Model-Targeted Poisoning Attacks: Provable Convergence and Certified Bounds

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Abstract

Machine learning systems that rely on training data collected from untrusted sources are vulnerable to poisoning attacks, in which adversaries controlling some of the collected data are able to induce a corrupted model. In this paper, we consider poisoning attacks where there is an adversary who has a particular target classifier in mind and hopes to induce a classifier close to that target by adding as few poisoning points as possible. We propose an efficient poisoning attack based on online convex optimization. Unlike previous model-targeted poisoning attacks, our attack comes with provable convergence to any achievable target classifier. The distance from the induced classifier to the target classifier is inversely proportional to the square root of the number of poisoning points. We also provide a certified lower bound on the minimum number of poisoning points needed to achieve a given target classifier. We report on experiments showing our attack has performance that is similar to or better than the state-of-the-art attacks in terms of attack success rate and distance to the target model, while providing the advantages of provable convergence, and the efficiency benefits associated with being an online attack that can determine near-optimal poisoning points incrementally.

1 Introduction

Training state-of-the-art machine learning models typically requires a large amount of labeled training data, which often depends on collecting data and labels from untrusted sources. A typical application is email spam filtering, where a spam detector filters out spam messages based on features (e.g., presence of certain words) and periodically updates the model based on newly received emails labeled by users. In such a setting, spammers can generate “non-spam” messages by injecting non-related words or benign words, and when models are trained on these “non-spam” messages, the filtering accuracy will drop significantly [11]. Such attacks are known as poisoning attacks, and any training process that collects labels or data from untrusted sources is potentially vulnerable to them.

Poisoning attacks can be categorized into objective-driven attacks and model-targeted attacks depending on whether a target model is considered in the attack process. Objective-driven attacks have a specific attacker objective and aim to achieve the attack objective by generating the poisoning points; model-targeted attacks have a specific target classifier in mind and aim to induce that target classifier by generating minimal number of poisoning points. Objective-driven attacks are most commonly studied in the existing literature. The attacker objective is typically one of two extremes: indiscriminate attacks, where the adversary’s goal is simply to decrease the overall accuracy of the model [2, 26, 18, 24, 9]; and instance-targeted attacks, where the goal is to produce a classifier that misclassifies a particular known input [21, 29, 8]. Recently, Jagielski et al. introduced a more realistic attacker objective known as a subpopulation attack, where the goal is to increase the error rate or obtain a particular output for a defined subpopulation of the data distribution [7].

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objectives for realistic attacks are diverse and designing a unified and effective attack strategy for
different attacker objectives is hard. Gradient-based local optimization is most commonly used to
construct poisoning points for a particular attacker objective [2, 26, 18, 8, 21, 29]. Although these
attacks can be modified to fit other attacker objectives, since they are based on local optimization
techniques they can easily get stuck into bad local optima and fail to find effective sets of poisoning
points [24, 9]. To circumvent the issue of local optima, Steinhardt et al. formulate an indiscriminate
attack as a min-max optimization problem and solve it efficiently using online convex optimization
techniques [24]. However, the strong min-max attack only applies to the indiscriminate setting.

In contrast, model-targeted attacks incorporate the attacker objective into a target model and hence,
the target model can reflect any attacker objective. Thus, the same model-targeted attack methods can
be directly applied to a range of indiscriminate and subpopulation attacks just by finding a suitable
target model. Mei et al. first introduced a target model into a poisoning attack [18], but their attack is
still based on gradient-based local optimization techniques and suffers from bad local optima [24, 9].
Koh et al. [9] proposed the KKT attack, which converts the complicated bi-level optimization into a
simple convex optimization problem utilizing the KKT condition, avoiding the local optima issues.
However, their attack only works for a special family of loss functions and does not provide any
guarantee on the number of poisoning points required to converge to the target classifier.

In this work, we focus on the model-targeted attacks and aim to understand the feasibility of a
poisoning adversary to induce any target model. In particular, we find both theoretical and empirical
bounds on the sufficient (and necessary) number of poisoning points to get close to a specific target
classifier.

**Contributions.** Our main contributions involve developing a principled and general model-targeted
poisoning attack strategy, and using it to establish a lower bound on the number of poisoning points
needed to reach a target model. Our poisoning method takes as input a target model, and produces a
set of poisoning points. We prove that the model induced by training on the original training data with
these points added, converges to the target classifier as the number of poison points increases, given
that the loss function is convex and proper regularization is adopted in training (Theorem 4.1). Our
attack works in an online fashion and can incrementally find poisoning points that are nearly optimal.
We then prove a lower bound on the minimum number of poisoning points needed to reach the target
model (Theorem 4.2). Our lower bound is a general result that applies to any loss function. However,
to compute the lower bound efficiently, we need the loss function to be convex. For non-convex
functions, we can still approximate a valid lower bound (Section 4.3). We run experiments to compare
our attack to the state-of-the-art model-targeted attack [9] and find that our attack outperforms it in the
convergence to the target classifier, for all the target classifiers that we tried. Then, we experimentally
evaluate the success rate of our attack, and find that it has superior performance to the best known
attacks in the more realistic subpopulation attack scenario, and comparable performance in the
conventional indiscriminate attack scenario (Section 5).

**Comparison to the KKT attack.** Our model-targeted attack has several advantages over the state-
of-the-art model-targeted attack, the KKT attack [9]. Our attack comes with a provable guarantee
that it converges to the target classifier, and also provides a certified lower bound on number of
poisoning points needed to reach a given target model. The KKT attack does not provide either of
these guarantees. The convergence property of our attack is applicable to the broader family of convex
loss functions while the validity of the KKT attack is limited to margin based loss functions. From the
practical side, our attack works in an online fashion without needing to know in advance the number of
poisoning points available, and hence is very efficient in incremental poisoning scenario. Empirically,
we also show that our attack has better performance than the KKT attack in the subpopulation attack
settings, and comparable performance in indiscriminate attack settings.

**2 Problem Setup**

We consider a binary prediction task, \( h : \mathcal{X} \rightarrow \mathcal{Y} \), where \( \mathcal{X} \subseteq \mathbb{R}^d \) and \( \mathcal{Y} = \{ +1, -1 \} \). We note that
the poisoning attack proposed in this paper also applies to multi-class prediction tasks or regression
problems, given some convexity conditions. The prediction model \( h \) is characterized by parameters
\( \theta \in \Theta \subseteq \mathbb{R}^d \). We define the non-negative convex loss on an individual point, \((x, y)\), as \( l(\theta; x, y) \)
We also assume the empirical risk minimization over a set of points \( A \) as 
\[
L(\theta; A) = \sum_{(x,y) \in A} l(\theta; x, y).
\]
We assume a true data distribution, \( p^* \), over \( \mathcal{X} \times \mathcal{Y} \).

We adopt the game-theoretic formalization of the poisoning attack process from Steinhardt et al. \cite{24} to describe our model-targetted attack scenario:

1. \( N \) data points are drawn uniformly at random from \( p^* \) and constitute the clean training set, \( \mathcal{D}_c \).
2. The adversary, with knowledge of \( \mathcal{D}_c \), the model training process and the model space \( \Theta \), generates a target classifier \( \theta_p \in \Theta \) that satisfies the attack goal.
3. The adversary produces a set of poisoning points, \( \mathcal{D}_p \), with the knowledge of \( \mathcal{D}_c \), model training process, \( \Theta \) and \( \theta_p \).
4. Model builder trains the model on \( \mathcal{D}_c \cup \mathcal{D}_p \) and produces a classifier, \( \theta_{atk} \).

The adversary’s goal is that the resulting classifier, \( \theta_{atk} \) is close to the desired target classifier \( \theta_p \) (see Section 4.2 for distance metric). Step 2 corresponds to the target classifier generation process, and we simply adopt the heuristic target classifier generation process from Koh et al. \cite{9} and also improve it for the indiscriminate attack with simple trick (Detail in Appendix C). The target model generation process from Mei et al. \cite{18} is not used because it is domain specific and is not generally applicable. Step 3 corresponds to our model-targeted poisoning attack, and is also the main contribution of the paper.

We also assume the model builder trains a model through the structural empirical risk minimization and the training process is known to the attacker:

\[
\theta_c = \arg \min_{\theta \in \Theta} \frac{1}{|\mathcal{D}_c|} L(\theta; \mathcal{D}_c) + C_R \cdot R(\theta)
\]

where \( R(\theta) \) is the regularization function (e.g., \( \frac{1}{2} \|\theta\|^2 \) for SVM model).

**Remark.** As shown in our formalization, we assume an adversary with full knowledge of training data, model space and training process, as with majority of prior works \cite{18,24,9,21}. The attacker only adds poisoning points into the training set (addition only attack), instead of modifying existing training data. Modifying existing points can form stronger attacks than the addition only attack, but is less practical because the modification typically requires administrative access to the system. We do not place any restrictions on the poisoning points (e.g., their features and labels), which enables us to perform the worse case analysis on the robustness of models against poisoning attacks. Although most previous works do not pose constraint on poisoning points \cite{1,18,24,9}, some works also put different restrictions on attacked points, which will be illustrated in detail in Section 3.

### 3 Related Work

The most commonly used poisoning strategy is gradient-based attack. Gradient-based attacks iteratively modify a candidate poisoning point \((\hat{x}, \hat{y})\) in the set \( \mathcal{D}_p \) based on the test loss defined on \( \hat{x} \) (keeping \( \hat{y} \) fixed). This kind of attack was first studied on SVM models \cite{2}, and later extended to linear and logistic regression \cite{18}, and recently to larger neural network models \cite{8,28,19,21,29,6}. In addition to classification tasks, gradient-based poisoning attacks are also applied to topic modeling \cite{17}, collaborative filtering \cite{10} and algorithmic fairness \cite{23}. While most of these works do not pose restrictions on the poisoning points, some works put various constraints on the poisoning points. The clean-label attack assumes adversaries can only perturb the features of the data, but the label is given by an oracle labeler \cite{8,21,29,6}. In label-flipping attack, adversaries are only allowed to change the labels \cite{1,26,27,7}. These restricted attacks are much weaker than the poisoning attacks without restrictions \cite{9,5}.

Besides the gradient-based attacks, researchers also utilize generative adversarial networks to craft poisoning points efficiently for larger neural networks, however, the effectiveness of the attack is limited \cite{29,20}. The strongest attacks so far are the KKT attack \cite{9} and the min-max attack \cite{24,9}. However, the KKT attack cannot scale for multi-class classification tasks, not suitable for broader family of loss functions. The min-max attack only works for indiscriminate attack setting, but additionally provides a certificate on worst case test loss (with fixed number of poisoning points), and our theoretical lower bound on number of poisons can treated as its dual problem. We are also
inspired by Steinhardt et al. [24] to adopt online convex optimization to instantiate our model-targeted attack, but now deals more general attack scenario. We also distinguish ourselves from the poisoning attack against online learning [25]. The attack against online learning considers a setting where training data arrives in a streaming manner while we consider the offline setting with training data being fixed. Another line of work studies provable “targeted” poisoning attacks where an adversary tries to increase the probability of an arbitrary “bad” property [14, 13, 15]. We are distinct from these works in that our attack tries to enforce a specific classifier, and not a bad property. Also, our attack could achieve its goal more efficiently, while still having provable guarantees.

4 Poisoning Attack with a Target Model

Our new poisoning attack determines a target model and selects poisoning points to achieve that target model. The target models are generated in a heuristic manner and we adopt the heuristic approach proposed by Koh et al. [9] and improve it for indiscriminate attack with simple trick. For the new poisoning attack, first, we show the algorithm that generates the poisoning points in Section 4.1 and then prove that the generated poisoning points, once added to the clean data, can produce a classifier that asymptotically converges to the target classifier in Section 4.2.

4.1 Model-Targeted Poisoning with Online Learning

The main idea of our model-targeted poisoning attack, as outlined in Algorithm 1, is to sequentially add a point into the training set that have maximum loss difference between the intermediate model obtained so far and the target model, and by training models on the updated training set, we actually minimize the gap in the loss of the intermediate classifier and the target classifier. Repeating the process then eventually generates classifiers that have similar loss distribution as the target classifier. We show in Section 4.2 why similar loss distribution implies convergence.

Algorithm 1 ModelTargetedPoisoning

| Input: | $D_c$, the loss functions ($L$ and $l$), $\theta_p$ |
| Output: | $D_p$ |
| 1: | $D_p = \emptyset$ |
| 2: | while stop criteria not met do |
| 3: | $\theta_t = \arg \max L(\theta; D_c \cup D_p)$ |
| 4: | $(x^*, y^*) = \arg \max_{x,y} l(\theta_t; x, y) - l(\theta_p; x, y)$ |
| 5: | $D_p = D_p \cup \{(x^*, y^*)\}$ |
| 6: | end while |
| 7: | return $D_p$ |

Algorithm 1 requires the input of clean training set $D_c$, the Loss function ($L$ for set of points and $l$ for individual point) and the target model $\theta_p$. The output from Algorithm 1 will be the set of poisoning points $D_p$. The algorithm is simple: first, adversaries train the intermediate model $\theta_t$ on the mixture of clean and poisoning points $D_c \cup D_p$ with $D_p$ an empty set in first iteration (Line 3). The adversary then searches for the point that maximizes the loss difference between $\theta_t$ and $\theta_p$ (Line 4). After the point of maximum loss difference is found, it is added to the poisoning set $D_p$ (Line 5). The whole process repeats until the stop condition is satisfied in Line 2. The stop condition is flexible and it can take various forms: 1) adversary has a budget $T$ on number of poisoning points, and the algorithm halts when the algorithm runs for $T$ iterations; 2) the intermediate classifier $\theta_t$ is closer to the target classifier (than a preset threshold $\epsilon$) in terms of the maximum loss difference, and more details regarding this distance metric will be introduced in Section 4.2; 3) adversary has some requirement on the accuracy and the algorithm terminates when $\theta_t$ satisfies the accuracy requirement. Since we focus on producing a classifier close to the target model, we adopt the second stop criteria that measures the distance with respect to the maximum loss difference, and report results based on this criteria in Section 5.

A nice property of Algorithm 1 is that the classifier $\theta_{atk}$ trained on $D_c \cup D_p$ is close to the target model $\theta_p$ and asymptotically converges to $\theta_p$. Details of the convergence will be shown in the next

2Although the provable attacks [14, 15] are shown to be polynomial time [12, 13], they are still far from practical.
section. The algorithm may appear to be slow, particularly for larger models due to requirement of repeatedly training a model in line 3. However, this is not an issue. First, as will be shown in next section, the algorithm is an online optimization process and line 3 corresponds to solving the online optimization problem exactly. However, people often use the very efficient online gradient descent method to approximately solve the problem and its asymptotic performance is the same [22]. Second, when we solve the optimization problem exactly, we can add multiple copies of \((x^*, y^*)\) into \(D_p\) each time. This reduces the overall iteration number, and hence reduces the number of times retraining models. The proof of convergence will be similar. For simplicity in interpreting the results, we do not use this in our experiments and add only one copy of \((x^*, y^*)\) each iteration. However, we also tested the performance by adding two copies of \((x^*, y^*)\) and find that the attack results are nearly the same while the efficiency being significantly improved. For example, for the experiments we tried on MNIST-17 dataset, by adding 2 copies of points, with similar number of poisoning points, the attack success rate decreases at most by 0.7% while the execution time is reduced approximately by half.

### 4.2 Convergence of Our Poisoning Attack

Before proving the convergence of Algorithm 1, we need to measure the distance of the model \(\theta_{atk}\) trained on \(D_c \cup D_p\) to the target model \(\theta_p\). First, we define a general closeness measure based on their prediction performance which we will use to state our convergence theorem:

**Definition 1 (Loss-based distance).** For two models \(\theta_1\) and \(\theta_2\), a space \(X \times Y\) and a loss function \(l(\theta; x, y)\), we define loss-based distance \(D_{l,X,Y}(\theta_1, \theta_2)\) as

\[
D_{l,X,Y}(\theta_1, \theta_2) = \max_{(x,y) \in X \times Y} l(\theta_1; x, y) - l(\theta_2; x, y).
\]

**Why is loss-based distance a meaningful notion of closeness?** We argue that this notion captures the “behavioral” distance between two models. Namely, if \(\theta_1\) is \(\epsilon\)-close (as measured by loss-based distance) to \(\theta_2\) and vice versa, then \(\theta_1\) and \(\theta_2\) would have almost equal loss on all the points, meaning that they have almost the same behavior across all the space. Note that our definition of distance does not have the symmetry property of metrics and hence is not a metric. However, it has some other properties of metrics in the space of attainable models. For example, if some model \(\theta\) is attainable, no model could have negative distance to it. To further show the value of this distance notion, in Appendix B we demonstrate an \(O(\epsilon)\) upper bound on the \(\ell_1\) norm of difference between two models that are \(\epsilon\)-close with respect to loss-based distance for the special case of Hinge loss.

Our convergence theorem uses the loss-based distance to establish that the attack of Algorithm 1 converges to the target classifier:

**Theorem 4.1.** After \(T\) steps, Algorithm 1 will produce the poisoning set \(D_p\) and the classifier trained on \(D_c \cup D_p\), \(\epsilon\)-close to \(\theta_p\), with respect to loss-based distance, \(D_{l,X,Y}\), for

\[
\epsilon = \frac{\alpha(T) + L(\theta_p; D_c) - L(\theta_c; D_c)}{T} \cdot \gamma
\]

where, \(\gamma\) is a universal constant determined only by the regularization factor \(C_R\) and \(\alpha(T)\) is the regret of the online algorithm when the loss function used for training is convex. Additionally, \(\alpha(T)\) is in the order of \(O(\log T)\) and we have \(\epsilon \leq O\left(\frac{\log T}{T}\right)\) when the regularizer \(R(\theta)\) is strongly convex, and \(\epsilon \to 0\) when \(T \to +\infty\).

**Remark 1.** \(\alpha(T)\) is also in the order of \(O(\log T)\) when the loss function used for training is strongly convex and the regularizer is convex.

**Proof idea.** The full proof of Theorem 4.1 is in Appendix A. Here, we only summarize the high level proof idea. The key idea is to frame the poisoning problem as an online learning problem. In this formulation, each step of the online learning problem corresponds to the \(i\)th poison point \((x_i, y_i)\). In particular, the loss function at iteration \(i\) of the online learning problem is set to \(l(\cdot; x_i, y_i)\). Then, we show that by defining the parameters of the online learning problem in a careful way, the output of the follow-the-leader (FTL) algorithm [22] at iteration \(i\) is a model that is identical to training a model on a dataset consisting of the clean points and the first \(i - 1\) poisoning points. On the other hand, the way the poisoning points are selected, we can show that at the \(i\)th iteration the maximum loss difference between the target model and the best induced model so far would be smaller than the
We are most interested in subpopulation attacks, since they correspond to the more realistic attacker. We first describe our experimental setup regarding the datasets, models, attacks and target classifiers. Appendix C. Our findings for the indiscriminate attacks are that the attack gradually and consistently with a known classifier. One easy choice is to replace \( \theta \)

### Theorem 4.2 (Certified Lower Bound)

The formula for the lower bound can be easily incorporated into Algorithm 1 to obtain tighter bounds for reaching a given target classifier and its gap to the number of poisoning points. The code for the comparison of attack success rates to state-of-the-art poisoning attacks, and the theoretical lower bound for the attacks. Next, we present the experimental results by showing the convergence of Algorithm 1, the comparison of attack success rates to state-of-the-art poisoning attacks, and the theoretical lower bound for reaching a given target classifier and its gap to the number of poisoning points. The code for evaluation is available at: [https://github.com/suyeecav/model-targeted-poisoning](https://github.com/suyeecav/model-targeted-poisoning)

### Implications of Theorem 4.1

The theorem says that the loss-based distance of the model trained on \( \mathcal{D}_c \cup \mathcal{D}_p \) to the target model correlates to the loss difference between the target model and the clean model \( \theta_c \) (trained on \( \mathcal{D}_c \)) on \( \mathcal{D}_c \), and correlates inversely with the number of poisoning points. In other words, if the target classifier \( \theta_p \) is closer to the clean model \( \theta_c \) (in terms of the loss on clean training data), then it is easier to achieve the target classifier. On the other hand, with more poisoning points, we can get closer to the target classifier, and our attack will be more effective. The theorem justifies the motivation behind the heuristic method [9] to select a target classifier with lower loss on clean data. For the indiscriminate attack scenario, we also improve the heuristic approach by adaptively updating the model and producing target classifiers with much lower loss on the clean set. This helps to empirically confirm our theorem. Details of the original and improved heuristic approach, and relevant experiments are in Appendix C.

### 4.3 Certified Lower Bound on the Number of poisoning Points

In this section, we first provide the theoretical lower bound on number of poisoning points required for producing the target classifier, and then explain how the lower bound estimation scheme can be incorporated into Algorithm 1.

Considering the learning problem defined in (1), in the theorem below, we show the minimum number of poisoning points required to produce \( \theta_p \) in the addition only poisoning scenario. The intuition behind the theorem is, when the number of poisoning points added to the clean training set is smaller than the certified lower bound, there always exists a classifier \( \theta \) that has lower loss compared to \( \theta_p \) and hence the target classifier cannot be achieved.

#### Theorem 4.2 (Certified Lower Bound).

Given a target classifier \( \theta_p \), to reproduce \( \theta_p \) by adding the poisoning set \( \mathcal{D}_p \) into \( \mathcal{D}_c \), the number of poisoning points \( |\mathcal{D}_p| \) cannot be lower than

\[
\sup_{\theta} c(\theta) = \frac{L(\theta_p; \mathcal{D}_c) - L(\theta; \mathcal{D}_c) + NC_R(R(\theta_p) - R(\theta))}{\sup_{x,y} (l(\theta; x, y) - l(\theta_p; x, y)) + C_R(R(\theta) - R(\theta_p))}.
\]

**Remark 2.** To obtain the actual value of the lower bound to achieve \( \theta_p \), one just needs to replace \( \theta \) with a known classifier. One easy choice is to replace \( \theta \) with model \( \theta_c \) trained on \( \mathcal{D}_c \).

**Remark 3.** Our theorem also applies to non-convex functions. However, in order to efficiently find the point with maximum loss difference for the non-convex scenario, one can use a convex function to upper bound the loss function difference and efficiently compute a conservative lower bound.

The formula for the lower bound can be easily incorporated into Algorithm 1 to obtain tighter theoretical lower bound. We simply need to check all of the intermediate classifier \( \theta_t \) produced during the attack process and replace \( \theta \) with \( \theta_t \). As long as \( L(\theta_p; \mathcal{D}_c) - L(\theta_t; \mathcal{D}_c) + NC_R(R(\theta_p) - R(\theta_t)) > 0 \), the lower bound can be computed for the pair of \( \theta_t \) and \( \theta_p \). Algorithm 1 then has an additionally returns the lower bound, which is the highest lower bound computed from our poisoning procedure.

### 5 Experiments

We first describe our experimental setup regarding the datasets, models, attacks and target classifiers for the attacks. Next, we present the experimental results by showing the convergence of Algorithm 1, the comparison of attack success rates to state-of-the-art poisoning attacks, and the theoretical lower bound for reaching a given target classifier and its gap to the number of poisoning points. The code for evaluation is available at: [https://github.com/suyeecav/model-targeted-poisoning](https://github.com/suyeecav/model-targeted-poisoning)

We are most interested in subpopulation attacks, since they correspond to the more realistic attacker goal of impacting the classifier outputs for a targeted subpopulation. For completeness, we also evaluate our attack in the indiscriminate poisoning scenario but defer details on those experiments to Appendix C. Our findings for the indiscriminate attacks are that the attack gradually and consistently
converges to the target model in terms of the maximum loss difference and the Euclidean distance to the target, with attack success rates that are comparable to the state-of-the-art attack (unlike the subpopulation attacks, where our attack produces superior results).

**Dataset, Model and Attacks.** For the subpopulation attack experiments, we use the Adult dataset [3]. This dataset was used for evaluation by the first subpopulation attack paper [7] and we directly use Jagielski et al.'s preprocessed version of the dataset. We conduct experiments on linear SVM model and compare our model-targeted poisoning attack in Algorithm 1 to the state-of-the-art KKT attack [9]. We do not include the model-targeted attack from Mei et al. [18] because it underperforms the KKT attack [9]. We also do not include objective-driven attacks because the attack goals are significantly different between the model-targeted and objective-driven attacks. However, model-targeted attacks can be compared to objective-driven attacks relatively fairly when some proper target models are provided. We provide more discussions on this in Appendix D.

Both our attack and the KKT attack take as input a target classifier and the original training data, and output a set of poisoning points selected with the goal that the induced classifier is as close as possible to the target classifier. We compare the effectiveness of the attacks in selecting poisoning points that converge to a given target classifier by testing the attacks using the same target model.

The KKT attack requires a target number of poisoning points as an input while our attack is more flexible and can either take a target number of poisoning points or a threshold for \( \epsilon \)-close distance to the target model. Since we do not know the number of poisoning points needed to reach some attacker goal in advance for the KKT attack, we first run our attack and produce a classifier that satisfies the selected \( \epsilon \)-close distance threshold. The loss function is set as the hinge loss since we target an SVM model in our experiments and we set \( \epsilon = 0.01 \) for all these experiments. Then, we use the size of the poisoning set returned from our attack (denoted by \( n_p \)) as the input to the KKT attack for the target number of poisoning points. We also compare the two attacks with varying numbers of poisoning points up to \( n_p \). For the KKT attack, its entire optimization process must be rerun whenever the target number of poisoning points changes. Hence, it is infeasible to evaluate the KKT attack on many different poisoning set sizes. In our experiments, we run the KKT attack five poisoning set sizes: \( 0.2 \cdot n_p, 0.4 \cdot n_p, 0.6 \cdot n_p, 0.8 \cdot n_p, \) and \( n_p \). In contrast, we simply run our attack for iterations up to the maximum number of poisoning points, collecting a data point for iteration up to \( n_p \).

**Subpopulations.** We identify the subpopulations for the Adult dataset using \( k \)-means clustering techniques (ClusterMatch [7]) to obtain different clusters (\( k = 20 \) in our case). For each cluster, we select instances with label “<=50K” to form the subpopulation (indicating all instances in the subpopulation are in low income group). This way of defining subpopulation is rather arbitrary (in contrast to a more likely attack goal which would select subpopulations based on demographic characteristics), but enables us to simplify the analysis. From the 20 subpopulations obtained, we select three subpopulations with the highest test accuracy on the clean model and they all have 100% test accuracy, indicating all instances in these subpopulations are correctly classified as low income. This enables us to use “attack success rate” and “accuracy” without any ambiguity on the subpopulation—for each of our subpopulations, all instances are originally classified as low income, and the simulated attacker’s goal is to have them classified as high income. For each subpopulation, we use the heuristic approach from Koh et al. [9] to generate a target classifier that that has 0% accuracy (100% attacker success) on the subpopulation, indicating that all subpopulation instances are now classified as high income.

**Convergence.** Figure 1 shows the convergence of Algorithm 1 using both maximum loss difference and Euclidean distance to the target. The maximum number of poisoning points \( (n_p) \) for the experiments is obtained when the classifier from Algorithm 1 is \( 0.01 \)-close to the target classifier. Our attack steadily reduces the maximum loss difference and Euclidean distance to the target model, in contrast to the KKT attack which does not seem to converge towards the target model reliably. Concretely, at the maximum number of poisoning points in Figure 1, both the maximum loss difference and Euclidean distance of our attack (to the target) is less than 2% of the corresponding distances of the KKT attack.

**Attack Success.** Next, we compare the classifiers induced by the two attacks in terms of the attacker’s goal of reducing the test accuracy on the subpopulation. Figure 2 shows the accuracy results for the three subpopulations. For each test, the maximum number of poisoning points is obtained by running our attack with a target of 0.01-closeness (in loss based distance). For the three subpopulations, at
Figure 1: Attack convergence (results shown are for the first subpopulation, Cluster 0). The maximum number of poisons is set using the 0.01-close threshold to target classifier.

Figure 2: Test accuracy for each subpopulation with classifiers induced by poisoning points obtained from our attack and the KKT attack.

Table 1: Theoretical lower bounds on the number of poisoning points needed to achieve classifiers induced from our attack for different subpopulations. All results are averaged over 4 runs, integer value in the cell means we get exactly same value for 4 runs and others are shown with the average and standard error.

Near Optimality of Our Attack. In order to show the optimality of our attack, we calculate a lower bound on the number of poisoning points needed to induce the model that is induced by the poisoning points that are found by our attack. We calculate this lower bound on the number of poisons using Theorem 4.2 (details in Section 4.3). Note that Theorem 4.2 provides a valid lower bound based on any intermediate model. In order to get a lower bound on the number of poisoning points, we only need to use Theorem 4.2 on some number of intermediate models and report the best one. We do this by running Algorithm 1 using the induced model (and not the previous target model) as the target model, terminating when the induced classifier is 0.01-close to the given target model. We then consider all the intermediate classifiers that the algorithm induced across the iterations. The Adult dataset contains many binary features and hence the search process for the point with maximum loss difference is a (non-convex) mixed integer programming problem. To obtain a valid lower bound, we relax the binary features into real valued features in range [0, 1] and then compute the lower bound. Our calculated lower bound shows that in some cases the gap between the lower bound and the number of used poison points is small. This means our attack is nearly optimal in terms of minimizing the number of poisoning points needed to induce a target classifier.
6 Conclusion

We propose a general poisoning framework with provable guarantees to reach any achievable target classifier, along with certified lower bound on the number of poisoning points needed. Our attack is a generic tool that first captures the goal of the adversary as a target model, and then focuses on the power of the attack to induce that model. This separation enables future work to explore the effectiveness of poisoning attacks corresponding to different adversarial goals. Our framework also applies in scenarios where adversaries can first remove points and then add new points into the training set. We have not considered defenses in this work, and it is important to study the effectiveness of our attack against data poisoning defenses. Defenses may be designed the limit the search space of the points with maximum loss difference, increasing the number of poisoning points needed.

One limitation of our framework is the requirement in the convexity of the difference of loss functions to efficiently search for its maximum value, which excludes other commonly used loss functions such as logistic loss. However, our approach might still be effective in these cases by using local optimization techniques to search for poisoning points with maximum loss difference. The convergence property of our attack holds for any convex loss function with bounded Lipschitz constant and the convergence does not depend on the loss difference to be convex.

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A Proof of Main Theorems

In this section, we provide the proofs of the main theorems shown in this paper. For convenience, we restate all the theorems below while also referencing to the main paper.

Before proving the main theorem, we introduce two new definitions and several lemmas to assist with the proof.

Definition 2 (Attainable models). We say $\theta$ is $C_R$-attainable with respect to loss function $l$ and regularization function $R$ if there exists a training set $D$ such that

$$\theta = \arg \min_{\theta \in \Theta} \frac{1}{|D|} \cdot L(\theta; D) + C_R \cdot R(\theta)$$

Lemma A.1. Let $\theta_1$ and $\theta_2$ be two $C_R$-attainable parameters for some $C_R > 0$ such that $R(\theta_1) > R(\theta_2)$. Then,

$$\sup_{x,y} \left( l(\theta_2; x, y) - l(\theta_1; x, y) \right) / \left( R(\theta_1) - R(\theta_2) \right) > C_R.$$ 

Proof. Consider any attainable pairs of $(\theta_1, \theta_2)$ such that $R(\theta_1) > R(\theta_2)$ and let $D_1$ to be training set that the training algorithm produces the unique minimizer $\theta_1$. Namely,

$$\theta_1 = \arg \min_{\theta} \frac{1}{|D_1|} \cdot L(\theta; D_1) + C_R \cdot R(\theta)$$

Since $\theta_1$ minimizes the total loss on $D_1$ uniquely, we have

$$\frac{1}{|D_1|} L(\theta_2; D_1) + C_R \cdot R(\theta_2) > \frac{1}{|D_1|} L(\theta_1; D_1) + C_R \cdot R(\theta_1)$$

By rearranging the above inequality and by an averaging argument, we have

$$\sup_{x,y} \left( l(\theta_2; x, y) - l(\theta_1; x, y) \right) \geq \frac{1}{|D_1|} L(\theta_2; D_1) - \frac{1}{|D_1|} L(\theta_1; D_1) > C_R \cdot \left( R(\theta_1) - R(\theta_2) \right).$$

Now since $R(\theta_1) > R(\theta_2)$ we have

$$\sup_{x,y} \left( l(\theta_2; x, y) - l(\theta_1; x, y) \right) / \left( R(\theta_1) - R(\theta_2) \right) > C_R.$$ 

Lemma A.2. Let $F$ be the family of all $C_R$-attainable models. For any $(\theta_1, \theta_2) \in F^2$, we have

$$\sup_{x,y} \left( l(\theta_2; x, y) - l(\theta_1; x, y) \right) + C_R \cdot (R(\theta_2) - R(\theta_1)) > \gamma \cdot \sup_{x,y} \left( l(\theta_2; x, y) - l(\theta_1; x, y) \right)$$

where $\gamma$ is a positive constant that only depends on $C_R$.

Proof. We prove the lemma for $\gamma = 1 - C_R/C$ for

$$C = \left( \inf_{(\theta_1, \theta_2) \in F^2} \sup_{x,y} \frac{l(\theta_2; x, y) - l(\theta_1; x, y)}{(R(\theta_1) - R(\theta_2))} \right).$$

First, note that by Lemma A.1 we have

$$C > C_R > 0.$$ 

which implies $\gamma$ is positive. Now we consider two subcases based on the sign of $R(\theta_2) - R(\theta_1)$:

Case 1: $R(\theta_2) - R(\theta_1) \geq 0$. In this case the inequality is straightforward:

$$\sup_{x,y} \left( l(\theta_2; x, y) - l(\theta_1; x, y) \right) + C_R \cdot (R(\theta_2) - R(\theta_1)) \geq \sup_{x,y} \left( l(\theta_2; x, y) - l(\theta_1; x, y) \right)$$

$$> (1 - C_R/C) \cdot \sup_{x,y} \left( l(\theta_2; x, y) - l(\theta_1; x, y) \right),$$

where $\gamma = 1 - C_R/C$.
where the last inequality is based on \(2\).

**Case 2:** \(R(\theta_2) - R(\theta_1) < 0\). From the definition of \(C\) we have
\[
R(\theta_1) - R(\theta_2) \leq \sup_{x,y} \left( l(\theta_2; x, y) - l(\theta_1; x, y) \right) / C.
\]
Equivalently, we can say
\[
R(\theta_2) - R(\theta_1) \geq -\sup_{x,y} \left( l(\theta_2; x, y) - l(\theta_1; x, y) \right) / C.
\]
Replacing \(R(\theta_2) - R(\theta_1)\) with the lower bound above completes the proof, namely
\[
\sup_{x,y} \left( l(\theta_2; x, y) - l(\theta_1; x, y) \right) + C_R(R(\theta_2) - R(\theta_1)) \geq (1 - C_R/C) \sup_{x,y} \left( l(\theta_2; x, y) - l(\theta_1; x, y) \right).
\]

With Definition 3 and the lemmas, we are ready to prove the theorem on convergence of Algorithm 1 (restating Theorem 4.1, from Section 4.2):

**Theorem 4.1** After \(T\) steps, Algorithm 1 will produce the poisoning set \(D_p\) and the classifier trained on \(D_c \cup D_p\) is \(\epsilon\)-close to \(\theta_\rho\), with respect to loss-based distance, \(D_{\theta; x, y, \gamma}\), for
\[
\epsilon = \frac{\alpha(T) + L(\theta_\rho; D_c) - L(\theta_c; D_c)}{T \cdot \gamma}
\]
where, \(\gamma\) is a universal constant determined only by the regularization factor \(C_R\) and \(\alpha(T)\) is the regret of the online algorithm when the loss function used for training is convex. Additionally, \(\alpha(T)\) is in the order of \(O(\log T)\) and we have \(\epsilon \leq O(\frac{\log T}{T})\) when the regularizer \(R(\theta)\) is strongly convex, and \(\epsilon \to 0\) when \(T \to +\infty\).

The goal of the adversary is to get \(\epsilon\)-close to \(\theta_\rho\) (in terms of the loss based distance) by injecting (potentially few) number of poisoned training data. The algorithm is in essence an online learning problem and we transform Algorithm 1 into the form of standard online learning problem and then utilize existing regret bounds to show the convergence.

We adopt the follow the leader (FTL) framework to describe Algorithm 1 in the language of standard online learning problem and then derive the desired logarithmic regret bound based on strong convexity of the loss function. We first describe the online learning setting considered in this paper and the notion of the regret.

**Definition 3.** Let \(\mathcal{L}\) be a class of loss functions, \(\Theta\) set of possible models, \(A: (\Theta \times \mathcal{L})^* \rightarrow \Theta\) an online learner and \(S: (\Theta \times \mathcal{L})^* \times \Theta \rightarrow \mathcal{L}\) a strategy for picking loss functions in different rounds of online learning (adversarial environment in the context of online convex optimization). We use \(\text{Regret}(A, S, T)\) to denote the regret of \(A\) against \(S\), in \(T\) rounds. Namely,
\[
\text{Regret}(A, S, T) = \sum_{j=0}^{T} l_j(\theta_j) - \min_{\theta \in \Theta} \sum_{j=0}^{T} l_j(\theta)
\]
where
\[
\theta_i = A((\theta_0, l_0), \ldots, (\theta_{i-1}, l_{i-1})) \quad \text{and} \quad l_i = S((\theta_0, l_0), \ldots, (\theta_{i-1}, l_{i-1}), \theta_i).
\]

With the online learning problem set up, we proceed to the main proof which first describes Algorithm 1 in the FTL framework and then shows the convergence utilizing the logarithmic regret bound of the FTL framework.

**Proof of Theorem 4.1** The FTL framework proceeds by solving all the functions incurred during the previous online optimization steps, namely, \(A_{\text{FTL}}((\theta_0, l_0), \ldots, (\theta_i, l_i)) = \arg\min_{\theta \in \Theta} \sum_{j=0}^{i} l_j(\theta)\).
Next, we describe how we design the $i$th loss function $l_i$ in each round of the online optimization. For the first choice, $A_{\text{FTL}}$ chooses a random model $\theta_0 \in \Theta$. In the first round (round 0), $S_{\theta_0}$ uses the clean training set $D_c$ and the loss is set as

$$S_{\theta_0}(\theta_0) = l_0(\theta) = L(\theta; D_c) + N \cdot C_R \cdot R(\theta).$$

According to the FTL framework, $A_{\text{FTL}}$ returns model that minimizes the loss on the clean training set $D_c$ using the structural empirical risk minimization. For the subsequent iterations ($i \geq 1$), the loss functions is defined as, given the latest model $\theta_i$, $S_{\theta_i}$ first finds $(x_i^*, y_i^*)$ that maximizes the loss difference between $\theta_i$ and a target model $\theta_p$. Namely,

$$(x_i^*, y_i^*) = \arg\max_{(x,y)} l(\theta_i; x, y) - l(\theta_p; x, y)$$

and then chooses the $i$th loss function as follows:

$$S_{\theta_i}((\theta_0, l_0), \ldots, (\theta_{i-1}, l_{i-1}), \theta_i) = l_i(\theta) = l(\theta; x_i^*, y_i^*) + C_R \cdot R(\theta).$$

Now we will see how FTL framework behaves when working on these loss functions at different iterations. We use $D_p^i$ to denote the set $\{(x_1^*, y_1^*), \ldots, (x_i^*, y_i^*)\}$. We have

$$\theta_i = A_{\text{FTL}}((\theta_0, l_0), \ldots, (\theta_{i-1}, l_{i-1})) = \arg\min_{\theta \in \Theta} \sum_{j=0}^{i-1} l_j(\theta)$$

$$= \arg\min_{\theta \in \Theta} L(\theta; D_c) + N \cdot C_R \cdot R(\theta) + \sum_{j=1}^{i-1} l(\theta; x_j^*, y_j^*) + C_R \cdot R(\theta)$$

$$= \arg\min_{\theta \in \Theta} L(\theta; D_c \cup D_p^{i-1}) + (N + i - 1) \cdot C_R \cdot R(\theta)$$

$$= \arg\min_{\theta \in \Theta} \frac{1}{|D_c \cup D_p^{i-1}|} L(\theta; D_c \cup D_p^{i-1}) + C_R \cdot R(\theta)$$

This means that $A_{\text{FTL}}$ algorithm, at each step, trains a new model over the combination of clean data and poison data so far ($i - 1$ number of poisons). Now we want to see what is the translation of the Regret$(A_{\text{FTL}}, S_{\theta_i}, T)$. If we can prove an upper bound on regret, namely if we show $\text{Regret}(A_{\text{FTL}}, S_{\theta_i}, T) \leq \alpha(T)$ for some function $\alpha$, then we have

$$\sum_{j=0}^{T} l_j(\theta_j) - \sum_{j=0}^{T} l_j(\theta_p) \leq \sum_{j=0}^{T} l_j(\theta_j) - \min_{\theta \in \Theta} \sum_{j=0}^{T} l_j(\theta) \leq \alpha(T)$$

which implies

$$\sum_{j=0}^{T} l_j(\theta_j) - \sum_{j=0}^{T} l_j(\theta_p) = L(\theta_c; D_c) - L(\theta_p; D_c) + N \cdot C_R \cdot (R(\theta_c) - R(\theta_p))$$

$$+ \sum_{j=1}^{T} l_j(\theta_j) - \sum_{j=1}^{T} l_j(\theta_p)$$

$$= L(\theta_c; D_c) - L(\theta_p; D_c) + N \cdot C_R \cdot (R(\theta_c) - R(\theta_p))$$

$$+ \sum_{j=1}^{T} \left[ \max_{x,y} (l(\theta_j; x, y) - l(\theta_p; x, y)) + C_R \cdot (R(\theta_j) - R(\theta_p)) \right]$$

$$\leq \alpha(T)$$
We simplify the summation notation with θ₂

Then, for any

Above inequality states that average of the maximum loss difference in all previous rounds is bounded from above. Therefore, using averaging argument, there exist j ∈ [T] such that the maximum loss difference θ_j is ϵ-close to θ_p with respect to the loss based distance where

\[ \epsilon = \frac{\alpha(T) + L(\theta_p; D_c) - L(\theta_c; D_c) + N \cdot C_R \cdot (R(\theta_p) - R(\theta_c))}{T \cdot \gamma}. \]

The Theorem states the convergence to the target classifier θ_p (in terms of loss based distance) by showing that ϵ ≤ O\left(\frac{\log T}{T}\right), where ϵ → 0 when T → +∞. Since \( L(\theta_c; D_c) - L(\theta_p; D_c) \) is fixed, to show ϵ ≤ O\left(\frac{\log T}{T}\right), we only need to show \( \alpha(T) \leq O(\log T) \).

Our FTL framework formulation can utilize the existing logarithmic regret bound of adaptive FTL algorithm when the objective functions are strongly convex with respect to some norm \( \| \cdot \| \), as illustrated in Section 3.7 in McMahan et al. [16]. For clarity in presentation, we restate their related results below.

**Setting 1** (Setting 1 in McMahan et al. [16]). Given a sequence of objective loss functions \( f_1, f_2, \ldots, f_i \) and a sequence of incremental regularization functions \( r_0, r_1, \ldots, r_1 \) we consider an algorithm that selects the response point based on

\[ \theta_1 = \arg\min_{\theta \in \mathbb{R}^d} r_0(\theta) \]

\[ \theta_{i+1} = \arg\min_{\theta \in \mathbb{R}^d} \sum_{j=1}^{i} f_j(\theta) + r_j(\theta) + r_0(\theta), \text{ for } i = 1, 2, \ldots \]

We simplify the summation notation with \( f_{1:i}(\theta) = \sum_{j=1}^{i} f_j(\theta) \). Assume that \( r_i \) is a convex function and satisfy \( r_i(\theta) \geq 0 \) for \( i \in \{0, 1, 2, \ldots\} \), against a sequence of convex loss functions \( f_i : \mathbb{R}^d \rightarrow R \cup \{\infty\} \). Further, letting \( h_{0;i} = r_{0;i} + f_{1;i} \) we assume dom \( h_{0;i} \) is non-empty. Recalling \( \theta_i = \arg\min_{\theta} h_{0;i-1}(\theta) \), we further assume \( \partial f_{i}(\theta_i) \) is non-empty. We denote the dual norm of a norm \( \| \cdot \| \) as \( \| \cdot \|_\ast \).

**Theorem A.3** (Restatement of Theorem 1 in McMahan et al. [16]). Consider Setting 1 and suppose the \( r_i \) are chosen such that \( r_{0;i} + f_{1;i+1} \) is 1-strongly-convex w.r.t. some norm \( \| \cdot \|_{(i)} \). If we define the regret of the algorithm with respect to a selected point \( \theta^\ast \) as

\[ \text{Regret}_T(\theta^\ast, f_i) = \sum_{i=1}^{T} f_i(\theta_i) - \sum_{i=1}^{T} f_i(\theta^\ast). \]

Then, for any \( \theta^\ast \in \mathbb{R}^d \) and for any \( T > 0 \), with \( g_i \in \partial f_i(\theta_i) \), we have

\[ \text{Regret}_T(\theta^\ast, f_i) \leq r_{0;T-1}(\theta^\ast) + \frac{1}{2} \| g_i \|_{(i-1)}^2. \]
Corollary A.3.1 (Formalization of FTL result in McMahan et al. [16]). In the FTL framework (no regularizers are used in the optimization procedure), suppose each loss function \( f_i \) is 1-strongly convex w.r.t. a norm \( \| \cdot \| \), then we have

\[
\text{Regret}_T(\theta^*, f_i) \leq \frac{1}{2} \sum_{i=1}^{T} \frac{1}{i} \| g_i \|_\star^2 \leq \frac{G^2}{2} (1 + \log T)
\]

with \( \| g_i \|_\star \leq G \).

**Proof.** The proof basically follows from Theorem A.3. Since we are considering the FTL framework, let \( r_i(\theta) = 0 \) for all \( i \) and define \( \| \theta \|_{(i)} = \sqrt{i} \| \theta \| \). Observe that \( h_{0,i} \) is 1-strongly convex w.r.t. \( \| \theta \|_{(i)} \) and based on Lemma 3 in McMahan et al. [16], \( \| \theta \|_{(i),\star} = \frac{1}{\sqrt{i}} \| \theta \|_\star \). Then by applying Theorem A.3 we have

\[
\text{Regret}_T(\theta^*, f_i) \leq \frac{1}{2} \sum_{i=1}^{T} \| g_i \|_{(i),\star}^2 = \frac{1}{2} \sum_{i=1}^{T} \frac{1}{i} \| g_i \|_\star^2
\]

Based on the inequality of \( \sum_{i=1}^{T} 1/i \leq 1 + \log T \) and if we further assume \( \| g_i \|_\star \leq G \), then we can have

\[
\frac{1}{2} \sum_{i=1}^{T} \frac{1}{i} \| g_i \|_\star^2 \leq \frac{G^2}{2} (1 + \log T)
\]

\( \square \)

The logarithmic regret of \( \text{Regret}(A_{\text{FTL}}, S_{\theta_p}, T) \) of our algorithm then naturally follows from the result of \( \text{Regret}_\star(\theta^*, f_i) \) in Corollary A.3.1. Specifically, based on the assumption, the objective function \( l_i \) in our algorithm are 1-strongly convex with respect to a norm \( \| \cdot \| \) (\( l_0 \) is an exception that is \( \sqrt{N} \)-strongly convex, but does not impact the subsequent results on \( l_{0,i} \)) and therefore \( l_{0,i} \) is 1-strongly convex to norm \( \| \cdot \| = \sqrt{N} + i \| \cdot \| \). The norm of the gradient \( g_i \in \partial l_i \) is also bounded in practice. For example, in the case of Hinge loss and \( \ell_2 \)-regularizer for the SVM model, \( \| g_i \|_\star \) is bounded by the Lipschitz constant of \( l_i \) w.r.t. \( \| \cdot \|_2 \) (Lemma 2.6 in Shalev-Shwartz et al. [22]), and \( l_i \) is Lipschitz continuous when the data point \( x \) has bounded norm. Therefore, utilizing Corollary A.3.1 we have the logarithmic regret bound of our algorithm as

\[
\text{Regret}(A_{\text{FTL}}, S_{\theta_p}, T) \leq \alpha(T) = \frac{G^2}{2} (1 + \log T) \leq O(\log T).
\]

\( \square \)

We next provide the proof of the certified lower bound (restating Theorem 4.2 from Section 4.3):

**Theorem 4.2** Given a target classifier \( \theta_p \) to reproduce \( \theta_p \) by adding the poisoning set \( D_p \) into \( D_c \), the number of poisoning points \( |D_p| \) cannot be lower than

\[
\sup_{\theta} c(\theta) = \frac{L(\theta_p; D_c) - L(\theta; D_c) + NC_R(R(\theta_p) - R(\theta))}{\sup_{x,y} (l(\theta; x, y) - l(\theta_p; x, y)) + C_R(R(\theta) - R(\theta_p))}.
\]

The main intuition behind the theorem is, when the number of poisoning points added to the clean training set is lower than the certified lower bound, for structural empirical risk minimization problem (shown in 11 in the main paper), then target classifier will always have higher loss than another classifier and hence cannot be achieved.

**Proof.** We first show that for all models \( \theta \), we can derive a lower bound on the number of poison points required to get \( \theta_p \). Then since these lower bounds all hold, we can take the maximum over all of them and get a valid lower bound. We first show that for any model \( \theta \), the minimum number of poisoning points cannot be lower than

\[
c(\theta) = \frac{L(\theta_p; D_c) - L(\theta; D_c) + NC_R(R(\theta_p) - R(\theta))}{\sup_{x,y} (l(\theta; x, y) - l(\theta_p; x, y)) + C_R(R(\theta) - R(\theta_p))}.
\]
Let us denote the point corresponding to the supremum of the loss difference between $\theta$ and $\theta_p$ as $(x^*, y^*)^\dagger$. Namely, $l(\theta; x^*, y^*) - l(\theta_p; x^*, y^*) = \sup_{x,y} (l(\theta; x, y) - l(\theta_p; x, y))$. Now suppose we can obtain $\theta_p$ with lower number of poisoning points $c < c(\theta)$. Assume there is a poisoning set $D_p$ with size $c$ such that when added to $D_c$ would result in $\theta_p$. We have

$$
sup_{x,y} (l(\theta; x, y) - l(\theta_p; x, y)) \geq \frac{1}{|D_c \cup D_p|} l(\theta; D_c \cup D_p) - \frac{1}{|D_c \cup D_p|} l(\theta_p; D_c \cup D_p) > C_R \cdot (R(\theta_p) - R(\theta)),
$$

implying $\sup_{x,y} (l(\theta; x, y) - l(\theta_p; x, y)) + C_R \cdot (R(\theta) - R(\theta_p)) > 0$. Based on the assumption that $c < c(\theta)$, and the fact that $\sup_{x,y} (l(\theta; x, y) - l(\theta_p; x, y)) + C_R \cdot (R(\theta) - R(\theta_p)) > 0$, we have

$$
\begin{align*}
&\zeta \cdot (l(\theta; x^*, y^*) - l(\theta_p; x^*, y^*) + C_R(R(\theta) - R(\theta_p))) \\
&< c(\theta) \cdot (l(\theta; x^*, y^*) - l(\theta_p; x^*, y^*) + C_R(R(\theta) - R(\theta_p))) \\
&= L(\theta_p; D_c) - L(\theta_p; D_c) + NC_R(R(\theta_p) - R(\theta)).
\end{align*}
$$

where the equality is based on the definition of $c(\theta)$. On the other hand, by definition of $(x^*, y^*)$ for any $D_p$ of size $\zeta$, we have

$$
L(\theta; D_p) - L(\theta_p, D_p) + \zeta \cdot (C_R \cdot R(\theta) - C_R \cdot R(\theta_p)) \\
\leq \zeta \cdot (l(\theta; x^*, y^*) - l(\theta_p; x^*, y^*) + C_R(R(\theta) - R(\theta_p))).
$$

The above two inequality imply that for any set $D_p$ with size $\zeta$ we have

$$
\frac{1}{|D_c \cup D_p|} L(\theta; D_c \cup D_p) + C_R \cdot R(\theta) < \frac{1}{|D_c \cup D_p|} L(\theta_p; D_c \cup D_p) + C_R \cdot R(\theta_p).
$$

which indicates that adding $D_p$ poisoning points into the training set $D_c$, the model $\theta$ has lower loss compared to $\theta_p$, which is a contradiction to the assumption that $\theta_p$ has lower loss on $D_c \cup D_p$ and can be achieved. Now, since $\theta_p$ needs to have lower loss on $D_c \cup D_p$ compared to any classifier $\theta \in \Theta$, the best lower bound is the supremum over all models in the model space $\Theta$. \hfill $\Box$

**B Relating $\epsilon$-closeness to closeness of parameters**

In theorem below, we show how one can relate the notion of $\epsilon$-closeness in Definition 1 in the main paper to closeness of parameters in the specific setting of hinge loss. We use this just as an example to show that our notion of $\epsilon$-closeness can be tightly related to the closeness of the models.

**Theorem B.1.** Consider the hinge loss function $l(\theta; x, y) = \max(1 - y \cdot \langle x, \theta \rangle, 0)$ for $\theta \in [-1, +1]^n$ and $x \in [-1, +1]^n$ and $y \in \{-1, +1\}$. For $\theta, \theta' \in [-1, +1]^n$ such that $||\theta||_1 \leq R$ and $||\theta'||_1 \leq R$ for some $R \geq 1$, if $\theta$ is $\epsilon$-close to $\theta'$ in the loss based distance, for some $\epsilon \leq 1$ then, $||\theta - \theta'||_1 \leq R \cdot \epsilon$.

**Remark 4.** In Theorem B.1 above, a naive upper bound on the norm of $\theta$ and $\theta'$ is $n$, however, the models that we care about in practice usually have much smaller norm because of the regularization.

**Proof.** We construct a point $x^*$ as follows:

$$
x_i^* = \begin{cases} 
-\frac{1}{R}, & \text{if } \theta_i > \theta_i', \ i \in [n] \\
+\frac{1}{R}, & \text{if } \theta_i \leq \theta_i', \ i \in [n]
\end{cases}
$$

Then we have

$$
\langle \theta - \theta', x^* \rangle = \frac{1}{R} \cdot ||\theta - \theta'||_1 \tag{3}
$$

Since $||\theta||_1 \leq R$ we have

$$
\langle x^*, \theta \rangle \geq -1 \tag{4}
$$

and similarly since $||\theta'||_1 \leq R$ we have

$$
\langle x^*, \theta' \rangle \geq -1. \tag{5}
$$

In practice, the data space $X$ is a closed convex set and hence, we can find $(x^*, y^*)$ using convex optimization. In other words, as we saw in experiments, calculating the lower bound is possible in practical scenarios.
Therefore by Inequalities (4) and (5) we have
\[
l(\theta; x^*, -1) - l(\theta'; x^*, -1) = \max(1 + \langle x^*, \theta \rangle, 0) - \max(1 + \langle x^*, \theta' \rangle, 0) = \langle \theta - \theta', x^* \rangle
\]
which by Equation (3) implies
\[
l(\theta; x^*, -1) - l(\theta'; x^*, -1) = \frac{1}{R} \cdot \| \theta - \theta' \|_1.
\]
Now since we know that, \( \forall x \in [-1, +1]^n \), the loss difference between \( \theta \) and \( \theta' \) is bounded by \( \epsilon \), the bound should also hold for the point \( (x^*, -1) \), meaning that
\[
\frac{1}{R} \cdot \| \theta - \theta' \|_1 \leq \epsilon
\]
which completes the proof. \( \square \)

C Indiscriminate Poisoning Attacks

In this section, we evaluate the attacks in the conventional indiscriminate attack setting, where the attacker’s goal is just to reduce the overall accuracy of the model.

Datasets and Models. For the indiscriminate attack, we use the MNIST-17 dataset, which consists of the digits 1 and 7 and is commonly used for evaluating indiscriminate poisoning attacks against binary classification [24, 2, 26]. The MNIST-17 dataset contains 13007 training data and 2163 test data. The dataset contains 784 features and all the features are normalized into range \([0, 1]\). For completeness, the Adult dataset used for subpopulation attack is downsampled to form a class-balanced dataset and contains 15682 training data and 7692 test data. The dataset contains 57 features and the features are also normalized into range \([0, 1]\) (except for the binary features). All of the processed datasets are included in the supplementary material. We still adopt linear SVM model in the indiscriminate attack scenario. All of the models for both datasets set the regularization parameter \( C_R = 0.09 \). We choose the hyperparameters by some simple grid search. All the experiments can be run on personal laptops without additional computing resources.

Target Classifiers. Accuracy of the clean MNIST-17 model has around 1% error rate on the test set. For our experiment, we aim to generate three target classifiers with overall test accuracies around 5%, 10% and 15%. To generate target classifiers with desired error rates, we follow the heuristic strategy proposed by Koh et al. [9] to generate multiple candidate target classifiers, and then among all the valid candidate models that satisfy the error rate requirement we choose the one with lowest loss on the clean training set. Using this approach, the final target classifiers induced have overall test accuracy of 94.0%, 88.8% and 83.3% respectively. (We describe a better way of finding the target classifiers in Appendix C.1, but for comparison purposes do not use those in the results here.)

Convergence. We show the convergence of Algorithm 1 by reporting the maximum loss difference and Euclidean distance between the classifier induced by the attack and the target classifier. Figures 3a and 3b summarize the results for the target classifier with a 10% error rate. The maximum number of
Figure 4: Test accuracy for each target model of given error rate with classifiers induced by poisoning points obtained from our attack and the KKT attack.

|               | 5% Error  | 10% Error | 15% Error |
|---------------|-----------|-----------|-----------|
| # of Poisons  | 1737      | 5458      | 6192      |
| Lower Bound   | 874       | 3850.4 ± 0.8 | 4904     |

Table 2: Theoretical lower bounds on the number of poisoning points needed to achieve classifiers induced from our attack for different target errors. All results are averaged over four runs. An integral value in a cell means we get exactly that same value for all four runs; for the one cell where we observe variation, we report the average and standard error.

|               | 5% Error  | 10% Error | 15% Error |
|---------------|-----------|-----------|-----------|
| # of Poisons  | 1737      | 5458      | 6192      |
| Lower Bound   | 856       | 4058.4+1.4 | 5031.4+4.8 |

Table 3: Theoretical lower bounds on the number of poisoning points needed to achieve classifiers induced from the KKT attack for different target errors. All results are averaged over 4 runs, integer value in the cell means we get exactly same value for 4 runs and others are shown with the average and standard error.

poisoning points in the figure is obtained when the classifier from Algorithm 1 is 0.1-close to the target classifier in the loss-based distance. In Figure 3, the classifier induced by our algorithm steadily converges to the target classifier both in the maximum loss difference and Euclidean distance, while the classifier induced by the KKT attack diverges initially and then starts to converge to the target model. At the maximum number of points, the maximum loss difference of KKT-induced classifier to the target is 0.46, compared to 0.1 for the classifier induced by our attack. For the Euclidean distance, the KKT-induced classifier is 0.16 away, compared to 0.07 for the classifier induced by our attack.

**Attack Success.** We next compare the classifier induced from our attack to the classifier induced by the KKT attack in terms of their overall test accuracy. Similarly, the maximum number of poisoning points in Figure 2 is obtained by running our attack with 0.1-closeness (in loss based distance) to the target as the input. In terms of the test accuracy, our attack has a comparable attack success rate compared to the KKT attack. Specifically, for target models of 5% and 10% error rates, both methods have almost identical performance, as shown in Figures 4a and 4b. For the target model of 15% error rate (test accuracy is 83.3%), the KKT attack is more successful than our attack, inducing models with 82.7% accuracy (17.3% attack success rate) compared to 85.9% accuracy (14.1% attack success rate) for our attack. Interestingly, in this case, the performance of models induced by the two attacks with fewer poisoning points our attack results in models with lower test accuracy (higher attacker success) than the KKT attack. To summarize, for the indiscriminate scenario, our attack produces classifiers that have much closer distance to the target models than the KKT attack, and has comparable attack success rates with the KKT attack.

**Lower Bound on Number of Poisons.** We next check the optimality of our attack in the indiscriminate attack scenario. Similar to the subpopulation attack setting, we still use Theorem 4.2 to compute
the lower bound of the induced classifier from our attack by using it as the input to Algorithm 1 and terminating when the induced classifier is 0.1-close to the given target model. Our calculated lower bound shows that there exists a relatively large gap between the number of poisoning points, especially for the induced classifier from our attack for the target model of 5% error rate, where the lower bound is only 50% of the actual number of poisoning points used. For the induced classifier for the target model of 15% error rate, the gap between the number of poisoning points and the lower bound is smallest, with the lower bound taking 79% of the number of poisoning points. The relatively large gap indicates that either the estimated lower bound is not tight or the attack itself is not close to optimal. To gain more insights into this problem, we further show the computed lower bound at each iteration when running Algorithm 1 and Figure 5 summarizes the results. From the Figure, we see that, the peak value of the curve (i.e., highest lower bound) always appears before the termination of the algorithm, indicating that the computed lower bound is likely to be tight and we may need to further improve the attack algorithm.

For completeness, we also repeat the lower bound computation process for classifiers induced from the KKT attack. The KKT induced classifiers are obtained by running the KKT attack with target classifiers of different error rates as target input. The target number of poisoning points of KKT attack is given by the size of poisoning set returned from our attack when our algorithm terminates when the induced classifier is 0.1-close to the target model of different error rates. Then the lower bound computation process is identical to the above – we simply send the KKT induced classifier as target input to Algorithm 1 and terminate it when the induced classifier from our algorithm is 0.1-close to the given target model (i.e., KKT induced classifier). The results are summarized in Table 3 and we observe that there also exists a relatively large gap between the lower bound and the number of poisoning points used by the attack. Similarly, we also plot the lower bound computed at different iterations of Algorithm 1 in Figure 6 and find that the peak value also appears before the termination of the algorithm, indicating that the lower bound might be tight and we need a stronger attack strategy to close the gap.
| Target Models | Test Acc (%) | Loss on Clean Set | # of Poisons |
|---------------|--------------|-------------------|-------------|
|               | Original     | Improved          | Original    | Improved    | Original | Improved |
| 5% Error      | 94.0         | 94.9              | 2254.6      | 1767.1      | 2170     | 1340     |
| 10% Error     | 88.8         | 88.9              | 4941.0      | 3233.1      | 5810     | 2432     |
| 15% Error     | 83.3         | 84.5              | 5428.4      | 4641.6      | 6762     | 3206     |

Table 4: Comparison of Two Target Generation Methods on Number of Poisoning Points Used to Reach 0.1-closeness to the Target. Original indicates the original target generation process from Koh et al. [9]. Improved denotes our improved target generation process with adaptive model updating.

Figure 7: Test Accuracy with classifiers obtained from our attack and KKT attack. Target model for KKT attack is generated from the original generation process and target model for our attack is generated from the improved generation process. Maximum number of poisoning points is obtained by running our attack with target model generated from the original process and resultant classifier is 0.1-close to the target.

C.1 Improved Target Generation Process

The original heuristic approach works by finding different quantiles of training points that have higher loss on the clean model, flipping their labels, repeating those points for multiple copies, and adding them to the clean training set. We find that, in the process of trying different quantiles and copies of high loss points, if we also adaptively update the model where the high loss points are found (instead of just always fixing it to be the clean model), we can generate a valid target classifier with even lower loss on the clean training. Such an improved generation process can significantly reduce the number of poisoning points needed to reach the same \( \epsilon \)-closeness (with respect to the loss based distance) to the target classifier, consistent with the claims in Theorem 4.1 in the main paper. In addition, we find that, if we compare our attack with improved generation process to the KKT attack with the original generation process [9], we can also reach the desired target error rate much faster using our attack.

Results. We first empirically validate Theorem 4.1 in the main paper. Obtaining the same \( \epsilon \)-closeness in loss-based distance, a target classifier with lower loss on the clean training set \( D_c \) requires fewer poisoning points. Therefore, when adversaries have multiple target classifiers that satisfy the attack goal, the one with lower loss on clean training set is preferred. This also reduces the risk of detection by the model owner.

For both the original and improved target generation methods, we generate three target classifiers with error rates of 5%, 10% and 15%. The original target classifier generation method returns classifiers with test accuracy of 94.0%, 88.8% and 82.3% respectively (also used in the previous experiments of indiscriminate attack). The improved target generation process returns target classifiers with approximately the same test accuracy (94.9%, 88.9% and 84.5%). However, for classifiers returned from the two generation methods of same error rate, the improved generation method produces classifiers with significantly lower loss compared to the original generation approach.

Table 4 compares the two target generation approaches by showing the number of poisoning points needed to get 0.1-close to the corresponding target model of same error rate. For example, for target models of 15% error rate, the model from the original approach has a total clean loss of 5428.4 while our improved method reduces it to 4641.6. With the reduced clean loss, getting 0.1-close to the target...
model generated from our improved process only requires 3206 poisoning points, while reaching the same distance from the target model produced by the original method would require 6762 poisoning points, a more than 50% reduction.

Figure 7 compares the two attacks in an end-to-end manner in terms of their test accuracy. With the improved target generation process, our attack can achieve the desired error rate more efficiently than the KKT attack with original process. For the KKT attack with target model from original process, we determine the target number of poisoning points using the size of poisoning set returned from running Algorithm 1 with 0.1-closeness and target model from original process as inputs. To run our attack with improved generation process, we terminate the algorithm when the size of the poisoning points is same as the number of poisoning points used by the KKT attack with original process. Such a termination criteria helps us to ensure that both attacks use same number of poisoning points and can be compared easily. We also evaluate the KKT attack on fractions of the maximum target number of poisoning points (0.2, 0.4, 0.6, and 0.8), as in the previous experiments. The accuracy plot shows that our attack (with improved target model) can achieve the desired error rate (e.g., 10%) much faster than the KKT attack (with original target model), especially for the target classifiers of error rates of 10% and 15%.

D Comparison of Model-Targeted and Objective-Driven Attacks

Model-targeted and objective-driven attacks have significantly different attack goals: the former aims to induce the target classifier while the latter aims to achieve the attacker objective such as increasing overall error rate in indiscriminate setting. Therefore, the two types of attacks are not directly comparable. However, the model-targeted attacks can also perform well in the attacker objective of objective-driven attacks, but requires providing a “better” target model. Here, we provide some observations of a “better” target model when one is interested in comparing the model-target attacks (e.g., the attack proposed in this paper) to some objective-driven attacks in indiscriminate attack scenario. In the indiscriminate setting, we observe that, in order for our model-targeted attacks to have same accuracy drops using less number of poisoning points, the target model needs to have larger error rate. For example in figures 7a and 7c if the adversary aims to get a classifier that has error rate around 5%, then choosing a target classifier of 5% error rate can reach the desired 5% error rate using approximately 1500 poisoning points. In contrast, if the adversary uses a target classifier of 15% error rate, the same 5% error rate can be achieved using only 500 poisoning points. Therefore, for our attack, choosing a target classifier of higher error rate can produce the classifier of the desired error rate using much lower number of poisoning points. Similar observation is also found for the KKT attack when comparing it with other strong objective-driven attacks (e.g., the min-max attack [24]) in indiscriminate setting [9]. Specifically, Koh et al. [9] generate multiple target models with varying error rates and run the KKT attack with each of the target models. Then among all the target models, they choose the one with which the KKT attack can reduce the overall classification accuracy most. Generally, the chosen target classifier at the end is the one with much larger error rate.