region at an angle $\theta$ with energy $E$, the wave functions in the different regions are constructed from the linear combination of corresponding eigenfunctions. For the FM region, the wave function is given by

$$\psi_{FM}(x) = \psi_{\uparrow, \uparrow}^F + r_{\uparrow, \uparrow}^I \psi_{\downarrow, \uparrow}^F + r_{\uparrow, \uparrow}^N \psi_{\uparrow, \uparrow}^N + r_{\uparrow, \uparrow}^R \psi_{\uparrow, \uparrow}^R + r_{\uparrow, \uparrow}^D \psi_{\uparrow, \uparrow}^D,$$

where the coefficients $r_{\uparrow, \uparrow}^I$, $r_{\uparrow, \uparrow}^N$, $r_{\uparrow, \uparrow}^R$, and $r_{\uparrow, \uparrow}^D$ correspond to the probability amplitudes of normal reflection, normal reflection with spin flip, novel AR, and usual AR processes, respectively. And the wave functions in the RSOC and I regions are respectively.
\[ \psi^{\text{RSOC}}(x) = a_1 \psi_{e,c,+}^{R,+} + a_2 \psi_{e,c,+}^{R,-} + a_3 \psi_{h,c,+}^{R,+} + a_4 \psi_{e,c,-}^{R,-} + a_5 \psi_{h,c,-}^{R,+} + a_6 \psi_{h,c,+}^{R,-} + a_7 \psi_{h,c,+}^{R,-} + a_8 \psi_{h,c,-}^{R,-} \]  

(4)

and

\[ \Psi^i(x) = b_1 \psi_{e,c,1}^{I,+} + b_2 \psi_{e,c,1}^{I,-} + b_3 \psi_{e,c,2}^{I,+} + b_4 \psi_{e,c,2}^{I,-} + b_5 \psi_{h,c,1}^{I,+} + b_6 \psi_{h,c,1}^{I,-} + b_7 \psi_{h,c,2}^{I,+} + b_8 \psi_{h,c,2}^{I,-} \]  

(5)

with \( a_n \) and \( b_n \) (\( n = 1 \)–8) indicating the propagating probability amplitudes of electrons and holes in the RSOC and I regions, respectively. For the SC region, the wave function is \( \Psi^{\text{SC}}(x) = t_1 \psi_{e,c,1}^{S,+} + t_2 \psi_{e,c,1}^{S,-} + t_3 \psi_{h,c,1}^{S,+} + t_4 \psi_{h,c,1}^{S,-} \) with the transmission probability amplitudes denoted by \( t_i \) (\( i = 1 \)–4). All the coefficients will be determined by matching the boundary conditions \( \Psi^{\text{FM}}(x)\big|_{x=-L} = \Psi^{\text{SC}}(x)\big|_{x=-L} \), \( \Psi^{\text{RSOC}}(x)\big|_{x=0} = \Psi(x)\big|_{x=0} \), and \( \Psi^i(x)\big|_{x=a} = \Psi^{\text{SC}}(x)\big|_{x=a} \). For spin-down (\( \downarrow \)) electrons incident on the interface at \( x = -L \), the coefficients can be similarly obtained by the DBDG equation and boundary conditions. After attaining the scattering coefficients, one can extend the BTK theory to calculate the tunneling conductance via

\[ G(eV) = \sum_\sigma G_{\sigma} \int d\theta \cos \theta \left( 1 - |r_{\sigma}^n|^2 \right) \frac{\cos \theta}{\cos \theta_0} (|s_{\sigma}^n|^2 + |s_{\sigma}^A|^2), \]  

(6)

where \( \sigma \) represents \( \uparrow \) or \( \downarrow \), \( \sigma \) is opposite to \( \sigma \), and \( G_{\sigma} = (2e^2/h)N_{\sigma}(eV) \) with \( N_{\sigma}(eV) = |eV + E_F + \sigma h|W/(eV \pi) \) the density of states in the graphene sheet of width \( W \), is the normal-state conductance for the incident spin-\( \sigma \) electron. The spin polarization degree \( P \) defined by \( (N_{\uparrow} - N_{\downarrow})/(N_{\uparrow} + N_{\downarrow}) \) in the usual FM reduces to \( h/E_F \) for \( h < E_F \) while \( E_F/h \) for \( h > E_F \) in the low bias regime \( (E_F \gg E) \) of the ferromagnetic graphene.

3. Results and discussions

3.1. An anomalous splitting ZBCP

The amplitude of the pair potential disappears for \( \alpha = \pi/4 \) (the \( x \) axis along the \{110\} direction, corresponding to a node in the \( d \)-wave superconducting order parameter), then in this context, the \( d \)-wave pair symmetry could lead to a sizable areal density of midgap states, which is the origin of the ZBCP observed in most high-\( T_c \) SC junctions [1, 2]. And thus the ZBCP is an experimental signature to identify the symmetry of the pair potential. In numerical calculations, we take \( E_F = 100 \Delta_0 \) and \( \alpha = \pi/4 \).

Now we turn our attention to the RSOC combined with the nodal relativistic low-energy Dirac fermions and anisotropic pair symmetry in a graphene-based FM/RSOC/\( d \)-wave SC junction. Figure 2 shows the normalized conductance \( G/G_0 \) at \( \lambda = 4 \Delta_0 \). For the FM/RSOC/\( d \)-wave SC structure with \( h = 0 \) given by the dotted line in figure 2(a), although a sharp ZBCP is observed, which is narrower compared with the one without the RSOC [13] and indicates that the AR is suppressed by the RSOC, no anomaly in the ZBCP is exhibited since the RSOC keeps time reversal symmetry. With increasing exchange field from \( h/E_F = 0 \) to \( h/E_F = 1 \), the ZBCP starts to turn into a splitting one with a ZBCD sandwiched in between two sub-peaks, as shown in figure 2(a), which could be just clearly observed from the small \( h \), namely \( 0.3E_F \). And not only the subpeaks but also the ZBCDs are widened with the enhancement of \( h \), companied by the gradual decrease of the zero-bias conductance for the ZBCD up to zero at \( h = E_F \). This is thoroughly different from that without RSOC in [14–17], where the ZBCPs always exist with its peak height modulated by \( h \). It follows therefore that the splitting ZBCP is originated from the interplay between \( h \) and RSOC, which induces triplet equal-spin AR or novel AR between the FM and SC with the incident electrons and the reflected holes coming from the same spin subband. The property of splitting for the ZBCP with the exchange field \( h \) increased is a little similar with that in the FM/FM/\( d \)-wave SC junctions with noncollinear magnetizations [11]. They both originate from the spin triplet pairing state induced by the BTRS, corresponding to the novel ARs. However, the mechanism of leading to the spin triplet one for the former is due to the RSOC combined with the nodal relativistic low-energy Dirac fermions, while that for the latter is thanks to the noncollinear magnetizations. And thus, this results in a considerable difference in the property that the ZBCP can split apparently for the former only under low spin polarization but for the latter only in the context of fairly high spin polarization.

For the exchange field \( h > E_F \) as shown in figure 2(b), however, with \( h \) increased, both the two subpeaks and ZBCD are narrowed and the zero-bias conductance is gradually increased at the same time, which is just contrary to the case for \( h < E_F \). When \( h \) is increased to a certain value, the two sub-peaks and ZBCD reach to zero width, and thus evolve into a sharp ZBCP as shown in the dotted line of figure 2(b) for \( h = 3E_F \). This stems
from the spin-polarized Andreev–Klein reflection, in which the AR of massless Dirac fermions is associated with the Klein tunneling through an exchange field p–n barrier [36]. This spin-polarized spin Klein tunneling leads to an enhancement of this specular AR for $h$ increased above $E_F$. As a result, the equal-spin AR is highly suppressed and the specular AR is dominated at $h > 2E_F$, leading to that the splitting of ZBCP is vanished. It is only in such ferromagnetic graphene $d$-wave SC junction that there can be this feature of splitting on the ZBCP induced by varying $h$. The feature is considerably distinctive since the situation $h > E_F$ can be easily actualized through proximity effect only in such 2D Dirac materials as graphene by experiment, bringing about the novel spin triplet AR accompanied by the Klein tunneling. In addition, the $d$-wave SC junctions based on the topological insulator (TI) are also investigated [37]. It is confirmed that only for the direction of the magnetization $m$ parallel to $z$ axis, the ZBCP is split upon increasing $m_z$. This strong sensitivity to the direction of $m$ is a new feature compared to the topologically trivial case which pertains directly to the anomalous band structure of the TI, specifically, $m_z$ opens the energy gap in the ferromagnetic region. This is thoroughly distinct from the situation for the ferromagnetic graphene with the gapless spin-up (down) subband, where the normal Fermi level at Dirac point ($E_F = 0$) is shifted upward (downward) for the spin-up (down) subband by the exchange field $h$, making the up- and down-spin carriers be electronlike n-type and holelike p-type, respectively. Particularly, with increasing the exchange field $h$ at the low energies, the evolution from the splitting ZBCP (the ZBCD) to the sharp ZBCP cannot be normally exhibited in the TI-based junction.

3.2. Novel spin-triplet AR due to the RSOC

In what follows, indeed it is extremely essential for us to study in detail how the triplet correlations affect the above-obtained feature on the ZBCP with varied $h$. For the total conductance given by equation (6), there exist two predominant parts, the novel AR contribution $G_{NAR} = 2 \int d\theta \cos \theta (|G_{r_{\uparrow}}|^2 + |G_{r_{\downarrow}}|^2)$ closely related to the formation of the spin rotation induced triplet component and the usual AR contribution $G_{AR} = 2 \int d\theta \cos \theta (|G_{r_{\uparrow}}|^2 \cos \theta_{\uparrow}^f + |G_{r_{\downarrow}}|^2 \cos \theta_{\downarrow}^f / \cos \theta_{\uparrow}^f)$, which are respectively shown in figures 3(a) and (b), belonging to the conductance in figure 2(a). For $h = 0$ corresponding to the NM/RSOC/d-wave SC hybrid structure, $G_{AR}$ is of maximum, forming a zero-bias peak, while $G_{NAR}$ vanishes, indicating no spin-triplet correlation as expected. With $h$ increased in the region of $h < E_F$, the curve of $G_{AR}$ shrinks with both the width

![Figure 2. The normalized charge conductance versus $eV/\Delta_0$ for different values of $h/E_F$ in graphene-based FM/RSOC/d-wave SC junction. Here, we set $E_1 = E_0 = 100\Delta_0, \lambda = 4\Delta_0, \alpha = \pi/4, L = 0.1\xi_S, d = 0, (a) h < E_F$ and (b) $h > E_F$.](image-url)
and height of the peak decreased due to the spin splitting energy \( h \) of FM, which will induce an extra momentum change \( 2\hbar /v_F \) of the reflected hole and thus diminishes the amplitude of AR. In contrast, with the increase of \( h \), the \( G_{\text{NAR}} \) increases and simultaneously produces two subpeaks separated by a zero-bias dip, whose curve is inflated with both the width and height of the subpeaks enhanced and the zero-bias conductance being always zero. At \( h/E_F = 1 \), the usual AR completely disappears and the novel AR gets the strongest, meaning the fully spin triplet and no spin singlet states. That \( G_{\text{NAR}} \) splits across at \( E = 0 \) arises from the interplay between the RSOC and BTRS induced by the exchange field. The behavior of \( G_{\text{NAR}} \) the same as that of the total conductance is a perfect signature that the novel AR is the origin of the ZBCP splitting. For \( h > E_F \), the outlines of the curves for the \( G_{\text{NAR}} \) and \( G_{\text{AR}} \) are the same as those for the \( h < E_F \), however with the increase of \( h \), the varied behavior of them are exactly contrary to those for the \( h < E_F \), which are not presented here for simplicity. The experimental observation of the splitting of ZBCP with varied \( h \), particularly for \( h > E_F \), will provide direct evidence for the existence of not only the RSOC but also the \( d \)-wave pairing symmetry both combined with the nodal relativistic low-energy Dirac fermions. Here, for \( h/E_F = 0 \), the FM region is reduced to a normal one without spin-splitting, and due to the formation of zero-energy states in the graphene-based \( d \)-wave SC junction, the perfect AR is produced at \( eV/\Delta_0 = 0 \), as a result, one can observe a ZBCP with twice the normal-state conductance as shown in figures 2(a) and 3(b), which normally appears in relative \( d \)-wave SC junctions [10–17].

In order to understand the anomalous zero-bias conductance and other features, it is necessary for us to present the four backscattering or reflection probabilities \( |r|_1^2, |r|_2^2, |r|_3^2 \) and \( |r|_4^2 \) versus the incident angle \( \theta \) at the FM/RSOC interface, as shown in figures 3(c) and (d) for an incident particle with spin-up, which are equivalent to the four transmission probabilities in determining the conductance. In figure 3(c) for \( h/E_F = 0.5 \), it is noticed that \( |r|_4^2 \) at smaller \( \theta \) approaches zero, while the situation for the \( |r|_3^2 \) is contrary. And at \( \theta \) beyond a critical value \( \theta_c = \arcsin(k_F^0/k_3^0) \), \( |r|_3^2 \) dramatically decreases to 0, indicating the corresponding process is no longer involved in the transport. In figure 3(d) for \( h/E_F = 1 \), we find that the novel AR dominates and its maximum value of \( |r|_3^2 \) can reach 0.9, while the usual AR is thoroughly forbidden due to the absence of the spin-down carriers in the half-metallic FM region. Therefore, the results for \( |r|_3^2 \) and \( |r|_4^2 \) with different \( h/E_F \) can explain the features of \( G_{\text{NAR}} \) and \( G_{\text{AR}} \) shown in figures 3(a) and (b), respectively, and then the ones of \( G \) in figure 2(a). Furthermore, compared with \( |r|_1^2, |r|_2^2 \) is large, especially, the former is also thoroughly suppressed for the half-metallic graphene case, however, they only contribute to modulation in the magnitude of \( G \). It follows that, \( |r|_4^2 \) can be generally enhanced by increasing \( P \), and consequently, the splitting of the ZBCP will be strengthened. As \( h/E_F > 1 \), the normal reflection with spin-flip and usual AR reappear. With increasing \( h \) from

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**Figure 3.** (a) The conductance \( G_{\text{NAR}} \) from novel AR contribution and (b) the conductance \( G_{\text{AR}} \) from usual AR contribution. Reflection probabilities versus incident angle \( \theta \) with spin-up at \( eV/\Delta_0 = 0.5 \) for (c) \( h/E_F = 0.5 \) and (d) \( h/E_F = 1 \). Here, the parameters are chosen to be the same as in figure 2.
\( h/E_F = 1 \), generally, the usual AR is strengthened while the novel AR is suppressed although they have different scattering probabilities for the different incident angle \( \theta \), particularly, at \( h/E_F \gg 1 \), the novel AR is completely suppressed, and thus the equal-spin pairing states are absent, which are just contrary to the situation for \( h/E_F < 1 \) and could similarly illustrate the futures of figure 2(b). The figures for the probabilities of the corresponding four backscattering processes at \( h/E_F > 1 \), which are accompanied with the so-called spin Klein tunneling, are not presented here for simplicity. It is pointed out here that, the total probability of the different reflection processes does not always hold due to the quasiparticle’s tunneling into the energy gap nodes of \( d \)-wave SC, which normally takes place in the relative \( d \)-wave SC hybrid structures and is much different from that for the \( s \)-wave case.

### 3.3. Influence of RSOC and FWVM

To get deep insight into the influence of the RSOC on the spin triplet correlations dominating the conductance features, we show the conductance spectra for a graphene-based ferromagnetic half metal/RSOC/\( d \)-wave SC tunnel junction as a function of \( eV/\Delta_0 \) in figure 4(a), where different values of RSOC strength \( \lambda \) are taken. This choice originates from considering that in the structure, the spin-singlet correlations are completely suppressed, as a result, the conductance is thoroughly determined by contribution from the spin triplet correlations, as above-mentioned, and its properties are the same as those of the \( G_{NAR} \); particularly, the zero-bias conductances for both of them are zero. In the absence of RSOC, a ZBCD without splitting is exhibited in figure 4(a) as shown by the solid line. As \( \lambda \) increases, the width of the subpeaks decreases while the height increases, which indicates that the splitting and novel ARs can be enhanced by increasing \( \lambda \) and is much different from that induced by varying \( h \). It is found that the RSOC cannot give rise to the variation of the features, and the curve keeps a splitting ZBCP with the zero-bias conductance being zero all through. Furthermore, it is shown that in a similar manner, for an anomalous Rashba metal/\( d_{xy} \)-wave SC junctions, the RSOC not only retains but also heightens the ZBCP by the enhancement of \( \lambda \), which contrasts sharply with the suppression of the ZBCP in a fully polarized ferromagnet metal/\( d_{xy} \)-wave SC junctions [38].

The effect of FWVM \( E_F' = E_F \) between the FM and SC regions on the the spin triplet correlations for the same structure, the graphene-based ferromagnetic half metal/RSOC/\( d \)-wave SC tunnel junctions, is also investigated, as illustrated in figure 4(b). It is shown that the feature of a splitting ZBCP with the zero-bias...
conductance being zero cannot be varied by the FWVM strength $E'_f/E_F$ and the behavior of the curve with the increase of $E'_f/E_F$ is the same as that induced by the increase of $h(>E_F)$, i.e. the curve shrinks, except for a little difference in magnitude of variation. The mechanism can be ascribed to the $d$-wave pair potential as a function of the angle $\theta_s$ between the quasiparticle wave vector and the interface normal, given by

$$d = \frac{D}{\cos^2 \frac{\theta_s}{2} - \sin^2 \frac{\theta_s}{2}}.$$ 

For $\alpha = \pi/4$, specifically, in the presence of FWVM, $\theta_s$ is suppressed, and thus the corresponding effective energy gap dependent on $\theta_s$ is reduced.

3.4. Interplay between the barrier strength and RSOC

Next, we investigate the impact of the interplay between the barrier strength and RSOC on splitting of the ZPCP for $\lambda = 4\Delta_0$. The barrier strength $\chi = V_0 d/\hbar v_F$, a dimensionless parameter, is used to describe the I layer, which is assumed much thin, i.e. $V_0 \to \infty$ and $d \to 0$, such that $\chi$ remains finite. Under the case of $\theta_f \to 0$ and $\theta_i \to 0$, one can attain $-k_i^d = (E - E_f + V_0) d/\hbar v_f \to \chi$ and $k_i^d = (E - E_f + V_0) d/\hbar v_f \to \chi$. Based on the same reason as in the above discussion, only for the graphene-based ferromagnetic half metal/RSOC/I/d-wave SC tunnel junction, do we present the voltage dependence of the normalized conductances with several values of $\chi$ in figure 5(a).

Figure 5. The normalized charge conductance in a graphene-based ferromagnetic half metal/RSOC/I/SC tunnel junction (a) versus $eV/\Delta_0$ for different values of barrier strength $\chi$ and (b) as a function of $\chi$ at $eV/\Delta_0 = 0.5$, where the other parameters are the same as in figure 2.

It is obvious that the zero-bias conductance is insensitive to any increase in $\chi$ due to the zero energy state formed at the junction interface [2, 3]. However, the variation of the non-zero bias conductance is non-monotonic with increasing $\chi$ from 0 to $\pi/2$ and is thoroughly different from the results of [13–17] for the absence of RSOC, where the conductance monotonously decreases. In order to unveil such a non-monotonic property, in figure 5(b), we give the plot of the conductance under the situation of $eV/\Delta_0 = 0.5$ as a function of $\chi$, which is corresponding to figure 5(a). It is clear seen that the $\pi$ periodic oscillating behavior is exhibited but anomalous compared with that in the graphene-based FM/I/d-wave SC junction [13–17], where the RSOC and novel spin-triplet AR are absent. The $\pi$ periodic oscillation is corresponding to the so-called Klein-like tunneling, which is originated from the relativistic low-energy Dirac fermions in graphene. In contrast with that in [13–17], the maximum of the conductance for $eV/\Delta_0 = 0.5$ is located at neither $\chi = n\pi$ nor $\chi = (n + 1/2)\pi$ with $n$ an integer (i.e. $n = 0, 1, 2, \ldots$). More importantly, the oscillating behaviors are not symmetric about the $\chi = n + \pi/2$ any more in the range from $n\pi$ to $(n + 1)\pi$ for $\chi$. The appearance of such an
anomalous oscillating behavior for the graphene-based ferromagnetic half metal/RSOC/SC junction in figure 5(b) is attributed to the novel AR due to the RSOC, which can offer direct evidence for the existence of not only the RSOC but also the $d$-wave pairing symmetry both combined with the nodal relativistic low-energy Dirac fermions.

3.5. Influence of tunable chemical potential

Since graphene has a tunable chemical potential or Fermi energy, being easily shifted in a controllable fashion (by doping or a gate voltage), it is better for us to investigate its influence on the conductance spectra together with anomalous spin-triplet pairing states in this structure. In figure 6 is illustrated the conductance spectra for the half-metallic FM case at different Fermi levels $E_F/\Delta_0 = 30, 50, 70, 100, \text{ and } 120$. As expected, the modulations of $E_F$ could not lead to the change of features. However, nonmonotonic behavior of the conductance spectra with $E_F$ is displayed, specifically, the height of the subpeak for spitting ZBCP first increases and then decreases from $E_F/\Delta_0 = 70$. Accordingly, the splitting of the ZBCP first strengthens and then weakens. Hence, the equal-spin triplet pairing states in this graphene-based $d$-wave SC junction are also controllable by tuning the Fermi levels, which is consistent with those of [30, 31].

3.6. Experimental realization

Finally, we comment the execution of the present graphene-based FM/RSOC/SC tunnel junction. The proximity-induced $d$-wave superconductivity in graphene could be attained by YBCO films grown on SrTiO$_3$ [19, 20]. As the YBCO films own very high $T_c$, one can study the proximity effect of graphene above the liquid nitrogen temperature, which favors the easily experimental performance. Particularly, it is shown that for the graphene used as a prototypical 2D system, its coupling to the model magnetic insulator (EuS) produces a substantial magnetic exchange field ($>14$ T) with the potential to reach hundreds of tesla [22]. Furthermore, recent developments have achieved large proximity-induced ferromagnetism and spin–orbit interactions in CVD grown graphene single layers coupled to an atomically flat yttrium iron garnet [27]. For the realization of our prediction, i.e. the splitting ZBCP, $h > 0.2E_g$ and $\lambda > 4\Delta$ are required. In terms of realistic parameters, we may choose the typical Fermi energy in graphene for the undoped case $E_g = 100$ meV, and the superconducting energy gap $\Delta = 1$ meV. To ensure that the splitting of ZBCP can be observed, we need the exchange filed and RSOC being at least $h \sim 20$ meV and $\lambda \sim 4$ meV, respectively, which can be achieved by the present experimental techniques. Thus, the fabrication of the present graphene hybrid structure is experimentally feasible and the anomalous ZBCPs could be probed by applying the scanning tunneling spectroscopy. Here, it is pointed out that the RSOC interaction can be also induced by an external electric field in this set-up. For the ideal graphene or the zero-buckling case, $\lambda$ increases linearly with field strength, even for the buckled graphene [39, 40]. $\lambda$ is about 50 $\mu$eV at 1.0 V Å$^{-1}$ for the ideal case, therefore, the electric field 80 V Å$^{-1}$ is needed to tune $\lambda$ up to 4 meV in the present work, while for 1% buckled graphene, only the electric field 1.0 V Å$^{-1}$ is required.
4. Conclusions

In summary, an extended BTK theory is utilized to investigate a graphene-based FM/RSOC/1/d-wave SC junction, which is found to exhibit anomalous splitting ZBCPs. We show that within the d-wave gap along the \{110\} direction, with increasing exchange energy from $h/E_F = 0$, the ZBCP begins to split into two subpeaks with a ZBCD sandwiched in between them, while from $h > E_F$, both the two subpeaks and ZBCD start to turn narrowed and finally evolve into a ZBCP. With increasing the strength of the RSOC, the width of the subpeaks is decreased but the height increased, whereas with FWVM, they simultaneously decrease, i.e. the curve shrinks. In particular, the periodic oscillating modulation for $\chi$ is shown but the curve at one period is not symmetric.

However, the modulations of the RSOC, FWVM, and barrier strength could not give rise to the variation of the features, for instance, a splitting peak with the zero-bias conductance being zero for ferromagnetic half metal case of graphene is remained all the time. All these peculiar features can be also used to experimentally identify and confirm not only the RSOC but also the $d$-wave pairing symmetry in graphene induced by proximity. It is expected that our findings will be demonstrated by the future experiments and lead to great interest in the designing and fabrication of graphene superconducting spintronic devices.

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Appendix

After solving the DBDG equation, we can obtain the eigenfunctions of each region. For the FM region, the eigenfunctions are

$$\psi_{\text{FM}}^{F_{\pm}}(x) = (1, \pm e^{\pm i\phi_{\pm}^F}, 0, 0, 0, 0, 0, 0)^T e^{\pm i k_{x_{\pm}}^{F} y_{\pm}},$$
$$\psi_{\text{FM}}^{E_{\pm}}(x) = (0, 0, 1, \pm e^{\pm i\phi_{\pm}^E}, 0, 0, 0, 0)^T e^{\pm i k_{x_{\pm}}^{E} y_{\pm}},$$
$$\psi_{\text{FM}}^{F_{\pm}}(x) = (0, 0, 0, 0, 0, 1, \mp e^{\pm i\phi_{\pm}^F}, 0, 0)^T e^{\pm i k_{x_{\pm}}^{F} y_{\pm}},$$
$$\psi_{\text{FM}}^{E_{\pm}}(x) = (0, 0, 0, 0, 0, 1, \mp e^{\pm i\phi_{\pm}^E}, 0, 0)^T e^{\pm i k_{x_{\pm}}^{E} y_{\pm}}.$$  \(7\)

Here, the $x$ components of the corresponding wave vectors are given by

$$k_{x_{\pm}}^{F} = (\hbar v_F)^{-1}(E + E_F + h) \cos \theta_{\pm}^F,$$
$$k_{x_{\pm}}^{E} = (\hbar v_F)^{-1}(E + E_F - h) \cos \theta_{\pm}^E,$$
$$k_{x_{\pm}}^{F_h} = (\hbar v_F)^{-1}(E - E_F - h) \cos \theta_{\pm}^{F_h},$$
$$k_{x_{\pm}}^{E_h} = (\hbar v_F)^{-1}(E - E_F + h) \cos \theta_{\pm}^{E_h}.$$  \(8\)

and the propagation angles can be expressed by

$$\theta_{F}^F = \arcsin \frac{\hbar v_F k_y}{E + E_F + h},$$
$$\theta_{E}^F = \arcsin \frac{\hbar v_F k_y}{E + E_F - h},$$
$$\theta_{F}^E = \arcsin \frac{\hbar v_F k_y}{E - E_F - h},$$
$$\theta_{E}^E = \arcsin \frac{\hbar v_F k_y}{E - E_F + h}.$$  \(9\)

where the junction width $W$ is assumed large enough so that the $y$ component of the wave vector $k_y$ is a conserved quantity upon the scattering. And the eigenfunctions for RSOC region are given by
where

\[ \theta^e = \arcsin \frac{\hbar v_\parallel k_y}{(E_f + E)g^e} \]
\[ \theta^h = \arcsin \frac{\hbar v_\parallel k_y}{(E_f - E)g^h} \]
\[ k_{1e}^{R,e} = (\hbar v_f)^{-1}(E_f + E)g^e \cos \theta^e \]
\[ k_{1h}^{R,h} = (\hbar v_f)^{-1}(E_f - E)g^h \cos \theta^h \]

and the definition of auxiliary parameters are \( g^e = \sqrt{1 + 2c_\lambda(E_f + E)^{-1}} \) with \( \theta^e = \pm \theta^h \) the electron and hole propagation angles in the region with spin–orbit interaction.

For the I region, the corresponding eigenfunctions can be expressed by

\[ \psi_{1e}^{I,e}(x) = (1, \pm e^{\pm i\theta_f}, 0, 0, 0, 0, 0, 0)^T e^{\pm ik^1_1 x} \]
\[ \psi_{1e}^{I,h}(x) = (0, 0, 0, 0, 0, 0, 0, 0)^T e^{\pm ik^1_1 x} \]
\[ \psi_{1h}^{I,h}(x) = (0, 0, 0, 0, 0, 0, 0, 0)^T e^{\pm ik^1_1 x} \]

The eigenfunctions for SC region can be obtained as

\[ \psi_{1e}^{S,e}(x) = (u(\theta_1), u(\theta_1)e^{i\theta_f}, 0, 0, \nu(\theta_1)e^{-i\theta_f}, \nu(\theta_1)e^{i\theta_f - i\theta_f}, 0, 0)^T e^{i\theta_1 \cos \theta^e} \]
\[ \psi_{1h}^{S,e}(x) = (0, 0, \nu(\theta_1), \nu(\theta_1)e^{i\theta_f}, 0, 0, \nu(\theta_1)e^{i\theta_f - i\theta_f}, 0)^T e^{i\theta_1 \cos \theta^h} \]
\[ \psi_{1h}^{S,h}(x) = (\nu(\theta_1), \nu(\theta_1)e^{i\theta_f}, 0, 0, u(\theta_1)e^{-i\theta_f}, u(\theta_1)e^{i\theta_f - i\theta_f}, 0, 0)^T e^{i\theta_1 \cos \theta^h} \]

where the wave vectors for the quasiparticle are given by \( k_{S,e} = \left[ E_f^e + \sqrt{E^2 - |\Delta(\theta_1)|^2} \right] / \hbar v_\parallel \) and \( k_{S,h} = \left[ E_f^e - \sqrt{E^2 - |\Delta(\theta_1)|^2} \right] / \hbar v_\parallel \), the coherence factors are, as usual, written as

\[ u(\theta) = \sqrt{1/2 + \sqrt{E^2 - |\Delta(\theta)|^2} / 2E} \]
\[ \nu(\theta) = \sqrt{1/2 - \sqrt{E^2 - |\Delta(\theta)|^2} / 2E} \]

\( \theta^* = \beta^* = 0 \) with \( \theta^e = \theta^h \) and \( e^{-i\theta_1} = e^{-i\theta_1} \Delta(\theta_1) / |\Delta(\theta_1)| \).

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