Optimal shutdown management

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Abstract.

The paper presents a novel approach for the synthesis of the open-loop pitch profile during emergency shutdowns. The problem is of interest in the design of wind turbines, as such maneuvers often generate design driving loads on some of the machine components.

The pitch profile synthesis is formulated as a constrained optimal control problem, solved numerically using a direct single shooting approach. A cost function expressing a compromise between load reduction and rotor overspeed is minimized with respect to the unknown blade pitch profile. Constraints may include a load reduction not-to-exceed the next dominating loads, a not-to-be-exceeded maximum rotor speed, and a maximum achievable blade pitch rate.

Cost function and constraints are computed over a possibly large number of operating conditions, defined so as to cover as well as possible the operating situations encountered in the lifetime of the machine. All such conditions are simulated by using a high-fidelity aeroservoelastic model of the wind turbine, ensuring the accuracy of the evaluation of all relevant parameters.

The paper demonstrates the capabilities of the novel proposed formulation, by optimizing the pitch profile of a multi-MW wind turbine. Results show that the procedure can reliably identify optimal pitch profiles that reduce design-driving loads, in a fully automated way.

1. Introduction

Emergency shutdowns [1] often generate design driving loads on some components of a wind turbine. Consider for example Figure 1, which shows the ranking of loads at the tower base (left) and blade root (right) for a 2.5 MW three-bladed up-wind variable-speed wind turbine. Each bar in the plots corresponds to the maximum load experienced during a transient design load case (DLC), as prescribed by certification guidelines [1]. Loads generated during shutdowns (DLCs 1.1-1.7, 2.1-2.2, 3.2, 4.2, 5.1) are depicted in yellow, loads generated in idling conditions (DLCs 6 and 7) in red, and loads obtained during closed-loop operation (DLCs 1.1, 1.3, 1.6 and 1.7) in green.

The figure shows that, for the tower base, the dominating loads are generated during shutdowns. These are followed in the ranking by idling loads; as the machine is not controlled in idling conditions, the reduction of loads in such cases is a design problem that can be dealt with by changing, for example, the rotor solidity and diameter, and/or the blade shape (with evident consequent tradeoffs with performance and loading on other components). Although the picture is specific of a given machine and restricted to tower base and blade root, this situation is quite typical of contemporary wind turbines, for which the sizing of at least some of their main components are often driven by shutdown loads. This is true also for crucial sub-components, as for example the drive-train [2].
The figure very clearly illustrates the basic idea pursued in this work: if one could find a way to conduct shutdowns such that the driving loads in the ranking of some components of interest were pushed to the level of the idling loads, then further load reductions could only be achieved by a redesign of the rotor. Hence, one would be achieving the maximum load reduction possible without a design change, i.e. one would be making the best possible use (from this point of view) of a given configuration. Notice that, as suggested by this design-centered view of the problem, in general there is no need to minimize shutdown loads as much as possible, but only to reduce them to the level of the next dominating loads in the ranking [3]. It should be added that the peak rotor speed throughout shutdown maneuvers should also be limited, as excessive speeds may induce potentially dangerous overloads on the electrical generator.

Emergency shutdowns are routinely conducted by a rapid pitch to feather of the blades, as the loss of connection to the grid implies that the generator can not be used to slow down the rotor. Furthermore, aerodynamic braking is used because, given the very high kinetic energy of modern large rotors, mechanical brakes can typically be engaged only once the rotor has reached a fairly low angular speed. Emergency devices can in principle be used in the first instants following the grid loss to power the generator, but the benefits of this additional help in the braking maneuver must be traded against the additional complexity and cost of the extra needed equipment.

The pitch maneuver has the effect of initially quickly reducing the aerodynamic torque and then changing its sign. This is illustrated in Figure 2 at left, which depicts the trajectory followed by the machine in the plane of the torque coefficient $C_Q$ vs. the tip-speed-ratio (TSR) $\lambda$ as the blade pitch angle $\beta$ is changed during the shutdown operation in a DLC 1.5 condition [1]. After the beginning of the gust, indicated in the plots by a circle, the wind initially drops and then starts rising. In response, the TSR first increases and then, as the rotor starts decelerating, rapidly decreases; during this phase the machine is governed by its closed-loop controller, which adjusts pitch and torque in response to the wind perturbation. At the instant of time when grid connection is lost, indicated in the plots by an asterisk and corresponding to the time of maximum wind speed gradient, the electrical torque drops suddenly and the rotor initially speeds up (moving to higher TSRs) before starting to slow down. In this phase of the maneuver, the machine is operated by an open-loop pitch profile, which rapidly reduces the aerodynamic torque towards large negative values.

As shown in Figure 2 at right, the pitch maneuver has also the effect of very rapidly reducing and then inverting the sign of the rotor thrust, which produces a violent pitch forward of the

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**Figure 1.** Ranking of maximum loads for typical DLCs [1], at the tower base (left) and blade root (right). Yellow: loads generated during shutdowns; red: loads generated in idling conditions; green: loads generated in closed-loop operation.
Figure 2. Trajectory followed during an emergency shutdown in the planes $C_Q - \lambda$ (at left, torque coefficient vs. TSR) and $C_T - \lambda$ (at right, thrust coefficient vs. TSR), as the blade pitch angle $\beta$ is changed. Circle: begin of gust; asterisk: grid loss instant; square: maximum fore-aft bending at tower base. The leg between two triangles is covered in 1 sec.

machine. Notice that this is not only due to the elastic restitution caused by the reduced push back of the rotor on the tower; in fact, the forward swinging is accentuated by the thrust reversal, which actually pulls the machine forward. As the wind turbine swings towards its maximum forward deflection, the maximum loading at the tower base is typically generated (square symbol in the plot). This is followed by rapidly damped fore-aft oscillations, with loads that are typically smaller than the ones of the first forward swing, until the machine comes to a full stop.

The loss of grid connection typically implies a limited operational capability of the on-board systems. While the pitch system is commonly powered by emergency battery packs, other systems and sensors are typically assumed to be unavailable [1]. Under this assumption, the pitch profile followed by the blades during a shutdown maneuver should not be computed in closed-loop (because of the assumed unavailability of feedback information), and consequently must be pre-computed off-line and then used in open-loop during the maneuver. As the same pitch profile must be used irrespectively of the conditions at the time of the maneuver initiation, typically quite simple blade pitch time histories are used, that rapidly pitch the blades forward at the maximum achievable pitch rate available with the emergency power system.

This description highlights the complexity of the problem of designing blade pitch profiles during emergency shutdowns. A very rapid and aggressive pitching means a fast slow down of the rotor with a reduced overspeed, but may imply large ensuing loads. On the other hand, a less aggressive policy may reduce the peak loads, but will in general lead to a larger rotor overspeed. The optimal compromise must be robust in the face of uncertainties, including the behavior of the wind speed throughout the maneuver, as well as the rotor azimuth and blade position at the maneuver initiation when the connection to the grid is lost. As one if facing the problem of identifying the best pitch profile, it may also be necessary to conduct other complex trade-off analyses, for example to size and verify the cost-effectiveness of an emergency pack for powering the generator in the first instants following the grid loss.

Given the complexity of the problem, the manual tuning of the open-loop pitch profile during emergency shutdowns is a complex and time consuming operation. Goal of the present paper is to develop a systematical method to optimize the pitch profile in a fully automated manner, while accounting for all desired tradeoffs and conditions. Such a tool can be used for rapidly synthesizing the optimal pitch profile of an existing machine, or to help in the quick exploration of the solution space during design.

The pitch profile synthesis is here formulated as an optimal control problem [4]. A cost
function expressing a compromise between load reduction and rotor overspeed is minimized with respect to the unknown blade pitch profile, subjected to inequality constraints that translate all other desired conditions. Constraints may include, among others, a load reduction not-to-exceed the next dominating loads (to avoid unnecessarily reducing shutdown generated loads), a not-to-be-exceeded maximum rotor speed, and a maximum achievable blade pitch rate.

Cost function and constraints are computed over a possibly large number of operating conditions, which typically include the one and fifty year extreme operating gusts (EOGs), extreme change directions (ECDs), normal turbulence models (NTMs), and possibly other conditions of interest, evaluated at multiple reference wind speeds and multiple grid loss times (for example, occurring at the maximum wind speed, at the maximum wind speed rate, etc.). These different conditions are defined so as to cover as well as possible all possible operating situations encountered in the lifetime of the machine, and are evaluated by using a high-fidelity aeroservoelastic model of the machine. As, during each optimization step, the computations of all such conditions are independent of one another, the various DLCs can be run in parallel, with substantial computational savings. The use of a sophisticated simulation model is central to the proposed procedure, as it ensures that the machine response is resolved to a sufficient level of accuracy.

The problem is solved numerically using a direct single shooting approach [5, 10]. By using a direct approach, one does not need to derive the governing optimal control equations, which represents a major simplification when high-fidelity simulation models are used. Furthermore, shooting allows for the use of small time steps without increasing the optimization problem size, which is again important when high-fidelity models of the machine are used, as these must resolve to a sufficient accuracy the fast scales of the solution. A single shooting approach is used in this case, as the system is stable and the time window over which the optimization is conducted is rather short; in fact, in general there is no need to cover the shutdown until the machine reaches a full stop, as peak loads and overspeed are generated during the first swing forward. The pitch time history is described in terms of suitable shape functions and associated discrete parameters, which represent the unknowns of the resulting non-linear programming problem (NLP). The discrete constrained optimization is finally solved using a sequential quadratic programming (SQP) method [7].

The paper has been organized as follows. At first, the mathematical formulation of the shutdown optimization problem is given, together with its numerical solution procedure. Next, the proposed procedures are demonstrated by optimizing the shutdown pitch profile of a multi-MW wind turbine. A comprehensive set of design DLCs are assumed, to investigate the robustness of the computed solution.

2. Optimal shutdown maneuver

The main goal of the optimization procedure is that of synthesizing an open-loop pitch profile yielding an ultimate tower root bending moment (and/or other relevant driving loads) during an emergency shutdown that is as close as possible to the next limit load case in the DLC ranking. This should be achieved within an acceptable overspeed or, if this is not possible, a best compromise between these two contrasting requirements should be found. To achieve this goal, the pitch profile design has been formulated in this paper as a constrained optimal control problem defined over a finite duration time window and over a set of relevant DLCs.

The optimal control pitch profile design problem is formulated as

\[
\min_{\beta(t)} J(\beta(t)) ,
\]

\[
\text{s.t.: } f(\dot{x}, x, \beta(t)) = 0, 
\]

\[
g_{\min} \leq g(\beta(t)) \leq g_{\max},
\]
where $J$ is the cost function, Eq. (1b) is the state space model of the wind turbine with states $x$ and unknown pitch input time history $\beta(t)$, while finally Eq. (1c) represents the design constraint inequality conditions.

Cost function $J$ is composed of three contributing terms, and writes

\[ J = \frac{1}{2} \sum_{i=1}^{N} \left( w_{M_y}(\max_{t} M_{y,i}(t))^2 + w_{\Omega}(\max_{t} \Omega_i(t))^2 + w_{\dot{\beta}} \int_{t_0}^{t_f} \dot{\beta}_i(t)^2 \, dt \right). \] (2)

The three terms have the following roles:

- The first component is the sum of the maxima over time $t$ of the fore-aft bending moment $M_y$ obtained from all $N$ considered DLCs. Notice that the optimization is performed for a collection of different operating conditions, which include different extreme gusts, different extreme turbulent wind cases and different instants of the fault occurrence. The adoption of an automated search method of the optimal pitch history provides for the ability to seamlessly extend the set of search load cases, at the cost of an increased computational load. In fact, besides the usual one-year gust with grid loss (DLC 1.5) required for certification, other conditions can be straightforwardly included, in order to increase the robustness and generality of the computed result.

- Similarly, the second term is the sum of the maxima of the rotor speed $\Omega$ from the same conditions. The use of the sum of the maxima instead of the absolute maximum over all considered DLCs leads to a smooth cost function, in turn allowing for the use of a gradient-based optimization method. In fact, by changing the pitch profile, the absolute maximum might suddenly switch from one DLC to another one, creating discontinuities in the cost. While very useful to ensure smoothness, the use of such a combined cost needs some care. Indeed, there is in general no guarantee that the absolute maximum will diminish even for a decreasing sum of maxima. This problem can be addressed by looking at the resulting leading terms in the cost and tuning the weights accordingly. At the beginning, uniform weights can be used. Next, after inspection of the results, higher weights can be associated with the dominating DLCs to have better control on them. The process can be repeated using a continuation technique, until a solution that is mostly driven by the actual maximum load(s) is found.

- The third term penalizes the pitch activity, where $\dot{\beta}(t) = \frac{d}{dt} \beta(t)$ is a derivative with respect to time. The initial time $t_0$ corresponds to the instant when the machine looses connection to the grid, while $t_f$ is the final time of the optimization window. As maximum loads and peak rotor speed take place at the first swing forward of the machine during the shutdown maneuver, the final time is not treated as an unknown (although this might be easily done) but rather set to a large enough value, to ensure that the optimization window covers the response of interest.

The terms $w_{M_y}$, $w_{\Omega}$ and $w_{\dot{\beta}}$ are suitable weights, chosen so as to make the three terms dimensionally coherent and to set their individual participation to the overall cost.

Optimal control problem (1) is subjected to differential and algebraic constraints. The differential constraint (1b) expresses the wind turbine dynamics. In this work, the dynamics are rendered through an aeroservoelastic model based on a geometrically exact multibody formulation, coupled to a BEM method and implemented in the software program $Cp$-$\Lambda$ [8, 9]. The use of a high fidelity model ensures that the dynamic response of the machine and the associated loads are resolved to a sufficient level of accuracy.
The algebraic inequality constraints (1c) are written as

\[
\begin{align*}
\max_t (\dot{\beta}(t)) & \leq \dot{\beta}_{\text{max}}, \\
\dot{\beta}_{f,\text{min}} & \leq \dot{\beta}(t_f) \leq \dot{\beta}_{f,\text{max}}, \\
\bar{M}_y & \leq \max_t \left( \max |M_{y,i}(t)| \right), \quad \bar{M}_y > 0, \\
\max_t \left( \max (\Omega_i(t)) \right) & \leq \Omega_{\text{max}}.
\end{align*}
\]

These conditions have the following meaning:

- Equation (3a) limits the maximum pitch-rate, which cannot exceed the \(\dot{\beta}_{\text{max}}\) value. This constraint is imposed in order to avoid an unfeasible control solution, as the pitch actuators always have a limited maximum actuation speed.
- Equation (3b) bounds the pitch rate at the end of the optimization window \(t_f\). At that time instant, even if the shutdown procedure is well underway, the turbine is usually still rotating, and the collective pitch of the blades has not yet reached the parking value. The optimized pitch history is then connected to a final constant-pitch-rate law, that drives the blades to the full feather condition with an assigned rate \(\dot{\beta}_f\). The bounds expressed by Eq. (3b) are then used to ensure a value of the pitch rate at the end of the optimization that allows for a smooth transition towards this final part of the maneuver.
- Equation (3c) maybe be used to avoid excessively aggressive maneuvers in terms of load reduction, which would then typically imply an increase in overspeed. The constraint limits the maximum fore-aft bending moment at the tower base not to exceed the next load in the ranking, \(\bar{M}_y\).
- Finally, Eq. (3d) expresses a constraint on the rotor speed, which is kept under a value \(\Omega_{\text{max}}\), chosen in order to avoid damage to the electrical and mechanical components of the machine.

The unknown pitch time history of optimal control problem (1) is discretized in terms of assumed basis functions \(n(t)\) and associated parameters \(p\), by writing

\[
\beta(t) = \beta(t_0) + n(t)^T p,
\]

where \(\beta(t_0)\) is the known pitch value at the instant of the grid loss. The parameterization of the unknown pitch profile was here based on a very simple fourth-order polynomial, although other choices are clearly possible, and the discretized profile is written as

\[
\beta(t) = \beta(t_0) + \sum_{k=1}^{4} a_k (t - t_0)^k.
\]

Notice that in this work a single extremal pitch profile is sought for the whole set of chosen operating conditions. This is not strictly necessary, as one could seek a family of pitch profiles that are scheduled in terms of relevant operating parameters (as wind, rotor speed, pitch setting, azimuthal position, or others). However, this approach was preferred here for two main reasons. Firstly, this avoids tailoring a specific control solution to one particular wind scenario or another, which would risk incurring in a diminished overall robustness. Secondly, this gives the simplest possible control law, which is independent from any scheduling parameters and which, even more importantly, does not require the ability to detect or distinguish a particular scenario from another. In fact, although in principle possible, the necessity of estimating wind or other parameters to drive the scheduling of a family of control laws would seem to considerably weaken
the robustness and reliability of such a critical component of the safety system of a wind turbine. Nonetheless, the extension of the present formulation to accommodate scheduled pitch profiles is straightforward and does not pose significant technical problems.

By the parameterization of the unknown control function, problem (1) is turned into the following optimization of the algebraic parameters $p$:

$$\min_p J(p),$$

s.t.: $f(\dot{x}, x, \beta(p)) = 0,$

$$g_{\min} \leq g(p) \leq g_{\max}.$$  

Problem (6) is solved by a direct shooting approach [5, 10], thereby avoiding the necessity of deriving the optimal control governing equations, which might be a very complex task when high-fidelity virtual models of the machine are used, as in the present case. Since stability is typically not an issue in wind turbine shutdown problems, a single shooting arc is used, so that no internal gluing constraints between arcs are necessary. At each instantiation of the design parameters during the optimization process, the pitch profile is known, and all necessary DLCs are run and then postprocessed; the results are used for evaluating terms either in the cost or the constraints, according to need. As no dependency among the various DLCs exists, all simulations can be performed in parallel, thereby significantly reducing the computational time. The resulting algebraic non-linear constrained optimization is here solved by a SQP algorithm [6], with gradients computed by finite differences and scaling of the equations to improve conditioning. Given the non-linearity of the problem, existence and uniqueness of the solution cannot be guaranteed in general, although experience has shown that this does not appeared to be a major concern in the use of the methodology for solving practical cases of engineering relevance.

3. Results

The proposed approach for the synthesis of pitch profiles during shutdowns was tested with reference to a 2.5 MW three-bladed upwind wind turbine. The model, implemented in Cp-Lambda, operates in closed-loop with a variable rotor speed pitch-torque controller. For this particular machine, the load envelope on the fore-aft tower base bending moment is dominated by DLC 1.5, obtained with a standard pitch-to-feather maneuver at constant rate after grid loss. The second-ranking DLC is an idling condition (DLC 6.2). Hence, in this scenario an optimized shutdown maneuver can have the effect of reducing the dominating loads on this component of the machine, with consequent possible design benefits [3].

3.1. Reference performance with standard pitch profile

At first, a reference performance for the machine during shutdowns was computed by considering a standard open-loop pitch profile, that drives the blades towards feather immediately after a grid loss with a pre-determined constant pitch-rate of $\dot{\beta}_f = 6$ deg/sec. After 2 sec after fault, the pitch-rate is decreased to 4 deg/sec, to simulate a partial discharge of the batteries powering the pitch actuators.

The turbine response was investigated in DLC 1.5 conditions, which correspond to 1-year extreme operating gusts (EOG-1yr) with concurrent grid loss. Following the prescriptions of GL design standards [1], 24 different conditions were simulated. These included four reference wind speeds: rated minus 2 m/sec, rated, rated plus 2 m/sec and cut-out. For each wind speed, six grid loss times within the gust time history were considered: minimum wind speed before the maximum speed peak, maximum positive gradient, half-way between the maximum positive gradient and the maximum speed, maximum speed, minimum negative gradient, and finally minimum speed after the maximum speed peak.
To better assess the performance of the machine during shutdowns, an additional set of conditions were also investigated with respect to tower base peak loads and rotor overspeed. These included:

- 50-year extreme operating gusts (EOG-50yr). Although GL standards do not contemplate faults for these already demanding conditions, a concurrent grid loss was nonetheless considered here, for the same wind speeds and faults times used in the EOG-1yr case.
- Extreme coherent gusts with direction change (ECD), in the presence of a grid loss. Simulations were run for the four reference wind speeds of the previous cases, each for both left and right misalignments between wind direction and rotor axis, and again each for five different grid loss times (at the beginning, half and final time of the direction change phase, as well as at two different times after a maximum misalignment had been reached).
- Real field-recorded time histories of very turbulent wind conditions. The hub-height recorded readings of the wind speed were used to compute three-dimensional wind time histories, that were in turn fed to the wind turbine simulation model. The synthetic three-dimensional wind fields were based on the Kaimal turbulence model, and computed so as to have the same turbulent intensity and average of the field measurements. Sixteen time histories were analyzed, and the instants with the occurrence of the strongest gusts were identified in each one. Close to each of these conditions, a grid loss was imposed at the instants of maximum positive gradient and of maximum wind.

Some of the above additional conditions turned out to be more demanding in terms of loads and rotor speed than DLC 1.5, while others were much less demanding. While other choice are indeed possible, it was decided here to discard conditions in excess of the GL demands. The rationale for such a choice is that current certification guidelines are widely accepted by industry and typically result in reasonable sizings, while the inclusion of other excessively extreme conditions might lead to oversizing. Therefore, after discarding the conditions in excess of the GL standards and the less demanding ones because of their little interest, eight conditions were retained. These conditions imply similar demands as DLC 1.5, but are generated in wind conditions that differ from the DLC 1.5 deterministic gust shape. Hence, these DLCs can be used for assessing the robustness of a single open-loop shutdown pitch profile in the face of changes in wind conditions that produce dominating loads. The eight retained DLCs include two EOGs-50yr and six ECDs, while all field-test wind scenarios were discarded because not demanding enough.

3.2. Optimal maneuvers

Next, the proposed optimization procedure was used for synthesizing an optimal pitch profile. An initial condition for the pitch parameters $p$ was chosen to describe a constant pitch-rate at $\dot{\beta}(t_0) = 6 \text{ deg/sec}$. The cost function weights were set by trial and error to deliver satisfactory results, although the solutions appeared to be quite robust with respect to small changes in these tuning parameters, while all terms and equations were non-dimensionalized by suitable reference values to improve conditioning.

Inequality constraints in (3) were set as follows. The maximum pitch-rate is $\dot{\beta}_{\text{max}} = 9 \text{ deg/sec}$, while $\dot{\beta}_{f,\text{min}}$ and $\dot{\beta}_{f,\text{max}}$ are equal to 6 deg/sec and 9 deg/sec, respectively. The tower fore-aft bending lower limit $M_y$ is the second dominating load, obtained from DLC 6.2.

The optimization time window lasts $t_f - t_0 = 5$ sec after the grid loss instant $t_0$. It was verified by inspection that such a time interval is sufficient to capture the load and overspeed peaks after grid loss.

The optimization procedure was performed using $N = 32$ simulations including the 24 DLC 1.5 as well as the additional 8 conditions that were previously described. Figure 3 shows the standard (solid line) and optimized (dash-dotted line) pitch profiles versus time. It may be
noticed that the optimized pitch profile features a lower pitch-rate than the reference one, more markedly after roughly 1.5 sec following the fault instant \( t_0 = 150 \) sec, and up to the end of the optimization window \( t_f = 155 \) sec.

**Figure 3.** Reference (solid line) and optimized (dash-dotted line) pitch profiles versus time.

Figure 4 shows the results that were obtained with this extended set of DLCs.

The figure on the left reports, for each one of the 32 considered cases, the normalized peak values of the fore-aft bending moment. The horizontal dashed line on the plot represents the second dominating load, \( \bar{M}_y \). The bar diagram shows that the dominating load (DLC 1.5) is reduced to the level of the next one (DLC 6.2). The worst case for the standard pitch profile is encountered in simulation 16, corresponding to a reference wind equal to the rated plus 2 m/sec, and a grid loss at maximum speed. On the other hand, the worst case for the optimized shutdown is encountered in simulation 15, corresponding to the same reference speed and a grid loss halfway between the wind maximum gradient and the gust peak. For both cases, the load reduction due to the optimized pitch profile is close to 12%, which highlights the effectiveness of an appropriate design of the shutdown maneuver. Notice that, as requested, the optimizer has done the bare minimum for reducing the shutdown load to the next dominating one, as further reductions would be of no value and would only imply larger overspeeds (although, in this particular case, there is still a good margin to the maximum allowed rotor speed, as shown immediately here below). Finally, notice that the optimized pitch history has a mitigating effect also on peak loads from load cases from 25 to 32, corresponding to various ECD and EOG-50yr with fault. In particular, two load peaks originally over the limit \( \bar{M}_y \) (numbers 27 and 31 on the left plot) fall well under the DLC 6.2 value.

The right part of Figure 4 reports the normalized rotor speed peak values, the horizontal dashed line indicating the not-to-be-exceeded threshold. It can be noticed that the optimized shutdown maneuver performs only slightly worse than the reference one, and the peak rotor values for all DLCs remains well below the limit.

4. **Conclusions**

The paper has presented a novel procedure for the synthesis of open-loop pitch control laws during shutdowns. The proposed formulation was demonstrated by studying the optimization of the shutdown pitch profile of a multi-MW wind turbine, with the goal of reducing the tower base fore-aft loads to the level of the next dominating non-shutdown case. In a more complete redesign exercise [3], one could leverage this to allow, for example, changes at the level of the rotor design.
Figure 4. Comparison of the results obtained from the reference (blue bars) and optimized (yellow bars) shutdown maneuvers in DLC 1.5, ECD with grid loss and EOG-50yr with grid loss conditions. At left, normalized maxima of absolute fore-aft bending moment at tower base; dashed horizontal line: next dominating load from DLC 6.2. At right, normalized maxima of rotor speed; dashed horizontal line: maximum rotor speed from specifications.

A comprehensive set of load conditions was considered, which included DLC 1.5 as well as other scenarios (ECD and EOG-50yr with fault) resulting in similar extreme loading on the machine, but with different operating and wind conditions. By considering this extended set of conditions, it was verified that the solutions generated by the proposed formulation are robust in the face of perturbations of the pitch profile design scenarios, which is clearly very important for the practical use of such an approach on board wind turbines.

Robust optimal control approaches, as opposed to the present purely deterministic one, could be considered for future developments of the present work in order to further hedge against uncertainties and directly consider the probabilistic nature of the problem.

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