Upscaling of absolute permeability for a super element model of petroleum reservoir

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Abstract. This paper presents a new method of local upscaling of absolute permeability for super element simulation of an oil reservoir. Upscaling was performed for each block of a super element unstructured grid. For this purpose, a set of problems of a one-phase steady-state flow was solved on a fine computational grid with the initial scalar field of absolute permeability with various boundary conditions. These conditions reflect the specific variants of filtrational flow through the super element and take into account the presence or absence of boreholes in the coarse block. The resulting components of the effective permeability tensor in each super element were found from the solution of the problem of minimizing the deviations of the normal flows through the super element faces, averaged on a detailed computational grid, from those approximated on a coarse super element grid. The results of using the method are demonstrated for reservoirs with river-type absolute permeability. The method is compared with the traditional methods of local upscaling.

1. Introduction
The super element model of petroleum reservoir simulation makes it possible to essentially speed up the computation process by using a coarse computational grid with a size of blocks being the same order as the distance between the wells [1]. The absolute (AP) and relative permeability (RP) functions contained in the equations of this model are obtained as a result of upscaling of the original characteristics by the scale of the coarse computational grid. Upscaling of RP in a super element model was considered earlier [2]. The present study focuses on AP upscaling. Simplified analytical formulas of upscaling prove to be applicable to a small number of idealized types of heterogeneity. Reservoirs with a more complex structure require using other averaging methods. Mathematical averaging tools for the problems of filter theory are described in [3] and allow us to estimate the effective parameters of porous media. Numerical methods of flow-based upscaling are currently the most widespread methods in the practice of reservoir simulation [4]. Both global [5] and more popular local methods [6, 4] are used for this purpose. It is obvious that upscaling must take into account not only the grid geometry, but also the computational scheme of the problem solved on a coarse grid. This paper proposes a specialized method for local upscaling of AP for a super element model of petroleum reservoir simulation.

2. Constitutive equations
The method of AP upscaling will use the mass balance equation for Darcy's flow neglecting the compressibility, gravitational and capillary forces:
\[ \text{div } \mathbf{u} = 0, \quad (1) \]
\[ \mathbf{u} = -\sigma \nabla p. \quad (2) \]

Here \( p(\mathbf{x}) \) is the pressure; \( \mathbf{u} \) is the flow rate; \( \sigma(\mathbf{x}) = k(\mathbf{x})/\mu \) is the specific conductivity of the reservoir; \( k(\mathbf{x}) \) is the scalar field of absolute permeability; \( \mu \) is the fluid’s dynamic viscosity, which will be assumed to be constant in the super element (SE) volume. The function \( k(\mathbf{x}) \) is defined on a geological grid, which is able to accurately reproduce the complex heterogeneous structure of the reservoir.

Equations (1)–(2) describe the flow in the domain \( \Omega \), bounded from above and below by the impermeable top and the bottom of the formation, and from one side–by the outer cylindrical surface, on which the hydrostatic pressure is maintained, as well as by the inner boundaries–perforated surfaces of wellbores with the radius \( r_w \), where the values of the bottomhole pressure \( p_w \) or the summarized flow \( q \) are set.

Let us consider the super element \( V \) with the outer boundary \( \Gamma \), consisting of faces \( \Gamma_j \), and with the inner boundary \( \gamma \). After integrating Equation (1), we obtain the relation

\[ Q_\Gamma + Q_\gamma = 0, \]
\[ Q_\Gamma = \int_{\Gamma} u_n \, d\Gamma = \sum_j \int_{\Gamma_j} u_{n,j} \, d\Gamma, \quad Q_\gamma = \int_{\gamma} u_n \, d\gamma = q, \]

where the sign for the average flow rate on the faces \( \Gamma_j \) is introduced:

\[ e, u_n = -\sigma \partial p/\partial n, \quad (3) \]

\( n \) is the outer normal direction. Let us determine the way of expressing velocities (3) through the average pressure

\[ \langle p \rangle = \frac{1}{|V|} \int_V p \, dV, \quad (4) \]

where \( |V| \) is the super element volume \( V \).

The set of values \( \langle p \rangle \) in the super elements determines the average pressure field in the reservoir. The use of a particular replenishment procedure for the grid function \( \langle p \rangle \) makes it possible to determine the gradient of the average pressure field and to use the relation similar to Darcy’s law for the average flow rate field (2):

\[ \mathbf{U} = -\Sigma \cdot \nabla \langle p \rangle. \]

Here \( \Sigma = k/\mu \); \( k \) is the effective tensor of absolute permeability in SE. Taking into the last formula, we can introduce approximation for the flow rate (3):

\[ u_n \approx U_n = \mathbf{U} \cdot \mathbf{n} = -\left( \Sigma \cdot \nabla \langle p \rangle \right) \cdot \mathbf{n} = -(k/\mu) \cdot \nabla \langle p \rangle = -\left( \frac{\partial \langle p \rangle}{\partial \mathbf{k}} \right), \quad (5) \]

where \( \mathbf{k} = \mathbf{n} \cdot \mathbf{K} \) is defined as the vector of full permeability on the super element edge, and \( \partial/\partial \mathbf{k} \) is the directional derivative, which is constructed by the colocations method [7, 2]:

\[ U_n \approx -\frac{k}{\mu} \frac{P_2 - P_1}{h_1 + h_2}, \quad k = \frac{\mathbf{K}_2 \cdot (h_1 + h_2)}{|\mathbf{K}_2|h_1 + |\mathbf{K}_2|h_2} \]

Here \( \mathbf{k}_1 = \mathbf{n} \cdot \mathbf{K}_1 \) and \( \mathbf{k}_2 = \mathbf{n} \cdot \mathbf{K}_2 \) are the full permeability vectors in the adjacent super elements with the AP effective tensors \( \mathbf{K}_1 \) and \( \mathbf{K}_2 \); \( P_1, P_2 \) are the average pressures at the distances \( h_1 \) and \( h_2 \) from the face on the directional lines \( k_1 \) and \( k_2 \), which are determined by the interpolation of values of the average pressure \( \langle p \rangle \). Thus, the colocation algorithm is a technique of the average field \( \langle p \rangle \) differentiation by the node values, which does not require an explicit replenishment of this function.
3. Upscaling algorithm

Coefficients of the effective AP tensors in each SE are found from the condition of the best approximation of the average normal flow rates (5). As an “exact” solution for the rates \( u_{nj} \) on the faces \( \Gamma_j \), we use the value (3), where the pressure \( p \) is obtained as the solution to the problem (1), (2) with the initial conductivity \( \sigma(x) \) on a fine enough grid in the SE volume.

To reflect various modes of flow through the SE, the following boundary conditions are set

\[
p^b \bigg|_{\partial \Omega} = \sum_{i} \delta^b_{ij} x', \quad b = 1..B,
\]

where \( x' \) are the coordinates at the outer boundary, \( \delta^b_{ij} \) is the Kronecker symbol, \( B \) is the number of various boundary conditions, equal to the problem dimension. For SE with the borehole \( \gamma \), we set the well discharge in addition to the conditions (6):

\[
\int_{\gamma} u_i \, d\gamma = q.
\]

The coefficients \( K^b \) of the tensor \( K \) are defined from the following condition

\[
R^2 = \frac{1}{B} \sum_{b=1}^{B} \rho^b \left( K^b \right) \rightarrow \min_{K^b}, \quad \rho^b \left( K^b \right) = \frac{1}{M} \sum_{m=1}^{M} \left( \Gamma_m \left( u^b_{nm} - U^b_{nm} \right) \right) \left( \max_{i = 1..M} \left( \Gamma_i \left| u^b_i \right| \right) \right)^{-1}.
\]

4. Testing

We will test the presented super element upscaling method (SEM) for a two-dimensional problem on the \((x, y)\) plane. Let the reservoir with the absolute permeability \( k = 0.001 \) have the high-permeable \((k = 1)\) channel. We will cover the rectangular area of the reservoir by a super element grid of 50 blocks (figure 1). The ratio of the channel width to the mean radius of the super elements is 0.25. Let us arrange two production (P-1, P-2) and two injection (I-1, I-2) wells with the same rates in absolute magnitude so that the high-permeable channels connect pairwise the wells I-1 with P-1 and I-2 with P-2. We will set the constant value of the hydrostatic pressure at the external boundaries of the area.

![Figure 1. Scheme of the reservoir with high-permeable channels.](image)

To evaluate the proposed upscaling method, the coefficients \( K^b \) of the effective AP tensors in each SE can be found in four ways. The simplest one is averaging by the volume method (VOL). The velocity averaging (VEL) and minimal dissipation (DIS) methods [4] with periodic and linear
boundary conditions are considered to be more reliable. The forth method is the presented super element upscaling method (SEM).

Figure 6 shows three typical types of SE, which differ in their location relative to the high-permeable channel: “A” demonstrates the channel which passes through the SE mainly in one direction, “B” shows the channel which changes its direction within the SE, and “C” illustrates the channel which does not pass through the SE. The AP upscaling for C-type SE gives the same results by all the listed methods: the effective tensor $K$ is isotropic, and its diagonal coefficients are equal to the reservoir permeability $k = 0.001$. The AP tensor components obtained by different methods for the SE types “A” and “B” are given in table 1.

| Method | Super element “A” | Super element “B” |
|--------|------------------|------------------|
| VOL    | $K^{xx}$ | 0.152 | 0.0000 | 0.0000 | 0.152 | 0.179 | 0.000 | 0.000 | 0.179 |
|        | $K^{yy}$ | 0.0000 | 0.0000 | 0.0000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
|        | $K^{xy}$ | 0.0000 | 0.0000 | 0.0000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| VEL    | $K^{xx}$ | 0.002 | 0.0015 | 0.0015 | 0.004 | 0.003 | 0.001 | 0.001 | 0.002 |
|        | $K^{yy}$ | 0.0015 | 0.0015 | 0.0015 | 0.003 | 0.003 | 0.001 | 0.001 | 0.002 |
|        | $K^{xy}$ | 0.0000 | 0.0000 | 0.0000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| DIS    | $K^{xx}$ | 0.057 | 0.0070 | 0.0070 | 0.155 | 0.103 | 0.062 | 0.062 | 0.060 |
|        | $K^{yy}$ | 0.0070 | 0.0070 | 0.0070 | 0.103 | 0.062 | 0.062 | 0.062 | 0.060 |
|        | $K^{xy}$ | 0.0000 | 0.0000 | 0.0000 | 0.062 | 0.062 | 0.062 | 0.062 | 0.060 |
| SEM    | $K^{xx}$ | 0.041 | 0.0820 | 0.0690 | 0.144 | 0.106 | 0.056 | 0.062 | 0.033 |
|        | $K^{yy}$ | 0.0820 | 0.0690 | 0.0690 | 0.106 | 0.056 | 0.062 | 0.062 | 0.033 |

Let us consider VEL results. The coefficients underrated by one or two orders on average are the result of using the periodic boundary conditions, which are unacceptable for the non-periodical structure of the $k(x)$ field. Calculation of the coefficients $K$ by SEM method for super elements with boreholes was carried out following the second scenario with the boundary conditions (7). As an example, we can compare the coefficients $K$ for the “A”-type SE (see table 1) and the coefficients for super elements with boreholes I-1: $K^{xx} = 0.235$, $K^{yy} = 0.404$, $K^{xy} = 0.404$, $K^{yx} = 0.702$.

Let us estimate the quality of the upscaling techniques by the values of deviations:

$$r_u = \frac{1}{M} \sum_{i=1}^{N} \left( \frac{(u_{ni} - U_{ni})}{u_i} \right)^2, \quad u_i = \max_{k=1..M} |u_{nk}|,$$

$$r_p = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{\langle p_i \rangle - P_i}{\Delta P} \right)^2, \quad \Delta P = \max_{i=1..N} \langle p_i \rangle - \min_{i=1..N} \langle p_i \rangle,$$

$$R_p = \max_{i=1..N} \left| \frac{\langle p_i \rangle - P_i}{\Delta P} \right|,$$

where $M$, $N$ are the total numbers of faces and blocks in the super element grid; the values $u_{ni}$ are calculated using Equation (3) for each SE face by the numerical integration of the solution obtained on a fine grid; the values $\langle p_i \rangle$ are obtained using Equation (4) for each SE; $U_{ni}$, $P_i$ are the average normal rate of flow through the super element faces and the average pressure in super elements, obtained from the solution on the course grid. The average fine grid step was about 1% of the average coarse grid step.

The values of these deviations for the considered methods are given in table 2. The best results demonstrate solutions built using DIS and SEM methods, though the latter method leads to smaller values of residual errors.

Minimizing these area-averaged residual errors is not the only criterion of AP upscaling methods quality. Equally important is to adequately describe, using a course grid, the structure of the flow determining the interwell interference. To analyze this description, we solve the problem of imitation of tracer injection alternately in each injector. The resulting tracer distribution in the reservoir will show the trajectory of the fluid injected into the wells, and the final concentrations in the producers will correspond to the degree of their interaction with the respective injectors. The exact solution of
the problem will be the solution built on a fine grid. Solution on a super element grid will be built using various upscaling methods.

**Table 2.** Average normal flow rate and average pressure deviations

| Deviation \ Method | VOL  | VEL  | DIS  | SEM  |
|-------------------|------|------|------|------|
| \( r_e \)         | 0.130| 0.222| 0.079| 0.063|
| \( r_p \)         | 0.158| 5.523| 0.186| 0.139|
| \( R_p \)         | 0.503| 18.76| 0.653| 0.434|

**Figure 2.** Tracer distribution from the injection well I-1 (on the left) and the well I-2 (on the right):

a) exact solution, b) VEL, c) VOL, d) DIS, e) SEM.

Figure 6 shows the tracer distribution in the reservoir at a fixed time moment, obtained in different ways. Superelements where the tracer concentration exceeds 20% are highlighted in color. For exact
solutions, such SE were determined by numerical integration of the concentration field on a detailed grid by the areas of SE.

It can be seen that AP upscaling by VEL method with the periodic boundary conditions does not give an adequate description of the flow structure. Among other methods, the super element upscaling method ensures the closest tracer distribution in the reservoir to the actual one. Upscaling performed using VOL and DIS methods does not provide a sufficient isolation of faces separating SE with the areas of two independent high-permeable channels. Due to this fact the tracer from the injector enters the producer, but they do not interact in reality. The final concentration $C$ of the tracer in the producer's liquid is shown in table 3.

| Calculation variant | Tracer injection into the well I-1 P-1 | Tracer injection into the well I-2 P-1 | Tracer injection into the well I-1 P-2 | Tracer injection into the well I-2 P-2 |
|---------------------|--------------------------------------|--------------------------------------|--------------------------------------|--------------------------------------|
| **exact solution**  | 0.375                                | 0.000                                | 0.000                                | 0.498                                |
| SEM                 | 0.291                                | 0.025                                | 0.039                                | 0.440                                |
| DIS                 | 0.233                                | 0.144                                | 0.084                                | 0.372                                |
| VEL                 | 0.164                                | 0.275                                | 0.148                                | 0.294                                |

5. Conclusion
We formulated a special method for local upscaling of absolute permeability, which makes it possible to minimize the error in flow calculation during super element simulation of oil reservoir development and accurately reflect the structure of the flow.

The comparative analysis showed a marked advantage of the presented upscaling method in comparison with the most popular methods of velocity averaging and minimum energy dissipation, used in rescaling of permeability field from a detailed geological grid to a course hydrodynamic one.

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