The Kinematics of Tracked Vehicles via the Power Dissipation Method

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Abstract—This paper develops a new quasi-static modeling framework for tracked robots based on the power dissipation method. Given a set of track speeds, this method predicts the vehicle’s instantaneous rigid body motion. We introduce three specific models: a model for tracked operation on flat ground, a model for vehicle motion when the track’s grouser tips touch the ground, and a model for operation on stairs. Experiments show that these models predict tracked vehicle motion more accurately than existing kinematic models, and predict phenomena which are not captured by other models. These novel models provide a basis for new feedback control approaches.

I. INTRODUCTION

Most robotic vehicles are propelled by wheels. However, tracked robots often have better mobility over uneven terrain, and they can potentially climb complex structures, such as stairs and rubble, that are impassable for wheeled robots [1]. While the complex interaction between the track and its supporting terrain provides superior traction, tracked vehicle propulsion is much more difficult to model, and potentially to control. A key issue is that, except for purely forward motion, portions of the track must be skidding over the terrain as the vehicle moves. This paper revisits the modeling of tracked vehicle motion using new methods. These newer models can lead to newer control approaches and better performance.

There is a long history of research on tracked vehicle modeling [2]–[4]. Early work endeavored to understand the mechanisms of traction [5], or the physical factors that affect the ability of a tracked vehicle to turn [6], [7]. Naturally, the analysis and modeling of the skidding process has received considerable attention, as a greater understanding can lead to better autonomous control of tracked vehicle motion [8]–[14]. The slipping process has also be considered from the viewpoint of tracked vehicle power use and efficiency [15].

Unfortunately, most terramechanics models are inconvenient for control design, as they do not formulate a concrete relationship between track inputs (torques or speeds) and vehicle motion. Instead, most models determine the forces on the vehicle given the vehicle’s speed or acceleration. Many practical approaches to tracked vehicle motion planning and control are based on a kinematic model that lumps the complex mechanics of track slip into a simple factors [16]. Shiller was one of the first to study a dynamic model of tracked vehicles [17] for purposes of motion planning. Shiller conceptualized the skid steer process as a dynamic nonholonomic constraint, which can then be used as a constraint in the motion planning process. While it represents a real advancement, Shiller’s work also relies upon the prediction of track forces given vehicle motion. Others have used online estimation to estimate some of the kinematic or dynamic model parameters [18]–[21].

Since tracks are one of the few practical alternatives to legs for stair climbing, several works have analyzed their operation and agility on stairs. These efforts have typically focused on the forces and geometry involved in stair climbing [22]. Others have managed the real-time control of stair climbing using simple differential drive equations [23]–[25]. Though simple enough to implement, we will show that these equations do not accurately portray vehicle motion on stairs. This paper gives one of the first methods to derive feedback equations of tracked vehicles on stairs.

This paper contains the following contributions. First, we apply the power dissipation methodology (PDM) to the modeling of tracked vehicles to yield a quasi-static model that estimates vehicle motion given the track speeds as input. Such models can support advances in feedback control design for tracked vehicles. We focus on a quasi-static model because we are most interested in cases where a tracked vehicle negotiates complex environments, which typically occurs at low speeds. Second, when a tracked vehicle operates on flat homogeneous ground, a version of the model with Coulomb friction predicts that the vehicle will move exactly like a differential drive vehicle, but with a fictitious wheel radius that depends nonlinearly on the track length and width. This gives justification for the simplified kinematic modeling approaches that have previously been used, but it improves them by giving a rigorous model for the lumped parameters coefficients that have previously been derived in ad-hoc ways.

Third, using the PDM, we model the motion of a tracked vehicle while it climbs a set of stairs. Importantly, we show that a model which predicts vehicle motion on flat ground gives erroneous motion predictions on stairs. As the vehicle climbs the stairs, particularly at an angle, the supporting contacts make discontinuous jumps. Our model predicts that, in the quasi-static modeling regime, the vehicle’s motion is discontinuous at these changes in the supporting contact configuration. In the true system, these discontinuities in contact will lead to motion perturbations.

We also apply the stair modeling framework to the study of tracks equipped with grousers. Grousers improve traction in soil [26]. However, when a tracked vehicle travels over hard
ground, only the grouser tips contact the terrain surface. Our model captures this situation with high fidelity.

Finally, we experimentally verify our theoretical predictions with data gathered from a Rover Robotics Flipper tracked vehicle while it drives on flat ground and while it climbs stairs. The models derived predict the vehicle’s motion more accurately than other simplified modeling methods and confirm the stair climbing motion predictions.

The next section reviews the power dissipation method. Section IV applies this method a tracked vehicle, yielding a novel set of input-output equations. Section III-C specializes the model to vehicle operation on stairs, while Section III-B analyzes the case of grousers on hard flat ground. Section V presents experimental results on flat ground as well as on stairs, and compares the results to our model predictions.

II. BACKGROUND ON THE POWER DISSIPATION METHOD

A quasi-static system is one in which the inertial forces are negligible, that is, either $m$ or $a$ is negligible in $F = ma$. Such an approximation of a system is useful when the dissipative forces are much greater than the inertial forces. Quasi-static models are often useful in practice, as they can abstract key relationships in a useful form. A principle question that we address in this paper is: “is it possible to develop a quasi-static model of a tracked vehicle?”

A. Minimum Power Principle

The principles of quasi-static modeling in this paper go back many decades. However, within the field of robotics, Peshkin and Sanderson [27] proposed the following principle of minimum power for quasi-static systems:

**Definition 1:** A quasi-static system chooses that motion, from among all motions satisfying the constraints, which minimizes the instantaneous power.

The above principle does not hold for all quasi-static systems. It holds for forces that are parallel to the velocities of the system’s particles, but that are independent of those velocity magnitudes (for example, Coulomb frictional forces). It also holds for forces that are independent of particle velocity (e.g., gravitational forces). Hence, the principle can be applied to the quasi-static sliding of a system of particles.

Alexander and Maddocks [28] introduced a similar notion for rolling motion. They considered the kinematics of wheeled mobile robots and showed that their motion is that of least power dissipation through frictional forces. This method is referred to as the Power Dissipation Methodology (PDM). Murphey and Burdick [29] considered the PDM from a control theoretic perspective. They determined the conditions under which a quasi-static model derived from the PDM is a rigorous kinematic reduction of a full Lagrangian mechanics model.

B. Power Dissipation Methodology

We assume that the system configuration, $q = (q_g, q_r) \in Q$, consists of states $q_g$ and control inputs $q_r$. $Q$ is written as a product of the state and control manifolds $Q_g$ and $Q_r$ respectively. The Power Dissipation Method is based on the notion of the Power Dissipation Function.

**Definition 1:** Given a system with configuration $q = (q_g, q_r) \in Q_g \times Q_r = Q$ and $q_r$ fixed, the power dissipation function, $D((q_g, q_r))$ for the system’s particles at configuration $q_g$ while the inputs $q_r$ are fixed.

The power dissipation methodology is based on the minimization of the power dissipation function:

**Proposition 1:** Given a power dissipation function, $D$, as defined in Definition 1, the system's motion, $\dot{q}_g$ at any given instant is the one that minimizes $D$ with respect to $\dot{q}_g$ while the inputs $q_r$ are held fixed:

$$\dot{q}_g^* = \arg\min_{\dot{q}_g \in T_{q_g}Q_g} D((q_g, q_r))((\dot{q}_g, \dot{q}_r))$$

A rigorous analysis of this methodology can be found in [29]. Practically, the PDM yields an input-output relationship $q_g = h(q_r)$. Note that the function $h(q_r)$ can be discontinuous, and set-valued at some system configurations.

Consider a system that contacts its surroundings at multiple points. The dissipation function quantifies the power lost while the system overcomes frictional contact forces during its motion. This paper considers Coulomb frictional forces, but other models are possible. If a contact is not slipping, its relative velocity is 0. If it slips, the relative velocity is given by $\omega(q)\dot{q}$ for some function $\omega(q)$. If the normal force at the $i^{th}$ contact is denoted by $N_i$ and $\mu_i$ is the friction coefficient, the dissipation function for $\kappa$ contacts is:

$$D(q)(\dot{q}) = \sum_{i=1}^{\kappa} \mu_i N_i |\omega(q)\dot{q}|$$

We use this principle below to obtain kinematic models of tracked robot motion on flat homogeneous ground and then on stairs. The models are not necessarily better than Lagrangian models. However, we obtain first order equations of motion that are easier to work with.

III. MODEL

Consider a the tracked vehicle model shown in Fig. 1. We assume a symmetric vehicle with identical tracks arranged symmetrically with respect to a right-handed body fixed reference frame, $B$. Each track has length $2L$ and width $2T$. The tracks are assumed to be driven by a sprocket with diameter $2D$. The distance between the track center lines is $2W$. The origin of $B$, is located at a height $D$ above the ground plane. Its $x$-axis points in the forward driving direction, the $y$-axis points to the left, and the $z$-axis points out of the plane.

We seek to compute the power dissipation function, Definition 1 for the vehicle in Fig. 1. To describe local coordinates on each track surface, define references frames $T_L$ and $T_R$ (Fig. 1) that are oriented parallel to $B$, with their origins located on the ground plane, a distance $\pm W$ from the origin of $B$ along $B$’s $y$-axis. In these local reference frames, each point on the right and left track surface (that
are in contact with the ground) respectively has coordinates 
\( [r_x \quad r_y] \) and 
\( [l_x \quad l_y] \). In the body-fixed frame, points on 
the track surfaces are located at:

\[
\rho_r^B = \begin{bmatrix} r_x \\ -W + r_y \end{bmatrix} \quad \rho_l^B = \begin{bmatrix} l_x \\ W + l_y \end{bmatrix}
\]

When the vehicle is on flat ground moving with velocity 
\( \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \) the velocities of the points on the left and right 
treads are (in frame \( B \)):

\[
v_r = v_r^B = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \dot{\theta} \begin{bmatrix} W - r_y \\ r_x \end{bmatrix} - S_r \begin{bmatrix} 1 \\ 0 \end{bmatrix}
\]

\[
v_l = v_l^B = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \dot{\theta} \begin{bmatrix} -(W + l_y) \\ l_x \end{bmatrix} - S_l \begin{bmatrix} 1 \\ 0 \end{bmatrix}.
\]

The right and left track speeds are denoted \( S_r \) and \( S_l \). Their 
signs are positive when the tracks move toward the rear of 
the vehicle (in a manner to propel the vehicle forward):

\[
S_r = -\dot{r}_x, \quad S_l = -\dot{l}_x, \quad \dot{r}_y = \dot{l}_y = 0.
\]

The total dissipated power is the integral over the track 
surfaces of the power dissipated at each point where the 
track touches the ground. According to the power dissipation 
methodology, the vehicles rigid body velocity due to given 
track inputs \( S_r \) and \( S_l \) is the one which minimizes the 
total dissipated power when \( S_r \) and \( S_l \) are held fixed:

\[
\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \arg\min_{\dot{x}, \dot{y}, \dot{\theta}} \int_{\rho_r \in \Omega_r} \mu_r(\rho_r)N(\rho_r)||v_r||d\rho_r \\
+ \int_{\rho_l \in \Omega_l} \mu_l(\rho_l)N(\rho_l)||v_l||d\rho_l
\]

where the tracks contact the ground in regions \( \Omega_r \) and \( \Omega_l \).

### A. Modeling on flat ground

(5) is impractical to solve in general. Consider a simpler case 
where the vehicle drives on flat, solid, homogeneous ground. We 
can reasonably assume that vehicle weight is evenly 
distributed across the two tracks, and that ground pressure 
is uniformly distribute across the treads: \( N(\rho_r) = N(\rho_l) = \) 
constant \( \forall \rho_r \in \Omega_r, \rho_l \in \Omega_l \). We also assume a Coulomb 
friction model for track/ground contact, with a uniform friction 
coefficient across tracks: \( \mu_r = \mu_l = \) constant \( \forall \rho_r \in \Omega_r, \rho_l \in \Omega_l \). Substituting these simplifications and (3) and (4) into the integrand of (5) yields the power dissipation function (in a body-fixed frame):

\[
D(\dot{x}, \dot{y}, \dot{\theta}, S_r, S_l) = \int_{\rho_r \in \Omega_r} ||v_r||d\rho_r + \int_{\rho_l \in \Omega_l} ||v_l||d\rho_l \\
= \int_{-T}^{T} \int_{-L}^{L} \sqrt{\left(\dot{x} + (W - r_y)\dot{\theta} - S_r\right)^2 + \left(\dot{y} + r_x\dot{\theta}\right)^2} dr_x dr_y \\
+ \int_{-T}^{T} \int_{-L}^{L} \sqrt{\left(\dot{x} - (W + l_y)\dot{\theta} - S_l\right)^2 + \left(\dot{y} + l_x\dot{\theta}\right)^2} dl_x dl_y
\]

The quasi-static equations of motion are obtained by mini-
mizing \( D \) with respect to \( \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \) for a given \( S_r \) and \( S_l \). This double integral can be computed numerically on a 
modest computer in faster-than real time for given values 
of \( L, W, T \). It can be similarly minimized with respect 
to body velocities in real-time using common minimization 
procedures.

Fig. 2 shows the values of \( \dot{x}, \dot{\theta} \) for varying lengths of 
track and track speeds. Note that \( \dot{y} \) has been excluded in 
the figure because it is 0 regardless of the dimensions and 
track speeds. Note that the relationship between \( \dot{x} \) and \( S_r, S_l \) 
remains the same as \( L \) varies. For \( \dot{\theta} \), however, the value varies 
as \( C(S_r - S_l) \). Here \( C \) is constant that varies with track 
geometry. When \( L = 0, C = \frac{1}{2\pi} \), i.e, the vehicle behaves 
like a differential drive robot.

![Variation in linear and angular velocity](image)

(a) \( \dot{x} \) vs. \( S_r, S_l \) for different \( L \) (b) \( \dot{\theta} \) vs. \( S_r, S_l \) for different \( L \).

### B. Modeling of grouser effects on flat ground

Most tracked vehicles employ grousers. On flat, hard 
surfaces, only the tips of the grousers make ground contact. Hence, instead of the double integral that we computed in the previous subsection, the power dissipation function becomes
an integral over a set of line contacts:

\[
D_{\text{grousers}}(\dot{x}, \dot{y}, \dot{\theta}, S_r, S_l) = \mu_r \int_{\rho_r \in \Omega_r} N(\rho_r) \| v_r \| d\rho_r \\
+ \mu_l \int_{\rho_l \in \Omega_l} N(\rho_l) \| v_l \| d\rho_l \\
+ \mu \int_{\rho \in \Omega} N(\rho) \| v \| d\rho
\]

\[
= \sum_{n=1}^{N_r} \int_0^T \alpha_{n,r}(\dot{x} + (W - r_y) \dot{\theta} - S_r)^2 \, dr_y \\
+ \sum_{n=1}^{N_l} \int_0^T \alpha_{n,l}(\dot{y} + p_{n,l}\dot{\theta})^2 \, dl_y
\]

where \( \alpha_{n,r} = \mu N(p_{n,r}) \) and \( \alpha_{n,l} = \mu N(p_{n,l}) \). The symbols \( p_{n,r/l} \) denote the \( x \)-component (in the body frame) of the \( n^{th} \) grouser contact between the right/left track and the ground. Similarly, \( N(p_{n,r/l}) \) denotes the normal force at \( p_{n,r/l} \), which is assumed to be uniform across the width of the grouser. One can arrive at this formula by substituting into the double integral a set of delta functions located at the distances where the grousers contact the ground.

Fig. 3: Side view of a tracked vehicle with grousers

The normal forces are calculated from a moment balance that balances the vehicle’s weight. The normal forces vary, based on where the grousers contact the ground. We assume the friction coefficients to be the same on all grouser tips. Again, the vehicle velocity can be predicted by minimizing the grouser-based power dissipation function, \( D_{\text{grousers}} \), with respect to the vehicle velocity. Note that the grouser positions on the right and left tracks may not be synchronized, so that the body velocity varies with changes in the relative grouser displacements. However, in practical vehicle designs, this variation is not significant. Section IV describes an approximate closed form solution to this case.

C. Modeling on stairs

Now we consider the important case where the track vehicle climbs (or descends) a set of stairs. The tracks are assumed to contact just the lip of the stair where the riser meets the tread. We assume that the stair edges are separated by a uniform distance \( D \), and that the stairs are inclined at an angle \( \phi \) with respect to the horizontal. We model the stair as a line contact, much like the grouser case. However, in this case, the line of contact is not necessarily aligned with body frame. So the dissipation function, must contain a dependence on the orientation of the body with respect to the stairs.

If the sum of the frictional reaction forces at the contacts are less than the gravitational force on the body, the body will accelerate in the direction of gravity and will violate the quasi-static approximation. We assume that this is not the case.

In addition to the body orientation dependence, the power dissipation function also includes a slope-dependent term that captures the effects of gravity on vehicle motion:

\[
D_{\text{stairs}}(\dot{x}, \dot{y}, \dot{\theta}, S_r, S_l) = \mu_r \int_{\rho_r \in \Omega_r} N(\rho_r) \| v_r \| d\rho_r \\
+ \mu_l \int_{\rho_l \in \Omega_l} N(\rho_l) \| v_l \| d\rho_l + mg \sin \phi (v_r + v_l)
\]

\[
= \sum_{n=1}^{N_r} \int_0^T \alpha_{n,r} \left[ (\dot{x} + (W - r_y \sec \theta) \dot{\theta} - S_r)^2 + (\dot{y} + (p_{n,r} + r_y \tan \theta) \dot{\theta})^2 \right] \, dr_y \\
+ \sum_{n=1}^{N_l} \int_0^T \alpha_{n,l} \left[ (\dot{x} + (W + l_y \sec \theta) \dot{\theta} - S_l)^2 + (\dot{y} + (p_{n,l} + l_y \tan \theta) \dot{\theta})^2 \right] \, dl_y
\]

Where \( \alpha_{n,r} = \mu N(p_{n,r}) + mg \sin \phi \) and \( \alpha_{n,l} = \mu N(p_{n,l}) + mg \sin \phi \). As before, \( p_{n,r/l} \) is the position (\( x \)-component) of the \( n^{th} \) contact between the right/left track and the stairs.

Once again, we calculate the normal forces by doing a moment balance. The normal forces change based on where the stairs contact the tracks. We assume the friction coefficients to be the same on all the stairs. The vehicle motion on a set of stairs is obtained by minimizing \( D_{\text{stairs}}(\dot{x}, \dot{y}, \dot{\theta}, S_r, S_l) \) with respect to the body velocities.

IV. Analysis

The power dissipation method [5] predicts the tracked vehicle’s body velocity for a given control input (track speeds). It is in general not possible to find a closed form algebraic solution to this minimization problem. While we have found the minimization described above is not burdensome, it is useful to express the state variables as a function
of the control variables \((S_r, S_l)\) given the vehicle geometry \((2L, 2W, 2T)\).

Since the power dissipation function is always positive, we can use a sum of squares formulation \([30]\) to approximate the dissipation function as a polynomial of order \(2k\). This coefficients of this polynomial can be found via least squares regression. Let \(D\) denote the sum of squares polynomial:

\[
\hat{D}(\dot{q}, u, A) = \nu^T A \nu
\]

where,

\[
\dot{q} = [\dot{x} \; \dot{y} \; \dot{\theta}]^T, \quad u = [S_r \; S_l]^T
\]

\[
\nu = [1 \; \dot{q}_x \; \dot{q}_y \; \dot{q}_\theta \; \ldots \; (\dot{q}^k)_x]^T
\]

\[
A \in \mathbb{R}^{m \times m}, \quad m = \binom{d+k}{k}.
\]

Here, \(d\) is the number of state and control variables, i.e., \(d = 5\). The polynomial fitting error is,

\[
E(\dot{q}, u, A) = \hat{D}(\dot{q}, u, A) - D_{min}(\dot{q}, u)
\]

where, \(D_{min}(\dot{q}, u)\) is the minimum value of \(D\) obtained from \([30]\). The polynomial fitting error minimization can be written as

\[
\min_A \sum_{i=1}^{n} E(\dot{q}_i, u_i, A) E^T(\dot{q}_i, u_i, A) \quad \text{s.t.}
\]

\[
\frac{\partial}{\partial \dot{q}} \hat{D}(\dot{q}_i, u_i, A) = 0 \quad \forall i \in \{1, 2, \ldots, n\}
\]

\[
A \succeq 0
\]

### A. Flipper robot on flat ground

In preparation for the experiments described in the next section, here we apply the principles described above to a model of the Rover Robotics Flipper tracked vehicle. The Flipper Rover has the dimensions given in Table [I].

On flat ground, we use a second-order approximation of the dissipation function, i.e., \(k = 1\). The polynomial coefficient matrix, \(A\), is found by solving the minimization \([30]\), yielding a second-order polynomial approximation of the dissipation function. A closed form approximation of the vehicle kinematics can be found by taking the partial derivative of the dissipation function w.r.t. \(\dot{q}\) and setting it to 0. This gives (with coefficients truncated to 3 decimal places),

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{bmatrix}
\approx
\begin{bmatrix}
0.5 & 0.5 \\
0 & 0 \\
1.5 & -1.5
\end{bmatrix}
\begin{bmatrix}
S_r \\
S_l
\end{bmatrix}
\]

### B. Flipper robot on flat ground - with grousers

We do a similar analysis as in Section [IV-A] for grousers that are about 0.04 m apart,

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{bmatrix}
\approx
\begin{bmatrix}
0.5 & 0.5 \\
0 & 0 \\
1.27 & -1.27
\end{bmatrix}
\begin{bmatrix}
S_r \\
S_l
\end{bmatrix}
\]

### C. Flipper robot on stairs

On stairs, the PDM gives a range of possible body velocities for a given \((S_r, S_l)\), as the motion depends upon the body orientation, \(\theta\), with respect to the stairs and the locations of the tread contacts on the stairs. If we know the robot’s initial configuration, we can predict its path on the stairs given \((S_r, S_l)\). Fig. 5 predicts one such path under three different motion models. For this set of track speeds, Fig. 6 plots \(\dot{q}\) as predicted by the stair model. Note how the speeds vary as the vehicle moves along the stairs and its points of contact of the tracks and stairs keep changing, along with the orientation \(\theta\). The velocities are discontinuous and hard to predict from a simple model such as the one on flat ground.

![Fig. 5: Comparison of predicted flipper robot motions under different models for constant track speeds \((S_r, S_l) = (0.21, 0.29)\) m/s.](image)

![Fig. 6: \(\dot{x}, \dot{y}, \dot{\theta}\) on stairs when \((S_r, S_l) = (0.21, 0.29)\) m/s.](image)

### V. Experiments

This section presents the results of experiments with the Flipper robot when it moves on flat ground and on stairs. We...
use the models obtained from the PDM, with the grouser and stair models computed in the previous section.

A. Setup

In each of the experiments, the Flipper’s rigid body motions and positions were recorded using an Optitrack motion capture system at Caltech. Fig. 7 shows the motion capture system and the stairs used in our experiments.

Fig. 7: Photograph of the stairs and motion capture system used in the "stair" experiments described in this section.

B. Results

We tested the Flipper robot’s movements on a flat circular path over hard solid ground. The Flipper was commanded via different motion models: (1) a differential drive model; (2) a "tuned" kinematic model provided by Rover Robotics; and (3) the power dissipation model, with grousers at the same spacing as found on the Flipper’s tracks. The tuned model includes traction coefficients and is specialized to this specific robot. We calculated the vehicle’s linear and angular velocities needed to follow the prescribed circle, and then used the different models to determine the necessary track speeds to obtain the circular path. The actual linear and angular velocity attained by the robot were obtained from the motion capture system. Because this is an open loop procedure, the errors between the commanded motion and actual motion are indicative of the errors in the underlying physics model of each approach. Fig. 8a shows the velocities attained by the different models. Fig. 8b shows the actual paths realized by the robot (with each path plot with respect to a common circular center location) under the three different models. The PDM model provided the greatest accuracy.

On stairs, it is difficult to compute a closed form solution of the kinematics for all inputs \((S_r, S_l)\) and vehicle orientations \(\theta\). Instead, we check the validity of our model by predicting how the vehicle should move on the stairs for fixed inputs \((S_r, S_l)\), and at different starting orientations of the vehicle with respect to the main axis of the stairs. We then command the Flipper vehicle with these same track speeds and record the actual trajectory. Fig. 9 shows three such trajectories. Note that the trajectories are symmetric, i.e., we get the same trajectories but in the opposite direction if we interchange the track speeds.

The PDM model predicts that the vehicle will drive straight when the track speeds are equal, regardless of the robot’s initial orientation. This behavior is seen in the experiments, see Fig. 9 With unequal track speeds, the robot will move with nonzero angular velocity. In this case the stair model predicts the path followed with some accuracy. In the specific case shown in Fig. 9 where \((S_r, S_l) = (0.21, 0.29)\) m/s, the model predicts that the robot will make a 90° turn before reaching the end of the stairs, i.e., it will never completely climb the stairs. This is exactly the behavior seen in the associated experiment. The differential drive model cannot predict this behavior on stairs as it does not take into account the interactions between the tracks and stairs. It predicts a circular motion on the stair, which is not observed.

VI. Conclusions

This paper applies, for the first time, the power dissipation method to tracked robotic vehicles. This method leads to
Fig. 9: Trajectory on stairs - PDM prediction (dashed line), differential drive prediction (dotted line), experimental data (thick line).

simple and efficient quasi-static models that can capture the surprisingly subtle effects of grousers on vehicle motion, as well as interaction with stairs. Experiments show that this model provides better motion predictions than other kinematic models for driving on flat ground and for climbing stairs.

There are many avenues for future work. An obvious next step is to use these equations within a feedback loop in order to improve trajectory tracking on flat ground and on stairs. The models presented in this paper make a simplistic assumption of Coulomb friction. The power dissipation methodology allows for other friction loss models, such as grouser/soil interaction models [2], [3], to be incorporated. This extension will provide useful motion prediction for tracked operation in soil. More importantly, this approach leads to a family of motion models on different terrain types that can be integrated within a hybrid or switching control framework to allow for adaptive behavior in different situations.

Finally, our models currently expect regular contact between the tracks and the ground or stairs. On uneven terrain it is practically impossible to know which portions of the track surfaces are in contact or not in contact with the supporting terrain. The power dissipation modeling approach should support an approach that has been taken in the quasi-static pushing literature [31]: bounds on the possible vehicle motions can be derived from the track geometry.

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