Isotope effects of trapped electron modes in the presence of impurities in tokamak plasmas

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Abstract
The trapped electron modes (TEMs) are numerically investigated in toroidal magnetized hydrogen, deuterium and tritium plasmas, taking into account the effects of impurity ions such as carbon, oxygen, helium, tungsten and others with positive and negative density gradients with the rigorous integral eigenmode equation. The effects of impurity ions on TEMs are investigated in detail. It is shown that impurity ions have substantially-destabilizing (stabilizing) effects on TEMs in isotope plasmas for $L_{ce} \equiv L_{ne}/L_{nc} > 0$ ($< 0$), opposite to the case of ion temperature gradient (ITG) driven modes. Detailed analyses of the isotope mass dependence for TEM turbulences in hydrogenic isotope plasmas with and without impurities are performed. The relations between the maximum growth rate of the TEMs with respect to the poloidal wave number and the ion mass number are given in the presence of the impurity ions. The results demonstrate that the maximum growth rates scale as $\gamma_{\text{max}} \propto M_i^{-0.5}$ in pure hydrogenic plasmas. The scale depends on the sign of its density gradient and charge number when there is a second species of (impurity) ions. When impurity ions have density profiles peaking inwardly (i.e. $L_{ce} \equiv L_{ne}/L_{nc} > 0$), the scaling also depends on ITG parameter $\eta_i$. The maximum growth rates scale as $\gamma_{\text{max}} \propto M_{\text{eff}}^{-0.5}$ for the case without ITG ($\eta_i = 0$) or the ITG parameter is positive ($\eta_i > 0$) but the impurity ion charge number is low ($Z \leq 5.0$). However, when $\eta_i > 0$ and the impurity ion charge number is moderate ($Z = 6.0 - 8.0$), the scaling law is found as $\gamma_{\text{max}} \propto M_{\text{eff}}^{-1.0}$. Here, $Z$ is impurity ion charge number, and the effective mass number, $M_{\text{eff}} = (1 - f_z)M_i + f_z M_Z$, with $M_i$ and $M_Z$ being the mass numbers of the hydrogenic and impurity ions, respectively, and $f_z = Z \eta_{qA}/\eta_{qI}$ being the charge concentration of impurity ions. In addition, with regard to the case of $L_{ce} < 0$, the maximum growth rate scaling is $\gamma_{\text{max}} \propto M_i^{-0.5}$. The possible relations of the results with experimental observations are discussed.

Keywords: isotope effect, scaling, trapped electron mode (TEM), ion temperature gradient (ITG) mode

(Some figures may appear in colour only in the online journal)
1. Introduction

The turbulence induced by drift instabilities, such as ion temperature gradient (ITG) mode and trapped electron mode (TEM), is a potential candidate responsible for the anomalous transports in magnetic fusion plasmas [1–4]. In recent years, the drift-micro-turbulence has been intensively investigated for tokamak plasmas [5–10] and other magnetic configurations, such as reversed field pinch (RFP), plasmas [11, 12]. It is also observed that hydrogenic isotope plasmas, such as deuterium or tritium plasmas exhibit certain advantages in energy confinement and other aspects in comparison with pure hydrogen plasmas. In many operational regimes the hydrogen isotopes give rise to highly different confinement behaviors. The isotope effect has been observed in many different tokamaks under different plasma conditions with a degree of confinement improvement in energy, particle, and momentum depending on plasma regimes. Bessenrodt-Weberpals et al. investigated the effect of isotopic mass on plasma parameters as observed on the ASDEX tokamak and revealed that the ion mass was a substantial and robust parameter, which affected all the confinement times (energy, particle and momentum) in the whole operational window [13]. Xu et al. provided the first direct experimental evidence for the importance of multi-scale physics for unraveling the isotope effect in fusion plasmas [14]. They emphasized that increasing the mass of the isotopes would weaken transport because that the characteristic step size of collisional transport and turbulent structures both decrease with the ion gyroradius $\rho_i$, which is consistent with the results of Ramisch et al. [15].

In addition, the theoretical studies of the isotopic mass dependence of transport are performed in [16–18]. Dong et al. numerically investigated isotope scaling of ITG mode growth rate in tokamak plasmas with impurities [18], which suggested that the maximum growth rate of ITG mode scaled as $M_i^{0.5}$ and $M_{eff}^{0.5}$ for plasmas of pure hydrogenic ions and with the presence of impurity ions, respectively, while it was $M_i^{0.5}$ for the impurity mode, where $M_i$, $M_{eff}$ are the mass numbers of the main and impurity ions, respectively, and $M_{eff} = (1 - f_c)M_i + f_c M_{Zi}$ is the effective mass number of the plasmas, $f_c = Zn_{io}/n_{io}$, where $Z$ and $n_{io}$ are the charge number and density of the impurity ions, respectively, while $n_{io}$ is the electron density.

An adiabatic electron response is assumed in the work of Dong et al. [18]. In fact, it has been well accepted that trapped electrons (TEs) play an essential role in plasma confinement and their contributions have to be taken into account in drift instability studies [19–21]. Studying the TEMs in hydrogen, deuterium and tritium plasma in detail and analyzing their isotope effects are of practical significance for the study of drift-wave-turbulence. Lately, many authors addressed this subject with different point of view in detail. For example, by applying an improved mixing length estimate to dissipative trapped electron mode (DT EM) turbulence, Tokar et al. [22] found that a favorable scaling with isotope mass might be realized, although this model fails to demonstrate empirical scaling with other parameters as argued by Waltz et al. [23]. In addition, to study the effect of primary ion species of differing charge and mass on drift wave instabilities and transport, linear and non-linear gyrokinetic simulations were carried out with GYRO code by Pusztai et al. [24], which revealed the significant effects of the different electron-to-ion mass ratio on ion scales or on the deviations from pure gyro-Bohm scaling. Very recently, ITG and TEM simulations were performed with the GENE (gyrokinetic electromagnetic numerical experiment) code for the three hydrogen isotopes by Bustos et al., they discovered that an isotope effect is clearly visible for some types of ITG/TEM and TEM turbulence [25]. These works remarkably push forward the studies of the isotope effects for drift wave turbulences.

It also needs to be noted that there are carbon (C), tungsten (W), oxygen (O) and other impurities deposited in the plasmas in tokamak devices where divertors are C or W-coated plasma facing components (PFCs), as well as limiters are used [26]. For instance, the tritium inventory in ITER is expected to be dominated by co-deposition with carbon. A strong degradation of the confinement is found for helium concentrations of about 5–10% in the main chamber [27]. Therefore, as was done in [18], impurities, in particular, carbon, helium (He), oxygen and tungsten ions, must be considered in the studies in order to make the results comparable with the experimental observations.

In this paper, we aim to study the characteristics and isotope effects of TEMs in the presence of impurities numerically and to attempt to conclude a systematic result. We focus on the collisionless trapped electron modes (CTEMs) because these modes are among the most plausible candidates for turbulent transports under reactor conditions. For this purpose, we deal with a model of collisionless toroidal plasmas which include TEM and the effect of a second ion species. This work is based on the developed comprehensive gyrokinetic dispersion equation [28] used for the study of low frequency drift-like instabilities, which now has been extended to include the contribution of impurities and the TE dynamics [29–31]. Hydrogen, deuterium and tritium are chosen as the work gases and the second ion species such as C, He, O, W are the impurities. The TEMs in the plasmas with different parameters are investigated in detail. As a result, we found that the isotope effects of TEM are more complicated than those of ITG mode. The reasons for such differences are discussed.

The remainder of this paper is organized as follows. In section 2, we present the gyrokinetic integral equations developed to include the contribution of impurity ions and the effects of the TEs. The numerical results are analyzed to study the isotope effects of TEM and the effects of impurities on TEM in section 3. Section 4 contains the conclusion and discussions.

2. Integral eigenvalue equations

In this work a toroidal geometry with circular cross section is considered for the tokamak magnetic configuration. We extend the gyrokinetic integral equation [30] for studying...
low-frequency drift modes to include impurity species and add the TE contributions. The kinetic characteristics of ions, such as Landau resonance, magnetic drift and finite Larmor radius are retained. The passing electron response is assumed to be adiabatic, and the finite Larmor radius effects of TEs are neglected. The dynamics of a low-frequency electrostatic perturbation in inhomogeneous plasmas is described by the quasineutrality condition

$$\rho_e = \rho_i + Zn_e,$$

with $Z$ being the charge number of the impurity ions. Here, the perturbed electron density $\rho_e$ consists of the perturbed densities of passing and TEs $\rho_{ep}$ and $\rho_{et}$, and $\rho_e = \rho_{ep} + \rho_{et}$. The perturbed density of passing electrons is assumed adiabatic, $\rho_{ep} = (1 - f_{et}) \frac{m_e}{T_e} \Phi$, and the perturbed density of TEs $\rho_{et}$ consists of the adiabatic response $\rho_{et,ad}$ and non-adiabatic response $\rho_{et,na}$ parts, and $\rho_{et,ad} = f_{et} \frac{m_e}{T_e} \Phi$, where $f_{et} = \sqrt{2\pi}$ is the fraction of TEs in tokamaks, with $\varepsilon = r/R$ being the ratio of the minor radius associated with the rational surface over the major radius of the torus. Note that the total value of the adiabatic component of perturbed density of electrons is $\rho_{et,ad} \equiv \rho_{ep} + \rho_{et,ad} = \frac{m_e}{T_e} \Phi$. The form of $\rho_{et,na}$ will be discussed later.

On the other hand, the perturbed main ion ($\bar{n}_i$) and impurity ion ($\bar{n}_z$) densities in a tokamak are given by [28, 29]

$$\bar{n}_i = -\frac{e n_{0i} \Phi}{T_i} + \int \frac{dV}{Rq} L_i(\alpha_i) h_i,$$

$$\bar{n}_z = -\frac{e n_{0z} \Phi}{T_z} + \int \frac{dV}{Rq} L_z(\alpha_z) h_z.$$

The non-adiabatic response $h_s$ ($s = i, z$) is determined by solving the following gyrokinetic equation,

$$\frac{\partial}{\partial \tau} h_s + (\omega - \omega_D) h_s = (\omega - \omega_{et,ad}) L_i(\alpha_i) F_{Mz} \frac{q_{Mz}}{T_z} \hat{\Phi}(\theta),$$

with

$$\omega_D = 2\pi T_i q_i B_{lin},$$

$$\omega_{et} = c k_\| T_i / q_i B_{lin}, \omega_{et,ad} = \omega_{et,0} \left[ 1 + \eta_i \left( \frac{v_i^2}{v_{Ti}^2} - \frac{3}{2} \right) \right],$$

$$\alpha_i = \sqrt{v_i^2 / \Omega_{ri}}, v_{Ti} = \sqrt{2T_i / m_i}.$$

We finally obtain the integral eigenvalue equation with TE response and including the effects of the impurity as following:

$$\int_{-\infty}^{+\infty} \frac{d\kappa}{\sqrt{2\pi}} K(k, \kappa) \hat{\Phi}(k) = \frac{T_e}{e n_{0e}} \hat{n}_{et,na},$$

with

$$\int_{0}^{+\infty} \omega_{et} \frac{\kappa^2}{2} \hat{\Phi}(k) = \Phi \left[ 1 + \pi(1 - f_j) + \tau (2T_j) \hat{\Phi}(k) \right] = \frac{\tau}{e n_{0e}} \hat{n}_{et,na}.$$
\[ \tilde{\omega}_{T_e} = -\frac{e\varphi_0}{T_e} \sqrt{\frac{2\pi}{\theta}} e^{-\gamma} \int_{0}^{1} d\omega \frac{\omega^2 - \omega^2_e}{\omega - \omega_d} \times \frac{d\omega}{2F(\kappa)} \sum_{j=-\infty}^{+\infty} g(\theta - 2\pi j, \kappa) \phi(\theta - 2\pi j) \]

with

\[ g(\theta, \kappa) = \int_{-\infty}^{0} \frac{\delta(\theta - \theta')}{\sqrt{\kappa^2 - \sin^2(\theta/2)}} \, d\theta' \]

\[ \omega^2_e = \omega^2_e \left[ 1 + \eta_0 \left( t - 3 \frac{2}{3} \right) \right] \]

\[ \omega_d = \omega_{de} \left[ \frac{1}{L_{de} / L_{Te}} \right] \]

\[ \omega_\eta = \omega_\eta \left[ \frac{1}{L_{de} / L_{Te}} \right] \]

\[ \tau = \int_{-\infty}^{+\infty} d\omega \left( \sigma \omega / \kappa + 1 \right) \]

\[ \sigma = \int_{-\infty}^{+\infty} d\omega \left( 1 - \frac{\omega^2}{\omega_d^2} \right) \]

3. Numerical results and analysis

The computer code HD7 for solving the integral eigenmode equation which has been updated by including the multiple ion species and TE effects is employed in this work. The TEM of hydrogen, deuterium and tritium plasmas with or without the second ion species are investigated with equation (4). In the calculations, we use the following typical parameters values: \( s = 1.5, q = 2.0, f_z = 0.2, \eta_0 = 0.2, \eta = \tau = 1, \varepsilon = 0.1 \), unless otherwise stated in addition to the varying parameters such as \( \eta_0, \eta_\eta, \) and \( L_{de} \). Note that the wave number is normalized to \( \rho_1 \), where \( \rho_1 = c / 2T_{01} / eB \) is the hydrogen ion Larmor radius, the mode growth rate and real frequency are both normalized to \( k_0 \rho_1 / \omega_{ce} \) in the following presentation.

3.1 TEM in pure hydrogenic plasmas

As the simple and archetypal cases we investigated the TEMs in pure hydrogen (H), deuterium (D) and tritium (T) plasmas first. The normalized growth rate and real frequency versus \( k_0 \rho_1 \) are plotted in figure 1. As presented in figure 1(a), the mode growth rates increase, reach maximum, and then decrease with the increment of the normalized wave number \( k_0 \rho_1 \) for \( \eta_0 = \eta_\eta = 0 \). The maximum growth rates are 0.154, 0.109 and 0.089, reached at \( k_0 \rho_1 = 1.7, 1.2 \) and 1.0 for H, D and T plasmas, respectively. It is clearly shown that the TEM has the highest maximum growth rate and the widest unstable \( k_0 \) range in hydrogen plasmas. It has the lowest maximum growth rate and the narrowest unstable \( k_0 \) range in tritium plasmas. It is in the deuterium plasmas. Here, it is interesting to note that the maximum growth rates satisfy the following relation,

\[ \gamma_{\text{maxH}} : \gamma_{\text{maxD}} : \gamma_{\text{maxT}} = 1 : 1 / \sqrt{2} : 1 / \sqrt{3}. \]
To study the isotope effects of TEMs in realistic tokamak plasmas, it is necessary to invoke impurity modification of TEM-turbulence. By means of introducing the impurity ions into the plasma, the atomic mass numbers of the primary and impurity ions both become explicit parameters in the drift wave turbulence. One can study the impacts of impurities in realistic plasma by taking all ion species into account individually. Since the impurity density gradient essentially affects the instability characteristics, we should investigate the two cases of the ratio of the electron density scale length over the impurity density scale length \( \frac{L_{ez}}{L_{nz}} \), which correspond to a positive and a negative impurity density gradient (inwardly and outwardly peaking impurity density profile), respectively.

### 3.2. TEM with impurities in the case of \( \eta_i = 0 \)

In the following we introduce a second ion species to study the isotope effects of the TEM in the presence of impurities. The impurity ions with inwardly peaked density profiles \( \eta_i = 0 \) are considered first.

#### 3.2.1. The case with \( \eta_i = 0 \)

The normalized growth rates and real frequencies as functions of poloidal wave number are given in figure 2 for hydrogen, deuterium and tritium plasmas with fully ionized carbon \((\text{C}^+6)\) or helium \((\text{He}^{+2})\) impurities. The effects of temperature gradients of ions and electrons are neglected here. The other parameters are \( L_{ez} = 2.0, f_c = 0.2, \varepsilon = 0.25 \). The solid lines correspond to the plasmas with carbon impurity, and the dotted lines are for helium impurity. Again, as shown in figure 2(a), the growth rates of the modes increases, reach maximum, and then decreases with the increment of the normalized wave number \( k_0 \rho_H \) for \( \eta_i = \eta_e = 0 \).

In addition, among the hydrogenic plasmas with carbon or helium impurities, the maximum growth rates of the modes are the highest and lowest in hydrogen and tritium plasmas, respectively. The maximum growth rates of the modes are in the between in deuterium plasmas.

The growth rates of the modes are higher in the H, D, and T plasmas with helium impurity than in the counterparts with carbon impurity for \( k_0 \rho_H < 1 \). However, the maximum growth rate is lower in tritium plasmas with \( \text{He}^{+2} \) impurity than that with \( \text{C}^+6 \) impurity for \( k_0 \rho_H > 1 \), although it is the opposite in hydrogen plasmas with \( \text{He}^{+2} \) and with \( \text{C}^+6 \) impurities. Meanwhile, the maximum growth rates are quite close to each other in the deuterium plasmas with helium and carbon impurities, although the former is slightly lower than the latter.

Figure 2(b) shows that the normalized real frequencies of TEMs are much higher in plasmas with carbon impurities than those with helium impurities. Among them, the maximum normalized real frequency is the highest in hydrogen plasmas, while it is the lowest in tritium plasmas.

#### 3.2.2. The case with \( \eta_i, \eta_e > 0 \)

The TEM characterizes some special properties for the case with moderate positive ITG. The variations of the normalized growth rate and real frequency with respect to the normalized poloidal wave number \( k_0 \rho_H \) are plotted in figure 3 for \( \eta_i = 1.5, \eta_e = 1.125 \), where different impurity species are considered.

For the cases with impurity ions of same or close charge numbers it is demonstrated in figure 3(a) that the unstable wave number spectrum is wider and the value of the maximum growth rate is higher when the mass number of the impurity ions is smaller. For example, for two cases with same impurity ion charge number (i.e. 2) in a hydrogen plasma, the mode growth rate with fully ionized helium ions (its mass number is 4) reaches the maximum value 0.142 at about \( k_0 \rho_H \approx 0.5 \),

![Figure 1. Normalized growth rate and real frequency of the TEM versus \( k_0 \rho_H \) for pure H, D, T plasmas. The other parameters are (a) and (b) for \( \eta_i = \eta_e = 0 \), (c) and (d) for \( \eta_i = \eta_e = 1.5 \).](image-url)
which is higher than that with $^{12}$C ions (whose mass number is 12), since the mode growth rate of the latter reaches the maximum value 0.105 at about $k_0 \rho_H \approx 0.3$. Nevertheless, as is shown in figure 3(c), the growth rates and widths of the unstable wave number spectrum are closer when the values of the mass numbers of two kind impurity ions are closer.

Comparing the normalized growth rates in the cases with $^{12}$C and $^{16}$O impurities, which are represented with the solid lines in figures 3(a) and (c), respectively, it is demonstrated that for the impurities with moderate mass numbers (e.g. carbon with the mass number 12), the higher the degree of ionization, the wider the unstable TEM wave number spectrum and the higher the value of the maximum growth rate. For example, in a hydrogen plasma, the normalized mode maximum growth rate reaches about 0.10 around the point $k_0 \rho_H = 0.28$ in the case with non-fully-ionized impurity ions $^{12}$C, while it reaches about 0.17 in the vicinity of $k_0 \rho_H = 0.52$ in the case with fully-ionized impurity $^{16}$O.
Also, we find from figure 3(c) that the maximum mode growth rates in hydrogen, deuterium and tritium plasmas with O$^{8+}$ ions are higher than those with C$^{6+}$ ions. This means that the charge number effect dominates the mass number effect in this case.

It is also demonstrated in figures 3(b) and (d) that the normalized real frequency increases with the normalized poloidal wave number $k_0\rho_H$ for all the cases, which implies that with moderate electron temperature gradient ($\eta_e = 1.125$) and somewhat higher TE parameter ($\varepsilon = 0.25$), the typical TEMs, propagating in electron diamagnetic drift direction, hold in hydrogenic plasmas with impurities.

Comparing the results of figure 2(a) with 1(a) and figure 3(a) with 1(c), the important result is that in $L_{ez} > 0$ case, impurity ions enhance the TEMs and mode growth rates increase. In other words, the effects of impurities on the TEMs are destabilizing, which is opposite to the case of ITG modes. One possible explanation is the dilution effect of impurity ions on hydrogenic ion effects [34]. Another reason may be that the impurity ions drift in the direction opposite to that of the electrons and, therefore, have reduced damping effect on the modes.

3.3. TEM with impurities in the case of $L_{ez} < 0$ Some new features of TEMs appear when the density profiles of impurity ions peak opposite to that of electrons, i.e. $L_{ez} \equiv L_{ne}/L_{nt} < 0$. For the case with impurities, however, the scaling law becomes complicated. In fact, as shown by the blue fitting lines in figure 4(a), above scaling hold for the case with $\eta_i = \eta_e = 0$. But there are two different results for moderate positive $\eta_i$ values. To be specific, as shown in figure 4(a), the fitting curves (red lines) of the maximum growth rates with respect to the effective mass number meet the scaling form (6) when the charge number of the impurity ions is low. Nevertheless, the fitting curves of the maximum growth rates (the solid lines in figure 4(b)) meet the following scaling law

$$\gamma_{\text{max}} \propto M_{\text{eff}}^{-1.0},$$

when the impurity ion charge numbers are moderate.

3.3.1. The case with $\eta_i = \eta_e = 0$. For $L_{ez} < 0$ cases we first study the case without temperature gradients. Figure 5 shows the variations of the normalized growth rates and real frequencies as functions of normalized poloidal wave number $k_0\rho_H$ in hydrogen, deuterium and tritium plasmas with fully ionized carbon impurity C$^{6+}$ and partially ionized tungsten impurity W$^{+6}$ for $L_{ez} = -2.0, \eta_i = \eta_e = 0$. The other parameters are $f_1 = 0.1, \varepsilon_n = 0.1, \varepsilon_i = 1$. From figure 5(a) it is easy to see that the mode growth rate is the highest in hydrogen plasmas, the lowest in tritium plasmas, and the intermediate in deuterium plasmas. In addition, the stabilization effect of mass number of the impurity ions is clearly demonstrated again in these cases. The real frequencies shown in figure 5(b) indicate that the modes propagate in the electron diamagnetic drift direction and that the modes have the widest, the narrowest and the intermediate ranges of unstable poloidal wave number spectra in hydrogen, tritium, and deuterium plasmas, respectively.

3.3.2. The case with $\eta_i, \eta_e > 0$. The growth rates and real frequencies of the modes are shown in figure 6 in the presence
of moderate ITG. Here, we choose the following parameters values: \( L_{\text{cc}} = -2.0, \eta_0 \) = 1.5, \( \eta_2 = 2.625, f_2 = 0.2, \epsilon_n = 0.1, \epsilon = 0.1 \).

**Figure 5.** (a) Normalized growth rate and (b) real frequency versus the normalized poloidal wave number \( k_\theta \rho_H \) for H, D, T plasmas with impurities, the solid lines are for full-ionized carbon \( C^{+6} \) ions, and the dotted lines are for non-fully-ionized tungsten \( W^{+6} \) impurity. The parameters are \( L_{\text{cc}} = -2.0, \eta_0 = 0, \eta_2 = 0.1, \epsilon_n = 0.1, \epsilon = 0.1 \).

**Figure 6.** Normalized growth rate (a) and real frequency (b) versus the normalized poloidal wave number \( k_\theta \rho_H \) for H, D and T plasmas with impurities, and the solid lines are for carbon \( C^{+6} \) impurity, and the dotted lines are for tungsten \( W^{+6} \) impurity. The parameters are \( L_{\text{cc}} = -2.0, \eta_0 = 1.5, \eta_2 = 2.625, f_2 = 0.2, \epsilon_n = 0.2, \epsilon = 0.1 \).

3.3.3. The isotope scaling (II). Shown in figure 7 are the maximum growth rates from figures 5 and 6 as functions of ion mass number, where the fitting curves are \( \gamma_{\text{max}} \sim M_i^{-0.5} \) with \( M_i \) being the mass number of the hydrogenic ions. The result is the same as the isotopic scaling of impurity-driven mode in [18]. Nevertheless, there is another scaling, \( \gamma_{\text{max}} \sim Z_{\text{eff}}^{1.5} \), for the latter (Here \( Z_{\text{eff}} \) is the effective charge number). However, no such \( Z_{\text{eff}} \) scaling evidences have been found in TEM case. The reason may be that TEMs in plasmas with impurity ions of outwardly peaked density profiles are not independent modes like impurity-driven modes and, therefore, have more sophisticated relationship with the effective charge numbers. More detailed study is certainly needed in this field.

3.4. Discussions

In previous sections it has been demonstrated that the TEM is rather robust (weak) and the growth rate of the mode is higher (lower) in plasmas with impurities of \( L_{\text{cc}} \equiv L_{\text{cc}}/L_{\text{ITG}} > 0 \) \((L_{\text{ITG}} < 0)\) than that in pure hydrogenic plasmas. The main results about the ion mass dependence of the maximum growth rate of TEMs are summarized in table 1.
The maximum growth rate of the TEMs scales as $\gamma_{\text{max}} \propto M_{i}^{0.5}$ in pure hydrogenic plasmas or when the impurity ion density profile peaks inwards, i.e. $L_{ez} > 0$, and $\eta_{i} = \eta_{e} = 0$, or $\eta_{i} \eta_{e} > 0$ but the charge number of the impurity ions is equal or less than 5. Here, $M_{\text{eff}} = (1 - f_{e})M_{i} + f_{e}M_{e}$ is the effective mass number of the primary and impurity ions, respectively. In particular, in the first case of pure hydrogenic plasmas the scaling law of the maximum growth rate of the TEMs can also be expressed by $\gamma_{\text{max}} \propto M_{i}^{0.5}$, which is consistent with and then verifies the previous conclusions [19, 22].

In addition, it scales as $\gamma_{\text{max}} \propto M_{\text{eff}}^{1.0}$ when the impurity ion density profile peaks inwards and $\eta_{i} \eta_{e} > 0$ and the charge number of the impurity ions is in the range of 6–8. The maximum growth rate of the TEMs scales as $\gamma_{\text{max}} \propto M_{i}^{0.5}$ when the impurity ion density profile peaks opposite to that of the electrons, i.e. $L_{ez} < 0$.

The isotopic effect has been observed in many tokamaks [13]. The general observation is that the energy confinement time is proportional to the power of 0.5 of the ion mass number, namely, $\tau_{E} \propto M_{i}^{0.5}$, here $M_{i}$ is the averaged ion mass [35, 36]. Taking into account heating conditions, the energy confinement time dependence on the ion mass number, $M_{i}$, may be modified gently [37]. Therefore, it is reasonable that, in some cases, the scaling law $\tau_{E} \propto M_{i}^{0.5} \rightarrow M_{i}^{0.0}$ holds. According to $\tau_{E} \sim \alpha^{2}/\chi$ (where $\chi$ being the thermal diffusivity), the empirical scaling $\propto M_{i}^{0.5} \rightarrow M_{i}^{1.0}$ thus also holds. By reviewing the previous discussion, we find that the relations between the maximum growth rate of the TEMs with respect to the poloidal wave number and the ion mass number given in this paper in the presence of the impurity ions, i.e. $\gamma_{\text{max}} \propto M_{i}^{0.5} \rightarrow M_{i}^{1.0}$ are similar to the above empirical scaling of confinement time. If one assumes, just as was done in [18], that the correlation time is determined by the single harmonic with the maximum growth rate, and that the correlation length is independent of the mode width of each single harmonic and determined by equilibrium plasma parameters, under which the plasma thermal conductivity has the same scaling as the maximum growth rate of the mode does, then we are possibly able to find a possible relation between our results and the isotope scaling of $\tau_{E}$. It can be found that if TEM turbulence controls the energy confinement of the bulk plasma, the results deduced from the isotope scaling of the maximum growth rate of the TEMs is consistent with the observed mass scaling of the confinement time.

### 4. Conclusions

By taking into account the kinetics of TEs and including a second ion species, the upgraded gyrokinetic integral eigenmode code HD7 is applied to investigate the TEMs in hydrogen, deuterium and tritium plasmas with or without impurity ions. The characteristics of TEMs in pure hydrogenic isotope plasmas and those with impurity ions are investigated and compared. In particular, the effects of impurity ions on TEMs in isotope plasmas are investigated in detail. The results suggest that the maximum growth rates of the TEMs are the highest, the intermediate and the lowest in hydrogen, deuterium and tritium plasmas, respectively, regardless of presence or absence of impurity ions. Correspondingly, the modes have the widest, the intermediate and the narrowest unstable $k_{0}$ spectra in hydrogen, deuterium and tritium plasmas, respectively, with or without impurity ions. In addition, the effects of impurities on the TEMs are substantially destabilizing in $L_{ez} \equiv L_{nz}/L_{nz} > 0$ case, which can be explained with the dilution effect of impurity ions on hydrogenic ion effects. In $L_{ez} < 0$ case impurity ions have stabilizing effects on TEMs, and the heavier the impurity ions, the stronger the effects. By means of introducing the impurity ions into the plasma, the atomic mass of the primary and impurity ions both become the explicit parameters in the drift wave turbulence.

On the other hand, a detailed analysis of the isotope mass dependence of TEMs is performed. The relations between the maximum growth rate of the TEMs with respect to the poloidal wave number and the ion mass number are given in the absence or presence of the impurity ions. It is demonstrated, as the first result, quantitatively speaking, that the maximum growth
rate scales with the ion mass as $\gamma_{\text{max}} \sim M_i^{-0.5}$ in pure hydrogenic plasmas, which is consistent with the previous results [19, 22]. Secondly, the scale depends on the sign of its density gradient and charge number when there is a second species of (impurity) ions. When the density gradient of the impurity ions has the same sign as that of electrons, i.e. $L_{ce} \equiv L_{ne}/L_{ac} > 0$, the scaling also depends on ITG parameter $\eta_i$. It is found that the maximum growth rates scale as $\gamma_{\text{max}} \propto M_{\text{eff}}^{-0.5}$ for the case without ITG ($\eta_i = 0$) or the ITG parameter is positive ($\eta_i > 0$) but the impurity ion charge number is lower ($Z \leq 5.0$). However, when $\eta_i > 0$ and the impurity ion charge number is moderate ($Z = 6.0 - 8.0$), the scaling law is found to scale as $\gamma_{\text{max}} \propto M_{\text{eff}}^{-1.0}$. Here, $M_{\text{eff}}$ is the effective mass number of total ions, $M_{\text{eff}} = (1 - f_i)M_e + f_iM_i$. Thirdly, it is also found that the maximum growth rate of the modes scales as $\gamma_{\text{max}} \propto M_i^{-0.5}$ when the impurity ion density profile peaks opposite to that of the electrons, i.e. $L_{ce} < 0$, where $M_i$ is the mass number of the primary ions.

There is evidently some modest difference on isotope scaling between ITG modes and TEMs. For $L_{ce} > 0$ case, in particular, as $\eta_i$ is non-zero and moderate, and the impurity ion charge number is moderate, the mode maximum growth rate scales as $\gamma \propto M_{\text{eff}}^{-1.0}$, which is in contrast with ITG case. This is not surprising and may provide a way to distinguish ITG and TEM in experiment.

We also discussed and compared the scaling and the experimental observations for plasma energy confinement time in an attempt to find a possible relation between them. It is found that there are some similar characteristics and a possible trend in their forms and this trend is consistent with the observed mass scaling of the confinement time if TEM turbulence controls the energy confinement of the bulk plasma.

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