Moment redistribution in concrete statically undetectable systems with FRP

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Abstract. Currently, engineers use, as a rule, the mechanics of elastic systems or take into account the inelastic properties of reinforced concrete without external reinforcement when performing calculations and designing the reinforcement of normal sections of reinforced concrete structures. It should be recognized that there is currently no model that takes into account the inelastic properties of normal sections of reinforced concrete elements with FRP. In this regard, an important issue is the evaluation of the influence of FRP on the redistribution of bending moments in a statically indeterminate system through a change in the stiffness of normal sections. This article analyzes the changes in the stiffness of normal sections, as well as the redistribution of bending moments in a statically indeterminate two-span beam on the basis of the existing model of inelastic deformations reinforced with external reinforcement sections with a model built on the basis of experiments (the actual stiffness of external reinforced normal sections).

1. Introduction
The system of external reinforcement based on carbon fibers to strengthen reinforced concrete structures has become firmly established in the everyday use of engineers. This system is well established by the results of almost 30 years of active use and currently, the developed standards cover the main area of application of these systems. At the same time, the majority of norms and standards are developed for statically definable constructions, for which a large number of experimental studies have been carried out and norms and recommendations have been developed accordingly [1-4]. Also, taking into account the development of monolithic concrete structures, the percentage of reinforced monolithic concrete structures has increased significantly. In fact, this means that the reinforced structures are statically indeterminate. Most of the standards currently in force do not consider the issue of redistribution of moments taking into account changes in the rigidity of reinforced concrete elements introducing General recommendations. At the same time, a number of authors [5-11] have analytically and experimentally proved the effect of redistribution of bending moments during reinforcement by external reinforcement.

At this stage of development of the theory of operation of normal sections reinforced by external reinforcement, as a rule, a modified method is adopted, which was developed for the calculation of reinforced concrete structures without reinforcement [2, 4]. Unfortunately, this technique did not initially imply to account for these gain parameters as: change of physico-mechanical characteristics of the binder in time, accounting for the anchoring of external reinforcement, creep of the outer reinforcement relative to the concrete surface, etc. Despite the fact that these methods take into account the redistribution of tensile forces between internal steel reinforcement and external reinforcement, this redistribution of moments requires clarification and better convergence with
experimental studies. In this case, it is necessary to pay attention to the ratio of stiffness and strength of steel reinforcement and external reinforcement (for example, the most applicable materials: A500 reinforcement and external reinforcement based on high-strength fibers):

- the strength of the external reinforcement (normative 3,600 MPa) is greater than the strength of steel (also normative 500 MPa) by 620%;
- the modulus of elasticity of steel and external reinforcement is almost comparable $2 \cdot 10^6$ MPa for steel and $2.45 \cdot 10^6$ MPa for external reinforcement, the difference is 22%.

This suggests, first of all, that the potential of external reinforcement is very difficult to implement, due to the fact that the deformation is significantly limited, including concrete. Most interesting in this case, high-modulus fibers (modulus of elasticity 390 – 760 HPA). However, these fibers are rarely used due to the high cost at present.

On the basis of the above it is possible to allocate the problem connected with the account of redistribution of the bending moments in the bent statically indeterminate designs connected with change of rigidity of normal beams after performance of strengthening by external reinforcement.

2. Analytical calculation of the change of stiffness of flexible element

At the moment, the determination of the stiffness of the normal cross-sections of the bent reinforced concrete elements is possible only by an adapted technique originally developed for reinforced concrete elements without reinforcement.

According to this theory, the rigidity of the bending element $D$ is determined by the formula (1) taking into account the moment of formation of cracks.

$$D = E_{b1} \cdot I_{red}$$ (1)

Where $E_{b1}$ is the modulus of deformation of compressed concrete taking into account the duration of the load;

$I_{red}$-the moment of inertia of the reduced section, determined by the formulas (2) and (3) taking into account the moment of formation of cracks:

- moment of inertia of the reduced section for the section without cracks:

$$I_{red} = I_b + \alpha \cdot I_s + \alpha' \cdot I'_s + \alpha_f \cdot I_f$$ (2)

- the moment of inertia of the reduced section relative to the center of gravity, taking into account the cross-sectional area only compressed zone of concrete (for cross-section with cracks):

$$I_{red} = \frac{bx^3}{12} + bx \left(\frac{x}{2} \right)^2 + \alpha \cdot A_s(x-a')^2 + \alpha' \cdot A'_s(x-a)^2 + \alpha_f \cdot A_f(h-x)^2$$ (3)

where are $I_b, I_s, I'_s, I_f$ - the moments of inertia of the cross-sectional areas ($A_b, A_s, A_s, A_f$) respectively of the compressed zone of concrete, stretched reinforcement, compressed reinforcement, external reinforcement by composite materials with respect to the center of gravity of the stretched zone of cross-section without concrete;

$\alpha, \alpha_f$ - coefficients of reduction of the stretched armature, the compressed armature and external strengthening by composite materials to concrete defined by the formula (4);

$b$ and $x$ – width and height of the compressed zone of concrete, determined by the formula (5)

$$\alpha = \frac{E_s}{E_b}, \alpha_f = \frac{E_f}{E_b}$$ (4)

$$x = h_0 \left[ \left( \mu_s \alpha_{s2} + \mu'_s \alpha_{s1} + \mu_f \alpha_f \right) + \frac{2 \left( \mu_s \alpha_{s2} + \mu'_s \alpha_{s1} \frac{a'}{h_0} + \mu_f \alpha_f \right)}{\left( \mu_s \alpha_{s2} + \mu'_s \alpha_{s1} + \mu_f \alpha_f \right)} \right]$$ (5)

Where

$$\mu_s = \frac{A_s}{bh_0}; \mu'_s = \frac{A'_s}{bh_0}; \mu_f = \frac{A_f}{bh_0}$$
A specific example based on a 150x300(mm) beam with the following characteristics will be considered next:
- concrete B25 (strength)
- armature A500S 3ø12
- for element with reinforcement, reinforcement by external reinforcement with carbon tape (strength 245 GPa)

The key point – the moment of formation of cracks determined by the formula (6) - will be used to construct the diagram.

\[ M_{cr,c} = R_{bt,ser} \cdot W_{red} \]  

(6)

Where \( R_{bt,ser} \) – strength of concrete;
\( W_{red} \) – inelastic moment

Figure 1 shows a calculated diagram of the dependence of the stiffness of the normal section on the bending moment.

The total change in stiffness in the presence of FFF gain is about 1%.
At the same time, for practical application, this change can actually be neglected.

The redistribution of moments will be considered in the construction presented in figure 2. This is a two-span reinforced concrete beam with a cross section of 150x300 with a uniformly distributed load.

The following are the moment plots calculated on the basis of analytically calculated stiffnesses of a given statically indeterminate two-span beam for the following situations:
- elastic work of the element (excluding changes in the stiffness of normal sections) - figure 3A
- elastic-plastic work of the beam (taking into account changes in stiffness) - figure 3b

a)

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33.8 kH
19.0 kH
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b)

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30.7 kH
20.1 kH
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**Figure 3.** Bending moment plots

Taking into account the difference in the stiffness of normal sections without reinforcement and with a gain of less than 1%, it makes no sense to consider separate situations with reinforcement from above and below.

In General, these plots can be distinguished:
- reduction of the reference torque by 9.1%;
- increase in flying torque by 5.7%.

3. **Experimental determination of element stiffness**

According to the results of experimental studies, other real parameters of stiffness are obtained. In this paper, the analysis of changes in the stiffness of reinforced concrete elements with reinforcement by external reinforcement based on carbon fibers was carried out on statically definable structures-single-span beams. A significant database of load-deflection diagrams has been accumulated for these structures.

On the basis of the analysis and comparison of the previously conducted experimental studies of bending structures, diagrams of changes in stiffness from the load (bending moment) are constructed. In General, the deformation graph of the samples is presented in figure 4.
According to the deformation graph shown in figure 4, for the averaged deflection values, a diagram of the stiffness change from the bending moment can be constructed. For this purpose the inverse problem of deflection determination is solved:

$$D = \frac{Pa(3l^2 - 4a^2)}{24f}$$

Thus, the "real" diagram of the change in the stiffness of the sample samples has the following form (figure 5):

![Figure 5. Diagram of the dependence of the stiffness of the normal section on the bending moment](image)

The graph (figure 5) highlights the lines on which further analysis will be carried out. These lines are averaged, more real lines are shown in dotted lines. The initial areas before the formation of cracks are shown in straight lines due to the fact that the deformations are very small and the error is high. The following preliminary conclusions can be drawn from this diagram:

- in General, the graph of stiffness reduction for the non-reinforced element can be adopted by the analytical model;
- diagram of reducing the stiffness of normal sections reinforced with external reinforcement is significantly different from the analytical, primarily due to the displacement of the moment of cracking

Based on the obtained data, it is possible to construct a diagram of the redistribution of moments for the cases presented in table 1, using real data of the stiffness of the samples (according to the graph in figure 5).
Table 1. Considered variants of amplification

| Option 1. | Option 2. | Option 3. |
|-----------|-----------|-----------|
| beam      | strengthening beams on the bottom | reinforcement the beam top and bottom |
| reinforcement from above | | |

All the bending moment plots in figure 6 below assume the following initial data:
- the cross-section is not damaged at the time of amplification;
- at the time of strengthening and strength gain binder structure is completely unloaded, i.e., the effective forces in the cross sections are 0;
- external reinforcement is not considered in the compressed zone;
- change of rigidity of sections from the bending moment are accepted on the diagram in figure 5.

Figure 6. Bending moment plots. a) Option 1. beam reinforcement from above; b) Option 2. Strengthening beams on the bottom; c) Option 3. Reinforcement the beam top and bottom
Table 2. Considered variants of amplification

| Strengthening                      | Analytical model of deformations | Experimental model of deformations |
|-----------------------------------|----------------------------------|------------------------------------|
|                                   | The elastic behaviour of the structure without reinforcement | Inelastic structural behaviour with increased |
|                                    | Upper face gain                  | Reinforcement at the bottom face    |
|                                    | Upper and lower face gain        |                                    |
| Moment at the base, kN m           | 33,88                            | 30,7                               |
|                                    | 34,6                             | 27,2                               |
|                                    | 32,4                             |                                    |
| Moment at the span, kN m           | 19,0                             | 21,5                               |
|                                    | 18,7                             | 21,5                               |
|                                    | 19,5                             |                                    |

Highlighting the individual options for strengthening the following conclusions can be made:
- reinforcement above the Central support increases the stiffness of the support sections, which leads to an increase in the support torque by 12.7%. This leads to the fact that it is necessary to calculate the gain on a large load. This gain does not lead to an increase in the flyover torque, respectively, does not require the strengthening of the flyover zone;
- strengthening the bottom of the span. This gain increases the rigidity of the lower part, which leads to the redistribution of the moments of the span without changing the maximum moment, the plot of moments becomes more flat. The reference torque is reduced by 11%;
- strengthening of the beam in the span and over the support leads to an increase in the reference torque by 5.5% and a decrease in the span by 9%.

4. Conclusion

On the basis of the numerical experiment the following conclusions can be drawn regarding the redistribution of moments in statically indeterminate constructions taking into account the reinforcement by external reinforcement

1. In the amplification of external reinforcement there is a change in the Flexural rigidity of normal sections. At the moment, the norms do not have a method for determining the stiffness of the reinforced concrete normal section, taking into account the external reinforcement based on carbon fibers, which allows to describe the process of changing the stiffness with satisfactory convergence.
2. Increasing the stiffness of the support sections causes an increase in the bending moment in this section. In this regard, it is expedient on the basis of additional special studies to consider the introduction of the coefficient of working conditions in the calculation of the load-bearing capacity of the normal cross-sections of the bent statically indeterminate elements reinforced by external reinforcement.

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