Unification without low-energy supersymmetry

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Abstract. Without supersymmetry, the gauge couplings in the standard model with five Higgs doublets unify around $10^{14}$ GeV. In this case, the trinified model, $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathbb{Z}_3$, with the minimal Higgs sector required for symmetry breaking, is the ideal grand-unified candidate. Small neutrino masses are generated via a radiative seesaw mechanism, without the need for intermediate scales, additional Higgs fields, or higher-dimensional operators. The proton lifetime is above the experimental limits, with the decay modes $p \to \bar{\nu}K^+$ and $p \to \mu^+K^0$ potentially observable. The split-SUSY version of the model, with one light Higgs doublet, is equally attractive.

Grand unification of the strong, weak, and electromagnetic interactions into a simple gauge group is a very appealing idea that has been vigorously pursued for many years. The fact that the three gauge couplings of the minimal supersymmetric standard model meet almost exactly at $M_U = 2 \times 10^{16}$ GeV, has made supersymmetric GUTs a beautiful framework for theories beyond the standard model (SM). However, if we go beyond $\mathrm{SU}(5)$, as suggested by neutrino experiments, we do not need single-step unification. Furthermore, unification is possible without supersymmetry if we extend the Higgs sector of the SM to five or six Higgs doublets [1].

In this talk, we present a viable candidate for a unified theory without (low-energy) supersymmetry [2]. Since the unification of the gauge couplings occurs at a lower scale, $M_U \approx 10^{14}$ GeV, we do not choose a simple GUT group, which would yield too rapid proton decay. Instead, we consider a product group, supplemented with a discrete symmetry to enforce the equality of the gauge couplings. The simplest theory of this type that has the same condition on the gauge couplings at the unification scale as $\mathrm{SU}(5)$ is $G_{\text{TR}} \equiv \mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \times \mathbb{Z}_3$ [3]. As we shall see, the minimal trinified model can have as many as five light Higgs doublets.

Minimal Trinification

We begin by briefly reviewing the minimal trinified model [2, 3]. The gauge bosons are assigned to the adjoint representation, the fermions to $\psi_L \oplus \psi_Q^c \oplus \psi_Q \equiv (1, 3, 3^*) \oplus (3^*, 1, 3) \oplus (3, 3^*, 1)$, where

$$
\psi_L = \begin{pmatrix}
\mathcal{E}' \\
\mathcal{N}_1 \\
\mathcal{E}^c \\
\mathcal{N}_2
\end{pmatrix}, \quad \psi_Q^c = \begin{pmatrix}
\mathcal{Q}^c \\
\mathcal{U}^c \\
\mathcal{D}^c
\end{pmatrix}, \quad \psi_Q = \begin{pmatrix}
d \\
u \nu_c \\
u^c
\end{pmatrix}.
$$
The field \((-d, u)\) is the (conjugate of the) usual quark doublet \(Q\), while \(B\) is an additional color-triplet, weak-singlet quark. The field \(u^c\) is the usual up-conjugate quark field, while \(\Phi^c\) and \(\bar{\Phi}^c\) have the quantum numbers of the down-conjugate quark field. The field \(e^c\) is the usual positron field, and the lepton doublet is a linear combination of \(\Phi\) with a mass at the unification scale, where \(N\) this may be corrected at one loop, via the “radiative see-saw” mechanism.

Active Dirac neutrino at the weak scale and a sterile Majorana neutrino at the eV scale; masses of the quarks and charged leptons. At tree level, however, the model yields an interference of the Higgs bosons, \(\Phi^Q\), \(\Phi_{Q^c}\), \(\Phi_Q\), and \(\Phi_{Q^c}\), and the cubic couplings of the Higgs fields. These interactions may be used to construct the one-loop diagram, displayed in the adjoining figure.

For simplicity, we set \(v_3 = n_3 = 0\); this does not have any effect on the qualitative aspects of the model [2]. Both \(v_1\) and \(v_2\) break \(SU(3)_L \times SU(3)_R\) to \(SU(2)_L \times SU(2)_R \times U(1)\), but the \(SU(2)_R \times U(1)\) are different. Together they break \(G_{\text{TR}}\) to the SM.

Of the six Higgs doublets (\(\Phi_i\), three each in \(\Phi_{1,2}^1\)), one linear combination is eaten by the gauge bosons that acquire unification-scale masses. If the remaining five doublets have weak-scale masses, then gauge-coupling unification results at \(M_{\text{GUT}} \simeq 10^{14}\) GeV without supersymmetry. In general it would take several fine-tunings to arrange this, so it is an even more acute form of the usual hierarchy problem.

When \(S_1^1\) and \(S_2^1\) acquire the vevs \(v_1\) and \(v_2\), respectively, \(B\) pairs up with \(B^c = c_\alpha \Phi^c + s_\alpha \bar{\Phi}^c\) (where \(\tan \alpha = \frac{v_1}{v_2}\) and \(s \equiv \sin, c \equiv \cos\)) to form a Dirac fermion with a mass at the unification scale, \(m_B = \frac{v_1^2 g_1}{\sqrt{2}} + \frac{v_2^2 g_2}{\sqrt{2}}\). Similarly \(E = -s_{\beta} \varphi + c_{\beta} \mathcal{L}\) and \(E^c\), where \(\tan \beta = \frac{h_1 v_1}{h_2 v_2}\), form a heavy Dirac state. The orthogonal linear combinations of \(\Phi^c\) and \(\bar{\Phi}^c\) as well as \(\mathcal{L}\) and \(\mathcal{L}\), which are \(d^c\) and \(L\), remain massless.

The electroweak symmetry is broken to \(U(1)_{\text{EM}}\) when the two Higgs doublets \(\Phi_1^1\) and \(\Phi_2^1\) acquire weak-scale vevs, \(u_1\) and \(u_2\). Then the light fermions acquire masses

\[
m_u = g_1 u_1, \quad m_d = g_1 u_1 s_\alpha, \quad m_e = h_1 u_1 s_\beta, \quad m_{\nu, N_1} = h_1 u_2, \quad m_{N_2} \approx \frac{h_1^2 u_1 u_2 s_\beta}{m_E},
\]

where \(N_1 = s_\beta N_1 - c_\beta N_2\) and \(N_2 = -c_\beta N_1 - s_\beta N_2\). The results for the fermion masses show that, even in the minimal model, there is no relation between the masses of the quarks and leptons, since they depend on five independent parameters \((g_1, h_1, \frac{u_1}{u_2}, s_\alpha, s_\beta)\). Thus the minimal trinification model is sufficient to describe the masses of the quarks and charged leptons. At tree level, however, the model yields an active Dirac neutrino at the weak scale and a sterile Majorana neutrino at the eV scale; this may be corrected at one loop, via the “radiative see-saw” mechanism.

**Radiative See-saw Mechanism.** Large radiative contributions to the masses of the sterile neutrinos occur due to the coupling of the neutral fermions to color-triplet Higgs bosons, \(\Phi_Q\) and \(\Phi_{Q^c}\), and the cubic couplings of the Higgs fields. These interactions may be used to construct the one-loop diagram, displayed in the adjoining figure.
This diagram is dominated by the quark that acquires a unification-scale mass, namely the heavy $B$ quark. It yields large one-loop contributions such that the effective neutrino mass matrix, in the $(\nu, N_1, N_2)$ basis, is

$$M_{N}^{1\text{-loop}} \simeq \begin{pmatrix} 0 & -h_1 u_2 & 0 \\ -h_1 u_2 & s_{\alpha-}\beta c_{\beta} g^2 F_B & (s_{2\beta}s_{\alpha} - c_{\alpha}) g^2 F_B \\ 0 & (s_{2\beta}s_{\alpha} - c_{\alpha}) g^2 F_B & c_{\alpha-}\beta s_{\beta} g^2 F_B \end{pmatrix},$$

where $F_B$ denotes the loop integral. This matrix has two eigenvalues,

$$m_{N_{1,2}} \sim g^2 F_B,$$

$$m_{\nu} \sim \frac{h_1^2 u_2^2}{g^2 F_B}.$$

Thus the two sterile neutrinos acquire unification-scale masses at one loop, while the active neutrino acquires a “radiative see-saw” Majorana mass. In order to obtain the correct values for the tau and top masses, we expect $h_1 \sim 0.1$, $g \sim 1$ and $u_2 \sim 100$ GeV. Since $F_B \simeq M_U/(4\pi)^2$, the mass of the light neutrino is then $\mathcal{O}(0.1$ eV), consistent with the experimental constraints [4].

There is also a one-loop diagram that couples $\nu$ to $N_{1,2}$ but with the heavy $B$ quark replaced by $d$. This diagram is comparable to the tree-level Dirac mass $h_1 u_2$, so it does not qualitatively change the radiative see-saw mechanism.

The radiative see-saw mechanism is absent in models with weak-scale supersymmetry, since the one-loop contributions are reduced to $\mathcal{O}(1\text{ TeV})$, but it is present if the mass difference between scalars and fermions is comparable to the grand-unified scale, as may be the case in split supersymmetry [5].

**Neutrino Hierarchy.** From the discussion above, we note that the neutrino hierarchy is related to the couplings in the quark sector. Since the one-loop contributions to the neutrino masses are proportional to the fermion mass in the loop, those with the heaviest quark, $B_3$, are dominant.

For our discussion, we assume that the matrices $g$ have the form [6]

$$g \sim \begin{pmatrix} \varepsilon^4 & \varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon & 1 & 1 \end{pmatrix}, \quad \varepsilon^2 \sim \frac{m_c}{m_t}, \quad \mathcal{G} = g_3 g_{j3} + g_j g_{3j} \sim \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & \varepsilon \\ \varepsilon^3 & \varepsilon^2 & 1 \\ \varepsilon & 1 & 1 \end{pmatrix}. \quad (1)$$

To obtain a hierarchical structure for the up and down quarks, both $g_1$ and $g_2$ will generally be hierarchical, so we expect the $B$ quarks to have a similar hierarchy. Then the three-generational mass matrix for the sterile neutrinos (both $N_1$ and $N_2$) has the eigenvalues $m_3^N \sim m_2^N \sim F_{B_3} \sim 10^{12}$ GeV and $m_1^N \sim \varepsilon^4 F_{B_3} \sim 10^8$ GeV; the latter is comparable to the mass of the lightest sterile neutrino in thermal leptogenesis.

The eigenvalues of the light neutrinos are proportional to $h^2/g^2$ due to the common loop-integral, where $g^2$ is given by the third column of the symmetric matrix $\mathcal{G}$ in Eq. (1). Since the hierarchy of $g^2$ is weak, the neutrino hierarchy is determined by the hierarchy of $h$ and we find either quasi-degenerate masses or a normal hierarchy.

**Proton Decay.** The gauge interactions conserve baryon number, and therefore do not mediate proton decay. Instead, proton decay is mediated by the colored Higgs fields.
These dimension-six operators are suppressed by the small Yukawa couplings. Hence, the flavor non-diagonal decay is dominant.

We can estimate the lifetime to be \( \tau \simeq \left( \frac{1}{gh} \right)^2 \times 10^{28} \) years; for details, see Ref. [2]. Using \( g \) as given in Eq. (1), we find the channel \( p \to \bar{\nu}K^+ \) be the dominant decay mode, followed by \( p \to \mu^+K^0 \). This result is similar to those in models with weak-scale supersymmetry, where the decay is dominated by dimension-five operators [7]. The decay might be observable in future experiments which aim to reach a lifetime of \( 10^{35-36} \) years [8]. In the split supersymmetry scenario, however, proton decay is unobservable due to the higher unification scale and the large sfermion masses.

Conclusion. The minimal trinified model is an interesting and viable candidate for non-supersymmetric unification. The breaking is achieved by only two \((1,3,3^*)\) Higgs fields, which include five Higgs doublets. Unlike other grand-unified theories, this model is able to correctly describe the fermion masses and mixing angles without the need to introduce intermediate scales, additional Higgs fields, or higher-dimensional operators.

Light, active neutrinos are naturally generated at one loop via the radiative seesaw-mechanism. The additional matter, which is either vectorlike or sterile, is superheavy with masses above \( 10^8 \text{ GeV} \). Thus no additional particles are present at the weak scale.

Proton decay is mediated by colored Higgs bosons with the dominant decay modes potentially observable in future experiments. Therefore the different types of models would be distinguished by the presence or absence of supersymmetric particles and the number of Higgs doublets at the weak scale, together with the observation of specific proton decay modes [7]. The smoking gun for minimal trinification would be the discovery of five Higgs doublets at the weak scale and the observation of proton decay into final states containing kaons.

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