String Interactions in PP-Waves

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Abstract

We argue that string interactions in a PP-wave spacetime are governed by an effective coupling $g_{\text{eff}} = g_s(\mu p^+\alpha')f(\mu p^+\alpha')$ where $f(\mu p^+\alpha')$ is proportional to the light cone energy of the string states involved in the interaction. This simply follows from generalities of a Matrix String description of this background. $g_{\text{eff}}$ nicely interpolates between the expected result ($g_s$) for flat space (small $\mu p^+\alpha'$ ) and a recently conjectured expression from the perturbative gauge theory side (large $\mu p^+\alpha'$).
1. Introduction

A recent important insight into the physics of large \( N, N = 4 \) Super Yang-Mills theory has been that one might be able to isolate the dynamics of subsectors of the full theory, for instance, those carrying large global quantum numbers \([1],[2]\). Rather remarkably, BMN \([3]\) conjectured that a well defined subset of the states in the gauge theory, carrying a large \( U(1) \) R-charge, are really the excitations of a Type IIB closed string in a PP-wave background geometry.

\[
\begin{align*}
 ds^2 &= dx^- dx^+ - \mu^2 \left( \sum_{i=1}^{8} (x^i)^2 \left( dx^+ \right)^2 + \sum_{i=1}^{8} (dx^i)^2 \right) \\
 F^{(5)}_{+1234} &= F^{(5)}_{+5678} = \mu. 
\end{align*}
\] (1.1)

Moreover, free string theory in this background is exactly solvable in lightcone gauge \([3]\). This allows one in principle to make direct comparisons between the gauge theory and the full string theory (beyond a supergravity limit) \([3]\).

In particular, the spectrum of the first quantised strings is given in terms of independent oscillators with the light cone energy of the \( n \)th oscillator being

\[
E_n = \mu \sqrt{1 + \frac{n^2}{(\mu p^+ \alpha')^2}}. 
\] (1.2)

The total light cone energy of a generic string state is \( E \equiv \mu f(\mu p^+ \alpha') = \sum_n N_n E_n \). The lightcone momentum \( p^+ \) is proportional to the \( U(1) \) R-charge \( J \) of the gauge theory

\[
\mu p^+ \alpha' = \frac{J}{\sqrt{\lambda}} = \frac{J}{g_{YM} \sqrt{N}}. 
\]

The energy \( E_n \) translates into the anomalous dimension in the gauge theory of the corresponding operator \((\Delta - J)_n = \sqrt{1 + \frac{n^2}{J^2}} \). For finite \( g_{YM}^2 = g_s \), the 'tHooft coupling blows up in the large \( N \) limit. However, \((\Delta - J)_n \) is finite provided one scales \( J \to \infty \) such that \( \frac{J^2}{N} \) is held fixed. In fact, there are good indications that, rather than \( \lambda \), it is \( \lambda' = \frac{\lambda}{J^2} = \frac{1}{(\mu p^+ \alpha')^2} \) which is the right expansion parameter for perturbative gauge theory in this sector \([1],[4]\).

A formalism for studying string interactions in the lightcone framework in this background has been developed by \([3]\) following the approach of \([3],[4]\) for flat space \([3]\). Interactions in a DLCQ framework are also being studied \([3]\). On the gauge theory side, it has

\[2\] A covariant formalism has also been proposed in \([8]\).
been observed by [10] [11] [12] that despite strictly taking $N \to \infty$, nonplanar diagrams survive, being suppressed by factors of $\frac{J^2}{N^2} = g_s (\mu p + \alpha')^2$. In other words, contributions from gauge theory diagrams of genus $g$ are weighted with a factor of $\frac{J^{2g}}{N^{2g}}$. Thus it might be natural to guess that the three string interaction which governs splitting and joining of strings is weighted by an effective string coupling of $\frac{J^2}{N} = g_s (\mu p + \alpha')^2$.

However, in this note we argue that the effective string coupling is actually $g_{eff} = g_s (\mu p + \alpha') f(\mu p + \alpha')$ where $E = \mu f(\mu p + \alpha')$ is the total light cone energy involved in the interaction. In the most generic case involving a few ($O(1)$) oscillators with $O(1)$ excitations we see from Eq.(1.2) that,

$$f(\mu p + \alpha') \sim \sqrt{1 + \frac{1}{(\mu p + \alpha')^2}}$$

\[\Rightarrow g_{eff} = g_s (\mu p + \alpha') \sqrt{1 + \frac{1}{(\mu p + \alpha')^2}} = \frac{J^2}{N} \sqrt{\frac{\lambda}{J^2}} (1 + \frac{\lambda}{J^2}) \tag{1.3}\]

which is quite different from the above natural guess.

Our reasoning is based on a second quantised Matrix String picture of interacting strings in the PP-wave background. The details of this Matrix String description will appear in a forthcoming publication [13]. Here we will obtain $g_{eff}$ simply by applying some scaling arguments that will need only the general features of the Matrix String picture.

The $g_{eff}$ obtained here is in pleasing accord with a couple of facts. Consider the generic case where $f$ is as in Eq.(1.3). Firstly, in the limit of small $\mu p + \alpha'$ (or large $\lambda'$) one recovers the flat space answer for $g_{eff}$, namely, simply $g_s$. Secondly, for large $\mu p + \alpha'$ (or small $\lambda'$), $g_{eff} \sim g_s (\mu p + \alpha')$. In their study from the perturbative gauge theory side, the authors of [12] were led to conjecture (for small $\lambda'$) precisely $g_s (\mu p + \alpha') = \frac{J^2}{N} \sqrt{\lambda'} = \frac{J}{\sqrt{N}} g_{YM}$ as the effective interaction, at least amongst a class of string states. Thus our $g_{eff}$ provides the interpolating behaviour between these two regimes. We note in passing that our formula implies that for any small but fixed $g_s$, (as well as fixed $(\mu p + \alpha')$) perturbation theory breaks down if one scatters high energy string states.

2. Matrix String Interactions

2.1. Flat Space

We briefly recapitulate the Matrix String description of flat space.
Matrix String theory [14][15][16] elegantly encapsulates the physics of weakly coupled second quantised strings in light cone gauge in terms of the IR dynamics of a (1 + 1) dimensional gauge theory. For instance, in the case of Type IIA in $R^{10}$ with a null circle $x^− \sim x^− + 2\pi R_0$ and $p^+ = \frac{J}{R_0}$, the gauge theory is the maximally supersymmetric (1 + 1) dimensional $U(J)$ Yang-Mills theory

$$S = \int d^2\sigma Tr_J \left(-\frac{1}{4}F_{\mu\nu}^2 + (D_\mu \Phi^i)^2 + g_{YM}^2[\Phi^i, \Phi^j]^2 + \text{fermions} \right).$$

The original parameters of the IIA lightcone string theory, namely, $R_0$ and the string coupling $g_{sIIA}$ translate into the gauge theory parameters [17] $g_{YM} = \frac{R_0}{g_{sIIA}\alpha'}$, $\Sigma_1 = \frac{\alpha'}{R_0}$

where $\Sigma_1$ is the radius of the spatial direction in the Yang-Mills.

The free string limit $g_{sIIA} \to 0$ corresponds to the IR (strong coupling) in the gauge theory. This is described by a free orbifold conformal field theory. The classical moduli space consists of commuting configurations $\Phi^I(\sigma) = \text{diag}\{\phi^I(\sigma)\}$, $(I = 1 \cdots J)$. Since the gauge symmetry includes the action by the Weyl group $S_J$, one can have long string configurations $\Phi^j(\sigma + 2\pi \Sigma_1) = g\Phi^j(\sigma)g^{-1}$, where $g$ is an element of the Weyl group and hence permutes the eigenvalues. The length of each permutation cycle in $g$ is the number of bits that go into forming a long string and is proportional to the $p^+$ it carries. The number of distinct cycles is the number of strings. In the limit $J \to \infty$ one can describe any number of free strings each carrying some arbitrary fraction of the total light cone momentum.

Similarly, for IIB string theory with $p^+ = \frac{J}{R_0}$ one first compactifies on a transverse circle of radius $R_1$, T-dualises to Type IIA, lifts to M-theory and interchanges $x^−$ and $x^{11}$, (the “9-11 flip”). The end result is a theory of $J$ D-0 branes on two transverse circles in a decoupling limit [18][17]. Or equivalently, the maximally supersymmetric $U(J)$ Yang-Mills in (2 + 1) dimensions with the parameters [17]

$$g_{YM}^2 = \frac{R_0(R_1)^2}{g_{sIIB}(\alpha')^2}, \quad \Sigma_1 = \frac{\alpha'}{R_0}, \quad \Sigma_2 = \frac{\alpha'}{R_0}g_{sIIB}^2,$$  

(2.1)

where $\Sigma_i$ are the radii of the two spatial directions.

The main difference from Type IIA arises in that even without going to weak coupling, if one takes $R_1 \to \infty$ (the noncompact limit of the original theory) one is at strong Yang-Mills coupling. The resulting theory at the origin of the moduli space is the interacting
maximally supersymmetric $(2+1)$ dimensional CFT. The massless modes are parametrised by commuting configurations $\Phi^j(\sigma_1, \sigma_2) = \text{diag}\{\phi^j_1(\sigma_1, \sigma_2)\}, \ (I = 1 \cdots J)$. Again they can obey twisted boundary conditions but now in both spatial directions: $\Phi^j(\sigma_1 + 2\pi \Sigma_1, \sigma_2) = g_1 \Phi^j(\sigma_1, \sigma_2)g_1^{-1}$ and similarly with indices $1, 2$ interchanged. To the extent that one can classically describe this interacting theory, these configurations can be thought of as long membranes wrapped some number of times on both the cycles of the torus.

The simplification at weak coupling, $g_s^{\text{IIB}} \to 0$, is that $\Sigma_2 \to 0$ and one has effectively a $(1+1)$ dimensional theory. The dimensional reduction is therefore the same orbifold CFT as above, the only difference being that the fermions now have opposite chirality. The massless modes are thus long strings as in the IIA case, but now originate from the higher dimensional configurations which are twisted only in the $\sigma_1$ direction.

2.2. The PP-Wave Spacetime

The matrix string description for the IIB PP-wave (1.1) closely follows the description for flat space [13]. We will study it as a limit $J, R_0 \to \infty \ (R_0 = \mu \sqrt{\lambda} \alpha')$ of a DLCQ theory with $p^+ = \frac{J}{R_0}$ fixed (see [19] [20] for an explicit realisation of the finite $R_0$ geometry).

Despite appearances the background Eq.(1.1) has eight commuting $U(1)$ isometries. One can compactify the Type IIB theory on one of these directions with radius $R_1$. T-duality takes one to a Type IIA background which can be lifted to M-theory [21]

$$ds^2_M = dx^- dx^+ - \mu^2 \left( \sum_{I=3}^8 (x^I)^2 + 4(x^9)^2 \right) (dx^+)^2 + \sum_{i=1}^9 (dx^i)^2$$

$$F_{129}^{(4)} = -2\mu; \quad F_{349}^{(4)} = 4\mu. \quad (2.2)$$

Reducing along $x^-$ gives a theory of $J$ D0-branes in a nontrivial curved background with two compact transverse directions $(x^1, x^2)$. The appropriate description of the decoupled theory is again a $(2 + 1)$ dimensional $U(J)$ Yang-Mills theory with the same field content as in the flat space case. But now with some number of additional pieces in the lagrangian

$$\Delta S = \int d^3\sigma (\Delta L_m + \Delta L_{\text{myers}} + \Delta L_{\text{ferm}});$$

$$\Delta L_m \propto \mu^2 Tr J(\Phi^i)^2; \quad \Delta L_{\text{myers}} \propto \mu M_{ijk} Tr J(\Phi^i \Phi^j \Phi^k). \quad (2.3)$$

It is easy to see the origin of these terms from Eq.(2.2). The mass terms come from the nontrivial part of the metric, while the Myers terms come from $F^{(4)}$. The corresponding mass and cubic terms involving fermions are determined by supersymmetry. This and other details, which will appear in [13], will not be crucial for our scaling argument.
2.3. String Interactions

Let us momentarily ignore the mass terms etc. in (2.3). The weak coupling picture of strings splitting and joining emerges beautifully from the orbifold CFT description [16] (which as we have seen is the $g_s \to 0$ limit of both IIA and IIB Matrix Strings, allowing for the differences in chirality). The leading irrelevant operator in the CFT is determined by the symmetries of the theory to be a dimension 3 operator, $V_{\text{int}} \propto \frac{1}{M} \int d^2 \sigma O^{(3)}$, built out of the twist fields of the CFT. The twist fields lead to precisely the splitting and joining of strings by permuting eigenvalues. A detailed study recovers the Mandelstam light-cone interaction vertex (delta function overlap) for these strings. We should think of this operator as arising, in the effective field theory language, from integrating out massive states whose dynamics we are not interested in. The mass parameter $M$ in $V_{\text{int}}$ is set by the lightest such state. In the Type IIB case, in reducing to the $(1+1)$ dimensional theory, we have ignored KK excitations, the lightest of which has mass $M = \frac{1}{J S_2}$. This comes from a configuration carrying fractional momentum in the $\sigma_2$ direction [22] [23].

Turning on the mass terms in (2.3) will not seriously disturb this effective field theory description provided we consider energies much less than $M$. To be precise, we want the quadratic part of the Hamiltonian (which we denote by $H_0$ and which includes the mass terms) to have energies much less than $M$. The relevant configurations will then continue to be diagonal matrices with the $S_J$ action. In other words, at the energy scales set by $H_0$, the states correspond to the second quantised Fock space of free strings in the PP-wave spacetime. The Matrix String action in this limit is essentially a (diagonal) matrix version of the Green-Schwarz action in the PP-wave background (as is appropriate for a description of multiple strings).

Since the eigenvalues of $H_0$ are the light cone energies of the free string excitations involved in the interaction (Eq. (1.2) and below), we need $E = \mu f(\mu p^+ \alpha') << M$. Using Eq. (2.1) this translates into $g_{\text{eff}} = g_s(\mu p^+ \alpha') f(\mu p^+ \alpha') << 1$ as the condition for the DVV picture of string interactions to hold.

Stated differently, from effective field theory reasoning, the dimensionless coupling that suppresses the higher dimension operator $O^{(3)}$ is really $g_{\text{eff}} = \frac{E}{M}$ and we can rewrite the above interaction vertex as,

$$V_{\text{int}} \propto \frac{g_{\text{eff}}}{E} \int d^2 \sigma O^{(3)}.$$

As mentioned in the introduction, this effective coupling $g_{\text{eff}}$ interpolates between flat space behaviour and that suggested by perturbative gauge theory.
We should remark here that the operator $O^{(3)}$ should be thought of as the DVV interaction operator but now dressed up by RG flow to energy scales $E$. The mass deformation preserves the original $S_J$ orbifold action. This and the constraint of supersymmetry should hopefully enable one to obtain an explicit expression for $O^{(3)}$ in terms of twisted sector modes of the free (but now massive) orbifold theory. This would be important for using the Matrix String description to do explicit string computations. We hope to address this issue in the future. It would also be very interesting to compare the results here with that from light cone string field theory as developed in [5].

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