Abstract

A recent proposal for quantizing gravity is investigated for self consistency. There are well-known difficulties in dealing with Einstein gravity when resorting to the perturbative techniques of quantum field theory. This however does not preclude the existence of a quantum form. This Letter is all about such a subtle but important difference.

January 1997
1 Introduction

The unification of gravity and the standard model describing strong and electroweak interactions, thirty years after the formulation of the latter, has yet to be achieved and is a formidable task. Conceptual progress was made by introducing an effective theory for processes at low energy (typically less than the Planck mass \( M_P \)). However, a fundamental theory of gravity is still lacking. Our ignorance is parameterized by the renormalized counter terms of the corresponding effective Lagrangian. At very low energy, the theory is provided by general relativity. The natural question to address is then the following: what about quantum corrections? Donoghue tackled this issue \(^2\) and we repeated his calculation of the first quantum correction to the Newtonian potential \(^3\). The validity of the abovementioned low-energy results holds, irrespectively of any definite proposal for the complete quantum theory of gravity.

In \(^4, 5\) an approach to quantizing gravity with a modified renormalization scheme was proposed, taking a massive scalar field interacting with gravity as an example. In this Letter we take up the issue of self consistency of a candidate theory for the perturbative quantization of gravity. For the sake of the simplicity, we focus on the theory of self-interacting massive scalar fields coupled to quantum gravity.

We plan this Letter in the following way.

We begin in Section 2 by describing the essential features of the perturbative quantization of gravity. We start out with a maximal gravity and then proceed to pin down physical criteria that lead us to Einstein gravity quantized. We then discuss in Section 3 the self consistency of this renormalized theory of quantum gravity for a scalar field. We finish this Letter in Section 4 by offering our remarks on the non-perturbative outlook.

2 Orthodox Gravity: the perturbative approach

A recent proposal for the perturbative quantization of gravity \(^4, 5\) suggests the quantization of an extended Lagrangian for gravity, such that the counter terms are all accommodated within the Lagrangian and so renormalization is formally achieved. In this Section, following and extending on this proposal, we introduce Einstein gravity quantized. Why the strange word-
ing? Having granted that Einstein gravity cannot be dealt with by resorting to the traditional perturbative quantum field theoretical techniques, this does not preclude the existence of a quantum form, and this Section deals with this subtle but important difference.

It is well known that the abovementioned difficulties with the quantization of Einstein gravity arise for the simple reason that the infinities cannot be accommodated within the starting Lagrangian. For the purpose of illustration we will be discussing minimal coupled gravity with massive scalar particles, as governed by the Lagrangian:

$$L = \sqrt{-g} \left( -2\Lambda + R + \frac{1}{2}g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) + \frac{1}{2}m^2\phi^2 \right)$$ (1)

The counter terms that carry the infinities cannot be accommodated back within this starting Lagrangian, and so the theory retains its divergent nature.

One often speaks of the starting Lagrangian as the classical Lagrangian arguing that this is the starting point for quantization, while the final Lagrangian, which is of the same form, is referred to as the quantized Lagrangian. This is a misleading notation, as the original Lagrangian is divergent, having taken up the counter terms, and the classical limit actually arises in the $\hbar \to 0$ limit from the final complete Lagrangian. This distinction will become especially poignant in what follows.

Having noted that the difficulty in the quantization of minimal gravity stemmed from the fact that the counter terms did not fall back into the starting Lagrangian, one can resort to extending the Lagrangian so as to ensure that the theory is ‘formally’ renormalizable. In this way we arrive at what we can call maximal gravity. This extended starting Lagrangian is constrained by symmetry to be:

$$L_0 = \sqrt{-g_0} \left( -2\Lambda_0 + R_0 + \frac{1}{2}p_0^2 + \frac{1}{2}m_0^2\phi_0^2 + \frac{1}{4!}\phi_0^4\lambda_0(\phi_0^2) + p_0^2\phi_0^2\kappa_0(\phi_0^2) + R_0\phi_0^2\gamma_0(\phi_0^2) + \cdots \right)$$

(2)

(using units where $16\pi G = 1$, $c = 1$)

where $p_0^2$ is shorthand for $g_0^{\mu\nu}\partial_\mu\phi_0\partial_\nu\phi_0$ and not the independent variable of Hamiltonian mechanics. $\lambda_0, \kappa_0, \gamma_0, a_0, b_0, \alpha_0, a_0 \ldots$ are arbitrary analytic functions, and the second line carries
all the higher derivative terms. Strictly this is formal in having neglected gauge fixing and the resulting presence of ghost particles.

Quantum anomalies arise from a conflict between symmetries, where only one can be maintained [6]. For this reason no such trouble is anticipated here. At this point some remarks are in order, to clarify why there is no trouble with a gravitational anomaly in our case. We are a bit cavalier on this point, but anomalies do not just turn up to break a symmetry (one can always fix things, so that the symmetry is restored). The trouble arises when there are two symmetries (say the conformal one also). Then one can restore any symmetry, except the fixing is different for each symmetry, and the two fixings tend to be in conflict. One could, for example, get rid of the conformal anomaly in massless gravity, but at the price of a worse anomaly. All this is detailed in Mann’s review [6].

The price for having achieved ‘formal’ renormalization, is that the theory (with its infinite number of arbitrary renormalized parameters) has only a limited predictive content. As explained in [2], the measurement of a finite number of parameters is needed, in order to make a class of predictions at a given accuracy for a specified energy scale. The failure to quantize has been rephrased from a problem of non-renormalizability to a difficulty in the predictability.

Despite this, after renormalization we are led to:

$$L = \sqrt{-g} \left( -2\Lambda + R + \frac{1}{2}p^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\phi^4\lambda(\phi^2) + p^2\phi^2\kappa(\phi^2) + R\phi^2\gamma(\phi^2) \right. \\
\left. + p^4a(p^2, \phi^2) + Rp^2b(p^2, \phi^2) + R^2c(p^2, \phi^2) + R_{\mu\nu}R^\mu\nu d(p^2, \phi^2) + \ldots \right) \quad (3)$$

However, there remain physical criteria to pin down some of these arbitrary factors. Since in general (except for special cases) the higher derivative terms lead to acausal classical behavior, their renormalized coefficient can be put down to zero on physical grounds. This is however not always correct, for there exist particular cases of Lagrangians having higher derivative terms, where the ghost poles cancel, the simplest such example being the Gauss-Bonnet term. Hence, there are higher derivative lagrangians which do not introduce propagating Weyl ghosts and so do not spoil unitarity. The corresponding terms in the action yield a topological invariant, whose metric variation vanishes identically, so one is left (in 4 dimensions) with a smaller number
of arbitrary renormalized coefficients, whose value can only be determined from experiments. To quote from the book by Birrell-Davies [7] (page 162): “In principle there is no reason why these renormalized quantities may not be set equal to zero, thus recovering Einstein’s theory. Quantum field theory merely indicates that terms involving higher derivatives of the metric are a priori expected.” The simplest approach is to force the corresponding renormalized coefficients to vanish.

This still leaves the three arbitrary functions: $\lambda(\phi^2)$, $\kappa(\phi^2)$ and $\gamma(\phi^2)$, associated with the terms $\phi^4$, $p^2\phi^2$, and $R\phi^2$ respectively. The last may be abandoned on the grounds that its coefficient must be extremely small, not to defy the equivalence principle. To see this, begin by considering the first term of the Taylor expansion, namely $R\phi^2$; this has the form of a mass term, hence one would be able to make local measurements of mass to determine the curvature, and so contradict the equivalence principle (charged particles, with their non-local fields, have this term present with a fixed coefficient). One could of course retain such terms, at the price of abandoning the equivalence principle, with a coefficient small enough, to be tolerable from the observational standpoint. We take, as explained above, the simplest approach. The same line of reasoning applies to the remaining terms, $R\phi^4$, $R\phi^6$, etc.

This leaves us the two remaining infinite families of ambiguities with the terms $\phi^4\lambda(\phi^2)$ and $p^2\phi^2\kappa(\phi^2)$. In the limit of flat space in 3+1 dimensions this will reduce to a renormalized theory in the traditional sense if $\lambda(\phi^2) = \text{constant}$, and $\kappa(\phi^2) = 0$. So one is led to proposing that the physical parameters should be

\begin{align}
\kappa(\phi^2) &= \gamma(\phi^2) = 0 \\
a(p^2, \phi^2) &= b(p^2, \phi^2) = c(p^2, \phi^2) = d(p^2, \phi^2) = \ldots = 0 \\
\lambda(\phi^2) &= \lambda = \text{scalar particle self coupling constant} \\
m &= \text{mass of the scalar particle}
\end{align}

\(4\)

Tests of the equivalence principle and bounds on the deviations from Newton’s inverse square law constrain also the antigravity fields advocated in $N = 2, 8$ supergravity [8, 9]. For a discussion of possible violations of the equivalence principle, see also [10].

Our approach does not make necessary to set the cosmological constant to zero. One can perhaps advocate only empirical reasons for doing so. An explanation of the shadowing of the cosmological constant effect by those of newtonian gravity has been given recently for superconformal invariant theories [11].

\(5\)
and so the renormalized theory of quantum gravity for a scalar field will have the Einstein form:

\[
L = \sqrt{-g} \left( -2\Lambda + R + \frac{1}{2}p^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\lambda\phi^4 \right)
\]  

(5)

This is a candidate for the long sought after Einstein gravity quantized, and not quantized Einstein gravity; and the classical theory arises in the $\hbar \rightarrow 0$ limit of this.

3 Self-consistent gravity

One might now worry about the renormalization group pulling the coupling constants around. Since we are interested only that the zeroed couplings remain so, we shall name them as external couplings, in so much as they belong to terms outside the final renormalised Lagrangian (eq. 5). The finite number remaining will naturally take up the designation of internal couplings.

When a coupling runs, its value at some energy scale must be specified. The beta functions then determines how the coupling varies for other scales. It can now be seen to be a trivial matter to stop the external couplings from running, namely by zeroing them at an infinite scale.

Notice that in the previous Section the coupling $G$ was not omitted, but set to unity. This is a matter of convenience and is not supposed to suggest that $G$ cannot run. So, strictly $G$ must be restored before embarking in any future calculation, a thing we are presently doing (with $G$ in the Feynman rules).

4 Non-perturbative perspective

The above argument was done completely within a perturbative context, and one might wonder if a non-perturbative perspective would lead to the same proposal, and then perhaps without the infinities of the perturbative approach.

4.1 Ashtekar variables approach

We wish here to just point out that there exists a very dynamic program, i.e. the Ashtekar variables approach \cite{12}, trying precisely to quantize gravity in a non-perturbative fashion.\footnote{This approach is not limited to the non-perturbative investigation.}
is at this point yet unclear, whether Einstein gravity can be quantized along these lines.

4.2 The high tension string

String theory might be thought of as another attempt to quantize gravity by generalizing away from point particle theory (supergravity having failed).

When viewing string theory as a higher derivative, infinitely large Lagrangian, one sees many similarities with orthodox gravity, excepting that string theory has only one, and not an infinity, of extra parameters in the form of the string tension. It then becomes very natural to wonder about the point particle limit of the superstring, when one anticipates the appearance of supergravity. There immediately arises a question of how to resolve the fact that supergravity is not renormalizable (at least using traditional quantum field theory techniques), but that the string in the high tension limit exists. The above investigation makes the resolution rather transparent in so much as the starting Lagrangian is not that of supergravity even in the limit, for one has no reason to suppose the higher derivative bare terms disappear in the high tension limit. Again, only the theory quantized reduces to supergravity.

In this way one might view orthodox gravity as the point particle (high tension) limit of the string, and as such it is a second confirmation of the existence of orthodox gravity as a candidate for gravity quantized.

4.3 Occam’s gravity

We generalised to supergravity and string theory because our former candidates failed to give us quantum gravity. But why go to the complexities of higher dimensions, or a new set of particles if we can locate a simpler candidate? Naturally, at the end of the day, it is not a choice for us to make, but rather a question to be put to nature.

We are aware of the fact that traditional power counting renormalizability can be replaced by a less restrictive one, where gravitational metric theories are considered as renormalizable. The appearance of the Einstein-Hilbert action within this framework is due to the suppression of higher derivative terms by powers of (possibly) the Planck mass. We paid special attention to the issue of unitarity, which is often lost for higher derivative gravity. We
stress that we are not truncating the higher derivative gravity Lagrangian, hence unitarity is not lost in our approach. In this sense we have gone beyond the approximation of the effective field theory approach, where higher order corrections are suppressed.

It is rather paradoxical that we have arrived at a minimalist proposal by having first resorted to a maximal theory. But it is satisfying in having added every ingredient to the broth and seeing it slim itself down on its own accord.

Being such a simple candidate, one can immediately go about calculating with this proposal.

Acknowledgement

Thanks are due to the referees of our Letter for useful comments. This work does not reflect the views of the High Energy Physics group at ICTP.

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