Model independent analysis of the simultaneous mixing of gauge bosons and mixing of fermions

Umberto Cotti* and Arnulfo Zepeda

Departamento de Física, Centro de Investigación y de Estudios Avanzados del IPN, A.P. 14-740, 07000 México D.F., México.

(November, 1996)

Abstract

We discuss the case of simultaneous mixing of gauge bosons and mixing of fermions in a model independent way and for a variety of extra-fermion representations. In this context we analyze a class of lepton family violating processes, namely $Z \to e\bar{\tau}$, $Z \to \mu\bar{\tau}$, $Z \to e\bar{\mu}$, $\mu \to e\bar{e}e$, $\tau \to e\bar{e}e$, $\tau \to \mu\bar{\mu}\bar{\mu}$, $\tau \to e\mu\bar{\mu}$ and $\tau \to \mu\bar{e}e$ in the presence of one neutral gauge boson, $Z'$, with universal, non-universal or family changing couplings. We derive bounds on the combined effect of $Z-Z'$ mixing and ordinary-exotic lepton mixing.

PACS numbers: 12.15.Ji, 12.60.Cn, 12.15.Mm, 14.60.Hi

1 Introduction

Tree level family changing neutral current (FCNC) interactions arise in extended models from three possible sources: (i) the exchange of family changing neutral gauge boson, (ii) the mixing between exotic and ordinary fermions and (iii) the existence of neutral scalars in the Higgs sector with family violating couplings. However, if the standard neutral $Z_0$ boson mixes with a boson which has a coupling which is either family changing or nonuniversal, its coupling to the light (that is the ordinary) fermions becomes family changing even in the absence of mixing between exotic and ordinary fermions.

In previous works an extensive research has been performed in the context of FCNC produced by the mixing of the standard neutral gauge boson with one which do not couple universally to fermion generation [1], or by the mixing between exotic and ordinary fermions [2, 3]. In this communication we show how this phenomenon arises in the general case of simultaneous mixing of neutral gauge bosons and mixing of ordinary fermions with exotic ones. We do not consider in this article FCNC arising from the exchange of scalars, nor additional indirect effects such as the shifts induced by the mixing between the neutral gauge bosons in the values of the weak angle $\theta_w$, the $\rho$ parameter and the Fermi coupling constant $G_F$ [4–6], since they are irrelevant for the present analysis. We apply the formalism in a model independent way to several lepton family violating processes in the $e-\mu$, $\mu-\tau$ and $e-\tau$ sectors considering several possible exotic fermionic representations. We obtain in each case bounds for the mixing parameters including the possibility that the contribution of the neutral gauge boson mixing and that of the fermion mixing are of the same order.

We describe in section 2 the formalism for dealing with FCNC which arise from simultaneous mixing of gauge bosons and mixing of fermions. This formalism is applied to the leptonic sector in section 3. In subsection 3.1 we describe how the mixing effects modify the diagonal couplings of the $Z$. In subsection 3.2 we apply the formalism to the $Z \to l_i\bar{l}_j$, $l_i \to l_j\bar{l}_j$ and $l_i \to l_jl_k\bar{l}_k$ decays and obtain constraints for the mixing parameters. These bounds are refined in section 4 considering special types of representations for the additional fermions.

*e-mail: ucotti@fis.cinvestav.mx
2 Mixing effects: the general formalism for simultaneous mixing of gauge bosons and mixing of fermions

To discuss the mixing of the massive neutral gauge bosons of a general theory we first divide them into two classes,

- The standard $Z^o$ gauge boson which is a linear combination of the $SU(2)_L \otimes U(1)_Y$ neutral bosons and has universal family diagonal (UFD) couplings determined by the eigenvalues $t_3$ and $q$ of the electroweak generators $T_3$ and $Q$.
- The extra $Z^o$ gauge bosons which can have either UFD or non universal family diagonal NUD or FC couplings. The last two types of couplings arise when the $Z^o$ gauge bosons are associated with horizontal interactions. Since the case where $Z^o$ has UFD couplings has already been discussed in the literature \cite{3,4,7}, we will concentrate our attention on the cases of NUFD \cite{1} and FC couplings.

To discuss the general mixing of fermions, including additional ones, we follow Langacker and London \cite{2} grouping all fermions of a given electric charge, $q$, and a given helicity, $a = L, R$, in a $n_a + m_a$ vector column of $n_a$ ordinary (O) and $m_a$ exotic (E) gauge eigenstates $\psi_a^0 = (\psi_O, \psi_E)^T$. The relation between the gauge eigenstates and the corresponding light ($l$) and heavy ($h$) mass eigenstates $\psi_a = (\psi_l, \psi_h)^T$, is given by

$$\psi_a^0 = U_a \psi_a. \quad (1)$$

where the unitary matrices $U_a$ have the block form

$$U_a = \begin{pmatrix} A_a & E_a \\ F_a & G_a \end{pmatrix}, \quad (2)$$

and the submatrices $A_a$ and $G_a$ are not unitary but satisfy the following conditions

$$(U^\dagger U)_a = \begin{pmatrix} A^\dagger A + F^\dagger F & A^\dagger E + F^\dagger G \\ E^\dagger A + G^\dagger F & E^\dagger E + G^\dagger G \end{pmatrix}_a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (3)$$

The term $(F^\dagger F)_a$, second order in the small exotic–ordinary fermion mixing, induces FC transitions in the light–light sector.

The neutral current term for the multiplet $\psi$ of a given electric charge, for the case when both types of mixings are present, is then

$$-L^{nc} = \frac{e}{s_{\theta_w}c_{\theta_w}} \sum_{a=L,R} \bar{\psi}_a^\dagger \gamma^\mu \left( D_a, H^+_a, \cdots, H^{n_a}_a \right) \psi_a \begin{pmatrix} Z^o_a \\ Z^\dagger_1 \\ \vdots \\ Z^\dagger_{n_a} \end{pmatrix}_\mu \quad (4a)$$

$$= \frac{e}{s_{\theta_w}c_{\theta_w}} \sum_{a=L,R} \bar{\psi}_a^\dagger \gamma^\mu \left( U^\dagger_a D_a U_a, U^\dagger_a H^+_a U_a, \cdots, U^\dagger_a H^{n_a}_a U_a \right) \psi_a R \begin{pmatrix} Z \\ Z_1 \\ \vdots \\ Z_{n_a} \end{pmatrix}_\mu \quad (4b)$$

where $s_{\theta_w}$ and $c_{\theta_w}$ are $\sin \theta_w$ and $\cos \theta_w$ respectively, $\theta_w$ is the weak mixing angle, $R$ is the $(n + 1) \times (n + 1)$ orthogonal matrix that diagonalizes the neutral boson mass matrix, $D_a$ is the $(n_a + m_a) \times (n_a + m_a)$ matrix that expresses the coupling of the $Z^o$ gauge boson to matter fields, and similarly $H^+_a$ are the $(n_a + m_a) \times (n_a + m_a)$ matrices that express the coupling of the NUFD and FC gauge bosons to matter. The electromagnetic part of $L^{nc}$ has not been displayed since its structure is not affected by the mixing effects.
In the simple case of only one extra neutral gauge boson, the R matrix is easily parametrizable as
\[
R = \begin{pmatrix}
\cos \Theta & -\sin \Theta \\
\sin \Theta & \cos \Theta
\end{pmatrix}
\] (5)
and the neutral current term is now,
\[
-\mathcal{L}^{nc} = \frac{e}{s_{\theta_w} c_{\theta_w}} \sum_{a=L,R} \bar{\psi}_a \gamma^\mu U_a^\dagger (D_a \cos \Theta + H_a \sin \Theta, H_a \cos \Theta - D_a \sin \Theta) U_a \psi_a \begin{pmatrix} Z \\ Z' \end{pmatrix}_\mu .
\] (6)
It should be obvious that the treatment of the general case, eqs. (4a) and (4b), is straightforward. From now on we restrict the discussion to the case of only one extra gauge boson just to simplify the notation. In eqs. (4a), (4b) and (6) the \(D_a\) matrices are given by
\[
D_a \equiv (T_3 - Q s^2_{\theta_w})_a.
\] (7)

They are diagonal by definition and in the case, which we assume for simplicity from now on, that the exotics of a given charge and helicity have a common eigenvalue \(t_{3Ea}\) of \(T_3\), they can be written as
\[
D_a = \begin{pmatrix} t_{3O} - q_{O} s^2_{\theta_w} & 0 \\
0 & t_{3E} - q_{E} s^2_{\theta_w} \end{pmatrix}_a,
\] (8)
where \(t_{3Oa}\) and \(t_{3Ea}\) are square matrices of dimension \(n_a\) and \(m_a\) respectively. They correspond to the ordinary and exotic part of the \(T_3\) operator and they are proportional to the unit matrix through the eigenvalues \(t_{3Oa}\) and \(t_{3Ea}\) of \(T_3\). This is the same situation for the \(q_{Oa}\) and \(q_{Ea}\) matrices in relation with the \(Q\) operator.

Contrary to the \(D_a\) matrices, the \(H\) ones are not diagonal in general but can however be written as
\[
H = \begin{pmatrix} H_O & 0 \\
0 & H_E \end{pmatrix},
\] (9)
where \(H_O\) and \(H_E\) represent the interactions of the \(Z_1^0\) with the ordinary and exotics fermions respectively. The point is that there are no \(H_{EO}\) nor \(H_{OE}\) terms in \(H\) (which would give rise to \(Z_1^0\) mediated transitions between exotic and ordinary fermions) as long as the horizontal group commutes with the Standard Model (SM) gauge group.

In the basis where the fermions are mass eigenstates, the form of \(D\) and \(H\) is
\[
(U^\dagger DU)_R = \begin{pmatrix} F^\dagger F & F^\dagger G \\
G^\dagger F & G^\dagger G \end{pmatrix}_R t_{3ER} - Qs^2_{\theta_w},
\] (10a)
\[
(U^\dagger DU)_L = \begin{pmatrix} F^\dagger F & F^\dagger G \\
G^\dagger F & G^\dagger G \end{pmatrix}_L t_{3EL} + \begin{pmatrix} A^\dagger A & A^\dagger E \\
E^\dagger A & E^\dagger E \end{pmatrix}_L t_{3OL} - Qs^2_{\theta_w},
\] (10b)
and by using the unitarity conditions (3) we can rewrite the last equation as
\[
(U^\dagger DU)_L = \begin{pmatrix} F^\dagger F & -A^\dagger E \\
G^\dagger F & -E^\dagger E \end{pmatrix}_L (t_{3EL} - t_{3OL}) + T_{3L} - Qs^2_{\theta_w}.
\] (10c)

From eq. (10a) one can see that for the light fermions, and in the absence of \(Z^0-Z_1^0\) mixing, the coupling of the \(Z^0\) to right handed FC and non-universal family diagonal neutral currents (NUFDNC) is possible only if \(t_{3ER} \neq 0\). Furthermore, from eqs. (10a) and (10c) it’s easy to see that sequential fermions do not induce FC nor NUFD couplings for the standard \(Z^0\) since their contribution to these currents is canceled out by that of the ordinary fermions.
On the other hand, no general statement can be made for the transformed H couplings:

\[(U^†HU)_a = \left( \begin{array}{ccc} A^†H_OA + F^†H_EF, & A^†H_OE + F^†H_EG \\ E^†H_OA + G^†H_EF, & E^†H_OE + G^†H_EG \end{array} \right)_a \equiv \left( \begin{array}{ccc} H_{ll} & H_{lh} \\ H_{jh}^† & H_{hh} \end{array} \right)_a. \] (11)

From the last equation one can see that in the presence of neutral gauge boson mixing, \( \Theta \neq 0 \), there will be in general FC couplings of the Z in the light sector, H\( _{ll} \) nondiagonal, even in the absence of mixing between exotic and ordinary fermions, F = 0 and even if the coupling of Z\( _{o1} \) to the ordinary fermions, H\( _O \), is diagonal but nonuniversal, since in general the mass and gauge eigenstates will not coincide in the light sector, A \( \neq 1 \).

2.1 The general neutral current lagrangian term in the light sector

From eqs. (10a), (10c) and (11) we obtain for the neutral-current lagrangian in the light–light sector the following expression:

\[-L_{nc} = \frac{e}{s_\theta c_\theta} \sum_{a=L,R} \bar{\psi}_{la} \gamma^\mu (K_a, K'_a) \psi_{la} \left( \frac{Z}{Z'} \right) \] (12)

where

\[K_L = \left( \frac{(F^†F)_L (t_{3EL} - t_{3OL}) + t_{3OL} - Qs_{\theta_w}^2}{(F^†F)_L (t_{3EL} - t_{3OL}) + t_{3OL} - Qs_{\theta_w}^2} \right) \cos \Theta + (H_{ll})_L \sin \Theta, \] (13a)

\[K_R = \left( \frac{(F^†F)_R t_{3ER} - Qs_{\theta_w}^2}{(F^†F)_R t_{3ER} - Qs_{\theta_w}^2} \right) \cos \Theta + (H_{ll})_R \sin \Theta, \] (13b)

\[K'_L = -\left( \frac{(F^†F)_L (t_{3EL} - t_{3OL}) + t_{3OL} - Qs_{\theta_w}^2}{(F^†F)_L (t_{3EL} - t_{3OL}) + t_{3OL} - Qs_{\theta_w}^2} \right) \sin \Theta + (H_{ll})_L \cos \Theta, \] (13c)

\[K'_R = -\left( \frac{(F^†F)_R t_{3ER} - Qs_{\theta_w}^2}{(F^†F)_R t_{3ER} - Qs_{\theta_w}^2} \right) \sin \Theta + (H_{ll})_R \cos \Theta. \] (13d)

From these eqs. it is easy to see that:

- There are two contributions to the FC couplings of the light fermions to the Z, proportional to \((F^†F)_a \cos \Theta \) and \((H_{ll})_a \sin \Theta \) respectively, which may be in principle of the same order;

- In the limit of no mixing between exotics and ordinary fermions (\( F_a = 0 \)) and no mixing between the Z and the extra gauge boson (\( \Theta = 0 \)) the SM couplings are recovered;

- In the absence of mixing with the exotic fermions, the FC couplings of the ordinary fermions (of a given helicity) to the Z may still survive through the term \((H_{ll})_a \sin \Theta \), provided that the family of ordinary fermions of the given helicity transforms nontrivially under the horizontal generator H\( _O \).

Further details of these couplings depend on the model and on the processes under consideration and are the subject of the next sections.

We may rewrite eqs. (13a) and (13b) as

\[K_L = \left( \Lambda_L + t_{3OL} - Qs_{\theta_w}^2 \right) \cos \Theta + \Xi_L \sin \Theta, \] (14a)

\[K_R = \left( \Lambda_R - Qs_{\theta_w}^2 \right) \cos \Theta + \Xi_R \sin \Theta, \] (14b)

where

\[\Lambda_L = \left( F^†F)_L (t_{3EL} - t_{3OL}), \] (15a)

\[\Lambda_R = \left( F^†F)_R t_{3ER}, \] (15b)

\[\Xi_a = \left( H_{ll})_a, \] (15c)

together with \( \Theta \), represent the physics beyond the SM.
2.1.1 Charged fermions

Since for the light charged fermions, the dimension of ψ_{IL} and ψ_{IR} are the same (there is an equal number of left and right handed fermions), we can rewrite the general lagrangian (12) as

\[ -\mathcal{L}^{nc} = \frac{c}{2s_\theta c_\theta} \bar{\psi}_l \gamma^\mu \left( g_V - g_A \gamma^5, g_V' - g_A' \gamma^5 \right) \psi_l \left( \frac{Z}{Z'} \right)_\mu, \]  

where

\[ g_V = K_L + K_R, \]  
\[ g_A = K_L - K_R, \]  

3 Applications to the leptonic sector

3.1 Constraints from the lepton family diagonal processes \( Z \to l_i \bar{l}_i \)

The effects of mixing between ordinary and and exotic fermions on the diagonal process \( Z \to l_i \bar{l}_i \) has been analyzed previously [2–4]. Likewise separate effect of mixing between the standard \( Z \) and a new one were discussed in Ref. [4, 5]. When both effects are present, the branching ratio \( B(Z \to l_i \bar{l}_i) \), in the \( M_Z > m_{l_i} \) approximation, is given by

\[ B(Z \to l_i \bar{l}_i) \approx \frac{1}{\Gamma_{tot}} \frac{G_F M_Z^3}{6\sqrt{2}\pi} \left( |g_V^i|^2 + |g_A^i|^2 \right) \]  

(18a)

\[ = \frac{1}{\Gamma_{tot}} \frac{G_F M_Z^3}{3\sqrt{2}\pi} \left( |\Lambda_{ii}^L + \Xi_{ii}^L \Theta - \frac{1}{2} + s_{\theta_w}^2|^2 \right) + \frac{1}{\Gamma_{tot}} \frac{G_F M_Z^3}{3\sqrt{2}\pi} \left( |\Lambda_{ii}^R + \Xi_{ii}^R (\Theta + s_{\theta_w}^2)|^2 \right) + O(\Theta^2). \]  

(18b)

Since the agreement of the SM predictions with the experimental data for these processes is better than 0.1 \% (the experimental value of \( \Gamma(Z \to l \bar{l}) \) is 83.83 ± 0.27 [8] against the theoretical one equal to 83.97 ± 0.07), the quantities \( \Lambda_{ii}^L + \Xi_{ii}^L \Theta \) are bounded practically by the experimental uncertainty in the data [8],

\[ B_{l_i \bar{l}_i} = B(Z \to l_i \bar{l}_i) = \begin{cases} B_{ee} = (3.366 \pm 0.008) \times 10^{-2} \\ B_{\mu \mu} = (3.67 \pm 0.013) \times 10^{-2} \\ B_{\tau \tau} = (3.360 \pm 0.015) \times 10^{-2}. \end{cases} \]  

(19)

We may also write

\[ |\Lambda_{ii}^L + \Xi_{ii}^L \Theta - \frac{1}{2} + s_{\theta_w}^2|^2 + |\Lambda_{ii}^R + \Xi_{ii}^R (\Theta + s_{\theta_w}^2)|^2 = c B_{l_i \bar{l}_i}, \]  

(20)

where \( c^{-1} = \left( \frac{1}{\Gamma_{tot}} \frac{G_F M_Z^3}{3\sqrt{2}\pi} \right) = 0.2675 \pm 0.0005 \) and from which we obtain, in a neighborhood of \( |\Lambda_{ii}^L + \Xi_{ii}^L \Theta| = 0 \) and with \( s_{\theta_w}^2 = 0.2237 \pm 0.0010 \), the bounds

\[ |\Lambda_{ii}^L + \Xi_{ii}^L \Theta| < \text{few} \times 10^{-3}. \]  

(21)

3.2 Constraints from lepton family violating processes

3.2.1 Constraints from \( Z \to l_i \bar{l}_j \)

With the approximation \( M_Z > m_{l_i}, m_{l_j} \), and taking into account that experimental limits exists only for the sum of the charge states of particles and antiparticles states, we should consider for \( i \neq j \)

\[ B(Z \to l_i \bar{l}_j) \approx \frac{2 B(Z \to l \bar{l})}{|g_V^i|^2 + |g_A^i|^2} \left( |g_V^j|^2 + |g_A^j|^2 \right) \]  

(22a)
where \( c^{-1} = \left(\frac{4B(Z \rightarrow l\bar{l})}{|g_V|^2 + |g_A|^2}\right) = 0.536 \) (using the conventional SM branching ratio 0.0337 for \( B_{ij} \) and the standard values for \( g_V \) and \( g_A \)) and where

\[
B_{l_i l_j} \equiv B(Z \rightarrow l_i l_j + \bar{l}_i l_j) = \begin{cases} 
B_{e\bar{\mu}} < 1.7 \times 10^{-6} & \equiv \bar{B}_{e\bar{\mu}} \\
B_{e\bar{\tau}} < 7.3 \times 10^{-6} & \equiv \bar{B}_{e\bar{\tau}} \\
B_{\mu\bar{\tau}} < 1.0 \times 10^{-5} & \equiv \bar{B}_{\mu\bar{\tau}},
\end{cases}
\]

(23)

according to the experimental limits [9, 10]. This means that the fermion mixing parameters \( \Lambda_{ij}^g \) are bounded to lie in a circular region centered at \((-\Xi_{ij}^L, -\Xi_{ij}^R)\) and of radius \( \sim 10^{-3} \).

It’s evident that the contribution of \( \Theta \) in the analysis of \( \Lambda_{ij}^g \) is non negligible when

\[
\Xi_{ij}^g \Theta \gtrsim c B_{l_i l_j}.
\]

(24)

This may be a common situation, since in general \( \Xi_{ij}^g = O(1) \) and the upper bounds for \( \Theta \), which are model dependent, are of the order of \( 10^{-1} \) to \( 10^{-3} \). Taking the limit \( \Theta \rightarrow 0 \) could lead to wrong conclusions: a contribution of \( \Xi_{ij}^g \Theta \sim 5 \times 10^{-3} \) is enough to give a completely new region of solutions for \( \Lambda_{ij}^g \). The results of this section are resumed in table 1.

### 3.2.2 Constraints from \( l_i \rightarrow l_j l_j \bar{l}_j \)

Assuming \( m_{l_i} \gg m_{l_j} \) and ignoring possible contributions from scalars, the branching ratio \( B(l_i \rightarrow l_j l_j \bar{l}_j) \) for \( i \neq j \) is

\[
\frac{B(l_i \rightarrow l_j l_j \bar{l}_j)}{B(l_i \rightarrow l_j \bar{l}_j m_{l_j})} = \frac{1}{2} \left[ 3 \left( |g_{ij}^V|^2 + |g_{ij}^A|^2 \right) \left( |g_{ij}^V|^2 + |g_{ij}^A|^2 \right) + 2\Re \left( g_{ij}^V g_{ij}^{V\ast} \right) 2\Re \left( g_{ij}^A g_{ij}^{A\ast} \right) \right] + \frac{M_Z^2}{M_Z^2} \Re \left[ 3 \left( g_{ij}^V g_{ij}^{V \ast} + g_{ij}^A g_{ij}^{A \ast} \right) \left( g_{ij}^V g_{ij}^{V \ast} + g_{ij}^A g_{ij}^{A \ast} \right) + \left( g_{ij}^V g_{ij}^{V \ast} + g_{ij}^A g_{ij}^{A \ast} \right) \left( g_{ij}^V g_{ij}^{V \ast} + g_{ij}^A g_{ij}^{A \ast} \right) \right] + \frac{1}{2} \frac{M_Z^2}{M_Z^2} \left[ 3 \left( |g_{ij}^V|^2 + |g_{ij}^A|^2 \right) \left( |g_{ij}^V|^2 + |g_{ij}^A|^2 \right) + 2\Re \left( g_{ij}^V g_{ij}^{V \ast} \right) 2\Re \left( g_{ij}^A g_{ij}^{A \ast} \right) \right] \]

(26a)

\[
\approx 4 \left[ 2 \left| -\frac{1}{2} + s_{\theta_w}^2 \right|^2 + |s_{\theta_w}^2|^2 \right] \left| \Lambda_{Lij}^g + \Xi_{ij}^L \Theta \right|^2 + \left( \left| -\frac{1}{2} + s_{\theta_w}^2 \right|^2 + 2 |s_{\theta_w}^2|^2 \right) \left| \Lambda_{Rij}^g + \Xi_{ij}^R \Theta \right|^2 + O (\Theta^2),
\]

(26b)
where we have assumed \( \left( \frac{M_T^2}{M_Z^2} \right)^2 \approx \Theta \), \( \Lambda_{ij}^2 \lesssim \Theta \) (remember that \( \Lambda_{ij}^2 \) is second order in the ordinary–exotic mixing) and we have taken into account the stringent limits obtained in eq. (21) from which

\[
\left| \Lambda_{ij}^L + \Xi_{ij}^L \Theta - \frac{1}{2} + s_\theta^2 \right| \simeq \left| \frac{1}{2} + s_\theta^2 \right|, \tag{27a}
\]

\[
\left| \Lambda_{ij}^R + \Xi_{ij}^R \Theta + s_\theta^2 \right| \simeq \left| s_\theta^2 \right|. \tag{27b}
\]

Using the experimental bounds [11–13]

\[
B_{l_i l_j l_i} = B(l_i \to l_j l_j l_i) = \begin{cases} 
B_{\mu e e} < 1.0 \times 10^{-12} \equiv \bar{B}_{\mu e e} \\
B_{\tau e e} < 3.3 \times 10^{-6} \equiv \bar{B}_{\tau e e} \\
B_{\tau \mu \mu} < 1.9 \times 10^{-6} \equiv \bar{B}_{\tau \mu \mu}
\end{cases} \tag{28}
\]

and \( s_{\theta_w}^2 = 0.2237 \), the constraints on the mixing parameters are

\[
0.203 \left| \Lambda_{ij}^L + \Xi_{ij}^L \Theta \right|^2 + 0.176 \left| \Lambda_{ij}^R + \Xi_{ij}^R \Theta \right|^2 < c_i \bar{B}_{l_i l_j l_j}, \tag{29}
\]

where \( c_i = \left( 4B(l_i \to l_j \bar{\nu}_l \nu_l) \right)^{-1} \) and

\[
B(l_i \to l_j \bar{\nu}_l \nu_l) = \begin{cases} 
B_{\mu e \nu_e \nu_e} \approx 1.00 \\
B_{\tau \mu \nu_e \nu_e} = 0.1735 \pm 0.0014 \\
B_{\tau \tau \nu_e \nu_e} = 0.1783 \pm 0.0008.
\end{cases} \tag{30}
\]

As in sec. 3.2.1, the contribution of \( \Theta \) in eq. (29) is important when

\[
\Xi_{ij}^L \Theta \geq \sqrt{\frac{c_i \bar{B}_{l_i l_j l_j}}{0.203}} \quad \text{and} \quad \Xi_{ij}^R \Theta \geq \sqrt{\frac{c_i \bar{B}_{l_i l_j l_j}}{0.176}}. \tag{31}
\]

The bounds for \( \Lambda_{ij}^e \pm \Xi_{ij}^e \Theta \) and \( \Lambda_{ij}^\mu \pm \Xi_{ij}^\mu \Theta \) obtained from eq. (29) are similar to those obtained from eq. (23). For the \( \mu e \) case eq. (29) is more stringent than eq. (23). The results of this section are resumed in table 2.

### 3.2.3 Constraints from \( l_i \to l_j l_k \bar{l}_k \)

Assuming \( m_i \gg m_{l_j}, m_{l_k}, \) ignoring possible contributions from scalars and neglecting the tree level diagrams which involve simultaneously two FCNC vertices, the branching ratio \( B(l_i \to l_j l_k \bar{l}_k) \) is

\[
\frac{B(l_i \to l_j l_k \bar{l}_k)}{B(l_i \to l_j \bar{\nu}_l \nu_l)} = \left( |g_{ij}^L|^2 + |g_{ij}^R|^2 \right) \left( |g_{ij}^L|^2 + |g_{ij}^R|^2 \right) \]

\[
+ \frac{M_Z^2}{M_Z^2} 2 \Re \left( g_{ij}^L g_{jk}^L g_{ij}^L + g_{ij}^R g_{jk}^L g_{ij}^L + g_{ij}^L g_{lk}^L g_{ij}^L + g_{ij}^R g_{lk}^L g_{ij}^L \right) \]

\[
+ \frac{M_Z^2}{M_Z^2} 2 \Re \left( g_{ij}^L g_{jk}^L g_{ij}^R + g_{ij}^R g_{jk}^L g_{ij}^R + g_{ij}^L g_{lk}^L g_{ij}^R + g_{ij}^R g_{lk}^L g_{ij}^R \right) \]

\[
+ \frac{M_Z^2}{M_Z^2} \left( |g_{ij}^L|^2 + |g_{ij}^R|^2 \right) \left( |g_{ij}^L|^2 + |g_{ij}^R|^2 \right) \]

\[
\simeq \left( \left| \frac{1}{2} + s_{\theta_w}^2 \right|^2 + \left| s_{\theta_w}^2 \right|^2 \right) \left( \left| \Lambda_{ij}^L + \Xi_{ij}^L \Theta \right|^2 + \left| \Lambda_{ij}^R + \Xi_{ij}^R \Theta \right|^2 \right) + O(\Theta^2). \tag{32a}
\]
where we made the same assumptions as in section 3.2.2. Using the experimental limits [12]

$$B_{l_1 l_2 l_3 l_4} \equiv B(l_i \rightarrow l_j l_k l_l) = \begin{cases} B_{\tau \nu \mu} & < 3.6 \times 10^{-6} = B_{\tau \nu \mu} \\ B_{\tau \mu \nu c} & < 3.4 \times 10^{-6} = B_{\tau \mu \nu c}, \end{cases} \quad (33)$$

the constraints on the mixing parameters are

$$0.126 \left( \left| \Lambda^{ij}_{L} \right|^2 + \left| \Xi^{ij}_{L} \right|^2 \right) + 0.126 \left( \left| \Lambda^{ij}_{R} \right|^2 + \left| \Xi^{ij}_{R} \right|^2 \right) < c_t B_{l_i l_j l_k l_l} \quad (34)$$

which are not of interest in our analysis since they are somewhat weaker than those of eq. (29).

### 4 Some SU(2)\(_L\) representation for additional fermions

Some improvement on the above derived bounds for the mixing parameters may be obtained with information about the SU(2)\(_L\) transformation properties of the additional fermions and for this reason we analyze here a few simple SU(2)\(_L\) representations in which new additional charged leptons may appear. In this analysis we will not consider any particular case for the \(\Xi^{ij}_{a}\) parameters, but we will assume that they are of O(1). What follows is valid for one or more additional families, independently of whether the extra families are fundamental or excited leptons in the context of composite models.

#### 4.1 No additional fermions

Equations (23) and (29) are valid even if no additional charged leptons are present in the extended theory. In this case \(F = \Lambda_L = \Lambda_R = 0\) and

$$\Xi_a = (A^\dagger H_O A)_a. \quad (35)$$

There are three subcases:

1. \(H_O\) is of the UFD type. Then Z does not couple to FCNC since \((A^\dagger A)_a = 1\) and therefore

$$\Xi^{ij}_{L} = \Xi^{ij}_{R} = 0 \quad \text{for} \ i \neq j, \quad (36)$$

2. \(H_O\) is of the NUFD type. Then there are two possibilities:

   (a) \(A = 1\) (no mixing among the ordinary leptons). Then Z does not couple to FCNC.

   (b) \(A \neq 1\). Then Z couples to FCNC.

3. \(H_O\) is of the FCNC type. Then Z couples to FCNC.

Therefore when FCNC exists, eqs. (23) and (29) read:

$$\left| \Xi^{ij}_{L} \Theta \right|^2 + \left| \Xi^{ij}_{R} \Theta \right|^2 < c_t B_{l_i l_j} = \begin{cases} c_t B_{\tau \nu} = 3.2 \times 10^{-6} \\ c_t B_{e} = 1.4 \times 10^{-5} \\ c_t B_{\mu} = 1.9 \times 10^{-5} \end{cases} \quad (37)$$

and

$$0.203 \left| \Xi^{ij}_{L} \Theta \right|^2 + 0.176 \left| \Xi^{ij}_{R} \Theta \right|^2 = c_t B_{l_i l_j} < \begin{cases} c_{\nu \mu} B_{\mu \nu} = 0.25 \times 10^{-12} \\ c_{\nu} B_{\nu \nu} = 4.8 \times 10^{-6} \\ c_{\mu} B_{\mu \mu} = 2.7 \times 10^{-6} \end{cases} \quad (38)$$

respectively. All the constraints are for the product \(\Xi^{ij}_{a} \Theta\) of the couplings of the light fermions to the Z' and the Z-Z' mixing angle. In particular \(\Xi^{ij}_{a} \Theta < 10^{-6}\).
4.2 Sequential fermions

\[
\begin{align*}
t_{3EL} &= -\frac{1}{2} \\
t_{3ER} &= 0
\end{align*}
\]
\[\implies \Lambda_L = \Lambda_R = 0,
\]

\[
\Xi_a = \left( A^\dagger H O A + F^\dagger H E F \right)_a.
\]

The situation is the same as that of no additional fermions. Eqs. (37) and (38) hold with the only difference that when \( F \neq 0 \) then \( \Xi_a \) contains the \( (F^\dagger H E F)_a \) contribution. As in the previous case the strongest constraint is for \( \Xi_\mu^\mu \Theta < 10^{-6} \).

4.3 Vector singlets

\[
\begin{align*}
t_{3EL} &= 0 \\
t_{3ER} &= 0
\end{align*}
\]
\[\implies \Lambda_R = 0 \quad \Lambda_L = \frac{1}{2} (F^\dagger F)_L,
\]

\[
\Xi_a = \left( A^\dagger H O A + F^\dagger H E F \right)_a.
\]

Therefore eqs. (23) and (29) now read

\[
\left| \Lambda_L^{ij} + \Xi_L^{ij} \Theta \right|^2 + \left| \Xi_R^{ij} \Theta \right|^2 < c\bar{B}_{i\bar{l}j}
\]

and

\[
0.203 \left| \Lambda_L^{ij} + \Xi_L^{ij} \Theta \right|^2 + 0.176 \left| \Xi_R^{ij} \Theta \right|^2 < c_{il} \bar{B}_{i\bar{l}j}
\]

respectively. The contribution to FCNC from the ordinary–exotic fermion mixing is only left handed. If \( \Xi_a^\mu \sim O(1) \), then the stringent bounds on \( \Theta \), consequence of \( \Xi_R^\mu \Theta < 10^{-6} \), imply an equally stringent bound on \( \Lambda_L^\mu \).

4.4 Vector doublets (homodoublets)

\[
\begin{align*}
t_{3EL} &= -\frac{1}{2} \\
t_{3ER} &= -\frac{1}{2}
\end{align*}
\]
\[\implies \Lambda_L = 0 \quad \Lambda_R = -\frac{1}{2} (F^\dagger F)_R,
\]

\[
\Xi_a = \left( A^\dagger H O A + F^\dagger H E F \right)_a.
\]

Thus eqs. (23) and (29) now read

\[
\left| \Xi_L^{ij} \Theta \right|^2 + \left| \Lambda_R^{ij} + \Xi_R^{ij} \Theta \right|^2 < c\bar{B}_{i\bar{l}j}
\]

and

\[
0.203 \left| \Xi_L^{ij} \Theta \right|^2 + 0.176 \left| \Lambda_R^{ij} + \Xi_R^{ij} \Theta \right|^2 < c_{il} \bar{B}_{i\bar{l}j}
\]

The contribution to FCNC from the ordinary–exotic fermions mixing is only right handed. The conclusions are the same as in the previous case with \( L \leftrightarrow R \).
4.5 Mirror fermions

\[
\begin{align*}
t_{3\text{EL}} &= 0 \quad \text{and} \quad t_{3\text{ER}} = -\frac{1}{2} \end{align*}
\]
\[
\Rightarrow \quad \Lambda_L = \frac{1}{2} (F^\dagger F)_L \quad \Lambda_R = -\frac{1}{2} (F^\dagger F)_R,
\]
(46a)
\[
\Xi_a = (A^\dagger H_O A + F^\dagger H_E F)_a.
\]
(46b)

Hence eqs. (23) and (29) are unchanged

\[
\begin{align*}
|\Lambda_{ij}^L + \Xi_{ij}^L \Theta|^2 + |\Lambda_{ij}^R + \Xi_{ij}^R \Theta|^2 < c_B_{i,j}
\end{align*}
\]
(47)

and

\[
\begin{align*}
0.203 |\Lambda_{ij}^L + \Xi_{ij}^L \Theta|^2 + 0.176 |\Lambda_{ij}^R + \Xi_{ij}^R \Theta|^2 < c_B_{i,j}
\end{align*}
\]
(48)

The contribution to FCNC from the ordinary–exotic fermions mixing is both left and right handed. As a consequence there are no stringent bounds on \( \Theta \) and the limits on \( \Lambda_{ij}^a \) and \( \Xi_a \) are strongly correlated.

4.6 Self conjugated triplets

\[
\begin{align*}
t_{3\text{EL}} &= -1 \quad \text{and} \quad t_{3\text{ER}} = -1 \end{align*}
\]
\[
\Rightarrow \quad \Lambda_L = -\frac{1}{2} (F^\dagger F)_L \quad \Lambda_R = -(F^\dagger F)_R,
\]
(49a)
\[
\Xi_a = (A^\dagger H_O A + F^\dagger H_E F)_a.
\]
(49b)

Hence eqs. (23) and (29) are unchanged

\[
\begin{align*}
|\Lambda_{ij}^L + \Xi_{ij}^L \Theta|^2 + |\Lambda_{ij}^R + \Xi_{ij}^R \Theta|^2 < c_B_{i,j}
\end{align*}
\]
(50)

and

\[
\begin{align*}
0.203 |\Lambda_{ij}^L + \Xi_{ij}^L \Theta|^2 + 0.176 |\Lambda_{ij}^R + \Xi_{ij}^R \Theta|^2 < c_B_{i,j}
\end{align*}
\]
(51)

As in the previous case the contribution to FCNC from the ordinary–exotic fermions mixing is both left and right handed. As a consequence there are no stringent bounds on \( \Theta \) and the limits on \( \Lambda_{ij}^a \) and \( \Xi_a \) are strongly correlated.

5 Conclusions

In a model independent way we obtained bounds for the strength of the FCNC, \( (\Lambda + \Xi \Theta)_a \), in the ordinary charged–leptons sector, produced both by the ordinary–exotic fermion mixing, \( \Lambda_{ij}^a \), and by the Z–Z̄ mixing, \( \Theta \). Giving that the experimental bounds on the decay \( \mu \to ee \) are more stringent than those for the FC decays of the \( \tau \) into three charged leptons and of the Z into two charged leptons, the bounds on the \( \mu–e \) coupling of the Z are stronger than those on the \( \tau–e \) and \( \tau–\mu \) couplings. We have shown also that in some cases, when the SU(2)_L representation of the additional fermions is relatively simple, the bounds may be refined. In other cases there may be a strong correlation between \( \Theta \) and \( \Lambda_{ij}^a \) and then it is not safe to take the limit \( \Theta \to 0 \). In the same way, if one consider specific extended models, e.g. \([1, 14–21]\), some additional statements may be drawn on the \( \Xi_a \). In this work we have concentrated our attention to LFV in decay processes. On the other hand, there may be LFV processes of a different type \([22]\) which will certainly put additional constraints on the LFV parameters.
Acknowledgements

This work was partially supported by CONACyT in Mexico. One of us (A.Z.) acknowledges the hospitality of Prof. J. Bernabeu and of the Theory Group at the University of Valencia as well as the financial support, during the 1995-1996 sabbatical leave, of Dirección General de Investigación Científica y Técnica (DGICYT) of the Ministry of Education and Science of Spain.

References

[1] T. Kuo and N. Nakagawa, Lepton-flavor-violating decays of Z⁰ and τ, Phys. Rev. D32, 306–307 (1985).
[2] P. Langacker and D. London, Mixing between ordinary and exotic fermions, Phys. Rev. D38, 886–906 (1988).
[3] E. Nardi, E. Roulet, and D. Tommasini, Global analysis of fermion mixing, Nucl. Phys. B386, 239–266 (1992).
[4] E. Nardi, E. Roulet, and D. Tommasini, Simultaneous analysis of Z' and new fermion effects: Global constrains in E₆ and SO(10) models, Phys. Rev. D46, 3040–3061 (1992).
[5] P. Langacker and M. Luo, Constraints on additional Z bosons, Phys. Rev. D45, 278–292 (1992).
[6] J. Layssac, F. M. Renard, and C. Verzegnassi, Model independent constraints on a heavy neutral vector boson from present and future LEP and SLC data, Z. Phys. C53, 97–114 (1992).
[7] E. Nardi and E. Roulet, Bounds on ordinary-exotic fermion mixing from LEP-1, Phys. Lett. B248, 139–145 (1990).
[8] Barnett et al., Review of particles properties: Particles Data Group, Phys. Rev. D54, 1 (1996).
[9] R. Akers et al., A Search for Lepton Flavor Violating Z⁰ Decays, Z. Phys. C67, 555–564 (1995), OPAL Collaboration.
[10] L3 Note 1798, Search for Lepton Flavour Violation in Z Decays, (1995), The L3 Collaboration.
[11] U. Bellgardt et al., Search for the decay µ → eee, Nucl. Phys. B299, 1 (1988), SINDRUM Collaboration.
[12] J. Bartelt et al., Search for neutrinoless decays of the tau lepton, Phys. Rev. Lett. 73, 1890–1894 (1994), CLEO Collaboration.
[13] H. Albrecht et al., Search for neutrinoless tau decays, Z. Phys. C55, 179–190 (1992), ARGUS Collaboration.
[14] A. Ilakovac and A. Pilaftsis, Flavor violating charged lepton decays in a GUT and superstring inspired standard model, Nucl. Phys. B437, 491 (1995).
[15] J. Bernabeu, A. Santamaria, J. Vidal, A. Mendez, and J. W. F. Valle, Lepton flavor nonconservation at high-energies in a superstring inspired standard model, Phys.Lett. 187B, 303 (1987).
[16] G. Eilam and G. Rizzo, Quark and Lepton Flavor Violating Z⁰ Decays in E₆, Phys. Lett. 188B, 91–94 (1987).
[17] J. Bernabeu and A. Santamaria, Lepton Flavor Violating Decay of the Z⁰ in the Scalar Triplet Model, Phys. Lett. 197B, 418 (1987).
[18] J. W. F. Valle, Resonant oscillations of massless neutrinos in matter, Phys.Lett. 199B, 432 (1987).
[19] F. Gabbiani, E. Gabrielli, A. Masiero, and L. Silvestrini, A complete analysis of FCNC and CP constraints in general SUSY extensions of the standard model, Nucl. Phys. B477, 321–352 (1996).

[20] E. Gabrielli, A. Masiero, and L. Silvestrini, Flavour changing neutral currents and CP violating processes in generalized supersymmetric theories, Phys. Lett. B374, 80–86 (1996).

[21] R. Gaitán-Lozano, A. Hernández-Galeana, S. A. Tomás, W. A. Ponce, and A. Zepeda, Signals of extra gauge bosons and exotic leptons in SU(6)L ⊗ U(1)Y, Phys. Rev. D51, 6474–6483 (1995).

[22] F. Sciulli and S. Yang, Lepton flavor violation searches, in Future Physics at HERA, edited by G. Ingelman, A. D. Roeck, and R. Klanner, page 260, DESY, 1996.

| $\left| \Lambda_{L}^{ij} + \Xi_{L}^{ij} \Theta \right|^2 + \left| \Lambda_{R}^{ij} + \Xi_{R}^{ij} \Theta \right|^2 < 1.87 \tilde{B}_{l_i l_j} = 1.87 \times$ | $1.7 \times 10^{-6} = \tilde{B}_{e\mu}$ |
| | $7.3 \times 10^{-6} = \tilde{B}_{e\tau}$ |
| | $1.0 \times 10^{-5} = \tilde{B}_{\mu\tau}$ |

Table 1: Bounds from the process $Z \to l_i \bar{l}_j$
Limits from $l_i \to l_j l_j \bar{l}_j$

\[
0.203 |\Lambda_L^{ij} + \Xi_L^{ij} \Theta|^2 + 0.176 |\Lambda_R^{ij} + \Xi_R^{ij} \Theta|^2 < c_l, \quad B_{l_i l_j l_j} = c_l \times \begin{cases} 
1.0 \times 10^{-12} \equiv \tilde{B}_{\text{neee}} \\
3.3 \times 10^{-6} \equiv \tilde{B}_{\text{teee}} \\
1.9 \times 10^{-6} \equiv B_{\tau \mu \mu},
\end{cases}
\]

Mixing $e-\mu$

\[
0.203 |\Lambda_L^{\mu \mu} + \Xi_L^{\mu \mu} \Theta|^2 + 0.176 |\Lambda_R^{\mu \mu} + \Xi_R^{\mu \mu} \Theta|^2 < 0.25 \times 10^{-12}
\]

|\Lambda_L^{\mu \mu} + \Xi_L^{\mu \mu} \Theta| < 1.1 \times 10^{-6} \quad |\Lambda_R^{\mu \mu} + \Xi_R^{\mu \mu} \Theta| < 1.2 \times 10^{-6}

Mixing $e-\tau$

\[
0.203 |\Lambda_L^{\tau \tau} + \Xi_L^{\tau \tau} \Theta|^2 + 0.176 |\Lambda_R^{\tau \tau} + \Xi_R^{\tau \tau} \Theta|^2 < 4.8 \times 10^{-6}
\]

|\Lambda_L^{\tau \tau} + \Xi_L^{\tau \tau} \Theta| < 4.9 \times 10^{-3} \quad |\Lambda_R^{\tau \tau} + \Xi_R^{\tau \tau} \Theta| < 5.2 \times 10^{-3}

Mixing $\mu-\tau$

\[
0.203 |\Lambda_L^{\mu \tau} + \Xi_L^{\mu \tau} \Theta|^2 + 0.176 |\Lambda_R^{\mu \tau} + \Xi_R^{\mu \tau} \Theta|^2 < 2.7 \times 10^{-6}
\]

|\Lambda_L^{\mu \tau} + \Xi_L^{\mu \tau} \Theta| < 3.6 \times 10^{-3} \quad |\Lambda_R^{\mu \tau} + \Xi_R^{\mu \tau} \Theta| < 3.9 \times 10^{-3}

Table 2: Bounds from the process $l_i \to l_j l_j \bar{l}_j$