A model of supernova feedback in galaxy formation

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ABSTRACT
A model of supernova feedback during disc galaxy formation is developed. The model incorporates infall of cooling gas from a halo, and outflow of hot gas from a multiphase interstellar medium (ISM). The star formation rate is determined by balancing the energy dissipated in collisions between cold gas clouds with that supplied by supernovae in a disc marginally unstable to axisymmetric instabilities. Hot gas is created by thermal evaporation of cold gas clouds in supernova remnants, and criteria are derived to estimate the characteristic temperature and density of the hot component and hence the net mass outflow rate. A number of refinements of the model are investigated, including a simple model of a galactic fountain, the response of the cold component to the pressure of the hot gas, pressure-induced star formation and chemical evolution. The main conclusion of this paper is that low rates of star formation can expel a large fraction of the gas from a dwarf galaxy. For example, a galaxy with circular speed $v_c \approx 50 \text{ km s}^{-1}$ can expel $60 \pm 80$ per cent of its gas over a time-scale of $\sim 1 \text{ Gyr}$, with a star formation rate that never exceeds $0.1 \text{ M}_\odot \text{ yr}^{-1}$. Effective feedback can therefore take place in a quiescent mode and does not require strong bursts of star formation. Even a large galaxy, such as the Milky Way, might have lost as much as 20 per cent of its mass in a supernova-driven wind. The models developed here suggest that dwarf galaxies at high redshifts will have low average star formation rates and may contain extended gaseous discs of largely unprocessed gas. Such extended gaseous discs might explain the numbers, metallicities and metallicity dispersions of damped Lyman $\alpha$ systems.

Key words: stars: formation – supernovae: general – supernova remnants – galaxies: formation – galaxies: ISM – galaxies: starburst.

1 INTRODUCTION
Since the pioneering paper of White & Rees (1978), it has been clear that some type of feedback mechanism is required to explain the shape of the galaxy luminosity function in hierarchical clustering theories. The reason for this is easy to understand; if the power spectrum of mass fluctuations is approximated as a power law $P(k) \propto k^n$, the Press–Schechter (1974) theory for the distribution of virialized haloes predicts a power-law dependence at low masses:

$$\frac{dN(m)}{dm} \propto m^{-(9-n)/6}.$$  (1)

For any reasonable value of the index $n$ [$n \approx -2$ on the scales relevant to galaxy formation in cold dark matter (CDM) models], equation (1) predicts a much steeper mass spectrum than the observed faint-end slope of the galaxy luminosity function, $dN(L)/dL \propto L^\alpha$, with $\alpha \approx -1$ (Efstathiou, Ellis & Peterson 1988; Loveday et al. 1992; Zucca et al. 1997). Furthermore, the cooling times of collisionally ionized gas clouds forming at high redshift are short compared to the Hubble time (Rees & Ostriker 1977; White & Rees 1978). Thus, in the absence of feedback, one would expect that a large fraction of the baryons would have collapsed at high redshift into low-mass dark matter haloes, in contradiction with observations.

In reality, there are a number of complex physical mechanisms that can influence galaxy formation, and these need to be understood if we are to construct a realistic model of galaxy formation. In the ‘standard’ CDM model (i.e., nearly scale-invariant adiabatic perturbations), the first generation of collapsed objects will form in haloes with low virial temperatures ($T \lesssim 10^5 \text{ K}$, characteristic circular speeds $v_c \approx 20 \text{ km s}^{-1}$). Molecular hydrogen is the dominant coolant at such low temperatures, and so an analysis of the formation of the first stellar objects requires an understanding of the molecular hydrogen abundance and how this is influenced by the ambient ultraviolet radiation field (Haiman, Rees & Loeb 1997; Haiman, Abel & Rees 2000). As the background UV flux rises, the temperature of the intergalactic
medium will rise to $\sim 10^4$ K (e.g., Gnedin & Ostriker 1997), and the UV background will reduce the effectiveness of cooling in low-density, highly ionized gas (Efstathiou 1992). A UV background can therefore suppress the collapse of gas in regions of low overdensity. It is this low-density photoionized gas that we believe accounts for the Ly$\alpha$ absorption lines (Cen et al. 1994; Hernquist et al. 1996; Theuns et al. 1998; Bryan et al. 1999). Photoionization can also suppress the collapse of gas in haloes with circular speeds of up to $v_c \sim 20-30$ km s$^{-1}$. However, numerical simulations have shown that a UV background cannot prevent the collapse of gas in haloes with higher circular speeds, although it can reduce significantly the efficiency with which low-density gas is accreted on to massive galaxies (Quinn, Katz & Efstathiou 1996; Navarro & Steinmetz 1997).

To explain the galaxy luminosity function, feedback is required in galaxies with circular speeds $v_c \gtrsim 50$ km s$^{-1}$ and characteristic virial temperatures of $\gtrsim 10^4$ K. Energy injection from supernovae is probably the most plausible feedback mechanism for systems with such high virial temperatures. Winds from quasars might also disrupt galaxy formation (Silk & Rees 1998) or, more plausibly, limit the growth of the central black hole (Fabian 1999). Here we will be concerned exclusively with supernova-driven feedback, and we will not consider feedback from an active nucleus. Simple parametric models of supernova feedback were developed by White & Rees (1978) and White & Frenk (1991), and form a key ingredient of semi-analytic models of galaxy formation (e.g., Kauffmann, White & Guiderdoni 1993; Lacey et al. 1993; Cole et al. 1994; Baugh, Cole & Frenk 1996; Baugh et al. 1998; Somerville & Primack 1999). In this paper we develop a more detailed model of the feedback process itself. Previous papers on supernova feedback include those of Larson (1974), Dekel & Silk (1986) and Babul & Rees (1992). These authors compute the energy injected by supernovae into a uniform interstellar medium (ISM) and apply a simple binding energy criterion to assess whether the ISM will be driven out of the galaxy. The feedback process in these models is explosive, operating on the characteristic time-scale of $\sim 10^9 - 10^{10}$ yr for supernova remnants to overlap. This is much shorter than the typical infall time-scale of hot gas in the halo, begging the question of how a reservoir of cold gas accumulated in the first place. The present paper differs in that we model the ISM as a two-phase medium consisting of cold clouds and a hot pressure-confining medium, i.e., as a simplified version of the three-phase model of the ISM developed by McKee & Ostriker (1977, hereafter MO77). The cold component contains most of the gaseous mass of the disc and is converted into a hot phase by thermal evaporation in expanding supernova remnants. In this type of model, the cold phase can be lost gradually in a galactic wind as it is slowly converted into a hot phase.

The main result of this paper is that low rates of star formation can expel a large fraction of the baryonic mass in dwarf galaxies over a relatively long time-scale of $\sim 1$ Gyr. We therefore propose that effective feedback can operate in an steady, unspectacular mode; strong bursts of star formation and superwind-like phenomena (e.g. Heckman, Armus & Miley 1990) are not required, although galaxies may experience additional feedback of this sort. In fact, hydrodynamic simulations suggest that nuclear starbursts are ineffective in removing the ISM from galaxies with gas masses $\gtrsim 10^{10} M_\odot$ (MacLow & Ferrara 1999; Strickland & Stevens 2000), because hot gas generated in the nuclear regions is expelled in a bipolar outflow without coupling to the cool gas in the rest of the disc. This result provides additional motivation for investigating a ‘quiescent’ mode of feedback. Silk (1997) describes a model which is similar, in some respects, to the model described here. However, the model described here is more detailed and allows a crude investigation of the radial properties of a disc galaxy during formation. A model of the evolution of a multiphase ISM and its consequences for the formation of dwarf galaxies is described by Norman & Spaans (1997) and Spaans & Norman (1997), although these authors concentrate on the effects of feedback processes on the efficiency of star formation rather than on outflows. Numerical simulations of the evolution of massive galaxies that include a multiphase model of the ISM are described by Thies, Burkert & Hensler (1992) and Sarma, Hensler & Thies (1997).

The layout of this paper is as follows. A simple model of star formation regulated by disc instabilities is described in Section 2. This is applied to ‘closed box’ (i.e., no infall or outflow of gas) models of disc galaxies neglecting feedback. Section 3 describes a model of the interaction of expanding supernova shells in a two-phase ISM. This section is based on the model of MO77, but instead of focusing on equilibrium solutions that might apply to our own Galaxy, we compute the net rate of conversion of cold gas to hot gas incorporating the model for self-regulating star formation. This yields the temperature and density of the hot phase as a function of time and radius within the disc. Section 4 revisits the model of Section 2, but includes simultaneous infall and outflow of gas. This model is extended in Section 5 to include a galactic fountain, the pressure response of the cold ISM to the hot phase, and a model of chemical evolution. Section 6 describes some results from this model and discusses the effects of varying some of the input parameters. In addition, the efficiency of feedback is computed as a function of the circular speed of the surrounding dark matter halo. Our conclusions are summarized in Section 7. Although we focus on disc galaxies in this paper, a similar formalism could be applied to the formation of bulges if the assumption that gas conserves its angular momentum during collapse is relaxed.

## 2 STAR FORMATION REGULATED BY DISC INSTABILITIES

### 2.1 Rotation curve for the disc and halo

The dark halo is assumed to be described by the Navarro, Frenk & White (1996, hereafter NFW) profile

$$
\rho_h(r) = \frac{\delta_c \rho_c}{(c^2 x)(1 + cx)^2}; \quad x = r/r_v,
$$

where $\rho_c$ is the critical density, $r_v$ is the virial radius at which the halo has a mean overdensity of 200 with respect to the background, and $c$ is a concentration parameter (approximately 10 for CDM models). The circular speed corresponding to this profile is

$$
v^2_h(r) = \frac{\delta_c}{x^2} \frac{1}{x} \frac{[\ln(1+cx) - cx/(1+cx)]}{\ln(1+c) - c/(1+c)}; \quad v^2_h = \frac{GM_r}{r_c},
$$

where $M_r$ is the mass of the halo within the virial radius.

We assume that the disc surface mass density distribution is described by an exponential,

$$
\rho_0(r) = \mu_0 \exp(-r/r_D), \quad M_D = 2\pi \mu_0 r_D^2,
$$

where $M_D$ is the total disc mass. The rotation curve of a cold exponential disc is given by (Freeman 1970)

$$
v^2_D(r) = 2\alpha^2 \left[ I_0(y) K_0(y) - I_1(y) K_1(y) \right],
$$

where $I_0(y)$ and $K_0(y)$ are modified Bessel functions of the first and second kind, respectively.

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y = 1 \frac{r}{2 r_0}, \quad v_c^2 = \frac{GM_\odot}{r_0}.

To relate the disc scalelength, $r_0$, to the virial radius of the halo $r_v$, we assume that the angular momentum of the disc material acquired by tidal torques is approximately conserved during the collapse of the disc (see Fall & Efstathiou 1980). This fixes the collapse factor

$$f_{\text{coll}} = \frac{r_v}{r_0} \quad (6)$$

in terms of the dimensionless spin parameter $\lambda_H = J |E|^{1/2} G^{-1} M_{\odot}^{-3/2}$ of the halo component. The spin parameter is found to have a median value of $\approx 0.05$ from $N$-body simulations (Barnes & Efstathiou 1987), and for the models described here this value is reproduced for collapse factors of around 50. A more detailed calculation of the collapse factor of the disc is given in Section 4.

### 2.2 Vertical scaleheight of the disc

The velocity dispersion of the cold gas clouds in the vertical direction is assumed to be constant and equal to $\sigma_z^2$. The equations of stellar hydrodynamics then give the following solution

$$\rho(z) = \frac{\mu_g}{2H_g} \text{sech}^2 \left( \frac{z}{H_g} \right), \quad (7)$$

where $\mu_g$ is the surface mass density of the gas, and the scale-height is given by

$$H_g = \frac{\sigma_z^2}{\pi G \mu_g}. \quad (8)$$

Equation (8) must be modified to take into account the stellar disc. We do this approximately by assuming ‘disc pressure equilibrium’ (Talbot & Arnett 1975)

$$H_g = \frac{\sigma_z^2}{\pi G \mu_*} \frac{1}{(1 + \beta/\alpha)}, \quad (9)$$

where the quantities $\alpha$ and $\beta$ relate the vertical velocity dispersion $\sigma_z^2$ and surface mass density $\mu_*$ of the stars to those of the gas clouds:

$$\sigma_z = \alpha \sigma_g, \quad (10a)$$

$$\mu_* = \beta \mu_g. \quad (10b)$$

### 2.3 Stability of a two-component rotating disc

The stability of rotating discs of gas and collisionless particles to axisymmetric modes has been analysed in classic papers by Toomre (1964) and Goldreich & Lynden-Bell (1965). Here we use the results of Jog & Solomon (1984), who analysed the stability of a rotating disc consisting of two isothermal fluids of sound speeds $c_1$ and $c_2$, and surface mass densities $\mu_1$ and $\mu_2$. These authors find that such a disc is stable to axisymmetric modes of wavenumber $k$ if

$$x = \frac{2\pi G \mu_1}{\kappa^2} \left( \frac{k}{1 + k^2 c_1^2 / \kappa^2} \right)^2 + \frac{2\pi G \mu_2}{\kappa^2} \left( \frac{k}{1 + k^2 c_2^2 / \kappa^2} \right)^2 < 1, \quad (11)$$

where $\kappa$ is the epicyclic frequency

$$\kappa = 2\omega \left( 1 + \frac{1}{2} \frac{d \omega}{d r} \right)^{1/2}. \quad (12)$$

#### Figure 1. The factor $g(\alpha, \beta)$ appearing in the stability criterion of equation (13) plotted against $\beta$ for three values of $\alpha$.

Equation (11) yields a cubic equation for the most unstable mode $k_0$. Solving this equation in terms of the parameters $\alpha$ and $\beta$ of equations (10), and ignoring the small differences between a gaseous and collisionless disc, we can write the stability criterion for a two-component system as

$$\sigma_g = \frac{\pi G \mu_g}{\kappa} g(\alpha, \beta). \quad (13)$$

This is identical to the Goldreich–Lynden-Bell criterion except for the factor $g(\alpha, \beta)$. This factor is plotted in Fig. 1 for various values of $\alpha$ and $\beta$.

### 2.4 Star formation and supernova energy input

We assume a stellar initial mass function (IMF) of the standard Salpeter (1955) form

$$\frac{dN_*}{dm} = A m^{-1.35}, \quad m_l < m < m_u, \quad x = 1.35, \quad (14)$$

$$m_l = 0.1 M_\odot, \quad m_u = 50 M_\odot,$$

and that each star of mass greater than $8 M_\odot$ releases $10^{51} E_{51} \text{erg}$ in kinetic energy in a supernova explosion. For the IMF of equation (14), one supernova is formed for every $125 M_\odot$ of star formation. The energy injection rate is therefore related to the star formation rate by

$$E_{51} = 2.5 \times 10^{51} E_{51} M_* \text{ erg s}^{-1}, \quad (15)$$

where $M_*$ is the star formation rate in $M_\odot$ per year.

### 2.5 Energy dissipated by cloud collisions

We assume cold clouds of constant density $\rho_c = 7 \times 10^{-23} \text{ g cm}^{-3}$ with a distribution of cloud radii

$$\frac{dN_{c0}}{da} = N_i a^{-4}, \quad a_l < a < a_u, \quad (16)$$

$$a_l = 0.5 \text{ pc}, \quad a_u = 10 \text{ pc},$$

(MO77). Following MO77, the clouds are assumed to have an isotropic Gaussian velocity distribution with velocity dispersion independent of cloud size and to lose energy through inelastic
collisions. The rate of energy loss per unit volume is given by

\[
\frac{dE_{\text{coll}}}{dr} = 24\pi^{3/2} \hat{\rho} N_{\text{cl}}^2 a^5 \sigma_{g5}^2 I_a,
\]

(17)

\[
I_a = \frac{1}{2} \int_{0}^{\infty} \int_{0}^{\infty} \frac{(x+y)^2}{(x^2+y^2)^{3/2}} \, dx \, dy,
\]

where \( N_{\text{cl}} \) is the local cloud density \( N_{\text{cl}} = N_0/3a^3 \). Integrating equation (17) over the vertical direction and using equation (9) for the scaleheight, the rate of energy loss per unit surface area \( E_{\text{coll}} \) is

\[
E_{\text{coll}}^0 = 5.0 \times 10^{20} \left( 1 + \frac{\beta}{\alpha} \right) \mu_{g5}^3 \sigma_{g5} \, \text{erg s}^{-1} \text{pc}^{-2},
\]

(18)

where \( \mu_{g5} \) is the surface mass density of the gas component in units of 5 M\( \odot \) pc\(^{-2} \), and \( \sigma_{g5} \) is the cloud velocity dispersion in units of 5 km s\(^{-1} \). These values are close to those observed in the local solar neighbourhood. To estimate the efficiency with which supernovae accelerate the system of clouds, we normalize to the observed net star formation rate of the Milky Way. Assuming that the gas distribution has a flat surface mass density profile to the scaleheight, the rate of energy loss per unit surface area \( E_{\text{coll}} \) is

\[
E_{\text{coll}} = \epsilon_c E_{\text{coll}}^0 = 0.004.
\]

(19)

An efficiency parameter of \( \epsilon_c = 0.01 \) produces a net star formation rate of 0.4 M\( \odot \) yr\(^{-1} \), which is reasonable for a Milky Way-like galaxy. We will therefore adopt a constant value of \( \epsilon_c = 0.01 \) in the models of the next subsection. The value of \( \epsilon_c \) will, of course, depend on the properties of the clouds, ISM and star formation rate. For example, in the model of MO77 the clouds are accelerated by interactions with the cold shells surrounding supernova remnants, and MO77 find efficiencies \( \epsilon_c \) of typically a few per cent. We investigate the effect of varying \( \epsilon_c \) in Section 6.

### 2.6 Self-regulating models without inflow or outflow

The equations derived above allow us to evolve an initially gaseous disc and to compute the local star formation rate, cloud velocity dispersion, etc. The system of stars and gas is constrained to satisfy the stability criterion of equation (13), which fixes the cloud velocity dispersion \( \sigma_c \). There is some empirical evidence that star formation in nearby galaxies is regulated by a stability criterion of this sort (e.g. Kennicutt 1998). The energy lost in cloud collisions (equation 18) is balanced against the energy input from supernovae, assuming a constant efficiency factor \( \epsilon_c = 0.01 \). We assume further that \( \alpha = 5 \) (equation 10a), i.e., that stars are instantaneously accelerated to higher random velocities than the system of gas clouds, and that the properties of the gas clouds (mass spectrum, internal density, etc.) are independent of time. These are clearly restrictive assumptions, but they allow us to generate simple models of self-regulating star formation with only one free parameter \( \epsilon_c \).

We study the evolution of two model galaxies with parameters listed in Table 1. Model MW has parameters roughly similar to those of the Milky Way, and model DW has parameters similar to those of a relatively high surface brightness dwarf galaxy.

![Figure 2](https://example.com/figure2.png)

**Figure 2.** The evolution of the gas (solid lines) and stellar (dashed lines) surface mass density distributions according to the simple self-regulating model described in this section. The results are shown for ages of 0, 0.1, 1, 3, 6 and 10 Gyr.

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**Table 1. Parameters of model galaxies.**

| Model    | \( v_c \) (km s\(^{-1} \)) | \( v_{\text{max}} \) (km s\(^{-1} \)) | \( v_c/v_{\text{max}} \) | \( r_D \) (kpc) | \( M_D \) (M\( \odot \)) | \( f_{\text{coll}} \) | \( c \) | \( \lambda_H \) |
|----------|----------------|----------------|----------------|---------------|----------------|----------------|-----|----------|
| Model MW | 280            | 212            | 0.45           | 3.0           | 5.5 \times 10^{10} | 50             | 10  | 0.065    |
| Model DW | 70             | 53             | 0.45           | 0.2           | 2.3 \times 10^{8}  | 50             | 10  | 0.065    |

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stellar disc is truncated at about the Holmberg (1958) radius \( r = r_D < 5 \); in rough agreement with observations. The truncation arises because the gas disc becomes thick at large radii (equation 9) and the rate of energy lost in cloud collisions can be balanced by a very low star formation rate.

The evolution of model DW is qualitatively similar, although the star formation rate is scaled down roughly in proportion to the disc mass. Half the gas is converted to stars by \( 3 \times 10^7 \) yr; and the gas fraction is 0.12 after \( 10^{10} \) yr, similar to the final gas fraction of 0.13 in model MW.

Neither of these models is satisfactory. The star formation rate in model MW is too high at early times to be compatible with deep number counts (see, e.g., Ellis 1997), which require more gentle star formation rates in typical \( L^* \) galaxies. Model SW converts most of its gas into stars on a short time-scale, and so does not solve the problem raised in the introduction of explaining the flat faint-end slope of the luminosity function in CDM-like models. As we will see in later sections, infall of gas provides the solution to the former problem, since this allows the disc to build up gradually on a cooling or dynamical time-scale. Outflow of hot gas heated by supernovae provides a solution to the latter problem.

### 3 EVOLUTION OF A TWO-PHASE ISM

In this section we consider the interaction of a multiphase ISM with expanding supernova remnants following the model of MO77, and discuss the conditions under which a protogalaxy can form a wind. The key ingredients of the model are as follows. Most of the cold gaseous mass is assumed to be in cold clouds with properties as given in Section 2.5. Supernovae explode, and their remnants propagate, evaporating some of the cold clouds and forming a low-density hot phase of the ISM. The star formation rate therefore determines the evaporation rate and hence the rate of conversion of the cold phase to a hot phase. A wind from the galaxy can result if the hot phase is (i) sufficiently pervasive (filling factor of order unity), (ii) low-density (so that radiative cooling is unimportant), and (iii) has a temperature exceeding the virial temperature of the galaxy. In this section we follow closely the theory of the ISM developed by MO77, and we use their notation where possible.

#### 3.1 Evaporation of cold clouds

An expanding supernova remnant will evaporate a mass of

\[
M_{ev} = 540 E_6^{6/5} \Sigma^{-3/5} n_h^{-1/5} M_\odot, \tag{20}
\]

where \( n_h \) is the density interior to the supernova remnant and \( \Sigma \) (in \( \text{pc}^2 \)) is the evaporation parameter introduced by MO77,

\[
\Sigma = \frac{\gamma}{4 \pi \alpha \Sigma \phi_k}. \tag{21}
\]

Here the parameter \( \gamma \) relates the blast wave velocity to the isothermal sound speed \( (\gamma = \gamma_0, \gamma = 2.5) \), and the parameter \( \phi_k \) quantifies the effectiveness of the classical thermal conductivity of the clouds \( (\phi_k = \phi_k) \), and so is less than unity if the conductivity is reduced by tangled magnetic fields, turbulence, etc. Using equations (7) and (9) to estimate the mean cloud density, we find

\[
\Sigma \approx 280 \left( \frac{\sigma^3}{\mu^3} \right) \frac{1}{(1 + \beta/\alpha)} \frac{1}{\phi_k} \text{pc}^2 = f_\Sigma \Sigma_\odot, \quad \Sigma_\odot \approx 95 \text{pc}^2, \tag{22}
\]

where \( \Sigma_\odot \) is the evaporation parameter characteristic of the local solar neighbourhood \( (\beta/\alpha = 2) \).

Evaluating equation (20), we find

\[
M_{ev} \approx 1390 E_6^{6/5} \Sigma^{-3/5} \phi_k^{-3/5} n_h^{-2} M_\odot, \tag{23}
\]

where \( n_h \) is \( n_h \) in units of \( 10^{-2} \) cm\(^{-2} \) (a characteristic value for the hot component). Thus, provided thermal conduction is not highly suppressed, a single supernova remnant can evaporate a much larger mass than the 125 M\( \odot \) formed in stars per supernova for a standard Salpeter IMF (Section 2.4). If a significant fraction
of this evaporated gas can escape in a wind, then star formation will be efficiently suppressed.

### 3.2 Temperature and density of the hot phase

To compute the properties of the hot phase, we assume that the disc achieves a state in which the porosity parameter $Q$ is equal to unity. The disc is then permeated by a network of overlapping supernova remnants. Ignoring cooling interior to the remnants (which will see is a reasonable approximation for an ISM with low metallicity), the age, radius and temperature of a supernova remnant when $Q = 1$ are given by

\[
t_o = 5.5 \times 10^6 S_{13}^{-5/11} \gamma^{-6/11} E_{51}^{-3/11} n_h^{3/11} \text{ yr},
\]

\[
R_o = 100 S_{13}^{-2/11} \gamma^{2/11} E_{51}^{1/11} n_h^{-1/11} \text{ pc},
\]

\[
T_o = 1.2 \times 10^4 S_{13}^{8/11} \gamma^{-6/11} E_{51}^{8/11} n_h^{-8/11} \text{ K},
\]

where $S_{13}$ is the supernova rate in units of $10^{-13}$ pc$^{-2}$ yr$^{-1}$. The density of a remnant at $t_o$ [$n_h^o = M_{\text{ev}}/(4/3 \pi R_o^3)$] gives an approximate estimate of the density of the ambient hot phase

\[
n_h^o \approx 4.3 \times 10^{-3} S_{13}^{0.36} \gamma^{-0.36} E_{51}^{0.61} J_{51}^{-0.39} \text{ cm}^{-3}.
\]

Inserting this estimate into equations (24), we find

\[
t_o = 1.2 \times 10^6 S_{13}^{-0.36} \gamma^{-0.64} (E_{51} J_{51})^{-0.11} \text{ yr},
\]

\[
R_o = 164(S_{13}/\gamma)^{-0.21} E_{51}^{0.04} J_{51}^{0.035} \text{ pc},
\]

\[
T_o = 6.6 \times 10^7 S_{13} E_{51} J_{51} / \gamma^{0.29} \text{ K},
\]

and the rate at which clouds are evaporated is

\[
M_{\text{ev}} = 2.7 \times 10^{-40} S_{13}^{0.71} \gamma^{-0.29} E_{51}^{0.71} J_{51}^{-0.29} \text{ M}_\odot \text{ pc}^{-2} \text{ yr}^{-1}.
\]

Integrating equation (27) over the scaleheight of the disc gives the evaporated mass per unit area,

\[
M_{\text{ev}}^{\text{OL}} \approx 1 \times 10^{-7} \frac{\sigma_{\gamma}^2}{\mu_{\gamma}(1 + \beta/\alpha)}
\]

\[
\times 10^{37} S_{13}^{0.71} \gamma^{-0.29} E_{51}^{0.71} J_{51}^{-0.29} \text{ M}_\odot \text{ pc}^{-2} \text{ yr}^{-1}.
\]

Adopting a cooling rate of $\Lambda = 2.5 \times 10^{-22} T_o^{-1.4}$ erg cm$^{-3}$ s$^{-1}$ for $10^5 \lesssim T \lesssim 10^6$ K for a gas with primordial composition, the ratio of $t_o$ to the cooling time $t_{\text{cool}}$ is

\[
\frac{t_o}{t_{\text{cool}}} \approx 0.5 T_5^{-2.4} J_5^{-0.5},
\]

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Thus, if the temperature of the hot phase is higher than about 10^5 K, the assumption that cooling can be neglected will be valid. A cooling function for a gas with primordial composition will be used throughout this paper. As the metallicity of the gas builds up, the cooling time of the hot component will shorten and more of the supernova energy will be lost radiatively. This effect will reduce the efficiency of feedback in galaxies with high metallicity, but is not included in this paper.

3.3 Simple self-regulating model with outflow

In this section we apply the results of the previous paragraphs to construct a simplified self-regulating model with outflow. The star formation rate is governed by the self-regulation algorithm as in Section 2.6 with the parameter $e_c = 0.01$. This provides an estimate of the local supernova rate per unit volume, which we insert in equations (26) to compute the properties of the hot phase, adopting a value $\phi_c = 0.1$ in equation (21) for the conduction efficiency parameter. The hot gas will be lost from the system if its specific enthalpy

$$\frac{1}{2} v^2 + \frac{5}{2} \rho$$

exceeds to within a factor of order unity its gravitational binding energy per unit mass. If the gas has an initial isothermal sound speed of $c_i = (kT/\mu_p)^{1/2} = 377^{1/2} \text{ km s}^{-1}$ (for a mean mass per particle of $\mu_p = 0.61m_p$), conservation of specific enthalpy implies that the wind will reach a bulk speed of $v_w = \sqrt{5c_i}$. Some of the thermal energy will be lost radiatively, and in fact the spherical steady wind solutions described in Appendix B suggest that a more accurate criterion for the wind to escape from a galaxy is $v_w = \sqrt{2.5c_i} > v_{esc}$, where $v_{esc}$ is the escape speed from the centre of the halo (neglecting the potential of the disc). If $\sqrt{2.5c_i} < v_{esc}$, the hot phase is returned instantaneously to the cold phase. This type of binding energy criterion for outflow has been adopted in previous studies (e.g. Larson 1974; Dekel & Silk 1986) and is clearly oversimplified, as are the assumptions of

Figure 5. Evolution of the star formation rate, gas fraction and gas cloud velocity dispersion for the models shown in Fig. 4.

Figure 6. Panel (a) shows the solution of the mass conservation equation (30) relating the initial halo radius $s_H = \sigma_H/r_0$ to the final disc radius $s_D = \sigma_D/r_D$. The solid line in panel (b) shows the derived rotation curve of the halo in units of $v_c = (GM_D/r_D)^{1/2}$, assuming conservation of specific angular momentum $h_H = h_0$. The dashed line shows the fitting function of equation (39).
instantaneous mass-loss and return of cold gas. These points will be discussed further in Section 6, but for the moment these assumptions will be adopted to illustrate the qualitative features of the model. As gas is lost from the system, the circular speed of the disc component (equation 5) is simply rescaled by the square root of the mass of the disc that remains.

The evolution of the surface mass densities for the two disc models is illustrated in Figs 4 and 5. In model MW, the evolution is similar to that without outflow shown in Fig. 2. With the simple prescription for mass-loss used here, no hot gas is lost unless the temperature of the hot phase exceeds $T_{\text{crit}} \approx 5 \times 10^5$ K. This does happen at early times when the star formation rate is high, and about 25 per cent of the galaxy mass is lost within $10^7$ yr. Thereafter, no more mass is lost, and a nearly exponential disc is built up with a gas distribution containing a central hole as in Fig. 2. The star formation rate in this model declines strongly with time, exceeding $100$ M$_\odot$ yr$^{-1}$ in the early phases of evolution.

The behaviour of model DW is qualitatively different. Here the critical temperature for mass-loss is much lower, $T_{\text{crit}} \approx 3 \times 10^3$ K; hence half the mass of the galaxy is expelled by $\sim 10^5$ yr and 66 per cent by 1 Gyr. After 1 Gyr, the temperature of the hot phase drops below $T_{\text{crit}}$ and the galaxy settles into a stable state with a low rate of star formation.

The wind prescription in these models, and particularly the assumption that gas below the critical temperature necessary for escape is returned instantaneously to the cold phase, is clearly oversimplified and so the mass-loss fractions should not be taken too seriously. A more detailed model is developed in Section 6. A more serious deficiency of the model presented here is that the entire gas disc is assumed to have formed instantaneously at $t = 0$. This is unrealistic and leads to high rates of star formation and gas ejection at early times. A simple infall model, similar to those adopted in semi-analytic models (White & Frenk 1991; Cole et al. 1994, hereafter C94) is included in the next section.

## 4 INFALL MODEL

### 4.1 Conservation of specific angular momentum

Following Fall & Efstathiou (1980), the gas is assumed to follow the spatial distribution of the halo component with the same distribution of specific angular momentum prior to collapse. The halo is assumed to rotate cylindrically with rotation speed $v_{\text{rot}}^D(\sigma_D)$, where $\sigma_D$ is the radial coordinate in the cylindrical coordinate system. The gas is assumed to conserve its specific angular momentum during its collapse, so that the final specific angular momentum of the disc at radius $\sigma_D$, $h_0 = \sigma_D v_{\text{rot}}^D(\sigma_D)$, is equal to the specific angular momentum of the halo $h_0 = \sigma_H v_{\text{tot}}^H(\sigma_H)$ at the radius $\sigma_H$ from which the gas originated. Mass conservation relates the radii $\sigma_H$ and $\sigma_D$:

$$\frac{\text{d} \sigma_H}{\text{d} \sigma_D} = \frac{\mu_0(\sigma_H) M_H(\sigma_H)}{\mu_0(\sigma_D) M_D(\sigma_D)}$$

(30)

where $M_D/M_H$ is the ratio of the halo to disc mass interior to the maximum infall radius of the disc (see Fig. 6a below), and $\mu_0$ is the projected surface mass density of the halo

$$\mu_0(\sigma) = 2 \int_0^\infty \rho_0 \left[ (\sigma^2 + z^2)^{1/2} \right] \, dz.$$  

(31)

The solution of equation (30) yields $\sigma_D(\sigma_H)$, and the rotation speed of the halo follows from the conservation of specific angular momentum, $v_{\text{rot}}^D = \sigma_D v_{\text{tot}}^H(\sigma_H)/\sigma_H$. The results for the parameters of models MW and DW are shown in Fig. 6, where we have used the notation $s = \sigma/r_D$. When expressed in the dimensionless units of Fig. 6, the solutions for models MW and DW are identical.

This prescription is guaranteed to form an exponential disc with the required parameters. The derived rotation velocity of the halo is almost independent of radius, in general agreement with what is found in N-body simulations (Frenk et al. 1988; Warren et al. 1992). The upturn in the halo rotation speed at $s_H(\text{max}) \approx 30$ is caused by the rapid decline in the mass of the input exponential disc at large radii, and is of little consequence in the discussion that follows. The values of the spin parameter quoted in Table 1 were derived from the mass and binding energy of the halo, assuming that the halo rotation velocity is constant at $0.095c$, at large radii.

### 4.2 Mass infall rate

To determine the gas infall rate we compute the free-fall time for a gas element at rest at radius $r$,

$$t_{\text{ff}} = \int_0^r \frac{\text{d} r}{\sqrt{2[\phi_0(r) - \phi_0(r_\text{vir})]^{1/2}}}.$$  

(32)

and the cooling time,

$$t_{\text{cool}} = \frac{3 k T_e \times 1.92}{2 \Lambda(T_e)n_e(r)},$$  

(33)

where $n_e(r)$ is the electron density. The temperature $T_e$ in equation (33) is set to the virial temperature derived from the equation of hydrostatic equilibrium, assuming that the temperature is slowly

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varying with radius:

\[ T_v \approx -\frac{\nu^2(r)}{k} \frac{d \ln r}{d \ln \rho_b(r)} , \]  

(34)

where we assume that the baryons follow the same spatial distribution as the halo. The infall rate is given by

\[ M_{\text{inf}} = 4\pi \rho_b(r_H) r_H^2 \left\{ \frac{d r_H(t_{\text{ff}} = t)/dt}{t_{\text{ff}} > t_{\text{cool}}} , \frac{d r_H(t_{\text{cool}} = t)/dt}{t_{\text{cool}} > t_{\text{ff}}} \right\} . \]  

(35)

Finally, conservation of specific angular momentum specifies the final radius in the disc for each gas element. Since the halo is assumed to rotate on cylinders, the gas near to the poles in an infalling shell has a lower specific angular momentum than the gas at the equator. The infalling material is therefore distributed through the disc according to

\[ 2\pi \sigma_D \mu_D \frac{\sigma_D}{r_D(r_H - \sigma_H)^{1/2}} = M_{\text{inf}} \frac{\sigma_H d\sigma_H}{r_H(r_H - \sigma_H)^{1/2}} , \]  

(36)

where \( \sigma_D \) and \( \sigma_H \) are related by the solution of equation (30).

Equations (32)–(36) specify the infall model. The free-fall and cooling times of the two model galaxies are shown in Fig. 7. In

the larger galaxy, gas within \( r_H/r_D \approx 10 \) infalls on the free-fall time-scale and ends up within one scalelength of the final disc. The material in the outer parts of the disc infalls on the cooling time-scale. In contrast, apart from a small amount of gas in the very central part of the halo with virial temperature \(<10^4 \text{ K}\), the gas in the dwarf galaxy infalls on a free-fall time-scale, because the cooling time is so short.

### 4.3 Simple self-regulating model with inflow and outflow

The models described in this section are exactly the same as those described in Section 3.3, except that we grow the discs gradually using the infall model in Sections 4.1 and 4.2. In the models described below, inflow and outflow are assumed to occur simultaneously. This is often assumed in semi-analytic models of galaxy formation (e.g. C94; Somerville & Primack 1999), and may not be completely unrealistic if the infalling gas is clumpy. The dark matter haloes will contain significant substructure (e.g. Moore et al. 1999), which may contain pockets of cooled gas. Furthermore, if the cooling time is short compared to the dynamical time, the infalling gas will be thermally unstable (Fall & Rees 1985) and will fragment into clouds. These will fall to the centre on a free-fall time-scale if they are sufficiently dense and massive that gravity dominates over the ram pressure of the wind. This

\[ \text{Figure 8. Figure 8. The left-hand panels show the evolution of the gas (solid lines) and stellar (dashed lines) surface mass density distributions for ages of 0, 0.1, 1, 3, 6 and 10 Gyr as in Fig. 2. The panels to the right show the radial distribution of density, temperature and ratio of overlap to cooling time-scales for the hot gas component.} \]  

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By guest on 27 July 2018
requires clouds with masses
\[ m_{\text{cloud}} \approx 9.5 \times 10^5 M_\odot \left( \frac{a_{\text{cloud}}}{1 \text{kpc}} \right) \left( \frac{r}{10 \text{kpc}} \right)^{-1} \times \left( \frac{M_w}{1 M_\odot \text{yr}^{-1}} \right) \left( \frac{v_w}{100 \text{km s}^{-1}} \right)^{-2}, \]
(37)

where \( a_{\text{cloud}} \) is the radius of the cloud. However, even if condition (37) is satisfied, the clouds may be sheared and disrupted into smaller clouds by Kelvin–Helmholtz instabilities on a time-scale of a few sound crossing times as they flow through the wind (e.g., Murray et al. 1993). The wind energy will be partially thermalized in shocks with the infalling clouds, and dissipated in evaporating small clouds. However, for the typical mass outflow rates expected from dwarf galaxies (\( M_w \approx 0.2 M_\odot \text{yr}^{-1} \)), the rate at which energy is supplied by the wind \( E_w = 1/2 M_w v_w^2 \) is much smaller than the energy lost in radiative cooling,
\[ \frac{E_{\text{cool}}}{E_w} \approx 50 \Lambda_{23} \left( \frac{v_w}{100 \text{km s}^{-1}} \right)^4 \left( \frac{r_{\text{cool}}}{10 \text{kpc}} \right)^{-1} \times \left( \frac{v_w}{100 \text{km s}^{-1}} \right)^{-2} \left( \frac{M_w}{1 M_\odot \text{yr}^{-1}} \right)^{-1}, \]
(38)

where \( r_{\text{cool}} \) is the radius at which the cooling time is equal to the age of the system.

The qualitative picture that we propose is as follows. In galaxies with a short cooling time, clouds formed by thermal instabilities will infall ballistically if condition (37) is satisfied. If (37) is not satisfied, the ram pressure of the wind will drive out the infalling gas, and infall will be suppressed. With infall suppressed, the star formation rate in the disc and the wind energy will decline until infall can begin again. The wind will be partially thermalized before reaching \( r_{\text{cool}} \), and completely thermalized at \( \sim r_{\text{cool}} \), but the energy supplied by the wind will be small compared to the energy radiated by the gas at \( r \approx r_{\text{cool}} \) and so cannot prevent radiative cooling. If (37) is satisfied, some of the outflowing gas may fall back down to the disc after shocking against infalling clouds. However, in the models described here the efficiency of converting infalling gas into stars is low in dwarf systems; so provided the gas does not cycle around the halo many times, neglecting return of some of the outflowing gas should not affect the qualitative features of the models. The global geometry of the system, e.g., if the wind is weakly collimated perpendicular to the disc, may also permit simultaneous inflow and outflow of gas.

The interaction of an outflowing wind with an inhomogeneous infalling gas clearly poses a complex physical problem. In reality, the process may be far from steady, with outflow occurring in bursts accompanied by infall from discrete subclumps containing cooled gas. In the models described below and in the rest of this paper, we will assume that the infall and outflow occur simultaneously, steadily and without any interaction between the inflowing and outflowing gas. As the discussion of the preceding two paragraphs indicates, this is obviously an over-simplification. It should be viewed as an idealization, on a similar footing to some of the other assumptions adopted in this paper (e.g., spherical symmetry, neglect of halo substructure and merging, steady star formation rates, etc.) designed to give some insight into how a quiescent mode of feedback might operate.

The analogues of Figs 4 and 5 for the models incorporating infall and outflow are shown in Figs 8 and 9. The discs build up from the inside out, as in the models of disc formation described by Fall & Efstathiou (1980) and Gunn (1982). Most of the star formation occurs in a propagating ring containing the most recently accreted gas. The most significant differences from the models in Section 3.3 are the net rates of star formation (Fig. 9) and the time-scale of outflow. The initial high rates of star formation in the models in Section 3.3 are suppressed in the models with infall, and the time-scale for outflow is now much longer because it is closely linked to the gas infall time-scale. Apart from these differences, the final states, gas fractions and mass-loss fractions are similar to those in the models without

![Figure 9](https://academic.oup.com/mnras/article-abstract/317/3/697/967513)
infall. In model MW some outflow occurs when \( t \leq 10^8 \) yr and the temperature of the hot gas is high enough that it can escape from the system. Thereafter, the hot component cannot escape, and the disc builds up without further outflow. About 17 per cent of the total galaxy mass is expelled in the early phases of evolution but, as we have described above, this could be an overestimate since some of this gas may be returned to the galaxy if the wind energy is thermalized before it reaches the virial radius. In contrast, model DW drives a wind until \( t \sim 1 \) Gyr and expels about 74 per cent of its mass. About half of the gas is lost within \( 3 \times 10^8 \) yr, i.e., on about the infall time-scale for most of the gas in the halo (see Fig. 7b).

5 REFINEMENTS OF THE MODEL

The models described in the previous sections contain a number of simplifications, which we will attempt to refine in this section. We do not address the problem of the interaction of a wind with the infalling gas, which is well beyond the scope of this paper. Instead, we introduce some simple improvements to the infall model (Section 5.1), model for mass-loss from the galactic disc (Sections 5.2 and 5.3), and the pressure response of the cold ISM to the hot phase (Sections 5.4 and 5.5).

5.1 Infall model

In Section 4 we used a simplified model of infall that guarantees the formation of an exponential disc if angular momentum is conserved during collapse. In this section we assume a specific functional form for the rotation velocity of the halo,

\[
v_{\text{rot}}(s) = c_1 v_0 \frac{(s/c)^2}{[1 + (s/c)^2]^{1 + (s/c)^2}}, \quad s = r/r_{D},
\]

with \( c_1 = 0.115, c_2 = 0.6, c_3 = 16 \) and \( c_4 = 0.25 \). The functional form and coefficients in equation (39) have been chosen to provide a good fit to the halo rotation profile derived in Section 4.1 from conservation of specific angular momentum, and is plotted as the dashed line in Fig. 6(b). As in the previous section, the gas is assumed to follow the same radial density distribution and rotation velocity as the halo component, but its final radius in the disc is computed by assuming conservation of specific angular momentum and self-consistently solving for the rotation speed of the disc component. The halo component is assumed to be rigid, and the contribution of the disc component to \( v_{\text{rot}}^2 \) is computed using the Fourier–Bessel theorem (see Binney & Tremaine 1987, Section 2.6):

\[
v^2(r) = -r \int_0^\infty S(k)J_1(kr)k \, dk,
\]

\[
S(k) = -2\pi G \int_0^{r_{D}} J_0(kr) \mu_0(r) r \, dr.
\]

Equation (40) is time-consuming to evaluate accurately, and in our application \( v^2(r) \) must be computed many times. A fast algorithm has therefore been developed as described in Appendix A. The epicyclic frequency \( \kappa \) is required in equation (13) to compute the instantaneous star formation rate and is derived by numerically differentiating the rotation speed.

With this formulation of the infall model, the infall rate is governed by the dark matter profile and the ratio of dark to baryonic mass within the virial radius, \( M_\text{c}/M_\text{D} = (v_0/v_\text{c})^2 f_{\text{coll}} \). For the models described here, we adopt \( M_\text{c}/M_\text{D} = 10 \), consistent with the parameters listed in Table 1. The final disc surface mass density will be close to an exponential by construction, since the halo rotation velocity (39) has been chosen to match the rotation profile derived by assuming an exponential disc and conservation of specific angular momentum.

5.2 Galactic fountain

In previous sections we have assumed that gas is lost from the disc if the bulk velocity of the wind, \( v_w \sim \sqrt{2 E_{\text{SNII}}} \), exceeds the escape speed \( v_{\text{esc}} \) from the halo, but is otherwise returned instantaneously to the ISM. More realistically, gas with \( v_w < v_{\text{esc}} \) will circulate in the halo along a roughly ballistic trajectory and will cool forming a galactic fountain (Shapiro & Field 1976; Bregman 1980). In the models described in this section, hot gas with \( v_w < v_{\text{esc}} \) is returned to the disc at the radius from which it was expelled after a time \( t_{\text{ret}} \),

\[
t_{\text{ret}} = 2t_{\text{ff}}(r_{\text{max}}), \quad v_w^2 = 2(\phi_{\text{H}}(r_{\text{max}}) - \phi_{\text{H}}(0)),
\]

i.e., we ignore the gravity of the disc and the angular momentum of the gas in computing the ballistic trajectory of a gas element.

5.3 Escape velocity of the wind

The detailed dynamics of the hot corona itself is complicated and beyond the scope of this paper. Type II supernovae at the upper and lower edges of the gas disc will be able to inject their energy directly into the hot gas, as will Type Ia supernovae forming in the thicker stellar disc. In addition, the hot component will interact with the primordial inflalling gas in a complicated way as sketched in Section 4.3.

In the absence of radiative cooling, the hot gas will extend high above the galactic disc in an extended corona. For an isothermal corona, the equation of hydrostatic equilibrium in the \( z \)-direction has the following approximate solution:

\[
\frac{\rho_H(\sigma, z)}{\rho_0(\sigma)} = \sinh^2 \frac{\sigma}{H_0} \sinh^2 \frac{z}{H_0} \exp \left[ -1 \frac{1}{c_1^2} \int_0^{\sigma} \frac{r_{\text{esc}}(r)}{r^2} \, dz \right],
\]

\[
r^2 = \sigma^2 + z^2,
\]

\[
\rho_0 = \frac{\mu_e \sigma_e^2 \sigma_\text{h}}{c_1^2 (\mu_e \sigma_e + \mu_\text{h} \sigma_\text{h})}, \quad \rho_s = \frac{\mu_e \sigma_e^2 \sigma_\text{h}}{c_1^2 (\mu_e \sigma_e + \mu_\text{h} \sigma_\text{h})},
\]

where we have assumed that both the stars and the gas follow \( \sinh^2 \) vertical distributions (equation 7), and \( c_1 \) is the isothermal sound speed of the hot gas. We define a characteristic scaleheight for the hot component, \( H_{\text{hot}}(\sigma) \), at which the density drops by a factor \( e \) according to equation (42). If radiative cooling were negligible, we would expect a sonic point in the flow at about \( z = H_{\text{hot}} \). It is interesting to compare some characteristic numbers for the coronal gas:

\[
E_{\text{in}} \approx 1.1 \times 10^{40} T_{\text{hot}} M_\text{c} \text{ erg s}^{-1},
\]

\[
E_{\text{SNII}} \approx 5.7 \times 10^{40} E_{\text{SNII}} E_{\text{SNIa}} M_\ast \text{ erg s}^{-1},
\]

\[
E_{\text{cool}} \approx 2 \times 10^{39} n_{\text{H}}^2 A_{-21} (H_{\text{hot}}/1\text{ kpc}) (R_{\text{hot}}/3\text{ kpc})^2 \text{ erg s}^{-1},
\]

where the rates \( M_{\text{c}}, M_\ast \) are in \( M_\odot \text{ yr}^{-1} \). Here \( E_{\text{in}} \) is the thermal energy injected into the hot corona by evaporating cold gas at a rate \( M_{\text{c}} \), \( E_{\text{SNII}} \) is the energy supplied to the corona by Type II supernovae forming above and below one scaleheight of the cold.
gas layer, and the parameter $\varepsilon_{\text{SNII}}$ expresses the efficiency with which this energy is coupled to the hot coronal gas. $E_{\text{cool}}$ is the rate of energy lost by a uniform density isothermal corona of scaleheight $H_{\text{cool}}$ within a cylinder of radius $R_{\text{gas}}$. For a large galaxy such as the Milky Way that can sustain an evaporation rate of $\sim 10 M_\odot \, \text{yr}^{-1}$, $E_{\text{cool}}$ is small compared to $E_{\text{in}}$, and it is a good approximation to neglect radiative cooling in the early stages of the flow (see Appendix B). However, in a dwarf galaxy $E_{\text{cool}}$ is typically larger than $E_{\text{in}}$. In this case, we expect that the hot component will develop a sonic point at a characteristic cooling scaleheight

$$\begin{align*}
H_{\text{cool}} \sim v_w t_{\text{cool}} \sim 11 \left( \frac{v_w}{100 \text{ km s}^{-1}} \right) T_{\text{inh}} n_h^{-2} \Lambda_{\text{cool}}^{-1} \text{kpc}. \tag{44}
\end{align*}$$

(see, e.g., Kahn 1991 and Appendix B), and that most of the thermal energy will be converted into kinetic energy by the time the gas flows to $H_{\text{cool}}$. We therefore ignore radiative cooling and estimate the bulk velocity of the wind at each radial shell in the disc from

$$\begin{align*}
\frac{1}{2} \dot{M}_{\text{esc}} v_w^2 = E_{\text{in}} + \dot{E}_{\text{SNII}}.
\end{align*}$$

and to close the equations we assume that

$$\dot{M}_{\text{out}} = \dot{M}_{\text{in}} = \dot{M}_{\text{esc}}. \tag{45b}$$

(Note that the numerical coefficient in equation 43a has been adjusted to give $v_w = \sqrt{2 \dot{M}_{\text{esc}}}$ if $E_{\text{SNII}} = 0$ so that the criterion for the wind to escape, $v_w > v_{\text{esc}}$, is the same as in the preceding section.)

Energy input from Type II supernovae exploding high above the cold gas layer will make a small contribution to the thermal energy of the hot coronal gas. For values of $E_{\text{SNII}} \sim 1$, $\dot{E}_{\text{SNII}}$ will be about 20 per cent or so of $E_{\text{in}}$, and cannot be higher because $\dot{M}_{\text{esc}}^2$ and the star formation rate are nearly proportional to each other (equation 27). Type Ia supernovae will also supply energy to the corona, with a time lag of perhaps $\gtrsim 1$ Gyr (Madau, Della Valle & Panagia 1998). However, this effect will also make a small perturbation to the energy budget of the corona, and so it is ignored here. Furthermore, in the models described here much of the gas is expelled on a time-scale of $\lesssim 1$ Gyr; thus feedback is likely to be more or less complete before energy injection from Type Ia supernovae becomes significant.

### 5.4 Pressure equilibrium and cold cloud radii

In the models in Sections 3 and 4, the cold cloud radii were kept constant, irrespective of the pressure of the confining hot phase. More realistically, the cold cloud radii will adjust to maintain approximate pressure equilibrium with the hot phase. Thus

$$\frac{a}{a_\odot} \approx 0.53 \left( \frac{T_c}{80 \text{ K}} \right)^{1/3} \left( \frac{n_{\text{H}_2} T_{\text{H}_2}}{\dot{M}_{\text{H}_2}} \right)^{-1/3}, \tag{46}$$

where $a_\odot$ denotes the cloud radii at the solar neighbourhood with values as given in equation (17), and $T_c$ is the internal temperature of the cold clouds. In our own Galaxy, photoelectric heating of dust grains is believed to be the main heating mechanism of the cold clouds (see, e.g., Wolfire et al. 1995), but other heating mechanisms may be important — for example, cosmic-ray heating (Field, Goldsmith & Habing 1969). We therefore expect that $T_c$ varies in a complex (and uncomputable) way as a galaxy evolves. To assess the effects of the pressure response of the cold clouds, $T_c$ will be assumed to remain constant at 80 K. The cloud radii are then determined solely by the pressure of the hot component via equation (46). The models described here are insensitive to the fiducial

### Table 2. Feedback efficiency: variation of input parameters.

| Model | MW1 | DW1 | MW2 | DW2 | MW3 | DW3 |
|-------|-----|-----|-----|-----|-----|-----|
| $\phi_c$ | 0.1 | 0.01 | 1.0 | 1.0 |
| $\varepsilon_{\text{SNII}}$ | 0.01 | 0.01 | 0.03 | 0.03 |
| $\varepsilon_{\text{SE}}$ | 0.0 | 0.0 | 1.0 | 1.0 |
| Section 5.4 | no | no | no | no |
| $M_1$ (M$_\odot$) | $2.8 \times 10^{10}$ | $4.2 \times 10^{10}$ | $2.3 \times 10^{10}$ | $7.1 \times 10^{10}$ | $2.3 \times 10^{10}$ | $3.6 \times 10^{10}$ |
| $M_2$ (M$_\odot$) | $5.4 \times 10^{10}$ | $1.3 \times 10^{10}$ | $5.4 \times 10^{10}$ | $9.9 \times 10^{10}$ | $6.9 \times 10^{10}$ | $1.3 \times 10^{10}$ |
| $M_3$ (M$_\odot$) | $4.6 \times 10^{10}$ | $2.6 \times 10^{10}$ | $7.9 \times 10^{10}$ | $3.1 \times 10^{10}$ | $6.5 \times 10^{10}$ | $3.0 \times 10^{10}$ |
| $f_{\text{SNII}}$ (Gyr) | 0.12 | 0.59 | 0.22 | 0.64 | 0.18 | 0.64 |
| $\tau_{\text{SNII}}$ (Gyr) | 0.25 | 0.82 | 0.40 | 1.8 | 0.30 | 1.24 |
| $f_\text{s}$ | 0.74 | 0.10 | 0.63 | 0.15 | 0.63 | 0.08 |
| $<Z/\text{Z}_{\odot}>$ | 0.65 | 0.03 | 0.57 | 0.04 | 0.56 | 0.02 |
| $<Z/\text{Z}_{\odot}>$ | 0.55 | 0.20 | 0.43 | 0.30 | 0.42 | 0.19 |
| $<Z/\text{Z}_{\odot}>$ | 0.29 | 0.11 | 0.38 | 0.14 | 0.26 | 0.09 |
| | | | | | | |
| Model | MW4 | DW4 | MW5 | DW5 | MW6 | DW6 |
| $\phi_c$ | 0.1 | 0.1 | 0.1 | 0.1 |
| $\varepsilon_{\text{SNII}}$ | 0.01 | 0.01 | 0.01 | 0.01 |
| $\varepsilon_{\text{SE}}$ | 1.0 | 1.0 | 1.0 | 1.0 |
| Section 5.4 | yes | no | yes | no |
| $M_1$ (M$_\odot$) | $2.5 \times 10^{10}$ | $4.3 \times 10^{10}$ | $2.4 \times 10^{10}$ | $4.3 \times 10^{10}$ | $2.5 \times 10^{10}$ | $4.5 \times 10^{10}$ |
| $M_2$ (M$_\odot$) | $2.1 \times 10^{10}$ | $3.5 \times 10^{10}$ | $4.9 \times 10^{10}$ | $9.5 \times 10^{10}$ | $1.9 \times 10^{10}$ | $2.9 \times 10^{10}$ |
| $M_3$ (M$_\odot$) | $7.1 \times 10^{10}$ | $3.4 \times 10^{10}$ | $6.8 \times 10^{10}$ | $3.1 \times 10^{10}$ | $7.2 \times 10^{10}$ | $3.4 \times 10^{10}$ |
| $f_{\text{SNII}}$ (Gyr) | 0.21 | 0.82 | 0.19 | 0.69 | 0.21 | 0.82 |
| $\tau_{\text{SNII}}$ (Gyr) | 0.30 | 1.19 | 0.30 | 1.19 | 0.21 | 1.12 |
| $f_\text{s}$ | 0.73 | 0.10 | 0.67 | 0.09 | 0.74 | 0.11 |
| $<Z/\text{Z}_{\odot}>$ | 0.62 | 0.03 | 0.59 | 0.02 | 0.64 | 0.02 |
| $<Z/\text{Z}_{\odot}>$ | 0.47 | 0.17 | 0.44 | 0.21 | 0.48 | 0.19 |
| $<Z/\text{Z}_{\odot}>$ | 0.26 | 0.09 | 0.27 | 0.10 | 0.27 | 0.10 |
temperature of 80 K adopted in equation (46), with ejected gas fractions varying by a few per cent as $T_c$ is varied from 40 to 160 K.

The energy lost through cloud collisions (equation 18) varies as $a^2$ (for fixed cloud masses). However, the cloud heating efficiency factor $e_c$ will also change as the cloud radii change in response to the pressure of the hot phase. In the model of MO77, the energy acquired from momentum exchange with cooling supernova shells varies as $a^4$. The net effect of these variations in the self-regulating star formation model is to introduce positive feedback, since a higher rate of star formation is required to balance energy lost through cloud collisions in regions where the ambient pressure is higher. This is modelled by assuming $E_{\text{coll}} \propto (a/a_{\odot})^2$ and $e_c = e_{\odot}(a/a_{\odot})^2$, where $e_{\odot}$ is a fiducial efficiency factor.

### 5.5 Induced star formation

The maximum mass for an isothermal cloud in pressure equilibrium with the confining medium of pressure $p_h$ is given by the Bonnor–Ebert criterion (Bonnor 1956; Ebert 1955; Spitzer 1968),

$$m_{\text{BE}} = 1.18 \left( \frac{k T_c}{\mu_p} \right)^2 \frac{G}{p_h}^{1/2} \approx 433 \left( \frac{T_c}{80 \text{ K}} \right) (n_h^{-2} T_h)^{-1/2} M_\odot. \quad (47)$$

For our own Galaxy (MO77), $n_h \approx 1.5 \times 10^{-3} \text{ cm}^{-3}$, $T_h \approx 4 \times 10^5 \text{ K}$ and $p_h \approx 3 \times 10^{-12} \text{ dyne cm}^{-2}$; hence $m_{\text{BE}} \approx 1700 M_\odot$. This is reasonably close to the upper mass limit, $m_u = 4300 M_\odot$, for the cold cloud mass spectrum adopted in this paper ($a_u = 10 \text{ pc}$ with $\rho_u = 7 \times 10^{-23} \text{ g cm}^{-3}$). Gravitational stability requires $m_u > m_{\text{BE}}$ and we will henceforth impose this condition in determining the upper mass limit of the cold cloud spectrum. An increase in the pressure of the hot phase will lead to a decrease in $m_u$, and hence to some pressure-induced star formation. If the over-pressured clouds fragment into stars with an efficiency $e_{\text{BE}}$, the induced star formation rate is given by

$$\frac{\text{d}M^0}{\text{d}t} = e_{\text{BE}} \frac{M_g^0}{2 p_h \text{ln}(m_u/m_L)} \left\{ \begin{array}{ll} \frac{\text{d}p_h}{\text{d}t} > 0 & \text{d}p_h/\text{d}t \\ 0 & \text{d}p_h/\text{d}t \leq 0 \end{array} \right., \quad (48)$$

where $m_L$ is the lower limit to the cloud mass spectrum, $m_L = 0.5 M_\odot$. This provides an additional source of positive feedback, since as the pressure of the hot component rises the star formation rate of the self-regulating model is enhanced by pressure-induced star formation.

### 5.6 Chemical evolution

It is straightforward to include chemical evolution in the models using the instantaneous recycling approximation. We distinguish
between ‘primordial’ infalling gas accreting at a rate \( \frac{d\mu}{dt} \) with metallicity \( Z_\text{i} \), and processed gas from the galactic fountain of metallicity \( Z_\text{f} \) accreted at a rate \( \frac{d\mu}{dt} \). The equation of chemical evolution is then

\[
\frac{dZ}{dt} = p\frac{d\mu}{dt} + (Z_\text{i} - Z) \frac{d\mu}{dt} + (Z_\text{f} - Z) \frac{d\mu}{dt},
\]

(49)

(see, e.g., Pagel 1997), where \( p \) is the yield. We adopt a yield of \( p = 0.02 \) and assume that the primordial gas has zero metallicity \( (Z_\text{i} = 0) \). Gas ejected in a galactic fountain is assumed to have the same metallicity as the ISM at the time that the gas was ejected. Within the disc, the ISM gas is assumed to be perfectly mixed at all times. We normalize the metallicities to the solar value, for which we adopt \( Z_\odot = 0.02 \).

\section{RESULTS AND DISCUSSION}

\subsection{6.1 Variation of input parameters}

In addition to the many simplifying assumptions introduced in previous sections, the model described here has four key parameters: (i) \( \phi_c \), determining the efficiency of heat conduction (equation 20); (ii) \( c_\odot \), controlling the star formation rate (equation 19); (iii) \( \varepsilon_{\text{SNR}} \), determining the efficiency with which energy from Type II supernovae couples directly to the gas (equation 45a); and (iv) \( \varepsilon_{\text{BE}} \), setting the efficiency with which over-pressured ISM clouds collapse to make stars (equation 48). In addition, the ISM, cloud radii can be allowed to vary in response to the pressure of the ISM, as described in Section 5.4.

The effects of varying these parameters are summarized in Table 2. Here we have run six models of galaxies MW and DW, varying the input parameters. We list the final stellar mass \( M_\ast \), gaseous disc mass \( M_\text{g} \), and ejected mass \( M_\text{ej} \) after 10 Gyr for model DW (there is very little evolution after this time), and after 15 Gyr for model MW. The parameters \( f_\text{ej} \) and \( f_\text{s} \) are the final ejected and stellar masses divided by the total baryonic mass \( (M_\text{ej} + M_\ast + M_\text{g}) \). \( \tau_\text{ej} \) is the time when half the final ejected mass is lost. The last three numbers list the final mean metallicities of the cold ISM, the stars and the ejected gas.

The most important result from this table is that the final parameters of the models are remarkably insensitive to variations of the input parameters. For models DW, the final stellar disc mass varies between \( \sim 4 \times 10^7 \) and \( \sim 7 \times 10^7 M_\odot \), and the gas ejection fraction varies from 0.59 to 0.82. For models MW, the final stellar disc mass varies between \( \sim 2.3 \times 10^{10} \) and \( \sim 2.8 \times 10^{10} M_\odot \), and the gas ejection fraction varies from 0.12 to 0.22. Fig. 10 shows the evolution of the radial density profiles of models MW1 and DW1, and Fig. 11 shows the time evolution of the star formation rates, gas fractions and gas velocity dispersions. The models in Table 2 behave in similar ways, and so these two figures are representative of the behaviour of all of the models. These figures are qualitatively similar to those of the simple model in Section 4 (Figs 8 and 9). The main differences are as follows.

(a) The gas discs have a sharper outer edge. This is a consequence of the infall model; the outer edge is determined by the final time of the model, which sets the maximum cooling radius within the halo (cf. Fig. 7).

(b) The radial profiles of models MW show oscillatory behaviour near their centres, and the star formation rates and gas fractions show oscillatory behaviour as a function of time. Both of these effects are a consequence of the galactic fountain.

In these models, the star formation rate begins to rise as the gas disc builds up from infalling gas. As the star formation rate rises, the cold ISM is converted efficiently into a hot phase, and this is either driven out of the halo or becomes part of the galactic fountain. In models MW, most of the gas that escapes from the system is lost within this early (\( \leq 0.2 \) Gyr) period of star formation when the net star formation rate is close to its peak of \( \sim 10 M_\odot \text{yr}^{-1} \). After about 0.2 Gyr, the temperature of the hot phase in models MW settles to \( \sim 10^8 K \), very nearly independent of radius (cf. Fig. 10), and so the galactic fountain cycles on a
characteristic time-scale of \( \sim 4 \times 10^8 \) yr. Models DW behave in much the same way as the simpler models in Section 4.3, except that infall, by construction, extends over a longer period of time. In these models, the escape criterion for the wind is satisfied over most of the lifetime of the disc, and hence the model of a galactic fountain is unimportant.

We discuss briefly the effects of varying the input parameters. 

- \( \phi_k \): The evaporation rate \( M_{\text{ev}} \) has a weak dependence on the evaporation parameter \( \phi_k (\propto \phi_k^{0.20} \text{, equation 28}) \) and obviously decreases as \( \phi_k \) is reduced. However, the temperature of the hot component is proportional to \( \phi_k^{-0.29} \), and hence rises as \( \phi_k \) is reduced. The net effect is that the mass of gas ejected is relatively insensitive to \( \phi_k \), but the mass of the final stellar disc increases as \( \phi_k \) is reduced.

- \( \epsilon_{\text{e0C}} \): Increasing this parameter reduces the star formation rate in the self-regulating model for a fixed gas surface density and velocity dispersion (equations 15 and 18). However, a lower past star formation rate leads to a higher gas surface density, which increases the star formation rate. These effects tend to cancel, and so the models are insensitive to variations in \( \epsilon_{\text{e0C}} \).

- \( \epsilon_{\text{SNII}} \): Setting this parameter to unity increases the temperature of the hot component slightly and hence increases the efficiency of feedback. As explained in Section 5.3, energy injection by Type II supernovae at large vertical scaleheights will always be small compared to the internal energy of the hot phase.

- \( \epsilon_{\text{BE}} \): Values of \( \epsilon_{\text{BE}} \approx 0.05 \) have little effect on the evolution. Provided that \( \epsilon_{\text{BE}} \) does not dominate the net star formation rate, pressure-enhanced star formation is self-limiting, because it increases the velocity dispersion of the cold clouds (reducing the star formation rate in the self-regulating model) and converts cold gas to hot gas. Both of these effects tend to reduce the net star formation rate.

**Pressure response of cold cloud radii:** Allowing the cold gas radii to respond to the pressure of the hot phase provides strong positive feedback in the very early stages of galaxy formation when the pressure of the hot phase is high. However, most of the cold ISM is ejected on a much longer time-scale (cf. values of \( \tau_{\text{ej}} \) in Table 2) when the typical pressure of the ISM is similar to that in our own Galaxy. The pressure response of the cold cloud radii therefore has little effect on the final feedback efficiency.

The models described here involve a complex set of coupled equations and a number of parameters. However, one of the most interesting aspects of this study is that the equations interact in such a way that the evolution of the models is insensitive to the parameters. Most importantly, the efficiency of feedback is insensitive to the thermal conduction parameter \( \phi_k \). The possible severe suppression of thermal conduction by tangled magnetic fields in astrophysical environments is a long-standing theoretical problem. However, our results show that even a reduction of \( \kappa \) by a factor of 100 or more will not significantly alter the efficiency of feedback.

### 6.2 Chemical evolution

In this section we summarize some of the results relating to chemical evolution in these models. Our intention is not to present a detailed model of chemical evolution in disc and dwarf systems along the lines of, for example, Lacey & Fall (1983, 1985), Gibson (1997) and Gibson & Matteucci (1997), but to investigate some of the general features of chemical evolution with physically

![Figure 12. The distribution of stellar metallicities at four radii in model MW1 at an age of 15 Gyr.](https://academic.oup.com/mnras/article-abstract/317/3/697/967513)
motivated models of inflow and outflow. The chemical evolution model is based on the instantaneous recycling approximation as described in Section 5.6. This is probably a reasonable approximation, since the time-scales of star formation and outflow are ~1 Gyr, but will overestimate the gas metallicities where the gas density is low. As in the previous section, results from models MW1 and DW1 are used to illustrate the general features of the models. The other models listed in Table 2 behave in very similar ways.

6.2.1 Stellar metallicity distribution
The final mean stellar metallicities are typically $Z_s/Z_\odot = 0.5$ in models MW and $<0.2$ in models DW. Models DW have a lower stellar metallicity, because a larger fraction of the ISM is expelled in a wind. The stellar metallicity distributions are shown in Figs 12 and 13. Fig. 12(c) is particularly interesting, because this radius is close to the solar radius. This metallicity distribution is quite similar to that of G-dwarfs in the solar cylinder (see, e.g., fig. 8.19 of Pagel 1997), showing that the infall model solves the ‘G-dwarf’ problem of closed box models of chemical evolution. The metallicity distributions of model DW1 plotted in Fig. 13 also show a lack of stars with low metallicities.

6.2.2 Metallicity gradients
Over most of the stellar disc, model MW1 has a fairly weak stellar metallicity gradient (Fig. 14a), except at the very outer edge where the stellar density and metallicity fall abruptly. This differs from the metallicity gradients seen in large disc systems, which show
The radial gas metallicity profiles are shown in Fig. 14(b). Model DW contains a gaseous disc extending well beyond the edge of the stellar disc. This gas disc has a low metallicity in the outer parts, with $Z/Z_\odot \approx 10^{-2}$ at $r \approx 2$ kpc. At these large radii, the star formation rate is always low and the gas disc can survive for much longer than a Hubble time without converting into stars. This is unlikely to happen in all galaxies for at least two reasons: (i) the energy injection from supernovae into this gas will not be uniform, as assumed in this paper; (ii) the extended gas disc is susceptible to external disturbances and so could be tidally stripped or transported towards the centre of the system in a tidal interaction. Nevertheless, it is possible that dwarf galaxies at high redshift possess extended gaseous discs, some of which survive to the present day.

6.2.3 Effective yields

According to the simple closed box model of chemical evolution, the metallicity of the ISM is related to the gas fraction according to

$$Z_g = -p \ln[M_g/(M_g + M_\ast)].$$  \hspace{1cm} (50)

It is well known that the yields derived from applying this relation to gas rich galaxies (usually dwarf systems) result in ‘effective yields’, $p_{\text{eff}}$, that are much lower than the yield expected from a standard Salpeter-like IMF. For example, Vila-Costas & Edmunds (1992) find effective yields in the range $p_{\text{eff}} \sim 0.004$–0.02, and that the effective yield decreases with increasing radius.

The solid line in Fig. 15 shows the final gas metallicity in radial rings in model DW1 plotted against the gas fraction within each ring. The dotted line shows equation (50) with an effective yield of 0.004 (i.e., one-fifth of the true yield). The strong outflows in this model suppress the effective yield well below the true yield, and produce a strong radial variation of the effective yield, in qualitative agreement with observations.

6.2.4 Metallicity of ejected gas

The last line in Table 2 lists the mean metallicity of the gas that escapes from the galaxy. The mean metallicity of the ejected gas is about $0.3Z_\odot$ for model MW1 and about $0.1Z_\odot$ for model DW1. In model DW1 this value is about 3 to 5 times higher than the mean metallicity of the final gas disc. The ejected gas in this model is therefore ‘metal-enhanced’ relative to the gaseous disc. The mechanism for this metal enhancement is physically different from that in the models of Vader (1986, 1987) and MacLow & Ferrara (1999). In the models of these authors, metal enhancement arises from incomplete local mixing between the supernova ejecta and the ISM. In our models, the gas is assumed to be well mixed locally, but metal enhancement arises because the gas is lost preferentially from the central part of the galaxy, which has a higher metallicity than the gas in the outer parts of the system.

6.3 Connection with damped Lyman $\alpha$ systems

The column density threshold for the identification of damped Ly$\alpha$ systems is $N(\text{H}\text{I}) \geq 2 \times 10^{20} \text{ cm}^{-2}$ (Wolfe 1995), corresponding to a neutral gas surface mass density of $\sim 1.6 \text{ M}_\odot\text{ pc}^{-2}$. Comparison with Fig. 10 shows that the extended cold gaseous discs around dwarf galaxies would be detectable as damped Ly$\alpha$ systems. Furthermore, in CDM-like models, such extended discs around dwarf galaxies would dominate the cross-section for the identification of damped Ly$\alpha$ systems at high redshift, because the space density of haloes with low circular speeds is high (Kauffmann & Charlot 1994; Mo & Miralda-Escudé 1994). If this is the case, then the metallicities of damped Ly$\alpha$ systems would be expected to be low at high redshift, $Z/Z_\odot \sim 10^{-2}$, with occasional lines of sight intersecting the central regions of galaxies where the metallicity rises to $Z/Z_\odot \sim 0.1$. At lower redshifts, the metallicities of damped systems would be expected to show a similarly large scatter, but with perhaps a trend for the mean metallicity to increase as disc systems with higher circular speeds form and the extended gaseous discs around dwarfs are disrupted by tidal encounters.

This is qualitatively in accord with what is observed (Pettini et al. 1997, 1999; Pettini 2000). These authors find that the typical metallicity of a damped Ly$\alpha$ system at $z \sim 2–3$ is about $0.08Z_\odot$, with a spread of about two orders of magnitude. Comparing the metallicities of high-redshift systems with those of 10 damped Ly$\alpha$ systems with redshifts $z = 0.4–1.5$, Pettini et al. (1999) find no evidence for evolution of the column density weighted metallicity. Whether these and other properties of the damped Ly$\alpha$ systems can be reproduced with the feedback model described here requires more detailed ‘semi-analytic’ calculations along the lines described by Kauffmann (1996). However, the key point that we wish to emphasise is that according to the models described here, most of the cross-section at any given redshift will be dominated by largely unprocessed gas in the outer parts of galaxies that does not participate in the star formation process. The metallicity distributions and the evolution of $\Omega_\text{HI}$ as a function of redshift are therefore more likely to tell us about feedback processes and the outer parts of dwarf galaxies than about the history of star formation. Attempts to use the properties of damped Ly$\alpha$ systems to constrain the cosmic star formation
In this section we investigate the efficiency of feedback as a function of halo circular speed and semi-analytic models of galaxy formation.

In this section we investigate the efficiency of feedback as a function of halo circular speed and semi-analytic models of galaxy formation. We have adopted the parameters of Table 1, and run a series of models varying the halo circular speed \( v_0 \). The virial radius of the halo is set to \( r_v = 150(v_0/126 \text{ km s}^{-1})^2 \text{kpc} \), the concentration parameter \( c = 10 \), and the ratio of gas to halo mass within the virial radius is set to 1/10. The halo rotation speed is set by equation (39), with the fiducial disc scalelength equal to \( r_v/50 \). With these parameters, the family of models has a constant value for the halo spin parameter of \( \lambda_H = 0.065 \).

The retained baryonic fraction, \( 1 - f_{\text{ej}} \), is plotted as a function of halo circular speed in Fig. 16. The dotted line shows the relation used by C94 in their semi-analytic models,

\[
1 - f_{\text{ej}} = \frac{1}{1 + \beta(v_0)}, \quad \beta(v_0) = \left( \frac{v_0}{v_{\text{hot}}} \right)^{-\alpha_{\text{hot}}},
\]

where \( f_{\text{ej}} \) is the fraction of the cooled gas that is reheated, and \( v_{\text{hot}} \) and \( \alpha_{\text{hot}} \) are parameters. C94 adopt a severe feedback prescription with \( \alpha_{\text{hot}} = 5.5 \) and \( v_{\text{hot}} = 140 \text{ km s}^{-1} \) to reproduce the flat faint-end slope of the B-band galaxy luminosity function in a critical-density CDM model. The C94 feedback model does not agree at all well with the models described here. There is a slight ambiguity in the appropriate value of \( v_0 \), to use in equation (51), because C94 adopt an isothermal rather than an NFW halo profile; the halo circular speed at \( r = 0.1r_v \) may be 20 per cent higher than the circular speed at the virial radius, but this is far too small a difference to reconcile the C94 feedback prescription with the models of this paper.

In fact, the dashed line in Fig. 16 shows that our models are reasonably well described by equation (51) with \( v_{\text{hot}} = 75 \text{ km s}^{-1} \) and \( \alpha_{\text{hot}} = 2.5 \). Our results therefore suggest a much gentler feedback prescription than assumed in C94. Note that with the C94 parameters, a Milky Way type galaxy with \( v_0 = 130 \text{ km s}^{-1} \) would have lost about 60 per cent of its baryonic mass in a wind. This is well outside the range found from our models for plausible choices of the input parameters (cf. Table 2).

Recently, Baugh et al. (1999) and Cole et al. (2000) describe semi-analytic models applied to \( \Lambda \)-dominated CDM cosmologies that employ a gentler feedback model. The prescription for their reference model is similar to that of equation (51) with \( v_{\text{hot}} = 150 \text{ km s}^{-1} \) and \( \alpha_{\text{hot}} = 2.0 \), but with \( v_0 \) replaced by the disc circular speed \( v_{\text{disc}} \). This model is closer to the results found here. Assuming angular momentum conservation, a halo with \( \lambda_H = 0.06 \) will produce a disc with a circular speed \( v_{\text{disc}} \approx 1.7v_0 \) (cf. Table 1), and so their model can be approximated by equation (51) with \( v_{\text{hot}} \approx 90 \text{ km s}^{-1} \) and \( \alpha_{\text{hot}} = 2.0 \). With these parameters, their model gives somewhat stronger feedback than found in our models, but is well within the range of physical uncertainties. Kauffmann et al. (1993) and Kauffmann, Guiderdoni & White (1994) also adopt a much less severe feedback prescription than C94 in their semi-analytic models. For a detailed analysis of the effects of varying the feedback prescription (and other parameters) in semi-analytic models see Somerville & Primack (1999).

The change from an Einstein–de Sitter CDM cosmology in C94 to a \( \Lambda \)-dominated CDM model in Cole et al. (2000) partly explains why the revised models provide a reasonable match to observations using less efficient feedback. However, the revised models predict a faint-end slope for the B-band luminosity function that is consistent with the observations of Zucca et al. (1997) but not with those of other authors (e.g. Loveday et al. 1992; Maddox et al. 1998). (The earlier paper of C94 attempted to reproduce the flat faint-end slope of the Loveday et al. luminosity function.) The observational differences in estimates of the faint-end slope of the optical luminosity function are not properly understood, and so it remains unclear whether a gentle feedback model, of the type proposed here and used in Cole et al. (2000), can account for galaxy formation in CDM-type models.

With the Cole et al. (2000) parametrization the efficiency of feedback depends, by construction, on the surface density of the galaxy and hence on the angular momentum of the parent halo. In their model, higher angular momentum haloes lead to more efficient feedback, because they form low surface density discs with low disc circular speeds. This is not what is found in our models. Table 3 lists the ejected gas fraction as a function of the halo spin parameter \( \lambda_H \). Here, the halo circular speed and virial radius, \( v_0 \) and \( r_v \), are the same as for model DW in Table 1, but the amplitude of the halo rotation speed (or equivalently the parameter \( f_{\text{coll}} \)) is adjusted to change the spin parameter of the halo. The parameters of the feedback model are the same as those for model DW1 in Table 2. The feedback efficiency depends weakly (and non-monotonically) on \( \lambda_H \), and is greater in systems with low values of \( \lambda_H \). This is because higher surface densities in low-\( \Lambda \)
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7 CONCLUSIONS

The main aim of this paper has been to show that supernova-driven feedback can operate in a quiescent mode, and that high rates of star formation are not required to drive efficient feedback. In dwarf galaxies, feedback occurs on an infalling time-scale and so can extend over a period of \( \sim 1 \) Gyr. In the feedback model developed here, cold gas clouds are steadily evaporated in expanding supernova remnants and converted into a hot component. Critically, the rate at which cold gas is evaporated can exceed the rate at which mass is converted into stars. If the temperature of the hot component is high enough, a wind will form and the hot gas can escape from the halo (provided the interaction with infalling gas can be ignored). If the temperature of the hot component is not high enough for it to escape from the halo, it will cool and fall back down to the disc in a galactic fountain. Some characteristic features of the models are as follows.

(i) In a Milky Way-type system, feedback from supernovae may drive out some of the gas from the halo in the early phases of evolution (\( t \lesssim 0.3 \) Gyr) when the star formation rate is high and the temperature of the hot phase exceeds \( \sim 5 \times 10^6 \) K. For plausible sets of parameters, perhaps 20–30 per cent of the final stellar mass might escape from the galaxy. At later times, the temperature of the hot phase drops to \( T \sim 10^6 \) K and the evaporated gas cycles within the halo in a galactic fountain.

(ii) In a dwarf galaxy with a circular speed \( v_c > 50 \) km s\(^{-1}\), expanding supernova remnants can convert the cold interstellar medium efficiently into a hot component with a characteristic temperature of a few times \( 10^6 \) K. This evaporated gas can escape from the halo in a cool wind. Typically, only about 10 per cent of the baryonic material forms stars. Gas accreted from the halo at \( t > 1 \) Gyr forms an extended gaseous disc which, according to the self-regulating star formation model used here, can survive for longer than a Hubble time without converting into stars.

(iii) The feedback model developed here is meant to provide a sketch of how feedback might operate in a multiphase interstellar medium. The model contains a number of obvious oversimplifications. For example, we have neglected any interaction of the outflowing gas with the infalling medium, we have not addressed the origin of the cold cloud spectrum,\(^1\) we have ignored any distinction between cold clouds and dense molecular clouds, and we have neglected any local dissipation of supernova energy in star-forming regions. These effects, and other processes, are undoubtedly important in determining the efficiency of feedback. Nevertheless, the simplified model presented here contains some interesting features. First, the model shows how positive feedback (via pressure-induced star formation) and negative feedback (via outflowing gas) can occur simultaneously. Secondly, the models are remarkably insensitive to uncertain physical parameters; in particular, thermal conduction would need to be suppressed relative to its ideal value by factors of much more than 100 to qualitatively change the model. If thermal conduction is highly suppressed, it may be possible to construct a qualitatively similar model to the one presented here in which cold gas is converted into hot gas in shocks.

(iv) The self-regulated star formation and feedback models described here provide physically based models for the star formation time-scale and feedback efficiency as a function of the parameters of the halo. The star formation time-scale and feedback efficiency (or time-scale) are taken as free parameters in semi-analytic models of galaxy formation (e.g. Kauffmann et al. 1994; Cole et al. 2000) and are critically important in determining some of the key predicted properties of these models, for example, the faint-end slope of the galaxy luminosity function and the star formation history at high redshifts (see, e.g., Somerville & Primack 1999). It is therefore important that we develop a theoretical understanding of these parameters (as attempted crudely here), and also that ways are found to constrain these parameters observationally. The results presented here show that supernova feedback is much less effective than assumed in some earlier semi-analytic models (Cole et al. 1994; Baugh et al. 1996), but is closer to the gentler feedback prescriptions used in more recent models (Cole et al. 2000; Somerville & Primack 1999).

The feedback model described in this paper has a number of consequences, and raises some problems which are summarized below.

(i) Evidence for outflows: According to the models described here, outflows with speeds of \( \sim 200 \) km s\(^{-1}\) should be common in high-redshift galaxies. There is evidence for an outflow of \( \sim 40 \) km s\(^{-1}\), a mass-loss rate of \( \sim 10^{-3} \) M\(_{\odot}\) yr\(^{-1}\), and a star formation rate of \( \sim 10^{-3} \) M\(_{\odot}\) yr\(^{-1}\) in the gravitationally lensed Lyman break galaxy MS1512-cB58 (Pettini et al. 2000). The outflow velocity in this galaxy is consistent with our models, but the star formation and mass-loss rates (which are very uncertain) are high. The most likely explanations are either that MS1512-cB58 is a massive galaxy driving an outflow that will remain bound to the system, or that it is a less massive system undergoing a burst of star formation. In addition to direct detection of outflowing gas, winds may have other observational consequences. The winds from dwarf galaxies will cool rapidly (see Appendix B). Wang (1995b) has suggested that photoionized gas clouds formed in the cooling wind might contribute to the Ly\(\alpha\) forest. Nulsen, Barcon & Fabian (1998) suggest that outflows caused by bursts of star formation in dwarf galaxies might even produce damped Ly\(\alpha\) systems.

(ii) Damped Ly\(\alpha\) systems: The extended gaseous discs that form around dwarf galaxies in our models have low metallicities, because they have low rates of star formation. If this is correct, then this largely unprocessed gas would dominate the cross-section for the formation of damped Ly\(\alpha\) absorbers. The metallicities of most of these systems would be low, but would...
show a large scatter because some lines of sight will pass close to the central regions of galaxies containing gas of high metallicity. This is broadly in agreement with what is observed. Extended gaseous discs would be vulnerable in tidal interactions. Some of the gas might be stripped, and some might be transported into the central regions to be converted into stars and hot gas. The evolution of $\Omega_{\text{HI}}$, determined from damped Ly$\alpha$ systems (e.g. Storrie-Lombardi, McMahon & Irwin 1996) might have more to do with infall, feedback and tidal disruption than with the cosmic star formation history.

(iii) **Angular momentum conservation:** In hydrodynamic simulations, gas is found to cool effectively in sub-units during the formation of a protogalaxy. These sub-units lose their orbital angular momentum to the halo as they spiral towards the centre and merge. Hence the gas does not conserve angular momentum during the formation of a massive galaxy. (Navarro & Benz 1991; Navarro & Steinmetz 1997; Weil, Eke & Efstathiou 1998; Navarro & Steinmetz 2000). In fact, in the absence of feedback it has proved impossible to form discs with angular momenta similar to those of real disc galaxies starting from CDM initial conditions. In the models described here, it has been assumed for simplicity that the specific angular momentum of the gas is conserved during collapse. This assumption could easily be relaxed. However, the feedback model described here suggests that the numerical simulations miss some important physics. First, it is preferentially the low angular momentum gas, infalling in the early stages of evolution, that is most likely to be ejected from a developing protogalaxy. Secondly, supernova-driven feedback may help to solve the angular momentum momentum problem by ejecting gas efficiently from sub-units. The ejected gas may then infall at later times when the halo is less substructured, approximately conserving its angular momentum (Weil et al. 1998; Eke, Efstathiou & Wright 2000).

(iv) **Implementing feedback in numerical simulations:** There have been a number of attempts to implement supernova feedback in gas dynamical numerical simulations (e.g. Katz 1992; Navarro & White 1993; Navarro & Steinmetz 2000). These involve either heating the gas around star-forming regions (which is ineffective because the energy is quickly radiated away) or reversing the flow of infalling gas. An implementation of the feedback model described here is well beyond the capabilities of present numerical codes. It would require modelling several gas phases, a cold interstellar medium, a hot outflowing medium and an infalling component, including mass transfer between each phase. It might be worth attempting simpler simulations in which cold, high-density gas is added to the halo beyond the virial radius at a rate that is determined by the local star formation rate.

(v) **Starbursts versus quiescent feedback:** It is likely that starbursts are more common at high redshift because of the increased frequency of galaxy interactions. Starbursts could contribute to supernova-driven feedback, in addition to the quiescent mode described here. However, at any one time, our models suggest that the cold gas component will have a mass of only 20–50 per cent of the mass of the stellar disc. Even if a substantial fraction of this gas is transported towards the centre of a galaxy in a tidal encounter (see, e.g., Barnes & Hernquist 1996) and is subsequently ejected in a superwind, this mode of feedback will be inefficient, because the mass of gas involved is a small fraction of the total gas mass ejected in the quiescent feedback mode over the lifetime of the galaxy.

(vi) **Metallicity ejection:** The mean metallicity of the gas ejected from a dwarf galaxy is typically about $Z/10$ in our models, and comparable to the mean metallicity of the stars in the final galaxy. Yet typically a dwarf galaxy is predicted to expel 5 to 10 times its residual mass in stars. Dwarf galaxies can therefore pollute the IGM with metals to a much higher level than might be inferred from their stellar content. The high metallicity of gas in the central regions of clusters $\sim Z/3$ (e.g. Mushotsky & Loewenstein 1997) may require a top-heavy IMF and gas ejection from massive galaxies (Renzini et al. 1993).

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APPENDIX A: FAST COMPUTATION OF THE ROTATION CURVE OF A THIN DISC

We begin with the expression for the potential of a thin disc at $z = 0$,

$$\phi(r, z = 0) = -2\pi G \int_0^\infty \int_0^\infty J_0(kr) J_0(kr') \mu(r) \, dr' \, dk.$$

(A1)

The integral over $k$ is a well-known discontinuous integral (e.g. Watson 1944)

$$\int_0^\infty J_0(kr) J_0(kr') \, dk = \frac{1}{2} \left( \frac{1}{r^2} K(r') / r' < r \right) + \frac{1}{2} \left( 1/r^2 K(r') / r' > r \right),$$

(A2)

where $K$ is the complete elliptic integral of the first kind. Differentiating equation (A2), we find

$$v^2(r) = 4Gr \left[ \int_0^r I_<(r, r') \mu(r') \, dr' + \int_r^\infty I_>(r, r') \mu(r') \, dr' \right],$$

(A3a)

$$I_<(r, r') = \frac{E(r'/r)}{(r^2 - r'^2)},$$

(A3b)

$$I_>(r, r') = \frac{K(r/r')}{rr'} - \frac{r' E(r'/r')}{rr'^2},$$

(A3c)

where $E$ is the complete elliptic integral of the second kind. This integral is convergent in the limit $\epsilon = 0$. The functions $I_<$ and $I_>$ can be evaluated once and stored, reducing the computation of $v^2(r)$ to a simple integral over the surface density of the disc multiplied by the tabulated functions. We evaluate the epicyclic frequency by differentiating equation (A3) numerically.

APPENDIX B: STEADY SPHERICAL WINDS

The equations governing a steady spherically symmetric wind are

$$\frac{1}{r^2} \frac{d}{dr} (prr^2) = q(r),$$

(B1a)

$$pr \frac{dv}{dr} = -\frac{dp}{dr} - \frac{d\Phi}{dr} - q(r)v,$$

(B1b)

$$\frac{1}{r^2} \frac{d}{dr} \left[ prv - \frac{1}{2} \frac{d}{dr} \left( pr^2 \right) \right] + pv \frac{d\Phi}{dr} = \mathcal{H} - \mathcal{C},$$

(B1c)

where $q(r)$ is the mass density injected per unit time, and $\mathcal{H}$ and $\mathcal{C}$ are the heating and cooling rates per unit volume (e.g. Burke 1968; Holzer & Axford 1970). We assume that the gravitational force is given by the NFW halo potential (equation 3), $d\Phi/dr = \hat{c}^2(r)/r$, and rewrite these equations as two dimensionless...
first-order equations:
\[
\frac{dv}{dx} = \frac{1}{2\pi x^2 (c^2 - v^2)} \times \left( -8\pi xc^2v^2 + 4\pi xc^2v_i^2 + \frac{4}{7} yc^2 + \gamma c_i^2 - \frac{2}{7}\kappa \right),
\]
\[
\frac{dc}{dx} = \frac{-1}{6\pi x^2 (c^2 - v^2) v_i^2} \left( -8\pi xc^2v^4 + 4\pi xc^2v_i^2 c_i^2 \right.
\]
\[
- \gamma (v^2 c^2 - \frac{1}{2} c^4 - \frac{2}{3} c_i^4) - \frac{4}{7} \gamma c_i^4 (c^2 - \frac{2}{7} v^2)
\]
\[
+ \kappa (c^2 - \frac{2}{7} v^2) \right),
\]
where \(c\) is the adiabatic sound speed, \(x = r/r_v\), and all velocities are expressed in units of \(v_v\). The quantities \(\gamma\) and \(\kappa\) in these equations are related to the mass injection and cooling rates according to
\[
\gamma(x, v) = \frac{q(r) M(r)}{\rho^2 v_v v_i^2}, \quad \kappa(x, c) = \frac{M(r) \Lambda(T)n_i^2}{\rho^2 v_i^2 r_v},
\]
\[
M(r) = 4\pi \int_0^r q(r)r^2 dr,
\]
and the injected gas is assumed to have a uniform initial isothermal sound speed of \(c_i = (kT_i/(0.61m_p))^{1/2}\).

We illustrate the behaviour of the wind solutions by studying two regimes. First, we assume that \(q = 0\) beyond an initial radius \(r_i = 0.04r_v\) defining the base of the flow (i.e., two disc scalelengths for \(f_{\text{coll}} = 50\)). Equations (B2) do not have a transonic point when \(q = 0\) (Wang 1995a; see also the discussion below), and so we begin the integrations at a Mach number slightly greater than unity with \(c = 5c_i^2/3\). We adopt the parameters of models MW and DW given in Table 1, and integrate the equations (B1) adopting \(M = 10 M_\odot\) yr\(^{-1}\) for model MW and \(M = 0.2 M_\odot\) yr\(^{-1}\) for model DW. These mass injection rates are close to the maximum rates at times \(t \gg t_i\) for the models described in Section 6. The curves in Fig. B1 show solutions for initial isothermal sound speeds of 0.75, 1.0 and 1.25 times the escape velocity from the centre of the halo (\(v_{\text{esc}} = 430\) km s\(^{-1}\) for model MW and 107 km s\(^{-1}\) for model DW).

The figure shows that the criterion \(v_w < \sqrt{\frac{5c_i}{2}} \approx v_{\text{esc}}\) is about right if the wind is to reach beyond the virial radius. For \(c_i < v_{\text{esc}}\) the wind in model MW begins at a high temperature of \(T_i = 1.4 \times 10^7\) K and cools almost adiabatically initially, reaching a

**Figure B1.** Steady wind solutions for models MW and DW including radiative cooling. The curves show the wind velocity (panels a and c) and adiabatic sound speed (panels b and d), assuming that the flow begins at \(r_i = 0.04r_v\) with a Mach number of unity. The numbers give the initial isothermal sound speed in units of the escape speed \(v_{\text{esc}}\) from the centre of the halo. These curves are for a mass injection rate of \(10 M_\odot\) yr\(^{-1}\) for model MW and \(0.2 M_\odot\) yr\(^{-1}\) for model DW.

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temperature of $\sim 1.5 \times 10^5$ K at the virial radius. The time-scale for the flow to reach the virial radius, $\sim 2 \times 10^6$ yr, is slightly longer than the cooling time at $r_v$. The behaviour of models DW is quite different. For $c_i < v_{esc}$ the initial temperature of the gas is $T_i < 8 \times 10^5$ K and cools to $\sim 10^4$ K by $r = 0.3 r_v$. As expected from the discussion in Section 6, cooling is important in outflows from dwarf galaxies (see, e.g., Kahn 1991 and Wang 1995a,b).

An investigation of transonic solutions of equations (B2) requires a model for $q(r)$. An example is illustrated in Fig. B2 for model DW, using

$$q(r) = \frac{\dot{M}(\infty)}{8\pi r_w^2} \exp(-r/r_w), \quad r_w = 0.04 r_v. \quad (B3)$$

In this solution, $\dot{M}(\infty) = 0.2 M_\odot$ yr$^{-1}$, and the central gas density was adjusted to obtain a critical solution for the case $c_i = v_{esc}$. If the gas is to escape from a dwarf galaxy, the transonic point must occur before cooling sets in. For such systems, the wind parameters would adjust so that a sonic point exists at a characteristic cooling scaleheight, as shown in Fig. B2. The wind will then cool radiatively just beyond the sonic point, forming a cold wind as discussed above. It is also likely that the wind will be heated to a temperature of $T \sim 10^4$ K by photo-ionizing radiation from the galaxy and the general UV background. These sources of heating have not been included in the models in Figs B1 and B2.

The wind will be thermally unstable when cooling sets in, and may form clouds. However, in the absence of a confining medium, the clouds would have a filling factor of order unity, so the wind is likely to maintain its integrity until it meets the surrounding IGM. The external pressure required to balance the ram pressure of the wind is

$$p_{\text{ext}} = 80 \left( \frac{\dot{M}}{0.2 M_\odot \text{yr}^{-1}} \right) \left( \frac{r}{10 \text{kpc}} \right)^{-2} \left( \frac{v_w}{100 \text{km} \text{s}^{-1}} \right) \text{cm}^{-3} \text{K},$$

which is about equal to the pressure of the IGM with a temperature of $10^4$ K and an overdensity of

$$\Delta \approx 4500 \left( \frac{2}{1+z} \right)^{3} T_{4}^{-1} \left( \frac{\dot{M}}{0.2 M_\odot \text{yr}^{-1}} \right) \times \left( \frac{r}{10 \text{kpc}} \right)^{-2} \left( \frac{v_w}{100 \text{km} \text{s}^{-1}} \right). \quad (B5)$$

Provided that the halo is devoid of high-pressure gas, the cool wind will propagate beyond the virial radius and will be halted either by the ram pressure of infalling gas or after sweeping up a few times its own mass. As pointed out by Babul & Rees (1992), if a dwarf galaxy is embedded in a group or cluster of galaxies with a pressure exceeding $\sim 100 \text{cm}^{-3} \text{K}$, the bulk motion of the outflowing gas would be thermalized in a shock, and the cooled shocked gas could fall back on to the galaxy, thereby generating a new burst of star formation. The efficiency of feedback is therefore likely to be a function of local environment.

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Figure B2. Critical solution for a wind in model DW with $c_i = v_{esc}$, $\dot{M}(\infty) = 0.2 M_\odot$ yr$^{-1}$, and $q(r)$ given by equation (B3).