Shielding of a small charged particle in weakly ionized plasmas

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Abstract—In this paper we present a concise overview of our recent results concerning the electric potential distribution around a small charged particle in weakly ionized plasmas. A number of different effects which influence plasma screening properties are considered. Some consequences of the results are discussed, mostly in the context of complex (dusty) plasmas.

Index Terms—Complex (dusty) plasmas, screening, potential distribution.

I. INTRODUCTION

Complex (dusty) plasmas are plasmas containing small charged particles of solid matter (dust grains). These particles are usually large enough to be observed individually, which allows experimental investigation with high temporal and spatial resolution. Hence, complex plasma is a valuable model system for studying various phenomena (e.g., phase transitions, self-organizations, waves, transport, etc.) at the most elementary kinetic level [1], [2], [3], [4], [5].

The character of the plasma-particle as well as the inter-particle interactions appears to be one of the most fundamental questions for understanding the physics behind the observed phenomena in laboratory complex plasmas [1], [2], [3], [4], [5] as well as in astrophysical complex plasmas [6], plasma of fusion devices [7], [8], [9], plasma processing [5], [10], [11], etc. Of particular importance are basic processes such as particle charging, electric potential distribution around a charged particle in plasmas, interparticle interactions, momentum and energy transfer between different complex plasma components etc. [12].

The main aim of this paper is to present a brief overview of our recent results (mostly theoretical) related to the electric potential distribution around a charged test particle in plasmas. A number of different effects which influence plasma screening properties are considered, such as plasma absorption on the particle surface, non-linearity of ion-particle interaction, ion-neutral collisions, plasma production and loss processes in the vicinity of the particle. These effects influence and often completely determine the shape of the electric potential around the particle, especially its long-range asymptote. This is important for a number of collective properties of complex plasmas (e.g., interparticle coupling, phase transitions, phase diagrams, transport, etc.). Below, we first discuss the role of these effects in isotropic plasma conditions (Section II) and then consider the case of anisotropic plasmas (Section III). This is followed by a short summary in Section IV.

II. ISOTROPIC PLASMAS

The distribution of the electric potential around a small individual spherical non-absorbing particle of radius $a$ and charge $Q$ in isotropic plasmas is often described by the Debye-Hückel (Yukawa) form

$$\phi(r) \approx \phi_s(a/r) \exp\left[-(r-a)/\lambda_D\right] \approx (Q/r) \exp(-r/\lambda_D),$$

where $\lambda_D$ is the linearized Debye length, $\lambda_D = \lambda_D/(1 + (\lambda_D/\lambda_D)^2)$. The ion (electron) Debye length is defined as $\lambda_D(e) = \sqrt{T_i(e)/4\pi e^2 n_i(e)}$, where $T_i(e)$ is the ion (electron) temperature, $n_i \approx n_e \approx n$ is the unperturbed plasma density, and $e$ is the elementary charge. The particle surface potential $\phi_s = (Q/a) \exp(-a/\lambda_D)$ is usually of the order of the electron temperature, $\phi_s \sim -T_e/e$, due to much higher electron mobility. The above expression can be obtained by solving the linearized Poisson equation with the assumption that ion and electron densities follow Boltzmann distributions and the condition $|e\phi_s/T_i(e)| \lesssim 1$ is satisfied. It is to be noted that usually in complex plasmas $T_e \gg T_i$ and the linearization is often invalid since ion-particle coupling is very strong close to the particle. Nevertheless, numerical solution of the non-linear Poisson-Boltzmann equation shows that the functional form of Eq. (1) still persists, but the actual value of the particle charge should be replaced by an effective charge which is somewhat smaller than the actual one in the absolute magnitude [13], [14].

The effect of plasma absorption on the particle surface strongly modifies the potential distribution. The continuous ion and electron fluxes from the bulk plasma to the particle make their distribution functions anisotropic in the velocity space. Although the deviations are negligible for repelled electrons [15], for attracted ions they are quite substantial. In the absence of plasma production and loss in the vicinity of the particle, conservation of the plasma flux completely determines the long-range asymptote of the potential and in collisionless plasmas it scales as $\phi(r) \propto r^{-2}$ [3], [15], [16]. Close to the particle (up to a distance of few Debye radii from its surface), the Debye-Hückel (DH) form works reasonably well. However, in the regime of strong ion-grain coupling, the linearized Debye length $\lambda_D$ should be replaced by the effective screening length $\lambda_{eff}$. The exact dependence of $\lambda_{eff}$ on plasma and grain parameters is not known for the general
case and so far it was determined using numerical simulations only for a limited number of special cases \[17\], \[18\], \[19\]. For a collisionless plasma with $T_e \gg T_i$, a fit based on numerical results of Ref. \[17\] has been recently proposed \[12\]. The corresponding expression is \(\lambda_{\text{eff}} \approx \lambda_D[1 + 0.105\sqrt{\beta} + 0.013\beta]\), where $\beta = |Q|e/(T_i\lambda_D)$ is the so-called scattering parameter \[20\], which is a natural measure of nonlinearity in ion-grain interaction. This fit has been used to calculate the ion drag force in a collisionless plasma with strong ion-grain coupling in Ref. \[21\] and to estimate the effect of polarization interaction on the propagation of dust acoustic waves in complex plasmas in Ref. \[22\].

Another important factor which influences the structure of the electric potential around an absorbing particle in plasmas is ion-neutral collisions. In the weakly collisional regime ($\ell_i \gtrsim \lambda_D$), where $\ell_i$ is the ion mean free path) the infrequent ion-neutral collisions create trapped ions which increase the ion density in the vicinity of the particle. This affects the ion flux collected by the particle \[23\], \[24\], \[25\] and, therefore, modifies the density distribution. In strongly collisional plasmas ($\ell_i \ll \lambda_D$) the electric potential is known to exhibit a Coulomb-like ($\propto r^{-1}$) decay \[26\], \[27\], \[28\], \[29\]. A transition from the DH to the unscreened Coulomb potential with increasing ion collisionality was observed in a numerical simulation \[30\].

Recently, a simple linear kinetic model has been proposed independently by Filippov et al. \[31\] and Khrapak et al. \[32\] which takes into account the combined effect of ion absorption on the particle and ion-neutral collisions. Using this model the electric potential distribution can be calculated in the entire range of ion collisionality. Below we present the general expression for the potential obtained in Ref. \[32\] and analyze some interesting limiting cases.

### A. Linear kinetic model

In this model a small (point-like) negatively charged individual grain immersed in a stationary isotropic weakly ionized plasma is considered. Plasma production and loss processes are neglected in the vicinity of the particle except on the particle surface, which is fully absorbing. The electron density satisfies the Boltzmann relation, and ions are described by the kinetic equation accounting for ion-neutral collisions and ion loss on the particle surface. Collisions are modeled by the Bhattacharyya-Gross-Krook (BGK) collision integral with a velocity independent (constant) effective ion-neutral collision frequency $\nu$. Ion loss is expressed through an effective (velocity dependent) collection cross section $\sigma$. The linear response formalism is used to solve the Poisson equation along with the corresponding equations for ions and electrons. The resulting expression for the electric potential is \[32\]

$$
\phi(r) = \frac{Q}{r} \exp(-k_Dr) - \frac{e}{r} \int_0^{\infty} \frac{\lambda_D \sinh(kr) f(\theta) d\theta}{k^2 + k_D^2} \equiv \Phi_1 + \Phi_2,
$$

where

$$
f(\theta) = \frac{8n}{\pi^{3/2} k_D} \int_0^{\infty} \sigma(\zeta) \zeta^2 \arctan(\zeta/\theta) \exp(-\zeta^2) d\zeta.
$$

Here $k_{D(e)}^{-1} = \lambda_D^{-1}[1 - 0.105\sqrt{\beta} + 0.013\beta]$ is the inverse ion (electron) Debye length, $k_D = \sqrt{k_{D(e)}^2 + k_{Dv}^2}$, $v_T_i = \sqrt{T_i/m_i}$ is the ion thermal velocity, $m_i$ is the ion mass, $\theta = (\nu/\sqrt{2kTv_i})$, and $\zeta^2 = v^2/2v_T_i^2$. The first term $\Phi_1$ in Eq. \(2\) is the familiar Debye-Hückel potential. The second term $\Phi_2$ appears due to ion absorption by the particle and accounts for ion-neutral collisions. For a non-absorbing particle $\sigma = 0$ only the conventional DH form survives, as expected. In this case ion-neutral collisions do not affect the potential distribution.

Let us now consider some important limiting cases.

1) **Collisionless limit:** In the collisionless (CL) limit one can use the OML collection cross section $\sigma(\zeta) = \pi a^2 \left[1 + (\zeta r) \zeta^2\right]$ to get \[29\]

$$
\Phi_2(r) = -\frac{e}{r} \frac{\pi a^2 n(1 + 2\zeta r)}{2k_D} F(k_D r),
$$

where $F(x) = [e^{-x} E_1(x) - e^x E_1(-x)]$ and $E_1(x)$ is the exponential integral. Here, $z = |Q|e/aT_e$ is the normalized particle charge and $\tau = T_e/T_i$ is the electron-to-ion temperature ratio. For sufficiently large distances $x \gg 1$, $F(x) \approx 2/x$ and the corresponding potential is

$$
\Phi_2(r) \approx -\frac{e}{r} \frac{\pi a^2 n}{2k_D} \left(1 + 2\tau \right),
$$

which coincides with the well known result of probe theory \[3\], \[15\], \[16\], \[33\]. The long-range asymptote of the potential scales as $\propto r^{-2}$ in this collisionless limit.

2) **Strongly collisional limit:** In the opposite strongly collisional (SC) regime the actual form of $\sigma(\zeta)$ is not important since the integral in $f(\theta)$ is directly expressed through the ion flux $J_i$ in this case. The resulting potential is

$$
\Phi_2(r) \approx -\frac{e}{r} \frac{J_i}{D_i k_D^2} \left[1 - \exp(-k_D r)\right],
$$

where $D_i = v_T_i^2/\nu$ is the diffusion coefficient of the ions. This expression coincides with the results obtained using the hydrodynamic approach \[34\], \[35\]. The long-range asymptote of the potential decays as $\propto r^{-1}$ in this case.

3) **Weakly collisional limit:** The most interesting regime relevant to many complex plasma experiments in gas discharges is the weakly collisional (WC) regime, $\ell_i \gtrsim \lambda_D$. In this case the functional form of $\sigma(\zeta)$ is required to calculate the potential distribution. A simple assumption of a constant cross section $\sigma(\zeta) = \sigma_0 = \sqrt{\pi/8}(J_i/\nu v_i T_e)$ was made in Ref. \[32\], which allowed to avoid divergence of the integrals in calculating $f(\theta)$. The result is

$$
\Phi_2(r) \approx -\frac{e}{r} \frac{J_i}{2k_D} \left\{F(k_D r) + \frac{3.34}{\ell_i k_D} \left[1 - \exp(-k_D r)\right]\right\}.
$$

The two terms in the curly brackets of Eq. \(6\) correspond to absorption induced “collisionless” and “collisional” contributions, respectively. The collisional contribution to the potential dominates over the collisionless one for $r \gtrsim 0.6\ell_i$.

Figure \[1\] demonstrates that the long-range asymptote of the potential is dominated by the combined effect of collisions and absorption. It exhibits Coulomb-like decay $\phi(r) \sim Q_{\text{eff}}/r$.
collisional plasmas. The effects of plasma production and loss processes are introduced in the ion continuity equation through the corresponding plasma source and loss terms. Electron impact ionization is usually considered as the main mechanism of plasma production. Plasma loss can be either due to electron-ion volume recombination [29] (which is relevant to high pressure plasmas) or due to ambipolar diffusion towards the discharge chamber walls and electrodes [37] (which occurs in low and moderate-pressure gas discharges). In addition, when many particles are present in the system, plasma losses can be associated with absorption on the particles themselves. This situation has been recently analyzed in detail in Ref. [38]. Let us consider these three cases separately.

1) Losses due to volume recombination: The plasma source and loss terms added to the ion continuity equation are in this case \( \nu_I n_e - \nu_R n_e n_i \), where \( \nu_I \) and \( \nu_R \) are the effective ionization and recombination constants, respectively. In the unperturbed state \( \nu_I = \nu_R n_i \). The standard linearization procedure yields the electric potential of the form

\[
\phi = (Q_+ / r) \exp(-r k_+) + (Q_- / r) \exp(-r k_-),
\]

where \( k_\pm = \frac{1}{2} \left( k_D^2 + \frac{\nu_I}{D_i} \right) \pm \frac{1}{2} \sqrt{\left( k_D^2 + \frac{\nu_I}{D_i} \right)^2 - \frac{4 \nu_I k_D e}{D_i}} \)

and

\[
Q_\pm = \pm Q \frac{k_\pm^2 - k_D^2 - \left( e J_i / Q D_i \right)}{k_\pm^2 - k_D^2}.
\]

The potential is screened exponentially but unlike in Debye-Hückel theory it is described by the superposition of the two exponentials with different inverse screening lengths \( k_+ \) and \( k_- \). Both these screening lengths depend on the strength of plasma production (ionization frequency \( \nu_I \)). The effective charges \( Q_+ \) and \( Q_- \) also depend on plasma production strength as well as on the ion flux collected by the grain. The long range asymptote of the potential is determined by the smaller screening constant \( k_- \) with effective charge \( Q_- \). Depending on the strength of plasma production two limiting cases can be considered: low and high ionization rate.

In the limit of low ionization rate, \( \nu_I / D_i \ll k_D^2 \), the screening length is dominated by ionization/recombination effects and the screening length \( \lambda_{De}(\ell, k_D) \sqrt{\nu / \nu_I} \) is considerably larger than the electron Debye length since \( (\ell, k_D) \gg \sqrt{\nu / \nu_I} \) in the considered regime. For distances \( \lambda_{De}(\ell, k_D) \sqrt{\nu / \nu_I} \) the potential behaves as Coulomb-like with the effective charge \( Q_- \approx -(e J_i / D_i k_D^2) \), i.e., we recover the result of the previous (no ionization/recombination) limit [Eq. (5)]. Thus, the distance \( \lambda_{De}(\ell, k_D) \sqrt{\nu / \nu_I} \) determines the length scale below which plasma production is not important and sets up the upper limit of applicability of the results obtained within the assumption of no ionization/recombination processes in the vicinity of the grain [Eq. (5)].

In the opposite limit of high ionization rate, \( \nu_I / D_i \gg k_D^2 \), the screening length is given by the electron Debye length \( \lambda_{De} \) and is independent of the ionization rate \( \nu_I \). The effective charge \( Q_- \approx Q - e J_i / \nu_I \) is somewhat larger in the absolute magnitude than the actual charge.

Fig. 1. Distribution of the normalized electric potential around a small individual charged particle in an isotropic weakly ionized plasma for different values of the ion collisionality index. The solid curves are obtained using the analytical approximation in the strongly collisional (collisionless) limit. The dash-dotted curve shows the Debye-Hückel potential with the surface potential calculated from the (collisionless) OML model. The inset shows a comparison between direct numerical integration of Eq. (2) (solid lines with symbols) and analytical approximation for the weakly collisional regime [Eqs. (2) and (9)].

where the effective charge \( Q_{\text{eff}} \) is determined by the plasma and particle parameters and increases monotonically in absolute magnitude with ion collisionality. At short distances the potential follows the DH form (1), but the actual particle charge \( Q \) shows a non-monotonic dependence on \( \ell, k_D \). In the WC regime \( |Q| \) decreases with increasing collisionality, while in the SC regime \( |Q| \) increases until it reaches a certain maximum value when both ion and electron collection by the particle becomes collision dominated [36]. Note that in the WC regime the transition from short-range DH to the long-range Coulomb-like asymptote can occur through an intermediate \( \propto r^{-2} \) decay, whilst in the SC regime the potential is Coulomb-like practically from the particle surface.

B. Effect of ionization/recombination

So far it has been assumed that there are no plasma sources and sinks in the vicinity of the particle except at its surface. Physically, this corresponds to the situation when the characteristic ionization/recombination length is considerably larger than the characteristic size of the plasma perturbation by the charged particle, i.e., compensation of plasma losses to the particle occurs very far from it. However, in real conditions some plasma production and loss processes always operate in the vicinity of the particle and therefore, it would be interesting to estimate the importance of these effects.

For an individual particle this has been done in Refs. [29], [37] using the hydrodynamic approach for the case of highly
2) Losses due to ambipolar diffusion: In this case the plasma source and loss terms are \( v_1 n_e - v_L n_i \), where \( v_L \) is the characteristic loss frequency \( (v_1 = v_L) \). The potential around the grain is [32],

\[
\phi = (Q_1/r) \exp(-k_{\text{eff}}r) + Q_2/r,
\]

where \( Q_1 = Q[1 - (v_1 - (e/Q) J)]/D_i k^2_{\text{eff}} \), \( Q_2 = Q - Q_1 = (Q v_1 - e J)/D_i k^2_{\text{eff}} \), and \( k^2_{\text{eff}} = k^2 + v_1/D_i \). Thus, in the considered case the potential is not completely screened, but has Coulomb-like long-range asymptote. The effective charge \( Q_2 \) depends both on the strength of ionization \( v_1 \) and the ion flux \( J \), collected by the grain. The analysis of the limits of low and high ionization rates is straightforward.

3) Losses due to absorption on other grains: In this case, the role of ion sink is played by a continuous, immovable, and uniform “particle medium”. The ion source and loss terms are essentially the same as in the previous case, with a minor difference that the ion loss frequency \( v_L \) may depend on the particle charge which varies with the ion and electron densities. The principal difference from the previous case is that \( n_i \neq n_e \) in the unperturbed state due to the presence of the charged particle medium. This has been shown to change the Coulomb term \( Q_2/r \) in Eq. \( (10) \) to \( (Q_2/r) \cos(k_0 r) \) as well as to change \( Q_{1,2} \) and \( k_{\text{eff}} \) [39], [40]. The parameter \( k_0 \) is

\[
k_0 = \sqrt{\frac{P z v_L}{\tau D_i [P + z + (d v_L/dz) (z/v_L)]}}
\]

where \( P = (n_e - n_i)/n_i \) is the Hames parameter (see, e.g., Eq. (27) of Ref. [40]). Equation \( (11) \) is derived for \( v_L/D_i \ll k^2_{\text{eff}}, \tau \gg 1, z \sim 1, (d v_L/dz)(z/v_L) \sim 1 \) and yields \( k_0 \sim 10^{-2} k_0 \) for typical experimental parameters (see also Fig. 3 of Ref. [19]).

However, it has been recently shown that this cosine-like potential cannot be observed in principle [38]. The reason is that such an ionization-absorption balanced plasma with \( n_i \neq n_e \) is unstable with respect to ion perturbations and the threshold wavenumber is exactly equal to \( k_0 \), whereas the attraction was derived by implicitly assuming the ion component to be in a stable equilibrium and considering the screening of a test particle as a static perturbation of this state.

This instability disappears for a constant ionization source [38]. However, for a constant ionization source the cosine in the potential changes to the exponent [41], [42].

III. ANISOTROPIC PLASMAS

Electric fields are often present in plasmas (e.g., in rf sheaths, positive column of dc discharge, dc discharge striations, ambipolar electric field in plasma bulk, etc.). This induces an ion drift and, hence, creates a perturbed region of plasma density downstream from the particle – the so-called “plasma wake”. One can apply the linear dielectric response formalism [43] to calculate the potential distribution in the wake. This approach is applicable provided ions are weakly coupled to the particle (i.e. the region of nonlinear ion-particle electric interaction is small compared to the plasma screening length). Note that higher ion drift velocities imply better applicability of the linear theory. The electrostatic potential created by a point-like charge at rest is defined in this approximation as

\[
\phi(r) = \frac{Q}{2\pi r^2} \int \frac{e^{ikr}dk}{k^2 z(0,k)},
\]

where \( \epsilon(\omega, k) \) is the plasma dielectric function. Using a certain model for the dielectric function, one can calculate the anisotropic potential distribution using analytical approach [44], [45], [46], [47], [48], [49], [50]. The potential profile can be also obtained from numerical simulations [51], [52], [53], [54], [55], [56], [57]. In general, the shape of the wake potential is sensitive to the ion flow velocity, plasma absorption on the particle, ion-neutral collisions, etc. Let us illustrate how the wake potential can depend on plasma conditions using few recent examples from analytic calculations.

The examples below deal with a homogeneous plasma with ion flow driven by an electric field, where the unperturbed velocity distribution of ions is determined by the balance of the electric field and collisions with neutrals (mobility-limited flow). Here, “homogeneous plasma” presumes that the inhomogeneity length (e.g. due to Boltzmann distribution of electrons in the field driving the flow) is large enough. Such a plasma has been shown to be stable with respect to the formation of ion plasma waves, except for the parameter range where the thermal Mach number \( M_T = u/v_n \) is larger than \( \approx 8 \) and the ratio of the ion-neutral collision frequency to the ion plasma frequency, \( v/\omega_{pi} \) is less than \( \approx 0.2 \) [38]. Here \( u \) is the ion flow velocity, \( v_n \) is the thermal velocity of neutrals, \( T_n \) is the neutral temperature, and the ion plasma frequency is \( \omega_{pi} = \sqrt{4\pi e^2 n/m_i} \). This justifies the use of the linear response formalism [42] outside the aforementioned instability range. However, note that the stability analysis was performed using the BGK collision term, whereas the use of the more realistic constant-mean-free-path collision term [59] may yield somewhat different instability thresholds.

A. Subthermal ion drifts

In this subsection we focus on the subthermal ion drift regime, \( M_T \lesssim 1 \).

The screening in the “almost collisionless” case has been investigated using the kinetic approach with the BGK collision term [50]. The “almost collisionless case” presumes that the ion-neutral collision length is much larger than the Debye length and distances where we want to find the potential. This case is treated by taking the limit \( \nu \to 0 \) at a fixed \( M_T \). In this limit the role of collisions is only to determine the non-Maxwellian form of the unperturbed velocity distributions of ions. The resulting potential for \( r \gg \lambda_D \) is [60]

\[
\phi(r) = Q \left[ \exp(-r/\lambda_D) - 2 \sqrt{2} \frac{M_T \lambda_D^2}{r^3} \cos \theta \right. \\
+ \left. (2 - \frac{\pi}{2}) \frac{M_T^2 \lambda_D^4}{r^3} (1 - 3 \cos^2 \theta) \right],
\]

where \( \theta \) is the angle between \( r \) and the ion flow, and \( \lambda_D = \sqrt{T_n/(4\pi n e^2)} \) (note a difference with respect to previous notation); the electron component is treated as a homogeneous neutralizing background which is not perturbed.
such a way that its directions are isotropically distributed in clouds. Here the most interesting case is the case of spherical averaged distribution of the electric potential in the screening field, whereas ions react instantaneously and the resulting much slower than the ion plasma frequency and the ion-neutral binary interactions in complex plasmas by applying electric shows:

\[ T \]

using Eq. (14) for the same parameters as for the CL case but with \( T_c = T_i \). The dash-dotted curve corresponds to the strongly collisional (SC) limit for a particle in the same highly collisional limit. Both last curves are calculated using Eq. (14) for the same parameters as for the CL case but with \( T_c = T_i \), \( a/\lambda_D = 0.02 \), \( \ell_i/\lambda_D = 0.1 \).

by the presence of the particle. Equation (13) is accurate to \( O(M_T/r^5) + O(M_T^2/r^3) \) at \( M_T \to 0, r \to \infty \). This result shows:

- At sufficiently large distances the potential has the \( r^{-3} \)-dependence, which is in agreement with the inverse third power law of screening in anisotropic collisionless plasmas [61].
- The resulting electric interaction between the particles is non-reciprocal \( \text{actio} \neq \text{reatio} \) since \( \phi(r) \neq \phi(-r) \). For instance, if two grains are aligned along the flow then the grain located downstream may experience attraction whereas the grain located upstream will be always repelled.
- The potential in the direction perpendicular to the flow does not have an attractive part. This is contrast to the case of a shifted Maxwellian distribution (see, e.g., Eq. (19) of Ref. [62], Fig. 2 of Ref. [63], or Fig. 3 of Ref. [64]) and shows the importance of accounting for the non-Maxwellian form of the ion distribution.

The wake effect has been proposed to be used to design binary interactions in complex plasmas by applying electric fields of various polarizations [60]. The idea is to apply an electric field oscillating with a frequency which is (i) much faster than that characterizing the particle dynamics and (ii) much slower than the ion plasma frequency and the ion-neutral collision frequency. In this case the particles do not react to the field, whereas ions react instantaneously and the resulting interaction between the particles is determined by the time-averaged distribution of the electric potential in the screening clouds. Here the most interesting case is the case of spherical polarization where the vector of the electric field rotates in such a way that its directions are isotropically distributed in 3D space but its absolute value remains constant. In this case the resulting potential is isotropic but the second and third terms in Eq. (13) are averaged out so that the effect of the field is in the rest term \( O(M_T/r^5) + O(M_T^2/r^3) \). The resulting potential was investigated numerically in Ref. [60] for finite \( M_T \) and found to have an attractive part.

The effect of ion absorption on the particle surface has been so far investigated only for subthermal ion flows in highly collisional plasmas using the hydrodynamic approach [65]. In this regime the absorption-induced ion rarefication behind the particle can overcome the effect of ion focusing and a negative space charge region develops downstream from the particle. This can have important consequences for particle motion in low-ionized plasmas [66] and, therefore, let us discuss this issue in some more detail.

A stationary negatively charged spherical point-like particle which is immersed in a quasineutral highly collisional plasma is considered. The electric field is sufficiently weak so that the ions are drifting with subthermal velocity while electrons form stationary background. Plasma absorption occurs on the grain surface, i.e. it acts as a plasma sink. Ionization/recombination processes in the vicinity of the grain are neglected. The potential distribution is obtained by solving Poisson equation coupled to the hydrodynamic equations for ions and Boltzmann equation for electrons using the standard linearization technique under the assumption \( M_T \ll k_D \ell_i \ll 1 \). Further, using the known asymptotic expression for the ion flux to an infinitesimally small grain (\( a \ll \lambda_D \)) in the continuum limit (\( \ell_i \ll a \)) [20], [28], [67] the expression for the potential downstream from the particle (\( \theta = 0 \)) can be written in the form [65]:

\[
\phi(r) = Q \left[ \exp \left( -\frac{r k_D}{D} \right) - \left( \frac{M_T k_D^2}{2 \ell_i k_D^4} \right) \frac{2 - (r^2 k_D^2 + 2r k_D + 2) \exp(-r k_D)}{r^2} + \left( \frac{k_D}{D} \right)^2 \frac{1 - \exp(-r k_D)}{r} + \left( 1 - \frac{2 \ell_i k_D}{k_D^2} \right) \left( \frac{M_T k_D^2}{2 \ell_i k_D^4} \right) \times 2 - \left( 1 - \frac{k_D^2}{k_D^2} \right)^{-1} \frac{r^2 k_D^2}{2r k_D + 2} \exp(-r k_D) \right] \frac{1}{r^2}.
\]

(14)

Let us briefly analyze the structure of the Eq. (14). The first term is the usual isotropic Debye-Hückel potential and the second term represents the anisotropic part of the potential behind a nonabsorbing particle. The third and fourth terms represent, respectively, the isotropic and anisotropic parts of the electric potential associated with the effect of ion absorption. In the absence of ion absorption the long-range asymptote of the electric potential can be written in the form

\[
\phi \approx -(Q \mu r^2/\omega_D^3)(k_D/\lambda_D)^4.
\]

(15)

Except of the factor \((k_D/\lambda_D)^4\) this expression is identical to that obtained in Ref. [68] for the potential behind a slowly moving non-absorbing test charge in collisional plasma. The difference appears because in Ref. [68] only electron screening was taken into account. Thus, in contrast to the collisionless
regime, the potential of a non-absorbing particle in collisional plasmas exhibits $\propto r^{-2}$ long-range decay. In the presence of absorption the long-range asymptote of the anisotropic part of the potential can be written as

$$\phi \approx -2(Qw/\omega_m^2r^2)(k_{D_1}/k_D)^4(k_{D_2}/k_D)^2.$$  

Thus, the effect of absorption induced ion rarefaction competes with the effect of ion focusing and the amplitude of the anisotropic part of potential decreases, provided $T_e > T_i$. More important is the contribution from the isotropic part associated with absorption. It is always negative (for a negatively charged particle) and can be written as

$$\phi \approx (Q/r)(k_{D_1}/k_D)^2.$$  

This contribution is most often dominant and completely determines the long-range behavior of the potential \[65\].

Figure 2 shows an example of calculating the electric potential downstream from the particle for the situations discussed above. The solid curve corresponds to the non-absorbing particle in collisionless plasma [Eq. (13)]. Ions are focused downstream from the particle and a positive space charge region exists. The dash-dotted curve shows the potential distribution for a non-absorbing particle in highly collisional plasmas [first two terms of Eq. (14)]. Here collisions enhance ion focusing \[69\] and the positive space charge region grows considerably. However, when absorption is included using Eq. (14), the absorption induced ion rarefaction plays a dominant role and the potential is negative (dashed curve), in contrast to the previous cases.

### B. Suprathermal ion drifts

In the highly suprathermal regime $M_T \gg 1$ the potential can be found analytically using the kinetic approach with the realistic constant-mean-free-path collision term \[70\]:

$$\phi(r) = \frac{2Q}{\pi \ell_i} \Re \int_0^\infty dt \exp[i t r_\parallel/\ell_i] \frac{1}{1 + (\ell_i/\lambda_{D,eff})^2} Y(t) \times K_0 \left( \frac{r_\perp}{\ell_i} \sqrt{t^2 + (\ell_i/\lambda_{D,eff})^2} X(t) \right).$$  

\[16\]

Here $r_\perp$ is the distance in the plane perpendicular to the flow, $r_\parallel$ is the distance along the flow ($r_\parallel > 0$ and $r_\parallel < 0$ along and against the flow, respectively), $\lambda_{D,eff} = [eE\ell_i/(4\pi n\ell_i^3)]^{1/2}$ is the effective Debye length, $E$ is the field which drives the flow [it is related to the flow velocity via $u = \sqrt{2eE\ell_i/(\pi m_i)}$, $K_0$ is the zero-order modified Bessel function of the second kind \[71\].

$$X(t) = 1 - \sqrt{1 + it},$$

$$Y(t) = \frac{2\sqrt{1 + it}}{it} \int_0^1 \frac{d\alpha}{\sqrt{[1 + it(1 - \alpha^2)]^2 - \frac{1}{it(1 + it)}}}.$$  

and the square roots must be taken with positive real part; the electrons are considered as a homogeneous background which is not perturbed by the presence of particle. Expression \[16\] works at all but very small angles with respect to the flow since it diverges logarithmically at $r_\perp \to 0$ for $r_\parallel > 0$ due to the neglect of the thermal spread of neutral velocities. The contour plot of potential \[16\] is shown in Fig. 3.

The derived potential applies to the screening of charged particles in plasma presheaths where the ion flow velocity is in between the thermal velocity of neutrals and the Bohm velocity. This potential has been shown in Ref. \[70\] to be in agreement with measurements of Konopka et al. \[72\].

The potential \[16\] at large distances is \[70\]:

$$\phi(r) = -\frac{Q\lambda_{D,eff}^2 \cos \theta}{\ell_i} \left( \frac{2}{1 + \cos^2 \theta} \right)^{3/2}$$  

\[18\]

which is accurate to $O(r^{-3})$ at $r \to \infty$. This is an unscreened dipole-like field with the dipole moment $|Q\lambda_{D,eff}^2/\ell_i|$ directed along the flow for $Q < 0$, although there is a difference from a pure dipole field due to the factor $[2/(1 + \cos^2 \theta)]^{3/2}$. The $r^{-2}$-dependence in Eq. \[18\] is again different from the inverse third power law decay in collisionless anisotropic plasmas because of a finite collision length. Note that for $\theta = 0$ Eq. \[18\] is identical to the result of hydrodynamic approach [Eq. \[15\]], provided we assume $T_e \to \infty$ and $u = eE/m_i\nu$ in Eq. \[15\]. In both cases the dipole moment is $\nu E/(m_i\omega_p^2)$.

### IV. SUMMARY

In this paper, we have summarized and discussed our recent results regarding the electric potential distribution around a small charged particle in weakly ionized plasmas. Different effects influencing the shape of the potential, including plasma absorption on the particle, ion-neutral collisions, plasma production and loss processes, and plasma anisotropy have been considered. The generic property of the results obtained so far is that sufficiently close to the particle the potential can be well approximated by the Debye-Hückel (Yukawa) form. At longer distances, the potential usually exhibits power-law decay $\propto c/r^n$. The value of $n$ ($n = 1, 2, 3$ or in the
cases investigated), as well as the sign and magnitude of the parameter $c$, depend on the plasma and particle properties. These results have important consequences for plasma-particle and interparticle interactions and related phenomena, including phase transitions, phase diagrams, transport, waves, etc.

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