Reliability of a capacitated system with parallel subsystems considering performance sharing mechanism

Ye Ma¹, Di Wu², Kaiye Gao¹, Yingchun Li¹, Rui Peng¹

¹School of Economics and Management, Beijing University of Technology, Beijing, China
²School of Management, Xi’an Jiaotong University, Xi’an, China
³School of Economics and Management, Beijing Information Science & Technology University, Beijing, China

Corresponding Author: Kaiye Gao. Email Address: kygao@foxmail.com.

Abstract. Performance sharing mechanism is widely applied in energy systems, computing systems and other types of industrial systems to increase system reliability. In reality, some systems may allow certain extent of performance deficiency instead of requiring every subsystem in the system satisfy its demand. Moreover, the subsystems in systems may combine with amounts of components, say that, a series of generators should be simultaneously used to provide the necessary energy for a given area. In this paper, we consider a system with performance sharing mechanism where the system remains functional as long as the sum weighted deficiency for subsystems after performance sharing is smaller than a predesigned reliable threshold. The amount of shared performance is further assumed to restricted by the bandwidth capacity of the system. We further consider the case where the surplus can be redistributed to maximize system reliability. Universal generating function technique is employed to model the reliability of the system and numerical examples are provided to illustrate the applications.

Keywords. Reliability, universal generating function, performance sharing mechanism, performance, generators

1. Introduction

Recently, there are amounts of studies concentrating on the reliability modelling of industrial systems with performance sharing mechanism (Xiao et al., 2015). Previous research widely assumed that the system will only function if all subsystems satisfy their demand individually (Peng et al., 2016). In fact, the surplus performance can be transmitted through a given bandwidth to the subsystems with deficiency for the sake of maximizing the reliability of whole system. Taking the energy system as an example, there will be a series of subsystems (areas) in an energy system allocated with different weights. The
goal of the system is to minimize the summed weighted deficiency of all subsystems in the system. The system is regarded as functional as long as the summed weighted deficiency is lower than some pre-specified threshold. Therefore, the surplus performance can be transmitted from low weight area to high weight area in order to make the system remain functional in case some subsystems suffer power deficiency. Similar examples can be found in production system, computing system and other types of engineering systems. Moreover, there will be amounts of components in the subsystem. For instance, in the energy system, areas should be equipped with amounts of generators to provide the necessary energy.

In light of this, this paper considers the reliability of a performance sharing system where the system fails if and only if the summed weighted deficiency of all subsystems is greater than a predesigned reliability threshold. Different from Wu et al. (2019) where each subsystem has only one component, the system is assumed to consist of several subsystems in parallel with different weights where each subsystem combines numbers of generators (components). The performance of a given subsystem equals to the summation of performance of all generators.

In addition, each generator has a random performance and each subsystem has a random demand to be satisfied. With the introduction of performance sharing bandwidth, the system will still be regarded as functional if the sum of weighted deficiency can be made to be lower than the threshold after certain types of redistribution strategies. Follow previous literature (M. El Falou et al,2009), we similarly employ universal generating function (UGF) to model the state distribution of generators, subsystems and system, based on which the system reliability is formulated.

The rest of this paper is organized as follows. Section 2 sets up the model. Section 3 introduces the application of UGF and discusses different performance sharing strategies. Section 4 presents numerical examples and Section 5 concludes.

2. Model Setup
Consider a system with $n$ subsystems in parallel with different weights $W_j, 0 < j \leq n$. For each subsystem $i$, there will be $n_i$ generators simultaneously working to provide the necessary energy. For simplification, we assume each subsystem has a random discrete demand $D_j$ with given probability mass function (PMF) and each generator belonging to subsystem $j$ has a random discrete performance $G_{jk}, 0 < k \leq n_j$ with given PMF. The predesigned reliability threshold of the system is assumed to be $T$ and the bandwidth of the performance sharing mechanism is denoted as $C$ with given PMF. To simplify the calculation, we use $m$ to denote the amounts of subsystems that satisfy their demand through their own generators’ performance and $n-m$ to represent the amounts of subsystems that suffer from performance deficiency. The system works as follows. If the minimal value between summed weighted deficiency of $n-m$ subsystems is smaller than or at least equal to $T$ even without performance sharing mechanism, the system is reliable. Otherwise, if the summed weighted deficiency of $n-m$ subsystems can be made to be smaller than or at least equal to $T$ after performance sharing, the system is also regarded as reliable. In this paper, the surplus redistribution mechanism proposed in paper Wu et al. (2019) is used for distributing the performance among subsystems. If the summed weighted deficiency of $n-m$ subsystems is smaller than or at least equal to $T$ after surplus redistribution, the system is still reliable. If the summed weighted deficiency of $n-m$ components after surplus redistribution is still larger than $T$, the system is taken as failed. The performance redistribution mechanism is called surplus redistribution (SR) since the system redistributes the surplus performance of $m$ component to compensate the deficiency of $n-m$ components. It is worth noting that the system will give priority to compensate the deficiency of components with the highest weight in performance sharing mechanism.
Since deficiency can actually be taken as the negative sufficiency, we sort the subsystems such that the subsystem 1 to subsystem $m$ are in descending order of deficiency and subsystem $m+1$ to subsystem $n$ are in descending order of their weights. The weighted sufficiency of first $m$ components can now be represented as $WS$ and the weighted deficiency of next $n-m$ components can be denoted as $WD$. We should further note that at this time the total weighted surplus performance and total weighted deficiency of the system are actually equal to $WS$ and $WD$, respectively. Further, the total sufficiency and deficiency can be represented as $TS$ and $TD$, respectively. It is easy to show that (1) 

$$WS = \sum_{j=1}^{m} (\sum_{k=1}^{n} G_{jk} - D_{j}) W_{j},$$

$$WD = \sum_{j=m+1}^{n} (D_{j} - \sum_{k=1}^{n} G_{jk} W_{j}),$$

$$TS = \sum_{j=1}^{m} (\sum_{k=1}^{n} G_{jk} - D_{j}),$$

$$TD = \sum_{j=m+1}^{n} (D_{j} - \sum_{k=1}^{n} G_{jk}).$$

Corresponding to four possible situations we mentioned, we now summarize the three cases theoretically.

A. The system is reliable if $WD < T$. Otherwise, go to cases B or C or D.

B. The maximal redistribution performance of the system is limited to the bandwidth and can be obtained by $\min(TS, C)$. The system is still reliable if $WD > T$ but $\min(TS, C) \geq TD$, say that, the minimal value between total sufficiency and bandwidth capacity can compensate the total deficiency to make the subsystems to have no deficiency. Otherwise, go to cases C or D.

C. When both $WD > T$ and $\min(TS, C) < TD$, the surplus cannot compensate the deficiency. The system under this case can always give priority to satisfy subsystems with the highest weight and employ the total surplus to compensate the unsatisfied subsystems with higher weights first. Since the subsystems are already in descending order, there should exist an integer $m+1 \leq h \leq n$ that makes the sufficiency just able to compensate the deficiency from component $m+1$ to component $h-1$ but cannot compensate the deficiency from component $h$ to component $n$. Therefore, the summed weighted deficiency after performance sharing, $WD'$, combines two parts: deficiency of component $h$ after redistribution and deficiency from component $h+1$ to component $n$. In other words, we can show that (5)

$$WD' = (D_{h} - \sum_{k=1}^{n} G_{hk} - (TS - \sum_{j=m+1}^{n} (D_{j} - \sum_{k=1}^{n} G_{jk})) W_{j})$$

$$+ \sum_{j=m+1}^{n} (D_{j} - \sum_{k=1}^{n} G_{jk} W_{j}).$$

The system is still reliable if $WD' \leq T$. Otherwise, go to case D.

D. When $WD > T, \min(TS, C) < T$ and $WD' > T$, the system failed.

The reliability of the whole system can now be calculated by combining the first three cases as (6)

$$R = \Pr(WD \leq T) + \Pr(WD > T, \min(TS, C) \geq T)$$

$$+ \Pr(WD > T, \min(TS, C) < T, WD' \leq T).$$

3. Universal Generating Function Technique
Since UGF can easily depict different performance distributions of components and systems through algebraic procedures, researchers commonly employ it to model the reliability of given multi-state systems. The UGF technique is therefore introduced as follows. The PMF of any discrete random variable \( X \) takes the form like

\[
q_j \cdot \Pr(X = x_{jm}) \cdot z^{x_{jm}}.
\]

where \( q_j \) is the possible values of \( X \) and \( \alpha_{jm} = \Pr(X = x_{jm}) \) is the corresponding probabilities. By introducing the composition operator \( \varphi \), we can obtain the UGF of the function of \( n \) independent random variables \( \varphi(X_1, \ldots, X_n) \) as follows.

\[
U(z) = \varphi(u_1(z), \ldots, u_n(z)) = \varphi\left(\sum_{m=1}^{q_j} q_j \cdot \alpha_{jm} \cdot z^{x_{jm}}\right) = \sum_{m=1}^{q_j} \sum_{m_2=1}^{q_j} \ldots \sum_{m_n=1}^{q_j} \left(\prod_{m=1}^{n} \alpha_{jm} \cdot z^{x_{jm}}\right).
\]

Following the common assumption in UGF, we similarly assume the random demand of subsystems \( D \) to take the value from a given set \( D = \{d_{j1}, d_{j2}, \ldots, d_{jd_j}\} \). Moreover, the random performances of generators \( G_{jk} \) are assumed to take the value from given sets \( G_{jk} = \{g_{jk1}, g_{jk2}, \ldots, g_{jk, M_{jk}}\} \). The PMF of the demand of any subsystem can be obtained by the UGF as

\[
w_j(z) = \sum_{q=0}^{Q_j} s_{jq} \cdot z^{d_{jq}},
\]

where \( s_{jq} = \Pr(D = d_{jq}) \). For any generators \( k \) in component \( j \), the PMF of the performance can be obtained by the UGF as

\[
u_{jk}(z) = \sum_{m=1}^{M_{jk}} b_{jk,m} \cdot g_{jk,m} \cdot z^{g_{jk,m}}.
\]

where \( b_{jk,m} = \Pr(G_{jk} = g_{jk,m}) \). The PMF of performance of any subsystem \( j \) can be obtained as

\[
u_j(z) = \prod_{i=1}^{M_{j1}} \sum_{m_{j1}=1}^{M_{j1}} b_{j1,m_{j1}} \cdot g_{j1,m_{j1}} \cdot z^{g_{j1,m_{j1}}} \ldots \prod_{i=1}^{M_{jn}} \sum_{m_{jn}=1}^{M_{jn}} b_{jn,m_{jn}} \cdot g_{jn,m_{jn}} \cdot z^{g_{jn,m_{jn}}}.
\]

We further denote the random capacity of the bandwidth which takes the value from a given set \( C = \{c_1, c_2, \ldots, c_c\} \) and the corresponding PMF can be denoted as

\[
\eta(z) = \sum_{i=1}^{c} \beta_i \cdot z^{\eta_i},
\]

where \( \beta_i = \Pr(C = c_i) \). The sufficiency and deficiency of each component can be calculated through combining the UGF of both \( D \) and \( G_{jk} \). Specifically, the different composition operator \( \varnothing \) should be employed, choosing the minimal value from the demand and performance and generating a new value from subtraction. Therefore, the UGF of the sufficiency/deficiency of the component \( \Delta_j(z) \) can be obtained by

\[
\Delta_j(z) = u_j(z) \varnothing \nu_j(z)
\]

\[
= \sum_{m=1}^{M_j} \sum_{q=1}^{Q_j} q_{jm} \cdot s_{jq} \cdot z^{d_{jm} - d_{jq}} = \sum_{m=1}^{M_j} \pi_{jm} \cdot z^{e_{jm} - d_{jm}}.
\]
where \( V_j = M_j \mathcal{Q}_j, \pi_{j,o} = q_j^{-1} s_j, m \pmod{(1, Q_j)} + 1, q_j' = q_j^{-1} \pmod{(1, Q_j)} + 1 \)

and \( d_{j,o} = d_{j,o} \pmod{(1, Q_j)} + 1 \). Here \( \lfloor \cdot \rfloor \) represents the maximum integer that is no bigger than the given number and \( \text{mod}(a, b) \) represents the remainder of the division of \( a \) by \( b \). The recursive procedure is commonly employed to obtain the UGF of the whole system and works as follows.

**Step 1.** Assign \( U_{\Omega}(z) = U_{\emptyset}(z) = z^0 \).

**Step 2.** For \( i = 1, \ldots, n \), do \( U_{\Omega^i}(z) = U_{\Omega}(z) \odot \Delta_i(z) \). Assign \( \Omega = \Omega \cup i \).

We should further note that the composition operator \( \odot \) requires the multiplication of coefficients and the union of the exponents for every pair of terms of the two UGFs. Therefore, the UGF considering both the performance and demand (sufficiency/deficiency) can be obtained by

\[
U_j(z) = U_{\emptyset}(z) \odot \Delta_j(z) \odot \cdots \odot \Delta_n(z)
\]

To calculate the reliability of the whole system, we further design a reliability indicator function to depict the reliability distribution as follows.

\[
f(g_{1,1}, \ldots, g_{n,1}, d_{1,1}, \ldots, d_{n,1}, w_1, \ldots, w_n) = \begin{cases} 0 & \text{otherwise} \\ 1 & \sum_{j=1}^{\nu_{j,1} - \sum_{l=1}^{\nu_{j,1}} (g_{j,1} - \sum_{l=1}^{\nu_{j,1}} g_{j,1}) w_j \leq T} \end{cases}
\]

According to Eq. (6), the reliability of the whole system without performance sharing can be obtained by

\[
R = \sum_{o_{1,1} = 1}^{v_{1,1}} \cdots \sum_{o_{n,1} = 1}^{v_{n,1}} \prod_{j=1}^{\nu_{j,1}} [\pi_{j,o_{j}}]
\times f(g'_{j,1}, \ldots, g'_{n,1}, d'_{1,1}, \ldots, d'_{n,1}, w_1, \ldots, w_n)
\]

To increase the reliability of the whole system, the surplus redistribution is now employed to reallocate the sufficiency to compensate the deficiency of component with the highest weight. This requires an updating UGF that eliminates all components that already satisfied their demand. Since the reliability indicator will equal to zero if the precondition is not held, we can multiply the one minus reliability indicator to the UGF to eliminate unnecessary terms. As a result, the updating UGF can be obtained by

\[
U'_j(z) = \sum_{o_{1,1} = 1}^{v_{1,1}} \cdots \sum_{o_{n,1} = 1}^{v_{n,1}} \prod_{j=1}^{\nu_{j,1}} [1 - f(g'_{j,1}, \ldots, g'_{n,1}, d'_{1,1}, \ldots, d'_{n,1}, w_1, \ldots, w_n)) \pi_{j,o_{j}}]
\times z^{(g'_{j,1} - \sum_{l=1}^{\nu_{j,1}} g'_{j,1}) (d'_{1,1} - \sum_{l=1}^{\nu_{j,1}} d'_{1,1})}
\]

Since the maximal transmitted performance is limited by the capacity of bandwidth, we should combine the updating UGF and \( \eta(z) \) to obtain the UGF incorporate with bandwidth. The construction of UGF incorporate with bandwidth requires the employment of \( \odot \) and takes the form like
Since the surplus redistribution requires the compensation from satisfied components to unsatisfied components, we can employ the judging criteria in case C in Section 2 and add the reliability augment from SR to the initial reliability. Thus, the reliability can now be obtained by

\[
\bar{U}_d(z) = U_d(z) \otimes \eta(z)
\]

\[
= \sum_{n_1 \in N_1} \sum_{n_2 \in N_2} \prod_{j=1}^{n_{para}} (1 - f(g_{1,n_1}^{*}, ..., g_{n_{para},n_{para}}^{*}, d'_{1,n_1}, ..., d'_{n_{para},n_{para}}, w_1, ..., w_n)) \tau_{j,n_j} \\
\times \prod_{j=1}^{n_{para}} (1 + \sum_{j=1}^{n_{para}} \beta_j z^{n_j})
\]

(18)

\[
= \sum_{n_1 \in N_1} \sum_{n_2 \in N_2} \prod_{j=1}^{n_{para}} (1 - f(g_{1,n_1}^{*}, ..., g_{n_{para},n_{para}}^{*}, d'_{1,n_1}, ..., d'_{n_{para},n_{para}}, w_1, ..., w_n)) \\
\times \prod_{j=1}^{n_{para}} (1 + \sum_{j=1}^{n_{para}} \beta_j z^{n_j}).
\]

Since the surplus redistribution requires the compensation from satisfied components to unsatisfied components, we can employ the judging criteria in case C in Section 2 and add the reliability augment from SR to the initial reliability. Thus, the reliability can now be obtained by

\[
R_{SR} = R + \sum_{n_1 \in N_1} \sum_{n_2 \in N_2} \prod_{j=1}^{n_{para}} (1 - f(g_{1,n_1}^{*}, ..., g_{n_{para},n_{para}}^{*}, d'_{1,n_1}, ..., d'_{n_{para},n_{para}}, w_1, ..., w_n)) \\
\times \prod_{j=1}^{n_{para}} (1 + \sum_{j=1}^{n_{para}} \beta_j z^{n_j}).
\]

(19)

4. Numerical Examples

Consider an energy system with two subsystems with weight \((w_1, w_2) = (6, 4)\). For components 1 and 2, there will be two generators providing the necessary energy. The PMF of the random demand for the subsystems and random performance for the generators are summarized in Table 1.

| Components | Value Set of Performance | Probability of Performance | Value Set of Demand | Probability of Demand |
|------------|--------------------------|---------------------------|---------------------|----------------------|
| 1          | \(g_1 = (4, 2)\)          | \(h_1 = (0.5, 0.5)\)      | \(d_1 = (0.8)\)     | \(a_1 = (0.6, 0.4)\) |
|            | \(g_2 = (6, 4)\)          | \(h_2 = (0.8, 0.2)\)      |                     |                      |
| 2          | \(g_3 = (8, 6)\)          | \(h_3 = (0.7, 0.3)\)      | \(d_2 = (0.4)\)     | \(a_2 = (0.5, 0.5)\) |
|            | \(g_4 = (5, 3)\)          | \(h_4 = (0.5, 0.5)\)      |                     |                      |

The capacity of bandwidth takes the PMF as \(c' = (4, 0)\) with \(\beta = (0.5, 0.5)\). The predesigned reliability threshold is assumed to be \(T = 2\). The UGF of generators in each component can be obtained by

\[
u_{i_1}(z) = 0.5z^4 + 0.5z^5, \quad \nu_{i_2}(z) = 0.8z^5 + 0.2z^6,
\]

\[
u_{i_3}(z) = 0.7z^6 + 0.3z^8, \quad \nu_{i_4}(z) = 0.5z^7 + 0.5z^8,
\]

respectively. Substituting the UGF of generators in Eq. (11), the UGF corresponds to the two subsystems can be calculated as

\[
u_1(z) = 0.4z^{10} + 0.5z^8 + 0.4z^9, \quad h_1(z) = 0.6z^{10} + 0.4z^8,
\]

\[
u_2(z) = 0.35z^{10} + 0.5z^9 + 0.15z^8, \quad h_2(z) = 0.5z^{12} + 0.5z^8,
\]

respectively. And the UGF for bandwidth can be similarly obtained by
\eta(z) = 0.5z^4 + 0.5z^6.

The UGF of the whole system can be calculated by combining the two subsystems as:

\[ U_d(Z) = 0.042Z^{[10,13],[10,12]} + 0.042Z^{[13,13],[13,4]} + 0.028Z^{[10,13],[8,12]} + 0.028Z^{[10,13],[8,4]} + 0.06Z^{[10,11],[10,12]} + 0.06Z^{[10,11],[10,4]} + 0.04Z^{[10,11],[10,12]} + 0.04Z^{[10,11],[8,12]} + 0.018Z^{[10,9],[10,12]} + 0.018Z^{[10,9],[10,4]} + 0.012Z^{[10,9],[10,12]} + 0.012Z^{[10,9],[8,12]} + 0.018Z^{[8,13],[10,12]} + 0.0525Z^{[8,13],[10,12]} + 0.035Z^{[8,13],[10,12]} + 0.035Z^{[8,13],[8,4]} + 0.075Z^{[8,9],[10,12]} + 0.075Z^{[8,9],[10,4]} + 0.05Z^{[8,9],[10,12]} + 0.05Z^{[8,9],[8,12]} + 0.0225Z^{[8,9],[8,12]} + 0.0225Z^{[8,9],[10,12]} + 0.015Z^{[8,9],[8,4]} + 0.015Z^{[8,9],[8,12]} + 0.0015Z^{[8,9],[10,12]} + 0.0075Z^{[6,13],[8,12]} + 0.0075Z^{[6,13],[8,4]} + 0.0015Z^{[6,13],[8,12]} + 0.015Z^{[6,11],[10,4]} + 0.015Z^{[6,11],[8,12]} + 0.01Z^{[6,11],[8,4]} + 0.0045Z^{[6,9],[10,12]} + 0.0045Z^{[6,9],[10,4]} + 0.003Z^{[6,9],[8,12]} + 0.003Z^{[6,9],[8,4]}

By introducing the reliability indicator function, the reliability of the whole system can be obtained by:

\[ R = R_{T-o} = 0.042 + 0.042 + 0.028 + 0.028 + 0.06 + 0.04 + 0.018 + 0.012 + 0.035 + 0.035 + 0.05 + 0.015 = 0.405 \]

The updated UGF can be obtained by eliminating the unnecessary terms in initial UGF and denoted as:

\[ \overline{U_d}(z) = 0.06Z^{[10,11],[10,12],[4]} + 0.04Z^{[10,11],[8,12],[4]} + 0.018Z^{[10,9],[10,12],[4]} + 0.012Z^{[10,9],[8,12],[4]} + 0.0525Z^{[8,13],[10,12],[4]} + 0.0525Z^{[8,13],[10,4],[4]} + 0.075Z^{[8,11],[10,12],[4]} + 0.075Z^{[8,11],[10,4],[4]} + 0.05Z^{[8,9],[10,12],[4]} + 0.0225Z^{[8,9],[10,12],[4]} + 0.0225Z^{[8,9],[10,4],[4]} + 0.015Z^{[8,9],[8,12],[4]} + 0.015Z^{[8,9],[8,4],[4]} + 0.0105Z^{[6,13],[10,12],[4]} + 0.0105Z^{[6,13],[10,4],[4]} + 0.0075Z^{[6,13],[8,12],[4]} + 0.0075Z^{[6,13],[8,4],[4]} + 0.0105Z^{[6,11],[10,12],[4]} + 0.0105Z^{[6,11],[10,4],[4]} + 0.01Z^{[6,11],[8,12],[4]} + 0.01Z^{[6,11],[8,4],[4]} + 0.0045Z^{[6,9],[10,12],[4]} + 0.0045Z^{[6,9],[10,4],[4]} + 0.03Z^{[6,9],[8,12],[4]} + 0.03Z^{[6,9],[8,4],[4]}

Employing the reliability indicator function again, and the updated reliability after SR can be addressed as:

\[ R_{SR\alpha=0} = R_{SR\alpha=2} = \frac{0.04 + 0.0525 + 0.075 + 0.0225 + 0.0105 + 0.022 + 0.01 + 0.0045 + 0.003}{2} = 0.525 \]

From the above results, performance sharing mechanism can improve system reliability.

5. Conclusions

In this paper, we consider a capacitated system with parallel subsystems connected by a performance sharing mechanism. Different from previous literatures, we relax the assumption that the system cannot tolerate the deficiency. In our proposed model, only if the summed weighted deficiency is larger than a predesigned reliability threshold, the system fails. Otherwise, the system is regarded as reliable. We further consider the case where each subsystem combines amounts of components (generators in our energy system example) with random performance. The existence of bandwidth allows the redistribution...
of performance among all subsystems, making the surplus able to somehow compensate the deficiency and thereby increase the reliability. Our results show that surplus redistribution is efficient and effective in remaining a functional system.

More works are needed. The system can actually employ the performance (not surplus) from the component with relative low weight to compensate the deficiency of component with relative high weight. It is also worth to analyze and compare other redistribution mechanisms.

Acknowledgements
The research was supported by the NSFC under grant numbers 71671016. The research was supported under Grant No.2019-00909-2-1. The research reported here was partially supported by National Natural Science Foundation of China (Grant no. 72001027)

References
[1] Wu D, Chi Y, Peng R and Sun M 2019 Reliability of capacitated systems with performance sharing mechanism Reliability Engineering & System Safety 189 335-44
[2] Xiao H, Shi, D.M., Ding, Y and Peng R 2015 Optimal Loading and Protection of Multi-state Systems Considering Performance Sharing Mechanism Reliability Engineering & System Safety 149 88-95
[3] Peng, R, Liu H and Xie M 2016 A Study of Reliability of Multi-state Systems with Two Performance Sharing Groups,” Quality &Reliability Engineering International 32(7) 2623-32
[4] Peng, R, Xiao, H and Liu, H 2017 Reliability of Multi-state Systems with a Performance Sharing Group of Limited Size Reliability Engineering & System Safety 166 164-70
[5] M. El Falou and E. Châtelet 2009 Availability assessment of stochastic multi-states systems based on UGF and taking into account data uncertainty 2009 IEEE International Conference on Industrial Engineering and Engineering Management 1170-74
[6] Diego Francisco Castillo Velázquez, Sebastián Plata Duarte and Enrique González Guerrero 2011 Unsatisfied Goal-Oriented Formations UGF 2011 6th Colombian Computing Congress (CCC)