Research on Heat Transfer in High Temperature Garments

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Abstract. This paper describes the Law of heat transfer between the layers of material and a method for determining the thickness of layers of material parameters of a high temperature protective clothing. Three important choice of protective clothing material and the air gap layer as the research object, based on the Fourier law and the energy conservation law to establish second order partial differential equation, solving the Crank-Nicolson method by second order partial differential equation, derived temperature, time, the relationship between the thickness. Finally, in the case where the heat transfer ANSYS software simulation under the above conditions, a comparative analysis with calculated transfer law, the error is 1.2%, higher accuracy results.

Keywords: Fourier’s law, Heat conduction equation, Clothing Design, ANSYS

1. Introduction

1.1. Problem Background

When working in a high temperature environment, people often need to wear special clothing to prevent burns. Special clothing is generally composed of three layers of textile material, i.e. I, II, III layer, wherein the I layer in direct contact with the external environment, there is a gap between the skin layer and the III, IV this gap is referred to as a layer. In order to design special clothing, the body temperature was controlled at 37°C in a laboratory mannequin is placed in a high temperature environment, to measure the temperature of the outer skin of the human body model. In order to
reduce development costs, shorten the development cycle, often we need to use a mathematical model to determine the temperature change of the outer skin of the manikin.

1.2. Temperature Distribution Model

The results herein use of existing data, the establishment of second order partial differential equation, solving the energy conservation law and the law of the Fourier based on the establishment of second order partial differential equation, by solving the second order partial differential equations Crank-Nicolson method to give a relationship between temperature, time and thickness.

1.3. Simulation

The temperature distribution of the four-layer material is simulated by ANSYS software after the system is stabilized, and the accuracy of the model is verified by comparing with the results calculated by the model.

2. Models and Methodology

2.1. Problem Analysis

During hot working, the inner garment is actually three-dimensional spatial temperature profile. Since air convection without considering the impact body and volume, so this problem is converted into a one-dimensional problem only related to the material thickness. As shown in Figure 2.1

![Problem Simplification Schematic](image)

Figure 1. Problem Simplification Schematic

Where: I, II and III are three-layer fabric material clothing, IV is an air layer between the skin layer and the iiiilayer, which is in direct contact with the outside world.

2.2. Basic Assumptions

- Four homogeneous medium, and remain isotropic.
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- The thermal conductivity of each of the same medium in all directions.
- In a fourth dielectric layer, the air convection is not considered.
- No external radiation.

2.3. Partial Differential Equation of Second Order

(i) Derivation of Second Order Partial Differential Equations

Firstly, the spatial oxyz Cartesian coordinate system, and remove a parallelepiped unit heat medium in the analysis, as shown in Figure 2.

According to the law of conservation of energy, The following thermal equilibrium exists at any time interval:

\[ dQ_{in} + dQ_{ge} = dQ_{out} + dQ_{add} \]  \hspace{1cm} (1)

where: \( dQ_{in} \) is the total heat flowing into the unit, \( dQ_{ge} \) is the total heat generated by the heat source in a unit, \( dQ_{add} \) is the total heat generated by the heat source in a unit.

Suppose \((\phi_x)_x\) is the value of heat flux component \((\phi_x)\) at \(x\), \((\phi_x)_{x+dx}\) is the value of heat flux component \((\phi_x)\) at \(x+dx\), similarly the x and z directions. By the Fourier’s law, to give the total heat flow into the cell is:

\[
\begin{align*}
    dQ_{in} &= dQ_{inx} + dQ_{iny} + dQ_{inz} \\
    dQ_{inx} &= (\phi_x)_x = -\lambda \left( \frac{\partial T}{\partial x} \right)_x dydz \\
    dQ_{iny} &= (\phi_y)_y = -\lambda \left( \frac{\partial T}{\partial y} \right)_y dxdz \\
    dQ_{inz} &= (\phi_z)_z = -\lambda \left( \frac{\partial T}{\partial z} \right)_z dxdy
\end{align*}
\]  \hspace{1cm} (2)

The total heat flow out of the unit is:

\[ dQ_{in} = dQ_{inx} + dQ_{iny} + dQ_{inz} \]  \hspace{1cm} (3)
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\[
\begin{align*}
\frac{dQ_{outx}}{dx} &= (\phi_x)_x + \frac{\partial \phi_x}{\partial x} dx = (\phi_x)_x + \frac{\partial}{\partial x} \left( -\lambda \frac{\partial T}{\partial x} dy dz \right) dx \\
\frac{dQ_{outy}}{dy} &= (\phi_y)_y + \frac{\partial \phi_y}{\partial y} dy = (\phi_y)_y + \frac{\partial}{\partial y} \left( -\lambda \frac{\partial T}{\partial y} dx dz \right) dy \\
\frac{dQ_{outz}}{dz} &= (\phi_z)_z + \frac{\partial \phi_z}{\partial z} dz = (\phi_z)_z + \frac{\partial}{\partial z} \left( -\lambda \frac{\partial T}{\partial z} dx dy \right) dz
\end{align*}
\]

The total heat generated by the heat source in the unit is:

\[dQ_{ge} = \varphi dx dy dz\]  \hspace{1cm} (6)

where: \(\varphi\) is the internal body per unit time in the heat source generates heat. The increment of internal energy of the unit is:

\[dQ_{add} = \rho c \frac{dT}{dt} dx dy dz\]  \hspace{1cm} (7)

where: \(\rho\) is the density of the unit body, \(c\) is the specific heat capacity of the unit body.

Substitute formula (2)(4)(6)(7) for formula (1), the general form of three-dimensional unsteady heat conduction differential equation in Cartesian coordinate system is obtained as follows:

\[\rho c \frac{dT}{dt} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda \frac{\partial T}{\partial z} \right) + \varphi\]  \hspace{1cm} (8)

In this problem, since the layers within the medium does not contain the heat source, so that \(\varphi=0\). And simplified model, each of the parallel layered considered infinite plane, considering only the temperature distribution in the X direction, the second order is simplified model partial differential equation is:

\[\frac{\partial T}{\partial t} - k \frac{\partial^2 T}{\partial x^2} = 0\]  \hspace{1cm} (9)

Where: \(k\) is the thermal diffusion coefficient of the medium, the expression is:

\[k = \frac{\lambda}{\rho c}\]  \hspace{1cm} (10)

where: \(\lambda\) is the thermal conductivity of textile material, \(\rho\) is the density of the textile material, \(c\) is the specific heat capacity of the fabric material.

Since the problem of heat conduction in a different textile material through four layers (layers I, II, III for clothing, and an air layer), the K segment should be expressed as:

\[
\begin{align*}
\lambda_1 & / \rho_1 c_1, \quad 0 < x < 0.6 \\
\lambda_2 & / \rho_2 c_2, \quad 0.6 < x < 6.6 \\
\lambda_2 & / \rho_3 c_3, \quad 6.6 < x < 10.2 \\
\lambda_2 & / \rho_4 c_4, \quad 10.2 < x < 15.2
\end{align*}
\]  \hspace{1cm} (11)
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Where: \(k_i (i = 1, 2, 3, 4)\) is NO.i layer material of constant fabric material. Second order partial differential equation model segment is thus obtained:

\[
\frac{\partial T}{\partial t} - k_i \frac{\partial^2 T}{\partial x^2} = 0 \quad i = 1, 2, 3, 4
\]  \(12\)

2.4. Determine the Relevant Conditions

2.4.1. Initial Conditions Initial time and temperature of the human body model consistent four-layer medium:

\[
T|_{t=0} = 37^\circ C
\]  \(13\)

2.4.2. Boundary Conditions The outer environment and the human body model as a constant temperature fluid environment, and between the I layer and the inner layer IV ambient thermal convection between the outside environment respectively, whereby the third boundary condition is determined:

(i) Left boundary condition (Dirichlet condition):

\[
-\lambda_1 \frac{\partial T}{\partial x}|_{x=0} = h_1 (T_{\text{outside}} - T|_{x=0})
\]  \(14\)

(ii) Right boundary condition (Robin condition):

\[
-\lambda_4 \frac{\partial T}{\partial x}|_{x=15.2} = h_1 (T_{15.2} - T|_{\text{inside}})
\]  \(15\)

The heat transfer coefficients of I and IV are \(h_1\) layer dielectric surface and \(h_4\) layer dielectric surface respectively.

2.4.3. Interface Conditions

(i) Temperature continuous conditions

\[
\lim_{x \to x_i^-} T = \lim_{x \to x_i^+} T (i = 1, 2, 3)
\]  \(16\)

(ii) Continuous heat flow density conditions

\[
\lim_{x \to x_i^-} \lambda_i \frac{\partial T}{\partial x} = \lim_{x \to x_i^+} \lambda_{i+1} \frac{\partial T}{\partial x} (i = 1, 2, 3)
\]  \(17\)

2.5. Determination of Surface Heat Transfer Coefficient

Using the idea of fitting to get \(h_1\) and \(h_4\). Assuming that the first temperature observation value is \(T_i\), the temperature value obtained by solving is recorded as \(T_i'\), and the Euclidean distance \(L\) is used as an index to evaluate the goodness of fit. The expression is as follows:

\[
l = \sqrt{\sum_{i=1}^{n} (T_i - T_i')^2}
\]  \(18\)

The \(h_1\) and \(h_4\) corresponding to the minimum Euclidean distance are the actual surface heat transfer coefficients.
3. Solution Based on Difference Method

The basic idea of difference method is: instead of using finite difference differential, instead of derivatives by finite difference quotient. Difference format into explicit and implicit, taking into account the implicit scheme converge stably solution process, it is solved using an implicit differential equation format.

Step1: Mesh generation for solving region: \( \Omega = \{(x, t)|0 \leq x \leq l, 0 \leq t \leq \tau \} \)

(i) Partition in the X direction:
Divide the layer \(i(i = 1, 2, 3, 4)\) medium into \(m_i\) and \(\Delta x_i\) is the step size of the layer I medium.

(ii) Partition in the t direction:
The interval \((0, \tau)\) is divided into \(f\) and \(\Delta t\) is the time step.

The partitioned grid is shown in Figure 3.

![Figure 3. Grid schematic](image)

Step2: Establish implicit difference scheme

(i) Second order partial differential equations:
Using the backward difference scheme of time \(t_{n+1}\) and the second-order central difference scheme of position \(x_{i,j}\), the following equations can be obtained:

\[
-rT_{n+1}^{(i,j)} + (1 + 2r)T_{n+1}^{(i+1,j)} - rT_{n+1}^{(i,j)} = T_n^{(i,j)}
\]

where: \( r = \frac{k_i \Delta t}{(\Delta x)^2} \)

(ii) Boundary conditions:

- Left boundary condition
The following equations can be obtained by forward difference of position \(x_{1,0}\):

\[
(h_1 + \frac{\lambda_1}{\Delta x_1})T_{n+1}^{(1,0)} - \frac{\lambda_1}{\Delta x_1}T_{n+1}^{(1,1)} = 75h_1
\]

(20)

- Right boundary conditions
The following equations can be obtained by backward difference of position \(x_{4,m_4}\):

\[
-\lambda_4 \frac{T_{n+1}^{(4,m_4)} - T_{n+1}^{(4,m_4-1)}}{\Delta x_4} = h_4(T_{n+1}^{(4,m_4)} - 37)
\]

(21)
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(iii) Continuous heat flux density conditions: Taking the continuity condition of heat flux density at the interface of I and II as an example, the following equations can be obtained by using forward and backward difference of position:

\[
\frac{\lambda_1}{\Delta x_1} T^{n+1}_{(1,m_1-1)} - \left( \frac{\lambda_1}{\Delta x_1} + \frac{\lambda_2}{\Delta x_2} \right) T^{n+1}_{(I,II)} + \frac{\lambda_2}{\Delta x_2} T^{n+1}_{(2,1)} = 0
\]  

(22)

Step3: Solution of Surface Heat Transfer Coefficient

Using simulated annealing particle swarm optimization to search for \( h_1 \) and \( h_4 \). The annealing process is shown in Figure 4.

\[ \text{Figure 4. Fitness curve of simulated annealing algorithm} \]

It can be seen from the graph that the simulated annealing particle swarm optimization algorithm has better convergence and robustness, and the solution results are more accurate. Finally, we can get \( h_1=122.3573, h_4=8.413 \).

Step4: Solution of Second Order Partial Differential Equations equations

By solving the equations in Step2 with the help of MATLAB, the temperature distribution is obtained as shown in Figure 5.

\[ \text{Figure 5. Temperature pattern} \]
From Figure 5, it can be seen that the temperature of heat source decreases gradually from high temperature to low temperature; from the beginning, with the progress of thermal diffusion, the temperature of each location increases gradually, and reaches thermal equilibrium after a period of time, and the temperature of each location remains unchanged.

4. Simulation

To test the accuracy of our model, we use the software ANSYS the heat transfer results in the simulation conditions, as shown in Figure 6.

![Thermodynamic simulation diagram based on ANSYS](image)

Figure 6. Thermodynamic simulation diagram based on ANSYS

5. Results and Analysis

In order to test the error between our model and the actual situation, we compare the temperature results of the stabilized four-layer material with the simulation results and find the relative error of the model, as shown in Table 1.

From Table 1, we can see that the temperature obtained by solving the four positions

| Number of layers | I       | II      | III     | IV       |
|------------------|---------|---------|---------|----------|
| Solved temperature | 73.6714 | 72.1615 | 64.7127 | 48.0859  |
| Simulated temperature | 73.6545 | 69.3481 | 63.0146 | 47.4858  |
| Error            | 0.2%    | 4.1%    | 2.7%    | 1.3%     |

is smaller than the temperature of the four positions obtained by the simulation, indicating that the model has higher accuracy.
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