The MSSM Higgs sector and $B - \overline{B}$ mixing for large $\tan \beta$

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Abstract. A systematic analysis of Higgs-mediated contributions to the $B_d$ and $B_s$ mass differences is presented in the MSSM with large values of $\tan \beta$. In particular, supersymmetric corrections to Higgs self-interactions are seen to modify the correlation between $\Delta M_b$ and $B(B_d \to \mu^+ \mu^-)$ for light Higgses. The present experimental upper bound on $B(B_s \to \mu^+ \mu^-)$ is nevertheless still sufficient to exclude noticeable Higgs-mediated effects on the mass differences in most of the parameter space.

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1 Introduction

If supersymmetry (SUSY) were exact, the two Higgs doublets of the Minimal Supersymmetric extension of the Standard Model (MSSM) would not be able to mix, and one of them only. $H_u = (h_u^0, h_u^\pm)$ would couple to up-type quark singlets while the other one, $H_d = (h_d^0, h_d^\pm)$, would interact with down-type quark singlets. As SUSY-breaking is required to be soft, this peculiar Yukawa structure actually holds at tree-level, and the dangerous flavour-changing neutral currents (FCNC) that can be generated after spontaneous electroweak symmetry breaking by the coupling of the quarks to the “wrong” Higgs are loop-suppressed:

$$\mathcal{L}_{FCNC}^{Higgs} = \kappa_{IJ} \overline{d}_L^I d_L^J (c_\beta h_u^0 - s_\beta h_u^\pm) + \kappa^*_{IJ} \overline{d}_R^I d_R^J (c_\beta h_u^0 - s_\beta h_u^\pm)$$

in the quark mass eigenstate basis, with the abbreviations $c_\beta \equiv \cos \beta$, $s_\beta \equiv \sin \beta$, and, under the Minimal Flavour Violation (MFV) assumption,

$$\kappa_{IJ} \approx \frac{m_t}{v} V_{tI} V_{tJ} t_\beta \varepsilon_Y,$$

with $\varepsilon_Y$, a loop factor, $t_\beta \equiv v_u/v_d$, the ratio of the two Higgs vacuum expectation values (VEV), and $v^2 = v_u^2 + v_d^2$. Note that the local effective interaction Eq.(1) supposes the scale hierarchy $M_{SUSY} \gg v$. The loop factor $\varepsilon_Y$ is then essentially driven by squark and higgsino intermediate states (see Eq.1a). Its effect is however non-decoupling in the limit $M_{SUSY} \to \infty$ as the induced effective operator has dimension-four (see e.g. Refs. [2–5] for details). For large $t_\beta$, one can see that the loop suppression is compensated, opening the door to large Higgs-mediated effects in flavour physics [2, 6].

A clean signature of this scenario was proposed in Ref. [3], which predicted a decrease of the mass difference in the $B_s - \overline{B}_s$ system, $\Delta M_s$, with respect to its Standard Model value, in direct correlation with an increase of the $B_s \to \mu^+ \mu^-$ branching fraction. Interestingly, as first noted in [1, 2], the a priori dominant Higgs-mediated contribution to $\Delta M_q$ (see Fig.2a, $q = d, s$),

$$\Delta M_q^{RR} \sim -\kappa_{bq}^2 \left( \frac{s_{\alpha - \beta}}{M_H} + \frac{c_{\alpha - \beta}}{M_h} - \frac{1}{M_A} \right),$$

where $\alpha$ denotes the CP-even Higgs mixing angle and $M_{H,h,A}$, the neutral Higgs masses, actually vanishes when tree-level Higgs mass relations are implemented. The aforementioned correlation was then derived flipping the chirality of one of the external $b$ quarks:

$$\Delta M_q^{LR} \sim -\kappa_{bq} \kappa_{qb} \left( \frac{s_{\alpha - \beta}}{M_H} + \frac{c_{\alpha - \beta}}{M_h^2} + \frac{1}{M_A^2} \right),$$

which costs a factor of $\kappa_{bq}^2/\kappa_{qb} = m_q/m_b$. The subject of the work reported here [7] is the systematic identification and computation of all contributions that present one suppression factor with respect to the superficially dominant term Eq.(3), and should thus be added to Eq.(4) before concluding on the correlation between $\Delta M_q$ and $B_q \to \mu^+ \mu^-$. 

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Consequently, for large \( \tan \beta \) that is to say, \( v_d \to 0 \) and fixed \( M_A \), \( B \mu \) tends to zero, and, as the \( h_0^0 \) FCNC coupling in Eq.(1) also vanishes, the two-Higgs doublet model (2HDM) composed of Eqs.(1) and (5) becomes invariant under the Peccei-Quinn-type symmetry with charge assignments \([4, 8, 9]\)\(^1\):

\[
U(1)_{PQ} : \quad Q(H_d) = Q(d_R^I) = 1, \quad Q(\text{other}) = 0. \tag{7}
\]

The cancellation in Eq.(3) now follows (at least in the \( v_d \to 0 \) limit) from the fact that the corresponding amplitude, with two right-handed external \( b \) quarks, requires a change of the PQ charge by two units (see Fig.2b), and therefore cannot be generated by tree-level Higgs exchanges.

Non-zero contributions are then obtained

- either allowing the conservation of the PQ charge, which can be done by (i) flipping the chirality of one of the external \( b \) quarks, as said before (Fig.2a) [3]; (ii) avoiding the suppressed \( \bar{b}_L q_R h_0^0 \) FCNC coupling but allowing for one loop in the effective 2HDM (Fig.3b). The diagram corresponding to this second possibility is readily computed from Eqs.(1) and (5) in the large \( \tan \beta \) limit, and found numerically small. Note that charged Higgs effects are suppressed under our approximations [3, 4].

- or providing a breaking of the PQ symmetry via (iii) sparticle-loop corrections to the tree-level effective potential \( V^{(0)} \) (Fig.3c); (iv) higher-dimensional quark-Higgs effective operators (Fig.3d). These cost a SUSY loop, like the dimension-four effective coupling of Fig.1a. Then, as the only place where this loop can be compensated by a large \( \tan \beta \) factor is the modification of the expression of the quark interaction eigenstates

\[\Delta M_q \text{ anatomy for large } \tan \beta\]

In order to properly identify the relevant contributions, let us have a closer look at the cancellation in Eq.(3). The tree-level Higgs mass matrices follow from the potential

\[
V^{(0)} = m_1^2 H_1^+ H_d + m_2^2 H_2^+ H_u + B \mu \{ H_u \cdot H_d + h.c. \} + \frac{\lambda_1}{8} (H_1^+ H_d - H_2^+ H_u)^2 + \frac{\lambda_2}{2} (H_1^+ H_u)(H_2^+ H_d), \tag{5}
\]

where \( m_{1,2}^2 \) and \( \lambda_{1,2} \) denote soft-breaking terms, \( \mu \) is the supersymmetric Higgs mass parameter, and \( H_u \cdot H_d = H_d^\dagger \varepsilon H_d \) with \( \varepsilon^{12} = +1 \). In particular, we have:

\[
M_A^2 = B \mu s_\beta^{-1} c_\beta^{-1}. \tag{6}
\]

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\[
\text{in terms of the quark mass eigenstates in the tree-level } \quad \bar{d}_R^0 Q_1^L \cdot H_d \text{ Yukawa vertex (here the quark fields are understood in the interaction eigenstate basis), we actually end up again, in good approximation, with a coupling of the type } \bar{b}_R q_L h_0^0 \text{ (in the mass eigenstate basis now), and the usual cancellation mechanism takes place.}
\]

Note that higher-order sparticle-loop effects in the four-dimensional Yukawa vertices would still lead to a vanishing \( \Delta M^{H_{RR}}_q \) at all orders in the \( t_{\text{ch}}^{-1} \) expansion for tree-level Higgs exchanges as the combination of Higgs fields appearing in Eq.(1) would be unchanged. Indeed, the occurrence of \( h_0^d \) or \( h_0^u \) is fixed by gauge symmetry for general dimension-four Yukawa interactions, and, replacing \( h_0^d \) by \( v_{u,d} \), one must obtain zero in the quark mass eigenstate basis.

Corrections (iii) have been analyzed recently \([10, 11]\). Their size is however subject to controversy. We thus go through them again in the next section.

\[\Delta M_q \text{ anatomy for large } \tan \beta\]

\[\text{3 SUSY corrections to the Higgs potential}\]

Sparticle loop corrections to \( V^{(0)} \) are determined at the one-loop level via a matching calculation on the most general dimension-four 2HDM potential for \( M_{SUSY} \gg v \)

\[
V^{(1)} = m_1^2 H_1^+ H_d + m_2^2 H_2^+ H_u + \{ m_{12}^2 H_u \cdot H_d + h.c. \} + \frac{\lambda_1}{2} (H_1^+ H_d)^2 + \frac{\lambda_2}{2} (H_1^+ H_u)(H_2^+ H_d) + \lambda_3 (H_1^+ H_u)(H_1^+ H_d) + \frac{\lambda_5}{2} (H_u \cdot H_d)^2 - \left[ \lambda_6 (H_1^+ H_d) + \lambda_7 (H_1^+ H_u) \right] (H_u \cdot H_d) + h.c., \tag{8}
\]

where \( m_{12}^2 \) and \( \lambda_{5-7} \) may be complex. Such a computation was actually already performed in the context of the corrections to the lightest Higgs mass \( M_h \). The explicit expressions for the \( \lambda_i \)'s available in the literature \([12]\), however, assume various approximations such as degenerate squark soft-breaking parameters or real trilinear terms. These were removed in the computation of the Higgs mass matrices \([13]\), but an updated list of \( \lambda_i \)'s including the effects of all sparticles for arbitrary flavour structure has to our knowledge not been published. Yet, \( \Delta M^{H_{RR}}_q \) in Eq.(3) takes a particularly transparent form when working in the Higgs interaction eigenstate basis, being directly related to the \( \Delta M^{(1)_{PQ}}_q \) violating Higgs self-couplings \( \lambda_5 \) and \( \lambda_7 \), see Eq.(13). We

\[\Delta M_q \text{ anatomy for large } \tan \beta\]
thus performed the matching again, keeping arbitrary $3 \times 3$ soft-breaking matrices. Particular attention was paid to the definition of the Higgs fields in the effective 2HDM, closely related to the definition of $t_\beta$, as we now briefly explain.

We chose to renormalize the MSSM parameters $m_1^2$ and $m_2^2$ such that the one-loop tadpoles are renormalized to zero at the matching scale, i.e., the tree-level VEV $v_{ud}^{(0)}$ still sit at the minimum of the potential (no finite counterterms are introduced for the parameters $m_3^2$ and $\lambda_1$ in the effective 2HDM). The actual one-loop VEV $v_{ud}^{(1)}$ must however take into account the field redefinition needed to cast the kinetic terms

$$L_{kin}^{(1)} = (1 + \delta Z_{udd})\partial_\mu H_u^\dagger \partial^\mu H_u + (1 + \delta Z_{udd})\partial_\mu H_u^\dagger \partial^\mu H_u - \{\delta Z_{udd}\partial_\mu H_u - \partial^\mu H_u + h.c.\}$$

induced by the matching of the two-point Green functions into the canonical form. We then have:

$$\left(\begin{array}{c} v_u^{(1)} \\ v_d^{(1)} \end{array}\right) = \left(1 + \frac{\delta Z_{ud}}{2}\right) \cdot \left(\begin{array}{c} v_u^{(0)} \\ v_d^{(0)} \end{array}\right),$$

where $\delta Z_{21} \equiv \delta Z_{udd}$, $\delta Z_{12} \equiv \delta Z_{udd}^*$, etc. Now, we take advantage of the freedom to change the Higgs basis [14],

$$\left(\begin{array}{c} H_u^c \\ -H_d^c \end{array}\right) = e^{i\delta H/2} \cdot \left(\begin{array}{c} H_u \\ -H_d \end{array}\right),$$

where $\delta H$ is an arbitrary $2 \times 2$ hermitian matrix and $H_d^c \equiv e\ H_d^\dagger$, to (i) keep the VEV real and positive (ii) more importantly, ensure that the corrections to $t_\beta$ are $t_\beta$-suppressed, or, in other words, that $v_{ud}$ does not receive any corrections from $v_{ud}$. This amounts to the following modification of Higgs field redefinition:

$$\delta Z \rightarrow \delta Z + i\delta H \over 2 = \left(\begin{array}{c} \delta Z_{ud} \\ i(1 - \beta) \Im(\delta Z_{ud}) \delta Z_{ud} \over 2 \end{array}\right).$$

The effects of the corrected Higgs masses and mixings on the “flipped” amplitude Eq.(4) are not essential, and we will ignore them here for simplicity. In the large $t_\beta$ limit, we then have:

$$\Delta M_{q}^{LR} \sim -\kappa_{bd}^2 \lambda_\beta^2 \delta Z_{ud} \over 2 M_A. $$

In contrast, these effects are of course crucial for the “non-flipped” amplitude Eq.(3), given for large $t_\beta$ to a good approximation by:

$$\Delta M_{q}^{RR} \sim \kappa_{bd}^2 \left(\lambda_5 - \lambda_2^2 \over \lambda_2\right) \delta Z_{ud} \over 2 M_A^2,$$

in the absence of new CP-violating phases. The above quantity is generated via the PQ-symmetry breaking brought about by the $\mu$ parameter at loop-level. To be explicit, in the case of $\lambda_5$, we obtain (within MFV and discarding the small contributions from the first two generations, as well as those proportional to $g'$):

$$\lambda_5 = -\frac{3y_t^4}{8\pi^2} a_t^2 \mu^2 \frac{m_t^2}{M_t^2} L_1 \left(\frac{M_{FB}^2}{M_t^2}\right) - \frac{3y_t^4}{8\pi^2} a_t^2 \mu^2 \frac{m_t^2}{M_t^2} L_1 \left(\frac{M_{FB}^2}{M_t^2}\right) \left(\frac{M_{FB}^2}{M_t^2}\right)^2,$$

$$\frac{y_t^4}{8\pi^2} a_t^2 \mu^2 \frac{m_t^2}{M_t^2} L_1 \left(\frac{M_{FB}^2}{M_t^2}\right) + \frac{3y_t^4}{8\pi^2} a_t^2 \mu^2 \frac{m_t^2}{M_t^2} L_1 \left(\frac{M_{FB}^2}{M_t^2}\right) + \frac{\mu^2}{M^2} \kappa_{bd}^2 \left(\lambda_5 - \lambda_2^2 \over \lambda_2\right) \delta Z_{ud} \over 2 M_A$$

with the loop function

$$L_1(x) = \frac{-1}{(1-x)^2} - \frac{(1 + x) \ln x}{2(1-x)^3}. $$

A typical contribution is depicted in Fig.1b.

In Ref. [10], the corrected masses and mixings in Eq.(3) were determined using the FeynHiggs package. We disagree numerically with the obtained results. We also do not reproduce the pole singularity for $M_t = M_H$ found in Ref. [11]. From our analysis, it emerges that the source of the non-vanishing of $\Delta M_{q}^{RR}$ is to be found in the Higgs self-couplings $\lambda_5$ and $\lambda_7$ for large $t_\beta$, related to CP-even Higgs mixing self-energies.

### 4 Numerical analysis

As we already mentioned, Eq.(13) is responsible for a decrease of the $B_s - \overline{B_s}$ mass difference, while $\Delta M_d$ is basically unaffected due to $\kappa_{db} \sim m_d$ [3]:

$$\Delta M_{q}^{LR} = C_{q}^{LR} X \left[\frac{m_q}{0.06 \text{GeV}}\right] \left[\frac{m_s}{13 \text{GeV}}\right] \left[\frac{P_{1/2}^{LR}}{2.56}\right]$$

with $C_{s}^{LR} = -14 \text{ ps}^{-1}$, $C_{d}^{LR} \sim 0 \text{ ps}^{-1}$, and

$$X = \frac{(\varepsilon_\gamma 16\pi^2)^2}{(1 + \tilde{\varepsilon}_3 t_\beta)^2} \frac{m_t^4}{(1 + \varepsilon_0 t_\beta)^2} \frac{M_{W}^2 M_{A}^2}{t_\beta^4} \left[\frac{\kappa_{bd}^2}{50}\right].$$

The loop factors $\varepsilon_0$, $\varepsilon_\gamma$ and $\tilde{\varepsilon}_3 \equiv \varepsilon_0 + y_t^2 \varepsilon_\gamma$ may be found in Refs. [5, 11], including the effects of the electroweak couplings $g$ and $g'$. The new contribution Eq.(14), on the other hand, increases both $\Delta M_s$ and $\Delta M_d$ (note that $\lambda_5$ and the bag factor $P_4^{SLL}$ are both negative):

$$\Delta M_{q}^{RR} = C_{q}^{RR} X \left[\frac{m_q}{3 \text{GeV}}\right]^2 \left[\frac{P_{1/2}^{SLL}}{1.06}\right] \times \frac{M_{W}^2}{M_{A}^2} \left[-\lambda_5 + \lambda_2^2 \lambda_5 \over 2\lambda_2\right] \left[-16\pi^2\right]$$

with $C_{s}^{RR} = +4.4 \text{ ps}^{-1}$ and $C_{d}^{RR} = +0.13 \text{ ps}^{-1}$. The numbers in Eqs.(17) and (19) have been obtained using $|V_{td} V_{ub}^\dagger| = 0.041$ [15], $|V_{td} V_{ub}^\dagger| = 0.0086$ [15], $F_{B_s} = 0.24 \text{ GeV}$ and $F_{B_d} = 0.2 \text{ GeV}$. These values suffer from large uncertainties, and are given here for the purpose of illustration (ratios are defined for actual numerical studies, see Fig.4). They correspond to the Standard Model central values $\Delta M_{SUSY}^2 = 20 \text{ ps}^{-1}$ and $\Delta M_{q}^{RR} = 0.59 \text{ ps}^{-1}$.

A first observation is that the typical effect of $\Delta M_{q}^{RR}$ is suppressed with respect to that of $\Delta M_{q}^{LR}$, which is due to a 1/2 symmetry factor and the small value of $P_4^{SLL}$. The effective couplings in Eq.(19) are also not very large. To get an idea of their size, the residual $\lambda_5$ value for $M_{SUSY} \rightarrow \infty$ is given by

$$\lambda_5 \sim -\frac{1}{2} \left(y_t^4 + y_t^4 + \frac{y_t^4}{16\pi^2} - g^4 \right)^{1 \over 16\pi^2}.$$

The “non-flipped” contribution $\Delta M_{q}^{RR}$ can still be relevant for small $M_A$ (i.e., $< 200 \text{ GeV}$). However, in that case, the
The present experimental upper bound on $B(B_s \rightarrow \mu^+ \mu^-)$ on $\Delta M_s$. The dark gray (blue) line is the theoretical prediction for $R_s \equiv \log_{10} [B(B_s \rightarrow \mu^+ \mu^-)/\Delta M_s]$, the light gray (red) lines indicate the size of SUSY effects in $\Delta M_s$, and the gray band shows the values of $R_s$ excluded experimentally [18]. The dashed line corresponds to $\Delta M_s = \Delta M_s^{SM} + \Delta M_s^{LR}$, while the plain line also takes $\Delta M_s^{LR}$ into account. Supersymmetric parameters have been fixed as follows: $t_\beta = 40$, $a_{t,b} = 2000$ GeV, $M_\chi = 1500$ GeV, $M_{\tilde{g}} = M_{\tilde{W}} = 1000$ GeV, $M_1 = 500$ GeV. Right: Analogue for the correlation between $\Delta M_d$ and $B(B_d \rightarrow \mu^+ \mu^-)$ (experimental values from [18, 19]). The bound on $B(B_d \rightarrow \mu^+ \mu^-)$ is at present not as efficient as the bound on $B(B_s \rightarrow \mu^+ \mu^-)$ to exclude Higgs-mediated effects on the mass differences, and $R_s (\simeq R_d)$ is preferably used.

The overall effect in $\Delta M_q$ (see Fig.4). In other words, the correlation between $B(B_q \rightarrow \mu^+ \mu^-)$ and $\Delta M_q$ can be modified, but this does not spoil the conclusion derived in Refs. [17] that the present data on $B(B_s \rightarrow \mu^+ \mu^-)$ already exclude visible effects in $\Delta M_q$ (it actually reinforces it, see Fig.4), while a similar conclusion can be reached for $\Delta M_d$.

Non negligible effects compatible with the $B_q \rightarrow \mu^+ \mu^-$ constraints are not excluded in some corners of parameter space, for large $\mu$ and large $a$-terms. However, they again require light Higgses, which is in any case disfavored (and partly excluded) by the observed $B \rightarrow \tau \nu$ branching fraction. A small window for very light CP-odd Higgs mass is still allowed for large $t_\beta$, but corresponds to the somewhat fine-tuned scenario where charged Higgs effects in $B \rightarrow \tau \nu$ interfere destructively with the Standard Model amplitude, and are about twice its value.

5 Conclusion

We have performed a systematic analysis of Higgs-mediated contributions to $\Delta M_q$ in the MFV-MSSM with large $\tan \beta$ and sparticles at the TeV scale. For $M_A > 200$ GeV, no new effect is found. For small $M_A$, SUSY loop corrections to the Higgs self-interactions can (moderately) modify the correlation between $\Delta M_q$ and $B(B_q \rightarrow \mu^+ \mu^-)$. The present experimental upper bound on $B(B_s \rightarrow \mu^+ \mu^-)$ is however still sufficient to exclude visible Higgs-mediated effects on $\Delta M_q$ in (practically) all parameter space. The precise measurements of $\Delta M_q$ are then to be used more as a normalization to avoid the large uncertainties related to $F_{B_s}$ and $V_{td}$ when using $B_q \rightarrow \mu^+ \mu^-$ to probe the MSSM in the large $t_\beta$ regime.

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Fig. 4. Left: constraint from $B(B_s \rightarrow \mu^+ \mu^-)$ on $\Delta M_s$. The dark gray (blue) line is the theoretical prediction for $R_s \equiv \log_{10} [B(B_s \rightarrow \mu^+ \mu^-)/\Delta M_s]$, the light gray (red) lines indicate the size of SUSY effects in $\Delta M_s$, and the gray band shows the values of $R_s$ excluded experimentally [18]. The dashed line corresponds to $\Delta M_s = \Delta M_s^{SM} + \Delta M_s^{LR}$, while the plain line also takes $\Delta M_s^{LR}$ into account. Supersymmetric parameters have been fixed as follows: $t_\beta = 40$, $a_{t,b} = 2000$ GeV, $M_\chi = 1500$ GeV, $M_{\tilde{g}} = M_{\tilde{W}} = 1000$ GeV, $M_1 = 500$ GeV. Right: Analogue for the correlation between $\Delta M_d$ and $B(B_d \rightarrow \mu^+ \mu^-)$ (experimental values from [18, 19]). The bound on $B(B_d \rightarrow \mu^+ \mu^-)$ is at present not as efficient as the bound on $B(B_s \rightarrow \mu^+ \mu^-)$ to exclude Higgs-mediated effects on the mass differences, and $R_s (\simeq R_d)$ is preferably used.