Un-Casimir effect

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In this paper we present the un-Casimir effect, namely the study of the Casimir energy related to the presence of an un-particle component in addition to the electromagnetic field contribution. We derive this result by considering modifications of the Feynman propagator in the unparticle sector. The contribution of unparticles turns out to be the integral of Casimir energies for particles at fixed mass with a weight depending on the scaling dimension. The distinctive feature of the un-Casimir effect is a fractalization of metallic plates. This fact emerges through a new dependence of the Casimir energy on the distance between plates, that scales with a continuous power controlled by the scaling dimension. More importantly the un-Casimir effect offers a reliable testbed for unparticle physics. We find bounds on the unparticle scale that are independent on the effective coupling constant describing the interaction between the scale invariant sector and ordinary matter. Therefore the un-Casimir effect, contrary to what found in previous unparticle physics situations (e.g. g-2 and a variety LEP/LHC data analyses), actually removes the ambiguity associated to the value of such a coupling. We also discuss some of the possible implications for unparticle physics when non-perfect conducting plates are considered.

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1 Introduction

Following a recent conjecture, there could be a sector of the Standard Model that, although massive, can preserve scale invariance properties upon the condition of exhibiting a non-integer number of particles. The topic has intersected a huge variety of fields, spanning from astrophysics to neutrino physics, AdS/CFT duality and quantum gravity. Such unlike kind of particles (shortly unparticles), being conventionally unobserved, has to be weakly interacting with the rest of the Standard Model. On these grounds, in the seminal paper [1], an effective field theory approach has been employed to derive, from a perturbative analysis, the couplings of the unparticle fields to the Standard Model fields.

At a very high energy scale $M_U$ the standard model fields interact with a field sector exhibiting a non-trivial infrared Banks-Zaks fixed point [2]

$$\mathcal{L} = \frac{1}{(M_U)^k} O_{SM} O_{BZ},$$

where the field operators have dimensions $d_{SM}, d_{BZ}$ and $k = d_{SM} + d_{BZ} - 4$ for dimensional consistency. Being the Banks-Zaks fields unobserved, their suppression requires that the scale $M_U$ is somewhere between the currently experimentally accessible scales and the Planck scale, with $k > 0$. At the fixed point, i.e., at a scale $\Lambda_U$ in between the observed scales and $M_U$, the Banks-Zaks sector has scale invariant properties and the particle number is controlled by a continuous parameter $d_U \neq d_{BZ}$. This is equivalent to saying that Banks-Zaks fields undergo a dimensional transmutation to become unparticles as

$$\frac{1}{(M_U)^k} O_{SM} O_{BZ} \sim \Lambda_U \lambda \frac{1}{(\Lambda_U)^{k_U}} O_U O_{SM}$$

where the unparticle operator $O_U$ has dimension $d_U$ and $k_U = d_U + d_{SM} - 4$. We notice that the resulting interaction term depends on a dimensionless coupling constant $\lambda = (\Lambda_U/M_U)^k$, that has to be smaller than 1.

Using unitarity constraint bounds on conformal field theory (CFT) it is possible to set the lower bound $d_U \geq 1$ [3]. Normally only operators with $d_U \leq 2$ are considered because for $d_U \geq 2$ the calculations become sensitive to the ultraviolet sector and therefore less predictive.

The phenomenology of unparticles and its associated signals have received quite a lot of attention in the literature during recent years. For more details than those that can be recalled here we refer the reader to the recent reviews in [4, 5]. In particular several authors have concentrated on the study of unparticle production at high energy colliders [4, 6]. We quote interesting recent analyses of LEP data which provide lower bounds on the scale $\Lambda_U > 20$ TeV for $d_U = 1.1$ down to $\Lambda_U > 0.2$ TeV for $d_U = 2.0$ [6]. Also after the start up of the CERN Large Hadron Collider (LHC) experiments, unparticle signals have extensively been searched: preliminary results provide further constraints to the parameter space, i.e., $(\Lambda_U, d_U)$ in Fig [7, 8] (left panel).
On the theoretical side, bounds on the same parameter space have been derived by computing the unparticle contribution to the muon anomaly \[9\]. Unparticles play also a relevant role in the field of short scale modifications of gravity. Black hole solutions have been derived for the case of scalar \[10, 11, 12\] and vector \[13\] unparticle exchange. As a major result, one finds a fractalization of the event horizon whose dimension turns to be controlled by the unparticle parameter \(d_U\). This feature has been confirmed by subsequent studies about the spectral dimension, as indicator of the short scale spacetime dimension perceived by an unparticle probe \[14\]. In addition, such kind of unparticle gravitational modifications offer compelling effects that can be experimentally exposed in the short scale measurement of Newton’s law \[15\].

Finally we might recall that it has recently been reported that, being the conformal dimension \(d_U\) in general not a half integer number, clockwise and counterclockwise interchange of un-particles do not lead to the same phase and time-reversal symmetry is broken spontaneously. This may have some relevance for experiments in the pseudo-gap phase of the cuprates \[16\].

Despite the great efforts devoted to these issues in a variety of fields, to the best of our knowledge the Casimir effect within the unparticle scenario has not been considered so far, although it has been discussed within various scenarios beyond the standard model such as in compactified extra dimensions \[17\] and in minimal length theories \[18, 19\]. With this paper we wish to fill this gap and we will discuss the Casimir effect assuming the existence of a scalar unparticle sector weakly coupled to the standard model fields. We will refer to the unparticle Casimir effect as the un-Casimir effect.

## 2 Un-particle contribution to the Casimir effect

The Casimir energy \[20\] is often described by the shift in the sum of the zero point energies of the normal modes of the electromagnetic field induced by the introduction of geometrical boundary conditions, like for example metallic plates at a given distance:

\[
\mathcal{E}^C = \frac{1}{2} \sum_n \left[ \omega_n \bigg|_{\text{boundary}} - \omega_n \bigg|_{\text{no-boundary}} \right].
\]  

(3)

This can also be written by means of the density of states \(dN/d\omega\) as

\[
\mathcal{E}^C = \frac{1}{2} \int d\omega \omega \left[ \frac{dN}{d\omega} \bigg|_{\text{boundary}} - \frac{dN}{d\omega} \bigg|_{\text{no-boundary}} \right].
\]  

(4)

On the other hand from quantum field theory (QFT) \[21, 22\] the density of states is related to the imaginary part of the trace of the Feynman propagator by

\[
\frac{dN}{d\omega} = -\frac{1}{\pi} \text{Im} \left[ \int dr \, \text{Tr} \, D(r, r; \omega) \right],
\]  

(5)
where the trace is intended over the spinor degrees of freedom of the field under consideration (assuming a boson field). Therefore we have

\[ \mathcal{E}^C = - \frac{1}{2\pi} \Im \left[ \int d\omega \omega \int d\mathbf{r} \times \right. \]
\[ \left. \mathrm{Tr} \left( \mathbf{D}(\mathbf{r}, \mathbf{r}; \omega) - \mathbf{D}^0(\mathbf{r}, \mathbf{r}; \omega) \right) \right] , \tag{6} \]

where \( \mathbf{D}(x, x') \) is the Green function of the field in the presence of boundary conditions, while \( \mathbf{D}^0(x, x') \) is the free field propagator.

The un-particle sector \[1\] is described by a modified Feynman propagator \[23, 14, 24\] which is given by the following representation

\[ \mathbf{D}_U(x, x') = \frac{A_{d_U}}{2\pi(A_{d_U}^2)^{d_U-1}} \int_0^\infty dm^2(m^2)^{d_U-2} \mathbf{D}(x, x'; m^2) \]
\[ A_{d_U} = \frac{16\pi^{3/2}}{(2\pi)^{2d_U} \Gamma(d_U + 1/2)} \frac{\Gamma(2 - d_U)}{\Gamma(d_U - 1) \Gamma(2d_U)} \tag{7} \]

\( i.e. \) it is a linear continuous superposition of Feynman propagators of fixed mass \( m \).

It is worth noting that when the conformal dimension tends to unity \((d_U \to 1)\) then the un-particle propagator reduces to that of an ordinary massless field. This can be seen readily from Eq. (7) by computing the unparticle propagator in momentum space. Considering for simplicity the scalar case and working in euclidean space one can use a Schwinger representation for \( D(p; m^2) \) to get:

\[ D_U(p^2) = \frac{A_{d_U}}{2\pi(A_{d_U}^2)^{d_U-1}} \int_0^\infty dm^2(m^2)^{d_U-2} \int_0^\infty ds e^{-s(p^2 + m^2)} \]
\[ = \frac{16\pi^{3/2}}{(2\pi)^{2d_U+1}(A_{d_U}^2)^{d_U-1} \Gamma(2d_U)} \frac{(p^2)^{d_U-2}}{2} \]
\[ \to \frac{1}{p^2} \text{ as } d_U \to 1 \tag{8} \]

The un-particle contribution to the Casimir energy is therefore:

\[ \mathcal{E}^C_{U} = - \frac{1}{2\pi} \Im \left[ \int d\omega \omega \int d\mathbf{r} \times \right. \]
\[ \left. \mathrm{Tr} \left( \mathbf{D}_U(\mathbf{r}, \mathbf{r}; \omega) - \mathbf{D}^0(\mathbf{r}, \mathbf{r}; \omega) \right) \right] . \tag{9} \]

Eq. (7) clearly holds as well for the Fourier components of \( \mathbf{D}_U(x, x'), \mathbf{D}(x, x') \) and \( \mathbf{D}^0(x, x') \) and one can thus write

\[ \mathcal{E}^C_{U} = - \frac{1}{2\pi} \Im \frac{A_{d_U}}{2\pi(A_{d_U}^2)^{d_U-1}} \times \]
\[ \int_0^\infty dm^2(m^2)^{d_U-2} \int d\omega \omega \int d\mathbf{r} \times \]
\[ \mathrm{Tr} \left[ \mathbf{D}(\mathbf{r}, \mathbf{r}; \omega) \bigg|_{m^2} - \mathbf{D}^0(\mathbf{r}, \mathbf{r}; \omega) \bigg|_{m^2} \right] . \tag{10} \]
Comparing with Eq. (6) we can evidently relate the unparticle Casimir energy \( E_{\text{UC}} \) with the standard Casimir energy of a particle field of fixed mass \( E_C(m^2) \):
\[
E_{\text{UC}} = \frac{A_{d_{\mu}}}{2\pi(\Lambda_{d_{\mu}}^2)^{d_{\mu}-1}} \int_0^\infty dm^2 (m^2)^{d_{\mu}-2} E_C(m^2) \nonumber
\]
\[
= \frac{A_{d_{\mu}}}{\pi(\Lambda_{d_{\mu}}^2)^{d_{\mu}-1}} \int_0^\infty dm m^{2d_{\mu}-3} E_C(m). \quad (11)
\]
This is our central result. We notice that Eq. (11) is formally equivalent to the Casimir effect in Randal Sundrum [25, 26] type II models, where the hidden 3-brane taken at infinity generate a continuous spectrum of Kaluza-Klein excitations [27].

3 Un-Casimir effect

We consider now the Casimir effect of a scalar particle in the geometry of the parallel plates separated by a distance \( a \). This particular case can be solved exactly for arbitrary masses and the result is [28, 29, 30]
\[
E_C(m) = -\frac{1}{8\pi^2} \frac{m^2}{a} \sum_{n=1}^{\infty} \frac{1}{n^2} K_2(2amn), \quad (12)
\]
where \( K_2(z) \) is a modified Bessel function of the second type. Then the un-particle Casimir energy becomes
\[
E_{\text{UC}}(a) = -\frac{1}{8\pi^2} \frac{1}{a} \frac{A_{d_{\mu}}}{\pi(\Lambda_{d_{\mu}}^2)^{d_{\mu}-1}} \times \nonumber
\]
\[
\sum_{n=1}^{\infty} \frac{1}{n^2} \int_0^\infty dm m^{2d_{\mu}-1} K_2(2amn). \quad (13)
\]
By integrating over \( m \) one finds
\[
\int_0^\infty dm m^{2d_{\mu}-1} K_2(2amn) = \frac{\Gamma(d_{\mu} - 1)\Gamma(d_{\mu} + 1)}{4(an)^{2d_{\mu}}} \quad (14)
\]
Finally upon introduction of the Riemann Zeta function \( \zeta(s) = \sum_{n=1}^{\infty} n^{-s} \) the scalar un-particle Casimir energy reads
\[
E_{\text{UC}}(a) = -\frac{1}{a^3} \frac{d_{\mu} \zeta(2 + 2d_{\mu})}{(4\pi)^{2d_{\mu}}} \frac{1}{(a\Lambda_{d_{\mu}})^{2d_{\mu}-2}}. \quad (15)
\]
It is interesting to note that the factor \( \Gamma(d_{\mu} - 1) \) appearing in Eq. (14), which diverges in the limit \( d_{\mu} \to 1 \), is actually canceled by the same factor in the denominator of \( A_{d_{\mu}} \) (see Eq. (7)) therefore making the final result in Eq. (17) perfectly finite. In addition
it can also be readily verified that in the limit $d_{U} \to 1$ Eq. (17) reproduces the ordinary result for the Casimir effect of the scalar massless field, *i.e.*, 

$$\lim_{d_{U} \to 1} \mathcal{E}_{U}^{C}(a) = -\frac{\pi^{2}}{1440 a^{3}}.$$  \hspace{1cm} (16)

We would also like to remark that, despite the similarities with the Casimir effect of RSII type [27], the final explicit result (17) bears also important differences. The unparticle contribution $\mathcal{E}_{U}^{C}$ depends not only on the new energy scale but, more importantly, also on the conformal dimension $d_{U}$. This makes the unparticle contribution sizable for $d_{U}$ approaching 1. On the contrary the RSII Casimir energy is always suppressed since it scales as $(\kappa a)^{-1} \approx 10^{-28}$, where $\kappa \sim 10^{19}$ GeV is the curvature parameter of the warped dimension and the separation lengths is typically $a \sim 1 \mu$m.

As a related remark, we notice that unparticles actually introduce a new, distinctive effect, *i.e.*, a fractalization of metallic plates. This turns to be evident by writing (17) as

$$\mathcal{E}_{U}^{C}(a) = -\frac{1}{a^{D+1}} \frac{d_{U} \zeta(2+2d_{U})}{(4\pi)^{2d_{U}}} \frac{1}{(\Lambda_{U})^{2d_{U}-2}}.$$  \hspace{1cm} (17)

In the conventional case, $d_{U} = 1$, one finds $D = D$, corresponding to the topological dimension $D = 2$ of the boundary (we recall that, on dimensional grounds, $\mathcal{E}(a)$ is an energy per unit of surface). On the other hand, when $d_{U} \neq 1$, the dimensional parameter $D$ departs from integer values, a typical feature of fractal surfaces. Specifically one finds that $D$ is completely controlled by the unparticle scale dimension and reads $D = 2d_{U}$. This result is in agreement to an equivalent fractalization of the horizon of a black hole solution obtained from scalar [15, 10, 12] and vector [13] uparticle exchange. The fractality encoded in unparticles has also been studied from a more general viewpoint. Fractals require the introduction of dimensional indicators like the spectral dimension, *i.e.*, the dimension perceived by a diffusive process or random walker. To this purpose it has been shown that the complete fractalization of plates, *i.e.*, $D = 2d_{U}$ is a general result descending from the spectral dimension for an unparticle field propagating on a manifold with topological dimension $D = 2$ [14].

Finally we find that, for two parallel metallic plates separated by a distance $a$, the total attractive energy reads

$$\mathcal{E}^{C}(a) = -\frac{\pi^{2}}{720 a^{3}} \left[ 1 + \frac{720 d_{U} \zeta(2+2d_{U})}{\pi^{2}(4\pi)^{2d_{U}}} \frac{1}{(a\Lambda_{U})^{2d_{U}-2}} \right].$$  \hspace{1cm} (18)

The above result exhibits an additional contribution to the standard electromagnetic Casimir effect (given by twice Eq. (16)). Eq. (18) allows the definition of the spectral dimension of plates in terms of the Casimir energy as

$$D = -\frac{\partial \log \left( \mathcal{E}^{C}(a) \right)}{\partial \log a} - 1.$$  \hspace{1cm} (19)
where \( E^C(a) \) and \( a \) play the role of the return probability and the diffusion time respectively. By using Eq. (18) one finds

\[
D = \frac{2 + (2d_U) L}{1 + L},
\]

(20)

where \( L = \frac{720 d_U \zeta (2 + 2d_U)}{\pi^2 (4\pi)^{2d_U}} \frac{1}{(a \Lambda_U)^{2d_U - 2}} \).

This formula shows, for \( d_U > 1 \), a dimensional flow interpolating the following two regimes. For large plate separation \( a \gg 1/\Lambda_U \) we recover the usual topological result, \( i.e., D \to 2 \); on the other hand in the unparticle dominated case \( a \ll 1/\Lambda_U \) the plate fractalization takes place, \( i.e., D \to 2d_U \). The conventional Casimir result, \( D = 2 \), is recovered by taking the limit of Eq. (19) for \( d_U \to 1 \). As expected \( \Lambda_U \) is the critical scale at which the transition between the two phases, ordinary matter and unparticles, occurs.

4 Discussion

The un-Casimir effect offers important phenomenological predictions. We can get an estimate of the unparticle scale \( \Lambda_U \) as follows. If \( \Delta_{\text{Cas}} \) is the relative error of the experimental measurement we obtain

\[
\frac{720 d_U \zeta (2 + 2d_U)}{\pi^2 (4\pi)^{2d_U}} \frac{1}{(a \Lambda_U)^{2d_U - 2}} \leq \Delta_{\text{Cas}}.
\]

(21)

Therefore we get (for \( d_U \neq 1 \))

\[
\Lambda_U \geq \frac{1}{a} \left[ \frac{720 d_U \zeta (2 + 2d_U)}{\pi^2 (4\pi)^{2d_U}} \frac{1}{\Delta_{\text{Cas}}} \right]^{\frac{1}{2d_U - 2}}
\]

(22)

We notice that there is a strong dependence on the parameter \( d_U \). In particular for values of \( d_U \) slightly above 1 the bound on \( \Lambda_U \) is very strong while as soon as \( d_U \) increases the bound exponentially decreases. Clearly the scale of the constraint is set by the separation between the plates. In Fig. 1 (left panel) we show the exclusion curves in the parameter plane \( \Lambda_U, d_U \) for different choices of the experimental resolution at a given plate separation and with different plate separation at a fixed experimental resolution (right panel).

We note that the final result for the unparticle contribution of the muon anomaly presents the same functional relation on the scale \( \Lambda_U \) as in our Eq. (21). More precisely the unparticle contribution to the muon anomaly is bounded by the difference of the experimental result with the Standard Model prediction \[31\], \( \Delta_\mu = \Delta a_\mu (\text{exp}) - \Delta a_\mu (\text{SM}) = 22 \times 10^{-10} \). Eq.(16) of \[9\] therefore translates in the bound :
Figure 1: (left panel) Lower bounds on $\Lambda_{U}$, the energy scale of the scalar unparticle sector, plotted from Eq. (22) assuming three different values of the relative precision of the Casimir effect measurement and a fixed separation of $a = 1\mu m$: the lower (dotted) line is for $\Delta_{\text{Cas}} = 10^{-2}$, the central (dashed) line is for $\Delta_{\text{Cas}} = 10^{-3}$ and the upper (solid) line is for $\Delta_{\text{Cas}} = 10^{-4}$. In the same plot we compare the un-Casimir bound with those from direct searches at high energy colliders: LEP [6] (blue) solid line with full dots and LHC [7] (green) solid line with full dots and also with those from the muon anomaly [9]. Note that the g-2 bounds given in the upper filled area refer to different choices of the coupling coefficient $\lambda$: the solid (grey) line is for $\lambda = 1$, the dashed (grey) line is for $\lambda = 0.1$ and the dotted (grey) line is for $\lambda = 0.05$. (right panel) Lower bounds on $\Lambda_{U}$, the energy scale of the scalar unparticle sector, plotted from Eq. (22) assuming three different combinations of the plates distance $a$ and the Casimir experiment relative error $\Delta_{\text{Cas}}$.

with $Z_{d_{U}} = A_{d_{U}}/(2 \sin(\pi d_{U})$ and $\lambda$ the unknown effective coupling of the unparticle field to the standard model operator. Eq. (23) can be compared directly with our unparticle bound on $\Lambda_{U}$, Eq. (21). One can easily see that they share various similarities. They have the same functional dependence on $\Lambda_{U}$ and in the limit $d_{U} \rightarrow 1$ they both reproduce the standard model result. It is also evident that while the bound from the muon anomaly is set by the muon mass $m_{\mu} \approx 105.7\text{MeV}/c^2$ the bound from the un-Casimir effect is set by the distance of the plates $a$ which is typically in micrometer range. The smallest
distances probed in Casimir experiments are about 100 nm, obtained in a sphere plate configuration consisting of a high quality nanomembrane resonator and a millimeter sized gold coated sphere [32].

In any case the comparison between the two results cannot proceed further. From Fig. 1 (left) we see that the bounds depend on the coupling coefficient $\lambda$, which is assumed to vary between 0.05 and 1. However we ignore the actual value of $\lambda$. Given the scale hierarchy $\Lambda_U < M_U < M_{Pl}$, the coupling might be smaller, with consequent decrease of predictivity of the muon anomaly analysis. This is not the case for the un-Casimir effect. The lower bound on $\Lambda_U$ derived from the Casimir effect does not depend on $\lambda$ (cf. Eq. (22)). This is in marked contrast with all the proposed bounds in the literature, such as the aforementioned case of the muon anomaly [9, 33], a variety of other particle physics phenomena [34, 35, 36], the predicted deviation of Newton’s law at short scales [15], other astrophysical bounds [37, 38] as well as bounds from atomic parity violation [39].

The peculiarity of the above bound is, however, based on an idealized condition of study of the Casimir effect, i.e., the perfect conductor approximation. As noticed in [22], Casimir energies are independent of the nature of the plates as well as of any particular interaction (e.g. the fine structure constant $\alpha_{EM}$, $\lambda$) when Dirichlet boundary conditions are perfectly met on metallic plates. In general this is not the case and deviations from the standard Casimir formula increase as the plate separation $a$ decreases. For the electromagnetic field one finds that deviations become relevant when

$$
\gamma \equiv \frac{1}{\omega_{pl}^2} \frac{\epsilon^2}{\hbar} \frac{c}{a^2} \sim \alpha_{EM},
$$

where $\omega_{pl}$ is the plasma frequency of the conductor described in terms of the Drude model. For a good conductor like copper and for typical plate separations, the r.h.s. of the above equation is of the order $\gamma \sim 10^{-5}$. In QED, being $\alpha_{EM} \approx 1/137 \gg \gamma = 10^{-5}$, one can safely employ the perfect conductor approximation. This is also the case in the Un-Casimir effect for $\lambda \gg 10^{-5}$ and accordingly one finds results for $\Lambda_U$ independent of such coupling constant as explained above.

There exist, however, two additional scenarios:

- For $\lambda \sim \gamma$ one has the critical case, in which the un-particle component is responsible for a breakdown of the validity of the Casimir formula. Experimentally one should observe a Casimir energy that depends on the nature of plates, with deviations with respect to Eq. (18) exceeding the experimental uncertainty.

- For $\lambda \ll \gamma$ the unparticle contribution is quadratically suppressed in agreement to the fact that the interaction Lagrangian (1) becomes negligible [22].

This kind of analysis suggests that larger plate separations (corresponding to smaller values of $\gamma$) could be used to restrict the space of parameters for which unparticles cannot give relevant contributions.
As a conclusion, by presenting the Un-Casimir effect, we offer some privileged testbeds for setting bounds on relevant regions of the parameters space governing unparticle physics.

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