Spin and Statistics in Quantum Gravity

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Abstract. We present a review of the spin and statistics of topological geons, particles in 3+1 quantum gravity. They can have half-odd-integral spin and fermionic statistics and since the underlying gravitational field is tensorial and bosonic, this is an example of “emergent” non-trivial spin and statistics as displayed by familiar non-gravitating objects such as skyrmions. We give the topological background and show that in a “canonical” quantization of gravity there is no spin-statistics correlation for topological geons. Allowing the topology of space to change, for example in a sum-over-histories approach, raises the possibility that a spin-statistics correlation can be recovered for geons. We review a conjectured set of rules powerful enough to give such a spin-statistics correlation for all topological geons. These would appear to rule out the possibility of parastatistics and may rule out spinorial and fermionic geons altogether.

I INTRODUCTION

In quantum mechanics and in quantum field theory in flat spacetime we have many examples of “emergent” fermionic statistics and spinorial (i.e., half-odd-integral) spin for objects built from entities which are fundamentally tensorial (i.e., integral spin) and bosonic. Can such a phenomenon occur in quantum gravity in which the dynamical variable, the spacetime metric, is tensorial and bosonic? The answer is yes, and in this review we will look at the best studied case, that of so-called “topological geons”, particles made of non-trivial spatial topology, in 3 + 1 dimensions.

The original work on spin-half states in quantum gravity was done by Friedman and Sorkin [1,2] and on fermionic states by Sorkin [3]. In section II we recall the basic features of topological geons, particles which exist by virtue of the non-trivial
topology of space, and show how they acquire their spin and statistics by a choice of unitary irreducible representation (UIR) of the so-called mapping class group (MCG). We will see that there is no spin-statistics correlation for quantum geons [3–6] and indeed there are always spin-statistics violating sectors for any species of geon [7,8]. Moreover, the presence of a particular type of symmetry between quantum geons, namely the “slides” which correspond to diffeomorphisms in which one geon slides through another, produces an extraordinary variety of quantum sectors [7,8].

The lack of a spin-statistics correlation for geons is perhaps not surprising. In the proofs of all existing spin-statistics theorems for extended objects like geons, the possibility of particle-anti-particle creation (and annihilation) is crucial. Indeed particle-anti-particle pair production and annihilation has been suggested (see [9,10]) as a unifying principle that might bring together the “topological” spin-statistics theorems, of which the Finkelstein-Rubenstein version [11] is the original, and the relativistic quantum field theory theorems (the conditions imposed — Lorentz invariance etc. — being supposedly just those that guarantee that antiparticles exist with the required possibilities for pair creation and annihilation).

Now, the process of geon-anti-geon pair production is a topology changing one and cannot be described within a formalism like canonical quantum gravity which assumes, a priori, that the spatial three-manifold is fixed. It has therefore been conjectured that in a formulation of quantum gravity which can accommodate topology change, the usual spin-statistics connection would be recovered for geons [4].

The sum-over-histories (SOH) for quantum gravity is such a formulation and there is indeed evidence that there’s a spin-statistics theorem for geons “trying to get out” in a SOH approach [12]. To prove a general spin-statistics theorem for all geons, extra assumptions are needed and in section III we review a set of rules which would achieve this [4]. The consequences of these rules are stronger than originally envisaged and there’s a possibility that they might rule out spinorial and fermionic geons altogether.

Section IV is a brief mention of some motivation for pursuing a geon spin-statistics theorem.

We will restrict ourselves to orientable three-manifolds throughout and will further assume that no handles, $S^2 \times S^1$, occur in the “prime decomposition” of the three-manifold (see section II).
II  HOW THE GEON GOT ITS SPIN AND STATISTICS

Topological geons are particles made from non-trivial spatial topology. We are interested in the situation of an isolated system of these particles and thus we will be dealing with a three-dimensional manifold $M$ which admits asymptotically flat metrics. Physically, $M$ is three-space at a “moment of time”, or, the “future boundary of truncated spacetime” [13].

A  The spatial three-manifold

There is a “Three-Manifold Decomposition Theorem” that identifies candidates for elementary geons, but in order to state this theorem we must first introduce the concepts of “connected sum” (denoted #) and “prime manifold.” To take the connected sum of two oriented three-manifolds $M_1$ and $M_2$, remove an open ball from each and identify the resulting two-sphere boundaries with an orientation-reversing diffeomorphism (henceforth, “diffeo”). Taking the connected sum of any three-manifold with $S^3$ gives a manifold diffeomorphic to the original one; taking it with $\mathbb{R}^3$ is topologically equivalent to deleting a point. A prime three-manifold, $P$, is a closed three-manifold that is not $S^3$ and such that whenever $P = P_1#P_2$, either $P_1$ or $P_2$ is $S^3$. Examples of primes are the three-torus, $T^3$, and the so-called spherical spaces, $S^3/G$, where $G$ is some discrete subgroup of $SO(4)$ acting freely on $S^3$.

The $M$ we are considering is $M = \mathbb{R}^3#K$ where $K$ is a closed three-manifold. The Decomposition Theorem states that any such $M$ can be decomposed into the connected sum of finitely many prime manifolds and this decomposition is unique:

$$M = \mathbb{R}^3#P_1#P_2\ldots#P_n.$$  (1)

We will assume that to each prime summand there corresponds an elementary quantum geon; with “correspond” being used in a suitable sense since there is a rather subtle relation between a particular piece of spatial topology and a physical particle—which subtlety has to do both with familiar “identical particle exchange effects” and unfamiliar effects due to the existence of diffeos known as “slides” [3,4].

For more details see [3,4,14]
B Wave functions

In canonical quantum gravity, for which the topology does not change, the configuration space, \( Q \), is the space of all three-geometries on \( M \),

\[
Q = \frac{\text{Riem}^\infty(\mathcal{M})}{\text{Diff}^\infty(\mathcal{M})},
\]

where \( \text{Riem}^\infty(\mathcal{M}) \) (\( R^\infty \) for short) is the space of asymptotically flat Riemannian metrics on \( M \) and \( \text{Diff}^\infty(\mathcal{M}) \) (\( D^\infty \) for short) is the group of diffeomorphisms of \( M \) that become trivial on approach to infinity. It can be shown that \( D^\infty \) acts freely on \( R^\infty \) and so \( Q \) is a manifold, \( R^\infty \) being a principal fibre bundle over \( Q \) with fibre \( D^\infty \). Thus, using the fact that \( R^\infty \) is convex and hence contractible to a point so that all its homotopy groups are trivial, we deduce that \( \pi_k(D^\infty) \cong \pi_{k+1}(Q) \).

Wave functions need not be single-valued on \( Q \) if \( Q \) contains non-contractible loops. Rather, the transformation of a wave function as such loops are traversed gives a representation of \( \pi_1(\mathcal{Q}) \). This is a special case of the general situation where wave functions are sections of a twisted vector bundle on \( Q \). Physical observables are invariant under \( \text{Diff}^\infty(\mathcal{M}) \) which means that the quantum state space reduces to a number of inequivalent and independent sectors each transforming under a different unitary irreducible representation of the group \( \pi_1(\mathcal{Q}) \). From the above we know that \( \pi_1(Q) \cong \pi_0(D^\infty) \), where \( G \equiv \pi_0(D^\infty) \) is known as the mapping class group (MCG) or the group of large diffeos of \( M \) and is the analogue for gravity of the group of large gauge transformations in a gauge theory.

C The mapping class group and UIR’s

Let us restrict attention to the manifold which is a connected sum of \( \mathbb{R}^3 \) and a number, \( n \), of identical primes, \( M = \mathbb{R}^3 \# \mathcal{P} \# \cdots \mathcal{P} \). \( G \) is generated by three types of large diffeomorphism, the exchanges, the internal diffeomorphisms and the slides. Each of these generators can be viewed as the result of a certain process (a “development” [3]), with the nature of the process being suggested by the name of the category. Thus, an exchange is the result of a process in which two identical primes continuously change their positions until they have swapped places. Similarly a slide is the result of a process in which one prime travels around a closed loop threading through one or more other primes, while an internal diffeo is a diffeo whose support is restricted to a single prime.

One very important internal diffeo is the \( 2\pi \)-rotation of a single prime. It is the result of a process in which the prime rotates around by \( 2\pi \). For particular primes, \( P \), this diffeo is deformable to the identity and so is not a non-trivial element of \( G \).
Geons based on these primes must have tensorial (integral) spin since a $2\pi$ rotation is equivalent to the identity and so cannot have a non-trivial effect on the quantum state. Of the known primes, only the handle (which we are explicitly excluding here) and the lens spaces are tensorial: the rest have non-trivial $2\pi$ rotation and are called spinorial primes.

The slides, internals and exchanges each form subgroups of $G$ and the exchanges generate a subgroup isomorphic to the permutation group, $S_n$. In [7,8] the literature on the MCG was sublimated into the following result

$$G = (\text{slides}) \ltimes (\text{internals}) \ltimes S_n,$$

where the symbol $\ltimes$ denotes semidirect product (with the normal subgroup on the left). What this says is that every element of $G$ is uniquely a product of three diffeomorphism-classes, one from each subgroup, and that each subgroup is invariant under conjugation by elements of the subgroups standing to its right in equation 3. See [15–17] for more details on the MCG.

The fact that $G$ is a semidirect product allows us to analyze its UIR’s in terms of representations of its factor groups and their subgroups. Let

$$G = N \ltimes K$$

be a semidirect product with $N$ being the normal subgroup. A finite dimensional UIR of $G$ is then determined by the following data

- $\Gamma$ = a UIR of $N$
- $T$ = a PUIR of $K_0 \subseteq K$,

where $K_0$ is the subgroup of $K$ that remains “unbroken by $\Gamma$” ($K_0$ is “the little group”) and “PUIR” stands for “projective UIR”, i.e., representation up to a phase. (The Schur multiplier for $T$ is determined by $\Gamma$.)

In seeking the UIR’s of this group there are two situations, depending on whether the slide subgroup is represented trivially or not. The results in the following two subsections are contained in [7,8].

**D The sectors with trivial slides**

In the simpler case of UIR’s of $G$ which annihilate the slides, in effect a complete classification is possible. In this case, the mathematical problem is reduced to finding the UIR’s of the quotient group, $G/(\text{slides})$, which by equation 3 is just the semidirect product.
Let us find the finite dimensional UIR’s of this group (sometimes called the “particle group” [17]). The normal group of internal diffeos is the direct product of $n$ copies of the MCG group, $H$, for a single prime $P$:

$$N = (\text{internals}) = H \times H \times \cdots \times H.$$  \hspace{1cm} (6)

The most general UIR of this normal subgroup $N$ is itself a product, namely the tensor product,

$$\Gamma = \Gamma_1 \otimes \Gamma_2 \otimes \cdots \Gamma_r,$$  \hspace{1cm} (7)

doing UIR’s of $H$, where the first $n_1$ factors are $\Gamma_1$, the next $n_2$ factors are $\Gamma_2$, etc. Physically, a given $\Gamma_i$ specifies a certain “internal structure” for the corresponding geon, and is therefore a “species parameter” or “quantum number”. Different $\Gamma_i$’s mean that the corresponding geons are not identical particles but are of different physical species. So here can we have a quantum breaking of indistinguishability.

With respect to the choice (7), the unbroken subgroup $K_0 \subseteq S_n$ reduces to a product of permutation groups,

$$K_0 = S_{n_1} \times S_{n_2} \times \cdots \times S_{n_r}.$$  \hspace{1cm} (8)

The statistics is then given by a UIR $T$ of $K_0$, that is to say by an independent UIR $T_1, T_2, \ldots$ for each of the subgroups $S_{n_1}, S_{n_2}, \ldots$. Each of these $T$’s in turn, can be specified by a choice of a Young tableau, and determines whether the corresponding geons will manifest Bose statistics, Fermi statistics or some particular parastatistics. Since there is no restriction on the choice of $T$, there is no restriction on which combinations of these possible statistics can occur.

To summarize, all possible sectors with trivial slides are accounted for by specifying

- a species for each geon (i.e. a UIR of $H$)
- a statistics for each resulting set of identical geons

E Some sectors with nontrivial slides

When the slide subgroup is represented nontrivially, a full classification of the possible UIR’s of $G$ does not exist. It is clear, however, that there is an extraordinary variety of possible UIR’s and in order to give an idea of this richness we
give an example of a UIR in a special case which avoids most of the complications which obstruct the full classification.

The prime in this special case is $\mathbb{R}P^3$, which can be visualized as a region of $M$ produced by excising a solid ball and then identifying antipodal pairs of points on the resulting $S^2$ boundary. The internal group is trivial for this prime. For each pair of $\mathbb{R}P^3$’s, one can slide one through the other, with the square of this slide being trivial (since $\pi_1(\mathbb{R}P^3) = \mathbb{Z}_2$), making a total of $n(n-1)$ independent order 2 generators. The complete group (slides) is then generated by products of these elementary slides.

Since for $\mathbb{R}P^3$, (internals) is trivial, the MCG reduces to

$$G = \langle \text{slides} \rangle \ltimes S_n. \quad (9)$$

Hence, according to the general scheme outlined earlier, we get a UIR of $G$ by choosing first a UIR, $\Gamma$, of $N = \langle \text{slides} \rangle$, and then a suitable PUIR, $T$, of the resulting unbroken subgroup $K_0 \subseteq S_n$. As before, we may interpret $K_0$ as describing the surviving indistinguishability of the geons, and $T$ as describing the statistics within each set of identical geons.

Consider abelian UIRs of $N$ in which, in effect, each ordered pair of primes is assigned a $\pm 1$. We can represent the various such UIR’s pictorially by drawing $n$ dots to represent the primes and an arrow to represent each ordered pair that receives a minus sign (meaning the a slide of the first prime through the second produces a phase-factor of $-1$). Each distinct diagram of this type then gives rise to a different class of UIR’s of (slides), and therefore furnishes a different building block for constructing UIR’s of $G$.

If we make a particular choice of UIR of (slides) by choosing a diagram in which the dots and arrows form a circle and the arrows point anticlockwise, say, this pattern leaves $\mathbb{Z}_n \subseteq S_n$ as the unbroken subgroup $K_0$. We acquire distinct UIR’s of $G$ corresponding to the possible UIR’s of $\mathbb{Z}_n$. Here, geon identity is expressed not by a permutation group, $S_n$, at all, but by the cyclic group $\mathbb{Z}_n$. With this new type of group comes a new type of statistics, in which a cyclic permutation of the geons produces the complex phase $q$ or $\bar{q}$, $q = 1^{1/n}$ being an $n$th-root of unity.

Another interesting possibility, arising when the UIR, $\Gamma$, of (slides) is non-abelian, is that of “projective statistics,” meaning a type of statistics expressed by a properly projective representation of the permutation group or one of its subgroups. To construct such an example, we would need at least four geons because $S_n$ possesses properly projective representations only for $n \geq 4$. 
F  No spin-statistics correlation

All the UIRs of the MCG are on a equal footing as far as the canonical theory goes. Given a fixed spatial three-manifold, there appears in the theory a set of, in principle unpredictable, new parameters, some discrete and some continuous, corresponding to Nature’s choice of a particular UIR amongst all the possibilities. Some of these UIRs violate the usual spin-statistics correlation in the grossest possible way. Subsection II D shows that if the basic prime is tensorial (the $2\pi$ rotation is trivial) then there is a UIR in which the permutation group is represented by the totally antisymmetric UIR. If the basic prime is spinorial then the quantum geon can be tensorial or spinorial and the permutation group be represented either by the totally symmetric or antisymmetric UIR. In other words there are UIRs in which the geons are spinorial bosons or tensorial fermions.

This lack of a correlation can be attributed to the fact that in a theory in which the topology is fixed (such as canonical quantization) there is no allowance for geon-anti-geon production. This is because a process in which a geon and anti-geon are created from $\mathbb{R}^3$ is a topology changing one (we know this from the decomposition theorem: one piece of non-trivial topology can’t “cancel” another). The known spin-statistics theorems for objects such as skyrmions and other kinks which have these emergent properties of spin and statistics all require, for their proofs, that the process of pair creation and annihilation be describable as a path in the configuration space ( [18] is a general such proof). This leads one to expect that in a formulation of quantum gravity in which topology change is naturally accommodated the spin statistics correlation might be recovered. The SOH is such a formulation and we therefore turn now to that approach.

III  SPACETIME APPROACH

In [12] it was shown that two non-chiral geons (i.e., one which are their own antiparticle) which are pair created in a topology changing process, satisfy a limited spin-statistics theorem which eliminates some of the spin-statistics violating sectors. Instead of reviewing that work, we will turn to the more general question of how a general spin-statistics theorem could be proved for all geons.

In the SOH framework the fundamental dynamical input is a rule attaching a quantum amplitude to each pair of truncated histories which “come together” at some “time” [19,13,20,21]. Let us call such a pair a “Schwinger history” and its underlying manifold a “Schwinger manifold”. In the case of quantum gravity, a truncated history is a Lorentzian manifold with final boundary (and possibly initial boundary depending on the physical context), and the “coming together” means the identification or “sewing together” of the final boundaries. In a framework
Now, without disturbing the classical limit of the theory or the local physics, we can multiply the amplitude of each Schwinger history by a “weight” $w$ depending only on the topology of the underlying manifold (and on the two initial metrics, if initial boundaries are present). We would like, somewhat analogously to [22], to argue that these weights carry some unitary representation of $G$, and that sets of weights belonging to disjoint representations “do not mix”. We cannot do so in the present case for several reasons. We do not know what $G$ is in a topology changing scenario precisely because the topology of space changes and so the MCG changes with it. Even if we could overcome this, we would be faced with the question of how to implement the higher-dimensional non-abelian UIR’s.

We will ignore these detailed questions here and instead sketch what a spin-statistics theorem might look like in a SOH framework.

Consider two Schwinger manifolds, $N$ and $N'$, that contribute to the SOH and which differ only in a compact region in which, in $N$ two identical primes swap places and in $N'$ one prime stays put and the other rotates around by $2\pi$. This is illustrated in figure 1 in which the “environment” depicted in grey is common to $N$ and $N'$. The lines represent the “worldlines” of the primes and are in reality framed curves in a four-dimensional manifold. (Given a framed curve in a four-dimensional space, there is a topologically unique way of attaching a prime to each point of that curve.) The corkscrew effect in $N'$ represents the frame rotating by $2\pi$.

The geons are bosons (fermions) if the weight of $N$ is equal to (minus) the weight of a third manifold, $N''$ which differs from $N$ only in that the two primes do not move at all in that same compact region. The geons are tensorial (spinorial) if the
weight of $N'$ is equal to (minus) the weight of $N''$. A spin statistics theorem would consist of the result that given any two Schwinger manifolds such as $N$ and $N'$, they must have the same weight in the SOH.

The rules that would achieve this result are the following [4]. We dub manifolds with the same weight “congruent.”

- Manifolds which are diffeomorphic, via a diffeomorphism which is the identity on the initial boundaries and at spatial infinity if these are present, are congruent.

- Let two manifolds, $N_1$ and $N_2$ differ only in a certain “compact region,” so $N_1 = R_1 \cup E$ and $N_2 = R_2 \cup E$, where $E$ is the “environment”. If $N_1$ and $N_2$ are congruent then $N'_1 = R_1 \cup E'$ and $N'_2 = R_2 \cup E'$ are also congruent.

- If $N$ is a manifold which admits a spacelike hypersurface which divides $N$ in half so that the second half is the time reverse of the first, then $N$ is congruent to the product manifold $V_0 \times [0,1]$ where $V_0$ is the initial boundary of $N$. (If there’s no initial boundary then $N$ is congruent to $S^4$).

Now, for any geon topology, $P$, there is a canonical manifold, $X$, which mediates geon-anti-geon annihilation. The third rule has the consequence that the manifold which consists of $X$ followed by its time reverse (pair annihilation followed by pair creation) is congruent to the manifold in which a prime and an anti-prime just sit there. This is illustrated in figure 2 where the lines again represent the worldlines of the primes and the arrows remind us that they are framed curves. The “anti-prime” is the mirror image, or “chiral conjugate,” of the prime.

This can be used to prove a spin-statistics theorem as illustrated in figure 3 which shows a sequence of congruent compact regions, to be thought of as embedded in
FIGURE 3. A sequence of congruent manifolds which establish a spin-statistics correlation for geons

some common environment. Again, the lines represent the (framed) worldlines of two identical primes. Region (i) contains a static prime and one which rotates by $2\pi$. “Untwisting” the rotation gives region (ii) which is diffeomorphic to (i). Notice that region (ii) contains a small region which is a pair-annihilation-pair-creation event and replacing that with the prime-anti-prime product, as we may do by rule 3, gives region (iii). This is in turn diffeomorphic to (iv). So (i) is congruent to (iv) which is the statement of a spin-statistics correlation in this framework.

It has now been realised that these rules are much more powerful than originally envisaged. In particular they allow only abelian UIR’s as potential weights and thus eliminate the possibility of parastatistics. (This seems to be a consequence of “topological” spin-statistics theorems in general [9,10].) A sketch of the proof of this result is given in figure 4 which shows a sequence of (compact regions of) congruent manifolds – the surrounding “boxes” are to be imagined.

In (i) is shown a manifold in which at early times there are a number of primes sitting in fixed positions, propagating in time: that’s denoted by the single vertical line which now stands for a collection of a number of framed worldlines. In the intermediate region a large diffeo, $g$, an element of the MCG of the three-manifold comprising those prime summands, is developed: that’s denoted by the shaded box labeled $g$. It’s useful to imagine an example in which, say, there are two identical primes and $g$ is the exchange diffeo so that diagram (i) is actually the manifold $N$ in figure 1. (i) is diffeomorphic to (ii) which is congruent to (iii) by rule 3. The line on the right in (iii) is now a spectator for the next few steps. Into the “closed loop” we may introduce the development of any diffeo $h$ followed by its inverse. $h$ is an element of the MCG not of the original multi-prime manifold but one which is its mirror image in which each original prime is replaced by its chiral conjugate. This can be seen by following carefully what is created and destroyed at the pair creation and annihilation events. Then the diffeos $h$ and $h^{-1}$ are eased along the
loop in opposite directions until they are adjacent to $g$ as shown in (v). In doing so $h$ becomes a new diffeo, $\hat{h}$, an element in the same MCG as $g$, related to $h$ in the obvious way. Similarly for $\hat{h}^{-1}$. Composing $\hat{h}$, $g$ and $\hat{h}^{-1}$ gives (vi) and the reverse of the first two steps results in (viii). This may be done for any $h$ and so (i) is congruent to (viii) for any $\hat{h}$ in the MCG.

This means that if we are able, in some effective sense, to implement the Laidlaw-Morette-DeWitt type of scheme, the weight of manifold (i) would have to be the same as that of manifold (viii). Thus, the weights can only depend on the conjugacy class of the elements of the mapping class group and so can only carry abelian UIR’s.

It might be seen as an advantage that only abelian UIR’s need be considered – the representation theory is radically simplified in this case and we feel that we understand how to attach mere phases as weights to the manifolds in the SOH.
But there is a potential, serious drawback. All spinorial geons known to us are non-abelian. That is, we do not know of a prime whose $2\pi$ rotation is non-trivial in an abelian UIR of the MCG. If no such primes do in fact exist, the restriction to abelian UIR's would eliminate spinorial and fermionic geons altogether and this seems much too high a price to pay for a spin-statistics theorem.

**IV MOTIVATION**

Why should we care whether or not topological geons obey a spin-statistics theorem? We can hope that by formulating rules that would give us a spin-statistics theorem we may glean clues about the nature of the underlying, more fundamental theory of quantum gravity which should eventually give rise to such rules. It is clear that the spin-statistics question is intimately connected to the question of the weights attachable to the different manifolds in the SOH, whose arbitrariness in the absence of something like these ideas seems a problem. Geons contain the potential to be the mechanism of the ultimate unification between spacetime and matter: perhaps spacetime is all there is. Bosonic fields could arise from Kaluza-Klein reductions and fermions could be quantum geons. Indeed Friedman and Higuchi showed that in pure Kaluza-Klein gravity there will be stable geons with the kinematical quantum numbers of the standard model, although obtaining chiral fermions is problematic [23]. (Showing that geons have the standard model masses is a problem of a different order, though one potential difficulty has been slightly alleviated since the discovery of a neutrino mass. There is still an enormous number, $m_{\text{planck}}/m_{\text{neutrino}}$, to explain but at least it’s finite.) As unlikely as this might seem, the payoff, were it to be true, is so high that it is worth bearing the possibility in mind. Then a geon spin-statistics theorem would give us the fundamental explanation of the perfect spin-statistics correlation we see in nature.

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