Application of a multi-input multi-output (MIMO) nonlinear non-minimum phase system control method to hydro turbine unit

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Abstract. In this paper, in view of the non minimum phase characteristics of hydraulic turbine unit, a method of designing the controller of multi-input multi-output (MIMO) nonlinear non-minimum phase system is proposed and applied to hydraulic turbine unit system described by double control input fifth order model. The turbine output electromagnetic power and generator power angle are stabilized by wicket gate opening and excitation combined control. Firstly, the method for designing the controller of multi-input multi-output (MIMO) nonlinear non-minimum phase system is introduced. The method is based on state feedback theory, pole assignment theory and Lyapunov stability theory. Secondly, the control law of hydro turbine unit is given according to the method. Results of simulation indicate that this nonlinear non-minimum phase controller has a good effect when applied to the hydro turbine unit.

1. Introduction
As a complex hydro-electromechanical system, the water hammer effect in the hydraulic turbine makes it a typical non-minimum phase system and the unstable zero dynamics in the hydraulic turbine unit system make it difficult to adjust. The approximate linearization method is a well-known system design method in the control field, and the most common one is to take the first-order approximation of the system. Although this method is simple and easy, it is only suitable for situations where the range of working points does not change much [1]. The PID control system with simple structure and easily adjustable parameters is commonly adopted in a hydraulic turbine unit regulation system. Under specific working conditions, proper selection of the proportion, integral and differential coefficient of unit speed deviation for linear combination can enable the system to achieve better dynamic performance and steady-state performance: In literature [2], PID control is applied to the single-input single-output turbine system, and good control results are obtained. However, for multi-input multi-output systems, because of the coupling between the control loops, it is not easy to adjust the PID parameters.

In the past 30 years, based on the theory of differential geometry, the accurate feedback linearization method for nonlinear systems has made great progress. This approach aims at transforming a nonlinear system into a linear system through a suitable set of coordinate transformation and an appropriate state feedback, then applying the linear system optimal control theory to design the controller of the linearized standard system. Finally, the obtained linear control law is brought back to the nonlinear state feedback expression, from which the control input of the original nonlinear system can be inversely solved. This method does not use any approximate
linearization method, so it can completely retain the non-linear characteristics of the system and overcome the limitations of the approximate linearization method. And this method can obtain a fully analytical nonlinear controller in the form of state feedback, which is easy to be realized in engineering. The optimality of the control law obtained by this approach has been proved in the literature [3]. In literature [4], a current-predictive control of permanent magnet synchronous motor based on input-output feedback linearization decoupling algorithm is proposed. Literature [5] combines sliding mode control based on feedback linearization with nonlinear disturbance observer to improve position control tracking accuracy of electro-hydraulic servo system. In Literature [6], accurate feedback linearization is applied to DC microgrid. Literature [7] applies the accurate feedback linearization method to the high speed UAV system.

Generally, the accurate state feedback linearization method transforms the original nonlinear system into two parts by differential homeomorphism mapping: the external dynamics which is described by the linear differential equations, and the internal dynamics (i.e., zero dynamics) which is described by the nonlinear differential equations. For nonlinear non-minimum phase systems with unstable zero dynamics, controller designed only for linear subsystems has the ability to control the external dynamics to meet the performance target, but it is difficult to ensure the internal zero dynamic stability of the system. When designing controllers for these nonlinear control systems with non-minimum phase characteristics, if the linear subsystem part is considered and the nonlinear subsystem part is neglected, under the external disturbance, the overshoot of the system is very likely to occur [8]. However, nonlinear systems with non-minimum phase characteristics exist widely in practical engineering. And hydraulic turbine unit is one of the typical non-minimum phase systems.

In view of the non minimum phase characteristics of turbine unit, in this paper, based on accurate feedback linearization method, the design method of single-input single-output (SISO) non-minimum phase system controller given in literature [9, 10] is improved and extended to the multiple input multiple output (MIMO) system and applied to the controller design of hydraulic turbine unit: the generator power angle and output electromagnetic power are adjusted by the wicket gate opening and excitation combined control. The specific design process is as follows: Firstly, the corresponding coordinate transformation are determined according to the relative order of the system, and through coordinate transformation, the linearized standard form of original nonlinear system is obtained which includes two independent linear subsystems and zero dynamics; Secondly, put zero dynamics into any linear subsystem to form a non-linear subsystem, and zero dynamics is adjusted in this non-linear subsystem. Then, use state feedback to stabilize the linear subsystem. State feedback and nonlinear compensation term are introduced to adjust the external dynamics and internal zero dynamics in the nonlinear subsystem respectively. In this paper, the simulation is carried out under different water hammer conditions, and the simulation results of non-minimum phase controller and PID controller are compared.

2. Controller design method of MIMO non-minimum phase system

2.1. Accurate feedback linearization method of MIMO nonlinear systems

The differential geometry theory is briefly reviewed in literature [11], and the accurate state feedback theory is introduced in literature [12]. The basic idea of accurate feedback linearization is to transform a nonlinear system into a linear system through a set of differential homeomorphism coordinate transformation and an appropriate state feedback, and then design the controller. For convenience, the following discussion will focus on dual-input and dual-output systems, and other multiple input and multiple output situations can be done with a similar simple deduction.

For dual input and dual output systems:

\[
\begin{cases}
\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2 \\
y_1 = h_1(x) \\
y_2 = h_2(x)
\end{cases}
\]  

(1)
is a \( n \)-dimensional state column vector, \( f(x), g_i(x), g_2(x) \) are \( n \)-dimensional smooth vector fields in the state space, \( u_1, u_2 \) are control scalar inputs, \( y_1, y_2 \) are scalar outputs, and \( h_1(x), h_2(x) \) are output functions.

If the set of relative orders of the system is \( r = \{r_1, r_2\} \) \([13]\) \((r = r_1 + r_2)\), you can take the differential homeomorphic mapping:

\[
\varphi(x) = \begin{bmatrix} z_1 \\ z_2 \\ \eta \end{bmatrix} = \begin{bmatrix} x_1(x) \\ x_2(x) \\ \vdots \\ x_{i_{n-r}}(x) \end{bmatrix} = \begin{bmatrix} h_1(x) \\ L_{i_{n-r}}h_1(x) \\ \vdots \\ h_2(x) \\ L_{i_{n-r}}h_2(x) \\ \vdots \\ \eta_1 \\ \vdots \\ \eta_{n-r} \end{bmatrix}
\]

(2)

\( z_1, z_2, \eta \) are state vectors of system after coordinate Transformation. \( L_{i}h(x) \) is Lie derivative operation. The coordinate transformation \( \phi_i(x) \) satisfies:

\[
L_{i}\varphi(x) = L_{i}\varphi_i(x) = 0, \quad 1 \leq i \leq n-r
\]

(3)

When the jacobian matrix of \( \varphi(x) \) at the system equilibrium point \( x = x_0 \) is nonsingular, the nonlinear system can be converted into the following standard type:

\[
\dot{z}_{1i} = z_{12} \\
\vdots \\
\dot{z}_{ih_{-1}} = z_{ih_{0}} \\
\dot{z}_{ih_0} = a_{i}(z, \eta) + b_{i1}(z, \eta)u_1 + b_{i2}(z, \eta)u_2 \\
\dot{z}_{i2} = z_{i22} \\
\vdots \\
\dot{z}_{i_{n-r}} = z_{i_{n-r}} \\
\dot{z}_{i_{n-r}} = a_{i}(z, \eta) + b_{21}(z, \eta)u_1 + b_{22}(z, \eta)u_2 \\
\dot{\eta} = q(z, \eta) \\
y_1 = z_{11} \\
y_2 = z_{21}
\]

(4)

Where:

\[
z = \begin{bmatrix} z_1, z_2 \end{bmatrix}^T, \quad z_1 = \begin{bmatrix} z_{11}, z_{12}, \cdots, z_{ih_0} \end{bmatrix}^T, \quad z_2 = \begin{bmatrix} z_{i21}, z_{i22}, \cdots, z_{i_{n-r}} \end{bmatrix}^T, \quad \eta = \begin{bmatrix} \eta_1, \eta_2, \cdots, \eta_{n-r} \end{bmatrix}^T
\]

\[
q(z, \eta) = \begin{bmatrix} q_1(z, \eta), \cdots, q_{n-r}(z, \eta) \end{bmatrix}^T, \quad q_i(z, \eta) = L_{i}\phi_i(\Phi^{-1}(z, \eta))
\]

(5)

Definition:

\[
\dot{\eta} = q(\theta, \eta)
\]

Is the zero dynamic equation of the nonlinear system, \( \eta \) represents the zero dynamics, \( z \) represents the external state, \( \theta \) represents the equilibrium point, when \( \eta \) is unstable, the nonlinear system is a nonlinear non-minimum phase system.
State feedback in (4) is written in matrix form:
\[
\begin{align*}
\dot{z}_1 &= a_1(z, \eta) + b_{11}(z, \eta)u_1 + b_{12}(z, \eta)u_2 = z_1
\dot{z}_2 &= a_2(z, \eta) + b_{21}(z, \eta)u_1 + b_{22}(z, \eta)u_2 = z_2 + \eta
\rightarrow \dot{A}(z, \eta) + B(z, \eta)u = v
\end{align*}
\] (6)

According to the definition of relative order, \(B(z, \eta)\) is a non-singular matrix, so its inverse matrix can be obtained. Then the external state \(z\) can be linearized by taking the control law \(u = B^{-1}(z, \eta)(v - A(z, \eta))\). And adopt linear system controller design method to design the controller \(v\) can make the external state stable.

However, it should be noted that in the standard form (4), the internal state \(\eta\) is completely independent of the controlled input \(u_1\) and \(u_2\), so it is not observable at this point. For systems with stable zero dynamics, the controller designed by the above method will not have any problems. For systems with unstable zero dynamics, under the control of such a controller, it is very likely to lead to the divergence of \(\eta\), so such a controller design method is not feasible for non-minimum phase systems. Therefore, it is necessary to adopt different control strategies for non-linear non-minimum phase systems.

2.2. Design of controller for MIMO non-minimum phase system

The linearization standard form (4) of the nonlinear system can be divided into two independent subsystems as shown in (7) below. The nonlinear subsystem (b) is obtained by putting zero dynamics \(\eta\) into linear subsystem whose order is \(r_1\).

\[
\begin{align*}
\dot{z}_{21} &= z_{22} \\
\vdots \\
\dot{z}_{2r_1} &= z_{2r_1} \\
\dot{z}_{2r_1} &= v_2 \\
\eta &= q(z, \eta)
\end{align*}
\] (a)

\[
\begin{align*}
\dot{z}_{11} &= z_{12} \\
\vdots \\
\dot{z}_{1r_1} &= z_{1r_1} \\
\dot{z}_{1r_1} &= v_1 \\
y_1 &= z_{11}
\end{align*}
\] (b)

For linear subsystem (a), it can be adjusted by adopting state feedback \(v_2 = -Kz_2\).

For nonlinear subsystem (b), according to the control method of the single-input single-output (SISO) non-minimum phase system given in literature [9, 10], the external dynamics can be adjusted by adopting state feedback, and zero dynamics can be stabilized by introducing the nonlinear regulation term. The specific steps are as follows:

First, do the first-order Taylor series expansion of the zero dynamics at equilibrium point \((z_0, \eta_0)\):

\[
\dot{\eta} = q(z, \eta) = A_1z_1 + A_\eta \eta + g(z, \eta)
\] (8)

Where:

\[
A_1 = \left. \frac{\partial q(z, \eta)}{\partial z_1} \right|_{(z, \eta) = (z_0, \eta_0)} A_\eta = \left. \frac{\partial q(z, \eta)}{\partial \eta} \right|_{(z, \eta) = (z_0, \eta_0)}
\]

Note that since these two subsystems are independent of each other, it can be considered that \(z_2\) in (a) has been adjusted to a steady state, and it will not have an impact on \(\eta\), so only taking the partial derivative for \(z_1\) \((z_0, \eta_0)\) is the corresponding system equilibrium point after coordinate transformation. \(g(z, \eta)\) is the high-order infinitesimal terms in Taylor expansion, and the second-order term can be used to replace it in practical operation, and the rest high-order terms can be eliminated. Since the elimination of high-order infinitesimal terms has little impact on the performance of the system, this practice is indeed feasible. The advantage of doing so is that it eliminates the situation where uncertainties lead to zero denominators. The specific form of the high-order infinitesimal terms \(g(z, \eta)\) can also be obtained by the formula: \(g(z, \eta) = q(z, \eta) - A_1z_1 - A_\eta \eta\).
Thus subsystem (b) can be rewritten as follows:

\[
\begin{bmatrix}
\dot{z}_i \\
\dot{\eta}
\end{bmatrix} = A \begin{bmatrix} z_i \\
\eta \end{bmatrix} + B v_i + \begin{bmatrix} 0 \\
g(z, \eta) \end{bmatrix}
\]

(9)

Where:

\[
A = \begin{bmatrix} A_i & 0 \\
A_i & A_i \end{bmatrix}, \quad B = \begin{bmatrix} 0 & \ldots & 0 \\
1 & 0 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & 1 \\
0 & 0 & \ldots & 0 \\
\end{bmatrix}
\]

\[
0_i, 0_{i-1} \text{ and } 0_n \text{ represent zero vector.}
\]

The feedback controller is set as follows:

\[
v_i = -G \begin{bmatrix} z_i \\
\eta \end{bmatrix} + v_{NL}
\]

(10)

Where the column gain vector \( G \) is used to adjust the linear part of subsystem (b), the nonlinear regulation term \( v_{NL} \) and is used to adjust the zero dynamics. Combined with (10), Equation (9) becomes:

\[
\begin{bmatrix}
\dot{z}_i \\
\dot{\eta}
\end{bmatrix} = (A - BG) \begin{bmatrix} z_i \\
\eta \end{bmatrix} + B v_i + \begin{bmatrix} 0 \\
g(z, \eta) \end{bmatrix}
\]

(11)

Let \( A_i = A - BG \). By selecting appropriate \( G \), \( A_i \) becomes a Hurwitz matrix \( G \) can be obtained by means of pole assignment and other methods.

Lyapunov function of the system can be selected as:

\[
L(z_i, \eta) = \begin{bmatrix} z_i^T \\
\eta^T \end{bmatrix} P \begin{bmatrix} z_i \\
\eta \end{bmatrix}
\]

(12)

\( P \) can be obtained through the following condition:

\[
\forall Q > 0, \exists P > 0, \text{ s.t. } A_i^T P + P A_i = -Q
\]

(13)

\( Q \) is a positive definite real symmetric matrix and is generally taken as a unit matrix. System stability requires \( L(z_i, \eta) < 0 \), that is:

\[
\dot{L}(z_i, \eta) = -\begin{bmatrix} z_i^T \\
\eta^T \end{bmatrix} Q \begin{bmatrix} z_i \\
\eta \end{bmatrix} + 2 \begin{bmatrix} 0_{i-1} \\
v_{NL} \\
g(z, \eta)^T \end{bmatrix} P \begin{bmatrix} z_i \\
\eta \end{bmatrix} < 0
\]

(14)

Obviously, the first term in the equation is negative, so the sufficient condition for the inequality (14) to hold is:

\[
\begin{bmatrix} 0_{i-1} \\
v_{NL} \\
g(z, \eta)^T \end{bmatrix} P \begin{bmatrix} z_i \\
\eta \end{bmatrix} = 0
\]

(15)

Then, \( v_{NL} \) can be obtained by solving the equation:

\[
v_{NL} = -\sum_{i=1}^{n} g_i(z, \eta) P_{i, i} \begin{bmatrix} z_i \\
\eta \end{bmatrix}
\]

(16)

Where:

\[
g(z, \eta) = \begin{bmatrix} g_1(z, \eta) & \ldots & g_{n-1}(z, \eta) \end{bmatrix}^T, \quad P = \begin{bmatrix} P_1^T & P_2^T & \ldots & P_{n-1}^T \end{bmatrix}^T
\]
In this way, feedback controller \( v_i \) is determined, and the system state is adjusted on the premise of ensuring the stability of system's internal zero dynamics. Finally, the original system's input control laws \( u_1 \) and \( u_2 \) can be inversely solved according to state feedback Expression (6).

### 3. MIMO non-minimum phase system controller’s application to hydraulic turbine unit

#### 3.1. Nonlinear model of hydraulic turbine unit

The hydraulic turbine unit adopts the approximate model single-machine-infinite model of third-order non-salient pole generator because the hydropower station is usually far away from the load center and is connected to the system through long-distance transmission lines \([14, 15]\). The regulating model of hydraulic turbine unit controlled jointly by the wicket gate opening and excitation can be expressed as [16]:

\[
\begin{align*}
\dot{\delta} &= \omega - \omega_b \\
\dot{\omega} &= \frac{a_p}{H} P_m - D (\omega - \omega_b) - \frac{a_p}{H} E'_s V_s x_{d\Sigma} \\
\dot{x}_d' &= \frac{x_{d\Sigma}}{d_{d\Sigma}} E' + \frac{x_d - x_d'}{T_{d0}} V_s \cos \delta + \frac{V_f}{T_{d0}} \\
P_m &= \frac{2}{T_w} \left[ -P_e + \mu - \frac{T_s}{T_w} (\mu_0 - \mu + u) \right] \\
\mu &= \frac{1}{T_v} (\mu_0 - \mu + u)
\end{align*}
\]

The meaning of the parameters is as follows:

- The system state variables are: generator power angle \( \delta \), generator rotor angular velocity \( \omega \), \( q \)-shaft transient potential \( E'_q \), hydraulic turbine mechanical power \( P_m \), wicket gate opening \( \mu \); Other parameters: wicket gate opening control quantity \( u \) and excitation control voltage \( V_f \), synchronous angular velocity \( \omega_b \), infinite bus voltage \( V_s \), water hammer time constant \( T_w \), servomotor reaction time constant \( T_s \), generator damping coefficient \( D \), generator inertia constant \( H \), line reactance \( x_L \), \( d \)-shaft synchronous reactance \( x_d \), \( d \)-shaft transient reactance \( x_d' \), transformer reactance \( x_r \), excitation winding time constant \( T_{d0} \).

- When the system is running in double-circuit, the reactance is calculated as follows:

\[
\begin{align*}
x_{d\Sigma} &= x_d + x_r + x_L / 2 \\
x_{d\Sigma}' &= x_d' + x_r + x_L / 2
\end{align*}
\]

System control input are wicket gate opening control quantity \( u \) and excitation control voltage \( V_f \). Electromagnetic power \( P_e \) and generator power angle \( \delta \) are the controlled output. Evaluation expression of \( P_e \) is:

\[
P_e = \frac{E_s' V_s}{x_{d\Sigma}} \sin \delta
\]

#### 3.2. Designing of hydraulic turbine unit controller

The concept of relative order of MIMO system can be referred to the literature \([13]\), which is not explained in detail here.

Take the derivative of the output \( P_e \) and \( \delta \) respectively until the control input is explicitly included:

\[
\dot{P_e} = \frac{V_f}{x_{d\Sigma}} \left[ \delta \cos \delta E' + \sin \delta \dot{E}' \right]
\]

Obviously, \( \dot{E}' \) explicitly contain control input \( V_f \), so the relative order \( r_i = 1 \).
Make the following transformation and derivation:
\[ z_{21} = \delta \rightarrow \dot{z}_{21} = \omega - \omega_0 \]
\[ z_{22} = \omega \rightarrow \dot{z}_{22} = \frac{\omega_0}{H} P_m - \frac{D}{H} (\omega - \omega_0) - \frac{a_0 E_a V_S}{H x_d^2} \sin \delta \]
\[ z_{23} = \dot{z}_{22} \rightarrow \ddot{z}_{23} = \frac{a_0}{H} \frac{\dot{P}}{H} - \frac{D}{H} (\omega - \omega_0) - \frac{a_0 V_S}{H x_d^2} [\delta \cos \delta E'_q + \sin \delta E'_q] \]

It is clear that in $\dot{z}_{23}$, both $E'_q$ and $\dot{P}$ contain control input explicitly. Let $Y = [\dot{P}_r \quad \dot{z}_{23}]^T$

Therefore, $\eta$ is selected as $\eta = P_m + 2\mu$. Take the derivative of $\eta$:
\[ \dot{\eta} = \frac{2}{T_e} (\mu - P_m) \frac{2}{T_e} (\eta - P_m) = \frac{1}{T_e} (\eta - 3P_m) \]

Further combined with coordinate transformation, $\dot{\eta}$ is expressed as:
\[ \dot{\eta} = \frac{1}{T_e} (\eta - \frac{3}{a_0} (Hx_{23} + D(\omega - \omega_0)) + \frac{a_0 E_a V_S}{H x_d^2} \sin \delta / x_d^3) = \eta - 18z_{23} - 30(\omega - 1.0) - 5.005991 \sin \delta \]

Select the feedback control $v_1 = \dot{P}_r$, $v_2 = \dot{z}_{23}$. Put zero dynamics $\eta$ into the third-order linear subsystem, and the system is transformed into the following two subsystems:

\[ \dot{P}_r = v_1 \quad (a) \]
\[ \dot{z}_{23} = v_2 \quad (b) \]

Due to the large number of system parameters, the algebraic calculation is relatively complex. In order to facilitate the calculation, the actual values of each parameter are brought directly in for numerical calculation, and the specific parameters’ values are explained in the simulation analysis. Subsystem (a) is a first-order system. The pole of the system (a) is assigned to -1 by adopting state feedback $v_1 = -(P_r - 0.8)$.

According to the following derivation:
\[ v_1 = \dot{P}_r = \frac{-P_r x_{d2}}{T_d V_S} + \frac{x_d - x'_d}{T_d V_S} \cos \delta + \frac{V_{f_{12}}}{T_d V_S} \sin \delta V_S \times (-0.418626 P_r + 0.355896 + 0.1V_f) \times 1.20762 \]

Control input $V_f$ is obtained:
\[ V_f = 8.280750V_1 + 4.18626P_r - 3.55896, \quad v_1 = -(P_r - 0.8) \]

It should be noted that since $\delta$ is regulated in subsystem (b), it is regarded as a constant when it appears in subsystem (a) and is taken as its steady-state value.

Perform first-order Taylor expansion on zero dynamics in subsystem (b) at the equilibrium point:
\[ \dot{\eta} = -4.393170(\delta - 0.5) - 30(\omega - 1.0) - 18z_{23} + (\eta - 2.4) + \omega(\delta, \omega, z_{23}, \eta) \]

Take the following feedback controller:
\[ v_2 = -[k_1 \quad k_2 \quad k_3 \quad k_4] \begin{bmatrix} \delta - 0.5 \\ \omega - 1.0 \\ z_{23} \\ \eta - 2.4 \end{bmatrix} + v_{NL} \]
In this paper, the specific value of $A_1$ can be obtained by assigning the poles of $A_1$ to $-2, -6, -10 \pm i$. $P$ can be obtained by solving the Lyapunov equation. Then the value of $v_{NL}$ can be obtained according to Equation (16):

$$v_{NL} = -\frac{\omega(\delta, \omega, z_{23}, \eta)P_4}{P_3}$$

(28)

$\omega(\delta, \omega, z_{23}, \eta)$ can be replaced by the second-order term in Taylor expansion, and of course, the specific form can also be taken according to Equation (26):

$$\omega(\delta, \omega, z_{23}, \eta) = \hat{\eta} + 4.39317(\delta - 0.5) + 30(\omega - 1.0) + 18(\omega - 2.4)$$

(29)

Further derivation:

$$z_{23} = \hat{\omega} = \frac{\omega_0}{H} P_m - \frac{D}{H}(\omega - \omega_0) - \frac{\omega_0 E'_s}{H \omega_{ix}} \sin \delta = \frac{1}{6} P_m - \frac{5}{3}(\omega - 1) - 0.27811 \sin \delta$$

(30)

From the system equation, it can be seen that:

$$P_m = 2[P_n - \mu - \frac{1}{5}(\mu_0 - \mu + u)]$$

(31)

In combination with (30) and (31), control input $u$ can be obtained when the water hammer time constant $T_w = 1$:

$$u = 6\mu - (5P_m + 15\nu_2 + 25z_{23} + \mu_0 + 4.17165(\omega - 1) \cos \delta)$$

(32)

4. Simulation analysis

The simulation in this paper is based on MATLAB platform. According to reference [16], the specific parameters of the system are as follows:

$$\omega_0 = 1.0, \mu_0 = 0.8, V_s = 1.0, T_s = 5, D = 10, H = 6, x_L = 0.04, x_d = 1.867, x'_d = 0.257, \omega = 0.12, T_{d0} = 10.$$  

The control target of the system is the output electromagnetic power $P_o = 0.8$. At this point, the power angle of the generator is $\delta_0 = 0.5$. The equilibrium point of the original system and the equilibrium point of the system after coordinate transformation:

$$\left(\delta_0, \omega_0, P_{n0}, \mu_0, E'_{0}\right) = (0.5, 1.0, 0.8, 0.8, 0.662459), \left(\delta_0, \omega_0, \mu_0, z_{23}\right) = (0.8, 0.5, 1.0, 2.4, 0).$$

The system simulation simulates the three-phase grounding short circuit fault in the single-machine infinite bus system: the double-circuit line of the system operates normally for 5 seconds, and then the grounding fault occurs. The fault is found after 0.1 seconds. The system switches to the single loop operation for 0.2 seconds, and then switches back to the double-circuit operation. The calculation of relevant reactance in the case of circuit fault can be referred to literature [17].

The simulation results are as follows:

Figure 1 to Figure 3 show the response curves of the system with different water hammer time constant $T_w$ under three-phase grounding short circuit fault: output electromagnetic power, generator power angle andshaft transient potential. It can be seen that the system can maintain stability under the control of this non-minimum phase controller. In the meanwhile, system has a fast response and small adjustment time.
Figure 1. Response curve of the output electromagnetic power $P_e$.

Figure 2. Response curve of generator power angle $\delta$.

Figure 3. Response curve of $q$-shaft transient potential $E'_q$.

Figure 4. Response curve of the output electromagnetic power $P_e$, when $T_r = 4$.

Figure 5. Response curve of generator power angle $\delta$, when $T_r = 4$.

Figure 6. Response curve of $q$-shaft transient potential $E'_q$, when $T_r = 4$. 
Figure 4 to Figure 6 show the comparison between the simulation results of the non-minimum phase controller and the PID controller. The simulation results show that when the non-minimum phase controller is adopted, the adjustment speed is faster and the time for the system to reach the stable running state is shorter.

5. Conclusions
In this paper, combined with the SISO non-minimum phase system control method presented in literature [9,10], a controller design method for MIMO nonlinear non-minimum phase system is proposed. The controller is applied to regulate the fifth-order non-salient pole hydro turbine generator unit with wicket gate opening and excitation control input and obtain good control effect.

The design of the non-minimum phase controller is based on accurate feedback linearization, pole assignment and Lyapunov stability theory. For how to configure the system poles, as well as the selection of Lyapunov function is still worth further study: how to select a set of suitable poles or to determine the feedback gain matrix $G$ by other methods, meanwhile, the selection of matrix $Q$ needs further study, thus positive definite matrix $P$ is determined to obtain the nonlinear state adjustment parameter $v_{NL}$, so that the control effect of the system can be further optimized.

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