Suppressing Proton Decay in the Minimal SO(10) Model

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We show that in a class of minimal supersymmetric SO(10) models which have been found to be quite successful in predicting neutrino mixings, all proton decay modes can be suppressed by a particular choice of Yukawa textures. This suppression works for contributions from both left and right operators for nucleon decay and for arbitrary tan β. The required texture not only fits all lepton and quark masses as well as CKM parameters but it also predicts neutrino mixing parameter U_{e3} and Dirac CP phase sin |δ_{MNS}| to be 0.07-0.09 and 0.3-0.7 respectively. We also discuss the relation between the GUT symmetry breaking parameters for the origin of these textures.

INTRODUCTION

Recent experiments in neutrino physics which have established that neutrino masses are in the sub-eV range have provided a new reason for taking supersymmetric grand unified theories (SUSY GUTs) seriously. This has to do with using the seesaw mechanism to understand the extreme smallness of neutrino masses compared to charged fermion masses[1]. The seesaw mechanism consists of extending the standard model by introducing three right-handed neutrinos with large Majorana masses M_R. Simple arguments based on the atmospheric oscillation data then tell us that at least one M_R is close to the conventional GUT scale M_U \sim 10^{16} \text{ GeV}, perhaps suggesting that the seesaw scale, M_R and GUT scale are one and the same.

The minimal GUT that unifies the right-handed neutrinos with the other fermions of the standard model and leads to the seesaw mechanism is the SO(10) model where all quarks and leptons are unified into one 16 dimensional spinor multiplet. This raises the possibility that the variety of masses and mixings of quarks and leptons can be understood in terms of a smaller number of parameters than in the standard model with three right-handed neutrinos. In fact, various recent works in the minimal SO(10) models with a single 10 and a single 126 Higgs multiplets[2, 3, 4, 5, 6] have substantiated this point of view. Unlike the generic seesaw models based on the extension of the standard model as well as right-handed neutrino extended SU(5) model, the above SO(10) models have fewer parameters and lead to predictions for neutrino mixings and phases in their minimal version without any extra symmetry assumptions. Their predictions for solar and atmospheric mixing angles are in agreement with present observations[2, 5] and that for U_{e3} is not far below the present upper limits, making the model testable in planned experiments[7]. Measurement of U_{e3} will therefore provide a crucial test of these models.

It has further been shown that the simplest way to accommodate CKM CP violation in these models is to include an additional Higgs field belonging to the 120 dimensional representation[8]. The model with 120 still remains predictive in the neutrino sector despite a small increase in the number of parameters[8, 9] and also leads to a solution to the SUSY CP problem.

Since the quarks and leptons are unified in a GUT, it predicts that proton is no longer stable and therefore proton decay becomes another test of any GUT such as SO(10). The present experimental bounds on the proton lifetime are in fact known to severely constrain some GUT models[10, 11] and one must therefore make sure that the above class of SO(10) models are in agreement with present experimental results.

In most generic SUSY GUTs, the dimension five operators induced by colored Higgsino[12] provide the dominant contribution to the proton decay amplitude rather than the dimension six operators induced by gauge bosons. Since the dimension five operators arise from diagrams involving Yukawa couplings, predictions for proton lifetime get related to fermion mass textures. For example, in simple GUT theories such as minimal SU(5), where by minimal we mean that only the most general renormalizable terms are included in the superpotential, the up- and down-type quark mass matrices are proportional to the Yukawa matrices, Y_u and Y_d respectively. In this case the proton decay rate, \tau_p^{-1} is directly proportional to |Y_{u,ij}Y_{d,kl}|^2 implying that \tau_p cannot be arbitrarily adjusted. That is why the minimal SU(5) theory is now ruled out by proton decay results[11].

One might argue that the minimal SU(5) model is anyway not realistic since it predicts wrong relations between fermion masses e.g. m_s = m_\mu and m_d = m_e. This fermion mass problem is however easily remedied in the class of minimal SO(10) models[2] with a single 10 and 126 field mentioned above. In view of the fact that it is also quite predictive in the neutrino sector[2, 4, 5, 6], it is tempting to consider this as the minimal GUT comparable to what SU(5) GUT was in the 1980’s. It is then important to look for its predictions for proton decay. To be sure, this model like minimal SU(5) (and the standard model) does not explain the origin of the Yukawa couplings; however if this model is confirmed by experiments, understanding the Yukawa sector will be the next order of business.
Unlike the SU(5) model, the $\Delta B = 1$ interactions in generic SO(10) models involve more GUT scale symmetry breaking parameters than just the color triplet Higgsino mass; therefore the situation for proton decay here is less restrictive. In particular, in the minimal SO(10) models of the type discussed in Ref. [5], there are four free parameters whereas there are about 16 decay modes which have lower bounds on their partial life times. It was shown through a numerical analysis (without including RRRR operators) that there exists a very small region in these parameter spaces for $LLLL$ operators, where all the present experimental constraints are satisfied for lower $\tan \beta$.

In this paper, we study the proton decay constraints on SO(10) models with $10 + \overline{126} + 120$ Higgs fields. We show that present proton decay constraints combined with fermion masses and mixings imply a specific relation among SO(10) breaking vacuum expectation values (VEVs) and a very specific form for the Yukawa textures. Roughly, they imply that the proton decay operators are proportional to the product of two up-type quark Yukawa couplings i.e. $Y_{u,ij}Y_{u,kl}$ instead of $Y_d Y_u$ as in SU(5). As a consequence, proton decay is not only suppressed for the left-handed quark contributions but also for right-handed ones and furthermore, the suppression works also for large $\tan \beta$ values as well which makes this model easily distinguishable from other simple GUT models. In addition, it leads to definite predictions for the neutrino mixing parameters $U_{e3}$ and the Dirac phase $\delta_{MNS}$. We also show the essential role played by the 120 in the suppression of proton decay when all operators are taken into account. These are the new results of this paper, which have important implications for the viability of the minimal SO(10) model for neutrino masses with CKM CP violation.

**FERMION MASS AND PROTON DECAY IN MINIMAL SO(10)**

We start by introducing the Yukawa interactions and the contents of Higgs fields in the SO(10) model. The Yukawa superpotential involves the couplings of 16-dimensional matter spinors $\psi_i$ ($i$ denotes a generation index) with $10$ ($H$), $\overline{126}$ ($\overline{\Delta}$), and $120$ ($D$) dimensional Higgs fields:

$$W_Y = \frac{1}{2} h_{ij} \psi_i \psi_j H + \frac{1}{2} f_{ij} \psi_i \psi_j \overline{\Delta} + \frac{1}{2} h'_{ij} \psi_i \psi_j D. \quad (1)$$

The Yukawa couplings, $h$ and $f$ are symmetric matrices, whereas $h'$ is an anti-symmetric matrix due to SO(10) symmetry. One 126 Higgs multiplet $\Delta$ is also introduced as a vector-like pair of $\overline{\Delta}$ whose VEV reduces the rank of SO(10) group. This helps to keep supersymmetry unbroken down to the weak scale. In order to break SO(10) symmetry down to the standard model, we employ one 210 Higgs field $(\Phi)$ which also contains a pair of Higgs doublets $(\Phi_u , \Phi_d)$. Altogether, we have six pairs of Higgs doublets: $\varphi_d = (H^u_1, D^1_1, D^2_1, \Delta_u, \Delta_d, \Phi_d)$, $\varphi_u = (H^u_1, D^1_1, D^2_1, \Delta_u, \Delta_d, \Phi_u)$, where superscripts $1, 2$ of $D_{u,d}$ stand for SU(4) singlet and adjoint pieces under the $G_{422} = SU(4) \times SU(2) \times SU(2)$ decomposition. The mass term of the Higgs doublets is given as $(\varphi_d, a) (M_D)^{ab} (\varphi_a)$, and the expression of the matrix $M_D$ is given in Ref. [17].

The mass matrix of the Higgs doublets is diagonalized by unitary matrices $U$ and $V$: $U M_D V^T = M_D^{diag}$. We assume that $\{(M_D^{diag})_{11} = \mu\}$, where $\mu$ is a Higgsino mass in the MSSM and the mass scale is much smaller than the GUT scale. Since we concentrate on the structure of Yukawa couplings, we do not need to specify the dynamical reason of the mass hierarchy in this letter, but we just require a fine-tuning such as in the case of minimal SU(5) model [10]. The MSSM Higgs doublets are given as linear combinations: $H_d = U_{12} \varphi_d a$, $H_u = V^*_{12} \varphi_a a$.

We use “Y diagonal basis” (or SU(5) basis) to describe the standard model decomposition of the SO(10) representation [17] [18]. The expression of the Yukawa interaction under the $G_{422}$ decomposition can be derived from Ref. [14]. The decomposed Yukawa interactions which give fermion masses are written as

$$W^{\text{Y doub}} = h H_{10}^d (q \ell + \epsilon e) + h H_{10}^u (q \ell + \epsilon e) \quad (2)$$
$$+ \frac{1}{\sqrt{3}} \Delta_d (q \ell - 3 \epsilon e) + \frac{1}{\sqrt{3}} \Delta_u (q \ell - 3 \epsilon e)$$
$$+ h' D_1^d (q \ell + \epsilon e) + h' D_1^u (q \ell + \epsilon e)$$
$$+ \frac{1}{\sqrt{3}} h' D_2^d (q \ell - 3 \epsilon e) - \frac{1}{\sqrt{3}} h' D_2^u (q \ell - 3 \epsilon e),$$

where $q, u, d, c, \ell, e, \nu$ are the quark and lepton fields for the standard model, which are all unified into one spinor representation of SO(10). We obtain the Yukawa coupling matrices for fermions as

$$Y_u = \tilde{h} + r_3 \tilde{f} + r_3 \tilde{h}, \quad (3)$$
$$Y_d = r_1 (\tilde{h} + \tilde{f} + \tilde{h}), \quad (4)$$
$$Y_e = r_1 (\tilde{h} - 3 \tilde{f} + c_6 \tilde{h}), \quad (5)$$
$$Y_\nu = \tilde{h} - 3 r_2 \tilde{f} + c_6 \tilde{h}, \quad (6)$$

where the subscripts $u, d, c, \ell, e, \nu$ denotes for up-type quark, down-type quark, charged-lepton, and Dirac neutrino Yukawa couplings, respectively and

$$\tilde{h} = V_{11} h, \quad r_1 = U_{11}/V_{11}, \quad r_2 = r_1 V_{15}/U_{14}, \quad (7)$$
$$r_3 = r_1 (V_{12} - V_{13}/\sqrt{3})/(U_{12} + U_{13}/\sqrt{3}), \quad (8)$$
$$\tilde{f} = U_{14}/(\sqrt{3} r_1), \quad \tilde{h}' = (U_{12} + U_{13}/\sqrt{3})/r_1 h', \quad (9)$$
$$c_6 = (U_{12} - \sqrt{3}U_{13})/(U_{12} + U_{13}/\sqrt{3}), \quad (10)$$
$$c_\nu = r_1 (V_{12} + \sqrt{3}V_{13})/(U_{12} + U_{13}/\sqrt{3}). \quad (11)$$

The Majorana mass matrices for both left- and right-handed neutrinos are proportional to the coupling $f$. In this letter, we will be using type II seesaw [20].
Next we consider dimension five operators induced by Higgs triplets. The dimension five operators (LLLL and RRRR operators),

$$- W_5 = \frac{1}{2} C_{L}^{ijkl} q_k q_i l_j + C_{R}^{ijkl} e_i^c u_i^c d_j^c,$$  \hspace{1cm} (12)

are obtained by integrating out the triplet Higgs fields, $\phi_T = (H_T, D_T, D_T^*, \Sigma_T, \Sigma_T^*, \Phi_T)$ and $\varphi_T = (H_T, D_T, D_T^*, \Sigma_T, \Sigma_T^*, \Phi_T)$. The quantum numbers under SU(3) × SU(2) × U(1) of the field $\varphi_T$ is (3, 1, −1/3). In the expression of $\varphi_T$, the fields with $i^i$ are decuplet, and the others are sextet under SU(4) decomposition. The RRRR operator, $C_R$, is also generated by other colored triplet, $\varphi_T = (D_T, \Sigma_T)$ and $\varphi_C = (D_C, \Sigma_C)$, where the quantum number of $\varphi_C$ is (3, 1, −4/3). The mass term of the Higgs triplets are given as $(\varphi_T^*)_{abc}(\varphi_T h) + (\varphi_C^*)_{abc}(\varphi_C h)$. The mass matrices, $M_T$ and $M_C$, are $7 \times 7$ and $2 \times 2$ matrices respectively, and their explicit forms are given in the literature [17]. The Yukawa couplings which cause proton decay are written as

$$W_{Y}^{\text{trip.}} = h H_T (q \ell + u^c d^c) + h H_T \left( \frac{1}{2} q q + e^c u^c \right)$$

$$+ f \Sigma_T (q \ell - u^c d^c) + f \Sigma_T \left( \frac{1}{2} q q - e^c u^c \right) + \sqrt{2} f \Sigma_T e^c u^c$$

$$+ \sqrt{2} h' D_T u^c d^c + \sqrt{2} h' D_T^* q \ell$$

$$- \sqrt{2} h' D_T e^c u^c + \sqrt{2} h' D_T^* e^c u^c$$

$$+ 2 f \Sigma_T d^c e^c + 2 h' D_T u^c d^c + 2 h' D_T^* u^c d^c.$$  \hspace{1cm} (13)

The dimension five operators are written by the Yukawa couplings $h$, $f$ and $h'$ as follow:

$$C_{L}^{ijkl} = c h_{ij} h_{kl} + x_1 f_{ij} f_{kl} + x_2 h_{ij} f_{kl} + x_3 f_{ij} h_{kl}$$

$$+ x_3 f_{ij} h_{kl} + x_5 h_{ij} f_{kl},$$  \hspace{1cm} (14)

$$C_{R}^{ijkl} = c h_{ij} h_{kl} + y_1 f_{ij} f_{kl} + y_2 h_{ij} f_{kl} + y_3 f_{ij} h_{kl}$$

$$+ y_4 h_{ij} h_{kl} + y_5 h_{ij} f_{kl} + y_6 h_{ij} f_{kl} + y_7 f_{ij} h_{kl}$$

$$+ y_8 h_{ij} h_{kl} + y_9 h_{ij} h_{kl} + y_{10} h_{ij} h_{kl}. $$  \hspace{1cm} (15)

The common coefficient $c$ is given as $c = (M_T^{-1})_{11}$. We obtain the other coefficients as the following:

$$\begin{align*}
(x_1, x_2, x_3, x_4, x_5) &= (M_{54}^{-1}, M_{51}^{-1}, M_{14}^{-1}, \sqrt{2} M_{13}^{-1}, \sqrt{2} M_{53}^{-1}), \\
(y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8) &= (M_{54}^{-1}, \sqrt{2} M_{64}^{-1} - M_{51}^{-1} - \sqrt{2} M_{61}^{-1} - M_{14}^{-1}, \\
\sqrt{2} M_{12}^{-1} - \sqrt{2} M_{21}^{-1} + 2 M_{22}^{-1} + \sqrt{2} (-M_{21}^{-1} + M_{31}^{-1}), \\
\sqrt{2} (M_{24}^{-1} - M_{34}^{-1}), 2(-M_{22}^{-1} + M_{32}^{-1})), 1 \hspace{1cm} (16)
\end{align*}$$
SUPPRESSION OF PROTON DECAY

Let us now investigate the conditions required to suppress the proton decay rate in the minimal SO(10) model described above. We first note that the four-fermion proton decay operators are produced by gaugino and Higgsino dressing of the dimension five operators. The four-fermion operators of LLLL type get dominant contribution from wino exchange and therefore retain the same flavor structure as that of the original dimension five supersymmetric operator (we have assumed universality of scalar masses for suppressing FCNC processes). However, as far as the RRRR operators are concerned, they receive contributions only from Higgsino exchange which can involve the top quark and tau lepton Yukawa couplings. For instance in the case of SU(5) model, the contribution to the decay mode \( p \to K \bar{\nu}_r \) gets so enhanced because of this that it exceeds the current experimental bound for any \( \tan \beta \) as long as stop mass is less than around 1 TeV\(^{[11]}\).

Since the original Yukawa couplings \( h, f, \) and \( h' \) are functions of fermion Yukawa couplings \( Y_u, Y_d \) and \( Y_e \) via Eqs. (3), their textures are roughly determined from the experimental inputs of quark and lepton masses and mixings. For instance, to fit the strange quark and muon masses, bottom-tau unification, relations among CKM mixings and also large mixings of neutrinos, the coupling \( f \) is almost determined to have the form \( f \sim \frac{\lambda^2}{\lambda^1} \frac{\lambda}{1} m_s/m_b, \) where \( \lambda \sim 0.2. \) A naive implication of this is that since up-type quark masses are more hierarchical than down-type ones (i.e. \( m_u/m_c \ll m_d/m_s, m_c/m_t \ll m_s/m_b \)), the expression for the up-type Yukawa matrix, \( Y_u, \) in Eq. (3) requires the following two typical choices: (a) there is cancellation among \( h, f, \) and \( h' \), or (b) \( h \) itself has a hierarchical form similar to the up-type quark masses. The first choice (a) is the case where \( [1,2] \) block of \( h_{ij} \) is not far smaller than \( f_{ij}, \) but \( r_{2,3} \) are chosen to be certain values to make \( m_u, m_c \) are hierarchically small. The second choice (b) corresponds to the case where \( r_{2,3} \sim 0. \) The second choice appears to be required to suppress the proton decay. Let us discuss the reason.

In order to suppress the decay rate, we need small couplings for first and second generations in the expressions in Eqs. (14-15). Clearly this would also require a cancellation among \( h, f \) and \( h' \) if we take the first choice (a). Since in general the coefficients \( r_2, r_3 \) in up-type Yukawa matrix and \( x_i \) and \( y_i \) in proton decay operators are unrelated, one must find a situation where both cancellations can be achieved in a satisfactory manner so as to be consistent with all data. However, if we take the choice (a), the cancellation cannot happen naturally due to the following reasons.

First, let us discuss the \( 126 \) Higgs contribution. Since there is an opposite signature for one of the coefficients in the LLLL and RRRR operators, \( y_3 = -x_3, \) the cancellation required to obtain small Yukawa coupling for \( Y_u \) by tuning \( r_2 f \) cannot simultaneously suppress both LLLL and RRRR operators. Thus, in general, it is hard to suppress proton decay rate by tuning \( 126 \) colored Higgs mixing \( X_{14}. \)

Next let us see the contribution from \( 120 \) Higgs field. The coefficient \( C_{ijkl}^{FF} \) is symmetric in the indices \( kl \) due to SU(3) × SU(2) contraction. Therefore \( h' \) contribution is absent for the \( q_S q_S \) part in LLLL operator, whereas it is of course present in the fermion masses. Thus, if the cancellation in \( Y_u \) happens by tuning \( r_3 h', \) such cancellation will not help in suppressing the LLLL operator.

The above discussions lead to the fact that the proton decay rate cannot be suppressed in natural way if we take the choice (a). As we will see numerical studies later, we require a fine-tuning to the level of 0.01% to suppress all the proton and neutron decay modes when we consider general parameter fitting in the choice (a). The choice (b) \( (r_{2,3} \sim 0) \) is necessary to achieve natural suppression of proton decay as a result.

We now show how the proton decay rate in the choice (b) is suppressed rather than the minimal SU(5). If \( r_{2,3} \approx 0, \) the LLLL dimension five operator can be a form as \( C_{ijkl}^{FF} \sim \gamma_{h'} x_i (Y_u)_{ikl} \) and in the RRRR operator \( C_{ijkl}^{RR} \), \( kl \) part is also related to \( Y_u. \) This will correspond to the case where \( X_{14}, Y_{15} \sim 0. \) We will give an example later where this can happen but first we note that for the case just noted, the RRRR contribution to \( p \to K \bar{\nu}_r \) mode is suppressed compared to the usual SU(5) models for the entire range of \( \tan \beta \) up to 50. Here, the dominant contribution is proportional to \( \lambda_u \) giving a suppression factor \( \lambda_u/\lambda_d \sim 1/100 \) for \( \tan \beta \sim 50 \) compared to minimal SU(5) case. Similarly, the \( kl \) part of \( C_L \) are also related to the \( Y_u \) instead of \( Y_d, \) the LLLL contribution to the \( p \to K \bar{\nu} \) is also suppressed even for \( \tan \beta \sim 50, \) compared to the SU(5) model (since \( \lambda_u/\lambda_d \sim 1/5). \) However as it turns out, these suppressions are not enough. We need to specify the Yukawa textures for the purpose as we see below.

Before describing the specific choice of Yukawa textures, let us show the numerical values for the pro-
The coefficients we need are determined from the fit to the vacuum expectation values and the Higgs couplings. According to our numerical studies, some of the mixing angles must be about a few percent in the case of tan β ∼ 50 to suppress the decay. However, the mixing angles can become larger as tan β becomes smaller. Details will be given in a future paper.\[22\]

Without the particular choice of texture, as mentioned above, the proton decay cannot be suppressed naturally in these models unless the tan β is very small.

The proton lifetime for p → Kν for this choice of texture can be larger than the current experimental bound, τp ∼ 2 · 10^{33} years for any tan β (using the lightest colored Higgsino mass) to be 2 · 10^{16} GeV and squark mass scale around 1 TeV). All other nucleon decay modes are suppressed as well. In our calculation, we use long- and short-distance renormalization factor, \( A_L = 1.43 \) and typically \( A_S = 1.8 \) similar to Ref.\[23\].

Given the above texture for Yukawa couplings, we find that \( U_{e3} \) is restricted to a range. In figure 1 we plot \( U_{e3} \) as a function of \( r_3 \) with \( f_{11,12} \rightarrow 0 \) (as required in the most preferred texture) and the value at \( |r_3| \sim 0.1 \) is most important. We find that \( U_{e3} \) is between 0.07 to 0.09. For this fit, \( \sin^2 2\theta_{23} \) is maximal and \( \tan^2 \theta_{SOLAR} \sim 0.4 \). The prediction for \( \sin \left| \delta_{MNS} \right| \) lies between 0.3 to 0.7 for \( |r_3| \sim 0.1 \) as shown in figure 2. The Yukawa matrices are assumed to be hermitian in order to keep the model free from SUSY CP problem.

The presence of \( h' \) is a necessity to suppress proton decay (suppress \( \tilde{A}_{hh} \)) and fit the fermion masses. This \( h' \) also helps to explain CKM CP violation.\[6\]

Finally, we show how the above proton decay suppression arises by an adjustment among different VEVs (or symmetry breaking parameters). In the above, we need to have \( r_3 \sim 0 \) in the Eq.\[6\]. Since this is satisfied in Eq.\[24\] for the SU(5) condition, it may be a hint that we stay close to the SU(5) symmetric vacuum. Secondly, we need \( r_3 \sim 0 \) while suppressing \( 126 \) colored Higgs contribution to proton decay, namely only \( U_{14} \) is enhanced while other mixings for the sub-multiplets in \( 126 \) are small.

\[ U_{14} \gg V_{15}, X_{14}, Y_{15}. \]
In order to explain how these conditions may be satisfied in the minimal SO(10) model, we denote the VEVs of the submultiplets in 210 multiplet that break SO(10) symmetry as follows: \( \Phi_1 : (1, 1, 1), \Phi_2 : (15, 1, 1), \Phi_3 : (15, 1, 3) \) (where numbers in the parenthesis denote \( G_{122} \) quantum numbers). The VEVs \( \Phi_i \) are around the unification scale of three gauge couplings. Recall that in the SU(5) symmetric vacua \( [17, 18] \), the \( \Phi_i \)'s satisfy the following relation: \( \sqrt{6}\Phi_1 = \sqrt{2}\Phi_2 = \Phi_3 \) (We have used the same normalization as in the Ref. \( [17] \)). They lead to the SU(5) relations \( [20, 28] \). Now we perturb the Higgs potential with a small coupling, \( \lambda_2 H\Delta \Phi \). We obtain \( r_2 \propto \lambda_1 (\sqrt{6}\Phi_1 - \Phi_3) \) (where \( \lambda_1 \) is associated with \( \lambda_1 H D\Phi \) term). If \( \sqrt{6}\Phi_1 = \Phi_3 \), we have \( r_3 \simeq 0 \). We obtain the Higgs mixings

\[
U_{14} \simeq -6\sqrt{5}\lambda_2/\eta \frac{\sqrt{2}\Phi_2 - \Phi_3}{\sqrt{6}\Phi_1 + \sqrt{2}\Phi_2 + 8\Phi_3} + \cdots, \quad (31)
\]

\[
X_{14} \simeq -2\sqrt{15}\lambda_2/\eta \frac{\sqrt{6}\Phi_1 - \sqrt{2}\Phi_2}{\sqrt{6}\Phi_1 + 3\sqrt{2}\Phi_2 + 6\Phi_3} + \cdots, \quad (32)
\]

where \( \eta \) is a coupling of \( \Phi \Delta \bar{\Delta} \) term. We also have similar terms for \( V_{15}, V_{15} \) and \( V_{16} \). All these terms have different denominators. All the Higgs mixing angles tend to zero in the limit \( \lambda_2 \to 0 \). However, suppose that \( \sqrt{6}\Phi_1 + \sqrt{2}\Phi_2 + 8\Phi_3 \sim 0 \) is satisfied, only \( U_{14} \) can be of finite value, all other mixing angles are zero and Eq. (30) is satisfied. On such a vacuum, the proper strange quark and the muon masses can be realized by an enhancement of \( U_{14} \), and Yukawa coupling of up-type quark, \( Y_u \), is almost proportional to \( h \). This is just an example, and in our detailed quantitative work, we keep all other terms in the Higgs potential and we satisfy Eqs. \( (23, 31) \) to suppress the proton potential and we satisfy Eqs. \( (23, 31) \).
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