ADIABATIC FOLLOWING CRITERION, ESTIMATION OF
THE NONADIABATIC EXCITATION FRACTION AND
QUANTUM JUMPS

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Abstract

An accurate theory describing adiabatic following of the dark, nonabsorbing
state in the three-level system is developed. An analytical solution for the
wave function of the particle experiencing Raman excitation is found as an
expansion in terms of the time varying nonadiabatic perturbation parameter.
The solution can be presented as a sum of adiabatic and nonadiabatic parts.
Both are estimated quantitatively. It is shown that the limiting value to which
the amplitude of the nonadiabatic part tends is equal to the Fourier compo-
nent of the nonadiabatic perturbation parameter taken at the Rabi frequency
of the Raman excitation. The time scale of the variation of both parts is
found. While the adiabatic part of the solution varies slowly and follows the
change of the nonadiabatic perturbation parameter, the nonadiabatic part
appears almost instantly, revealing a jumpwise transition between the dark
and bright states. This jump happens when the nonadiabatic perturbation
parameter takes its maximum value.

I. INTRODUCTION

Stimulated Raman adiabatic passage (STIRAP) resulting in the population transfer be-
tween the states which are not directly coupled [1], electromagnetically induced transparency
(EIT) via adiabatic following of the dark, non-absorbing state [2], nonresonant pulse exci-
tation of the two-level atom [3] are just a short list of phenomena in quantum optics where
adiabatic processes are considered. Generally, one can find in any part of physics problems
concerned with adiabaticity, which are treated almost similarly. Among them we can men-
tion multiphoton resonances induced in atoms and molecules by a strong low-frequency field
[4]-[7], wave-packet dynamics in physics and chemistry or, so-called, "femto-chemistry" [8]-
[10] and slow atomic and molecular collisions [11]-[22].

A fully adiabatic process takes place if the Hamiltonian of the problem is diagonal. Then
any time dependence of this diagonal Hamiltonian does not change the state of the quantum
system except its phase. In general, the Hamiltonian, describing the interacting systems, has
both diagonal and nondiagonal components. In most cases dealing with adiabatic excitation,
the problem can be reduced to the consideration of the two-level system perturbed by the diagonal and nondiagonal interactions. Then, both components of the Hamiltonian, i.e., the two-level splitting and the nondiagonal perturbation, are time dependent. One can find instantaneous eigenstates of such a Hamiltonian. These states are called adiabatic states if the system, being initially in one of them, follows this state. In contrast, the initial states, where the transition takes place, are called diabatic states.

The diagonalization of the instantaneous Hamiltonian takes into account the diabatic states coupling, incorporating the interaction parameter into the instantaneous eigenvalues. However, a new coupling appears again because of the time dependence of the transformation from the initial, diabatic basis to the adiabatic basis (see, for example, [10]). Inasmuch as the coupling between instantaneous eigenstates does not vanish, the following of the adiabatic states can never be perfect. Since it is almost impossible to get rid of the state coupling by the time dependent transformation of the basis functions, the choice of the adiabatic basis is aimed to minimize the transition probability between the states, almost approaching in this sense to the fully adiabatic Hamiltonian (i.e., the diagonal one) when the coupling can be neglected. The estimation of the fraction of the system that is transferred from one adiabatic state to another during the interaction (nonadiabatic correction) is a main problem in this approach. This nonadiabatic correction is a measure of the nonadiabaticity of the process.

It should be obvious that the searching for the adiabatic basis is an approximate approach. It was developed for the first time to treat the behavior of the two-level system subject to an excitation that couples the states if both the two-level splitting and the excitation are time dependent. Only a few models of such a two-level system have analytical solutions. Among them are the Rosen–Zener model with $H_{\text{diag}} = \text{const}$ and $H_{\text{nond}} = V_0 \sec h(rt)$, the Demkov–Kunike model with $H_{\text{diag}} = E + E_0 \tanh(rt)$ and $H_{\text{nond}} = V_0 \sec h(rt)$ or $H_{\text{nond}} = \text{const}$, the Landau–Zener model with $H_{\text{diag}} = \lambda t$ and $H_{\text{nond}} = \text{const}$, and the parabolic model with $H_{\text{diag}} = at^2 + b$ and $H_{\text{nond}} = \text{const}$. Although the last model has not an exact solution, it has been studied comprehensively (see, for example, Ref. [23]). Here, $H_{\text{diag}}$ is the amplitude of the diagonal component of the Hamiltonian (i.e., the two-level splitting) and $H_{\text{nond}}$ is the amplitude of the nondiagonal component (i.e., the coupling perturbation). The symbols $V_0$, $E$, $E_0$, $r$ can represent arbitrary constants and $t$ is time. One can find these models and their solutions in, for example, [10].

The advantage of these models comes from the possibility to reduce their Schrödinger equations to second order differential equations (with time dependent coefficients) whose solutions are known. For example, the Schrödinger equation for the Rosen–Zener model can be reduced to an equation whose solution is the hypergeometric function and the Schrödinger equation for the Landau-Zener model can be solved in terms of parabolic cylinder functions. For an arbitrary Hamiltonian, the quasi-classical approach based on the Wentzel-Kramers-Brillouin (WKB) approximation was developed by Dykhne [24]–[25]. In this method, the amplitude of the nonadiabatic transition is calculated under the assumption that the adiabatic levels cross in the complex time plane and the main contribution of the nonadiabatic coupling is given at the time of the level crossing. The latter is usually the branch point of the phase integral of the energies. The result becomes universal, i.e., independent of the nonadiabatic coupling model and it is sufficient to take the leading term of the coupling at the crossing point.

In this paper we consider adiabatic following of the dark, nonabsorbing state in the
three-level system excited by two resonant fields. The adiabatic following of the dark state results in STIRAP (population transfer). It was proposed in Ref. [26] where the so-called counterintuitive Raman pulse sequence emerged as a result of the search for the generalization of the Liouville equations for the $N$-level system using $SU(N)$ coherence vector theory [27]–[26]. This pulse sequence consists of two fields coupling two low energy levels 1 and 2 with one common excited state 3. If one of the levels (for example, 2) is initially empty, then applying first the field which couples this state with level 3, and then the field coupling the populated state (for example, 1) with 3, it is possible to transfer the population of state 1 to state 2, without appreciably populating the intermediate state 3.

Later, the importance of the adiabaticity in the counterintuitive pulse sequence development was realized in [29]–[31]. There is a particular superposition of states 1 and 2 that does not interact with the coupling fields and, if the development of the field amplitudes in time is properly chosen, this superposition state changes from state 1 to state 2. If the three-level atom follows this superposition state, the atom population is transferred from state 1 to state 2 by the pulse sequence without populating the intermediate excited state 3. This noncoupled superposition state was first introduced by Arimondo [32] to explain qualitatively the dark resonance as population trapping in this state [33]–[35]. The noncoupled state is often referred to as a dark state.

It is obvious that, if the dark state changes in time, a process must exist which tends to empty this state. The condition minimizing the dark state depopulation is formulated in [29]. This is done ad hoc, without an estimation of the nonadiabatic correction for the excited probability amplitude. However, numerical calculations show that, if this condition is satisfied, the adiabatic population transfer $1 \to 2$ is almost perfect. Some attempts were undertaken to find a rigorous justification of the intuitively found adiabaticity condition and to estimate the amplitude of the excited state during the STIRAP pulse sequence (see, for example, Refs. [36]–37]). In [36], the so-called ramp pulses were considered, which allow an exact solution. Furthermore, with the help of the method developed for the two-level system by Dykhne (see the discussion above and Refs. [38]–[39]), the nonadiabatic amplitude of the excited state 3 is estimated for Gaussian and hyperbolic secant pulses. This is possible because the Liouville equation for the two-level atom in terms of the $SU(2)$ coherence vector (Bloch-vector model, see, for example, [40]) coincides with the Schrödinger equation for the state probability amplitudes of the three-level atom excited by two resonant fields (see, for example, [31], [36], [2]). However, the authors of [36] admit that the analytical approximations for the nonadiabatic corrections ”have been introduced ad hoc without derivation” and they ”would really like to see more detailed investigations of the analytic behavior of the system discussed”.

Fleischhauer with coauthors [37] developed a different approach introducing higher-order trapping states. They define a $n$-th-order generalized adiabatic basis, which is similar to the superadiabatic basis introduced for the two-level system in [41]–[44]. By successive transformations from the diabatic basis to the adiabatic basis, then from this adiabatic basis, which can be considered as the adiabatic basis of the first order, to the next, i.e., the second order adiabatic basis, etc., the solution is presented as an infinite product of transformations. The general expression for the $n$-th transformation matrix is presented. However, it is hard to implement this scheme for an arbitrary pulse sequence.

In this paper, we develop a new method that allows us to calculate the adiabatic and nona-
The diabatic components of the solution with controllable accuracy. The adiabatic component describes the part of the atomic probability amplitude visiting the excited atomic state during the pulse train and coming back to the dark state when the pulses are gone. The atom evolution following the adiabatic component of the solution resembles the excitation–deexcitation process induced by a soliton in the two-level atom. The nonadiabatic component is that part which is lost from the dark state and describes the fraction of the atomic probability amplitude that is left excited after the pulse train. We compare our result with the previous theories reported in [36] and [37]. We found corrections to the theory presented in [36] and show that our result corresponds to the calculation of the infinite number of transformations proposed in [37]. As an example of the validity of the method, we present also the solution of the Rosen–Zener and Landau-Zener models by our method (see the Appendix and the end of Sections VI and VII).

The paper is organized as follows. In Sec. II we present the general formalism employed in the description of the three-level atom excited by two resonant fields. The transformation to the basis of the bright and dark states is derived. It is shown that the system evolves between two states, i.e., "bright" and "common" (they are specified in Section II). In Sec. III we consider the time evolution of the atomic state vector for the Λ-scheme of excitation if the field amplitudes are time dependent. Bloch-like equations are derived. In Sec. IV the adiabatic following approximation is presented. The adiabatic solution for the stimulated Raman adiabatic passage (STIRAP) is found in Sec. V. Nonadiabatic corrections are described in Sec. VI. The case when the Raman Rabi frequency changes in time is considered in Sec. VII-IX.

II. THREE-LEVEL ATOM INTERACTING WITH TWO RESONANT FIELDS.
GENERAL FORMALISM

We consider a three-level atom shown in Fig. 1 (a). The arrows indicate the transitions induced by the coherent fields. This excitation scheme is classified as the Λ scheme. We define the level that is common for both transitions as 3. The others are designated by the numbers 1 and 2, level 2 being of higher energy than 1 and initially not populated. The dynamic part of the Hamiltonian of this three-level atom, excited by two resonant fields $E_1(t) = E_1 \cos(\Omega_1 t + \varphi_1)$ and $E_2(t) = E_2 \cos(\Omega_2 t + \varphi_2)$, is

$$H = \sum_{n=1}^{3} \omega_n \hat{P}_{nn} - \left( B_1 \hat{P}_{13} e^{i\Omega_1 t + i\varphi_1} + B_2 \hat{P}_{23} e^{i\Omega_2 t + i\varphi_2} + h.c. \right),$$

where $\omega_n$ is the energy of the state $n$. Planck’s constant is set equal one ($\hbar = 1$) for simplicity. The operators $\hat{P}_{mn}$ are defined by $\hat{P}_{mn} = |m\rangle \langle n|$, where $\langle n|$ and $|m\rangle$ are bra and ket vectors of the states $n$ and $m$ in the Schrödinger representation. The interaction constant (Rabi frequency) $B_n = (\mathbf{d}_{n3} \cdot \mathbf{E}_n) / 2 = (\mathbf{d}_{3n} \cdot \mathbf{E}_n) / 2$ depends on the dipole-transition matrix element, taken real so, $\mathbf{d}_{n3} = \mathbf{d}_{3n}$, and on the field amplitude $E_n$. The rotating wave approximation is taken into account.

If the fields $E_1(t)$ and $E_2(t)$ are in exact resonance with the relevant transitions, the Hamiltonian, Eq. (1), can be made slowly varying by transforming it into the interaction
representation (IR). This representation is defined by a canonical transformation by means of the unitary operator

\[ T = \exp \left( i \sum_{n=1}^{3} \omega_n \hat{P}_{nn} t \right). \]  

(2)

The wave function of the atom in the IR is defined by \( |\Phi(t)\rangle = T |\Psi(t)\rangle \), where \( |\Psi(t)\rangle \) is the wave function in the Schrödinger representation. \( |\Phi(t)\rangle \) satisfies the Schrödinger equation with the effective Hamiltonian \([45]\)

\[ \mathcal{H} = THT^{-1} + i \dot{T} T^{-1}. \]  

(3)

This Hamiltonian has the explicit form

\[ \mathcal{H} = -B_1 \hat{P}_{13} e^{i\varphi_1} - B_2 \hat{P}_{23} e^{i\varphi_2} + \text{h.c.}, \]  

(4)

or in matrix notation

\[ \mathcal{H} = - \begin{pmatrix} 0 & 0 & B_1 e^{i\varphi_1} \\ 0 & 0 & B_2 e^{i\varphi_2} \\ B_1 e^{-i\varphi_1} & B_2 e^{-i\varphi_2} & 0 \end{pmatrix}. \]  

(5)

The \( \hat{P}_{MN} \)-operators are defined for the vectors \( |M\rangle \) and \( \langle N| \) of the interaction representation differing from the states \( |m\rangle \) and \( \langle n| \) by the phase factors \( \exp(-i\omega_m t) \) and \( \exp(i\omega_n t) \).

Assume that the field amplitudes \( B_1, B_2 \) and the phases \( \varphi_1, \varphi_2 \) are constant. Then the Hamiltonian, Eqs. (4)–(5), is diagonalized by a unitary transformation

\[ \overline{\mathcal{H}} = Q \mathcal{H} Q^{-1} = \sqrt{B_2^2 + B_2^2} \left( \hat{P}_{33} - \hat{P}_{11} \right), \]  

(6)

where

\[ Q = \frac{1}{\sqrt{2}} \begin{bmatrix} e^{-i\varphi_1} \sin \alpha & e^{-i\varphi_2} \cos \alpha & 1 \\ \sqrt{2} e^{i\varphi_2} \cos \alpha & -\sqrt{2} e^{i\varphi_1} \sin \alpha & 0 \\ e^{-i\varphi_1} \sin \alpha & e^{-i\varphi_2} \cos \alpha & -1 \end{bmatrix}. \]  

(7)

and \( \tan \alpha = B_1/B_2 \). The new ket vectors \( |\overline{1}\rangle \) are related to the former ones \( |N\rangle \), defined in the interaction representation, as follows \( |\overline{\Phi}\rangle = Q |\Phi\rangle \). The explicit relations are

\[ |\overline{1}\rangle = \frac{\sqrt{2}}{2} \left( e^{i\varphi_1} \sin \alpha |1\rangle + e^{i\varphi_2} \cos \alpha |2\rangle + |3\rangle \right), \]  

(8)

\[ |\overline{2}\rangle = e^{-i\varphi_2} \cos \alpha |1\rangle - e^{-i\varphi_1} \sin \alpha |2\rangle, \]  

(9)

\[ |\overline{3}\rangle = \frac{\sqrt{2}}{2} \left( e^{i\varphi_1} \sin \alpha |1\rangle + e^{i\varphi_2} \cos \alpha |2\rangle - |3\rangle \right). \]  

(10)
The basis $|\pi\rangle$, in which the Hamiltonian is diagonal, is called the basis of the quasi-energy states [46], [47]. It coincides with the basis of the dressed states if the limit of the classical field is taken for them.

States $|\pi\rangle$ and $|\pi\rangle$ and $|\pi\rangle$ are mixtures of all unperturbed states $|1\rangle$, $|2\rangle$ and $|3\rangle$, whereas state $|\pi\rangle$ does not contain the common state $|3\rangle$. If the atom is in state $|\pi\rangle$, a mixture of the ground state sublevels 1 and 2, the atom does not leave this state since it is an eigenstate of the interaction Hamiltonian, Eq. (6). The bichromatic field $E(t) = E_1(t) + E_2(t)$ does not interact with such an atom, and, consequently, it is not excited. Therefore, we call this state a dark state and designate it $|d\rangle$. This state was introduced for the first time by Arimondo in Ref. [32], who called this state a non-coupled state. He introduced the coupled state as well, which interacts with the bichromatic field $E(t)$. Following Arimondo we define the state

$$|b\rangle = e^{i\varphi_1} \sin \alpha \ |1\rangle + e^{i\varphi_2} \cos \alpha \ |2\rangle,$$

which is orthogonal to the dark state. We call this state a bright state, since for the $\Lambda$-scheme, if the atom is in this state, it is excited by the bichromatic field and then the luminescence from state $|3\rangle$ may follow. Excitation can take place because the bright state is not an eigenstate of the interaction Hamiltonian, Eq. (6).

The states $|d\rangle$, $|b\rangle$, and $|3\rangle$ are mutually orthogonal and can be chosen as a new basis. We call this basis $dbc$, designating the state $|3\rangle$ by the letter $|c\rangle$, since it is common for the induced transitions. Because we will often refer to these states, they are presented below in a common set of equations to simplify further citation.

$$|d\rangle = e^{-i\varphi_2} \cos \alpha \ |1\rangle - e^{-i\varphi_1} \sin \alpha \ |2\rangle,$$

$$|b\rangle = e^{i\varphi_1} \sin \alpha \ |1\rangle + e^{i\varphi_2} \cos \alpha \ |2\rangle,$$

$$|c\rangle = |3\rangle.$$

The interaction Hamiltonian, Eqs. (4)–(5), is transformed in this basis as follows

$$\mathcal{H}_{dbc} = S \mathcal{H} S^{-1} = -B \left( \hat{P}_{bc} + \hat{P}_{cb} \right),$$

or in a matrix form

$$\mathcal{H}_{dbc} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & B & 0 \end{bmatrix},$$

where $B = \sqrt{B_1^2 + B_2^2}$ and the unitary operator of the canonical transformation is

$$S = \begin{bmatrix} e^{i\varphi_2} \cos \alpha & -e^{i\varphi_1} \sin \alpha & 0 \\ e^{-i\varphi_1} \sin \alpha & e^{-i\varphi_2} \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

We call this basis the $dbc$-representation.
Figure 1 (b) shows the excitation scheme in the $dbc$-basis where only the $b$ and $c$ states are coupled. The $b$ and $c$ states do not correspond to a defined energy. To present schematically the $dbc$ states and the transitions between them, we choose the state population before the excitation for their relative position in the diagram. So, the vertical scale in Fig. 1 (b) represents the initial population of the states counted from the bottom to the top.

The Hamiltonian in the $dbc$-basis, Eq. (15), resembles the interaction Hamiltonian of the two-level system $bc$, excited by one resonant field with an effective interaction constant $B$. The dynamic evolution of the atom is described by the Schrödinger equation with this Hamiltonian containing only the transition operators between states $b$ and $c$. If, for example, initially the $d$ and $b$ states are populated and $c$ is not, 

$$|\Phi_{bc}(0)\rangle = C_d |d\rangle + C_b |b\rangle, \quad (18)$$

the dynamics of the three-level atom follows the very simple formula 

$$|\Phi_{bc}(t, \pm)\rangle = C_b \left[ \cos \left( \frac{\chi_R t}{2} \right) |b\rangle + i \sin \left( \frac{\chi_R t}{2} \right) |c\rangle \right] + C_d |d\rangle, \quad (19)$$

where $\chi_R = 2B$ is the Raman Rabi frequency. This Rabi frequency is defined as twice the interaction constant in the Hamiltonian, Eq. (15), according to the conventional definition adopted for the two-level system [40]. The initial condition specified in Eq. (18) has a simple relation to the IR basis

$$|\Phi_{bc}(0)\rangle = (C_b e^{i\phi_1} \sin \alpha + C_d e^{-i\phi_2} \cos \alpha) |1\rangle + (C_b e^{i\phi_2} \cos \alpha - C_d e^{-i\phi_1} \sin \alpha) |2\rangle. \quad (20)$$

If the initial condition is $|\Phi_{bc}(0)\rangle = |1\rangle$, then $C_b = e^{-i\phi_1} \sin \alpha$ and $C_d = e^{i\phi_2} \cos \alpha$, which means that the phase relation between the amplitudes of $b$ and $d$ states is fixed as $C_b = C_d e^{-i(\phi_1 - \phi_2)} \tan \alpha$.

Suppose that the pulse duration of the bichromatic field, $t_{2\pi}$, is chosen such that the effective two-level system $bc$ experiences a so called $2\pi$-pulse, i.e., $\chi_R t_{2\pi} = 2\pi$. During the first half of the pulse, the population of level $b$ is completely transferred to state $c$. During the second half of the pulse, the population of the excited state $c$ is transferred back to state $b$. The state vector of the atom at the end of the pulse is 

$$|\Phi_{bc}(t_{2\pi})\rangle = -C_b |b\rangle + C_d |d\rangle. \quad (21)$$

Equation (21) shows that the $2\pi$-pulse results in a $\pi$ phase shift of the $b$ state probability amplitude. Usually, for the simple two-level atom, this $\pi$ phase shift of the ground state after a $2\pi$-pulse is not revealed in the observables. In our case, apart from the two levels $b$ and $c$, we have the third level $d$ whose phase is well defined with respect to the phase of state $b$. The phase definition is determined by the phase relation of the components of the bichromatic field [see comments after Eq. (20)]. In section IV we show the importance of this point.

### III. TIME DEPENDENT AMPLITUDES

If during the pulse excitation the field amplitudes $E_1$, $E_2$ are time dependent and they have the same time evolution satisfying the condition $B_1(t)/B_2(t) = \tan \alpha = const$, then
the developments of the \( |d\rangle, |b\rangle \) states in the ket-vectors \( |1\rangle, |2\rangle \) have constant coefficients and the \( S \)-transformation is time independent. In this case the solution of the Schrödinger equation is simple and the development coefficients of the state vector \( |\Phi_{dbc}\rangle \),

\[
|\Phi_{dbc}(t)\rangle = C_d \, |d\rangle + C_b \, |b\rangle + C_c \, |c\rangle,
\]

are \( C_d(t) = C_d(0) \), \( C_b(t) = C_b(0) \cos[\theta(t)/2] \) and \( C_c(t) = iC_b(0) \sin[\theta(t)/2] \), where \( C_d(0) \), \( C_b(0) \) are the initial values of the probability amplitudes and \( \theta(t) = 2 \int_{-\infty}^{t} B(\tau) \, d\tau \) is the pulse area of the bichromatic field. This case corresponds to so called matched pulses [48]–[49].

If there is a time shift \( T \) between the pulses \( E_1(t) \) and \( E_2(t) \), one can again introduce the bright and dark states, employing the time dependent \( S \)-transformation. Then the evolution of the state vector of the atom in the \( dbc \)-basis is given by

\[
|\Phi_{dbc}(t)\rangle = S(t) \, |\Phi(t)\rangle,
\]

where \( |\Phi(t)\rangle \) is the state vector in the interaction representation. Taking the time derivative of the equation (23), one can obtain the Schrödinger equation

\[
\frac{d|\Phi_{dbc}\rangle}{dt} = -i \, \mathcal{H}_{dbc} \, |\Phi_{dbc}\rangle
\]

with the modified Hamiltonian

\[
\mathcal{H}_{dbc} = \mathcal{H}_{dbc} + i \, S \, S^{-1},
\]

where \( \mathcal{H}_{dbc} \) is defined in Eq. (15) and

\[
i \, S \, S^{-1} = i \, \dot{\alpha} \left[ \hat{P}_{bd} e^{-i(\varphi_1 + \varphi_2)} - \hat{P}_{db} e^{i(\varphi_1 + \varphi_2)} \right]
\]

The first part, \( \mathcal{H}_{dbc} \), of the modified Hamiltonian induces transitions between the \( |b\rangle \) and \( |c\rangle \) states with the rate \( \chi_R = 2B \), and the second part, \( i \, S \, S^{-1} \), induces transitions between states \( |d\rangle \) and \( |b\rangle \) with the rate \( 2 \, \dot{\alpha} \) (see Fig. 2).

The development coefficients of the state vector \( |\Phi_{dbc}\rangle \) satisfy the equations

\[
\dot{Z}_d = -\dot{\alpha} \, Y_b
\]

\[
\dot{Y}_b = -B \, X_c + \dot{\alpha} \, Z_d
\]

\[
\dot{X}_c = B \, Y_b
\]

where the substitution \( Z_d = C_d \exp(-i\varphi_2), \ Y_b = C_b \exp(i\varphi_1), \ X_c = -iC_c \exp(i\varphi_1) \) is made to deal with real numbers. Here the phases \( \varphi_1 \) and \( \varphi_2 \) are assumed to be constant throughout the excitation process. Equations (27)–(29) remarkably coincide with the Bloch equations for an abstract two-level system \( g-e \) excited by a field with frequency \( \omega \) slightly tuned from resonance (\( g \) and \( e \) being ground and excited states, respectively) [2]. Expressed in terms of the Bloch-vector components, which are the following combinations of the density matrix
of the two-level system \( \rho \):  
\[ u - iv = 2\rho_{ge} \exp(-i\omega t) \quad \text{and} \quad w = \rho_{gg} - \rho_{ee}, \]
these equations are, Ref. [40],

\[ \dot{w} = -\chi v, \tag{30} \]

\[ \dot{v} = -\Delta u + \chi w, \tag{31} \]

\[ \dot{u} = \Delta v. \tag{32} \]

Here \( \Delta \) is the detuning from resonance and \( \chi \) is the Rabi frequency. If one makes the substitution \( w = Z_d \), \( v = Y_b \), \( u = X_c \) for the variables and \( \chi = \dot{\alpha} \), \( \Delta = B \) for the parameters, both sets of equations are identical.

The coincidence of the equations enables us to use the adiabatic following approach developed by Crisp [3] to describe nonresonant excitation of the two-level system. A similar approach was applied by Laine and Stenholm (LS) [36] based on the ideas of the adiabatic following developed by Dykhne [7], [25] for the two-level system with time dependent splitting and coupling parameters. In the LS approach, [36], the instantaneous eigenstates of the three-level atom excited by two resonant pulses are found. This basis is called the adiabatic representation. The transformation to the instantaneous basis [see Eq. (7)] and the instantaneous Hamiltonian diagonal in this basis [see Eq. (6)] are time dependent. Therefore, the adiabatic states coupling, \( i \dot{Q} Q^{-1} \), also appears in their consideration. In spite of being different in structure, our Schrödinger equation in the changing \( dbc \)-basis and the LS equation in the instantaneous eigenstate basis can both be reduced to the equation for the two-level system. However, our equations (27)–(29) are in a one to one correspondence to the Bloch equations while the LS equations match the equations for the two-level density matrix elements \( \rho_{eg}, \rho_{ge} \) and \( \rho_{gg} - \rho_{ee} \). Although this difference is not crucial, it brings, however, some convenience in our case, because our equations are expressed for real quantities.

**IV. ADIABATIC FOLLOWING APPROXIMATION**

Crisp gave in his paper [3] a qualitative and quantitative description of the adiabatic following approximation for the two-level atom excited by a nonresonant field. The qualitative explanation employs the Bloch-vector model as follows. The Bloch equations can be written in the form

\[ \dot{\mathbf{R}} = \mathbf{H}_{\text{eff}} \times \mathbf{R}, \tag{33} \]

if the relaxation times \( T_1 \) and \( T_2 \) are very long compared with the pulse duration. The Bloch vector \( \mathbf{R} \) is given by

\[ \mathbf{R} = Xe_x + Ye_y + Ze_z, \tag{34} \]

and the effective field \( \mathbf{H}_{\text{eff}} \) is

\[ \mathbf{H}_{\text{eff}} = -\chi e_x + \Delta e_z. \tag{35} \]
Equation (33) has the geometric interpretation that the Bloch vector tries to precess about the effective field as \( H_{\text{eff}} \) varies both in magnitude and direction. If the condition \(|\Delta| \gg |\chi|\) is satisfied, the precession frequency of the Bloch-vector, \( \sqrt{\chi^2 + \Delta^2} \), will be large compared with the rate of change of the field \( \chi(t) \). If initially the Bloch-vector points parallel or antiparallel to \( e_z \) \([X(0) = Y(0) = 0 \text{ and } Z(0) = \pm 1]\), then precessing with high frequency \(|\Delta|\) about the effective field, the Bloch-vector remains nearly parallel or antiparallel to the effective field as it moves adiabatically. Qualitatively, adiabatic movement of the vector \( H_{\text{eff}} \) means that the Bloch-vector does not slip off from the direction of the effective field if the rotation of the Bloch-vector around the \( H_{\text{eff}} \)-field is much faster than the change of the effective field direction. Quantitatively, this sets a condition that the precession period \( T_p \approx 2\pi / |\Delta| \) is to be much shorter than the time scale of the effective field variation. In this limit, the angle between the Bloch vector and the effective field will remain very small. The deviation of the Bloch vector from the effective field is described by the nonadiabatic corrections. In the next section we consider how small this deviation angle is for particular pulses.

The approximation of the adiabatic following allows also a quantum mechanical interpretation. The Hamiltonian in the \( dbc \) time varying representation, Eq. (25), consists of two parts. One part couples two, initially unpopulated states, \( b \) and \( c \). The coupling parameter \( B \) is time varying. The other part couples the populated state \( d \) with the unpopulated state \( b \). The coupling parameter \( \alpha \) is also time varying. In order that the atom is subject to the adiabatic following of the changing \( d \) state (the atom stays in this state), one has to choose the pulse train with the Raman Rabi frequency \( B \) much larger than \( \alpha \) when the latter takes its maximum value \([2]\). Also the time variation scale of \( \alpha \) must be much longer than the Raman Rabi period. These conditions can be made plausible by the following speculations.

Since initially only state \( d \) is populated, the atom starts to evolve when \( \alpha(t) \) becomes nonzero. When the slow process \( \alpha \) brings a small fraction of the population to state \( b \), the fast Rabi oscillation via the \( 2\pi \)-cycle (see the discussion at the end of Section II) changes the sign of the \( Y_b \)-amplitude from plus to minus, reversing back the population transfer process from \( d \rightarrow b \) to \( b \rightarrow d \). As a result, the population of the \( b \)-state does not increase and remains small. One can roughly estimate its mean value by solving the equations (27)–(29) with the initial condition \( X_c(0) = Y_b(t) = 0, Z_d(0) = 1 \) and assuming that \( \chi_R \) and \( \alpha \) are constant, i.e., neglecting the derivatives of these parameters,

\[
Z_d(t) = \frac{B^2}{B^2 + \dot{\alpha}^2} + \frac{\dot{\alpha}^2}{B^2 + \dot{\alpha}^2} \cos \sqrt{B^2 + \dot{\alpha}^2} t, \tag{36}
\]

\[
Y_b(t) = \frac{\dot{\alpha}}{\sqrt{B^2 + \dot{\alpha}^2}} \sin \sqrt{B^2 + \dot{\alpha}^2} t, \tag{37}
\]

\[
X_c(t) = \frac{\dot{\alpha} B}{B^2 + \dot{\alpha}^2} \left( 1 - \cos \sqrt{B^2 + \dot{\alpha}^2} t \right). \tag{38}
\]

The solution shows that the change of the amplitude of the \( d \)-state is small if \( B = \sqrt{B_1^2 + B_2^2} \gg \dot{\alpha} \). This is exactly the condition of the adiabatic following formulated in Ref. [29] for STIRAP.
V. ADIABATIC SOLUTION

The adiabatic following approximation can be applied to the consideration of STIRAP and electromagnetically induced transparency (EIT) because in both cases the atom follows the dark, noncoupled state. A first attempt to study adiabatic following in EIT was undertaken in Ref. [2]. In this section we consider the application of this approach to STIRAP.

The stimulated Raman adiabatic passage assumes that before the application of the $E_1(t)$ and $E_2(t)$ pulses the atom is in state $|1⟩$. The duration of the excitation as well as the pulse sequence must be chosen such that at the end of the pulse train the atom is left in state $|2⟩$. It is expected that during this process the atom stays in the dark state. However, this state itself changes since the coefficients $\cos \alpha$ and $\sin \alpha$ of the development of the dark state in the vectors $|1⟩$ and $|2⟩$ [see Eq. (12)] change in time. The parameter $\alpha$ rises from zero to $\pi/2$, so $|d⟩ = |1⟩$ before the excitation and $|d⟩ = -e^{-i(\varphi_1 - \varphi_2)}|2⟩$ after it. Since $\tan \alpha = B_1/B_2$, the condition imposed on $\alpha$ means that the $B_1(t)$-pulse must be delayed with respect to the $B_2(t)$-pulse. We have to emphasize that the phase of the final state 2 after the pulse train is related to the phase of the initial state 1 according to Eq. (12).

If we choose two identical, bell-shaped, delayed pulses having a hyperbolic secant shape, then the interaction constants evolve in time as follows

$$B_n(t) = B_0 \sec h[r(t - t_n)],$$

where $n = 1$ or $2$, $t_n$ is the time when the $n$-pulse has maximum amplitude, and $r$ is the rise and fall rate of the pulse edges. The mixing parameter $\alpha(t)$ increases monotonously if the condition $t_1 > t_2$ is satisfied. This pulse sequence was considered by Laine and Stenholm in Ref. [36].

Let us analyze the constraints imposed on the parameters $r$, $t_1$ and $t_2$ to have $\alpha$ changed from zero to $\pi/2$. The mixing angle $\alpha$ varies according to the relation $\tan \alpha(t) = B_1(t)/B_2(t)$. For the pulse sequence specified above, an explicit form of this relation is

$$\tan \alpha(t) = \frac{1 + D \tanh[r(t - t_0)]}{1 - D \tanh[r(t - t_0)]},$$

where $D = \tanh(rT/2)$, $t_0 = (t_1 + t_2)/2$ is the mean time of the maxima and $T = t_1 - t_2$ is the time interval between the maxima of the pulses. Suppose that, initially, the atom is in the ground state 1 and we start the atom evolution from the initial mixing parameter $\alpha_{in}(-\infty)$ satisfying the condition $\tan(\alpha_{in}) = 0.01$, which means that essentially $C_d(-\infty)e^{-i\varphi_2} = \cos(\alpha_{in}) \approx 1$ and $C_b(-\infty)e^{i\varphi_1} = \sin(\alpha_{in}) = 10^{-2}$. We stop the atom evolution at $\tan[\alpha_{fin}(+\infty)] = 100$. So, if the final state coincides with state 2 then $C_d(+\infty)e^{-i\varphi_1} = \sin(\alpha_{fin}) \approx 1$ and $C_b(-\infty)e^{i\varphi_2} = -\cos(\alpha_{fin}) = -10^{-2}$, where the phase change of state 2 is taken into account. From Eq. (40) it follows that $\tan \alpha(\pm \infty) = (1 \pm D)/(1 \mp D)$ and at the condition imposed on $\alpha_{in}$ and $\alpha_{fin}$ we have $rT = 4.6$. Since state $b$ is strongly coupled.

It should be emphasized that the higher time derivatives of the mixing parameter $\alpha(t)$, neglected above, play a crucial role in the adiabatic population transfer. However, the solution (36)–(38) gives some qualitative hint on the choice of the relation between $B$ and $\dot{\alpha}$.
with state \(c\) [the coupling is \(B(t)\)], we have to keep the initial population of state \(b\) as small as possible \([C_b(-\infty)\) must be close to zero]. Otherwise, the probability amplitude \(C_b(-\infty)\), if not infinitely small, will be spread among the \(b\) and \(c\) states by the pulse train and population transfer \(1 \rightarrow 2\) will be imperfect. Therefore, on the one hand, the initial value of the mixing parameter \(\alpha\) must be as small as possible to have complete population transfer \(1 \rightarrow 2\). For a small initial mixing angle \(\alpha_{in}\), the relation between \(\alpha_{in}\) and \(T\) becomes simple, i.e., \(\tanh(rT/2) \approx 1 - 2\alpha_{in}\). The smaller the initial value of the mixing angle \(\alpha\), the larger the product \(rT\) or the pulse separation \(T\) is. On the other hand, if the distance between the pulses is large, the value of the coupling \(B(t)\) at \(t_0\) becomes small: the larger the distance, the smaller the coupling. However, the adiabatic following demands a large coupling \(B\) at \(t_0\). Therefore, one has to choose the optimum value of the pulse spacing satisfying two conditions simultaneously. The distance between pulses is to be as large as possible to have a small value of \(\alpha_{in}\) and, at the same time, \(B(t_0)\) should be kept as large as possible. Below we give some arguments how to find this optimum value.

If \(rT \rightarrow 0\), then according to Eq. (40) we have \(\alpha \rightarrow \pi/4\) throughout the excitation and the dark state does not change in time. It has the probability amplitude \([C_d] = \sqrt{2}/2\). State \(b\), having the initial population \([C_b(-\infty)]^2 = 1/2\), is depopulated with the rate \(B\) \([C_b(t) = C_b(-\infty) \cos(\theta(t)/2), C_c(t) = iC_b(-\infty) \sin(\theta(t)/2), \) see the definition of \(\theta(t)\) in Section III]. For small values of \(rT\), the time interval of the \(\alpha\) change is small. For example, if \(rT = 0.5\), then \(\cos(\alpha_{in}) = 0.855, \sin(\alpha_{in}) = 0.519\) and \(\cos(\alpha_{fin}) = 0.519, \sin(\alpha_{fin}) = 0.855\), which corresponds to the change of \(\alpha\) from \(\alpha_{in} = 0.174\pi\) to \(\alpha_{fin} = 0.326\pi\) during the pulse train. In this case only the fraction \((0.855)^2 = 0.731\) of the atomic population is transferred to state \(|2\) if the atom adiabatically follows the dark state. Another fraction \((0.515)^2 = 0.269\) of the atomic population participates in the process of excitation and de-excitation between states \(b\) and \(c\). This means that the population transfer via the change of the amplitude of the dark state components is possible only if the time interval between the pulses exceeds a certain value. We choose the value \(rT = 5\) since in this case the initial population of state \(b\) is \(4.5 \times 10^{-5}\) so that we neglect this population in our further consideration.

Figure 3 (a) shows the pulse train with \(rT = 5\) along with the change of the mixing parameter \(\alpha\) during the excitation. On the same plot the dependence of the derivative

\[
\dot{\alpha}(t) = \frac{rD \sec h^2(r(t - t_0))}{1 + D^2 \tanh^2[r(t - t_0)]} = \frac{r \sinh(rT)}{1 + \cosh(rT) \cosh(2r(t - t_0))}
\]

(41)

is shown. This derivative takes a maximum value \(\dot{\alpha}_{\text{max}}(t_0) = rD\) at \(t = t_0\). For example, if \(rT = 5\), then \(\dot{\alpha}_{\text{max}}(t_0) = 0.987r\). In Fig. 3 (b) we compare the time evolution of the Rabi frequency \(\chi_R/2 = B(t) = \sqrt{B_1^2(t) + B_2^2(t)}\) with the evolution of the derivative \(\dot{\alpha}(t)\).

The Rabi frequency determines the transition rate between states \(b\) and \(c\), whereas the derivative of the state mixing angle \(\alpha\) specifies the transition rate between states \(d\) and \(b\) [see Eqs. (27)–(29)]. Since at \(t = t_0\) the parameter \(\dot{\alpha}(t)\) takes its maximum value and \(B(t)\) has its minimum, the adiabatic following condition is \(B(t_0) \gg \dot{\alpha}(t_0)\) or, explicitly, \(\sqrt{2}B_0 \gg r \sinh(rT/2)\). Figure 4 (solid lines) shows the time dependence of the amplitudes \(X_c\) (plot a), \(Y_b\) (plot b) and \(Z_d\) (plot c) obtained by the numerical solution of the equations (27)–(29) for the pulse train with \(t_1 = 2.5/r, t_0 = 0, t_2 = -2.5/r, T = 5/r, B_0 = 42.8r\). The relation between \(B_0\) and \(r\) corresponds to the ratio \(B(t_0)/\dot{\alpha}(t) = 10\). We see that the deviation of the \(Z_d\) amplitude from its initial value \((\sim 0.005)\) and the maximum absolute
value of the $Y_b$-amplitude ($\sim 0.01$) are different from the values predicted by the simplified solution, Eqs. (36)–(38). Only the maximum value of $X_c$ ($\sim 0.1$) fits the value of the simplified solution.

To estimate the actual values of the probability amplitudes of the $dbc$ states during the excitation and find the borders within which the adiabatic following of the dark state takes place, we follow the theory developed by Crisp [3] for the case when the condition $B(t_0) \gg \dot{\alpha}(t)$ is well satisfied.

In this case we can use the expansion in a power series of the parameter $\dot{\alpha}(t)$ for the solution of the equations (27)–(29). Then the first term of the expansion is found by setting $Z_d(t) = 1$ in the equations (28),(29) and then solving them. The solution is

$$Y_b(t) = \int_{-\infty}^{t} \cos \left[ \int_{\tau}^{t} B(\tau_1) d\tau_1 \right] \dot{\alpha}(\tau) d\tau,$$

$$X_c(t) = \int_{-\infty}^{t} \sin \left[ \int_{\tau}^{t} B(\tau_1) d\tau_1 \right] \dot{\alpha}(\tau) d\tau.$$  

This is the part of the general solution that is linear in $\dot{\alpha}(t)$. $Z_d$ satisfies the equation

$$Z_d(t) = 1 - \int_{-\infty}^{t} \dot{\alpha}(\tau) X_b(\tau) d\tau,$$

which is the formal solution of Eq.(27). Therefore the change of the dark state amplitude $Z_d$ is nonlinear in $\dot{\alpha}(t)$ and can be presented in products of $\dot{\alpha}$-s. The first contribution of $\dot{\alpha}(t)$ to $Z_d$ can be found as the square of $\dot{\alpha}$, i.e., in the second term of the expansion. Substituting the corrected $Z_d$ into Eqs. (28),(29) (instead of $Z_d = 1$), one can find the next term in the expansion of $X_b$ and $Y_c$. Then the substitution of the thus found $X_b$ into Eq. (44) gives the next term of the expansion of $Z_d$, etc. In this paper we consider only the linear corrections to $Y_b$ and $X_c$.

Our solution, given by the equations (42)–(44), differs from Crisp’s solution in the $B$-time dependence. He considered the case when $B$ is constant and $\dot{\alpha}$ is time dependent. Fig. 4 (a–c) shows the comparison of the numerical solution of the equations (27)–(29) with Eqs. (42)–(44). They are indistinguishable and shown by the same solid lines.

We find the adiabatic and nonadiabatic components of the analytical solution by applying two different procedures. The adiabatic components of the equations (42)–(43) are calculated by integrating them by parts. For example, the first step of the $X_c(t)$ calculation is

$$X_c(t) = \int_{-\infty}^{t} \frac{\dot{\alpha}(\tau)}{B(\tau)} d \left( \cos \left( \int_{\tau}^{t} B(\tau_1) d\tau_1 \right) \right) = \frac{\dot{\alpha}}{B} \int_{-\infty}^{t} \cos \left( \int_{\tau}^{t} B(\tau_1) d\tau_1 \right) \left( \frac{\dot{\alpha}}{B} \right)' d\tau.$$  

where $(\dot{\alpha}/B)'_\tau$ is the derivative with respect to $\tau$. Repeating these steps several times, we obtain

$$Y_b(t) = \alpha_2 - \alpha_4 + \alpha_6 - \alpha_8 + ...,$$

$$X_c(t) = \alpha_1 - \alpha_3 + \alpha_5 - \alpha_7 + ...,$$
where $\alpha_n = \alpha_{n-1} / B$, $\alpha_0 = \alpha$ and it is assumed that $\dot{\alpha}(t)$ is a bell-shaped function with first and higher derivatives equal zero at $t = -\infty$. To find a similar expansion of equation (44), we introduce the function $\Omega(t) = \int_0^t B(\tau)d\tau$. Then the equation for $Z_d(t)$ takes the form

$$Z_d(t) = 1 - A_c(t) - A_s(t), \quad (48)$$

with

$$A_c(t) = \int_{-\infty}^{t} d\tau_1 \dot{\alpha}(\tau_1) \cos \Omega(\tau_1) \int_{-\infty}^{\tau_1} d\tau_2 \dot{\alpha}(\tau_2) \cos \Omega(\tau_2), \quad (49)$$

$$A_s(t) = \int_{-\infty}^{t} d\tau_1 \dot{\alpha}(\tau_1) \sin \Omega(\tau_1) \int_{-\infty}^{\tau_1} d\tau_2 \dot{\alpha}(\tau_2) \sin \Omega(\tau_2). \quad (50)$$

The functions $A_c(t)$ and $A_s(t)$ are reduced to the single integrals

$$A_c(t) = \frac{1}{2} \left( \int_{-\infty}^{t} d\tau_1 \dot{\alpha}(\tau_1) \cos \Omega(\tau_1) \right)^2, \quad (51)$$

$$A_s(t) = \frac{1}{2} \left( \int_{-\infty}^{t} d\tau_1 \dot{\alpha}(\tau_1) \sin \Omega(\tau_1) \right)^2. \quad (52)$$

This can be done since they have the structure

$$A_{c,s}(t) = \int_{-\infty}^{t} d\tau \ f_{c,s}(\tau) f_{c,s}(\tau) = \frac{1}{2} f_{c,s}^2(t), \quad (53)$$

where $f_{c,s}(\tau)$ is

$$f_{c,s}(t) = \int_{-\infty}^{t} d\tau \dot{\alpha}(\tau) \begin{Bmatrix} \cos \Omega(\tau) \\ \sin \Omega(\tau) \end{Bmatrix}, \quad (54)$$

and the index $c$ stands for the cosine function and index $s$ for sine. Integrating these integrals by parts, we obtain

$$Z_d(t) = 1 - \frac{1}{2} \left[ (\alpha_1 - \alpha_3 + \alpha_5 + \ldots)^2 + (\alpha_2 - \alpha_4 + \alpha_6 + \ldots)^2 \right], \quad (55)$$

Equations (46,47) and (55) give the probability amplitudes of the dark, bright and common states. They coincide with those one obtains if the successive transformations $S_n S_{n-1} \ldots S_1 S$ to the new set of $d_n b_n c_n$-states are performed, as was done by Fleischhauer and coauthors in Ref. [37], i.e., $| \Phi_n \rangle = S_n \ldots S_1 S | \Phi_0 \rangle$, where $| \Phi_0 \rangle = C_1 | 1 \rangle + C_2 | 2 \rangle + C_3 | 3 \rangle$ is the initial state $| C_1(-\infty) = 1, C_2(-\infty) = C_3(-\infty) = 0 \rangle$. The transformation $S$ is defined in Eq. (17) and the other transformations ($S_i$) are specified below. These transformations have a simple meaning. Since the $d-b$ and $b-c$ transitions are excited simultaneously by the $\dot{\alpha}$ and $B$ “fields”, one can make a $S_1$-transformation to the new set of dark, bright and common states, $d_1$, $b_1$ and $c_1$, where $d_1$ is a particular mixture of the former $d$ and $c$ states,
$b_1$ is a state orthogonal to state $d_1$, and $c_1$ coincides with state $b$. The new mixing angle is $\alpha_1 = \arctan(\dot{\alpha} / B)$. Repeating this procedure $n$-times, one can get our solution if the conditions $\arctan(\dot{\alpha}_n / B) \approx \dot{\alpha}_n / B$ and $\sqrt{\dot{\alpha}_n^2 + B^2} \approx B$ are applied at each step.

Figure 4 (a-c) shows the comparison of the numerical solution (solid lines) with the expansions given by the equations (46), (47) and (55) (dashed line s). The parameters of the pulse train are specified above and they are the same as in Fig. 3 (a,b). Only the first two terms of the expansions are taken into account for each plot, which is justified because $|\alpha_1| \gg |\alpha_2| \gg |\alpha_3| \gg |\alpha_4| \gg ....$. For the $X_c$ and $Y_b$ components, the maximum absolute values of the second terms of the expansions (i.e., the adiabatic terms) are already comparable with the amplitude of the oscillations (i.e., the nonadiabatic contribution, coming from the summation of an infinite number of the expansion terms). Of course, nonadiabatic oscillations are not described by the main part of the adiabatic solution presented by a few leading terms of the expansion.

Concluding this section, we refer to a particular case when $B(t)$ and $\dot{\alpha}$ have the same time dependence. This is again the case of matched pulses (see the beginning of Section III), however in the $dbc$ basis. Fleischhauer and coauthors, Ref. [37], classify this case as second order matched pulses. The solution of the equations (27)–(29) is trivial since these equations in terms of a new variable $\zeta = \int_{-\infty}^{t} F(\tau)d\tau$ can be reduced to a set of differential equations with constant coefficients, where $F(t) = B(t)/B(t_0) = \dot{\alpha}(t)/\dot{\alpha}(t_0)$. For the first time, this analytically solvable model was considered by Vitanov and Stenholm in Ref. [50]. We would classify the case as nonadiabatic, however in the second order $d_1b_1c_1$-basis, where the transition takes place. If the generalized pulse area is properly chosen in this basis, the population transfer between the diabatic states 1 and 2 is complete. For these particular pulse areas, there are no nonadiabatic corrections, which is typical for the resonant nonadiabatic transitions.

VI. NONADIABATIC CORRECTIONS

All adiabatic terms tend to zero at $t \to +\infty$, which secures for the three-level atom the adiabatic following of a particular state coinciding with state $d$ at $t \to +\infty$. However, as it will be shown below, if we sum all these infinitely small terms, the result will be finite. The net value of the small contributions of each adiabatic term is a nonadiabatic contribution, which specifies the excited probability amplitude left by the pulse train. To estimate this value, we rewrite the solution, Eqs. (42) and (43), as follows

\begin{align}
Y_b(t) &= f_c(t) \cos \Omega(t) + f_s(t) \sin \Omega(t), \\
X_c(t) &= f_c(t) \sin \Omega(t) - f_s(t) \cos \Omega(t).
\end{align}

When $t \to +\infty$, the function $f_c(t)$ tends to a finite value whereas $f_s(t)$ tends to zero since in the corresponding integrals [see Eq. (54)] $\dot{\alpha}(\tau)$ is an even function of time and $\Omega(\tau)$ is an odd function. As a result, at the end of the pulse train $Y_b(t)$ and $X_c(t)$ oscillate as $\cos \Omega(t)$ and $\sin \Omega(t)$, respectively, and they have constant amplitudes $f_c(+\infty)$. The value of $f_c(+\infty)$ defines the probability amplitude left by the pulse sequence in states $b$ and $c$. 

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The excitation process is adiabatic if the nonadiabatic part $f_c(+\infty)$ is small. If the nonadiabatic part becomes comparable with the main term $\alpha_1(t_0)$ of the adiabatic expansion, then we cannot describe the excitation process as adiabatic. Figure 5 (a) shows the numerically found dependences of the $\alpha_1(t_0)$ term (dashed line) and the nonadiabatic contribution $f_c(+\infty)$ (solid line) on the maximum amplitude $B_0$ of the pulses for the pulse train with $Tr = 5$. The adiabatic and nonadiabatic parts of the solution become comparable if $B_0 < 15r$.

The semilogarithmic plot of $f_c(+\infty)$ versus $B_0$, Figure 5 (a), clearly demonstrates that the nonadiabatic part decreases exponentially with the increase of $B_0$. Laine and Stenholm [36] also found an exponential decrease of the nonadiabatic deviation from the ideal population transfer with increase of $B_0T$ ($B_0$ is the amplitude of each pulse at the maximum and $T$ is the interpulse distance, fixed in our case by the relation $Tr = 5$, so $B_0/r$ is a variable). They followed the calculation procedure proposed by Davis and Pechukas [38] employing the Dykhne model [24]. According to Ref. [38], one has to find the zeros of the function $B(t)$ in a complex plane $t$ and take the one, $t_c$, that is nearest to the real axis. Then the population of the dark state after the pulse train is

$$|d(+\infty)|^2 \propto \exp \left[ -2 \text{Im} \int_{t_0}^{t_c} B(t) \, dt \right]. \quad (58)$$

We propose another calculation procedure of the nonadiabatic deviation. It will be shown that there are two nonadiabatic contributions, one coming from the Rabi frequency $B(t)$ and another from the mixing parameter derivative $\dot{\alpha}(t)$. Only the cooperative contribution of both determines the net nonadiabatic correction, while the Pechukas-Dykhne theory, taking into account only the $B(t)$-change, underestimates the nonadiabatic correction.

To show this, we express $f_c(t)$ via the Fourier transform of $\dot{\alpha}(t)$:

$$a(\omega) = \int_{-\infty}^{+\infty} d\tau \dot{\alpha}(\tau) e^{i\omega \tau}, \quad (59)$$

$$f_c(t) = \int_{-\infty}^{t} d\tau \cos \Omega(\tau) \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega a(\omega) e^{-i\omega \tau}. \quad (60)$$

Let us consider first the case if the Raman Rabi frequency is constant, i.e., $B(t) = \beta_0 = \text{const}$. Then $\Omega(\tau) = \beta_0 \tau$ and

$$f_c(+\infty) = \int_{-\infty}^{+\infty} d\omega \frac{1}{2} [\delta(\omega + \beta_0) + \delta(\omega - \beta_0)] \left. a(\omega) \right|_{\omega = \beta_0}, \quad (61)$$

where $\delta(x)$ is the Dirac delta function and $a(\omega)$ is an even function of $\omega$. The Fourier transform of the mixing parameter derivative for a secant hyperbolic pulse train can be found, for example in Ref. [51]. This function is

$$a(\omega) = \pi \frac{\sinh \left[ \frac{\pi}{2\beta} \arctan \left( \frac{\sinh(rT)}{\beta} \right) \right]}{\sinh \left( \frac{\pi \omega}{2\beta} \right)}. \quad (62)$$

If we take $\beta_0 = B(t_0) = \sqrt{2}B_0 \sec h(rT/2)$, which is the value of the Raman Rabi frequency at time $t = t_0$ when $\dot{\alpha}(t)$ takes its maximum, then for our numerical example specified
above we obtain the amplitude of the nonadiabatic contribution \( f_e(\infty) = 1.263 \times 10^{-3} \). This value is four times smaller than the amplitude of the nonadiabatic oscillations on the right tail of the functions \( X_e(t) \) and \( Y_e(t) \) (which is \( \sim 5 \times 10^{-3} \)), shown in Figures 4 (a,b), i.e., four times smaller than the nonadiabatic contribution given by the numerical calculations of the solution of Eqs. (27)–(29).

To explain this difference and clarify the origin of the nonadiabatic contribution, we recall the interaction Hamiltonian in the \( dbc \)-basis, Eq. (25) [see also Eqs. (15) and (26)]. This Hamiltonian resembles the interaction representation Hamiltonian of the three-level system excited by two resonant “fields” with amplitudes \( B(t) \) and \( \alpha(t) \) (see Fig. 2). Assume, first, that the couplings \( B(t) \) and \( \alpha(t) \) are absent and the system is in state \( d \). Then, states \( d, b \) and \( c \) can be considered as having the same energies in the interaction representation. Switching on the coupling \( \alpha(t) = a \Theta(t) \) [here \( \Theta(t) \) is the Heaviside step function and \( a \) is an arbitrary constant] mixes states \( d \) and \( b \) or in other words induces the transition \( d \to b \). If the \( B \)-field is also present and its amplitude \( \beta_0 \) is constant, i.e., \( B(t) = \beta_0 \Theta(t) \), this \( B \)-field mixes states \( b \) and \( c \) producing a new couple of states \( b' \) and \( c' \), which are the states \([1] \) and \([3] \) [see Eqs. (8) and (10)]. This couple is split by the energy gap \( 2\beta_0 = 2B = 2\sqrt{B_1^2 + B_2^2} \) [see Eq. (6)]. This is the so called Mollow splitting [52] or quasi-energy splitting [46], [47]. Assume that without the \( B \)-field the \( \alpha \)-field is in resonance with levels \( d \) and \( b \). The switching on of the \( B \)-field mixes levels \( b \) and \( c \), producing an additional splitting. Level \( b \) moves on the frequency \( B \) out of resonance with the \( \alpha \)-field. If the \( \alpha \)-field would have a \( \delta \)-spectrum (in the case specified above it has only a zero frequency component), then it would not interact with the atom. However, because of the finite spectral width of the \( \alpha \)-field, its spectrum has a component with frequency \( B \) on the far tail which is in resonance with the new position of level \( b \). Only this spectral component excites the atom if the \( B \)-field is on. With increase of the \( B \)-field, the component of the \( \alpha \)-field spectrum, which interacts with the atom, shifts further to the tail of the spectrum. If the \( B \)-field amplitude changes in time, several spectral components of the \( \alpha \)-field interact with the atom since at each instant of time some particular spectral component is in resonance. The process of the sweeping of the splitting \( B(t) \) along the tail of the \( \alpha \)-field spectrum involves a broader band of the \( \alpha \)-spectrum in the interaction. To find the net atom excitation in this case, we have to calculate the integral \( f_e(\infty) \) where the change of \( B(t) \) is taken into account. This is done in the next two sections.

Concluding this section, we apply our spectral method to find the approximate solution of the Rosen-Zener model. The comparison of our solution with the exact analytical solution helps to find the condition when our approximation is valid. Referring to the Bloch equations (30)–(32) discussed in Section III, we define the splitting of the two-level system as \( \Delta = \text{const} \) and the coupling field amplitude as \( \chi = 2V_0 \sec h(\pi t) \), where \( V_0 \) is the amplitude of the secant hyperbolic pulse of width \( r \). According to the exact solution (see, for example, Ref. [10]), the population of the excited state \( e \) after the pulse is

\[
\rho_{ee}(\infty) = \sin^2 \left( \frac{\pi V_0}{r} \right) \sec h^2 \left( \frac{\pi \Delta}{2r} \right),
\]

(63)

if initially the system is in the ground state \( g \), i.e., \( \rho_{gg}(-\infty) = 1 \).

Employing the correspondence between the Bloch equations for the two-level system and the Schrödinger equation for the three-level system discussed in Section III, we make the
substitution \( \dot{\alpha} = \chi \) and \( B = \Delta \) in Eq. (54) where \( \Omega(t) \) becomes a simple function of time, i.e., \( \Omega(t) = \Delta t \). Then, if \( t \to +\infty \), the solution is \( Y_e(t) = f_c(+\infty) \cos \Delta t, \ X_e(t) = f_c(+\infty) \sin \Delta t \) [see Eqs. (56)–(57) and the discussion following these equations]. \( f_s(+\infty) = 0 \) since \( \Omega(t) \) is the odd function. \( f_c(+\infty) \) is simply the Fourier transform \( \chi(\Delta) \) [defined as a nondimensional value according to Eqs. (59) and (61)] of the function \( \chi(t) \) at the frequency \( B = \Delta \), i.e., \( f_c(+\infty) = \chi(\Delta) \), [see Eqs. (61) and (59)], or explicitly

\[
f_c(+\infty) = 2V_0 \int_{-\infty}^{+\infty} dt \sec h(rt) \cos \Delta t. \tag{64}
\]

This integral also can be found in Ref. [51]: \( f_c(+\infty) = 2\pi(V_0/r) \sec h(\pi\Delta/2r) \).

The population of the excited state \( e \) of the two-level system can be expressed as follows \( \rho_{ee} = (1-w)/2 \). Since there is a reciprocity between the Bloch-vector components of the two-level system and the state probability amplitudes of the three-level system \( (w = Z_d, \ v = Y_b, \ u = X_c) \), the excited state population is \( \rho_{ee} = (1 - Z_d)/2 \). The probability amplitudes are normalized, \( X_c^2 + Y_b^2 + Z_d^2 = 1 \), and hence \( Z_d = \sqrt{1 - X_c^2 - Y_b^2} = \sqrt{1 - f_c^2(+\infty)} \). Assuming that \( f_c(+\infty) \ll 1 \), we obtain

\[
\rho_{ee}(+\infty) \simeq \left( \frac{\pi V_0}{r} \right)^2 \sec h^2 \left( \frac{\pi\Delta}{2r} \right). \tag{65}
\]

Thus, our approximate result, Eq. (65), coincides with the exact solution, Eq. (63), if the population change is small and the sine function can be represented by the first term of its expansion in a power series. These conditions specify the validity of the adiabatic approximation. What is remarkable here is that the dependence of the excited state population on the two-level splitting \( \Delta \) in the approximate solution coincides with that in the exact solution. This shows that our spectral approach in the description of the nonadiabatic corrections is valid.

If \( \Delta \to 0 \), the nonadiabatic transition between states \( g \) and \( e \) takes place according to the straightforward solution of the Bloch-equation for the two-level system: \( \rho_{ee}(t) = \sin^2[\theta(t)/2] \), where \( \theta(t) = \int_{-\infty}^{t} \chi(\tau) d\tau \). At \( t \to +\infty \) we have \( \rho_{ee}(+\infty) = \sin^2(\pi V_0/r) \), which reproduces the exact Rosen-Zener solution (63) if \( \Delta \to 0 \).

VII. NONADIABATIC TRANSITION AS A QUANTUM JUMP: BASIC ARGUMENTS

If the Raman Rabi frequency changes during the development of the mixing parameter \( \dot{\alpha}(t) \), those Rabi frequencies sweeping the frequency bandwidth of \( \dot{\alpha}(t) \) contribute to the nonadiabatic corrections. To take this process into account, we have to convolute the spectral content of both the mixing parameter and the Raman Rabi frequency. In general, this calculation is nontrivial. For the case of secant hyperbolic pulses [see the time development of the pulses, the Raman Rabi frequency and the mixing parameter in Fig. 3 (a,b)], it is possible to simplify the problem. Since \( B(t) \) has a minimum value of \( \beta_0 \) at \( t_0 \) where \( \dot{\alpha}(t) \) has its maximum, only the spectral components of \( a(\omega) \) with \( |\omega| \geq \beta_0 \) contribute. Because time and frequency domains are inversely proportional, the main part of the nonadiabatic contribution appears in a short time interval around \( t_0 \), corresponding to the far tail of
the spectral distribution \(a(\omega)\). This means that the time scale of the nonadiabatic change is shorter than \(1/\beta_0\), while, according to the condition imposed on the parameters, the time variation of \(\dot{\alpha}(t)\) is much longer than the Rabi oscillation period \(\sim 1/\beta_0\). Thus, a nonadiabatic transition takes place almost jumpwise compared to the time scale of the variation of \(\dot{\alpha}(t)\). Due to this circumstance, we can simplify the calculation of the integral \(f_c(+\infty)\) by expanding \(B(t)\) in a power series of \(t\) near \(t_0\) and retaining only the first two terms of the expansion

\[
B(t) \approx \beta_0 \left[ 1 + g(t - t_0)^2 \right],
\]

where \(g = r^2 \left[ 3 \tanh^2(rT/2) - 1 \right]/2\). We verified this approximation by comparing numerically two integrals \(f_c(+\infty)\) and \(f_{cA}(+\infty)\) calculated with \(B(t)\) and its approximated value, Eq. (66), respectively. The comparison is shown in Fig. 5 b, where \(f_{cA}(\infty)\) (dashed line) is the approximation. The dependences of both integrals on the amplitude \(B_0\) are indistinguishable.

The approximation of \(B(t)\) by a parabolic function helps to express \(f_c(+\infty)\) via the Airy integral. Further, to simplify the notations we set \(t_0 = 0\). Then the phase \(\Omega(t)\) is

\[
\Omega(t) = \beta_0 \left( t + \frac{g}{3} t^3 \right),
\]

and \(f_{cA}(+\infty)\) is expressed as

\[
f_{cA}(+\infty) = \gamma \int_{-\infty}^{+\infty} d\omega \, a(\omega) e^{-i\omega t}. \tag{68}
\]

Evaluating the time integral in Eq. (68), we obtain

\[
f_{cA}(+\infty) = \gamma \int_{-\infty}^{+\infty} d\omega \, a(\omega) \text{Ai} \left[ \gamma(\beta_0 + \omega) \right], \tag{69}
\]

where \(\text{Ai}(x)\) is Airy integral and \(\gamma = 1/\sqrt{g\beta_0}\). We derived this equation assuming that \(a(\omega)\) is an even function. The Airy integral has a different dependence for positive and negative arguments (see, for example Ref. [53]). If \(\beta_0 + \omega \geq 0\), we have

\[
\gamma \text{Ai} \left[ \gamma(\beta_0 + \omega) \right] = \frac{1}{3} \sqrt{\frac{\beta_0 + \omega}{3g\beta_0}} \left\{ I_{-\frac{1}{3}} \left[ 2 \sqrt{\frac{(\beta_0 + \omega)^3}{g\beta_0}} \right] - I_{\frac{1}{3}} \left[ 2 \sqrt{\frac{(\beta_0 + \omega)^3}{g\beta_0}} \right] \right\}, \tag{70}
\]

where \(I_{\pm\frac{1}{3}}(x)\) is the modified Bessel function of order \(\pm\frac{1}{3}\). If the argument is negative, \(\beta_0 + \omega < 0\), then

\[
\gamma \text{Ai} \left[ \gamma(\beta_0 + \omega) \right] = \frac{1}{3} \sqrt{\frac{\beta_0 + \omega}{3g\beta_0}} \left\{ J_{-\frac{1}{3}} \left[ 2 \sqrt{\frac{|\beta_0 + \omega|^3}{g\beta_0}} \right] + J_{\frac{1}{3}} \left[ 2 \sqrt{\frac{|\beta_0 + \omega|^3}{g\beta_0}} \right] \right\}, \tag{71}
\]

where \(J_{\pm\frac{1}{3}}(x)\) is the Bessel function of order \(\pm\frac{1}{3}\). For large arguments both functions have a simple asymptotic behavior.
\[ \gamma A_i [\gamma (\beta_0 + \omega)] \approx \frac{\exp \left[ -\frac{2}{3} \sqrt{\frac{\beta_0 + \omega}{g\beta_0}} \right]}{2\sqrt{\pi} \left[ g\beta_0 (\beta_0 + \omega) \right]^{\frac{1}{4}}}, \]  

(72)

if \( \beta_0 + \omega \) is positive, and

\[ \gamma A_i [\gamma (\beta_0 + \omega)] \approx \frac{\cos \left( \frac{2}{3} \sqrt{\frac{\beta_0 + \omega}{g\beta_0}} - \frac{\pi}{4} \right)}{\sqrt{\pi} \left( g\beta_0 |\beta_0 + \omega| \right)^{\frac{1}{4}}}, \]

(73)

if \( \beta_0 + \omega \) is negative. If the argument is zero, \( \beta_0 + \omega = 0 \), this function is

\[ \gamma A_i [\gamma (\beta_0 + \omega)] \mid_{\omega = -\beta_0} = \frac{\Gamma (\frac{1}{3})}{2\pi^{\frac{1}{2}} \sqrt{3} g\beta_0}, \]  

(74)

where \( \Gamma (\frac{1}{3}) \approx 2.679 \) is the gamma function.

\section*{VIII. Rabi Chirping Only}

If the \( \dot{\alpha} \)-field does not depend on time and has a value \( \dot{\alpha} (t) = \dot{\alpha} (t_0) \), then \( a(\omega) = 2\pi \dot{\alpha} (t_0) \delta (\omega) / r \) and the main contribution to the nonadiabatic part is given by the Airy integral

\[ f_{cA}(+\infty) = 2\pi \gamma A_i (\gamma \beta_0) \dot{\alpha} (t_0) / r, \]  

(75)

which has the explicit form

\[ f_{cA}(+\infty) = \tanh \left( \frac{rT}{2} \right) \sqrt{\frac{\pi}{\beta_0 \sqrt{g}}} \exp \left( -\frac{2}{3} \beta_0 \sqrt{\frac{g}{3}} \right), \]  

(76)

expressed via approximation (72). In this section we show how the approximation, specified above, is related to the Dykhne-Pechukas model [24], [38].

For large \( \beta_0 \), the Airy integral can be calculated by the saddle point methods as shown, for example, in Ref. [54]. First, the stationary or saddle point \( t_s \) is found where the phase \( \Omega (t) \) becomes stationary: \( \dot{\Omega} (t_0) = 0 \). In this point, the Raman Rabi frequency becomes zero since \( \dot{\Omega} (t) = B(t) \). For the case of the positive argument of the Airy integral, the saddle point is in the complex plane and \( t_s \) has only an imaginary component \( \text{Re} (t_s) = 0 \), i.e.,

\[ t_s = -\frac{i}{\sqrt{g}} = -\frac{i}{r} \sqrt{\frac{2}{3 \tanh^2 (rT/2) - 1}}. \]  

(77)

In fact, there are two saddle points: \( t_s = \pm i/\sqrt{g} \). The negative sign is chosen to avoid exponentially increasing numbers. Then the method of stationary phase (one of the saddle point methods) is applied. This method is applicable to integrals of the form

\[ I(\beta_0) = \int_C e^{i\beta_0 f(z)} dz, \]  

(78)
where $\beta_0$ is large and $C$ is a path in the complex plane such that the ends of the path do not contribute significantly to the integral. The idea of the method is to deform the contour $C$ so that the region of most of the contribution to $I(\beta_0)$ is compressed into as short a space as possible. This compression occurs at the saddle point.

Since the main contribution to this integral comes from the vicinity of the saddle point, one may conclude that in the adiabatic limit the atom abruptly makes a nonadiabatic transition between adiabatic states. This is very much like crossing a Stokes line for the asymptotic series, and resembles the turning point problem in the WKB theory (see, for example, Refs. [10, 54]). In the WKB method, the one-dimensional Schrödinger equation is considered for a particle with mass $m$ in a potential $V(x)$:

$$\frac{d^2\Psi}{dx^2} = -\frac{2m}{\hbar^2}[E - V(x)]\Psi.$$  \hfill (79)

The turning point is the one where the energy of the particle coincides with the potential: $E - V(x) = 0$. If $2m[E - V(x)]/\hbar^2 \propto x$, the solution of the equation (79) can be expressed via the Airy integral [54]. In this approach the WKB connection formula, relating the exponentially small solution on one side of a turning point to an oscillatory solution on the other side, is derived quite naturally.

The Pechukas-Dykhne [38] recipe of the calculation of the nonadiabatic contribution is similar to the method described above since the definitions of the saddle point $t_s$ and the crossing point $t_c$, where $B(t_c)$ is zero, are the same. However, in our consideration we substituted the expression for the Raman Rabi frequency

$$B(t) = 2B_0\sqrt{1 + \cosh(rT)\cosh(2rt)} \over \cosh(rT) + \cosh(2rt),$$ \hfill (80)

by the expansion near time $t_0$, Eq. (66). Therefore, we have only two saddle or crossing points, (79), whereas equation (80) has an infinite number of crossing points in the complex plane. According to Pechukas, one has to take the crossing point that is nearest to the real axis. For the secant hyperbolic pulse train, this point, as shown by Stenholm [36], has only an imaginary part $\text{Im}(t_c)$, i.e.,

$$t_c = \frac{i}{r}\tan^{-1}\left[\coth\left(\frac{rT}{2}\right)\right].$$ \hfill (81)

Reformulating the Dykhne approach [24], we conclude: if there are no spectral components of $a(\omega)$ matching the frequency gap between the quasi-energy levels split by the $B$-field, the $\alpha$-field comes to resonance with the $d-c$ transition at the imaginary time $t_c$ when $B$ is zero.

This approach disregards the spectral content of $\hat{\alpha}(t)$. If we take $\hat{\alpha}(t) = \hat{\alpha}(t_0)$ throughout the excitation, this method gives also an underestimated value of the nonadiabatic contribution. For our numerical example, equation (76) gives $f_cA(+\infty) = 7.131 \times 10^{-4}$, which is 7 times smaller than the value of the nonadiabatic contribution given by the numerical calculation of the Schrödinger equation (27)–(29). Figure 6 (a) shows the comparison of the numerically calculated nonadiabatic component $f_c(+\infty)$ (solid line), Eq. (54), with that calculated for the case if the time dependence of the $\hat{\alpha}(t)$-field is neglected (dashed line). The latter is given by Eq. (76). The plots obviously demonstrate that this approximation underestimates the nonadiabatic contribution.
Concluding this Section, we demonstrate the application of our method to the Landau-Zener model. In this model the parameters of the corresponding two-level system are $\Delta = 2\lambda t$ and $\chi = 2V_0$. Here, the level crossing takes place at $t = 0$ and the nonadiabatic correction appears in the vicinity of the crossing point. After the substitution specified in Section III and repeating the same procedure as described at the end of Section IV, we find that

$$f_c(+\infty) = 2V_0 \int_{-\infty}^{+\infty} dt \cos(\lambda t^2), \quad (82)$$

$$f_s(+\infty) = 2V_0 \int_{-\infty}^{+\infty} dt \sin(\lambda t^2). \quad (83)$$

The value $f_s(+\infty)$ is not zero for this model since $\Omega(t) = \lambda t^2$ is even function of time. The integrals in Eqs. (82)-(83) are expressed via Fresnel integral asymptotes (see, for example, [53]): $f_c(+\infty) = f_s(+\infty) = V_0 \sqrt{2\pi/\lambda}$. Recall that $Z_d(+\infty) = \sqrt{1 - f_c^2(+\infty) - f_s^2(+\infty)}$. Then in the adiabatic limit, $V_0^2 \ll \lambda/2\pi$, we obtain

$$\rho_{ee}(+\infty) = \frac{1 - Z_d(+\infty)}{2} \simeq \pi \frac{V_0^2}{\lambda}. \quad (84)$$

This expression coincides with the leading term of the exact solution, Ref. [10], expansion, that is

$$\rho_{ee}(+\infty) = 1 - \exp\left(-\pi \frac{V_0^2}{\lambda}\right) \simeq \pi \frac{V_0^2}{\lambda}. \quad (85)$$

We have to emphasize that our solution is valid only if the atom adiabatically follows the ground state $g$. Therefore, the nonadiabatic correction (84), specifying the population probability of the excited state $e$, coincides with the Landau-Zener solution, Eq. (85), if $V_0^2 \ll \lambda/2\pi$, i.e., when the population of the $e$ and $g$ states does not change appreciably. In this case our three-level system $dbc$, equivalent to the corresponding two-level system $eg$, also follows adiabatically state $d$. The opposite case is considered in the Appendix. There the new adiabatic states $d_1b_1c_1$ are introduced whose population also does not change appreciably. This transformation is necessary if $V_0^2 \gg \lambda/2\pi$, because the reciprocal three-level system $dbc$ does not follow state $d$, making a transition from state $d$ to state $c$ and back at the level crossing.

The necessity of the transformation can also be formulated in a different way. To apply the adiabatic solution, we need a certain relation between the interaction parameters $B$ and $\alpha$. Then the expansion in $\alpha$’s, where each next term of the expansion is smaller than the previous one [see Eqs. (42)-(55)], is applicable. If $V_0^2 \ll \lambda/2\pi$, this expansion can be made in the $dbc$-basis. In the opposite case, this expansion is not applicable. Therefore, we have to find new $d_1b_1c_1$-basis where the new interaction parameters $\tilde{B}$ and $\tilde{\alpha}_1$ are related such that the expansion in $\alpha$’s converges.
 IX. COOPERATIVE CONTRIBUTION OF RABI CHIRPING AND THE TIME DEPENDENT COUPLING, THE APPLICATION TO STIRAP BY SECANT HYPERBOLIC PULSES

In previous sections we showed that the contribution to the nonadiabatic corrections of the Rabi frequency chirping or the time dependence of the $\dot{\alpha}(t)$-field, being taken into account separately, was not enough to have the right nonadiabatic correction for STIRAP induced by secant hyperbolic pulses. Only both processes taken together, i.e., the Rabi frequency $B(t)$ chirping and the participation of all spectral components $a(\omega)$ of the $\dot{\alpha}(t)$-field engaged by this chirping to excite the atom, give the right value of the nonadiabatic component. This value is given by Eq. (69). To calculate analytically the convolution integral of the two spectra, $a(\omega)$ and the Airy integral describing the process of the Rabi frequency sweeping, we make two approximations. First, we take the approximation of the Airy integral, given by Eq. (72) in the frequency domain $\omega \geq -\beta_0$. This gives a small overestimation of the integrand near $-\beta_0$. For this reason, we start the integration from this value, not from $-\infty$. The oscillating part of the Airy integral for $\omega < -\beta_0$ gives a much smaller contribution than the main part, which is located between $-\beta_0$ and zero, $(-\beta_0,0)$. So, neglecting the part $(-\infty,-\beta_0)$, we compensate the overestimation near $-\beta_0$. Second, we approximate the spectrum $a(\omega)$ in the domain $(-\beta_0,0)$ by

$$a(\omega) = \pi \exp(-R|\omega|), \quad (86)$$

where $R = (\pi - \text{arctan}[\sinh(rT)])/2r \approx \pi/4r$. This approximation also gives a slight overestimation of the integrand near $\omega \sim 0$. To compensate this, we stop the integration at $\omega = 0$. The integrand has a maximum between $\omega = -\beta_0$ and $\omega = 0$. To calculate the contribution of this part, we use the modified method of the saddle point method. Usually, in the method of the saddle point, the integration near the point is extended to $\pm\infty$. In our case, to avoid the overestimation of the integrand we limit the integration by finite boundaries. The calculation of these boundaries for the deformed integration contour $C$ is simple because the deformed $C$ stays on the real axis. The result of the integration is

$$f_{cA}(+\infty) = \pi Q \exp[-\beta_0 R \left(1 - \frac{1}{3}gR^2\right)], \quad (87)$$

where $Q$ is a correction factor, which takes into account the finite integration boundaries to avoid the overestimation of the integral. Explicitly, we have

$$Q = \frac{1}{2} \left[\text{erf}(h_{\text{max}}) + \text{erf}(h_{\text{min}})\right], \quad (88)$$

$$h_{\text{max}} = \sqrt{\beta_0 \left[\frac{2}{3}\sqrt{g} - \left(\frac{2}{3}\right)^{\frac{4}{3}} R + \frac{1}{3}R^3g\right]}, \quad (89)$$

$$h_{\text{min}} = \sqrt{\frac{\beta_0 gR^3}{3}}, \quad (90)$$
and \( \text{erf}(x) \) is the error function. Figure 6 (b) shows the comparison of the true nonadiabatic contribution \( f_c(+\infty) \) (solid line) with our approximate calculation of \( f_{cA}(+\infty) \), given by Eq. (87) (dashed line). Figure 6 (c) compares the same dependencies if the correction factor is \( Q = 1 \) (dash dotted line).

For our numerical example specified above, the approximated value of the nonadiabatic contribution is \( f_{cA}(+\infty) = 5.822 \times 10^{-3} \). As was discussed above, this contribution appears in a very short time interval around \( t_0 \). Therefore, we can approximate the solution of the Schrödinger equations (27)–(29) by

\[
X_{bA}(t) \approx \alpha_2 - \alpha_4 + \Theta(t - t_0) f_{cA}(+\infty) \cos \Omega(t),
\]

\[
X_{cA}(t) \approx \alpha_1 - \alpha_3 + \Theta(t - t_0) f_{cA}(+\infty) \sin \Omega(t).
\]

where the index \( A \) stands for the approximation, \( \Theta(t - t_0) \) is the Heaviside step function, \( \Omega(t) = \int_0^t B(\tau)d\tau \) and \( B \) has its exact value [not the approximation (66)]. Figures 7 (a,b) show the comparison of the numerically found solutions of the Schrödinger equation (solid lines) with the approximation (91)–(92) (dashed lines). The fit of the solutions is striking. This means that the nonadiabatic contribution really appears in a quite short time range around \( t_0 \).

X. CONCLUSION

The introduction of the basis of the dark and bright states facilitates the understanding of the physical processes in the three-level atom excited by a bichromatic field. The dynamic evolution of the atom between the dark, bright and common states is as simple as the dynamics of the two-level atom excited by one field with Rabi frequency \( B \). This is because the evolution of the three-level atom can be effectively reduced to the evolution between two states, i.e., the bright and common states. The reduction of the three-level model to the two-level one allows also the application of the Bloch-vector model and Bloch equations for the treatment of the three-level atom excitation by the bichromatic field.

This quite simple algebra is applicable for the case of matched pulses. If the pulses do not match in shape and have different time dependencies, one can also reduce the consideration to the Bloch-vector model since there is a similarity between the Schrödinger equations for the probability amplitudes of the dark, bright and common states and the Bloch equation for an effective two-level system. The effective detuning of the two-level system from resonance is \( B(t) \) and the Rabi frequency is \( \dot{\alpha}(t) \), which is the derivative of the mixing angle in the dark state development in states of the reciprocal three-level system. This similarity allows a simple interpretation of the physical processes in the three-level system in case of adiabatic following of the dark state. In terms of the Bloch-vector model, the adiabatic following criterion is formulated resulting in three conditions. First, the initial state must coincide with the Bloch-vector aligned along an effective field at \( t = -\infty \). This field is determined by the value \( \sqrt{B^2(t) + \dot{\alpha}^2(t)} \). Second, the effective field makes a small angle with its initial position when the reciprocal parameter \( \dot{\alpha}(t) \) responsible for the atom-field interaction takes its maximum. Third, the time scale of the change of the ”pulse”, determined by the scale
of the variation $\dot{\alpha}(t)$, is much longer than the period of oscillations induced by the effective field $B(t)$. If all three conditions are met, the dynamic evolution of the three-level atom excited by a sequence of pulses is described by simple algebra. The adiabatic population transfer has a nice interpretation in terms of dark, bright and common states.

We developed an approximation describing the adiabatic interaction of the three-level atom with two resonant pulses. The method of estimation of the nonadiabatic correction is presented. It is applied to the case of two secant hyperbolic pulses. The adiabatic part of the solution describes the excitation and de-excitation processes of the three-level atom. The time dependence of the adiabatic part is smooth and follows the derivatives of the mixing angle $\alpha(t)$. Both parts, excitation and de-excitation, are symmetric in time with respect to $t = 0$ where $\dot{\alpha}(t)$ takes its maximum and they exactly compensate each other. In this respect the adiabatic following resembles the soliton-like interaction with the field. The nonadiabatic part appears in a short time interval in the vicinity of the maximum of the mixing parameter $\dot{\alpha}(t)$. It contains the information about the excitation left in the atom by the pulses because of the imperfect following of the dark state. Since this excitation lasts only a short time, we consider the nonadiabatic process as a transition between the ground and excited states, which takes place like a jump. The origin of the transition has a simple interpretation. The parameter $B$ defines the coupling strength of the bright ($b$) and common ($c$) states of the three-level system in the $dbc$–basis. The parameter $\dot{\alpha}(t)$ defines the coupling strength of the bright and dark ($d$) states. It is assumed that, initially, the system is in the dark state and any time we have $B \gg \dot{\alpha}(t)$. The $B$–coupling moves the bright state from the resonance with the $\dot{\alpha}(t)$–coupling to the value determined by the frequency $B$. If the spectral content of the $\dot{\alpha}(t)$–coupling has a component with frequency $B$, the nonadiabatic transition takes place. If $B$ changes in time, several spectral components of the $\dot{\alpha}(t)$–coupling contribute to the transition. Our results are compared with those of Laine-Stenholm [36] and Fleischhauer et al. [37].

We applied also our method to the Rosen-Zener and Landau-Zener models. The comparison of our approximate solution with the exact solutions available for these models is quite encouraging. We obtained exponentially small nonadiabatic corrections for the case of a symmetric time dependence of the excitation parameters $B(t)$ and $\dot{\alpha}(t)$ with respect to $t = 0$ (our model, the Rosen-Zener model and the Landau-Zener model with $V_0^2 \gg \lambda/2\pi$), and a small power dependent nonadiabatic correction for the case of an asymmetric time dependence of $B(t)$ (Landau-Zener model with $V_0^2 \ll \lambda/2\pi$). We considered also the excitation of the three-level system by asymmetric pulses and found a power dependent nonadiabatic correction. The results on asymmetric pulses, exciting the three-level atom, will be presented in another paper.

The simplified algebra developed in this paper could be useful for the description of the atom state manipulation by coherent fields.

XI. ACKNOWLEDGMENTS

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XII. APPENDIX

In this section we compare our approximate solution with an exact solution of the Landau-Zener model if the parameters of the corresponding two-level system $\Delta = 2\lambda t$ and $\chi = 2V_0$, satisfy the condition $V_0^2 \gg \lambda/2\pi$. This condition is opposite to the one considered at the end of Section VII. There, since the coupling $V_0$ is small compared to the sweeping rate $\lambda$ of the two-level system splitting $\Delta \ll \lambda/2\pi$, the system adiabatically follows the ground state $g$ and the net population of the excited state $e$ is small for $t \to +\infty$. If $V_0^2 \gg \lambda/2\pi$, states $g$ and $e$ are not an adiabatic basis. To apply the adiabatic approximation in this case, we have to choose a new basis that is adiabatic. This can be done if we use again the similarity between the two-level and three-level systems discussed in Section II. We consider the three-level system $dbc$ described by the Hamiltonian (25) with the parameters $B = \Delta = 2\lambda t$ and $\alpha = \chi = 2V_0$. Then we make a $S_1$–transformation to the new basis $d_1 b_1 c_1$

$$
|d_1\rangle = \sin \alpha_1 |d\rangle + i \cos \alpha_1 |c\rangle,
$$

$$
|b_1\rangle = i \cos \alpha_1 |d\rangle + \sin \alpha_1 |c\rangle,
$$

$$
|c_1\rangle = |b\rangle,
$$

where $\alpha_1 = \arctan(\Delta/\chi)$ and the new Rabi frequency is $B = \sqrt{\Delta^2 + \chi^2}$. A new mixing angle $\alpha_1$ takes the values $\pm \pi/2$ at $t = \pm \infty$. Thus, it varies from $-\pi/2$ to $+\pi/2$ with time and, hence, state $d_1$ coincides with $-|d\rangle$ and $+|d\rangle$ before and after the crossing of levels $g$ and $e$, which takes place at $t = 0$. If the system adiabatically follows the changing state $d_1$, no transition to the $b_1$ state takes place and, finally, the system is left in the initial state $d$ but with the $\pi$-shift of its phase. This phase shift originates from the transition $d \to c \to d$ in the $dbc$–basis. One can find that, according to Eq. (93), the system is in state $d$ at $t = \pm \infty$ when $\alpha_1 = \pm \pi/2$, and in state $c$ at $t = 0$ when $\alpha_1 = 0$ if the system follows state $d_1$.

We know from the exact solution of the Landau-Zener model that $\rho_{gg}(+\infty) = \exp(-\pi V_0^2/\lambda)$ and $\rho_{cc}(+\infty) = 1 - \exp(-\pi V_0^2/\lambda)$ if $\rho_{gg}(-\infty) = 1$ [see Eq. (85)]. Thus, if $\pi V_0^2 \gg \lambda$, the population of the ground state $g$ of the corresponding two-level system is exponentially small at $t \to +\infty$. With the help of our method for the estimation of the nonadiabatic correction, we can find the probability amplitudes of states $b_1$ and $c_1$ left by the interaction $\chi$ (coupling states $d$ and $b$) after the level crossing of the equivalent two-level system. These probability amplitudes allow to estimate the extent of the depopulation of state $d_1$. If the system adiabatically follows the $d_1$ state, the state-vector changes from $-|d\rangle$ to $+|d\rangle$. Since the development coefficient $C_d$ of the state vector in the $dbc$–basis corresponds to the $Z$–component of the Bloch-vector of the equivalent two-level system, the change of the $C_d$ sign means the flip of the $Z$–component. This flip means a population inversion of the two-level system, i.e., if before level crossing (at $t = -\infty$) the system is in the ground state $g$, after the level crossing (at $t = +\infty$) it is in the excited state $e$. We can estimate the amount of population left in the ground state, which is the nonadiabatic correction.

To find the nonadiabatic correction, we apply the $S_1$–transformation

$$
S_1 = \begin{bmatrix}
\sin \alpha_1 & 0 & -i \cos \alpha_1 \\
-i \cos \alpha_1 & 0 & \sin \alpha_1 \\
0 & 0 & 0
\end{bmatrix}
$$

(96)
to the Hamiltonian (25) with the parameters $B = 2\lambda t$ and $\alpha = 2V_0$. The result is

$$\mathcal{H}_{d_1b_1c_1} = \mathcal{H}_{d_1b_1c_1} + i \hat{S}_1 S^{-1}_1,$$

where

$$\mathcal{H}_{d_1b_1c_1} = -\sqrt{\Delta^2 + \chi^2} \left( \hat{P}_{b_1c_1} + \hat{P}_{c_1b_1} \right),$$

and

$$i \dot{S}_1 S^{-1}_1 = -\alpha_1 \left( \hat{P}_{b_1d_1} + \hat{P}_{d_1b_1} \right).$$

Here, for simplicity, we set the phases [see Eq. (1),(4)] $\varphi_1$ and $\varphi_2$ equal to zero. The development coefficients $C_{d_1}$, $C_{b_1}$, $C_{c_1}$ of the state vector $|\Phi_{d_1b_1c_1}\rangle$ in the $d_1b_1c_1$-basis satisfy equations that are similar to equations (27)–(29) if the substitution $Z_1 = -C_{d_1}$, $Y_1 = iC_{b_1}$ and $X_1 = C_{c_1}$ is made. These equations are

$$\dot{Z}_1 = -\alpha_1 Y_1,$$

$$\dot{Y}_1 = -\tilde{B} X_1 + \alpha_1 Z_1,$$

$$\dot{X}_1 = \tilde{B} Y_1.$$

The derivative of the mixing parameter $\alpha_1$ has an explicit form

$$\ddot{\alpha}_1 = \frac{\lambda}{V_0} \frac{1}{1 + \left( \frac{\Delta}{V_0}\right)^2}.$$}

This function has a bell shape with maximum value $\lambda/V_0$ at $t = 0$. The modified Rabi frequency $\tilde{B} = 2V_0 \sqrt{1 + (\lambda t/V_0)^2}$ has a minimum value $2V_0$ at $t = 0$ and increases to infinity at $t = \pm \infty$. Qualitatively, the time dependence of the interaction parameters resembles what we have for the STIRAP induced by the secant hyperbolic pulses, i.e., $B(t)$ and $\alpha(t)$. Therefore, we can apply the same procedure of the calculation of the nonadiabatic correction and the same approximations.

First, we approximate the Rabi frequency by its expansion in a power series of $(\lambda t/V_0)^2$, retaining only the first two terms

$$\tilde{B}(t) \approx \tilde{\beta}_0 \left( 1 + \tilde{g} t^2 \right),$$

where $\tilde{\beta}_0 = 2V_0$ and $\tilde{g} = (\lambda/V_0)^2/2$ [see Eq. (66)]. Rescaling time as $\tau = \lambda t/V_0$, we can define the nonadiabatic correction $f_{CA}(+\infty)$ as follows

$$f_{CA}(+\infty) = \int_{-\infty}^{+\infty} \frac{\cos \Omega(\tau)}{1 + \tau^2} d\tau,$$

where
\[ \Omega(\tau) = 2\eta(\tau + \frac{g_1}{3}\tau^3), \] (106)

and \( \eta = V_0^2/\lambda \), \( g_1 = 1/2 \) [compare the function \( \Omega(\tau) \) with the one in Eq. (67)]. The Fourier transform

\[ a_1(\omega) = \int_{-\infty}^{+\infty} d\tau \, \dot{\alpha}_1(\tau)e^{-i\omega\tau}, \] (107)

of the coupling parameter \( \dot{\alpha}_1(\tau) \) is

\[ a_1(\omega) = \pi \exp(-|\omega|). \] (108)

Following the same procedure as the one described in Section VII, we obtain

\[ f_{cA}(+\infty) = \int_{-\infty}^{+\infty} a_1(\omega)\eta^{-\frac{4}{3}} Ai\left[\eta^{-\frac{1}{3}}(2\eta + \omega)\right] d\omega. \] (109)

Employing the same approximation of the Airy function by equation (72) for positive arguments and taking into account only the negative wing of the function \( a_1(\omega) \), we limit the integration in Eq. (109) by the boundaries \((-2\eta, 0)\). These boundaries are taken to reduce the overestimation of the integrand introduced by the approximation applied to the Airy function and \( a_1(\omega) \) function. The arguments justifying this procedure are absolutely similar to those that are applied in Section IX to the case of the secant hyperbolic pulses.

After some algebra, we obtain

\[ f_{cA}(+\infty) = \pi Q_1 \exp\left(-\frac{5}{3}\eta\right), \] (110)

where

\[ Q_1 = \frac{1}{2} \left[ \text{erf}(h_{\text{max}}) + \text{erf}(h_{\text{min}}) \right], \] (111)

and

\[ h_{\text{max}} = \sqrt{2\eta \left[ \frac{2\sqrt{2}}{3} - \left( \frac{2}{3} \right)^{\frac{4}{3}} + \frac{1}{6} \right]}, \] (112)

\[ h_{\text{min}} = \sqrt{\frac{\eta}{3}}. \] (113)

This result can be obtained from Eqs. (87)–(90) by the straightforward substitution \( R = 1 \), \( \beta_0 = 2\eta \) and \( g = g_1 \).

We take \( Z_1(-\infty) = 1 \) or \( C_{d_1}(-\infty) = -1 \) and \( C_d(-\infty) = 1 \) as initial condition, which means that the reciprocal two-level system is initially in the ground state \( g \). The final state is defined by the relation \( C_d(+\infty) = C_{d_1}(+\infty) = -Z_1(+\infty) \). Since \( Z_1 \) keeps its sign, \( C_d \) changes sign. The population difference of the equivalent two-level system \( w = \rho_{gg} - \rho_{ee} \) is
related to \( C_d \) as \( w = C_d \). If \( C_d \) becomes negative, then \( \rho_{ee} > \rho_{gg} \). In this case, to estimate the population fraction left in the ground state \( g \), we have to use the expression

\[
\rho_{gg}(+\infty) = \frac{1 + C_d(+\infty)}{2} = \frac{1 - Z_1(+\infty)}{2}. \tag{114}
\]

After the substitution \( Z_1(+\infty) = \sqrt{1 - f_c^2(+\infty)} \) into Eq. (114) and expanding the square root in a power series, we find the nonadiabatic correction for the Rosen-Zener model

\[
\rho_{gg}(+\infty) \approx \frac{1}{4} f_{cA}(+\infty) = \frac{\pi^2 Q_1^2}{4} \exp \left( -\frac{10}{3} \eta \right). \tag{115}
\]

Figure 8 shows the comparison of our approximate solution, Eq. (115), with the exact solution of the Landau-Zener model

\[
\rho_{gg}(+\infty) = \exp (-\pi \eta). \tag{116}
\]

The difference is small and it originates from the approximations made. To estimate the accuracy of our result, we compare Eqs. (115) and (116) in logarithmic scale as follows

\[
\ln [\rho_{ee}(+\infty)] = -\eta \pi \approx -\eta \left[ \frac{10}{3} - \frac{2}{\eta} \ln \left( \frac{\pi Q_1}{2} \right) \right]. \tag{117}
\]

For large \( \eta \), the accuracy of the exponential factor calculation, expressed in the square brackets in Eq. (117), is 6%. 

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Fig. 1. The excitation scheme of the three-level atom by two resonant fields with interaction constants $B_1$ and $B_2$: $\Lambda$-scheme (a), and the same excitation scheme in the basis of dark $d$, bright $b$ and common $c$ states (b). $B = \sqrt{B_1^2 + B_2^2}$ is the generalized interaction constant for the Raman excitation. The vertical scale in diagram b is not the energy and represents the initial population of the levels before the excitation. The higher populated level is put at the bottom and the lower or unpopulated level is at the top.

Fig. 2. The bichromatic excitation scheme of the three-level atom in the $dbc$ basis if the state developments of $|d\rangle$ and $|b\rangle$ change in time. The coupling parameter of states $d$ and $b$ is $\hat{\alpha}$, the derivative of the mixing angle (see the text).

Fig. 3. (a) The pulse train with corresponding interaction parameters $B_1(t)$ and $B_2(t)$ evolving in time (bold lines). They are normalized by the maximum value $B_0$. The delay between pulses is $r\tau = 5$, $t_0 = 0$. The time is scaled in units of $r$. The time dependence of the mixing parameter $\alpha$ is shown by the thin line. The dashed line shows the dependence of the mixing parameter derivative $\hat{\alpha}$ normalized by $r$. In (b), the time dependence of the bichromatic Rabi frequency $\chi/2 = B(t)$ (solid line) and the mixing parameter derivative (dashed line) are shown for comparison.

Fig. 4. (a) The evolution of the amplitudes $X_c$ (plot a), $Y_b$ (plot b), $Z_d$ (plot c) for a pulse train $B_1(t)$, $B_2(t)$ with the parameters $B_0 = 42.8r$, $t_1 = 2.5/r$, $t_2 = -2.5/r$. Solid lines are the numerical solution of equations (27)–(29). Dashed lines are analytical approximations given by the first two terms of equations (46,47) and the first two terms in each of the parentheses of equation (55).

Fig. 5. (a) Comparison of the dependence of the adiabatic $\alpha_1(t_0)$ (dashed line) and nonadiabatic $f_c(+\infty)$ (solid line) parts of the analytical solution of Eqs. (27)–(29) on the maximum pulse amplitudes $B_0$. In (b), the comparison of the nonadiabatic part, numerically calculated using the actual Rabi frequency $B(t)$ (solid line) and its parabolic approximation, Eq. (66) (dashed line), are shown. $f_c(+\infty)$ is the true nonadiabatic part and $f_{cA}(+\infty)$ is an approximation.

Fig. 6. (a) Comparison of the dependencies of the nonadiabatic contributions $f_c(+\infty)$ (solid line) and $f_{cA}(+\infty)$ (dashed line) versus the pulse amplitude $B_0$. The first is calculated with and the second without taking into account the time dependence of the mixing parameter derivative $\hat{\alpha}(t)$. (b) The plots of the true, $f_c(+\infty)$, and the approximate, $f_{cA}(+\infty)$, nonadiabatic contributions versus $B_0$, where $f_{cA}(+\infty)$ (long dashed line) is calculated taking into account the time dependence of $\hat{\alpha}(t)$ using the approximation described in the text. (c) The same plots as in (b), except $f_{cA}(+\infty)$ (dot dashed line), where the correction factor $Q$ is dropped (see the text).

Fig. 7. Numerical solution of the Schrödinger equation (solid lines) for the amplitudes $X_c(t)$ (plot a), $Y_b(t)$ (plot b) and the approximation given by equations (87)–(88) (dashed lines) for the amplitudes $X_{cA}(t)$ (plot a), $Y_{bA}(t)$ (plot b).

Fig. 8. Comparison of the exact solution of the Landau-Zener model (dashed line) with our approximate solution (solid line) for the ground state population of the reciprocal two-level system after level crossing. The dependence of the exact solution of the Landau-Zener model for the ground state population, $\rho_{gg}(\eta)_{LZ}$, on the parameter $\eta = V_0^{\alpha}/\lambda$ is shown by the dashed line [see Eq. (116)]. The same dependence of our approximate solution, $[\rho_{gg}(\eta)]_A$,
is shown by the solid line. Our solution is given by Eq. (115).
Fig. 1
Fig. 2
Fig. 3 (a,b)
Fig. 4 (a-c)
Fig. 5 (a,b)
Fig. 7 (a,b)
Fig. 8

\[ \frac{[\rho_{gg}(\eta)]_A}{[\rho_{gg}(\eta)]_{LZ}} \]

\[ \eta \]

\[ \begin{array}{c}
0.1 \\
0.01 \\
1 \times 10^{-3} \\
1 \times 10^{-4} \\
1 \times 10^{-5} \\
1 \times 10^{-6} \\
1 \times 10^{-7} \\
1 \\
2 \\
3 \\
4 \\
5 \\
\end{array} \]