A Universal Form of Power Law Relationships for River and Stream Channels

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Abstract. The description of the geomorphic characteristics in power law forms has been the subject of research, over the past 70 years, and has become the cornerstone of regime theory. However, just why power functions should represent such geomorphic relationships remains poorly understood. Hence, differences in the values of the regime exponents observed for different river systems remain largely unexplained. To address this issue, we derived generic forms of the power law relationships without postulating any power functions of the discharge. The theoretical approach accurately captures the systematic variations of the regime exponents shown by a number of large data sets from previous research. We also explain how frictional resistance is responsible for the systematic variability of regime exponents. Overall, our study provides a robust mechanism to describe the variations of the exponents, along with a deductive explanation of the power laws at the core of fluvial hydraulic geometry.

1. Introduction

The evolution of rivers changes surficial landforms and hence influences the life, economy, and well-being of a large proportion of the human population. One of the crucial questions is to understand what constitutes a stable channel shape and how this might change, as flow conditions change. Not surprisingly, the ideal channel shape has been extensively studied and is the basis of so-called “regime theory.” Typically, power law relationships have been used to describe the hydraulic geometry of rivers, since they were first proposed by Leopold and Maddock (1953). On the basis of field measurements, empirical relationships have been proposed:

\[
\begin{align*}
B & \propto Q^b \\
H & \propto Q^f \\
U & \propto Q^m
\end{align*}
\]  

(1)

where \(B\), the bankfull width; \(H\), the mean or hydraulic depth; and \(U\), the mean velocity, are related to the discharge, \(Q\) through power law relationships with constant exponents \(b\), \(f\), and \(m\) (\(b + f + m = 1\) given \(Q = BHU\)). Such descriptions of the hydraulic geometry without considering temporal discharge variability and bars have been the subject of extensive investigation (Andreadis et al., 2013; Gleason, 2015; Millar, 2005; Parker et al., 2007). Efforts have been made to theoretically derive these power functions and provide a deductive explanation of their existence (Cao & Knight, 1996; Finnegans et al., 2005; Huang et al., 2002; Julien & Wargadalam, 1995; Langbein, 1963; Savenije, 2003). However, it was usually found that the available hydraulic equations yielded more unknowns than equations and hence were insufficient for a solution (Gleason, 2015; Leopold & Langbein, 1962; Savenije, 2003). This put the focus of many theoretical studies on seeking rational constraints to make the problem mathematically determinate. For instance, a rectangular or trapezoidal cross-sectional shape was often assumed, and the variables, \(B\), \(H\), and \(U\), were usually directly substituted in power law forms into the hydraulic equations (Cao & Knight, 1996; Huang et al., 2002; Langbein, 1963). Although these approaches did lead to solutions of the exponents in the power laws, it remained unexplained why power functions should necessarily represent the relationships between such variables and the discharge (Richards, 1973). Overall, as pointed out...
by Gleason (2015), after almost 70 years of research, “The utility of power laws is unquestioned, but its elevation from empirically observed relationship to physical principle is not complete, despite universal acceptance of its existence. It may be that such a transition is never made, in which case future research of hydraulic geometry will focus almost exclusively on its applications.”

Another problem related to the lack of a deductive/theoretical framework is that observational data indicate pronounced variability in the value of the exponents that define a regime channel (Balister et al., 2018; Knighton, 1974; Park, 1977; Phillips & Harlin, 1984; Rinaldo et al., 2006, 2014). Figure 1a shows a large data set of regime exponents representing the downstream hydraulic geometry, including the fitted values from over 150 natural stream channels (Park, 1977; Rhodes, 1987) and those proposed by previous researchers (Ackers, 1964; Blench, 1957; Cao & Knight, 1996; Chang, 1992; Fahnstock, 1963; Glover & Florey, 1951; Hey & Thorne, 1986; Kellerhals, 1967; Lacey, 1930; Lapturev, 1969; Nixon et al., 1959; Savenije, 2003; Sherwood & Huitger, 2005; Simons et al., 1960; Stevens, 1989; Williams, 1978; Wolman & Brush, 1961; Xu, 2004), which do not show any clear trend in the ternary diagram (see Table 1 for the specific values).

Researchers have attempted to eliminate these differences by proposing averaged exponents (Cao & Knight, 1996; Finnegan et al., 2005; Huang et al., 2002; Langbein, 1963; Savenije, 2003) or by nondimensionalizing the variables (Francalanci et al., 2020; Millar, 2005; Parker et al., 2007). However, a physical explanation addressing the variability in the exponents for different river systems is still lacking. In fact, it remains ambiguous whether these variations were due to observational errors or physical differences. The consequence is the inevitable limitation of induction, whereby any number of observational data sets cannot theoretically prove or explain the existence of power law relationships (Gleason, 2015; Park, 1977; Savenije, 2003).

In this study, we draw inspiration from the works of Leopold et al. (1964), Richards (1973), and Ferguson (1986), who observed the influence of frictional resistance on the regime exponents. The regime relationships, Equations 1, imply a power law-formed frictional closure characterized by the ratio \( m/f \) (as, e.g., Park, 1977; this ratio is denoted by \( q \) hereafter). The variations of the exponents \( b, f, \) and \( m \) with the ratio \( q \) (Figure 1b) show more systematic distribution than the ternary diagram (Figures 1a). The second-order polynomial regression lines clearly exhibit a decrease in \( b \), a relatively fast decrease in \( f \), and an increase in \( m \) with the ratio \( m/f \).

Based on this observation, we hypothesize that such systematic variability in the value of the exponents largely depends on the variations in frictional resistance between different river systems. An idealized stream channel cross section is developed attempting to derive the explicit functional form of the regime

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**Figure 1.** (a) Ternary diagram of the fitted exponents \( b, f, \) and \( m \) collected from over 150 downstream geometry of natural stream channels by Park (1977) (black circles) and Rhodes (1987) (blue triangles). The 34 red squares show the power law relationships collated from previous research (see Table 1). (b) Variations of the regime exponents shown in panel (a) with the ratio \( m/f \). The lines indicate the polynomial regressions of the scatters. The regime exponents systematically indicate a decrease in \( b \), a relatively fast decrease in \( f \), and an increase in \( m \) with the ratio \( m/f \).
exponents $b$, $f$, and $m$ without previously postulating them as power functions of the discharge. The hypothesis is then investigated by comparative analysis of theoretical and observational exponents of the power functions. By addressing these points, we are able to provide a theoretical foundation for regime relationships of fluvial stream channels.

## 2. Methods

The hydrodynamics of the idealized stream channel cross section is governed by a general momentum balance in a Cartesian coordinate system:

$$\rho ghS + \tau = \frac{\partial}{\partial y} \int h \tau_{yx} \, dz = 0,$$

where $x$ and $y$ are the longitudinal and the lateral coordinates (m), $\tau$ is the bed shear stress ($\text{kg/m/s}^2$), $\rho$ is the water density ($\text{kg/m}^3$), $g$ is the gravity ($\text{m/s}^2$), $h$ is the water depth (m), $\tau_{yx}$ is the turbulent shear stress.
(kg/ms²), and S is the dimensionless longitudinal water surface slope, which is assumed to be constant in the lateral direction. The frictional resistance is described using the Manning-Strickler formulation:

\[ u = \frac{u^* h}{n \sqrt{g}} \]  

(3)

where \( u \) is the local depth-averaged velocity (m/s), \( u^* = \sqrt{\tau/\rho} \) is the friction velocity (m/s), and \( n \) and \( q \) are constant parameters. Based on the variability of \( m \) and \( f \) indicated in the field data, we assume that \( q \) can take any value between 0 and 1 (i.e., \( f > m \)). Notice that resistance equations in logarithmic form (e.g., Colebrook-White formula) are not considered, since they are not conducive to analytical solution. On the other hand, the power law-formed frictional closure does not determine the functional form of the regime relationships a priori. The lateral momentum exchange is approximated using an eddy viscosity closure:

\[ \int_0^h \tau_y d z = \rho \nu_e \partial u / \partial y \]  

(4)

where \( \nu_e \) is the depth-averaged eddy viscosity coefficient (m²/s) expressed as \( \nu_e = \Lambda u^* \) (Fischer, 1973), where \( \Lambda \) is a dimensionless coefficient. The momentum balance is valid for wide and gently curved cross sections, which is common in fluvial circumstance.

The morphodynamic evolution of the idealized stream channel cross section is determined by a constant threshold bed shear stress \( \tau_e \) for simplicity. Values of local shear stress \( \tau \) that are larger and smaller than this threshold lead to local erosion and deposition, respectively:

\[ \frac{\partial h}{\partial t} = Q_{e0} \left( \frac{\tau}{\tau_e} - 1 \right) \]  

(5)

where \( Q_{e0} \) is a reference erosion/deposition rate (m/s). This implies that the morphodynamic equilibrium is characterized by the threshold with bed shear stress evenly distributed. The decrease of the threshold shear stress at the bank region resulting from the lateral bed slope is neglected, since we focus on an ideal equilibrium state associated to the energy dissipation of the stream flow (as, e.g., Cao & Knight, 1996; Huang et al., 2002; Langbein, 1963). The constant threshold value can be considered as the lowest-order approximation (in the sense of a Taylor series) of the observed uneven distribution (Parker, 1978). The morphodynamic evolution governed by Equations 2-5 allows the stream channel cross section to self-organize into its equilibrium state. This process can be simulated numerically considering a constant discharge (as, e.g., Pizzuto, 1990; Xu et al., 2019).

3. Results
3.1. Equilibrium Channel Profile

Figure 2 shows the simulated morphodynamic evolution of the channel profile as well as the redistribution of the bed shear stress \( \tau \) with a with discharge \( Q = 30 \text{ m}^3/\text{s} \), the ratio \( m/f = 1/6 \), the Manning’s coefficient \( n = 0.012 \text{ m}^{1/3} / \text{s} \), the nondimensional eddy viscosity \( \Lambda = 0.2 \) (Shiono & Knight, 1991; Vionnet et al., 2004), critical shear stress \( \tau_c = 0.4 \text{ Pa} \), and the reference erosion/deposition rate \( Q_{e0} = 4 \times 10^{-8} \text{ m/s} \) (as, e.g., Lanzoni & D’Alpaos, 2015). Notice that \( Q_{e0} \) merely affects the rate at which the system converges to the equilibrium state. The numerical simulation starts from a flat bed and perturbed by a small incision in the central area, and the model is run until the profile converges to a channelized equilibrium shape. The equilibrium profile shows a curvilinear channelized shape and the bed shear stresses \( \tau \) over the cross section approach a threshold value \( \tau_e \), such that the lateral gradients in bed level changes vanish. Different initial perturbations
have also been tested, but their details do not affect the equilibrium cross-sectional shape (see Movie S1 in the supporting information). The equilibrium profile is thus considered to be convergent.

The numerical simulation confirms that the equilibrium profile is characterized by an even distribution of the bed shear stresses (Figure 2a). As shown in the momentum balance (Equation 2), the bed shear stress is determined the hydraulic pressure $\rho g S h$ and the turbulent diffusion $\frac{3}{2} \frac{\partial h}{\partial y} \gamma_{\text{rms}} dz$. Since the water surface slope $S$ is assumed to be constant in the lateral direction, the contribution of hydraulic pressure is relatively large in deep water. This gives rise to a positive feedback, causing the stream flow to concentrate in the deeper region and further reinforces the local erosion (Parker, 1978). On the other hand, such asymmetric distribution is compensated by the turbulent diffusion term, which results in a negative feedback enhancing the stream flow and related scour near the channel edge (Xu et al., 2019). The counter-balance between these two feedbacks results in the convergence of the idealized system to the equilibrium state.

Substituting Equations 3 and 4 and the equilibrium condition $\tau = \tau_e$ into the momentum equation 1, one can show that the equilibrium channel profile is governed by a second-order nonlinear differential equation:

$$1 - \frac{h}{h_0} - \frac{A}{\sqrt{g}} \frac{d}{dy} \left( \frac{h^3}{h_0^3} \right) = 0,$$

where $h$ is the local water depth (m) and $h_0 = \tau_e/\rho g S$ is a reference water depth (m). Taking $\mu(h) = (dh/dy)^2$, an implicit solution for Equation 6 can be derived, which governs the equilibrium profile:

$$\mu(h) = -\frac{2\sqrt{g}}{q A h_0^2} \left( \frac{h}{3 + q} - \frac{h_0}{2 + q} \right).$$

The equilibrium profile predicted by Equation 7 is in a good agreement with the numerical simulation (compare the red circles and the black lines in Figure 2). We refer the reader to the supporting information for further comparisons with different parameters.

### 3.2. Power Law Regime Relationships

Although the equilibrium channel profile governed by Equation 7 does not admit an explicit formulation, the functional form of the bankfull width $B$, the cross-sectional area $A$, and the discharge $Q$ can be derived through integration. Taking the bankfull width $B$, for example:

$$B = \int_0^B dy = 2 \int_0^{h_{\text{ax}}} dy dh = 2 \int_0^{h_{\text{ax}}} \frac{1}{\sqrt{\mu(h)}} dh$$

where $h_{\text{ax}}$ is the water depth at the channel axis, where the bed slope is zero. Hence, taking $\mu(h) = 0$ in Equation 7, one can show that $h_{\text{ax}} = (3 + q) h_0/(2 + q)$ (see the black solid dots in Figure 2). Substituting Equation 7 into Equation 8, the functional form of the width can be derived:

$$B = \sqrt{2q \frac{(3 + q)^{\frac{1}{2}} + \frac{q}{2}}{(2 + q)^{\frac{3}{2}}}} \int \frac{A}{\sqrt{g}} \left( 1 + \frac{q}{2} \right) h_0^{1 + \frac{q}{2}},$$

where the Beta function, $\beta$, is constant only depending on the value of $q$. Hence, the resultant power functions are $B \propto h_0^{(2 + q)/2}$, $A \propto h_0^{(4 + q)/2}$, and $Q \propto h_0^{(4 + 3q)/2}$, which leads to a generic and concise functional form of the power law relationships:

$$\begin{cases} B \propto Q^{\frac{1 + q}{2}} \\ H \propto Q^{\frac{3 + q}{2}} \\ U \propto Q^{\frac{4 + 3q}{4}} \end{cases}$$

We refer the reader to the supporting information for a detailed derivation.

The functional form of the power law regime relationships (Equation 10) indicates that the regime exponents are simply determined by the ratio $q$. In particular, imposing $q = 1/6$ in Equation 10 gives
This set of values agrees well with those proposed by Finnegan et al. (2005), who derived that \((b, f, m) \approx (0.47, 0.47, 0.06)\) using the same value of the ratio \(q\). The variation of the exponents \(b, m, f\) with \(q\), as predicted by Equation 10, is further compared with the data sets consisting of the empirical, theoretical, and fitted values collated from previous research (Table 1 and Figure 3). The data and Equation 10 exhibit a decrease in \(b\), a relatively fast decrease in \(f\), and an increase in \(m\); \(b > f > m\) as \(q\) tends to 1; and \(b \approx f\) as \(q\) tends to 0. Moreover, the theoretical predictions (bold lines in Figures 3a and 3b) keep close to the polynomial regressions (thin lines in Figures 3a and 3b) throughout the domain of \(q\) from 0 to 1. The root-mean-square differences of \(b, f, m\) are small, with values of 0.038, 0.029, and 0.028 for Park (1977) and 0.015, 0.021, and 0.01 for Rhodes (1987), respectively. This consistency further supports our hypothesis on the systematic variability of the regime exponents. Notice that only mean annual downstream hydraulic characteristics are used to verify the derived regime relationships, since a stable regime discharge is assumed in the theoretical approach.

The coefficients of the regime relationships can also be derived as shown in Equation 9. However, the coefficients show more complicated functional forms, which are jointly determined by \(q, A\), and \(n\). This coincides with the large diversity in the fitted values of the regime coefficients, which range from \(10^3\) to \(10^5\) (Gleason & Smith, 2014; Rhoads, 1991; Stewardson, 2005). However, this also implies the regime coefficients are inherently more difficult to investigate and more data, often not available in existing data sets, are needed to determine the additional parameters.
4. Discussion

As summarized in section 1, there is a large body of work devoted to the utility and applicability of the regime relationships observed in natural rivers. Equally, there is, as yet, no theoretical basis for these relationships. However, just why power laws provide such a good representation of the hydraulic geometry observed in rivers warrants further investigation. In this study, we employ a generic momentum equation with a term accounting for the energy dissipation due to the turbulent diffusion, which has been usually neglected in regime studies (e.g., Cao & Knight, 1996; Finnegan et al., 2005; Huang et al., 2002; Langbein, 1963). In addition, the lateral bed-slope sediment transport is neglected, and the relevant processes, such as the frictional resistance and the turbulent diffusion, are approximated using simple theoretical-empirical formulations (Equations 3 and 4). Although these simplifications limit the depiction of the specific shape of the channel profile, the improvement is that we can show analytically that a river system, albeit idealized, inherently abides by a power law regime relationship. Moreover, the resultant functional forms (Equation 10) accurately capture the systematic variations of the regime exponents indicated by the measured data sets (Figure 3), which cover different types of bed material. Therefore, the present derivation provides a robust theoretical basis to mechanistically explain the existence of power law-formed regime relationships.

The theoretical approach also helps to explain why the parameter estimates reported in the literature show systematic variations. The ratio q characterizes the frictional resistance (Equation 3) and affects the turbulent diffusion, since the eddy viscosity $\nu_*$ is relevant to the frictional velocity $u_*$ (Fischer, 1973). This means that the implied values of ratio q derived from the reported values (see Table 1) reflect the regime characteristics of the systems. The concise functional forms (Equation 10) provide an alternative approach to the prediction of the regime exponents, which are mutually consistent from a theoretical perspective. However, to determine suitable values of q based on appropriate field data, new approaches will need to be developed. This will require the characterization of roughness based on, for example, sediment type, viscosity, and suspended sediment concentration, so that values of q can be estimated using parameters that are readily observed.

Finally, we emphasize that the present study does not fully resolve the variability of the regime exponents in natural systems. Considerable divergence of the data can still be observed around the theoretical lines even after the transformation with respect to q (Figures 3a and 3b). Besides, there are also some differences between the present derivation and those previously proposed (Figure 3c). For instance, many authors suggested values of $b \approx 0.5$ regardless of the variation of q (see black line in Figure 3c). In contrast, the present derivation suggests that $b$ gently decreases from 0.5 to 0.43, as q increases from 0 to 1 (see black line in Figure 3c). This is not surprising, since those reported exponents (dots in Figure 3c) result from far more complicated natural settings than the ideal configurations used in this study. Some of the variability in the reported exponents is possibly due to uncertainties in field observations and other factors that may affect the cross-sectional shape, such as inhomogeneity of bed material (Leopold et al., 1964), lateral sediment transport (Baar et al., 2019), and bank strength (Eaton & Church, 2007; Eaton & Giles, 2009). The effects of these processes on the functional forms of the regime relationships deserve future studies.

5. Conclusions

To summarize, we derived a universal form of power law relationships and investigated the systematic variations in the regime exponents between different river systems. The so far unexplained distribution of the exponents, $b$, $f$, and $m$ in the ternary diagram is found to be dependent on the frictional resistance through the exponent $m/f$. The data sets collected from the downstream geometry of over 150 rivers and the 34 power law relationships collated from previous research indicate a decrease in $b$, a relatively fast decrease in $f$, an increase in $m$, and $b > f > m$, as $m/f$ increases and $b \approx f$ as $m/f$ tends to 0. The consistency between the theoretical prediction and the data suggests that power law relationships are largely determined by the balance between the pressure gradient and the turbulent diffusion in the stream flow. Overall, this study provides a robust theoretical explanation that brings new insights into the foundation of regime theory.

Data Availability Statement

Data sets for this research are compiled by Park (1977) and Rhodes (1987). The code developed in this study is available online (through the https://doi.org/10.5281/zenodo.3825909).
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