About the equilibrium speed of sound in a liquid with gas-vapor bubbles

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Abstract. The general expression of an equilibrium velocity of a sound in vapor-gas-liquid mixtures is presented and influence of concentration of vapor and a volume content of bubbles on the received expression is analyzed. In special cases, for gas-liquid and vapor-liquid mixtures expressions of an equilibrium velocity are presented and the satisfactory consent of the received expressions with known experimental data is discovered.

1. Introduction
It is known that the presence of vapor or gas bubbles in the liquid significantly changes its acoustic properties. There is a considerable amount of works on this problem. Propagation of small disturbances in a liquid with bubbles of vapor or gas in a particular formulation is theoretically investigated in works [1] - [5], with the gas-vapor bubbles – in works [6] - [9]. Some of them can be found in [10], [11].

The results of experiments on the study of the propagation of low-frequency pressure waves in the vapor-liquid flow moving through the close-packed layer of solid spherical particles are presented in [12], [13]. The experimental results allowed to determine the characteristic parameters and conditions under which the speed of propagation of the pressure waves coincides with the thermodynamic equilibrium speed of sound in the liquid-vapor mixture. Thus, it has been found experimentally that the rate of low-frequency disturbances in a vapor-water environment may be a few meters per second, and takes a value close to the sound velocity of Landau. When the carrier phase is a gas or vapor, and the disperse - droplets or particles expressions of equilibrium sound of velocity are obtained in [14], [15].

In this paper we analyzed the expression of the equilibrium speed received by us in [9].

2. The analytical expression.
The expression of the equilibrium speed of sound \(C_e\) for the vapor-gas-liquid mixture, resulting in [9], is written as follows

\[
C_e^2 = \frac{p_0}{\alpha_0 \rho_0^2} \frac{\xi_1}{\xi_1 + \xi_2 - \xi_3}
\]

\[
\xi_1 = k_G \alpha_1 \alpha_2 (E T_0 + G R_1), \xi_2 = (1 - G) \frac{m l_0 \alpha_2}{c_{pl}}
\]
\[ \xi_3 = \frac{k_{G0} \alpha_{20} p_0 m}{\rho_{20}^0 c_{p1}} \left( \frac{R_1}{T_0} + E \right), \quad \xi_4 = \frac{m l_0}{c_{p1}} + m_1 k_{G0} E T_0 \]

\[ E = \frac{R V_0 R_{G0}}{R^2_{20}}, \quad G = k_{V0} \frac{R V_0}{R_{20}}, \quad R_{20} = k_{V0} R_{V0} + k_{G0} R_{G0} \]

\[ m_1 = 1 + m_2 \frac{c_{p2}}{c_{p1}} R_1 = \left( \Delta R + \frac{1 - m^0}{k_{G0}} \right) T_0, \quad m = \frac{\alpha_{20}}{\alpha_{10}} m^o, \quad \rho_2 = \frac{\rho_{20}^0}{\rho_{10}^o} \]

\[ \Delta R = \frac{R V_0 - R_{G0}}{R_{20}}, \quad \rho_2 = k_{V0} \rho_{V0}^0 + k_{G0} \rho_{G0}^0 \]

Here \( \rho_{V0}^0, \rho_{G0}^0 \) – the density of vapor and gas, \( R_{V0}, R_{G0} \) – vapor and gas constants, \( k_{V0}, k_{G0} \) – mass concentration of vapor and gas, the index 1 refers to the carrier phase, index 2 – to disperse, \( \alpha \) – volume content, \( T_0 \) – temperature, \( p_0 \) – pressure, \( c_p \) – specific heat, \( l_0 \) – latent heat of vaporization.

Expression of equilibrium speed of sound \( C_e \) for the vapor-liquid mixture is obtained from the formula (1) in the limit \( k_{V0} \to 1 \), and is written as follows:

\[ C_e^2 = \frac{p_0 \rho_0 m \rho_{V0}^0}{(m p_0 - c_{p1} m_1 T_0 \rho_{V0}^0) (m^o - 1) \alpha_{20} \alpha_{10} \rho_{V0}^0} \]  \( (2) \)

\[ m_1 = 1 + m_2 \frac{c_{p2}}{c_{p1}}, \quad m = \frac{\alpha_{20}}{\alpha_{10}} m^o, \quad m^o = \frac{\rho_{V0}^0}{\rho_{10}^o} \alpha_{10} + \alpha_{20} = 1 \]

In the case where the liquid contains only the gas bubbles \( k_{V0} \to 0 \), the equilibrium sound velocity becomes by Gubaidullin and Fedorov [5], Shagapov and Sarapulova [16]

\[ C_e^2 = \frac{p_0}{\alpha_{10} \alpha_{20} \rho_{10}^o} \]  \( (3) \)

3. The results of calculations.

**Figure 1.** The dependences of the equilibrium speed of sound on the volume of gas or vapor concentration at different values of the initial concentration of vapor.

**Figure 2.** Dependences of the equilibrium speed of sound on the initial concentration of vapor at various volume content of bubbles.

Fig. 1 shows the dependences of the equilibrium speed of sound on the volume content for the mixture of water with vapor-air bubbles at different values of the initial concentration of vapor.
water vapor. Pressure of mixture \( p_0 = 0.1 \) MPa. The bottom curve \( (k_V = 1) \) is built according to the formula (2) and corresponds to a vapor bubble, the upper curve \( (k_V = 0) \) according to the formula (3) and corresponds to the air bubbles, the other curves by the formula (1) and correspond to vapor-air bubbles.

It is evident that a small amount of gas in the gas-vapor bubbles affects the character of the curve of the equilibrium speed. If for the gas or gas-vapor bubbles curves of equilibrium speed are non-monotonic, then for pure vapor bubbles we observe a monotonically increasing curve.

The following Fig. 2 shows the results of a comparison of the dependency of equilibrium sound speed on the initial concentration of vapor in the bubbles at various volumetric content. The calculations were performed using the formula (1). For the construction of the theoretical curves, the following approximate dependences of thermodynamic parameters on the concentration of vapor were used:

\[
T_0 = -68.75k_V^2 + 123.75k_V + 315.3125, \quad l_0 = 10^6(0.875k_V^2 - 1.2975k_V + 2.721)
\]

\[
cp_V = 1000(-0.1281k_V^2 + 0.2719k_V + 1.8861), \quad cp_G = 1010
\]

\[
cp_l = 1000(-0.0219k_V^2 + 0.0606k_V + 4.1772), \quad cp_2 = k_Vcp_V + k_Gcp_G
\]

\[
\rho_1^2 = 37.5k_V^2 - 70k_V + 992.625, \quad \rho_3^2 = 0.0781k_V^2 - 0.4469k_V + 0.9439
\]

\[
\rho_4^2 = -0.2969k_V^2 + 0.8531k_V + 0.0177, \quad \rho_5^2 = k_V\rho_V + k_G\rho_G, \quad R_V = 460, \quad R_G = 287
\]

In particular, we see that the volume content of bubbles \( \alpha_{20} = 0.01 \) and vapor concentration \( k_V = 0 \) (air bubbles in the water), the equilibrium speed of sound is about 100 m/s. This fact is also theoretically shown, for example, by Commander and Prosperetti [1], Temkin [17] and experimentally by Silberman [18], Karplus [19]. Further, with an increase of the initial concentration of vapor in bubbles at different volumetric contents equilibrium speed decreases and, if the vapor concentration is equal 1, reaches practically the same minimum value.

4. Comparison with experiment.

![Figure 3](image-url) Comparison of the dependence of equilibrium speed of sound on the volume gas or vapor content with the experimental data.

![Figure 4](image-url) Comparison of the dependences of the equilibrium speed of sound on the volume vapor content with the experimental data.

Fig. 3 shows a comparison of dependences of equilibrium speed of sound on the volume gas or vapor content with the experimental data: 1, 2 – [12], 3 – [19]. The curve 4 is
constructed for an air-water mixture under the pressure \( p_0 = 0.1 \) MPa by formula (3), the curve 5 – for vapor-water mixture under the pressure \( p_0 = 0.2 \) MPa by the formula (2). The characteristic frequency of the input disturbance in the experiment was 1.5 - 2.5 Hz. We see that the curve of the equilibrium velocity for the gas-liquid mixture differs significantly from the curve for the vapor-liquid mixture, what is confirmed by experimental data. The authors of the experiment [12] found that when the volume content of water vapor increases velocity of the low-frequency disturbances almost doesn’t differ and tends to the speed of sound Landau. The shown theoretical curve 5 is increasing, but, in general, agrees satisfactorily with the experimental data.

The following Fig. 4 shows a comparison of the dependence of the equilibrium speed of sound on the volume vapor content with experimental data [20]. The curve 1 is built for R404A refrigerant at saturation temperature of 293 K, the curve 2 – for R134A refrigerant at saturation temperature 303 K. The calculations were performed using the formula (2). The characteristic frequency of the input disturbance in the experiment was 0.2 – 5 Hz. For these mixtures taking into account phase transformations in the field of moderate vapor content the speed of sound also takes quite low values, and that is about 5 m/s at a volume content \( \alpha_2 = 0.4 \). With increasing of volume vapor content equilibrium speed increases and with approximation of the volume content to unit tends to the speed of sound in a clean vapor. The theoretical curves are in good agreement with the experimental data for the vapor content to 85-90%. At high vapor content experimental values of the equilibrium velocity accept higher values compared with the theoretical.

5. Conclusions

1. When the concentration of vapor in bubbles increases the equilibrium speed of sound decreases.

2. The presence of gas in the bubbles leads to a nonmonotonic dependence of the equilibrium speed of sound on a volume content.

3. Satisfactory agreement of obtained expression with the experimental data was disclosed.

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References

[1] Commander K W and Prosperetti A 1989 J. Acoust. Soc. Am. 85 732
[2] Ardron K H and Duffey R B 1978 Int. J. Multiphase Flow 4 303
[3] Nigmatulin R I, Gubaidullin D A and Fedorov Yu V 2013 Doklady Physics 58 261
[4] Gubaidullin D A, Gubaidullina D D and Fedorov Yu V 2013 Fluid Dynamics 48 781
[5] Gubaidullin D A and Fedorov Yu V 2013 Journal of Applied Mathematics and Mechanics 77 532
[6] Gubaidullin D A and Nikiforov A A 2010 High Temperature 48 170
[7] Gubaidullin D A, Nikiforov A A and Gafiyatov R N 2012 High Temperature 50 250
[8] Gubaidullin D A, Nikiforov A A and Gafiyatov R N 2015 High Temperature 53 240
[9] Gubaidullin D A and Fedorov Yu V 2015 Fluid Dynamics 50 61
[10] Varaksin A Yu 2013 High Temperature 51 377
[11] Varaksin A Yu 2015 High Temperature 53 423
[12] Pokusaev B G, Tairov E A and Vasilyev S A 2010 Acoustical Physics 56 306
[13] Pokusaev B G, Tairov E A, Safarov A S and Nekrasov D A 2014 Thermal Science 18 591
[14] Gubaidullin D A, Teregulova E A and Gubaidullina D D 2015 High Temperature 53 714
[15] Gubaidullin D A and Fedorov Yu V 2012 Fluid Dynamics 47 593
[16] Shagapov V Sh and Sarapulova V V 2015 Acoustical Physics 61 37
[17] Temkin S 1992 Phys. Fluids A 4 2399
[18] Silberman E 1957 J. Acoust. Soc. Am. 29 925
[19] Karpus H B 1961 Research and Development/Report ARF-4132-12. Atomic Energy Commission.
[20] Kuczynski W 2013 Int. J. Heat and Fluid Flow 40 135