An approach for configuration of the equivalence relation

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Abstract. This paper presents an approach for feature extraction using the equivalence relation. The equivalence relation is formed using features permutations of primary data. Every feature of new representation matches to optimal permutation, that minimizes some loss function. A loss function choice depends on the kind of problem. In this paper, we consider copy-move forgery detection problem and demonstrate the effectiveness of our approach in solving this problem.

1. Introduction
One of the most challenging tasks in computer science is finding a good representation of data. From one hand, it is necessary to use a description that is robust to some transformations, and from another side, it has to be sensitive to others. It is almost impossible to find a simple universal feature extraction algorithm for every problem.

The most popular approach nowadays for most of computer science problems is using deep learning algorithms [1]. This approach is efficient because a feature extraction part and the task itself are solved simultaneously. Some difficulties occur when using such a powerful tool. Sometimes it is impossible to describe a problem as a supervised task. Other challenges are the need for a useful data set and enough computational power.

Other methods can be used to find a good representation of an image for many types of tasks [2, 3]. Some of these methods use the idea that primary data is possible to present as relationships between features. The example of such approach is local binary patterns [4]. We offer the equivalence relation that exploits the method of using relationships between primary data features.

We organize the rest of this paper as follows. Section 2 describes the main terms and the approach to configuring the equivalence relation. Section 3 tells how to use the presented equivalence relation to image for feature extraction. The fourth section describes a problem to solve and contains the experimental results. The final section summarizes the results and tells about further studies.

2. Equivalence relation configuration
Consider a set of $N$ real vectors with $n$ features:
This set can be presented as the matrix $\mathbf{G}$:

$$
\mathbf{G} = \begin{pmatrix}
- & - & - & - & - \\
- & - & - & - & - \\
- & - & - & - & - \\
- & - & - & - & - \\
- & - & - & - & - \\
\end{pmatrix}
$$

(2)

Let $L(\sigma, \mathbf{G})$ be the loss function that depends on the order of vectors in the matrix. So, it is possible to find the optimal permutation of vectors $\sigma'_{LG} \in \Sigma_{N}$ that minimizes $L$:

$$
\sigma : \mathbf{Z}_{N-1} \rightarrow \mathbf{Z}_{N-1},
$$

(3)

where $\mathbf{Z}_{N-1}$ is a set of non-negative numbers $\{0,1,2,...,N-1\}$ and $\Sigma_{N}$ is a set of all possible permutations. Cardinality of the set $\Sigma_{N} = |\Sigma_{N}| = N!$ as the number of all possible permutations of vectors in $\mathbf{G}$.

Let $\chi(\sigma)$ be the code or the index of permutation:

$$
\chi : \Sigma_{N} \rightarrow \mathbf{Z}_{N},
$$

(4)

Let $L^{\min}(\sigma, \mathbf{G})$ be the minimum of loss function $L$:

$$
L^{\min}(\sigma, \mathbf{G}) = \min_{\sigma \in \Sigma_{N}} L(\sigma, \mathbf{G}).
$$

(5)

It is possible that not only one permutation can have $L^{\min}(\sigma, \mathbf{G})$ value, so we consider the set $\Sigma^{\text{em}}_{N}$:

$$
\Sigma^{\text{em}}_{N} = \left\{ \sigma : L(\sigma, \mathbf{G}) = L^{\min}, \sigma \in \Sigma_{N} \right\}.
$$

(6)

Now we can define optimal permutation $\sigma'_{LG}$:

$$
\sigma'_{LG} = \arg \min_{\sigma \in \Sigma^{\text{em}}_{N}} \chi(\sigma).
$$

(7)

Instead of using the value $\chi(\sigma'_{LG})$ directly, sometimes it is convenient to use some hash function $\kappa(\chi(\sigma'_{LG}))$, that is surjection:

$$
\kappa : \mathbf{Z}_{N} \rightarrow \mathbf{Z}_{K} (K < N!).
$$

(8)

$\kappa(\chi(\sigma'_{LG}))$ is used as a feature of new data representation.

**Example 1.** Consider matrix $\mathbf{G}$ be $\mathbf{G} = \begin{pmatrix} 26 & 33 & 17 & 1 & 18 \end{pmatrix}$ and loss function as:

$$
L(\sigma, \mathbf{G}) = \sum_{j=0}^{N-2} \left| g_{\sigma(j+1)} - g_{\sigma(j)} \right|.
$$

(9)

For the above loss function the set $\Sigma^{\text{em}}_{N}$ contains permutations that arrange elements of $\mathbf{G}$ in ascending or descending order:

$$
\Sigma^{\text{em}}_{N} = \left\{ \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 0 & 1 \\ 1 & 0 & 4 & 2 & 3 \end{pmatrix} \right\}.
$$

(10)

The value of $\sigma'_{LG}$ as $\chi(\sigma'_{LG})$ depends on the order of iterated permutations.

As was mentioned in Introduction, feature extraction algorithms have to be invariant to some undesirable transformations. For example, in many computer visions tasks, the representation of an image has to be robust to changing lighting conditions. Using our approach, we can define a loss function that could be invariant to such transformations.
Let $\Psi$ be the set of mappings with $\psi \in \Psi$, where $\psi : \mathbb{R} \rightarrow \mathbb{R}$. Such mappings may be bijective functions $\Psi_p$, monotonically increasing functions $\Psi_m$, or linear increasing functions $\Psi_l$.

**Definition 1.** The loss function $L$ is invariant to set of mappings $\Psi$ or $\Psi^\ast$-invariant if:

$$\forall \psi \in \Psi, \forall G \in \mathbb{R}^{mN}, \forall \sigma_1, \sigma_2 \in \sum_n N$$

$$L(\sigma_1, G) < L(\sigma_2, G) \Rightarrow L(\sigma_1, \psi(G)) < L(\sigma_2, \psi(G)),$$

where $\psi(G) = \begin{pmatrix} \psi(g_{0,0}) & \psi(g_{0,1}) & \ldots & \psi(g_{0,N-1}) \\ \psi(g_{1,0}) & \psi(g_{1,1}) & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ \psi(g_{n-1,0}) & \ldots & \ldots & \psi(g_{n-1,N-1}) \end{pmatrix}$. Using Definition 1, it is easy to show that

$$\sigma^\ast_{L,G} = \sigma^\ast_{L,\psi(G)}.$$

**Definition 2.** Matrices $G_1, G_2$ are equivalent or in the same equivalence class in respect to loss function $L$ if $\sigma^\ast_{L,G_1} = \sigma^\ast_{L,G_2}$.

We formed the equivalence relation that organizes $\mathbb{R}^{mN}$ into $N!$ disjoint groups:

$$J(\sigma) = \{ G : \sigma^\ast_{L,G} = G, G \in \mathbb{R}^{mN} \} \subseteq \mathbb{R}^{mN}.$$

The approach of defining classes $J(\sigma)$ looks similar to the method presented in [5]. It follows from Definitions 1 and 2 that it is possible to organize equivalence classes into disjoint subclasses with respect to mappings set $\Psi$:

$$J(G^*) = \{ G : G = \psi(G^*), \psi \in \Psi \}.$$

Then:

$$J(\sigma) = \bigcup_{G^* \in J(\sigma)} J(\sigma, G^*),$$

where $J^*(\sigma)$ is the set of primitive matrices.

Consider loss functions that are based on the $p$-norm:

$$L(\sigma, G)^p = \sum_{j=0}^{N-2} \| \tilde{g}_{j+1} - \tilde{g}_j \|_p^p,$$

where the $p$-norm is defined as:

$$\| \tilde{k} \|_p^p = \left( \sum_{j=0}^{N-1} |k_j|^p \right)^{1/p} (p \in N).$$

It is possible to prove that the loss functions (15) are $\Psi^\ast$-invariant and $L(\sigma, G)^p$ are $\Psi_m$-invariant functions.

Let us prove that $L(\sigma, G)^p$ are $\Psi^\ast$-invariant loss functions. Every function $\psi \in \Psi$ is possible to present as $\psi(x) = ax + b, a > 0$, so:

$$L(\sigma, \psi(G))^p = \sum_{j=0}^{N-2} \| \tilde{g}_{j+1} - \tilde{g}_j \|_p^p = \sum_{j=0}^{N-2} \left( \sum_{j=0}^{N-1} \| \tilde{g}_{\sigma(j+1,j)} + b - a g_{\sigma(j+1,j)} - b \|_p \right)^p,$$

$$= \sum_{j=0}^{N-2} \sqrt{a \sum_{j=0}^{N-1} g_{\sigma(j+1,j)}^2 + g_{\sigma(j+1,j)} + b} = aL(\sigma, \psi(G))^p.$$

Using the fact that $a > 0$, the relation order between values of $L(\sigma, G)^p$ remains the same.

As monotonically increasing functions do not change the relation order, it is easy to see that $L(\sigma, G)^p$ are $\Psi_m$-invariant functions.

3. Feature extraction algorithm

In this section, we describe a feature extraction algorithm for images, using the proposed approach based on the equivalence relation. We present all the steps of the algorithm in the following list:
1) Choosing the way of image region scanning.
2) Configuring the feature vectors for matrix $G$.
3) Choosing the loss function to optimize.
4) Using a normalization algorithm (if necessary) to get additional robustness to undesired transformations.
5) Specifying the hash function.

It is possible to get several representations of primary data, using different methods of arranging pixels into vectors or specifying various loss functions. There can be used procedures like hash code concatenation, getting histogram values to get final descriptors [6, 7, 8].

4. Copy-move forgery detection

Copy-move forgery attacks consist of three stages: copy selected image fragment from one place, transform it using some image processing algorithm and paste it to another place of the same image [9, 10]. Processing algorithms may include geometric transformations like rotation, scale, brightness changes. Brightness changes may be linear functions or monotonically-increasing functions. In this section, we consider fragments with rotation transformation and use the described approach in Section 3 to find such fragments.

We present all the steps of our algorithm to find copied fragments with geometric transformations below. The size of the analyzed fragment is $19 \times 19$. The origin of coordinates is placed to the fragment center, i.e. $m_1, m_2 = -9, ..., 9$.

1) The way of region scanning is presented in Figure 1.
2) The way of forming feature vectors is presented in Figure 1. We use polar coordinates to form vectors. The number of vectors is 8.
3) The loss function is:

$$F(G, \sigma) = \sum_{j=0}^{N-2} \| \bar{g}_{\sigma(j,i)} - \bar{g}_{\sigma(j)} \|_2.$$  

4) The optimal permutation normalization procedure is defined as follows:

$$\sigma^*_{G,F} = \text{arg min}_{k \in \mathbb{Z}} \sigma_{G,F}((i + k^*) \mod N),$$

$$k^* = \text{arg min}_{k \in \mathbb{Z}} \sigma_{G,F}((i + k^*) \mod N).$$

5) The vector feature is the following:

$$\chi(\sigma_{G,F}) \in \mathbb{N}_8(8! = 40320).$$

![Figure 1](image_url)

**Figure 1.** The way of image region scanning and forming feature vectors.

We present results of experiments in Figures 2-5. Figure 2 contains the original image. Figure 3 shows an image containing some duplicates. Figure 4 consists of an image with features, and the last figure contains the found duplicates. We rotated some duplicates by 60 degrees.
5. Conclusions
This paper presented a novel approach for feature extraction based on the equivalence relation. We considered some properties of the formed equivalence relation and told how it could be applied to image representation in a copy-move forgery detection task. Experimental results showed that our method has a robust ability to geometric transformations. We conduct the following studies to use our approach in other computer vision problems.

6. References
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Acknowledgments
The reported study was funded by RFBR according to the research projects 18-01-00748, 17-29-3190.