Efficient, Verifiable and Privacy-Preserving Combinatorial Auction Design

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Abstract—We propose a construction to efficiently and securely compute a combinatorial auction (also referred as combinatorial auction) which is able to forbid participants (both auctioneer and the bidders) from learning unnecessary information except those implied in the output of the auction. The auctioneer and bidders are assumed to be untrusted, and they may misbehave throughout the protocol to illegally increase their own benefit. Therefore, we need to either prevent the misbehavior or detect the misbehavior. We achieve this goal by introducing a payment mechanism to control bidders’ behaviors game-theoretically, and we further introduce a blind signature scheme to let bidders verify the authenticity of their payment reported by the auctioneer. Although a third-party signer is involved, he only signs a value blindly (i.e. without knowing the value) and is also untrusted. Moreover, our construction requires at most $O(mn^2)$ rounds of the communication between bidders and the auctioneer where $m$ is the total number of goods and $n$ is the total number of bidders, and the extra computation overhead incurred by our design is very efficient.

I. INTRODUCTION

Auction mechanisms have been widely adopted in many fields. Spectrum auction ([15][18]), cellular networks ([13]), ad hoc networks ([24]), cloud Computing ([26]), cognitive radio networks ([8][48]) and web advertisement ([2]), are good examples. In contrast with simple auctions which sell one good only, a Combinatorial Auction (CA hereafter) sells multiple heterogeneous goods simultaneously and let the bidders bid on any combination of the goods. Such auctions have been researched extensively recently ([10][41][23][42]), in part due to the generality of it, and in part due to growing applications in which combinatorial bidding is necessary ([29][44][38][32][21]). Despite its merits, CA is hard to use in a real life due to the following two issues: infeasibility and security implications.

A. Infeasibility of Combinatorial Auction

Even with assumptions which limit bidders’ bidding behaviors (e.g., assuming single-minded bidders ([23][30])), CA typically requires to solve one or more NP-hard optimization problems, which leads to infeasible generic theoretical designs ([27][47]). Therefore, we replace the exact optimization problems by approximate ones for seek of feasibility. However, this raises another problem: traditional mechanism design in CA assumes the goods are allocated optimally, and its property disappears otherwise (e.g., truthfulness). Therefore, we not only exploit existing efficient approximation algorithm, but also improve an existing mechanism ([23]) which preserves truthfulness (defined later) of the auction in this paper.

B. Security Implications in Combinatorial Auction

Although the market for online auctions is increasingly growing, there are many security and privacy issues that prevent users from readily participate in them ([33]), in particular for online auctions where a long-term relationship between bidders and auctioneers does not exist. Bidders are reluctant to disclose their bid information to others as well as the auctioneer since this may reflect their true valuation on the goods, and they are not willing to disclose their goods selection to other bidders as well since this reveal their preferences. Due to similar reasons, non-winning bidders do not want to report their goods selection to the auctioneer. Ideally, the auctioneer should be able to determine the winners of the auction without knowing individuals’ goods selections.

On the other hand, the payment determined by the auctioneer has to be verifiable by individual bidders. Otherwise, auctioneer may try to report a higher payment than the winner deserves to illegally increase his revenue.

C. Main Challenges

One of the main challenges in our privacy-preserving auction is to let the auctioneer determine the winners without knowing losers’ selection of goods. This seems to be contradictory since a bidder’s selection should be examined by the auctioneer to decide whether he is the loser.

Next one is to preserve the truthfulness of the CA after replacing the exact optimization in the winner determination with an approximated one.

The last main challenge is to verify the payment reported by the auctioneer when the auction terminates. It is highly possible that an untrusted third party auctioneer may misbehave to achieve illegal monetary gain. Therefore, a winning bidder should be able to verify the payment without knowing other irrelevant information.

Besides, the confidentiality of the bid or the goods selection is somewhat obviously required in any privacy-preserving auction design and thus not highlighted, but the privacy is indeed preserved throughout our privacy-preserving CA construction.

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The rest of the paper is organized as follows. More backgrounds of CA as well as related works are presented in Section [1] to show why our work is viable compared to them, and the preliminaries are described in Section [V]. The privacy-preserving CA is modelled in Section [III], and the actual design and the preliminaries are described in Section [IV]. The privacy grounds of CA as well as related works are presented in Section [II] to show why our work is viable compared to them, and the actual design and the preliminaries are described in Section [IV].

II. BACKGROUNDS & RELATED WORK

A. Brief Introduction

Among various types of combinatorial auctions ([10][39]), we shall consider the most common type in this work, one-stage, sealed-bid and single-sided CA.

B. Auction Design

Auction design is composed of two parts. Firstly, winners of the auction are chosen based on their submitted bundles and bids (winner determination), then each winner’s payment is determined by some mechanism (payment determination). Note that winners’ payment may not be equal to their bids.

The standard goal of auction design is to maximize the social efficiency ([23][12][50]), which is the sum of winners’ reported valuations (i.e., their bids) on their allocated goods. However, to find an allocation to fulfil such goal is NP-hard ([40]), and it is in fact APX-hard. That is, no known polynomial-time algorithm can approximate the optimum result within a constant factor. It is shown that the optimal allocation can be approximated within a factor of \(O(m^{2/3})\) but not to a factor of \(O(m^{2-\epsilon})\) for any \(\epsilon > 0\) ([23][41]), where \(m\) is the number of total goods. The efficient allocation algorithm is presented in Section [V]. In a nutshell, each submitted bundle is sorted by the norm \(\frac{b}{\sqrt{|S|}}\), where \(b\) is the bid and \(|S|\) is the number of goods in the bundle, and allocate the bundle if possible in the sorted order.

C. Payment Determination and the Truthfulness

Each bidder’s bid may not truly reflect his valuation of the bundle. The payment is determined by all bidders’ bids, therefore bidders may try to report a fake valuation to decrease their payment.

Definition 1. An auction is truthful if reporting a true valuation is a weakly dominant strategy for every bidder, and utility of any honest bidder is non-negative.

That is, no bidder can increase his benefit by lying no matter other bidders lie or not. Naturally, the payment mechanism determines whether the auction is truthful. The one in the famous Generalized Vickrey Auction (GVA, [27]) guarantees the truthful auction, but determining one bidder’s payment requires finding an optimum allocation without him, which is already shown to be NP-hard. Therefore, it is infeasible to implement the GVA in reality. Therefore, we should mainly review another mechanism which guarantees the truthfulness in conjunction with the aforementioned greedy allocation.

A truthful mechanism for the greedy allocation is introduced in [23]. A bidder’s utility cannot be increased by behaving dishonestly in [23], but an honest bidder’s utility may be negative in some cases. We consider the following mechanism with slight modification which guarantees the truthfulness and the non-negativeness of honest bidders’ utility (corresponding proofs are followed in Section [VI]).

Let \(L\) be the sorted list of bundles in the greedy allocation (sorted by the norm \(\frac{b}{\sqrt{|S|}}\)). For any bundle \(i\), denote the first bundle \(j\) in \(L\) which would be allocated if \(i\) were denied at first as candidate of \(i\). Then, \(i\)’s payment is \(\frac{b'}{\sqrt{|S'|}} \cdot \sqrt{|S|}\) where \(b'\) is the bid of the candidate bundle, \(S'\) is the candidate bundle, and \(S\) is the allocated bundle \(i\). This payment guarantees the truthfulness of the auction, which will be proved later in this paper (Section [VI]).

D. Privacy-preserving Combinatorial Auctions

Various approaches are proposed to achieve a private sealed-bid auction ([5][3][45][43][33]), but much less attention is paid to the combinatorial auction. In general, recently proposed approaches for the secure multi-agent combinatorial auction can be divided into two classes: Homomorphic Encryption (HE) based one and Secure Multi-party Computation (SMC) based one.

Yokoo et al. [49][46] implemented a secure multi-agent dynamic programming based on HE, which is in turn used to design the privacy-preserving combinatorial auction. Pan et al. [87] also designed a combinatorial auction based on HE.

Yokoo et al. [50] also designed a secure Generalized Vickrey Auction (GVA) based on the Secure Multi-party Computation (SMC), and Palmer et al. [24] proposed a garbled circuit based solution, which is essentially a SMC based solution.

However, they all aim at achieving the optimal solution, and this led to exponential-time protocols w.r.t the number of bidders in all aforementioned works, therefore it is desirable to concentrate on achieving approximated result efficiently.

III. PRIVACY-PRESERVING COMBINATORIAL AUCTION MODEL

A. Auction Model

A set of \(m\) goods \(G = \{g_1, \cdots, g_m\}\) are auctioned to \(n\) bidders \(\{B_1, \cdots, B_n\}\) in the CA, and the combinatorial
characteristic comes from each bidder $B_i$'s bid and valuation on a bundle $S_i \subseteq G$ instead of each good. $B_i$ proposes his bid $b_i(S_i)$ (i.e., maximum willingness to pay) on the a bundle $S_i$, and the bid might be different from his true valuation $v_i(S_i)$. A set $W$ of winners are chosen by the auctioneer as follows:

$$W = \arg\max x \sum_{B_i \in X} b_i(S_i) \ s.t. \ \bigcap_{B_i \in X} S_i = \emptyset$$

I.e., a set of conflict-free bidders whose social welfare is maximized, and the corresponding allocation set $A^*$ is $A = \bigcup_{B_i \in W} S_i$. After the winners are chosen, each winner $B_i$'s payment $p_i$ is determined by the auction mechanism based on all bidders bids. Then, bidders’ utility is defined as:

$$u_i = \begin{cases} 
 v_i(S_i) - p_i & B_i \text{ is a winner} \\
 0 & \text{Otherwise}
\end{cases}$$

We assume bidders are single-minded. That is, each of them cares only about one specific set of goods, and if they do not get the desired set, their valuation on the result is 0. Formally, for any $B_i$’s desired set $S_i$, $B_i$’s valuation on a set $S'$ is $v_i$ if and only if $S_i \subseteq S'$. This assumption is equivalent to the restriction that each bidder is limited to one bid only. Therefore, we simplify the notation $v_i(S_i)$ as $v_i$ and $b_i(S_i)$ as $b_i$ hereafter.

| TABLE I FREQUENTLY USED NOTATIONS |
|------------------------------------|
| Auctioneer | i-th bidder | $B_i$'s bundle | $S_i$ | $B_i$'s bid | $b_i$ |

B. Adversarial Model

When the auction terminates, $A$ is supposed to know only the winners, their bundles and corresponding payments. Each bidder $B_i$ only learns whether he is the winner when the auction terminates, and he is informed of the payment if he is chosen as a winner. He does not learn anything about others’ bids or bundles except what implied in his payment (which is very limited information).

The auctioneer is assumed to be curious, malicious and ignorant. He is interested in bidders’ bids and bundles to improve his business (called “curious”). For example, he may infer bidders’ preferences and rivalry relationship based on the bids and the bundles. The auctioneer may also report a fake payment to the winners to illegally increase his revenue (called “malicious”), but he is not aware of bidders’ side information such as distribution of bid values or bidders’ preferences on goods (called “ignorant”).

We also assume every bidder is selfish, curious and non-cooperative. Their objective is to maximize their own utilities, and bidders will report fake valuations if the utility is increased by doing so (called “selfish”). On the other hand, bidders are interested in others’ bids and bundles to improve the decision making (called “curious”). However, they will not collude with other bidders or the auctioneer (called “non-cooperative”).

In addition, we assume no probabilistic polynomial time algorithm (PPTA) can efficiently solve the discrete logarithm problem. This is a necessary condition to let MPEP [20] work privately, and no auction can be run privately without this condition as proven by Brandt et al. [4]. This will be further discussed in Section VI.

C. Privacy Definitions

Definition 2. Given all the communication strings $C$ during the auction and the output of the auction Output, an adversary’s advantage over the bidder $B_i$’s bid $b_i$ is defined as

$$adv_{b_i} = Pr[b_i|C, Output] - Pr[b_i|Output]$$

where $Pr[b_i]$ is the probability that a correct $b_i$ is inferred.

Definition 3. Given all the communication strings $C$ during the auction and the output of the auction Output, an adversary’s advantage over the bidder $B_i$’s bundle $S_i$ is defined as

$$adv_{S_i} = Pr[S_i|C, Output] - Pr[S_i|Output]$$

where $Pr[S_i]$ is the probability that any information about $S_i$ is inferred.

Informally, these advantages measure how much extra information an adversary gained during our privacy-preserving auction by measuring the increased probabilities.

IV. PRELIMINARIES

Before we introduce the privacy-preserving auction construction, we first introduce several building blocks as well as preliminary techniques used in the construction.

A. Homomorphic Encryption

Homomorphic encryption allows specific computations to be directly carried on ciphertexts while preserving their decryptability. El Gamal encryption [14], Goldwasser Micalli cryptosystem [16], Benaloh cryptosystem [11] and Paillier cryptosystem [35] are good examples, and we employ the Paillier cryptosystem to implement a one-way privacy-preserving auction.

Paillier Cryptosystem

System parameters: two prime numbers $p, q$.

Public key: modulus $n = pq$ and a random number $g \in \mathbb{Z}_n^*$

Private key: $\lambda = \text{LCM}(p - 1, q - 1)$

Encryption: $c = E(m, r) = g^{m+n+r} \mod n^2$

where $r \in \mathbb{Z}_n^*$ is a random number.

Decryption: $m = D(c) = L(c^{\lambda-1} \mod n^2) \mod n$

where $L(x) = x \mod n$

Self-blinding: $E(m, r) \cdot g^{nr} = E(m, r + r')$

$$\left\{ \begin{array}{ll} E(m_1, r_1) \cdot E(m_2, r_2) = E(m_1 + m_2, r_1 + r_2) \\ E(m_1) \cdot g^{nr} = E(m_1 + m_2, r_1) \\ E(m_1, r_1)^{m_2} = E(m_1 \cdot m_2, r_1 \cdot m_2) \end{array} \right.$$
B. Digital Signature

In our work, we employ a signer who is involved only to generate a signature of each bidder’s value. Many works can be considered ([17], [28], [34]), but we use the blinded Nyberg-Rueppel scheme in [6] (Table), which is a blind signature scheme. In a blind signature scheme, the signer can generate a signature of a value without ‘seeing’ it. In our work, we use the mechanism design to guarantee the truthfulness of the bidding, and use a blind signature scheme to verify the payment is calculated correctly. Since the authenticity of the bids are guaranteed by the truthful mechanism, we do not need the signer to verify the authenticity of them, therefore we use the Blinded Nyberg-Rueppel scheme, where signers do not see the signed values, as a building block. Notably, the recipient can recover the message from the signature in this scheme.

Scheme 1 Blinded Nyberg-Rueppel Scheme

**System parameter**: a prime number \( p \), a prime factor \( q \) of \( p - 1 \), and an element \( g \in \mathbb{Z}_p^* \) of order \( q \).

**Key Generation**: signer picks a random number \( x \in \mathbb{Z}_q \).
Then, he picks a random number \( h \in \mathbb{Z}_p^* \) such that \( g = h^{\frac{x}{q}} \not\equiv 1 \pmod{p} \). Then, the secret parameter is \( x \) and the public parameters are \( g, g^x \pmod{p} \).

**Signing**: signer blindly signs signee’s message \( m \).

1. The signer randomly selects \( \tilde{k} \in \mathbb{Z}_q \) and sends \( \tilde{r} = g^{k} \pmod{p} \) to the signee.
2. The signee randomly selects \( \alpha \in \mathbb{Z}_q, \beta \in \mathbb{Z}_p^* \), computes \( r = m^{\alpha} \pmod{p} \) and \( \tilde{m} = r^{\beta \alpha^{-1}} \pmod{p} \). Then, he sends \( \tilde{m} \) to the signer.
3. The signer computes \( \tilde{s} = \tilde{m}x + \tilde{k} \pmod{p} \) and sends \( \tilde{s} \) to the signee.
4. The signee computes \( s = \tilde{s}\beta + \alpha \pmod{p} \), and the pair \((r, s)\) is the Nyberg-Rueppel signature for \( m \).

**Verification**: check whether \( m = g^{-s}y^r r \pmod{p} \).

Not all blind signature schemes can be used. The ones using homomorphic encryption is often subject to this problem as follows:

\[
\text{Sig}(v_1)^{v_2} = E_{EK}(v_1)^{v_2} = E_{EK}(v_1v_2) = \text{Sig}(v_1v_2)
\]

where ‘Sig’ stands for the signature. Then, the signature of value \( v_1v_2 \) is illegally generated from \( \text{Sig}(v_1) \). In addition, the DSA signature scheme mentioned in [6] is also subject to forgery. Given a signature \( \text{Sig}(m) = (s, r) \) for \( m \), an attacker can create a fake signature \( \text{Sig}(km) = (ks, \frac{k}{\gamma}) \).

Information such as the ID of value owner, signature generation time etc. is embedded in a signature reality, but for the sake of simplicity, we denote the signature of \( m \) as \( \text{Sig}(m) \) hereafter and assume the recipient receives relevant information when he receives the signature.

V. PRIVACY-PRESERVING COMBINATORIAL AUCTION MECHANISM

We design a privacy-preserving winner determination scheme and a privacy-preserving verifiable payment determination scheme in this section.

A. Privacy-preserving Winner Determination

**Algorithm 2 Greedy Winner Determination**

1: \( A := \emptyset, W := \emptyset \). For each \( B_i \), computes \( \psi_i = \frac{b_i}{\sqrt{|S_i|}} \).
2: Sort the instances in the non-increasing order of \( \psi_i \).
Denote the sorted list as \( L \).
3: For each \( B_i \in L \) (in the sorted order), check whether \( A \cap S_i = \emptyset \). If true, \( A := A \cup S_i \), \( W := W \cup B_i \).
4: Announce \( W \) as the winners. Finally allocated goods are \( A^* \).

The above approximation algorithm for the winner determination guarantees an approximation ratio of at least \( \frac{O(\sqrt{m})}{|S_i|} \) ([23]), where \( m \) is the number of total goods. To guarantee each bidder’s privacy, the sorting based on \( \psi_i = \frac{b_i}{\sqrt{|S_i|}} \) should be conducted in a privacy-preserving manner in order to keep the confidentiality of each bidder’s bid to the auctioneer as well as other bidders. In addition, the allocation check \( A \cap S_i = \emptyset \) at the Step 3 in Algorithm 1 should be conducted with privacy preserving as well to guarantee that auctioneer does not learn about any bundle of non-winners, and on the other hand, to guarantee that no bidders learn about other’s bundles.

**Sorting**

The basic idea is to conduct a rank-preserving transformation \( \psi_i \to \psi_i' \) and let the auctioneer sort the bidders using the transformed weights. The process is described in detail in Protocol 3.

**Protocol 3 Privacy-preserving Sorting**

1: Each bidder \( B_i \) randomly picks a positive number \( \delta_i \) and sends it to all other bidders.
2: W.l.o.g. we assume \( n \) (number of bidders) is even. Every bidder \( B_i \) computes \( \psi_i' = (\sum_{k=1}^{n/2} (\delta_{2k-1} + \delta_{2k})\psi_k^i) + \delta_n \) locally, and achieves the signature \( \text{Sig}(\psi_i') \) blindly from the signer \( T \).
3: Every bidder \( B_i \) sends the following ciphertext encrypted with \( A \)’s public key to the auctioneer \( A \): \( E_{A}(\text{Sig}(\psi_i')) \).
4: \( A \) decrypts all the ciphertexts and recovers each \( \psi_i' \) using the signature (blinded Nyberg-Rueppel scheme). Then, he sorts the bidders based on the transformed weights.

The transformed norm \( \psi_i' \) preserves \( B_i \)’s rank among all bidders, and it does not reveal \( \psi_i \) to the auctioneer due to the random numbers.

Note that the signed messages in Nyberg-Rueppel scheme must be an integer. This is also required in the blinded version [6]. Therefore, we need to use the rounded norms \( \psi_i := \lfloor \psi_i \cdot 10^K \rfloor \) instead, where \( K \) is chosen appropriately for various auctions so that the norms’ ranks are preserved. This will lead to a auctioneer’s small revenue loss in the order of \( 10^{1-K} \) (Section VI). Typically, \( K \) can be chosen between \([3, 10]\) for practical purpose.
Allocation Check

After the bidders are sorted in a privacy-preserving manner, the allocation check should be conducted with privacy preserving as well. We use an m-dimension binary vector A to represent the allocation status, where the k-th bit a_k = 1 if the k-th good g_k is allocated already and 0 otherwise. Similarly, we use another vector S_i to represent B_i’s bundle S_i. S_i’s k-th bit s_{i,k} = 1 if g_k ∈ S_i and 0 otherwise.

Then, A ∩ S_i = ∅ if and only if

\[ A \cdot S_i = 0 \iff \sum_{k=1}^{m} a_k s_{i,k} = 0 \]

If the scalar product is 1, that means B_i’s bundle S_i includes some good already allocated so far and thus at least one term a_k s_k in the scalar product is equal to 1. We use the following privacy-preserving scalar product to let the auctioneer compute the above sum without knowing the vector S_i.

Protocol 4 Privacy-preserving Scalar Product

1. \( A \) picks a pair of Paillier cryptosystem key: \( PK_A = (n, g), SK_A = \lambda \) (Section IV).
2. \( A \) encrypts every bit a_k homomorphically and sends its ciphertext \( E_A(a_k) \) to the bidder \( B_i \) whose bundle is being checked.
3. Upon receiving \( m \) ciphertexts, \( B_i \) conducts following operations:

\[ \forall k : c_k = E_A(a_k)^{x_i,k} = E_A(a_k s_{i,k}) \]

Then, he computes the following and sends to \( A \):

\[ c = \prod_{k=1}^{m} c_k = E_A(\sum_{k=1}^{m} a_k s_{i,k}) \]

4. \( A \) decrypts the received ciphertext using his secret key, which is the scalar product \( A \cdot S_i \).

If \( A \cdot S_i = 0 \), \( A \) tells the \( B_i \) that he is the winner. \( B_i \) reports his bundle to \( A \) and the \( A \) is updated accordingly.

B. Privacy-preserving Verifiable Payment Determination

In the truthful auction mechanism mentioned in Section II, the auctioneer determines a winner \( B_i \)’s payment as follows. He finds the first bidder \( B_j \) after \( B_i \) in the list \( L \) whose bundle would be allocated if \( B_i \) were not the winner (i.e., the candidate of \( B_i \)). Then, \( B_i \)’s payment is

\[ p_i = \frac{b_j}{\sqrt{|S_j|}} \cdot \sqrt{|S_i|} \]

Three parties are engaged here: auctioneer \( A \), winner \( B_i \) and \( B_i \)’s candidate \( B_j \). The auctioneer \( A \) needs to know \( p_i \) without knowing \( B_j \)’s bundle or bid; the winner \( B_i \) needs to know \( p_i \) without knowing \( B_j \)’s bundle or bid, and he should not even know who is the \( B_j \); the bidder \( B_j \) does not need to know anything from this whole process. Furthermore, both the auctioneer and the winner should be able to verify the payment.

We present the privacy-preserving verifiable payment determination protocol (Protocol 5) which fulfills above requirements as follows. Note that all communication is conducted via secured channel (i.e., encrypted).

Protocol 5 \( B_i \)’s Verifiable Payment Determination

1. The auctioneer \( A \) examine the following term \( (A - S_i) \cup S_j \) with every bidder \( B_j \) via aforementioned privacy-preserving allocation check until he finds the bidder \( B_j \) who returns an empty set (i.e., \( B_i \)’s candidate).
2. B_j sends his integer vector \( \hat{n}_j = \frac{b_j}{\sqrt{|S_j|}} \cdot 10^K \) to \( A \).
3. \( A \) calculates \( p_i = \hat{n}_j \cdot 10^{-K} \cdot \sqrt{|S_i|} \) and sends \( p_i \) as well as the \( \text{Sig}(\psi'_j) \) to \( B_i \).
4. \( B_i \) recovers \( \psi'_j \) from \( \text{Sig}(\psi'_j) \). Then, using all random numbers \( \{\delta_1, \cdots, \delta_n\} \), he solves the following \( n/2 \)-degree equation:

\[ \psi'_j = (n/2) \sum_{k=1}^{n/2} (\delta_2k-1 + \delta_{2k})\psi'^2_j + \delta_n \]

\[ = (\delta_n + \delta_{n-1})\psi'^2_j + \cdots + (\delta_2 + \delta_1)\psi_j + \delta_n \]

Since all coefficients are positive, the polynomial is monotonically increasing in the positive domain. Therefore, \( B_i \) can use a binary search to find \( \psi_j \).

5. \( B_i \) knows that the payment is altered by the auctioneer if \( p_i \geq \psi_j \cdot 10^{-K} \cdot \sqrt{|S_i|} \).

Since \( A \) uses privacy-preserving allocation check, he does not learn about \( B_i \)’s bundle, and therefore he does not learn \( b_j \) from \( \hat{n}_j = \frac{b_j}{\sqrt{|S_j|}} \cdot 10^K \). The winner \( B_i \) does not know \( b_j \) due to the same reason, and he also does not know who is \( B_j \) since he does not even communicate with \( B_j \). On the other hand, owing to the signature \( \text{Sig}(\psi'_j) \) generated by \( T \), \( A \) is convinced that \( B_i \) did not report a fake lower \( \hat{n}_j \) to harm \( A \)’s business, and the winner \( B_i \) believes \( A \) did not tell a fake higher \( p_i \) to illegally increase \( A \)’s revenue.

VI. THEORETICAL PROPERTIES OF OUR PROTOCOLS

A. Small Deviation Caused by the Roundness

We use the rounded norm \( |\psi_j| \cdot 10^K \) throughout the auction, and use \( |\psi_j \cdot 10^K| \cdot 10^{-K} \) in the payment calculation, which leads to the following difference between the original payment \( p_i \) with the original norm and new payment \( \hat{p}_i \) with the rounded norm for a winner \( B_i \), where \( B_j \) is \( B_i \)’s candidate:

\[ p_i - \hat{p}_i = \psi_j \sqrt{|S_j|} - 10^{-K} \cdot |10^K \psi_j| \sqrt{|S_i|} \]

\[ \leq \sqrt{|S_i|} (|\psi_j| - 10^{-K} \cdot |10^K \psi_j|) < 10^{-1-K} \cdot \sqrt{|S_i|} \]

which also shows the upper bound of the revenue loss of the auctioneer, which is at the order of \( 10^{1-K} \).

B. Truthfulness and the Payment Mechanism

The winner \( B_i \)’s payment is determined by his candidate \( B_j \) whose bundle would be allocated if \( B_i \) were not. His payment is

\[ p_i = \frac{b_j}{\sqrt{|S_j|}} \cdot S_i \]

Theorem 1. Any honest bidder’s utility is non-negative.

Proof: \( v_i = b_i \) for a honest bidder \( B_i \). If \( B_i \) is not a winner, his utility is 0. Otherwise, his utility is
\[ u_i = v_i - p_i = b_i - \frac{b_j}{\sqrt{|S_i|}} \cdot \sqrt{|S_i|} \]

Since \( B_j \) is behind \( B_i \) in the sorted list \( L \), \( B_i \)'s norm is greater than \( B_j \)'s one. Then,
\[ \frac{b_i}{\sqrt{|S_i|}} \geq \frac{b_j}{\sqrt{|S_j|}} \Rightarrow b_i \geq \frac{b_j}{\sqrt{|S_j|}} \cdot |S_i| \Rightarrow u_i \geq 0 \]

Therefore, any honest bidder’s utility is non-negative.

**Theorem 2.** Any bidder’s utility is not increased when he bids dishonestly.

**Proof:** Any bidder \( B_i \)'s utility is
\[ u_i = \begin{cases} v_i - \frac{b_j}{\sqrt{|S_i|}} \cdot \sqrt{|S_i|} & \text{if } B_i \text{ is a winner} \\ 0 & \text{Otherwise} \end{cases} \]

We discuss two different cases of \( B_i \)'s valuation \( v_i \). Again, \( B_j \) is \( B_i \)'s candidate.

**Case 1:** \( v_i < \frac{b_j}{\sqrt{|S_i|}} \cdot \sqrt{|S_i|} \)

In this case, if \( B_i \) bids honestly, \( B_j \) becomes the winner, and \( B_i \) gains 0 as his utility. This is same when he reports a lower bid than \( v_i \), or he reports a higher bid than \( v_i \) but not high enough to beat \( B_j \) and becomes a winner. However, if he bids dishonestly by reporting a higher bid than \( v_i \) and becomes the winner, his utility is
\[ u_i = v_i - \frac{b_j}{\sqrt{|S_i|}} \cdot \sqrt{|S_i|} \]

Since \( B_i \) is not a winner if \( b_i = v_i \), we have:
\[ \frac{v_i}{\sqrt{|S_i|}} < \frac{b_j}{\sqrt{|S_i|}} \Rightarrow v_i < \frac{b_j}{\sqrt{|S_i|}} \cdot \sqrt{|S_i|} \Rightarrow u_i < 0 \]

Therefore, bidders do not gain benefit by lying in this case.

**Case 2:** \( v_i \geq \frac{b_j}{\sqrt{|S_i|}} \cdot \sqrt{|S_i|} \)

In this case, if \( B_i \) bids honestly, he is the winner and achieves a non-negative utility (Theorem 2). This is same when he reports a higher bid than \( v_i \), or he reports a lower bid than \( v_i \) but not low enough to lose the auction. However, if he bids dishonestly by reporting a lower bid than \( v_i \) and loses the auction, his utility becomes 0. Therefore, bidders do not gain benefit by lying in this cases.

In conclusion, bidders do not increase their utility by reporting a fake valuation as bid in this payment mechanism.

Combining the Theorem 1 and 2, we can conclude that our payment mechanism guarantees the truthfulness of the auction.

**C. Intractability Assumption**

We assume the discrete logarithm problem is intractable in our work as in similar cryptographic works ([19], [51], [25], [14], [51]). We prove that no one is able to achieve privacy-preserving winner determination without this assumption.

Chor and Kushilevitz [9] gave a necessary condition for the unconditionally private computability (without intractability assumption) of \( n \)-party functions, and it is developed to the following lemma by Brandt et al. [4].

**Lemma 3.** Given \( \overline{x} = (x_1, \cdots, x_{n-1}) \), \( \overline{y} = (y_1, \cdots, y_{n-1}) \), \( x_n \) and \( y_n \), where \( x_i, y_i \) are input from \( n \) parties, it is impossible to privately compute a \( n \)-party function \( f \) if \( f(\overline{x}, x_n) = f(\overline{x}, y_n) = f(\overline{y}, x_n) = a \) but \( f(\overline{y}, y_n) \neq a \) for some input.

The above lemma is based on the private computability for two-party function cases in Kushilevitz [22] and Chor and Kushilevitz [9]. We omit the details due to space limit.

**Theorem 4.** Without the intractability assumption, the sorting in the winner determination is not privately computable.

**Proof:** Lets say \( f'(\overline{x}, x_n) \) returns \( x_n \)'s rank among \( x_1, \cdots, x_n \) (i.e., ranking function) and \( f'(\overline{y}, \cdots, \overline{y}_{n-1}, y_n) \) returns \( \{i_1, \cdots, i_{n-1}\} \cup \{i_n\} \)'s feasibility (i.e., feasibility check function).

In the setting \( \overline{x} = (3, 1, 1), \overline{y} = (5, 1, 1), x_4 = 4 \) and \( y_4 = 6 \). \( f'(\overline{x}, x_4) = f'(\overline{x}, y_4) = f'(\overline{y}, x_4) = 1 \) but \( f'(\overline{y}, y_4) = 2 \). According to the Lemma 1, this ranking function is not privately computable. Since ranking is a necessary condition of the sorting, sorting is not privately computable either.

Therefore, we design our privacy-preservin winner determination scheme based on the widely used assumption that the discrete logarithm problem is hard.

**D. Random Numbers in Sorting**

We assumed an infinite number domain in our framework, but in fact, all computation is conducted in a finite cyclic integer group in a real implementation. Suppose the integer group we choose is a subset of an integer group \( \mathbb{Z}_p \) (i.e., \( \{1, \cdots, p-1\} \)) and corresponding modulo operations are followed after all arithmetic operations, then it becomes important to find ‘good’ random numbers so that
\[ \forall i \in \{1, \cdots, n\} : \psi_i = \sum_{k=1}^{n} \delta_i \psi_i^{k-1} < p \]

Otherwise, the rank of each bidder is not preserved after modulo operations. On the other hand, we also need to guarantee that the random numbers are large enough to securely mask all \( \psi_i \)'s.

Suppose the bit lengths of integer norm \( \psi_i \) is \( \text{bit}_\psi \) and the bit lengths of every \( \delta \) \( \text{bit}_\delta \). Then, the bit length of \( \psi_i \) is determined by the greatest term \((\delta_{n-1} + \delta_n)\psi_i^{n/2}\):
\[ \text{bit}_\psi + 1 + \text{bit}_\psi \cdot \frac{n}{2} \]

which should be less than or equal to \( \text{bit}_\psi - 1 \) (\( \text{bit}_p \) is \( p \)'s bit length) to guarantee the correctness of the sorting. Therefore, in a real implementation, one needs to find an integer group of order \( p \) of bit length \( \text{bit}_p \) which is at least as large as
\[ \text{bit}_\delta + \text{bit}_\psi \cdot \frac{n}{2} + 2 \]

Typically, \( n \) is small with value at most in the order of thousands.
E. Privacy of the Auction

**Theorem 5.** An adversarial auctioneer $A$’s advantage $adv_S$ is less than $\frac{1}{2[n-1]}$ for every $i$, where $G$ is the set of all goods.

**Proof:** Every winner’s bundle $S_i$ is given to $A$, therefore we have:
$$adv_{S_i} = Pr[S_i|Output] - Pr[S_i|Output] = 1 - 1 = 0$$
if $B_i$ is a winner.

It is already shown that the Paillier cryptosystem is semantically secure ([35]). Therefore, all that an adversarial $A$ learns about any loser’s $S_i$ during the allocation check (Section [V]) is whether his bundle is compatible with currently allocated goods. Then, the auctioneer’s probability to guess $B_i$’s correct $S_i$ given all communication strings and the output of the auction is:
$$Pr[S_i|Output] = \frac{1}{2^{[A]} - 1} \cdot \frac{1}{2^{[G]}}$$

On the other hand, the probability to guess the correct $S_i$ given only the output of the auction (i.e., the finally allocated bundles) is:
$$Pr[S_i|Output] = \frac{2^{[A]} - 1}{2^{[G]}}$$

Then, the advantage $adv_{S_i}$ is:
$$adv_{S_i} = \frac{1}{2^{[G]}} \cdot \left( \frac{2^{[A]} - 1}{2^{[G]}} - \frac{2^{[A]} - 1}{2^{[G]}} \right)$$

The first and second inequalities hold because the partial derivatives $\frac{\partial adv_{S_i}}{\partial [A]}$ and $\frac{\partial adv_{S_i}}{\partial [G]}$ are both positive when neither $A^*$ nor $S_1^*$ is an empty set, therefore $adv_{S_i}$ is maximized when $|A^*|, |A|$ are maximized. In conclusion, $A$’s advantage $adv_{S_i}$ is less than $\frac{1}{2[n-1]}$.

**Theorem 6.** An adversarial auctioneer’s advantage $adv_{b}$ is less than $\frac{1}{2[n-1]}$ for the candidates, and $adv_{b} = 0$ for other bidders. $G$ and $B$ are the set of total goods and the bid space respectively.

**Proof:** First of all, in the sorting of the winner determination (Protocol 2), bidders’ norms are masked by the randomized polynomial where each coefficient is independently chosen by each bidder, therefore the auctioneer only knows each norm’s rank. Also, because the transformed norm is a polynomial, it is hard to get the pair-wise difference or ratio between any two norms, and it is impossible to solve those polynomial since the auctioneer can achieve at most $\frac{n}{2}$ norms from the candidates (when $\frac{n}{2}$ are winners and rest of the bidders are all candidates) while the degree of the polynomial is $\frac{n}{2} + 1$.

In the payment determination of the winner $B_i$, his candidate $B_i$’s norm $\sqrt{\frac{b_i}{G}}$ is disclosed to the auctioneer $A$. However, $A$’s advantage on inferring $B_i$’s bundle is proven to be less than $\frac{1}{2[n-1]}$. Therefore, all he can guess about $|S_j|$ is that it is an integer value between $[1, \cdots, |G|]$, and $A$’s probability to guess a correct bid $b_j$ given all communication strings and the output is:
$$Pr[b_j|Output] = \frac{1}{|G|}$$

If $A$ gets only the output of the auction, he learns the norm $\psi_j$ based on $B_i$’s payment $p_i = \psi_j \cdot \sqrt{|S_i|}$ and his requested bundle $S_i$, but he does not know the owner of the norm, therefore his probability becomes:
$$Pr[b_j|Output] = \frac{n - 1}{n} \cdot \frac{1}{|B|} + \frac{1}{n} \cdot \frac{1}{|G|}$$

Therefore, an adversarial $A$’s advantage on the candidate $B_j$’s bid is:
$$adv_{b_j} = \frac{1}{|G|} - \frac{n - 1}{n} \cdot \frac{1}{|B|} - \frac{1}{n} \cdot \frac{1}{|G|} < \frac{1}{|G|} - \frac{1}{|B|}$$

For other losers or the winners, $A$ does not see their norms at all, therefore the advantages on their bids are all 0.

**Theorem 7.** An adversarial bidder $B_k$’s advantages $adv_{b_i}$ and $adv_{S_i}$ are all equal to 0 for all $i \neq k$.

**Proof:** For an adversarial bidder, he does not learn side information during our auction no matter he is a winner or not. All he learns from our privacy-preserving auction is included in the valid auction output $Output$. Therefore, his advantages $adv_{b_i} = adv_{S_i} = 0$ for all $i$.

The final advantages of adversarial auctioneer and bidders are summarized in the following table.

| Adversarial | $adv_{S_i}$ | $adv_{b_i}$ |
|-------------|-------------|-------------|
| $A$         | $\frac{1}{2[n-1]}$ | $\frac{1}{2[n-1]}$ |
| $B_i$ is a candidate | $0$ | $\frac{1}{2[n-1]}$ |
| Otherwise   | $0$ | $0$ |

F. Limitation of Our Construction

We have some minor limitations in our construction needs to be further improved.

First of all, a malicious winner try to report a fake bundle to the auctioneer. We kept the actual bundles secret from the auctioneer during the whole allocation check, therefore the bidder may report a ‘bigger’ bundle after he learns that he is a winner. Currently, we are not able to let the auctioneer verify the winner’s bundle while keeping the bundle secret during the allocation check, which seems a contradiction. However, a malicious winner’s misbehavior will be detected if he illegally includes a good which is already allocated, therefore winners cannot always lie successfully since they do not know about the currently allocated bundles.
Secondly, our construction allows a winner to calculate the true payment based on the signatures. This further allows the winner to know whether the auctioneer charged more than what he deserves. But, if the winner’s candidate reported a fake lower norm to the auctioneer in the first place, the winner would notice the true payment is higher than what is charged from the auctioneer, and naturally he will keep silent. In this case, auctioneer does not receive what he deserves. Nevertheless, we claim this is not likely to happen due to the following reason. The candidate did not win the auction because of the winner, therefore he will unlikely report a fake lower norm to help the winner’s business. Thus we assume there is no collusion among bidders. If there are collusions among bidders, we need different mechanisms (e.g., [7]).

VII. PERFORMANCE EVALUATION

In this section, we analyze the communication and computation overhead to show that our construction is both scalable and efficient. Most of the complexities are linear to the number of bidders or the number of goods, which allows huge number of bidders or the goods, and the extra data transmission and the run time introduced by our auction is almost negligible.

A. Communication Overhead

The communication overhead in terms of the data transmission is depicted in the following table, where $\text{bit}_p$ is the bit length of the $p$ (i.e., order of the integer group $\mathbb{Z}_p$).

| Winner Determination | Receive | Send |
|----------------------|--------|------|
| Auctioneer           | $O(n \cdot \text{bit}_p)$ | $O(n \cdot \text{bit}_p)$ |
| Per bidder           | $O(n \cdot \text{bit}_p)$ | $O(n \cdot \text{bit}_p)$ |

| Payment Determination | Receive | Send |
|----------------------|--------|------|
| Auctioneer           | $O(n \cdot \text{bit}_p)$ | $O(n^2 \cdot \text{bit}_p)$ |
| Per Winner           | $O(n \cdot \text{bit}_p)$ | 0 |
| Per Candidate        | $O(m \cdot \text{bit}_p)$ | $O(b \cdot \text{bit}_p)$ |

Note that the allocation check in the payment determination is executed until the auctioneer finds the candidate for each winner. Therefore, the average communication rounds for the auctioneer should be much less than $O(mn^2)$, which means the practical communication overhead will be much less than the worst case $O(mn^2 \cdot \text{bit}_p)$.

B. Computation Overhead

To evaluate the computation overhead, we implemented our privacy-preserving combinatorial auction in Ubuntu 12.04 using the GMP library (gmplib.org) based on C in a computer with Intel i3-2365M CPU @ 1.40 GHz ×4, Memory 4GB and SATA Hard Drive 500GB (5400RPM). To exclude the communication overhead from the measurement, we generated the average run time. Every operation or protocol is run 1000 times to measured the average run time.

In general, the auction is composed of the winner determination part and the payment determination part. The winner determination includes bidders’ blind signature generation, the auctioneer’s sorting, and the allocation check; the payment determination includes allocation check, the auctioneer’s payment calculation, and the bidders’ norm calculation from the polynomial. Next, we analyze their run time one by one.

Blind Signature Generation

The signer’s run time for blindly generating one pair of the Nyberg-Rueppel signature is $10\mu s$ (microseconds) and the bidder’s one is $20ms$ on average.

Sorting

One bidder’s run time of computing the $n/2$-degree polynomial in privacy-preserving sorting (Protocol 2) is in the order $\mu s$ ($70\mu s$ when there are 150 bidders), and thus the overall run time of the sorting is dominated by the signature generation, which is shown as Figure 2(b). The auctioneer’s run time is depicted as Figure 2(a). We use the Quicksort to sort the bidders after the auctioneer receives all $\psi'_j$’s.

Allocation Check

The number of bidders is fixed at 100, then the run time of the allocation check for various number of goods is shown in Figure 2(c) and Figure 2(d).

Payment Calculation

For each winner $B_i$ and his candidate $B_j$, the auctioneer calculates the $p_i = \psi_j \cdot 10^{-K} \cdot \sqrt{|S_j|}$. A single calculation needs $0.6\mu s$ on average, and the overall computation overhead is linear to the number of winners.

Norm Calculation

Upon receiving the polynomial $\psi'_j$, the winner uses a binary search to find the only positive root of the Equation [1]. We use the binary search on the norm space to find the $\psi_j$ from it, which needs $13.26ms$ on average.

VIII. CONCLUSION & FUTURE DIRECTION

In this paper, we presented a privacy-preserving combinatorial auction design which efficiently yet approximately determines the winners to maximize the social efficiency. Our construction allows untrusted bidders to participate in an auction held by an untrusted auctioneer, while bidders are able to detect the fake payment reported by the auctioneer and the payment mechanism enforces a truthful bidding. We extensively discussed and analyzed the security of our construction to show that unnecessary information leakage is very small, and our implementation also shows that it is efficient both communicationally and computationally.

One of the future directions is to maximize the auctioneer’s revenue instead of the social efficiency. The revenue-maximizing algorithm is shown in [31], and it requires that the auctioneer know each bidder’s bid distribution in advance. However, this increases an adversarial auctioneer’s advantage to infer the bids, and reducing such threat is the main challenge of the revenue maximization.

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