Stable vortex-antivortex molecules in mesoscopic superconducting triangles

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(April 8, 2022)

A thermodynamically stable vortex-antivortex pattern has been revealed in mesoscopic type I superconducting triangles, contrary to type II superconductors where similar patterns are unstable. The stable vortex-antivortex “molecule” appears due to the interplay between two factors: a repulsive vortex-antivortex interaction in type I superconductors and the vortex confinement in the triangle.

PACS numbers: 74.60.Ec; 74.55.+h; 74.60.-w; 74.20.De

Symmetrically-confined vortex matter in superconductors, superfluids and Bose-Einstein condensates offers unique possibilities to study the interplay between the $C_\infty$ symmetry of the magnetic field and the discrete symmetry of the boundary conditions. More specifically, superconductivity in mesoscopic equilateral triangles, squares etc. in the presence of a magnetic field nucleates by conserving the imposed symmetry ($C_3$, $C_4$) of the boundary conditions and the applied vorticity. As a result, in an equilateral triangle, for example, in an applied magnetic field $H$ generating two flux quanta, $2\Phi_0$, superconductivity appears as the $C_3$-symmetric combination $3\Phi_0 - \Phi_0$ (further on denoted as $3 - 1$) of three vortices and one antivortex in the center. These symmetry-induced antivortices can be important not only for superconductors but also for symmetrically confined superfluids and Bose-Einstein condensates. Since the order parameter condensates reported in Refs. \cite{1} have been obtained in the framework of the linearized Ginzburg-Landau (GL) theory, this approach is valid only close to the nucleation line $T_c(H)$. Can then these novel symmetry-induced vortex-antivortex patterns survive deep in the superconducting state? Several attempts have been already made to answer this crucial question. In the limit of an extreme type II superconductor ($\kappa \gg 1$), it has been shown that for a thin-film square, a configuration of one antivortex in the center and four vortices on the diagonals of the square is unstable away from the phase boundary \cite{2}. According to the analysis based on the coupled nonlinear GL equations, the vortex-antivortex pairs are unstable and no antivortices appear spontaneously at the $T_c(H)$ line \cite{2}. Possible scenarios of penetration of a vortex into a mesoscopic superconducting triangle with increasing magnetic field have been studied in Ref. \cite{4}. Two different states were considered: a single vortex state and a state in the form of a symmetric combination of three vortices and an antivortex with vorticity $L_{\text{av}} = -2$ ("3 – 2" combination). The calculations \cite{4} have shown that while a single vortex enters the triangle through a midpoint of one side, the "3 – 2" combination turns out to be energetically favorable when the vortices are close to the center of the triangle. Equilibrium is achieved when a single vortex is in the center of the triangle. When approaching the phase boundary, the free energy of a single-vortex state tends to coincide with the energy of the "3 – 2" combination \cite{4}, thus confirming conclusions \cite{2,4} that formation of antivortices is possible in the close vicinity to the phase boundary.

The previous inferences \cite{2,4} on vortex-antivortex states in mesoscopic structures seem to give us no hope to find stable vortex-antivortex configurations deeper in the superconducting state, considering them just as the features appearing in materials with $\kappa \gg 1$ at the phase boundary together with superconductivity. Here we propose the new solution demonstrating the stability of the vortex-antivortex patterns. This solution is based on the simple conjecture made by one of the authors (VVM, \cite{5}): the main source of the vortex-antivortex pattern instability, namely vortex-antivortex attraction, can be removed by taking instead of type II – type I superconductors, where vortex-antivortex interaction becomes repulsive. Indeed, when passing through the dual point $\kappa = 1/\sqrt{2}$, the vortex-vortex interaction changes the sign \cite{6,7} and becomes attractive at $\kappa < 1/\sqrt{2}$. At the same time, the vortex-antivortex interaction becomes repulsive. Therefore, one can expect that presence of antivortices, together with confinement of vortices and antivortices due to a potential barrier at the boundaries, can stabilize novel vortex-antivortex patterns in a mesoscopic sample of type I superconductor. Optimizing the geometry and the sizes of mesoscopic samples, one can therefore fulfill the conditions necessary for the existence of stable vortex-antivortex configurations. For instance, presence of sharp corners is known \cite{8,9} to lead to a strongly inhomogeneous distribution of the superconducting order parameter in a mesoscopic sample. Enhanced superconducting condensate density at the corners prevents vortices from leaving the sample. Altogether, a triangular type I superconducting sample seems to be an appro-
appropriate candidate to search for a stable vortex-antivortex configuration.

![Diagram of vortex cores overlap first](image)

**FIG. 1.** Schematics of unstable (type II) and stable (type I) vortex-antivortex molecules in a mesoscopic superconducting triangle.

Fig. 1 further illustrates this idea. In a type I superconductor with $\xi > \lambda$, vortex cores overlap first when vortices approach each other. This triggers vortex-vortex attraction (and vortex-antivortex repulsion). In contrast with that, in a type II superconductor with $\xi < \lambda$, local fields created by vortices overlap first when vortices approach each other, thus inducing vortex-vortex repulsion (and vortex-antivortex attraction). As a result, a vortex-antivortex combination, consisting of three vortices ($L_{3v} = 3$) and one antivortex with vorticity $L_{av} = -1$ (‘3−1’ combination, or $3\nu + 1$ av molecule) is stable or unstable, respectively, in a mesoscopic type I or type II superconducting triangle.

To verify these intuitive considerations we investigate a mesoscopic equilateral triangle of a type I superconductor in an applied magnetic field. The side of the triangle is chosen ($a = 1 \mu m$) to be larger than both characteristic lengths of the GL theory, $\xi$ and $\lambda$. In order to provide a sample to be a type I superconductor, we should consider in fact a triangular *prison* with a height $h \gg \xi$ (and also $h \gg \lambda$). This allows us to avoid an increase of $\kappa$ resulting in a known effect when a mesoscopic sample made of material, which is type I superconductor in bulk, becomes effectively type II superconductor [14]. For comparison, a mesoscopic triangle of type II superconductor will be also considered. In our calculations, we use the GL parameters of Pb (type I): $\xi_{\text{Pb}} = 82 \text{ nm}$, $\lambda_{\text{Pb}} = 39 \text{ nm}$, $\kappa_{\text{Pb}} = 0.48$, and of Nb (type II): $\xi_{\text{Nb}} = 39 \text{ nm}$, $\lambda_{\text{Nb}} = 50 \text{ nm}$, $\kappa_{\text{Nb}} = 1.28$ [15].

In the description of the superconducting properties of mesoscopic triangles we rely upon the GL equations for the order parameter $\psi$ and the vector potential $A$ of the magnetic field $H = \text{rot} A$ [4, 10]. In the dimensionless form, when keeping the temperature dependence explicitly, the GL equations are

\[
(-i \nabla - A)^2 \psi - \psi \left[ \left(1 - \frac{T}{T_c}\right) - |\psi|^2 \right] = 0,
\]

\[
\kappa^2 \Delta A = \frac{i}{2} (\psi^* \nabla \psi - \psi \nabla \psi^*) + A |\psi|^2.
\]

Here the GL parameter $\kappa = \lambda(T)/\xi(T)$, and $\xi(0)$ serves as the unit length. The imposed boundary condition is

\[
n \cdot (-i \nabla - A) \psi|_{\text{boundary}} = 0.
\]

Topological characteristics of solutions of the GL equations are determined by (anti)vortex core lines. One revolution along any closed path around such a line changes the phase of the order parameter by $2\pi L$, where $L$ is the winding number (vorticity) of a vortex or antivortex.

Close to the sample, the local magnetic field is distorted as compared to the applied magnetic field $H$. However, far away from the sample the distortion is negligible, and the symmetric gauge of the vector potential can be applied: $A = [H \times r]/2$. The boundary condition, which equates the vector potential of the local field with that of the applied field, is justified at the boundaries of the simulation region, which is taken large enough to provide that all the changes of the magnetic field occur inside this region (see, for example, Refs. [4, 17, 18]). Further, we choose the $z$-axis coinciding with the direction of the applied magnetic field, which is normal to the triangular base. Because $h \gg \lambda$, a non-zero $z$-component of the magnetic field is appreciable in the vicinity of the bases, where the field skirts the prism partially penetrating it. Near the central cross-section in the $xy$-plane, the magnetic field and the order parameter are uniform in the $z$-direction, and the problem becomes effectively two-dimensional. The third dimension is taken into consideration by imposing that, inside the simulation region, the total magnetic flux through any cross-section, perpendicular to the $z$-axis, is the same [17, 19]. The GL equations (1), (2) with the above boundary conditions are solved numerically, using the finite-difference method, on a square mesh with the density of 200 nodes per side of the triangle.

The calculations are performed for temperature $T/T_c = 0.96$. First, we find the magnetic field regions, in which states with antivortices are expected to take place in a type I superconducting triangle. The free energy calculations show that at low fields, from $H_0 = 0$ to $0.14 H_c(0)$, where $H_c(0)$ is the thermodynamical critical field [14, 15] at zero temperature, no vortices penetrate the triangle. The order parameter distribution changes continuously from homogeneous at $H_0 = 0$ to
a strongly inhomogeneous one, characterized by concentration of the superconducting phase in the corners. At $H_0 \approx 0.14H_c(0)$ one vortex enters the triangle. For vorticity $L = 1$, the equilibrium is achieved when the vortex is in the center of the triangle. For magnetic fields within the region from $H_0 = 0.25$ to $0.38H_c(0)$, a giant vortex state with the total vorticity $L = 2$ is energetically preferable. Further on, we focus on $H_0 = 0.32H_c(0)$, which corresponds to the states with the total vorticity $L = 2$. This state can be represented by two possible configurations: (i) two vortices in the form of a multivortex or a giant vortex state; (ii) a symmetric combination of three vortices on the triangle bisectors and one antivortex in the center (“3−1” combination, or $3v + 1av$ molecule). (Symmetric combinations with a larger number of vortices and antivortices such as “6−4”, “9−7” etc. possess a higher free energy than the “3−1” combination and are not considered.)

FIG. 2. The free energy $F_s - F_n$ [measured in $H_c^2(0)/4\pi$] as a function of the distance $d_v$ from the center of a mesoscopic type I superconducting triangle for a symmetric combination of three vortices and one antivortex (the “3−1” combination, or $3v + 1av$ molecule), at $T/T_c = 0.96$, $H_0 = 0.32H_c(0)$, $\kappa = 0.48$.

According to our calculations, it is the “3−1” combination that minimizes the free energy in case of a type I superconductor. In Fig. 2, the free energy for this combination is shown as a function of the distance $d_v$ counted from the center of the triangle along the bisectors to vortices. There are three minima of the free energy as a function of $d_v$. The first minimum, which is at $-1.29\xi(0)$ from the center of the triangle, corresponds to a configuration when vortices are situated between the center of the triangle and the midpoints of the sides of the triangle. This is a saddle point for the free energy as a function of the coordinates $(x, y)$ in the plane of the triangle, and the state is unstable. The second minimum is reached when all the vortices are in the center of the triangle, and the vortex-antivortex combination degenerates to a giant vortex $L_{2v} = 2$. This local minimum represents a metastable state.

FIG. 3. The distribution of the squared modulus of the order parameter $|\psi(x, y)|^2$ for the states with the total vorticity $L = 2$ in mesoscopic superconducting triangles at $T/T_c = 0.96$, $H_0 = 0.32H_c(0)$: $3v + 1av$ molecule in a type I superconducting triangle with $\kappa = 0.48$ (a); a giant vortex state in a type II superconducting triangle $\kappa = 1.28$ (b); a stable multivortex state in a type II superconducting triangle (c).

The absolute minimum is reached when three vortices
are situated between the center and the apexes of the triangle at 1.83\(\xi(0)\) from the antivortex in the center (Fig. 2). This vortex-antivortex molecule is thermodynamically stable. Its stability can be understood in the following way. The distribution of the squared modulus of the order parameter \(|\psi(x, y)|^2\), which relates to the above stable vortex-antivortex molecule, is shown in Fig. 3a. Four zeros of \(|\psi(x, y)|^2\) correspond to three vortices and one antivortex. The distribution of the magnetic field \(H(x, y)\) consistent with the above pattern of \(|\psi(x, y)|^2\) will be presented elsewhere [20]. The function \(|\psi(x, y)|^2\) reaches its maximum value in the corners. These “islands” of the superconducting phase in the corners prohibit vortices, which are repelled by an antivortex in the center, from leaving the triangle through the corners. Thus, vortices, being confined in a type I superconducting triangle and interacting with an antivortex, form a stable vortex-antivortex molecule.

It is worth noting, that for our type I sample a strongly enhanced nucleation field appears due to the confinement of the superconducting condensate in the mesoscopic triangle. In fact, this provides the “soft” scenario for the nucleation of the order parameter, like in bulk type II superconductors.

The stable vortex-antivortex patterns are qualitatively different in case of a type II superconducting triangle. In Fig. 4, the free energy of the “3 – 1” combination is shown (curve “a”) as a function of the distance \(d_v\) from the center of the triangle to vortices. The lowest minimum is reached when all the vortices are in the center of the triangle. This means that a giant vortex with vorticity \(L_{2v} = 2\) is energetically more favorable in a type II superconducting triangle. (The local minimum at \(d_v = 11.57\xi(0)\) represents an unstable state.) The corresponding distribution of the squared modulus of the order parameter \(|\psi(x, y)|^2\) is plotted in Fig. 3b. Although a giant vortex state with \(L_{2v} = 2\) has a lower free energy than the “3 – 1” combination, the equilibrium is reached for a multivortex state when two vortices are at two different bisectors (cf. Ref. [3]) of the triangle (see Fig. 4, curve “b”). The order parameter pattern corresponding to this stable two-vortex state is shown in Fig. 3c.

In conclusion, we have found deep in the superconducting state a thermodynamically stable vortex-antivortex configuration for a mesoscopic type I superconducting triangle, although until now it has been thought that vortex-antivortex patterns are unstable and they can manifest themselves only in the close vicinity to the phase boundary. Vortex-antivortex arrays become unstable in a type II superconducting triangle, in accordance with previous reports. The stability of the vortex-antivortex molecules in type I superconducting triangles is due to the change of the sign in the vortex-vortex and vortex-antivortex interaction forces when passing through the dual point \(\kappa = 1/\sqrt{2}\), combined with the condensate confinement by the boundaries of the mesoscopic triangle.

This work has been supported by GOA BOF UA 2000, IUAP, the FWO-V projects Nos. G.0306.00, G.0274.01, WOG WO.025.99N (Belgium), and the ESF Programme VORTEX. Useful discussions with L. Van Look, M. Morelle and G. Teniers are acknowledged.

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