Interferometric determination of the $s$- and $d$-wave scattering amplitudes in $^{87}\text{Rb}$

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We demonstrate an interference method to determine the low-energy elastic scattering amplitudes of a quantum gas. We linearly accelerate two ultracold atomic clouds up to energies of 1.2 mK and observe the collision halo by direct imaging in free space. From the interference between $s$- and $d$-wave scattering amplitudes in the differential scattering pattern we extract the corresponding phase shifts. The method does not require knowledge of the atomic density. This allows us to infer accurate values for the $s$- and $d$-wave scattering amplitudes from the zero-energy limit up to the first Ramsauer minimum using only the Van der Waals $C_6$ coefficient as theoretical input. For the $^{87}\text{Rb}$ triplet potential, the method reproduces the scattering length with an accuracy of 6%.

PACS numbers: 34.50.-s, 32.80.Pj, 03.75.-b, 03.65.Sq

The scattering length $a$, the elastic scattering amplitude in the zero-energy limit, is the central parameter in the theoretical description of quantum gases [1, 2, 3]. It determines the kinetic properties of these gases as well as the bosonic mean field. Its sign is decisive for the condensate as accurate

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In our experiments, we load about one billion $^{87}\text{Rb}$ atoms in the (fully stretched) $|F=2, m_F=2\rangle$ hyperfine level of the electronic ground state from a magneto-optical trap (MOT) into a Ioffe-Pritchard quadrupole trap ($21 \times 477\text{Hz}$) with an offset field of $B_0 = +0.9\text{G}$. We pre-cool the sample to about 6 $\mu\text{K}$ using forced radio-frequency (RF) evaporation. The cloud is split in two by applying a rotating magnetic field and ramping $B_0$ down to a negative value $B_0^{-}$. This results in two Time-averaged Orbiting Potential (TOP) traps loaded with atoms [17]. By RF-evaporative cooling we reach Bose-Einstein condensation with about $10^5$ atoms in each...
cloud and a condensate fraction of ~60%.

We then switch off the TOP fields and ramp $B_0$ back to positive values, thus accelerating the clouds until they collide with opposite horizontal momenta at the location of the trap center. The collision energies $E = 2\mu_B B_0^2 = \frac{kT^2}{m}$ (with $\mu_B$ the Bohr magneton and $m$ the mass of $^{87}$Rb) range from 138 μK to 1.23 mK with an overall uncertainty of 3% (RMS). Approximately 0.5 ms before the collision we switch off the trap. A few ms later we observe the scattering halo by absorption imaging. Fig. 1a (upper part) displays the $s$-wave-dominated scattering halo (averaged over 20 pictures) of fully entangled pairs (see 1) obtained for a collision energy of $E/k_B = 138(4)$ μK. In Fig. 2a (upper part), taken at $E/k_B = 1.23(4)$ mK the halo is entirely different, showing a $d$-wave-dominated pattern. The lower halves of Fig. 1 and Fig. 2 show the theoretical column densities $n_2(x, z) = \int n(x, y, z) dy$, where $n(x, y, z)$ is the calculated density of the halo.

As the atoms are scattered by a central field, the scattering pattern must be axially symmetric around the (horizontal) scattering axis (z-axis). As pointed out by the Weizmann group, this allows a computerized tomography transformation to reconstruct the radial density distribution of the halo in cylindrical coordinates,

$$n(\rho, z) = \frac{1}{4\pi} \int_{-\infty}^{\infty} \tilde{n}_2(\kappa_x, z) J_0(\kappa_x \rho) |\kappa_x| d\kappa_x.$$  \hspace{1cm} (1)

Here $\rho = (x^2 + y^2)^{1/2}$, $\tilde{n}_2(\kappa_x, z)$ is the 1D Fourier transform along the $x$-direction of the optical density with respect to $z$, and $J_0(\rho)$ is the zero-order Bessel function.

The transformed plots corresponding to the images of Fig. 1a,d are shown as Fig. 1b,e respectively.

To obtain the angular scattering distribution $W(\theta)$ the tomography pictures are binned in 40 discrete angular sectors. For gas clouds much smaller than the diameter of the halo, $W(\theta)$ is directly proportional to the differential cross section $\sigma(\theta) = 2\pi |f(\theta) + f(\pi - \theta)|^2$. Here, the Bose-symmetrized scattering amplitude is given by a summation over the even partial waves, $f(\theta) + f(\pi - \theta) = (2/k) \sum_{l=\text{even}}(2l+1) \exp (i\eta_l) \sin \eta_l$. Note that unlike in the total elastic cross section $\sigma = \int d^2 \theta \sigma(\theta)$, the interference between the partial waves is prominent in the differential cross section. Given the small collision energy in our experiments, only the $s$- and $d$-wave scattering amplitudes contribute, $f_s(\theta) + f_d(\pi - \theta) = (2/k) \exp (i\eta_s) \sin \eta_s$ and $f_d(\theta) + f_d(\pi - \theta) = (2/k)(5/2) \exp (3\cos^2 \theta - 1) \sin \eta_d$. Therefore the differential cross section is given by

$$\sigma(\theta) = \frac{8\pi}{k^2} \sin^2 \eta_s \left[ 1 + 5 \cos(\eta_s - \eta_d) u + \frac{25}{4} u^2 \right],$$  \hspace{1cm} (2)

where $u \equiv (\sin \eta_d / \sin \eta_s)(3 \cos^2 \theta - 1)$.

To obtain the phase shifts, we plot the measured angular distribution $W(\theta)$ as a function of $3 \cos^2 \theta - 1$ as suggested by Eq. 2. The results for Fig. 1a and Fig. 1d are shown as the solid dots in Fig. 1c and Fig. 1f, respectively. A parabolic fit to $W(\theta)$ directly yields a pair $(\eta^{\text{exp}}_s(k), \eta^{\text{exp}}_d(k))$ of asymptotic phase shifts (defined modulo $\pi$) corresponding to the two partial waves involved. The absolute value of $W(\theta)$ depends on quantities that are hard to measure accurately (like the atom number) so we leave it out of consideration. We rather emphasize that the measurement of the phase shifts is a complete determination of the (complex) $s$- and $d$-wave scattering amplitudes at a given energy.

The radial wavefunctions corresponding to scattering at different (low) collision energies and different (low) angular momenta should all be in phase at small interatomic distances. This so-called accumulated phase common to all low-energy wavefunctions can be extracted from the full data set $\{\eta^{\text{exp}}_s(k), \eta^{\text{exp}}_d(k)\}$ mentioned above. In practice, we use the experimental phase shifts $\eta^{\text{exp}}_s(k)$ and $\eta^{\text{exp}}_d(k)$ as boundary conditions to integrate inwards - for given $E$ and $l$ - the Schrödinger equation $h^2 d^2 \chi(r)/dr^2 + p^2 \chi(r) = 0$, and obtain the radial wavefunctions $\chi(r)/r$ down to radius
of $k$ outwards and compute uncertainty.

condition on the optimization of $\Phi$ for individual images. The full black lines is calculated from the accumulated phase $\Phi_{\text{opt}}$ optimized from all data points. The grey lines show the influence of the uncertainty of $\pm \pi \times 0.025$ on $\Phi_{\text{opt}}$. (The vertical dotted line indicates the condition $\eta_0 = \eta_2$). $s-d$ interference is only observed in the gray areas. The first $s$-wave Ramsauer-Townsend minimum is found at $E_{RT} = 2.1(2)$ mK.

$\eta_{\text{exp}}(k_i)$ obtained from the parabolic fit of $W(\theta)$ from individual images. The three open circles correspond to measurements for which the sign of the phase shifts could not be established. Refinements to the present data analysis may include the occurrence of multiple scattering as well as the influence of the spatial extension of the colliding clouds taking into account the non condensed fraction.

Knowing the phase shifts, we can infer all the low-energy scattering properties. Our results for the elastic scattering cross section are shown in Fig. 3. The (asymmetric) $d$-wave resonance emerges pronouncedly at 300(70) $\mu$K with an approximate width of 150 $\mu$K (FWHM). Most importantly, the scattering length follows from the $k \rightarrow 0$ limiting behavior, $\eta_0(k \rightarrow 0) = -ka$. We find $a = +102(6) a_0$, whereas the state-of-the-art value is $a = 98.99(2) a_0$.  

Comparison with the precision determinations shows that our method readily yields fairly accurate results, relying only on input of the $C_6$ coefficient. We used the value $C_6 = 4.698(4) \times 10^3$ a.u. In the present case, one does not need to know $C_6$ to this accuracy. Increasing $C_6$ by 10% results in a 1 %-change of the scattering length. Clearly, the systematic error in $\Phi_{\text{opt}}$ accumulated by integrating the Schrödinger equation inward with a wrong $C_6$ largely cancels when integrating back outward. However, in the case of a $s$-wave resonance other atomic species may reveal a stronger influence of $C_6$ on the calculated scattering length. Simple numerical simulations show that the value of $C_6$ becomes critical only when the (virtual) least-bound state in the interaction potential has an extremely small (virtual) binding energy (less than $10^{-2}$ level spacing). Hence our method...
should remain accurate in almost any case.

This method can therefore be applied to other bosonic or fermionic atomic species, provided the gases can be cooled and accelerated in such a way that the lowest-order partial-wave interference can be observed with good energy resolution. We speculate that the accuracy of the method can be strongly improved by turning to smaller optical-density clouds and fluorescence detection. It will enable higher collision energies and observation of higher-order partial-wave interference. The use of more dilute clouds and longer expansion times will also eliminate multiple-scattering effects and finite-size convolution broadening of the interference pattern. Finally it will enable precision measurements of the scattered fraction, which in the case of $^{87}$Rb will allow us to pinpoint the location of the $d$-wave resonance to an accuracy of 10 $\mu$K or better. In combination with state-of-the-art theory such improvements are likely to turn our approach into a true precision method.

Similar experiments were reported during the final stage of completion of this Letter [27].

The authors acknowledge valuable discussions with S. Kokkelmans, D. Petrov, G. Shlyapnikov, S. Gensemer and B. Verhaar. This work is part of the research programme of the ‘Stichting voor Fundamenteel Onderzoek der Materie (FOM), supported by the ‘Nederlandse organisatie voor Wetenschappelijk Onderzoek (NWO)’. JL acknowledges support from a Marie Curie Intra-European Fellowship (MEIF-CT-2003-501578).

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[21] This procedure breaks down in the marginal case $\eta_0 = \eta_2$, where the expression in the square brackets in Eq. 2 becomes phase-shift independent.

[22] We checked that, within the range $15 a_0 < r < 25 a_0$, the exact choice of the inner radius of the integration interval is of no influence for the results presented in this Letter.

[23] The determination of $\Phi_{\text{opt}}$ is done by minimizing $\left(\sum_{i=1}^{N} \sum_{x=0}^{2} (\eta^{\exp}_x(k_i) - \eta^{\text{opt}}_x(k_i))^2\right)^{1/2}$, where $N$ is the number of data points, and the $\eta^{\text{opt}}_x(k_i)$ are the phase shifts computed from $\Phi_{\text{opt}}$.

[24] The $g$-wave ($\ell = 4$) phase shift can also be computed from the same $\Phi_{\text{opt}}$, but the error made by assuming a constant accumulated phase increases like $(l+1)$, and the resulting $g$-wave would be accordingly less accurate.

[25] Ramsauer-Townsend minima are observed around collision energies where a phase shift crosses zero (see Fig. 4).

[26] Since Eq. 2 is unchanged under reversal of the sign of $\eta_0^{\exp}(k)$ and $\eta_2^{\exp}(k)$, the result of the parabolic fit is not unique. We eliminate this ambiguity by comparing the difference in accumulated phases obtained for $\eta_0^{\exp}(k)$ and $\eta_2^{\exp}(k)$ with that obtained for $-\eta_0^{\exp}(k)$ and $-\eta_2^{\exp}(k)$. We find that in almost all cases, this difference - which should be vanishingly small - is much larger for one choice of signs than for the other. We conclude that the correct sign for a pair $(\eta_0^{\exp}(k), \eta_2^{\exp}(k))$ is the one for which the difference between the accumulated phases is the smallest. In all but three cases (at the same collision energy), this criterion is conclusive. These three measurements (the open circles in Fig. 2) are left out of the procedure used to compute $\Phi_{\text{opt}}$. In hindsight they turn out to correspond to the marginal case mentioned in [21].

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