Weapon-Target Assignment Problem Using Modified Water Wave Optimization Algorithm

Yuanfei Wei¹, Qifang Luo² and Yongquan Zhou²*  
¹Xiangsihu College of Guangxi University for Nationalities, Nanning, 532100, China  
²College of Artificial Intelligence, Guangxi University for Nationalities, Nanning 530006, China  
*Corresponding author email: yongquanzhou@126.com

Abstract. The weapon-target assignment (WTA) is a classic problem. The WTA mathematical model is that warship formations are reasonably equipped with weapons resources for each weapon system to attack the air threaten targets. The purpose of targets optimization is to maximize combat effectiveness, that is to say, the mathematical expectation is maximum. We adopt the greedy strategy and improved propagation operation is to strengthen the water wave optimization (WWO) search performance. This article elaborates a modified water wave optimization (MWWO) to solve the WTA problem, which can detect optimized allocation decision matrix and search for the maximum mathematical expectation. Based on parameter optimization, the overall performance of the MWWO is more stable, the search speed is accelerated and the accuracy is improved. The experiment results indicate that the MWWO are verified and avoids local optimum, and can be more convenient for solving the WTA and obtain better performance.

1. Introduction
In recent years, heuristic optimization technology develops very rapidly; especially meta-heuristic optimization algorithm has achieved success in solving complex problems. In general, the meta-heuristics algorithms are mainly divided into evolutionary algorithm (EA), swarm-based algorithm, physics-based algorithm. EAs mimic the concept of natural evolution. For example, Genetic Algorithm (GA) [1], Differential Evolution (DE) [2], Evolutionary Strategy (ES) [3], Evolutionary Programming (EP) [4], and Biogeography-based Optimization (BBO) [5]. Swarm intelligent-based have been widely used in science and engineering. For example, Artificial Bee Colony (ABC) [6], Particle Swarm Optimization (PSO)[7], Cuckoo Search Algorithm (CS) [8], Firefly Algorithm (FA) [9], Bat algorithm (BA) [10], Grey Wolf Optimizer (GWO) [11], Dolphin Echolocation (DE) [12], Whale Optimization Algorithm (WOA) [13], Fruit-fly Optimization Algorithm (FOA) [14], and Chimp Optimization Algorithm (ChOA) [15], and so on. Physics-based algorithm, such as, Gravity Search Algorithm (GSA) [16], Charged System Search (CSS) [17], Magnetic Optimization Algorithms (MOA) [18], Harmony Search (HS) [19], and water wave optimization (WVO) [20] are all physics-based optimization algorithms.  
In the military, air strikes and anti-air raids have become the main mode of combat in modern warfare. The weapon-target assignment (WTA) in a modern combat command and decision support system, command automation system is an indispensable key component. The main task of the WTA is to destroy the incoming aircraft, unmanned aerial vehicles, to intercept the enemy cruise missiles, anti-ship missiles and radiation missiles. This requires the commander to consider how to rationally allocate firepower, quickly and efficiently to complete the air defense mission to ensure that the air defense efficiency of warship formation, improve the viability of the warship formation.
The WTA [21] is essentially an integer type nonlinear combination optimization decision problem. The WTA based on the purpose of combat, battlefield situation and weapons performance and other factors will be a certain type and quantity of weapon units to a reasonable allocation of certain criteria to attack a certain number of air threaten targets. The result of the optimization is that the combat effectiveness is maximized. The WTA is the key technology in military operations. There are many intelligent technology are applied in many fields of military affairs, such as grey wolf optimizer (GWO) [22], bat search algorithm (BA) [23], flower pollination algorithm (FPA) [24], sine cosine algorithm (SCA) [25], PSO [7], Social spider optimization algorithm(SSO)[26], Quantum-inspired satin bowerbird algorithm(QSBA)[27], water wave optimization (WWO) [20]. To improve WWO performance, the WTA model was established by Ruan [28]. In this paper, the MWWO algorithm is used to solve WTA problems, which not only can balance exploration and exploitation abilities, and the MWWO algorithm finds the optimal decision matrix, so that it has the maximum combat effectiveness in WTA. This indicates the MWWO algorithm has a stable performance and a strong search capability.

2. Related Works

2.1. Problem Formulation

The WTA, using most effective allocation of weapon units to intercept multiple batches of air threaten targets, is a very important issue in modern warfare and an important auxiliary decision in the command automation system. The WTA is a dynamic allocation process in warship formation antiaircraft that can be depicted as Fig.1.

Figure 1. Schematic diagram of WTA in warship formation antiaircraft.

2.1.1. Assumption Description. The WTA is a complex and vital issue, which makes full use of weapon units of different weapon systems to combat air threaten targets and mathematical expectation is maximum. In order to better describe the WTA problem, some assumptions are defined:

**Assumption 1** There are $K$ different types of weapons systems in warship formation antiaircraft. Each weapon system has a certain number of weapon units and air threaten targets $T$. A weapon system is distributed effectively for air threaten target in the effective area and the time of action, otherwise no distribution.

**Assumption 2** The total number of weapon units $W$ and targets $T$. A weapon system can application multiple weapon units to attack a target.

**Assumption 3** A weapon unit can only assail an air threaten target once. The number of weapon units distributed for each weapon cannot more than the number of allocated weapon unit resources in the combat time.

**Assumption 4** The probability of damage $P_{ij}$ that each type weapon system strikes the air threaten targets and the threat coefficient $\mu_j$ that menace of target to warship formation are known. Each weapon system damage probability is $P_{ij}$, which indicates that the $i$th weapon system engage $j$th air threaten target.

**Assumption 5** To achieve maximum operational effectiveness, when faced with multiple batches of air threaten targets, the available resources for each weapon system are fully allocated within effective areas of
action and time of action.

2.1.2. Mathematical Model of WTA. There are $T$ air threaten targets and $K$ weapon systems. In the effective warfare time and combat area, the number of resources by a weapon system is $W_i$. The objective of WTA optimization is that the mathematical expectation of the damage target is maximized by reasonably assigning weapon units to the air threaten targets. Therefore, the mathematical model of the WTA can be established.

**Definition 1** The WTA mathematical model is defined:

$$f = \max \sum_{j=1}^{T} \mu_j \left[ 1 - \prod_{i=1}^{K} (1 - P_{ij})^{x_{ij}} \right]$$

(1)

**Definition 2** The $P_{ij} \in [0,1]$ is damage probability of the weapon systems, which declares the $i$th ($i=1,2,\ldots,K$) weapon system strikes the $j$th ($j=1,2,\ldots,T$) target.

**Definition 3** The $\mu_j \in [0,1]$ represents threat coefficient matrix for each air threaten target $j=1,2,\ldots,T$.

**Definition 4** The decision matrix $X = [x_{ij}]_{K \times T}$ can be given:

$$X = [x_{ij}]_{K \times T} = \begin{bmatrix} x_{11} & x_{12} & \ldots & x_{1T} \\ x_{21} & x_{22} & \ldots & x_{2T} \\ \vdots & \vdots & \ddots & \vdots \\ x_{K1} & x_{K2} & \ldots & x_{KT} \end{bmatrix}$$

(2)

where $x_{ij} \in Z$ ($i=1,2,\ldots,K; j=1,2,\ldots,T$) indicates weapon units number are assigned to the $j$th air threaten target by $i$th weapon system.

**Definition 5** Constraints conditions stated as:

$$\sum_{i=1}^{K} x_{ij} \geq 1, \ \forall j = 1,2,\ldots,T \quad (3)$$

$$\sum_{j=1}^{T} x_{ij} \geq 1, \ \forall i = 1,2,\ldots,K \quad (4)$$

$$\sum_{i=1}^{K} \sum_{j=1}^{T} x_{ij} = K \quad (5)$$

$$\sum_{j=1}^{T} \mu_j = 1 \quad (6)$$

The Eq.(3) express at least one weapon unit is arranged to each incoming threaten target. Eq.(4) express each weapon unit concentrated against only one incoming threat target. Eq.s(5-6) express parameters $x_{ij}$ and $\mu_j$ range, respectively.

2.2. WWO Algorithm

The WWO is obtained the objective function optimal value by wave propagation, refraction and breaking operations. Population initialization, a water wave population is constructed that the wave height $h$ and wavelength $\lambda$ of each water wave are set to $h_{\text{max}}$ and 0.5, respectively. For the uneven of seabed, the fitness value is variable with wave propagation, refraction and breaking. The fitness value of the long wave with low energy is much smaller than that of the short wave with high energy from Fig.2.

**Figure 2.** Diagram of the difference between deep wave and shallow water shapes.
2.2.1 Propagation. The each wave is subjected to a propagation in each iteration process. The new wave \( x' \) location update:

\[
x'(d) = x(d) + \text{rand}(-1,1) \cdot \lambda L(d)
\]  

Here \( d \in [1,D] \) is the problem space dimension, \( x(d) \) and \( x'(d) \) represents the original water wave and the new water wave position, respectively, \( \text{rand}(-1,1) \) is a random number, \( L(d) \) expresses the \( d \)-dimension space. If the new wave position is beyond bound, it’s randomly generated new position. In wave propagation, the fitness value of the original wave and that of the new wave are calculated. If \( f(x') > f(x) \), \( x \) is replaced by \( x' \) and wave height of the new wave is \( h_{\text{max}} \). Otherwise, \( x \) is retained and wave height is reduced by one that represents the loss of water wave energy. The update formula for the wavelength \( \lambda \) is:

\[
\lambda = \lambda \cdot \alpha \cdot e^{-((f(x') - f_{\text{max}} + \varepsilon)/(f_{\text{max}} - f_{\text{min}} + \varepsilon))}
\]  

where \( \alpha \) indicates the attenuation coefficient for the water wave wavelength, \( f_{\text{max}} \) and \( f_{\text{min}} \) are the maximum and minimum fitness values, respectively, \( \varepsilon \) is a very small positive integer.

2.2.2 Breaking. The seabed is rugged, the water wave may become much higher and higher, that is to say, the water wave energy increase. Finally, the water wave breaks into a series of solitary waves. Once a new optimal wave is found in the global search space, the new water wave will execute the breaking operation, which improves the solution accuracy. The operation is randomly selected \( k \in [1,D] \) dimension from \( D \) and each dimension is updated:

\[
x'(d) = x(d) + N(0,1) \cdot \beta L(d)
\]  

Here the parameter \( \beta \in [0.001,0.1] \) is broken wave coefficient. If the fitness value of \( x' \) is much better than all the solitary waves, \( x' \) is remained; otherwise, \( x \) is replaced.

2.2.3 Refraction. The refraction operation is to remove the energy depleted water waves, which effectively avoids the search stagnation. The wave height gradually decays to zero, meaning that a wave has not been improved after many propagation operations. In that case, the water wave refraction is:

\[
x'(d) = N \left( \frac{(x^*(d) + x(d))}{2}, \frac{|x^*(d) - x(d)|}{2} \right)
\]  

The \( x^* \) represents the best wave in the population, in other words, the current position is the optimal solution, \( N(\mu,\sigma) \) represents Gaussian random number, \( \mu \) and \( \sigma \) are mean and standard deviation respectively. After refraction, wave height \( x' \) is also reset to \( h_{\text{max}} \) and the fitness value is inversely proportional to the wavelength.

\[
\lambda' = \lambda \cdot \frac{f(x)}{f(x')}
\]  

In short, we solving the object problem is to conduct by propagation, refraction and breaking operations.

3. Proposed Modified WWO Algorithm

The water wave superior process regarded as the process of the deep water moving to shallow area. The propagation operation makes that the larger fitness value exploits in a small range and the lower fitness value explores in large water area. Similar to the PSO [21], the formula (7) is added to the inertia weight to strengthen population search ability. The improved propagation operator can be provided as follows.

\[
w = w_{\text{max}} - (w_{\text{max}} - w_{\text{min}}) \cdot \frac{\text{iter}}{\text{iter}_{\text{max}}}
\]  

\[
\rho = \frac{1}{1 + e^{a \cdot \sigma - \sigma}} \cdot \text{rand}(-1,1)
\]  

\[
x'(d) = w \cdot x(d) + \rho \cdot \lambda L(d)
\]  

where the \( w_{\text{min}} = 0.4 \) and \( w_{\text{max}} = 1.5 \) express the minimum and maximum inertia weights, respectively.
the $iter$ is current iterations number, $a = 0.02$ and $b = 20$ are given constants.

3.1 Greedy Strategy
To get a better optimal solution in the water wave movement process, the greedy strategy is applied to improving the exploitation ability of the WWO. The $x_{i,j}$ represents greed solution, the formula (15) and formula (16) are as follows

$$x_{i,j} = x_{i,j} + \varphi_{i,j} C_{i,j} x_{i,j}$$

$$C_{i,j} = c_{\min} + \exp\left(-e^{\frac{iter}{iter_{\max}}} (c_{\max} - c_{\min})\right)$$

where the $x_{i,j}$ is the $i$th greedy solution of the $j$th decision variables. $\varphi_{i,j} \in [-1,1]$ is a random number, $iter$ is the current iterations, $iter_{\max}$ is the maximum iterations $c_{\min} = 0.1$, $c_{\max} = 0.5$, $c = 60$ and $d = 5$ are related control parameters.

3.2 The MWWO to solve WTA problem

Algorithm 1. The pseudo-code of MWWO to solve the WTA problem

Step 1: Initialize population $P$ of $n$ waves (solutions), the parameters $iter_{\max}$, $\alpha$, $\beta$, $\lambda$;

Step 2: Establish the WTA optimization model, define distribution principle, weapon systems $K$, air threaten targets $T$, weapon units $W$, set up constraint conditions according to Eqs. (1), (3), (4), (5) and (6).

Step 3: Compute the fitness function $f$ value of the population by the decision matrix $X$.

Step 4: When the termination condition is not met do,

Step 5: For each wave $x \in P$ do

Step 5.1: The greedy strategy add to waves based on Eqs. (15) and (16).

Step 5.2: The wave $x$ is subjected to propagation operation to obtain a new wave each individual $x'$ use Eqs (12), (13) and (14).

Step 5.3: If the fitness function $f(x') > f(x)$ then

Step 5.3.1: If the fitness function $f(x') > f(x^*)$, the wave $x'$ performs breaking operation according to Eq. (9), the current optimal wave $x^*$ will be replaced by wave $x'$.

Step 5.3.2: The original wave $x$ is replaced by the new wave $x'$ in the population.

Step 5.4: Otherwise, wave height $h$ of wave $x$ is reduced one by one. If $h = 0$, the wave $x$ will perform refraction operation using Eqs. (10) and (11).

Step 5.5: Updating the wavelength of all waves using Eq.(8).

Step 6: According to the MWWO update process, a new decision matrix $X$ is obtained and the fitness value of the new population is calculated.

Step 7: Let $iter = iter + 1$.

Step 8: If $iter < iter_{\max}$, go Step 5.

Step 9: Output optimal value $f$ and relevant WTA decision matrix $X$.

4. Experiment Results and Discussion

4.1. The Solve the WTA Problem Using MWWO
Assume that there are 10 air threaten targets from different directions and 7 different types of weapons systems in warship formation antiaircraft, this are $K = 7$ and $T = 10$. The number of available resources for each weapon system is defined as $W = [4,5,4,5,4,5,4]$ in the specified combat time. For the sake of comparison, targets’ threat coefficient $\mu_{i,j}$ ($j = 1,2,...,T$) and weapons’ damage probability $p_{i}^{j} (i = 1,2,...,K,j = 1,2,...,T)$ are list in Table 1 and 2, respectively.
Table 1. The targets’ threat coefficient.

| Target | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Threat coefficient | 0.09 | 0.12 | 0.14 | 0.06 | 0.05 | 0.10 | 0.08 | 0.09 | 0.15 | 0.12 |

Table 2. The weapons’ damage probability.

| Weapon systems | Target batch |
|----------------|--------------|
|                | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
| 1              | 0.82 | 0.85 | 0.78 | 0.75 | 0.52 | 0.88 | 0.44 | 0.39 | 0.82 | 0.56 |
| 2              | 0.56 | 0.72 | 0.88 | 0.46 | 0.72 | 0.56 | 0.68 | 0.45 | 0.48 | 0.75 |
| 3              | 0.45 | 0.61 | 0.54 | 0.73 | 0.84 | 0.84 | 0.78 | 0.42 | 0.53 | 0.65 |
| 4              | 0.56 | 0.42 | 0.76 | 0.84 | 0.73 | 0.83 | 0.86 | 0.62 | 0.78 | 0.82 |
| 5              | 0.45 | 0.58 | 0.38 | 0.44 | 0.36 | 0.59 | 0.78 | 0.77 | 0.65 | 0.81 |
| 6              | 0.46 | 0.61 | 0.55 | 0.68 | 0.75 | 0.83 | 0.73 | 0.66 | 0.82 | 0.48 |
| 7              | 0.66 | 0.71 | 0.65 | 0.44 | 0.86 | 0.79 | 0.44 | 0.85 | 0.53 | 0.39 |

The proposed MWWO performances are compared with that of the other intelligent algorithms, such as PSO [7], BA [10], GWO [11], FPA [24], SCA [25], and WWO [20]. The primary parameters are given (see Table 3).

Table 3. The seven algorithms parameters

| Algorithms | Parameter values |
|------------|------------------|
| GWO        | The population size is 30. |
| BA         | \( A = 0.25 \), \( f \in [0,2] \), \( r = 0.5 \), the population size is 30. |
| FPA        | \( \rho = 0.8 \), the population size is 30. |
| SCA        | \( r_s \in [0.2\pi] \), \( a = 2 \), \( r_s \in [0,1] \), the population size is 30. |
| PSO        | \( c_1 = 1.4962 \), \( c_2 = 1.4962 \), \( \omega = 0.7298 \), the population size is 30. |
| WWO        | \( \lambda = 0.5 \), \( h_{\max} = 12 \), \( \alpha = 1.0026 \), \( \beta \in [0.01,0.25] \), \( k_{\max} = \min(12,D/2) \), the population size is 30. |
| MWWO       | \( \lambda = 0.5 \), \( h_{\max} = 12 \), \( \alpha = 1.0026 \), \( \beta \in [0.01,0.25] \), \( k_{\max} = \min(12,D/2) \), \( w_{\min} = 0.4 \), \( w_{\max} = 1.5 \), \( a = 0.02 \), \( b = 20 \), \( c_{\min} = 0.1 \), \( c_{\max} = 0.5 \), \( c = 60 \), \( d = 5 \); The population size is 30. |

The objective of WTA is to find the best decision matrix \( X \) and achieve the optimal combat effectiveness, that is to say, the mathematical expectations of the damage targets is maximized. The weapon systems effectively strikes the air threaten targets, which is reasonably assigned to the weapon units. There are three weapon units be arranged to first target and one weapon unit is arranged to second target in first weapon system, some targets have been stricken by several weapon units. For the third weapon system, there are for weapon units be arranged to third, fourth, fifth and fifth target. For the seventh weapon system, there are second, second, fifth and sixth target can be attacked by three weapon units, separately. To better
demonstrate the performance of the MWWO, the comparison result between MWWO and other algorithms are given in Table 4.

Table 4. The comparison results between MWWO and the other six algorithms.

| Objective function | Algorithm | Best      | Worst     | Mean      | Std        | Rank |
|--------------------|-----------|-----------|-----------|-----------|------------|------|
| f                  | GWO       | 0.773019  | 0.39      | 0.628297  | 0.097394   | 6    |
|                    | BA        | 0.773019  | 0.39      | 0.679762  | 0.106242   | 7    |
|                    | FPA       | 0.861542  | 0.6279    | 0.742109  | 0.066069   | 5    |
|                    | SCA       | 0.773019  | 0.6279    | 0.656924  | 0.05904    | 4    |
|                    | PSO       | 0.861542  | 0.861542  | 0.861542  | 4.52E-16   | 1    |
|                    | WWO       | 0.955849  | 0.955084  | 0.981475  | 0.011098   | 2    |
|                    | MWWO      | 0.995849  | 0.955084  | 0.981475  | 0.011098   | 2    |

From the Table 4, we can obtain that mean fitness function value of the MWWO is the best, which shows the MWWO has strong stable performance and balances exploration and exploitation. The optimal fitness value of the MWWO is better than that of other algorithms, which represents the MWWO has fast convergence speed and accurate accuracy.

Figure 3. Fitness functions evolution curves
Figure 4. ANOVA test

As is revealed in the Fig.3, the MWWO compared with GWO, BA, FPA, SCA, PSO and WWO algorithms, the MWWO convergence speed is the fastest, that is to say, it achieves the optimal combat effectiveness or obtains maximum mathematical expectation. The MWWO search ability is better than that of the original WWO, because it is adopted improved propagation operator and greedy strategy. So the MWWO calculation accuracy is much better than that of the GWO, BA, FPA, SCA, PSO and WWO algorithms. In Fig.4, we can seen the standard deviation of the MWWO is much worse than that of the PSO, but the MWWO gains maximum combat effectiveness. The standard deviation of the MWWO is better than other algorithms.

5. Conclusion
This paper, an improve MWWO is proposed, which can better balance exploration and exploitation ability. The optimal fitness value of the MWWO is much better than that of other intelligent optimization algorithms. The MWWO is used to solve the WTA problem, which can find the optimal decision matrix \( X \) and maximum mathematic expectation. The experimental results show that proposed MWWO outstanding performance than other the swarm intelligent algorithms in terms of the performance measures.

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