Simple method to improve comb frequency characteristic in passband and folding bands

G. Jovanovic Dolecek
Department of Electronics, Institute INAOE, Enrique Erro 1, 72740 Tonantzintla, Puebla, MEXICO
E-mail: gordana@ieee.org

Abstract. The most simple decimation filter is comb filter, which has all coefficients equal to unity. The filter must attenuate aliasing which occur in the bands around the comb zeros, called folding bands. Additionally, it must have flat magnitude characteristic in the passband in order to avoid the deterioration of the decimated signal. However, comb filter has pure attenuation in the folding bands and droop in the passband of interest. The attenuation in the folding bands can be increased by cascading comb filters. Nevertheless, this process increases comb passband droop. We present here a simple method to improve simultaneously comb magnitude characteristic in both: passband and folding bands. The cascade of combs is used to improve the attenuation in the folding bands, and the compensator, cascaded with combs, improve the passband characteristic. Two cases are elaborated depending if the passband of interest is a wideband or narrowband. The comparisons with some methods from the literature confirm the benefit of the proposed method.

1. Introduction
Decimation is decreasing of sample rate by an integer [1]. This process generates aliasing, which must be eliminated by a filter, called decimation filter. The most simple decimation filter is comb filter which does not require multipliers because all its coefficients are equal to unity. The system function of comb is given as:

$$H(z) = \left[ \frac{1}{M} \frac{1 - z^{-M}}{1 - z^{-1}} \right]^K,$$

where $M$ is decimation factor and $K$ is the number of cascaded combs, also known as comb order. Comb filter must have high attenuation in folding bands, where aliasing occur, and a flat passband characteristic to avoid the deterioration of decimated signal.

Simplest way to increase attenuation in folding bands is achieved by increasing the number of cascaded combs. However, the passband droop increases with the increase of the comb parameter $K$. As a result, the cascade of combs is usually limited to five. The comb passband droop is corrected using compensator filters proposed in [2-6]. Similarly, different methods have been proposed to improve aliasing rejection, like [7-10]. In [11] is introduced sharpening technique to simultaneously improve comb magnitude characteristic in passband and folding bands. The five comb corrector filters

1 Any correspondence address to: gordana@ieee.org
are introduced in [12], each for a given value of $K$, $K=1,\ldots,5$, to simultaneously improve passband and folding band comb magnitude characteristic.

We propose here to use a high value of parameter $K$ to improve alias rejection in folding bands, and to use compensator to compensate for a high passband droop. We consider two cases depending if the passband of interest is wideband or narrowband. The rest of the paper is organized in the following way. Next Section describes wideband case, while the Section 3 is dedicated to a narrowband case. The comparisons are presented in Section 4.

2. Wideband case

In this section we discuss the comb magnitude characteristic improvement for wide passband of interest, which is defined by the passband edge frequency [11]:

$$\omega_p = \frac{\pi}{2M}. \quad (2)$$

First, the choice of compensator is discussed and followed by the proposed method, which is illustrated with two examples and compared with the original comb filter.

2.1. Choosing the compensator

The magnitude response of the comb compensator has to approximate the inverse of the comb magnitude response in the comb passband. As a result, the cascade of combs and compensator should have the magnitude characteristic approximately equal to 1, in the passband [6]. Our goal here is that the compensated comb has an maximum absolute value of passband deviation equal or less than 0.1 dB. To this end, we consider here the compensator [6], recently proposed in literature with the following magnitude response, at low rate:

$$G(e^{j\omega}) = [1 + A \sin^4 (\omega/2)][1 + B \sin^2 (\omega/2)]. \quad (3)$$

where $A$ and $B$ are the amplitude of sinewave functions which depend on the comb parameter $K$.

The system function of the compensator is given as:

$$G(z) = \{z^{-4}A[1 + z^{-4} - 2(z^{-1} + z^{-3}) + (2^2 + 2)z^{-2}] + z^{-2}\} \times \{2^{-2}B[-1 + 2z^{-1} - z^{-2}] + z^{-1}\}. \quad (4)$$

2.2. Proposed method

We proposed here using the comb parameter $K=6$ and the compensator from [6], requiring 9 adders. As a difference to method in [6], using MATLAB simulation we find the parameters $A$ and $B$ both equal to unity. This choice requires 9 adders keeping the maximum of the absolute value of the passband deviation lesser than 0.1dB. As a result, the proposed compensator has the following system function:

$$G_p(z) = \{z^{-4}[1 + z^{-4} - 2(z^{-1} + z^{-3}) + (2^2 + 2)z^{-2}] + z^{-2}\} \times \{2^{-2}[-1 + 2z^{-1} - z^{-2}] + z^{-1}\}. \quad (5)$$

The method is illustrated in two examples.

Example 1:
Consider $M=20$ and $K=6$. Fig.1 compares magnitude responses of comb and the proposed filter. The zooms in the passband and the first folding bands are also shown. Note that the passband requirement is satisfied, and that the worst case attenuation is about 60dB.

Example 2:
In this example we chose $M=32$ and $K=6$, to show that the choice of $A=B=1$ does not depend on decimation factor $M$. Fig.2 confirms that the maximum of absolute value of passband deviation and the worst case of attenuation are similar to those, obtained in Example 1.

3. Narrowband case

In this section we discuss the comb magnitude characteristic improvement taking narrow passband, which is defined by the passband edge frequency [11]:
Lake in wideband case, we use the compensator from \[6\] with parameters \(A=B=1\). However, in this case we consider \(K=6,\ldots,8\), to get more choices for the worst case attenuation, keeping the maximum absolute value of passband deviation lesser than 0.1 dB. We also consider the value of \(K=9\). Yet, in this case the maximum absolute value of the passband deviation is about 1.6 dB. The parameters are shown in Table 1. The method is illustrated in the following two examples.

Example 3:
We chose \(M=15\) and the minimum attenuation in the first folding band of 150 dB. From Table 1 it follows \(K=7\). Fig. 3 contrasts magnitude responses of comb and the proposed filter, including the corresponding first folding bands and the passbands.

\[
\omega_p = \frac{\pi}{8M}.
\]  

\(^{6}\)
Table 1. The worst case attenuations for different values of $K$.

| $K$ | $A$ [dB] |
|-----|----------|
| 6   | 142      |
| 7   | 165      |
| 8   | 190      |
| 9   | 213      |

Figure 3. Magnitude responses in Example 3.

Example 4:
In this example we chose the same value for decimation factor as in Example 3, $M=15$. However, here the minimum required attenuation is of 200 $\text{dB}$ and the absolute value of the passband deviation is lesser than 0.2 $\text{dB}$. From Table I we chose $K=9$. Figure 4 shows the overall magnitude responses of comb and the proposed filter. The zooms in the first folding banda and passbands are also shown.

4. Some comparisons
In this section we compare the proposed method with two methods, proposed so far, for simultaneously improvement of passband and folding bands of comb filter.

4.1. Comparison with sharpening method
The sharpening method to simultaneously improve passband and folding bands in comb filter, (narrowband case), using the sharpening polynomial in the form: $H_{sh} = 3H^2 - 2H$, where $H$ is the prototype filter, is introduced in [11]. For a sake of comparison we take $M=16$, and the comb filter with the parameter $K_1=3$, as a prototype filter. In the proposed method the parameter $K=9$. The corresponding magnitude responses are contrasted in Fig.4. Note that the folding bands are similar, while the proposed method provides better passband compensation. However, the sharpening method is more complex.
Figure 4. Magnitude responses in Example 4.

Figure 5. Comparison with Sharpening method in [11].

4.2. Comparison with Corrector filter method
The five corrector filters for simultaneous improvement of comb magnitude responses in wide passband and folding bands, for values of \( K_c = 1, \ldots, 5 \), are introduced in [12].

For the sake of comparison with method in [12] we choose \( K_c = 4 \). The corrector filter has coefficients [12]:

\[ [1, 1, -2, -8, 1, 24, 24, 1, -8, -2, 1, 1] \]

In the proposed method \( K = 6, A = B = 1 \). The magnitude responses are compared in Fig. 6. The proposed method has better attenuation in even folding bands and also has better passband compensation. The attenuations in the first folding bands are similar. The method in [12] requires 11 adders, (two more than in proposed method. Additionally, the method in [12] is limited to even decimation factors, while the proposed method has limited value of \( K \).
Figure 6. Comparison with Corrector filter method.

Acknowledgement
This work is supported by Conacyt Mexico.

References
[1] Jovanovic Dolecek G 2002 Multirate Signal Processing: Design and Application (Hershey: IGP).
[2] Jovanovic Dolecek G and Mitra S K 2008 Electronics Letters Simple method for compensation of CIC decimation filter 44 1162-1163.
[3] Jovanovic Dolecek G 2009 Electronics Letters Simple wideband CIC compensator 45, 1270-1272.
[4] Molnar G Vucic M 2011 IEEE Transactions on Circuits and Systems II: Express Brief Closed-form design of CIC compensators based on maximally flat error criterion 58 926-930.
[5] Jovanovic Dolecek G and Fernandez-Vazquez A 2014 Trigonometrical approach to design a simple wideband comb compensator International Journal for Electronics and Communications (AEUE) 68 437–441.
[6] Jovanovic Dolecek G, Garcia Baez R, Molina Salgado G and de la Rosa JM 2016 Novel multiplierless wideband comb compensator with high compensation capability. Circuits, Systems and Signal Processing Published on line August 2016, 1-19
[7] Presti L 2000 IEEE Transactions on Circuits and Systems II: Express Brief Efficient modified sinc filters for sigma-delta A/D converters 47 1204–1213.
[8] Laddomada M 2007 IEEE Trans on Circ. and Syst.-I Generalized comb decimation filters for sigma-delta A/D converters 54, 994–1005.
[9] Baez R and Jovanovic Dolecek G 2015 International Journal of Electronics 4 569-582.
[10] Jovanovic Dolecek G, Garcia Baez R, and Laddomada M 2017 Design of efficient multiplierless modified cosine-based comb decimation filters: Analysis and implementation. IEEE Transactions on Circuits and Systems I: Regular Papers 5 2031–2049.
[11] Kwentus A and Wilson Jr A 1997 IEEE Trans. on Signal Processing Application of filter sharpening to cascaded integrator-comb decimation filters 45, 457-467.
[12] Jovanovic Dolecek G Fernandez-Vazquez A Novel droop-compensated comb decimation filter with improved alias rejections International Journal for Electronics and Communication (AEUE) 67, 387-396.