A new bound on the Dirac neutrino magnetic moment from the plasma induced neutrino chirality flip in a supernova

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Abstract. The neutrino chirality flip process under the conditions of the supernova core is investigated in detail with the plasma polarization effects in the photon propagator taken into account, in a more consistent way than in earlier publications. It is shown in part that the contribution of the proton fraction of plasma dominates. A new upper bound on the Dirac neutrino magnetic moment is obtained from the limit on the supernova core luminosity for $\nu_R$ emission: $\mu_\nu < (0.7-1.5) \times 10^{-12} \mu_B$. The best upper bound on the neutrino magnetic moment from SN1987A is improved by the factor of 2.

Keywords: supernova neutrinos, neutrino properties
1. Introduction

Non-vanishing neutrino magnetic moment leads to various chirality flipping processes where the left-handed neutrinos produced in the stellar interior become the right-handed ones, i.e. sterile with respect to the weak interaction, and this can be important e.g. for the stellar energy loss. In the standard model extended to include the neutrino mass $m_\nu$, the well-known result for the neutrino magnetic moment is \( \mu^{(\text{SM})}_\nu = \frac{3e G_F}{8\pi^2\sqrt{2}} m_\nu \approx 3.20 \times 10^{-19} \left( \frac{m_\nu}{1\text{eV}} \right) \mu_B, \) (1)

where $\mu_B = e/2m_e$ is the Bohr magneton. Thus, it is unobservably small given the known limits on neutrino masses. On the other hand, non-trivial extensions of the standard model such as left–right symmetry can lead to more significant values for the neutrino magnetic moment.

First attempts at exploiting the mechanism of the neutrino chirality flipping were connected with the solar neutrino problem, and two different scenarios were analysed. The first one, based on the neutrino magnetic moment rotation in a stellar magnetic field, was investigated in the papers [3]–[5]. In the second scenario, a neutrino changed the chirality due to the electromagnetic interaction of its magnetic moment with plasma [6, 7]. For a more extended list of references see e.g. [8]. In all these cases the effect appeared to be small to have an essential impact on the solar neutrino problem, if $\mu_\nu < 10^{-10} \mu_B$.

More significant constraints on $\mu_\nu$ are provided by other stars. For example, the cores of low mass red giants are about $10^4$ times denser than the Sun, and non-standard neutrino losses would have a more essential effect there, delaying the ignition of helium. Thus, a limit was obtained [9, 10]:

$$\mu_\nu < 0.3 \times 10^{-11} \mu_B.$$ (2)
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An independent constraint on the magnetic moment of a neutrino was also obtained from the Early Universe [11, 12]:

$$\mu_\nu < 6.2 \times 10^{-11} \mu_B,$$

where spin flip collisions would populate the sterile Dirac components in the era before the decoupling of the neutrinos. Thus, it doubles the effective number of thermally excited neutrino degrees of freedom and increases the expansion rate of the Universe, causing the overabundance of helium.

A considerable interest in the neutrino magnetic moment arose after the great event of SN1987A, in connection with the modelling of a supernova explosion, where gigantic neutrino fluxes define in fact the process energetics. This means that such a microscopic neutrino characteristic as the neutrino magnetic moment would have a critical influence on macroscopic properties of these astrophysical events. Namely, the left-handed neutrinos produced inside the supernova core during the collapse could convert into the right-handed neutrinos due to the magnetic moment interaction. These sterile neutrinos would escape from the core leaving no energy to explain the observed neutrino luminosity of the supernova. Thus, the upper bound on the neutrino magnetic moment can be established.

This matter was investigated by many authors in different aspects [13]–[17]. We will mainly focus on the paper by Barbieri and Mohapatra [15] which now looks to have the most reliable instant constraint on the neutrino magnetic moment from SN1987A, according to [18]. The authors [15] considered the neutrino spin flip via both $\nu_L e^- \rightarrow \nu_R e^-$ and $\nu_L p \rightarrow \nu_R p$ scattering processes in the inner core of a supernova immediately after the collapse. Imposing for the $\nu_R$ luminosity $Q_{\nu_R}$ the upper limit of $10^{53}$ ergs s$^{-1}$, the authors obtained the upper bound on the neutrino magnetic moment:

$$\mu_\nu < (0.2 - 0.8) \times 10^{-11} \mu_B.$$  \hfill (4)

However, the essential plasma polarization effects in the photon propagator were not considered in [15], and the photon dispersion was taken in a phenomenological way, by inserting an ad hoc thermal mass into the vacuum photon propagator. A detailed investigation of this question was performed in the papers by Ayala et al [19, 20], who used the formalism of the thermal field theory to take into account the influence of hot dense astrophysical plasma on the photon propagator. The upper bound on the neutrino magnetic moment compared with the result of the paper [15] was improved in [19, 20] by the factor of 2:

$$\mu_\nu < (0.1 - 0.4) \times 10^{-11} \mu_B.$$  \hfill (5)

However, looking at the intermediate analytical results of the authors [19, 20], one can see that only the contribution of plasma electrons was taken into account there, while the proton fraction was omitted. This is despite the fact that the electron and proton contributions to the neutrino spin flip process were evaluated in [15] to be of the same order. It should be mentioned also that the improvement of the bound (5) with respect to the bound (4) was based in part on the enhancement by the factor of 2 of the supernova core volume made in [19, 20] if compared with [15], while the density was taken to be the same, $\rho_c \simeq 8 \times 10^{14}$ g cm$^{-3}$. This means that the core mass appeared to be in [19, 20] of the order of 3 $M_\odot$, which is nearly twice the mass of the supernova remnant believed to be typical.
Thus, a reason exists to reconsider the neutrino spin flip processes in the supernova core more attentively. In this paper, we perform such an analysis, and we show in part that the proton contribution to the photon propagator is not less essential than the electron contribution.

We consider the Dirac neutrinos only, because in this case the neutrino magnetic moment interaction (both diagonal and non-diagonal) with a photon transforms the active left-handed neutrinos into right-handed neutrinos which are sterile with respect to the weak interaction. We do not consider the Majorana neutrinos, because the produced right-handed antineutrino states are not sterile in this case.

We begin in section 2 with calculations of the amplitude of the neutrino spin flip process due to the neutrino scattering off plasma components. We formulate a general expression for the rate of creation of the right-handed neutrino with the fixed energy. Some details of calculations are presented in appendices A and B. In section 3 we calculate the supernova core luminosity for \( \nu_R \) emission and we obtain the upper limit on the neutrino magnetic moment.

2. Neutrino interaction with background

2.1. The neutrino chirality flip amplitude

The neutrino chirality flip is caused by the scattering via the intermediate photon (plasmon) off the plasma electromagnetic current presented by electrons, \( \nu_L e^- \rightarrow \nu_R e^- \), protons, \( \nu_L p \rightarrow \nu_R p \), etc. The total process Lagrangian consists of two parts; the first one is the interaction of a neutrino having a magnetic moment \( \mu^\nu \) (both diagonal and transition) with photons, while the second part describes the plasma interaction with photons:

\[
\mathcal{L} = - \frac{i}{2} \sum_{i,j} \mu^\nu_{ij} (\bar{\nu}_i \sigma_{\alpha\beta} \nu_j) F^{\alpha\beta} - e J_\alpha A^\alpha, \tag{6}
\]

where \( \sigma_{\alpha\beta} = (1/2) (\gamma_\alpha \gamma_\beta - \gamma_\beta \gamma_\alpha) \), \( F^{\alpha\beta} \) is the tensor of the photon electromagnetic field, \( e > 0 \) is the elementary charge, \( J_\alpha = -(\bar{e} \gamma_\alpha e) + (\bar{p} \gamma_\alpha p) + \cdots \) is an electromagnetic current in the general sense, formed by different components of the medium, i.e. free electrons and positrons, protons, free ions, etc. Here we will consider the diagonal neutrino magnetic moment \( \mu_\nu \). An extension to the case of the transition magnetic moment \( \mu^\nu_{ij} \) is straightforward.

With the Lagrangian (6), the process is described by the Feynman diagram shown in figure 1.

The technique of calculation of the neutrino spin flip rate is rather standard. The invariant amplitude for the process of the neutrino scattering off the \( k \)th plasma component can be written in the form

\[
\mathcal{M}^{(k)} = -ie \mu_\nu J^\nu_{(k)} G_{\alpha\beta}(Q) J^\beta_{(k)}, \tag{7}
\]

where \( J^\nu_{(k)} \) is the Fourier transform of the neutrino magnetic moment current,

\[
J^\alpha_{(\nu)} = [\bar{\nu}_R(p') \sigma^{\mu\nu} \nu_L(p)] Q_\mu.
\]
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Figure 1. The Feynman diagram for the neutrino spin flip scattering via the intermediate plasmon $\gamma^*$ on the plasma electromagnetic current $J$.

$J^\beta_{(k)}$ is the Fourier transform of the $k$th plasma component electromagnetic current, and $Q = (q_0, \mathbf{q})$ is the 4-momentum transferred. The only principal point is to use the photon propagator $G_{\alpha\beta}(Q)$ with the plasma polarization effects taken into account. We use the straightforward way of taking account of these effects by summation of the Feynman diagrams of the forward photon scattering off plasma particles. Similarly to the vacuum case, this summation leads to the Dyson equation which provides a correct result for the photon propagator in plasma in the region where the photon polarization operator is real, in the form

$$G_{\alpha\beta}(Q) = \frac{i \rho_{\alpha\beta}(t)}{Q^2 - \Pi_t} + \frac{i \rho_{\alpha\beta}(\ell)}{Q^2 - \Pi_{\ell}},$$

where $\Pi_{t,\ell}$ are the eigenvalues of the photon polarization tensor $\Pi_{\alpha\beta}$ for the transverse and longitudinal plasmon,

$$\Pi_{t,\ell} = -\Pi_t \rho_{t,\ell}(t) - \Pi_{\ell} \rho_{t,\ell}(\ell),$$

and $\rho_{t,\ell}(t, \ell)$ are the corresponding density matrices

$$\rho_{t,\ell}(t) = -\left(g_{\alpha\beta} - \frac{Q_\alpha Q_\beta}{Q^2} - \frac{L_\alpha L_\beta}{L^2}\right),$$

$$\rho_{t,\ell}(\ell) = -\frac{L_\alpha L_\beta}{L^2},$$

$$L_\alpha = Q_\alpha (u Q) - u_\alpha Q^2,$$

$u_\alpha$ is the 4-vector of the plasma velocity. The density matrices $\rho_{\alpha\beta}(\lambda)$ with $\lambda = t, \ell$ have properties of the projection operators:

$$\rho_{\alpha\mu}(\lambda) \rho_{\beta}(\lambda') = -\delta_{\lambda\lambda'} \rho_{\alpha\beta}(\lambda).$$
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In the region where the eigenvalues $\Pi_{t,\ell}$ of the photon polarization tensor develop imaginary parts, they can be written as

$$\Pi_\lambda = R_\lambda + i I_\lambda,$$  \hspace{1cm} (14)

where $R_\lambda$ and $I_\lambda$ are the real and imaginary parts, containing the contributions of all components of the active medium. For extracting the imaginary parts $I_{t,\ell}$, it will suffice to make an analytical extension $q_0 \to q_0 + i \epsilon$ corresponding to the retarded polarization operator.

The eigenvalues $\Pi_{t,\ell}$ of the photon polarization tensor are presented in appendix A both in the general form and in some particular cases.

2.2. The rate of creation of the right-handed neutrino

The value of physical interest is the rate of creation of the right-handed neutrino $\nu_R$, $\Gamma_{\nu_R}(E')$, with the fixed energy $E'$, by all the left-handed neutrinos. This function can be obtained by integration of the amplitude (7) squared over the states of the initial left-handed neutrinos and over the states of the initial and final plasma particles forming the electromagnetic current $J^\beta_{(k)}$:

$$\Gamma_{\nu_R}(E') = \sum_k \Gamma_{\nu_R}^{(k)}(E'),$$ \hspace{1cm} (15)

$$\Gamma_{\nu_R}^{(k)}(E') = \frac{1}{16 (2\pi)^5 E'} \int \sum_{s,s'} |\mathcal{M}^{(k)}|^2 \delta^{(4)}(p' + \mathcal{P}' - p - \mathcal{P}) \times \frac{d^3 \mathcal{P}}{E} f_k(E') \frac{d^3 \mathcal{P}'}{E'} [1 \mp f_k(E') \frac{d^3 \mathcal{P}}{E} f_{\nu}(E').$$ \hspace{1cm} (16)

Here, $p^\alpha = (E, p)$ and $p'^\alpha = (E', p')$ are the 4-momenta of the initial and final neutrinos, $\mathcal{P}^\alpha = (\mathcal{E}, \mathcal{P})$ and $\mathcal{P}'^\alpha = (\mathcal{E}', \mathcal{P}')$ are the 4-momenta of the initial and final plasma particles; $\sum_{s,s'}$ means the summation over the spins of these particles, the index $k = e, p, i, \ldots$ corresponds to the type of the plasma particles (electrons, protons, free ions, etc) with the distribution function $f_k(E)$, which can be both fermions (the upper sign in $[1 \mp f_k(E')]$) and bosons (the lower sign); $f_{\nu}(E) = (e^{(E-\bar{\mu}_{\nu})/T} + 1)^{-1}$ is the Fermi–Dirac distribution function for the initial left-handed neutrinos in the plasma rest frame, $\bar{\mu}_{\nu}$ is the neutrino chemical potential.

It is convenient to pass in equation (16) from integration over the initial neutrino momentum $p$ to the integration over the virtual plasmon momentum $p' - p' = Q = (q_0, q)$, $|q| \equiv q$, using the relation

$$\frac{d^3 \mathcal{P}}{E} f_{\nu}(E) = \frac{2 \pi}{E'} q dq \theta(-Q^2) \theta(2E' + q_0 - q) f_{\nu}(E' + q_0).$$

Substituting the amplitude (7) squared into equation (16), one obtains

$$\Gamma_{\nu_R}(E') = \frac{\mu_{\nu}^2}{8 \pi^2 E'^2} \int_{-E'}^{+E'} dq_0 \int_{|q_0|}^{2E' + q_0} q dq f_{\nu}(E' + q_0) j_{(\nu)}^{\alpha} j_{(\nu)}^{\alpha'},$$

$$\times \sum_{\lambda,\lambda'} \frac{\rho_{\alpha \beta}(\lambda) \rho_{\alpha' \beta'}(\lambda')}{(Q^2 - \Pi_\lambda)(Q^2 - \Pi_{\lambda'})} T^{\beta \beta'},$$ \hspace{1cm} (17)
where the following tensor integral is introduced:

\[
T_{\alpha \beta} = \frac{e^2}{32 \pi^2} \sum_k \sum_{s, s'} \int J_{(k)}^\alpha J_{(k)}^{\beta*} \, d\Phi,
\]

\[
d\Phi = \frac{d^3P \, d^3P'}{E \, E'} f_k(P) [1 + f_k(P')] \delta^{(4)}(P' - P - Q).
\]

The detailed calculation of the tensor \( T_{\alpha \beta} \) is presented in appendix B. It is remarkable that the result is expressed in terms of the density matrices (10), (11):

\[
T_{\alpha \beta} = \left[ -I_t \rho^{\alpha \beta}(t) - I_\ell \rho^{\alpha \beta}(\ell) \right] [1 + f_\gamma(q_0)],
\]

where \( I_t, \ell \) are the imaginary parts of the eigenvalues \( \Pi_t, \Pi_\ell \) of the photon polarization tensor; \( f_\gamma(q_0) \) is the Bose–Einstein distribution function for a photon.

Substituting (19) into (17), using the orthogonality of the tensors \( \rho^{\alpha \beta}(t) \) and \( \rho^{\alpha \beta}(\ell) \), see equation (13), and taking into account the expressions for the contractions of the neutrino current with these tensors:

\[
\langle j_\nu^\alpha \rangle j_\nu^{\beta*} \rho_{\alpha \beta}(t) = Q^4 \left( \frac{(2E' + q_0)^2}{q^2} - 1 \right),
\]

\[
\langle j_\nu^\alpha \rangle j_\nu^{\beta*} \rho_{\alpha \beta}(\ell) = -Q^4 \frac{(2E' + q_0)^2}{q^2},
\]

one finally obtains for the rate of creation of the right-handed neutrino

\[
\Gamma_{\nu_R}(E') = \frac{\mu_\nu^2}{16 \pi^2 E'^2} \int_{-E'}^\infty dq_0 \int_{|q_0|}^{2E' + q_0} q^3 \, dq \, f_\nu(E' + q_0)(2E' + q_0)^2
\]

\[
\times \left( 1 - \frac{q_0^2}{q^2} \right)^2 [1 + f_\gamma(q_0)] \left[ 1 - \frac{q^2}{(2E' + q_0)^2} \right] \rho_t - \rho_\ell.
\]

Here, the plasmon spectral densities are introduced:

\[
\varrho_\lambda = \frac{-2 I_\lambda}{(Q^2 - R_\lambda)^2 + I_\lambda^2},
\]

which are defined by the eigenvalues (14) of the photon polarization tensor (9).

The formula (20) presents our main result. We note that it is in agreement, to notation, with the rate obtained by Elmfors et al [12] from the retarded self-energy operator of the right-handed neutrino. However, extracting from our general expression the electron contribution only, we obtain the result which is larger by the factor of 2 than the corresponding formula in the papers by Ayala et al [19, 20]. It can be seen that an error was made there just in the first formula defining the production rate \( \Gamma \) of a right-handed neutrino.

Our formula, being obtained for the process of the neutrino interaction with virtual photons, has in fact a more general sense, and can be used for neutrino–photon processes in any optically active medium. We only need to identify the photon spectral density functions \( \varrho_\lambda \). For example, in the medium where \( I_\ell \to 0 \) in the space-like region \( Q^2 < 0 \) corresponding to the refractive index values \( n > 1 \), the spectral density function is transformed to a \( \delta \)-function, and we can reproduce the result of the paper by Grimus.
and Neufeld [21] devoted to the study of the Cherenkov radiation of transverse photons by neutrinos.

If one formally takes the limit $I_\ell \to 0$, the result obtained by Mohanty and Sahu [22] can be reproduced, namely, the width of the Cherenkov radiation and absorption of longitudinal photons by neutrinos in the space-like region $Q^2 < 0$. However, the limit $I_\ell \to 0$ itself is irrelevant for $Q^2 < 0$ in the real astrophysical plasma conditions considered by those authors and leads to the strong overestimation of a result.

2.3. Contributions of plasma components to the neutrino scattering process

As was mentioned above, an analysis of the neutrino chirality flip process has to be performed taking account of the neutrino scattering off various plasma components: electrons, protons, free ions, etc. For the first step we consider the contribution of the neutrino scattering off electrons to the right-handed neutrino production rate. This means that we take into account the electron contribution only to the function $I_\lambda$ in the numerator of equation (21). It should be stressed however, that the functions $R_\lambda$ and $I_\lambda$ in the denominator of equation (21) contain the contributions of all plasma components. At this point our result for the neutrino scattering off electrons differs from the result of Ayala et al [19, 20], where the electron contribution only was taken both in the numerator and in the denominator of the plasmon spectral densities.

As the analysis shows, see appendix A, the electron and proton contributions to the imaginary parts $I_\lambda$ of the eigenvalues $\Pi_\lambda$ of the photon polarization tensor are of the same order of magnitude and have the same sign for $\lambda = t$ and for $\lambda = t'$; see figures A.2 and A.4. This in itself should lead to a decreasing of the electron contribution to the function $\Gamma\nu_R(E')$. On the other hand, it is seen from figure A.1, that the electron and proton contributions to the real part $R_\ell$ of the eigenvalue $\Pi_\ell$ are of the same order of magnitude but have the opposite signs in the region where the imaginary part of the electron contribution to the numerator of equation (21) is relatively large. As a result, the contribution of the neutrino scattering off electrons to the right-handed neutrino production rate, obtained by us, appears to be close to the result of Ayala et al, besides the above-mentioned factor of 2.

It is possible to consider similarly the contribution of the neutrino scattering off protons to the right-handed neutrino production rate. In this case, we take the proton contribution to the functions $I_\lambda$ (A.11) and (A.13) in the numerator of equation (21).

The results of our numerical analysis of the separate contributions of the neutrino scattering off electrons and protons, as well as the total $\nu_R$ production rate in the typical conditions of the supernova core, are presented in figure 2.

The plotted function $F(E')$ is defined by the expression

$$\Gamma\nu_R(E') = \frac{\mu^2}{32\pi} \Gamma(0) F(E').$$

For comparison, the result by Ayala et al [20] is also shown in figure 2, illustrating a strong underestimation of the neutrino chirality flip rate made by those authors.

We consider also the contribution of the neutrino scattering off free ions to the $\nu_R$ production rate. While the ions are believed to be absent in the supernova core, a significant fraction of them could be presented e.g. in the upper layers of the supernova envelope. It should be mentioned that longitudinal virtual plasmons give the main
Figure 2. The function $F(E')$ defining the electron contribution (dashed line), the proton contribution (dash–dotted line) to the $\nu_R$ production rate, and the total rate (solid line) for the plasma temperature $T = 30$ MeV. The dotted line shows the result of Ayala et al [20].

contribution to the $\nu_R$ production rate in this case. As is seen from equations (A.15), the function $I^{(i)}_n$ differs from zero only in the narrow area $\Delta x$ of the variable $x = q_0/q$, namely, $\Delta x \sim \sqrt{T/m_i} \ll 1$, where $m_i$ is the ion mass. This allows one to perform calculations of the ion contribution to the $\nu_R$ production rate analytically, to obtain

$$
\Delta \Gamma^{(i)}_{\nu_R}(E') = \mu_{\nu}^2 \alpha Z_i^2 n_i f_\nu(E') \left( \ln \frac{4E'^2 + m_D^2}{m_D^2} - \frac{4E'^2}{4E'^2 + m_D^2} \right),
$$

(23)

where $\alpha$ is the fine structure constant, $e Z_i$ and $n_i$ are the charge and the density of ions, $m_D$ has the meaning of the Debye screening radius inverse, $m_D^2 = \sum_k R^{(k)}_D(q_0 = 0)$. We recall that the summation is performed over all plasma components.

It is interesting to note that equation (23) obtained in the approximation of heavy ions describes rather satisfactorily the proton contribution.

Given the function $\Gamma_{\nu_R}(E')$, one can calculate the total number of right-handed neutrinos emitted per 1 MeV per unit time from the unit volume, i.e. the right-handed neutrino energy spectrum:

$$
\frac{dn_{\nu_R}}{dE'} = \frac{E'^2}{2 \pi^2} \Gamma_{\nu_R}(E').
$$

(24)

This value is presented in figure 3 for two values of the plasma temperature.

One can see from equation (24) that a very narrow peak of the function $\Gamma_{\nu_R}(E')$ at small neutrino energy, which was analysed in detail in [20], does not provide a huge
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Figure 3. The number of right-handed neutrinos (for $\mu = 10^{-12} \mu_B$) emitted per 1 MeV of the energy spectrum per unit time from the unit volume for the plasma temperature $T = 60$ MeV (solid line) and for $T = 30$ MeV (dashed line).

amount of soft right-handed neutrino production, as was declared in [20], because of the factor $E''^2$.

The right-handed neutrino energy spectrum (24) can be useful for investigations of possible mechanisms of the energy transfer from these neutrinos to the outer layers of the supernova envelope. For example, a process is possible for inverse conversion of a part of right-handed neutrinos into left-handed ones, with their subsequent absorption. Just these processes were proposed by Dar [23] and then investigated in [24]–[26] as a possible mechanism for the stalled shock wave revival in the supernova explosion.

3. Limits on the neutrino magnetic moment

As a possible application of the formulae obtained, we can establish the upper limit on the neutrino magnetic moment, by comparison of the supernova core luminosity computed from the $\nu_R$ energy spectrum (24) with the left-handed neutrino luminosity $Q_{\nu_L} \sim 10^{52} - 10^{53}$ ergs s$^{-1}$ [27]; for a recent review see e.g. [28].

The supernova core luminosity for $\nu_R$ emission can be computed as

$$Q_{\nu_R} = V \int_0^\infty \frac{d\nu_{\nu_R}}{dE'} E' dE' = \frac{V}{2\pi^2} \int_0^\infty E'^3 \Gamma_{\nu_R}(E') dE',$$

(25)

where $V$ is the plasma volume.

The physical conditions inside the supernova core are rather uncertain; they are model dependent and vary in time [28]. To compare our results with the previous
estimations [15, 19, 20], we use the same supernova core conditions (plasma volume $V \sim 4 \times 10^{18}$ cm$^3$, temperature range $T = 30–60$ MeV, electron chemical potential range $\bar{\mu}_e = 280–307$ MeV, neutrino chemical potential $\bar{\mu}_\nu = 160$ MeV). These conditions could exist in the time interval before one second after the collapse; see [27], pp 397–401. We found

$$Q_{\nu_R} = \left( \frac{\mu_\nu}{\mu_B} \right)^2 (0.38–2.2) \times 10^{77} \text{ ergs s}^{-1}. \quad (26)$$

This value should be compared with the corresponding formula (48) of the paper [20] to see that our result for the luminosity is greater by the factor of 10. This discrepancy can be explained by the following features: (i) the factor of 2 was lost in the electron contribution in the papers [19, 20]; (ii) the proton contribution was omitted there. The neutrino scattering off protons appears to give an even more essential contribution to the luminosity because of shift of the rate $\Gamma_{\nu R}(E')$ maximum into the region of larger energies; see figure 2.

Assuming that the right-handed neutrino luminosity is less than the left-handed neutrino luminosity at the time $\sim$0.1 s after the collapse, $Q_{\nu_R} < 10^{53}$ ergs s$^{-1}$, we obtain from equation (26) the upper limit on the neutrino magnetic moment

$$\mu_\nu < (0.7–1.5) \times 10^{-12} \mu_B. \quad (27)$$

As the analysis shows, the limit obtained appears to be rather stable with respect to the variation of the supernova core parameters, when the product of the average value of the electron fraction $Y_e$ on the core mass is fixed.

By this means, we improve the best upper bound on the neutrino magnetic moment from SN1987A obtained by Ayala et al [19] by the factor of 2.

4. Summary

We have investigated in detail the neutrino chirality flip process under the conditions of the supernova core. The plasma polarization effects caused both by electrons and protons were taken into account in the photon propagator. The rate $\Gamma_{\nu R}(E')$ of creation of the right-handed neutrino with the fixed energy $E'$, the energy spectrum, and the luminosity have been calculated.

From the limit on the supernova core luminosity for $\nu_R$ emission, we have obtained the upper bound on the neutrino magnetic moment $\mu_\nu < (0.7–1.5) \times 10^{-12} \mu_B$. Thus, we have improved the best upper bound on the neutrino magnetic moment from SN1987A by the factor of 2.

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1 We take the volume as in [15], which corresponds to the core mass of the order of $1.5 M_\odot$ in the conditions presented.
Appendix A. Eigenvalues of the photon polarization tensor

The expressions for the contributions of a charged fermion to the polarization functions $\Pi_{t,\ell}$ in the hard thermal loop approximation can be found e.g. in [29] and have the form

$$\Pi_{t} = \frac{4\alpha}{\pi} \int_{0}^{\infty} \frac{dP \mathcal{P}^2}{E} \left[ f_0(E) + \bar{f}_0(E) \right] \left( \frac{q_0^2}{q^2} - \frac{q_0^2 - q^2}{q^2} \frac{q_0}{2vq} \ln \frac{q_0 + vq}{q_0 - vq} \right),$$  \hspace{1cm} (A.1)$$

$$\Pi_{\ell} = \frac{4\alpha}{\pi} \frac{q_0^2 - q^2}{q^2} \int_{0}^{\infty} \frac{dP \mathcal{P}^2}{E} \left[ f_0(E) + \bar{f}_0(E) \right] \left( \frac{q_0}{vq} \ln \frac{q_0 + vq}{q_0 - vq} - \frac{q_0^2 - q^2}{q_0^2 - v^2q^2} - 1 \right),$$  \hspace{1cm} (A.2)$$

where $v = \mathcal{P}/E$, and the Fermi–Dirac distribution functions for the fermions and antifermions are

$$f_0(E) = \frac{1}{e^{(E-\mu)/T} + 1}, \quad \bar{f}_0(E) = \frac{1}{e^{(E+\bar{\mu})/T} + 1},$$  \hspace{1cm} (A.3)$$

$\bar{\mu}$ is the fermion chemical potential.

For the supernova core conditions, the main contribution comes from the plasma electrons and protons:

$$R_{t,\ell} \simeq R_{t,\ell}^{(e)} + R_{t,\ell}^{(p)}, \quad I_{t,\ell} \simeq I_{t,\ell}^{(e)} + I_{t,\ell}^{(p)}. \hspace{1cm} (A.4)$$

In these conditions, it is a good approximation to consider the electron fraction as the relativistic plasma $\langle \tilde{\mu}_e, T \gg m_e \rangle$.

The real and imaginary parts (A.4) of the electron contributions to the photon polarization functions take the following forms:

$$R_{t}^{(e)} = m_{\gamma}^2 \left( x^2 + \frac{x(1-x^2)}{2} \ln \left| \frac{1+x}{1-x} \right| \right),$$  \hspace{1cm} (A.5)$$

$$I_{t}^{(e)} = -\pi \frac{m_{\gamma}^2}{2} x \left( 1 - x^2 \right),$$  \hspace{1cm} (A.6)$$

$$R_{\ell}^{(e)} = 2 m_{\gamma}^2 \left( 1 - x^2 \right) \left( 1 - \frac{x}{2} \ln \left| \frac{1+x}{1-x} \right| \right),$$  \hspace{1cm} (A.7)$$

$$I_{\ell}^{(e)} = \pi m_{\gamma}^2 x \left( 1 - x^2 \right),$$  \hspace{1cm} (A.8)$$

where $x = q_0/q$, $|x| < 1$, $m_{\gamma}$ is the so-called photon thermal mass,

$$m_{\gamma}^2 = \frac{2\alpha}{\pi} \left( \tilde{\mu}_e^2 + \frac{\pi^2 T^2}{3} \right). \hspace{1cm} (A.9)$$

For the proton contributions, the situation appears to be more complicated. For the real and imaginary parts of the proton contribution to the polarization functions (A.1), (A.2), for the conditions $\tilde{\mu}_p \gg T$, where $\tilde{\mu}_p$ is the proton chemical potential, one obtains

$$R_{t}^{(p)} = \frac{4\alpha}{\pi} \int_{0}^{\infty} \frac{dP \mathcal{P}^2}{E (e^{(E-\tilde{\mu}_p)/T} + 1)} \left( x^2 + \frac{x(1-x^2)}{2v} \ln \left| \frac{x+v}{x-v} \right| \right),$$  \hspace{1cm} (A.10)$$

$$I_{t}^{(p)} = -2\alpha x \left( 1 - x^2 \right) \int_{\mathcal{P}_{\min}}^{\infty} \frac{dP \mathcal{P}}{e^{(E-\tilde{\mu}_p)/T} + 1}, \quad \mathcal{P}_{\min} = \frac{m_p|x|}{\sqrt{1-x^2}},$$  \hspace{1cm} (A.11)$$

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\begin{equation}
R^p_\ell = \frac{4\alpha}{\pi} \left( 1 - x^2 \right) \int_0^\infty \frac{dP P^2}{\mathcal{E}(e^{(\mathcal{E} - \tilde{\mu}_p)/T} + 1)} \left( 1 + \frac{1 - x^2}{v^2} - \frac{x}{v} \ln \left| \frac{x + v}{x - v} \right| \right),
\end{equation}

\begin{equation}
I^p_\ell = -2 I_t^{(p)} + 2\alpha m_p^2 x \left[ \exp \left( \frac{m_p}{T \sqrt{1 - x^2}} - \frac{\tilde{\mu}_p}{T} \right) + 1 \right]^{-1},
\end{equation}

where $m_p$ is the effective proton mass in plasma [30] (in numerical calculations we take $m_p \simeq 700$ MeV, corresponding to the nuclear density $3 \times 10^{14}$ g cm$^{-3}$).

The proton chemical potential $\tilde{\mu}_p$ is defined from the equation

\begin{equation}
N_p \simeq N_e \simeq \frac{\tilde{\mu}_p^3}{3 \pi^2} = \frac{1}{\pi^2} \int_0^\infty \frac{dP P^2}{e^{(\mathcal{E} - \tilde{\mu}_p)/T} + 1}.
\end{equation}

As the analysis of equation (A.14) shows, the difference $\tilde{\mu}_p - m_p$ appears to be of positive sign at the temperatures $T \simeq 30$–$60$ MeV, and of the same order of magnitude as the temperature. Thus, in the supernova core conditions both the approximations of the degenerate Fermi gas and of the classical Boltzmann gas should be, in general, hardly applicable for protons. However, we have verified by direct calculation that the observables computed in section 3 such as the luminosity (25) appear to be rather stable with respect to the choice of the approximation for the proton distribution function.

In figures A.1, A.2, A.3, and A.4 we present for the sake of illustration the electron and proton contributions to the eigenvalues $\Pi_{\ell,t}$ for the longitudinal and transverse plasmon. The importance of taking into account the proton contribution is evident.

Together with electrons and protons, in general, a small fraction $Y_{i}$ of the free ions could also present in plasma. This fraction can be considered with a good accuracy as the
Figure A.2. Electron contribution (dotted line) and proton contribution (dashed line) at $T = 30$ MeV to the imaginary part of $\Pi_\ell$.

classical Boltzmann gas. The real and imaginary parts of the corresponding polarization functions have the form

$$R_i^\ell = 4\pi \alpha \frac{Z_i^2 n_i}{T} \left[ 1 - \phi \left( \frac{x}{x_0} \right) \right],$$

$$I_i^\ell = 8\pi^{3/2} \alpha Z_i^2 n_i \frac{1}{x_0 q} \sinh \frac{q_0}{2T} \exp \left( \frac{q^2}{8m_i T} \right) \exp \left( -\frac{x_0^2}{x_0^2} \right), \quad (A.15)$$

where $x_0 = \sqrt{2T/m_i}$, and a function is introduced:

$$\phi(y) = \frac{2}{\sqrt{\pi}} |y|^3 \int_0^\infty u \ln \left| \frac{1+u}{1-u} \right| e^{-y^2u^2} du. \quad (A.16)$$

As is seen from equation (A.15), the function $I_i^\ell$ differs from zero only in the narrow area of the variable $x = q_0/q$, namely, $x \lesssim x_0 \sim \sqrt{T/m_i} \ll 1$.

The functions $R_i^\ell$ and $I_i^\ell$ for the transverse plasmon are of the order $\alpha Z_i^2 n_i/m_i$ and thus are suppressed by the large mass of the ion in the denominator. Thus, only the contribution of the neutrino scattering off free ions via the longitudinal plasmon ($\lambda = \ell$) is essential.

The ion contribution (A.15) comes with the factor $Z_i^2 Y_i$, and it is negligibly small in the supernova core conditions, because of the smallness of $Y_i$. However, it could be essential in the upper layers of the supernova envelope, which are believed to be rich in iron.
Appendix B. Integration over the initial and final plasma particles

Here we present the detailed calculation of the tensor integral (18):

\[
T^{\alpha\beta} = \frac{e^2}{32 \pi^2} \sum_k \sum_{s,s'} \int J_\alpha^{(k)} J_\beta^{* (k)} \, d\Phi,  
\]
\[
d\Phi = \frac{d^3P \, d^3P'}{E \, E'} \, f_k(P) \left[ 1 \mp f_k(P') \right] \delta^{(4)}(P' - P - Q).  
\]

To use the covariant properties of the tensor \( T^{\alpha\beta} \), one should write the distribution functions \( f_k(P) \) in the arbitrary frame

\[
f_k(P) = \left[ \exp \left( \frac{\langle Pu \rangle - \bar{\mu}}{T} \pm 1 \right) \right]^{-1},  
\]

where \( u_\alpha \) is the 4-vector of the plasma velocity. This vector and the 4-vector \( Q_\alpha \) are the building bricks for constructing the tensor \( T^{\alpha\beta} \). This tensor is symmetric because the electromagnetic current \( J_\alpha^{(k)} \) is real. The tensor is also orthogonal to the 4-vector \( Q_\alpha \) because of the electromagnetic current conservation. There exist only two independent structures having these properties, which are the density matrices (10) and (11), and thus one can write

\[
T^{\alpha\beta} = \mathcal{A}^{(t)} \rho_{\alpha\beta}(t) + \mathcal{A}^{(\ell)} \rho_{\alpha\beta}(\ell).  
\]
Because of orthogonality of the tensors $\rho_{\alpha\beta}(t)$ and $\rho_{\alpha\beta}(\ell)$, see equation (13), one obtains

\[ A(t) = \frac{1}{2} T^{\alpha\beta} \rho_{\alpha\beta}(t) = \frac{e^2}{64 \pi^2} \rho_{\alpha\beta}(t) \sum_k \sum_{s,s'} \int J^s_{\alpha(k)} J^s_{\beta(k)} d\Phi; \quad (B.4) \]

\[ A(\ell) = T^{\alpha\beta} \rho_{\alpha\beta}(\ell) = \frac{e^2}{32 \pi^2} \rho_{\alpha\beta}(\ell) \sum_k \sum_{s,s'} \int J^s_{\alpha(k)} J^s_{\beta(k)} d\Phi. \quad (B.5) \]

As we show below, just these integrals (B.4) and (B.5) define the widths of absorption (at $q_0 > 0$) and creation (at $q_0 < 0$) of a plasmon by the plasma particles. Really, let us consider for definiteness the width of absorption of the transverse plasmon by plasma particles forming the electromagnetic current $J^3_{(k)}$. The amplitude of the process has the form

\[ M^{(k)}(t) = -e \varepsilon_\alpha(t) J^\alpha_{\beta(k)}; \quad (B.6) \]

where $\varepsilon_\alpha(t)$ is the unit polarization 4-vector. Performing standard calculations, one obtains for the width of the plasmon absorption by all the components of plasma

\[ \Gamma_{\text{abs}}^{(\ell)} = \frac{1}{32 \pi^2 q_0} \frac{1}{2} \sum_\tau \sum_k \sum_{s,s'} \int |M^{(k)}(t)|^2 d\Phi; \quad (B.7) \]

where the summation is made both over the $k$th types of the plasma particles and over the polarizations of all particles participating in the process, $\tau$ for a plasmon and $s, s'$ for plasma particles.
Substituting the amplitude (B.6) into (B.7),
\[ \Gamma_{\text{abs}}(t) = \frac{e^2}{64 \pi^2 q_0} \rho_{\alpha\beta}(t) \sum_k \sum_{s,s'} \int J^\alpha_{(k)} J^{\beta*}_{(k)} \, d\Phi, \]  
\( \text{(B.8)} \)

where \( \rho_{\alpha\beta}(t) = \sum_{\tau=1}^{2} \varepsilon_{\tau}^\alpha(t) \varepsilon_{\tau}^\beta(t) \), and comparing it with equation (B.4), one can find the value
\[ A(t) = q_0 \Gamma_{\text{abs}}(t). \]  
\( \text{(B.9)} \)

Using the known relation [31] between the width of absorption of the transverse plasmon and the imaginary part \( I_t \) of the eigenvalue \( \Pi_t \) of the photon polarization tensor \( \Pi_{\alpha\beta} \),
\[ I_t(q_0) = -q_0 \left( 1 - e^{-q_0/T} \right) \Gamma_{\text{abs}}(t), \]  
\( \text{(B.10)} \)

we express the value \( A(t) \) in terms of \( I_t \):
\[ A(t) = -\frac{I_t}{1 - e^{-q_0/T}} = -I_t \left[ 1 + f_\gamma(q_0) \right], \]  
\( \text{(B.11)} \)

where \( f_\gamma(q_0) = \left( e^{q_0/T} - 1 \right)^{-1} \) is the Bose–Einstein distribution function for a photon. This relation obtained in the case \( q_0 > 0 \) is also correct for the case \( q_0 < 0 \), which corresponds to the transverse plasmon creation with the energy \( \omega = -q_0 > 0 \). The connection should be used here between the imaginary part \( I_t \) and the width of creation of the transverse plasmon:
\[ I_t(\omega) = -\omega \left( e^{\omega/T} - 1 \right) \Gamma_{\text{cr}}(t). \]  
\( \text{(B.12)} \)

It is essential also that the function \( I_t \) is odd:
\[ I_t(-q_0) = -I_t(q_0), \]  
\( \text{(B.13)} \)

and this is the feature of the retarded polarization operator.

Performing similar calculations, one can see that the relation (B.11) is valid for the longitudinal plasmon also. It is necessary to remember that \( \rho_{\alpha\beta}(\ell) = -\varepsilon_{\alpha}(\ell) \varepsilon_{\beta}(\ell) \), and
\[ I_t(q_0) = q_0 \left( 1 - e^{-q_0/T} \right) \Gamma_{\text{abs}}(t). \]  
\( \text{(B.14)} \)

Finally, we obtain the tensor \( T^{\alpha\beta} \) in the form
\[ T^{\alpha\beta} = \left[ -I_t \rho^{\alpha\beta}(t) - I_t \rho^{\alpha\beta}(\ell) \right] \left[ 1 + f_\gamma(q_0) \right]. \]  
\( \text{(B.15)} \)

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