Determination of critical current density in melt-processed HTS bulks from levitation force measurements

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Abstract

A simple approach to describe the levitation force measurements on melt-processed HTS bulks was developed. A couple of methods to determine the critical current density \( J_c \) were introduced. The averaged \( ab \)-plane \( J_c \) values for the field parallel to this plane were determined. The first and second levitation force hysteresis loops calculated with these \( J_c \) values coincide remarkably well with the experimental data.

1 Introduction

Superconducting systems with magnetic levitation have long been known and the discovery of high temperature superconductors (HTS) highly stimulated their investigation but a real interest in them for large scale applications appears only with the successful development of the melt-processed (MP)
technology [1]. The use of MP HTS in large scale systems such as flywheels for energy storage, electric motors and generators, permanent magnets, etc. is the most promising HTS application now [2]. In this applied region the levitation force measurements can be considered in two roles: as an information source to know more about levitation systems and as a quick technique to test HTS samples [3]. In many earlier works [4, 5] it has been shown that the forces between a PM and HTS sample are closely related with HTS magnetization curves. Vertical levitation force versus vertical distance $F_z(z)$ is the nearest analog to $M(H)$ dependencies with their major and minor hysteresis loops but the complexity of a field configuration in such large scale PM-HTS systems makes it very difficult to directly correlate them in general case. The problem can be solved by numerical approaches and some of them have been successfully used [6]. The numerical approaches are undoubtedly useful to evaluate the real system parameters but usually need too much computer resources to be applicable to direct HTS sample investigation. To perform such an investigation an analytical evaluation is more wished for.

Two limiting cases of HTS structure have been considered as analytical to calculate the dynamic parameters of an idealized system, a point magnetic dipole over an infinite flat superconductor. The first one is the case of ‘granular superconductor’ which can be modeled by a set of small isolated superconducting grains [7]. It was shown the usual granular HTS obtained by standard sintering techniques can be described very well within this model [7, 8]. The second one is the case of an ‘ideally hard superconductor’ [9, 10, 11, 12]. It was shown recently [9, 10, 11] that dynamics in a wide variety of levitation systems can be described in terms of surface screening currents which screen alternating magnetic field component due to PM displacements like it would be for an ideally hard superconductor, a superconductor with infinite pinning forces. For an infinite flat superconductor the frozen-image method was introduced [9] as an illustration of simple analytical calculation of forces acting in the system. A perfect agreements with experiments were found by us for small PM resonance oscillations frequencies [10] and, recently, by Hull and Cansiz [13], for both vertical and lateral force components.
2 Approach description

The feasibility of the ‘ideally hard superconductor’ approach is that the penetration depth $\delta$ of alternating magnetic field is much less than system dimensions $[9, 10]$. To calculate the stiffness or resonance frequencies the limit $\delta \to 0$ can be used, but it was shown that taking into account the finite values of $\delta$ it is possible to calculate the a.c. loss $[11]$ and even recover critical current density profiles within $\delta$ depth from a.c. loss measurements $[12]$. In this paper we present such an approach of levitation force calculation (including its hysteresis behavior) for superconductor with finite values of critical current density $J_c$ and simple methods to obtain $J_c$ values from levitation force experimental data.

A PM placed over ideally hard superconductor induces at its surface the screening currents $j = (c/4\pi)n \times b_r$, where $n$ is the surface normal and $b_r$ is the tangential magnetic field component at the surface (the normal component $b_n$ at the surface is zero). From the symmetry, for an infinite flat surface $[10]$

$$b_r = 2b_{ar},$$

where $b_a$ is the variation of the PM magnetic field $B_a$ due to its displacements in respect to initial field cooled (FC) position: $b_a = B_a - B_{a FC}$ $[10]$. For the $z$-axial symmetric configuration $r = (rsin\theta, rcos\theta, z)$ where only $j_\theta$ component is induced, for the vertical force acting on PM from the screening currents one can write

$$F_{id} = \int_0^{\infty} rb_{ar}^2(r) dr.$$  \hspace{1cm} (2)

This is ideal force which can be readily calculated just from known tangential component of PM field. The Eq.(2) is obtained from zero-depth screening currents approach that we will call a zero approximation of real PM-HTS systems. Within this approximation any configuration of such systems can be calculated numerically $[14]$ but to describe hysteresis phenomena next-order approximations have to be considered.
In the second stage (a first approximation) we will examine a model where: (i) $\delta$ is finite but still much less than system dimensions $L$, (ii) the critical state model is applicable to these samples, and (iii) critical current density is constant. The applicability of the critical state model to melt-processed HTS has been proven in many experiments \cite{3, 11, 12} and is quite acceptable here. The first condition on $\delta$ can be written as

$$\delta(r) \ll B_{ar}(r) \left( \frac{dB_{ar}(r)}{dr} \right)^{-1} \sim L, \tag{3}$$

and, because of $\delta \propto b_r$, can be satisfied anyway limiting the minimum distance between PM and HTS surface. One can estimate $L \approx z + d/2$, where $z$ is, here and below, the distance between PM and HTS surface and $d$ is the PM thickness.

As for condition on $J_c$, it is far from reality by itself since the critical current density usually depends on both magnetic field and space coordinate. But as it will be shown below we can accept this condition for levitation force measurements. This just means that in the next relation for $j$, the surface density of screening currents,

$$j(r, z) = \delta(r, z)J_c = \frac{c}{2\pi}b_{ar}(r, z), \tag{4}$$

$J_c$ can be treated as a coefficient between $j$ and $\delta$, a coarse-grained flux penetration depth averaged over $L$ scale. The $j(r)$ function does not depend on field history but only on PM position $z$ in the same way as $b_{ar}(r)$, the distribution at the HTS surface of the PM field variation, that after cooling is a function of $r$ and $z$. Thus, in the considering approximation, the function $\delta(r, z)$ formally does not depend on field history but means the flux penetration depth at the first PM descent only.

Next, if we use a protocol of PM motion according to which it moves between two points: the initial or FC point $z_{max}$ that is included in $b_{ar}(r, z)$ function as a condition $b_{ar}(r, z_{max}) = 0$ ($z_{max} = \infty$ for ZFC case), and the lowest point $z_{min}$, the current distributions in the depth $z$ of superconductor are the following. After the PM first stop and beginning to go up (the first ascent), the depth of the layer where currents flow remains constant and equal to its maximum value $\delta_{max} \equiv \delta(r, z_{min})$ but there are two regions with
opposite currents. The opposite flowing current penetrates from the top at the depth $\delta_\up$ that can be obtained from (4)

$$\delta_\up(r, z, z_{\text{min}}) = \frac{1}{2}(\delta(r, z_{\text{min}}) - \delta(r, z)).$$

(5)

Its maximum value is $\delta_{\up\text{max}} \equiv \delta_\up(r, z_{\text{max}}, z_{\text{min}}) = \delta(r, z_{\text{min}})/2$, so during the second descent there are three regions with $+J_c$ for $0 < \zeta < \delta_\down$, $-J_c$ for $\delta_\down < \zeta < \delta_{\up\text{max}}$, and $+J_c$ for $\delta_{\up\text{max}} < \zeta < \delta_\text{max}$, where $\delta_\down(r, z) = \delta(r, z)/2$ also does not depend on $z_{\text{min}}$. If one can neglect flux creep for times greater than the descent-ascent time, any other ascents are equal to the first one and any other descents are equal to the second one. Any other current distributions for other protocols, for example to describe minor hysteresis loops, can also readily be obtained within the scheme above.

Applying this scheme to calculate the vertical forces during the first descent $F(z)$, the first and the next ascents $F_\up(z, z_{\text{min}})$ and the second and the next descents $F_\down(z, z_{\text{min}})$ one can write

$$F(z) = \frac{2\pi}{c} J_c \int_0^\infty \int_0^{\delta(r, z)} rdr d\zeta b_{\text{ar}}(r, z + \zeta).$$

(6)

$$F_\up(z, z_{\text{min}}) = \frac{2\pi}{c} J_c \int_0^\infty \int_0^{\delta_{\up\text{max}}} rdr \left[ \int_0^{\delta_\up} d\zeta - \int_0^{\delta\down} d\zeta \right] b_{\text{ar}}(r, z + \zeta).$$

(7)

$$F_\down(z, z_{\text{min}}) = \frac{2\pi}{c} J_c \int_0^\infty \int_0^{\delta_{\up\text{max}}/2} rdr \left[ \int_0^{\delta_{\up\text{max}}/2} d\zeta - \int_0^{\delta/2} d\zeta + \int_0^\delta d\zeta \right] b_{\text{ar}}(r, z + \zeta).$$

(8)

The functions $\delta(r, z)$ depend on $J_c$ in according to the above equations (4), (5) and below) and for $J_c \to \infty$ all these forces become equal to $F_{\text{id}}(z)$.

Remaining within the condition (i) we can approximate the integrals over $z$ from the formulas (6)-(8) by multiplying the depth of the layer where current flows by the field bar in its center. It is easy to show that within the above approximation the formula (6), for example, can be rewritten as
\[ F(z) = \int_0^\infty rb_{ar}(r, z)b_{ar}(r, z + \delta) r dr , \]  

which highly increases the calculation speed.

## 3 Experiment

To check the applicability of the above consideration to real MP HTS we used a standard experimental setup on levitation force measurements. The SmCo\textsubscript{5} disk shape PM was 15 mm in diameter and 8 mm in thickness (the effective thickness with ferromagnetic holder that was evaluated from real PM field configuration was 12.7 mm) with averaged axial magnetization of \( 4\pi M = 9236 \) G (the field measured by Hall probe in its center at the distance of 0.8 mm from its bottom surface was 3350 G). The magnetic field of the PM was calculated as field of a coil with the same dimensions and with lateral surface current density \( J = cM \). All measured samples were melt-processed HTS of 30±0.5 mm in diameter and 17.5±0.5 mm in thickness. The distance \( z \) between PM bottom surface and HTS top surface varied from \( z_{\text{max}} = 400 \) mm (that can be considered as ZFC case) to its minimum value \( z_{\text{min}} = 0.5 \) mm. The minimum step of PM motion was 75 mm. The accuracy of force detecting was 15 mN. Within this accuracy the experimental data were reproducible for every sample. Fig.1 represents the first and second hysteresis loops (the first and second descent and ascent) for two samples.

Within the above approximation we have only one parameter, \( J_c \), for forces \( (6)-(8) \) (or \( 9 \) and analogous ones) to be fitted to the experimental ones \( F_{\text{exp}}(z) \). To do this, we have to choose one of these functions and one point \( z_i \), and solve the equation

\[ F(J_c, z_i) = F_{\text{exp}}(z_i) . \]

The forces calculated from formulas \( (6)-(8) \) with the \( J_c \) values obtained from \( (1) \) in \( z_{\text{min}} \) point are also represented in Fig.1 by solid lines. The forces calculated from the formula \( (9) \) and from analogous ones practically
coincide with the above in the $F(z)$ plot scale. A good agreement between the experimental and calculated $F(z)$ dependencies demonstrates the above approximation is correct.

Nevertheless, the discrepancy between the experimental and calculated forces still exists and is larger than the experimental accuracy. One of the most likely reasons is a variation of $J_c$ with depth and field. Fig. 2 shows the values of $J_c$ versus maximum value of $B_r(z)$ at the HTS surface for two HTS samples. The data were obtained by solving Eq. (10). Open symbols represent the solution for the function (6), and solid symbols represent the solution for the function (9). The solid line in Fig. 2 with respect to right axis represents the dependence of $B_{r_{max}}(z)$. For a perfectly uniform sample with $c$-axis exactly perpendicular to the surface such a dependence of $J_c(B_{r_{max}})$ would be uniquely determined by the dependence of $J_{c}^{ab}(B_{ab})$, the critical current density flowing in $ab$-plane versus the magnetic field parallel to this plane. But for real melt processed samples, it is more reasonable to assume that the dependencies of $J_c(B_{r_{max}})$ in Fig. 2 are mostly caused by space variations of critical current density. The steep slope of the curves in Fig. 2 at low field, which caused the maximum in the upper curve, is related with the finite diameter of the HTS samples and shows the lower field limit for given configuration.

4 Another simple method

There is a possibility here to introduce a visual simple method to evaluate $J_c$. It is understandable that in spirit of the above consideration a shift $\Delta z$ (see Fig. 1) of the first descent experimental curve with respect to ideal one has to be proportional to an average penetration depth. From the condition $F_{id}(z + \Delta z) = F(J_c, z)$ and Eq. (9) one can readily obtain

$$\delta \approx 4\Delta z, \quad J_c \approx \frac{c}{8\pi} \frac{b_{ar}}{\Delta z}.$$  \hspace{1cm} (11)

The values of $J_c$ evaluated in such way are also represented in Fig. 2 and show a good agreement with ones determined before. The experimental error $\sigma_{J_c}$ that is shown here was estimated from the formula $\sigma_{J_c}/J_c = \sigma_F(dF/dz)^{-1}/\Delta z$ which assumes the maximum error is caused by the force measuring: $\sigma_F \approx 30$ mN.
5  Conclusions

In summary, we have considered the approach, which we call the "first approximation", to describe levitation force data. The term "first" implies that we consider a case in which such parameters as flux penetration depth $\delta$ or normal component of magnetic field at HTS surface $b_n$ are already not zero, as it is for ideally hard superconductor \cite{9}, but small enough: $\delta \ll L$, $b_n \ll b_r$. Within this condition the methods to calculate $J_c$, the critical current density, which we have introduced in the paper are exact. Remarkably, the approach works well even beyond this condition, when $\delta \sim L$, $b_n \sim b_r$. In this region the methods become empirical. The $J_c$ value that can be obtained by the methods is averaged over $L$ scale critical current density in $ab$-plane for field parallel to this plane: $J_c = \langle J_{c}^{ab}(B \parallel ab) \rangle$. $L$ scale depends on the size of a magnet we use.

6  Acknowledgments

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7 Figures Captions

Fig. 1. Experimental (symbols) and calculated (solid lines) data on the first and second hysteresis loops of the vertical levitation force vs. distance $z$ between PM and HTS surface. Dashed line represents the force for an ideal superconductor.

Fig. 2. The values of averaged critical current density versus maximum value of $B_r$, the magnetic field tangential component at the HTS surface, obtained by different methods. The solid line with respect to right axis represents the dependence of $B_{r_{\text{max}}}(z)$. 

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