Cosmological Sphaleron from Real Tunneling
and Its Fate

Shuxue Ding

Department of Physics, Tokyo Institute of Technology,
Oh-Okayama, Meguro, Tokyo 152, Japan

ABSTRACT

We show that the cosmological sphaleron of the Einstein-Yang-Mills system can be produced from real tunneling geometries. The sphaleron will tend to roll down to the vacuum or pure gauge field configuration, when the universe evolves in the Lorentzian signature region with the sphaleron and the corresponding hypersurface being the initial data for the Yang-Mills field and the universe, respectively. However, we can also show that the sphaleron, although unstable, can be regarded as a pseudostable solution because its lifetime is even much greater than that of the universe.

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*Electronic address: sding@th.phys.titech.ac.jp
1 Introduction

Since Bartnik and McKinnon [1] found a series of static solutions of the Einstein-Yang-Mills equation, great interest has been paid to the coupled system of the Yang-Mills field and gravity [2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. What is interesting is that the Bartnik and McKinnon (BK) solutions were shown to be some analogues of a sphaleron in the pure Yang-Mills theory or the Weinberg-Salam model [12, 13]. We call them EYM sphalerons. This means that the (Lorentzian) solutions sit on the top of the potential barrier between topologically distinct vacua in the field configuration space. If the collapse of BK solutions form black holes with color, it will shed some light on the no-hair theorem of black hole [9]. Some authors also discussed anomalous productions of fermions [9, 10], with the sphalerons as backgrounds.

The sphaleron solutions in the EYM system were also considered in the context of cosmology [10]. Previously several kinds of cosmological solutions were found [14, 15]; however, it has recently been shown that some of the solutions are relevant to sphalerons [11]. One such sphaleron is a spacetime with $S^3$ spatial section and homogeneous Yang-Mills field [10].

In this paper we will discuss how and what kind of EYM sphalerons can appear in the context of cosmology and how they develop later. As we know a sphaleron is an unstable configuration which corresponds to a saddle
point on the energy functional, sitting on the top of the energy barrier. If a spacetime has only a Yang-Mills field as the source, classically, we cannot find a mechanism to make the system on the top of the barrier statically although the state exists theoretically, because the system always take the least energy state (Of course here we are only concerned with the case of zero-temperature, and we can say nothing about the existence of such a mechanism at high temperature). However, as we shall see, the cosmological sphaleron can be created by real tunneling geometries, i.e., by the nucleation of the universe from nothing represented by a Riemannian manifold with a single totally geodesic boundary or by a quantum tunneling between two Lorentzian signature regions which sandwich a Euclidean signature region with the boundary surfaces being totally geodesic \[16, 17\]. Such events are relevant to the WKB approximation of the Wheeler-DeWitt equation.

In Sec. 2, we shall give a review of the decomposition of the Einstein-Yang-Mills equation which is suitable for our purpose. In Sec. 3, we will solve the constraint equations with the real tunneling conditions both for the spacetime and the Yang-Mills field configurations. In Sec. 4, we discuss how the spacetime and the Yang-Mills field configuration develop in the Lorentzian signature region with the the real tunneling geometry and sphaleron as the initial conditions. In Sec. 5 we draw some conclusions and give some discussions.
2 3+1 Decomposition of Einstein-Yang-Mills Equation

Our aim is to discuss the time development of the universe and Yang-Mills field configuration in the Lorentzian signature region, with a real tunneling hypersurface Σ as an initial spatial section and with a purely magnetic Yang-Mills field configuration on the hypersurface Σ as initial data. Here we only consider the Yang-Mills field with $SU(2)$ as a gauge group, for simplicity. The results in the papers can be generalized to any Yang-Mills field with arbitrary compact gauge group.

As is well known, there are constraint equations in the Einstein-Yang-Mills theory (the 3+1 decompositions of the EYM equations are nontrivial [11, 18, 19]). The allowed initial data are restricted to the constraint submanifold Σ defined by [11, 18, 19]

\begin{align*}
G^a &\equiv 4[\sqrt{h}\nabla^i(E^a_i/\sqrt{h}) + \epsilon^{abc}A^{bi}E^c_i] = 0, \quad (2.1) \\
H_0 &\equiv \sqrt{h}[-(3R - 2\Lambda) + \frac{1}{h}(\pi_{ik}\pi^{ik} - \frac{1}{2}\pi^2)] + \\
&\quad \kappa \left[\left(\frac{2}{\sqrt{h}}\right)E^a_iE^i_a + \sqrt{h}F^a_{ik}F^i_{ak}\right] = 0, \quad (2.2) \\
H_i &\equiv -2\sqrt{h}\nabla_k(\pi^k_i/\sqrt{h}) + 2\kappa F^a_{ik}E^k_a = 0. \quad (2.3)
\end{align*}

at each point $x \in \Sigma$ [18, 11, 19]. Here $E^a_i = \sqrt{h}F^a_{i\theta}$ is the electric field of the Yang-Mills field, $\nabla_k$ is the covariant derivative operator on $\Sigma$ compatible
with the 3-metric $h_{ik}(i, k = 1, 2, 3)$, and $^3R$ denotes the scalar curvature with respect to $h_{ik}$. $\kappa$ stands for $8\pi G$, the coupling constant between matter field and gravity.

As a matter of fact, the constraint (2.1) is just the orthogonal component of the Yang-Mills field equation

$$(*D * F)_\perp = 0,$$

(2.4)

in the formalism of differential form, where the asterisk denotes is the Hodge dual.

Here the metric is decomposed as

$$ds^2 = -(N^2 - N^i N_i)dt^2 + 2N_i dt dx^i + h_{ik} dx^i dx^k,$$

(2.5)

in the Lorentzian signature region. $\pi^{ik}$ is the momentum canonically conjugate to $h^{ik}$ which is related to the extrinsic curvature $K^{ik}$ of $\Sigma$,

$$\pi^{ik} = \sqrt h (K^{ik} - h^{ik} K).$$

(2.6)

The extrinsic curvature is defined as

$$K_{ik} = \frac{1}{2N}(h_{ik,0} - \nabla_k N_i - \nabla_i N_k),$$

(2.7)

where $K$ is the trace of $K_{ik}$.

The evolution equations of geometry and Yang-Mills field are given by

$$\mathcal{L}_0 E^a_k = N[D^i F^a_{ik} + \frac{1}{2} K E^a_k + F^b_k \epsilon^{abc} A^c_{0i}] + \nabla^i N F^a_{ik} + \mathcal{L}_N E^a_k,$$

(2.8)
\[ \mathcal{L}_0 K^i_k = N[R^i_k + \delta K^i_k + 2\kappa(E^{ai}E^a_k + F^{ai}F^{al}_k) - \kappa \delta^i_k (E^{al}E^a) - \frac{1}{2} F^{i}_{lm} F^{alm}] - \nabla^i \nabla_k N + \mathcal{L}_N K^i_k. \]  

(2.9)

Here \( D^i \) stands for the covariant derivative compatible both with Yang-Mills and spacetime connections and \( \mathcal{L} \) stands for the Lie derivative. The constraint equation (2.1) together with the evolution equation (2.8) are equivalent to the Yang-Mills equation. Similarly the constraint equations (2.2), (2.3) and the evolution equation (2.9) are equivalent to the Einstein equation.

It is worthy of noticing that the evolution equation (2.8) is the \( \Sigma \) component of the Yang-Mills field equation

\[ (\ast D \ast F)_\Sigma = 0. \]  

(2.10)

in the formalism of differential form.

3 Real Tunneling Geometry and Yang-Mills Field

Real tunneling geometries arise in the WKB approximation for the solution of the Wheeler-DeWitt equation as a special case of the general situation, in which one considers “complex paths” which are spacetimes with complex metrics. Real tunneling geometries are partially Lorentzian and partially Riemannian [10, 17], as a special complex geometry. Alterna-
tively the Lorentzian and Riemannian portions may be regarded as different real slices of a complex spacetime $M_c$, which have a common boundary $\Sigma$ (spacelike). This hypersurface $\Sigma$ acts as an initial Cauchy surface for the Lorentzian spacetime and the matter field on it as the initial matter field of the Lorentzian spacetime.

It was shown [16, 17] that the extrinsic curvature $K_{ik}$ and matter field must be continuous across $\Sigma$ to make the action finite. The continuity of these require that on $\Sigma$

$$K_{ik} = 0, \quad (3.1)$$

and momentum of the matter field vanishes, to our case

$$E^a_k = 0. \quad (3.2)$$

Let us find a solution which satisfies the above conditions and constraint equations (2.1), (2.2), and (2.3) on the initial hypersurface $\Sigma$. Substituting the conditions (3.1) and (3.2) into the constraints (2.1), (2.2), and (2.3), we find the first and the third constraints are automatically satisfied, and the Hamiltonian (2.2) constraint of gravity becomes

$$3R = \frac{\kappa}{2} F^{a}_{ik} F_{a}^{ik} + 2\Lambda \quad (3.3)$$

This can be regarded as a constraint equation for the curvature of spatial section if we are given a pure magnetic field. The simplest but nontrivial
situation will be the spatial section with a constant curvature. Because, however, the right hand side of the equation (3.3) is always positive if the cosmological constant is positive, there exists only a positive constant curvature solution, i.e., $S^3$, for spatial section. For simplicity of discussions, we introduce a four-dimensional Euclidean space with the induced metric

$$d\Omega^3 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 + (dx^4)^2,$$

for a three-dimensional sphere $S^3$,

$$(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2 = 1.$$  

(3.5)

Because there exists a natural imbedding of $S^3$ into the group manifold $SU(2)$, $g = x^4 + ix^i\sigma^i$, the left invariant one-form of the gauge group is given by

$$e^i = 2(x^4 dx^i - x^i dx^4 + \epsilon_{ijk} x^j dx^k), \quad i = 1, 2, 3.$$  

(3.6)

Here the one-form $e^i$ obeys the Maurer-Cartan structure equation

$$de^i = \frac{1}{2} \epsilon_{ijk} e^j \wedge e^k.$$  

(3.7)

There is a natural ansatz for the gauge field on $S^3$ which takes the form

$$A = -\frac{1}{e} f e^i \sigma^i,$$  

(3.8)

where $f$ is a constant to be determined. The field strength associated with (3.8) is

$$F = \frac{1}{2e} f (f - 1) \epsilon_{ijk} e^j \wedge e^k \sigma^i.$$  

(3.9)
This is a kind of homogeneous Yang-Mills field. From the expression (3.9) or directly from (3.8), one observes that \( f = 0 \) and \( f = 1 \) correspond to vacuum or pure gauge field configurations.

To get a complete solution with a given initial value on the hypersurface \( \Sigma \), only considering the constraint equations is not enough, and we have to take advantage of the evolution equations. In terms of differential form, the evolution equation (2.8) becomes

\[
(\partial_0 E^a_k) e^k \sigma^a = [\tilde{e}^i F_{ik}^a - \frac{1}{2} \epsilon_k^{ij} F_{ij}^a + e \epsilon^{abc}(A^a_{ij} F_{jk}^b - A^a_{0k} E^b_k)] e^k \sigma^a.
\]  

(3.10)

where

\[
\tilde{e}^i = \frac{1}{2}(x^4 \frac{\partial}{\partial x^i} - x^i \frac{\partial}{\partial x^4} + \epsilon^i_{jk} x^j \frac{\partial}{\partial x^k}),
\]  

(3.11)

is the left invariant vector field dual to (3.6). By the real tunneling condition (3.2) together with our ansatz (3.8) and the field strength (3.9), the evolution equation for the Yang-Mills field now becomes

\[
\frac{1}{e} f (2f - 1)(f - 1) \epsilon_{ijk} e^j \wedge e^k \sigma^i = 0.
\]  

(3.12)

We immediately notice that, \( f = 1/2 \) is a nontrivial static solution of the equation (3.12). The Chern-Simons index is easy to be calculated, which is

\[
N_{CS} = 3 f^2 [1 - (2/3)f] = 1/2.
\]  

(3.13)

Then this static solution is a sphaleron.
At this initial moment, the universe is just equivalent to the Hosotani type universe [14]. However we shall see in the next section that the spacetime and Yang-Mills field will evolve to make the universe leave the Hosotani universe due to the instability of the sphaleron.

The Yang-Mills field configuration, i.e., the sphaleron, is corresponding to a local maximum point in the viewpoint of the Lorentzian signature region, while it is corresponding to a local minimum point in the viewpoint of Euclidean signature region [20]. It is just from this reason that, we can extend the solution to the whole Euclidean signature region, i.e.,

\[ F^i = -\frac{1}{8}\epsilon^{ijk}e^j e^k \sigma^i / 2. \]

Our spacetime is endowed with the metric

\[ ds^2 = a_0^2(d\tau^2 + d\Omega^3) = a_0^2(-d\eta^2 + d\Omega^3), \tag{3.14} \]

where \( a_0^2 \) is any constant having dimension \([L^2]\), \( \tau \) is the Euclidean time and \( \eta \) is the conformal time in the Lorentzian signature region, which have a relation with the comoving time \( t \) defined in the next section. A Wick rotation \( \tau = i\eta \) will act on the initial hypersurface \( \Sigma \) because the hypersurface may be defined as the time slice with \( \tau = \eta = 0 \). With the viewpoint of the Euclidean signature region, the hypersurface \( \Sigma \) corresponds to the Yang-Mills wormhole [21] at the throat.
4 Evolution in Lorentzian Signature Region

Let us discuss how the spacetime and the Yang-Mills field develop with (3.8), (3.9), and (3.14) being the initial data. It is reasonable to suppose that the spatial section still has the symmetry of $S^3$ and the Yang-Mills field homogeneously distributes in the spatial section $\Sigma$ at a later time, because there is no mechanism to break such a symmetry in our model. However, the radius of $\Sigma$ and then $f$ in the ansatz (3.8) will change in time. Then following Gibbons and Steif [10], the ansatz for the Yang-Mills connection one-form is

$$A = -\frac{1}{e} f(\eta) e^i \sigma^i. \quad (4.1)$$

This will make the electric field nonzero,

$$F = F^i \sigma^i, \quad F^i = -\frac{\dot{f}}{e} d\eta \wedge e^i + \frac{1}{2} e f(f - 1) \epsilon_{ijk} e^j \wedge e^k. \quad (4.2)$$

The metric is supposed to be

$$ds^2 = a^2(\eta)(-d\eta^2 + d\Omega^3) = -dt^2 + a^2(t)d\Omega^3, \quad (4.3)$$

based on the symmetry of the spacetime. Here the cosmic time $t$ and the conformal time $\eta$ are related by $\partial \eta/\partial t = a^{-1}$.

We will solve the constraint equations and evolution equation with the initial values

$$f(0) = \frac{1}{2}, \quad f'(0) = 0. \quad (4.4)$$
and
\[ a^2(0) = a_0^2. \]  
(4.5)

We first consider the constraints (2.1), (2.2), and (2.3). Again the constraints (2.1) and (2.3) are automatically satisfied by the ansatz (4.1) and (4.3).

From the constraint (2.2), with (4.1), (4.2) and (4.3), we obtain
\[ \frac{(\dot{a})^2}{a^2} = \frac{\Lambda}{3} a^2 + 4\kappa e^{-2} \left[ \frac{1}{2} (\dot{f})^2 + 2f^2(f-1)^2 \right] \frac{1}{a^2} - 1. \]  
(4.6)

Now let us turn to the evolution equations (2.8) and (2.9). By the ansatz (4.3), we have \( N = a(\eta) \) and \( N_i = 0 \). The evolution equation (2.8) is relevant to the developing of the Yang-Mills field. Because the Yang-Mills equation is conformal invariant, we can discuss the equation (2.8) in a conformally transformed metric \( ds'^2 = -d\eta^2 + d\Omega^2 \), i.e., \( N' = 1 \) and \( N'_i = 0 \). The evolution equation (2.8) can be simplified into the form
\[ \frac{d^2f}{d\eta^2} = -\frac{\partial V}{\partial f}, \quad V(f) = 2f^2(f-1)^2, \]  
(4.7)
or
\[ \frac{1}{2} f^2 + V(f) = E, \]  
(4.8)
where \( E \) is a constant to be determined.

The evolution equation (2.9) can be written as
\[ a\ddot{a} + 2a^2 = 2\kappa e^2 a^{-2} + \Lambda a^2 - 2. \]  
(4.9)
The general solution of the equation (4.8) can be written as an elliptic function. However, from the initial condition (4.4), we have $E = 1/8$. For such a special case the solution of the equation is simply given by

$$f(\eta) = \frac{1}{2} \pm \frac{\sqrt{2} \exp(\sqrt{2}\eta)}{1 + \exp(2\sqrt{2}\eta)}.$$  

(4.10)

This solution was given by Gibbons and Steif [10], taken as a special example for a rolling down cosmological sphaleron. However, we give it as an inevitable result if the sphaleron comes from real tunneling.

We can see that at $\eta = -\infty$, the Yang-Mills field becomes the sphaleron configuration, while as $\eta \to \frac{\sqrt{2}}{2} \ln(\sqrt{2} - 1)$ the Yang-Mills field approaches the vacuum or pure gauge configuration. From the equation (4.10), we see that the lifetime of the sphaleron to roll down is infinite.

By the equation (4.8) with $E = 1/8$, the equation (4.6) can be simplified as

$$a^{-2}(\dot{a})^2 = \frac{\Lambda}{3} a^2 + \frac{1}{2} \kappa e^{-2} a^{-2} - 1.$$  

(4.11)

which is independent of the specific form of $f(\eta)$. To contrast this with a Newtonian particle moving on a potential, we can write the equation (4.11) as

$$\frac{1}{2} (\dot{a})^2 + U(a) = E_a$$  

(4.12)

where $E_a = \frac{\kappa}{4e^2}$ is a constant analogous to the energy and $U(a) = \frac{1}{2}(a^2 - \frac{\Lambda}{3} a^4)$ the potential of classical particle. When the cosmological constant $\Lambda$
vanishes, the “potential” is simply $\frac{1}{2}a^2$ and the solution can easily be got

$$a^2(\eta) = \frac{\kappa}{2e^2} \sin^2(\eta + \eta_0) = \frac{\kappa}{2e^2} (1 - (1 - \frac{t}{\sqrt{2e^2}})^2).$$

(4.13)

From the equation (4.13), we see the lifetime of the universe is $\Delta \eta = \pi$. However, from the equation (4.10), the lifetime of the sphaleron to roll down to the vacuum or pure gauge configuration is, infinite, much longer than the lifetime of the universe. As we know a sphaleron solution is an unstable solution generally. However, for our case the sphaleron solution can be regarded as a pseudostable solution because it will roll down very slowly compared with the evolution of the universe.

As is well known, a cosmological constant, if it exists and is positive, is always likely to drive the universe to expand longer. Now let us see if a sphaleron can complete a whole process of rolling down to the vacuum or pure gauge configuration, provided that $\Lambda > 0$. For this sake, we solve the equation (4.11) and change it into the form

$$\frac{da}{d\eta} = \sqrt{\frac{\Lambda}{3}} \sqrt{(a^2 - A)(a^2 - B)}.$$

(4.14)

Here

$$A = \frac{3}{2\Lambda} (1 + \sqrt{1 - \frac{2\kappa\Lambda}{3e^2}})$$

and

$$B = \frac{3}{2\Lambda} (1 - \sqrt{1 - \frac{2\kappa\Lambda}{3e^2}})$$
are two constants.

The solution of the equation (4.14) can be written as an elliptic function generally. If the universe initiates from the hypersurface with radius $a_0 < \sqrt{A}$, the situation is similar to those when the cosmological constant is vanishing. If the universe initiates from a hypersurface with $a_0 > \sqrt{A}$, the universe will expand forever. By the real tunneling, it is possible for a universe to start from a finite region [22], and what is more the potential $U(a)$ in the equation (2.11) is very similar to those discussed by Blau et al. with a domain wall emerging from a finite size by quantum tunneling [23]. In this case the solution for the equation (4.14) is

$$a(\eta) = \sqrt{A}dc\left(\sqrt{\frac{A\Lambda}{3}}(\eta - \eta_0)\frac{B}{A}\right), \quad (4.15)$$

where $dc(x|k)$ is one of the elliptic functions [24]. Here $\eta_0$ is a constant determined by

$$a_0 = a(\eta = 0) = \sqrt{A}dc\left(\sqrt{\frac{A\Lambda}{3}}(-\eta_0)\frac{B}{A}\right). \quad (4.16)$$

As a matter of fact, the solution of the equation (4.11) can be gotten more easily in the coordinate $t$,

$$a^2(t) = \frac{3\kappa\Lambda}{\Lambda c^3 e^2} - \frac{1}{4}(1 - c \cdot exp(-2\sqrt{\frac{\Lambda}{3}t}))^2 \cdot exp(2\sqrt{\frac{\Lambda}{3}t}). \quad (4.17)$$

Here $c$ is a constant determined by the initial condition (4.3),

$$c = \left[1 - \frac{2\Lambda}{3}a_0^2 + \sqrt{(1 - \frac{2\Lambda}{3}a_0^2)^2 + \frac{2\kappa\Lambda}{3e^2} - 1}\right]. \quad (4.18)$$
Because the solution for the Yang-Mills field configuration (4.10) is written as a function of $\eta$, we have to compare the lifetimes for the sphaleron to roll down and the universe to develop by $\eta$. By the equation (4.15), the scale factor $a$ will get infinity when $\eta \to 2B/A$. That is to say, although the universe may expand to infinity, the lifetime is finite in the coordinate of $\eta$. Because the rolling from the sphaleron will take infinite time ($\eta$), this whole procedure cannot be finished within one period of the evolution of the universe. In this case the sphaleron can also be regarded as a pseudostable solution.

5 Conclusions and Discussions

We have obtained the cosmological Yang-Mills field which describes the rolling down from the cosmological sphaleron in the potential picture. The cosmological sphaleron is produced by a real tunneling geometry from nothing or from another Lorentzian signature region emerging from an Euclidean signature region. It is natural to ask such a question as how probable the real tunneling is? To answer this question, the amplitude for a hypersurface to emerge from a real tunneling with sphaleron on it should be calculated,

$$\Psi(S^3, \text{Sphaleron}) = Ne^{-Se} = N,$$  

(5.1)
because the Euclidean action vanishes for the sphaleron states. Here N is a prefactor in WKB approximation which in principle calculable. This amplitude is a local maximum point because the action is a local minimum point in the Euclidean signature region. We can see this fact from the equation (4.7) by making the Wick rotation, $\eta = -i\tau$. The potential becomes $-2f^2(f - 1)$, and $f = 1/2$ is a minimum value of it. As a matter of fact, this is very similar to the discussion for the BK solutions corresponding to saddle points for the Euclidean energy functional [7].

After the real tunneling, the solution for $f(t)$ [or $f(\eta)$] and $a^2(t)$ still describes a homogeneous Yang-Mills field and the homogeneous and isotropic universe with $S^3$ as a spatial section $\Sigma$. The electrical field is produced during the rolling and evolution for the Yang-Mills field and the universe, respectively.

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