Neutron-Proton Collisions

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A theoretical model describing neutron-proton scattering developed by Majorana as early as in 1932, is discussed in detail with the experiments that motivated it. Majorana using collisions’ theory, obtained the explicit expression of solutions of wave equation of the neutron-proton system. In this work two different models, the unpublished one of Majorana and the contemporary work of Massey, are studied and compared.

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INTRODUCTION

In early 1932 a set of experimental phenomena revealed that the neutron plays an important role in the structure of nucleus like the proton, electron and α-particle and can be emitted by artificial disintegration of lighter elements. The discovery of the neutron is one of the important milestones for the advancement of contemporary physics. Its existence as a neutral particle has been suggested for the first time by Rutherford in 1920, because he thought it was necessary to explain the formation of nuclei of heavy elements. This idea was supported by other scientists that sought to verify experimentally its existence. Because of its neutrality it was difficult to detect the neutron and then to demonstrate its existence, hence for many years research stopped, and eventually, in between 1928-1930, the physics community started talking again about the neutron. For instance in a model was developed in which the neutron was regarded as a particle composed of a combination of proton and electron. At the beginning of 1930 there were experiments on induced radioactivity, which were interpreted as due to neutrons. Indeed in 1930 Bothe and Becker found that light nuclei bombarded by α particles produced radiation having higher penetrating power than γ radiation. In 1932 I. Curie and F. Juliet discovered the first artificial radiative substances bombarding light elements, Beryllium and Boron, by α particle (doubly ionized helium nuclei obtained from spontaneous disintegration of polonium).

In the same period Chadwick made experiments on the radioactivity and the experimental results obtained in these experiments and in those of I. Curie and F. Juliet, Bothe and Becker were explained by assuming that the radiation consisted of a new type of particle of mass nearly equal to that of a proton and with no net charge, i.e. the neutron.

The discovery of the neutron, raised a number of problems to be analyzed:
1) the relation of neutron with electrons and protons;
2) emission of γ rays associated with the neutron;
3) laws of interaction between neutron and nuclei.

This motivated the experiments of 1932 on disintegration of nuclei of light elements with fast neutrons, on the conversion of their kinetic energy in emission of γ radiation, on distribution of speed in neutron scattering. After discovery of the neutron in different laboratories the study of neutron interactions with matter, in particular proton and electron, continued. In fact the interaction of neutrons with electrons, the collision of neutrons with nitrogen nuclei, the concentration of slow neutrons in the atmosphere were examined and, in Italy, Rasetti was experimenting on Beryllium.

In this paper we attempt to give a short summary review about some of these experiments including contributions of Feather and Dee. As a matter of fact in order to understand these experiments one has to take into account the laws of collisions of neutrons with the matter.

Massey and Majorana, separately, proposed two different models about the disintegration mechanism by neutrons.
In the two models proposed by Massey and Majorana, the spins of the particles have not been considered, while later on Majorana proved that this is significant in these interactions \[13\].

The paper is organized as follows. In Sect. 1 we consider the experiments by Chadwick, Feather, Dee, in Sect. 2 we give a brief review on collision theory. In Sect. 3 we outline the proposal of models, made by Massey and Majorana separately, for a description of the passage of neutrons through matter in terms of the collision theory. Finally in Sect. 4 we draw our conclusions.

**COLLISIONS OF NEUTRONS WITH ATOMIC NUCLEUS. EXPERIMENTS ON NEUTRONS AND THEIR PASSAGE IN THE MATTER**

The nature and properties of the neutron are of the interest because, as for the proton, it is important to understand the structure of matter.

In fact the discovery of the neutron by Chadwick \[6\] is followed by different experiments involving neutrons as projectiles, to analyze nuclear structure. In the same period Feather used neutrons as projectiles and found that they could disintegrate the nitrogen nucleus \[8\], while Dee studied its interaction with electrons \(e^-\) \[7\].

The discovery by Chadwick of the neutron with mass approximately that of the proton motivates different experiments to determine the nature of this particle, i.e., the nature of the field surrounding the particle. These experiments have investigated the properties of the neutron through interactions of the neutron with material particles such as proton and atomic electrons. It is therefore of interest to analyze the experiments on scattering of neutron with the proton and neutron, and the theoretical calculations of the behavior of neutrons in this respect. Hence in the next two sections we are going to examine the principal experimental and theoretical contributions about the neutron collisions.

In this section we are going to analyze the experiments developed in the same period by Chadwick, Feather and Dee to examine the properties of the neutron which is taken into account in models that we describe.

**Experiments of Chadwick**

In 1930, the German physicists Bothe and Becker \[4\] bombarded the light metal beryllium with particles, and noticed that a very penetrating radiation was emitted. This neutral radiation was non-ionising, and they assumed it consisted of rays. In 1932 Irène and Frédéric Joliot-Curie \[5\] investigated this Bothe’s penetrating radiation in France. They let this neutral radiation, generated by polonium-beryllium sources irradiated by alpha particles, hit a block of paraffin wax, and found it caused the wax to emit high speed protons (3-9 cm/s). Because of the high speed of these protons, the \(\gamma\) rays would have to be incredibly energetic to knock them from the wax. They interpreted the resulting atomic recoils as Compton effect. At the same time Chadwick \[6\] reported the Joliot-Curie’s experiment to Rutherford, who did not believe that gamma rays could account for the protons from the wax. He and Chadwick proposed that the beryllium was emitting a stream of neutrons, which have nearly the same mass as protons, and hence should knock protons from a wax block fairly easily. James Chadwick repeated this experiment. The alpha-particles from the radioactive source hit the beryllium nuclei and transformed them into carbon nuclei, leaving neutral radiation (one free neutron). Then he used this radiation to bombard hydrogen and nitrogen in the wax and it could knock a free proton. He concluded that this neutral radiation was absolutely not a gamma-radiation. Because gamma-radiation had no momentum to produce proton from atom, i.e. Compton Scattering by Gamma Rays would violate conservation laws, it was reasonable that this radiation from beryllium was a kind of neutral particle with a mass similar to the proton. The particle mass was estimated by combining information from paraffin and nitrogen recoils and nuclear decay measurements.

The first (I) step was to obtain a stream of neutrons and the second step (II) was to detect it and then analyze its properties (III).
Because of its neutrality the neutron has a great penetrating power and it could be detected indirectly by ionization measurements of recoiling nuclei, i.e. by collision of neutrons in passage through the matter with an atomic nucleus. Although the collisions were so infrequent, their number in a beam could be estimated from the frequency of the collisions and the angular distribution of the struck protons and electrons. Hence in its passage through the matter the neutron is deflected from its path because of the internal field of the nucleus. The struck nucleus recoils and acquires energy to produce ions which can be detected by an ionization chamber connected to an amplifier and oscillograph.

The probability of a collision between a neutron and an atomic nucleus depends on the number of neutrons and on their velocity.

In fact some experiments with slower neutrons suggested that the radius for the proton-neutron collisions increased as much as the velocity of the neutron decreased.

I) The first step is to obtain neutrons realized in a nuclear reaction by a new nucleus formed by bombardment of polonium $\alpha$-particles captured by neutron source, all the light elements up to aluminum (lithium, beryllium, boron, neon, fluorine, sodium, aluminum, magnesium), with exceptions of helium, nitrogen, carbon, oxygen. The nuclear process according to the nuclear reaction

\[
(Z, A) + \alpha \rightarrow (Z + 2, A + 3) + n
\]  

consists in the capture of the $\alpha$-particle into the atomic nucleus with the formation of a new nucleus and the release of a neutron. The elements from which Chadwick obtained neutrons are lithium, beryllium, boron, fluorine, neon, sodium, magnesium and aluminum. The elements of higher atomic number up to argon produced neutron if $\alpha$-particles had sufficient energy.

Consequently, in most experiments, beryllium and boron have been used as sources of neutrons and the dependence of emission of the neutrons on the velocity of the bombarding $\alpha$-particles has been analyzed.

II) The second step is to detect the neutron. Chadwick examined the dependence of the neutron emission on the velocity of the bombarding $\alpha$-particles. Moreover, neutrons can be detected only in an indirect way, by the observation of the recoiling atoms, and Chadwick found that the probability of a collision between an emitted neutron and a nitrogen atom in the chamber depends on the velocity of the neutron, with less energy of the recoil atoms when the neutron is slower.

In particular, Chadwick observed that the neutrons emitted from beryllium by polonium $\alpha$-particles of velocity $1.6 \times 10^9$ cm/sec consisted of at least two groups: the slower with velocity $2.8 \times 10^9$ cm/sec (energy of $4.1 \times 10^6$ eV) and the faster group with velocity grater than $4 \times 10^9$ cm/sec (energy $> 8 \times 10^6$ eV) according to the reaction

\[
Be_9^4 + He_4^2 \rightarrow C_6^{12} + n_0^1,
\]  

and corresponding to these two groups of $n$ there are two groups of recoil atoms.

Furthermore, the experiments of Chadwick provided evidence for the emission of neutrons of energy up to about $12 \times 10^6$ eV. Moreover, he showed that the capture of an $\alpha$-particle by the beryllium nucleus results in a complete breakdown of the nucleus, with the emission of an $\alpha$-particle, a neutron $n$, and a $\gamma$-radiation. He measured that the mass of the neutron was about the same as that of the proton, lying in between 1.0058 and 1.0070, observing the momenta transferred in collisions of neutron with atomic nucleus. Anyway, for an accurate estimate of the mass of the neutron Chadwick used the energy relation in the disintegration in which the neutron is ejected from an atomic nucleus. In fact, assuming the conservation of energy and momentum in the disintegration of the nucleus of the known mass, the neutron mass is given from the measurement of the kinetic energy of the neutron liberated by $\alpha$-particles of known energy.

III) The third step is to analyze the nature and the properties of the neutron. In this respect Chadwick obtained the confirmation that the mass of the neutron was very close to that of the hydrogen atom. This could be consistent with a model of the neutron structure in which the neutron is combination of a proton and an electron with binding energy corresponding to $1 \times 10^6$ eV. This argument could be in favor of the complex nature (model) of the neutron. Hence it was necessary to understand the nature of the neutron and its properties, realizing experiments of nucleus collisions with neutrons, and if there was observation of the splitting of the neutron into a proton and an electron to have the most direct proof of
the complex nature of the neutron. In 1935 the missing observation of the above property, the arguments of quantum mechanics and relativistic mechanics and statistics, and the measurement of nuclear spins, provided experimental proof that the neutrons didn’t contain electrons. All this supported the idea that a neutron could be an elementary particle and the hydrogen represented the only possible combination of a proton and an electron. Before of this time, in 1932, different theoretical models of the neutron had been proposed and different experiments on the nuclear collision with neutron were made to confirm the right nature, i.e. model, of the neutron.

In this context one should insert the experimental contributions of Feather and Dee, and the theoretical contributions of Massey and Majorana, that we will analyze in the next subsections.

In the next sections we will analyze the experiments of Feather and Dee, which analyze the neutron interaction with protons and electrons.

Experiments on collisions of neutrons of Feather and Dee

In this section we analyze some experiments involving the neutron collisions with proton and electron by which it was possible to understand the neutron properties and nature. In these experiments the neutron excited in light elements under $\alpha$-particle bombardment interacts with the matter by means of the expansion chamber or Wilson chamber.

For example, as we have said in the previous section, Curie and Joliot, in an expansion chamber, observed recoil tracks of proton and helium nuclei from paraffin by neutrons from Beryllium. Contemporary Rasetti and Auger observed tracks of protons produced in the same way. On the other hand, Faether and Dee reported observations of proton and electron tracks.

The neutrons have some very interesting properties described in the papers of Chadwick, Feather, and Dee. The penetrating power of neutrons shows that they have no electric charge, and the experiments proved their loss of energy is due to the collisions with atomic nuclei and more rarely with the electrons.

Moreover, these experiments confirmed that the neutron is a particle with mass $M$ nearly equal to that of the hydrogen atom, and investigated the relation between the scattering of neutrons and their velocity. In particular, it was experimentally proved that the faster neutrons are more easily stopped than the slower ones, because the faster neutrons make more inelastic collisions with the nuclei.

Feather, for the first time, made experiments of disintegration of fluorine, nitrogen, oxygen, carbon nuclei. In these experiments he measured the ranges of the atoms recoiling and he found that the inelastic collisions were less frequent than the elastic ones. Feather employed an automatic expansion chamber, indispensable for such investigations on neutron interactions, filled with nitrogen and traversed by neutrons produced by beryllium under $\alpha$-polonium particle bombardment with energies distributed over a wide range. In these experiments Feather investigated collisions between neutrons and nitrogen nuclei by stereoscopic photography of the produced recoil and disintegration tracks. The tracks observed by Feather were the result of elastic and inelastic collisions between neutrons of mass 1 and nitrogen nuclei. Feather observed about 130 cases of neutron-nitrogen nucleus interaction, which are 109 elastic recoil tracks (elastic collisions) of nitrogen nucleus projected at an angle $\theta$ with the direction of incident neutron of a definite initial velocity and the other 32 are disintegration tracks (inelastic collisions). The inelastic collisions result in artificial disintegration of nuclei under neutron bombardment, and Feather measured they are of two main types:

1) in 12 occasions the neutron is captured with $\alpha$-particle emission;
2) in 20 occasions there is no captured neutron.

The capture cases explored have been

\[
\begin{align*}
    n + N^{14} & \rightarrow B^{11} + H e^4 \\
    n + N^{14} & \rightarrow C^{13} + H^2 \\
    n + N^{14} & \rightarrow C^{14} + H^1
\end{align*}
\]
the first one has been observed to take place in the forward and reverse directions in about half the cases
\[ n + N^{14} \leftrightarrow B^{11} + He^4, \] (4)
and \( \gamma \)-rays were emitted with the capturing of the neutron and \( \alpha \)-particle emission. He found that the lengths of the recoil tracks were in agreement with the neutron hypothesis of Chadwick. Moreover, electronic tracks were also produced in the Wilson chamber due to the passage of neutrons.

The non-capture cases explored have been
\[ n^1 + N^{14} \rightarrow B^{10} + He^4 + n \]
\[ n^1 + N^{14} \rightarrow C^{12} + H^2 + n \]
\[ n^1 + N^{14} \rightarrow C^{13} + H^1 + n. \] (5)

Then in [18] Feather investigated the action of neutrons also on fluorine. He analyzed the disintegration of oxygen in a Wilson chamber where neutron capture and \( \alpha \)-emission occurred:
\[ n^1 + O^{16} \rightarrow C^{13} + He^4 \] (6)

Before the beginning of 1932 the only known nuclear disintegration was that produced by \( \alpha \)-particles. Instead both \( \alpha \)'s and neutrons could disintegrate the nucleus and light elements, in most cases, captured by the incident particle, and ejected another one with emission of \( \gamma \)-rays.

Thus, the experiments of Feather supported Chadwick in interpreting the penetrating radiation from beryllium as consisting of neutrons.

In this artificial disintegration a general phenomenon is the emission of \( \gamma \)-radiation. Moreover Feather undertook general considerations of the energy changes in the various nuclear processes and deduced the kinetic energy from measurements of lengths of cloud tracks with range-velocity curves.

He also investigated range-velocity curves for the recoil atoms of fluorine and carbon [15]. In all these experiments there is the evidence of recoil protons due to neutrons produced in the resonance disintegration of beryllium and also of carbon recoil atoms due to neutrons of high energy.

While Feather investigated, in an automatic expansion chamber, the neutron collisions in its passage through matter with atomic nuclei, producing recoil atoms of short range and great ionizing power, on the other hand Dee [7] examined the interaction of the neutrons, emitted by beryllium bombarded by \( \alpha \)-particles of polonium, with electrons in a Wilson chamber filled with Nitrogen atoms (gas of Nitrogen). The main conclusions which Dee could draw from these experiments was that the production of electron recoils by neutrons is a rare occurrence compared with the production of recoil nitrogen atoms.

He obtained that the primary ionization along the path of a neutron of velocity \( 3 \times 10^9 \text{ cm/sec} \) was of order of 1 ion pair in 3 meters of air, if the effective radius for a neutron-nitrogen collision was \( r = 5 \times 10^{-13} \text{ cm} \). The main conclusion from these experiments was that the production of electrons recoiled by neutrons was rare compared with proton or nitrogen atoms recoil. Unlike the behavior of a proton which dissipates its energy almost entirely in electron collisions, the probability of interaction of a neutron with an electron is less than 1% of the probability of interaction with a proton or a nitrogen nucleus.

These experiments show that the collision of a neutron with an atomic nucleus is much more frequent than with an electron, depending on the electric field between a neutron and a nucleus small except at distances of the order of \( 10^{-12} \text{ cm} \).

Then they give information about the neutron radius. In fact if the effective radius of neutron-nitrogen collision was of order \( 5 \times 10^{-13} \text{ cm} \) then the ionization produced by a recoil nitrogen atom along the path of neutron is less than 1 ion pair per 3 meters of air with a neutron, ejected from a beryllium nucleus, of velocity \( 3 \times 10^9 \text{ cm per second} \), while if the radius was \( 5 \times 10^{-12} \text{ cm} \) the ionization can be 1 ion pair per 3 cm of air. He observed the first case confirming that the neutron radius is of order \( 5 \times 10^{-13} \text{ cm} \).

**NEUTRON MODELS AND THEORY**

Taking into account the experimental results, the great penetrating power of the neutron and its failure to interact with the electron different models [11], [18], [17] have been developed to report on proton-neutron
interaction.

To understand the effect of the passage of the neutron through the matter it is necessary to develop a theory with the help of wave mechanics, assuming a possible potential neutron interaction (neutron field) with other particles in accordance with the experimental results. Hence at the beginning of 1932 many theoretical physicists have tried to explain the existence of the neutron and to establish a model that describes it.

One of the problems of quantum theory is to understand what is the nature of the neutron, in order to explain the phenomena of induced and natural radioactivity, artificial disintegration. In 1932 one option explored was to consider the neutron not as an elementary particle, but as a combination of a proton and an electron.

However there were conditions imposed by experiments:

a) the first hypothesis of Chadwick, following Rutherford [1] that the neutron consisted of a proton and an electron;

b) the size of \( n \) is of order \( 10^{-12} - 10^{-13} \) cm;

c) the mass of the neutron obtained from nuclear reaction \( B^{11} + \alpha \rightarrow N + N^{11} \), \( m_n \) is in the \( \sim 1.005 - 1.007 \);

d) at the beginning of 1932 it was not yet clear whether or not the neutron possessed. [29]

On the basis of these conditions different models were proposed. These models can be summarized as follows: (A) a dipole model of strength \( a \sim 10^{-13} e \) (like a dumbbell, with a positive and negative charge separated by a small distance with their effects cancelled), but this seems to be less probable than (B); (B) a proton imbedded in an electron or an elastic spherically symmetric model.

One of the models proposed represents the neutron as a dipole (A) [12] formed by two opposite charges at distance \( l \), where the positive particle is the proton and the negative particle is the electron with a potential:

\[
V = \frac{e l \cos \theta}{r^2}
\]  

(7)

and a magnetic moment \( m = le \) that had to be measured experimentally.

Another model [16] (B) (a proton imbedded in an electron, like an onion, with a sphere of one kind of electricity surrounded by a layer of the other kind, so that again the charge is cancelled) is a kind of reverse Thomson atom with positive charge + at center in the negative charge density \( \rho(r) \) distribution. It has a spherical symmetry and represents well the neutron suggested by Rutherford, and the potential at \( r \) is:

\[
V = \frac{e}{r} - \frac{q(r)}{r},
\]

(8)

where \( q(r) = \int \rho(r)dr \) or type

\[
V = f(r)e^{-kr}
\]

(9)

where \( f(r) \) varies slowly with \( r \).

Hence in an elastic spherically symmetrical model the neutron is like a hydrogen atom in a nearly zero quantum state, as was discussed in 1920 by Rutherford [1]. This model was discussed for the first time from the point of view of quantum theory by Langer and Rosen [19]. For such a system they supposed the interaction between a material particle and a possible neutron would be of the form \( e^2 e^{-\lambda r} \), where the parameter \( \lambda \) is connected with the binding energy.

All these models should take into account the problem of transition of neutron through the matter, obtaining different theoretical results depending on the model.

Furthermore, one analyzes theoretically the phenomenon of passage of the neutron through matter because the main effects of neutrons are due to collisions of the neutron with atomic nuclei, more rarely with electrons. As we have seen from the experimental point of view [7, 8, 18], there are different types of nuclear reactions involving neutrons (neutron interactions with the matter) to reveal the neutron:
1) absorption of neutrons (calculation their absorption rate);
2) electron and proton scattering from a neutron (calculation of the number of scattered neutrons distributed over the tracks);
3) disintegration by a neutron;
4) ionization from the neutron then calculating the loss of ionization energy \(-dT/dx = f(T)\), which is a function of kinetic energy \(T\) (calculation of the number of ions per unit path and then the number of neutrons produced, depending on the model of neutron and method of calculation).

Thus it must be a theory describing the phenomenon of neutron collision with the matter, to explain the experimental results \[7, 8\]. We must bear in mind that the theoretical results relatively to experimental phenomena exhibit differences for models of neutrons that are proposed.

Massey \[11\] estimated \(f(T)\) in the case of spherical model (B) where the neutron is considered as a particle composed of a spherical symmetry with a mass about that of the proton and with a potential field outside \(V(r) = \xi e^{-kr}\) applying the theory of Born collisions, which we briefly review in the next section. He then obtained the effective cross-section and evaluated the energy loss with Bethe’s method.

There is another model proposed by Majorana in his unpublished work in which the neutron is considered as an elementary particle.

We will analyze these two models in next sections, with a brief review on the theory of collisions that is at the basis of the models of Massey and Majorana.

Hence a study of the angular distribution of recoil tracks leads to important data for a theory of the field of the neutron. In fact the results of the experiments made to determine the field force consisting in the observations of the collisions of neutrons with material particles such as protons and electrons, have to be interpreted. All this requires the development of a theory of such collisions. The smallness of the field interaction between a neutron and a charged particle leads to the possibility of applying the approximate quantum theory of collisions of Born \[21\] in elastic scattering of neutron with particles, as Massey did.

In the next section we will review the basic properties of collisions theory that Massey applied to elastic collisions of neutrons with material particles. In fact we will highlight in the work of Massey that, knowing the laws of collisions of neutrons with matter, he could interpret the experimental data to determine the field of a neutron in nuclear collisions, confirming that the radius of such a particle is less than \(2 \times 10^{-13}\) cm in the case of a particular model of neutron.

**QUANTUM COLLISION THEORY**

To understand the experimental results on the collisions it is necessary to use scattering theory. In this section we give a brief summary about collision theory that is at the base of neutron interactions models and of models proposed by Massey and Majorana. Suppose that we have a stream of \(N\) particles per unit area per second incident with velocity \(v\) on particle-target. The number of particles deflected between angles \(\theta\) and \(\theta + d\theta\) is \(2\pi N I(\theta) \sin \theta d\theta d\phi\), and the collision cross-section \(Q\) is given in terms of \(I(\theta)\).

In fact collisional processes are described quantitatively in terms of cross sections and to study them one needs the quantum mechanics. One can distinguish between elastic and nonelastic collisions depending on whether or not translational momentum and energy are conserved. In this section we recall the main steps of the quantum collision theory in the case of two elastic interacting particles. As we said the main observable quantity involved in a scattering process is the collision cross-section \(Q\) given by

\[
Q = \int_0^{2\pi} \int_0^\pi I(\theta, \phi) \sin \theta d\theta d\phi = \int_0^{2\pi} \int_0^\pi |f(\theta, \phi)|^2 \sin \theta d\theta d\phi
\]

where the function \(I(\theta)\) has the dimension of area and, in a scattering of a particle of mass \(m\) and velocity \(v\) by a potential \(V(r)\), is given by the exact formula

\[
I(\theta) = |f(\theta, \phi)|^2 = \frac{1}{k^2} \left| \sum_n (2n + 1)(e^{i\delta_n} - 1)P_n(\cos \theta) \right|^2,
\]

\[11\]
where \( k = \frac{2\pi mv}{\hbar} \) is the wave number and \( \delta_n \) are the unknown phases, which are important for the purpose of evaluating \( Q \). Hence in the elastic collision theory one considers the wave equation of Schrodinger for the relative motion of two interacting particles of comparable masses \( M_1, M_2 \)

\[
\nabla \psi + \left\{ k^2 - \frac{8\pi^2 m}{\hbar^2} V(r) \right\} \psi = 0,
\]

(12)

where \( M = \frac{M_1 M_2}{M_1 + M_2} \) is the reduced mass of the system of colliding particles. In the collision theory one requires that a proper solution of the equation (12) has the asymptotic form \[22\]

\[
\psi \sim e^{ikr \cos \theta} + f(\theta, \phi) e^{ikr},
\]

(13)

and to obtain such a solution \( \psi \) is expanded

\[
\psi = \sum_n \psi_n(r) P_n(\cos \theta),
\]

(14)

where \( \psi_n \) must then satisfy the wave equation subject to finiteness at the origin

\[
\frac{d^2 r \psi_n}{dr^2} + \left\{ k^2 - \frac{8\pi^2 m}{\hbar^2} V(r) - \frac{n(n+1)}{r^2} \right\} (r \psi_n) = 0,
\]

(15)

while having the asymptotic form \[22\]

\[
\psi_n \sim \frac{A_n}{kr} \sin(kr - \frac{1}{2}n\pi + \delta_n).
\]

(16)

From \[16\] Faxen and Holtsmark \[22\] obtained the following expression for the collision cross-section \( Q \)

\[
Q = \frac{4\pi}{k^2} \sum_n (2n+1)\sin^2 \delta_n.
\]

(17)

Thus to obtain \( Q \) it is necessary to calculate the phases \( \delta_n \). In collision theory there are two methods to calculate the phases: the method of Jeffrey for \( \delta_n \) greater than unity, and the method of Born when \( \delta_n \) is less than unity.

Hence if the scattering potential field \( V(r) \) is small compared with the centrifugal force term, i.e.

\[
\frac{8\pi^2 m}{\hbar^2} V(r) \ll \frac{n(n+1)}{r^2}
\]

(18)

for \( r \) such that \( kr \sim n + \frac{1}{2} \), for large \( n \) and the phase \( \delta_n \) is small (\( \delta_n \ll 1 \)) hence one is in the validity regime of Born’s approximation \[30\], \[21\] and the exponential in (11) can be expanded in series and one obtains the first approximation of Born method. In fact under these conditions Mott \[23\] showed that \( \delta_n \) has the following approximate expression

\[
\delta_n = \frac{4\pi^3 m}{h^2} \int_0^\infty V(r) J^2_{n+\frac{1}{2}}(kr) r dr
\]

(19)

where \( J_{n+\frac{1}{2}} \) are the Bessel functions of half-odd order. Then assuming \( \sin \delta_n \sim \delta_n \) (for \( \delta_n \ll 1 \)) it is possible to sum the series \[17\] for \( Q \) to give

\[
Q = \frac{64\pi^4 M^2}{h^4} \int_0^\pi \left| \int_0^\infty V(r) \frac{\sin(2krsin\frac{1}{2}\theta)}{2krsin\frac{1}{2}\theta} r dr \right|^2 \sin\theta d\theta
\]

(20)

which is the expression of \( Q \) in the approximation due to Born \[21\] (the formula of Born). In this approximation the total collision cross-section \( Q \) is finite if \( V(r) \) vanishes at infinity faster than \( r^{-3} \), the
The same condition is for the exact formula (11), because for $n$ sufficiently large the exact and approximate series (11), (17) converge together by virtue of (18) and (19). In the case in which the Born approximation is not applicable ($\delta_n \gg 1$), i.e. for small $n$, for $\frac{8\pi^2 m}{h^2}V(r) \gg \frac{n(n+1)}{r^2}$, the phases $\delta_n$ can be calculated by using an approximation based on classical theory given by Jeffreys [24] (method of Jeffreys or JWKB method). This method gives for the solutions of the equation (15) the following asymptotic forms

$$
\psi_n \sim \sin \left( \frac{\pi}{4} + \int_{r_0}^{\infty} f^{1/2}(r) \, dr \right), \quad \sin \left( \frac{\pi}{12} + \int_{r_0}^{\infty} f^{1/2}(r) \, dr \right)
$$

(21)

where $f(r) = k^2 - \frac{8\pi^2 m}{h^2}V(r) - \frac{n(n+1)}{r^2}$ and $r_0$ is the largest zero of $f(r)$, and only the first is the required solution because it is finite at the origin. Comparing (21) with (16) the phase $\delta_n$ is

$$
\delta_n = \frac{1}{2} \pi + \frac{1}{4} \pi + \int_{r_0}^{\infty} [f^{1/2}(r) - k] \, dr.
$$

(22)

For the case of intermediate phase, $\delta_n$ can be obtained by interpolation of the two previous methods.

In conclusion, the conditions on $V(r)$ imply the use of one of two methods, hence they provide a way to choose the mutual field force $V(r)$.

In a system which satisfies certain symmetry properties it is possible to have some simplification in computation of the phases. For example Mizushima was the first to find the solution for the scattered wave [26] in an elastic sphere model (collisions between rigid spheres) with the interaction energy $V(r)$

$$
V = \begin{cases} 
\infty, & \text{for } r < R, \\
0, & \text{for } r > R,
\end{cases}
$$

and it is given by

$$
\psi = -i C \sum_{n=0}^{\infty} (2n + 1) \frac{J_{n+\frac{1}{2}}(kR)}{H_{n+\frac{1}{2}}^{(2)}(kR)} P_n(\cos \theta) \frac{R e^{-ikr}}{r}
$$

(23)

where $k = \frac{2\pi m V}{h} = 2\pi \lambda$, $R$ is interpretable, in a spherical model for interaction between neutron and nucleus, as the nuclear radius plus neutron radius, and $J_{n+\frac{1}{2}}$ is a Bessel function and $H_{n+\frac{1}{2}}^{(2)}$ is a Hankel function of the second kind, while $P_n(\cos \theta)$ is a Legendre polynomial. In particular Massey and Mohr in the appendix of their paper [27] obtained the cross section $Q$ in this model and using the general solution of the wave equation (15)

$$
r^{-1/2} \psi = AJ_{n+\frac{1}{2}}(kr) + BJ_{n-\frac{1}{2}}(kr)
$$

(24)

the phases are

$$
\delta_n = (-1)^{n+1} \arctan \frac{B}{A},
$$

(25)

obtained from the condition that the solution (21) is zero at $R$. Substituting the expression of $\delta_n$ (25) in (17) one obtains $Q$.

Application of Collision Theory to neutron scattering in a model proposed by Massey

In this section we outline the model formulated by Massey to describe the neutron behavior in the collisions with the other particles. Massey applied the quantum theory of collision due to Born [21], Faxen and Holtsmark [22] to the collision of neutrons with atomic nuclei and with electrons. He considered the model of neutron in which the neutron is viewed like an atom consisting of a proton and an electron (the
spherical model (B) in the previous section), suggesting a complex nature of the neutron, i.e. a hydrogen atom in a zero quantum state, in which the electron moves in a field given by

\[ V(r) = e^2 \left( \frac{1}{r} + \frac{Z}{a_0} \right) e^{-\frac{Zr}{a_0}} \]  

(26)

where \( Z \) is the effective nuclear charge, \( a_0 \) is the Bohr radius, and \( a_0/Z \) is the neutron radius. Massey assumed that the electron and proton behave as point charges and considered the collisions of neutrons with these particles as the scattering of particles by a potential field \( V(r) \), i.e. the potential interaction field of a proton-neutron can be compared to a deep hole of small radius. He showed that the model assumed for the neutron was able to explain the experimental results such as that its radius must be less than \( 2.0 \times 10^{-13} \) cm and the probability of disintegration of a neutron by nuclear collision is very small for this model.

In particular Massey, applying the collision theory, found that, if the proton behaves as an elementary charge at small distances of interaction, the collision cross-section is given by

\[ Q \sim \frac{16\pi^5 M^2 e^4 a_0^4}{h^4 Z^4} \]  

(27)

where \( M \) is the proton mass, and he found that the collision radius in this model must be less than \( 1.4 \times 10^{-14} \) cm, against the experimental observations which indicate a greater value than this as Feather measured it.

Hence this could induce to think that the proton could not behave as a unit charge for distances of interaction less than \( 10^{-13} \) cm, either the energy of interaction between neutrons and nuclei at large distances increases less rapidly, or the neutron could be a point neutral particle.

In this context it is possible to insert the contribution of Majorana, that we will analyze in the next section. To conclude this section we outline the study of Massey about the neutron-electron collision. He found that the number of ions produced per centimeter of path by neutrons is very small, because in its passage through air, containing \( 5.3 \times 10^{20} \) electrons per \( cm^3 \), the total number of neutron-electron collisions per centimeter path is of order \( 3.0 \times 10^{-13} \) per cm, from which the total number of ions formed per centimeter path could be less than 1 ion per \( 10^{10} \) cm, explaining therefore only the negative results of experiments of Dee about the electron-neutron collision, but not the numerical correspondence as Feather measured.

The spherical model of Massey is valid as long as the Born formula is valid and the wave length associated with the collision is long compared with the size of the neutron, and the field of the neutron must vary gradually.

Another model alternative to this one, that Massey investigated, was a dipolar model (model (A) in the previous section) in which the neutron could behave as a dipole with potential field

\[ V(r) = \alpha \frac{e^2}{r^2} \]  

(28)

It could be possible to distinguish the spherical or dipolar distribution from the experimental point of view by measuring if the interaction falls off more rapidly than \( r^{-1} \) or \( r^{-2} \) for large \( r \).

Massey deduced that the spherical model was better suited for describing some experimental results than the dipolar. The "complex structure" of neutron proposed by Massey could be at the base of some negative theoretical results as we said above.

In fact a proof that supported the validity of the model of complex nature of the neutron could be the observation of the splitting of the neutron into an electron and a proton in a nuclear scattering, but this wasn’t observed in the experiments. Moreover a difficulty of this model, if one considered the spin of particles, was the inconsistency of statistics and spin.
The model of neutron of a complex particle, proton-electron combination, used by Massey in its calculations seems to be in disagreement with some experimental observations (see for example [7], [8], [28]).

The direction in which either of the models A, B of the neutron would eject protons was calculated and it was found that the dumbbell type should eject them all perpendicularly to their own path, while the onion type would eject some straight ahead, with about ten times as many being thrown off perpendicularly.

Experiments with neutrons did not confirm either of these models and hence the neutron is not built according to either of the accepted models.

Thus, motivated by these results, one could regard the neutron as an elementary particle rather than as a composite particle, as like the electron and proton.

Indeed it is possible to formulate another model in which the neutron is considered an elementary particle and there is another type of exchange interaction between neutron and proton.

Thus, the innovative intuition of Majorana was to develop a first theoretical model of neutron different from models (A), (B), in which in the first instance the spin of proton and neutron was neglected (a neutron model as an elementary particle of neutron with spin was analyzed by Majorana in the published paper of 1933 [13]) and proton and neutron are considered as elementary particles, in order to analyze the relative motion between a neutron and a proton in a scattering process. In fact this intuition will be confirmed from experimental results that are explained if the neutron is not a mere close combination of electron and proton acting like a fundamental particle of nature, but it actually is an elementary particle itself. He studied the following radial wave equation for relative motion of the neutron and nucleus:

$$u'' + \frac{2}{r} u' + \left(\frac{2m}{\hbar^2} (E - V) - \frac{l(l+1)}{r^2}\right) u = 0$$

(29)

where $m \sim 1/2M_N$ is the reduced mass of the system.

He assumed that the field of interaction between a neutron and the proton was a square potential well unlike the choice of Massey [26]:

$$V = \begin{cases} -A & \text{for } r < R, \\ 0 & \text{for } r > R. \end{cases}$$

For $r < R$, a solution of Eq. (29), regular at the origin, was

$$u = \frac{1}{\sqrt{r}} I_{l+1/2} \left(\sqrt{\frac{2m}{\hbar^2} (E + V)} r\right), \quad r < R,$$

(30)

$I_{l+1/2}$ are the modified Bessel functions of first kind) while for $r > R$, the solutions were linear combinations of half-odd order Bessel and Neumann functions

$$\frac{1}{\sqrt{r}} I_{l+1/2} \left(\sqrt{\frac{2m}{\hbar^2} E} r\right),$$

$$\frac{1}{\sqrt{r}} N_{l+1/2} \left(\sqrt{\frac{2m}{\hbar^2} E} r\right),$$

(31)

with the boundary condition that the solution reduced to (30) at $r = R$. He obtained the following solution of Eq. (29), which was regular at the origin:

$$u_l = \begin{cases} \frac{C_l}{\sqrt{r}} I_{l+1/2}(k_0 r) & \text{for } r < R, \\ \frac{C_l}{\sqrt{r}} \left(aI_{l+1/2}(kr) + bN_{l+1/2}(kr)\right) & \text{for } r > R. \end{cases}$$

(32)
and \( a = a(x), b = b(x) \) were:

\[
a = \frac{\pi x}{2} \left( I_{l+1/2}(kr)N_{l+1/2}'(kr) - \frac{k_0}{k} I_{l+1/2}(kr)N_{l+1/2}(kr) \right),
\]

\[
b = \frac{\pi x}{2} \left( \frac{k_0}{k} I_{l+1/2}(kr)N_{l+1/2}'(kr) - I_{l+1/2}(kr)N_{l+1/2}(kr) \right).
\]

(33)

The arbitrary constants \( C_l \) were determined in such a way that, far from the origin, the solution is a combination of Hankel functions of the first kind, i.e., the quantity \( u = \sum_{l=0}^{\infty} u_l P_l(\cos \theta) \) describes a plane wave \( I \)

\[
I = \sum_{l=0}^{\infty} i^l (2l + 1) \sqrt{\frac{\pi}{2k}} I_{l+1/2}(kr) P_l(\cos \theta),
\]

(34)

plus a diverging wave \( S = u - I \)

\[
S = \sum_{l=0}^{\infty} i^l (2l + 1) \frac{\epsilon_l}{\sqrt{r}} H_{l+1/2}^1(\sqrt{r}) P_l(\cos \theta),
\]

(35)

with \( H_{l+1/2}^1(\sqrt{r}) = I_{l+1/2}(kr) + iN_{l+1/2}(kr) \), hence he obtained

\[
C_l = \frac{i^l}{a + ib} (2l + 1) \sqrt{\frac{\pi}{2k}},
\]

\[
\epsilon_l = \frac{-2ib^l (2l + 1)}{a + ib} \sqrt{\frac{\pi}{2k}}
\]

\[
= (e^{2ibl} - 1)i^l \frac{2l + 1}{2} \sqrt{\frac{\pi}{2k}}
\]

(36)

where the angle \( \theta_l \) is the relative phase between \( u_l \) and \( I_{l+1/2} \) at large distances, which determines the effect of the scattering center on the \( l \)-th order

\[
\tan \theta_l = -\frac{b_l}{a_l},
\]

(37)

and it gives information about the collision cross-section as we have seen in previous section.

**CONCLUSION**

As emerged from the above Majorana, as early as 1932, motivated by the experiments of that period, aimed at describing neutron-proton scattering developed a model of neutron without spin, forerunner of the upcoming model of neutron plus spin [13]. In Sect. 1 we have given a review of the principal experiments on the neutron collisions, in Sect. 2 we have made a summary on the collision theory. In Sect. 3 we have outlined the proposal of models of neutrons. Hence we have analyzed the model of complex neutron proposed by Massey to describe the passage of neutrons through matter using theory of collision in presence of a particular expression of potential. Furthermore we have exhibited the work of Majorana which considered the elementary nature of the neutron unlike Massey, obtaining the explicit expression of solutions of wave equation in a square potential well. We have stressed his original contribution about the analysis of a model for the neutron as an elementary particle with respect to the suggestion of Rutherford that there might exist a neutral particle formed by the close combination of a proton and an electron, because it was at first natural to suppose that the neutron might be such a complex particle. Majorana took into account the difficulties raised by the experimental results on neutron structure: is the neutron particle composed of one proton plus one electron closely related, or an elementary neutral particle? He
tried to establish the interaction law between the proton and neutron based on criteria of simplicity, chosen in such a way as to allow you to display as correctly as possible the properties of more general characteristics of the neutron interactions.

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