D3 Brane(s) in $AdS_5 \times S^5$ and $\mathcal{N} = 4, 2, 1$ SYM

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Abstract

Recently, Douglas and Taylor proposed to identify the Higgs vev of the Coulomb branch of the 4 dimensional $\mathcal{N} = 4$ super Yang-Mills theory with the positions of D3 branes in the $AdS_5 \times S^5$ string theory. We extend this identification to more general configurations that preserve less supersymmetry. We show that a single D3 brane in $AdS_5 \times S^5$ string theory can break the spacetime supersymmetry to 1/4 or even 1/8. Such configurations, interpreted as backgrounds of the SYM, also break the $\mathcal{N} = 4$ superconformal symmetry of SYM to 1/4 or 1/8, giving $\mathcal{N} = 2$ or $\mathcal{N} = 1$. We discuss the implications of this correspondence. We also present other BPS D3-brane configurations that preserve 1/8 supersymmetry corresponding to $\mathcal{N} = 1$ on the SYM side.
1 Introduction

A year ago, a remarkable proposal has been put forwarded by Maldacena [1] which conjectures that the type IIB string theory on $AdS_5 \times S^5$ is dual to the four dimensional $\mathcal{N} = 4$ supersymmetric Yang-Mills theory (SYM) with gauge group $SU(N)$. A precise recipe of relating type IIB sugra in $AdS_5 \times S^5$ to the SYM on the $AdS$ boundary has been given in [2, 3] and it was demonstrated that this duality is holographic in nature. This holographic property has been further studied in [3, 4, 5, 6, 7, 8, 9] and in particular in the presence of branes in $AdS_5 \times S^5$ in [3, 10, 11, 12, 13, 14, 15, 16, 17, 18].

Recently, Douglas and Taylor [15] proposed that the large $N$ limit of the $\mathcal{N} = 4$ $SU(N)$ SYM in the Coulomb branch, i.e. with nonzero vacuum expectation values for the scalar fields, corresponds to branes in the bulk of $AdS_5 \times S^5$ string theory. In the large $N$ limit, one can replace $SU(N)$ by $U(N)$ and these authors considered the following configuration of Higgs fields

$$X^m = \begin{pmatrix} x^m & 0 & 0 \\ 0 & \tilde{x}^m & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (1)$$

They provided some evidence that $\mathcal{N} = 4$ SYM in the vacuum (1) does describe two D3-branes in the $AdS$ bulk. This proposal is in accord with the principle of holography. The D3-branes these authors considered have worldvolumes parallel to the $AdS$-boundary. One may wonder what about more general D3-branes configurations in the $AdS$ bulk? How are they described in the SYM picture? This paper is a first attempt to answer these questions. We propose that the identification of SYM configurations with configurations of D3-branes in $AdS_5 \times S^5$ indeed holds true even for non-constant backgrounds.

Like in most other checks of various dualities, the study of BPS configurations is a valuable tool. On one hand it allows one to perform a manageable analysis of the problem, on the other hand it can often tell us a lot about the duality and provide non-trivial checks. In this paper, we construct BPS configurations preserving 1/2, 1/4 and 1/8 of the relevant supersymmetries both for the $\mathcal{N} = 4$ SYM theory and for the 10 dimensional $AdS_5 \times S^5$ string theory. The constructions are independent of each other and a priori, there is no relation between these BPS configurations. We then check the above generalized form of the duality proposal by showing that there is a precise and consistent matching of supersymmetry if one identifies the non-constant Higgs values and YM field strength with the positions and the Born-Infeld field strength of the D3-brane(s) in $AdS_5 \times S^5$.

The organization of the paper is as follows. First, within the SYM framework, we study the constant Higgs configurations proposed by [15] in more detail. These configurations

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1 To avoid confusion let us insist that initially we have 32 supersymmetries. This is obvious on the IIB side. On the SYM side one has an $\mathcal{N} = 4$ Poincaré supersymmetry (16 supersymmetries), and at the superconformal point also 16 special superconformal symmetries, hence also a total of 32. The notation $\mathcal{N} = 4$ SYM thus is a bit misleading at the conformal point.
preserve $\mathcal{N} = 4$ conformal supersymmetry in four dimensions (32 susys) or only $\mathcal{N} = 4$ supersymmetry (16 susys), depending on whether the Higgs vevs are all zero or not. Then we present Higgs configurations that preserve $\mathcal{N} = 2$ or $\mathcal{N} = 1$ supersymmetry in four dimensions. This is achieved by allowing two (or four) of the Higgs scalars to depend holomorphically on two of the worldvolume coordinates (see condition (13) below). Finally we present other different ways of getting $\mathcal{N} = 1$. This is achieved by adding a self-dual YM configuration on top of the $\mathcal{N} = 2$ Higgs configuration. We show that these new configurations break $7/8$ of the supersymmetries and give rise to $\mathcal{N} = 1$.

Section 3 contains an independent analysis of the supersymmetry preserved by the corresponding D3-brane in $AdS_5 \times S^5$. We first recall the supersymmetry preserving condition for D$p$-branes. This condition can be derived from the $\kappa$-symmetric formulation of D$p$-branes. A flat D3 brane preserves 1/2 of the supersymmetries. Then we present a novel brane configuration that preserves 1/4 supersymmetries. This D3-brane configuration is indeed a direct transcription of the Higgs configuration in section 2 by interpreting the latter as the position of the D3-brane in $AdS_5 \times S^5$. We show that the resulting D3-brane in fact preserves the same amount of supersymmetry as is preserved on the gauge theory side. A precise and consistent 1-1 mapping between the 4 dimensional supersymmetry and the 10 dimensional $AdS$ supersymmetry is established. We also show that our results imply that, by allowing for two D3-brane probes, one can break the spacetime supersymmetry down to 1/8. We then discuss that by allowing a “self-dual” Born-Infeld configuration on the worldvolume of a single D3-brane, one can also reduce the residual supersymmetry by half and this single D3-brane breaks 7/8 of the spacetime supersymmetries. Such “self-dual” configurations make sense with Euclidean signature but their Minkowski interpretation is less clear.

Single D3-brane configurations which preserve 1/4 of the spacetime supersymmetry were first constructed in [19] by wrapping the D3 over certain cycles in a Calabi-Yau. Here, we construct new D3-brane configurations that preserve 1/4 or 1/8 of the spacetime supersymmetry. One of the differences of our construction with theirs is that the D3-brane we construct is not wrapped on any cycle [20, 21, 19] but infinitely extended. For the 1/4 supersymmetries case, we found that the condition for it to satisfy the equations of motion is precisely the same as the condition for it to preserve 1/4 supersymmetry. On the SYM side this corresponds to an extra condition for the gauge theory Higgs configuration which is not necessary in the classical limit. For the 1/8 supersymmetries case, we again find that the brane configuration is slightly more restrictive then the corresponding classical YM configurations. The reason and significance of this are discussed in section 4, where we summarize our results and present some directions for further works.
2 BPS Configurations in SYM

Consider $\mathcal{N} = 4$ SYM living in (3+1) dimensions with coordinates $\sigma^i, i = 0, 1, 2, 3$. For our purpose here, it is convenient to think of the 4 dimensional $\mathcal{N} = 4$ SYM theory as obtained through dimensional reduction of the 10 dimensional SYM theory and use the 10 dimensional $\Gamma$ matrix notation. The field content consists of the gauge fields $\hat{A}_i$, the six scalars $\hat{X}_m, (m = 4, \cdots, 9)$ and the 16 fermions $\hat{\Psi}$. $\hat{\Psi}$ is a Majorana-Weyl spinor in 10 dimensions and it decomposes into four Majorana spinors in 4 dimensions. All fields are Hermitian and are in the adjoint representation of the gauge group. The theory is invariant under 16 Poincaré supersymmetries $\delta$ and 16 special supersymmetries $\tilde{\delta}$ given by

$$\delta \hat{X}_m = i \xi \Gamma_m \hat{\Psi}, \quad (2)$$
$$\delta \hat{\Psi} = \left( \frac{i}{2} [\hat{X}_m, \hat{X}_n] \Gamma^{mn} + D_i \hat{X}_m \Gamma^{im} + \frac{1}{2} \hat{F}_{ij} \Gamma^{ij} \right) \xi, \quad (3)$$
$$\delta \hat{A}_i = \xi \Gamma_i \hat{\Psi}, \quad (4)$$

and

$$\tilde{\delta} \hat{X}_m = i \bar{\zeta} \bar{\Phi} \Gamma_m \hat{\Psi}, \quad (5)$$
$$\tilde{\delta} \hat{\Psi} = \left( \frac{i}{2} [\hat{X}_m, \hat{X}_n] \Gamma^{mn} + D_i \hat{X}_m \Gamma^{im} + \frac{1}{2} \hat{F}_{ij} \Gamma^{ij} \right) \bar{\Phi} \zeta + 2 \hat{X}_m \Gamma^m \zeta, \quad (6)$$
$$\tilde{\delta} \hat{A}_i = \bar{\zeta} \bar{\Phi} \Gamma_i \hat{\Psi}. \quad (7)$$

The $\Gamma$-matrices are $16 \times 16$ dimensional, Hermitian and satisfy the Clifford algebra

$$\{ \Gamma_M, \Gamma_N \} = 2 \eta_{MN}, \quad M, N = 0, \cdots, 9, \quad (8)$$

where the metric is $\eta_{MN} = diag(-, +, \cdots, +)$.

2.1 $\mathcal{N} = 4$ and $\mathcal{N} = 2$ Configurations

In this paper, we will only consider diagonal Higgs configurations $\hat{X}^m$ with the non-vanishing eigenvalues all different from each other. The theory reduces to a number of non-interacting sectors and one can apply the analysis to each $U(1)$ sector individually. The analysis becomes harder when one allows for coinciding eigenvalues. The corresponding brane picture will be a set of overlapping D3-branes described by a non-Abelian Born-Infeld action. The later hasn’t been fully understood\footnote{Our convention is to put a hat on the symbols for the adjoint $U(N)$ matrices. Below the same symbols without hats will be used for a certain diagonal element of these matrices.}.
Throughout this paper, we will restrict ourselves to bosonic configurations \((\Psi = 0)\), and until further notice also to \(\hat{F}_{ij} = 0\). Let us consider Higgs configurations that are diagonal with only the first diagonal element non-vanishing
\[
\hat{X}^m = \begin{pmatrix} X^m & 0 \\ 0 & 0 \end{pmatrix}.
\] (9)

In the following, \(X^m\) will denote the non-vanishing diagonal entry of the matrix \(\hat{X}^m\), and similarly \(\Psi\) and \(F_{ij}\) will denote the corresponding diagonal elements of \(\hat{\Psi}\) and \(\hat{F}_{ij}\).

Consider first the

**Higgs Config. 1 \((\mathcal{N} = 4)\)**

\[
X^m = c^m \quad \text{where } c^m \text{ are constants and not all zero } \quad (m = 4, \ldots, 9).
\] (10)

It is easy to check that this configuration breaks all the special supersymmetries and preserves all the 16 Poincaré supersymmetries of SYM. This is the configuration considered in [15] which is proposed to describe a D3-brane sitting parallel to the boundary of \(AdS_5 \times S^5\). In the brane picture, as we will check below, such a D3-brane breaks half of the 32 \(AdS_5 \times S^5\) supersymmetries and the picture is indeed consistent.

We are interested in having \(\mathcal{N} = 2\) supersymmetry in 4 dimensions. The original \(\mathcal{N} = 4\) vector multiplet will split into an \(\mathcal{N} = 2\) vector multiplet (two real scalars) and an \(\mathcal{N} = 2\) hyper-multiplet (four real scalars). The R-symmetry is broken as
\[
SO(6)_R \rightarrow SU(2)_R \times U(1)_R.
\] (11)

This suggests to try the following configuration,

**Higgs Config. 2 \((\mathcal{N} = 2)\)**

\[
X^{4,5,6,7} = \text{constants, and } X^{8,9} = X^{8,9}(\sigma_1, \sigma_2) \text{ satisfying}
\]
\[
\partial_1 X^8 = \pm \partial_2 X^9, \quad \partial_2 X^8 = \mp \partial_1 X^9.
\] (12)

(Obviously, we exclude the trivial case where \(X^{8,9}\) both vanish.) Due to the explicit \(\sigma_i\) dependence in (12), it is easy to see that with this Higgs configuration the special supersymmetries are generally all broken even if all of the \(X^{4,5,6,7}\) vanish. So to preserve 1/4 of the total of 16 + 16 supersymmetries, we will try to find conditions on \(X^{8,9}\) such that 1/2 of the 16 Poincaré supersymmetries are preserved. The latter act as
\[
\delta \Psi = \Gamma^{18}(\partial_1 X^8 + \Gamma^{12} \partial_2 X^8) M \xi,
\] (14)

where
\[
M = 1 + (\partial_1 X^8 + \Gamma^{12} \partial_2 X^8)^{-1}(\partial_1 X^9 + \Gamma^{12} \partial_2 X^9) \Gamma^{89}.
\] (15)
Notice that \( (\partial_1 X^8 + \Gamma^{12} \partial_2 X^8) \) is invertible as long as \( \partial_1 X^8 \) or \( \partial_2 X^8 \) is non-vanishing. Thus in the present case, (13) is well defined. It is not hard to show that \( M \) is a matrix of half rank iff

\[
\partial_1 X^8 = \pm \partial_2 X^9, \quad \partial_2 X^8 = \mp \partial_1 X^9,
\]

i.e. \( X^8, X^9 \) satisfy the Cauchy-Riemann condition. This condition also guarantees that \( X^{8,9} \) satisfy the equations of motion. In this case, \( M \) is equal to

\[
M = 1 \pm \Gamma^{1289}
\]

and hence only that half of the Poincaré supersymmetries \( \xi \) which satisfy

\[
(1 \pm \Gamma^{1289}) \xi = 0
\]

are preserved. We will see in the next section that this Higgs configuration corresponds in the brane picture to having a D3-brane in \( AdS_5 \times S^5 \) whose embedding satisfies the Cauchy-Riemann condition (13).

Notice that a condition similar to (13) has also appeared in the construction of the three-brane soliton \([23]\) on the M5 brane worldvolume. There, the M5-brane occupies the \( \sigma^{0,1,2,3,4,5} \) directions and the transverse scalars are \( X^{6,7,8,9,10} \). It was shown that a three-brane soliton preserving 1/2 of the M5-brane worldvolume supersymmetry can be constructed if

\[
\partial_4 X^6 = \pm \partial_5 X^{10}, \quad \partial_5 X^6 = \mp \partial_4 X^{10},
\]

and with the other three scalars unexcited. The three-brane soliton has worldvolume in the 0123 directions and the condition is imposed entirely in the transverse direction. This is different from our case.

The most general configuration with two Higgs scalars excited is

**Higgs Config. 2'** \( X^{8,9} = X^{8,9}(\sigma_0, \sigma_1, \sigma_2, \sigma_3) \) and \( X^{4,5,6,7} = \text{constants} \).

It is possible to show that this configuration satisfies the equations of motion and preserves half of the Poincaré supersymmetries iff it reduces to the Higgs configuration \([3]\) and hence the same holomorphicity condition (13) applies.

### 2.2 \( \mathcal{N} = 1 \) Configurations

From the discussion above, it should be clear that one way to get \( \mathcal{N} = 1 \) is to consider a Higgs configuration of the form

**Higgs Config. 3** (\( \mathcal{N} = 1 \))

\[
\hat{X}^m = \begin{pmatrix} X^m & 0 & 0 \\ 0 & \tilde{X}^m & 0 \\ 0 & 0 & 0 \end{pmatrix}
\]

(20)
with

\[ X^{8,9}(\sigma_1, \sigma_2) \text{ satisfying the Cauchy-Riemann condition} \]  \hspace{1cm} (21)

\[ \tilde{X}^{6,7}(\sigma_1, \sigma_2) \text{ satisfying the Cauchy-Riemann condition} \]  \hspace{1cm} (22)

\[ \tilde{X}^{8,9}, X^{6,7}, X^{4,5}, \tilde{X}^{4,5} = \text{constants} \]  \hspace{1cm} (23)

The condition (21) on \( X^{8,9} \) gives rise to the condition

\[ (1 \pm \Gamma_{1289}) \xi = 0 \]  \hspace{1cm} (24)

for the unbroken supersymmetries \( \xi \). Similarly, (22) gives rise to

\[ (1 \pm \Gamma_{1267}) \xi = 0 \]  \hspace{1cm} (25)

and the two projectors in (24) and (25) commute. The two conditions together break the Poincaré supersymmetry to 1/4 and only a \( \mathcal{N} = 1 \) supersymmetry is left unbroken.

There are also other Higgs configurations with \( \mathcal{N} = 1 \) by trivially permuting the coordinates \( \sigma_i \) and/or the Higgs fields \( X^m \) as long as we obtain commuting projectors as in (24), (25). For example, a configuration with \( X^{8,9}(\sigma_1, \sigma_2) \) and \( \tilde{X}^{6,7}(\sigma_1, \sigma_3) \) satisfying Cauchy-Riemann condition (all the other \( X, \tilde{X} \)'s being constant) wouldn’t work and will break all supersymmetry as \( \Gamma_{1289} \) and \( \Gamma_{1367} \) anticommute rather than commute with each other.

The same configuration with \( X^7 \) replaced by \( X^9 \) has \( \mathcal{N} = 1 \) unbroken supersymmetry as \( \Gamma_{1289} \) and \( \Gamma_{1369} \) commute with each other.

We will see that in the brane picture, the Higgs configuration 3 corresponds to having two D3-branes in \( \text{AdS}_5 \times S^5 \), each satisfying a holomorphic embedding condition.

It is known that an (anti-)instanton background breaks half of the special supersymmetries and breaks half of the Poincaré supersymmetries. In this paper, we are working with Lorentzian signature, so we should look at the configuration that is obtained from the Euclidean instanton after continuation to Minkowski space, namely

\[ \hat{F}_{ij} = \frac{c}{2} \varepsilon_{ijkl} \hat{F}^{kl}, \quad c = \pm i . \]  \hspace{1cm} (26)

The case \( c = -i \) corresponds to a (Euclidean) YM instanton and \( c = i \) corresponds to a (Euclidean) anti-instanton. We will simply refer to these “self-dual” or “anti-self-dual” Minkowskian configurations as (Minkowskian) instantons. Let us first embed an instanton in the non-abelian part of \( \hat{F} \) and consider the following configuration

\[ \text{It is obvious that the Minkowski (anti) self-duality equation (26) leads to certain components of } F_{ij} \text{ that are purely imaginary. The proper way to think about these configurations is that they make sense when continued to Euclidean signature. However, since throughout this paper we work with Minkowski signature, we will formally use (26).} \]
Higgs-Instanton Config. 4 ($\mathcal{N} = 1$)

\[
\hat{X}^m = \begin{pmatrix} X^m & 0 \\ 0 & 0 \end{pmatrix}, \quad \hat{F}_{ij} = \begin{pmatrix} 0 & 0 \\ 0 & F_{ij} \end{pmatrix}
\]

where $X^m$ is the Higgs configuration 2 above and $F_{ij}$ is an (anti-)instanton.

As we will see shortly, the unbroken supersymmetry is $\mathcal{N} = 1$.

Next, we would like to consider a configuration where the non-trivial gauge field lies in the same $U(1)$ factor as the non-trivial Higgs component. It is well-known that one cannot embed an instanton into the $U(1)$ factors of a gauge group. However, one can consider the following (anti) self-dual configuration

\[
\hat{F}_{ij} = \begin{pmatrix} F_{ij} & 0 \\ 0 & 0 \end{pmatrix},
\]

\[
F_{ij} = \frac{c}{2} \epsilon_{ijkl} F^{kl}, \quad c = \pm i
\]

and consider the following

Higgs-Gauge Config. 5 ($\mathcal{N} = 1$)

\[
\hat{X}^m = \begin{pmatrix} X^m & 0 \\ 0 & 0 \end{pmatrix}, \quad \hat{F}_{ij} = \begin{pmatrix} F_{ij} & 0 \\ 0 & 0 \end{pmatrix}
\]

where $X^m$ is the Higgs configuration 2 above and $F_{ij}$ is the self-dual or anti self-dual configuration (28).

For both configurations 4 and 5 above, one easily checks that the equations of motion are satisfied and the special supersymmetries are all broken. From eq. (3) we see that the preserved Poincaré supersymmetries must satisfy

\[
(1 + c \Gamma_{0123}) \xi = 0
\]

\[
(1 \pm \Gamma_{1289}) \xi = 0
\]

and hence there is an unbroken $\mathcal{N} = 1$ supersymmetry left. The corresponding brane picture will be discussed in the next section. In particular, we will see there that the self-dual configuration 5 corresponds to a single D3-brane with a self-dual Born-Infeld field strength. Note that such constructions with a non-vanishing $\hat{F}$ field could also be done for the Higgs configuration 1 resulting in a $\mathcal{N} = 2$ SYM. The case $X^m = 0$ was the subject of interest in [10].

3 D3-Branes in $AdS_5 \times S^5$

D-branes are basic objects in the nonperturbative formulation of string theory. They are endpoints of open strings and one can learn much of their properties by studying the
boundary (S)CFT of the open strings ending on it. Along this line, [21, 19] used the SCF worldsheets to derive the embedding condition for a D-brane to preserve supersymmetry. There has been some progress [24, 25, 26, 27] in constructing $AdS_5 \times S^5$ string theory. However, a satisfactory string worldsheet description including non-trivial RR backgrounds is still lacking. Therefore, although one would like to carry out a microscopic SCFT analysis similar to [21, 19], to see how a D3-brane in $AdS_5 \times S^5$ breaks the spacetime supersymmetry, we will be contented in this paper with a low energy analysis, using a $\kappa$-symmetric formulation of D-branes.

We will present configurations of a single D3-brane in $AdS_5 \times S^5$ which preserve $1/2$, $1/4$ or $1/8$ of the spacetime supersymmetries. Until now, the only known way for a single D-brane to break more than half of the spacetime supersymmetries was to wrap the brane over a Cayley submanifold [19]. As these authors have shown, such a brane saturates a BPS condition and hence attains its minimal energy. For $AdS_5 \times S^5$, there is no appropriate cycle on which the D3-brane could be wrapped. So our construction has to be different. In fact, we find that the desired configuration ((68)-(70) below) preserving $1/4$ of the spacetime supersymmetries is non-compact. Since the brane is not wrapped on anything, one also needs to make sure it satisfies the equations of motion. Maybe somewhat surprisingly, we find that the condition for it to satisfy the equations of motion is precisely the same condition for it to preserve supersymmetry. The configuration (68)-(70) provides the only known example of a single, unwrapped D3-brane preserving $1/4$ of the spacetime supersymmetries. By turning on an appropriate “self-dual” Born-Infeld field strength $\mathcal{F}$ on a single D3-brane, we will also get a configuration preserving $1/8$ of the supersymmetries.

### 3.1 Supersymmetry Preserving Condition

**$\kappa$-symmetry on a $Dp$-brane**

In order to be self-contained, we briefly review here the necessary part of the $\kappa$-symmetric formulation for a $Dp$-brane coupled to a supersymmetric background. For details of the formulation as an action, see [28, 29, 30, 31, 32]; and for the formulation as equations of motion, see [33, 34]. For a pedagogical review we refer to [35], while the equivalence of these approaches is discussed in [36]. We will not need the details here. What we need is that the $\kappa$-symmetry takes the simple covariant form

$$\delta \theta = (1 + \Gamma) \kappa,$$

(33)

where $\theta$ is the spacetime spinor depending on the worldvolume coordinates $\sigma$, $\kappa(\sigma)$ is a local parameter for the $\kappa$-transformation and $\Gamma$ is the “pull-back” $\Gamma$-matrix which depends on the worldvolume fields and satisfies $\Gamma^2 = 1$ (see below).

For our present use, we recall the explicit form of $\Gamma$ [37] in the case that all fermionic
fields vanish. It is
\[ \Gamma = e^{-a/2} \Gamma'_0 e^{a/2}, \]
where \( a \) is a matrix given below and
\[ \Gamma'_0 = \begin{cases} (\Gamma_{11})^{p+2} \Gamma_0 & \text{IIA}, \\ (\sigma_3) \frac{p+2}{2} i \sigma_2 \otimes \Gamma_0 & \text{IIB}, \end{cases} \]
and
\[ \Gamma_0 = \frac{1}{(p+1)! \sqrt{|g|}} \epsilon_{i_1 \cdots i_{(p+1)}} \partial_{i_1} X^{M_1} \cdots \partial_{i_{(p+1)}} X^{M_{(p+1)}} \Gamma'_{M_1 \cdots M_{(p+1)}}. \]
As usual, the matrix \( \Gamma'_{M_1 \cdots M_{(p+1)}} \) is the antisymmetrized product of the \( \Gamma'_M \) with the \( \Gamma'_M \) being the 10 dimensional \( \Gamma \)-matrices in the coordinate basis defined by
\[ \Gamma'_M := E_M^A \Gamma_A, \]
where the \( \Gamma_A \) are flat space \( \Gamma \)-matrices. The metric \( g_{ij} \) is the induced worldvolume metric
\[ g_{ij} = \partial_i X^M \partial_j X^N G_{MN} \]
and \( |g| \) is its determinant. To define the matrix \( a \) appearing in (34) we need to introduce the modified 2-form field strength \( \mathcal{F} \) which is related to the Born-Infeld field strength \( F = dA \) by
\[ \mathcal{F} = F - \mathcal{B}, \]
where \( \mathcal{B} \) is the pull-back of the target space NS-NS 2-form potential to the worldvolume. The matrix \( a \) depends only on the worldvolume Born-Infeld field strength and is given by
\[ a = \begin{cases} -\frac{1}{2} Y_{jk} \gamma^{jk} \Gamma_{11} & \text{IIA}, \\ \frac{1}{2} Y_{jk} \sigma_3 \otimes \gamma^{jk} & \text{IIB}, \end{cases} \]
the \( \gamma^{jk} \) being worldvolume \( \gamma \) matrices,
\[ \gamma_i = \partial_i X^M \Gamma'_M \]
and \( Y \) is a function of \( \mathcal{F} \). The relation in the frame basis of the worldvolume (underlined indices) is
\[ Y'_{ik} := \tan^{-1} \mathcal{F}'_{ik}. \]
One can show that \( \Gamma \) satisfies,
\[ \text{tr} \Gamma = 0, \quad \Gamma^2 = 1. \]
We will be interested in the case of a D3-brane in a IIB background. A general D3-brane configuration will break part or all of the supersymmetries of the 10 dimensional background. The surviving supersymmetries must satisfy
\[ (1 - \Gamma) \xi = 0 \]
since only then can the \( \kappa \)-symmetry \(^{(53)}\) compensate for the transformation induced by \( \xi \), i.e. \( \xi = - (1 + \Gamma) \kappa \) for some \( \kappa \). The existence and the amount of unbroken supersymmetry impose severe conditions on the brane configuration. In this paper, the D3-brane is treated as a brane probe in the \( AdS_5 \times S^5 \) background (this is appropriate in the limit of large \( N \)) and so \( \xi \) are just the 32 supersymmetries of this background.

The \( AdS_5 \times S^5 \) background

The \( AdS_5 \times S^5 \) background of IIB relevant to Maldacena’s proposal \(^{(1)}\) is given by the metric of \( AdS_5 \times S^5 \) and a nontrivial RR 5-form field strength. Denote the 10 dimensional coordinates by \( X^0, \cdots, X^9 \). The metric is

\[
\frac{1}{R^2} ds^2 = \frac{1}{V^2} (- (dX^0)^2 + \cdots + (dX^3)^2) + \frac{1}{V^2} (dV^2 + V^2 d\Omega_5^2),
\]

where

\[
V^2 = (X^4)^2 + \cdots + (X^9)^2
\]

and the \( AdS \)-radius is \( R/\sqrt{\alpha'} = (4\pi g_s^2 N)^{1/4} \). The \( AdS_5 \) is described by the coordinates \( X^{0,1,2,3} \) and \( V \) while the \( S^5 \) is parameterized by the five angles of \( \Omega_5 \). The non-vanishing RR 5-form field strength is

\[
F_5 = \frac{4}{R} (\epsilon_{AdS_5} + \epsilon_{S^5}),
\]

where \( \epsilon_{AdS_5} \) and \( \epsilon_{S^5} \) are the volume forms on the \( AdS_5 \) and \( S^5 \) respectively. Explicitly, for example

\[
F_{0123} = 4R^4 \frac{V^3}{V^5}.
\]

For convenience, we will adopt a unit of \( R = 1 \) from now on. Explicit factors of \( R \) can be put back easily by simple dimensional arguments. This background can be thought of as the near horizon limit of the sugra solution of a D3-brane. See for example \(^{(38)}\) for a review on BPS branes in supergravity.

The gravitino \( \psi_M \) (complex-Weyl) transforms as

\[
\delta \psi_M = D_M \xi + \frac{i}{480} \Gamma^{M_1 \cdots M_5} F_{M_1 \cdots M_5} \Gamma'_M \xi,
\]

where \( \xi \) is a 32 component complex spinor of positive chirality,

\[
(1 - \Gamma^{11}) \xi = 0, \quad \Gamma^{11} := \Gamma^{01 \cdots 9}.
\]

One can take the following representation for the 10 dimensional \( \Gamma \)-matrices which is adapted to \( AdS_5 \times S^5 \),

\[
\Gamma_{\mu} = \sigma_1 \otimes 1_{4 \times 4} \otimes \gamma_{\mu}, \quad \mu = 0, 1, \cdots, 4,
\]

\[
\Gamma_m = \sigma_2 \otimes \gamma_m \otimes 1_{4 \times 4}, \quad m = 5, 6, \cdots, 9,
\]
where $\sigma_i$ are Pauli matrices and $\gamma_\mu (\gamma_m)$ are the 5 dimensional Dirac matrices satisfying
\[
\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}, \quad \{\gamma_m, \gamma_n\} = 2\delta_{mn},
\]
with $\gamma_4 := -i\gamma_{0123}$, $\gamma_9 := \gamma_{5678}$.

In this representation,
\[
\Gamma^{01\cdots 9} = \sigma_3 \otimes 1_{4\times 4} \otimes 1_{4\times 4},
\]
and (50) is solved by spinors of the form
\[
\xi = \left(\begin{array}{c} 1 \\ 0 \end{array}\right) \otimes \epsilon \otimes \eta
\]
with $\epsilon$ and $\eta$ being arbitrary $SO(1,4)$ and $SO(5)$ spinors. The IIB background has 32 supersymmetries of the form,
\[
\xi_1 = \left(\begin{array}{c} 1 \\ 0 \end{array}\right) \otimes \frac{1}{\sqrt{V}} \epsilon_0^+ \otimes \eta,
\]
\[
\xi_2 = \left(\begin{array}{c} 1 \\ 0 \end{array}\right) \otimes (\sqrt{V} + \frac{1}{\sqrt{V}} \hat{\phi}) \epsilon_0^- \otimes \eta,
\]
where $\epsilon_0^\pm$ are 4 components constants spinors satisfying
\[
-i\gamma_{0123}\epsilon_0^\pm = \pm \epsilon_0^\pm,
\]
so that $\epsilon_0^+ / \sqrt{V}$ and $(\sqrt{V} + \hat{\phi} / \sqrt{V})\epsilon_0^-$ are the Killing spinors of $AdS_5$, and $\eta$ is the Killing spinor of $S^5$. It has the form
\[
\eta = \Omega \eta_0,
\]
where $\eta_0$ is an arbitrary constant spinor and $\Omega$ is a certain rotational matrix whose details can be found in [39].

The question of preserved supersymmetries then simply boils down to finding out which of the $\xi_1$ and $\xi_2$ satisfy eq. (44), namely $\Gamma\xi = \xi$.

### 3.2 1/2 and 1/4 BPS Brane Configurations

We are now ready to analyze the supersymmetry preserved by a bosonic D3-brane in $AdS_5 \times S^5$. Note that the NS-NS two-form $B$ vanishes for our background.

We will consider $F_{ij} = 0$ as suggested by the SYM analysis above and hence
\[
\mathcal{F}_{ij} = 0
\]
so that the matrix $a$ vanishes. It is convenient to work with the complex spinor formalism in which the factor of Pauli matrices $\sigma_i$, for example in (35) and (40), can be avoided.
For a spinor \( \psi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \) the complex spinor is \( \psi_c = \alpha + i\beta \), and the correspondence is: \( i\sigma_2\psi \leftrightarrow -i\psi_c, \sigma_3\psi \leftrightarrow \psi^*_c \). Eqs (34) and (35) simply become
\[
\Gamma = i\sigma_2 \otimes \Gamma_{(0)} \leftrightarrow -i\Gamma_{(0)} .
\] (61)
All the D3 brane configurations we will consider will always be in the physical gauge
\[
X^i = \sigma^i, \; i = 0, 1, 2, 3 .
\] (62)
We start with a D3-brane sitting parallel to the AdS-boundary,

**Brane Config. 1 (1/2 BPS)**
\[
X^m = c^m \quad \text{where } c^m \text{ are constants and not all zero, } (m = 4, \cdots, 9).
\] (63)
It is trivial that the brane wave equation of motion is satisfied. The \( \kappa \)-symmetry \( \Gamma \) matrix (61) reduces in this case to
\[
\Gamma = -i\Gamma_{0123} .
\] (64)
Thus by eqs (44), (51) and (58) we see that \( \xi_1 \) is preserved and \( \xi_2 \) is projected out. The corresponding SYM theory is in the Coulomb branch with 16 preserved (Poincaré) supersymmetries. All the special supersymmetries are broken by the Higgs vev. Thus, we see the following identification of supersymmetries,

\[
\begin{array}{c|c}
\text{SYM} & \text{AdS}_5 \times S^5 \\
\hline
\text{Poincaré supersymmetry } \xi & \xi_1 \\
\text{Special supersymmetry } \zeta & \xi_2
\end{array}
\] (65)
The identification of supersymmetry between that of \( \mathcal{N} = 4 \) SYM and that of the \( \text{AdS}_5 \times S^5 \) sugra has been pointed out previously in [11]. We will see it in more detail below. We next consider

**Brane Config. 1′ (Unbroken SUSY)**
\[
X^m = 0 \quad (m = 4, \cdots, 9).
\] (66)
This means \( V = 0 \). It is interesting to note that if we let the D3-brane go to the AdS-boundary, \( V \to 0 \), then
\[
(1 + i\Gamma_{0123})\xi_2 \sim \frac{1}{\sqrt{V}}(1 + i\gamma_{0123})\phi \epsilon_0^- \\
= \frac{1}{\sqrt{V}}\phi (1 - i\gamma_{0123})\epsilon_0^- \\
= 0
\] (67)
due to (58). Thus \( \xi_2 \) is no longer projected out and we recover the full spacetime supersymmetry. This is consistent with the SYM picture of going back to the superconformal phase by letting the Higgs vev \( X^m \) go to zero. The special supersymmetries are recovered in this limit. Note that the structure of the “\( AdS_5 \)” part of the Killing spinor \( \xi_2 \) (i.e. \( (\sqrt{V} + \frac{1}{\sqrt{V}} \phi) \epsilon_0^- \)) is crucial here for this recovery of the special supersymmetries. It will not work, for example, for the Killing spinor of a flat metric or a multi-centered solution.

Next we consider the more general case with the D3-brane embedded as

**Brane Config. 2 \((1/4 \text{ BPS})\)**

\[
X_i = \sigma_i, \quad i = 0, 1, 2, 3, \quad (68)
\]
\[
X^{4,5,6,7} = 0, \quad (69)
\]
\[
X^8 = X^8(\sigma_1, \sigma_2), \quad X^9 = X^9(\sigma_1, \sigma_2), \quad (70)
\]

with \( X^8, X^9 \) satisfying the condition \((13)\).

Let’s first start with the slightly more general condition

\[
X^{4,5,6,7} = e^{4,5,6,7} \quad \text{arbitrary constants} \quad (71)
\]

instead of \((69)\). We will see shortly that we need \( X^{4,5,6,7} \) to be zero for two different reasons. We need it for the brane to preserve 1/4 supersymmetry, and we also need it for the brane to satisfy the equations of motion. We will comment on the significance of this requirement on the SYM side in the discussion section.

We first look at the supersymmetry preserving condition. Substituting \((68), (70)\) and \((71)\) into the definition of \( \Gamma_{(0)} \), we get,

\[
\Gamma_{(0)} = \frac{1}{4! \sqrt{|g|}} \epsilon^{i_1 \cdots i_4} \partial_{i_1} X^{M_1} \cdots \partial_{i_4} X^{M_4} \Gamma'_{M_1 \cdots M_4} \quad (72)
\]
\[
= \frac{1}{V^4 \sqrt{|g|}} (Q_0 + \alpha_1 Q_1 + \alpha_2 Q_2 + \alpha^2 Q_3), \quad (73)
\]

where

\[
\alpha_i := \partial_i X^8, \quad i = 1, 2, \quad (74)
\]
\[
\alpha^2 := \alpha_1^2 + \alpha_2^2 \quad (75)
\]

and

\[
g_{00} = g_{33} = \frac{1}{\sqrt{V}}, \quad g_{11} = g_{22} = \frac{1}{\sqrt{V}} (1 + \alpha^2), \quad (76)
\]
\[
V^4 \sqrt{|g|} = 1 + \alpha^2. \quad (77)
\]
The \( Q \)'s are numerical matrices

\[
\begin{align*}
Q_0 & := \Gamma_{0123}, & Q_3 & := \Gamma_{0389}, \\
Q_1 & := \Gamma_{0183} \mp \Gamma_{0923}, & Q_2 & := \Gamma_{0823} \pm \Gamma_{0193}.
\end{align*}
\]

where the \( \pm \) signs are correlated with the choice of sign in the condition (I3) and satisfy

\[
\begin{align*}
Q_0^2 &= Q_3^2 = -1, & Q_1^2 &= Q_2^2 = -2(1 \pm Q_4), \\
Q_0 Q_1 + Q_1 Q_0 &= Q_0 Q_2 + Q_2 Q_0 = 0, \\
Q_1 Q_2 + Q_2 Q_1 &= Q_1 Q_3 + Q_3 Q_1 = Q_2 Q_3 + Q_3 Q_2 = 0, \\
Q_0 Q_3 &= Q_3 Q_0 = Q_4,
\end{align*}
\]

with

\[
Q_4 := \Gamma_{1289}.
\]

With this \( \Gamma_{(0)} \), and remembering that \( \Gamma = -i\Gamma_{(0)} \), it is not hard to see that \( \xi_2 \) cannot satisfy the condition for unbroken supersymmetry (I4) and hence the corresponding supersymmetries are all broken. Notice that \( \xi_1 \) satisfies

\[
(1 + i\Gamma_{0123})\xi_1 = 0 \iff \xi_1 = -iQ_0\xi_1
\]

because of (58) and thus the condition (44) reads

\[-i(\alpha_1 Q_1 + \alpha_2 Q_2)\xi_1 = \alpha^2(1 + iQ_3)\xi_1.
\]

Apply \((\alpha_1 Q_1 + \alpha_2 Q_2)\) to the l.h.s. of (84) to get

\[
(\alpha_1 Q_1 + \alpha_2 Q_2)^2\xi_1 = 0,
\]

where the facts that \( Q_3 \) anticommutes with \( Q_1, Q_2 \) and \( Q_4^2 = -1 \) have been used. Using (80) and (82), we have

\[
(1 \pm Q_4)\xi_1 = 0.
\]

One can check that this is also the sufficient condition for \( \xi_1 \) to satisfy (14). Note that \( Q_4 \) is a constant matrix, while \( \xi_1 \) is a spinor that involves the Killing spinor \( \eta \) of \( S^5 \) at the point on \( S^5 \) where the D3 brane intersects it. Thus, in general, it depends nontrivially on the values of \( X^{4,5,6,7,8,9} \), and in particular on the worldvolume coordinates \( \sigma^{1,2} \). This dependence comes in through the factor \( \sqrt{V} \), but more importantly through the matrix \( \Omega \) that relates the \( S^5 \) Killing spinor \( \eta \) to the constant spinor \( \eta_0 \). It is thus clear that in general, (88) will not have any solution and there will be no unbroken supersymmetry, unless \((1 \pm Q_4)\) commutes with this \( \Omega \) matrix. This will be the case only if

\[
X^{4,5,6,7} = 0,
\]
since then $\Omega$ is reduced to a simple rotational matrix in the $X^8 - X^9$ plane and hence commutes with $\Gamma_{1289}$. So if one imposes also the condition

$$(1 \pm \Gamma_{1289})\epsilon^+_0 \otimes \eta_0 = 0$$

(90)

on the constant spinor $\epsilon^+_0 \otimes \eta_0$, then we can solve (88) for half of the $\xi_1$ and hence the brane configuration 2 preserves 1/4 of the supersymmetries. Notice that the two conditions satisfied by $\epsilon^+_0 \otimes \eta_0$,

$$(1 + i\Gamma_{0123})\epsilon^+_0 \otimes \eta_0 = 0, \quad (1 \pm \Gamma_{1289})\epsilon^+_0 \otimes \eta_0 = 0$$

(91)

are similar to the usual 1/4 supersymmetry conditions [40] which appear for orthogonally intersecting D-branes in flat target spacetime. Here however, the background is curved and we have a single D3-brane.

Note that the two conditions

$$(1 + i\Gamma_{0123})\xi = 0, \quad (1 \pm \Gamma_{1289})\xi = 0$$

(92)

satisfied by the unbroken supersymmetries $\xi$ are precisely the same as the supersymmetry preserving conditions in the four-dimensional SYM. The first eq. (92) says that $\xi_1$ is kept and $\xi_2$ is projected out. Using the mapping (65), this is translated to the condition that only the Poincaré supersymmetry $\xi$ can be unbroken. The second eq. (92) says that $\xi_1$ is annihilated by $(1 \pm \Gamma_{1289})$, which is precisely the same condition (18) on the SYM side. Thus we see that there is a precise matching of unbroken supersymmetries between certain backgrounds in $\mathcal{N} = 4$ SYM and certain configurations of a (single) D3-brane in $AdS_5 \times S^5$. Not just the amount of unbroken supersymmetries is equal, but even the part which is preserved can be identified consistently.

Finally we check that the brane configuration 2 satisfies the equations of motion. Naively, one might expect a simple Laplace equation in $AdS_5 \times S^5$ for the fields $X^n$ (suitably amended with terms to describe the interaction with the RR-background). This is suitable for describing a scalar propagating in a fixed geometry but is not suitable for the $X^n$, which are themselves coordinates of the target space. In our case, since we are using the physical gauge $X^{0,1,2,3} = \sigma^{0,1,2,3}$, the correct equations of motion are derived from the gauge fixed form of the Dirac-Born-Infeld action including the WZ-term,

$$I = \int d^4\sigma \sqrt{-\det(g_{ij} + F_{ij})} + \int C := I_{DBI} + I_{WZ},$$

(93)

where

$$g_{ij} = G_{ij} + G_{mn}\partial_i X^m \partial_j X^n$$

(94)

is the induced worldvolume metric in the physical gauge and $C$ is the pullback to the worldvolume of the RR 4-form potential. The field strength of $C$ is given by (47). The
equations of motion obtained from (93) are nonlinear in the field $X^n$ and are much more complicated than a simple linear Laplace equation. We don’t know of a general way of solving them but we can check whether they are satisfied. Define for convenience

$$I_n := \delta I / \delta X^n - \partial_j (\delta I / \delta (\partial_j X^n)).$$

(95)

For configurations satisfying (70), we simply get

$$I_{DBI,n} = -\frac{4X^n}{V^6} (1 + \alpha^2), \quad n = 4, 5, 6, 7,$$

(96)

$$I_{DBI,n} = -\frac{4X^n}{V^6}, \quad n = 8, 9,$$

(97)

and for $n = 4, \cdots, 9$

$$I_{WZ,n} = \frac{\partial X^{m_0}}{\partial \sigma_0} \cdots \frac{\partial X^{m_3}}{\partial \sigma_3} F_{nm_0 \cdots m_3}$$

$$= \frac{X^n}{V} F_{V0123}$$

$$= \frac{4X^n}{V^6}$$

(98)

where eqs (17) and (18) have been used. Hence

$$I_n = -\frac{4\alpha^2 X^n}{V^6}, \quad n = 4, 5, 6, 7,$$

(99)

$$I_n = 0, \quad n = 8, 9.$$

(100)

The Euler-Lagrange equations $I_n = 0$ for all $n$ thus demand that $X^{4,5,6,7} = 0$, exactly the same condition as we got for preserving 1/4 of the supersymmetries.

Before we move on, it is important to stress that in performing the checks of this paper, we have made the identification (46), instead of the one

$$U^2 = (X^4)^2 + \cdots + (X^9)^2.$$  

(101)

used in [13]. Their $U$ is our $1/V$. For example, consider the brane configurations 1 and 1’ of a D3-brane sitting parallel to the boundary. Had we used (101), then the conformal point ($X^n = 0$) of SYM will be identified with $U = 0$, (i.e. $V = \infty$ in our notation). Then in this limit $\xi_2$ would be $\sqrt{V \epsilon_0}$ (rather than our $\frac{1}{\sqrt{V \epsilon_0}}$) and the whole $\xi_2$ would be projected out in the limit. Our identification (101) was also used in [13] to reproduce the YM instanton measure from the D-instanton action in $AdS_5 \times S^5$.  

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3.3 1/8 BPS Brane Configurations

From the discussion above, it should be obvious that the following configuration is 1/8 BPS,

**Brane Config. 3 (1/8 BPS)**

*Two D3-brane probes in AdS$_5 \times S^5$, one satisfying the conditions (68)-(70) and the other satisfying the analogous conditions with the roles of $X_{8,9}$ and $X_{6,7}$ interchanged.*

The reason is obvious, the two D3-branes give rise to the following conditions respectively

\[
(1 + i \Gamma_{0123}) \epsilon_0^+ \otimes \eta_0 = 0, \quad (1 \pm \Gamma_{1289}) \epsilon_0^+ \otimes \eta_0 = 0,
\]

\[
(1 + i \Gamma_{0123}) \epsilon_0^+ \otimes \eta_0 = 0, \quad (1 \pm \Gamma_{1267}) \epsilon_0^+ \otimes \eta_0 = 0.
\]

As a result, the brane system preserves 1/8 of the spacetime supersymmetries. The three conditions (102), (103) on the constant spinor $\epsilon_0^+ \otimes \eta_0$ are precisely the same as the conditions one would get for three D3-branes in flat space-time, and with worldvolumes in the 0123, 0389 and 0367 directions respectively.

It is instructive to recall that in the case of flat target spacetime, one can break the spacetime supersymmetry to 1/8 by having a triple intersection of D3-branes. Here, we produced the mathematically similar structure (102), (103) by having two D3-branes in AdS$_5 \times S^5$. This suggests to look at other ways to get the same amount of supersymmetry but with a D3-brane and another D$p$-brane, for example a D-instanton. The desired combination of a D-instanton and a D3-brane (Brane Config. 4) can be accounted for by the Higgs-Instanton configuration 4 of the last section. The D-instanton corresponds to the non-abelian YM instanton configuration [10], while the D3-brane corresponds to the Higgs configuration.

As suggested by the Higgs-Gauge configuration 5, we now discuss a non-vanishing (abelian) Born-Infeld field strength on the D3-brane. In particular, we consider the “(anti) self-dual” Born-Infeld field $F$ satisfying

\[
F_{ij} = \frac{c}{2} \epsilon_{ijkl} F^{kl},
\]

with $c = -i$ for the self-dual and $c = i$ for the anti self-dual case. This is the simplest and most natural definition one can adopt in a nonlinear theory like the Born-Infeld one. The same remarks concerning the Euclidean continuation as in the last section apply.

Now consider the following *single* D3-brane with

\[\text{Notice that for the IIB background we are interested in, } B \text{ is zero and so the Born-Infeld field strength is } F = dA.\]
Brane Config. 5 \((1/8 \text{ BPS})\)

The positions \(X^m\) of the D3-brane satisfy the conditions of brane configuration 2 and the Born-Infeld \(\mathcal{F}\) is self-dual: \((104)\) with \(c = -i\).

In this case, \(\Gamma\) is given by
\[
\Gamma = e^{-\sigma_3 \otimes \Delta} (\sigma_2 \otimes \Gamma_{(0)}) e^{\sigma_3 \otimes \Delta}
\]
(105)

where
\[
\Delta := Z \cdot (1 - c \Gamma_{(0)})
\]
(106)

where \(\Gamma_{(0)}\) is given by \((72)\) and \(Z\) is some matrix depending on the Born-Infeld field strength whose details are not important to us. Remember that
\[
\sigma_3 \otimes \Delta \xi \leftrightarrow \Delta \xi^*
\]
(107)

So if we impose the reality condition on \(\xi\)
\[
\xi = \xi^*
\]
(108)

and impose
\[
(1 - c \Gamma_{(0)}) \xi = 0
\]
(109)

then the supersymmetry preserving condition
\[
(1 - \Gamma) \xi = 0
\]
(110)

becomes
\[
(1 + i \Gamma_{(0)}) \xi = 0.
\]
(111)

Equations \((109)\) and \((111)\) are only compatible for \(c = -i\), so only then can one have any unbroken supersymmetry \((1/8\) of the original 32). We will comment on this in the discussion section. Note that such a construction with a non-vanishing \(\mathcal{F}\) field could also be done for the brane configurations 1 and 1’ resulting in 1/4 or 1/2 preserved supersymmetries.

Summarizing, we propose the following duality map between backgrounds in SYM and brane configurations in \(AdS_5 \times S^5\):

\[
\begin{align*}
\text{SYM} & \\ 
\text{Higgs scalars } X^m & \leftrightarrow \text{position } X^m \text{ of D3-branes} \\ 
\text{YM field strengths } F_{ij} & \leftrightarrow \text{Born-Infeld strength } \mathcal{F}_{ij} \text{ on the D3-branes}
\end{align*}
\]
(112)
4 Discussion

In this paper, we presented D3-brane configurations that preserve 1/n (with \(n = 2, 4, 8\)) of the original 32 spacetime supersymmetries. These are in perfect correspondence with the SYM Higgs/Gauge configurations that preserve \(\mathcal{N} = 4, 2, 1\) supersymmetry of the original \(\mathcal{N} = 4\) conformal supersymmetry. We demonstrated that when one identifies the values of the Higgs scalars and the YM field strength \(F_{ij}\) with the positions and the Born-Infeld field strength \(\mathcal{F}_{ij}\) of the D3-branes in \(AdS_5 \times S^5\), one can match not just the amount of unbroken supersymmetry, but also establish consistently a 1-1 mapping between the 4 dimensional Poincaré and special supersymmetries on the one side, and the spacetime supersymmetries of \(AdS_5 \times S^5\) on the other side. In particular, for example, we have shown that a single unwrapped D3-brane in \(AdS_5 \times S^5\) satisfying the Cauchy-Riemann condition (70) breaks 3/4 of the spacetime supersymmetries.

Given the nice matching of supersymmetries for the examples studied here, it is natural to think that this matching and the proposal of [15] will work for the more general cases with general non-vanishing YM and Born-Infeld field strengths, and probably even to full generality with nonvanishing fermion fields.

We saw in the last subsection that while the D3 brane with a self-dual Born-Infeld configuration preserved 1/8 supersymmetry, the anti self-dual one breaks all of it. Let us now comment on this. It has been shown [16] that a fundamental (F) string or a D-string extended in the \(\sigma_i\) directions can be represented by an electric or magnetic flux of the Born-Infeld field strength. Hence the self-dual brane configuration 5 can be identified with a F1 - D1 - D3 system, while the anti self-dual one can be identified with a F1 - anti-D1 - D3 system. So one can also understand from this point of view why the first preserves some supersymmetry, while the latter breaks it all.

Due to its importance, we stress again that the condition (69), \(X_{4,5,6,7} = 0\) is crucial for the brane configuration 2 to preserve supersymmetry and to satisfy the equations of motion. However, on the SYM side, there seems to be no need to impose this condition in order to preserve 1/4 supersymmetry. All we need is the holomorphicity condition (13). We recall [2, 3] that in the AdS/CFT correspondence, it is the classical tree level sugra action that becomes the generating functional for the full quantum gauge theory correlation functions in the large \(N\) limit with \(g_{YM}^2 N\) fixed but large. For example as studied in [3, 43], the Chern-Simon couplings on the sugra side generate the anomalous correlation functions for the R-symmetry current of the four dimensional SYM. Our check of supersymmetry in section 2 is a classical analysis. This is expected to be modified in a full quantum analysis. Upon quantizing the SYM theory in our non-trivial background Higgs configuration, it could well be that a scalar potential is generated due to infrared effects. Another possible modification to the classical equations of motion is through higher derivative terms in the effective action. The point is that, for example, a hypothetical term \(\sim (X^4)^2(\partial X^8)^4\) usually plays no role in low-energy considerations, but with our background it would
typically generate a mass-like term $\sim (X^4)^2$ and hence would be relevant. The fact the single condition \[(39)\] serves two different purposes (supersymmetry and equations of motion) is not obvious a priori. This adds to our confidence that it will eventually also arise on the SYM side.

Similar phenomena appear in the cases of 1/8 supersymmetry with non-vanishing field strength. We recall that with the gauge field turned on, on the YM side, one can preserve 1/8 supersymmetry with a self-dual as well as an anti self-dual configuration, while on the D3-brane side, we saw that the configuration can preserve 1/8 supersymmetry only with a self-dual but not an anti self-dual Born-Infeld field. The situation could be brought back into harmony if we remember (again) that the analysis we did on the SYM side is a classical one. We expect that if one does a careful job of quantization, then the anti self-dual gauge theory configuration can be seen to break all supersymmetries. A similar situation occurs in the $\mathcal{N} = 2$ Seiberg-Witten theory where only instantons (no anti-instantons) contribute to the prepotential.

For the case of our brane configuration 2 of a single D3-brane preserving 1/4 of the supersymmetries, an intuitive way to think about the pair of conditions \[(71)\] is that before one takes the large $N$ limit, one has in fact a D3-brane with worldvolume in the 0389 directions intersecting a large stack of D3-branes with worldvolume in the 0123 directions and sitting at $X^m = 0$. It would be interesting to see how the Cauchy-Riemann condition \[(13)\] emerges \[(11)\] when one takes the near horizon limit of the sugra solution for this system of branes.

In this paper, we have treated the D3-brane as a brane probe in the $AdS_5 \times S^5$ background. This is appropriate for the large $N$ limit of the AdS/gauge theory correspondence. However, as an independent question, it would be interesting to work out the sugra solution with the D3-brane as a brane source corresponding to the various cases we considered here. The metric should be asymptotically $AdS_5 \times S^5$ and the RR-flux should jump as one crosses the brane \[(15)\]. It is particularly interesting to obtain the sugra solution corresponding to the “holomorphically” embedded case.

The agreement of supersymmetry is of course a necessary condition for the desired duality to work. Having obtained $\mathcal{N} = 2, 1$ four dimensional gauge theories, one may wonder what can one learn about their correlation functions from studying the corresponding sugra picture or vice versa. Many 3-point and 4-point correlations function of the $\mathcal{N} = 4$ superconformal Yang-Mills has been computed \[(11, 17, 48, 19, 50)\] and precise agreements were found with the direct gauge theory calculations.

Quantum modifications of equations of motion and BPS conditions in $\mathcal{N} = 2$ SYM have been studied in \[(44)\].
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