Hidden conformal symmetry of extreme and non-extreme Einstein-Maxwell-Dilaton-Axion black holes

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Abstract: The hidden conformal symmetry of extreme and non-extreme Einstein-Maxwell-Dilaton-Axion (EMDA) black holes is addressed in this paper. For the non-extreme one, employing the wave equation of massless scalars, the conformal symmetry with left temperature \( T_L = \frac{M}{2\pi a} \) and right temperature \( T_R = \frac{\sqrt{M^2-a^2}}{2\pi a} \) in the near region is found. The conformal symmetry is spontaneously broken due to the periodicity of the azimuthal angle. The microscopic entropy is derived by the Cardy formula and is fully in consistence with the Bekenstein-Hawking area-entropy law. The absorption cross section in the near region is calculated and exactly equals that in a 2D CFT. For the extreme case, by redefining the conformal coordinates, the duality between the solution space and CFT is studied. The microscopic entropy is found to exactly agree with the area-entropy law.
1. Introduction

Holographic duality that reflects the relation of the quantum gravity theory and quantum field theory plays an important role in the modern physics [1]. One successful example of this duality is the AdS/CFT correspondence. The rudiment of this correspondence was proposed by Maldacena in the late 1990s [2]. In his work, the equivalence between type IIB string theory compactified on $\text{AdS}_5 \times S^5$ and four-dimensional supersymmetric Yang-Mills theory was showed. Ever since then, it has attracted great interest and other examples of the correspondence have been found. One can refer to [3] and references therein for reviews.

In [4], Brown and Henneaux showed that any consistent theory of quantum gravity in three-dimensional anti-de Sitter space ($\text{AdS}_3$) is holographically dual to a two-dimensional (2D) conformal field theory (CFT). Inspired by this work [4], Guica, Hartman, Song and Strominger replaced $\text{AdS}_3$ with the near-horizon extreme Kerr (NHEK) geometry and put forward the Kerr/CFT correspondence [5]. After specifying the boundary conditions at the asymptotic infinity of the NHEK, they showed the asymptotic symmetry group is one copy of the conformal group and has a central charge $c_L = 12J$. This implies that the quantum gravity in the NHEK geometry is dual to a 2D CFT. The microscopic entropy was derived from Cardy formula with the CFT temperatures and central charges. It turns out that this entropy precisely agrees with the Bekenstein-Hawking area-entropy law. This result is a significant step to the evidence of the holographic duality. The generalizations to other extreme black holes can be found in [6].

Since the duality of the extreme black holes has been investigated, it is natural to extend this work to near-extreme and non-extreme black holes. In [6], the authors studied the superradiance of the near extreme black hole and showed the holographic duality can also exist in this black hole. Very recently, the holographic duality of the non-extreme Kerr black hole has been explored by Castro, Maloney and Strominger [7]. After introducing the conformal coordinates $(\omega^+, \omega^-, y)$ and the local vector fields, they found that there is a conformal symmetry in the solution space of a massless scalar field equation in the near...
region of the Kerr black hole. However, this symmetry is local and would be spontaneously broken due to the existence of the periodicity \(2\pi\) of the azimuthal angle \(\phi\). Furthermore, the absorption cross section in the near region is found to be precisely the finite-temperature one in a 2D CFT. Moreover, the entropy, obtained from the Cardy formula with central charges \(c_L = c_R = 12J\), agrees with the Bekenstein-Hawking area-entropy law. Their results provide more confidence to the duality of generic Kerr/CFT. Subsequently, this work is extended to other non-extreme black holes \([9, 10, 11, 12, 13, 15]\). By introducing a new set of conformal coordinates and vector fields, similar methods are applied to the extreme and near extreme black holes in \([16]\).

The aim of this paper is to extend the work to dilatonic black holes. We examine the hidden conformal symmetry of the extreme and non-extreme EMDA black holes. Our result shows that there does exist a conformal symmetry in the near region. This symmetry is spontaneously broken under CFT temperatures due to the \(2\pi\) periodicity of the azimuthal angle \(\phi\) of the metric. To clearly show the duality, we calculate the microscopic entropy by the Cardy formula with central charges \(c_L = c_R = 12J\). We find a full coincidence with the Bekenstein-Hawking area-entropy law. Meanwhile, the absorption cross section in the near region of the non-extreme EMDA black hole is derived. This cross section is exactly equal to the finite-temperature one in a 2D CFT. We find that the CFT temperatures are \(T_L = \frac{M}{2\pi a}\) and \(T_R = \frac{\sqrt{M^2-a^2}}{2\pi a}\) for the non-extreme EMDA black hole. While the temperatures are \(T_L = \frac{r^+}{2\pi a}\) and \(T_R = 0\) for the extreme one.

The reminder of this paper is outlined as follows. In sect. 2, we discuss the hidden conformal symmetry of the non-extreme EMDA black hole and give the microscopic description. The black hole entropy and the absorption cross section are derived. Sect. 3 gives the hidden symmetry of the extreme EMDA black hole. Sec. 4 is our discussions and conclusions.

2. The hidden conformal symmetry of the non-extreme EMDA Black hole

The EMDA black hole is a solution of the Einstein-Maxwell-Dilaton-Axion field equations. From the action

\[
S = \int d^4x \sqrt{-g} \left[ R - 2g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} e^{4\phi} g^{\mu\nu} \partial_\mu \kappa \partial_\nu \kappa - e^{-2\phi} F^\mu_\nu F^{\mu\nu} - \kappa F_{\mu\nu} \tilde{F}^{\mu\nu} \right],
\]

(2.1)

where \(\phi\) is the dilaton and \(\kappa\) is the axion scalar field, Garcia, Galtsov and Kechkin \([17]\) derived the axially symmetrical solution in 1995, the EMDA metric,

\[
ds^2 = -\frac{\Delta - a^2 \sin^2 \theta}{\Sigma} dt^2 - \frac{2a \sin^2 \theta (r^2 + 2br + a^2 - \Delta)}{\Sigma} dt d\phi + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{(r^2 + 2br + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma} \sin^2 \theta d\phi^2,
\]

(2.2)

where
\[
\Sigma = r^2 + 2br + a^2 \cos^2 \theta, \\
r_{\pm} = M \pm \sqrt{M^2 - a^2}, \\
\Delta = r^2 - 2Mr + a^2 = (r - r_+) (r - r_-). 
\] (2.3)

\(r_+\) and \(r_-\) are the outer and inner horizons. The parameters \(a\) and \(b\) represent the angular momentum and dilaton constant per unit mass, respectively. The mass \(M\) is related to the ADM mass of the black hole as \(M_A = M + b\). \(J\) is the angular momentum and relates to the black hole mass as \(J = (M + b)a\). There is a lot of work appeared to study thermodynamic properties of this black hole. Here we simply give the thermodynamic parameters, the area \(A\), Hawking temperature \(T\) and angular velocity \(\Omega\) at the outer horizon,

\[
A = 4\pi \left( r_+^2 + 2br_+ + a^2 \right), \\
T = \frac{r_+^2 - a^2}{4\pi r_+ \left( r_+^2 + 2br_+ + a^2 \right)}, \\
\Omega = \frac{a}{r_+^2 + 2br_+ + a^2}. 
\] (2.4)

We first use Klein-Gordon equation to explore the wave function of a massless scalar field on the background spacetime of the black hole,

\[
\frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} g^{\mu\nu} \partial_\nu \Phi \right) = 0. 
\] (2.5)

Due to the existence of two Killing vectors (\(\partial_t\) and \(\partial_\varphi\)) in the EMDA spacetime, we can carry out separation of variables as

\[
\Phi (t, r, \theta, \varphi) = e^{-i\omega t + im\varphi} R (r) S (\theta). 
\] (2.6)

Substituting the inverse metric and eqn.(2.6) into the Klein-Gordon equation, one gets two sectors, the angular sector

\[
\left[ \frac{1}{\sin \theta} \partial_\theta (\sin \theta \partial_\theta) - \frac{m^2}{\sin^2 \theta} + a^2 \omega^2 \cos^2 \theta \right] S (\theta) = -\lambda_{lm} S (\theta),
\] (2.7)

and the radial sector

\[
\left[ \partial_r \Delta \partial_r + (2(M + b) r_+ \omega - ma)^2 \right] R (r) = \lambda_{lm} R (r),
\] (2.8)

where \(\lambda_{lm}\) is the constant of variable separation. If the term included \(\omega^2\) vanishes in eqn.(2.7), one obtains the standard Laplacian on the 2-sphere and the solutions of eqn.(2.7)
are spherical harmonics with the corresponding value $\lambda_{lm} = l(l + 1)$. We follow [8] and only discuss the case of the near region defined by $r\omega \ll 1$. In this case, the terms included $\omega^2$ should be neglected and the equations of the angular part as well as the radial one can be rewritten as

$$
\left[ \frac{1}{\sin \theta} \partial_\theta (\sin \theta \partial_\theta) - \frac{m^2}{\sin^2 \theta} \right] S(\theta) = -\ell (\ell + 1) S(\theta), \quad (2.9)
$$

and

$$
\left[ \partial_r \triangle \partial_r + \frac{(2(M + b)r_+\omega - ma)^2}{(r - r_+)(r_+ - r_-)} - \frac{(2(M + b)r_-\omega - ma)^2}{(r - r_-)(r_+ - r_-)} \right] R(r) = \ell (\ell + 1) R(r). \quad (2.10)
$$

In the following, we can find that the solution of the radial part eqn.(2.10) is related to $SL(2, R)$, which implies the existence of a hidden conformal symmetry. Following [8], we define the conformal coordinates $(\omega^\pm, y)$ in terms of $(t, r, \phi)$ by

$$
\begin{align*}
\omega^+ &= \sqrt{\frac{r - r_+}{r - r_-}} e^{2\pi T_L \phi}, \\
\omega^- &= \sqrt{\frac{r - r_+}{r - r_-}} e^{2\pi T_R \phi} \\
y &= \sqrt{\frac{r_+ - r_-}{r - r_-}} e^{\pi (T_L + T_R) \phi} - \frac{i}{2(M + b)},
\end{align*}
$$

(2.11)

with $T_L = \frac{r_+ + r}{4\pi a}, T_R = \frac{r_+ - r}{4\pi a}$. The local vector fields are defined as

$$
\begin{align*}
H_1 &= i \partial_+ , \\
H_0 &= i \left( \omega^+ \partial_+ + \frac{1}{2} y \partial_y \right), \\
H_{-1} &= i \left( \omega^+ \partial_- + \omega^- y \partial_y - y^2 \partial_- \right),
\end{align*}
$$

(2.12)

and

$$
\begin{align*}
\tilde{H}_1 &= i \partial_-, \\
\tilde{H}_0 &= i \left( \omega^- \partial_- + \frac{1}{2} y \partial_y \right), \\
\tilde{H}_{-1} &= i \left( \omega^- \partial_+ + \omega^+ y \partial_y - y^2 \partial_+ \right),
\end{align*}
$$

(2.13)

which obey the $SL(2, R)$ Lie bracket algebra

$$
[H_0, H_{\pm}] = \mp i H_{\pm}, \quad [H_{-1}, H_1] = -2i H_0. \quad (2.14)
$$

Similar relations are hold for $(\tilde{H}_0, \tilde{H}_\pm)$. The corresponding quadratic Casimir is
\[ H^2 = \mathcal{H}^2 = -H_0^2 + \frac{1}{2} (H_1 H_{-1} + H_{-1} H_1) \]
\[ = \frac{1}{4} (g^2 \partial_y^2 - y \partial_y) + y^2 \partial_+ \partial_. \]  
(2.15)

Inserting eqn. (2.11) into eqn. (2.15) yields
\[ H^2 = \partial_r \Delta \partial_r - \frac{(a \partial_\phi + 2 (M + b) r_+ \partial_\phi)^2}{(r - r_+)(r_+ - r_-)} + \frac{(a \partial_\phi + 2 (M + b) r_- \partial_\phi)^2}{(r - r_-)(r_+ - r_-)}. \]  
(2.16)

Thus eqn. (2.10) can be written as
\[ H^2 R(r) = \mathcal{H}^2 R(r) = \ell (\ell + 1) R(r), \]  
(2.17)

which means that the scalar Laplacian can reduce to the \( SL(2, R) \) Casimir and the conformal symmetry exists here. However, due to the existence of the \( 2\pi \) periodicity of the azimuth angle \( \phi \) and the local definition of the vector fields, the \( SL(2, R)_L \times SL(2, R)_R \) symmetry is spontaneously broken.

In the following, we give some microscopical interpretation. As analysis in [8], we find that there exists a relation between Minkowski \( \omega^\pm \) and Rindler \( t^\pm \) coordinates in non-extreme case,
\[ \omega^\pm = e^{\pm t^\pm}, \]  
(2.18)

where
\[ t^+ = 2\pi T_R \phi, \quad t^- = \frac{t}{2(M + b)} - 2\pi T_L \phi. \]  
(2.19)

This indicates that in the \( SL(2, R)_L \times SL(2, R)_R \) invariant Minkowski vacuum, the observer in Rinder space can observe the thermal bath of Unruh radiation. Hence, the non-extreme EMDA black hole is dual to the left and right temperatures \( (T_L, T_R) \) in a the 2D CFT.

Moreover, one can find the EMDA entropy. It has been known that the Bekenstein-Hawking area-entropy of black holes can be obtained by the Cardy formula. However, the central charges \( C_L \) and \( C_R \) of the non-extreme EMDA black hole are unknown. In Ref. [8, 8, 11, 12, 13], the central charges near the extreme case are adopted and are regarded as those of the non-extreme case. The reason is that there is a smooth limit from near-extremal to extremal solution and probably the hidden conformal symmetry connects smoothly to that of extreme limit. Therefore we can get the central charges from the extreme black holes,
\[ c_L = c_R = 12J. \]  
(2.20)

Using the Cardy formula with the central charges and temperatures, we obtain the microcosmic entropy
\[ S = \frac{\pi^2}{3} (c_L T_L + c_R T_R) = 2\pi (M + b) r_+, \]  
(2.21)

which is the same as the Hawking-Bekenstein area-entropy of the black hole. We now discuss the solution of the radial sector. There are many papers appeared to investigate such object \[18\]. The difference here is that our attention is focused on the near region \( r\omega \ll 1 \). After introducing a new coordinate \( z = \frac{r-r_-}{r_+-r_-} \), eqn.(2.8) becomes

\[
(1 - z) z \partial_z^2 R(z) + (1 - z) \partial_z R(z) 
+ \left[ \frac{2 (M + b) r_+ \omega - ma}{z (r_+ - r_-)^2} - \frac{2 (M + b) r_- \omega - ma}{(r_+ - r_-)^2} \right] \frac{\ell (\ell + 1)}{1 - z} R(z) = 0. 
(2.22)
\]

Considering the outer boundary, the solution of eqn.(2.22) takes the form

\[ R \simeq A r^\ell + B r^{-\ell-1}, \]  
(2.23)

where

\[
A = \frac{\Gamma \left( 1 - i 2 (M + b) \frac{r_+ \omega - ma}{r_+ - r_-} \right) \Gamma \left( 1 + 2\ell \right)}{\Gamma \left( 1 + \ell - i 2 (M + b) \omega \right) \Gamma \left( 1 + \ell - i 2 (M + b) \frac{(r_+ + r_-) \omega - 2ma}{r_+ - r_-} \right)}. \]  
(2.24)

Therefore the absorption cross section is

\[
P_{\text{abs}} \sim |A|^2 \sim \sinh \left( \frac{4\pi (M + b) r_+ \omega - 2\pi ma}{r_+ - r_-} \right) \left| \Gamma \left( 1 + \ell - i 2 (M + b) \omega \right) \right|^2 \times \left| \Gamma \left( 1 + \ell - i 2 (M + b) \frac{(r_+ + r_-) \omega - 2ma}{r_+ - r_-} \right) \right|^2. 
(2.25)
\]

To compare with the finite-temperature absorption cross section for a 2D CFT, we introduce the conjugate charges \( \delta E_L \) and \( \delta E_R \), which satisfy the relation

\[ \delta S = \frac{\delta E_L}{T_L} + \frac{\delta E_R}{T_R}. \]  
(2.26)

The first law of thermodynamics of the black hole tells us

\[ T \delta S = \delta M_A - \Omega \delta J, \]  
(2.27)

where \( T, S, M_A \) and \( \Omega \) are the Hawking temperature, the area-entropy, the ADM mass and the angular velocity of the black hole respectively. Combining eqn.(2.24) and (2.27), one finds

\[
\delta E_L = \frac{\delta M_A}{a} (M + b) (r_+ + r_-),
\]
\[
\delta E_R = \frac{\delta M_A}{a} (M + b) (r_+ + r_-) - \delta J. \]  
(2.28)
The left and right moving frequencies are defined as

\[ \omega_L \equiv \delta E_L = \frac{\omega}{a} (M + b) (r_+ + r_-), \]
\[ \omega_R \equiv \delta E_R = \frac{\omega}{a} (M + b) (r_+ + r_-) - m, \] (2.29)

where \( \omega = \delta M_A \) and \( m = \delta J \) were introduced. Now applying the conformal weights \((h_L, h_R) = (\ell, \ell)\) and substituting eqn (2.29) into (2.25), one has

\[ P_{\text{abs}} \sim T^2_{L} T^2_{R} \sinh \left( \frac{\omega_L}{2T_L} + \frac{\omega_R}{2T_R} \right) \left| \Gamma \left( h_L + i \frac{\omega_L}{2\pi T_L} \right) \right|^2 \left| \Gamma \left( h_R + i \frac{\omega_R}{2\pi T_R} \right) \right|^2, \] (2.30)

which is just the finite-temperature absorption cross section for a 2D CFT. This gives further evidence to the duality of the non-extreme EMDA black hole.

3. The hidden conformal symmetry of the extreme EMDA black hole

In this section, we address the conformal symmetry of the extreme EMDA black hole. In the extreme case, the outer horizon and inner horizon are coincident with each other. This implies that the conformal coordinate \( y \), right temperature and the denominator in eqs. (2.8), (2.10) and (2.16) are equal to zero. The local vector fields do not obey the \( SL(2, R) \) algebra in eqn. (2.14). Thus one has to re-construct the conformal coordinates. In the extremal case, the radial wave function is different from that of non-extreme case, eqn. (2.10). It is now

\[ \left[ \partial_r \Delta \partial_r + \frac{4 (M + b) \omega ((M + b) r_+ \omega - ma)}{(r - r_+)^2} + \frac{(2 (M + b) r_+ \omega - ma)^2}{(r - r_+)^2} \right] R(r) = \ell (\ell + 1) R(r). \] (3.1)

We also consider the near region in this section. The above equation can be rewritten as

\[ \left[ \partial_r \Delta \partial_r + \frac{4 (M + b) \omega ((M + b) r_+ \omega - ma)}{(r - r_+)^2} + \frac{(2 (M + b) r_+ \omega - ma)^2}{(r - r_+)^2} \right] R(r) = \ell (\ell + 1) R(r). \] (3.2)

Due to the coincidence of the outer and inner horizons, \( y \) in the conformal coordinates vanishes. It is necessary to introduce a new set of conformal coordinates [16]:

\[ \omega^+ = \frac{\gamma_1}{2} \left( \frac{1}{a \phi} - \frac{1}{r - r_+} \right), \]
\[ \omega^- = \frac{1}{2} \left( e^{r_+ \phi/a-(M+b)t/2} - \frac{2}{\gamma_1} \right), \]
\[ y = \sqrt{2 (r - r_+)} e^{r_+ \phi/a-(M+b)t/2}. \] (3.3)
The new coordinate transformations have an undetermined parameter $\gamma_1$. This degree of freedom is free to take arbitrary values (to get the right Casimir operator), based on particular black holes and they do not affect the physics. The local vector fields are then defined by eqs.(2.12) and (2.13). These generators again satisfy the $SL(2, R)$ algebra, eqs.(2.14) and (2.15). Inserting the above equations into (2.15), one derives the Casimir

$$\mathcal{H}^2 = \partial_r \triangle \partial_r - \frac{4(M + b) \partial_t (2(M + b) r_+ \partial_t + a \partial_\phi)}{(r - r_+)} - \frac{(2(M + b) r_+ \partial_t + a \partial_\phi)^2}{(r - r_+)^2}.$$  

(3.4)

Hence the radial wave function can be written as

$$\mathcal{H}^2 R(r) = \hat{\mathcal{H}}^2 R(r) = \ell (\ell + 1) R(r),$$  

(3.5)

Thus, the conformal symmetry exists in the extreme EMDA black hole. Similarly, this symmetry is spontaneously broken due to the the existence of the periodicity of the azimuth angle ($\phi - \phi + 2\pi$). In the extreme case, as indicated by the second equation in eqn.(3.3), there only exists one relation $\omega^{-} = e^{-t^{-}}$ between Minkowski $\omega^{-}$ and Rindler $t^{-}$, where $t^{-} = (M + b) t/2 - 2\pi T_L \phi$, and $T_L = r_+/2\pi a$. It implies that an observer in the Rindler space can observe the thermal bath of Unruh radiation and the extremal EMDA black only dual to the left temperature in the 2D CFT. Therefore, the CFT left temperature is $T_L = \frac{r_+}{2\pi a}$ and the right temperature vanishes.

The asymptotic symmetry group (ASG) plays an important role in the studies of Kerr/CFT correspondence. Every consistent set of boundary conditions specifies an associated ASG for the near horizon extreme black holes. The Virasoro algebra is obtained from the generators of the ASG. The corresponding central charge is related to the angular momentum as $c_L = 12\pi J$. Using the Cardy formula with the central charges derived, the microscopic entropy is

$$S = \frac{\pi^2}{3} c_L T_L = 2\pi(M + b) r_+,$$

(3.6)

which is agreement with the Bekenstein-Hawking area-entropy.

4. Conclusions

In conclusion, we have discussed the hidden conformal symmetry in the solution space of the massless scalar for the extreme and non-extreme EMDA black holes. We found that there is a spontaneously symmetry breaking due to the periodicity ($2\pi$) of the azimuth angle $\phi$. The Bekenstein-Hawking area-entropies of the black holes were recovered by the Cardy formula with the central charges. We also found that the CFT temperatures $T_L = \frac{M}{2\pi a}$ and $T_R = \frac{\sqrt{M^2 - a^2}}{2\pi a}$ for the non-extreme EMDA black hole and that $T_L = \frac{r_+}{2\pi a}$ and $T_R = 0$ for the extreme one. Finally, we investigated the absorption cross section of the non-extreme EMDA black hole in the near region. This cross section is equal to the finite-temperature one in a 2D CFT. Our results provide evidence for the duality between the near region of the EMDA background spacetime and the two-dimensional CFT.
The investigation of the hidden conformal symmetry in this paper is limited to the scalar fields. It is natural to expect that the symmetry also exists in the solution space of higher spin fields. Thus, it is of interest to conduct research on the spinor fields or vector fields in the future work.

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