Carbon forestry compensation on estate level

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Abstract
The expense of carbon sequestration in terms of capital return deficiency is investigated at estate level, in the case of a fertile boreal estate dominated by spruce forest. Thinnings from below result as a high expense of increased rotation age, thinnings from above as a small expense. The expense of increased timber stock is greater than any proportional carbon rent based on present carbon prices. Application of non-proportional carbon rent is proposed.

Keywords
Capitalization; capital return rate deficiency; expected value; carbon storage; timber stock; carbon rent

1. Introduction
Boreal forests constitute a potentially significant carbon sink. A particular benefit of boreal regions is significant carbon storage in soil. It has been approximated that the amount of soil carbon may exceed the carbon storage in living biomass [1, 2, 3, 4]. However, living biomass produces the litter resulting in soil carbon accumulation. The rate of carbon storage depends on the rate of biomass production on the site. Biomass production rate in turn is related to the amount of living biomass [5, 6, 2].
In the occurrence of clearcutting and soil preparation, net release of carbon from the soil to the atmosphere begins [7, 8, 9, 10]. Correspondingly, maintenance of canopy cover possibly is essential in carbon sequestration.

In general, the amount of productive biomass is not constant within any forest estate. Neither is the canopy cover constant, but subject to change in time. A natural reason for the variation in time is that any estate has some variety of stand ages and stand biomass densities. The development of the state of any particular estate can be designed in terms of some sort of dynamic programming [11].

From the viewpoint of generic instructions, or policy actions, it might be beneficial to reduce the variety of initial estate states by adopting some kind of unifying boundary conditions. A tempting candidate is the normal forest principle [12]. This principle simply refers to postulating that stand ages are evenly distributed and stand characteristics are uniquely determined by stand age. Such a postulation, even if often departing from reality, simplifies many treatments significantly, producing idealized systems that are stationary in time.

Within the normal forest principle, quantities like rotation age and average stand trunk volume per area unit become well defined, not only for a single stand but also on estate level. Correspondingly, microeconomic discussion regarding such quantities becomes simplified. One can even state that the normal forest principle allows determination of microeconomically optimal rotation age, as well as expected value of stand volume.

Microeconomically optimal rotation age and expected value of stand volume are not necessarily optimal from the viewpoint of national economics, neither from the viewpoint of carbon sequestration. It has been recently shown that microeconomics often favors solutions with relatively low capitalization [11, 13, 14]. Low capitalization has adverse effects on volumetric growth rate, as well as litter accumulation rate [13, 15].

In this paper, our primary interest in the microeconomics of actions indisputably favorable from the carbon sequestration viewpoint. These are an increment of biomass density on the one hand, and extension of semiclosed canopy cover on the other hand. In terms of rotation forestry, the latter corresponds to extension of rotation age.
First, we will present our experimental materials, and related computational methods. Then, we will introduce microeconomic methods, as well as carbon rent formulae. Third, results are reported, separately for systems with thinnings from above and below. Finally, the outcome is discussed, and a functional carbon sequestration subsidy system is proposed.

2. Materials and methods

Eleven circular plots of area 314 square meters were taken from typical spots of 11 spruce-dominated forest stands in November 2018 at Vihtari, Eastern Finland. Seven of the stands had experienced only young stand cleaning, whereas four of the stands were previously thinned commercially. Breast-height diameters were recorded, as well as tree species, and a quality class was visually determined for any measured tree.

On measured plots on sites without any previous commercial thinning, basal area of tree trunks at breast height varied from 32 to 48 m²/ha, and stem count from 1655 to 2451 per hectare. On measured plots on stands previously thinned commercially, the basal area varied from 29 to 49 m²/ha, and the stem count from 891 to 955 per hectare. Further details of the experimental stands are reported in earlier papers [13].

The particular estate is characterized by measurements from the 11 sample plots. However, such a sample does not necessarily represent the entire estate accurately. More importantly, the sampling does not confirm to the normal forest principle, with assumptions of constant age distribution and stand characteristics uniquely determined by stand age. In this study, we utilize the normal forest principle in terms of establishing a “normal stand” on the basis of the observations, and then approximating the development of this “normal stand” as a function of stand age.

The normal stand is established on the bases of two never-thinned sample plots of medium site fertility and younger range of stand age, among the empirical observations. The two sample plots were combined into one experimental plot of area 628 square meters. Within the representative sample plot, the basal area of acceptable-quality trees was 35.3 m²/ha, and corresponding stem count 1401 per hectare. The age of the normal stand was 35 years, and dominating tree height 15 m.
For prognostication of further development of the normal stand, some kind of a growth model is needed. The growth model of Bollandsås et al. [16, 13, 14] is adopted, discussing not only growth but also mortality and recruitment. Trees are discussed in diameter classes of 50 mm, and each class is represented by its central tree. Growth is computationally implemented in terms of the probability of a tree to transfer to the next diameter class [16, 13, 14].

Breast-height diameters being measured for any sample plot, and diameter growth being approximated by the above-mentioned growth model, some kind of technique is needed for determining stem volumes. For any diameter class, the volumetric amount of two assortments, pulpwood and sawlogs, is clarified according to an appendix given by of Rä mö and Tahvonen [17, 18]. In order to discuss financial issues, value growth needs to be addressed, instead of merely volumetric growth. The monetary value of the assortment volumes is taken as national stumpage prices from year 2000 to 2011, as given by Rä mö and Tahvonen [17]. A bare land value of 600 Euros/ha is used, correspond to local circumstances at the time of writing.

Description of stand development until the time of observation at 2018 requires another kind of an approach. According to approximation by local professionals, 2017 annual volume growth rate was 16 m$^3$/ha. Using this growth rate, as well as the 2018 commercial volume estimate 256 m$^3$/ha in an exponential growth function resulted as a stand volume history with a small but finite initial volume.

Possibly the simplest way to approximate financial history is to determine an internal rate of operative return for the period from stand establishment to 2018. In other words, we require

$$V(t_3)e^{-r_3} - R(t_1)e^{-r_1} - C(t_2)e^{-r_2} = 0$$  \hspace{1cm} (1),

where $R$ is regeneration expense at regeneration time $t_1$, $C$ is young stand cleaning expense at cleaning time $t_2$, and $V$ is stumpage value of trees at observation time $t_3$. In this study, the observation time $t_3$ corresponds to November 2018, regeneration time is clarified according to known stand age, and young stand cleaning is assumed to have occurred ten years after regeneration. It is assumed that prices and expenses do not evolve in real terms, and thus presently valid expenses can be used in Eq. (1). The regeneration expense is taken as 1250 Eur/ha, and young stand cleaning 625 Eur/ha, occurring ten years after regeneration.
It is worth noting that the operative internal rate of return $s$ in Eq. (1) does not correspond to capital return rate in the entire activity, as the latter depends on non-operative capitalization like bare land value.

In order to determine a momentary capital return rate, we need to discuss the amount of financial resources occupied [24, 15, 13]. This is done in terms of a financial potential function, defined in terms of capitalization per unit area $K$. The momentary capital return rate becomes

$$r(t) = \frac{d\kappa}{K(t)dt} \quad (2),$$

where $\kappa$ in the numerator considers value growth, operative expenses, interests and amortizations, but neglects investments and withdrawals. In other words, it is the change of capitalization on economic profit/loss – basis. $K$ in the denominator gives capitalization on balance sheet – basis, being directly affected by any investment or withdrawal.

Eq. (2) gives a momentary capital return rate, not necessarily sufficient for management considerations. By definition, the expected value capitalization per unit area is

$$\langle K \rangle = \int_{-\infty}^{\infty} p(K)KdK \quad (3),$$

where $p(K)$ is the probability density function of capitalization $K$. By change of variables we get

$$\langle K \rangle = \int_{0}^{\infty} p(K)K \frac{dK}{da} = \int_{0}^{\infty} p(a)K(a)da \quad (4),$$

where $a$ is stand age (or time elapsed since latest regeneration harvesting), and $\tau$ is rotation age.

The expected value of the change rate of capitalization is

$$\left\langle \frac{d\kappa}{dt} \right\rangle = \int_{0}^{\infty} p(a) \frac{d\kappa(a)}{dt} da \quad (5).$$

Correspondingly, the expected momentary rate of relative capital return is

$$\left\langle \frac{r(t)}{\langle K \rangle} \right\rangle = \int_{0}^{\infty} p(a) \frac{r(a,t)}{K(a,t)} da \quad (6).$$

We find from Eq. (6) that the expected value of capital return rate within an estate generally evolves in time as the probability density of stand ages evolves. However, Eq. (6) can be simplified to be independent of time by adopting the normal forest principle, where stand age
probability density is constant [12]. In addition, constancy of the expected value of capital return rate in time requires that prices and expenses do not evolve in real terms. Then, the expected value of the capital return rate becomes

$$\langle r \rangle = \frac{\int_0^\tau \frac{d\kappa(a)}{dt} da}{\int_0^\tau K(a) da} = \frac{\int_0^\tau K(a) r(a) da}{\int_0^\tau K(a) da}$$

(7).

It has been recently shown that Eq. (7) corresponds to the ratio of the partition functions of change rate of capitalization and capitalization itself [19]. It also has been recently shown that maximization of net present value of future revenues may result in financially devastating consequences [20]. Momentary capital return rate as given in Eq. (2) was introduced in 1860 [24]; an expected value was mentioned in 1967 [25, 26], however applications have been introduced only recently [19, 20, 13, 14].

High capital return rates are gained by an improvement harvesting including diameter-limit cutting to the transition diameter between pulpwood and sawlogs [13]. It is possible to retain a state of high capital return for decades, implementing further diameter-limit cuttings frequently, provided there is an abundant supply of pulpwood-size stems of an at least semi-shade-tolerant tree species [13]. In this investigation, however, the focus is in procedures inducing an increment of biomass density on the one hand, and extension of semiclosed canopy cover on the other hand. The former can simply be achieved by applying cutting diameter limits greater than the transition diameter between pulpwood and sawlogs.

In the context of the improvement harvesting, 20% of the stemcount of good-quality trees were removed in all diameter classes due to establishment of striproads.

The stumpage value is determined in terms of roadside price, deducted by harvesting expense. We here use the roadside prices recently applied by Parkatti et. al. [21], 34.04 Eur/m$^3$ for spruce pulpwood and 58.44 Eur/m$^3$ for sawlogs. We further use the same harvest-expense function as Parkatti et al. [21], stated to be based on a productivity study of Nurminen et al. [22], however with one correction. Model parameter $C_5$ value is taken as 2, instead of 1. With this correction, the expense function corresponds to present local circumstances, including transfer expense of machinery. In addition to the expense function, we include a fixed harvesting entry expense
per hectare. Justification of the latter is that some sites require at least partial pre-harvest cleaning. The entry expense is approximated as 200 Eur/ha. Again, is assumed that prices and expenses do not evolve in time. In other words, the capital return rate is discussed in real terms.

The growth model operates in five-year time steps [16, 13, 14], as discussed above. Consequently, an eventual harvesting entry may take place every five years. According to recent investigations, it often is favorable to harvest every five years [13]. However, the fixed harvesting entry cost of 200 Eur/ha restricts low-yield harvesting entries. Numerical investigations indicated that it is not reasonable to harvest if the yield would be less than 20-30 m²/ha. Such an entry limit is in concert with practices applied in the area. It is worth noting that maximization of capital return rate within any five-year period does not lead to maximization of the expected value of capital return rate. The reason is that provided the capital return rate is greater than the accumulated expected value, increased capitalization increases the expected value according to Eq. (7).

In addition to high thinnings intended to maximize capital return rate [19, 20, 13, 14], consequences of following semi-official silvicultural guidance commonly applied in the area [28] are discussed. Thinnings are predominantly applied from below, and any rotation is terminated in clearcutting. Clearcutting expenses are lower than thinning harvesting costs, according to Parkatti et al. [21], stated to be based on a productivity study of Nurminen et al. [22]. In addition, a 15% clearcutting premium for the roadside price of sawlogs is applied, in accordance to local tradition.

The last issue in this section of methods regards carbon trade and carbon renting. It has been recently shown that policies based on carbon rent are equivalent to policies based on carbon sequestration subsidies and taxes [27]. Unbiased carbon sequestration trade would require a huge initial investment; correspondingly mostly carbon rent procedures are practically feasible [27]. We will here present a brief derivation of the equivalency of the two principles of subsidies, however adopting boundary conditions possibly less restrictive than those of Lintunen et al. [27].

Let us establish a carbon sequestration subsidy system at a particular time \( \tau_1 \). Within a time range up to time \( \tau_2 \) the total carbon trade compensation is
\[ p_\gamma C_\gamma + \int_{\tau_1}^{\tau_2} \frac{d(pC)}{dt} dt = p_\gamma C_\gamma + \int_{\tau_1}^{\tau_2} \left[ p \frac{dC}{dt} + C \frac{dp}{dt} \right] dt \]  \hspace{1cm} (8),

where \( p_\gamma \) is carbon price at time \( t \), and \( C_\gamma \) is carbon inventory at time \( t \). On the other hand, the revenue from carbon rentals is

\[ \int_{\tau_1}^{\tau_2} uC dt \]  \hspace{1cm} (9),

where \( u \) is rent rate per carbon unit. Now, in order to establish equivalency between the carbon storage trade and rent, Eqs. (8) and (9) must become equal. The Equality naturally should apply in any possible circumstance. One of the circumstances is that the time change rate of prices as well as inventories is zero. In such a case the latter term of Eq. (8) vanishes. Consequently, a long-term flow of carbon rents should equal a one-time initial storage purchase payment. If the duration of the rent payments extends towards infinity, the only possibility is that the present value of rent payments forms a contracting series. One possibility of such contracting series is

\[ u_\gamma C_\gamma \int_0^\infty e^{-qt} dt = p_\gamma C_\gamma \]  \hspace{1cm} (10),

where \( q \) is discount rate. The corresponding solution for the carbon rent rate is

\[ u_\gamma = qp_\gamma \]  \hspace{1cm} (11).

One can readily show that Eq. (11) applies not only to steady state of Eqs. (8) and (9), but also to any incremental carbon price and inventory.

3. Results

31. Thinnings from above

Thin drawings in Fig. 1 show capital return rate according to Eq. (2) for any five-year period after the improvement harvesting for three different cutting limit diameters. The lower is the cutting limit diameter, the higher is capital return rate. Non-smoothness of the curves is due to irregular harvesting: skipping harvesting at the end of any five-year period increases capital return rate due to missing harvesting entry expense. The absence of harvesting however reduces the capital return rate during the following five-year period due to greater capitalization.
Thick drawings in Fig. 1 show expected value of capital return rate accumulated according to Eq. (7). These curves appear rather flat: expected value of capital return rate is not sensitive to rotation age, at least not after 70 years. We find that increasing the cutting limit diameter decreases the expected value of the capital return rate by 10% to 30%.

![Graph showing capital return rate and expected value accumulated over stand age](image)

Fig. 1. Annual capital return rate (thin drawings) and expected value accumulated over stand age (thick drawings) for three different cutting diameter limits (200, 250 and 300 mm).

Thin drawings in Fig. 2 show standing commercial trunk volume per hectare during any five-year period after the improvement harvesting. Differences are rather significant: diameter-limit cutting to the transition diameter between pulpwood and sawlogs reduces the stand volume rapidly, whereas 100 mm larger cutting limit retains a high stand volume. The same goes for expected (average) values integrated over the stand age.
Fig. 2. Commercial stem volume per hectare (thin drawings) and expected value accumulated over stand age (thick drawings) for three different cutting diameter limits (200, 250 and 300 mm).

Fig. 3 shows capitalization in Euros per hectare, thin drawings for any five-year period, and thick drawings for any accumulated expected value. The capitalization in Fig. 3 shows many of the features as stand volume in Fig. 2 does, however with one significant difference: expected values of capitalization are higher, in relation capitalizations within any five-year period. The reason for this difference is that regeneration expenses induce a significant initial capitalization on any stand, whereas initial stand volume is small. Amortization of regeneration and young stand cleaning expenses is done at the end of any rotation.
Fig. 3. Capitalization per hectare (thin drawings) and expected values accumulated over stand age (thick drawings) for three different cutting diameter limits (200, 250 and 300 mm).

Fig. 4 shows that net growth rate very significantly depends on the cutting diameter limit. Diameter limit cutting to the transition diameter between pulpwood and sawlogs, resulting as a high capital return rate as shown in Fig. 1, results as a small stand volume (Fig. 2) and correspondingly a low volumetric growth rate (Fig. 4). The growth rate also declines rapidly along with decreasing stand volume (Figs. 4 and 2). However, the volumetric growth rate in Fig. 4 is less sensitive to cutting limit diameter than the stand volume in Fig. 2 and capitalization in Fig. 3.
Fig. 4. Net growth rate per year and hectare (thin drawings) and expected values accumulated over stand age (thick drawings) for three different cutting diameter limits (200, 250 and 300 mm).

Fig. 1 showed that the greatest expected value of capital return rate is gained with diameter limit cuttings to the transition diameter between pulpwood and sawlogs. We also found that the capital return rate is not very sensitive on rotation age, but the maximum is reached at rotation age of 75 years. Greater cutting limit diameters induce capital return rate deficiency according to Fig. 1. The deficiency also depends on rotation age. The deficiency is replotted in Fig. 5. It is found that increment of standing volume by greater cutting diameter limit (Fig. 2) induces a capital return rate deficiency that often is in excess of a percent unit (Fig 5). Doubling the expected value of stand volume would correspond to roughly 25% reduction in the capital return rate (Figs. 1, 2 and 5).
The capital return rate deficiency in terms of percentages (Fig. 5) can be converted to Euros per hectare and year by multiplying by the capitalization appearing in Fig. 3. The result is shown in Fig. 6. It is found that the deficiency may be more than one hundred Euros per hectare and year. Doubling the standing volume (Fig. 2) would induce a capital return deficiency in the order of 100 Euros per hectare and year (Figs 2 and 6).
From the viewpoint of any carbon rent policy, or any other carbon sequestration policy, it is of interest what is the capital return rate deficiency due to any excess stand volume. First, the excess commercial stand volume in cubic meters per hectare is plotted in Fig. 7. This plot actually is a more detailed analysis of the data appearing in Fig. 2. The expected value of stand volume (Fig. 2) corresponding to the maximum value of expected capital return rate (Fig. 1) is 80 m³/(ha*a). Fig. 7 reveals that the excess stand volume achievable may be more than that.

Fig. 7. Excess commercial stem volume per hectare, in comparison to the treatment schedule corresponding the greatest expected value of capital return rate. The three curves correspond to three different cutting diameter limits (200, 250 and 300 mm).

The annual capital return rate deficiency per excess stand volume is shown in Fig. 8. Fig. 8 simply is the deficiency per hectare shown in Fig. 6 divided by the excess volume per hectare shown in Fig. 7. It is found that cutting diameter limit of 300 mm results as a deficiency in the order of 1.5 Eur/(excess m³*a). In the case of the lower cutting diameter limits 250 mm, the deficiency depends strongly on rotation age. The Figure displays also negative deficiencies. They are due to negative excess volumes, which however are not relevant from the viewpoint of carbon sequestration.
Fig. 8. Capital return rate deficiency in Euros per excess commercial volume and year, for different rotation ages and three different cutting diameter limits (200, 250 and 300 mm).

32. Thinnings from below

The second set of treatment procedures correspond to thinnings from below. The treatments are implemented according to semiofficial silvicultural instructions applicable in the area [28]. Thinning entries are triggered by given basal area of trees per hectare, and another value of basal area is required after thinning. The limit basal areas depend on the dominant height of trees. The first thinning from below, combined with quality thinning, as well as opening striproads, was conducted to basal area 21 m²/ha. The second thinning was implemented when the basal area exceeded 32 m²/ha, and it was conducted to basal area 25 m²/ha. The latter basal area limits were used for all further thinning.

Fig. 9 shows the capital return rate for the normal stand if thinned from below, thin drawings for any five-year period, and thick drawings for accumulated expected values. During the first five-year period the capital return rate is greater than the expected value; in later periods the expected value is greater, with one exception. A thinning at stand age 100 years is considered the last thinning feasible for the stand. According to local tradition there is a 15% premium in the price of sawlogs from clearcuttings. The high value of capital return rate is due to this price increment, along with lower harvesting expense in clearcutting. However, the momentary peak has a small contribution to the expected value, which indicates a clearcutting possibly should occur at a rather young age. One must recognize that the last thinning, triggering the
clearcutting price premium, as well as clearcutting harvesting expenses, would occur earlier with younger rotation ages.

Fig. 9. Capital return rate per year (thin drawing) and expected values accumulated over stand age (thick drawing), with thinnings from below.

Fig. 10 shows the development of stand volume per hectare, five-year average with thin markings, accumulated expected value with thick markings. The effect of thinnings is very clearly visible in the five-year average volume, unlike the expected value, and unlike the capital return rate in Fig. 9.

Fig. 10. Commercial volume per hectare (thin drawing) and expected values accumulated over stand age (thick drawing), with thinnings from below.
Fig. 11 shows the development of capitalization per hectare, five-year average with thin markings, accumulated expected value with thick markings. Again, there are similarities between Figs. 10 and 11. However, the capitalization in Fig. 11 is less sensitive to thinning from below than the stand volume in Fig. 10. There also is the same difference as between Figs. 2 and 3: expected values of capitalization are higher, in relation capitalizations within any five-year period, due to capitalization induced by regeneration expenses. It is also worth noting that there is hardly any net growth after the age of 105 years in Fig. 10, whereas capitalization increases to 110 years in Fig. 11.

Fig. 11. Capitalization per hectare (thin drawing) and expected values accumulated over stand age (thick drawing), with thinnings from below.

Fig. 12 shows the net growth per hectare, five-year average with thin markings, accumulated expected value with thick markings. After the first thinning from below, the growth rate is greater than after improvement harvesting to the transition limit of pulpwood and sawlogs, but smaller than after gentle thinnings from above (cf. Fig. 4). In Fig. 12, the expected value of net annual growth reaches a weak maximum at the age of 50 years.
Fig. 12. Net growth rate per year and hectare (thin drawing) and expected values accumulated over stand age (thick drawing), with thinnings from below.

The expected capital return rate deficiency increases with rotation age (Fig. 13). It appears in Fig. 13 that there would be a short-term decrement at the age of 100 years. Again, with younger rotation ages, the last thinning resulting as the clearcutting premium in sawlog prices and reduction in harvesting expenses would occur earlier. In comparison to Fig. 5, the capital return rate deficiency is only slightly higher below rotation age of 50 years, but then increases roughly proportionally to stand age.

Fig. 11. Capital return rate deficiency in percentage per year, with thinnings from below.
The capital return rate deficiency in terms of Euros per hectare and year also increases monotonically, considering the variable timing of the last thinning inducing the transition to clearcutting sawlog prices and harvesting expenses (Fig. 14). In comparison to Fig. 6, the capital return rate deficiency again is only slightly higher below the stand age of 50 years, but then strongly increases.

Fig. 12. Capital return rate deficiency in Euros per hectare and year, with thinnings from below.

The expected excess commercial volume, in comparison to that providing the greatest capital return rate in Fig. 1, is shown as a function of rotation age in Fig. 13. Unlike in the case of thinnings from above (Fig. 6), it increases monotonically.
Fig. 13. Excess commercial volume per hectare with thinnings from below, in comparison to the treatment schedule corresponding the greatest expected value of capital return rate.

The annual capital return rate deficiency per excess stand volume is shown in Fig. 14. At rotation ages up to 50 years, the deficiency appears to decrease. That situation however would change if clearcutting price premium, as well as clearcutting harvesting expenses would be applied right after the first thinning. Along with increasing rotation age, the capital return deficiency per excess volume increases. In comparison to Fig. 8, the deficiencies are higher with thinnings from below, particularly with high rotation age.

Fig. 14. Capital return rate deficiency in Euros per excess commercial volume and year with thinnings from below, in comparison to the treatment schedule corresponding the greatest expected value of capital return rate.
4. Discussion

Issues related to increment of biomass density on the one hand, and extension of semiclosed canopy cover on the other hand have been discussed on estate level, with application to fertile boreal spruce estates.

Extending rotations induces a large capital return rate deficiency if thinnings from below are applied (Figs. 9 and 14)). With thinnings from above, the capital return rate is insensitive to rotation age (Fig. 1). However, intensive thinnings from above tend to reduce the amount of living biomass, and correspondingly capital return rate deficiency per standing cubic meter may depend on rotation age (Fig. 8).

Manipulation of biomass density is possible even if the rotation age would not be manipulated. The possibilities for this are particularly pronounced if thinnings are done from above (Fig. 2). It is possible to double the density of living biomass, in comparison to the financially optimal treatment schedule (Fig. 2), and consequently growth rate is increased (Fig. 4). However, another consequence is deficiency in capital return rate (Figs. 1 and 5).

It is possible to allocate the capital return rate deficiency to the excess value of commercial stand volume. With prices applicable at the time of writing, the deficiency per excess volume is in the order of 1.5 Euros per excess standing cubicmeter, if the biomass density is doubled (Fig. 8). With smaller excess volume, the specific deficiency is less (Figs. 2 and 8).

An applicable carbon rent can be derived from carbon storage market price according to Eq. (11). At the time of writing, the market price of carbon dioxide emissions is in the order of 25 Euros per ton. This inserted to Equation (11) together with a 3% discount rate, often applied in Forestry [29, 30, 31], results as an annual carbon rent of 0.75 Euros per ton of carbon dioxide.

In coarse terms, a cubicmeter of commercial trunk volume in boreal forest stores a ton of carbon dioxide (in living biomass, litter, and soil) [1, 6, 2, 4, 32, 33]. Correspondingly, considering the prices valid at the time of writing, one might reasonably expect a carbon rent of 0.75 Euros per standing cubicmeter. In comparison to Fig. 8, that is not enough. The deficiency of capital return rate per standing cubicmeter is greater than an appropriate rent.
It has recently been shown that a carbon sequestration trade policy is equivalent to a carbon rent policy [27]. The same treatment, abbreviated and with somewhat less restrictive boundary conditions, is given in Eqs. (8) to (11) of this paper. There are reasons why any carbon sequestration trade compensation must be proportional to the carbon storage [27]. If it would not be, agents with large carbon inventories initially would suffer heavy and unjustified release taxes. Applying the Equivalency of Eq. (10) would thus indicate that also the carbon rent arrangement would be proportional to the total carbon inventory.

As mentioned above, the carbon rent derived from present carbon dioxide market price (Eq. (11)) is not enough to compensate for the capital return deficiency due to increased biomass density. However, there is no particular reason why the carbon rent should be proportional (unlike the carbon trade). If the carbon rent is made non-proportional, the marginal rent may reasonably exceed the capital return rate deficiency.

All the quantitative results in this paper are estate-specific. It is of interest how they would change if the properties of the estate would change. Changing soil fertility, as well as changing temperature sum, would change the values of all time derivatives. Correspondingly, capital return rates, as well as annual capital return deficiencies would change. Greater fertility would result as greater change rates with respect to time, and vice versa. Capital return rates and capital return rate deficiencies would change correspondingly. Effect of adopting other dominating tree species but Norway spruce on the estate are somewhat more difficult to estimate.

It has been recently proposed that on a national level, sites of low fertility and regions of cool climate are most suitable for carbon sequestration, while regions with high production capacity are best suitable for wood raw material supply [34]. Consequently one might think that carbon rent derived from present emission prices not being enough to compensate for capital return deficiency on a fertile spruce estate would not necessarily be very detrimental. However, maximizing capital return rate results as low capitalization, and correspondingly as reduced growth (Figs. 1, 2, 3 and 4), which in turn reduces timber supply. Any carbon sequestration subsidy would increase capitalization, growth, and timber supply.
Many of the quantitative results in this paper depend on the market prices of roundwood assortments, bare land, silviculturar and harvesting expenses, carbon emission, as well as the discount rate applied in Eqs. (10) and (11). Considering eventual changes of the latter two is straightforward in terms of Eqs. (10) and (11). Changes in the former quantities contribute in a trivial manner if they are proportional. In the case of significant non-proportional changes of prices, most of the Figures of this paper will have to be redrawn.

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