Self-consistent theory of stellar electron capture rates

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Abstract. Recently a novel theory framework has been established for description of electron capture rates at temperatures and densities in stellar environment, based on the relativistic energy density functional. The model includes finite temperature relativistic mean field to determine single-particle basis and the corresponding thermal occupation factors of target nucleus, while relevant charge-exchange excitations are described by the self-consistent finite temperature relativistic random phase approximation (FTRRPA). The calculated electron capture rates for $^{54,56}$Fe are shown in comparison with other advanced theory approaches.

1. Introduction
Electron capture on nuclei plays an important role in the evolution of core-collapse supernova [1]. During the precollapse phase of the star evolution, it reduces the number of electrons available for the electron degeneracy pressure support, while beta decay goes in the opposite direction. At the same time, the neutrinos produced by the electron capture freely escape from the star at densities $\leq 10^{11}$ g/cm$^3$, removing energy and entropy from the core[1, 2]. For the initial electron-to-baryon ratio $Y_e \approx 0.5$, $\beta^-$ decay processes can be effectively hindered by the electron degeneracy, but become competitive when the nuclei become more neutron-rich.

At lower central stellar densities ($\leq 10^{10}$ g/cm$^3$) and temperature range 300 - 800 keV, electron capture occurs mainly on nuclei in the mass range $A \sim 60$ [1]. Since the electron chemical potential becomes of the same order of magnitude as the nuclear $Q$-value, the respective rates are sensitive to the detailed structure of Gamow-Teller (GT) transition strength. However, for higher densities and temperature, nuclei with mass numbers $A > 65$ become quite abundant, the electron chemical potential is noticeably larger than the $Q$-value, thus the electron capture rates are mainly determined by the overall GT strength and its centroid energy. When reaching even higher densities ($\rho > 10^{11}$ g/cm$^3$), the electron chemical potential becomes larger than $\approx 20$...
MeV. Therefore, in addition to the GT strength, forbidden transitions should also be taken into account in modeling the electron capture rates at higher densities [1, 2].

Various approaches have so far been employed in calculations of stellar electron capture rates, often strongly dependent on the empirical data. The first tabulation of the rates for nuclei between $A = 21$ and 60 has been accomplished by Fuller, Fowler, and Newman (FFN), based on the independent particle model for allowed transitions, supplemented by the empirical values of the Gamow-Teller (GT) matrix elements [3]. Within the shell model Monte Carlo method (SMMC) applied for fp-shell nuclei, the thermal effects and GT transitions have been calculated in a microscopic way [4]. The large-scale shell-model diagonalization (LSSM) approach provided tabulation of weak interaction rates for more than 100 nuclei in the mass range $45 \leq A \leq 65$ [5]. In order to overcome the limitations of the shell model, in the study of nuclei beyond fp-shell, a hybrid model of the SMMC and random phase approximation (RPA) has been employed for large number of nuclei involved in stellar core collapse [6, 7]. The electron capture rates have also been studied with the proton-neutron quasiparticle RPA (QRPA) employing the Nilsson model and separable GT forces [8]. More recently, thermal QRPA (TQRPA) has been introduced, based on the Woods-Saxon potential and separable multipole and spin-multipole particle-hole interactions, with temperature taken into account by the thermofield dynamics [9].

Most of currently available models of the stellar electron capture rates have to be adjusted to the experimental data for the structure properties of involved nuclei. Therefore, extrapolations toward target nuclei away from the valley of stability are rather unreliable. Feasible systematic description of nuclear ground state properties and excitations over the nuclide chart currently can be provided only by the nuclear energy density functionals. Recently two self-consistent frameworks have been introduced for modeling the electron capture cross sections and rates, based on i) Skyrme functionals [10] and ii) relativistic energy density functionals [11]. In this proceedings we give an overview of recent progress in self-consistent modeling of the stellar electron capture rates in the relativistic framework [11].

2. Electron capture rates based on relativistic energy density functional

The relativistic density functional theory, realized in terms of the relativistic mean field (RMF) and the relativistic Hartree Bogoliubov (RHB) model, has achieved great success in the description of ground-state properties and excitations in nuclei all over the periodic table, including those far away from the stability line [12, 13, 14, 15]. For the study of astrophysical processes, temperature effects have recently been taken into account in the self-consistent relativistic RPA, which has been used to investigate excitations without charge exchange, in particular the low-energy monopole and dipole response in nuclei at finite temperature [16].

Since the electron capture on nuclei involves transitions with charge exchange, recently the self-consistent finite temperature relativistic random phase approximation (FTRRPA) has been extended for the implementation in the charge-exchange channel [11]. The properties of the nuclear initial state, i.e. single nucleon basis and the corresponding thermal occupation probabilities, are obtained within the RMF at finite temperature, formulated using the effective Lagrangian with medium-dependent meson-nucleon couplings with DD-ME2 parameterization [17]. The FTRRPA equations are derived using the single-nucleon basis of the RMF at finite temperature [16]. In a self-consistent approach, the residual interaction terms in the FTRRPA matrix are obtained as the derivative of the single particle Dirac Hamiltonian from the same Lagrangian as in the RMF framework. In the relativistic RPA, the configuration space includes not only proton-neutron pairs in the Fermi sea but also pairs formed from the fully or partially occupied states in the Fermi sea and the empty negative-energy states from the Dirac sea. The spin-isospin-dependent residual interaction terms are generated by the exchange of $\pi$ and $\rho$ mesons. For the $\rho$-meson density-dependent coupling strength we choose the same functional form used in the effective interaction in the RMF [17]. The particle-hole residual interaction
includes thermal occupation probabilities for protons and neutrons obtained from the self-consistent RMF calculations at finite temperature.

The electron capture rate is evaluated using the cross section and distribution of electrons $f(E_e, \mu_e, T)$ at given temperature [11]. In stellar conditions present in the collapsing core of supernova, electrons are well described by the Fermi-Dirac distribution, with temperature $T$ and electron chemical potential $\mu_e$ [7]. As an example, in Fig. 1 we show the calculated electron capture rates for $^{54,56}$Fe for different conditions associated with the initial phase of the core-collapse supernova.

**Figure 1.** Electron capture rates as functions of temperature $T$ for $^{54,56}$Fe at selected densities $\rho Y_e$ (g cm$^{-3}$), calculated with the FTRRP A based on DD-ME2 interaction, in comparison with the LSSM [5] and TQRP A [9] results. Taken from Ref. [11].

The FTRRP A rates are displayed as functions of temperature $T_9$ ($T_9 = 10^9$ K) for $^{54,56}$Fe and selected densities $\rho Y_e$ (g cm$^{-3}$), in comparison with the results from studies based on LSSM and TQRP A/5, 9/. In general, the electron capture rates increase with temperature and electron density. For high electron densities, the rates increase slower, and in the case of $\rho Y_e = 10^{10}$ g/cm$^3$ the temperature dependence almost vanishes. At high electron densities, when the electron chemical potential is not small, high-energy electrons are able to excite most or even all the GT$^+$ transitions even at lower temperatures. For $^{54}$Fe, at lower densities, e.g. $\rho Y_e = 10^7$, $10^8$ g/cm$^3$, the FTRRP A agrees with the shell model calculations very well. At higher densities, e.g. $\rho Y_e = 10^9$ g/cm$^3$, with electron chemical potential about 5 MeV that is similar as the threshold energy, at lower temperatures the FTRRP A yields a higher rate than the shell model. One can understand this difference in the fragmentation of the shell model GT$^+$ strength over the energy range $0 \sim 10$ MeV[2].

When the electrons at lower temperature excite all the GT$^+$ strength at $E \simeq 2$ MeV in FTRRP A, only a fraction of the shell model strength could be excited [11]. Since part of the shell model GT$^+$ strength is located at higher energies than in the FTRRP A, the resulting rate is smaller than in the FTRRP A case. When the electron density becomes even higher,
e.g. $\rho Y_e = 10^{10}$ g/cm$^3$ with chemical potential about 11 MeV, the high-energy electrons at lower temperatures are gradually sufficient enough to excite all the transition strength in the shell model approach as well, thus the discrepancy with the FTRRP reduces. However, as the GT$^+$ centroid energy of the FTRRP is lower than that of the shell model, in the former case the rates are somewhat lower. For $^{56}$Fe, at densities $\rho Y_e = 10^7, 10^8$ g/cm$^3$ the FTRRP results agree better with the TQRPA, while the LSSM gives lower rates. At higher densities, the FTRRP yields a more similar trend with the shell model than the TQRPA, but the values are still above the shell model results. In general, the discrepancy between the FTRRP and the shell model rates is larger in $^{56}$Fe than $^{54}$Fe. The main reason lies in the difference between the GT$^+$ centroid energy between the two models, that is larger in $^{56}$Fe [11].

3. Conclusion

Recently a novel self-consistent theory framework has been introduced for modeling the electron capture rates in the initial phase of supernova core collapse, based on the relativistic energy density functional [16]. It is realized in terms of the finite temperature RMF and FTRRP, allowing inclusion of all relevant charge-exchange transitions induced in the electron capture process. The electron capture rates have been studied for $^{54}, ^{56}$Fe target nuclei in different stellar environments characteristic for the core collapse supernova, and compared with the shell model and the TQRPA. It is shown that the FTRRP results in similar trend of temperature dependence as other approaches, but the overall values are somewhat larger, due to differences in the structure of GT$^+$ strength distributions. It is important to note that the finite temperature RMF and FTRRP provide universal framework for description of stellar weak-interaction processes at finite temperature in a self-consistent way. This approach is especially valuable for those nuclei not accessible to the shell model diagonalization approach, and it is advantageous in comparison to other models which necessitate input from the experimental data on nuclei under consideration, which may not always be available.

Acknowledgments

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