Accelerating universe in higher dimensional space time: an alternative approach

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Received: 13 June 2021 / Accepted: 14 July 2021
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Abstract We have discussed here a higher dimensional cosmological model and explained the recent acceleration with a Chaplygin type of gas. Dimensional reduction of extra space is possible in this case. Our solutions are general in nature because all the well known results of 4D Chaplygin driven cosmology are recovered when $d = 0$. We have drawn the best fit graph using the data obtained by the differential age method and it is seen that the graph favours only one extra dimension. That means the Chaplygin gas is apparently dominated by a 5D world. Relevant to point out that the final equation in this case are highly nonlinear in nature. Naturally it is not possible to obtain explicit solution of the 4D scale factor with time. To circumvent this difficulty, we consider a first order approximation of the key equation which has made it possible to get time explicit solution of 4D scale factor in exact form as well as the expression of extra dimensions. It may be pointed out that for large four dimensional scale factor this solution mimics $\Lambda$CDM model. An analysis of flip time is also studied both analytically and graphically in some detail. It clearly shows that early flip occurs for higher dimensions. It is also seen that the rate of dimensional reduction is faster for higher dimensions. So we may conclude that the effect of compactification of extra dimension helps the acceleration.

1 Introduction

There has been a resurgence of interests in models where the present universe seems to be undergoing an accelerated expansion. Gravitational force being always attractive in nature, this finding is contrary to our intuition. However, detail investigations of redshifts of type Ia distant nabulae as well as cosmic microwave background anisotropy measurements did suggest the accelerated type of expansion. Several explanations present themselves—introduction of higher derivative theories [1], a variable cosmological constant in Einstein’s field equation [2], flavor oscillations of axions [3], inhomogeneity in space time structure.
In this context, the authors of the present article have been, of late, struggling with the idea of explaining the late acceleration as a higher dimensional (HD) phenomena \[9,10\]. In the framework of higher dimensional cosmology, we have been able to show, though in a rather naive way, that the acceleration can be explained as a consequence of the presence of the extra spatial dimensions and this effect has been coined as ‘dimension driven’ accelerating model. In fact, here the effective Friedmann equations contain additional terms resulting from the presence of extra dimensions which may be interpreted as a ‘fluid’ causing the late acceleration. So, in this work, we attempt to incorporate the phenomenon of acceleration within the framework of higher dimensional spacetime itself without invoking a mysterious scalar field with large negative pressure by hand. Moreover, the origin of the extra fluid responsible for the acceleration is geometric in origin having strong physical foundation and more in line with the spirit of general relativity as proposed by Einstein \[11,12\] and later developed by Wesson and his collaborators \[13\]. In an earlier work, Milton \[14\] has shown that quantum fluctuations in 4D spacetime do not generate the dark energy but rather a possible source of the dark energy is the fluctuations in the quantum fields including quantum gravity inhabiting extra compactified dimensions. This has led a number of workers to concentrate on ideas relating to higher dimensional space in its attempts to unify gravity with other forces of nature, interpretation of different brane models, space-time-matter (STM) proposal \[13\] and also the dimension driven quintessential models \[15\]. The present investigation is primarily motivated by two considerations. While we have plenty of multidimensional cosmological models in literature \[16,17\] and also some sporadic works of brane models \[18,19\] with Chaplygin type of fluid, but scant attention has been paid so far to explain the cosmic acceleration either by extra dimensions themselves or by Chaplygin type of matter field \[20,21\].

The present work essentially contains two parts. We have taken a (d+4) dimensional homogeneous spacetime with two scale factors and a perfect fluid as a source field. Here we have taken a Chaplygin type of matter field with higher dimensional spacetime. The solution of our key Eq. (13) in closed form cannot be obtained because integration yields an elliptical solution only and we get hypergeometric series. In any case, certain inferences can always be drawn in the extreme cases and our analysis shows that an initially decelerating model transits to an accelerating one as in 4D. An interesting result in this section is the fact that the effective equation of state (EOS) at the late stage of evolution contains some additional terms coming from extra dimensions. This finding has marked similarity with the EOS obtained by Guo et al. \[22\] for a variable Chaplygin gas model. Depending upon the presence of extra dimensions, the cosmology then evolves as ΛCDM or Phantom type. This is definitely at variance with the usual 4D models which essentially ends up in a de Sitter phase with time. Though not exactly similar, this points to the ‘k-essence’ type of models which lead to cosmic acceleration today for a wide range of initial conditions without fine-tuning and without invoking an anthropic argument.

We adopt here χ² minimization technique to obtain constraints imposed by cosmological observations. We use Type Ia Supernova data and the predictions of CMB, BAO in constraining the cosmological models. Defining a total χ² function, we analyse cosmological models using the \(H(z) - z\) OHD data (Table 1). The constraints on the \(Ω_m\) and \(m\) (to account for dimensional reduction \(m > 0\)) are determined by drawing contour plots at different confidence levels. We have drawn the best fit graph using the data obtained by the differential age method (CC) and it is seen that the graph favours only one extra dimension. That means the Chaplygin gas apparently mimics a 5D world.
Table 1 The latest Hubble parameter measurements $H(z)$ (in units of $\text{km s}^{-1} \text{Mpc}^{-1}$) and their errors $\sigma$ at redshift $z$ obtained from the differential age method (CC)

| $z$   | $H(z)$ | $\sigma$ | References |
|-------|--------|----------|------------|
| 0.0700| 69.00  | $\mp 19.6$ | [41]       |
| 0.1000| 69.60  | $\mp 12.00$ | [34]       |
| 0.1200| 68.60  | $\mp 26.20$ | [41]       |
| 0.1700| 83.00  | $\mp 8.00$ | [34]       |
| 0.1797| 75.00  | $\mp 4.00$ | [36]       |
| 0.1993| 75.00  | $\mp 5.00$ | [36]       |
| 0.2000| 72.90  | $\mp 29.60$ | [41]       |
| 0.2700| 77.00  | $\mp 14.00$ | [34]       |
| 0.2800| 88.80  | $\mp 36.60$ | [41]       |
| 0.3519| 83.10  | $\mp 14.00$ | [36]       |
| 0.3802| 83.45  | $\mp 13.50$ | [42]       |
| 0.4000| 95.00  | $\mp 17.00$ | [34]       |
| 0.4004| 77.00  | $\mp 10.20$ | [42]       |
| 0.4247| 87.10  | $\mp 11.20$ | [42]       |
| 0.4497| 92.80  | $\mp 12.90$ | [42]       |
| 0.4700| 89.00  | $\mp 34.00$ | [43]       |
| 0.4783| 80.90  | $\mp 9.00$ | [42]       |
| 0.4800| 97.00  | $\mp 60.00$ | [34]       |
| 0.5929| 104.00 | $\mp 13.00$ | [36]       |
| 0.6797| 92.00  | $\mp 8.00$ | [36]       |
| 0.7812| 105.00 | $\mp 12.00$ | [36]       |
| 0.8754| 125.00 | $\mp 17.00$ | [36]       |
| 0.8800| 90.00  | $\mp 40.00$ | [34]       |
| 0.9000| 117.00 | $\mp 23.00$ | [34]       |
| 1.0370| 154.00 | $\mp 20.00$ | [36]       |
| 1.3000| 168.00 | $\mp 17.00$ | [34]       |
| 1.3630| 160.00 | $\mp 33.60$ | [44]       |
| 1.4300| 177.00 | $\mp 18.00$ | [34]       |
| 1.5300| 140.00 | $\mp 14.00$ | [34]       |
| 1.7500| 202.00 | $\mp 40.00$ | [34]       |
| 1.9650| 186.50 | $\mp 50.40$ | [44]       |

It is to be mentioned here that dimensional reduction of extra space is possible in this case. But we can not explain the impact of compactification of extra dimensions on present acceleration or the evolution of scale factor of the universe because the key Eq. (13) is not amenable to obtain an explicit solution, so we have to study the extremal cases only. This type of incompleteness may be remedied via an alternative approach [23] where the higher order terms of the binomial expansion of RHS of Eq. (13) are neglected. The main reason behind it is that the 4D scale factor should be large enough at zero pressure era and it may not be inappropriate if we take only the first order terms of the binomial expansion of RHS of the Eq. (13) which was shown in Eq. (29). In the process we have got an exact solution through which we study an explicit time dependent solution (Fig. 1).
The region plot of $d$ versus $m$ is shown in this figure with the conditions ‘$k > 0$’ and ‘$md < 2$’

2 Higher dimensional field equations

The Einstein field equation in $d-$ dimension is given by

$$R_{AB} - \frac{1}{2} g_{AB} R = \kappa T_{AB}$$  \hspace{1cm} (1)

where $A, B$ are $(0, 1, 2, 3, \ldots d)$, $R_{AB}$ and $R$ are the Ricci tensor and Ricci scalar respectively. We consider the line element of (d+4)-dimensional spacetime

$$ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) - b^2(t) \gamma_{\alpha\beta} dy^\alpha dy^\beta$$  \hspace{1cm} (2)

where $y^\alpha$ ($\alpha, \beta = 4, \ldots, 3 + d$) are the extra dimensional spatial coordinates and the 3D and extra dimensional scale factors $a(t)$ and $b(t)$ depend on time only and the compact manifold is described by the metric $\gamma_{ab}$. We consider the manifold $M^1 \times S^3 \times S^d$ the symmetry group of the spatial section is $O(4) \times O(d + 1)$. The stress tensor whose form will be dictated by Einstein’s equations must have the same invariance leading to the energy momentum tensor as [24]

$$T_{00} = \rho, \hspace{0.5cm} T_{ij} = - p(t) g_{ij}, \hspace{0.5cm} T_{\alpha\beta} = - p_d(t) g_{\alpha\beta}$$  \hspace{1cm} (3)

where the rest of the components vanish. Here $\rho$ is the isotropic 3-pressure and $p_d$, that in the extra dimensions. Considering,

$$b(t) = a(t)^{-m}$$  \hspace{1cm} (4)

where $m$ is any positive number so that dimensional reduction is ensured a priori. For the matter field we here assume an equation of state given by the Chaplygin type of gas in 3D space only [25] which is

$$p = - \frac{B}{\rho}$$  \hspace{1cm} (5)

The field equations are given by [9]

$$\rho = \frac{k \dot{a}^2}{2 a^2}$$  \hspace{1cm} (6)

Fig. 1  The region plot of $d$ versus $m$ is shown in this figure with the conditions ‘$k > 0$’ and ‘$md < 2$’
\[-p = (2 - md) \frac{\ddot{a}}{a} + \frac{1}{2} \frac{m^2 \left(d + 1\right) + 2 \left(1 - md\right)}{a^2} \dot{a}^2 \quad (7)\]

\[-p_d = (3 - md + m) \frac{\ddot{a}}{a} + \frac{1}{2} \frac{m^2 \left(d - 1\right) \left(md - 4\right)}{a^2} \dot{a}^2 \quad (8)\]

where \( k = m^2 d(d - 1) + 6(d - md) \). For a positive energy density, \( k \) must be greater than zero which implies \( m < \frac{3d - \sqrt{3d(d+2)}}{d(d-1)} \) or, \( m > \frac{3d + \sqrt{3d(d+2)}}{d(d-1)} \).

The conservation equation is given by

\[\dot{\rho} + 3 \left(\rho + p\right) \frac{\dot{a}}{a} + d \left(\rho + p_d\right) \frac{\dot{b}}{b} = 0 \quad (9)\]

Now using Eqs. (7), (8) and (9) we get

\[\dot{\rho} + \frac{k}{2 - dm} \frac{\dot{a}}{a} \left\{1 + \frac{2dm(m+1)}{k}\right\} \rho - B \rho^{-1} = 0 \quad (10)\]

Solving Eq. (10) we get

\[\rho = \left[\frac{Bk}{M} + \frac{c}{a^{\frac{2m}{1-2dm}}} \right]^{\frac{1}{2}} (11)\]

where

\[M = k + 2d \left(m + 1\right) (12)\]

and \( c \) is the integration constant. From physical considerations we determine the restriction on \( m \) as \( m < \frac{3d - \sqrt{3d(d+2)}}{d(d-1)} \) for \( d \neq 1 \) otherwise \( m < 1 \) for \( d = 1 \). It may be mentioned here that a detail analysis was given by two of us in Ref. [9] in a Modified Chaplygin gas cosmology. With the constraint (12), the last term in the right hand side is obtained from higher dimensional contribution which is absent in 4D \((d = 0)\). Thus the density of the universe at the present epoch in the framework of a higher dimensional is less compared to a 4-dimensional universe. We note the following:

(i) when \( m = -1 \) we get a universe with \( b(t) = a(t) \), which permits a universe with expansion in all dimensions. In this case the observed universe with desirable feature of dimensional reduction to get an isotropic expansion is not obtained.

(ii) when \( m = 0 \), a universe with flat extra space in \((d + 3)\) dimensions is obtained. In this case also we get similar scenario that is obtained in a 4D universe which was reported in Ref. [26]. In fact the similarity is a direct consequence of a known theorem of Campbell that any analytic N-dimensional Riemmanian manifold can be locally embedded in a higher dimensional Ricci-flat manifold [27].

(iii) when \( d = 0 \), we recover the 4D metric with all the known features of 4D cosmology.

Using Eqs. (6) and (11), we get

\[\frac{\dot{a}^2}{a^2} = \frac{2}{k} \left[\frac{Bk}{M} + \frac{c}{a^{\frac{2m}{1-2dm}}} \right]^{\frac{1}{2}} (13)\]

The known form of the scale factor can be obtained from the above equation. The solution of the Eq. (13) in closed form cannot be obtained because integration yields an elliptical solution only and we get hypergeometric series. However the Eq. (13) gives significant information under extremal conditions as briefly discussed here.
3 Cosmological dynamics

Now we have discussed the cosmological behaviour of the Chaplygin gas equation of state in higher dimensional spacetime. We have expressed the relevant equations with the help of deceleration parameter. In what follows we shall see from the observational data, the best fit graph favours a 5-dimensional interpretation of the cosmological dynamics.

3.1 Deceleration parameter

At the early stage of the cosmological evolution when the scale factor $a(t)$ of the universe is relatively small, the second term of the right hand side of the Eq. (13) dominates which has been discussed in the literature [28]. Using the expression of the deceleration parameter, $q$ we get

$$q = -1 + \frac{M}{2(2-md)} - \frac{kB}{2(2-md)} \left( \frac{Bk}{M} + \frac{c}{a^{\frac{M}{2-md}}} \right)^{-1}$$

one again using Eq. (11) we obtain

$$q = -1 + \frac{M}{2(2-md)} - \frac{kB}{2(2-md)} \frac{1}{\rho^2}$$

Again at flip time, i.e. when $q = 0$ the scale factor becomes

$$a_{flip} = \left[ \frac{c}{\frac{kB}{M} - \frac{Bk}{M}} \right]^{\frac{2-md}{M}}$$

where $a_{flip}$ signifies the sign change of the deceleration parameter. Again, in terms of redshift parameter $(1 + z = \frac{a_0}{a})$ where $a_0$ is the scale factor of the present universe, we can re-write the Eq. (16) as

$$q = -1 + \frac{M}{2(2-md)} - \frac{Bk}{2(2-md)} \left[ \frac{Bk}{M} + c \left( \frac{1+z}{a_0} \right)^{\frac{2-md}{2md}} \right]^{-1}$$

and the redshift parameter at the flip epoch ($z_f$) is given by

$$z_f = \left[ 1 - c \left( \frac{Bk}{M - 2(2-md)} - \frac{Bk}{M} \right)^{\frac{2-md}{2md}} a_0 - 1 \right]$$

As the universe expands the energy density $\rho$ decreases with time such that the last term in the Eq. (15) increases indicating a sign flip when the density attains a critical value given by

$$\rho = \rho_{flip} = \left[ \frac{Bk}{M - 2(2-md)} \right]^{\frac{1}{2}}$$

It is evident that for $M > 2(2-md)$ one gets a universe with normal matter. This is a consistent result for a realistic $z_f$. 

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In the next section we discuss the extremal cases to understand the evolution of the universe. Similar cases in 4 dimensional universe with modified Chaplygin gas is discussed in [29,30].

**CASE A:** In the early phase when the scale factor \(a(t)\) is very small, the Eq. (16) reduces to

\[
q = -1 + \frac{M}{2(2 - dm)} = \frac{1}{2} + \frac{dm(m + 1)}{2(2 - dm)} - \frac{dm}{2}
\]  

(21)

representing a dust dominated universe. It is found that \(q = \frac{1}{2}\) for \(d = 0\), i.e., in a 4-dimensional space time, which is in consonance with well known 4D results.

**CASE B:** In the later epoch of evolution, i.e., for a large size of the universe, we get from the Eq. (16)

\[
q = -\frac{1}{2} \quad (22)
\]

which is similar to that one finds in a \(\Lambda CDM\) model. It further gives the effective EoS using the Eq. (14),

\[
\mathcal{W} = \frac{p}{\rho} = -\left[1 + \frac{2dm(m + 1)}{k}\right] 
\]  

(23)

It is interesting to note that the effective EoS is not time dependent. In what follows we shall find that at the later stage of evolution of the universe as \(a(t) \rightarrow \infty\), \(\mathcal{W} \rightarrow -1\) so in the asymptotic region it can be expressed as \(p = -\rho\) even one begins with a modified Chaplygin gas which corresponds to an empty universe. It corresponds to a universe with a cosmological constant from Eq. (22) it is evident that the deceleration parameter, \(q\) reduces to \(-1\). Again in the presence of extra dimensions \((d \neq 0)\), Eq. (23) points to a phantom type \((\mathcal{W} < -1)\) but in the analogous 4D case \((d = 0)\), it mimics a \(\Lambda CDM\) model. This striking difference results from the appearance of extra terms coming from additional dimensions in EoS.

### 3.2 Observational constraints on the model parameters

In this section the observational data \([31]\) will be used to analyze cosmological model estimating the constraints imposed on the model parameters. We use Type Ia Supernova data and the predictions of CMB, BAO in constraining the cosmological models. It is difficult to integrate the expression to determine the exact temporal behaviour of the scale factor we use an alternative way to express the expansion rate of the universe as a function of redshift, i.e., \(H(z)\) \([32]\) in the data analysis. In our case the observed Hubble data (OHD) set, the most direct and model independent observable of the dynamics of the universe will be used here in the model. Naturally, the \(H(z)\) dataset here shows the fine structure of the expansion history of the universe. One can not get the Hubble \(H(z)\) data directly from a tailored telescope. Instead, one may get it from two different methods. First is to calculate the differential ages of galaxies \([33,34]\), usually called cosmic chronometer (CC), other is to the deduction from the radial BAO peaks in the galaxy power spectrum \([35,36]\) or from the BAO peak using the \(Ly\)-\(\alpha\) forest of QSOs \([37]\) based on the clustering of galaxies or quasars. We analyze the cosmological model using the compilation of OHD data points collected by Magana et al. \([38]\) and Geng et al. \([39]\), the \(H(z)\) data reported in various surveys so far. The 31 CC \(H(z)\) data points are listed in Table 1.

The Hubble parameter depending on the differential ages as a function of redshift \(z\) can be written in the form of

\[
H(z) = -\frac{1}{1 + z} \frac{dz}{dt} 
\]  

(24)
The variation of Hubble parameter $H(z)$ with Redshift $z$

due to Eq. (24) $H(z)$ can be found directly once $\frac{dz}{dt}$ is known [32]. We consider the Hubble parameter $H$ and the three space scale factor $a$, then the Eq. (6) may be expressed as

$$\rho = k_{\Omega m} \frac{2}{3} H^2.$$ 

Using the present value of the scale factor normalised to unity, i.e., $a = a_0 = 1$, we get a relation of the Hubble parameter with the redshift parameter $z$. If $\rho_0$ be the density at present epoch then the well known density parameter can be written as $\Omega_m = \frac{c^2}{M c + c}$ [40].

Now using Eq. (11), we can express the three space matter density as

$$\rho = \rho_0 \left\{ 1 - \Omega_m + \Omega_m (1 + z) \frac{2 M}{z - \Omega_m} \right\}^{\frac{1}{2}}$$

and the Hubble parameter

$$H(z) = H_0 \left\{ 1 - \Omega_m + \Omega_m (1 + z) \frac{2 M}{z - \Omega_m} \right\}^{\frac{1}{2}}$$

where $H_0 = \left( \frac{2 \rho_0}{3} \right)^{\frac{1}{2}}$ is the present value of the Hubble parameter. The Eq. (26) shows the evolution of Hubble parameter $H(z)$ as a function of redshift parameter $z$. The graphical presentation of (26) is shown in Fig. 2b where it has been compared with best fit curve. We draw a best fit curve of redshift against Hubble parameter in the $1\sigma$ confidence region from the data given by the Table 1. The dots in Fig. 2a indicate the recent observable values. Here the value of $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ with $\Omega_m = 0.18$ is taken. In the Fig. 2b we compared the best fit curve with the theoretical expression of $H(z)$ obtained in Eq. (26). It is evident that for $d = 1$ graph almost coincides with the best fit graph for $m = 0.54$ (Fig. 3). Thus it gives hint of a universe with one extra dimension which is more acceptable with $m = 0.54$.

The apparently small uncertainty of the measurement naturally increases its weightage in estimating $\chi^2$ statistics. We define here the $\chi^2$ as

$$\chi^2_H = \sum_{i=1}^{30} \frac{[H^{obs}(z_i) - H^{th}(z_i, H_0, \theta)]^2}{\sigma_H^2(z_i)}$$

where $H^{obs}$ is the observed Hubble parameter at $z_i$ and $H^{th}$ is the corresponding theoretical Hubble parameter given by Eq. (26). Also, $\sigma_H(z_i)$ denotes the uncertainty for the $i$th data point in the sample and $\theta$ is the model parameter. In this work, we have used the latest observational $H(z)$ dataset consisting of 31 data points in the redshift range, $0.07 \leq z \leq 1.965$, larger than the redshift range that is covered by the type Ia supernova. It should be noted that the confidence levels $1\sigma (68.3\%), 2\sigma (95.4\%)$ and $3\sigma (99.7\%)$ are taken proportional to

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Fig. 3  The contour of $\Omega_m$ versus $m$ are shown in this figure.

Fig. 4  Likelihood

$\Delta \chi^2 = 2.3$, 6.17 and 11.8 respectively, where $\Delta \chi^2 = \chi^2(\theta) - \chi^2(\theta^*)$ and $\chi^2_m$ is the minimum value of $\chi^2$. An important quantity which is used for data fitting process is

$$\chi^2 = \frac{\chi^2_m}{dof}$$

where subscript $dof$ is the degree of freedom, and it is defined as the difference between all observational data points and the number of free parameters. If $\chi^2_m/dof \leq 1$, we get a good fit and the observed data are consistent with the considered model (Fig. 4).

$\chi^2_{min}$ and the best-fit values of the model parameters obtained by using the SN Ia dataset and Planck-2015 results in the $1\sigma$ confidence region is shown in Table 2.

The range of $\Omega_m$ and $m$ are respectively (0.1257, 0.3553) and (–0.5862, 0.5660) in $1\sigma$ confidence region. It is seen that the value of $m$ may be both positive or negative. To
Table 2 The $\chi^2_{min}$ value and corresponding $\Omega_m$ & $m$

| Parameter | $\chi^2_{min}$ | $\Omega_m$ | $m$ |
|-----------|---------------|-------------|-----|
| Min values | 14.62 | 0.24 | 0.44 |
| Min values | 14.62 | 0.24 | $-0.20$ |

get a dimensional reduction of extra dimensions, we consider the positive value of $m$ only. For $m = 0.54$, we use the corresponding value of $\Omega_m = 0.18$ which are lying in the $1\sigma$ confidence region.

In a higher dimensional model with extra d-dimensions it is noted that comparing the data obtained by the differential age method (CC), the model with Chaplygin gas favours a 5D universe for a given value of $m$. It is to be mentioned here that from Eq. (4) in the framework of dimensional reduction the 4-dimensional scale factor increases. But we can not explain the impact of compactification of extra dimensions on the present acceleration of the universe.

As pointed earlier the key Eq. (13) is not amenable to obtain an explicit solution which is a function of time in known simple form. In this case the variation of cosmological variables like scale factor, flip time, dependence on extra dimensions etc. can not be explicitly obtained. To avoid such a difficulty of obtaining solution in known form to determine the flip time and other physical features of cosmology we adopt here an alternative approach in the next section.

4 An alternative approach

In the late evolution the universe is big enough and the second term of the right hand side (RHS) in the Eq. (13) is almost negligible compared to the first term. We know that the Chaplygin gas equation of state explains only from dust dominated era to present accelerating universe. As the 4D scale factor is large enough it may not be inappropriate to consider only the first order approximation of the binomial expansion of RHS of the Eq. (13). We obtain an exact solution of the first order approximation in the Eq. (13). Now from Eq. (13) we determine the late stage of evolution of the universe neglecting the higher order terms which is given by

$$\frac{\dot{a}^2}{a^2} = \left( \frac{4B}{kM} \right)^{\frac{1}{n}} + \frac{cM}{B^{\frac{1}{2}}k^{\frac{1}{2}}a^{\frac{2-dm}{2}}} \left( \frac{M}{2M} \right)^{\frac{1}{2}}$$

the Eq. (29) yields as first integral an expression of the scale factor

$$a(t) = a_0 \sinh^n \omega t$$

where, $a_0 = \left\{ \frac{cM}{2Bk} \right\}^{\frac{2-dm}{2M}}$; $n = \frac{2-dm}{2M}$ and $\omega = \sqrt{\frac{2}{k} \left( \frac{Bk}{M} \right)^\frac{1}{2}} \frac{M}{\Sigma-M}$. In Fig. 5, it is evident that the evolution of the scale factor $a(t)$ and the reduction of extra dimensional scale factor $b(t)$ with time $t$ is determined by different values of $d$. This shows that the desirable feature of dimensional reduction of extra scale factor is possible. It is also seen that the rate of growth of 4D scale factor depends on the number of dimensions and it is higher as the number of extra dimension increases. Again the reduction rate of extra dimensional scale factor is faster for higher dimensions. So it is physically realistic to consider the presence of extra dimension which enhances the acceleration. In this context, it is to be remembered that the observational results hold only for one extra dimension in our model (Fig. 6).
Fig. 5 The variation of $a$, $b$ with $t$ for different values of $d$ with $B = 1.0$ & $c = 1.0$.

Now using Eqs. (6), (7) and (30) we can write the expression of $p$ and $\rho$ as follows.

\[ \rho = \left( \frac{B k}{M} \right)^{1/2} \coth^2 \omega t \]  
\[ p = - \left( \frac{B M}{k} \right)^{1/2} (2 - \coth^2 \omega t) \]  

The effective equation of state is given by

\[ w(t) = \frac{p}{\rho} = - \frac{M}{k} (2 \tanh^2 \omega t - 1) \]  

here $w(t)$\(^1\) is a function of time.

From Eqs. (14) and (33) we get the deceleration parameter

\[ q = 1 - \frac{n \cosh^2 \omega t}{n \cosh^2 \omega t} \]  

The Eq. (34) gives that the exponent $n$ which determines the evolution of $q$. A numerical analysis using Eq. (34) shows that (i) if $n > 1$ one gets only acceleration, no flip occurs in this condition. But the eq. (20) leads to a physically unrealistic matter field for $n > 1$. (ii) Again, if $0 < n < \frac{2}{3}$ it gives early deceleration and late acceleration, so the desirable feature of flip occurs which agrees with the observational analysis.

Figure 7 shows the variation of $q$ with $t$ for different values of $d$ where flip occurs. It is seen that the flip occurs faster for more dimensions.

\[ t_f = \frac{1}{\omega} \cosh^{-1} \left( \sqrt{\frac{1}{n}} \right) \]  

\(^1\) By rescaling of time parameter the Eq. (33) may also be written as $w(t) = - \frac{2M}{k} \tanh^2 \omega (t - t_0)$, when $t = t_0$, $w(t) = 0$, i.e., $p = 0$ implying dust dominated universe.
Using Eq. (35) we have drawn the Fig. 8 where the variation of $t_f$ with $d$ is shown. It is seen that the flip time is lower for higher dimensions, i.e., it gives the early acceleration for higher dimensions.

5 Summary

In the paper we present a higher dimensional cosmological model to explain the recent acceleration with a Chaplygin type of gas. The salient features of this model are briefly as follows:

The most important thing is that depending on some initial conditions, the effective equation of state during late evolution interpolates between $\Lambda CDM$ and Phantom type of expansion. In this respect our work recovers the effective equation of state (for large scale factor) for an analogous work of Guo et al. [22] where a very generalised Chaplygin type of gas is
taken. One may mention that our solutions are quite general in nature because all the well known results of 4D Chaplygin driven cosmology are recovered when $d = 0$.

While working on any higher dimensional model one always looks for situation where dimensional reduction takes place and the cosmology eventually becomes 4D one. Interesting to point out that our present model satisfies this important criteria for positive $m$. It is to be noted that with the help of observational data and following $\chi^2$ minimization programme we find the range of $\Omega_m$ and $m$ are respectively $(0.1257, 0.3553)$ and $(-0.5862, 0.5660)$ in $1\sigma$ confidence region. One takes the value of $m = 0.54$ and the corresponding $\Omega_m = 0.18$ which are lying in the $1\sigma$ confidence region. The best fit graph is drawn from the observational data and it is seen that the graph favours only one extra dimension. That means the Chaplygin gas is apparently dominated by a 5D world.

To end the section a final remark may be in order. Being highly nonlinear one can not get a solution of the key Eq. (13) in a closed form forcing us to look for solutions in the asymptotic regions only. So we can not explain the evolution of 4D scale factor or reduction of extra dimensions etc. in a general way. To compensate for these incompleteness an alternative approach is suggested where only the first order terms of the binomial expansion are considered. By the above approach we get the time explicit solution of 4D scale factor $a(t)$ as well as the expression of extra dimensions $b(t)$. It is also seen that the rate of dimensional reduction is higher for higher dimensions. So we may conclude that the effect of compactification of extra dimension helps the acceleration. We also investigate dimensional dependence on the deceleration parameter $q$ and flip time $t_f$. It clearly shows that early flip occurs for higher dimensions.

Acknowledgements DP acknowledges financial support of Sree Chaitanya College, Habra for a Minor Research project vide no SCC/MRP/2019-20/03.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: There are no associated data available.]

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