Evolutionarily stable strategy analysis of Quantitative investment strategy

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Abstract. Based on Chinese Stock Index Future, this study focuses on two types of market price changes, trends and shake. How investors’ strategies evolve to influence the market from trend to shake or from shake to trend to obtain the “evolutionary stable strategy” (ESS) of the market is conducted. The results show that trend strategies are more favourable in most market environments, and find a single strategy cannot survive in the market, and trading frequency should be appropriately reduced to reduce transaction costs, so as to survive in the market for a long time.

1. Introduction
Stock market and future market as secondary market in china has good liquidity. There are numerous institutional investors and individual investors taking part in these markets and therefore they can be assumed that are perfectly competitive. In stock market and future market, buyers and sellers implement their strategies in trading and they affect the last price of markets, their counterparts’ next move, the state of assets and so on. These futures satisfy prerequisites of a game.

Logically speaking, all market price changes can be classified as two types: trends and shakes, which is established in figure1. Correspondingly, investment strategies can be categorized into two types: trend strategies that follow the direction of the market price, and shake strategies that sell at high price and buy at low price.

From the perspective of evolutionary game theory, this paper analyses how investors’ strategies evolve to influence the market from trend to shake or that from shake to trend, so as to obtain the “evolutionary stable strategy” (ESS) of the market.

2. Related Literature
In recent years, it is a new direction to study the financial capital market by relating to the application of evolutionary game. At present, there are a lot of literature that can be referred, which can either reach a conclusion through pure theoretical derivation assuming several hypothesis, or conduct numerical simulation after assuming a model, or reach statistical conclusions through organizing the participants to conduct behavioral experiments.

Yan Xu [1] defined some assumptions that constructed a dynamic evolution model of investors, the results show that the utility function of both sides in the game, the wealth of the investors, the style of the capital market, the initial status of the population, rational transaction costs, and system risks inherited in capital market, all affect market evolution. Xingji Wei et al [2] studied the stability of long-term assets investment strategy of the capital market based on evolutionary game, the article proved that investors simply follow Kelly rules would not be able to completely dominate the market,
and they put forward and proved the conditions for Kelly rules evolve to "evolutionary stable strategy" (ESS), and finally they applied it to study dynamic evolution process of investors and their wealth. Weidong Meng, Yongping Wang [3] using evolutionary game theory and empirical research, find that the evolution of investors' trading behavior satisfied neither traditional finance, nor behavioral finance which assumes only a single path and equilibrium, but is more likely to have multiple equilibrium characteristics, which is determined by the investors complementary strategy behavior. Friedman and Fama, among others, maintained that noise traders would encounter a counterbalance of rational arbitrageurs, bringing asset prices close to fundamental value. In this process, noise traders would suffer losses, while arbitrageurs would keep profits, which would eventually cause noise traders to disappear from the market [4,5]. However, behavioral financial scientist Delong et al. [6,7] believed that when rational arbitrageurs conducted arbitrage, they would not only face the risk of changes in basic factors, but also face the risk of changes in irrational expectations of noise traders.

This paper is different from the literatures above that assume investor behavior and return of asset distribution to build models. Instead, it differentiates "shake strategy" and "trend strategy" from the perspective of market performance, and analyzes the "evolutionary stable strategy" (ESS) of the two type strategies from both theoretical and empirical perspectives based on evolutionary game theory.

3. Theoretical analysis

No matter how many changes there are in the market price, fundamentally speaking, they can be divided into two kinds: first, the price stays in a certain range up and down, which is called “shake”, as shown in Figure 1(a); The second way is that the price breaks through the shaking range, thus the overall price rises or falls, which is called the trend, as shown in Figure 1(b).

![Figure 1](attachment:image1.png)

(a) The shake of market  (b) The trend of market

Figure 1 Two types of market price change

For these two kinds of markets performance, there are two corresponding strategies. In the shake market, short at the upper bound, long at the lower bound, you can get the spread income, this is the shake strategy. In the trend market, follow the downward trend and short, follow the upward trend and long. By this way, we can also get the corresponding gains, this is the trend strategy.

Therefore, in the market changes, either think that the next trend will be contrary to the trend before, and thus take shake strategy, or think that the next trend will continue the trend before, and adopt trend strategy, all the trade can be classified as these two basic strategies. Hence, the following payoff matrix is formed as shown in table 1. A,B,C and D represent payoff. Generally, when the trend strategy is consistent with another trend strategy, the trend will be enhanced if the trading direction is superimposed, so A>0. When the shake strategy is consistent with another shake strategy, the market direction is reversed, so the shake strategy can also profit, so B>0. When the shake strategy is conflicted with the trend strategy, the market direction depends on the strength of the two strategies, so the sign symbol of C and D are uncertain.
Table 1. Payoff matrix

| Trend strategy | Shake strategy |
|----------------|----------------|
| Trend strategy | A, A           |
| Shake strategy | D, C           |
|                | C, D           |
|                | B, B           |

Assuming $T^t$ as the proportion of the trend strategy in the whole market, then the average fitness of the trend strategy is as equation (1).

$$f_{\text{trend}} = T^t \times A + (1 - T^t) \times D$$  \hspace{1cm} (1)

The average fitness of the shake strategy is equation (2).

$$f_{\text{shake}} = C \times T^t + B \times (1 - T^t)$$  \hspace{1cm} (2)

If the proportion of trend strategy increases, it needs to meet equation (3).

$$f_{\text{trend}} > f_{\text{shake}}$$  \hspace{1cm} (3)

Thus, it can be solved as follows equation (4):

$$T^t > \frac{B - D}{A - C + B}$$  \hspace{1cm} (4)

Conversely, if the proportion of trend strategy decreases, it needs to meet equation (5).

$$T^t < \frac{B - D}{A - C + B}$$  \hspace{1cm} (5)

Formula (4) can be discussed in the following four cases.

Case 1: $A + B > C$ and $B > D$  \hspace{1cm} (6)

Case 2: $A + B > C$ and $B \leq D$  \hspace{1cm} (7)

Case 3: $A + B < C$ and $B > D$  \hspace{1cm} (8)

Case 4: $A + B < C$ and $B \leq D$  \hspace{1cm} (9)

In case 3 and 4, it means that the return when the trend strategy is consistent plus the return when the shake strategy is consistent is less than the return when the two strategies are conflicted. This is not quite in line with the practical experience, because usually when the strategy is consistent, the trend will increase towards the direction of the strategy, while when the two strategies are conflicted, they will cancel each other and then decline profit even if there is no loss. As case 3 and 4 are not practical, we will not discuss it for the time being.

For case 1, this situation represents the case where the shake range is large and therefore the return of the shake strategy is large but the trend is not significant after the trend strategy breaks the shake range. See Figure 2.

![Figure 2: Shake range is big and trend is small](Image)
The attractor basin of case 1 can be depicted as shown in Figure 3. At this point, \( \frac{B - D}{A - C + B} \) in equation (4) is a positive number. When equation (4) is true, the strategy type in the market evolves towards a complete trend strategy, while if equation (4) is true, the strategy type in the market evolves towards a complete shake strategy.

For case 2, this situation corresponds to the situation where the shake range is small and the return of shake strategies is small but when the trend strategy breaks through the shake range, the trend is larger. See Figure 4.

The attraction basin of case 2 can be described as shown in Figure 5. At this time, there is a negative number \( \frac{B - D}{A - C + B} \) in Equation (4), and the proportion of trend strategy \( T^t \) in the market cannot be less than 0. Therefore, in this case, the overall strategy type of the market is bound to evolve to trend strategy, and the trend strategy will eventually dominate the market.

To sum up, the analysis of the above two situations 1 and 2 shows that only when equation (5) and (6) are true, will the type of strategy in the market evolve into shake strategies; otherwise, in most cases, the dominant strategy in the market will evolve into trend strategies.

4. **empirical analysis**

4.1. **Experimental procedure**

This empirical simulation data adopts the 1-minute market price data of main contract of CSI 300 Stock Index futures (futures code IF) from April 16, 2010 to June 19, 2020, as shown in Figure 6.
Since the market performance of stock index futures may be particularly favorable to a kind of strategies at a particular period, in order to maintain the generality of the research, data fragments of random length were randomly selected from the IF series from April 16, 2010 to June 19, 2020 for strategy testing. At the same time, due to the different market details presented by different frequencies, it may have different effects on the returns of the same strategy. Therefore, for the data fragments of random length extracted, frequency conversion is carried out to extract the original data of 1 minute into 5 minutes, 15 minutes, 30 minutes and 60 minutes.

The strategies are set as two types: shake strategy and trend strategy. The shake strategy is as follows: take the direction of market price change from Time T-1 to time T as the benchmark; if the price rises from time T-1 to time T, the short-seller holds the position until time T +1 to close the position; on the contrary, if the price falls from time T-1 to time T, the long-seller holds the position until time T +1 to close the position. The trend strategy is as follows: take the direction of market price change from Time T-1 to time T as the benchmark; if the trend rises from time T-1 to time T, the long-seller holds the position until time T +1 to close the position; if the trend falls from time T-1 to time T, short-seller holds the position until time T +1 to close the position.

Apply the above shake strategy and trend strategy to a certain market segment, and test the strategy respectively in the market segment of 1 minute, 5 minutes, 15 minutes, 30 minutes and 60 minutes. The annualized Sharpe ratio, annualized return rate and max drawdown of 5 trading frequencies of these two strategies were compared. 5000 segments were randomly selected and the experiment was repeated for 5000 times to reach a general conclusion.

4.2. Experimental results
First of all, the annual Sharpe ratio distributions of shake strategy and trend strategy at different 5 frequencies in 5000 experiments were statistically analyzed, as shown in Figure 7. “Shake 1min” represents the annualized Sharpe distribution of the shake strategy at the 1-minute frequency, while “trend 60min” represents the annualized Sharpe distribution of the trend strategy at the 60-minute frequency, and so on. As you can see, most strategies have a negative Sharpe ratio, only the 1-minute shake strategy and the 60-minute trend strategy have a positive Sharpe ratio.
Then, the correlation coefficients of Sharpe ratio of 5000 experiments of shake strategies and trend strategies in different frequencies were observed, as shown in Table 2. S1 represents the shake strategy of the 1-minute frequency, and T60 represents the trend strategy of the 60-minute frequency. It can be seen from Table 1 that, in most cases, the Sharpe ratio of the shake strategy and the trend strategy is inversely correlated, and only in some cases, there is a positive correlation between them. However, according to experience, this is usually the case where both strategies lose money.

Table 2 the correlation coefficients of Sharpe ratio of 5000 experiments of shake strategies and trend strategies in different frequencies

|     | s1   | s5   | s15  | s30  | s60  | t1   | t5   | t15  | t30  | t60  |
|-----|------|------|------|------|------|------|------|------|------|------|
| s1  | 1    | 0.40070 | 0.33768 | -0.2683 | 0.19756 | -0.9999 | -0.3820 | -0.3140 | 0.29390 | -0.1720 |
| s5  | 0.40070 | 1    | 0.20718 | -0.0782 | 0.12395 | -0.3969 | -0.9988 | -0.1868 | 0.09831 | -0.101  |
| s15 | 0.33768 | 0.20718 | 1    | 0.10869 | 0.40767 | -0.3414 | -0.2153 | -0.9990 | -0.1107 | -0.4089 |
| s30 | -0.2683 | -0.0782 | 0.10869 | 1    | 0.10383 | 0.27051 | 0.07995 | -0.1077 | -0.9989 | -0.1007 |
| s60 | 0.19756 | 0.12395 | 0.40767 | 0.10383 | 1    | -0.2000 | -0.1288 | -0.4067 | -0.1057 | -0.9986 |
| t1  | -0.9999 | -0.3969 | -0.3414 | 0.27051 | -0.2000 | 1    | 0.37884 | 0.31825 | -0.2955 | 0.17517 |
| t5  | -0.3820 | -0.9988 | -0.2153 | 0.07995 | -0.1288 | 0.37884 | 1    | 0.19687 | -0.0980 | 0.10811 |
| t15 | -0.3140 | -0.1868 | -0.9990 | -0.1077 | -0.4067 | 0.31825 | 0.19687 | 1    | 0.11179 | 0.41025 |
| t30 | 0.29390 | 0.0983 | -0.1107 | -0.9989 | -0.1057 | -0.295 | -0.0980 | 0.11179 | 1    | 0.1050  |
| t60 | -0.172  | -0.101 | -0.408  | -0.100  | -0.9986 | 0.17517 | 0.10811 | 0.41025 | 0.105004 | 1     |

The third step is to observe the return of all historical data of the two strategies at 5 frequencies, as shown in Figure 8. Among them, the 1-minute shake strategy returns extremely well, which is then discussed separately. It can be seen from Figure 9 that the overall performance of shake strategy earnings are poor, in which the 5-minute shake strategy slightly improves after the stock market crash, while other frequency shake strategies are basically dominated by losses. Trending strategies perform well on the 15-minute, 30-minute, and 60-minute scales, with the 60-minute trend strategy performing best.
Next, pick the 60-minute trend strategy with the best earnings performance and the 1-minute shake strategy with the best earnings performance for an in-depth look.

First, observe the return of 1-minute fluctuation strategy, as shown in Figure 9, the yield is as high as 2.5e4 times. Such a rate of return is obviously contrary to experience, so in order to make the experiment more realistic, transaction costs are introduced for further verification. Stock index futures day closing charge is 3.45%%, opening charge is 0.23%%, because 1 minute trading frequency is short, if you want to ensure the immediate transaction, usually need to accept the counterparty price, which means to lose 1-2 slip points. To sum up, the transaction cost parameter is set as 5%%.

After the restriction of transaction costs is introduced, the return of 1 min shake strategy is shown in Figure 10. As shown in Figure 10, the 1 min shake strategy changes from a 2.5e4 gain to a -2.2e4 loss. It can be seen that the trading frequency is too high, and the strategy cannot cover the transaction cost.
Figure 10 return of 1 min shake strategy with transaction cost

Similarly, the transaction cost of 5%% is also introduced into the 60 min trend strategy, and the results are shown in Figure 11. Like the 1 min shake strategy, the 60 min trend strategy also produces a large loss once transaction costs are taken into account, but the overall loss is less than the 1 min shake strategy.

Figure 11 return of 60 min trend strategy with transaction cost

Shake strategies and trend strategies that introduce transaction costs into other frequencies also result in losses.

5. Conclusion
From the theoretical analysis and empirical results of this paper, the conclusions are obtained as below:

(1) From the theoretical analysis in the second part of this paper, it can be seen that the market environment is conducive to the evolution of trend strategy in most cases, and only in a few cases can it adapt to the shake strategy. This can be seen from the empirical analysis in the third part, except for
the 1-minute shake strategy, the returns of most trend strategies under different frequency are positive, while most shake strategies under different frequency are loss-making. This empirical conclusion confirms the results of theoretical analysis that trend strategies are more favorable in most market environments.

(2) Although some frequency trend strategy and 1 min shake strategy are profitable, once transaction costs are introduced, all strategies will lose money. This means that in the real market environment, due to the objective existence of transaction costs, a single strategy cannot survive in the market.

(3) In real capital market, complementary shake and trend strategies should be used comprehensively, and trading frequency should be appropriately reduced to reduce transaction costs, so as to survive in the market for a long time.

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