Quantum Statistical Processes in the Early Universe *

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Abstract

We show how the concept of quantum open system and the methods in non-equilibrium statistical mechanics can be usefully applied to studies of quantum statistical processes in the early universe. We first sketch how noise, fluctuation, dissipation and decoherence processes arise in a wide range of cosmological problems. We then focus on the origin and nature of noise in quantum fields and spacetime dynamics. We introduce the concept of geometrodynamic noise and suggest a statistical mechanical definition of gravitational entropy. We end with a brief discussion of the theoretical appropriateness to view the physical universe as an open system.

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1 Introduction and Overview

The aim of this talk is to show how the concept of quantum open systems and the methods in non-equilibrium statistical mechanics can be usefully applied to the study of quantum statistical processes in the early universe. By the early universe we refer to the period from the Planck time \(10^{-43}\) sec down to, say, the GUT time \(10^{-34}\) sec, a period depicted by semiclassical cosmology. Semiclassical gravity theories assume that the gravitational field can be treated as classical while the matter fields are quantized. Semiclassical cosmology refers to those classes of universes which are solutions to the semiclassical Einstein equations with backreaction (the sources are usually taken to be the vacuum expectation values of the energy momentum tensor of the matter quantum fields). It includes many versions of inflationary cosmology where the universe is driven by a quantum vacuum or stochastic source. Prior to the Planck time is the realm of quantum cosmology, where the gravitational field has also to be quantized.

The key concept we use is that of a quantum open system. The paradigm we adopt is that of a quantum Brownian model in a general environment. The method we use is the Feynman-Vernon influence functional technique.

The key processes we study are noise and fluctuation on the one hand, and dissipation and diffusion on the other. On the level of the influence functional one can obtain explicit forms of the noise and dissipation kernels corresponding to different types of environment and system-environment couplings; and from the stochastic equations of motion (the master equation, Fokker-Planck equation, or Langevin equation) derivable from the influence action one can examine the processes of dissipation and diffusion at work. Notice that on the level of noise and fluctuation, the description is microscopic and stochastic, whereas on the level of dissipation and diffusion the description is macroscopic and deterministic. Indeed it is the degradation of information in the environment which engenders the dissipative dynamics of the system. The three steps – separation of system and environment, coarse-graining the environment and the backreaction of the environment on the system – turns the reversible dynamics of a closed system into the irreversible dynamics of an open system, an elemental theory into an effective theory. Many common issues in theoretical physics which interpolate between the microscopic and the macroscopic, the stochastic and the deterministic, the random and the structured, can be studied with such a conceptual scheme and paradigm.
Some examples of current interest are:

a) Relation of quantum and thermal fluctuations \([5, 6]\). This occurs in e.g., gravitational wave detectors, early universe phase transitions, black hole radiance.

b) Quantum to classical transition \([7, 8, 9]\). The conditions of decoherence (towards consistent histories) and correlation (peaking in phase space) leading to a classical description are related to the diffusion process as a consequence of noise.

c) Emergence of persistent structures in nonlinear dissipative systems. \([10, 11, 12]\).

Examples of quantum processes in semiclassical and quantum cosmology which require non-equilibrium statistical field-theoretical considerations are:

1. Galaxy formation from primordial quantum fluctuations
2. Entropy generation from quantum stochastic and kinetic processes
3. Phase transitions in the early universe as vacuum or noise-induced processes
4. Anisotropy dissipation from particle creation and other backreaction processes
5. Dissipation in quantum cosmology and the issue of the initial state
6. Validity of the minisuperspace approximation as a backreaction problem
7. Decoherence, backreaction and the semiclassical limit of quantum gravity
8. ‘Birth’ of the universe as a spacetime fluctuation and tunneling phenomenon
9. Topology change and quantum coherence problems
10. Gravitational entropy, singularity and time asymmetry

As a general introduction to these ideas, a range of topics was covered in my talk, but at varying details:

A. Introduction–Issues, Concepts and Methodology
B. Noise, Fluctuation and Structure in Inflationary Universe
C. Dissipation and Backreaction in Semiclassical Cosmology
D. Quantum Cosmology in the Light of Statistical Mechanics
E. Spacetime Noise and Gravitational Entropy
F. Discussion

What follows is a progress report of recent work on these problems carried out by Calzetta, Paz, Sinha, Zhang and myself. It is not meant to be a review of all the work done in this field, as there are many. Amongst researchers in quantum cosmology there is no consensus on the interpretation of conceptual issues. The viewpoint I take here is probably as subjective as others’.

Since I have discussed some of these topics in general terms in other occasions, where written reports are available, I do not want to overburden this report here. I would rather focus on one or two themes and discuss some new ideas, specifically, on Topic E above. Let me, however, mention where discussions of some of these topics can be found in our previous work:

The adoption of open-system concepts to quantum cosmological processes naturally emerged on the one hand from our earlier systematic studies of particle creation backreaction problems in semiclassical cosmology [13], and on the other from seeking a quantum field-theretical formulation of kinetic theory [14]. A statistical mechanical meaning of dissipation in the cosmological backreaction problem was explored and a fluctuation-dissipation relation for quantum field processes in a dynamical spacetime was proposed in [15]. The appearance of dissipation in the equations of quantum cosmology (when certain sectors of superspace are coarse-grained, as in the minisuperspace approximation) was brought up in [16, 17, 18, 19, 20] and its implications on the specificity of the initial conditions are discussed.

Dissipation is only half of the story. In the closed-time-path (CTP) or Schwinger-Keldysh effective action [21, 22] we used to describe these processes it comes from the real part only. The imaginary part which we obtained (e.g., in the problem of a scalar field in Bianchi Type-I universe [13]) depicts the noise. We did not quite appreciate its physical meaning until we saw it in the context of the Feynman-Vernon influence functional description of quantum Brownian motion. (This is a problem studied by many people, notably [23, 24, 25].) To see this, the framework has to be extended from the treatment of wave functions to density matrices (more precisely, reduced density matrices for open systems). Formally, an intermediate step which leads from the effective action which incorporates vacuum fluctuations to the influence action which incorporates the averaged effects of an environment is the so-called coarse-grained effective action [26]. Indeed the in-in CGEA turns out to be just the influence action applied to vacuum states.
It is the noise kernel which is responsible for processes like decoherence. Studies of decoherence in quantum mechanics and quantum cosmology have seen some interesting development from the early 80’s (see talks by Zurek and Hartle in this Conference). Structurally, dissipation (relaxation) and decoherence (diffusion) are two interrelated processes both induced by the noise and fluctuations in the environment. For us it was from the recognition that the closed-time-path effective action (which subtotals our investigations of the cosmological backreaction problem) is equivalent to the influence action (which is used for the derivation of the stochastic equations) that we confirmed our earlier speculation on the statistical mechanical meaning of these cosmological processes. The perturbation techniques we used in the derivation of effective actions have become immediately useful for the treatment of noise and fluctuation in nonlinear problems, as we will show in the next section. This methodology was used to derive master equations for nonlinear couplings \[27, 28\] and generalized fluctuation-dissipation relations for quantum Brownian motions in a general environment in \[28, 29, 30\].

A unified viewpoint was advocated for studying the statistical aspects of quantum cosmology in \[31\], where it is urged that the statistical processes of decoherence and correlation, dissipation and backreaction, noise and fluctuation be studied in an integrated and interconnected way, not in isolation, for the discussion of quantum to classical transition, semiclassical limits, and related issues. (Anderson \[32\], and Habib and Laflamme \[33\] first noticed the importance of decoherence in the WKB Wigner function as an indication of correlation in phase space, Paz and Sinha \[34, 35\] showed how a decohered WKB branch obeys the semiclassical Einstein equations.) The application of statistical mechanical ideas and methods for such studies in quantum cosmology, including the minisuperspace approximation problem mentioned above \[19\] is described in greater details in \[36, 21\]. The study of how fluctuation and dissipation processes relate to irreversibility in cosmology is discussed in \[37\].

As expressed in \[31\], while many of these statistical processes have been explored in some detail in their respective contexts, the study of noise which underlies these processes is largely untouched, at least in the context of cosmology. Extending the treatment of quantum Brownian motion \[27, 28, 29\] to quantum fields in Minkowsky \[38\] and then to de Sitter spacetimes \[39\], we have recently begun an investigation of noise, fluctuation and structure in the inflationary universe \[40\]. We showed from first principles how one can
derive classical stochastic dynamics from quantum field theory (in curved spacetime, if for cosmological purposes). We showed how the coarse-graining of an environment field (e.g., in Starobinsky’s stochastic inflation model [3], it is the high frequency modes inside the de Sitter horizon) leads to noise and dissipation in the (functional) stochastic equations. With these equations one can study how readily the different sectors of the spectrum decohere, and check on the validity of the commonly invoked assumption that the long wavelength modes behave classically. In this setting, we also proposed a new mechanism of colored noise generation from nonlinear coupling of the inflation field with the bath. These results are detailed in [40]. Colored noise would give rise to non-Gaussian fluctuations. The implication of these effects on galaxy formation is under investigation [41]. The above is a sketch of topics A to D of my talk. In the following I will focus on the ideas in topic E, i.e., origin of noise in quantum fields and spacetime dynamics, and suggest an alternative definition of gravitational entropy. We shall examine the soundness of viewing the physical universe as a quantum open system in the Discussion.

2 Noise from Coarse-Grained Interacting Quantum Fields

We have worked out four examples, one in field theory: 1) a $\lambda\phi^4$ theory where two self-interacting scalar fields are coupled bi-quadratically [38]; one for semiclassical cosmology: 2) a free scalar field propagating in a Bianchi Type I background spacetime [13]; and two in quantum cosmology: 3) a minisuperspace consisting of the Robertson-Walker (RW) scale factor and the lowest mode of a $\lambda\phi^4$ scalar field is coupled to the higher scalar modes [13]; and 4) the fully quantum version of the scalar field in Bianchi Type I (BI) universe problem [35]. The two quantum cosmological examples have the same coupling types as the two corresponding problems in field theory and semiclassical cosmology, therefore we shall use these two examples as models to show how noise can be associated with the coarse-graining of quantum fields and spacetimes.

In [28] we used a quantum Brownian model where a system (of one harmonic oscillator) is coupled nonlinearly to a bath (of many oscillators) to
show how one can obtain different types of colored noise from different couplings (we analyzed a polynomial type) and from different baths (we analyzed both ohmic and nonohmic spectral densities). In extending this work to field theory [38], we discussed in particular the case of bi-quadratic coupling, and a zero temperature bath. Hence the noise is originated from vacuum fluctuations. This brings the present statistical field problem even closer to that of the quantum field problem studied before via CTP effective actions.

Consider two independent self-interacting scalar fields: $\phi(x)$ depicting the system, and $\psi(x)$ depicting the bath. The classical action for these two fields are given respectively by:

$$S[\phi] = \int d^4x \left\{ \frac{1}{2} \partial_\nu \phi(x) \partial^\nu \phi(x) - \frac{1}{2} m_\phi^2 \phi^2(x) - \frac{1}{4!} \lambda_\phi \phi^4(x) \right\}$$

$$S[\psi] = \int d^4x \left\{ \frac{1}{2} \partial_\mu \psi(x) \partial^\mu \psi(x) - \frac{1}{2} m_\psi^2 \psi^2(x) - \frac{1}{4!} \lambda_\psi \psi^4(x) \right\} = S_0[\psi] + S_I[\psi]$$

where $m_\phi$ and $m_\psi$ are the bare masses of $\phi(x)$ and $\psi(x)$ fields respectively. Both fields have a quartic self-interaction with the bare coupling constants $\lambda_\phi$ and $\lambda_\psi$. In (2.2) we have written $S[\psi]$ in terms of a free part $S_0$ and an interacting part $S_I$ which contains $\lambda_\psi$. Assume these two scalar fields interact via a bi-quadratic coupling

$$S_{int} = \int d^4x \left\{ -\lambda_{\phi\psi} \phi^2(x) \psi^2(x) \right\}$$

Assume also that all three coupling constants $\lambda_\phi$, $\lambda_\psi$ and $\lambda_{\phi\psi}$ are small parameters of the same order. The total classical action of the combined system plus bath field is then given by

$$S[\phi, \psi] = S[\phi] + S[\psi] + S_{int}[\phi, \psi]$$

The total density matrix of the combined system plus bath field is defined by

$$\rho[\phi_f, \psi_f, \phi'_i, \psi'_f, t] = <\phi_f, \psi_f | \hat{\rho}(t) | \phi'_i, \psi'_f >$$

where $|\phi >$ and $|\psi >$ are the eigenstates of the field operators $\hat{\phi}(x)$ and $\hat{\psi}(x)$,

$$\hat{\phi}(\vec{x}) |\phi >= \phi(\vec{x}) |\phi >, \quad \hat{\psi}(\vec{x}) |\psi >= \psi(\vec{x}) |\psi >$$
Since we are interested in the behavior of the system, and the environment only to the extent of how it influences the system, the quantity of relevance is the reduced density matrix defined by

$$\rho_r[\phi, \phi', t] \equiv \int d\psi \int d\psi' \delta(\psi - \psi') \rho[\phi, \psi, \phi', \psi', t]$$  

(7)

Here \( \int d\psi (\vec{s}) \) denotes the functional integral over the Hilbert space of all quantum states of the \( \psi \) field. For technical convenience, let us assume that the total density matrix at an initial time is factorized, i.e., that the system and bath are statistically independent,

$$\hat{\rho}(t_0) = \hat{\rho}_\phi(t_0) \times \hat{\rho}_\psi(t_0)$$  

(8)

where \( \hat{\rho}_\phi(t_0) \) and \( \hat{\rho}_\psi(t_0) \) are the initial density matrix operator of the \( \phi \) and \( \psi \) field respectively, the former being equal to the reduced density matrix \( \hat{\rho}_r \) at \( t_0 \) by this assumption.

The influence functional \( F[\phi, \phi'] \) which summarizes the averaged effect of the bath on the system is defined as

$$F[\phi, \phi'] = \int d\psi_f(\vec{x}) \int d\psi_i(\vec{x}) \rho_{\psi}[\psi_i, \psi_i', t_0] \int_{\psi_i(\vec{x})}^{\psi_f(\vec{x})} D\psi \int_{\psi_i'(\vec{x})}^{\psi_f'(\vec{x})} D\psi' \times \exp i\left\{ S[\psi] + S_{int}[\phi, \psi] - S[\psi'] - S_{int}[\phi', \psi'] \right\}$$  

(9)

Here \( \int D\psi \) denotes the functional path integral over all possible histories of the \( \psi \) field under some boundary conditions. The influence action \( \delta A[\phi, \phi'] \) and the effective action \( A[\phi, \phi'] \) are defined as

$$F[\phi, \phi'] = \exp i\delta A[\phi, \phi']; \quad A[\phi, \phi'] = S[\phi] - S[\phi'] + \delta A[\phi, \phi']$$  

(10)

The quantum average of a physical variable \( Q[\psi, \psi'] \) over the unperturbed action \( S_0[\psi] \) can be expressed in terms of \( F^{(1)}[J_1, J_2] \), the influence functional of the free bath field, assumed to be linearly coupled with external sources \( J_1 \) and \( J_2 \). For a zero temperature bath, the bath field \( \psi \) is in a vacuum state,

$$\hat{\rho}_\psi(t_0) = |0><0|$$  

(11)
then the influence functional \( F^{(1)}[J_1, J_2] \) is the same as the closed-time-path (CTP) or ‘in-in’ vacuum generating functional \([21, 22]\) and the associated influence action is the usual CTP or in-in vacuum effective action. In such cases, the two-point functions of the bath fields are just the well-known Feynman, Dyson and positive-frequency Wightman propagators for a free scalar field.

If \( \lambda \phi \psi \) and \( \lambda \psi \) are assumed to be small parameters, the influence functional can be calculated perturbatively by making a power expansion of \( \exp i \{ S_{int} + S_I \} \) in orders of \( \lambda \) and \( \bar{\hbar} \). This is similar to the perturbation calculation for \( \lambda \phi^4 \) theory in the CTP formalism carried out before for quantum fluctuations \([13]\) and for coarsed-grained fields \([26]\). The 1-loop effective action up to \( O(\lambda^2) \) is \([29, 38]\):

\[
A[\phi, \phi'] = \{ S[\phi] + \delta S_1[\phi] + \delta_2[\phi] \} - \{ S[\phi'] + \delta S_1[\phi'] + \delta_2[\phi'] \} + \delta A[\phi, \phi']
\]

\[
= S_{ren}[\phi] + \int d^4 x \int d^4 y \, \frac{1}{2} \lambda_{\phi\psi}^2 \phi^2(x)V(x-y)\phi^2(y)
\]

\[
- S_{ren}[\phi'] - \int d^4 x \int d^4 y \, \frac{1}{2} \lambda_{\phi\psi}^2 \phi'^2(x)V(x-y)\phi'^2(y)
\]

\[
- \int_{t_0}^t ds_x \int d^3 \vec{x} \int_{s_x}^{s_y} ds_y \int d^3 \vec{y} \lambda_{\phi\psi}^2 \left[ \phi^2(x) - \phi'^2(x) \right] \times \eta(x-x') \left[ \phi^2(y) + \phi'^2(y) \right]
\]

\[
+ i \int_{t_0}^t ds_x \int d^3 \vec{x} \int_{s_x}^{s_y} ds_y \int d^3 \vec{y} \lambda_{\phi\psi}^2 \left[ \phi^2(x) - \phi'^2(x) \right] \times \nu(x-y) \left[ \phi^2(y) - \phi'^2(y) \right]
\]

(12)

where \( S_{ren}[\phi] \) is the renormalized action of the \( \phi \) field, now replaced by the physical mass \( m_{\phi r}^2 \) and physical coupling constant \( \lambda_{\phi r} \), namely,

\[
S_{ren}[\phi] = \int d^4 x \left\{ \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} m_{\phi r}^2 \phi^2 - \frac{1}{4!} \lambda_{\phi r} \phi^4 \right\}
\]

(13)

and the kernel for the non-local potential in (2.12) is

\[
V(x-y) = \mu(x-y) - sgn(s_x - s_y)\eta(x-y)
\]

(14)

which is symmetric and \( \eta, \nu, \mu \) are real and nonlocal kernels.
\[ \eta(x - y) = \frac{1}{16\pi^2} \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} \pi \sqrt{1 - \frac{4m^2}{p^2}} \theta(p^2 - 4m^2) \times isgn(p_0) \] (15)

\[ \nu(x - y) = \frac{2}{16\pi^2} \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} \pi \sqrt{1 - \frac{4m^2}{p^2}} \theta(p^2 - 4m^2) \] (16)

\[ \mu(x - y) = -\frac{2}{16\pi^2} \int \frac{d^4p}{(2\pi)^4} e^{ip(x-y)} \int_0^1 d\alpha \ln |1 - i\epsilon - \alpha(1 - \alpha)\frac{p^2}{m^2}| \] (17)

\[ \eta(x - y) = \frac{1}{16}\pi \int d^4p \left\{ \frac{1}{(2\pi)^4} e^{ip(x-y)} \pi \right\} \sqrt{1 - \frac{4m^2}{p^2}} \theta(p^2 - 4m^2) \times isgn(p_0) \] (22)
In the Langevin field equation, the dissipative force is

\[ F_\gamma(x) \sim \left\{ \int d^4y \, \eta(x - y) \phi^2(y) \right\} \phi(x) \]  

(23)

As discussed in \[29, 38\], we find that a fluctuation-dissipation relation exists between the dissipation kernel (2.15) and the noise kernel (2.16):

\[ \nu(x) = \int d^4y \, K(x - y) \eta(y) \]  

(24)

where

\[ K(x - y) = \delta^3(\vec{x} - \vec{y}) + \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} e^{i\omega(s_x - s'_y)} [\omega] \]

\[ = \delta^3(\vec{x} - \vec{y}) + \int_0^{+\infty} \frac{d\omega}{\pi} \omega \cos(\omega)(s_x - s_y) \]

Apart from the delta function \(\delta^3(\vec{x} - \vec{x}')\), the convolution kernel for quantum fields has exactly the same form as for the quantum Brownian harmonic oscillator with linear or nonlinear dissipations at zero temperature.

### 3 Geometrodynamic Noise

We now use the results of the above example to discuss the problem of noise in quantum cosmology. In the minisuperspace model discussed by Sinha and Hu \[19\] (SH) the system is made up of the scale factor \(a\) of a Robertson-Walker Universe and the homogeneous mode \(\chi\) of a massive \((m)\) interacting \((\lambda)\) scalar field. The higher inhomogeneous modes \(f_k\) make up the environment. Since the equation of motion for the linearized gravitational perturbations (the Lifshitz equations) has the same form as a massless minimally coupled scalar field, they can be used to mimic the gravitational modes customarily ignored when one takes the minisuperspace approximation. The interaction action is

\[ S_{\text{int}} = -\frac{1}{2} \int d\eta [m^2 a^2 f_k^2 - \frac{\lambda}{4!} (6 \sum_k \chi_0^2 f_k^2 + \ldots)] \]  

(1)
where $\eta = \int dt/a$ is the conformal time, and the dots denote higher order $f$ terms, whose effect is ignored here. The coupling between the system $(a, \chi_0)$ and the bath $(f_k)$ is of a biquadratic kind. The framework which Sinha and Hu used is in terms of the in-in effective action for wave functions of the universe. To discuss noise one needs to extend the framework to that of influence functionals dealing with reduced density matrices constructed from the wavefunctions of the minisuperspace. However, using prior experience in interpolating between these two formalisms, we can almost guess the result. For a zero temperature bath consisting of vacuum fluctuations of the higher scalar field modes, we expect the noise kernel be the same as (2.16) in the example of Sec. 2. This noise we can call a minisuperspace noise, it is actually the noise experienced in the minisuperspace sector from coarse-graining all the other modes.

Since the SH model is meant to mimic the gravitational interactions, let us see how this can be applied to gravitation coarse-grainings. For a purely gravitational problem where the system consists of the homogeneous universe (e.g., Robertson-Walker universe or Bianchi universes) and the environment consists of the inhomogeneous modes (or perturbations of the homogeneous background spacetime), such as that studied by Halliwell and Hawking [43], one can deduce a noise in the minisuperspace arising from coarse-graining the inhomogeneous gravitational modes and call it, appropriately, a ‘geometrodynamical’ or ‘spacetime’ noise. Of course, in the case of gravitational interaction, the coupling between the minisuperspace and the other gravitational modes is different from the SH model. In the perturbative case, they are of the derivative type. \[1\]

Another well-studied example which we can use to deduce noise associated with quantum field- and spacetime- coarse-grainings is that of a scalar field in a Bianchi Type I universe. The system is the BI universe with line element

$$ds^2 = a^2(d\eta^2 - \epsilon_{ij}^2dx^i dx^j) \equiv g_{\mu\nu}dx^\mu dx^\nu$$

(2)

where $\eta$ is the conformal time and $\beta_{ij}$ is the (traceless) anisotropy matrix. The bath is made up of a massless conformal scalar field with classical action

$$S_m = \frac{1}{2} \int d^4x(g^{\mu\nu}\partial_\mu \phi \partial_\nu \phi - \frac{1}{6}R\phi^2).$$

(3)

They come from separating the scalar curvature term $R$ in the Einstein action into two groups $g^{00} + h^1$, the background spacetime and the gravitational perturbations, and $R$ has two derivatives in $g$.\[1\]
where $R$ is the scalar curvature. Calzetta and Hu have derived the form of the Schwinger-Keldysh effective action. The real and imaginary parts of it one can identify as the dissipation and noise kernels. Before, we only concentrated on the former part as we were interested in the dissipative effects on the dynamics of spacetime due to the backreaction of particle creation in the scalar field. Here we would like to explore the noise aspect. The extension to the framework of reduced density matrices has been carried out by Paz and Sinha. They used this model in quantum cosmology to illustrate the effects of decoherence, correlation and backreaction for attaining the semiclassical limit. One can use the QBM paradigm illustrated in the previous section to treat this problem in quantum cosmology. Paz and Sinha derived the influence functional for the BI universe. From it one can examine the noise kernel and derive the distribution function associated with the noise force. Details of this investigation can be found in.

The noise in this model of course originates from the scalar field. But, by the same reasoning, one can construct similar models where the environment is of gravitational origin. Gravitational perturbations of a BI universe is an example. (The classical backreaction problem was studied by Hu.) The Gowdy universe is another interesting model where the system can be taken to be the homogeneous mode and the environment the inhomogeneous modes. One can calculate the form of noise associated with the gravitational wave modes. Sinha and I are in the process of calculating the geometrodynamical noises for these quantum cosmological models (e.g., the minisuperspace RW models, the Bianchi I model, the midi-superspace model of perturbed RW universe, and the Gowdy models).

4 Gravitational Entropy

In the general framework of statistical field theory as exemplified by the quantum Brownian model discussed above it is natural to define an entropy associated with the reduced density matrix of the system with the coarse-grained effect of the environment taken into consideration. It is given formally by

$$S \equiv -Tr \rho_{red} \ln \rho_{red} \quad (1)$$

For the cases where the environment is a coarse-grained quantum field, this can be regarded as the entropy of a quantum field; whereas for the
cases where the environment is a coarse-grained spacetime in the form of inhomogeneous cosmological modes or gravitational waves, this can be rightfully called spacetime or gravitational entropy. (For a related definition see [47, 48].) It is a well-known result in statistical mechanics that any entropy function associated with a closed system remains constant in time but the entropy functions constructed from the reduced density matrix of a subsystem with coarse-graining in its environment increases with time. For quantum cosmology, there is no intrinsic time. Operationally, time is a parameter chosen to impart ‘dynamics’ for the evolution of the system. One can use the WKB time associated with the semiclassical limit. The direction of time associated with the expansion of the universe defines a cosmological arrow of time. (Often in quantum cosmology the volume of the universe is chosen as the time parameter.) Information in an open system is degraded by the environment and flows in the direction of entropy increase. The increase of entropy (with respect to this cosmological time) defines a thermodynamic arrow of time.

Let me first distinguish the meaning of gravitational entropy defined in our present context from that used previously in different contexts. Penrose [49] first introduced this concept as a way to describe how many distinguishable configurations or degrees of freedom a gravitational field or spacetime geometry has. He suggested that some integrated form of the square of the Weyl curvature tensor $C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}$ can serve as a measure of how ‘coarse’ or ‘irregular’ the spacetime is. This is a reasonable suggestion as the Weyl curvature measures the gravitational wave components of spacetime (e.g., the RW universe which is isotropic and homogeneous has $C = 0$.) Penrose believes that the universe begins in a highly regular gravitational state, and ends with a highly irregular one. He proposed this Weyl Curvature Hypothesis, i.e., the increase of gravitational entropy proportional to the square of the Weyl curvature, to describe the increase of observed gravitational clumping as the universe expands.

Hu [50] discussed gravitational entropy in the extended context when quantum field processes like particle creation are important. He brought in the effect of backreaction of quantum matter on the dynamics of geometry.

\[\text{Since we are not dealing with ordinary matter, but with coarse-grainings in spacetimes, perhaps one should be careful in using the word thermodynamic here: it need not refer to matter, even though the high-frequency gravitational waves can indeed be viewed as a form of matter.}\]
and showed that if one adopts Penrose’s geometric notion of gravitational entropy, particle creation in an anisotropic or inhomogeneous universe (which is proportional to the square of the Weyl curvature tensor) can act as an efficient process near the Planck time to ‘convert’ gravitational entropy (associated with spacetime) to matter entropy (associated with the created particles).

The gravitational entropy defined here in a statistical mechanical sense is very different from Penrose’s geometric definition both in terms of syntax and context. Choosing the Weyl curvature as a measure of the entropy of a gravitational field is a descriptive definition but it lacks a sound theoretical basis. By contrast the present definition of gravitational entropy adheres to the formal premises of (non-equilibrium) statistical mechanics. But one wants to know how relevant it is in describing reality?–How good is it in helping us understand the nature of physical laws involving gravity, and the behavior of our universe? The questions we need to address are thus two-fold: 1) the physical meaning of such a definition and its relation with the more intuitive and descriptive definition of Penrose. 2) Since the definition of entropy here depends on how the reduced density matrix is constructed, and hence on how the system is defined with respect to its environment, a deeper question to ask is: ‘Who’ decides what should be called the system and how is this decision justified? The first issue relates to the specific context of gravity and cosmology, the second issue points to the basic tenets of statistical mechanics. We will have space here only to mention some ideas on these important problems. Detailed discussions are to be given elsewhere.

Despite the difference in syntax from which these definitions are introduced, and the difference in emphasis they attach– Penrose’s (79) definition is of a geometrical nature, Hu’s (83) incorporates quantum fields, and this one (93) is based on statistical mechanics concepts– they are nevertheless interrelated with each other. Hu’s (83) is related to Penrose’s in that the Weyl tensor which measures the ‘disorder’ of geometry in Penrose’s definition of gravitational entropy also measures the amount of particle creation in the field; and it is due to the backreaction of created particles that gravitational

\[3\]The underlying logic runs somewhat like: 1) One knows from second law of thermodynamics that entropy increases with time (does it apply equally well to entropy of gravitational field?); 2) History of the universe shows (rather, one believes) that spacetime evolves from a highly regular state to a highly irregular state, the Weyl tensor increasing in time correspondingly; 3) Hence pick the Weyl tensor (squared) as a definition of gravitational entropy.
entropy changes (indeed, decreases). Now, the present definition uses the same but extended theoretical framework as the Hu’s 1983 work, as we explained above, so they are consistent with each other. The essential physics described in both places lies in the dissipation and noise kernels. The 83 paper focuses on the former aspect while here is added the latter aspect. For quantum processes in anisotropic and inhomogeneous spacetimes, both are proportional to the Weyl curvature tensor (squared). Thus, in the conception of [15], the noise associated with vacuum fluctuations of the field (or spacetime) when parametrically amplified gives rise to particle creation which is measured by the Weyl curvature squared, while the dissipative term in the Langevin equation for the probability distribution of the geometry variables, which measures in this capacity the damping of anisotropy in the dynamics of spacetime, is also proportional to $C^2$. This is not surprising as the noise and dissipation are connected by a fluctuation -dissipation relation [28, 44]. (See [51, 52] for such relations in spacetimes with event horizons.) Indeed, as emphasized in [15], the backreaction problem in cosmology, i.e., the mutual influence between quantum matter in the form of particle creation and geometry through the laws of geometrodynamics, can be seen as a manifestation of a generalized fluctuation-dissipation relation between matter and spacetime.

For other discussions of gravitational entropy, see [18].

5 The Physical Universe as a Quantum Open System

To close, let me now say a few words about the second issue above, i.e., how appropriate is it to view the universe as an open system and to what extent can the basic tenets of non-equilibrium statistical mechanics be applied to cosmological problems. In describing some physical entity as an open system the criteria in the separation of the system and environment and the choice of coarse-graining enter in a fundamental way. One can ask the following questions:

1) How natural?

In cosmology many physical problems of interest have ‘natural’ separations of system and environment – event horizon, particle horizon, causal
boundaries, multiply connectedness, etc. So an open system description of cosmological problems is not just an artificial scheme, but can be meaningful. (See last section of [47].) However, how ‘natural’ the separation is can vary with the degree of accuracy (fineness of description) or level of structure (hierarchy of compositeness) one restricts one’s attention to. (See last section of [37].) For example, a background field or spacetime separation gives meaningful description in the semiclassical regime, but can become meaningless as the interaction between matter and gravitational degrees of freedom becomes highly nonlinear or nonlocal. A black hole in the full quantum gravity regime may evaporate completely, and with the disappearance of the event horizon, the semiclassical description premised upon such a separation is no longer meaningful. But at each definite level it is permissible to consider the system as open. In this sense a theoretically closed system which requires a large amount of detailed information for its complete description can in principle be approximated successively by a hierarchy of ordered open systems.

2) How open?

If, say, an event horizon used for the separation of the system and environment changes with time for some observer, does it mean that the system sees different things at different moments, as the system and noise both change. The answer is yes, but it is really not so alarming. (This is indeed the case underlying the workings of ‘stochastic inflation’ in the de Sitter universe, where the higher modes, which make up the noise, leave the horizon at every moment.) As for the particle horizon, this is happening to our physically observable (causally-connected) universe at every moment. Our ever widening horizon is no cause for concern even though the treatment of quantum fields with moving boundaries (changing constituents in the Hilbert space) is not exactly easy.

3) How sensitive is the behavior of the system to changes in the environment and variations of coarse-grainings?

Is coarse-graining a subjective or objective effect? Are there particular sets of coarse-graining which can lead to stable or persistent structures? It is believed that that part of the classical world which possesses a definite structure should be largely independent of the choice and influence of its surroundings. This is the case when the system is, say, in its hydrodynamic (long wavelength, low energy) regime. In the linear coupling regime microscopic derivations of the transport coefficients are also largely independent of variations in the environment. These are familiar examples of the con-
ditions upon which robust structures can arise. This basic issue in quantum and statistical mechanics certainly merits more serious studies [37].

4) How good is the choice of the homogeneous cosmology as the system and viewing the inhomogeneous sector and the matter fields as the environment?

This is, of course, more often than not, a matter of convenience rather than principle. However, one may ponder why the physical universe is indeed adequately described by a homogeneous cosmology in a theoretical sense. This has been attempted in a few directions before, such as:

a) viewing the prominence of the homogeneous modes as a result of infrared dominance and dimensional reduction, in a sense similar to the Kaluza-Klein idea. This was proposed in [18].

b) viewing the gravitational excitations as collective modes. The distinction between the prominent modes which constitute the ‘system’ and the others (inhomogeneous modes) which are relegated to the ‘environment’ may be determined at a higher level in the hierarchy of spacetime structures. (For example, the rotational collective modes of the nucleus are not fundamental variables, but they are more appropriate in the description of macroscopic motions of the nucleus than quarks and gluons. An open system model of the collective variables can adequately describe the dissipative dynamics for the nucleus – it is impractical and unnecessary to invoke the details of the nucleon wavefunctions, but this will be a wrong model to use if one wants to probe into the nuclear effects of quark-gluon interactions.) The choice of an open system is not arbitrary, but should be guided by the physical conditions which defines the problem one wishes to study. More discussions of these issues can be found in the last section of [37].

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