Observation of Cosmic Acceleration and Determining the Fate of the Universe

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Current observations of Type Ia supernovae provide evidence for cosmic acceleration out to a redshift of \( z \lesssim 1 \), leading to the possibility that the universe is entering an inflationary epoch. However, inflation can take place only if vacuum-energy (or other sufficiently slowly redshifting source of energy density) dominates the energy density of a region of physical radius \( 1/H \). We argue that for the best-fit values of \( \Omega_\Lambda \) and \( \Omega_m \) inferred from the supernovae and other data, one must confirm cosmic acceleration out to at least \( z \approx 1.8 \) to infer that our portion of the universe is in principle capable of inflating; but no measurement will be able to confirm or falsify that inference until \( \Omega_\Lambda \) rises to 0.96.

Recent direct measurements of the cosmic expansion using supernova at redshifts from \( z = 0 - 1.2 \), suggest that the expansion of our portion of the universe is accelerating. This supports earlier indirect evidence leading to the same conclusion. This is unlike the deceleration expected in a universe dominated by the energy density of ordinary or dark matter. It implies that the expansion is driven by the energy density of the vacuum. This expansion has variously been ascribed to a cosmological constant in the Einstein equations, to the zero-point fluctuations of quantum fields, so-called vacuum energy, to the potential energy density of dynamical fields, quintessence, and to a network of cosmic strings.

In exploring the implications of these explanations, it has widely been assumed that the energy density driving the accelerating expansion is homogeneous. If so, then unless the field dynamics are chosen to avoid it, the universe has entered on an extended period of rapid growth – a new epoch of inflation – in which objects currently within our observable universe will soon begin to leave it. If correct, this result could have dramatic implications for our understanding of fundamental processes underlying the Big Bang. However, one must be careful to separate the observational results from the assumptions which are built into the standard interpretations of them.

The observational situation can be briefly summarized: measurement of the light curves of several tens of Type Ia supernovae have allowed accurate measurements of the scale factor as a function of redshift \((a(z))\) out to \( z \lesssim 1 \). When the supernova data in conjunction with CMBR anisotropy data are fit to a family of cosmological models parametrized by a homogeneous vacuum energy density and a homogeneous matter energy density (characterized by the ratios \( \Omega_\Lambda \) and \( \Omega_m \) of these energy densities to the critical energy density \( \rho_c = 3H_0^2/8\pi G \), all at the present epoch), then the measured functional form of \( a \) is most consistent with \( \Omega_\Lambda \approx 0.8 \) and \( \Omega_m \approx 0.2 \). Notwithstanding any debates about systematic uncertainties, for the purposes of this paper let us accept this data as reported. Still, the data do not imply that the observable universe out to the last scattering surface and beyond is vacuum-energy dominated. We could be living in a bubble of high vacuum energy density surrounded by much lower, even zero, vacuum energy density. For example, the vacuum energy density could be due to a scalar field which locally deviates from the minimum of its potential.

The question now arises – will the local vacuum-dominated region inflate or will it not? The answer to that question depends on what one means by “inflate”. The effective scale factor of our local corner of the universe is apparently experiencing a period of accelerated growth. However, the essence of inflation is not local acceleration, but acceleration over a large enough region to affect the causal relationships between (comoving) observers. In particular, if the acceleration is taking place over a sufficiently large region, then unless the acceleration is halted, comoving observers from whom one was previously able to receive signals, will disappear from view. To be precise, much as if we were watching them fall through the horizon of a black hole, we will see them freeze into apparent immobility, the signals from them declining indefinitely in brightness and energy.

The question we address in the balance of this letter is how far out must one look to infer that the patch of the universe in which we live is inflating?

In any Friedman-Robertson-Walker (FRW) cosmology, there exists for each comoving observer a sphere centered on that observer, on which the velocity of comoving objects is the speed of light. When sources inside that sphere emit radially inward-directed light-rays, the photons approach the observer; when sources outside that sphere emit radially inward-directed light-rays, the physical distance between the photons and the observer increases. This sphere is the minimal anti-trapped surface (MAS). For a homogeneous universe, the physical radius of the MAS is \( 1/H \), since the physical velocity \( v \) of a comoving observer at a physical distance \( x \) is \( v = cHx \).

In a matter or radiation dominated epoch, when the dominant energy density in the universe scales with the scale factor \( a \) as \( \rho_{\text{dom}} \propto a^{-n} \) with \( n > 2 \), the comoving...
radius of the MAS grows – the inward-directed photons outside the MAS, which were making negative progress in their journey to the observer, eventually find themselves inside the MAS, and reach the observer. New objects are therefore constantly coming into view. If, on the other hand, the dominant energy density in the universe scales as the scale factor to a power greater than \(-2\) then the comoving radius of the MAS is contracting. For an equation of state with \(\lim_{t \to \infty} n < 2\), the MAS contracts to zero comoving radius in finite conformal time, \(\eta_{\text{max}}\), corresponding to time-like infinity. The interior of the past null cone of the observer at \(\eta_{\text{max}}\) is the entire portion of the history of the universe that the observer can see, what we shall call hereafter the observer’s visible history of the universe (VHU). Since the null cone is contracting, comoving objects cross out of the null cone, and thus disappear from view (see Fig. 1). More precisely, the history of the objects after the time when they cross out of the null cone is unobservable. Again, much as when watching something fall through the horizon of a black hole, the observer continues to receive photons from the disappearing source until \(\eta_{\text{max}}\), but the source appears progressively redder and dimmer, and its time evolution freezes at the moment of horizon-crossing.

The comoving contraction of our MAS, and particularly the motion of comoving sources out of our VHU is the essence of inflation. These sources can never again be seen, unless the equation of state changes so that \(n > 2\) and the MAS grows once more, i.e. inflation ends. (Then, the sources never actually crossed out of the VHU, merely out of the apparent VHU – the VHU one would have inferred without the change in equation of state.) By examining the Raychaudhuri equation governing the evolution of the divergence of geodesics, Vachaspati and Trodden recently showed that at any time \(\eta_e\) a contracting anti-trapped surface cannot exist in the universe unless a region of radius greater than \(1/H(\eta_e)\) is vacuum dominated and homogeneous. (The result depends on the validity of the weak energy and certain other conditions. In the present application, the conditions seem reasonable.) Thus no inflation will have occurred unless a region of size \(1/H(\eta)\) remains vacuum dominated long enough for the MAS to begin collapsing. We apply this bound to determine our ability to infer the current and future state of our patch of the universe.

The geometry of a homogeneous and isotropic universe is described by the FRW metric:

\[
\begin{equation}
 ds^2 = a^2 \left[ d\eta^2 - \frac{dr^2}{1 - kr^2} - r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right]
\end{equation}
\]

(1)

\(a(\eta)\) encodes the changing relationship between coordinate (comoving) distances and physical distance. (The conformal time \(\eta\) is related to the proper time \(t\) measured by comoving observers by \(a(\eta)d\eta = dt\).) The evolution of \(a\) is determined by the mean energy density of the universe \(\rho\) and by the curvature radius of the geometry, characterized by \(k\), via the Friedmann equation:

\[
\left( \frac{\dot{a}}{a^2} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2},
\]

(2)

where \(\dot{a} \equiv (da/d\eta)\), and \((\dot{a}/a^2) \equiv H\) is the Hubble parameter; currently \(H \approx 72\) km/s/Mpc.

FIG. 1. Spacetime diagram showing the MAS position \((r = 1/aH)\) as a function of conformal time \(\eta\) where the universe eventually does (solid line) and does not (dashed line) undergo inflation. On our present lightcone we observe supernovae (SN) and the cosmic microwave background (CMB). If the universe inflates, the MAS curve turns around at \(\eta_c\) and the interior of the past light-cone of an observer at future infinity \(\eta_{\text{max}}\) only covers a portion of the spacetime (triangular area under the dashed line marked VHU). In this case, the first object to leave causal contact with us will do so at \(\eta_d\) through the point P.

The energy density \(\rho\) can receive many distinct contributions, but what is crucial to the nature of the solution to (3) is the fraction of the critical density \(\rho_c = 3H_0^2/8\pi G\) comprised by each species, and how each of these contributions scales with \(a\). The two most important possible contributions today, non-relativistic matter and vacuum energy, scale respectively as \(a^{-3}\) and \(a^0\).

Since the metric (3) describes a homogeneous and isotropic space, we can choose, without loss of generality, to locate ourselves at the origin of coordinates, \(r = 0\). Consider then a source located at a comoving distance \(r\) from us. At conformal time \(\eta_e\) the source emits a photon directed toward us; the photon is received, and the source therefore observed, at \(r = 0\) at our present conformal time \(\eta_0\). If the source is a standard candle of known luminosity \(\mathcal{L}\), then by measuring the observed flux \(\mathcal{F}\) of

\[
\begin{equation}
 \mathcal{F} \sim \frac{\mathcal{L}}{r^2},
\end{equation}
\]

(4)

If the source is a variable astrophysical object, then

\[
\begin{equation}
 \mathcal{F} \sim \frac{\mathcal{L}(\eta)}{r^2},
\end{equation}
\]

(5)
the source, one can determine its luminosity distance:

\[ d_L = \left( \frac{L}{4\pi F} \right)^{1/2} \]  

(3)

If one can measure the redshift \( z \) of the source, then one can constrain \( \Omega_M \) and \( \Omega_\Lambda \) using the relationship

\[ d_L(z) = cH_0^{-1}(1+z)|\Omega_k|^{-1/2}\sinh[|\Omega_k|^{1/2} \int_0^z dz'[1+(1+\Omega_M z') - z'(2+z')\Omega_\Lambda]^{-1/2}] \]  

(4)

Here \( \Omega_k \equiv 1 - \Omega_M - \Omega_\Lambda \) and \( \sinh(x) \) is \( \sin(x) \) if \( k > 0 \) and \( \sin(x) \) if \( k < 0 \). Repeating this for a variety of sources at different comoving distances allows one to determine \( \Omega_M \) and \( \Omega_\Lambda \). In Fig. 2, we plot the difference in apparent magnitude between a flat \((k=0)\) homogeneous vacuum-energy dominated cosmology \((\Omega_\Lambda = 0.4, 0.6, 0.8)\) and the best fit vacuum-energy free cosmology, a negatively-curved \((k<0)\) universe with \( \Omega = 0.3 \).

**FIG. 2.** The difference in apparent magnitude between a flat FRW universe \((\Omega_\Lambda + \Omega_{\text{matter}} = 1)\) for \( \Omega_\Lambda = 0.4 \) (dashed curve), 0.6 (dotted curve) and 0.8 (solid curve), and an \( \Omega_\Lambda = 0 \), negatively curved universe with \( \Omega = 0.3 \), vs. the redshift \( z \). The dashed-dotted curve shows \( z_{\text{MAS}} \) for the different values of \( \Omega_\Lambda \).

This approach has recently been applied to Type Ia supernovae by two independent groups [1,2]. Looking at tens of supernovae out to a redshift of \( z < 0.83 \), both groups find that the best-fit values of \( \Omega_\Lambda \) and \( \Omega_m \) are approximately 0.8 and 0.2 respectively.

The physical distance between the emitting supernova and the observer at time the time of emission \( \eta_c \) was

\[ d(\eta_c) = a(\eta_c) \int_{\eta_c}^{\eta_0} d\eta. \]  

(5)

where, for simplicity, we have now restricted our attention to the case when the geometry of the universe is flat \((k=0)\). We are seeing our MAS if \( d(\eta_c) = 1/H(\eta_c) \), i.e.

\[ \sqrt{1 + \alpha(1+z)^\alpha} \int_1^{1+z} \frac{dy}{\sqrt{1 + \alpha y^\alpha}} = 1, \]  

(6)

where \( \alpha \equiv (\Omega_\Lambda^{-1} - 1) \). In Fig. 3, we plot \( z_{\text{MAS}} \), the redshift of the MAS, vs. \( \Omega_\Lambda \). (In Fig. 2, the dash-dotted line shows both the redshift \( z_{\text{MAS}} \) and the associated magnitude difference from the best fit negatively-curved universe.) We find that for the best fit value \( \Omega_\Lambda = 0.8 \), we will see the MAS if we look out to \( z = 1.8 \). If observations continue to find cosmic acceleration with \( \Omega_\Lambda = 0.8 \) out to \( z = 1.8 \), then we can infer that our MAS is contracting, i.e. our patch of the universe is inflating.

Fig. 3 shows the redshifts out to which one must necessarily ascertain cosmic acceleration for any given value of vacuum energy before one can be confident that inflation is a possible fate of the universe.

**FIG. 3.** \( z_{\text{MAS}} \) versus \( \Omega_\Lambda \). The curve specifies the redshift out to which it is necessary to confirm cosmic acceleration so that it might lead to an inflationary universe.

![Graph showing the relationship between redshift and \( \Omega_\Lambda \)]
us living inside a vacuum bubble? The answer is likely model dependent. We believe the dominant bubble contributions to the CMBR fluctuations will come from the time evolution of the bubble wall and the integrated Sachs-Wolfe effect, and so depend on the wall velocity, energy-density profile, shape, etc. This is a fertile subject for future investigations.

Let us return to Fig. 1 in which, because comoving distance and conformal time are the coordinates, light rays propagate at 45°. We see that during the matter and radiation dominated phases the MAS grows, so that new objects at fixed comoving radius r are constantly entering the interior of the MAS. More importantly, for the observer at r = 0, new objects are constantly coming into view. As the universe becomes vacuum dominated, the expansion of the MAS decelerates and it eventually begins to contract. We can readily find the value of the scale factor at which that happens by setting

\[ 0 = \dot{r}_{\text{MAS}} \propto \frac{d}{d\eta} \frac{da}{da} \left[ \sqrt{\frac{8\pi G \rho_m \Omega_m}{3} a^2 + \frac{1 - \Omega_\Lambda a_0^3}{\Omega_\Lambda} a} \right]. \]  

(7)

We have assumed k = 0 and ignored the energy density in radiation. Eq. (7) is satisfied when \( a = a_c \) with

\[ \frac{a_0}{a_c} = \left( \frac{2\Omega_\Lambda}{1 - \Omega_\Lambda} \right)^{1/3}. \]  

(8)

We see that the MAS begins to contract at an epoch \( \eta_c \) when \( \Omega_\Lambda(\eta_c) = 1/3 \). For a present value of \( \Omega_\Lambda = 0.8 \) this occurs at \( a(\eta_c) = a_0/2 \).

When we look out to \( z_{\text{MAS}} \), will we see this contraction? The answer is not immediately clear, since from Fig. 1 we see that, well after \( \eta_c \), the past null cone of the observer at \( r = 0 \) intersects the observer’s MAS below the turnover. Defining \( \eta_c \) (\( a_c \)) to be the time (scale factor) when the turnover in the MAS comes into view, we see that \( \eta_c \) is given by

\[ a_c \int_{\eta_0}^{\eta_c} d\eta = H(\eta_c)^{-1}. \]  

(9)

This can easily be rewritten as

\[ \int_{a_0/a_c}^{1} \frac{dx}{\sqrt{1 + 2x^3}} = \frac{1}{\sqrt{3}} \]  

(10)

(including only matter and vacuum energy contributions to \( H \)), which can be solved numerically to give \( a_c \approx 4.369 a_c \). Combining this with equation (8), we see that

\[ \frac{a_c}{a_0} = 4.369 \left( \frac{1 - \Omega_\Lambda}{2\Omega_\Lambda} \right)^{1/3} \]  

(11)

For \( \Omega_\Lambda \geq 0.96, a_c \leq a_0 \). Thus only for \( \Omega_\Lambda \geq 0.96 \) is it possible to look out far enough to infer the contraction of our MAS. Since all observations agree that \( \Omega_\Lambda < 0.96 \), it is currently impossible to make observations of a contracting MAS.

The contraction of the MAS is in some sense the onset of inflation; indeed we could reasonably define it as such. This is because it is the projected contraction of our MAS to zero comoving radius in finite conformal time which is the cause of the eventual loss of contact with comoving observers. We see in Fig. 1, that our VHU is an inverted cone whose apex is at the intersection of our MAS with our worldline, \( r = 0 \). However, if the loss of contact with previously visible objects is what we consider the defining characteristic of inflation, then this can begin either before or after \( \eta_c \), depending on the details of the transition from expanding to contracting MAS.

The value of \( \eta_d \) is easily obtained, since \( \eta_d = \eta_{\text{mas}}/2 \). For \( \Omega_\Lambda = 0.8, \Omega_m = 0.2 \), we find \( a_0/a_c \approx 3 \). Since this is greater than \( a_0/a_c \), if we look out to \( z_{\text{MAS}} \) and see the contraction of our MAS, then objects may have already left our apparent VHU.

As we implied above, while the value of \( \eta_d \) may be of some philosophical interest, there is no measurement that one can make that guarantees that objects have left the VHU, since that takes an infinite amount of proper time for an observer to observe. If inflation is driven by the metastable or unstable potential energy of some quantum field, then the MAS could eventually start expanding once again and objects which were almost out of view, could come back into view.

In conclusion, if present observations of cosmic acceleration with \( \Omega_\Lambda \) (suggested to be \( \approx 0.7 - 0.8 \)) do not extend to a redshift of \( z_{\text{MAS}}(\Omega_\Lambda) \) (equal to 1.8 for \( \Omega_\Lambda = 0.8 \)), then either we live in a sub-critical vacuum bubble which cannot, on its own, support inflation, or fine-tuned field dynamics led to a rare period of growth in the vacuum energy immediately preceding its domination of the energy density. If, on the other hand, future observations confirm the acceleration up to and beyond a redshift of 1.8, this still will not show a contracting MAS, because the accelerated expansion is not yet sufficiently advanced for the MAS to contract — the universe is not yet inflating. Either way we will have discovered exotic fundamental physics; however, we will never be able to tell for certain if objects are moving out of our causal horizon - because that would require observations out to infinite redshift.

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