Study on Low Characteristic Formulation Optimization of Solid Propellant Based on Improved Particle Swarm Optimization

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Abstract. The combustion product of solid propellant produces strong plume flow field downstream of nozzle outlet, which improves plume characteristics, reduces plume characteristic signals, and improves stealth characteristics and survivability of missile. In this paper, an intelligent algorithm is proposed to optimize the formulation of solid propellant with low characteristic signal. An improved multi-objective particle swarm optimization algorithm is used to solve the formulation optimization model, and several feasible formulations for realizing low characteristic signal are obtained. On this basis, considering the energy performance, a formula screening method is given to calculate various characteristic signals and energy properties of the feasible formulation. The comprehensive measurement can not only take into account the optimization target requirements of propellant energy performance and plume characteristic signals, solve the conflict problem of reducing multiple characteristic signals, but also improve the efficiency of formulation design, and provide a reference for formulation designers.

1. Introduction

With the wide application of solid propellant in all kinds of strategic and tactical missile weapons, the diversified requirements of future operational tasks put forward more complex requirements for propellant performance. During the working process of rocket motor, the combustion products of solid propellant produce strong plume flow field downstream of nozzle exit, accompanied by a variety of complex physical and chemical phenomena, resulting in a variety of characteristic signals. Plume is a complex flow field, and the formation process is affected by many factors. Therefore, the plume characteristics are improved and the plume characteristic signals are reduced. [1] [2] Designing propellant formulation by experiencing and experimenting or adjusting oxygen balance of formulation, or using less or no smoke-producing ingredients, or replacing all or part of smoke-producing ingredients with new materials, or adding inhibitors, or adjusting the composition ratio of formulation components, often requires many tests and repeated tests, which is costly and inefficient. At present, it is only used to reduce smoke. The single characteristic signal is difficult to solve the problem of reducing the conflict of multiple characteristic signals. In addition, most of the theoretical modeling methods such as test and thermodynamic calculation are only preliminarily applied to the analysis and prediction of plume characteristic signals [3].

In this paper, an intelligent algorithm is proposed to optimize the formulation of solid propellant with low characteristic signal, which is a new research direction in the field of Engineering application.
The comprehensive measurement of energy performance can not only take into account the optimization objective requirements of propellant energy performance and plume characteristic signals, solve the problem of reducing the conflict of multiple characteristic signals, but also improve the efficiency of formulation design, and provide a reference for formulation designers.

2. Multi-objective particle swarm optimization algorithm

To overcome the shortcomings of particle swarm optimization (PSO) which can not be directly applied to solving multi-objective multi-constrained problems and is easy to fall into local optimum, this paper proposes an improved multi-objective particle swarm optimization (MPSO) algorithm.[3][4][5] Taking the minimization problem as an example, the specific steps to improve the algorithm are as follows:

(1) Initialize algorithm parameters.

The parameters of the algorithm include population particle number \(N\), upper limit of external file particle number \(W\), acceleration coefficient \(c_1\) and \(c_2\), inertia weight \(\omega\), maximum number of iterations \(M_{\text{itera}}\), particle mutation probability \(P_m\), penalty coefficient \(d_{ip} = [d_{ip1}, d_{ip2}, L, d_{ipL}, d_{ipn}]\), threshold \(\delta\) of average convergence degree of particles in external file, judgment of condition interval algebra \(\Delta t\) and threshold \(\tau\), and random method is used to generate particles \(x\) and their velocities \(v\), which are limited to given boundary constraints when generating particles. In the range of \([L, U]\), e. g. (1), the maximum \(V_{\text{min}}\) and minimum \(V_{\text{max}}\) velocities of particles, e. g. (2) are set.

\[
x = L + \text{rand}(U - L)
\]
\[
v = \frac{\text{rand} - \text{rand}(U - L)}{2}
\]

Among them, \(\text{rand} \in [0, 1]\).

\[
V_{\text{min}} = \frac{L - U}{2}
\]
\[
V_{\text{max}} = \frac{U - L}{2}
\]

(2) Computing the fitness of population particles. The penalty function method is used to take the inequality constraints (except boundary constraints) as penalty term, and fitness function is added. The substitution method is used to replace one decision variable in the equality constraints in the optimization model with other decision variables, and the fitness function is substituted. The maximization objective function in the optimization model is transformed into the minimization objective function to calculate the fitness of particles. The corresponding value is shown in equation (3).

\[
eval_i(x') = f_i(x') + d_{ip} \cdot \max(g_p(x') - a_p, 0) \text{ or } d_{ip} \cdot \min(g_p(x') - a_p, 0)
\]

Among them, \(x'\) represents the vector of decision variables after dealing with equality constraints \(h_q(x) = b_q\) by the substitution method, the fitness function \(eval_i(x')\) with the first objective \(x'\), and the mathematical model \(f_i(x')\) with the first objective \(x'\). At that time \(g_p(x) \leq a_p\), the penalty item was \(d_{ip} \cdot \max(g_p(x') - a_p, 0)\), at that time \(g_p(x) \geq a_p\), the penalty item was \(d_{ip} \cdot \min(g_p(x') - a_p, 0)\), \(d_{ip} = [d_{ip1}, d_{ip2}, L, d_{ipL}, d_{ipn}]\) indicating the coefficient of the penalty item.

(3) Establish external archives to store non-dominant solutions. When a new particle is generated, it is compared with the non-dominant relationship of the particle in the external file. If the new particle is dominated by more than two particles in the external file, it is discarded and the particle is regenerated for comparison again. If the new particle is the non-dominant solution of the particle in the current external file or is dominated by only one particle in the external file, it is stored in the
external file. At the same time, discarding the particles dominated by the particle in the external archives can properly improve the diversity of particles in the external archives; if the number of particles $C_W$ in the external archives exceeds its upper limit $W$ after adding new particles, discarding the particles with the highest density in the external archives and maintaining the uniform distribution of the non-dominated frontier. Finally, calculating the relationship between the new particles and the remaining particles in the external archives. Crowding distance [97], e.g. (4).

$$D(x, x_n) = \sqrt{\sum_{i=1}^{n} (eval_i(x) - eval_i(x_n))^2}$$ \hspace{1cm} (4)

Among them, $x$ represents the new particle and $x_n$ represents the particle $w$ in the external file.

$$M(x_i) = \frac{1}{\min(D(x_i, x_1), D(x_i, x_2), \ldots, D(x_i, x_{n-1}), D(x_i, x_n), \ldots, D(x_i, x_{C_W})))$$ \hspace{1cm} (5)

Among them, $M(x_w)$ represents the density of the particle $w$.

(4) Choose the best location for the whole and the individual [96]. Based on the non-dominant relationship and spatial distance of the particles in the external archives, the global best position $g_b$ of the population is selected. Based on the non-dominant relationship between the current position of the particles in the population and the historical best position $p_b$ and the utility function value of the fitness value, the best position of the individuals in the population is selected.

When choosing the best position of the population, we first calculate the spatial distance $\frac{C_W}{3}$ of the particles in the external archives, such as formula (6). Then we rank the particles in the external archives according to the spatial distance from large to small, select the first particle, and use roulette to select the best position of the population. That is, if the random number rand $\in [0, 1]$ is within the probability of the spatial distance of a particle, we regard the particle as the best population in the whole. Location.

$$D_1 = \min(D(x_w, x_1), D(x_w, x_2), \ldots, D(x_w, x_{n-1}), D(x_w, x_n), \ldots, D(x_w, x_{C_W}))$$

$$D_2 = \min\{D(x_w, x_1), D(x_w, x_2), \ldots, D(x_w, x_{n-1}), D(x_w, x_n), \ldots, D(x_w, x_{C_W})\} - \{D_1\}$$

$$D_w = (D_1 + D_2) / 2$$ \hspace{1cm} (6)

Among them, $D_1$ represents the minimum crowding distance between the particle $w$ and other particles in the external file, $D_2$ represents the second minimum crowding distance between the particle $w$ and other particles in the external file, and the spatial distance $D_w$ between the particle $w$ in the external file. When choosing the best position of an individual, if the current position updated by a particle in the population is in a dominant position with a certain position in its historical best position, it will be regarded as the best position of the individual; if the current position updated by a particle is not dominated by its historical best position, the position with the minimum utility function value of fitness value will be regarded as the particle. The best position of an individual, such as equation (7), where the initial particle or step (7) is the same as its historical best position in the first iteration of the particle obtained by mutation operation.

$$U_{value} = \sum_{i=1}^{n} eval_i(x')$$ \hspace{1cm} (7)
Among them, $U_{value}$ is the utility function of the fitness value of the particle $x'$.

(5) Update the velocity and position of particles $x$. Particles are updated according to the velocity and displacement formulas, such as formula (3.1), to adjust the particles beyond the boundaries $[L \ U ]$ of decision variables, maximum $v_{max}$ or minimum $v_{min}$ velocities;

When the particle's velocity $v_j$ exceeds the maximum $v_{max}^j$ or minimum $v_{min}^j$ velocity, then $v_j = v_{max}^j$ or $v_j = v_{min}^j$, when the particle exceeds the boundary constraint range $[L \ U ]$, then $x_j = L_j$ or $x_j = U_j$, and $v_j = -v_j$.

where $v = [v_1, v_2, \ldots, v_j, \ldots, v_D]$ , $v_{max} = [v_{max}^1, v_{max}^2, L, v_{max}^j, L, v_{max}^D]$ , $v_{min} = [v_{min}^1, v_{min}^2, L, v_{min}^j, L, v_{min}^D]$ .

$j = 1,2, \ldots, D$ , $D$ is the number of decision variables.

(6) Update the fitness value of the particle and the non-dominant solution in the external file. According to step (2), the fitness value of the newly generated particles in the population is calculated. According to step (3), the external files are maintained and the current non-dominant solution is updated.

(7) Maintaining population diversity. The average convergence degree $C$ of particles in external files is calculated as feedback information of the convergence degree of the algorithm. If it is less than the threshold value $\delta$, the mutation operation is carried out by randomly selecting the particles based on the external files, and then the fitness value of the mutated particles in the population is calculated according to step (2), otherwise step (8) is carried out directly.

The specific process of particle mutation operation is as follows: Firstly, the average convergence degree $C$ of particles in external files is calculated as feedback information of algorithm convergence, such as formula (8). If it is less than the threshold value $\delta$, particle $\frac{C_W}{20}$ in the external file is selected randomly. If the random number $\text{rand} \in \{0,1\}$ is less than the given probability $P_m$, then particle $\frac{C_W}{20}$ is selected again for mutation operation, such as formula (9). Then a mutated particle $\frac{C_W}{30}$ and an unmutated particle $\frac{C_W}{60}$ are used to replace the particle $\frac{C_W}{20}$ with the highest density in the population.

The mutated particle $\frac{C_W}{60}$ can keep the diversity of the population appropriately and unchanged. Particles with different operations can provide more information for searching non-dominant frontiers and improve the efficiency of the algorithm.

$$C = \sum_{j=1}^{C_W} \sum_{i=1}^{n} (eval_i(x'_j) - eval_{i, min}^j)^2$$  \hspace{1cm} (8)

Among them, $eval_{i, min} = \min(eval_i(x'_1), eval_i(x'_2), \ldots, eval_i(x'_{c_n}))$.

(8) If the population reaches the maximum number of iterations $M_{iter}$ or the difference $\Delta C$ between the average convergence degree $\Delta t$ of particles in the adjacent two generations of external archives is less than the threshold value $\tau$, then the particles in the external archives are output, i.e. the non-dominant solution is obtained, otherwise the search is continued in step 5.
3. Engineering application
The formulation optimization model for reducing the radiation intensity and primary smoke resistance of solid propellant in 2 μm-5 μm infrared band is established. The calculation and analysis are carried out as an example, e.g. (9). Among the HMX/Al/AP/HTPB propellants, the lower boundary values \( L_{Al}, L_{AP}, L_{HMX}, L_{HTPB} \), \( L_{fer} \) of aluminium powder, AP, HMX, HTPB, ferrocene mass fraction are 4%, 60%, 0, 10%, respectively, while the upper boundary values \( U_{Al}, U_{AP}, U_{HMX}, U_{HTPB} \), \( U_{fer} \) are 18%, 75%, 1.7%, 20%, 11%. The values of aluminium powder, ammonia perchlorate, ferrocene, HMX, HTPB mass fraction, \( I(T_E)_{max} \) and \( AGARDP_{max} \) are determined by the optimum target requirements of different types of missile weapons. The theoretical specific impulse of the agent HMX/Al/AP/HTPB ranges from 262s to 264s. In this case, the lower limit \( S_{min} \) of theoretical specific impulse is 262s and the lower limit \( A_{L} \) of oxygen coefficient is 0.55.

\[
\begin{align*}
\text{min} & \quad (AGARDP, I(T_E)) \\
s.t. & \quad x_{max} + x_{Al} + x_{AP} + x_{HMX} + x_{HTPB} + x_{fer} = 1 \\
& \quad 0.89 < x_{max} + x_{Al} + x_{AP} + x_{fer} < 0.90 \\
& \quad AGARDP \leq 1 \\
& \quad I(T_E) \leq 70000 \\
& \quad I_T \geq 262 \\
& \quad 0.9 \leq A_L \leq 1 \\
& \quad 0 \leq x_{max} \leq 20\% \\
& \quad 4\% \leq x_{Al} \leq 18\% \\
& \quad 60\% \leq x_{AP} \leq 75\% \\
& \quad 10\% \leq x_{HMX} \leq 11\% \\
& \quad 0 \leq x_{fer} \leq 1.7\% 
\end{align*}
\]  

Figure 1. Approximate Non-dominated Frontier
It can be seen from Figure 1 that the reduction of primary smoke and infrared radiation intensity of solid propellant is conflicting, but the obtained approximate non-dominant front solves the problem of reducing the conflict between them. The corresponding formulations are all feasible formulations for realizing low characteristic signal, which can reduce primary smoke and infrared radiation intensity to the greatest extent on the basis of satisfying certain theoretical specific impulse. According to the different influence degree of primary smoke and infrared radiation intensity of solid propellant on missile weapon, the feasible formula $C_{EPj}$ is sorted by formula selection method according to the order from big to small, and finally the largest formula $C_{EPj}$ is screened out, which can provide reference for formula designer.

4. Conclusion
Firstly, this paper introduces the basic principle of standard algorithm, the general model of formula optimization design and the related concepts of non-dominant relationship. The equality constraints, boundary constraints and inequality constraints of decision variables are dealt with by substitution method, random generation method and penalty function method, respectively, to form fitness function, which solves the problem that particle swarm optimization can not deal with constraints directly.

Secondly, based on the internal population and the established external archives, the global and individual best positions are selected respectively, which solves the problem that the particle swarm optimization algorithm can not directly solve the multi-objective optimization problem. In addition, a particle mutation operation method based on the external archives technology is proposed, which keeps the diversity of the population, avoids the algorithm falling into the local optimum, and maximizes the number of iterations and external archives. The change of the average convergence degree of the middle particle is used as the double judgment basis for the termination of the algorithm, which improves the efficiency of the algorithm. Thus, an improved multi-objective particle swarm optimization algorithm is formed to calculate the feasible formula for realizing the low characteristic signal.

Finally, the classical example is solved by the algorithm, and how to set the maximum iteration times of the algorithm is given for the specific example. The comparison results show that the convergence and uniformity of the proposed algorithm have obvious advantages, which verifies the effectiveness of the improved algorithm.

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