Sensitivity for tau neutrinos at PeV energies and beyond with the MAGIC telescopes

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The MAGIC telescopes, located at the Roque de los Muchachos Observatory (2200 a.s.l.) in the Canary Island of La Palma, are placed on the top of a mountain, from where a window of visibility of about 5 deg in zenith and 80 deg in azimuth is open in the direction of the surrounding ocean. This permits to search for a signature of particle showers induced by earth-skimming cosmic tau neutrinos in the PeV to EeV energy range arising from the ocean. We have studied the response of MAGIC to such events, employing Monte Carlo simulations of upward-going tau neutrino showers. The analysis of the shower images shows that air showers induced by tau neutrinos can be discriminated from the hadronic background coming from a similar direction. We have calculated the point source acceptance and the expected event rates, assuming an incoming tau neutrino flux consistent with IceCube measurements, and for a sample of generic neutrino fluxes from photo-hadronic interactions in AGNs. The analysis of about 30 hours of data taken toward the sea leads to a point source sensitivity for tau neutrinos at the level of the down-going point source analysis of the Pierre Auger Observatory.

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1. Introduction

A conventional approach for the detection of neutrinos with energies in the PeV range is based on detectors which use large volumes of ice (IceCube) or water (ANTARES). They sample Cherenkov light from muons produced by muon neutrinos, or from electron and tau lepton induced showers initiated by the charged current interactions of electron and tau neutrinos. An alternative technique is based on the observation of upward going extensive air showers produced by the leptons originating from neutrino interactions below the surface of the Earth, the so-called earth-skimming method [1, 2]. Neutrino induced showers could be detected using a variety of methods: surface particle detector arrays and air fluorescence telescopes, like in the Pierre Auger Observatory and the Telescope Array1, or radio detectors like ANITA2. Another possibility is to use the technique of Imaging Atmospheric-Cherenkov Telescopes (IACTs) which is widely used in contemporary gamma-ray astronomy. An IACT system capable to detect the neutrino-induced signature should look to the ground, e.g. the side of a mountain or the sea surface [1, 3, 4].

In this paper we study the possibility to use the MAGIC (Major Atmospheric Gamma Imaging Cherenkov) telescopes to search for air showers induced by tau neutrinos (τ-induced showers) in the PeV-EeV energy range. MAGIC is a system of two IACTs located at the Roque de los Muchachos Observatory (28.8° N, 17.9° W), in the Canary Island of La Palma (Spain). They are placed 85 m apart, each with a primary mirror of 17 m diameter. The MAGIC telescopes, with a field of view (FOV) of 3.5°, are able to detect cosmic γ-rays in the energy range 50 GeV - 50 TeV [5].

In order to use MAGIC for tau neutrino searches, the telescopes need to be pointed in the direction of the tau neutrinos escaping first from the Earth crust and then from the ocean, i.e. at the horizon or a few degrees below. More precisely, at a zenith angle of 91.5° the surface of the sea is 165 km away, thus the telescopes can monitor a large volume in their FOV resulting in a space angle area (defined as the intersection of the telescopes FOV and the sea surface) of about a few km². In [6], the effective area for up-going tau neutrino observations with the MAGIC telescopes was calculated analytically and found to reach ∼ 10³ m² (at 100 TeV) and 10⁵ m² (at 1 EeV) for an observation angle of about 1.5° below the horizon, rapidly diminishing with higher inclination. However, the sensitivity for diffuse neutrinos was found to be very poor because of the limited FOV, the observation time, and the low expected neutrino flux.

On the other hand, if flaring or disrupting point sources such as gamma ray bursts (GRBs) or active galactic nuclei (AGNs) are being pointed at, one can expect neutrino fluxes producing an observable number of events [3] and [6]. Also, for sites with different orographic conditions, the acceptance for upward-going tau neutrinos is increased by the presence of mountains, which shield against cosmic rays and star light and serve as target for neutrino interaction leading to an enhancement in the flux of emerging tau leptons.

From the observational point of view, it is worth noting that the time that can be dedicated to this kind of observations almost does not interfere with regular MAGIC gamma-ray observations of the sky, because the sea can be pointed even in the presence of optically thick clouds above the MAGIC site (high clouds). In fact, high-altitude clouds prevent the observation of gamma-ray

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1https://www.auger.org/ and http://www.telescopearray.org/
2http://www.phys.hawaii.edu/~anita/new/html/science.html
Figure 1: Example of simulated shower images for 1 PeV protons (upper panels) injected at the top of the atmosphere (detector-to-proton distance of about 800 km) and 1 PeV tau lepton (lower panels) decaying close to the detector (about 50 kms) for zenith angle of $\theta \approx 86^\circ$, as seen by one of the MAGIC telescopes. (A) muon boundles producing a Cherenkov part of a ring and gamma-electron showers induced by the secondary muon decay in flight; (B) a ring from a high energetic muon; (C) muons interacting via radiative processes. For tau lepton images are shown for different tau decay channels: $\tau^- \rightarrow e^- \nu_e \nu_\tau$ (D), $\tau^- \rightarrow \pi^- \pi^- \pi^- \pi^0 \nu_\tau$ (E), and $\tau^- \rightarrow \pi^- \nu_\tau$ (F).

Table 1: Summary of data taken at very large-zenith angles by the MAGIC telescopes. * for this zenith angle, we expected the maximal acceptance for tau neutrinos [6].

|                | seaOFF | seaON | Roque |
|----------------|--------|-------|-------|
| Zenith angle $\theta$ (°) | 87.5   | 92.5* | 89.5  |
| Azimuth $\phi$ (°)       | -30    | -30   | 170   |
| Observation time (hrs)  | 9.2    | 31.5  | 7.5   |

2. MAGIC observations and Monte Carlo simulations

Recently, the MAGIC telescopes has taken data at very large zenith angles ($85^\circ < \theta < 95^\circ$) in the direction of the sea (seeON), just above the sea (seeOFF) and towards the Roque de los Muchachos mountain, as listed in Table 1. 91% of the data were taken during nights characterized by optically thick high cumulus clouds, when normal $\gamma$-ray observations are usually worthless. Thus, Table 1 also demonstrates, that a large amount of data ($\sim 39$ hrs) can be accumulated during such conditions, enhancing the overall duty cycle of MAGIC. In order to study the signatures expected from neutrino-induced showers by MAGIC, a full Monte Carlo (MC) simulation chain was set up, which consists of three steps. First, the interaction of a given neutrino flux with the Earth and propagation of the resulting charged lepton through the Earth and the atmosphere is simulated using an extended version [8] of the ANIS code [9]. Second, the shower development of $\tau$-induced showers and its Cherenkov light production is simulated with CORSIKA [10].

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Footnotes:

1. For this zenith angle, we expected the maximal acceptance for tau neutrinos [6].
2. MAGIC site there are up to about 100 hours per year where high clouds are present [7].
3. CORSIKA (version 6.99) was compiled with the CERENKOV, CURVED-EARTH, TAULEP, and THIN options.
Search for tau neutrinos at PeV energies and beyond with the MAGIC telescopes

J. Palacio

Figure 2: (A),(C): normalized distribution of Hillas parameters for deep $\tau$-induced showers with zenith angle $\theta = 87^\circ$ and impact distances less than 0.3 km (signal) and data taken toward the seaOFF direction (about $2.2 \cdot 10^5$ background events). (B): scatter plot of the Hillas *Length* parameter as a function of the Hillas *Size* parameter. Dashed line indicates the selection cut used in this work to calculate the identification efficiency for deep $\tau$-induced showers. Note that the region below the discrimination cut is characterized by the absence of background events. (C): The one-dimensional distribution of seaON, seaOFF and signal MC obtained in the direction perpendicular to the selection cut. The new coordinate for data were obtained from the following formula: $\log_{10}(Y') = \log_{10}(\text{Size}[\text{p.e.}]) \cdot \cos(\alpha) - \log_{10}(\text{Length}[\text{deg}]) \cdot \sin(\alpha)$, where $\alpha = 63.435^\circ$. Note that above the selection cut ($\log_{10}(Y') > 2.35$) zero neutrino candidates are found. For showers with the larger impact distances (0.3-1.3 km) a slightly relaxed cut was used: $\log_{10}(Y') > 2.10$.

results of the CORSIKA simulation are used as inputs for the last step, i.e. the simulation of atmospheric extinction and the MAGIC detector response [11]. It has to be said that, we could not simulate showers with zenith angle $\theta > 90^\circ$ when combining CORSIKA and MAGIC simulations due to unsolved technical subtleties with the MAGIC simulations. Therefore, here we have used simulations for zenith angles in the range between $86^\circ$ and $90^\circ$ to study properties of upward-going tau neutrino showers. This is a reasonable assumption, because the response of IACTs to Cherenkov light from showers of a same energy and equal column depth only slightly depend on the zenith angle, as has been already demonstrated in [4].

In order to compare simulated images on the MAGIC camera plane with those obtained from data, we also simulated inclined showers induced by protons. Proton showers can mimic the Cosmic Rays (CRs) background for Cherenkov telescopes. In case of showers induced at the top of the atmosphere and observed at large zenith angles, the hadronic and electromagnetic component of extensive air showers (EAS) are almost absorbed because of the deep horizontal column depth while the tau decay is simulated with PYTHIA.
ranging from \( \approx 10^4 \) to \( 5 \times 10^4 \) g cm\(^{-2} \) at such directions. Thus, for proton induced showers with lower energies (tens of GeVs) only a few pixels will be triggered by the camera yielding dimmer and smaller images. Only for higher energy CRs (above \( 1 \times 10^{15} \) eV), high energetic muons (tens to hundreds of GeV) from the first stages of the shower development in the atmosphere or muon bundles from later stages can reach the detector and produce larger shower images, see Figure 1 (A). On the contrary, for high energetic muons the shower image in the camera will mostly contain a single muon ring, if the muon hits the telescope mirror, or an incomplete ring (arc), see Figure 1 (B). At larger zenith angles, for highest energetic muons (\( > 1 \) TeVs) the interaction length due to \( e^+e^- \)-pair production, bremsstrahlung and photonuclear scattering is comparable to the depth of the atmosphere, thus we also expect a few shower events per night, coming from interacting muons via radiative processes \([12, 13]\). As our simulation shows, if one of these sub-showers from radiative interactions are induced close to a detector, this can lead to a strong flash of Cherenkov light and a bright image on the camera (see Figure 1 (C) as an example). All classes of simulated events discussed above were also identified in the MAGIC data taken at very large zenith angles.

In case of tau leptons, the expected signature on the camera depends on the tau decay channel. The tau lepton decays mostly into electrons, muons or charged and neutral pions \([14]\). As an example, in Figure 1 (D)-(F) we show simulated shower images for the 1 PeV tau lepton decaying into an electron, pion (or several pions) close to the detector. In general, for such a geometry, the shower images on the camera have a much larger size and contain many more photons compared to the proton images. The tau lepton can also decay into muons. However, at high energies (\( > 1 \) PeV) the muon has a large interaction length, more than a few thousand kilometers in air, thus it mostly interacts with the atmosphere through secondary bremsstrahlung processes, which blur the muon image and make the ring only poorly visible.

### 3. Discrimination of \( \tau \)-induced showers

Each simulated event recorded and calibrated consists of a number of photoelectrons (p.e.) per camera pixel. The cleaned camera image is characterized by a set of image parameters introduced by M. Hillas in \([15]\). These parameters provide a geometrical description of the images of showers and are used to infer the energy of the primary particle, its arrival direction, and to distinguish between \( \gamma \)-ray and hadron induced showers. In the following we study these parameters also in the case of deep \( \tau \)-induced simulated showers and compare the corresponding distributions with the data.

As previously mentioned the MAGIC telescopes took data also at zenith angle 87.5\(^\circ\) (seeOFF). For this zenith angle we expect a negligible signal (neutrino events) contribution compared to the sea. Thus seeOFF data were used to estimate the background and to construct the selection criterion to indentify tau-neutrino showers. In addition, the rate of stereo seeOFF events is about 27 times larger (\( \sim 4.6 \) Hz) than for seaON (\( \sim 0.17 \) Hz) observations. Thus these observations provide high-statistics background estimates for about 30 hours of seaON data.

In Figure 2 (A), (C) the distribution of the Hillas parameter Size and Length for deep \( \tau \)-induced simulated showers are shown, in comparison to seeOFF data. In general, these parameters depend on the geometrical distance of the shower maximum to the detector, which for deep \( \tau \)-induced showers is much smaller (a few tens of kilometers) than for showers from CRs interacting at the
top of the atmosphere (a few hundred kilometers). This geometrical effect leads to a very good separation of close (\(\tau\)-induced) and far-away (data) events in the Hillas parameter phase space.

Figure 2 (B) shows the scatter plot of the \textit{Length} parameter as a function of the \textit{Size} parameter for our MC signal simulations and seeOFF data. We can see that with these two parameters only, we can easily identify a region with no background events. This plot shows that MAGIC can discriminate deep \(\tau\)-induced showers from the background of high zenith angles hadronic showers. In Figure 2 (D), we show the one-dimensional distribution of seeOFF, seeON and MC signal simulation projected onto the line perpendicular to our selection cut. As it is seen we did not find any neutrino candidate, if the selection cut is applied to all seeON data.

It is also worth noting that the shape of the Hillas distributions for \textit{Size} and \textit{Length} agree reasonably well with our MC background simulation, i.e. with simulated proton induced showers. In addition the shape of these distributions for seeOFF data and seeON data is very similar, indicating a universal behaviour of Hillas distributions at these zenith angles. This universality also confirms our previous assumption when performing MC simulations only at large zenith angles, in order to study the response of MAGIC to upward-going \(\tau\)-induced showers.

4. Aperture and event rate calculations

The total observable event rates (number of expected events) were calculated as \(N = \Delta T \times \int E_{\text{th}}^\text{Emax} A^{\text{PS}}(E_{\nu\tau}) \times \Phi(E_{\nu\tau}) \times dE_{\nu\tau}\) where \(\Delta T\) is the observation time, and \(A^{\text{PS}}(E_{\nu\tau})\) the point source acceptance and \(\Phi(E_{\nu\tau})\) the expected neutrino flux. The detector acceptance for an initial neutrino energy \(E_{\nu\tau}\) is calculated as: \(A^{\text{PS}}(E_{\nu\tau}, \theta, \phi) = N^{-1}_{\text{gen}} \times \sum_{i=1}^{N_{\text{FOV cut}}} P(E_{\nu\tau}, E_\tau, \theta) \times A_i(\theta) \times T_{\text{eff},i}(E_{\tau}, r, d, \theta)\) where \(\theta, \phi\) are the simulated zenith and azimuth pointing angles of the MAGIC telescope, \(N_{\text{gen}}\) is the number of generated neutrino events, \(N_{\text{FOV cut}}\) is the number of \(\tau\) leptons with energies \(E_\tau\) larger than the threshold energy \(E_{\text{th}} = 1\) PeV and with estimated position of the shower maximum in the FOV of the MAGIC telescope. In addition the impact distance of showers should be smaller than 1.3 km. \(P(E_{\nu\tau}, E_\tau, \theta)\) is the probability that a neutrino with energy \(E_{\nu\tau}\) and zenith angle \(\theta\) produces a lepton with energy \(E_\tau\), this probability was used as a "weight" of the event. \(A_i(\theta)\) is the physical cross-section area of the interaction volume seen by the neutrino, simulated by a cylinder with radius of 50 km and height 10 km. \(T_{\text{eff},i}(E_{\tau}, r, d, \theta)\) is the trigger and identification efficiency for \(\tau\)-lepton induced showers with the decay vertex position at distance \(r\) to the telescope and impact distance \(d\). The trigger efficiency in an energy range interval \(\Delta E\), is defined as the number of simulated showers with positive trigger decision and fulfilling our selection criterion shown in Figure 2 (D) over the total number of generated showers for fixed zenith angle \(\theta\), initial energy of primary particle \(E_\tau\), and the impact distance range 4. As an example Figure 2 (A) gives the trigger/identification efficiency as a function of the distance to the detector. For smaller distances \((d < 20\) km) the trigger/identification efficiency drops due to the fact that the shower maximum is too close to the detector or the shower does not reach yet the maximum of shower development, decreasing the amount of Cherenkov light seen by the telescopes. In general, the plot provides an estimate of the typical distance for \(\tau\)-induced showers seen by MAGIC. In Figure 2 (B) we show an estimate for the MC point-source acceptance to tau neutrinos. The simulation of the aperture includes the density profile of the Earth and a 3 km deep water layer around the La Palma island.

\footnote{The impact distance of simulated showers was randomized in CORSIKA simulations (by using CSCAT option) and later Cherenkov telescope orientation for such showers was randomized over the MAGIC camera FOV.}
Search for tau neutrinos at PeV energies and beyond with the MAGIC telescopes

J. Palacio

Figure 3: Left panel: trigger/identification efficiency for MAGIC as a function of the distance to the telescope. This is an average over simulated showers with impact distance smaller than 1.3 km and for zenith angle 86°. Right panel: MC acceptance for point sources, $A_{PS}(E_{\nu})$, to earth-skimming tau neutrinos as estimated for the MAGIC site. For MAGIC pointing at $\theta = 92.5^\circ$ and $\phi = -30^\circ$, the local orographic condition are included.

The water layer is important because it leads to about a factor two (in the energy range 10-100 PeV) smaller acceptance than for the spherical Earth calculations with the rock density of about 2.65 g/cm². In Figure 3 (B) we also show the acceptance, when a cloud layer is included in our simulation i.e. the quasi-stable sea of cumulus between 1500 and 1900 m a.s.l., usually present at MAGIC site due to the temperature inversion. As we can see from the plot this cloud cut leads also to a smaller (about factor two) acceptance.

As we already mentioned, Cherenkov telescopes can be sensitive to tau neutrinos from fast transient objects like GRBs or AGNs. In other words flaring sources, including GRBs, Tidal Disruption Events or the so-called Low Luminosity GRB (LLGRBS), can provide a boosted flux of neutrinos. In Table 2 the expected event rates for MAGIC for fluxes from AGN benchmark models, shown in [4], are listed. The rate is calculated for tau neutrinos assuming that the source is in the MAGIC telescope FOV for a period of 3 hours. For Flux-3 and Flux-4 (i.e. those models covering the energy range beyond $\sim 1 \times 10^8$ GeV) the event rate is at the level of $4 \times 10^{-5}$. For neutrino fluxes covering the energy range below $\sim 5 \times 10^7$ GeV (Flux-1, Flux-2, Flux-5), the number of expected events is not larger than $1 \times 10^{-5}$.

From the estimated acceptance with cloud cut, the sensitivity for an injected spectrum $K \times \Phi(E_{\nu})$ with a known shape $\Phi(E_{\nu})$ was calculated. The 90% C.L. on the value of $K$, according to [16] is $K_{90\%} = 2.44/N_{\text{Events}}$, with the assumption of negligible background, zero neutrino events being observed by the MAGIC during sea observations, and in case of the flux $\Phi(E_{\nu}) = 1 \times 10^{-8}E^{-2}$ [GeV cm⁻² s⁻¹], the sensitivity for a point source search is $E_{\nu}^2 \Phi_{PS}(E_{\nu}) < 1.7 \times 10^{-4}$ [GeV cm⁻² s⁻¹] in the range from 2 to 1000 PeV. The sensitivity is calculated for the expected number of tau neutrino events equal to $N_{\text{Events}} = 1.4 \times 10^{-4}$, based on the result listed in Table 2 for Flux-5, and for 30 hours of observation time. The sensitivity can be improved about two orders of magnitude for larger observation time ($\sim 300$ hrs) and in case of a strong flare with a neutrino flux given by Flux-4, reaching the value $E_{\nu}^2 \Phi_{PS}(E_{\nu}) < 5.8 \times 10^{-6}$ [GeV cm⁻² s⁻¹] i.e. the level of the down-going analysis of the Pierre Auger Observatory [22].

5To estimate the effect of this we assumed that all decaying tau leptons below 1500 a.s.l. are discarded in the acceptance calculations. Thus we assume that for such induced shower all Cherenkov light is absorbed when it passes the range from 1.5-1.9 km a.s.l.
**Table 2:** Expected event rates for the MAGIC detector in case of AGN flares. Flux-1 and Flux-2 are predictions for neutrinos from γ-ray flares of 3C 279 [7]. Flux-3 and Flux-4 are predictions for PKS 2155-304 in low-state and high-state, respectively [8]. Flux-5 corresponds to a prediction for 3C 279 calculated in [19] and is at a similar level in the PeV energy range like the flux reported by IceCube for astrophysical high-energies neutrinos [20].

| Flux      | 1.3 | 0.7 | 0.42 | 4.2 | 1.4 |
|-----------|-----|-----|------|-----|-----|
| Flux-1    | 10^{-5}/3 hrs | (×10^{-5}/3 hrs) | (×10^{-5}/3 hrs) | (×10^{-5}/3 hrs) | (×10^{-5}/3 hrs) |
| Flux-2    | 10^{-5}/3 hrs | (×10^{-5}/3 hrs) | (×10^{-5}/3 hrs) | (×10^{-5}/3 hrs) | (×10^{-5}/3 hrs) |
| Flux-3    | 10^{-5}/3 hrs | (×10^{-5}/3 hrs) | (×10^{-5}/3 hrs) | (×10^{-5}/3 hrs) | (×10^{-5}/3 hrs) |
| Flux-4    | 10^{-5}/3 hrs | (×10^{-5}/3 hrs) | (×10^{-5}/3 hrs) | (×10^{-5}/3 hrs) | (×10^{-5}/3 hrs) |
| Flux-5    | 10^{-5}/3 hrs | (×10^{-5}/3 hrs) | (×10^{-5}/3 hrs) | (×10^{-5}/3 hrs) | (×10^{-5}/3 hrs) |

5. **Summary and conclusion**

Taking into account our sensitivity estimate, the observational program for tau neutrino searches seems to be challenging, but in principle not impossible to pursue. We would like to stress that observation time can be accumulated during periods with high clouds, when those instruments are not used for gamma-ray observations. Note also, that this kind of search, as shown in this proceeding, is basically background free, so the tau neutrino sensitivity increases linearly with the observation time. Finally, the next-generation Cherenkov telescopes, i.e. the Cherenkov Telescope Array, will exploit its much larger FOV (in extended observation mode), and much larger effective areas.

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