Stochastic expansion planning of interconnected distribution networks with renewable sources considering uncertainties and power transfer capability

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Abstract: This study proposes a stochastic optimisation model for interconnected distribution network planning (IDNP) in the presence of renewable sources (RSs) considering power transfer capability after an \(N-1\) contingency. Two types of scenarios, feeder contingency, and substation transformer contingency, are formulated in the IDNP problem. Uncertainties of the RS output and load fluctuation are also integrated. The planning method provides planners with the decisions of feeder reformation, new tie lines, new feeders, substation expansion, new substations and new load allocation. The IDNP model is a chance-constrained mixed integer non-linear programming problem, which is solved by dynamic niche differential evolution algorithm. A case study carried out on a modified 104-bus distribution network demonstrates the effectiveness of those techniques. Compared with the traditional planning approach, the IDNP method could exploit asset utilisation by optimising existing networks efficiently as well as improve system economy and security simultaneously.

1 Introduction

Traditional distribution network planning (DNP) problems involve the determination of the placement, size and time of allocation of a new resource, such as a feeder or a substation transformer, based on the existing network and load forecasting result [1]. Generally, the objective of the DNP is to minimise total investment and operation costs on the premise of power supply security and reliability [2]. However, traditional planning may not obtain the most economical planning scheme because it may result in some unnecessary constructions (e.g. adding new substations and feeders), even though the existing distribution network is well developed. On the one hand, this planning measure can lower asset utilisation; on the other hand, new substations and feeders cost a lot of money.

Distribution networks can be divided into two types of topologic-al structures: radial and interconnected. However, in recent years, the distribution networks in urban areas tend to be interconnected. The interconnections among substation transformers provide a restoration path for fault power when an \(N-1\) contingency occurs at a substation transformer or a feeder. Furthermore, with the wide application of distribution automation and management, remote switch control would become much faster [3] and feeder loading can be detected by using a feeder terminal unit [4]. This progress makes it possible for power to transfer among different substations flexibly [5]. Meanwhile, the high integration of renewable sources (RSs), such as wind turbine generations (WTGs) and photovoltaic generations (PVGs), greatly increases the uncertainty of substation power supply radius [6]. Thus, it is necessary to find a new DNP method to handle all the above features.

The DNP problem can be usually divided into two sub-problems [7]: optimal allocation of distribution substations and planning of distribution networks. To address the two sub-problems, numerous studies have been reported in the literature [8–11]. The authors of [8] proposed a planning algorithm to optimise the site, size and power supply area of medium-voltage/low-voltage transformer substations by utilising reference network models. Nahman and Peric [9] present an optimal DNP method based on combinations of the steepest descent and simulated annealing technique. A long-term multi-objective planning model in [10] is proposed to maximise the benefits of distribution feeder reconfiguration and RS allocation in distribution networks. The authors of [11] formulated a multi-objective multi-stage DNP problem to minimise total cost and non-supplied energy simultaneously.

All the above methods help a lot in the DNP problem. However, to the authors’ knowledge, there is no printed literature considering power transfer capability of interconnected networks after a sub-station transformer contingency when planning distribution networks. This neglected work may result in; on the one hand, ignorance of the potential supply capability from interconnected substation transformers, on the other hand, violation of \(N-1\) security guideline under the planning scheme.

To alleviate the above drawbacks, an interconnected DNP method is proposed considering power transfer capability. The uncertainties of the RS output and load fluctuation are also integrated by using a scenario-based modelling approach. The dynamic niche differential evolution algorithm (DNDEA) is utilised to solve the DNP problem. Simulation results show that the proposed method can exploit the potential supply capability by optimising existing networks as well as improve economy and security of the planning scheme simultaneously.

2 \(N-1\) security evaluation method for distribution networks

2.1 Concepts on steady-state security distance (SSD)

(i) Operating points: in distribution systems, an operating point is the minimum set of state variables representing system \(N-1\) security, which can be stated as

\[ W_\varepsilon = (S_1^\varepsilon, S_2^\varepsilon, \ldots, S_k^\varepsilon, \ldots, S_n^\varepsilon), \]
where $W_i$ is the operating point vector and $S_{W_i}$ is the load of feeder section $F_i$.

(ii) Steady-state security region (SSR): the SSR for distribution networks is defined as a set of operating points which guarantee system $N-1$ security [12]. The basic constraints of SSR are conductor thermal constraints and bus voltage constraints when an $N-1$ contingency occurs at a feeder or substation transformer. Thus, the SSR model can be formulated as

$$\Omega_{SSR} = \{ W_i | g(W_i) \leq 0 \},$$

where $\Omega_{SSR}$ represents the SSR, $g(W_i)$ is a series of network security constraints after an $N-1$ contingency, which can be described as

$$\begin{aligned}
R_i &= U \in R^{n-1} | U_i^{\min} \leq U_i \leq U_i^{\max}, \forall i \in \psi_i, \\
R_i &= I_b \in R^{n-1} | 0 \leq I_b \leq I_b^{\max}, \forall i \in \psi_b,
\end{aligned}$$

where $\psi_i$ is the set of load buses, $\psi_b$ is the set of branches, $U_i$, $U_i^{\min}$ and $U_i^{\max}$ are, respectively, the voltage, lower and upper voltage limits at bus $i$; $I_b$, $I_b^{\min}$ and $I_b^{\max}$ are, respectively, the current and upper current limit at branch $i$; $R_i$ is the bus voltage constraint; $R_i$ is the branch current constraint, including feeder capacity constraint and substation transformer capacity constraint.

(iii) SSR boundary: for a given distribution system, the SSR boundary in the operating space between secure and insecure points exists, which can be searched by the $N-1$ approximation method. The boundary is approximately linear and can be depicted by several hyperplanes [12].

(iv) SSD: based on the SSR model, the location of an operating point can reflect the $N-1$ security of distribution systems and SSD is defined to evaluate it. SSD is the minimum distance from an operating point to different SSR boundaries, which can be described as [12]

$$V_{SSD}^k = \min_{W_i \in \Omega_{SSR}}\| W_i - W_{f0} \|, B_k \in \partial \Omega_{SSR},$$

s.t. $W_i \in \Omega_{SSR}$,

where $V_{SSD}^k$ is the security distance of $F_k$; $W_{f0}$ is any operating point on SSR boundary $B_k$.

2.2 $N-1$ security margin indices

The following security margin indices are defined based on the SSD in this study.

(i) The expectation of SSD ($E_{SSD}$): $E_{SSD}$ can make a general estimation of security distance. When $E_{SSD}$ is great, it means that it is hard for the present operating point to cross the security region boundary. Its mathematical expression is as follows:

$$E_{SSD} = \frac{\sum_{k=1}^{n} V_{SSD}^k}{n},$$

where $n$ is the number of feeder sections.

(ii) The standard deviation of SSD ($\sigma_{SSD}$): $\sigma_{SSD}$ is mainly formulated to evaluate the fluctuation of security distance. When $\sigma_{SSD}$ is small, the probability of the violation of the substation transformer or feeder $N-1$ contingency constraint under uncertainties is small. Its mathematical expression is as follows:

$$\sigma_{SSD} = \sqrt{\frac{\sum_{k=1}^{n} (V_{SSD}^k - E_{SSD})^2}{n}}.$$
(iv) $C_{S1}$: new transformers are allocated to the reserved locations in existing substations. $C_{S1}$ can be calculated as

$$C_{S1} = \sum_{s \in \Theta_S} u_{S1}^{s} c_{S1}, \quad (12)$$

where $u_{S1}^{s}$ is a binary variable, $u_{S1}^{s}$ takes the value 1 if substations $S_2$ is constructed and 0 otherwise; $c_{S1}$ is the investment cost to expand substations; $\Theta_S$ is the set of substations expansion.

(v) $C_{S2}$: new substations should be constructed when existing substations are so overloading that new load cannot be allocated to the networks. $C_{S2}$ can be calculated as

$$C_{S2} = \sum_{s \in \Theta_S} u_{S2}^{s} c_{S2}, \quad (13)$$

where $u_{S2}^{s}$ is a binary variable, $u_{S2}^{s}$ takes the value 1 if $S_2$ is constructed and 0 otherwise; $c_{S2}$ is the investment cost to construct substations; $\Theta_S$ is the set of new substations.

(vi) $C_{OM}$

$$C_{OM} = \sum_y \sum_i \sum_j \sum_{i,j} N_{p} p_{DG, i,j}^{grid} c_{OM}, \quad (14)$$

where $\eta = 1/(1 + dy)$ is the present value factor, $d$ is the discount rate, $y$ is the index of the planning period; $N_{SC}$ is the number of the $s$th scenary; $p_{DG, i,j}^{grid}$ is the active power output of RSs at bus $i$, scenario $j$, and year $y$; $c_{OM}$ is the unit operation and maintenance cost of RSs at bus; $\Theta_p$ is the set of RS types; $\Theta_m$ is the set of RS installation buses of the $h$th type RS; subscript $sc$ indicates the value of the corresponding scenario.

(vii) $C_{P}$

$$C_{P} = \sum_y \sum_i \sum_j p_{grid, i,j}^{grid} c_{P}, \quad (15)$$

where $p_{grid, i,j}^{grid}$ is the active power injected from the upstream grid at bus $i$, scenario $j$ and year $y$; $c_{P}$ is the unit operation and maintenance cost of RSs.

(viii) $C_{CE}$: the carbon emission cost mainly originates from the upstream grid because RSs are the zero carbon emission generations. $C_{CE}$ can be calculated as

$$C_{CE} = \sum_y \sum_i \sum_j p_{grid, i,j}^{grid} c_{CE}, \quad (16)$$

where $c_{grid}$ is the unit carbon emission price; $\lambda$ is the carbon emission intensity of the upstream grid.

To sum up, the investment cost includes $C_{T1}$, $C_{TL}$, $C_{T2}$, $C_{S1}$, and $C_{S2}$ while the operation cost includes $C_{OM}$, $C_{P}$ and $C_{CE}$ for the proposed DNP. It should be pointed out that costs can differ from various planning measures in DNP. Thus, there exist priorities of planning measures for economical concern.

3.2 Constraints

(i) Power flow equation constraint

$$\begin{align*}
P_{i,j,k} - U_{i,j,k} \sum_{j=1}^{N_{bus}} U_{j,k} (G_{i,j} \cos \theta_{i,j,k} + B_{i,j} \sin \theta_{i,j,k}) &= 0, \\
Q_{i,j,k} - U_{i,j,k} \sum_{j=1}^{N_{bus}} U_{j,k} (G_{i,j} \cos \theta_{i,j,k} + B_{i,j} \sin \theta_{i,j,k}) &= 0, \\
\end{align*} \quad (17)$$

where $P_{i,j,k}$ and $Q_{i,j,k}$ are, respectively, the active and reactive power at bus $i$; $G_{i,j}$ and $B_{i,j}$ are, respectively, the conductance and admittance of branch $ij$; $\theta_{i,j,k}$ is the phase difference between the voltage of buses $i$ and $j$; $N_{bus}$ is the number of buses.

(ii) Power transfer constraint

$$V_{SD, i,j}^d \geq V_{SD, min}^d \quad (18)$$

where $V_{SD, min}^d$ is the lower SSD limit at $F_k$.

(iii) Bus voltage chance constraint

$$P \left\{ U_{i,k}^{\min} \leq U_{i,k} \leq U_{i,k}^{\max} \right\} \geq \beta_k, \quad (19)$$

where $P$ is the probability of the incident in $\cdot$; $\beta_k$ is the specified probability satisfying bus voltage constraint.

(iv) Topology structure constraint

$$\sum_{b \in \Theta_b} u_{b} = N_{bus} - N_{root}, \quad (20)$$

where $u_{b}$ is a binary variable, $u_{b}$ takes the value 1 if branch $b$ is closed and 0 otherwise; $N_{root}$ is the number of substation buses.

Equations (17) and (20) can ensure radiality in distribution networks. However, when there are some distributed generations (DGs), this conclusion may be insufficient. Thus, some additional loop elimination methods are taken to guarantee the feasible topology of distribution networks for all conditions [13].

3.3 Solving algorithm

The planning problem is a chance-constrained mixed integer non-linear programming model. The set of decision variables includes feeder reformation, new tie lines, new feeders, substation expansion, new substations and new load allocation. Thus, it is hard and time consuming to solve the optimal planning scheme because of big data. To avoid the above difficulty, DNEA [14] is put forward to solve the planning problem.

Two tournament selection, two-point crossover, and one-point uniform mutation are adopted as genetic operators. The probability of crossover and mutation is variable and the elitism strategy is implemented in DNEA. When the Hamming distance between two individuals is less than the specified distance criterion, the individual whose fitness is smaller would be punished with niche elimination operations.

4 Simulation analysis

The modified 104-bus distribution network is used to demonstrate the validity of the proposed model and solving algorithm. Three types of loads, residential, commercial and industrial customer, exist in the case. The new load data are shown in Table 1. The rated capacity of each WTG is 0.5 MW. The cut-in wind speed, rated wind speed, and cut-out wind speed are, respectively, 3, 13 and 20 m/s. The WTG operation and maintenance cost is 30$/MWh. The rated capacity of each PVG is 0.5 MW. The rated illumination intensity is 0.5 kW/m$^2$. The PVG operation and maintenance cost is 40$/MWh. The carbon emission intensity of the

| Table 1 Data of new loads |
|---------------------------|
| New load no. | No. of loads | Load, MVA | Customer type |
|---------------|--------------|-----------|----------------|
| 1, 5, 12, 13, 16, 19, 23, 24 | 8 | 0.296 | residential |
| 3, 6, 8, 14, 15, 17, 22 | 7 | 0.407 | industrial |
| 9, 10, 21 | 3 | 0.611 | commercial |
| 2, 4, 7, 11, 18, 20, 25 | 7 | 0.336 | commercial |
upstream grid is 629.2 kg/MWh while the unit carbon emission cost is 10 $/t. The discount rate is 0.06 and the planning horizon is 5 years. The annual load growth rate is assumed to be 1%. The yearly profiles of wind speed, illumination intensity and loads are shown in Fig. 1. The program is compiled in MATLAB R2013a and runs on a PC with 3.60 GHz CPU and 8 GB RAM.

4.1 Results of proposed planning

The schemes of proposed planning and new allocation are, respectively, shown in Table 2 and Fig. 2. It can be seen from Fig. 2 that feeders $F_{8}$ and $F_{11}$, feeders $F_{4}$ and $F_{16}$ as well as feeders $F_{7}$ and $F_{18}$ are, respectively, exchanged, which means that feeder reformation is conducted in the proposed planning. The corresponding total cost is $3.03 \times 10^{7}$.

The SSD of each feeder after planning is presented in Fig. 3. As a contrast, Fig. 3 also shows the SSD of each feeder before planning and the corresponding total cost is $2.96 \times 10^{7}$. As can be seen in Fig. 3, the SSD of feeders $F_{8}$, $F_{11}$ and $F_{15}$ before planning is negative while the SSD of all the feeders after planning is positive. This phenomenon indicates that feeder reformation can guarantee system $N-1$ security with the assistance of power transfer capability and exploit the potential supply capability of the existing network to postpone upgrade. Besides, the differential system security demand can be satisfied by specifying the lower SSD limit at each feeder.

Compared with the results before planning, although the total cost after planning increases $6.84 \times 10^{5}$ ($2.3\%$), the variation coefficient of SSD after planning decreases 0.18 ($19.6\%$). Thus, the proposed planning can dramatically increase $N-1$ security margin at a cost of a small but acceptable increase in the total cost.

4.2 Comparison with traditional planning

The traditional planning scheme is shown in Fig. 4. As shown in Fig. 4, two new transformers of 12 and 16 MVA are constructed, which means that 28 MVA substation capacities should be increased in the traditional planning. The planning result comparison is listed in Table 3.

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**Table 2** New load allocation scheme

| No. | Feeder | No. | Feeder | No. | Feeder | No. | Feeder |
|-----|--------|-----|--------|-----|--------|-----|--------|
| 1   | $F_{8}$ | 8   | $F_{4}$ | 15  | $F_{5}$ | 22  | $F_{20}$ |
| 2   | $F_{9}$ | 9   | $F_{11}$ | 16  | $F_{3}$ | 23  | $F_{19}$ |
| 3   | $F_{11}$ | 10  | $F_{13}$ | 17  | $F_{14}$ | 24  | $F_{20}$ |
| 4   | $F_{8}$ | 11  | $F_{16}$ | 18  | $F_{14}$ | 25  | $F_{19}$ |
| 5   | $F_{3}$ | 12  | $F_{12}$ | 19  | $F_{16}$ |  --- |  --- |
| 6   | $F_{10}$ | 13  | $F_{13}$ | 20  | $F_{12}$ |  --- |  --- |
| 7   | $F_{8}$ | 14  | $F_{3}$ | 21  | $F_{6}$ |  --- |  --- |

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**Fig. 1** Initial yearly profiles of wind speed, illumination intensity and loads

**Fig. 2** Proposed planning scheme

**Fig. 3** SSD of each feeder before and after planning

**Fig. 4** Traditional planning scheme

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The proposed DNDEA algorithm has the best performance in terms of optimisation results and algorithm speed, as shown in Table 4. It can be observed from Table 4 that the DNDEA is superior to PSO and GA in reducing the total cost and computation time.

5 Conclusions

This study proposes an interconnected DNP model in which loads can be flexibly re-distributed among feeders through tie lines with advanced distribution automation. Two types of contingency scenarios, feeder, and substation transformer contingency and uncertainties of RSs and loads are integrated into the planning. The planning problem is a chance-constrained mixed integer non-linear programming problem, which is solved by DNDEA.

The case study carried out on a modified 104-bus distribution network illustrates that the solution can not only improve system economy and security simultaneously but also exploit asset utilisation by optimising the existing network. Reasonable feeder reformation makes it possible for the operating point to move into a safer part. The planning scheme has the self-healing feature, which can postpone the network upgrade.

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7 References

[1] Haffner S., Pereira L.F.A., Pereira L.A., et al.: ‘Multistage model for distribution expansion planning with distributed generation—part 1: problem formulation’, IEEE Trans. Power Deliv., 2008, 23, (2), pp. 915–923.
[2] Keane A., Ochoa L.F., Borges C.L.T., et al.: ’State-of-the-art techniques and challenges ahead for distributed generation planning and optimization’, IEEE Trans. Power Syst., 2013, 28, (2), pp. 1493–1502.
[3] de Groot R.J.W., Moren J., Slootweg J.G.: ‘Smart integration of distribution automation applications’. Proc. IEEE PES Int. Conf. on Exhibition, Berlin, Germany, October 2012, pp. 1–7.
[4] Li Y.J., Wang F.R.: ‘Implementation of smart grid software system based on configurable feeder terminal unit’. Proc. IEEE Symp. Electrical and Electronics Engineering Conf., Kuala Lumpur, Malaysia, June 2012, pp. 277–280.
[5] Staszewski D.M., Craig D., Befus C.: ‘Advanced feeder automation is here’, IEEE Power Energy Mag., 2015, 3, (5), pp. 56–63.
[6] Chen P., Salcedo R., Zhu Q., et al.: ‘Analysis of voltage profile problems due to the penetration of distributed generation in low-voltage secondary distribution networks’, IEEE Trans. Power Deliv., 2012, 27, (4), pp. 2020–2028.
[7] Gonen T., Ramirez-Rosado J.I.: ‘Review of distribution system planning models: a model for optimal multistage planning’, IEEE Proc. Gener. Transm. Distrib., 1986, 133, (7), pp. 397–408.
[8] Gonzalez-Sotres L., Domingo C.M., Sanchez-Miralles A., et al.: ‘Large-scale MV/LV transformer substitution planning considering network costs and flexible area decomposition’, IEEE Trans. Power Deliv., 2013, 28, (4), pp. 2245–2253.
[9] Nahman J.M., Peric D.M.: ‘Optimal planning of radial distribution networks by simulated annealing technique’, IEEE Trans. Power Syst., 2008, 23, (2), pp. 790–795.
[10] Zidan A., Shaaban M.F., El-Saadany E.F.: ‘Long-term multi-objective distribution network planning by DG allocation and feeders’ reconfiguration’, Electr. Power Syst. Res., 2013, 93, (105), pp. 95–104.
[11] Junior B.R.P., Cossi A.M., Contreras J., et al.: ‘Multiobjective multistage distribution system planning using tabu search’, IET Gener. Transm. Distrib., 2014, 8, (11), pp. 35–45.
[12] Xiao J., Zuo Q., Gong X.X., et al.: ‘Model and topological characteristics of power distribution system security region’, J. Appl. Math., 2014, 2014, pp. 1–13, Art. ID 327078.
[13] Lavorato M., Franco J.F., Rider M.J., et al.: ‘Imposing radial constraints in distribution system optimization problems’, IEEE Trans. Power Syst., 2012, 27, (1), pp. 172–180.
[14] Li Y., Wang Y., Li B.: ‘A hybrid artificial bee colony assisted differential evolution algorithm for optimal reactive power flow’, Int. J. Electr. Power Energy Syst., 2013, 52, pp. 25–33.