On the estimation of the radiant heating surface temperature and the heat transfer rate calculation: New transient simplified analytical model

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Abstract. In this paper, a new semi-analytical model based on a hybrid approach that combines a numerical approach and a sensitivity analysis is developed to evaluate the surface temperature of the underfloor heating system and analyse its thermal behaviour under transient conditions. The model is validated by measurements which are performed in a full-scale test cell. Furthermore, a good agreement is obtained by comparing the developed semi-analytical model with two multidimensional numerical models based on the finite difference method and the finite volume method that are previously developed. The sensitivity analysis, based on the Design of experiments method, indicated that the most influencing parameters on the thermal response of the heating slab, are respectively the specific heat capacity and the heating water flowrate.

1. Introduction
The floor heating systems (FHS) aroused renewed interest because of their energy efficiency. They allow a good indoor thermal comfort since they provide a homogenous surface temperature and a low value of the operative temperature [1,2]. The purpose is to maintain the upper surface of the floor at a maximum temperature of 28/29 °C. In fact, the heat exchange occurs mainly by radiation. Thanks to uniformity of heat distribution, this technique provides better thermal comfort compared to other heating systems [3].

In the literature, several methods has been described to develop heat transfer models associated with the radiant slab namely: numerical, analytical and semi-analytical approaches. The numerical approaches mainly use the finite difference method (FDM), the finite element method (FEM), or the finite volume method (FVM) [4-6]. These approaches are intended to simulate both local and global heat transfer with a good accuracy, but it require a substantial CPU time [6-10]. In the other hand, the analytical approaches, that mainly use Separation of Variables method, Fourier decomposition method or Laplace transformation method, are reliable, accurate and fast [11,12-15]. However, they are complex to solve especially when it concerns multi-dimensional descriptions under transient conditions [16-22]. As a hybrid approach, semi-analytical methods take the advantages of both previous methods. In fact, they are based on simplified analytical models involving parameters that are obtained from sensitivity analysis based on numerical models [23].

The purpose of this work is to develop and validate a simplified semi-analytical model to predict the thermal response of an anhydrite radiant slab involving the main thermophysical and design
parameters. The design of experiments (DoE) method is employed in conjunction with the measurements on a full-scale test cell and a 2D finite difference model is developed and validated.

2. Full-scale experiments and data collection
As shown in Figure 1, the test cell is heated by a heat pump (1) with supplies hot water to a variable air volume system (4), radiators (5), and a radiant slab (6). Several sensors are used to monitor both inlet and outlet hot water temperatures, surface temperatures and depth temperatures of the radiant slab, ambient air temperature, and radiative temperature. An experimental scenario was applied in this study regarding the inlet water temperature, which was kept constant at 31.5 °C.

3. Transient simplified semi-analytical modelling
3.1. Semi-Analytical modelling
If we assumed that the water temperature evolving in the FHS has a logarithmic profile, its transient thermal behaviour can be studied using the Pierson and Padet approach [23]. Therefore, the surface temperature equation is a time-dependent function with a time constant $\tau$ a delay time $t_d$ that could be experimentally estimated. This function may be expressed as follows:

$$T_s(t) = \begin{cases} T_{s,0} & t < t_d \\ T_{s,\infty} + (T_{s,0} - T_{s,\infty})e^{\frac{(t-t_d)}{\tau}} & t \geq t_d \end{cases}$$  \hspace{1cm} (1) [9]

where $T_{s,0}$ is the average surface temperature at $t = 0$; $T_{s,\infty}$ is the asymptotic average surface temperature.

To express the steady-state surface temperature $T_{s,\infty}$, we can consider the calculation details that are reported in [24] considering the whole heat transfer between the heated slab and the indoor environment. $T_{s,\infty}$ is then derived from the heat energy balance equation as follows:

$$T_{s,\infty} = \frac{(T_t - T_a)}{R_a + R_{conv} + R_p + R_{cond}} R_a + T_a$$  \hspace{1cm} (2) [18]

The heat flux rate between the hot water and the indoor environment is calculated using the logarithmic mean temperature difference [28] as follows:

$$\phi = \frac{(T_{f,a} - T_{a,0}) - (T_{f,i} - T_{a,i})}{ln\left(\frac{T_{f,a} - T_{a,i}}{T_{f,i} - T_{a,0}}\right)} = \dot{m}_f C_p f(T_{f,i} - T_{f,0})$$  \hspace{1cm} (3) [19]
with \( U = \frac{1}{\sum_{n} \rho_n} \) is the total heat transfer coefficient; \( T_{f,i} \) and \( T_{f,o} \) are the inlet and the outlet hot water temperatures; \( T_{a,i} \) and \( T_{a,o} \) are the ambient air temperatures at \( x = 0 \) and \( L \), that are considered equal to \( T_a \); \( \Delta T \) is the mass flowrate of the water; and \( C_{p,f} \) is the specific heat of the water.

From the previous equation, the outlet temperature of the hot water can be calculated:

\[
T_{f,o} = T_a + (T_{f,i} - T_a) e^{-\frac{us}{m_f C_{p,f}}} \tag{4} \tag{20}
\]

The temperature \( T_x \) of the water at position \( x \) of the pipe is:

\[
T_x = T_a + (T_{f,i} - T_a) e^{-\frac{us S_x}{m_f C_{p,f}}} \tag{5} \tag{21}
\]

where \( U_x \) and \( S_x \) are the overall heat transfer coefficient and heat exchange surface at position \( x \) of the pipe.

The sum of \( T_x \) over the total length of the tube \( L \) allows the evaluation of the water average temperature of water \( \overline{T_f} \):

\[
\overline{T_f} = \frac{1}{L} \int_{0}^{L} T_x \, dx = T_a + (T_{f,i} - T_a) \left( \frac{1-e^{-\zeta}}{\zeta} \right) \tag{6} \tag{22}
\]

where \( \zeta = \frac{us}{m_f C_{p,f}} \).

3.2. Determination of \( \tau \) and \( t_d \) using the DoE method

The Design of Experiments (DoE) technique is a statistical method used to approximate the mathematical relationship between different factors affecting several response variables, and most often one response variable. It could be used to simplify parametric studies by reducing significantly the required number of experiments or simulations. The obtained mathematical models, also known as meta-models, could be used instead of numerical simulation tools to simplify and accelerate the parametric studies to find optimal solutions and to analyze the effect of each factor on the response variable and the interaction between factors. In our case study, we need to estimate the time constant \( \tau \) and the delay time \( t_d \), by using the validated 2D FDM numerical model. Merabtine et al. [24] have developed and validated both 2D FDM and 3D FVM models with regards to the given experimental scenario.

Numerous factors influencing \( \tau \) and \( t_d \) need to be considered: those related to the ambient air properties \( \rho_a, \lambda_a, C_{p,a}, \) and \( h_{c,a}, h_e \); the anhydrite slab thermo-physical properties \( \rho_c, \lambda_c, C_{p,c}, \) and \( \varepsilon \); the thermo-physical properties of the hot water \( \rho_e, \lambda_e, C_{p,e}, h_{c,e}, \) and \( \dot{V} \); and the geometric parameters \( e \) and \( D_l \).

To simplify the process, a number of assumptions have been made. The ambient air temperature was set to be \( T_a = 16 \, ^\circ C-28 \, ^\circ C \). We assumed that \( \rho_a, \lambda_a, \) and \( C_{p,a} \) remain essentially constant in this temperature range and, as a result, the heat transfer coefficients are kept constant. The water temperature was set between 27.5 \, ^\circ C and 31.5 \, ^\circ C. Table 1 depicts the range variation of the all parameters based on the recommendations of the French standards.
be considered as a useful tool for the estimation and analysis of the thermal behavior of a rad
of the model, which represents a significant advantage when looking for fast and reliable results, it can
under

FDM and the 3D FVM models

Table 2

| Factors influencing τ and td | Labels | Levels          |
|-----------------------------|--------|-----------------|
| Slab thickness, e (m)       | A      | 0.04 - 0.06     |
| Thermal conductivity of the slab, λ (W.m-1.K-1) | B | 1.2 - 2.6     |
| Slab density, ρ (kgm-3)     | C      | 1500 - 2500     |
| Specific heat of the slab, C_p (J.kg-1.K-1) | D | 1000 - 2000     |
| Volume flow rate, V̇ (L.s-1) | E | 0.02 - 0.06     |
| Tube inner diameter, D_i (m) | F | 0.012 - 0.02    |

A full factorial plan was used to generate all the numerical data from the DoE, which require 2^6 = 64 simulations, including all interactions between parameters. Furthermore, statistical data was obtained by implementing each factor combination in the FDM model. The temperature profiles obtained from the 2D FDM model were then post-treated using a nonlinear regression method to obtain numerical values of τ and t_d. Once all the values of τ and t_d were obtained, the meta-models of τ and t_d were generated. The reduced statistical meta-models of τ and t_d obtained by the full factorial DOE for the factors in Table 4 and in the given ranges of variation, are expressed, respectively, as follows:

\[ \tau = (22.16 - 98 e - 1.34 \lambda + 0.00006 \rho + 0.0016 C_p - 23.3 \dot{V} + 1257 D_i + 0.37 e \rho + 0.47 e C_p - 15465 e D_i + 0.000008 \rho C_p - 0.76 \rho D_i - 1.02 C_p D_i)^2 \]  

(7) [23]

\[ t_d = 1532 + 6586 e + 342 \lambda - 0.0745 \rho - 0.0543 C_p + 3109 \dot{V} + 5390 D_i - 6936 e \lambda + 5.14 e \rho + 8.88 e C_p + 7533 \lambda D_i - 9.02 C_p D_i - 366133 \dot{V} D_i \]  

(8) [24]

3.3 Validation of the semi-analytical model

Considering the logarithmic profile of the measured surface temperature, τ corresponds to the time required for the temperature profile to reach \((1 - \frac{1}{e})\sim 63\%\) of its value at the steady state. Also, the obtained meta-models (Eqs. 23 and 24) allow to calculate the time constant τ and the delay time t_d values. After that, such values are substituted into Eq. 9 to obtain the average surface temperature of the radiant slab. Table 2 compares the calculated τ and d t_d with the experimental ones.

Table 2. Comparison between calculated and measured time constant and delay time

| Parameter      | Measurements (s) | Meta-model (s) | Relative deviation (%) |
|----------------|------------------|----------------|------------------------|
| Time constant τ | 9353             | 9188           | 1.76 %                 |
| Delay time t_d | 503              | 527            | 4.77 %                 |

As observed in Figure 2 the semi-analytical model presents a good agreement with both the 2D FDM and the 3D FVM models and the experimental data with regard to the average surface temperature under transient conditions. This deviation of 4% (1.1 °c) is mainly related to the assumptions of the model, which consider a logarithmic profile for the surface temperature. However, given the simplicity of the model, which represents a significant advantage when looking for fast and reliable results, it can be considered as a useful tool for the estimation and analysis of the thermal behavior of a radiant slab.
4. Conclusion
In this study, a semi-analytical model based on a hybrid approach to simulate the thermal behavior of the radiant heating slab was developed. Measurements were carried out on a full-scale experimental heating slab to validate the developed model. The comparison between the simulation results and the experimental data has shown a good agreement. The semi-analytical that we present here is a promising tool that allows us to compute the optimal design parameters of the slab for each given material that can be used for the screed of the radiant slab.

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