Explanation of the Tao effect

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In a series of experiments Tao and coworkers[1, 2, 3] found that superconducting microparticles in the presence of a strong electrostatic field aggregate into balls of macroscopic dimensions. No explanation of this phenomenon exists within the conventional theory of superconductivity. We show that this effect can be understood within an alternative electrodynamic description of superconductors recently proposed that follows from an unconventional theory of superconductivity. Experiments to test the theory are discussed.

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The phenomenon was discovered by Tao and coworkers in 1999 in samples of high T_c superconducting materials, and initially attributed to special properties of high T_c cuprates, in particular their layered structure[4]. However, subsequent experiments reported in 2002 for conventional superconducting materials (Pb, V, V_3Ga, Nb - N and Nb_3Sn) all showed the same behavior[5, 6], and the same behavior was observed in MgB_2[7]. Briefly the remarkable observation is: when the applied electric field exceeds a critical value, typically of order 0.5-1 kV/mm, millions of superconducting microparticles each of size ~ 5µm spontaneously aggregate into spherical balls of size of order mm. As the electric field increases further the size of the balls decreases, and above a second critical field, of order 1.5-2 kV/mm the balls disintegrate and the microparticles fly to the electrodes and cling to them. The conventional theory of superconductivity predicts that superconductors respond to applied electrostatic fields in the same way as normal metals do[8, 9]. Because normal metallic microparticles do not aggregate into spherical balls upon application of electrostatic fields nor do they cling to the electrodes, Tao’s observation represents a fundamental puzzle within the conventional understanding of superconductivity. For high T_c materials, Tao’s findings have been independently confirmed[10].

We have proposed an unconventional theory of superconductivity to describe both high T_c[11] and conventional superconductors[12] that is based on the fundamental charge asymmetry of matter[13]. This theory leads to a new description of the electrodynamic properties of superconductors[14]. Here we show that this new formulation provides an explanation for the Tao effect.

In this theoretical framework, negative charge is expelled from the interior of the material towards the surface when the transition to superconductivity occurs. The resulting charge density obeys the differential equation

\[ \rho(\vec{r}) = \rho_0 + \lambda_L^2 \nabla^2 \rho(\vec{r}) \]  

(1)

in the interior of the superconductor, with \( \lambda_L \) the London penetration depth and \( \rho_0 \) a positive constant that is function of the parameters of the material and of the dimensions of the sample. Eq. (1) predicts that the charge density is \( \rho_0 \) deep in the interior of the superconductor. By charge neutrality the average charge density is negative near the surface (= \( \rho_- \)) and is approximately related to \( \rho_0 \) by \( \rho_- \lambda_L = -\rho_0 V \) with \( S \) and \( V \) the surface area and volume of the body. Energetic arguments show that \( \rho_- \) is independent of the volume of the sample for samples of dimensions much larger than the London penetration depth. The electrostatic potential \( \phi \) obeys the equation

\[ \phi(\vec{r}) = -4\pi \lambda_L^2 \rho(\vec{r}) + \phi_0(\vec{r}) \]  

(2)

in the interior of the superconductor, with \( \phi_0 \) the electrostatic potential due to a uniform positive charge density \( \rho_0 \). Justification of these equations is given in Ref.[10].

In the exterior, \( \phi(\vec{r}) \) obeys Laplace’s equation \( \nabla^2 \phi = 0 \), and furthermore we assume that \( \phi \) and its normal derivative are continuous at the surface of the superconductor, hence that no surface charge exists.

Under these conditions a unique solution for the charge distribution and electrostatic potential exists for given sample geometry. For samples of high symmetry (e.g. spherical, infinite cylinder or infinite plane) no electric field exists in the exterior of the superconductor. However for samples of general shape these equations predict that ‘spontaneous’ electric fields exist outside the sample near the surface.

Consider samples of ellipsoidal shape. Figure 1 shows the electric field lines obtained for prolate and oblate ellipsoids of eccentricity \( |e| \) = 0.745, corresponding to ratio of axis \( b/a = 1.5 \) and \( a/b = 1.5 \) respectively, and London penetration depth \( \lambda_L = 0.2 \) in units where \( a^2 b = 1.5 \). For other values of the penetration depth the results are very similar. In both cases electric field lines go out in the region of low surface curvature and go in in regions of high surface curvature. We find the same behavior in samples of other shapes. The magnitude of these quadrupolar electric fields increases as the eccentricity increases.

Examples of the charge distribution inside the superconductor that gives rise to these electric fields are given in ref.[11]. The reason that macroscopic charge inhomogeneity and differences in electric potential in the interior of superconductors can exist is because superfluid electrons will compensate for the difference in potential energy in different regions with corresponding changes.
in their kinetic energy\cite{10}.

Figure 2 shows the behavior of the electrostatic potential on the surface of the ellipsoids of Fig. 1. In the prolate (oblate) case the maximum potential occurs at the equator (poles). Fig. 2 also shows the potential for a quadrupole moment at the origin of the magnitude obtained from integration over the interior charge distribution

\[ Q = \int_V d^3r \rho(\vec{r})(3z^2 - r^2) \] (3)

At distances further away the actual potential rapidly approaches the one generated by a pure quadrupole of value given by Eq. (4), as seen in Fig. 2.

We calculate the electrostatic energy associated with these electric fields and charge distribution

\[ U_E = \int d^3r \frac{\vec{E}^2}{8\pi} = \frac{1}{2} \int_V d^3r \rho(\vec{r})\phi(\vec{r}) \] (4)

with $\vec{E} = -\vec{\nabla}\phi$. The first integral is over all space, the second over the volume of the sample. The contribution to the electrostatic energy associated with fields outside the sample is given by the surface integral

\[ U_{out} = -\int_S dS \frac{\partial\phi}{\partial n} \] (5)

and the energy from the electric field inside of the sample is $U_{in} = U_E - U_{out}$. We assume that for samples of different shapes and fixed volume, $\rho_-$ is constant, and calculate these energies for ellipsoids of fixed volume and varying eccentricity. Results are shown in Fig. 3. The minimum energy occurs for a spherical shape, where only an electric field in the interior exists. As the ellipsoid deviates from spherical shape the energy due to the outside electric fields increases rapidly, while the one from the interior electric field decreases somewhat, giving rise to an increase in the total electrostatic energy which goes linearly with the increase in surface area of the sample and coefficient close but somewhat larger than unity.

We infer from these results that there is an energetic advantage for superconducting bodies to adopt a spherical shape, in order to eliminate the electric fields outside the sample that raise the electrostatic energy. This is similar to the effect of the surface tension in liquids, which causes liquid drops to be spherical. However because superconducting bodies unlike liquid drops have a rigid structure they cannot spontaneously deform into spherical shapes.

Consider now the effect of an applied electrostatic field. The phenomenon of electric field induced coalescence of conducting or dielectric fluid droplets is well known\cite{12, 13}. For such liquid drops in suspension, application of a uniform electrostatic field causes an induced dipole moment in droplets and an attraction between droplets along the field direction. Upon contact, the surface tension deforms the elongated droplet into a spherical droplet of larger size than the original ones.

We can envisage a similar scenario for superconducting microparticles. Figure 4a shows four spherical microparticles aligned due to the application of an electrostatic field. When they come into contact, charge will redistribute to conform to the new shape, and the electro-
static energy will increase due to the generation of the external electric fields. We can approximate the resulting electric field by that corresponding to a prolate ellipsoid of aspect ratio $b/a = 4$, with a 28% larger surface area than the sphere of equal volume.

This physics clearly favors clustering of microparticles into spherical shapes as seen in the Tao experiments, aided by an applied electric field that will induce dipole moments on the microparticles causing them first to align as in Fig. 4a. However we believe there is more to the Tao phenomenon than this. Note that in the electrical coalescence of fluid droplets the size distribution changes so that the average drop size increases somewhat, however there is not an aggregation of a large number of droplets into a single large drop. We believe the much stronger Tao phenomenon requires the existence of an actual force between particles that are not in contact that can act not only in direction parallel to the applied electric field (as occurs due to the induced electric dipoles) but also in direction perpendicular to the field, so as to favor a compact arrangement. We now show that such a force indeed exists in our case.

In fact, the first question to ask is: why doesn’t aggregation of spherical microparticles occur even in the absence of an applied electric field? The predicted electrostatic fields around nonspherical particles shown in Fig. 1 should give rise to an attractive force between close-by microparticles in the proper relative orientation, namely high curvature region of one particle close to low curvature region of another. Only if the microparticles are perfectly spherical will no electric field exist in the exterior, so why then don’t microparticles of random shapes spontaneously aggregate to form spherical arrangements?

We believe the answer to this question is that in fact in the absence of an applied electric field a microparticle cannot sustain differences in electrostatic potential between two surface points larger than the work function of the material. If such large potential difference on the surface exists it will become energetically advantageous for electrons to ‘pop out’ of the superconducting condensate in the region of low electric potential and migrate to the region of high potential, and the resulting electronic layer outside the superconductor will screen the outgoing electric field lines. This then implies that electric field lines that start and end at points on the surface with electric potential difference larger than the work function, typically a few electron volts, will be screened. We estimated the magnitude of the electric fields near the surface of superconductors at about $10^6\text{V/cm}$, which for a microparticle of dimension $5\mu$ and eccentricity as in Fig. 1 would give a maximum potential difference between points on the surface of about $25\text{V}$, much larger than the work function. We conclude that such particles will be ‘coated’ by a layer of charge on the surface so that the electric field in the exterior will be screened out.

We propose then that the role of the applied electric field in Tao’s experiment is to remove the electronic ‘coating’ that hides these electric fields so that they become observable. The coating electrons have already payed the work function price, i.e. they are outside the surface ‘double layer’ that gives rise to the work function potential. Hence they can be removed by an applied electric field strong enough to overcome the force due to the electric field of the superconductor. As these coating electrons are removed, the electric field lines shown in Fig. 1 start to become visible and give rise to forces between microparticles.

We assume then that the applied electric field has removed the electronic coating, and calculate the total electric field around a microparticle, which is a superposition of the external plus internal field plus the field generated by the electric dipole that is induced as for a normal metallic particle. The differential equation to be solved is still Eq. (2), except that now the boundary condition for the potential changes. The electrostatic potential is given by

$$\phi(\vec{r}) = -E\hat{z} + \int_V \frac{d^3\rho(\vec{r}')}{|\vec{r} - \vec{r}'|}$$

with $E$ the applied electric field along the $z$ direction. The numerical procedure is analogous to that described in Ref. [11].

Figure 5 shows electric field lines for a prolate and an oblate ellipsoid in the presence of an applied electric field in the vertical direction for two values of the applied field strength. In the units used, the maximum spontaneous electric field that we obtain at the surface is about unity; hence the applied fields in Fig. 5 correspond to 420 V/mm and 1250 V/mm. As the electric field increases the field lines that start and end on the sample get eliminated. Note that some electric field lines come out and go in the ellipsoids in directions perpendicular to the applied field, due to the effect of the spontaneous fields. As a consequence a prolate and an oblate ellipsoid at short distance from each other arranged in the orientation shown in Fig. 5 will experience a strong attractive force in direction perpendicular to the applied electric field. An attractive
yields a force between microparticles of order $\sim 10^{-2} \text{dyn}$, much smaller than the force that we estimate to hold the microparticles together, so indeed the balls should be able to survive such impacts.

Furthermore, once the applied electric field has removed the ‘coating layer’ on the surface of the microparticles, the spontaneous electric field of the microparticles will exert a strong force on electric charges in the electrodes, causing microparticles to ‘cling’ to the electrodes as observed by Tao et al. This ‘clinging’ should increase as the applied electric field and hence the charge on the electrodes increases and exist also beyond the field strength where the aggregation into balls no longer occurs, as observed experimentally.

Experimental test of our scenario should be possible. For particles of sufficiently small size ($< 0.5 \mu$) differences in the electrostatic potential on the surface should be smaller than the work function, hence screening of the exterior electric fields should not occur and spontaneous aggregation should occur in the absence of an applied electric field. For larger particles, it should be possible to measure the resulting field configuration in the neighborhood of the surface upon application of a strong electric field that ‘uncoats’ the particles, and consistency with the predictions of our theory can be checked by solution of the differential equation (2). In an inhomogeneous applied strong electric field, the force and torque acting on a particle will undergo distinct changes between the normal and superconducting state that can be calculated for given shape and orientation of the particle.

Normal metal microparticles under applied electric fields align in elongated arrangements due to induced electric dipoles but do not form spherical balls. The conventional London-BCS theory of superconductivity predicts that the static dielectric response function of superconductors is the same as that of normal metals[5,14], however Tao’s experiment shows that the response is in fact qualitatively different. Our alternative theory was shown here to be consistent with Tao’s observations. We predict a definite relation between the shape of a superconducting particle and the magnitude and sign of resulting electric fields near its surface, a manifestation of the fundamental charge asymmetry of matter.

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