Composite plate analysis made in an unsymmetric configuration

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Abstract. The work presents a thin-walled plate element with the central rectangular cut-out which can be use as an elastic or load-bearing element. Plates were made of carbon epoxy laminate and subjected to uniform compression. Plates were simply supported on shorter edges, and loaded axial load. The study included analysis of the critical and weakly post-critical behavior using experimental and numerical methods. Numerical analysis was performed with using linear analysis of eigenvalue problem to determination critical loads. The second step connected nonlinear analysis of structure with initiated geometrically imperfection corresponding to the flexural-torsional buckling mode of the plate. To the numerical calculations the commercial ABAQUS program was used.

1. Introduction
Nowadays, share of composites in new constructions is still increasing, what is connected high strength and durability of these materials, with a relatively low weight [1-3]. The uniform, flat plates belong to the group of structural elements relatively cheap to manufacture, and due to the low flexural stiffness can move relatively small loads [4]. They quite easy loss of stability when they are subjected to compression or shearing [5-8] and buckling has a flexural buckling character (figure 1a) [9-13]. More information connected with this topic and showing failure analysis are presented in articles [14-19]. To increase the range of transferred loads it is proposed to force plate work according to higher flexural-torsional form of buckling (Figure 1b). But this form is still unstable.

Figure 1. The concept of using thin-walled plate elements as the elastic elements a) 1th buckling mode – flexural form, b) higher buckling mode – flexural-torsional form (unstable), c) higher buckling mode – flexural-torsional form (stable).
However, there is a way to improve the carrying capacity of this type of construction, through the execution of the central cut-out and by use unsymmetrical configuration of laminate which forces the construction work according to the higher, bending-torsional form of buckling.

The research was conducted towards to disturbance of the unsymmetry of the composite layer arrangement. A wide description of stiffness matrix couplings of the unsymmetrical laminate can be found in the works of York [20, 21], Altenbach [22] or Samborski [23]. The aims of the study, partially revealed in this paper demand thorough analysis of the B matrix form effect on the coupled composites behavior, rather than the two other matrices. The specific arrangement of layers used in the analysis and presented in this article are some mix-up and modify of the layer configuration tested by York [20, 21].

This study incudes experimental and numerical analysis of thin-walled composite plates in unsymmetric arrangement of layers weakened by cut-out. The scope of research will include issues of linear and nonlinear analysis of composite plate structures subjected to uniform compression. The research were lead with using numerical calculations basing on the finite element method, which currently has a very wide application [24–32], with the ABAQUS software. Obtained numerical results were validated with experimental results obtained for physical models of the tested structures.

2. Object of the study
The research was performed on rectangular plate weakened by central cut-out with geometric parameters presented in Figure 2, with 45 angles of fibre layout in unsymmetrical configuration.

![Figure 2. Geometry of: a) analysed plates, b) real sample.](image)

The main object of this study is compare results from experimental research with numerical results. This work is a part of the authors’ research [33-37], which concludes that plates with a forced higher form of buckling are characterised by stable, progressive paths of post-critical equilibrium, enabling their use as elastic elements.

The laminate has 12 plies unsymmetric to the mid-plane, each play has a thickness of 0.105 mm (figure 3). The analysed configuration presented in table1. The ply orientations used in the analysis and presented in this paper is combination and modification of the lamina ply orientations tested by York [20] and is based on the use of matrix couplings B. More information about choosing suitable laminate configuration it can be found in previous article of author [33-37].
Table 1. Investigated unsymmetric laminate.

| Number of plies $n$ | Employed ply orientations/couplings | The final laminate configuration |
|---------------------|-------------------------------------|---------------------------------|
| 12                  | $A_S B_T D_s \quad A_S B_T D_s$ | $[\theta/-\theta/\theta/0/0]\quad [\theta/-\theta/0/0/\theta/\theta/\theta/0]$ |

where: $\theta$ - the angle of ply orientation

Analysed plates had cut-out in the middle of the plate, which dimensions constituted geometrical parameters of the structure having a decisive effect on the characteristics of configuration in the loaded condition. Plates were manufactured of carbon-epoxy laminate and the elastic material properties ($E_1=143530$ [MPa], $E_2=5826$ [MPa], $G_{12}=3845$ [MPa], $\nu_{12}=0.36$) of the analysed composite were determined according to test standards defined by ISO.

The scope of work included performing numerical simulations using the finite element method in the commercial ABAQUS® program.

3. Research methodology

Experimental tests concerned the axial compression process of analysed plate and were performed on the Instron – universal testing machine with constant crosshead velocity of 2 mm/min, in room temperature (Figure 4). To provide articulated support of plate, the specially grips were designed. During the experimental tests, the compressive force, plate shortening and deflection in the perpendicular to the plate direction (in the half of the plate vertical stripes height, where the deflection value was the highest) were measured.

Numerical analyses were performed by a commercial ABAQUS® software package. The plates under investigation were discretised with six-node shell elements with 6 degrees of freedom at each node. To model the laminate structure, a technique called “Layup-Ply” has been used, which allowed defining thickness and material properties of each layer separately. The properties of the composite material were described by defining a model of orthotropic material in a flat stress state.

The plate was simply supported and loaded by a compressive force distributed along the top of the edge. The discrete plate model with boundary conditions is shown in Figure 5.
Numerical calculations concerned the linear stability analysis and included a critical state analysis of rectangular plates with cut-outs. The solution of the eigenvalue problem concerned the determination of critical load values and the corresponding form of the loss of stability. This issue was solved by the minimum potential energy criterion.

In the second stage of calculations non-linear, static analysis using stable Newton-Raphson algorithm was performed. The non-linear stability issue were carried out on models with initial geometric imperfection corresponding to the flexural-torsional buckling form of the structure.

4. Results
Numerical and experimental tests allowed to assess the critical and post-critical state of the tested composite plates.

The performed numerical and experimental analysis showed that the applied concept of the plate with an unsymmetrical arrangement of layers allows for obtaining the natural, flexural-torsional form of buckling as the lowest (Figure 6). This is a confirmation that the asymmetric configuration of the used composite is able to ensure stable work of the structure in the postcritical range.
In the numerical analysis, based on the solution of the linear eigen problem, based on the criterion of minimum potential energy, apart from the loss of stability, the corresponding value of the critical force was also determined: $P_{crFEM} = 115.5$ N. In the case of experimental studies, this value was determined using the second order Koiter’s method [38,39]. The approximation process with the second order polynomial was carried out on the force-deflection characteristic, selecting the effective range of the approximated curve in such a way that the highest possible correlation coefficient was maintained at the level of $R^2 \geq 0.95$. A broader description of the approximation methods and the method of selecting the approximation range have been described in the works [40, 41]. The determined critical load value with used the approximation function was presented in Figure 7.

**Table 2.** Critical load – compare the results.

| Critical load [N] | Experimental test [N] | FEM analysis [N] | Diffrence [%] |
|-------------------|-----------------------|-----------------|---------------|
|                   | 118.48                | 115.50          | 2.58          |

![Figure 7. Experimental post-buckling path and approximation line from Koiter’s method.](image)

The value of the critical load, obtained on the basis of the used approximation method, is the constant term of the approximation function (second order polynomial). The experimentally determined critical load was $P_{crexp} = 118.48$ N. The error difference between the experimentally determined critical force and the numerically determined one was 2.58% (table 2). The obtained results prove the correctness of the prepared numerical model in relation to the conducted experimental research.

The analysis of the structure behavior subjected to compression made it possible to determine the post-critical equilibrium paths (Figure 8).

![Figure 8. Post-critical equilibrium path.](image)
It has been shown that the post-critical equilibrium paths remain stable, which means that the increase of the thin-walled structure deflections is accompanied by the increase of the compressive load (despite the phenomenon of loss of stability).

The obtained results make it possible to assess the correctness of the numerical model preparation, which was verified by the results of experimental tests.

5. Conclusions
The paper presents the results of experimental and numerical tests of thin-walled composite plate weakened by cut-out, with an unsymmetrical arrangement of layers subjected to compression. The analysis showed that the applied concept of the plate with an unsymmetrical arrangement of composite layers allows for obtaining the lowest flexural-torsional buckling form, showing its stable features in the full range of the loading process.

As part of the linear stability analysis, where the approximation Koiter’s method was used, it was shown that the error difference in the values of critical loads does not exceed 3% - which proves the high compliance of the experimental tests with the developed numerical model.

The presented results confirm the adequacy of the developed numerical model, enabling the simulation of the critical and post-critical state of compressed flat plates with a central cut-out.

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