Abstract

We discuss a model in which the third generation fermions undergo a different $SU(2)$ weak interaction from the first two generation fermions. In general, a flavor changing neutral current interaction is expected. Constrained by the precision low energy data, the mass ($M_{Z'} = M_{W'}$) of the heavy gauge bosons is bounded from below to be about $1.1$ TeV for $\alpha_s = 0.125$ and about $1.3$ TeV for $\alpha_s = 0.115$, at the $3\sigma$ level. This model favors a larger $R_b$ and a smaller $R_c$ as compared with the Standard Model, but it does not explain the $R_c$ data. If one takes the $R_b$ data seriously, then $M_{Z'}$ is bounded, at the $3\sigma$ level, to be $462$ GeV $< M_{Z'} \cos \phi < 1481$ GeV, where $\cos \phi$ is the mixing angle between the two $SU(2)$’s in the model. Effects predicted for high energy experiments at the Tevatron, LEP140, LEP-II, LHC, and future linear colliders are also discussed.

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1 Introduction

With the exception of a few measurements, all the data agree with the predictions of the Standard Model (SM). Among those measurements are \( R_b \) and \( R_c \) at LEP with deviations of about 3.5\( \sigma \) and 2.5\( \sigma \), respectively \([1]\), and the excess of large \( E_t \) jets at the Tevatron \([2]\). If one takes the above measurements seriously, one can advocate specific types of new physics which tackle these experimental concerns, such as the studies done in Ref. \([3]\).

In this paper we are not restricting our motives to just explaining the measurements discussed above, but more generally, we are driven by a theoretical observation, namely the hierarchy of the fermion mass spectrum. The relatively large mass of the third generation fermions may suggest a dynamical behavior different from that of the first two generations. Here, we consider a model \([4]\) in which the third generation undergoes a different flavor dynamics from the usual weak interactions proposed in the SM. We assume this flavor dynamics to be associated with a new \( SU(2) \) gauged symmetry. Therefore, a new spectrum of gauge bosons emerges in this model. No modifications to QCD interactions are considered here; this case has been discussed elsewhere \([5]\).

2 The Model

This model is based on the electroweak symmetry group \( SU(2)_l \times SU(2)_h \times U(1)_Y \), where the third generation of fermions, (the top and bottom quarks; \( t \) and \( b \), the tau lepton and its neutrino; \( \tau \) and \( \nu_{\tau} \)) can experience a strong flavor interaction instead of the weak interaction advocated by the SM. In that case, the first two generations only feel the weak interactions supposedly equivalent to the SM case. The quantum numbers of the fermions are

For the first two generations,
- Left-handed quarks: \((2, 1)_{1/3}\),
- Left-handed leptons: \((2, 1)_{-1}\).

For the third generation
- Left-handed quarks: \((1, 2)_{1/3}\),
- Left-handed leptons: \((1, 2)_{-1}\).

For all the right-handed fermions we have
- Right-handed quarks and leptons: \((1, 1)_Q\),

where \( Q_e \) is the electric charge of the right-handed fermions.

The prescribed model is similar to the Ununified Standard Model \([6]\) in the gauge sector. The difference between the two models lies in the fermionic quantum numbers under the gauge group. Following the same notation in Ref. \([6]\) the covariant derivative may be written as

\[
\partial^\mu + ig_l T_l^a W_l^a + ig_h T_h^a W_h^a + ig' Y B^\mu, \tag{1}
\]

where \( T_l^a \) and \( T_h^a \), \( a = 1 \) to \( 3 \) are the \( SU(2) \) generators and \( Y \) is the hypercharge.
The gauge couplings may be written as
\[ g_l = \frac{e}{\sin \theta \cos \phi}, \quad g_h = \frac{e}{\sin \theta \sin \phi}, \quad g' = \frac{e}{\cos \theta}, \quad (2) \]
where \( \theta \) plays the role of the usual weak mixing angle and \( \phi \) is a new parameter in this model.

The symmetry breaking of the gauge group into the electromagnetic group \( U(1)_{\text{em}} \) is a two-stage mechanism: first, \( SU(2)_l \times SU(2)_h \times U(1)_Y \) breaks down into \( SU(2)_{l+h} \times U(1)_Y \) at some large mass scale; second, \( SU(2)_{l+h} \times U(1)_Y \) breaks down into \( U(1)_{\text{em}} \) at a scale of the order of the SM electroweak symmetry-breaking scale. This breakdown can be accomplished by introducing two scalar matrix fields: \( \Sigma = \sigma + i\pi a \tau^a \) and \( \Phi \), with the transformations \( \Sigma \sim (2, 2)_0 \) and \( \Phi \sim (2, 1)_1 \). The \( \tau^a \)’s are the Pauli matrices which satisfy \( \text{Tr}(\tau^a \tau^b) = 1/2 \delta_{ab} \). When the \( \Sigma \) field acquires a vacuum expectation value (v.e.v.) \( u \), the symmetry \( SU(2)_l \times SU(2)_h \times U(1)_Y \) is broken into the group \( SU(2)_{l+h} \times U(1)_Y \), and the symmetry-breaking scale is set by the v.e.v \( u \). Then, at a lower energy scale \( v \), the scalar \( \Phi \) acquires a v.e.v \( v \) and the symmetry is finally reduced to \( U(1)_{\text{em}} \). The scalar fields, except for the two remaining neutral Higgs particles, become the longitudinal components of the physical gauge bosons.

To obtain the gauge boson mass eigenstates, we first write the gauge bosons in the following basis:
\[ W^{\pm}_1 = \cos \phi W^\pm_{1 \mu} + \sin \phi W^\pm_{h \mu}, \quad W^{\pm}_2 = -\sin \phi W^\pm_{1 \mu} + \cos \phi W^\pm_{h \mu}, \quad (3) \]
\[ Z_1 = \cos \theta (\cos \phi W^3_{1 \mu} + \sin \phi W^3_{h \mu}) - \sin \theta B_\mu, \quad (4) \]
\[ Z_2 = -\sin \phi W^3_{1 \mu} + \cos \phi W^3_{h \mu}, \quad (5) \]
\[ A_\mu = \sin \theta (\cos \phi W^3_{1 \mu} + \sin \phi W^3_{h \mu}) + \cos \theta B_\mu, \quad (6) \]
where \( W^\pm_{1 \mu} = (W^1_{1 \mu} \mp iW^2_{1 \mu})/\sqrt{2} \), and similarly for \( W^\pm_{h \mu} \). The gauge field \( A_\mu \) is massless, corresponding to the physical photon field.

So far, \( g_h, g_l \), and \( x \) are free parameters. In this paper we mainly concentrate on the case where \( g_h > g_l \) (equivalently \( \tan \phi < 1 \)) but with \( g_h^2 \leq 4\pi \) (which implies \( \sin^2 \phi \geq g^2/(4\pi) \sim 1/30 \)) so that the perturbation theory is valid. Similarly, for \( g_h < g_l \), we require \( \sin^2 \phi \leq 0.96 \). Furthermore, we focus on the region where \( x \gg 1 \), though another region of interest could be \( x \sim 1 \) \((u \sim v)\), but in this later case the one-loop level contributions due to the heavy gauge bosons should also be included because they are of the same order as the SM one-loop contributions.

To order \( 1/x \) the eigenstates of the light gauge bosons are
\[ W^\pm_\mu = W^\pm_{1 \mu} + \frac{\sin^3 \phi \cos \phi}{x} W^\pm_{2 \mu}, \quad Z_\mu = Z_{1 \mu} + \frac{\sin^3 \phi \cos \phi}{x \cos \theta} Z_{2 \mu}. \quad (7) \]
While for the heavy gauge bosons we find

$$W'_{\mu} = - \frac{\sin^3 \phi \cos \phi}{x} W_1^\pm + W_2^\pm\mu, \quad Z'_\mu = - \frac{\sin^3 \phi \cos \phi}{x \cos \theta} Z_{1\mu} + Z_{2\mu}. \quad (8)$$

To the same order, the gauge boson masses are

$$M^2_{W^\pm} = M^2_0 (1 - \frac{\sin^4 \phi}{x}), \quad M^2_Z = M^2_0 \frac{\cos^2 \theta}{x} (1 - \frac{\sin^4 \phi}{x}), \quad (9)$$

$$M^2_{W'^\pm} = M^2_{Z'} = M^2_0 \left( \frac{x}{\sin^2 \phi \cos^2 \phi} + \frac{\sin^2 \phi}{\cos^2 \phi} \right), \quad (10)$$

where, $M_0 = ev/(2 \sin \theta)$. Notice that the heavy gauge bosons are degenerate up to this order, i.e. $M_{Z'} = M_{W'^\pm}$. The left-handed fermion couplings to the light gauge bosons may be written as,

$$\frac{e}{\sin \theta} \left( T^\pm_h + T^\pm_l + \frac{\sin^2 \phi}{x} \left( \cos^2 \phi T^\pm_h - \sin^2 \phi T^\pm_l \right) \right), \quad (11)$$

$$\frac{e}{\sin \theta \cos \theta} \left( T^3_h + T^3_l - Q \sin^2 \theta + \frac{\sin^2 \phi}{x} \left( \cos^2 \phi T^3_h - \sin^2 \phi T^3_l \right) \right). \quad (12)$$

While the left-handed fermion couplings to the heavy gauge bosons are

$$\frac{e}{\sin \theta} \left( \frac{\cos \phi}{\sin \phi} T^\pm_h - \frac{\sin \phi}{\cos \phi} T^\pm_l - \frac{x}{\sin \phi \cos \phi} \left( T^\pm_h + T^\pm_l \right) \right), \quad (13)$$

$$\frac{e}{\sin \theta} \left( \frac{\cos \phi}{\sin \phi} T^3_h - \frac{\sin \phi}{\cos \phi} T^3_l - \frac{x}{\sin \phi \cos \phi} \left( T^3_h + T^3_l - Q \sin^2 \theta \right) \right). \quad (14)$$

The right-handed fermion couplings to the neutral gauge bosons $Z$ and $Z'$ are, respectively,

$$\frac{e}{\sin \theta \cos \theta} \left( -Q \sin^2 \theta \right), \quad (15)$$

$$\frac{e}{\sin \theta} \left( -Q \sin^2 \theta \frac{\sin^3 \phi \cos \phi}{x \cos^2 \theta} \right). \quad (16)$$

The fermion couplings to the photon are the usual electromagnetic couplings. As we see, if $g_h > g_l$, the heavy gauge bosons would couple strongly to the third generation and weakly to the first two generations, and vice versa.

The first and second generations acquire their masses through the Yukawa interactions to the $\Phi$ field just as in the SM. For the third generation we cannot generate their masses through the usual Yukawa terms (dimension four operators), as it is not allowed by gauge invariance. It is only through higher dimension operators that we can generate these fermion masses. (Another possible model is to introduce an additional
Higgs doublet as done in Ref. [4]. This may be attributed to the strong flavor dynamics which may be evident at adequately high energies where the interactions become strong. However, we do not offer an explicit scenario in this paper for such a picture.

Once we generate the fermion mass matrices, we can diagonalize them by using bilinear unitary transformations, and then obtain the physical masses. Since the third family interacts differently from the first two families, we expect in general that Flavor Changing Neutral Currents (FCNC) will occur at tree level. For the lepton sector we introduce the unitary matrices \( L_e \), and \( R_e \) with the transformations,

\[
e_i^L \rightarrow L^{ij}_e e_i^L, \quad e_i^R \rightarrow R^{ij}_e e_i^R.
\]

Hence, the physical mass matrix is given by

\[
M_e^{\text{diag.}} = L_e^\dagger M_e R_e.
\]

We see that new features are manifest in this model, e.g. lepton mixing. This is an exciting possibility. Even though the neutrinos are massless, they can still mix in this model due to the different interactions of different family neutrinos. This might be connected to the solar neutrino problem. For the quark sector, we find that the measurements of \( R_b \) and \( R_c \) from the SM favor no mixing in the neutral sector. Thus we consider FCNCs only in the lepton sector, but not in the quark sector.

### 3 Low Energy Constraints

To test this model by low energy data, it is convenient to consider the form of the four-fermion current-current interactions at zero momentum transfer. The four-fermion charged-current weak interactions are

\[
\frac{2}{v^2}(j_l^\pm + j_h^\pm)^2 + \frac{2}{u^2}j_h^\pm^2,
\]

and the neutral current four-fermion interactions are

\[
\frac{2}{v^2}(\tilde{j}_l^3 + j_h^3 - \sin^2 \theta_{j_{em}})^2 + \frac{2}{u^2}(j_h^3 - \sin^2 \phi \sin^2 \theta_{j_{em}})^2,
\]

where \( j_{l,h}^\pm \) are the left-handed charged currents corresponding to the first two generations and the third generation, respectively. Similarly, \( \tilde{j}_{l,h}^3 \) refers to the left-handed \( T^3 \) currents, while \( j_{em} \) represents the full electromagnetic current of the three families. We conclude that if there is no mixing in the lepton families then all leptonic decays are identical to the SM, e.g. the \( \tau \) lifetime can not furnish any new information about this model. However, it is more general to allow mixing in the leptonic families, so we will investigate this possibility more carefully. Because of the almost vanishing branching ratio of the decay \( \Gamma(\mu^- \rightarrow e^- e^+ e^-) \), \( BR \leq 10^{-12} \), we will only allow mixing of \( \mu \) and
observables as follows, \( \sin \delta O \) and \( \tau \) measure of \( O \). The constraints on \( \sin^2 \beta \) come from: the ALEPH measurement (in terms of the effective couplings ratio \( g_\tau/g_\mu \), cf. Table 1) of the branching fraction for \( \tau \) decay into \( \mu \) and the determination of the \( \tau \) lifetime \([8]\), the lepton number violation decay of \( \tau \rightarrow \mu \mu \mu \), with a branching ratio \( BR < 4.3 \times 10^{-6} \) (at 90% C.L.) \([7]\), and the FCNC search at LEP with \( BR(Z \rightarrow \mu^\pm \tau^\mp) < 1.7 \times 10^{-5} \) (at 95% C.L.) \([9]\). All other fermionic processes at zero momentum transfer, such as the \( \mu \) decay, \( K-K \) mixing, and \( B-B \) mixing, are identical to the SM predictions.

In this model, the low energy predictions depend on the values of \( 1/x \), \( \sin^2 \phi \), and \( \sin^2 \beta \) in addition to the measured values of \( \alpha_{em}(M_Z) \), \( G_F \), and \( M_Z \). Using the most recent LEP measurements \([8]\) (the total width of the \( Z \) boson, \( R_e \), \( R_\mu \), \( R_\tau \), the vector \( g_V \) and axial-vector \( g_A \) couplings of \( e \), the ratios \( g_V(\mu, \tau)/g_V(e) \), \( g_A(\mu, \tau)/g_A(e) \), the lepton forward-backward asymmetries, the \( \tau \) and \( e \) polarization asymmetry, the hadronic pole cross section \( \sigma_h^0 \), and the ALEPH measurement of \( g_\tau/g_\mu \) \([8]\)) combined with the FCNC measurements of \( \tau^- \rightarrow \mu^- \mu^- \mu^+ \) and \( Z \rightarrow \mu^- \tau^+ \) we determine the allowed values of \( \sin^2 \phi \), \( \sin^2 \beta \), and \( M_Z \). We do not include the controversial observables \( R_\tau \) and \( R_e \) as a part of our fit. Instead, we treat them as a prediction and discuss later whether our model is able to explain the anomaly in these measurements. The experimental values of the electroweak observables \([8]\) and their SM prediction \([10]\) are given in Table 1.

We calculate the changes in the relevant physical observables relative to their SM values to leading order in \( 1/x \), i.e.

\[
O = O^{SM}(1 + \delta O),
\]

where \( O^{SM} \) is the SM value for the observable \( O \) including the one-loop SM correction, and \( \delta O \) represents the new physics effect to leading order in \( 1/x \). We list the calculated observables as follows,

\[
\Gamma_Z = \Gamma_{Z}^{SM} \left(1 + \frac{1}{x} \left[-0.896 \sin^4 \phi + 0.588 \sin^2 \phi\right]\right),
\]

\[
R_e = R_e^{SM} \left(1 + \frac{1}{x} \left[0.0794 \sin^4 \phi + 0.549 \sin^2 \phi\right]\right),
\]

\[
R_\mu = R_\mu^{SM} \left(1 + \frac{1}{x} \left[0.0794 \sin^4 \phi + 0.549 \sin^2 \phi - 2.139 \sin^2 \beta \sin^2 \phi\right]\right),
\]

\[
R_\tau = R_\tau^{SM} \left(1 + \frac{1}{x} \left[0.0794 \sin^4 \phi + 0.549 \sin^2 \phi - 2.139 \cos^2 \beta \sin^2 \phi\right]\right),
\]

\[
A_{FB}^e = (A_{FB}^e)^{SM} \left(1 + \frac{1}{x} \left[10.44 \sin^4 \phi\right]\right),
\]

\[
A_{FB}^\mu = (A_{FB}^\mu)^{SM} \left(1 + \frac{1}{x} \left[10.44 \sin^4 \phi + 12.14 \sin^2 \beta \sin^2 \phi\right]\right),
\]

\[
A_{FB}^\tau = (A_{FB}^\tau)^{SM} \left(1 + \frac{1}{x} \left[10.44 \sin^4 \phi + 12.14 \cos^2 \beta \sin^2 \phi\right]\right),
\]
\[ A_e = A_e^{\text{SM}} \left( 1 + \frac{1}{x} \left[ 5.22 \sin^4 \phi \right] \right), \]  
\[ A_\tau = A_\tau^{\text{SM}} \left( 1 + \frac{1}{x} \left[ 5.22 \sin^4 \phi + 12.14 \cos^2 \beta \sin^2 \phi \right] \right), \]  
\[ \sigma_h^0 = (\sigma_h^0)^{\text{SM}} \left( 1 + \frac{1}{x} \left[ -0.01 \sin^4 \phi - 0.628 \sin^2 \phi \right] \right), \]  
\[ M_W = M_W^{\text{SM}} \left( 1 + \frac{1}{x} \left[ 1 + 0.215 \sin^4 \phi \right] \right), \]  
\[ \frac{g_\tau}{g_\mu} = \left( \frac{g_\tau}{g_\mu} \right)^{\text{SM}} \left( 1 + \frac{1}{x} \left[ 0.50 \sin^2 \beta \cos^2 \beta \right] \right), \]  
\[ \text{BR}(\tau^- \to \mu^- \mu^+) = 0.045 \frac{\sin^2 \beta \cos^2 \beta}{x^2} \left( \sin^2 \beta - 4 \sin^2 \theta \sin^2 \phi \right)^2, \]  
\[ \Gamma(Z \to \mu^- \tau^+) = 0.167 \text{ GeV} \left( \frac{\sin^2 \phi \sin \beta \cos \beta}{x} \right)^2. \]  

In Figure 1 we show the fit result, at the 3\sigma level, of the \( Z' \) mass as a function of \( \sin^2 \phi \), for \( \alpha_s = 0.125 \) and for three values of the mixing parameter \( \sin^2 \beta = 0 \) (dashed line), 0.5 (dot-dashed line) and 1 (solid line). In the case of \( \sin^2 \beta = 0 \) we find a lower bound on \( M_{Z'} \) of approximately 1.1 TeV. For \( \sin^2 \beta = 1, M_{Z'} \) is approximately 1.4 TeV. For \( \sin^2 \beta = 0.5, M_{Z'} \) is required to be larger for smaller \( \sin^2 \phi \) (< 0.1) due to the strong constraint from the lepton number violating process \( \tau \to \mu \mu \mu \). As shown in Figure 1, as \( \sin^2 \phi \) increases the lower bound on \( M_{Z'} \) increases, and increase in \( M_{Z'} \) is slow for \( \sin^2 \phi < 0.5 \) and fast in the other case. This indicates that a relatively light \( Z' \) prefers strong interactions with the third family fermions. If we consider a 2\sigma fit, then the lower bound on \( M_{Z'} \) is about 1.4 TeV for \( \sin^2 \beta = 0 \) and 1.8 TeV for \( \sin^2 \beta = 1 \). For \( \alpha_s = 0.115 \) we find that \( M_{Z'} \geq 1.3 \text{ TeV} \) for \( \sin^2 \beta = 0 \) and \( M_{Z'} \geq 2.1 \text{ TeV} \) for \( \sin^2 \beta = 1 \) at the 3\sigma level.

The LEP measured quantities \( R_b = \frac{\Gamma_b}{\Gamma_h} \) and \( R_c = \frac{\Gamma_c}{\Gamma_h} \) are not consistent with the SM prediction. One possibility to explain the anomaly in these quantities is to consider new physics which can affect the \( b \) and \( c \) quarks’ couplings to the \( Z \) boson. The question now is whether our model is able to give any insight regarding these measurements. The observed value \( R_b^{\text{exp}} = 0.2219 \pm 0.0017 \) \([1]\) is higher than the SM value \( R_b^{\text{SM}} = 0.2157 \) \([10]\) by about 3.5\sigma. On the other hand, \( R_c^{\text{exp}} = 0.1543 \pm 0.0074 \) is smaller than the SM value \( R_c^{\text{SM}} = 0.1721 \) by about 2.5\sigma. With the allowed region of our parameter space being determined, we investigate which part of the allowed space is able to explain the anomaly in \( R_b \). Because the measured value of \( R_b \) is different from the SM value by more than 3\sigma, we expect to be able to constrain the smallest and largest \( Z' \) mass by requiring that the new physics effect shifts the theoretical value of \( R_b \) to be within the 3\sigma range of the measured value. In our model \( R_b \) is given by

\[ R_b = R_b^{\text{SM}} \left( 1 + \frac{1}{x} \left[ -0.0149 \sin^4 \phi + 1.739 \sin^2 \phi \right] \right). \]
Thus, the $Z'$ mass can be constrained to be

$$462 \text{ GeV} < M_{Z'} \cos \phi < 1481 \text{ GeV}.$$  \hspace{1cm} (37)

Therefore, if we assume the anomaly in $R_b$ is mainly due to this type of new physics, then there is an upper bound on $M_{Z'}$ which depends on the gauge coupling (equivalently $\sin \phi$). For example, for $\sin^2 \phi = 0.04$, the upper bound (which is independent of $\sin^2 \beta$) on $M_{Z'}$ obtained from $R_b$ is $\sim 1.5$ TeV.

For $R_c$ we find that the new modification to the SM model shifts $R_c$ in the correct direction, i.e. it decreases the theoretical value as desired. However, the amount of shift is too small to account for its anomaly, e.g. with the lower bound on the heavy mass coming from 1.1 TeV, we find that the theoretical value of $R_c$ is still outside the 2$\sigma$ range of the measured value.

From these results we conclude that this model can account for the deviation in $R_b$ from the SM at the 3$\sigma$ level. Even though $R_c$ is shifted in the needed direction, the predicted value is still outside the 2$\sigma$ range of the data. Therefore, we cannot explain the anomaly in $R_c$ entirely based on the proposed model. Furthermore, $A_{\text{LR}}$ in this model is identical to $A_e$. Thus, this model cannot explain the discrepancy between the the SLD measurement $A_{\text{LR}} = 0.1551 \pm 0.0040$ and the LEP measurement $A_e$ \cite{10}.

4 High Energy Experiments

LEP was operating at the $Z$-pole with large production rates, it is therefore unlikely to better test this model at other high energy colliders at the scale of $M_Z$. We have checked that the measurements of $W^\pm$ and $Z$ properties at the Tevatron by CDF and DØ groups \cite{11} do not further constrain the allowed parameters in Figure 1. To study the possible effects due to the heavy $W'$ and $Z'$ bosons, we shall concentrate on physics at energy scales larger than $M_Z$. In this study, the interference effects from $\gamma$, $Z$ and $Z'$ in neutral channels and the interference of $W$ and $W'$ in charged channels are all included. To simplify our discussion, we shall consider two sets of parameters for $(x, \sin^2 \phi, \sin^2 \beta)$ : (7,0.04,0) and (20.6,0.14,0.5) which correspond to $(M_{Z'},\Gamma_{Z'})$ equal to (1083,291) GeV and (1050,76) GeV, respectively. Our conclusions, however, will not significantly depend on the details of the parameters chosen from Figure 1.

At the Tevatron, it is possible to reach the high energy region where the $W'$ or $Z'$ effects can be important. CDF has reported the result of searching for new gauge bosons by measuring the number of excess di-lepton events with large transverse mass \cite{12} or invariant mass \cite{13}. We find that those results do not further constrain the parameters shown in Figure 1. For the Tevatron with Main Injector (a $\bar{p}p$ collider at $\sqrt{S} = 2$ TeV with a 2 fb$^{-1}$ luminosity), the excess in the $e^-e^+$ or $e^+\nu_e$ rates from this model is generally not big enough to be easily observed. Since the third family leptons can strongly couple to the new gauge bosons, the rate of $\tau$ lepton production can in principle be quite different from that of $e$ or $\mu$. Furthermore, if $\sin \beta$ is not zero, the production rates of
\( \bar{p}p \rightarrow W', W' \rightarrow \ell \nu_\ell \) or \( \bar{p}p \rightarrow \gamma, Z, Z' \rightarrow \ell \ell' \) will be different for \( \ell = e \) and \( \mu \). However, even with the maximal mixing between \( \tau \) and \( \mu \) (i.e. \( \sin \beta = 1 \)) this difference at the Tevatron can only exceed a 3\( \sigma \) effect for a 10 fb\(^{-1} \) of integrated luminosity. At the LHC (a pp collider with \( \sqrt{S} = 10 \) TeV and a luminosity of 100 fb\(^{-1} \)), this excess cannot be mistaken. Furthermore, at the LHC, the excess in the production rates of the \( \ell^+ \ell^- \) events can also be individually tested. Thus, it is much easier to either find such new effects or constrain parameters of the model at the LHC than at the Tevatron. We note that this conclusion holds for either a small or large \( \sin^2 \phi \). Although with a large \( \sin^2 \phi \), the new physics effects to light family fermions will be large, because of the large \( W' \) and \( Z' \) masses, the net effect of the new physics to the production of di-lepton pairs does not significantly depend on \( \sin^2 \phi \).

Another signature of the model is an excess in the top quark production, however, this excess cannot be observed at the Tevatron because of large background from the QCD processes \( q \bar{q}, gg \rightarrow t \bar{t} \). At the LHC, the excess in the \( t \bar{t} \) pair productions can easily be seen in the invariant mass distributions. The extra gauge bosons can also produce an excess of di-jet events in the large invariant mass region, but the parameter space remaining after imposing low energy constraints does not allow a big enough effect to explain the results reported by CDF [2].

Another possible interesting signature is the production of \( \mu^+ \tau^- \) pairs, which is unconstrained by current LEP data. At the Tevatron for \( (x, \sin^2 \phi, \sin^2 \beta) \) equal to \((20.6, 0.14, 0.5)\), the most favorable scenario for observing this signal, we find a total of about 20 events for 2 fb\(^{-1} \) of integrated Luminosity, assuming no cuts are imposed. It is interesting to notice that this implies that the upgraded Tevatron can provide a better constraint on this FCNC type of event than LEP can. At the LHC, the cross section is big, about 170 fb for this choice of parameters.

At high energy electron colliders, the detection of the above new signatures becomes much easier as long as there are enough of them produced in the collisions. In this model, neither LEP140 or LEP-II can see them, so we shall concentrate on the future high energy Linear Collider (LC) [14]. Consider the proposed \( e^+e^- \) LC at center of mass (CM) energy \( \sqrt{s} = 500 \) GeV with an integrated luminosity of 50 fb\(^{-1} \). For \( m_t = 175 \) GeV the SM production rate \( \sigma^{SM}_{(e^+e^- \rightarrow t\bar{t})} \) is 558 fb. Thus a large number of \( t\bar{t} \) pairs is expected at the NLC. Considering the set of parameters \( (x, \sin^2 \phi, \sin^2 \beta) = (7.0, 0.04, 0.0) \), we find that \( \sigma_{(e^+e^- \rightarrow t\bar{t})} = 709 \) fb, i.e. there is about 27\% increase in the total production rate compared to the SM. At the LC it is expected to measure the \( t\bar{t} \) cross section, for \( \ell + \) jets decay modes, to within a few percent. With the assumption that the expected measurement is within 3 standard deviation from the SM, we can constrain the parameters to those which produce \( M_{Z'} \geq 2.3 \) TeV. We note that the same constraints hold for different choices of \( \sin^2 \phi \) and \( x \) but with almost the same ratio \( \sin^2 \phi/x \), especially for small \( \sin \phi \), since in the cross section the two parameters enter as a ratio. Because only the left-handed couplings of the top quark are significantly modified in this model, measuring the angular distribution of \( t \) in the \( t\bar{t} \) CM frame, or
its production rate from a polarized $e^\pm$ beam, can further improve these bounds if no new signal is found.

Although the $e^+e^-$ LC is suitable to probe the model under study, we notice that the $\mu^+\mu^-$ collider is also interesting because of the possible mixing between $\mu$ and $\tau$ leptons. For small mixing the $e^+e^-$ and the $\mu^+\mu^-$ colliders lead to similar production rates as expected. For large $\sin\beta$ the total production rate of $\sigma(\mu^+\mu^- \to t\bar{t})$ becomes smaller than the SM rate which shows the opposite effect to the production of the $e^+e^- \to t\bar{t}$ events predicted by this model. For the same reason, it is easy to observe the difference in the production rates of $e^-e^+$ and $\mu^-\mu^+$ (or $\tau^+\tau^-$) pairs at the LC. Furthermore, at the LC, if the FCNC event $e^-e^+ \to \mu^\pm\tau^\mp$ occurs, it can be unmistakably identified. For a 500 GeV LC with a 50 fb$^{-1}$ luminosity, we expect an order of 300 such events to be observed for $(x, \sin^2\phi, \sin^2\beta)$ equal to $(20.6, 0.14, 0.5)$. Figure 2 shows the FCNC event numbers at the LC for a few choices of parameters, assuming no cuts are imposed.

In summary, we find that due to the strong constraints to this model implied from low energy data (including $Z$-pole data) it is not easy to find events with new signatures predicted for Tevatron or LEP-II. However, at the LHC and the LC, it becomes easy to detect deviations from the SM in the productions of the third family or second family (in case of large mixing between $\tau$ and $\mu$ lepton) fermions. We have also checked the possible excess in the $W^+W^-$ or the $W^\pm Z$ productions at future high energy colliders. It turns out that the branching ratios for $Z'$ or $W'$ to the pure gauge boson modes are always small, so the gauge boson pair productions are not good channels for testing this model.

In the process of preparing for this paper, we noticed that another similar work was done in Ref. [15]. Our conclusions on the allowed parameters of the model and the predictions on the event yields for electron or hadron colliders are different from theirs.

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Table Captions

Table. 1.
Experimental [1,8] and predicted values of electroweak observables for the SM [10] and the proposed model (with different choices of parameters) for $\alpha_s = 0.125$ with $m_t = 175$ GeV and $m_H = 300$ GeV.

Figure Captions

Fig. 1.
The lower bound on the heavy $Z'$ mass as a function of $\sin^2 \phi$ at the 3$\sigma$ level, for $\sin^2 \beta = 0$ (solid), $\sin^2 \beta = 1$ (dashed), and $\sin^2 \beta = 0.5$ (dot-dashed) with $\alpha_s = 0.125$.

Fig. 2.
The number of $\mu^\pm \tau^\mp$ events produced at the LC, an $e^+e^-$ collider at $\sqrt{s} = 500$ GeV with an integrated luminosity of 50 fb$^{-1}$ as a function of $\sin^2 \beta$, for two choices of parameters: $(M_{Z'}, \sin^2 \phi, \Gamma_{Z'}) = (1700$ GeV, 0.1, 174 GeV) and (1700 GeV, 0.4, 55 GeV).
a: $\sin^2 \beta = 0, \sin^2 \phi = 0.04, M_{Z'} = 1.1 \text{ TeV}, \Gamma_{Z'} = 288 \text{ GeV}$.

b: $\sin^2 \beta = 1, \sin^2 \phi = 0.04, M_{Z'} = 1.4 \text{ TeV}, \Gamma_{Z'} = 370 \text{ GeV}$.

c: $\sin^2 \beta = 0, \sin^2 \phi = 0.80, M_{Z'} = 3.0 \text{ TeV}, \Gamma_{Z'} = 287 \text{ GeV}$.

d: $\sin^2 \beta = 1, \sin^2 \phi = 0.80, M_{Z'} = 3.3 \text{ TeV}, \Gamma_{Z'} = 316 \text{ GeV}$.
Table 1: Experimental [1,8] and predicted values of electroweak observables for the SM [10] and the proposed model (with different choices of parameters) for $\alpha_s = 0.125$ with $m_t = 175$ GeV and $m_H = 300$ GeV.

| Observables | Experimental data | SM | The model |
|-------------|-------------------|----|-----------|
|             |                   | a  | b  | c  | d  |
| $g_V(e)$    | $-0.0368 \pm 0.0017$ | $-0.0367$ | $-0.0367$ | $-0.0372$ | $-0.0371$ |
| $g_A(e)$    | $-0.50115 \pm 0.00052$ | $-0.5012$ | $-0.5012$ | $-0.5005$ | $-0.5006$ |
| $g_V(\mu)/g_V(e)$ | $1.01 \pm 0.14$   | $1.00$ | $1.00$ | $1.05$ | $1.00$ | $1.04$ |
| $g_A(\mu)/g_A(e)$ | $1.0000 \pm 0.0018$ | $1.0000$ | $1.0034$ | $1.0000$ | $1.0030$ |
| $g_V(\tau)/g_V(e)$ | $1.008 \pm 0.071$  | $1.0000$ | $1.073$ | $1.0000$ | $1.047$ | $1.000$ |
| $g_A(\tau)/g_A(e)$ | $1.0007 \pm 0.0020$ | $1.0000$ | $1.0055$ | $1.0000$ | $1.0036$ | $1.0000$ |
| $\Gamma_Z$  | $2.4963 \pm 0.0032$ | $2.4978$ | $2.5054$ | $2.5025$ | $2.4967$ | $2.4969$ |
| $R_e$       | $20.797 \pm 0.058$ | $20.784$ | $20.848$ | $20.823$ | $20.830$ | $20.822$ |
| $R_\mu$    | $20.796 \pm 0.043$ | $20.784$ | $20.848$ | $20.671$ | $20.830$ | $20.690$ |
| $R_\tau$   | $20.813 \pm 0.061$ | $20.831$ | $20.648$ | $20.870$ | $20.717$ | $20.869$ |
| $\sigma_h^0$| $41.488 \pm 0.078$ | $41.437$ | $41.293$ | $41.348$ | $41.343$ | $41.359$ |
| $A_e$       | $0.139 \pm 0.0089$ | $0.1439$ | $0.1441$ | $0.1440$ | $0.1461$ | $0.1457$ |
| $A_\tau$   | $0.1418 \pm 0.0075$ | $0.1439$ | $0.1537$ | $0.1440$ | $0.1523$ | $0.1457$ |
| $A_{FB}^e$ | $0.0157 \pm 0.0028$ | $0.0157$ | $0.0157$ | $0.0157$ | $0.0162$ | $0.0161$ |
| $A_{FB}^\mu$| $0.0163 \pm 0.0016$ | $0.0157$ | $0.0157$ | $0.0164$ | $0.0162$ | $0.0167$ |
| $A_{FB}^\tau$| $0.0206 \pm 0.0023$ | $0.0157$ | $0.0168$ | $0.0157$ | $0.0169$ | $0.0161$ |
| $g_\tau/g_\mu$ | $0.9943 \pm 0.0065$ | $1.0000$ | $1.0000$ | $1.0000$ | $1.0000$ |
| $R_b$       | $0.2219 \pm 0.0017$ | $0.2157$ | $0.2178$ | $0.2170$ | $0.2170$ | $0.2168$ |
| $R_c$       | $0.1543 \pm 0.0074$ | $0.1721$ | $0.1716$ | $0.1718$ | $0.1718$ | $0.1718$ |
| $M_W$       | $80.26 \pm 0.16$ | $80.32$ | $80.32$ | $80.32$ | $80.37$ | $80.36$ |
| $A_{LR}$   | $0.1551 \pm 0.0040$ | $0.1439$ | $0.1441$ | $0.1440$ | $0.1461$ | $0.1457$ |
