Multiplicity Fluctuations in the Pion-Fireball Gas

M.I. Gorenstein\textsuperscript{1,2} and O.N. Moroz\textsuperscript{1}

\textsuperscript{1}Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine
\textsuperscript{2}Frankfurt Institute for Advanced Studies, Frankfurt, Germany

Abstract

The pion number fluctuations are considered in the system of pions and large mass fireballs decaying finally into pions. A formulation which gives an extension of the model of independent sources is suggested. The grand canonical and micro-canonical ensemble formulations of the pion-fireball gas are considered as particular examples.

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During several decades the models of hadron productions in high energy collisions from decays of the fireballs \([1]\), droplets \([2]\), strings \([3]\), quark-gluon bags \([4]\) have been considered. We will use the name fireball for all these objects with high mass \(m\). A typical estimate is \(m > 2 \text{ GeV}\). Fireballs may crucially influence the thermodynamical properties of the system. For example, their presence leads to the limiting temperature due to the exponential mass spectrum of fireballs \([1]\) and strings \([3]\), or to the phase transitions in the gas of quark-gluon bags \([4]\). It is assumed that the fireballs decay into stable hadrons, mainly to pions. A comparison with the data on hadron multiplicities at high energy collisions does not indicate a presence of massive fireballs (see however recent discussion in Ref. \([5]\)). Thus their contribution to the hadron average multiplicities is probably rather small, if it exists. On the other hand, even for small contributions of fireballs to average pion multiplicity, there is a chance to find the signatures of large mass fireballs by measuring the multiplicity fluctuations. This will be discussed in the present paper. The study of event-by-event fluctuations in high energy nucleus-nucleus collisions may also open new possibilities to investigate the phase transition between hadronic and partonic matter as well as the QCD critical point (see, e.g., the reviews \([6, 7, 8, 9]\)).

The scaled variance

\[
\omega = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle},
\]

is a useful measure to describe the pion multiplicity fluctuations. In Eq. (1), \(N\) is the total pion number after fireballs decay. The event-by-event averaging is defined by the pion probability distribution \(W(N)\),

\[
\langle \ldots \rangle = \sum_{N=0}^{\infty} \ldots W(N).
\]

If there is only one type of fireballs, and they are the only sources of pions, the scaled variance \(\omega\) is equal to (see, e.g., \([6]\)):

\[
\omega = \omega_f + \pi_f \omega_f^*,
\]

where \(\pi_f\) and \(\omega_f\) is, respectively, the average number of pions and the scaled variance of pion multiplicity fluctuations from one-fireball decay. In Eq. (3), \(\omega_f^*\) is the scaled variance for the fluctuations of the number of fireballs. The Eq. (3) corresponds to the contributions
of independent sources (fireballs). In this paper we present the extension of the model of independent sources in two directions. First, we consider both fireballs and primordial pions on an equal footing. Thus, only a part of the final pion multiplicity (probably a small part) is due to the fireball decays. Another part is just the number of the primordial pions. Second, only the decays of fireballs (sources) are assumed to be independent, while their production may include different type of correlations. These correlations are absent in the standard model of independent sources.

2. We introduce the probability distribution function \( P(N_0, N_1, \ldots, N_k) \) to describe the production of fireballs and primordial pions. The distribution function \( \Gamma_i(n) \) describes the decays of \( i \)-th fireball into pions. The probability \( W(N) \) to find \( N \) pions in the system of primordial pions and fireballs decaying finally into pions is then calculated as:

\[
W(N) = \sum_{N_0, N_1, \ldots, N_k} P(N_0, N_1, \ldots, N_k) \prod_i \prod_j \sum_{n_{ij}=0}^{\infty} \Gamma_i(n_{ij}) \delta \left( N - \sum_i \sum_j n_{ij} \right). \tag{4}
\]

In Eq. (4), \( i = 0 \) corresponds to pions and \( i = 1, \ldots, k \) to different types of fireballs, \( N_i \) is the number of \( i \)-th species, and \( n_{ij} \) is the number of pions from the decay of fireball of type \( i \) with the number \( j = 1, \ldots, N_i \). The normalization conditions,

\[
\sum_{N_0, N_1, \ldots, N_k} P(N_0, N_1, \ldots, N_k) = 1, \quad \sum_{n=0}^{\infty} \Gamma_i(n) = 1, \tag{5}
\]

are assumed, and they lead to \( \sum_N W(N) = 1 \). The following assignment will be used:

\[
\langle \ldots \rangle_T \equiv \sum_{N_0, N_1, \ldots, N_k} \ldots P(N_0, N_1, \ldots, N_k). \tag{6}
\]

According to (4) the fireballs decay independently, i.e. the number of pions from the decays of two different fireballs do not correlate. One then finds,

\[
\sum_{n_{ij}=0}^{\infty} \Gamma_i(n_{ij}) n_{ij} = \pi_i, \quad \sum_{n_{ij}, n_{pk}=0}^{\infty} \Gamma_i(n_{ij}) \Gamma_p(n_{pk}) n_{ij} n_{pk} = \pi_i \pi_p + \delta_{ip} \delta_{jq} \omega_i \pi_i, \tag{7}
\]

where

\[
\omega_i \equiv \frac{1}{\pi_i} \sum_{n=0}^{\infty} (n - \overline{n}_i)^2 \Gamma_i(n), \tag{8}
\]
with $\pi_0 = 1$ and $\omega_0 = 0$. For convenience the pions are considered as fireballs decaying into themselves, i.e. $\Gamma_0(n_{ij}) = \delta(n_{ij} - 1)$. Introducing the average numbers, $\langle N_i \rangle_T \equiv \overline{N}_i$, and correlators, $\langle \Delta N_i \Delta N_j \rangle_T$, where $\Delta N_i \equiv N_i - \langle N_i \rangle_T$, one obtains:

$$
\langle N \rangle = \sum_{i=0}^{k} \overline{N}_i \overline{n}_i, \quad \langle (\Delta N)^2 \rangle = \sum_{i=1}^{k} \omega_i \overline{N}_i \overline{n}_i + \sum_{i,j=0}^{k} \overline{n}_i \overline{n}_j \langle \Delta N_i \Delta N_j \rangle_T,
$$

where $\Delta N \equiv N - \langle N \rangle$. In Eq. (9) the terms $\omega_i \overline{N}_i \overline{n}_i$ describe the pion fluctuations at fixed number of fireballs $\overline{N}_i$ due to the probabilistic character of fireball decays. The next terms for $\langle (\Delta N)^2 \rangle$ in Eq. (9) present the contribution due to the fluctuations and correlations of $N_i$ numbers. Using Eq. (9), the scaled variance (1) is presented as:

$$
\omega = \frac{\sum_{i=1}^{k} \omega_i \overline{N}_i \overline{n}_i + \sum_{i,j=0}^{k} \overline{n}_i \overline{n}_j \langle \Delta N_i \Delta N_j \rangle_T}{\sum_{i=0}^{k} \overline{n}_i \overline{N}_i}.
$$

The same structure of equation for $\omega$ was previously obtained for the equilibrium hadron-resonance gas within the canonical ensemble [10] and microcanonical ensemble [11]. The form of Eqs. (9) and (10) appear to be rather general and their validity do not require any thermal equilibrium, i.e. the averaging $\langle \ldots \rangle_T$ may include arbitrary dynamical (non-thermal) effects.

3. If no fireballs exist, Eq. (10) is reduced to

$$
\omega = \frac{\langle (\Delta N_0)^2 \rangle_T}{\overline{N}_0} \equiv \omega_0^*.
$$

If there is only one type of fireballs and no primordial pions, i.e. $k = 1$ and $\overline{N}_0 = 0$, Eq. (10) reads:

$$
\omega = \omega_1 + \overline{n}_1 \frac{\langle (\Delta N_1)^2 \rangle_T}{\overline{N}_1} \equiv \omega_1 + \overline{n}_1 \omega_1^*,
$$

and corresponds to the model of independent sources (3).

Let us consider now the system when both the primordial pions and fireballs are present. For one type of fireballs and no correlations between fireballs and primordial pions, i.e. $\langle \Delta N_0 \Delta N_1 \rangle_T = 0$, the scaled variance (10) reads:

$$
\omega = 1 + \frac{(\omega_1^* - 1) \overline{N}_0}{\overline{N}_0 + \overline{n}_1} + \overline{n}_1 (\omega_1 - 1) \overline{N}_1 \overline{n}_1.
$$

The Eq. (13) is reduced to (11) and (12) at $\overline{n}_1 = 0$ and $\overline{N}_0 = 0$, respectively. Let us fix the ratio $R$ of $\pi$-mesons from the fireball decays to the total number of $\pi$-mesons,

$$
R = \frac{\overline{n}_1 \overline{N}_1}{\overline{N}_0 + \overline{n}_1 \overline{N}_1}.
$$
Using Eq. (14), the scaled variance (13) can be rewritten as,

\[
\omega = \omega^*_0 (1 - R) + (\omega_1 + \bar{n}_1 \omega^*_1) R.
\]  

(15)

The Eq. (15) includes the results for the pure pion system (11) and pure fireball system (12) as the special cases at \( R = 0 \) and \( R = 1 \) respectively. Assuming

\[
\bar{n}_1(m) = \gamma m,
\]  

(16)

where \( \gamma^{-1} \) has a meaning of the average pion energy from the decay of a fireball, one finds that \( \omega \) increases linearly with fireball mass. This is shown in Fig. 1, where the parameters in Eqs. (15,16) are fixed as \( \omega^*_0 = \omega^*_1 = \omega_1 = 1 \) and \( \gamma^{-1} = 0.5 \) GeV. If all pions come from the fireball decays, i.e. \( R = 1 \), the scaled variance \( \omega \) becomes more than 10 times larger than \( \omega^*_0 = 1 \) at \( m \approx 5 \) GeV. Even for very small average pion multiplicity from fireball decays, i.e. when \( R \ll 1 \), the fireball contribution to \( \omega \) always dominates at large \( m \).

![Graph showing the scaled variance as a function of fireball mass](image)

**FIG. 1:** The scaled variance (15) as the function on \( m \) is shown at \( R = 0, 0.05, 0.1, 0.25, 0.5, \) and 1. The lower bound for the fireball mass is taken as \( m = 2 \) GeV. The parameters are fixed as \( \omega^*_0 = \omega^*_1 = \omega_1 = 1 \) and \( \gamma^{-1} = 0.5 \) GeV.
4. The statistical mechanics of pion-fireball gas in the grand canonical ensemble (GCE) gives an example when \( \langle \Delta N_0 \Delta N_1 \rangle_T \) vanishes. This leads to Eq. (13). In the Boltzmann approximation the distributions of \( N_0 \) and \( N_1 \) in the GCE are the Poissonian ones, hence \( \omega_0 = \omega_1 = 1 \), like they are fixed in Fig. 1. The average particle numbers \( N_0 \) and \( N_1 \) are:

\[
N_0(V, T) = \frac{V g_0 m_0^2 T}{2\pi^2} K_2 \left( \frac{m_0}{T} \right) \approx \frac{3g_0}{\pi^2} VT^3, \tag{17}
\]

\[
N_1(V, T) = \frac{V g_1 m_1^2 T}{2\pi^2} K_2 \left( \frac{m_1}{T} \right) \approx g_1 \left( \frac{mT}{2\pi} \right)^{3/2} \exp \left( -\frac{m}{T} \right), \tag{18}
\]

where \( V \) and \( T \) are the system volume and temperature, \( m_0, g_0 \) and \( m, g_1 \) are the mass and the degeneracy factor of the pion and fireball respectively. The energies \( E_0, E_1 \) and specific heats \( C_0, C_1 \) of the pions and fireballs are respectively,

\[
E_0 = T^2 \frac{\partial N_0}{\partial T} \approx 3T N_0, \quad E_1 = T^2 \frac{\partial N_1}{\partial T} \approx \left( m + \frac{3}{2} T \right) N_1, \tag{19}
\]

\[
C_0 = \frac{\partial E_0}{\partial T} \approx 12 N_0, \quad C_1 = \frac{\partial E_1}{\partial T} \approx \left[ \left( \frac{m}{T} \right)^2 + 3 \frac{m}{T} + \frac{15}{4} \right] N_1. \tag{20}
\]

The non-relativistic approximation \( m/T \gg 1 \) for fireball thermodynamical functions and ultra-relativistic approximations \( m_0 = 0 \) for pions are presented at the last steps in Eqs. (17-20). They will be used further to obtain simple analytical estimates. The thermodynamical functions (18-20) of the fireball component include the exponential suppression factor \( \exp(-m/T) \) at \( m/T \gg 1 \) and thus go to zero at large mass. To have the finite values of \( R \) at \( m/T \gg 1 \) one needs large fireball effective degeneracy factor \( g_1 \). Using Eq. (17,18), from Eq. (14) at fixed \( R \) one finds \( g_1 \sim m^{-5/2} \exp(m/T) \) at \( m/T \gg 1 \). This behavior resembles the fireball mass spectrum \( \rho(m) = Cm^a \exp(m/T_H) \) assumed in the Hagedorn model \[1]. The integration over fireball masses with spectrum \( \rho(m) \) leads to the limiting temperature \( T = T_H \) in the Hagedorn model. Our model consideration in the present paper is rather different. The fireball mass \( m \) is assumed to be fixed. There is no integration over the fireball mass spectrum and thus there is no limiting temperature in the system.

For thermal massless pions the average energy per particle equals to \( 3T \). The typical temperature of the hadron gas is about 160 MeV. Thus the average energy \( \gamma^{-1} = 0.5 \) GeV of pion from the fireball decay assumed in Fig.[1] is close to the average thermal energy of the primordial pions. However, other relations are also possible. The resonance decays in the hadron-resonance thermal gas at different \( T \) give the examples when the energy of pions from resonance decays can be both larger and smaller than the thermal energy of the primordial pion.
The GCE formulation of the pion-fireball gas leads to a linear increase of ω with m for fixed value of R. Thus large values of ω are possible even at small R if fireball mass is large enough. This is connected with unusual thermodynamic properties of this system. At finite values of R the ratio $E_1/E_0$ is also finite. It is proportional to R at $R \ll 1$ and thus is close to zero. On the other hand, $C_1/C_0$ is large and goes to infinity at large m and fixed R. Thus fireball contribution to the total energy can be rather small (i.e. $E_1/E_0 \ll 1$ when $R \ll 1$), but the specific heat of fireball component always dominates, $C_1/C_0 \gg 1$, at large m.

5. In the GCE, the average energy is fixed and thermal energy fluctuations around this average value are admitted. In the micro-canonical ensemble (MCE)\(^1\) the total energy is fixed for each microscopic state of the pion-fireball gas. This exact energy conservation generates specific correlations between particle numbers. The MCE correlators $\langle \Delta N_i \Delta N_j \rangle_{mce}$ can be presented as $[11]$:

$$\langle \Delta N_i \Delta N_j \rangle_{mce} = \delta_{ij} \sum_p v_{p,i}^2 - \frac{\sum_p v_{p,i}^2 \epsilon_{p,i} \sum_p v_{p,j}^2 \epsilon_{p,j}}{\sum_p v_{p,i}^2 \epsilon_{p,i}} ,$$

where $\epsilon_{p,i} = \sqrt{p^2 + m_i^2}$ and $v_{p,i}^2 = \tilde{\pi}_{p,i}(1 \pm \pi_{p,i})$ with plus sign for bosons and minus sign for fermions. The values $\tilde{\pi}_{p,i}$ are the average occupation numbers of states, labeled by 3-momentum $p$. In the Boltzmann approximation the above expressions are simplified: $v_{p,i}^2 = \pi_{p,i} = \exp(-\epsilon_{p,i}/T)$. In the thermodynamic limit of large volume $V$, the sums $\sum_p \ldots$ in Eq. (21) can be replaced by the integrals and presented in terms of the GCE thermodynamical functions $[17][20]$:

$$\sum_p v_{p,i}^2 = \frac{g_i V}{2\pi^2} \int_0^\infty p^2 dp \exp\left(-\sqrt{p^2 + m_i^2}/T\right) = \overline{N}_i(V, T) ,$$

$$\sum_p v_{p,i}^2 \epsilon_{p,i} = \frac{g_i V}{2\pi^2} \int_0^\infty p^2 dp \sqrt{p^2 + m_i^2} \exp\left(-\sqrt{p^2 + m_i^2}/T\right) = \overline{E}_i(V, T) ,$$

$$\sum_p v_{p,i}^2 \epsilon_{p,i}^2 = \frac{g_i V}{2\pi^2} \int_0^\infty p^2 dp \left(p^2 + m_i^2\right) \exp\left(-\sqrt{p^2 + m_i^2}/T\right) = T^2 \overline{C}_i(V, T) .$$

Using Eqs. (22)-(24), the correlators (21) can be presented as

$$\langle \Delta N_i \Delta N_j \rangle_{mce} = \delta_{ij} \overline{N}_i - \frac{\overline{E}_i \overline{E}_j}{T^2 \sum_i \overline{C}_i} .$$

\(^1\) It can be also named the grand micro-canonical ensemble $[12]$ as no conserved charges are introduced.
The second term in the r.h.s. of Eq. (25) comes due to the exact energy conservation in the MCE. It is absent in the GCE, \( \langle \Delta N_i \Delta N_j \rangle_{\text{gce}} = \delta_{ij} N_i \). Note that being different from the GCE values the MCE correlators (25) can be expressed in terms of the GCE quantities. The temperature parameter \( T \) is fixed by the requirement that GCE average energy is equal to the fixed value of the MCE energy. Under this condition the average values \( N_i \) and \( E_i \) are the same in the MCE and GCE. The average final pion multiplicity \( \langle N \rangle \) and the pion number fluctuations expressed in \( \langle (\Delta N)^2 \rangle \) or \( \omega \) are given by general formulae (9) and (10). In Eqs. (9,10) the correlators \( \langle \Delta N_i \Delta N_j \rangle_T \) should be substituted by the MCE ones (25).

Let us start with an example of the pure pion gas with no fireballs. The scaled variance \( \omega \) then reads:

\[
\omega = \frac{\langle (\Delta N_0)^2 \rangle_{\text{mce}}}{N_0} = 1 - \frac{E_0^2}{T^2 C_0^2 N_0},
\]

(26)

instead of \( \omega = 1 \) for the Boltzmann pion gas in the GCE. The second term in r.h.s. of Eq. (26) leads to a suppression of the pion number fluctuations due to the exact energy conservation in the MCE. For massless pions, Eq. (26) gives \( \omega = 1/4 \). This result was obtained for the first time in Ref. [13]. If there are only fireballs with mass \( m \) and no primordial pions (i.e. \( N_0 = 0 \)), using Eq. (10), in the MCE one finds:

\[
\omega = \omega_1 + \bar{\pi}_1 \left( 1 - \frac{E_1^2}{T^2 C_1^2 N_1} \right) \equiv \omega_1 + \bar{\pi}_1 \omega_1^* \cong \omega_1 + \gamma m \frac{3}{2} \left( \frac{T}{m} \right)^2 \cong \omega_1,
\]

(27)

where the relation \( \bar{\pi}_1 = \gamma m \) (16) and non-relativistic approximations for \( N_1, \bar{E}_1, \) and \( C_1 \) (18, 20) are used. The Eqs.(26) and (27) give the MCE realization of Eqs. (11) and (12) respectively.

The scaled variance for the fireball number fluctuations in the MCE is \( \omega_1^* \cong 3(T/m^2)^2/2 \). This was obtained for the first time in Ref. [13]. For \( m/T \to \infty \) the scaled variance \( \omega_1^* \) goes to zero as \( m^{-2} \) in the MCE. Thus \( \bar{\pi}_1 \omega_1^* \) goes to zero as \( m^{-1} \), and the pion scaled variance \( \omega \) approaches \( \omega_1 \) at large \( m \), i.e. it is defined by the only fluctuations due to the one-fireball decay. This behavior is completely different from the GCE result. In the GCE, \( \omega_1^* = 1 \) and the term \( \bar{\pi}_1 \omega_1^* \) increases linearly with \( m \). Thus the contribution due to the fireball number fluctuations dominates in GCE at large \( m \).

For the case when both the primordial pions and one type of fireballs of mass \( m \) exist one finds, using Eq. (14):

\[
\omega \cong 1 + R (\omega_1 - 1) + \frac{6 \gamma T (1 - R) R (2 \gamma T - 1) + \frac{3}{2} \gamma T (R^2 + 6 R - 6) \cdot (T/m)}{R + [12 \gamma T (1 - R) + 3 R] \cdot (T/m)},
\]

(28)
where the relation $\bar{n}_1 = \gamma m$ (16), non-relativistic approximations for $\bar{N}_1$, $\bar{E}_1$, and $C_1$ (18,20) and ultra-relativistic approximation for $\bar{N}_0$, $\bar{E}_0$, and $C_0$ (17,19,20) of the pion component are used.

![Graphs showing the scaled variance $\omega$ as a function of fireball mass $m$ at different values of $\gamma T$.](image)

**FIG. 2**: The scaled variance $\omega$ (28) in the MCE for the pion fireball gas as the function of fireball mass $m$ at different values of $\gamma T$. The lower bound for the fireball mass is taken as $m = 2$ GeV. The solid line corresponds to $R = 0.05$, dashed line to $R = 0.25$, and dotted line to $R = 0.75$. The parameters are fixed as $T = 160$ MeV and $\omega_1 = 1$.

The dependence of $\omega$ on particle mass $m$ is presented in Fig. 2. One finds that $\omega$ approaches to finite value at $m/T \to \infty$. At small $\gamma T$ the limiting values of $\omega$ is reached at moderate values of $m$, as it is seen in the upper panel of Fig. 2. Most rapidly the asymptotic value of $\omega$ is reached at $\gamma T = 0.25$. It requires much larger $m$ at large $\gamma T$ as the lower panel of Fig. 2 demonstrates. The Fig. 2 also shows that the smaller is $R$ the larger values of $m$ are needed to reach the limiting values of $\omega$. The energy of pion from fireball decay should be larger than pion
mass. This leads to the restriction $\gamma < 1/m_\pi$. As the typical temperature is $T = 160$ MeV, the largest physically allowed value of $\gamma T$ is approximately equal to 1.

The limiting values of $\omega$ for $m/T \to \infty$ at fixed non-zero $R$ are equal to:

$$\omega = 1 + R (\omega_1 - 1) + 6\gamma T (1 - R)(2\gamma T - 1). \quad (29)$$

The dependence of the scaled variance (29) on the dimensionless parameters $\gamma T$ at different $R$ and on $R$ at different $\gamma T$ is presented in Fig. 3 left and right respectively. The parameter $\omega_1$ is fixed in Fig. 1 as $\omega_1 = 1$.

![Graph showing the dependence of the scaled variance $\omega$ on $\gamma T$ for different values of $R$.](image)

**FIG. 3:** The dependence of the scaled variance $\omega$ on $\gamma T$ for different values of $R = 0.01, 0.25, 0.5, 1$ (left) and on $R$ at $\gamma T = 0.25, 0.5, 0.75, 1$ (right). The Eq. (29) is used with $\omega_1 = 1$.

The minimum of (29) corresponds to $\gamma T = 1/4$, and the minimal value is $\omega = 1/4 + R(\omega_1 - 1/4)$. If $\omega_1 < 1/4$, the minimal value of $\omega$ is for $R = 1$, and it equals to $\omega = 0$ at $\omega_1 = 0$. If $\omega_1 > 1/4$, the minimal value of $\omega$ is for $R \to 0$ and it equals $\omega = 1/4$.

At $\gamma T \to 0$, it follows from Eq. (29), $\omega = 1 + R(\omega_1 - 1)$. This is shown in Fig. 4 left. If $R \to 0$ one obtains $\omega = 1$ for all values of $\omega_1$. In Fig. 3 the value of $\omega_1$ is fixed as $\omega_1 = 1$. Thus $\omega = 1$ at $\gamma T \to 0$ for all values of $R$.

The simultaneous limit $m \to \infty$ and $R \to 0$ leads to the uncertainty. Introducing $mR/T \equiv \alpha = const$ one finds at $m \to \infty$:

$$\omega = 1 + \frac{\alpha(2\gamma T - 1)6\gamma T - 9\gamma T}{\alpha + 12\gamma T}. \quad (30)$$

Changing $\alpha$ in the last formula from 0 to $\infty$ one obtains the values of $\omega$ from $1/4$, which is the MCE value for the massless pion gas in the Boltzmann approximation, to $1 + 6\gamma T(2T\gamma - 1)$.
FIG. 4: Left: The dependence of the scaled variance $\omega$ on $R$ at $\gamma T \to 0$ for different values of $\omega_1 = 0, 0.25, 1$. Right: The dependence of the scaled variance $\omega$ (30) on $\gamma T$ at different $\alpha$. The lower solid line $\omega = 1/4$ corresponds to $\alpha = 0$ and the upper line $1 + 6\gamma T(2T\gamma - 1)$ to $\alpha = \infty$. Possible values of $\omega$ in the simultaneous limit $m \to \infty$ and $R \to 0$ lie between these two lines in the dashed region.

This is shown in Fig. 4 right. The fireballs do not contribute to the pion multiplicity at $R \to 0$. However, their presence affects $\omega$.

6. Note that the numbers of positively $N_+$ and negatively $N_-$ charged pions are usually measured. Effects for the fluctuations of $N_+$ and $N_-$ due to neutral vector mesons decaying into $\pi^+\pi^-$-pairs have been discussed in Ref. [14]. Fireballs decaying into large numbers of $\pi^+\pi^-$-pairs make these effects very strong. Assuming that fireballs have zero electric charge one finds for $Q \equiv N_+ - N_-$ and $N_{ch} = N_+ + N_-$ in the GCE:

$$\langle (\Delta Q)^2 \rangle = (1 - R) \langle N_{ch} \rangle, \quad \langle (\Delta N_{ch})^2 \rangle = [1 + (\omega_{ch} - 1)R + \bar{n}_{ch}R] \langle N_{ch} \rangle,$$  \hspace{1cm} (31)

where $R$ is the ratio of the number of charged pions from the decays of fireballs to the total number of charged pions, $\bar{n}_{ch} \sim m$ and $\omega_{ch} \sim 1$ are respectively the average number and scaled variance of charged pions from one fireball decay. A presence of fireballs leads to the strong enhancement of the $N_{ch}$ fluctuations and to the suppression of the $Q$ fluctuations. As seen from Eq. (31), the ratio

$$\frac{\langle (\Delta Q)^2 \rangle}{\langle (\Delta N_{ch})^2 \rangle} = \frac{1 - R}{1 + (\omega_{ch} - 1)R + \bar{n}_{ch}R}$$  \hspace{1cm} (32)
is sensitive to both $R$ and $\pi_{ch}$ and it goes to zero if $R$ is fixed and $m \to \infty$. The GCE analysis may be applicable in high energy pion production if the following conditions for the rapidity intervals are satisfied \[14\],

$$Y_{\text{tot}} \gg Y_{\text{acc}} \gg 1,$$

(33)

where $Y_{\text{tot}}$ is the total rapidity interval allowed in high energy collision and $Y_{\text{acc}}$ is the rapidity interval for the accepted pions. The first of these inequalities is needed to relax the energy conservation effects and the second inequality ensures that the pion rapidity spreading from fireball decay is much smaller than the accepted interval.

7. The pion number fluctuations have been considered in the system of pions and fireballs decaying into pions. Our model formulation gives a generalization of the model of independent sources. Both statistical and dynamical effects in the production of primordial pions and fireballs can be included within the present scheme. The statistical equilibrium within the grand canonical and micro-canonical ensemble formulations for the pion-fireball gas are considered in details as particular examples. Important result of these studies is a strong suppression effect for the pion number fluctuations in the MCE due to the energy conservation. In the GCE the scaled variance of the fireball number fluctuations is $\omega_1^* = 1$, and this gives a linear increase of the scaled variance $\omega$ of pion number fluctuations with fireball mass $m$ due to a large number of pions from one-fireball decay, $\bar{n}_1 \sim m$. In the MCE the $\omega_1^*$ is strongly suppressed at high masses $m$. This leads in the limit $m \to \infty$ to finite values of $\omega$ depending of the model parameters $R$ and $\gamma T$.

Even for small contributions of fireballs to the average pion multiplicity there is a chance to find the signals of large mass fireballs by measuring the event-by-event multiplicity fluctuations. If final pions are detected in a small part of the phase space the exact energy conservation becomes not so much important. This leads to a strong enhancement of the pion multiplicity fluctuations and linear increase of the scaled variance $\omega$ with fireball mass $m$. Both conditions \[33\] can be simultaneously satisfied at high collision energies. They are only approximately fulfilled in nucleus-nucleus collisions at RHIC, and can be much stringent in the future LHC experiments.
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