Development of fragility curves for road embankments exposed to perpendicular debris flows

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ABSTRACT

Debris flows cause recurrent interruptions and permanent damages in rural road networks, representing significant economic losses. Embankments are road assets frequently exposed to debris flows, especially in mountainous areas. The potential risk of infrastructure exposed to debris flows can be assessed in terms of the probability of expected damage based on fragility curves. The aim of the study is to develop fragility curves for road embankments exposed to debris flows representative to the expected physical damage and capacity loss. Two conceptual models are proposed to describe the impact and erosion of debris flows that run perpendicularly to road embankments. Probabilistic models are then developed in terms of limit state functions and the simulation of potential scenarios. The resulting models are finally fit to log-normal distributions and compared to historical data. Headcut erosion has higher probabilities of occurrence compared to sliding failure caused by the impact of debris flow. Lower road embankments present higher probabilities of failure to the impact of debris flows, especially for dense flows. Whereas, longer interaction times of less dense flows increase the probability of headcut erosion. Two-lane roads may present 50\% more probabilities of headcut erosion compared to multilane roads for similar debris flows intensity.

ARTICLE HISTORY

Received 13 November 2020
Accepted 22 May 2021

KEYWORDS

Debris flows; road embankments; rural roads; perpendicular impact; erosion; fragility curves

1. Introduction

Debris flows are sediment and water mixtures driven by gravity transporting debris of various sizes (Takahashi 2014; Jakob and Hungr 2005). This phenomena are considered the third most lethal and destructive natural hazard, after earthquakes and...
floods (Thouret et al. 2020). Prieto et al. (2018) emphasize that these phenomena represent a significant part of the global economic losses generated by hydrological hazards, often affecting human settlements and infrastructure located at valley bottoms. The consequences of debris flows include loss of human lives, destruction of homes and facilities, damage to railway lines, destruction and interruption of roads, among other indirect consequences such as loss of productivity and social impact (Jakob and Hungr 2005; Tacnet et al. 2012). One of the most exposed infrastructures to these phenomena are road networks, as they adapt to the topographies in which they are located. In 2004 in Scotland a series of debris flows affected the road network, resulting in approximately US$2.5 million of direct costs for clearing, repairing, and replacing of damaged infrastructure (Winter et al. 2016). In 2006, an event affected the motorway connecting Sweden and Norway where losses of EUR$1.2 million were estimated due to road damage and disruption (Jenelius 2010). In 2015, several debris flows affected the North of Chile where damages to homes, buildings and road infrastructure were valued at US$1.5 billion (SERNAGEOMIN, Servicio Nacional de Geología y Minería 2017).

Bridges and embankments are road assets most commonly exposed to debris flows (Argyroudis et al. 2019). The effects of flows depend on whether the runoff moves parallel or perpendicular to the exposed road element, with the understanding that in most cases the approximation may also flow in other directions. Bridges have a high exposure to debris flows since they are designed to cross river sections, mostly perpendicular to the flow. The main effects of these flows on bridges result from the riverbed scouring on the foundation of piers and abutments, as well as deck sliding depending on the intensity of the event (Kim et al. 2017; Dagá et al. 2018; Zou et al. 2018; Argyroudis et al. 2019). Embankments are also affected when they are exposed to perpendicular flows passing through smaller water runoff areas, where sewers or culverts usually exist, as well as when they provide access to bridges. This impact can partially or completely damage embankments as a result of thrust force and the erosion of the substructure (Zou et al. 2018; Argyroudis et al. 2019). Also, when roads are placed parallel to or near river plains, embankments may be eroded by the flow. In addition to physical consequences of embankment damage, flows may temporarily or permanently affect traffic and the level of service of a road, decreasing their operational capacity (Suweda 2016; Contreras-Jara et al. 2018; Argyroudis et al. 2019).

The stochastic nature of the natural hazard and susceptibility that the road infrastructure has to damage requires the development of probabilistic models, or fragility curves (Liang and Xiong 2019). Fragility curves estimate the probability of exceeding different limit damage states as function of an intensity measure of the hazard (Shinozuka et al. 2000; Schultz et al. 2010; Pitilakis et al. 2014; Porter 2016). Tsubaki et al. (2016) developed fragility curves to assess the damage probability of rail embankments due to overtopping flood using flow height simulations and field records. Martinović et al. (2018) proposed a methodology for developing fragility curves for rainfall induced landslides on transport networks and applied it to a typical slope case on the Irish rail network. Through a reliability and finite element analysis Kim et al. (2017) developed fragility curves representing multiple types of bridge failure due to flood. Winter et al. (2013, 2014) developed fragility curves for roads...
associated with traffic disruption resulting from the deposit of debris flow material on the platform. Dagá et al. (2018) developed failure curves of bridges exposed to lahar flows of volcanoes, analytically modeling the overturning pier/abutment, as well as the bridge deck sliding. Liang and Xiong (2019) quantified the damage of bridge substructure exposed to the impact of debris flows, obtaining fragility curves through finite element simulation and definition of a damage index.

Based on probabilities of damage or failure obtained from fragility curves, the potential risk of infrastructure exposed to debris flows can be assessed. Utasse et al. (2016) developed two independent approaches to assess the vulnerability and the theoretical risk of road networks exposed to debris flows, and compared the results to current management practices in the French Alps. The study estimated a deterministic vulnerability in terms of loss of territorial accessibility, not considering the physical effects of debris flows and the probabilities of damage of the road network. Winter and Wong (2020) proposed a methodology to quantify road users risk in terms of potential fatalities, when they are exposed to debris flows. The study was applied to two road networks in Scotland with high debris flows exposure.

Several authors have described and modelled the effects of debris flows over bridges, railways and buildings; however, limited literature is available regarding the probability of failure due to physical impacts on road embankments and their operational effects. The aim of the study is to develop fragility curves for road embankments exposed to debris flows representative to the expected physical damage and their effect in road capacity loss. Two conceptual models are proposed to describe the impact and erosion of debris flows that run perpendicularly to road embankments. The scope is to understand the mechanisms of failure of embankments exposed perpendicularly to debris flows, which is a common phenomenon observed in mountainous areas. The effects of debris flows running parallel to embankments, however, are not part of the scope of the present research, being a future challenge to study.

2. Effects of debris flows on road embankments

2.1. Debris flows characteristics

Debris flows can be triggered by intense rainfall, snowfall, earthquakes, and volcanic eruptions (Nettleton et al. 2005). Conditions that favor their occurrence are the availability of sediments and debris, saturation of surface soils, absence of vegetation coverage, steep slopes, among others (Skilodimou and Bathrellos 2016). Debris flows are composed of two solid-liquid phases, where fine particles and water form a relatively uniform grout, while thicker particles form the solid phase of the flow (He et al. 2016). It has been shown that due mainly to their density, debris flows are able to transport rocks, trees, and more forceful elements such as cars or parts of construction (Contreras et al. 2015). To simplify the analysis, regardless of its triggering factor, this study considers from hyper-diluted flows, associated with densities of 1300 $\text{kg m}^{-3}$, to hyper-concentrated flows associated with densities close to 2500 $\text{kg m}^{-3}$ (Takahashi 2014), which is reflected in the variability assigned to weight specific flow later. The intensity at which the destructive capacity of these flows can be measured relates to the hydraulic characteristics that define them, which generally depend on the velocity,
discharge rate and height of the flow (Thouret et al. 2020). These intensities in turn will depend on the conditions of the particular area in which debris flow is developing. However, defining this in a deterministic way is complex and limits the study. To consider more than one possible scenario, the study includes areas with average slopes of 0.17 \( m/m \) (Sepúlveda et al. 2006, 2014) and Manning roughness values between 0.03 and 0.16 (Molinas et al. 2001; Fazarinc et al. 2006). For the debris, given that it can be practically any type of object, it was decided to consider a gravel block with an average diameter of 1.5 m (Sepúlveda et al. 2006).

### 2.2. Exposure of road embankments to debris flows

A road embankment is a volume of earth laid and compacted over the natural soil, raising the level of a road to improve geometric and structural characteristics. It consists of three parts, described bottom-up: base of the embankment, core corresponding to the central body, and the crown or surface. The embankment core is commonly designed with relatively homogeneous non-cohesive granular soil (Wu et al. 2012). The height of the embankment depends on several factors such as design, location, and functionality. However, to simplify the analysis and based on the experience of the authors, embankments with heights from 1 m to 6 m will be considered in this study. As can be seen in Figure 1, some conditions are activated once the height of the flow exceeds the height of the embankment. Therefore, to consider these effects in the models, it is decided to discretize in 3 ranges of embankment height, 1–2.5 m, 2.5–4 m, and 4–6 m. This division tried to group embankments that behaved similarly, in addition to not losing practicality in the analysis.

The effects of debris flows on road embankments depend on the relative exposure to the flow. Embankments that run adjacent and relatively parallel to the flow may present erosion of the exposed slope, this is especially critical in absence of fluvial defenses. Embankments that are exposed perpendicularly to debris flows commonly act as a body that obstructs the flow (see Figure 1(a)) demanded by three major stresses: flow impact on the body of embankment, flow shear stress and weight on the crown of embankment, and erosion resulting from the flow over the base of the embankment.
downstream slope. Table 1 describes in detail these phenomena and involved forces. Overturning of embankments is not typical given their non-cohesive nature.

Predominant processes affecting embankments are three-dimensional (Robinson and Hanson 1994). To simplify modelling, however, the analyses will be two-dimensional considering plane strain conditions and a unit width of debris flow $L_{\text{flow}}$ (m) as recommended by several authors (Robinson and Hanson 1994; Hanson et al. 2001; Zhao et al. 2015).

### 2.3. Physical and operational effects on embankments

Two independent physical models resulting from the combined demanding stresses acting over embankments are proposed. The first is related to the impact that flow exerts on the embankment body, which can cause complete sliding of the latter, mainly when flows are dense. Embankments often have sewers or culverts, which are usually located transversely to the road and to the runoff of surrounding waters. The second model, known as headcut or headcut erosion, corresponds to downstream erosion of the embankment caused by the flow falling over the foot of the downstream slope; this erosion may be caused by all flow types, even clear water flows (Figure 3). The physical model that will be triggered in a specific scenario will depend on the embankment characteristics, geography and geology of the site, as well as the type of flow, whose characteristics and properties will be assigned as variables in section 4 to include the possible scenarios in which debris flows interact with road embankments. Rural road pavements are normally flexible (asphalt) or rigid (concrete), and their thickness design depends on factors like traffic and climate, among others. In addition, embankments are generally built-in layers with different physical and mechanical properties. However, for the analysis of this study, the road is considered as a body composed of homogeneous granular material, because the weakest interface of the embankment is between the culvert and granular material identified

| Demanding stresses | Description |
|--------------------|-------------|
| Impact on the embankment body | This is divided into the impact of liquid phase of flow containing sediments diluted in it resulting in a hydrodynamic force $\text{1}$ and the impact of debris that flow can carry resulting in a collision force $\text{2}$ (Zanchetta et al. 2004; Liang and Xiong 2019). |
| Shear stress and weight on the embankment | When height of debris flows exceeds the height of embankment, flow exerts force and passes over the embankment crown, exerting a shear stress $\text{3}$ and adding a hydrostatic force $\text{4}$ produced by the weight of fluid over the infrastructure (Zanchetta et al. 2004). |
| Erosion downstream of the embankment | When the flow passes over the crown, it falls downstream of the embankment with a phenomenon known as headcut, where there is local erosion at the foot of the slope, which causes a rupture or sliding surface in the slope by the loss of soil and weight $\text{5}$ (Hanson et al. 2001; Zhu et al. 2006, 2008). |
as a zone of discontinuity, regardless of the materiality or properties of the surface layers. The main difference between the types of roads considered are the widths of platform and lateral clearance as described below.

Operational consequences, such as traffic disruptions due to embankment damage must be accounted when assessing risk of road networks. The expected impacts will depend on traffic flows and road capacity, which is defined as maximum hourly rate at which vehicles reasonably can be expected to traverse a point or a uniform section of a lane or roadway during a given time period under prevailing roadway, traffic, and control conditions (TRB 2010). Two types of roads representative to rural roads capacity observed in mountainous areas are considered in the study. These are: two-lane roads, having a single lane per direction, and multilane roads, with two lanes per direction (TRB 2010). For both cases the platform width considers lane width and lateral clearance. Where lateral clearance is the sum of the outer shoulder width, platform crown over-width and platform base over-width. In addition, for multilane roads the inner shoulder width and median width are considered. The model triggered by debris flows impact considers that the embankment displacement will cause surface failure and, therefore, complete capacity loss. For the headcut erosion model, three damage states are considered in terms of capacity loss ex-post the occurrence of the event as described in Table 2. Capacity loss will depend on the proportion of affected embankment crown, platform width and the number of lanes.

3. Conceptual models of embankment damage

3.1. Limit state function

Two conceptual analytical models are developed for the construction of fragility curves (Pitilakis et al. 2014; Porter 2016). The basis of this method is the definition of a limit state function that considers most critical variables involved in the model (Schultz et al. 2010). Function $S(\hat{x}_j)$ represents debris flows forces and function $R(\hat{x}_j)$ the embankment resistant forces, where the difference between both defines the limit state function $G(\hat{x}_j)$ presented in Equation (1).

$$G(\hat{x}_j) = R(\hat{x}_j) - S(\hat{x}_j)$$

If $G(\hat{x}_j)$ takes a value greater than 0 it means that there is still residual resistance; whereas, if the function is less than 0, the system is in a failed state (Schultz et al. 2010). Variables $\hat{x}_j$ that determine the demand and resistant forces can be deterministic or probabilistic introducing randomness to the analysis. Each value that takes the limit state function is obtained for a certain hazard intensity measure, depending on the characteristics and severity of the natural phenomenon and its effects on the infrastructure (Pitilakis et al. 2014). Most common intensity measures observed in literature for debris flows are the momentum flux (height by squared velocity) (Jakob et al. 2012; Prieto et al. 2018; Liang and Xiong 2019), debris flow volume (Winter et al. 2014), and flow height (Quan Luna et al. 2011; Totschnig et al. 2011; Dagá et al. 2018). The three demanding stresses defined in Table 1 are dependent of flow height, whereas, flow characteristics can be parametrized in terms of this variable. Given this,
flow height was selected as the hazard intensity measure for the development of fragility curves.

### 3.2. Embankment sliding conceptual model

Embankment sliding failure occurs when debris flow forces defined in Equation (1) as $S(x_j)$ exceed the frictional force between culvert material and the soil at the embankment base related to $R(x_j)$, considering in the analysis a unit portion of width on the out of plane axis of Figure 2. Demanding forces $S(x_j)$ are hydrodynamic force $F_{\text{hydro}}(N)$ and debris impact force $F_{\text{deb}}(N)$, while the resistant force $R(x_j)$ is friction $F_{\text{fric}}(N)$ between embankment base and the contact surface with the culvert or natural subgrade. When flow height $h_{\text{flow}}(m)$ exceeds embankment height $h_{\text{emb}}(m)$, the flow weight is added to friction force, defined as $F_{\text{fric}}(N)$ and adding the force associated with shear force $F_{\text{shear}}(N)$ on the embankment crown. The forces mentioned are

| Damage state               | Two-lane rural road                                      | Multilane road                                      |
|----------------------------|---------------------------------------------------------|-----------------------------------------------------|
| Slight damage              | Erosion causes damage within the lateral clearance       | Erosion causes damage within the lateral clearance   |
| Low to null probability of capacity reduction |                                                         |                                                     |
| Moderate damage            | Erosion causes damage in the downstream lane resulting in partial or complete closure of one traffic direction | Erosion causes damage in one or both downstream lanes resulting in partial or complete closure of one traffic direction |
| Probability of capacity reduction up to 50% |                                                         |                                                     |
| Extensive damage           | Erosion causes damage in both lanes resulting in partial or complete closure of both traffic directions | Erosion causes damage in more than two lanes resulting in partial or complete closure of both traffic direction |
| Probability of capacity reduction greater than 50% |                                                         |                                                     |

Figure 2. Free body diagram for embankment sliding delimited by green dotted line. The sliding zone is located between the base of the embankment and the upper zone of the culvert. A car is shown just to contextualize the diagram.
illustrated in Figure 2, in which the embankment body is defined as a free-body diagram delimited by a green dotted line. In case of no drainage, phenomenon results in evaluating the friction generated between the compacted soil of the embankment and the natural soil, which in practice is a similar analysis to what is considered for friction between embankment base and drainage.

Limit state function $G_1(\tilde{x}_j)$ is defined in terms of whether the flow exceeds embankment height.

$$G_1(\tilde{x}_j) = \begin{cases} F_{fric_1} - F_{hydro} - F_{deb}, h_{flow} \leq h_{emb} \\ F_{fric_2} - F_{hydro} - F_{deb} - F_{shear}, h_{flow} > h_{emb} \end{cases}$$  \hspace{1cm} (2)

For forces $F_{hydro}$, $F_{deb}$, $F_{fric_1}$, $F_{fric_2}$ and $F_{shear}$ physical and geometric parameters of debris flows are defined. For calculation of average flow velocity, Manning’s equation is considered using a variation in the term of roughness called ‘pseudo Manning’, which considers the rheological differences between water and debris flows (Jin and Fread 1999). Being $n'(\cdot)$ Manning’s ‘pseudo’ roughness, flow height $h_{flow}(m)$ and $i(m)_m$ channel slope.

$$v_{flow} = \frac{1}{n'} \left( \frac{h_{flow} * L_{flow}}{2 * h_{flow} + L_{flow}} \right) ^ {\frac{2}{3}} \sqrt{i} \hspace{1cm} (3)$$

The model proposed by AASHTO (2012) was used for hydrodynamic pressure, which depends on the flow velocity $v_{flow}(m/s)$, gravity acceleration $g(m/s^2)$, flow specific weight $\gamma_{flow}(N/m^3)$ and a drag coefficient $C_d(\cdot)$ equal to 1.4 (AASHTO 2012; Dagá et al. 2018). In addition, an inverted triangular pressure distribution was considered. This is the pressure on a perpendicular surface, however, road embankments have a slope, so the resulting maximum pressure must be expressed in terms of the angle $\alpha(\cdot)$ of the embankment slope. The resulting force is calculated as the maximum pressure applied to the surface of the embankment slope, where $\beta(\cdot)$ is the slope inclination of the surface where the flow drains resulting in the hydrodynamic force $F_{hydro}(N)$ as presented in Equation (4).

$$F_{hydro} = \frac{C_d v_{flow}^2 h_{flow} L_{flow} \cos(90^\circ - \alpha) \cos(\beta)}{2g \sin \alpha} \hspace{1cm} (4)$$

For debris force, the equation of Haechnel and Daly (2004) adapted by Stolle et al. (2018) with a correction factor equal to 0.52 was used (Equation (5)). This considers flow velocity $v_{flow}(m/s)$, debris average diameter $D_{deb}(m)$, and specific weight of gravel blocks $\gamma_{deb}(kN/m^3)$ equal to 25.97 (Wang et al. 2018). Contact stiffness of the impacting debris $k(MN/m)$ is equal to 14 for similar analysis in steel elements (Haechnel and Daly 2004; AASHTO 2012; Dagá et al. 2018).
$F_{deb} = \nu_{flow} \sqrt{0.52k \left( \frac{4}{3} \pi (0.5D_{deb})^3 \gamma_{deb} \right)}$ (5)

Shear force $F_{shear}$ (Equation (6)) applied on the surface of embankment crown is defined in terms of a shear stress resulting from the flow liquid phase $\tau_{flow}$ and a shear stress caused by thick grains $\tau_{grain}$, as these interact with the surface through durable contacts such as frictional sliding and rolling (Jakob and Hungr 2005). The contact surface where stress is applied depends on the unit width of flow $L_{flow}$ and the width of the platform crown $b_{platform}$.

$$F_{shear} = (\tau_{flow} + \tau_{grain})L_{flow}b_{platform}$$ (6)

The stress $\tau_{flow}$ is associated with basal shear that flows exert on riverbeds (Equation (7)), which depends on height $h_1$ (m), flow specific weight $\gamma_{flow} (\frac{N}{m^3})$ and inclination $s(-)$ of the surface where stress is applied (Takahashi 2014; Julien and Paris 2010; Haas and Woerkom 2016). For flow height $h_1$ (m) it is considered that the embankment works similarly to a thick-walled hydraulic spillway, calculated as defined by Durán (2004) in Equation (8).

$$\tau_{flow} = \gamma_{flow} h_1 s$$ (7)

$$h_1 = \frac{2}{3} (h_{flow} - h_{emb})$$ (8)

Equation (9) describes the shear stress of suspended solid grains $\tau_{grain}$, with friction angle $\phi(-)$ of the coarse fraction of the granular material (Jakob and Hungr 2005).

$$\tau_{grain} = \gamma_{flow} h_1 \tan(\phi)$$ (9)

When flow height exceeds embankment height, the friction force increases, because the flow weight is added as a supplementary weight (Equation (11)). $V_e (m^3)$ is embankment volume, $\gamma_e (\frac{N}{m^3})$ specific weight of the compacted soil of embankment, $L_{flow} (m)$ the width of flow and $\mu_e(-)$ static friction coefficient.

$$F_{fric_1} = \mu_e\gamma_e V_e, \sin h_{flow} \leq h_{emb}$$ (10)

$$F_{fric_2} = \mu_e(\gamma_e V_e + h_1 L_{flow} b_{platform} \gamma_{flow}), \sin h_{flow} > h_{emb}$$ (11)

3.3. Headcut erosion conceptual model

Two processes are triggered from headcut erosion. First erosion is generated as a result of downstream flow of the embankment, weakening the slope foot due to material loss (1 in Figure 3), then, a sliding surface parallel to the embankment downstream slope is generated as a result of the erosion (2 yellow dotted line in Figure 3).
In this case, the limit state function $G_2(x_j)$ is defined in Equation (12) in terms of sliding of ABCD block but considers $b_{AB}$ width of involved forces, related to the loss of embankment length and depends on the erosive process and limit equilibrium analysis of ABCD block.

$$G_2(x_j) = (F_{\text{soil}} + \cos(z)P_2) - (F_{\text{shear}}\cos(x) + W_w\sin(z) + W_s\sin(z))$$  \hspace{1cm} (12)

Partheniades (1965) proposed an erosion model (Equation (13)) which considers an erosion rate $\varepsilon$ (m/s) in cohesive soils, but has been extended to be applied to granular soils (Midgley et al. 2012).

$$B = \varepsilon \Delta t = k_d (\tau_o - \tau_c) \Delta t$$  \hspace{1cm} (13)

On Equation (13), $B$ (m) is the eroded distance in a time interval $\Delta t$ (s), $k_d$ (m$^3$N$^{-1}$s$^{-1}$) is erodibility coefficient considered by Midgley et al. (2012) as $k_d = 2 \times 10^{-7} \tau_c^{-0.5}$, $\tau_o$ (Pa) is average of shear stress exerted by the fluid, and $\tau_c$ (Pa) is the critical shear stress of soil being eroded, defined by Briaud et al. (2019) as $\tau_c = D_{50}$, depending on the particle average diameter considered equal to 2 mm as characteristic of granular soils (MOP 2019b).

Headcut erosion begins unstable when weakening reaches a critical degree; however, for simplicity it has been taken at the angle between $\overline{DC}$ and surface as $45^\circ$ explained by Zhu et al. (2008). On the ABCD block a shear force $F_{\text{shear}}$ (N) is applied by the flow over $\overline{AB}$, $W_w$ (N) is the weight of the flow material over the $\overline{AB}$ surface, $W_s$ (N) the weight of the ABCD block, $F_{\text{soil}}$ (N) is the shear force on the sliding surface $\overline{AD}$ and $P_2$ (N) the resulting force of the pressure exerted by the downstream flow of the embankment, which Hanson et al. (2001) define as $h_2 = \frac{1}{3} h_{\text{emb}}$.

$$F_{\text{shear}} = \gamma_{\text{flow}} h_1 s + \gamma_{\text{flow}} h_1 \tan(\phi)$$  \hspace{1cm} (14)
\[ W_w = \gamma_{\text{flow}} h_1 L_{\text{flow}} w_f \]  
\[ W_s = \gamma_{\text{soil}} v_{o1} \text{vol}_{ABCD} \]  
\[ v_{o1} \text{vol}_{ABCD} = h_{\text{emb}} w_f - \frac{w_f^2 \tan(x)}{2 + 2\tan(x)} \]  
\[ F_{\text{soil}} = \gamma_{\text{soil}} h_{\text{emb}} \cos(x) \tan(\theta) \frac{h_{\text{emb}}}{\sin(x)} L_{\text{flow}} \]  
\[ P_2 = \frac{1}{2} \frac{h_2^2}{\gamma_{\text{flow}} \sin(45^\circ)} \]  

The goal is to find the width that triggers sliding of the ABCD block. If the equilibrium of the block is considered as a limit condition, as raised by Robinson and Hanson (1994), the minimum width \( w_f \) is obtained by solving the quadratic equation presented in Equation (20).

\[ a_1 w_f^2 + b_1 w_f + c_1 = 0 \]  
\[ a_1 = -\frac{\gamma_{\text{soil}} \tan(x)}{2\tan(x) + 2} \]  
\[ b_1 = \gamma_{\text{flow}} h_1 + \gamma_{\text{soil}} h_{\text{emb}} - \tan(x) \gamma_{\text{flow}} \]  
\[ c_1 = \frac{\tan(x) \gamma_{\text{flow}} h_2^2}{2} \]  

The minimum width \( w_f \) for sliding is compared against the width \( B \) that erodes, where the largest value is selected as \( b_{AB} \). If \( B \leq w_f \), eroded width is smaller than the required for the ABCD block unbalance, i.e. the total width that is lost is \( b_{AB} = w_f \). On the other hand, if \( \geq w_f \), eroded width is greater than the required for the ABCD block to lose its equilibrium, then the width is \( b_{AB} = B \).

4. Methods and results

4.1. Methods

4.1.1. Monte Carlo simulations for construction fragility curves

Simulation of different hazard scenarios, embankment geometries and their potential consequences are considered to capture the variability and uncertainty of the proposed conceptual models. Monte Carlo simulations (SMC) were carried for this purpose, sampling each variable \( \tilde{x}_j \) of the limit state function \( G_i(\tilde{x}_j) \) and evaluating whether the limit state is exceeded or not. The experiment is repeated \( N \) times with a
randomly generated vector $\tilde{x}_j$ based on the probability distributions assigned to each variable (Melchers and Beck 2018). The probability of exceeding a damage state is defined in Equation (24).

$$p_f \approx P(G_i(\tilde{x}_j) \leq 0|H = h_{\text{flow}}) = \frac{n(G_i(\tilde{x}_j) \leq 0)}{N}$$ (24)

where $n(G_i(\tilde{x}_j) \leq 0)$ is the number of times the $N$ evaluations took a value $\leq 0$. In total, 100,000 simulations were considered for each debris flows height $h_{\text{flow}}$, ranging from 0.25 m to 12 m. The upper limit was chosen based on events registered in Chile (Sepúlveda et al. 2014, Sepúlveda et al. 2006; Marín et al. 2017; Muñoz 2018) and collated with international cases. Higher flow heights are unlikely to be observed in the practice. The increase of 0.25 m allows to obtain a smooth fragility curve. To assess convergence of probabilities, the Coefficient of Variation (CV) was calculated for small flow heights (which would also have low probability values) obtaining coefficients of variation less than 5%.

4.1.2. Defining variables for simulation

Table 3 presents all variables $\tilde{x}_j$ considered in the limit state functions $G_1(\tilde{x}_j)$ and $G_2(\tilde{x}_j)$. Deterministic value or probabilistic distribution is presented as recommended in literature, goodness of fit tests were undertaken to determine probability functions. As described in Equation (13), a time interval $\Delta t$ that ranged from 0 to 5 hours was considered to simulate a broad range of debris flows scenarios.

4.1.3. Curves calibration and adjustment

The last step of modeling is to calibrate curves in terms of a probability distribution that describes a damage state for any height of debris flows. Most literature considers log-normal distribution is appropriate for the adjustment of fragility curves, mainly for its simplicity, as it has a simple parametric shape using only two parameters, and it has zero probability density for below zero variables (Nazari and Bargi 2012; Porter 2016; Dagá et al. 2018). The Maximum Likelihood Estimator (EMV) was used to adjust the curves, defining the maximum likelihood function $L$ for the log-normal probability function, where $F(\cdot)$ represents the fragility curve for each damage state and $x_i$ takes the value of 1 or 0 depending on whether the limit state is exceeded (Shinozuka et al. 2000). The method looks for parameters $\mu, \sigma$ that maximize the function $L$ through the derivation of the function, reaching the Equation (28), which obtains the parameters of the log-normal distribution that best fits to the data.

$$L = \prod_{i=1}^{N} \left[ F(h_{\text{flow}}) \right]^{x_i} \left[ 1 - F(h_{\text{flow}}) \right]^{1-x_i}$$ (25)

$$F(h_{\text{flow}}) = \Phi \left[ \frac{\ln \left( \frac{h_{\text{flow}}}{\mu} \right)}{\sigma} \right]$$ (26)
Table 3. Variables $\hat{x}_j$ considered in the limit state functions.

| Variable | Name | Unit | Probabilistic distribution* | Value reference |
|----------|------|------|-----------------------------|-----------------|
| $n'$     | Manning pseudo roughness | –    | Uniform (0.03, 0.16)         | Molinas et al. (2001); Fazáric et al. (2006) |
| $i$      | Bed slope | m    | Log-normal (0.17, 2.43)      | Sepúlveda et al. (2006); Sepúlveda et al. (2014) |
| $\alpha$ | Embankment slope angle | o    | Uniform (33, 56)             | MOP (2019a) |
| $\gamma_{flow}$ | Debris flow specific weight | kN/m  | Log-normal (19.32, 1.15)     | Takahashi (2014); Valdés-Pineda et al. (2017) |
| $D_{debris}$ | Debris average diameter | m    | Log-normal (1.52, 2.17)      | Sepúlveda et al. (2006) |
| $h_{emb}$ | Embankment height | m    | Uniform (1.00, 2.50) Uniform (2.50, 4.00) Uniform (4.00, 6.00) | Assumption according to analysis |
| $s$      | Slope carriageway | –    | Uniform (0.025, 0.04)        | MOP (2019a) |
| $\varphi$ | Friction angle of solid grains in debris flow | o    | Uniform (40, 45)             | Briaud (2013) |
| $b_{lane}$ | Two-lane rural road width | m    | Uniform (2, 3.5)             | MOP (2019a) |
| $b_{shoulder-out}$ | Two-lane rural road outside shoulder width | m    | Uniform (0, 1)               | MOP (2019a) |
| $b_{shoulder-inner}$ | Multilane road width inner side shoulder | m    | Uniform (0.60, 1)            | MOP (2019a) |
| $b_{over-width}$ | Platform crown over-width | m    | Uniform (0.50, 1)            | MOP (2019a) |
| $b_{median}$ | Median width | m    | Uniform (0, 2)               | MOP (2019a) |
| $\mu_e$  | Static roughness coefficient | –    | Uniform (0.20, 0.80)         | Jara and Fort-López (2009) |
| $\gamma_s$ | Soil specific weight | kN/m  | Log-normal (16.45, 1.19)     | Terzaghi et al. (1996) |
| $\theta$ | Embankment soil friction angle | rad  | Log-normal (0.59, 1.18)      | Terzaghi et al. (1996) |

*Two types of distributions are considered: Log-normal ($\mu$, $\sigma$), with $\mu$ the mean and $\sigma$ standard deviation and Uniform $(a, b)$ with $a$ the min extreme value and $b$ the max extreme value. References for each variables values are presented in the table.

\[
\frac{d\ln L}{d\mu} = \frac{d\ln L}{d\sigma} = 0
\]  

\[
\{\hat{\mu}, \hat{\sigma}\} = \text{argmax} \left( \sum_{i=1}^{N} \left( n_i \ln \left( \Phi \left( \frac{\ln(h_{flowi}) - \mu}{\sigma} \right) \right) \right) \right) + (N_i - n_i) \ln \left( 1 - \Phi \left( \frac{\ln(h_{flowi}) - \mu}{\sigma} \right) \right)
\]
Through a goodness-of-fit test $\chi^2$, distributions obtained with a confidence level of 99.5% were checked. Considering an increase of 0.25 m, with a flow height from 0 to 12 m, 48 points were obtained ($12 \times 4$), and, with 2 estimated parameters, 45 degrees of freedom were obtained. For all the adjustments made, a value of $\chi^2$ less than the

Figure 4. Embankment sliding failure probability curves for different embankment heights ($h_{emb}$).

Figure 5. Headcut erosion fragility curves for two-lane rural roads with embankment heights of 1–2.5 m.
corresponding statistic was obtained, with 0.34 the highest obtained value; so, it is concluded that the calibrated data fits the log-normal distribution in all cases.

### 4.2. Results

The curves obtained from the simulations and fitted log-normal distributions are presented in Figures 4–6. Also, Tables 4 and 5 present the parameters of the probability distribution for headcut erosion fragility curves and embankment model curves downstream.

**Table 4. Log-normal distribution parameters for embankment sliding fragility curves.**

| Road type                        | h<sub>emb</sub> range [m] | μ   | σ   |
|----------------------------------|--------------------------|-----|-----|
| Two-lane rural and multilane     | 1–2.5                    | 1.39| 0.68|
|                                  | 2.5–4                    | 2.08| 0.60|
|                                  | 4–6                      | 2.55| 0.55|

**Table 5. Estimated log-normal distribution parameters for embankment model curves downstream.**

| Road type                        | h<sub>emb</sub> range [m] | Damage state | μ   | σ   |
|----------------------------------|--------------------------|--------------|-----|-----|
| Two-lane rural                   | 1–2.5                    | Slight       | 0.83| 0.27|
|                                  |                          | Moderate     | 1.19| 0.27|
|                                  |                          | Extensive    | 1.65| 0.22|
|                                  | 2.5–4                    | Slight       | 1.46| 0.15|
|                                  |                          | Moderate     | 1.76| 0.16|
|                                  |                          | Extensive    | 2.04| 0.16|
|                                  | 4–6                      | Slight       | 1.89| 0.13|
|                                  |                          | Moderate     | 2.16| 0.15|
|                                  |                          | Extensive    | 2.35| 0.15|
| Multilane                        | 1–2.5                    | Slight       | 1.25| 0.28|
|                                  |                          | Moderate     | 2.26| 0.24|
|                                  | 2.5–4                    | Slight       | 1.89| 0.18|
|                                  |                          | Moderate     | 2.67| 0.20|
|                                  | 4–6                      | Slight       | 2.32| 0.17|
distribution for different embankment heights. It must be noted that in the case of the embankment sliding model curves represent probabilities of failure rather than fragilities.

For headcut erosion model, curves presented in Figures 5 and 6 represent embankment heights of 1 to 2.5 m, since these heights represent the most fragile scenarios. However, simulations and adjustments of log-normal probability distributions were performed for three ranges of embankment heights, as shown in Table 5.

5. Discussion of results and comparison with historical data

5.1. Discussion of conceptual models and developed curves

5.1.1. Discussion of conceptual model

The damage models were developed to include triggered phenomena when road embankment is exposed perpendicularly to the debris flow. Although the models developed are analyzed independently of each other, the possibility that these models occur at the same time once the flow height exceeds the embankment height cannot be ruled out. On the other hand, the models were developed under the hypothesis of a homogeneous soil for the embankment. However, it is expected that if there are variability of materials (usually in horizontal layers), the impact in the results will be minor, and the models presented are a good approximation to reality.

Both the debris flow and the response of the road embankment are highly variable. The same study site can experience two events with very different characteristics from one year to the next, which makes it difficult to define a case study. For this, it was decided to explore variability to most of the variables involved in the models, based mostly on bibliography.

5.1.2. Discussion of failure and fragility curves

For the embankment sliding model, only one curve per embankment height was obtained for two types of roads (Figure 4). When flow exceeds embankment height, the shear stress and its associated loading force on the platform predominate above the embankment own weight and its associated resistant force, no generating variations in small or large platform widths $b_{platform}$ (two-lane rural or multilane). Comparing embankment sliding curves shows in Figure 4, it observed that the fragility of road embankments is inversely proportional to their height, so higher embankments are more robust and less fragile to the impact of debris flows. Figures 5 and 6 show probabilities of exceeding damage states. To obtain the probability that the road remains with a certain state of damage, it is enough to subtract the probability of exceedance between this damage state with its upper damage state. For example, for intensity of 4 m, a two-lane rural road (Figure 5) has a 25% of probability being left with slight damage, 65% of probability being left with moderate damage, and 10% of probability of extensive damage. The same analysis for the multilane road (Figure 6), which has a 70% of probability being left with slight damage and no chance of moderate or extensive damage. If the exceedance probabilities are now analyzed, for the same flow height, the probability of exceeding slight damage is 100% for two-lane rural roads and 66% for multilane roads. The above indicates that disruption and decrease in capacity on multilane roads affected by debris flows is an unlikely
phenomenon, while for two-lane rural roads a decrease of at least 50% of capacity is expected with certainty.

Considering the long-term consequences of both models, for the embankment sliding, any displacement will affect safety and therefore completely reduce road capacity. While, for headcut erosion model of the embankment, the scenario changes in the long term. If the platform is free of debris flow material, the physical damage which the infrastructure suffers will allow, depending on the level of damage, the passage of vehicles with relevant precautionary measures. For a flow height of 4 m, the embankment sliding has a 50% of probability of failure and 100% slight damage probability for downstream erosion, indicating that is more likely that a road embankment damage for headcut erosion than the sliding. In the embankment sliding model, an impact failure is verified, which is independent of the duration of the interaction between flow and embankment. This feature is key as it is a mode of failure unique to debris flows because they have a higher specific weight than water and can carry various types of debris associated with a higher impact force. On the other hand, the headcut erosion model of the embankment is defined in terms of the erosive process at the foot of the slope. This phenomenon is not unique to debris flows, but rather is a process that can be triggered by any type of fluid. In this sense, the analysis developed for this model could be extended to the clear water flows. Finally, graphic comparison of simulated and adjusted curves shows great similarity, committing to a confidence level of 99.5%. This indicates that the calculated log-normal probability distributions conform to the simulated data and corroborating the usefulness of this type of distribution in fragility curves.

5.2. Comparison between developed curves and historical data

From meetings with field teams from Highways Agency of the Ministry of Public Works of Chile, data associated with embankment damage was collected by filling in an excel spreadsheet that contained the flow and embankment height, interaction time, damage state, or failure road, among others. Data was obtained just for two-lane rural roads. It was requested to separate the data by damage model and to declare the damage state that the embankment had remained with (as defined in Table 2). For the sliding model, 10 data from damaged embankments were obtained,
of which 6 were discarded due to lack of information about flow height or because they correspond to embankments with a height less than 1 m. For the headcut erosion model, 12 data were obtained, discarding 6 for the same reasons as above. The amount of data obtained was very limited, so a binary comparison was made. This results in plotting deterministic points as 0 or 1, depending on whether the failure or damage state was reached. From the \( \mu \) and \( \sigma \) obtained through adjusting log-normal distribution to the simulated curves, the 5th–95th and 16th–84th percentiles were calculate considered as those representative percentiles in the literature (Mina et al. 2020; Simões et al. 2020). For the embankment sliding model, empirical data of a failure that occurred in a bridge access embankment (Figure 7) was added, considering a flow height of 11 m based on the mark that the flow wave left on the railway bridge located meters upstream of the failed embankment (Marín et al. 2017; Muñoz 2018).

It was confirmed that downstream erosion of the embankment is also triggered by clear water flows. In this case, the data obtained were associated only with this type of event, which meant in the analysis to obtain curves by changing the specific weight of the flow \( \gamma_{\text{flow}} \) and removing the grain shear stress \( \tau_{\text{grain}} \) exerted on section \( \overline{AB} \). This same damage model depends on the interaction time, so curves adjusted to the times of the data collected were obtained for slight damage with 12 hours of interaction (grouping 3 data associated with 10, 12, and 14 hours) and moderate damage with 48 hours of interaction (grouping 3 data of 48, 52 and 72 hours). For the latter case, it was verified that for durations greater than 48 hours the curves become staggered and do not present a major difference between them.
For embankment sliding (Figure 8) and slight damage in the headcut erosion model (Figure 9(a)), the empirical data are within the 16th and 84th percentiles, with a square error of 0.003 and 0.012, respectively. For the case of moderate damage (Figure 9(b)), the data are within the 5th percentile, with a square error of 0.066. Representing a good initial approximation to the models identified.

It is clear to note that the procedure described above is only an initial comparison of the developed curves with empirical data. The quantity and quality of the data collected do not allow to generate a robust empirical validation, so it is a future challenge to carry out this work, as well as to advance in protocols for the collection of damage data.

6. Conclusions

The aim of the study was to develop fragility curves for road embankments exposed to debris flows representative of the expected physical damage and road capacity loss. There are important advances in the development of fragility models for elements of the road network exposed to debris flows, however, in the case of road embankments there is still no in-depth research. The following findings were derived from the study:

1. A preliminary comparison was made between the developed curves and a limited amount of empirical data. It is necessary to advance protocols that ensure the collection of damage data associated with elements of the road network affected by natural hazards to facilitate a robust validation of the models developed.
2. Embankment sliding model is labeled as a result of flow impact on the embankment, associated with a complete failure, while headcut erosion model is caused by flow fall to the foot of the slope, generating intermediate damage states to the failure. Given the magnitude of lateral force required to generate embankment
sliding, the first model is associated exclusively with debris flows, while the second model can be used for debris flow and it was proposed its extension and applicability to clear water flows.

3. Embankment sliding model resulting from the impact of flow against the embankment behaves similarly on two-lane rural and multilane roads, which is explained by compensation between shear stress on the platform surface and embankment weight.

4. In the headcut erosion model, two-lane rural roads may present 50% more probabilities of damage compared to multilane roads for similar debris flows intensities. The above indicates that two-lane rural roads are more fragile than multilane for similar demanding flows.

5. Fragility of road embankments depends on their height, mainly because there are phenomena that are activated only when debris flow exceeds embankment height. Lower embankment height to be more fragile to the impact of debris flows, especially for dense flows.

6. For the same flow height, the embankment sliding model has a lower chance of damage than the headcut erosion model, so the second model would predominate over the first. The analyses presented consider that both damage models are independent; however, it could be the case that headcut erosion model occurs and weakens the embankment, and then causes it to fail because of the embankment sliding model. This would require an analysis of embankment behavior over time to identify whether there is a coupling between two damage mechanisms considered.

7. Longer interactions time of less dense flows increase the probability of headcut erosion. In turn, the dependence that headcut erosion model has with interaction time is a difficult calculation in practice since curves would be needed for each \( \Delta t \) possible. For high time intervals, such as 48 hours, it was observed that curves already had greater variability; so, they could be used for any longer time. However, special care must be taken with time intervals under 12 hours, since in these cases the effects of erosion on defined damage states have great variability; so, this is a future line of research.

8. This research provides input data for risk management of rural road networks to compare possible mitigation strategies to reduce the effects of debris flows on road embankment. However, a future challenge is to complement failure and damage models defined in this research with those involved when debris flows running parallel to embankments.

Acknowledgments

Authors would like to acknowledge the FONDEF Project ‘Research and Development of Models to Quantify and Mitigate the Risk of Natural Hazards in the National Road Network’ (Grant no. ID14I20309/Fondef/ANID), the FONDECYT Project (Grant no. 1181754/ FONDECYT/ANID), and the Research Center for Integrated Disaster Risk Management (CIGIDEN) (Grant no. ANID/FONDAP/15110017) and the Highways Agency of the Ministry of Public Works of Chile and their regional offices.
Funding

This study was supported by the National Agency of Research and Development of Chile (ANID) through funding the FONDEF Project ‘Research and Development of Models to Quantify and Mitigate the Risk of Natural Hazards in the National Road Network’ (Grant no. ID14I20309/Fondef/ANID), the FONDECYT Project (Grant no. 1181754/FONDECYT/ANID), and the Research Center for Integrated Disaster Risk Management (CIGIDEN) (Grant no. ANID/FONDAP/15110017).

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Data availability statement

The authors confirm that the data supporting the findings of this study are available within the article.

Disclosure statement

The authors declare that they have no conflicts of interest.

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