Observation of Intensity-Intensity Correlation Speckle Patterns with Thermal Light

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In this Letter, we propose a modified Hanbury-Brown and Twiss (HBT) scheme to change thermal correlations for observing the intensity-intensity correlation speckle for thermal light. Our scheme, same as two-photon speckle, is different from those based on ghost imaging. The thermal photons in our case pass through a common transmission mask (TM), and the light source here is thermal light not the entangled two-photon source.

We first briefly discuss the traditional HBT scheme, see Fig. 1. The light passes through the TM, and then it is divided into two paths by the beam splitter (BS). It is known that, for thermal or incoherent sources obeying Gaussian statistics, the intensity-intensity correlation $C_T(x_1, x_2)$ is expressed by Siegert relation [19]

$$C_T(x_1, x_2) = \langle I_T(x_1) \rangle \langle I_T(x_2) \rangle + |W_T(x_1, x_2)|^2,$$

where $\langle I_T(x_j) \rangle$ ($j = 1, 2$) are the average intensities on the output planes, $W_T(x_1, x_2)$ is the cross-spectral density between the two output planes, and they are respectively given by [15]

$$\langle I_T(x_j) \rangle = \int \int W_i(\nu_1, \nu_2)h^*_j(\nu_1, x_1)h_j(\nu_2, x_2)d\nu_1d\nu_2,$$

$$W_T(x_1, x_2) = \int \int W_i(\nu_1, \nu_2)h^*_i(\nu_1, x_1)h_i(\nu_2, x_2)d\nu_1d\nu_2.$$

Here $W_i(\nu_1, \nu_2) \equiv \langle E_i^\dagger(\nu_1)E_i(\nu_2) \rangle$ is the initial cross-spectral density of the input random light fields $E_i(\nu)$ at

![FIG. 1: (color online). The traditional HBT scheme. The TM is in front of the beam splitter (BS), and the intensities on the output planes (OPs) 1 and 2 are correlated by a correlator. Optical paths 1 (2) from the TM to the OP’s 1 (2) are characterized by the $2 \times 2$ ray transfer matrices.](attachment:image.png)

In this Letter, we propose a modified Hanbury-Brown and Twiss (HBT) scheme to change thermal correlations for observing the intensity-intensity correlation speckle for thermal light. Our scheme, same as two-photon speckle [5,7], is different from those based on ghost imaging. The thermal photons in our case pass through a common transmission mask (TM), and the light source here is thermal light not the entangled two-photon source.
the TM. The impulse response functions $h_j(\nu, x_j)$, from the Collins' formula, can be expressed as  

$$h_j(\nu, x_j) = t(\nu) \left( -\frac{i}{\lambda B_j} \right)^2 e^{-i \frac{2\pi}{\lambda B_j}} (A_j \nu^2 - 2\nu x_j + D_j x_j^2)$$  

(4) 

under the paraxial approximation, where $\lambda$ is the wavelength, $A_j$, $B_j$, and $D_j$ are the elements of the $2 \times 2$ ray transfer matrices $\begin{pmatrix} A_j & B_j \\ C_j & D_j \end{pmatrix}$ describing the linear optical systems from the TM to the output planes, and $t(\nu)$ is the complex transmission coefficient of the TM.

For simplicity, both optical paths 1 and 2 are assumed to be within the range of Fraunhofer diffraction \[21\], i. e., $A_j = 0$. Meanwhile, the input light is a thermal or incoherent source, i. e., $W_i(u_1, u_2) = I_0 \delta(u_1 - u_2)$ with $I_0$ a constant. Therefore, $C_T(x_1, x_2)$ can be written as

$$C_T(x_1, x_2) = \langle I_T(x_1) \rangle \langle I_T(x_2) \rangle [1 + \mu_T(x_1, x_2)]$$  

(5) 

where

$$\mu_T(x_1, x_2) = \frac{1}{N_0^2} \left| F_1 \left[ \langle t(\nu)^2 \rangle \left( \frac{x_2}{\lambda B_2} - \frac{x_1}{\lambda B_1} \right) \right] \right|^2$$  

(6) 

is the normalized phase-insensitive shape function. This shape function is only related to $\langle t(\nu)^2 \rangle$, $\langle I_T(x) \rangle = I_0 N_0 (\lambda |B_j|)^{-1}$ with $N_0 = \int \langle t(\nu)^2 \rangle d\nu$, and $F_1$ denotes the one-dimensional Fourier transform of $\langle t(\nu)^2 \rangle$ with the argument of $\frac{x_2}{\lambda B_2} - \frac{x_1}{\lambda B_1}$. It is clear that $\mu_T(x_1, x_2)$ contains only the partial information of $t(\nu)$ [i. e., the amplitude of $t(\nu)$], and it does not have any phase information of $t(\nu)$. Therefore, the thermal intensity-intensity correlations based on the traditional HBT scheme are essentially phase insensitive \[24\]. It should be emphasized that both $\langle I_T(x_1) \rangle$ and $\langle I_T(x_2) \rangle$ are uniform and have no any information of $t(\nu)$ for completely incoherent fields.

In order to overcome the limit of the traditional HBT-based scheme, we design a new optical system to fulfill the phase-sensitive intensity-intensity correlation scheme for thermal light, as shown in Fig. 2. The thermal fields first pass through the optical systems in Fig. 2(a), for generating the modified thermal source at the incident plane $\nu$ of the TM [in Fig. 2(b)]. A forward non-degenerated phase conjugation (PC) device \[23\] is inserted into the upper path in Fig. 2(a), and it generates the PC waves with wavelength $\lambda_p$ (here $\lambda_p \neq \lambda$). When $\lambda d_1 = \lambda_p d_2$, where $d_1$ ($d_2$) are the distances from the input plane $u$ (the PC device) to the PC device (the TM), then the random light at the TM via the upper path forms a conjugated image of the input light, i. e., $E_{\mathrm{up}}(\nu) = \alpha E_1(\nu)$ \[24\], where $\nu$ is the coordinate on the incident plane of the TM, and $\alpha$ is the rate of generating the PC light. In the lower path of Fig. 2(a), it consists of two pairs of $4-f$ optical systems \[25\] \[26\] with the same focus length $f_L$. Thus, the light at the TM via the lower optical path is the same as the input field, i. e., $E_{\mathrm{down}}(\nu) = \alpha E_1(\nu)$ \[24\], and $\nu$ is the coordinate on the incident plane of the TM, and $\alpha$ is the rate of generating the PC light. In the lower path of Fig. 2(a), it consists of two pairs of $4-f$ optical systems \[25\] \[26\] with the same focus length $f_L$. Thus, the light at the TM via the lower optical path is the same as the input field, i. e., $E_{\mathrm{down}}(\nu) = \alpha E_1(\nu)$ \[24\]. In Fig. 2(b), it displays the measurement diagram of the intensity-intensity correlation, and the total light fields from both two paths of Fig. 2(a) pass through the common TM. The subsystems from the TM to two output planes 1 and 2 also lie in Fraunhofer region [i. e., $A_j = 0$] \[21\], and they are the same as those in Fig.I except for the additional optical filters. The filters 1 and 2 transmit the light fields of wavelength $\lambda_p$ and $\lambda$, respectively, while blocking the remainder in each arm. Therefore, the intensity-intensity correlation in the modified system can also be derived from its definition: $C_M(x_1, x_2) = \langle I_M(x_1) I_M(x_2) \rangle$ \[27\], where $I_M(x_1, 2)$ are the instantaneous intensities on each output plane. It is the correlation between the original random light fields and their PC fields that leads to the phase-sensitive term. Thus, $C_M(x_1, x_2)$ now can be written as

$$C_M(x_1, x_2) = \langle I_M(x_1) \rangle \langle I_M(x_2) \rangle [1 + \mu_M(p(x_1, x_2))]$$  

(7) 

where $\mu_M(p(x_1, x_2)) = |W_M(p(x_1, x_2))^2|/\langle I_M(x_1) \rangle \langle I_M(x_2) \rangle$ is the normalized phase-sensitive shape function and it is dependent on the detailed configuration of the optical system containing the TM [see Fig. 2(c)], and $W_M(p(x_1, x_2)) = \alpha \int W(v_1, v_2) h(v_1, x_1) h(v_2, x_2) dv_1 dv_2$ is the phase-sensitive cross-spectral density between the two output planes in Fig. 2(b). Actually, $\mu_M(p(x_1, x_2)$
determines the main behavior of $C_M(x_1, x_2)$ since the common factor $\langle I_M(x_1) \rangle \langle I_M(x_2) \rangle$ is separable.

Next we present the results for three configurations with thermal light, as shown in Fig. 2(c), demonstrating the similar features as two-photon speckle patterns [3], although the calculation is tedious but straightforward.

In the configuration (i), the TM is located at the common imaging position of both paths of Fig. 2(a). In this case, $\mu_M^{(p)}(x_1, x_2)$ in Eq. (7) is given by [29]

$$
\mu_M^{(p)}(x_1, x_2) = \frac{1}{N_0^2} \left[ F_1 \left[ t^2(\nu) \right] \left( \frac{x_1}{\lambda_B} + \frac{x_2}{\lambda_B} \right)^2 \right], \quad (8)
$$

It is clear that $\mu_M^{(p)}(x_1, x_2)$ has a different form compared to Eq. (6) as $|t(\nu)|^2$ is replaced by $t^2(\nu)$. The modified intensity-intensity correlation in this case naturally contains all phase-sensitive information of $t(\nu)$. Here the average output intensities are $\langle I_M(x_1) \rangle = I_0 N_0 (\lambda/|B_1|)^{-1}$ and $\langle I_M(x_2) \rangle = I_0 N_0 (\lambda/|B_2|)^{-1}$, which are constants and can also be subtracted from the measurement of $C_M(x_1, x_2)$. When $\lambda_B = \lambda B_2$, Eq. (8) becomes $\mu_M^{(p)}(x_1, x_2) = N_0^{-2} \left[ F_1 \left[ t^2(\nu) \right] \left( \frac{x_1 + x_2}{\lambda_B} \right)^2 \right]$, i.e., a function of the sum coordinate $x_1 + x_2$. This property is also at ease that of the two-photon speckle for the configuration (a) in Ref. [3].

In the configuration (ii), the TM is placed at the exit plane of a 2-f Fourier optical system with the focus length $f_c$. [25] 26, so that $\mu_M^{(p)}(x_1, x_2)$ is given by [29]

$$
\mu_M^{(p)}(x_1, x_2) = \frac{\lambda}{\lambda_B N_0^2} \left[ F_1 \left[ \Omega(\nu) \right] \left( \frac{x_2}{\lambda_B} - \frac{x_1}{\lambda_B} \right)^2 \right], \quad (9)
$$

From Eq. (9), the phase sensitive effect comes from the Fourier transformation of $\Omega(\nu)$. The average intensities here are the same as that of the configuration (i). Different from the previous case, $B_2 = \lambda B_2$, Eq. (7) can be rewritten as $\mu_M^{(p)}(x_1, x_2) = \frac{\lambda}{\lambda_B N_0^2} \left[ F_1 \left[ \Omega(\nu) \right] \left( \frac{x_2 - x_1}{\lambda_B} \right)^2 \right]^2$, which is a function of the difference coordinate $x_2 - x_1$. This property is also similar to that of the two-photon speckle for the configuration (b) in Ref. [3].

For the configuration (iii), two TMs are placed at the incident and exit planes of the 2-f Fourier optical system with the same $f_c$. As pointed out in Ref. [3], this configuration mimics a volume interferometer. By a tedious but straightforward calculation, $\mu_M^{(p)}(x_1, x_2)$ is given by [29]

$$
\mu_M^{(p)}(x_1, x_2) = \frac{F_2 \left[ \Theta_{\nu} \right] \left( \frac{x_1}{\lambda_B} \frac{x_2}{\lambda_B} \right)^2}{S(x_1)S(x_2)}, \quad (10)
$$

where $F_2$ denotes the two-dimensional Fourier transform, $\Theta_{\nu} = \eta \theta_1(v_1) \theta_2(v_2) F_1 \left[ t_c(\nu) \right] \left( \frac{x_1}{x_{fc}} + \frac{x_2}{x_{fc}} \right)$ with $\eta = \frac{1}{x_{fc}} \left( \frac{\lambda}{\lambda B_1} \right)^{-1/2}$, and $S(x_j) = F_2 \left[ \Theta_{\nu,\nu} \right] \left( \frac{x_1}{x_{\nu,\nu}} \frac{x_2}{x_{\nu,\nu}} \right)$ with $\Theta_{\nu,\nu} = \lambda_{\nu}(v_1, v_2)$.

![Figure 3](image-url)

**FIG. 3.** (Color) Dependence of $\mu_M^{(p)}(x_1, x_2)$ on the phase of one double slits. (a) $\phi = 0$ or $\pi$, (b) $\phi = \pi/4$, (c) $\phi = \pi/2$, and (d) $\phi = 3\pi/4$. Other parameters are $\lambda_B = 0.25$ mm$^2$, $a = 0.5$ mm, and $b = 1.0$ mm.
\( \mu_M^{(p)}(x_1, x_2) \) for three different diffusers in the configuration (i). The random amplitude and phase distributions of three diffusers are correspondingly shown at the upper parts in Figs. 4(a)-4(c). Note that the values of \(|t(\nu)| \) in Figs. 4(a)-4(c) are the same, while their phase magnitudes are totally different. It is seen that the patterns of \( \mu_M^{(p)}(x_1, x_2) \) vary with changing the phase distributions of \( t(\nu) \), and the more randomness of the phase distributions may lead to the more homogeneous interference speckle patterns with the smaller average speckle size.

In Fig. 5, we demonstrate the patterns of \( \mu_M^{(p)}(x_1, x_2) \) for the diffusers in (a) the configuration (ii), and (b-c) the configuration (iii). The functions of the TMs in these simulations are the same as that in Fig. 4(c). Comparing with Fig. 4(c), the pattern in Fig. 5(a) is along with the difference coordinate \( x_1 - x_2 \) not along with the sum coordinate \( x_1 + x_2 \). Such changes are similar to the cases in two-photon speckle \[5\], and they cannot happen in the traditional HBT scheme with thermal light. From Figs. 5(b-c), for the configuration (iii), the patterns of \( \mu_M^{(p)}(x_1, x_2) \) mimic the volume scatterer, and the nonfactorizable features in the correlation patterns are clearly seen. For a small value of \( f_c \) in Fig. 5(c), the correlation speckle spots in the pattern of \( \mu_M^{(p)}(x_1, x_2) \) are elongated along the difference coordinate of \( x_1 - x_2 \). This can be understood from the fact that the second diffuser is illuminated with the far-field patterns of the first diffuser. Within the same area of the second diffuser, the smaller of \( f_c \), the less information from the first diffuser can be projected. This can be seen from the form of the function \( \Theta_M(\nu_1, \nu_2) \). Therefore, we can conclude that the modified HBT scheme with thermal light can provide the phase-sensitive intensity-intensity correlation speckle.

Lastly, we discuss the possibility of experimentally realizing our scheme. The key challenge of our scheme in Fig. 2 is to generate the non-degenerate PC fields of thermal light. For demonstrating our predicted result, one can employ the pseudothermal light source (produced via the random scattering when a laser field passes through a ground glass) as the input light. The PC light of the pseudothermal light can be generated via the conventional PC technologies, such as the four-wave mixing processes (e.g., Refs. [32-35]) and the stimulated scattering processes (e.g., Refs. [36,39]). For example, the nondegenerate PC light is generated by using a Pr\(^{3+}\):Y\(_2\)SiO\(_5\) crystal based on the electromagnetically induced transparency effect [40]. Meanwhile, the fidelity of the PC fields may have an influence on the correlations between the input and PC fields, and this will in turn affect the intensity-intensity correlations. In another scheme, we can use the novel digital PC technology [41-44], which does not involve the nonlinear processes and can even generate the high-quality PC waves for the weak, incoherent fluorescence signal [45], to verify this effect. In fact, if the filters in Fig. 2(b) are removed or disabled (when \( \lambda_p = \lambda \)), both the phase-sensitive and phase-insensitive terms will occur in Eq. (7), which only increases the complexity to determine the phase-sensitive patterns.

In summary, we have presented the phase-sensitive intensity-intensity correlation speckle effect of thermal light in the modified HBT scheme. This scheme is based on introducing the PC light to change the correlations between the two optical paths. It is revealed that the phase-sensitive and nonfactorizable features can be seen in thermal intensity-intensity correlation speckle. Finally, the discussion on the experimental realization is presented. This scheme is different from those thermal ghost imaging and diffraction [8-10,31,46], and the unbalanced interferometer-based scheme via the direct intensity measurements [47], since all thermal photons in our case pass through the common sample. Our scheme can also be used to recover the phase information in the thermal-like temporal intensity-intensity correlation cases [48]. This modified HBT scheme may have important applications for developing the intensity-intensity correlation speckle.
and imaging technologies of thermal or incoherent light sources.

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