Non-orientable genus of a knot in punctured Spin 4-manifolds

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Abstract. For a closed 4-manifold $M$ and a knot $K$ in the boundary of punctured $M$, we define $\gamma_M^0(K)$ to be the smallest first Betti number of non-orientable and null-homologous surfaces in punctured $M$ with boundary $K$. Note that $\gamma_{S^4}^0$ is equal to the non-orientable 4-ball genus and hence $\gamma_M^0$ is generalization of the non-orientable 4-ball genus.

While it is very likely that for given $M$, $\gamma_M^0$ has no upper bound, it is difficult to show it. In fact, even in the case of $\gamma_{S^4}^0$, its non-boundedness was shown for the first time by Batson in 2012.

In this talk, we show that for any Spin 4-manifold $M$, $\gamma_M^0$ has no upper bound.