What is the future of causal models of cosmic structure formation?∗

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Abstract

Recent research has severely constrained the standard “defect” models of cosmic structure formation. Here I discuss the nature of the problems with defect models, and place this discussion in the context of the big picture of cosmic structure formation. In particular, I classify models of cosmic structure formation as either “causal” or “acausal”, and ask whether the problems with the defect models extend to all other causal models. I argue that determining the causal nature of the primordial perturbations is within the reach of modern cosmology, and that such a determination would yield deep insights into the very early Universe.

1 Causality and cosmic structure

There is now overwhelming evidence that the Universe is extremely homogeneous on large scales. This fact, combined with the tendency for gravity to make matter more clumpy as time goes on means that the early Universe was very smooth indeed. Still, the early Universe must have had some very small primordial inhomogeneities in order to seed the process of gravitational collapse and produce the observed structure in the Universe. Because

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of the small amplitude required of these initial inhomogeneities, it has become common in cosmology to speak of the almost perfect homogeneity and the primordial perturbations as two separate features of the early Universe.

The popular “inflationary cosmology” offers one explanation for the origin of both these features. During a period of cosmic inflation whatever initial homogeneities are present are pushed to such large scales that they are unobservable. The fluctuations on observable scales are predicted based on well-defined calculable processes which take zero-point quantum fluctuations in the quantum fields and amplify them into what ultimately become large scale classical perturbations in the cosmic matter. If the amplitude of these perturbations is tuned to be sufficiently small, the inflationary models predict that the “Standard Big Bang” (SBB) epoch which follows inflation will start out with the required homogeneity and primordial perturbations.

Another popular paradigm, typified by the cosmic defect models, starts with a perfectly homogeneous universe which is already experiencing SBB evolution. At some point, physical processes (such as a phase transition) then produce inhomogeneities which can seed cosmic structure. The origin of the initial perfect homogeneity might still be inflation (for example some speculate that the fine tuning problem of the inflationary perturbation amplitude might actually be solved in nature by producing an absolutely infinitesimal perturbation amplitude\[1, 2\], resulting in an essentially perfectly homogeneous start to the SBB). Because this paradigm operates entirely within the SBB, the causality structure of the SBB is respected. In particular, matter cannot be moved around outside the causal horizon, and this severely constrains the nature of the perturbations on large scales. By contrast, the inflationary models have a very different causality structure, which allows in principle for arbitrary adiabatic perturbations to be produced on all relevant scales by the time the SBB epoch begins. Models such as inflation, for which outside-horizon perturbations are present at the start of the SBB are called, by convention, “acausal” models. Models which start with a homogeneous SBB and produce the perturbations in accordance with SBB causality are called “causal” models.

Causal and acausal models of cosmic structure offer strikingly contrasting pictures, and it is thus of quite general interest to attempt to completely rule out one or the other on the basis of observations. In this article I pursue this goal by investigating the extent to which the known problems with the defect models reflect more general problems with other causal models. In
the process I will introduce a very interesting class of causal models (the “Causal White Noise” models) which still are just allowed by the data, but which ultimately will be highly distinguishable from active models.

2 The problems with defects

The defect models have for a long time been considered the primary alternative to the inflationary origin of the cosmological seeds. They are classic examples of causal models, in that the SBB is assumed to start with perfect homogeneity which is then broken by the formation of defects in a cosmic phase transition. A “domain coarsening” process then follows in which the dynamics steadily reduces the number of defects, but at any finite time after the phase transition there are typically some defects left. This coarsening process is usually expected to obey a simple scaling law, in which the mean defect separation scales linearly with time, subject to some “transient” behavior right after the phase transition and during the radiation-matter transition. For some defect types (strings and textures are favourites) the scaling property allows the defect energy density to keep at a constant fraction of the total matter density, and this appears to be roughly what is required to produce the observed cosmic structure.

However, there has been a growing understanding that the defect models have difficulties matching all the current observations. These problems came to a head over the past year, as new calculations appeared which greatly reduced the overall uncertainties in the predictions from some models[3], and showed that the uncertainties in a wide class of other models did not help circumvent conflicts with some particular observations[4].

My collaboration[4] has emphasized that the major source of conflict with the data was the “$b_{100}$ problem”. This is illustrated in Fig. 1 where the predictions from a defect model for the density field power spectrum are compared with the data as presented by Peacock and Dodds[5]. The overall normalization is fixed by COBE normalizing the Cosmic Microwave Background (CMB) angular power spectrum which is shown for the same model in Fig. 2.

A number of discrepancies between theory and data can be observed in these figures, but we have argued[6] that by far the most robust of these problems is the large gap apparent between the theory and data around
Figure 1: The power spectrum of the dark matter perturbations for the standard cosmic string model (solid) plotted with the current observational data, the standard CDM curve (dotted). The two dashed curves give the partial contributions from two time windows to either side of $z = 100$.

Figure 2: The (COBE normalized) angular power spectrum of CMB anisotropies for the same models and windows shown in Fig. 1, plotted with the data.
100$h^{-1}$ scales in the matter power spectrum. (We found[4], for example, that it was easy to exploit uncertainties in the model to boost power in the CMB anisotropies at high $l$'s where it appears lacking in Fig. 4.) We chose to express the main difficulty in terms of the “$b_{100}$ problem”. The value of $b_{100}$ is the bias required for the theory and data to match on 100$h^{-1}$Mpc scales, and $b_{100} = 5.4$ for the model pictured. The extent to which there is a $b_{100}$ problem is the extent to which such large values of the bias are excluded on those scales. Most people seem to be convinced that present data strongly exclude such large values of $b_{100}$ (and favor $b_{100} \approx 1$), but a minority still feel it is too early to tell.

The two “time windows” illustrated in Figs 1 and 2 tell an important part of the story. The defects produce perturbations throughout time, and the perturbations on a given scale are produced predominantly by the defect motions during a finite period of time. The “window” curves in these two figures show that the COBE normalization and matter fluctuations which are relevant to $b_{100}$ are produced during two different time windows, to either side of the redshift $z = 100$. This means that the $b_{100}$ problem has a lot to do with how the defect motions in the two different time windows are related. In the calculations shown in Figs 1 and 2 this relationship was provided by simply assuming the standard scaling law. We have shown[4, 6] that extreme deviations from scaling were required to resolve the $b_{100}$ problem. But scaling is only one factor. When we did violate scaling sufficiently to solve the $b_{100}$ problem, we found that CMB power for $l > 100$ was greatly overproduced. The problems with defects are connected both with their scaling properties and their tendency to overproduce CMB anisotropies relative to density perturbations. These issues are discussed in more detail elsewhere[6].

3 Other causal models

It is interesting to note that in contrast to the models presented above, the nearly scale invariant adiabatic spectrum predicted by most inflation models has no trouble with $b_{100}$. Contrasting this with how difficult it is for the defect models to reproduce this feature certainly boosts the standing of the inflation-based models.

To what extent do the problems with standard defect models extend to other causal models? Right now there are a number of “workable” alternative
causal models. The proposals of Turok¶ and Durrer and Sakellariadou¶ consider causal perturbations which obey standard scaling laws, but which are designed to produce less power in the CMB anisotropies, per matter power, than the standard defect models. At the moment these models are phenomenological, and do not correspond to a specific microphysical picture. Still, I believe they are well worth considering. There are a number of ways these models can distinguish themselves from acausal models, and polarisation is probably a particularly good discriminator¶.

One can expect, however, that many causal processes will produce power in the CMB and matter power spectra in similar proportions to the defect models. Thus, it is also interesting to investigate how well deviations from the scaling law can achieve better agreement with the data in these models. One such example is the defect models in non-Einstein-De Sitter cosmologies. These are expected to exhibit significant deviations from scaling which might prove sufficient to produce a viable model¶,¶. In the next section I will discuss another type of non-scaling causal model which I find particularly intriguing: The “Causal White Noise” models.

4 Causal white noise

The defect models are often called “active” models, because they involve a component of matter (the defects) which evolves in a highly random non-linear manner throughout time, seeding perturbations all along. By contrast inflationary models are “passive” models, which evolve a set of initial perturbations in an essentially linear manner until non-linear gravitational effects set in at late times.

One characteristic of causal active perturbations is that at any given time, they have been unable to move matter around outside the causal horizon which applies to that process. While a completely random process can produce white noise \((P(k) \propto k^0)\) on large scales, the overall mass is constrained to have a \(P(k) \propto k^4\) behavior on large scales for a causal random processes.

While investigating various non-scaling causal models, we have made the following remarkable discovery: A causal white noise spectrum passes the COBE normalized \(b_{100}\) test beautifully. This means that any causal process which has a sufficiently small maximum causal horizon, and which gets the
Figure 3: The (COBE normalized) angular power spectrum of CMB anisotropies for the same models and windows shown in Fig. 1, plotted with the data.

COBE normalization right, will fit the galaxy clustering data on $100 h^{-1}$Mpc scales.

Figure 3 illustrates this point. To produce Fig. 3 we used the same “standard string” model which was described in [6] but we turned the string sources on and off (in a causal way) so that they were only active during a finite time window [12]. Figure 3 shows the COBE normalized matter power spectra for different time windows. We have found that as long as we turn the sources off before some critical redshift ($z_c \approx 100$) the power spectrum passes nicely through the large scale data. In order to fit the smaller scale data, more details need to be specified. Both the specifics of the non-linear processes on scales inside the horizon and the type of dark matter are crucial, and we have already found a number of combinations which provide a good fit.

However, Fig. 4 shows that these Causal White Noise (CWN) models fare much worse when it comes to the CMB anisotropies. There is an excess of power for large $l$ which can be remedied quite well by choosing a non-standard ionization history [12]. The low $l$ behavior does not look particularly good either, but there is some hope that low $\Omega_m$ models could produce a better result.
Figure 4: The (COBE normalized) angular power spectrum of CMB anisotropies for the same CWN models depicted in Fig.3 (The upturn exhibited by some curves at very low $l$ is a numerical artifact.) The excess power for large $l$ can be adequately suppressed by a suitable ionization history, and the small $l$ behavior may produce a better fit in open or $\Lambda$ models. Standard CDM is also shown.
The CWN models are far from being concrete models at this stage. We are still working with them at the phenomenological level of trying to determine what ionization histories and background spacetimes give these models the best chance of success. Then there is the matter of finding interesting candidates for the active sources which produce the noise. (There have already been some interesting developments on this front[13, 14].)

The interesting point at this stage is that CWN models appear to have what it takes for causal models to avoid a $b_{100}$ problem, and CWN models reflect very generic processes. It will be interesting to see if a concrete viable model can emerge from these ideas. Again, there should be ample opportunity to discriminate between CWN models and acausal models. One striking difference is the slope of the matter power spectrum on scales just larger than those on which data presently exist (Fig. 2). On these scales the slope of the power spectrum is much steeper for CWN models than for the standard scale invariant acausal models.

5 Conclusions

The distinction between causal and acausal models of cosmic structure formation is a very interesting one, which reflects two very different pictures of the very early Universe. Recent progress in calculating the predictions from defect models have brought a wide class of causal models into conflict with the data. Some interesting causal models still remain viable, but there appear to be sufficiently many discriminating features that future experiments should be able to determine quite clearly the causal properties of cosmic structure formation.

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