Magnetic moments of heavy baryons in the Skyrme model

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Abstract

We calculate the magnetic moments of heavy baryons in the Skyrme model in the limit of infinite heavy quark mass. We show that the Skyrme model yields the same limit as the nonrelativistic quark model when heavy vector mesons are treated properly. The essential role of the magnetic moment coupling terms in the electromagnetic interactions of heavy mesons is discussed.

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The structure of hadrons containing a single heavy quark (or antiquark) becomes independent of the spin and flavor of the heavy quark as its mass \( m_Q \) becomes sufficiently larger than the typical scale of the strong interactions. \[1\] A consequence of such heavy quark symmetries can be found in the spectrum of heavy hadrons; that is, the hadrons come in degenerate doublets \[2\] with total spin \( j_\pm = j_\ell \pm \frac{1}{2} \) (unless \( j_\ell = 0 \)) with \( j_\ell \) being the total angular momentum of the light degrees of freedom. In the Skyrme model (SM), heavy baryons with a single heavy flavor are described by bound states of heavy mesons and a soliton of a chiral lagrangian. \[3\] Recently, it has been shown that such heavy quark symmetry can be consistently incorporated into this picture by introducing heavy vector meson fields. \[4–9\] There, the heavy vector mesons play an essential role in making the \( \Sigma^*_Q \) and \( \Sigma_Q \) states become degenerate in the infinite mass limit.

The magnetic moments also provide important informations on the baryon structure. Although there are no experimental data for the heavy baryon magnetic moments, naive predictions have been made in the phenomenological models such as nonrelativistic quark model (NRQM) \[10,11\], bag model \[12\] and Skyrme Model \[13,14\]. In NRQM, the magnetic moment of hadrons can be read off from their wave functions as a vector sum of the contributions from their constituent quarks. The resulting magnetic moments with arbitrary number of colors \((N_c = 2k + 1)\) are summarized in Table I for the heavy baryons of our concern. There, \( \mu_u \), \( \mu_d \) and \( \mu_Q \) denote the corresponding magnetic moment of the constituent quarks. Assuming that they are given by the charge-to-mass ratio, the magnetic moment of the heavy quark \( \mu_Q \) goes to 0 as \( m_Q \) goes to infinity.

In the Skyrme model, the bound state approach has been shown to work well in reproducing magnetic moments of the strange hyperons. \[13,15\] It is further applied to calculating the magnetic moments of charm baryons \[13\] and bottom baryons \[14\]. Surprisingly, the Skyrme model predictions are qualitatively very similar to those of NRQM. In SM, the magnetic moments of heavy baryons are expressed in terms of four calculable quantities, \( \mu^\text{sol}_s \), \( \mu^\text{sol}_v \), \( \mu^\text{hm}_s \) and \( \mu^\text{hm}_v \), as given in Table I. The first two quantities come from the soliton configuration, while the rest two come from the bound heavy mesons and vanish when the heavy quark (thus the heavy mesons) becomes infinitely heavy. (See Ref. \[13\] for the details on these quantities.) That is, in their works, both \( \mu^\text{hm}_s \) and \( \mu^\text{hm}_v \) are of order of \( 1/m_Q \) while they are of order of \( 1/N_c \) counting. On the other hand, comparing the NRQM and SM results, one can easily find the relations between these quantities and the constituent quark magnetic moments as

\[
\begin{align*}
\mu^{\text{sol}}_s &= \mu_u + \mu_d, \\
\mu^{\text{sol}}_v &= \frac{2k+3}{4} (\mu_u - \mu_d), \\
\mu^\text{hm}_s &= 2\mu_Q, \\
\mu^\text{hm}_v &= -\frac{1}{4} (\mu_u - \mu_d).
\end{align*}
\]

From these relations, one can see that as far as the \( 1/N_c \) order counting is concerned, \( \mu^\text{hm}_s \) and \( \mu^\text{hm}_v \) of Refs. \[13,14\] are consistent with NRQM. But there seems to be a discrepancy in \( 1/m_Q \) order counting between the two models; NRQM predicts \( \mu^\text{hm}_v \) to be \( \mathcal{O}(N_c^0 m_Q^0) \) while it is \( \mathcal{O}(N_c^0 m_Q^{-1}) \) in SM. (\( \mu^\text{hm}_s \) is consistently of \( 1/m_Q \) order in both models.) However, in Refs. \[13,14\], the calculations have been done in the model \[3\] where the heavy vector meson fields are integrated out in favor of the heavy pseudoscalar meson field, which breaks the heavy quark symmetry seriously. So it will be interesting to see whether the above mentioned discrepancy is an artifact of such an approximation. In this paper, we show that the SM
and the NRQM have the same infinite heavy quark mass limit of the heavy baryon magnetic moments when the heavy vector mesons are treated properly.

We will work with the effective lagrangian constructed by Wise \cite{16} for the heavy mesons interacting with Goldstone bosons, which reads

\[
\mathcal{L} = \mathcal{L}_{\Sigma}^{(0)} - iv^\mu \text{Tr}(\mathcal{D}_\mu^{(0)} H \tilde{H}) - g \text{Tr}(H \gamma_5 \gamma^\mu A_\mu^{(0)} \tilde{H}).
\]  

(2)

Here, \( \mathcal{L}_{\Sigma}^{(0)} \) denotes the chiral lagrangian for the Goldstone boson fields \( \Sigma = \exp (i \tau \cdot \pi / f_\pi) \):

\[
\mathcal{L}_{\Sigma}^{(0)} = \frac{f_\pi^2}{4} \text{Tr}(\partial^\mu \Sigma \partial^\mu \Sigma) + \cdots,
\]

(3)

where \( f_\pi \) is the pion decay constant and higher derivative terms are abbreviated by the ellipses. The pseudoscalar \((j^\pi = 0^-)\) and vector \((1^-)\) heavy mesons fields, \( P \) and \( P^* \), are combined into a \( 4 \times 4 \) matrix field \( H(x) \),

\[
H(x) = \frac{1 + v^\mu \gamma^\mu}{2} (P \gamma_5 - P^* \gamma^\mu),
\]

(4)

where the subscript \( v \) of the field operator denotes that they are the fields moving with a four-velocity \( v^\mu \). The field \( H(x) \) has an antidoublet structure in the isospin space, and transforms as

\[
H \xrightarrow{\chi} H h^\dagger,
\]

(5)

under the chiral transformation, while

\[
\Sigma \xrightarrow{\chi} L \Sigma R^\dagger, \quad \xi \equiv \sqrt{\Sigma} \xrightarrow{\chi} L \xi h^\dagger = h R \xi h^\dagger,
\]

(6)

with \( L \in SU_L(2), R \in SU_R(2) \) and \( h \) being an \( SU(2) \) matrix depending on \( L, R \) and \( \xi \). The chiral covariant derivative \( \mathcal{D}_\mu^{(0)} \) acting on \( H \) is defined as

\[
\mathcal{D}_\mu^{(0)} H = \partial_\mu H + H V_\mu^{(0)}\dagger.
\]

(7)

The vector and axial vector fields, \( V_\mu^{(0)} \) and \( A_\mu^{(0)} \), are defined in terms of \( \xi \) as

\[
V_\mu^{(0)} = \frac{i}{2} (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger),
\]

(8)

\[
A_\mu^{(0)} = \frac{i}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger).
\]

The superscript ‘(0)’ is adopted to distinguish the objects from those after electromagnetic couplings. Finally, \( g \) is a universal constant for the heavy meson couplings to the pions.

Referring to the details in Ref. \cite{3}, we first briefly describe the bound state approach. With a suitable stabilizing term, the nonlinear lagrangian \( \mathcal{L}_{\Sigma}^{(0)} \) supports a classical soliton solution in the form of \( \Sigma_0(r) = \exp [i \tau \cdot \hat{r} F(r)] \) with \( F(r) \) satisfying the boundary conditions \( F(0) = \pi \) and \( F(r) \xrightarrow{r \to \infty} 0 \). It provides static potentials to the heavy mesons so that they form a bound object which carries a baryon number due to the soliton configuration and a heavy flavor coming from the bound heavy mesons. In \( m_Q \to \infty \) limit, the heavy mesons just sit at
the center of the soliton and the binding energy of this system can be naively estimated as $\frac{3}{2}gF'(0)$ with $F'(0)$ being the slope of $F(r)$ at the center.

Because of the hedgehog configuration the isospin ($I_h$) and the angular momentum ($L$) of the heavy mesons become correlated, while the heavy quark spin ($S_Q$) decouples as a consequence of the heavy quark symmetry. Thus, the soliton–heavy-meson bound states come out as eigenstates of the ‘light quark grand spin’ $[K\ell \equiv (S - S_Q) + L + I_h$ with the heavy meson spin $S]$, heavy quark spin and parity. For example, the wave function of the eigenmode with $k_\ell = 0$, $s_Q = \pm \frac{1}{2}$ which is the lowest bound state is obtained as

$$H_{0,0,\pm\frac{1}{2}}(r) = f(r)\frac{1 + \gamma_0}{8\sqrt{2\pi}\phi_\pm[\gamma_5 - \gamma \cdot \tau]}(r \cdot r),$$

(9)

where $f(r)$ is a radial function that is strongly peaked at the origin and $\phi_\pm$ is the isospin basis for the antidoublet structure of the heavy mesons. (See Refs. [5,9] for details.) The heavy meson field can be expanded in terms of these eigenmodes as

$$H(x) = \sum_n H_n(r)e^{-i\varepsilon_nt}a_n,$$

(10)

with the heavy meson annihilation operator $a_n$ ($n = k_\ell, k_{\ell,3}, s_Q$). We will denote a single-particle Fock state as $|n\rangle = a_n^\dagger|\text{vac}\rangle$, where the heavy mesons occupy the corresponding bound state of the specified quantum numbers.

The quantization can be done by introducing collective coordinates to the zero modes associated with the invariance under simultaneous isospin rotation of the soliton field together with the heavy meson field:

$$\xi(r, t) = C(t)\xi_0(r)C^\dagger(t),$$

$$H(r, t) = H_{bf}(r, t)C^\dagger(t).$$

(11)

Here, $H_{bf}$ represents the heavy meson field in the rotating frame. Assuming sufficiently slow collective rotation, we can expand it in terms of the unchanged classical eigenmodes. In this collective coordinate quantization scheme, the isospin of the heavy meson field is transmuted into the part of the spin; the isospin operator $I$ and the spin operator $J$ of the soliton–heavy-meson bound system are given by

$$I_a = D_{ab}(C)R_b,$$

$$J = R + K_{bf}.$$

(12)

Here, $R$ is the ‘rotor-spin’ operator associated with the collective rotation, $D_{ab}(C)[\equiv \frac{1}{2}\text{Tr}(\tau_aC\tau_bC^\dagger)]$ is the $SU(2)$ adjoint representation associated with the collective variables and $K_{bf}$ is the grand spin operator of the heavy meson fields in the isospin co-moving system. The wave functions of the rotor spin states are the Wigner $D$-functions, $\sqrt{2i + 1}D^{(i)}_{m_1 m_2}(C)$ which satisfy

$$R^2D^{(i)}_{m_1 m_2}(C) = i(i + 1)D^{(i)}_{m_1 m_2}(C),$$

$$R_3D^{(i)}_{m_1 m_2}(C) = -m_2D^{(i)}_{m_1 m_2}(C),$$

$$I_3D^{(i)}_{m_1 m_2}(C) = m_1D^{(i)}_{m_1 m_2}(C).$$

(13)
Since the heavy quark spin decouples, it is convenient to classify the heavy baryons by the spin of the light degrees of freedom. The corresponding operator $J_\ell$ is given by

$$J_\ell = J - S_Q = R + K_\ell. \quad (14)$$

The heavy baryon states with quantum numbers $j_\ell, j_{\ell,3}$ (spin of light degrees of freedom) and $i, i_3$ (isospin) are obtained by linear combinations of direct products of the rotor-spin states $|i; m_1, m_2\rangle$ and the single-particle Fock state $|k_{\ell}, k_{\ell,3}, s_Q\rangle$. For example, $j_\ell=0$ and $j_\ell=1$ states are obtained by combining $i=0$ and 1 rotor-spin states to $k_{\ell}=0$ heavy meson bound state, respectively:

$$|j_\ell=0, 0, s_Q; i=0, 0\rangle = D_{0,0}^{(0)}|k_{\ell}=0, 0; s_Q\rangle,$$

$$|j_\ell=1, m_\ell, s_Q; i=1, i_3\rangle = \sqrt{3}D_{i_3, m_\ell}^{(1)}|k_{\ell}=0, 0; s_Q\rangle. \quad (15)$$

Conventional $\Lambda_Q$, $\Sigma_Q$ and $\Sigma_Q^*$ states are obtained by combining the heavy quark spin and $j_\ell$.

The magnetic moments of the heavy baryons can be obtained by taking the expectation value of the corresponding operator

$$\mu = \frac{1}{2}\int d^3r \, r \times j^{em}, \quad (16)$$

with respect to the states given in Eq. (15). Here, $j^{em}$ is the electromagnetic current, which can be derived by gauging the electromagnetic $U(1)$ symmetry of the lagrangian. Under the electromagnetic $U(1)$ transformation, the chiral field and the heavy meson field transform as

$$\Sigma \xrightarrow{U(1)} e^{iQ\lambda} \Sigma e^{-iQ\lambda},$$

$$H \xrightarrow{U(1)} e^{iQ\lambda} H e^{-iQ\lambda}, \quad (17)$$

and similar equation for $\xi$. Here, $Q$ is the charge matrix associated with the light quark doublet, $Q = \text{diag}(\frac{2}{3}, -\frac{1}{3})$, and $Q'$ is the charge of the heavy quark $Q$ ($Q'(Q = c) = \frac{2}{3}$, $Q'(Q = b) = -\frac{1}{3}$). The minimal coupling of the electromagnetic field $\alpha_\mu$ can be achieved by replacing all the plain derivatives in the lagrangian (3) by the corresponding covariant derivatives:

$$\partial_\mu \Sigma \Rightarrow D_\mu \Sigma = \partial_\mu \Sigma - i\alpha_\mu [Q, \Sigma],$$

$$\mathcal{V}_\mu^{(0)} \Rightarrow V_\mu = \frac{1}{2}(\xi\xi' + \xi'\xi) = \mathcal{V}_\mu^{(0)} + i\alpha_\mu (Q - Q_V),$$

$$\mathcal{A}_\mu^{(0)} \Rightarrow A_\mu = \frac{1}{2}(\xi\xi' + \xi'\xi) = \mathcal{A}_\mu^{(0)} + i\alpha_\mu Q_A,$$

$$D_\mu^{(0)} H \Rightarrow D_\mu H = \partial_\mu H + HV_\mu - i\alpha_\mu (Q' H - H Q)$$

$$= D_\mu^{(0)} H - i\alpha_\mu (Q' H - H Q_V), \quad (18)$$

where $Q_V = \frac{1}{2}(\xi\xi' + \xi'\xi)$ and $Q_A = \frac{1}{2}(\xi\xi' - \xi'\xi)$. Note that the $SU(2)$ flavor symmetry is broken by the electromagnetic interactions. However, the charge operator $Q$ has an equal mixture of $3_L$ and $3_R$, since the electromagnetic interactions conserve parity.
However, such a minimal coupling cannot incorporate the radiative transition like $P^* \rightarrow P \gamma$, because $V_\mu (A_\mu)$ contains only an even (odd) number of pions interacting electro-magnetically; that is, the kinetic term of Eq. (2) gives rise to contact terms with one photon and even-number pion emissions, while the interaction term yields those with odd-number of pions. The lowest order interaction term that contributes to $P^* \rightarrow P \gamma$ is \[ L_{\text{mag}} = \frac{\kappa}{2} F_{\mu \nu} \text{Tr}(H \sigma^{\mu \nu} Q_V \bar{H}), \] (19)

where $F_{\mu \nu} = \partial_\mu \alpha_\nu - \partial_\nu \alpha_\mu$ and $\sigma_{\mu \nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$. It comes from the “anomalous” magnetic moment of the heavy vector mesons due to their internal structure. If the heavy mesons are so strongly bound that they have zero bound-state radius, the magnetic moment of the heavy vector mesons will take the canonical value $e/m_\rho$ \[ \text{[20]} \] and would vanish at the infinite heavy meson mass limit. On the other hand, the NRQM provides a simple prediction for $\kappa$ \[ \text{[17]}: \]

\[ \kappa = \frac{e}{2m_u} = \frac{3}{2} \mu_u, \] (20)

which is in the range of the fitted value given in Ref. \[ \text{[18]} \].

Now, the electromagnetic current can be directly read off from the lagrangian (2) and (19) as

\[
j_{\text{em}}^\mu = \frac{1}{48\pi^2} \varepsilon^{\mu \lambda \rho \sigma} \text{Tr}(\Sigma^\dagger \partial_\sigma \Sigma^\dagger \partial_\lambda \Sigma + \Sigma^\dagger \partial_\rho \Sigma) + i e \frac{f}{2} \text{Tr}(\Sigma^\dagger Q \partial^\mu \Sigma + \Sigma Q \partial^\mu \Sigma^\dagger) + \cdots \\
- e Q' v^\mu \text{Tr}(H \bar{H}) + ev^{\mu \nu} \text{Tr}(H Q_V \bar{H}) - eg \text{Tr}(H \gamma_5 \gamma^\mu Q_A \bar{H}) + \kappa \partial_\nu [\text{Tr}(H \sigma^{\mu \nu} Q_V \bar{H})],
\] (21)

where we have included the “baryon number” current as the isoscalar component of the electromagnetic current coming from the soliton. The ellipsis denotes the contributions of higher derivative terms in the chiral lagrangian $L_{\Sigma}^{(0)}$. The magnetic moment operator can be obtained by substituting the space component of the electromagnetic current into Eq. (16) with the ‘rotating fields’ of Eq. (11). Since $v$ is of order of $1/m_Q$ and the bound heavy mesons are strongly peaked at the origin where $Q_A$ vanishes, the contribution of heavy mesons to the magnetic moments comes only from the last term. Finally, we are led to

\[ \kappa = e/(2m_u) = 3/2 \mu_u, \] (20)

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\[ \text{In Ref. [17], the authors work with the conventional pseudoscalar and vector fields to describe the heavy mesons with finite masses. One can easily obtain Eq. (19) by substituting} \]

\[
P = e^{-im_p v \cdot x} \frac{1}{\sqrt{m_p}} P_\mu, \\
P^* = e^{-im_p v \cdot x} \frac{1}{\sqrt{m_p^*}} P^*_{\mu \lambda},
\]

into their lagrangian $L_{\Sigma}^{(2)}$ and keeping the leading order terms in $1/m_P$. In Ref. \[ \text{[18]} \], the “bare” charge matrix $Q$ is adopted instead of the “dressed” one $Q_V$. Although both yield the same transition rates for the process $P^* \rightarrow P \gamma$ at tree level, using $Q_V$ seems to give the correct result in our work. There also can be a term $F_{\mu \nu} \text{Tr}(H \sigma^{\mu \nu} Q_V \bar{H})$ in $L_{\text{mag}}$, but it is suppressed by $1/m_Q$. \[ \text{[17,18]} \]
\[
\mu_3 = \mu_{s}^{\text{sol}} R_3 - 2\mu_{v}^{\text{sol}} D_{33}(C) - 2\kappa \int d^3 r \text{ Tr}[H_{bf}(-\frac{1}{2}\sigma_3)C^\dagger Q_v C \bar{H}_{bf}].
\] (22)

(See, e.g., Ref. [13] for explicit forms of \(\mu_{s,v}^{\text{sol}}\).) When the expectation value of the last term with heavy meson field operators is taken with respect to the single-particle Fock state \(|k_\ell = 0, 0, s_Q\rangle\) of Eq. (15), the magnetic moment operator is reduced to a form of

\[
\mu_3 = \mu_{s}^{\text{sol}} R_3 - 2(\mu_{v}^{\text{sol}} + \mu_{v}^{\text{hm}}) D_{33}(C),
\] (23)

where

\[
\mu_{v}^{\text{hm}} = -\frac{1}{4}\kappa,
\] (24)

which acts on the rotor-spin states in Eq. (15). In this formula, it can be seen that the heavy mesons do not contribute to the isoscalar part of the magnetic moment so that \(\mu_{v}^{\text{hm}}\) is \(\mathcal{O}(N_c m_Q^{-1})\), which is consistent with the NRQM. The expectation value of \(R_3\) and \(D_{33}(C)\) with respect to the rotor spin states \(D^{(1)}_{m_1 m_2}(C)\) are \(-m_2\) and \(m_1 m_2/2\), respectively. The magnetic moments of the conventional heavy baryons such as \(\Sigma_Q\) and \(\Sigma_Q^*\) can be obtained by multiplying a factor that appears in combining the heavy quark spin to \(j_\ell\), which yields exactly the same results of Table I but with vanishing \(\mu_{v}^{\text{hm}}\) in the infinite heavy quark mass limit. Therefore, \(\mathcal{L}_{\text{mag}}\) of Eq. (19) gives a contribution of \(\mathcal{O}(N_c^0 m_Q^0)\) that was missing in the earlier calculations [13,14]. Note also that, with the NRQM estimation of \(\kappa\) given by Eq. (20), we can reproduce the last relation of Eq. (1). When the two parameters \(\mu_{s}^{\text{sol}}\) and \(\mu_{v}^{\text{sol}}\) are adjusted to fit the nucleon magnetic moments, both models predict on the heavy baryon magnetic moments as

\[
\mu(\Lambda_Q) = 0, \quad \mu(\Sigma_Q^0) = \frac{2}{9}\mu_p, \quad \mu(\Sigma_Q^{+1}) = \frac{8}{9}\mu_p, \quad \mu(\Sigma_Q^{-1}) = -\frac{4}{9}\mu_p,
\] (25)

in the infinite heavy quark mass limit. \([\mu(\Sigma_Q^*)'s\) are given by the relation \(\mu(\Sigma_Q^*) = \frac{3}{2}\mu(\Sigma_Q)\).]

As a summary we have shown that the Skyrme model could yield the same heavy baryon magnetic moments as NRQM in the limit of the heavy quark mass going to infinity if heavy vector mesons are treated properly. This study tells us that the “anomalous” coupling term(s) in the Lagrangian for the electromagnetic transition of the heavy vector mesons can play a nontrivial role in the Skyrme model calculations on these physical quantities. Their contributions to the isovector part of the magnetic moments are of the next leading order in \(1/N_c\) counting but of the leading order in the \(1/m_Q\) expansion. Actually, in the literature [13,21], the Skyrme model estimations on the baryon magnetic moments have suffered from too small isovector part. The inclusion of the vector mesons [22] into the model, which has been expected to cure the problem, could not solve this problem completely although it improved the model predictions. So it will be interesting to see how much the incorporation

\[2\text{If one has used } Q \text{ in Eq. (19) instead of } Q_V, \text{ he would have obtained } \mu_{v}^{\text{hm}} = +\frac{12}{\kappa}.\]
of the “anomalous” coupling terms (e.g., $K^* K\gamma$ and $\rho \pi \gamma$ terms) into the magnetic moment calculations improves the results.

In this work, we have worked with finite $N_c$ but with infinite heavy quark mass. In Refs. [23,24], it was shown that the kinetic effects of the heavy mesons give nontrivial corrections in the mass spectrum. In order to have more realistic predictions on the magnetic moments of the heavy baryons with finite masses, one should take into account the finite mass corrections on these physical quantities. Such a work is in progress and the results will be presented in a further publication [25].

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REFERENCES

[1] M.B. Voloshin and M.A. Shifman, Sov. J. Nucl. Phys. 45 (1987) 292; Sov. J. Nucl. Phys. 47 (1988) 511;
   N. Isgur and M.B. Wise, Phys. Lett. B 232 (1989) 113; Phys. Lett. B 237 (1990) 527.
[2] N. Isgur and M.B. Wise, Phys. Rev. Lett. 66 (1991) 1130.
[3] C.G. Callan and I. Klebanov, Nucl. Phys. B 262 (1985) 365;
   M. Rho, D.O. Riska and N.N. Scoccola, Phys. Lett. B 251 (1990) 597; Z. Phys. A 341 (1992) 343.
[4] E. Jenkins, A.V. Manohar and M.B. Wise, Nucl. Phys. B 396 (1993) 27.
[5] Y. Oh, B.-Y. Park and D.-P. Min, Phys. Rev. D 50 (1994) 3350.
[6] E. Jenkins and A.V. Manohar, Phys. Lett. B 294 (1992) 273;
   Z. Guralnik, M. Luke and A.V. Manohar, Nucl. Phys. B 390 (1993) 474.
[7] M.A. Nowak, M. Rho and I. Zahed, Phys. Lett. B 303 (1993) 130;
   H.K. Lee and M. Rho, Phys. Rev. D 48 (1993) 2329;
   H.K. Lee, M.A. Nowak, M. Rho and I. Zahed, Ann. Phys. (N.Y.) 227 (1993) 175.
[8] K.S. Gupta, M.A. Momen, J. Schechter and A. Subbaraman, Phys. Rev. D 47 (1993) 4835;
   A. Momen, J. Schechter and A. Subbaraman, Phys. Rev. D 49 (1994) 5970.
[9] For a review, see, for example, D.-P. Min, Y. Oh, B.-Y. Park and M. Rho, Int. J. Mod. Phys. E 4 (1995) 47.
[10] D.B. Lichtenberg, Phys. Rev. D 15 (1977) 345.
[11] G. Karl and J.E. Paton, Phys. Rev. D 30 (1984) 238.
[12] S.K. Bose and L.P. Singh, Phys. Rev. D 22 (1980) 773.
[13] Y. Oh, D.-P. Min, M. Rho and N.N. Scoccola, Nucl. Phys. A 534 (1991) 493.
[14] M. Björnberg, K. Dannbom, D.O. Riska and N.N. Scoccola, Nucl. Phys. A 539 (1992) 662.
[15] J. Kunz and P.J. Mulders, Phys. Lett. B 231 (1989) 335; Phys. Rev. D 41 (1990) 1578;
   E.M. Nyman and D.O. Riska, Nucl. Phys. B 325 (1989) 593;
   D.-P. Min, Y.S. Koh, Y. Oh and H.K. Lee, Nucl. Phys. A 530 (1991) 698.
[16] M.B. Wise, Phys. Rev. D 45 (1992) 2188.
[17] H.-Y. Cheng et al., Phys. Rev. D 47 (1993) 1030.
[18] P. Cho and H. Georgi, Phys. Lett. B 296 (1992) 408; (E) 300 (1993) 410.
[19] J.F. Amundson et al., Phys. Lett. B 296 (1992) 415.
[20] S.J. Brodsky and J.R. Hiller, Phys. Rev. D 46 (1992) 2141.
[21] G.S. Adkins, C.R. Nappi and E. Witten, Nucl. Phys. B 228 (1983) 552;
   G.S. Adkins and C.R. Nappi, Phys. Lett. B 137 (1984) 251.
[22] U.-G. Meissner, Phys. Rep. 161 (1988) 213 and references therein.
[23] Y. Oh, B.-Y. Park and D.-P. Min, Phys. Rev. D 49 (1994) 4649;
   Y. Oh and B.-Y. Park, SNU preprint SNUTP-94/131, [hep-ph/9501350], Phys. Rev. D 51 (in press).
[24] J. Schechter and A. Subbaraman, Phys. Rev. D 51 (1995) 2311;
   J. Schechter, A. Subbaraman, S. Vaidya and H. Weigel, Syracuse preprint SU-4240-606,
   [hep-ph/9503307] (1995).
[25] Y. Oh, B.-Y. Park and D.-P. Min, work in progress.
TABLES

TABLE I. Magnetic moment of heavy baryons. The superscripts (0,±1) are the isospin projections, therefore, $\Sigma^+_Q$ means $\Sigma_b^+$ for $Q=b$ and $\Sigma^+_c$ for $Q=c$.

| Particle | NRQM ($N_c=2k+1$) | SM [13] |
|----------|------------------|---------|
| $p$      | $\frac{1}{2}[(k+3)\mu_u - k\mu_d]$ | $\frac{1}{2}\mu^s_{sol} + \frac{2}{3}\mu^v_{sol}$ |
| $n$      | $\frac{1}{2}[(k+3)\mu_d - k\mu_u]$ | $\frac{1}{2}\mu^s_{sol} - \frac{2}{3}\mu^v_{sol}$ |
| $\Lambda_Q$ | $\mu_Q$ | $\frac{1}{2}\mu^h_{hm}$ |
| $\Sigma^+_Q$ | $\frac{1}{2}[(k+3)\mu_u - (k-1)\mu_d] - \frac{1}{3}\mu_Q$ | $\frac{1}{6}(4\mu^s_{sol} - \mu^h_{hm}) + \frac{2}{3}(\mu^v_{sol} + \mu^h_{hm})$ |
| $\Sigma^-_Q$ | $\frac{1}{2}(\mu_u + \mu_d) - \frac{1}{3}\mu_Q$ | $\frac{1}{6}(4\mu^s_{sol} - \mu^h_{hm}) - \frac{2}{3}(\mu^v_{sol} + \mu^h_{hm})$ |
| $\Sigma^+_Q$ | $\frac{1}{2}[(k+3)\mu_d - (k-1)\mu_u] - \frac{1}{3}\mu_Q$ | $\mu^s_{sol} + \frac{1}{2}\mu^h_{hm} + (\mu^v_{sol} + \mu^h_{hm})$ |
| $\Sigma^0_Q$ | $\mu_u + \mu_d + \mu_Q$ | $\mu^s_{sol} + \frac{1}{2}\mu^h_{hm}$ |
| $\Sigma^-_Q$ | $\frac{1}{2}[(k+3)\mu_u - (k-1)\mu_d] + \mu_Q$ | $\mu^s_{sol} + \frac{1}{2}\mu^h_{hm} - (\mu^v_{sol} + \mu^h_{hm})$ |