ABSTRACT

The links between the internal structure of galaxy groups and clusters and cosmological parameters are reviewed here. The mass density profiles of clusters, inferred from both optical analyses of the galaxy surface number density profile coupled with internal kinematics, and from weak gravitational lensing, appear cuspy with inner density profiles at least as steep as $r^{-1}$, as is expected from high-resolution cosmological $N$-body simulations starting with scale-free or CDM initial conditions. The high level of substructure both in the galaxy distribution and in the diffuse hot gas seen in X-rays is consistent with dynamically young clusters and a cosmological density parameter $\Omega_0 \gtrsim 0.5$. Despite the importance of cluster-cluster merging witnessed by these substructures, the spherical top-hat cosmology appears to give good indications of the underlying physics of clusters and groups, including a fundamental evolutionary track in a space of observational parameters for groups. Dynamical processes operating in groups and clusters are reviewed. Previrialization of groups and clusters before their full collapse is not expected on theoretical grounds, and the ensemble of positions of compact groups, relative to the fundamental track is a potentially useful constraint on previrialization.

1 Introduction

This review focuses on the links between the richest hierarchies, groups and clusters, and global cosmological properties of the Universe, principally the density parameter, $\Omega_0$, and the primordial density fluctuation spectrum, $P(k)$. We begin with the observational facts, derive constraints on cosmological parameters, and focus in the end on dynamical and cosmological evolution of structures. The review is written for non-experts, so the specialist should be patient with some very obvious definitions given here. We will not touch upon issues of the long term evolution of galaxies in groups/clusters or the baryon content of these systems.

2 Observational Facts

2.1 Small-scale structures of the Universe

The attractive nature of gravity pushes matter to concentrate in overdense regions, up to large scales, because of gravity’s long range. To first order, galaxies are a tracer of the total mass content of the Universe (departures of which are called bias). And
so, one expects and indeed sees that galaxies tend to agglomerate in a hierarchy of structures as shown in Table 1.

Table 1: Hierarchy of galaxy systems

| System          | $N_{\text{gal}}$ | $m_{\text{faint}}$ | Isolation | Scale        | $(M/L)$ | $P_{\text{gal}}$ |
|-----------------|------------------|---------------------|-----------|--------------|---------|-----------------|
| Clusters        | 30–300           | $m_3 + 2$           | None      | 1.5 $h^{-1}$ Mpc | 300 $h$ | 10%             |
| Loose Groups    | 3–30             | $m_1 + 3$           | None      | 1.5 $h^{-1}$ Mpc | 150 $h$ | 50%             |
| Compact Groups  | 4–8              | $m_1 + 3$           | 3$r$      | < 0.2 $h^{-1}$ Mpc | 50 $h$ | 0.1%            |
| Binaries        | 2                | $m_1 + 3$           | 5$r_{12}$ | < 0.2 $h^{-1}$ Mpc | 50 $h$ | 10%             |
| Isolated        | 1                |                     |           |              |         | 30%             |

Notes: $N_{\text{gal}}$ is the number of galaxies per system within the given range of magnitudes ($m = -2.5 \log_{10} \text{flux} + \text{cst}$). $M/L$ is the mass-to-light ratio, in solar units, assuming dynamical equilibrium, $P_{\text{gal}}$ is the fraction of galaxies in a given type of system, and $h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$ is the dimensionless Hubble constant, measuring the current rate of expansion of the Universe. The criteria for clusters [1], compact groups [2], and binaries [3], have been slightly modified here.

Originally, galaxy systems have been cataloged through eye-ball searches of galaxy positions projected on the sky. Spectroscopy of galaxies provide redshifts, measuring galaxy radial velocities (through the Doppler effect), which are the sum of the Hubble expansion velocity (proportional to distance through the Hubble law $v = H_0D$) and the peculiar velocity relative to this Hubble flow. Except for nearby galaxies, the Hubble flow term dominates, so that redshift is a first-order distance estimator. Recent catalogs of galaxy systems are defined in 3D (projected positions and depth inferred from the redshift) in automated fashion with well defined criteria.

One should understand that galaxy systems probably span a continuous range of hierarchies, and therefore not put too much weight in the special categories defined here, whose properties depend strongly on the adopted selection criteria. From Table 1, most galaxies lie within fairly rich environments, and we shall see in § 3.1 that they are dense enough to be, using cosmological terms, in the quasi-linear to non-linear regimes of the growth of the primordial density fluctuations by gravitational instability. Their properties are thus function of both the underlying cosmological parameters and the dynamical processes operating within them.

2.2 Density profiles

Surprisingly little is known on the galaxy number or total mass density profiles in groups and clusters? The galaxy number density profiles of groups and clusters are poorly known because of

- small number statistics (most clusters have at best a few hundred galaxies with radial velocities confirming their membership)
2 OBSERVATIONAL FACTS

• their unknown center
• contamination from foreground and background interlopers

For example, in clusters, model profiles with homogeneous cores fit the data [4], but so do models with cuspy cores [5], and the two are hard to distinguish given the limits on the data [3]. Homogeneous cores are the best fits when one superposes the information from different clusters, rescaling to the same size [6], but when one uses the X-ray barycenter or the position of the giant cD galaxy as the cluster center instead of the center of the galaxy positions, the clusters unequivocally show cuspy cores [6]. So homogeneous cores were the result of poor choices for the cluster centers. The cuspsiness of clusters has been recently confirmed [7] in an analysis of 1500 suspected members of a single nearby rich cluster (Coma), for which the number space-density profile is $\nu \sim r^{-1}$. The presence of possible interlopers in the sample can only wash out the central density cusp, so that, in reality, the slope of the 3D number density profile may be even steeper.

If the situation in galaxy clusters is difficult, it should be impossible to conclude anything on groups of galaxies. Nevertheless, by averaging over groups in the NBG [8] catalog, and rescaling the groups to some unit size, it has been noted [9] that these were closer to homogeneous than to $\nu \sim r^{-2}$. Averaging over the best defined compact group sample [2], again with rescaling, compact groups catalog were found to be significantly centrally concentrated [10], although the slope of the underlying profile is not given. A reanalysis [11] of that compact group sample, after removal of groups with interlopers in redshift space [12], and starting with the distribution of absolute pair separations, shows that the distribution of galaxy separations within compact groups is consistent with a unique absolute density profile, falling off as $\nu \sim r^{-2.4}$ in the envelope and with a small homogeneous core of size $18 h^{-1}$ kpc, that is half the median pair separation within compact groups. It is not yet clear that this result is caused by high central concentration or by the presence of tight binaries within less concentrated density profiles (as seems to be the case in loose groups [3]).

What about the total mass density profiles? One can resort to either optical data or X-ray data. In both cases, one writes the equation of hydrostatic equilibrium (or Jeans equation), which in its general spherically symmetric form is

$$\frac{dP}{dr} = -\rho \frac{d\Phi}{dr} = \rho \frac{GM_{\text{tot}}(r)}{r^2},$$

(1)

where $\rho$ is the density (in mass or number) of whatever tracer we use to measure the pressure $P$. It is then easy to obtain the mass density profile

$$\rho = \frac{1}{4\pi r^2} \frac{dM_{\text{tot}}}{dr}.$$  

(2)

Now, from equation (1), the Jeans equation for the ‘gas’ of galaxies in a spherical cluster is

$$\nu \frac{d\sigma_r^2}{dr} + 2\beta_{\text{anis}} \frac{\nu \sigma_r^2}{r} = -\nu \frac{GM_{\text{tot}}(r)}{r^2},$$

(3)
where \( \nu \) is the space number density of galaxies in the cluster, \( \sigma_r \) is the radial velocity dispersion (standard deviation of the velocity distribution) of the cluster, and \( \beta_{\text{anis}} = 1 - \sigma_t^2/\sigma_r^2 \) is the velocity anisotropy, given that \( \sigma_t \) is one tangential component of the cluster velocity dispersion.

The problem with the optical analysis is that the variation of anisotropy with radius is unknown, even if there are good reasons to believe that, in the central regions of clusters, the system should be isotropic, since the two-body relaxation time of galaxy-galaxy encounters is short (see §4 below). The anisotropy of the outer envelope can be either radial, if the envelope evolution is dominated by near-radial infall of galaxies, or tangential if the cluster is experiencing an off-center collision with a smaller one. Note that the reversed problem of fixing the mass density profile and computing the anisotropy profile is solvable in quadrature [13].

The advantage of the X-ray method is that there is no pressure anisotropy term to include in the Jeans equation. Using the equation of state of an ideal hot X-ray emitting gas, the total mass profile is [14]

\[
M_{\text{tot}}(r) = -\frac{kT}{G\mu m_p} \left( \frac{d \ln n}{d \ln r} + \frac{d \ln T}{d \ln r} \right), \tag{4}
\]

where \( n \) and \( T \) are the local gas number density and temperature, while \( \mu m_p \) is the mean particle mass in the hot plasma. Since both the galaxy system and the gas react to the same overall potential, and when both systems are isothermal, and when the galaxy system is also isotropic, one finds [15] (eqs. [3] and [4]) \( n_{\text{gas}} \sim n_{\text{gal}}^{\beta} \), where \( \beta = \sigma_t^2/(kT/\mu m_p) \) is the ratio of kinetic energies of the galaxies and gas. Now, using equations (2) and (4), one can compare the total mass density profile to the gas density profile, where the emissivity of the gas locally scales as the square of its density. It turns out that the dark matter is significantly more centrally peaked than the diffuse X-ray emitting gas [16, 17, 18, 19]. However, the dark matter density profile is similar to the galaxy number density profile [17, 19].

The optical analysis of the Coma cluster, assuming isotropy of the velocity distribution, yields [7] a total mass density \( \rho \sim r^{-3} \) steeper than the galaxy number density given above. Putting in radial anisotropy at large radii produces a broken total mass density profile varying as \( r^{-3} \) in the center but only \( r^{-1} \) in the envelope [7]. It seems difficult to have the inner mass density profile fall as slowly as the galaxy number density profile with an anisotropy profile consistent with dynamical and cosmological principles. If the Coma cluster is indeed regular, the X-rays may be telling us something about the radial variation of the velocity anisotropy.

Gravitational optics is rapidly becoming a promising alternative method for deriving projected mass distributions. Indeed, the amplification due to an elliptical gravitational lens is [20]

\[
A = \frac{1}{(1 - \kappa)^2 - \gamma^2}, \tag{5}
\]
where the matter (or Ricci) term $\kappa$ can be expressed in terms of a critical surface mass density $\Sigma_c$:

$$\kappa = \frac{\Sigma_{\text{mass}}(R)}{\Sigma_c},$$

(6)

$$\Sigma_c = \frac{c^2}{4\pi G D_l(1 - D_l/D_s)}\left(1 - \frac{D_l}{D_s}\right),$$

where $D_l$ and $D_s$ are the distances to the lens and to the source, respectively, and where the shear (or Weyl) term is

$$\gamma = \kappa - \frac{2}{R^2\Sigma_c} \int_0^R \Sigma(x) x dx.$$

(7)

Equations (5) and (6) indicate that gravitational amplification of background galaxies by the potential of a galaxy group cluster is directly related to the surface mass density.

When the surface mass density of the cluster is close to $\Sigma_c$, background galaxies are lensed into a giant arc, arclt, or are just slightly elongated, in a sequence of decreasing alignment of the background galaxy with the caustics in the plane of the foreground cluster [21]. While giant arcs have been observed [22, 23], there only comes one or two per cluster and thus cannot constrain the cluster density profile. On the other hand, the elongations of lensed background galaxies, caused by the shear term (eq. [7]), allow one to map the surface density of a cluster, by measuring and inverting the shear field [24]. With such methods, one finds cuspy surface density profiles at least as steep as $\Sigma \sim r^{-1}$ [25, 26]. Surprisingly, there often seems to be more mass at a given radius than inferred from the X-rays (eq. [4]) by a factor 2 to 3 [27, 28], but the discrepancy disappears if one allows for small-scale fluctuations in the radial variation of the gas temperature [26].

2.3 Substructure

The naïve view that clusters of galaxies are regular structures with little substructure can be taken as justification for the dynamical analyses of their underlying mass content. Recent studies point to the opposite picture of very irregular clusters. These are based upon analyses of the projected distribution of galaxies in clusters without [24, 30, 31] or with [32] the radial velocities of the member galaxies, as well as on X-ray [33] analyses. In particular, at least 30% of clusters show significant substructure from optical 3D analyses [34], including the once canonical regular Coma cluster [35]. Similarly, the X-ray analyses indicate that at least 22% of clusters show bimodal substructure [33], and optical analyses indicate that roughly half [36] or as much as 90% [37] of the clusters that are regular in the X-rays show small-scale substructure (the latter analysis is based upon wavelets).
In loose groups, again the small-number statistics make it very difficult to assess statistically significant substructure. In a system of say 8 galaxies, small-scale substructure would show up as galaxy pairs (subgroups of triplets or quartets of galaxies would show up as separate groups in the catalogs). For the NBG [8] group catalog, 1.5 pairs are found on average per group [9], although a fraction three times lower has been claimed [38] for groups in an unpublished catalog.

2.4 Fundamental plane

Three parameters are most easily measured in astronomical systems: angular size, velocity dispersion (or temperature), and flux. In a subsample of systems at known distances, angular size provides the physical size, and flux provides the (intrinsic) luminosity (i.e., power). Thus, astronomical systems are well characterized by the parameters, \( R \) (size), \( V \) (velocity dispersion), and \( L \) (luminosity). Whether these systems are star clusters, galaxies, or galaxy clusters, people have attempted to find a plane in \( R, V, L \) space in which the objects of a sample all fit. Such fundamental planes have indeed been found for galaxy clusters [39] with the relation \( L \sim R^{0.9} V^{1.3} \), not very different from that of elliptical galaxies (with no disks or spiral structure) or globular star clusters.

Fundamental planes are useful for two reasons: 1) They teach us about how the systems form and evolve. 2) They serve as distance indicators, independently of the Hubble law (\( V_r = H_0 D \), where \( V_r = c z \) is the radial velocity, while \( c \) is the velocity of light, and \( z \) the redshift). This is important, because the Hubble law is only uniform on average on very large scales, but on smaller scales is perturbed by local overdensities of matter (where the expansion rate is lower than the average one, and becomes even negative when matter falls into a galaxy, group, or cluster) and underdensities (where the expansion rate is larger). Therefore, outliers from the fundamental plane are usually systems for which the distance was poorly estimated, and the correct distance is obtained by forcing the system to lie on the plane. In other words, if \( L \simeq \text{Cst} R^\alpha V^\beta \) (no relation with the previous \( \alpha \)'s and \( \beta \)'s), then the correct distance to an outlier is

\[
D \simeq \left( \frac{\text{Cst} \theta^\alpha V^\beta}{4\pi f} \right)^{1/(2-\alpha)},
\]

where the angular size \( \theta = R/D \), and the measured flux \( f = L/(4\pi D^2) \).

2.5 Internal kinematics

The motions of galaxies in clusters probe the potentials of these systems, when these are near dynamical equilibrium. For example, Jeans’ equation [3] specifies the radial variation of the second moment of the velocity distribution, the squared velocity dispersion. The measured line-of-sight velocity dispersion is the emissivity-weighted
average of the velocity dispersion, where the geometry becomes a little complicated in the presence of velocity anisotropy \[13\]. In any event, the radial variation of the line-of-sight velocity dispersion provides the knowledge of the velocity anisotropy, assuming that one knows the potential \[13\], or of the potential if one knows the anisotropy.

The early realization that the velocity dispersion decreases towards the center of the Coma cluster \[40\] (as well as outwards in the envelope) is interpreted as either a cuspy total mass density profile with a core that has isotropic velocities \[41\] or a homogeneous core with tangential (nearly circular) velocity dispersions \[42\]. A recent systematic study of a large number of clusters \[42\] reveals that roughly 15% of clusters have such inverted velocity dispersion profiles, while roughly 40% have velocity dispersions decreasing outwards everywhere, the remaining profiles being either flat or unclassifiable. The clusters with cuspy velocity dispersion profiles indicate cuspy steep density profiles \(\rho \sim R^{-\gamma}\), where \(\gamma = 3.5 - 4\) \[42\], with at best very small \(< 40 \, h^{-1} \text{kpc}\) homogeneous cores \[12\].

3 Cosmological Constraints

3.1 The spherical top-hat approximation

Consider a constant finite spherical overdensity in a homogeneous Universe. The evolution of this top-hat perturbation is governed by the simple equation

\[
\ddot{R} = -\frac{GM}{R^2}.
\]

(8)

It will expand with the Hubble expansion then reach maximum expansion at the turnaround time and collapse at twice the turnaround time. Subsequently, the matter may reach dynamical equilibrium, and become virialized as it satisfies the virial theorem \(2\langle T\rangle + \langle W\rangle = 0\), where \(T\) and \(W\) are its kinetic and potential energies, respectively.

It is easy to show (without cosmological arguments, simply integrating eq. \[8\]) that the turnaround time can be expressed in terms of the density at turnaround

\[
\tau_{ta} = \left(\frac{32G\rho_{ta}}{3\pi}\right)^{-1/2},
\]

(9)

where \(\rho_{ta}\) is the density at turnaround. Once the top-hat collapses, the surrounding regions will collapse onto it in a gradual fashion, each reaching maximum expansion following equation \(9\) where \(\rho_{ta}\) is now the mean density within the turnaround region at the epoch of turnaround \[13\]. This process of so-called secondary infall has been studied in detail \[13\], and for \(\Omega = 1\), one obtains simple scaling laws for the matter within the turnaround radius, beyond which matter is expanding:

\[
M_{ta} \sim t^{2/3} \quad R_{ta} \sim t^{8/9} \quad \bar{\rho}_{ta} \sim t^{-2}.
\]

(10)
Reversing equation (9) one can then obtain the present radius of turnaround of cosmological objects in the top-hat approximation, and these are given in Table 2 from

\[ R_{\text{ta}} = \left( \frac{8GM_{\text{ta}}t_0^2}{\pi^2} \right)^{1/3}, \]  

(11)
independent of \( \Omega_0 \). Here, \( t_0 \) is the age of the Universe.

| System            | \( M_{\text{ta}}(M_\odot) \) | \( R_{\text{ta}} \) (Mpc) |
|-------------------|-------------------------------|----------------------------|
| Small galaxy      | \( 3 \times 10^{11} \)       | 0.4                        |
| Large galaxy      | \( 3 \times 10^{12} \)       | 0.8                        |
| Small group       | \( 3 \times 10^{13} \)       | 1.7                        |
| Small cluster     | \( 3 \times 10^{14} \)       | 3.6                        |
| Large cluster     | \( 3 \times 10^{15} \)       | 7.8                        |

Within the turnaround radius, matter is infalling up to the region of shell crossing inside which matter is well mixed (see [44] for a graphical example). The radius of mixing is approximately the radius of second turnaround or rebound radius, which is roughly one-third of the turnaround radius [45]. For an \( \Omega = 1 \) universe, the density is critical at \( \rho_c = (6\pi Gt_0^2)^{-1} \), and the mean overdensity within the turnaround radius is (using eq. [11]) \( 9\pi^2/16 = 5.55 \). If matter virializes at collapse and settles, from the virial theorem, at half its turnaround radius at epoch 2 \( \tau_{\text{ta}} \), its density will be 8 times larger than it was at turnaround, while the density of the Universe will decreased by a factor (eq. [10]) 4 (for \( \Omega = 1 \)), so that the overdensity at virialization is \( 18\pi^2 = 178 \). Loose groups of galaxies are defined with density contrasts of 20 [47] to 80 [48], and so must still be feeling the collapse. Combining the virial theorem with this cosmological minimum density yields a small maximum size of \( \simeq 200 \, h^{-1} \) kpc and minimum velocity dispersion of \( \simeq 250 \, \text{km s}^{-1} \) for groups [49].

Recent cosmological collisionless N-body simulations [46] with power-law primordial density fluctuation spectra \( P(k) \sim k^n \) indicate that the spherical top-hat approximation is very good, in that the radius where the density is 178 times the mean density delimits the infalling region from the quasi-static region. Nevertheless, the top-hat approximation cannot represent all the cosmological aspects of clusters and groups, because the Universe is filled with structures and substructures (§ 2.3).

### 3.2 Density profiles

What should one expect for the density profiles of clusters? In simple terms, the final density profile of an object will be set in part by whether the object collapses at once or in a slower smoother fashion. The details are set by the primordial density.
fluctuation spectrum, i.e., the variation of the rms overdensity fluctuations $\sigma_8$ with (comoving) scale $R$.

In the top hat approximation ($\S$ 3.1), it is easy to derive density profiles using the relations of equation (10), yielding $\rho \sim r^{-9/4}$ and $\sigma_v \sim r^{-1/8}$ [50]. The $-9/4$ slope of the density profile has been verified with more detailed semi-analytical calculations [51, 45] and with $N$-body simulations [52].

Consider now a spherical top-hat perturbation in an open ($\Omega < 1$) Universe. Because the Universe is less dense than critical, there will be a lack of matter falling onto the perturbation at late times, so that the envelope of the evolving system should be steeper than $\rho \sim r^{-9/4}$ [53], as checked with $N$-body simulations [54, 55]. However, the regions that are accreted at early times came from a Universe whose density was closer to critical, thus resembling more an $\Omega = 1$ Universe.

Now reality is not as simple as spherical top-hat perturbations in an otherwise homogeneous Universe. First, perturbations may not be spherical, which then produces shallower slopes [47]. Moreover, fluctuations in the primordial density field occur everywhere on a variety of scales. One can then show analytically [50] that the density profiles have slopes of $\mathrm{Min}[−2, −3(3 + n)/(4 + n)]$, given a scale-free primordial density fluctuation spectrum $P(k) \sim k^n$, and again, this has been checked with $N$-body simulations [56, 54, 53]. $N$-body simulations with CDM spectra (which have spectral index $n$ increasing with scale from $−3$ to $1$, and is roughly $−2$ on the scale of galaxies and $−1$ on the scale of clusters) show a varying a slope from $−1$ inside to $−4$ [58] or $−3$ [59] outside (the latter study includes dissipative gas). Interestingly, high-resolution scale-free simulations [46] reproduce the density profile with slope varying from $−1$ to $−3$ found in low resolution CDM simulations with gas [59]. All the high resolution simulations with CDM or scale-free spectra show cuspy cores, with a core radius smaller than the numerical resolution, which can be as small as a few kpc [58].

3.3 Substructure

In a low density Universe, structure formation tends to freeze at redshift $z = 1/\Omega_0$ [60], because there is simply not enough matter to keep accreting onto already formed structures. In contrast, a critical density Universe sees accretion at all times. Therefore, the level of substructure in galaxy clusters seems to be an excellent way to constrain the density parameter $\Omega_0$.

The approach is to predict the fraction of galaxy clusters with given level of substructure, as a function of $\Omega$ and compare with the observations ($\S$ 2.3). One way is the analytical computation of the distribution of collapse times of structures of given mass [61]. Alternatively, one builds Monte-Carlo realizations of the history of structures and their mergers, based upon the Press-Schechter mass function of structures at given epoch [62, 63]. All studies yield $\Omega_0 \gtrsim 0.5$, although the precise lower limit to the density parameter is still subject to debate.
These predictions are insensitive to the primordial density fluctuation spectrum \cite{63} and to the cosmological constant \cite{61}.

For example, bimodal substructure is expected for a critical density universe in 15\% \cite{64}, 24\% \cite{61}, and 28\% \cite{63}, depending on how long the bimodal substructure can survive within the cluster (assumed at 0.08 times the present age of the Universe, for the first two studies and 0.1 \( t_0 \), respectively), compared to > 22\% observed (\S \ref{2.3}). For \( \Omega_0 = 0.2 \) the corresponding predicted fractions are 4 to 6 times lower, and clearly discrepant with the observations.

Half of all clusters are predicted to harbor 10\% level small-scale substructure \cite{64} if \( \Omega_0 = 1 \), which compares well with the smaller of the two observed estimates \cite{36}. If instead 90\% of all clusters harbor small-scale substructure \cite{37}, one would need either a higher \( \Omega_0 \) or a survival time longer than 0.08 \( t_0 \). A recent study \cite{65} does provide a 50\% longer survival time for small-scale substructure caused by accreted small clusters, but slightly less, when the small-scale substructure if caused by the dense cores of two similar mass clusters detaching from their parents when these two merge together \cite{66}.

N-body simulations with gas \cite{67,68} confirm the semi-analytical predictions given above and illustrate very well how the gaseous distribution in clusters in a low-\( \Omega \) Universe is considerably smoother on small-scales than observed in the X rays.

### 3.4 Previrialization

Most analytical approaches to cosmology apply the top-hat approach, in which the collapse is radial. In principle, non-radial motions can arise as angular momentum can be pumped into the system by external tidal torques or by gravitational interactions on small scales within the system. The existence of significant non-radial motions is known as Previrialization \cite{69}. How important is this effect? The first numerical study \cite{70} argued for a very strong effect from the outer environment, so that the critical density for collapse, expressed in units of present day linearly evolved primordial density, is found to be of order 5, instead of the canonical value of 1.69 for \( \Omega_0 = 1 \) \cite{71} or 1.61 for \( \Omega_0 = 0.1 \) \cite{63}. A more realistic set of cosmological N-body simulations \cite{72} with \( P(k) \sim k^{-1} \), showed that power on small scales within clusters does not alter its collapse. Moreover, implicit in this study is that external tidal torques bring only low angular momentum \( J = 0.07 J_{\text{circ}} \) just within the turnaround radius of clusters. A very recent analytical study \cite{73}, using perturbative expansions in the quasi-linear regime, suggests that for \( n \approx -1 \), the collapse is as one expects from linear theory, hence with insignificant non-radial motions. The cold dark matter spectrum, still considered to be a close approximation to the true power spectrum \cite{74}, has a slope close to \(-1\) for the infalling regions of groups and clusters, hence previrialization should not be important in modeling these systems.
4 Dynamical Evolution

There are quite a few dynamical processes competing in the evolution of groups and clusters of galaxies. What is written below is largely taken from a previous review [75].

The simplest timescale in a self-gravitating system is the circular orbital time:

$$\tau_{\text{orb}} = \frac{2\pi R}{V_{\text{circ}}(R)} = \left[ \frac{3\pi}{G\bar{\rho}(R)} \right]^{1/2},$$

(12) since $V_{\text{circ}}^2 = GM(R)/R$.

4.1 Relaxation

As a test particle undergoes scattering collisions within a sea of field particles, it will progressively forget its initial conditions. This two-body relaxation time can be defined in at least three ways:

$$\tau_{2-\text{rel}} \equiv \left\langle \frac{1}{v^2} \frac{dV^2}{dt} \right\rangle^{-1} \text{ or } \left\langle \frac{1}{E} \frac{dE}{dt} \right\rangle^{-1} \text{ or } \left\langle \frac{d}{dt} \sin^2 \Delta \alpha \right\rangle^{-1},$$

where $\Delta \alpha$ is the deflection angle in an encounter. This can be written as [76]

$$\tau_{2-\text{rel}} = \frac{v^3}{G^2 m_f^2 n f(v/\sigma_v) \ln \Lambda},$$

where $v$ is the velocity of the test particle, $m_f$, $n$, and $\sigma_v$ are the mass, number density, and 1D velocity dispersion of the field particles, respectively, $f(x)$ is the fraction of particles traveling faster than $x \sigma_v$, and $\ln \Lambda$ is called the Coulomb logarithm, with typical values of 2 to 10, where $\Lambda$ is the ratio of maximum to minimum impact parameter. For a system of galaxies and dark matter particles, one finds that the galaxies relax by galaxy-galaxy collisions, but not by collisions with individual dark matter particles (whose masses are too low). Similarly, the dark matter particles relax mainly by collisions with individual galaxies.

A collective relaxation time has been derived, not by summing up the encounters but by computing the collective response of the system [77]

$$\tau_{N-\text{rel}} = \text{Cst} \frac{v}{Gm_f nm^{2/3}}.$$

This collective relaxation turns out to be somewhat more efficient than two-body relaxation in clusters and loose groups but not in dense groups. In general, only the cores of rich clusters are relaxed.
Particles that evolve in a rapidly time varying potential can rapidly forget their initial conditions [78]. This violent relaxation occurs in a timescale

$$\tau_{v-rel} \sim \tau_{ff} \sim \tau_{dyn} \quad \text{when} \quad |\frac{\partial \phi}{\partial t}| > |v \cdot \nabla \phi|,$$

where $\tau_{ff}$ is the free-fall time, and $\phi$ is the global potential. This applies for example to collapsing systems, as is often the case in cosmology. For example, elliptical galaxies are thought to form by dissipationless collapse or by mergers of disk galaxies [79] [44], and since both phenomena perturb the potential in a rapid violent manner, the cores of elliptical galaxies appear relaxed although their 2-body (and collective) relaxation times are much longer than the age of the Universe.

Energy exchanges during encounters lead to energy equipartition, and the more massive objects tend to move slowly and fall to the center of their systems, leading to mass segregation. The timescale is similar to that of 2-body relaxation.

Occasionally, collisions will pump sufficient energy into a particle that it’s velocity will be larger than the escape velocity of its system, and, barring subsequent encounters that may reduce its kinetic energy, the particle will escape. If during one relaxation time the distribution of particle velocities reaches a Maxwellian, the fraction of unbound particles is roughly 1% so that the timescale for evaporation is roughly 100 times the relaxation time [80].

### 4.2 Dynamical friction, orbital decay and circularization

Field particles are deflected by the mass of the test particle. Hence, in the frame of the test particle, the field particle density is higher behind the test particle than in front of it. This leads to a drag force known as dynamical friction [81], which plays a major role in group and cluster dynamics. For an infinite homogeneous medium, the timescale for dynamical friction is [81]

$$\tau_{df} \equiv \left( \frac{1}{v_{||}} \frac{dv_{||}}{dt} \right)^{-1} = \frac{v^3}{2\pi G^2(m + m_f) \rho f(v/\sigma_v) \ln(1 + \Lambda^2)}.$$  \hspace{1cm} (13)

where $\rho$ is the local mass density of field particles, and where $f$ and $\Lambda$ are defined in § 4.1.

Perhaps more physical is the timescale for orbital decay defined as

$$\tau_{od} \equiv \left( \frac{1}{R} \frac{dR}{dt} \right)^{-1} = \left( \frac{R dE/dR}{mv^2} \right) \tau_{df} = \frac{3}{2} \left( \frac{\rho}{\rho} + \frac{1}{3} \right) \tau_{df}.$$  \hspace{1cm} (14)

Dynamical friction and orbital decay can lead to misleading answers:

- No orbital decay is predicted in zero density environments, whereas a satellite galaxy sitting just outside its parent galaxy will see its orbit decay, because of resonances with its parent [82].
• Although orbital decay should be slowed by tidal effects that reduce the test particle’s mass, the contrary may occur with a satellite galaxy circling its parent, as the tides from the latter remove stars from the former, and these carry off energy and angular momentum, thus accelerating the orbital decay [83].

• For a particle radially falling into a medium with a outwards decreasing density profile, the dynamical friction time computed from equation (13) is longer than the time on which the particle sees an increasing density. A system, dense enough to survive the tidal forces from the primary into which it falls, will nearly stick to the core of the primary after its second passage, if its mass is more than 10% of the primary’s mass [84].

For circular orbits, combining equations (12), (13), and (14), one obtains

\[
\frac{\tau_{\text{od}}}{\tau_{\text{orb}}} = \frac{(1 + \frac{1}{3}\bar{\rho}/\rho) M/m}{2\pi f(1) \ln(1 + A^2)(1 + m/M)^{1/2}} \approx \frac{2}{\pi} \frac{M/m}{(1 + m/M)^{1/2} \ln [1 + (M/m)^{2/3}]} ,
\]

where \( f \approx 1/2, p_{\text{min}} \sim r, p_{\text{max}} \sim R \), primary and secondary have the same mean density \( (M/m) \sim [R/r]^3 \), and for singular isothermal density profiles \( (\rho \sim r^{-2}) \), \( \bar{\rho} = 3\rho \). The term \((1 + m/M)^{1/2}\) is a correction for non-negligible secondary to equations (12) and (13). Table 3 below gives the ratios from equation (15) for typical astrophysical ratios.

Table 3: Number of circular orbits for orbital decay

| Secondary    | \( M/m \) | \( \tau_{\text{orb}}/\tau_{\text{od}} \) |
|--------------|-----------|------------------|
| Galaxy       | 10 000    | 1000             |
| Group        | 100       | 20               |
| Small cluster| 10        | 3.5              |
| Moderate cluster | 3   | 1.5              |

Notes: Examples of secondaries are for a rich cluster primary, but should be considered as academic, since the secondaries are not expected to acquire sufficient angular momentum as to orbit in near circular orbits as they enter the cluster (§3.4).

From table 3, dynamical friction and orbital decay are not important, in terms of number of near-circular orbits, unless the mass ratio of secondary to primary is \( \gtrsim 0.1 \), i.e., small clusters falling into larger ones, or galaxies falling into small groups. In absolute terms, the circular orbital time is \( \tau \approx 0.4t_0 \) for a secondary orbiting a \( 10^{15}M_\odot \) cluster at \( 1 \, h^{-1}\text{Mpc} \) from its center. Hence, the timescale for orbital decay in rich clusters is greater than the age of the Universe for systems with \( m < 10^{14}M_\odot \), orbiting at \( 1 \, h^{-1}\text{Mpc} \).
Another outgrowth of dynamical friction is orbital circularization, whose timescale can be defined as the rate a test particle acquires angular momentum from interactions with other particles:

\[ \tau_{oc} = \left( \frac{1}{J_{\text{circ}}(E)} \frac{dJ}{dt} \right)^{-1}, \]

and is found [85] to be shorter than the orbital decay time outside of the core of a cluster.

### 4.3 Tides

Tidal forces act on particles in a system relative to the full system itself. As such there are two types of tides acting on galaxies in groups and clusters: those caused by close encounters with other galaxies and those caused by variations in the gradient of the global group/cluster potential. The first type of tides (collisional stripping) has a timescale

\[ \tau_{cs} \equiv \left( \frac{1}{\frac{dm}{dt}} \right)^{-1} = \langle (\Delta m/m) n(\sigma v) \rangle^{-1} = \frac{Cst}{nr^2v_g}, \]

where \( \sigma \) is the collisional stripping cross-section, and the outer stars are assumed to follow elongated orbits [86, 87].

Global potential tides depend strongly on the galaxy’s orbit around the cluster. If the galaxy is phase locked in a nearly circular orbit around the cluster, it will feel a roughly constant tidal shear, and its tidal radius will be obtained by equating the tidal shear in the galaxy with the gravitational pull that the full galaxy exerts on a star at that radius, plus an inertial term:

\[ \Delta \left( \frac{GM(r)}{R^2} \right) = \frac{Gm(r)}{r^2} + \Omega^2 r, \]

yielding for \( r \ll R \)

\[ \bar{\rho}_g(r_t) = \bar{\rho}_{cl}(R) \left[ 2 - 3 \frac{\rho_{cl}(R)}{\bar{\rho}_{cl}(R)} + \frac{V_p^2(R)}{V_{\text{circ}}^2(R_p)} \right], \]

i.e., the galaxy is tidally truncated at a radius \( r_t \) where its mean density is of the order of the mean cluster density within the radius \( R_p \) of closest approach of the galaxy (where \( V_p \) and \( V_{\text{circ}} \) are the pericentric and circular velocities, respectively).

If the orbits are elongated, the instantaneous tide obtained from equation (16) is short lived and the galaxy experiences a tidal shock [88]. Using the impulse approximation [89], in which the perturber moves with a constant relative velocity \( \mathbf{V} \), one can show [44], again for \( r \ll R \), that

\[ \bar{\rho}_g(r_t) = Cst \bar{\rho}_{cl}(R_p)f(\epsilon), \]
where $R_p$ is the pericentric of the galaxy’s orbit, and $f(\epsilon)$ is a function of order unity of the galaxy’s orbital eccentricity. This criterion is similar to that for circular orbits, but the constants are higher, because at given pericenter, a galaxy in a circular orbit must feel a more effective tide, since it is long-lived \[90\]. Numerical simulations \[91\] confirm this result although other simulations \[92\] suggest that the tide is most efficient for some intermediate elongation at given pericenter, when this is within the nearly homogeneous region of the cluster. Note that the timescales for global potential tides are basically the orbital timescales divided by the typical mass-loss per passage through the cluster core.

The effectiveness of a tide is related to the maximum strength of the tide times the duration of this maximum tide. So, from equation (17) one gets

$$\Delta v \sim F_{\text{tid}} \Delta t \sim \rho_g \Delta t \sim \frac{2 - 3\rho_{cl}/\rho_{cl} + V_p^2/V_{\text{circ}}^2}{V_p/V_{\text{circ}}}$$

$$\sim 3 \left(1 - \frac{\rho_{cl}}{\rho_{cl}}\right) - \left(1 - 3\rho_{cl}/\rho_{cl}\right) \left(\frac{V_p}{V_{\text{circ}}} - 1\right) \text{ for } V_p \gtrsim V_{\text{circ}}.$$  

Hence, the $N$-body simulations showing an intermediate orbit elongation for maximum global potential tides \[92\] are understood, since when the cluster region is nearly homogeneous, the effective tide increases with increasing pericenter velocity, but not when the cluster density profile decreases sharply as outside the core of the Modified Hubble model used in the simulations.

4.4 Mergers

Because galaxies have their own internal energy, galaxy collisions are often inelastic enough to lead to merging. The timescale for merging may be estimated from a merging cross-section, again as

$$\tau_m = n \langle \sigma v \rangle^{-1}.$$  

Using a numerical experimental cross-section \[93\], and integrating over a Maxwellian velocity distribution, the merger time can be written \[14\]

$$\tau_m = \text{Cst} \left[ nr_g^2 v_g K(v_{cl}/v_g) \right]^{-1},$$

where $n$ is the number density of galaxies, $r_g$ and $v_g$ are the galaxy half-mass radius and internal velocity dispersion, respectively, and the dimensionless merging efficiency $K$ is optimum for groups ($v_{cl} \approx v_g$), while for clusters it falls off as $v_{cl}^3$ \[14\]. In groups as dense as compact groups \[4\] appear to be, merging ought to be extremely efficient, and the relatively low fraction of ellipticals indicates that chance alignments are contaminating the compact group catalogs \[14\]. Despite their high velocity dispersions, rich clusters are able to produce the right amount of mergers to produce elliptical morphologies \[14\], and moreover, merging is able to account \[14\] for the morphology-density \[94\] and morphology-radius \[95\] relations.
5 Cluster evolution

5.1 Dynamical evolution

The physical processes described in §4 compete in the evolution of the galaxy system. For example, merging leads to increased merger cross-sections, hence to a merging instability [96]. However, this instability is slowed down by tidal processes which are usually thought to truncate galaxies of their outlying particles which become unbound [97]. Yet, if the merging cross-section is related to the galaxy half-mass radius [97], and since the tidal processes for galaxies on elongated orbits or from collisions pump energy into the system, then the half-mass radius of those particles that remain bound to the galaxy should increase. The question remains whether the new half-mass radius is then greater or smaller than the old value, but this reviewer is not aware of any numerical study that has addressed this question yet.

Similarly, if galaxies possess huge halos when they enter clusters (as one can infer from Table 2), they should feel strong dynamical friction and orbital decay. But once they pass near the cluster core, these halos should be severely stripped by the global tide of the cluster potential, after which the galaxy is less massive and will be no longer subject to much dynamical friction.

The interplay of the various dynamical processes and the difficulty in analytical modeling of tidal effects render necessary to run numerical $N$-body simulations to see how groups and clusters evolve. In collisionless particle simulations, galaxies rapidly dissolve in clusters [98], a problem known as overmerging, which is often ascribed to numerical effects. In particular, two-body relaxation between galaxy particles and cluster particles has been blamed for overmerging [99]. Two analytical estimates differ on the importance of this effect. Whereas, one [100] finds a short timescale for the two-body relaxation between galaxy particles and cluster particles: $\tau_{cl-g} = (v_g/v_{cl})^2 \tau_{cl-cl} \simeq 0.1 \tau_{cl-cl}$, the other study [101] considers the time $\tau = U/\dot{U} = U/(N\Delta U_{imp, apx})$, where $U$ is the internal energy of the galaxy and $\dot{U}$ is the rate of collisions, and finds a timescale typically longer than the Hubble time. This last study [101] also checks that evaporation is not responsible for overmerging.

Overmerging can also be ascribed to physical effects such as the tides from the global potential of the cluster or dense group or two-body relaxation between galaxies. In semi-analytical calculations of the passage through a cuspy ($\rho \sim r^{-2}$ density profile) primary potential, of a smooth secondary, also with a cuspy potential, one finds [65] that, for reasonably elongated orbits, the tides of the primary pump in considerably more energy into the secondary than its own binding energy. Hence, secondaries such as galaxies dissolve at first passage. It remains to be seen whether these secondaries are completely disrupted or whether energy exchanges within it allow a dense core to survive. Recent simulations [101] of multi-particle secondaries moving along elongated orbits in a given isothermal potential show that tidal disruption is relatively slow, although it is not clear whether their “isothermal” primary is the singular $\rho \sim r^{-2}$ model or the non-singular isothermal (that thus has an homogeneous core), for which
one indeed expects softer tidal effects.

In any event, overmerging is much less evident in simulations where gas is included \[102, 103\], presumably because the gas sinks to the bottom of the galaxy potential wells and deepens these wells, which thus avoid merging with one another. The latter study emphasizes the importance of star formation in their gaseous galaxies in reducing overmerging.

Dense groups of galaxies witness rapid merging and coalesce into a single elliptical galaxy \[104, 105, 90, 106\]. A detailed comparison of the results on groups \[107\] showed that the different numerical studies of groups produced comparable rates of merging. The differences arise in part from the initial conditions and from the fact that the merger rate in dense groups is decreased when most of the dark matter is distributed in a common envelope, rather than in individual galaxy halos \[108, 105, 90, 109\]. Indeed, with large individual halos, galaxy merging is direct, while the presence of an important intergalactic background causes galaxies to dissipate orbital energy by dynamical friction, suffer orbital decay and finally merge together at the group center \[90\]. It may be that overmerging is occurring in all these dense group simulations, and that with the inclusion of gas dynamics, the galaxies in dense groups may survive longer.

5.2 Cosmological evolution

Spherical top-hat cosmology (§ 3.2), can be applied to a galaxy system bathing in an empty or uniform universe. An homogeneous isolated system should see its size evolve as shown in Figure 1a. It first follows the Hubble expansion, then decouples from this expansion and turns around, collapses and subsequently virializes. If a system is in dynamical equilibrium, one can apply the virial theorem, and so derive a virial mass \(M_{\text{vir}} = RV^2/G\). While the time evolution of the radius \(R\), velocity dispersion \(V\), ratio of virial mass to true mass \(M_{\text{vir}}/M\) and ratio of crossing time to running time \(t_{\text{cr}}/t = R/(Vt)\) parameters have been studied before \[110\] (though simple \(N\)-body simulations), and an analytical version is shown in Figures 1b, 1c, and 1d, one gains considerable insight in plotting the evolution of \(M_{\text{vir}}/M\) versus \(t_{\text{cr}}/t\) as is done in Figure 2a. The dotted track is for groups made of point mass galaxies, while the solid track is for extended galaxies, which reach a terminal velocity at group collapse (because the smoothed potential is flat at the center), and after virialization, dissipate their orbital energy by dynamical friction against their common massive halo (merged from their individual halos after group collapse).

The solid track in Figure 2a can be considered as a fundamental track that galaxy systems should follow, similar in concept to a Hertzprung-Russell diagram for cosmological galaxy systems. Unfortunately, true masses of galaxy systems are not known. To compare with observed parameters, we must make an assumption on the true mass, and the simplest one is to assume that the true mass follows light, i.e., \(M/L = \text{cst.}\) In Figures 2b, c, and d, we plot the observable parameters, \(M_{\text{vir}}/L_B\) vs. \(H_0t_{\text{cr}}\), for loose groups \[111\] of different multiplicities, and superpose the theoretical evolution-
Figure 1: Time evolution of bias in observed radius (a), velocity dispersion (b), mass (c), and crossing time (d), relative to virial equilibrium (VE), where $T_{ta}$ is the turnaround time. The dotted curves show the evolution for point-masses, while the full curves show the effects of softened potentials and orbital energy dissipation by dynamical friction (starting at $t = 3 T_{ta}$).

ary track, adjusting the $y$-axis with the high multiplicity groups of Figure 2b, while the $x$-axis scaling is imposed by theory.

The high-multiplicity groups fit the theoretical tracks very well. A one proceeds to lower multiplicities, the statistical noise in the mass-to-light ratio and crossing time estimates increases, but so does the probability for chance alignments, which make the groups appear smaller while conserving on the average their velocity dispersion. Although precise assignments of group cosmo-dynamical states is difficult because of statistical noise, one can nevertheless get a handle on which groups are unbound (above theoretical track), which are still in their expansion phase (upper-right handle of track), which are near turnaround (lower-right handles of track), which are collapsing (central handle), which are near maximum collapse (first lower-left handle),
Figure 2: Mass assuming dynamical equilibrium, scaled to total mass (a) or total blue luminosity (b, c, and d), versus crossing time (in units of the age of the Universe for $\Omega = 1$, while for $\Omega = 0.2$ the points should be displaced to the left by 0.1 decade). The polygons (b, c, and d) represent the loose groups. The thin curves are the theoretical point-mass evolutionary tracks, while the thick curves are the same for softened potentials and allowing for orbital energy dissipation after virialization. In (b, c, and d), these curves are scaled to mass-to-light ratios assuming that all groups have a true $M/L = 440\,h$. The typical error bar for each data point is shown.

and those that are virialized (second lower-left handle). The theoretical fundamental track in observable space $M_{\text{vir}}/L$ vs. $t_{\text{cr}}/t_0$ thus represents a slice through the fundamental surface (which is curved) of groups, where the third axis can be the scale of the system, such as its total optical luminosity. In contrast, an empirical fundamental plane has been reported [112] for loose groups, similar to that of elliptical galaxies and clusters (see §2.4).

The true $M/L$ is obtained by extrapolating to the early virialized state (before dissipation of orbital energy, which occurs at nearly constant velocity dispersion since the
common halo should have near constant circular velocity). The loose groups then have $M_{\text{true}}/L = 440\, h$, much higher than the median $M/L = 130\, h$, for the groups of $N \geq 4$ members (the mass estimate used here is the median of the non-weighted virial, weighted virial, and projected masses). In other words, the mass-to-light ratios of groups are severely underestimated because most groups are still relatively near their turnaround phase. This points to $\Omega \simeq 0.3$ obtained by extrapolating the galaxy luminosity function to $(M/L)_{\text{closure}} = 1560\, h$.

In any event, no groups in the loose group catalog has yet completed its collapse, not even the Virgo cluster included in the catalog, whose outer members are still collapsing onto the virialized core. The dynamical youth of loose groups suggests a high value (close to unity) for $\Omega_0$. Details of this analysis are still in preparation.

The fundamental tracks for groups can been computed in a hierarchical binary approximation, with a galaxy merging with an already merged pair. The precise position of the track depends slightly on the mass ratio of the merging pair. Any system can be represented at a given hierarchy. For example, the NGC 2300 group, in which X-rays from the intergalactic gas were first discovered, is a loose triplet that has just turned around from its maximum Hubble expansion, while the tight binary around which the X-rays are roughly centered appears on the evolved part of the fundamental track (late collapse or rebound). The fundamental surface analysis can be easily applied to pairs, but the error bars in the position of a pair on the diagram are relatively important, and a detailed statistical analysis is required. Conversely, clusters suffer little from statistical uncertainties of the observed parameters, but when a subcluster is merging into a cluster, the fundamental surface analysis on the primary cluster may break down as the secondary can exert non-negligible tidal stresses that alter the collapse of the primary.

In the previrialization scenario, structures acquire angular momentum at maximum expansion, and do not fully collapse to reach a rapid virial equilibrium. The fundamental track, should then be slightly altered, as it will follow the standard track at early epochs (and high observed crossing times, to the right of Fig. 2), and up to $t_{cr}/t \simeq 0.03$, then slightly rebound to the virialized horizontal wing (the thin horizontal track to the left of Fig. 2). If the previrialized system coalesces by dynamical friction, its track will then follow the soft collapse track, and its overall behavior will be similar to the standard track. However, a coalescence is somewhat akin to a collapse, but slower. Hence, the the decrease of the track to the lower left of the diagram is delayed with respect to the standard track.

In Figure 3 are shown the locations of compact groups in the fundamental track diagram. The groups on the upper left of the diagram are high velocity dispersion groups close to the fundamental track. Conversely, the groups on the lower right of the diagram are low velocity dispersion groups too far from the fundamental track to be explained by standard evolution. Instead, they are best explained as arising from chance alignments of galaxies along the line of sight, either within larger loose groups that are still collapsing, or within long ($\sim 7\, h^{-1}\, \text{Mpc}$) filaments.
[116], often seen in cosmological simulations, and some of which may be expanding (the spherical equivalent being the upper right track). Thus, the higher velocity dispersion half of this sample of compact groups are real 3-D dense systems, either terminating their collapse or rebounding from it, or in their final coalescence stage, while the lower velocity dispersion half are caused by chance alignments. If previrialization is not followed by rapid coalescence, then as much as 90% of the compact groups would lie too far from the fundamental track to be real and would be ascribed to chance alignments. This is inconsistent with the detection of X rays, probing the potential of truly dense groups, in roughly half of these compact groups [117] (typically the high velocity dispersion ones [117, 118]). If coalescence does occur after previrialization, the data of Figure 3 is not good enough to distinguish between the two scenarios, but a richer compact group catalog, or the use of X-rays to infer in a more secure fashion the velocity group dispersion should help settle between them.

Figure 3: Same as Figures 2bcd for compact groups (crosses).

References
[1] G.O. Abell Ap. J. Supp. Ser. 3 (1958), 211.
[2] P. Hickson Ap. J. 255 (1982), 382.
[3] E.L. Turner Ap. J. **208** (1976), 20.
[4] N.A. Bahcall Ap. J. **198** (1975), 249.
[5] A. Yahil Ap. J. **191** (1974), 623.
[6] T. Beers and J.L. Tonry Ap. J. **300** (1986), 557.
[7] D. Merritt and K. Gebhardt (1994) in XIVth Moriond Astrophysics Meeting *Clusters of Galaxies*, ed. F. Durret, A. Mazure and J. Tran Thanh Van (Frontières, Gif-sur-Yvette, 1994), p. 11, astro-ph/9510091.
[8] R.B. Tully Ap. J. **321** (1987), 280.
[9] D.G. Walke and G.A. Mamon Astr. Ap. **225** (1989), 291.
[10] P. Hickson, Z. Ninkov, J.P. Huchra and G.A. Mamon in *Clusters and Groups of Galaxies*, ed. F. Mardirossian, G. Giuricin, and M. Mezzetti (Reidel, Dordrecht, 1984), p. 367.
[11] M.-L. Montoya, R. Domínguez-Tenreiro, G. González-Casado, G.A. Mamon and E. Salavador-Solé in *From Galaxies to Galaxy Systems* also Ap. Lett. Comm. (1996), in press.
[12] P. Hickson, C. Mendes de Oliveira, J.P. Huchra and G.G.C. Palumbo Ap. J. **399** (1992), 353.
[13] J. Binney and G.A. Mamon *M.N.R.A.S.* **200** (1982), 361.
[14] D.M. Fabricant, M. Lecar and P. Gorenstein Ap. J. **241** (1980), 552.
[15] A. Cavaliere Mem. Soc. Astron. It. **44** (1974), 571.
[16] M.J. Henriksen Ph.D. Thesis, *Univ. of Maryland* (1985).
[17] J.P. Hughes Ap. J. **337** (1989), 21.
[18] D. Gerbal, F. Durret, M. Lachièze-Rey and G. Lima-Neto Astr. Ap. **253** (1992), 77.
[19] F. Durret, D. Gerbal, M. Lachièze-Rey, G. Lima-Neto and R. Sadat Astr. Ap. **287** (1994), 733.
[20] R. Bourassa and R. Kantowski Ap. J. **195** (1975), 13.
[21] S.A. Grossman and R. Narayan Ap. J. **324** (1988), L77.
[22] G. Soucail, B. Fort, Y. Mellier and J.-P. Picat Astr. Ap. **172** (1987), L14.
[23] R. Lynds and V. Petrosian *Bull. Am. Astr. Soc.* **18** (1986), 1014.
[24] N. Kaiser and G. Squires Ap. J. **404** (1993), 441.
[25] A.J. Tyson and P. Fischer Ap. J. **446** (1995), L55, astro-ph/9503119.
[26] G. Squires, N. Kaiser, A. Babul, G. Fahlman, D. Woods, D.M. Neumann and H. Böhringer Ap. J., submitted (1995), astro-ph/9507008.
[27] A. Loeb and S. Mao Ap. J. **435** (1994), L109, astro-ph/9406030.
[28] J. Miralda-Escudé and A. Babul Ap. J. **449** (1995), 18, astro-ph/9405063.
[29] G.O. Abell, J. Neyman and E. Scott Astr. J. **69** (1964), 529.
[30] F.W. Baier Astr. Nach. **298** (1977), 151.
[31] M.J. Geller and T.C. Beers *P.A.S.P.* **94** (1982), 421.
[32] G. Bothun, M.J. Geller, T.C. Beers and J.P. Huchra Ap. J. **268** (1983), 47.
[33] C. Jones and W. Forman in *Clusters and Superclusters of Galaxies*, ed. A.C. Fabian (Kluwer, Dordrecht, 1992), p. 49.
[34] A. Dressler and S.A. Shectman Astr. J. **95** (1988), 985.
REFERENCES

[35] F.W. Baier Astr. Nach. 305 (1984), 175.
[36] E. Salvador-Solé, G. González-Casado and J.M. Solanes Ap. J. 410 (1993), 1.
[37] E. Escalera, A. Biviano, M. Girardi, G. Giuricin, F. Mardirossian, A. Mazure and M. Mezzetti Ap. J. 423 (1994), 539, astro-ph/9309057.
[38] P. Hickson and H.J. Rood Ap. J. 331 (1988) L69.
[39] R. Schaeffer, S. Maurogordato, A. Cappi and F. Bernardeau M.N.R.A.S. 263 (1992), L21.
[40] S.M. Kent, J.E. Gunn Astr. J. 87 (1982), 945.
[41] J. Binney M.N.R.A.S. 200 (1982), 951.
[42] R. den Hartog, P. Katgert M.N.R.A.S. (1995), submitted.
[43] J.E. Gunn, J.R. Gott Ap. J. 176 (1972), 1.
[44] G.A. Mamon Ap. J. 401 (1992), L3.
[45] E. Bertchinger Ap. J. Supp. Ser. 58 (1985), 39.
[46] S. Cole and L. Cacey M.N.R.A.S., submitted (1995), astro-ph/9510147.
[47] M.J. Geller and J.P Huchra Ap. J. Suppl. bf 52 (1983), 61.
[48] M. Ramella, M.J. Geller and J.P Huchra Ap. J. bf 344 (1989), 57.
[49] G.A. Mamon in XIVth Moriond Mtg. Clusters of Galaxies, ed. F. Durrell, A. Mazure and J. Tran Van Thanh (Gif-sur-Yvette, Frontières, 1994), 291, astro-ph/9406043.
[50] J.R. Gott Ap. J. 201 (1975), 296.
[51] J.A. Fillmore, P. Goldreich Ap. J. 281 (1984), 1.
[52] F. Moutarde, J.-M. Alimi, F.R. Bouchet, R. Pellat Ap. J. 441 (1995), 10.
[53] J.E. Gunn Ap. J. 218 (1977), 592.
[54] W.H. Zurek, P.J. Quinn, J.K. Salmon Ap. J. 330 (1988), 519.
[55] M.M. Crone, A.E. Evrard, D.O. Richstone Ap. J. 434 (1994), 402, astro-ph/9404030.
[56] Y. Hoffman, J. Shabam Ap. J. 297 (1985), 16.
[57] G. Efstrathiou, C.S. Frenk, S.D.M. White, M. Davis M.N.R.A.S. 235 (1988), 715.
[58] J. Dubinski and R.G. Carlberg Ap. J. 378 (1991), 496.
[59] J.F. Navarro, C.S. Frenk and S.D.M. White M.N.R.A.S. 275 (1995), 720, astro-ph/9408069.
[60] J.R. Gott and M. Rees Astr. Ap. 45 (1975), 365.
[61] D. Richstone, A. Loeb and E.L. Turner Ap. J. 393 (1992), 477.
[62] W.H. Press and P. Schechter Ap. J. 187 (1974), 425.
[63] C. Lacey and S. Cole M.N.R.A.S. 262 (1993), 627.
[64] G. Kauffmann and S.D.M. White M.N.R.A.S. 261, 921.
[65] G. González-Casado, G.A. Mamon and E. Salvador-Solé Ap. J. 433 (1994), L61, astro-ph/9406066.
[66] G. González-Casado, J.M. Solanes and E. Salvador-Solé Ap. J. 410 (1994), 15.
[67] A.E. Evrard, J.J. Mohr, D.J. Fabricant and M.J. Geller Ap. J. 419 (1993), L9.
[68] J.J. Mohr, A.E. Evrard, D.J. Fabricant and M.J. Geller Ap. J. 447 (1995), 8, astro-ph/9501011.
[69] M. Davis and P.J.E. Peebles Ap. J. Supp. 34 (1977), 425.
[70] P.J.E. Peebles Ap. J. 365 (1990), 27.
[71] P.J.E. Peebles Large Scale Structure of the Universe, Princeton, Princeton University Press (1980), chap. 19.
[72] A.E. Evrard and M.M. Crone Ap. J. 394, L1.
[73] E.L. Lokas, R. Juszkiewicz, F.R. Bouchet and E. Hivon Ap. J. in press (1996), astro-ph/9508032.
[74] M. White, D. Scott, J. Silk and M. Davis M.N.R.A.S., in press (1995), astro-ph/9508009.
[75] G.A. Mamon in N-Body Problems and Gravitational Dynamics. ed. F. Combes and E. Athanassoula, (Meudon, Obs. de Paris, 1993), 188, astro-ph/9308032.
[76] S. Chandrasekhar Principles of Stellar Dynamics (1942), New York, Dover.
[77] V.G. Gurzadyan and G.K. Savvidy Astr. Ap. 160 (1986), 203.
[78] D. Lynden-Bell M.N.R.A.S. 136 (1967), 101.
[79] A. Toomre in The Evolution of Galaxies and Stellar Populations, ed. B.M. Tinsley and R.B. Larson (New Haven, Yale Univ. Press, 1977), p. 401.
[80] V.A. Ambartsumian Ann. Leningrad State Univ. 22 (1938), 19, translated in IAU Symp. 113 Dynamics of Star Clusters, ed. J. Goodman and P. Hut (Dordrecht, Reidel, 1985), p. 521.
[81] S. Chandrasekhar Ap. J. 97 (1943), 255.
[82] D.N.C. Lin and S.D. Tremaine Ap. J. 264 (1983), 364.
[83] P. Prugniel and F. Combes Astr. Ap. 259 (1992), 25.
[84] R. Chan, G.A. Mamon and D. Gerbal Astr. Ap. (1995), submitted.
[85] D. Merritt Ap. J. 289 (1985), 18.
[86] D.O. Richstone Ap. J. 200 (1975), 535.
[87] A. Dekel, M. Lecar and J. Shaham Ap. J. 241 (180), 946.
[88] J.P. Ostriker, L. Spitzer and R.A. Chevalier Ap. J. 176 (1972), L51.
[89] L. Spitzer Ap. J. 127 (1958), 17.
[90] G.A. Mamon Ap. J. 321 (1987), 622.
[91] A.J. Allen and D.O. Richstone Ap. J. 325 (1988), 583.
[92] D. Merritt and S.D.M. White in IAU Symp. 117 Dark Matter in the Universe, ed. J. Kormendy & G.R. Knapp (Dordrecht, Reidel, 1987) 283.
[93] N. Roos and C.A. Norman Astr. Ap. 95 (1979), 349.
[94] M. Postman and M.J. Geller Ap. J. 281 (1984), 95.
[95] B.C. Whitmore and D. Gilmore Ap. J. 367 (1991), 64.
[96] J.P. Ostriker and M. Hausman Ap. J. 217 (1977), L125.
[97] S. Aarseth and S.M. Fall Ap. J. 236 (1980), 43.
[98] S.D.M. White, M. Davis, G. Efstathiou and C.S. Frenk Nature 330 (1987), 451.
[99] R.G. Carlberg Ap. J. 433 (1994), 468.
[100] E. van Kampen M.N.R.A.S. 273 (1995), 295.
[101] B. Moore, N. Katz and G. Lake Ap. J. (Lett.), submitted (1995), astro-ph/9503083.
[102] N. Katz and S.D.M. White Ap. J. 412 (1993), 455.
[103] A.E. Evrard, F. Summers and M. Davis Ap. J. 422 (1994), 11.
[104] P. Carnevali, A. Cavaliere and P. Santangelo, Ap. J. 249 (1981), 449.
[105] J. Barnes, M.N.R.A.S. 215 (1985), 517.
[106] J. Barnes, Nature 338 (1989), 123.
[107] G.A. Mamon in IAU Coll. 124, Paired and Interacting Galaxies, ed. J.W. Sulentic & W.C. Keel (Washington, NASA, 1990), 609.
[108] A. Cavaliere, P. Santangelo, G. Tarquini and N. Vittorio in Clustering in the Universe, ed. D. Gerbal & A. Mazure (Gif-sur-Yvette, Frontières, 1982) p. 25.
[109] P.W. Bode, H.N. Cohn and P.M. Lugger, Ap. J. 416 (1993), 17.
[110] G. Giuricin, P. Gondolo, F. Mardirossian, M. Mezzetti and M. Ramella, Astr. Ap. 199 (1988), 85.
[111] E. Gourgoulhon, P. Chamaraux and P. Fouqué, Astr. Ap. 255 (1992), 69.
[112] D. Burstein, R. Bender, S.M. Faber and R. Nolthenius, Ap. Lett. Comm. 31 (1995), 95.
[113] J. Loveday, B.A. Peterson, G. Efstathiou and S.J. Maddox, Ap. J. 390 (1992), 338.
[114] J.S. Mulchaey, D. Davis, R.F. Mushotzky and D. Burstein, Ap. J. 404 (1993), L9.
[115] G.A. Mamon, Ap. J. 307 (1986), 426.
[116] L. Hernquist, N. Katz and D.H. Weinberg, ApJ 442 (1995), 57, astro-ph/9407059.
[117] J.S. Mulchaey, D. Davis, R.F. Mushotzky and D. Burstein, Ap. J. in press (1995).
[118] G.A. Mamon and M.J. Henriksen, in preparation.