Confronting Tri-direct CP-symmetry models to neutrino oscillation experiments

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Tri-direct CP symmetry is an economical neutrino model building paradigm, and it allows red for the description of neutrino masses, mixing angles and CP violation phases in terms of four free parameters. Viability of a class of tri-direct CP models is examined with a comprehensive simulation of current and future neutrino oscillation experiments. The full parameter space of four independent parameters is carefully scanned, and the problem of parameter degeneracy appears for the constraints from one group of neutrino oscillation experiments. Two benchmark models which are promising from a model building point of view are also examined. Complementary roles from accelerator neutrino experiments (e.g., T2HK and DUNE) and reactor neutrino experiments (e.g., JUNO) are crucial to break the degeneracy and nail down the fundamental neutrino mixing parameters of the underlying theory.

I. INTRODUCTION

Neutrinos in the standard model (SM) of particle physics are strictly massless. Neutrino oscillation requires mass-squared differences and non-zero neutrino masses, which is striking new physics beyond the SM and calls for new degrees of freedom. In the framework of the three-generation neutrino oscillation paradigm, we have two mass-squared differences ($\Delta m_{21}^2$, $\Delta m_{31}^2$), three mixing angles ($\theta_{12}$, $\theta_{13}$ and $\theta_{23}$) and Dirac CP phase $\delta_{CP}$ [1]. The precision of measuring $\theta_{13}$ is dominated by reactor neutrino experiments [2–4], $\theta_{12}$ and $\Delta m_{21}^2$ are dominated by solar and reactor neutrino experiment KamLAND [5–8], as well as $\theta_{23}$ and $|\Delta m_{31}^2|$ are dominated by atmospheric neutrino experiments [4, 9, 10]. A global analysis of different experiments provides the precise values of mixing parameters at the percentage level [1]. However, the mass ordering $\Delta m_{31}^2 > 0$ or $\Delta m_{31}^2 < 0$ and the value of $\delta_{CP}$ remains unclear, although hints exist from experiments which are currently running.

Many models have been proposed to accommodate massive neutrinos without violating overwhelming constraints from previous experimental results. The origin of neutrino masses, flavor mixing and CP violation is a longstanding open question in particle physics. It turns out that a broken flavour symmetry based on a discrete group is particularly suitable to explain the structure of the leptonic mixing matrix, see Refs. [11–15] for review. If the discrete flavor symmetry is extended to also involve CP as a symmetry, the CP violation phases in the quark sector (observed) and lepton sector can be predicted [16–18]. Recently a new discrete flavor symmetry model building approach called tri-direct CP was proposed [19, 20], and it is dictated by residual symmetries such that it is quite predictive. The light neutrino mass matrix only depends on four real free parameters to describe the entire neutrino sector (three neutrino masses as well as the lepton mixing matrix). Moreover the CP violations in neutrino oscillations and leptogenesis generally arise from the same phase in the tri-direct CP model, consequently they are closely related to each other.

Precision measurements of neutrino oscillation parameters will guide us to the new physics domain. While in the quark sector, the precision is at the sub-percentage level [21], in the neutrino sector, the parameter uncertainties remain at the percentage level [1]. New physics might be hidden in the uncertainties of measured neutrino mixing parameters, as neutrino oscillations bridge neutrino mixings and other factors which affect...
the propagation of coherent states. The new models to accommodate massive neutrinos intend to bring new fundamental symmetries, new particles and their new interactions beyond the standard model. It is promising to conduct precision measurements in accelerator neutrino experiments to search for new physics, including non-standard interactions, and neutrino decays (e.g. Ref. [22–32]). It is a question of whether we are able to test different flavor and CP-violation models directly in running accelerator neutrino experiments like T2K [4, 33] and NOvA [9, 34, 35], and future neutrino oscillation experiments like DUNE [36] and T2HK [37]. In this paper, we shall determine the potential of current and upcoming neutrino facilities to test the tri-direct CP approach, and the sensitivity regions of oscillation parameters will be presented.

The paper is organized as follows: we firstly review the tri-direct CP symmetry in Sec. II. In Sec. III, we investigate the precision measurements of oscillation parameters, either represented by standard three neutrino mixing parameters or denoted by the benchmark model parameters, in running experiments such as T2K and NOvA. We expect better sensitivities in future neutrino experiments, such as T2HK, DUNE and Jiangmen Underground Neutrino Observatory (JUNO). In Sec. IV, we show our simulation results. We study the precision of model parameters for different experimental configurations, and then discuss a degeneracy problem, which can be resolved by including JUNO data. We further study how to constrain oscillation parameters with the restriction of the tri-direct CP model, before we discuss two benchmark models. Finally, we summarize our results in Sec. V.

II. REVIEW OF TRI-DIRECT CP SYMMETRY MODELS

Let us firstly recapitulate a benchmark tri-direct CP model proposed in [19]. This model is based on the $S_4$ flavor symmetry and CP symmetry. The flavor group $S_4$ and CP are broken to the subgroups $Z_3^T \times Z_2^{STST} \times X_{\text{atm}}$ and $Z_2^T \times X_{\text{sol}}$ in the charged lepton, atmospheric neutrino and solar neutrino sectors, respectively, where $S$, $T$, $U$ are the generators of $S_4$ and $X_{\text{atm}} = SU$ and $X_{\text{sol}} = U$ denote the residual CP symmetry. In the generic tri-direct CP paradigm, the structure of the neutrino and charged lepton mass matrices essentially arise from the vacuum alignment of flavon fields which are fixed by the residual symmetry. In the working basis of [19], the residual flavor symmetry $Z_3^T$ enforces that the charged lepton mass matrix is diagonal [19]. The atmospheric and solar flavon vacuum alignments are determined to be $\langle \phi_{\text{atm}} \rangle \propto (1, \omega^2, \omega)^T$ and $\langle \phi_{\text{sol}} \rangle \propto (1, x, x^2)^T$, where $\omega = e^{2\pi i/3}$ is a cube root of unity and the parameter $x$ is real because of the imposed CP symmetry. As a result, the Dirac neutrino mass matrix reads as

$$m_D = \begin{pmatrix} y_a & y_s \\ \omega y_a & \omega^2 y_a \\ \omega^2 y_a & \omega y_a \\ \end{pmatrix}.$$  

The right-handed neutrino Majorana mass matrix is diagonal

$$m_N = \begin{pmatrix} M_{\text{atm}} & 0 \\ 0 & M_{\text{sol}} \end{pmatrix}.$$  

The light effective left-handed Majorana neutrino mass matrix is given by the seesaw formula:

$$m_{\nu} = m_a \begin{pmatrix} 1 & \omega & \omega^2 \\ \omega & 1 & \omega^2 \\ \omega^2 & \omega & 1 \end{pmatrix} + e^{i\eta} m_s \begin{pmatrix} 1 & x & x^2 \\ x & 1 & x \\ x^2 & x & 1 \end{pmatrix},$$  

where $m_a = |y_a^2/M_{\text{atm}}|$, $m_s = |y_s^2/M_{\text{sol}}|$, and the only physically important phase $\eta$ depends on the relative phase between $y_a^2/M_{\text{atm}}$ and $y_s^2/M_{\text{sol}}$. It is noteworthy that only four parameters $m_a$, $m_s$, $\eta$ and $x$ are involved to describe both neutrino masses and lepton mixing parameters. As a consequence, this model is quite predictable. The low-energy phenomenology of this model has been studied both numerically and analytically in [19]. Although $x$ and the relative phase $\eta$ are free parameters in the general setup of tri-direct CP, they can be fixed to some particular values through the vacuum alignment technique in a discrete flavor symmetry model. It is found that a quite good fit to the experimental data can be obtained for certain choices of $x$ and $\eta$, two benchmark examples
are \( x = -7/2, \eta = \pi \) and \( x = -4, \eta = 5\pi/4 \). In these benchmark models, the corresponding vacuum alignments take a simple form such that they can be easily realized in concrete models [19, 20]. Moreover, the neutrino mass matrix as well as neutrino masses and mixing parameters only depend on two free parameters \( m_a \) and \( m_s \) in the benchmark models and the experimental data can be accommodated very well.

The neutrino mass spectrum is predicted to be normal ordering in this model, and the lightest neutrino is massless \( m_1 = 0 \). The other two non-vanishing neutrino masses \( m_2 \) and \( m_3 \) are expressed in terms of the input parameters as follows.

\[
\begin{align*}
m_2^2 &= \frac{1}{2} \left[ \left| y \right|^2 + \left| w \right|^2 + 2|z|^2 \right] - \sqrt{\left( \left| w \right|^2 - \left| y \right|^2 \right)^2 + 4|y^* z + w^* z|^2}, \\
m_3^2 &= \frac{1}{2} \left[ \left| y \right|^2 + \left| w \right|^2 + 2|z|^2 \right] + \sqrt{\left( \left| w \right|^2 - \left| y \right|^2 \right)^2 + 4|y^* z + w^* z|^2},
\end{align*}
\]

where

\[
\begin{align*}
y &= \frac{5x^2 + 2x + 2}{2(x^2 + x + 1)} (m_a + e^{i\eta} m_s), \\
z &= -\frac{\sqrt{5}x^2 + 2x + 2}{\sqrt{5}x^2 + 2x + 2} \left( x + 2 \right) m_a - x(2x + 1) e^{i\eta} m_s, \\
w &= \frac{1}{2(x^2 + x + 1)} \left( x + 2 \right)^2 m_a + x^2 (2x + 1)^2 e^{i\eta} m_s.
\end{align*}
\]

As regards the predictions for lepton flavor mixing, the first column of the mixing matrix is determined to be proportional to \((\sqrt{3} x, \sqrt{x^2 + x + 1}, \sqrt{x^2 + x + 1})^T\), the other two columns are uniquely fixed by the input parameters, and the lepton mixing matrix is of the form [19]

\[
U = \frac{1}{\sqrt{2}} \left( \frac{\sqrt{5}x}{\sqrt{5}x^2 + 2x + 2} \right) \begin{pmatrix}
2i \sqrt{\frac{x^2 + x + 1}{x^2 + 2x + 2}} \cos \theta & - \sin \psi \\
2i \sqrt{\frac{x^2 + x + 1}{x^2 + 2x + 2}} \sin \theta & \cos \psi
\end{pmatrix}.
\]

The angles \( \theta \) and \( \psi \) are specified by

\[
\begin{align*}
\sin \psi &= \frac{3}{\left| y^* z + w^* z \right|}, & \cos \psi &= \frac{\Re \left( y^* z + w^* z \right)}{\left| y^* z + w^* z \right|}, \\
\sin 2\theta &= \frac{2y^* z + w^* z}{\sqrt{\left( \left| w \right|^2 - \left| y \right|^2 \right)^2 + 4\left| y^* z + w^* z \right|^2}}, \\
\cos 2\theta &= \frac{\left| w \right|^2 - \left| y \right|^2}{\sqrt{\left( \left| w \right|^2 - \left| y \right|^2 \right)^2 + 4\left| y^* z + w^* z \right|^2}}.
\end{align*}
\]

As a consequence, we find the exact expressions for the mixing angles are

\[
\begin{align*}
\sin^2 \theta_{13} &= \frac{2(x^2 + x + 1) \sin^2 \theta}{5x^2 + 2x + 2}, \\
\sin^2 \theta_{12} &= 1 - \frac{3x^2 + 2(x^2 + x + 1) \cos^2 \theta}{3x^2}, \\
\sin^2 \theta_{23} &= \frac{1}{2} + \frac{x \sqrt{3(5x^2 + 2x + 2) \sin 2\theta \sin \psi}}{2 [3x^2 + 2(x^2 + x + 1) \cos^2 \theta]}.
\end{align*}
\]

We see that the solar and reactor mixing angles satisfy the following sum rule

\[
\cos^2 \theta_{12} \cos^2 \theta_{13} = \frac{3x^2}{5x^2 + 2x + 2}.
\]
Moreover, from the first column of the mixing matrix in Eq. (6), we can obtain a sum rule for $\cos \delta_{\text{CP}}$ in terms of the lepton mixing angles

$$
\cos \delta_{\text{CP}} = \frac{\cot 2\theta_{23} \left[ 3x^2 - (4x^2 + x + 1) \cos^2 \theta_{13} \right]}{\sqrt{3} |x| \sin \theta_{13} \sqrt{(5x^2 + 2x + 2) \cos^2 \theta_{13} - 3x^2}}.
$$

(10)

If the atmospheric mixing angle is maximal, this sum rule implies that the Dirac CP phase would be maximal (i.e. $\delta_{\text{CP}} = \pm \pi/2$) as well. Furthermore, the result for the Jarlskog invariant is

$$
J_{\text{CP}} = \frac{\sqrt{3}x (x^2 + x + 1) \sin 2\theta \cos \psi}{2 (5x^2 + 2x + 2)^{3/2}},
$$

(11)

from which we can extract the value of $\sin \delta_{\text{CP}}$,

$$
\sin \delta_{\text{CP}} = \pm \csc 2\theta_{23} \sqrt{1 + \frac{(x^2 + x + 1)^2 \cot^2 \theta_{13} \cos^2 \delta_{\text{CP}}}{3x^2 \left( 3x^2 \tan^2 \theta_{13} - 2(x^2 + x + 1) \right)}},
$$

(12)

with “+” for $x \cos \psi > 0$ and “−” for $x \cos \psi < 0$. The above results for $\cos \delta_{\text{CP}}$ and $\sin \delta_{\text{CP}}$ allow us to fix the value of $\delta_{\text{CP}}$. Comprehensive numerical analyses showed that the allowed region of the parameters $x, \eta, r = m_a/m_s$ and $m_a$ are $-5.475 \leq x \leq -3.370$, $0.455 \leq \eta \leq 1.545\pi$, $0.204 \leq r \leq 0.606$ and $3.343 \text{ meV} \leq m_a \leq 4.597 \text{ meV}$ respectively in order to accommodate the experimental data on neutrino masses and lepton mixing angles [38]. It is remarkable that both solar mixing angle and Dirac CP phase are predicted to lie in a narrow range $0.329 \leq \sin^2 \theta_{12} \leq 0.346$ and $1.371 \pi \leq \delta_{\text{CP}} \leq 1.629\pi$ in this model.

For the benchmark values of the vacuum parameters $x = -7/2$ and $\eta = \pi$, the effective light neutrino mass matrix in Eq. (3) only depends on two free parameters $m_a$ and $m_s$. Using the general results presented above, we find that the lepton mixing matrix is of the form

$$
U = \frac{1}{5\sqrt{6}} \begin{pmatrix}
\sqrt{2} & \sqrt{2} & 2 \sqrt{13} \sin \theta \\
\sqrt{26} & 7i \cos \theta - 5\sqrt{3} \sin \theta & -7 \sin \theta + 5\sqrt{3} i \cos \theta \\
\sqrt{26} & 7i \cos \theta + 5\sqrt{3} \sin \theta & -7 \sin \theta - 5\sqrt{3} i \cos \theta
\end{pmatrix},
$$

(13)

where

$$
\sin 2\theta = \frac{10|14r - 1|}{13\sqrt{4 + 32r + 289r^2}}, \quad \cos 2\theta = \frac{3(8 + 57r)}{13\sqrt{4 + 32r + 289r^2}},
$$

(14)

with $r = m_s/m_a$. The lepton mixing angles read

$$
\sin^2 \theta_{13} = \frac{26}{75} \sin^2 \theta, \quad \sin^2 \theta_{12} = \frac{26 \cos^2 \theta}{62 + 13\cos 2\theta}, \quad \sin^2 \theta_{23} = \frac{1}{2},
$$

(15)

and the Jarlskog invariant is

$$
J_{\text{CP}} = -\frac{91}{750\sqrt{3}} \sin 2\theta,
$$

(16)

which implies that the Dirac CP phase is exactly maximal, i.e.

$$
\delta_{\text{CP}} = -\pi/2.
$$

(17)

Notice that both $\theta_{23}$ and $\delta_{\text{CP}}$ are predicted to be maximal and they are favored by the latest data from T2K [39] and NOνA [9, 40], the reason is the neutrino mass matrix of Eq. (3) fulfills the $\mu - \tau$ reflection symmetry in this case. The lightest neutrino masses as functions of $m_a$ and $r$ are

$$
m_1^2 = 0,
$$

$$
m_2^2 = \frac{9}{8} m_a^2 \left( 4 - 18r + 289r^2 - |2 - 17r| \sqrt{4 + 32r + 289r^2} \right),
$$

$$
m_3^2 = \frac{9}{8} m_a^2 \left( 4 - 18r + 289r^2 + |2 - 17r| \sqrt{4 + 32r + 289r^2} \right).
$$

(18)
In order to accommodate the experimental values of the mixing angles and neutrino mass splittings $\Delta m_{21}^2$ and $\Delta m_{31}^2$ \cite{131}, we find that $m_\alpha$ and $r$ are constrained to lie in rather narrow regions $3.560\text{meV} \leq m_\alpha \leq 3.859\text{meV}$ and $0.5282 \leq r \leq 0.5904$. Accordingly the allowed regions of the reactor and solar mixing angles are strongly constrained $0.02206 \leq \sin^2 \theta_{13} \leq 0.02349$ and $0.3310 \leq \sin^2 \theta_{12} \leq 0.3319$.

Then we proceed to discuss the second representative values of the vacuum parameters $x = -4$ and $\eta = 5\pi/4$, the lepton mixing matrix reads as

$$U = \frac{1}{\sqrt{74}} \begin{pmatrix} 4\sqrt{3} & -i\sqrt{26} \cos \theta & -i\sqrt{26} e^{i\psi} \sin \theta \\ 2i\sqrt{6} \cos \theta - \sqrt{37} e^{-i\psi} \sin \theta & 2i\sqrt{6} e^{i\psi} \sin \theta + \sqrt{37} \cos \theta \\ 2i\sqrt{6} \cos \theta + \sqrt{37} e^{-i\psi} \sin \theta & 2i\sqrt{6} e^{i\psi} \sin \theta - \sqrt{37} \cos \theta \end{pmatrix},$$

where a Majorana phase matrix is omitted, and the parameters $\theta$ and $\psi$ are functions of the mass ratio $r$,

$$\tan 2\theta = \frac{2\sqrt{37} \sqrt{4225r^2 + 9 \left(\sqrt{2} - 25r + 154\sqrt{2} r^2\right)^2}}{15 \left(-7 + 3\sqrt{2} r + 781r^2\right)},$$

$$\tan \psi = \frac{65r}{3 \left(\sqrt{2} - 25r + 154\sqrt{2} r^2\right)}.$$

The expressions of the mixing angles are

$$\sin^2 \theta_{13} = \frac{13}{37} \sin^2 \theta, \quad \sin^2 \theta_{12} = \frac{26 \cos^2 \theta}{61 + 13 \cos 2\theta}, \quad \sin^2 \theta_{23} = \frac{1}{2} - \frac{2\sqrt{222} \sin 2\theta \sin \psi}{61 + 13 \cos 2\theta}.$$

The Jarlskog CP invariant takes the form

$$J_{CP} = \frac{13}{74} \sqrt{\frac{6}{37}} \sin 2\theta \cos \psi.$$

The sum rules for the Dirac CP phase in terms of lepton mixing angles are given by

$$\cos \delta_{CP} = \frac{(35 - 61 \cos 2\theta_{13}) \cot 2\theta_{23}}{8 \sin \theta_{13} \sqrt{111 \cos 2\theta_{13}} - 33},$$

$$\sin \delta_{CP} = -\csc 2\theta_{23} \sqrt{1 - \frac{169 \cot^2 \theta_{13} \cos^2 2\theta_{23}}{96(13 - 24 \tan^2 \theta_{13})}}.$$

It noteworthy that all the lepton mixing angles as well as $\delta_{CP}$ only depend on the parameter $r$ through $\theta$ and $\psi$ in this case. Moreover, the results for the light neutrino masses are

$$m_1^2 = 0,$$

$$m_2^2 = \frac{1}{2} m_\alpha^2 \left(9 - 25\sqrt{2}r + 1089r^2 - \sqrt{81 - 450\sqrt{2}(1 + 121r^2)r + [(1089r^2 - 1052)r^2]} \right),$$

$$m_3^2 = \frac{1}{2} m_\alpha^2 \left(9 - 25\sqrt{2}r + 1089r^2 + \sqrt{81 - 450\sqrt{2}(1 + 121r^2)r + [(1089r^2 - 1052)r^2]} \right).$$

In order to describe the experimentally measured values of both lepton mixing angles and neutrino mass squared differences, we find the allowed ranges of the input parameters are $3.568\text{meV} \leq m_\alpha \leq 3.871\text{meV}$ and $0.3983 \leq r \leq 0.4473$. As a consequence, the solar and reactor mixing angles are constrained to lie in the narrow intervals $0.02254 \leq \sin^2 \theta_{13} \leq 0.02280$ and $0.3362 \leq \sin^2 \theta_{12} \leq 0.3364$, and the atmospheric mixing angle is predicted to be in the second octant $0.5559 \leq \sin^2 \theta_{23} \leq 0.5636$. The predicted values of $\delta_{CP}$ are distributed around $3\pi/2$, namely $1.582\pi \leq \delta_{CP} \leq 1.594\pi$.

Since the model is very predictive and the mixing angles as well as Dirac CP phase are constrained to lie in rather narrow regions, in particular we have $0.329 \leq \sin^2 \theta_{12} \leq 0.346$ for the most general case, we expect the benchmark tri-direct model could be excluded in future neutrino experiments. If $\theta_{23}$ and $\delta_{CP}$ are measured precisely enough, the two values $x = -7/2$, $\eta = \pi$ and $x = -4$, $\eta = 5\pi/4$ may be distinguished from each other. It will be nice to probe these features in detailed simulations of current and future neutrino oscillation experiments.
TABLE I: The best fit values of the lepton mixing angles, CP violation phases, neutrino masses and the effective Majorana mass $m_{ee}$ for the benchmark values $(x, \eta) = (-7/2, \pi), (-4, 5\pi/4)$ of the tri-direct CP model.

| $x$  | $\eta$ | $m_a$(meV) | $r$ | $\chi^2_{\text{min}}$ | $\sin^2 \theta_{12}$ | $\sin^2 \theta_{13}$ | $\sin^2 \theta_{23}$ | $\delta_{\text{CP}}/\pi$ | $\beta/\pi$ | $m_2$(meV) | $m_3$(meV) | $m_{ee}$(meV) |
|------|--------|-------------|-----|------------------|-------------------|-------------------|-------------------|----------------|--------|-----------|-----------|-------------|
| $-\frac{7}{2}$ | $\pi$     | 3.716       | 0.557 | 17.524          | 0.0227            | 0.331            | 0.5               | $-0.5$          | 0       | 8.611     | 50.232    | 1.647       |
| $-4$    | $\frac{5\pi}{2}$ | 3.723       | 0.421 | 5.168            | 0.0226            | 0.336            | 0.560             | $-0.412$         | 0.264  | 8.603     | 50.242    | 2.840       |

III. IMPLEMENTATION OF NEUTRINO EXPERIMENTS IN SIMULATION

In this section, we will introduce the current and future experiments – T2K, NO$\nu$A, T2HK, DUNE and JUNO. All sensitivities in experiments are simulated in a state-of-the-art tool GLoBES [41, 42] where the experimental details can be very nicely implemented by an Abstract Experimental Design Language (AEDL) file. As soon as the publicly available signal and background spectra are reproduced, we can safely claim the expected sensitivities in the precision measurements. In the simulation, input values of neutrino mixing parameters are taken as the best fit values of the latest NuFit4.0 [1]: $\sin^2 \theta_{12} = 0.310$, $\sin^2 \theta_{13} = 0.0224$, $\sin^2 \theta_{23} = 0.580$, $\delta_{\text{CP}} = 215^\circ$, $\Delta m^2_{21} = 7.39 \times 10^{-5}$ eV$^2$, $\Delta m^2_{31} = 2.525 \times 10^{-3}$ eV$^2$. In the current study, we will choose a normal mass hierarchy as a demonstration. In the meantime, the Preliminary Reference Earth Model (PREM) density profile is considered in the numerical calculations [43]. We are using two methods to present our results:

- Standard three neutrino oscillations expressed by $\theta_{12}, \theta_{13}, \theta_{23}, \delta_{\text{CP}}, \Delta m^2_{21}$, and $\Delta m^2_{31}$ are taken as the truth in Nature, we expect that precision measurements of mixing parameters are correlated, and uncertainties of current global fit results are taken into account. For given oscillation parameters, we define a set of parameters:

$$\Theta = \{\theta_{12}, \theta_{13}, \theta_{23}, \delta_{\text{CP}}, \Delta m^2_{21}, \Delta m^2_{31}\}$$

(25)

and predict the expected event rate in the bin $i$ $\mu_i(\Theta)$. We suppose a given experiment reconstructs neutrino spectra in $N$ bins sequentially. The event rate in the bin $i$ is recorded as $n_i$. We can build a $\chi^2(\Theta)$ to quantify the sensitivity:

$$\chi^2(\Theta) = \sum_{i=1}^{N} \frac{\left[\mu_i(\Theta) - n_i\right]^2}{\sigma_i^2}.$$  

(26)

The final results come from a minimization of the summation of the $\chi^2(\Theta)$ in every oscillation channel of all experiments over a set of parameters, or the so-called marginalization.

- Once we fit the model parameters, the number of degrees of freedom is reduced from six to four, as shown in the previous section. We consider the following parameters from the tri-direct CP symmetry models: $x$, $\eta$, $m_a$ and $r$. In this case, we have to change the oscillation parameters predicted by the specific model:

$$\mathcal{M} = \{x, \eta, m_a, r\}$$

(27)

Other steps in the likelihood analysis will follow the same strategy as the above method, but replace the equation Eq. (26) by

$$\chi^2(\mathcal{M}) = \sum_{i=1}^{N} \frac{\left[\mu_i(\Theta(\mathcal{M})) - n_i\right]^2}{\sigma_i^2}.$$  

(28)

with the PMNS parameters as functions of model parameters $\Theta(\mathcal{M})$. We can expect better measurements of input parameters after a combination of experimental results and symmetry-induced constraints from the theory.
A. T2K

T2K stands for Tokai to Kamioka a long-baseline experiment in Japan. In Tokai, muon neutrinos or antineutrinos are produced by bombarding a 30 GeV proton beam onto a graphite target station in the J-PARC accelerator center. The neutrino beams are detected firstly at the near detectors which are 280 meters away from the target station. The far detector which reconstructs oscillated neutrino/antineutrino signals is Super-Kamiokande which has a fiducial mass of 22.5 kilotons and is 295 kilometers away with an off-axis angle of 2.5° from the beam direction. With the carefully chosen off-axis angle, the neutrino beam energy is peaked at about 0.6 GeV and matches the first maximum in the neutrino oscillation channels: $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$.

In 2011, the T2K collaboration published their first result on $P(\nu_\mu \rightarrow \nu_e)$ with $1.43 \times 10^{20}$ Protons On Target (POT). It is the first hint of non-zero $\theta_{13}$ at $2.5\sigma$ confidence level (C.L.) [33]. In 2012, they presented an analysis of neutrino oscillation for $P(\nu_\mu \rightarrow \nu_\mu)$ based on the same POT data, where we have best-fit values of $\Delta m^2_{32} = 2.63 \times 10^{-3}$ eV$^2$ and $\sin^2 2\theta_{23} = 0.98$ in the three-neutrino mixing framework [44]. In 2016, the first antineutrino result was published based on $4.01 \times 10^{20}$ POT, where we have best-fit values of $\Delta m^2_{32} = 2.51 \times 10^{-3}$ eV$^2$ and $\sin^2 2\theta_{23} = 0.45$. The latest results of searching for CP violation in neutrino and antineutrino oscillations by T2K are based on $2.2 \times 10^{21}$ POT [4]. In our simulation, we equally split $7.8 \times 10^{21}$ POT into two modes for T2K as the final total POT number.

B. NO$\nu$A

NO$\nu$A is a long-baseline neutrino oscillation experiment in the United States. Muon neutrinos or antineutrinos are produced by the NuMI beam at Fermilab. The experiment also adopts an off-axis angle of 14.6 mrad to reach the first neutrino oscillation maximum at a peak energy of 2 GeV, since the far detector using 14 kt active scintillator is 810 km away from the target station. The far detector is on the surface. An identical detector with a mass of 290 ton scintillator is 100 meter deep at a distance of 1 km in order to monitor the neutrino flux and cancel the systematic uncertainties.

In 2016, the NO$\nu$A collaboration published their first result in the $\nu_\mu \rightarrow \nu_e$ channel [34] and in the $\nu_\mu \rightarrow \nu_\mu$ channel [35] based on $2.74 \times 10^{20}$ POT. In 2017, they updated results on the electron neutrino appearance channel based on $6.05 \times 10^{20}$ POT [45]. The degeneracy of $\theta_{23}$ shows up at $2.6\sigma$ C.L. The latest results in a combination of neutrino and antineutrino runs are given in Ref. [9]. In our simulation, we assume total $36 \times 10^{20}$ POT for $\nu$ and $\bar{\nu}$ modes until 2024 for NO$\nu$A.

C. T2HK

An evolution of Water Cherenkov detectors from Kamiokande to Hyper-Kamiokande makes it possible to conduct an upgrade of T2K to T2HK [37]. The HyperK detector will have 560 kt fiducial mass to reconstruct neutrino oscillation spectra. T2HK shares the same baseline of 295 km as T2K while the offaxis beam remains in the same direction with an upgraded proton beam at 1.3 MW. We assume T2HK is running in the neutrino mode in 2.5 years and in the antineutrino mode in 7.5 years. The second far detector in Korea is actively under consideration. In our simulation, we will keep the conservative option without the second far detector.

D. DUNE

DUNE is the next-generation accelerator neutrino oscillation experiment with a baseline of 1300 km from FNAL to the underground laboratory in South Dakota. The experiment will search for CP violation in the leptonic sector and conduct precision measurements using appearance and disappearance channels by $\nu_\mu$ and $\bar{\nu}_\mu$ beams. DUNE is going to reconstruct oscillated neutrino spectra with a detector complex of four 10-kt Liquid Argon Time Projection Chamber (LArTPC). We adopt an AEDL file provided by Ref. [36]. We assume the experiment will be running in the neutrino/antineutrino mode in 3.5 years, and adopt the 3-horn optimised beam design, which consists of the 62.5 GeV proton beam with a power of $1.83 \times 10^{21}$ POT per year [46, 47].
E. JUNO

JUNO is a multi-purpose underground neutrino experiment, which will build a 20 kt liquid scintillator detector in South China and is planned to be online in 2021 [48]. The primary goal of JUNO [49] is to determine the neutrino mass ordering and precision measurement of oscillation parameters using the reactor electron neutrino disappearance channel thanks to the unprecedented energy resolution of 3%/√E. Regarding the precision measurement of |Δm^2_{31}|, Δm^2_{21}, and sin^2θ_{12}, JUNO can reach the levels of 0.44%, 0.59% and 0.67% respectively, after six years of data taking. Moreover, the determination of the neutrino mass ordering at reactors is free from the contamination of matter effects [50] and possible new physics [51]. JUNO will be rather robust when combined with accelerator neutrino experiments. The sub-percent level precision for three of all six standard oscillation parameters will certainly be powerful for selecting the flavor-symmetry models. In our simulation, we use the standard precision levels as our input priors to combine with accelerator neutrino experiments using the state-of-the-art GLoBES tool.

IV. MODEL TESTING WITH NEUTRINO OSCILLATION EXPERIMENTS

In this section, we show our simulation results with the experiments introduced in Sec. III. The configurations we considered are the synergy of T2K and NOνA, DUNE, T2HK, the combination of all LBLs, and the interplay of LBLs and the reactor experiment JUNO. In Sec. IV A, we will firstly investigate the precision of four model parameters – x, η, r, and m_a. One will see in those results, there is a degeneracy problem. In Sec. IV B, we will explain how this degeneracy problem appears, and propose a way to resolve this problem. In the following section, we will study how the uncertainties are changed for oscillation parameters by tri-direct CP models. The above subsections are based on the general tri-direct CP model. In Tab. I, we show two benchmark models. Finally, we will predict how these two benchmark models can be tested in future experiments.

A. Precision measurement of model parameters

In Fig. 1, we show the Δχ^2 values for each model parameter. We use the true values for the model parameters; (x, η, r, m_a) = (−3.65, 1.1π, 0.5, 3.7meV), which is the best fit for the NuFit4.0 result. We also include the prior according to the NuFit4.0 result. We consider the configurations: DUNE (dashed-blue), T2HK (dashed-green), the combination of NOνA and T2K (dashed-brown), the synergy of all LBLs (dashed-grey), and the optimised configuration by combing all LBLs and the JUNO experiment (solid-red). Except for the m_a result, we see a great improvement in the DUNE result compared to the T2K-and-NOνA combination. T2HK further improves the measurements, and its performance is similar to the combination of all LBLs. This demonstrates the fact that T2HK dominates the contributions. The feature that the performance of T2HK is better than that of DUNE reflects the well-known result that T2HK works better than DUNE with fixed mass ordering, which is naturally imposed by the tri-direct CP model. In more detail, for x (the upper-left panel) the 3σ uncertainty improves from the T2K-and-NOνA combination (∼ [−4.8, −3.5]) to DUNE (∼ [−4.2, −3.5]) and T2HK (∼ [−3.8, −3.5]). The combination of all LBLs performs similarly to T2HK.

Features and tendencies of each Δχ^2 curves against r (the lower-left panel) are similar to the result for x. The uncertainties at 3σ for the T2K-and-NOνA combination, DUNE, T2HK are ∼ [0.3, 0.6], ∼ [0.4, 0.6], ∼ [0.45, 0.6], respectively. The 3σ uncertainty for combining all LBLs is almost the same as that for T2HK. The relative symmetry is seen in the result for η. The size of the 3σ uncertainty for the T2K-and-NOνA combination, DUNE and T2HK are about 0.3π, 0.2π, 0.15π, respectively. The correlation between η and r worsens the sensitivity for η smaller than the assumed true value. Thanks to the high precision of T2HK and combing all LBLs, the degeneracy problem can be resolved when η is very close to the true value. Therefore, we see a twist around η = π for these two configurations. Details about this degeneracy will be introduced in Sec. IV B.

For the above three parameters x, η and r, it is hard to see the improvement by including the data of JUNO to those of all LBLs. Data from JUNO is important for the m_a measurement. We see the overlapping of all curves for all LBL configurations (dashed-blue, dashed-green, dashed-brown, black curves) in the m_a result. The
uncertainty is mainly contributed from $\Delta m_{21}^2$, which is not measured well by LBLs. As a result, we see a great improvement by including data from JUNO, which well measures $\Delta m_{21}^2$.

We also show the $3\sigma$ ($\Delta \chi^2 = 11.83$) contour between any two model parameters in Fig. 2. We see some correlations among $x$, $\eta$ and $r$ for all configurations on $x-\eta$, $x-r$ and $\eta-r$ planes. This correlation is consistent with what we see in Eq. (3), in which $m_a$ is less dependent on the other three parameters. We discover a degeneracy problem related to this correlation for all possible LBL configurations—the combination of NOvA and T2K, DUNE, T2HK, and all-LBL synergy. This degeneracy is mainly caused by the poor measurement of $\theta_{12}$. More details about this degeneracy can be seen in Sec. IV B. These correlations are not removed even if we include JUNO, but combing LBLs and JUNO data can resolve the degeneracy problem.

B. Breaking degeneracies

The degeneracy in Fig. 2 can be understood by the equal-oscillation-parameter-value contours on different planes as shown in Fig. 3. In Fig. 3, we show these contours on the $x-\eta$, $x-r$, and $\eta-r$ planes. In the upper panels, we set the model parameters at the true values $(x, \eta, r, m_a) = (-3.65, 1.1\pi, 0.5, 3.7 \text{meV})$, which predicts the value for oscillation parameters $\theta_{12} \sim 35.3^\circ$ (grey), $\theta_{13} \sim 8.6^\circ$ (short-dashed-green), $\theta_{23} \sim 47^\circ$ (short-dashed-blue), $\delta \sim 279^\circ$ (short-dashed-red), $\Delta m_{21}^2 \sim 7.4 \times 10^{-5} \text{eV}^2$ (orange), and $\Delta m_{31}^2 \sim 2.52 \times 10^{-3} \text{eV}^2$.
FIG. 2: Precision measurements of any two model parameters at the 3σ confidence level in the framework of three-neutrino oscillations taking uncertainties of the NuFit4.0 results. True values for the model parameters are used \((x, \eta, r, m_a) = (-3.65, 1.1\pi, 0.5, 3.7\text{meV})\). We present the expected results from DUNE (light blue), T2HK (green), the combination NOνA and T2K (pink) and the synergy of all LBLs (light grey) and the interplay of LBLs and JUNO (brown). The black dot denotes the best fit values, while the magenta triangle is for the local minimum, where \(r \sim 0.1, \eta \sim 1.84\pi, x \sim -8\) and \(m_a \sim 3.81\text{meV}\). (short-dashed-black). The contours are shown with these conditions. Therefore, the intersection of all contours is at the assumed true values. In the lower panels, we focus on the degeneracy region: for the left, middle, and the right panels, we set \(r \sim 0.1, \eta \sim 1.84\pi, x \sim -8\) and \(m_a \sim 3.81\text{meV}\). We see that the local minimum of the degeneracy region (magenta triangles) takes place where the green, blue, red, orange and black curves meet together or go very close. LBL experiments are not sensitive to \(\theta_{12}\) (grey curve) and \(\Delta m_{21}^2\) (orange curve). As a
result, these LBL experiments cannot exclude this region by improving precision. This also explains why once we include reactor data that is sensitive to $\theta_{12}$, the degeneracy region is excluded. One may notice that the different curves do not intersect at the magenta triangle in the $x-r$ plane and there is the gray curve of $\theta_{12}$ in the last panel for $\eta-r$. The reason is that the triangle presents a local minimum at which $11.83 > \Delta \chi^2 > 9$. Though that is a local minimum, it does not need to cross all curves. The grey curve of $\theta_{12}$ is below $r=0.07$, where the bottom of the panel is. We did not show it in this panel because the main feature here crossing or going close to $\theta_{13}, \theta_{23}, \delta$ and $\Delta m^2_{31}$ results in the degeneracy issue for LBLs.

C. Standard oscillation parameters under tri-direct CP symmetry model

In Fig. 4, we show $\Delta \chi^2$ values against all oscillation parameters for the combination of NO$\nu$A and T2K (brown), DUNE (dashed-blue), T2HK (dashed-green), the synergy of these four LBLs (grey), and including all LBLs and JUNO (red), assuming the tri-direct model. We see that under the tri-direct assumption, the combination of NO$\nu$A and T2K performs the worst, while DUNE performs much better, except for $\theta_{13}$ and $\Delta m^2_{21}$. T2HK works slightly better than DUNE, and dominates the performance of the combination of all LBLs. The asymmetry for $\Delta m^2_{31}$ comes from the asymmetry behaviour of $x$ and $r$ in Fig. 1 through Eq. (4). Obviously, the twist behaviour
for η is passed to those for θ_{13}, θ_{23} and δ by the tri-direct model. We note that even LBL experiments are not sensitive to θ_{12}, the uncertainty can be improved by precisely measuring other oscillation parameters within the tri-direct model. We further point out a great improvement by including JUNO data which can be seen in the result for Δm^2_{21}.
FIG. 5: The points on the $3\sigma$ sphere in the 4-dimension model-parameter space projected on $\theta_{23}$-$\Delta m^2_{31}$ (upper-left), $\theta_{13}$-$\Delta m^2_{31}$ (upper-right), $\theta_{12}$-$\Delta m^2_{21}$ (lower-left), $\theta_{13}$-$\delta$ (lower-right) for the synergy of these four LBLs (grey), and including all LBLs and JUNO (red). We also compare these results to those without the restrictions from tri-direct models for LBLs synergy (grey contour) and combing all experiments (red contour). Because of the nonlinear relations between model parameters and standard parameters, the data do not spread uniformly. We also compare them with those without the restriction from the tri-direct CP model: the grey curve is for including all LBLs, while the dashed black curve is for a combination of LBLs and JUNO. There is a discontinuity when $\theta_{13}$ is larger than $\sim 8.8^\circ$, because of the degeneracy with $\theta_{23}$. These results show that assuming tri-direct CP improves the key measurements for future experiments.
D. A discrimination of two benchmark models

In Fig. 6, we show $\Delta \chi^2$ against $r$ (left) and $m_a$ (right) assuming model A (solid curve; Eq. (13)) and model B (dotted curve; Eq. (19)), for DUNE (blue), T2HK (green), the combination of NOvA and T2K (brown), the synergy of four LBLs (black) and the interplay of LBLs and JUNO data (red). These two models, shown in Tab. 1, assume different values for $x$ and $\eta$: $(x, \eta) = (-7/2, \pi)$ for model A and $(x, \eta) = (-4, 5\pi/4)$ for model B. The corresponding best fits with the global fit result are given $(r, m_a) = (0.557, 3.716 \text{ meV})$ for model A and $(r, m_a) = (0.421, 3.723 \text{ meV})$ for model B. We see that based on one model, the better way to exclude the other one is by precision measurement of $r$. The experimental configuration does not affect the uncertainty for $r$. This uncertainty is $\sim 0.2$ at 3$\sigma$ under both models. Two models predict very similar values for $m_a$. As a result, it is impossible to exclude the wrong model by measuring this parameter alone. Moreover, the uncertainty of $m_a$ depends on the model and the experimental configuration. The precision under Model A is generally better than that for Model B, expect for the combination of LBLs and JUNO data. The rank of precision from the worst to the best experimental configuration is the combination of NOvA and T2K, DUNE, T2HK, the synergy of all LBLs, and combining all LBLs and JUNO. Both $m_a$ and $m_s$ will be determined precisely by experiments.

V. SUMMARY

The tri-direct CP symmetry model offers fruitful features to accommodate neutrino masses and explain neutrino mixing and oscillations. The more powerful aspect is the model predicted correlations of standard neutrino mixing parameters preserved by an underlying symmetry. We looked into a probe of the tri-direct CP symmetry model by simulating the current and future neutrino oscillation experiments, including T2K, NOvA, T2HK, DUNE and JUNO. We found that the degeneracy problem cannot be avoided at a single long baseline experiment in the precision measurement of model parameters while a combination of long-baseline and reactor experiments will resolve the problem. This fact highlights the complementarity of different neutrino oscillation experiments. In addition, we scanned the standard neutrino mixing parameters expressed by the underlying model “true” values in order to determine how powerful precision measurements in the traditional analysis will be. It seems that shape of contours in the projected parameter space can give us hints of the underlying theory but the information
remains limited by a multiple-channel analysis in a single experiment. This limitation points to a combined analysis by multiple experiments with different beams and baseline configurations. Finally, we can discriminate benchmark models after a discovery of CP violation in the leptonic sector by any one of these experiments.

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Appendix A: Physics performance of different configurations

In this section, we demonstrate the experimental potential for different configurations by showing the $1\sigma$, $2\sigma$ and $3\sigma$ contours on $\theta_{23}$-$\delta$ and $\theta_{23}$-$\Delta m_{31}^2$ planes. These measurements are the main goals for current and future LBLs. For current running experiments NO$\nu$A (upper) and T2K (middle), we show their expected final performance in Fig. 7. For NO$\nu$A, we assume total $36\times10^{20}$ POT for $\nu$ and $\bar{\nu}$ modes until 2024, while for T2K we equally split $7.8\times10^{21}$ POT into two modes. We also show the combination of these two experiments in the lower panels of Fig. 7. In Fig. 8, we show the performance of DUNE (upper) and T2HK (lower). For DUNE, we consider the 3-horn-optimised design with $1.83\times10^{21}$ POT per year, and we adopt 3.5 years for each mode. For T2HK, we assume a 1.3 MW proton beam for the neutrino source, and run $\nu$ and $\bar{\nu}$ modes for 2.5 and 7.5 years respectively. More details about these experiments can be seen in Secs. III.
FIG. 7: The contours on $\theta_{23}$-$\delta$ (left) and $\theta_{23}$-$\Delta m_{31}^2$ (right) for NO$\nu$A (upper) planes, T2K (middle) and their combination (lower) at 1$\sigma$ (red), 2$\sigma$ (blue) and 3$\sigma$ (green) precision. The true values are $\theta_{12} \sim 35.3^\circ$, $\theta_{13} \sim 8.6^\circ$, $\theta_{23} \sim 47^\circ$, $\delta \sim 279^\circ$, $\Delta m_{21}^2 \sim 7.4 \times 10^{-5}$ eV$^2$, and $\Delta m_{31}^2 \sim 2.52 \times 10^{-3}$ eV$^2$. These results include the NuFit4.0 results as priors.
FIG. 8: The contours on $\theta_{23}$-\(\delta\) (left) and $\theta_{23}$-$\Delta m^2_{31}$ (right) for DUNE (upper) and T2HK (lower) at 1$\sigma$ (red), 2$\sigma$ (blue) and 3$\sigma$ (green) precision. The true values are $\theta_{12} \sim 35.3^\circ$, $\theta_{13} \sim 8.6^\circ$, $\theta_{23} \sim 47^\circ$, $\delta \sim 279^\circ$, $\Delta m^2_{21} \sim 7.4 \times 10^{-5}$ eV$^2$, and $\Delta m^2_{31} \sim 2.52 \times 10^{-3}$ eV$^2$. These results include the NuFit4.0 results as priors.
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