Spotted hyena optimizer for well-profile energy optimization

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Abstract. In recent years, directional drilling becomes more popular in petroleum industry due to its more exposure to reservoir. During wellbore trajectory design for directional drilling, more importance should be given on safety issues. Well-profile energy is the key parameter which can assure a safe and efficient wellbore trajectory through proper optimization. In this work, the Spotted Hyena Optimizer (SHO) is proposed and implemented for optimizing the well-profile energy and compared with another state of art method named Particle Swarm Optimization (PSO) algorithm. The trajectory is mathematically formulated by using radius curvature method (RCM) considering 17 variables on which well-profile energy is depended. The SHO successfully obtained the optimum values of the 17 design variables which eventually given the minimum well-profile energy. The optimum well-profile energy obtained by SHO is 207.00 which is 18.28\% better than PSO. Additionally, the sensitivity of the algorithm has been analysed by changing different operational parameter of SHO. It is observed that the efficiency of SHO increased with the increment in the number of search agents (hyenas). The minimum well-profile energy achieved through SHO ensure a less complex and safe wellbore trajectory.

Keywords: Wellbore trajectory design, Spotted hyena optimizer, Well-profile energy, Directional drilling, Drilling optimization.

1. Introduction

The increasing demand of hydrocarbon energy around the world is promoting the advancement of some technologies, for example directional drilling \cite{1}, \cite{2}. Directional drilling and its automation are becoming a widely used technique in the petroleum industry for obstacle avoiding, approaching to an inaccessible surface location, multiple wells, fault controlling, sidetracking, relief wells, salt domes, and horizontal drilling \cite{3}. Though the advancement in the automation of directional drilling is not eye touching comparing to other industries. Wellbore trajectory design is one of the crucial elements for automating the directional drilling processes \cite{4}. Wellbore trajectory design depends on a series of drilling related parameters and optimizing these parameters make the drilling processes safe, efficient,
and cost effective. However, energy is an important parameter which depends on different variables and directly related to the safety of a drilling processes.

The concept of wellbore energy as a wellbore trajectory optimizing parameter was introduced by Samuel (2009) depending on mathematical reasoning in lieu of geometrical reasoning [5]. According to his research, well-profile energy is different from the wellbore induced strain energy and simply it is considered as the complexity or difficulty index of a well path. The lower value of wellbore profile energy reduces the possibility of accidents during the drilling processes. Therefore, optimization of the wellbore profile energy is the key to reduce the drilling risk.

To optimize the wellbore profile energy several approaches were conducted by different researchers [6], [7]. Liu and Samuel (2016) developed a model for wellbore trajectory design using minimum well profile energy profile criterion. Their model was for the returning path of a deviated well from the planned well path. In some other approaches, well-profile energy has been optimized with other parameters during multi-objective optimization processes [8], [9]. In multi objective optimizations researcher used cellular particle swarm optimization (CPSO) and dynamic differential search algorithm (d-DS) for optimizing well-profile energy with other parameters [10]–[12]. In case of CPSO the incorporated cellular automata concept helped the PSO to explore more area [12]. But it took more computational time to hit the target while the DS faced a problem with premature convergence. However, in this research, the optimization of well-profile energy is done for wellbore trajectory design by spotted hyena optimizer and the mathematical formulations are derived by radius curvature method. SHO algorithm has been implemented to find out the optimum values for 17 design variables which eventually has given optimum well profile energy. Later, SHO is compared with PSO to evaluate the performance of SHO. Additionally, the sensitivity of SHO with respect to the changes in behavioral parameters has been analyzed. It is observed that SHO has outperformed over the PSO and shown better performance with increasing number of search agents.

2. Problem formulation
The wellbore-profile energy minimization plays an important role in the effective design of wellbore trajectories and improves the safety of the drilling operations. However, the optimization of wellbore-profile energy needs mathematical formulations for wellbore trajectory length and energy. The formulation of these two objective functions and the details of the proposed SHO algorithm for well-profile energy optimization are discussed in the following section.

2.1. True measured depth (TMD) calculation
The trajectory length is computed using radius curvature (RCM) method [13]–[15]. Various variables such as hold angle, true vertical depth, vertical inclination, azimuth angle, dogleg severity, and lateral length should be known for the TMD calculation. The expression for radius curvature and constant curvature between two points in RCM is as:

\[ a = \frac{1}{\Delta M} \sqrt{(\theta_2 - \theta_1)^2 \sin^4 \left(\frac{\phi_2 + \phi_1}{2}\right) + (\phi_2 + \phi_1)^2} \]  

\[ r = \frac{1}{a} = \frac{180 \times 100}{\pi \times T} \]  

\[ \Delta M = r \times \sqrt{\left((\theta_1 - \theta_2)^2 \sin^4 \left(\frac{\phi_1 - \phi_2}{2}\right) + (\phi_1 - \phi_2)^2\right)} \]

Here, \( \Delta M \) is the 3D well path between two points, \( r \) and \( a \) are the radius and constant of the curvature, respectively, \( \theta_i \) and \( \phi_i \) represent the azimuth and inclination angle, respectively and \( T \) is the dogleg severity.
The basic equation of TMD contains seven segments as shown in Figure 1 and equation 4.

\[ TMD = D_{kop} + D_1 + D_2 + D_3 + D_4 + D_5 + HD \]  

TMD is the summation of seven segments and these are the length of kick of point \( D_{kop} \), three build and drop \( (D_1, D_3 \text{ and } D_5) \), tangent \( (D_2) \), \( (D_4) \) is the length of hold, and the horizontal or target layer (HD). These segments are calculated through equations (5) to (9). The details of the variables used in these equations are given in Appendix A.

\[
D_1 = r_1 * [(\theta_2 - \theta_1)^2 \sin^4 \left(\frac{\varphi_1}{2}\right) + (\varphi_1)^2]^{0.5} \\
D_2 = D_D - D_{kop} - D_1 * \left(\frac{\sin(\theta_1) - \sin(\theta_0)}{\varphi_1 - \varphi_0}\right) / \cos(\varphi_1) \\
D_3 = r_4 * [(\theta_3 - \theta_1)^2 \sin^4 \left(\frac{\varphi_2 + \varphi_1}{2}\right) + (\varphi_1 - \varphi_2)^2]^{0.5} \\
D_4 = D_B - D_D - D_3 * \left(\frac{\sin(\theta_2) - \sin(\theta_1)}{\varphi_2 - \varphi_1}\right) / \cos(\varphi_2) \\
D_5 = r_4 * [(\theta_6 - \theta_5)^2 \sin^4 \left(\frac{\varphi_3 + \varphi_2}{2}\right) + (\varphi_3 - \varphi_2)^2]^{0.5}
\]

Here, \( r_1, r_2 \text{ and } r_3 \) are the curvature radius of \( D_1, D_2 \text{ and } D_3 \) segments, \( \varphi_i \) and \( \theta_i \) are the inclination and azimuth angle, respectively.

2.2. Total well-profile energy

The complexity of the wellbore trajectory is estimated with the strain energy of the well profile. The energy of the wellbore trajectory is computed as the arch length integral with the curvature squared.

\[ E = \int_0^l k(x)^2 \, dx \]  

The well-profile energy for the wellbore trajectory model is discussed in Figure 1 is estimated as

\[ E = D_1 \varphi_1^2 + D_3 (\varphi_2 - \varphi_1)^2 + D_5 (\varphi_3 - \varphi_2)^2 \]
2.3. Constraints
The wellbore trajectory design is bounded by two types of constraints which are operational and range of tuning variables. True vertical depth (TVD) to the target and casing setting depth \( (C_1) \) are the two operational constraints for this problem. The vertical depth from the surface to the drilling target is measured by TVD. To have a vigil whether the bore to the target has reached, the operational constraint TVD must be satisfied. The TVD is calculated as in equation (12).

\[
TVD = Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6
\] (12)

Where, \( Y_{i=1,6} \) is the subsection vertical depth at each drop off point of drilling. This section makes a vertical perpendicular with the hypotenuse of this section as shown in Fig. 1. This hypotenuse is the measured drilling depth of that section which is a calculated parameter based upon the tuned variable \( \theta_{i=1,3} \). So, in a way, tuning variables are also bounded by TVD.

![Survey calculation of deviated well trajectory](image)

**Figure 2** Survey calculation of deviated well trajectory  

Figure 2 shows the deviated direction of well trajectory to the north, to the east and vertical sides. The radius of the curvature method is used to derive the offset distance in three specific directions.

\[
\Delta North = \frac{\Delta M(\cos(\theta_1) - \cos(\theta_2)).(\sin(\theta_2) - \sin(\theta_1))}{(\theta_2 - \theta_1).(\theta_2 - \theta_1)}
\] (13)

\[
\Delta East = \frac{\Delta M(\cos(\theta_1) - \cos(\theta_2)).(\cos(\theta_1) - \cos(\theta_2))}{(\theta_2 - \theta_1).(\theta_2 - \theta_1)}
\] (14)

\[
\Delta Vertical = \frac{\Delta M(\sin(\theta_2) - \sin(\theta_1))}{(\theta_2 - \theta_1)}
\] (15)

The offset distance calculated through the equation (13) to (15) provide the north-south, east-west, and True Vertical Depth (TVD) at any point along with the specific curved segments of a wellbore trajectory. The larger casing increases the installing cost of a wellbore. So, casing setting depth constraints are also calculated as an operational constraint. In a nutshell, the operational constraints are:

\[
TVD_{min} < TVD < TVD_{max}
\]

\[
C_{1, min} < C_1 < C_{1, max}
\]
The constraints of tuning variables are tabulated in Table 1 which gives the maximum and minimum limit of tuning variables.

### Table 1. Constraints for wellbore trajectory optimization

| Variables | Constraints Limit |
|-----------|-------------------|
| Target True Vertical Depth (TVD) | Min.= 10850 ft. and Max.= 10950 ft. |
| Lateral Section length (LSL) HD. | 2500 ft. |
| Dogleg Severity | $T_1 \leq \frac{5^0}{100 \text{ ft.}}; T_2 \leq \frac{5^0}{100 \text{ ft.}}; T_3 \leq \frac{5^0}{100 \text{ ft.}}; T_4 \leq \frac{5^0}{100 \text{ ft.}}; T_5 \leq \frac{5^0}{100 \text{ ft.}}. $ |
| Minimum value of inclination | $\phi_1 = 10^0; \phi_2 = 40^0; \phi_3 = 90^0$ |
| Maximum value of inclination | $\phi_1 = 20^0; \phi_2 = 70^0; \phi_3 = 95^0$ |
| Minimum value of azimuth angles | $\theta_1 = 270^0; \theta_2 = 270^0; \theta_3 = 270^0; \theta_4 = 330^0; \theta_5 = $ |
| Maximum value of azimuth angles | $\theta_1 = 280^0; \theta_2 = 280^0; \theta_3 = 280^0; \theta_4 = 340^0; \theta_5 = $ |
| Kick off point depth (TVD) | Min. $D_{kop} = 600 \text{ ft.}$. Max. $D_{kop} = 1000 \text{ ft.}$ |
| Second build point depth (TVD) | Min. $D_p = 6000 \text{ ft.}$. Max. $D_p = 7000 \text{ ft.}$ |
| Third build point depth (TVD) | Min. $D_p = 10000 \text{ ft.}$. Max. $D_p = 10200 \text{ ft.}$ |
| Casing setting depth after first build | Min. $C_1 = 1800 \text{ ft.}$. Max. $C_1 = 2200 \text{ ft.}$ |
| Casing setting depth after second build | Min. $C_2 = 200 \text{ ft.}$. Max. $C_2 = 8700 \text{ ft.}$ |
| Casing setting depth after third build | Min. $C_3 = 10300 \text{ ft.}$. Max. $C_3 = 11000 \text{ ft.}$ |

### 3. Methodology

Metaheuristic algorithms became very popular in engineering applications due to its easy implementations. Spotted hyena optimizer (SHO) is a recently developed popular metaheuristic algorithms where social relationships among hyenas are the main inspiration [16]. The dominant members of spotted hyenas’ family are the female members. The spotted hyenas use their natural power of sight, hearing and smell to track the prey. During the search process for a new food source, spotted hyenas produce a sound to communicate with each other. For hunting they rely on a group of trusted friends which is consist of 100 hyenas approximately. Mathematical formulation of SHO algorithms are discussed in the following sections.

#### 3.1. Encircling prey

Since the search space is unknown, the target prey is considered as the current best solution. The surrounding search agents (i.e. hyenas) try to update their positions with respect to the target. Mathematical formulation of this behaviour can be formulated as follows.

$$\vec{M}_h = |\vec{A}_{\vec{P}}(x) - \vec{Q}(x)|$$  \hspace{1cm} (16)

$$\vec{Q}(x + 1) = \vec{Q}_p(x) - \vec{P} \cdot \vec{M}_h$$  \hspace{1cm} (17)
Here, $\vec{M}_h$ is the distance between the prey and spotted hyena, $x$ is the current iteration, $\vec{A}$ and $\vec{F}$ are the coefficient vectors, $\vec{Q}_p$ and $\vec{Q}$ are the position vector of prey and hyena. However, ($\|\|$) and (.) are the absolute value and multiplication of vector, respectively.

The vectors $\vec{A}$ and $\vec{F}$ are calculated as follows
\begin{align*}
\vec{A} &= 2.r_1 \vec{r} \\
\vec{F} &= 2\vec{h}.r_2 - \vec{h}
\end{align*}

(18) (19)

\[ \vec{h} = 5 - (\text{Iteration} \times \frac{\text{MaxIteration}}{5}) \] (20)

Here, Iteration =1,2,3,........ MaxIteration, $r_1$ and $r_2$ are random vectors in [0,1]

By adjusting values of $\vec{A}$ and $\vec{F}$ vector, there are a different number of places which can be reached. By using equation (16) and (17), a hyena can update its position around the prey.

3.2. Hunting
Usually hyenas use the information provided by the trusted friends and prey tracking ability during hunting. The hyena which has the best information about the location of prey is defined as the best search agent. Therefore, other search agents make a group and follow the best search agent to update their positions.

\[ \vec{M}_h = |\vec{A}.(\vec{Q}_h - \vec{Q}_k)| \] (21)
\[ \vec{Q}_k = \frac{\vec{Q}_h + \vec{F} \cdot \vec{M}_h}{\text{count}} \] (22)
\[ \vec{D}_h = \vec{Q}_h + \vec{Q}_{k+1} + \ldots \] (23)

Where, $\vec{Q}_h$ and $\vec{Q}_k$ is the position of first best spotted hyena and other spotted hyenas, respectively. $N$ indicates the number of spotted hyenas which is computed as follows:

\[ N = \text{count}_{\text{nos}}(\vec{Q}_h, \vec{Q}_{h+1}, \vec{Q}_{h+2}, \ldots ) \] (24)

Where, $\vec{G}$ is a random vector in [0.5,1], nos defines total number of solutions after adding $\vec{G}$, and $\vec{D}_h$ is a group of N number of optimal solutions.

3.3. Exploitation
At the time of exploitation, the value of $\vec{h}$ is gradually decreased from 5 to 0. It also assisted by the variation in $\vec{F}$. When the value become $|\vec{F}|<1$ then the algorithm enables the hyenas to attack the target. It can be formulated as follows:

\[ \vec{Q}(x + 1) = \frac{\vec{D}_h}{N} \] (25)

Where, $\vec{Q}(x + 1)$ is the record of best position, also helps to update the position of other search agent according to the best search agent’s position.

3.4. Exploration
The hyenas which are the members of vector $\vec{D}_h$ help the algorithm to search in a diverse search region. During exploration the value of $\vec{F}$ become $|\vec{F}|>1$ or $|\vec{F}|<-1$. It allows the search agent to remain away from the target. Usually it enables the hyenas for global search. $\vec{A}$ vector also plays an important role during exploration by providing the random weights of the target. By taking these assistance SHO avoids the local optima trap.

**Algorithm 1** Spotted Hyena Optimizer.

| Input: number of populations $Q_i$ ($i = 1,2,\ldots,n$) |
| Output: optimum energy |
| 1. procedure SHO |


2. $h,A,F$ and $N$ initialization
3. objective function is calculation for each hyena
4. $Q_h = \text{best hyena}$
5. $D_h = \text{all optimal solution's group}$
6. \textbf{while} (x < \text{Max number of iterations}) \textbf{do}
7. for each hyena \textbf{do}
8. current hyena’s position update
9. \textbf{end for}
10. Update $h,A,F$ and $N$
11. hyena’s position adjustment if it goes beyond the area
12. calculation of objective function for each search agent
13. checking for better solution and updating of $Q_h$
14. updating of $D_h$
15. $x = x + 1$
16. \textbf{end while}
17. return $Q_h$
18. \textbf{end procedure}

4. Result analysis
The quantitative analysis of the proposed method is discussed in this section. For the comparative analysis, SHO is compared with PSO algorithm with respect to well profile energy optimization [17]. Table 2 has shown the operational parameters for both SHO and PSO. SHO has used its exploration and exploitation capability to find out the optimum design parameters with respect to minimum well profile energy that reduced the complexity of wellbore trajectory design. Table 3 has represented the obtained values of design parameter by both SHO and PSO. From Table 3 and Figure 3, it is found that SHO performed better that PSO. Besides, SHO has converged more rapidly than PSO. It is because during exploration the value of $F$ becomes $|F|>1$, which eventually helps it to avoid the local optima. The reason behind SHO’s faster convergence is also the value of $\tilde{F}$. At that time its value becomes $|\tilde{F}|<1$.

\begin{table}[h]
\centering
\begin{tabular}{|l|l|l|}
\hline
\textbf{Algorithms} & \textbf{Name of parameter} & \textbf{Value} \\
\hline
SHO & Number of hyenas & 40 \\
 & Control Parameter $h$ & [5,0] \\
 & Generation’s number & 100 \\
 & Value of $\tilde{G}$ & [0.5,1] \\
 & Trials & 20 \\
\hline
\end{tabular}
\caption{Values of algorithm’s parameters during optimization}
\end{table}
| PSO Population | 100 |
|----------------|-----|
| Acceleration coefficient $C_1$ | -0.41 |
| Acceleration coefficient $C_2$ | 2.35 |
| Trials | 20 |

Table 3. Optimum values of 17 design variable after optimization

| Design Parameters | SHO | PSO | Design Parameters | SHO | PSO |
|-------------------|-----|-----|-------------------|-----|-----|
| $\phi_1$          | 20.00 | 14.68 | $D_{kap}$         | 1000.00 | 933.10 |
| $\phi_2$          | 40.00 | 40.00 | $D_D$             | 6000.00 | 6981.30 |
| $\phi_3$          | 90.00 | 90.04 | $D_B$             | 10200.00 | 10048.00 |
| $\theta_1$        | 270.00 | 280.00 | $T_1$             | 1.01 | 1.35 |
| $\theta_2$        | 280.00 | 270.67 | $T_2$             | 0.01 | 0.26 |
| $\theta_3$        | 270.00 | 270.00 | $T_3$             | 5.00 | 5.00 |
| $\theta_4$        | 330.00 | 332.66 | $T_4$             | 5.00 | 2.96 |
| $\theta_5$        | 330.00 | 339.89 | $T_5$             | 5.00 | 2.58 |
| $\theta_6$        | 355.00 | 355.02 | Well profile energy | 207.00 | 253.29 |

Figure 3. A comparison between SHO and PSO for optimizing well profile energy
It is found from Figure 3 that during early iteration PSO has converged more rapidly than SHO but later SHO converged rapidly. This is due to the premature convergence of PSO. In case of sensitivity analysis, the performance of SHO is analysed for different numbers of hyenas. From Figure 4, it is found that SHO shows better performance when the number of hyenas is higher. Because it can explore more search space. In that scenario it can avoid local optima. The results depicted in Table 4 is also showing similar phenomenon where A, B, and C are representing the dependency of well-profile energy on search agents (number of hyenas). From Table 4 it is found that with 20 hyenas SHO has obtained a minimum well profile energy of 219.03 but when the number of hyenas increased it has obtained a better optimum value which is 207.00 .From Figure 4, it is found that with a smaller number of hyenas it converges more rapidly during early iterations. But it failed to give the best minimum value due to weakness in exploration capability. It may fall under local optima trap due to less exploration capability.

![Figure 4. Effects of number of hyenas on well profile energy optimization](image)

**Table 4. Sensitivity analysis of SHO**

| Parameters          | A      | B      | C      |
|---------------------|--------|--------|--------|
| Number of hyenas    | 20     | 30     | 40     |
| Number of iterations| 100    | 100    | 100    |
| Well-profile energy | 219.03 | 213.00 | 207.00 |

Figure 5 is representing the box plot of 17 design variables. It gives an idea about the range of optimum values for each design variables which are used to obtain the optimum well profile energy. Statistical distribution of the used values for each of the seventeen variables has shown by this plot. In box plot, red line represents the median value of distribution while the middle two quartiles of distribution are represented by the box. Whisker are represented by red crosses which are outside of the outlier. It is found that SHO has given optimum values for all the constraints variables within the constraints limit. It is due to the stochastic initialization of SHO which is less chaotic in nature. The initial function has ensured that all the search agents remain within the feasible region. All the
infeasible solutions have a fitness score equals to zero which does not allow them to enter within the best solutions.
5. Conclusion
Optimization of wellbore trajectory plays an important role in petroleum industry by producing oil and gas from the reservoirs where conventional drilling methods do not work properly. However, the optimization depends on several parameters and variables based on different calculation methods. Safety is an important parameter during trajectory design which can be evaluated by an index called minimum well profile energy. In this work, SHO has been used as an efficient algorithm to obtain the minimum well profile energy for wellbore trajectory design and compared with PSO. SHO has outperformed and shown better convergence than PSO during the optimization. From result analysis, it is observed that SHO performs better with higher number of search agents but took more computational time. If the number of search agents become less, it may fall into local optima trap. However, this minimum well profile energy reduces the complexity of wellbore trajectory design. Besides it may also help to reduce the torque and drag during optimization.

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