WINTGEN INEQUALITY FOR STATISTICAL SURFACES

MUHITTIN EVREN AYDIN AND ION MIHAI

Abstract. The Wintgen inequality (1979) is a sharp geometric inequality for surfaces in the 4-dimensional Euclidean space involving the Gauss curvature (intrinsic invariant) and the normal curvature and squared mean curvature (extrinsic invariants), respectively. In the present paper we obtain a Wintgen inequality for statistical surfaces.

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REFERENCES

[1] S. AMARI, Differential-Geometrical Methods in Statistics, Springer-Verlag, 1985.
[2] M. E. AYDIN, A. MIHAI, I. MIHAI, Some inequalities on submanifolds in statistical manifolds of constant curvature, Filomat 29, 3 (2015), 465–477.
[3] M. E. AYDIN, A. MIHAI, I. MIHAI, Generalized Wintgen inequality for statistical submanifolds in statistical manifolds of constant curvature, Bull. Math. Sci. 7 (2017), 155–166.
[4] B. Y. CHEN, Geometry of Submanifolds, New York, M. Dekker, 1973.
[5] B. Y. CHEN, On Wintgen ideal surfaces, in: Riemannian Geometry and Applications – Proceedings RIGA 2011 (Eds. A. Mihai and I. Mihai), Ed. Univ. Bucuresti, Bucharest, 2011, pp. 59–74.
[6] B. Y. CHEN, Mean curvature and shape operator of isometric immersions in real-space-forms, Glasg. Math. J. 38 (1996), 87–97.
[7] B. Y. CHEN, Classification of Wintgen ideal surfaces in Euclidean 4-space with equal Gauss and normal curvatures, Ann. Global Anal. Geom. 38 (2010), 145–160.
[8] P. J. DE SMET, F. DILLEN, L. VERSTRAELEN, L. VRANCENK, A pointwise inequality in submanifold theory, Arch. Math. (Brno), 35 (1999), 115–128.
[9] F. DILLEN, K. NOMIZU, L. VRANCENK, Conjugate connections and Radon’s theorem in affine differential geometry, Monatsh. Math. 109 (1990), 221-235.
[10] H. FURUHATA, Hypersurfaces in statistical manifolds, Diff. Geom. Appl. 27 (2009), 420–429.
[11] H. FURUHATA, Statistical hypersurfaces in the space of Hessian curvature zero, Diff. Geom. Appl. 29 (2011), 586–590.
[12] J. GE, Z. TANG, A proof of the DDVV conjecture and its equality case, Pacific J. Math. 237 (2008), 87–95.
[13] I. V. GUADALUPE, L. RODRIGUEZ, Normal curvature of surfaces in space forms, Pacific J. Math. 106 (1983), 95–103.
[14] S. HAESEN, L. VERSTRAELEN, Natural intrinsic geometrical symmetries, SIGMA Symmetry Integrability Geom. Methods Appl. 5 (2009), 15, paper 086.
[15] A.-M. LI, U. SIMON, G. ZHAO, Z. HU, Global Affine Differential Geometry of Hypersurfaces, 2nd revised and extended ed. (English), De Gruyter Expositions in Mathematics 11, Berlin: De Gruyter (ISBN 978-3-11-026667-2/hbk), 410 p. (2015).
[16] R. LOPEZ, Parabolic surfaces in hyperbolic space with constant Gaussian curvature, Bull. Belg. Math. Soc. Simon Stevin 16 (2009), 337–349.
[17] Z. LU, Normal scalar curvature conjecture and its applications, J. Funct. Anal. 261 (2011), 1284–1308.
[18] A. MIHAI, An inequality for totally real surfaces in complex space forms, Kragujevac J. Math. 26 (2004) 83–88.
[19] A. Mihai, Geometric inequalities for purely real submanifolds in complex space forms, Results Math. 55 (2009), 457–468.

[20] I. Mihai, On the generalized Wintgen inequality for Lagrangian submanifolds in complex space forms, Nonlinear Analysis 95 (2014), 714–720.

[21] I. Mihai, On the generalized Wintgen inequality for Legendrian submanifolds in Sasakian space forms, Tohoku J. Math. 69 (2017), 43–53.

[22] C. R. Min, S. O. Choe, Y. H. An, Statistical immersions between statistical manifolds of constant curvature, Glob. J. Adv. Res. Class. Mod. Geom. 3 (2014), 66–75.

[23] K. Nomizu, T. Sasaki, Affine Differential Geometry, Cambridge University Press, 1994.

[24] B. Opozda, A sectional curvature for statistical structures, Linear Algebra Appl. 497 (2016), 134–161.

[25] B. Opozda, Bochner’s technique for statistical structures, Ann. Global Anal. Geom. 48 (2015), 357–395.

[26] B. Rouxel, Sur une famille des A-surfaces d’un espace Euclidien $E^4$, Österreichischer Mathematiker Kongress, Innsbruck, 1981, p. 185.

[27] U. Simon, Affine Differential Geometry, in Handbook of Differential Geometry (Eds. F. Dillen and L. Verstraelen), Vol. I, 905–961, North-Holland, Amsterdam, 2000.

[28] K. Uohashi, A. Ohara and T. Fujii, 1-conformally flat statistical submanifolds, Osaka J. Math. 37 (2000), 501–507.

[29] P. W. Vos, Fundamental equations for statistical submanifolds with applications to the Bartlett connection, Ann. Inst. Statist. Math. 41, 3 (1989), 429–450.

[30] P. Wintgen, Sur l’inégalité de Chen-Wilmore, C. R. Acad. Sci. Paris Sér. A–B, 288 (1979), A993–A995.