DRAFT: TOWARDS REDUCED ORDER MODELS OF SMALL-SCALE ACOUSTICALLY SIGNIFICANT COMPONENTS IN GAS TURBINE COMBUSTION CHAMBERS

Suhas A Kowshik,
Indian Institute of Science
Bengaluru, India
Email: suhasakowshik@gmail.com

Sumukha Shridhar
RV College of Engineering
Bengaluru, India
Email: sumukhas.me15@rvce.edu.in

N. C. W. Treleaven
STFS, TU-Darmstadt
Darmstadt, Germany
Email: treleaven@stfs.tu-darmstadt.de

ABSTRACT
Gas turbine combustion chambers contain numerous small-scale features that help to dampen acoustic waves and alter the acoustic mode shapes. This damping helps to alleviate problems such as thermoacoustic instabilities. During computational fluid dynamics simulations (CFD) of combustion chambers, these small-scale features are often neglected as the corresponding increase in the mesh cell count augments significantly the cost of simulation while the small physical size of these cells can present problems for the stability of the solver. In problems where acoustics are prevalent and critical to the validity of the simulation, the neglected small-scale features and the associated reduction in overall acoustic damping can cause problems with spurious, non-physical noise and prevents accurate simulation of transients and limit cycle oscillations. Low-order dynamical systems (LODS) and artificial neural networks (ANNs) are proposed and tested in their ability to represent a simple two-dimensional acoustically forced simulation of an orifice at multiple frequencies. These models were built using compressible CFD, using OpenFOAM, of an orifice placed between two ducts. The acoustic impedance of the orifice has been computed using the multi-microphone method and compared to a commonly used analytical model. Following this, the flow field downstream of the orifice has been modelled using both a LODS and ANN model. Both methods have shown the ability to closely represent the simulated dynamical flows at much lower computational cost than the original CFD simulation. This work opens the possibility of models that can dynamically predict the flow through, for instance, acoustic liners, dilution ports and fuel injectors in real engines during thermoacoustic instabilities without having to mesh and simulate these small-scale features directly. Such models may also assist in the accurate simulation of flame quenching due to cooling flows or the design of effusion cooled aerodynamic surfaces such as nozzle guide vanes (NGVs) and turbine blades.

NOMENCLATURE

- $Z$: Impedance.
- $K_R$: Rayleigh Conductivity.
- $D$: Diameter of the orifice.
- $L$: Length of the orifice.
- $\omega$: Cyclic frequency.
- $c$: Speed of sound.
- $I_1$: Bessel function of first kind.
- $K_1$: Bessel function of second kind.
- $St$: Strouhal number.
- $\nu$: Kinematic viscosity.
- $R$: Reflection coefficient.
- $T$: Transmission coefficient.
- $\alpha$: Absorption coefficient.
- $\tau_{vis}$: Loss coefficient.
- $\hat{\cdot}$: Complex magnitude of fluctuating component.
- $\vec{\cdot}$: Fluctuating component.
- $\bar{\cdot}$: Mean of the component.
INTRODUCTION

Gas turbine combustion systems remain susceptible to thermoacoustic instabilities that can lead to customer discomfort, or component fatigue and failure. These instabilities are caused by a feedback loop between the flame heat release rate and acoustic waves inside the combustor. Instabilities occur often at frequencies that correspond to an acoustic eigenmode of the system. Modelling strategies often use a divide and conquer approach which aims to separate the acoustic field from the convective and reflecting field. The acoustic field is solved using linearised Navier-Stokes [1], Euler [2] or Helmholtz [3] solvers or low-order acoustic network models [4]. These models often assume that only components of the same order of magnitude in size as the acoustic wavelength are important and so large cell sizes are used to reduce the cell count of the mesh and speed up convergence.

Strategies for reducing the prevalence of instabilities include de-tuning the flame-combustor system by seeking either to alter the shape and/or frequency of the acoustic modes in the combustor [5], or by seeking a change in the system damping. The Rayleigh criterion [6] gives a measure of whether acoustic energy will be generated or damped by the flame and is ultimately dependent on the phase difference between the flame heat release fluctuations and the acoustic pressure. The stability of the combustion chamber is not just dependent on the value of the Rayleigh criterion however, it is also dependent on how much acoustic energy is absorbed by small scale components such as cooling holes and acoustic liners. There is an open question as to how to model these small scale features: They affect the acoustics but because they are so much smaller than the acoustic wavelength, small scale features, such as cooling holes, are removed from the domain because they have minimal effect on the frequency of the acoustic eigenmodes and because including them results in fine meshes, the cost of simulations increases greatly.

In LES simulations of combustion chambers, it is also assumed that the small scale features are unimportant however fluctuating mass flow rates of cooling flows could adversely effect the emissions performance of combustion chambers [7][8]. Small scale features are also important for other components in engines: In turbines, cooling flows disrupt the flow over aerodynamic surfaces and must be taken into account during design. These cooling flows are subjected to a fluctuating external pressure field caused by acoustic waves, convecting structures flowing downstream from the combustion chamber or vane wakes [9][10]. Including these small structures in the mesh means very high cell counts and very small or distorted cells. Very small cells lead to problems with CFL number limits while distorted cells cause problems with solver stability and accuracy. It has already been shown that the reduction in cell count and geometric complexity of replacing a fuel injector with simple model can reduce simulation cost by a factor of 2.5-7 [11][12].

The ideal solution would be a simple model that captures the response of small scale features to pressure fluctuations in the time domain. One possible methodology is the use of reduced order models (ROMs). Two different classes of models are tested in this study in their ability to reconstruct an acoustically forced flow through an orifice: The first is a model based on the weak-form of the Navier-Stokes equations, where the basis functions are derived from simulation data using proper orthogonal decomposition (POD) to compress the response of the orifice into a handful of modes whose evolution is governed by a set of ordinary differential equations (ODEs). The second is a method where artificial neural networks (ANNs) are trained to predict the evolution of the POD modes in time without a set of governing equations.

POD ROMs have been used in a number of academic and industrially interesting flows for flow prediction and control such as boundary layers [13], cylinder flows [14] and cavities [15][16]. They have also been used to reconstruct turbulent inlet flows: The POD-LODS (low-order dynamical system) model [17] formulates the model as a set of unknown matrix coefficients, which when multiplied by the current model state, reproduces the acceleration vector so that the future model state can be calculated. These matrix coefficients are computed from simulation data using the least-squares method.

ANNs have been used to predict, for example, the temporal evolution of and a quasi-one dimensional continuously variable resonance combustor [18] and the temporal evolution of the viscous Burgers equation [19]. ANNs may be used in a variety of ways, however in this paper, the focus is on equation free approaches applied to the evolution of the POD temporal coefficients. The two methods tested are a sequential network model (POD-ANN-SN), which aims to predict the next time step value based on the current one; and a residual network model (POD-ANN-RN), which seeks to predict the difference between the current and next time step value based on the current value. In both the POD-ROMs and POD-ANNs, the time dependent value of the pressure gradient in the orifice normal direction is used as a
In this paper, the acoustic impedance for a laminar orifice flow is calculated using two dimensional OpenFoam simulations at several different frequencies. The acoustic impedance is calculated using the Belluci model and compared to the multimicrophone method from CFD data while Howe’s model for the Rayleigh conductivity is used to validate the reflection, transmission and absorption coefficients. In the modelling section, two data driven models (POD-LODS and ANNs) are built and tested in their ability to reproduce the temporal evolution of the flow. The equation free ANN model is trained with POD data with two different frameworks: a sequential and a residual network approach.

**TEST CASE**

An isothermal flow with density $= 1.18 \text{ kg/m}^3$ at temperature $T = 300\text{K}$ is considered. The inlet velocity of the flow is taken to be $\bar{u} = 0.074 \text{ m/s}$, which corresponds to $\bar{u} = 3.4 \text{ m/s}$ at the orifice aperture, with Mach number in the orifice $M \approx 0.01$ and the Reynolds number $Re \approx 220$.

Ten different simulations with frequencies ranging from 100 to 1000 Hz, with imposed mean pressure of 101300 Pa at the outlet were conducted to investigate the acoustic parameters of the orifice. This corresponds to a Strouhal number $St = \omega d / \bar{u}$ ranging from 0.092 to 0.92. The acoustic signal of the form $p_{\text{in}} = \hat{p}|\sin(\omega t)|$ is forced from the outlet using an upstream travelling wave with an amplitude of 50 Pa.

**SIMULATION METHOD**

**Computation domain**

Figure 1 shows the schematic view of the 2-D numerical domain. The domain has a wall length of $L = 60 \text{ mm}$ and a breadth of $b = 6 \text{ mm}$. The orifice diameter is $D = 2d = 1 \text{ mm}$ with a thickness of $h = 1 \text{ mm}$. The flow is initiated from left side of the domain and acoustic forcing is introduced from right side of the domain. Quad Meshing was done in ANSYS 13.0 Figure 2. The grid was refined until doubling the number of grid points yielded less than 1% difference in the solution, therefore the given grid size is 0.09 mm.

**Numerical method**

A finite volume based unsteady laminar flow computational methodology is used to build a module to study the compressible flows within the open source CFD framework OpenFOAM [20]. Initially, fully developed flow was achieved using pimple-foam with time step $\Delta t = 0.0001s$ and simulation run time of $t = 1s$. Later starting from the developed flow, acoustic forcing is introduced using the solver which uses the PISO algorithm of [21], where the pressure correction loop is solved multiple times within each time step. Within each time step the velocity equation is solved first, followed by the pressure correction loop to ensure mass continuity. The time step for acoustic forcing simulation was $\Delta t = 0.000025s$ and simulation run time $t = 1 \text{ to } t = 1.05s$. The energy equation is also solved within this pressure correction loop to account for variations of temperature. Pressure, density and temperature are linked by the ideal gas equation. This is a very similar methodology to Su et al. [22].

**Boundary conditions**

The boundary conditions at the inlet and outlet of the flow are given as a velocity inlet and static pressure outlet. Walls of the domain including orifice walls are modelled as using the no-slip condition. The upstream and downstream boundaries for acoustics are treated using characteristic boundary conditions with the locally-one dimensional (LODI) approach as detailed in [23][22]. At the inlet, the in-going characteristic wave magnitude is set to zero and the outgoing wave magnitude determined using a first order Euler approach leading to an acoustically non-reflective condition. At the downstream boundary the outgoing characteristic is set in a similar way however the in-going characteristic $(\delta W_5)$ is set to inject sinusoidal acoustic waves into the domain with:

$$\delta W_5 = -\frac{2|\hat{p}|(\sin(\omega(n+1)\delta t) - \sin(\omega n\delta t))}{\rho c}$$  \hspace{1cm} (1)

Where $\hat{p}$ is the desired magnitude of the acoustic wave, $n$ is the time step number and $\delta t$ is the time step.

**Analytical model for impedance**

The acoustic impedance is defined as the ratio of complex amplitude of acoustic pressure across the orifice to the volume velocity through it.

$$Z = \frac{p^\prime}{u^\prime} = R + iX$$  \hspace{1cm} (2)

Howe [24] proposed a linear model to calculate Rayleigh conductivity for flows through infinitely thin aperture with ($Re \gg 1$):
\[ K_R = \frac{-i\omega \rho_0 A_t'}{\rho} \]  
(3)

\[ K_R = D(\gamma - i\delta) \]  
(4)

\[ \gamma - i\delta = 1 + \frac{A}{B} \]  
(5)

\[ A = \pi/2I_1(St) \exp((-St) - iK_1(St) \sinh(St)) \]  
(6)

\[ B = St[\pi/2I_1(St) \exp(-St) + iK_1(St) \cosh(St)] \]  
(7)

However, this model was derived for an orifice within an infinitely thin wall. Since this case involves an orifice with finite length and diameter, it is more appropriate to consider the model of Belluci [25] for the acoustic impedance. This model includes the effects of the mean flow and is applicable for both short and long orifices. This model also takes into account an end correction, where the duct length L is replaced by the effective length \( l = L + D\Delta \), where \( \Delta = (8/3\pi) \):

\[
Z_s = \rho_0 \left( \frac{U_c}{C_d} + \frac{2L}{D} \sqrt{2\nu\omega} + i \left( \frac{L + 8D}{3\pi} \omega + \frac{2L}{D} \sqrt{2\nu\omega} \right) \right)
\]  
(8)

**Multi-Microphone Method**

Measurements of unsteady pressure within the ducts were obtained using 4 probes in the downstream duct and 2 probes in the upstream duct. All the probes were located at the same height. The probes were placed at sufficient distances from the orifice so that the orifice effect does not influence the probe measurements.

\[
\tilde{p}(x, t) = \tilde{p}_e \exp \left( i\omega t - ik_+ x \right) + \tilde{p}_r \exp \left( i\omega t + ik_- x \right)
\]  
(9)

\[
k_\pm = \omega / (1 \pm M)
\]  
(10)

The acoustic velocity can be found as:

\[
\tilde{u}(x, t) = \frac{\tilde{p}_e \exp(\omega t - ik_+ x) - \tilde{p}_r \exp(\omega t + ik_- x)}{\rho c}
\]  
(11)

Measurements from the 4 downstream probes are used to find the acoustic impedance according to the multi-microphone method proposed by [26]. This leads to a system of linear equations \( Ax = B \) which are evaluated by a least square approach to determine the complex impedance of the orifice.

\[
\begin{bmatrix}
\exp(i\omega t - ik_+ x_1) \\
\vdots \\
\exp(i\omega t - ik_+ x_4)
\end{bmatrix}
\begin{bmatrix}
\tilde{p}_1(x_1) \\
\vdots \\
\tilde{p}_4(x_1)
\end{bmatrix}
= 
\begin{bmatrix}
\tilde{p}_1 \\
\vdots \\
\tilde{p}_4
\end{bmatrix}
\]  
(12)

**Reflection, transmission and absorption**

The reflection, transmission and absorption coefficients are related to Rayleigh conductivity [3] as follows [27]:

\[
R = \frac{-ikb^2}{2KR} \left( 1 - \frac{ikb^2}{2KR} \right)^{-1}
\]  
(13)

\[
T = \left( 1 - \frac{ikb^2}{2KR} \right)^{-1}
\]  
(14)

\[
\alpha = 1 - |R^2| - |T^2|
\]  
(15)

The calculations of reflection, transmission and absorption coefficients from simulation are done by using the unsteady pressure measurements obtained by the probes placed at the downstream (d) and upstream (u) ducts.

The reflection coefficient is defined as the ratio of downstream (+) and upstream (-) travelling wave amplitudes in the downstream side of the orifice. The transmission coefficient is defined as the ratio of upstream travelling (-) wave amplitudes in the downstream and upstream side of the orifice. The absorption coefficient is again given by Eq. (15).

\[
R = \left| \frac{\tilde{p}_{d+}}{\tilde{p}_{d-}} \right|
\]  
(16)

\[
T = \left| \frac{\tilde{p}_{u-}}{\tilde{p}_{u+}} \right|
\]  
(17)

**REDUCED ORDER MODELLING**

**Proper orthogonal decomposition (POD)**

In order to significantly reduce the number of equations that need to be solved as the flow develops, the number of basis functions that define the flow solution should be reduced. One option for this data-reduction, and the method chosen in this work is the Proper Orthogonal Decomposition (POD). The POD was introduced in [28] and aims to minimise the flow reconstruction error with the smallest number of orthogonal modes. This allows the velocity field to be reconstructed with:

\[
u(x, t) = \bar{u}(x, t) + \sum_{k=1}^{N_o} a_k^d(t) \phi_k(x)
\]  
(18)

Where \( N_o \) is the number snapshots extracted from the original simulation and \( a_k^d \) is the temporal coefficient of the \( k \)th POD mode. The method of snapshots [29] can be used to generate this optimal basis through computation of the co-variance matrix whose elements are constructed using:

\[
C_{i,j} = \frac{1}{N_i} \int \nu(x, t_i) \cdot \nu(x, t_j) dx
\]  
(19)

The temporal coefficients of the POD modes are found through an eigenvalue decomposition of the co-variance matrix that splits...
the matrix into an optimal set of orthogonal modes whose contribution to the input data is proportional to each modes’ eigenvalue \( \lambda \). In this sense the POD modes can seen to be optimal in terms of the reproduction of the kinetic energy of the original flow. In order to simplify the calculation of the POD modes, the volume of the computational cells were assumed to be constant. Furthermore, while the POD temporal coefficients are typically scaled such that:

\[
\frac{1}{N_s} \sum_{k=1}^{N_s} a_k^i a_j^k = \lambda_i \delta_{ij}
\]  

(20)

In this work, the temporal coefficients have been normalised such that:

\[
\sum_{k=1}^{N_s} a_i(t_k) a_j(t_k) = \delta_{ij}
\]  

(21)

and therefore the velocity can be reconstructed using:

\[
\mathbf{u}(x, t) = \mathbf{\bar{u}}(x, t) + N_s \sum_{k=1}^{N_s} \sqrt{\lambda_i} a_k(t) \phi_k(x)
\]  

(22)

This normalisation of the temporal coefficients improves the stability of the LODS reduced order model as the coefficient magnitudes remain lower than 1 and hence errors introduced in the calculation of the matrix \( Q_{l,j,k} \) are less likely to destabilise the ROM.

**Low-order dynamical system (LODS)**

In the present study, temporal evolution of the velocity fluctuations are modeled through the POD projection coefficients \( a_n(t) \). A polynomial low-order dynamical system (LODS) [13] is derived in order to model the temporal behavior of the \( N_s \) most energetic POD modes of the flow. It is formulated on the basis that the dynamics of the temporal POD coefficients \( a_n(t) \) are described by a set of ordinary differential equations (ODEs). The model employed in our study, utilizes polynomial ODEs of quadratic order which is equivalent to the result obtained through the use of a Galerkin projection of the momentum equation:

\[
\dot{a}_i = D_i + L_{ij} a_j + Q_{ijk} a_i a_j + \mathcal{P}(t)
\]  

(23)

Since the problem is associated with a flow that is acoustically forced, the time dependent pressure gradient in the orifice normal direction is set as a time varying system control. This control is included in the model by considering the Galerkin projection of the pressure gradient term in the Navier-Stokes momentum equation:

\[
\mathcal{P}(t) = -\int_V \nabla p \cdot \phi_k \, dV
\]  

(24)

In incompressible flows, the pressure gradient can be related directly to the velocity fields due to mass conservation and the properties of the POD modes. The pressure gradient is therefore already included in Eqn. 23, however in compressible flows, this is no longer the case. The pressure gradient term is assumed to be of the form:

\[
\mathcal{P}(t) = P_l \frac{\partial p}{\partial x}(x_0, t)
\]  

(25)

where \( P_l \) is a coefficient that needs to be set for each POD mode and \( x_0 \) is a reference location in the flow. Thus the final form of LODS becomes:

\[
\dot{a}_i = D_i + L_{ij} a_j + Q_{ijk} a_i a_j + \mathcal{P}(t)
\]  

(26)

\( D_i, L_{ij}, Q_{ijk} \) and \( P_l \) coefficients are obtained by solving the least square system of form \( Ax = B \).

\[
\begin{bmatrix}
1 & a_j(t_1) & a_j(t_1) a_k(t_1) & \frac{\partial p}{\partial x}(t_1) \\
\vdots & \vdots & \vdots & \vdots \\
1 & a_j(t_p) & a_j(t_p) a_k(t_p) & \frac{\partial p}{\partial x}(t_p) \\
\vdots & \vdots & \vdots & \vdots \\
1 & a_j(t_N_p) & a_j(t_N_p) a_k(t_N_p) & \frac{\partial p}{\partial x}(t_N_p)
\end{bmatrix}
\begin{bmatrix}
D_i \\
L_{ij} \\
Q_{ijk} \\
P_l
\end{bmatrix}
= 
\begin{bmatrix}
\dot{a}_i(t_1) \\
\vdots \\
\dot{a}_i(t_p) \\
\vdots \\
\dot{a}_i(t_N_p)
\end{bmatrix}
\]  

(27)

The temporal coefficients are then modeled by performing a fourth-order Runge–Kutta integration with initial conditions \( a_i(t_0) \). The time derivatives are calculated by second order finite difference,

\[
a_n(t_p - 1/2) = \frac{a_n(t_p - 1) - 2a_n(t_p) + a_n(t_p + 1)}{\Delta t}
\]  

(28)

The temporal coefficients are averaged to ensure that they are all at the same phase as the time derivatives:

\[
a_n(t_p - 1/2) = \frac{a_n(t_p - 1) + a_n(t_p)}{2}
\]  

(29)
Neural Networks

As done in LODS, time dependent pressure gradient and $N_r$ most energetic POD modes of the flow are used as inputs to the network. Thus, the network maps from input space of dimension $N_r + 1$ to output space with dimension of $N_r$, which also maps coefficients values at time $t \rightarrow t+1$ in each mode.

The coefficient values are normalised to the range (0,1) before they are fed to the network. The model comprises of a hyperbolic tangent (tanh) activated input layer, linearly activated output layer with 7 hidden layers activated by Rectified Linear Units (relu) and the number of neurons in the hidden layers vary from 32 to 144. The model is trained and validated by splitting the data in 80:20 ratio for training and validation. The time taken by the model for training and prediction are 50 and 20 seconds respectively. The predicted values are again scaled up to get the original temporal coefficient. Hyperparameter tuning was performed for optimal values of hyperparameter using Bayesian Optimisation algorithm [30]. The algorithm takes in a range hyperparameters like epochs, batch size, hidden layers, learning rate etc. and maximises the validation accuracy. The model is implemented using the Tensorflow library [31] available in Python.

The studies were conducted by utilising two different neural network architectures namely, a sequential network [30] which predicts the coefficient values at time $t+1$ based on the value at time $t$, and a residual network [32] which seeks to predict the difference between the coefficient value at time $t+1$ and $t$, based on the value at time $t$. A diagram of these two ANN architectures is shown in Fig. 3.

RESULTS AND DISCUSSION

POD and the flow field

Figure 4 shows the magnitude of the mean velocity field for the simulation forced at 600 Hz. It shows the jet from the orifice spreading out downstream and a recirculation zone forming either side of the jet. The POD was carried out for $N$ time snapshots of $u'$ with forcing frequencies ranging from $f = 100$ to
$f = 1000 \text{ Hz}$ with intervals of $100\text{Hz}$. A total of 2000 snapshots is obtained from a particular frequency simulation from time $t = 1s$ to $t = 1.05s$. This leads to co-variance matrix $C_{i,j}$ of size $2000 \times 2000$. Figure 5 shows energetic modes obtained from eigen-values $\lambda$ of the co-variance matrix $C_{i,j}$ matrix, which is defined as,

$$E_k = \frac{\sum_{n=1}^{k} \lambda_n}{\sum_{n=1}^{N} \lambda_n} \times 100$$

(34)

where the number of snapshots is set to $N = 2000$ in this study. It can be seen in Fig. 5 that a small number of modes are enough to capture the temporal dynamics of the flow. Therefore the models were built using the $N_r = 3$ largest POD modes capturing 91.84\% of the energy.

These three most energetic modes are shown in Fig. 6, the first mode can be seen to be mostly representative of the fluctuating magnitude of the orifice jet while the second mode is more representative of the recirculation zone.

**Validation of acoustic effects**

The complex impedance has been calculated and expressed as resistance and reactance. The comparison has been made for impedance calculated from the CFD data using the multi-microphone method and the theoretical model Eq. (8) as a function of frequency shown in Fig. 7. The resistance calculated from the CFD results shows an excellent agreement with the model but a much lower reactance, this was similarly seen in [22] for low frequencies.

The acoustic reflection, transmission and absorption coefficients has been calculated and compared using theoretical models as presented in Eqn. 13 to 15 and CFD data as shown in Figure 8. The $|T|$ and $|\alpha|$ coefficients calculated with the model are in good agreement with the CFD. The reflection coefficient from CFD data are close to theoretical model for lower frequencies ($< 500\text{Hz}$) than at higher frequencies.

**LODS and ANN**

The reduced order modelling and ANN training is performed for the flow downstream of the orifice. The temporal coefficients of the leading POD modes are subsequently used for LODS and ANN training with the pressure gradient as a control parameter.

The temporal dynamics calculated are compared with LODS, the sequential network and the residual network model for the frequencies 300, 600 and 900Hz as shown in Figures 9.
FIGURE 8: ACOUSTIC EFFECTS AS A FUNCTION OF FREQUENCY.

TABLE 1: SIMULATION COST

| Model                  | Degrees of freedom | Time  |
|------------------------|--------------------|-------|
| Acoustic Flow Simulation | 89453              | 480 hrs |
| POD-LODS               | 3                  | 10s   |
| POD-ANN                | 3                  | 20s   |

**FUTURE WORK**

Table 1 shows a comparison between the integration times of the CFD simulation and the ROMs. It is clear that the significant reduction in the degrees of freedom in the ROMs means that they are orders of magnitude faster than the full-order CFD model. As with all ROMs, their use in real world applications is dependent on their stability and their generality. Future work will focus on improving the model performance and stability during transients, longer integration times, multiple frequency forcing and more engine representative Mach and Reynolds numbers.

**CONCLUSIONS**

Two-dimensional laminar acoustic flow simulations through an orifice were carried out for a fixed amplitude and varied frequencies. The study and analysis conducted here has shown that the calculated reactance from CFD is lower than what is expected from theory. Acoustic effects are in good agreement with the CFD calculation, except for the theoretical reflection coefficient which is higher at frequencies $>500\text{Hz}$. The performance of 2 POD enabled models for temporal evolution prediction have been investigated: At first, a lower order dynamical system, where system of ODEs are solved, has been developed.
Secondly, an ANN model has been developed through data-driven learning to predict the evolution of the POD modes. The ANN model was trained for 2 different frameworks: (a) a sequential network (SN), where the modes are predicted directly, and (b) a residual network (RN) where the model is trained to reproduce the difference between its future and current state. The POD-ANN approach provides significantly better results compared to POD-LODS. Furthermore, POD-ANN-RN has the advantage of requiring less memory and being more stable than the sequential network. Some of the key observations made during this study are listed below:

- The POD-LODS approach may lead to instabilities while solving the ODEs.
- Despite the benefits of POD-ANN approach, it needs a large training dataset for good generalizability.
- More advanced techniques can be used to train neural networks for better familiarization (not in scope of present study), which would lead to more stable methods for modelling.

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