Landau Ginzburg theory of the d-wave Josephson junction

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Abstract

This letter discusses the Landau Ginzburg theory of a Josephson junction composed of on one side a pure d-wave superconductor oriented with the (110) axis normal to the junction and on the other side either s-wave or d-wave oriented with (100) normal to the junction. We use simple symmetry arguments to show that the Josephson current as a function of the phase must have the form $j(\phi) = j_1 \sin(\phi) + j_2 \sin(2\phi)$. In principle $j_1$ vanishes for a perfect junction of this type, but anisotropy effects, either due to a-b axis asymmetry or junction imperfections can easily cause $j_1/j_2$ to be quite large even in a high quality junction. If $j_1/j_2$ is sufficiently small and $j_2$ is negative, local time reversal symmetry breaking will appear. Arbitrary values of the flux would then be pinned to corners between such junctions and occasionally on junction faces, which is consistent with experiments on grain boundary junctions.

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A Josephson junction that is constructed with d-wave superconductor oriented with the (110) axis normal to the junction and either an s-wave superconductor or a d-wave superconductor oriented with the (100) axis normal to the junction has a reflection symmetry in the plane of the junction which suppresses the conventional Josephson coupling. Calculation involving all orders of the tunneling process shows that this leaves an anomalous part that is doubly periodic in the Josephson phase. This effect has been further explored by a number of authors through the Bogoliubov-de Gennes equations and interpreted in terms of Andreev levels.

A number of recent papers have independently explored the possibilities of exotic local order such as $(s + id)$ or $d_{x^2-y^2} + id_{xy}$ appearing near a bicrystal junction. These latter scenarios have suggested that coupling to these order parameters can be important in the vicinity of the junction and can lead to spontaneously broken time reversal near the junction. Different couplings and mechanisms have been proposed to explain the effect.

This paper shows that symmetry alone forces all these features to be present in the Landau Ginzburg theory of a pure $d_{x^2-y^2}$ superconductor without assuming the existence of any other local order parameters or any specific mechanism.

The geometry of the junction we will consider is given in Fig.[1]. Our detailed discussion will assume there are identical d-wave superconductors on both sides, but the arguments apply equally well to an asymmetric junction with s-wave on the left side and a d-wave on the right. (Fig.[2]) We will consider system with a uniform junction along the y-axis so the only spatial dependence is along x.

The free energy of the superconductor is given by a bulk term $F = F_{bulk} + F_{jct}$ where

$$F_{bulk} = \int_{-\infty}^{\infty} (-|\psi(x)|^2 + \frac{1}{2} |\psi(x)|^4 + |d\psi/dx|^2)$$

with the important terms in the junction energy given by $F_{jct} = \int \delta(x)f_{jct} dx$ where $f_{jct} =$

$$\alpha \left( |\psi_+|^2 + |\psi_-|^2 \right) - \beta (\psi_+^* \psi_- + CC) - \frac{1}{2}\gamma \left( (\psi_- \psi_+^*)^2 + CC \right)$$

We have normalized the length in units of coherence length, and the order parameter to its value at $\pm \infty$. We use the notation $\psi_\pm = \lim_{x \to 0_\pm} \psi(x)$ so that $\psi_\pm$ indicates the order parameter on either side of the junction. Other fourth order terms at the junction such as $|\psi_- \psi_+|^2$ can be neglected assuming “$\alpha$” is large compared to $|\psi_\pm|^2$.

The Josephson equations are then

$$0 = - \frac{d^2}{dx^2} \psi(x) - \psi(x) + |\psi(x)|^2 \psi(x)$$

$$0 = - \left( \frac{d}{dx} \psi(x) \right)_{x \to 0_-} + \alpha \psi_- - \beta \psi_+ - \gamma \psi_+^* (\psi_+)^2$$

$$0 = \left( \frac{d}{dx} \psi(x) \right)_{x \to 0_+} + \alpha \psi_+ - \beta \psi_- - \gamma \psi_-^* (\psi_-)^2$$

where the second and third term include a derivative from integration by parts in addition to terms explicitly derived from the junction free energy.

For most weakly coupled junctions, the “$\gamma$” term can safely be ignored, since $|\psi_\pm|^2$ in the junction is small and the $\beta$ term will totally dominate the Josephson coupling. However, when the junction has the symmetry of Fig. [1] or Fig [2], the junction becomes symmetric.
under reflections $y \rightarrow -y$. For this symmetry operation $(\psi_-, \psi_+) \rightarrow (\psi_-, -\psi_+)$ so that in this case $\beta$ must vanish and the fourth order term will determine the residual coupling.

For a symmetric junction, we can take $\psi_+ = e^{i\phi} \psi_-; multiplying the second term by $\psi_-$ and taking the imaginary part, we find that the Josephson current-phase relation is given by

$$j(\phi) = j_1 \sin(\phi) + j_2 \sin(2\phi)$$

where

$$j_1/j_2 = \beta / (\gamma |\psi_-|^2)$$

Demanding that $j(\phi) = 0$ implies that either $\phi = 0, \pi$ or $\cos(\phi_c) = -\beta / (2\gamma |\psi_-|^2)$. If the ordinary Josephson coupling term $\beta$ is sufficiently small, this equation will have a symmetric pair of nontrivial solutions. In the limit of weak Josephson current, we can approximate the boundary condition governing the magnitude of $\psi$ by $-\frac{d}{dx} |\psi_-| + \alpha |\psi_-| = 0$.

Comparing the junction free energies of the three solutions we find that $\delta F_{jct}$, the portion of the junction energy that varies with $\phi$ are

$$\delta F_{jct}(\phi = 0) = |\psi_-|^2 (-2\beta - \gamma |\psi_-|^2)$$

$$\delta F_{jct}(\phi = \pi) = |\psi_-|^2 (2\beta - \gamma |\psi_-|^2)$$

$$\delta F_{jct}(\phi = \phi_c) = \beta^2 / (2\gamma) + \gamma |\psi_-|^4$$

We see that if $\gamma < 0$, the anomalous solutions $\phi_c$ will dominate when $\beta$ is sufficiently small and the junction will favor a zero-current state with broken time reversal symmetry. In the limit $\beta = 0$ the current is $\pi$-periodic rather than $2\pi$-periodic as would be expected in a conventional Josephson junction. We note that these conclusions hold also for a junction where one side of the junction is a d-wave superconductor with the junction along the (110) axis and the other side of the junction is pure s-wave.

For a Josephson junction without the special inversion symmetry, the “$\beta$” term does not vanish by symmetry. However, when the relative crystal orientations are close to $\pi/4$ but microscopic arguments indicate that it may still be small. Whether or not this is actually the case can in principle be measured in a phase-current experiment.

In the experiments of Kirtley et al. triangular and hexagonal inclusions of crystal whose (100) is axis rotated by $\pi/4$ relative to the surrounding crystals had apparently arbitrary values of flux condensed at the corners. Our scenario suggests that for each such junction face there is a phase shift across each interface given by different values of $\phi_c$, which can take arbitrary values across the interface if $\gamma < 0$. The value of the flux condensed at each corner is simply the difference of the values of $\phi_c$ across the two faces that meet at the corner. There is also the possibility of flux being pinned at an arbitrary location on the interface boundary corresponding to a defect where the Josephson phase changes between the two degenerate solutions. The flux trapped on such a face can be any of the values $2n\pi \pm 2\phi_c$.

Whether or not this scenario can explain Kirtley’s experiments can be further tested experimentally through phase-current measurements across individual junctions with the same junction geometry that occur in individual faces in the Kirtley experiments. In order
to be consistent with the above scenario, a triangular inclusion that shows nonintegral flux condensation at two or more corners must have at least one face that independently shows a Josephson current with sufficiently large deviations from \( \sin(\phi) \) behavior to have zero current at two values of phase in addition to the solutions that are multiples of \( \pi \) that occur since the Landau-Ginburg equations are time reversal symmetric.

A phase-current experiment such as the one recently made by Il’ich [12] measures precisely Eq. (4). Eq. (5) suggests, however, that the most dramatic deviations from pure sinusoidal behavior could be quite difficult to reproduce from sample to sample, because the ratio \( j_1/j_2 \) depends on properties such as the barrier width and deviations from inversion symmetry in the junction plane which could be hard to control. The tendency of the system to generate broken local time reversal symmetry however, should be measurable through the sign of \( j_2 \) which must be negative for such symmetry breaking to occur.

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FIG. 1. The geometry of a symmetric $(0, \pi/4)$ Josephson junction. The system is symmetric under reflections through the $x$ axis while the order parameter on the right changes sign.

FIG. 2. An $s$-$d$ Josephson junction with reflection symmetry through the $x$-axis.