Bell-state correlations in current of local Fermi liquid

Rui Sakano\textsuperscript{1,*} Akira Oguri\textsuperscript{2}, Yunori Nishikawa\textsuperscript{2}, and Eisuke Abe\textsuperscript{3}

\textsuperscript{1}Institute for solid state physics, the university of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, Chiba, 277-8581 Japan
\textsuperscript{2}Department of physics, Osaka city university, 3-3-138 Sugimoto Sumiyoshi-ku, Osaka-shi, 558-8585 Japan
\textsuperscript{3}Spintronics Research Center, Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522, Japan

(Dated: November 1, 2018)

We study Bell-state correlations for quasiparticle pairs excited in nonlinear current through a double quantum dot in the Kondo regime. Exploiting the renormalized perturbation expansion in the residual interactions of the local Fermi liquid and Bell’s inequality for cross correlation of spin currents through distinct conduction channels, we derive an asymptotically exact form of Bell’s correlation for the double dot at low bias voltages. We find that pairs of quasiparticles and holes excited by the residual exchange interaction can violate Bell’s inequality for the spin currents.

PACS numbers: 71.10.Ay, 71.27.+a, 72.15.Qm

Quantum dots with magnetic moments that strongly interact with conduction electrons in connected lead electrodes exhibit the Kondo effect, which has been a central issue of the condensed matter physics over the 50 years [1]. The low energy properties of the Kondo effect are described well by the local Fermi liquid theory. The local Fermi liquid is an extension of Landau’s Fermi liquid to cover quantum impurities, in which free quasiparticles and residual interactions account for the underlying physics [2–8]. In electric current through the Kondo dot at low applied bias voltages, residual interactions excite quasiparticle pairs that have an effective charge of 2e [5, 9–15]. This doubly-charged state has been observed as enhancements of the shot noise [16–22].

This letter will explore the nature of the correlation between the quasiparticles that are excited by the residual interactions within the current. In a previous work of ours [23], we found that the residual exchange interaction of a quantum dot excites spin-entangled quasiparticles and holes in a nonlinear current. We investigate the nature of the spin entanglement by exploiting Bell’s inequality. Bell’s theorem draws an essential distinction between the correlations found in quantum mechanics and those found in classical mechanics. As a no-go theorem, Bell’s theorem places limits on physical possibility [24–30]. Bell-state correlation of electrons involved in tunneling currents through mesoscopic devices has been studied for the past 20 years [31–34]. Several studies have focused on Bell-state correlations of electrons that are entangled by many-body effects. For example, Bell-state correlations of superconducting electron pairs have been studied with the Cooper pair splitter formed by a Y-shaped junction of superconductor and semiconductor [35–37]. Bell-state correlations have also been predicted for electrons scattered by the Kondo exchange interaction at temperatures near the Kondo temperature [38]. In this letter, we propose a new way to investigate the entanglement of the Kondo state using Bell’s test.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Schematic of the double dot and quasiparticle pairs excited within the two channels of the current. The filled and unfilled circles represent quasiparticles and holes, respectively, and the arrows attached to them indicate their spin degrees of freedom. Cyan and yellow indicate channels 1 and 2, respectively.}
\end{figure}

\textbf{Model} Consider the double dot illustrated in Fig. 1. The system is described by the action of the Anderson impurity model given as \( S = \sum_\mu \int_{-T/2}^{T/2} dt \left( \sigma_3 \right)^{\mu\mu} L_\mu^\mu \), where the Lagrangian is given as \( L_\mu^\mu = L_0^\mu + L_T^\mu + L_1^\mu \) with

\begin{align}
L_0^\mu &= \sum_{\alpha m \sigma} \int_0^D d\bar{\epsilon} \bar{\psi}_{\alpha m \sigma} \left( \frac{i}{\hbar} \frac{\partial}{\partial \bar{\epsilon}} - \epsilon \right) \psi_{\alpha m \sigma} \nonumber \\
&\quad + \sum_{m \sigma} \bar{\psi}_{m \sigma} \left( \frac{\partial}{\partial \bar{\epsilon}} - \epsilon_d \right) \psi_{m \sigma}, \quad (1) \\
L_T^\mu &= \sum_{\alpha m \sigma} \left[ i \bar{\psi}_{m \sigma} \psi_{\alpha m \sigma} + u^* \bar{\psi}_{\alpha m \sigma} \psi_{m \sigma} \right], \quad (2) \\
L_1^\mu &= U \sum_m n_{d m \uparrow}^\mu n_{d m \downarrow}^\mu + W n_{d1}^\mu n_{d2}^\mu + 2JS_{d1}^\mu S_{d2}^\mu. \quad (3)
\end{align}

Here, \( T \) is a long time interval, \( \sigma_3 = ((1, 0)^t, (0, -1)^t) \) is the third element of the Pauli matrix \( \sigma \), and the superscripts \( \mu = - \) and \( + \) represent the forward and backward paths of the Keldysh contour, respectively. Note that, throughout this letter, the time argument \( t \) in the Lagrangian and the Grassmann numbers are suppressed.
$L^\mu$ represents electrons in the lead electrodes and the double dot. $c^\mu_{\alpha \sigma m}$ and $\bar{c}^\mu_{\alpha \sigma m}$ are the Grassmann numbers for electrons with spin $\sigma = \uparrow, \downarrow$ and energy $\epsilon$ in the conduction band of the lead and right leads $\alpha = L, R$ of channel $m = 1, 2$. $d^\mu_{m \sigma}$ and $\bar{d}^\mu_{m \sigma}$ are the Grassmann numbers for electrons with spin $\sigma$ in level $\epsilon_d$ of dot $m$. $L^\mu_T$ represents electron tunneling between the leads and the dots. They are connected by tunneling matrix element $v$ through $\psi^\mu_{\alpha \sigma m} := \int_D d\epsilon \sqrt{\epsilon} c_{\epsilon \alpha \sigma m}$ and $\bar{\psi}^\mu_{\alpha \sigma m} := \int_D d\epsilon \sqrt{\epsilon} \bar{c}_{\epsilon \alpha \sigma m}$, where $D$ is the half width of the conduction band and $\rho_c = \frac{1}{2 \pi D}$ is the density of state for the conduction electrodes. Electron tunneling causes an intrinsic linewidth of the dot levels to be $\Gamma = 2 \pi \rho_c |v|^2$. $L^\mu$ represents interactions between the electrons in the double dot. $U$ and $W$ are the intra- and inter-dot Coulomb interactions, respectively, and $J$ is the exchange interaction. The Grassmann number corresponding to the electron occupations and the total spin in dot $m$ are given by $n_{d m \sigma} = \bar{d}_{m \sigma} d_{m \sigma}$, $n_{d m} = \sum_\sigma n_{d m \sigma}$, and $S_{d m} = \frac{1}{2} \sum_{\sigma, \sigma'} \bar{d}_{m \sigma} \sigma' d_{m \sigma'}$. We impose the particle-hole symmetry $\epsilon_d = -\frac{\epsilon}{2} = -W$ and the absolute zero temperature $T = 0$ to eliminate the thermal and partition noises and maximize the effect of $J$. The bias voltage $eV$ is applied symmetrically: the chemical potentials of the left and right leads are $\mu_L = +\frac{e}{2}V$ and $\mu_R = -\frac{e}{2}V$, respectively. With no loss of generality, a positive bias voltage $eV > 0$ can be assumed. We also use the natural units $\hbar = \hbar_k = 1$.

Bell’s inequality for current correlations We investigate quasiparticles that become correlated across the two channels. In the original argument of Bell’s theorem, the spin correlation of two particles was studied [39]. However, one-by-one detection of every spin of the quasiparticles in a quantum-scale current is still difficult to be achieved in solid-state devices. Thus, we exploit Bell’s inequality for two correlated currents, derived by Chtchelkatchev et al. [36]. This approach is outlined below.

The key idea of Bell’s theorem is that determinism with a hidden variable is assumed to describe any correlations in the world. The violation of this assumption gives a sufficient condition for the quantum entanglement. For our double dot, the correlation between channel 1 and 2 is assumed to be described by a hidden variable $\eta$. Then, the density matrix of the whole system can be written in the form

$$\rho_{\text{HVT}} = \int d\eta f(\eta) \rho_1(\eta) \otimes \rho_2(\eta),$$  
(4)

where the distribution function for the hidden variable is satisfied with $f(\eta) \geq 0$ and $\int d\eta f(\eta) = 1$, and $\rho_\mu(\eta)$ is the density matrix for channel $m$.

Integration of the current can give the average of spin angled to $\theta$ per a particle in the current of channel $m$ in a long time interval from $t - \frac{T}{2}$ to $t + \frac{T}{2}$:

$$\bar{A}_m(t, \eta) = \frac{\int_{t-T/2}^{t+T/2} dt' \text{tr}[\rho_m(t', \eta)J_m^\mu(t')]}{\int_{t-T/2}^{t+T/2} dt' \text{tr}[\rho_m(t', \eta)J_m^\mu(t')]}.$$  
(5)

$J_m^\mu = J_{m\theta} - J_{m\theta+\pi}$ and $J_m = J_{m\theta} + J_{m\theta+\pi}$ are the spin and charge current, respectively, where $J_{m\theta}$ is the current with spin angled to the $\theta$ direction in channel $m$. For a current which effectively carries the spin correlation, the average spin is normalized as $|\bar{A}_m(t, \eta)| \leq 1$. Then, the conventional derivation of Bell’s inequality for two incident entangled particles is applicable to the averaged spin in the currents through the two channels. We obtain the Clauser-Horne-Shimony-Holt Bell’s inequality for two correlated currents as

$$0 \leq C \leq 2,$$  
(6)

where the Bell’s correlation is given in the form

$$C = |F(\theta, \varphi) - F(\theta', \varphi') + F(\theta', \varphi') - F(\theta, \varphi')|.$$  
(7)

Here, $F(\theta, \varphi) = h^\mu(\theta, \varphi)/\hbar^\mu$ is given by a cross-correlation of the spin current

$$h^\mu(\theta, \varphi) := \int dt \langle J_{10}^\mu(t) J_{2\varphi}^\mu(0) \rangle_{\text{HVT}},$$  
(8)

and that of the charge current,

$$h^e := \int dt \langle J_{10}^e(t) J_{2\varphi}^e(0) \rangle_{\text{HVT}},$$  
(9)

with the average by the density matrix of the hidden variable theory, $\langle \cdots \rangle_{\text{HVT}} := \text{tr} \rho_{\text{HVT}} \cdots$. Therefore, violation of Eq. (6) for Bell’s correlation $C_{\text{QM}}$ calculated with the fully quantum mechanical density matrix $\rho_{\text{QM}}$ gives a sufficient condition for quantum correlation.

Current correlations—In our dot, the current of the electrons with spin angled to the $\theta$ direction in channel $m$ is given as $I_{m\gamma} = -i(e d_{m\gamma} \psi_{Rm\gamma} + e^* \psi_{Rm\gamma} d_{m\gamma})$. To calculate current correlation, we introduce the source term

$$L^\mu_{\text{son}}(\lambda) = -i \sum_{m\gamma} \langle e^{i \lambda_{m\gamma} \psi_{Rm\gamma}^lu^* \bar{\psi}_{Rm\gamma} d_{m\gamma}} \rangle.$$  
(10)

in $L^\mu_{\text{A}}$. Here, $\lambda_{m\gamma} = (\sigma_3)^{\mu \mu} \lambda_{m\gamma}$ is a contour-dependent source field, and $\gamma = (\theta, \theta + \pi)$ is the spin index defined with respect to the $\theta$ direction. The Grassmann number for an electron in the dot with spin $\gamma$ can be given by a rotational transformation as $d^\mu_{m\gamma} = \cos \frac{\theta}{2} d^\mu_{m\gamma} - \sin \frac{\theta}{2} d^\mu_{m\gamma+\pi}$, $\bar{d}^\mu_{m\gamma} \psi_{Rm\gamma}^l$, and $\bar{\psi}_{Rm\gamma}^l$ are also defined in the same manner. Current correlations can be calculated by differentiating the generating function $Z(\lambda)$ with the corresponding source fields. The partition function is given in the form

$$Z(\lambda) = \int \mathcal{D}(\bar{c}_{\text{qoma}}) \mathcal{D}(c_{\text{qoma}}) \mathcal{D}(\bar{d}_{m\sigma}) \mathcal{D}(d_{m\sigma}) e^{S(\lambda)}.$$  
(11)

with $S(\lambda) = \sum_{\mu} \int_{-T/2}^{T/2} dt \langle (\sigma_3)^{\mu \mu} [L^\mu_{\text{A}} + L^\mu_{\text{son}}(\lambda)] \rangle_{[23]}$. 

\newpage
Renormalized perturbation theory To take electron correlations into account, we use the renormalized perturbation theory [40–42]. At low energies perturbation expansion in $U, W$, and $J$, provides an exact result if all the terms in the series are accounted for. However, this expansion is difficult, except for some special cases. Below, employing the idea of the renormalized perturbation theory, we reorganize the perturbation expansion and effectively carry out all-order calculations at low energies.

First, we formulate the quasiparticle’s Lagrangian $\tilde{L}_{QM}$ by replacing $\epsilon_{d}, \tilde{v}, U, W, J, d_{\mu \sigma}$, and $\tilde{d}_{\mu \sigma}$ of $L_{A}$ with the renormalized parameters and the Grassmann numbers of the quasiparticle given by $\tilde{\epsilon}_{d}, \tilde{\tilde{v}}, \tilde{U}, \tilde{W}, \tilde{J}, \tilde{d}_{\mu \sigma}$, and $\tilde{d}_{\mu \sigma}$. These renormalized parameters and Grassmann numbers that are defined by sets of perturbation series given by the self-energy and the four vertex at $T = eV = 0$ [23]. Note that the renormalized linewidth given by $\tilde{\Gamma} := 2\pi \rho_{0} |\tilde{v}|^{2}$ corresponds to the characteristic energy scale, namely, the Kondo temperature: $T_K = \pi \tilde{\Gamma}/4$. We can evaluate $\tilde{\epsilon}_{d}, \tilde{\tilde{v}}, \tilde{U}, \tilde{W}$, and $\tilde{J}$ by using the numerical renormalization group (NRG) approach [42–44]. The nonequilibrium effects at low bias voltages $eV \ll T_K$ arise through perturbation expansions in the residual interactions.

As a part of the interaction effects are taken into account ab initio in the quasiparticle’s Lagrangian during renormalized perturbation expansion, a counter term has to be introduced to avoid overcounting in the perturbation expansion. In the other words, the total Lagrangian has to be satisfied with $L_{A}^{\mu} = \tilde{L}_{QM}^{\mu} + L_{CT}^{\mu}$. The counter term $L_{CT}^{\mu}$, can be expressed in terms of the renormalized parameters and the renormalized Grassmann numbers, which are determined by the renormalized condition for the renormalized self-energy and the renormalized four-vertex. In the particle-hole symmetric case, the perturbation expansion up to only the second order in the residual interactions provides an asymptotically exact form of the self-energy at $T = 0$ up to the second order in $\omega$ and $V$, and asymptotically exact forms of currents and current correlations up to order $V^{3}$. Then, any higher orders terms in the residual interactions do not yield contribution of order $V^{3}$ [15, 20, 21, 45]. We shall calculate the current correlations using perturbation expansion in the residual interactions.

Results and discussion.— Let us calculate $C_{QM}$ in terms of the quasiparticle parameters. Since $\langle I_{m}^{s} \rangle = 0$ in our model, the correlation of the spin currents given by Eq. (8) can be rewritten into the correlation of spin current fluctuations $\delta I_{m}^{s} = I_{m}^{s, \theta} - \langle I_{m}^{s, \theta} \rangle$ as

$$h_{QM}^{s}(\theta, \varphi) = \lim_{T \to \infty} \int_{-\frac{T}{2}}^{\frac{T}{2}} dt \langle \delta I_{m}^{s, \theta}(t) \delta I_{m}^{s, \varphi}(0) \rangle. \quad (12)$$

Thus, differentiation of $\ln Z(\lambda)$ with the source fields

$$h_{QM}^{s}(\theta, \varphi) = -\frac{V}{2\pi} \left(V \frac{1}{\tilde{\Gamma}} \right)^{2} \left(1 - \frac{1}{3} \bar{\tilde{W}} \tilde{J} \right) \cos(\theta - \varphi) + O(V^{5}), \quad (13)$$

where $\bar{\tilde{W}} = \frac{\tilde{W}}{\pi \tilde{\Gamma}}$ and $\tilde{J} = \frac{J}{\pi \tilde{\Gamma}}$. Note that the spin correlation measured by $h_{QM}^{s}(\theta, \varphi)$ comes from only a portion of the entangled quasiparticle pairs within the current. As seen in the specific form of $\ln Z(\lambda)$ [23], the residual interactions can excite four types of the quasiparticle pairs in the current (See Fig. 1). As TABLE I shows, the spin and charge current correlations of these pairs have different signs from each other. Consequently, some of the spin and charge correlations due to these pairs are independently canceled in the full current. Therefore, the correlation of the charge current that effectively carries the spin current correlation must be calculated, rather that of the full current $I_{m}^{s}$ given by $h_{e}^{\text{CC}} = \int_{-\infty}^{\infty} dt \langle I_{1}^{s}(t) I_{2}^{s}(0) \rangle$ with $I_{m}^{s} = \sum \gamma I_{m\gamma}^{\gamma}$. The current correlation given by Eq. (9) can be written in terms of current fluctuation of $I_{m}^{s}$ as

$$h_{QM}^{c} = H_{QM}^{c} + \mathcal{T} \langle I_{1}^{c} \rangle \langle I_{2}^{c} \rangle, \quad (14)$$

where $H_{QM}^{c} = \int_{-\infty}^{\infty} dt \left( \delta I_{m}^{c}(t) \delta I_{m}^{c}(0) \right)$ with $\delta I_{m}^{c}(t) = I_{m}^{c}(t) - \langle I_{m}^{c} \rangle$. Although an explicit expression of $I_{m}^{c}$ is not easy to derive, the correlation can be evaluated readily using the terms of spin correlated carriers in $\ln Z(\lambda)$:

$$H_{QM}^{c} = -\frac{V}{2\pi} \left(V \frac{1}{\tilde{\Gamma}} \right)^{2} \left(1 - \frac{1}{3} \bar{\tilde{W}} \tilde{J} \right) + O(V^{5}). \quad (15)$$

The leading term of the charge current is of the third order in the applied bias voltage, $\langle I_{m}^{c} \rangle \propto eV \left(\frac{V}{\tilde{\Gamma}}\right)^{2}$. Thus there are two regions in $h_{QM}^{c}$. One is $T^{-1} \gg eV \left(\frac{V}{\tilde{\Gamma}}\right)^{2}$, where $H_{QM}^{c} \ll \mathcal{T} \langle I_{1}^{c} \rangle \langle I_{2}^{c} \rangle$. Then, the correlation function can be given simply as $h_{QM}^{c} \sim \mathcal{T} \langle I_{1}^{c} \rangle \langle I_{2}^{c} \rangle$. This results in $C_{QM} \sim 0$, and $C_{QM}$ never violates Bell’s inequality in this region. In the opposite region $T^{-1} \ll eV \left(\frac{V}{\tilde{\Gamma}}\right)^{2}$, the correlation of the fluctuations is dominant, namely, $H_{QM}^{c} \gg \mathcal{T} \langle I_{1}^{c} \rangle \langle I_{2}^{c} \rangle$, which leads to $h_{QM}^{c} \sim H_{QM}^{c}$. Then, Bell’s correlation is given in the form

$$C_{QM} \sim K(\theta, \theta'; \varphi, \varphi') \quad (16)$$

| TABLE I. Signs of spin/charge current correlations of particle-particle(p-p), hole-hole(h-h) and particle-hole (p-h) pairs with parallel and antiparallel spins. The pairs excited in the current are shown in Fig. 1. |
|---|---|---|---|
| p-p or h-h pairs | p-h pair |
| parallel spin | (i) +/+ | (ii) −/− |
| antiparallel spin | (iii) −/+ | (iv) +/− |
with
\[ K(\theta, \theta'; \varphi, \varphi') = |\cos(\theta - \varphi) - \cos(\theta' - \varphi) + \cos(\theta - \varphi') + \cos(\theta' - \varphi')|, \] (17)

Since \( K(\theta, \theta'; \varphi, \varphi') \) is bounded in \([0, 2\sqrt{2}]\), it is concluded that the exchange interaction of the Fermi liquid can violate Bell’s inequality.

However, \( C_{QM} \) may be difficult to measure experimentally, because \( h_{QM}^c \) is the current correlation of the carriers that effectively carry the correlated spins. Next we suggest a measurable form of Bell’s correlation. It is obtained by replacing \( h^c \) with the cross correlation of the full charge currents through the two channels \( h_{QM}^{rcc} \) as
\[ C^* \leq 2r \] (18)
with \( r = |h^c/h_{QM}^{rcc}| \), and
\[ C^* := |F^*(\theta, \varphi) - F^*(\theta', \varphi) + F^*(\theta, \varphi') + F^*(\theta', \varphi')| \] (19)
with \( F^*(\theta, \varphi) = h^s(\theta, \varphi)/h_{QM}^{rcc} \). For the quantum mechanical density of states, \( C^* \) and \( r \) take the forms
\[ C^*_{QM} = r_{QM} K(\theta, \theta'; \varphi, \varphi'), \] (20)
\[ r_{QM} = \left| \frac{h_{QM}^c}{h_{QM}^{rcc}} \right| = \left| 1 - \frac{1}{2} \left( \frac{\tilde{w}}{\tilde{w}_f} \right)^2 \right| \] (21)
Since \( C^* \) is simply given by a product of \( C \) and \( r \), \( C^*_{QM} \) can also violate Bell’s inequality given by Eq. (18). The maximum value of \( C^*_{QM} \) is given by \( C^*_{QM, \text{max}} = 2\sqrt{2} r_{QM} \), which corresponds to the Tselson’s bound [46] in our model. This bound gives the upper limit for the correlation in the quantum regime. \( C^*_{QM, \text{max}} \) and \( 2r_{QM} \) are plotted as a function of \( J \); \( J \leq 0 \) and \( J \geq 0 \) for \( U = W = 3.0\pi T \) in Fig. 2 (a) and Fig. 3 (a), respectively. A critical point appears at \( J = J_c > 0 \) [44, 47]. For \( J > J_c \), two electrons occupying in the double dot form an isolated singlet state and decouple from the conduction electrons, and then no charge currents can flow through the double dot. Thus, we focus on the region \( J < J_c \), in which the low-energy state is accounted for by the local Fermi-liquid, and electric current flows through the dot. The region between \( C^*_{QM, \text{max}} \) and \( 2r_{QM} \) represents a sufficient condition that the correlation of spin currents across the two channels is quantum mechanical in nature.

For the finite strength of \( J > 0 \), the value \( C^*_{QM, \text{max}} \) takes a local minimum to zero, where the excited quasiparticles pairs with parallel and antiparallel contributions to the spin correlation cancel each other out. Thus, Bell’s test is not applicable with this value of \( J \). Experimentally, the violation of Bell’s inequality can be confirmed through observation with values of \( C^*_{QM} \) larger than the theoretically calculated value of \( 2r_{QM} \). This parameter \( 2r_{QM} \) depends on the strength of \( U, W \), and \( J \), which can be evaluated using NRG calculations. \( 2r_{QM} \) is plotted as a function of \( J \) for several choices of \( U \) and \( W \) for \( J \leq 0 \) and \( J \geq 0 \) in Fig. 2 (b) and Fig. 3 (b), respectively.

For \( |J| \gg T_K \), the values of \( h_{QM}^{rcc} \) coincide with \( h_{QM}^c \), which results in \( r_{QM} \rightarrow 1 \) and the \( r_{QM} \) independent form of Bell’s inequality is recovered. In this region, therefore, Bell’s test can be examined without the need for any numerical calculations of \( r_{QM} \).

Finally, we discuss the causal locality of Bell’s theorem in our model. Bell-state correlations in our model are induced by entangled quasiparticles that are excited by the residual exchange interaction that is scaled by \( T_K \). Therefore, for the causal locality to hold, the two measurements in channel 1 and 2 must be separated by a distance \( d \gg c t_K \), where \( t_K = \frac{\hbar}{K_B T_K} \) is the Kondo time scale and \( c \) is the speed of light. For a typical Kondo temperature of quantum dots \( T_K \sim 1K \), \( d \) must be much larger than \( c t_K \sim 4.58 \times 10^{-2} \mathrm{m} \).

RS thanks Shiro Kawabata, Taro Wakamura, Thierry Martin, and Kensuke Kobayashi for the helpful discussions, and thanks Yuya Shimazaki for the inspiring discussions. This work was partially supported by JSPS KAKENHI Grant Numbers JP26220711, JP15K05181, JP16K17723, and JP18K03495.
The thin dotted line indicates the value of the local maximum $J$ normalized by the critical value $J_c$. The gray area is covered by the hidden variable theory, and the yellow area represents the sufficient condition for the quantum correlation. (b) $2\tau_{\text{QM}}$ as a function of ferromagnetic $J$ for $U = W = 3.0\pi\Gamma$ and several choices of $W = 3.0\pi\Gamma, 2.9\pi\Gamma, 2.8\pi\Gamma, 2.0\pi\Gamma,$ and $U = W = 0$. The thin dotted line indicates the value of the local maximum $\tau_{\text{QM}} = 1 - \frac{\sqrt{3}}{2}\approx 0.528$.

* sakano@issp.u-tokyo.ac.jp

[1] A. C. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge University Press, 1993).
[2] P. Nozières, J. Low Temp. Phys. 17, 31 (1974).
[3] K. Yamada, Prog. Theor. Phys. 53, 970 (1975).
[4] A. Oguri, Phys. Rev. B 64, 153305 (2001).
[5] C. Mora, C. P. Moca, J. von Delft, and G. Zarand, Phys. Rev. B 92, 075120 (2015).
[6] A. Oguri and A. C. Hewson, Phys. Rev. Lett. 120, 126802 (2018).
[7] A. Oguri and A. C. Hewson, Phys. Rev. B 97, 045406 (2018).
[8] M. Filippone, C. P. Moca, A. Weichselbaum, J. von Delft, and C. Mora, Phys. Rev. B 98, 075404 (2018).
[9] A. O. Gogolin and A. Komnik, Phys. Rev. Lett. 97, 016602 (2006).
[10] E. Sela, Y. Oreg, F. von Oppen, and J. Koch, Phys. Rev. Lett. 97, 086601 (2006).
[11] P. Vitushtinsky, A. A. Clerk, and K. Le Hur, Phys. Rev. Lett. 100, 036603 (2008).
[12] C. Mora, X. Leyronas, and N. Regnaut, Phys. Rev. Lett. 100, 036604 (2008).
[13] C. Mora, P. Vitushtinsky, X. Leyronas, A. A. Clerk, and K. Le Hur, Phys. Rev. B 80, 155322 (2009).
[14] T. Fujii, J. Phys. Soc. Jpn. 79, 044714 (2010).
[15] R. Sakano, Y. Nishikawa, A. Oguri, A. C. Hewson, and S. Tarucha, Phys. Rev. Lett. 108, 266401 (2012).
[16] O. Zarchin, M. Zaffalon, M. Heiblum, D. Mahalu, and V. Umansky, Phys. Rev. B 77, 241303 (2008).
[17] T. Delattre, C. Feuillet-Palma, L. G. Herrmann, P. Morfin, J.-M. Berroir, G. Fève, B. Plaçais, D. C. Glattli, M.-S. Choi, C. Mora, and T. Kontos, Nat. Phys. 5, 208 (2009).
[18] Y. Yamauchi, K. Sekiguchi, K. Chida, T. Arakawa, S. Nakamura, K. Kobayashi, T. Ono, T. Fujii, and R. Sakano, Phys. Rev. Lett. 106, 176601 (2011).
[19] M. Ferrer, T. Arakawa, T. Hata, R. Fujiiwara, R. Delagrange, R. Weil, R. Debloch, R. Sakano, A. Oguri, and K. Kobayashi, Nat. Phys. 12, 230 (2016).
[20] R. Sakano, T. Fujii, and A. Oguri, Phys. Rev. B 83, 075440 (2011).
[21] R. Sakano, A. Oguri, T. Kato, and S. Tarucha, Phys. Rev. B 83, 241301 (2011).
[22] M. Ferrer, T. Arakawa, T. Hata, R. Fujiiwara, R. Delagrange, R. Debloch, Y. Teratani, R. Sakano, A. Oguri, and K. Kobayashi, Phys. Rev. Lett. 118, 196803 (2017).
[23] R. Sakano, A. Oguri, Y. Nishikawa, and E. Abe, Phys. Rev. B 97, 045127 (2018).
[24] S. J. Freedman and J. F. Clauser, Phys. Rev. Lett. 28, 938 (1972).
[25] A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. 49, 1804 (1982).
[26] G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. 81, 5039 (1998).
[27] T. Scheidl, R. Ursin, J. Kofler, S. Ramelow, X.-S. Ma, T. Herbst, L. Ratschbacher, A. Fedrizzi, N. K. Langford, T. Jennewein, and A. Zeilinger, Proc. Natl. Acad. Sci. USA 107, 19708 (2010), http://www.pnas.org/content/107/46/19708.full.pdf.
[28] M. Giustina, A. Mech, S. Ramelow, B. Wittmann, J. Kofler, J. Beyer, A. Lita, B. Calkins, T. Gerrits, S. W. Nam, R. Ursin, and A. Zeilinger, Nature 497, 227 (2013).
[29] B. G. Christensen, K. T. McCusker, J. B. Altepeter, B. Calkins, T. Gerrits, A. E. Lita, A. Miller, L. K. Shalm, Y. Zhang, S. W. Nam, N. Brunner, C. C. W. Lim, N. Gisin, and P. G. Kwiat, Phys. Rev. Lett. 111, 130406 (2013).
[30] B. Hensen, H. Bernien, A. E. Dr’eau, A. Reiserer, N. Kalb, M. S. Blok, J. Ruitenberg, R. F. L. Vermeulen, R. N. Schouten, C. Abell, W. Amaya, V. Pruneri, M. W. Mitchell, M. Markham, D. J. Twitchen, D. Elkouss, S. Wehner, T. H. Taminiau, and R. Hanson, Nature 526, 682 (2015).
[31] C. W. J. Beenakker, C. Emary, M. Kindermann, and J. L. van Velsen, Phys. Rev. Lett. 91, 147901 (2003).
[32] P. Samuelsson, E. V. Sukhorukov, and M. B"{u}ttiker, Phys. Rev. Lett. 91, 157002 (2003).
[33] P. Samuelsson, E. V. Sukhorukov, and M. B"{u}ttiker, Phys. Rev. Lett. 92, 026805 (2004).
[34] A. V. Lebedev, G. B. Leos, and G. Blatter, Phys. Rev. B 71, 045306 (2005).
[35] S. Kawabata, J. Phys. Soc. Jpn. 70, 1210 (2001).
[36] N. M. Chtrekchekov, G. Blatter, G. B. Leos, and T. Martin, Phys. Rev. B 66, 161320 (2002).
[37] L. Hofstetter, S. Csonka, J. Nygard, and C. Schonenberger, Nature 461, 960 (2009).
[38] A. T. Costa and S. Bose, Phys. Rev. Lett. 87, 277901 (2001).
[39] N. Brunner, D. Cavalcanti, S. Pironio, V. Scarani, and S. Wehner, Rev. Mod. Phys. 86, 419 (2014).
[40] A. C. Hewson, Phys. Rev. Lett. 70, 4007 (1993).
[41] A. C. Hewson, J. Phys.: Condens. Matter 13, 10011 (2001).
[42] Y. Nishikawa, D. J. G. Crow, and A. C. Hewson, Phys. Rev. B 82, 115123 (2010).

[43] A. C. Hewson, A. Oguri, and D. Meyer, Eur. Phys. J. B 40, 177 (2004).

[44] Y. Nishikawa, D. J. G. Crow, and A. C. Hewson, Phys. Rev. Lett. 108, 056402 (2012).

[45] A. Oguri, J. Phys. Soc. Jpn. 74, 110 (2005).

[46] B. S. Cirel’son, Lett. Math. Phys. 4, 93 (1980).

[47] Y. Nishikawa, D. J. G. Crow, and A. C. Hewson, Phys. Rev. B 86, 125134 (2012).