Correlators of Vertex Operators for Circular Strings with Winding Numbers in $AdS_5 \times S^5$

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Abstract

We compute semiclassically the two-point correlator of the marginal vertex operators describing the rigid circular spinning string state with one large spin and one winding number in $AdS_5$ and three large spins and three winding numbers in $S^5$. The marginality condition and the conformal invariant expression for the two-point correlator obtained by using an appropriate vertex operator are shown to be associated with the diagonal and off-diagonal Virasoro constraints respectively. We evaluate semiclassically the three-point correlator of two heavy circular string vertex operators and one zero-momentum dilaton vertex operator and discuss its relation with the derivative of the dimension of the heavy circular string state with respect to the string tension.

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1 Introduction

From the AdS/CFT correspondence [1] the correlation functions in the $\mathcal{N} = 4$ super Yang-Mills theory (SYM) can be calculated both at weak and at strong coupling. Using the relation between the generating functionals for the correlation functions and the AdS string partition functions, the three-point correlation functions of protected operators have been derived at strong coupling in the supergravity approximation [2, 3].

The AdS/CFT correspondence suggests that each on-shell string state or marginal string vertex operator in $AdS_5 \times S^5$ string theory should be associated with a local gauge-invariant operator in the $\mathcal{N} = 4$ SYM theory with definite conformal dimension [4]. There has been a prescription for computing a two-point correlation function of vertex operators that describe the string states with large spins and are dual to the non-protected states with large quantum numbers [5]. The stationary point solution obtained by inserting appropriate vertex operators for the two-point function should coincide with the classical string solution carrying the same charges. The semiclassical computation of the two-point function by demanding the 2d conformal invariance should lead to the same relation between energy and spins as constructed for the corresponding classical string solution by requiring the Virasoro constraints.

For the case that the associated string is the folded string with large $AdS_5$ spin [6] this connection has been demonstrated in the flat space limit [5], in the global coordinates [7] where the dependence of vertex operators on the $AdS_5$ boundary points is ignored, and in the Poincare coordinates [8] where the locations of vertex operators in the four-dimensional boundary space are specified and the two-point function shows the scaling behavior of the strong-coupling limit of the two-point function of single trace minimal twist operators in gauge theory. In ref. [7] the two-point function associated with the string solution with large two equal spins $J_1 = J_2$ and two equal winding numbers $m_1 = m_2$ in $S^5$ has been discussed where the vertex operator includes an additional factor expressed in terms of the T-dual variables.

There have been various semiclassical studies about the calculations of correlators involving Wilson loops [9, 10, 11, 12, 13].

Using appropriate wave functions the two-point correlation function has been computed [14], where a circular heavy string state with spin $S$ in $AdS_5$ and equal spin $J(=S)$ in $S^5$ [15, 16] propagates between two $AdS_5$ boundary points, and has been shown to produce right scaling behavior with the scaling dimension which is determined by the saddle point in the integraion over the modular parameter.

This approach has been extended to computation of a three-point correlation function which consists of two non-BPS operators dual to heavy string states and one chiral primary operator dual to a supergravity field [17]. The three-point correlator has been derived by evaluating semiclassically a Witten diagram with a supergravity field propagating from the $AdS_5$ boundary to the heavy string worldsheet. Specially for one gauge theory Lagrangian dual to the massless dilaton field and two non-BPS operators dual to several string configurations such as the circular spinning string with two equal spins $S = J$ in $AdS_5 \times S^5$ or $J_1 = J_2$ in $S^5$ [15] and the giant magnon in $S^5$ [18], the three-point correlators have been computed and the associated three-point couplings, namely, the OPE coefficients have been
shown to agree with the values predicted by renormalization group arguments.

On the other hand there has been a construction of a three-point correlator [19] for two operators dual to the folded spinning string with two spins $J_1, J_2$ in $S^5$ [20] and one primary scalar operator dual to the supergravity massless scalar field that is a mixture of the metric with RR four-form [3, 21]. The three-point correlator takes the form of a vertex operator in the coordinate representation [4, 5] integrated over the classical string worldsheet.

From the vertex operator prescription [5, 7, 8] various three-point correlators of two heavy vertex operators and one light vertex operator have been computed [22]. The heavy vertex operator represents the folded spinning string with two light vertex operators describing this string state on massive string levels. Further there have been constructions superconformal primary scalar state, the massive state on the leading Regge trajectory and a special singlet string state on massive string levels. Further there have been constructions of the three-point correlators between several choices of one light vertex operator that represent dilaton, the rigid circular string with three spins $J_1 = J_2$ and $J_3$ [15] which includes the small string limit, while there are various choices of light vertex operators that represent dilaton, the superconformal primary scalar state, the massive state on the leading Regge trajectory and a special singlet string state on massive string levels. Further there have been constructions of the three-point correlators between several choices of one light vertex operator and the two heavy vertex operators representing the circular spinning string with one $AdS_5$ spin, three different $S^5$ spins and the corresponding winding numbers [24].

It is desirable to elucidate further the relations between the vertex operators and various string solutions in $AdS_5 \times S^5$. We are interested how to construct the vertex operators representing the circular spinning string states with winding numbers. Using the vertex operator prescription we will compute a two-point correlation function of the vertex operators which describe the rigid circular spinning string in $AdS_5 \times S^5$ [16] carrying spin $S$ with winding number $n$ in $AdS_5$ as well as spin $J$ with winding number $m$ in $S^5$. The extension to the circular spinning string including three pairs of spin and winding number $(J_k, m_k), k = 1, 2, 3$ in $S^5$ and the $AdS_5$ quantum numbers $(S, n)$ will be performed. We will consider a three-point correlation function which consists of two heavy vertex operators describing this circular multi-spin $(S, J_k)$ string and one zero-momentum dilaton vertex operator. From the three-point correlator the three-point coupling will be constructed explicitly in terms of the relevant quantum numbers.

## 2 Two-point correlator

We consider a two-point correlation function of the string vertex operators which are associated with the circular spinning string solution in the Poincare coordinates of $AdS_5 \times S^5$ with spins and winding numbers $(S, n)$ in $AdS_5$ and $(J, m)$ in $S^5$.

The embedding coordinates $Y_M$ $(M = 0, \cdots, 5)$ for the Minkowski signature $AdS_5$ are expressed in terms of the global coordinates $(t, \rho, \theta, \phi_1, \phi_2)$ as

\[
Y_5 + iY_0 = \cosh \rho e^{it}, \quad Y_1 + iY_2 = \sinh \rho \cos \theta e^{i\phi_1}, \quad Y_3 + iY_4 = \sinh \rho \sin \theta e^{i\phi_2}, \\
Y^M Y_M = -Y_5^2 + Y^m Y_m + Y_4^2 = -1, \quad Y^m Y_m = -Y_0^2 + Y_i Y_i \tag{1}
\]

with $m = 0, 1, 2, 3, i = 1, 2, 3$. These coordinates are related with the Poincare coordinates $(z, x^m), ds^2 = z^{-2}(dz^2 + dx^m dx_m)$

\[
Y_m = \frac{x_m}{z}, \quad Y_4 = \frac{1}{2z}(-1 + z^2 + x^m x_m), \quad Y_5 = \frac{1}{2z}(1 + z^2 + x^m x_m) \tag{2}
\]
with \( x^m x_m = -x_0^2 + x_i^2 \). For \( S^5 \) the embedding coordinates are defined by

\[
X_1 \equiv X_1 + iX_2 = \sin \gamma \cos \psi e^{i\varphi_1} = r_1 e^{i\varphi_1}, \quad X_2 \equiv X_3 + iX_4 = \sin \gamma \sin \psi e^{i\varphi_2} = r_2 e^{i\varphi_2}, \\
X_3 \equiv X_5 + iX_6 = \cos \gamma e^{i\varphi_3} = r_3 e^{i\varphi_3}, \quad \sum_{k=1}^{3} r_k^2 = 1.
\]

(3)

Following the procedure of ref. [5], we rotate the worldsheet time \( \tau \) as well as the global AdS time \( t \) to the Euclidean ones simultaneously

\[
\tau_e = i\tau, \quad t_e = it.
\]

(4)

We perform the following conformal transformation to map the Euclidean 2d cylinder \((\tau_e, \sigma)\) into the complex plane \((\xi, \bar{\xi})\) with two punctures at \( \xi = \xi_1 (\tau_e = \infty), \xi = \xi_2 (\tau_e = -\infty) \)

\[
e^{\tau_e + i\sigma} = \frac{\xi - \xi_2}{\xi - \xi_1},
\]

(5)

where \( \xi_1 \) and \( \xi_2 \) are regarded as the points where two vertex operators are inserted. The Euclidean rotation \( t_e = it \) in (4) leads to the similar rotations for the time-like coordinates

\[
Y_{0e} = iY_0, \quad x_{0e} = ix_0.
\]

(6)

In general the integrated marginal vertex operator represents a string state and is described in terms of four coordinates of a point on the boundary of the Euclidean Poincare patch of \( AdS_5 \) space as

\[
V(x') = \int d^2 \xi V(x(\xi) - x', \cdots)
= \int d^2 \xi [z(\xi) + z^{-1}(\xi)(x_m(\xi) - x_m')^2]^{-\Delta} U[x_m(\xi) - x_m', z(\xi), X_k(\xi)],
\]

(7)

where we shift \( x_m = (x_{0e}, x_i) \) by a constant vector \( x_m' \). The explicit expressions of \( U \) are specified by the quantum numbers other than the 4d dimension \( \Delta \) of the string state in \( AdS_5 \times S^5 \).

Let us consider the vertex operator which describes the circular spinning closed string state with quantum numbers like spins \((S, J)\) and winding numbers \((n, m)\) [16]

\[
t = \kappa \tau, \quad \rho = \rho_0, \quad \theta = 0, \quad \phi_1 = \omega \tau - n \sigma \equiv \phi, \quad \varphi = w \tau + m \sigma.
\]

(8)

Its energy-spin relation is given by

\[
E = J + S + \frac{\lambda}{2 J^2} (m^2 J + n^2 S) + \cdots.
\]

(9)

We propose a vertex operator

\[
V(a) = c \int d^2 \xi [z + z^{-1}((x_{0e} - a)^2 + x_i^2)]^{-\Delta} (Y_1 + iY_2)^S (X_1 + iX_2)^J,
\]

(10)
where the location of the vertex operator in the boundary is chosen by \( x'_m = (a, 0, 0, 0) \). The expression \( Y_1 + iY_2 \) is described in terms of Euclidean Poincare coordinates as

\[
Y_1 + iY_2 = \frac{x_1 + ix_2}{z} = \frac{r}{z} e^{i\phi}.
\]

(11)

From (3) with \( \gamma = \pi/2 \), \( \psi = 0 \) and \( \varphi_1 \equiv \varphi \) the \( S^5 \) part is expressed as \( X_1 + iX_2 = e^{i\varphi} \). The proposed vertex operator shows a symmetric expression such that the \( AdS_5 \) factor \((Y_1 + iY_2)^S\) takes the same form as the \( S^5 \) factor \((X_1 + iX_2)^J\) including no derivatives with respect to the worldsheet coordinates. Due to winding numbers the string solution has no short string limit, that is, no flat space limit so that the expression (10) takes a different form from the vertex operator in flat space which includes the derivatives.

We use this vertex operator involving the winding number dependencies implicitly through the angular coordinates to compute a two-point function

\[
\langle V_{\Delta, S, n, J, m}(a(V_{\Delta, S, n, J, m}(0))^* \rangle
\]

(12)
in large spins of order of string tension \( T = \sqrt{\lambda/2\pi} \gg 1 \). The Euclidean continuation (4) of (8) is given by

\[
t_e = \kappa \tau_e, \quad \rho = \rho_0, \quad \theta = 0, \quad \phi = i\omega\tau_e + ns, \quad \varphi = i\omega\tau_e - ns,
\]

(13)

where we change the signs of \( \phi \) and \( \varphi \). The corresponding embedding coordinates (1) are expressed as

\[
Y_5 = \cosh \rho_0 \cosh \kappa \tau_e, \quad Y_{0e} = \cosh \rho_0 \sinh \kappa \tau_e, \quad Y_4 = 0, \quad Y_1 = \sinh \rho_0 \cosh(\omega\tau_e - in), \quad Y_2 = i \sinh \rho_0 \cosh(\omega\tau_e - in), \quad Y_3 = 0,
\]

(14)

which are transformed to the Euclidean Poincare coordinates in the form

\[
\begin{align*}
    &z = \frac{1}{\cosh \rho_0 \cosh \kappa \tau_e}, \quad x_{0e} = \tanh \kappa \tau_e, \\
    &x_\pm = x_1 \pm ix_2 = re^{\pm i\phi} = \frac{\tanh \rho_0}{\cosh \kappa \tau_e} e^{\pm (in - \omega \tau_e)}, \\
\end{align*}
\]

(15)

which satisfy \( z^2 + x_+ x_- + x_{0e}^2 = 1 \).

For the large spin limit \( \kappa \approx \omega \gg 1 \) the complex world surface given by (15) approaches the boundary \( z \to 0 \) at \( \tau_e \to \pm \infty \) with \( x_{0e}(\pm \infty) = \pm 1 \). By making a dilatation and a translation such that \( x_{0e}(\infty) = a, \ x_{0e}(-\infty) = 0 \) at the boundary we have

\[
\begin{align*}
    &z = \frac{a}{2 \cosh \rho_0 \cosh \kappa \tau_e}, \quad x_{0e} = \frac{a}{2} (\tanh \kappa \tau_e + 1), \\
    &x_\pm = x_1 \pm ix_2 = re^{\pm i\phi} = \frac{a \tanh \rho_0}{2 \cosh \kappa \tau_e} e^{\pm (in - \omega \tau_e)}. \\
\end{align*}
\]

(16)

The end points of the Euclidean world cylinder for the transformed configuration are specified by

\[
\begin{align*}
    \tau_e &\to \infty : \quad z \to 0, \quad x_{0e} \to a, \quad r \to 0, \quad x_+ \to 0, \quad x_- \to a \tanh \rho_0 e^{-in}, \\
    \tau_e &\to -\infty : \quad z \to 0, \quad x_{0e} \to 0, \quad r \to 0, \quad x_+ \to a \tanh \rho_0 e^{in}, \quad x_- \to 0. \\
\end{align*}
\]

(17)
When we compute semiclassically the two-point correlator \( \langle \rangle \), the Euclidean action accompanied with the vertex contributions is given by

\[
A_e = \frac{\sqrt{\lambda}}{\pi} \int d^2 \xi \left[ \frac{1}{z^2} (\partial z \bar{\partial} z + \partial x_{0e} \bar{\partial} x_{0e} + \partial r \bar{\partial} r + r^2 \partial \phi \bar{\partial} \phi) + \partial \varphi \bar{\partial} \varphi \right]
- \Delta \int d^2 \xi \left[ \delta^2(\xi - \xi_1) \ln \frac{z}{z^2 + r^2 + (x_{0e} - a)^2} + \delta^2(\xi - \xi_2) \ln \frac{z}{z^2 + r^2 + x_{0e}^2} \right]
- s \int d^2 \xi \left[ \delta^2(\xi - \xi_1) \ln \frac{r e^{i\phi}}{z} + \delta^2(\xi - \xi_2) \ln \frac{r e^{-i\phi}}{z} \right]
- \int d^2 \xi [\delta^2(\xi - \xi_1) \ln e^{i\varphi} + \delta^2(\xi - \xi_2) \ln e^{-i\varphi}].
\] (18)

We will show that the transformed complex solution \( \langle \rangle \) with \( \varphi \) in \( \langle \rangle \) becomes the stationary trajectory in the presence of the vertex operators as source terms. The equation of motion for \( \phi \) is obtained by

\[
\partial \left( \frac{r^2}{z^2} \bar{\partial} \phi \right) + \bar{\partial} \left( \frac{r^2}{z^2} \partial \phi \right) = -\frac{i\pi S}{\sqrt{\lambda}} \left[ \delta^2(\xi - \xi_1) - \delta^2(\xi - \xi_2) \right].
\] (19)

The inversion of the conformal transformation \( \langle \rangle \) is given by

\[
\tau_e = \frac{1}{2} \ln \frac{(\xi - \xi_2)(\bar{\xi} - \bar{\xi}_2)}{(\xi - \xi_1)(\bar{\xi} - \bar{\xi}_1)}, \quad \sigma = \frac{1}{2i} \ln \frac{(\xi - \xi_2)(\bar{\xi} - \bar{\xi}_1)}{(\xi - \xi_1)(\bar{\xi} - \bar{\xi}_1)},
\] (20)

which leads to

\[
(\partial \bar{\partial} + \bar{\partial} \partial) \tau_e = \pi [\delta^2(\xi - \xi_2) - \delta^2(\xi - \xi_1)], \quad (\partial \bar{\partial} + \bar{\partial} \partial) \sigma = 0.
\] (21)

Taking account of (21) and (16) we see that the equation (19) is satisfied only when \( S \) is given by

\[
S = \omega \sinh^2 \rho_0 \sqrt{\lambda}.
\] (22)

We turn to the equation of motion for \( x_{0e} \)

\[
\bar{\partial} \frac{\partial x_{0e}}{z^2} + \bar{\partial} \frac{\partial x_{0e}}{z^2} = \frac{2\pi \Delta}{\sqrt{\lambda}} \left[ \frac{x_{0e} - a}{z^2 + r^2 + (x_{0e} - a)^2} \delta^2(\xi - \xi_1) + \frac{x_{0e}}{z^2 + r^2 + x_{0e}^2} \delta^2(\xi - \xi_2) \right].
\] (23)

which becomes through (16)

\[
\kappa \cosh^2 \rho_0 (\partial \bar{\partial} + \bar{\partial} \partial) \tau_e = \frac{\pi \Delta}{\sqrt{\lambda}} [\delta^2(\xi - \xi_2) - \delta^2(\xi - \xi_1)].
\] (24)

Owing to (21) this equation leads to

\[
\Delta = \kappa \cosh^2 \rho_0 \sqrt{\lambda}.
\] (25)

The equation of motion for \( r \) is expressed as

\[
r \bar{\partial} \frac{\partial r}{z^2} + r \bar{\partial} \frac{\partial r}{z^2} - 2 \frac{r^2}{z^2} \bar{\partial} \phi \partial \phi = -\frac{\pi S}{\sqrt{\lambda}} [\delta^2(\xi - \xi_1) + \delta^2(\xi - \xi_2)]
+ \frac{2\pi \Delta}{\sqrt{\lambda}} \left[ \frac{r^2}{z^2 + r^2 + (x_{0e} - a)^2} \delta^2(\xi - \xi_1) + \frac{r^2}{z^2 + r^2 + x_{0e}^2} \delta^2(\xi - \xi_2) \right].
\] (26)
Since $\tau_e \to \pm \infty$ corresponds to $\xi \to \xi_{1,2}$ it reads
\[
-k[2\kappa \partial_\tau_e \bar{\partial}_\tau_e + \tanh \kappa \tau_e (\partial \bar{\partial} + \bar{\partial} \partial)] - 2(n^2 - \omega^2) \partial_\tau_e \bar{\partial}_\tau_e = \pi(2\kappa - \omega)[\delta^2(\xi - \xi_1) + \delta^2(\xi - \xi_2)],
\]
(27)
where we use $\partial \sigma \bar{\partial} \sigma = \partial_\tau_e \bar{\partial}_\tau_e$, $\partial \sigma \partial_\tau_e + \bar{\partial} \sigma \partial_\tau_e = 0$, (22) and (25). In view of the non-singular $\partial_\tau_e \bar{\partial}_\tau_e$ terms in (27) we have
\[
\omega^2 = \kappa^2 + n^2
\]
(28)
and the remaining singular terms provide
\[
k[\tanh \kappa \tau_e \delta^2(\xi - \xi_1) - \tanh \kappa \tau_e \delta^2(\xi - \xi_2)] = (2\kappa - \omega)[\delta^2(\xi - \xi_1) + \delta^2(\xi - \xi_2)].
\]
(29)
Owing to
\[
\tanh \kappa \tau_e|_{\xi \to \xi_1} = 1, \quad \tanh \kappa \tau_e|_{\xi \to \xi_2} = -1
\]
(30)
the equation (29) is satisfied to the leading order in the large spin limit $\kappa \approx \omega \gg 1$.

Combining the equation of motion for $z$ and that for $r$ (26) we have
\[
\frac{1}{2} \frac{\partial}{\partial z} (z^2 + r^2) + \frac{1}{2} \frac{\partial}{\partial z} (2z^2 + r^2) + \frac{2}{z^2} \partial x_{0c} \bar{\partial} x_{0c} = \frac{\pi \Delta}{\sqrt{\lambda}} [\delta^2(\xi - \xi_1) + \delta^2(\xi - \xi_2)],
\]
(31)
which takes the following compact form
\[
k \cosh^2 \rho_0 (\partial \bar{\partial} + \bar{\partial} \partial) \tau_e = -\frac{\pi \Delta}{\sqrt{\lambda}} \left[ \frac{\delta^2(\xi - \xi_1)}{\tanh \kappa \tau_e} + \frac{\delta^2(\xi - \xi_2)}{\tanh \kappa \tau_e} \right].
\]
(32)
Because of (30) this equation coincides with the equation of motion for $x_{0c}$ (24). The remaining equation of motion for $\varphi$ reads
\[
(\partial \bar{\partial} + \bar{\partial} \partial) \varphi = -\frac{i\pi J}{\sqrt{\lambda}} [\delta^2(\xi - \xi_1) - \delta^2(\xi - \xi_2)].
\]
(33)
It holds if $J$ is given by
\[
J = w \sqrt{\lambda}.
\]
(34)

Now the two-point correlation function can be calculated semiclassically by evaluating the string action with the source terms on the stationary string trajectory. It is convenient to go back to the Euclidean 2d cylinder $(\tau_e, \sigma)$ coordinates for computing the string action in (18)
\[
A_{str} = \frac{\sqrt{\lambda}}{4\pi} \int_{\tau_e, -\infty}^{\tau_e, \infty} d\tau_e \int_0^{2\pi} d\sigma \left[ \frac{1}{z^2} \left( (\partial_{\tau_e} z)^2 + (\partial_{\tau_e} x_{0c})^2 + (\partial_{\sigma} r)^2 + r^2 ((\partial_{\tau_e} \varphi)^2 + (\partial_{\sigma} \varphi)^2) \right) + (\partial_{\tau_e} \varphi)^2 + (\partial_{\sigma} \varphi)^2 \right],
\]
(35)
with
\[
\tau_{e, \pm \infty} = \frac{1}{2} (\ln |\xi - \xi_2|^2 - \ln |\xi - \xi_1|^2) |_{\xi \to \xi_{1,2}}.
\]
(36)
The substitution of the stationary solution into (35) yields

\[ A_{str} = \frac{\sqrt{\lambda}}{2} [\kappa^2 \cosh^2 \rho_0 + (n^2 - \omega^2) \sinh^2 \rho_0 + m^2 - w^2] (\tau_{e,\infty} - \tau_{e,-\infty}). \]  

(37)

We neglect the one-point function divergence (\( \sim \ln 0 \)) to obtain

\[ A_{str} = \frac{\sqrt{\lambda}}{2} [\kappa^2 \cosh^2 \rho_0 + (n^2 - \omega^2) \sinh^2 \rho_0 + m^2 - w^2] \ln |\xi_1 - \xi_2|^2, \]  

(38)

while the source terms in (18) are also evaluated using the delta-function as

\[ A_{sour} = 2\Delta \ln a + (-\Delta \kappa + S\omega + Jw) \ln |\xi_1 - \xi_2|^2 \]  
\[ + (Jm - Sn) \ln \frac{\xi_1 - \xi_2}{\widetilde{\xi}_1 - \widetilde{\xi}_2}. \]  

(39)

When two spins \( S, J \) obey a symmetric relation

\[ Sn = Jm, \]  

(40)

which is associated with the symmetric form of the vertex operator (10), we derive a 2d conformal invariant expression for the two-point correlation function

\[
< V_{\Delta,S,n,J,m}(a)(V_{\Delta,S,n,J,m}(0))^* \geq \int d^2\xi_1 d^2\xi_2 e^{-A_{str} - A_{sour}} \\
\approx \frac{1}{a^{2\Delta}} \int d^2\xi_1 d^2\xi_2 \frac{1}{|\xi_1 - \xi_2|^{\Delta - \sqrt{\lambda} + \sinh^2 \rho_0 (\omega^2 + n^2) - w^2 - m^2}}.
\]  

(41)

This expression shows the scaling \( \sim a^{-2\Delta} \) which is characterized by the scaling dimension \( \Delta \) of the vertex operator and is expected from the 4d conformal invariance of string action under the rescaling \( z \to az, x_m \to ax_m \). From the marginality condition of vertex operator the two-point correlator (41) should take a behavior \( |\xi_1 - \xi_2|^{-4} \) which in the large spin leads to

\[ \frac{\kappa \Delta}{\sqrt{\lambda}} - \sinh^2 \rho_0 (\omega^2 + n^2) - w^2 - m^2 = 0. \]  

(42)

Using (22), (25), (28) we eliminate \( n \) and \( \rho_0 \) to obtain

\[ 2\kappa \Delta - 2\omega S - \kappa^2 = J^2 + m^2 \]  

(43)

with \( S = S\sqrt{\lambda}, J = J\sqrt{\lambda} \).

In ref. [16] the circular two-spin \( (S,J) \) string solution with the corresponding winding numbers \( (n,m) \) was constructed to be specified by the same relations between the relevant parameters as (22), (25), (28), (34) where the off-diagonal Virasoro constraint gives the same relation as (10) and the diagonal Virasoro constraint is presented by

\[ 2\kappa \mathcal{E} - 2\omega S - \kappa^2 = 2\sqrt{m^2 + \nu^2 J - \nu^2} \]  

(44)

8
with $J = \sqrt{m^2 + \nu^2}$.

The marginality condition (13) becomes the same expression as (14) when the parameter $\nu$ is eliminated and the dimension $\Delta$ is identified with the string energy $E = \mathcal{E} \sqrt{\lambda}$. Therefore the dimension $\Delta$ is determined as (9). We observe that the stationary trajectory of path integral representing the two-point function of vertex operators is provided by conformally mapped Euclidean continuation of the circular two-spin $(S, J)$ string solution (16), whose energy-spin relation (9) is derived from the Virasoro constraints for the worldsheet conformal invariance. Thus the two-point correlation function (41) shows the right scaling behavior with the dimension $\Delta$ being the corresponding string energy (9).

Now we consider the two-point correlator of the vertex operators representing the rigid circular spinning string which has spin $S$ with winding number $n$ in AdS$_5$ and three spins $J_k$, $k = 1, 2, 3$ with the corresponding winding numbers $m_k$ in $S^5$. We take the following vertex operator

$$V(a) = c \int d^2 \xi [z + z^{-1}((x_{0e} - a)^2 + x_i^2)]^{-\Delta}(Y_1 + iY_2)^S X_1^J X_2^J X_3^J.$$ (45)

The Euclidean action with the source terms from the vertex operators is also expressed as

$$A_e = \frac{\sqrt{\lambda}}{\pi} \int d^2 \xi \left[ \frac{1}{z^2} \left( \partial z \partial \bar{z} + \partial x_{0e} \partial \bar{x}_{0e} + \partial r \partial \bar{r} + r^2 \partial \phi \partial \bar{\phi} \right) + \frac{1}{2} \sum_{k=1}^{3} (\partial X_k \partial \bar{X}_k + \bar{\partial} X_k \partial \bar{X}_k) \right]$$

$$- \Delta \int d^2 \xi \left[ \frac{\delta^2(\xi - \xi_1)}{z^2 + r^2 + (x_{0e} - a)^2} + \frac{\delta^2(\xi - \xi_2)}{z^2 + r^2 + x_{0e}^2} \right]$$

$$- S \int d^2 \xi \left[ \frac{\delta^2(\xi - \xi_1)}{z} + \frac{\delta^2(\xi - \xi_2)}{z} \right]$$

$$- \sum_{k=1}^{3} J_k \int d^2 \xi [\delta^2(\xi - \xi_1) \ln r_k e^{i\phi_k} + \delta^2(\xi - \xi_2) \ln r_k e^{-i\phi_k}].$$ (46)

This circular spinning string (16) has the following Euclidean continuation

$$t_e = \kappa \tau_e, \quad \rho = \rho_0, \quad \theta = 0, \quad \phi = i\omega \tau_e + n\sigma,$$

$$r_k = \text{const}, \quad \varphi_k = iw_k \tau_e - m_k \sigma.$$ (47)

The equations of motion for $\varphi_k$ are given by

$$r_k^2 (\partial \bar{\partial} + \bar{\partial} \partial) \varphi_k = - \frac{i\pi J_k}{\sqrt{\lambda}} [\delta^2(\xi - \xi_1) - \delta^2(\xi - \xi_2)], \quad k = 1, 2, 3,$$ (48)

which yield

$$J_k = w_k r_k^2 \sqrt{\lambda}.$$ (49)

The equations of motion for $r_k$, or for $\psi$ and $\gamma$, have additional contributions to the singular part. Here we assume that they are still satisfied by constant $r_k$. The string action evaluated on this multi-spin stationary solution is expressed as

$$A_{str} = \frac{\sqrt{\lambda}}{2} \kappa^2 \cosh^2 \rho_0 + (n^2 - \omega^2) \sinh^2 \rho_0 + \sum_{k=1}^{3} r_k^2 (m_k^2 - w_k^2) \ln |\xi_1 - \xi_2|^2,$$ (50)
where the one-point function divergence is ignored. The source terms are computed by

\[
A_{\text{source}} = 2\Delta \ln a + (-\Delta \kappa + S\omega + \sum_{k=1}^{3} J_k w_k) \ln |\xi_1 - \xi_2|^2 \\
+ \left( \sum_{k=1}^{3} J_k m_k - Sn \right) \ln \frac{\xi_1 - \xi_2}{\xi_1 - \xi_2}. \tag{51}
\]

When there is a relation

\[
S n = \sum_{k=1}^{3} J_k m_k, \tag{52}
\]

we obtain the following two-point function that is consistent with 2d conformal symmetry

\[
<V_{\Delta,S,n,J_k,m_k}(a)V_{\Delta,S,n,J_k,m_k}(0)^\ast > \approx \frac{1}{a^{2\Delta}} \int d^2\xi_1 d^2\xi_2 \frac{1}{|\xi_1 - \xi_2|^{2\Delta}} \frac{1}{\sqrt{\lambda - 2\omega S - \kappa^2 + \sum_k r_k^2 (w_k^2 + m_k^2)}}. \tag{53}
\]

The marginality condition of the vertex operator implies

\[
\frac{2\kappa \Delta}{\sqrt{\lambda}} - 2\omega S - \kappa^2 = \sum_{k=1}^{3} r_k^2 (w_k^2 + m_k^2). \tag{54}
\]

The circular multi-spin \((S,J_k)\) string solution with winding numbers \((n,m_k)\), \(k = 1,2,3\) was constructed \[16\] from the off-diagonal Virasoro constraint that agrees with \(52\) and the diagonal Virasoro constraint

\[
2\kappa \mathcal{E} - 2\omega S - \kappa^2 = 2 \sum_{k=1}^{3} \sqrt{m_k^2 + \nu^2} J_k - \nu^2. \tag{55}
\]

with \(J_k = r_k^2 w_k, w_k = \sqrt{m_k^2 + \nu^2}\). We use an identity \(\nu^2 = \sum_k r_k^2 \nu^2 = \sum_k r_k^2 (w_k^2 - m_k^2)\) to see that the two relations \(54\) and \(55\) coincide for \(\Delta = \mathcal{E} \sqrt{\lambda}\). These agreements imply that the symmetric vertex operator \(15\) is a natural candidate describing the circular spinning string with quantum numbers like spins \((S,J_k)\) and winding numbers \((n,m_k)\) in \(AdS_5 \times S^5\).

### 3 Three-point correlator

Let us consider the three-point correlation function of two heavy vertex operators \(V_H\) representing a large spin string state and one light vertex operator \(V_L\) representing a massless string state \[22\]

\[
<V_{H_1}(x_1)V_{H_2}(x_2)V_{L}(x_3) >. \tag{56}
\]

We choose \(V_{\Delta,S,n,J_k,m_k}(x_1)(V_{\Delta,S,n,J_k,m_k}(x_2))^\ast\) as the product of two heavy vertex operators \(V_{H_1}(x_1)V_{H_2}(x_2)\), where the vertex operators are located on the boundary points \(x_1^m = (1,0,0,0)\) and \(x_2^m = (-1,0,0,0)\) so that we use the multi-spin \((S,J_k)\) string Euclidean
solution \textbf{[15]} with \textbf{[17]} in stead of \textbf{[16]} to evaluate the three-point correlator \textbf{[56]} semiclassically. Fixing the location of $V_L(x_3)$ as $x_3^m = (0,0,0)$ we express the light vertex operator in the same form as \textbf{[7]}

$$V_L(0) = \int d^2\xi (Y_+)^{-\Delta_L} U[x_m(\xi), z(\xi), X_k(\xi)]$$

(57)

with $Y_+ = Y_4 + Y_5 = z(\xi) + z^{-1}(\xi)x_m^2(\xi)$. The three-point correlator \textbf{[56]} is estimated semiclassically by integrating over the worldsheet of the heavy string as

$$<V_{H_1}(x_1)V_{H_2}(x_2)V_L(0)> \approx \int d^2\xi (Y_+)^{-\Delta_L} U[x_m^d(\xi), z^d(\xi), X_k^d(\xi)]$$

$$= \int d^2\xi (z^d)^{\Delta_L} U[x_m^d(\xi), z^d(\xi), X_k^d(\xi)],$$

(58)

where the heavy string configuration \textbf{[15]} satisfies $z^2 + x_m^2 = 1$ and the superscript represents the stationary point solution for the two-point function of two heavy vertex operators. The structure coefficient $C_{123}$ is obtained by

$$C_{123} = \frac{<V_{H_1}(x_1)V_{H_2}(x_2)V_L(0)>}{<V_{H_1}(x_1)V_{H_2}(x_2)>} = c_{\Delta_L} \int d^2\xi (z^d)^{\Delta_L} U[x_m^d(\xi), z^d(\xi), X_k^d(\xi)],$$

(59)

where $c_{\Delta_L}$ depends on the normalization of the light vertex operator.

As the unintegrated light vertex operator for $V_L$ we choose the massless string vertex operator dual to the dilaton field with $\Delta_L \equiv \Delta$

$$V_j^{(dil)} = (Y_+)^{-\Delta}(X_3)^2 \sqrt{\lambda} \left( \partial Y_M \bar{\partial} Y^M + \frac{1}{2} \sum_{k=1}^{3} (\partial X_k \bar{\partial} \bar{X}_k + \bar{\partial} X_k \partial \bar{X}_k) \right),$$

(60)

which represents a highest weight state of $SO(2,4) \times SO(6)$ and is proportional to the string Lagrangian including the string tension $\sqrt{\lambda}/2\pi$. The marginality condition for $V_j^{(dil)}$ gives $\Delta = 4 + j$ to the leading order in the large $\sqrt{\lambda}$ expansion. The corresponding dual gauge theory operator is $tr[F_{mn}^2, Z^j + \cdots]$ which becomes the SYM Lagrangian when the KK momentum $j$ of the dilaton field is zero.

Here we devote ourselves to the $j = 0$ case. The three-point function coefficient $C_{123}$ is given by

$$C_{123} = \frac{c_{\Delta} \sqrt{\lambda}}{4} \int_{-\infty}^{\infty} \int_{0}^{2\pi} d\tau_\epsilon d\sigma z^4 \sum_{a=\tau_\epsilon,\sigma} \left[ \frac{(\partial_a x_m)^2 + (\partial_a z)^2}{z^2} + \frac{1}{2} \sum_{k=1}^{3} (\partial_a X_k \partial_a \bar{X}_k + \partial_a X_k \partial_a \bar{X}_k) \right].$$

(61)

We substitute the stationary multi-spin $(S, J_k)$ string Euclidean solution \textbf{[15]} with \textbf{[17]} into \textbf{[61]} to have

$$C_{123} = \frac{c_{\Delta} \sqrt{\lambda}}{4} \int_{-\infty}^{\infty} d\tau_\epsilon \int_{0}^{2\pi} d\sigma \kappa^2 + \sum_k r_k^2 (m_k^2 - w_k^2)\frac{\kappa^2}{(cosh \rho_0 \cosh \kappa \tau_\epsilon)^4},$$

(62)

whose integration over $\tau_\epsilon$ leads to a finite value

$$C_{123} = \frac{2c_{\Delta} \pi}{3} \frac{1}{cosh^4 \rho_0} \frac{\kappa^2 - \nu^2}{\kappa} \sqrt{\lambda}.$$  

(63)
The diagonal Virasoro constraint (55) together with
\[ \frac{E}{\kappa} - \frac{S}{\omega} = 1 \] (64)
can be expressed as
\[ \kappa^2 + \nu^2 = \frac{2n^2}{\kappa^2 + n^2}S + 2\sum_k \sqrt{\nu^2 + m_k^2 J_k}. \] (65)

The equation
\[ \sum_k r_k^2 = \sum_k \frac{J_k}{\sqrt{\nu^2 + m_k^2}} = 1 \] (66)
and (65) are solved by using the large \( J \) expansion with \( J \equiv \sum_k J_k \)
\[ \kappa = J + \frac{1}{2J^2}(\sum m_k^2 J_k + 2n^2 S) \]
\[ - \frac{1}{8J^3}(J \sum m_k^4 J_k + 4n^4 S J + 8n^2 S \sum m_k^2 J_k + 12n^4 S^2) + \cdots, \]
\[ \nu = J - \frac{1}{2J^2} \sum m_k^2 J_k + \frac{1}{8J^5}[3J \sum m_k^4 J_k - 4(\sum m_k^2 J_k)^2] \cdots. \] (67)

Combining together we have the energy-spin relation in the \( \lambda/J^2 \) expansion with total spin \( J = J\sqrt{\lambda} \)
\[ E = J + S + \frac{\lambda}{2J^2}(\sum m_k^2 J_k + n^2 S) \]
\[ - \frac{\lambda^2}{8J^3}(J \sum m_k^4 J_k + n^4 S J + 4n^2 S \sum m_k^2 J_k + 4n^4 S^2) + \cdots. \] (68)

The substitution of the expansion (67) into (63) yields
\[ C_{123} \approx \frac{4\epsilon_\Delta \pi}{3} \frac{1}{\cosh^2 \rho_0} \left[ \frac{\lambda}{J^2}(\sum m_k^2 J_k + n^2 S) \right. \]
\[ - \frac{\lambda^2}{2J^5}(J \sum m_k^4 J_k + n^4 S J + 4n^2 S \sum m_k^2 J_k + 4n^4 S^2) + \cdots \right]. \] (69)

The factor \( \cosh^2 \rho_0 = 1 + S/\sqrt{\kappa^2 + n^2} \) is expressed in the \( \lambda/J^2 \) expansion by using (67) as
\[ \cosh^2 \rho_0 = 1 + S/\sqrt{\kappa^2 + n^2} \]
\[ \left( n^2 S + \frac{S}{J}(\sum m_k^2 J_k + 2n^2 S) \right) + \cdots. \] (70)

Making the \( \lambda \)-derivative of the heavy state dimension \( \Delta \), that is, the string energy \( E \) (68) under the fixed quantum numbers like \( S, n, J_k, m_k \) we have
\[ \sqrt{\lambda} \frac{\partial \Delta}{\partial \sqrt{\lambda}} = \frac{\lambda}{J^2}(\sum m_k^2 J_k + n^2 S) \]
\[ - \frac{\lambda^2}{2J^5}(J \sum m_k^4 J_k + n^4 S J + 4n^2 S \sum m_k^2 J_k + 4n^4 S^2) + \cdots. \] (71)
We observe that the resulting expression (71) is contained in the three-point coupling $C_{123}$ (69).

If we use the string Lagrangian instead of the zero-momentum dilaton vertex operator which includes a $z^4$ factor, the three-point coupling $C_{123}$ has no $1/cosh^4 \rho_0$ factor but the integration over $\tau_1$ needs some IR cutoff. Here in the expression (62) the IR divergence is regularized by the $z^4$ factor and we show that the three-point coupling $C_{123}$ includes $\sqrt{\lambda} \frac{\partial}{\partial \sqrt{\lambda}} \Delta$ by using the large $J$ expansion. This kind of proportionality was presented by using the vertex operator procedure, where the light state is described by the zero-momentum dilaton vertex operator and the associated heavy state is given by the circular string solution with two equal spins $J_1 = J_2$ [22] or further two different spins $J_1 \neq J_2$ [24]. The relations between the derivative of the heavy state dimensions over the 't Hooft coupling $\lambda$ and the three-point couplings for the gauge theory Lagrangian and two non-BPS operators dual to heavy string states were studied at strong coupling [17] by using the wave function procedure of ref. [14] and the equality between the gauge theory generating functional for the correlators and the string partition function, where the associated heavy states are provided by the circular string solution with two equal spins $S = J$ or $J_1 = J_2$, and the giant magnon in $S^5$.

Let us make the $\sqrt{\lambda}$-derivative of the following expression obtained from (64)

$$\Delta = \kappa \sqrt{\lambda} + \frac{\kappa S}{\sqrt{\kappa^2 + n^2}}$$

keeping the quantum numbers $S$ and $n$ fixed to have

$$\frac{\partial \Delta}{\partial \sqrt{\lambda}} = \kappa + \frac{\partial \kappa}{\partial \sqrt{\lambda}} \sqrt{\lambda} \left( 1 + \frac{n^2 S}{\sqrt{\lambda} (\kappa^2 + n^2)^{3/2}} \right),$$

where $\kappa$ is regarded as a function of $\lambda$. The equation (65) is rewritten by

$$\kappa^2 + \nu^2 = \frac{2n^2}{\lambda (\kappa^2 + n^2)} S + 2 \sum_k \sqrt{\nu^2 + m_k^2} J_k,$$

whose derivative over $\sqrt{\lambda}$ under the fixed quantum numbers yields

$$\kappa \frac{\partial \kappa}{\partial \sqrt{\lambda}} \left( 1 + \frac{n^2 S}{\sqrt{\lambda} (\kappa^2 + n^2)^{3/2}} \right) + \nu \frac{\partial \nu}{\partial \sqrt{\lambda}} \left( 1 - \frac{1}{\sqrt{\lambda}} \sum_k \frac{J_k}{\sqrt{\nu^2 + m_k^2}} \right)$$

$$= -\frac{n^2 S}{\lambda \sqrt{\kappa^2 + n^2}} - \frac{1}{\lambda} \sum_k \sqrt{\nu^2 + m_k^2} J_k,$$

where $\partial \nu/\partial \sqrt{\lambda}$ can be computed from (66) but its coefficient factor already vanishes. Combining (75) with (73) we have

$$\sqrt{\lambda} \frac{\partial \Delta}{\partial \sqrt{\lambda}} = \frac{1}{\kappa} \left[ \kappa^2 \sqrt{\lambda} - \left( \frac{n^2 S}{\sqrt{\kappa^2 + n^2}} + \sum_k \sqrt{\nu^2 + m_k^2} J_k \right) \right],$$

for which we use (74) again to derive

$$\sqrt{\lambda} \frac{\partial \Delta}{\partial \sqrt{\lambda}} = \sqrt{\lambda} \frac{\kappa^2 - \nu^2}{2 \kappa}.$$
Thus without resort to the $1/J$ expansion we show exactly that $\sqrt{\lambda} \frac{\partial}{\partial \lambda} \Delta$ is contained in the relevant three-point coupling (33).

4 Conclusion

Based on the vertex operator prescription [5, 7, 8] we have constructed the two-point correlation function of the vertex operators representing the circular spinning string solution [16] with spins $(S, J)$ or $(S, J_k)$, $k = 1, 2, 3$ as well as winding numbers $(n, m)$ or $(n, m_k)$ in $AdS_5 \times S^5$. We have chosen an appropriate corresponding vertex operator and shown that the analytically continued version of this multi-spin string solution mapped onto complex plane is the same as the stationary point solution for the two-point function by demonstrating that it indeed solves the relevant equations of motion on the complex plane with the delta-function sources at two insertion points of vertex operators.

We have observed that the marginality condition of the vertex operator for the 2d scaling behavior of the semiclassically evaluated two-point function yields the same relation among energy (dimension), spins and winding numbers as is obtained from the diagonal Virasoro constraint, while the requirement for the two-point function to have the 2d conformal invariant expression gives the same relation between the spin winding-number pairs $(S, n)$ and $(J_k, m_k)$ as follows from the off-diagonal Virasoro constraint. These two coincidences confirm the validity of the proposed symmetric vertex operator with no derivatives over the worldsheet coordinates.

We have computed the three-point coupling from evaluating the three-point correlation function of two heavy string vertex operators representing the circular multi-spin $(S, J_k)$ string with winding numbers $(n, m_k)$ and one zero-momentum dilaton vertex operator by using the stationary string surface saturating the two-point correlation function of two heavy string vertex operators. By using the $1/J$ expansion we have demonstrated that the three-point coupling contains a factor representing the derivative of the dimension of the heavy string state with respect to the string tension. We have further confirmed this relation exactly by manipulating various relations between the relevant parameters and the quantum numbers of the rigid circular multi-spin string solution.

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