Note on helicity amplitudes in $D \to V$ semileptonic decays

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Motivated by the recent extraction of the helicity amplitudes for the $D^+ \to K^{*0} \mu^+ \nu$ decay, done by the FOCUS collaboration, we determine helicity amplitudes for the $D^+ \to K^{*0} \ell \nu$, $D^+ \to \rho^0 \ell \nu$ and $D_s^+ \to \phi \ell \nu$ semileptonic decays using the knowledge of the relevant form factors. The vector and axial form factors for $D \to V \nu\ell$ decays are parameterized by including contributions of charm meson resonances and using the HQET and SCET limits. In the case that the vector form factor receives contributions from two poles while axial form factors are dominated by a single pole for $D^+ \to K^{*0} \ell \nu$, we obtain better agreement with the experimental result then when all of them are dominated by single poles.

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Recently the FOCUS \textsuperscript{1} collaboration has presented first non-parametric determination of helicity amplitudes in the semileptonic decay $D^+ \to K^{*0} \mu^+ \nu$. This measurement allows for more detailed analysis of the $D \to V$ form factors, especially it enables the studying of the shapes of the form factors.

We have recently proposed a generalization of the Bećirević-Kaidalov (BK) form factor parameterization \textsuperscript{2} for the semileptonic $H \to V$ form factors based on HQET and SCET scaling predictions \textsuperscript{3}. Furthermore we have calculated the $D \to P$ and $D \to V$ form factors shapes within a model which combines properties of the heavy meson chiral Lagrangian by taking into account known and predicted charm resonances and utilizing the general form factor parameterizations \textsuperscript{3}\textsuperscript{,}4. In this note we determine helicity amplitudes for the $D \to V$ semileptonic decays and compare our model predictions for the shapes of the form factors with the new experimental results coming from FOCUS for the $D^+ \to K^{*0} \mu^+ \nu$ decay.

The standard decomposition of the current matrix elements relevant to semileptonic decays between a heavy pseudoscalar meson state $|H(p_H)|$ with momentum $p_H$ and a light polarization meson $|V(p_V,\epsilon_V)|$ with momentum $p_V$ and polarization vector $\epsilon_V$ is in terms of four form factors $V$, $A_0$, $A_1$ and $A_2$, functions of the exchanged momentum squared $q^2 = (p_H - p_V)^2$ \textsuperscript{3}. Here $V$ denotes the vector form factor and is expected to be dominated by vector meson resonance exchange, the axial $A_1$ and $A_2$ form factors are expected to be dominated by axial resonances, while $A_0$ denotes the pseudoscalar form factor and is expected to be dominated by pseudoscalar meson resonance exchange \textsuperscript{3}. In order that the matrix elements are finite at $q^2 = 0$, the form factors must also satisfy the well known relation $A_0(0) + A_1(0)(m_H + m_V)/2m_V - A_2(0)(m_H - m_V)/2m_V = 0$.

Next we follow the analysis of Ref. \textsuperscript{2}, where the $F_+$ form factor in $H \to P$ transitions is given as a sum of two pole contributions, while the $F_0$ form factor is written as a single pole, based on form factor dispersion properties as well as known HQET \textsuperscript{6} and SCET \textsuperscript{7,8,9} scaling limits near zero and maximum recoil momentum respectively. Utilizing the same approach we have proposed a general parametrization of the heavy to light vector form factors, which also takes into account all the known scaling and resonance properties of the form factors. The details of the analysis are outlined in Ref. \textsuperscript{3} and we only give the results for the derived form factor parameterizations:

$$
V(q^2) = \frac{c_H''(1-a)}{(1-x)(1-ax)},
$$

$$
A_1(q^2) = \xi c_H'(1-a)\frac{1-b'x}{1-\frac{1}{b}x},
$$

$$
A_0(q^2) = \frac{c_H''(1-a')}{(1-y)(1-a'y)},
$$

$$
A_2(q^2) = \frac{c_H''}{(1-b'x)(1-b''x)},
$$

where $c_H'' = [(m_H + m_V)\xi c_H'(1-a) + 2m_V c_H'(1-a')]/(m_H - m_V)$ is fixed by the relation between the form factors at $q^2 = 0$ while $\xi = m_H^2/(m_H + m_V)^2$ is the proportionality factor between $A_1$ and $V$ from the SCET relation. Variables $x = q^2/m_H^2$, and $y = q^2/m_V^2$ ensure, that the $V$ and $A_0$ form factors are dominated by the physical $1^-$ and $0^-$ resonance poles, while $a$ and $a'$ measure the contributions of higher states, parameterized by additional effective poles. On the other hand $b'$ in $A_1$ and $A_2$ measures the contribution of resonant states with spin-parity assignment $1^+$ which are parameterized by the effective pole at $m_{H,h}^2 = m_H^2/b'$ while the scaling properties and form factor relations require an additional effective pole for the $A_2$ form factor. At the end we have parameterized the four $H \to V$ vector form factors in terms of the six parameters $c_H''$, $a$, $a'$, $b'$, $c_H'$ and $b''$.  

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We determine the above parameters via heavy meson chiral theory (HMχT) calculation of the form factors near $q^2_{\text{max}} = (m_H - m_V)^2$. We use the leading order heavy meson chiral Lagrangian in which we include additional charm meson states. The details of this framework are given in [3] and [4]. We first calculate values of the form factors in the small recoil region. The presence of charm meson resonances in our Lagrangian affects the values of the form factors at $q^2_{\text{max}}$ and induces saturation of the second poles in the parameterizations of the $F_+(q^2)$, $V(q^2)$ and $A_0(q^2)$ form factors by the next radial excitations of $D^*_0$ and $D^*_1$ mesons respectively. Using HQET parameterization of the current matrix elements [3], which is especially suitable for HMχT calculations of the form factors near zero recoil, we are able to extract consistently the contributions of individual resonances from our Lagrangian to the various $D \rightarrow V$ form factors. We use physical pole masses of excited state charmed mesons in the extrapolation, giving for the pole parameters $a = m_{H^*}^2/m_{H^*}^0, a' = m_{H}^2/m_{H^*}^0$ and $b' = m_{H^*}^2/m_{H^*}^0$. Although in the general parameterization of the form factors the extra poles in $V$ and $A_{0,1,2}$ parameterize all the neglected higher resonances beyond the ground state heavy meson spin doublets ($0^-, 1^-$), we are saturating those by a single nearest resonance. The single pole $q^2$ behavior of the $A_1(q^2)$ form factor is explained by the presence of a single $1^+$ state relevant to each decay, while in $A_2(q^2)$ in addition to these states one might also account for their next radial excitations. However, due to the lack of data on their presence we assume their masses being much higher than the first $1^+$ states and we neglect their effects, setting effectively $b'' = 0$.

The values of the unknown HMχT parameters appearing in $D \rightarrow V l \nu_l$ decay amplitudes [3] are determined by fitting the model predictions to known experimental values of branching ratios and partial decay width ratios.

In order to compare our model predictions with recent experimental analysis performed by FOCUS collaboration, following [10] we introduce helicity amplitudes $H_{+,-,0}$:

\[
H_{+}(y) = + (m_H + m_V)A_1(m_H^2y) + \frac{2}{m_H + m_V}A_0(m_H^2y)
\]

\[
H_0(y) = + \frac{m_H + m_V}{2m_H m_V \sqrt{y}} [m_H^2(1 - y) - m_V^2] A_1(m_H^2y) - \frac{2m_H}{m_V(m_H + m_V) \sqrt{y}} A_2(m_H^2y)
\]

where $y = q^2/m_H^2$ and the three-momentum of the light vector meson is given by:

\[
|\vec{p}_V(y)|^2 = \frac{(m_H^2(1 - y) + m_V^2)^2}{4m_H^2} - m_V^2.
\]

Because of the arbitrary normalization of the form factors in [1], we fit our model predictions for a common overall scale in order to compare the results. We plot the $q^2$ dependence of the predicted helicity amplitudes and compare them with the experimental results of FOCUS, scaled by an overall factor determined by the least square fit of our model predictions, on FIGs. 1 and 2. The scale factor is common to all form factors.

![FIG. 1: Our model predictions (double pole in solid line and single pole in dashed line) for the $q^2$ dependence of the helicity amplitude $H_+^2(q^2)$ in comparison with scaled FOCUS data on $D^+ \rightarrow K^{\ast0}$ semileptonic decay.](image)

![FIG. 2: Our model predictions (double pole in solid line and single pole in dashed line) for the $q^2$ dependence of the helicity amplitude $H_0^2(q^2)$ in comparison with scaled FOCUS data on $D^+ \rightarrow K^{\ast0}$ semileptonic decay.](image)

In addition to the two pole contributions we calculate helicity amplitudes in the case when all the form factors exhibit single pole behavior. Putting contributions of higher charm resonances to be zero we fit the remaining model parameters to existing branching ratios and partial decay ratios. We obtain the values for the following parameter combinations as explained in [3]:

\[
\hat{\alpha} \hat{\mu} = 0
\]

\[
\alpha' \zeta = -0.180 \text{ GeV}^{3/2}
\]

\[
\alpha' \mu = -0.00273 \text{ GeV}^{1/2}
\]

\[
\alpha_1 = -0.203 \text{ GeV}^{1/2}
\]
mined the helicity amplitudes for the \( FOCUS \) collaboration. In addition we have deter-
mine parameters are determined at \( q^2 = 0 \) would
\( H_0 \) helicity amplitude near \( q^2 = 0 \) would
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As shown on FIGs. 1 and 2 the experimental data for \( H_\pm \)
do not favor such a parametrization, while in the case
of \( H_0 \) helicity amplitude there is almost no difference
since the \( H_0 \) helicity amplitude is defined via the \( A_{1,2} \)
form factors, which are in our approach both effectively
dominated by a single pole. The agreement between the
FOCUS results and our model predictions for the \( q^2 \) de-
pendence of the helicity amplitudes is good, although as
noted already in [1], the uncertainties of the data points
are still rather large. On FIGs. 4 and 5 we present hel-
city amplitudes for the \( D^+ \to \rho^0 l\nu_l \) and \( D_s^+ \to \phi l\nu_l \)
decays. Both decay modes are most promising for the
future experimental studies. We make predictions for the
shapes of helicity amplitudes for both cases: where two
poles contribute to the vector form factor and a single
pole to the axial form factors, and the second case where
all form factors exhibit single pole behavior.

In principle one can apply the above procedure to the
\( B \to \rho l\nu_l \) semileptonic decays. However, due to the much
broader leptons invariant mass dependence in this case,
our procedure is much more sensitive to the values of
the form factors at \( q^2 \approx 0 \). In addition, the semileptonic
decay rates in our model fit are numerically dominated by
the longitudinal helicity amplitude \( H_0 \) which has a broad
\( 1/\sqrt{q^2} \) pole [11]. This is true especially for \( D \to V \) but
to minor extent also for \( B \to V \) transitions. Since our
model parameters are determined at \( q^2_{\text{max}} \), this gives a
poor handle on the dominating effects in the overall decay
rate. Thus, accurate determination of the magnitude and
shape of the \( H_0 \) helicity amplitude near \( q^2 = 0 \) would
contribute much to clarifying this issue.

We can summarize: we have investigated the predic-
tions of the general \( H \to V \) form factor parametrization
combined with HMX\( \chi \)T calculation for the \( D^+ \to K^{*0} \)
semileptonic helicity amplitudes, recently determined by
the FOCUS collaboration. In addition we have deter-
mained the helicity amplitudes for the \( D^+ \to \rho^0 l\nu_l \) and
\( D_s^+ \to \phi l\nu_l \) decays. In all three cases that we have con-
sidered we used two approaches: one with a two poles
shape for the vector form factor and single pole for the
axial form factors, andsecondly the usually assumed sin-
gle pole behavior of all three relevant form factors. Our
study indicates that the two pole shape for the \( V(q^2) \)
form factor in \( D^+ \to K^{*0} \) transition is favored over the
single pole shape, when compared to the FOCUS result.

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