Statistical Model Description of $K^+$ and $K^-$ Production between 1 - 10 $A$·GeV

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Abstract

The excitation functions of $K^+$ and $K^-$ mesons in heavy ion collisions are studied within a statistical model assuming chemical and thermal equilibrium with exact strangeness conservation. At low incident energies the associate production of kaons, i.e. the production of a $K^+$ together with a hyperon and the production of a $K^-$ together with a $K^+$, implies specific features: different threshold energies and different dependences of $K^+$ and $K^-$ yields on baryon number density. It is shown that the experimentally observed equality of the $K^+$ and $K^-$ rates at energies $\sqrt{s} - \sqrt{s_{th}} \leq 0$ is due to a crossing of the two excitation functions. Furthermore, the independence of the $K^+$ to $K^-$ ratio on the number of participating nucleons observed at 1 and 10 $A$·GeV is consistent with this model.
Central heavy ion collisions at relativistic energies present an ideal tool to study nuclear matter at high densities and high temperatures. However, these collisions are complex and in order to interpret the results, two strategies are commonly used: (i) to describe the time evolution of the collisions using transport models and (ii) to use a statistical concept assuming thermal and chemical equilibrium and common freeze out parameters for all particles.

In this Letter the second procedure, the statistical concept, will be followed. Of special interest here is the production of $K^+$ and $K^-$ below and above the respective $NN$ thresholds. The experimental results have attracted much interest as the measured $K^+$ to $K^-$ ratios in heavy ion collisions differ strongly from the ratios obtained in $NN$ reactions [1,2]. These findings have lead to the proposal that in heavy ion collisions the “effective masses” of $K^+$ and $K^-$ are changed as predicted for dense nuclear matter. The aim of this Letter is to discuss the $K^+$ and $K^-$ production within a statistical model. This model describes the condition at freeze out using masses of free particles.

The production of strange particles has to respect strangeness conservation. The attempts to describe the measured particle ratios including strange hadrons at AGS and SPS using a strangeness fugacity $\lambda_s$ is quite successful [3–7]. However, the usual grand-canonical treatment is not sufficient, if the number of strange particles is small [8]. This requires exact strangeness conservation which is done in the statistical model using the canonical formulation of strangeness conservation [9]. Consequently, the abundance of $K^+$ mesons is suppressed since together with each $K^+$ also another strange particle, e.g. a $\Lambda$ hyperon is produced via $NN \rightarrow N\Lambda K^+$. And for $K^-$ the corresponding channel is $NN \rightarrow NNK^-K^+$. While the pion multiplicity per $A_{\text{part}}$ is approximately given by a simple Boltzmann factor (neglecting resonance contributions and isospin asymmetry),

$$\frac{M_\pi}{A_{\text{part}}} \sim \exp \left( -\frac{E_\pi}{T} \right),$$  

(1)

the multiplicity of positively charged kaons is given by

$$\frac{M_{K^+}}{A_{\text{part}}} \sim \exp \left( -\frac{E_{K^+}}{T} \right) \left[ g_\Lambda V \int \frac{d^3p}{(2\pi)^3} \exp \left( -\frac{(E_\Lambda - \mu_B)}{T} \right) \right],$$  

(2)

with the temperature $T$, the baryo-chemical potential $\mu_B$, the degeneracy factors $g_i$, the volume $V$ (see [9]) and the energies $E_i$ of the particles $i$ and integrating over momentum $p$.

The formula above, simplified for demonstration purpose, neglects higher order terms in $V$ [9], quantum statistics and other processes leading to the production of $K^+$. The corresponding formula for $K^-$ production is similar, but does
not depend on $\mu_B$,

$$\frac{M_{K^-}}{A_{\text{part}}} \sim \exp \left( -\frac{E_{K^-}}{T} \right) \left[ g_{K^+}V \int \frac{d^3p}{(2\pi)^3} \exp \left( -\frac{E_{K^+}}{T} \right) \right]. \quad (3)$$

From Eqs. (1) - (3) it is obvious that the exact strangeness conservation implies a reduction of $K^+$ and $K^-$ yields as compared to the values calculated without exact strangeness conservation [9].

In addition, since the volume in Eqs. (2) - (3) is proportional to the number of participants $A_{\text{part}}$, the $K^+$ and $K^-$ multiplicities are expected to rise (for low $T$ and small $V$) quadratically with $A_{\text{part}}$ while $M_\pi$ increases linearly with $A_{\text{part}}$. These properties are in remarkable agreement with the experimental observations [9–11].

The measured yields (or particle ratios) can be described in this statistical concept by lines in the $T$ and $\mu_B$ plane. All particle ratios measured around 1 A-GeV (besides $\eta/\pi_0$) intersect within the experimental errors reflecting common values for $T$ and $\mu_B$ for all particles at freeze out [9]. Surprisingly, even the measured $K^+/K^-$ ratio fits into this picture and the calculated ratio does not depend on the choice of the volume $V$. However, the $A_{\text{part}}$ dependence enters in the ratios of strange to non-strange particles, e.g. in $K^+/\pi^+$. 
Figure 1 shows the $K^+/K^-$ ratios measured by the KaoS Collaboration [1,2], by the FRS Group [12] and by the E866/E917 Collaboration at AGS [13] as a function of $\sqrt{s}$. To obtain the theoretical results, shown as dashed line, we start from the universal freeze-out curve suggested in [14]. Together with the measured systematics for the pion multiplicities, relations for $T$ and $\mu_B$ as functions of $\sqrt{s}$ are obtained [15]. Within this approach, the $K^+/K^-$ ratios are given as a dashed line in Fig. 1. The observed rise towards low incident energies reflects the fact that the two kaon species have different threshold energies due to their associate production.

The $K^+/K^-$ ratios measured in heavy ion collisions by the KaoS Collaboration show that the $K^-$ yield compared to the $K^+$ cross section is much higher than expected from $NN$ collisions [1,2]. This is especially evident, if the kaon multiplicities are plotted as a function of $\sqrt{s} - \sqrt{s_{th}}$ where $\sqrt{s_{th}} - 2m_N$ is the energy needed to produce the corresponding particles taking into account the mass of the produced partners ($\sqrt{s_{th}}(K^+)-2m_N = 0.67$ GeV, $\sqrt{s_{th}}(K^-)-2m_N = 0.987$ GeV). The measured $K^+$ and $K^-$ yields in heavy ion collisions are about equal for $\sqrt{s} - \sqrt{s_{th}} \leq 0$ while the $K^+$ yield in $NN$ collision exceeds the $K^-$ yields by a factor of 10 – 100 close to threshold.

In Fig. 2 we show in the upper part the multiplicities of $K^+$ and $K^-$ divided by $A_{part}$ as a function of $\sqrt{s} - \sqrt{s_{th}}$ over a large energy range from SIS up to AGS. The full and dashed lines refer to the statistical model results for $K^-$ and $K^+$ respectively. At values of $\sqrt{s} - \sqrt{s_{th}}$ less than zero the two excitation functions cross. They differ at AGS energies by a factor of five which is in good agreement with the result for central collisions of Au+Au at 10.8 A-GeV [16]. The model calculations depend on the choice of the system, here Ni+Ni collisions. At SIS energies, only inclusive measurements for Ni+Ni are available. The values for $K^+$ are from Ref. [1]. The results for $K^-$ are from Ref. [12] and corrected for the angular distribution [17]. $A_{part}$ is chosen as $A$ which is based on estimates from the mean $A_{part}$ for $K$ production as kaons originate more from central collisions. At AGS energies the choice of the system has little influence which allows to plot the results for Au+Au collisions at 10.8 A-GeV as well. This figure evidences that the similarity of the $K^+$ and $K^-$ yield observed around $1 - 2$ A-GeV arises from the difference in the rise of the two excitation functions. This difference can be understood by the approximate formulae given in Eqs. (2) and (3). The density of $K^+$ contains the term $E_k - \mu_B$ while the $K^-$ density has $E_{K^+}$ in the exponent. As these two values are different, the excitation functions, i.e. their variation with $T$, exhibit different slopes.

Furthermore, Eqs. (2) and (3) evidence that for a low temperature $T$ and a small volume $V$ the dependence of the $K^+$ and $K^-$ multiplicity on $A_{part}$ is quadratic which is in very good agreement with data [9–11]. Small variations from the quadratic dependence can occur due to a change of $T$ and $\mu_B$ with
Fig. 2. Upper part: Calculated $M_K/A_{\text{part}}$ ratios in the statistical model as a function of $\sqrt{s} - \sqrt{s_{\text{th}}}$. The dashed (solid) line refers to $K^+ (K^-)$. Open (full) symbols represent measured $K^+ (K^-)$ multiplicities. Lower part: $K^+/K^-$ ratios from the excitation functions above together with results from Ni+Ni collisions ($\times$) at SIS energies and Au+Au at AGS energies ($\diamond$).

It is interesting to note that also hydrodynamical models predict a variation of $K^+$ and $K^-$ with $A_{\text{part}}$ [18,19]. Transport models, on the other hand, show an increase with $A_{\text{part}}^\alpha$ where $\alpha$ is approximately 1.4 - 1.6 [20]. In these models the kaons are produced in multi-step processes which are more likely in central collisions where the density is higher. As already mentioned, the statistical model predicts that the variation of the $K^+$ and of the $K^-$ yields with $A_{\text{part}}$ are equal. Hence, for a given collision the $K^+/K^-$ ratio is expected not to vary with centrality. Indeed, this is in accordance with the data for Au+Au collisions at 10.2 A·GeV [21]. Figure 3 shows the results together with the prediction of the statistical model. It has even been observed at SIS energies for Ni+Ni collisions at 1.93 A·GeV [17]. This is remarkable as the $K^+$ production is above and the $K^-$ production below their respective $NN$ thresholds.
Fig. 3. Ratio of $K^+$ to $K^-$ as a function of the number of participants $A_{part}$ from [21] together with the statistical model (dashed line) evidencing the independence of $A_{part}$.

In summary, the statistical model using exact strangeness conservation is able to describe most of the measured particle ratios from SIS up to SPS energies. Within this framework the equality of $K^+$ and $K^-$ multiplicities as a function of $\sqrt{s} - \sqrt{s_{th}}$ is a consequence of two excitation functions with different slopes crossing at values $\sqrt{s} - \sqrt{s_{th}}$ below zero. This model is also able to describe the dependence of the kaon yields on $A_{part}$ being quadratic around 1 $A\cdot$GeV. This effect fades away with increasing incident energy. The $K^+/K^-$ ratio is predicted in the considered model as being independent of $A_{part}$ and this is, indeed, observed from SIS up to AGS energies.

The statistical model presented here uses a unique freeze out for all particles. Detailed experimental studies on pion production show evidence for a time evolution of the pion emission with high-energy pions being emitted earlier [24]. Such effects, however, are not visible on the level of total particle multiplicities since these involve integrals over the whole phase space. Despite the apparent success of the statistical model of particle production under the assumption of thermal and chemical equilibration and using masses of free particles, the present understanding of hadronic interactions contradicts chemical equilibrium for strange particles [22,23] This discrepancy seem to put into question our present understanding of interactions at the high densities reached in heavy ion collisions. Indeed, already at and above twice nuclear matter densities, nucleons are hardly “free” individual particles. This “in-medium” effect clearly deserves further studies.

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