Two models for identification and predicting behaviour of an induction motor system

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Abstract. System identification or modelling is the process of building mathematical models of dynamical systems based on the available input and output data from the systems. This paper introduces system identification by using ARX (Auto Regressive with eXogeneous input) and ARMAX (Auto Regressive Moving Average with eXogeneous input) models. Through the identified system model, the predicted output could be compared with the measured one to help prevent the motor faults from developing into a catastrophic machine failure and avoid unnecessary costs and delays caused by the need to carry out unscheduled repairs. The induction motor system is illustrated as an example. Numerical and experimental results are shown for the identified induction motor system.

1. Introduction

System identification deals with the problem of building the mathematical models of dynamical systems according to the input and output data. This technique is important in a diversity of fields such as manufacturing process, communication, economics, and system dynamics and control. For an unknown system, it is mostly required to identify the system model before one can perform the control design. Shah et al. [1] proposed an algorithm which balances a data fidelity term with a norm induced by the set of single pole filters to identify the unknown system from noisy linear measurements. Smith [2] used a nuclear norm minimization based method for frequency domain subspace identification. The induction machines are widely used in industry, but the possibility of faults is unavoidable. Fault identification and diagnosis schemes are intended to provide advanced warnings of incipient faults, so that appropriate maintenance action can be taken at an early stage [3, 4]. In this paper, the induction motor system is identified by both ARX and ARMAX model methods. Through the identified system model, the predicted output could be compared with the measured one to help prevent the motor faults from developing into a catastrophic machine failure and avoid unnecessary costs and delays caused by the need to carry out unscheduled repairs.
2. Model Structures

A finite-dimensional, linear, discrete-time, time-invariant stochastic system can be expressed as:

\[ x_{k+1} = Ax_k + Bu_k + w_k, \quad y_k = Cx_k + v_k \]  

(1)

where \( x \in \mathbb{R}^{n_x} \), \( u \in \mathbb{R}^{n_u} \), \( y \in \mathbb{R}^{n_y} \) are state, input and output vectors, respectively; \([A,B,C]\) are the state-space system matrices. The sequences of process noise \( w \in \mathbb{R}^{n_w} \) and measurement noise \( v \in \mathbb{R}^{n_v} \) are assumed white, Gaussian, and zero mean. The noises \( w_k \) and \( v_k \) are also assumed uncorrelated with covariance \( Q \) and \( R \), respectively.

2.1. ARX Model

If one defines the error between the actual output \( y_k \) and the estimated output \( \hat{C} \hat{x}_k \) as residual \( \hat{e}_k \), equation (1) can also be expressed as

\[ \hat{x}_{k+1} = A\hat{x}_k + Bu_k + AK\hat{e}_k, \quad y_k = C\hat{x}_k + \hat{e}_k. \]  

(2)

where \( \hat{x} \) is the estimated state vector and \( K \in \mathbb{R}^{n_y \times n_x} \) is the steady-state Kalman filter gain. The relation between signal input and output with zero initial condition could be described as

\[ y_k = \sum_{i=1}^{\infty} C\bar{A}^{i-1}AKy_{k-i} + \sum_{i=1}^{\infty} C\bar{A}^{i-1}Bu_{k-i} + \hat{e}_k \]  

(3)

where \( \bar{A} = A - AK \). Since \( \bar{A} \) is asymptotically stable, \( \bar{A}^{i-1} \approx 0 \) if \( i \geq q \) for a sufficient large number \( q \) [5]. Thus, equation (3) becomes

\[ y_k = \sum_{i=1}^{q} a_i y_{k-i} + \sum_{i=1}^{q} b_i u_{k-i} + \hat{e}_k, \]  

(4)

where

\[ a_i = C\bar{A}^{i-1}K, b_i = C\bar{A}^{i-1}B, i = 1,2,...,q. \]  

(5)

The model described by equation (4) is the open-loop ARX model. \( q \) is the open-loop system ARX model order, and the ARX model parameters \( a_i \) and \( b_i \) can be estimated through least squares method. Suppose that there are \( N \) data points of \( y_k \) and \( u_k, k = 0,1,...,N-1 \), are given. The batch least squares solution for estimating the parameters \( a_i \) and \( b_i \) is

\[ \phi = YY^T(YY^T)^{-1} \]  

(6)

where \( Y = [y_0 \ y_1 \ ... \ y_q \ ... \ y_{N-1}] \), \( \phi = [b_1 \ a_1 \ ... \ b_q \ a_q] \)

\[ V = \begin{bmatrix} 0 & u_0 & ... & u_{q-1} & ... & u_{N-2} \\ 0 & y_0 & ... & y_{q-1} & ... & y_{N-2} \\ ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & u_0 & ... & u_{N-q-1} \\ 0 & 0 & 0 & y_0 & ... & y_{N-q-1} \end{bmatrix} \]  

(7)

The open-loop system Markov parameters \( Y_j(k) = CA^{k-j}B \), and the open-loop Kalman filter Markov parameters \( Y_j(k) = CA^{k-j}AK \), \( k = 1,2,...,q \) can be obtained from the coefficients \( a_i \) and \( b_i \)

\[ Y_j(k) = b_k + \sum_{i=1}^{k} a_i Y_j(k-i), Y_j(k) = a_k + \sum_{i=1}^{k-1} a_i Y_j(k-i) \]  

(8)

where \( Y_j(0) = 0 \) and \( Y_j(0) = I \) which is the identity matrix.

Then, system matrices \( [A,B,C] \) and Kalman filter gain \( K \) can be realized by the Eigensystem Realization Algorithm (ERA) [6].
2.2. ARMAX Model

A modified model structure is formulated to reduce the requirement of model order. To add and subtract the term \( M_y \) to the right-hand side of the state equation in equation (2) to yield

\[
\dot{x}_{k+1} = A\dot{x}_k + Bu_k + AK_e + MY_k - M_y_k = (A + MC)\dot{x}_k + (AK + M)e_i + Bu_k - M_y_k
\]

\[
y_k = C\dot{x}_k + e_k
\]

Now the new relationship of reference input and output becomes

\[
y_k = \sum_{i=1}^{N} C\tilde{A}^{-1}(-M)y_{k-i} + \sum_{i=1}^{N} C\tilde{A}^{-1}Bu + \sum_{i=1}^{N} C\tilde{A}^{-1}\tilde{M}e_{k-i} + e_k
\]

where \( \tilde{A} = A + MC, \tilde{M} = M + AK \)

Make \( \tilde{A} \) asymptotically stable, \( \tilde{A}^{-1} \approx 0 \) if \( i \geq p \) for a sufficient large number \( p \), equation (10) becomes

\[
y_k = \sum_{i=1}^{N} h_i y_{k-i} + \sum_{i=1}^{N} t_i u_{k-i} + \sum_{i=1}^{N} s_i e_{k-i} + e_k
\]

Equation (12) is an ARMAX model containing the dynamics of residual which is different from equation (4).

Define \( \Theta = [t_1, h_1, \ldots, t_p, h_p], \Psi = [s_1, s_2, \ldots, s_p] \),

\[
W = \begin{bmatrix} 0 & \varepsilon_0 & \varepsilon_1 & \ldots & \varepsilon_{p-1} & \varepsilon_{N-2} \\ 0 & 0 & \varepsilon_0 & \ldots & \varepsilon_{p-2} & \varepsilon_{N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & \varepsilon_0 & \varepsilon_{N-p-1} \end{bmatrix}, R = [e_0, \varepsilon_1, \ldots, \varepsilon_{N-1}] \]

where \( Y \) and \( \Psi \) are shown in equation (7). Then equation (12) could be written in a matrix form

\[
Y = \Theta V + \Psi W + R
\]

The least squares solution of the parameters \( \hat{\Theta} \) and \( \hat{\Psi} \) that minimizes the cost function \( J = \sum_{k=0}^{N-1} \varepsilon_k^2 \) is

\[
\begin{bmatrix} \hat{\Theta} \\ \hat{\Psi} \end{bmatrix} = Y[V^T W]^+ Y[V^T W]^{-1} W V^T W^{-1}
\]

The estimated parameters could also be expressed as

\[
\hat{\Theta} = Y\Lambda - \hat{\Psi} W \Lambda, \hat{\Psi} = Y(I - \Lambda V)W^T \hat{\Lambda}^{-1}
\]

Exercising the first term of \( \hat{\Theta} \) in equation (15) which is the ordinary least squares solution in open loop system; and in the second term one may consider it as a bias term. One can write, hence, \( \hat{\Theta} = \Theta^{LS} - \Theta^{bias} \)

where \( \Theta^{LS} = Y\Lambda = YV^T(VV^T)^{-1}, \Theta^{bias} = \hat{\Psi} W \Lambda = \hat{\Psi} W V^T (VV^T)^{-1} \).

The parameter matrix, \( \hat{\Psi} \), in Equation (15) could also be written as

\[
\hat{\Psi} = Y(I - \Lambda V)W^T \hat{\Lambda}^{-1}
\]

\[
= (Y - \hat{\Theta}^{LS} V)W^T [W V^T - W V^T (VV^T)^{-1} V W^T]^{-1}
\]

\[
= \hat{\varepsilon}^{LS} W^T [W V^T - W V^T (VV^T)^{-1} V W^T]^{-1}
\]

where \( \hat{\varepsilon}^{LS} \) denotes the colored residual between the measurement \( Y \) and the least squares estimation \( \Theta^{LS} V \).
2.3. Iterative Procedure for Identification

Step 1. An initial estimate of the parameter $\theta$, denoted by $\hat{\theta}^{LS}$, is computed from the ordinary least squares solution. It is assumed that an estimate of the parameter matrix, $\Psi$, denoted by $\hat{\Psi}_0$ is zero.

$$\hat{\theta}^{LS} = YV^T (VV^T)^{-1}, \hat{\Psi}_0 = 0.$$  

Step 2. Calculate the colored residual sequence $\hat{e}^{LS}$ which corresponds to the initial estimate ARMAX model parameter $\hat{\theta}^{LS}$. $\hat{e}^{LS} = Y - \hat{\theta}^{LS}V$. This is also an initial estimate of the white residual sequence denoted by $\hat{e}_0 = \hat{R}_0$.

Step 3. Construct the residual matrix $\hat{W}_0$ and then the parameter $\hat{\Psi}$ could be updated by equation (17)

$$\hat{\Psi}_1 = \hat{e}^{LS}\hat{W}_0^T [\hat{W}_0\hat{W}_0^T - \hat{W}_0V^T (VV^T)^{-1}V\hat{W}_0^T]^{-1}$$

The updated parameter $\hat{\Psi}_1$ is used to correct the initial bias as follows

$$\hat{\theta}_1^{bias} = \hat{\Psi}_1\hat{W}_0\Lambda = \hat{\Psi}_1\hat{W}_0V^T (VV^T)^{-1}, \hat{\theta}_1 = \hat{\theta}^{LS} - \hat{\theta}_1^{bias}$$

Step 4. Compute the new whitened residual sequence $\hat{R}_1$ by using the estimated parameters $\hat{\theta}_1$ and $\hat{\Psi}_1$, as follows: $\hat{R}_1 = Y - \hat{\theta}_1V - \hat{\Psi}_1\hat{W}_0$.

Step 5. Iterate the procedure from step 3 to step 4 by generating the new residual matrix $\hat{W}_1$ and using the updated parameters $\hat{\theta}_1$ and $\hat{\Psi}_1$. The next cycle is to calculate $\hat{\Psi}_2$, $\hat{\theta}_2^{bias}$, and $\hat{\theta}_2$.

After having obtained the ARMAX model estimated parameters $\theta$ and $\Psi$ [7], one can use the estimated coefficients to construct the open-loop system Markov parameter $\gamma(k) = CA^{k-1}B$, Kalman filter Markov parameters $Y_s(k) = CA^{k-1}AK$, and $Y_m(k) = CA^{k-1}M$, $k = 1,2,...,p,p+1...$, as follows:

$$\gamma(k) = t_k + \sum_{i=1}^{k} h_sY_s(k-i), Y_m(k) = -h_k + \sum_{i=1}^{k-1} h_sY_m(k-i),$$

$$Y_s(k) = h_k + s_k + \sum_{i=1}^{k-1} h_sY_s(k-i)$$

$\{A, B, C, M, K\}$ can also be realized by the Eigensystem Realization Algorithm (ERA) [6].

3. Experimental Setup

The experimental apparatus used in this study consists of a 3-hp, 1800-rpm (i.e. $f_r = 30$ Hz), 4-pole three phase induction machine driving a 5-kW DC generator via a flexible coupling, as shown in figure 1. The generator is used to absorb the energy generated by the motor. A piezo-electric accelerometer is mounted on the housing of the induction electrical machine to measure the vibration signal. Three current sensors were used to record the three phase current signals from the inverter. All measured signals were sampled at 6 kHz via a real-time data acquisition device (Type NI 6062E) under the motor no-load condition. The frequency of inverter varies from 1 Hz to 60 Hz.
4. Numerical Validation

The inputs consist of three currents into the motor and the output is the vibration signal from the accelerometer. Each data set contains 122,880 points. Two sets of input-output data are recorded, one set is implemented for system identification, and the other is used for comparison between the predicted outputs and the true one. Figure 2 shows new test data (red) and predicted outputs of ARX (blue) and ARMAX (green) from 8,000 to 8,500 points. (Here, both identified models use model order=15, realized states=4 and the number of iterations for ARMAX=4.) Figure 3 shows the frequency plot of the real (blue) and predicted outputs (ARX (blue) and ARMAX (green)) from Fourier Transform.

Figure 1. Experimental setup.

Figure 2. New test data (red), ARX (blue) and ARMAX (green) predicted outputs from 8,000 to 8,500 points.
Figure 3. The frequency plot of the real (red), ARX (blue) and ARMAX (green) predicted outputs from Fourier Transform.

5. Conclusion
In this paper, both the ARX and ARMAX system models show an effectiveness approach for identifying the induction motor system. The ARMAX model has a better result compared with the ARX model when the model order is small; however, there is not much difference between these two when the model order is large enough. Actually, for the cases with high level of noise, the most significant error is from the original estimated parameters which have been corrupted by the disturbance. Hence, it is desirable to design a filter that can reduce the influence of disturbances so that the system identification will have a more accurate model.

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