Colour Screening, Quark Propagation in Nuclear Matter and the Broadening of the Momentum Distribution of Drell-Yan Pairs

J. Dolejší\textsuperscript{1,2}, J. Hüfner\textsuperscript{1}, B.Z.Kopeliovich\textsuperscript{1,3}

\textsuperscript{1} Inst. of Theoretical Physics, University of Heidelberg
Philosophenweg 19, D-6900 Heidelberg

\textsuperscript{2}Dept. of Nuclear Physics
Fac. of Mathematics and Physics, Charles University
V Holešovičkách 2, CS-18000 Praha 8

\textsuperscript{3} Laboratory of Nuclear Problems
Joint Institute for Nuclear Research
Head Post Office, P.O. Box 79
101000 Moscow, Russia

Abstract

We calculate the broadening of the transverse momentum distribution of a quark propagating through nuclear matter. Colour screening plays a fundamental role in that it cuts off quark-nucleon interactions with soft gluons. The mean transverse momentum of the quark acquired along its trajectory, observed via Drell-Yan pairs, is related to it the ratio of the total inelastic meson-nucleon cross section it to the meson mean squared radius. Parameter-free calculations agree with the data.
Nuclei may be used as micro-laboratories for the observation of processes which cannot be studied otherwise, e.g. strong interaction dynamics at short-time intervals before or after an interaction. This information is completely lost in the scattering on a nucleon. Nuclei provide a unique opportunity to study properties of hadronic systems at early stages of their formation. Propagation of a quark (antiquark) through nuclear matter is one of the examples. The momentum distribution of such a quark after the multiple interaction with the target nucleons can be measured in a Drell-Yan process of lepton-antilepton pair production. Since the lepton pair does not interact on its way out of the nucleus, it carries the undistorted information about the interactions of the quark.

In this paper, we study the observable of the transverse momentum, particularly the broadening of the transverse momentum distribution of Drell-Yan pairs produced on nuclei, as compared to a nucleon target. Empirically, the increase of the mean squared transverse momentum $\langle p_T^2 \rangle$ is proportional to $L_A$, the mean length of the quark path in a nucleus before creation of the Drell-Yan pair.

$$\delta \langle p_T^2 \rangle_{DY}^A = \langle p_T^2 \rangle_{DY}^A - \langle p_T^2 \rangle_{DY}^p = \kappa \rho_A L$$

with $\kappa$ being a dimensionless and $A$-independent constant; $\rho_A$ is the nuclear density. Relation (1) resembles broadening originating from a “random walk”-process. It is the purpose of our letter to derive eq. (1) and to calculate the constant $\kappa$, starting from basic available information about strong interaction dynamics.

Confinement is a fundamental aspect of strong interactions. The colour of a quark is always compensated by the colours of other accompanying partons. While this fact is unimportant at high momentum transfers, it is crucial for soft processes. Here the colour screening cuts off all low-momentum gluons whose wavelength is longer than the screening-length scale. Mathematically, this is equivalent to a cut-off of undesirable infrared divergences.

We aim at calculating the transverse momentum distribution of a high-energy quark, which has traversed nuclear matter. We call this quark, originating from a parent high-energy hadron, the tagged quark in distinction to the screening partons which we call for simplicity “antiquarks”. At sufficiently high energies, the quark-antiquark separation is “frozen” by the Lorentz transformation, while the two partons pass the nucleus. These physical ideas can be technically realized as follows. The individual amplitudes of quark-nucleon scattering convolute to a hadron-nucleus multiple-interaction amplitude, with the tagged quark having in the final state an additional transverse momentum $\vec{q}$, relative to the initial hadron momentum. The corresponding diagram is shown in fig. 1. Let us start with the $n$-fold scattering amplitude. Its square summed over all final states of nucleons participating in the interaction and integrated over all the transferred momenta, fixing the tagged-quark
final momentum $\vec{q}$, has the form:

$$\left( \frac{d\sigma}{d^2 q} \right)_n = (2\pi)^2 \int_0^1 d\alpha \prod_{i=1}^n \frac{d^2 k_i}{(2\pi)^2} f(k_i) \left[ \sum_{m=0}^n \binom{n}{m} (-1)^{n-m} \phi\left( \vec{q} - \sum_{i=1}^m \vec{k}_i |\alpha \right) \right]^2. \quad (2)$$

While $n$ gluons are exchanged between the target nucleons and the incoming $q\bar{q}$ system, only $m$ ($\leq n$) gluons couple to the tagged quark (c.f. fig. 1). The wave function $\phi(\vec{k}|\alpha)$ of the incoming hadron depends on the transverse momentum $\vec{k}$ of the tagged quark, and the relative longitudinal momentum $0 < \alpha < 1$. The function $f(k)$ in eq. (2) includes the two-gluon-nucleon coupling and gluon propagators

$$f(k) = \frac{8}{3} \frac{\alpha^2(k)}{(k^2 + m_g^2)^2} \left[ 1 - F_N(k^2) \right], \quad (3)$$

where $F_N(k^2)$ is the two-quark form factor of a nucleon: $F_N(k^2) = \langle N | \exp(i\vec{k}(\vec{r}_1 - \vec{r}_2)) | N \rangle$. The effective gluon mass $m_g$ is introduced to take into account confinement and is inessential at large $k$. We use the one-loop approximation for the QCD coupling constant $\alpha_s(k)$ at large $k$, and freeze it at small $k$. The details can be found in [1].

After transforming the wave functions $\phi(\vec{k}|\alpha)$ in eq. (2) to the mixed $\vec{\rho}, \alpha$-representation, where $\vec{\rho}$ is a quark-antiquark separation in the impact parameter plane, eq. (2) changes into

$$\left( \frac{d\sigma}{d^2 q} \right)_n = \frac{1}{(2\pi)^2} \int_0^1 d\alpha \int d^2\rho d^2\rho' \ e^{i\vec{q}(\vec{\rho} - \vec{\rho}')} \phi(\vec{\rho}|\alpha) \phi^*(\vec{\rho}'|\alpha) \left[ \sigma(\vec{\rho}, \vec{\rho}') \right]^n. \quad (4)$$

Here

$$\sigma(\vec{\rho}, \vec{\rho}') = \int \frac{d^2 k}{(2\pi)^2} f(k) \left( 1 - e^{-i\vec{k}\vec{\rho}} \right) \left( 1 - e^{i\vec{k}\vec{\rho}'} \right). \quad (5)$$

Note that $\sigma(\vec{\rho}, \vec{\rho}')$ can be represented as

$$\sigma(\vec{\rho}, \vec{\rho}') = \frac{1}{2} \sigma(\vec{\rho}) + \frac{1}{2} \sigma(\vec{\rho}') - \frac{1}{2} \sigma(\vec{\rho} - \vec{\rho}'), \quad (6)$$

where

$$\sigma(\vec{\rho}) = 2 \int \frac{d^2 k}{(2\pi)^2} f(k^2) \left( 1 - e^{-i\vec{k}\vec{\rho}} \right) \quad (7)$$

is the inelastic interaction cross section of $\bar{q}q$-pair with separation $\rho$, with a nucleon [2].

We sum eq. (4) over $n$ with weight factors $T^n(b)/n!$, where $T(b)$ is the nuclear thickness function at the impact parameter $b$, $T(b) = \int_{-\infty}^{\infty} \rho_A(b, z) dz$. In the Drell-Yan process one has to include the term with $n = 0$, which corresponds to no
initial state interaction of the incoming hadron in the nucleus before lepton-pair
production. One also has to take into account the absorptive corrections, which in b-
representation have a simple form, \( \exp\left\{-\frac{1}{2}[\sigma(\vec{\rho}) + \sigma(\vec{\rho}')] \right\} \) being the probability amplitude to have no inelastic interactions in the initial and final state.

Putting the factors together and using eq. (6) we get

\[
\frac{d\sigma}{d^2q} = 1 \left(\frac{2\pi}{\rho}\right)^2 \int d^2b \int_0^1 d\alpha \int d^2\rho d^2\rho' e^{i\vec{q}(\vec{\rho}-\vec{\rho}')} \phi(\vec{\rho}|\alpha) \phi^*(\vec{\rho}'|\alpha) e^{-\frac{1}{2}\sigma(\vec{\rho}-\vec{\rho})} T(b). \tag{8}
\]

The integration over \( \vec{q} \) at fixed impact parameter gives 1, since \( \sigma(0) = 0 \), eq. (7). This result could have been anticipated since the quark does not disappear in the integration of all momenta.

Expression eq. (8) is the main technical result of this paper. It is of more general nature than the double-gluon approximation used as an input. One can use any QCD-inspired model for \( \sigma(\vec{\rho}) \). Nevertheless we will use the results of the two-gluon approximation with running coupling \( \alpha_s(k) \), of ref. [1]. For small \( \rho^2, \rho^2 \ll 1 \text{ fm}^2 \), where perturbative QCD is valid, one gets

\[
\sigma(\vec{\rho}) = C\rho^2,
\]

with a dimensionless constant \( C = 3.2 \). The limit of small \( \rho \) is given in Drell-Yan production of heavy lepton pair to be discussed below, since the virtuality of the tagged quark is of the order of the squared mass of the pair.

In addition we use a Gaussian approximation for the hadron wave function in transverse direction \( \phi(\vec{r}) = \sqrt{\mu_h^2/\pi} \exp(-\frac{1}{2}\mu_h^2 r^2) \), we obtain the cross section eq. (8) in the explicit form

\[
\frac{d\sigma}{d^2q} = \int d^2b \frac{1}{\pi[\mu_h^2 + 2CT(b)]} e^{-q^2/[\mu_h^2 + 2CT(b)]}. \tag{10}
\]

With the help of eq. (10), one can derive an expression for the cross section of the Drell-Yan process on a nucleus. We combine the transverse momentum distribution of the tagged projectile quark after it has traversed the nuclear thickness \( T(b, z) = \int_{-\infty}^{z} dz' \rho_A(b, z') \) with the transverse momentum distribution of an antiquark in a target nucleon to obtain

\[
\frac{d\sigma_{hA}^{hN}}{d^2q} = \sigma_{hN}^{hN} \int d^2b \int_{-\infty}^{\infty} dz \rho_A(b, z) \frac{1}{\pi[<p_T^2>_{hN}^{hN} + 2CT(b, z)]} e^{-q^2/[<p_T^2>_{hN}^{hN} + 2CT(b, z)]}. \tag{11}
\]

Here, \( \sigma_{hN}^{hN} \) is the total cross section of the Drell-Yan process on a nucleon and \( <p_T^2>_{hN}^{hN} \) the mean squared transverse momentum of the lepton pair produced in a \( hN \) collision.
One deduces from eq. (11) that the increase of the mean squared transverse momentum of the lepton pair produced on a nucleus compared to a nucleon as target, is

$$\delta <p_T^2>_{hA}^{DY} = <p_T^2>_{hA}^{DY} - <p_T^2>_{hN}^{DY} = C <T>,$$

(12)

where \(<T> = \frac{1}{A} \int d^2b T^2(b)\), is the mean nuclear thickness. The result eq. (12) confirms the empirical observation mentioned earlier eq. (1), that the broadening of \(<p_T^2>_{hA}^{DY}\) of a lepton-pair produced on a nucleus is proportional to the mean length of path of the projectile quark in the nucleus, \(L_A = <T> / 2\rho_A\). Moreover,

$$\kappa = 2C.$$  

(13)

Eq. (13) relates the constant of proportionality \(\kappa\) in eq. (1) for the mean squared momentum transfer \(\delta <p_T^2>\) to the proportionality constant \(C\), which governs the inelastic cross section \(\sigma(\rho) = C\rho^2\) eq. (1) for the interaction of a \(q\bar{q}\) pair transverse separation \(\rho\) with a nucleon. It is remarkable that according to eq. (13) the constant \(\kappa\) is independent of properties of the colliding hadrons and of the kinematics of the Drell-Yan process.

The experiment NA10 [3] has measured a value

$$\delta <p_T^2>_{\pi p}^{DY} = 0.15 \pm 0.03^{\text{stat}} \pm 0.03^{\text{syst}} \text{(GeV/c)}^2$$  

(14)

for incoming pions at 140 and 286 GeV, while the experiment E772 with 800 GeV protons [4] reports a value

$$\delta <p_T^2>_{pW}^{DY} = 0.113 \pm 0.016 \text{(GeV/c)}^2.$$  

(15)

The two values coincide within the error bars which implies that no dependence on the type of incident hadron and its energy is visible, as expected from our expression eq. (13). The expression eq. (12) predicts a value

$$\delta <p_T^2>_{hW}^{DY} = 0.17 \text{(GeV/c)}^2$$  

(16)

for \(C = 3.2\) from [1] in fair agreement with the data.

We calculate the ratio of differential cross sections of the Drell-Yan process on nucleus to nucleon targets, \(R(A/N)\), normalized with atomic weight \(A\). We use expression (11) with \(<p_T^2>_{hN}^{DY}\) from the systematics of [3] of D-Y data.

$$<p_T^2>_{DY} = \frac{4}{\pi} <p_T>^2.$$  

(17)

Here, \(s\) is the square of the total c.m. energy in GeV. Assuming a Gaussian form for the \(p_T\)-distribution, we use \(<p_T^2> = \frac{4}{\pi} <p_T>^2\).
The results of calculations of $R(A/N)$ are compared with experimental data [3,6] on fig. 2. One can see that our parameter-free calculations provide a good description of experimental data. The flattening of the $p_T$-dependence of $R(A/N)$ at high energies observed experimentally is a result of the increase of $\langle p_T^2 \rangle_{DY}$ as a function of energy.

Conclusions

- The phenomenon of broadening of the transverse momentum distribution of a quark propagating through nuclear matter crucially depends on the effect of colour screening in soft interactions. While the cross section $\sigma(\rho)$ for the individual quark-nucleon scattering decreases with decreasing screening radius $\rho$, the mean transverse momentum picked up in each collision is inversely proportional to $\rho$. The product of $\langle p_T^2 \rangle \cdot \sigma(\rho)$ in each collision is therefore independent of $\rho$ and equals the constant $2C$. The smaller the screening radius, the rarer the quark rescatters, but the larger is mean transverse momentum in each rescattering.

- We calculate the cross section $\sigma(\rho)$ in the one-gluon exchange approximation for the hadron inelastic scattering amplitude. This approach is justified because of the high virtuality of the quarks participating in the Drell-Yan reaction.

- Analogous consideration may be applied to the hadroproduction of charmonia on nuclei which predominantly proceed via gluon fusion. We expect an approximate doubling of the broadening effect due to the additional factor $9/4$ in the cross section of the gluon-nucleon interaction. Experimental data on $J/\Psi$ production on nuclei at 200 GeV confirm the expected increase and give values for $\kappa_{J/\Psi}/\kappa_{DY} = 1.9 \pm 0.4$ [3] and $1.6 \pm 0.25$ [7]. However, for the $\Upsilon$ production at 800 GeV [4] the experimental ratio $\kappa_{J/\Psi}/\kappa_{DY} = 5.9 \pm 1.3$ is much larger than what we would expect and needs another explanation.

- The process of inclusive production of hadrons with high transverse momenta off nuclei (“Cronin effect”) has much to do with the colour screening effect. Indeed the high transverse momentum is built up of many soft scatterings in a way similar to the case treated in this paper for the Drell-Yan process.

- The broadening of the transverse momentum of a quark propagating through nuclear matter, might be relevant also to the process of inclusive production of leading hadrons in deep inelastic scattering on nuclei. However, it occures
only if the hadron is produced outside the nucleus, i.e. the length of the formation zone exceeds the nuclear radius. Otherwise, the broadening becomes much weaker than in the Drell-Yan process, as the colourless hadron should reinteract in the nucleus with a small elastic cross section.

Acknowledgements: J. D. and B. Z. K. gratefully acknowledge the financial support of their visit in Heidelberg by the Max-Planck-Institut für Kernphysik. The work has been supported in part by the Bundesministerium für Forschung und Technologie (BMFT) under contract number 06 HD 710.

References

[1] B. Z. Kopeliovich and B. G. Zakharov: Phys. Rev D 44 (1991) 3466.

[2] A. B. Zamolodchikov, B. Z. Kopeliovich, L. I. Lapidus: JETP Lett. 33 (1981) 595.

[3] P. Bordalo et al.: Phys. Lett. B 193 (1987) 373.

[4] D. M. Alde et al.: Phys. Rev. Lett. 66 (1991) 2285.

[5] C. Grosso-Pilcher and M. J. Shocket: Ann. Rev. Nucl. Part. Sci. 36 (1986) 1

[6] D. M. Alde et al.: Phys. Rev. Lett. 64 (1990) 2479, M. J. Leitch et al.: Nucl. Phys. A522 (1991) 351c.

[7] C. Baglin et al.: Phys. Lett. B 268 (1991) 453.

Figure captions

Fig. 1: The inelastic interaction of a quark $q$ and a antiquark $\bar{q}$ with $n$ nucleons of target nucleus.

Fig. 2: The ratios $R(A/B)$ of the Drell-Yan cross sections $d\sigma^{hA}_{DY}/d^2p_T$ and $d\sigma^{hB}_{DY}/d^2p_T$ as a function of the transverse momentum $p_t$ of the D-Y pair. The data from experiments with various projectiles, targets and energies are compared with the calculation based on eq. (11).