On the Degrees of Freedom of Full-Duplex Cellular Networks

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Abstract

Full-duplex (FD) cellular networks are considered in which a FD base station (BS) simultaneously supports a set of half-duplex (HD) downlink (DL) users and a set of HD uplink (UL) users. The transmitter and the receiver of the BS are equipped with reconfigurable antennas, each of which can choose its transmit or receive mode from several preset modes. Under the no self-interference assumption arisen from FD operation at the BS, the sum degrees of freedom (DoF) of FD cellular networks is investigated for both no channel state information at the transmit side (CSIT) and partial CSIT. In particular, the sum DoF is completely characterized for the no CSIT model and an achievable sum DoF is established for the partial CSIT model, which improves the sum DoF of the conventional HD cellular networks. For both the no CSIT and partial CSIT models, the results show that the FD BS with reconfigurable antennas can double the sum DoF even in the presence of user-to-user interference as both the numbers of DL and UL users and preset modes increase. It is further demonstrated that such DoF improvement indeed yields the sum rate improvement at the finite and operational signal-to-noise ratio regime.

Index Terms

Blind interference alignment, degrees of freedom (DoF), full-duplex (FD), interference management, reconfigurable antennas.

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I. INTRODUCTION

To meet soaring wireless demand with limited spectrum, there has been considerable researches for boosting utilization of wireless resources. Recently, full-duplex (FD) radios have emerged as a potential way of improving spectral efficiency by enabling simultaneous transmission and reception at the same time with the same wireless spectrum. Because of such simultaneous transmission and reception, FD has a potential to double the spectral efficiency compared to the conventional half-duplex (HD) mode such as frequency division duplex (FDD) and time division duplex (TDD). Nonetheless, FD involves the practical issue of suppressing high-powered self-interference arisen from simultaneous transmission and reception [1]–[4]. In recent researches, there has been remarkable progress on analog and digital domain self-interference suppression techniques, showing that the point-to-point bidirectional FD system can achieve nearly twice higher throughput than the corresponding HD system, which demonstrates the possibility of implementing FD radios in practice [2]–[4].

Unlike the point-to-point bidirectional FD system, we cannot simply argue that the network throughput can be doubled for cellular systems even under the ideal assumption that self-interference is perfectly suppressed. In particular, consider the cellular system in Figure 1 in which a FD base station (BS) simultaneously supports a set of HD downlink (DL) users and a set of HD uplink (UL) users, one of the feasible scenarios of FD radios considering compatibility with legacy HD users in the current communication systems. For such case, a new source of interference from UL users to DL users appears, which does not exist in HD cellular systems where DL and UL traffic is orthogonalized by frequency or time domain. The impact of such user-to-user interference in FD cellular systems has been widely discussed in several researches [5]–[9]. They showed that if interference from UL users to DL users is not properly mitigated, the network throughput may be degraded even though self-interference is perfectly suppressed. Therefore, efficient interference management from UL users to DL users is a key challenge to boosting the network throughput of cellular systems by adapting FD operation at BSs [5]–[9].

In order to understand fundamental limits of FD radios in cellular networks, there have been several recent researches on characterizing the degrees of freedom (DoF) of FD cellular networks [10]–[13]. In particular, a single-cell FD cellular network has been studied in [12], [13], in which a FD BS with perfect self-interference suppression supports both HD DL and UL users.
as seen in Fig. 1. In [12], the authors characterized the sum DoF of the single-cell FD cellular network assuming that global channel state information (CSI) is available at the BS, i.e., full CSI at the transmit side (CSIT). They showed that FD operation at the BS can double the sum DoF compared to HD operation when both the numbers of DL and UL users become large even in the presence of user-to-user interference, concurrently reported in [13]. However, asymptotic interference alignment (IA) techniques proposed in [12], [13] require perfect CSIT and an arbitrarily large number of time extension to achieve the optimal sum DoF, which is quite challenging in practice due to feedback delay, system overhead and complexity, and etc [14]–[20].

To resolve such practical restrictions for interference management, the concept of blind IA has been recently proposed, which aligns multiple interfering signals into the same signal space at each receiver without any CSIT. In particular, various blind IA techniques have been proposed for both heterogeneous block fading models where certain users experience smaller coherence time/bandwidth than others [21] and homogeneous block fading models where all users experience independent block fading with the same coherence time, but different offsets [22]–[24]. In [25], Wang, Gou, and Jafar have first observed that reconfigurable antennas can artificially create channel correlation across time in a certain structure letting blind IA be possible for multiple-input and multiple-output (MIMO) broadcast channels [25], [26]. Reconfigurable
antennas are capable of dynamically adjusting its radiation patterns in a controlled and reversible manner through various technologies such as solid state switches or microelectromechanical switches (MEMS) without additional RF-chains, which take a dominant factor for hardware complexity [27], [28]. That is, reconfigurable antennas can choose its transmit or receive mode among several preset modes at each time instant, see also [26, Section I] for the concept of reconfigurable antennas. Subsequently, blind IA using reconfigurable antennas has been extended to general MIMO broadcast channels characterizing linear sum DoF, i.e., the maximum sum DoF achievable by linear coding schemes [29] and also applied to a class of single-input and single-output (SISO) and multiple-input and single-output (MISO) interference channels consisting of receivers equipped with reconfigurable antennas [30], [31]. From the recent results in [25], [26], [29]–[31] together with the advantage of reconfigurable antennas on hardware complexity [27], [28], blind IA using reconfigurable antennas has been considered as a promising solution for boosting the DoF of practical wireless systems with no CSIT.

Motivated by such advantages of FD radios and reconfigurable antennas, we consider FD cellular networks in which a FD BS equipped with reconfigurable transmit and receive antennas supports HD DL and UL users simultaneously in the same frequency spectrum. For comprehensive understanding on the impact of FD radios and CSI conditions in the context of IA or blind IA using reconfigurable antennas, we consider two different CSI models: For the no CSIT case, both the BS and each UL user do not know their CSIT; For the partial CSIT case, the BS only knows its CSIT. For both models, we assume that CSI at the receive side (CSIR) is available. Similar to the previous full CSIT models in [10]–[13], the primary aim is to characterize whether the sum DoF can be doubled or not with partial or no CSIT by FD operation at the BS equipped with reconfigurable antennas. The main contributions of this paper are as follows:

- For the no CSIT model, we completely characterize the sum DoF of FD cellular networks. We propose a novel blind IA technique, which perfectly aligns user-to-user interference at each DL user while preserving intended signal space at the BS, and establish the converse showing the optimality of the proposed scheme in terms of the sum DoF. The result shows that the sum DoF is asymptotically doubled if both the numbers of UL users and preset modes at the receiver of the BS increase, which is the first result demonstrating the benefit of FD radios on cellular networks under no CSIT.
- For the partial CSIT model, we establish an achievable lower bound on the sum DoF
of FD cellular networks, which characterizes the sum DoF for a broad class of network topologies. We propose a novel blind IA technique combined with zero-forcing beamforming based on partial CSIT, which partially aligns user-to-user interference at each DL user while preserving intended signal space at the BS. The result shows that the sum DoF is doubled if there exist two DL and two UL users and two preset modes at the transmitter and the receiver of the BS. For the single-antenna case, our result for the partial CSIT model extends the previous achievability result in [13] to a general antenna configuration assuming different numbers of preset modes at the transmitter and receiver of the BS.

- We further demonstrate that such DoF improvement indeed yields the sum rate improvement at the finite and operational signal-to-noise ratio (SNR) regime, which presents the benefit of blind IA using reconfigurable antennas compared with the previous works [10]–[13].

The rest of this paper is organized as follows. In Section II, we introduce the network model and DoF metric considered throughout the paper. In Section III, we state the main results of this paper, the sum DoF of FD cellular networks, and remark several observations possibly deduced from the main results. We present achievability and converse proofs of the main results in Section IV and Section V respectively. We finally conclude in Section VI.

II. PROBLEM FORMULATION

In this section, we introduce FD cellular networks consisting of a FD BS and HD DL and HD UL users and then formally define the sum DoF metric, which will be analyzed throughout the paper.

A. Notation

For integer numbers $a$ and $b$, $a \backslash b$ and $a \| b$ denote the quotient and the remainder respectively when dividing $a$ by $b$. For integer numbers $a$ and $b$, $[a : b] = \{a, a + 1, \cdots , b\}$ when $a \leq b$ and $[a : b] = \emptyset$ when $a > b$. For matrices $A$ and $B$, $A \otimes B$ is the Kronecker product of $A$ and $B$. For a matrix $A$, denote the Frobenius norm, transpose, and conjugate transpose of $A$ by $\| A \|$, $A^T$, and $A^H$, respectively. For a set of matrices $\{A_i\}_{i \in [1:n]}$, $\text{diag}(A_1, \cdots , A_n)$ denotes the block-diagonal matrix consisting of $A_i$ as the $i$th diagonal block. For natural numbers $a$ and $b$, $I_a$, $1_{a \times b}$, and $0_{a \times b}$ denote the $a \times a$ identity matrix, the $a \times b$ all-one matrix, and the $a \times b$ all-zero matrix respectively. Let $e_a(b)$ be the $b$th column vector of $I_a$ where $b \in [1 : a]$. 
B. Full-Duplex Cellular Networks

We consider a FD cellular network depicted in Fig. 2 in which a FD BS simultaneously supports $K_d$ HD DL users and $K_u$ HD UL users. Both the transmitter and receiver of the BS are equipped with reconfigurable antennas. In particular, the transmitter of the BS is equipped with a reconfigurable antenna able to switch among $M_d$ preset modes at each time and the receiver of the BS is equipped with a reconfigurable antenna able to switch among $M_u$ preset modes at each time. Notice that $M_d = 1$ (or $M_u = 1$) corresponds to the case where the transmitter (or the receiver) of the BS is equipped with a conventional antenna. Each DL and UL user is equipped with a conventional antenna.

We assume block fading in this paper, i.e., each channel coefficient remains the same in a consecutive time slots of coherence time and is drawn independently in the next consecutive time slots of coherence time. The length of the coherence time is assumed to be sufficiently large. We further assume that self-interference within the BS due to FD operation is assumed to be perfectly suppressed. Let $h_i(k) \in \mathbb{C}$ be the channel from the transmitter of the BS to the $i$th DL user when the BS selects its transmit mode as the $k$th preset mode, where $i \in [1 : K_d]$ and $k \in [1 : M_d]$. Similarly, let $f_j(l) \in \mathbb{C}$ be the channel from the $j$th UL user to the receiver...
of the BS when the BS selects its receive mode as the \(l\)th preset mode, where \(j \in [1 : K_u]\) and \(l \in [1 : M_u]\). Let \(g_{ij} \in \mathbb{C}\) be the channel from the \(j\)th UL user to the \(i\)th DL user. All channel coefficients are assumed to be independent and identically distributed (i.i.d.) drawn from a continuous distribution.

Denote the transmit mode and the receive mode of the BS at time \(t\) by \(\alpha(t) \in [1 : M_d]\) and \(\beta(t) \in [1 : M_u]\), respectively. Then the received signal of the \(i\)th DL user at time \(t\) is given by

\[
y_{di}(t) = h_i(\alpha(t)) x_d(t) + \sum_{j=1}^{K_u} g_{ij} x_{uj}(t) + z_{di}(t)
\]

for \(i \in [1 : K_d]\) and the received signal of the BS at time \(t\) is given by

\[
y_u(t) = \sum_{j=1}^{K_u} f_j(\beta(t)) x_{uj}(t) + z_u(t)
\]

where \(x_d(t)\) is the transmit signal of the BS at time \(t\), \(x_{uj}(t)\) is the transmit signal of the \(j\)th UL user at time \(t\), \(z_{di}(t)\) is the additive noise of the \(i\)th DL user at time \(t\), and \(z_u(t)\) is the additive noise of the BS at time \(t\). The additive noises are assumed to be i.i.d. drawn from \(\mathcal{CN}(0,1)\) and independent over time. The BS and each UL user should satisfy the average power constraint \(P\), i.e., \(\mathbb{E} ||x_d(t)||^2 \leq P\) and \(\mathbb{E} ||x_{uj}(t)||^2 \leq P\) for all \(j \in [1 : K_u]\).

For notational convenience, from (1) and (2), we define the length-\(n\) time-extended input–output relation as

\[
y_{di} = H_i(\bar{\alpha}) x_d + \sum_{j=1}^{K_u} G_{ij} x_{uj} + z_{di},
\]

\[
y_u = \sum_{j=1}^{K_u} F_j(\bar{\beta}) x_{uj} + z_u
\]

where

\[
\bar{\alpha} = [\alpha(1), \cdots, \alpha(n)]^T, \quad \bar{\beta} = [\beta(1), \cdots, \beta(n)]^T,
\]

\[
H_i(\bar{\alpha}) = \text{diag} (h_i(\alpha(1)), \cdots, h_i(\alpha(n))),
\]

\[
F_j(\bar{\beta}) = \text{diag} (f_j(\beta(1)), \cdots, f_j(\beta(n))), \quad G_{ij} = g_{ij} I_n,
\]

\[
y_{di} = [y_{di}(1), \cdots, y_{di}(n)]^T, \quad y_u = [y_u(1), \cdots, y_u(n)]^T,
\]

\[
x_d = [x_d(1), \cdots, x_d(n)]^T, \quad x_{ui} = [x_{ui}(1), \cdots, x_{ui}(n)]^T,
\]

\[
z_{di} = [z_{di}(1), \cdots, z_{di}(n)]^T, \quad z_u = [z_u(1), \cdots, z_u(n)]^T.
\]
For comprehensive understanding on the DoF improvement achievable by reconfigurable antennas at the FD BS, we consider the following two different scenarios for CSI assumption:

- **No CSIT model** (CSIT is not available):
  The BS knows its receive side CSI, \( \{ f_j(k) \} \) \( j \in [1:K_u], k \in [1:M_d] \); The \( i \)th DL user knows its receive side CSI, \( \{ h_i(k) \} \) \( k \in [1:M_d] \) and \( \{ g_{ij} \} \) \( j \in [1:K_u] \); The \( j \)th UL user does not know any CSI.

- **Partial CSIT model** (CSIT is only available at the BS):
  The BS knows both its transmit and receive side CSI, i.e., \( \{ h_i(k) \} \) \( i \in [1:K_d], k \in [1:M_d] \) and \( \{ f_j(k) \} \) \( j \in [1:K_u], k \in [1:M_d] \); The \( i \)th DL user knows its receive side CSI, \( \{ h_i(k) \} \) \( k \in [1:M_d] \) and \( \{ g_{ij} \} \) \( j \in [1:K_u] \); The \( j \)th UL user does not know any CSI.

For the no CSIT model, both the BS and each UL user do not know their transmit side CSI, i.e., no CSIT. Hence, the first model corresponds to the no CSIT assumption that have been considered for the previous works studying blind IA using reconfigurable antennas [26], [29], [30]. For the partial CSIT model, on the other hand, we allow for the BS to know its transmit side CSI, i.e., \( \{ h_i(k) \} \) \( i \in [1:K_d], k \in [1:M_d] \). As will be seen in Section III acquiring CSIT for DL transmission might be more useful than acquiring CSIT for UL transmission because joint processing at the transmit side (i.e., the BS) is required for downlink transmission. For this reason, we also consider the second model allowing CSIT for DL transmission at the BS.

**C. Degrees of Freedom**

For the network model stated in Section II-B, we define a set of length-\( n \) block codes and its achievable DoF. Let \( W_{di} \in [1 : 2^{nR_d}] \) and \( W_{uj} \in [1 : 2^{nR_u}] \) be the \( i \)th DL message and the \( j \)th UL message respectively, where \( i \in [1 : K_d] \) and \( j \in [1 : K_u] \). For the no CSIT model, a \( (2^{nR_d}, \ldots, 2^{nR_d}, 2^{nR_u}, \ldots, 2^{nR_u}; n) \) code consists of the following set of encoding and decoding functions:

- **Encoding:**
  For \( t \in [1 : n] \), the encoding function of the BS at time \( t \) is given by
  \[
  (x_d(t), \alpha(t)) = \phi_t (W_{d1}, \ldots, W_{dK_d}, y_u(1), \ldots, y_u(t - 1), \{ f_j(k) \} \mid j \in [1:K_u], k \in [1:M_d]).
  \]
  For \( t \in [1 : n] \), the encoding function of the \( j \)th UL user \( (j \in [1 : K_u]) \) at time \( t \) is given by
  \[ x_{uj}(t) = \varphi_{jt} (W_{uj}). \]
Finally, the sum DoF for the no CSIT model is defined as
\[ \text{tuple } W \text{ is given by } \]
\[ \sum_{i=1}^{K_d} d_i, d_k, d_u, \ldots, d_{uK_u} \rightarrow 0 \text{ as } n \text{ increases for all } i \in [1 : K_d] \text{ and } j \in [1 : K_u], \text{ a rate tuple } (R_{d1}, \ldots, R_{dK_d}, R_{u1}, \ldots, R_{uK_u}) \text{ is said to be achievable. Then the achievable DoF tuple is given by } \]
\[ (d_{d1}, \ldots, d_{dK_d}, d_{u1}, \ldots, d_{uK_u}) = \lim_{P \to \infty} \left( \frac{R_{d1}}{\log P}, \ldots, \frac{R_{dK_d}}{\log P}, \frac{R_{u1}}{\log P}, \ldots, \frac{R_{uK_u}}{\log P} \right). \]
Finally, the sum DoF for the no CSIT model is defined as
\[ d_{\Sigma, \text{noCSIT}} = \max_{(d_{d1}, \ldots, d_{dK_d}, d_{u1}, \ldots, d_{uK_u}) \in \mathcal{D}} \left\{ \sum_{i=1}^{K_d} d_i + \sum_{j=1}^{K_u} d_j \right\} \]
where \( \mathcal{D} \) denotes the achievable DoF region.

For the partial CSIT model, the encoding and decoding functions of the BS are replaced as
\[
(x_d(t), \alpha(t)) = \phi_t(W_{d1}, \ldots, W_{dK_d}, y_u(1), \ldots, y_u(t-1), \{h_i(k)\}_{i \in [1 : K_d], k \in [1 : M_d]}, \{f_j(k)\}_{j \in [1 : K_u], k \in [1 : M_u]}),
\]
\[ \hat{W}_{uj} = \chi_j(y_u, W_{d1}, \ldots, W_{dK_d}, \{h_i(k)\}_{i \in [1 : K_d], k \in [1 : M_d]}, \{f_j(k)\}_{j \in [1 : K_u], k \in [1 : M_u]}), \]
respectively. Then the sum DoF can be defined in the same manner. Let \( d_{\Sigma, pCSIT} \) denote the sum DoF for the partial CSIT model.

For the rest of this paper, we characterize the sum DoF of the FD cellular network under both the no CSIT model and the partial CSIT model.
III. MAIN RESULTS

In this section, we state our main results, the sum DoF of the FD cellular network for both no CSIT and partial CSIT models, and provide a numerical example for demonstrating the benefit of FD operation and reconfigurable antennas at the BS.

For the no CSIT model, we completely characterize the sum DoF of the FD cellular network in the following theorem.

**Theorem 1:** For the FD cellular network with no CSIT,

\[
d_{\Sigma, \text{noCSIT}} = 2 - \frac{1}{\min(K_u, M_u)}.
\]  

**Proof:** We refer to Section IV-A for the achievability proof and Section V for the converse proof.

**Remark 1:** From Theorem 1, \(d_{\Sigma, \text{noCSIT}}\) is independent of the parameters \(K_d\) and \(M_d\). That is, for the no CSIT case, equipping a reconfigurable antenna at the transmitter of the BS cannot increase the sum DoF and similarly a single DL user is enough to achieve the optimal sum DoF. More importantly, \(d_{\Sigma, \text{noCSIT}}\) is asymptotically doubled if both \(K_u\) and \(M_u\) increase. Therefore, for the no CSIT case, arbitrarily large numbers of UL users and preset modes at the receiver of the BS are required to double the sum DoF by FD operation at the BS.

For the partial CSIT model, we establish an upper and achievable lower bounds on the sum DoF of the FD cellular network in the following theorem.

**Theorem 2:** For the FD cellular network with partial CSIT,

\[
d_{\Sigma, pCSIT} \leq \min \left\{ 2, \max \left( 1 + \frac{K_u(K_d - 1)}{K_d}, 1 + \frac{K_d(K_u - 1)}{K_u} \right) \right\}
\]  

and

\[
d_{\Sigma, pCSIT} \geq \min \left\{ 2, \max \left( 1 + \frac{\min(K_u, M_u)(\min(K_d, M_d) - 1)}{\min(K_d, M_d)}, 1 + \frac{\min(K_d, M_d)(\min(K_u, M_u) - 1)}{\min(K_u, M_u)} \right) \right\}.
\]

**Proof:** We refer to the converse in [12, Theorem 1] for the proof of the upper bound in (6). In particular, [12] considers the FD BS equipped with conventional multiple transmit and receive antennas (instead of reconfigurable antennas each of which can choose its transmit or receive mode from several preset modes) and assumes that full CSI is available at the BS and each user. The upper bound in (6) is attained from [12, Theorem 1] by assuming a single transmit and
receive antenna at the BS. We can easily see that the converse argument in [12, Theorem 1] is applicable to the reconfigurable antenna model in Fig. 2 for the full CSIT case. Hence (6) can be an upper bound on $d_{\Sigma,\text{pCSIT}}$. We refer to Section IV-B for the proof of the achievable lower bound in (7).

**Corollary 1:** For the FD cellular network with partial CSIT,

$$
\begin{align*}
    d_{\Sigma,\text{pCSIT}} &= \begin{cases} 
        2 & \text{if } K_d, K_u, M_d, M_u \geq 2, \\
        1 + \frac{K_u - 1}{K_u} & \text{if } K_u = 1, M_u \geq K_u, \\
        1 + \frac{K_d - 1}{K_d} & \text{if } K_u = 1, M_d \geq K_d.
    \end{cases}
\end{align*}
$$

**Proof:** By comparing the upper and lower bounds on $d_{\Sigma,\text{pCSIT}}$ in Theorem 2, (8) can be straightforwardly obtained.

For the single-antenna case, Theorem 1 and Corollary 1 extend the previous achievability result for the partial CSIT model in [13] to a general antenna configuration assuming different numbers of preset modes at the transmitter and receiver of the BS.

**Remark 2:** From Theorem 2 and Corollary 1 $d_{\Sigma,\text{pCSIT}}$ is asymptotically doubled if both $K_u$ and $M_u$ increase when $\min(K_d, M_d) = 1$ or both $K_d$ and $M_d$ increase when $\min(K_u, M_u) = 1$. Hence, similar to the no CSIT case, arbitrarily large numbers of users and preset modes are
required to double the sum DoF by FD operation only at the DL or UL side. On the other hand, \( d_{\Sigma,p\text{CSIT}} \) is doubled if \( M_d, M_u, K_d, K_u \geq 2 \). That is, only two DL and UL users and the FD BS equipped with reconfigurable antennas having two preset modes are enough to double the sum DoF if the BS can attain its downlink CSI. Lastly, unlike the no CSIT case in which reconfigurable antennas are only beneficial at the receiver of the BS, reconfigurable antennas are equally beneficial at the transmitter and receiver of the BS for the partial CSIT case.

In summary, from Theorems 1 and 2, the sum DoF is doubled even in the presence of user-to-user interference by FD operation at the BS. Furthermore, reconfigurable antennas can effectively improve the sum DoF under both partial and no CSIT cases. The following example plots the sum DoFs in Theorems 1 and 2 for the symmetric case.

Example 1: For comparison, consider the symmetric case where \( K_d = K_u := K \) and \( M_d = M_u := M \). Then, from Theorems 1 and 2,

\[
d_{\Sigma,\text{noCSIT}} = 2 - \frac{1}{\min(K, M)}
\]

and

\[
\min(K, M, 2) \leq d_{\Sigma, p\text{CSIT}} \leq \min(K, 2).
\]

Fig. 3 plots (9) and (10) with respect to \( K \). Obviously, if the BS operates as the conventional HD operation, i.e., serving either DL users or UL users, the sum DoF is limited by one. From (9) and the lower bound in (10), the sum DoF is still one if the FD BS is equipped with conventional non-reconfigurable antennas, i.e., \( M = 1 \). For the partial CSIT case, \( K = M = 2 \) is enough to double the sum DoF. On the other hand, arbitrarily large \( K \) and \( M \) are required to double the sum DoF in the case of no CSIT.

In Section IV-C, we further demonstrate that the above sum DoF improvement achievable by FD operation and reconfigurable antennas at the BS yields the sum rate at the finite and operational SNR regime, which presents the benefit of blind IA using reconfigurable antennas compared with the previous works [10]–[13].

IV. ACHIEVABILITY

In this section, we establish the achievability in Theorems 1 and 2 and then present the achievable sum rates of the proposed schemes at the finite SNR regime. For notational convenience,
let

\[ L_d = \min(K_d, M_d) \quad \text{and} \quad L_u = \min(K_u, M_u) \]

for the rest of this section. In the following, the \( n \)-point inverse discrete Fourier transform (IDFT) matrix \( \Omega_n \in \mathbb{C}^{n \times n} \) will be used for transmit precoding matrices [32], which is given by

\[
\Omega_n = \frac{1}{\sqrt{n}} \begin{bmatrix}
1 & 1 & 1 & \cdots & 1 \\
1 & \omega & \omega^2 & \cdots & \omega^{n-1} \\
1 & \omega^2 & \omega^4 & \cdots & \omega^{2(n-1)} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & \omega^{n-1} & \omega^{2(n-1)} & \cdots & \omega^{(n-1)(n-1)}
\end{bmatrix}
\]

where \( \omega = e^{j2\pi/n} \) [32]. Note that \( \Omega_n \) is an orthonormal matrix, i.e.,

\[ \Omega_n^H \Omega_n = I_n. \tag{11} \]

A. Achievability for Theorem 1

In this subsection, we establish the achievability of Theorem 1 showing that the sum DoF of \( 2 - \frac{1}{L_u} \) is achievable for the no CSIT model. In particular, the BS sends \( L_u - 1 \) information symbols to only the first DL user and \( K_u \) UL users send a single information symbol each to the BS during \( L_u \) time slots.

Let \( s_{d1} \in \mathbb{C}^{(L_u-1) \times 1} \) be the information symbol vector for the first DL user satisfying that \( \mathbb{E}[\|s_{d1}\|^2] = L_u - 1 \) and \( s_{uj} \in \mathbb{C} \) be the information symbol for the \( j \)th UL user, \( j \in [1 : K_u] \), satisfying that \( \mathbb{E}[|s_{uj}|^2] = 1 \). These information symbols will be delivered by \( L_u \) symbol extension, i.e., beamforming over \( L_u \) time slots. In particular, let \( W_1 \in \mathbb{C}^{L_u \times (L_u-1)} \) be the submatrix consisting of the first through \( (L_u - 1) \)th column vectors of the \( L_u \)-point IDFT matrix and \( w_2 \in \mathbb{C}^{L_u \times 1} \) be the \( L_u \)th column of the \( L_u \)-point IDFT matrix. That is, \( [W_1^T, w_2^T]^T = \Omega_{L_u} \).

The BS and the \( j \)th UL user set their length-\( L_u \) time-extended transmit signal vectors as

\[
\begin{align*}
x_d &= \sqrt{\frac{P L_u}{L_u - 1}} W_1 s_{d1}, \\
x_{uj} &= \sqrt{P L_u} w_2 s_{uj} \quad \text{for} \; j \in [1 : K_u],
\end{align*}
\]

each of which satisfies the average power constraint \( P \), i.e., \( \mathbb{E}(\|x_d\|^2) = L_u P \) and \( \mathbb{E}(\|x_{uj}\|^2) = L_u P \) for \( j \in [1 : K_u] \). Here, \( W_1 \) is used as the transmit precoding matrix for sending \( s_{d1} \) and \( w_2 \).
is used as the transmit precoding vector for sending $s_{uj}$, which is the same for all $j \in [1 : K_u]$. During signal transmission, the BS fixes its transmit mode, i.e., $\alpha(t) = 1$ for all $t \in [1 : L_u]$. During signal reception, on the other hand, the BS sets its receive mode differently at each time, i.e., $\beta(t) = t$ for all $t \in [1 : L_u]$. Denote the above transmit mode vector and receive mode vector by $\bar{\alpha}_1$ and $\bar{\beta}_1$, respectively.

Then, from (3) and (12), the length-$L_u$ time-extended input–output relation is given by

$$y_{d1} = \sqrt{\frac{P L_u}{L_u - 1}} H_1(\bar{\alpha}_1) W_1 s_{d1} + \sqrt{P L_u} \sum_{j=1}^{K_u} G_{1j} w_2 s_{uj} + z_{d1},$$  \hfill (13)

$$y_u = \sqrt{P L_u} \sum_{j=1}^{K_u} F_j(\bar{\beta}_1) w_2 s_{uj} + z_u.$$ \hfill (14)

For decoding its DL message, the first DL user multiplies $W_1^H$ to its length-$L_u$ time-extended received signal vector $y_{d1}$, given by

$$W_1^H y_{d1} = W_1^H \left( \sqrt{\frac{P L_u}{L_u - 1}} H_1(\bar{\alpha}_1) W_1 s_{d1} + \sqrt{P L_u} \sum_{j=1}^{K_u} G_{1j} w_2 s_{uj} + z_{d1} \right)$$

$$= (a) \sqrt{\frac{P L_u}{L_u - 1}} H_1(\bar{\alpha}_1) W_1^H W_1 s_{d1} + \sqrt{P L_u} \sum_{j=1}^{K_u} G_{1j} W_1^H w_2 s_{uj} + W_1^H z_{d1}$$

$$= (b) \sqrt{\frac{P L_u}{L_u - 1}} h_1(1) s_{d1} + W_1^H z_{d1}$$ \hfill (15)

where $(a)$ follows from the facts that $H_1(\bar{\alpha}_1) = I_{L_u} \otimes h_1(1)$ and $G_{1j} = g_{1j} I_{L_u}$ and $(b)$ follows from (11). Then, the first DL user estimates its information symbols based on (15). Since

$$\mathbb{E} \left( W_1^H z_{d1} \left( W_1^H z_{d1} \right)^H \right) = W_1^H W_1 = I_{L_u - 1},$$

from (15), the achievable rate of the first DL user is given by

$$R_{d1} = \frac{L_u - 1}{L_u} \log P + o(\log P).$$ \hfill (16)

Hence, the achievable DoF of the first DL user is given by

$$d_{d1} = \lim_{P \to \infty} \frac{R_{d1}}{\log P} = 1 - \frac{1}{L_u}.$$ 

Now consider decoding of $L_u$ UL messages at the BS. We first introduce the following lemma, which will be used for decoding at the BS.
Lemma 1: Denote

\[ Q(\bar{\beta}_1) = \left[ F_1(\bar{\beta}_1)w_2, F_2(\bar{\beta}_1)w_2, \ldots, F_{K_u}(\bar{\beta}_1)w_2 \right] \in \mathbb{C}^{L_u \times K_u}. \]  

(17)

Then \( \text{rank}(Q(\bar{\beta}_1)) = L_u \) almost surely.

Proof: Recall that \( \bar{\beta}_1 = [1, \cdots, L_u]^T \) so that the channel coefficients in \( \{F_j(\bar{\beta}_1)w_2\}_{j \in [1:L_u]} \) are i.i.d. drawn from a continuous distribution. Therefore, \( \{F_j(\bar{\beta}_1)w_2\}_{j \in [1:L_u]} \) consists of \( L_u \) vectors obtained by randomly projecting \( w_2 \in \mathbb{C}^{L_u \times 1} \) into \( \mathbb{C}^{L_u \times 1} \) signal space. As a consequence, \( \{F_j(\bar{\beta}_1)w_2\}_{j \in [1:L_u]} \) are linearly independent almost surely, guaranteeing that \( \text{rank}(Q(\bar{\beta}_1)) \geq L_u \) almost surely. From the dimension of \( Q(\bar{\beta}_1) \), \( \text{rank}(Q(\bar{\beta}_1)) \leq L_u \), which completes the proof of Lemma 1.

From (14) and (17), \( y_u \) can be represented by

\[ y_u = \sqrt{P L_u} Q(\bar{\beta}_1)s_u + z_u \]  

(18)

where \( s_u = [s_{u1}, \cdots, s_{uK_u}]^T \). Notice that the BS can estimate its UL information symbols without interference from DL transmission because self-interference is assumed to be perfectly suppressed. From (18), the achievable sum rate of the \( K_u \) UL users is given by

\[ \sum_{i=j}^{K_u} R_{u_j} = \frac{\text{rank}(Q(\bar{\beta}_1))}{L_u} \log P + o(\log P). \]  

(19)

Since \( \text{rank}(Q(\bar{\beta}_1)) = L_u \) almost surely from Lemma 1, the achievable sum DoF of the \( K_u \) UL users is given by

\[ \sum_{j=1}^{K_u} d_{u_j} = \lim_{P \to \infty} \frac{\sum_{j=1}^{K_u} R_{u_j}}{\log P} = 1. \]

Consequently, the sum DoF of \( 2 - \frac{1}{L_u} \) is achievable for the no CSIT model, which completes the achievability proof of Theorem 1.

B. Achievability for Theorem 2

In this subsection, we establish the achievability of Theorem 2, i.e.,

\[ d_{\Sigma, p_{\text{CSIT}}} \geq \min \left\{ 2, \max \left( 1 + \frac{L_u(L_d - 1)}{L_d}, 1 + \frac{L_d(L_u - 1)}{L_u} \right) \right\}. \]

We first introduce the following lemma.
Lemma 2: The following equality holds:

\[
\max_{n_d \in [1 : L_u], n_u \in [1 : L_d]} \left\{ \frac{n_d}{L_u} + \frac{n_u}{L_d} \right\} = \min \left\{ 2, \max \left( 1 + \frac{L_u(L_d - 1)}{L_d}, 1 + \frac{L_d(L_u - 1)}{L_u} \right) \right\}.
\]

(20)

Theorem 2: Hence, for the rest of this subsection, we will show that the sum DoF of the original optimization problem in (20). For the relaxed linear program, either \((n_d, n_u) = (L_u, \min(L_d(L_d - 1), L_d))\) or \((n_d, n_u) = (\min(L_d(L_u - 1), L_u), L_d)\) maximizes the object function \(\frac{n_d}{L_u} + \frac{n_u}{L_d}\), each of which achieves \(\min\left( 2, 1 + \frac{L_u(L_d - 1)}{L_d} \right)\) and \(\min\left( 2, 1 + \frac{L_d(L_u - 1)}{L_u} \right)\), respectively. Note that they are also achievable by the original optimization problem in (20). As a result,

\[
\max_{n_d \in [1 : L_u], n_u \in [1 : L_d]} \left\{ \frac{n_d}{L_u} + \frac{n_u}{L_d} \right\} = \max \left\{ \min \left( 2, 1 + \frac{L_d(L_u - 1)}{L_u} \right), \min \left( 2, 1 + \frac{L_u(L_d - 1)}{L_d} \right) \right\}
\]

\[
= \min \left\{ 2, \max \left( 1 + \frac{L_d(L_u - 1)}{L_u}, 1 + \frac{L_u(L_d - 1)}{L_d} \right) \right\},
\]

which completes the proof of Lemma 2.

Proof: Obviously, the solution of the following linear program is an upper bound on that of the original optimization problem in (20).

\[
\max_{1 \leq n_d \leq L_u, 1 \leq n_u \leq L_d, n_d + n_u \leq L_d L_u} \left\{ \frac{n_d}{L_u} + \frac{n_u}{L_d} \right\},
\]

which relaxes the range of control variables from integers to real numbers. For the relaxed linear program, either \((n_d, n_u) = (L_u, \min(L_d(L_d - 1), L_d))\) or \((n_d, n_u) = (\min(L_d(L_u - 1), L_u), L_d)\) maximizes the object function \(\frac{n_d}{L_u} + \frac{n_u}{L_d}\), each of which achieves \(\min\left( 2, 1 + \frac{L_u(L_d - 1)}{L_d} \right)\) and \(\min\left( 2, 1 + \frac{L_d(L_u - 1)}{L_u} \right)\), respectively. Note that they are also achievable by the original optimization problem in (20). As a result,

\[
\max_{n_d \in [1 : L_u], n_u \in [1 : L_d]} \left\{ \frac{n_d}{L_u} + \frac{n_u}{L_d} \right\} = \max \left\{ \min \left( 2, 1 + \frac{L_d(L_u - 1)}{L_u} \right), \min \left( 2, 1 + \frac{L_u(L_d - 1)}{L_d} \right) \right\}
\]

\[
= \min \left\{ 2, \max \left( 1 + \frac{L_d(L_u - 1)}{L_u}, 1 + \frac{L_u(L_d - 1)}{L_d} \right) \right\},
\]

which completes the proof of Lemma 2.

From Lemma 2, it is enough to show that the sum DoF of \(\frac{n_d}{L_u} + \frac{n_u}{L_d}\) is achievable for any \((n_d, n_u)\) satisfying that \(n_d \in [1 : L_u]\), \(n_u \in [1 : L_d]\), and \(n_d + n_u \in [2 : L_d L_u]\) for the achievability of Theorem 2. Hence, for the rest of this subsection, we will show that the sum DoF of \(\frac{n_d}{L_u} + \frac{n_u}{L_d}\) is achievable assuming the above three constraints.

Let \(s_{di} \in \mathbb{C}^{n_d \times 1}\) be the information vector for the \(i\)th DL user, \(i \in [1 : L_d]\), satisfying that \(\mathbb{E}[\|s_{di}\|^2] = n_d\). Let \(s_{uj} \in \mathbb{C}^{n_u \times 1}\) be the information vector for the \(j\)th UL user, where \(j \in [1 : K_u]\), satisfying that \(\mathbb{E}[\|s_{uj}\|^2] = n_u\). These information symbols will be delivered by \(L_d L_u\) symbol extension, i.e., beamforming over \(L_d L_u\) time slots. Let \(U_i \in \mathbb{C}^{L_d L_u \times n_d}\) be the transmit precoding matrix for sending \(s_{di}\), where \(i \in [1 : L_d]\), satisfying that \(\sum_{i=1}^{L_d} \|U_i\|^2 = 1\) and \(V \in \mathbb{C}^{L_d L_u \times n_u}\) be the transmit precoding matrix for sending \(s_{uj}\), which is same for all \(j \in [1 : K_u]\), satisfying that \(\|V\|^2 = 1\). We will discuss designing of transmit precoding matrices of the BS and the UL users later. The BS and the \(j\)th UL user set their length-\((L_d L_u)\) time extended transmit signal
vector as
\[ x_d = \sqrt{PL_d L_u} \sum_{j=1}^{L_d} U_j s_{d_j}, \]
\[ x_{uj} = \sqrt{PL_d L_u} V s_{uj} \text{ for } j \in [1 : K_u], \]
each of which satisfies the average power constraint \( P \), i.e., \( \mathbb{E}(\|x_d\|^2) = L_d L_u P \) and \( \mathbb{E}(\|x_{uj}\|^2) = L_d L_u P \) for \( j \in [1 : K_u] \).

During signal transmission and reception, the BS sets its transmit and receive mode differently at each time with cycle of \( L_d \) and \( L_u \) respectively, i.e., \( \alpha(t) = (t-1)\!L_d + 1 \) and \( \beta(t) = (t-1)\!L_u + 1 \) for \( t \in [1 : L_d L_u] \). Denote the above transmit mode vector and receive mode vector by \( \bar{\alpha}_2 \) and \( \bar{\beta}_2 \), respectively.

Then, from (3) and (21), the length-\( (L_d L_u) \) time-extended input–output relation is given by
\[
y_{dj} = \sqrt{P L_d L_u} H_j(\bar{\alpha}_2) \sum_{j=1}^{L_d} U_j s_{d_j} + \sqrt{P L_d L_u} \sum_{j=1}^{K_u} G_{ij} V s_{uj} + z_{dj}
\]
\[
= \sqrt{P L_d L_u} H_j(\bar{\alpha}_2)[U_1, \cdots, U_{L_d}]s_d + \sqrt{P L_d L_u} \sum_{j=1}^{K_u} G_{ij} V s_{uj} + z_{dj},
\]
\[
y_u = \sqrt{P L_d L_u} \sum_{j=1}^{K_u} F_j(\bar{\beta}_2) V s_{uj} + z_u
\]
\[
= \sqrt{\frac{P L_d L_u}{n_u}} [F_1(\bar{\beta}_2) V, \cdots, F_{K_u}(\bar{\beta}_2) V] s_u + z_u
\]
where \( s_d = [(s_{d1})^T, \cdots, (s_{dL_d})^T]^T \) and \( s_u = [(s_{u1})^T, \cdots, (s_{uK_u})^T]^T \).

Now consider designing of the DL transmit precoding matrix \( U_j \) for \( j \in [1 : L_d] \) and the UL transmit precoding matrix \( V \). Let \( W_3 \in \mathbb{C}^{L_d L_u 	imes n_d} \) be the submatrix consisting of the first through \( n_d \)th columns of \( \Omega_{L_d L_u} \) and \( W_4 \in \mathbb{C}^{L_d L_u 	imes n_u} \) be the submatrix consisting of the \( (n_d + 1) \)th through \( (n_d + n_u) \)th columns of \( \Omega_{L_d L_u} \). The following lemma is used for designing the transmit precoding matrices of the BS and the UL users.

**Lemma 3:** Suppose that \( n_d \in [1 : L_u] \), \( n_u \in [1 : L_d] \), and \( n_d + n_u \in [2 : L_d L_u] \). Denote
\[
P(\bar{\alpha}_2) = \begin{bmatrix} W_3^H H_1(\bar{\alpha}_2) \\ : \\ W_3^H H_{L_d}(\bar{\alpha}_2) \end{bmatrix} \in \mathbb{C}^{L_d n_d \times L_d L_u},
\]
\[
Q(\bar{\beta}_2) = \begin{bmatrix} F_1(\bar{\beta}_2) W_4, \cdots, F_{K_u}(\bar{\beta}_2) W_4 \end{bmatrix} \in \mathbb{C}^{L_d L_u \times K_u n_u}.
\]
Then \( \text{rank}(P(\bar{\alpha}_2)) = L_d n_d \) and \( \text{rank}(Q(\bar{\beta}_2)) \geq L_u n_u \) almost surely.

**Proof:** We refer to the Appendix for the proof.

We set the transmit precoding matrices of the BS and the UL users as

\[
[U_1, \cdots, U_{L_d}] = \frac{(P(\bar{\alpha}_2))^\dagger}{\| (P(\bar{\alpha}_2))^\dagger \|},
\]

\[
V = \frac{1}{\sqrt{n_u}} W_4
\]  

(24)

where \( (P(\bar{\alpha}_2))^\dagger = P(\bar{\alpha}_2)^H (P(\bar{\alpha}_2)P(\bar{\alpha}_2)^H)^{-1} \) is the right inverse matrix of \( P(\bar{\alpha}_2) \) satisfying that \( P(\bar{\alpha}_2)(P(\bar{\alpha}_2))^\dagger = I_{L_d n_d} \), which exists almost surely from Lemma 3.

For decoding its DL message, the \( i \)th DL user multiplies \( W_3^H \) to its length- \( (L_d L_u) \) time-extended received signal vector \( y_{di} \), given from (22) and (24) by

\[
W_3^H y_{di} = W_3^H \left( \frac{\sqrt{P L_d L_u}}{\| (P(\bar{\alpha}_2))^\dagger \|} H_i(\bar{\alpha}_2)(P(\bar{\alpha}_2))^\dagger s_d + \frac{\sqrt{P L_d L_u}}{n_u} \sum_{j=1}^{K_u} G_{ij} W_4 s_{uj} + z_{di} \right)
\]

\[
= \frac{\sqrt{P L_d L_u}}{\| (P(\bar{\alpha}_2))^\dagger \|} W_3^H H_i(\bar{\alpha}_2)(P(\bar{\alpha}_2))^\dagger s_d + \frac{\sqrt{P L_d L_u}}{n_u} \sum_{j=1}^{K_u} G_{ij} W_3^H W_4 s_{uj} + W_3^H z_{di}
\]

(25)

where the last equality holds from the definition of \( P(\bar{\alpha}_2) \) in (23) and the property of the IDFT matrix in (11). Then, the \( i \)th DL user estimates its information symbols based on (25). As a result, the achievable rate of the \( i \)th DL user \( (i \in [1 : L_d]) \) is given by

\[
R_{di} = \frac{n_d}{L_d L_u} \log P + o(\log P)
\]

Hence, the achievable sum DoF of the \( L_d \) DL users is given by

\[
\sum_{i=1}^{L_d} d_{di} = \lim_{P \to \infty} \sum_{i=1}^{L_d} R_{di} = \frac{n_d}{L_u}
\]

Now consider decoding of the UL messages at the BS. From (22), the achievable sum rate of the \( K_u \) UL users is given by

\[
\sum_{i=1}^{K_u} R_{ui} = \frac{\text{rank}(Q(\bar{\beta}_2))}{L_d L_u} \log P + o(\log P)
\]

Since \( \text{rank}(Q(\bar{\beta}_2)) \geq L_u n_u \) almost surely in Lemma 3, the achievable sum DoF of the \( K_u \) UL users is given by

\[
\sum_{i=1}^{K_u} d_{ui} = \lim_{P \to \infty} \sum_{i=1}^{K_u} R_{ui} \geq \frac{n_u}{L_d}
\]
Consequently, the sum DoF of $\frac{n_d}{L_u} + \frac{n_u}{L_d}$ is achievable assuming that $n_d \in [1 : L_u]$, $n_u \in [1 : L_d]$, and $n_d + n_u \in [2 : L_d L_u]$, which completes the achievability of Theorem 2.

**C. Sum Rate Comparison**

In this subsection, we compare the achievable sum rates of the proposed schemes with the cases where the BS operates in the conventional HD mode. To evaluate the sum rates of the HD BS, we assume that the half fraction of time (or bandwidth) is allocated for DL transmission and the rest half fraction is allocated for UL transmission. For fair comparison, we assume that the average transmit power of $2P$ is available at the BS for DL transmission and at each UL user for UL transmission. We further assume that all UL users simultaneously transmit to the BS to maximize the UL sum rate and, on the other hand, the BS transmits to a single DL user to maximize the DL sum rate [33].

For the no CSIT model, the following ergodic sum rate is achievable when the BS operates in HD mode:

$$\sum_{i=1}^{K_d} R_{d_i} + \sum_{i=1}^{K_u} R_{u_i} = \mathbb{E} \left\{ \frac{1}{2} \log \left( 1 + 2|h_1(1)|^2P \right) + \frac{1}{2} \log \left( 1 + \max_{j \in [1: M_u]} \sum_{i=1}^{K_u} 2|f_i(j)|^2P \right) \right\}. \quad (26)$$

That is, the BS selects a single DL user at random (for example, the first DL user) since it does
not know its DL channels but the BS can choose its receive mode among $M_u$ preset modes to maximize the UL sum rate.

For the partial CSIT model, the following ergodic sum rate is achievable when the BS operates in HD mode:

$$
\sum_{i=1}^{K_d} R_{di} + \sum_{i=1}^{K_u} R_{ui}
$$

$$
= \mathbb{E}\left\{ \frac{1}{2} \log \left( 1 + \max_{i \in [1:K_d], j \in [1:M_d]} 2|h_i(j)|^2 P \right) + \frac{1}{2} \log \left( 1 + \max_{j \in [1:M_u]} \sum_{i=1}^{K_u} 2|f_i(j)|^2 P \right) \right\}.
$$

The only difference from the no CSIT model is the fact that the BS can choose its single serving DL user and the corresponding transmit mode to maximize the DL sum rate.

Now, we derive the ergodic sum rates of the proposed schemes introduced in Sections IV-A and IV-B. From (15) and (18), the ergodic sum rate of the proposed scheme for the no CSIT model is given by

$$
\sum_{i=1}^{K_d} R_{di} + \sum_{i=1}^{K_u} R_{ui}
$$

$$
= \mathbb{E}\left\{ \frac{1}{L_u} \log \det \left( \mathbf{I}_{L_u-1} + \frac{PL_u|h_1(1)|^2}{L_u-1} \mathbf{I}_{L_u-1} \right) + \frac{1}{L_u} \log \det \left( \mathbf{I}_{L_u} + P L_u Q(\bar{\beta}_1) Q(\bar{\beta}_1)^H \right) \right\}.
$$
From (22) and (25), the ergodic sum rate of the proposed scheme for the partial CSIT model is given by

$$\sum_{i=1}^{K_d} R_{di} + \sum_{i=1}^{K_u} R_{ui}$$

$$= E \left\{ \frac{n_d}{L_u} \log \left( 1 + \frac{P L_d L_u}{\| (P(\bar{\alpha}_2))^H \|^2} \right) + \frac{1}{L_d L_u} \log \det \left( I_{L_d L_u} + \frac{P L_d L_u}{n_u} Q(\bar{\beta}_2) Q(\bar{\beta}_2)^H \right) \right\}.$$  \hspace{1cm} (29)

Fig. 4 and 5 plot the ergodic sum rates of the proposed schemes ((28) for no CSIT and (29) for partial CSIT) and the corresponding HD systems ((26) for no CSIT and (27) for partial CSIT) with respect to $P$ when $K_d = K_u = 2$, $M_d = M_u = 3$ in Fig. 4 and $K_d = K_u = 5$, $M_d = M_u = 3$ in Fig. 5. In simulation, we assume that all channel coefficients are i.i.d. drawn from the circularly symmetric complex Gaussian distribution, i.e., $CN(0,1)$. As seen in the figures, the FD operation at the BS outperforms the conventional HD systems and the rate gap increases as SNR increases. Hence the DoF gain achievable by the proposed schemes actually yields the sum rate gain at the finite and operational SNR regime.

V. CONVERSE

In this section, we establish the converse of Theorem 1. We first introduce the following key lemma.

**Lemma 4:** For the FD cellular network with no CSIT, any achievable DoF tuple must satisfy the following inequality:

$$\sum_{i=1}^{K_d} d_{di} + \frac{1}{\min(K_u, M_u)} \sum_{j=1}^{K_u} d_{uj} \leq 1.$$  \hspace{1cm} (30)

**Proof:** We refer to Section V-A for the proof. \hspace{1cm} \blacksquare

For notational convenience, let $d_d = \sum_{i=1}^{K_d} d_{di}$, $d_u = \sum_{j=1}^{K_u} d_{uj}$, and $L_u = \min(K_u, M_u)$. Then (30) is rewritten as

$$d_d + \frac{1}{L_u} d_u \leq 1.$$  \hspace{1cm} (31)

Trivially, from the sum DoF of the multiple-access channel, we have

$$d_u \leq 1.$$  \hspace{1cm} (32)

Therefore, any achievable $(\sum_{i=1}^{K_d} d_{di}, \sum_{j=1}^{K_u} d_{uj})$ pair should be located inside the shaded region in Fig. 6 from (31) and (32). In conclusion, $d_{\Sigma, noCSIT} \leq 2 - \frac{1}{L_u}$, which completes the proof of Theorem 1. For the rest of this section, we prove Lemma 4.
Fig. 6. Feasible region of \((\sum_{i=1}^{K_u} d_{ui}, \sum_{j=1}^{K_d} d_{dj})\).

\[
\sum_{j=1}^{K_u} d_{uj} = 1, \quad 1 - \frac{1}{L_u} \rightarrow d_{\Sigma, \text{noCSIT}} \leq 2 - \frac{1}{L_u}
\]

Fig. 7. Extended networks having \(M_d\) tx antennas and \(M_u\) rx antennas at the BS.

A. Proof of Lemma 4

1) Extended networks: To prove Lemma 4, we first introduce the extended network in Fig. 7 consisting of \(M_d\) and \(M_u\) conventional antennas at the transmitter and the receiver of the BS, instead of reconfigurable antennas each of which can choose a single transmit and receive mode from \(M_d\) and \(M_u\) preset modes. Obviously, the achievable DoF region of the original network
is included in that of the extended network.

More specifically, the received signal of the $i$th DL user at time $t$ and the received signal vector of the BS at time $t$ are given respectively by

$$
y_{di}(t) = h_i x_d(t) + \sum_{j=1}^{K_u} g_{ij} x_{uj}(t) + z_{di}(t),
$$

$$
y_u(t) = \sum_{j=1}^{K_u} f_j x_{uj}(t) + z_u(t)
$$

(33)

where $x_d(t) \in \mathbb{C}^{M_d \times 1}$ is the transmit signal vector of the BS at time $t$, $x_{uj}(t) \in \mathbb{C}$ is the transmit signal of the $j$th UL user at time $t$, $h_i \in \mathbb{C}^{1 \times M_d}$ is the channel vector from the transmitter of the BS to the $i$th DL user, $g_{ij} \in \mathbb{C}$ is the channel from the $j$th UL user to the $i$th DL user, and $f_j \in \mathbb{C}^{M_u \times 1}$ is the channel vector from the $j$th UL user to the receiver of the BS. The elements in additive noises $z_{di}(t) \in \mathbb{C}$ and $z_u(t) \in \mathbb{C}^{M_u \times 1}$ are i.i.d. drawn from $\mathcal{CN}(0, 1)$. The BS and each UL user should satisfy the average power constraint $P$, i.e., $\mathbb{E}[\|x_d(t)\|^2] \leq P$ and $\mathbb{E}[\|x_{uj}(t)\|^2] \leq P$ for all $j \in [1 : K_u]$. In the same manner as in Section II-B, we assume that all channel coefficients are i.i.d. drawn from a continuous distribution and CSIT is not available at the BS and each UL user (no CSIT model). Then we can define the sum DoF of the extended model in the same manner as in Section II-C.

From (33), the length-$n$ time-extended input–output relation is given by

$$
y_{di} = H_i x_d + \sum_{j=1}^{K_u} G_{ij} x_{uj} + z_{di},
$$

$$
y_u = \sum_{j=1}^{K_u} F_j x_{uj} + z_u
$$

where

$$
H_i = I_n \otimes h_i, \quad F_j = I_n \otimes f_j, \quad G_{ij} = g_{ij} I_n,
$$

$$
y_{di} = [y_{di}(1), \ldots, y_{di}(n)]^T, \quad y_u = [y_u(1)^T, \ldots, y_u(n)^T]^T,
$$

$$
x_d = [x_d(1)^T, \ldots, x_d(n)^T]^T, \quad x_{ui} = [x_{ui}(1), \ldots, x_{ui}(n)]^T,
$$

$$
z_{di} = [z_{di}(1), \ldots, z_{di}(n)]^T, \quad z_u = [z_u(1)^T, \ldots, z_u(n)^T]^T.
$$

2) DoF upper bound: We will prove that any DoF tuple achievable for the extended network in Fig. 7 satisfies (31). Let $F = [f_1, \ldots, f_{K_u}] \in \mathbb{C}^{M_u \times K_u}$ be the compound channel matrix from
The $K_u$ UL user to the receiver of the BS. In order to establish (31), we decompose $y_u(t)$, $z_u(t)$, and $F$ as follows:

- Decompose $y_u(t)$ into $y_u\alpha(t) \in \mathbb{C}^{L_u \times 1}$ and $y_u\beta(t) \in \mathbb{C}^{(M_u - L_u) \times 1}$ such that
  \[ y_u(t) = [y_u\alpha(t)^T, y_u\beta(t)^T]^T. \]
- Decompose $z_u(t)$ into $z_u\alpha(t) \in \mathbb{C}^{L_u \times 1}$ and $z_u\beta(t) \in \mathbb{C}^{(M_u - L_u) \times 1}$ such that
  \[ z_u(t) = [z_u\alpha(t)^T, z_u\beta(t)^T]^T. \]
- Decompose $F$ into $F_\alpha \in \mathbb{C}^{L_u \times K_u}$, and $F_\beta \in \mathbb{C}^{(M_u - L_u) \times K_u}$ such that
  \[ F = [F_\alpha^T, F_\beta^T]^T. \]

Furthermore, we define

\[ \tilde{y}_{u\alpha}(t) = y_{u\alpha}(t) + Qx_d(t) \quad (34) \]

and $\tilde{y}_{u\alpha} = [\tilde{y}_{u\alpha}(1)^T, \cdots, \tilde{y}_{u\alpha}(n)^T]^T$, where all coefficients in $Q \in \mathbb{C}^{L_u \times M_d}$ are i.i.d. drawn from the distribution of the channel coefficients. For convenience, let the set of all channel coefficients, the set of DL messages, and the set of UL messages by $\mathcal{H} = \{\{h_{ij}\}_{i \in [1:K_d]}, \{g_{ij}\}_{i \in [1:K_d], j \in [1:K_d]}, \{f_j\}_{j \in [1:K_a]}, Q\}$, $\mathcal{W}_d = (W_{d1}, \cdots, W_{dK_d})$, and $\mathcal{W}_u = (W_{u1}, \cdots, W_{uK_u})$, respectively.

We are now ready to prove (30) under the extended network. From Fano’s inequality, we have

\[ R_{di} - \epsilon_n \leq \frac{1}{n} \mathcal{I}(W_{di}; y_{di} | \mathcal{H}, W_{d1}, \cdots, W_{di-1}) \]
\[ = \frac{1}{n} \mathcal{I}(W_{di}; y_{di} | \mathcal{H}, W_{d1}, \cdots, W_{di-1}) \]

where $\epsilon_n \geq 0$ converges to zero as $n$ increases. Here the equality holds from the fact that the conditional probability distribution of $y_{di}$ is the same for all $i \in [1:K_d]$ when $(\mathcal{H}, W_{d1}, \cdots, W_{di})$ is given. Subsequently,

\[ \sum_{i=1}^{K_d} R_{di} - K_d\epsilon_n \leq \frac{1}{n} \sum_{i=1}^{K_d} \mathcal{I}(W_{di}; y_{di} | \mathcal{H}, W_{d1}, \cdots, W_{di-1}) \]
\[ = \frac{1}{n} \mathcal{I}(W_d; y_{d1} | \mathcal{H}) \]
\[ = \frac{1}{n} h(y_{d1} | \mathcal{H}) - \frac{1}{n} h(y_{d1} | \mathcal{H}, \mathcal{W}_d) \]
\[ \leq \log P - \frac{1}{n} h(y_{d1} | \mathcal{H}, \mathcal{W}_d) + o(\log P) \quad (35) \]
where the last inequality holds since $h(y_d | \mathcal{H}) \leq n (\log P + o(\log P))$.

Again, from Fano’s inequality, we have

$$R_{u_j} - \epsilon_n \leq \frac{1}{n} I(W_{u_j}; y_u | \mathcal{H}, W_{u_1}, \cdots, W_{u_{j-1}})$$

yielding that

$$\sum_{j=1}^{K_u} R_{u_j} - K_u \epsilon_n \leq \frac{1}{n} \sum_{j=1}^{K_u} I(W_{u_j}; y_u | \mathcal{H}, W_{u_1}, \cdots, W_{u_{j-1}})$$

$$= \frac{1}{n} I(W_u; y_u | \mathcal{H})$$

$$(a) \leq \frac{1}{n} I(W_u; y_u | \mathcal{H}, W_d)$$

$$= \frac{1}{n} h(y_u | \mathcal{H}, W_d) - \frac{1}{n} h(y_u | \mathcal{H}, W_d, W_u)$$

$$(b) \leq \frac{1}{n} h(y_u | \mathcal{H}, W_d) + o(\log P)$$

$$(c) \leq \frac{1}{n} h(\tilde{y}_{ua} | \mathcal{H}, W_d) + o(\log P)$$

(36)

where $(a)$ follows from the fact that $W_d$ is independent of $(W_u, y_u)$, $(b)$ follows from the fact that $h(y_u | \mathcal{H}, W_d, W_u) \leq n \cdot o(\log P)$, and $(c)$ follows since

$$h(y_u | \mathcal{H}, W_d)$$

$$= \sum_{t=1}^{n} h(y_u(t) | \mathcal{H}, W_d, y_u(1), \cdots, y_u(t-1))$$

$$= \sum_{t=1}^{n} h(y_{ua}(t) | \mathcal{H}, W_d, y_u(1), \cdots, y_u(t-1))$$

$$+ \sum_{t=1}^{n} h(y_{u\beta}(t) | \mathcal{H}, W_d, y_u(1), \cdots, y_u(t-1), y_{ua}(t))$$

$$(a) \leq \sum_{t=1}^{n} h(y_{ua}(t) | \mathcal{H}, W_d, y_u(1), \cdots, y_u(t-1)) + n \cdot o(\log P)$$

$$(b) \leq \sum_{t=1}^{n} h(\tilde{y}_{ua}(t) | \mathcal{H}, W_d, y_u(1), \cdots, y_u(t-1), \tilde{y}_{ua}(1), \cdots, \tilde{y}_{ua}(t-1)) + n \cdot o(\log(P))$$

$$(c) \leq \sum_{i=1}^{n} h(\tilde{y}_{ua}(i) | \mathcal{H}, W_d, \tilde{y}_{ua}(1), \cdots, \tilde{y}_{ua}(t-1)) + n \cdot o(\log(P))$$

$$= h(\tilde{y}_{ua} | \mathcal{H}, W_d) + n \cdot o(\log(P))$$
where (a) follows from the fact that if \( K_u < M_u \), then
\[
y_{u\beta}(t) - F_{\beta}F_{\alpha}^{-1}y_{u\alpha}(t) = z_{u\beta}(t) - F_{\beta}F_{\alpha}^{-1}z_{u\alpha}(t)
\]
and otherwise, \( y_{u\beta}(t) \) does not exist from its definition, (b) follows since \( \tilde{y}_{u\alpha}(t) \) is a function of \( \{H, W_d, y_u(1), \cdots, y_u(t-1), y_{u\alpha}(t)\} \) for \( t \in [1:n] \) from the definition in (34), and (c) follows from the fact that conditioning reduces differential entropy.

Let \( \tilde{y}_{uai}(t) \in \mathbb{C} \) for \( i \in [1:L_u] \) be the \( i \)th element of \( \tilde{y}_{u\alpha}(t) \) and \( \tilde{y}_{uai} = [\tilde{y}_{uai}(1), \cdots, \tilde{y}_{uai}(n)]^T \). From the definition of \( \tilde{y}_{u\alpha}(t) \) in (34), the conditional probability distribution of \( y_{d1} \) is identical with that of \( \tilde{y}_{uai} \) for all \( i \in [1:L_u] \) when \( (H, W_d) \) is given. Consequently, from (36)
\[
\sum_{j=1}^{K_u} R_{uj} - K_u \epsilon_n \leq \frac{1}{n} h(\tilde{y}_{uai}|H, W_d) + o(\log P)
\]
\[
\leq \frac{1}{n} \sum_{i=1}^{L_u} h(\tilde{y}_{uai}|H, W_d) + o(\log P)
\]
\[
= \frac{1}{n} L_u h(y_{d1}|H, W_d) + o(\log P)
\]
(37)
where the second inequality follows from the fact that conditioning reduces differential entropy.

Then, multiplying \( \frac{1}{L_u} \) to (37) and adding it to (35), we have
\[
\sum_{i=1}^{K_d} R_{di} + \frac{1}{L_u} \sum_{j=1}^{K_u} R_{uj} \leq \log P + o(\log P) + \left( K_d + \frac{1}{L_u} K_u \right) \epsilon_n.
\]
(38)
By dividing both sides of (38) by \( \log P \) and letting \( n \) and \( P \) to infinity, we have
\[
\sum_{i=1}^{K_d} d_{di} + \frac{1}{L_u} \sum_{j=1}^{K_u} d_{uj} \leq 1
\]
because \( \epsilon_n \) converges to zero as \( n \) increases, which completes the proof of Lemma 4.

VI. CONCLUSION

In this paper, we studied the sum DoF of FD cellular networks consisting of a FD BS, HD DL users and HD UL users. In particular, we completely characterized the sum DoF of FD cellular networks for the no CSIT model and established an achievable sum DoF for the partial CSIT model. Our results demonstrated that reconfigurable antennas only at the FD BS can improve the sum DoF and eventually double the sum DoF as both the numbers of DL and UL users and preset modes increase in the presence of user-to-user interference. We further demonstrated that such
DoF improvement yields the sum rate improvement compared to the conventional HD cellular networks at the finite SNR regime. Beyond this work, the impact of multiple reconfigurable antennas at FD BSs will be a promising future research topic.

APPENDIX

PROOF OF LEMMA 3

In this appendix, we prove Lemma 3. First, we show that \( \text{rank}(\mathbf{P}(\tilde{\alpha}_2)) = L_d n_d \) almost surely. Recall that \( \tilde{\alpha}_2(t) = (t-1)|L_d + 1 \) for \( t \in [1 : L_d L_u] \) and the definition of \( \mathbf{P}(\tilde{\alpha}_2) \) is given in (23). Let us permute the columns of \( \mathbf{P}(\tilde{\alpha}_2) \) as in the following order and denote the resultant matrix as \( \mathbf{A} \):

\[
\{1, 1 + L_d, \cdots, 1 + (L_u - 1)L_d, 2, 2 + L_d, \cdots, 2 + (L_u - 1)L_d, \cdots, L_d, 2L_d, \cdots, L_u L_d \}.
\]

From the definition of \( \tilde{\alpha}_2(t) \), \( \mathbf{A} \in \mathbb{C}^{L_d n_d \times L_u} \) is then given by

\[
\mathbf{A} = \begin{bmatrix}
    h_1(1) \mathbf{W}_3^H (\mathbf{I}_{L_u} \otimes \mathbf{e}_{L_d}(1)) & h_1(2) \mathbf{W}_3^H (\mathbf{I}_{L_u} \otimes \mathbf{e}_{L_d}(2)) & \cdots & h_1(L_d) \mathbf{W}_3^H (\mathbf{I}_{L_u} \otimes \mathbf{e}_{L_d}(L_d)) \\
    \vdots & \vdots & \ddots & \vdots \\
    h_{L_d}(1) \mathbf{W}_3^H (\mathbf{I}_{L_u} \otimes \mathbf{e}_{L_d}(1)) & h_{L_d}(2) \mathbf{W}_3^H (\mathbf{I}_{L_u} \otimes \mathbf{e}_{L_d}(2)) & \cdots & h_{L_d}(L_d) \mathbf{W}_3^H (\mathbf{I}_{L_u} \otimes \mathbf{e}_{L_d}(L_d))
\end{bmatrix}.
\]

Let \( \mathbf{A}_i = \mathbf{W}_3^j (\mathbf{I}_{L_u} \otimes \mathbf{e}_{L_d}(i)) \in \mathbb{C}^{n_d \times L_u} \) for \( i \in [1 : L_d] \). Since any submatrix of the IDFT matrix is a full-rank matrix \([34]\) and \( n_d \leq L_u \), \( \text{rank}(\mathbf{A}_i) = n_d \) so that it is right invertible. Denoting the right inverse matrix of \( \mathbf{A}_i \) by \( \mathbf{A}_i^+ = \mathbf{A}_i^{-1} \mathbf{A}_i \mathbf{A}_i^{-1} \), the following relation holds:

\[
\mathbf{A} \text{diag}(\mathbf{A}_1^+, \cdots, \mathbf{A}_{L_d}^+) = \begin{bmatrix}
    h_1(1) & h_1(2) & \cdots & h_1(L_d) \\
    \vdots & \vdots & \ddots & \vdots \\
    h_{L_d}(1) & h_{L_d}(2) & \cdots & h_{L_d}(L_d)
\end{bmatrix} \otimes \mathbf{I}_{n_d} \in \mathbb{C}^{L_d n_d \times L_d n_d}
\]

Since every element in \( \mathbf{H} \) is i.i.d. drawn from a continuous distribution, \( \mathbf{H} \) is a full-rank matrix almost surely so that \( \text{rank}(\mathbf{H} \otimes \mathbf{I}_{n_d}) = L_d n_d \) almost surely. Because \( \text{rank}(\mathbf{A}) \geq \text{rank}(\mathbf{H} \otimes \mathbf{I}_{n_d}) \), we finally have \( \text{rank}(\mathbf{P}(\tilde{\alpha}_2)) = \text{rank}(\mathbf{A}) \geq L_d n_d \) almost surely. Obviously, \( \text{rank}(\mathbf{P}(\tilde{\alpha}_2)) \leq L_d n_d \) from the dimension of \( \mathbf{P}(\tilde{\alpha}_2) \). Therefore, \( \text{rank}(\mathbf{P}(\tilde{\alpha}_2)) = L_d n_d \) almost surely.

Next, we show that \( \text{rank}(\mathbf{Q}(\tilde{\beta}_2)) \geq L_u n_u \) almost surely. Recall that \( \tilde{\beta}_2(t) = (t-1)|L_u + 1 \) for \( t \in [1 : L_d L_u] \) and the definition of \( \text{rank}(\mathbf{Q}(\tilde{\beta}_2)) \) is given in (23). Let \( \mathbf{Q}_{\text{sub}}(\tilde{\beta}_2) = \)
Let $B$ be a submatrix of the IDFT matrix that is full-rank and right invertible. Denoting the right inverse matrix of $B$ as $B^\dagger = B^H (B B^H)^{-1}$, the following relation holds:

$$
\begin{bmatrix}
  f_1(1) & f_2(1) & \cdots & f_L(1) \\
  \vdots & \vdots & \ddots & \vdots \\
  f_L(1) & f_2(1) & \cdots & f_L(1)
\end{bmatrix}^{T} \otimes I_{nu} \in \mathbb{C}^{L_d \times L_d \times n_u}
$$

Then, $\text{rank}(Q_{\text{sub}}(\hat{z}_2)) = \text{rank}(B) = L_u n_u$ almost surely, which completes the proof of Lemma 3.

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