Pressure characteristic analysis and estimation of effective length of a hybrid diffuser and modified hybrid diffusers

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Abstract. In this paper, performance simulation of a hybrid diffuser, hybrid diffuser with suction and hybrid diffuser with fence have been carried out. The two-dimensional steady differential equations for conservation of mass and momentum have been solved for the Reynolds number ranging from 100 to 600. During the simulation, a fixed aspect ratio of 2 and a fixed value of half divergence angle of 3.5\textdegree have been considered for all the diffuser geometries. At the inlet, fully developed velocity profile has been used. The effect of Reynolds number on average static pressure have been studied in detail, and compared between three diffuser geometries. The effective diffuser lengths for achieving maximum average static pressure at the diffuser exit have been computed for the considered Reynolds numbers.

1. Introduction
Diffuser is used in many industrial applications to convert the kinetic energy of flow into potential energy exhibited in pressure rise. Diffuser is considered to be an important component in many flow devices, such as, wind tunnels, heat exchangers, turbo machines, gas turbine stationary power plant, fluidic actuators etc. One very important application of a diffuser is to use it in between the compressor and the combustor of an aircraft gas turbine plant. The primary function of a combustor diffuser is to enhance the static pressure so as to ensure deep penetration of the air jet into the flame tube, and side by side to reduce the velocity of air at the exit from the diffuser, to such a value that helps to sustain a stationary flame in the combustor. Due to space limitations and the high power demands, the diffusion process needs not only to be highly effective but the process needs to be executed in a very short space. Since space is an important criterion for an aircraft engine, it, therefore, has led researchers to carry out an extensive research on the performance of the short length, highly efficient diffusers.

From the review of literatures, it has been observed that, flow analysis of a fluid passing through the configuration of a sudden expansion has been carried out experimentally [1] and numerically [2] by some researchers. Some modifications of sudden expansion configuration have been found in literatures. Among these, Chakrabarti et al. [3] have presented the results of their numerical simulation on the performance of a sudden expansion with suction viewed as a diffuser.
2. Mathematical formulation

2.1 Governing equations

The schematic diagrams of the computational domains of the hybrid diffuser, hybrid diffuser with suction and hybrid diffuser with fence have been illustrated in Figs 1, 2 and 3 respectively. In all the considered cases, the same lengths of the inlet section (L_1), sudden expansion section (L_2) and the diverging section (L_3) has been assigned.

The flow under consideration has been assumed to be steady, two-dimensional, laminar and axisymmetric. The fluid has been considered to be incompressible and Newtonian.

The following dimensionless variables are defined to obtain the governing conservation equations in the non-dimensional form.

Lengths: \( r^* = r / D_1 \), \( z^* = z / D_1 \), \( L_1^* = L_1 / D_1 \), \( L_2^* = L_2 / D_1 \), \( L_3^* = L_3 / D_1 \), \( L_4^* = L_4 / D_1 \),

\[
D_1^* = D_2^* / D_1 , \quad D_3^* = D_3 / D_1 .
\]

Velocities: \( u_r^* = u_r / U \), \( u_z^* = u_z / U \).

Pressure: \( p^* = p / \rho U^2 \).

With the help of these variables, the continuity and Navier-Stokes equations in two-dimensional cylindrical coordinates \((r,z)\) can be written in the differential form as follows:

Continuity equation:

\[
\frac{\partial}{\partial r^*} (r^* u_r^*) + \frac{\partial}{\partial z^*} (u_z^*) = 0
\]

(1)

r-direction momentum equation:

\[
\frac{\partial (r^* u_r^*)}{\partial r^*} + \frac{\partial (u_z^*)}{\partial z^*} = -\frac{\partial p^*}{\partial r^*} + \frac{1}{\text{Re}} \left( \frac{\partial}{\partial r^*} \left( r^* \frac{\partial (r^* u_r^*)}{\partial r^*} \right) + \frac{\partial^2 u_r^*}{\partial z^*^2} \right)
\]

(2)

z-direction momentum equation:

\[
\frac{\partial (r^* u_r^*)}{\partial r^*} + \frac{\partial (u_z^*)}{\partial z^*} = -\frac{\partial p^*}{\partial z^*} + \frac{1}{\text{Re}} \left( \frac{\partial}{\partial r^*} \left( r^* \frac{\partial (u_z^*)}{\partial r^*} \right) + \frac{\partial^2 u_z^*}{\partial z^*^2} \right)
\]

(3)
Where, the flow Reynolds number, \( \text{Re} = \frac{\rho U D_1}{\mu} \).

2.2 Boundary conditions
In the present analysis, five different types of boundary conditions have been applied. They are as follows,

(i) At the walls: No slip and no penetration conditions, \( i.e., u_r^* = 0, u_z^* = 0 \).

(ii) At the bleed slot: The negative uniform axial velocity is specified and the transverse velocity has been set to zero, \( i.e., \text{specified } u_z^* = 0, u_r^* = 0 \).

(iii) At the inlet: Axial velocity has been specified and the transverse velocity has been set to zero, \( i.e., \text{specified } u_z^* = 0, u_r^* = 0 \). Fully developed flow condition has been specified at the inlet, \( i.e., u_z^* = 2.0 [1 - (2r^*)^2] \).

(iv) At the exit: Constant pressure boundary has been adopted.

(v) At the line of symmetry: The normal gradient of the axial velocity and the transverse velocity have been set to zero, \( i.e., \frac{\partial u_r^*}{\partial z} = 0, u_r^* = 0 \).

2.3 Numerical procedure
The governing equations (1), (2) and (3) have been solved numerically by an in-house CFD code developed using integral approach of the finite volume method on a non-uniform staggered grid following SIMPLER algorithm (Patankar [8]). The third-order upwind scheme has been used for advective part. The discretised equations have been solved using Tri-diagonal Matrix Algorithm (TDMA) with Alternate Direction Implicit (ADI) scheme. The convergence of the iterative scheme has been considered to be achieved when the normalized residuals for mass and momentum equations summed over the entire calculation domain falls below \( 10^{-8} \).

In the present work, the non-dimensional lengths of the inlet section, sudden expansion section and the diverging section have been chosen as 2, 50 and 100 respectively. The non-dimensional inlet diameter has been taken as 1. The aspect ratio has been defined as the ratio of diameter of the sudden expansion section to the inlet diameter \( (D_2/D_1) \). A fixed aspect ratio of 2 has been chosen for all the considered cases. The half divergence angle \( (\theta) \) has been taken as 3.5\(^\circ\), following the experimental work of Reneau et al. [7].
A grid independence test has been carried out for the hybrid diffuser with staggered and non-uniform distribution of grid nodes in both coordinate directions. Following the results of the grid independence test, a numerical mesh with 260 × 186 grid divisions in z and r directions respectively has been employed for the present work.

3. Results and discussion
The important results of the present study are reported in this section. The Reynolds number has been varied from 100 to 600. In case of hybrid diffuser with suction a fixed bleed fraction of 6% has been considered and for the case of hybrid diffuser with fence, \( L_f \) (distance of the fence from throat) has been held constant at 1.0.

3.1 Variation of average static pressure along the length of the diffuser
In the present work, the following expression has been used to determine the average static pressure at any cross-section.

\[
p_{av} = \frac{\int p dA}{dA}
\]

Figure 4 represents the variation of average static pressure along the length of the diffuser for the case of hybrid diffuser for a typical Reynolds number of 400. The figure shows that a steep fall of average static pressure in the inlet section occurs in the throat region as represented by segment a-b of the plot. This phenomenon can easily be understood from the expression of average static pressure, that just after the throat, the cross sectional area increases suddenly resulting in a sudden increase in value of the denominator of the right hand side of equation (4) without any significant static pressure recovery in this region for compensating the sharp fall of average static pressure in the throat region. As the fluid moves downstream in the post throat region, there is significant increase in diffusion rate resulting in higher pressure recovery which is reflected in the increasing trend of the average pressure curve in segment b-c. As the fluid moves further downstream, the rate of increase in the average static pressure is decayed by the fluid friction at the diffuser walls. Therefore, in the sudden expansion zone, the average static pressure first gradually increases due to prevailing fluid diffusion and attains a maximum value (point c of the plot) followed by a gradual decrease in the intentionally considered large extra section in sudden expansion zone due to dominating frictional effect as shown by segment c-d.

In the diverging section, the cross-sectional area increases gradually and the fluid once again undergoes diffusion, it assists to increase the pressure and side by side the enhancement of friction. Therefore, in the diverging section, the average static pressure curve first exhibits a marginal increase as represented by segment d-e, due to further fluid diffusion, then the curves slowly falls in small magnitude due to dominating frictional effect as shown by segment e-f. This is more clearly visible in the inset of figure 4. Thus, for maximum benefit in terms of average static pressure, the effective length of the sudden expansion section should be assigned as \( L_{p1} \) as shown in figure 4 and the length of the gradual expansion section should be restricted to \( L_{p2} \) so as to get the maximum benefit in terms...
of average static pressure at the exit of the diffuser. It is worth mentioning here that the magnitudes of both \( L_{p1} \) and \( L_{p2} \) are expected to vary with Reynolds number as the pressure enhancement by diffusion and the frictional pressure loss both being supposedly vary with Reynolds number.

### 3.2 Possible effective length of diffuser for maximum static pressure at the exit

Figure 5 shows the variation of average static pressure along the length of the diffuser for the three considered diffuser geometries for different Reynolds numbers. This figure shows that incorporation of suction or fence in a hybrid diffuser is beneficial in terms of maximum static pressure rise. Fence offers higher benefit up to the Reynolds number of 200. For the Reynolds number 300 and above, suction offers higher benefit as compared to fence which is also reflected in table 1.

From figure 5 it may be noted that the magnitude and axial location of the maximum static pressure vary with the increase in Reynolds number. For all the considered diffuser geometries, the average static pressure curves exhibit a drooping characteristic after reaching a peak at \( L_{p1} \) in the sudden expansion section and at \( L_{p2} \) in the diverging section. It may therefore be concluded that for achieving highest benefit in terms of maximum average static pressure at the diffuser exit, the lengths of the sudden expansion section and the diverging section have to be considered as \( L_{p1} \) and \( L_{p2} \) respectively which varies with Reynolds number. Therefore, for the considered diffuser geometries the effective lengths of the sudden expansion section, diverging section, and the total length have been computed for different Reynolds numbers and are shown in Table 1.

From Table 1 it can be observed that the effective length of the sudden expansion section (\( L_{p1} \)) increases with the increase in Reynolds number. It has also been observed that for Reynolds number 100, the dimensionless length of \( L_{p2} \) is zero. This may be due to the reason that for lower Reynolds numbers frictional pressure loss exceeds the pressure enhancement by diffusion in the diverging section, which causes a gradual fall of the average static pressure plots in that section without reaching a peak value. On the other hand, for higher Reynolds numbers (200 and above), a positive value of \( L_{p2} \) has been obtained for each Reynolds number and the value of \( L_{p2} \) has been expectedly found to increase with the increase in Reynolds number.
Table 1: Effective lengths of sudden expansion section ($L_{p1}$), diverging section ($L_{p2}$) and total length ($L$) of the considered diffuser geometries for different Reynolds numbers.

| Re  | Diffuser type | Sudden expansion section | Diverging section | Diffuser length ($L$) $(L_1 + L_{p1} + L_{p2})$ |
|-----|---------------|--------------------------|-------------------|---------------------------------|
|     |               | $P_{av\ max}$ | $L_{p1}$ | $P_{av\ max}$ | $L_{p2}$ |                                      |
| 100 | HY            | 0.2654       | 6.375    | -              | 0       | 8.375                                |
|     | HY-S          | 0.2903       | 6.375    | -              | 0       | 8.375                                |
|     | HY-F          | 0.2996       | 6.225    | -              | 0       | 8.225                                |
| 200 | HY            | 0.2724       | 12.75    | -0.06387       | 13.5    | 28.25                                |
|     | HY-S          | 0.2977       | 12.75    | -0.02061       | 12.5    | 27.25                                |
|     | HY-F          | 0.3012       | 12.50    | -0.03752       | 13.5    | 28.0                                 |
| 300 | HY            | 0.2740       | 19.25    | 0.1167         | 22.5    | 43.75                                |
|     | HY-S          | 0.2995       | 19.25    | 0.1493         | 20.5    | 41.75                                |
|     | HY-F          | 0.2963       | 19.17    | 0.1370         | 21.5    | 42.67                                |
| 400 | HY            | 0.2750       | 25.75    | 0.2053         | 27.5    | 55.25                                |
|     | HY-S          | 0.3004       | 25.75    | 0.2331         | 25.5    | 53.25                                |
|     | HY-F          | 0.2926       | 25.28    | 0.2215         | 28.5    | 55.78                                |
| 500 | HY            | 0.2756       | 32.25    | 0.2553         | 38.5    | 72.75                                |
|     | HY-S          | 0.3010       | 32.25    | 0.2807         | 35.5    | 69.75                                |
|     | HY-F          | 0.2900       | 31.39    | 0.2686         | 37.5    | 70.89                                |
| 600 | HY            | 0.2760       | 38.75    | 0.2829         | 47.5    | 88.25                                |
|     | HY-S          | 0.3015       | 38.75    | 0.3074         | 44.5    | 85.25                                |
|     | HY-F          | 0.2881       | 38.06    | 0.2942         | 46.5    | 86.56                                |

4. Conclusions

The numerical analysis carried out in the present work leads to the following important conclusions:

(i) When the Reynolds number is below 200, the diverging section of a hybrid diffuser does not give any benefit as far as the maximum static pressure is concerned. A sudden expansion diffuser with a proper diffuser length of $L_{p1}$, depending upon the Reynolds number, can be chosen in order to get benefit in terms of material and space savings.

(ii) When the Reynolds number is 200 and above, the diverging section of the hybrid diffuser gives further enhancement of pressure, which helps to obtain higher static pressure at the diffuser exit. Therefore, a hybrid diffuser with properly assigned lengths of the sudden expansion section ($L_{p1}$) and the diverging section ($L_{p2}$) may be chosen in consideration with the flow Reynolds number. Allowing extra length in any of the above sections will deliver lower average static pressure at the diffuser exit.

(iii) Incorporation of suction or fence in a hybrid diffuser offers additional benefit in terms of maximum static pressure rise. Fence gives higher benefit up to the Reynolds number of 200, while suction offers more benefit when the Reynolds number is 300 and above.

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**Nomenclature**

| Symbol | Description |
|--------|-------------|
| A      | area at any section |
| A*     | aspect ratio, given by D2/D1 |
| dA     | elemental area |
| D1     | diameter of the inlet duct |
| D2     | diameter of the duct downstream of sudden expansion |
| D3     | diameter of the duct at the end of gradual expansion |
| L1     | length of the inlet section |
| L2     | length of the sudden expansion section |
| L3     | length of the gradual expansion section |
| Lf     | distance of the fence from throat |
| Lp1    | distance of the location of maximum static pressure in sudden expansion section from throat |
| Lp2    | distance of the location of maximum static pressure in diverging section from the beginning of diverging section |
| Lah   | Width of the bleed slot |
| Lr     | reattachment length |
| p      | static pressure |
| pav    | average static pressure |
| Re     | Reynolds number |
| U      | average velocity |
| ur     | velocity in z-direction |
| ur     | velocity in r-direction |
| V̇      | velocity vector |
| z, r   | cylindrical co-ordinates |
| μ      | dynamic viscosity |
| ρ      | density |
| ψ      | stream function |
| ϕ      | fence subtended angle |
| θ      | half divergence angle |

**Superscripts**

* dimensionless terms

**Subscripts**

1-1   | inlet |
2-2   | exit |
e     | pertaining to section e-e |