A CLASS OF DEDUCTIVE THEORIES THAT CANNOT BE DETERMINISTIC

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Abstract. Although the concept of an observing device with memory is very simple and definable in terms of algorithmic machine, it is not compatible with a predictive theory involving this concept. This is sufficient to show that Physics as a whole cannot be deterministic; moreover, using the Conway-Kochen Free Will Theorem [1] and the simple logical proof of it we gave in [2], we show, without any need of free will or observer's freedom, that neither Quantum Mechanics nor any extension of this theory can be deterministic.

I - Definitions
1). By deductive theory, we mean a theory that is based on – not necessarily completely formal – logical reasoning, as e.g. mathematics, logic itself of course, or more casually physical science, theoretical or applied. If F is the theory and ⊢ the symbol of deduction, F ⊢ A means that A is deducible from (or can be proved in) the theory F.

2). By deterministic theory, we mean that in the deductive theory a time variable t is defined, t belonging as usual to an interval of real numbers (however an ordered set is sufficient here), and that any true event E(t) expressible in the theory and occurring at time t can be deduced 'in advance' in the theory i.e. there exists t' < t such that

(a) F(t') ⊢ E(t)

where in F appear only events that occurred at time ≤ t' (this can be formulated precisely in logical terms, but is not necessary here). In such case the event E(t) is said to be predicted or (pre)determined at time t' by theory F.

3). By an observing machine M with memory we mean a machine e.g. a Turing machine, a computer or a human brain, that can observe (or read) any finitely coded sentence or formula expressible in the theory, that appears on a tape or in the coded observational field of the machine; moreover, the machine stores this sentence or a part of it in its memory, i.e., the finite set of already stored sentences since a given time (the machine, of course, can observe only a finite number of times). In particular, the machine M can observe the second member of formula (a) above relative to the event E(t) as soon as the formula has appeared among (or in the list of) deducible formulas in F i.e. at time t' < t , and stores the predicted event E(t) in its memory.

4). Let us write M(t, x(t), m(t)) for “at time t M observes x(t) and stores it in its memory m(t)”. By definition above, memory has obviously the following property:

(b) (b) if t' < t then m(t') ⊆ m(t).

This corresponds to the very meaning of the notion of memory.
II. Main Results

**Theorem** The existence of an observing machine with memory based on a deductive theory $F$ is inconsistent with the theory $F$ being deterministic.

**Proof.** Suppose the contrary. Since the theory $F$ is deterministic, in particular in what concerns $M$, at some time $t' < t$ the behaviour of the machine $M$ at time $t$ is predetermined in the theory, i.e., $F(t') \vdash M(t, x(t), m(t))$. Then already at time $t'$, $M$ observes the predicted event and stores it in its memory $m(t')$, thus

(c) $M(t, x(t), m(t)) \nvdash m(t')$.

Now, when later on, time $t$ indeed occurs, if the deterministic prediction is correct, the expected event $M(t, x(t), m(t))$ should occur as well, but since $t' < t$, because of (b) we have $m(t') \nvdash m(t)$, and therefore, $M(t, x(t), m(t)) \nvdash m(t)$, which is impossible since $m(t)$ is finitely coded.

**Commentary.** What we need is that the interval of time between $t'$ and $t$, be large enough for $M$ to have observed and stored (possibly at time $t''$ with $t' < t'' \leq t$) the predicted sentence $M(t, x(t), m(t))$ before (or at the latest at) time $t$. Technically for such simple a task, a machine nowadays needs a very small fraction of time, while a deterministic theory, if all needed information is available, should calculate and predict an event, in principle, 'sufficiently' in advance. Otherwise, if there is no time to know a predicted result before it happens, there is of course no prediction and the determinism would have no more value than the religious belief that 'everything is already written' in some Book for ever unknown. In any case, if we accept determinism in the ordinary meaning of the word, so that the predicted event can be recorded as such, the theorem shows that it is not compatible with observation and memory.

**Corollary 1** In the deterministic part of classical physics, an observing machine with memory cannot be defined (and therefore constructed).

Now, since an observing machine with memory certainly exists in our physical world e.g. as the brain of human being or, more elementary, as a modern computer, we have

**Corollary 2** Actual Physics cannot be completely deterministic.

**Corollary 3** Quantum Mechanics, if consistent, are not deterministic.

**Proof.** In [2], we have given a formal proof (improving the Conway – Kochen Free Will Theorem [1]) that if an observer of spin components is free in this observation, then Quantum Mechanics are not deterministic; but our main theorem here proves that the behaviour of an observer or machine with memory cannot be predetermined, i.e., is necessarily free.

The proof shows that no extension, e.g. by hidden variables, of Q.M. can be a deterministic theory.

If observed, Nature cannot be deterministic; actually, when observed, Nature is observing itself and clearly, since it includes the observing device, it has memory; but this, as we have shown, yields non-determinism. Extending Einstein's famous saying, we should say: God doesn't play dice, but lets the dice play.

**References**

[1] J.Conway and S.Kochen, The Strong Free Will Theorem, *AMS*, vol 56/2, p.226-232, Providence, February 2009.
[2] I. Reznikoff, A Logical Proof of the Free Will Theorem, arXiv: 1008.3661v1 [quant-ph] 21 Aug 2010 (http://arxiv.org/abs/1008.3661).

[3] I. Reznikoff, The Consistency of Quantum Mechanics Implies Its Non-Determinism, arXiv: 1010.4020v1 [quant-ph] 19 Oct 2010 (http://arxiv.org/abs/1010.4020). (In this paper some proofs are not clearly explained and a logical assumption is actually unsatisfactory; the paper given here proves, in a much more powerful way, the same main conclusion).