The study of oscillations excitation patterns in the process of milling with portable equipment

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Abstract. This paper examines the mechanism of excitation of vibrations in the milling process with low-rigid metal-cutting equipment. The results of theoretical and practical experiments are presented. The mechanism of self-adjustment of the technological system and the possibility of damping of its oscillations, not associated with increased stiffness of equipment or tool, are examined.

1. Introduction
With the development of mechanical engineering, the use of compact metal-cutting equipment becomes the more urgent task. Its most common representatives are compact multi-axis milling machines (they are also called 3D-milling) and portable milling equipment for the processing of large parts (when the machine is installed on a part). The peculiarity of the use of such equipment is the presence of the low-rigid technological system.

The well-known fact that during milling, the mechanism of occurrence of oscillations is quite complicated due to the intermittent nature of the cutting process, and the major cause of their increase is a regenerative excitement of the track on the cutting surface. The vibrations arising in the process of milling, are a powerful deterrent to increase its productivity. Therefore, the special attention is paid to the study of the mechanism of this type of excitation, as well as the possible impact on it.

2. Theoretical study
The system for theoretical studies of the mechanism of the regenerative vibration excitation includes one reduced mass having one degree of freedom in the direction perpendicular to the cutting speed (Figure 1). For it, the parameters of the tool subsystem of the milling machine in the direction of the feed axis were adopted, as this subsystem is most exposed to the trace of the cut surface [1, 2].

The variability of the slice thickness due to ongoing vibration movements and vibrations of the track is taken as the main reason of the oscillations excitation of the adopted system. In order not to take into account changing of the slice thickness versus cutter turning during the rotation (because they do not influence the regenerative process of excitation of oscillations), the treatment process was ‘expanded’ in linear: the cutting surface in its nominal position (without wave-shaped track) was adopted as flat and continuous, the movement of the teeth relative to it as straight and forward. It was also accepted that teeth are located in parallel and secured in a common unit (as in the gang
processing). It has been accepted that the inertial properties of the oscillating system are concentrated in the cutter block, and they are represented in the diagram by weight $m$.

![Figure 1](image)

**Figure 1.** The oscillatory system scheme: $X_t$ – mass displacement at a given time, m; $X_{t-r}$ – track oscillation on the surface of the cutting, m; $C_{yc}$ – stiffness of the oscillating system, N/m; $h$ – damping of oscillating system N·s/m; $a_n$ – nominal slice thickness, m; $Z_t$ – coordinate characterizing the position of the tooth along axis $z$, m.

Teeth are displaced along the x-axis relative to each other by the amount equal to the slice thickness, and along the $z$-axis – by the amount equal to circumferential distance between them in the tool. Impact on the system formed by the tooth is determined by the thickness of the shear layer by the hardness of the cutting:

$$F_t = K_{pt} a = K_{pt} (a_n + X_t - X_{t-r}), \quad (1)$$

$K_{pt}$ – hardness of the cutting in the direction of main movement, N/m; $a$ – instantaneous slice thickness, m; $F_t$ – tangent component of the cutting force, N.

To ease the calculation, it was assumed that the cutting stiffness does not depend on the thickness of the shear layer. Other components of the cutting force were expressed through coefficients:

$$F_r = K_r K_{pt} a = K_r F_t, \quad F_0 = K_0 K_{pt} a = K_0 F_t.$$  

The equation of system motion according to Figure 1 can be written as:

$$m \ddot{X}_t = -C_{yc} X_t - K_p (X_r - X_{t-r}) + h \dot{X}_t, \quad (2)$$

$K_p$ – hardness of the cutting, N/m.

The mass in the direction of the x-axis is influenced only by a radial force, so:

$$K_p = K_{pt} K_r.$$

Let us mark the tooth, leaving the trace on the cutting surface as the ‘tooth 1’, and the tooth, cutting this track – as ‘tooth 2’. Let these teeth move along the $z$-axis at constant speed $v_0$, and the equations of motion in the direction of cutting movement of the first and second teeth are, respectively:

$$Z_1(t) = v_0 t; \quad Z_2(t) = Z_{12} + Z_1(t), \quad (3)$$

$Z_1(t)$, $Z_2(t)$ – position of respectively the first and second teeth along axis $z$ at time moment $t$, m; $v_0$ – velocity of cutting movement, m/s; $Z_{12}$ – distance from tooth 1 to tooth 2 in the direction of the main movement of the cutting, m; $Z_{12} < 0$. 
When moving along the surface of the cutting, tooth 2 is sequentially positioned at some positions, characterized by coordinate \( Z_2(t) \). Preceding tooth 1 was also positioned at these positions some time ago, and left a trace on the surface. When the considered tooth passes through this position, it cuts off this trace, i.e. the so-called ‘amount of lag’ \( \tau_{21} \), is determined by the time between the moments when two consecutive teeth pass the same position along the direction of the main movement (i.e. along the \( z \)-axis). At constant velocity \( v_0 \) along the \( z \)-axis, value \( \tau_{21} \) is determined by path length \( Z_{21} \). In this case, path length \( Z_{21} \) does not contain the whole number of waves on the surface of cutting:

\[
Z_{21} = Z_{12} > 0; \quad Z_{21} = L(k+i); \quad L = \frac{v_0}{f_x} = \text{const}; \quad \tau_{21} = (k+i) \cdot T_x,
\]

\( L \) – length of one period of oscillations on the surface of the cutting, \( m \); \( k \) – number of oscillation periods that fall entirely within the length of the path \( Z_{21} \); \( i \) – ratio of the rest of the period of oscillation that does not fit in to the oscillation period of a single-mass system along \( x \)-axis; \( f_x \) — vibration frequency of the system along axis \( x \), Hz; \( T_x \) – period of oscillation of the system along axis \( x \), s.

A condition to determine the magnitude of the lag is:

\[
Z_2(t) = Z_1(t - \tau_{21}).
\]

And taking into account (3):

\[
\tau_{21} = \frac{Z_{12}}{v_0} = \frac{Z_{21}}{v_0}.
\]

Thus, at a constant speed of main movement \( v_0 \) the time delay is a constant value. The value of the phase difference corresponds to the time delay - this is the same interval but measured in phase (i.e. in phase angles):

\[
\varphi_{21x} = \omega_x \cdot \tau_{21} = \omega_x \cdot (k+i) \cdot T_x = (k+i) \cdot 2\pi, \quad (4)
\]

\( \varphi_{21x} \) – the phase difference between the oscillations of the system along axis \( x \) when passing two adjacent teeth through the given point in space, rad; \( \omega_x \) – angular frequency of oscillations along axis \( x \), rad/s.

Thus, the value of phase difference \( \varphi_{21x} \) may include different numbers of oscillation periods. The whole part of this number determines the number of full oscillations and fractional remainder \( 2\pi i \) – the phase shift \([1–3]\).

3. Results of the practical experiment

A feature of the milling process is the presence of periodic inputs-outputs of the teeth from contact with the workpiece. This leads to intense torsional vibration of the main movement drive, therefore, to the change of the cutting speed. When machining at a variable cutting speed, the phase shift is changing constantly, and the level of vibration should change depending on the instantaneous value of this angle.

![Figure 2. Equipment for the experiment.](image)
To confirm the amplifying effect of the regenerative effect on the level of vibrations when milling with portable equipment, a series of experiments was made. The experiments were carried out at the facility, which is a small milling machine with the CNC system, mounted on non-rigid supports (Figure 2). The processing was conducted in various modes by the end-cylindrical milling cutter with a diameter of 8 mm and teeth number 5 according to procedures [4]. Vibration measurements were carried out using device VibXpert II. Data processing was carried out in software OMNITREND.

![Figure 3. Wave form of system vibration in the direction of z-axis for various treatment modes.](image)

A parameter that does not depend on phase change, but affects the level of regenerative oscillation, can be the lag time. Another parameter influencing the level of oscillation is the phase shift in the beginning of cutting on the trace. The minimum level of vibration occurs when the initial shear is equal to $\pi/2$, and is maximum at $-\pi/2$. Data analysis of vibrodiagnostics (Figure 3) shows that if the initial phase shift is different from $\pi/2$, the part of the vibration energy is spent on setting up the system, and the fluctuations are damped quite rapidly. If the initial phase shift equals $-\pi/2$ or is close to this value, the system immediately adjusts to the correct phase shift, and steady oscillations occur in
the system. Due to the presence of friction in the system, these oscillations are gradually damped and the decay time depends on the initial phase shift.

![Vibration spectra of the experimental installation in the z-axis direction at different cutting modes.](image_url)

**Figure 4.** Vibration spectra of the experimental installation in the z-axis direction at different cutting modes.

Figure 4 shows the most characteristic spectra of the oscillatory process for different values of the phase shift between the track and system oscillations. It should be noted that the oscillatory process, in this case, has several components. There are forced resonant oscillations in the system, characterized by pronounced pulses from the entry and exit of the teeth and by the intense transition process. The level of these oscillations is quite high, and the presence of regenerative effect leads to a resonant increase in the amplitude of the harmonic with frequency corresponding to the natural frequency of the system. As a result, there are two frequencies that are in dominated in the spectrum (Figure 4): the frequency of the driving force and multiplied by its frequency is close to the frequency of the system;
in addition, the spectrum contains many other harmonics that are multiples to the frequency of the driving force. There are also self-oscillations that occur in the form of a beat. Therefore, the frequency of the driving force and the natural frequency of flexural vibrations of the instrument subsystem shifted up due to the influence of the stiffness of cutting are also present in the spectrum (Figure 4). The level of vibration velocity from self-oscillations is lower than in case of forced resonant oscillations.

Figures 3 and 4 show that the system tends in all cases to a state, when vibration velocity fluctuations are behind a trace at the angle of $\pi/2$ (i.e., $2\pi i = -\pi/2$), so they are always $i = -1/4$ or $i = 3/4$.

If we consider the trace on the cutting surface as driving force with frequency $p$, the phase of the forced oscillations of the system should lag behind the phase of the perturbing force for the value:

$$2\pi i_p = -\arctan \frac{2\varepsilon \gamma}{1 - \varepsilon \gamma},$$

$\varepsilon$ – dimensionless damping coefficient; $\gamma$ – ratio of the frequency of driving force $p$ to its own frequency $f_c$.

In case of equality of its own and driving frequencies, the phase shift does not depend on $\varepsilon$ and can only be equal to $\pi/2$. More specifically this value of phase shift corresponds to the conditions of resonance occurrence. Hence, the increase in the level of oscillations in the perturbation on the trace in the first place is due to resonance or near-resonance phenomena due to the coincidence of the own frequency of the system with the wave frequency on the cutting surface.

Consequently, the value of phase difference $\varphi_{21c}$ calculated in equation (4) determines its value only at the initial moment of cutting in trace because after a while the phase of oscillations will be self-set with a new value. At the same time, the time of phase adjustment does not depend on its initial value and is equal to several periods of oscillation of the driving force. This self-adjusting of the system is due to the change of the oscillation period of the system itself, $T_c$. It follows that time $\tau_{21c}$ always includes the whole number of $T_c$ periods, determined by the own frequency of the system or the duration of the transition, i.e. $i = 0$, and at this value of phase shift, the effect of regenerative excitation is minimal [1, 5]. Thus, the limit of stability of the cutting process does not depend on the disturbance on the trace.

In the analysis of vibration spectra (Figure 4), there is a soft excitement of regenerative vibrations. This implies that the system is stable when cutting on the trace but with a small stiffness coefficient; and it may become unstable when cutting on the trace, when the stiffness ratio exceeds a certain value [6–8].

4. Conclusion

Thus, if the magnitude of the impact force is small, the increase in vibrations level is caused by resonance phenomena with a trace on the cutting surface and, over time, due to the absence of the external periodic source, these vibrations are damped. If the magnitude of the impact force exceeds a certain value, there is a sharp increase in the amplitude of the oscillations which take the form of continuous beating. These fluctuations can not be attributed to the forced resonance, because they occur only for sufficiently large force action and outwardly behave as self-oscillation. This self-adjusting of the system always occurs at phase shift $\pi/2$. As a result, the part of the oscillation energy is consumed not to unbalance the system but to adjust it. The more energy will be spent on the adjustment, the less of it will remain on the excitation of vibrations. It follows that one of the effective ways to damp regenerative oscillations can be to create such conditions of its operation in which it would be necessary to perform this reconfiguration constantly. Thus, if the wave period for the cutting surface will not coincide with the period of self-oscillations, the resonance will not occur, and the vibration level will be much less [9, 10]. The search for such conditions and determination of their interrelationships with its parameters is a goal for further research.

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