Research Article

Seepage Grouting Mechanism for Foundations in Goaf Sites considering Diffusion Paths

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Grouting treatment is the main technology to reduce or eliminate the residual deformation and activation deformation of foundations in the goaf sites. Under the influences of the overburden of the mining, the distribution of grouting in goaf foundation is quite different from that of conventional grouting mechanism in porous media. In this paper, considering the time-dependent viscosity and diffusion path of the slurry, the conventional permeation diffusion mechanism of Bingham fluid is derived based on the seepage motion equation in porous media. The theoretical formula is modified according to the fracture distribution characteristics of caving zone and fault zone of the goaf foundation and the superposition effect of porous grouting. Combined with the laboratory test, the theoretical formulas for four working conditions ((i) only considering the time-dependent viscosity, (ii) considering the time-dependent viscosity and diffusion path, (iii) combining the fracture distribution characteristics of goaf foundation, and (iv) combining the fracture distribution characteristics and the superposition effect of porous grouting) are verified, respectively. The results of theoretical formula are used to compare with the design scheme of an engineering example. The research results have an important engineering significance for revealing the mechanism of seepage grouting in goaf foundation and designing the optimal spacing between grouting holes.

1. Introduction

According to incomplete statistics, mining subsidence area China has exceeded 8000 km² and is still expanding at the speed of 200 km²/a. For the safety of buildings or structures above the goaf, some technical measures must be taken, in which grouting treatment is an important technology. When applying this method, holes are drilled into the surface and the grouting pipelines penetrate through the collapse zone and fault zone to 1~2m below the floor of the goaf. The main functions of grouting are to fill the empty holes, voids, and cracks and reinforce the fractured and broken rock mass. The aims are to reduce or eliminate the residual deformation of goaf site and the activated deformation of goaf foundation, as shown in Figure 1. Therefore, the study on the slurry diffusion process of permeation grouting in goaf foundation is of great engineering significance to reveal the mechanism of slurry filling and cementing cracks and to strengthen grouting technology in goaf foundation. Many experts and scholars at home and abroad have done a lot of research on the grouting reinforcement mechanism and the grouting treatment and achieved some research results.

In terms of grouting reinforcement mechanism, Hu et al. [1] comprehensively summarized the research status in the field of grouting in fractured rock mass, which pointed out that the existing theory lagged behind the engineering practice. Based on the rheological equation and time-dependent viscosity equation of Bingham slurry, Yang et al. [2, 3] deduced the column-hemispherical infiltration grouting mechanism of Bingham slurry considering time-varying; however, there was a gap between the calculated value and the actual value because the influence of diffusion path was
not considered. Barenblatt et al. [4–6] established a numerical model and gave an analytical solution according to the fluid percolation theory in fractured rocks. Ruan et al. [7–9] established a grouting diffusion model of stable slurry considering the circuitous effect of pore channels; however, because the physical meaning of the parameters was not clear, it was difficult to meet the needs of practice and ensure the grouting effect.

In terms of grouting treatment of goaf foundation, Zhao et al. [10] carried out the research on the solidification mechanism of a new grouting material by scanning electron microscope and interactive mixing test. Zhu et al. [11] carried out the flow diffusion test of adjacent grouting to fill goaf; however, there were some limitations in promoting this method. Wang et al. [12–14] adopted the numerical calculation method to simulate the diffusion of grouting fluid in goaf and analyzed the influence of grouting time and grouting amount on the spatial diffusion of slurry, but it was difficult to obtain the parameters required for simulation. Wang et al. [15] analyzed the design theory of belt grouting filling in the old goaf according to the characteristics of expressway crossing goaf in strips. Shi et al. [16] studied the influence of slurry volume, grouting pressure, porosity, and water-cement ratio on slurry diffusion by using the designed experimental system for fracture development in goaf.

At present, the research results of grouting technology at home and abroad mainly focus on cracks or a single grouting hole. However, less research was carried out on the actual grouting engineering with large construction area, many grouting holes, complicated grouting procedures, and complex geological conditions, especially the interaction between grouting holes is not considered. Due to its own characteristics, the permeability of slurry in the collapse zone and fault zone has certain particularity. In this paper, based on Bingham fluid, considering the time-dependent viscosity and diffusion path, the seepage movement equation of slurry was established. Combined with the distribution characteristics of mining-induced fractures and the superposition effect of porous grouting, the theoretical formula was modified. Its feasibility and rationality were verified by an indoor test. Finally, it was used in a project example for scheme design. The research results can provide theoretical guidance for solving practical problems such as excessively depending on experience, serious material waste, and poor reinforcement effect.

2. The Diffusion and Seepage Theory of Grouting Slurry

2.1. Basic Assumptions

1. The slurry is regarded as isotropic and incompressible homogeneous, and the change of slurry flow type during grouting is not considered
2. The flow type of slurry in the fractures is laminar flow, and it conforms to the continuity equation
3. The effects of gravity and hydrostatic pressure are not considered
4. The fracture is a single horizontal fracture, and it conforms to the no-slip boundary condition
5. The fluid-structure interaction is ignored; that is, the deformation of fractures during grouting is ignored
2.2. Slurry Diffusion Mode. The seepage and diffusion form of slurry in the injected medium depend on the grouting method: (i) During point grouting, it is spherical diffusion, as shown in Figure 2(a); (ii) when grouting through complete holes or from bottom to top, a cylindrical diffusion mode is formed, as shown in Figure 2(b); and (iii) when grouting through incomplete holes or from top to bottom, a column-hemispherical diffusion mode is formed, as shown in Figure 2(c). The grouting in goaf foundation belongs to the second mode.

2.3. Rheological Equation considering Time Variability. The slurry enters the overburden of the goaf. Affected by the distribution of overburden fractures, the diffusion mode of the slurry in different rock and soil is different. Due to the small grouting pressure of the goaf foundation, there is basically no splitting grouting. There are mainly two flow diffusion modes: (i) the seepage diffusion along pores and fractures in the overburden collapse accumulation area and fault zone and (ii) the free diffusion of Newtonian-like fluid in a cavity or tunnel. This paper mainly studies the first mode, that is, the mechanism of infiltration and diffusion in pores or fractures.

The cement slurry and cement composite slurry with water-cement ratio of 0.8–1.0 commonly used in engineering belong to typical Bingham fluid. The rheological equation considering time-dependent viscosity [17] is:

\[ \tau = \tau_0 + \mu_0 e^{k_t} \gamma, \]  

(1)

where \( \tau \) is the shear stress, \( \tau_0 \) is the yield stress, \( \mu_0 \) is the initial viscosity of the slurry, \( P_s \); \( \gamma \) is the shear rate; \( t \) is the grouting time, s; and \( k \) is the coefficient of time-dependent viscosity.

2.4. Motion Equation of Seepage considering Time Variability. In order to discuss the time-varying of seepage equation of Bingham fluid, the flow diagram in a circular tube is established, as shown in Figure 3. The radius of the circular tube is \( r_0 \). Take a section of fluid column microelement with the tube axis as the symmetry axis in the circular tube. The radius is \( r < r_0 \) and the length is \( dl \), the pressure on both sides is \( p \) and \( p + dp \), respectively, and the shear stress is \( \tau \).

Without considering the gravity of slurry, the stress of the microelement of fluid column is analyzed, and the following equation can be obtained:

\[ \pi r^2 dp + 2 \pi r r dl = 0. \]  

(2)

This leads to:

\[ \tau = -\frac{rdp}{2dl}. \]  

(3)

It can be seen from Equation (3) that \( t = 0 \) when \( -(rdp/2dl) \leq \tau_0 \); if the radius of the fluid column microelement is \( r_1 \), then when \( 0 \leq r \leq r_1 \); the fluid column microelement is stationary relative to the adjacent fluid and has no relative movement; and when \( r_1 \leq r \leq r_0 \), the fluid column microelement moves relatively to the adjacent fluid.

Substituting \( r = r_1 \), \( \tau = \tau_0 \) into Equation (3), we can get:

\[ r_1 = -\frac{2\tau_0}{\mu_0 e^{k_t}} dp/dl. \]  

(4)

Substituting Equation (3) into Equation (1), we can get:

\[ \gamma = -\frac{dv}{dr} = -\frac{\tau_0 + (r/2)(dp/dl)}{\mu_0 e^{k_t}}. \]  

(5)

Integrating both sides of Equation (5), we can obtain:

\[ v = \left( \tau_0 r + (r^2/4)(dp/dl) \right) + c, \]  

(6)

where \( c \) is a constant.

\( r_1 \leq r \leq r_0 \), according to the boundary conditions of \( r = r_0 \) and \( v = 0 \), Equation (6) can be transformed into:

\[ v = \left( \tau_0 r + (r^2/4)(dp/dl) \right) - \left( \tau_0 r_0 + (r_0^2/4)(dp/dl) \right). \]  

(7)

According to the analysis, the flow of the grouting tube is composed of two parts, which are shear zone \( (r_1 \leq r \leq r_0) \) and piston zone \( (0 \leq r \leq r_1) \); therefore, the total flow is:

\[ q = \pi r_1^2 v_1 + \int_{r_1}^{r_0} 2\pi rvdr. \]  

(9)

Substituting Equations (7) and (8) into Equation (9), we can get:

\[ q = \frac{-\pi \tau_0 r_0^3/3 + (\pi r_0 r_1^3/3) - (\pi r_0^3/8)(dp/dl) + (\pi r_1^3/8)(dp/dl)}{\mu_0 e^{k_t}}. \]  

(10)

Substituting Equation (4) into Equation (10), we can get:

\[ q = \frac{-\pi \tau_0 r_0^4}{\mu_0 e^{k_t} 8} \left( \frac{dp}{dl} \right) \left[ -\frac{4}{3} \left( \frac{2\tau_0}{r_0} \right) - \frac{\left( 2\tau_0 r_0^4 / (dp/dl) \right)^4}{3} + 1 \right]. \]  

(11)
In order to make Bingham fluid flow in porous media channels, it is necessary to overcome the yield stress $\tau_0$. Namely, letting $q = 0$ in Equation (11), we can get \((2\tau_0/r_0)/(-(dp/dl)) = 1\). The initial pressure gradient of the fluid along the diffusion path is as follows:

$$-\frac{dp}{dl} = \frac{2\tau_0}{r_0} = \lambda.$$  \hspace{1cm} \text{(12)}$$

Then, the average velocity of fluid in the cross section of grouting tube is:

$$\bar{v} = \frac{q}{\pi r_0^2} = \frac{-r_0^2}{8 \mu_0 \varepsilon_0} \left(\frac{dp}{dl}\right) \left[ -\frac{4}{3} \frac{2\tau_0/r_0}{-dp/dl} + \frac{1}{3} \left(\frac{2\tau_0/r_0}{-dp/dl}\right)^4 + 1 \right].$$  \hspace{1cm} \text{(13)}$$

Figure 2: Slurry diffusion modes.

Figure 3: Flow diagram of Bingham fluid in circular tube.
According to $V = \varphi \bar{v}$, the seepage equation of Bingham fluid considering time variation can be obtained as follows:

$$V = -\frac{\varphi r^3}{8\mu_0 e^{\lambda t}} \left( \frac{dp}{dl} \right) \left[ -\frac{4}{3} \frac{2\tau_0/\varphi}{(-dp/dl)} + \frac{1}{3} \frac{2\tau_0/\varphi}{(-dp/dl)} \right]^4 + 1]. \tag{14}$$

2.5. Motion Equation of Seepage considering Diffusion Path. Equation (14) does not consider the influence of diffusion path. In fact, the channel of slurry diffusion is tortuous. The research results based on the hypothesis of linear pore channel often deviate from the actual value. In order to make the theoretical calculation results closer to reality, Equation (14) is modified considering the influence of slurry diffusion path on permeability mechanism in this paper.

Taking a single hole grouting as an example, investigate 1/2 of the grouting area, as shown in Figure 4. $l$ is the linear length of porous media and $l_i$ is the actual channel length of porous media.

Take a microunit within the grouting range, with length of $l$, width of $1$, and height of $m$. The pore channel is shown in Figure 5. According to Hagen-Poiseuille formula:

$$q = -\frac{\pi dp r^4}{8 dl_i \mu}, \tag{15}$$

where $q$ is the flow through a single pore channel, $-dp/dl_i$ is the pressure gradient of the fluid along the diffusion path, and $r$ is the radius of a single pore channel.

Assuming that there are $N$ pore channels and considering the time-dependent viscosity, Equation (14) can be modified as:

$$Q(t) = -N \frac{\pi dp}{8 dl_i} \frac{r^4}{\mu(t)}. \tag{16}$$

According to Darcy’s law:

$$Q(t) = -\frac{KA}{\mu(t)} \frac{dp}{dl_i}, \tag{17}$$

where $A$ is the cross-sectional area of unit and $K$ is the permeability of unit.

Defined by porosity:

$$\begin{align*}
\varphi &= \frac{V_{\text{pore}}}{V_{\text{total}}} \\
\frac{dV_{\text{pore}}}{dV_{\text{total}}} &= N\pi r^2 dl_i. \tag{18}
\end{align*}$$

The following equation can be obtained from Equations (16) to (18):

$$\frac{8K}{r^2 \varphi} = \frac{l}{l_i}. \tag{19}$$

The pressure gradient after grouting is much greater than the initial pressure gradient, that is, $(2\tau_0/r_0) \ll (-dp/dl_i)$, then Equation (14) can be simplified as:

$$V = -\frac{\varphi r^3}{8\mu_0 e^{\lambda t}} \left( \frac{dp}{dl_i} \right) + \frac{4}{3} \lambda. \tag{20}$$

Because of $q = VA$, Equation (20) is transformed into:

$$q = -\frac{\varphi Ar_0^3}{8\mu_0 e^{\lambda t}} \left( \frac{dp}{dl_i} \right) + \frac{4}{3} \lambda. \tag{21}$$

Separating the variables according to Equation (21), we can get:

$$dp = \left[ \frac{8\mu_0 e^{\lambda t}}{\varphi Ar_0^3} + \frac{4}{3} \lambda \right] dl_i. \tag{22}$$

Because the radius of the circular tube is $r_0$, the radius of the pore channel is $r_0$. Namely, Equation (19) can be transformed into:

$$\frac{l}{l_i} = \frac{8K}{r^2 \varphi}. \tag{23}$$

Substituting Equation (23) into Equation (22), and considering $\eta = l/r_0, \lambda = 2\tau_0/r_0$, we can get:

$$dp = -\left( \frac{\mu_0 e^{\lambda t} q}{2\eta ml} + \frac{\tau_0 \varphi}{3\eta K} \right) dl. \tag{24}$$

Substituting the cylindrical diffusion formula $A = 2\pi ml$ into Equation (24) and integrating on both sides, we can obtain:

$$p = -\left( \frac{\mu_0 e^{\lambda t} q \ln l + \frac{l^2}{2} \times \frac{\tau_0 \varphi}{3\eta K} \right) + c. \tag{25}$$

Considering the grouting boundary conditions, when $p = p_1$, $l = l_0$, and when $p = p_0$, $l = l_i$, The slurry pressure loss $\Delta p$ is:

$$\Delta p = p_1 - p_0 = \frac{q\mu_0 e^{\lambda t} \ln l + \left( \frac{l^2}{2} \right) \times \frac{\tau_0 \varphi}{3\eta K}}, \tag{26}$$

Where $p_1$ is the grouting pressure, Pa; $p_0$ is the pressure of groundwater at the grouting point, Pa; $l_0$ is the radius of grouting pipe, m; $t$ is the grouting time, s; and $l_i$ is the diffusion radius of fluid when the grouting time is $t$, m.

Grouting amount is $Q = \varphi ml^2 m = qt$, then:

$$q = \frac{\varphi ml^2 m}{t}. \tag{27}$$
Substituting Equation (27) into Equation (26), we can get:

$$
\Delta p = p_1 - p_0 = \phi l_2 \mu_0 e^{k t} \ln \frac{l_1}{l_0} + \frac{(l_1^2 - l_0^2)}{2} \times \frac{\tau_0 \varphi}{3 \eta K}.
$$

Equation (28) is the permeation diffusion mechanism of Bingham fluid considering time-dependent viscosity and diffusion effects.

When diffusion effect is not considered, that is $l = l_t$, the slurry pressure loss $\Delta p$ can be obtained as follows:

$$
\Delta p = p_1 - p_0 = \frac{4l_1^2 \mu_0 e^{k t}}{\tau_0} \ln \frac{l_1}{l_0} + \frac{4}{3} \lambda (l_1 - l_0).
$$

### 3. Diffusion Mechanism of Infiltration

**Grouting considering the Characteristics of Goaf Foundation**

When the underground mineral resources are mined, many cavities are left, the upper rock strata lose their support, and the equilibrium conditions are destroyed. When the working face advances, the roof strata collapse and break under the gravity of the overlying strata, and the rock mass failure is transmitted from bottom to top until a new mechanical balance is reached. According to the damage degree of overburden, it is divided into collapse belt, fault zone, and bending band from bottom to top.

As shown in Figure 6, the fault zone belongs to broken structure, and the rock strata are cut by irregular structural planes such as joints and fissures, and the physical property is characterized by heterogeneous anisotropic
body. The collapse belt belongs to loose structure, in fragment and granular form, containing a large number of pores, voids, and cavities, and its physical property is characterized by homogeneous isotropic body [18]. According to the assumptions which were put forward in the comparative literature [7] on the cylindrical seepage grouting mechanism of Bingham fluid considering viscosity and time-varying, the collapse belt basically meets the requirements, but some parameters need to be modified for the fault zone.

3.1. Considering the Influence of Roughness. The fracture structure in goaf foundation is very complex, and the roughness has an important impact on its mechanical properties and permeability. It is generally measured by the flat surface formed by the filling of rough sidewall. The empirical formula of roughness is as follows:

\[
\kappa = \frac{1}{1+6(\Delta/b)^{1.5}},
\]  

where \( \kappa \) is roughness; \( \Delta \) is absolute roughness, \( m \); \( b \) is the crack opening, \( m \); and \( \Delta/b \) is the relative roughness.

Considering the influence of fracture roughness on permeability coefficient, Equation (28) is corrected as:

\[
\Delta p = \left[ \frac{\phi_i^3 \mu_i e_i}{2Kt} \ln \frac{l_1}{l_0} + \frac{(l_2 - l_0)}{2} \times \tau_0 \frac{\phi}{3\eta K} \right] \times \frac{1}{\kappa},
\]  

Although the crack opening is difficult to measure, it can be inferred from borehole data such as porosity and RQD value of rock mass quality index. The empirical formula is as follows:

\[
\begin{align*}
\text{RQD} &= 115 - 10\Lambda \\
\phi &= 3b\Lambda
\end{align*}
\]

where \( \phi \) is porosity and \( \Lambda \) is fracture density.

3.2. Consider the Superposition Effect of Porous Grouting. In this paper, the superposition effect of porous grouting is considered according to the rectangular layout form in Figure 7, and the interaction factor of porous grouting is deduced by establishing the grouting hole distribution model.

As shown in Figure 7, four grouting holes are arranged according to four vertices of the rectangle, and designed minimum grouting radius \( R_b \) can be obtained from the following formula:

\[
R_b = \sqrt{\frac{a^2 + b^2}{2}}.
\]

As shown in Figure 8, the diffusion distance \( R_2 \) on the central axis is the minimum of the radius in the flow field, while the diffusion distance \( R_1 \) on the main streamline is the maximum of the radius in the flow field.
where $\xi$ is the interaction factor of porous grouting and $R_{\text{single}}$ is the grouting diffusion radius without considering the influence of porous interaction, m.

Based on the above analysis and considering the superposition effect of porous grouting, Equation (31) is modified as follows:

$$
\Delta p = \left[ \frac{\varphi \rho K_0 e^{kt}}{2Kt} \ln \frac{l_1}{l_0} + \frac{(l_1^2 - l_0^2)}{2} \times \frac{t_0 \varphi}{3\eta K} \right] \times \frac{1}{\kappa} \times \frac{1}{\xi} \tag{38}
$$

4. Verification of Theoretical Formula

In this paper, the theoretical formulas for four working conditions are compared with the relevant penetration grouting test results carried out by Yang et al. [19] in the early stage. They are separated as follows: (i) only considering time-dependent viscosity (Equation (29)); (ii) considering time-dependent viscosity and diffusion path (Equation (28)); (iii) combined with the overburden characteristics of goaf foundation, only the influence of roughness is considered (Equation (31)); and (iv) combined with the overburden characteristics of goaf foundation and considering the influence of roughness and the superposition effect of porous grouting (Equation (38)).

Yang et al. [19] selected two kinds of pure cement slurry with water-cement ratio of 0.80 and 1.00 separately for penetration grouting tests. The cement slurry with water-cement ratio of 0.8 injected into the gravel (sand) with particle size of 5~10 mm is called composite material 1, and the cement slurry with water-cement ratio of 1.0 injected into the gravel (sand) with particle size of 3~5 mm is called composite material 2 (see Tables 1~3 for the test scheme).

Referring to [20], the rheological equation of Bingham fluid considering time-dependent viscosity is shown in Table 4.

The calculated values, according to Equation (29), Equation (28), Equation (31), and Equation (38), are compared with the relevant penetration grouting test results. The values and deviation analysis of diffusion radius are shown in Table 5. The length ratios $\eta$ of pore channel for porous media of composite materials 1 and 2 are 0.39 and 0.52, respectively. According to reference [21], the range of relative roughness $\Delta/k$ of roof and both sides for tunnel is 0.03~0.10. In this paper, it is 0.065, at this time $k=1.1$. According to reference [22], the nonuniformity coefficient of superposition effect for four holes grouting in rectangular arrangement is 0.5. Considering the superposition effect of porous grouting, the value range of influence coefficient is 0.7~0.9, which is taken as 0.8 in this paper.

According to the analysis of Table 5, the calculated value of diffusion radius of Bingham fluid considering diffusion path (Equation (28)) is closer to the test value than the calculated value without considering diffusion path (Equation (29)). This is consistent with the conclusion of literature [23]. Therefore, considering the diffusion path can better reflect the diffusion form and rule of Bingham fluid in porous media. The calculated value of diffusion radius of Bingham fluid considering the influence of roughness (Equation (31)) is larger than that of Equation (29), which
5.1. Engineering Background. The study site is the mined out area of civilian bauxite mine, which is dominated by tunnel mining. The mining is random, and the tunnel distribution is irregular. Let it collapse after mining. Many collapse pits can be seen on the surface of the area, which are round and scattered, as shown in Figure 9. The proposed 5° and 6° buildings are located in the subsidence area of bauxite mine. In order to ensure the safety and stability of buildings, grouting treatment shall be carried out for the goaf foundation. The structure of grouting hole is shown in Figure 10.

5.2. Calculation of Diffusion Radius. The diffusion radius of slurry plays an important role in the spacing and row spacing of grouting holes. Too large spacing will lead to slurry filling insufficiently, and too small spacing will lead to excessive cost and waste. The grouting treatment of goaf foundation does not need to strengthen the bearing stratum of buildings, so the strength requirements of grouting materials are low. In addition, the amount of grouting is large; therefore, cheap fly ash, waste brick powder, and soil are usually added to the cement in order to realize the recycling of construction waste. The grouting material of bauxite goaf foundation is cement fly ash slurry with water-solid ratio (water: solid material) of 1:1.2 and solid ratio (cement: fly ash) of 3:7. According to the research results of literature [24], the addition of fly ash admixture will not change the rheological type of slurry, but will change the rheological parameters, namely, yield stress and slurry viscosity. For the rheological equation of cement fly ash slurry, there are great differences due to the different specification degree of operators, material ratio, and types of additives [25–28]. Based on the analysis of multiple groups of rheological regression equations of cement fly ash slurry, this paper multiplies the rheological parameters of pure cement by ash slurry, there are great differences due to the different specification degree of operators, material ratio, and types of additives [25–28]. Based on the analysis of multiple groups of rheological regression equations of cement fly ash slurry, this paper multiplies the rheological parameters of pure cement slurry with correction coefficient to reduce the error. Since the water-solid ratio of the test material is approximately equal to 0.8, the rheological equation of water cement ratio of 0.8 is

is because the fracture opening of the collapse zone in the goaf foundation is larger, and there are more pores, cavities, and holes. Compared with the porous medium with uniform particle grading, the flow velocity of slurry is faster and the penetration radius is larger. At the same time, the calculated value of diffusion radius of Bingham fluid (Equation (38)) considering the distribution characteristics of overburden fractures in goaf foundation and the superposition effect of porous grouting is relatively smaller compared with Equation (31), which is because porous grouting affects each other. For a single hole, adjacent grouting improves the stone rate of porous media, which will inevitably affect the diffusion speed and radius of the slurry in porous media. In addition, the deviation between the composite material 2 and the test value is larger than that of the composite material 1, which is due to the unstable performance of the cement slurry with water-cement ratio of 1.0 prepared during the test.

Table 1: Design parameters of cement slurry.

| Composite material | Water-cement ratio | Grouting quantity (ml) | Grouting pressure (Δp) | Grouting duration (t/s) |
|--------------------|--------------------|------------------------|------------------------|-------------------------|
| 1                  | 0.80               | 2026                   | 10.19                  | 28.3                    |
| 2                  | 1.00               | 2051                   | 6.12                   | 21.7                    |

Table 2: Design parameters of porous media.

| Composite material | Particle gradation | Permeability coefficient (%) | Porosity (%) | Water content (%) |
|--------------------|--------------------|------------------------------|--------------|------------------|
| 1                  | 5–10               | 8.94                         | 50.74        | 10.26            |
| 2                  | 3–5                | 2.11                         | 45.05        | 14.86            |

Table 3: Design parameters of infiltration grouting.

| Composite material | \( r_0 \) (mm) | \( \beta \) | \( \lambda \) | \( l_0 \) (mm) | \( \mu \) (pa*s) |
|--------------------|----------------|-------------|---------------|----------------|----------------|
| 1                  | 0.04           | 13.55       | 266.05        | 7.5            | \( 1.01 \times 10^{-3} \) |
| 2                  | 0.02           | 7.155       | 156.3         | 7.5            | \( 1.01 \times 10^{-3} \) |

Note: \( \mu \) is the viscosity of water at 20°C.

The stratum lithology is shown in Figure 11. The loess of Quaternary middle and upper Pleistocene (Q3 and Q2) exposed within 110 m depth is grayish yellow silt and silty clay mixed with calcareous nodules and gravel (soil). The thickness of this layer is about 20~40 m. The underlying bedrock is Carboniferous (C2+3) sandstone, limestone, mudstone, and bauxite (ore), and the bottom is Ordovician limestone. For the bauxite in the study area, the roof clay rock is a low-strength rock mass, with the depth of 50~90 m and the thickness of 3~30 m.
The drilling and coring results are shown in Figure 12. Taking the ZK01 borehole exploration data of the research site as an example, the depth is 97.7 m; the vertical cracks are developed in the rock core at 50.0~60.0 m; the maximum joint fissure is 40 cm, which is characteristic of the fault zone; and water leakage is found at 57~59 m. The rock core at 60.1~67.8 m is in black powder and fragment shape, which is characterized by collapse zone. The values of permeability coefficient and porosity are shown in Table 6 based on the characteristics of collapse zone and fault zone, considering field measurement results and relevant laboratory tests [29].

Table 5: The calculated values, the test values and deviation analysis of diffusion radius.

| Composite material | Equation (29) (mm) | Equation (28) (mm) | Equation (31) (mm) | Equation (38) (mm) | The test value (mm) |
|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| 1                  | 29.8               | 29.0               | 30.4               | 27.4               | 27.0               |
| Deviation Δ        | 10.4%              | 7.40%              | 12.6%              | 1.48%              |                    |
| 2                  | 22.5               | 25.0               | 26.2               | 23.7               | 29.0               |
| Deviation Δ        | 22.4%              | 13.8%              | 9.7%               | 18.3%              |                    |

Considering distribution characteristics of cracks, the diffusion radius $l_1 = 7.4$ m can be obtained by substituting the grouting design parameters into Equation (31). Considering the superposition effect of porous grouting, the diffusion radius $l_1 = 6.7$ m can be obtained by substituting the grouting design parameters into Equation (38). According to the engineering data, the drilling layout scheme is shown in Figure 13. The grouting holes are arranged in quincunx shape, the spacing between holes is 10 m, and the optimal diffusion radius is OB = 7.1 m, which is in good agreement with the theoretical calculation results.

6. Discussion

Grouting treatment is the main technology to solve the foundation deformation in the goaf. However, due to the
complex distribution of fractures in the goaf, the existing theory is far behind the engineering practice, and it is difficult for scientific research results to be applied to the actual grouting engineering. There are mainly the following difficulties:

(1) The fractured rock mass in the goaf foundation in different areas varies greatly. For example, the permeability coefficient and fracture degree of the recomacted area in the middle of the goaf are significantly lower than those of the fracture development areas on both sides of the goaf, and there is even a difference of 2 orders of magnitude [30]. It is difficult to put forward a general calculation method in theoretical analysis.

(2) There are great differences between indoor test and field construction. The conditions of indoor test are easy to control, and the properties of medium and slurry are relatively stable, but site operation conditions are difficult to guarantee. The medium has unknown and uncertainty, and it is necessary to determine the grouting material and slurry ratio according to the characteristics of goaf. For example, the cement fly ash slurry is usually used as the grouting material in the caving goaf. However, due to the large crack sizes and many cavities in the cavitating goaf, it is necessary to select the slurry mixed by cement, fly ash, sand, and fine stone as the grouting material in order to form the stone body. These materials have special properties. In addition, the grouting sequence of site construction also affects the rules of slurry diffusion.

(3) The rheological equations of pure cement grout and cement-based composite slurry have their own particularities due to the influence of proportion, material, and environment and are limited by the specification degree of operators, types of additives, and laboratory control conditions. The theoretical calculation results are different from the laboratory.
test data. In addition, the rheological equations determined by references [20, 30] are slightly different, which will cause a certain impact on the verification of the theoretical formula.

4. In fact, the fractured rock mass in the caving zone of the goaf has a large fracture rate and strong permeability, which is a highly heterogeneous porous medium [29], which is not completely consistent with the assumed conditions of theoretical derivation, so the calculation result in this paper is an approximate value in the actual working conditions.

7. Conclusions

Based on the seepage equation of porous media, the seepage diffusion mechanism of Bingham fluid considering time-dependent viscosity and diffusion effect was derived in this paper. Combined with the fracture distribution characteristics of the collapse zone and fault zone in the goaf and the superposition effect of porous grouting, the theoretical formula was modified. The research conclusions are as follows:

1. Based on the time-dependent viscosity, the diffusion formula of cylindrical infiltration grouting of Bingham fluid is deduced. Based on the pore tortuosity effect, the infiltration grouting mechanism of Bingham fluid considering diffusion path is revealed. Considering the fracture distribution characteristics of the collapse zone and fault zone in the goaf, the cylindrical diffusion formula of permeability grouting is modified by roughness. Combined with the superposition effect of porous grouting, the cylindrical diffusion formula of seepage grouting is modified by using the interaction factor of porous grouting.

2. Combined with the indoor test, the theoretical calculation formulas of the proposed four working conditions are carried out and compared with the test values. The results show that the calculation result considering the diffusion effect is better than that without considering the diffusion effect. The calculation result of the modified formula considering the crack distribution is larger, and the calculation result of the modified formula considering the grouting superposition effect is smaller, which is finally close to the test values.

3. Considering the influence of fly ash on the rheological properties of cement mortar, the rheological parameters of cement slurry are modified. Combined with drilling and coring results, the theoretical formula is applied to an engineering example for verification. The results are in good agreement with the optimal diffusion radius of engineering design.

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**Table 6: Grouting design parameters of the example.**

| $r_0$ (mm) | $\eta$ | $\mu$ (pa·s) | $l_0$ (m) | $\Delta P$ (MPa) | $t$ (s) | $K$ (cm/s) | $\varphi$ (%) |
|-----------|-------|-------------|-----------|-----------------|-------|-----------|-------------|
| 0.06      | 0.30  | $1.01 \times 10^{-3}$ | 0.05      | 1.0             | 980   | 5.167     | 36.89       |

*Note: $\mu$ is the viscosity of water at 20°C.*
Data Availability

The study did not report any data.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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