Switching of Capacitors in an Asymmetric Induction Motor with Single-Phase Power, Triggering Several Applied Loads

Prado Wellington¹, Marcos Antonio¹ and Geraldo Caixeta Guimarães²
1. IFG, Electrotechnical Coordination, Itumbiara, GO, Brazil
2. Electrical Machinery Laboratory, Faculty of Electrical Engineering, Federal University of Uberlândia, Campus Santa Mônica, Uberlândia, MG, Brazil

Abstract: This study has as its objective to collaborate with the expansion in the market of electric energy in rural areas, offering as such an innovative prospect to the solution of associated problems through use of the asymmetric three-phase induction motor, supplied by a single-phase source. In this system, capacitor switching is applied during operation, while theoretical and practical results are presented for the application of this switching in a three-phase asymmetric induction motor of 20 hp.

Key words: Startup current, capacitor switching, three-phase induction motor, asymmetric motor, power factor.

1. Introduction

The aim of the present study is to benefit rural locations, where the electric energy distribution system, which is almost entirely single-phase, is presented as strong ally of the rural resident by improving life quality. These qualities are seen in the use of energy for domestic use or as an aggregate added to the production process, thus contributing to obtaining production and quality indexes in their products.

Currently, the development of new techniques concerning irrigated planting and even the benefit of agricultural products in the very places of production, according to Ref. [1], impose on the rural producer the need to increase the consumption of electric energy, mainly in terms of maximum demand. However, with availability of electric energy based on the single-phase system, the rural producer is bound to have the peculiarities imposed by such a system.

The proposal of this study is to demonstrate that the asymmetric induction motor is apt for operations with variable loads, using capacitor switching according to the load applied on the machine, as the asymmetric motor only presents adequate operation if it operates with a nominal load. The result of this study will show that the asymmetric motor can be used in the rural scenario, where there exist variable loads.

Within this context, this paper will present a study that is eminently technical-scientific, as an important point that still has not been covered is the performance of the three-phase induction motor with a single-phase supply from a variable load, which is from a no load condition until 100% of load. This study seeks to create an analysis for a number of different load conditions through the implementation of capacitor switching with the aim of minimizing the torque oscillations during operation.

2. Mathematical Modeling of the Three-Phase Asymmetric Induction Motor

In order to obtain better precision in the theoretical calculation of the electromagnetic torque, the modeling in the time domain includes the spatial harmonics of the magnetic field that arise from the machine windings.
Fig. 1  Representation of the phases for the windings of the asymmetric three-phase induction machine.

Consistent with Ref. [2], this inclusion is part of the harmonic inductance concept.

According to Refs. [3, 4], the three-phase asymmetric induction motor possesses a squirrel cage rotor and ferromagnetic structure of the stator identical to that of the symmetric three-phase induction motor. This possesses only one change on the stator winding that goes on to have a number of different turns per phase, thus maintaining the angular shifts between phases at 120 electrical degrees.

As the previously developed modeling refers to the phases, an equivalent three-phase winding should represent the squirrel cage. As such, one uses nomenclature “a, b, c” for the three-phase winding of the stator and the phases “A, B, C” for the equivalent three-phase winding of the cage rotor. Fig. 1 schematically shows the relative position of the axes from the different phases of the machine, where one finds that $\theta_R$ is the angle that defines the instantaneous position of the rotor in relation to the stator.

As the desire is to apply the previous modeling in the case of the asymmetric machine, the phases “i” and “j” being generic, they, in this case, represent all the phases a, b, c, A, B, C. As such, the angles $\alpha_i$ and $\alpha_j$ take the values:

$$\alpha_a = 0 \quad (1)$$
$$\alpha_b = \frac{2\pi}{3} \quad (2)$$

$$\alpha_c = -\frac{2\pi}{3} \quad (3)$$
$$\alpha_A = \theta_R \quad (4)$$
$$\alpha_B = \theta_R + \frac{2\pi}{3} \quad (5)$$
$$\alpha_C = \theta_R - \frac{2\pi}{3} \quad (6)$$

3. Modeling of the Asymmetrical Motor on Their Connection

The asymmetrical machine follows the same line as the conventional symmetrical motor, with alterations only in the number of turns per phase, as proposed by Refs. [5-7]. Under these conditions, the machine can be supplied by a single-phase system, which for the Y connection is indicated in Fig. 2. Noted here is the presence of an additional capacitor between the extremities “B” and “C”, which has its aim to resolve the problem of machine startup, as well as improve the performance in nominal steady state.

Consider the three-phase asymmetric induction motor connected in Y and supplied by a single-phase voltage system supported by a capacitor, as shown in Fig. 2.

From the circuit in Fig. 2, one can write:

$$\dot{I} = -\dot{I}_{\text{Cap}} \quad (7)$$

$$V_b - V_c = -jX_{\text{Cap}} I_{\text{Cap}} \quad (8)$$

$$I_a + I_b + I_c = 0 \quad (9)$$

$$Vb - Vc = \left(Z0*Ia + Z1*Ia + Z2*Ia_n\right) \quad (10)$$

where:

$$Z0 = \left(\frac{1}{b} - \frac{1}{c}\right) * Z_{sa}$$

$$Z1 = \left(\frac{\alpha^2}{b} - \frac{\alpha}{c}\right) * Z_{AP}$$
It is possible from mathematical manipulation to obtain the relationship \( I_{a_N}/I_{a_P} \), denominated as the imbalance factor \( F \), in Eq. (11).

\[
\hat{F} = \frac{I_{aN}}{I_{aP}} = \frac{Z_{1} - \hat{Z}_{0} \gamma_{1} + jX_{\text{cap,b}} \gamma_{1} - \alpha^{2}}{Z_{0} \gamma_{2} - \hat{Z}_{2} + jX_{\text{cap,b}} (\alpha - \gamma_{2})}
\]  

(11)

where:

\[
\gamma_{1} = \frac{1 + b, \alpha^{2} + c, \alpha}{1 + b + c}
\]

\[
\gamma_{2} = \frac{1 + b, \alpha + c, \alpha^{2}}{1 + b + c}
\]

In function of \( \hat{F} \), from the previous expressions, one can obtain the currents and voltages of the stator phases for the asymmetrical motor, in Eqs. (12)-(17).

\[
\hat{I}_{a} = \frac{\hat{V}}{Z} \left( 1 - \gamma_{1} + \hat{F} \left( 1 - \gamma_{2} \right) \right)
\]  

(12)

\[
\hat{I}_{b} = \frac{b, \hat{V}}{Z} \left( \alpha^{2} - \gamma_{1} + \hat{F} \left( \alpha - \gamma_{2} \right) \right)
\]  

(13)

\[
\hat{I}_{c} = \frac{c, \hat{V}}{Z} \left( \alpha - \gamma_{1} + \hat{F} \left( \alpha - \gamma_{2} \right) \right)
\]  

(14)

\[
\hat{V}_{a} = \frac{\hat{V}}{Z} \left( -Z_{sa} \gamma_{1} + \hat{F} \gamma_{2} + \hat{Z}_{ap} + \hat{Z}_{aN} \right)
\]  

(15)

\[
\hat{V}_{b} = \frac{\hat{V}}{h, Z} \left( -Z_{sa} \gamma_{1} + \hat{F} \gamma_{2} + \alpha^{2}, \hat{Z}_{ap} + a, \hat{Z}_{aN} \right)
\]  

(16)

\[
\hat{V}_{c} = \frac{\hat{V}}{c, Z} \left( -Z_{sa} \gamma_{1} + \hat{F} \gamma_{2} + a, \hat{Z}_{ap} + a^{2}, \hat{Z}_{aN} \right)
\]  

(17)

where:

\[
\hat{Z} = Z_{4} - Z_{3} \gamma_{1} + \left( Z_{5} - Z_{3} \gamma_{2} \right) \hat{F}
\]

\[
\hat{Z}_{3} = \left( \frac{1}{c} - 1 \right) \hat{Z}_{sa}
\]
Switching of Capacitors in an Asymmetric Induction Motor with Single-Phase Power, Triggering Several Applied Loads

\[ \dot{Z}_4 = \left( \frac{\alpha}{c} - 1 \right) \dot{Z}_{a_p} \]
\[ \dot{Z}_5 = \left( \frac{\alpha^2}{c} - 1 \right) \dot{Z}_{a_N} \]

The single-phase supply has its voltage (V) and current (I_a) respectively. If one takes V in the reference, the angle of I_a provides the power factor on the supplier, that is:

\[ \cos \phi = \frac{-\dot{V}}{Z} \left[ 1 - \gamma \frac{1}{1 + F (1 - \gamma) 2} \right] \]

Consequently, the active electric power that the feeder supplies to the motor is given by:

\[ P_e = V^2 \cdot \text{parte real} \left[ -\frac{1}{Z} \left[ 1 - \gamma \frac{1}{1 + F (1 - \gamma) 2} \right] \right] \]

For the motor torque calculation, one can use the relationship between the power supplied to the rotor (Pfr) and synchronous velocity (Ws). The power supplied to the rotor per phase is equal to the difference between the powers delivered to the rotor, due to the impedances of the circuits of positive sequence, \( Z_{a_p} \), and negative, \( Z_{a_N} \). Therefore, mathematically one has:

\[ P_{fr} = 3 \left[ \text{parte real} \left( Z_{a_p} \right) I_{a_p}^2 - \text{parte real} \left( Z_{a_N} \right) I_{a_N}^2 \right] \]

(20)

From the definition of \( F \) and the previous expressions, Eq. (20) becomes Eq. (21).

\[ P_{fr} = 3 \left[ \text{parte real} \left( Z_{a_p} \right) - \text{parte real} \left( Z_{a_N} \right) F^2 \right] \frac{V^2}{Z^2} \]

(21)

Using Eq. (22) and the procedure of the previously presented calculation, one obtains the electromagnetic torque developed by the motor T, from Eq. (22).

\[ T = \frac{3 V^2}{W_s} \left[ \text{parte real} \left( Z_{a_p} \right) - \text{parte real} \left( Z_{a_N} \right) F^2 \right] \frac{V^2}{Z^2} \]

(22)

Mechanical power is given by Eq. (23).

\[ P_{mech} = 3 V^2 (1 - s) \left[ \text{parte real} \left( Z_{a_p} \right) - \text{parte real} \left( Z_{a_N} \right) F^2 \right] \frac{V^2}{Z^2} \]

(23)

Yield is given by Eq. (24).

\[ R_{end} = \frac{3 (1 - s) \left[ \text{parte real} \left( Z_{a_p} \right) - \text{parte real} \left( Z_{a_N} \right) F^2 \right] \frac{V^2}{Z^2}}{\text{parte real} \left[ 1 - \gamma \frac{1}{1 + F (1 - \gamma) 2} \right]} \]

(24)

4. Capacitor Switching

The switching of capacitors consists of connecting capacitors as requested by the load, that is, for each load variation on the motor shaft a different nominal capacitor should be switched. As in each load state there is a magnetomotive force distribution in the air gap of the machine, and consequently an internal unbalance, which in turn favors oscillating electromagnetic torque, one seeks through this switching to produce a reduction in this unbalance.

The study [8-10] demonstrated the good performance of the asymmetric motor under nominal conditions through the use of a fixed capacitor in steady state, but for half load conditions, for example, this particular capacitor does not provide a beneficial effect, as the electromagnetic torque oscillations are greater than those reached under nominal conditions. As such, different capacitance values will be implemented for each load condition.

4.1 Digital Simulation

In Figs. 3 and 4, one can clearly see the load
variation over the operation of the three-phase asymmetric induction motor, as well as the reduction in electromagnetic torque oscillations with the use of switching capacitors.

From Fig. 3, one notes that only under nominal condition the capacitor produces a fitting reduction from the oscillation of the electromagnetic torque, and in the loads outside the nominal condition, large oscillations are presented, even when maintaining the same number of turns per phase and with the same capacitor for the nominal condition, one obtains a reduction in the torque oscillation for different nominal loads, which serves well enough for use with a capacitor switching that is adequate for each load condition.

Fig. 4 shows the behavior of the electromagnetic torque under various load conditions, while considering only the fundamental harmonics, with the use of capacitor switching during operation.

Table 1 supplies the data for the capacitors used by the switching to each load state.

![Fig. 3 Electromagnetic torque of the asymmetric motor with load variation and without capacitor switching.](image)

![Fig. 4 Electromagnetic torque of the asymmetric motor with load variation and with use of capacitor switching.](image)

Table 1  Load and capacitor variation.

| Load (%) | 100% | 75%  | 50%  | 25%  | 0  |
|----------|------|------|------|------|----|
| Cap (μF) | 290 μF | 210 μF | 100 μF | 50 μF |
5. Results from Experiments Using Capacitor Switching

The tests were performed taking as a reference with the load factors (percentage values in relation to nominal load) of 0 hp, 10 hp, 10 hp and 20 hp. For each load, the useful power load was measured, and consequently, the yield from the pump/motor arrangement.

In order to reach the load percentage values, the strategy employed was to adjust the opening of “a stopcock”, highlighted in Fig. 5, to control the water entering the pump.

The power of the pump was determined through the measuring of the pump shaft, manometric height (mca) water outflow and pump yield.

For the electric power absorbed by the motor, an energy analyzer was employed that allows for the measuring of the following electric magnitudes: voltages, currents, electric powers (active, reactive and apparent), power factor. The equipment allows for the recording of these magnitudes on a computer in order that future analyses can be performed.

6. Yield Analysis: Motor-Load

Initially, the studies were directed to evaluate the yields for the motor/pump. This study is founded on demonstrating that an asymmetric motor can operate under a low nominal power, while managing a performance similar to that of a motor (traditional) of the same power, by only altering the value of the capacitors, as we can see the capacitor bank in Fig. 6.

The decision was made that the power for testing the asymmetric motor would be of powers 10, 15 and 20 hp, while being reminded that our asymmetric motor is a motor of 20 hp. Through adjusting the stopcock, we managed a flow rate from the pump. With this flow rate and the values obtained from Fig. 7, we managed to determine the working power of the pump.

Table 2, respectively presents, the values obtained in the experimental measurements used from the motor/pump arrangement, through these, the check and proof was sought into the power that the pump is receiving from the motor to execute its designated task.
### Table 2  Calculation of the pump power for 10, 15 and 20 hp.

| Parameter                                      | 10 CV  | 15 CV  | 20 CV  |
|------------------------------------------------|--------|--------|--------|
| V2 Repression speed (data)                     | 9.3 m/s| 10.9 m/s| 12.2 m/s|
| V1 Suction speed (calculated)                  | 9.3 m/s| 10.9 m/s| 12.2 m/s|
| D1 Suction diameter (data)                     | 0.1 m  | 0.1 m  | 0.1 m  |
| D2 Repression diameter (data)                  | 0.1 m  | 0.1 m  | 0.1 m  |
| E + D Vertical suction length (Fig. 7)         | 0 m    | 0 m    | 0 m    |
| A Horizontal suction length (Fig. 7)           | 1 m    | 1 m    | 1 m    |
| F Vertical length pump to tank (Fig. 7)        | 2 m    | 2 m    | 2 m    |
| B + C Horizontal length pump to tank (Fig. 7)  | 1 m    | 1 m    | 1 m    |
| D VP depth (Fig. 7)                            | 0 m    | 0 m    | 0 m    |
| F + E Tank height to water level (Fig. 7)      | 2 m    | 2 m    | 2 m    |
| VR Equivalent length check valve               | 0 m    | 0 m    | 0 m    |
| VP Equivalent length foot valve                | 0 m    | 0 m    | 0 m    |
| VG Equivalent length globe valve               | 42.3 m | 42.3 m | 42.3 m |
| Suction curve Total equivalent length suction curves | 0 m | 0 m | 0 m |
| Repression curve Total equivalent length repression curves | 4.3 m | 4.3 m | 4.3 m |
| Output Equivalent output length                | 1 m    | 1 m    | 1 m    |
| f1 Suction friction factor (Moody Diagram)     | 0.019  | 0.019  | 0.019  |
| f2 Repression friction factor (Moody Diagram)  | 0.016  | 0.016  | 0.016  |
| L eq 1 Suction equivalent length (calculated)  | 1 m    | 1 m    | 1 m    |
| L eq 2 Repression equivalent length (calculated) | 4 m | 4 m | 4 m |
| Flow rate Outflow (calculated)                 | 0.073005 m³/s | 0.085565 m³/s | 0.09577 m³/s |
| Point m Mass flow (calculated)                 | 73.005 kg/s | 85.565 kg/s | 95.77 kg/s |
| Delta e1 Loss of energy in suction (calculated) | 8.21655 m³/s² | 11.28696 m³/s² | 14.1398 m³/s² |
| Delta e2 Loss of energy in repression (calculated) | 27.6768 m³/s² | 38.0192 m³/s² | 47.6288 m³/s² |
| Pump power Pump power in watts (calculated)    | 7,237.595 | 11,013.17 | 14,958.18 |
| Pump power Pump power in CV (calculated)       | 9.85   | 14.98  | 20.35  |
| A1 Suction pipe area (calculated)              | 0.00785 m² | 0.00785 m² | 0.00785 m² |
| A2 Repression pipe area (calculated)           | 0.00785 m² | 0.00785 m² | 0.00785 m² |

#### 7. Current Analysis

In this stage, evaluations are made into the behavior of the electric currents on each defined power of 10, 15 and 20 hp, all with the capacitor value defined for each power. The study of this magnitude has significant importance as it exercises an influence over the sizing of the suppliers, system protection and energy losses.

The unbalance noted on the currents is due to the unbalance of the three-phase supply voltages. In terms of the asymmetric motor, it is seen as normal that its currents are unbalanced.

Fig. 8 presents the currents on the asymmetric three-phase motor during the process with capacitor switching to 120 μF, the current is verified as being in agreement with the power for 10 hp.

Noted here is that in Fig. 9 for the power referring to 15 hp with switching for 210 μF, one notes a current in steady state of 28 A, while maintaining the efficiency of the arrangement as was demonstrated.

Noted from Fig. 10 is that for the power referring to 20 cv, which is nominal of the asymmetric motor with switching for 290 μF, one notes a current in steady state of 38 A and maintaining the efficiency of the apparatus as was demonstrated.

#### 8. Power Factor

The power for an electric installation is associated to the reactive power consumption necessary for
Fig. 8  Current under load of 10 hp and capacitor of 120 µF.

Fig. 9  Current under load of 15 hp and capacitor of 210 µF.

Fig. 10  Current under load of 20 hp and capacitor of 290 µF.
magnetization of the magnetic cores. The consequences of a low power factor are well known to the academic community as any effort to decrease the circulation of this power by suppliers and transformers, and equally, the increase of energy efficiency on the installations, will always be an objective to pursue.

Induction motors are the main consumers of reactive power and as such responsible for the low power factor on electric networks and industrial installations. In light of the aforementioned, it becomes necessary to evaluate the consumption of reactive power on the three-phase asymmetric motor, with the aim of checking for an improved efficiency in the conversion process of electric energy into mechanical.

Table 3 presents the values of the power factor for each load condition of the motors.

Note that for any loading situation, the asymmetric motor presents a lower power factor, which is normally found on other electric motor models of the same power.

9. Conclusions

Through tests, the dynamic characteristics were found for the asymmetric motor, these were then verified experimentally in a series of tests with different capacitor values. Those values that provided the best motor operation for a given load, while taking as a foundation the unbalance of currents and motor operation with and without an applied load, determined the best capacitor for the power supplied by the motor.

Theoretical verifications showed that the capacitors found experimentally can be seen through the symmetric components analysis, using as input data the experimental results from a single test on the motor, while operating in single-phase on any particular capacitor. As such, the capacitance values that minimized the unbalance on the currents were sought. This type of situation was expected theoretically.

The behavior of the motor was compared, which was experimentally obtained through theory, and attained through the parameters of the equivalent circuit, for operation with the capacitor found in the calculation of the motor when varying the capacitor according to the power supplied by the motor.

Finally, it is noteworthy that the aspects relevant to the merit of this study are associated to the fact that the comparisons performed are grounded on experimental results through measurements of electric and mechanical magnitudes. Through use of equipment denominated as the asymmetrical motor a new option is provided for activating elevated power loads in locations with only single-phase electric networks available, such as in rural installations.

References

[1] De Carvalho, G. B. 2004. “Dimensionamento e Simulação Hidráulica da Irrigação Localizada sob Condição Variável de Setores de Operação.” MSc thesis, USP. (in Portuguese)

[2] Martins Neto, L. 1990. “Motor de Indução Trifásico Assimétrico Com Alimentação Monofásica.” In Proceedings of I Seminário Internacional de Distribuição de Energia Elétrica, Belo Horizonte, MG, Outubro. (in Portuguese)

[3] Lynce Ribeiro Chaves, M. 1987. “Desenvolvimento e Construção de Sistemas Estáticos Para Alimentação de Cargas Trifásicas a Partir de Redes Monofásicas.” MSc thesis, UFU. (in Portuguese)

[4] El-Maghraby, M. H., Thejel, R. H., and Ibrahim, M. M. 1992. “New Approach for the Analysis of a Three-Phase Induction Motor of Different Ratings Connected to a Single-Phase Supply.” Electric Power Applications, IEE Proceedings B 139 (3): 145-54.

[5] Martins Neto, L. 1980. “Motor de Indução Assimétrico Funcionando Como Conversor de Número de Fases.” PhD thesis, USP. (in Portuguese)

[6] Neto, L. M., Camacho, J. R., and Mendonca, R. G. 1998. “Asymmetrical Three-Phase Induction Motors under Single-Phase Feeding: Oscillating Torque, Theoretical and Experimental Analysis—Harmonic Effects.” In Proceedings of International Conference on Electrical Machines, Istanbul, Turkey, September 1998.
Switching of Capacitors in an Asymmetric Induction Motor with Single-Phase Power, Triggering Several Applied Loads

[7] Mendonca, R. G., Teixeira, H. B., and Neto, L. M. 2000. “Asymmetrical Three-Phase Induction Motor Performance through the Use of Dynamic Capacitance Switching.” *International Conference on Electrical Machines* 1: 28-30.

[8] Wellington, P., de Mendonca Roberlam, G., and Neto Luciano, M. 2015. “Comparative Performance Analysis of a Standard Three-Phase Induction Motor and an Asymmetric Three-Phase Induction Motor Fed from a Single-Phase Network.” *Electric Power Systems Research* 70: 211-9.

[9] Enjeti, P. N., Sulistyono, W., and Choi, S. 1994. “The Starting of a Three-Phase Induction Motor Connected to a Single-Phase Supply System.” In *Proceedings of the Power Electronics Specialists Conference*, Vol. 2, pp. 1173-9.

[10] Brown, J. E., and Jha, C. S. 1959. “The Starting of a 3-Phase Induction Motor Connected to a Single-Phase Supply System.” *Proceedings of the IEE Part A: Power Engineering* 106 (26): 183-90.