HEAVY BARYONS IN A RELATIVISTIC QUARK MODEL WITH A NONLOCAL INTERACTION OF LIGHT QUARKS

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Abstract
Semileptonic decays of bottom and charm baryons are considered within a relativistic three-quark model with the Gaussian shape for the baryon-three-quark vertex and standard quark propagators.

1 Introduction
This paper is focused on the actual problem of heavy flavor physics, exclusive s.l. decays of low-lying bottom and charm baryons. Recently, activity in this field has obtained a great interest due to the experiments worked out by the CLEO Collaboration [1] on the observation of the heavy-to-light s.l. decay $\Lambda_c^+ \rightarrow \Lambda c^+ \nu_c$. Also the ALEPH [2] and OPAL [3] Collaborations expect in the near future to observe the exclusive mode $\Lambda_b \rightarrow \Lambda c\ell\nu$.

In ref. [4] a model for QCD bound states composed from light and heavy quarks was proposed. Actually, this model is the Lagrangian formulation of the NJL model with separable interaction [5] but its advantage consists in the possibility of studying of baryons as the relativistic systems of three quarks. The framework was developed for light mesons [4] and baryons [4, 6], and also for heavy-light hadrons [7].

The purpose of present work is to give a description of properties of baryons containing a single heavy quark within framework proposed in ref. [4] and developed in ref. [6, 7]. Namely, we report the calculation of observables of semileptonic decays of bottom and charm baryons: Isgur-Wise functions, asymmetry parameters, decay rates and distributions.

2 Model
Our approach [4] is based on the interaction Lagrangians describing the transition of hadrons into constituent quarks and vice versa:

$$\mathcal{L}_B^{\text{int}}(x) = g_B \bar{B}(x) \int dy_1 ... \int dy_3 \delta \left( x - \frac{\sum_i m_i y_i}{\sum_i m_i} \right) F \left( \sum_{i<j} \frac{(y_i - y_j)^2}{18} \right) J_B(y_1, y_2, y_3) + \text{h.c.}$$

with $J_B(y_1, y_2, y_3)$ being the 3-quark current with quantum numbers of a baryon $B$:

$$J_B(y_1, y_2, y_3) = \Gamma_1 q^a_1(y_1) q^a_2(y_2) \text{CT}_2 q^a_3(y_3) \varepsilon^{a_1 a_2 a_3}$$
Here $\Gamma_{1(2)}$ are the Dirac matrices, $C = \gamma^0\gamma^2$ is the charge conjugation matrix, and $a_i$ are the color indices. We assume that the momentum distribution of the constituents inside a baryon is modeled by an effective relativistic vertex function which depends on the sum of relative coordinates only $F \left( \frac{1}{\text{Re}} \sum_{i<j} (y_i - y_j)^2 \right)$ in the configuration space where $y_i$ ($i=1,2,3$) are the spatial 4-coordinates of quarks with masses $m_i$, respectively. They are expressed through the center of mass coordinate ($x$) and relative Jacobi coordinates ($\xi_1, \xi_2$). The shape of vertex function is chosen to guarantee ultraviolet convergence of matrix elements. At the same time the vertex function is a phenomenological description of the long distance QCD interactions between quarks and gluons. In the case of light baryons we shall work in the limit of isospin invariance by assuming the masses of $u$ and $d$ quarks are equal each other, $m_u = m_d = m$. Breaking of the unitary SU(3) symmetry is taken into account via a difference of strange and nonstrange quark masses $m_s - m \neq 0$. In the case of heavy-light baryonic currents we suppose that heavy quark is much larger than light quark ($m_Q \gg m_{q_1, q_2}$), i.e. a heavy quark is in the c.m. of heavy-light baryon. Now we discuss the model parameters. First, there are the baryon-quark coupling constants and the vertex function in the Lagrangian $\mathcal{L}^\text{int}_B(x)$. The coupling constant $g_B$ is calculated from the compositeness condition that means that the renormalization constant of the baryon wave function is equal to zero, $Z_B = 1 - g_B^2 \Sigma_B(M_B) = 0$, with $\Sigma_B$ being the baryon mass operator. The vertex function is an arbitrary function except that it should make the Feynman diagrams ultraviolet finite, as we have mentioned above. We choose in this paper a Gaussian vertex function for simplicity. In Minkowski space we write $F(k_1^2 + k_2^2) = \exp[(k_1^2 + k_2^2)/\Lambda_B^2]$ where $\Lambda_B$ is the Gaussian range parameter which is related to the size of a baryon. It was found that for nucleons ($B = N$) the value $\Lambda_N = 1.25$ GeV gives a good description of the nucleon static characteristics (magnetic moments, charge radii) and also form factors in space-like region of $Q^2$ transfer up to 1 GeV$^2$. In this work we will use the value $\Lambda_{Bq} \equiv \Lambda_N = 1.25$ GeV for light baryons and consider the value $\Lambda_{Bq}$ for the heavy-light baryons as an adjustable parameter. As far as the quark propagators are concerned we shall use the standard form of light quark propagator with a mass $m_q$

$$< 0|T(q(x)\bar{q}(y))|0> = \int \frac{d^4k}{(2\pi)^4} e^{-ik(x-y)} S_q(k), \quad S_q(k) = \frac{1}{m_q - k}$$

and the form

$$S(k + v\Lambda_{(q_1 q_2)}) = \frac{(1 + y')}{2(v \cdot k + \Lambda_{(q_1 q_2)} + i\epsilon)}$$

for heavy quark propagator obtained in the heavy quark limit (HQL) $m_Q \to \infty$. The notation are the following: $\Lambda_{(q_1 q_2)} = M_{(q_1 q_2)} - m_Q$ is the difference between masses of heavy baryon $M_{(q_1 q_2)} \equiv M_{Bq}$ and heavy quark $m_Q$ in the HQL, $v$ is the four-velocity of heavy baryon. It is seen that the value $\Lambda_{(q_1 q_2)}$ depends on a flavor of light quarks $q_1$ and $q_2$. Neglecting the SU(2)-isotopic breaking gives three independent parameters: $\Lambda_u \equiv \Lambda_{uu} = \Lambda_{dd} = \Lambda_{du}$, $\Lambda_s \equiv \Lambda_{us} = \Lambda_{ds}$, and $\Lambda_{us}$. Of course, the deficiency of such a choice of light quark propagator is lack of confinement. This could be corrected by changing the analytic properties of the propagator. We leave that to a future study. For the time being we shall avoid the appearance of unphysical imaginary parts in the Feynman diagrams by
restricting the calculations to the following condition: the baryon mass must be less than the sum of constituent quark masses $M_B < \sum_i m_{q_i}$. In the case of heavy-light baryons the restriction $M_B < \sum_i m_{q_i}$ trivially gives that the parameter $\bar{\Lambda}_{(q_1q_2)}$ must be less than the sum of light quark masses $\bar{\Lambda}_{(q_1q_2)} < m_{q_1} + m_{q_2}$. The last constraint serves as the upper limit for a choice of parameter $\bar{\Lambda}_{(q_1q_2)}$. Parameters $\Lambda_{BQ}$, $m_s$, $\bar{\lambda}$ are fixed in this paper from the description of data on $\Lambda^+_c \to \Lambda^0 + e^+ + \nu_e$ decay. It is found that $\Lambda_{BQ} = 2.5$ GeV, $m_s = 570$ MeV and $\bar{\Lambda} = 710$ MeV. Parameters $\bar{\lambda}$ and $\bar{\Lambda}_{(ss)}$ cannot be adjusted at this moment since the experimental data on the decays of heavy-light baryons having the strange quarks (one or two) are not available. In this paper we use $\bar{\lambda} = 850$ MeV and $\bar{\Lambda}_{(ss)} = 1000$ MeV.

3 Results

In this section we give the numerical results for the observables of semileptonic decays of bottom and charm baryons: the baryonic Isgur-Wise functions, decay rates and asymmetry parameters. We check that $\xi_1$ and $\xi_2$ functions are satisfied to the model-independent Bjorken-Xu inequalities. Also the description of the $\Lambda^+_c \to \Lambda^0 + e^+ + \nu_e$ decay which was recently measured by CLEO Collaboration \[1\] is given. In what follows we will use the following values for CKM matrix elements: $|V_{bc}| = 0.04$, $|V_{cs}| = 0.975$.

In our calculations of heavy-to-heavy matrix elements we are restricted only by one variant of three-quark current for each kind of heavy-light baryon: Scalar current for $\Lambda_Q$-type baryons and Vector current for $\Omega_Q$-type baryons [8, 7].

The functions $\zeta$ and $\xi_1$ have the upper limit $\Phi_0(\omega) = \frac{\ln(\omega + \sqrt{\omega^2 - 1})}{\sqrt{\omega^2 - 1}}$. It is easy to show that $\zeta(\omega) = \xi_1(\omega) = \Phi_0(\omega)$ when $\bar{\lambda} = 0$. The radii of $\zeta$ and $\xi_1$ have the lower bound $\zeta \geq 1/3$ and $\xi_1 \geq 1/3$. Increasing of the $\bar{\lambda}$ value leads to the suppression of IW-functions in the physical kinematical region for variable $\omega$. The IW-functions $\xi_1$ and $\xi_2$ must satisfy two model-independent Bjorken-Xu inequalities [9] derived from the Bjorken sum rule for semileptonic $\Omega_b$ decays to ground and low-lying negative-parity excited charmed baryon states in the HQL

$$1 \geq B(\omega) = \frac{2 + \omega^2}{3} \xi_1(\omega) + \frac{(\omega^2 - 1)^2}{3} \xi_2(\omega) + \frac{2}{3} (\omega - \omega^3) \xi_1(\omega) \xi_2(\omega)$$

(1)

$$\rho_{\xi_1} \geq \frac{1}{3} - \frac{2}{3} \xi_2(1)$$

(2)

The inequality (2) for the slope of the $\xi_1$-function is fulfilled automatically because of $\rho_{\xi_1} \geq 1/3$ and $\xi_2(1) > 0$. From the inequality (1) one finds the upper limit for the function $\xi_1(\omega)$: $\xi_1(\omega) \leq \sqrt{3/(2 + \omega^2)}$

In Fig.1 we plot the $\zeta$ function in the kinematical region $1 \leq \omega \leq \omega_{max}$. For a comparison the results of other phenomenological approaches are drawn. There are data of QCD sum rule [10], IMF models [14, 14], MIT bag model [13], a simple quark model (SQM) [17] and the dipole formula [14]. Our result is close to the result of QCD sum rules [10].

In Table 1 our results for total rates are compared with the predictions of other phenomenological approaches: constituent quark model [11], spectator quark model [12], nonrelativistic quark model [14].
Table 1. Model Results for Rates of Bottom Baryons (in $10^{10}$ sec$^{-1}$)

| Process       | Ref. [12] | Ref. [16] | Ref. [11] | Our results |
|---------------|-----------|-----------|-----------|-------------|
| $\Lambda^+_b \to \Lambda^0 e^- \bar{\nu}_e$ | 5.9       | 5.1       | 5.14      | 5.39        |
| $\Xi^0_b \to \Xi^+ e^- \bar{\nu}_e$     | 7.2       | 5.3       | 5.21      | 5.27        |
| $\Sigma^+_b \to \Sigma^{*+} e^- \bar{\nu}_e$ | 4.3       |           | 2.23      |             |
| $\Sigma^+_b \to \Sigma^{*+} e^- \bar{\nu}_e$ |           | 4.56      |           |             |
| $\Omega^+_b \to \Omega^0 e^- \bar{\nu}_e$ | 5.4       | 2.3       | 1.52      | 1.87        |
| $\Omega^-_b \to \Omega^- e^- \bar{\nu}_e$  | 3.41      |           |           | 4.01        |

Now we consider the heavy-to-light semileptonic modes. Particular the process $\Lambda^+_c \to \Lambda^0 + e^+ + \nu_e$ which was recently investigated by CLEO Collaboration [1] is studied in details. At the HQL ($m_C \to \infty$), the weak hadronic current of this process is defined by two form factors $f_1$ and $f_2$ [11, 16]. Supposing identical dipole forms of the form factors (as in the model of Körner and Krämer [11]), CLEO found that $R = f_2/f_1 = 0.25 \pm 0.14 \pm 0.08$. Our form factors have different $q^2$ dependence. In other words, the quantity $R = f_2/f_1$ has a $q^2$ dependence in our approach. In Fig.10 we plot the results for $R$ in the kinematical region $1 \leq \omega \leq \omega_{\text{max}}$ for different magnitudes of $\Lambda$ parameter. Here $\omega$ is the scalar product of four velocities of $\Lambda^+_c$ and $\Lambda^0$ baryons. It is seen that growth of the $\Lambda$ leads to the increasing of ratio $R$. The best fit of experimental data is achieved when our parameters are equal to $m_s = 570$ MeV, $\Lambda_Q = 2.5$ GeV and $\bar{\Lambda} = 710$ MeV. In this case the $\omega$-dependence of the form factors $f_1$, $f_2$ and their ratio $R$ are drawn in Fig.11. Particularly, we get $f_1(q^2_{\text{max}}) = 0.8$, $f_2(q^2_{\text{max}}) = 0.18$, $R = 0.22$ at zero recoil ($\omega = 1$ or $q^2 = q^2_{\text{max}}$) and $f_1(0) = 0.38$, $f_2(0) = 0.06$, $R = 0.16$ at maximum recoil ($\omega = \omega_{\text{max}}$ or $q^2 = 0$). One has to remark that our results at $q^2_{\text{max}}$ are closed to the results of nonrelativistic quark model [16]: $f_1(q^2_{\text{max}}) = 0.75$, $f_2(q^2_{\text{max}}) = 0.17$, $R = 0.23$.

Also our result for $R$ weakly deviate from the experimental data [1] $R = -0.25 \pm 0.14 \pm 0.08$ and the result of nonrelativistic quark model (Ref. [16]). Our prediction for the decay rate $\Gamma(\Lambda^+_c \to \Lambda^0 e^+ \nu_e) = 7.22 \times 10^{10}$ sec$^{-1}$ and asymmetry parameter $\alpha_{\Lambda_c} = -0.812$ also coincides with the experimental data $\Gamma_{\exp} = 7.0 \pm 2.5 \times 10^{10}$ sec$^{-1}$ and $\alpha_{\Lambda_c}^{\exp} = -0.82^{+0.09+0.06}_{-0.06-0.03}$ and the data of Ref. [16] $\Gamma = 7.1 \times 10^{10}$ sec$^{-1}$. One has to remark that the success in the reproducing of experimental results is connected with the using of the $\Lambda^0$ three-quark current in the $SU(3)$-flavor symmetric form. By analogy, in the nonrelativistic quark model [16] the assuming the $SU(3)$ flavor symmetry leads to the presence of the flavor-suppression factor $N_{\Lambda_c \Lambda} = 1/\sqrt{3}$ in matrix element of $\Lambda^+_c \to \Lambda^0 e^+ \nu_e$ decay. If the $SU(3)$ symmetric structure of $\Lambda^0$ hyperon is not taken into account the predicted rate for $\Lambda^+_c \to \Lambda^0 e^+ \nu_e$ became too large (see, discussion in ref. [11, 16]). Finally, in Table 2 we give our predictions for some modes of semileptonic heavy-to-lights transitions. Also the results of other approaches are tabulated.

Table 2. Heavy-to-Light Decay Rates (in $10^{10}$ s$^{-1}$).

| Process       | Quantity | Ref. [12] | Ref. [16] | Ref. [20] | Our   | Experiment     |
|---------------|----------|-----------|-----------|-----------|-------|----------------|
| $\Lambda^+_c \to \Lambda^0 e^+ \nu_e$ | $\Gamma$ | 9.8       | 7.1       | 5.36      | 7.22  | 7.0 $\pm$ 2.5 |
| $\Xi^0_b \to \Xi^+ e^- \bar{\nu}_e$  | $\Gamma$ | 8.5       | 7.4       |           | 8.16  |                |
| $\Lambda^0_b \to pe^- \bar{\nu}_e$   | $\Gamma/|V_{be}|^2$ | 6.48 x $10^2$ |           | 7.47 x $10^2$ |       |                |
| $\Lambda^+_c \to ne^+ \nu_e$        | $\Gamma/|V_{cd}|^2$ |           |           | 0.26 x $10^2$ |       |                |
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References

[1] CLEO Collaboration, G.Crawford, et al, Phys.Rev.Lett. 75 (1995) 624.
[2] ALEPH Collaboration, D.Decamp et al., Phys.Lett. B278 209 (1992).
[3] OPAL Collaboration, P.D.Acton et al., Phys.Lett. B281 394 (1992).
[4] I.V.Anikin et al., Phys.At.Nucl. 57, 1082 (1994); Z.Phys. C65, 681 (1995).
[5] T.Goldman and R.W.Haymaker, Phys.Rev. D24, 724 (1981).
[6] M.A.Ivanov, M.P.Locher, V.E.Lyubovitskij, Preprint PSI-PR-96-08 (to be publ. in Few Body Syst.)
[7] M.A.Ivanov and V.E.Lyubovitskij, Proceedings of ”HADRON-95”, p.396.
[8] E.V.Shuryak, Nucl.Phys. B198, (1982) 83.
[9] Q.P.Xu, Phys. Rev. D48 (1993) 5429.
[10] A.G.Grozin and O.I.Yakovlev, Phys.Lett. B291 (1992) 441.
[11] J.G.Körner, D. Pirjol and M. Krämer, Prog.Part.Nucl.Phys. 33 (1994) 787.
[12] R.Jr.Singleton, Phys.Rev. D43 (1991) 2939.
[13] X.-H.Guo, P.Kroll, Z. Phys. C59 (1993) 567.
[14] B.König, J.G.Körner, et al., Preprint DESY 93-011 (1993).
[15] M.Sadzikowski and K.Zalewski, Z.Phys. C59 (1993) 677.
[16] H.-Y.Cheng & B.Tseng, Phys.Rev. D53 (1996) 1457.
[17] B.Holdom, M.Sutherland and J.Mureika, Phys.Rev. D49 (1994) 2359.
[18] M.Sutherland, Z.Phys. C63 (1994) 111.
[19] Y.-B.Dai, et al., Preprint AS-ITP-96-08 (1996).
[20] A. Datta, Preprint UH-511-825-95 (1995).
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Fig. 1 $\zeta(\omega)$ form factor
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2. Grozin & Yakovlev [10]
3. Our result
4. Dipole formula [11]
5. Sadzikowski & Zalewski [15]
6. Körner et al. [11]
7. Guo & Kroll [13]

Fig. 2 Form factors $f_1, f_2$ and ratio $R$ for $\Lambda_c^+ \to \Lambda^0 e^+\nu$ decay