Noise thermometry at low temperatures: MFFT measurements between 1.6 K and 1 mK

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Abstract. Reliable and traceable thermometry is a demanding task at low temperatures. Recently, we have developed a dc SQUID-based noise thermometer, the magnetic field fluctuation thermometer (MFFT), for practical thermometry in the low temperature range \cite{1}. Its operational principle is based on the Nyquist theorem ensuring a linear characteristic over a wide range of temperatures. This makes the MFFT capable to replace a variety of secondary thermometers, which are normally used in the low temperature range.

For the first time, we report on MFFT measurements over more than 3 decades in temperature from about 1.6 K down to below 1 mK. The deviations from a high-accuracy realization of the PLTS-2000 are found to be $\approx 1\%$.

1. Introduction
For practical thermometry a multitude of secondary thermometers is available in the low temperature region. Most of them exhibit a non-linear characteristic, which limits the useful range of application. To cover broader temperature ranges with sufficient precision and accuracy usually several different thermometers must be used. Especially in the millikelvin region problems arise from the dissipation of heat by the measurement process in the thermometer and the growing thermal resistances between the sensing element and the object, which temperature is to be measured. This may lead to a thermal decoupling of the thermometer as the temperature is lowered.

A solution to the mentioned problems is the MFFT operated in secondary mode. Secondary mode operation avoids the expensive and time-consuming determination of all parameters of the measurement chain that would be required for operation in primary mode.

In this paper we describe the application of a fast, compact, and easy to use MFFT system comprising a metallic temperature sensor with a SQUID gradiometer, a data acquisition unit and a software package.

2. MFFT set-up
The principle layout of the MFFT is shown in Figure 1. In the MFFT, the noise resistor $R$ is represented by the metallic sensor body made of high purity copper. The thermally activated noise currents inside the copper body cause magnetic-field fluctuations above its surface. These fluctuations are inductively detected by a SQUID gradiometer as thermal magnetic flux noise (TMFN). As the power spectral density (PSD) of noise is proportional to
thermodynamic temperature according to Niquist’s theorem, the PSD of the TMFN can be described by the following empirical relation [2]:

\[ S_\Phi(f, T) = \frac{S_0(T)}{(1 + (f/f_c)^2a)^b}, \]  

where \( S_0(T) \) is the zero-frequency value of \( S_\Phi(f, T) \), \( f_c \) is the characteristic fall-off frequency of the TMFN, and \( a \) and \( b \) are constants of the order of one. For a fixed configuration of the SQUID gradiometer and the temperature sensor, \( S_0(T) \) is proportional to temperature and electrical conductivity of the MFFT sensor. Because the sensor body is made of high purity copper containing no magnetic impurities, its conductivity can be assumed to be constant in the temperature range of interest here. In this case, \( f_c \), \( a \), and \( b \) are also constant and \( S_\Phi \) shows a “low-pass-like” frequency dependence which is independent of temperature. The fall-off frequency is determined by the geometry of the metallic temperature sensor, its distance to the gradiometer and electrical conductivity [2].

The sensor-gradiometer configuration was optimized to obtain both a high TMFN signal and a high \( f_c \) [3] for measurements at very low temperatures. This was achieved by gluing a 120 \( \mu m \) thick SQUID-gradiometer chip directly onto the sensor body. The part of the sensor body, which is sensed by the SQUID gradiometer (Figure 1(b, c)), is about 200 \( \mu m \) thin. With a length of about 35 mm the total volume of the copper sensor body amounts to approximately 1200 mm\(^3\). The temperature sensor of the MFFT has no electrical connections to the SQUID (Figure 1(a)) and the whole set-up is encapsulated in an Nb cylinder for parasitic noise reduction.

![Figure 1](image.png)

**Figure 1.** Schematic diagram (a) and principle layout (b) of the MFFT sensor body made of copper with the SQUID gradiometer. \( R(f) \) and \( L(f) \) denote the frequency dependent resistance and inductance of the metallic temperature sensor, which is inductively coupled to the SQUID by the mutual inductance \( M(f) \). In the cross-sectional view (c) the arrow is directed to the thin part of the sensor body directly underneath the SQUID gradiometer. A thread allows tight metallic connection to the object to be measured.

The SQUID gradiometer is connected to a SEL-1 electronics from MAGNICON [4] and operated in flux locked loop mode. The output of the SQUID electronics is fed to the input of a DAQ box equipped with a 16Bit digital to analog converter based on a National Instruments I/O card. A software package is used to set up the working point of the SQUID and control the data acquisition by the DAQ box. The noise measurements are carried out by recording time traces of the fluctuating output voltage of the SQUID electronics caused by the TMFN in the SQUID gradiometer. Then, the time traces are Fourier transformed and averaged to obtain the PSD of the TMFN. The software also includes a fit functionality to extract the temperature information from the PSD of the TMFN. In addition, for the experimental results presented below, we have also used a separate software with extended fitting possibilities.
3. Experiments

All experiments were carried out in a nuclear demagnetization cryostat providing temperatures from above 1 K to well below 1 mK. The cryostat is equipped with a $^3$He melting pressure thermometer for the high-accuracy realization of the PLTS-2000, $T_{2000}$, with typical combined standard uncertainties of a few ten µK [5]. Above 1 K a copy of the ITS-90 maintained on a rhodium iron thermometer was used as temperature reference $T_{90}$.

For the temperature measurements we have operated the MFFT in secondary mode. As the shape of the PSD of the TMFN is independent of temperature for a fixed sensor-gradiometer configuration and a constant electrical conductivity of the sensor material, all device characteristic parameters of the PSD can be obtained from a fit to equation (1) at a stabilized reference temperature $T_{\text{ref}}$. The determination of the MFFT parameters $S_0(T_{\text{ref}})$, a, b, and $f_c$ is equivalent to a calibration of the MFFT at $T_{\text{ref}}$. Then, keeping these parameters unchanged an unknown temperature $T$ can be calculated using the relation:

$$T = T_{\text{ref}} \frac{S_0(T)}{S_0(T_{\text{ref}})}, \quad \text{(2)}$$

where $S_0(T)$ is obtained from a fit of the corresponding TMFN spectrum measured at $T$.

For practical reasons, we have chosen as $T_{\text{ref}}$ the tungsten superconductive transition at 15.5 mK. This allowed us to stabilize the temperature within a few µK over long periods of time to take a sufficient number of averages for the calibration measurements. The temperature values $T_{2000}$ were recorded simultaneously during the MFFT calibration.

![Figure 2. Power spectral density of thermal magnetic flux noise recorded at different temperatures between 1 mK and 1.6 K. The spectra with higher plateau values correspond to higher temperatures as indicated by the right axis.](image)

The comparison between the MFFT and the temperature scales PLTS-2000 and ITS-90 was carried out in the temperature range from below 1 mK up to 1.6 K. The values of the MFFT temperatures, $T_{\text{MFFT}}$, were obtained from fits of the TMFN spectra using equations 1–2. The corresponding PSDs shown in Figure 2 were taken over at least 40 minutes at stabilized temperature plateaus above 15 mK and during slow temperature drifts of no more than several µK below 15 mK. Before fitting, the peaks at 50 Hz and higher harmonics were removed from all spectra.

In order to extract from the noise spectra only the temperature relevant information, we have subtracted from the measured PSD the noise generated by the SQUID itself, a constant noise contribution which was determined in independent measurements. In our MFFT set-up, the SQUID is in direct contact with the sensing element. As we have applied the MFFT at
extremely low temperatures, low-frequency excess noise of the SQUID is an issue [6] as can be seen in Figure 2. Even though its influence is noticeable only in the low frequency part of the PSD at the lowest temperatures, for fitting the PSDs we have added to equation (1) a term inverse proportional to frequency. The value of this term was also determined during the MFFT calibration. Finally, the fits were restricted to a useful frequency range from 50 Hz to 3000 Hz.

![Linear fit to MFFT temperatures](image1)

**Figure 3.** Linear fit to MFFT temperatures $T_{\text{MFFT}}$ in dependence on $T_{2000}$ and $T_{90}$ between 1 mK and 1.6 K. The green arrow indicates the reference temperature for the MFFT calibration at 15.5 mK.

![Relative deviation](image2)

**Figure 4.** Relative deviation of $T_{\text{MFFT}}$ from $T_{2000}$ and $T_{90}$ between 1 mK and 1.6 K. The relative standard uncertainty ($k=2$) of $T_{2000}$ is indicated by blue lines. The ±1% deviation band is shown by green lines.

The results of the comparison of $T_{\text{MFFT}}$ with $T_{2000}$ and $T_{90}$ are shown in Figure 3 and Figure 4. The uncertainties are marked by error bars or are less than the point size. The MFFT temperatures follow closely the reference temperatures with deviations of about ±1% from 1.6 K down to below 1 mK, except for one outlier data point. Because the temperature sensing element was in excellent thermal contact to the demagnetization stage, we didn’t observe any problems caused by overheating or thermal decoupling of the MFFT down to the lower end of the PLTS-2000 at 902 μK. We also have measured MFFT temperatures as low as 400 μK. Work is underway to establish a reference scale in that region to confirm these results.

4. Conclusions

We have shown that noise thermometry using a MFFT operated in secondary mode can cover the temperature range from above 1 K down to the μK region. A simple calibration at one reference temperature ensured traceability to the PLTS-2000. Applied in this way, the MFFT is capable to replace a number of secondary thermometers usually used in that broad temperature range. Moreover, the MFFT set-up used for the experiments presented is commercially available from MAGNICON [4].

References

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