Low lying $qqqq\bar{q}$ states in the baryon spectrum

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Abstract

The coupling to light mesons leads to large widths and shifts of the energy of the excited states in the baryon spectrum from the predictions of the constituent quark model with three valence quarks. This coupling may be modelled by admixtures of sea-quark $qq(q\bar{q})^n$ configurations in the resonances. A schematic flavor and spin dependent interaction model, similar to the one which reproduces the low lying part of the experimental baryon spectrum in the valence quark model, is shown to bring the lowest $qqqq\bar{q}$ states with $L = 1$ and positive parity below the states with $L = 0$. Because of the partial overlap with the corresponding $qqq$ states this suggests that most of the low lying states in the baryon resonance spectrum have substantial $qqqq\bar{q}$ admixtures.
1. Introduction

The large color limit of QCD provides a simple and phenomenologically satisfactory approach to analyze the baryon spectrum [1]. This limit may either be realized through the constituent quark model or through topological soliton models, the generic version of which is the Skyrme model [2]. Both approaches lead to a baryon spectrum, which has the same ordering of positive and negative parity states as the empirical spectrum, provided that the hyperfine interaction between the constituent quarks in the quark model is assumed to have the flavor and spin dependence characteristic of pion and vector meson exchange interaction between quarks [3, 4, 5]. In particular, both approaches predict that the lowest excited states of the nucleon and the $\Delta(1232)$ resonance are positive parity states, which correspond to the $N(1440)$, $\frac{1}{2}^+$ and $\Delta(1600)$, $\frac{3}{2}^+$ resonances, respectively, whereas the lowest excited states in the spectrum of the $\Lambda$ hyperon have negative parity.

The description of these states is nevertheless quite different in the two approaches. The constituent quark model with 3 valence quarks describes them as a single quark excitation to the excited S-state, whereas the Skyrme type model describes them as collective breathing modes [6]. The large widths of these states do indeed indicate strong coupling to the open mesonic channels, which in the quark model would be viewed as sea-quark admixtures. In the meson sector similar sea-quark admixture was noted by Jaffe [7], who noted that $(q\bar{q})^2$ states may have energies that are close to those of excited $q\bar{q}$ states. In the baryon sector there is recent empirical evidence for a dual character of at least the $N(1440)$ [8].

The lowest negative parity states in the baryon spectrum are the $\Lambda(1405)$, $\frac{1}{2}^-$ and $\Lambda(1520)$, $\frac{3}{2}^-$ resonances, which in the quark model with 3 valence quarks are described as a flavor singlet multiplet [9]. The $qqq$ valence quark model with the pion exchange type hyperfine interaction correctly predicts the low energy of this multiplet, but cannot explain its spin-orbit splitting [4]. The bound state version of the Skyrme model predicts both the low energy and the spin-orbit splittings of this multiplet correctly, however [10, 11]. This suggests that this multiplet should have a relatively large $\bar{q}qqq\bar{q}$ component. The $\Lambda(1405)$ may even have a dominant $\bar{K}N$ component [12], and a separate
The lowest negative parity state in the nucleon spectrum is the N(1535), $\frac{1}{2}^-$ state, the energy of which is correctly predicted by the $qqq$ model. Yet the $N(1535) \rightarrow N\gamma$ transition form factor falls off at a slower rate with momentum transfer than the nucleon form factor, a feature that would be very hard to describe by the quark model with 3 valence quarks [14], in which it has to be an $L = 1$ state which a priori should be more spatially extended than the $L = 0$ nucleon. This feature, along with its peculiarly strong $\eta$ decay branch, suggests that this state should have significant sea-quark admixtures. The same observation concerning strong $\eta$ decay modes very near the threshold for $\eta$ decay also applies to the $\Lambda(1670)$, $\frac{1}{2}^-$ and the $\Sigma(1750)$, $\frac{1}{2}^-$ states [15].

Significant $qqqq\overline{q}$ admixtures in the baryon resonances should be expected to occur where the energies and level splittings in the $qqqq\overline{q}$ states are close to those of the $qqq$ states. We here compare the energies of the low lying baryon resonances that are obtained with $qqq$ and $qqqq\overline{q}$ states using the same schematic flavor spin interaction, which is known to yield a spectrum for the $qqq$ states which agrees well with the empirical baryon spectrum up to $\sim 1700$ MeV.

In this schematic model the confining interaction is described as harmonic, mainly for the purpose of analytic integrability of the Hamiltonian. The strength of the confining interaction for the $qq\overline{q}$ interaction and the $qq$ interaction is suggested by the conventional color dependence $\lambda_i^c \cdot \lambda_j^c$ of the confining interaction. For the hyperfine interaction between quarks we adopt the simple form

$$H_\chi = -C_\chi \sum_{i<j}^N \lambda_i^F \cdot \lambda_j^F \sigma_i \cdot \sigma_j ,$$

(1.1)

where $C_\chi$ is a constant. The hyperfine interaction between quarks and antiquarks is taken to be negligible, as suggested by the small hyperfine splittings in the empirical meson spectrum, with exception of the light pseudoscalar octet mesons. In view of their small mass the latter are better described as the Goldstone bosons of the spontaneously broken approximate chiral sym-
metry of QCD than as \( q\bar{q} \) states.

If the constant \( C_\chi \) in Eq. (1.1) is chosen so that both the \( N(1440), \frac{1}{2}^+ \) and \( N(1535), \frac{1}{2}^- \) resonances can be described as \( qqq \) states with additional \( qqqq\bar{q} \) components the lowest \( qqqq\bar{q} \) states are states with positive parity. The energy difference between the lowest lying negative and positive parity \( qqqq\bar{q} \) states is, due to different group theoretical structures of the states, larger for the nucleon-like states than for the \( \Delta \)-like (and \( \Omega^- \)-like) states, while the difference for the \( \Lambda \)-, \( \Sigma \)- and \( \Xi \)-like states is much smaller. The low lying \( qqqq\bar{q} \) states provide a discrete approximation to the open \( qqq \)-meson channels that are the cause of the large width of the empirical low lying baryon resonances. The most interesting feature of this rich spectrum are the discrete low lying states, which in many cases overlap with the corresponding low lying excited \( qqq \) states, and thus suggest significant \( qqqq\bar{q} \) admixtures in several of the latter states. The higher lying \( qqqq\bar{q} \) states, on the other hand are so dense as to almost form a near continuum of states.

The main result of this investigation for the structure of the nucleon is that the \( qqqq\bar{q} \) system has a well separated low lying \( \frac{1}{2}^+ \) state, which overlaps with the \( N(1440) \), and supports the view of substantial collectivity of that resonance. The \( qqqq\bar{q} \) system also has a low lying \( \frac{1}{2}^- - \frac{3}{2}^- \) doublet, which similarly indicates that the corresponding empirical states \( N(1535) \) and \( N(1520) \) have strong \( qqqq\bar{q} \) admixtures.

The lowest positive parity \( qqqq\bar{q} \) state in the spectrum of the \( \Delta \) resonance is a triplet of \( \frac{1}{2}^+, \frac{3}{2}^+ \) and \( \frac{5}{2}^+ \) states. The \( \frac{3}{2}^+ \) state partly overlaps with the empirical \( \Delta(1600) \) resonance, and the low lying negative parity states overlap with the empirical \( \Delta(1620) \) and \( \Delta(1700) \) states, thus suggesting substantial sea-quark admixture in those states and a qualitative explanation for the large widths of these states.

In the case of the spectrum of the \( \Lambda \)-hyperon the outstanding feature of the \( qqqq\bar{q} \) spectrum is that in the present model the lowest lying state is a \( \frac{1}{2}^+ - \frac{3}{2}^+ \) doublet state but that it also has a well separated low lying \( \frac{1}{2}^- \) state, which may suggest that the \( \Lambda(1405) \) is partly a 5-quark state (an \( \bar{K}N \) molecule) \[12\]. About \( \sim 65 \) MeV above this state lies a \( \frac{1}{2}^- - \frac{3}{2}^- \) multiplet.
which for the $\frac{3}{2}^-$ overlaps with the $\Lambda(1520)$, $\frac{3}{2}^-$ resonance. The lowest lying positive and negative parity $qqqq\bar{q}$ states are close in energy, and this difference is very sensitive to the value of the parameter $C_\chi$. A slightly smaller $C_\chi$ would switch the ordering of the lowest positive and negative $\Lambda$-like states, and also of the $\Sigma$- and $\Xi$-like states, while for the $N$, $\Delta$- and $\Omega^-$-like states a positive parity state would still be the lowest in energy. In the case of the strange hyperons the predicted $qqqq\bar{q}$ spectrum as expected corresponds well to the spectrum predicted with the bound state version of the topological soliton model [11, 16].

This paper falls into 6 sections. In section 2 the Hamiltonian for the $qqqq\bar{q}$ states, which is formed of a harmonic confining term and a flavor-spin dependent hyperfine interaction, is presented along with the symmetry classification of its eigenstates with $L = 0$ and 1. In section 3 the energies of the $qqqq\bar{q}$ states that have the flavor quantum numbers of the nucleon and the $\Delta(1232)$ resonances are calculated. The corresponding energies of the $qqqq\bar{q}$ states in the spectrum of the $\Lambda$ and $\Sigma$ hyperons are given in section 4 and those of the $\Xi$ and $\Omega^-$ are given in section 5. Section 6 contains a concluding discussion.

2. The $qqqq\bar{q}$ system

A translationally invariant Hamiltonian model for the $qqqq\bar{q}$ system, the simplest version of which would be the harmonic oscillator Hamiltonian, may be written as

$$H = \sum_{i=1}^{5} \frac{\vec{p}_i^2}{2m_i} - \frac{\vec{P}^2}{2M} + \sum_{i<j}^{5} V_{\text{conf}}(r_{ij}) + \sum_{i=1}^{5} m_i \ . \quad (2.1)$$

Here $m_i$ denotes the constituent masses of the quarks (and the antiquark), and $\vec{P}$ and $M$ are the total momentum and mass of the $qqqq\bar{q}$ system respectively. The confining potential $V_{\text{conf}}(r_{ij})$ will be taken to have the form

$$V_{\text{conf}}(r_{ij}) = -\frac{3}{8} \chi_i^C \cdot \chi_j^C (C[r_i - r_j]^2 + V_0) \ . \quad (2.2)$$
In Eq. (2.2) the color dependence of the potential is indicated by \( \lambda^C_i \cdot \lambda^C_j \), and \( C \) and \( V_0 \) are constants. The presence of negative constants such as \( V_0 \) is required for a realistic description of the baryon spectrum [5]. The above form for the confining potential is chosen in order to achieve agreement with Regge phenomenology that indicates that the string tension between quarks and antiquarks in mesons is twice that between two quarks in a baryon. Thus, with a confining interaction described by Eq. (2.2), \( < \lambda^C_i \cdot \lambda^C_j > = -8/3 \) for a \( qq \)-pair in a baryon, while \( < \lambda^C_i \cdot \lambda^C_j > = -16/3 \) for \( q\bar{q} \) in mesons. The situation for \( qqqq\bar{q} \) states is, however, different, as has been shown in Ref. [17]. Due to the color symmetry structure of the \( qqqq\bar{q} \)-system, \( < \lambda^C_i \cdot \lambda^C_j > = -4/3 \) for both \( qq \)- and \( q\bar{q} \)-pairs. It is thus possible to rewrite the Hamiltonian (2.1) as

\[
H = \sum_{i=1}^{5} \frac{\vec{p}_i^2}{2m_i} - \frac{\vec{P}^2}{2M} + \frac{1}{2} \sum_{i<j} (C[r_i - r_j]^2 + V_0) + \sum_{i=1}^{5} m_i . \tag{2.3}
\]

Note that the color dependence of the confining interaction is conventional and mainly motivated by fact that the shell spacing in the experimental meson spectrum is larger (\( \sim 700 \) MeV) than it is in the experimental baryon spectrum (\( \sim 400 \) MeV). The recent quenched lattice calculation of the effective confining interaction in heavy \( q\bar{q} \) and heavy \( qq \) systems suggests that there is no color dependence in the confining interaction [18]. The result above concerning the strength of the effective confining interaction between an antiquark and a 4-quark system is by the result of Ref. [17] independent of the color (in)dependence of the confining interaction.

If the constituent masses of the quarks (and the antiquark) are taken to be equal the translationally invariant Hamiltonian (2.3) may be rewritten as a sum of 4 uncoupled harmonic oscillator Hamiltonians by the following change of variables:

\[
\tilde{R} = \frac{1}{\sqrt{5}}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 + \vec{r}_5) ,
\]

\[
\tilde{\xi}_1 = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) ,
\]
\[ \vec{\xi}_2 = \frac{1}{\sqrt{6}} (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3), \]
\[ \vec{\xi}_3 = \frac{1}{\sqrt{12}} (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 - 3\vec{r}_4), \]
\[ \vec{\xi}_4 = \frac{1}{\sqrt{20}} (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 - 4\vec{r}_5). \quad (2.4) \]

Here $\vec{R}$ is the c.m. coordinate. The resulting uncoupled Hamiltonian then takes the form

\[ \tilde{H} = \frac{1}{2m} \sum_{i=1}^{4} \eta_i^2 + \frac{5}{2} C \sum_{i=1}^{4} \xi_i^2 + 5V_0 + 5m. \quad (2.5) \]

Here $\vec{\eta}_i$ is the momentum operator, which is canonically conjugate to the operator $\vec{\xi}_i$ ($\vec{\eta}_i = -i\vec{\nabla} \vec{\xi}_i$). The Hamiltonian (2.5) describes 4 uncoupled harmonic oscillators, with the oscillator frequency $\omega = \sqrt{\frac{5C}{m}}$. Note that the choice of variables (2.4) is sufficient to reduce the Hamiltonian (2.3) to a set of uncoupled oscillators also in the more general case, where the string tension is different for the different quark pairs.

When one or more of the quarks in the $qqqq\bar{q}$ system is strange, this Hamiltonian is only approximate. The main effect of the difference between the constituent masses of the light flavor and strange quarks is to shift the energy upwards by multiples of the number of strange quarks. This effect will be taken into account here. The energy levels of the Hamiltonian (2.5) are

\[ E_0 = 6\omega + N\omega + 5V_0 + 5m, \quad (2.6) \]

where $N$ is the number of excited quanta of the oscillators.

The excited spectrum of the oscillator Hamiltonian is organized in shells, the ground state shell of which $(N = 0)$ is formed of states with negative parity because of the negative parity of the antiquark. The lowest positive parity states occur in the first excited P-shell and have excitation energies $\omega$, with a quark or an antiquark in a p-state. Because of the negative parity of the ground state band it is clear that the $qqqq\bar{q}$ states represent excited and not ground state baryons. The key point is, however, that the hyperfine
interaction (1.1) is strong enough to bring the lowest $L = 1$ states down to or below the lowest states with $L = 0$ in the spectra of the nucleon and the $\Delta(1232)$ resonances, and therefore the lowest $qqqq\bar{q}$ states form a band with positive parity states. This situation is analogous to the description of the spectrum of the strange hyperons in the bound state version of the topological soliton model [11, 16].

The matrix element of the hyperfine interaction $H_x$ can be calculated as [19, 20]

$$< [f]^{SU(6)}[f]^{SU(3)}[f]^{SU(2)}| \sum_{i<j} \vec{\lambda}_i \cdot \vec{\lambda}_j \sigma_i \cdot \sigma_j | [f]^{SU(6)}[f]^{SU(3)}[f]^{SU(2)} >$$

$$= 4C_2^{(6)} - 2C_2^{(3)} - \frac{4}{3}C_2^{(2)} - 8N , \quad (2.7)$$

where $C_2^{(n)}$ is the matrix element of the quadratic Casimir operator of $SU(n)$, with $n = 6, 3, \text{ and } 2$, and $N$ is the number of quarks, i.e. 4 in the subsystem of four quarks. The matrix element $C_2^{(n)}$ is given by [20]

$$C_2^{(n)} = \frac{1}{2} \left( f'_1 (f'_1 + n - 1) + f'_2 (f'_2 + n - 3) + \ldots \right.$$

$$\left. + f'_{n-1} (f'_{n-1} - n + 3) \right) - \frac{1}{2n} \left( \sum_{i=1}^{n-1} f'_i \right)^2 , \quad (2.8)$$

where $f'_i = (f_i - f_n)$, with $f_i$ being the length of the $i$th row of the corresponding Young pattern. The numerical values for $C_2^{(n)}$ are given in Table 1.

The state in which the 4 quarks are in their lowest s-states is completely symmetric in the position coordinates ($[4]_X$). The Pauli principle demands that the corresponding color-flavor-spin state be completely antisymmetric ($[1111]_{CFS}$). This symmetry is realized if the color state has the mixed symmetry $[211]_C$ and therefore the flavor-spin state has the mixed symmetry $[31]_{FS}$. In Table 2 the matrix elements of the hyperfine interaction Hamiltonian (1.1) are listed for all the combinations of flavor and spin states, which have this mixed symmetry character.
The lowest state with \( L = 1 \) has one quark in the p-state, and consequently the symmetry of the orbital state is \([31]_X\). The corresponding color-flavor-spin state has accordingly to have the mixed symmetry \([211]_{CFS}\), and since the symmetry of the color state is again \([211]_C\), the possible symmetries are \([4]_{FS}, [31]_{FS}, [22]_{FS}\) and \([211]_{FS}\). The calculated matrix elements of the hyperfine interaction (1.1) for states with these 4 flavor-spin symmetries are listed in Table 3. The relative positions of these states with respect to those that have \( L = 0 \) is obtained by addition of the orbital excitation energy \( \omega = \sqrt{5/6} \). Since positive parity \( L = 1 \) states can also be formed if the four quarks are in their lowest s-states \([4]_X\) while the antiquark is in a p-state the states with the four-quark flavor-spin symmetry \([31]_{FS}\) will be repeated at the energy \( \omega \) above the corresponding \( L = 0 \) states.

In order to compare the calculated \( qq\bar{q}q\bar{q}\) spectrum with the corresponding \( qqqq\) spectrum for the baryons, the value of the confining constant \( C \) is determined from the oscillator parameter in the three quark baryon system, \( \omega_0 = \sqrt{\frac{5}{6}} \omega \), which in turn is determined by the empirical splitting between the N(1440), \( \frac{1}{2}^+ \) resonance and the nucleon to be \( \omega_0 = 250 \) MeV. The splitting between states with the same flavor-spin symmetry in the (lowest) \( L = 1 \) and \( L = 0 \) bands of the \( qq\bar{q}q\bar{q}\) system is then \( \omega = \sqrt{\frac{5}{6}} \omega_0 \approx 228 \) MeV. The parameter \( C \chi \) in the hyperfine quark-quark interaction (1.1) is then adjusted to the value \( C \chi = 21 \) MeV to allow for \( qq\bar{q}q\bar{q}\) admixtures in the lowest lying positive and negative parity nucleon resonance states. With these parameter values the ground state of the \( qq\bar{q}q\bar{q}\) system will be the \([31]_X[4]_{FS}[22]_F[22]_S\) positive parity state \([2]_S\), which then for the nucleon mixes with the N(1440), \( \frac{1}{2}^+ \) resonance.

Assessment of the possibility for significant admixture of \( qq\bar{q}q\bar{q}\) configurations into the \( qq \) states requires determination of the spin flavor quantum numbers in the \( qq\bar{q}q\bar{q}\) multiplets for comparison with those of the \( qq \) multiplet. The latter correspond well to the measured quantum numbers of the states in the empirical baryon spectrum.

The isospin and strangeness content of the \( qq\bar{q}q\bar{q}\) states are most simply analyzed by considering the \( qqq \) subsystem first, and subsequently adding the antiquark. The flavor multiplet \([4]_F\), which corresponds to the \( SU(3)\)
flavor representation $15'$, is then combined with an antiquark in the flavor representation $\mathbf{3}$, giving $15' \times 3 = 10 + 35$. The two resulting representations are shown in $(I_3, Y)$ diagrams in Fig. 1, where $I_3$ is the third component of the isospin $I$, and $Y$ is the hypercharge, defined as the sum of the baryon number $B$ and the strangeness number $S$. The $qqqq\bar{q}$ states in the decuplet that results from $[4]_F$ have the correct quantum numbers to describe mixing with a three-quark decuplet.

The flavor multiplet $[31]_F$, on the other hand, can be described by the representation $15$ which, when combined with an antiquark, gives $15 \times 3 = 8 + 10 + 27$. These representations are shown in Fig. 2. The states in the octet and decuplet of the resulting $qqqq\bar{q}$ states can in this case be mixed with the three-quark flavor octet and decuplet states. The $[22]_F$ flavor multiplet corresponds to the representation $\mathbf{3}$, which yields $\mathbf{3} \times 3 = 8 + 10$ when combined with an antiquark. These representations are shown in Fig. 3. Mixing with a three-quark flavor octet state is thus possible. Finally, the $[211]_F$ flavor multiplet, described by the representation $\mathbf{3}$ will, when combined with an antiquark, yield $\mathbf{3} \times 3 = 1 + 8$ (Fig. 4) and mixing with the three-quark flavor singlet and octet states, respectively, is thus possible.

To summarize, the flavor decuplet states ($\Delta, \Sigma^*, \Xi^*, \Omega^-$) can be derived from the $[4]_F$ and $[31]_F$ four-quark subsystems, the flavor octet states ($N, \Lambda, \Sigma, \Xi$) from $[31]_F$, $[22]_F$ and $[211]_F$, and, finally, the flavor singlet state ($\Lambda'$) from $[211]_F$. To derive some of the states, e.g. the nucleon from the four quark symmetry $[211]_F$, it is, however, necessary to assume that the states contain $s\bar{s}$ states. Since in this model the hyperfine splittings of the states are determined by the flavor-spin structure of the four-quark subsystem different energy states will therefore be denoted below by their four-quark $[f]_F[f]_S[f]_F[f]_S$ structure.

The total angular momentum $J$ of the $qqqq\bar{q}$ system can be calculated for states of different spin symmetry in the four-quark subsystem by first adding the spins of the four quarks to reach a state $[f]_S$, and then adding the spin of the antiquark ($s = \frac{1}{2}$) and, finally, adding the orbital angular momentum if $L \neq 0$. The results for the different states are given in Table 4, where the parity $P$ of the states has also been given. Note that in the $qqqq\bar{q}$ system the ground state configuration will have $P = -1$ due to the presence
of a \( q\bar{q} \) pair and, subsequently, the first excited state will have positive parity.

3. 5 quark components in the nucleon and \( \Delta \) resonance states

States with the quantum numbers of the nucleon resonances ([21]_F) appear in the \( qqqq\bar{q} \) states that belong to the flavor multiplets [31]_F, [22]_F, and [211]_F of the four-quark subsystem. The symmetry [211]_F has, however, to contain a strange quark, and to have total strangeness zero the antiquark has to be \( \bar{s} \) in this \( qqqq\bar{q} \) state. Nucleon-like states derived from the states with four-quark symmetries [31]_F and [22]_F may be both states without and with hidden strangeness. Among the \( qqqq\bar{q} \) states with zero strangeness that can mix with the nucleon there are thus many states with \( ss \) pairs, and which therefore represent strangeness components of the nucleon. Experimental signatures for such \( ss \) components in the nucleon have been a topic of intense experimental interest recently [22, 23]. Here it has been assumed that the explicit \( qqqq\bar{q} \) configurations have energies that lie above the lowest excited nucleon resonance energy. Consequently signatures of these configurations are expected to be most visible in the decay patterns of the nucleon. The main effect of \( SU(3)_F \) breaking is a shift of the energies of these \( ss \) configurations upwards by \( \sim 2\delta m \), where \( \delta m \) is the difference between the constituent masses of the strange and light flavor quarks. We shall take \( \delta m \) to be \( \delta m = m_s - m_u = 120 \text{ MeV} \) [4]. The \( L = 0 \) state \( [31]_{FS}[211]_F[22]_S \) that otherwise would be the lowest lying negative parity state, will for the nucleon be lifted up \( 2\delta m \) due to the flavor symmetry structure [211]_F of the four-quark system.

We interpret the \( qqqq\bar{q} \) states as sea-quark admixtures of the excited \( qqq \) states, and therefore, as mentioned earlier, take the energy of the lowest nucleon resonance, the \( N(1440) \), as the reference point for the energy of the positive parity ground state of the \( qqqq\bar{q} \) system. The \( N(1440), \frac{1}{2}^+ \) state has a complex structure and is expected to have large sea-quark and possible more exotic components [24]. The expression for the mass of the lowest positive parity \( qqqq\bar{q} \) state in the nucleon spectrum is
Using the parameter values above and taking $m = 340$ MeV and $V_0 = -269$ MeV the energy of this state is 1365 MeV, which is the empirical value for the real part of the pole position of the N(1440) resonance. The real part of the pole position is the appropriate energy to which compare quark models, as it takes into account the shift caused by the coupling to the open mesonic channels [25]. Smaller values for $V_0$ would shift the energy of the lowest positive parity $qqqq\bar{q}$ state, and consequently also the energy of other $qqqq\bar{q}$ states, upwards.

The symmetry classification of those $qqqq\bar{q}$ states which have the same quantum numbers as the nucleon resonances, which are predicted to have energies below 1900 MeV, along with their estimated energies are listed in Tables 5 and 6 for the states with negative and positive parity respectively. Since there is no hyperfine interaction between the quarks and antiquarks in the present model, states with the same four-quark symmetry as the ground state but with an antiquark in a p-state will repeat themselves in the positive parity spectrum at the same energy as states with one quark in a p-state and with similar flavor-spin symmetry in the $L = 1$ band. For convenience the corresponding $qqqq\bar{q}$ states, which describe $\Delta$ resonance excitations are listed in the same tables. There are no nucleon resonances with positive parity in the [4]$_F$ multiplet, and no $\Delta$ resonances in the flavor multiplets [22]$_F$ and [211]$_F$.

In Fig. 5 the presently known empirical spectrum of nucleon resonances with energy between 1.4 GeV and 2.5 GeV and known spin-parity $J^P$ is shown for $J \leq \frac{7}{2}$. In Tables 5 and 6 those known nucleon resonances, which may have significant $qqqq\bar{q}$ components with the symmetry structure indicated are also listed. Since the empirical SD shell of nucleon and $\Delta$ resonances is not yet complete, we have indicated the probable $qqq$ states, which have corresponding $qqqq\bar{q}$ admixtures, where the empirical confirmation is still lacking. For the most part there is a good correspondence between the predicted low lying $qqqq\bar{q}$ states and the empirical nucleon spectrum in the negative parity sector. In the positive parity sector all low lying empirical states lie close to predicted $qqqq\bar{q}$ states, but there are, however, some low lying $qqqq\bar{q}$ states

\[
E\{[4]_F S[22]_F[22]_S\} = 7\omega - 28C_\chi + 5V_0 + 5m. \tag{3.1}
\]
that are not seen in the nucleon spectrum, especially in the \( \frac{3}{2}^+ \) sector, the lowest state of which empirically is the \( N(1720) \) resonance.

States with quantum numbers of the \( \Delta \) resonances ([3]_F) are contained in the [4]_F and [31]_F flavor multiplets of the four-quark subsystem. Some of these states may also contain hidden strangeness, i.e. \( s\bar{s} \)-pairs. Such states then lie \( 2\delta m \) above corresponding states without \( s\bar{s} \)-pairs. The calculated spectrum of states with \( J^P \) up to \( \frac{7}{2}^+ \) and energy up to 2.5 GeV is shown in Fig. 6 along with the presently empirically known \( \Delta \) resonances. The numerical values of the calculated energies of the negative and positive parity \( \Delta \) resonance states with calculated energy below 1900 MeV are listed in Tables 5 and 6 respectively. In the column ”emp” those empirically known and predicted \( qqqq \) \( \Delta \)-resonance states, which are likely to have corresponding \( qqqq\bar{q} \) admixtures are also indicated. As can be seen from Fig. 6 several of the empirical positive parity states can be found close to corresponding \( qqqq\bar{q} \) states, e.g. the \( \Delta(1600) \), \( \frac{3}{2}^+ \) resonance. The empirical positive parity states \( \Delta(1910) \), \( \frac{1}{2}^+ \), \( \Delta(1920) \), \( \frac{3}{2}^+ \) and \( \Delta(1905) \), \( \frac{5}{2}^+ \) may have admixtures of the \([31]_FS[31]_F[31]_S \) multiplet, while the ”one star” resonance \( \Delta(1750) \), \( \frac{1}{2}^+ \) seems to have admixtures of several \( qqqq\bar{q} \) states that are predicted to have energies around 1745 - 1785 MeV. In the negative parity sector the \( \Delta(1620) \), \( \frac{1}{2}^- \) resonance seems to have a substantial admixture of a \([31]_FS[31]_F[31]_S \) multiplet at 1613 MeV, and maybe also some admixture of a \([31]_FS[31]_F[22]_S \), \( \frac{1}{2}^- \) singlet state at \( \sim 1560 \) MeV. The \( \Delta(1700) \), \( \frac{3}{2}^- \) resonance (with the real part of the pole position at 1660 MeV) also seems to have admixtures of several \( qqqq\bar{q} \) states in an energy range of about 1610 - 1780 MeV.

4. 5 quark components in the \( \Lambda \) and \( \Sigma \) resonances

The \( qqqq\bar{q} \) states with strangeness \(-1\) contain at least one strange quark, and accordingly their spectrum begins at a level that is shifted about \( \delta m \) above the corresponding level for non-strange \( qqqq\bar{q} \) states. Combination of the \( qqqq \) flavor multiplets [3]_F, [22]_F and [211]_F with an antiquark leads to states with the quantum numbers of the \( \Lambda \) hyperon resonances. Of these only the [211]_F multiplet can combine with an antiquark to form a flavor singlet
state. The states derived from the four-quark symmetries $[31]_F$ and $[211]_F$ are states both with one $s$ quark, and with $ss\bar{s}$-combinations, while from the four-quark symmetry $[22]_F$ only $\Lambda$-like states with one strange quark can be derived. The states containing $ss\bar{s}$-combinations lie $2\delta m$ above the corresponding states with one $s$-quark.

In Tables 7 and 8 we list the calculated energies of the $\Lambda$ hyperon resonances in the $qqqq\bar{q}$ spectrum up to 2 GeV, along with their symmetry character. The empirically known $\Lambda$ hyperon resonances, which have similar energies and the same quantum numbers are also indicated. These are the states, which are expected to have correspondingly strong $qqqq\bar{q}$ admixtures. The calculated $qqqq\bar{q}$ $\Lambda$ hyperon spectrum up to 2.5 GeV and $J^P = \frac{3}{2}^-$ with $L = 0$ and 1 is shown in Fig. 7.

The outstanding feature of the $\Lambda$ hyperon spectrum that can be formed of 5 quark states is the appearance of both a low lying $\frac{1}{2}^-$ flavor singlet state and a low lying $\frac{1}{2}^+ - \frac{3}{2}^+$ doublet state. The former state may in this model give a small $qqqq\bar{q}$ contribution the low lying isolated $\Lambda(1405)$ state, which conventionally has been described as a $K\bar{N}$ molecule [12] or in other words - a 5 quark state.

Above this low lying flavor singlet $\frac{1}{2}^-$ state are found several $\frac{1}{2}^- - \frac{3}{2}^-$ $qqqq\bar{q}$ doublets, which are likely to be mixed into the $\Lambda(1670), \frac{1}{2}^-$ and $\Lambda(1690), \frac{3}{2}^-$ resonances, and in the case of the lowest negative parity doublet also probably with the $\Lambda(1520), \frac{3}{2}^-$ state. Above these doublets further appear a singlet $\frac{1}{2}^-$ state, and several $\frac{1}{2}^- - \frac{3}{2}^-$ doublets, which are likely to be mixed into the empirical $\Lambda(1800), \frac{1}{2}^-$, and an expected but still "missing" $\frac{3}{2}^-$ state near 1800 MeV. There is also a $\frac{3}{2}^- - \frac{5}{2}^-$ doublet, which is strongly mixed into the $\Lambda(1830), \frac{5}{2}^-$. The lowest positive parity $qqqq\bar{q}$ state in the spectrum of the $\Lambda$ hyperon is the $\frac{1}{2}^+ - \frac{3}{2}^+$ multiplet $[4]_{FS}[22]_F[22]_S$, which is found to have an energy of 1485 MeV. This multiplet is not seen empirically. It may, however, form a small $qqqq\bar{q}$ admixture in the empirical $\Lambda(1600), \frac{1}{2}^+$ resonance. About 140 MeV above this $qqqq\bar{q}$ state lies the 5 quark multiplet $[4]_{FS}[31]_F[31]_S$, which has states with $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ and $\frac{5}{2}^+$. The first one of these is probably mixed
into the $\Lambda(1600)$. Above this multiplet appear several multiplets that may be mixed into the $\Lambda(1810)$, $\frac{1}{2}^+$, $\Lambda(1890)$, $\frac{3}{2}^+$ and $\Lambda(1820)$, $\frac{5}{2}^+$ resonances.

The spectrum of the $\Sigma$ hyperon does in all versions of the quark model have the same structure as the spectra of the nucleon and the $\Delta(1232)$ combined. Hence all the $qqqq$ flavor multiplets $[4]_F$, $[31]_F$, $[22]_F$ and $[211]_F$ can be combined with an antiquark to states that have the quantum numbers of the $\Sigma$ resonances, all but the states derived from the symmetry structure $[211]_F$ containing one $s$-quark and $ss\bar{s}$-combinations. States derived from the symmetry structure $[211]_F$ contain only states with one $s$-quark and no $ss\bar{s}$-combinations. In Tables 7 and 8 we list the energies and symmetry character of those $qqqq\bar{q}$ $\Sigma$ hyperon resonances that in the present model have energies below 2 GeV. In Fig. 8 the $qqqq\bar{q}$ $\Sigma$ states with $J^P \leq \frac{7}{2}^+$ and $L = 0, 1$ have been plotted for energies up to 2.5 GeV.

The identification of which $\Sigma$ hyperon resonances should have strong 5 quark admixtures is difficult because of the still very incomplete empirical information of even the low energy part of the $\Sigma$ hyperon spectrum. Whether e.g. the "one-star" resonance $\Sigma(1480)$ is a real resonance, and the $qqqq\bar{q}$ analog of the low lying $\Lambda(1405)$, $\frac{1}{2}^-$ state is not known. On the other hand, in the present model there is a low lying positive parity $\frac{1}{2}^+ - \frac{3}{2}^+$ multiplet with the energy 1485 MeV. Similarly, the quantum numbers of the $\Sigma(1560)$ remain unknown. If indeed the $\Lambda(1405)$ is partly a 5 quark state, it would be natural to expect the $\Sigma(1560)$ to be the analog of the $\Lambda(1405)$, and thus that it has $J^P = \frac{1}{2}^-$. This is indeed what the structure of the $qqqq\bar{q}$ spectrum shown in Table 7 indicates. This would also explain why the usual quark model description of the baryons as 3 quark states cannot predict sufficiently low energies for the $\Lambda(1405)$ and $\Sigma(1560)$ [4]. The $\Sigma(1480)^\ast$ may then, on the other hand, contain admixtures of the lowest lying positive parity $qqqq\bar{q}$ state, thus having $J^P = \frac{1}{2}^+$ or $J^P = \frac{3}{2}^+$. Several of the negative parity $qqqq\bar{q}$ $\Sigma$ hyperon resonance states that lie above the lowest $[31]_F [211]_F [22]_S$ state have energies that are close to empirically established negative parity states (Table 7). Thus the $\Sigma(1620)$, $\frac{1}{2}^-$ "two-star" state lies close to the predicted energy of both the $[31]_F [211]_F [31]_S$ and the $[31]_F [22]_F [31]_S qqqq\bar{q}$ multiplets, and may be expected to have a substan-
tial 5 quark component, while the former state is also probably mixed with the \( \Sigma(1580), \frac{3}{2}^- \) ("two-star") state. Similarly the \( \Sigma(1670), \frac{3}{2}^- \) lies close to the predicted energy of the \([31]_{FS}[22]_F[31]_S\) multiplet, and may also get a small \(qqqq\bar{q}\) admixture from the \([31]_{FS}[31]_F[31]_S\) multiplet, and the \(\Sigma(1690), \frac{1}{2}^-\) ("two-star") state has an energy close to that of the \([31]_{FS}[31]_F[22]_S\) singlet state. The predicted energy for the \([31]_{FS}[31]_F[31]_S\) multiplet lies close to the \(\Sigma(1750)\), \(\frac{1}{2}^+\) state, while the \(\Sigma(1940), \frac{3}{2}^-\) is close in energy to several \(qqqq\bar{q}\) multiplets. Consequently these resonances may very well be expected to have substantial 5 quark components.

The lowest \(qqqq\bar{q}\) \(\Sigma\) hyperon state with positive parity is the \([4]_{FS}[22]_F[22]_S\) multiplet (Table 8). In the present model its energy is found to be at 1485 MeV, which is 175 MeV below the \(\Sigma(1660), \frac{4}{2}^+\) resonance. Whether this state form an admixture in the "one-star" \(\Sigma(1480)\) state is not clear, but it may in any case form a small admixture in the \(\Sigma(1660)\), in analogy with the corresponding admixture in the \(\Lambda(1600)\). The following state above this \(qqqq\bar{q}\) multiplet is the \([4]_{FS}[31]_F[31]_S\) multiplet, which may form a significant admixture in the \(\Sigma(1660), \frac{1}{2}^+\), as its energy is found to be \(\sim 1625\) MeV. Above that lies a \(qqss\bar{s}\) state and a \(qqqs\bar{q}\) state with energies of 1725 MeV and 1737 MeV, respectively. Also these states may form admixtures in the \(\Sigma(1660), \frac{1}{2}^+\), and in the "one-star" \(\Sigma(1770), \frac{1}{2}^+\) resonance. Above these are several \(qqqq\bar{q}\) multiplets, which in the present model form significant admixtures in the one- and two-star states \(\Sigma(1840), \frac{3}{2}^+\) and \(\Sigma(1880), \frac{1}{2}^+\), respectively, as well as states close in energy to the empirical \(\Sigma(1915), \frac{5}{2}^+\) resonance.

5. \(qqqq\bar{q}\) components in the \(\Xi\) and \(\Omega^-\) resonances

The \(\Xi\) hyperons have strangeness \(-2\) and therefore contain at least two strange constituent quarks. States derived from the \([211]_F\) and \([22]_F\) symmetries contain only two \(s\)-quarks, while states derived from the \([4]_F\) and \([31]_F\) symmetries have both states with two \(s\)-quarks and \(ss\bar{s}\)-combinations. Their spectrum, while otherwise similar to that of the \(\Sigma\) hyperon both in the 3 quark and 4 quark + 1 antiquark model, with the exception mentioned
above, therefore begins at an energy $2\delta m = 2(m_s - m_u)$ above that of the nucleon and $\delta m$ above that of the $\Sigma$ hyperon spectrum.

The experimental spectrum of the $\Xi$ hyperon is rich, but poorly understood, in fact of the resonances above the ground state band only the $\Xi(1820)$ is known to be a $\frac{3}{2}^-$ state. This makes it impossible to make definite suggestions for which $\Xi$ resonances may have significant $qq\bar{q}\bar{q}$ configurations. The calculated $qq\bar{q}\bar{q}$ $\Xi$ hyperon spectrum up to 2.5 GeV and $JP = \frac{7}{2}^-$ with $L = 0$ and 1 is shown in Fig. 9 and the numerical values for the energies for states up to 2.1 MeV are given in Tables 9 and 10.

The lowest in energy of the $qq\bar{q}\bar{q}$ $\Xi$ resonances with negative parity is the $[31]_{FS}[21][22]_S \frac{1}{2}^-$ state, which in the present model is found to be at $\sim 1730$ MeV, and above this is found a $qq\bar{q}\bar{q} \frac{1}{2}^- - \frac{3}{2}^-$ multiplet at 1685 MeV. If the $\Xi(1690)$ resonance is confirmed as a $\frac{1}{2}^-$ or a $\frac{3}{2}^-$ state it is natural to expect that it has a significant $qq\bar{q}\bar{q}$ component. Also the negative parity $\Xi(1820)$ resonance may have admixtures of several 5 quark states. As there is a negative parity $qq\bar{q}\bar{q}$ state close in energy to the $\Xi(1950)$ this state may then be either a $\frac{3}{2}^-$ or a $\frac{5}{2}^-$ state with a significant $qq\bar{q}\bar{q}$ component.

The lowest lying positive parity $qq\bar{q}\bar{q}$ state in the $\Xi$ spectrum is the $\frac{1}{2}^+$ - $\frac{3}{2}^+$ multiplet $[4]_{FS}[22][22]_S$. This is predicted to have an energy of 1605 MeV, which is suggestively close to the one-star state $\Xi(1620)$, which is then likely a positive parity state. If so the pattern of the other flavor sectors of finding $qq\bar{q}\bar{q}$ states with positive parity repeats itself also in the $\Xi$ spectrum. The $\Xi(1620)(\ast)$ should in any case be expected to have a substantial $qq\bar{q}\bar{q}$ component. The $\Xi(2030)$, with $JP \leq \frac{5}{2}$ could also be a positive parity state, with either $JP = \frac{5}{2}^+$ or $\frac{7}{2}^+$, containing significant $qq\bar{q}\bar{q}$ admixtures.

The symmetry structure of the $sssq\bar{q}$ states in the spectrum of the $\Omega^-$ hyperon is the same as that of the $qq\bar{q}\bar{q}$ states in the spectrum of the $\Delta(1232)$ resonance. From the $[4]_F$ symmetry also states containing only $s$-quarks may be derived, while the symmetry $[31]_F$ implies only states that contain three $s$-quarks. This spectrum should begin at an energy $3\delta m$ above that of the corresponding $\Delta$ spectrum. The symmetry structure of the negative and positive parity $sssq\bar{q}$ $\Omega^-$ resonances, with energies that are predicted to fall
below 2.4 GeV is listed in Tables 11 and 12 respectively. In the absence of empirical quantum number assignments for hitherto found Ω⁻ resonances it requires pure speculation to indicate which type of sssq̅ admixture those states are likely to have. In Fig. 10 the calculated qqqq̅ Ω⁻ states with \( J^P \leq \frac{7}{2}^+ \) and \( L = 0, 1 \) have been plotted for energies up to 2.5 GeV.

Both the negative and positive parity sectors of the excited Ω⁻ sssq̅ spectrum begin, according to the present model, at around 1.9 GeV, with the positive parity multiplet \([4]_{FS}[31]_F[31]_S\) falling somewhat lower than the corresponding negative parity multiplet \([31]_{FS}[31]_F[22]_S\). The corresponding 3 valence quark model for the Ω⁻ in contrast suggests that the lowest excited Ω⁻ state is the negative parity \([21]_{FS}[3]_F[21]_S\ \frac{1}{2}^− - \frac{3}{2}^-\) multiplet, with an energy in the range 1950 - 1990 MeV \([4, 26]\). The lowest sssq̅ resonance with positive parity in the present model falls at 1865 MeV, while the lowest negative parity state lies at \( \sim 1920 \) MeV. The low lying part of the sssq̅ Ω⁻ spectrum does in any case resemble the corresponding excited spectrum of the 3 valence quark model. Hence it is natural to expect considerable sea-quark admixture in the spectrum of the Ω⁻ hyperon.

The only well established Ω⁻ resonance is the Ω(2250)⁻. In the 3 valence quark model this has to be a positive parity state. In the present model the positive parity state \([31]_{FS}[31]_F[31]_S\) has an energy of \( \sim 2200 \) MeV. This would then suggest that this resonance is likely to be partly a 5 quark state.

6. Discussion

Besides the 5 quark states that have been considered above, there will also exist exotic qqqq̅ states with quantum numbers that are excluded for pure qqq configurations. Such exotic states can be formed from the flavor representations \(35, 27\) and \(10\), the four-quark flavor substructure of which have the symmetries \([4]_F, [31]_F\) and \([22]_F\), e.g. states with strangeness \(S = 1\) and isospin \(I = 2, 1\) and \(0\). The \(35\) representation also includes other ”exotic” states, such as \((S, I) = (0, \frac{2}{3}), (-1, 2), (-2, \frac{3}{2}), (-3, 1)\) and \((-4, \frac{1}{2})\). The representation \(27\), on the other hand, contains ”exotic” states with \((S, I) = (1, 1), (-1, 2), (-2, \frac{3}{2})\) and \((-3, 1)\) and, finally, the \(10\) representa-
tion contains the "exotic" states \((S, I) = (1, 0)\) and \((-2, \frac{3}{2})\). Baryon states with \(S = +1\), with the structure \(qqq\bar{s}\), where \(q\) represents \(u\) or \(d\) quarks, are the strange analogs of the \(C = -1\) pentaquarks, which may be stable against strong decay \([21, 27, 28, 29]\). Strange pentaquarks in contrast do decay strongly.

The conventional view that the baryon resonances should be described as 3 quark states is mainly due to the remarkably successful description that the 3 valence quark model provides for not only the energies, but the magnetic moments and axial couplings as well of the ground state baryons in all flavor sectors of the baryon spectrum. No such simple description has yet been found for the baryon resonances, although the spectrum itself can be qualitatively described by interaction models with the form considered here. As noted above there are strong empirical and phenomenological indications for large sea-quark admixtures in the low lying negative and positive parity states. The main result of the present study is that the chiral constituent quark model, a quark model with a flavor-spin dependent hyperfine interaction, will bring the lowest \(L = 1\) positive parity \(qqq\bar{q}\) states, with quantum numbers that correspond well with the lowest lying positive parity baryon resonances above the ground state band, below the lowest \(L = 0\) negative parity \(qqq\bar{q}\) states. Although the present study is based on a somewhat schematic model for the flavor-spin dependent hyperfine interaction, it nevertheless suggests that most of the lowest lying baryon resonances in all flavor sectors of the baryon spectrum have strong sea-quark components of the form \(qqq\bar{q}\). What is really needed to understand the spectrum of baryon excitations in the framework of this paper is a model for the mixing between ordinary 3-body excitations and these \(qqq\bar{q}\) states.

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Figure Captions

Figure 1: The representations (a) 10 and (b) 35 of $15' \times 3$ in flavor $SU(3)$. Here $Y$ is the hypercharge, $I$ isospin and $I_3$ the third component of the isospin.

Figure 2: The representations (a) 8, (b) 10 and (c) 27 of $15 \times 3$ in flavor $SU(3)$. Here $Y$ is the hypercharge, $I$ isospin and $I_3$ the third component of the isospin.

Figure 3: The representations (a) 8 and (b) $\overline{10}$ of $6 \times 3$ in flavor $SU(3)$. Here $Y$ is the hypercharge, $I$ isospin and $I_3$ the third component of the isospin.

Figure 4: The representations (a) 1 and (b) 8 of $3 \times 3$ in flavor $SU(3)$. Here $Y$ is the hypercharge, $I$ isospin and $I_3$ the third component of the isospin.

Figure 5: Empirical N states (real part of pole position) compared to $qqqq\bar{q}$ states with four-quark flavor symmetries [31]$_F$, [22]$_F$ and [211]$_F$. Above E denotes the energy in GeV, $J$ total spin and $P$ parity.

Figure 6: Empirical $\Delta$ states (real part of pole position) compared to $qqqq\bar{q}$ states with four-quark flavor symmetries [4]$_F$, [31]$_F$ and [22]$_F$. Above E denotes the energy in GeV, $J$ total spin and $P$ parity.

Figure 7: Empirical $\Lambda$ states compared to $qqqq\bar{q}$ states with four-quark flavor symmetries [31]$_F$, [22]$_F$ and [211]$_F$. Above E denotes the energy in GeV, $J$ total spin and $P$ parity.

Figure 8: Empirical $\Sigma$ states compared to $qqqq\bar{q}$ states with four-quark flavor symmetries [4]$_F$, [31]$_F$, [22]$_F$ and [211]$_F$. Above E denotes the energy in GeV, $J$ total spin and $P$ parity.

Figure 9: Empirical $\Xi$ states compared to $qqqq\bar{q}$ states with four-quark flavor symmetries [4]$_F$, [31]$_F$, [22]$_F$ and [211]$_F$. Above E denotes the energy in GeV, $J$ total spin and $P$ parity.
Figure 10: Empirical $\Omega^-$ state compared to $qqq\bar{q}$ states with four-quark flavor symmetries $[4]_F$ and $[31]_F$. Above $E$ denotes the energy in GeV, $J$ total spin and $P$ parity.
Table 1

Matrix elements of the squared Casimir operator $C_2^{(n)}$ for different symmetries $[f]$ of the four-quark system.

| $[f]$ | $C_2^{(6)}$ | $C_2^{(3)}$ | $C_2^{(2)}$ |
|-------|-------------|-------------|-------------|
| [4]   | $\frac{50}{3}$ | $\frac{28}{3}$ | 6           |
| [31]  | $\frac{38}{3}$ | $\frac{16}{3}$ | 2           |
| [22]  | $\frac{32}{3}$ | $\frac{10}{3}$ | 0           |
| [211] | $\frac{26}{3}$ | $\frac{4}{3}$  | –           |
Matrix elements of the chiral hyperfine interaction (1.1) for the ground state of the four-quark system with different flavor-spin symmetries $[31]_{FS}[f][f][f][S]$.

| $[4]_X[1111]_{CFS}[211]_C$ | $[f]_{FS}[f][f][f][S]$ | $\langle H_\chi \rangle$ |
|---------------------------|------------------------|------------------|
| $[31]_{FS}[211]_{F}[22]_S$ |                       | $-16 \ C_\chi$ |
| $[31]_{FS}[211]_{F}[31]_S$ |                       | $-\frac{40}{3} \ C_\chi$ |
| $[31]_{FS}[22]_{F}[31]_S$  |                       | $-\frac{28}{3} \ C_\chi$ |
| $[31]_{FS}[31]_{F}[22]_S$  |                       | $-8 \ C_\chi$ |
| $[31]_{FS}[31]_{F}[31]_S$  |                       | $-\frac{16}{3} \ C_\chi$ |
| $[31]_{FS}[31]_{F}[4]_S$   |                       | $0$ |
| $[31]_{FS}[4]_{F}[31]_S$   |                       | $\frac{8}{3} \ C_\chi$ |
Table 3

Matrix elements of the chiral hyperfine interaction (1.1) for the first excited state ($L = 1$) of the four-quark system with different flavor-spin symmetries $[f]FS[f]F[f]S$.

| $[31]_X[211]_{CFS}[211]_C$ | $< H_X >$ | $[31]_X[211]_{CFS}[211]_C$ | $< H_X >$ |
|-----------------------------|-----------|-----------------------------|-----------|
| $[4]_{FS}[22]_{F}[22]_S$   | $-28 \ C_X$ | $[211]_{FS}[211]_{F}[22]_S$ | $0$       |
| $[4]_{FS}[31]_{F}[31]_S$   | $-\frac{64}{3} \ C_X$ | $[31]_{FS}[31]_{F}[4]_S$ | $0$       |
| $[31]_F[211]_F[22]_S$     | $-16 \ C_X$ | $[211]_{FS}[211]_{F}[31]_S$ | $\frac{8}{3} \ C_X$ |
| $[31]_F[211]_F[31]_S$     | $-\frac{40}{3} \ C_X$ | $[22]_{FS}[31]_{F}[31]_S$ | $\frac{8}{3} \ C_X$ |
| $[31]_F[22]_F[31]_S$      | $-\frac{28}{3} \ C_X$ | $[31]_{FS}[4]_{F}[31]_S$ | $\frac{8}{3} \ C_X$ |
| $[31]_F[31]_F[22]_S$      | $-8 \ C_X$ | $[22]_{FS}[22]_{F}[4]_S$ | $4 \ C_X$ |
| $[4]_{FS}[4]_{F}[4]_S$    | $-8 \ C_X$ | $[211]_{FS}[22]_{F}[31]_S$ | $\frac{20}{3} \ C_X$ |
| $[22]_{FS}[211]_{F}[31]_S$ | $-\frac{16}{3} \ C_X$ | $[211]_{FS}[211]_{F}[4]_S$ | $8 \ C_X$ |
| $[31]_F[31]_F[31]_S$      | $-\frac{16}{3} \ C_X$ | $[211]_{FS}[31]_{F}[22]_S$ | $8 \ C_X$ |
| $[22]_{FS}[22]_{F}[22]_S$ | $-4 \ C_X$ | $[22]_{FS}[4]_{F}[22]_S$ | $8 \ C_X$ |
|                            |            | $[211]_{FS}[31]_{F}[31]_S$ | $\frac{32}{3} \ C_X$ |
Table 4

Total angular momentum $J$ and parity $P$ of the $qqqq\bar{q}$ system for different spin symmetries $[f]_S$ of the four-quark subsystem. Below $S(qqqq)$ is the spin of the four-quark system, $S(qqqq\bar{q})$ the spin and $L$ the angular momentum of the $qqqq\bar{q}$ system.

| $[f]_{S(qqqq)}$ | $S(qqqq)$ | $S(qqqq\bar{q})$ | $J(L = 0)^P$ | $J(L = 1)^P$ |
|----------------|-----------|------------------|--------------|--------------|
| $[4]_S$ | 2 | $\frac{5}{2}$ | $\frac{5^-}{2}$ | $\frac{7^+}{2}, \frac{5^+}{2}, \frac{3^+}{2}$ |
| | | $\frac{3}{2}$ | $\frac{3^-}{2}$ | $\frac{5^+}{2}, \frac{3^+}{2}, \frac{1^+}{2}$ |
| $[31]_S$ | 1 | $\frac{3}{2}$ | $\frac{3^-}{2}$ | $\frac{5^+}{2}, \frac{3^+}{2}, \frac{1^+}{2}$ |
| | | $\frac{1}{2}$ | $\frac{1^-}{2}$ | $\frac{3^+}{2}, \frac{1^+}{2}$ |
| $[22]_S$ | 0 | $\frac{1}{2}$ | $\frac{1^-}{2}$ | $\frac{3^+}{2}, \frac{1^+}{2}$ |
The negative parity \((L = 0)\) \(qqqq\bar{q}\) states that have the quantum numbers of the nucleon and \(\Delta\) resonance states, which are predicted to have energies below 1900 MeV, and therefore are candidates for mixing with the corresponding \(qqq\) states. The corresponding empirically known negative parity resonances, which are likely to have much admixtures are also listed.

Table 5

| The negative parity \((L = 0)\) \(qqqq\bar{q}\) states that have the quantum numbers of the nucleon and \(\Delta\) resonance states, which are predicted to have energies below 1900 MeV, and therefore are candidates for mixing with the corresponding \(qqq\) states. The corresponding empirically known negative parity resonances, which are likely to have much admixtures are also listed. |
| \([4]_{X[111]}\) | Energy (MeV) | \(J^P\) | N (emp.) | \(\Delta\) (emp.) |
|-----------------|--------------|----------|----------|-----------------|
| \([31]_F S[22]_F [31]_S\) | 1529 | \(\frac{1}{2}^-\) | N(1535) | – |
| \([31]_F S[31]_F [22]_S\) | 1557 | \(\frac{1}{2}^-\) | N(1535) | \(\Delta(1620)\) |
| \([31]_F S[31]_F [31]_S\) | 1613 | \(\frac{3}{2}^-\) | N(1650) | \(\Delta(1620)\) |
| \([31]_F S[211]_F [22]_S (qqqs\bar{s})\) | 1629 | \(\frac{1}{2}^-\) | N(1650) | – |
| \([31]_F S[211]_F [31]_S (qqqs\bar{s})\) | 1685 | \(\frac{3}{2}^-\) | N(1670) | – |
| \([31]_F S[31]_F [4]_S\) | 1725 | \(\frac{3}{2}^-\) | N(1670) | \(\Delta(1700)\) |
| \([31]_F S[22]_F [31]_S (qqqs\bar{s})\) | 1769 | \(\frac{1}{2}^-\) | – | – |
| \([31]_F S[4]_F [31]_S\) | 1781 | \(\frac{3}{2}^-\) | – | \(\Delta(1900)(**)\) |
| \([31]_F S[31]_F [22]_S (qqqs\bar{s})\) | 1797 | \(\frac{3}{2}^-\) | – | \(\Delta(1700)\) |
| \([31]_F S[31]_F [31]_S (qqqs\bar{s})\) | 1853 | \(\frac{3}{2}^-\) | – | \(\Delta(1900)(**)\) |
Table 6

The positive parity \((L = 1)\) states that have the quantum numbers of the nucleon and the \(\Delta\) resonance states, which are predicted to have energies below 1900 MeV, and therefore are candidates for mixing with the corresponding \(qqq\) states. The corresponding empirically known positive parity resonances, which are likely to have much admixtures are also listed. The corresponding predicted \(qqq\) states in this energy range, which have not yet been identified empirically, are also shown, with a question mark. The symbol + indicates that there is a corresponding state with the antiquark in the p-state and the four quarks in a state with symmetry \([4]_X[1111]_{CFS}[211]_{C}[31]_{FS}[f]_F[f]_S\) at the indicated energy.
| $[31]_X [2111]_{CFS} [211]_C$ | Energy (MeV) | $J^P$ | N (emp.) | $\Delta$ (emp.) |
|----------------|--------------|--------|---------|-----------------|
| $[4]_{FS}[22]_F[22]_S$ | 1365 | $\frac{1^+}{2}$ | N(1440) | $-$ |
| | | $\frac{3^+}{2}$ | $-$ | $-$ |
| $[4]_{FS}[31]_F[31]_S$ | 1505 | $\frac{1^+}{2}$ | N(1440) | $-$ |
| | | $\frac{3^+}{2}$ | $-$ | $\Delta$(1600) |
| $[4]_{FS}[22]_F[22]_S \ (qqqs\bar{s})$ | 1605 | $\frac{1^+}{2}$ | N(1710) | $-$ |
| | | $\frac{3^+}{2}$ | N(1720) | $-$ |
| $[4]_{FS}[31]_F[31]_S \ (qqqs\bar{s})$ | 1745 | $\frac{1^+}{2}$ | N(1710) | $\Delta$(1750)(*), $\Delta$(1600) |
| | | $\frac{3^+}{2}$ | N(1720) | $\Delta$(?), $\Delta$(1905) |
| $[31]_{FS}[22]_F[31]_S$ | 1757 | $\frac{1^+}{2}$ | N(1710) | $-$ |
| | | $\frac{3^+}{2}$ | N(1720) | $-$ |
| | | $\frac{5^+}{2}$ | N(1680) | $-$ |
Table 6, continued

| Scenario | Energy (MeV) | $J^P$ | N (emp.)   | $\Delta$ (emp.) |
|----------|-------------|------|------------|-----------------|
| $[31]_F[31]_S$ | 1785        | $\frac{1}{2}^+$ | N(1710) | $\Delta(1750)$ (*) |
|           |             | $\frac{3}{2}^+$ | N(1720) |                 |
| $[4]_F[4]_S$  | 1785        | $\frac{1}{2}^+$ | $-$     | $\Delta(1750)$ (*) |
|           |             | $\frac{3}{2}^+$ | $-$     |                 |
|           |             | $\frac{5}{2}^+$ | $-$     | $\Delta(1905)$ |
|           |             | $\frac{7}{2}^+$ | $-$     |                 |
| $[31]_F[31]_S$ | 1841        | $\frac{1}{2}^+$ | N(?)    | $\Delta(1910)$ |
|           |             | $\frac{3}{2}^+$ | N(?)    | $\Delta(1920)$ |
|           |             | $\frac{5}{2}^+$ | N(?)    | $\Delta(1905)$ |
| $[31]_F[211]_S$ (qqq\bar{s}) | 1857        | $\frac{1}{2}^+$ | N(?)    | $-$             |
|           |             | $\frac{3}{2}^+$ | N(?)    | $-$             |
| $[22]_F[22]_S$ | 1869        | $\frac{1}{2}^+$ | N(?)    | $-$             |
|           |             | $\frac{3}{2}^+$ | N(?)    | $-$             |
Table 7

The negative parity ($L = 0$) $qqqq\bar{q}$ states that have the quantum numbers of the $\Lambda$ and $\Sigma$ hyperon resonance states, which are predicted to have energies up to 2000 MeV, and therefore are candidates for mixing with the corresponding $qqq$ states. The corresponding empirically known negative parity resonances, which are likely to have much admixtures are also listed. The corresponding predicted $qqq$ states in this energy range, which have not yet been identified empirically, are also shown, with a question mark.

| $[4]_{X}[111]_{CFS}[211]_{C}$ | Energy (MeV) | $J^P$ | $\Lambda$ (emp.) | $\Sigma$ (emp.) |
|-----------------------------|--------------|-------|-----------------|-----------------|
| $[31]_{FS}[211]_{F}[22]_{S}$ | 1509         | $\frac{1}{2}^-$ | $\Lambda(1405)$ | $\Sigma(1560)$(**) |
| $[31]_{FS}[211]_{F}[31]_{S}$ | 1565         | $\frac{3}{2}^-$ | $\Lambda(1520)$ | $\Sigma(1580)$(**) |
| $[31]_{FS}[22]_{F}[31]_{S}$ | 1649         | $\frac{1}{2}^-$ | $\Lambda(1670)$ | $\Sigma(1620)$(**) |
|                          |              | $\frac{3}{2}^-$ | $\Lambda(1690)$ | $\Sigma(1670)$ |
| $[31]_{FS}[31]_{F}[22]_{S}$ | 1677         | $\frac{1}{2}^-$ | $\Lambda(1670)$ | $\Sigma(1690)$(**) |
| $[31]_{FS}[31]_{F}[31]_{S}$ | 1733         | $\frac{3}{2}^-$ | $\Lambda(1690)$ | $\Sigma(1750)$ |
|                          |              | $\frac{3}{2}^-$ | $\Lambda(1690)$ | $\Sigma(1670)$, $\Sigma(?)$ |
| $[31]_{FS}[211]_{F}[22]_{S}$ (qqss\bar{s}) | 1749         | $\frac{1}{2}^-$ | $\Lambda(1800)$ | $-$ |
| $[31]_{FS}[211]_{F}[31]_{S}$ (qqss\bar{s}) | 1805         | $\frac{3}{2}^-$ | $\Lambda(1800)$ | $-$ |
| \([4]_X[1111]_{\text{CFS}}[211]_C\) | \([f]_{\text{FS}}[f]_F[f]_S\) | Energy (MeV) | \(J^P\) | \(\Lambda\) (emp.) | \(\Sigma\) (emp.) |
|---|---|---|---|---|---|
| \([31]_{\text{FS}}[31]_F[4]_S\) | 1845 | \(1/2^-\) | \(3/2^-\) | \(\Sigma(1940)\) | |
| \([31]_{\text{FS}}[22]_F[31]_S\) (qqss\bar{s}) | 1889 | \(1/2^-\) | \(3/2^-\) | \(\Lambda(1830)\) | \(\Sigma(1775)\) |
| \([31]_{\text{FS}}[4]_F[31]_S\) | 1901 | \(1/2^-\) | \(3/2^-\) | – | \(\Sigma(1940)\) |
| \([31]_{\text{FS}}[31]_F[22]_S\) (qqss\bar{s}) | 1917 | \(1/2^-\) | \(3/2^-\) | – | \(\Sigma(1940)\) |
| \([31]_{\text{FS}}[31]_F[31]_S\) (qqss\bar{s}) | 1973 | \(1/2^-\) | \(3/2^-\) | – | \(\Sigma(1940)\) |
The positive parity \((L = 1)\) states that have the quantum numbers of the \(\Lambda\) and the \(\Sigma\) hyperon resonance states, which are predicted to have energies up to 2000 MeV, and therefore are candidates for mixing with the corresponding \(qqq\) states. The corresponding empirically known positive parity resonances, which are likely to have much admixtures are also listed. The corresponding predicted \(qqq\) states in this energy range, which have not yet been identified empirically, are also shown, with a question mark. The symbol + indicates that there is a corresponding state with the antiquark in the p-state and the four quarks in a state with symmetry \([4]_X[1111]_{\text{CFS}}[211]_{\text{C}}[31]_{\text{FS}}[f]_{\text{P}}[f]_{\text{S}}\) at the indicated energy.

### Table 8

| Energy (MeV) | States | Description |
|-------------|--------|-------------|
| [31] \(X[211]CFS[211]C\) \([f]_{FS}[f]_{F}[f]_{S}\) | Energy (MeV) | \(J^P\) | \(\Lambda\) (emp.) | \(\Sigma\) (emp.) |
|----------------|--------------|--------|----------------|----------------|
| \([4]_{FS}[22]_{F}[22]_{S}\) | 1485 | \(1/2^+\) | \(\Lambda(1600)\) | \(\Sigma(1480)(*), \Sigma(1600)\) |
| \(3/2^+\) | \(\Sigma(1480)(*?)\) |
| \([4]_{FS}[31]_{F}[31]_{S}\) | 1625 | \(1/2^+\) | \(\Lambda(1600)\) | \(\Sigma(1660)\) |
| \(3/2^+\) | \(\Sigma(1480)(*?)\) |
| \([4]_{FS}[22]_{F}[22]_{S} (qqss\bar{s})\) | 1725 | \(1/2^+\) | – | \(\Sigma(1660), \Sigma(1770)(*\)) |
| \(3/2^+\) | – | \(\Sigma(?)\) |
| \([31]_{FS}[211]_{F}[22]_{S}\) | 1737 | \(1/2^+\) | \(\Lambda(1810)\) | \(\Sigma(1660), \Sigma(1770)(*\)) |
| \(3/2^+\) | \(\Lambda(1890)\) | \(\Sigma(?)\) |
| \([31]_{FS}[211]_{F}[31]_{S}\) | 1793 | \(1/2^+\) | \(\Lambda(1810)\) | \(\Sigma(1770)(**)\) |
| \(3/2^+\) | \(\Lambda(1890)\) | \(\Sigma(?) , \Sigma(1840)(*\)) |
| \(5/2^+\) | \(\Lambda(1820)\) |
| \([4]_{FS}[31]_{F}[31]_{S} (qqss\bar{s})\) | 1865 | \(1/2^+\) | \(\Lambda(1810)\) | \(\Sigma(1880)(**)\) |
| \(3/2^+\) | \(\Lambda(1890)\) | \(\Sigma(1840)(*\)) |
| \(5/2^+\) | \(\Lambda(1820)\) | \(\Sigma(?) , \Sigma(1915)\) |
| \([31]_{FS}[22]_{F}[31]_{S}\) | 1877 | \(1/2^+\) | \(\Lambda(1810)\) | \(\Sigma(1880)(**)\) |
| \(3/2^+\) | \(\Sigma(1840)(*), \Sigma(?)\) |
| \(5/2^+\) | \(\Sigma(1915)\) |
| $[31]_{X}[211]_{CFS}[211]_{C}$ | Energy (MeV) | $J^P$ | $\Lambda$ (emp.) | $\Sigma$ (emp.) |
|----------------|-------------|-------|-----------------|-----------------|
| $[31]_{FS}[31]_{F}[22]_{S}$ | 1905 | $\frac{1}{2}^+$ | $\Lambda(1810)$ | $\Sigma(1880)$($**$), $\Sigma(?)$ |
| | | $\frac{3}{2}^+$ | $\Lambda(?)$ | $\Sigma(?)$ |
| $[4]_{FS}[4]_{F}[4]_{S}$ | 1905 | $\frac{1}{2}^+$ | $\Sigma(1880)$($**$), $\Sigma(?)$ |
| | | $\frac{3}{2}^+$ | $\Sigma(?)$ |
| | | $\frac{5}{2}^+$ | $\Sigma(1915)$ |
| | | $\frac{7}{2}^+$ | $\Sigma(1915)$ |
| $[22]_{FS}[211]_{F}[31]_{S}$ | 1961 | $\frac{1}{2}^+$ | $\Lambda(?)$ | $\Sigma(?)$ |
| | | $\frac{3}{2}^+$ | $\Lambda(?)$ |
| | | $\frac{5}{2}^+$ | $\Lambda(?)$, $\Sigma(1915)$ |
| $[31]_{FS}[31]_{F}[31]_{S}$ | 1961 | $\frac{1}{2}^+$ | $\Lambda(?)$ | $\Sigma(?)$ |
| | | $\frac{3}{2}^+$ | $\Lambda(?)$ |
| | | $\frac{5}{2}^+$ | $\Lambda(?)$, $\Sigma(1915)$ |
| $[31]_{FS}[211]_{F}[22]_{S}$ ($qqss\bar{s}$) | 1977 | $\frac{1}{2}^+$ | $\Lambda(?)$ | $\Sigma(?)$ |
| | | $\frac{3}{2}^+$ | $\Lambda(?)$ | $\Sigma(?)$ |
| $[22]_{FS}[22]_{F}[22]_{S}$ | 1989 | $\frac{1}{2}^+$ | $\Lambda(?)$ | $\Sigma(?)$ |
| | | $\frac{3}{2}^+$ | $\Lambda(?)$ | $\Sigma(?)$ |
Table 9

The negative parity ($L = 0$) $qqqq$ states that have the quantum numbers of the $\Xi$ hyperon resonance states, which are predicted to have energies up to 2100 MeV, and therefore are candidates for mixing with the corresponding $qqq$ states. The corresponding predicted $qqq$ states in this energy range, which have not yet been identified empirically, are also shown, with a question mark.
| $[4]_X[1111]_{CFS}[211]_C \ [f]_{FS}[f]_F[f]_S$ | Energy (MeV) | $J^P$ | $\Xi$ (emp.) |
|---|---|---|---|
| $[31]_{FS}[211]_F[22]_S$ | 1629 | $\frac{1}{2}^-$ | $\Xi(1690)$? |
| $[31]_{FS}[211]_F[31]_S$ | 1685 | $\frac{1}{2}^+$ | $\Xi(1690)$? |
| $[31]_{FS}[22]_F[31]_S$ | 1769 | $\frac{1}{2}^-$ | $\Xi(?)$ |
| $[31]_{FS}[31]_F[22]_S$ | 1797 | $\frac{1}{2}^-$ | $\Xi(?)$, $\Xi(1820)$ |
| $[31]_{FS}[31]_F[31]_S$ | 1853 | $\frac{3}{2}^-$ | $\Xi(1820)$, $\Xi(?)$ |
| $[31]_{FS}[31]_F[4]_S$ | 1965 | $\frac{3}{2}^-$ | $\Xi(1950)$? |
| $[31]_{FS}[4]_F[31]_S$ | 2021 | $\frac{1}{2}^-$ | $\Xi(1950)$? |
| $[31]_{FS}[31]_F[22]_S \ (qsss\bar{s})$ | 2037 | $\frac{1}{2}^-$ | |
| $[31]_{FS}[31]_F[31]_S \ (qss\bar{s})$ | 2093 | $\frac{1}{2}^-$ | |
Table 10

The positive parity ($L = 1$) states that have the quantum numbers of the $\Xi$ hyperon resonance states, which are predicted to have energies up to 2100 MeV, and therefore are candidates for mixing with the corresponding $qqq$ states. The corresponding predicted $qqq$ states in this energy range, which have not yet been identified empirically, are also shown, with a question mark. The symbol + indicates that there is a corresponding state with the antiquark in the p-state and the four quarks in a state with symmetry $[4]_X[1111]_{CFS}[211]_C[31]_{FS}[f]_F[f]_S$ at the indicated energy.
| $[31]_X [211]_{CFS} [211]_C$ | Energy (MeV) | $J^P$ | $\Xi$ (emp.) |
|--------------------------------|--------------|-------|-------------|
| $[4]_F S [22]_F [22]_S$       | 1605         | $^{1+}_2$ | $\Xi(1620)(*)?$ |
|                               |              | $^{3+}_2$ | $\Xi(1620)(*)?$ |
| $[4]_F S [31]_F [31]_S$       | 1745         | $^{1+}_2$ | $\Xi(?)$     |
|                               |              | $^{3+}_2$ |             |
|                               |              | $^{5+}_2$ |             |
| $[31]_F S [211]_F [22]_S$     | 1857         | $^{1+}_2$ | $\Xi(?)$     |
|                               |              | $^{3+}_2$ |             |
| $[31]_F S [211]_F [31]_S$     | 1913         | $^{1+}_2$ | $\Xi(?)$     |
|                               |              | $^{3+}_2$ |             |
|                               |              | $^{5+}_2$ |             |
| $[4]_F S [31]_F [31]_S$ (qsss) | 1985         | $^{1+}_2$ | $\Xi(?)$     |
|                               |              | $^{3+}_2$ |             |
|                               |              | $^{5+}_2$ |             |
| Energy (MeV) | $J^P$ | $\Xi$ (emp.) |
|---------------|-------|---------------|
| [31]$_{FS}[22]_F[31]_S$ | 1997 | $1^+$ $\frac{1}{2}^+$ | $\Xi(?)$ |
| [31]$_{FS}[31]_F[22]_S$ | 2025 | $1^+$ $\frac{3}{2}^+$ | $\Xi(?)$ |
| [4]$_{FS}[4]_F[4]_S$ | 2025 | $1^+$ $\frac{5}{2}^+$ | $\Xi(2030)$? |
| [22]$_{FS}[211]_F[31]_S$ | 2081 | $1^+$ $\frac{7}{2}^+$ | $\Xi(2030)$?, $\Xi(?)$ |
| [31]$_{FS}[31]_F[31]_S$ | 2081 | $1^+$ $\frac{9}{2}^+$ | $\Xi(?)$ |
The negative parity ($L = 0$) $qqq\bar{q}$ states that have the quantum numbers of the $\Omega^-$ hyperon resonance states, which are predicted to have energies up to 2400 MeV, and therefore are candidates for mixing with the corresponding $qqq$ states. The corresponding predicted $qqq$ states in this energy range, which have not yet been identified empirically, are also shown, with a question mark.

| $[4]_X[1111]_{CFS}[211]_C$ | Energy (MeV) | $J^P$ | $\Omega^-$ (emp.) |
|-----------------|-------------|-------|-------------------|
| $[31]_{FS}[31]_F[22]_S$ | 1917         | $\frac{1}{2}^-$ |                 |
| $[31]_{FS}[31]_F[31]_S$ | 1973         | $\frac{1}{2}^-$ | $\Omega^-(?)$   |
| $\Omega^-(?)$     | 2085         | $\frac{3}{2}^-$ | $\Omega^-(?)$   |
| $[31]_{FS}[31]_F[4]_S$ | 2085         | $\frac{3}{2}^-$ |                   |
| $[31]_{FS}[4]_F[31]_S$ | 2141         | $\frac{1}{2}^-$ |                   |
| $[31]_{FS}[4]_F[31]_S$ | 2381         | $\frac{1}{2}^-$ | $\Omega^-(2380)(**)$? |
| $[31]_{FS}[4]_F[31]_S$ | 2381         | $\frac{3}{2}^-$ | $\Omega^-(2380)(**)$? |
The positive parity \((L = 1)\) states that have the quantum numbers of the \(\Omega^-\) hyperon resonance states, which are predicted to have energies up to 2400 MeV, and therefore are candidates for mixing with the corresponding \(qqq\) states. The corresponding predicted \(qqq\) states in this energy range, which have not yet been identified empirically, are also shown, with a question mark. The symbol + indicates that there is a corresponding state with the antiquark in the p-state and the four quarks in a state with symmetry \(\{4\}_X[1111]_{CFS}[211]_C[31]_{FS}[f]_F[f]_S\) at the indicated energy.
| $[31]_X[211]_{CFS}[211]_C$ | Energy (MeV) | $J^P$ | $\Omega^-$ (emp.) |
|--------------------------|--------------|------|------------------|
| $[4]_{FS}[31]_F[31]_S$  | 1865         | $\frac{1}{2}^+$ |                  |
|                          |              | $3^+$     |                  |
|                          |              | $5^+$     |                  |
| $[31]_{FS}[31]_F[22]_S$ | 2145         | $\frac{1}{2}^+$ |                  |
|                          |              | $\frac{3}{2}^+$ |                  |
| $[4]_{FS}[4]_F[4]_S$    | 2145         | $\frac{1}{2}^+$ | $\Omega^-$ (?)   |
|                          |              | $\frac{3}{2}^+$ | $\Omega^-$ (?)   |
|                          |              | $\frac{5}{2}^+$ | $\Omega^-$ (?)   |
| $[31]_{FS}[31]_F[31]_S$ | 2201         | $\frac{1}{2}^+$ | $\Omega^-$ (2250)? |
|                          |              | $\frac{3}{2}^+$ | $\Omega^-$ (2250)? |
|                          |              | $\frac{5}{2}^+$ | $\Omega^-$ (2250)? |
| \([31]_X{[211]}_{CF}{[211]}_C\) \([f]_{FS}[f]_F[f]_S\) | Energy (MeV) | \(J^p\) | \(\Omega^-\) (emp.) |
|---|---|---|---|
| \([31]_FS[31]_F[4]_S\) | 2313 | \(1^+\) |  |
| | + | \(\frac{3}{2}^+\) |  |
| | | \(\frac{5}{2}^+\) |  |
| \([22]_FS[31]_F[31]_S\) | 2369 | \(1^+\) |  |
| | + | \(\frac{3}{2}^+\) |  |
| | | \(\frac{5}{2}^+\) |  |
| \([31]_FS[4]_F[31]_S\) | 2369 | \(1^+\) |  |
| | + | \(\frac{3}{2}^+\) |  |
| | | \(\frac{5}{2}^+\) |  |
| \([4]_FS[4]_F[4]_S\) \((ssss)\) | 2385 | \(1^+\) | \(\Omega^-\) (2380)(**)? |
| | | \(\frac{3}{2}^+\) | \(\Omega^-\) (2380)(**)? |
| | | \(\frac{5}{2}^+\) | \(\Omega^-\) (2380)(**)? |
| | | \(\frac{7}{2}^+\) | \(\Omega^-\) (2380)(**)? |
Fig. 1
Fig 10
Fig. 2
Fig. 5
Fig. 6
Fig. 7
Fig. 9
Fig. 3
Fig. 4