Three-Parton Contributions to $B \rightarrow M_1 M_2$ Annihilation at Leading Order

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Abstract

We compute annihilation amplitudes for charmless $B$ decays that are proportional to the three-parton twist-3 light meson distribution amplitude $\phi_{3M}(x_1, x_2)$ with an active gluon. Due to an enhancement from a quark propagator at the scale $p^2 \sim m_b \Lambda_{\text{QCD}}$ these terms occur at the same parametric order in $\alpha_s(m_b)$ and $1/m_b$ as the known leading order annihilation involving $f_B$ and twist-2 meson distributions. With our calculation the leading order annihilation amplitude is now complete. At lowest order in $\alpha_s$ the new amplitudes are real and only $O_{5-8}$ contribute. Using simple models we find that the three-parton and two-parton terms are of comparable size.
The nonleptonic charmless decay channels $B \rightarrow M_1 M_2$ provide a wealth of information about the standard model, including the study of CP violation and the strong interactions. Since many amplitudes for these decays are loop dominated, it is possible for new physics to give a significant contribution. However, except for the simplest observables, testing for new physics requires an understanding of the standard model background. Predicting standard model decay rates and CP asymmetries with quantum chromodynamics (QCD) is difficult, but the task is simplified by the use of the soft-collinear effective theory (SCET) and factorization theorems.

An interesting experimental observable is the relative “strong” phase between standard model amplitudes multiplying the CKM factors $V_{ub}^* V_{cf}$ and $V_{cb}^* V_{df}$ ($f = d, s$), since these phases are measured to be large in the $B \rightarrow \pi\pi$ and $B \rightarrow K\pi$ channels. There are two competing standard model explanations for these phases, sizeable charm penguin loops or sizable annihilation amplitudes in which the initial state “spectator” quark is Wick-contracted with a quark field in the effective Hamiltonian. In this paper we report on a new contribution to the leading annihilation amplitudes.

Our notation follows that of Ref. where factorization theorems for the leading order $B \rightarrow M_1 M_2$ amplitudes were derived with an expansion in $\Lambda/Q$ where $\Lambda$ is a hadronic scale and $Q \sim m_b \sim m_c \sim E_M$. We restrict our discussion to non-isosinglet mesons ($M_i = \pi, K, \rho, \ldots$) which can not be produced solely by gluons, for which the annihilation amplitudes are power suppressed by $\sim \Lambda/Q$. Recently these power corrections were classified according to their perturbative order and source for strong phases. The annihilation amplitude is

$$A_{\text{ann}} = A_{\text{hard}} + A_{\text{hard-coll}}(1) + A_{\text{hard}}(2) + \ldots ,$$

where the superscript denotes the order in $\Lambda/Q$. A subscript “hard” denotes annihilation amplitudes generated by propagators offshell by $p^2 \sim m_b^2$. At lowest order in $\alpha_s$ these include the standard leading order amplitudes $A_{\text{hard}}(1) \sim [\alpha_s(m_b) f_B f_{M_1} f_{M_2} \phi_{M_1} \phi_{M_2}]$ as well as the chirally enhanced amplitudes $A_{\text{hard}}(2) \sim [\alpha_s(m_b) f_B f_{M_1} f_{M_2} \phi_{M_1} \phi_{M_2}^{pp} \mu_{M_2}/m_b]$. These have been studied in earlier analyses, and at this order, factorization in rapidity reveals that they are real. Here the $f$’s are decay constants, and the chiral enhancement factors are $\mu_{\pi} = m_\pi^2/(m_u + m_d)$ and $\mu_K = m_K^2/(m_u + m_s)$. $\phi_{M_i}$ are twist-2 distribution functions, and $\phi_{M_i}^{pp}$ is a two-parton twist-3 distribution. In Eq. the amplitudes $A_{\text{hard-coll}}(1)$ have hard-collinear propagators, which are offshell by an amount $\mu_i^2 \sim m_b \Lambda$. A non-perturbative phase is first generated by soft exchange between the two mesons as an order $\alpha_s^2(\mu_i)/\pi$ suppressed term in $A_{\text{hard-coll}}(1)$. The ellipsis in Eq. denotes the fact that the full set of $A_{\text{hard-coll}}(2)$ amplitudes have not yet been classified.

In this paper we compute the leading term in the perturbative expansion of $A_{\text{hard-coll}}(1)$, which has the form

$$A_{\text{hard-coll}}(1) \sim \alpha_s(m_b) \frac{H(x_1, y_1, y_2)}{k} \otimes f_B \phi_{B}^{*}(k) f_{M_1} \phi_{M_1} \phi_{M_2} \phi_{M_2}^{pp} \phi_{M_2}(y_1, y_2).$$
Here $H$ is a calculable hard-scattering kernel, $\phi_{3M}$ is a three-parton twist-3 distribution, and $f_{3M}$ is the corresponding decay constant. The amplitude in Eq. (2) occurs at the same order in $1/m_b$ and $\alpha_s(m_b)$ as $A_{\text{hard}}^{(1\text{ann})}$ and should be included for a complete leading order annihilation amplitude. Unlike $A_{\text{hard}}^{(1\text{ann})}$ its convolution integrals converge without using rapidity factorization. Furthermore, the LO annihilation involves $B$-meson information beyond $f_B$, thus demonstrating that annihilation is more complicated than the short distance picture leading to a scaling $\sim f_B/m_b$ that is often used in parametric estimates [19].

In QCD at the scale $m_b$, flavor changes are mediated by the weak effective Hamiltonian. For $B \to M_1 M_2$ with $\Delta S = 0$,

$$H_W = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pd}^* \left( C_1 O_1^p + C_2 O_2^p + \sum_{i=3}^{10,7,8g} C_i O_i \right). \quad (3)$$

Most of these operators have spin $(V-A) \otimes (V-A)$, such as $O_1^u = (\bar{u}b)_{V-A} (\bar{d}u)_{V-A}$. We will prove below that all such operators give vanishing contribution to Eq. (2), so that only

$$O_5 = \sum_{q'} (\bar{d}b)_{V-A} (\bar{q}' q')_{V+A}, \quad O_6 = \sum_{q'} (\bar{d}_b b_\alpha)_{V-A} (\bar{q}'_\beta q'_\beta)_{V+A},$$

$$O_7 = \sum_{q'} 3e_{q'} \left( \frac{1}{2} (\bar{d}b)_{V-A} (\bar{q}' q')_{V+A} \right), \quad O_8 = \sum_{q'} 3e_{q'} \left( \frac{1}{2} (\bar{d}_b b_\alpha)_{V-A} (\bar{q}'_\beta q'_\beta)_{V+A} \right), \quad (4)$$

are relevant for our analysis. Here $\alpha$ and $\beta$ are color indices, $e_{q'}$ are electric charges, and the sum is over flavors $q' = u, d, s, c, b$. Results for $\Delta S = 1$ transitions are obtained by replacing $d \to s$ in Eqs. (3) and (4), and likewise in the equations below. The coefficients in Eq. (3) are known at next-to-leading-log order [20]. (We have $O_1^p \leftrightarrow O_2^p$ relative to [20].) In the NDR scheme with $m_b = 4.8 \text{GeV}$, the coefficients are $C_{5-8}(m_b) = \{0.010, -0.040, 4.9 \times 10^{-4}, 4.6 \times 10^{-4}\}$. They are considerably smaller than $C_1(m_b) = 1.08$ and $C_2(m_b) = -0.18$, but can give important contributions in penguin observables because $C_{1,2}$ only contribute through loops [14].

To separate the mass scales below $m_b$ we match $H_W$ onto operators in SCET. The amplitude for $B \to M_1 M_2$ is most easily calculated in the $B$ rest frame where soft fields with typical momenta $\sim \Lambda$ interpolate for the initial state $B$. The final state hadrons $M_1$ and $M_2$ are back-to-back energetic charmless pseudoscalar or vector mesons. Collinear fields in the light-like direction $n$ interpolate for one light meson, and collinear fields in the direction $\bar{n}$ interpolate for the other. These fields have typical momenta $(n \cdot p, \bar{n} \cdot p, p_\perp) \sim Q(\eta^2, 1, \eta)$ and $Q(1, \eta^2, \eta)$, respectively, in terms of the power counting parameter $\eta \sim \Lambda/Q$. The vectors $n$ and $\bar{n}$ satisfy $n \cdot \bar{n} = 2$, and we work in a frame where the $B$-meson four-velocity $v$ has $n \cdot v = \bar{n} \cdot v = 1$. To calculate the amplitude in Eq. (2) we first match QCD onto operators in SCET$_1$ where the hard-collinear modes with $p^2 \sim m_b \Lambda$ are still propagating and then match these onto operators in SCET$_{11}$ which has only non-perturbative modes with $p^2 \sim \Lambda^2$ [21].

Before presenting the details of the calculation of Eq. (2) we complete our review of Ref. [17] with a discussion of the required ingredients and an overview of the matching
procedure. Let $Q^{(0)}$ denote the leading SCET$_{1}$ weak operators. Annihilation contributions require either (a) an $n$-quark $\bar{n}$-antiquark pair produced by hard interactions giving a $Q^{(k \geq 2)}$ six-quark operator, or (b) a time-ordered product of two mixed Lagrangians $L_{\xi_{q}}^{(k)}L_{\xi_{n}}^{(k)}$ to produce the $n$-$\bar{n}$ pair by soft exchange. Annihilation also requires a mechanism for connecting the “spectator” to the weak operator, either by (i) having a soft field directly in the weak operator, or (ii) having a time-ordered product with an $L_{\xi_{q}}^{(k)}$. Case (a,i) gives operators $Q^{(4)}$ which contribute only to $A^{(1 \text{ann})}_{\text{hard}}$. Case (b,i) vanishes at this order. Case (b,ii) involves three hard-collinear gluons and is $\sim \alpha_{s}^{2}(\mu_{i})$. This leaves case (a,ii). Here $Q^{(2)}_{\xi_{q}}L_{\xi_{n}}^{(1)}$ can contribute at $\mathcal{O}(\alpha_{s}(m_{b}))$ if the gluon from $L_{\xi_{q}}^{(1)}$ is uncontracted. Since the uncontracted gluon costs an extra power when matched onto SCET$_{II}$, it is only this class of operators that contributes with an external gluon, not $Q^{(3)}L_{\xi_{q}}^{(1)}$, $Q^{(2)}L_{\xi_{q}}^{(2)}$, etc. Thus the calculation of $A^{(1 \text{ann})}_{\text{hard-coi}}$ involves finding SCET$_{1}$ operators of the form

$$Q^{(2)}_{\xi_{q}} \propto \left[ \bar{q}_{n',\omega_{n}} \Theta_{u_{n}} b_{u_{n}} \right] \left[ \bar{d}_{n,\omega_{n}} \Theta_{\bar{q}_{n}} q_{n,\omega_{n}} \right] \left[ \bar{q}_{n,\omega_{n}} \Theta_{n} q'_{n',\omega_{n}} \right] ,$$  

(5)

where $\Theta_{u_{n}} \otimes \Theta_{\bar{q}_{n}} \otimes \Theta_{n}$ are color and spin structures, $q$ and $q'$ are flavors, and the collinear direction $n' = n$ or $\bar{n}$. The fermion fields are gauge invariant with large label momenta specified by the subscripts $\omega$, for example $q_{n,\omega_{n}} = \delta(\omega_{n} - \bar{n} \cdot \mathcal{P})W_{n}^{T (q)}$ where $W_{n}$ is a Wilson line. At tree level these operators arise from the full-theory diagrams in Fig. 2 with three light $n'$-collinear quarks and two collinear in the other direction, $\bar{n}'$. They have Wilson coefficients of $\mathcal{O}(\alpha_{s}(m_{b}))$. We identify $n$ as the collinear direction of the pair-produced quark of flavor $q$ and sum over all $n$ in the SCET$_{1}$ weak Hamiltonian. We will see shortly that the flavor structure is as in Eq. (5), and that the matching requires $T^{A}$ color structures for two of the $\Theta$'s.

To finalize our description of the calculation we consider matching the time-ordered product $T[Q^{(2)}L_{\xi_{q}}^{(1)}]$ onto SCET$_{II}$ with diagrams as shown in Figure 1. $Q^{(2)}$ has an excess of $n'$-collinear fermions since only two are needed to interpolate for a collinear meson. The subleading Lagrangian $L_{\xi_{q}}^{(1)} = \bar{q}_{n} ig B_{n} q'_{n'}$ removes an $n'$-collinear fermion and provides the soft field that interpolates for the light anti-quark in the $B$ meson. Here
FIG. 2: Tree-level annihilation graphs for $B \to M_1 M_2$ decays. The gluon and the fermion propagator connecting it to the weak vertex are both offshell by $p^2 \sim m_b$. Matching on to SCET$_{II}$, these graphs give rise to the six-quark operators $Q^{(2)}$, the filled circle at the center of Fig. 1.

\[ ig \mathcal{B}_{n',\omega}^{\perp} = \left[ 1/(\bar{n}' \cdot \mathcal{P}) W_{n'}^{\dagger} \left[ i \bar{n} \cdot D_{n'}, i D_{n',\perp}^{\mu} \right] W_n \delta(\omega - \bar{n}' \cdot \mathcal{P}^\dagger) \right], \]

and the form of the SCET$_{II}$ operators is

\[ O_{id}^{(1T)} \propto \frac{1}{n' \cdot k} \left[ \bar{q}_s \gamma_i k \Gamma_n b_{\nu} \right] \left[ \bar{d}_n \Gamma_n q_{\nu} \right] \left[ \bar{q}_n \Gamma_n q_{\nu}' \right] i g \mathcal{B}_{n'}^{\perp \beta}, \]

with $\Gamma_s \otimes \Gamma_{\bar{n}} \otimes \Gamma_n$ containing spin and color structures. The collinear gluon field strength $ig \mathcal{B}_{n'}^{\perp} \sim \eta$, interpolates for gluons in a final state meson, so there is no perturbative suppression from the factor of $g$. At tree level, integrating out the hard-collinear quark propagator in Fig. 1 induces an inverse factor $1/(n' \cdot k)$ of the soft momentum which will be convoluted with the $B$-distribution, $\phi_k(n' \cdot k)$. In Eq. (6) this compensates the $\eta$ suppression from $ig \mathcal{B}_{n'}^{\perp}$ to make $O^{(1T)}$ the same order as the six-quark operators for the hard annihilation, which is $O(\eta^7)$. We have checked that operators with more $ig \mathcal{B}_{n'}^{\perp}$'s or with soft gluon field strengths do not occur at this order in $1/m_b$ and $\alpha_s(m_b)$.

Note that SCET$_{II}$ time-ordered products (T-products) do not contribute at $O(\eta^7)$. To see this, recall that our process has a soft initial state and $n$ and $\bar{n}$-collinear final states. An example of an SCET$_{II}$ Lagrangian that connects these sectors [23] has two-collinear quarks and two-soft quarks [24], $\bar{q}_s \gamma_i k \xi_n, \sim \eta$. In these operators the two $n$-collinear particles conserve the large $p^- \sim \eta^0$ momenta, and the two soft particles conserve the $p^+ \sim \eta$ momenta. Thus this operator, as well as analogous operators with gluons, only support scattering, $ns \to ns$, and not annihilation such as $nn \to ss$ or $ss \to nn$. Another example is $L_\text{II} \sim \xi_n A_n A_n \xi_n \sim \eta^2$, where analogous statements hold for $n$ and $\bar{n}$. Weak operators, like $O^{(1T)}$, that have the same $n-\bar{n}$-s structure as the initial and final states are already $O(\eta^7)$, so T-products with them are power suppressed. The above considerations rule out the majority of T-products. An example of an annihilation T-product in SCET$_{II}$ that survives these criteria is $L_\text{II}$, with a weak operator with fields $(\bar{q}_s h_v \xi_n \xi_n) \sim \eta^5$. These T-products involve at least one loop momentum $\ell^\mu$ where, due to the double multipole expansion, $\ell^\pm$ must be smaller than the conserved $p^- \sim \eta^0$ and $p^+ \sim \eta^0$, see Eq.(25) of Ref. [25]. As a contour integral in $\ell^+$ or $\ell^-$ we have $\geq 2$ poles that are all on the same side of the axis, and therefore the loop gives zero. At $O(\eta^7)$ this is sufficient to rule out possible annihilation T-products, including those with more than one SCET$_{II}$ Lagrangian. Note that in Ref. [24] a T-product contribution was identified for $B^0 \to D^0 \pi^0$, however in that scattering process the integral did not satisfy the same pole criteria as we find here.
Constructing the Operator Bases

Next we construct a full basis for the operators $Q^{(2)}$ and $O^{(1T)}$ in the SCET$_I$ and SCET$_II$ weak effective Hamiltonians, respectively. (The matching calculations beginning on page 7 can be understood without the details of this somewhat technical construction, the results of which are Eqs. (15) and (16).) General symmetry arguments allow us to reduce the operator bases to the small subset relevant to our calculation of $A^{(1ann)}_{\text{hard-col}}$, and for this reason it is convenient to construct the bases for SCET$_I$ and SCET$_II$ simultaneously. First consider spin in SCET$_I$. For light fermion fields of definite handedness, a complete basis of Dirac structures for the individual bilinears in Eq. (5) is

$$\Theta_{us/n'} = \{1, \gamma^a_\perp, \gamma^\mu_\perp\}, \quad \Theta_{\bar{n}} = \{\psi, \gamma^\mu_\perp\}, \quad \Theta_n = \{\bar{\psi}, \gamma^\mu_\perp\}.$$  

(7)

Using these bases, we must construct a complete set of $Q^{(2)}$ spin structures with chiralities inherited in perturbative matching from the full-theory fields in $O_{1-10}$ and the produced $q\bar{q}$ pair. To make a Lorentz scalar, the spin structure must have zero $\gamma_\perp$’s or two $\gamma_\perp$’s contracted with $g^{\alpha\beta} = g^{\alpha\beta} - n^\alpha n^\beta/2 - n^\beta n^\alpha/2$. Note that contracting with $\gamma^\alpha_\perp = n^\mu n^\nu \epsilon^{\alpha\beta\mu\nu}/2$ does not yield an independent operator since for example $ie^{\mu}_{\perp} \xi_n \bar{\psi} \gamma^\perp \gamma^\perp \xi_n = \frac{e^{\perp}_{\perp}}{n^\mu} \bar{\psi} \gamma^\perp_\perp \gamma_\perp \xi_n = \frac{e^{\perp}_{\perp}}{n^\mu} \bar{\psi} \gamma^\perp_\perp \xi_n$. For $O_{1-4,9,10}$ the only allowed chiral structure is $(LH)(LR)(LL)$ where $L$ and $R$ refer to the handedness for the light quarks in the bilinears in the order shown in Eq. (5). We cannot assign a handedness to the heavy quark denoted here by $H$. This chiral structure is realized as the spin structures

$$\Theta_{us/n} \otimes \Theta_{\bar{n}} \otimes \Theta_n = 1 \otimes \bar{\psi} \otimes \bar{\psi}, \quad \Theta_{us/\bar{n}} \otimes \Theta_{\bar{n}} \otimes \Theta_n = 1 \otimes \psi \otimes \psi.$$  

(8)

We have ruled out the chirality $(LH)(LR)(RL)$ corresponding to a spin structure $1 \otimes \bar{\psi} \gamma^\perp_\perp \otimes \bar{\psi} \gamma^\perp_\perp$ by using $P_R \bar{\psi} \gamma^\perp_\perp \otimes P_L \gamma^\perp_\perp = 0$. This equation encodes the helicity flip argument of Ref. [15]. Similarly, for $O_{5-8}$ the chirality $(LH)(RR)(RR)$ is also realized as the spin structures Eq. (8), whereas $(LH)(RL)(LR)$ is not allowed since $P_L \bar{\psi} \gamma^\perp_\perp \otimes P_R \gamma^\perp_\perp = 0$. We will show momentarily, however, that using SCET$_II$ the terms in Eq. (8) are not needed to compute $A^{(1ann)}_{\text{hard-col}}$. For $O_{5-8}$ we can also have

$$\Theta_{us/n} \otimes \Theta_{\bar{n}} \otimes \Theta_n = \gamma^\alpha_\perp \otimes \bar{\psi} \otimes \bar{\psi} \gamma^\perp_\perp, \quad \Theta_{us/\bar{n}} \otimes \Theta_{\bar{n}} \otimes \Theta_n = \gamma^\alpha_\perp \otimes \psi \otimes \psi \gamma^\perp_\perp.$$  

(9)

corresponding to chiralities $(RH)(LL)(LR)$ and $(RH)(LR)(RR)$, respectively, and thus the flavor structure shown in Eq. (5), namely $(q'b)(\bar{d}q)(\bar{q}q')$. The second structure in Eq. (8) is related to the second structure in Eq. (9) by a Fierz transformation swapping $\bar{d}_n$ and $\bar{q}_\bar{n}$ quarks and we will choose the latter for our operator basis. The complete set of spin structures in Eqs. (8) and (9) contains neither $\Theta_{us/n} \otimes \Theta_{\bar{n}} \otimes \Theta_n = \gamma^\alpha_\perp \otimes \bar{\psi} \gamma^\perp_\perp \otimes \bar{\psi}$ nor $\Theta_{us/\bar{n}} \otimes \Theta_{\bar{n}} \otimes \Theta_n = \gamma^\alpha_\perp \otimes \psi \otimes \psi \gamma^\perp_\perp$. These possibilities are excluded by the projection relation $\Theta_{us/n'} = \bar{\psi} \gamma^\perp \Theta_{us/n'} \psi/4$ and the helicity flip equation.

Now consider spin and chirality in SCET$_II$. The allowed $O^{(1T)}$ spin structures must respect the handedness inherited from the SCET$_I$ fields in the perturbative matching of
\[ T[Q^{(2)}L^{(1)}_{\xi q}] \] For \( n' = \bar{n} \) in \( Q^{(2)} \), taking either one of the \( \bar{n} \)-collinear anti-quark fields soft yields an annihilation operator. For \( n' = n \), however, the field \( \bar{q}_a \) in the third bilinear was pair produced and does not contribute to the annihilation amplitude when made soft by \( L^{(1)}_{\xi q} \). So given the SCET\(_1\) spin structures Eqs. (8) and (9) corresponding to chiralities described in the text, we need to consider \( O^{(1T)} \) chiralities \((LH)(LL)(LL), (LH)(RR)(RR), (RH)(LL)(LR), \) and \((RH)(LR)(RR)\) with bilinears in the order shown in Eq. (10), i.e. soft – \( \bar{n} - n \). With the first bilinear purely soft, a complete basis of Dirac structures for the individual bilinears is

\[ \Gamma_s = \{ \bar{\psi} \gamma_\mu \}, \quad \Gamma_{\bar{n}} = \{ \bar{\psi} \gamma^\mu \}, \quad \Gamma_n = \{ \bar{\psi} \gamma_\mu \}. \] (10)

A Lorentz scalar \( O^{(1T)} \) has an odd number of \( \gamma_\perp \)'s since one must be contracted into the \( n–\bar{n} \)-collinear field strength \( B_\perp^\beta \). For chiralities \((LH)(LL)(LL) \) and \((LH)(RR)(RR)\) the allowed Dirac structure is

\[ (\Gamma_s \otimes \Gamma_{\bar{n}} \otimes \Gamma_n)B^\beta_{n',\perp} = (\gamma_\beta \otimes \bar{\psi} \otimes \bar{\psi})B^\beta_{n',\perp} \] (11)

with \( n' = n \) or \( \bar{n} \), but the corresponding operators \( O^{(1T)} \) have \( \bar{q}_a \gamma_\mu b_a \) and do not contribute for \( B \) decays. Since \((LH)(LL)(LL)\) is the only \( O^{(1T)} \) chirality corresponding to the \((V-A)(V-A)\) operators \( O_{1-4,9,10} \), this proves that only \( O_{5-8} \) can contribute to Eq. (12). Furthermore since all \((LH)\) terms are ruled out, the soft quark can only be \( q' \), and not a \( d \)-quark.

This leaves the \((RH)(LL)(LR)\) and \((RH)(LR)(RR)\) structures from \( O_{5-8} \) with soft quark flavor \( q' \), for which we have the additional spin structures,

\[ n' = n : \quad \Gamma_s \otimes \Gamma_{\bar{n}} \otimes \Gamma_n B^\beta_{n',\perp} = \{ \bar{\psi} \otimes \bar{\psi} \otimes \bar{\psi} B_{n',\perp}, \bar{\psi} \otimes \bar{\psi} \gamma_\beta \otimes \bar{\psi} B^\beta_{n',\perp} \}, \]
\[ n' = \bar{n} : \quad \Gamma_s \otimes \Gamma_{\bar{n}} \otimes \Gamma_n B_{n',\perp} = \{ \bar{\psi} \otimes \bar{\psi} B_{n',\perp} \otimes \bar{\psi}, \bar{\psi} \otimes \bar{\psi} \gamma\beta \otimes \bar{\psi} B^\beta_{n',\perp} \}, \] (12)

plus those with \( \bar{\psi} \leftrightarrow \bar{\psi} \) in \( \Gamma_s \). While these eight are all allowed by chirality and Lorentz invariance, six can be ruled out by considering the spin and factorization properties of our time-ordered product. The matching from SCET\(_1\) to SCET\(_H\) does not affect the spin and color structure of the \( \bar{n}' \)-collinear bilinear at this order in the power expansion, since once a jet direction is chosen the collinear fields in the opposite direction are decoupled. Here \( \bar{n}' \) is the opposite of \( n' \). From Eqs. (8) and (9) the allowed \( \Theta_{n'} \) structures have no \( \gamma_\perp \)'s, and therefore the second structure on each line of Eq. (12) does not appear at any order in the perturbative matching. Also, the allowed structures Eqs. (8) and (9) are invariant under \( \Theta_{us} \rightarrow \Theta_{us} \bar{\psi}' /2 \) and only power-suppressed interactions couple the \( b \)-quark to the \( n' \) sector. Therefore, \( \Gamma_s \) should not vanish under \( \Gamma_s \rightarrow \Gamma_s \bar{\psi}' /2 \), and the operators with \( \bar{\psi} \leftrightarrow \bar{\psi} \) mentioned below Eq. (12) are ruled out. In perturbation theory this just corresponds to the appearance of an \( \bar{\psi}' \) from the \( n' \)-collinear propagator next to the \( b \)-quark. This leaves only the operators with a \( B_\perp \) in Eq. (12).

Finally consider color. In SCET\(_1\) the operators \( Q^{(2)} \) are color singlets, but each bilinear
on its own could be singlet or octet. A complete set of color structures includes

$$\Theta_{us/n'} \otimes \Theta_{\bar{n}} \otimes \Theta_n = \left\{ T^a \otimes 1 \otimes T^a, \ T^a \otimes T^a \otimes 1, \ 1 \otimes 1 \otimes 1, \right.$$

$$\left. 1 \otimes T^a \otimes T^a, \ T^a \otimes T^b \otimes T^c f^{abc}, \ T^a \otimes T^b \otimes T^c d^{abc} \right\}.$$  \hspace{1cm} (13)

Once again we can reduce this set using the factorization properties of SCET. As argued for spin, an SCET operator with color structure $\Theta_{\bar{n}'}$ matches onto a SCET operator with the same structure $\Gamma_{\bar{n}'}$ in its $\bar{n}'$ bilinear. So $\Theta_{\bar{n}'}$ cannot be a color octet, and the allowed structures are

$$\Theta_{us/n} \otimes \Theta_{\bar{n}} \otimes \Theta_n = \{ 1 \otimes 1 \otimes 1, \ T^a \otimes 1 \otimes T^a \}$$

$$\Theta_{us/n} \otimes \Theta_{\bar{n}} \otimes \Theta_n = \{ 1 \otimes 1 \otimes 1, \ T^a \otimes T^a \otimes 1 \}.$$  \hspace{1cm} (14)

In SCET each of the three bilinears interpolates for a color singlet meson and therefore each bilinear must separately be a color singlet, $\Gamma_s \otimes \Gamma_{\bar{n}} \otimes \Gamma_n = 1 \otimes 1 \otimes 1$.

**Matching onto SCET and SCET**

We now present the matching from $H_W$ in Eq. (3) onto the SCET operators $Q^{(2)}$ and then the matching of the SCET time-ordered product $T[Q^{(2)}L_{\xi q}^{(1)}]$ onto SCET operators $O_{id}^{(1T)}$. The hadronic matrix elements of $O_{id}^{(1T)}$ will give the factorization formula for $A_{\text{hard-col}}^{(1am)}$. From the arguments presented above, the complete basis of SCET operators $Q^{(2)}$ is

$$Q_{1d}^{(2)} = \frac{2}{m_b^3} \sum_{q,q'} \left[ \bar{q}_{n,\omega_5} P_L \gamma_\perp \gamma^a T^a b_e \right] \left[ \bar{d}_{\bar{n},\omega_2} P_L q_{\bar{n},\omega_3} \right] P_R q'_{n,\omega_4},$$

$$Q_{2d}^{(2)} = \frac{2}{m_b^3} \sum_{q,q'} \left[ \bar{q}_{n,\omega_5} P_L \gamma_\perp \gamma^a T^a b_e \right] \left[ \bar{d}_{\bar{n},\omega_2} P_L q_{\bar{n},\omega_3} \right] P_R q'_{n,\omega_4},$$

$$Q_{3d,4d}^{(2)} = \frac{3 e_q}{2}.$$  \hspace{1cm} (15)

with sums over $q, q' = u, d, s$, plus analogous operators $Q^{(2)}_{5d-8d}$ which have color structure $1 \otimes 1 \otimes 1$. The electroweak penguin operators $O_{7,8}$ induce the two operators $Q_{3d,4d}^{(2)}$, which have the same spin and flavor structures as $Q_{1d,2d}^{(2)}$, but with a factor of the quark electric charge $e_q$ included under the summation. Combining the pieces in SCET, a complete basis for the $O(\eta^7)$ operators with one $igB_\perp$ that contribute to $B$ decays is

$$O_{1d}^{(1T)} = \frac{1}{m_b^3} \sum_{q,q'} \left[ \bar{q}_{s,-k^+} P_L \gamma_\perp \gamma^a T^a b_e \right] \left[ \bar{d}_{\bar{n},\omega_2} P_L q_{\bar{n},\omega_3} \right] \left[ \bar{q}_{n,\omega_4} P_R q'_{n,\omega_4} \right],$$

$$O_{2d}^{(1T)} = \frac{1}{m_b^3} \sum_{q,q'} \left[ \bar{q}_{s,-k^-} P_L \gamma_\perp \gamma^a T^a b_e \right] \left[ \bar{d}_{\bar{n},\omega_2} P_L q_{\bar{n},\omega_3} \right] \left[ \bar{q}_{n,\omega_4} P_R q'_{n,\omega_4} \right],$$

$$O_{3-4d}^{(1T)} = O_{1-2d}^{(1T)} \frac{3 e_q}{2}.$$  \hspace{1cm} (16)

Here $\bar{q}_{s,-k^+} = (\bar{q}'_{s}\delta S_n) \delta(k^+ + n \cdot \mathcal{P}^\dagger)$ and $\bar{q}_{s,-k^-} = (\bar{q}'_{s}\delta S_{\bar{n}}) \delta(k^- + \bar{n} \cdot \mathcal{P}^\dagger)$ and the direction for the soft Wilson lines $S_n$ and $S_{\bar{n}}$ are determined by the matching from SCET. Just like the local
annihilation operators, we see that the \(O_i^{1T}\)'s can not create transversely polarized vector mesons. The basis for \(\Delta S = 1\) decays, \(O_i^{1T}\) switches \(\bar{d}_n \rightarrow \bar{s}_n\).

Next, we carry out the perturbative matching onto the bases in Eqs. (13) and (16), and derive the factorization theorem. The SCET\(_1\) weak Hamiltonian with Wilson coefficients \(a_i^{hc}\) for the operators \(Q_{id}^{(2)}\) is

\[
H_W = \frac{4G_F}{\sqrt{2}} (\lambda_{u}^{(d)} + \lambda_{c}^{(d)}) \sum_{n, \bar{n}} \int [d\omega_1 d\omega_2 d\omega_3 d\omega_4 d\omega_5] \sum_{i=1-8} a_i^{hc}(\omega_j) Q_{id}^{(2)}(\omega_j). \tag{17}
\]

Since only the penguin operators \(O_{5-8}\) contribute, we pulled out the common CKM factor with \(\lambda_{u}^{(d)} = V_{ub} V_{ud}^\star\) and \(\lambda_{c}^{(d)} = V_{cb} V_{cd}^\star\). The analogous result for \(\Delta S = 1\) has the same \(a_i^{hc}\) coefficients. To match onto the \(a_i^{hc}\) at tree level we first do a spin Fierz on the full theory \(O_{5-8}\) operators to obtain spin structures \(P_L \otimes P_R\), and then compute the graphs in Fig. 2. Only graphs c) and d) are nonzero and we find [at \(\mu = m_b\)]

\[
a_1^{hc}(x, y, \bar{y}) = \frac{\pi \alpha_s(m_b)}{N_C} \left\{ \frac{2C_F C_5 + C_6}{y[x(1-y) - 1]} + \frac{(2C_F - C_A)C_5 + C_6}{(1-x)[y(1-y)]} \right\},
\]

\[
a_2^{hc}(x, \bar{x}, y) = \frac{\pi \alpha_s(m_b)}{N_C} \left\{ -\frac{(2C_F - C_A)C_5 + C_6}{\bar{x}[1-x)(1-y) - 1]} - \frac{2C_F C_5 + C_6}{\bar{x}y(1-x)} \right\}. \tag{18}
\]

The coefficients \(a_{3,4}^{hc}\) are identical to \(a_{1,2}^{hc}\) respectively with the replacements \(C_{5,6} \rightarrow C_{7,8}\). \(a_{5-8}^{hc}\) also begin at \(O(\alpha_s(m_b))\) but give \(\alpha_s(\mu_i)\)-suppressed contributions when matched onto SCET\(_{II}\), so we do not list their values. These coefficients are “polluted” in that one-loop \(O(\alpha_s(m_b)^2)\) contributions proportional to \(C_{1,2}\) could compete numerically with the results in Eq. (18). Here \(x, \bar{x}, y, \) and \(\bar{y}\) are defined in Fig. 2, namely \(y = \omega_1/m_b, \bar{y} = -\omega_4/m_b, \) \(x = \omega_2/m_b, \bar{x} = -\omega_3/m_b\). For \(n' = n\) as in \(a_{1,3}\), we have \(\bar{x} = 1 - x\), but \(\bar{y} \neq 1 - y\) since the momentum is shared between three \(n\)-collinear partons. Likewise, for \(n' = \bar{n}\) as in \(a_{2,4}^{hc}\) we have \(\bar{y} = 1 - y\) but \(\bar{x} \neq 1 - x\).

Having constructed the operators \(Q^{(2)}\) and determined their Wilson coefficients, it is straightforward to match the time-ordered products \(T[Q^{(2)} L^{(1)}_{\xi q}]\) onto the SCET\(_{II}\) operators \(O_i^{1T}\). For odd indices \(i\) and even indices \(i'\) we find that integrating out the hard-collinear quark propagator, shown as the dashed line inside the gray region in Fig. 1, gives

\[
i \int d^4 x T[Q^{(2)}_{id}(\omega_j)](0)L^{(1)}_{\xi q}(x) = \frac{-1}{N_c} \int dk^+ O^{(1T)}_{id}(k^+, \omega_j),
\]

\[
i \int d^4 x T[Q^{(2)}_{i'd}(\omega_j)](0)L^{(1)}_{\xi q}(x) = \frac{-1}{N_c} \int dk^- O^{(1T)}_{i'd}(k^-, \omega_j). \tag{19}
\]

At \(O(\alpha_s^2)\) in perturbation theory this matching would include non-trivial jet functions. For example, in the first line a \(\int d\omega_{1,4} J(k^+, \omega_{1,4}, \omega_{1,4})\) with \(\omega_{1,4}\) taking the place of \(\omega_{1,4}\) in \(O^{(1T)}_{id}\). However at this order additional time-ordered products and non-perturbative functions become relevant so we stick to \(O(\alpha_s)\) in our analysis. Together Eqs. (18, 19) complete the
three-body distributions. Now take the matrix element of $O^{(1T)}_{id}$ using

$$
\langle \pi^+_n(p)|\bar{u}_{n,\omega_1}\not{p}P_Lp_{n,\omega_4}|0\rangle = \frac{\pm i f_{p}}{2}\delta_{n_1}\delta(n\cdot p-\omega_1+\omega_4)\phi_{p}(y),
$$

$$
\langle \rho^+_{n_1}(p,\varepsilon)|\bar{u}_{n,\omega_1}\not{p}P_Lp_{n,\omega_4}|0\rangle = \frac{i f_{V}m_{V}n\cdot \varepsilon}{2 n\cdot p}\delta_{n_1}\delta(n\cdot p-\omega_1+\omega_4)\phi_{\rho}(y),
$$

and the three-body distributions

$$
\langle \pi^+_n(p)|\bar{u}_{n,\omega_1}\not{p}(ig\not{B}_{\pm})_{n,\omega_5}P_Rp_{n,\omega_4}|0\rangle = \frac{f_{3p}}{\omega_5}\delta_{n_1}\delta(n\cdot p-\omega_1-\omega_5+\omega_4)\phi_{3p}(y,\bar{y}),
$$

$$
\langle \rho^+_{n_1}(p,\varepsilon)|\bar{u}_{n,\omega_1}\not{p}(ig\not{B}_{\pm})_{n,\omega_5}P_Rp_{n,\omega_4}|0\rangle = \frac{f_{3V}m_{V}n\cdot \varepsilon}{\omega_5 n\cdot p}\delta_{n_1}\delta(n\cdot p-\omega_1-\omega_5+\omega_4)\phi_{3V}(y,\bar{y}).
$$

Our convention for the vector meson matrix element has been chosen to simplify the final result for the amplitude and is related to that of [26] by $f_{3V} = m_{V}f_{V}^{T}$ and $\phi_{3V} = -T/2$. Permutations in the flavors give the definitions for other meson channels, and we use the phase convention in [27]. The soft matrix element is

$$
\langle 0|\bar{q}^{(f)}_{s_{n},-n',k}P_L\not{p}'S_{n'}c_{v}|B\rangle = i\frac{f_{B}m_{B}}{2}\phi_{B}^{+}(n',k).
$$

Combining these pieces the factorization theorem with tree-level jet functions is

$$
A^{(1ann)}_{hard-collin} = \frac{-G_{F}f_{B}m_{B}}{\sqrt{2}m_{b}N_{c}}(\lambda_{u}(d)+\lambda_{c}(d))\int_{0}^{\infty}dk\frac{\phi_{B}^{+}(k)}{k}
$$

$$
\times \left\{ f_{3M_{1}}f_{M_{2}}\int_{0}^{1}dx\int_{0}^{1}dy\int_{0}^{1-y}d\bar{y}\frac{H_{hc1}^{M_{1}M_{2}}(x,y,\bar{y})}{1-y-\bar{y}}\phi_{3M_{1}}(y,\bar{y})\phi_{M_{2}}(x) + \eta_{M_{1}}f_{M_{1}}f_{3M_{2}}\int_{0}^{1}dy\int_{0}^{1}dx\int_{0}^{1-x}d\bar{x}\frac{H_{hc2}^{M_{1}M_{2}}(x,\bar{x},y)}{1-x-\bar{x}}\phi_{M_{1}}(y)\phi_{3M_{2}}(x,\bar{x}) \right\},
$$

where $\eta_{M} = -1$ or $+1$ for a pseudoscalar or vector meson, respectively. The hard coefficients $H_{hc1}^{M_{1}M_{2}}$ and $H_{hc2}^{M_{1}M_{2}}$ for different $B \rightarrow M_{1}M_{2}$ channels are listed in Table II in terms of coefficients in the SCET weak Hamiltonian. The amplitude contains the three-body distribution function as promised. The convolutions in Eq. (23) are real, and assuming the standard endpoint behavior for the distribution functions they converge without the rapidity factorization of [18].

We conclude by comparing our result parametrically and numerically to $A^{(1ann)}_{hard}$ and $A^{(2ann)}_{hard}$ as defined in Ref. [17]. For this comparison it is useful to define moment parameters

$$
\beta_{hc1}^{M_{1}M_{2}}, \beta_{hc3}^{M_{1}M_{2}} = \int dx dy dy \frac{a_{hc1}^{M_{1}M_{2}}(x,y,\bar{y})}{1-y-\bar{y}}\phi_{3M_{1}}(y,\bar{y})\phi_{M_{2}}(x),
$$

$$
\beta_{hc2}^{M_{1}M_{2}}, \beta_{hc4}^{M_{1}M_{2}} = \int dy dx dx \frac{a_{hc4}^{M_{1}M_{2}}(x,\bar{x},y)}{1-x-\bar{x}}\phi_{M_{1}}(y)\phi_{M_{2}}(x,\bar{x}), \quad \beta_{B} = \frac{1}{3} \int dk \frac{k}{\phi_{B}^{+}(k)},
$$

10
where $\beta_B = \lambda_B^{-1}/3$ has mass dimension $-1$. First we compare the leading-power annihilation amplitudes in $B \to \pi^+ K^-$. Dropping terms proportional to the tiny Wilson coefficients $C_{7-8}$, we have

$$R_1(\pi^+ K^-) \equiv \frac{A_{(1)\text{ann}}^{(\text{hard-col})}(\pi^+ K^-)}{A_{(1)\text{ann}}^{(\text{hard})}(\pi^+ K^-)} = \frac{G_F f_B m_B}{\sqrt{2} m_N c} (\lambda_u^{(s)} + \lambda_c^{(s)}) 3 \beta_B \left[ -f_{3K} f_{K} \beta_{hc1}^{K} + f_{3K} f_{\pi K} \beta_{hc2}^{K} \right].$$  \hspace{1cm} (25)

Parametrically, the moments in $R_1$ have $\beta_{4u} \sim \beta_{hc1} \sim O(\alpha_s(m_b))$, and the power counting of the prefactor is $f_{3K}B/f_K \sim \alpha$. Also there is no suppression from the hierarchy in the $C_i$’s since $\beta_{4u}$ involves $C_3$, and $C_3 \approx C_5 \approx C_6$. Thus, we have shown that for consistency in the $\alpha_s$ and $1/m_b$ expansion, the contributions $A_{(1)\text{ann}}^{(\text{hard-col})}$ need to be included with the local contributions $A_{(1)\text{ann}}^{(\text{hard})}$ in the leading annihilation amplitude. Similarly we can compare the new hard-collinear annihilation amplitude to the chirally enhanced annihilation contribution in $B^0 \to \pi^+ K^-$. Isolating the terms proportional to the large coefficients $C_5$ and $C_6$ we have

$$R_2(\pi^+ K^-) \equiv \frac{A_{(2)\text{ann}}^{(\text{hard-col})}(\pi^+ K^-)}{A_{(2)\text{ann}}^{(\text{hard})}(\pi^+ K^-)} = \frac{G_F f_B m_B}{\sqrt{2} m_N c} (\lambda_u^{(s)} + \lambda_c^{(s)}) 3 \beta_B \left[ -f_{3K} f_{K} \beta_{hc1}^{K} + f_{3K} f_{\pi K} \beta_{hc2}^{K} \right].$$  \hspace{1cm} (26)
Parametrically $\beta_\chi \sim \beta_{hc} \sim \alpha_s(m_b)$, and $R(\pi^+K^-) \sim m_B\beta_B f_3 \pi / f_3 \mu_\pi \sim m_b / \mu_\pi \sim m_b / \Lambda$ as expected.

We conclude with a brief numerical analysis of the ratios $R_1$ and $R_2$. The $C_i$'s are quoted below Eq. (24), and we use $\alpha_s(m_b) = 0.22$, $f_K = 0.16$ GeV, and $f_\pi = 0.13$ GeV. $f_B = 0.22$ GeV is taken from a recent lattice determination [28], the three-body decay constants $f_{3K} \simeq 4.5 \times 10^{-3}$ GeV$^2$ and $f_{3\pi} \simeq 4.5 \times 10^{-3}$ GeV$^2$ come from QCD sum rules [29] and $\beta_B \simeq 1 / (4$ GeV$)$ was determined in a fit to nonleptonic data [30]. To model the nonperturbative meson distributions we truncate the conformal partial wave expansions [31] as

$$\phi^M(x) = 6x(1-x)[1 + a^M_1(6x - 3) + 6a^M_2(1 - 5x + 5x^2)],$$

$$\phi^{3M}(x, \bar{x}) = 360x\bar{x}(1 - x - \bar{x})^2[1 + \frac{w_{3M}}{2}(7(1 - x - \bar{x}) - 3)].$$

Eq. (27) has convergent convolution integrals for these distribution functions. To estimate the moments $\beta$ and the ratios $R$ we vary the coefficients in Eq. (27) in a conservative range inferred from recent lattice results [32] for the $a^M_i$'s and QCD sum rules [29] for the $w_{3M}$'s. Specifically we take $a^+_{1K} = 0$, $a^+_{1\pi} = 0.05 \pm 0.02$, $a^+_{2\pi} = 0.2 \pm 0.2$, and $w_{3\pi, K} = -1 \pm 1$. A Gaussian scan of the model parameters gives

$$\beta^K_{hc1} = -1.4 \pm 0.4, \quad \beta^K_{hc2} = 0.3 \pm 0.1, \quad R_1(\pi^+K^-) = 0.3 \text{ to } 1.2,$$

$$\beta^K_{hc1} = -1.4 \pm 0.5, \quad \beta^K_{hc2} = 0.1 \pm 0.1, \quad R_2(\pi^+K^-) = -0.1 \text{ to } 0.1.$$  

The denominators of Eq. (25) and (26) can vanish, giving large departures from Gaussian statistics. So for $R_1$ and $R_2$ we quote the range that contains an equivalent number of points as one standard deviation for a Gaussian distribution. Eq. (28) demonstrates that numerically the three-parton contributions to $A^{(1ann)}$ could be of the same size or larger than the local piece $A^{(1ann)}_{hard}$. Numerically, $m_B\beta_B f_3 \pi / (f_3 \mu_\pi) \sim 0.2$ causing some suppression in $R_2(\pi^+K^-)$. It would be interesting to examine the size of these three-parton contributions in the $k_T$-approach of Ref. 4.

In this paper we computed the final missing term of the leading order annihilation amplitude in $B \rightarrow M_1 M_2$ decays. These terms involve a three-parton distribution and need to be included for a complete analysis of annihilation.

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[1] C. W. Bauer, S. Fleming, and M. E. Luke, Phys. Rev. D63, 014006 (2001), hep-ph/0005275
[2] C. W. Bauer, S. Fleming, D. Pirjol, and I. W. Stewart, Phys. Rev. D63, 114020 (2001), hep-ph/0011136.
[3] C. W. Bauer and I. W. Stewart, Phys. Lett. B516, 134 (2001), hep-ph/0107001.
[4] C. W. Bauer, D. Pirjol, and I. W. Stewart, Phys. Rev. D65, 054022 (2002), hep-ph/0109045.
[5] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Nucl. Phys. B591, 313 (2000), hep-ph/0006124.
[6] Y.-Y. Keum, H.-n. Li, and A. I. Sanda, Phys. Lett. B504, 6 (2001), hep-ph/0004004.
[7] J.-g. Chay and C. Kim, Phys. Rev. D68, 071502 (2003), hep-ph/0301055.
[8] C. W. Bauer, D. Pirjol, I. Z. Rothstein, and I. W. Stewart, Phys. Rev. D70, 054015 (2004), hep-ph/0401188.
[9] J. Charles et al., Eur. Phys. J. C41, 1 (2005), URL http://ckmfitter.in2p3.fr/.
[10] M. Ciuchini, E. Franco, G. Martinelli, and L. Silvestrini, Nucl. Phys. B501, 271 (1997), hep-ph/9703353.
[11] P. Colangelo, G. Nardulli, N. Paver, and Riazuddin, Z. Phys. C45, 575 (1990).
[12] A. R. Williamson and J. Zupan (2006), hep-ph/0601214.
[13] C.-D. Lu, K. Ukai, and M.-Z. Yang, Phys. Rev. D63, 074009 (2001), hep-ph/0004213.
[14] M. Beneke, G. Buchalla, M. Neubert, and C. T. Sachrajda, Nucl. Phys. B606, 245 (2001), hep-ph/0104110.
[15] A. L. Kagan, Phys. Lett. B601, 151 (2004), hep-ph/0405134.
[16] A. Khodjamirian, T. Mannel, M. Melcher, and B. Melic, Phys. Rev. D72, 094012 (2005).
[17] C. M. Arnesen, Z. Ligeti, I. Z. Rothstein, and I. W. Stewart (2006), hep-ph/0607001.
[18] A. V. Manohar and I. W. Stewart (2006), hep-ph/0605001.
[19] B. Blok, M. Gronau, and J. L. Rosner, Phys. Rev. Lett. 78, 3999 (1997), hep-ph/9701396.
[20] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996).
[21] C. W. Bauer, D. Pirjol, and I. W. Stewart, Phys. Rev. D67, 071502 (2003), hep-ph/0211069.
[22] M. Beneke and T. Feldmann, Phys. Lett. B553, 267 (2003), hep-ph/0211358.
[23] R. J. Hill and M. Neubert, Nucl. Phys. B 657, 229 (2003), hep-ph/0211018.
[24] S. Mantry, D. Pirjol, and I. W. Stewart, Phys. Rev. D 68, 114009 (2003), hep-ph/0306254.
[25] I. W. Stewart (2003), hep-ph/0308185.
[26] A. Hardmeier, E. Lunghi, D. Pirjol, and D. Wyler, Nucl. Phys. B682, 150 (2004).
[27] M. Gronau, O. F. Hernandez, D. London, and J. L. Rosner, Phys. Rev. D50, 4529 (1994).
[28] A. Gray et al., Phys. Rev. Lett. 95, 212001 (2005), hep-lat/0507015.
[29] P. Ball, V. M. Braun, and A. Lenz, JHEP 05, 004 (2006), hep-ph/0603063.
[30] C. W. Bauer, I. Z. Rothstein, and I. W. Stewart (2005), hep-ph/0510241.
[31] A. R. Zhitnitsky, I. R. Zhitnitsky, and V. L. Chernyak, Sov. J. Nucl. Phys. 41, 284 (1985).
[32] V. M. Braun et al. (2006), hep-lat/0606012.