CONFINEMENT MECHANISM
IN VARIOUS ABELIAN PROJECTIONS
OF LATTICE GLUODYNAMICS

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ABSTRACT
We show that the monopole confinement mechanism in lattice gluodynamics may be a particular feature of the maximal abelian projection. We give an explicit example of the $SU(2) \rightarrow U(1)$ projection (the minimal abelian projection), in which the confinement is due to topological objects other than monopoles. We also discuss the string representation of the abelian projected $SU(2)$ gluodynamics.

1. Introduction

In his well known–paper, 't Hooft suggested a partial gauge fixing procedure for the $SU(N)$ gluodynamics which does not fix the $[U(1)]^{N-1}$ gauge group. Under the abelian transformations, the diagonal elements of the gluon field transform as gauge fields; the nondiagonal elements transforms as matter fields. Due to the compactness of the $U(1)$ gauge group, the monopoles exist, and if they are condensed, the confinement of color can be explained in the framework of the classical equations of motion. The string between the colored charges is formed as the dual analogue of the Abrikosov string in a superconductor, the monopoles playing the role of the Cooper pairs.

Many numerical experiments (see e.g. the review) confirm the monopole confinement mechanism in the $U(1)$ theory obtained by the abelian projection from the $SU(2)$ lattice gluodynamics. The string tension $\sigma_{U(1)}$ calculated from the $U(1)$ Wilson loops (loops constructed only from the abelian gauge fields) coincides with the full $SU(2)$ string tension; the monopole currents satisfy the London equation for a superconductor. Recently it has been shown that the $SU(2)$ string tension is well reproduced by the contribution of the abelian monopole currents. Numerical study of the effective monopole action shows

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that the entropy of the monopole loops dominates over the energy, and therefore, there exists the monopole condensate in the zero temperature $SU(2)$ lattice gluodynamics. All these remarkable facts, however, have been obtained only for the so called maximal abelian (MaA) projection \cite{10}. Other abelian projections (such as the diagonalization of the plaquette matrix $U_{x,12}$) do not give evidence that the vacuum behaves as the dual superconductor. Below we give two relevant examples.

First, it turns out \cite{11} that the fractal dimensionality of the monopole currents extracted from the lattice vacuum by means of the maximal abelian projection is strongly correlated with the string tension. If monopoles are extracted by means of other projections, this correlation is absent (cf. Fig.2 and Fig.4 of ref.\cite{11}). Another example is the temperature dependence of the monopole condensate measured on the basis of the percolation properties of the clusters of monopole currents \cite{12}. For the maximal abelian projection the condensate is nonzero below the critical temperature $T_c$ and vanishes above it. For the projection which corresponds to the diagonalization of $U_{x,12}$, the condensate is nonzero at $T > T_c$, and it is not the order parameter for the phase transition. The last result has been obtained by the authors of \cite{12}, but is unpublished.

In the present talk we discuss the dependence of the confinement mechanism on the type of the abelian projection. We find that the monopole confinement mechanism is natural for the MaA projection (Section 2), and we give an explicit example of the abelian projection \cite{13} in which confinement is due to topological defects which are not monopoles (Section 3).

### 2. Maximal Abelian Projection and Compact Electrodynamics

The MaA projection \cite{10} corresponds to the gauge transformation that makes the link matrices diagonal “as much as possible”. For the $SU(2)$ lattice gauge theory, the matrices of the gauge transformation $\Omega_x$ are defined by the following the maximization condition:

\[
\max_{\{\Omega_x\}} R(U') ,
\]

\[
R(U') = \sum_{x,\mu} Tr(U'_{x\mu} \sigma_3 U'_{x+\mu}^\dagger \sigma_3) , \quad U'_{x\mu} = \Omega_x^+ U_{x\mu} \Omega_x + \hat{e}_\mu .
\]

For the standard parametrization of the $SU(2)$ link matrix, we have $U_{x\mu}^{11} = \cos \phi_{x\mu} e^{i\theta_{x\mu}}$; $U_{x\mu}^{12} = \sin \phi_{x\mu} e^{i\chi_{x\mu}}$; $U_{x\mu}^{22} = U_{x\mu}^{11*}$; $U_{x\mu}^{21} = -U_{x\mu}^{12*}$; $0 \leq \phi \leq \pi/2$, $-\pi < \theta, \chi \leq \pi$; condition (1) has the form:

\[
\max_{\{\Omega_x\}} \sum_{x,\mu} \cos 2\phi'_{x\mu} .
\]

The $U(1)$ gauge transformations, which leave invariant the gauge conditions (1), (3), show that after the abelian projection $\theta$ becomes the abelian gauge field and $\chi$ is the vector goldstone field, which carry charge two in the continuum limit:

\[
\theta_{x\mu} \rightarrow \theta_{x\mu} + \alpha_x - \alpha_{x+\hat{\mu}} ,
\]
Here we have set:

\[ \chi_{x\mu} \rightarrow \chi_{x\mu} + \alpha_x + \alpha_{x+\hat{\mu}}. \]  

(5)

It is instructive to consider the plaquette action in terms of the angles \( \phi, \theta \) and \( \chi \):

\[ S_P = \frac{1}{2} \operatorname{Tr} U_1 U_2 U_3^+ U_4^+ = S^a + S^n + S^i, \]  

(6)

where

\[
S^a = \cos \theta_P \cos \phi_1 \cos \phi_2 \cos \phi_3 \cos \phi_4, \\
S^n = -\cos(\theta_3 + \theta_4 - \chi_1 + \chi_2) \cos \phi_3 \cos \phi_4 \sin \phi_1 \sin \phi_2 \\
+ \cos(\theta_2 + \theta_4 - \chi_1 + \chi_3) \cos \phi_2 \cos \phi_4 \sin \phi_1 \sin \phi_3 \\
+ \cos(\theta_1 - \theta_4 + \chi_2 - \chi_3) \cos \phi_1 \cos \phi_4 \sin \phi_2 \sin \phi_3 \\
+ \cos(\theta_2 - \theta_3 - \chi_1 + \chi_4) \cos \phi_2 \cos \phi_3 \sin \phi_1 \sin \phi_4 \\
+ \cos(\theta_1 + \theta_3 + \chi_2 - \chi_4) \cos \phi_1 \cos \phi_3 \sin \phi_2 \sin \phi_4 \\
- \cos(\theta_1 + \theta_2 + \chi_3 - \chi_4) \cos \phi_1 \cos \phi_2 \sin \phi_3 \sin \phi_4, \\
S^i = \cos \chi_P \sin \phi_1 \sin \phi_2 \sin \phi_3 \sin \phi_4;
\]

(7)

here we have set:

\[ \theta_P = \theta_1 + \theta_2 - \theta_3 - \theta_4, \]  

(8)

\[ \chi_P = \chi_1 - \chi_2 + \chi_3 - \chi_4, \]  

(9)

and the subscripts \( 1, \ldots, 4 \) correspond to the links of the plaquette: \( 1 \rightarrow \{x, x + \hat{\mu}\}, \ldots, 4 \rightarrow \{x, x + \hat{\nu}\} \). Note that \( S^a \) is proportional to the Wilson plaquette action of compact electrodynamics for the “gauge” field \( \theta \); the corresponding action \( S^i \) for the “matter” field \( \chi \) contains the unusual combination \( \chi_P \), which is invariant under the gauge transformations \( \chi \). Action \( S^n \) describes the interaction of the fields \( \theta \) and \( \chi \).

Due to condition (8), in the MaA projection the angle \( \phi \) fluctuates near zero, and we can expect that the largest contribution to the total action (8) comes from \( S^a \), and that \( S^a > S^n > S^i \). This conjecture is confirmed by numerical calculations. We use the standard heat bath method to simulate \( SU(2) \) gluodynamics on the \( 10^4 \) lattice, we study 15 values of \( \beta, 0.1 \leq \beta \leq 3.5 \); at each value of \( \beta \) we used 15 field configurations separated by 100 of heat bath sweeps. To obtain the MaA projection, we performed 800 gauge fixing sweeps through the lattice for each field configuration. It occurs that \( < S^a > \) is close to the total action, the maximal difference between \( < S_P > \) and \( < S^i > \) is at \( \beta \approx 2.2 \), where \( < S^a > \approx 0.82 < S_P >; \) \( S^i \) is unexpectedly small: \( < S^i > \approx -0.001 \pm 0.0004 \) at \( \beta = 2.2 \), at other values of \( \beta \) the absolute value of \( < S^i > \) is even smaller. It is clear that if we neglect the fluctuations of the angle \( \phi \), as well as the Faddeev-Popov determinant, the \( SU(2) \) action in the maximal abelian gauge is well approximated by the \( U(1) \) action: \( S_P \approx \cos \theta_P \), with the renormalized constant \( \tilde{\beta} = \beta \cos^4 \phi \).

Since in the compact electrodynamics the confinement is due to the monopole condensation, it is not surprising that in numerical experiments the vacuum of gluodynamics behaves
in the MaA projection as the dual superconductor. Of course, this is only an intuitive argument. The confinement in the $U(1)$ theory exists in the strong coupling region, in which the rotational invariance is absent. Therefore, in order to explain the confinement at large values of $\beta$ in $SU(2)$ gluodynamics, we have to study in detail some special features of the gauge fixing procedure (such as the Faddeev-Popov–determinant, fluctuations of the angle $\phi$, etc.).

The fact that $< S^a >$ is close to $< S_P >$ is very interesting; it means that in the MaA projection there is a small parameter in the $SU(2)$ lattice gluodynamics, which is $\varepsilon = \frac{< S_P > - < S^a >}{< S_P >}$; at all values of $\beta$, we have $\varepsilon \leq 0.18$. The meaning of this parameter is simple: it is the natural measure of closeness between the diagonal matrices and the link matrices after the gauge projection. Therefore the lattice $SU(2)$ gauge theory in MaA projection is very close to an abelian theory. In Appendix we show how the abelian theory with an arbitrary action can be represented as a string theory. The strings carry the electric flux, and confine electric charges.

3. $SU(2)$ Gluodynamics in the Minimal Abelian Projection

The minimal abelian (MiA) projection [13] is defined similarly to the MaA projection [1] by

$$\min_{\{\Omega_x\}} R(U') ,$$

where $R(U')$ is defined by [2]. In this projection the largest part of the plaquette action (1) is $S^i$, and the term which is most important for the dynamics is $\cos \chi_P$ (rather than $\cos \theta_P$ as it is in the MaA projection). The fields in the MiA projection can be transformed into the fields in the MaA projection by the following gauge transformation:

$$\Omega(x) = -i\sigma_2 \cdot \frac{(-1)^{x_1+x_2+x_3+x_4} + 1}{2} + \mathbb{1} \cdot \frac{(-1)^{x_1+x_2+x_3+x_4} - 1}{2}.$$  \hspace{1cm} (11)

Thus $\Omega(x)$ is equal to the unity in the “odd” sites of the lattice, and to $-i\sigma_2$ in the “even” sites; this gauge transformation becomes singular in the continuum limit. The angles $\phi$, $\theta$ and $\chi$, which parametrize the link matrix $U_l$, transform under this gauge transformation in the following way. If the link starts at an even point, $((-1)^{x_1+x_2+x_3+x_4} = 1)$, then $U_l \to (-i\sigma_2)U_l$ and

$$\phi \to \frac{\pi}{2} - \phi, \ \theta \to -\chi, \ \chi \to (\pi - \theta) \mod 2\pi.$$  \hspace{1cm} (12)

If the link starts at an odd point, then $U_l \to U_l(-i\sigma_2)^+$ and

$$\phi \to \frac{\pi}{2} - \phi, \ \theta \to (\pi + \chi) \mod 2\pi, \ \chi \to \theta.$$  \hspace{1cm} (13)

Since $Tr(U'_{x\mu}\sigma_3 U'_{x\mu}\sigma_3) = \cos 2\phi'$, it follows that under this gauge transformation

$$Tr(U'_{x\mu}\sigma_3 U'_{x\mu}\sigma_3) \to -Tr(U'_{x\mu}\sigma_3 U'_{x\mu}\sigma_3),$$

since
and the fields in the MaA projection are transformed into fields in the MiA projection (and vice versa). Moreover, the monopoles extracted from the field $\theta$ in the MaA projection turn, in the MiA projection, into some topological defects constructed from the “matter” fields $\chi$. We call these topological defects “minopoles”.

Minopoles can be extracted from a given configuration of gauge fields similarly to monopoles: from the angles $\chi$ we construct gauge invariant plaquette variables $\chi_{\bar{P}} = \bar{d}\chi \mod 2\pi$, where $\bar{d}\chi$ is defined by (9). From these plaquette variables we construct the variables attached to the elementary cubes $^*j = \frac{1}{2\pi}\bar{d}\chi_{\bar{P}}$; for $^*j \neq 0$ the link dual to the cube carries the minopole current. We use the notation $\bar{d}$ (instead of $d$), since the gauge transformations of $\chi$ given by (5) differ from the gauge transformations of $\theta$ given by (4), and the construction of the plaquette variable from the link variables and that of the cube variable from the plaquette variables differ in an obvious way from the standard construction. For example, $d\theta$ is defined by (8) and $\bar{d}\chi$ is defined by (9). In Fig.1 we illustrate the standard construction of the monopoles from the field $\theta$, and the construction of the minopoles from the field $\chi$.

Since monopoles, which exist in the MaA projection become minopoles in the MiA projection, than if in the MaA projection the confinement phenomenon is due to condensation of monopoles (constructed from the field $\theta$), then in the MiA projection the confinement is due to other topological objects (minopoles), constructed from the “matter” field $\chi$. We thus conclude that in the MiA projection the confinement is not due to monopoles and the vacuum is not an analogue of the dual superconductor. It should be stressed that monopoles still exist in the MiA projection; they can be extracted from the fields $\theta$ in the usual way, but they are not at all related to the dynamics. To illustrate this simple fact we plot in Fig.2 the space–time asymmetry of the monopole currents \(^a\) for the $SU(2)$ gauge theory on the $10^3 \times 4$ lattice for the MiA projection. In the same figure we also show the asymmetry of the minopole currents in the MiA projection. It is well known that the temperature phase transition is at $\beta \approx 2.3$ for the $10^3 \times 4$ lattice. It is clearly seen that the asymmetry of the minopole currents is the order parameter for the temperature phase transition, while the asymmetry of the monopole currents is not. Since the monopole currents and the minopole currents are interchanged when the fields are transformed from the MiA to the MaA projection, Fig.2 also shows that for the MaA projection the asymmetry of the monopole currents is the order parameter, whereas the asymmetry of minopole currents is not an order parameter. These results have been obtained by averaging over 10 statistically independent field configurations for each value of $\beta$, and 500–800 of gauge fixing sweeps have been performed for each configuration.

Minopoles are to some extend the lattice artifacts, since the gauge fields in the MaA and the MiA projections are related by the gauge transformation, which becomes singular in the continuum limit. We discuss minopoles, since they clearly illustrate the dependence of the confinement mechanism on the lattice, upon the type of the abelian projection.

\(^a\)The definition of this asymmetry is obvious: $A = <(J_t - J^S)/J_t >$, where $J^S = (J_x + J_y + J_z)/3$, $J_\mu$ is the monopole current in the direction $\mu$. 

5
4. Conclusions

If monopoles are responsible for the confinement in the MaA projection and minopoles are responsible for the confinement in the MiA projection, what are the important topological excitations in a general abelian projection? If both diagonal and nondiagonal gluons are not suppressed, then, in plus to monopoles and to minopoles, string-like topological defects can also be important for the dynamics of the system \cite{15}. The idea is: nondiagonal gluons transform under the $U(1)$ gauge transformations as matter fields, diagonal gluons transform as gauge fields, and an analogue of the Abrikosov–Nielsen–Olesen strings exists in gluodynamics after the abelian projection. Between strings made of condensed nondiagonal gluons (which carry the $U(1)$ charge 2) and the test quark of the charge 1, there exists topological interaction \cite{16, 17}, which is the analogue of the Aharonov–Bohm effect. Thus, in the effective $U(1)$ action of the $SU(2)$ gluodynamics there probably exists a very specific topological interaction. We describe an analytical and numerical study of this interaction in a separate publication.

The topological defects discussed above may be a reflection of some $SU(2)$ gauge field configuration. For example, monopoles and minopoles may be the abelian projection of $SU(2)$ monopoles \cite{18}. In ref. \cite{19} it is found that the “extended monopoles” \cite{11} may be important for the confinement mechanism in different abelian projections of the 3D $SU(2)$ gluodynamics. Finally, we note that it was found recently \cite{20} that the contribution of the Dirac sheets to the abelian Polyakov loops plays the role of the order parameter for finite temperature lattice gluodynamics; it is interesting that this fact holds not only for the MaA projection but also for others unitary gauges. It means that in the considered unitary gauges monopoles are important for the dynamics, other topological excitations (minopoles and strings) may be also important. Note that in the MiA projection monopoles are substituted by minopoles, and in this projection we expect that the contribution of the minopole “Dirac sheets” is correlated with the expectation values of the Polyakov loops.

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Appendix A

First we perform the analogue of the Berezinski–Kosterlitz–Thauless (BKT) transformation \[21\] for an arbitrary $U(1)$ action $S[d\theta], S[\ldots, X_P + 2\pi, \ldots] = S[\ldots, X_P, \ldots]$, where $P$ denotes any plaquette of the original lattice. For the sake of convenience we use the differential forms formalism on the lattice (see Ref.\[22\] for the introduction). We start from the partition function of the compact $U(1)$ theory

$$Z = \int_{-\pi}^{+\pi} D\theta \exp\{-S[d\theta]\}. \quad (14)$$

The expansion of the function $e^{-S(X)}$ in the Fourier series yields:

$$Z = \text{const.} \cdot \int_{-\pi}^{+\pi} D\theta \sum_{n(c_2) \in \mathbb{Z}} F(n) e^{i(n, d\theta)}, \quad (15)$$

where $n$ is an integer–valued two-form,

$$F(n) = \text{const.} \cdot \int_{-\pi}^{+\pi} DX \exp\{-S[X] - i(n, X)\}. \quad (16)$$

Let us insert the unity

$$1 = \int_{-\infty}^{+\infty} DG \delta(G - n) \quad (17)$$

into the sum (15):

$$Z = \int_{-\pi}^{+\pi} DA \exp\{-S_P(dA)\} = \text{const.} \cdot \int_{-\pi}^{+\pi} DA \int_{-\infty}^{+\infty} DG \sum_{n(c_2) \in \mathbb{Z}} \delta(G - n) F(G) e^{i(G, dA)} \quad (18)$$

and use the Poisson summation formula

$$\sum_{n(c_2) \in \mathbb{Z}} \delta(G - n) = \sum_{n(c_2) \in \mathbb{Z}} e^{2\pi i(G,n)} , \quad (19)$$

the result is:

$$Z = \text{const.} \cdot \int_{-\pi}^{+\pi} D\theta \int_{-\infty}^{+\infty} DG \sum_{n(c_2) \in \mathbb{Z}} F(G) e^{i(d\theta + 2\pi n, G)}. \quad (20)$$

Here $G$ is a real–valued two–form.

Now we perform the BKT transformation with respect to the integer–valued 2-form $n$:
\[ n = m[j] + dq, \quad dm[j] = j, \quad dj = 0, \quad (21) \]

where \( q \) and \( j \) are one– and three–forms respectively. First change the summation variable,

\[ \sum_{n(c_2) \in \mathbb{Z}} = \sum_{j(c_3) \in \mathbb{Z}} \sum_{q(c_1) \in \mathbb{Z}} dj = 0. \]

Using the Hodge–de–Rham decomposition we adsorb the d–closed part of the 2–form \( n \) into the compact variable \( \theta \):

\[ d\theta + 2\pi n = d\theta_{n.c.} + 2\pi \delta \Delta^{-1} j, \quad \theta_{n.c.} = \theta + 2\pi \Delta^{-1} dm[j] + 2\pi q. \quad (22) \]

Substituting eq. (22) in eq. (20) and integrating out the noncompact field \( \theta_{n.c.} \) we get the following representation of the partition function:

\[ Z = \text{const.} \cdot \int_{-\infty}^{+\infty} DG \sum_{\ast j(c_1) \in \mathbb{Z}} \delta \ast j = 0 F(G) \exp \{ 2\pi i (G, \delta \Delta^{-1} j) \} \delta(\delta G). \quad (23) \]

Let us consider the lattice with the trivial topology (e.g. \( \mathbb{R}^4 \)). Then the constraint \( \delta G = 0 \) can be solved as \( ^b G = \delta H \) where \( H \) is a real valued 3–form. Substituting this solution into eq. (23) we obtain on the dual lattice the final expression for the BKT–transformed action:

\[ Z = \text{const.} \cdot \sum_{\ast j(c_1) \in \mathbb{Z}} e^{-S_{mon}(\ast j)}. \quad (24) \]

where

\[ S_{mon}(\ast j) = -\ln \left( \int_{-\infty}^{+\infty} DHF(\delta H) \exp \{ 2\pi i (\ast H, \ast j) \} \right), \quad (25) \]

we used the relation \( d\delta \Delta^{-1} j \equiv j, \forall j : dj = 0. \) Therefore for the general \( U(1) \) action \( S[d\theta] \) the monopole action (23) is nonlocal and is expressed through the integral over all lattice \( ( \int_{-\infty}^{+\infty} DH) \). It is well known that for the Villain form of the \( U(1) \) action

\[ \exp \{-S[d\theta]\} = \sum_{n(c_2) \in \mathbb{Z}} \exp \left\{ -\beta \| d\theta + 2\pi n \|^2 \right\}, \quad (26) \]

the monopole action has the simple form: \( S_{mon}(\ast j) = 4\pi^2 \beta (\ast j, \Delta^{-1} \ast j) \).

The partition function (24,25) can be rewritten as:

\[ Z = \text{const.} \cdot \int_{-\infty}^{+\infty} DH \int_{-\pi}^{+\pi} D\theta F(\delta H) \sum_{\ast j(c_1) \in \mathbb{Z}} \exp \left\{ i(d\ast \theta + 2\pi \ast H, \ast j) \right\}, \quad (27) \]

\(^b\)If we consider the space with a nontrivial topology, arbitrary harmonical forms must be added to the r.h.s. of this equation.
where we introduced the compact fields \( \vartheta \) to represent the closeness of the monopole currents \( *j \). The use of the Poisson formula leads to

\[
Z = \text{const} \cdot \int_{-\infty}^{+\infty} DH \int_{-\infty}^{+\infty} DC \int_{-\pi}^{+\pi} D\vartheta F(\delta H) \sum_{*j(c_1) \in \mathbb{Z}} \exp \left\{ i(d^*\vartheta + 2\pi^*H + 2\pi^*j, *C) \right\},
\]

(28)

where the \( \delta \)-functions are represented as the integral over the new field \( C \). Let us perform the BKT transformation with respect to the integer–valued one–form \( *j \). Repeating all the transformations which led to eq.(22) we get

\[
d^*\vartheta + 2\pi^*j = d^*\vartheta_{n.c.} + 2\pi\delta\Delta^{-1}*\sigma, \quad \delta\sigma = 0,
\]

(29)

\( \vartheta_{n.c.} \) is new noncompact field. Substituting eq.(29) in eq.(28) and integrating over the field \( \vartheta_{n.c.} \) we get the constraint on the field \( C \), \( \delta^*C \equiv *dC = 0 \), which can be resolved by the introduction of new noncompact field \( C = dB \). Using the identity \( (\delta\Delta^{-1}*\sigma, *dB) = (\sigma, B) \) (which is valid if \( \delta\sigma = 0 \)) we get:

\[
Z = \text{const} \cdot \int_{-\infty}^{+\infty} DH \int_{-\infty}^{+\infty} DB \sum_{\sigma(c_2) \in \mathbb{Z} \atop \delta\sigma = 0} F(\delta H) \exp \left\{ 2\pi i(H, dB) + 2\pi(\sigma, B) \right\}
\]

\[
= \text{const} \cdot \int DB \sum_{\sigma(c_2) \in \mathbb{Z} \atop \delta\sigma = 0} \exp \left\{ -S_{KR}(B) + 2\pi i(\sigma, B) \right\}
\]

\[
= \text{const} \cdot \sum_{\sigma(c_2) \in \mathbb{Z} \atop \delta\sigma = 0} \exp \left\{ -S_{str}(\sigma) \right\},
\]

(30)

where \( S_{KR}(B) \) is the action for the Kalb–Ramond lattice fields \( B \),

\[
S_{KR}(B) = -\ln \left( \int_{-\infty}^{+\infty} DHF(\delta H) \exp \left\{ 2\pi i(H, dB) \right\} \right),
\]

(31)

the term \( (\sigma, B) \) represents the interaction between the string world sheet \( \sigma \) and the Kalb–Ramond fields \( B \).

The string action results from the integration over the field \( B \):

\[
S_{str}(\sigma) = -\ln \left( \int_{-\infty}^{+\infty} DB \exp \left\{ -S_{KR}(B) + 2\pi i(\sigma, B) \right\} \right).
\]

(32)

It can be shown that the strings \( \sigma \) carry electric fluxes. Therefore the partition function of the compact electrodynamics is reduced to the sum over the electric strings. The string
action is complicated in general case, but if we start with the Villain $U(1)$ action we get the simple expression for the string partition function:

$$Z = \text{const.} \cdot \sum_{\sigma(c_2) \in \mathbb{Z}} \exp \left\{ -\frac{1}{4\beta} ||\sigma||^2 \right\}. \quad (33)$$

**Figure Captions**

Fig.1. Construction of a monopole from the field $\theta$ and minopole from the field $\chi$.

Fig.2. Asymmetry of the monopole currents (circles) and the minopole currents (crosses) in the minimal abelian projection.
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Fig. 1
Fig. 2