Rare $\Omega^{-} \to \Xi^{0}(1530)\pi^{-}$ decay in the Skyrme model

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(Dated: November 18, 2018)

Rare nonleptonic $\Omega^{-} \to \Xi^{0}(1530)\pi^{-}$ decay branching ratio is estimated by means of the QCD enhanced effective weak Hamiltonian supplemented by the SU(3) Skyrme model used to estimate the nonperturbative matrix elements. Using mean values for experimental input parameters and the Skyrme charge $e = 4.75$ we obtain the rate which is in a good agreement with data.

PACS numbers: PACS number(s): 12.39.Dc, 12.39.Fe, 13.30.Eg

It is well known that the nonleptonic weak decays of baryons can be reasonably well described in the framework of the Standard Model [1, 2]. Recently both s-, p-wave nonleptonic hyperon and p-wave $\Omega^{-}$ decay amplitudes were quite successfully reproduced by the SU(3) extended Skyrme model with the QCD enhanced effective weak Hamiltonian [3, 4]. The decay amplitudes were described through the current-algebra commutator, the ground-state baryon pole terms and factorizable contributions. The nonperturbative quantities, i.e. the baryon six dimensional operator matrix elements, were determined using the SU(3) extended Skyrme model. For the s-wave hyperon decay amplitudes correct relative signs and absolute magnitudes were obtained. For the p-waves all relative signs were correct, with their relative magnitudes roughly following the experimental data. One is thus faced with an obvious question: could an analogous approach work equally well for the rare nonleptonic $\Omega^{-} \to \Xi^{0}_{3/2^{+}}(1530)\pi^{-}$ weak decay? Such a question should be considered in connection with the measurements of the $\Omega^{-}$ lifetime and branching ratios [2]:

$$\tau(\Omega^{-})_{\text{exp}} = (82.1 \pm 1.1) \text{ ps} \quad (1)$$

$$\Gamma(\Omega^{-} \to \Xi^{0}\pi^{-})/\Gamma_{\text{tot}} = (6.4^{+5.1}_{-2.0}) \times 10^{-4} \quad (2)$$

In this report our goal is to test whether the effective weak Hamiltonian and the SU(3) Skyrme model are able to predict the rare nonleptonic $\Omega^{-} \to \Xi^{0}\pi^{-}$ decay following the method in [3, 4]. To this end we shall employ the minimal number of couplings concept of the Skyrme model to estimate the nonperturbative matrix elements of the 4-quark operators [3, 4, 5] for decuplet-decuplet transitions. Throughout this report we use the arctan ansatz for the Skyrme profile function $F(r)$ [5], which allows to calculate the pertinent overlap integrals analytically with accuracy of the order of $\lesssim 1\%$ with respect to the exact numerical results.

The starting point in such an analysis is the effective weak Hamiltonian in the form of the current $\otimes$ current interaction, enhanced by QCD,

$$H_{\text{eff}}^{\text{fj}}(\Delta S = 1) = \sqrt{2}G_{F}V_{ud}^{*}V_{us}\sum c_{i}O_{i} \quad (3)$$

where $G_{F}$ is the Fermi constant and $V_{ud}^{*}V_{us}$ are the Cabibbo-Kobayashi-Maskawa matrix elements. The $O_{i}$ are the 4-quark operators and the $c_{i}$ factors are the QCD-short distance Wilson coefficients [2, 9]: $c_{1} = -1.90 - 0.61\zeta$, $c_{2} = 0.14 + 0.020\zeta$, $c_{3} = c_{4}/5$, $c_{4} = 0.49+0.005\zeta$, with $\zeta = V_{us}^{*}V_{cs}/V_{ud}^{*}V_{us}$. For the purpose of this work we neglect the so called Penguin operators since their contributions are proven to be small [4]. Without QCD corrections, the Wilson coefficients would have the following values: $c_{1} = -1$, $c_{2} = 1/5$, $c_{3} = 2/15$, $c_{4} = 2/3$. In this paper we consider both possibilities and compare the resulting rates.

The techniques used to describe nonleptonic $\Omega^{-}$ decays (in this work we have only the $3/2^{+} \to 3/2^{+} + 0^{-}$ reaction) are known as a modified current-algebra (CA) approach. The general form of the decay amplitude reads:

$$\langle \pi(q)B'(p')|H_{\text{eff}}^{\text{fj}}|B(p)\rangle = \frac{\Gamma(\Omega^{-} \to \Xi^{0}_{3/2^{+}}(p') + \pi^{-}_{0}(q))}{\Gamma_{\text{tot}}} \quad (4)$$

The $W(p)$ denotes the Rarita-Schwinger spinor. The parity-violating amplitudes $\mathcal{A}$ correspond to the s-wave and parity-conserving amplitudes $\mathcal{B}$ correspond to the p-wave $\Omega^{-}$ decays, respectively. Since the decay of $\Xi^{0}\pi^{-}$ is strongly limited by phase space (momentum transfer $\simeq 1$ MeV at the peak value of the $\Xi^{0}$ mass), we will neglect the amplitudes $\mathcal{C}$ and $\mathcal{D}$ (d- and f- waves). The decay probability $\Gamma(\Omega_{3/2^{+}}^{-}(p) \to \Xi^{0}_{3/2^{+}}(p') + \pi^{-}_{0}(q))$ is:

$$\Gamma = \frac{|q|m_{f}}{18\pi m_{\Omega}} \left\{ \left( \frac{E'}{m_{f}} + 1 \right) \left[ \frac{E'^{2}}{m_{f}^{2}} + \frac{E'}{m_{f}} + \frac{5}{2} \right] |\mathcal{A}|^{2} \right. + \left. \left( \frac{E'}{m_{f}} - 1 \right) \left[ \frac{E'^{2}}{m_{f}^{2}} - \frac{E'}{m_{f}} + \frac{5}{2} \right] |\mathcal{B}|^{2} \right\}, \quad (5)$$

$$|q|^{2} = \left[ (m_{\Omega}^{2} - m_{f}^{2} + m_{\phi}^{2})/2m_{\Omega} \right]^{2} - m_{\phi}^{2}, \quad (6)$$

$$E' = (m_{\Omega}^{2} + m_{f}^{2} - m_{\phi}^{2})/2m_{\Omega}. \quad (7)$$

Here $\Omega^{-}$ is at rest, $m_{f}$ denotes the final baryon mass and $m_{\phi}$ is the mass of the emitted meson.
We calculate the decay amplitudes by using the so-called tree-diagram approximation at the particle level, i.e. factorizable and pole diagrams plus the commutator term. The amplitudes receive contributions from the commutator term and the pole diagram, respectively:

\[ A_{\text{Comm}} = -\frac{1}{\sqrt{2} f_\pi} a_{\Xi^-, \Omega}, \]  
\[ B_P = \frac{-g_{\Xi^0 \Xi^+ \pi^-}}{m_\Omega - m_{\Xi^+}} a_{\Xi^-, \Omega}. \]  

However, another state rather strongly coupled to \( \Xi^+ \pi \) and with a mass close to the \( \Omega^- \) exists, the \( \Xi(1820, J^P = 3/2^-) \) resonance \[10\], whose mass fits nicely to the Gell-Mann-Okubo formula for an octet of \( 3/2^- \) baryons. Therefore, there is a pole term in the s-wave amplitudes, \( A_P \):

\[ A_P = \frac{g_{\Xi^+ \pi^-}}{m_\Omega - m_{\Xi^+}} b_{\Xi^-, \Omega}. \]  

The commutator and the baryon-pole amplitudes contain weak matrix elements defined as

\[ \begin{pmatrix} a_{\Xi^-, \Omega} \\ b_{\Xi^-, \Omega} \end{pmatrix} = \sqrt{2} G_F V_{ud} V_{us} \begin{pmatrix} \langle \Xi^+ | c_i O_P^{PC} | \Omega \rangle \\ \langle \Xi^+ | c_i O_{PV}^{PC} | \Omega \rangle \end{pmatrix}, \]  

where the important parts are the Wilson coefficients and the 4-quark operator matrix elements.

The factorizable contributions to s- and p-waves, are calculated by inserting the vacuum states; it is therefore a factorized product of two current matrix elements, where the decuplet-decuplet matrix element of the vector and the axial-vector currents reads:

\[ \langle \Xi^+ (p') | \begin{pmatrix} V_{\mu} \\ A_{\mu} \end{pmatrix} | \Omega^- (p) \rangle = g_{\Xi^+ \Omega} \langle \Xi^0 | \bar{W}_e (p) \gamma_{\mu} (1/\gamma_5) W^\mu (p') \rangle. \]  

Summing over all factorizable contributions gives the following expressions for the amplitudes:

\[ \begin{pmatrix} A \\ B \end{pmatrix} = G_F \frac{\sqrt{2}}{3} V_{ud} V_{us} (m_\Omega + m_{\Xi^+}) f_\pi \] 
\[ \times g_{\Xi^+ \Omega} [c_1 - 2(c_2 + c_3 + c_4)], \]  

where the \( g_{\Xi^+ \Omega} \) represents the form-factor of the spatial component of the vector-current.

The total theoretical amplitudes are:

\[ A_{\text{th}}(m_\pi^2) = A_{\text{Comm}}(0) + A_P(m_\pi^2) + A_S(m_\pi^2), \]  
\[ B_{\text{th}}(m_\pi^2) = B_P(m_\pi^2) + B_S(m_\pi^2), \]  

were the relative signs between commutator, pole and factorizable contributions are determined via SU(3) and the generalized Goldberger-Treiman relation.

In order to estimate the 4-quark operator matrix elements entering \[11\], we take the Skyrme model where baryons emerge as soliton configurations of the field \( U \) of pseudo-scalar mesons \[11\] \[12\] \[13\] \[14\] \[15\] \[16\]. The SU(3) extended Skyrme model action is \( \mathcal{L} = \mathcal{L}_\sigma + \mathcal{L}_\text{Sk} + \mathcal{L}_W + \mathcal{L}_{BW}, \) where \( \mathcal{L}_\sigma \), \( \mathcal{L}_\text{Sk} \), \( \mathcal{L}_W \), and \( \mathcal{L}_{BW} \) denote the \( \sigma \)-model, Skyrme, symmetry breaking (SB), and Wess-Zumino (WZ) terms, respectively \[16\] \[17\] \[18\]. Extension of the model to the strange sector \[13\] \[14\] \[16\] is done by an isospin embedding of the static hedgehog ansatz into an SU(3) matrix, which is a subject of a time dependent rotation \( U(r, t) = A(t)U(r)A(t) \) by a collective coordinate matrix \( A(t) \in SU(3) \). The generalized velocities are defined by \( A'(t)A(t) = \frac{1}{2} \sum_{a=1}^8 \lambda_a \alpha_a \) and the profile function is interpreted as a chiral angle that parameterizes the soliton. The collective coordinates \( \alpha_a \) are canonically quantized to generate the states that possess the quantum numbers of the physical strange baryons. In order to account for a non-zero strange quark mass the appropriate symmetry breaking terms should be included.

In this work we will use the SU(3) extended action \( \mathcal{L} \) with the following set of parameters \[18\] \( \bar{x} = 36.97, \beta^t = -28.6 \text{MeV}^2, \beta^t = 4.12 \times 10^7 \text{MeV}^4 \), determined from the masses and decay constants of the pseudo-scalar mesons. The \( \bar{x} \) term is responsible for the baryon mass splittings and the admixture of higher representations in the baryon wave functions. However, we use the SU(3) symmetric baryon wave functions in the spirit of the perturbative approach to SB. Indeed, we have shown in \[3\] \[4\], that the WZ and SB contributions through the weak operator matrix elements are small and introduce few % uncertainty of the decay amplitudes dominated by the Skyrme term which scales like \( 1/e \).

Since the coupling \( f_\pi \) is equal to its experimental value, for the evaluation of rare nonleptonic \( \Omega^- \) decay rate the only remaining free parameter, as in \[3\] \[4\], is the Skyrme charge \( e \). It has been shown in Figure 12 of \[18\] that for \( 4.0 \leq e \leq 5.0 \) the mass spectrums of \[8\] \[10\] and \[16\]-plets are reasonably well described. This is the reason we are using that particular range of \( e \) further on.

The 4-quark operator matrix element contribution to the commutator \[8\] and to the baryon pole term \[10\], evaluated as a function of the Skyrme charge \( e \), in units of \( 10^{-8} \text{ GeV} \), is

\[ a_{\Xi^-, \Omega^-} = \begin{pmatrix} \text{QCD off} & \text{QCD on} \\ -1.86 & -3.52 \\ -1.31 & -2.75 \\ -1.22 & -2.55 \end{pmatrix} \]  
\[ e \]  
\[ 4.00 \]  
\[ 4.75 \]  
\[ 5.00 \]  

The size of the strong coupling from pole term \[10\], determined via generalized Goldberger-Treiman relation,

\[ g_{\Xi^+ \Xi^0 \pi^-} = \frac{m_{\Xi^+} + m_{\Xi^0}}{2f_\pi} g_{\Xi^+ \Xi^0 \pi^-} \approx 9.11 g_{\Xi^+ \Omega}^{\text{pm}}, \]  

is close to the estimate given in \[19\].

The strong coupling \( g_{\Xi^+ \Xi^0 \pi^-} \), which enters in the calculation of \( A_P \), we extract from experimental value for
partial width $\Gamma(\Xi^{*+} \to \Xi^+\pi)$. Using strong effective Lagrangian for spin $3/2^-$ baryon $\Xi^{*+}$

$$\mathcal{L}_{\text{int}}(\Xi^{*+} \to \Xi^+\pi) = g_{\Xi^{*+}\Xi^+\pi} \bar{W}_\mu(p') W^\mu(p)\phi_\pi(q).$$

(18)

where $W^\mu$ denotes the $3/2^-$ Rarita-Schwinger spinor, we obtain for $\Xi^{*+}$ at rest,

$$\Gamma(\Xi^{*+} \to \Xi^+\pi) = \frac{g_{\Xi^{*+}\Xi^+\pi}^2}{36\pi m_\Xi^2 m_{\Xi^+}} (E' + m_{\Xi^+})$$

(19)

$$\times \left[ 2E'^2 + 2E'm_{\Xi^+} + 5m_{\Xi^+}^2 \right].$$

The $E'$ and $|q|$ are given in (4) and (7). From experiment we have $\Gamma_{\text{tot}}(\Xi^{*+}) = (24 \pm 6)$ MeV and

$$\Gamma(\Xi^{*+} \to \Xi^+\pi)/\Gamma_{\text{tot}}(\Xi^{*+}) = (0.30 \pm 0.15).$$

(20)

The generalized Goldberger-Treiman relation applied to $\Omega$ gives the product $b_{\Xi^{*+}\Xi^+\pi} g_{\Xi^{*+}\Xi^+\pi}$, which does not depend on the phase of the exchanged $\Xi^{*+}$ in the pole diagram. It has been found in (20) that the sign of an axial-vector matrix element, between particles of the opposite parity, is negative. Than from (10) and (20) we find $g_{\Xi^{*+}\Xi^+\pi} = - (0.48 \pm 0.14)$, a small negative value with large uncertainty.

The weak matrix elements of the same spin but opposite parity baryons are found to be about the same as the ones including ground states only (20), i.e. $b_{\Xi^{*+}\Xi^+\pi} \cong a_{\Xi^{*+}\Xi^+\pi}$. This together with (8) and (10) gives

$$A_P = -(0.44 \pm 0.12) A_{\text{Comm}},$$

(21)

up to the sign, similar to the conclusion in (19). Since the resonance $\Xi^{*+}$ is known with quite a large error, the same error is also present in (21).

We proceed with the computation of the vector and axial-vector form-factors $g_{\nu V(A)}^{\Xi^{*+}\Xi^+\pi}$, in terms of $g_{\nu V(A)}^{pn}$, via SU(3) Clebsh-Gordan coefficients. Using from experiment, $g_{A}^{pn}/g_{V}^{pn} = 1.26$, we find an agreement with the SU(6) result (18), as we should:

$$g_{V(A)}^{\Xi^{*+}\Xi^+\pi} \cong g_{V(A)}^{\Xi^{*+}\Xi^+\pi} \cong \left( \begin{array}{c} 1.1 \\ 0.7 \end{array} \right).$$

(22)

Factorizable amplitudes $A_S$ and $B_S$ are very slowly changing functions, via the $g_{V(A)}^{pn}$, with respect to the Skyrme charge $e$, (for details see Fig’s 2-4 in (18)). For QCD off/on we found from (13) and (22):

$$A_S^{\text{off}} = -5.65 \times 10^{-8}, A_S^{\text{on}} = -6.32 \times 10^{-8},$$

$$B_S^{\text{off}} = -90.1 \times 10^{-8}, B_S^{\text{on}} = -100.8 \times 10^{-8}. \quad (23)$$

The mean values of theoretical amplitudes $A_{\text{Comm}}, A_P$ and $B_P$ for QCD off/on, as a functions of the Skyrme charge $e$, are presented in Table 1.

Rare nonleptonic two-body $\Omega^-$ decay partial width formulae

$$\Gamma(\Omega^- \to \Xi^{*0}\pi^-) = \left\{ 9.000617 |A_S + A_{\text{Comm}} + A_P|^2 + 0.000148 2.697 \times 10^{-4} \text{ GeV}, \quad (24) \right.$$
with the matrix element $a_{\Omega^-\Omega^-} = -2.75 \times 10^{-8}$ GeV and factorizable contributions from Table 1 in [25].

produces the $\Omega^-$ nonleptonic decay amplitudes: $B_{\Omega K} = 6.02, B_{\Omega K}(\Omega^-) = 1.48$ and $B_{\Omega K}(\Omega^0) = 1.29$, in units of $[10^{-6}$ GeV$^{-1}]$, which are closer to the conclusions in [21] and in better agreement with experiment than the ones obtained in [4]. The same should hold for the p-wave nonleptonic hyperon decays [2].

Altho our dynamical scheme is the same as the one in [19], situation has changed during the past twenty years. The important improvements happened in the areas of QCD corrections to the effective weak Hamiltonian and in the further development of the Skyrme model. The latter becomes a good candidate for the correct description of the higher SU(3) representation multiplets [18, 22] and a good tool for evaluation of the nonperturbative quantities like the 4-quark operator matrix elements between different baryon states. Comparing the result [25] with Table 1 from [19], we conclude that the dynamical scheme is working well in both cases. In this work we are using well accepted result for the NLO computations of the QCD corrections [4]. According to [5] the errors generated by different choice of renormalization scheme and scale are less than 10%. Overall QCD corrections are more stable and smaller today; penguins are about one order of magnitude smaller and also $c_1 \simeq -3 \rightarrow -1.9$ [5].

Skyrme model provide us with 4-quark operator matrix elements, in different way than the MIT bag model did in [1, 19], producing rough agreement with measurement [20]. Finally, according to (e), as a bonus we obtain better agreement with experiment for the nonleptonic $\Omega^-$ and the p-wave hyperon decays.

Obviously, not all details are under full control. For example $m_s$ corrections are neglected and the uncertainties contained in the input parameters [24] are unfortunately large. Nevertheless, the QCD-corrected weak Hamiltonian $H^{\text{eff}}_{\text{QCD}}$, supplemented by the Skyrme model, for the mean values of the input parameters, lead to the correct description of the rare $\Omega^- \rightarrow \Xi^0(1530)\pi^-$ decay.

This work was supported by the Ministry of Science of the Republic of Croatia under the contract No. 00980102.

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