Searching for cold spots in multipion systems

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Abstract

Local fluctuations of pion density in momentum space may lead to Bose-Einstein condensation. Conditions for this phenomenon to occur in high-energy collisions and possibilities of its experimental investigation are discussed.

1. The possibility of Bose-Einstein condensation in a dense multipion system was first indicated in the pioneering work by Pratt [1]. Recently, the general solution of the problem has been obtained for gaussian distributions [2] and also in the general framework of uncorrelated production [3, 4, 5]. In the present note, using the results of [2]-[5] we discuss the conditions for creation of such a pion condensate in high-energy collisions and the possibilities of its experimental discovery.

The main conclusion of this investigation is that, although it seems rather unlikely that the condensate may include all pions produced in the collision, the possibility of its creation in a limited region of phase-space is not excluded and thus worth experimental investigation. The specific characteristics of the condensate which may help in its experimental identification are

(i) rather small relative momenta of the pions ($\Delta p < 100$ MeV);

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(ii) multiplicity distributions showing much stronger fluctuations than those expected from the Poisson distribution;

(iii) large fluctuations in the charged/neutral ratio.

The first property requires the "temperature" of the condensate to be much smaller than that of its environment (characterized by the average transverse momentum of about 350 MeV). Thus the search for the condensate is in fact a search for "cold spots" in the pion system.

The actual existence of such "cold spots" is supported by the observation of "intermittency" [6] showing that large non-poissonian fluctuations in very small momentum intervals do indeed exist and, moreover, that these fluctuations are strongly related to the Bose-Einstein interference [7].

We thus propose to undertake a systematic search for the "cold spots" in multiparticle production and, once they are identified, to investigate their specific properties. We believe that such a program is feasible and can provide interesting information on the structure of the systems created in high-energy collisions.

2. The study of a general system of identical pions, which exhibits only correlations due to the quantum interference [3, 4, 5] allowed to formulate the necessary and sufficient condition at which Bose-Einstein condensation takes place. Denoting by $\nu$ the assumed average multiplicity of pions before the quantum interference effects are taken into account, the multiplicity distribution with quantum interference included is described by the generating function of the form

$$
\Phi(z) = \prod_m \frac{1 - \nu \lambda_m}{1 - \nu \lambda_m z},
$$

(1)

where $\lambda_m$ are eigenvalues of the single pion density matrix, $\rho^{(0)}(q, q')$ satisfying the normalization condition

$$
\sum_m \lambda_m = 1.
$$

(2)

It is clear from this formula that the distribution becomes singular when

$$
\nu \lambda_0 \rightarrow 1,
$$

(3)

where $\lambda_0$ is the largest eigenvalue. This is precisely the point of condensation: the average multiplicity tends to infinity and, moreover, almost all particles occupy a single quantum state (corresponding to eigenvalue $\lambda_0$) [4].

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It is also seen from (1) that in the limit (3) the first factor in the product dominates and the distribution approaches the geometrical one

\[ P(n) \to (1 - \nu \lambda_0)[1, \nu \lambda_0, (\nu \lambda_0)^2, \ldots] \] (4)

with the average

\[ < n > \to \frac{\nu \lambda_0}{1 - \nu \lambda_0}. \] (5)

Thus we conclude that close to the condensation point the multiplicity distribution becomes very broad and differs drastically from the original Poisson distribution of independently produced pions. This broad distribution implies of course that the fluctuations in the observed particle number must be very large. It was realized already by Pratt [1] (see also the recent discussion in [8]) that this can be a good signal for observing this phenomenon.

However, since the limit (3) requires – strictly speaking – infinite average multiplicity, and thus can never be reached in practice, it is important to investigate what is the chance to observe this new regime in real experimental conditions, i.e. at finite multiplicity. It is thus necessary to discuss the approach to the condensation limit.

To quantify this problem, it is convenient to consider the cumulants of the distribution, which are easily derived from (1) [3]

\[ K_p = (p - 1)! \sum_m \left( \frac{\lambda_m \nu}{1 - \lambda_m \nu} \right)^p. \] (6)

In the limit (3) we have

\[ \frac{K_p}{< n >^p} \to (p - 1)!, \] (7)

whereas for the Poisson distribution all \( K_p \) vanish.

It is now fairly clear that the approximation (4), (7) can be reasonable at finite multiplicities only if the difference between the largest and the second largest eigenvalue is not too small. In view of the normalization condition (2), the sufficient condition for this is that \( \lambda_0 \) is close enough to 1.

Even if several terms contribute substantially to (3), and thus the approximation (7) is not adequate, one may still have fairly strong deviations from the original Poisson distribution describing uncorrelated emission. Such deviations may serve as an indicator of the approach to the condensation point. It is therefore interesting to discuss them in more detail.
3. To estimate these effects of finite multiplicity, we have calculated the multiplicity distribution following from the generating function (1) in the case of a Gaussian density matrix of the form

$$\rho^{(0)}(q, q') = \frac{1}{\sqrt{(2\pi \Delta)^3}} \exp\left(-\frac{(\vec{q}^+)^2}{2\Delta^2}\right) \exp\left(-\frac{R^2(\vec{q}^-)^2}{2}\right), \quad (8)$$

where

$$\vec{q}^+ = \frac{\vec{q} + \vec{q}'}{2}; \quad \vec{q}^- = \vec{q} - \vec{q}'.$$

(9)

It is not difficult to verify that

$$\langle \vec{q}^2 \rangle = 3\Delta^2; \quad \langle \vec{r}^2 \rangle = 3R^2, \quad (10)$$

which explains the physical meaning of these parameters. It is also not very difficult to obtain the formula for $\lambda_0$ [3]

$$\lambda_0 = \left(\frac{2}{1 + 2R\Delta}\right)^3. \quad (11)$$

In Fig. 1 the multiplicity distributions are plotted for $\langle n \rangle = 3$ and several values of $\lambda_0$. One sees radical deviations from the Poisson distribution even for fairly small $\lambda_0$. The distributions obtained are much broader and extend to rather large multiplicities. We conclude that, as soon as $R\Delta$ becomes close to .75 (or smaller), one may expect strong (and thus perhaps observable) effects on multiplicity distributions. We have chosen a rather small average multiplicity of 3 identical pions to emphasize that we would like to consider phenomena local in phase-space. For larger multiplicities the effects are stronger, because one is closer to the condensation point. One should keep in mind, however, that these results ignore the energy-momentum conservation which tends to cut large multiplicities and thus to reduce the deviations from the Poisson distribution.

4. It is seen from (11) that the condition $R\Delta \leq .75$ implies

$$\sqrt{\langle \vec{q}^2 \rangle} \sqrt{\langle \vec{r}^2 \rangle} \leq \frac{9}{4}. \quad (12)$$

\footnote{One should remember, however, that this means 9 pions on the average}
Let us first discuss if a "standard" system created in a collision at high energy may satisfy this condition. Taking the average transverse momentum of about 350 MeV as approximate measure of $\sqrt{<q^2>}$:

$$<(q_t)^2> = \frac{2}{3} <q^2>$$

(13)

we obtain from (12)

$$\sqrt{<\vec{r}^2>} \leq \frac{\sqrt{27/8}}{350MeV} \approx 1 fm.$$  

(14)

It seems unlikely that such a small region can contain more than few pions at "freeze-out" and therefore the analysis cannot apply to the total multiplicity of an event. Moreover, even if one considers a fraction of the produced particles, the meaningful comparison between the distributions shown in Fig. 1, can only be performed if multiplicities significantly larger than average are not reduced by requirement of a a certain maximal density of pions at freeze-out.

\textbf{Figure 1.} Multiplicity distributions of identical bosons for $\langle n \rangle = 3$. 


On the other hand, one also sees from (12) that even for a fairly large volume of the system, the effects may be strong, provided $\langle \vec{q}^2 \rangle$ is small enough. For example, for $\sqrt{\langle \vec{q}^2 \rangle}$ smaller than, say, 100 MeV - strong deviations from the Poisson distribution appear even for average radius $\sqrt{\langle \vec{r}^2 \rangle}$ as large as 5 fm. Of course, such a small relative momentum inside a group of pions is a rare phenomenon. However, once such a "cold spot" appears, it should have a very unusual multiplicity distribution of identical particles.

5. The following comments are in order.

(i) The rather broad distribution of identical pions implies also very large fluctuations in the charged/neutral ratio. Such large fluctuations were suggested some time ago [9] as a possible signal for the disoriented chiral condensate. It was also shown that the DCC is characterised by very small relative momenta of pions [10]. We thus conclude that it may be fairly difficult to distinguish between DCC and "cold spots" solely on the basis of measurements of charged/neutral ratio.

(ii) A clear difference between DCC and the BE effect we discuss here is in the distributions of identical particles. Indeed, the identical pions emitted from DCC are expected to have a distribution close to the Poisson one, whereas, as discussed in the present paper, this in not the case for "cold spots". Consequently, the simultaneous measurements of charged and neutral distributions gives a chance to distinguish between the two phenomena.

(iii) A search for "cold spots" in multiparticle systems created in high-energy collisions can probably be performed with the help of the existing cluster algorithms. In view of the arguments presented here, it seems worthwhile to undertake such a systematic search.

(iv) A particularly attractive place to look for "cold spots" is the region of small transverse momentum, where an enhancement in the pion density was observed, particularly in heavy ion collisions. It is believed that the majority of those pions are coming from resonance decays [8]. We would like to point out, however, that the origin of the pions is irrelevant for our argument, provided the region of their emission does not exceed the limit given by (12). It is therefore certainly interesting to verify what is the multiplicity distribution of pions in the region of the "low $p_t$ enhancement".

(v) It was suggested [1, 3] that the phenomenon of BE condensation could be perhaps at the origin of the so-called "centauro" and "anticentauro" events.
However, since the observed "centauro" events show a rather large relative momentum (of the order of 1 GeV) between the particles belonging to the group, the present analysis implies that this cannot be the case. Thus another explanation of these exotic events is needed.

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