A low complexity hybrid combiner design for the ill-conditioned multiuser mmWave massive MIMO uplink

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Abstract
This paper presents a simple and robust hybrid combiner for the multiuser massive multi-input multiple-output (MIMO) uplink. The hybrid beamforming is used in massive MIMO systems to reduce the number of radio frequency chains. Radio frequency chain consists of power hungry electronic components like mixer, analog-to-digital converter / digital-to-analog converter etc. This paper proposes a Householder based hybrid beamforming (HHBF) design to simultaneously reduce the beam domain inter-user interference and increase the sum-rate performance of massive MIMO uplink. The Householder reflectors generate the orthogonal analog combiner matrix which not only reduces inter-user interference but also helps reduce the dimension of the digital combiner. It has been shown that the proposed HHBF scheme provides better sum-rate performance than the Gram–Schmidt based hybrid beamforming design in the real-World ill-conditioned MIMO channel. Simulation results show that the beam domain orthogonality error remains less than 0.5 with 0.97 probability and the proposed HHBF provides 25.95% increased sum-rate at 10 dB SNR.

1 | INTRODUCTION

The key essence of ultra high speed 5G wireless networks lies in the use of high-frequency mmWave band ranging from 3 to 300 GHz which opens the door for massive multiple-input-multiple-output (MIMO) systems [1]. However, the mmWave frequencies suffer from high pathloss and have low penetration power. These shortfalls are compensated by the massive MIMO beamforming technique. Usually massive MIMO is deployed at the base-station to get the benefits of massive MIMO precoding in downlink and combining in the uplink. The conventional MIMO techniques use one RF chain (analog-to-digital converter (ADC)/digital-to-analog converter (DAC), mixer, data converters) which is infeasible in massive MIMO scenario because of large capital cost (CAPEX) and operational cost (OPEX). In order to reduce the number of RF chains, researchers take the advantage of mmWave channel's sparsity and split the beamforming into two stages: analog beamforming (AB) and digital beamforming (DB). This method is called hybrid beamforming (HBF). In the uplink, the received signal is first processed by the phase shifter network in analog (or RF) domain; then, the pre-processed signal is fed to the low dimension digital beamformer. In this way, mmWave massive MIMO system becomes the key enabler of next-generation 5G communications. There is a lot of research work on the design of hybrid beamforming.

The authors of [2] propose a two stage hybrid beamforming with perfect channel state information (CSI), in order to maximize the sum-rate in a single cell downlink multi-user massive MIMO system and avoid the loss of information at each stage. It uses alternating optimization method to maximize the information rate over analog and digital precoders. In alternating optimization method, digital precoder is fixed and we need to find the analog precoder and vice versa. This method has high computational complexity.

In [3], the authors proposed a study on joint radio resource allocation and hybrid precoding in decoupled mmWave multiuser-MIMO downlink. A resource allocation algorithm aiming to maximize the proportional fairness (PF) spectral efficiency under the per-sub-channel power and the beamforming rank constraints was presented.
The authors of [4] propose a decoupled two-stage hybrid beamforming design for multi-user MIMO systems using eigenvectors based analog precoder and combiner to maximize the energy efficiency. It uses singular value decomposition based analog combiner [5, (37)] in the case of multiuser uplink of a mmWave massive-MIMO base station (BS). A heuristic hybrid precoding design for the fully connected and the partially connected structures aiming to maximize the weighted sum rate in the downlink using practical number of RF chains is proposed in [6]. Due to the heuristic iterative algorithm, this design has high computational complexity. A max–min SINR fairness problem is formulated for a hybrid analog-digital beamforming with discrete-phase shifters for multiuser MISO downlink transmission in mmWave channels [7]. The proposed design only maximizes the SINR of the worst-case user, that is, it maximizes the SINR of the user which has the minimum SINR. The optimization problem is solved over the analog steering vectors using inexact majorization–minimization method at each iteration, which is computationally intensive. A joint precoding and combining design for the downlink hybrid beamforming systems with partially connected subarray architecture is proposed in [8]. The study provides solutions to the sum-rate maximization problem for multiuser massive MIMO system using iterative alternating optimization for analog precoder and combiner. The analog beamformer is obtained by the successive interference cancellation based on suboptimization problems, each of which considers one partially connected subarray. All the above-mentioned work consider the downlink hybrid beamforming in massive MIMO systems.

A hybrid analog/digital beamformers design for uplink connection in MIMO systems under imperfect channel state information (CSI) is proposed in [5]. It uses computationally intensive iterative method to find the analog combiner. The authors of [9] use the concept of uplink/downlink duality to transform complex downlink optimization problems into equivalent dual uplink problems to ease their resolution. The applied minimum mean square error precoder, which takes into account residual hardware impairments in a single cell massive MIMO system, is reported to minimize the sum mean square error under a sum power constraint.

Two low-complexity uplink multiuser receiver design schemes are proposed in [10]. Only users' angles of arrival (AOAs) are supposed to be available at base station. A single-beam scheme, each subarray serves a specific user. In the multi-beam scheme, any subarray's beamforming signal potentially points at all users. Then, all subarray signals are combined by digital beamforming followed by equalization. It uses exhaustive search method to maximize the sum-rate performance. The channel covariance [11] and Discrete Fourier transform (DFT)-based analog beamforming designs utilize the \( N \times N \) orthogonal analog beamforming matrices but require an additional users selection algorithm to select \( K \) out of \( N \) orthogonal columns [12]. This paper uses Ricean fading channel instead of mmWave channel. The asymptotic rate is maximized and \( K \) users are selected as columns of \( N \times N \) DFT matrix. Due to the fixed structure of DFT matrix, only suboptimal solution can be obtained. In [13], the hybrid beamforming design for broadband mmWave transmissions is based on the minimum sum-MSE criterion of the transmission reliability. Two more criteria are used for the design, namely the weighted sum-MSE and the spectral efficiency-based design for digital beamformer with eigenvalue based analog beamforming design. It uses alternating minimization method to optimize the hybrid transmit and receive beamformers. Then, a low complexity general Eigenvalue decomposition based scheme is presented. The fixed structure of analog beamformers leads to suboptimal solution. The paper [14] proposes a design of a two-stage analog combiner in a hybrid analog/digital beamforming for MIMO systems with array response vectors and discrete Fourier transform matrix which is near optimal only for very large antenna array [15]. The DFT-based analog combining uses a fixed beam structure; hence, it can only provide suboptimal solution. To improve the suboptimality of the fixed beam structure in [14], Khammari et al. [15] proposes an unsupervised K-mean algorithm to select the columns of the DFT matrix that maximize the sum-rate performance. Papers [11–14] investigate the uplink hybrid combining but utilize the fixed beam structure of analog combiner. Directed beams are selected by iterative alternating optimization methods which have high computational complexity. An HBF algorithm for the uplink multiuser scenario with low computational complexity is reported in [16]. The inter-user interference reduction is performed for both analog beamforming and digital beamforming. This hybrid beamforming design aims to jointly increase the sum throughput and inter-user interference. Authors use Gram–Schmidt method to orthogonalize the channel matrix to obtain an analog beamformer. Authors in [17] extend the work of [16] for the multi-carrier case. They consider a low-complexity hybrid beamforming for uplink multiuser mmWave MIMO systems. The proposed schemes, simulated in both ideal Rayleigh fading channels and sparsely scattered mmWave channels, are reported to approach the full digital algorithm in multi-carrier case. Unlike [16], it obtains analog beamformer by maximizing each user's equivalent channel gain. Last two works are more related to our work but unlike these papers, we use real-World ill-conditioned channel matrix and analyze the Householder-based hybrid combiner both for fully connected and partially connected architectures.

In order to improve the hybrid beamforming design, it is imperative to enhance the beam domain performance. The mmWave channel measurements have shown that diffuse scattering exists in mmWave [18]. This phenomenon leads to common scatterers shared by different users with the same angle of arrival. This results in severe inter-user interference in the beam domain. In the proposed analog combiner design, we orthogonalize the channel vectors and use the orthonormal steering vectors as analog combiner to mitigate the beam domain inter-user interference. Due to the sparsity of the mmWave massive MIMO channel, there exist only few multipath components. This results in a rank deficient and ill-conditioned channel matrix [19]. This paper proposes a Householder reflector [20] based orthogonal analog combiner design to efficiently reduce the inter-user interference in the analog domain in ill-conditioned mmWave massive MIMO channel. Gram–Schmidt
method is the most commonly used technique for the matrix orthogonalization [21], but with the ill-conditioned matrix, the Householder gives better orthogonality among the given vectors as compared to the Gram–Schmidt and modified Gram–Schmidt methods [22].

The combiner with orthogonal column vectors also reduces the effective channel dimension to help design a low dimensional regularized zero-forcing (RZF) digital combiner.

Contribution:
To summarize, the paper makes the following specific contributions:

• At first, we show that the sum-rate performance of mmWave multiuser massive MIMO system depends on the condition number of the estimated MIMO channel. A well-conditioned channel matrix (0 < condition number < 10 dB) allows reliable multiuser MIMO communication. Matrix with condition number > 20 dB does not allow multiuser transmission. Unlike most of the previous work, we consider an ill-conditioned channel matrix to model the real-world environment.

• A two-stage hybrid combining scheme is proposed to maximize the sum-rate for the uplink of multiuser massive MIMO systems. We design a Householder orthogonalization based analog combiner to reduce the beam domain inter-user interference caused by the common scatterers. Simulation results show that the proposed hybrid combiner outperforms the recent low complexity schemes using Gram–Schmidt [16], DFT [14], Eigenvalue [13], and Singular value based analog beamformers.

• Both fully connected and partially connected hybrid beamforming architectures are evaluated with the proposed combiner design unlike [16] and [14] which only evaluate the fully connected architectures.

• Finally, we derive close-form expressions for the upper bound on the sum-rate maximization in case of number of RF chains equal to the number of users and the case when the number of users are greater than the available number of RF chains.

2 | SYSTEM AND CHANNEL MODEL

2.1 | System model

We consider an uplink of a single cell massive MIMO system. There is one BS in the cell and is equipped with \( N \) antennas and \( N_{RF} \) RF chains to serve \( K \) single antenna users such that \( N > N_{RF} \geq K \) as shown in Figure 1. At BS, each antenna is connected with all the RF chains through an independent phase shifter to form a fully connected combiner structure [1]. The RF chains are connected to a digital combiner which outputs \( N_{s} \) data streams. We assume \( N_{s} = K \); then, the information signal \( s \in \mathbb{C}^{K \times 1} \) at the input of the BS transceiver is,

\[
s = [s_1, s_2, \ldots, s_K]^T,
\]

where \( s \sim \mathcal{CN}(0, \mathbf{I}_K) \) is the normalized transmitted signal vector with \( \mathbb{E}\{ss^H\} = \mathbf{I}_N \) or \( \mathbb{E}\{|s|^2\} = 1 \). The information symbol vector is \( s \in \mathbb{C}^{N_{s} \times 1} \), where \( N_{s} \) is the number of symbols. The received signal vector \( y \in \mathbb{C}^{N_{s} \times 1} \) at the BS is given as

\[
y = \mathbf{H} \mathbf{P} s + z,
\]

where \( s = [s_1, \ldots, s_K]^T \) is the transmit signal vector, \( \mathbf{P} = \text{diag}(\sqrt{p_1}, \ldots, \sqrt{p_K}) \in \mathbb{C}^{K \times K} \) is a diagonal matrix with \( p_k \) being the transmit power of user \( k \), \( \mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_K] \sim \mathcal{CN}(0, \mathbf{G}) \) is \( N \times K \) channel matrix, and \( z \sim \mathcal{CN}(0, \mathbf{I}_N) \) is the \( N \times 1 \) noise vector. \( \mathbf{G} = \text{diag}(\mathbf{g}_1, \ldots, \mathbf{g}_K) \) where \( \mathbf{g}_k \) is the average power gain between the BS and the \( k \)-th user.

We use the statistical channel inversion power control scheme [23], that is, \( \rho_k \mathbf{g}_k = \rho \). With this control scheme, the effective channel gain between the BS and users is constant and the average received signal-to-noise ratio (SNR) per antenna and user is \( \rho \).

Upon receiving the signal block, the BS first uses an analog combiner \( \mathbf{W}_{AB} \in \mathbb{C}^{N_{s} \times N_{RF}} \) and then, a low dimensional digital combiner \( \mathbf{W}_{DB} = [\mathbf{w}_{1,DB}, \ldots, \mathbf{w}_{K, DB}] \in \mathbb{C}^{N_{RF} \times K} \) is used to detect the \( K \) data streams as shown in Figure 1. The hybrid combiner can be written as,

\[
\mathbf{W}_B = \mathbf{W}_{AB} \mathbf{W}_{DB} \in \mathbb{C}^{N \times K}.
\]

The hybrid beamforming at the BS recovers the transmitted signal with,

\[
s = [s_1, \ldots, s_K] = \mathbf{W}_B^H y
\]

\[
= \mathbf{W}_{DB}^H \mathbf{W}_{AB}^H y
\]

\[
= \mathbf{W}_{DB}^H \mathbf{W}_{AB}^H \left( \mathbf{H} \mathbf{P} s + z \right).
\]

We assume \( N_{RF} = K \), as it is the minimum possible number of RF chains for linearly decoding the received signal [5].

\[\text{Beamforming and combining terms are used interchangeably.}\]
2.2 Channel model

In order to model the mmWave channel we use extended Saleh-Valenzuela model, which accurately captures the mathematical structure present in mmWave channels [24]. For simplicity, we assume that each scattering cluster around the transmitter and receiver contributes few propagation paths [25]. Geometrical channel model describes the physical propagation between transmit array and receive array. Due to near optical line-of-sight (LOS) wave propagation at mmWave frequencies, the mmWave channels are expected to have limited scattering, say, \( L \). The \( N \times K \) MIMO channel matrix at the BS can be written as

\[
H = \sqrt{KN} \sum_{l=1}^{L} \alpha_l a(\phi_l),
\]

where \( \alpha_l \) represents the complex gain of the \( l \)th path with i.i.d. \( \mathcal{CN}(0,1) \). Moreover, \( a(\phi_l) \) is the receive steering vector. The variable \( \phi_l \in [0,2\pi) \) is the \( l \)th path’s azimuth angles (boresight angles in the receive array) of arrival. The receive steering vector is given by

\[
a(\phi_l) = \frac{1}{\sqrt{N}} [a_1(\phi_l), ..., a_N(\phi_l)].
\]

The elements of receive steering vectors are given by

\[
a_i(\phi_l) = e^{-\jmath \omega \tau_i}, \quad \tau_i = \frac{\omega}{\lambda} d \sin(\phi_l), \quad i = 1, 2, ..., N,
\]

where \( \lambda \) is the wavelength, \( \omega = \frac{2\pi}{\lambda} \) is the beamforming delay, and \( d \) is the antenna spacing at the BS.

2.3 Effect of channel condition number

The mathematical model of MIMO communications consists of a set of linear equations \( Ax = b \). This is analogous to signals and systems model, where, transmit antennas generate input signals \( s \), passing through the system (channel matrix) \( H \), and producing output signals \( y \) as given in (2). The solution of (2) requires the estimate of channel matrix \( H \). Multiplication of \( H \) inverse with \( y \) gives the transmit signal vector \( s \). The condition number measures the sensitivity of the solution \( x \) of the linear system \( Ax = b \). Perturbation in \( A \) or \( b \) may give rise to relative changes in \( x \) which are proportional to \( \text{cond}(A) \) [26]. A well-conditioned channel matrix allows reliable multiusers MIMO communications. An ill-conditioned matrix prevents this or at the very least makes it difficult. The condition number of a channel matrix \( H \) is calculated as [19]

\[
\text{Cond}(H) = \frac{\sigma_{\text{max}}}{\sigma_{\text{min}}} \geq 1,
\]

where \( \sigma_{\text{max}} \) is the largest singular value and \( \sigma_{\text{min}} \) is the smallest singular value in the channel matrix \( H \). In real-world MIMO communications, logarithmic condition numbers in the range \( 0 - 10 \) dB are considered to be very good; on the other hand if the condition number is above 20 dB, then multiusers MIMO communication is not possible [27].

3 PROBLEM FORMULATION

Assume that perfect CSI is available at the BS, then the rate expression for user \( k \) is given as

\[
R_k = \log_2 \left( 1 + \frac{\gamma_k}{\Gamma} \right),
\]

where \( \Gamma \) is the SNR gap between Shannon capacity and the performance obtained by the employed modulation and coding scheme in practical wireless channel. For M-QAM modulation and target bit error rate of \( \epsilon \), \( \Gamma = -(2/3)/n(5\rho) \) [28].

The received signal-to-interference-and-noise ratio (SINR) \( \gamma_k \) of the user \( k \) is given as

\[
\gamma_k = \frac{|w^{\text{dB}}_{k,\text{DB}} w^{\text{dB}}_{k,\text{dB}} h_k|^2}{\sum_{j \neq k} |w^{\text{dB}}_{k,\text{DB}} w^{\text{dB}}_{j,\text{dB}} h_j|^2 + \frac{1}{\rho} ||W_{\text{AB}} w_{k,\text{DB}}||_2^2},
\]
Our objective is to design a hybrid combiner to maximize the sum-rate for multiuser massive MIMO system. The sum-rate is given by

$$ R_{\text{sum}} = \sum_{k=1}^{K} \log_2 \left( 1 + \frac{|w_{k, DB}^H w_{AB} h_k|^2}{\sum_{j \neq k} |w_{k, DB}^H w_{AB} h_j|^2 + \frac{1}{\rho} \|W_{DB} w_{k, DB}\|^2} \right). $$

(11)

The above sum-rate can be written as,

$$ R_{\text{sum}} = \log_2 \left| I_K + Q^{-1} w_{DB}^H w_{AB}^H H H^H w_{AB} w_{DB} \right|, $$

(12)

where $Q = Q_l + Q_N$ is the noise plus interference matrix, with $Q_l = \sum_{j=1,j \neq k}^{K} w_{k, DB}^H w_{AB}^H h_j h_j^H w_{AB} w_{k, DB}$ is the interference matrix and $Q = \frac{1}{N} w_{DB}^H w_{AB}^H w_{AB} w_{DB}$ is the covariance matrix of noise. Therefore, the design of hybrid combiner can be described as an optimization problem $P$:

$$ \begin{align*}
\text{maximize} & \quad R_{\text{sum}} \\
\text{s.t.} & \quad |W_{AB}|_{i,j} = \frac{1}{\sqrt{N}}, \forall 1 \leq i \leq N, \forall 1 \leq j \leq N_{RF}. 
\end{align*} $$

(13)

### 3.1 Beamforming decomposition: Hybrid beamforming

Hybrid beamforming has separate digital beamformer and analog beamformer stages. The digital beamformer consists of a digital transceiver (RF chain). Each transceiver contains a digital-to-analog/analog-to-digital converter, data converter, mixer etc. On the other hand, the analog beamformer is generally implemented by the time-delay elements (phase shifters) and analog adders. These particular components impose stringent constraints on the design of the analog beamformer, for example, all elements of the analog beamforming matrix must have unity gain.

#### 3.1.1 Fully connected architecture

In fully connected architecture each RF chain is connected to $N$ phase shifters; hence, there are $N_{RF} N$ phase shifters $[1]$ as shown in Figure 2(a). In designing analog beamforming for fully connected architecture, we use $N \times N_{RF}$ matrix with a constraint of $|W_{AB}|_{i,j} = \frac{1}{\sqrt{N}}, \forall 1 \leq i \leq N, \forall 1 \leq j \leq N_{RF}$. When $N_{RF} = K$, each column corresponds to one user. The $N$-dimensional column vector is the user’s signal passing from $N$ antennas and $N$ phase shifters. This architecture provides optimal channel capacity for $N_{RF}$ users, but it requires $N_{RF} N$ number of phase shifters.

#### 3.1.2 Partially connected architecture

The partially-connected architecture has lower hardware complexity of $N$ phase shifters as shown in Figure 2(b), but this comes at the cost of $\frac{1}{N_{RF}}$ times beamforming gain as compared to the fully connected architecture. The partially connected analog beamformer design consists of an analog beamforming matrix $W_{AB} \in \mathbb{C}^{N \times N_{RF}}$ as,

$$ W_{AB} = \begin{bmatrix}
    w_{AB,1} & 0 & \cdots & 0 \\
    0 & w_{AB,2} & \ddots & \vdots \\
    \vdots & \ddots & \ddots & 0 \\
    0 & \cdots & 0 & w_{AB,N_{RF}}
\end{bmatrix}, $$

(14)

where $w_{AB,j}$ is $N/N_{RF}$-dimensional column vector associated with the $j^{th}$ RF chain.
4 | UPLINK HYBRID COMBINER DESIGN FOR MULTIUSER MASSIVE MIMO SYSTEM

The main challenge in hybrid beamforming design is high complexity and the power consumption, especially with the large number of RF chains. A well-known approach to handle this problem is to split the beamforming matrix into an analog and a low dimensional digital beamforming matrices. The conventional MIMO techniques like minimum mean square error (MMSE) or zero-forcing (ZF) cannot be applied on the analog beamforming design because of the unity modulus constraint on each element of the analog matrix. We propose Householder-based analog combiner to efficiently reduce the inter-user interference (by making orthonormal analog beamforming matrix) and improve the sum-rate performance of the system. Based on the effective channel $W_{AB}H$, a low-dimensional regularized zero-forcing (RZF) digital beamformer $W_{DB}$ is designed. RZF has been widely used for the realization of digital beamformer [4, 7, 12, 29]. We use the RZF because of its simple analytical expression to get the closed form expression for the sum-rate maximization problem. It has been shown that RZF and MMSE have similar performance (less than 1 dB performance gap) when the number of antennas are large ($N/K \geq 6$) [30]. Figure 3 shows the methodology of the proposed scheme.

### Algorithm 1

| Line | Description |
|-----|-------------|
| 1:   | Inputs |
| 2:   | $H$: Channel matrix |
| 3:   | $K$: Number of users |
| 4:   | $N_{RF}$: Number of RF chains |
| 5:   | $N$: Number of receive antennas |
| 6:   | $Q = I_N$: Analog Beamforming Design |
| 7:   | for $k = 1$ to $K$ do |
| 8:   | $z = h_k \cdot N, k$ |
| 9:   | $v = [-\text{signum}(z(1)) ||z|| - z(1)] - z(2 : end)$ |
| 10:  | for $j = 1$ to $N$ do |
| 11:  | $Q_k \cdot N, j = Q_k \cdot N, j - 2v(Q_k \cdot N, j v)$ |
| 12:  | end for |
| 13:  | end for |
| 14:  | $Q = Q^H$ |
| 15:  | $Q_\cdot 1 : K$ |
| 16:  | for $i = 1$ to $K$ do |
| 17:  | for $j = 1$ to $N$ do |
| 18:  | $W_{AB i, j} = \frac{1}{\sqrt{N}} Q_i \cdot 1 \cdot Q^* i, j$ |
| 19:  | end for |
| 20:  | end for |
| 21:  | $H_{ij} = W_{AB i} H$ |
| 22:  | $W_{DB} = H_{ij} (H_{ij}^H H_{ij} + I_k)^{-1}$ |
| 23:  | Output $W_{AB}$, $W_{DB}$ |

#### 4.1 Analog combiner design

In order to improve the hybrid beamforming design, it is imperative to enhance the beam domain performance. The mmWave channel measurements have shown that diffuse scattering exists in mmWave [18]. This phenomenon leads to common scatterers shared by different users with same angle of arrival. This results in severe inter-user interference in beam domain. In the proposed analog combiner design, we try to orthogonalize the channel vectors and use the orthonormal steering vectors as analog combiner to mitigate the beam domain inter-user interference. A step by step design of the proposed hybrid beamformer for fully connected architecture is presented in the Algorithm 1. In the analog beamforming design step, Householder reflection method [20] is utilized to orthogonalize the multiuser channel column vectors. The Householder gives better orthogonality among the given vectors as compared to the Gram–Schmidt and modified Gram–Schmidt methods [22]. In order to satisfy the unity modulus constraint of the practical phase shifters, the obtained orthogonal column vectors are normalized element-wise in line 16.

The same algorithm can also work for the partially connected structure. By inserting, $W_{RF} = W_{RF} \cdot (I_{N_{RF}} \otimes 1)$ after the line 18 makes it executable for the partially connected architecture. Here, $1$ is a $N/N_{RF} \times 1$ all ones column vector.
low-dimensional effective channel $\mathbf{H}_{eff}$ at the input of digital combiner is obtained as

$$\mathbf{H}_{eff} = \mathbf{W}_{AB}^H \mathbf{H} \in \mathbb{C}^{N_R \times K}. \quad (15)$$

The low-dimensional RZF digital combiner is defined as

$$\hat{\mathbf{W}}_{DB} \triangleq \mathbf{H}_{eff} \left( \mathbf{H}_{eff}^H \mathbf{H}_{eff} + \mathbf{I}_K \right)^{-1}, \quad (16)$$

then,

$$\hat{\mathbf{W}}_{DB} = \mathbf{W}_{AB}^H \mathbf{H} \left( \mathbf{H}_{AB}^H \mathbf{W}_{AB} \mathbf{W}_{AB}^H \mathbf{H} + \mathbf{I}_K \right)^{-1}. \quad (17)$$

It has been proven [6, Lemma 1] that for a large-scale antenna array, $\mathbf{W}_{AB} \mathbf{W}_{AB}^H \simeq N \mathbf{I}_N$, therefore,

$$\hat{\mathbf{W}}_{DB} = \mathbf{W}_{AB}^H \mathbf{H} \left( \mathbf{N} \mathbf{H}_{AB}^H \mathbf{H} + \mathbf{I}_K \right)^{-1}. \quad (18)$$

The energy scaling of $\hat{\mathbf{W}}_{DB}$ is arbitrary; for simplicity, it is chosen as

$$\hat{\mathbf{W}}_{DB} = \frac{1}{\sqrt{\alpha}} \hat{\mathbf{W}}_{DB}, \quad (19)$$

where $\alpha \triangleq \| \mathbf{W}_{AB} \hat{\mathbf{W}}_{DB} \|^2_F = tr(\mathbf{W}_{AB} \hat{\mathbf{W}}_{DB}^H \hat{\mathbf{W}}_{DB} \mathbf{W}_{AB})$. Substituting the value of $\hat{\mathbf{W}}_{DB}$ from (18),

$$\alpha = tr(\mathbf{W}_{AB} \mathbf{W}_{AB}^H \mathbf{H} (\mathbf{N} \mathbf{H}_{AB}^H \mathbf{H} + \mathbf{I}_K)^{-1} \mathbf{H}^H \mathbf{W}_{AB} \mathbf{W}_{AB}^H). \quad (20)$$

Using the approximation $\mathbf{W}_{AB} \mathbf{W}_{AB}^H \simeq N \mathbf{I}$ and the property $\mathbf{X}^{-1} \mathbf{X}^{-1} = (\mathbf{X} \mathbf{X})^{-1}$, we get

$$\alpha = tr(\mathbf{N} \mathbf{H} ((\mathbf{N} \mathbf{H}_{AB}^H \mathbf{H} + \mathbf{I}_K) (\mathbf{N} \mathbf{H}_{AB}^H \mathbf{H} + \mathbf{I}_K)^{-1} \mathbf{H}^H \mathbf{N}). \quad (21)$$

Applying the cyclic property of trace, we have

$$\alpha = tr(N^2 ((\mathbf{N} \mathbf{H}_{AB}^H \mathbf{H} + \mathbf{I}_K) (\mathbf{N} \mathbf{H}_{AB}^H \mathbf{H} + \mathbf{I}_K)^{-1} \mathbf{H}^H \mathbf{H})). \quad (22)$$

By assuming negligible inter-user interference due to the RZF combiner, the sum-rate is given as

$$R_{sum} = \sum_{k=1}^{K} \log_2 \left( 1 + \frac{| \mathbf{w}_{k, DB} \mathbf{W}_{AB}^H \mathbf{h}_{k} |^2}{\rho \| \mathbf{W}_{AB} \mathbf{W}_{DB} \|^2_2} \right). \quad (23)$$

The post-processed sum-rate after the low dimensional digital combiner is given as

$$R_{sum} = K \log_2 \left( 1 + \frac{\tilde{\rho}}{\| \hat{\mathbf{W}}_{DB} \mathbf{W}_{AB}^H \|^2_2} \right), \quad (24)$$

where $\tilde{\rho} = \rho / \| \mathbf{W}_{AB} \mathbf{W}_{DB} \|^2_2$. Substituting trace for square of Frobenius norm in (24), we get
$R_{sum} = K \log_2 \left( 1 + \frac{\tilde{\rho}}{tr(N^2((NH^H + I_K)(NH^H + I_K)^{-1}H^H H))} \right)$ (25)

Using the Lemma 1, we get the following upper bound on the sum-rate,

$R_{sum} \leq K \log_2 \left( 1 + \frac{\tilde{\rho}}{N^2tr((NH^H + I_K)tr(H^H H)^{-1})} \right)$, (26)

taking $N$ common,
The optimization problem on the maximization of $R_{\text{sum}}$ can be solved by maximizing the RHS of (28). The increase in the RHS of (28) increases the lower bound on RHS of (27).

Since $(H^H H + \frac{I_K}{N})$ is a $K \times K$ symmetric matrix, we can decompose it using Eigenvalue decomposition, as

$$H^H H + \frac{I_K}{N} = QA_K Q^T,$$

where $Q$ is a $K \times K$ square matrix whose columns are the eigenvectors of $(H^H H + \frac{I_K}{N})$, and $A_K$ is the diagonal matrix whose diagonal elements are the corresponding eigenvalues. We know that

$$\left(H^H H + \frac{I_K}{N}\right)^2 = QA_K^2 Q^T.$$  \hspace{1cm} (30)

Using the cyclic property of the matrix trace, we can get

$$tr\left(\left(H^H H + \frac{I_K}{N}\right)^2\right) = tr(A_K^2 Q^T Q)$$

$$= tr(A_K^2).$$ \hspace{1cm} (31)

Hence,

$$K \log_2 \left(1 + \frac{\tilde{\rho}}{N}tr(H^H H + \frac{I_K}{N})^2tr(H^H H)^{-1}\right)$$

$$\geq K \log_2 \left(1 + \tilde{\rho} \frac{tr(A_K^2)}{tr(A)}\right),$$  \hspace{1cm} (32)

where $A$ is the diagonal matrix from the eigenvalue decomposition of $H^H H$.

In case of $K > N_{RF}$, intuitively, the $R_{\text{sum}}$ can be maximized by selecting the $N_{RF}$ eigenvectors associated with the highest eigenvalues in $A_K$.

**Lemma 1.** For symmetric positive definite matrices $A_{(N \times N)}$ and $B_{(M \times M)}$, $tr(A^{-1}B) \geq \frac{N}{tr(A)} tr(B^{-1})$.

**Proof.** Using inequality of the trace of matrices $tr(XY) \leq tr(X)tr(Y)$, we can get $tr(A^{-1}) \geq \frac{N}{tr(A)}$. Further, $tr(A^{-1}) = tr(A^{-1}BB^{-1}) \leq tr(A^{-1}B)tr(B^{-1})$ and we get $\frac{N}{tr(A)} \leq tr(A^{-1}B)tr(B^{-1})$. Therefore, $tr(A^{-1}B) \geq \frac{N}{tr(A)} tr(B^{-1})$. \hspace{1cm} □

## 5 | SIMULATION RESULTS

In this section, performance of the proposed hybrid combiner is evaluated. The sum-rate performance of Gram–Schmidt based hybrid beamforming (GSHBF) [16], eigenvalue decomposition based HBF [35], and singular value decomposition based HBF [36] are also provided for comparison. As a benchmark, the digital-only combiner is also included in the graphs.

We consider a single cell where base-station is equipped with $N = 128$ uniform linear array (ULA) antenna elements and there are $K = 8$ single antenna users. The ULA has $\lambda/2$ inter-element distance, where $\lambda$ is the transmission wavelength. The logarithmic condition number 11.5 dB is used to characterize the ill-conditioned MIMO channel matrix. The number of multipath $L = 5$ [16] is taken in the simulations. In simulation, the results are obtained by averaging over 1000 channel realizations. Though we use a geometrical channel model to describe the physical propagation in mmWave wireless channel, however, a more detailed radio link channel model...
based on Doppler power density spectrum [37] can also be employed.

In order to see the effect of channel condition number on the sum-rate performance, we plot average sum-rate as a function of channel condition number in Figure 5. In general, well-conditioned channel matrix (small values of Cond($\mathbf{H}$)) is suitable for the multi-user MIMO communications and provides higher sum-rate. It can be seen that as compared to the other schemes, HHBF has better performance even at the higher values of the condition number (ill-conditioned channel matrix).

The cumulative distribution function (CDF) of orthogonality errors of GS- and HH-based analog beamforming is plotted in the Figure 6. The orthogonality error is defined as $||\mathbf{W}_{\text{AB}}^H\mathbf{W}_{\text{AB}} - \mathbf{I}||$. The CDF graph shows that 97% HHBF orthogonality error remains less that 0.5, whereas 80% of the time, the orthogonality error of the GSHBF reaches to 2.4.

The sum-rate performance with received SNR for fully connected architecture is given in Figure 7. It shows that sum-rate increases with the SNR but saturates at high SNR because of the increased interference from other users’ transmit power. It can be seen that the proposed HHBF design outperforms the Gram–Schmidt based HBF (GSHBF) for all values of SNR. The performance gap between the two increases with SNR and reaches 25.95% increase at 10 dB. The SVD and EVD based HBF performances are lower because they use only first $N_{RF}$ (out of $N$) left-singular vectors and eigenvectors, respectively, to form the vectors of analog combining matrix. Performance of SVD-based HBF is better than the EVD-based HBF because the singular vectors are always orthonormal, whereas eigenvectors in EVD are not necessarily orthogonal. The HHBF performance is close to the ideal digital beamforming in low SNR regime.

Figure 8 depicts the uplink average sum-rate with the number of users. For this graph, the SNR is fixed at $\rho = 10$ dB. The sum-rate increases with the number of users between $K = 8$ and $K = 16$. As the number of users increases, performance of GSHBF deteriorates rapidly as compared to the HHBF because with the increase in number of users, the inter-user interference increases and GSHBF is more affected due to the higher orthogonality error. The HHBF gives 37.5% better performance than the GSHBF when $K = 16$.

The mmWave massive MIMO transmission employs large number of antennas. In Figure 9, the average sum-rate is plotted against the number of receive antennas in the uplink transmission for received SNR of $\rho = 0$ dB and $\rho = 10$ dB. The two sets $\rho = 0$ dB and $\rho = 10$ dB are in compliance with the graphs in Figure 7 which shows that performance gap between HHBF and GSHBF is small at low SNR and it is large at high SNR regime. For a fixed number of users and received SNR, HHBF outperforms the GSHBF. An interesting observation is the decrease in performance gap between HHBF and GSHBF with the increase in antennas number. However, the rate of improvement in GSHBF is very small and is negligible in the practical range of antennas number.

The partially connected architecture with the proposed HHBF also outperforms the other designs for sum-rate versus SNR performance as shown in Figure 10. As compared to the fully connected architecture, it provides less performance gain, for example, 6.5% increase in sum-rate as compared to the GSHBF at 10 dB SNR. At 10 dB SNR, HHBF exhibits 2.28 and 2.49 times better performance than SVD- and EVD-based HBF, respectively. Figure 11 shows sum-rate with increasing number of users in partially connected architecture. The HHBF gives better performance than the other designs. Due to the low-dimensional non-zero columns $\mathbf{w}_{\text{AB},i}$ of analog combiner matrix, both HHBF and GSHBF have reduced orthogonality error and give closer performance. Finally, the average sum-rate is increasing with the number of antennas per RF chain.
The advantage of GS-based orthogonality is that GSHBF algorithm provides the orthogonal columns iteratively, whereas the HHBF algorithm gives all the orthogonal columns together at the end of the algorithm. In our application, earlier generation of few orthogonal columns does not give any advantage because the complete analog beamforming matrix can be used to get effective channel matrix; therefore, HHBF is more suitable for hybrid beamforming.

6 | CONCLUSIONS

This paper studied the two-stage hybrid beamforming for decomposing the combining matrix at the base-station. We propose a Householder-based analog combiner design to reduce the dimension of the digital combiner as well as the inter-user interference in the beam domain for an ill-conditioned MIMO uplink. The Householder reflectors generate the orthogonal analog combiner matrix which provides better sum-rate performance than the Gram–Schmidt based hybrid beamforming design. Simulation results show that the proposed HHBF provides 25.95% increased sum-rate at 10 dB SNR as compared to the GSHBF, and it has very small performance gap with ideal digital beamforming at low SNR regime.

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