Kaon Physics with Light Sgoldstinos and Parity Conservation

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Abstract

Superpartners of goldstino — scalar and pseudoscalar sgoldstinos — interact weakly with ordinary particles. One or both of them may be light. We consider a class of supersymmetric extensions of the Standard Model in which interactions of sgoldstinos with quarks and gluons conserve parity but do not conserve quark flavor. If the pseudoscalar sgoldstino $P$ is light, $m_P < (m_K - 2m_\pi)$, and the scalar sgoldstino is heavier, $m_S > (m_K - m_\pi)$, an interesting place for experimental searches is the poorly explored area of three-body decays of kaons, $K_{S,L}^0 \rightarrow \pi^+ \pi^- P, K_{S,L}^0 \rightarrow \pi^0 \pi^0 P$ and $K^+ \rightarrow \pi^+ \pi^0 P$, with $P$ subsequently decaying into $\gamma\gamma$, possibly $e^+e^-$, or flying away from the detector. We evaluate the constraints on the flavor-violating coupling of sgoldstino to quarks which are imposed by $K^0_{S,L} - K^0_S$ mass difference and CP-violation in neutral kaon system, and find that these constraints allow for fairly large $\text{Br}(K \rightarrow \pi\pi P)$. Depending on the phase of sgoldstino-quark coupling, most sensitive to light pseudoscalar sgoldstino are searches either for decays $K_{L}^0 \rightarrow \pi\pi P$ or $K^+ \rightarrow \pi^+ \pi^0 P$ and $K^0_S \rightarrow \pi\pi P$. Generally speaking, there are no bounds on $\text{Br}(K_L^0 \rightarrow \pi\pi P)$. For most values of the phase, branching ratio of $K^+ \rightarrow \pi^+ \pi^0 P$ is about three orders of magnitude smaller than $\text{Br}(K_L^0 \rightarrow \pi\pi P)$ and the branching ratios of $K^0_S \rightarrow \pi\pi P$ are very small. However, for a certain phase the situation is opposite. We find that the most interesting ranges of branching ratios start at

$$\text{Br}(K_L^0 \rightarrow \pi\pi P) \sim 10^{-3}, \quad \text{Br}(K^+ \rightarrow \pi^+ \pi^0 P) \sim 10^{-4}, \quad \text{Br}(K^0_S \rightarrow \pi\pi P) \sim 10^{-3}.$$  

These searches for light pseudoscalar sgoldstino would be sensitive to the supersymmetry breaking scale $\sqrt{F}$ in the 100 TeV range and above, provided MSSM flavor violating parameters are close to their experimental bounds. We also briefly discuss the cases of light scalar sgoldstino and relatively heavy sgoldstinos.

1. In supersymmetric models of particle physics, spontaneous supersymmetry breaking results in the appearance of a Goldstone fermion — goldstino — which becomes the longitudinal component of gravitino. There should exist also superpartners of goldstino, pseudoscalar $P$ and scalar $S$, both neutral under all gauge interactions. The masses of $P$ and $S$ are in general different; their values are model-dependent and may well be lower than a few GeV or even a few MeV. These bosons — sgoldstinos — are indeed light in various versions of both gravity mediated theories \cite{1,2} and gauge mediated models (see, e.g., Ref. \cite{3} and references therein). It is certainly of interest to search for sgoldstinos at colliders \cite{4,5,6} and in rare decays \cite{7,8,9}.

Interactions of sgoldstinos with ordinary quarks, leptons and gauge bosons are suppressed by the scale, traditionally denoted by $\sqrt{F}$, at which supersymmetry is broken in the underlying theory. On the one hand, this means that sgoldstinos are naturally weakly coupled to ordinary particles. On the other hand, sgoldstinos, in similarity to gravitinos, are potential sources of information about this fundamental scale, which otherwise enters low energy physics indirectly, through soft supersymmetry breaking masses and couplings of ordinary particles and their superpartners.

Below the electroweak scale, interactions of sgoldstinos with quarks and gluons may or may not conserve parity. If parity is not conserved, there is no real distinction between the pseudoscalar sgoldstino $P$ and scalar sgoldstino $S$ insofar

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as their couplings to hadrons are concerned. If parity is conserved, the situation is different: we will see that the low energy phenomenology of the light pseudoscalar sgoldstino is not completely standard. In this paper we will mostly consider the case of parity-conserving sgoldstino interactions, and comment on the opposite case in appropriate places.

Parity conservation in sgoldstino interactions with quarks and gluons (as well as with leptons and photons) may not be accidental. As an example, it is natural in theories with spontaneously broken left-right symmetry, as we discuss in Appendix 1. We note in this regard that left-right symmetric extensions of MSSM (for a review see, e.g., Ref. 9) not only are aesthetically appealing but also provide a solution \[11\] to the strong CP-problem, which is a viable alternative to the Peccei–Quinn mechanism. It is likely that sgoldstino interactions will conserve parity in supersymmetric versions of other models (see, e.g., Ref. 12) designed to solve the strong CP-problem without introducing light axion.

Parity-conserving low energy interactions of pseudoscalar sgoldstino \( P \) with quarks are written\[12\] as follows\[13\] \[1\],

\[
\mathcal{L}_{P,q} = -P \cdot (h_{ij}^{(D)} \cdot \bar{d}_i i\gamma^5 d_j + h_{ij}^{(U)} \cdot \bar{u}_i i\gamma^5 u_j),
\]

where

\[
d_i = (d,s,b), \quad u_i = (u,c,t).
\]

In general, the coupling constants \( h_{ij}^{(D,U)} \) receive contributions from various terms in the Lagrangian of an underlying theory. In particular, there are always contributions proportional to the left-right soft terms in the matrix of squared masses of squarks.\[1\]

\[
h_{ij}^{(D)} = \frac{1}{\sqrt{2}} \frac{\tilde{m}_{ij}^{(LR)2}}{F},
\]

\[
h_{ij}^{(U)} = \frac{1}{\sqrt{2}} \frac{\tilde{m}_{ij}^{(LR)2}}{F}.
\]

We will use Eqs. \(2\) and \(3\) later on to estimate the sensitivity of low energy experiments to the scale of supersymmetry breaking, \(\sqrt{F}\).

The low energy interactions of scalar sgoldstino \( S \) are governed by the same coupling constants (again assuming parity conservation),

\[
\mathcal{L}_{S,q} = -S \cdot (h_{ij}^{(D)} \cdot \bar{d}_i d_j + h_{ij}^{(U)} \cdot \bar{u}_i u_j).
\]

It has been pointed out in Refs. 6, 8 that sgoldstino interactions generically violate quark flavor and CP. Flavor changing processes and CP-violation occur due to off-diagonal elements in the (Hermitian) matrices of couplings \( h_{ij}^{(D,U)} \); these off-diagonal elements are generally complex. In this note we will be primarily interested in kaon physics, in which case the relevant flavor-violating terms in the low energy Lagrangian are

\[
\mathcal{L}_{P,q} = -P \cdot (h_{12}^{(D)} \cdot \bar{d}_1 i\gamma^5 s + \text{h.c.}) ,
\]

\[
\mathcal{L}_{S,q} = -S \cdot (h_{12}^{(D)} \cdot \bar{d}_1 s + \text{h.c.}) .
\]

These terms induce two major effects in kaon physics. First, sgoldstino exchange contributes to \(K_L^0 - K_S^0\) mass difference and CP-violation in the system of neutral kaons, as shown in fig. 3. Second, if sgoldstinos are sufficiently light, they can

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1More realistically, loop effects induce non-zero parity-violating terms in the low energy sgoldstino-quark Lagrangian even if such terms are absent at the tree level. In the context of left-right models, we find in Appendix 1 that these loop contributions are small.

2If parity is not conserved in sgoldstino-quark interactions, the particle \( P \) couples to both pseudoscalar and scalar densities, \( \bar{q}_i i\gamma^5 q_j \) and \( \bar{q}_i q_j \). If the scalar coupling is considerable, low energy phenomenology of \( P \) is similar to that of \( S \). The latter will be discussed towards the end of this paper.

3One of the conditions ensuring parity conservation in sgoldstino-fermion interactions is that the left-right soft mass matrices are Hermitian, \([\tilde{m}_{ij}^{(LR)2}]^\dagger = \tilde{m}_{ij}^{(LR)2}\), and similarly for \( \tilde{m}_{ij}^{(LR)2} \) and \( \tilde{m}_{ij}^{(LR)2} \), where \( \tilde{m}_{ij}^{(LR)2} \) refers to sleptons.
Figure 1: Sgoldstino contribution to $K_L^0 - K_S^0$ mass difference and CP-violation in the system of neutral kaons.

be produced in kaon decays. Let us discuss these two effects in turn.

2. We begin with the pseudoscalar sgoldstino $P$. If it is light, $m_P \lesssim m_K$, its contribution to $K_L^0 - K_S^0$ mass difference is readily calculated in chiral theory

$$\Delta m_K \equiv m_{K_L^0} - m_{K_S^0} = \left[ (\text{Re} h_{12}^{(D)})^2 - (\text{Im} h_{12}^{(D)})^2 \right] \frac{B_0^2 f_K^2}{m_K (m_K^2 - m_P^2)},$$

where $f_K = 160$ MeV and the constant $B_0$ is related to quark condensate, $\langle 0 | \bar{q} q | 0 \rangle = -\frac{1}{4} B_0 f_\pi^2$, $f_\pi = 130$ MeV, that is $B_0 = M_K^2 / (m_d + m_s) = 1.9$ GeV. Neglecting the mass of $P$ and requiring that sgoldstino contribution does not exceed the actual value of $\Delta m_K$, we obtain in the case of light pseudoscalar sgoldstino

$$| (\text{Re} h_{12}^{(D)})^2 - (\text{Im} h_{12}^{(D)})^2 | < 5 \cdot 10^{-15}.$$  

(6)

In what follows we will not assume any cancellation between $(\text{Re} h_{12}^{(D)})^2$ and $(\text{Im} h_{12}^{(D)})^2$, so we estimate

$$| h_{12}^{(D)} | \lesssim 7 \cdot 10^{-8}.$$  

(7)

The contribution of the pseudoscalar sgoldstino exchange into CP-violating term $m'$ that mixes $K_1$ and $K_2$ (we use the standard notations [14]) is also straightforwardly evaluated in chiral theory,

$$\Delta m' = \text{Re} h_{12}^{(D)} \cdot \text{Im} h_{12}^{(D)} \cdot \frac{B_0^2 f_K^2}{m_K (m_K^2 - m_P^2)}.$$  

Requiring that the corresponding contribution into the parameter $\epsilon$ of CP-violation in kaon system is smaller than its measured value (and again neglecting sgoldstino mass), we find

$$| \text{Re} h_{12}^{(D)} \cdot \text{Im} h_{12}^{(D)} | < 1.5 \cdot 10^{-17}.$$  

(8)

Figure 2: Kaon decay into sgoldstino and pions.

Now, light pseudoscalar sgoldstino can be produced in kaon decays, as shown in fig. 2. Parity conservation implies

that these decays involve at least two pions in the final state,

$$K \rightarrow \pi \pi P.$$
The dominant amplitudes come from non-derivative couplings of mesons and $P$, so the pions are in $s$-wave state within this approximation. Then it is straightforward to see that the charged kaon decay $K^+ \rightarrow \pi^+\pi^0P$ is forbidden at this level.\footnote{As the only relevant term in the effective Lagrangian is given by Eq. (8), sgoldstino $P$ behaves in the process $K \rightarrow \pi\pi P$ as a component of an isodoublet. By Bose statistics, $s$-wave state of two pions has either isospin 0 or isospin 2. In the case of $K^+ \rightarrow \pi^+\pi^0P$, the isospin-0 state of two pions is impossible, so the total isospin of the final state is at least 3/2. Hence, $K^+ \rightarrow \pi^+\pi^0P$ is forbidden, in the leading order of derivative expansion, by the conservation of total isospin.}

On the other hand, the non-derivative couplings of $K^0$, pions and $P$ is straightforward to calculate in chiral theory,

$$\mathcal{L}_{P,K^0,\pi,\bar{\pi}} = \frac{2B_0}{3f_\pi} \cdot P \cdot \left[ h^{(D)}_{12} \left( K^0 \pi^+\pi^- + \frac{1}{2} K^0\pi^0\pi^0 \right) + h^{(D)}_{12} \left( \bar{K}^0 \pi^+\pi^- + \frac{1}{2} \bar{K}^0\pi^0\pi^0 \right) \right].$$

Neglecting for a moment the Standard Model CP-violation in the neutral kaon system, we find the partial widths of $K_L^0$ and $K_S^0$,

$$\Gamma(K_L^0 \rightarrow \pi^+\pi^-P) = 2\Gamma(K_L^0 \rightarrow \pi^0\pi^0P) = (\text{Re } h^{(D)}_{12})^2 \frac{m_K B_0^2}{576\pi^3 f_\pi^2} \cdot F(m_P, m_\pi, m_K),$$

$$\Gamma(K_S^0 \rightarrow \pi^+\pi^-P) = 2\Gamma(K_S^0 \rightarrow \pi^0\pi^0P) = (\text{Im } h^{(D)}_{12})^2 \frac{m_K B_0^2}{576\pi^3 f_\pi^2} \cdot F(m_P, m_\pi, m_K),$$

where $F(m_P, m_\pi, m_K)$ is a correction factor accounting for finite masses of pions and $P$; at $m_P \approx 0$ it is equal to $F \approx 0.3$.

Decays of charged kaons, $K^+ \rightarrow \pi^+\pi^0P$, are due to isospin violation as well as chiral loops and derivative couplings in the effective meson-sgoldstino Lagrangian. These are numerically small $[1, 3, 12]$, and the decay amplitudes are somewhat suppressed. A chiral theory estimate (see Appendix 2) gives at small $m_P$

$$\text{Br}(K^+ \rightarrow \pi^+\pi^0P) \sim 8.5 \cdot 10^{-10} \cdot |h^{(D)}_{12}|^2.$$ 

The ranges of $\text{Br}(K \rightarrow \pi\pi P)$, allowed by constraints $[5]$ and $[8]$, depend on the phase of $h^{(D)}_{12}$. Hence, we have to consider three cases.

(i) General phase of $h^{(D)}_{12}$, i.e., $\text{Im } h^{(D)}_{12} \sim \text{Re } h^{(D)}_{12}$. We make use of the constraint $[8]$ to obtain the following bounds

$$\text{Br}(K_L^0 \rightarrow \pi^+\pi^-P) \lesssim 2 \cdot 10^{-3}, \quad \text{Br}(K_L^0 \rightarrow \pi^0\pi^0P) \lesssim 1 \cdot 10^{-3},$$

$$\text{Br}(K_S^0 \rightarrow \pi^+\pi^-P) \lesssim 3 \cdot 10^{-6}, \quad \text{Br}(K_S^0 \rightarrow \pi^0\pi^0P) \lesssim 1.5 \cdot 10^{-6},$$

$$\text{Br}(K^+ \rightarrow \pi^+\pi^0P) \lesssim 1.5 \cdot 10^{-6}. \quad (9)$$

We note that in this case the decays of $K_S^0$ are not particularly interesting, whereas the branching ratio of $K^+ \rightarrow \pi^+\pi^0P$ is about three orders of magnitude lower than Br($K_L^0 \rightarrow \pi\pi P$),

$$\frac{\text{Br}(K^+ \rightarrow \pi^+\pi^0P)}{\text{Br}(K_L^0 \rightarrow \pi^+\pi^-P)} = \frac{1}{2} \frac{\text{Br}(K^+ \rightarrow \pi^+\pi^0P)}{\text{Br}(K_S^0 \rightarrow \pi^0\pi^0P)} \sim 10^{-3}. \quad (10)$$

(ii) Small phase of $h^{(D)}_{12}$, i.e., $\text{Im } h^{(D)}_{12} \approx 0$. In this case the constraint $[8]$ is irrelevant. The constraint $[5]$ does not imply any meaningful bounds on Br($K_L^0 \rightarrow \pi\pi P$) but gives for $K^+$-decay

$$\text{Br}(K^+ \rightarrow \pi^+\pi^0P) \lesssim 4 \cdot 10^{-4}. \quad (11)$$

The relation between the branching ratios, Eq. (11), still holds. The decays $K_S^0 \rightarrow \pi\pi P$ occur due to CP-violation in the Standard Model and are suppressed by the square of the SM parameter $\epsilon$. Bounds on Br($K_S^0 \rightarrow \pi\pi P$) are very strong,

$$\text{Br}(K_S^0 \rightarrow \pi^+\pi^-P) < 6 \cdot 10^{-9}, \quad \text{Br}(K_S^0 \rightarrow \pi^0\pi^0P) < 3 \cdot 10^{-9},$$

so in this case search for light pseudoscalar sgoldstinos in $K_S^0$-decays is hopeless.

(iii) Phase of $h^{(D)}_{12}$ is close to $\pi/2$, i.e., $\text{Re } h^{(D)}_{12} \approx 0$. Again, the constraint $[8]$ is irrelevant. In this case the decays $K_S^0 \rightarrow \pi\pi P$ are unsuppressed, whereas the decays $K_L^0 \rightarrow \pi\pi P$ are suppressed by $\epsilon^2$, as they originate from the CP-violation
in the Standard Model. The constraint (9) then implies the following bounds

\[
\begin{align*}
\text{Br}(K_L^0 \to \pi^+\pi^- P) & \lesssim 3 \cdot 10^{-6}, \\
\text{Br}(K_L^0 \to \pi^0\pi^0 P) & \lesssim 1.5 \cdot 10^{-6}, \\
\text{Br}(K_S^0 \to \pi^+\pi^- P) & \lesssim 1 \cdot 10^{-3}, \\
\text{Br}(K_S^0 \to \pi^0\pi^0 P) & \lesssim 0.5 \cdot 10^{-3}, \\
\text{Br}(K^+ \to \pi^+\pi^0 P) & \lesssim 4 \cdot 10^{-4},
\end{align*}
\]

Thus, unlike the previous two cases, the search for light pseudoscalar sgoldstino in three-body decays of $K_S^0$ and $K^+$ is of particular interest for $\text{Re } h_{12}^{(D)} \approx 0$.

To complete the picture of $K \to \pi\pi P$, let us briefly discuss the decays of sgoldstinos. Light sgoldstino decays into two photons or into a pair of charged leptons due to the following terms in the effective low-energy Lagrangian \[12, 13, 5\],

\[
\mathcal{L}_{P,\gamma} = g_\gamma \cdot P \cdot \frac{1}{2} \epsilon_{\mu\nu\lambda\rho} F^{\mu\nu} F^{\lambda\rho}, \quad \mathcal{L}_{P,l} = -h_{ij}^{(L)} \cdot P \cdot \bar{l}_i \gamma^5 l_j,
\]

where

\[
g_\gamma = \frac{1}{2\sqrt{2}} \frac{M_{\gamma\gamma}}{F},
\]

$M_{\gamma\gamma}$ is of the order of the photino mass, and the contribution to $h_{ij}^{(L)}$ related to soft slepton masses is

\[
h_{ij}^{(L)} = \frac{1}{\sqrt{2}} \frac{h_{ij}^{(LR)2}}{F}.
\]

Scalar sgoldstino interacts with photons and leptons in a similar way. Almost everywhere in the parameter space, two-photon decays of sgoldstinos dominate, although there exist regions where sgoldstinos decay mostly into $e^+e^-$-pair \[8\]. Depending on $g_\gamma$ and $h_{ij}^{(L)}$, sgoldstino decays either inside or outside the detector: as an example, at $M_{\gamma\gamma} \sim 100$ GeV and $\sqrt{F} \sim 1$ TeV, sgoldstino flies away from the detector if $m_P \lesssim 10$ MeV; otherwise it decays inside the detector into two photons. At $M_{\gamma\gamma} \sim 100$ GeV and $\sqrt{F} \sim 10$ TeV the borderline is at $m_P \sim 200$ MeV. Given the uncertainties in supersymmetry breaking parameters, no reliable estimate of sgoldstino lifetime can be presently made.

To summarize, very interesting probe of physics of supersymmetry breaking is the search for processes

\[
K_L^0 \to \pi^+\pi^- P \to \gamma\gamma, \quad K_L^0 \to \pi^0\pi^0 P \to \gamma\gamma,
\]

\[
K_L^0 \to \pi^+\pi^- P, \quad K_L^0 \to \pi^0\pi^0 P, \quad \text{invisible } P
\]

Generally, $K_L^0 - K_S^0$ mass difference does not impose any upper bounds on the branching ratios of these decays, but from Eq. (9) we infer that it is most interesting to search for these decays at the level $\text{Br}(K_L^0 \to \pi\pi P) \sim 10^{-3}$ and below. Search for

\[
K_L^0 \to \pi^+\pi^- P \to e^+e^-, \quad K_L^0 \to \pi^0\pi^0 P \to e^+e^-
\]

at the same level, is also of interest. For most values of the phase of the sgoldstino-quark coupling $h_{12}^{(D)}$, the branching ratio of the decay $K^+ \to \pi^+\pi P$ is about three orders of magnitude lower than $\text{Br}(K_L^0 \to \pi\pi P)$, whereas the decays $K_S^0 \to \pi\pi P$ are strongly suppressed. In the special case $\text{Re } h_{12}^{(D)} = 0$, however, the most promising places to search for light pseudoscalar sgoldstino are three-body decays of $K^+$ and $K_S^0$. The interesting ranges of the branching ratios of the decays

\[
K_S^0 \to \pi\pi P, \quad (P \to \gamma\gamma, \text{invisible}, e^+e^-)
\]

and

\[
K^+ \to \pi^+\pi^0 P, \quad (P \to \gamma\gamma, \text{invisible}, e^+e^-)
\]
start in that case at
\[ \text{Br}(K^+ \rightarrow \pi^+ \pi^0 P) \sim 4 \cdot 10^{-4}, \quad \text{Br}(K_S^0 \rightarrow \pi \pi P) \sim 10^{-3}, \]
while \( \text{Br}(K_L^0 \rightarrow \pi \pi P) \) is three orders of magnitude smaller than \( \text{Br}(K_S^0 \rightarrow \pi \pi P) \), and two orders of magnitude smaller than \( \text{Br}(K^+ \rightarrow \pi^+ \pi^0 P) \).

To the best of authors’ knowledge, there exists only one experimental limit [17] directly related to the processes under discussion,
\[ \text{Br}(K^+ \rightarrow \pi^+ \pi^0 X(X \rightarrow \text{invisible})) \lesssim 4 \cdot 10^{-5}, \quad m_X < 80 \text{ MeV}; \]
for heavier \( X \)-particle the limit is weaker. This result is important for the models with light pseudoscalar goldstino escaping from the detector. Indeed, the limit [14] is one order of magnitude stronger than the bound from kaon mixing in the cases \((ii)\) and \((iii)\) (see Eqs. (11) and (12)). Besides this direct constraint, there are two other experimental results which are of interest in our context. These are measurements of \( \text{Br}(K_L^0 \rightarrow \pi^+ \pi^- e^+ e^-) = 3.5 \times 10^{-7} \) and \( \text{Br}(K_S^0 \rightarrow \pi^+ \pi^- e^+ e^-) = 5.1 \times 10^{-5} \) [18]. These results demonstrate possible sensitivity of search for goldstino decaying inside a detector into \( e^+ e^- \) pair, although no limits on the partial widths of \( K_L^0,S \rightarrow \pi^+ \pi^0 X(X \rightarrow e^+ e^-) \) have been obtained yet.

3. To get an idea of the sensitivity of searches for pseudoscalar goldstino in kaon decays to the fundamental parameter of supersymmetric theories, \( \sqrt{F} \), let us make use of Eq. (3). We recall that the off-diagonal entries \( \tilde{m}_{12}^{(LR)2} \) in the matrix of squared masses of squarks are constrained irrespectively of light goldstinos. These constraints again come from FCNC processes and CP-violation, but now occurring due to the exchange of superpartners of ordinary particles [20]. The bounds depend on the masses of squarks, \( \tilde{m}_Q \), and gluinos, \( M_3 \). As an example, at
\[ \tilde{m}_Q = M_3 = 500 \text{ GeV} \]
one has [21]
\[
\sqrt{|\text{Re } [(\delta_1^{(D)})^2]|} = \frac{\sqrt{|\text{Re } \tilde{m}_{D,12}^{(LR)2} - (\text{Im } \tilde{m}_{D,12}^{(LR)2})^2|}}{m_Q^2} \lesssim 2.7 \cdot 10^{-3} \quad \text{from } \Delta m_K, \tag{16}
\]
\[
\sqrt{|\text{Im } [(\delta_1^{(D)})^2]|} = \frac{\sqrt{2|\text{Re } \tilde{m}_{D,12}^{(LR)2} \text{ Im } \tilde{m}_{D,12}^{(LR)2}|}}{m_Q^2} \lesssim 3.5 \cdot 10^{-4} \quad \text{from } \epsilon, \tag{17}
\]
\[ |\text{Im } (\delta_1^{(D)})| = \frac{|\text{Im } \tilde{m}_{D,12}^{(LR)2}|}{m_Q^2} \lesssim 2.0 \cdot 10^{-5} \quad \text{from } \epsilon'/\epsilon. \]

Note that the limits (16), (17) apply precisely to those combinations of \( \text{Re } \tilde{m}_{D,12}^{(LR)2} \) and \( \text{Im } \tilde{m}_{D,12}^{(LR)2} \) which enter Eqs. (4) and (8), respectively.

In the case of maximum CP-violation in the squark sector, i.e., at \( \text{Im } (\delta_1^{(D)}) \approx \text{Re } (\delta_1^{(D)}) \), one may take, as the best case, \( \delta_1^{(D)} \sim 2 \cdot 10^{-5} \); then the constraint (8) implies
\[ \sqrt{F} > 30 \text{ TeV}. \]

Searches for decays \( K_L^0 \rightarrow \pi \pi P \) would be sensitive to larger \( \sqrt{F} \). The values of \( \sqrt{F} \) accessible to these searches are comparable to astrophysical and reactor limits [3] which, however, apply only to models with very light goldstinos, \( m_P \lesssim 1 \text{ MeV} \). Also for comparison, current collider searches for goldstinos of masses \( m_{P,S} \lesssim 200 \text{ GeV} \) are sensitive to \( \sqrt{F} \) at the level of 1 TeV [3, 4, 5].

If CP is not significantly violated in the squark sector, \( \text{Im } \tilde{m}_{12}^{(LR)2} \approx 0 \), then larger values of flavor violating squark masses are allowed, see Eq. (16). With the above values of squark and gluino masses, Eq. (15), \( \delta_1^{(D)} \) may be as large as
3 \cdot 10^{-3}$. If this is so, searches for pseudoscalar sgoldstino in kaon decays are sensitive to even higher values of the scale of supersymmetry breaking, $\sqrt{F} \gtrsim 85$ TeV.

We present these estimates for illustration purposes only, as the allowed range of $\delta^{(D)}_{12}$ depends substantially on the parameters of superpartners in MSSM. It is worth noting that the branching ratios of $K \to \pi\pi P$ decrease rather mildly as $\sqrt{F}$ increases: they scale as $(\sqrt{F})^{-4}$.

4. Effective interactions lead also to direct mixing of neutral kaons and pion with the pseudoscalar goldstino,

$$
\mathcal{L}_{K^0P-\text{mixing}} = B_0 f_K \cdot \mathcal{P}(h_{12}^{(D)*} K^0 + h_{12}^{(D)} \bar{K}^0) \\
\mathcal{L}_{\pi P-\text{mixing}} = \frac{1}{\sqrt{2}} B_0 f_\pi (h_{11}^{(U)} - h_{11}^{(D)}) \cdot P \pi^0
$$

If sgoldstino is relatively light, these mixing terms also give rise to rare kaon decays. Namely, $K^0-P$-mixing induces decays

$$
K^0_S \to \gamma\gamma, \quad K^0_S \to e^+e^-, \quad K^0_S \to \mu^+\mu^-, \\
K^0_L \to \gamma\gamma, \quad K^0_L \to e^+e^-, \quad K^0_L \to \mu^+\mu^-,
$$

with rates proportional to $(h_{12}^{(D)})^2$ and depending on couplings of sgoldstinos to photons and leptons, see Eq. (11). To illustrate the situation with these decays, let us take $\sqrt{F} = 1$ TeV, $M_{\gamma\gamma} = 100$ GeV and $m_{L,11}^2 = m_e A_0$, $m_{L,22}^2 = m_\mu A_0$, $A_0 = 100$ GeV. Making use of the constraints (3, 8) one finds that the branching ratios of these decays must be fairly small,

$$
\text{Br}(K_L \to \gamma\gamma, e^+e^-, \mu^+\mu^-) < 4.5 \cdot 10^{-11}, \quad \text{Re } h_{12}^{(D)} \sim \text{Im } h_{12}^{(D)}, \\
\text{Br}(K_L \to \gamma\gamma, e^+e^-, \mu^+\mu^-) < 1.5 \cdot 10^{-8}, \quad \text{Im } h_{12}^{(D)} \approx 0, \\
\text{Br}(K_L \to \gamma\gamma, e^+e^-, \mu^+\mu^-) < 7.5 \cdot 10^{-14}, \quad \text{Re } h_{12}^{(D)} \approx 0.
$$

At larger $\sqrt{F}$ the allowed branching ratios are even smaller. The branching ratios of $K^0_S$ decays are below the sensitivity of current experiments. In other words, available data on the decays do not add extra constraints on $\text{Br}(K^0_S \to \pi\pi P)$. On the other hand, current experimental data (11) on leptonic decays of $K^0_L$

$$
\text{Br}(K^0_L \to e^+e^-) = (9.1 \pm 6) \cdot 10^{-12}, \\
\text{Br}(K^0_L \to \mu^+\mu^-) = (7.15 \pm 0.16) \cdot 10^{-9}.
$$

are comparable and even below the right hand sides of Eqs. (24). This means that Eqs. (22) and (23) may give stronger bounds on sgoldstino-quark coupling, as compared to Eqs. (1) and (8). The bounds coming from Eqs. (22), (23), however, strongly depend on unknown parameters like $\sqrt{F}$, $m_{L,11}^2$ and sgoldstino masses; in fact, in most part of the parameter space the analysis of decays $K^0_L \to e^+e^-$ and $K^0_L \to \mu^+\mu^-$ either does not lead to new constraints, or leads to weak bounds on the branching ratios of the three-body decays of kaons. As an example, with $\sqrt{F} = 1$ TeV, $m_{L,11}^2 = m_e A_0$, $A_0 = 100$ GeV, and assuming $\text{Br}(P \to e^+e^-) \sim 1$, one finds from Eq. (22) the following new bound

$$
\text{Br}(K^0_L \to \pi^+\pi^-P(P \to e^+e^-)) < 3 \cdot 10^{-4}.
$$

This is to be compared to Eq. (1); we see that new bound is indeed not much stronger than that coming from Eq. (8). In more realistic case of higher $\sqrt{F}$, the bound implied by Eq. (22) is weaker than Eq. (1).

The most interesting consequence of $\pi^0-P$ mixing, Eq. (16), are the decays

$$
K^+ \to \pi^+ P, \quad K^0 \to \pi^0 P,
$$

(24)
which involves ordinary weak interaction as shown in fig. 3. As the mixing term (13) does not involve the flavor-violating coupling $h_{12}^{(D)}$, the constraints (11), (12) do not apply here. It is straightforward to see that current searches for two-body decays (14) with $P$ subsequently decaying into $\gamma\gamma$, $e^+e^-$, $\mu^+\mu^-$ or flying away from the detector, are sensitive to the scale of supersymmetry breaking up to $\sqrt{F} \sim 1/10$ TeV, depending on squark masses, provided that $m_P < (m_K - m_\pi)$.  

5. For completeness, let us now discuss kaon physics in the case when, instead of pseudoscalar sgoldstino $P$, scalar sgoldstino $S$ is light. If $m_S < (m_K - m_\pi)$, both charged and neutral kaons can decay into $\pi S$, the rates being  

$$\Gamma(K^+ \to \pi^+ S) = |h_{12}^{(D)}|^2 \frac{B_0^2}{16\pi m_K} \cdot F'(m_P, m_\pi, m_K),$$

$$\Gamma(K_0^0 \to \pi^0 S) = (\text{Re} h_{12}^{(D)})^2 \frac{B_0^2}{16\pi m_K} \cdot F'(m_P, m_\pi, m_K),$$

$$\Gamma(K_0^0 \to \pi^0 S) = (\text{Im} h_{12}^{(D)})^2 \frac{B_0^2}{16\pi m_K} \cdot F'(m_P, m_\pi, m_K).$$

where $F'$ is again a correction factor; $F' \approx 0.9$ at $m_P \approx 0$.

Search for scalars in two-body decays of kaons, $K \to \pi S$, with $S$ either flying away or decaying into two photons or lepton pair inside the detector, is well explored area of experimental kaon physics. In particular, depending on the channel of $S$-decay, the existing limits on $K^+$-decay are in the range $\text{Br}(K^+ \to \pi^+ S) < 10^{-7}$ to $\text{Br}(K^+ \to \pi^+ S) < 10^{-9}$. These limits are much stronger than the bounds analogous to Eqs. (1), (2), so the consideration of $K_0^0 - K_0^0$ mass difference and CP-violation in kaon system is not relevant if $S$ is light. The sensitivity to $\sqrt{F}$ of searches for rare kaon decays $K^+ \to \pi^+ S$ is in the range up to $10^3 - 10^4$ TeV, provided that the scalar sgoldstino mass is smaller than $(m_K - m_\pi)$ (see Ref. [8] for details). Similar analysis applies to $K_0^0 \to \pi^0 S$ decay, and searches for rare neutral kaon decays have sensitivity to $\sqrt{F}$ of the same order.

Let us again note that if parity is violated substantially in sgoldstino-quark interactions, the discussion of this section applies to $P$ as well. The best place to search for both $P$ and $S$ (if any of them is lighter than $(m_K - m_\pi)$) is then two-body decays $K \to \pi P$, $K \to \pi S$.

6. Finally, as mentioned in Ref. [8], sgoldstinos heavier than kaons, though cannot be observed in kaon decays, still contribute to $K_0^0 - K_0^0$ mass difference through the diagrams of fig. 3. If $m_{S,P}$ are larger than the hadronic scale, sgoldstino exchange induces effective four-quark interactions at low energies,

$$\mathcal{L} = \frac{1}{m_S^2} \left( h_{ij}^{(D)} \bar{d}_i d_j + h_{ij}^{(U)} \bar{u}_i u_j \right) \left( h_{ij}^{(D)\ast} \bar{d}_i d_j + h_{ij}^{(U)\ast} \bar{u}_i u_j \right) + \frac{1}{m_P^2} \left( h_{ij}^{(D)} \bar{d}_i \gamma^5 d_j + h_{ij}^{(U)} \bar{u}_i \gamma^5 u_j \right) \left( h_{ij}^{(D)\ast} \bar{d}_i \gamma^5 d_j + h_{ij}^{(U)\ast} \bar{u}_i \gamma^5 u_j \right).$$

Making use of the Fierz identities one obtains, within vacuum insertion approximation,

$$\Delta m_K = \frac{1}{6} \text{Re} \left( h_{12}^{(D)} \right)^2 \frac{J_{K}\gamma_{K}}{m_K} \left( \frac{11}{m_P^2} - \frac{1}{m_S^2} \right).$$

This implies a constraint on $h_{12}^{(D)}$ similar to Eq. (3) but now depending on $m_S, m_P$. Similarly, the mass differences of neutral $D$- and $B$-mesons constrain $h_{12}^{(U)}$ and $h_{13}^{(D)}$, respectively. These constraints are summarized in Table 1 where we

\footnote{At $M_3 = m_Q = 1$ TeV one would have $\sqrt{F} \gtrsim 120$ TeV and 240 TeV instead of 30 TeV and 85 TeV, respectively.}
Table 1: Constraints on SUSY models from measurements of mass differences of neutral mesons at various $m_P$; flavor violating terms $\text{Re} \left( \delta_{12}^{(D)} \right) = \text{Re} \left[ \left( \tilde{m}^{LR}_{D_{12}} / \tilde{m}^2_{Q} \right)^2 \right]$, $\text{Re} \left( \delta_{13}^{(D)} \right) = \text{Re} \left[ \left( \tilde{m}^{LR}_{D_{13}} / \tilde{m}^2_{Q} \right)^2 \right]$, $\text{Re} \left( \delta_{ij}^{(U)} \right) = \text{Re} \left[ \left( \tilde{m}^{LR}_{U_{ij}} / \tilde{m}^2_{Q} \right)^2 \right]$ are set equal to their current limits (see text) at equal masses of squarks and gluino, $M_3 = \tilde{m}_Q = 500$ GeV.

To illustrate the sensitivity of meson mass differences to $\sqrt{F}$, we again choose parameters according to Eq. (13), set $\text{Re} \left( \delta_{ij}^{(U,D)} \right) = \text{Re} \left[ \left( m^{(LR)}_{U,D_{ij}} / m^2_{Q} \right)^2 \right]$ equal to their experimental limits [20], namely, $|\text{Re} \left( \delta_{12}^{(D)} \right)|^2 / 2 = 2.7 \times 10^{-5}$, $|\text{Re} \left( \delta_{12}^{(U)} \right)|^2 / 2 = 3.1 \times 10^{-2}$, $|\text{Re} \left( \delta_{13}^{(D)} \right)|^2 / 2 = 3.3 \times 10^{-2}$, at $m_Q = M_3 = 500$ GeV and under these assumptions transform the limits on $h_{ij}$ into limits on $\sqrt{F}$. The results are also presented in Table 1.

Heavy sgoldstino exchange would contribute also to CP-violation in neutral meson systems. The corresponding constraints on $\text{Im} \left( h_{12}^{(D)} \right)$ and $\text{Im} \left( h_{13}^{(D)} \right)$ are summarized in Table 2, where we again assume that $P$ is lighter than $S$.

Table 2: Constraints on SUSY models from CP-violation in $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ systems at various $m_P$; assumptions entering the bounds on $\sqrt{F}$ are presented in the text.

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Appendix 1

A prototype model with natural tree-level parity conservation in sgoldstino-quark and sgoldstino-gluon interactions is the supersymmetric left-right theory whose gauge group is $SU(3)_c \times SU(2)_L \times SU(2)_R$. Besides gauge superfields, it contains goldstino superfield (cf. Ref. [13]) $Z = \frac{1}{\sqrt{2}} (S + iP), \psi_z, F_z$, which is a gauge singlet, quark superfields $Q$ and $Q^c$ (the generation index is suppressed) transforming as (2, 1) and (1, 2) under $SU(2)_L \times SU(2)_R$, two Higgs bidoublets, both in (2, 2) representation, plus other fields which are required to break left-right symmetry spontaneously (see Refs. [21], [8], [10].
for details). For convenience, let us denote the two Higgs bidoublets as \( \Phi^{(U)} \) and \( \Phi^{(D)} \). The relevant terms in the effective Lagrangian are determined by a superpotential

\[
W = FZ + \frac{\sigma}{6}Z^3 + Y^{(i)}Q^T \tau_2 \Phi^{(i)} \tau_2 Q^c + Y^{(i)}ZQ^T \tau_2 \Phi^{(i)} \tau_2 Q^c + \ldots, \quad i = U, D, 
\]

(25)

the gauge kinetic functions

\[
f_3 = \frac{1}{g_3^2}(1 + 2\eta_3 Z + \ldots),
\]

(26)

\[
f_{L,R} = \frac{1}{g_{L,R}^2}(1 + 2\eta_{L,R} Z + \ldots),
\]

(27)

and the Kähler potential

\[
K = K_{\text{can}} + K_{\text{non-ren}},
\]

the latter containing canonical kinetic terms for chiral superfields and also non-renormalizable terms,

\[
K_{\text{non-ren}} = -\alpha_Z \frac{|Z|^4}{4} - B_{Qc} |Z|^2 Q Q^c - B_{Qc} |Z|^2 Q^c Q^c + \ldots
\]

The supersymmetric Lagrangian is then

\[
L = \int \frac{d^2 \theta}{4} f_a W_a W_a + W + \int d^2 \bar{\theta} \left( \frac{1}{4} f^* a W^* a W^* a + W^* a \right) + \int d^2 \theta \, d^2 \bar{\theta} \left( \frac{1}{4} f^* a W^* a W^* a + W^* a \right)
\]

(28)

Left-right symmetry is realized as an involution, with the interchange of \( SU(2)_L \leftrightarrow SU(2)_R \),

\[
\begin{align*}
\theta & \leftrightarrow \bar{\theta} \\
Q & \leftrightarrow Q^c \\
\Phi^{(i)} & \leftrightarrow \Phi^{(i)*} \\
Z & \leftrightarrow Z^* \\
W_L & \leftrightarrow W_R^* \\
W_{SU(3)} & \leftrightarrow W_{SU(3)}^*
\end{align*}
\]

This symmetry imposes the following constraints on the parameters of the model,

\[
\begin{align*}
F &= F^* \\
\sigma &= \sigma^* \\
Y^{(i)} &= Y^{(i)*} \\
A^{(i)} &= A^{(i)*}, \quad i = U, D \\
\eta_3 &= \eta_3^* \\
\eta_{L,R} &= \eta_{R,L}^* \\
B_{Qc} &= B_{Qc}^*
\end{align*}
\]

(29)

where

\[
A^{(i)} = F Y^{(i)}.
\]

To solve the strong CP-problem, one also requires

\[
\eta_L = \eta_L^*, \quad \eta_R = \eta_R^*.
\]

(30)
All these conditions are valid above the scale $M_R$ at which the left-right symmetry is spontaneously broken. Below this scale, the model reduces to MSSM with the sgoldstino superfield and with relations (29) valid at the tree level.

There exists a local supersymmetry breaking minimum of the scalar potential at which the $F$-component of the sgoldstino superfield has non-zero value,

$$\langle F_z \rangle = F,$$

Due to supersymmetry breaking, scalar and pseudoscalar components of $Z$ acquire masses,

$$m_Z^2 = \alpha_z F^2 + \sigma F, \quad m_{Z}^2 = \alpha_z F^2 - \sigma F.$$

Soft masses of squarks and gauginos, as well as trilinear soft terms are also generated. In particular, the gluino mass is

$$M_{\tilde{g}} = \eta_3 F,$$  \hspace{1cm} (31)

and trilinear soft terms involve

$$\mathcal{L}_{soft} = A^{(i)} \tilde{Q}^i \tau_2 \Phi^{(i)} \tau_2 \tilde{Q}^c, \quad i = U, D$$  \hspace{1cm} (32)

where $\tilde{Q}$ and $\tilde{Q}^c$ are squark fields.

Finally, breaking of $SU(2)_L$ is arranged in such a way that $\Phi^{(U,D)}$ obtain real vacuum expectation values [10].

$$\Phi^{(U)} = \begin{pmatrix} 0 & 0 \\ v_U & 0 \end{pmatrix}, \quad \Phi^{(D)} = \begin{pmatrix} v_D & 0 \\ 0 & 0 \end{pmatrix}, \quad v_{U,D} = \text{real}.$$  \hspace{1cm} (33)

The soft term (32) then produces left-right entries in the matrix of squared masses of squarks,

$$\tilde{m}^{(LR)}_D = A^{(D)} v_D, \quad \tilde{m}^{(LR)}_U = A^{(U)} v_U.$$  \hspace{1cm} (34)

Equations (31) and (34) illustrate the relations between the gaugino mass and sgoldstino couplings to gauge bosons, cf. Eq. (29). Note that the relations (29) indeed ensure parity conservation at the tree level.

The interactions of sgoldstinos $S$ and $P$ with ordinary quarks and gluons are read off from Eq. (28). Sgoldstino-gluon interaction is due to the second term in Eq. (26),

$$\mathcal{L}_{S,P,G} = - \frac{1}{2 \sqrt{2}} \eta_3 S G_{\mu\nu} G^{\mu\nu} + \frac{1}{4 \sqrt{2}} \eta_3 P \epsilon_{\mu\nu\lambda\rho} G^{\mu\nu} G^{\lambda\rho}.  \hspace{1cm} (35)$$

Due to Eq. (33), the matrices $\tilde{m}^{(LR)}_U, \tilde{m}^{(LR)}_D$ are Hermitian above the scale $m_R$, and we come to Eqs. (1), (2), (4). Hence, parity is conserved in sgoldstino-quark interactions at the tree level.

Below the scale $m_R$ of left-right symmetry breaking, relations (29) are no longer valid. In particular, $A^{(U,D)}$ receive non-Hermitian contributions in loops, and parity-violating couplings are generated. At the one-loop level, contributions
to the non-Hermitian parts of $A^{(U,D)}$ come from diagrams involving (s)quarks and Higgs(ino)s (assuming the validity of Eq. (23)). One obtains for the non-Hermitian part of $A^{(D)}$

$$\frac{A^{(D)} - A^{(D)\dagger}}{2} = \frac{1}{32\pi^2}(Y^{(U)\dagger} A^{(D)} + A^{(U)} Y^{(U)\dagger} Y^{(D)} - \text{h. c.}) \cdot \log \frac{M_R^2}{M_{\text{weak}}^2},$$

where $M_{\text{weak}}$ is the electroweak scale. The tree-level values of $Y^{(U,D)}$ and $A^{(U,D)}$ entering the right hand side are Hermitian; by unitary rotations of quarks one makes $A^{(D)}$ diagonal, $Y^{(D)} = Y^{(D)\text{diag}}$, whereas $Y^{(U)}$ takes the form

$$Y^{(U)} = V Y^{(U)\text{diag}},$$

where $V$ is the CKM matrix.

Parity-violating coupling relevant for kaon decays is proportional to $(A^{(D)} - A^{(D)\dagger})_{12}$. The contribution to this term that does not contain small Yukawa couplings of $d$, $u$, $s$- and $c$-quarks is

$$\frac{1}{2}(A^{(D)} - A^{(D)\dagger})_{12} = \frac{Y_t^2}{32\pi^2} \left(V_{13} V_{13}^{*} A^{(D)}_{12} - A^{(D)*}_{11} V_{13}^{*} V_{13}^{*}_{32}\right) \cdot \ln \frac{M_R^2}{M_{\text{weak}}^2}.$$  

Assuming that $A_{12}^{(D)}$ is not particularly small compared to $A_{11}^{(D)}$ and $A_{22}^{(D)}$, we find that the largest contributions here are

$$\tilde{h}_{12}^{(D)} = \frac{(A^{(D)} - A^{(D)\dagger})_{12}}{2A_{12}^{(D)}} = \frac{Y_t^2}{32\pi^2} \left(V_{cb} V_{ts} e^{i\phi_1} + \left|A_{13}^{(D)}\right| V_{cB} V_{ts} e^{i\phi_2}\right) \cdot \ln \frac{M_R^2}{M_{\text{weak}}^2},$$

where the phases $\phi_{1,2}$ are irrelevant for our estimates. With $Y_t \sim 1$ and $|A_{13}^{(D)}| \sim |A_{12}^{(D)}|$ we obtain

$$\frac{\tilde{h}_{12}^{(D)}}{|A_{12}^{(D)}|} \sim 10^{-3}.$$  

Assuming, by analogy to quark mixing, mild hierarchy $|A_{13}^{(D)}|/|A_{12}^{(D)}| \lesssim (\text{a few}) \cdot 10^{-2}$ we would find instead

$$\frac{\tilde{h}_{12}^{(D)}}{|A_{12}^{(D)}|} \lesssim (\text{a few}) \cdot 10^{-5}.  

(35)$$

Similar contributions to $\tilde{h}_{12}^{(D)}$ are expected from threshold effects. In terms of the branching ratios of kaon decays, this means

$$\begin{align*}
\text{Br}(K^0 \to \pi^0 \pi^0) / \text{Br}(K_L^0 \to \pi^0 \pi^0) & \lesssim 10^{-7}, \\
\text{Im} h_{12}^{(D)} & \sim \text{Re} h_{12}^{(D)}, \\
\text{Br}(K^+ \to \pi^+ \pi^0) / \text{Br}(K\bar{K} \to \pi^+ \pi^0) & \lesssim 3 \cdot 10^{-5}.
\end{align*}$$

Of course, these estimates depend on many unknown parameters, but they do suggest that the two-body decays of kaons are suppressed as compared to three-body decays.

Yet another source of parity violation in sgoldstino-quark interactions is the radiatively induced non-Hermitian part in the Yukawa matrix $Y^{(D)}$. Its effect, however, can be shown to be even smaller than the estimate (35); cf. Ref. 22.

**Appendix 2**

Charged kaon decay into pseudoscalar sgoldstino and pions is suppressed in chiral theory. There are two types of contributions into this process. The first one is due to isospin violation and originates from tree-level diagram proportional to the mass difference between up- and down-quarks,

$$\mathcal{L} = \frac{B_0 (m_u - m_d)}{2\sqrt{2}f_\pi} K^+ \bar{K}^0 \pi^- - \pi^0.$$  

(36)

If Eq. (30) does not hold at the tree level, there are also one-loop contributions involving gauginos. These contributions, however, are proportional to quark Yukawa couplings, $\Delta A^{(D)} \propto Y^{(D)}$, and hence are diagonal in the basis where $Y^{(D)}$ is diagonal. As the kaon decays occur due to non-zero $A_{12}^{(D)}$, the latter contributions are irrelevant for our purposes.
This leads to $K^+ \to \pi^- \pi^0 P$ decay via $K^0 - P$ mixing,
\[
\mathcal{L} = \frac{B_0^2 f_K h_{12}^{(D)} (m_u - m_d)}{2\sqrt{2} f_\pi^2 m_K^2} K^+ \pi^- \pi^0 P \\
\Gamma(K^+ \to \pi^0 P) = \frac{B_0^4 f_K^2 (m_u - m_d)^2}{8 \sqrt{2} f_\pi^3 f_K^2 m_K^4} |h_{12}^{(D)}|^2 F(m_P, m_\pi, m_K)
\]
where $F'(0, 0, m_K) = 1$, $F(0, m_\pi, m_K) = 0.3$. For $m_d - m_u = 5$ MeV we obtain
\[
\text{Br}(K^+ \to \pi^- \pi^0 P) = 1 \cdot 10^{10} |h_{12}^{(D)}|^2.
\]

Similar contribution from kinetic term
\[
\mathcal{L} = \frac{1}{\sqrt{2} f_\pi^2} \left( \left( \pi^- \partial_\mu \pi^0 - \pi^0 \partial_\mu \pi^- \right) \left( K^+ \partial_\mu K^0 - \tilde{K}^0 \partial_\mu K^+ \right) + \frac{\sqrt{3}}{4} \left( \left( \pi^0 \partial_\mu K^+ - \tilde{K}^0 \partial_\mu \pi^- \right) \left( K^0 \partial_\mu \pi^0 - \tilde{K}^0 \partial_\mu \pi^0 \right) \right) \right)
\]
is suppressed at least by an order of magnitude as compared to Eq. (37).

The second type of contributions occurs in the next-to-leading order in momenta. Generically, the next-to-leading amplitudes are sums of chiral loops and tree level contributions due to explicit higher order terms in the chiral Lagrangian [15],
\[
\mathcal{L}_{\text{int}} = L_5 \text{Tr} \left[ \partial_\mu U^\dagger \partial_\mu U \left( iU^\dagger h^{(D)} - i\dot{h}^{(D)} U \right) \right] \cdot 2B_0 P,
\]
where $3 \times 3$ matrix $U$ describes light mesons. The dimensionless coupling constant $L_5$ depends on the normalization point $\mu$; it runs according to
\[
L_5(\mu_2) = L_5(\mu_1) + \frac{3}{128\pi^2} \log \frac{\mu_1}{\mu_2}
\]
and its value at the $\rho$-meson mass is $L_5(m_\rho) = (1.4 \pm 0.5) \cdot 10^{-3}$ [16]. The dependence of $L_5$ on $\mu$ is cancelled out in physical quantities by chiral loops [15].

The part of $\mathcal{L}_{\text{int}}$ relevant for the decay $K^+ \to \pi^+ \pi^0 P$ is
\[
\mathcal{L}_{K^+, \pi^+, \pi^0, \pi^-} = L_5 \frac{8\sqrt{2} B_0}{f_\pi^2} h_{12}^{(D)} P \partial_\mu K^+ \left( \pi^- \partial_\mu \pi^0 - \pi^0 \partial_\mu \pi^- \right).
\]

The calculation of chiral loops is performed by making use of general technique developed in Ref. [13]. In standard notations [14] the amplitude reads
\[
A = \frac{B_0 h_{12}^{(D)}}{64\sqrt{2} \pi^2 f_\pi^3} \left( \tilde{J} \left( K, \eta, (p_P + p_{\pi^+}) \right) \left( m_K E_{\pi^0} + \frac{1}{2} \left( m_K^2 - m_\pi^2 \right) \left( m_\pi^2 - m_K^2 \right) \right) \right) \left( m_K E_{\pi^+} + \frac{1}{2} \left( m_K^2 - m_\pi^2 \right) \left( m_\pi^2 - m_K^2 \right) \right) + \frac{3}{2} \left( m_K^2 - m_\pi^2 \right) \left( m_\pi^2 - m_K^2 \right)
\]
\[
+ 2 \left( E_{\pi^+} - E_{\pi^0} \right) \left( 5k(K, \pi, \mu) + k(K, \eta, \mu) - 512\pi^2 L_5(\mu) \right)
\]
where
\[
\tilde{J} \left( A, B, s \right) = 2 + \left( \frac{\Sigma}{s} - \frac{\Sigma}{\Delta} \right) \ln \frac{m_A^2}{m_B^2} - \frac{\alpha}{s} \ln \frac{(s + \nu)^2 - \Delta^2}{(s - \nu)^2 - \Delta^2}
\]
\[
\Sigma = m_A^2 + m_B^2, \quad \Delta = m_B^2 - m_A^2, \quad \nu^2 = s^2 + m_A^4 + m_B^4 - 2s \left( m_A^2 + m_B^2 \right) - 2m_A^2 m_B^2,
\]
\[
k(A, B, \mu) = \frac{m_A^2 \ln \frac{m_A^2}{m_B^2} - m_B^2 \ln \frac{m_A^2}{m_B^2}}{m_A^2 - m_B^2}.
\]
and $E_A$ is energy of particle $A$ in the rest frame of decaying kaon. The dependence on the normalization point $\mu$ indeed cancels out. In the limit of massless $P$ one obtains

$$\text{Br}(K^+ \to \pi^- \pi^0 P) = 7.5 \cdot 10^{10} |\alpha^{(D)}_{12}|^2$$

We note further that the isospin violation, Eq. (36), gives rise to $s$-wave amplitude, whereas the chiral loops together with the term (38) lead to the $p$-wave amplitude. Thus, these two amplitudes do not interfere, and we merely sum up the contributions (37) and (39). We obtain finally

$$\text{Br}(K^+ \to \pi^- \pi^0 P) = 8.5 \cdot 10^{10} |\alpha^{(D)}_{12}|^2.$$  

We recall that this result is valid at small mass of the pseudoscalar goldstino $P$.

References

[1] J. Ellis, K. Enqvist, D. Nanopoulos, Phys. Lett. B147 (1984) 99; J. Ellis, K. Enqvist, D. Nanopoulos, Phys. Lett. B151 (1985) 357.
[2] T. Bhattacharya, P. Roy, Phys. Rev. D38 (1988) 2284.
[3] G. Giudice, R. Rattazzi, , Phys. Rep. 322 (1999) 419; S. Dubovsky, D. Gorbunov, S. Troitsky, Phys.-Usp. 42 (1999) 623.
[4] D. Dicus, S. Nandi, J. Woodside, Phys. Rev. D41 (1990) 2347; D. Dicus, S. Nandi, Phys. Rev. D56 (1997) 4166.
[5] E. Perazzi, G. Ridolfi, F. Zwirner, Nucl. Phys. B574 (2000) 3; B590 (2000) 287.
[6] P. Abreu et al. [DELPHI Collaboration], Phys. Lett. B494 (2000) 203.
[7] A. Brignole and A. Rossi, Nucl. Phys. B587 (2000) 3.
[8] D. Gorbunov, Light Goldstino: Precision Measurements versus Collider Searches, hep-ph/0007325.
[9] R. N. Mohapatra, Supersymmetric grand unification: An update, hep-ph/9911274.
[10] R. Kuchimanchi, Phys. Rev. Lett. 76 (1996) 3486; R. N. Mohapatra and A. Rasin, Phys. Rev. Lett. 76 (1996) 3490; R. N. Mohapatra and A. Rasin, Phys. Rev. D54 (1996) 5835; R. N. Mohapatra, A. Rasin and G. Senjanovic, Phys. Rev. Lett. 79 (1997) 4744.
[11] S. H. Tye, Phys. Rev. Lett. 47 (1981) 1035; V. A. Rubakov, JETP Lett. 65 (1997) 621.
[12] T. Bhattacharya, P. Roy, Phys. Lett. B206 (1988) 655.
[13] A. Brignole, E. Perazzi, F. Zwirner, JHEP 9909 (1999) 002.
[14] Particle Data Group, Eur. Phys. J. C15 (2000) 1.
[15] J. Gasser and H. Leutwyler, Nucl. Phys. B250 (1985) 465; 517.
[16] A. Pich, Effective Field Theory (Lectures at the 1997 Les Houches Summer School "Probing the Standard Model of Particle Interactions"), hep-ph/9806303.
[17] S. Adler et al. [E787 Collaboration], Phys. Rev. D63 (2001) 032004.
[18] KTeV Collaboration (J. Adams et al.); Phys. Rev. Lett. 80 (1998) 4123; Y. Takeuchi et al., Phys. Lett. B443 (1998) 409.
[19] NA48 Collaboration, Talk given by V. Kekelidze at XXXth International Conference on High Energy Physics, July 27 - August 2, 2000, Osaka, Japan.; A. Lai et al. [NA48 Collaboration], Phys. Lett. B496 (2000) 137.
[20] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B477 (1996) 321; M. Ciuchini et al., JHEP 9810 (1998) 008.
[21] R. Kuchimanchi and R. N. Mohapatra, Phys. Rev. D48 (1993) 4352; R. Kuchimanchi and R. N. Mohapatra, Phys. Rev. Lett. 75 (1995) 3989; C. S. Aulakh, K. Benakli and G. Senjanovic, Phys. Rev. Lett. 79 (1997) 2188; C. S. Aulakh, A. Melfo and G. Senjanovic, Phys. Rev. D57 (1998) 4174.
[22] M. E. Pospelov, Phys. Lett. B391 (1997) 324.