Identification of energy storage rate components. Theoretical and experimental approach

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Abstract. The subject of the present paper is decomposition of energy storage rate into terms related to different mode of deformation. The stored energy is the change in internal energy due to plastic deformation after specimen unloading. Hence, this energy describes the state of the cold-worked material. Whereas, the ratio of the stored energy increment to the appropriate increment of plastic work is the measure of energy conversion process. This ratio is called the energy storage rate. Experimental results show that the energy storage rate is dependent on plastic strain. This dependence is influenced by different microscopic deformation mechanisms.

It has been shown that the energy storage rate can be presented as a sum of particular components. Each of them is related to the separate internal microscopic mechanism. Two of the components are identified. One of them is the storage rate of statistically stored dislocation energy related to uniform deformation. Another one is connected with non-uniform deformation at the grain level. It is the storage rate of the long range stresses energy and geometrically necessary dislocation energy. The maximum of energy storage rate, that appeared at initial stage of plastic deformation is discussed in terms of internal micro-stresses.

1. Introduction

When metals deform plastically an energy conversion occurs; a part of the mechanical work $w_p$ done during the single cycle of plastic straining is converted into a heat $q_d$. The rest of it remains in the strained material and it is known as the stored energy $e_s$.

\begin{equation}
e_s = w_p - q_d,
\end{equation}

where $e_s$, $w_p$, and $q_d$ are specific quantities.

The stored energy characterises the cold-worked state of the deformed material and it represents the change in internal energy measured at external stress free state. Thus, the stored energy depends on deformation history. The measure of energy conversion at each instant of the deformation process is the rate of energy storage $de_s/dw_p$. It has been shown experimentally, that at the initial stage of plastic deformation of annealed polycrystalline materials the dependence of the $de_s/dw_p$ on the work $w_p$ expended during plastic deformation has a maximum [1, 2]. Such results can be related to different influence of particular micro-scale deformation mechanisms on the energy stored rate at different deformation stage. Thus $de_s/dw_p$ is macroscopic quantity that depends on number of internal...
parameters $H_1, H_2, \ldots, H_n$. Each of them is related to individual mechanism or deformation mode. Since, there is a coupling between different mechanisms, the function can not be, in general, written in the form: $\sum_{i=1}^{n} e_s^{(i)}(H_i)$. The similar remark concerns also the energy storage rate. The question appears. Is it possible to distinguish the influence of the given internal parameter on the rate of energy storage? In order to find the answer, the theoretical analysis of energy storage rate, on the basis of phenomenological thermodynamics of plastic deformation has been performed. The results of the analysis have been used to decompose the total energy storage rate, obtained for the initial stage of uniaxial tension of austenitic steel, into two components. The results of our previous work [4] and work by Szczepiński [5] were used for such decomposition.

2. Decomposition of the energy storage rate

In the course of loading plastic strain is a monotonic function of time $t$, $e^p(t)$. Hence, the time $t$ can be replaced by the plastic strain $e^p$. In effect, all internal parameters $H$ (independently of their physical meaning) are some functions of $e^p$,

$$H = H\left(e^p\right). \quad (2.1)$$

The difference between incremental plastic work and stored energy can be presented in the form:

$$dw_p - de_s(H) = dw_p(1 - Z), \quad Z = \frac{de_s(H)}{dw_p}, \quad (2.2)$$

where $Z$ is the total energy storage rate. It is quantity describing energy conversion at a given instant of deformation. For infinitesimal increment of stored energy:

$$de_s(H) = \sum_{i=1}^{n} \left(\frac{\partial e_s}{\partial H_i}\right) dH_i. \quad (2.3)$$

Taking into consideration that $H$ is function of $e^p$ and $dw_p = \frac{1}{\rho_0} \tau de^p$, the energy storage rate $Z$, during uniaxial tensile deformation, can be written in the form:

$$Z = \frac{de_s}{dw_p} = \frac{\rho_0}{\tau} \frac{de_s}{de^p} = \frac{\rho_0}{\tau} \sum_{i=1}^{n} \frac{\partial e_s}{\partial H_i} \frac{dH_i}{de^p} = Z_1 + Z_2 + \ldots + Z_n, \quad (2.4)$$

$\tau$ is Kirchhoff stress.

From (2.4) it follows that the rate of energy storage can be presented as a sum of particular components. Each of them describes the contribution of some microscopic mechanism (that is represented by $H_i$) to the energy conversion.

3. Determination of the energy storage rate

The energy storage rate can be obtained differentiating the stored energy $e_s$ as a function of the plastic work $w_p$. The experimental method of stored energy determination, as in the previous works by Oliferuk et al. [2 - 4], was employed. The work $w_p$ was derived from the load-elongation curve. The dissipated energy (equal to $q_d$, see Eq. 1.1) was determined by simulating the process of specimen heating during deformation using a controlled electrical power supply (related to the unit mass) in such a way that the temperature increase over time during the simulation was identical to that measured during tensile testing. When the straining and the simulation are conducted under identical conditions then the heat, which would have been transferred to the surroundings if the temperature of
the unloaded sample has returned to the initial value, is the same in both cases and can be calculated from the measurable electrical quantities. The method gives the same order of magnitude of the stored energy as that obtained by X-ray and calorimetric measurements [6 - 8].

3.1. Components of the energy storage rate

The stored energy is the energy of stress field that remains in the deformed material after unloading. It can be divided into at least two parts: the energy of stress field connected with uniform and non-uniform deformation at the grain level. Thus, the energy storage rate can be also divided into two appropriate terms. The stored energy $E_{st}$ connected with non uniform deformation is a sum of a lattice stretch energy, energy of geometrically necessary dislocations and the energy of long-range internal stresses due to heterogeneous distributions of dislocations in the gauge part of the specimen. In our previous work [4] it has been shown that during uniaxial tension this part of the stored energy can be estimated from experimentally obtained load–displacement curve and it is equal to the $AEB$ area (Figure 1). This energy can be calculated also from stress-strain curve. The specific energy $e_{s1}$ is equal to the product of the $A E'B'$ area and mass density $\rho_0$ of a tested material (Figure 2).

![Figure 1. Generalized load versus generalized displacement curve. $W_e$ is elastic energy.](image)

![Figure 2. Stress-strain curve at the initial stage of tensile deformation of austenitic steel.](image)

From the load-displacement dependence the $E_{st}$ as a function of plastic work can be determined and then $Z_1 = de_{s1} / dw_p$ can be calculated. Therefore, the remaining term $Z_2 = de_{s2} / dw_p$ of the total energy storage rate is related to uniform deformation. $e_{s2}$ is the specific energy of statistically stored dislocations accumulated by their mutual trapping. It can be determined as a difference between experimentally measured total energy storage rate $de_1 / dw_p$ and the energy storage rate $de_{s1} / dw_p$ related to non-uniform deformation.

4. Experimental procedure and results

The experiments were performed on the 304L austenitic stainless steel with mean grain size 7 $\mu$m. All specimens were strained using the MTS testing machine at the constant strain rate $\dot{\varepsilon} = 4.3 \times 10^{-3}$ $s^{-1}$. During the tensile test the temperature distribution on the surface of the specimen was measured by IR camera. Simultaneously, the stress and strain were determined. Figure 2 shows the typical stress-strain curve for the tested steel. On the basis of the thermo-mechanical characteristics, the stored energy as a function of the true strain was determined. It was assumed that before plastic deformation, at annealed state, the energy stored in the specimen is zero. The experimentally determined total stored energy as a function of the plastic work for the tested material is presented in Figure 3. In the same figure the part $e_{s1}$ of the stored energy calculated from the stress-strain curve is shown. Differentiating the total stored energy $e_s$, and the stored energy $e_{s1}$ as a function of plastic work, the total energy storage rate
$Z = \frac{d e_s}{d w_p}$ and the $Z_i = \frac{d e_s}{d w_p}$ are determined. Results of such operation is shown in Figure 4. According to result of the theoretical analysis (see paragraph 3.1) the rate $Z_2$ of energy accumulated in statistically stored dislocations was calculated as the difference between $Z$ and $Z_1$.

![Figure 3](image1.png) ![Figure 4](image2.png)

**Figure 3.** The total stored energy $e_s$ and the energy $e_{s1}$ connected with non-uniform deformation as a function of plastic work at the initial stage of tensile deformation.

**Figure 4.** Decomposition of the total energy storage rate $Z$ into two components $Z_1$ and $Z_2$. $Z = Z_1 + Z_2$.

### 5. Discussion

It is seen that the maximum of the total energy storage rate ($Z$) is mainly determined by the energy storage rate ($Z_1$) related to non-uniform deformation at the grain level (Figure 4). Just after loading, different orientation of grains leads to the incompatible slip, what generates long-range internal stresses and $Z_1$ increases. To ensure compatible deformation of different parts of plastically non-uniform material geometrically necessary dislocations (GND-s) are generated, what reduces the existing long-range internal stresses and creates a new ones, because distribution of GND-s is non-uniform [9, 10]. This means that plastic deformation is a continuous process ensuring compatible strain of different material part until deformed specimen will be broken. Microstructure evolution during plastic deformation tends to formation of low energy dislocation structures (LEDSs) what causes a decline of the energy storage rate [2, 11]. At the initial stage of plastic deformation of the tested steel, $Z_2$ manifests such tendency. But it should be noticed that formation of LEDSs concerns also GND-s.

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