Time Dependent Solution in Open Bosonic String Field Theory

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ABSTRACT: In this paper we present time dependent solution of the open bosonic string field theory describing the motion of the tachyon on unstable D-brane.

KEYWORDS: String field theory

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1. Introduction

Time dependent solutions of the open string field theory have been studied very intensively in the last few months. In particular, the solution describing rolling of the tachyon field on non-BPS D-brane or D-brane anti-D-brane pair away from the unstable minimum of the potential to the stable one has been studied very intensively in series of remarkable papers by A. Sen [1, 2, 3]. This problem has been also discussed in [4, 5] and very recently in two papers [24, 25].

In this paper we will study this problem from the point of view of the open bosonic string field theory (SFT) [10] (For review and extensive list of references, see [6, 7, 8, 9].). We present an exact time dependent solution of SFT that has many properties that we should expect from the rolling tachyon solution. In particular, we will show that in the asymptotic future there are not any states corresponding to the open string fluctuation modes. We will also show that this solution is characterised by initial position and velocity of the tachyon field with agreement with [1, 2, 3, 24, 25].

In our calculation we will proceed as follows. We will presume that there is solution of the SFT equation of motion that can be written using string field identity field [17] and any operator of the ghost number one that acts on it. This presumption is crucial for our analysis since then we can easily find an exact solution of the SFT equation of motion and the form of the modified BRST operator. The similar approach for searching of the exact solutions of SFT has been used recently in [11, 12, 13, 14, 15, 16]. Success of this approach strongly support our conjecture that the identity field I is the fundamental object in the string field theory and deserve further study. In particular, this string field theory identity field has been studied recently in many interesting papers [18, 19, 20, 21, 22].

This paper is organised as follows. In the next section (2) we perform explicit calculation and we find time dependent solution of the SFT. In (3) section we show
that there are not any plane-wave modes that propagate about the new background given by the rolling tachyon solution. And finally, in (3) we outline our results and suggest further open questions and problems.

2. Calculation

In this section we will solve the string field theory equation of motion. We show that there is an exact time dependent solution that describes rolling of the tachyon from its initial value \( \lambda \) to the minimum of the potential in the limit \( x^0 \to \infty \).

To begin with, we firstly review basic facts about bosonic string field theory, following mainly [6]. Gauge invariant string field theory is described with the full Hilbert space of the first quantized open string including \( b, c \) ghost fields subject to the condition that the states must carry ghost number one, where \( b \) has ghost number \(-1\), \( c \) has ghost number 1 and \( SL(2, \mathbb{C}) \) invariant vacuum \( |0\rangle \) carries ghost number 0. We denote \( \mathcal{H} \) the subspace of the full Hilbert space carrying ghost number 1. Any state in \( \mathcal{H} \) will be denoted as \( |\Phi\rangle \) and corresponding vertex operator \( \Phi \) is the vertex operator that creates state \( |\Phi\rangle \) out of the vacuum state \( |0\rangle \)

\[
|\Phi\rangle = \Phi |0\rangle .
\]

Since we are dealing with open string theory, the vertex operators should be put on the boundary of the world-sheet. In the following we will also use double cover of the upper half plane and then we will consider holomorphic fields only. The string field theory action is given [10]

\[
S = -\frac{1}{g_0^2} \left( \frac{1}{2\alpha'} \langle I \circ \Phi(0) Q_B \Phi(0) \rangle + \frac{1}{3} \langle f_1 \circ \Phi(0) f_2 \circ \Phi(0) f_3 \circ \Phi(0) \rangle \right) ,
\]

where \( g_0 \) is open string coupling constant, \( Q_B \) is BRST operator and \( <> \) denotes correlation function in the combined matter ghost conformal field theory. \( I, f_1, f_2, f_3 \) are conformal mapping exact form of which is reviewed in [6] and \( f_i \circ \Phi(0) \) denotes the conformal transformation of \( \Phi(0) \) by \( f_i \). For example, for \( \Phi \) a primary field of dimension \( h \), then \( f_i \circ \Phi(0) = (f'_i(0))^h \Phi(f_i(0)) \). In the abstract language pioneered in [10], the open string field theory action (2.2) has a form (For more details, see [6, 7, 8, 9].)

\[
S = -\frac{1}{g_0^2} \left( \frac{1}{2\alpha'} \int \Phi \star Q \Phi + \frac{1}{3} \int \Phi \star \Phi \star \Phi \right) .
\]

Let us presume that the solution of the string field theory equation of motion

\[
\frac{1}{\alpha'} Q \Phi_0 + \Phi_0 \star \Phi_0 = 0 ,
\]

can be written as

\[
\Phi_0 = \mathcal{T}_L(\mathcal{I}) ,
\]
where $\mathcal{I}$ is string field algebra $\star$ identity field [17] that obeys
\[ \mathcal{I} \star X = X \star \mathcal{I} = X , \] (2.6)
for any string field $X$. The ghost number one operator $\mathcal{T}$ is defined as
\[ \mathcal{T}(\Phi(w)) = \frac{1}{2\pi i} \oint_C dz(t)\Phi(w) , \] (2.7)
where the integration contour $C$ encircles the point $w$ of the complex plane counterclockwise and we use the convention that $t(z)$ is holomorphic field defined over the whole complex plane via the doubling trick. In upper expression $\Phi(w)$ is any CFT operator. For $\mathcal{T}_L$ acting on any wedge state $|n\rangle$ the situation is slightly more complicated, for precise discussion of these problems, see very nice papers [22, 21]. For our purpose the definition (2.7) will be sufficient.

Using results given in [17] and recently discussed in [22] we can easily show that the field (2.5) obeys
\[ \Phi_0 \star \Phi_0 = \mathcal{T}_L(\mathcal{I}) \star \mathcal{T}_L(\mathcal{I}) = -\mathcal{T}_L(\mathcal{I}) \star \mathcal{T}_R(\mathcal{I}) = -\mathcal{T}_L(\mathcal{I}) = -\frac{1}{2} \{ \mathcal{T}_L, \mathcal{T}_L \} (\mathcal{I}) . \] (2.8)

In the same way we get
\[ Q\Phi_0 = (Q_R + Q_L)(\mathcal{T}_L(\mathcal{I})) = Q_L(\mathcal{T}_L(\mathcal{I})) - \mathcal{T}_L(Q_R(\mathcal{I})) = \{ Q, \mathcal{T} \} L (\mathcal{I}) , \] (2.9)
where we have used the fact that $Q_R$ anticommutes with $\mathcal{T}_L$ and the relation [17]
\[ Q_L(\mathcal{I}) = -Q_R(\mathcal{I}) . \] (2.10)

In these formulas the subscript $R$ means the integration over right side of the string. And finally, we use the notation
\[ [X_L, Y_L] = [X, Y] L . \] (2.11)

Let us presume that the solution of (2.4) can be written as
\[ \mathcal{T}_L(\mathcal{I}) = T_L(\mathcal{I}) + \delta T_L(\mathcal{I}) , Q(T_L(\mathcal{I})) = \{ Q, T \} L (\mathcal{I}) = 0 \] (2.12)
and $\delta T$ obeys
\[ \{ Q, \delta T \} L - \frac{1}{2} \{ T, T \} L = 0 , \{ \delta T, \delta T \} L = 0 = \{ \delta T, T \} = 0 . \] (2.13)
Then it is clear that (2.12) is exact solution of the string field theory equation of motion.

Let us consider $T$ in the form

$$T = \frac{1}{2\pi i} \int dwt(X^0(w))c(w) , t(X^0(w)) = \sum_n a_n X^0(w)^n ,$$

(2.14)

where the normal ordering between $X^0$ is understood. In the following we use the convention $[6, 23]$}

$$T_m(z) = -\frac{1}{\alpha'} \partial_z X^\mu(z) \partial_z X^\nu \eta_{\mu\nu} ,$$

$$X^\mu(z)X^\nu(w) \sim -\frac{\alpha'}{2} \eta_{\mu\nu} \ln(z - w) ,$$

$$c(z)b(w) \sim -\frac{1}{z - w} ,$$

$$Q = \frac{1}{2\pi i} \int dzc(z) \left[ T_m(z) + \frac{1}{2} T_{\text{ghost}}(z) \right] ,$$

$$T_{\text{ghost}}(z) = -2b(z)\partial c(z) - \partial(b(z))c(z) ,$$

$$T_{\text{ghost}}(z)c(w) \sim -\frac{1}{(z - w)^2} c(w) + \frac{\partial c(w)}{z - w} .$$

(2.15)

Then we obtain from (2.12)

$$0 = \{Q, T\}_L =$$

$$= \left\{ \frac{1}{2\pi i} \int_{C_1} dzc(z) \left[ T_m(z) + \frac{1}{2} T_{\text{ghost}}(z) \right] , \frac{1}{2\pi i} \int_{C_2} dwt(X^0(w))c(w) \right\} =$$

$$= \frac{1}{2\pi i} \int_{C_1} dw\partial c(w)c(w) \left[ \frac{\alpha'}{4} t''(X^0(w)) - t(X^0(w)) \right] ,$$

(2.16)

where $C_1, C_2$ are integration contours in the complex plane corresponding to the integration over left side of the string. Finally, we have defined $t''(X^0(w)) = \frac{d^2t(w)}{dt^2}$. From the requirement of the vanishing (2.16) we immediately get two linearly independent solutions

$$t_1(w) = e^{\frac{\alpha'}{2\alpha} X^0(w)} , t_2(w) = e^{-\frac{\alpha'}{2\alpha} X^0(w)} .$$

(2.17)

Now we can ask the question whether the linear combination of these two solutions

$$T = AT_1 + BT_2$$

(2.18)

is exact solution of the SFT equation of motion. It is clear that this operator obeys

$$\{Q, T\}_L = 0$$

(2.19)
and hence it is solution of the SFT equation of motion in the linearised approximation \[4\]. In order to determine whether the string field \(T_L(\mathcal{I})\) is solution of \((2.24)\) we must calculate following expression

\[
\frac{1}{2} \{T_L, T_L\} = \frac{1}{2} \left\{ \frac{1}{2\pi i} \int_{C_1} dz c(z) t(X^0(z)), \frac{1}{2\pi i} \int_{C_2} dw c(w) t(X^0(w)) \right\}, \quad (2.20)
\]

where \(C_1, C_2\) are the same integration contours as in the previous example. Now we rewrite \((2.20)\) as

\[
\frac{1}{4\pi i} \int_{C_1} dw \left[ \frac{1}{2\pi i} \oint_{C'} c(z) t(X^0(z)) c(w) t(X^0(w)) \right] c(w), \quad (2.21)
\]

where the integration contour \(C'\) encircles the point \(w\) counterclockwise. From upper expression it is clear that we must determine Operator product expansion (OPE) between \(t(z)\), \(t(w)\). Using the well known formula \[23\] for the calculation of the OPE between two normal ordered operators \(F : \cdot ; G : \cdot \)

\[
\mathcal{F}(z) :: G(w) := \exp \left( -\frac{\alpha'}{2} \int dz_1 dz_2 \ln(z_1 - z_2) \eta^{\mu\nu} \frac{\delta \mathcal{F}}{\delta X^\mu(z_1)} \frac{\delta G}{\delta X^\nu(z_2)} \right) : \mathcal{F}(z) G(w) : \quad (2.22)
\]

we get

\[
t_1(z)t_1(w) =: e^{\frac{2}{\alpha'} X^0(z)} :: e^{\frac{2}{\alpha'} X^0(w)} := \exp \left( -\frac{\alpha'}{2} \int dz_1 dz_2 \ln(z_1 - z_2) \frac{2}{\sqrt{\alpha'}} \delta_{\mu} \delta(z - z_1) \times \right.
\]

\[
\left. \frac{2}{\sqrt{\alpha'}} \delta_{\nu}(w - z_2) \delta^{\mu\nu} \right) e^{\frac{2}{\alpha'} X^0(z)} e^{\frac{2}{\alpha'} X^0(w)} := (z - w)^2 : e^{\frac{2}{\alpha'} X^0(z)} e^{\frac{2}{\alpha'} X^0(w)} :, \quad (2.23)
\]

\[
t_2(z)t_2(w) =: e^{-\frac{2}{\alpha'} X^0(z)} :: e^{-\frac{2}{\alpha'} X^0(w)} := (z - w)^2 : e^{-\frac{2}{\alpha'} X^0(z)} e^{-\frac{2}{\alpha'} X^0(w)} :, \quad (2.23)
\]

where we have used

\[
\frac{\delta}{\delta X^\mu(z_1)} t_1(z) = \frac{\delta}{\delta X^\mu(z_1)} e^{\frac{2}{\alpha'} X^0(z)} = \delta(z - z_1) \frac{2}{\sqrt{\alpha'}} \delta_{\mu} e^{\frac{2}{\alpha'} X^0(z)},
\]

\[
\frac{\delta}{\delta X^\nu(z_2)} t_2(w) = \frac{\delta}{\delta X^\nu(z_2)} e^{-\frac{2}{\alpha'} X^0(w)} = -\delta(w - z_2) \frac{2}{\sqrt{\alpha'}} \delta_{\nu} e^{-\frac{2}{\alpha'} X^0(w)}. \quad (2.24)
\]

We see from \((2.23)\) that the OPE between \(t_1(z)t_1(w)\), \(t_2(z)t_2(w)\) are non-singular and consequently

\[
\{T_1, T_1\}_L = 0, \\
\{T_2, T_2\}_L = 0. \quad (2.25)
\]

\[\text{In the following we introduce the symbol} : F : \text{for the normal ordered operator} F.\]
We can also show that
\[
(t_1(z)t_2(w)) = e^{\frac{z}{\sqrt{\alpha'}} X^0(z)} \cdot e^{-\frac{z}{\sqrt{\alpha'}} X^0(w)} := (z - w)^{-2} : e^{\frac{z}{\sqrt{\alpha'}} X^0(z)} e^{-\frac{z}{\sqrt{\alpha'}} X^0(w)} + \frac{1}{(z - w)^2} \cdot \partial_w X^0(w)(z - w)e^{\frac{z}{\sqrt{\alpha'}} X^0(w)} e^{-\frac{z}{\sqrt{\alpha'}} X^0(w)} : + \ldots =
\]
\[
= \frac{1}{(z - w)^2} + \frac{2}{z - w \sqrt{\alpha'}} \partial_w X^0(w) + \ldots .
\]
(2.26)

Using (2.23), (2.25), (2.26) we get
\[
\frac{1}{2} \{T_L, T_L\} = AB \{T_{1L}, T_{2L}\} = AB \frac{1}{2\pi i} \int_{C_1} dw \left[ \frac{1}{2\pi i} \int_{C_0} c(z) t_1(X^0(z)) t_2(X^0(w)) \right] c(w) =
\]
\[
= AB \frac{1}{2\pi i} \int_{C_1} dw \left[ \frac{1}{2\pi i} \int_{C_0} dz \left( \frac{1}{(z - w)^2} + \frac{2}{\sqrt{\alpha'} z - w} \partial_w X^0(w) \right) c(z) \right] c(w) =
\]
\[
= AB \frac{1}{2\pi i} \int_{C_1} dw \partial_w c(w) c(w) + c(w) \frac{2}{\sqrt{\alpha'}} \partial_w X^0(w) c(w) = AB \frac{1}{2\pi i} \int_{C_1} dw \partial_w c(w) c(w) \neq 0
\]
(2.27)

and hence the linear combination $AT_1 + BT_2$ is not an exact solution of the string field theory equation of motion. As in (2.12) we introduce following term
\[
\delta T = -AB \alpha' \frac{1}{2\pi i} \int_{C_1} dz c(z).
\]
(2.28)

From the upper expression we immediately obtain
\[
\{\delta T, \delta T\}_L = 0 , \{\delta T, T_{1,2}\} = 0 .
\]
(2.29)

We can also easily see that
\[
\frac{1}{\alpha'} \{Q, \delta T\}_L = \left\{ \frac{1}{4\pi i} \int_{C_1} dz c(z) T_{9}(z), \frac{1}{2\pi i} \int_{C_2} dw c(w) \right\} =
\]
\[
= -AB \frac{1}{4\pi i} \int_{C_1} dw \left[ \frac{1}{2\pi i} \int_{C_0} c(z) \left( \frac{1}{(z - w)^2} c(w) + \frac{1}{z - w} \partial_w c(w) \right) \right] =
\]
\[
= -AB \frac{1}{4\pi i} \int_{C_1} dw \left\{ -\frac{\partial}{\partial_w} \left[ \int_{C_0} dz c(z) \frac{1}{z - w} \right] c(w) + c(w) \partial_w c(w) \right\} =
\]
\[
= AB \frac{1}{2\pi i} \int_{C_1} dw \partial_w c(w) c(w) .
\]
(2.30)

From (2.27) and (2.30) we immediately get
\[
\frac{1}{\alpha'} \{Q, \delta T\}_L -\frac{1}{2} \{T, T\}_L = 0
\]
(2.31)
so that
\[
\Phi_0 = \mathcal{T}_L(\mathcal{I}) \cdot \mathcal{T} = \frac{1}{2\pi i} \oint dz c(z) \left( Ae^{\sqrt{\frac{\alpha'}{\lambda}} X^0(z)} + Be^{-\sqrt{\frac{\alpha'}{\lambda}} X^0(z)} - \alpha' AB \right) .
\] (2.32)
is exact solution of the SFT equation of motion.

Let us consider following initial condition \(|1\)
\[
\mathcal{T}(X^0 = 0) = \lambda c(z), \quad \mathcal{T}'(X^0 = 0) = 0 .
\] (2.33)
From the second condition we get
\[
0 = \frac{d\mathcal{T}(X^0)}{dX^0}(X^0 = 0) = c(z)(A - B) \Rightarrow A = B ,
\] (2.34)
and when we insert this result into the first condition in (2.33) we immediately obtain
\[
\mathcal{T}(X^0 = 0) = c(z)(-\alpha' AB + A + B) = c(z)\lambda \Rightarrow -\alpha' A^2 + 2A - \lambda = 0 \Rightarrow
\]
\[
A_1 = \frac{1}{\alpha'} \left(1 + \sqrt{1 - \alpha'\lambda}\right) , \quad A_2 = \frac{1}{\alpha'} \left(1 - \sqrt{1 - \alpha'\lambda}\right) .
\] (2.35)
For \(\lambda \ll 1/\alpha'\) we get
\[
A_1 \sim \frac{1}{\alpha'} \left(1 + (1 - \alpha'\lambda/2)\right) = \frac{1}{\alpha'} \left(2 - \alpha'\lambda/2\right) . \quad A_2 \sim \frac{1}{\alpha'} \left(1 - (1 - \alpha'\lambda/2)\right) = \frac{\lambda}{2} .
\] (2.36)
For \(\lambda \to 0\), \(A_2\) goes to zero and hence the solution \(\Phi_0\) vanishes. On the other hand for \(\lambda \to 0\) \(A_1\) goes to \(2/\alpha'\). In this case the tachyon field would start to roll even if its initial value and velocity are zero. We mean that in the classical string field theory there is no reason for such a behaviour so that we will not consider the first root \(A_1\).

To obtain the string field theory action for the fluctuation fields about the classical solution \(\Phi_0\) we insert the general string field \(\Phi = \Phi_0 + \Psi\) into (2.3). Then we get the string field theory action for the string field \(\Psi\) which has the same form as (2.3) however the original BRST operator \(Q\) is replaced with the new one
\[
Q' (\Psi) = Q(\Psi) + \Phi_0 \star \Psi - (-1)^{|\Psi|} \Psi \star \Phi_0 = Q(\Psi) - \mathcal{T}(\Psi) ,
\] (2.37)
using \([17, 22]\)
\[
\Phi_0 \star \Psi = \mathcal{T}_L(\mathcal{I}) \star \Psi = -\mathcal{I} \star \mathcal{T}_R(\Psi) = -\mathcal{T}_R(\Psi) ,
\]
\[
-(-1)^{|\Psi|} \Psi \star \Phi_0 = (-1)^{|\Psi|} \Psi \star \mathcal{T}_R(\mathcal{I}) = -\mathcal{T}_L(\Psi) \star \mathcal{I} = -\mathcal{T}_L(\Psi) ,
\]
\[
\mathcal{T}_L(\mathcal{I}) = -\mathcal{T}_R(\mathcal{I}) , \quad \mathcal{T}(\Psi) = \mathcal{T}_L(\Psi) + \mathcal{T}_R(\Psi) .
\] (2.38)
As a result we obtain the new BRST operator $Q'$ in the form

$$Q' = \frac{1}{2\pi i} \oint dzc(z) \left[ T_m(z) - Ae^{\frac{2}{\sqrt{\alpha}}X^0(z)} - Be^{-\frac{2}{\sqrt{\alpha}}X^0(z)} + \alpha' AB + \frac{1}{2} T_{gh}(z) \right] =$$

$$= \frac{1}{2\pi i} \oint dzc(z) \left[ T_m(z) - 2A \cosh \left( \frac{2}{\sqrt{\alpha}}X^0(z) \right) + \alpha' A^2 + \frac{1}{2} T_{gh}(z) \right].$$

(2.39)

In the next section we will study the fate of the fluctuation modes about the solution $\Phi_0 = T_L(I)$.

### 3. Absence of the perturbative states in the asymptotic future

In this section we will analyse fluctuation modes about the solution (2.32) in the limit $X^0 \rightarrow \infty$. According to very nice analysis given in [3], there should not be any plane-wave perturbative states in the limit $X^0 \rightarrow \infty$. We will argue that the same result holds for the fluctuation fields about the string field solution $T_L(I)$. We hope that this conclusion strongly supports our conjecture that (2.32) really describes rolling the tachyon in the string field theory.

To begin with, we propose following form of the fluctuation field. We conjecture that any fluctuation mode about the solution $\Phi_0 = T_L(I)$ has the form

$$\Psi = F(c^\dagger, b^\dagger, a^\dagger)(T_L(I)),$$

(3.1)

where $F(c^\dagger, b^\dagger, a^\dagger)$ is ghost number zero operator. The proposal (3.1) is mainly motivated by recent works [26, 27] where the fluctuation mode about sliver state in VSFT has been studied. Other aspects of the fluctuation modes about sliver state have been discussed in [28, 29, 30, 31, 32, 33].

In CFT language the fluctuation field (3.1) can be written as

$$\Psi = \frac{1}{2\pi i} \oint dwf(X(w), \partial X(w)) \left( \frac{1}{2\pi i} \int_{C_1} dz \mathcal{T}(z)(I) \right) =$$

$$= \frac{1}{2\pi i} \int_{C_1} dz \left[ \frac{1}{2\pi i} \oint_{C_1} dwf(w) \mathcal{T}(z) \right] (I) = [F, T]_L(I),$$

(3.2)

using

$$F_L(I) = -F_R(I).$$

(3.3)

Now we will try to find spectrum of the fluctuation modes. By definition, the fluctuation modes obey the linearised string field theory equation of motion formulated about the new background

$$Q' ([F, T]_L(I)) = 0 \Rightarrow \{Q, [F, T] \}_L = \{T, [F, T] \}_L = 0.$$
To solve previous equation, we need following formulas

\[
\begin{align*}
\{Q, [F, \mathcal{T}]\}_L - [F, \{\mathcal{T}, Q\}]_L - \{\mathcal{T}, [Q, F]\}_L = 0 & \Rightarrow , \\
\Rightarrow \{Q, [F, \mathcal{T}]\}_L = [F, \{\mathcal{T}, Q\}]_L + \{\mathcal{T}, [Q, F]\}_L = \\
= \{F, \{\delta \mathcal{T}, Q\}\}_L + \{\mathcal{T}, [Q, F]\}_L = \{\mathcal{T}, [Q, F]\}_L ,
\end{align*}
\]

(3.5)

where we have used the fact that \(F\) is the ghost number zero operator and hence has trivial OPE with any ghost field so that \([F, \{Q, \delta \mathcal{T}\}] = 0\).

In the same way we get

\[
\{\mathcal{T}, [F, \mathcal{T}]\}_L = \{\delta \mathcal{T} + T_1 + T_2, [F, T_1 + T_2]\}_L = \\
= \{T_1, [F, T_2]\} + \{T_2, [F, T_1]\}_L = \\
= \{F, \{T_1, T_2\}\}_L = 2 \{F, \{Q, \delta \mathcal{T}\}\}_L = 0 .
\]

(3.6)

Then (3.4) reduces to

\[
0 = Q' (F (\mathcal{T}_L (\mathcal{I})) = \{\mathcal{T}, [Q, F]\}_L \Rightarrow [Q, F]_L = 0 .
\]

(3.7)

We are interested in the limit \(X^0 \to \infty\) so that \(\mathcal{T} \to \frac{A}{2\pi i} \oint_{C_1} dz c(z) e^{\frac{2}{\sqrt{\alpha'}} X^0 (z)}\). Now we would like to show, in the same way as in [3], that in the asymptotic future there are not any plane-wave states around the tachyon vacuum. Let us start with the operator

\[
F = \frac{1}{2\pi i} \oint dw f(w) = \frac{1}{2\pi i} \oint dw e^{2i k_\mu X^\mu (w)} .
\]

(3.8)

Then (3.4) gives the condition

\[
\left( \alpha' k_i k_j \eta^{ij} - \alpha' k_0^2 \right) = -k_\mu k_\nu \eta^{\mu \nu} = 0 .
\]

(3.9)

Note that there is not any contribution from the ghost sector since \(F\) is the ghost number zero operator and hence has trivial OPE with \(T_{\text{ghost}}\). According to (3.2) we must calculate following commutator

\[
[F, \mathcal{T} (X^0 \to \infty)]_L = \left[ \frac{1}{2\pi i} \int_{C_1} dz e^{2i k_0 X^0 (z)} , \frac{A}{2\pi i} \int_{C_2} dw c(w) e^{\frac{2}{\sqrt{\alpha'}} X^0 (w)} \right] = \\
= \frac{1}{2\pi i} \int_{C_1} dw c(w) \left[ \frac{A}{2\pi i} \oint_{C'} dz e^{2i k_0 X^0 (z)} e^{\frac{2}{\sqrt{\alpha'}} X^0 (w)} \right] .
\]

(3.10)

In order to obtain nontrivial result we should consider \(k_0\) in the form

\[
k_0 = \frac{i n}{\sqrt{\alpha'}} , \quad n = 1, 2 . . .
\]

(3.11)
so we get

\[ e^{-\frac{2n}{\sqrt{\alpha'}} X^0(z)} : e^{\frac{2n}{\sqrt{\alpha'}} X^0(w)} := \exp\left(\frac{\alpha'}{2} \int dz_1 dz_2 \ln(z_1 - z_2) \frac{-2n}{\sqrt{\alpha'}} \delta(z_1 - z) \frac{2}{\sqrt{\alpha'}} \delta(z_2 - w)\right) \times \]

\[ \times : e^{-\frac{2n}{\sqrt{\alpha'}} X^0(z)} e^{\frac{2n}{\sqrt{\alpha'}} X^0(w)} := \frac{1}{(z - w)^{2n}} : e^{-\frac{2n}{\sqrt{\alpha'}} X^0(z)} e^{\frac{2n}{\sqrt{\alpha'}} X^0(w)} : . \]

(3.12)

When we insert (3.12) into (3.10) we obtain well defined operator and hence well defined fluctuation mode. On the other hand, the condition (3.9) implies

\[ k_i k^i = -\frac{1}{\alpha'} n^2 , n = 1, 2, \ldots \] (3.13)

so that the spatial part of the momentum is imaginary. This result clearly implies that there are not any fluctuation modes that propagate as plane-wave solutions about tachyon vacuum with agreement with the analysis performed in [3]. It is easy to extend this arguments to other perturbative modes

\[ F = \frac{1}{2\pi i} \int dw O(\partial_w X(w), \ldots) e^{ik\mu X^{\mu}(w)} , \] (3.14)

where now the condition \([Q, F]_L = 0\) implies

\[ -k_0^2 + k_i k^i = -\frac{1}{\alpha'} N \Rightarrow k_i k^i = -\frac{1}{\alpha'} (n^2 + N) . \] (3.15)

As in previous case the requirement of the well defined operator leads to the condition (3.11) which again implies that spatial components of the momentum are imaginary and hence there are not present any plane-wave excitations. We mean that this result strongly supports our conjecture that the solution given in the previous section really describes rolling tachyon [1, 2, 3].

4. Conclusion

In this short note we have found an exact solution of the open bosonic string field theory that represents the time dependent flow to the tachyonic vacuum. We have seen that this solution is characterised by one parameter \(\lambda\) so that despite the non-locality of the string field theory action, it does admit time dependent solutions labelled by initial position and velocity of the tachyon field, with complete agreement with recent papers [1, 2, 3, 24, 25]. We have also shown that in the asymptotic future there are not any physical fluctuations about this solution again with the agreement with [3].

There are many problems and questions that deserve further study. Since we have argued that the solution given in this paper corresponds to the flow of the system
to the tachyonic vacuum, we should expect that the string field theory formulated about this solution will have many common with the vacuum string field theory. In particular, we believe that there exist such a string field redefinition that maps the shifted BRST operator \(2.39\) in the limit \(X^0 \to \infty\) to the pure ghost BRST operator in VSFT \[34, 35, 36, 37\].

We have seen that there are not any perturbative fluctuations corresponding to the open string modes about the rolling tachyon solution. On the other hand, it is possible that there could be solutions of the string field theory formulated about rolling tachyon background in the limit \(X^0 \to \infty\) that represent closed string excitations or D-branes of various dimensions. If our presumption is correct that these solutions should be related to the sliver states in VSFT. We hope to return to these important and exciting problems in the future.

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