Radiative corrections to the Higgs boson decay rate 
\( \Gamma(H \to ZZ) \) in the minimal supersymmetric model

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We consider radiative corrections to the decay rate \( \Gamma(H \to ZZ) \) of the heavy CP-even Higgs boson of the minimal supersymmetric model to two Z bosons. We perform a one loop Feynman diagram calculation in the on-mass-shell renormalization scheme, and include the third generation of quarks and squarks. The tree level rate is suppressed by a mixing angle factor and decreases as \( 1/M_H \) for large \( M_H \). The corrected rate overcomes this suppression and increases with \( M_H \) for \( M_H \gtrsim 500 \) GeV. The corrections can be very large and depend in detail on the top squark masses and \( A \)-term, as well as the supersymmetric Higgs mass parameter \( \mu \).

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I. INTRODUCTION

One of the least attractive features of the standard model (SM) is the existence of the naturalness problem. Roughly speaking this means that when one computes corrections to the Higgs boson mass one finds quadratically divergent contributions. This situation implies that input parameters must be extremely fine-tuned at high energies to yield the low energy physics that we observe, a situation that is unappealing especially in connection with GUTs.

One way to control the naturalness problem is to consider supersymmetric (SUSY) extensions of the standard model. Here the quadratic divergences are cancelled by loop diagrams involving the superpartners of the SM particles. We know that SUSY must be broken in the real world, and yet the scale of supersymmetry breaking must not be too large or the hierarchy problem will be reintroduced. Thus, although superparticles must be sufficiently heavy to have avoided detection at present colliders, they cannot be much heavier than a few TeV if we are to meet the naturalness criterion.

In this work we will be concerned with the simplest supersymmetric extension of the standard model (MSSM) [1,2]. In the MSSM we need two Higgs doublets $H_1$ and $H_2$ to give masses to up and down type fermions and to assure cancellation of anomalies. The neutral Higgs spectrum consists of two $CP$-even Higgs scalar particles $H$ and $h$ (where $M_H > M_h$), one $CP$-odd particle $A$, and a Goldstone boson $G$ which is “eaten by” and gives mass to the $Z$ boson. The Higgs sector of the MSSM is highly constrained. At tree level the Higgs boson masses and couplings are determined by two input parameters. We take these to be the mass of the $CP$-odd Higgs boson $M_A$ and an angle $\beta$ which at tree level is given by $\tan \beta = v_2/v_1$ where $v_2$ and $v_1$ are the vacuum expectation values of the two Higgs boson fields $H_2$ and $H_1$. The tree level masses of the $CP$-even Higgs bosons are then given by

$$M_{H, h}^2 = \frac{1}{2} \left( M_A^2 + M_Z^2 \pm \sqrt{(M_A^2 + M_Z^2)^2 - 4M_Z^2M_A^2 \cos^2(2\beta)} \right).$$

(1)

The above equation implies the inequalities $M_h < M_Z$, $M_H > M_Z$ and the sum rule $M_{H}^2 + M_{h}^2 = M_Z^2 + M_A^2$. 

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Recently it was shown that one loop corrections involving top-quark and squark loops can significantly modify the sum rule \([3]\) and also violate the bound \(M_h < M_Z\) \([4,5]\). For 1 TeV squark masses the correction to the light Higgs boson mass is of the order 20 (50) GeV for a top mass of 150 (200) GeV. Corrections to the neutral Higgs boson mass sum rule due to the gauge-Higgs and gaugino-higgsino sectors were considered earlier \([6]\) and were found to be generically small.

In this work we consider corrections to the decay rate \(\Gamma(H \rightarrow ZZ)\) which is relevant for the detection of the heavy Higgs boson at a proton supercollider such as the SSC via the “gold-plated” mode \(H \rightarrow ZZ \rightarrow \ell^+\ell^-\ell^+\ell^-\), where \(\ell\) is e or \(\mu\). We confine ourselves to corrections due to third family (top and bottom) quark and squark loops. Previous work on this subject has appeared in Ref.’s \([7]\) and \([8]\) where the effective potential and the renormalization group methods are used. We perform a Feynman diagram calculation utilizing the on-mass-shell renormalization scheme, and present explicit analytic results. The structure of the paper is as follows: in Section 2 we present our renormalization procedure, in Section 3 we discuss our results, Section 4 lists briefly our conclusions, and in the Appendix we present the necessary explicit formulas.

II. FORMALISM FOR RADIATIVE CORRECTIONS

Due to the presence of mixing in the \(CP\)-even and \(CP\)-odd sectors the renormalization of the Higgs sector of the MSSM presents a few complications when compared to the standard model. Therefore, in this section we present in detail our renormalization procedure. We follow the approach of Aoki et al. \([9]\) adapted to the MSSM.

The Higgs potential in the MSSM is

\[
V = \frac{g^2 + g'^2}{8} \left( H_1^* H_1^i - H_2^{*i} H_2^i \right)^2 + \frac{g'^2}{2} |H_1^{*i} H_2^i|^2 \\
+ (m_1^2 + \mu^2) H_1^* H_1^i + (m_2^2 + \mu^2) H_2^{*i} H_2^i - \left( m_3^2 \epsilon_{ij} H_1^i H_2^j + h.c. \right),
\]

where \(g(g')\) is the \(SU(2)_L(U(1)_Y)\) gauge coupling, the \(m_i\)'s, \((i = 1,2,3)\) are the soft supersymmetry breaking Higgs sector mass parameters, and \(\mu\) is the supersymmetric Higgs mass.
parameter. We can absorb $\mu^2$ in Eq.(1) by redefining $m_1^2 + \mu^2 \to m_1^2$ and similarly for $m_2^2$. $H_1$ and $H_2$ are given in terms of the shifted (but unrotated) fields by

$$ H_1 = \frac{1}{\sqrt{2}} \left( v_1 + S_1 - iP_1 \right), \quad H_2 = \frac{1}{\sqrt{2}} \left( \sqrt{2}H_+ v_2 + S_2 + iP_2 \right). $$

In order to discuss the tadpole and mixing structure of the theory we need the terms that are linear and quadratic in $S_1$, $S_2$ and quadratic in $P_1$, $P_2$. These are given by

$$ V_s = \left( \frac{g^2 + g'^2}{8} (v_1^2 - v_2^2) v_1 + m_1^2 v_1 - m_3^2 v_1 \right) S_1 + \left( \frac{g^2 + g'^2}{8} (v_2^2 - v_1^2) v_2 + m_2^2 v_2 - m_3^2 v_1 \right) S_2 
+ \left( \frac{g^2 + g'^2}{16} (3v_1^2 - v_2^2) + \frac{m_1^2}{2} \right) S_1 S_2 
- \left( \frac{g^2 + g'^2}{4} v_1 v_2 + m_3^2 \right) S_1 S_2 
+ \left( \frac{g^2 + g'^2}{8} (v_2^2 - v_1^2) + \frac{m_2^2}{2} \right) P_1^2 
+ \left( \frac{g^2 + g'^2}{16} (v_2^2 - v_1^2) + \frac{m_2^2}{2} \right) P_2^2 
-m_3^2 P_1 P_2. $$

(3a)

(3b)

We now define the coefficients of $S_1$ and $S_2$ in Eq.(3a) to be

$$ T_1 = \frac{\alpha^2 + \beta^2}{8} (v_1^2 - v_2^2) v_1 + m_1^2 v_1 - m_3^2 v_2, $$

(4a)

$$ T_2 = \frac{\alpha^2 + \beta^2}{8} (v_2^2 - v_1^2) v_2 + m_2^2 v_2 - m_3^2 v_1. $$

(4b)

Eliminating $m_1^2$, $m_2^2$ in favor of $T_1$, $T_2$ from Eqs.(4) and substituting back in Eqs.(3) we obtain, using a matrix notation

$$ V_s = (S_1 \ S_2) \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} + \frac{1}{2} (S_1 \ S_2) \begin{pmatrix} T_{\alpha} \\ v_1 \\ v_2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \end{pmatrix} 
+ \frac{1}{2} (S_1 \ S_2) \begin{pmatrix} \frac{\alpha^2 + \beta^2}{4} v_1^2 + m_3^2 v_1 \\ -\frac{\alpha^2 + \beta^2}{4} v_1 v_2 - m_3^2 v_1 \\ -\frac{\alpha^2 + \beta^2}{4} v_1 v_2 + m_3^2 v_2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \end{pmatrix}. $$

(5a)

$$ V_p = \frac{1}{2} (P_1 \ P_2) \begin{pmatrix} T_{\beta} \\ v_1 \\ v_2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} + \frac{1}{2} (P_1 \ P_2) \begin{pmatrix} \frac{m_3^2 v_1}{v_1} \\ -m_3^2 v_1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \end{pmatrix}. $$

(5b)

The next step is to introduce rotation matrices $O(\alpha)$ and $O(\beta)$ such that the part of the $CP$-even and $CP$-odd mass matrices that does not depend on $T_1$, $T_2$ is diagonalized. Specifically, by defining

$$ \begin{pmatrix} S_1 \\ S_2 \end{pmatrix} = O(\alpha) \begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix} $$

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we find that

\[
V_s = (H\ h) \left( \begin{array}{c} T_H \\ T_h \end{array} \right) + \frac{1}{2}(H\ h)O(-\alpha) \left( \begin{array}{cc} \frac{T_1}{v_1} & 0 \\ 0 & \frac{T_2}{v_2} \end{array} \right) O(\alpha) \left( \begin{array}{c} H \\ h \end{array} \right) + \frac{1}{2}(H\ h) \left( \begin{array}{cc} M_H^2 & 0 \\ 0 & M_h^2 \end{array} \right) \left( \begin{array}{c} H \\ h \end{array} \right)
\]

(6a)

\[
V_p = \frac{1}{2}(G\ A)O(-\beta) \left( \begin{array}{cc} \frac{T_1}{v_1} & 0 \\ 0 & \frac{T_2}{v_2} \end{array} \right) O(\beta) \left( \begin{array}{c} G \\ A \end{array} \right) + \frac{1}{2}(G\ A) \left( \begin{array}{cc} 0 & 0 \\ 0 & M_A^2 \end{array} \right) \left( \begin{array}{c} G \\ A \end{array} \right).
\]

(6b)

Here we have defined

\[
\left( \begin{array}{c} T_1 \\ T_2 \end{array} \right) = O(\alpha) \left( \begin{array}{c} T_H \\ T_h \end{array} \right).
\]

The parameters \(\beta, \alpha, M_H, M_h\) and \(M_A\) are related to the original fundamental parameters \(v_1, v_2\) and \(m_3^2\) by the following formulas

\[
\tan \beta = \frac{v_2}{v_1}, \quad M_A^2 = m_3^2 (\tan \beta + \cot \beta), \quad \tan 2\alpha = \frac{M_H^2 + M_Z^2}{M_H^2 - M_Z^2} \tan 2\beta,
\]

(7)

as well as Eq.(1). Here we used \(M_Z^2 = \frac{g^2 + g^2}{4}(v_1^2 + v_2^2)\). Carrying out the remaining matrix multiplications involving the tadpole contributions to the mass matrices we obtain the final result

\[
V_s = HT_H + hT_h + \frac{1}{2}(H\ h) \left( \begin{array}{cc} M_H^2 + b_{HH} & b_{HH} \\ b_{HH} & M_h^2 + b_{hh} \end{array} \right) \left( \begin{array}{c} H \\ h \end{array} \right)
\]

(8a)

\[
V_p = \frac{1}{2}(G\ A) \left( \begin{array}{cc} b_{GG} & b_{GA} \\ b_{GA} & M_A^2 + b_{AA} \end{array} \right) \left( \begin{array}{c} G \\ A \end{array} \right)
\]

(8b)

with

\[
b_{HH} = \frac{2}{v \sin 2\beta} \left( T_H (\cos^3 \alpha \sin \beta + \sin^3 \alpha \cos \beta) + T_h \sin \alpha \cos \alpha \sin(\alpha - \beta) \right)
\]

\[
b_{HH} = \frac{\sin 2\alpha}{v \sin 2\beta} \left( T_H \sin(\alpha - \beta) + T_h \cos(\alpha - \beta) \right)
\]

(8c)

\[
b_{hh} = \frac{2}{v \sin 2\beta} \left( T_H \cos \alpha \sin \alpha \cos(\alpha - \beta) + T_h (\cos^3 \alpha \cos \beta - \sin^3 \alpha \sin \beta) \right)
\]
and
\[ b_{GG} = \frac{1}{v} \left( T_H \cos(\alpha - \beta) - T_h \sin(\alpha - \beta) \right) \]
\[ b_{GA} = \frac{1}{v} \left( T_H \sin(\alpha - \beta) + T_h \cos(\alpha - \beta) \right) \]
\[ b_{AA} = \frac{2}{v \sin 2\beta} \left( T_H(\sin^3 \beta \cos \alpha + \cos^3 \beta \sin \alpha) + T_h(\cos^3 \beta \cos \alpha - \sin^3 \beta \sin \alpha) \right) \tag{8d} \]

The terms linear in \( H \) and \( h \) are to be thought of as counterterms for the tadpoles. To each order in the loop expansion we require that the total tadpole contribution vanishes. At tree level this implies \(-iT_H = 0 = -iT_h\). This then gives the conventional tree level masses. At one loop \(-iT_H (-iT_h)\) must cancel the one loop \( H \) (\( h \)) tadpole diagrams \( i\tau_H \) (\( i\tau_h \)) (Fig.1). These conditions determine \( T_H \) and \( T_h \) and Eqs.(8) determine their contribution to the one loop mass matrices.

Taking as renormalized inputs \( \tan \beta \) and \( M_A \) we calculate the physical masses \( M_H, M_h \) and the decay rate \( \Gamma(H \to ZZ) \) at one loop. It follows that the measurement of any two of the physical quantities \( M_A, M_H, M_h \) and \( \Gamma(H \to ZZ) \) will allow us to make a prediction for the other two. We stress that \( \beta \) is only to be viewed as a useful parametrization of physical observables. Since by itself \( \beta \) has no physical meaning we can renormalize it in any suitably convenient way. We explain our renormalization prescription for \( \beta \) below.

From this point on we adopt the following notation conventions: a quantity such as a field, coupling, or mass with a subscript 0 indicates a bare quantity, renormalized quantities have a subscript \( r \), and physical observables such as the pole of a propagator do not have subscripts. The bare tree Lagrangian contains
\[
\mathcal{L} \supset \frac{1}{2} \partial_\mu H_0 \partial^\mu H_0 + \frac{1}{2} \partial_\mu h_0 \partial^\mu h_0 - \frac{1}{2} (M^2_{H_0} + b_{HH}) H_0^2 - \frac{1}{2} (M^2_{h_0} + b_{hh}) h_0^2 - b_{Hh} H_0 h_0 \tag{9} \]

where \( M^2_{H_0} \) and \( M^2_{h_0} \) are taken to be functions of \( M_{A_0}, \beta_0 \) and \( M_{Z_0} \) as given by equation (1). We now write the bare parameters in terms of renormalized parameters and shifts
\[
\beta_0 = \beta_r + \delta \beta, \quad M^2_{A_0} = M^2_{A_r} + \delta M^2_A, \quad M^2_{Z_0} = M^2_{Z_r} + \delta M^2_Z \tag{10} \]
and also introduce wave function renormalization

\[ H_0 = Z_{HH}^\frac{1}{2} H_r + Z_{hh}^\frac{1}{2} h_r, \quad h_0 = Z_{hh}^\frac{1}{2} h_r + Z_{HH}^\frac{1}{2} H_r. \] (11)

Note that \( Z_{HH}^\frac{1}{2} = 1 + \mathcal{O}(\alpha) \), \( Z_{hh}^\frac{1}{2} = 1 + \mathcal{O}(\alpha) \) while \( Z_{HH}^\frac{1}{2}, Z_{hh}^\frac{1}{2}, b_{HH}, b_{hh}, \) and \( b_{hh} \) are all \( \mathcal{O}(\alpha) \). Substituting equations (10) and (11) into (9) we obtain the one loop renormalized two-point functions

\[ i\Gamma_{HH}(p^2) = (Z_{HH}^\frac{1}{2})^2(p^2 - M_{Hr}^2) - \frac{\partial M_{Hr}^2}{\partial x_{ir}}\delta x_i - b_{HH} + \Pi_{HH}(p^2) \]

\[ i\Gamma_{hh}(p^2) = (Z_{hh}^\frac{1}{2})^2(p^2 - M_{hr}^2) - \frac{\partial M_{hr}^2}{\partial x_{ir}}\delta x_i - b_{hh} + \Pi_{hh}(p^2) \] (12)

\[ i\Gamma_{Hh}(p^2) = Z_{Hh}^\frac{1}{2}(p^2 - M_{Hr}^2) + Z_{hH}^\frac{1}{2}(p^2 - M_{hr}^2) - b_{Hh} + \Pi_{Hh}(p^2), \]

where \( x_{ir} = M^2_{A_r}, M^2_{Z_r}, \beta_r \) and the \( \Pi \)'s are the scalar self-energies (Fig.2). The on-shell renormalization conditions are

\[ i\Gamma_{HH}(M_{Hr}^2) = i\Gamma_{Hh}(M_{hr}^2) = i\Gamma_{Hh}(M_{Hr}^2) = i\Gamma_{hh}(M_{hr}^2) = 0 \]

\[ i \left. \frac{\partial \Gamma_{HH}}{\partial p^2} \right|_{p^2 = M_{Hr}^2} = 1 = i \left. \frac{\partial \Gamma_{hh}}{\partial p^2} \right|_{p^2 = M_{hr}^2} \] (13)

Here \( M_H \) and \( M_h \) are the physical masses of \( H \) and \( h \). Making the definitions \( \delta M_{Hr}^2 = \Pi_{HH}(M_{Hr}^2) - b_{HH} \) and similarly for \( \delta M_{hr}^2 \), we obtain from Eqs.(12) and (13)

\[ M_{Hr}^2 = M_{Hr}^2 + \frac{\partial M_{Hr}^2}{\partial x_{ir}}\delta x_i - \delta M_{Hr}^2 \] (14a)

\[ M_{hr}^2 = M_{hr}^2 + \frac{\partial M_{hr}^2}{\partial x_{ir}}\delta x_i - \delta M_{hr}^2 \] (14b)

\[ Z_{HH}^\frac{1}{2} = 1 - \frac{1}{2} \Pi_{HH}'(M_{Hr}^2) \] (14c)

\[ Z_{hh}^\frac{1}{2} = 1 - \frac{1}{2} \Pi_{hh}'(M_{hr}^2) \] (14d)

\[ Z_{hH}^\frac{1}{2} = \frac{1}{M_{Hr}^2 - M_{hr}^2} \left( -\Pi_{Hh}(M_{Hr}^2) + b_{Hh} \right) \] (14e)
\[
Z_{Hh}^{\frac{1}{2}} = \frac{1}{M_{Hr}^2 - M_{hr}^2} \left( -\Pi_{Hh}(M_{hr}^2) + b_{Hh} \right),
\]
(14f)

where the prime in Eqs.(14c,d) indicates differentiation with respect to \( p^2 \). Note that \( M_{Hr}^2 \) and \( M_{hr}^2 \) have the same functional form as in Eq.(1) except that they are functions of renormalized quantities, i.e.

\[
M_{Hr,hr}^2 = \frac{1}{2} \left( M_{A_r}^2 + M_{Z_r}^2 \pm \sqrt{(M_{A_r}^2 + M_{Z_r}^2)^2 - 4M_{A_r}^2 M_{Z_r}^2 \cos^2(2\beta_r)} \right).
\]
(15)

We now drop the subscript \( r \) on \( M_{Z_r}, M_{A_r} \) and \( \beta_r \). Eqs.(14a,b) determine the physical \( CP \)-even Higgs boson masses in terms of self energies, tadpole contributions, and shifts of the inputs parameters \( \delta x_i \). We now determine the shifts. The shift \( \delta M_A^2 \) is defined so that \( M_A \) is equal to the physical \( A \) mass. An analysis similar to that of the \( CP \)-even sector yields

\[
\delta M_A^2 = \Pi_{AA}(M_A^2) - b_{AA}.
\]
(16)

Additionally, we find for the shift in the \( Z \)-boson mass

\[
\delta M_Z^2 = \Pi_{ZZ}^T(M_Z^2)
\]
(17)

where \( \Pi_{ZZ}^T \) is the transverse part of the \( Z \) boson self energy, \( \Pi_{ZZ}^{\mu\nu} = g^{\mu\nu} \Pi_{ZZ}^T + \frac{g^{\mu\nu}}{p^2} \Pi_{ZZ}^L \). At this point it is worth noting that if we are only interested in the sum \( M_{H_r}^2 + M_{h_r}^2 \) we do not need a specification for \( \delta \beta \). When Eqs.(14a) and (14b) are added the terms proportional to \( \delta \beta \) cancel leaving

\[
M_{H_r}^2 + M_{h_r}^2 = M_{A_r}^2 + M_{Z_r}^2 - \Pi_{HH}(M_{Hr}^2) - \Pi_{hh}(M_{h_r}^2) + \Pi_{AA}(M_A^2) + \Pi_{ZZ}^T(M_Z^2)
\]
\[+ b_{HH} + b_{ hh} - b_{AA}
\]
(18)

This is just the renormalization of the neutral Higgs boson mass sum rule and the divergences in Eq.(18) implicit in the \( \Pi \)'s and \( b \)'s cancel leaving behind a finite correction. Since we demand that \( M_H \) and \( M_h \) are physical masses they must be individually finite. Equivalently, since \( M_{H_r}^2 + M_{h_r}^2 \) is finite we must have that \( M_{H_r}^2 - M_{h_r}^2 \) is also free of divergences. This latter requirement gives
\[
\frac{\partial \Delta}{\partial \beta} \delta \beta + \frac{\partial \Delta}{\partial M_Z^2} \delta M_Z^2 + \frac{\partial \Delta}{\partial M_A^2} \delta M_A^2 - \delta M_H^2 + \delta M_h^2 = \text{finite} \tag{19}
\]

where \( \Delta = \sqrt{(M_A^2 + M_Z^2)^2 - 4M_A^2M_Z^2 \cos^2(2\beta)} \). The above equation clearly determines only the “infinite” part of \( \delta \beta \). By “infinite” we mean the part that is proportional to \( C_{UV} = \frac{1}{\epsilon} - \gamma \log 4\pi \) in dimensional regularization. To fully specify \( \delta \beta \) we take a \( \overline{MS} \)-type approach and define \( \delta \beta \) to be purely “infinite” so that Eq.(19) becomes

\[
\frac{\partial \Delta}{\partial \beta} \delta \beta = - \left( \frac{\partial \Delta}{\partial M_Z^2} \delta M_Z^2 + \frac{\partial \Delta}{\partial M_A^2} \delta M_A^2 - \delta M_H^2 + \delta M_h^2 \right) \bigg|_{\infty} \tag{20}
\]

where the subscript \( \infty \) on a quantity indicates the “infinite” part of that quantity. Eq.(20) implies

\[
\delta \beta = \frac{1}{2M_A^2 M_Z^2 \sin(4\beta)} \times \left( (M_Z^2 \delta M_A^2 + M_A^2 \delta M_Z^2) \cos^2(2\beta) - M_H^2 \delta M_h^2 - M_h^2 \delta M_H^2 \right) \bigg|_{\infty}. \tag{21}
\]

This definition of \( \beta \) at one loop gives renormalized \( CP \)-even Higgs boson masses in close agreement with those obtained using the effective potential \([\text{I}]\). This shift in \( \delta \beta \) induces a shift in \( \alpha \) through equation (7)

\[
\delta \alpha = \sin(4\alpha) \left( \frac{\delta \beta}{\sin(4\beta)} \left( \frac{M_A^2 \delta M_Z^2 - M_Z^2 \delta M_A^2}{2(M_A^4 - M_Z^4)} \right) \right). \tag{22}
\]

We now come to the renormalization of the \( HZZ \) coupling. The bare \( HZZ \) and \( hZZ \) couplings are given by

\[
\lambda_0^{HZZ} = \frac{e_0 M_Z^3}{M_W (M_Z^2 - M_W^2)^{\frac{3}{2}}} \cos(\beta_0 - \alpha_0), \quad \lambda_0^{hZZ} = \lambda_0^{HZZ} \tan(\beta_0 - \alpha_0).
\]

Defining

\[
e_0 = e_r + \delta e, \quad Z_H^{\frac{1}{2}} = 1 + \delta Z_H^{\frac{1}{2}}, \quad (Z_Z^2)^{\frac{1}{2}} = 1 + \delta Z_{ZZ} \tag{23}
\]

(here \( Z_{\mu_0} = Z_Z^2 Z_{\mu_r} + Z_A^2 A_{\mu_r} \) where \( Z_{\mu_0}(Z_{\mu_r}) \) is the bare (renormalized) \( Z \) boson field and \( A_{\mu_r} \) is the renormalized photon field) we obtain for the renormalized one loop 3-point function

\[
\Gamma^{HZZ}_{\mu\nu} = (\lambda_r^{HZZ} + \lambda_{CT}^{HZZ}) g_{\mu\nu} + \Delta \Gamma^{HZZ}_{\mu\nu} \tag{23}
\]
where $\lambda_r^{HZZ} = \frac{e_r M_Z^2 \cos(\beta_r - \alpha_r)}{M_W r (M_Z^2 - M_W^2)^2}$ and

$$\lambda_{CT}^{HZZ} = \lambda_r^{HZZ} \left( \frac{\delta e}{e} + \frac{3 \delta M_Z^2}{2 M_Z^2} - 2 \frac{\delta M_Z^2 - \delta M_W^2}{M_Z^2 - M_W^2} - \frac{1}{2} \frac{\delta M_W^2}{M_W^2} \right) - \tan(\beta_r - \alpha_r) (\delta \beta - \delta \alpha) + \delta Z_{HH}^7 + \delta Z_{ZZ}^7 + \frac{1}{2} \tan(\beta_r - \alpha_r)$$

and $\Delta\Gamma_{\mu\nu}^{HZZ}$ is the explicit one loop Feynman diagram contribution (Fig.3). The angle $\alpha_r$ is defined as in Eq.(7), but with the right hand side written in terms of renormalized quantities. The expressions for $\delta M_Z^2$, $\delta \beta$, $\delta \alpha$, $Z_{HH}^7$ and $Z_{ZZ}^7$ in terms of self energies and tadpole contributions are given in Eqs.(17), (21), (22) and (14c,e). We simply state the results for the remaining shifts $\delta e$, $\delta M_W^2$ and $\delta Z_{ZZ}^7$. We have

$$\frac{\delta e}{e} = \frac{1}{2} \Pi_{\gamma\gamma}^T(0) + \left( \frac{4 c_w^2}{4 s_w c_w} \right) \frac{\Pi_{Z\gamma}^T(0)}{M_Z^2},$$

$$\delta M_W^2 = \Pi_{WW}^T(M_W^2), \quad \delta Z_{ZZ} = -\Pi_{ZZ}^T(M_Z^2)$$

where $c_w = M_W/M_Z$ and $s_w = \sqrt{1 - c_w^2}$. We note that $\Pi_{Z\gamma}^T(0)$ vanishes in our case. The $H-h$ mixing gives a contribution to $\Gamma_{HZZ}^{\mu\nu}$ through the term proportional to $Z_{hH}^7$. The quantity on the R.H.S. of Eq.(23) is given as a sum of terms which are individually divergent. In the full sum the divergences must of course cancel. We checked both analytically and numerically that this is indeed the case. The renormalizability of the theory requires that the definition of $\delta \beta$ which renders the $CP$-even Higgs boson masses finite also gives finite couplings.

The explicit one loop Feynman diagrams shown in Fig.3 give a contribution to the three-point function which can be expanded in terms of form factors as

$$\Delta\Gamma_{HZZ}^{\mu\nu} = D_0 g^{\mu\nu} + D_1 p_1^\mu p_1^\nu + D_2 p_2^\mu p_2^\nu + D_3 p_1^\mu p_2^\nu + D_4 p_2^\mu p_1^\nu$$

(a form factor proportional to $\epsilon^{\mu\nu\alpha\beta} p_1 \alpha p_2 \beta$ vanishes by $CP$ invariance). The formula for the decay rate at one loop is

$$\Gamma = \frac{\sqrt{1 - 4r}}{128 \pi r^2 M_H} \left\{ (1 - 4r + 12 r^2) \left( (\lambda_r^{HZZ})^2 + 2 \lambda_r^{HZZ} \text{Re}(\lambda_{CT}^{HZZ} + D_0) + |\lambda_{CT}^{HZZ} + D_0|^2 \right) \right. \right.$$  

$$\left. + M_H^2(1 - 2r)(1 - 4r) \left( \lambda_r^{HZZ} \text{Re}(D_4) + \text{Re}[(\lambda_{CT}^{HZZ} + D_0)D_4^\ast] \right) \right. \right.$$  

$$\left. + M_H^4 \left( \frac{1}{2} - 2r \right)^2 |D_4|^2 \right\}$$

(27)
where \( r = M_Z^2 / M_H^2 \) and we list \( D_0 \) and \( D_4 \) in the Appendix.

We note that the terms in the above expression which do not involve \( \lambda_r^{HZZ} \) are formally of \( O(g^6) \). Nevertheless we find that for large Higgs boson mass \((M_H \gg M_Z)\) they are numerically important. This is because \( \lambda_r^{HZZ} \) is proportional to \( \cos(\alpha - \beta) \) which is proportional to \( 1/M_H^2 \) for large \( M_H \) and hence small. Keeping these \( O(g^6) \) terms is consistent: the terms in the amplitude that are of \( O(g^5) \) which arise at two loop level also give a contribution of \( O(g^6) \) in the decay rate, but these two loop \( O(g^6) \) terms are proportional to \( \cos(\alpha - \beta) \) and are thus suppressed when \( M_H \gg M_Z \), in precisely the region where the \( O(g^6) \) terms in our one loop expression become large.

### III. RESULTS

In the MSSM at tree level the decay rate \( \Gamma(H \rightarrow ZZ) \) is suppressed relative to the same decay rate in the standard model by the factor \( \cos^2(\alpha - \beta) \). The “gold-plated” decay mode \( H \rightarrow ZZ \rightarrow 4\ell \) has great discovery potential for a standard model Higgs boson at a proton super collider such as the SSC for Higgs boson masses \( 130 \text{ GeV} \lesssim M_\phi \lesssim 800 \text{ GeV} \) [10]. The discovery potential for the heavy Higgs boson of the MSSM in this mode is not as promising due to the above mentioned suppression factor. However, the “gold-plated” mode may be the only discovery mode for the heavy Higgs boson at a hadron collider [11]. The discovery potential is improved when radiative corrections are taken into account.

We discuss our numerical results below. We have checked our numerics in a number of ways. First, we checked the cancellation of divergences as mentioned in the last section. Second, we found our result for the correction to the neutral Higgs boson mass sum rule agreed very closely with that of Ref. [3]. Lastly, we checked that our calculation, when modified to give the correction to the standard model Higgs boson decay rate to two \( Z \)'s due to an extra heavy fermion doublet, agrees with the results of Ref. [14].

In Fig.4a we show the tree level and radiatively corrected decay rate versus the heavy Higgs mass for \( \tan \beta=5 \) and a top quark mass of 160 GeV. In this figure we have not included
mixing effects, \textit{i.e.} $A_t = A_b = \mu = 0$ and the squark masses are all equal. We show the corrected rate for the two squark mass choices $M_{sq} = 300$ GeV and $M_{sq} = 1000$ GeV. We see in Fig.4a the importance of keeping corrections which are of $\mathcal{O}(g^6)$ in the rate. The one loop corrections which contribute $\mathcal{O}(g^4)$ to the rate fall with $M_H$ (as they multiply the tree level coupling). However, the one loop corrections which contribute $\mathcal{O}(g^6)$ to the rate increase as $M_H$ increases. Hence, these terms eventually dominate the rate as $M_H$ becomes large.

In Fig.4a the corrected rate is dominated by the $\mathcal{O}(g^4)$ terms for small $M_H$, and hence it initially falls as $M_H$ increases beyond the kinematic suppression. Eventually, however, the terms of order $\mathcal{O}(g^6)$ become larger than the $\mathcal{O}(g^4)$ terms and the rate then rises with $M_H$. This begins to occur for values of $M_H$ of about 500 GeV.

In Fig.4b we show the rate versus $\tan \beta$ for a Higgs boson mass of 300 GeV, a top quark mass of 160 GeV, a squark mass of 1 TeV, and again for no mixing. We see that the corrected rate is approximately twice as large as the tree level value, almost independent of $\tan \beta$. As we will discuss below, the rate depends dramatically on $\tan \beta$ once mixing is included.

In Fig.5 the ratio of the radiatively corrected rate to the tree level rate is shown versus the top quark mass, for the same set of parameters as Fig.4b, and $\tan \beta=5$. Fig.5 illustrates that the corrected rate depends strongly on two parameters in the case of no mixing. Clearly the rate depends on the value of the top quark mass. But note for $M_H=1$ TeV that even for a top quark mass as small as 100 GeV the corrected rate is still over a factor of two larger than at tree level. Thus the relative size of the correction depends greatly on the value of $M_H$ as well. Note, however, that when the top quark mass is less than around 120 GeV we expect that the corrections from other sectors will be of the same order of magnitude as the correction due to the quark/squark sector included here.

When mixing is included the parameter space increases. We will choose a point in mixing space and examine the effect of mixing in deviations from that point. We choose $A-$terms $A_t = A_b = 600$ GeV and squark masses $\tilde{m}_{t_1} = \tilde{m}_{b_1}=600$ GeV, and $\tilde{m}_{t_2} = \tilde{m}_{b_2}=300$ GeV. Additionally, we will consider the two cases $\mu = \pm 400$ GeV. In all three of the figures 6, 7 and 8 the heavy Higgs boson mass is set to 300 GeV and the top quark mass is 160

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GeV. In order to isolate the effect of mixing we will plot the ratio of the corrected rate including mixing to the corrected rate with no mixing (where the common squark mass is set to 600 GeV). In Figs.6 we plot this ratio vs. the squark mass $\tilde{m}_{t_1}$. We find that the effect due to mixing is strongly dependent on $\tan \beta$ and $\mu$. For large values of $\tan \beta$ the effects of mixing are greatly enhanced. As shown in Figs.6, the inclusion of mixing can change the rate by a factor 1.3 for $\tan \beta=2$ and for $\tan \beta=20$ by a factor 2.7 or 0.3, for $\mu=-400$ GeV or $\mu=+400$ GeV, respectively.

Similar ratios are seen in Figs.7, where the ratio of the corrected rate including mixing to the corrected rate with no mixing is shown vs. $A_t$, the top squark mixing parameter. As in Figs.6 the two curves for $\mu = \pm 400$ GeV are similar when $\tan \beta=2$; the rate can be increased by 50% or decreased by 25%. If $\tan \beta=20$ the effects of mixing are more pronounced and the ratio varies between roughly 1/3 and 3. The $\mu=400$ GeV curve in Fig.7a (and the $\tan \beta=2$ curve in Fig.8) does not span the entire ordinate axis shown because an unphysical region of the squark mixing parameter space is encountered. In Fig.8 we plot the (mixing) to (no mixing) ratio vs. the supersymmetric Higgs mass parameter $\mu$. We see there is little dependence on $\mu$ for small $\tan \beta$, while for larger values of $\tan \beta$ the dependence is quite significant. If $\tan \beta=20$ the ratio varies between 4 and 1/36 as $\mu$ varies from -750 to 750 GeV. Finally, we note that there is very little dependence on the bottom squark masses and $A-$term $A_b$ for the mixing configurations considered.

IV. CONCLUSIONS

To summarize, we have computed the one loop corrections to the decay rate $\Gamma(H \rightarrow ZZ)$ in the MSSM including third family quark and squark loops. We perform a Feynman diagram calculation in the on-mass-shell renormalization scheme. As the tree level rate falls like $1/M_H$ for large $M_H$ and we find corrections that grow with $M_H$, the corrected rate may be many times the tree level rate. For example, at $M_H = 1$ TeV the corrected rate may be 13 times the uncorrected rate for $m_t=200$ GeV (with no squark mixing). The corrected rate depends
very strongly on the squark mixing parameters. For example, for the mixing configuration considered here, the rate varies by two orders of magnitude as the Higgs mass parameter $\mu$ varies between $\pm 750$ GeV. Indeed, the squark mixing parameters $\mu$, $A_t$, and the top squark masses, in addition to the top quark mass, must be measured in order to test the Higgs sector of the MSSM.

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In this Appendix we give explicit analytic expressions for the self energies, tadpoles, and form factors introduced in the text. Our expressions are given in terms of the standard $A, B, C$ functions introduced by Passarino and Veltman [12] which appear in one loop calculations. We adopt the metric (1,-1,-1,-1), which is different than that of Ref. [12]. Explicit analytic formulas for these functions appear in Ref. [13].

To make the equations more concise we adopt the following conventions. $N_c$ denotes the number of quark colors. The index $\alpha$ runs over the top and bottom sectors while the indices $i$, $j$, and $k$ run over squark mass eigenstates. Thus, $m_\alpha$ denotes a quark mass while $\tilde{m}_{\alpha i}$ denotes a squark mass. For the $A$ and $B$ functions we define $A_\alpha = A(m_\alpha^2)$, $\tilde{A}_{\alpha i} = A(\tilde{m}_{\alpha i}^2)$, $B_{0\alpha} = B_0(p^2, m_\alpha^2, m_\alpha^2)$, $\tilde{B}_{0\alpha ij} = B_0(p^2, \tilde{m}_{\alpha i}^2, \tilde{m}_{\alpha j}^2)$ and similarly for the rest of the $B$’s. A $C$ function has six arguments: three external squared momenta and the three squared masses of the particles which appear in loop of the 3-point diagram. We thus define $\tilde{C}_{0\alpha ij k} = C_0(M_Z^2, M_Z^2, M_H^2, \tilde{m}_{\alpha i}^2, \tilde{m}_{\alpha j}^2, \tilde{m}_{\alpha k}^2)$ and $C_{0\alpha} = C_0(M_Z^2, M_Z^2, M_H^2, m_\alpha^2, m_\alpha^2, m_\alpha^2)$ with analogous definitions for the rest of the $C$’s.

First we give expressions for the Higgs boson self energies.
\[ \Pi_{HH}(p^2) = N_c \sum_{\alpha ij} (V_{\alpha ij}^H)^2 \tilde{B}_{0aij} - N_c \sum_{\alpha} \tilde{U}_{aai}^{HH} \tilde{A}_{ai} \]  

\[ -12N_c \sum_{\alpha} (V_{\alpha}^H)^2 \left( m_{a}^2 B_{0a} + p^2 (B_{21a} - B_{1a}) + \frac{1}{48\pi^2} (m_{a}^2 - \frac{p^2}{6}) \right). \]  

The various \( V \) and \( U \) vertex factors are shown in Fig.9 and explicit expressions appear in Refs. [11][14]. However, the \( H - h - \tilde{q}_{kL} - \tilde{q}_{kL} \) and \( H - h - \tilde{q}_{kR} - \tilde{q}_{kR} \) vertices given in Ref. [10] are incorrect. In the notation of Ref. [10] the above couplings are respectively \( \left( D_{up} = 1/\sin^2 \beta, D_{down} = -1/\cos^2 \beta \right) \).

\( \Pi_{hh} \) is given as \( \Pi_{HH} \) with \( \bar{V}_{\alpha ij}^H \to \tilde{V}_{\alpha ij}^h, \bar{U}_{aai}^{HH} \to \bar{\tilde{U}}_{aaii}^{hh}, \) and \( V_{\alpha}^H \to V_{\alpha}^h. \) \( \Pi_{hh} \) is given as \( \Pi_{HH} \) with \( (\bar{V}_{\alpha ij}^H)^2 \to \tilde{V}_{\alpha ij}^h \tilde{V}_{\alpha ij}^h, \bar{U}_{aai}^{HH} \to \tilde{U}_{aaii}^{hh}, \) and \( (V_{\alpha}^H)^2 \to V_{\alpha}^h V_{\alpha}^h. \) \( \Pi_{AA} \) is given as \( \Pi_{HH} \) with \( \tilde{V}_{\alpha ij}^h \to \tilde{V}_{\alpha ij}^A, \tilde{U}_{aaii}^{HH} \to \tilde{U}_{aaii}^{AA}, V_{\alpha}^H \to V_{\alpha}^A, \) and \( B_{0a} \to \frac{1}{3} B_{0a}. \) Next we list the transverse part of the gauge boson self energies.

\[ \Pi_{ZZ}^T(p^2) = N_c \sum_{ai} \tilde{U}_{aaii}^{ZZ} \tilde{A}_{ai} \]  

\[ -2N_c \sum_{aij} (\bar{V}_{\alpha ij}^Z)^2 \left( \tilde{m}_{ai}^2 \tilde{B}_{0aij} - (\tilde{m}_{ai}^2 - \tilde{m}_{aj}^2 + p^2) \tilde{B}_{1aij} + p^2 \tilde{B}_{2aij} \right) + \frac{1}{16\pi^2} \left( \frac{\tilde{m}_{ai}^2 + \tilde{m}_{aj}^2}{2} - \frac{p^2}{6} \right) \]  

\[ -8N_c \sum_{\alpha} \left( (V_{5a}^Z)^2 m_{a}^2 B_{0a} + ((V_{\alpha}^Z)^2 + (V_{5a}^Z)^2) p^2 (B_{21a} - B_{1a}) \right) \]  

\[ \Pi_{WW}^T(p^2) = N_c \sum_{ai} \tilde{U}_{aaii}^{WW} \tilde{A}_{ai} \]  

\[ -2N_c \sum_{aij} (\bar{V}_{\alpha ij}^W)^2 \left( \tilde{m}_{ii}^2 \tilde{B}_{0} - (\tilde{m}_{ii}^2 - \tilde{m}_{bi}^2 + p^2) \tilde{B}_{1} + p^2 \tilde{B}_{2} \right) + \frac{1}{16\pi^2} \left( \frac{\tilde{m}_{ii}^2 + \tilde{m}_{bi}^2}{2} - \frac{p^2}{6} \right) \]  

\[ -8N_c (V_{\alpha}^W)^2 \left( m_{i}^2 B_{0}^W - (m_{i}^2 - m_{b}^2 + 2p^2) B_{1}^W + 2p^2 B_{2}^W \right) \]  

where \( \tilde{B}_{0} = B(p^2, \tilde{m}_{ii}^2, \tilde{m}_{bi}^2) \) and \( B_{0}^W = B(p^2, m_{i}^2, m_{b}^2) \). \( \Pi_{\gamma\gamma}^T \) is given as \( \Pi_{ZZ}^T \) with \( \tilde{V}_{\alpha ij}^Z \to \tilde{V}_{\alpha ij}^\gamma, \tilde{U}_{aaii}^{ZZ} \to \tilde{U}_{aaii}^{\gamma\gamma}, V_{\alpha}^Z \to V_{\alpha}^\gamma, V_{5a}^Z \to 0. \) The heavy Higgs boson tadpole contribution is given by

\[ T_H = N_c \left( 4 \sum_{\alpha} V_{\alpha}^H m_{a} A_{a} - \sum_{ai} \tilde{V}_{aaii}^{H} \tilde{A}_{ai} \right). \]  

\( T_h \) is given as \( T_H \) with \( \tilde{V}_{aaii}^{H} \to \tilde{V}_{aaii}^{h} \) and \( V_{\alpha}^H \to V_{\alpha}^h. \) Lastly, the two three-point Feynman diagram form factors which are relevant for calculating the Higgs boson decay rate are
\[ D_0 = 8N_c \sum_{\alpha} m_\alpha V^H_\alpha ((V^{Z}_\alpha)^2 + (V^{Z}_{5\alpha})^2) \]
\[ \times \left( 4C_{24\alpha} + (M_H^2 - 2M_Z^2)C_{12\alpha} + 2M_Z^2C_{11\alpha} - \frac{M_Z^2}{2}C_{0\alpha} - B_0(M_Z^2, m^2_\alpha, m^2_{5\alpha}) \right) \]
\[ + 8N_c \sum_{\alpha} m_\alpha V^H_\alpha (V^{Z}_{5\alpha})^2 \left( (M_H^2 - 4m^2_\alpha)C_{0\alpha} - 2B_0(M_Z^2, m^2_\alpha, m^2_{5\alpha}) \right) \]
\[ - 8N_c \sum_{\alpha ijk} \tilde{V}^H_{\alpha ki} \tilde{V}^Z_{\alpha ij} \tilde{V}^Z_{\alpha jk} \tilde{C}_{24\alpha ijk} + N_c \sum_{\alpha ij} \tilde{V}^H_{\alpha i j} \tilde{U}^{ZZ}_{\alpha i j} B_0(M_H^2, \tilde{m}^2_{\alpha i}, \tilde{m}^2_{\alpha j}) \]  
\[ \text{(A5)} \]

and

\[ D_4 = 8N_c \sum_{\alpha} m_\alpha V^H_\alpha ((V^{Z}_\alpha)^2 + (V^{Z}_{5\alpha})^2) \left( 4C_{23\alpha} + C_{0\alpha} - 4C_{12\alpha} \right) - 2(V^{Z}_{5\alpha})^2 (C_{11\alpha} - C_{12\alpha}) \]
\[ - 8N_c \sum_{\alpha ijk} \tilde{V}^H_{\alpha ki} \tilde{V}^Z_{\alpha ij} \tilde{V}^Z_{\alpha jk} \left( \tilde{C}_{23\alpha ijk} - \tilde{C}_{12\alpha ijk} \right). \]
\[ \text{(A6)} \]
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FIGURES

FIG. 1. The Higgs boson tadpole diagram.

FIG. 2. The Higgs boson and gauge boson self energy diagrams.

FIG. 3. The one loop $H \rightarrow ZZ$ Feynman diagrams included in our calculation. The dashed loops represent squarks, the solid loop represents quarks.

FIG. 4. The tree level and one loop corrected decay rate $\Gamma (H \rightarrow ZZ)$. Fig. 4a shows the rates vs. the heavy Higgs boson mass for $\tan \beta = 5$, $m_t=160$ GeV, and no mixing. Fig. 4b shows the decay rates vs. $\tan \beta$ for the same parameters as in Fig. 4a, with $M_H=300$ GeV.

FIG. 5. The ratio of the corrected decay rate to the tree level rate vs. the top quark mass. The parameters are the same as in Fig. 4a, with the Higgs boson mass set to 300 GeV and 1 TeV.

FIG. 6. The ratio of the corrected rate including mixing to the corrected rate without mixing vs. the top squark mass $\tilde{m}_t_1$ as explained in the text. The top quark mass is 160 GeV.

FIG. 7. The ratio of the corrected rate including mixing to the corrected rate without mixing vs. the top mixing parameter $A_t$, as explained in the text. The top quark mass is 160 GeV. In Fig. 7a the point where the $\mu=400$ GeV curve stops corresponds to an unphysical point in squark mixing parameter space.

FIG. 8. The ratio of the corrected rate including mixing to the corrected rate without mixing, as explained in the text, vs. $\mu$. The heavy Higgs boson mass is 300 GeV, and the top quark mass is 160 GeV. The curve for $\tan \beta = 2$ stops at an unphysical point in the squark mixing parameter space.

FIG. 9. The vertices used in the Appendix are displayed. The values of the vertex factors may be found in Refs. [1,10]. Note that the value of $U^{Hh}_{\alpha ij}$ listed in Ref. [10] is incorrect (see the Appendix for the correct values of these vertices).