Effect of the anisotropy of monocrystalline silicon mechanical properties on the dynamic characteristics of a micromechanical gyroscope

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Abstract. The aim of the research was to determine the effect of temperature on mechanical properties of a micromechanical gyroscope with the sensing element mounted on a silicon wafer, with the crystallographic orientation of (100) (110) (111). The research is of current relevancy since the metrological characteristics that depend on the eigenfrequencies over the full temperature range are to be controlled. The temperature-modal analysis of the micromechanical gyroscope model was performed with ANSYS program. The temperature dependence for eigenfrequencies was obtained. The dependence of the scale factor on temperature for the most temperature-independent variant of sensor positioning on the wafer was determined. The developed mathematical model was used to find the forms of the output oscillations of the gyroscope.

1. Introduction
The perspectives of modern instrument making are related to the development of the devices of low weight, small dimensions, low cost, power consumption and high reliability. Microelectromechanical sensors (MEMS) meet these requirements. The main advantages of the devices are as follows.

- **Small parameter spread within the finished product.** Manufacturing of components in a single process cycle allows production of components with almost similar parameters.
- **High manufacturability and repeatability.** Well-proven and controlled microelectronics process technologies are used in MEMS manufacturing, which allow the product to obtain the desired characteristics. Lack of assembly operations improves manufacturability of complex systems.
- **Subminiature.** Chip technology makes micromechanical components much smaller than those made with traditional technologies.
- **High functionality.** Miniature products and the ability to manufacture sensors processing circuits and actuators in one device enable to create complete systems of sufficiently high complexity in a single chip, comparable in dimensions to the integrated circuit (IC).
- **Improved performance of operation.** Electronics and the electrical channels for connection between sensors and mechanics based on integrated technology and small dimensions of the chip can improve the characteristics of the gyro such as the operating frequency, signal / noise ratio, etc.
- **Low power consumption.**
- **Low cost.** The cost of MEMS-based devices is much lower than that of the devices made without using integrated technology. Low cost of the MEMS-based final product depends on its serial production and MEMS components in the product.

The areas of MEMS application are expanding. The market in the field of manufacturing of MEMS inertial sensors - microaccelerometers and microgyroscopes, the most complex MEMS devices, is expected to exponentially grow in the next few years [1]. The sensing element of the micromechanical gyro (MMG) is a movable structure made of single-crystal silicon.

2. **Description and architecture of MMG**

The effect of the anisotropy of mechanical properties of single-crystal silicon on the dynamic characteristics of an LL-type micromechanical gyro [2] was investigated under stationary thermal fields.

![Figure 1. Schematic diagram of the silicon micromechanical gyroscope sensor.](image)

The gyro contains an electromechanical silicon sensor (figure 1) and an electronic unit. The sensor includes frame 3 mounted on elastic suspensions 10 in base 9. The frame contains two sensitive masses 1 and 6 with mutually perpendicular suspension axes. The frame with sensitive masses produces primary oscillations along Z axis, which are excited by comb vibrodrive 13 or by a planar vibrodrive in the form of two planar electrodes located under and above the inertial mass.

One of the electrodes in the form of metal deposition is located on the basis, and the other one is represented by frame 3. Presence of angular velocities of the object rotation causes the Coriolis forces. The forces are proportional to these angular velocities and cause secondary informative oscillations of the sensitive mass along X and Y axes. The oscillation amplitude of sensitive mass 6 along Y axis is proportional to the angular velocity of the object around X axis. The oscillation amplitude of mass 1 along sensitive X axis is proportional to the angular velocity of the object around Y axis.

Comb structures 4 and 8 are used to obtain data on the angular velocity of the object. Comb structures 5 and 7 are used to correct the errors of the gyroscope. Comb structure 14 is used to measure the amplitude of primary oscillations in the system controlling the gyroscope primary oscillations. Thus, MMG has two measuring axes, and represents a system of three oscillators.
3. **Properties of single-crystal silicon**

The silicon structure is a diamond-type lattice with tetrahedral bonds [3]. Each atom has four nearest neighboring atoms. These atoms interact due to forces of the covalent bonds. Packing of atoms in plane (111) (or direction (111)) in the complex silicon structure is more dense compared to that in plane (100) (direction (100)). Physical properties of silicon in different crystallographic directions are different [4-5]. Quantitative relationships for the elastic modulus, shear modulus and Poisson's ratio in different crystallographic directions are determined by the generalized Hooke's law for anisotropic media.

Metrological characteristics of MMG considerably depend on the material of the oscillator. Oscillators of high-Q and high amplitude oscillating provide high precision measurements of the angular velocity of the base. Therefore, the maximum match operating frequency of the model with a desired frequency is to be achieved, when calculating eigenfrequencies and modes of the sensor.

Silicon, like all materials changes its mechanical properties depending on temperature. This can significantly affect the dynamic characteristics of MMG which in many ways depend on the stability of the scale factor. Stable relationship between the construction eigenfrequencies is required to make the scaling factor constant. Slight deviation of the oscillator eigenfrequencies results in a significant change in the scale factor. The eigenfrequencies of the device depend on the hardness of the elastic suspension; therefore, the stability of this parameter should be maintained with high accuracy.

Analytically, the natural frequency of the oscillator is related to the stiffness of the elastic suspension by the expression

$$\omega_i = \sqrt{\frac{G}{m}},$$

where G is the stiffness of elastic suspension, m is the inertial mass.

The temperature effect causes the drift of the silicon elastic characteristics and changes the linear dimensions of the structure. This causes the internal stresses, the geometry of the construction changes, temperature imbalance arises, and the dynamic characteristics of MMG change [6].

To estimate the changes in the eigenfrequencies of the oscillator in the operating temperature range from minus 50 to plus 80, the temperature-modal analysis of MMG was performed. The material of the construction was single-crystal silicon with anisotropy of the mechanical and thermal properties [7]. The research was carried out for five the most commonly used variants of sensor positioning on the wafer depending on the plane and the orientation of the wafer (table 1).

| Wafer orientation | Variant 1 (100) | Variant 2 (100) | Variant 3 (110) | Variant 4 (110) | Variant 5 (111) |
|-------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Orientation of crystalllographic axis | X <100> | X <110> | X <100> | X <111> | X <1-12> |
| | Y <010> | Y <011> | Y <011> | Y <21> | Y <1-10> |
| | Z <001> | Z <001> | Z <0-11> | Z <0-11> | Z <111> |

The mechanical and thermal characteristics of silicon, such as Young's modulus, shear modulus and temperature coefficient of Young's modulus were set for each variant of its positioning in an explicit form [8]. Due to the cubic symmetry of single-crystal silicon, the anisotropic mechanical properties can be determined by the tensor of second rank with three independent stiffness coefficients or crystal compliance coefficients. Appropriative mechanical and thermal characteristics of the crystal for each variant can be found by converting the tensor components of the initial coordinate system to the tensor components of the target coordinate system using the expression [9]:

$$T_i^j = a_{ik}a_{j\ell}T_{\ell k},$$
where $T^i_j$ is tensor components of the target coordinate system, $a_{ik}, a_{jl}$ are the matrices of direction cosines, $T^k_l$ is tensor components of the initial coordinate system.

### 4. Research and results

The research was carried out using ANSYS. The finite element model (FE) of the test MMG is shown in figure 2.

![MMG Silicon sensor](image)

**Figure 2.** MMG Silicon sensor.

When the sensor geometry had been designed, the modal analysis was performed. figures 3, 4 and 5 show the first vibrational modes of the sensor. As can be seen, that the movable sensor parts make the required movements along X, Y and Z axes with the eigenfrequencies which differ in no more than 5%.

![Sensor and inertial body](image)

**Figure 3.** The first mode of sensor eigenfrequencies ($f_1 = 5032.57$ Hz).

The first form of the gyro eigenfrequency corresponds to its informative vibrations due to the presence of angular velocity $\Omega$. While vibrating, the inertial body 6 makes a linear movement along Y axis (figure 3).
The first form of the gyro eigenfrequency corresponds to its informative vibrations due to the presence of angular velocity $\Omega_y$. While vibrating, the inertial body 1 makes a linear movement along X axis (figure 4).

The third form of the gyro eigenfrequencies (figure 5) corresponds to its forced vibrations under electrostatic forces generated by the vibrodrive. Along Z axis, the frame and inertial bodies move as one body. The difference between the frequencies of the first three modes of vibration is no more than 1.33%.

The initial operating temperature of the sensor corresponding to the nominal dimensions of the gyro sensor and its initial mechanical properties was accepted as 20 °C. The operating temperature range was from −50 °C to + 80 °C.

The dependences for the change in the MMG eigenfrequencies due to changes in the thermoelastic state of the structure are shown in figures 6, 7 and 8.
Figure 6. The first mode of eigenfrequencies (Y axis).

Figure 7. The second mode of eigenfrequencies (X axis).
Figure 8. The third mode of eigenfrequencies (Z axis).

Absolute and relative changes in the eigenfrequencies for five variants of sensor positioning on the plate are shown in table 2 and figure 9, respectively.

Table 2. Absolute changes in gyro eigenfrequencies.

| Mode | Variant 1 (100), Hz | Variant 2 (100), Hz | Variant 3 (110), Hz | Variant 4 (110), Hz | Variant 5 (111), Hz |
|------|---------------------|---------------------|---------------------|---------------------|---------------------|
| 1    | 0.60384             | 0.60384             | 1.18252             | 1.0064              | 1.15736             |
| 2    | 0.2517              | 0.256734            | 0.55374             | 1.10748             | 1.08231             |
| 3    | 1.63168             | 1.5297              | 1.17277             | 3.5693              | 3.31435             |

Figure 9. Relative changes in gyro eigenfrequencies.

The wafer with crystallographic orientation (100) is found to be the least sensitive over the full range of temperature change (figure 9).

To obtain the temperature dependence for scale factor the mathematical model was made. It consists of two nonlinear differential equations of second order, which reflects the dynamic behaviors of MMG. The mathematical model contains temperature dependence for eigenfrequencies, stiffness coefficients and modal mass approximated by second-order polynomials.

The resulting dependence of the scale factor vs temperature is shown in figure 10.
Figure 10. Dependence of scale factor vs temperature.

To reduce the effect of temperature on the accuracy of the angular velocity measurement the temperature stabilization system is to be used. The system is a combination of capacitive sensors to create positive or negative stiffness. The dependences reflecting the drift of the eigenfrequencies are virtually linear. This simplifies the thermal stabilization algorithm.

As a result of the research, direction (100) is found to be the most suitable sensor positioning on the silicon wafer. It causes the least deviation of the eigenfrequencies and the scale factor in the operating temperature range.

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