A Coordinated Direct AF/DF Relay-Aided
NOMA Framework for Low Outage

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Abstract

This paper investigates the performance of a framework for low-outage downlink non-orthogonal multiple access (NOMA) signalling using a coordinated direct and relay transmission (CDRT) scheme with direct links to both the near-user (NU) and the far-user (FU). Both amplify-and-forward (AF) and decode-and-forward (DF) relaying are considered. In this framework, NU and FU combine the signals from BS and R to attain good outage performance and harness a diversity of two without any need for feedback. For the NU, this serves as an incentive to participate in NOMA signalling. For both NU and FU, expressions for outage probability and throughput are derived in closed form. High-SNR approximations to the outage probability are also presented. We demonstrate that the choice of power allocation coefficient and target rate is crucial to maximize the NU performance while ensuring a desired FU performance. We demonstrate performance gain of the proposed scheme over selective decode-and-forward (SDF) CDRT-NOMA in terms of three metrics: outage probability, sum throughput and energy efficiency. Further, we demonstrate that by choosing the target rate intelligently, the proposed CDRT NOMA scheme ensures higher energy efficiency (EE) in comparison to its orthogonal multiple access counterpart. Monte Carlo simulations validate the derived expressions.

Index Terms

Non-orthogonal multiple access (NOMA), coordinated direct and relay transmission (CDRT), energy efficiency (EE), amplify-and-forward (AF), decode-and-forward (DF).

I. INTRODUCTION

The proliferation of internet of things (IoT) and massive machine type communication has augmented the demand for higher data rates, low latency and high bandwidth efficiency in beyond 5G (B5G) and

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6G networks [1]–[4]. Multiple access techniques used in older generations of wireless communication networks are not sufficient to fulfill these stringent requirements on user density and network traffic. In order to meet the requirements of future wireless networks, several multiple access technologies were studied in the last decade. Among them non-orthogonal multiple access (NOMA) has emerged as one of the most promising technology for future wireless networks due to its high spectral efficiency (SE), low latency, and its ability to facilitate massive connectivity [5]–[7].

In contrast to traditional orthogonal multiple access (OMA) schemes, NOMA facilitates concurrent transmission to multiple users over the same spectral/time/spreading code resource using power domain multiplexing at the transmitter side and successive interference cancellation (SIC) at the receiver side. In downlink NOMA, the user with a poor channel condition is termed as the far user (FU), and is allocated a larger fraction of the transmit power in comparison to the user with a better channel condition i.e. the near user (NU) [8]. The quality of service at the FU (as well as the coverage area) can be further enhanced by using relayed NOMA (R-NOMA) [9]–[12] and cooperative NOMA (C-NOMA) [13]–[16]. In C-NOMA, NU assists the information transmission to FU whereas in R-NOMA, dedicated nodes are used for relaying to FU. In [9], [17], the performance of amplify-and-forward (AF) and decode-and-forward (DF) relaying schemes was compared for different relay selection criteria in an R-NOMA framework. Further, in [18], the block error performance for short packet communication in a C-NOMA network was compared to that in conventional NOMA assuming Rayleigh flat fading channels. To improve the SE, successive user relaying in C-NOMA was discussed in [19].

The use of NOMA in coordinated direct and relay transmission (CDRT) [20]–[25] is another promising approach to improve the QoS at FU. In a typical NOMA CDRT signalling scheme, the base station (BS) shares a direct link to the NU, while communication to the FU is assisted by a dedicated relay. In [20], the capacity of NOMA CDRT was analyzed. In [21], [22], capacity gain at FU was achieved along with improved SE. IoT-assisted NOMA CDRT was analyzed in terms of outage probability and ergodic sum capacity in [23]. In [24], the SE and user fairness index of CDRT NOMA were derived considering perfect and imperfect SIC at NU. Coordinated uplink transmission for cooperative NOMA system was studied in [25]. To date, existing literature on NOMA CDRT focuses on AF as well as DF relaying in the absence of the BS-FU direct link. The availability of BS-FU direct link not only enhances the quality of service (QoS) at FU but also lowers the required energy consumption at the relay. In [26], DF relay-aided CDRT NOMA in presence of BS-FU link has been investigated for three different cases a) fixed relaying where the DF relay always forwards the incoming signal from BS, b) selective DF where the relay only forwards after successful decoding
of the incoming signal, and c) incremental selective DF relaying where the relayed link is used opportunistically, and requires a three-bit feedback. Further, in [27], performance of fixed-gain AF relay based CDRT NOMA was investigated in the presence of the BS-FU link assuming sub-optimal selection combining at the FU.

A. Motivation and Contribution

Due to participation in NOMA signalling, NU needs to decode the FU symbol first to perform SIC (complexity of the NU receiver clearly increases) before it can decode its own symbol. Depending on the channel condition as well as the target rate and power allocation factor, NU might occasionally fail to decode its own symbol due to insufficient signal-to-noise ratio (SNR) or unsuccessful decoding of the FU symbol. In such a scenario the BS has to re-transmit the superimposed symbol, which causes reduction in SE as well as energy efficiency (EE). In C-NOMA NU incurs loss in SE (due to allocation of power to the FU symbols, and use of two signalling phases), and EE as well (it is required to expend energy for relaying the FU symbol).

All the aforementioned works [10], [13], [15]–[17], [21]–[24], [26], [27] have laid a solid foundation for the use of C-NOMA and CDRT to improve the FU’s performance, but there has been little effort to improve the NU’s performance. Several multimedia-type applications impose a low outage QoS, and it might not be possible to satisfy NU’s constraints using these CDRT approaches (they only guarantee a diversity of one at the NU [21]). However, considering an AF or DF relay to assist FU as well as NU with optimal combining at both users not only improves the NU diversity, but also results in improved QoS at both the users. The NU will be required to perform combining with SIC (in a framework discussed in this paper), but attains much better performance, which is required in many scenarios. We also discuss optimization of NU performance while ensuring a desired performance at FU. The key contributions of this paper are as follows:

- Considering both AF and DF relaying, we investigate the performance of a new framework for CDRT based downlink NOMA network with direct link to both NU and FU. In the considered framework, BS transmits NOMA signal to relay, NU and FU in the first signalling phase, while in the second signalling phase, R communicates to both NU as well as FU. The NU combines the incoming signals from BS and relay to enhance its performance as an incentive for its participation in NOMA signalling.

- For both NU and FU, considering both AF and DF relaying, we present expressions for outage probability and throughput in closed-form. Unlike other works [28], [29] on AF CDRT NOMA
where (for analytical simplicity) the harmonic to min approximation is used, we derive the exact outage probability in closed form.

- Our simulations show that the performance of NU with AF and DF relaying is almost the same throughout the range of SNRs. For FU, in the presence of BS-FU link, DF performs better than that of AF at low SNRs and at high SNRs, both achieve similar performance. However, DF always outperforms AF in the absence of the BS-FU link. We also demonstrate that CDRT NOMA with AF relaying allows a wider range of valid power allocations in comparison to DF relaying for both NU and FU.

- We also derive highly accurate high-SNR expressions for the outage probability to demonstrate that combining direct and relayed signals at NU as well as FU (while performing SIC at NU) allows both NU and FU to harness a diversity of 2 with both AF and DF relaying. Clearly, the NU is rewarded for participation in NOMA signalling. However, in the absence of the BS-FU link, FU attains a diversity of only one.

- Next, we show that how the choice of power allocation coefficient and target symbol rates are crucial to maximize the NU throughput while guaranteeing a desired target throughput at FU. We also present an approximate closed-form expression for power allocation coefficient that maximizes NU throughput for a given FU throughput requirement. In addition to this, we also formulate a NU throughput maximization problem and determine the optimal NU and FU target rate pair.

- We also observe that in DF-assisted CDRT NOMA (unlike in the AF case), both BS and R transmit the superimposed symbols (combination of NU and FU symbols) in first and second phase of signalling, respectively. However, the power allocation at BS significantly affects the system performance in comparison to the power allocation at R.

- Further, we observe that the proposed scheme (without using any feedback) always outperforms SDF CDRT-NOMA [26] in terms of outage probability, sum throughput and energy efficiency by a huge margin and also achieves better diversity at both the users.

- Moreover, we compare the performance of the proposed CDRT-NOMA framework to its OMA counterpart (with relaying) and observe that the proposed system ensures higher energy efficiency (EE). We further observe that the availability of BS-FU link helps in achieving a higher EE in comparison to the case when BS-FU link is absent. Also, optimal rate selection is important to maximize the EE.

The rest of this paper is structured as follows. Section II elaborates on the system model for AF
as well as DF relay assisted CDRT NOMA framework. Section III analyzes the performance with the proposed framework in terms of outage probability. Section IV discusses NU throughput and energy efficiency maximization. Numerical results based on mathematical analysis are compared to computer simulations in Section V. Finally, Section VI concludes this paper.

Notations: $\mathcal{CN}(0,\sigma^2)$ represents a zero-mean complex Gaussian distribution with variance $\sigma^2$. $f_X(x)$ and $U(\cdot)$ respectively denote the probability density function (PDF) of a random variable (RV) $X$ and the unit step function. $E_1(\cdot), K_1(\cdot, \cdot)$ and $\Gamma(a, x; b)$ denote the exponential integral of type 1, the modified Bessel function of the second kind and the generalized incomplete gamma function, respectively.

II. System Model

As depicted in Fig. 1, we consider a CDRT downlink NOMA framework consisting of a base station $B$, a near user $U_N$, a far user $U_F$ and a relay station $R$. Both $U_N$ and $U_F$ have stringent outage QoS constraints. All nodes operate in the half-duplex (HD) mode and are equipped with a single antenna. Communication to $U_N$ and $U_F$ takes place in two signalling phases. In the first phase, $B$ communicates to $U_N$, $R$ and $U_F$ over direct links, while in the second phase communication from $R$ to $U_N$ as well as $U_F$ takes place using either AF or DF mode of relaying.

The channel coefficients $h_{ij} \sim \mathcal{CN}(0, 1/\lambda_{ij})$ with $i \in \{B,R\}, j \in \{R,N,F\}$ are assumed to be independent and of quasi-static Rayleigh fading type, where $\lambda_{ij} = d_{ij}^m$ and $m$ is the path-loss exponent. The additive zero-mean complex Rayleigh noise at all the receiving nodes is assumed to be of variance $\sigma^2$. Superscript "I" and "II" are used to represent first and second phase quantities.

Fig. 1: (a) System Model. (b) An illustration of the proposed CDRT transmission process.

A. Amplify-and-Forward Relaying

In the first phase, $B$ transmits a superposition of unit-energy symbols $s_N$ and $s_F$ (of information rates $R_N$ and $R_F$) intended respectively for $U_N$ and $U_F$. The transmit power $P_B$ is apportioned to
U_N and U_F symbols in the ratio $\alpha_b : (1 - \alpha_b)$. Thus, the superimposed symbol can be expressed as $s_b = \sqrt{P_B \alpha_b} s_N + \sqrt{P_B (1 - \alpha_b)} s_F$. The sampled matched filter outputs at U_N, R and U_F are

$$y^I_N = s_b h_{BN} + w^I_N, \quad y^I_R = s_b h_{BR} + w^I_R \quad \text{and} \quad y^I_F = s_b h_{BF} + w^I_F \quad (1)$$

respectively, wherein $w^I_N$, $w^I_R$ and $w^I_F$ are the respective additive Gaussian noise samples. In accordance with the concept of NOMA, U_N first decodes the FU symbol $s_F$, and then performs SIC to decode the NU symbol $s_N$. Using (1), the signal-to-interference-plus-noise ratio (SINR) $\Gamma_{NF}^I$ to decode $s_F$, and the signal-to-noise ratio (SNR) $\Gamma_{NN}^I$ to decode $s_N$ at U_N after SIC, are expressed as

$$\Gamma_{NF}^I = \frac{(1 - \alpha_b) P_B |h_{BN}|^2}{\alpha_b P_B |h_{BN}|^2 + 1}, \quad \Gamma_{NN}^I = \frac{\alpha_b P_B |h_{BN}|^2}{\alpha_b P_B |h_{BN}|^2 + 1}, \quad \text{provided} \ \Gamma_{NF}^I \geq \gamma_F, \quad (2)$$

where $\rho_B = P_B/\sigma^2$ represents the transmit SNR at B and $\gamma_F = 2^{R_F} - 1$ represents the threshold SNR at U_F. Using (1), the SINR $\Gamma_{FF}^I$ to decode $s_F$ at U_F can be expressed as

$$\Gamma_{FF}^I = \frac{(1 - \alpha_b) P_B |h_{BF}|^2}{\alpha_b P_B |h_{BF}|^2 + 1}. \quad (3)$$

Let $P_R$ denote the available transmit power at R. In the second phase, R amplifies the incoming signal from the first phase and then forwards $s_R = \beta y^I_R$ to U_N and U_F as shown in Fig. [b], where $\beta = \sqrt{\rho_R/|h_{BR}|^2+1}$ with $\rho_R = P_R/\sigma^2$. The signals received at U_N and U_F in the second phase are given by

$$y^I_N = \beta y^I_R h_{BN} + w^I_N \quad \text{and} \quad y^I_F = \beta y^I_R h_{RF} + w^I_F \quad (4)$$

respectively, where $w^I_N$ and $w^I_F$ are the additive noise samples at U_N and U_F. Similar to the first phase, using (4) the SINRs $\Gamma_{NF}^I$ and $\Gamma_{NN}^I$ to decode $s_F$ and $s_N$ (after SIC) are expressed as

$$\Gamma_{NF}^I = \frac{\rho_B \beta^2 (1 - \alpha_b) |h_{BR}|^2 |h_{BN}|^2}{\rho_B \beta^2 \alpha_b |h_{BR}|^2 |h_{BN}|^2 + |h_{BN}|^2 \beta^2 + 1} = \frac{\rho_B \rho_R (1 - \alpha_b) |h_{BR}|^2 |h_{BN}|^2}{\rho_B \rho_R \alpha_b |h_{BR}|^2 |h_{BN}|^2 + \rho_R |h_{BN}|^2 + \rho_B |h_{BR}|^2 + 1}, \quad (5)$$

$$\Gamma_{NN}^I = \frac{\rho_B \beta^2 \alpha_b |h_{BR}|^2 |h_{BN}|^2}{\beta^2 |h_{BN}|^2 + 1} = \frac{\rho_B \rho_R \alpha_b |h_{BR}|^2 |h_{BN}|^2}{\rho_R |h_{BN}|^2 + \rho_B |h_{BR}|^2 + 1}, \quad \text{provided} \ \Gamma_{NF}^I > \gamma_F. \quad (6)$$

Using (4), the SINR to decode $s_F$ at U_F is given by

$$\Gamma_{FF}^I = \frac{\rho_B \beta^2 (1 - \alpha_b) |h_{BR}|^2 |h_{RF}|^2}{\rho_B \beta^2 \alpha_b |h_{BR}|^2 |h_{RF}|^2 + |h_{RF}|^2 \beta^2 + 1} = \frac{\rho_B \rho_R (1 - \alpha_b) |h_{BR}|^2 |h_{RF}|^2}{\rho_B \rho_R \alpha_b |h_{BR}|^2 |h_{RF}|^2 + \rho_R |h_{RF}|^2 + \rho_B |h_{BR}|^2 + 1}. \quad (7)$$

The signals from first and second phases are combined at U_F. In this paper, we use an approach that deviates from all existing works to date and enable U_N to combine the signals in the two phases at each stage of the SIC. We show that this enables U_N to attain very good performance and harness a diversity of two without any need for feedback. This allows the network to be used for multimedia and other applications that impose strict outage QoS constraints. It is emphasized that none of the techniques suggested so far [21], [26] can ensure a diversity of two at the near-user in the absence of
any feedback. This improvement in diversity as well as in throughput performance also serves as an incentive for $U_N$ to participate in NOMA signalling. Clearly, $U_N$ cannot attain this performance using traditional OMA. In this respect, this work is quite different from all existing works.

$U_F$ combines $y^I_F$ and $y^{II}_F$ to decode $s_F$ with SINR $\Gamma^{COM}_{FF} = \Gamma^I_{FF} + \Gamma^{II}_{FF}$. If $s_F$ is decoded successfully in the first phase, $U_N$ cancels the interference from $y^I_N$ and then combines the first and second phase signals to decode $s_N$ with SNR $\Gamma^{COM}_{NN} = \Gamma^I_{NN} + \Gamma^{II}_{NN}$. However, when $U_N$ fails to decode $s_F$, it combines $y^I_N$ and $y^{II}_N$ to first decode $s_N$ with SINR $\Gamma^{COM}_{NF} = \Gamma^I_{NF} + \Gamma^{II}_{NF}$, cancels interference, and then decodes $s_N$ with SNR $\Gamma^{COM}_{NN} = \Gamma^I_{NN} + \Gamma^{II}_{NN}$.

**B. Decode-and-Forward relaying**

As in AF relaying, B transmits the superposed signal $s_B$ in the first phase. Therefore, the SINRs $\Gamma^I_{NF}$, $\Gamma^I_{NN}$, and $\Gamma^I_F$ remains same. Different from AF relaying, the DF relay decodes $s_F$ and $s_N$ by implementing SIC. The SINRs $\Gamma^I_{RF}$ and $\Gamma^I_{RN}$ (after SIC) to decode $s_F$ and $s_N$ at R can be expressed as

$$\Gamma^I_{RF} = \frac{(1 - \alpha_R)p_B|h_{BR}|^2}{\alpha_b \rho_B|h_{BR}|^2 + 1} \quad \text{and} \quad \Gamma^I_{RN} = \alpha_R \rho_R|h_{BR}|^2$$

provided $\Gamma^I_{RF} \geq \gamma_F$. (8)

In the second phase, based on the decoding status of $s_N$ and $s_F$, R transmits either superimposed signal $s_R$ or only the far user symbol $s_F$ as shown in Fig1(b). If R decodes both $s_N$ and $s_F$ successfully, then it forwards the superimposed signal $s_R = \sqrt{\rho_R} \alpha_R s_N + \sqrt{\rho_R} (1 - \alpha_R) s_F$ to both $U_N$ and $U_F$, where $\alpha_R$ and $(1 - \alpha_R)$ denotes the portion of power allocated to $U_N$ and $U_F$, respectively. Note that DF relaying (unlike its AF counter part) allows different NOMA power allocations at B and R. The signals received at $U_N$ and $U_F$ can be expressed as $y^I_{N-F} = s_R h_{RN} + w^I_N$ and $y^I_{F-F} = s_R h_{RF} + w^I_F$, where $-S$ in the subscript is used to emphasize that a superposed signal is transmitted by R. Using $y^I_{N-F}$, the respective SINRs before and after SIC ($\Gamma^{II}_{NF-S}$ and $\Gamma^{II}_{NN-S}$) to decode $s_F$ and $s_N$ at $U_N$ are

$$\Gamma^{II}_{NF-S} = \frac{(1 - \alpha_R)p_R|h_{RN}|^2}{\alpha_R \rho_R|h_{RN}|^2 + 1} \quad \text{and} \quad \Gamma^{II}_{NN-S} = \alpha_R \rho_R|h_{RN}|^2$$

provided $\Gamma^{II}_{NF-S} \geq \gamma_F$. (9)

Using $y^I_{F-F}$, the SINR $\Gamma^{II}_{FF-S}$ at F to decode $s_F$ is

$$\Gamma^{II}_{FF-S} = \frac{(1 - \alpha_R)p_R|h_{RF}|^2}{\alpha_R \rho_R|h_{RF}|^2 + 1}.$$ (10)

However, if R fails to decode $s_N$ then it forwards only $s_F$. Now, the received signals at $U_N$ and $U_F$ are expressed as $y^I_{N-F} = \sqrt{\rho_R} s_F h_{RN} + w^I_N$ and $y^I_{F-F} = \sqrt{\rho_R} s_F h_{RF} + w^I_F$ respectively, where the subscript $-F$ is used to emphasize that R merely transmits $s_F$. Using $y^I_{N-F}$ and $y^I_{F-F}$, the respective SINRs $\Gamma^{II}_{NF-F}$ and $\Gamma^{II}_{FF-F}$ to decode $s_F$ at $U_N$ and $U_F$ can be expressed as

1 Since the suggested combining based SIC at $U_N$ improves performance when the direct links to $U_N$ fails, the performance advantage is expected to be retained in the case when feedback is available.

2 In CDRT NOMA with DF relaying, if R successfully decodes $s_N$ and $s_F$, the power allocation takes place at both B and R, thus, DF provides more flexibility and a performance improvement can be expected in comparison to AF relaying.
with that the RVs $X, Y, Z, W, V$ are independent and exponentially distributed with PDF $\rho_x|h_{xn}|^2$ and $\Gamma_{f-f}^{II} = \rho_r|h_{rf}|^2$. (11)

$U_F$ combines $y_f^I$ with $y_f^{II}$ when $s_n$ and $s_f$ are decoded at R, or with $y_f^{II}$ otherwise. The combined SINRs are $\Gamma_{f-f-s}^{COM} = \Gamma_f^I + \Gamma_f^{II}$ or $\Gamma_{f-f-s}^{COM} = \Gamma_f^I + \Gamma_f^{II}$ respectively. However, if $s_f$ is decoded successfully in the first phase, $U_N$ cancels interference from $y_{N-S}^{II}$ or $y_{N-F}^{II}$ and then combines the first and second phase signals to decode $s_n$ with SNR $\Gamma_{N-N-S}^{COM} = \Gamma_{N-N}^{I} + \Gamma_{N-N-S}^{II}$ or $\Gamma_{N-N}^{II}$. When $U_N$ fails to decode $s_f$, it combines $y_f^I$ and $y_{N-S}^{II}$ or $y_{N-F}^{II}$ to first decode $s_f$ with SINR $\Gamma_{N-F}^{COM} = \Gamma_{N-F}^{I} + \Gamma_{N-F}^{II}$ or $\Gamma_{N-F}^{II}$. Subsequently, after cancelling the interference due to $s_f$, decoding of $s_n$ takes place with SNR $\Gamma_{N-N-S}^{COM} = \Gamma_{N-N}^{I} + \Gamma_{N-N-S}^{II}$ or $\Gamma_{N-N}^{II}$. In this paper we compare performance of the proposed scheme with a relayed OMA scheme described below.

C. Relayed OMA Scheme

In the considered relayed OMA system, B transmits $x_f$ to $U_F$, $U_N$ and R in the first signalling phase. In the second signalling phase R forwards either the amplified (with AF relaying) or the decoded (with DF relaying) FU symbol, while B transmits $x_n$ to NU. Using the decoded FU symbol in the first phase, NU cancels the interference caused by transmission by R before decoding. However, if NU fails to decode $x_f$ in the first phase, then the signal received from R in the second phase acts as interference at $U_N$ and $x_n$ transmitted from B acts as interference at FU. Note that NU receives symbols in both time-slots in this OMA scheme with relaying. It can therefore be expected to attain high throughput. We observe in Section □ that the proposed scheme yields much better throughput performance as compared to its relayed OMA counterpart.

III. OUTAGE PROBABILITY ANALYSIS

In this section, the outage performance of coordinated direct AF/DF relay assisted downlink NOMA is analyzed. We derive closed-form expressions of near and far user outage probabilities considering AF or DF relaying. In order to determine the diversity order for $U_N$ and $U_F$, the asymptotic expressions of user outage probabilities are also derived. For ease of exposition, we define $|h_{BR}|^2 = X (\lambda_{BR} = \lambda_x)$, $|h_{RF}|^2 = Y (\lambda_{RF} = \lambda_y)$, $|h_{BN}|^2 = Z (\lambda_{BN} = \lambda_z)$, $|h_{BF}|^2 = W (\lambda_{BF} = \lambda_w)$, and $|h_{RN}|^2 = V (\lambda_{RN} = \lambda_v)$. Note that the RVs $X, Y, Z, W, V$ are independent and exponentially distributed with PDF $f_\Omega(\omega) = \lambda_\omega e^{-\lambda_\omega \omega}$ with $\Omega \in \{X,Y,Z,W,V\}$, $\omega \in \{x,y,z,w,v\}$.

A. Amplify-and-Forward Relaying

1) Far user outage probability: The outage probability of $U_F$ can be written as $p_f^{AF} = \Pr \{ \Gamma_{f}^{COM} < \gamma_f \}$, where $\Gamma_{f}^{COM} = \Gamma_{f}^{I} + \Gamma_{f}^{II}$. Using (3) and (7), $p_f^{AF}$ can be expressed as $p_f^{AF} = \Pr \{ \Gamma_{f}^{COM} < \gamma_f \} = \Pr \left\{ \frac{(1 - \alpha_g)\rho_g W}{\alpha_g \rho_g W + 1} + \frac{\rho_g \rho_r (1 - \alpha_g)XY}{\rho_g \rho_r \alpha_g XY + \rho_r Y + \rho_g X + 1} < \gamma_f \right\}$
wherein \( \phi = 1 - \alpha_B (1 + \gamma_F) \). The sign of \( (\gamma_F - \rho_B W) \) depends on the range of \( W \) and different values of \( \phi \). Depending on \( (\gamma_F - \rho_B W) \geq 0 \), the conditions for outage on ranges of \( X, Y \) and \( W \) with \( \phi > 0 \) \( (\alpha_B < \frac{1}{1 + \gamma_F}) \), \( \alpha_B < 1 < \phi < 0 \) \( (\frac{1 + \gamma_F}{1 + \gamma_F} \leq \alpha_B < \frac{2}{1 + \gamma_F}) \) and \( \phi < \alpha_B - 1 \) \( (\alpha_B \geq \frac{2}{1 + \gamma_F}) \) are listed in Table I, wherein

\[
\xi_{f_1}(x, w) = \frac{1}{\alpha_p \rho_B (1 - \alpha_B + \phi)} \left( \frac{(\gamma_F - \rho_B W) (1 + \rho_B x)}{\rho_B (\phi + \alpha_B \rho_B W (1 - \alpha_B + \phi) - (\gamma_F - \rho_B W))} \right), \quad \xi_{f_1}(w) = \frac{(\gamma_F - \rho_B W)}{\rho_B (\phi + \alpha_B \rho_B W (1 - \alpha_B + \phi))}, \quad \chi_f = \frac{\gamma_F}{\phi_B}
\]

and \( \xi_1 = \frac{\alpha_B \rho_B (1 - \alpha_B + \phi)}{\alpha_p \rho_B (1 - \alpha_B + \phi)} \). Now, \( p^{AF}_F \) can be expressed as

\[
p^{AF}_F = \begin{cases} 
\int \int_{o \xi_{f_1}(w)}^{\infty} \left[ 1 - e^{-\lambda \xi_{f_1}(x, w)} \right] \lambda x e^{-\lambda x} \lambda w e^{-\lambda w} dxdw + \int \int_{o}^{\infty} \left[ 1 - e^{-\lambda \xi_{f_1}(w)} \right] \lambda w e^{-\lambda w} dw & \text{if } \alpha_B < \frac{1}{1 + \gamma_F} \\
1 - e^{-\lambda \xi_{f_1}} + \int \int_{\xi_1 \xi_{f_1}(w)}^{\infty} \left[ 1 - e^{-\lambda \xi_{f_1}(x, w)} \right] \lambda x e^{-\lambda x} \lambda w e^{-\lambda w} dxdw + \int \int_{\xi_1}^{\infty} \left[ 1 - e^{-\lambda \xi_{f_1}(w)} \right] \lambda w e^{-\lambda w} dw & \text{if } \frac{1}{1 + \gamma_F} \leq \alpha_B < \frac{2}{2 + \gamma_F} \\
1 & \text{otherwise,}
\end{cases}
\]

After some mathematical rearrangement (13) can be expressed as

\[
p^{AF}_F = \begin{cases} 
1 - e^{-\lambda \xi_{f_1}} - \int \int_{o \xi_{f_1}(w)}^{\infty} \exp \left( -\frac{\lambda \xi_{f_1}(w) (1 + \rho_B x)}{\rho_B (x - \xi_{f_1}(w))} \right) \lambda x e^{-\lambda x} \lambda w e^{-\lambda w} dxdw & \text{if } \alpha_B < \frac{1}{1 + \gamma_F} \\
1 - \int \int_{\xi_1 \xi_{f_1}(w)}^{\infty} \exp \left( -\frac{\lambda \xi_{f_1}(w) (1 + \rho_B x)}{\rho_B (x - \xi_{f_1}(w))} \right) \lambda x e^{-\lambda x} \lambda w e^{-\lambda w} dxdw & \text{if } \frac{1}{1 + \gamma_F} \leq \alpha_B < \frac{2}{2 + \gamma_F} \\
1 & \text{otherwise.}
\end{cases}
\]

We first solve for \( I_1 \) as follows:

\[
I_1 = \int \int_{o \xi_{f_1}(w)}^{\infty} \exp \left( -\frac{\lambda \xi_{f_1}(w) (1 + \rho_B x)}{\rho_B (x - \xi_{f_1}(w))} \right) \lambda x e^{-\lambda x} \lambda w e^{-\lambda w} dxdw.
\]

**TABLE I**: \( U_F \) outage - ranges of \( Y, X \) and \( W \) for different values of \( \alpha_B \)

| \( \alpha_B \) | \( W < \chi_F \) | \( W > \chi_F \) |
|---|---|---|
| \( \frac{1}{1 + \gamma_F} \leq \alpha_B < \frac{2}{\gamma_F + \gamma_F} \) | \( X < \xi_{f_1}(W), Y > 0 \) | \( X \geq \xi_{f_1}(W), Y < \xi_{f_1}(X, W) \) |
| \( \frac{2}{\gamma_F + \gamma_F} \leq \alpha_B < \frac{2}{\gamma_F + \gamma_F} \) | \( W < \xi_{f_1} \) | \( X < \xi_{f_1}(W), Y > 0 \) |
| \( \frac{2}{\gamma_F + \gamma_F} \) | \( W \geq \xi_{f_1} \) | \( X \geq \xi_{f_1}(W), Y < \xi_{f_1}(X, W) \) |
| \( \alpha_B \geq \frac{2}{\gamma_F + \gamma_F} \) | Entire range of \( W, X \) and \( Y \) |
Substituting \( x = x - \zeta_{1}(w) \) in the above and re-arranging terms, we get

\[
I_1 = \int_{\xi}^{\chi_F} \exp \left( -\zeta_{1}(w) \left( \lambda_{x} + \frac{\lambda_{y} \rho_{b}}{\rho_{r}} \right) \right) \int_{0}^{\infty} \lambda_{x} \exp \left( -\lambda_{x} x - \frac{\lambda_{y} \zeta_{1}(w) \left[ 1 + \rho_{b} \zeta_{1}(w) \right]}{x \rho_{r}} \right) \lambda_{w} e^{-\lambda_{w} w} dx dw. \tag{16}
\]

Solving for the inner integral using [31, 3.324.1], we obtain

\[
I_1 = \int_{\xi}^{\chi_F} \exp \left( -\zeta_{1}(w) \left( \lambda_{x} + \frac{\lambda_{y} \rho_{b}}{\rho_{r}} \right) \right) 2 \mu_{F_1}(w) K_1(2 \mu_{F_1}(w)) \lambda_{w} e^{-\lambda_{w} w} dw,
\tag{17}
\]

where \( \mu_{F_1}(w) = \frac{\lambda_{x} \lambda_{y} \zeta_{1}(w)[\rho_{b} \zeta_{1}(w)+1]}{\rho_{r}} \). Following a similar approach, we derive an expression for \( I_2 \). \( p_{AF}^{F} \) can then be expressed as

\[
p_{AF}^{F} = \begin{cases} 
1 - e^{-\lambda_{w} \chi_{F}} & \text{if } \alpha_{B} < \frac{1}{1+\gamma_{F}} \\
1 - \int_{\xi}^{\chi_{F}} \exp \left( -\zeta_{1}(w) \left( \lambda_{x} + \frac{\lambda_{y} \rho_{b}}{\rho_{r}} \right) \right) 2 \mu_{F_1}(w) K_1(2 \mu_{F_1}(w)) \lambda_{w} e^{-\lambda_{w} w} dw & \text{if } \frac{1}{1+\gamma_{F}} \leq \alpha_{B} < \frac{2}{2+\gamma_{F}} \\
1 & \text{otherwise},
\end{cases}
\tag{18}
\]

where \( \mu_{F_1}(w) = \frac{\lambda_{x} \lambda_{y} \zeta_{1}(w)[\rho_{b} \zeta_{1}(w)+1]}{\rho_{r}} \). Unfortunately, the integrals in (18) do not admit a closed form.

We therefore present a highly accurate approximate expression for \( p_{AF}^{F} \) in the following theorem.

**Theorem 1.** An accurate closed-form expression for FU outage probability with AF relaying is given by

\[
p_{AF}^{F} = \begin{cases} 
1 - e^{-\lambda_{w} \chi_{F}} - e^{-\xi_{1}(\rho_{b}+\frac{1}{\rho_{r}})} [\Gamma(1, -\lambda_{w} \xi_{1}; \vartheta_{6}) - \Gamma(1, \lambda_{w}(\chi_{F} - \xi_{1}); \vartheta_{6})] & \text{if } \alpha_{B} < \frac{1}{1+\gamma_{F}} \\
1 - e^{-\xi_{1}(\rho_{b}+\frac{1}{\rho_{r}})} \sqrt{4 \vartheta_{6} K_1(\sqrt{4 \vartheta_{6}})} & \text{if } \frac{1}{1+\gamma_{F}} \leq \alpha_{B} < \frac{2}{2+\gamma_{F}} \\
1 & \text{otherwise}.
\end{cases}
\tag{19}
\]

**Proof.** Refer to Appendix [A].

Using the expression obtained in (19), it is extremely difficult to establish the diversity order. We therefore derive a high-SNR approximation to \( p_{AF}^{F} \) in the following Lemma.

**Lemma 1.** A high SNR approximation to \( p_{AF}^{F} \) is given by

\[
p_{AF}^{F} \approx \begin{cases} 
\left( \lambda_{x} + \frac{\lambda_{y} \rho_{b}}{\rho_{r}} \right) \xi_{1} \left[ 1 - e^{-\lambda_{w} \chi_{F}} + (\xi_{1} - \chi_{F}) \lambda_{w} e^{-\lambda_{w} \xi_{1}} \left[ \mathcal{E}_{1}(\lambda_{w} \xi_{1}) - \mathcal{E}_{1}(\lambda_{w} \xi_{1} + \lambda_{w} \chi_{F}) \right] \right] & \text{if } \alpha_{B} < \frac{1}{1+\gamma_{F}} \\
1 - e^{-\lambda_{w} \xi_{1}} \frac{1 + \lambda_{x} + \lambda_{y}}{1 + \xi_{1} \lambda_{x} + \lambda_{y}} & \text{if } \frac{1}{1+\gamma_{F}} \leq \alpha_{B} < \frac{2}{2+\gamma_{F}} \\
1 & \text{otherwise}.
\end{cases}
\tag{20}
\]

**Proof.** Refer to Appendix [B].
Remark 1. In the absence of direct link to $U_F$, the outage probability of $U_F$ can be readily derived by substituting $W = 0$ into (12). After using a procedure similar to that used to derive (18), we obtain an exact closed-form expression for $p_{AF}^F$ as follows:

$$
p_{AF}^F = 1 - \exp\left(-\frac{X_F}{\rho_r}(\lambda_x\rho_r + \lambda_y\rho_b)\right) 2\sqrt{\mu_F K_1(2\sqrt{\mu_F})}, \quad \text{if } \alpha_b < \frac{1}{1+\gamma_F}
$$

(21)

where $\mu_F = \frac{\lambda_x\lambda_y N_F(1+\rho_b X_F)}{\rho_r}$. The above equality holds only for $\alpha_b < \frac{1}{1+\gamma_F}$, otherwise $p_{AF}^F = 1$. Note that for higher values of $\rho_b$ and $\rho_r$, $\mu_F$ and $\lambda_F$ are very small. Therefore, applying $K_1(\theta) \simeq \frac{1}{2}\Gamma(\frac{\theta}{2})^{-1}$ [32 9.6.9] in (21), we obtain

$$
p_{AF}^F \simeq 1 - \exp\left(-X_F \left[\lambda_x + \frac{\lambda_y\rho_b}{\rho_r}\right]\right).
$$

(22)

Using $e^{-\theta} \simeq 1 - \theta$ in the above, a high SNR approximation to $p_{AF}^F$ is given by

$$
p_{AF}^F \simeq \frac{\gamma_F [\rho_r \lambda_x + \rho_b \lambda_y]}{\rho_r \rho_b \phi}.
$$

(23)

2) Near user Outage Probability: Let $\Gamma_{NF}^{COM} = \Gamma_{NF}^I + \Gamma_{NF}^{II}$ and $\Gamma_{NN}^{COM} = \Gamma_{NN}^I + \Gamma_{NN}^{II}$. Due to the manner in which SIC is performed with combining, the outage probability of $U_N$ is

$$
p_{AF}^N = 1 - \Pr\left\{\Gamma_{NF}^{COM} \geq \gamma_F, \Gamma_{NN}^{COM} \geq \gamma_N\right\}.
$$

(24)

Theorem 2. The near user outage probability can be written as follows:

$$
p_{AF}^N = \left\{\begin{array}{ll}
\frac{1}{1} & \frac{1}{1} \\
\left[\frac{1 - \Pr\{Z > \max(\chi_F, \chi_N)\}}{I_3} - \Pr\{V > \zeta_{N_1}(X,Z), X > \chi_{N_1}(Z), \chi_N > Z > \chi_F\}\right] & I_4 \\
\left[-\Pr\{V > \zeta_{N_1}(X,Z), X > \chi_{N_1}(Z), \min(\chi_F, \chi_N) > Z\}\right] & I_5 \\
1 - \Pr\{Z > \max(\chi_F, \chi_N)\} & I_6 \\
\left[-\Pr\{V > \zeta_{N_2}(X,Z), X > \chi_{N_2}(Z), \chi_F > Z > \chi_N\}\right] & I_7 \\
1 - \Pr\{V > \zeta_{N_2}(X,Z), X > \chi_{N_2}(Z), Z > \xi_1\} & I_8 \\
\end{array}\right. \quad \text{if } \alpha_b < \frac{1}{2+\gamma_F}
$$

(25)

where $\chi_N = \frac{\gamma_N}{\alpha_b \rho_b}$, $X_F = \frac{\gamma_N - \alpha_b \rho_b}{\rho_b \alpha_b}$, $\chi_{N_1}(Z) = \frac{\chi_{N_1}(Z)(\rho_b X + 1)}{\rho_r (X - \chi_{N_1}(Z))}$, $\chi_{N_2}(Z) = \frac{\chi_{N_2}(Z)(\rho_b X + 1)}{\rho_r (X - \chi_{N_2}(Z))}$.

Proof. Using (2), (5) and (6) into (24), we obtain
Note that the signs of \( \min(\chi_F, \chi_N) > Z \) can be expressed as

\[
\Pr[V \rho_B'(1 - \alpha_b)X - \rho_B\alpha_bXV + \rho_BV + \rho_BX + 1 \geq \gamma_N] = 1 - \Pr[V \rho_B'(\phi + \alpha_b \rho_B Z(1 - \alpha_b + \phi)) - (\gamma_f - \phi_B Z)] \geq (\gamma_f - \phi_B Z)(1 + \rho_B X)],
\]

In the above, depending on values of \( \alpha_b \) and \( \gamma_f \), two ranges of \( \alpha_b \) exist i.e. \( \alpha_b < \frac{1}{1 + \gamma_f} \) or \( \alpha_b \geq \frac{1}{1 + \gamma_f} \).

Note that the signs of \( \gamma_F - \phi_B Z) \) and \( \gamma_N - \alpha_b \rho_B Z \) depend on the range of \( Z \). Based on the ranges of RVs \( V, X \) and \( Z \), the conditions for non-outage probability of \( U_N \) for different values of \( \alpha_b \) are listed in Table II. Now, \( p_{NAF}^F \) can be expressed as

\[
p_{NAF}^F = 1 - \Pr \left\{ V > \xi_{N_2}(X, Z), X > \chi_{N_2}(Z), Z > \xi_1 \right\} U(-\phi) - \left[ \Pr \left\{ Z > \max(\chi_N, \chi_F) \right\} \right. \\
+ \Pr \left\{ V > \xi_{N_1}(X, Z), X > \chi_{N_1}(Z), \chi_N > Z > \chi_F \right\} U(\chi_N - \chi_F) \right\} + \Pr \left\{ V > \xi_{N_2}(X, Z), X > \chi_{N_2}(Z), \chi_F > Z > \chi_N \right\} U(\chi_F - \chi_N) + \Pr \left\{ V > \xi_{N_1}(X, Z), X > \chi_{N_1}(Z), Z < \min(\chi_N, \chi_F) \right\} U(\phi),
\]

where \( \chi_{N_1}(Z), \chi_{N_2}(Z), \xi_{N_1}(X, Z), \xi_{N_2}(X, Z), \chi_F, \chi_N \) and \( \phi \) are defined in the line following (25). After performing mathematical rearrangements in (27) we obtain (25).

**Theorem 3.** An accurate closed-form expression for \( p_{NAF}^F \) is given by

\[
p_{NAF}^F \approx \begin{cases} 
1 - e^{-\lambda_z \max(\chi_N, \chi_F)} - \lambda_c e^{-\chi_N C_l} \left[ 1 - e^{-\xi_1(\chi_F + \chi_N)} + e^{C(\min(\chi_F, \chi_N) - \chi_N)} - e^{-\chi_N} \right] & \text{if } \alpha_b < \frac{1}{2 + \gamma_f} \\
1 - e^{-\lambda_z \max(\chi_N, \chi_F)} - \lambda_c e^{-\chi_N C_l} \left[ e^{C(\min(\chi_F, \chi_N) - \chi_N)} - e^{-\chi_N} \right] - e^{\xi_1(C_l + \lambda_z)} \left[ e^{-(\xi_1 + \chi_N)} - e^{-(\xi_1 + \chi_F)} \right] & \text{if } \frac{1}{2 + \gamma_f} \leq \alpha_b < \frac{1}{1 + \gamma_f} \\
1 - e^{-\xi_1(C_l + \lambda_z)} \left[ e^{2\xi_1} - \lambda_c \frac{\partial_2 e^{\lambda_z \xi_2}}{2} \left[ (\xi_1 + \partial_2) - (\xi_1 + \partial_2) \right] \right] & \text{if } \frac{1}{1 + \gamma_f} \leq \alpha_b < \frac{1}{2 + \gamma_f} \\
1, & \text{otherwise},
\end{cases}
\]

where \( \partial_2 = \xi_1 \left( \xi_1 + \frac{\gamma_f}{\rho_B \phi} \right) C_l, C = C_l - \lambda_c \) and \( C_l = \lambda_c + \frac{\lambda_B \rho_B}{\rho_R} \).
Lemma 2. The diversity orders $D^\text{AF}_F$ and $D^\text{AF}_N$ achieved at $U_F$ and $U_N$ are given by

$$D^\text{AF}_F = \begin{cases} 
2, & \text{if } \alpha_B < \frac{1}{1 + \gamma_F} \\
1, & \frac{1}{1 + \gamma_F} \leq \alpha_B < \frac{2}{2 + \gamma_F} \\
0, & \text{otherwise}
\end{cases}$$

$$D^\text{AF}_N = \begin{cases} 
2, & \text{if } \alpha_B < \frac{1}{1 + \gamma_F} \\
1, & \frac{1}{1 + \gamma_F} \leq \alpha_B < \frac{2}{2 + \gamma_F} \\
0, & \text{otherwise}
\end{cases}$$  \hspace{1cm} (29)

Proof. To derive the diversity order for $U_F$ and $U_N$, we use the high-SNR expression of $p^\text{AF}_F$ and $p^\text{AF}_N$ given by (20) and (28), respectively. We consider $\rho_B = \rho_R = \rho$ without loss of generality. The diversity orders attained by $U_F$ and $U_N$ are given by $D^\text{AF}_j = -\lim_{\rho \to \infty} \frac{\log_2 p^\text{AF}_j(\rho)}{\log_2 \rho}$, $j \in \{N, F\}$. For $\rho \to \infty$, we use $e^{-\theta} = 1 - \theta$ and substitute for $\chi_F$ and $\xi_1$. After neglecting higher order terms of $1/\rho$, we obtain diversity order as expressed in (29). The detailed proof is omitted due to paucity of space. \hspace{1cm} ■

Remark 2. The fact that the near-user also attains a diversity of two is significant, and motivates it to participate in NOMA signalling.

Remark 3. In the absence of direct link to $U_F$, the diversity order $D^\text{AF}_F$ is obtained by using $\rho_B = \rho_R = \rho$ into (23). The diversity order $D^\text{AF}_F$ can be expressed as

$$D^\text{AF}_F = \begin{cases} 
1, & \text{if } \alpha_B < \frac{1}{1 + \gamma_F} \\
0, & \text{otherwise}
\end{cases}$$  \hspace{1cm} (30)

However, the diversity order for $U_N$ remains the same as expressed in (29).

B. Decode-and-Forward Relaying

1) Far user outage probability: Let $\Gamma^\text{COM}_{FF-S} = \Gamma^\text{I}_{FF} + \Gamma^\text{II}_{FF-S}$ and $\Gamma^\text{COM}_{FF-F} = \Gamma^\text{I}_{FF} + \Gamma^\text{II}_{FF-F}$. FU is not in outage when a) both $s_F$ and $s_N$ are decoded at $R$ ($\Gamma^\text{I}_{RF} \geq \gamma_F$, $\Gamma^\text{I}_{RN} \geq \gamma_N$) and FU SNR after combining is sufficient ($\Gamma^\text{COM}_{FF-S} \geq \gamma_F$), b) $s_F$ can be decoded at $R$, but not $s_N$ but SNR after combining is sufficient at FU ($\Gamma^\text{COM}_{FF-F} \geq \gamma_F$), and c) $s_F$ cannot be decoded at $R$ but $\Gamma^\text{I}_{FF} \geq \gamma_F$. The outage probability for $U_F$ can be formulated as

$$p^{DF}_F = 1 - \Pr_A \left\{ \Gamma^\text{I}_{RF} \geq \gamma_F, \Gamma^\text{I}_{RN} \geq \gamma_N, \Gamma^\text{COM}_{FF-S} \geq \gamma_F \right\} - \Pr_{A_2} \left\{ \Gamma^\text{I}_{RF} \geq \gamma_F, \Gamma^\text{I}_{RN} < \gamma_N, \Gamma^\text{COM}_{FF-F} \geq \gamma_F \right\} - \Pr_{A_3} \left\{ \Gamma^\text{I}_{RF} < \gamma_F, \Gamma^\text{I}_{FF} \geq \gamma_F \right\}$$  \hspace{1cm} (31)

In the following, we present closed-form expressions for FU outage.

Theorem 4. The exact closed-form expression of the outage probability for $U_F$ is given by
where $\phi = 1 - \alpha_R (1 + \gamma_f)$. $\vartheta_1 = \rho_B (\alpha_B (1 - \alpha_R) + \alpha_R \phi)$, $\zeta_1 = \frac{\lambda_w \phi_B}{\vartheta_1}$ and $\zeta_2 = \chi_F + \frac{\phi_B}{\vartheta_1}$.

**Proof.** Refer to Appendix [D].

The expressions obtained in (32) involve the generalized incomplete gamma function due to which the diversity cannot readily be established from them. We therefore present a highly accurate high-SNR approximation to $p_{DF}^N$ in the following Lemma.

**Lemma 3.** A high SNR approximation to $p_{DF}^N$ is given by

\[
p_{DF}^N \approx \begin{cases} 
1 + e^{-\lambda_w \chi_F} (1 - e^{-\lambda_w \chi_N}) - e^{-\lambda_w \chi_N} \left[ \exp \left( \zeta_1 + \frac{\lambda_w \phi_B}{\rho_R \vartheta_1} - \max (\zeta_1, 0) \right) + e^{-\lambda_w \chi_F} \left( 1 - \exp \left( \frac{\lambda_w \phi_B}{\rho_R \vartheta_1} \right) \right) \right] & \text{if } \alpha_B < \frac{\gamma_N}{\gamma_N + \gamma_f + \gamma_f \gamma_N} \\
1 - e^{-\lambda_w \chi_F} (1 - e^{-\lambda_w \chi_N}) - e^{-\lambda_w \chi_N} \left[ \exp \left( \zeta_1 + \frac{\lambda_w \phi_B}{\rho_R \vartheta_1} - \max (\zeta_1, 0) \right) \right] & \text{if } \frac{\gamma_N}{\gamma_N + \gamma_f + \gamma_f \gamma_N} \leq \alpha_B < \frac{1}{1 + \gamma_f} \\
1 & \text{otherwise,}
\end{cases}
\]

(32)

**Remark 4.** In the absence of the direct link to $U_F$, we can use $W = 0$ in (31) to get

\[
p_{DF}^N = 1 - \Pr \{ X > \max (\chi_N, \chi_f), Y_F > \rho \} - \Pr \{ \chi_F < X < \chi_N, Y_F > \gamma_f \}.
\]

(34)

After solving (34) as in Theorem 4, an exact expression for $p_{DF}^N$ is given by

\[
p_{DF}^N = \begin{cases} 
1 - \exp \left( - \frac{\lambda_w \gamma_f}{\rho_R \phi_B} - \lambda_w \chi_N \right) - \exp \left( - \frac{\lambda_w \gamma_f}{\rho_R} \right) e^{-\lambda_w \chi_F} & \text{if } \alpha_B < \frac{\gamma_N}{\gamma_N + \gamma_f + \gamma_f \gamma_N} \\
1 - \exp \left( - \frac{\lambda_w \gamma_f}{\rho_R \phi_B} - \lambda_w \chi_f \right) & \text{if } \frac{\gamma_N}{\gamma_N + \gamma_f + \gamma_f \gamma_N} \leq \alpha_B < \frac{1}{1 + \gamma_f} \\
1 & \text{otherwise.}
\end{cases}
\]

(35)
Using $e^{-\theta} \simeq 1 - \theta$ in above and neglecting higher order terms of $\frac{1}{\rho_R}$, a high-SNR approximation to $p_{DF}^F$ is given by

$$p_{DF}^F \simeq \begin{cases} \frac{\lambda_s \gamma_F}{\rho_R \phi_R} + \lambda_s \chi_F \ & \text{if} \ \alpha_b < \frac{1}{1 + \gamma_F} \\ 1 \ & \text{otherwise}. \end{cases} \quad (36)$$

2) Near user outage probability: NU is not in outage when a) $R$ decodes both $s_F$ and $s_N$ ($\Gamma_{RF}^l \geq \gamma_F, \Gamma_{RN}^l \geq \gamma_N$) and SNRs at NU (after combining) are sufficient to decode both $s_F$ and $s_N$ ($\Gamma_{NF-S}^l \geq \gamma_F, \Gamma_{NN-S}^l \geq \gamma_N$), b) $R$ decodes $s_F$ but fails to decode $s_N$ ($\Gamma_{RF}^l \geq \gamma_F, \Gamma_{RN}^l < \gamma_N$), and SNRs to decode $s_F$ (after combining) and $s_N$ (from first phase) are sufficient at NU ($\Gamma_{NF-F}^l \geq \gamma_F, \Gamma_{NN}^l \geq \gamma_N$), and c) $R$ fails to decode $s_F$ ($\Gamma_{RF}^l < \gamma_F$), but the first phase SNRs are sufficient to decode both $s_F$ and $s_N$ at NU ($\Gamma_{NF}^l \geq \gamma_F, \Gamma_{NN}^l \geq \gamma_N$).

The outage probability for $U_N$ can be formulated as

$$p_{N}^{DF} = 1 - \Pr \left\{ \Gamma_{RF}^l < \gamma_F, \Gamma_{NF}^l \geq \gamma_F, \Gamma_{NN}^l \geq \gamma_N \right\} - \Pr \left\{ \Gamma_{RF}^l \geq \gamma_F, \Gamma_{RN}^l \geq \gamma_N, \Gamma_{COM}^l \geq \gamma_F, \Gamma_{COM}^l \geq \gamma_N \right\} - \Pr \left\{ \Gamma_{RF}^l \geq \gamma_F, \Gamma_{RN}^l \geq \gamma_N, \Gamma_{COM}^l \geq \gamma_F, \Gamma_{COM}^l \geq \gamma_N \right\}, \quad (37)$$

where $\Gamma_{COM}^l_{NF-F} = \Gamma_{NF}^l + \Gamma_{II}^l_{NF-F}, \Gamma_{COM}^l_{NF-S} = \Gamma_{NF}^l + \Gamma_{II}^l_{NF-S}$ and $\Gamma_{COM}^l_{NN-S} = \Gamma_{N}^l + \Gamma_{II}^l_{NN-S}$.

Theorem 5. The exact closed-form expression of the outage probability for $U_N$ is given by

$$p_{DF}^N = \begin{cases} 1 - e^{-\lambda_s \chi_F} (1 - e^{-\lambda_s \chi_N}) - e^{-\lambda_s \chi_N} \left[ e^{-\lambda_s \chi_N} - \frac{\lambda_s \alpha_b \rho_b e^{\frac{\lambda_s \gamma_N}{\rho_R \phi_R}}}{\lambda_s \alpha_b \rho_b - \lambda_s \alpha_b \rho_R} \left[ \exp \left( \max \left( 0, \frac{\gamma_N}{\beta_5} \right) \right) \lambda_s \rho_b - \lambda_s \max \left( 0, \frac{\gamma_N}{\beta_5} \right) \right] \right] \ & \text{if} \ \alpha_b < \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \\ 1 - e^{-\lambda_s \chi_F} (1 - e^{-\lambda_s \chi_N}) - e^{-\lambda_s \chi_F} \left[ e^{-\lambda_s \chi_F} - \frac{\lambda_s \alpha_b \rho_b e^{\frac{\lambda_s \gamma_N}{\rho_R \phi_R}}}{\lambda_s \alpha_b \rho_b - \lambda_s \alpha_b \rho_R} \left[ \exp \left( \max \left( 0, \frac{\gamma_N}{\beta_5} \right) \right) \lambda_s \rho_b - \lambda_s \max \left( 0, \frac{\gamma_N}{\beta_5} \right) \right] \right] \ & \text{if} \ \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \leq \alpha_b < \frac{1}{1 + \gamma_F} \\ 1 \ & \text{otherwise}, \end{cases} \quad (38)$$

where $\beta_4 = \frac{\beta \alpha_b + \gamma_N \alpha_b + \gamma_N \alpha_b (1 - \alpha_b) - \alpha_b \phi_R}{\rho_b \alpha_b (\beta \alpha_b + \alpha_b (1 - \alpha_b))}, \ \beta_5 = -\frac{\phi_R}{\beta_1}, \ \beta_1 = \rho_b (\beta \alpha_b + \alpha_b (1 - \alpha_b)), \ \chi_F = \frac{\lambda_s \lambda_s \phi_R (\chi_F + \delta \beta)}{\beta_1}$.
and $\chi_N = \min(\chi_f, \chi_N, \theta_4)$.

**Proof.** Refer to Appendix F. ■

In the following lemma, we present a high-SNR approximate expression for $p_N^{DF}$ to obtain the diversity order at NU.

**Lemma 4.** A high-SNR approximation to $p_N^{DF}$ is given by

\[
p_N^{DF} \approx \begin{cases} 
1 - e^{-\lambda_N (1 - e^{-\lambda_N})} & \text{if } \alpha_b < \frac{1}{1 + \gamma_F}, \alpha_R < \frac{1}{1 + \gamma_F} \\
1 - e^{-\lambda_N (1 - e^{-\lambda_N})} - \frac{\lambda_N e^{-\lambda_N} }{\epsilon_3 \alpha_R \rho_b - \lambda_N} \left[ \exp(\epsilon_3 \gamma_N - \lambda_N \chi_N) - \exp \left( \frac{\epsilon_3 \alpha_R \gamma_F - \lambda_N \chi_f}{\phi} \right) \right] & \text{if } \alpha_b < \frac{\gamma_N}{\gamma_N + \gamma_F + \gamma_F \gamma_N} \\
1 - e^{-\lambda_N (1 - e^{-\lambda_N})} - \frac{\lambda_N e^{-\lambda_N} }{\epsilon_3 \alpha_R \rho_b - \lambda_N} \left[ \exp(\epsilon_3 \gamma_N - \lambda_N \chi_N) - \exp \left( \frac{\epsilon_3 \alpha_R \gamma_F - \lambda_N \chi_f}{\phi} \right) \right] & \text{if } \alpha_b < \frac{1}{1 + \gamma_F} \\
1 & \text{otherwise.}
\end{cases}
\]

where $\epsilon_3 = \frac{\lambda_N}{\alpha_R \rho_R}$.

**Proof.** Proof is similar to that used to derive (33), and is therefore omitted. ■

**Lemma 5.** The diversity orders $D_F^{DF}$ and $D_N^{DF}$ attained by $U_F$ and $U_N$ can be expressed as

\[
D_F^{DF} = \begin{cases} 
2 & \text{if } \alpha_b < \frac{1}{1 + \gamma_F}, \alpha_R < \frac{1}{1 + \gamma_F} \\
1 & \text{if } \alpha_b < \frac{1}{1 + \gamma_F}, \alpha_R \geq \frac{1}{1 + \gamma_F} \\
0 & \text{otherwise.}
\end{cases}
\]

\[
D_N^{DF} = \begin{cases} 
2 & \text{if } \alpha_b < \frac{1}{1 + \gamma_F}, \alpha_R < \frac{1}{1 + \gamma_F} \\
1 & \text{if } \alpha_b < \frac{1}{1 + \gamma_F}, \alpha_R \geq \frac{1}{1 + \gamma_F} \\
0 & \text{otherwise.}
\end{cases}
\]

**Proof.** The diversity orders for $U_F$ and $U_N$ are obtained by using high-SNR expressions of $p_F^{DF}$ and $p_N^{DF}$ given by (33) and (39), respectively. We substitute $\rho_b = \rho_R = \rho$ and solve for $D_j^{DF} = - \lim_{\rho \to \infty} \frac{\log_2 p_j^{DF}(\rho)}{\log_2 \rho}$, $j \in \{N, F\}$. Details are omitted due to paucity of space. ■

**Remark 5.** In the absence of direct link to $U_F$, the diversity order $D_F^{DF}$ is obtained by using $\rho_b = \rho_R = \rho \to \infty$ into (35) and using linear approximations to the exponential terms. The diversity order $D_F^{DF}$ can be expressed as

\[
D_F^{DF} = \begin{cases} 
1 & \text{if } \alpha_b < \frac{1}{1 + \gamma_F}, \alpha_R < \frac{1}{1 + \gamma_F} \\
0 & \text{otherwise.}
\end{cases}
\]

However, the diversity order for $U_N$ remains the same as in (40).
IV. NU THROUGHPUT AND ENERGY EFFICIENCY MAXIMIZATION

NU needs to decode the FU symbol and perform SIC to decode its own symbol, and might therefore incur some performance loss. For this reason, maximizing the NU throughput while guaranteeing a desired FU target throughput $\tilde{\tau}$ is well motivated. We do so in this section. The FU and NU throughput in bits per channel use (bpcu) are defined as

$$\tau^i_j = \frac{1}{2} R_j (1 - p^i_j), \quad j \in \{N,F\}, i \in \{AF,DF\}$$

where $p^AF, p^NF, p^DF$ and $p^DF_N$ are as in (19), (28), (32) and (38) respectively.

Careful choice of power allocation coefficient $\alpha_B$ and symbol rates $R_F$ and $R_N$ is crucial to attaining good throughput at $U_N$ and $U_F$. With DF relays, $\alpha_B$ can also be chosen. However it will be demonstrated in Section V that a large range of $\alpha_B$ values result in the same throughput performance. The reason for this is that $\alpha_B$ is only used when both $s_F$ and $s_N$ are decoded correctly at R (which in turn depends on $\alpha_B$). We therefore focus only on choice of $\alpha_B, R_N$ and $R_F$.

We first consider the case when the B-U_F link is absent. Noting that the NU benefits when the maximum value of $\alpha_B$ is chosen, we derive $\alpha_B^{opt}$ by using (23) and (36) into (42) to get

$$\alpha_B^{AFopt} \approx \frac{1}{1 + \gamma_F} - \frac{\gamma_F R_F (\lambda_x \rho_R + \lambda_y \rho_B)}{\rho_B \rho_R (1 + \gamma_F) (R_F - 2 \tilde{\tau}_F)}.$$

$$\alpha_B^{DFopt} \approx \frac{\gamma_F \lambda_y \rho_B - (\alpha_R + \alpha_R \gamma_F - 1) (\gamma_F \lambda_x + (\frac{2\tilde{\tau}}{R_F} - 1) \rho_B) \rho_R}{\rho_B (1 + \gamma_F) (\gamma_F \lambda_y - \rho_R (\frac{2\tilde{\tau}}{R_F} - 1) (\alpha_R + \alpha_R \gamma_F - 1))}.$$

The NU throughput for AF and DF relaying schemes can be obtained by substituting $\alpha_B = \alpha_B^{opt}$ into expressions for $\tau^i_N$. Clearly, $\tau^i_N$ is now a function only of symbol rates $R_N$ and $R_F$ (and therefore $\gamma_N$ and $\gamma_F$). Thus, careful choice of symbol rates is essential for maximizing the NU throughput. Let $R_N^{opt}$ and $R_F^{opt}$ denote the optimal symbol rates that maximize $\tau^i_N$. Now, the optimization problem is formulated as

$$R_N^{opt}, R_F^{opt} = \arg \max_{R_N, R_F} \tau^i_N(R_N, R_F)$$

Solving the above joint optimization problem analytically is difficult. However, numerical techniques can be used to pick the optimal $R_N$ and $R_F$.

In the presence of the B-U_F link, a three-dimensional search is required to determine $\alpha_B, R_N$ and $R_F$ while ensuring that $\tau^i_F \geq \tilde{\tau}$.

Energy efficiency is an important performance metric for any communication system. It is defined as [14]:

$$\eta^i_E = \frac{\text{Sum throughput}}{\text{Total energy consumed}}, \quad i \in \{AF,DF\},$$

where

$$\tau^i_j = \frac{1}{2} R_j (1 - p^i_j), \quad j \in \{N,F\}, i \in \{AF,DF\}$$

and $p^AF, p^NF, p^DF$ and $p^DF_N$ are as in (19), (28), (32) and (38) respectively.
where 'sum throughput' represents the sum of $\tau_i^N$ and $\tau_i^F$. Let $T$ denote the signalling duration. Let $E_B = \frac{P_B T}{2}$ and $E_R = \frac{P_R T}{2}$. Using $\tau_i^N$, and $\tau_i^F$ from (42), the EE of the system is expressed as

$$\eta_i^E = \frac{\tau_i^{\text{sum}}}{E_B + E_R}$$

where $\tau_i^{\text{sum}} = \tau_i^F + \tau_i^N$. (47)

Since the sum throughput $\tau_i^{\text{sum}}$ is function of symbol rates $R_N$ and $R_F$, careful choice of symbol rates is essential for maximizing the EE. Let $R_N^{\text{opt}}$ and $R_F^{\text{opt}}$ denote the optimal symbol rates that maximize $\eta_i^E$. The EE optimization problem can be formulated as

$$R_N^{\text{opt}}, R_F^{\text{opt}} = \arg \max_{R_N, R_F} \eta_i^E(R_N, R_F)$$

(48)

Obtaining a closed-form solution to the above joint optimization problem is extremely difficult. However, a two dimensional search can be used to find the optimal $R_N$ and $R_F$.

V. SIMULATIONS AND NUMERICAL RESULTS

In this section we present computer simulations to confirm accuracy of the derived analytical expressions and derive useful insights. Unless mentioned otherwise, the considered system parameters are as follows: $d_{BR} = d_{RF} = d_{RN} = 2$, $d_{BN} = 1.2$, $d_{BF} = 4$, and $m = 4$. Also $\sigma^2$ is normalized to unity [15]. Further, we assume that $P_B = P_R = P (\rho_B = \rho_R = \rho)$ [14] and symbol rates $R_N = R_F = R (\gamma_N = \gamma_F = \gamma_T)$. We also compare the performance of the proposed schemes with its relayed OMA counterpart described in Section II.

In Fig 2 and Fig 3, we plot $p_F^{AF}$ and $p_N^{AF}$ versus transmit SNR ($\rho$) for different values of $\alpha_B$ and $R$. The accuracy of derived analytical expressions in (19), (21) and (28) can be clearly confirmed by simulations. Also the derived high-SNR approximations in (23) and (20) are quite accurate. With decreasing $\alpha_B$, more power is allocated to $U_F$, and $p_F^{AF}$ therefore continuously improves as shown.

Fig. 2: $p_F^{AF}$ vs. $\rho$.

Fig. 3: $p_N^{AF}$ vs. $\rho$.

Fig. 4: $p_F^{DF}$ vs. $\rho$. 
in Fig. 5 it is evident from Fig. 3 that an initial increase in $\alpha_B$ significantly improves $p_{DF}^N$ due to successful decoding of $x_F$ and $x_N$ at $U_N$, while further increase in $\alpha_B$ deteriorates $p_{DF}^N$ as $U_N$ fails to decode $x_F$. From Fig. 2 and Fig. 3 it is clear that (unlike in all CDRT schemes proposed so far) combining of incoming signals from B and R at $U_N$ as well as $U_F$ in the proposed CDRT framework ensures a full diversity of 2 to be harnessed (as derived in Lemma 2 for the case of AF relaying) by both users. However, in the absence of the $B-U_F$ link, diversity order of 1 is attained at $U_F$ (as noted in Remark 3). The switching of diversity order from 2 to 1 and 0 can also be observed when changing $\alpha_B$ from 0.1, 0.3 and 0.4 for $R = 2$. Therefore, a careful choice of power allocation coefficient is important to attain full diversity at both users.

Fig. 4 and Fig. 5 depict the variation of $p_{DF}^N$ and $p_{DF}^F$ versus $\rho$ for different values of $\alpha_B$, $\alpha_R$ and $R$. Simulations confirm the accuracy of the analytical expressions derived in (32), (35) and (38) respectively. Also, the approximate high-SNR expressions for outage probability derived in (36) and (33) are quite accurate. A decrease in $\alpha_B$ and $\alpha_R$ signifies that more power is allocated to $U_F$, and $p_{DF}^F$ therefore continuously improves with decreasing $\alpha_B$ and $\alpha_R$ as shown in Fig. 4. Clearly, from Fig. 5 $p_{DF}^N$ significantly improves with increasing $\alpha_B$ initially (due to successful decoding of $x_F$ and $x_N$ at $U_N$), while further increase in $\alpha_B$ degrades $p_{DF}^N$ ($U_N$ fails to decode $x_F$). Similar to the case of AF relaying, full diversity order of two can be attained (derived in Lemma 5) for both $U_N$ and $U_F$ in the presence of the direct link. While in the absence of direct link to $U_F$, a diversity of order 1 can be achieved (as noted in Remark 5), while diversity of 2 is achieved by $U_N$.

Fig. 6 shows the variation of $\tau_{AF}^F$ versus $\rho$ at $R = 1, 2$. Here, we compare both DF and AF relaying schemes considering optimal power allocation coefficients $\alpha_B$ and $\alpha_R$. In the presence of the $B-U_F$ link, performance with DF is superior to that of AF for lower SNRs. At high SNRs, the performance of both AF and DF is comparable. However, in the absence of $B-U_F$ link, performance with DF is
always superior. AF relays are often preferred due to their simplicity, and can be seen to result in good performance except at low SNRs. Fig. 7 depicts $\tau^N_i$ versus $\rho$ for $R = 1, 2$. For fair comparison of AF and DF relaying, optimal power allocation coefficients are used at $B$ and $R$. We observe that both AF and DF achieve almost similar performance in terms of near-user throughput.

Fig. 8: $\tau^N_F$ or $\tau^F_F$ vs. $\alpha_B$ with $\rho = 30$ dB. 

Fig. 8 depicts the variation of $\tau^N_F$ or $\tau^F_F$ versus $\alpha_B$ for $R = 2, 3$. When $\alpha_B$ is very small, more power is allocated to $U_F$ and $\tau^F_F$ therefore is maximum at $\alpha_B = 0$. With increase in $\alpha_B$, $\tau^F_F$ gradually decreases and becomes 0 for $\alpha_B \geq \frac{1}{1+\gamma_F}$. Therefore, the presence of $B-U_F$ link significantly improves the FU performance and allows a wider range of valid power allocations. On the other hand $\tau^N_F$ increases initially and attains a maximum value (at $\alpha_B = \frac{1}{1+\gamma_F}$). Further increase in $\alpha_B$ decreases the probability of successful decoding of $s_F$ at $U_N$, and $\tau^N_F$ therefore also decreases.

In Fig. 9, $\tau^N_F$ or $\tau^F_N$ is plotted versus $\alpha_R$ (considering optimal $\alpha_B$) for $R = 1.5, 2, 2.5$. As with AF relaying, $\tau^F_N$ is maximum at $\alpha_B = 0$ and then decreases to 0 with increasing $\alpha_B$. $\tau^N_N$ increases initially, attains a maximum value at $\alpha_B = \frac{\gamma_N}{\gamma_N+\gamma_F+\gamma_F^2}$, and then decreases to 0 (the SINR to decode $s_F$ at $NU$ decreases). It can be observed that presence of the $B-U_F$ significantly improves the FU throughput. Also, both $\tau^F_F$ and $\tau^N_N$ change significantly with $\alpha_B$ while change in $\alpha_R$ does not significantly change the throughput. Hence, an optimal choice of $\alpha_B$ is important. For this reason, we study variation of energy efficiency w.r.t $\alpha_B$ only. From Fig. 8 and Fig. 9 it can be clearly seen that CDRT NOMA with AF relaying provides wider range of valid power

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3 Although DF relay R offers additional degree of freedom in terms of $\alpha_R$ (power allocation at R), AF achieves performance almost similar to it. This is because in CDRT NOMA with DF relaying, power allocation at R takes place if and only if R decodes both FU and NU symbols successfully.

4 Since transmission from R depends on successful decoding of $s_F$, power allocation at S is crucial. Also at R power allocation is required only if both $s_F$ and $s_N$ get successfully decoded, an optimal choice of $\alpha_R$ is therefore more important in comparison to $\alpha_B$. 


allocations in comparison to DF relaying for both UN and UF. The presence of the $B-U_F$ widens the valid range of power allocations for UF with AF relaying (but not with DF relaying).

In Fig[10] we plot sum throughput $\tau_{\text{sum}}^i = \tau_F^i + \tau_N^i$ versus $R = R_N = R_F$ using optimal $\alpha_B$ and $\alpha_R$. With increase in SNR $\tau_{\text{sum}}^i$ also increases and $\tau_{\text{sum}}^i$ is a quasi-concave function of $R$, and a unique value $R_{\text{opt}}^p$ exists at which $\tau_{\text{sum}}^i$ is maximum. Clearly, CDRT NOMA with DF outperforms that with AF in terms of sum throughput.

Fig[11] illustrates the effect of transmit power ($P_B = P_R = P$) on the energy efficiency of AF or DF relay-aided CDRT NOMA. It can be observed that at $\rho = 30$ dB and $R = 2$, with $P = 2W$ $\eta_{E}^{AF} = 0.955$ and $\eta_{E}^{DF} = 0.945$, while at $P = 3W$, $\eta_{E}^{AF} = 0.637$ and $\eta_{E}^{DF} = 0.630$. Thus, EE improves with decreasing $P$. In addition to this we observe that for mid-SNR range DF outperforms AF, while at low as well as at high SNRs both achieve similar performance in terms of EE. However, in the absence of the $B-U_F$ link, DF outperforms AF for a wider range of SNRs and the gap is more as compared to the case when the $B-U_F$ link is present.

In Fig[12] we plot $\eta_{E}^{AF}$ and $\eta_{E}^{DF}$ versus $\alpha_B$ with $R = 1$ and $\rho = 30$ dB for $P = 1W$ and $P = 2W$. We also compare the EE of NOMA CDRT with its relayed OMA counterpart. It is evident that EE of both AF and DF relay-aided NOMA in CDRT are higher than that of relayed OMA. This is largely because of the fact that the signals from both B and R are combined at NU as well as FU. It can be seen that in the OMA case, EE with DF is better than that with AF, whereas in CDRT NOMA, AF is more energy efficient. We also show that the presence of the $B-U_F$ link significantly improves the EE (the gap is larger with AF as compared to DF).

In Fig[13] we plot the variation of $\eta_{E}^{AF}$ and $\eta_{E}^{DF}$ versus $\alpha_B$ for fixed transmit power ($P = 3W$), $\rho = 30$ dB and $R = 1,2,R_{\text{opt}}^p$. Clearly, the energy efficiencies of AF as well as DF increase as we increase the

![Fig. 10: $\tau_{\text{sum}}^i$ or $\tau_{\text{sum}}^{DF}$ vs. $R$.](image1.png)

![Fig. 11: $\eta_{E}^{AF}$ or $\eta_{E}^{DF}$ vs. $\rho$.](image2.png)

![Fig. 12: $\eta_{E}^{AF}$ or $\eta_{E}^{DF}$ vs. $\alpha_B$.](image3.png)
symbol rate from 1 to 2, and the improvement is huge when the optimal symbol rate ($R_{opt}^F$) is used. Further, this is true for the entire range of the power allocation parameter. We also observe that EE of relay-aided NOMA CDRT is significantly higher than that of OMA.

Fig. 14 and Fig. 15 depict $\tau^AF_N$ and $\tau^DF_N$ versus $R_N$ for NOMA in CDRT with AF and DF relaying, respectively for different $\tau_F = \tau$ and $\rho = 25$ dB. It can be seen that $\tau_N$ increases with decreasing $\tau$. In case of both DF and AF aided NOMA in CDRT, we observe that by optimally choosing $R_N$, the NU throughput can be maximized while guaranteeing the desired FU throughput $\tau$. Also, selecting an optimal rate ($R_F = R_{opt}^F$) helps to provide huge gain in NU throughput in comparison to any fixed rate $R_F$. Therefore, the NU throughput attains a maximum performance at $(R_{opt}^N, R_{opt}^F)$. Clearly, jointly optimizing $R_N$ and $R_F$ (as in Section IV) is of utmost importance. It can also be observed that presence of the $B - U_F$ significantly improves the NU performance.

In Fig. 16, Fig. 17 and Fig. 18 we compare the performance of proposed AF as well as DF relay-aided...
CDRT NOMA with selective decode and forward (SDF) CDRT NOMA of [26] in terms of outage probability, sum throughput and energy efficiency, respectively. It is clear from Fig. [16] that for optimal power allocation, the proposed AF as well as DF relay-aided CDRT NOMA outperform the SDF CDRT NOMA framework of [26] by a huge margin. The SDF CDRT NOMA achieves diversity order of one, whereas, proposed schemes can achieve a diversity of two at both the users. It is clear from Fig. [17] that the proposed scheme almost doubles the sum throughput in comparison to SDF-CDRT NOMA framework of [26]. Fig. [18] illustrates that the proposed scheme is also more energy efficient.

VI. Conclusion

In this paper, we analyzed the performance of new framework for downlink non-orthogonal multiple access (NOMA) in a coordinated direct and relay transmission (CDRT) scheme with direct link to both near-user (NU) and far-user (FU). In this framework, NU performs combining of direct and relayed signals while performing successive interference cancellation, which considerably improves NU outage performance, and enables it to attain a diversity of two without feedback. This scheme is very useful in multimedia and other applications where the NU has stringent outage constraints. The FU also can attain a diversity of two. Considering either amplify-and-forward (AF) or decode-and-forward (DF) relaying, closed-form expressions for outage probability and throughput were derived for both NU and FU. In spite of the fact that DF relaying allows for different choice of power allocation at the relay, performance with simpler AF relays is comparable to that with DF relays. We demonstrate that careful choice of power allocations coefficient and the the target rates is important to maximize throughput and energy efficiency.

Appendix A

Proof of Theorem 1: Substituting \( \zeta_{\rho_1}(w) \) from line following (13) into (17), \( I_1 \) can be expressed as

\[
I_1 = \int_0^{\chi_F} \lambda w \exp \left( \frac{-\xi_1(w - \chi_F)(\lambda x \rho_R + \lambda y \rho_B)}{\rho_R (w - \xi_1)} \right) \frac{-2\xi_1(w - \chi_F)\lambda x \lambda y}{\rho_R (w - \xi_1)} K_1 \left( \frac{-2\xi_1(w - \chi_F)\lambda x \lambda y}{\rho_R (w - \xi_1)} \right) e^{-\lambda w w} dw. \tag{49}
\]

For large values of \( \rho_B \) and \( \rho_R \), the argument inside the Bessel function becomes very small throughout the range of SNRs (as verified in Section V). We therefore use \( K_1(\theta) \approx \frac{1}{2} \Gamma(1)(\frac{\theta}{2})^{-1} \) \[32, 9.6.9\]. Using \( \lambda w (w - \xi_1) = w \) we obtain

\[
I_1 \approx e^{-\xi_1(w + \frac{1}{\rho_R}) \lambda w (\chi_F - \xi_1)} \int_{-\xi_1 \lambda w} \exp \left( -w - \frac{\lambda w \xi_1 (\xi_1 - \chi_F)(\lambda x \rho_R + \lambda y \rho_B)}{\rho_R w} \right) dw. \tag{50}
\]

Using \( \int_a^b e^{-t} dt = \Gamma(a, x; b) \) \[33, eq. (4)\] in the above, we obtain
where $\vartheta_6 = \frac{\lambda_w \xi_1 - \chi_F}{\rho_R}$. Similarly, we solve for $I_2$ as

$$I_2 \simeq e^{-\xi_1(\lambda_w + \frac{1}{R})} \int_0^\infty \exp\left(-w - \frac{\lambda_w \xi_1 - \chi_F}{\rho_R} \right) dW.$$  

Using $\int_0^\infty \exp(-\frac{b}{4t} - at) dt = \sqrt{\frac{b}{a}} K_1(\sqrt{ab})$ [31, 3.324.1] in the above, we obtain

$$I_2 \simeq e^{-\xi_1(\lambda_w + \frac{1}{R})} \sqrt{\frac{b}{a}} e^{\frac{1}{2} K_1(\sqrt{ab})}.$$  

After substituting $I_1$ and $I_2$ from (51) and (53) into (14), we obtain (19).

**APPENDIX B**

**Proof of Lemma 1** After some mathematical arrangements, (16) can be expressed as

$$I_1 = \int \int_{0}^{\chi_F} \exp\left(-\xi_{F1}(w)\left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right) \right) \int_{0}^{\infty} \exp\left(-\lambda_x x\right) \frac{\lambda_w e^{-\lambda_w w} dx}{x} dx,$$

For large $\rho_B$ and $\rho_R$, we use the linear approximation to the exponential term in denominator to obtain

$$I_1 \simeq \int \int_{0}^{\chi_F} \exp\left(-\xi_{F1}(w)\left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right) \right) \int_{0}^{\infty} \lambda_x e^{-\lambda_x x} dx - \xi_2(w) \int_{0}^{\infty} \frac{\lambda_x e^{-\lambda_x x}}{x + \xi_2(w)} dx \lambda_w e^{-\lambda_w w} dw,$$

where $\xi_2(w) \simeq \frac{\lambda_x \xi_{F1}(w)[1 + \rho_B \xi_{F1}(w)]}{\rho_B}$. Using [31, 3.352.6] in the above, we obtain

$$I_1 \simeq \int \int_{0}^{\chi_F} \exp\left(-\xi_{F1}(w)\left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right) \right) \left(1 - \xi_2(w)\lambda_x e^{\xi_2(w)\lambda_x} E_1(\xi_2(w)\lambda_x)\right) \lambda_w e^{-\lambda_w w} dw.$$  

Further using $e^{\xi_2(w)\lambda_x} E_1(\xi_2(w)\lambda_x) = \frac{1}{1 + \xi_2(w)\lambda_x}$ [32, 5.1.19] and simplifying, we obtain

$$I_1 \simeq \int \int_{0}^{\chi_F} \frac{\lambda_w e^{-\lambda_w w}}{1 + \xi_2(w)\lambda_x} \exp\left(-\xi_{F1}(w)\left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right) \right) dw.$$  

Substituting for $\xi_{F1}(w)$ from the line following (18) and neglecting higher-order terms of $1/\rho_B$, we get

$$I_1 \simeq 1 - e^{-\lambda_w \chi_F} - \left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right) \xi_1 \left[ \int_{0}^{\chi_F} \lambda_w e^{-\lambda_w w} dw + (\xi_1 - \chi_F) \lambda_w \int_{0}^{\chi_F} \frac{\lambda_w e^{-\lambda_w w}}{w - \xi_1} dw \right].$$  

We use [31, 3.352.1] to obtain

$$I_1 \simeq 1 - e^{-\lambda_w \chi_F} - \left(\lambda_x + \frac{\lambda_y \rho_B}{\rho_R}\right) \xi_1 \left[ 1 - e^{-\lambda_w \chi_F} + (\xi_1 - \chi_F) \lambda_w e^{-\lambda_w \xi_1} \left[ E_1(\lambda_w, \xi_1) - E_1(-\lambda_w \xi_1 + \lambda_w \chi_F) \right] \right].$$
Similarly, the expression for $I_2$ is given by

$$I_2 \simeq \frac{e^{-\lambda_x \xi_1}}{1 + \xi_1 (\lambda_x + \lambda_z)}.$$  \hspace{1cm} (60)

After substituting for $I_1$ and $I_2$ from (59) and (60) into (14), the high SNR approximation to $p_F^{AF}$ is given by (20).

**APPENDIX C**

*Proof of Theorem* \[^3\] Using (25), $p_F^{AF} = 1 - (I_5 + I_3)U(\phi) - I_4U(\phi - \alpha_b) - I_6[U(\phi) - U(\phi - \alpha_b)] - I_7U(-\phi)$. We first solve for $I_4 = \Pr \{V > \xi_{N_1}(X,Z), X > \chi_{N_1}(Z), \chi_f < Z < \chi_N \}$ as follows:

$$I_4 = \int_0^\chi_N \int_{\chi_f N_1(z)}^{\chi_N} f_Y(v)f_X(x)f_Z(z)dvdxdz = \int_0^\chi_N \int_{\chi_f N_1(z)}^{\chi_N} \lambda_x \lambda_z \exp \left(-\lambda_x \xi_{N_1}(x,z) - \lambda_z x - \lambda_z z \right) dx dz. \hspace{1cm} (61)

Substituting $x = x - \chi_{N_1}(z)$ in the above and rearranging, we obtain

$$I_4 = \int_0^\chi_N \int_{\chi_f N_1(z)}^{\chi_N} \exp \left(-\chi_{N_1}(z) \left[\lambda_x + \frac{\lambda_z \rho_z}{\rho_R} \right] \right) \frac{\lambda_x e^{-\lambda_x \chi_{N_1}(z)}}{\lambda_x e^{-\lambda_z \chi_{N_1}(z)}} dx dz; \hspace{1cm} (62)

where $\mu_{N_1}(z) = \frac{\lambda_x \lambda_z \xi_{N_1}(z) \rho_z (\chi_{N_1}(z) + 1)}{\rho_R}$. Solving the above using \[^{[31]}\ 3.324.1\], we obtain

$$I_4 = \int_0^\chi_N e^{-\chi_{N_1}(z)} \left(\lambda_x + \frac{\lambda_z \rho_z}{\rho_R} \right) 2 \mu_{N_1}(z) K_1(2 \mu_{N_1}(z)) \frac{\lambda_x e^{-\lambda_z \chi_{N_1}(z)}}{\lambda_x e^{-\lambda_z \chi_{N_1}(z)}} dz. \hspace{1cm} (63)

The above integral does not admit a closed form. To derive a highly accurate closed-form expression (as with $I_1$) we use $K_1(\theta) \simeq \frac{1}{2} \Gamma(1) \left(\frac{\theta}{2} \right)^{-1} \left[ \frac{3}{2} \right.$ \[^{[32]}\] 9.6.9] to obtain

$$I_4 \simeq \lambda_x e^{-C/\chi_N} \int_0^\chi_N e^{Cz} dz, \hspace{1cm} (64)

where $C = C_\ell - \lambda_z$ with $C_\ell = \lambda_x + \frac{\lambda_z \rho_z}{\rho_R}$. After solving the above integral, $I_4$ is obtained as

$$I_4 \simeq \lambda_x e^{-\chi_{N_1}(z)} \left[ \frac{1 - e^{C(\chi_{N_1} - \chi_N)}}{C} \right]. \hspace{1cm} (65)

Using a similar approach, $I_5$, $I_6$ and $I_7$ are obtained as

$$I_5 \simeq \lambda_x e^{-\chi_{N_1}(z)} \left[ e^{C(\chi_{N_1} - \chi_N)} - e^{-C\chi_N} \right], \hspace{1cm} (66)

I_6 \simeq \lambda_x e^{-\chi_{N_1}(z)} \left[ e^{-(\xi_1 + \gamma_{N_1})} e^{-(\xi_1 + \chi_N) \lambda_x \theta_2} e^{\lambda_z \theta_2} \left[ E_1 \left( \lambda_z (\chi_N + \xi_1 + \theta_2) \right) - E_1 \left( \lambda_z (\chi_N + \xi_1 + \theta_2) \right) \right] \right], \hspace{1cm} (67)

and $I_7 \simeq \lambda_x e^{-\chi_{N_1}(z)} \left[ e^{-2\xi_1} - \lambda_z \theta_2 e^{\lambda_z \theta_2} \left[ E_1 \left( \lambda_z (2\xi_1 + \theta_2) \right) \right] \right], \hspace{1cm} (68)$

respectively, where $\theta_2 = \xi_1 \left(\xi_1 + \frac{\chi_{N_1}}{\rho_R \theta} \right) C_\ell$. Finally, $I_3$ can be derived as
Since the RVs \( I_3 = \Pr \{ Z > \max(\chi_F, \chi_N) \} = e^{-\lambda \max(\chi_F, \chi_N)}. \) (69)

After substituting the derived expressions of \( I_3, I_4, I_5, I_6 \) and \( I_7 \) from (69), (65), (66), (67) and (68), respectively, into (25), \( p_{AF}^N \) is expressed as in (28).

**APPENDIX D**

**Proof of Theorem 4** Using (3), (8) and (10) into (31), \( \Pr \{ A_1 \} \) can be expressed as

\[
\Pr \{ A_1 \} = \Pr \left\{ \frac{(1 - \alpha_R)p_B X}{\alpha_B p_B X + 1} \geq \gamma_F, \alpha_B p_B X \geq \gamma_N, \frac{(1 - \alpha_R)p_B Y}{\alpha_B p_B Y + 1} + \frac{(1 - \alpha_R)p_B W}{\alpha_B p_B W + 1} \geq \gamma_F \right\}
\]

\[
= \Pr \{ X \geq \max(\chi_F, \chi_N) \} \left[ \Pr \{ W \geq \chi_F \} + \Pr \{ \chi_F > W \geq \max \left( 0, \frac{-\phi_R}{\rho_B (\alpha_B (1 - \alpha_R) + \alpha_R \phi)} \right) \right]
\]

\[
Y \geq \frac{(\gamma_F - \phi_B W)}{\rho_B (\phi_B + (1 - \alpha_R)\alpha_B p_B W + \alpha_R \phi_B W)} \right\}. \tag{70}
\]

Since the RVs \( W, X \) and \( Y \) are independent and have an exponential distribution, we obtain

\[
\Pr \{ A_1 \} = e^{-\lambda \max(\chi_F, \chi_N)} \left[ e^{-\lambda \chi_F} + \int_{\max(0, -\phi_B \gamma_F/R)}^{\chi_F} \lambda \exp \left( \frac{\lambda \phi_B (w - \chi_F)}{\rho_B \phi_B (w + \phi_B/R)} \right) e^{-\lambda w} dw \right]. \tag{71}
\]

After substituting \( \lambda w(w + \phi_B/R) = w \) and solving, we obtain

\[
\Pr \{ A_1 \} = e^{-\lambda \max(\chi_F, \chi_N)} \left[ e^{-\lambda \chi_F} + \frac{\lambda \phi_B}{\rho_B \phi_B} \int_{\max(0, -\phi_B \gamma_F/R)}^{\chi_F} \exp \left( -\frac{\lambda \gamma \phi_B (w + \phi_B/R)}{\rho_B \phi_B (w + \phi_B/R)} \right) e^{-w} dw \right]. \tag{72}
\]

Using an approach similar to that used to derive (50), we obtain

\[
\Pr \{ A_1 \} = e^{-\lambda \max(\chi_F, \chi_N)} \left[ \exp \left( \frac{\lambda \phi_B}{\rho_B \phi_B} \right) + \exp \left( \frac{\lambda \gamma \phi_B}{\rho_B \phi_B} \right) \left\{ \Gamma \left( 1, \max \left( \frac{\lambda \phi_B}{\rho_B \phi_B}, 0 \right) \right) \right\} - \Gamma \left( 1, \frac{\lambda \phi_B}{\rho_B \phi_B} \right) \right] e^{-\lambda \chi_F}. \tag{73}
\]

Similarly, the expression for \( \Pr \{ A_2 \} \) is obtained as

\[
\Pr \{ A_2 \} = (e^{-\lambda \chi_F} - e^{-\lambda \chi_N}) \left[ \exp \left( -\frac{\lambda \gamma \phi_B}{\rho_B} \right) + \exp \left( \frac{\lambda \gamma \phi_B}{\alpha_B \rho_B} \right) \left\{ \Gamma \left( 1, \frac{\lambda \gamma \phi_B}{\alpha_B \rho_B} \right) \right\} - \Gamma \left( 1, \frac{\lambda \gamma \phi_B}{\rho_B} \right) \right] \frac{\phi_B}{\alpha_B \rho_B}. \tag{74}
\]

Further, using (3) and (8), \( \Pr \{ A_3 \} \) of (31) can be expressed as

\[
\Pr \{ A_3 \} = e^{-\lambda \chi_F} (1 - e^{-\lambda \chi_F}). \tag{75}
\]

Using (73), (74) and (75) in (31), \( p_{DF}^F \) is obtained as in (32).
APPENDIX E

Proof of Lemma 3: For higher values of $\rho_B$ and $\rho_R$, we use $e^{-\frac{a}{x}} = \frac{1}{1 + \frac{a}{x}}$ in (72) to obtain

$$
\Pr\{A_1\} \simeq e^{-\lambda_x \max(\chi_F, \chi_N)} \left[ e^{-\lambda_x \chi_F} + \exp\left(\frac{\lambda_w \phi_R}{\gamma_1} + \frac{\lambda_y \phi_B}{\gamma_1}\right) \int_{\max(\frac{\lambda_w \phi_R}{\phi_1},0)}^{\lambda_x(\chi_F + \phi_R)} \left(1 - \frac{\kappa}{w + \kappa} \right) e^{-w} d\kappa \right],
$$

(76)

where $\kappa = \frac{\lambda_w \phi_R}{\gamma_1} + \chi_F$. Solving the above integral using [31, 3.352.3], we obtain

$$
\Pr\{A_1\} \simeq e^{-\lambda_x \max(\chi_F, \chi_N)} \left[ \exp\left(\frac{\lambda_w \phi_R}{\gamma_1} + \frac{\lambda_y \phi_B}{\gamma_1}\right) \right] \left[ \left(1 - \frac{\kappa}{\max(\frac{\lambda_w \phi_R}{\phi_1},0)} \right) e^{-\lambda_x \chi_F} \right].
$$

(77)

At high SNRs, $\kappa \propto \frac{1}{\rho_B}$. We therefore neglect the product term of $\kappa \chi_F$. Now $\Pr\{A_1\}$ is expressed as

$$
\Pr\{A_1\} \simeq e^{-\lambda_x \max(\chi_F, \chi_N)} \left[ \exp\left(\frac{\lambda_w \phi_R}{\gamma_1} + \frac{\lambda_y \phi_B}{\gamma_1} - \max(\frac{\lambda_w \phi_R}{\phi_1},0)\right) + e^{-\lambda_x \chi_F} \left(1 - \frac{\kappa}{\rho_B} \right) \right].
$$

(78)

Similarly, solving for $A_2$, we obtain

$$
\Pr\{A_2\} \simeq \left( e^{-\lambda_x \chi_F} - e^{-\lambda_x \chi_N} \right) \left[ \exp\left(\frac{\lambda_w \phi_R}{\gamma_1} + \frac{\lambda_y \phi_B}{\gamma_1}\right) \right] \left[ (1 - \frac{\kappa}{\rho_B}) \left(1 - \frac{\lambda_x \chi_F}{\rho_B} \right) \right].
$$

(79)

Substituting for $\Pr\{A_1\}$ and $\Pr\{A_2\}$ from (77) and (79) into (31), we obtain $p_f^{DF}$ given by (33).

APPENDIX F

Proof of Theorem 5: Using (2) and (8) in (37), $\Pr\{B_1\}$ can be expressed as

$$
\Pr\{B_1\} = \Pr\left\{ \Gamma_R^I < \gamma_F, \Gamma_{NF}^I \geq \gamma_F, \Gamma_{NN}^I \geq \gamma_N \right\} = \Pr\left\{ \frac{(1 - \alpha_B) \rho_B X}{B} \leq \gamma_F, \frac{(1 - \alpha_B) \rho_B Z}{B} \geq \gamma_N \right\}
$$

(80)

Using (37), $\Pr\{B_2\}$ is given by

$$
\Pr\{B_2\} = \Pr\left\{ \Gamma_R^I \geq \gamma_F, \Gamma_{RN}^I < \gamma_N, \Gamma_{COM}^I \geq \gamma_F, \Gamma_{NN}^I \geq \gamma_N \right\}
$$

(81)

To satisfy $\chi_N > \chi_F$, $\max\left(\chi_N, \frac{\gamma_F - \rho_B V}{\rho_B + \alpha_B V}\right) = \chi_N$. Using this fact, the above can be simplified as

$$
\Pr\{B_2\} = \Pr\left\{ \chi_F < \chi_N, Z \geq \chi_N \right\} = e^{-\lambda_x \chi_N} \left( e^{-\lambda_x \chi_N} - e^{-\lambda_x \chi_N} \right) U\left( \phi - \frac{\alpha_B \gamma_F}{\gamma_N} \right).
$$

(82)

Further, the expression for $\Pr\{B_3\}$ is calculated as
Using the above, the conditions for outage on ranges of $V, X$ and $Z$ with $\phi > 0$ and $\phi < 0$ are listed in Table III. Now, $\Pr\{B_3\}$ can be expressed as

$$
\Pr\{B_3\} = \Pr\left\{ \begin{array}{l}
\Gamma_{RF}^t \geq \gamma_f, \Gamma_{RN}^t \geq \gamma_N, \Gamma_{NF}^t \geq \gamma_f, \Gamma_{NN}^t \geq \gamma_N \end{array} \right\} \\
= \Pr\left\{ \begin{array}{l}
X > \max(\chi_F, \chi_N), \rho_R V \left( 1 - \alpha_B \right) (1 + \alpha_B \rho_B Z) - \alpha_R (\gamma_f - \phi \rho_B Z) > (\gamma_f - \phi \rho_B Z), V \geq \frac{\gamma_N - \alpha_B \rho_B Z}{\alpha_R \rho_R}, z < \chi_N \end{array} \right\},
$$

(83)

Using the above, the conditions for outage on ranges of $V, X$ and $Z$ with $\phi > 0$ and $\phi < 0$ are listed in Table III. Now, $\Pr\{B_3\}$ can be expressed as

$$
\Pr\{B_3\} = \Pr\left\{ \begin{array}{l}
X > \max(\chi_F, \chi_N), \rho_R V \left( 1 - \alpha_B \right) (1 + \alpha_B \rho_B Z) - \alpha_R (\gamma_f - \phi \rho_B Z) > (\gamma_f - \phi \rho_B Z), V \geq \frac{\gamma_N - \alpha_B \rho_B Z}{\alpha_R \rho_R}, z < \chi_N \end{array} \right\} \\
= \Pr\left\{ \begin{array}{l}
X > \max(\chi_F, \chi_N), \rho_R V \left( 1 - \alpha_B \right) (1 + \alpha_B \rho_B Z) - \alpha_R (\gamma_f - \phi \rho_B Z) > (\gamma_f - \phi \rho_B Z), V \geq \frac{\gamma_N - \alpha_B \rho_B Z}{\alpha_R \rho_R}, z < \chi_N \end{array} \right\}.
$$

(84)

where $\chi_N^0(Z) = \frac{\gamma_f - \phi \rho_B Z}{\rho_R (\phi_R + \rho_B Z (\alpha_R + \alpha_B (1 - \alpha_B)))}$ and $\chi_N^1(Z) = \frac{\gamma_f - \alpha_B \rho_B Z}{\alpha_R \rho_R}$. After averaging over the PDFs of RVs $V, X$ and $Z$, we obtain

$$
\Pr\{B_3\} = e^{-\lambda_z \max(\chi_F, \chi_N)} \left[ e^{-\lambda_z \max(\chi_F, \chi_N)} + \int_{\chi_F}^{\chi_N} \frac{e^{-\lambda_N (\gamma_f - \alpha_B \rho_B z)}}{\alpha_R \rho_R} \lambda_z e^{-\lambda_z z} dz + \int_{\max(\chi_N, -\frac{\phi_B}{\alpha_R})}^{\min(\chi_F, \chi_N)} \frac{e^{-\lambda_N (\gamma_f - \phi \rho_B z)}}{\rho_R (\phi_R + \rho_B \omega)} \lambda_z e^{-\lambda_z z} dz \right],
$$

(85)

where $\omega = \frac{-\rho_R (\phi_R + \rho_B z (\alpha_R + \alpha_B (1 - \alpha_B)))}{\rho_R (\phi_R + \rho_B z (\alpha_R + \alpha_B (1 - \alpha_B)))}$, $\chi_N = \min(\chi_F, \chi_N, \phi_3)$. After some mathematical rearrangement, the above can be expressed as

$$
\Pr\{B_3\} = e^{-\lambda_z \max(\chi_F, \chi_N)} \left[ e^{-\lambda_z \max(\chi_F, \chi_N)} + \int_{\chi_F}^{\chi_N} \frac{e^{-\lambda_z z}}{\alpha_B \rho_B} \lambda_z e^{-\lambda_z z} dz + \int_{\max(\chi_N, -\frac{\phi_B}{\alpha_R})}^{\min(\chi_F, \chi_N)} \frac{e^{-\lambda_z z}}{\rho_R \phi_1} \lambda_z e^{-\lambda_z z} dz \right].
$$

(86)
Substituting (80), (82) and (89) into (37), $I_8$ and $I_9$ are obtained as

$$I_8 = \exp \left( \frac{\phi_R \lambda_c - \rho_R + \lambda_c \phi_R}{\rho_R \vartheta_1} \right) \left[ \Gamma \left( 1, \lambda_c \max(0, \chi_N + \frac{\phi_R}{\vartheta_1}) \chi_F \right) - \Gamma \left( 1, \lambda_c \max(0, \chi_N + \frac{\phi_R}{\vartheta_1}) \chi_F \right) \right],$$  

(87)

$$I_9 = \exp \left( \frac{\phi_R \lambda_c - \rho_R + \lambda_c \phi_R}{\rho_R \vartheta_1} \right) \left[ \Gamma \left( 1, \lambda_c \max(0, \theta_4 + \frac{\phi_R}{\vartheta_1}) \chi_F \right) - \Gamma \left( 1, \lambda_c \max(0, \chi_N + \frac{\phi_R}{\vartheta_1}) \chi_F \right) \right],$$  

(88)

where $\chi_F = \frac{\gamma_c \phi_R \rho_R (\chi_F - \rho_R)}{\vartheta_1}$. Using $I_8$ and $I_9$ from (87) and (88) into (86), $\Pr \{ B_3 \}$ is expressed as

$$\Pr \{ B_3 \} = e^{-\lambda_c \max(\chi_F, \chi_N)} e^{-\lambda_c \max(\chi_F, \chi_N)} + \lambda_c e^{-\lambda_c \max(\chi_F, \chi_N)} \exp(\varepsilon_3 \alpha_B \rho_B \chi_N - \lambda_c \chi_N) - \exp(\varepsilon_3 \alpha_B \rho_B \chi_N - \lambda_c \chi_F) \frac{\exp(\varepsilon_3 \alpha_B \rho_B \chi_N)}{\exp(\max(0, \frac{\phi_R}{\vartheta_1} \lambda_c)} \frac{\exp(\varepsilon_3 \alpha_B \rho_B \chi_N)}{\exp(\max(0, \frac{\phi_R}{\vartheta_1} \lambda_c)} + \exp(\phi_R \lambda_c - \rho_R + \lambda_c \phi_R) \left[ \Gamma \left( 1, \lambda_c \max(0, \chi_N + \frac{\phi_R}{\vartheta_1}) \chi_F \right) \right] - \Gamma \left( 1, \lambda_c \max(0, \theta_4 + \frac{\phi_R}{\vartheta_1}) \chi_F \right) + \Gamma \left( 1, \lambda_c \max(0, \chi_N + \frac{\phi_R}{\vartheta_1}) \chi_F \right) - \Gamma \left( 1, \lambda_c \max(0, \chi_N + \frac{\phi_R}{\vartheta_1}) \chi_F \right).$$  

(89)

Substituting (80), (82) and (89) into (37), $p_{DF}^N$ is obtained as in (38).

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