Impulsive synchronization of a class of chaotic systems

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This paper deals with the impulsive synchronization problem of a class of chaotic systems. By employing the comparison principle and the linear matrix inequalities approach, some less conservative and easily verified sufficient conditions for impulsive synchronization of this class of chaotic systems are derived, these new sufficient conditions can be applied to analyze the impulsive synchronization of the Chua’s oscillators. Moreover, the numerical simulation of Chua’s oscillators under impulsive control shows the effectiveness of the proposed theory, and obtains better estimation of the boundary of the stable region than the existing approaches.

Keywords: impulsive synchronization; chaotic system; linear matrix inequality; Chua’s oscillators

1. Introduction

Chaotic systems have complex dynamical behaviors which own some special performances, such as being extremely sensitive to tiny variations of initial conditions, having bounded trajectories in the phase space with a positive leading Lyapunov exponent, and so on. In recent decades, synchronization of coupled chaotic systems has been of ongoing interest due to its potential applications for secure communication. The phenomenon of chaos synchronization was first revealed by Pecora and Carroll (1990). Since then, numerous methods have been proposed for synchronization of chaotic systems, which include complete synchronization (Mahmoud & Mahmoud, 2010), phase synchronization (Ge & Chen, 2004), generalized synchronization (He & Cao, 2009), adaptive synchronization (Chen & Chang, 2006), lag synchronization (Bhowmick, Pal, Roy & Dana, 2012), impulsive control (He, Qian, Cao & Han, 2011; Li, Liao & Zhang, 2005; Sun, Zhang & Wu, 2002; Yang & Chua, 1997; Yang, Yang & Yang, 1997), etc.

It is common knowledge that impulsive control (Sun, Zhang & Wu, 2003; Yang, 1999, 2001) is characterized by the abrupt changes in the system dynamics at certain instants, which is an advantage in reducing the amount of information transmission and improving the security and robustness against disturbances especially in telecommunication network and power grid, orbital transfer of satellite. In addition, impulsive control allows the stabilization and synchronization of chaotic systems using only small control impulses. Thus, it has been widely used to stabilize and synchronize chaotic systems. Recently, several impulsive synchronization schemes have been reported in the literature (He et al., 2011; Li et al., 2005; Sun et al., 2002; Yang & Chua, 1997; Yang et al., 1997). Wang-Li He, etc in (He et al., 2011) have obtained some sufficient conditions for the synchronization of two nonidentical chaotic systems with time-varying delay via impulsive control. In Li et al. (2005), the authors used impulsive theory and the linear matrix inequality (LMI) technique derived from some less conservative and easily verified criteria for impulsive synchronization of chaotic systems. The stabilization and synchronization of Lorenz systems (Lorenz, 1995) via impulsive control are studied in Sun et al. (2002).

Inspired by the former, we further study the impulsive synchronization of a class of chaotic systems in this paper. Some less conservative and easily verified criteria for impulsive synchronization of chaotic systems are derived via impulsive control theory and LMI. We then use the LMI toolbox in MATLAB to obtain the synchronization conditions and estimate the stable region of synchronized systems. The proposed method is also applied to the original Chua’s oscillators to demonstrate the effectiveness.

The organization of this paper is as follows. In Section 2, the theory of impulsive differential equations is given. Some new synchronization criteria are obtained in Section 3. In Section 4, the simulation results of the impulsive synchronization of the original Chua’s oscillators are presented. Finally, conclusions are given in Section 5.

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2. Model description and preliminaries

Consider a class of chaotic systems which can be described by

\[ \dot{x}(t) = Ax(t) + f(x(t)) + J, \]  

(1)

where \( x(t) = (x_1(t), \ldots, x_n(t))^T \) is the state variable, \( A \in \mathbb{R}^{n \times n} \), \( J \in \mathbb{R}^n \) is the constant input vector, and \( f(x) = (f_1(x_1), \ldots, f_n(x_n))^T \) is the nonlinear vector-value function.

Throughout the paper, we assume that:

(H1) Each function \( f_i \) is continuous, and there exist scalars \( l_i^- \) and \( l_i^+ \) such that

\[ l_i^- \leq \frac{f_i(a) - f_i(b)}{a - b} \leq l_i^+ \]

for any \( a, b \in \mathbb{R}, a \neq b \), where \( l_i^- \) and \( l_i^+ \) can be positive, negative or zero.

We set

\[
L_1 = \text{diag}(l_1^-, l_2^-, \ldots, l_n^-) \quad \text{and} \quad L_2 = \text{diag}(l_1^+, l_2^+, \ldots, l_n^+). 
\]

Definition 1 In the usual Lipschitz condition, it is assumed \( l_i^- = -l_i^+ \). Clearly, the condition (H1) is more general than the usual Lipschitz condition and it has been adopted in Khalil (1996).

We take system (1) as the drive system, and the corresponding response system is given by

\[ \dot{y}(t) = Ay(t) + f(y(t)) + J, \]  

(2)

where \( y(t) \) is the state variable of the response system.

At discrete time \( t_k \), the state variables of the drive system are transmitted to the response system as the control input such that the state variables of the response system are suddenly changed at these instants. Therefore, the impulsive controlled response system can be written as (3),

\[
\begin{align*}
\dot{y}(t) &= Ay(t) + f(y(t)) + J, \quad t \neq t_k, \\
\Delta y(t_k) &= y(t_k^+) - y(t_k^-) = H(y(t_k^-) - x(t_k^-)), \\
t &= t_k, k = 1, 2, \ldots,
\end{align*}
\]

(3)

where \( \Delta y(t_k) \) denotes the state jumping at impulsive time instant \( t = t_k \), \( y(t_k^+) \) and \( y(t_k^-) \) are the right-hand and left-hand limits of the functions \( y(t) \) and \( x(t) \) at \( t_k \), respectively. Moreover, \( y(t) \) and \( x(t) \) are both left-hand continuous at \( t = t_k \), i.e. \( y(t_k) = y(t_k^-) \) and \( x(t_k) = x(t_k^-) \). \( H \) is the impulsive matrix. Suppose that the discrete instant set \( \{t_k\} \) satisfies \( 0 < t_1 < t_2 < \cdots \) and \( \lim_{k \to \infty} t_k = \infty \). For simplicity, it is assumed that \( y(t) \) is continuous at \( t_0 \), and \( y(t_0^+) = y_0 \).

Defining the synchronization error as \( e(t) = y(t) - x(t) \), then we have the following error system:

\[
\begin{align*}
\dot{e}(t) &= Ae(t) + g(e(t)), \quad t \neq t_k, \\
\Delta e(t_k) &= e(t_k^+) - e(t_k^-) = He(t_k^-), \quad t = t_k, k = 1, 2, \ldots, \\
e(t_0^-) &= e_0, \quad t_0 = 0,
\end{align*}
\]

(4)

where \( g(e(t)) = f(y(t)) - f(x(t)) \).

To begin with, we introduce some notation and recall some basic definitions.

In general, the impulsive functional differential equation can be described by

\[
\begin{align*}
\dot{x}(t) &= f(t, x(t)), \quad t \neq t_k, \quad t > 0, \\
\Delta x(t_k) &= x(t_k^+) - x(t_k^-) = U(k, x(t_k^-)), \\
x(t_0) &= 0, \\
k &= 1, 2, \ldots,
\end{align*}
\]

(5)

where \( f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n \) is continuous with \( f(t, 0) = 0 \), \( x \in \mathbb{R}^n \) is the state variable, \( U(k, x(t_k^-)) : S_{\rho} \to \mathbb{R}^n \) with \( U(k, 0) = 0 \).

Definition 1 (Yang, 2001) Let \( V : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}_+ \), if

(i) \( V \) is continuous in \( (t_{k-1}, t_k] \times \mathbb{R}^n \) and for each \( x \in \mathbb{R}^n, k = 1, 2, \ldots, \lim_{\delta \to 0^+} (\{ t \} \times \{ x \} ) \}\ V(t, y) = V(t_k^+, x) \) exists;

(ii) \( V \) is locally Lipschitzian in \( x \).

Definition 2 For \( (t, x) \in (t_{k-1}, t_k] \times \mathbb{R}^n \), we define the right and upper Dini’s derivative of \( V \) in \( \mathcal{V}_0 \) with respect to the time variable

\[
D^+ V(t, x) \equiv \lim_{h \to 0^+} \sup \frac{1}{h} \{ V[t + h, x + hf(t, x)] - V(t, x) \}.
\]

(6)

Definition 3 (Yang, 2001) Let \( V \in \mathcal{V}_0 \) and assume that

\[
D^+ V(t, x) \leq g(t, V(t, x)), \quad t \neq t_k, \\
V(t, x + U(k, x)) \leq \psi_k(V(t, x)), \quad t = t_k, k = 1, 2, \ldots,
\]

(7)

where \( g : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R} \) is continuous and \( g(t, 0) = 0 \), \( \psi_k : \mathbb{R}_+ \to \mathbb{R}_+ \) is nondecreasing. Then, the system

\[
\begin{align*}
\dot{\omega} &= g(t, \omega), \quad t \neq t_k, \\
\omega(t_k^+) &= \psi_k(\omega(t_k)), \quad k = 1, 2, \ldots, \\
\omega(t_0^-) &= \omega_0 \geq 0.
\end{align*}
\]

(8)

is called the comparison system of system (5).


**Definition 4**  
$S_{\rho} = \{ x \in \mathbb{R}^{n} \mid \|x\| < \rho \}$, where $\| \cdot \|$ denotes the Euclidean norm on $\mathbb{R}^{n}$.

**Definition 5**  
A function $\alpha$ is said to belong to $\mathcal{K}$, if $\alpha \in [R_{+}, R_{+}]$, $\alpha(0) = 0$, and $\alpha(x)$ is strictly increasing in $x$.

**Lemma 1**  
(Li, Wen & Soh, 2001) Let $g(t, \omega) = \hat{\lambda}(t)\omega, \lambda \in C^{1}[R_{+}, R_{+}], \psi_{0}(\omega) = d_{k}\omega, d_{k} \geq 0, k = 1, 2, \ldots$. Then, the origin of system (5) is asymptotically stable if the following conditions hold:

(i)  
\[
\sup_{k}[d_{k}\exp(\lambda(t_{k}+1) - \lambda(t_{k}))] = \varepsilon_{0} < \infty; \tag{9}
\]

(ii)  
there exists a $r > 1$ such that  
\[
\lambda(r_{k}+1) + \ln(r\cdot d_{k}+d_{k-1}) \geq \lambda(r_{k}+1) \tag{10}
\]

holds for all $d_{k} \geq 0, k = 0, 1, \ldots$;

(iii)  
$\dot{\lambda}(t) \geq 0$;

(iv)  
there exists $\beta(\cdot)$ and $\beta(\cdot)$ in class $\mathcal{K}$ such that \[
\beta(\|x\|) \leq V(t, x) \leq \alpha(\|x\|).
\]

### 3. Main Results

In the section, we investigate the impulsive synchronization of the chaotic system by the stability analysis of impulsive functional differential equation.

**Theorem 1**  
Assume that hypothesis (H1) holds. For any constant scalars $\theta > 0, 0 < \kappa < 2$, the origin of the error system (4) is asymptotically stable if there exist a symmetric and positive definite matrix $P \in \mathbb{R}^{n \times n}$, and constant scalars $0 < \mu < 1, \xi > 0, \gamma > 1$ such that

(C1)  
\[
\left[ A^{T}P + PA - \frac{2\theta}{2-\kappa}L_{2} + \frac{\theta}{2-k}L_{1} - \xi P \right] \leq 0, \tag{C1}
\]

(C2)  
\[
\left[ \mu P \begin{array}{c}
I + H^{T}P
\end{array} \right] \geq 0, \tag{C2}
\]

(C3)  
$\max_{k}[t_{k} - t_{k-1}] \leq -\ln \gamma \mu / \xi, k = 1, 2, \ldots$,

where the notation $*$ always denotes the symmetric block in one symmetric matrix.

**Proof**  
Construct a Lyapunov functional as follows:

\[
V(t, e(t)) = e^{T}(t)Pe(t) \tag{11}
\]

it follows (11) that $\lambda_{m}(P)e^{T}(t)e(t), \lambda_{M}(P)e^{T}(t)e(t) \in \mathcal{K}, \lambda_{m}(P)e^{T}(t)e(t) \leq V(t, e(t)) \leq \lambda_{M}(P)e^{T}(t)e(t), \lambda_{m}(P)$ and $\lambda_{M}(P)$ denote the smallest and largest eigenvalues of $P$, respectively.

According to (H1), it is easy to see that

\[
0 \leq \sum_{i=1}^{n} (l_{i}^{+}e_{i}(t) - g_{i}(e_{i}(t)))(g_{i}(e_{i}(t)) - l_{i}^{-}e_{i}(t)) = e^{T}(t)L_{1}g(e(t)) - g^{T}(e(t))g(e(t)) - e^{T}(t)L_{2}e(t),
\]

For $t \neq t_{k}, k = 1, 2, \ldots$, the derivative of $V(t, e(t))$ along the solution of (4) is

\[
D^{+}V(t, e(t)) = e^{T}(t)(A^{T}P + PA)e(t) + g^{T}(e(t))Pe(t) + e^{T}(t)Pg(e(t)) \leq e^{T}(t)(A^{T}P + PA)e(t) + \theta g^{T}(e(t))g(e(t)) + \frac{1}{\theta}e^{T}(t)P^{T}Pe(t) \leq e^{T}(t)\left[ A^{T}P + PA + \frac{1}{\theta}P^{2} \right] e(t) + \theta(e^{T}(t)L_{1}g(e(t)) - e^{T}(t)L_{2}e(t)) = e^{T}(t)\left[ A^{T}P + PA + \frac{1}{\theta}P^{2} - \theta L_{2} \right] e(t) + \theta e^{T}(t)L_{1}g(e(t))
\]

notice that

\[
e^{T}(t)L_{1}g(e(t)) \leq \frac{1}{2\kappa}e^{T}(t)L_{1}^{T}e(t) + \kappa \theta g^{T}(e(t))g(e(t)) \leq \frac{1}{2\kappa}e^{T}(t)L_{1}^{T}e(t) + \kappa \frac{1}{2}e^{T}(t)L_{1}g(e(t)) - e^{T}(t)L_{2}e(t),
\]

which implies

\[
e^{T}(t)L_{1}g(e(t)) \leq e^{T}(t)\left( \frac{1}{2\kappa - \kappa^{2}} - \frac{\kappa}{2 - \kappa}L_{2} \right) e(t),
\]

then

\[
D^{+}V(t, e(t)) \leq e^{T}(t)\left[ A^{T}P + PA + \frac{1}{\theta}P^{2} - \theta L_{2} \right] e(t) + \theta e^{T}(t)\left( \frac{1}{2\kappa - \kappa^{2}}L_{2}^{T} - \frac{\kappa}{2 - \kappa}L_{2} \right) e(t) + \frac{\theta}{2\kappa - \kappa^{2}}L_{2}^{T} e(t) + \frac{\theta}{2\kappa - \kappa^{2}}L_{2} e(t) = e^{T}(t)\left[ A^{T}P + PA + \frac{1}{\theta}P^{2} - \frac{2\theta}{2 - \kappa}L_{2} \right] e(t) + \frac{\theta}{2\kappa - \kappa^{2}}L_{2}^{T} e(t) + \frac{\theta}{2\kappa - \kappa^{2}}L_{2} e(t) + \frac{\theta}{2\kappa - \kappa^{2}}L_{2}^{T} e(t) + \frac{\theta}{2\kappa - \kappa^{2}}L_{2} e(t),
\]
From condition (C1) and Schur complement (Boyd, Ghaoui, Feron & Balakrishnan, 1994), we obtain $D^+ V(t, e(t)) \leq \xi V(t, e(t))$. For $t = t_k, k = 1, 2, \ldots$,
\[ V(t_k, e(t_k) + \Delta e(t_k)) = e^T(t_k)(I + H)^TP(I + H)e(t_k) = e^T(t_k)(I + H)^TP(I + H)e(t_k). \]

From condition (C2), i.e., $\left[ \begin{array}{c} \mu P \ (I + H)^TP \ 0 \\ 0 \end{array} \right] \geq 0$, we have
\[ \left[ \begin{array}{c} I \ -(I + H)^TP \ 0 \\ 0 \end{array} \right] \left[ \begin{array}{c} \mu P \ (I + H)^TP \ 0 \\ 0 \end{array} \right] \geq 0, \]

it follows that
\[ \mu P - (I + H)^TP(I + H) \geq 0, \]
which yields $V(t_k, e(t_k) + \Delta e(t_k)) \leq \mu V(t_k, e(t_k))$. Let $\hat{\lambda}(t) = \xi, \delta = \mu, k = 1, 2, \ldots$. From Lemma 1, the origin of the error system (4) is asymptotically stable. The proof of Theorem 1 is completed. \hfill \Box

Remark 2 When $\theta, \kappa, \xi, \mu$ are chosen, conditions (C1) and (C2) in Theorem 1 are LMI, which can be solved numerically and very efficiently using the interior point algorithms (Boyd et al., 1994).

When $l_1^- = -l_1^+ = -L < 0$ in (H1), we have the following corollary holds.

Corollary 1 Assume that hypothesis (H1) holds. For any constant scalars $\theta > 0, 0 < \kappa < 2$, if there exist a symmetric and positive definite matrix $P \in \mathbb{R}^{n \times n}$ and constant scalars $0 < \mu < 1, \xi > 0, \delta > 0, \gamma > 1$ such that

(C1')
\[ \left[ \begin{array}{c} A^TP + PA + \frac{\theta}{2 - \kappa} L_1^2 - \xi P \sqrt{\frac{1}{\theta}} P \\ 0 \end{array} \right] \leq 0; \]

(C2)
\[ \left[ \begin{array}{c} \mu P \ (I + H)^TP \ 0 \\ 0 \end{array} \right] \geq 0; \]

(C3) $\delta = \max \{t_k - t_{k-1}\} \leq -\ln \gamma \mu / \xi, k = 1, 2, \ldots$;

then, the origin of the error system (4) is asymptotically stable.

Remark 3 It is worth noting that the Theorem 3 in Li et al. (2005) is a special form of Corollary 1 with appropriate $\theta, \kappa$, thus our method is more general than those given out in Li et al. (2005).

In particular, when $l_1^- = 0, l_1^+ > 0$ in (H1), the following corollary holds

Corollary 2 Assume that hypothesis (H1) holds. For any constant scalars $\theta > 0, 0 < \kappa < 2$, if there exist a symmetric and positive definite matrix $P \in \mathbb{R}^{n \times n}$ and constant scalars $0 < \mu < 1, \xi > 0, \delta > 0, \gamma > 1$ such that

(C1')
\[ \left[ \begin{array}{c} A^TP + PA + \frac{\theta}{2 - \kappa} L_1^2 - \xi P \sqrt{\frac{1}{\theta}} P \\ 0 \end{array} \right] \leq 0; \]

(C2) $(I + H)^TP(I + H) - \mu P \leq 0$;

(C3) $\delta = \max \{t_k - t_{k-1}\} \leq -\ln \gamma \mu / \xi, k = 1, 2, \ldots$;

then, the origin of the error system (4) is asymptotically stable.

The proof of Corollaries 1 and 2 is direct, so it is omitted.

4. Numerical experiments

In this section, we use the proposed method to study the impulsive synchronization of Chua’s oscillators. The dimensionless form of a Chua’s oscillator is given by
\begin{equation}
\begin{align*}
\dot{x}_1 &= \alpha[x_2 - x_1 - g(x_1)] \\
\dot{x}_2 &= x_1 - x_2 + x_3 \\
\dot{x}_3 &= -\beta x_2,
\end{align*}
\end{equation}

where $g(x_1)$ is the piecewise-linear characteristics of the Chua’s diode, which is given by
\begin{equation}
g(x_1) = b_1 x_1 + 0.5(a_1 - b_1)(|x_1 + 1| - |x_1 - 1|),
\end{equation}

where $a_1 < b_1 < 0$ are two constants.

Let $\alpha = (x_1, x_2, x_3)$, then we can rewrite the Chua’s oscillator equation in the form as
\begin{equation}
\dot{\alpha} = Ax + f(x) + J,
\end{equation}
where
\[ J = (0, 0, 0)^T, \quad A = \begin{bmatrix} -\alpha - ab_1 & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix}, \]
\[ f(x) = \begin{bmatrix} 0 \\ 0 \\ -0.5\alpha(a_1 - b_1)(|x_1 + 1| - |x_1 - 1|) \end{bmatrix}. \]

In the following numerical simulation, we choose the parameters of system (12) as $\alpha = 9.2156, \beta = 15.9946, a_1 = -1.24905, b_1 = -0.75735$. The initial conditions for drive and response systems are given by $x(0) = (0.15, 0.1, 0.2)^T$ and $y(0) = (0.5, 0.3, -0.5)^T$, respectively. The phase diagram of the drive system is shown in Figure 1, which is the Chua’s double-scroll attractor.
Choose $\theta = 1, \kappa = 1, H = \text{diag}(K, K, K)$, then $-2 < K < 0$ and note that $L_1 = \text{diag}(4.5313, 0, 0)$, $L_2 = \text{diag}(0, 0, 0)$. From condition 1 in Theorem 1, we find the approximate smallest value of control gain, $\xi = 10.4053$. Then using Matlab LMI toolbox, we can obtain the following feasible solution to LMIs in Theorem 1:

$$P = \begin{bmatrix} 7.0484 & -0.5010 & 2.7907 \\ -0.5010 & 9.9859 & 0.7248 \\ 2.7907 & 0.7248 & 1.9347 \end{bmatrix}$$

with eigenvalues $\text{eigen}(P) = [0.6266, 8.2613, 10.0811]^T$.

Let $\delta$ be the impulsive interval, then estimates of bounds of stable regions are given by

$$0 \leq \delta \leq -\frac{\ln \gamma + \ln(K + 1)^2}{10.4053}, -2 < K < 0.$$
5. Conclusion

In this paper, the impulsive synchronization of a class of chaotic systems have been investigated, in which an impulsive control scheme has been proposed and some new sufficient conditions have been established by means of the impulsive comparison principle and LMIs approach. Moreover, the drive system and the response system can be synchronized to within a desired control matrix. Finally, the numerical results have confirmed that these new sufficient conditions are less conservative than the existing ones, and our method can obtain better estimation of the boundary of the stable region than the existing approaches.

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