Reconsideration of quantum measurements

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(Dated: October 26, 2018)

Abstract

The usual conjectures of quantum measurements approaches, inspired from the traditional interpretation of Heisenberg’s ("uncertainty") relations, are proved as being incorrect. A group of reconsidered conjectures and a corresponding new approach are set forth. The quantum measurements, regarded experimentally as statistical samplings, are described theoretically by means of linear integral transforms of quantum probability density and current, from intrinsic into recorded readings. Accordingly, the quantum observables appear as random variables, valuable, in both readings, through probabilistic numerical parameters (characteristics). The measurements uncertainties (errors) are described by means of the intrinsic-recorded changes for the alluded parameters or for the informational entropies. The present approach, together with other author’s investigations, give a natural and unified reconsideration of primary problems of Heisenberg’s relations and quantum measurements. The respective reconsideration can offer some nontrivial elements for an expected re-examination of some disputed subsequent questions regarding the foundations and interpretation of quantum mechanics.

PACS numbers: 03.65.Ta, 03.65-w, 03.65.Ca, 01.70+w

Keywords: quantum measurements, Heisenberg’s relations, uncertainty indicators, quantum randomness.
I. INTRODUCTION

In connection with the foundation and interpretation of quantum mechanics (QM) the problem of quantum measurements (QMS) description is often regarded [1] as: "probably the most important part of the theory". The respective problem consists in the search for theoretical descriptions of the measurements regarding the observables of quantum microparticles. It was approached in a large number of works published during the history of QM, particularly [1] in the last decades. Many of the mentioned approaches were founded on conjectures, which somehow are inspired from the early orthodox views on QM. Especially were referred the Heisenberg’s ("uncertainty") relations (HR), currently regarded as cornerstone for foundation and interpretation of QM. On the other hand nowadays [2]: "the idea that there are defects in the foundation of orthodox quantum theory is unquestionable present in the conscience of many physicists".

The spirit of the alluded idea motivated [3, 4, 5, 6] a careful reinvestigation of the significance of HR. The reinvestigation shows [3, 4, 5, 6] that HR must be reinterpreted in a more natural manner and, especially, deprived of all the attributes usually assumed by the QMS approaches. Then it directly appears the necessity of a reconsideration of QMS problem. Such a reconsideration is the aim of the present paper.

For our aim in the next section, we present the main alluded conjectures and their corresponding shortcomings. Subsequently in Sec.III we present a new approach of the QMS problem. The respective approach is inspired from a view [7, 8] about the measurements of classical (non-quantum) random observables. The new approach is detailed through a simple exemplification in Sec.IV. We end our considerations in Sec.V with some conclusions.

II. CONJECTURES AND SHORTCOMINGS

In the afferent publications, one finds a multitude of diverse approaches of QMS (for a comprehensive bibliography see [1]). A careful examination of the things shows that, in their essence, many of the respective approaches imply somehow one or more of the following conjectures(C):

- **C.1:** The description of QMS must be regarded and developed as an enlargement of the traditional interpretation of HR (TIHR).
• **C.2:** Between quantum and classical measurements there exists a fundamental distinction, due to the exclusive existence of HR in quantum cases.

• **C.3:** The description of QMS must take into account the non-null and unavoidable jumps in the state of the measured system.

• **C.4:** A QMS consists in a single detection act representable as collapse (reduction) of the wave function.

(Note that the conjectures **C.2-3** are inferable in part from **C.1** -see below).

The conjectures **C.1-3** are inspired from TIHR. Then, for a pertinent analysis of them, let us remind the main elements of TIHR. Therefore firstly we note that for two observables A and B the HR are known in the QM-theoretical version:

\[ \Delta_{\Psi} A \cdot \Delta_{\Psi} B \geq \frac{1}{2} \left| \langle [\hat{A}, \hat{B}] \rangle_{\Psi} \right| \]  

(1)

respectively in the thought experimental (TE) version

\[ \Delta_{TE} A \cdot \Delta_{TE} B \sim \hbar \]  

(2)

In (1) \([\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}\) denotes the commutator of the operators \(\hat{A}\) and \(\hat{B}\), while the other usual QM notations are those specified below in relations (3)-(4). In (2) \(\Delta_{TE} A\) and \(\Delta_{TE} B\) signify the so-called TE-uncertainties for simultaneous measurement of two canonically conjugated (or complementary) observables A and B while \(\hbar\) is the Planck’s constant.

Based on the relations (1)-(2) TIHR promoted the following main assertions (**Ass.**):

• **Ass.1:** The quantities \(\Delta_{\Psi} A\) and \(\Delta_{TE} A\) from (1) and (2) have the same significance of measuring uncertainty for A. Consequently, the respective relations refer to the description of a characteristic of QMS.

• **Ass.2:** In QMS a single observable can be evaluated without any uncertainty while two observables A and B are compatible (i.e. simultaneously measurable without uncertainties) respectively incompatible as they are commutable or not (i.e. as \([\hat{A}, \hat{B}] = 0\) respectively \([\hat{A}, \hat{B}] \neq 0\)).

• **Ass.3:** The relations (1)-(2) do not have analogues in classical physics because they imply the quantum Planck’s constant \(\hbar\).
It is easy to see that the assertions **Ass.1-3** of THIR are irrefutably contradicted by the following arguments (**Arg.**):

- **Arg.1:** The relation (1) is in fact only a restricted consequence of the more general formula

\[ \Delta_{\psi}A \cdot \Delta_{\psi}B \geq \left| \langle \delta_{\psi}A \hat{\psi}, \delta_{\psi}B \hat{\psi} \rangle \right| \] (3)

(For the significance of the QM notations see below the relations (3)-(6)). Particularly there are situations in which (1) is inapplicable and only (3) remains valid in the trivial form \( 0 = 0 \). Such is the case of pairs \( L_z - \varphi \) (angular momentum-angle) and \( N - \phi \) (number-phase) in eigenstates of \( L_z \) respectively of \( N \).

- **Arg.2:** The observables implied in (1) and (3) have the characteristics of random variables. They are similar with the macroscopic observables from classical statistical physics. The respective similarity is pointed out by the existence of some fluctuation formulae which are completely analogous with (1) and (3). The alluded classical observables and formulae describe the macroscopic bodies themselves (i.e. their intrinsic characteristic) but not the aspects of the measurements on such bodies. Similarly, relations (1) and (3) and the afferent quantities must be regarded as referring to the intrinsic properties (fluctuations) of quantum microparticles but not to the qualities (uncertainties) of QMS.

- **Arg.3:** The relations (2) are founded only on the features of TE. However, as it is noted in there is not known any real experiment capable to attest TIHR with a convincing accuracy. On the other hand, the convincingness of the relations (2) is troubled by their provisional character. The respective character is related with the fact that the relations (2) were founded on old classical limitative criteria (due to Abe and Rayleigh). However, in modern experimental physics super-resolution techniques which overstep the mentioned criteria are known. Then it is easy to see that (2) can be replaced by some super-resolution TE (SRTE) relations of the form

\[ \Delta_{SRTE}A \cdot \Delta_{SRTE}B < \hbar \] (4)
where $\Delta_{STEA}$ and $\Delta_{STEB}$ denote the corresponding uncertainties. Evidently the relations of type (1) incriminate the whole philosophy of TIHR. Now one can note that the above mentioned facts argue for the conclusion that relations (2) are only fictional formulas without any real value or significance for the description of QMS.

**Arg.4:** Conjointly with the facts noted in **Arg.1-3** one finds [3, 4, 5, 6] that the Planck’s constant $\hbar$ does not play a role of finite lower bound for the products of measuring uncertainties. Moreover [3, 20, 21] $\hbar$ proves oneself to be in the posture of a generic indicator of stochasticity (randomness) for quantum observables. In a similar posture, for classical macroscopic observables, there is the Boltzmann’s constant $k$ [12, 20].

**Arg.5:** The attunement of the energy-time pair with **Ass.2** (and consequently with TIHR doctrine) is in fact [3, 4, 5, 6] an impossible task. It generates only unproductive and interminable disputes. For the respective pair it must adopt a reasonable and natural regard. Such a regard can be obtained if, in quantum context, energy is considered as a random observable (endowed with possible fluctuations) while time is taken as a deterministic variable. Then for the energy time pair (1) is not applicable and only (3) remains valid in the trivial form $0 = 0$.

**Arg.6:** The assertion **Ass.2** irrevocably fails in some non-trivial situations regarding [3, 4, 5, 6] single observables respectively pairs of commutable observables.

**Arg.7:** It is notable [3, 4, 5, 6] the fact that **Ass.2** can not offer any reasonable base for the interpretation of some natural generalizations of the relations (1). Such is the case of bitemporal, many-observable and macroscopic-quantum-statistical generalizations.

The arguments **Arg.1-7** indubitably invalidate the whole class of assertions **Ass.1-3**. Consequently, TIHR must be denied as an unjustified doctrine. In addition, the HR (1) and (2) must be reinterpreted. In a genuine conception relation (1) appears [3, 4, 5, 6] as referring to intrinsic characteristics of quantum microparticles and belongs to a more large class of fluctuation formulas form both quantum and classical physics. In the same conception the relations (2) must be disregarded as fictitious formulas without any scientific significance. The mentioned denial of TIHR leaves without any base the conjectures **C.1-2**.
Such a fact is an insurmountable shortcoming for all the QMS approaches, which imply the respective conjectures.

As regards to the conjectures C.3 the following facts are notable. The respective conjecture was not inferred directly from the main assertions of TIHR. However, it was promoted adjacently in discussions generated by TIHR. Firstly, it was said that the measurements uncertainties are due to the interactions between measured systems and measuring devices. Secondly, it was added that the respective interactions cause jumps in the states of the measured systems. Then it was accredited the supposition that, in contrast with the classical situations, in QMS the mentioned uncertainties, interactions and jumps have an unavoidable character. Subsequently it was promoted the idea that the alluded measuring jumps must be taken into account in the description of QMS. In spite of its genesis, the above mentioned idea is proved to be incorrect by the following indubitable opinion [22]: "it seems essential to the notion of a measurement that it answers a question about the given situation existing before measurement. Whether the measurement leaves the measured system unchanged or brings about a new and different state of that system is a second and independent question". The natural acceptance of the quoted opinion brings the conjecture C.3 in an insurmountable shortcoming.

The conjecture C.4 is contradicted by natural and authorized views from both physics and mathematics. From physical viewpoint, the measurement of a random observable must have the same general features, independently of its quantum or classical nature. However, in the classical context (e.g. in the study of fluctuation [7, 8, 9, 10, 23]) the measurement of a macroscopic random observable is not viewed as a single detection act, associated with some collapse (reduction) of the corresponding probability distribution. More exactly such a measurement is regarded [23] as a statistical sampling, i.e. as an ensemble of great number of individual detection acts. The respective ensemble gives a nontrivial set of values belonging to the spectrum of the considered observable. In addition, from a mathematical viewpoint [24] a random variable must be evaluated not by a unique value but through a statistical set of values. Then it directly results that because QMS regards observables with random characteristics they must be viewed as statistical sampling (in the above-mentioned sense). Consequently there are no reason to represent (describe) a QMS as a collapse (reduction) of a wave function. The mentioned result and consequence incontestably invalidate the conjecture C.4. So one finds an insurmountable shortcoming for the respective conjecture.
The above presented aspects of the conjectures C.1-4 show that in fact many of the proposed QMS approaches have important and unavoidable shortcomings. Then it results that the problem of QMS description is still an open question that requires further investigations. The actuality of the alluded investigations is evidenced also by the nowadays scientific publications (see [1] and references), gray literature [25, 26] respectively meetings [27, 28]. In such a context, we think that the new approach that we present in the next section can be of nontrivial interest.

III. A NEW APPROACH

It is known that each approach of QMS description resorts (more or less explicitly) to some conjectures. Then, for the new approach aimed here, we suggest the set of the following reconsidered conjectures (RC):

- **RC.1** Any measurement search for information regarding the pre-existent state of the investigated system, independently of the quantum or classical nature of the respective system.

- **RC.2** Due to the randomness of quantum systems a QMS must consists obligatory in a statistical sampling i.e. in a great number of individual detection acts.

- **RC.3** QM refers to the intrinsic properties of the quantum systems and, consequently, a description of QMS must contain some extra-QM elements regarding the measuring devices and procedures.

- **RC.4** Because, in the last analysis, the results supplied by QMS refer to the measured quantum systems they must be evaluated in terms of QM.

In mind with **RC.1-4**, we develop the announced approach as follows. We consider a spin-less quantum microparticle with own orbital characteristics described by the intrinsic (I) wave function \( \Psi_I \). From theoretical viewpoint \( \Psi_I \) can be regarded as solution of the corresponding Schrödinger equation. In the following probabilistic considerations, the microparticle is regarded as equivalent with a statistical ensemble of its own replica taken at the same instant of time and described by the same wave function \( \Psi_I \). Therefore, for our purposes, the
time $t$ appears as a "decorative" variable and $\Psi_I$ will be written as a function only of the radius vector $\vec{r}$, i.e. $\Psi_I = \Psi_I(\vec{r})$. The specific observables $A_j (j = 1, 2, \ldots , n)$ of the microparticle are described by the usual QM operators (e.g. $\hat{x}_\mu = x_\mu$, and $\hat{p}_\mu = -i\hbar \frac{\partial}{\partial x_\mu} \ (\mu = 1, 2, 3)$ for Cartesian coordinates and momenta, $\hat{p} = -i\hbar \nabla$ and $\hat{L} = -i\hbar \vec{r} \times \nabla$ for momentum and angular momentum vectors or $\hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r})$ for Hamiltonian).

Because $A_j$ have random properties, as in probability theory [24], for practical purposes they are described by means of the so-called numerical parameters (or characteristics). In QM the mostly used such parameters are: the mean values $\langle A_j \rangle_I$, the correlations $C_I(A_j, A_l)$, respectively the standard deviations $\Delta_I A_j$. Note that, from a probabilistic perspective, the mentioned numerical parameters are lower order entities. Additionally, as in probability theory [24], can be used also higher order numerical parameters (e.g. higher order correlations and moments). However, such parameters are not usual in QM literature. As it is known the alluded lower order numerical parameters are defined by the relations:

\[
\langle A_j \rangle_I = \langle \Psi_I | \hat{A}_j \Psi_I \rangle = \int \Psi_I^* (\vec{r}) \hat{A}_j \Psi_I (\vec{r}) \, d^3 \vec{r} \tag{5} 
\]

\[
C_I(A_j, A_l) = \langle \delta_I \hat{A}_j \Psi_I | \delta_I \hat{A}_l \Psi_I \rangle \ , \ \delta_I \hat{A}_j = \hat{A}_j - \langle A_j \rangle_I \tag{6} 
\]

\[
\Delta_I A_j = \sqrt{C_I(A_j, A_j)} \tag{7} 
\]

In (5) and (6) $\langle f_a | f_b \rangle$ denotes the scalar product of the functions $f_a$ and $f_b$. From measurements perspective the intrinsic parameters (5)-(7) must be compared with the corresponding recorded (R) parameters:

\[
\langle A_j \rangle_R , \quad C_R(A_j, A_l) , \quad \Delta_R A_j \tag{8} 
\]

In experimental approach the parameters (8) can be obtained by statistical processing of the data from the real measurements about the observables. On the other hand, if one wishes to operate with a description of the measurements, the parameters (8) must be regarded as pieces of an adequate theoretical model. For such a model we consider that the parameters (8) are defined similarly with (5)-(7) by means of a "recorded" wave function $\Psi_R$ and with the same QM operators, i.e.

\[
\langle A_j \rangle_R = \langle \Psi_R | \hat{A}_j \Psi_R \rangle = \int \Psi_R^* (\vec{r}) \hat{A}_j \Psi_R (\vec{r}) \, d^3 \vec{r} \tag{9} 
\]
\[ C_R(A_j, A_l) = \langle \delta_R \hat{A}_j \Psi_R | \delta_R \hat{A}_l \Psi_R \rangle, \quad \delta_R \hat{A}_j = \hat{A}_j - \langle A_j \rangle_R \] (10)

\[ \Delta_R A_j = \sqrt{C_R(A_j, A_j)} \] (11)

Our above consideration is motivated by the known fact that, in theoretical descriptions, the randomness of a quantum microparticle is incorporated in its wave function but not in operators of its observables. Properties of various states of a microparticle are described by different wave functions but with the same operators. A similar situation exists in the case of classical statistical systems for which the randomness is incorporated in the probability densities but not in the expressions of macroscopic random variables. In the alluded cases the properties of various states of a system are also described with different probability densities but with the same expressions for the macroscopic random variables. In classical case a measurements described similarly \[ [7, 8] \] by appealing to a ”recorded” density of probability. Note that in both quantum and classical cases the appeals to ”recorded” entities (wave function or probability density) must not be regarded as a description of collapse (reduction) for the corresponding intrinsic entities.

By adopting the relations (9)-(11) the task of our approach becomes to express \( \Psi_R \) (or related quantities) in terms of \( \Psi_I \) (or associated entities) and of some elements regarding the measuring devices. For such a task, firstly we show that the parameters (5)-(7) and (9)-(11) can be expressed in terms of certain quantities connected with \( \Psi_Y \) \( (Y = R, I) \) and having ordinary probabilistic significance in the probability theory sense \[ [24] \]. So we transcribe \( \Psi_Y \) in the form \( \Psi_Y = |\Psi_Y| \exp(i\Phi_Y) \), where \( |\Psi_Y| \) and \( \Phi_Y \) denote the modulus respectively the argument of \( \Psi_Y \). As such ordinary quantities we take firstly the probability densities associated with \( \Psi_Y \) and defined by

\[ \rho_Y = |\Psi_Y|^2 \] (12)

Other quantities with ordinary probabilistic significance are the probability currents (or probability fluxes per unit-area):

\[ \vec{J}_Y = -\frac{i\hbar}{2m} (\Psi_Y^* \nabla \Psi_Y - \Psi_Y \nabla \Psi_Y^*) = \frac{\hbar}{m} |\Psi_Y|^2 \cdot \nabla \Phi_Y \] (13)

\( (m \) denotes the mass of microparticle).
Now let us show that the parameters (5)-(7) and (9)-(11) can be expressed in terms of \(\rho_Y\) and \(\vec{J}_Y\). Then we observe that if an operator \(\hat{A}\) does not depend on \(\nabla\), i.e. \(\hat{A} = \hat{A}(\vec{r})\) in (3) and (9) can be used the substitutions:

\[
\Psi_Y^* \hat{A} \Psi_Y = A(\vec{r}) \rho_Y \tag{14}
\]

On the other hand if \(\hat{A}\) depends on \(\nabla\), i.e. \(\hat{A} = \hat{A}(\nabla)\), by taking \(\Psi_Y = |\Psi_Y| \exp(i\Phi_Y)\) and using (12)-(13) in (3) and (9) one can resort to the substitutions like:

\[
\Psi_Y^* \nabla \Psi_Y = \frac{1}{2} \nabla \rho_Y + \frac{im}{\hbar} \vec{J}_Y \tag{15}
\]

\[
\Psi_Y^* \nabla^2 \Psi_Y = \rho_Y^{1/2} \nabla^2 \rho_Y^{1/2} + \frac{im}{\hbar} \nabla \vec{J}_Y - \frac{m^2}{\hbar^2} \vec{J}_Y^2 \rho_Y \tag{16}
\]

The existence of substitutions (14)-(16) suggests that the description of QMS can be completed by adequate considerations about the quantities \(\rho_Y\) and \(\vec{J}_Y\). As the respective quantities have ordinary probabilistic significance for the alluded completion we resort to the model used [7, 8] in the description of measurements of classical random observables. We also take into account the fact that \(\rho_Y\) and \(\vec{J}_Y\) refer to the positional respectively motional aspects of probabilities. Or, from an experimental perspective, the two aspects can be regarded as measurable by independent devices and procedures. Then the alluded completion must consists in giving independent relationships between \(\rho_R\) and \(\rho_I\) on the one hand respectively between \(\vec{J}_R\) and \(\vec{J}_I\) on the other hand. The mentioned relationships can be expressed formally by the following generic formulas:

\[
\rho_R = \hat{G} \rho_I \tag{17}
\]

\[
J_{R,\mu} = \sum_{\nu=1}^{3} \hat{\Lambda}_{\mu,\nu} J_{I,\nu} \tag{18}
\]

\((J_{Y,\mu} \text{ with } Y = R, I \text{ and } \mu = 1, 2, 3 = x, y, z \text{ denote the Cartesian components of vectors } \vec{J}_Y)\). In (17) and (18) \(\hat{G}\) and \(\hat{\Lambda}_{\mu,\nu}\) signify the measurements operators. They must comprise obligatory characteristics of measuring devices and procedures. So \(\hat{G}\) and \(\hat{\Lambda}_{\mu,\nu}\) must contain some extra-QM elements, i.e. elements that do not belong to the usual QM description of the intrinsic properties of the measured microparticles.
For measuring devices with linear and stationary characteristics, similarly with the classical case \[7, 8\], the relations (17)-(18) can be written as:

\[ \rho_R(\vec{r}) = \int G(\vec{r}, \vec{r}') \rho_I(\vec{r}') \, d^3\vec{r}' \] (19)

\[ J_{R,\mu}(\vec{r}) = \sum_{\nu=1}^{3} \int \Lambda_{\mu\nu}(\vec{r}, \vec{r}') J_{I,\nu}(\vec{r}') \, d^3\vec{r}' \] (20)

The kernels \( G(\vec{r}, \vec{r}') \) and \( \Lambda_{\mu\nu}(\vec{r}, \vec{r}') \) are supposed to satisfy the conditions:

\[ \int G(\vec{r}, \vec{r}') \, d^3\vec{r} = \int G(\vec{r}, \vec{r}') \, d^3\vec{r}' = 1 \] (21)

\[ \int \Lambda_{\mu\nu}(\vec{r}, \vec{r}') \, d^3\vec{r} = \int \Lambda_{\mu\nu}(\vec{r}, \vec{r}') \, d^3\vec{r}' = 1 \] (22)

These conditions show the one-to-one probabilistic correspondence between the intrinsic quantities \( \rho_I \) and \( J_I \) respectively the recorded ones \( \rho_R \) and \( J_R \). Parameters (9)-(11), evaluated by means of the relations (14)-(16) and (17)-(22), incorporate randomness of both intrinsic and extrinsic nature, corresponding to the own properties of the investigated microparticle respectively to the measuring devices. As evaluated the mentioned parameters have a theoretical significance. Their adequacy must be tested by comparing with the corresponding parameters obtained by statistical processing of the real experimental data. If the test is affirmative both descriptions, of intrinsic QM properties respectively of QMS, can be accepted as adequate. However, if the test invalidates the theoretical results, at least one of the respective descriptions must be regarded as inadequate.

From the origins of their history, the QMS approaches are concerned with the problem of quantitative evaluation for measuring uncertainties (i.e. for errors induced by the measurements in the values of the measured quantum observables). That is why it is of interest to discuss the respective problem in connection with the here promoted approach. Our discussion starts by pointing out the fact that quantum observables have a random character. Consequently, the uncertainties of such an observable must be evaluated through indicators, which comprise information from the whole its spectrum. It is easy to see that indicators of the alluded kind can be introduced by means of the numerical parameters defined by relations (5)-(7) and (9)-(11). That is why we suggest that, conjointly with the above–presented
approach of QMS, the measuring uncertainties to be evaluated through the following error
(or uncertainty) indicators:

\[ \delta \left( \langle A_j \rangle \right) = |\langle A_j \rangle_R - \langle A_j \rangle_I | \quad (23) \]

\[ \delta (C(A_j, A_l)) = |C_R(A_j, A_l) - C_I(A_j, A_l)| \quad (24) \]

\[ \delta (\Delta A_j) = |\Delta_R A_j - \Delta_I A_j| \quad (25) \]

These indicators have a restricted significance for a system (microparticle), because they
refer to some particular observables of the respective system. A more generic uncertainty
indicators, regarding a system in the whole, can be introduced by means of informational
entropies.

In connection with the applicability of the informational entropies for quantum cases a
recent opinion is known [29]. According to the respective opinion the Shannon entropy is
inadequate for the respective cases because of the contrast between the quantum and classical
situations. The contrast is associated with ideas such are: (i) QMS [29]: "cannot be claimed
to reveal a property of the individual quantum system existing before the measurement is
performed" or (ii) The description of QMS must be considered joint with the quantum
complementarity.

One can see that the mentioned ideas are reminiscences of the QMS approaches based on
TIHR. However, as we have argued above such approaches are incorrect. On the other hand
the quantities \( \rho_Y \) and \( \vec{J}_Y \) used in our approach have all the essential characteristics of classical
probabilities. Therefore, for our purposes, we can operate with entropies of Shannon type.
Note that for other purposes the informational entities (different from Shannon entropy)
proposed or reminded in [29] can be of real utility. Here we use the following informational
entropies of Shannon type:

\[ H_Y = - \int \rho_Y \ln \rho_Y \, d^3 \vec{r} \quad (26) \]

\[ \tau_Y = - \int |\vec{J}_Y| \ln |\vec{J}_Y| \, d^3 \vec{r} \quad (27) \]

Here \( H_Y \) and \( \tau_Y \) can be called positional respectively motional informational entropies. Then
the alluded generic uncertainty indicators can be defined as

\[ \delta H = H_R - H_I \quad (28) \]
\[ \delta \tau = \tau_R - \tau_I \] (29)

It is interesting to note the fact that within the above-presented description of QMS the indicator \( \delta \mathcal{H} \) is a nonnegative quantity (i.e. \( \delta \mathcal{H} \geq 0 \)). The respective fact can be proved, similarly with the classical situation \[7, 8\], by means of the relations (19) and (21). So by taking into account the respective relations, the normalization of both \( \rho_I \) and \( \rho_R \), and the evident formula \( \ln y \leq y - 1 \) \((y > 0)\) one can write:

\[
\delta \mathcal{H} = \mathcal{H}_R - \mathcal{H}_I =
\begin{align*}
&= - \int d^3 \vec{r} \int d^3 \vec{r}' G(\vec{r}, \vec{r}') \rho_I(\vec{r}')^2 \ln \frac{\rho_R(\vec{r})}{\rho_I(\vec{r})} \\
&\geq - \int d^3 \vec{r} \int d^3 \vec{r}' G(\vec{r}, \vec{r}') \rho_I(\vec{r}') \left[ \frac{\rho_R(\vec{r})}{\rho_I(\vec{r})} - 1 \right] = 0
\end{align*}
\] (30)

The above considerations give a natural description of QMS in which one finds, in adequate positions, all the essential elements. The respective elements include: (i) the intrinsic numerical parameters \((5)-(7)\), (ii) the model represented by \((17)-(22)\) for describing the influences of measuring devices, (iii) the recorded numerical parameters \((9)-(11)\) and \((23)-(25)\) or \((28)-(29)\).

In the end of this section we note that the description of QMS presented here, as well as the one discussed in \[7, 8\] for classical measurements, can be regarded formally from the perspective of information theory. In such a perspective, a measurement appears as a process of information transmission. The source of information is the measured system and the intrinsic values of its random characteristics (probability density and current, or observables) represent the input information. The chain of measuring devices play the role of channel for information transmission. The recorded data about the measured random characteristics represent the output information. Then the measurement uncertainties can be regarded as alterations of the transmitted information.

IV. A SIMPLE EXEMPLARYIFICATION

To illustrate the above-introduced QMS approach let us refer to the following simple exemplification. We consider a quantum microparticle in a one-dimensional motion along the \(x\)-axis. Its own properties are supposed to be described by the intrinsic wave function
\( \Psi_I(x) = |\Psi_I(x)| \exp(i\Phi_I(x)) \) with:

\[
\Psi_I(x) = \left( \alpha \sqrt{2\pi} \right)^{-\frac{1}{2}} \exp \left\{ -\frac{(x-x_0)^2}{4\alpha^2} \right\}, \quad \Phi(x) = kx
\]  

Then the intrinsic probability density and current defined by (12) and (13) are:

\[
\rho_I(x) = \frac{1}{\alpha \sqrt{2\pi}} \exp \left\{ -\frac{(x-x_0)^2}{2\alpha^2} \right\}
\]

respectively

\[
J_I(x) = \frac{\hbar k}{m \alpha \sqrt{2\pi}} \exp \left\{ -\frac{(x-x_0)^2}{2\alpha^2} \right\}
\]

So the intrinsic characteristics of the microparticle are described by the parameters \( x_0, \alpha \) and \( k \).

Considering that the errors of QMS are small in (19) and (20), one can operate with the one-dimensional kernels of Gaussian forms given by:

\[
G(x, x') = \frac{1}{\sigma \sqrt{2\pi}} \exp \left\{ -\frac{(x-x')^2}{2\sigma^2} \right\}
\]

\[
\Lambda(x, x') = \frac{1}{\lambda \sqrt{2\pi}} \exp \left\{ -\frac{(x-x')^2}{2\lambda^2} \right\}
\]

Here \( \sigma \) and \( \lambda \) describe the error characteristics of the measuring devices (see below).

By using (34)-(35) in the one-dimensional versions of the relations (19)-(20) one finds:

\[
\rho_R(x) = \frac{1}{\sqrt{2\pi}(\alpha^2 + \sigma^2)} \exp \left\{ -\frac{(x-x_0)^2}{2(\alpha^2 + \sigma^2)} \right\}
\]

\[
J_R(x) = \frac{\hbar k}{m \sqrt{2\pi}(\alpha^2 + \lambda^2)} \exp \left\{ -\frac{(x-x_0)^2}{2(\alpha^2 + \lambda^2)} \right\}
\]

One can see that in the case when \( \sigma \to 0 \) and \( \lambda \to 0 \) the kernels \( G(x, x') \) and \( \Lambda(x, x') \) degenerate into the Dirac function \( \delta(x-x') \). Then \( \rho_R(x) \to \rho_I(x) \) and \( J_R(x) \to J_I(x) \). Such a case corresponds to an ideal measurement. Alternatively the cases with \( \sigma \neq 0 \)and/or \( \lambda \neq 0 \) are associated with non-ideal measurements.

As observables of interest, we consider the coordinate \( x \) and momentum \( p \) described by the operators \( \hat{x} = x \cdot \) and \( \hat{p} = -i\hbar \frac{\partial}{\partial x} \). Adequately we use the expressions (32)-(33) and (36)-(38)
in the relations (5)-(6) and (9)-(11). Then, by using (14)-(16), for the mentioned observables one finds the following intrinsic (I) respectively recorded (R) numerical parameters:

\[ \langle x \rangle_I = \langle x \rangle_R = x_0, \quad \langle p \rangle_I = \langle p \rangle_R = \hbar k \] (38)

\[ C_I(x, p) = C_R(x, p) = \frac{i\hbar}{2} \] (39)

\[ \Delta_I x = \alpha, \quad \Delta_R x = \sqrt{\alpha^2 + \sigma^2} \] (40)

\[ \Delta_I p = \frac{\hbar}{2\alpha} \] (41)

\[ \Delta_R p = \hbar \sqrt{\frac{k^2(\alpha^2 + \sigma^2)}{\sqrt{\alpha^4 - \lambda^2 + 2\sigma^2(\alpha^2 + \lambda^2)}} - k^2 + \frac{1}{4(\alpha^2 + \sigma^2)}} \] (42)

Then for the considered observables \( x \) and \( p \) the error indicators (23)-(25) become:

\[ \delta (\langle x \rangle) = 0, \quad \delta (\langle p \rangle) = 0, \quad \delta (C(x, p)) = 0 \] (43)

\[ \delta (\Delta x) = \sqrt{\alpha^2 + \sigma^2} - \alpha \] (44)

\[ \delta (\Delta p) = \hbar \left\{ \sqrt{\frac{k^2(\alpha^2 + \sigma^2)}{\sqrt{\alpha^4 - \lambda^2 + 2\sigma^2(\alpha^2 + \lambda^2)}} - k^2 + \frac{1}{4(\alpha^2 + \sigma^2)}} - \frac{1}{2\alpha} \right\} \] (45)

These relations show that for the considered association microparticle-QMS the numerical parameters \( \langle x \rangle, \langle p \rangle \) and \( C(x, p) \) are not affected by errors. However, for the same association the parameters \( \Delta x \) and \( \Delta p \) are troubled by the measurement, the corresponding non-null error indicators being given by (14)-(16). Then, within the discussed model for the pair of non-commutable observables \( x \) and \( p \) one finds the relations:

\[ \delta (\langle x \rangle) \cdot \delta (\langle p \rangle) = 0 \] (46)

\[ \delta (\Delta x) \cdot \delta (\Delta p) = \hbar \varepsilon \] (47)

where \( \varepsilon \) can be identified from (14)-(16).
Relations \((46)-(47)\) can be offered as pieces for a natural reconsideration of the above-mentioned assertion Ass.2 of TIHR regarding the non-commutable observables. Of course that the offer is accompanied by the observation that, as can be see from \((44),(45)\) and \((47)\), \(\varepsilon \to 0\) when \(\sigma \to 0\) and \(\lambda \to 0\) (i.e. for ideal measurements). Then it results that on principle the products \(\delta(\langle x \rangle) \cdot \delta(\langle p \rangle)\) and \(\delta(\Delta x) \cdot \delta(\Delta p)\) does not have non-null lower bounds. But such a fact contradicts the essential idea of TIHR that for non-commutable observables the measuring uncertainties (errors) are mutually lower bounded, independently of the concrete performances of the experiments.

Now, for the here discussed model of QMS description, let us search the entropic error indicators defined by the relations \((26)-(29)\). By using the expressions \((32)-(36)\) one finds:

\[
\delta \mathcal{H} = \frac{1}{2} \ln \left(1 + \frac{\sigma^2}{\alpha^2}\right) \tag{48}
\]

\[
\delta \tau = \frac{\hbar k}{2m} \ln \left(1 + \frac{\lambda^2}{\alpha^2}\right) \tag{49}
\]

If in \((31)\) we choose \(x_0 = 0, k = 0\) and \(\alpha = \sqrt{\frac{\hbar}{2m\omega}}\), our system is just a quantum oscillator with mass \(m\) and pulsation \(\omega\) situated in its ground state. The corresponding statistical estimators and error indicators for observables \(x\) and \(p\) can be obtained from \((38)-(43)\) respectively \((44)-(46)\) by mentioned choosing. However, in the case of oscillator it is interesting to point out the measuring characteristics for another observable, described by the Hamiltonian:

\[
\hat{H} = \frac{1}{2m} p^2 + \frac{m\omega^2}{2} x^2 \tag{50}
\]

Then, by using \((18)-(20)\) and \((33)\), for the numerical parameters of oscillator energy one finds:

\[
\langle H \rangle_I = \frac{\hbar \omega}{2}, \quad \Delta_I H = 0 \tag{51}
\]

\[
\langle H \rangle_R = \frac{\omega \left[ \hbar^2 + (\hbar + 2m\omega\sigma^2)^2 \right]}{4(\hbar + 2m\omega\sigma^2)} \tag{52}
\]

\[
\Delta_R H = \frac{2m\omega^2\sigma^2(\hbar + m\omega\sigma^2)}{\sqrt{2}(\hbar + 2m\omega\sigma^2)} \tag{53}
\]

16
The corresponding uncertainty (error) indicators are:

\[
\delta (\langle H \rangle) = \frac{\omega \left[ h^2 + (h + 2m\omega \sigma^2)^2 \right]}{4(h + 2m\omega \sigma^2)} - \frac{h\omega}{2} 
\] (54)

\[
\delta(\Delta H) = \frac{2m\omega^2 \sigma^2(h + m\omega \sigma^2)}{\sqrt{2}(h + 2m\omega \sigma^2)} 
\] (55)

V. CONCLUSIONS

The problem of QMS description persists in our days as an open and disputed question. Many of its approaches are founded on fictitious conjectures, mainly inspired from TIHR. We aimed here a new approach started with an investigation of the correctness of the alluded conjectures. So we found in fact a true incorrectness, the respective conjectures being affected by insurmountable shortcomings. Such a finding motivates our interest for a new set of reconsidered and natural conjectures. Consequently, we proposed a set of four such conjectures. Guided by that proposal we developed a new approach for the description of QMS.

Our approach is founded on the usual probabilistic conception of QM. Therefore, for a quantum microparticle we operate with probability density, probability current and QM operators. We opine that, because in practice a correct QMS must consist in a statistical sampling, from a theoretical viewpoint a QMS must be represented as processing of the mentioned probabilistic entities. Similarly, with the description of classical (non-quantum) measurements, the alluded processing must be pictured as changes of the respective entities. We opine that for a wide class of situations such changes can be modelled as linear integral transforms. Therefore, both probability density and probability current appear in intrinsic respectively recorded posture. In the first posture, they regard the own characteristics of the measured microparticle, while in the second posture they comprise information related both with the respective microparticle and with the measuring devices.

Together with the mentioned features of QMS the quantum observables must be naturally evaluated through the numerical parameters such are: mean values, correlations and standard deviations. Within the discussed approach, the respective parameters are characterized by intrinsic respectively recorded values. Then a natural description of measuring uncertainties for quantum observables is expressible in terms of differences between the
mentioned recorded and intrinsic values. Another description of measurements uncertainties, more generic (i.e. not associated with some particular observables), can be done in terms of informational entropies.

The here recapitulated features of our QMS approach are detailed from a general perspective in Section III, while in Section IV they are illustrated by means of a simple exemplification.

We remind here that our QMS approach is quite different from the approaches founded on (or inspired from) TIHR. The difference is evidenced on the one hand by the idea that QMS must be regarded as statistical samplings but not as individual detection acts. Consequently, we can avoid completely the controversial conception of wave function collapse (reduction). On the other hand, the alluded difference is pointed out by our natural presumption that the description of QMS must be regarded as distinct and independent task comparatively with the QM investigations. Accordingly, with the respective regards the description of measurement must be considered and discussed as a scientific branch self-determined and additional comparatively with the quantum or classical chapters of physics. The mentioned chapters, as in fact is well established by the scientific practice, investigate only the intrinsic properties of the physical systems.

The views promoted here and in \[3, 4, 5, 6\] give a natural and unified reconsideration of the prime problems regarding HR and QMS. The problems refer to the interpretation of HR respectively to the description of QMS and the reconsideration is founded on an argued denial of TIHR. On the other hand HR and QMS are also implied in many subsequent questions. The questions regard the foundations and interpretation of QM and have been largely disputed in the last years (for an actualized bibliography see the works \[1, 2, 30, 31, 32\] with their references as well as the gray literature sources \[23, 26\]. In the respective questions HR and QMS are taken as pre-existing entities with properties inferred from various presumptions. Surprisingly, in many disputed cases the alluded presumptions originate (more or less explicitly) from TIHR. Therefore, due to the mentioned denial of TIHR, it is thinkable that our reconsideration of HR and QMS can offer nontrivial elements for an expected re-examination of some from the alluded subsequent questions. Of course that the respective elements can (and must) be complemented with other considerations regarding the QM problems.
Acknowledgments

I wish to express my deep gratitude to the World Scientific Publishing Company for putting at my disposal a copy of the monumental book [1].

The investigations reported in the present text benefited partially from some facilities of a grant from the Romanian Ministry of Education and Research.

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