Theory of spin-current-induced auto-oscillations in biaxial antiferromagnets

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We study the dynamics of the Néel order in biaxial antiferromagnets under the influence of an antidamping-like torque exerted by a spin current. Large spin currents can produce steady-state precession in a plane perpendicular to spin polarization. When the spin-current is polarized along the hard-anisotropy axis, the Néel order revolves in the most stable ‘easy plane’ with dynamical equation of a damped-driven pendulum. The two-dimensional parameter space of drive and damping is explored to determine the regimes of periodic and stationary solutions, and the hysteretic region is identified. Analytic solutions for oscillations in angular velocity are derived when damping and drive are large, and a model for the fundamental mode is conceived. The oscillations are coherent for large drive and spiky for large damping near the threshold. We also propose experimental setup for electrical control and detection of terahertz auto-oscillations in thin films. The in-plane antiferromagnet overlaid with lateral spin-valve structure and the perpendicular antiferromagnet overlaid with spin-Hall structure form compatible geometries. The current drive after initiating oscillations can be lowered by exploiting the hysteresis to mitigate excessive Joule heating in the metal and to tune the oscillations to detectable frequencies. Room-temperature antiferromagnetic insulators such as in-plane NiO and perpendicular Cr2O3 are considered to calculate the current density, oscillation frequency and spin-pumping voltage.

I. INTRODUCTION

In antiferromagnetic materials, injection of a spin current can excite steady-state precession of alternating spin moments in a plane perpendicular to spin polarization at terahertz frequencies. An oscillating coherent signal or Dirac comb can originate from the reciprocal process of spin pumping in a compact geometry at room-temperature. Terahertz sources have potential applications in imaging and sensing, while spike generators are the basis of neuromorphic computing.

Auto-oscillations in antiferromagnets with biaxial anisotropy have been theoretically considered for spin current polarized along the ‘easy axis’ and along the ‘hard axis’ using a spin-Hall structure. For easy-axis spin polarization, the critical current density and the accompanying Joule heating are undesirably large in strongly in-plane materials like NiO, and the tunability of frequencies is limited. For hard-axis spin polarization, the dynamics is established to be equivalent to a classical damped pendulum in gravity driven with a constant torque, and the problem has been studied numerically. However, a general formulation of the problem, derivation of analytic solutions and proposal of anisotropy-specific geometries are lacking.

In this work, we begin with spin-current-induced dynamics of biaxial antiferromagnets, and narrow our focus to hard-axis spin polarization to obtain the reduced equation of motion in the ‘easy plane’ (Sec. II). We analyze the non-linear dynamics and find closed-form expressions for angular velocity and fundamental mode (Sec. III). We propose experimental setup for electrical control and detection of terahertz auto-oscillations in thin-film antiferromagnetic insulators, which involve pure spin currents without direct flow of electrons making them attractive for low-dissipation magnonics (Sec. IV).

II. THEORY

Antiferromagnets with a collinear bipartite ordering are composed of two interpenetrating square lattices A and B possessing oppositely aligned moments. For such ordering, the thermodynamic state of the antiferromagnet under mean-field approximation is represented by the unit vectors m_A and m_B for the sublattice spin moments, each of which has a magnetization M_s. There are three significant contributions to the free energy density of an antiferromagnet: the exchange coupling of neighboring spins that originates from Pauli exclusion principle, the magnetocrystalline anisotropy due to spin-orbit interaction, and the Zeeman interaction of magnetic moments with an external magnetic field.

Biaxial-anisotropy antiferromagnets are characterized by a direction of minimum energy which is ‘easy’ for spins to orient along, and an orthogonal direction of maximum energy which is ‘hard’. Consider a macrospin film (the spin ordering is spatially invariant) of thickness d_A with the easy and the hard axes oriented along the unit vectors u_e and u_h, respectively. If the antiferromagnetic ex-
change coupling between the sublattice moments is much stronger than the constraints of anisotropy, the average net moment \( \mathbf{m} = (\mathbf{m}_A + \mathbf{m}_B)/2 \) and the Néel order \( \mathbf{n} = (\mathbf{m}_A - \mathbf{m}_B)/2 \) satisfy the relation \( \mathbf{n}^2 = 1 - \mathbf{m}^2 \approx 1 \). The total free energy density in the presence of an external magnetic field \( H \) applied along the unit vector \( \mathbf{h} \) up to lowest, non-vanishing order is written as\(^{11,12} \)

\[
\mathcal{F} = J\mathbf{m}^2 + K_h(n \cdot u_h)^2 - K_e(n \cdot u_e)^2 - 2Z\mathbf{m} \cdot \mathbf{h},
\]

where the exchange coupling \( J \), the anisotropy coefficients \( K_i \) and the Zeeman energy density \( Z = \mu_0 M_h H \) are positive numbers, and obey the limit \( J \gg K_i, Z \).

In the limit of large exchange, the problem of coupled dynamics of the sublattice moments is virtually reduced to motion of the Néel order on a unit sphere.\(^{13,14} \)

Starting from the Landau-Lifshitz-Gilbert equation augmented with the antidamping-like torque induced by a spin current\(^{15,16} \) for each sublattice moment, the equation of motion of the Néel order is defined as\(^{12} \)

\[
\mathbf{n} \times \left[ \frac{M_s}{\gamma J} \mathbf{n} + \alpha \mathbf{n} + \frac{\gamma}{M_s} (K_h(n \cdot u_h)u_h - K_e(n \cdot u_e)u_e) + \frac{Z}{J} (2\mathbf{n} \times \mathbf{h} + \gamma \mu_0 H (n \cdot \mathbf{h}) \mathbf{h}) + \frac{\gamma J_s}{M_s d_a} (n \times u_e) \right] = 0, \quad (2)
\]

where an overdot denotes derivative with respect to time \( t \), \( \alpha \) is the Gilbert damping, \( \gamma \) is the absolute gyromagnetic ratio, and \( J_s \) is the injected spin-current density polarized along the unit vector \( u_e \).\(^{17} \) The net moment\(^{12} \)

\[
\mathbf{m} = \frac{M_s}{\gamma J} (\mathbf{n} \times \mathbf{n}) + \frac{Z}{J} [\mathbf{h} - (\mathbf{n} \cdot \mathbf{h}) \mathbf{n}] \quad (3)
\]

becomes a dependent variable, so that the Néel order dynamics (2) can be solved alone, as long as \( \mathbf{m}^2 \ll 1 \).

When the external field is much smaller than the spin-flop field \( (Z \ll \sqrt{JK_e}) \)\(^{9} \), the Zeeman energy terms can be dropped from Eq. 2. In the timescale of antiferromagnetic resonance\(^{10} \) \( \tau = \gamma \sqrt{JK_e}/M_s \), the Néel order dynamics is condensed to the dimensionless form

\[
\mathbf{n} \times [n'' + \beta n' + \kappa (n \cdot u_h)u_h - (n \cdot u_e)u_e + \Gamma (n \times u_e)/2] = 0, \quad (4)
\]

where a prime denotes \( \tau \) derivative, and the unitless parameters are defined as \( \beta = \alpha \sqrt{J/K_e} \), \( \kappa = K_h/K_e \) and \( \Gamma = 2J_s/(K_e d_a) \).

Consider a spherical coordinate system where the polar angle \( \theta \) is measured from the direction of spin-current polarization \( u_h \) and the azimuthal angle \( \phi \) is measured from an arbitrary axis in the plane perpendicular to \( u_e \). The unit vectors are \( \mathbf{n} = [\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta] \) and \( \mathbf{u}_e = [-\sin \phi, \cos \phi, 0] \). For a nontrivial solution to Eq. (4), the tangential components of the Néel order inside the square bracket must vanish, which evaluates to the coupled equations

\[
\theta'' + \beta \theta' + \kappa (n \cdot u_h) (\theta \cdot u_h) - (n \cdot u_e) (\theta \cdot u_e) - \phi'^{2} \sin \theta \cos \theta = 0, \quad (5a)
\]

\[
\phi'' \sin \theta + 2 \phi' \phi' \cos \theta + \beta \phi' \sin \theta + \kappa (n \cdot u_h) (\phi \cdot u_h) - (n \cdot u_e) (\phi \cdot u_e) = (\Gamma/2) \sin \theta. \quad (5b)
\]

For large \( \Gamma \), it can be found from numerical simulation and perturbation that the stable solutions are \( \theta \lesssim \pi/2 \) and \( 2\phi' \approx \Gamma/\beta \); meaning a sufficiently large spin current can produce steady-state precession of Néel order in a plane perpendicular to spin polarization.

When the spin-current is polarized along the hard axis, the Néel order can precess in the perpendicular plane consisting of the easy axis, referred as the ‘easy plane’. If \( \theta = \pi/2 - \theta \ll 1 \) denotes the perturbation from the easy plane and the azimuth \( \phi \) is measured from the easy axis, then Eqs. (5a) and (5b) are simplified to

\[
\theta'' + \beta \theta' + (\phi'^2 + \kappa + \cos^2 \phi) \theta = 0, \quad (6a)
\]

\[
\phi'' + \beta \phi' + \sin \phi \cos \phi = \Gamma/2. \quad (6b)
\]

The sign of the coefficient of \( \theta \) (indicative of spring constant) is positive, so the dynamics is restored to the easy plane regardless of the precession frequency. For a different direction of the spin polarization, the circular motion would have to acquire enough speed for stability of the plane.\(^{18} \)

The dynamics of the Néel order for hard-axis-polarized spin-current is reduced to angular motion in the easy-plane (6b), which by replacing \( \phi = 2\theta \) is recast into

\[
\psi'' + \beta \psi' + \sin \psi = \Gamma, \quad (7)
\]

the equation of motion of a simple gravity pendulum in viscous damping \( \beta \) driven with a constant torque \( \Gamma \) (mass, length and gravity are equal to one).\(^{19} \)

III. ANALYSIS OF SOLUTIONS

The solution of the damped-driven pendulum equation (7) depends only on two dimensionless parameters \( \beta \) and \( \Gamma \). The steady-state dynamics \( (\tau \to \infty) \) of this non-linear system yields either a stationary solution \([\psi'(\tau) = 0]\), or a periodic solution \([\psi'(\tau + T) = \psi'(\tau) \neq 0]\) with a period \( T \). Ref. 19 presents a qualitative description of the dynamics, and numerically examines the boundaries of periodic and stationary solutions in the parameter space. We run transient simulation with initial conditions \( \psi_0 = 180^\circ \) and \( \psi_0 = 0 \) over the parameter space, and obtain the time \( T \) to complete a revolution in steady state \( (\tau > T/\beta) \). A contour plot of the fundamental mode \( 1/T \) is produced in Fig. 1 depicting the regions of different types of solutions.

In the hysteretic region, the steady-state solution can be periodic or stationary depending on the initial condition.

For example, if the pendulum was released vertically upside down \( (\phi_0 = 180^\circ) \), then it revolves continually; but if it was released horizontally from a point where the gravity opposes the drive \( (\phi_0 = 90^\circ) \), then it dips and settles to equilibrium at \( \sin \phi_{eq} = \Gamma \). The hysteretic region is defined as: \( f_h(\beta) < \Gamma(\beta) < 1 \) on
Hysteretic

\[ \phi \]

the domain \( 0 < \beta < \beta_h \), where \( f_h \) determines the homoclinic bifurcation.\(^{20}\) From inspection of the data, we obtain \( \beta_h = 1.2 \) and the cubic-polynomial fit

\[
f_h(\beta) = a_0 \beta + \left( \frac{3 - 2a_0 \beta_h}{\beta_h^2} \right) \beta^2 + \left( \frac{a_0 \beta_h - 2}{\beta_h^3} \right) \beta^3,
\]

where \( \beta \ll 1 \), the slope is \( a_0 = 4/\pi \).\(^{19}\) The minimum drive needed to sustain revolutions (solid cyan line) is expressed as

\[
\Gamma_m(\beta) = \begin{cases} f_h(\beta) & \beta < \beta_h \\ 1 = \Gamma_{th} & \beta \geq \beta_h \end{cases},
\]

where \( \Gamma_{th} \) denotes the threshold that can initiate revolutions independent of the damping (dashed cyan line).

In the overdrive limit \( \Gamma \gg \Gamma_m \), the fundamental mode is calculated by integrating Eq. 7 over a period and ignoring the role of gravity (\( \sin \phi \) term) to obtain

\[
\Omega_{O\beta} = \frac{1}{2\pi} \frac{\Gamma}{\beta},
\]

This is consistent with the slope of the contour lines seen in Fig. 1 in regions far above the threshold. The time-dependence of the steady-state angular velocity \( \omega = \dot{\phi} \) is estimated from Eq. 7 by truncating \( \sin \phi \simeq \sin(\Gamma \tau/\beta) \), multiplying by \( e^{\beta \tau} \), integrating and letting the transients decay to give

\[
\omega_{O\beta}(\tau) \simeq \frac{\Gamma}{\beta} - \frac{\beta}{\sqrt{3 + 4\Gamma^2}} \sin \left( \frac{\Gamma \tau}{\beta} - \xi_{O\beta} \right),
\]

where \( \xi_{O\beta} = \arctan(\Gamma/\beta^2) \).

In the overdamping limit \( \beta \gg 1 \), Eq. 7 is reduced to \( \dot{\phi} = (\Gamma - \sin \phi)/\beta \), which is separable and has a closed-form integral. The periodic solution of the angle is expressed as\(^{19}\)

\[
\Gamma \tan \frac{\varphi_{O\beta}(\tau)}{2} = 1 + \sqrt{\Gamma^2 - 1} \tan \left( \frac{\sqrt{\Gamma^2 - 1}}{2\beta} \tau - \xi_{O\beta} \right),
\]

where \( \xi_{O\beta} = \arctan(1/\sqrt{\Gamma^2 - 1}) \). The fundamental mode is obtained as

\[
\Omega_{O\beta} = \frac{1}{2\pi} \sqrt{\Gamma^2 - 1},
\]

and the steady-state angular velocity is calculated as

\[
\omega_{O\beta}(\tau) = \frac{\Gamma}{\beta} - \frac{2}{\beta} \frac{\tan(\varphi_{O\beta}/2)}{1 + \tan^2(\varphi_{O\beta}/2)}.
\]

It can be checked that superposing the overdrive limit results in convergence of solutions.

For moderate drive \( \Gamma \gtrsim \Gamma_m \) and damping \( \beta \ll 1 \), it is not possible to derive analytic solutions.\(^\text{19,21}\) However, the fundamental mode is found to obey the model

\[
\Omega(\Gamma, \beta) = \frac{1}{2\pi} \left( \frac{\Gamma^p - (\Gamma_m)^p}{\beta} \right)^{1/p} \begin{cases} \Gamma_m = \Gamma_m(\beta) \\ \beta = p(\beta) \end{cases},
\]

conceived by generalizing the overdamping result (12). The functional relationship \( p(\beta) \) and the accuracy of the model are depicted in Fig. 2. The inverse dependence of \( p \) on \( \beta \) implies that a weakly damped system \( \beta \ll 1 \) is much easier to oscillate than a strongly damped \( \beta \gg 1 \) for a fixed threshold-to-drive ratio.

To compare periodic solutions across the parameter space, the first-order term \( t_{O\Gamma} = (\Gamma_m/\Gamma)^p/p \) in the expansion of \( 2\pi \beta \Omega/\Gamma \) is used to measure nearness to overdrive; the relative increment \( \epsilon_{th} = (\Gamma - \Gamma_m)/\Gamma_m \) is used to measure nearness to threshold. The waveforms of the angular velocity for limiting cases of damping and drive are juxtaposed in Fig 3. The closed-form solutions agree with the numerical ones; except when the drive is near threshold and the damping is weak, which violates the apriori assumption. The oscillations in angular velocity are coherent for overdrive, and Dirac-delta-like for overdamping near the threshold.
FIG. 3. Waveforms of the angular velocity $\omega(\tau)$ for (a) overdrive $\epsilon_{\text{OF}} = 0.05$ and underdamping $\beta = 0.2$, (b) overdrive $\epsilon_{\text{OF}} = 0.05$ and overdamping $\beta = 5$, (c) near-threshold drive $\epsilon_{\text{th}} = 0.05$ and underdamping $\beta = 0.2$, (d) near-threshold drive $\epsilon_{\text{th}} = 0.05$ and overdamping $\beta = 5$. Dark-color plot is the exact numerical solution and light-color plot is the approximate closed-form solution. Dotted horizontal line denotes the average value equal to $2\pi\Omega$ (14). At time $\tau = 0$, the pendulum is at the lowest point $\varphi = 0$.

IV. ELECTRICAL CONTROL AND DETECTION

The magnetic anisotropy of thin films and multilayers can be significantly different from that observed in bulk magnetic materials due to epitaxial strain from substrate and reduced local symmetry of surface atoms. The magnetic anisotropy energy is augmented by magnetostrictive and surface anisotropies which can elicit an in-plane or a perpendicular spin orientation independent of the specific growth direction of the film. In the bi-axial description, the in-plane anisotropy represents an in-plane easy axis and a perpendicular hard axis, while the perpendicular anisotropy represents a perpendicular easy axis and an in-plane hard axis, with effective coefficients $K_e, K_h$.

Pure spin current through an insulating antiferromagnetic layer can be generated from electric current flowing in an overlaid lateral spin-valve or spin-Hall structure. In the lateral spin-valve structure shown in Fig. 4a, electric current across the ferromagnetic reference layers (whose magnetization orientation is fixed) injects spin accumulation in the normal metal and spin current in the antiferromagnet, polarized parallel to the reference-layer magnetization. In the spin-Hall structure shown in Fig. 4b, electric current in the heavy metal with strong spin-orbit coupling produces a perpendicular spin current polarized transverse to electric current in the film plane. Large spin-Hall angle materials like topological insulators can be very efficient in converting electric current to spin, but the spin-polarization is constrained to in-plane direction.

For generating spin polarization along the hard axis of the antiferromagnetic layer, the viable geometries are the in-plane-anisotropy antiferromagnet with the lateral spin-valve structure made of a perpendicular reference-layer magnetization, and the perpendicular-anisotropy antiferromagnet with the spin-Hall structure in which the in-plane electric current is transverse to the in-plane hard axis. For both the geometries, the input is a constant-current source, the back diffusion (“backflow”) of injected spins is considered small, and the output is detected under open-circuit condition.

To excite and control Néel-order precession, the spin current must exert adequate antidamping-like torque. The threshold spin-current density that initiates precession is obtained from the condition $\Gamma_{\text{th}} = \frac{1}{\epsilon}$ (9) as $J_{s,\text{th}} = \frac{K_e d_{\text{s}}}{2}$. The corresponding electric-current density depends on the specific geometry. For the in-plane spin-valve geometry, assuming low spin-memory loss in the normal metal, the conversion from electric current to spin is determined by the conductance of majority and
minority-spin electrons \( g_M \) and \( g_m \), respectively, and the spin-mixing conductance at the interface of normal metal and antiferromagnet \( g_t \). The threshold electric-current density is calculated as

\[
J_{c,th}^{\text{ip-valv}} = \frac{2e}{\hbar} \frac{(g_t + g_M + g_m)(g_M + g_m)}{g_M - g_m} J_{s,th}. \tag{15}
\]

For the perpendicular spin-Hall geometry, the conversion from electric current to spin is determined by the spin-Hall angle \( \Theta_s \), the layer thickness \( d_n \), the spin diffusion length \( \lambda \) and the conductivity \( \sigma \) of the heavy metal, and the spin-mixing conductance at the interface of heavy metal and antiferromagnet \( g_t \). The threshold electric-current density is given as

\[
J_{c,th}^{\text{pe-Hall}} = \frac{e}{\hbar \lambda g_t} \coth(\frac{d_n}{2\lambda}) J_{s,th}. \tag{16}
\]

If the effective damping \( \beta \ll 1 \), the minimum current for sustaining precession is about \( 1/\beta \) times lower as seen from Fig. 1 and Eq. 9. Lowering the current drive after initiation mitigates excessive Joule heating in the metal, and allows for tunability of oscillation to sub-terahertz frequencies detectable by microelectronic circuits.

Precessing Néel order can reciprocally pump time-varying spin current back into the adjacent metal, and experience a damping-like backaction.\(^{29}\) This virtually enhances the Gilbert damping expressed as \( \alpha = \alpha_0 + \alpha_s \), where \( \alpha_0 \) is the intrinsic damping constant and the enhancement\(^{28}\)

\[
\alpha_s = \frac{\hbar^2 \gamma g_t}{2e^2 M_s d_n}. \tag{17}
\]

The pumped spin current is converted into voltage under open-circuit condition via spin filtering for the in-plane spin-valve geometry and via inverse spin-Hall effect for the perpendicular spin-Hall geometry. The voltage signal generated at the output terminal of each geometry is calculated as\(^{6,27}\)

\[
V_{\text{out}}^{\text{ip-valv}}(t) = \frac{\hbar \gamma \sqrt{K_c}}{2e M_s} \frac{g_t (g_M - g_m) \omega(t)}{(g_t + g_M + g_m)(g_M + g_m)}, \tag{18}
\]

\[
V_{\text{out}}^{\text{pe-Hall}}(t) = \frac{\hbar \gamma \sqrt{K_c}}{2e M_s} \frac{\lambda g_t \Theta_s \tanh\left(\frac{d_n}{2\lambda}\right) \omega(t)}{\sigma}, \tag{19}
\]

where \( \omega \) is the dimensionless angular velocity (Sec. III).

Materials such as NiO and Cr\(_2\)O\(_3\) are room-temperature antiferromagnetic insulators, whose bulk magnetic properties are well-studied (Table I). For thin films, it is known experimentally that NiO(001) on SrTiO\(_3\) substrate has an in-plane anisotropy,\(^{30}\) while Cr\(_2\)O\(_3\)(001) on Al\(_2\)O\(_3\) substrate has a perpendicular anisotropy,\(^{31}\) but measurement of magnetic anisotropy of antiferromagnetic films is virtually unexplored.\(^{32}\) Nonetheless, we consider the bulk properties to estimate specifications for the proposed setup. For characteristic values of geometry parameters (Table II), the critical current density \( J_{c,th}^{\text{ip-valv}} \approx 2 \times 10^7 \) A/cm\(^2\) and \( J_{c,th}^{\text{pe-Hall}} \approx 10^8 \) A/cm\(^2\); the effective damping \( \beta^{\text{ip-valv}} = 0.18 \) and \( \beta^{\text{pe-Hall}} = 0.24 \); the frequency scale is 1.4 THz for NiO and 0.5 THz for Cr\(_2\)O\(_3\); the amplitude of alternating output signal \( v_{\text{out}}^{\text{ip-valv}} = 70 \) µV and \( v_{\text{out}}^{\text{pe-Hall}} = 0.4 \) µV at the threshold current.

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**TABLE I.** Properties of bulk NiO and Cr\(_2\)O\(_3\) at 300 K

| Parameter | NiO | Ref. | Cr\(_2\)O\(_3\) | Ref. |
|-----------|-----|------|----------------|------|
| \( J \) (J/m\(^3\)) | \(3.4 \times 10^8\) | 33.2 | \(9.5 \times 10^7\) | 34.35 |
| \( K_e \) (J/m\(^3\)) | \(2.2 \times 10^4\) | 33.2 | \(3.2 \times 10^3\) | 34.35 |
| \( K_M \) (J/m\(^3\)) | \(5.5 \times 10^5\) | \(33.2\) | \(0\) | \(34.35\) |
| \( M_s \) (A/m) | \(3.5 \times 10^5\) | 36.2 | \(1.9 \times 10^5\) | 34.35 |
| \( \alpha_0 \) | \(6 \times 10^{-4}\) | 36.2 | \(2 \times 10^{-4}\) | 34.37 |

**TABLE II.** Characteristics of geometry

| Parameter | Ref. | Parameter | Ref. |
|-----------|------|-----------|------|
| \( g_M = 10^{10} \) (S/m\(^2\)) | 27 | \( d_n = 5 \) nm | 2 |
| \( g_n = 10^9 \) (S/m\(^2\)) | 27 | \( d_n = 20 \) nm | 2 |
| \( g_t = 10^{14} \) (S/m\(^2\)) | 38 | \( \lambda = 7 \) nm | 2 |
| \( \sigma = 10^6 \) (S/m) | 2 | \( \Theta_s = 0.1 \) | 2 |
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