Comments on Works of Sudarshan in the 1950’s at Rochester

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Abstract. I shall discuss some works done by Sudarshan in the 1950’s at Rochester. Especially, the topic of magnetic moments of Σ-baryons as well as their electromagnetic mass differences was explained with some historical background. Also, works on weak interaction theory after the discovery of the V-A theory are discussed.

This paper is dedicated to the 75th birthday of Professor E.C. George Sudarshan with whom I coauthored 7 joint papers [1-7] in the 1950’s at the University of Rochester, and I will review some of these papers. Although they are now perhaps only of historical interest, it may offer some lesson of interest especially for the younger generation of physicists who are less familiar with the history of the subject. Except for the first paper [1], the rest of them [2-7] deal with further development of the pivotal V-A theory of Sudarshan and Marshak [8] in 1957 and subsequently by Feynman and Gell-Mann [9]. First, I will discuss the earliest joint paper [1] on electromagnetic mass differences among Σ-baryons.

1. Magnetic Moment of Baryons and Electromagnetic Mass Differences

1.1. Historical Background

In the early 1930’s, Heisenberg [10] for the nucleon and then Kemmer [11] for pions have proposed the notion of the isotopic spin invariance based upon SU(2) group, which has been supposed to be a exact symmetry group for strong interactions. The experimentally observed small mass differences between the proton, $p$, and the neutron, $n$ for the nucleon, $N$ as well as between $\pi^+$, $\pi^0$, and $\pi^-$ for pions have been attributed to be due to effects of electromagnetic self-energies, although the lowest order perturbation theory will lead to a incorrect sign for the mass difference, $M(n) - M(p) \simeq 1.29$ MeV. Feynman and Speisman [12] have, nevertheless, observed that we could explain the correct mass difference, if we take into account the anomalous magnetic moments of the nucleon with very sharp electromagnetic form factors. Although the latter assumption on the form factors was found later to be incorrect from studies of $e-p$ elastic scatterings in the late 1960’s, it was not yet known in 1950. Soon after the discovery of the Σ baryon, $\Sigma = (\Sigma^+, \Sigma^0, \Sigma^-)$ which belongs to the isotopic spin 1 realization of the SU(2) group, it has been noted that they have also small mass differences among them. The current experimental values [13] are

$$M(\Sigma^-) - M(\Sigma^+) \simeq (8.08 \pm 0.08) \text{ MeV},$$
Sudarshan and Marshak [14] in 1956 have proposed that these mass differences may similarly be explained by anomalous magnetic moments of these baryons, and George decided to compute them.

### 1.2. Magnetic Moment Sum Rule

Although Sakata [15] has proposed the so-called Sakata model in 1956 in which all hadrons are assumed to be bound states of three fundamental objects of $p$, $n$, and $\Lambda$, the idea was not so well-known, until Gell-Mann [16] and Zweig [17] in 1964 transformed it by introducing the quark model based upon three fractionally charged quark, $u$, $d$, and $s$ instead. (See reference 18 for some brief historical account on this subject.) At any rate, the most particle theorists around the 1950’s and early 1960’s thought that all hadrons such as $N$, $\Lambda$, $\Sigma$, $\Xi$, $\pi$, and $K$ are elementary particles with Yukawa interactions of form:

\[
H_\pi = i \left\{ g_{N\pi\pi} \left( \bar{N} \gamma_5 \Sigma N \right) + g_{\Sigma\Lambda\pi} \left( \bar{\Sigma} \gamma_5 \Sigma \Lambda + h.c. \right) \right. \\
+ g_{\Sigma\Sigma\pi \Sigma} \left( \bar{\Sigma} \times \gamma_5 \Sigma \right) + g_{\Xi\Xi\pi \Xi} \left( \bar{\Xi} \gamma_5 \Xi \right) \left\} \right. \\
\]

for pion and

\[
H_K = i \left\{ g_{NK\Lambda} \left( \bar{N} \gamma_5 \Lambda \right) K + g_{NK\Sigma} \left( \bar{N} \gamma_5 \Sigma \cdot \bar{\Sigma} \right) K \right. \\
+ g_{\Xi\Lambda\Lambda \Xi} \left( \bar{\Xi} \gamma_5 \Xi \right) i\tau_2 K + g_{\Xi\Sigma\Lambda \Xi} \left( \bar{\Xi} \cdot \bar{\Sigma} \gamma_5 \Xi \right) i\tau_2 K \left\} + h.c. \right. \\
\]

for kaon interactions.

In the lowest order perturbation calculations for these coupling constants $g_{\Sigma\Sigma\pi \Sigma}$, $g_{NK\Lambda}$ etc., George (Sudarshan) found a interesting sum rule

\[
\mu \left( \Sigma^+ \right) + \mu \left( \Sigma^- \right) = 2\mu \left( \Sigma^0 \right) \\
\]

among magnetic moments $\mu \left( \Sigma \right)$’s.

Because the relation does not appear to be accidental and seems to be of more general validity, Professor Marshak organized special research study seminars on the problem. He himself conjured up a Bohr-like model of the $\Sigma$-hyperon in which the $\Sigma$’s are regarded as bound states of core $\Lambda$ or $N$ with circulating $\pi$ or $\bar{K}$. On the basis of such a model, he could derive the sum rule of Eq. (4). However, we could find the following more satisfactory explanation. In 1954, Gell-Mann [19] and Nakano-Nishijima [20] have proposed the formula for electric charge $Q$ of hadrons in terms of the baryon number $B$ and the strangeness quantum number $S$ by

\[
Q = I_3 + \frac{1}{2} (B + S) \\
\]

where $I_3$ is the 3rd component of the iso-spin $I = (I_1, I_2, I_3)$. This formula was very successful to explain the semi-stability (against strong interactions) of these newly discovered hadrons. Corresponding to Eq. (5), the electromagnetic current operator $j^{\text{em}}_{\mu}(x)$ and hence the quantum mechanical magnetic moment operator $\mu$ must have the same iso-spin structure, i.e.,

\[
\mu = V^{(I=1)}_3 + V^{(I=0)}_0 \\
\]
where $V_3^{(I=1)}$ is the 3rd component of some iso-vector operator $V^{(I=1)}$ and $V_0^{(I=0)}$ stands for an iso-scalar one. Then, the experimentally observed magnetic moment $\mu(I_3)$ of a baryon with isotopic spin $I$ can be computed by

$$\mu(I_3) = <I, I_3|\mu|I, I_3> = aI_3 + b$$

(7)

according to the well known angular momentum theory [21]. If $I \geq 1$, then Eq. (7) will give a sum rule

$$\mu(I_3 = +1) + \mu(I_3 = -1) = 2\mu(I_3 = 0)$$

(8)

since we calculate

$$\mu(I_3 = \pm 1) = \pm a + b, \quad \mu(I_3 = 0) = b,$$

noting that constants $a$ and $b$ are independent of $I_3$. Applying this formula for the $\Sigma$'s with $I = 1$, it reproduces the sum rule Eq. (4) which holds valid independently of any dynamical details, as long as the strong interaction is invariant under the SU(2) group.

Remark 1

In 1959, Weinberg and Treiman [22] have applied the same technique to prove the validity of

$$M(\Delta^{++}) - M(\Delta^-) = 2\left\{M(\Delta^+) - M(\Delta^0)\right\}$$

for electromagnetic mass differences among the $\Delta$ resonances with $I = \frac{3}{2}$.

Remark 2

Let us consider the flavor SU(3) symmetry based upon three quarks $u, d, s$. Then, the major mass difference among them (ignoring the small mass difference between $M(u)$ and $M(d)$) behaves as a $T^3_3$ component of a octet tensor $T^\mu_\nu(\mu, \nu = 1, 2, 3)$ under the SU(3) group. This leads to the SU(3) mass formula [18] for hadrons composed of these quarks. However, we can proceed also as in the following way by using the concept of the $U$-spin [23]. In the $U$-spin basis, the mass operator $M$ with the mass splitting operator $T^3_3$ can now be expressed as

$$M = M^{(U=1)}_3 + M^{(U=0)}_0$$

(9)

analogously to Eq. (6). Then just as Eq. (7), we calculate

$$M(U_3) = <U, U_3|M|U, U_3> = a'U_3 + b'$$

(10)

for the masses of hadron multiplets with the $U$-spin $U$. Especially for $U = \frac{3}{2}$, this yields

$$M\left(U_3 = \frac{3}{2}\right) - M\left(U_3 = \frac{1}{2}\right) = M\left(U_3 = \frac{1}{2}\right) - M\left(U_3 = -\frac{1}{2}\right)$$

$$= M\left(U_3 = -\frac{1}{2}\right) - M\left(U_3 = -\frac{3}{2}\right).$$

Applying this to the baryon decouplet of $(\Omega^-, \Xi^*, Y, \Delta)$, we find the well known mass formula
\[ M (\Omega^-) - M (\Xi^-) = M (\Pi^-) - M (Y^-) = M (Y^-) - M (\Delta^-) \]

which established the SU(3) theory into a firmer ground.

**Remark 3 (Global Symmetry)**

Although this is a unsuccessful attempt, it was perhaps one of the most popular models of strong interactions before the SU(3) theory. In this model, we assume

\[ g = g_{NN\pi} = g_{\Sigma\Lambda\pi} = g_{\Sigma\Sigma\pi} = g_{\Xi\Xi\pi} \] (11)

among coupling constants of \( H_\pi \) in Eq. (2). If we set

\[
\begin{align*}
N_1 &= \begin{pmatrix} p \\ n \end{pmatrix}, & N_2 &= \begin{pmatrix} \Sigma^+ \\ \frac{1}{\sqrt{2}}(\Lambda - \Sigma^0) \end{pmatrix}, \\
N_3 &= \begin{pmatrix} \frac{1}{\sqrt{2}}(\Lambda + \Sigma^0) \\ \Sigma^- \end{pmatrix}, & N_4 &= \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix},
\end{align*}
\] (12)

then \( H_\pi \) can be rewritten (see e.g. ref. 24 and earlier references quoted therein) as

\[ H_\pi = ig \left\{ \bar{N}_1 \gamma_5 N_1 + \bar{N}_2 \gamma_5 N_2 + \bar{N}_3 \gamma_5 N_3 + \bar{N}_4 \gamma_5 N_4 \right\} \pi, \] (13)

which has a very large symmetry group of SU(2) \( \otimes \) U(4), where U(4) stands for the four-dimensional unitary group among \( N_1, N_2, N_3, \) and \( N_4. \) Although the kaon Hamiltonian \( H_K \) of Eq. (3) does not possess such a nice symmetry, it was believed at that time that \( H_K \) is small in comparison to \( H_\pi \) and can be ignored in the first order approximation. As we shall show shortly, this model in this approximation will then predict

\[
\begin{align*}
M (\Sigma^+) - M (\Sigma^-) &= 0, \\
M (\Xi^-) - M (\Xi^0) &= M(p) - M(n) \quad (14)
\end{align*}
\]

for electromagnetic mass differences [25], as well as

\[
\begin{align*}
\mu(p) &= \mu (\Sigma^+) = -\mu (\Sigma^-) = -\mu (\Xi^-), \\
\mu(n) &= -\mu (\Xi^0), \quad \mu (\Sigma^0) = \mu (\Lambda) = 0 \quad (15)
\end{align*}
\]

for magnetic moments. To demonstrate the validity of these relations, we utilize the concept of the Weyl symmetry group, which is the 4-dimensional symmetric group \( S_4 \) of 4-objects for the present problem. We note first that the electromagnetic interaction is also invariant under the Weyl symmetry of \( N_1 \leftrightarrow N_2 \) and also of \( N_3 \leftrightarrow N_4, \) this immediately proves the validity of \( \mu(p) = \mu(\Sigma^+), \) and \( \mu(\Sigma^-) = \mu(\Xi^-). \) In order to prove the rest of the relations, we consider antiparticle states \( N'_3 \) and \( N'_4 \) of \( N_3 \) and \( N_4 \) with

\[
\begin{align*}
N'_3 &= i\tau_2 C\bar{N}_3, \\
N'_4 &= i\tau_2 C\bar{N}_4
\end{align*}
\]
where $C$ is the particle-antiparticle (or charge conjugate) operator. Then $H_\pi$ is rewritten as

$$H_\pi = ig \left\{ N_1 \gamma_5 N_1 + N_2 \gamma_5 N_2 - N'_3 \gamma_5 N'_3 - N'_4 \gamma_5 N'_4 \right\} \pi.$$

Although its invariance group is no longer U(4) but the non-compact one of U(2,2), both $H_\pi$ and electromagnetic interaction are invariant under the modified Weyl reflection of $N_1 \leftrightarrow N'_3, \ N_2 \leftrightarrow N'_4, \ \pi \rightarrow -\pi$.

Since the magnetic moment of antiparticle differs in sign in comparison to that of the particle, this gives the rest of the relations in Eq. (15). Such a method also explains the validity of Eq. (14) for electromagnetic mass differences [25].

We note that relations, Eqs. (14) and (15) are very badly satisfied in comparison to the experimental values. For example, Eq. (14) implies $6.48 \text{ MeV} = -1.29 \text{ MeV}$, which is wrong in sign. These are one reason why we abandoned the global symmetry model in favor of the SU(3) theory. Note that the Coleman-Glashow relation [26] based upon the SU(3) symmetry predicts

$$\mu(p) = \mu(\Sigma^+), \quad \mu(\Sigma^-) = \mu(\Xi^-), \quad \mu(\Xi^0) = \mu(n) = 2\mu(\Lambda),$$

in contrast to Eq. (15), which are reasonably consistent with the present experimental values [13]. Also, it predicts the mass relation of

$$M(\Xi^-) - M(\Xi^0) = M(\Sigma^-) - M(\Sigma^+) + M(p) - M(n),$$

i.e.,

$$6.48 \text{ MeV} = 8.08 \text{ MeV} - 1.29 \text{ MeV} (= 6.73 \text{ MeV})$$

again in a reasonable agreement with the experiment. We remark that these SU(3) relations can be more easily derived from Weyl symmetry group $S_3$ of the SU(3) group. The same method has been extensively used by Levison, Lipkin, Meshkov [27] and by Dullemond, McFarlane, and Sudarshan [28] for various other processes. However, these topics will be covered by the talk of Professor Meshkov.

**Remark 4 (Quark Model)**

In the 1960’s, a significant paradigm change has occurred in particle physics. First, it was the quark model in which the SU(3) octet baryon can be realized as

$$p = (uud), \quad n = (ddu), \quad \Sigma^+ = (uus), \quad \Sigma^- = (dds), \quad \Xi^0 = (ssu), \quad \Xi^- = (ssd) \quad (18)$$

as bound states of three quarks $u, d,$ and $s$. Moreover, the iso-spin group SU(2) may also be violated by a small mass difference between masses of $u$ and $d$ quarks. In the naive quark model, the mass difference among hadrons are simply calculated accordingly as

$$M(n) - M(p) = M(d) - M(u),$$

$$M(\Sigma^-) - M(\Sigma^+) = 2(M(d) - M(u)),$$

$$M(\Xi^-) - M(\Xi^0) = M(d) - M(u)$$
from Eq. (18). Note also that \( M(d) = 3 \sim 7 \text{ MeV} \), \( M(u) = 1.5 \sim 3.0 \text{ MeV} \), \( M(s) \approx 95 \pm 25 \text{ MeV} \) with \( M(u)/M(d) \approx 0.3 \sim 0.6 \). Although these predict the correct sign for mass differences, it leads to a very bad relation of

\[
M(\Xi^-) - M(\Xi^0) = M(n) - M(p) = \frac{1}{2} \{ M(\Sigma^-) - M(\Sigma^+) \}.
\]

This fact shows the deficiency of the naive quark model. Moreover, a better Coleman-Glashov relation of Eq. (17) indicates the necessity of taking into account the electromagnetic interactions.

1.3. Concluding Remark

Presently, no particle theorist will doubt that these electromagnetic mass differences as well as magnetic moments of baryons can be explained from calculations based upon the lattice QCD theory. However the computation is notoriously complicated and this has not yet been achieved. Hence, the problem posed by George Sudarshan in 1956 has not yet been answered even after 50 years. Perhaps we have to wait at least another 10 years for the final answer.

2. \( \Delta I = \frac{1}{2} \) Selection Rule for Semi-leptonic Hadron Decays

This section deals with two papers in references [2] and [4]. We first note:

2.1. Historical Background

In the present standard model of the weak interaction, the weak Hamiltonian responsible for semi-leptonic hadronic decays involving the first family of particles; \( u, d, s, \mu, e, \nu_\mu \) and \( \nu_e \) is given by

\[
H_W = \frac{1}{\sqrt{2}} G \left\{ \cos \theta J_\mu^{(\Delta S=0)} + \sin \theta J_\mu^{(\Delta S=1)} \right\} \{ \bar{\nu}_\mu (1 + \gamma_5) \nu_\mu + \bar{\nu}_e (1 + \gamma_5) \nu_e \}.
\]

where the strangeness-conserving current \( J_\mu^{(\Delta S=0)} \) and the strangeness violating current \( J_\mu^{(\Delta S=1)} \) are expressed as

\[
J_\mu^{(\Delta S=0)} = \bar{\mu} \gamma_\mu (1 + \gamma_5) d,
\]

\[
J_\mu^{(\Delta S=1)} = \bar{\mu} \gamma_\mu (1 + \gamma_5) s.
\]

Note that they have particular isotopic spin structures of \( I = 1 \) and \( I = \frac{1}{2} \), respectively. However, even after the pivotal discovery of the V-A theory in 1957, by Sudarshan and Marshak, such a simple isotopic spin behavior for \( J_\mu^{(\Delta S=0)} \) and \( J_\mu^{(\Delta S=1)} \) were completely unknown. This is mostly due to the fact that the quark model did not appear until 1964. As a matter of fact we simply expressed \( J_\mu^{(\Delta S=1)} \) for example to be

\[
J_\mu^{(\Delta S=1)} = g_1 \bar{\mu} Q_\mu \Lambda + g_2 \bar{\mu} Q_\mu \Sigma^0 + g_3 \bar{\mu} Q_\mu \Sigma^- + g_4 \bar{\mu} \Xi^+ Q_\mu \Xi^- + g_5 \bar{\mu} \Sigma^0 Q_\mu \Xi^- + g_6 \bar{\mu} \Sigma^- Q_\mu \Xi^- + \ldots
\]

with \( Q_\mu = \gamma_\mu (1 + \gamma_5) \) in terms of supposedly elementary objects \( p, n, \Sigma^+, \Sigma^0, \Sigma^-, \Lambda, \Xi^0, \) and \( \Xi^- \) etc. just as Eqs. (2) and (3) for strong interaction Hamiltonians with so many unknown coupling constants \( g_1, g_2, \ldots \).
In the summer of 1958, Professor R.E. Marshak had organized a discussion seminar on this problem at Rochester by inviting very young promising theorists, Dr. S. Weinberg from Columbia University and Dr. E.C.G. Sudarshan from Harvard University. Note that Sudarshan has finished his Ph.D thesis at Rochester under Marshak in 1957 and went to Harvard as a postdoctoral fellow. As for me, I was still a graduate student at Rochester because of a problem with my thesis advisor. At any rate, after several discussions, we produced two joint papers (see ref. 2 and 3). The idea was simply that $J_{\mu}(\Delta S = 1)$ has the isotopic spin property of $I = \frac{1}{2}$.

Although this idea is now so trivial, it was regarded as quite a novel approach at that time. In fact, we could derive the so called $\Delta Q = \Delta S$ rule for $J_{\mu}(\Delta S = 1)$ from our hypothesis as follows. From the Gell-Mann-Nakano-Nishijima relation, Eq. (5), we observe

$$\Delta Q = \Delta I_3 + \frac{1}{2}(\Delta B + \Delta S) = \Delta I_3 + \frac{1}{2}\Delta S$$

(22)

since $J_{\mu}(\Delta S = 1)$ preserves the baryonic number so that $\Delta B = 0$. However, we must have $\Delta Q = \pm 1$, $\Delta S = \pm 1$ for $J_{\mu}(\Delta S = 1)$ with $\Delta I_3 = \pm \frac{1}{2}$. There are only two solutions consistent with Eq. (22) of $\Delta Q = \Delta S = 1$ and $\Delta I_3 = \frac{1}{2}$ or $\Delta Q = \Delta S = -1$ and $\Delta I_3 = -\frac{1}{2}$, both of which satisfies $\Delta Q = \Delta S$. Again, this rule is now so obvious from the structure of $J_{\mu}(\Delta S = 1)$ given by Eq. (20b). However, it was again a very new discovery at that time.

### 2.2. $\Delta I = \frac{1}{2}$ Selection Rule

Since the isotopic spin group SU(2) is expected to be a good symmetry for strong interactions, any consequence from the isospin $\frac{1}{2}$ structure of $J_{\mu}(\Delta S = 1)$ will constitute a good test for the standard model. Using the Wigner-Eckart theorem, we can then readily find the following relations

(i)

$$M\left(K^+ \rightarrow \pi^0 \ell \nu\right) = \frac{1}{\sqrt{2}} M\left(K^0 \rightarrow \pi^- \ell \nu\right),$$

$$M\left(K^0 \rightarrow \pi^- \ell \nu\right) = 0,$$

(23)

for decay rates of these semi-leptonic modes. Especially, this implies

$$M\left(K^+ \rightarrow \pi^0 \ell \nu\right) = M\left(K^0_2 \rightarrow \pi^- \ell \nu\right) = M\left(K^0_1 \rightarrow \pi^- \ell \nu\right).$$

(24)

Experimentally, this yields

$$\frac{B\left(K^0_{L} \rightarrow \pi^+ e^+ \nu\right)}{\tau\left(K^0_{L}\right)} = \frac{B\left(K^0_S \rightarrow \pi^+ e^+ \nu\right)}{\tau\left(K^0_S\right)} = \frac{2B\left(K^+ \rightarrow \pi^0 e^+ \nu\right)}{\tau\left(K^+\right)}$$

for branching rates $B$ and decay lifetime $\tau$. Experimentally, it is quite well satisfied to be given by

$$7.94 \times 10^6 \text{sec}^{-1} = 7.85 \times 10^6 \text{sec}^{-1} = 7.90 \times 10^6 \text{sec}^{-1}.$$

(ii) Similarly, for muon decays

$$\frac{B\left(K^0_{L} \rightarrow \pi^+ \mu^+ \nu\right)}{\tau\left(K^0_{L}\right)} = \frac{2B\left(K^+ \rightarrow \pi^0 \mu^+ \nu\right)}{\tau\left(K^+\right)},$$
which should be compared experimentally
\[ 5.29 \times 10^6 \text{sec}^{-1} = 5.32 \times 10^6 \text{sec}. \]

(iii) We would get
\[
M \left( \Sigma^+ \rightarrow n\bar{\ell}\nu \right) = 0,
M \left( \Xi^0 \rightarrow \Sigma^-\bar{\ell}\nu \right) = 0.
\]

The current experimental limit for decay rates are
\[ \frac{R(\Sigma^+ \rightarrow ne^+\nu)}{R(\Sigma^- \rightarrow ne\bar{\nu})} < 0.9 \times 10^{-2}. \]

(iv) We also find
\[
M \left( \Xi^0 \rightarrow \Sigma^+\bar{\ell}\nu \right) = \sqrt{2}M \left( \Xi^- \rightarrow \Sigma^0\ell\nu \right)
\]
which gives
\[
\frac{B(\Xi^0 \rightarrow \Sigma^+e\bar{\nu})}{\tau(\Xi^0)} = \frac{2B(\Xi^- \rightarrow \Sigma^0e\bar{\nu})}{\tau(\Xi^-)}
\]
or
\[ 1.0 \times 10^6 \text{sec}^{-1} = 0.93 \times 10^6 \text{sec}^{-1} \]

(v) S. Oneda has noted the validity of
\[
M \left( K^+ \rightarrow \pi^+\pi^+e\bar{\nu} \right) = 0,
M \left( K^0 \rightarrow \pi^+\pi^0e\bar{\nu} \right) = 0,
\]
as well as a sum rule
\[
\sqrt{2}M \left( K_2^0 \rightarrow \pi^0\pi^-e^+\nu \right) = \sqrt{2}M \left( K_1^0 \rightarrow \pi^0\pi^-e^+\nu \right)
= \frac{1}{\sqrt{2}}M \left( K^+ \rightarrow \pi^0\pi^+e^+\nu \right) + M \left( K^+ \rightarrow \pi^+\pi^-e^+\nu \right)
\]
on the basis of the \( \Delta I = \frac{1}{2} \). The latter can give a triangle inequality of
\[
\left| R(\pi^+\pi^-e^+\nu) - \frac{\sqrt{2}}{2}R(\pi^0\pi^+e^+\nu) \right|
\leq \sqrt{R(\pi^+\pi^-e^+\nu)} \leq \sqrt{R(\pi^+\pi^-e^+\nu)} + \sqrt{\frac{\sqrt{2}}{2}R(\pi^0\pi^+e^+\nu)}
\]
for the decay rates. This should be compared to the experimental values
\[ 36.6 - 29.3 \leq 57.1 \leq 36.6 + 29.3 \ (= 65.9). \]
2.3. Conclusion
The $I = \frac{1}{2}$ current hypothesis for $J^{(\Delta S=1)}_\mu$ is now just a simple consequence of the standard model of Weinberg-Salam-Glashow theory of the weak interaction. Nevertheless, it provides good experimental tests of the theory in good agreements with currently available data. In this sense, our 1958 paper has stood a test of 50 years of time.

3. Conserved Current Hypothesis for Vector Part of $J^{(\Delta S=1)}_\mu$
This is based upon two papers of ref. 3 and 7.

3.1. Historical Background
Now, we know that $\Delta S = 0$ vector current $V^{(\Delta S=0)}_\mu = \pi \gamma_\mu d$ will satisfy the conservation law

$$\partial^\mu V^{(\Delta S=0)}_\mu = 0$$

in the SU(2) limit in which we ignore the small mass difference of $M(d) - M(u)$. This leads to no renormalization theorem for various $\Delta S = 0$ semi-leptonic decays. We wondered in 1958, whether a similar situation exists for the vector part of $\Delta S = 1$ current, i.e.,

$$\partial^\mu V^{(\Delta S=1)}_\mu = \partial^\mu (\pi \gamma_\mu s) = 0$$

in the current terminology. We now know that this is possible for the SU(3) limit of $M(s) = M(u)$, which is far worse than that of the SU(2) limit. However, in 1958, the situation was moot, and we computed its possible consequence for semi-leptonic decay modes $K \rightarrow \pi \ell \nu$.

3.2. Conclusion
As is expected, the hypothesis yields a pole in the variable $q^2$ ($q$ is the momentum transfer between $K$ and $\pi$ mesons) for one of the decay form factors. Now we know that this is an unrealistic consequence of the conservation law. The correct approach is to compute these decay form factors directly from the lattice QCD and we have nothing more to say about the subject.

4. Asymmetry Parameters of $\Xi$ and $\Lambda$ Decays
The paper of ref. 6 was concerned with the asymmetry parameters of non-leptonic decays of $\Xi \rightarrow \Lambda \pi$ followed by $\Lambda \rightarrow N \pi$. If the parity is violated in these hadronic decays, the decay matrix element of $\Lambda \rightarrow p \pi^-$ for example will be given by

$$M(\Lambda \rightarrow p \pi^-) = A + B \sigma \cdot n$$

where $n = \frac{p}{|p|}$ for the momentum $p$ of the decaying pion in the rest frame of $\Lambda$. The asymmetry parameters $\alpha$, $\beta$, $\gamma$ of the decay are defined by

$$\alpha = \frac{2 \Re(A^* B)}{|A|^2 + |B|^2}, \quad \beta = \frac{2 \Im(A^* B)}{|A|^2 + |B|^2},$$

$$\gamma = \frac{|A|^2 - |B|^2}{|A|^2 + |B|^2}$$

with $\alpha^2 + \beta^2 + \gamma^2 = 1$. The parity violation implies $AB \neq 0$. Especially, if $\alpha \neq 0$ or $\beta \neq 0$, the parity is violated in the decay.
Now consider the successive decay mode of $\Xi^- \to \Lambda \pi^-$ followed by $\Lambda \to p \pi^-$. Then, we discovered [6] that the angular correlation of two decaying pions has a simple form of

$$P(\theta) = 1 + \alpha_\Lambda \alpha_\Xi \cos \theta$$

(31)

where

$$\cos \theta = \frac{p_1 \cdot p_2}{|p_1| \cdot |p_2|}$$

is the cosine of the angle $\theta$ for two pion moments $p_1$ and $p_2$. Here the second pion momentum $p_2$ must be measured in the rest frame of $\Lambda$. The important point is that the formula $P(\theta)$ does not depend upon polarization of the initial $\Xi$. Hence, by measuring $P(\theta)$, we can experimentally determine $\alpha_\Lambda \alpha_\Xi$. The currently available experimental values [13] are

$$\begin{align*}
\alpha_{\Lambda \to p \pi^-} & = 0.64, \\
\alpha_{\Lambda \to n \pi^0} & = 0.65, \\
\alpha_{\Xi^- \to \Lambda \pi^-} & = -0.46, \\
\alpha_{\Xi^0 \to \Lambda \pi^0} & = -0.41
\end{align*}$$

which are non-zero, indicating the parity violations for all these hadronic decays.

5. V-A Theory and $\Lambda$-Decay

In ref. 5, we attempted to compute the decay rate and the asymmetry parameter $\alpha_\Lambda$ for $\Lambda \to p \pi^-$ decay from the first principle based upon the V-A theory. We used a dispersion theoretical calculation to estimate the effect of the final state interaction. But the attempt was definitely premature. We have yet to wait for the correct calculation based upon the lattice QCD.

5.1. Acknowledgement

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