Extension of two-dimensional DDA on co-seismic slope
stability analysis considering dynamic shearing strength

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Abstract. In this study, a new model for evaluating the dynamic shear strength of slope material has been proposed in DDA for earthquake-induced landslide simulation. The effect of fatigue and accumulated damage in evaluating the dynamic strength of rocks has been considered to judge the failure of joint contacts in the DDA block system. The accuracy of the extended DDA method is verified by simulating several examples, such as Single-Degree-Of-Freedom (SDOF) system and practical landslide model. The simulation results show that the new model considering the dynamic effect is more accurate and feasible to dynamic slope failure under seismic waves. Compared with the original DDA, the extended DDA method can be better applied to analyze earthquake-induced landslides with dynamic shearing strength considered.

1. Introduction
Earthquake-induced landslides frequently occur around the world, which is one of the most catastrophic disasters. Discontinuous deformation analysis (DDA) is becoming more and more popular for earthquake-induced landslide analysis because of its advantage in simulating the deformation and motivation of the fractured and jointed rock masses.

However, the static shear strength, which is obtained by in situ rock shear tests under monotonic loading, is always used to judge the joint contacts between blocks for both static and dynamic analysis in the constitutive model of the DDA [1]. The use of the static shear strength is reasonable for static slope stability analysis, while it is probably inappropriate for seismic conditions because the structure materials like rock always behave a dynamic shear strength under dynamic cyclic loading or irregular seismic wave [2-5]. Meanwhile, the previous researches on earthquake resistance design indicated that the peak amplitude of a seismic event has no significance on the deformation of slopes during earthquakes from the studies on the effect of an acceleration-time response. It is the total that includes amplitude, frequency, and duration of the event that needs to be considered in the analysis [6]. For the seismic condition analysis in the original DDA, the peak loading force resulted from the PGA of seismic wave is a key factor dominating contact failure, which plays an important role in slope stability in simulation analysis. But in recent earthquake-induced landslide events, many phenomena that earthquake ground motions with small PGA can induce large-scale landslide are observed. Particularly, the pulse-like ground motions (PLGM) that have low frequency and long-period pulse are...
considered as one of the key factors causing severe damage in the earthquake [7-9]. For example, a strong earthquake (M7.3) occurred in Kumamoto prefecture, Japan in 2016, called the Kumamoto earthquake. While the large-scale landslides occurred in the area with smaller PGA and no landslide occurred in the area with larger PGA [10]. The original DDA cannot accurately evaluate the initiation of earthquake-induced landslides under PLGM with small PGA because the static shear strength is used. How to analyse the earthquake-induced landslide with PLGM considered is important but unsolved. The dynamic strength should be considered in the quantitative analysis of earthquake-induced landslides. An accurate model for evaluating the dynamic failure of slopes subjected to seismic waves, especially the PLGM, is important and necessary for accessing the earthquake-induced landslide problems.

In this study, to simulate the landslide evaluation process under the dynamic seismic wave, especially the pulse-like ground motions in the near-fault region, a model that considers the dynamic strength of the material has been proposed for earthquake-induced landslide simulation in DDA. The model can evaluate the dynamic strength of joint contact by considering the effect of fatigue and accumulated damage under dynamic seismic loading. The Single-Degree-Of-Freedom (SDOF) system is simulated by proposed DDA to validate the dynamic failure of joint contact in shear direction, respectively. A practical landslide triggered by PLGM in the Kumamoto earthquake is also analyzed to verify the availability and accuracy of the proposed model for dynamic failure simulation.

2. Theory of DDA

2.1. Interaction Between Blocks in original DDA

In the DDA method, individual blocks are connected and form a block system by joint contacts between blocks and displacement constraints on single blocks. As shown in Figure 1, normal and tangential penalty springs are respectively used to join blocks into a block system. The interaction force between adjacent blocks is governed by equations of contact detection and contact treatment. Based on the procedure of contact detection, the contact penalty springs in either the normal or tangent direction of the boundary between two blocks will be added or removed. By adding and removing the contact penalty springs in either the normal or tangent direction of the boundary between two blocks, the blocks can be coupled and separated to simulate shearing or tensile failure.

![Figure 1. The contact penalty springs between blocks.](image)

After solving the governing equations, the open-close iteration (OCI) is then implemented to determine the valid system state. The final states of all contacts after solving the governing equations are tested and compared to those assumed before. The system state is considered valid only if all the calculated contact states are consistent with the initial ones. The OCI follows the criteria that the shear spring force cannot exceed the maximum friction, and the distance serves as evidence to determine open and close:

\[
k_n d_n > 0 \text{ and } k_n d_n \leq k_n d_n \tan \varphi + \tau l \rightarrow \text{locked} \quad \text{MERGEFORMAT (1)}
\]

\[
k_n d_n > 0 \text{ and } k_n d_n > k_n d_n \tan \varphi + \tau l \rightarrow \text{sliding} \quad \text{MERGEFORMAT (2)}
\]
where $\tau_c$, $\phi$, $\sigma_t$, and $l$ are cohesion, friction angle, tensile stress, and contact length of the contact. $k_n$ and $k_t$ respectively denote the stiffness of normal spring and shear spring. $d_n$ is the normal penetration distance and $d_t$ is the tangential relative displacement.

3. A DDA dynamic shear constitutive model

To evaluate the dynamic strength of rocks in dynamic response analysis, Tetsuji and Tomohiro (2019) proposed a mathematical model based on the effects of fatigue, loading rate, and cumulative damage [4]. For the effect of fatigue, the cyclic fatigue function $f_1$ has been determined:

$$\tau_f = \frac{\tau_{f,Nf}}{\tau_{f,Nf-1}} = 1 - \alpha \log_{10} N_f$$ \hspace{1cm} (* MERGEFORMAT (4))

where $N_f (\geq 1)$ is the number of loading cycles at the time of failure, $\tau_{f,Nf}$ is the shear strength at the time of failure after loading $N_f$, $\tau_{f,Nf-1}$ is the shear strength under monotonic loading ($N = 1$), and $\alpha$ is a parameter defining the slope of the function. For the effect of cumulative damage, the cyclic damage function $f_2$ representing the effects of damage owing to cyclic loading before failure can be expressed:

$$f_2(N) = \frac{\tau_{d,Nf}}{\tau_{f,Nf-1}} = 1 - d (N-1)$$ \hspace{1cm} (* MERGEFORMAT (5))

where $d$ is the slope of the initial and failure points, and $\tau_{d,Nf}$ is the cyclic residual shear strength exerted after $N$ cycles of loading. The linear cumulative damage rule is adopted in this study. The pre-peak damage of hard rocks exhibits a nonlinear relationship [11-12], the nonlinear cumulative damage rule might be required instead of Eqs. However, the linear cumulative damage rule tends to at the side of safe, and it is not an important study for the proposed evaluation method in this research. As the linear cumulative damage rule is used, and then $f_1(N_f) = f_2(N_f)$, therefore the determination of parameter $d$ can be obtained as follows:

$$d = \begin{cases} \frac{\alpha \log_{10} N_f}{N_f - 1}, & (N_f > 1) \\ 0, & (N_f = 1) \end{cases}$$ \hspace{1cm} (* MERGEFORMAT (6))

As the loading is applied, the residual shear strength ratio will decrease. Once it reaches a particular value $R_{res} = 0.7$, as expressed in Tetsuji and Tomohiro's (2019) study, suggesting that the failure occurs. However, the proposed model is based on the cyclic loading tests, which can be applied effectively to evaluate the dynamic strength under cyclic loading. For seismic wave, which is considered as irregular loading, several cyclic loading tests with different stress amplitudes should be performed at a single frequency. Based on the mathematical model proposed by Tetsuji and Tomohiro (2019), a new model considering the evaluation of dynamic strength for earthquake problem analysis is incorporated into DDA, which is named as DDA-Dynamic in this study. Owing to insufficient empirical data on this issue, some assumptions for simplification are made in DDA: (1) when the loading stress of seismic wave is larger than 60% of the static strength of the rock, the impact will be considered as one-time effective loading [5]; (2) When $\tau_{d,Nf} / \tau_{f,Nf-1}$ reaches a critical value $R_{res}$ such as 0.7, the failure occurs. Of course, the evaluation of dynamic strength in DDA will be more accurate as more sufficient experiment data are obtained. More experimental studies on this issue will be carried in our future study to improve the accuracy and effectiveness of this dynamic model in DDA. With the consideration of the effects of fatigue in the close-open algorithm of DDA, the residual shear strength ratio after irregular seismic loading can be expressed as follows:
where $N$ is the number of effective loading under seismic loading. The determination of parameter $N_f$ can be obtained from the experiment. It also has a close relationship with the loading frequency, which can be calculated by the calculation time step in DDA. The larger time step it is, the less effective loading times $N_f$ it requires in DDA calculation. In the dynamic conditions, those parameters $\alpha$, $N_f$, $R_{res}$ for dynamic failure should be obtained based on the experiment data of the dynamic fatigue test. Therefore, the dynamic failure model is proposed in DDA to evaluate the slope failure under seismic conditions. As shown in Figure 2, the Open-close iteration process of dynamic failure model for evaluation of dynamic shear failure can be expressed as follows.

![Figure 2. Open-close iteration of dynamic shear failure model](image)

4. Validation examples

4.1. SDOF model

SDOF is a fundamental case in structural dynamics. Therefore, the SDOF system is applied to validate the dynamic shear and tensile failure evaluation of the proposed DDA-Dynamic.

![Figure 3. Model of SDOF](image)

As shown in Figure 3, an SDOF model that a square block rests over a bigger base block is constructed. The joint contacts that hold the two blocks together can be separated under the seismic wave loading force. The seismic waves are respectively applied in the horizontal and vertical direction of the bottom base block to simulate the dynamic shear and tensile failure. To validate the effective consideration of fatigue in the proposed DDA-Dynamic, the SDOF model is calculated with both the original DDA and the proposed DDA-Dynamic. As shown in Figure 4, the two seismic waves that one is PLGM with PGA of 0.67g and the other is Non-PLGM with PGA of 0.7g are applied in this simulation. Figure 5 shows the energy of the applied PLGM and Non-PLGM, which is represented by...
the cumulative squared velocity of seismic wave [13-14]. It can be seen that PLGM has significantly larger energy than Non-PLGM, suggesting the more serious damage it can cause. The critical joint failure strength parameters are as follows: \( c = 17142 \) Pa, \( \phi = 0^\circ \), \( T = 1 \) Mpa. The residual strength ratio at the critical failure state is assumed to be 0.7, namely \( R_{res} = 0.7 \). The dynamic failure parameters for fatigue effect are as follows: \( a = 0.1 \), \( N_f = 100 \).

![Figure 4. Ground motions of Non-PLGM and PLGM.](image1)

![Figure 5. Energy of Non-PLGM and PLGM.](image2)

In the original DDA, the joint contact failure between two blocks occurs when the shear force is larger than the peak shear strength, which indicates that the PGA of a seismic wave dominates the failure behavior of structures in seismic response analysis. As shown in Fig. 6 (a), the top block of the SDOF model separates from the bottom block and generates the crack shown in the red line when subjected to Non-PLGM while it can remain stable under PLGM because the Non-PLGM has a larger PGA. However, the PLGM in Figure 4(b) is a kind of ground motion with velocity pulse and large energy, which is proved to be able to cause larger damage compared with the Non-PLGM with less energy [8-10]. The original DDA failure model controlled by PGA is inappropriate for seismic conditions. As shown in Figure 6 (b), the simulation results in the I-DDA with dynamic failure model are obtained. The two blocks separate and generate the crack when it is subjected to PLGM but it remains stable under Non-PLGM. With the dynamic failure model incorporated, the proposed DDA-Dynamic can not only consider the peak amplitude of a seismic wave but also take the whole duration of a seismic wave into account. The simulated results show that the effect of fatigue of dynamic seismic waves has been effectively considered in the proposed DDA-Dynamic, which can be better applied to evaluate the dynamic failure in seismic response analysis.
5. Practical application to earthquake-induced landslide with PLGM

To verify the availability and effectiveness of the proposed DDA-Dynamic model for earthquake-induced landslides with PLGM, a practical model is constructed based on the longitudinal cross-section of the Donghekou landslide, including two parts: the base block and sliding mass. The sliding mass is randomly discretized into multiple smaller discrete deformable blocks by the Voronoi method. The recorded seismic waves PLGM and Non-PLGM in Figure 4 are respectively applied to the landslide analysis by using the proposed DDA-Dynamic. It is clear that the slope internal factors including slope physical characteristics and geometry are also closely related to landslide initiation. However, in this study we focus on the effect of earthquake trigger forces, especially the difference between the PLGM and Non-PLGM by using the dynamic failure model to verify the effectiveness of DDA-Dynamic. The simulation analysis is conducted by using the same model with the same parameters but different seismic waves to exclude the effect of slope internal factors. The material properties are taken as follows: density of 2000 kg/m³, Young’s modulus of 5.4 GPa, Poisson’s ratio of 0.27, unit weight of 20000 N/m³. The cohesion strength of the interface between blocks is 5.0 MPa and the friction angle is 14.5°. The dynamic failure parameters for the model are as follows: the residual strength ratio at the critical failure state $R_{res} = 0.7$, slope of fatigue function $a = 0.1$, Number of loading cycles $N_f = 1000$. 

5.1 Results of the O-DDA

5.2 Results of the I-DDA

Figure 6. Ground motions of Non-PLGM and PLGM.
As shown in figure 7, the simulation sequences of Donghekou landslide under PLGM and Non-PLGM are obtained by using the proposed DDA-Dynamic with dynamic failure model. In Figure 7(a), the landslide is hard to be triggered and remain stable in whole process under Non-PLGM with a large PGA because of the small energy it contains. While in Figure 7(b) when subjected to PLGM with large energy and smaller PGA, the slope is stable before the seismic velocity pulse and starts to slide after the main velocity pulse. The final deposition of the simulation results in DDA-Dynamic is in good accordance with the actual topography after the landslide, which suggests the effectiveness of the DDA-Dynamic that considers the dynamic failure strength in earthquake-induced landslide post-failure analysis, especially for the slope subjected to PLGM with a long period. Therefore, DDA-Dynamic with dynamic failure model considering the effect of the fatigue can be better applied for evaluating the earthquake-induced landslides.

**6. Conclusion**

In this study, a model considering the dynamic failure strength of slope has been proposed for earthquake-induced landslide analysis with PLGM in DDA, named DDA-Dynamic. This model can effectively evaluate the dynamic failure of joint contacts in the DDA method by considering the effect of fatigue and accumulated damage under seismic dynamic loading. The Single-Degree-Of-Freedom (SDOF) systems under PLGM and Non-PLGM have been simulated by using the proposed DDA-
Dynamic and O-DDA to validate the accuracy of the proposed model in describing the dynamic failure of contact joints. Compared with O-DDA, the proposed DDA-Dynamic has more reasonable results for seismic conditions. Meanwhile, a practical landslide under both PLGM and Non-PLGM has also been analyzed to further verify the availability and effectiveness of the proposed model for dynamic failure simulation by using DDA-dynamic. The final deposition of the simulation results with PLGM in DDA-Dynamic was in good accordance with the actual topography after the landslide, which suggests that DDA-Dynamic can be well applied for simulating the earthquake-induced landslides, especially for those triggered by PLGM.

7. References

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