The $\bar{D}^*\bar{D}^*\Sigma_c$ Three-Body System

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The discovery of two hidden charm pentaquark-like resonances by the LHCb [1], the $P_c(4380)$ and $P_c(4450)$, begs the question of what is their nature. These resonances are indeed different: while the $P_c(4380)$ is relatively broad, the $P_c(4450)$ is narrow and is located close to a few meson-baryon thresholds, namely the $\bar{D}\Lambda_c(2595)$, $D^*\Sigma_c$, $D^*\Sigma_c^*$ and $J/\psi\eta$ thresholds. This coincidence makes the $P_c(4450)$ a good candidate for a hadronic molecule, a type of exotic hadron theorized a few decades ago [2, 3]. In particular the $P_c(4450)$ has been suggested to be a $D^*\Sigma_c$ [4, 5], a $D^*\Sigma_c^*$ molecule [6, 7] (in these two cases probably with a small admixture of $\Delta\Lambda_c(2595)$ [8, 9]), and a $\chi_{c1}p$ molecule [10]. Of these possibilities, the most natural one probably is $D^*\Sigma_c$. First, the interaction between a heavy baryon and antimeson is expected to be mediated by the exchange of light mesons, providing a mechanism to justify the existence of attraction. Second, the binding energy of the $\Sigma_cD^*$, $B_{3} = 12 \pm 3$ MeV, translates into a bound state that is not excessively compact, which is compatible with the molecular hypothesis. The expected size is of the order of $1/\sqrt{\mu B_3} \sim 1.2$ fm, with $\mu$ is the reduced mass of the molecule. Besides the molecular hypothesis there are other competing explanations for the nature of the $P_c^*$, which include a genuine pentaquark [12, 13], threshold effects [18, 19] (for a detailed discussion see Ref. [20]), baryocharmonia [21], a molecule bound by colour chemistry [22] or a soliton [23].

If the $P_c(4450)$ — the $P_c^*$ from now on — is indeed a $J^P = \frac{3}{2}^+$ $\bar{D}^*\Sigma_c$ bound state, its location will provide useful information about the dynamics of this two-body system. This information can be used to deduce the existence of new molecules. For instance, from heavy quark spin symmetry it is sensible to expect the existence of a $J^P = \frac{3}{2}^+$ $\bar{D}^*\Sigma_c^*$ partner of the $P_c^*$ with a mass of 5515 MeV [24]. In this manuscript we argue that the dynamics of the $D^*\Sigma_c$ two-body system imply the existence of a series of $\bar{D}^*\bar{D}^*\Sigma_c$ three-body bound states. Concrete calculations in a contact-range theory lead to the following predictions:

(i) a $J = \frac{3}{2}$, $I = 0$ trimer with $B_3 \sim 3 - 5$ MeV,

(ii) a $J = \frac{5}{2}$, $I = 1$ trimer with $B_3 \sim 1 - 3$ MeV,

(iii) a $J = \frac{7}{2}$, $I = 0$ trimer with $B_3 \sim 14 - 16$ MeV,

(iv) a $J = \frac{9}{2}$, $I = 1$ trimer with $B_3 \sim 3 - 5$ MeV,

where the trimer binding energy $B_3$ is relative to the hadron-dimer threshold, i.e. the $D^*P_c^*$ threshold. There are a series of uncertainties related to the previous predictions of which the most crucial one is whether the $P_c^*$ is really a $D^*\Sigma_c$ bound state. Until the nature of the $P_c^*$ is settled, these trimers will remain a theoretical possibility. Conversely the experimental production of these trimers or their detection in the lattice will strongly suggest a molecular nature for the $P_c^*$. It is worth noticing that analogous trimer predictions have been made for other molecular candidates. For instance, from the hypothesis that the $X(3873)$ is a $D^*D$ molecule it is possible to theorize about $D^*\bar{D}K$, $D^*\bar{D}K$ [25], $D^*D^*\bar{D}$ and $D^*D^*\bar{D}^*$ trimers [26]. Within more phenomenological approaches there has also been a growing interest in the possibility of three-body systems in the heavy sector, for instance $D^*D^*\rho$ [27], $B^*B^*\rho$ [28], $BDD$, $BDD$, $B^*D\bar{D}$, etc. [29, 30], just to mention a few recent works.

The manuscript is structured as follows: in Sect. I we write the Faddeev equations for the $D^*\bar{D}^*\Sigma_c$ system. In Sect. II we consider the $D^*\bar{D}^*\Sigma_c$ system in the unitary limit, i.e. when the binding energy of the $D^*\Sigma_c$ system approaches zero. In Sect. III we present our predictions of three body states. Finally in Sect. IV we summarize the results of this manuscript.

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II. FADDEEV EQUATIONS FOR THE $\Sigma_c\bar{D}^*\bar{D}^*$ SYSTEM

In this section we write the Faddeev decomposition and equations for the $\bar{D}^*\bar{D}^*\Sigma_c$ three body system. We will consider the S-wave three body states only, as they have the highest chances of binding. We will make the following assumptions:

(i) the $\bar{D}\bar{D}$, $\bar{D}\bar{D}^*$ and $\bar{D}^*\bar{D}^*$ pairs do not interact,

(ii) the $\bar{D}^*\Sigma_c$ only interacts in the $P_c^*$ channel,

(iii) the $\bar{D}^*\Sigma_c$ interaction in the $P_c^*$ channel can be described in terms of a contact-range interaction.

This leads to a considerable simplification of the Faddeev equations. Equivalently we can consider the $\bar{D}^*\bar{D}^*\Sigma_c$ system from the effective field theory point of view. In this case we will say that the $\bar{D}^*\Sigma_c$ contact-range interaction is leading order, while all the other interactions are subleading and can be ignored at leading order. We will review the previous set of assumptions at the end of the manuscript.

A. The Equations for $\bar{D}^*\bar{D}^*\Sigma_c$

We begin by writing the three body wave function in terms of Faddeev components for the $\bar{D}^*\bar{D}^*\Sigma_c$ system

$$\Psi_{3B} = \sum_\beta \left[ \phi_\beta(\vec{k}_{23}, \vec{p}_1) + \zeta_\beta \phi_\beta(\vec{k}_{13}, \vec{p}_2) \right] \times |S_{12} \otimes \frac{1}{2}S| I_{12} \otimes 1 \rangle_I,$$

where we have labeled the two $D^*$ mesons as particles 1 and 2 and the $\Sigma_c$ as particle 3. The summation index $\beta = (S_{12}, I_{12})$ refers to the spin and isospin of the $\bar{D}^*\bar{D}^*$ subsystem. The spin and isospin piece of the wave function is indicated by

$$|S_{12} \otimes \frac{1}{2}S| I_{12} \otimes 1 \rangle_I,$$

where $S$ and $I$ are the total spin and isospin of the $\bar{D}^*\bar{D}^*\Sigma_c$ three body system. The sign $\zeta_\beta = \pm 1$ indicates whether the spatial part of the $\bar{D}^*\bar{D}^*$ wave function is symmetric or antisymmetric. We will ignore the $\zeta_\beta = -1$ configurations: they require the orbital angular momentum of the 12 subsystem to be $L_{12} \geq 1$, which implies that they are suppressed for the positive parity states (they depend on the P-wave $\bar{D}^*\Sigma_c$ interaction).

We define the Jacobi momenta as follows

$$\vec{k}_{ij} = \frac{m_i \vec{k}_i - m_j \vec{k}_j}{m_i + m_j},$$

$$\vec{p}_k = \frac{1}{M_T} \left[ (m_i + m_j) \vec{k}_k - m_k (\vec{k}_i + \vec{k}_j) \right],$$

with $m_1, m_2, m_3$ the masses of particles 1, 2, 3, $M_T = m_1 + m_2 + m_3$ the total mass and $ijk$ an even permutation of 123. In particular we have $m_1 = m_2 = m(\bar{D}^*)$ and $m_3 = m(\Sigma_c)$. At this point it will be helpful to make the following observation about the Faddeev components: in principle there are up to three Faddeev components for each channel $\beta$. Of these, the first two Faddeev components are symmetric or antisymmetric (as indicated by the sign $\zeta_\beta$) because particles 1 and 2 are identical: $\phi_\beta(\vec{k}, \vec{p})$ denotes these components. Finally the third Faddeev component vanishes because we have considered that the $\bar{D}^*\bar{D}^*$ subsystem does not interact.

We assume a contact-range $\bar{D}^*\Sigma_c$ potential of the type

$$V_{\Sigma_c, \bar{D}} = C_{\Sigma} g(k) g(k'),$$

where $C_{\Sigma}$ is a coupling constant and $g(p)$ is a regulator function. The subscripts $\Sigma$ and $\bar{D}$ indicate the spin and isospin channel of the $\bar{D}^*\Sigma_c$ subsystem, i.e. $\Sigma = J_{23}$ and $\bar{D} = I_{23}$. The $\bar{D}^*\Sigma_c$ T-matrix can be written as

$$T_{23} = t_{\Sigma\bar{D}}(Z) g(k) g(k'),$$

depending on the spin and isospin channel, where $Z$ represents the energy of the few-body system with respect to the few-body mass threshold. With this two-body T-matrix, the Faddeev component $\phi_\beta$ admits the ansatz

$$\phi_\beta(k, p) = \frac{g(k)}{Z - \frac{\alpha^2}{2m_{23}} - \frac{\beta^2}{2m_1}} a_\beta(p),$$

with the reduced masses defined as

$$\frac{1}{\mu_{ij}} = \frac{1}{m_i} + \frac{1}{m_j}, \quad \frac{1}{\mu_k} = \frac{1}{m_k} + \frac{1}{m_i + m_j}.$$
Regarding the $D^*\Sigma_c$ potential, the only $\sigma\tau$ channel for which we know the interaction is the $P_c^*$ channel, i.e. $\sigma = \frac{1}{2}$ and $\tau = \frac{1}{2}$. For simplicity we will only consider the interaction in the $P^*_c$ channel and neglect the interaction in all other channels

$$t_{\sigma\tau}(Z) = t_{P^*_c}(Z) \delta_{\sigma \frac{1}{2}} \delta_{\tau \frac{1}{2}}.$$  

(13)

Equivalently, in the effective field theory language this means that we are considering the $P^*_c$ interaction as leading order and the interaction in all the other channels as subleading corrections. This point is important as it will entail a simplification in the Faddeev equations: coupled-channel equations will become single-channel ones.

To understand this simplification, we first notice that there are nine possible quantum numbers for the $S$-wave $D^*D^*\Sigma_c$ system: three spin states $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ compounded by three isospin states $I = 0, 1, 2$. Seven of these configurations involve a single channel, i.e. a single possible $\beta = (S_{12}, I_{12})$ configuration for the spin and isospin of the $D^*D^*$ subsystem. Then there are two quantum numbers that involve coupled channels: $J = \frac{1}{2}, \frac{3}{2}$ with $I = 1$. Now if we ignore all the $D^*\Sigma_c$ interactions with the exception of the $P^*_c$ channel, we can simply make the substitution

$$\lambda_{\sigma\tau} \to \lambda_{P^*_c} t_{P^*_c}.$$  

(14)

The matrix $\lambda_{P^*_c}$ can be diagonalized, in which case the coupled-channel equation reduces to a single-channel one:

$$a(p_1) = \lambda_{P^*_c} \tau(Z_{23}) \int \frac{d^3p_2}{(2\pi)^3} B^0_{12}(\vec{p}_1, \vec{p}_2) a(p_2),$$  

(15)

where $\lambda$ is one of the eigenvalues of $\lambda_{P^*_c}$. In fact this is the equation that we will use in all cases to find the binding energies of the $D^*D^*\Sigma_c$ trimers. The factors $\lambda$ for the different trimer quantum numbers can be found in Table III.

III. THE EFIMOV EFFECT IN THE $D^*D^*\Sigma_c$ SYSTEM

Here we will consider the unitary limit of the previous set of Faddeev equations. The unitary limit refers to the limit in which a two-body system is bound at threshold, which is interesting from a theoretical perspective because of its relation with the Efimov effect [33]. As already seen, the eigenvalue equation of the $D^*D^*\Sigma_c$ system always reduces to

$$a(p_1) = \lambda \tau(Z_{23}) \int \frac{d^3p_2}{(2\pi)^3} B^0_{12}(\vec{p}_1, \vec{p}_2) a(p_2),$$  

(16)

with $\lambda$ the factor listed in Table III which ranges from $\frac{1}{\sqrt{2}}$ to 1 for the isoscalar and isovector trimers. Now we take the unitary limit, where we have

$$\tau(Z_{23}) \to -\frac{2\pi}{\mu_{23}} \sqrt{\frac{\mu_{23}}{\mu_1}} \frac{1}{p_1},$$  

(17)

$$\int \frac{d^3p_2}{4\pi} B^0_{12} \to -\frac{m_3}{2p_1p_2} \log \left[ \frac{p_1^2 + p_2^2 + \frac{2m}{m + m_3} p_1 p_2}{p_1^2 + p_2^2 - \frac{2m}{m + m_3} p_1 p_2} \right],$$  

(18)

with $m = m(D^*)$, $m_3 = m(\Sigma_c)$ and where $\mu_{23}$ and $\mu_1$ are defined in Eqs. 8 and 9. From this we arrive to

$$p_1 a(p_1) = \frac{\lambda}{2\pi} \frac{m_3}{\mu_{23}} \int \frac{dp_2 a(p_2)}{p_1} \int_0^\infty dp_2 \int_0^\infty dx \frac{b(xp)}{x} \times \log \left[ \frac{1 + x^2 + \frac{2m}{m + m_3} x}{1 + x^2 - \frac{2m}{m + m_3} x} \right].$$  

(19)

This equation admits power-law solutions of the type $b(p) = p^s$, in which case we end up with an eigenvalue equation for $s$

$$1 = \frac{\lambda}{2\pi} \frac{m_3}{\mu_{23}} \int_0^\infty dx \frac{1}{x} \times \log \left[ \frac{1 + x^2 + \frac{2m}{m + m_3} x}{1 + x^2 - \frac{2m}{m + m_3} x} \right].$$  

(20)

This integral can be evaluated analytically [31, 32], in which case we arrive at the eigenvalue equation

$$1 = \lambda J_{\text{Efimov}}(s, \alpha),$$  

(22)

where $J_{\text{Efimov}}$ is given by

$$J_{\text{Efimov}}(s, \alpha) = \frac{1}{\sin 2\alpha} \frac{2}{s} \frac{\sin \alpha s}{s},$$  

(23)

and with the angle $\alpha$ determined as

$$\alpha = \arcsin \left( \frac{1}{1 + \frac{m}{m_3}} \right),$$  

(24)

with $m$ and $m_3$ the masses of the charmed meson and baryon, respectively.

The Efimov effect [33] happens when the eigenvalue equation, i.e. Eq. (22), admits complex solutions of the type $s = \pm is_0$, which leads to a $b(p)$ that oscillates:

$$b(p) \propto \sin \left( s_0 \log \left( \frac{p}{ \lambda_3} \right) + \phi \right),$$  

(25)
TABLE I: The coupling factors $\lambda_{\beta\gamma}^{\alpha\delta}$ to be used in the coupled-channel Faddeev equations. The $P_{\sigma}^c$ channel corresponds to $\sigma = \frac{1}{2}$ and $\tau = \frac{1}{2}$, i.e. $\lambda_{\frac{1}{2}\frac{1}{2}}^{\frac{1}{2}\frac{1}{2}}$. In the last column we write the $\lambda$ to be included in the single-channel Faddeev equation we use in this work, Eq. (28). For uncoupled channels $\lambda = \lambda_{\frac{1}{2}\frac{1}{2}}$, while for coupled channels $\lambda$ is given by the eigenvalues of $\lambda_{\beta\gamma}^{\alpha\delta}$.

with $\Lambda_3$ a momentum scale and $\varphi$ a phase. From the form of $b(p)$ we deduce that the system is invariant under the discrete scaling transformation $p \rightarrow e^{\pi/s_0}p$, which for the case of the binding energy reads as $B_3 \rightarrow e^{2\pi/s_0}B_3$. The condition for having the Efimov geometric spectrum is

$$\lambda \geq \lambda_c = \frac{\sin 2\alpha}{2\alpha},$$  

which for the $\bar{D}^*\bar{D}^*\Sigma_c$ system give us $\lambda_c \approx 0.861$. From this we deduce that the $J = \frac{3}{2}$, $I = 0$ trimer is potentially Efimov-like, while all the others are not (but only for the set of assumptions laid out at the beginning of Sect[I]). For the Efimov-like trimer we have $s_0 \approx 0.363$, from which the discrete scaling factor is $e^{\pi/s_0} \approx 5711$ for momenta and $e^{2\pi/s_0} \approx 3.262 \cdot 10^7$ for the trimer binding energy. Even with a molecular $P_{\frac{1}{2}}^c$ close to the unitary limit, the factors are too big to be realistically observed. Thus the analysis presented here is mostly academical, except for one thing: if the forces binding the trimer are short-ranged, the possibility of the Efimov effect is related to the necessity of a three body force for the system to be properly renormalized [34, 35]. The reason is the relation between the Efimov effect and the Thomas collapse [36]; that is, as the range of the two-body potential shrinks, the trimer binding energy grows, eventually diverging. However for the $\bar{D}^*\bar{D}^*\Sigma_c$ trimer, the observation of this effect requires interaction ranges that are orders of magnitude smaller than the typical hadron size.

**IV. PREDICTIONS**

The basic building block of the three body calculation is the $\bar{D}^*\Sigma_c$ interaction in the channel with the quantum numbers of the $P_{\frac{1}{2}}^c$: $J^P = \frac{3}{2}^+$ and $I = \frac{1}{2}$. We will describe the $\bar{D}^*\Sigma_c$ system in terms of a contact-range potential of the type

$$V(\bar{D}^*\Sigma_c) = C(\Lambda) f(\frac{k}{\Lambda}) f(\frac{k'}{\Lambda}),$$  

where $C(\Lambda)$ is a coupling constant and $f(\Lambda)$ a regulator function. Here we will use a Gaussian regulator $f(x) = e^{-x^2}$ and a cut-off window $\Lambda = 0.5 - 1.0$ GeV. In the molecular interpretation the $P_{\frac{1}{2}}^c$ is a $\bar{D}^*\Sigma_c$ bound state with a binding energy of $B_2 = 12 \pm 3$ MeV. We can determine the strength of the coupling $C(\Lambda)$ from the condition of reproducing the binding energy $B_3$. For this we use the two-body eigenvalue equation

$$1 + C(\Lambda) \int \frac{d^3q}{(2\pi)^3} \frac{f^2(q)}{B_2 + \mu_{23}} = 0,$$  

where $B_2$ is the two-body binding energy and $\mu_{23}$ the reduced mass of the $\bar{D}^*\Sigma_c$ system.

Once the contact-range coupling is obtained from the two-body eigenvalue equation, we can solve the three-body eigenvalue equation with $C_{\frac{3}{2}^+} = C(\Lambda)$, $g(k) = f(k/\Lambda)$ and the appropriate factor $\lambda$. Concrete calculations lead to the predictions of Table [II] where the binding energy $B_3$ is shown for different trimer configurations. The binding energy $B_3$ is defined with respect to the dimer-particle threshold, i.e. the mass of the trimers is

$$M = 2m + m_3 - B_2 - B_3,$$  

with $m$ and $m_3$ the mass of the $\bar{D}^*$ meson and $\Sigma_c$ baryon, respectively. The binding energy does not directly depend on the quantum numbers of the trimer, but indirectly by means of the factor $\lambda$ as can be appreciated in Table [II] This dependence on the coefficient $\lambda$ is shown explicitly in Figure [II].

The trimer binding energies are affected by a series of uncertainties, which we will discuss below. The results of Table [II] already contain two error sources, the binding energy of the $P_{\frac{1}{2}}^c$ ($B_2 = 12 \pm 3$ MeV) and the cut-off window ($\Lambda = 0.5 - 1.0$ GeV). Assuming that the $P_{\frac{1}{2}}^c$ is indeed molecular, the next most important source of uncertainty is the the choice of which interactions are leading and subleading. Previously we have assumed that the only leading order interaction is the $\bar{D}^*\Sigma_c$ short-range potential in the $P_{\frac{1}{2}}^c$ channel. We will review this assumption in detail in the following lines, where the different possibilities will be named scenarios A, B and C.
We begin with the $\bar{D}^*D^*$ interaction. Törnqvist pointed out [23] that the flavour exotic configurations of this system are less likely to bind in general. But this conclusion is probably incomplete because it relies on one-pion exchange while ignoring the short-range contributions to the $\bar{D}^*D^*$ interaction. In this regard several works [33, 34, 41, 42] have indicated the possibility of a shallow isoscalar flavour exotic $1^+$ tetraquark below (above) the $D\bar{D}^*$ threshold. If close enough to the $D^*\bar{D}^*$ threshold, it might contribute to the dynamics of this two-body system, suggesting a strong attraction in the $J = 1$, $I = 0$ channel. In turns out that this contribution will reduce/increase the binding of the isovector $J^P = \frac{3}{2}^-/\frac{5}{2}^+$ trimer. The reason for this reduction/increase of the binding energy are the $\lambda_{0;2}^n$ coefficients in Table II for the isovector $J^P = \frac{3}{2}^-/\frac{5}{2}^+$ three-body state, attraction in the isoscalar $\bar{D}^*\bar{D}^*$ subsystem forces the trimer into a configuration that has less/more overlap with the $P_c^*$ channel of the $D^*\Sigma_c$ subsystem. Scenario A will represent the possibility of a shallow isoscalar $J = 1$ $D^*\bar{D}^*$ bound state located at threshold, where predictions for the binding energy of the two affected trimers can be found in Table III.

Next we consider the $D^*\Sigma_c$ interaction in channels different than the $P_c^*$ ($I = \frac{1}{2}$, $J^P = \frac{3}{2}^-$). Phenomenology, in particular the hidden-gauge model [43], indicates that the interaction in the $I = \frac{1}{2}$, $J^P = \frac{3}{2}^-$ channel could very well be as attractive as in the $\bar{P}_c^*$ channel. Scenario B will refer to this possibility. If this is the case the three isoscalar trimers will be degenerate, having the same binding energy as the isoscalar $J^P = \frac{3}{2}^+$ trimer, see Table III. The $D^*\Sigma_c$ interaction in the $I = \frac{1}{2}$ channel has received less attention, though it could also be strong and attractive [19]. Scenario C will consider that there is a $I = \frac{3}{2}$ $D^*\Sigma_c$ bound state at threshold for both $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ (while also assuming scenario B). In this scenario the isovector trimers will be degenerate and more bound than expected, see Table III. The degeneracy is again a consequence of the $\lambda_{0;2}^n$ coefficients of Table II, the isovector trimers can always be in a configuration with the same relative contribution from the $I = \frac{1}{2}$ and $\frac{3}{2}$ $D^*\Sigma_c$ channels. In addition the hypothesis of $D^*\Sigma_c$ spin degeneracy in scenario B means that the trimers are effectively decoupled of the isoscalar $J = 1$ $D^*\bar{D}^*$ interaction. That is, the combination of scenarios A and C leads to the same predictions as scenario C alone, see again Table III.

Another aspect of the $\bar{D}^*\Sigma_c$ interaction is its long-range piece, which is given by one-pion exchange (OPE). Here we have considered OPE to be subleading. This assumption can be analyzed with the formalism of Refs. [46, 47], which investigated the range of momenta for which OPE is perturbative in the heavy meson-meson and heavy baryon-baryon systems. By adapting the methods of Refs. [46, 47] to the heavy meson-baryon case, i.e. to $D^*\Sigma_c$, we arrive at the conclusion that in the $P_c^*$ channel OPE is perturbative for $p < \Lambda_{OPE} \approx 270 - 450$ MeV in the chiral limit ($m_\pi = 0$, with $m_\pi$ the pion mass). The uncertainty is a consequence of the axial coupling constant of the pion with the charmed meson $\Sigma_c$, which is not known experimentally, see Ref. [47] for a discussion. For the physical pion mass, $m_\pi \approx 140$ MeV, we expect $\Lambda_{OPE} \approx 410 - 710$ MeV, see Refs. [46, 47] for a detailed explanation. If we take into account that the binding momentum of a molecular $P_c^*$ is about 200 MeV, it is natural to expect OPE to be subleading, although it will provide important corrections to the leading order description. In principle we expect this to be also the case in the three-body sector, in agreement with our original assumption.

Finally, though the isoscalar $\frac{3}{2}^+$ trimer depends only on the interaction in the $P_c^*$ channel, it actually contains an additional source of uncertainty. The Efimov effect [32] can happen in this trimer, as shown by the

| $J^P$ | $I$ | $B_3(\Lambda = 0.5 \text{ GeV})$ | $B_3(\Lambda = 1.0 \text{ GeV})$ | $\lambda$ |
|-------|-----|---------------------------------|---------------------------------|-------|
| $\frac{1}{2}^+$ | 0 | $4.8^{+1.5}_{-1.4}$ | $3.1^{+1.1}_{-1.0}$ | $\frac{3}{2}^+$ |
| $\frac{3}{2}^+$ | 1 | $2.6^{+1.0}_{-0.9}$ | $1.3^{+0.6}_{-0.5}$ | $\frac{5}{2}^+$ |
| $\frac{3}{2}^+$ | 0 | - | - | $\frac{3}{2}^+$ |
| $\frac{5}{2}^+$ | 1 | $0.5^{+0.3}_{-0.2}$ | - | $\frac{5}{2}^+$ |
| $\frac{5}{2}^+$ | 0 | $14 \pm 3$ | $16^{+3}_{-4}$ | $\frac{3}{2}^+$ |
| $\frac{5}{2}^+$ | 1 | $4.8^{+1.5}_{-1.4}$ | $3.1^{+1.1}_{-1.0}$ | $\frac{3}{2}^+$ |

TABLE II: Predictions for the binding energy $B_3$ of the $D^*\bar{D}^*\Sigma_c$ trimers for different quantum numbers and for a cut-off $\Lambda = 0.5 - 1.0$ GeV, where $B_3$ is relative to the $D^*P_c^*$ threshold (i.e. $B_3 > 0$ indicates that the trimer binds). The errors in $B_3$ are a consequence of the uncertainty in the $D^*\Sigma_c$ binding energy, $B_2 = 12 \pm 3$ MeV.

FIG. 1: Binding energy $B_3$ of the $D^*\bar{D}^*\Sigma_c$ trimer as a function of the coefficient $\lambda$, which parametrizes the overlap of the trimer quantum numbers with the $D^*\Sigma_c$ system in the $P_c^*$ channel. The bars represent the error coming from the uncertainty in the binding energy of the $P_c^*$, $B_2 = 12 \pm 3$ MeV.
TABLE III: Different scenarios for the predictions for the binding energy $B_3$ of the $D^* D^* \Sigma_c$ trimers. Scenario A refers to the existence of a $D^* D^*$ bound state at threshold. Scenario B is when the $I = \frac{1}{2}$ and $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^+$ $D^* \Sigma_c^*$ are degenerate. Scenario C assumes that the $I = \frac{3}{2}^- D^* \Sigma_c^*$ interaction is strong, generating a bound state at threshold for both $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$. For each scenario only the quantum numbers affected are shown.

| Scenario | $J^P$ | $I$ | $B_3$ |
|----------|-------|-----|-------|
| A        | $\frac{1}{2}$ | 1   | 0.5 − 1.0 |
| A        | $\frac{3}{2}$ | 1   | 0.1 − 0.9 |
| B        | $\frac{1}{2}$ | 0   | 14 − 16 |
| B        | $\frac{3}{2}$ | 0   | 14 − 16 |
| C/A+C    | $\frac{1}{2}$ | 1   | 6.7 − 7.3 |
| C/A+C    | $\frac{3}{2}$ | 1   | 6.7 − 7.3 |

analysis of the $D^* D^* \Sigma_c$ system in the unitary limit, see Sect. III The $D^* \Sigma_c$ two-body system in the $P_c^*$ channel is actually far from the unitary limit, but the analysis is still relevant because of the relation between the Efimov effect and Thomas collapse[36]. The idea is that, even though a molecular $P_c^*$ is not in the unitary limit, for $\Lambda \to \infty$ the isoscalar $\frac{5}{2}^+$ trimer will collapse, i.e. its binding energy will diverge ($B_3 \to \infty$). This collapse can be prevented with the inclusion of a three-body force, without which the trimer binding energy predictions will not be formally cut-off independent [34, 55]. In practice, owing the large discrete scaling factor of $e^{\pi/s_0} \approx 5711$, the divergence of the trimer binding requires fantastically large cut-offs to be noticed. For instance, the cut-off required for the first nonphysical isovector $J^P = \frac{5}{2}^+$ trimer to appear is $\Lambda \sim 62$ GeV, which is pretty large. Thus it is not surprising that the cut-off uncertainty for this trimer is not particularly big, about 2 MeV in the $\Lambda = 0.5 - 1.0$ GeV cut-off window. Yet caution is advised because the formal requirement of a three-body force, even if the related divergence happens at really large cut-offs, indicates the existence of systematic uncertainties that are not being taken into account.

V. CONCLUSIONS

The hypothesis that the $P_c^*$ is a $I = \frac{1}{2}^+$, $J^P = \frac{3}{2}^-$ $D^* \Sigma_c$ molecule implies the existence of a few $D^* D^* \Sigma_c$ trimers. Calculations in a contact-range theory indicate that the most bound of these trimers has the quantum numbers $I = 0$, $J^P = \frac{5}{2}^+$ and a binding energy $B_3 \sim 14 - 16$ MeV. There are other three or four trimer configurations that are likely to bind, but they are expected to be considerably less bound: two states with $B_3 \sim 3 - 5$ MeV with quantum numbers $I = 0$, $J^P = \frac{3}{2}^+$ and $I = 1$, $J^P = \frac{5}{2}^+$, a state with $B_3 \sim 1 - 3$ MeV and quantum numbers $I = 1$, $J^P = \frac{3}{2}^+$ and maybe a state on the verge of binding with $I = 1$, $J^P = \frac{5}{2}^+$, see Table III for details. These predictions are affected by a series of uncertainties, which mostly stem from the fact that we do not know too much about the $D^* D^*$ and $D^* \Sigma_c$ interactions, which are summarized in Table III. From the previous considerations we arrive to the conclusion that the most solid prediction is that of the isoscalar $J^P = \frac{5}{2}^+$ trimer.

We have also considered the $D^* D^* \Sigma_c$ system in the unitary limit, i.e. when the $P_c^*$ is located at the $D^* \Sigma_c$ threshold. In this situation the $J^P = \frac{5}{2}^+$ trimer will be Efimov-like and will have a geometric spectrum with a scaling factor of $3.3 \cdot 10^7$ for the binding energies. This scaling factor is fantastically large, which implies that there would be no practical way to observe it even if we could tune the $D^* \Sigma_c$ scattering length. Yet the possibility of the Efimov effect in the isoscalar $J^P = \frac{5}{2}^+$ trimer implies that a three-body force should be included in the calculations at leading order. The practical importance of this three-body force might be tangential, as reflected by the mild cut-off dependence of the isoscalar $J^P = \frac{5}{2}^+$ trimer binding in the cut-off window chosen in this work, $\Lambda = 0.5 - 1.0$ GeV.

Without knowing the nature of the $P_c^*$, the predictions of this work will remain theoretical. The experimental production of these trimers is expected to be difficult, with the lattice probably providing a more convenient way to investigate them. Finally we notice that the existence of $\Sigma_c \Sigma_c D^*$ bound trimers is also likely, but their location will be subjected to even larger uncertainties as a consequence of the $\Sigma_c \Sigma_c$ interaction, which is not particularly well-known but probably strong.

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