Searching for the Most Suitable Loss Model Set for Subsonic Centrifugal Compressors Using an Improved Method for Off-Design Performance Prediction

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Abstract: This work investigates loss model sets based on empirical loss correlations for subsonic centrifugal compressors. These loss models in combination with off-design performance prediction algorithms make up an essential tool in predicting off-design behaviour of turbomachines. This is important since turbomachines rarely work under design conditions. This study employs an off-design performance prediction algorithm based on an iterative process from Galvas. Modelling of ten different loss mechanisms and physical phenomena is involved in this approach and is thoroughly described in this work. Geometries of two subsonic compressors were reconstructed and used in the evaluation of individual loss correlations in order to obtain a suitable loss model. Results of these variations are compared to experimental data. In addition, 4608 loss model sets were created by taking all possible combinations of individual loss estimations from which three promising candidates were selected for further investigation. Finally, off-design performance of both centrifugal compressors was computed. These results were compared to experimental data and to other loss model sets from literature. The newly composed loss model set No. 2137 approximates experimental data over a 21.2% better in relative error than the recent Zhang set and nearly a 36.7% better than the outdated Oh’s set. Therefore, set No. 2137 may contribute to higher precision of centrifugal turbomachines’ off-design predictions in the upcoming research.

Keywords: centrifugal compressor; performance prediction; empirical loss correlations; Eckardt’s compressors

1. Introduction

Compressors are machines serving to deliver compressed air. Compressed air is useful in various industrial applications e.g., gas transportation, protective atmosphere of food, wastewater treatment [1] or combustion engines, where compressed air enhances power output through turbocharging. Based on the airflow direction change over the rotor (rotating part of the compressor), they can be divided into two groups–axial and centrifugal [2]. Centrifugal compressors change airflow direction significantly, specifically from a general axial direction to radial or the other way around. For axial compressors, the general idea is to have the airflow both coming and exiting in the axial direction. Their compression ratio per stage is lower and therefore is often designed as multistage, i.e., with multiple rows of rotor and stator blades according to [3]. This is generally a much more complicated task than designing a centrifugal compressor. Apart from providing a higher pressure ratio per one stage, they can operate in a wide range of mass flows [4], which plays a huge role in aircraft industry.

Due to the nature of fluid flow and governing Navier–Stokes equations, three-dimensional CFD simulations of internal aerodynamics are still a complex task today, which is very
time consuming and computationally demanding. Thus, preliminary one-dimensional design still has its importance. This leads to the effort of developing simplified tools for performance prediction outside of the design point. These methods are based on three-dimensional phenomena approximation by empirical loss correlations. In literature, individual loss mechanisms are described by many different relations and authors. For axial turbocompressors, Barbosa in [5] presented an off-design performance prediction algorithm. Peyvan in [6] introduced another approach, where single-stage axial compressor performance was investigated. Aerodynamic analysis and performance prediction for transonic and three-stage subsonic compressors were made by Li in [7].

In the case of centrifugal compressors, Coppage in [8] introduced the first performance prediction procedure where some loss mechanisms were merged into one. Some other loss mechanisms were not taken into account at all. The first published program for performance prediction was presented by Galvas in [9], where more loss correlations were included and also showed comparisons with the experiment. Thanapandi and Prasad in [10] found an optimal loss model set for submersible pumps with low specific speed. Whitfield and Baines in [11] reviewed empirical loss correlations and introduced generalized one-dimensional performance prediction for centrifugal turbomachinery. Aungier provided a comprehensive mean streamline aerodynamic performance prediction procedure for centrifugal compressor stages in [12], where many empirical loss correlations were presented and tested on various compressor geometries. The first attempt to find an optimal set of loss correlations was made by Oh in [13]. Furthermore, new loss correlation for recirculation was developed, and it was shown that it should be included into the new loss model set. Possible combinations were tested on Eckardt’s compressors. Performance prediction using a dynamic compressor model is presented in [14], where only a few loss mechanisms are considered. Li in [15] reviewed current loss correlations and presented a loss model set which outperformed both Galvas’s and Oh’s sets. Gutierrez in [16] also found a loss model set where several loss mechanism correlations were varied. Zhang in [17] presented a comprehensive study about loss correlations, but some empirical relations were excluded from review. A new approach for compressor categorization was introduced based on non-dimensional parameters such as relative Mach number at inducer tip and specific speed.

In this paper, a performance prediction methodology is presented including a complete empirical loss correlations review. In order to find the most suitable correlation set, all possible combinations of loss model sets are tested. In contrast with Zhang in [17], there are no correlations excluded a priori. In addition, there is no need to determine the friction coefficient empirically, and this parameter is instead determined by using well known relations established by Blasius in [18] and Prandtl in [19]. In order to classify a compressor as subsonic, a method based on an inlet tip relative to Mach number and specific speed as presented in [17] is adopted. Eckardt’s subsonic compressors published in [20] were chosen as test geometries for this paper; thus, shock loss is excluded. Performance parameters of these compressors are evaluated in a vaneless diffuser configuration as in [21], thus vaned diffuser loss mechanisms and correlations are excluded.

2. Methodology

Off-design performance prediction of a centrifugal compressor is presented in this paper. For computation, the knowledge of main geometrical parameters of the centrifugal compressor, information about impeller blades and thermodynamic parameters of fluid at the inlet is essential. A part of these input parameters is visualised in Figure 1.

Methodology presented in this paper is based on principles described by Galvas in [9]. Since it was validated by multiple authors e.g., Zhang in [17], Oh in [13] and more, this algorithm is widely used. Individual losses are estimated using empirical loss correlations, which are described later in detail. When inlet mass flow rate \( \dot{m} \), rotational speed RPM and compressor geometry are known, inlet velocity \( C_0 \) can be computed through an iterative
process until convergence in mass flow $\dot{m}$ is reached. Equations explaining this loop are described in [9].

![Figure 1](image1.png)

**Figure 1.** (left) Sketch of a centrifugal impeller with important dimensions in meridional cross-section; (right) example of velocity triangles along the height of impeller’s blade tip.

Velocity triangles along the blade tip at the inlet can be determined since impeller blade geometry is known as well. At the impeller inlet, an incidence loss occurs. This enthalpy loss $\Delta h_{in}$ serves to determine total pressure just inside the blade row $p'_{11}$ in [9] as

$$p'_{11} = p_{10} \exp\left(-\frac{\Delta h_{in}}{T_{2r}}\right). \quad (1)$$

Total relative temperature at impeller exit $T'_{12}$ which serves to calculate total temperature $T_{12}$ can be computed according to [9] as follows:

$$T'_{12} = T'_{11} + 0.5 \frac{U_{12}^2 - U_{12}^2}{c_p}. \quad (2)$$

The enthalpy rise throughout the impeller, total temperature and density at the impeller exit are initially approximated in [9]. The impeller exit density is obtained through iteration of the impeller loss equations and resulting state properties. During this process, slip factor and velocity triangles at the impeller exit are determined as it can be seen in Figure 2.

![Figure 2](image2.png)

**Figure 2.** Example of velocity triangles at an impeller outlet with a breakdown into different velocity components.
Subsequently, static and total temperatures are calculated. Dynamic viscosity $\mu_2$ is computed using Sutherland’s law from [22] and friction coefficient $C_f$ is determined using Blasius solution published in [18] or Prandtl’s one seventh power law from [19]. Theoretical enthalpy rise in the impeller is calculated as in [9] by the following equation:

$$\Delta h_{eu} = c_p T_{t0} \left( \frac{T_{t2}}{T_{t0}} - 1 \right).$$

(3)

Internal losses such as choke, entrance diffusion, blade loading, skin friction, clearance and mixing loss (in summary $\Delta h_{int}$) are evaluated using empirical loss correlations. In the end of the iterative process, state properties are calculated in [9] as

$$p_{t2} = p_{t2}^* \left( \frac{\eta_{eu} \Delta h_{eu}}{c_p T_{t0}} + 1 \right)^{-\frac{1}{\gamma}}, \text{ where } \eta_{eu} = \frac{\Delta h_{eu} - \Delta h_{int}}{\Delta h_{eu}}.$$  

(4)

Exit density is then determined using equations of gas dynamics and ideal gas equation of state. The procedure continues until convergence in density is reached. When state properties at the impeller exit are calculated, parasitic losses $\Delta h_{par}$ such as recirculation and disc friction are taken into account. To calculate loss in the vaneless diffuser, it is necessary to determine exit total pressure. Pressure loss can be obtained through the solution of equation derived in [9] as

$$\frac{p_{3}'}{p_{4}'} = 1 + \frac{\gamma C_f}{\cos(\alpha_3)} \frac{r_3}{b_3} \int_{M_3}^{R} R dR \frac{M_3}{\left( \frac{p_{3}'}{p_{4}'} \right)_{3}}.$$  

(5)

where integral on the right side is evaluated by the trapezoidal numerical method. When total pressure loss is known, vaneless diffuser loss $\Delta h_{vld}$ is calculated.

When thermodynamic parameters at the outlet and decrements in enthalpy due to loss mechanisms are known, compressor performance is evaluated by Oh in [13] in the following manner:

$$\eta = \frac{\Delta h_{R} - \Delta h_{int} - \Delta h_{vld}}{\Delta h_{R} + \Delta h_{par}},$$  

(6)

$$PR = \left( \frac{\eta}{c_p T_{t0}} + 1 \right)^{-\frac{1}{\gamma}}.$$  

(7)

For clarity, the main parts of the presented algorithm are shown in Figure 3.

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**Figure 3.** Flowchart of the off-design performance prediction algorithm.
3. Empirical Loss Correlations

An essential part of almost every mean streamline aerodynamic performance prediction and also analysis of centrifugal compressors are the empirical loss correlations. These predictions are often unavoidable in the development of a new turbomachine. Those correlations are empirical approximations of aerodynamic losses, which take place in different regions of centrifugal compressors. Furthermore, they also differ in the principle of physical origin. These correlations are commonly divided into two basic groups, which are internal and external losses. The main difference between these two groups, as it was proposed by Oh in [13], is that the internal losses give rise to impeller discharge stagnation enthalpy with corresponding increase in static pressure. On the other hand, external losses only give rise to the impeller discharge stagnation enthalpy without any increase in static pressure. Those two groups are further subdivided into several categories based on their physical origin.

As this work deals with centrifugal compressors with vaneless diffusers, a separate category is considered for losses taking place in vaneless diffusers.

The flow cannot be perfectly guided through the impeller with a finite number of blades. It is said to slip, resulting in modified velocity triangles at the impeller outlet. Magnitude of the tangential velocity component is reduced, leading to delivered pressure ratio decrease. In order to quantify the compressor’s slip, a slip factor $SF$ is defined by Whitfield in [11] as

$$ SF = 1 - \frac{C_{SF}}{U_2}. $$

(8)

Many slip factor relations are published in literature. Stodola in [23] proposed the first slip factor based on the concept of relative eddy. Busemann in [24] developed a more accurate theoretical solution which was later published by Wislicenus in [25]. Wiesner in [26] approximated the original Busemann results with the following equation:

$$ SF = 1 - \frac{\sqrt{\cos(\tilde{\beta}_2)}}{\tilde{\beta}_2}. $$

(9)

In present work, slip factor with the limit of the blade solidity described in [26] is adopted.

3.1. Internal Losses

3.1.1. Incidence Loss

Incidence loss considers change of flow angle due to fluid siding to the inducer blade at the impeller inlet. This means that fluid near the inducer blade experiences rapid and nearly instant change in angle and velocity due to inequality in angles of the blade and incoming flow. When the angle change is significant, flow separates and losses are generated through the boundary layer separation phenomenon.

Stanitz in [27] assumed an incompressible working fluid and introduced equation for optimal inducer incidence angle of turbomachines with axial flow inlets. The optimum flow angle is obtained from the inlet velocity triangle and from the blade characteristics including the blade thickness at the root mean square diameter. Therefore, the incidence loss is simply

$$ \Delta h_{in} = \frac{W^2_\ast}{2}, $$

(10)

where the velocity correction $W_\ast$ necessary to change the flow angle is defined as

$$ W_\ast = W_{1\xi} \sin |\beta_{opt} - \beta_{1\xi}| $$

(11)
and the optimal flow angle is defined as follows:

$$\beta_{opt} = \arctan \left( \frac{\pi D_1}{\pi D_1 - Zt_1} \tan(\tilde{\beta}_1) \right). \quad (12)$$

Conrad in [28] introduced a coefficient $\sigma$ to Equation (10) which adjusts the value of the incidence loss. Therefore, Conrad’s incidence loss formula reads

$$\Delta h_{in} = \sigma \frac{W_1^2}{2}, \quad \sigma \in (0.5 - 0.7), \quad (13)$$

with a common value for $\sigma$ being 0.6.

Later, Aungier in [12] developed following equations which are applied at the hub, shroud, and root mean square diameter. Aungier’s equations therefore have the following form:

$$\Delta h_{in,i} = 0.4 \left( W_{1,i} - \frac{C_{m1,i}}{\sin(\tilde{\beta}_{1,i})} \right)^2, \quad i \in \{h, \xi, s\}, \quad (14)$$

where the absolute velocity components are defined as

$$C_{m1,h} = C_{m1} \left( 1 - \frac{k_1 h_1}{2} \right), \quad (15)$$

$$C_{m1,s} = C_{m1} \left( 1 + \frac{k_1 h_1}{2} \right). \quad (16)$$

The three incidence losses are then combined in a weighted average, with the mean streamline value weighted 10 times as heavy as the hub and shroud value. Therefore, the final incidence loss reads

$$\Delta h_{in} = \frac{\Delta h_{in,h} + 10 \Delta h_{in,\xi} + \Delta h_{in,s}}{12}. \quad (17)$$

Another incidence loss correlation was developed by Gravdahl in [29]. The following equation is derived using velocity triangle considerations in the following form:

$$\Delta h_{in} = \frac{1}{2} \left( U_{1,\xi} - \frac{\cot(\tilde{\beta}_{1,\xi}) \eta}{\rho_1 A_1} \right)^2. \quad (18)$$

3.1.2. Choke Loss

Choke loss correlation includes losses that occur in the compressor’s aerodynamic throat during increased mass flows. These losses are caused by the presence of a normal shock wave, which starts appearing when the flow in the throat reaches supersonic state.

Aungier in [12] modeled the choking loss by a contraction ratio $C_r$ and by an onset criterion $x$, which decides whether the flow is choked. The final loss is defined as the following conditional and polynomial expression:

$$\Delta h_{ch} = \begin{cases} W_{1,\xi} (0.05 x + x^2) / 2 & \text{if } x > 0, \\ 0 & \text{otherwise}, \end{cases} \quad (19)$$

where the onset criterion is defined as

$$x = 10 (1.1 - A_{th}^* / (C_r A_{th})). \quad (20)$$
The sonic throat area $A_{th}^*$ is defined by gas-dynamic equations in [30]. The throat contraction ratio is obtained as

$$C_r = \sqrt{A_1 \sin(\tilde{\beta}_{1z}) / A_{th}}.$$  

(21)

### 3.1.3. Entrance Diffusion Loss

Equations (10), (13), (17) and (18) for estimation of incidence losses often underestimate the entrance loss at positive incidence angles. In some cases, the flow entrance angle adjustment between the inducer leading edge and the impeller throat is the prevailing effect. The entrance diffusion loss, which was introduced by Aungier in [12], corrects this phenomenon by introducing

$$\Delta h_{\text{edf}} = 0.4(W_1 - W_{th})^2 - \Delta h_{\text{in}},$$

and if $\Delta h_{\text{edf}} < 0 \Rightarrow \Delta h_{\text{edf}} = 0,$  

(22)

where the relative inducer throat velocity $W_{th}$ is obtained by solving a system of three nonlinear algebraic equations published by Kosuge in [31]. Kosuge also found out that the flow diffusion from the blade inlet to the throat is a primary indicator of inducer stall. Moreover, the specific stall criterion was determined from available data sets. The criterion reads as

$$W_{1s} > 1.75W_{th}.$$  

(23)

When this condition is satisfied, the entrance diffusion loss is unlike in Equation (22) calculated as

$$\Delta h_{\text{edf}} = \begin{cases} 0.4(W_1 - W_{th})^2 - \Delta h_{\text{in}} & \text{if } \Delta h_{\text{edf}} > 0.5(W_{1s} - 1.75W_{th})^2 - \Delta h_{\text{in}}, \\ 0.5(W_{1s} - 1.75W_{th})^2 - \Delta h_{\text{in}} & \text{otherwise.} \end{cases}$$

(24)

### 3.1.4. Blade Loading Loss

Blade loading loss arises from the nature of boundary layers which develop throughout the compressor. Negative gradient of velocity in the boundary layers leads to fast growth of the boundary layers, which can also eventually result in boundary layer separation and stall.

Coppage in [8] formed the blade loading loss using the diffusion factor. The final form of the blade loading loss by Coppage reads

$$\Delta h_{\text{bl}} = 0.05D_f^2U_2^2.$$  

(25)

where diffusion factor $D_f$ is determined as

$$D_f = 1 - \frac{W_2}{W_{1s}} + \frac{0.75\Delta h_{\text{cu}}}{\frac{W_1}{W_2} \left( \frac{z}{\pi} \left( 1 - \frac{D_{1s}}{D_2} \right) + 2 \frac{D_{1s}}{D_2^2} \right) U_2^2}.$$  

(26)

Aungier in [12] suggested that the blade loading loss should be computed as the mixing loss derived from the integrated difference between averaged and mass-averaged relative velocity squared for the distorted profiles, so the formula reads

$$\Delta h_{\text{bl}} = \frac{\Delta W^2}{48},$$

(27)

where $\Delta W$ is determined from standard irrotational flow relations as

$$\Delta W = \frac{2\pi D_2 C_{n2}}{zL}.$$  

(28)
3.1.5. Skin Friction Loss

The skin friction loss arises on surfaces of the impeller due to turbulent friction. This loss is calculated by considering equivalent hydraulic diameter \( d_h \) of the impeller defined by Jansen in [32] as

\[
d_h = \frac{\cos(\beta_2)}{\left( \frac{\pi}{\pi + \frac{D_2 \cos(\beta_2)}{\beta_2}} \right)} D_2 + \frac{1}{\left( \frac{\pi}{\pi + \frac{D_1 \cos(\beta_1)}{\beta_1}} \right)} \left( \frac{\cos(\beta_1) + \cos(\beta_1)}{2} \right)
\]

and using the equations for loss calculation in pipes.

Coppage in [8] stated that approximation using pipe friction correlation is suitable. Later, the concept of averaged relative velocity was used. Thus, the skin friction loss reads as

\[
\Delta h_{sf} = 5.6 C_f \frac{L}{d_h} \left( \frac{W}{U_2} \right)^2 U_2^2,
\]

where the squared averaged relative velocity is

\[
\left( \frac{W}{U_2} \right)^2 = \frac{1}{2} \left[ \frac{C_{m1}^2}{U_2^2} + \frac{D_{1s}^2}{D_2^2} + \frac{W_2}{W_{1s}^2} \left( \frac{C_{m1}^2}{U_2^2} + \frac{D_{1s}^2}{D_2^2} \right) \right].
\]

Later, Jansen in [32] approximated this loss using relations for a fully developed flow in a pipe. Diameter and length of this fictive pipe would be equal to average hydraulic diameter and length of impeller flow passage, respectively. Moreover, effects of non-uniform velocity distribution are excluded. The skin friction loss then has the following form:

\[
\Delta h_{sf} = 2C_f \frac{\bar{L}}{d_h} \bar{W}^2,
\]

where the averaged relative velocity \( \bar{W} \) is computed as

\[
\bar{W} = \frac{2W_2 + W_{1s} - W_{1h}}{4}.
\]

Aungier in [12] used Equation (32) and suggested that the averaged relative velocity should be calculated by a slightly altered relation as

\[
\bar{W} = \sqrt{\frac{W_{1s}^2 + W_2^2}{2}}.
\]

3.1.6. Clearance Loss

Clearance loss considers a fraction of mass flow escaping from the space between pressure and suction side of two neighboring blades through the clearance between the blades and compressor shroud. This phenomenon results from the pressure difference between the suction and pressure sides.

Krylov and Spunde in [33] modeled clearance loss using geometrical parameters of the compressor as

\[
\Delta h_{cl} = 2 \frac{\epsilon}{D_2} \left( \frac{D_{1h} + D_{1s}}{2D_2} - 0.275 \right) U_2^2.
\]

Rodgers in [34] showed that compressor efficiency increases with decreasing clearance up to a limit after which further clearance reduction brings essentially no improvement in efficiency. This result was explained by the presence of large boundary layer accumulation
under an adverse pressure gradient, so that the effective clearance gap would be smaller than the physical gap. The clearance loss is approximated as

\[ \Delta h_{cl} = 0.1 \frac{\epsilon}{b_2} U_{2}^2. \]  

(36)

Jansen in [32] came up with a model of clearance loss which estimates the pressure difference across the clearance gap by characteristic blade loading. Total pressure losses are obtained by assigning standard loss factors to sudden contraction and expansion processes. It can be written as

\[ \Delta h_{cl} = 0.6 \left( \frac{\epsilon}{b_2} \right) C_{u2} \sqrt{\frac{2\pi (D_{1c}^2 - D_{1h}^2)}{b_2 z (D_2 - D_{1h}) (1 + \rho_2/\rho_1) C_{u2} C_{m1} \xi}}. \]  

(37)

Finally, Aungier in [12] defined a clearance loss formula as a function of clearance mass flow and average pressure difference. It can be expressed as

\[ \Delta h_{cl} = \frac{\dot{m}_{cl} \Delta P_{cl}}{\dot{m} \rho}, \]  

(38)

where the clearance mass flow is estimated as

\[ \dot{m}_{cl} = \frac{(\rho_1 + \rho_2) z \rho L \dot{m}}{2}. \]  

(39)

The velocity of the clearance flow through the gap is given by

\[ U_{cl} = 0.816 \sqrt{2 \Delta P_{cl}/\rho_2}, \]  

(40)

where the average pressure difference across the gap is estimated from the change in fluid angular momentum through the impeller as

\[ \Delta P_{cl} = \frac{\dot{m} (D_2 C_{a2} - D_{1c} C_{a1c})}{\overline{z} \overline{D} \overline{b} \rho L \dot{m}}, \quad D = \frac{D_{1c} + D_2}{2}, \quad \overline{b} = \frac{b_1 + b_2}{2}. \]  

(41)

3.1.7. Mixing Loss

As the air flow goes through impeller’s passage, the characteristic flow pattern is formed. This pattern is called “jet and wake”, and it consists of two different regions. Jet is a high energy region while wake is a region with low momentum. These two different streams will inevitably mix after the impeller’s outlet.

Johnston and Dean in [35] assumed that the mixing process is a rapid expansion, so that the jet and wake pattern disappears almost immediately after the flow exits the impeller. Another assumption proposed by Johnston and Dean is that the jet and wake exit velocity distribution is composed from two square waves and the exit static pressure is equal in both zones. The mixing loss correlation suggested by Johnston and Dean in [35] can be expressed as

\[ \Delta h_{mix} = \frac{1}{1 + \tan^2(\alpha_2)} \left( \frac{1 - \xi - \lambda}{1 - \xi} \right)^2 C_{m2}^2 \frac{2}{\xi}, \]  

(42)

where the wake width \( \xi \) is obtained from two-zone modeling procedure published by Oh in [36].

Aungier in [12] assumes that the wake contains stagnant fluid, which mixes with free stream fluid. It is suggested to compute the mixing loss as

\[ \Delta h_{mix} = 0.5 (W_{sep} - W_{out})^2, \]  

(43)
where the free stream velocity $W_{sep}$ is determined by the following condition

$$W_{sep} = \begin{cases} \frac{W_2}{W_{2,D_{out}}} & \text{if } D_{eq} \leq 2, \\ \frac{W_2}{W_d} & \text{otherwise}, \end{cases}$$

where $D_{eq} = \frac{W_{max} - W_2 + \Delta W}{2}$. \hfill (44)

Velocity $\Delta W$ is determined using Equation (28) and the velocity $W_{out}$ is defined by

$$W_{out} = \sqrt{\left(\frac{C_{m2}A_2}{\pi D_2 b_2}\right)^2 + W_{u1}^2}. \hfill (45)$$

### 3.2. External Losses

#### 3.2.1. Recirculation Loss

This loss is associated with recirculation of the fluid from the diffuser back to the tip area of the impeller. This recirculation is caused by the increased flow angle at the exit and inability of the fluid to withstand the pressure gradient, which makes part of the energy given to the fluid return to the impeller.

Coppage in [8] estimated recirculation loss by a functional dependence on the exit absolute flow angle $\alpha_2$ and the diffusion factor $D_f$ as

$$\Delta h_{rc} = 0.02 \sqrt{\tan(\alpha_2)D_f^2 U_2^2}, \hfill (46)$$

with $D_f$ defined in Equation (26).

Rodgers came up with recirculation loss correlation in [34] that omits the diffusion factor and employs empirical constant. This formula can be written as

$$\Delta h_{rc} = 0.032 \left(\frac{U_2^2}{C_{m1}^2}\right)^2. \hfill (47)$$

Aungier in [12] published a modified approach to computing recirculation loss. This formula is only used when the equivalent diffusion factor $D_{eq}$ is greater than a certain number representing the stall of the impeller. Lieblein in [37] estimated this number as being equal to 2. Thus, when this condition is satisfied, Aungier’s recirculation loss formula reads

$$\Delta h_{rc} = \left(\frac{D_{eq}}{2} - 1\right) \left(\frac{W_{u2}}{C_{m2}} - 2 \tan(\beta_2)\right). \hfill (48)$$

Oh in [13] presented a new form of the recirculation loss correlation by putting higher power to the flow angle contribution. In addition, in order to connect the loss distributions smoothly between low and high flow regions, a hyperbolic function was employed. Therefore, the recirculation loss is estimated as

$$\Delta h_{rc} = 0.00008 \sinh(3.5\alpha_2^2)D_f^2 U_2^2. \hfill (49)$$

#### 3.2.2. Disk Friction Loss

Disk friction loss occurs due to frictional torque of shear forces on the back surface of the rotor (rotating disk) induced by the fluid surrounding it. This loss is the same for a given disk size whether it is used for a radial inflow compressor, a radial outflow compressor, or a radial inflow turbine.

Shepherd in [38] formulated disk friction loss using correlation with Reynolds number and empirically found constant, forming the relation

$$\Delta h_{df} = 0.01356 \frac{\rho U_2^2 D_2^2}{m Rc_{D2}^2}, \hfill (50)$$
where the Reynolds number $Re_2$ is determined as

$$Re_2 = \frac{U_2 D_2}{\nu_2}. \quad (51)$$

Daily and Nece in [39] studied the rotation of a smooth plane disk enclosed within a right-cylindrical chamber both experimentally and theoretically. They established the following loss correlation formula based on flow regime as

$$\Delta h_{df} = 0.25 K_f \bar{\rho} U_2^3 \frac{L_3^2}{4\eta}, \quad \bar{\rho} = \frac{\rho_1 + \rho_2}{2}. \quad (52)$$

The coefficient $K_f$ is determined based on the Reynolds number as follows:

$$K_f = \begin{cases} 
3.7 \left( \frac{2 \epsilon}{D_2^2} \right)^{0.1} & \text{if } Re_2 \leq 30,000, \\
0.102 \left( \frac{2 \epsilon}{D_2^2} \right)^{0.1} \frac{Re_2^{0.2}}{\nu_2^2} & \text{otherwise.}
\end{cases} \quad (53)$$

Aungier in [12] came up with disk friction loss model, which assumes an empirical coefficient correction to Equation (52). It is based on extensive study and measurement of hundreds of compressor stages as it is said in [12]. The final form of the loss reads as

$$\Delta h_{df} = C_{MD} \rho_2 U_2^3 D_2^2 \frac{L_3^2}{8\eta}. \quad (54)$$

The coefficient $C_{MD}$ can be obtained as

$$C_{MD} = 0.8 C_M, \quad (55)$$

where the torque coefficient $C_M$ is estimated as maximum value over four flow regimes for which Daily and Nece derived expressions in [39]. This process is detailed in [40].

Boyce in [41] suggested disk friction loss to be computed with a clearance gap neglected and formulated disk friction loss based on experimental work of Watabe in [42,43] as

$$\Delta h_{df} = K_f \left( 1 + \frac{P_2}{P_1} \right) \frac{\Delta h_{eu} C_{12} D_2^2}{4 U_2 D_1 s} \left( 1 - \frac{D_2^2}{D_1^2} \right), \quad (56)$$

where he used $K_f$ from Equation (53).

3.3. Vaneless Diffuser Loss

A vaneless diffuser is an integral part of every radial compressor. It is located directly after the impeller, and it is a space where the flow continually slows down. The losses in a vaneless diffuser are occurring mainly due to friction and diffusion.

Stanitz developed the mass, momentum and energy conservation for vaneless diffusers in [27]. The flow solution is obtained by the integration of those equations and therefore the relation for enthalpy loss in a vaneless diffuser can be written as

$$\Delta h_{vld} = c_p T_{12} \left[ \left( \frac{P_3}{P_{13}} \right)^{\frac{\gamma-1}{\gamma}} - \left( \frac{P_3}{P_{12}} \right)^{\frac{\gamma-1}{\gamma}} \right]. \quad (57)$$

Coppage in [8] simplified the general equations into semi-empirical alternation of the relation between velocity and radius to help account for the compressibility. Furthermore, the relation between Mach number and radius is sufficiently approximated. Lastly, a constant profile of impeller outlet velocity is assumed. The vaneless diffuser loss was derived as

$$\Delta h_{vld} = 2 \left( \frac{D_2}{D_3} \right)^{\gamma} \left( \frac{C_f D_2}{8 b_2} \right) \frac{C_2^2}{\cos^2(\alpha_2)}. \quad (58)$$
4. Subsonic Eckardt’s Compressors

Unlike simple design algorithms presented e.g., by Kovar in [44] and Schiff in [45], which require only limited information about the impeller, presented off-design performance prediction requires knowledge of full three-dimensional compressor geometry.

Two well described compressors were chosen as test cases–Eckardt’s impeller type O with radial blades at the outlet and backswept type A. A full description of both geometries can be found in [20,46]. The main dimensions of both compressors are shown in following sketches in Figure 4. The vaneless diffuser radius is 338 mm as noted in [21].

In literature, these geometries are usually used in a simplified form with chosen blade thickness distribution and heel radii are not taken into account as in [47,48]. In the presented paper, however, original blade thicknesses were obtained from velocity distribution figures in [20,21] at individual cross-sections of compressors as indicated in Figure 4. These points were approximated using NURBS [49] as it can be seen in Figure 5.

**Figure 4.** A detailed sketch of Eckardt’s compressors (both type A and O) with their corresponding dimensions: (left) Main dimensions of the impeller; (right) position of individual cross-sections.

**Figure 5.** Eckardt type O blade thickness reconstruction: (left) Span-wise thickness distribution; (right) stream-wise thickness distribution.
These approximations are then interpolated using NURBS in order to obtain continuous thickness distribution along blade height. Close to heel radii, thickness distribution was approximated instead, as it is shown in Figure 5. Finally, reconstructed geometry of studied compressors can be seen in Figure 6.

Figure 6. Studied compressor’s geometry: (left) Eckardt O; (right) Eckardt A.

5. Algorithm Input Parameters

For the purpose of one-dimensional off-design performance prediction, some geometrical parameters need to be determined. In Table 1, there is an input parameters list of Eckardt’s compressors. Geometrical angles of blade $\beta_{1h,\xi,s}$ are defined in the same manner as shown in Figure 1.

Parameter $L$ is the blade camber line length, and $z$ stands for number of blades. Blade thickness at the impeller inlet is marked as $t_1$.

Table 1. Algorithm input parameters.

| Parameter          | RPM  | $m$  | $D_{1h}$ | $D_{1\xi}$ | $D_{1s}$ | $\chi_{1h}$ | $\chi_{1\xi}$ | $\chi_{1s}$ |
|--------------------|------|------|----------|------------|----------|-------------|-------------|-------------|
| Unit               | [min$^{-1}$] | [kg $\cdot$ s$^{-1}$] | [mm] | [mm] | [mm] | [°] | [°] | [°] |
| Type O             | 14,000 | 5.31 | 90       | 208        | 280      | 5.6         | 2.1         | 0           |
| Type A             | 14,000 | 4.54 | 120      | 215.4      | 280      | 12.4        | 5           | 0           |

| Parameter          | $\kappa_{1h}$ | $\kappa_{1\xi}$ | $\kappa_{1s}$ | $z$ | $\beta_{1h}$ | $\beta_{1\xi}$ | $\beta_{1s}$ | $\beta_2$ |
|--------------------|---------------|-----------------|----------------|----|--------------|-----------------|--------------|----------|
| Unit               | [m$^{-1}$]    | [m$^{-1}$]      | [m$^{-1}$]     | [1] | [°]         | [°]             | [°]         | [°]       |
| Type O             | 0.0071        | 0.0027          | 0              | 20 | 31.7        | 55             | 62.6        | 0         |
| Type A             | 0.0071        | 0.0029          | 0              | 20 | 39.3        | 56             | 62.6        | 30        |

| Parameter          | $t_1$ | $\epsilon$ | $D_2$ | $b_2$ | $L_z$ | $L_m$ | $L$ | $D_3$ |
|--------------------|-------|-------------|-------|-------|-------|-------|-----|-------|
| Unit               | [mm]  | [mm]        | [mm]  | [mm]  | [mm]  | [mm]  | [mm]| [mm]  |
| Type O             | 2.7   | 0.5         | 400   | 26    | 130   | 173.9 | 204.8 | 676   |
| Type A             | 3.4   | 0.5         | 400   | 26    | 130   | 169.3 | 204.8 | 676   |
6. Loss Model Suitability Evaluation

In order to evaluate the best loss model set, all possible combinations of individual empirical loss correlations described in Section 3 are taken into account. These loss model sets are then used in algorithm from Section 2 to evaluate compressor performance. This section presents a method for evaluating correctness of these loss model sets by comparing them to experimental results obtained from [20,21]. Performance parameters measured by Eckardt, specifically pressure ratio \( PR_{\text{exp}} \) and efficiency \( \eta_{\text{exp}} \), are then compared with calculated outputs of the presented one-dimensional algorithm \( PR_{\text{calc}} \) and \( \eta_{\text{calc}} \). Individual errors in \( PR \) and \( \eta \) are computed as Euclidean distance between experimental and calculated performance parameters. Average relative error of an individual loss model set in pressure ratio \( E_{PR}^s \) is then computed as follows:

\[
E_{PR}^s = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left( \frac{PR_{\text{exp},i} - PR_{\text{calc},i}}{PR_{\text{exp},i}} \right)^2}, \quad i = \{1, 2, ..., N\}, s = \{1, 2, ..., M\}, \quad (59)
\]

where \( i \) stands for index of mass flow point, and \( s \) denotes the index of a loss model set with overall set count \( M = 4608 \), which are all possible combinations of all individual loss models from Section 3 and are summed up later in Table 2. In order to compare individual errors in efficiency and pressure ratio, efficiency error was scaled with ratio \( K^s \)

\[
E_{\eta}^s = \left( \frac{\eta_{\text{exp},i} - \eta_{\text{calc},i}}{\eta_{\text{exp},i}} \right)^2, \quad K^s = \max_i \left( \frac{PR_{\text{exp},i} - PR_{\text{calc},i}}{PR_{\text{exp},i}} \right) \max_i \left( \frac{\eta_{\text{exp},i} - \eta_{\text{calc},i}}{\eta_{\text{exp},i}} \right), \quad (60)
\]

For all loss models and both geometries normalized relative error, \( E_{\text{norm},k}^s \) is calculated in the following manner:

\[
E_{\text{norm},k}^s = \left( \sqrt{E_{PR}^s + E_{\eta}^s} \right)^k, \quad k = \{O, A\}, \quad (61)
\]

where index \( k \) refers to compressor type O or type A. In an effort to find a loss model set, which describes both compressors’ off-design performance the best, relative error of a loss model set averaged over both geometries \( E_{\text{norm}}^s \) is evaluated using

\[
E_{\text{norm}}^s = \sqrt{E_{\text{norm},O}^s + E_{\text{norm},A}^s}. \quad (62)
\]

Table 2. Individual loss model correlations from Section 3 sorted according to physical origin of the loss. This gives 4608 possible combinations of different loss model sets.

| Loss Origin | Correlation According to |
|-------------|--------------------------|
| Incidence (4) | Aungier [12], Conrad [28], Gravdahl [29], Stanitz [27] |
| Choke (1) | Aungier [12] |
| Entrance diffusion (1) | Aungier [12] |
| Blade loading (2) | Aungier [12], Coppage [8] |
| Skin friction (3) | Coppage [8], Jansen [32], Aungier [12] |
| Clearance (4) | Aungier [12], Jansen [32], Krylov and Spunde [33], Rodgers [34] |
| Mixing (2) | Aungier [12], Johnston [35] |
| Recirculation (3) | Coppage [8], Coppage [8], Oh [13] |
| Disk friction (4) | Aungier [12], Boyce [41], Daily [39], Shepherd [38] |
| Vaneless diffuser (2) | Coppage [8], Stanitz [27] |
7. Results

In this section, results of previously discussed methodology and empirical correlations for individual loss mechanisms in the centrifugal compressor in comparison with experimental data by Eckardt [20,21] are presented.

Eckardt performed these experiments on a test rig where the centrifugal compressor was driven by a DC-motor with a maximum power input of about 1500 kW. The measured quantities necessary to determine $PR$ and $\eta$ experimentally were total pressures $p_{t0}$, $p_{t3}$ and total temperatures $T_{t0}$ and $T_{t3}$. Eckardt measured the ambient quantities marked as $0$ at a sufficient distance in front of the impeller using conventional total pressure and total temperature probes. The quantities at the vaneless diffuser outlet marked as $3$ were measured differently due to the instantaneous (time-dependent) flow pattern behind the impeller. These instantaneous measurements were time-weighted by Eckardt using a special pneumatic probe described in [50]. Eckardt then reduced the indicated mean pressure to the time-weighted value by means of a correlation method, since the dynamic part of the oscillating pressure at the same measuring point was known. Estimated errors during these measurements were guaranteed to be less than 1%.

7.1. Loss Model

In order to find the best fitting loss model set, all possible combinations of individual empirical loss correlations as they are described in Section 3 are taken into account. A summary of empirical loss correlations from this section is presented in Table 2.

These loss model sets are then used in an algorithm from Section 2 to evaluate compressor performance. The evaluated performance criteria $PR$ and $\eta$ are compared to experiments performed by Eckardt using the error evaluation as described in Section 6. Experimental performance parameters of considered compressors are taken from measured compressor maps published in [20,21] and rotational speed of 14,000 min$^{-1}$ is chosen, with mass flow from ranges $\langle 4.3; 6.09 \rangle$ for type O and $\langle 3.78; 6.75 \rangle$ in case of type A with $N = 6$ points for both geometries. Mass flow points were determined in two different ways. Firstly, three mass flow points for the O-geometry and two mass flow points for the A-geometry are explicitly mentioned in [20,21], respectively. Secondly, three points for the O-geometry and four points for the A-geometry are obtained using intersections of pressure ratio lines at constant RPM with efficiency isocurves presented by Eckardt in [20,21]. As it can be seen in Figure 7, a wide variety of loss model sets show higher normalized error and only a few are acceptable.

Figure 7. Histograms of loss model sets split according to their deviation from experiment, which is evaluated using Equation (61): (left) Eckardt O; (right) Eckardt A.
Based on described error estimation, loss model set No. 1759 reports the lowest error norm for O-geometry. On an A-geometry loss model, set No. 2042 turns out to be the most successful. It is however quite inaccurate for O-geometry, where it is ranked far behind the best sets as it can be seen from Figure 7. The most successful set according to described ranking method is No. 2137. As Figure 7 shows, this set ranks among the best on both geometries.

Figure 8 shows reported errors of mentioned loss model sets in comparison with loss model sets taken from [13,17].

![Loss model set comparison](image)

Figure 8. Bar graph of error norms for candidate loss model sets. Error evaluation is defined by Equations (61) and (62).

Individual empirical loss correlations used in the most successful loss model sets are listed in Table 3.

Table 3. Breakdown of individual loss models for candidate loss model sets.

| Loss                  | Set No. 1759 | Set No. 2042 | Set No. 2137 | Set Oh [13] | Set Zhang [17] |
|-----------------------|--------------|--------------|--------------|-------------|----------------|
| Incidence             | Conrad [28]  | Conrad [28]  | Conrad [28]  | Conrad [28] | Aungier [12]   |
| Choke                 | Aungier [12] | Aungier [12] | Aungier [12] | ×           | Aungier [12]   |
| Entrance diffusion    | Aungier [12] | Aungier [12] | Aungier [12] | ×           | Aungier [12]   |
| Blade loading         | Aungier [12] | Aungier [12] | Aungier [12] | Coppage [8] | Aungier [12]   |
| Skin friction         | Cappage [8]  | Jansen [32]  | Aungier [12] | Jansen [32] | Jansen [32]    |
| Clearance             | Jansen [32]  | Rodgers [34] | Jansen [32]  | Jansen [32] | Jansen [32]    |
| Mixing                | Aungier [12] | Aungier [12] | Aungier [12] | Johnston [35]| Aungier [12]   |
| Recirculation         | Cappage [8]  | Cappage [8]  | Cappage [8]  | Oh [13]     | Cappage [8]    |
| Disk friction         | Boyce [41]   | Shepherd [38]| Shepherd [38]| Daily [39]  | Daily [39]     |
| Vaneless diffuser     | Stanitz [27] | Cappage [8]  | Stanitz [27] | Stanitz [27]| Stanitz [27]   |

7.2. Performance of Candidate Loss Model Sets

Performance prediction results using the most suitable loss correlation sets for chosen geometries are visualized in Figures 9 and 10. Results of loss model sets presented by other authors in [13,17] are also shown for comparison. Loss model set from Zhang ranks better than Oh, since it does an excellent job on O-geometry and reports almost the same error on A-geometry. The biggest disadvantage of an Oh loss model set is that it does not incorporate choke and entrance diffusion losses from [12], therefore largely overestimating efficiency near choke point. It can be seen that efficiency estimation is a very complex task, mainly for higher mass flows i.e., close to choking point of the compressor. Even though
some loss correlations for choke loss exist and were also used in the loss model sets, they fail to capture this phenomenon precisely. The main reason for this to happen is that the aerodynamic choke point depends on more parameters (geometrical and other) than loss correlations usually work with. Pressure ratio is generally captured more accurately. In the case of O-geometry, it is apparent that loss model set No. 1759 outperformed other sets in prediction of both performance parameters. It however does a poor job in predicting the pressure ratio for A-geometry. Such results are very similar to those of the Zhang loss model set, but in terms of error norms, it outperforms it in every way. Set No. 2042 does the best job on A-geometry, predicting both efficiency and pressure ratio with the lowest error. On O-geometry, however, it is way off, since it dramatically overestimates both performance parameters. The go-to loss model set according to the presented ranking method would be set No. 2137. Among the five investigated sets, it ranks 3rd on O-geometry and 2nd on A-geometry while having the lowest $E_{\text{norm}}$ out of all sets.

Figure 9. Eckardt O compressor map ($\text{RPM} = 14,000 \text{ min}^{-1}$) for chosen loss model sets in comparison to experimental data: (left) Efficiency; (right) pressure ratio.

Figure 10. Eckardt A compressor map ($\text{RPM} = 14,000 \text{ min}^{-1}$) for chosen loss model sets in comparison to experimental data: (left) Efficiency; (right) pressure ratio.

Figures 11 and 12 show compressor maps for a complete range of RPM, showing a comparison between loss correlation sets from Table 3 and experimental data measured by Eckardt in [20,21] using both geometry types.
Sets No. 1759 and 2137 are in good agreement with experimental data. The most difficult part for all loss model sets to cope with is the case of A-geometry with higher RPM (14 and 16k). To reiterate some statements from a previous paragraph, loss set No. 1759 reports similar behaviour to the Zhang [17] loss model set, while being closer to experimental data in most cases. The go-to loss model set No. 2137 shows results in very good agreement with experiment on O-geometry, for some RPM even better in efficiency prediction than set No. 1759.

Figure 11. Eckardt O compressor map for chosen loss model sets in comparison to experimental data: (left) Efficiency; (right) pressure ratio.

Figure 12. Eckardt A compressor map for chosen loss model sets in comparison to experimental data: (left) Efficiency; (right) pressure ratio.
8. Conclusions

This paper presents a methodology and the algorithm for subsonic centrifugal compressors’ off-design performance prediction based on empirical loss correlations. A great emphasis is put on an extensive literature review of empirical loss correlations and the physical origin of loss mechanisms in centrifugal compressors. These loss mechanisms are described in detail, which is necessary for understanding individual formulas from various authors with the main idea of empirical formulas summed up.

Two Eckardt’s subsonic centrifugal compressors are presented and described. These compressors were chosen since they are well-described and a lot of information is available in literature. The topic of using these compressors in their simplified form, i.e., with constant thicknesses, is discussed. The issue of proper geometry reconstruction using NURBS is also outlined. Geometrical input parameters for the performance prediction algorithm are obtained and displayed in the table for further clarity.

The composition process of the new suitable loss models is described. The search process is composed of varying previously mentioned empirical loss correlations and an error estimation method. Compressor maps were computed for both compressors and 4608 loss model sets. This process led to acquisition of three candidate loss model sets No. 1759, 2042 and 2137. Results were presented in compressor maps for both compressors at four different speed lines. This includes maps for experimental data sets, loss model sets defined by Oh in [13], Zhang in [17] and the candidate loss model sets.

Firstly, set No. 1759 outperformed all sets on O-geometry, where its relative error value is 0.0104. On the other hand, set No. 2042 outperformed all sets on A-geometry, where its relative error value is 0.0278. Lastly, No. 2137 came out as a reasonable compromise between the two compressors which predicts both off-design performances very well with combined relative error value 0.0353. For comparison, Zhang’s and Oh’s combined relative error values were 0.0428 and 0.0479, respectively. These values indicate that our set No. 2137 outperforms both sets from literature for the tested geometries. More precisely, our set predicts off-design performance 21.2% better than Zhang’s set and a 36.7% better than Oh’s set. That is a significant improvement in off-design performance prediction of centrifugal turbomachines, which can lead to enhancement in turbomachine efficiency, while reducing the price for development of aerodynamic design and computational fluid dynamics simulations. The main scientific contributions of this work can be summarized as follows. An exact specification of loss model set comparison methodology is presented in Section 6, which can be re-used later, for more sets or test geometries. Using this comparison method, a new loss model set was found, which outperformed any set published in the literature on tested geometries. In addition, as to the authors’ knowledge, there exists no clear verbal explanation of the thermodynamical meaning of off-design performance algorithm code from [9]. Section 2 aims to correct that. Further work should include implementation of the off-design prediction algorithm and the newly obtained loss model sets into various applications after testing these candidate loss model sets on more geometries. The biggest aim of the following work is to create a time-efficient optimization algorithm that would optimize impellers’ shapes without any need of time-demanding computational fluid dynamics’ simulations. Another application of this existing algorithm and loss models should be an addition of the three-dimensional effects to one-dimensional and two-dimensional meridional computational fluid dynamics simulation in order to increase accuracy of these simulations.

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Nomenclature

\( A \) Area (excluding blade area)  
\( C \) Absolute velocity  
\( C_f \) Friction coefficient  
\( C_{SF} \) Slip velocity  
\( D \) Diameter  
\( D_{eq} \) Equivalent diffusion factor  
\( D_f \) Diffusion coefficient  
\( L \) Length  
\( M \) Mach number  
\( PR \) Pressure ratio  
\( Re \) Reynold’s number  
\( SF \) Slip factor  
\( T \) Thermodynamic temperature  
\( U \) Circumferential speed  
\( W \) Relative velocity  
\( a \) Speed of sound  
\( b \) Impeller channel width  
\( c_p \) Specific heat capacity at constant pressure  
\( d_h \) Hydraulic diameter  
\( h \) Enthalpy  
\( \dot{m} \) Mass flow  
\( p \) Pressure  
\( r \) Specific gas constant  
\( t \) Blade thickness  
\( z \) Number of blades  
\( \alpha, \beta \) Flow angles (absolute, relative), respectively  
\( \gamma \) Specific heat ratio  
\( \Delta \) Variable change  
\( e \) Blade tip clearance  
\( \zeta \) Wake width  
\( \eta \) Efficiency  
\( \kappa \) Curvature  
\( \lambda \) The ratio of the vaneless diffuser inlet width to the impeller exit width  
\( \mu, \nu \) Viscosity (dynamic, kinematic), respectively  
\( \rho \) Density  
\( \chi \) Streamline angle  
\( ( )_{eu} \) Theoretical parameter  
\( ( )_f \) Flow parameter  
\( ( )_{int,par} \) Losses (internal, parasitic/external), respectively  
\( ( )_{in,ch,ed,bl,cl,mix} \) Internal losses (incidence, choke, entrance diffusion, blade loading, skin friction, clearance, mixing), respectively  
\( ( )_m \) Meridional parameter  
\( ( )_{rc,df} \) External losses (recirculation, disk friction), respectively  
\( ( )_{sh,h,\xi} \) Blade positions (shroud, hub, root mean square), respectively
Total condition

Throat parameter

Tangential parameter

Vaneless diffuser

Axial direction

Positions of interest (Ambient atmosphere, Impeller inlet, Impeller outlet/vaneless diffuser inlet, Vaneless diffuser outlet), respectively

Blade parameter

Variable in the relative system

Sonic condition

References

1. Dalbert, P.; Ribi, B.; Kmecl, T.; Casey, M.V. Radial compressor design for industrial compressors. Proc. Inst. Mech. Eng. Part C J. Mech. Eng. Sci. 1999, 213, 71–83. [CrossRef]

2. Cumpsty, N. Compressor Aerodynamics; Krieger Publishing Company: Malabar, FL, USA, 2004.

3. Farokhi, S. Compressor Aerodynamics; John Wiley & Sons Ltd.: Hoboken, NJ, USA, 2014.

4. Xu, C. Centrifugal compressor design considerations. In Proceedings of the Fluids Engineering Division Summer Meeting, Miami, FL, USA, 17–20 July 2006; Volume 47519, pp. 217–225.

5. Barbosa, J. Streamline Curvature Computational Programme for Axial Compressor Performance Prediction; Cranfield University: Cranfield, UK, 1987.

6. Peyvan, A.; Benisi, A. Axial-Flow Compressor Performance Prediction in Design and Off-Design Conditions through 1D and 3D Modeling and Experimental Study. J. Fluids Eng. 2016, 9, 2149–2160.

7. Li, T.; Wu, Y.; Ouyang, H.; Qiang, X. Axial compressor performance prediction using improved streamline curvature approach. In Turbo Expo: Power for Land, Sea, and Air; American Society of Mechanical Engineers: New York, NY, USA, 2018; Volume 51012, p. V02CT42A010.

8. Coppage, J.; Dallenbach, F. Study of Supersonic radial Compressors for Refrigeration and Pressurization Systems; Technical Report; Garrett Corp Los Angeles CA AiResearch MFG DIV: Los Angeles, CA, USA, 1956.

9. Galvas, M.; NASA Lewis Research Center Cleveland. FORTRAN Program for Predicting Off-Design Performance of Centrifugal Compressors; NASA Technical Note; National Aeronautics and Space Administration: Washington, DC, USA, 1973.

10. Thanapandi, P.; Prasad, R. Performance prediction and loss analysis of low specific speed submersible pumps. Proc. Inst. Mech. Eng. Part A J. Power Energy 1990, 204, 243–252. [CrossRef]

11. Whitfield, A.; Baines, N.C. Design of Radial Turbomachines; John Wiley and Sons Inc.: New York, NY, USA, 1990.

12. Aungier, R.H. Mean Streamline Aerodynamic Performance Analysis of Centrifugal Compressors. J. Turbomach. 1995, 117, 360–366. [CrossRef]

13. Oh, H.W.; Yoon, E.S.; Chung, M.K. An optimum set of loss models for performance prediction of centrifugal compressors. Proc. Inst. Mech. Eng. Part A J. Power Energy 1997, 211, 331–338. [CrossRef]

14. Gravdahl, J.T.; Willems, F.; De Jager, B.; Egeland, O. Modeling of surge in free-spool centrifugal compressors: Experimental validation. J. Propuls. Power 2004, 20, 849–857. [CrossRef]

15. Li, P.; Gu, C.; Song, Y. A new optimization method for centrifugal compressors based on 1D calculations and analyses. Energies 2015, 8, 4317–4334. [CrossRef]

16. Velásquez, E.I.G. Determination of a suitable set of loss models for centrifugal compressor performance prediction. Chin. J. Aeronaut. 2017, 30, 1644–1650. [CrossRef]

17. Zhang, C.; Dong, X.; Liu, T.; Sun, Z.; Wu, S.; Gao, Q.; Tan, C. A method to select loss correlations for centrifugal compressor performance prediction. Aerosp. Sci. Technol. 2019, 93, 105335. [CrossRef]

18. Blasius, H. The Boundary Layers in Fluids with Little Friction; National Advisory Committee for Aeronautics: Moffett Field, CA, USA, 1950.

19. Prandtl, L. Bericht über Untersuchungen zur ausgebildeten Turbulenz. ZAMM-J. Appl. Math. Mech. Angew. Math. Mech. 1925, 5, 136–139. [CrossRef]

20. Eckardt, D. Flow field analysis of radial and backswept centrifugal compressor impellers. I—Flow measurements using a laser velocimeter. In Proceedings of the ASME Twenty-Fifth Annual International Gas Turbine Conference and Twenty Second Annual Fluids Engineering Conference on Performance Prediction of Centrifugal Pumps and Compressors, New Orleans, LA, USA, 9–13 March 1979; pp. 77–86.

21. Eckardt, D. Instantaneous Measurements in the Jet-Wake Discharge Flow of a Centrifugal Compressor Impeller; American Society of Mechanical Engineers: New York, NY, USA, 1975.

22. Sutherland, W. LII. The viscosity of gases and molecular force. Lond. Edinb. Dublin Philos. Mag. J. Sci. 1893, 36, 507–531. [CrossRef]

23. Stodola, A. Steam and Gas Turbines: With a Supplement on the Prospects of the Thermal Prime Mover; McGraw-Hill: New York, NY, USA, 1927; Volume 2.
