THE LANDSCAPE OF SOFTWARE FOR TENSOR COMPUTATIONS

CHRISTOS PSARRAS∗, LARS KARLSSON‡, JIAJIA LI†, AND PAOLO BIENTINESI‡

Abstract. Tensors (also commonly seen as multi-linear operators or as multi-dimensional arrays) are ubiquitous in scientific computing and in data science, and so are the software efforts for tensor operations. Particularly in recent years, we have observed an explosion in libraries, compilers, packages, and toolboxes; unfortunately these efforts are very much scattered among the different scientific domains, and inevitably suffer from replication, suboptimal implementations, and in many cases, limited visibility. As a first step towards countering these inefficiencies, here we survey and loosely classify software packages related to tensor computations. Our aim is to assemble a comprehensive and up-to-date snapshot of the tensor software landscape, with the intention of helping both users and developers. Aware of the difficulties inherent in any multi-discipline survey, we very much welcome the reader’s help in amending and expanding our software list, which currently features 80 projects.

Key words. tensor software, multi dimensional arrays, contractions, decompositions

AMS subject classifications. 68N15 68N20

1. Introduction. Similar to matrices, tensors arise in a multitude of disciplines in engineering and science—for instance, in computational chemistry, computational physics, chemometrics, data science, signal processing, and machine learning [50, 78, 18, 11, 76, 17]—and naturally, significant effort goes into the development of numerical software. However, in sharp contrast to the software landscape for (dense) matrix computations, which is nicely layered and organized, that of tensor computations is fragmented and largely unstructured. Indeed, the tensor counterparts to the universally-used libraries such as BLAS—collection of building blocks—and LAPACK—collection of solvers—are still missing.

When surveying the landscape of libraries, packages, compilers, and toolboxes for tensor computations, a massive replication of effort becomes apparent, and the absence of building blocks libraries is certainly one of the main causes for this. Other reasons are to be associated to the fact that tensor software is mostly driven by applications, and is therefore scattered among different communities and scientific outlets. It could be argued that matrix computations are possibly even more widespread, yet a community effort made it possible to create “collection” libraries and standardize interfaces already in the 1970s. The profound difference is that while the language (and notation) for linear algebra (i.e., matrix computations) is quite consistent across disparate disciplines, the same cannot be further from the truth when it comes to the language of multi-linear algebra and multi-dimensional arrays (i.e., tensor computations). Even the most basic concepts, such as the number and the length of the “axes” of a tensor, have entirely different (and often conflicting) names in different disciplines.

In short, the development of tensor software has often been carried out independently in different communities, and even within the same community there has not

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∗Aachen Institute for Advanced Study in Computational Engineering Science, RWTH Aachen University, Aachen, Germany (psarras@aiaces.rwth-aachen.de).
†Pacific Northwest National Laboratory, Richland, Washington and College of William & Mary, Williamsburg, Virginia (Jiajia.Li@pnnl.gov).
‡Department of Computing Science, Umeå University, Umeå, Sweden (larsk@cs.umu.se, pauldj@cs.umu.se).

1For the sake of simplicity, in the rest of this manuscript we will refer to all of these types of software simply as “packages”.

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been any real coordinated effort. Motivated by this observation, we set out to survey the software landscape for tensor computations, aiming to a) create awareness among users of the existing packages, and b) guide the development at large. We see this survey as an essential step towards finding common ground between different applications, and towards identifying possible divisions of concerns, with the ultimate goal of defining a set of fundamental computational building blocks.

This survey takes up from a variety of communities, which often do not code in the same programming language, and, perhaps more importantly, present notable differences in the symbols and nomenclature they use to describe tensor operations. In this document, we do not address those differences, nor aim to rank packages qualitatively, in any way. Instead, we are merely attempting to put this diverse and large set of software packages on the map, with a loose classification of the functionality they provide.

This document is very much a work in progress, and we plan to keep it up-to-date by uploading new versions with some regularity. To this end, we welcome and encourage input, contributions and corrections, to help create a more complete and fair snapshot of the current tensor software landscape. We invite readers to send us contributions via email, and would greatly appreciate consulting the questionnaire in Appendix A.

2. Software list. We present a list of packages that support some form of tensor computations. To be considered for inclusion, a package must offer functionality in at least one of the following categories.

- **Data Manipulation** (DatM): Any operation related to the layout or storage of tensors, such as tensor transposition, reshaping, conversion between different storage formats, . . .
- **Element-Wise Operations** (EWOps): Any kind of element-by-element operation such as addition/subtraction, and/or reductions such as norms, min, max, . . .
- **Contractions** (Con): General contractions between two or more tensors. Currently the survey does not differentiate between binary, ternary, or hypercontractions.
- **Specific Contractions** (SpecCon): Specific operations that qualify as specific contractions, e.g., Tensor Times Vector (TTV), Tensor Times Matrix (TTM), Matricized Tensor Times Khatri-Rao Product (MTTKRP), . . .
- **Decompositions** (Decomp): At least one tensor decomposition, including but not limited to the Canonical Polyadic Decomposition (CPD or CP, also known as PARAllel FACtors analysis, PARAFAC), the Tucker Decomposition, Tensor Train, and their variants.

The following are the key aspects of a package that we aim to focus on.

- **Language**: What language is it written in, and, in the case of compilers/transpilers, what language does it generate code in (denoted by a \(\rightarrow\))?  
- **Tensor type**: What type of tensor does it operate on? (e.g., Dense (D), Sparse (S), BlockSparse (BS), symmetric, supersymmetric, . . .)  
- **Target system**: What types of computing architecture does it target? (e.g., CPU (C), GPU (G), Distributed Memory (D), . . .). Note: For CPU, there is currently no distinction between single-threaded and multi-threaded implementations.
- **Functionality**: Which of the categories mentioned above does it support?  

In the following tables, packages are listed alphabetically. For each package,
Table 1 provides when available a hyperlink to the source code (click on the package name), and a reference to a publication or website. Additionally, each package is listed with an ID number.

Tables 2, 3, and 4 group all packages according to the categories DatM, Con, and Decomp, respectively.

Finally, Table 5 attempts to gather the “more complete” packages, i.e., those that offer support for at least four out of the five categories described above.
| ID | Package Name         | Functionality |       |       |       |       |       |       |       |       | Platform | Language |
|----|----------------------|---------------|-------|-------|-------|-------|-------|-------|-------|-------|----------|----------|
| 0  | Acrotensor [28]      |               | ✓     | ✓     | ✓     | ✓     |       |       |       |       | D, C, G  | C++      |
| 1  | AdaTM [54]           |               | ✓     |       |       | ✓     |       |       |       |       | S, C     | C        |
| 2  | Boost.uBlas.Tensor [6]| ✓             | ✓     | ✓     | ✓     | ✓     |       |       |       |       | D, C     | C++      |
| 3  | BTAS [73]            | ✓             | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | nan      | C++      |
| 4  | COGENT [48]          |               | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, G     | Python → CUDA |
| 5  | COMET [86]           |               | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | S, C     | C++, CUDA |
| 6  | CoTenGra [34]        |               | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, C, G  | Python   |
| 7  | CP-CALS [75]         |               | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, C, G  | C++, MATLAB |
| 8  | CSTF [9]             |               | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | S, D     | Scala    |
| 9  | CuTensor [64]        | ✓             | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, G     | C, CUDA  |
| 10 | cuTT [41]            | ✓             | ✓     |     |     | ✓     |       |       |       |       | D, G     | C++, CUDA |
| 11 | Cyclops [81]         | ✓             | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | S, D     | C++, CUDA |
| 12 | D-Tucker [43]        |               | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, C     | Matlab   |
| 13 | DFacTo [15]          |               | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, C     | C++      |
| 14 | Eigen Tensor [16]    | ✓             | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, C, G  | C++      |
| 15 | ExaTN [60]           | ✓             | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, C, G  | C++, Python |
| 16 | Fastor [74]          | ✓             | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, C     | C++      |
| 17 | FTensor [52]         | ✓             | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, C     | C++      |
| 18 | Genten [72]          |               | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, S, C  | C, G     |
| 19 | GigaTensor [45]      |               | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | S, C, D  | Unknown  |
| 20 | HPTT [85]            | ✓             | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, C     | C++, Python, C++ |
| 21 | ITensor [29]         | ✓             | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, BS    | C, G     |
| 22 | libtensor [42]       |               | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, BS    | C++      |
| 23 | Ltensor [2]          |               | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, C     | C++      |
| 24 | MATLAB [58]          | ✓             | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, C     | Matlab   |
| 25 | MultiArray [30]      | ✓             | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, C     | C++      |
| 26 | multiway [38]        |               | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, C     | R        |
| 27 | N-way toolbox [4]    |               | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, C     | Matlab   |
| 28 | NCON [69]            |               | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, C     | Matlab   |
| 29 | netcon [70]          |               | ✓     | ✓     | ✓     | ✓     | ✓     |       |       |       | D, C     | Matlab   |
| ID  | Package Name          | Functionality | Tensor Type | Platform | Language |
|-----|-----------------------|---------------|-------------|----------|----------|
| 30  | NumPy [36]            | ✓ ✓ ✓ ✓       | D C         | Python   |
| 31  | Ocean [88]            | ✓ ✓ – –       | D C, G      | C, Py    |
| 32  | ParCube [65]          | – – – ✓       | S C         | Matlab   |
| 33  | ParTensor [56]        | ✓ – – ✓       | D C, G      | C++      |
| 34  | ParTT! [55]           | ✓ ✓ ✓ –       | S C, G      | C, CUDA, Mat |
| 35  | PLANC [46]            | – – – ✓       | D C, D      | C++      |
| 36  | PLS toolbox [90]      | – – – ✓       | D C         | Matlab   |
| 37  | Pytensor [91]         | ✓ ✓ ✓ ✓       | D S C       | Python   |
| 38  | PyTorch [66]          | ✓ ✓ ✓ ✓ ✓     | D S C, G    | Python, C++, CUDA |
| 39  | quimb [33]            | – – ✓ –       | D C, D, G   | Python   |
| 40  | rTensor [53]          | ✓ ✓ ✓ ✓ ✓     | D C         | Python   |
| 41  | rTensor (randomized)  [25] | – – – ✓       | D C         | Python   |
| 42  | scikit-tensor [62]    | ✓ ✓ ✓ ✓ ✓     | D S C       | Python   |
| 43  | Scikit-TT [31]        | – – – ✓       | D C         | Python   |
| 44  | SPALS [14]            | – – – ✓       | S C         | C++      |
| 45  | SPARTan [68]          | – – – ✓       | S C         | Matlab   |
| 46  | SPLATT [80]           | – – ✓ –       | S C, D      | C, C++, Oct, Mat |
| 47  | SuSMoST [3]           | – – – ✓       | D C         | Python   |
| 48  | T3F [63]              | ✓ ✓ ✓ ✓ –     | D C, G      | Python   |
| 49  | TACO [49]             | ✓ ✓ ✓ ✓ –     | D S C, G    | C++, C++ → C++ |
| 50  | TAL_SH [57]           | ✓ ✓ ✓ ✓ –     | D C, G      | C, C++, Fort |
| 51  | TBlis [59]            | ✓ ✓ ✓ ✓ –     | D C         | C++      |
| 52  | TCCG [82]             | – – ✓ –       | D C         | C++      |
| 53  | TCL [83]              | – – ✓ ✓ –     | D C         | C++, Python |
| 54  | TDALAB [92]           | – – – ✓       | D S C       | Matlab, GUI |
| 55  | TeNPy [37]            | – – – ✓       | D C         | Python   |
| 56  | Tensor Fox [23]       | – – – ✓       | D S C       | Python, Matlab |
| 57  | Tensor package [19]   | – – – ✓       | D C         | Matlab   |
| 58  | Tensor Toolbox [10]   | ✓ ✓ ✓ ✓ ✓     | D S C       | Matlab   |
| 59  | tensor_decomposition  [79] | – – – ✓       | D C, D      | Python   |
| ID | Package Name | Functionality | Tensor Type | Platform | Language |
|----|--------------|---------------|-------------|----------|----------|
|    |              | DatM EWOps SpecCon Con Decomp |             |          |          |
| 60 | TensorBox [71] | – – – – ✓ ✓ | D, S C | Matlab |          |
| 61 | TensorD [35]  | – – – – ✓ ✓ | D, C, G | Python |          |
| 62 | TensorFlow [1] | ✓ ✓ ✓ ✓ | D, S C, D, G | C++, Python |          |
| 63 | TensorLab [89] | – – – – ✓ ✓ | D, S C | Matlab |          |
| 64 | TensorLab+ [39] | – – – – ✓ ✓ | D, S C | Matlab |          |
| 65 | TensorLy [51] | ✓ ✓ ✓ ✓ ✓ ✓ | D, C, G | Python |          |
| 66 | TensorNetwork [77] | – – ✓ ✓ ✓ ✓ ✓ | D, S C, G | Python |          |
| 67 | TensorOperations.jl [44] | ✓ ✓ ✓ ✓ ✓ ✓ | D, C, G | Julia |          |
| 68 | TensorTrace [26] | – – ✓ ✓ ✓ ✓ ✓ | D, C | GUI → Py, Jul, Mat |          |
| 69 | Three-Way [32] | – – ✓ ✓ ✓ ✓ ✓ | D, C | R |          |
| 70 | TiledArray [12] | ✓ ✓ ✓ ✓ ✓ ✓ | D, BS C, D | C++ |          |
| 71 | tncontract [20] | – – ✓ ✓ ✓ ✓ ✓ | D, C | Python |          |
| 72 | TNR [27] | – – – – ✓ ✓ ✓ | D, C | Matlab |          |
| 73 | TorchMPS [61] | – – ✓ ✓ ✓ ✓ ✓ | D, C | Python |          |
| 74 | TT-Toolbox [24] | ✓ ✓ ✓ ✓ ✓ ✓ | D, C, D*, G* | Matlab, Python |          |
| 75 | TTC [84] | ✓ ✓ ✓ ✓ ✓ ✓ | D, C | Python → C++ |          |
| 76 | TVM [13] | – – ✓ ✓ ✓ ✓ ✓ | D, C | C++ |          |
| 77 | Uni10 [47] | ✓ ✓ ✓ ✓ ✓ ✓ | D, S C, G | Python |          |
| 78 | xerus [40] | ✓ ✓ ✓ ✓ ✓ ✓ | D, S C | C++ |          |

Table 1: Main list. Packages are sorted alphabetically by name. We use an i superscript to denote an interface to the specific language. We use the x superscript to denote an experimental feature.
| ID | Name                          | DatM | EWOps | SpecCon | Con | Decomp |
|----|-------------------------------|------|-------|---------|-----|--------|
| 2  | Boost.uBlas.Tensor            | ✓    | ✓     | ✓       | ✓   | –      |
| 3  | BTAS                          | ✓    | ✓     | ✓       | ✓   | ✓      |
| 9  | CuTensor                      | ✓    | ✓     | ✓       | ✓   | –      |
| 10 | cuTT                          | ✓    | –     | –       | –   | –      |
| 11 | Cyclops                       | ✓    | ✓     | ✓       | ✓   | –      |
| 14 | Eigen Tensor                  | ✓    | ✓     | ✓       | ✓   | –      |
| 15 | ExaTN                         | ✓    | ✓     | ✓       | ✓   | ✓      |
| 16 | Fastor                        | ✓    | ✓     | ✓       | ✓   | –      |
| 17 | FTensor                       | ✓    | ✓     | ✓       | ✓   | –      |
| 20 | HPTT                          | ✓    | –     | –       | –   | –      |
| 24 | MATLAB                        | ✓    | ✓     | ✓       | ✓   | –      |
| 25 | MultiArray                    | ✓    | –     | –       | –   | –      |
| 30 | NumPy                         | ✓    | ✓     | ✓       | ✓   | –      |
| 31 | Ocean                         | ✓    | ✓     | –       | –   | –      |
| 34 | ParTI!                        | ✓    | ✓     | –       | –   | ✓      |
| 37 | Pytensor                      | ✓    | ✓     | ✓       | ✓   | ✓      |
| 38 | PyTorch                       | ✓    | ✓     | ✓       | ✓   | –      |
| 40 | rTensor                       | ✓    | ✓     | ✓       | –   | ✓      |
| 42 | scikit-tensor                 | ✓    | ✓     | –       | –   | ✓      |
| 48 | T3F                           | ✓    | ✓     | –       | –   | ✓      |
| 49 | TACO                          | ✓    | ✓     | ✓       | ✓   | –      |
| 50 | TAL.SH                        | ✓    | ✓     | ✓       | ✓   | –      |
| 51 | TBlis                         | ✓    | ✓     | ✓       | ✓   | –      |
| 58 | Tensor Toolbox                | ✓    | ✓     | ✓       | ✓   | ✓      |
| 62 | TensorFlow                    | ✓    | ✓     | ✓       | –   | –      |
| 65 | TensorLy                      | ✓    | ✓     | ✓       | ✓   | –      |
| 67 | TensorFlow.jl                 | ✓    | ✓     | ✓       | ✓   | –      |
| 70 | TiledArray                    | ✓    | ✓     | ✓       | ✓   | –      |
| 74 | TT-Toolbox                    | ✓    | –     | –       | –   | ✓      |
| 75 | TTC                           | ✓    | –     | –       | –   | –      |
| 78 | Uni10                         | ✓    | ✓     | ✓       | ✓   | –      |

Table 2: Packages that support Data Manipulation (DatM).
| ID | Name                  | DatM | EWOps | SpecCon | Con | Decomp |
|----|-----------------------|------|-------|---------|-----|--------|
| 0  | Acrotensor            |      | ✓     |         |     | −      |
| 2  | Boost.uBlas.Tensor    | ✓    | ✓     | ✓       |     | −      |
| 3  | BTAS                  | ✓    | ✓     | ✓       |     | ✓      |
| 4  | COGENT                |      |       | ✓       |     | −      |
| 5  | COMET                 |      |       | ✓       |     | −      |
| 6  | CuTenGra              |      |       | ✓       |     | −      |
| 9  | CuTensor              | ✓    | ✓     | ✓       |     | −      |
| 11 | Cyclops               | ✓    | ✓     | ✓       |     | −      |
| 14 | Eigen Tensor          | ✓    | ✓     | ✓       |     | −      |
| 15 | ExaTN                 | ✓    | ✓     | ✓       |     | ✓      |
| 16 | Fastor                | ✓    | ✓     | ✓       |     | −      |
| 17 | FTensor               | ✓    | ✓     | ✓       |     | −      |
| 21 | ITensor               |      |       | ✓       |     | ✓      |
| 22 | libtensor             |      |       | ✓       |     | −      |
| 23 | Ltensor               |      |       | ✓       |     | −      |
| 24 | MATLAB                | ✓    | ✓     | ✓       |     | −      |
| 28 | NCON                  |      |       | ✓       |     | −      |
| 29 | netcon                |      |       | ✓       |     | −      |
| 30 | NumPy                 | ✓    | ✓     | ✓       |     | −      |
| 37 | Pytensor              | ✓    | ✓     | ✓       |     | ✓      |
| 38 | PyTorch               | ✓    | ✓     | ✓       |     | −      |
| 39 | quimb                 |      |       | ✓       |     | −      |
| 49 | TACO                  | ✓    | ✓     | ✓       |     | −      |
| 50 | TAL_SH                | ✓    | ✓     | ✓       |     | −      |
| 51 | TBlis                 | ✓    | ✓     | ✓       |     | −      |
| 52 | TCCG                  |      |       | ✓       |     | −      |
| 53 | TCL                   |      |       | ✓       |     | −      |
| 58 | Tensor Toolbox        | ✓    | ✓     | ✓       |     | ✓      |
| 62 | TensorFlow            | ✓    | ✓     | ✓       |     | −      |
| 65 | TensorLy              | ✓    | ✓     | ✓       |     | ✓      |
| 66 | TensorNetwork         |      |       | ✓       |     | −      |
| 67 | TensorOperations.jl   | ✓    | ✓     | ✓       |     | −      |
| 68 | TensorTrace           |      |       | ✓       |     | −      |
| 69 | Three-Way             |      |       | ✓       |     | ✓      |
| 70 | TiledArray            | ✓    | ✓     | ✓       |     | −      |
| 71 | tncontract            |      |       | ✓       |     | −      |
| 73 | TorchMPS              |      |       | ✓       |     | −      |
| 78 | Uni10                 | ✓    | ✓     | ✓       |     | −      |
| 79 | xerus                 |      |       | ✓       |     | ✓      |

Table 3: Packages that support Contractions (Con).
| ID | Name                  | CP | Tucker | TensorTrain | Other |
|----|-----------------------|----|--------|-------------|-------|
| 1  | AdaTM                 | ✓  |        |             | −     |
| 3  | BTAS                  | ✓  | ✓      |             | −     |
| 7  | CP-CALS               | ✓  | −      | −           | −     |
| 8  | CSTF                  | −  | −      | −           | ✓     |
| 12 | D-Tucker              | −  | ✓      | −           | −     |
| 13 | DFacTo                | ✓  | −      | −           | −     |
| 15 | ExaTN                 | −  | −      | ✓           | −     |
| 18 | Genten                | ✓  | −      | −           | −     |
| 19 | GigaTensor            | ✓  | −      | −           | −     |
| 21 | ITensor               | −  | −      | ✓           | −     |
| 26 | multiway              | ✓  | ✓      | −           | ✓     |
| 27 | N-way toolbox         | ✓  | ✓      | −           | ✓     |
| 32 | ParCube               | ✓  | −      | −           | −     |
| 33 | ParTensor             | ✓  | −      | −           | −     |
| 34 | ParTI!                | ✓  | ✓      | −           | −     |
| 35 | PLANC                 | ✓  | −      | −           | −     |
| 36 | PLS toolbox           | ✓  | ✓      | −           | −     |
| 37 | Pytensor              | −  | ✓      | −           | −     |
| 40 | rTensor               | ✓  | ✓      | −           | ✓     |
| 41 | rTensor (randomized)  | ✓  | −      | −           | −     |
| 42 | scikit-tensor         | ✓  | ✓      | −           | ✓     |
| 43 | Scikit-TT             | −  | −      | ✓           | −     |
| 44 | SPALS                 | ✓  | −      | −           | −     |
| 45 | SPARTan               | −  | −      |            | ✓     |
| 46 | SPLATT                | ✓  | −      | −           | −     |
| 47 | SuSMoST               | −  | −      | ✓           | ✓     |
| 48 | T3F                   | −  | −      | ✓           | −     |
| 54 | TDALAB                | ✓  | −      | −           | −     |
| 55 | TeNPy                 | −  | −      | ✓           | −     |
| 56 | Tensor Fox            | ✓  | −      | −           | −     |
| 57 | Tensor package        | ✓  | −      | −           | −     |
| 58 | Tensor Toolbox        | ✓  | ✓      | −           | ✓     |
| 59 | tensor_decomposition  | ✓  | ✓      | −           | −     |
| 60 | TensorBox             | ✓  | ✓      | −           | ✓     |
| 61 | TensorD               | ✓  | ✓      | −           | −     |
| 63 | TensorLab             | ✓  | ✓      | −           | ✓     |
| 64 | TensorLab+            | ✓  | −      | −           | ✓     |
| 65 | TensorLy              | ✓  | ✓      | ✓           | −     |
| 69 | Three-Way             | ✓  | ✓      | −           | −     |
| 72 | TNR                   | −  | −      | −           | ✓     |
| 74 | TT-Toolbox            | −  | −      | ✓           | −     |
| 79 | xerus                 | −  | −      | ✓           | −     |

Table 4: Packages that support Decompositions (Decomp).
2.1. Notable omissions. Certain packages that are well known in the community for offering tensor contractions and other operations include TAMM [5] and TCE [8]. These packages were not included in the list, since both are implemented as components of a larger project, NWChem [87] (a software primarily targeted towards computational chemistry), and are not usable independently. Furthermore, while many tensor operations can be cast in terms of BLAS and LAPACK calls, e.g., [21, 22, 67], in this survey we only focus on packages that support multi-dimensional arrays.

3. Conclusion. We provide a survey of tensor packages which arise in a wide range of fields and applications. Most of the packages are written in (or target) one of a handful of well-known programming languages, and are standalone, i.e., they do not depend on one another. This means that many packages (re-)implement, often sub-optimally, the same or similar functionality within their own codebase (e.g., tensor transposition, and specific operations such as MTTKRP and TTV).

With this list we aim to help both (new) users in finding a suitable package for their needs, and developers in identifying opportunities for cooperation, modularity, and optimization. Ultimately, our goal is to create awareness about the level of redundancy that permeates the software landscape of tensor computations, and the

| ID | Name                          | Language | DATM | EWOps | SpecCon | Con | Decomp |
|----|-------------------------------|----------|------|-------|---------|-----|--------|
| 2  | Boost.uBlas.Tensor            | C++      | ✓    | ✓     | ✓      | ✓   |        |
| 3  | BTAS                          | C++      | ✓    | ✓     | ✓      | ✓   | −      |
| 9  | CuTensor                      | C, CUDA  | ✓    | ✓     | ✓      | ✓   | −      |
| 11 | Cyclops                       | C++      | ✓    | ✓     | ✓      | ✓   | −      |
| 14 | Eigen Tensor                  | C++      | ✓    | ✓     | ✓      | ✓   | −      |
| 15 | ExaTN                         | C++, Py3  | ✓    | ✓     | ✓      | ✓   | ✓      |
| 16 | Fastor                        | C++      | ✓    | ✓     | ✓      | ✓   | −      |
| 17 | FTensor                       | C++      | ✓    | ✓     | ✓      | ✓   | −      |
| 21 | iTensor                       | C++, Julia| −    | ✓     | ✓      | ✓   | ✓      |
| 24 | MATLAB                        | Matlab   | ✓    | ✓     | ✓      | ✓   | −      |
| 30 | NumPy                         | Python   | ✓    | ✓     | ✓      | ✓   | −      |
| 34 | ParTi!                        | C, CUDA, Matlab | ✓    | ✓     | ✓      | −   | ✓      |
| 37 | Pytensor                      | Python   | ✓    | ✓     | ✓      | ✓   | −      |
| 38 | PyTorch                       | Python, C++, CUDA | ✓    | ✓     | ✓      | −   | ✓      |
| 40 | rTensor                       | R        | ✓    | ✓     | ✓      | ✓   | −      |
| 42 | scikit-tensor                 | Python   | ✓    | ✓     | ✓      | ✓   | −      |
| 49 | TACO                          | C++, C++ → C++ | ✓    | ✓     | ✓      | −   | −      |
| 50 | TAL_SH                        | C, C++, Fort | ✓    | ✓     | ✓      | ✓   | −      |
| 51 | TBlis                         | C++      | ✓    | ✓     | ✓      | ✓   | −      |
| 58 | Tensor Toolbox                | Matlab   | ✓    | ✓     | ✓      | ✓   | ✓      |
| 62 | TensorFlow                    | C++, Python | ✓    | ✓     | ✓      | ✓   | −      |
| 65 | TensorLy                      | Python   | ✓    | ✓     | ✓      | ✓   | ✓      |
| 67 | TensorOperations.jl           | Julia    | ✓    | ✓     | ✓      | −   | −      |
| 70 | TiledArray                    | C++      | ✓    | ✓     | ✓      | ✓   | −      |
| 78 | Uni10                         | C++      | ✓    | ✓     | ✓      | ✓   | −      |
potential implications on software quality, performance, and productivity.

Furthermore, we see this survey as a first step to pave the way towards a set of universal, optimized, building blocks, which shall play the same role as the one that BLAS and LAPACK have played (and are playing) in the domain of numerical linear algebra.

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Appendix A. Questionnaire.

Any cross-disciplinary investigation of this size is bound to be incomplete and to contain mistakes. We therefore kindly ask the reader to help us by emailing corrections and additions to pauldj@cs.umu.se. When providing information about a package, please consider the following questions.

1. What is the name of the package?
2. Where is the source code located?
3. Would you please provide a reference (preferably in BibTeX format) to a publication, preprint, or website about the package?
4. In which programming language(s) is the package written?
5. Which programming language(s) do users have to code in to use the package?
6. Is the package standalone? Alternatively, does it depend on another package that is either in the list or that belongs in the list?
7. What is the target computing architecture (CPUs, GPUs, Distributed Memory, others)?
8. Does the package support layout-related operations, such as tensor transpositions, or reshaping, . . . ?
9. Does it support element-wise operations, such as addition/subtraction, reductions, . . . ?
10. Does it support general binary contractions? If not, what are the limitations?
11. Does it support only a specific, subset of contractions? (e.g., TTV, TTM, MTTKRP)
12. Does it support other types of contractions? Which one(s)?
13. Does it support tensor decompositions? If so, please provide a comma separated list of decompositions supported.
14. Would you please describe in 1-2 sentences what the package is about, what target problem it addresses, and what functionalities it provides?
15. Is there any other information about the package that you deem essential in order to describe its functionality?

Thank you for your help!

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