Photoelasticity Ray-casting for Residual Stresses in the Axisymmetric Glass Cylinder

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Abstract. Residual stresses are important index to evaluate the precision molded glass plane. Residual stresses inside a glass cylinder was simulated by Finite Element Method. The circular polariscope was used to compare retardation and isoclinic angle between the simulated residual stresses and the thermal treated glass cylinder. The retardation and isoclinic angle of simulated residual stresses were calculated by Jones calculus. The comparison results show a good agreement.

1. Introduction

As an alternative fabrication method to traditional glass devices fabrication process, precision compression molding of glass optics has attracted more and more attention because of its high output and net molding fabrication capability [1]. Glass molding is a kind of hot forming process. The heated glass blank is pressed into required geometry by a pair of optical polishing molds. The heat pressed glass is held in the mold for a predetermined time at the molding temperature to release the stress caused by the pressing. During the cooling stage, cooling rate is controlled to keep residual stresses and thermal shrinkage at or below the required levels.

Residual stresses are important index to evaluate the precision molded optical glass devices. When the glass solidifies completely in the cooling process, the residual stresses freezes in the molded glass plane. The residual stresses will cause the change of refractive index, which will result in unnecessary light deviation and intensity change. The unnecessary light deviation and intensity change will lead to the deterioration of image quality. Photoelastic tomography is the most commonly used method to measure the residual stresses inside glass pieces[2-4]. This method utilizes the photoelasticity of a glass sample when polarized light pass through, and is based on the birefringence of the glass when it is stressed.

In this research, the birefringence characteristics of the molded glass cylinder are simulated by Jones calculus. First, Finite element method (FEM) was employed to simulate the thermal treatment process[5] and then the residual stresses were calculated by the numerical simulation. Then the simulated residual stresses were used as source to simulate the circular polariscope outputs. The isoclinic angle and optical relative retardation calculated. By comparing isoclinic angle and retardation between the simulation and experiment, the results showed that FEM simulations are confirmed with the residual stresses inside glass lenses by glass molding.
2. Numerical Simulation of residual stresses of a Glass Cylinder

2.1. FEM Model of a Glass Plane
Numerical simulation of the heat treatment process of BK7 glass cylinder was performed by using the FEM software MSC/MARC[1]. The glass cylinder selected in this research is high 7 mm, and diameter of 7 mm. In the heat treat experiment, the glass cylinder sit on a ceramic plane with 4 mm height and 15 mm diameter. Then the glass cylinder and the ceramic plane were placed in an oven to perform heat treatment. The two-dimensional (2D) axisymmetric model is adopted by using the rotational symmetric geometry of the mold and cylinder, as shown in Fig. 1. There are two steps in the numerical simulation: (1) heating the glass cylinder and mold to the molding temperature, (2) at a certain cooling rate, cooling the molded glass plane and mold to room temperature.

![Figure 1 FEM model of the glass cylinder](image)

2.2. Simulated Residual Stresses
Fig. 2 shows the distribution of residual stresses in the glass cylinder in cylindrical coordinates. The figures display half of the normal cross section of the cylinder. In the stress distribution figures, compressive stress is represented by negative value and tensile stress is represented by positive value.

![Figure 2 Simulated distributions of residual stresses of the normal cross section of the glass cylinder in cylindrical coordinates](image)

3. Photoelasticity Ray Casting
According to Jones Calculus, the light exporting from the circular polariscope can be expressed as [2, 6]:

\[
\sigma_z, \sigma_r, \tau_z, \sigma_\theta
\]
\[
\begin{pmatrix}
E_x \\
E_y
\end{pmatrix} = \begin{pmatrix}
\cos^2 \beta & \cos \beta \sin \beta \\
-\cos \beta \sin \beta & \sin^2 \beta
\end{pmatrix} \times \begin{pmatrix}
i \cos^2 \gamma + \sin^2 \gamma & (i-1) \sin \gamma \cos \gamma \\
(i-1) \sin \gamma \cos \gamma & i \sin^2 \gamma + \cos^2 \gamma
\end{pmatrix} \\
\times J(\Delta, \varphi) \times \begin{pmatrix}
i \cos^2 \varepsilon + \sin^2 \varepsilon & (i-1) \sin \varepsilon \cos \varepsilon \\
(i-1) \sin \varepsilon \cos \varepsilon & i \sin^2 \varepsilon + \cos^2 \varepsilon
\end{pmatrix} \times \begin{pmatrix}0 \\
1\end{pmatrix}
\]

(1)

\[
J(\Delta, \varphi) = \begin{pmatrix}
\cos \varphi & -\sin \varphi \\
\sin \varphi & \cos \varphi
\end{pmatrix} \begin{pmatrix}
e^{i\Delta} & 0 \\
0 & 1
\end{pmatrix} = \begin{pmatrix}
e^{i\Delta} \cos^2 \varphi + \sin^2 \varphi & (e^{i\Delta} - 1) \sin \varphi \cos \varphi \\
(e^{i\Delta} - 1) \sin \varphi \cos \varphi & e^{i\Delta} \sin^2 \varphi + \cos^2 \varphi
\end{pmatrix}
\]

(2)

Along the light path, the object can be divided into \(n\) layers with a thickness of \(dz\). The relation between the outgoing light \(E_{out}\) and the incident light \(E_{in}\) is expressed as \([3, 4]\):

\[
E_{out} = \sum_{k=1}^{n} (1 + A_k dz) E_{in}
\]

(3)

where, \(E = \begin{pmatrix} E_x \\ E_y \end{pmatrix}\), \(A = -\frac{1}{2} iC_0 \begin{pmatrix} \sigma_{xx} - \sigma_{yy} & 2\tau_{xy} \\ 2\tau_{xy} & -(\sigma_{xx} - \sigma_{yy}) \end{pmatrix}\). \(\sigma_{xx}, \sigma_{yy}\) and \(\tau_{xy}\) are stress components in \(XY\) plane, \(i\) is imaginary.

Then, the Jones matrix of the photoelastic object is obtained as:

\[
J_{stress} = \sum_{k=1}^{n} (1 + A_k dz) = J(\Delta, \varphi)
\]

(4)

Figure 3 The intensity images of the glass cylinder from simulated stresses by Jones calculus

With the simulated intensity images of the glass cylinder shown in Fig.3, the isoclinic angle \(\varphi\) and optical retardation \(\Delta\) of the glass cylinder can be obtained as follows:

\[
\varphi = \frac{1}{2} \arctan \left( \frac{I_5 - I_3}{I_4 - I_6} \right)
\]

(5)
Therefore, the optical retardation and isoclinic angle of the glass cylinder can be calculated through intensity images in Fig. 4. In Fig. 4, the isoclinic angle retardation are expressed in radian.

\[
\Delta = \arctan \left( \frac{(I_5 - I_3) \sin 2\varphi + (I_4 - I_6) \cos 2\varphi}{I_1 - I_2} \right)
\]

(6)

Figure 4 Optical retardation and isoclinic angle of the glass cylinder expressed in radian, and calculated from intensity images: (a) optical retardation, (b) isoclinic angle.

4. Results and Discussion

Due to the containment and the residual stress effect of the glass tank, intensities shown in Fig. 5 are hard to tell by eyes. According to Eqs. 5 and 6, the retardation and isoclinic angle can be calculated by the figures in Fig. 5. The isoclinic angle in radian and the retardation in nm, calculated from the intensity images in Fig. 5, are shown in Fig. 6 (a) and Fig. 6 (b) respectively [4].

Figure 5 Experiment intensity images of the glass cylinder
5. Conclusions
Residual stresses of a glass cylinder were simulated by using FEM with a structural relaxation model. At the same time, the light intensity images of glass cylinder from the circular polariscope under the same experimental conditions are measured. The comparison of the retardation and isoclinic angle from simulation by simulated residual stresses and experiments demonstrates that residual stresses calculated in numerical simulation have a reasonable conformance with experimental results.

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