The forward-backward asymmetry with $Z'$ effects
in the process $e^+ e^- \rightarrow \mu^+ \mu^-$

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Abstract

Having taken the QED radiative correction and one-loop weak corrections into account, we have computed the unpolarized forward-backward asymmetry in the process $e^+ e^- \rightarrow \mu^+ \mu^-$ at a very reasonable accuracy. Special attention to the effects around the LEP 200 energy, induced by the possible $Z'$ boson of the left-right symmetric model $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, is paid. The numerical results of the directly measurable asymmetry $A_{FB}^{\mu}$ versus the CMS energies are presented in figures. One may be convinced by the results that precise measurements on the asymmetry in future colliders will possibly present some indirectly evidence of the $Z'$ boson.

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Up to now there has been no evidence which shows any discrepancy between the predictions of the standard model $SU_c(3) \times SU_L(2) \times U_Y(1)$ (SM) and the measurements at the nowadays best experimental accuracy. Nevertheless, the SM still should be considered as a low energy limit of a more fundamental theory, such as a unification of the electroweak and strong interactions at much higher energy scale, due to the fact that there are so many parameters in the model, which should be understood. Being one of the efforts in the direction, the left-right symmetric model $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ (LRM) is a comparative natural extension of the SM and has some predictabilities. An additional neutral gauge boson $Z'$ is one of its important predictions. If favorable parameters are taken by nature, the boson may have not too heavy mass, so that it is accessible at future colliders\[1\]. The neutral gauge boson $Z'$ is to couple to the fermion-antifermion pair so must effect the measurable forward-backward asymmetry in fermion pair to fermion pair processes.

In this report, we will focus lights on the effects due to the boson. Pursuing the accuracy of a few percents, which may be accessible in theoretical calculations and experimental measurements both. The theoretical calculations to the accuracy just correspond up to the QED radiative correction and the electroweak one-loop correction level. We are to analyze the unpolarized forward-backward asymmetry for muons ($A_{FB}^\mu$) in the $e^+ e^- \rightarrow \mu^+ \mu^-$ process at the accurate level in two extreme cases of LRM in the paper. To see the effects clearly we also compare the results with those obtained from SM theory. Note that in the calculations, the most recent constraints on the $Z'$ mass by experiments[2] have been concerned. The results of our analysis are presented by figures, the measurable asymmetry ($A_{FB}^\mu$) versus the CMS energies of collisions. As expected, the QED radiative correction is important and smears the asymmetry effects strongly. However our results still show certain possibilities that one may find some evidence of the new gauge boson $Z'$ at LEP 200 or at future $e^+ e^-$ colliders if analyzing the measured $A_{FB}^\mu$ against the calculated one carefully. How significant the evidence to be depends on the value of the $Z'$ mass
(if it exists) and the accuracy of the experimental measurements on the asymmetry as well.

In the LRM a right-handed $SU(2)_R$ gauge interaction is extended from SM. Although in principle a mixing between $W^+_L$ and $W^+_R$ in LRM is possible, we suppose it can be neglected, that is indicated by experiments e.g. the charge currents of quarks and leptons are of $V - A$, as indicated by experiments within the accuracy of measurements on them. In the model the possible right-handed neutrinos are required very heavy by experimental measurements too, and the mixing between them and the left-handed ones is so tiny that it can do nothing what we are interested in here. The parameter $\alpha_{LR}$ is used to describe the couplings of the heavy boson to fermions.

$$\alpha_{LR} = \sqrt{\frac{\cos^2 \theta_w}{\lambda^2 \cdot \sin^2 \theta_w} - 1}. \quad (1)$$

We define $\lambda = g_L/g_R$, $g_L$ and $g_R$ represent the $SU(2)_{L,R}$ coupling constants. When $\alpha_{LR}$ may take the value either $\sqrt{\frac{2}{3}}$ or 1.53, which corresponds to its lowest and upper bounds respectively. For $\lambda = 1$ (i.e. $\alpha_{LR} \simeq 1.53$ for $\sin^2 \theta_w = 0.23$), it is just the case often used in LRM, while when $\alpha_{LR} = \sqrt{2/3}$, the LRM is identical to the $\chi$ model, attributed from the superstring inspired $E_6$ gauge model[3]. In the following we will compute the effects in the two extreme cases.

The neutral-current lagrangian in LRM is given by[4]

$$L = igJ_{3L}W_{3L} + i\frac{g}{\lambda}J_{3R}W_{3R} + ig'J_{B-L}B \quad (2)$$

where

$$g = g_L = \frac{e}{\sin \theta_w}. \quad (3)$$

Here we would define

$$y = \sqrt{\frac{\cos^2 \theta_w}{\lambda^2} - \sin^2 \theta_w}. \quad (4)$$
for later usages. $W_{3L}, W_{3R}$ and $B$ are the $SU(2)_L, SU(2)_R$ and $U(1)_{B-L}$ gauge fields; $g, g/\lambda$ and $g'$ are the coupling constants of gauge fields to the fermion currents:

$$J_{3L}^\mu = \bar{\psi} \gamma^\mu I_{3L} \psi, \quad J_{3R}^\mu = \bar{\psi} \gamma^\mu I_{3R} \psi$$

$$J_{B-L}^\mu = \bar{\psi} \gamma^\mu \frac{B - L}{2} \psi$$

As $Z_L, Z_R$ are not the mass eigenstates in general, the corresponding mass eigenstates $Z^0$ and $Z'$ are of a mixture of them as follows:

$$
\begin{pmatrix} Z \\ Z' \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} Z_L \\ Z_R \end{pmatrix}
$$

From equation (2), (5), (6) and (7), we may read out the vertices of $\gamma \mu \mu, Z^0 \mu \mu$ and $Z' \mu \mu$ etc are in the form $i g \gamma^\mu (g^{(V)} - g^{(A)} \gamma_5)[6]$, where

$$g^{(V)}_{\gamma \mu \mu} = - \sin \theta_w,$$

$$g^{(A)}_{\gamma \mu \mu} = 0,$$

$$g^{(V)}_{Z \mu \mu} = \frac{1}{4} \left[ (- \cos \theta_w + 3 \sin \theta_w \tan \theta_w) \cos \phi + \left( -\frac{y}{\cos \theta_w} + \frac{2 \sin \theta_w \tan \theta_w}{y} \right) \sin \phi \right]$$

$$= \frac{1}{4 \cos \theta_w} [V_1 \cos \phi + V_2 \sin \phi],$$

$$g^{(A)}_{Z \mu \mu} = \frac{1}{4 \cos \theta_w} (- \cos \phi + y \sin \phi),$$

$$g^{(V)}_{Z' \mu \mu} = \frac{1}{4} \left[ (- \frac{y}{\cos \theta_w} + \frac{2 \sin \theta_w \tan \theta_w}{y}) \cos \phi - (- \cos \theta_w + 3 \sin \theta_w \tan \theta_w) \sin \phi \right]$$

$$= \frac{1}{4 \cos \theta_w} (V_2 \cos \phi - V_1 \sin \phi),$$

$$g^{(A)}_{Z' \mu \mu} = \frac{1}{4 \cos \theta_w} [y \cos \phi + \sin \phi],$$

and with

$$V_1 = 4 \sin^2 \theta_w - 1, \quad V_2 = \frac{2 \sin^2 \theta_w}{y} - y.$$
The typical Feynman diagrams which contribute to the process $e^+e^- \rightarrow \mu^+\mu^-$ at the tree level in LRM are shown in Fig. 1. The differential cross sections in Born approximation in the framework of SM is given in references[4,5]. Here to calculate out the differential cross section in LRM is necessary. At tree level and neglecting the external fermion masses, we may write the cross section in the same way as in SM, i.e. both are in the form as Eq.(11) of Refs.[6,7]:

$$\frac{d\sigma^{SM,LRM}}{d\Omega} = \frac{\alpha^2}{4s}[(1 - z^2)G_1^{SM,LRM}(s) + 2zG_3^{SM,LRM}(s)], \quad (11)$$

here $z = \cos \theta$, only the functions, $G_1(s)$ and $G_3(s)$, are different from each other for SM and LRM. They have the formulae as follows respectively:

$$G_1^{SM}(s) = 1 + 2v_e v_\mu Re(D_Z(s)) + (v_e^2 + a_e^2)(v_\mu^2 + a_\mu^2)|D_Z(s)|^2, \quad (12a)$$

$$G_3^{SM}(s) = 2a_e v_\mu Re(D_Z(s)) + 4v_e a_e v_\mu a_\mu|D_Z(s)|^2, \quad (12b)$$

for those of SM, and

$$G_1^{LRM}(s) = 1 + \frac{B}{8} \cdot \frac{s - M_Z^2}{s} \cdot |D_Z(s)|^2(V_1 \cos \phi + V_2 \sin \phi)^2$$

$$+ \frac{B^2}{256} |D_Z(s)|^2[(V_1 \cos \phi + V_2 \sin \phi)^2 + (-\cos \phi + y \sin \phi)^2]^2$$

$$+ \frac{B^2}{256} |D_Z(s)|^2[(V_2 \cos \phi - V_1 \sin \phi)^2 + (y \cos \phi + \sin \phi)^2]^2$$

$$+ \frac{B(s - M_Z^2)}{8s} |D_Z(s)|^2(V_2 \cos \phi - V_1 \sin \phi)^2$$

$$+ \frac{B^2}{128} \cdot \frac{(s - M_Z^2)}{s} \cdot |D_Z(s)|^2 \cdot |D_Z(s)|^2 \cdot |D_Z(s)|^2$$

$$\cdot [(V_1 \cos \phi + V_2 \sin \phi) \cdot (V_2 \cos \phi - V_1 \sin \phi)$$

$$+ (-\cos \phi + y \sin \phi)(y \cos \phi + \sin \phi)]^2, \quad (13a)$$

$$G_3^{LRM}(s) = \frac{B}{8} \cdot \frac{s - M_Z^2}{s} \cdot |D_Z(s)|^2(-\cos \phi + y \sin \phi)^2$$

$$+ \frac{B^2}{64} |D_Z(s)|^2(V_1 \cos \phi + V_2 \sin \phi)^2(-\cos \phi + y \sin \phi)^2$$

$$+ \frac{B^2}{64} |D_Z(s)|^2(V_2 \cos \phi - V_1 \sin \phi)^2(y \cos \phi + \sin \phi)^2$$

$$+ \frac{B^2}{128} \cdot \frac{(s - M_Z^2)}{s} \cdot |D_Z(s)|^2 \cdot |D_Z(s)|^2 \cdot |D_Z(s)|^2$$

$$\cdot [(V_1 \cos \phi + V_2 \sin \phi) \cdot (V_2 \cos \phi - V_1 \sin \phi)$$

$$+ (-\cos \phi + y \sin \phi)(y \cos \phi + \sin \phi)]^2, \quad (13b)$$
\[ \text{for those of LRM, here we have taken the notation:} \]

\[ B = \frac{1}{\sin^2 \theta_w \cos^2 \theta_w}, \quad (14a) \]

\[ D_Z(s) = \frac{s}{s - M_Z^2 + iM_Z \Gamma_0 Z}, \quad (14b) \]

\[ D_{Z'}(s) = \frac{s}{s - M_{Z'}^2 + iM_{Z'} \Gamma_0 Z'}. \quad (14c) \]

According to the definition, the unpolarized forward-backward asymmetry \( A_{FB}^\mu \) is properly related to the cross section:

\[ A_{FB}^\mu = \frac{2\pi (\int_0^{\pi/2} \frac{d\sigma}{d\theta} d\theta - \int_{\pi/2}^{\pi} \frac{d\sigma}{d\theta} d\theta)}{\sigma_T} = \frac{\sigma_{FB}}{\sigma_T}. \quad (15) \]

In Born approximation, the \( A_{FB}^\mu \) is formulated simply

\[ A_{FB}^\mu = \frac{3}{4} \cdot \frac{G_{SM,LRM}^S(s)}{G_{SM,LRM}^1(s)}. \quad (16) \]

It is known that the mass of neutral boson \( Z' \) in LRM is located far beyond the energy range of the colliders LEP-1 and LEP 200 [2], therefore the effects from the \( Z' \)-exchange diagrams cannot be expected to be great. In order to detect the expected small \( Z' \) effects from quite great background of SM, we need to calculate the background to a higher order level precisely. The weak corrections up to one-loop in SM is needed here. The corrections in SM, including propagator and vertex corrections as well as the box diagram contributions, are given by Wolfgang F. L. Hollik in Ref.[5]. To the accurate level, the differential cross section still may
write in the form as Eq.(11), but the functions $G_1$ and $G_3$ now will depend on
the two Mandelstam variables $t$ as well as $s$, thus when we consider the one-loop
weak corrections of SM alone to the cross section, we may just simply use the
invariant functions $G_1(s, t)$ and $G_3(s, t)$ specified in the equation (6.34) of Ref.[5]
and to replace them properly into the above formula Eq.(11). As for the radiative
corrections of LRM, since the $Z'$ mass is probably much larger than the energy range
of LEP 200, where we are considering in the paper, the $Z'$-exchange contribution is
rather small, so that it is accurate enough to neglect safely the $Z - Z'$ mixing, the
interference between the pure weak loop correction and the $Z'$-exchange diagram
and the contribution from those loops contained one or more $Z'$ boson propagators
as well at this moment. The extra Higgs sector correction in LRM is also neglected
for simplification.

However, the QED radiative corrections, especially those from the colinear and/or
soft photon emissions, are great due to a tiny mass of electrons. Additionally it
makes that the precise measured value of the asymmetry depends on the experi-
mental conditions. As expected, among all the QED corrections, those of the initial
states are the largest[8]. We use the convolution formula to consider the initial state
radiation effects[9,10,11,12]:

$$
\frac{d\sigma}{d\Omega} = \int_0^\Delta dk \frac{d\sigma^w(s')}{d\Omega} R_T^e(k),
$$

(17)

where $s' = (1-k)s$, $\Delta = \frac{E_{\text{max}}}{E_{\text{beam}}}$, $\sigma^w(s')$ is the cross section involving the one-loop weak
corrections. The function $R_T^e$ contains two parts. One is to take the soft (real and
virtual) photon radiation into account, $S^e(\epsilon)$, and the other is the hard and colinear
photon radiation $H^e(k)[9,10,11,12]$:

$$
R_T^e(k) = \delta(k)[1 + S^e(\epsilon)] + \theta(k - \epsilon)H^e(k).
$$

(18)

To the first order, we have

$$
S^e(\epsilon) = \beta_e(\ln \epsilon + \frac{3}{4}) + \frac{\alpha}{\pi} e^2 \left[ \frac{1}{3} \pi^2 - \frac{1}{2} \right]
$$

(19a)
and

\[ H^e(k) = \beta_e \frac{1 + (1 - k)^2}{2k}, \]  

(19b)

where

\[ \beta_e = \frac{2\alpha}{\pi} c_e^2 (\ln \frac{s}{m_e^2} - 1) \]  

(20a)

and

\[ e_e = -1. \]  

(20b)

Numerically we take \( M_{Z'} \) as low as possible constrained by experimental limits, e.g. 395 GeV, 514 GeV and 682 GeV respectively, and calculate the asymmetry parameter \( A_{FB}^\mu \) in the two extreme cases mentioned above, to see the general tendency of the asymmetry varying with the possible masses of the \( Z' \) boson. The values \( \alpha = \frac{1}{137}, \sin^2 \theta_w = 0.2325, \epsilon = 100MeV, M_{H^0} = 200GeV \) and \( M_t = 150GeV \) are adopted in the calculation as well. The results show that the effects to the asymmetry \( A_{FB}^\mu \) from pure weak loop corrections are small, roughly in the order of \( 10^{-3} \) at the energy region above \( Z_0 \) resonance, but the QED radiative corrections from the initial state influence the asymmetry \( A_{FB}^\mu \) strongly. The curves of forward-backward asymmetry versus CMS energies \( \sqrt{S} \), with various \( Z' \) masses and \( \alpha_{LR} = 1.53 \) without QED and weak radiative corrections but in SM and LRM both are plotted in Fig.2. The values of \( A_{FB}^\mu \) with various \( Z' \) masses but \( \alpha_{LR} = \sqrt{2/3} \) in LRM at tree level are nearly the same with that in SM. The discrepancies between those values are less than \( 10^{-3} \) in the LEP 200 energy range, and the dependence on the extra neutral boson \( Z' \) mass is not strong. We may see the fact from Fig.2 that the \( Z' \) effects to \( A_{FB}^\mu \) is increasing with increasing of \( \alpha_{LR} \) and decreasing of the \( Z' \) mass. The \( A_{FB}^\mu \) versus \( \sqrt{S} \) curves with QED radiative corrections and weak one-loop corrections for SM and LRM both, are plotted in Fig.3, when taking \( \alpha_{LR} = 1.53 \), but in Fig.4 when taking \( \alpha_{LR} = \sqrt{2/3} \), respectively. They keep a similar tendency as
shown in Fig.2. In order to see the $A_{FB}^{\mu}$ shifts caused by the LRM physics clearly, we define

$$\delta A_{FB}^{\mu} = A_{FB}^{\mu, SM} - A_{FB}^{\mu, LRM}$$

(21)

and plot it versus the CMS energy of collision with $M_{Z'} = 514 GeV$ in Fig.5.

In summary, we have calculated the forward-backward-asymmetry of the process $e^+e^- \rightarrow \mu^+\mu^-$ in two extreme cases of LRM at the tree level and electroweak loop correction levels. Comparisons on the asymmetry of LRM with those of SM are made precisely. We find that the QED corrections influence the results strongly comparing with the weak corrections. However having them concerned we still may conclude that probably same indirect indication about the $Z'$ extra boson predicted by LRM may be obtained through precisely measuring $A_{FB}^{\mu}$, as long as the measurements on the asymmetry may reach at an accuracy to a few percents and the mass of the boson $Z'$ is not too heavy. In this paper, although we focus on the process $e^+e^- \rightarrow \mu^+\mu^-$ only, for the other processes, such as the process $e^+e^- \rightarrow b\bar{b}$, the asymmetry have a similar tendency in fact[13]. Finally one thing should be noted is that in the LRM with $\alpha_{LR} = \sqrt{2/3}$, that of $E_6$ inspired one, the $Z'$ effects to the asymmetry $A_{FB}^{\mu}$ are smaller than that with $\alpha_{LR} = 1.53$, the exact symmetric one of left and right hand. That is because in the $\alpha_{LR} = 1.53$ case the coupling constant ratio $g_R/g_L$ is larger than that in the $\alpha_{LR} = \sqrt{2/3}$ case, so the $Z'$ effect in $A_{FB}^{\mu}$ is enhanced comparing with that in the later case.

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Figure Captions

**Figure 1** All the Feynman diagrams which contribute to process $e^+e^- \to \mu^+\mu^-$ at the tree level.

**Figure 2** The forward-backward-asymmetry $A_{FB}^\mu$ versus CMS energy $\sqrt{s}$ without radiative corrections in SM and LRM (in $\alpha_{LR} = 1.53$ case). The full line is of SM; the short-dashed line is of LRM with $M_{Z'} = 395GeV$; the dotted line is of LRM with $M_{Z'} = 514GeV$; the long-dashed line is of LRM with $M_{Z'} = 682GeV$.

**Figure 3** The forward-backward-asymmetry $A_{FB}^\mu$ versus CMS energy $\sqrt{s}$ with QED radiative correction and one-loop weak corrections in SM and LRM (in $\alpha_{LR} = 1.53$ case). The full line is of SM; the short-dashed line is of LRM with $M_{Z'} = 395GeV$; the dotted line is of LRM with $M_{Z'} = 514GeV$; the long-dashed line is of LRM with $M_{Z'} = 682GeV$.

**Figure 4** The forward-backward-asymmetry $A_{FB}^\mu$ versus CMS energy $\sqrt{s}$ with QED radiative correction and one-loop weak corrections in SM and LRM (in $\alpha_{LR} = \sqrt{2/3}$ case). The full line is of SM; the short-dashed line is of LRM with $M_{Z'} = 395GeV$; the dotted line is of LRM with $M_{Z'} = 514GeV$; the long-dashed line is of LRM with $M_{Z'} = 682GeV$.

**Figure 5** Plot of $\delta A_{FB}^\mu = A_{AB}^{SM} - A_{AB}^{LRM}$ versus $\sqrt{s}$ with $M_{Z'} = 514GeV$ (here the QED radiative correction and one-loop weak corrections are included in the forward-backward-asymmetry $A_{AB}^{SM}$ and $A_{AB}^{LRM}$). The full line is for $\alpha_{LR} = 1.53$ and dashed line is for $\alpha_{LR} = \sqrt{2/3}$. 

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