The physical content of long tensor modes in cosmology

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Abstract. We analyze the physical content of squeezed bispectra involving long-wavelength tensor perturbations, showing that these modes cannot be gauged away, except for the exact \((unphysical) limit of infinite wavelength, \(k = 0\). This result has a direct implication on the validity of the Maldacena consistency relation, respected by a subclass of inflationary models. Consequently, in the squeezed limit, as in the case of the scalar-scalar-scalar bispectrum, squeezed mixed correlators could be observed by future experiments, remaining a key channel to study Early Universe physics and discriminate among different models of inflation.

Keywords: cosmological perturbation theory, gravitational waves / theory, inflation, non-gaussianity

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1 Introduction

The study of primordial non-Gaussianity (PNG) is one of the most important avenues to probe inflation, trying to resolve the large degeneracy among different models still present after analyzing Cosmic Microwave Background (CMB) data. As well-known, the amount of non-Gaussianity in standard single-field slow-roll inflation is very tiny, being of the order of the slow-roll parameters ([1–6]), yet non-vanishing; on the other hand, a large class of multi-field theories leads to quite different predictions ([7–10]), as well more general single-field models of inflation [11–14].

The strength of non-Gaussianity $f_{NL}$ is the key parameter to quantify the phenomenon ([14]), being related to the bispectrum, which vanishes for a perfectly Gaussian field. In the case of single-field “standard inflation”, $f_{NL}$ contains in particular a local contribution, which is maximum for squeezed bispectrum triangles, where one wave-number is much shorter than the other two. In this context, one of the main results is the so-called Maldacena consistency relation ([5, 15–20]), stating that in the squeezed limit the bispectrum (for any single-field model of inflation) becomes simply a product of two power-spectra

$$\lim_{k_1 \to 0} \langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) \rangle = - (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) (n_s - 1) P_\zeta(k_1) P_\zeta(k_2),$$

where $\zeta$ is the comoving curvature perturbation,\footnote{In our convention, $\zeta$ is defined from the Ricci scalar curvature $R_S$ of an hypersurface of equation $S(x) = \text{const.}$ where $S$ is a four-dimensional scalar, according with $R_S = - \frac{4}{a^2} \nabla^2 \zeta$, at first order in perturbations (where $a$ is the scale factor defined in (2.1)).} $P_\zeta$ its power spectrum and $n_s$ the scalar spectral index. This consistency relation has been derived, using different approaches, such as path-integration ([21]), exploiting the residual symmetries of the gauge-fixed action for $\zeta$ ([22–25]), BRST symmetry ([26, 27]) and holography ([28, 29]).

A similar consistency condition is valid for any type of bispectra in (single-field) inflation, including both the curvature perturbation and the tensor modes ([5, 24, 30]). There are however models for which the consistency relation is violated. For example, it has been
explicitly shown that some inflationary models involving more than one scalar field ([31]), a non-attractor phase ([32–34]), an unstable background ([35, 36]) or breaking of space-time diffeomorphism invariance ([37–40]) violate the consistency condition. This implies that, from the phenomenological point of view, the consistency relation is a very interesting channel to study Early Universe physics, given that it links observable quantities: any deviation from it would rule out all single-field models of inflation and could imply that one of the previous assumptions is valid. The \textit{Planck} analysis of the CMB temperature and E-mode polarization provided, among the various results, the following limit on the non-Gaussianity strength for the local shape ([14]) of curvature bispectrum: \( f_{\text{NL}}^{\text{local}} = -0.9 \pm 5.1 \) at 68\% C.L. When compared to the spectral index, \( n_s = 0.9652 \pm 0.0042 \) (68\% C.L.), it is clear that the consistency relation is far from being tested from an experimental point of view.

In the last decade a debate has emerged ([41–47]) on the observability of squeezed bispectra: various groups have claimed that the consistency relations can be gauged away by a suitable rescaling of the spatial coordinates and, as a result, they cannot be considered as physical observables. In particular, the key-ingredient to cancel squeezed bispectra is the passage to the so-called Conformal Fermi Coordinates (CFC) frame ([42, 44]). The very same technique was used to cancel any squeezed \( \zeta \)-related quantity and, as a consequence, also the halo bias scale-dependence, as far as the so-called “GR-contribution” (see [48]) is concerned ([43, 49–51]). Moreover, tensor fossils ([52–61]) in single-field inflationary models have been claimed to be not genuine physical quantities [35, 42].

In this paper we argue that the gauge freedom used to cancel squeezed correlation functions is only valid if the squeezed momentum is \textit{exactly zero}. As shown in [62], when the squeezed momentum is finite, the gradient expansion restores the consistency relations. In [62] the analysis was limited only to the scalar sector, but here we argue that the same result applies to the tensor sector.

This paper is organized as follows. In section 2 we discuss the transformation of the metric components under a gauge transformations involving long-wavelength modes, for a more generic transformation than the one discussed in [62] which in particular accounts for tensor modes. In section 3 we show that under this deformed space dilations, the tensor perturbation of the metric tensor is unaffected and no shift is present for any finite value of the wave-number \( k \). In section 4 we discuss the transformation properties of a generic bispectrum under such a transformation. We conclude by summarizing our main results in section 5.

2 Deformed dilation

Let us consider a perturbed Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime\(^2\)

\[
 ds^2 = -dt^2 + a^2 \delta_{ij} dx^i dx^j + h_{\mu\nu} dx^\mu dx^\nu .
\]  

Under an infinitesimal coordinate transformation of the following type,

\[
 x^\mu \rightarrow \tilde{x}^\mu = x^\mu + \epsilon^\mu ,
\]

\(^2\)We take the spatial \( \kappa \) curvature zero for simplicity, the results could be easily extended to the case of \( \kappa \neq 0 \).
the induced change $\Delta h = \hat{h}(x) - h(x)$ in the metric perturbation (gauge transformation) is given by ([63])

$$\Delta h_{00} = 2\epsilon^0;$$
$$\Delta h_{0i} = \partial_i \epsilon^0 - a^2 \epsilon^i;$$
$$\Delta h_{ij} = -2a \omega^0 \delta_{ij} + a^2 \partial^i \epsilon^1 - a^2 \partial_j \epsilon^1.$$

(2.2)

Exploiting rotational invariance, it is convenient to decompose the metric perturbation into scalars, vectors and tensors (SVT) under SO(3); namely, we decompose $\epsilon^\mu$ according to

$$\epsilon^\mu = \left(\epsilon^0, \partial^i \epsilon + \epsilon_i^\nu\right) \quad \text{where} \quad \partial_i \epsilon_i^\nu = 0$$

(2.3)

and $h_{\mu\nu}$ as

$$h_{00} = -2\phi,$$
$$h_{0i} = a(\partial_i F + G_i),$$
$$h_{ij} = a^2 (-2\psi \delta_{ij} + \partial_i \partial_j B + \partial_j C_i + \partial_i C_j + D_{ij}) \quad \partial_i C_i = \partial_j D_{ij} = \delta_{ij} D_{ij} = 0.$$

(2.4)

In this way, we get the following standard transformation rules for linear perturbations ([63])

$$\Delta \phi = \epsilon^0,$$
$$\Delta F = \frac{1}{a} \epsilon^0 - a \dot{\epsilon},$$
$$\Delta G_i = -a \dot{\epsilon}^i,$$

$$\Delta \psi = H \epsilon^0,$$
$$\Delta B = -2\epsilon,$$
$$\Delta C_i = -\epsilon_i^V,$$
$$\Delta D_{ij} = 0.$$

(2.5)

(2.6)

Consider now the following transformation

$$\epsilon^i = \lambda x^i + \omega^i_j x^j, \quad \omega^i_i = 0,$$

(2.7)

where $\lambda$ is a constant and $\omega$ a constant $3 \times 3$ matrix, traceless by definition.\footnote{The trace part of $\omega$, one can always absorb it in $\lambda$. Indeed}

This can be interpreted as the leading contribution in a derivative expansion of $\epsilon^i$. Using rules (2.2), the change of the spatial metric perturbation is given by

$$\Delta h_{ij} = -a^2 \left(2 \lambda \delta_{ij} + \omega^i_j + \omega^i_i\right).$$

(2.8)

Thus, only the symmetric part $\omega^S$ of $\omega$ contributes to the transformed metric. Moreover, one can easily realize that the transformation (2.8) can be reproduced by the following 3-parameter family of transformations of the scalar, vector and tensor parts defined in (2.4):

$$\Delta \psi = \alpha \lambda,$$
$$\Delta B = \lambda (\alpha - 1) x^i x^j + \gamma \omega^S_{ij} x^i x^j,$$
$$\Delta C_i = (\beta - 1) \omega^S_{ij} x^j,$$
$$\Delta D_{ij} = -2 (\beta + \gamma) \omega^S_{ij},$$

(2.9)

with $\alpha, \beta, \gamma \in \mathbb{R}$. One should stress that the degeneracy in the above transformation rule is due the ambiguity of the decomposition (2.4) for the transformed metric (2.8) and it is removed as soon as the coordinates transformation (2.7) contains terms quadratic in $x^i$, or equivalently $\lambda$ and $\omega_{ij}$ becomes space-dependent (in the general case $\lambda$ and $\omega$ are functions of $x^i$). A popular choice ([42, 44, 45]) is to argue that a scalar perturbation $\psi_L$ and the
tensor perturbation $D_L$ with a very long wavelength can always be gauged away by setting $\alpha = \beta = 1$ and $\gamma = 0$, thus
\[
\Delta \psi_L = \lambda, \quad \Delta D_{ij} = 2 \omega_{ij}^S, \quad \Delta B = \Delta C_i = 0.
\] (2.10)
Besides the fact that such a choice is only one among the infinitely many possible, it works only to gauge away a genuinely constant mode which is not physical.\footnote{In Fourier basis this is equivalent to have a scalar or a tensor perturbation proportional to $\delta^{(3)}(k)$.}

Consider now to split the scalar and tensor part of the metric perturbation in their long and short parts
\[
\psi = \psi_L + \psi_S, \quad D_{ij} = D_{ij}^{(L)} + D_{ij}^{(S)},
\] (2.11)
by using a suitable window function $W(k) = W_k$ such that
\[
\psi_L(x) = \frac{1}{(2 \pi)^3} \int d^3 k e^{i \mathbf{k} \cdot \mathbf{x}} W_k \psi(k);
\] (2.12)
and similarly for the tensor part. In [42] it was claimed that under a class of coordinates transformations\footnote{This class of transformations are similar to the ones used in the transition from comoving to conformal Fermi coordinates ([42, 44]). See the appendix.}
\[
x^i \rightarrow (1 - \psi_L) x^i + \frac{1}{2} D_{ij}^{(L)} x^j,
\] (3.1)
(basically the same of (2.7) when $\psi_L$ and $D_{ij}$ are constant) the “long wavelength” part in (2.12) can be removed by using the transformation rules (2.10). The problem is that
\begin{itemize}
  \item the choice that leads to (2.10) is by no means unique; for instance, by taking $\beta = -\gamma$ and $\alpha = 0$ in (2.9), then (2.10) is no longer valid: the transformations of the scalar $B$ and of the transverse vector $C_i$ reproduce (2.8) with $\psi$ unchanged;
  \item the cancellation can take place only in the peculiar case of purely constant $\psi_L$ and $D_{ij}^{(L)}$ and this is not the case in any reasonable coordinate transformation.
\end{itemize}
As it will be shown in the next section 3, when the splitting (2.11) between long and short parts is done by a physical window function, the ambiguity (2.9) disappears and the standard transformation rules (2.6) are recovered; thus no shift is present when a proper gradient expansion is considered. As already discussed in [62], the transformation rules (2.6) are precisely the ones that guarantee the gauge invariant character of scalars related to $\psi$-like fields, the comoving curvature perturbation $\zeta$ and $D_{ij}$ itself.

We conclude this section by underlining that the ambiguity just described was used by Weinberg to show that in the large-scale limit, under a number of technical assumptions, there is a least a conserved adiabatic mode [63, 64] by exploiting the residual gauge-invariance of the perturbed FLRW metric in the Newtonian gauge. But we remark that this is valid only in the exact $k = 0$ limit, when $\lambda$ and $\omega$ are pure constants and so the ambiguity (2.9) is still present.

3 Restoring the SVT decomposition: a discontinuity in the gradient expansion
Let us consider a deformation of (2.7) in the sense that now both $\lambda$ and $\omega_{ij}$ can depend on the space point $\vec{x}$, namely
\[
e^i = \lambda(x) x^i + \omega^i_j(x) x^j, \quad \omega^i_i = 0, \quad \partial_i \omega^i_j = 0.
\] (3.1)
To be as general as possible, we consider $\omega$ to be transverse and traceless, but not symmetric. By introducing a suitable window function $W_k$, in Fourier space $\lambda$ and $\omega$ are written as

$$\lambda = \frac{1}{(2\pi)^d} \int d^3k \, e^{i \mathbf{k} \cdot \mathbf{x}} \, W_k \, \zeta_k, \quad \omega^{ij} = \frac{1}{(2\pi)^d} \int d^3k \, e^{i \mathbf{k} \cdot \mathbf{x}} \, W_k \, \Omega^{ij}_k. \quad (3.2)$$

In the main text, the $\lambda$ and $\omega$ Fourier transform will be denoted by

$$\lambda_k = W_k \, \zeta_k, \quad \omega_k = W_k \, \Omega_k, \quad (3.3)$$

implicitly assuming the presence of the window function $W$. A very common choice for the window function $W_k$ is

$$W_k = \theta \left[ \frac{1}{H} (k_c - k) \right], \quad (3.4)$$

where $k_c$ is a reference scale for the long-short modes splitting: modes with a wavelength smaller than $k_c > 0$ are considered long and they do not contribute. However, keep in mind that the rest of this section is independent of the particular choice of $W_k$. Notice also that we have taken the function $\lambda_k$ such that it depends only on $k = |\mathbf{k}|$, being (for our purposes) related to the Fourier transform of $\zeta$ on super-horizon scales (\([62]\)).

Using the transformation (3.1) in (2.2), the variation of $h_{ij}$ results in

$$\Delta h_{ij} = -a^2 \left( \partial_l \epsilon^l + \partial_j \epsilon^j \right) = -a^2 \left[ 2 \delta_{ij} \lambda + 2 \omega^3_{ij} + x^i \partial_j \lambda + x^j \partial_i \lambda + x^\ell \left( \partial_i \omega^\ell_j + \partial_j \omega^\ell_i \right) \right]. \quad (3.5)$$

The Fourier transform of $x^i \partial_j \lambda$ entering in (3.5), can be written as follows (\([62]\)) by using integration by parts:

$$x^i \partial_j \lambda = -\frac{1}{(2\pi)^3} \int d^3k \, e^{i \mathbf{k} \cdot \mathbf{x}} \partial_k \left( k^j \lambda_k \right) + \text{BT}. \quad (3.6)$$

The boundary term BT is evaluated at very large $k$, where the window function vanishes: thus, terms of such a type do not contribute. Similar considerations apply to the Fourier transform of $x^i \partial_\ell \omega^\ell_j$. As a result, in Fourier space eq. (3.5) reads

$$\Delta h_{ij}(k) = a^2 \left[ 2 \frac{k^i k^j}{k} \lambda'_k + k^i \partial_k \omega^j_k + k^j \partial_k \omega^i_k \right]. \quad (3.7)$$

where $\lambda'_k = \frac{d\lambda_k}{dk}$. Thus, as claimed, no shift neither in $\psi_k$ nor in $D_{ij}(\vec{k})$ is present and, by comparison with eq. (2.2), one gets the following gauge variations

$$\Delta \psi(k) = 0, \quad \Delta B(k) = -\frac{2}{k} \lambda'_k, \quad \Delta C_i(k) = \partial_k \omega^i_k, \quad \Delta D_{ij}(k) = 0. \quad (3.8)$$

The SVT decomposition is restored and no ambiguity is present: the would-be shift of $\psi$ is actually a gradient term involving the transformation $B$, while the would-be shift of $D_{ij}$ is turned into a gradient of $\omega_{ij}$ by using

$$k^i \partial_k \omega^j_k = \partial_k \left( k^i \omega^j_k \right) = 0; \quad (3.9)$$

the first equality is obtained by considering the traceless condition $\delta_{ij} \omega^i_k = 0$. As a result, the shifts in (2.9) are just the artifact of the very special form (2.7) where $\lambda$ and $\omega_{ij}$ are taken to be constant. Whenever $\lambda$ and $\omega$ acquire a space dependence, the shifts disappear and the gauge transformation cannot be used to cancel a physical long mode (that is not proportional to $\delta^{(3)}(\vec{k})$).
4 Gauge variation of a correlator

We are interested in the correlation function of an operator $O(x_1, \ldots, x_N)$ built out of $\zeta$ and the tensor mode $D_{ij}$ taken as quantum field operators and evaluated by using the in-in formalism, see for instance [65]; namely

$$O(x_1, \ldots, x_N) = \zeta(x_1) \ldots \zeta(x_M) D(x_{M+1}) \ldots D(x_N).$$

(4.1)

In Fourier space, the case $N = 3$ gives various types of bispectra. The infinitesimal coordinate transformation (3.1) can be generalized at the non-linear level by

$$\bar{x}^i = e^\lambda x^i + (1 - e^\omega)|_{ij} x^j, \quad g_{ij} = a^2 e^{2\zeta} \delta_{ij} + h_{ij} + \frac{1}{2} h_{il} h_{lj} + \frac{1}{6} h_{im} h_{mj},$$

(4.2)

with $\zeta$, $\lambda$ and $\omega$ dependent on the spacetime point. Such a transformation represents the non-linear extension of the linear deformed dilatation used to connect the comoving gauge\(^7\) with CFC-like reference frame. Let us consider the gauge variation of a correlator as the difference of the expectation value of the operator $O'(x_1, \ldots, x_N)$ in the new coordinates defined by (4.2) and the original operator $O(x_1, \ldots, x_N)$.

$$\Delta O_{\text{gauge}} = \langle O'(x_1, \ldots, x_N) \rangle - \langle O(x_1, \ldots, x_N) \rangle.$$  

(4.3)

The action describing gravity and the inflaton field is invariant under a coordinate transformation and, up to a boundary term, it can be written in the ADM form as [5, 66]

$$S = \int d^4x \sqrt{h} N \left[ R^{(3)} + K_{ij} K^{ij} - K^2 + \mathcal{L}_m \right] \equiv \int d^4x \sqrt{h(x)} S(x),$$

(4.4)

where $h$ is the spatial metric determinant and $K_{ij}$ is the extrinsic curvature tensor of the hypersurface (see footnote 1) of equation $t = \text{constant}$, while $\mathcal{L}_m$ is the Lagrangian for the inflaton field $\phi$. The 3-scalar $S$ in (4.4) can be expanded as

$$\bar{S}(\bar{x}) \equiv S(x) = \bar{S}(t) + S^{(1)}(x) + S^{(2)}(x) + \ldots,$$

(4.5)

where $\bar{S}(t)$ contains only background quantities, while $S^{(n)}(x)$ is of order $n$ in perturbations. It is convenient to define the following gauge variation

$$\Delta S = \sqrt{\bar{h}(x)} \bar{S}(x) - \sqrt{h(x)} S(x),$$

(4.6)

which gives for the change of the action $\Delta \text{action} = \int d^4x \Delta S$. Splitting also $\Delta S$ into background and $n$-th order perturbations as done for $S$ in (4.5), the change of the spatial coordinates induces the following variation $\Delta S$ up to third order

$$\begin{align*}
\Delta S &= 0, \\
\Delta S^{(1)} &= a^3 \partial_i \left( \bar{S} \lambda x^i \right), \\
\Delta S^{(2)} &= -a^3 \partial_i \left[ \left( S^{(1)} + 3 \bar{S} \zeta \right) \left( \lambda x^i + \omega_j x^j \right) \right], \\
\Delta S^{(3)} &= -a^3 \partial_i \left[ \left( \frac{9}{2} \bar{S} \zeta^2 + 3 S^{(1)} \zeta + S^{(2)} \right) \left( \lambda x^i + \omega_j x^j \right) \right],
\end{align*}$$

(4.7)

\(^6\)In Fourier space it is convenient to strip out the overall delta function according to

$$\langle O(\vec{k}_1, \ldots, \vec{k}_N) \rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \ldots + \vec{k}_N) B_C(k_1, \ldots, k_N).$$

\(^7\)In our convention the comoving gauge is defined as the one in which $B$ and the peculiar velocity are set to zero.
where for simplicity we have omitted\(^8\) all the quadratic and cubic terms in \(\lambda - \omega\). The bottom line is that all the new terms in the cubic action introduced by the deformed dilation can be written as total spatial derivatives. As shown in [62] the gauge variation \(\Delta O_{\text{gauge}}\) can be written as the commutator of \(O(\vec{x}_1, \ldots, \vec{x}_N)\) with \(\Delta_{\text{action}}\) which vanishes being \(\Delta S\) a total spatial derivative. In conclusion, one cannot gauge away the long modes of the fields at least at first order in perturbation theory, so the squeezed bispectrum cannot be canceled.

5 Conclusions

Measuring the primordial non-Gaussianity remains one of the most important goals to study the physics of the Early Universe. In single-field inflationary model the squeezed limit is completely fixed in a model independent way thanks to the consistency relation, but its physical observability has been criticized, limiting the contributions to observed correlations to projection effects such as gravitational lensing and redshift perturbations ([42]). As discussed in [62], in this debate it is crucial to determine how a very long perturbation affects the quantities of physical interest. In this paper we have analyzed carefully the transformation properties of cosmological observables such as the curvature perturbation \(\zeta\), the tensor perturbation \(D\) and their correlation functions, thereby generalizing the results of [62] where the analysis was done for correlators involving only \(\zeta\)'s. In this case the infinitesimal diffeomorphism is generalized by equation (2.7) and, in the same way, the result is that no shift both in \(\zeta\) and in \(D\) is found, independently of the filter used to select long modes, excluding the case of infinitely long-wavelength (hence non-physical) perturbations. The latter is the main ingredient often stated for canceling tensor-scalar \(f_{\text{NL}}\), but we have seen that this is not consistent with a CFC-like transformation. We think that the problem is the role played by a constant spatial dilatation in single-field inflation.\(^9\) The transformation rules for the SVT elements in Fourier space presented in [62] have been generalized in eq. (3.8), showing once again that no shift is present neither in \(\psi\) nor in \(D\). It has also been shown explicitly that the equations of motion are not affected by a gauge transformation of type (2.7), implying that the correlator is unaffected according to eq. (4.3). Indeed, to cancel the bispectrum it has been used that this difference gives \(- \langle O(\vec{x}_1, \ldots, \vec{x}_N) \rangle\) ([41, 42, 44, 45]), but we have shown this to be not the case.

The outcome of our study is that all the squeezed bispectra, involving both \(\zeta\) and \(D\), cannot be gauged away and they remain physical observables, analogously to what obtained in [62] for \(B_{\zeta\zeta\zeta}\). This has a remarkable impact on future observations of a primordial gravitational-wave background. Consistency relations remain a very important tool to study Early Universe physics.

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\(^8\)Being \(\lambda - \omega\) defined by long modes only, \(\lambda^n - \omega^n\) (\(n > 1\)) vertices should imply correlators with two and three squeezed momenta that are not relevant for the consistency relation.

\(^9\)It is well-known that single-field cubic interactions are conformally symmetric. In [24], it is shown how to extract the scalar consistency relations by using dilatation symmetry itself and the related Ward identities.
A CFC transformation

In this appendix we give the structure of $\lambda$ and $\omega$ related to the CFC expansion. At first order of the CFC transformation, $\lambda$ and $\omega$ reduce to be exactly $\zeta$ and $D_{ij}$, as in eq. (2.13). However, the first-order analysis gives rise to two main issues:

1. as just demonstrated, we get a discontinuity in the gradient expansion;
2. the typical structure of the CFC metric ($g_{ij}^F \sim O(x_F^2)$) is not reproduced.

For this reason, we are forced to consider the transformation to the CFC frame up to third order in the CFC series. The scalar part has already been discussed in [62], so we can consider only the tensor perturbation part of the transformation:

$$\Delta x_F^k = \Delta x^k + \frac{1}{2} D^k_i \Delta x^i \bigg|_p + \frac{1}{4} \Delta x^i \Delta x^j (\partial_i D^k_j + \partial_j D^k_i - \partial^k D_{ij}) \bigg|_p + \frac{1}{12} \Delta x^i \Delta x^j \Delta x^l \left( \partial_l \partial_i D^k_j + \partial_l \partial_j D^k_i - \partial^k \partial_l D_{ij} \right) \bigg|_p,$$

(A.1)

where $\Delta x = x(\tau) - p(\tau)$ is the deviation from a central world-line and $\tilde{\Delta}$ is its background value. The transformation (A.1) can be SVT decomposed in Fourier space as follows,

$$\epsilon_k = -\frac{1}{12} \frac{1}{k^2} \sum_{s=\pm 2} \left( D_k^{(s)} - k D_k^{(s)'s} \right),$$

(A.2)

where we considered $D^{(s)}_{ij} = \sum_{s=\pm 2} \epsilon_{ij}^{(s)} D_k^{(s)}$ in the standard convention for spin-2 polarization tensors, $\epsilon_{ij}^{(s)} \epsilon_{ij}^{(s)'} = 2 \delta_{ss'}$ and

$$\epsilon_V = -\frac{i}{12} \left( 10 \partial_i D^i_j + 2k_m \partial_k \partial_l D_{ij} \right).$$

(A.3)

This allows the extension of the results presented in [62], where only the scalar sector was considered. Notice that, since $\partial_i \epsilon_V = 0$, (A.3) can be always rewritten as $\epsilon_V = A^i_k x^k$ with $A$ depending on the space-time point but transverse and traceless, so playing the role of $\omega$ in (3.1). Thus, to reproduce the proper local structure of the metric tensor $g_{ij}^F \sim O(x_F^2)$ we find an interesting mixing: the tensor degrees of freedom start to influence both the scalar and vector sectors.

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