Quark-Gluon Mixed Condensate $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$ from Lattice QCD

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We study the quark-gluon mixed condensate $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$, which is another chiral order parameter, in SU(3), lattice QCD with the Kogut-Susskind fermion at the quenched level. Using 100 gauge configurations on the $16^4$ lattice with $\beta = 6.0$, we measure $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$ at 16 points in each gauge configuration for each current-quark mass of $m_q = 21, 36, 52$ MeV. From the 1600 data for each $m_q$, we find $m_0^2 \equiv g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle/\langle \bar{q}q \rangle \simeq 2.5 \text{ GeV}^2$ at the lattice scale of $a^{-1} \simeq 2\text{GeV}$ in the chiral limit. The large value of $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$ suggests its importance in the operator product expansions in QCD. We also show our preliminary results of $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$ at finite temperature.

1. The importance of the quark-gluon mixed condensate $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$

In order to understand the non-perturbative structure of the QCD vacuum, it is important to study various condensates such as $\langle \bar{q}q \rangle$ and $\langle G_{\mu\nu}G^{\mu\nu} \rangle$. Among various condensates, we emphasize the importance of the quark-gluon mixed condensate $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \equiv g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}^A G_{\mu\nu}^\lambda q \rangle$. First, the mixed condensate represents a direct correlation between quarks and gluons in the QCD vacuum. In this point, the mixed condensate differs from $\langle \bar{q}q \rangle$ and $\langle G_{\mu\nu}G^{\mu\nu} \rangle$ even at the qualitative level. Second, this mixed condensate is another chiral order parameter of the second lowest dimension, because the chirality of the quark flips as $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}^A q \rangle = g\langle \bar{q}_R (\sigma_{\mu\nu}G_{\mu\nu}^A) q_L \rangle + g\langle \bar{q}_L (\sigma_{\mu\nu}G_{\mu\nu}^A) q_R \rangle$. Third, the mixed condensate plays an important role in various QCD sum rules, especially in the baryons [1, 2], the light-heavy mesons [3] and the exotic mesons [4]. In the QCD sum rules, the value $m_0^2 \equiv g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}^A q \rangle/\langle \bar{q}q \rangle \simeq 0.8 \pm 0.2 \text{ GeV}^2$ has been proposed as a result of the phenomenological analyses [5]. However, in spite of the importance of $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}^A q \rangle$, there was only one preliminary lattice QCD work [6], which was performed with very little statistics (only 5 data) using a small ($8^4$) and coarse lattice ($\beta = 5.7$).

Therefore, we present the calculation for $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}^A q \rangle$ in lattice QCD with a larger ($16^4$) and finer ($\beta = 6.0$) lattice and with high statistics (1600 data). We perform the measurement of $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}^A q \rangle$ as well as $\langle \bar{q}q \rangle$ in the SU(3)$_c$ lattice at the quenched level, using the Kogut-Susskind (KS) fermion to keep the chiral symmetry. We generate 100 gauge configurations and pick up 16 points for each configuration to calculate the condensates. With this high statistics of 1600 data for each quark mass, we perform reliable estimate for the ratio $m_0^2 \equiv g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}^A q \rangle/\langle \bar{q}q \rangle$ at the lattice scale in the chiral limit [6].

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2. Lattice Formalism

We first emphasize that both of the condensates $\langle \overline{q}q \rangle$ and $g\langle \overline{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$ work as the chiral order parameter, and therefore to keep chiral symmetry is essential for our study. From this viewpoint, we adopt the KS-fermion, which can preserve the explicit chiral order parameter, and therefore to keep chiral symmetry is essential for our study.

The action of the KS-fermion is described by spinless Grassmann fields $\overline{\chi}, \chi$ and the gauge link-variable $U_\mu \equiv \exp[-ia\gamma_5 A_\mu]$. In the absence of the gauge field, the SU(4)$_f$ quark-spinor field $q$ with spinor $i$ and flavor $f$ is expressed by $\chi$ as

$$q_i^f(x) = \frac{1}{8} \sum_\rho (\Gamma_\rho)_{if} \chi(x + \rho), \quad \Gamma_\rho \equiv \gamma^1_i \gamma^2_j \gamma^3_k \gamma^4_l, \quad \rho \equiv (\rho_1, \rho_2, \rho_3, \rho_4)$$

where $\rho$ with $\rho_\mu \in \{0, 1\}$ runs over the 16 sites in the $2^4$ hypercube. When the gluon field is turned on, we insert additional link-variables in Eq. (1) to respect the gauge covariance. Hence, the flavor-averaged condensates are expressed as

$$a^3 \langle \overline{q}q \rangle = -\frac{1}{4} \sum_f \text{Tr} \left[ \langle q_i^f(x)\overline{q}^\dagger_i^f(x) \rangle \right] = -\frac{1}{2^8} \sum_\rho \text{Tr} \left[ \Gamma_\rho \Gamma_\rho^\dagger \langle \chi(x + \rho)\overline{\chi}(x + \rho) \rangle \right],$$

$$a^5 g\langle \overline{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle = -\frac{1}{4} \sum_\mu \sum_\nu \text{Tr} \left[ \langle q_i^f(x)\overline{q}^\dagger_i^f(x) \rangle \sigma_{\mu\nu} G_{\mu\nu} \right] = -\frac{1}{2^8} \sum_\mu \sum_\nu \text{Tr} \left[ U_{\pm\mu,\pm\nu}(x + \rho) \Gamma_\rho \Gamma_\rho^\dagger \langle \chi(x + \rho)\overline{\chi}(x + \rho) \rangle \sigma_{\mu\nu} G_{\mu\nu}^\text{lat}(x + \rho) \right],$$

where $\rho'$ is defined as $\rho' \equiv \rho \pm \mu \pm \nu$, and the sign $\pm$ is taken such that the sink point $(x + \rho')$ belongs to the same hypercube of the source point $(x + \rho)$. Here, $U_{\mu,\nu}(x) \equiv \frac{1}{2} \left[ U_{\mu}(x)U_{\nu}(x + \mu) + U_{\nu}(x)U_{\mu}(x + \nu) \right]$ is introduced to keep the gauge covariance. We adopt the clover-type definition of the gluon field strength $G_{\mu\nu}$ on the lattice as

$$G_{\mu\nu}^\text{lat}(s) = \frac{i}{16} \sum_A \sum_{s' = s, s - \mu, s - \nu, s - \mu - \nu} \lambda^A \text{Tr} \left[ \lambda^A \{U_{\mu\nu}(s') - U_{\nu\mu}(s')\} \right],$$

which has no $O(a)$ discretization error. (This benefit is not available in Ref. [3].)

3. The lattice QCD results

We calculate the condensates $\langle \overline{q}q \rangle$ and $g\langle \overline{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$ using the SU(3)$_c$ lattice QCD at the quenched level. We use the standard Wilson action at $\beta = 6.0$ on the $16^4$ lattice. The lattice unit $a \simeq 0.10$ fm is obtained so as to reproduce the string tension $\sigma = 0.89$ GeV/fm$^2$. We use the quark mass $m = 21, 36, 52$ MeV (i.e. $ma = 0.0105, 0.0184, 0.0263$). For the fields $\chi, \overline{\chi}$, the anti-periodic condition is imposed. We measure the condensates on 16 different space-time points $x$ in each configuration as $x = (x_1, x_2, x_3, x_4)$ with $x_\mu \in \{0, 8\}$ in $R^4$ in the lattice unit. For each $m$, we calculate the flavor-averaged condensates, and average them over the 16 space-time points and 100 gauge configurations.

Figure 1 shows the bare condensates $\langle \overline{q}q \rangle$ and $g\langle \overline{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$ against the quark mass $ma$. We emphasize that the jackknife errors are almost negligible, due to the high statistics of 1600 data for each quark mass $m$. Since both of $\langle \overline{q}q \rangle$ and $g\langle \overline{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$ show a clear linear
Figure 1. The bare condensates $\langle \bar{q}q \rangle$ and $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$ plotted against the quark mass $ma$. The dashed lines denote the best linear extrapolations, and the cross symbols correspond to the values in the chiral limit. The jackknife errors are hidden in the circles.

Table 1

| $ma$     | $a^3\langle \bar{q}q \rangle$ | $a^5g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$ |
|----------|--------------------------------|--------------------------------------------------------|
| $0.0263$ | $-0.042397(16)$               | $-0.018820(15)$ |
| $0.0184$ | $-0.032470(15)$               | $-0.014979(14)$ |
| $0.0105$ | $-0.022124(16)$               | $-0.010884(14)$ |
| chiral  | $-0.008721(17)$               | $-0.005652(14)$ |

response to $m$, we fit the data linearly and determine the condensates in the chiral limit. The obtained data are summarized in Table 1. To check the finite volume artifact, we calculate the condensates imposing the periodic boundary condition on $\chi, \bar{\chi}$, instead of the anti-periodic condition as before. The results with different boundary conditions almost coincide within about 1% deviation. Therefore, we conclude that the physical volume $V \sim (1.6 \text{ fm})^4$ in our simulations is large enough to avoid the finite volume artifact.

The values of the condensates in the continuum limit are to be obtained through the renormalization, which, however, suffers from uncertainty of the non-perturbative effect. As a more reliable quantity, we provide the ratio $m_0^2 \equiv g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle/\langle \bar{q}q \rangle$, which is free from the uncertainty from the wave function renormalization of the quark.

Now, we present the result of $m_0^2$ using our bare results of SU(3)$_c$ lattice QCD as

$$m_0^2 \equiv g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle/\langle \bar{q}q \rangle \simeq 2.5 \text{ GeV}^2 \quad (\beta = 6.0 \text{ or } a^{-1} \simeq 2 \text{GeV}).$$

We see that $m_0^2$ is rather large, which suggests the importance of the mixed condensate in OPE. We note this bare result itself is determined very precisely.

4. Discussion and Outlook

For comparison with the standard value in the QCD sum rule, we rescale our result from $\mu \simeq \pi/a$ to $\mu \simeq 1 \text{ GeV}$ corresponding to the QCD sum rule. Following Ref. [4], we first take the bare values of the condensates as the starting point of the flow, and then rescale
Figure 2. The quark-gluon mixed condensate $g \langle \bar{q} \sigma_{\mu \nu} G_{\mu \nu} q \rangle_T / g \langle \bar{q} \sigma_{\mu \nu} G_{\mu \nu} q \rangle_{T=0}$ plotted against the temperature $T$. The jackknife errors are hidden in the triangles. The vertical dashed line denotes the critical temperature $T_c \approx 280\text{MeV}$ at the quenched level.

the condensates perturbatively. We adopt the anomalous dimensions at the one-loop level [9], and choose the parameters $\Lambda_{\text{QCD}} = 200 - 300\text{MeV}$ and $N_f = 0$ corresponding to quenched lattice QCD. We obtain $m_0^2\big|_{\mu=1\text{GeV}} \equiv g \langle \bar{q} \sigma_{\mu \nu} G_{\mu \nu} q \rangle / \langle \bar{q} q \rangle\big|_{\mu=1\text{GeV}} \sim 3.5 - 3.7\text{GeV}^2$. Comparing with the standard value of $m_0^2 = 0.8 \pm 0.2\text{GeV}^2$ in the QCD sum rule, our calculation results in a rather large value. (Note that the instanton model have made a slightly larger estimate as $m_0^2 \approx 1.4\text{GeV}^2$ at $\mu \approx 0.6\text{GeV}$ [10].) For the improvement, the non-perturbative renormalization scheme may be desired.

Finally, considering the importance of finite-temperature QCD in the RHIC project, we investigate the thermal effect on the mixed condensate $g \langle \bar{q} \sigma_{\mu \nu} G_{\mu \nu} q \rangle$ using the $16^3 \times N_t$ lattices with $N_t = 16, 8, 6, 4$ at $\beta = 6.0$. In figure 4, we show our preliminary results for the mixed condensate at finite temperature. We find a drastic change of the mixed condensate around the critical temperature $T_c$, which reflects the chiral-symmetry restoration.

In summary, we have studied the quark-gluon mixed condensate $g \langle \bar{q} \sigma_{\mu \nu} G_{\mu \nu} q \rangle$ using SU(3)$_c$ lattice QCD with the KS-fermion at the quenched level. For each quark mass of $m_q = 21, 36, 52\text{MeV}$, we have generated 100 gauge configurations on the $16^4$ lattice at the quenched level. For each quark mass of $m_q = 21, 36, 52\text{MeV}$, we have generated 100 gauge configurations on the $16^4$ lattice with $\beta = 6.0$. Using the 1600 data for each $m_q$, we have found $m_0^2 \equiv g \langle \bar{q} \sigma_{\mu \nu} G_{\mu \nu} q \rangle / \langle \bar{q} q \rangle \approx 2.5\text{GeV}^2$ in the chiral limit at the lattice scale corresponding to $\beta = 6.0$ or $a^{-1} \approx 2\text{GeV}$. We have also shown our preliminary results of $g \langle \bar{q} \sigma_{\mu \nu} G_{\mu \nu} q \rangle$ at finite temperature.

REFERENCES

1. B. L. Ioffe, Nucl. Phys. B188 (1981) 317, Erratum-ibid. B191 (1981) 591.
2. H.G. Dosch, M. Jamin and S. Narison, Phys. Lett. B220 (1989) 251.
3. H.G. Dosch and S. Narison, Phys. Lett. B417 (1998) 173 and references therein.
4. J.I. Latorre, P. Pascual and S. Narison, Z. Phys. C34 (1987) 347.
5. V.M. Belyaev and B.L. Ioffe, Sov. Phys. JETP 56 (1982) 493.
6. M. Kremer and G. Schierholz, Phys. Lett. B194 (1987) 283.
7. T. Doi, N. Ishii, M. Oka and H. Suganuma, hep-lat/0211039 (2002).
8. T.T. Takahashi et al., Phys. Rev. D65 (2002) 114509.
9. S. Narison and R. Tarrach, Phys. Lett. B 125 (1983) 217.
10. M.V. Polyakov and C. Weiss, Phys. Lett. B387 (1996) 841.