The decoherence dynamics of multipartite entanglement in a non-Markovian environment

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Abstract
We consider four two-level atoms interacting with independent non-Markovian reservoirs with detuning. We mainly investigate the effects of the detuning and the length of the reservoir correlation time on the decoherence dynamics of multipartite entanglement. We find that the time evolution of the entanglement of atomic and reservoir subsystems is determined by a parameter which is a function of the detuning and the reservoir correlation time. We also find that the decay and revival of the entanglement of the atomic and reservoir subsystems are closely related to the sign of the decay rate. We also show that the cluster state is the most robust to decoherence compared with Dicke, GHZ and W states for this decoherence channel.

1. Introduction
Entanglement plays a central role in quantum information processing. Since the maintenance of entangled states is very important in quantum information processing systems, the study of the effect of decoherence on entangled states would be of considerable importance from a theoretical as well as an experimental point of view. Recently, entanglement of open quantum systems has attracted considerable attention due to its significance for both fundamentals and applications of quantum information processing [1–7]. Yu and Eberly [3] found that the decay of a single qubit coherence can be slower than the decay of entanglement, and they named the abrupt disappearance of entanglement at a finite time entanglement sudden death (ESD), which has been experimentally tested very recently [8]. Later López et al [5] revealed that when the bipartite entanglement suddenly disappears, the entanglement of the corresponding reservoir suddenly and necessarily appears, which is called entanglement sudden birth (ESB). Although bipartite entanglement is well understood in many aspects, until now multipartite entanglement is far from clear and thus deserves profound investigation. Several important applications of multipartite entangled states, such as quantum error correction, quantum computing, etc have been found (for recent reviews see [9–11] and references therein). Recently, researchers investigated the decoherence dynamics of multipartite entangled states; for instance, Gühne, Bodoky and Blaauboer [12] discussed some multipartite entanglement properties under the influence of a global dephasing process using the geometric measure of entanglement, and they showed that the Dicke state is the most robust to decoherence compared with GHZ, cluster and W states. Borras et al [13] investigated the robustness of highly entangled multiqubit states under different decoherence channels [14], and later, they further studied the geometry of robust entangled multiparticle states under decoherence [15].

In realistic physical systems the assumption of Markovian dynamics can only be an approximation. Generally speaking the quantum system of interest interacts with the reservoirs, whose spectral density strongly varies with frequency, which is called a non-Markovian open quantum system [16]. Non-Markovian dynamics is characterized by the existence of a memory time scale during which information that has been transferred from the system to the environment can flow back into the system. Non-Markovian systems appear in many branches of physics, such as quantum optics [16–18], solid state physics [19] and quantum chemistry [20]. In quantum information processing, the non-Markovian character of decoherence has to be considered [21]. Non-Markovian dynamics has drawn great attention including continuous-variable [22–28] and discrete-variable systems [29–37]. Very recently N qubits initially in the mixed
GHZ-type and W-type states interacting with independent structured reservoirs have been investigated and it is found that the N-qubit entanglement revivals are related to the qubit number N and the purity of the initial state of N qubits [35]. The measure of the degree of non-Markovian behaviour [38] and a basic relation between the quantum Fisher information flow and non-Markovianity have been proposed for quantum dynamics of open systems [39].

In this paper, we are interested in the decoherence dynamics of multipartite entanglement under the influence of a non-Markovian reservoir. We mainly investigate the effects of the detuning and the length of the reservoir correlation time on the decoherence dynamics of the multipartite entanglement. We consider four noninteracting two-level atoms interacting with individual non-Markovian reservoirs with detuning. We let the four atoms initially be prepared in different four-particle entangled states (cluster, Dicke, GHZ and W states). We find that the dynamical behaviours of the entanglement of the atomic and reservoir subsystems for various initial states are determined by a complex parameter which is a function of the detuning and the reservoir correlation time. The real part of this parameter is closely related to the decay of the entanglement and the reservoir correlation time. The imaginary part of this parameter is a function of the detuning and the length of the reservoir correlation time. We find that the cluster state is the most robust to decoherence, by the relation

\[ \psi_t = v(t)|1_a\rangle|0_r\rangle + \sum_{k=1}^N \sum_{r=1} d_k(t)|0_r\rangle|1_k\rangle, \]

where the state \(|1_k\rangle\) represents the reservoir having one excitation in mode \(k\).

Similar to the method used in [16], we can obtain a closed equation for the coefficient \(v(t)\) in equation (2),

\[ \dot{v}(t) = -\int_0^t f(t - t_1)v(t_1)dt_1, \]

where the kernel \(f(t - t_1) = \int d\omega J(\omega) \exp[i(\omega - \omega)(t - t_1)]\) is related to the spectral density \(J(\omega)\) of the reservoir. We take a Lorentzian spectral density of the reservoir \([16, 38]\)

\[ J(\omega) = \frac{1}{2\pi} \frac{\gamma_0\lambda^2}{(\omega_0 - \delta - \omega)^2 + \lambda^2}, \]

the centre of which is detuned from the transition frequency \(\omega_0\) of the two-level atom by an amount \(\delta\). And the parameter \(\lambda\) defines the spectral width of the coupling, which is associated with the reservoir correlation time by the relation \(\tau_B = \lambda^{-1}\) and the parameter \(\gamma_0\) is related to the relaxation time scale \(\tau_R\) by the relation \(\tau_R = \gamma_0^{-1}\).

By making the Laplace transformation of equation (3), we can obtain the solution of \(v(t)\) with initial condition \(v(0) = 1\),

\[ v(t) = e^{-(\lambda - i\delta)t/2} \left[ \cos(\chi t/2) + \frac{\lambda - i\delta}{\chi} \sinh(\chi t/2) \right], \]

where

\[ \chi = \sqrt{(\lambda - i\delta)^2 - 2\gamma_0\lambda}. \]

Furthermore, if we take the form of a collective state of the reservoir, namely, letting \(|1_r\rangle = (1/\mu(t)) \sum_{k=1}^N d_k(t)|1_k\rangle\), with \(\mu(t) = \sqrt{1 - |v(t)|^2}\), equation (2) can be rewritten as

\[ |\psi_t\rangle = v(t)|1_a\rangle|0_r\rangle + \mu(t)|0_a\rangle|1_r\rangle. \]

There are many entanglement measures to quantify the bipartite entanglement such as von Neumann entropy, negativity [40, 41], concurrence [42], relative entropy [43], etc. However, generally speaking, for multipartite system the definition of entanglement measure is difficult. Up to now, there are some entanglement measures proposed to quantify the multipartite entanglement, which include Schmidt measure [44], the geometric measure of entanglement [45], the global entanglement measure [46, 47], etc. In this paper, we will use a very popular entanglement measure [13, 15] which is averaged over all possible bipartitions. The mathematical definition of this measure is

\[ E = \frac{1}{[N/2]} \sum_{m=1}^{[N/2]} E^{(m)}, \]

where
where

\[ E^{(m)} = \frac{1}{N_{\text{bipart}}} \sum_{i=1}^{N_{\text{bipart}}} E(i). \]  

(9)

\( E(i) \) indicates the entanglement connected to single bipartition of the \( N \)-qubit system, and \( E^{(m)} \) denotes the average entanglement over all nonequivalent bipartitions \( N_{\text{bipart}} \) between subsets of \( m \) qubits and the remaining \( N-m \) qubits. If one uses the linear entropy \( S_L \) of the reduced density matrix of the smaller bipartitions to compute \( E(i) \), it will reduce to the well-known Meyer–Wallach multipartite entanglement measure [46]. Because in this paper we will deal with mixed states, we take the negativity to measure the bipartite entanglement. The normalized negativity is defined as [15]

\[ E(i) = \frac{2}{2^m - 1} \sum |\alpha_i|, \]  

(10)

where \( \alpha_i \) is the negative eigenvalue of the partial transpose matrix for \( m \) and the remaining \( N-m \) bipartition.

3. Results and discussion

We suppose that four two-level atoms are initially prepared in some famous multipartite entangled states (cluster, Dicke, GHZ and W states), and the corresponding reservoirs are initially prepared in the vacuum state. For W state the initial state of the atom–reservoir system is

\[ |\phi_0\rangle = (|0001\rangle_{a_1a_2a_3a_4} + |0010\rangle_{a_1a_2a_3a_4} + |0100\rangle_{a_1a_2a_3a_4}
+ |1000\rangle_{a_1a_2a_3a_4})\bar{0}\bar{0}\bar{0}\bar{0}/2, \]  

(11)

where the subscripts \( a_i(i = 1, 2, 3, 4) \) represent the atom, and \( r_i(i = 1, 2, 3, 4) \) refer to the corresponding reservoir. From equation (7), the evolution of the total system can be obtained as

\[ |\phi_{\tau}\rangle = |0001\rangle_{a_1a_2a_3a_4}\bar{0}\bar{0}\bar{0}\bar{0}r_1r_2r_3r_4\mu(t)|0\rangle_{a_1}+\mu(t)|0\rangle_{a_1}|1\rangle_{r_1}+\mu(t)|0\rangle_{a_1}|0\rangle_{r_1}+\mu(t)|0\rangle_{a_1}r_1r_2r_3r_4\mu(t)|0\rangle_{a_1}|1\rangle_{r_1}+\mu(t)|0\rangle_{a_1}|1\rangle_{r_1}|1\rangle_{r_1} r_2r_3r_4/2. \]  

(12)

So the reduced density operator of the atomic subsystem \( \rho_a(t) = \text{Tr}_r(|\phi_0\rangle\langle\phi_0|) \) and the reduced density operator of the reservoir subsystem \( \rho_r(t) = \text{Tr}_a(|\phi_0\rangle\langle\phi_0|) \) can be obtained. Then from equations (8)–(10) we can obtain the degree of the entanglement of the atomic and the reservoir subsystems. For convenience, we denote the degree of the entanglement of the atomic subsystem and the corresponding reservoir subsystem by \( E_a \) and \( E_r \), respectively. Here for the W state, \( E_a \) and \( E_r \) can be obtained as

\[ E_a = [16 - 16|\nu(t)|^2 - 6(\sqrt{7}|\nu(t)|^4 - 8|\nu(t)|^2 + 4 + 4\sqrt{2}|\nu(t)|^4 - 2|\nu(t)|^2 + 1)]/24, \]  

(13)

and

\[ E_r = [16|\nu(t)|^2 - 6(\sqrt{7}|\nu(t)|^4 - 6|\nu(t)|^2 + 3 + 4\sqrt{2}|\nu(t)|^4 - 2|\nu(t)|^2 + 1)]/24. \]  

(14)

Figure 1. In the Markovian regime (\( \lambda = 10\gamma_0 \), \( \delta = 0 \)) \( E_a \) and \( E_r \) as functions of scaled time \( \gamma_0 t \) for various initial states: (a) the atomic subsystem; (b) the reservoir subsystem.

Similarly we can obtain \( E_a \) and \( E_r \) for the other initial states, cluster state \( |CL_4\rangle = (|0000\rangle + |0011\rangle + |1100\rangle - |1111\rangle)/2 \), Dicke state \( |D_4\rangle = (|0011\rangle + |0101\rangle + |1001\rangle + |1100\rangle + |1010\rangle)/\sqrt{6} \) and GHZ state \( |GHZ_4\rangle = (|0000\rangle + |1111\rangle)/\sqrt{2} \), respectively.

First we consider the resonant case, i.e., \( \delta = 0 \). (i) \( \lambda = 10\gamma_0 \), which corresponds to the Markovian regime. In figure 1 we plot the entanglement evolution of the atomic and reservoir subsystems for initial cluster, Dicke, GHZ and W states. From figure 1(a) we can see that the cluster state is the most robust against decoherence. The result is quite different from that of [12], in which the Dicke state is the most robust against decoherence among GHZ, cluster and W states. This is because the decoherence channel used in [12] is different from ours. In [12] the global dephasing channel is considered, while in this paper we consider the amplitude damping channel. This means that the most robust multipartite entangled state might be different for different decoherence channels. It should be noted that the entanglement measure different from ours is used in [12], but we use our entanglement measure to recalculate the entanglement of [12], and the result is the same as that of [12]. From figure 1 we can see that the entanglement of the atomic subsystem for all the initial states decreases monotonically to
In the non-Markovian regime with relatively short reservoir correlation time ($\lambda = 0.1 \gamma_0$, $\delta = 0$) $E_a$ and $E_r$ as functions of scaled time $\gamma_0 t$ for various initial states: (a) the atomic subsystem; (b) the reservoir subsystem.

The steady maximum, and the entanglement contained initially in the atomic subsystem is finally transferred into the reservoir subsystem.

(ii) $\lambda = 0.1 \gamma_0$, which corresponds to the non-Markovian regime with relatively short reservoir correlation time. In figure 2 we plot the entanglement evolution of the atomic and reservoir subsystems for initial cluster, Dicke, GHZ and W states with $\lambda = 0.1 \gamma_0$ and $\delta = 0$. The dynamical behaviours of the entanglement of the atomic and reservoir subsystems in the non-Markovian regime are quite different from that in the Markovian regime. It can be seen from figures 1(a) and 2(a) that the common feature for different initial states in the non-Markovian regime is that the entanglement of the atomic subsystem decreases to zero much more slowly than that in the Markovian regime, and in the non-Markovian regime after the entanglement of the atomic subsystem decays to zero it can revive at a later time, which is quite different from the Markovian case. The reason is that the information, which the atomic subsystem loses to the reservoir, is later recovered by the atomic subsystem due to the reservoir non-Markovian memory. It is noted that in both the Markovian and non-Markovian regimes all the initial entanglement of atomic subsystem $E_a$ will decay and is eventually lost for long times, and the entanglement of the reservoir subsystem $E_r$ gradually increases to the steady maximum from zero, which can be seen from figures 1 and 2. It can be seen from figures 1(b) and 2(b) that in the non-Markovian regime the entanglement of the reservoir subsystem $E_r$ at first shows oscillations as a function of time for all the initial atomic states, and finally the steady maximum entanglement is achieved, while in the Markovian regime $E_r$ increases to the steady maximum monotonically. The distinction between the entanglement properties in the Markovian regime and those in the non-Markovian regime is induced by the non-Markovian memory. In other words, in the Markovian regime the information flow is one directional, namely from atoms to reservoirs, while in the non-Markovian regime the information flow is bidirectional, namely the exchange of information back and forth between the atomic and reservoir subsystems, which causes the oscillations of the entanglement of the atomic and reservoir subsystems.

(iii) $\lambda = 0.01 \gamma_0$, which corresponds to the non-Markovian regime with the relatively long reservoir correlation time. In figure 3 we plot the entanglement evolutions of atomic and reservoir subsystems for the four initial states with $\lambda = 0.01 \gamma_0$ and $\delta = 0$. Comparing figures 2(a) and 3(a), we can...
find that the revival of $E_a$ with relatively long reservoir correlation time is more obvious than that with relatively short reservoir correlation time, i.e., the amplitude of revival with relatively long reservoir correlation time is much larger than that with relatively short reservoir correlation time. For the reservoir subsystem, compared with the case with relatively short reservoir correlation time, it is more difficult to achieve the steady maximum of entanglement with relatively long reservoir correlation time, which can be seen from figures 2(b) and 3(b). This can be understood as follows: increasing the reservoir correlation time means that the memory effect of the reservoir becomes stronger, and then the amount of information exchanged between the atomic and the reservoir subsystems will be enhanced. So the atomic subsystem can obtain more information from the reservoir subsystem in the case of relatively long reservoir correlation time and the revival is stronger, and because of the enhanced information exchange back and forth the reservoir subsystem will need more time to achieve the final maximum entanglement.

Now we consider the off-resonant case, i.e., $\delta = 8\lambda$. (i) $\lambda = 0.1\gamma_0$. In figure 4 we plot the entanglement evolution of atomic and reservoir subsystems for the four initial states with $\delta = 8\lambda$ and $\lambda = 0.1\gamma_0$. In the off-resonant case the entanglement of the atomic subsystem $E_a$ decays to zero with small oscillations, and during each oscillation $E_a$ cannot collapse to zero. And the overall decay rate is smaller than that in the corresponding resonant case, which can be seen from figures 4(a) and 2(a). The entanglement of the reservoir subsystem $E_r$ at first increases with very small amplitude oscillations in a very short period of time and then increases monotonically to the steady entanglement, and the overall increasing rate is smaller than that in the corresponding resonant case, which can be seen from figures 4(b) and 2(b). This can be easily understood: when the value of the detuning $\delta$ increases, the effective coupling between the atomic and reservoir subsystems decreases. So the exchange of information between the atomic subsystem and the reservoir subsystem is neither effective nor adequate. (ii) $\lambda = 0.01\gamma_0$. In figure 5 we plot the entanglement evolution of atomic and reservoir subsystems for the four initial states with $\delta = 8\lambda$ and $\lambda = 0.01\gamma_0$. Comparing figures 5(b) and 3(b) we can find that due to the increasing of $\delta$, the exchange of information is not effective, the oscillations of $E_r$ are not adequate, more specifically $E_r$ cannot achieve its maximum during each oscillation. From figures 5(a) and 3(a) it can be found that by
increasing the detuning $\delta$, the period of the revival is shortened and the amplitude of the revival increases. This result is very interesting. As we have mentioned above, increasing the detuning $\delta$ will make the exchange of information less effective; then why does the revival become stronger? Now we analyse the decoherence dynamics of the multipartite entangled states in detail.

To gain insight in the physical processes characterizing the decoherence dynamics for different initial states, we consider the parameter $\chi$, and find that all the above phenomena can be uniformly explained by this parameter. From figures 1–5 it is easy to find that the dynamical behaviours of the entanglement for different initial states are very similar. For simplicity in the following we will take W state as an example. From equation (6) we can see that generally $\chi$ is a complex number, and we will show that the real part $\text{Re} \chi$ is responsible for the decay of $E_a$ and the imaginary part $\text{Im} \chi$ is responsible for the oscillations associated with the revival. From equation (13) we can see that the degree of entanglement is a function of $|\nu(t)|^2$, which means that all the decoherence dynamics of the entanglement entirely depends on $\nu(t)$. And it is easy to see from equation (5) that in the long time limit $\nu(t)$ is dominated by the terms containing the factor $e^{-\gamma(t)/2}$.

From numerical calculations we find that $|\text{Re} \chi|$ increases with $\delta$ and is always less than $\lambda$, and $|\text{Im} \chi|$ also increases with $\delta$. From equations (5) and (13), roughly speaking, $\lambda - |\text{Re} \chi|$ determines the decay of the entanglement, which we call as the decay exponent to be distinguished from the decay rate $\gamma(t)$ [16], and $|\text{Im} \chi|$ determines the basic frequency of the oscillations in the revivals (it is noted that the overall phase factor $e^{\nu(t)/2}$ in $\nu(t)$ does not make any contributions to the entanglement). When $\lambda > 2\gamma_0$ and $\delta = 0$, from equation (6) $\chi$ is a real number, i.e., $|\text{Im} \chi| = 0$, which corresponds to the Markovian regime. Hence $E_a$ will decay exponentially to zero without oscillations, and the revival cannot appear. It is easy to prove that when $\lambda > 2\gamma_0$, the decay exponent $\lambda - \sqrt{\lambda^2 - 2\gamma_0 \lambda}$ is a decreasing function of $\lambda$, and approaches $\gamma_0$ with the increasing $\lambda$, the maximum value of which is $2\gamma_0$ occurring at $\lambda = 2\gamma_0$. When $\delta = 0$ and $\lambda < 2\gamma_0$, which is corresponding to the non-Markovian regime, $\chi$ is a pure imaginary, and the oscillations appear. In this case the decay exponent is just $\lambda$. That is why the entanglement decay for $\lambda = 0.1\gamma_0$ corresponding to the non-Markovian regime (figure 2(a)) is slower than that for $\lambda = 10\gamma_0$ corresponding to the Markovian regime (figure 1(a)). Also that is why the entanglement with $\lambda = 0.01\gamma_0$ and $\delta = 0$ (figure 3(a)) decays much slower than that with $\lambda = 0.1\gamma_0$ and $\delta = 0$ (figure 2(a)). Remember that $|\text{Re} \chi|$ and $|\text{Im} \chi|$ increase with $\delta$, so the decay exponent $\lambda - |\text{Re} \chi|$ increases with the increasing of $\delta$. In this way the envelope of $E_a(t)$ decays more and more slowly with the increasing of $\delta$, so that during each revival the amplitude achieved increases with the increasing of $\delta$. This explains why with the increasing detuning $\delta$ the period of the revival is shortened, and the amplitude of the revival increases (see figures 3(a) and 5(a)). Now we consider the dispersive regime, i.e., $\delta \gg \lambda, \gamma_0$, and in this case in the long time limit $\nu(t) \approx 1 - i\lambda^2/4\delta^2$ and the steady entanglement of the corresponding atomic subsystem $E_a \approx |6\sqrt{3} + 4 + (20 + 6\sqrt{3})\lambda^2/16\delta^4|/24$ can be achieved. This means that in the dispersive regime the decay of entanglement $E_a$ is strongly inhibited. We also calculate the degree of entanglement for the initial five (and six) particle W states, and we find that the entanglement dynamics of the atomic subsystem for the initial five (or six) particle W state is almost the same as that of the initial four-particle W state, more specifically the influence of the detuning and the length of the reservoir correlation time on the dynamical behaviour of the entanglement for the initial five (or six) particle W state is almost the same as that for the initial four-particle W state. Whenever the degree of entanglement for the four-particle case increases, the degree of entanglement for the corresponding five (or six) particle case also increases, and whenever the degree of entanglement for the four-particle case decreases, the degree of entanglement for the five (or six) particle case also decreases. And the time, at which the entanglement reaches the maximum (or the minimum), is almost the same for all the three cases. This can be understood. Because we find that the entanglement of the atomic subsystem for five (or six) particle case is also a function of $|\nu(t)|^2$, which means that the dynamical behaviour of the entanglement for the five (or six) particle is also decided by the real part and imaginary part of $|\chi|$. It is well known that in the most general form of a time-local master equation for the reduced density operator, the decoherence is induced by the Lindblad (jump) operator with a decay rate $\gamma(t)$. If the decay rate $\gamma(t)$ is always positive, this describes the so-called time-dependent Markovian process [29, 48, 49], but if at least during a period of time the decay rate $\gamma(t)$ is negative, the non-Markovian process emerges. Now we also take W state as an example to show the relation between the decoherence dynamics of the entanglement and the decay rate $\gamma(t)$. For simplicity we let $\lambda = 0.01\gamma_0$ and $\delta = 0$, and in this case the decay rate $\gamma(t)$ can be expressed as $\gamma(t) = -2\text{Re}[\nu(t)/\nu(t)]$ [16], where $\nu(t)$ is obtained by choosing $\delta = 0$ in equation (5). In figure 6 we plot $E_a$, $E_r$ and $\gamma(t)$ as functions of scaled time $\gamma_0 t$ for $\lambda = 0.01\gamma_0$ and $\delta = 0$. From figure 6, it is obvious to see that whenever $\gamma(t)$ (dotted line) takes negative values, $E_a$ (solid line) begins to
revive and increase monophonically, and the corresponding $E_r$ begins to decrease monophonically; when $\gamma(t)$ takes positive values, $E_a$ will begin to decrease monophonically, and $E_r$ begins to increase monophonically. This can be easily understood: when $\gamma(t)$ is positive, the information flow is from atomic subsystem to reservoir subsystem, which means that $E_a$ will decay, and $E_r$ will increase; when $\gamma(t)$ is negative corresponding to the memory effect of the reservoir, the information flow is from the reservoir subsystem to atomic subsystem, so $E_a$ will revive and $E_r$ will decay.

4. Conclusions

In this paper, we have considered four atoms with initial entanglement interacting with independent non-Markovian reservoirs. We have analysed the decoherence dynamics for various initial states in Markovian ($\lambda = 10\gamma_0$), weak non-Markovian ($\lambda = 0.1\gamma_0$) and strong non-Markovian ($\lambda = 0.01\gamma_0$) regimes, with and without the detunings. We have found that the decoherence dynamics of the atomic and reservoir subsystems strongly depends on a parameter which is decided by the detuning and the reservoir correlation time, and all the phenomena can be explained by this parameter. The real part of this parameter determines the decay of the entanglement and the imaginary part of it determines the oscillations of the revival. We also have found that whenever $\gamma(t)$ takes negative values, $E_a$ begins to revive, and the corresponding $E_r$ begins to decrease; when $\gamma(t)$ takes positive values, $E_a$ will begin to decay, and $E_r$ will begin to increase. We have also found that for this decoherence channel the cluster state is the most robust to decoherence compared with Dicke, GHZ and W states.

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