Evaporation and condensation of rarefied gas between two parallel condensed phases

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Abstract. Evaporation and condensation processes between two parallel condensed phases are simulated using the kinetic models (S-model and BGK model). The compressible Navier-Stokes equations subjected to the temperature and pressure jump conditions on the liquid-vapor interphase are also solved numerically. The results obtained by the different models are compared. It was found that the Navier-Stokes model subjected to the jumps conditions provide the results close to the kinetic simulations results.

1. Introduction
The evaporation and condensation phenomena are interesting not only from the fundamental point of view [1], these phenomena are also largely present in various industrial applications. The thin layer, Knudsen layer is formed on the liquid-vapour interface. In this layer a gas is in non-equilibrium state even at atmospheric conditions. This why the correct and detailed description of physical processes of evaporation and condensation requires the application of the kinetic theory based approaches [2], [3], [4]. However, the implementation of the kinetic methodology for the practical problems is not so easy. This is why it is interesting to use the classical Navier-Stokes equations with the temperature and pressure jumps boundary conditions [5], [6] to simulate the evaporation-condensation problems.

The main objective of this study is the development of the mathematical model to simulate the behavior of the vapor phase at the liquid-gas interface. First the S-model [7] and BGK kinetic equations are used to simulate the evaporation-condensation phenomena appearing between two parallel condensed phases maintained at different temperatures. Then, these kinetic results are compared with that obtained from the Navier-Stokes equations subjected to the special temperature and pressure jumps boundary conditions [5]. These conditions were derived to take into account the non-equilibrium behaviors of a gas during the evaporation and condensation on the vapor-liquid interface. The comparison between two solutions allows us to determine the domain of the applicability of the Navier-Stokes equations with the jump conditions.

2. The problem statement
We consider two parallel plane gas - liquid interfaces at rest, maintained at temperature \( T_1 \) and \( T_2 \), \( T_2 > T_1 \) at the bottom \( (y = 0) \) and top \( (y = H) \) interfaces, respectively, see Fig. 1. We investigate the behavior of the steady gas flows caused by evaporation and condensation on the condensed phases...
on the basis of kinetic theory. Let \( p_1 \) be the saturation gas pressure at temperature \( T_1 \) and \( p_2 \) corresponds to temperature \( T_2 \). The problem is characterized by the parameters \( p_2/p_1 \) and \( T_2/T_1 \) and by the rarefaction parameter \( \delta \). The saturation gas pressure is a function of the saturation gas temperature given by the Clausius-Clapeyron equation [8].

\[ \frac{\partial f}{\partial t} + v \frac{\partial f}{\partial r} = Q(f, f), \quad (1) \]

where \( f(t, r, v) \) is the velocity distribution function of gas molecules, \( v \) is the molecular velocity vector, \( r \) is the position vector, \( t \) is a time. In the frame of S-model kinetic equation the collision term \( Q(f, f) \) has the form

\[ Q(f, f) = \nu (f^S - f), \quad (2) \]

where \( \nu \) is the collision frequency, \( f^S \) is an equilibrium distribution function defined by

\[ f^S(r, v, t) = f^M \left[ 1 + \frac{2m(v-u)q}{15n(r)(kT(r))^2} \left( \frac{m(v-u)^2}{2kT(r)} - \frac{5}{2} \right) \right], \quad (3) \]

here \( T(r) \) is a gas temperature, \( n(r) \) is a gas number density, \( u \) is a bulk velocity, \( q \) is a heat flux, \( m \) is a mass of gas molecule, \( k \) is the Boltzmann constant.

The boundary conditions for the plane vapor-liquid interface can be written as

\[ f^i = (\alpha n_i + (1-\alpha)n_{wi}) f^i_{wi}, \quad i = 1, 2, \quad (4) \]

\[ f^i_{wi}(v) = \left( \frac{m}{2\pi kT_i} \right)^{3/2} \exp\left( -\frac{mv^2}{2kT_i} \right), \quad (5) \]
is the number density of the saturated vapor near the plane interface, it can be calculated from the equation of state using the values of the saturated pressure at the saturated temperature as 

\[ n_i = \frac{p_i}{(kT_i)}. \]

\( \alpha \) is the evaporation and condensation coefficient, here we supposed that both coefficients are equal. The number density \( n_{wi} (i = 1,2) \) can be found from the relation:

\[ n_{wi} \sqrt{\frac{kT_i}{2\pi m}} = \int_{v=0}^{v_i} f_{v}dv, \quad i = 1,2. \] (6)

If the function \( f \) is known the macroscopic parameters can be calculated:

\[ n(r,t) = \int f(r,v,t)dv, \] (7)

\[ T(r,t) = \frac{m}{3kn} \int f(r,v,t)v^2dv, \] (8)

\[ q(r,t) = \frac{m}{2} \int f(r,v,t)V^2Vdv, \] (9)

here \( V = v - u \). The mass flow rate can be calculated as

\[ J_N = \int v_yf(r,v,t)dv. \] (10)

The distance \( H \) between the two interfaces is taken as the characteristic dimension of the problem. The Knudsen number and the rarefaction parameter are defined as following:

\[ Kn = \frac{\ell}{H}, \quad \delta = \frac{1}{Kn}, \quad \ell = \frac{H\sqrt{2RT_i}}{p_i}, \] (11)

here \( \ell \) is the equivalent mean free path, \( \mu_i \) is the dynamic viscosity of the vapor phase and \( \sqrt{2RT_i} \) is the most probable molecular speed, both at the temperature \( T_i \), \( R \) is the specific gas constant.

3. Results

The evaporation-condensation between two condensed planes is simulated by two kinetic models (BGK and S-model) and by using the compressible Navier-Stokes equation with the temperature and pressures jumps boundary conditions [2], [4] for several sets of the rarefaction parameter (Knudsen number), temperature and pressure ratios.

As an example the evaporation and condensation of Argon is considered between two parallel condensed phases. The temperature and pressure ratios are equal to \( T_2/T_1 = 1.036 \) and \( p_2/p_1 = 1.459 \), respectively. The temparture and pressure are interrelated through the Clausius-Clayperon relation [8]. The comparison between the results obtained with the kinetic models (S-model and BGK model) and the Navier-Stokes equations with the temperature and pressure jumps boundary conditions are provided on Figure 2. The rarefaction parameter \( \delta \) is equal to 100 which corresponds to the slip flow regime. Relatively good agreement is found between all three approaches.

Under several temperature and pressure ratios the inverse temperature gradient phenomenon [1] can be observed: the vapor temperature near the colder (condensed phase) is hotter than that near the hotter phase.

Further simulations are needed to find the conditions of the applicability of the Navier-Stokes equations with the jump boundary conditions.
4. Conclusion

The evaporation-condensation problem between two parallel condensed phases are simulated using the kinetic approach. The profiles of the macroscopic parameters such as temperature, pressure and number density of the vapor are obtained. The system of compressible Navier-Stokes equations completed by the energy conservation equation is solved also numerically with the pressure and temperature jumps boundary conditions. It was shown that the continuum modeling results agree relatively good with the kinetic solution when the rarefaction parameter is large enough.

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