Temperature dependence of transport coefficients of QCD in high-energy heavy-ion collisions

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Using our developed new relativistic viscous hydrodynamics code, we investigate the temperature dependence of shear and bulk viscosities from comparison with the ALICE data; single particle spectra and collective flows at Pb+Pb $\sqrt{s_{NN}} = 2.76$ TeV collisions at the Large Hadron Collider. We find that from the comprehensive analyses of centrality dependence of single particle spectra and collective flows, we can extract the detailed information of the QGP bulk property, without being smeared by the final state interactions.

I. INTRODUCTION

Since the success of production of the strongly interacting quark-gluon plasma (QGP) at Relativistic Heavy Ion Collider (RHIC) \cite{1}, a relativistic viscous hydrodynamic model has been widely used for the description of space-time evolution of the hot and dense matter created after collisions. Now at RHIC as well as at the Large Hadron Collider (LHC) high-energy heavy-ion collisions are performed and many experimental data are reported. Because the relativistic viscous hydrodynamic equation has close relation to an equation of state (EoS) and transport coefficients of the QCD matter, analyses of experimental data at RHIC and the LHC based on relativistic viscous hydrodynamic model can provide an insight into the detailed information of QGP bulk property.

Recent development of lattice QCD calculation for EoS at vanishing chemical potential is remarkable. Two groups Wuppertal-Budapest and hotQCD Collaborations report almost the same (pseudo-)critical temperature, $T_c = 155 \pm 6$ MeV \cite{3} and $T_c = 154 \pm 9$ MeV \cite{4}, respectively. On the other hand, the evaluation of the shear viscosity to entropy density ratio $\eta/s$ of the hadronic phase and the QGP phase is investigated based on the Boltzmann equation \cite{5,6}. Due to existence of the KSS bound, the lower bound of $\eta/s$ \cite{6}, $\eta/s$ takes the minimum around the critical temperature \cite{7}. The bulk viscosity to entropy density ratio $\zeta/s$ has a peak around the critical temperature from the sum rule analysis with the lattice QCD EoS \cite{8,9}. However, there is not conclusive understanding for quantitative information of the transport coefficients of the QCD matter.

Therefore phenomenological analyses of transport coefficients from comparison with experimental data at RHIC and the LHC are indispensable \cite{10,17}. It turns out that the value of $\eta/s$ at the LHC is larger than that at RHIC, which suggests that temperature dependent $\eta/s$. At RHIC elliptic flow $v_2$ is sensitive to the $\eta/s$ of the hadronic phase, whereas at the LHC it depends on $\eta/s$ both of the hadronic phase and the QGP phase \cite{15,21}. Simultaneous analyses of $v_2$ at RHIC and the LHC give constraint to temperature dependence of $\eta/s$ \cite{15,21}. Even in one collision energy, we can explore the temperature dependence of shear and bulk viscosities from centrality and/or rapidity dependence of observables. At peripheral collisions the effect of $\eta/s$ of the hadronic phase is dominant, compared with that of the QGP phase \cite{19,21}. At the forward rapidity where the temperature becomes small, behavior of $\eta/s$ in the hadronic phase affects the elliptic flow \cite{20,22}.

Now not only the shear viscosity but also the bulk viscosity are included in relativistic viscous hydrodynamic simulation \cite{23,33}. Generally bulk viscosity reduces the growth of radial flow in hydrodynamic expansion. In the computation with IP-Glasma initial condition, finite bulk viscosity is important for explanation of experimental data, for example, mean $p_T$ \cite{24,31,33}. However the evaluation of effect of bulk viscosity in calculation of particle distribution in the Cooper-Frye formula is not fixed yet. Furthermore from the application of Bayesian analyses to model-to-data comparison the temperature dependence of shear and bulk viscosities are investigated \cite{32}. The results support that $\eta/s$ takes the minimum value around the critical temperature and increases with temperature in the QGP phase and $\zeta/s$ takes the maximum value around the critical temperature.

Furthermore, to achieve the quantitative analyses of the transport coefficients of the QCD matter from comparison with high statistics and high precision experimental data, we need to perform numerical calculations for relativistic viscous hydrodynamics with high accuracy. We have developed a new relativistic viscous hydrodynamics code optimized in the Milne coordinates \cite{34}. The code is constructed based on a Riemann solver with the lattice QCD EoS \cite{8,9}. Using our new developed hydrodynamics code we investigate the temperature dependence of shear and bulk viscosities from comprehensive analyses of centrality and rapidity dependence of particle distributions and higher flow harmonics at the LHC.

This paper is organized as follows. We begin in Section

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by showing the relativistic viscous hydrodynamic equations and numerical algorithm for solving them briefly. In Section [II] we explain the phenomenological model; initial conditions, equation of states used in the relativistic viscous hydrodynamic equations, freezeout process, and final state interactions based on UrQMD. We discuss the temperature dependence of shear and bulk viscosities in Section [IV]. In Section [V] we show our numerical results of particle distributions and collective flows at the LHC. We end in Section [VI] with our conclusions.

II. RELATIVISTIC VISCOS HYDRODYNAMIC EQUATION AND ALGORITHM

In a hydrodynamic model, we numerically solve the relativistic viscous hydrodynamic equation which is based on the conservation equations, \( T^\mu{}_{\nu} = 0 \), where \( T^\mu{}_{\nu} \) is the energy-momentum tensor. In the Landau frame, the energy-momentum tensor of the viscous fluid is decomposed as \( T^\mu{}_{\nu} = eu^\mu u^\nu - (p + \Pi)\Delta^\mu{}^\nu + \pi^{\mu\nu} \), where \( \Pi \) is the bulk pressure and \( \pi^{\mu\nu} \) is the shear tensor [38]. The relativistic extension of Navier-Stokes theory in non-relativistic fluid usually is not an easy task because of a problem of acausality and instability [39–41]. The problem can be resolved by introducing the second-order terms of the viscous tensor and the derivative of fluid equations [42, 43]. Here we use the relativistic viscous hydrodynamic equation derived from the Boltzmann equation based on the method of moments [48–50]. The relaxation equations for the bulk viscous pressure \( \Pi \) and the shear-stress tensor \( \pi^{\mu\nu} \) read

\[
\tau_{\Pi}\Pi + \Pi = -\zeta \theta - \delta_{HH}\Pi\Pi + \lambda_{\Pi\Pi}\pi^{\mu\nu}\sigma_{\mu\nu},
\]

\[
\tau_{\pi}\pi^{\mu\nu} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \varphi_{\gamma\gamma}(\pi^{\mu\nu})^{\alpha} - \tau_{\pi\pi}\pi^{(\mu}(\sigma^{\nu)\alpha} + \lambda_{\Pi\Pi}\Pi\sigma^{\mu\nu},
\]

where \( \tau_{\Pi} \) and \( \tau_{\pi} \) are the relaxation times and \( \delta_{HH} \), \( \lambda_{\Pi\Pi} \), \( \delta_{\pi\pi} \), \( \varphi_{\gamma\gamma} \), \( \tau_{\pi\pi} \), and \( \lambda_{\Pi\Pi} \) are the transport coefficients. To analyze high-energy heavy-ion collisions where the strong longitudinal expansion exists we perform numerical computation in the Milne coordinates [51]. For details, see Refs. [33–50].

In our algorithm [34], we split the conservation equation into two parts, an ideal part and a viscous part using the Strang splitting method [51]. It is also applied to evaluate the constitutive equations of the viscous tensors Eqs. (1) and (2). We decompose them into the following three parts, the convection equations, the relaxation equations, and the equations with source terms. In numerical simulation of relativistic hydrodynamic equation, a time-step size \( \Delta\tau \) is usually determined by the Courant-Friedrichs-Lewy (CFL) condition. However in the relativistic dissipative hydrodynamics, one needs to determine the value of \( \Delta\tau \) carefully. To save computational cost, we use the Piecewise Exact Solution (PES) method [35–37], in stead of using a simple explicit scheme. If, however, the relaxation times are larger than \( \Delta\tau \) determined by the CFL condition, the PES method is not applied. We have checked the energy and momentum conservation in one-dimensional expansion of high-energy heavy ion collisions [55] and the correctness of our code in the following test problems; the viscous Bjorken flow for one-dimensional expansion and the Israel-Stewart theory in Gubser flow regime for the three-dimensional calculation [34].

III. MODEL

For an initial condition of our hydrodynamic model, we use the TRENTo [52–53]. In the parametric model TRENTo, the initial entropy density \( s(x, \eta) \) is given by a function on the transverse plane at midrapidity \( f(x) \) and a rapidity-dependent function \( g(x, \eta) \): \( s(x, \eta) \propto f(x) \times g(x, \eta) \). The function \( f(x) \) is given by

\[
f(x) \propto \left( \frac{T_A^p + T_B^p}{2} \right)^{1/p},
\]

where \( T_A \) is the nucleus thickness function expressed by proton thickness function \( T_p \),

\[
T_A(x) = \sum_{i=1}^{N_{\text{part}}} w_i T_p(x - x_i),
\]

\[
T_p(x) = \frac{1}{2\pi w^2} \exp \left( -\frac{x^2}{2w^2} \right),
\]

\( w_i \) is random weight for introduction of negative binomial distribution of produced particles \( p \) is Gaussian nucleon width. A parameter \( p \) in the function \( f(x) \) can interpolate among different types of entropy schemes such as wounded nucleon model \( p = 1 \), KLN model \( p \sim -0.67 \), IP-Glasma and EKRT \( p \sim 0 \). Throughout our calculations we fix the parameter \( p \) to \( p = 0 \), which is suggested by the Bayesian analyses [53]. The initial distribution in the rapidity direction is described by \( g(x, \eta) = g(x, y) dy/d\eta \), where \( g(x, y) \) is constructed by the inverse Fourier transform of its cumulant-generating function

\[
g(x, y) = \mathcal{F}^{-1} \{ g(x, k) \} = \mathcal{F}^{-1} \left\{ \exp \left( i\mu k - \frac{1}{2}\sigma^2 k^2 - \frac{1}{6}\gamma^2\sigma^3 k^3 \right) \right\}.
\]

The first three cumulants \( \mu, \sigma \), and \( \gamma \) of the local rapidity distribution are parametrized by three corresponding coefficients \( \mu_0, \sigma_0 \), and \( \gamma_0 \) which characterize rapidity distribution’s shift, width, and skewness, respectively. \( dy/d\eta \) is determined by Jacobian parameter \( J \). The initial entropy density contains the following parameters
other than the parameter $p$; normalization $N$, nucleon width $w$, multiplicity fluctuation shape $k$, rapidity distributions’ shift $\mu_0$, width $\sigma_0$, and skewness $\gamma_0$, and pseudorapidity Jacobian parameter $J$. We set initial flows and initial values of viscous tensors to be vanishing at $\tau_0 = 0.6 \text{ fm}$.

We use a realistic parametrized EoS [54] based on continuum-extrapolated lattice QCD results in the physical quark mass limit [55], which is also combined with an hadron resonance gas model [66, 67] at low temperature. In the parametrization, the sound velocity takes the minimum value at $T_c \sim 167 \text{ MeV}$ [54].

At the switching temperature $T_{SW} = 150 \text{ MeV}$ the hydrodynamic expansion terminates and the UrQMD starts for the description of space-time evolution [58, 59]. We sample produced particles from the fluid, using the Cooper-Frye formula [60]

$$E \frac{dN_i}{dp} = \frac{g_i}{2\pi} \int_\Sigma f_i(x, p) \rho^\mu d^3\sigma_\mu,$$  \hspace{1cm} (7)

where $i$ is an index over particle species, $f_i$ is the distribution function, and $d^3\sigma_\mu$ is a volume element of the isothermal hypersurface $\Sigma$ defined by $T_{SW}$. We introduce the shear viscous correction to the distribution function based on Ref. [61]. We neglect the bulk viscosity correction, because the ambiguity in estimate exists [28]. Both bulk viscosity in hydrodynamic expansion and the bulk viscous correction to the distribution function reduce the mean transverse momentum [24, 25, 27, 29-31, 52]. We find the particlization hypothesis is disfavored [28]. Both bulk viscosity in hydrodynamic expansion terminals and the UrQMD [64, 65]. In UrQMD all produced hadrons move along classical trajectories, including their scattering, resonance formations, and decay processes until interactions among them stop.

IV. TEMPERATURE DEPENDENT TRANSPORT COEFFICIENTS

One of the pioneer works of analyses of temperature dependent $\eta/s$ is done in Ref. [18]. More comprehensive analyses are performed in Refs. [20, 21]. In Ref. [22] they show that a temperature independent $\eta/s$ is disfavored from comparison with experimental data at RHIC and event-by-event flow as a function of rapidity is useful for constraint of the temperature dependence of shear viscosity. The first analyses of experimental data with both shear and bulk viscosities of the hadronic phase are carried out in Ref. [26]. Effect of bulk viscosity on higher flow harmonics is also investigated [28, 29]. In addition, inclusion of bulk viscosity is preferable for better description of the data of ultracentral relativistic heavy-ion collisions at the LHC [60].

Here we use the same parametrization as that in

\begin{align}
T(\text{MeV}) & \hspace{1cm} \eta/s \hspace{1cm} \begin{cases}
(d) \eta/s=0.17 \ b=40 \\
(f) \eta/s\text{min}=0.08 \ c_1=10 \ c_2=0.7 \\
(g) \eta/s\text{min}=0.08 \ c_1=0 \ c_2=0.7 \\
(h) \eta/s\text{min}=0.08 \ c_1=10 \ c_2=0
\end{cases} \\
\zeta/s & = b \frac{\eta}{s} \left( \frac{1}{3} - c_s^2 \right)^2,
\end{align}

FIG. 1 (color online). The temperature dependence of shear viscosity to entropy density ratio $\eta/s$ (top) and bulk viscosity to entropy density ratio $\zeta/s$ (bottom).

Refs. [49, 66] for bulk viscosity,

where $c_s$ is the sound velocity and $b$ is a parameter. We parametrize the $\eta/s(T)$ [21, 22],

$$\frac{\eta}{s}(T) = \left( \frac{\eta}{s} \right)_{\text{min}} + c_1(T_c-T)\theta(T_c-T)+c_2(T-T_c)\theta(T-T_c),$$  \hspace{1cm} (9)

where $c_1$ and $c_2$ are parameters and $T_c = 167 \text{ MeV}$. We compare our calculated results with experimental data, in the case of following parameter sets (Fig. 1); (d) $\eta/s = 0.17, b = 40$, (f) $(\eta/s)\text{min} = 0.08, c_1=10, c_2=0.7, b = 40$, (g) $(\eta/s)\text{min} = 0.08, c_1=0, c_2=0.7, b = 40$, and (h) $(\eta/s)\text{min} = 0.08, c_1=10, c_2=0, b = 40$. The values of $c_1$ and $c_2$ are given in GeV$^{-1}$.

V. NUMERICAL RESULTS

A. Parameters in Computations

In the next subsections we explore transport coefficients of QGP through quantitative analyses of the LHC data such as one particle distributions and collective flows
at Pb+Pb $\sqrt{s_{NN}} = 2.76$ TeV collisions. First in the subsection $V_B$ we shall discuss an appropriate value of constant shear viscosity without including bulk viscosity and then in the subsection $V_C$ we shall argue the effect of bulk viscosity on physical observables. In the subsection $V_D$ we shall investigate the temperature dependence of the shear viscosity. Finally in the subsection $V_E$ we shall discuss the final state interactions to $p_T$ spectra and collective flows. We determine parameters in the initial condition TRENTo, using pseudorapidity distributions of charged hadrons at central collision [67][68]. Our parameters are the same as those in Ref. [63], except for the normalization $N$ and $\sigma_0$.

We fix the centrality based on initial entropy densities. First we produce initial entropy distributions of a certain number of events with minimum bias, using TRENTo. Then we arrange the events in decreasing order of total entropy $dS/\sigma y=0$. For example, for the centrality 0-10 % we pick up the largest 10 % events from entire events. To save computational time, we perform numerical calculation only for our focusing centralities. For the following analysis of experimental data, we prepare 2000 minimum bias events using TRENTo.

In the top panel of Fig. 2 we show pseudorapidity distributions of charged particles in 0-5 %, 10-20 %, 30-40 %, and 50-60 % centralities in the case of $\eta/s = 0.08$, 0.17, and 0.24, neglecting the bulk viscosity. The middle panel shows the computational results with finite bulk viscosities and $\eta/s = 0.17$. In the bottom panel we include the temperature dependence of $\eta/s$. For all the cases we reproduce centrality dependence of pseudorapidity distributions of charged particles very well (Fig. 2). In the Table I, we list the values of $N$ and $\sigma_0$ which are used in the subsections $V_B$ (constant shear viscosity: (a) $\eta/s = 0.08$, (b) $\eta/s = 0.17$, and (c) $\eta/s = 0.24$), $V_C$ (finite bulk viscosity: (d) $b=40$ and (e) $b=60$), and $V_D$ (temperature dependent shear viscosity: (f) $c_1 = 10$, $c_2 = 0.7$, (g) $c_1 = 0$, $c_2 = 0.7$, and (h) $c_1 = 10$, $c_2 = 0$). In the case of (d) and (e), we need to choose smaller $\sigma_0$ because finite bulk viscosity increases the width of pseudorapidity distributions. We carry out the numerical computation with spatial grid sizes $\Delta x = \Delta y = 0.2$ fm, $\Delta y = 0.3$ and time step $\Delta \tau = 0.5 \Delta x$ fm.

| $N$ | 116 | 110 | 105 |
| $\sigma_0$ | 2.9 | 2.9 | 2.9 |
| $\eta/s$ | 17 | 0.8 | 0.08 |

**TABLE I.** The values of normalization $N$ and $\sigma_0$ in the TRENTo.

B. Constant shear viscosity with vanishing bulk viscosity

First we extract a suitable value of the shear viscosity $\eta/s$ from comparison with experimental data of $p_T$ distributions and collective flows $v_2$ and $v_3$, neglecting bulk viscosity. In Fig. 3 the $p_T$ distributions for $\pi^+$, $K^+$, and $p$ are shown, together with the ALICE data [69]. The differences among calculated results of $p_T$ spectra with $\eta/s = 0.08$, 0.17, and 0.24 are very small, which suggests that $p_T$ spectra themselves are not sensitive to the value of $\eta/s$. For all cases we reproduce experimental data reasonably well, though in 0-5 % and 10-20 % centralities we observe the deviation from experimental data above

![Fig. 2](image-url)
$p_T \sim 1.5 \text{ GeV.}$

To understand the effect of the shear viscosity on the $p_T$ distributions, we show the time evolution of the spatial averaged radial flow $v_T = \sqrt{v_x^2 + v_y^2}$ of fluid cells whose temperatures are $T > 150 \text{ MeV}$ at $\eta_s = 0$ in 0-5 % centrality. The values are taken the average over 20 events. The solid line, the dashed line and the dashed-dotted line stand for $\eta/s = 0.17$, $\eta/s = 0.08$, and $\eta/s = 0.08$ with $N = 116$, respectively.

Figure 5 shows the longitudinal flow $v_{\eta_s}$ as a function of $\eta_s$ at $\tau = 7 \text{ fm}$ in 0-5 % centrality. The solid line stands for $\eta/s = 0.17$ and the dashed line stands for $\eta/s = 0.08$. Both of them are computed with the normalization $N = 110$ for 1 event.

FIG. 5 (color online). The longitudinal flow $v_{\eta_s}$ as a function of $\eta_s$ at $\tau = 7 \text{ fm}$ in 0-5 % centrality. The solid line stands for $\eta/s = 0.17$ and the dashed line stands for $\eta/s = 0.08$. Both of them are computed with the normalization $N = 110$ for 1 event.
FIG. 6 (color online). The elliptic and triangular flows of charged hadrons as a function of $p_T$ in 0-5 % and 30-40 % centralities, together with the ALICE data (the open circles) [70]. The orange dashed line, black solid line, and the blue dashed-dotted line stand for $\eta/s = 0.08$, 0.17, and 0.24, respectively.

FIG. 7 (color online). The $p_T$ distributions for $\pi^+$, $K^+$, and $p$ in 0-5 %, 10-20 %, 30-40 %, and 50-60 % centralities, together with the ALICE data (the open circles) [69]. The black dashed line, the green solid line, and the yellow dashed-dotted line stand for vanishing bulk viscosity, $b = 40$, and $b = 60$, respectively.

viscosity.

Figure 6 shows the elliptic flow $v_2$ and triangular flow $v_3$ of charged hadrons as a function of $p_T$ in 0-5 % and 30-40 % centralities. We compute the flow harmonics $v_n$ from the two particle cumulant, using the Q-cumulant method [71]. The same $p_T$ and rapidity cuts as those
of the ALICE data \cite{70} are applied. In contrast to the $p_T$ distribution Fig. 3, there is clear $\eta/s$ dependence in behavior of collective flows. The larger $\eta/s$, the smaller $v_2$ and $v_3$. The existence of shear viscosity suppresses the growth of anisotropy of the flow on the transverse plane.

In the case of constant shear viscosity, we conclude that $\eta/s = 0.17$ is the most suitable value as shear viscosity of the QGP, which is consistent with Ref. \cite{53}.

Here we give a short summary for the constant shear viscosity. The $p_T$ spectra of $\pi^+$, $K^+$, and $p$ are insensitive to the value of $\eta/s$ and in 0-5 % and 10-20 % centralities our computed $p_T$ spectra overestimate above $p_T > 1.5$ GeV, which suggests that the mean $p_T$ is larger. We find the clear $\eta/s$ dependence in $v_2$ and $v_3$, i.e., the larger $\eta/s$, the smaller $v_2$ and $v_3$ of charged hadrons.

C. Effect of bulk viscosity

Next we investigate how the effect of bulk viscosity appears in $p_T$ spectra and collective flows $v_2$ and $v_3$. We introduce the bulk viscosity through Eq. \ref{eq:8}, fixing the value of shear viscosity to $\eta/s = 0.17$.

Figure 7 shows the transverse momentum distributions for $\pi^+$, $K^+$, and $p$ in 0-5 %, 10-20 %, 30-40 %, and 50-60 % centralities. The slopes of $p_T$ spectra of $\pi^+$, $K^+$, and $p$ are steeper, if the value of bulk viscosity is larger. The growth of the transverse flow becomes small due to the existence of bulk viscosity.

To understand the detailed feature of the behavior of $p_T$ spectra, we investigate the time evolution of the transverse flow in Fig. 8. From comparison between the red solid line ($\eta/s = 0.17$, $N = 110$) and the green dashed line ($\eta/s = 0.17$, $b = 40$, $N = 110$) we can see the bulk viscosity effect. The existence of bulk viscosity delays the growth of the transverse flow until around $\tau = 7.0$ fm. After $\tau = 7.0$ fm, the $\langle v_T \rangle$ of the green dashed line is larger than that of the red solid line, in turn. On the other hand, if we change the normalization $N$ from...
Here we can see that in the case of finite bulk viscosity the fraction of fluid elements whose temperature is around the critical temperature becomes smaller (larger) at low (high) \( p_T \). For the triangular flow, we do not find clear dependence of bulk viscosity in 0-5 % centrality, however, in 30-40 % centrality we observe enhancement of \( v_3 \) with larger bulk viscosity.

For the finite bulk viscosity, we obtain the following results. The slope of \( p_T \) spectra of \( \pi^+ \), \( K^+ \), and \( p \) becomes steeper in the finite bulk viscosity, which suggests the small mean \( p_T \). The elliptic flow \( v_2 \) becomes small at low \( p_T \), whereas above \( p_T > 2 \) GeV it becomes large. The triangular flow \( v_3 \) is enhanced for the larger bulk viscosity in 30-40 % centrality. Furthermore we find the bulk viscosity effect as the enhancement of the profile function around the critical temperature which may affect physical observables.
\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure13}
\caption{(color online). The $p_T$ distributions for $\pi^+$, $K^+$, and $p$ in 0-5 \%, 10-20 \%, 30-40 \%, and 50-60 \% centralities together with the ALICE data (the open circles) \cite{ALICE}. The red dashed line, the blue dashed-dotted line, and the purple dotted line stand for (f) $c_1 = 10$, $c_2 = 0.7$, (g) $c_1 = 0$, $c_2 = 0.7$, and (h) $c_1 = 10$, $c_2 = 0.7$, respectively. See the details in the text.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure14}
\caption{(color online). The mean $p_T$ of $\pi^+$ (open circles), $K^+$ (open triangles), and $p$ (open squares) as a function of centrality, together with the ALICE data \cite{ALICE}. We evaluate the mean $p_T$ for whole region of calculated $p_T$, $p_T > 0 \text{ GeV}$.}
\end{figure}

D. Temperature dependent $\eta/s(T)$ and $\zeta/s(T)$

We investigate the temperature dependence of shear and bulk viscosities from comparison with the ALICE data. Here we set the value of $b$ to $b = 40$. First in Fig. 12 we show the $p_T$ distributions for $\pi^+$, $K^+$, and $p$ in 0-5 \%, 10-20 \%, 30-40 \%, and 50-60 \% centralities, together with the ALICE data. From the $p_T$ spectra we can mainly extract the bulk viscosity effect. The $p_T$ spectra for (f), (g), and (h) are almost identical, which means that the bulk viscosity effect during hydrodynamic expansion in the three cases is the same. Also, compared with those in Fig. 7, the slope of our computed $p_T$ spectra becomes flat and shows better agreement with experimental data.

Introduction of the temperature dependence of $\eta/s$ reduces average value of $\zeta/s$ during hydrodynamic expansion through Eq. (8) (Fig. 1).

In Fig. 13 we show the centrality dependence of mean $p_T$ of $\pi^+$, $K^+$ and $p$, together with the ALICE data \cite{ALICE}. All computational results show reasonable agreement with experimental data, except the constant $\eta/s = 0.17$ with vanishing bulk viscosity. In the case of (b), our calculated values of $\langle p_T \rangle$ are larger than experimental data, which implies that in our calculation radial flow grows stronger. In particular, the large deviation from experimental data exists in radial flow of protons. Centrality dependence of mean $p_T$ of our computational results shows steeper decrease with centrality, compared with experimental data. If bulk viscosity is included, our calculated results of (d), (f), (g), and (h) become close to experimental data, because bulk viscosity suppresses the growth of radial flow \cite{ALICE, ALICE2}. Our calculations of (d) show good agreement with experimental data up to centrality 10-20 \%, however they deviate from the experimental data at peripheral collisions. In peripheral collisions the suppression due to bulk viscosity is too strong and the mean $p_T$ shows rapid decrease with centrality. We find the mean $p_T$ is not sensitive to the differences of the temperature dependence of $\eta/s$ (cases (f), (g), and (h)), as expected from $p_T$ spectra of $\pi^+$, $K^+$, and $p$.

Figure 14 shows the elliptic and triangular flows of charged hadrons as a function of $p_T$ in 0-5 \% and 30-40 \% centralities. From collective flows we can investigate both shear and bulk viscosities. The results of $p_T$ spectra and mean $p_T$ suggest that the cases (f), (g), and (h) have almost the same bulk viscosity effect during hydrodynamic expansion. Therefore we can understand
The average value of shear viscosity over fluid cells is largest, which leads the smallest amplitude of elliptic flow among them. There are small differences between cases (g) and (h) in the elliptic flows. This is because in the current parametrization of $\eta/s(T)$ the average values of shear viscosity are almost the same. Furthermore, compared with Fig. 6 enhancement of $v_2$ due to finite bulk viscosity is observed above $p_T > 2$ GeV. For the triangular flow we find the same tendency in 0-5 % centrality, in spite of errors. In 30-40 % centrality all (f), (g), and (h) cases show good agreement with the experimental data, which is realized by enhancement of $v_3$ due to finite bulk viscosity.

Figure 15 shows centrality dependence of integrated $v_n$ ($n = 2, 3$). The results of case (b) are consistent with the ALICE data [70]. However this agreement is accidentally achieved by larger $\langle p_T \rangle$ and smaller $v_2(p_T)$. The integrated $v_n$ is affected not only by $v_n(p_T)$ but also by $p_T$ spectra through integration over $0.2 < p_T < 5$ GeV. The computed $v_2$ for the case (d) is smaller than experimental data except for the central collision, because the mean $p_T$ is smaller than the experimental data especially in peripheral collisions. Cases (f), (g), and (h) show good agreement with the experimental data, which is a consequence of consistent with experimental data for $v_2(p_T)$ and $p_T$ spectra. In 50-60 % centrality $v_2$ of case (g) is larger than experimental data, which suggests that in the centrality the shear viscosity of hadronic phase is...
important.

We examine the viscosity effects of elliptic flows as a function of pseudorapidity for 0-5%, 10-20%, 30-40%, and 50-60% centralities in Fig. 16. If we input only the constant $\eta/s$ in the calculations, our results of $v_2$ are larger than experimental data and the slope of $v_2(\eta)$ at peripheral collisions is gentler than that of the ALICE data. On the other hand, if we add bulk viscosity (case (d)), the absolute value of $v_2(\eta)$ becomes small and approaches to the experimental data. Because in the computation of $v_2(\eta)$ $p_T$ integration is performed, the mean $p_T$ for cases (b) and (d) affects amplitude of $v_2(\eta)$. For cases (f), (g), and (h), their centrality dependence of mean $p_T$ as well as behavior of $v_2(p_T)$ are almost the same, which means that there should be small differences among the behavior of $v_2(\eta)$. However we find the interesting viscosity effect in centrality dependence of $v_2(\eta)$. For the three cases the average bulk viscosity effect during hydrodynamic expansion is almost the same. It means that different behavior of $v_2(\eta)$ among them originates mainly from different temperature dependence of shear viscosity. In 10-20% centrality, $v_2(\eta)$ of case (h) becomes the largest. In the centrality shear viscosity of the QGP phase is dominant. In 30-40% centrality, $v_2(\eta)$ of case (f) shows the smallest value, which suggests that shear viscosity of the QGP and hadronic phases important. In 50-60% centrality $v_2(\eta)$ of case (g) becomes the largest, which suggests that shear viscosity of the hadronic phase is important. The centrality dependence of $v_2(\eta)$ reveals the detail temperature dependence of $\eta/s$ and $\zeta/s$. The deviation between our results and the experimental data becomes large at forward rapidity.

Here we make a comment on behavior of $v_2(\eta)$ at RHIC, which also shows rapid decrease at forward and backward $\eta$. It is understood by introduction of temperature dependent shear viscosity in the hadronic phase. On the other hand, in our results for the LHC we do not find the clear difference between the slope of $v_2(\eta)$ of cases (b) and (h), because in our parametrization average value of shear viscosity during hydrodynamic expansion between the two cases is almost the same. The shape of the $v_2(\eta)$ is determined not only by the shear viscosity of the hadronic phase but also by the average value of shear viscosity of fluid. Furthermore since $v_2(\eta)$ is evaluated by the correlation between particles at mid rapidity and those at forward rapidity, decorrelation between them may need to be considered.

In Fig. 17 we show $v_2$ of charged hadrons as a function of pseudorapidity in 0-5%, 10-20%, 30-40%, and 50-60% centralities. The errors in the triangular flows are larger than those in the elliptic flows. The case (b) shows...
good agreement with the experimental data, however this agreement is realized from combination of large mean $p_T$ (Fig. 12) and small $v_3(p_T)$ (Fig. 6). The smaller value of $v_3(\eta)$ of case (d) comes from smaller mean $p_T$. Due to the large errors in the triangular flows cases (f), (g), and (h) are not distinguishable. To reach the conclusive results for the temperature dependence of $\eta/s$ from $v_3(\eta)$, we need to perform calculations with more statistics.  In 50-60 % centrality $v_3(\eta)$ for all cases are larger than the experimental data.

E. Final state interactions

Finally we investigate the effect of the final state interactions on $p_T$ spectra and collective flows $v_2$ and $v_3$ in detail. Here we focus on case (f) ($c_1 = 10$, $c_2 = 0.7$ and $\eta/s=0.17$). Figure 18 shows the $p_T$ spectra of $\pi^+, K^+$, and $p$ in 0-5 % (left panel) and 30-40 % (right panel) centralities, together with the ALICE data [69]. The $p_T$ spectra just from hydrodynamics at the switching temperature ($T_{SW} = 150$ MeV), the slopes of them are almost the same as those of the experimental data, however, yields of them are much less than the experimental data. Once we include the resonance decay, the $p_T$ spectra move to close to the experimental data. In particular, for $\pi^+$ we observe the significant decay effect at low $p_T$. Furthermore during final state interactions, particles earn the transverse momentum so that the slope of $p_T$ spectra including rescattering becomes flat compared with that without rescattering. We can see the same
(1/2πpT)dN/dydpT

FIG. 18 (color online). The p_T distributions for π^+, K^+, and p in 0-5 % (left panel) and 30-40 % (right panel) centralities, together with the ALICE data (the open circles) [69]. The red solid line, the green dashed line, and the blue dashed-dotted line stand for computed p_T spectra with rescattering, without rescattering (only resonance decay), and without UrQMD (just from hydrodynamic evolution), respectively.

FIG. 19 (color online). The mean p_T for π^+, K^+, and p with rescattering (the red solid circles), without rescattering (the green solid triangles), and without UrQMD (the blue solid squares) in 0-5 % (left panel) and 30-40 % centralities (right panel), together with the ALICE data (the open circles) [69]. We evaluate the mean p_T in the region of p_T > 0 GeV.

TABLE II. The values of mean p_T of π^+, K^+, and p with rescattering, without rescattering, and without UrQMD in 0-5 % and 30-40 % centralities.

|                | 0-5%       |           | 30-40%     |           |
|----------------|------------|-----------|------------|-----------|
|                | π^+        | K^+       | p          | π^+       | K^+       | p          |
| w/ rescattering| 0.560 ± 0.002 | 0.929 ± 0.005 | 1.462 ± 0.013 | 0.502 ± 0.002 | 0.800 ± 0.007 | 1.225 ± 0.016 |
| w/o rescattering| 0.546 ± 0.002 | 0.837 ± 0.005 | 1.168 ± 0.010 | 0.493 ± 0.002 | 0.749 ± 0.006 | 1.631 ± 0.013 |
| w/o UrQMD      | 0.617 ± 0.001 | 0.888 ± 0.002 | 1.208 ± 0.005 | 0.553 ± 0.001 | 0.794 ± 0.003 | 1.070 ± 0.008 |

In Fig. [19] we show the values of mean ⟨p_T⟩ of π^+, K^+, and p with rescattering, without rescattering, and without UrQMD in 0-5 % (left panel) and 30-40 % (right panel), together with the ALICE data [69]. We also list the values of ⟨p_T⟩ in Tab. II. In the case of π^+, the resonance decay at low p_T is so large that the slope of p_T is steeper. As a result ⟨p_T⟩ of with rescattering and without rescattering becomes small, compared with that without UrQMD. On the other hand, for ⟨p_T⟩ of p due to the rescattering in the final state interactions the slope of p_T spectra becomes flat, which leads to growth of mean ⟨p_T⟩.

In Fig. [20] we investigate the effect of resonance decay and final state interactions from the elliptic flow of charged hadrons in 30-40 % centrality. We find that most part of the elliptic flow develops during the hydrodynamic evolution. It indicates that the elliptic flow reflects the feature of QGP fluid such as the EoS and transport coefficients without being smeared by the final state interactions. If the decay effect is included, the elliptic flow of charged hadrons becomes large to be close to the experimental data at low p_T. Particles through decay process tend to be produced in the same direction.
as that of their parent particles [74], which enhances the amplitude of the elliptic flow [75]. On the other hand, we only find small effect from the rescattering on \( v_2 \), comparing \( v_2 \) with rescattering (the red solid line) and that without rescattering (the green dashed line). The change is small, however, elliptic flow with rescattering shows the best agreement with the experimental data.

Finally, in Fig. 21 we compare the elliptic flows of charged hadrons as a function of \( p_T(GeV) \) in 0-5 % and 30-40 % centralities, together with the ALICE data. The red solid line stands for \( v_2 \) with rescattering, the green dashed line stands for \( v_2 \) without rescattering, and the blue dashed-dotted line stands for \( v_2 \) without UrQMD.

VI. SUMMARY

We have investigated the temperature dependence of shear and bulk viscosities from comparison with the ALICE data; single particle spectra and collective flows at Pb+Pb \( \sqrt{s_{NN}} = 2.76 \) collisions.

First we have studied the constant shear viscosity with comparison between our calculated results and the ALICE data. The \( p_T \) spectra of \( \pi^+ \), \( K^+ \), and \( p \) are insensitive to the value of \( \eta/s \) and in 0-5 % and 10-20 % centralities our computed \( p_T \) spectra overestimate above \( p_T > 1.5 \) GeV. We find the clear \( \eta/s \) dependence in \( v_2 \) and \( v_3 \), i.e., the larger \( \eta/s \), the smaller \( v_2 \) and \( v_3 \) of charged hadrons.

For the finite bulk viscosity, we obtain the following results. The slope of \( p_T \) spectra of \( \pi^+ \), \( K^+ \), and \( p \) becomes steep in the finite bulk viscosity, which suggests the small mean \( p_T \). The elliptic flow \( v_2 \) becomes small.
at low $p_T$, whereas above $p_T > 2$ GeV it becomes large. The triangular flow $v_2$ is enhanced for the larger bulk viscosity in 30-40 % centrality. Furthermore we find the bulk viscosity effect as the enhancement of the profile function around the critical temperature which may affect physical observables.

Furthermore we have argue consequences of the temperature dependence of $\eta/s$ and $\zeta/s$. In the parametrization of $\eta/s(T)$ and $\zeta/s(T)$, $p_T$ spectra are not sensitive to the parametrization $\eta/s$, but their slopes depend on the average value of $\zeta/s$ during the hydrodynamic expansion. They are steeper with larger $\zeta/s$. From amplitude of mean $p_T$, we can extract the bulk-viscosity effect. The collective flows $v_2$ and $v_3$ are affected by both shear and bulk viscosities. The shear viscosity reduces the amplitude of $v_2$ and $v_3$. On the other hand, the bulk viscosity reduces $v_2$ and $v_3$ at low $p_T$ and enhances them at high $p_T$, which is related with change of $p_T$ slope. Our parametrization of (f), (g), and (h) shows the best agreement with the experimental data compared with other cases ((b) and (d)), which implies that the parametrization $\eta/s$ and $\zeta/s$ in (f), (g), and (h) is one of the most suitable choices. Furthermore we point out that the agreement with the experimental data found in $\langle v_n \rangle$ with constant $\eta/s$ realizes just from combination of smaller $v_2(p_T)$ and larger $\langle p_T \rangle$. We find the effect of the temperature dependence of $\eta/s$ in centrality dependence of $v_n$. In the central collisions (peripheral collisions), viscosities of hadronic (QGP) phase becomes important.

Finally we have investigated the effect of the final state interactions. The resonance decays increase yields and the slope of $p_T$ spectra becomes flat during final state interactions. Besides, the mean $p_T$ of $\pi^+$ becomes small, whereas that of $p$ becomes large. For elliptic flow as a function of $p_T$, the most part of the elliptic flow develops during the hydrodynamic evolution, though the elliptic flow still continues to grow a little through resonance decays. It indicates that the elliptic flow reflects the feature of QGP fluid such as the EoS and transport coefficients. In $v_2(\eta)$, through integration of $p_T$, $v_2(\eta)$ without UrQMD shows the largest value and the resonance decay only changes the magnitude of $v_2(\eta)$. From the comprehensive analyses of centrality dependence of single particle spectra and collective flow, we can extract the detailed information of the QGP bulk property, without being smeared by the final state interactions.

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