Differential Game Research on the Development of New Energy Automobile Enterprises under the Background of Subsidy Retreat

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Abstract. Under the background of the government’s subsidy for new energy vehicles, this paper takes a secondary energy supply chain composed of a new energy automobile enterprise and an online car-hailing platform as the research object. Considering the long-term dynamic characteristics of cooperation between new energy automotive enterprise and online car-hailing platform, this paper constructs a differential game model of automotive enterprise and online car-hailing platform under decentralized decision and centralized decision, and obtains their equilibrium strategy and optimal profit function under the two situations, as well as the optimal trajectory of automotive carbon emission reduction over time. The research finds that: compared with decentralized decision, under the centralized decision, new energy automobile enterprise increase its efforts in emission reduction, and the operation efforts of online car-hailing platform increase. Pareto improvement is achieved in the profit of supply chain, and the carbon emission reduction of automobiles is improved. Finally, the model is verified and analyzed by numerical experiments.

1. Introduction
Under the pressure of energy and environmental protection, the new energy vehicle guided by the concept of ‘low carbon economy’ has become the development direction of the future car. However, China’s new energy vehicle subsidy policy has been declining in 2019, and the subsidy standards for new energy passenger cars, new energy buses and new energy vehicles have been reduced. This is a difficult problem for new energy auto companies [1]. In recent years, due to the B2C business model and Internet properties, the network car company has been concerned by many car brands, such as the cooperation between Didi Travel and Volkswagen Group and the car home [2]. Under the background of the government subsidies, does the cooperation between new energy auto companies and the online car-hailing platform help them out of the development dilemma? How should the two companies cooperate to maximize the profit growth of both parties?

The development of new energy vehicles is one of the hottest research fields. At present, scholars at home and abroad have had many research results. Zhong and Du [3] explored the adverse selection problems in the subsidy process between the government and new energy auto companies by constructing a signal transmission game model. Huang and Pu [4] analyzed the new energy car rental market with the characteristics of ‘internet + timeshare’ leasing in the market by constructing an evolutionary game model. Liu et al. [5] studied the current status and future development of the main
technology pathways of new energy vehicles in China. Yu et al. [6] explored the evolution of China’s new energy industry through the use of three-dimensional models of technology, markets and policies. The literature [3, 4, 5, 6] has studied the government subsidies, leasing market, technology path and industrial development of new energy vehicles, but there is no literature to quantify the cooperation between new energy auto companies and online car-hailing platforms. The differential game is an infinite dynamic game with continuous time. Compared with static games or other dynamic game methods to study enterprise cooperation problems, it is more practical to use differential game model to study such problems [7]. This paper considers the dynamic continuity of enterprise cooperation, and uses the differential game method to study the cooperation between new energy auto companies and online car-hailing platforms, enriching the research results of predecessors and providing a certain reference for enterprise decision.

2. Problem description and model hypothesis

2.1. Problem Description

This paper takes a two-layer supply chain system consisting of a new energy automobile enterprise and an online car-hailing platform as the research object. New energy auto companies increase vehicle emission reductions through equipment upgrades, technology research and development, etc., and the online car-hailing platform increases the amount of new energy vehicle networks through operations and marketing. The emission reduction efforts of new energy automobile companies directly affect the emission reduction of automobiles. The operation efforts of the online car-hailing platform directly affect the network revenue of new energy vehicles. This paper compares the profit levels of new energy auto companies and online car-hailing platforms in both decentralized and centralized situations.

2.2. Symbol Description

- \( m(t) \): the level of emission reduction efforts of new energy vehicle company at time \( t \);
- \( r(t) \): the operational effort level of the online car-hailing platform at time \( t \);
- \( G(t) \): each vehicle emission reductions at time \( t \);
- \( \omega_m \): marginal profit for new energy auto company, \( \omega_m > 0 \);
- \( \omega_r \): marginal profit of the online car-hailing platform, \( \omega_r > 0 \).

2.3. Model hypothesis

(1) The emission reduction of automobiles is positively related to the emission reduction efforts of new energy automobile companies. As time goes by, due to factors such as aging of manufacturing equipment, there is a natural attenuation of automobile emission reductions [11], so the process of changing the emission reduction of each car can be expressed by the following differential equation:

\[
\dot{G}(t) = \alpha m(t) - \beta G(t)
\]  

Among them, \( G(t) \) represents the unit emission reduction amount at the time of \( t \), and the initial emission reduction amount is \( G(0) = G_0 > 0 \); \( \alpha (\alpha > 0) \) The degree of impact of new energy vehicle emission reduction efforts on unit vehicle emission reductions; \( \beta (\beta > 0) \) represents the attenuation factor of unit vehicle emission reductions.

(2) Since the energy-saving efforts cost of new energy vehicle company is \( C_m(t) \) related to its emission reduction effort \( m(t) \), the operating cost of the online car-hailing platform is \( C_r(t) \) and it related to its operations efforts \( r(t) \), drawing on the hypothesis of effort cost from the literature [12] and [13], setting the emission reduction effort cost of the new energy vehicle enterprise and the operation effort cost of the online car-hailing platform respectively

\[
C_m(t) = \frac{\eta_m}{2} m^2(t), \quad C_r(t) = \frac{\eta_r}{2} r^2(t),
\]
Among them, $\eta_m(\eta_m>0)$ represents the emission reduction effort cost coefficient of new energy automobile enterprises, and $\eta_r(\eta_r>0)$ represents the operation effort cost coefficient of car-hailing platform.

(3) Since the car-hailing quantity of new energy vehicles on the online car-hailing platform is affected by both vehicle emission reduction and the operation efforts of the online car-hailing platform. Reference to the literature [14], it is advisable to set the car-hailing quantity as

$$Q(t) = \lambda + \mu r(t) + \delta G(t)$$

Among them, $\lambda$, $\mu$, $\delta>0$, $\lambda$ is the potential ride-hailing quantity of new energy vehicles when the emission reduction of new energy vehicles is zero and the online ride-hailing platform is not put into operation; $\mu$ is the influence coefficient of online ride-hailing platform operation efforts on ride-hailing quantity; $\delta$ is the influence coefficient of car emission reduction on ride-hailing quantity.

(4) Assume that in the infinite time range, both the new energy vehicle company and the online car-hailing platform have the same discount factor $\rho$, and $\rho > 0$ [15].

3. Model analysis

This section is divided into two parts. Firstly, it studies the decision behaviors and supply chain profits of new energy auto company and online car-hailing platform under decentralized decision. Secondly, it studies the decision behaviors and supply chain profits of both parties under centralized decision. New energy Auto Company, online car-hailing platform and supply chain are indicated by subscripts m, r and s respectively.

3.1. Decentralized decision

In decentralized decision, new energy Auto Company and online car-hailing platform make decisions separately. The superscript N is used to indicate the decision behaviors of new energy Auto Company and online car-hailing platform under decentralized decision. At this time, the decision problems of new energy Auto Company and online car-hailing platform are respectively

$$\max_m J^N_m (G,t) = \int_0^\infty e^{-\rho t} (\omega_m Q_m - C_m) dt,$$

$$\max_r J^N_r (G,t) = \int_0^\infty e^{-\rho t} (\omega_r Q_r - C_r) dt.$$  

The mathematical model in this paper is solved by the Hamilton-Jacobi-Bellman equation (HJB equation for short). Due to the difficulty of solving under dynamic parameters, the reference [16] refers to the assumption that the parameters in the model are time-independent constants. For the convenience of writing, the time $t$ is omitted later. The result is as follows:

**Proposition 1** Under decentralized decision, the equilibrium strategy of new energy Auto Company is $m^N^\ast$ and the equilibrium strategy of the online car-hailing platform is $r^N^\ast$,

$$m^N^\ast = \frac{\alpha \delta \omega_m}{\eta_m (\rho + \beta)}, \quad r^N^\ast = \frac{\mu \omega_r}{\eta_r}.$$  

Further, the profit optimal function of the supply chain is

$$J^N_s (G,t) = e^{-\rho t} (a_s G + b_s),$$

Among them,
\[ a_3 = \frac{\delta(\omega_m + \omega_r)}{\rho + \beta}, \quad b_3 = \frac{\alpha^2 \delta^2 \omega_m (\omega_m + 2\omega_r) + \mu^2 \omega_r (2\omega_m + \omega_r) + \lambda(\omega_m + \omega_r)}{2\rho \eta_r} \].

Proof: Solving by inverse induction. After the time of \( t \), the profit optimal function of the online car-hailing platform is

\[ J^N_r(G,t) = e^{-\rho t} V^N_r(G). \]

According to the optimal control theory, for any \( G \geq 0 \), \( V^N_r(G) \) satisfies the HJB equation, ie

\[ \rho V^N_r(G) = \max_r \left[ \omega_r (\lambda + \mu r + \delta G) - \frac{\eta_r}{2} r^2 + V^N_r(G)(\alpha m - \beta G) \right]. \tag{2} \]

Take the first partial derivative of (2) with respect to \( r \) and set the partial derivative equal to 0, then

\[ r = \frac{\mu \omega_r}{\eta_r}. \tag{3} \]

In the same way, after the time \( t \), the profit optimal function of the new energy auto company is

\[ J^N_m(G,t) = e^{-\rho t} V^N_m(G), \tag{4} \]

For any \( G \geq 0 \), \( V^N_m(G) \) satisfies the HJB equation, ie

\[ \rho V^N_m(G) = \max_m \left[ \omega_m (\lambda + \mu r + \delta G) - \frac{\eta_m}{2} m^2 + V^N_m(G)(\alpha m - \beta G) \right]. \tag{5} \]

Take the first partial derivative of (5) with respect to \( m \) and set the partial derivative equal to 0, then

\[ m = \frac{\alpha V^N_m(G)}{\eta_m}. \tag{6} \]

Substitute (3) and (6) into (5),

\[ \rho V^N_m(G) = (\omega_m \delta - \beta V^N_m(G))G + \frac{\alpha^2 [V^N_m(G)]^2}{2 \eta_m} + \omega_m \omega_r \mu^2 + \lambda \omega_m. \tag{7} \]

According to the characteristics of the differential equation (7), it is speculated that the linear optimal value function for \( G \) is the solution of the HJB equation, so the linear expression of the function \( V^N_m(G) \) is set for

\[ V^N_m(G) = a_1 G + b_1. \tag{8} \]

Among them, \( a_1, b_1 \) are pending parameters. Substituting (8) into (7),

\[ \rho(a_1 G + b_1) = (\omega_m \delta - \beta a_1)G + \frac{\alpha^2 a_1^2}{2 \eta_m} + \frac{\mu^2 \omega_m \omega_r}{\eta_r} + \lambda \omega_m. \]
Comparing the coefficients of the same term at both ends of the equation

\[ a_1 = \frac{\delta \omega_m}{\rho + \beta}, \]

\[ b_1 = \frac{\alpha^2 \delta^2 \omega_m^2}{2 \rho \eta_m (\rho + \beta)^2} + \frac{\mu^2 \omega_m^2}{\rho \eta_m} + \frac{\lambda \omega_m}{\rho}. \]

Substitute (9) into (6), then

\[ m = \frac{\alpha \delta \omega_m}{\eta_m (\rho + \beta)}. \]

Substituting (8) into (4), the profit optimal function of the new energy auto company is

\[ J_m(G, t) = e^{-\rho t} (a_G G + b_1). \]

Substitute (3) and (10) into (2)

\[ \rho V_r^N(G) = (\omega_r \delta - \beta V_r^N(G))G + \frac{\alpha^2 a G V_r^N(G)}{\eta_m} + \frac{\omega_r^2}{2 \eta_r} + \omega_r \lambda. \]

According to the characteristics of the differential equation (11), it is speculated that the linear optimal value function for G is the solution of the HJB equation, so the linear expression of the function \( V_r^N(G) \) is set for

\[ V_r^N(G) = a_2 G + b_2. \]

Among them, \( a_2, b_2 \) are pending parameters. In the same way, we can get the profit optimal value function of the online car-hailing platform

\[ J_r^N(G, t) = e^{-\rho t} (a_G G + b_2). \]

Among them, \( a_2 = \frac{\delta \omega_r}{\rho + \beta}, b_2 = \frac{\alpha^2 \delta^2 \omega_m \omega_r}{\rho \eta_m (\rho + \beta)^2} + \frac{\mu^2 \omega_r^2}{2 \eta_r} + \frac{\lambda \omega_r}{\rho}. \)

Therefore, the profit optimal function of the supply chain is

\[ J_s^N(G, t) = e^{-\rho t} (a_G G + b_3) = J_m^N(G, t) + J_r^N(G, t). \]

According to (3), (10), and (12), Proposition 1 is established.

**Proposition 2** Under the decentralized decision, the optimal trajectory of the emission reduction of unit new energy vehicle is

\[ G^N(t) = (G_0 - A^N)e^{-\rho t} + A^N, \]

and \( A^N = \frac{\alpha^2 \delta \omega_m}{\beta \eta_m (\rho + \beta)}. \)

Proof: Bring (10) into the equation of state (1)
\[ G(t) = \frac{\alpha^2 \delta \omega_m}{\eta_m (\rho + \beta)} - \beta G(t). \]

Also \( G(0) = G_0 \), then
\[ G(t) = \left[ G_0 - \frac{\alpha^2 \delta \omega_m}{\beta \eta_m (\rho + \beta)} \right] e^{-\beta t} - \frac{\alpha^2 \delta \omega_m}{\beta \eta_m (\rho + \beta)} . \]

Let \( A^N = \frac{\alpha^2 \delta \omega_m}{\beta \eta_m (\rho + \beta)} \), then the conclusion of Proposition 2 is established.

When \( t \to \infty \), the limit of \( G^N \) is \( A^N \), \( A^N \) is the stable value of emission reduction per unit of new energy vehicles under decentralized decision.

### 3.2 Centralized decision

Under the centralized decision, the new energy automobile enterprise and the online car-hailing platform all take the supply chain benefit maximization as the common decision goal. Use the superscript \( C \) to represent centralized decision making. At this time, the profit of the supply chain is composed of the profit of the new energy auto company and the profit of the online car-hailing platform. Then the decision problem of the whole supply chain is

\[ \max_{m, r} J^C_s (G, t) = \int_0^\infty e^{-\beta t} [(\omega_m + \omega_r) Q - C_m - C_r] dt. \]

Consistent with the previous section, the mathematical model in this section is solved using the HJB equation. The result is as follows:

**Proposition 3** Under the centralized decision, the equilibrium strategy of the new energy auto company \( m^C \) and the equilibrium strategy of the online car-hailing platform \( r^C \) are respectively

\[ m^C = \frac{\alpha \delta (\omega_m + \omega_r)}{\eta_m (\rho + \beta)}, \quad r^C = \frac{\mu (\omega_m + \omega_r)}{\eta_r}. \]

Further, the optimal profit function of the supply chain system is

\[ J^C_s (G) = e^{-\beta t} (a_4 G + b_4), \]

Among them,

\[ a_4 = \frac{\delta (\omega_m + \omega_r)}{\rho + \beta}, \quad b_4 = \frac{\alpha^2 \delta^2 (\omega_m + \omega_r)^2}{2 \rho \eta_m (\rho + \beta)^2} + \frac{\mu^2 (\omega_m + \omega_r)^2}{2 \rho \eta_r} + \frac{\lambda (\omega_m + \omega_r)}{\rho} . \]

Proof: After the time \( t \), the profit optimal function of the supply chain system is

\[ J^C_s (G, t) = e^{-\beta t} V^C_s (G), \quad (13) \]

For any \( G \geq 0 \), \( V^C_s (G) \) satisfies the HJB equation, ie

\[ \rho V^C_s (G) = \max_{m, r} \left[ (\omega_m + \omega_r) (\lambda + \mu r + \delta G) - \frac{\eta_m}{2} m^2 - \frac{\eta_r}{2} r^2 + V^C_s (G)(\alpha m - \beta G) \right]. \quad (14) \]
On the right side of the equation (14), find the first-order partial derivatives for m and r respectively, and let the partial derivative equal 0, then

\[ m = \frac{\alpha V_s^C(G)}{\eta_m}, \quad (15) \]

\[ r = \frac{\mu(\omega_m + \omega_r)}{\eta_r}. \quad (16) \]

Substitute (15) and (16) into (14),

\[ \rho V_s^C(G) = [(\omega_m + \omega_r)\delta - \beta V_s^C(G)]G + \frac{\alpha^2 [V_s^C(G)]^2}{2\eta_m} + \frac{\mu^2 (\omega_m + \omega_r)^2}{2\eta_r} + \lambda (\omega_m + \omega_r). \quad (17) \]

According to the characteristics of the differential equation (17), it is speculated that the linear optimal value function for G is the solution of the HJB equation, so the linear expression of the function \( V_s^G(G) \) is set for

\[ V_s^C(G) = a_4 G + b_4. \quad (18) \]

Among them, \( a_4, b_4 \) are pending parameters. Substituting (18) into (17),

\[ \rho (a_4 G + b_4) = [(\omega_m + \omega_r)\delta - \beta a_4]G + \frac{\alpha^2 a_4^2}{2\eta_m} + \frac{\mu^2 (\omega_m + \omega_r)^2}{2\eta_r} + \lambda (\omega_m + \omega_r). \]

Comparing the coefficients of the same term at both ends of the equation

\[ a_4 = \frac{\delta (\omega_m + \omega_r)}{\rho + \beta}, \quad (19) \]

\[ b_4 = \frac{\alpha^2 \delta^2 (\omega_m + \omega_r)^2}{2\rho \eta_m (\rho + \beta)^2} + \frac{\mu^2 (\omega_m + \omega_r)^2}{2\rho \eta_r} + \frac{\lambda (\omega_m + \omega_r)}{\rho}. \]

Substitute (19) into (15)

\[ m = \frac{\alpha \delta (\omega_m + \omega_r)}{\eta_m (\rho + \beta)}. \quad (20) \]

Substituting (18) into (13) gives the profit optimal function of the supply chain as

\[ J_s^C(G, t) = e^{-\rho t} (a_4 G + b_4). \quad (21) \]

Proposition 3 is established according to (16), (20), and (21).

**Proposition 4** Under the centralized decision, the optimal trajectory of the vehicle emission reduction of the new energy automobile enterprise is

\[ G_s^C(t) = (G_0 - A_s^C) e^{-\rho t} + A_s^C. \]
Among them, \( A^C = \frac{\alpha^2 \delta (\omega_m + \omega_r)}{\beta \eta_m (\rho + \beta)} \).

Proof: Substituting (20) into the equation of state (1). According to the state equation boundary condition \( G(0) = G_0 \geq 0 \), the optimal trajectory of emission reduction of new energy vehicles per unit is obtained. The proof method is similar to that of proposition 2, and is not described again.

Obviously, when \( t \to \infty \), the limit of \( G^C \) is \( A^C \), \( A^C \) is the stable value of emission reduction of each vehicle under centralized decision.

4. Comparative analysis and expansion

Inference 1 can be obtained from Proposition 1, Proposition 2, Proposition 3, and Proposition 4.

\[
(1) m^C - m^N = \frac{\alpha \delta \omega_r}{\eta_m (\rho + \beta)} > 0,
\]

\[
(2) r^C - r^N = \frac{\mu \omega_m}{\eta_r} > 0,
\]

\[
(3) J^C_s (G) - J^N_s (G) = \frac{\alpha^2 \delta^2 \omega_r^2}{2 \rho \eta_m (\rho + \beta)^2} + \frac{\mu^2 \omega_m^2}{2 \rho \eta_r} > 0,
\]

\[
(4) G^C(t) - G^N(t) = \frac{\alpha^2 \delta \omega_r}{\beta \eta_m (\rho + \beta)} (1 - e^{-\beta t}) \geq 0.
\]

According to the inference 1, compared with the decentralized decision, the new energy vehicle enterprise under the centralized decision have made greater efforts to reduce emissions, and the online car-hailing platform has invested more in operational efforts, the supply chain has higher profits, and the emission reduction of each car is higher. Because under the decentralized decision, new energy Auto Company and online car-hailing platform have their own interests to maximize the goal, it is easy to cause double marginal effects. Under the centralized decision, both partners aim at maximizing the supply chain profit, which strengthens the cooperation intensity of the two parties, reduces the impact of the double marginal effect, and reduces the transaction costs of both parties, can let the enterprise will be more money for efforts and operation, thus increase the car’s emissions and supply chain profits.

5. Numerical Simulation

Through the assignment of exogenous variables, the numerical simulation analysis of the conclusions of Part 3 and Part 4 using Matlab R2017b software can more intuitively show the trends of the profit and emission reduction of the supply chain under decentralized decision and centralized decision. Set the baseline parameter to: \( \alpha=2, \beta=1, \delta=2, \lambda=4, \mu=1, \eta_m=3, \eta_r=2, \omega_m=6, \omega_r=4.5, \rho=0.9, G_0 = 0 \).
As can be seen from Figure 1, compared with the decentralized decision, the profit of the supply chain under the centralized decision is significantly increased, and supply chain profit achieves Pareto improvement. The profits of new energy auto companies and online car-hailing platforms can be further distributed through negotiation.

It can be seen from Figure 2 that under centralized decision, the emission reduction of each vehicles is significantly higher than that of each vehicles under decentralized decision.

6. Conclusion and Outlook
From the perspective of long-term dynamics, this paper uses the differential game method to analyze the decision behaviors and supply chain profits of new energy Auto Company and online car-hailing platform under decentralized decision and centralized decision under the background of government subsidies, and finally pass numerical experiments. After further analysis, it is concluded that under the
centralized decision, new energy Auto Company will invest more efforts in reducing emissions, and the online car-hailing platform will invest more operational efforts, and the supply chain profits will increase. The emission reduction of automobiles has increased significantly.

It should be noted that this paper focuses on the analysis of cooperation between new energy auto companies and online car-hailing platforms, and does not consider the impact of government subsidies. In future research, the impact of government subsidies on companies will be considered.

References

[1] New energy vehicle subsidies retreat, crisis or turnaround? http://sh.Qihoo.com/pc/9ec30d36a580aa1ad? cota=4sign=360e39369d1referscene=so1

[2] Cross-border cooperation, investment acquisition, network car company has become a new focus of the automotive-related industry - News - Global IC Trade Starts Here Free Join, http://www.sohu.com/a/242352940236016

[3] Zhong Taiyong, Du Rong. Research on New Energy Vehicle Subsidy Strategy Based on Game Theory [J]. CMS, 2015 (S1): 817-822.

[4] Huang Yixiang, Pu Yongjian. Research on Evolutionary Game Model of New Energy Vehicle Timeshare Rental Market Competition [J]. China Management Science, 2018.

[5] Liua Z, Haoa H, Chenga X, et al. Critical issues of energy efficient and new energy vehicles development in China [J]. Energy Policy, 2018, 115:92-97.

[6] Yu P, Zhang J, Yang D, et al. The Evolution of China’s New Energy Vehicle Industry from the Perspective of a Technology Market Policy Framework [J]. Sustainability, 2019, 11.

[7] Ban Yunhao. Research on cooperative differential game problem [D]. Dongbei University of Finance and Economics, 2009.

[8] Zhao Dao-zhi, Xu Chunqiu, Wang Qinpeng. Combined countermeasures for retailer emission reduction and low-carbon propaganda considering retailer competition [J]. Control and Decision, 2014, 29 (10): 1809-1815.

[9] Hong Jiangtao, Huang Pei. Research on Dynamic Coordination Mechanism of Quality Control in Two Level Supply Chain [J]. Journal of Industrial Engineering and Engineering Management, 2011, 25 (02): 62-65+94.

[10] Tian Houping, Liu Changxian. A Dual-Objective Mixed Incentive Model for Sales Channels under Dual Information Asymmetry [J]. Journal of Management Sciences, 2011, 14 (3): 34-47.

[11] Lambertini L, Orsini R. Process and product innovation in a vertically differentiated monopoly [J]. Economics Letters, 2000, 68: 333 - 337.

[12] Wang Qinpeng, Zhao Daozhi. Research on Supply Chain Revenue Sharing Contract under Consumers’ Low Carbon Preference [J]. China Management Science, 2014, 22 (9): 106-113.

[13] Jorgensen S, Zaccour G. Retail promotions with negative brand image effects: Is cooperation possible? [J]. European Journal of Operational Research, 2003, 150 (2): 395-405.