Background Gluon Effects on $B \to X_s \gamma \gamma$

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Abstract

We consider non-perturbative QCD effects on the energy spectrum of either one of the photons in $B \to X_s \gamma \gamma$. These are due to the subprocesses in which a charm quark loop interacts with a self-consistently produced background static QCD field. The magnitude is estimated to be a few percents in $B \to X_s \gamma \gamma$, but can be quite substantial in $B_s \to \gamma \gamma$. An extension of the Euler-Heisenberg Lagrangian is given.

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I. Introduction

The flavor changing neutral current inclusive transition $B \to X_s \gamma$ has been a subject of intense interest during the last few years. The basic theoretical framework is the Standard electroweak Model at scales $\sim m_W$ or $m_t$. QCD short distance corrections [1] are then incorporated in it via renormalization group technique to yield an effective weak interaction Hamiltonian valid at scales $\sim m_b$ relevant for $B$ decay processes. The special role that $B \to X_s \gamma$ enjoys is related to its being rather clean and as such is much more model independent compared to exclusive $B$-decays. Its rate has recently been measured [2] with increased accuracy and is in remarkable agreement with theoretical estimates. This implies that revelation of New Physics will have to wait and that further confrontations have to be mounted. In light of these developments, it is natural to consider other inclusive channels which as a whole will separate out contributions from various operators in the effective Hamiltonian. The branching ratios may be somewhat lower in other novel processes, but they will still be amenable to experiments in the new facilities either under construction or being proposed for the not too distant future. One such process is the inclusive process $B \to X_s \gamma \gamma$, which is expected to be $\sim 10^{-2}$ smaller in its branching ratio relative to $B \to X_s \gamma$. Just like $B \to X_s \gamma$, however, it is relatively clean after some proper precaution to take out effects due to strong resonances such as $\eta_c$ at its peak to the two photon spectrum. It will then provide further opportunities for testing the whole technology of weak decays, or better yet in pointing towards some clues of New Physics.

There have been some theoretical activities for the process $B \to X_s \gamma \gamma$, which corresponds at the quark level to the transition $b \to s \gamma \gamma$. Calculations were first done on the basis of pure electroweak theory [3-6] and subsequently improved to include the leading order renormalization group improved QCD effects [7-9]. These investigations (like most of the investigations for $B \to X_s \gamma$) are mostly based on the free quark decay of $b$, the justification of which is from the heavy quark effective theory (HQET) [10]. According to the argument given, corrections to the free quark results in inclusive processes are suppressed by powers of $(\frac{\Lambda_{QCD}}{m_b})^2$. Recently, however, Voloshin [11] has shown that corrections which scale like $(\frac{\Lambda_{QCD}}{m_c})^2$ should also exist. This last number is $\sim 1$ and thus has the potential of damaging all free quark estimates of inclusive $B$ decays. For the leading process $B \to X_s \gamma$, however, it has been shown that the Voloshin type of corrections have small coefficients multiplied to $(\frac{\Lambda_{QCD}}{m_c})^2$ and the overall effects are about $3\%$ of the main term [12-15]. In fact, a more positive view towards this kind of corrections is to interprete them as systematic accounts of the long distance non-resonant contributions of $c\bar{c}$ intermediate states. We shall subscribe to this constructive point of view and the present article is an investigation of the related effects in the parallel process $B \to X_s \gamma \gamma$. We shall show that while the
corresponding corrections apparently have terms which scale like \((\frac{\Lambda_{QCD} \cdot k}{m_c})^2\), where \(k\) is a typical photon momentum, in addition to corrections of the \((\frac{\Lambda_{QCD}}{m_c})^2\) type, the overall effects are also only a few percents, just as in \(B \to X_s \gamma\).

The plan of this article is as follows: in the next section, we shall briefly summarize some perturbative results of \(b \to s \gamma \gamma\) as a way to introduce our notation. In section III, we shall discuss the relevant diagrams which give rise to \(\frac{1}{m_c^2}\) and \(\frac{1}{m_c^4}\) corrections to this process. They come from a charm quark loop, from which an almost static gluon is emitted in addition to the two photons. Explicit formulae will be given and matrix elements by HQET will be used to estimate their contributions to the decay amplitude. Some numerical work will be presented in the last section, followed by concluding remarks.

II. \(b \to s \gamma \gamma\) in Standard Model with leading QCD corrections

Radiative \(b \to s\) processes is best described in the framework of the following effective Hamiltonian

\[
H_{\text{eff}}(1) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{bs}^* \sum_i C_i O_i, \tag{1}
\]

where \(V_{ij}\)'s are the CKM matrix elements, \(G_F\) is the Fermi constant, \(C_i\) are the QCD improved Wilson coefficients, and the \(O_i\)'s are local operators:

\[
O_1 = -\bar{s}_\alpha \gamma^\mu L c_\beta \cdot \bar{c}_\beta \gamma^\mu L b_\alpha, \\
O_2 = -\bar{s}_\alpha \gamma^\mu L c_\alpha \cdot \bar{c}_\beta \gamma^\mu L b_\beta, \\
O_{3,5} = -\bar{s}_\alpha \gamma^\mu L b_\alpha \cdot \sum_q \bar{q}_\beta \gamma^\mu (L, R) q_\beta, \\
O_{4,6} = -\bar{s}_\alpha \gamma^\mu L b_\beta \cdot \sum_q \bar{q}_\beta \gamma^\mu (L, R) q_\alpha, \\
O_7 = \frac{e}{16\pi^2} \bar{s}_\alpha \sigma^{\mu\nu} (m_b R + m_s L) b_\alpha \cdot F_{\mu\nu}, \\
O_8 = \frac{g_s}{16\pi^2} \bar{s}_\alpha \sigma^{\mu\nu} (m_b R + m_s L) (\frac{\lambda^a}{2})_{\alpha\beta} b_\beta \cdot G_{\mu\nu}^a. \tag{2}
\]
**Fig. 1:** One particle irreducible diagram (a) and one particle reducible diagrams (b) and (c) for $b(p) \rightarrow s(p')\gamma(k_1)\gamma(k_2)$. Diagrams with $(k_1, \mu) \leftrightarrow (k_2, \nu)$ should be added for (b) and (c).

Upon using this effective Hamiltonian, the basic diagrams for the process $b \rightarrow s\gamma\gamma$ are shown in fig.(1). Pure electroweak theory can be reclaimed by retaining only the $C_2O_2$ term in fig.(1a) and $C_7O_7$ term in figs.(1b-1c), with the $C_{2,7}$ given by Inami and Lim [16]. As expected, figs.(1b-1c) exhibit infra-red divergences, which are cancelled by the virtual radiative corrections to the process $b \rightarrow s\gamma$ as far as any observables are concerned. Alternatively, a practical measurable rate for the $b \rightarrow s\gamma\gamma$ can be formed by making a cut on the lower end of the energy of either of the two photons. This rate in the pure electroweak theory is known to be dominated by the one particle reducible diagrams of figs.(1b-1c); inclusion of QCD effects enhances their dominance, because $C_7$ becomes even bigger. Therefore, in what follows we will assume that the principal diagrams for $b \rightarrow s\gamma\gamma$ are figs.(1b-1c), with only $C_7O_7$ insertions. We will compare the 'non-perturbative' gluon corrections to be discussed in the next section relative to this contribution.

The $b \rightarrow s\gamma\gamma$ amplitude generated by figs.(1b-1c) strictly speaking cannot be described by an effective local Hamiltonian. However, for a fixed value of the invariants $p \cdot k_1$, $p \cdot k_2$ and $k_1 \cdot k_2$, we can formally write an effective Hamiltonian of the form $\bar{b} \Gamma_{\mu\nu} s A^\mu A^\nu$, where $\Gamma_{\mu\nu}$ carries $\gamma$-matrices and momenta. After summing over the polarizations of the s-quark and the photons, the transition rate can be represented as $< B|T(H_{eff}^\dagger(1)H_{eff}(1))|B >$,
where the following transition operator is defined
\[
< B|T(H_{eff}^+(1)H_{eff}(1)))|B > = (-\frac{e^2 G_F \lambda_t Q_d C_7}{\sqrt{2} \pi^2})^2 < B|\bar{b}(W_7^{-1})_{\mu\nu} m_{s} \rightarrow \gamma^\nu (W_7)_{\mu\nu} b|B >, \tag{3}
\]
where \( \lambda_t = V_{tb} V_{bs}^* \) and
\[
(W_7)_{\mu\nu} = \frac{1}{2} \left[ - \frac{1}{2p' \cdot k_2} k_1 \gamma_\mu (m_b R + m_s L)(m_b - \gamma + k_2) \gamma_\nu \\
+ \frac{1}{2p' \cdot k_2} \gamma_\mu (m_s - \gamma + k_1) k_1 \gamma_\mu (m_b R + m_s L) \\
+ (k_1, \mu \leftrightarrow k_2, \nu) \right]. \tag{4}
\]
A salient feature of the spectrum generated by eqs.(3-4) is that as a function of the invariant mass of the two photons, it peaks at small values. This point will become useful in simplifying the one-gluon corrections, which we now immediately turn to.

III. \( \frac{1}{m_c^2} \) and \( \frac{1}{m_c^4} \) corrections to \( b \rightarrow s\gamma\gamma \)

Because the coefficient \( C_2 \) is much bigger than \( C_{4,6} \), we shall discard effects due to \( O_{4,6} \) in our discussion. As for the operator \( O_2 \), we rewrite it by a Fierz transformation as
\[
O_2 = \frac{1}{3} O_1 + 2\tilde{O}_1, \tag{5}
\]
where
\[
\tilde{O}_1 = -\bar{c} \gamma_\mu \frac{\lambda^a}{2} L_c \cdot \bar{s} \gamma^\mu \frac{\lambda^a}{2} L_b. \tag{6}
\]

**Fig.2:** (a) A charm quark loop which generates an effective \( bs\gamma g \) vertex; (b) and (c) represent one particle reducible diagramss, which involve the \( bs\gamma g \) vertex above. Diagrams with \( (k_1, \mu \leftrightarrow (k_2, \nu) \) should be added.

The operator \( \tilde{O}_1 \) generates a b-s transition with a c quark loop. Hooking onto it a
photon and a gluon, fig.(2a) leads to an effective $bs\gamma g$ Voloshin local effective vertex

$$O_2 \to O_{Vol} = \frac{-e g_s}{72\pi^2 m_c^2} \bar{s}_\rho \gamma^\rho \frac{\lambda^a}{2} L b G^a_{\alpha\beta} \partial^\beta \tilde{F}^\rho_{\alpha},$$

(7)

in the soft gluon limit, where $\tilde{F}^\rho_{\alpha} = \frac{1}{2} \epsilon^{\rho\alpha\lambda\kappa} F_{\lambda\kappa}$, $\epsilon^{0123} = 1$) is the dual tensor to the electromagnetic field, and $G^a_{\alpha\beta}$ is the gluon field tensor. In this approximation which was used successfully and justified in ref.[11-15], the gluon inside the B-meson has been treated as a static field. The effective $bs\gamma g$-vertex in eq.(7) generates IPR $b \to s\gamma\gamma g$ amplitudes as shown in figs.(2b-2c). They cannot strictly be written as local effective interactions. However, as in the case of figs.(1b-1c) discussed before, for a given set of values for the momentum variables, the matrix element of figs.(2b-2c) (plus those with $k_1, \mu \leftrightarrow k_2, \nu$) can be formally looked upon as arising out of a Hamiltonian

$$H_{eff}(2) = (-\frac{4G_F}{\sqrt{2}} \lambda_t) (-\frac{e^2 Q_d C_2}{72\pi^2 m_c^2}) O_R,$$

(8)

with $(G_{\alpha\beta} \equiv g_s G^a_{\alpha\beta} \frac{\lambda^a}{2})$

$$O_R = \bar{s} \Gamma_{\alpha\beta}^{\mu\nu} G^\alpha_b A_\mu(k_1) A_\nu(k_2),$$

(9)

and

$$\Gamma_{\alpha\beta}^{\mu\nu} = \frac{1}{2p' \cdot k_2} \gamma^\nu(m_s - p' + k_1) \epsilon^\mu_{\rho\alpha} k_1^\lambda \gamma^\rho L k_1^\beta - \frac{1}{2p \cdot k_2} \epsilon^\mu_{\rho\alpha} k_1^\lambda \gamma^\rho L k_1^\beta(m_b - p' + k_2) \gamma^\nu + (k_1, \mu \leftrightarrow k_2, \nu).$$

(10)
Fig.3: A one particle irreducible diagram due to a charm quark loop, which generates an effective $b \to s \gamma \gamma g$ vertex. Five other diagrams due to permutations of $\gamma$’s and $g$ should be added.

Finally, fig.(3) represents an irreducible diagram for $b \to s \gamma \gamma g$ transition generated by the charm loop, again via $\tilde{O}_1$ of eq.(6), which scales like $1/m_c^2$. Unlike the vertex in fig.(2a), only the vector part of $\tilde{O}_1$ now gives a non-vanishing contribution to it. Fig.(3) can be written as an effective Hamiltonian, which is basically the photon-photon scattering amplitude. The general expression for arbitrary $k_1$, $k_2$ and $k_3$ is quite complicated [17], but, in the same spirit taken earlier, the gluon inside the B-meson can be treated as a background static field. We first form the gluon field tensor and then consider the limit with its momentum $k_3 \to 0$. In this provision, we obtain

$$H_{eff}(3) = \left( \frac{Q_w^2 e^2 g_s C_2}{16\pi^2} \right) \left[ -\frac{4G_F \lambda_t}{\sqrt{2}} \right] \times \left( G_\alpha^{\delta \kappa} F_{\kappa \mu} F_{\nu \delta}^\mu H_\nu^a \left[ (-\frac{112}{m_c^2 t} - \frac{2}{m_c^2 t^2}) + \frac{64}{m_c^2} - \frac{8}{t^2} \right] I_{00}^1 \right)$$

$$+ G_\alpha^{\delta \kappa} F_{\kappa \mu} H_\nu^a F_{\nu \delta}^\mu \left[ \frac{40}{t^2} - \frac{16}{m_c^2 t^2} \right] I_{00}^1$$

$$+ G_\alpha^{\delta \kappa} F_{\kappa \mu} H_\nu^a F_{\nu \delta}^\mu \left[ \frac{40}{t^2} - \frac{16}{m_c^2 t^2} \right] I_{00}^1$$

$$+ G_\alpha^{\delta \kappa} F_{\kappa \mu} F_{\nu \delta}^\mu H_{\nu \delta}^a \left[ \frac{32}{t^2} \right] \left[ \frac{1}{3m_c^2 t^2} \right] + \left[ -\frac{16}{m_c^2} + \frac{4}{t} \right] I_{00}^1$$

$$+ G_\alpha^{\delta \kappa} H_{\delta \kappa} F_{\nu \delta}^\mu F_{\nu \delta}^\mu \left[ \frac{11}{t^2} \right] \left[ \frac{1}{2m_c^2 t^2} \right] - \frac{8}{m_c^2 t^2} I_{00}^1$$

$$+ \left( \partial \lambda \partial \alpha F_{\mu \nu} \right) (\partial \delta F_{\mu \nu}) H_\alpha^a G_\delta^{\lambda \alpha} \left[ \frac{192}{t^2} \right] + \left[ -\frac{96}{m_c^2} + \frac{16}{t^2} \right] I_{00}^1$$

$$+ (-2304 \frac{m_c^4}{t^5} - 1152 \frac{m_c^2}{t^4} - \frac{144}{t^3}) I_{00}^1 \right).$$
where

\[ I_{00}^1 = \int_0^1 \frac{dx_1 dx_2 dx_3 q(1 - x_1 - x_2 - x_3)}{m_c^2 + x_1 x_2 t} = \frac{1}{2t} \ln^2 \left( \frac{\sqrt{4m_c^2 + t} + \sqrt{t}}{\sqrt{4m_c^2 + t} - \sqrt{t}} \right), \]

and

\[ f = \left( \frac{t}{t + 4m_c^2} \right)^{3/2} \ln \left( \frac{\sqrt{\frac{t}{m_c^2} + \frac{4m_c^2}{t}}}{2} \right), \]

\[ H_\mu^a = \bar{s}_\gamma \mu \lambda^a \frac{b}{2}, \quad H_{\mu
u} = \partial_\mu H_{\nu}^a - \partial_\nu H_{\mu}^a. \quad (12) \]

have been defined for \( t \equiv 2k_1 \cdot k_2 > 0 \). The prescription \( m_c^2 - i\epsilon \) should be taken to analytically continue to the physical region \( t < 0 \). As we mentioned in the last section, the region of interest for the spectrum \( b \rightarrow s\gamma\gamma \) turns out to be when one of the momenta of the two photons becomes soft. Then the expression in eq.(11) is further simplified into a local one

\[ H_{\text{eff}}(3) = \left( \frac{Q_s^2 e^2 g_s C_2}{16\pi^2 m_c^4} \right) \left( -\frac{4G_F\lambda_t}{\sqrt{2}} \right) \left[ \frac{4}{15} G_{\kappa}^a \mathcal{F}_{\kappa \mu} F^{\mu\nu} H_{\nu}^a \right] \left( -\frac{14}{45} \right) + G_{\kappa}^a \mathcal{F}_{\kappa \mu} H_{\nu}^{\mu \nu} \left( -\frac{7}{45} \right)

+ G_{\kappa}^a \mathcal{F}_{\kappa \mu} H_{\nu}^{\mu \nu} \left( \frac{1}{9} \right) + G_{\kappa}^a H_{\kappa \mu}^{\nu} F^{\mu\nu} \left( \frac{1}{18} \right), \quad (13) \]

which is in momentum space

\[ H_{\text{eff}}(3) = \left( -i \frac{Q_s^2 e^2 C_2}{16\pi^2 m_c^4} \right) \left( -\frac{4G_F\lambda_t}{\sqrt{2}} \right) \mathcal{S}(O_{IR})_{\mu \nu} G_{\delta} b A^\mu(k_1) A^\nu(k_2), \quad (14) \]

where

\[ (O_{IR})_{\mu \nu} \equiv \frac{1}{45} \left[ 3 k_1^\mu k_1^\nu ((k_1 + k_2)^\delta \gamma^\kappa - (k_1 + k_2)^\kappa \gamma^\delta) + 14(k_2^\mu \gamma ^\nu - k_2^\nu \gamma ^\mu) (k_1^\mu k_2^\kappa - k_1^\kappa k_2^\mu) + 7(g^{\mu \nu} k_1^\delta k_2^\kappa - k_1^\kappa k_2^\mu) (k_1 + k_2) \cdot \gamma + 3g^{\mu \nu} ((k_1 + k_2)^\kappa \gamma^\delta - (k_1 + k_2)^\delta \gamma^\kappa) k_1 \cdot k_2

- 7(g^{\mu \delta} g_\nu^\kappa - g^{\mu \nu} g_\kappa^\delta) k_1 \cdot k_2 (k_1 - k_2) \cdot \gamma

+ 3(g^{\mu \nu} k_1^\delta - g^{\mu \delta} k_1^\nu) k_1 \cdot k_2 \cdot \gamma + 3(g^{\nu \mu} k_2^\delta - g^{\nu \delta} k_2^\mu) k_2 \cdot k_1 \cdot \gamma

+ 3(g^{\mu \delta} k_1^\kappa - g^{\mu \kappa} k_1^\delta) \gamma^\nu k_1 \cdot k_2 + 3(g^{\nu \delta} k_2^\kappa - g^{\nu \kappa} k_2^\delta) \gamma^\mu k_1 \cdot k_2

+ 7(g^{\mu \delta} k_2^\kappa - g^{\mu \kappa} k_2^\delta) k_1 \cdot k_2 + 7(g^{\nu \delta} k_2^\kappa - g^{\nu \kappa} k_2^\delta) k_1 \cdot k_2

+ 14(g^{\mu \nu} k_1^\delta - g^{\mu \delta} k_1^\nu) \gamma^\nu k_1 \cdot k_2 + 14(g^{\nu \mu} k_2^\delta - g^{\nu \delta} k_2^\mu) \gamma^\mu k_1 \cdot k_2 \right] L. \quad (15) \]

The quarks inside the B-meson are in constant interaction with each other via soft gluon exchanges. The \( B \rightarrow X_s \gamma \gamma \) transition can thus be regarded as the quark transition \( b \rightarrow s\gamma\gamma \) in the presence of a background gluon field. This is the approach of Voloshin,
who further regarded this background gluon field as static. To first order in the strong QCD coupling, the effective Hamiltonians $H_{\text{eff}}(2)$ of eq.(8) and $H_{\text{eff}}(3)$ of eq.(14), in which a zero momentum gluon is emitted from a c-quark loop, would then implement this dynamics. The corresponding amplitudes so generated would add coherently with the main amplitude of eq.(3). These correction terms are small (to be justified a posteriori), so that the addition to the principal transition operator would be the interference terms of amplitudes of fig.(2-3) with those of figs.(1b-c).

The interference terms between $H_{\text{eff}}(1)$ due to $C_7 \gamma_7$ and $H_{\text{eff}}(2)$ are

$$< B | T(H_{\text{eff}}^\dagger(1)H_{\text{eff}}(2))|B> = (-\frac{e^2 Q_d G_F \lambda_t C_7}{\sqrt{2} \pi^2})(\frac{-4 G_F \lambda_t}{\sqrt{2}})(\frac{-e^2 Q_d C_2}{72 \pi^2 m_c^2})$$

$$< B | \bar{b}(W_7^\dagger)_{\mu\nu}\frac{m_s - \not{\gamma}}{2} \Gamma_{\alpha\beta} G_{\alpha\beta} b|B>,$$

and the complex conjugate. Corresponding to the interference of $H_{\text{eff}}(1)$ and $H_{\text{eff}}(3)$, we have

$$< B | T(H_{\text{eff}}^\dagger(1)H_{\text{eff}}(3))|B> = (-\frac{e^2 Q_d G_F \lambda_t C_7}{\sqrt{2} \pi^2})(\frac{-i Q_d^2 e^2 C_2}{16 \pi^2 m_c^4})(\frac{-4 G_F \lambda_t}{\sqrt{2}})$$

$$< B | \bar{b}(W_7^\dagger)_{\mu\nu}\frac{m_s - \not{\gamma}}{2} (O_{IR})_{\delta\epsilon} G^{\delta\epsilon\gamma} b|B>,$$

and its complex conjugate. Eqs.(3, 16 and 17) give the total transition amplitude

$$< T > = < B | T(H_{\text{eff}}^\dagger(1)H_{\text{eff}}(1))|B> + ( < B | T(H_{\text{eff}}^\dagger(1)H_{\text{eff}}(2))|B> + c.c. )$$

$$+ ( < B | T(H_{\text{eff}}^\dagger(1)H_{\text{eff}}(3))|B> + c.c. )$$

We rely on HQFT to evaluate the matrix elements [18]

$$< B(v)|\bar{b} \Gamma b|B(v)> = \frac{1}{2} Tr[\frac{1 - \not{\gamma}}{2} \Gamma],$$

and

$$< B(v)|\bar{b} \Gamma G_{\alpha\beta} b|B(v)> = \frac{\lambda_2}{2} Tr[\frac{1 - \not{\gamma}}{2} \Gamma \frac{1 - \not{\gamma}}{2} \sigma_{\alpha\beta}],$$

where $v$ is related to the momentum of $b$ by $p = m_b v$, $\Gamma$ is any Dirac structure, and $\lambda_2$ is related to the $B^* - B$ mass splitting with a numerical value $\lambda_2 = 0.12(Gev)^2$. We have normalized $< B(v)|B(v)> = 1$. The rate for $b(p) \rightarrow s(p') \gamma(k_1)\gamma(k_2)$ is

$$d\Gamma = \frac{1}{2(2\pi)^5} \delta^4(p - p' - k_1 - k_2) \frac{d^3 p'}{(p')^0} \frac{d^3 k_1}{2\omega_1} \frac{d^3 k_2}{2\omega_2} < T > .$$

IV. Numerical Results and Discussion
We would like to estimate the relative contributions of the three terms in eq.(18). As discussed earlier, we shall interprete corrections from the second and the third term as an indication of the long distance non-perturbative effects away from the peaks of the appropriate $c\bar{c}$ resonances. In this regard, the case for $\eta_c$ was addressed by the authors in ref.(8). They concluded that its effects on the spectrum of the two photon invariant mass are very localised. Our study here is on a different aspect of the same kind of issues.

We find it convenient to present our numerical work in a quantity introduced by these authors. This is the spectrum of the photon with the lower energy and that of the photon with the higher energy, defined by

$$\frac{d\Gamma^{L,H}}{dk_1} = \int \frac{d\Gamma}{dk_1 dk_2} \theta(\pm k_2 \mp k_1) dk_2,$$

where the integration domain, as in ref.(8), is restricted by the requirements that the energy of each photon be larger than $E_{\gamma}^{min} = 100 Mev$, and that the angles between any two outgoing particles be bigger than 20 degrees. These make it experimentally practical to distinguish from $b \rightarrow s\gamma$. We fix $\mu = m_b = 4.8 Gev$, $m_c = 1.5 Gev$ and $m_s = 450 Mev$, which lead to $C_2 = 1.09$ and $C_7 = -0.31$. Figs.(4-5) exhibit $50\times$ and $100\times$ the two interference terms separately vs. the main term of the spectrum from Eq.(18). Note that the photon energy $k_1$ is measured in unit of $m_b$. Also, all the curves have been normalized to $\frac{d\Gamma^L}{dk_1}$ of the main term at $k_1 = 0.086$. The results show that despite the $\frac{1}{m_c^2}$ or $\frac{1}{m_c^4}$ scaling the static gluon corrections from the charm loop is at most a few percents and hence not experimentally observable at this time. This is similar to the results for the one photon process, where however the corrections scale only as $\frac{1}{m_c^2}$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4}
\caption{The spectra of higher energy photon $\frac{d\Gamma^H}{dk_1}$ (HI) and lower energy photon $\frac{d\Gamma^L}{dk_1}$}
\end{figure}
due to the first main term of Eq.(18) and 50\times of the corresponding quantities due to the second term, denoted as HICOR and LOCOR, respectively, are shown. $k_1$ is measured in unit of $m_b$. All curves have been devided by the value of LO at $k_1 = 0.086$.

Fig.5: The spectra of higher energy photon $\frac{d\Gamma}{dk_1}$ (HI) and lower energy photon $\frac{d\Gamma}{dk_1}$ (LO) due to the first main term of Eq.(18) and 100\times of the corresponding quantities due to the third term, denoted as HICOR and LOCOR, respectively, are shown. $k_1$ is measured in unit of $m_b$. All curves have been devided by the value of LO at $k_1 = 0.086$.

The fact that a correction term which apparently goes like $(\frac{k_1 \cdot k_2}{m_c})^2$, where k’s are the typical photon momenta, is small is obviously because the two photon invariant mass stays small in the regions with appreciable rate. This situation would change dramatically for the exclusive process $B_s \to \gamma \gamma$ where the invariant mass $-2k_1 \cdot k_2 = m_b^2$. One can expect contributions from fig.(3) to be of order $(\frac{m_b}{m_c})^4$. On the other hand, $B_s \to \gamma \gamma$ is an exclusive channel, and therefore the same technique applied here and in refs.[11-15] cannot be directly relied upon. We are presently engaged in evaluating these effects.

Before closing, we would like to point out that eq.(13) is in agreement with the Euler-Heisenberg Lagrangian [19] and that eq.(11) is an exact extension of the photon-photon scattering amplitude to a situation when one of the photons ($G^a_{\mu\nu}$ in this case) is static while two on-shell photons can have any four momenta. The last photon is off-shell to give non-trivial kinematics.

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References

[1] B. Grinstein, R. Springer, and M. B. Wise, Phys. Lett. B202, 138 (1988); Nucl. Phys. B339, 269 (1990);
R. Grigjanis, P. J. O. Donnel, M. Sutherland, and H. Navelet, Phys. Lett. B213, 355 (1998); B286, 413 (E) (1992);
M. Misiak, Phys. Lett. B269, 161 (1991); Nucl. Phys. B393, 23 (1993); B439, 461 (E) (1995);
K. Adel and Y.-P. Yao, Mod. Phys. Lett. A8, 1679 (1993); Phys. Rev. D49, 4945 (1994);
M. Chiuchini, E. Franco, G. Martinelli, L. Reina, and L. Silvestrini, Phys. Lett. B316, 127 (1993); Nucl. Phys. B421, 41 (1994);
G. Cella, G. Curci, G. Ricciardi, and A. Viceré, Phys. Lett. B325, 227 (1994); Nucl. Phys. B431, 417 (1994);
K. G. Chetyrkin, M. Misiak, and M. Münz, Phys. Lett. B400, 206 (1997);
C. Greub and T. Hurth, Phys. Rev. D56, 2934 (1997);
A. Buras, A. Kwiatkowski, and N. Pott, hep-ph/9710336.

[2] M. S. Alam et al., Phys. Rev. Lett. 74, 2885 (1995).

[3] G.-L. Lin, J. Liu, and Y.-P. Yao, Phys. Rev. Lett. 64, 1498 (1990); Phys. Rev. D42, 2314 (1990).

[4] H. Simma and D. Wyler, Nucl. Phys. B344, 283 (1990).

[5] S. Herrlich and J. Kalinowski, Nucl. Phys. B381, 501 (1992).

[6] L. Reina, G. Ricciardi and A. Soni, Phys. Lett. B396, 231 (1997).

[7] C.-H. V. Chang, G.-L. Lin, and Y.-P. Yao, Phys. Lett. B415, 395 (1997).

[8] L. Reina, G. Ricciardi, and A. Soni, Phys. Rev. D56, 5805 (1997).

[9] G. Hiller and E. O. Iltan, hep-ph/9704388.

[10] J. Chay et al. Phys. Lett. B247, 399 (1990); A. F. Falk, M. Luke, and M. J. Savage, Phys. Rev. D49, 3367 (1994).

[11] M. B. Voloshin, Phys. Lett. B397, 275 (1997).

[12] A. K. Grant, A. G. Morgan, S. Nussinov, and R. D. Peccei, Phys. Rev. D56, 3151 (1997).

[13] Z. Legeti, L. Randall, and M. B. Wise, Phys. Lett. B402, 178 (1997).

[14] A. Khodjamirian, R. Rükle, G. Stoll, and D. Wyler, Phys. Lett. B402, 167 (1997).

[15] G. Buchalla, G. Isidori, and S.-J. Rey, hep-ph/9705253.

[16] T. Inami and C. S. Lin, Prog. Theor. Phys. 65, 297 (1981); 65, 1772 (1981).

[17] R. Karplus and M. Neuman, Phys. Rev. 80, 380 (1950).

[18] For a review, see M. Neubert, Phys. Reports 245, 260 (1994).
[19] H. Euler, Ann. Phys. (Leipzig) 26, 398 (1936); W. Heisenberg and H. Euler, Zeit. Phys. 98, 714 (1936).