Are heavy scalars natural in minimal supergravity?

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Abstract

It has been recently claimed that very large values of a universal soft mass term $m_0$ for sfermions and higgs bosons become natural when $M_t$ is close to 175 GeV if tan $\beta \approx 10$. We show that very large values of $m_0$ require accidental cancellations not guaranteed by experimental data or theoretical assumptions, and consequently an unnatural fine-tuning of the parameters.

While supersymmetric particles continue to be elusive, it has been suggested that a very heavy universal scalar mass parameter $m_0$ should be considered ‘natural’, so that all sfermions and non-SM higgses could have multi-TeV masses above the LHC discovery reach. The claim is based on the observation that for values of the pole top mass $M_t$ around its experimental value and for moderately large tan $\beta$ the MSSM RGE equations for the soft terms with minimal supergravity (mSUGRA) boundary conditions can exhibit a peculiar behavior named ‘focus point’ in [2]: $m_{h_u}(Q)$ (the soft mass term of the higgs $h_u$ coupled to up-quarks) renormalized at a scale $Q \sim$ TeV has a negligible dependence on its initial value at $Q \sim M_{\text{GUT}}$.

It is easy to understand what a ‘focus point’ is: RGE effects trigger electro-weak symmetry breaking (EWSB) by converting a positive value of $m_{h_u}^2(M_{\text{GUT}})$ into a negative value of $m_{h_u}^2(Q)$ if the top Yukawa coupling is sufficiently large. On the contrary, $m_{h_u}^2(Q)$ remains positive if the top Yukawa coupling is too small. Consequently, $m_{h_u}^2(Q)$ vanishes for some appropriate intermediate value

$$\lambda_t \sim 4\pi/\ln^{1/2}(M_{\text{GUT}}^2/M_Z^2) \sim 1$$

of the top Yukawa coupling. Starting with mSUGRA boundary conditions (universal gaugino masses $m_{1/2}$ and universal scalar masses $m_0$ at $M_{\text{GUT}} \approx 2 \times 10^{16}$ GeV; for ease of illustration we assume a vanishing $\Lambda_{\mu}$-term), we can write

$$m_{h_u}^2(Q) = a_0 \cdot m^2 + a_{1/2} \cdot m_{1/2}^2.$$

In absence of radiative corrections $a_0 = 1$ and $a_{1/2} = 0$. For an appropriate value of $\lambda_t(M_{\text{GUT}})$ close to 1/2 the coefficient $a_0$ vanishes and a large $m_0$ can coexist with a small $M_Z$. Such a cancellation had already been noticed when the notion of fine-tuning (FT) had been introduced (see fig. 1a of [4]). If the scalar soft terms are non-universal the value of $\lambda_t$ giving the analogous cancellation is different. An experimentally acceptable top mass $M_t \approx v_\lambda \sin \beta$ can be obtained with an appropriate choice of tan $\beta$. With universal soft terms, $a_0$ can vanish for moderately large values of tan $\beta$ (a regime where sin $\beta \approx 1$ is fixed).

Unfortunately such cancellation, even if taking place, would not allow to improve the present unsatisfactory ‘naturalness status’ of mSUGRA models [4, 5], mainly determined by the radiative contribution to $M_Z^2$ proportional to the squared gluino mass $M_{\tilde{g}}^2$, a few times larger than $M_Z^2$ itself. On the contrary, the $m_0^2$ contribution to $M_Z^2$ is not problematic so that suppressing it would not help (see appendix B). However, the possibility that mSUGRA does not become less natural when $m_0 \gg M_Z$ would certainly have implications for model-building and experiments.

Is a cancellation between the $m_0^2$ contribution to $M_Z^2$ and the radiative corrections to it more ‘natural’ than a cancellation between different soft terms? A FT analysis says that a cancellation in the $m_0^2$ contribution allows to have a heavy $m_0$ without a large $F_T$ of the soft terms, but with a large FT of the couplings (mainly $\lambda_\tau$ and $\alpha_3$). The FT associated with the couplings is sometimes included, omitted or neglected in the various definitions of FT employed in the literature. This choice is usually irrelevant (because the fine-tuning with respect to the $\mu$-term is often the strongest one), but not in this case. The FT-parameter used in [3] does not include the FT associated with the couplings. In the following, we will discuss why and how it has to be included, making too large values of $m_0$ unnatural.

The real issue does not consist of computing a number that should quantify “how much we like” the cancellation necessary to have a large $m_0$. The problem of ‘unnatural situations’ (like a strong accidental cancellation) is that they are unlikely, because they happen only in a small percentage of the available parameter space.

Consequently, in order to assess if $m_0 \gg M_Z$ is natural in minimal supergravity with tan $\beta \approx 10$, what we should actually determine is whether the experimental determination of $M_t$ implies that the necessary cancellation is happening i.e. if the coefficient $a_0$ is forced to be much smaller than its
Figure 1: The lighter regions show the uncertainties on the \((a_0, \tan \beta)\) (fig. 1a) and \((\lambda_t(M_{\text{GUT}}), \tan \beta)\) (fig. 1b) plane induced by 1\(\sigma\) uncertainties on \(\alpha_3\) and \(M_t\) and by the uncertainty on the sparticle spectrum. In fig. 1a we have fixed \(m_0 = 3\) TeV. Despite the top mass is precisely known, the values of \(a_0\) and \(\lambda_t(M_{\text{GUT}})\) are still quite uncertain. Even if \(M_t\) were perfectly measured, the uncertainties would be only partially reduced (inner darker regions). Only if \(a_0\) lies inside the thin light region the correction to \(M_Z^2\) proportional to \(m_0^2\) is smaller than \(10M_Z^2\).

Even if \(\lambda_t\) and \(g_t\) were measured with negligible error at the \(Z\)-scale, unknown threshold corrections would still induce a \(\sim 0.1\) uncertainty on \(a_0\). We illustrate this uncertainty in fig. 1b, where we show the allowed region of the \((a_0, \tan \beta)\) plane corresponding to

\[
M_t = (175 \pm 5)\text{ GeV}, \quad \alpha_3(M_{\text{GUT}}) = 0.120 \pm 0.003
\]

\(m_0 = 3\) TeV, \(300\text{ GeV} < M_3 < 1\) TeV (lighter region). The inner darker region has been plotted assuming \(M_t = 175\) GeV in order to show that a significant uncertainty on \(a_0\) would be present even if \(M_t\) were perfectly known. Shown are also the values of \(a_0\) for which the \(m_0^2\) contribution to \(M_Z^2\) is smaller than \(10M_Z^2\). For completeness, in fig. 1b, we show the allowed regions of the \((\lambda_t(M_{\text{GUT}}), \tan \beta)\) plane corresponding to the same parameter ranges, except for \(m_0\), now varied between 200 GeV and 1 TeV.

To summarize, fig. 1b shows that there is no experimental evidence that \(\lambda_t\) is very close to the value that gives \(a_0 = 0\) — i.e. that a cancellation is suppressing the \(m_0^2\) contribution to the \(Z\) mass. Although such a suppression is not excluded, it would require a FT of the relevant parameters inside their experimental ranges. As a consequence, \(m_0\) can be heavy only if some cancellation is forced: between \(m_0^2\) and the radiative corrections to it (by fine-tuning the couplings), or between \(m_3^2\) and other soft terms (for example by fine-tuning the \(\mu\) term), or both. In both cases a significant cancellation is unlikely and therefore unnatural.

Having explained our main point, we rediscuss it in a more quantitative way. In order to compute the naturalness upper bound on \(m_0\) we have to estimate how unlikely is the cancellation necessary to allow large values of \(m_0\).

\[\tan \beta \approx 10 \quad |a_0(M_t, \alpha_3(M_Z), \text{sparticle spectrum})| \sim 0.2. \]  

The answer is no. The experimental uncertainty on \(M_t\), on \(\alpha_3(M_Z)\) and on the sparticle spectrum induces an uncertainty on \(a_0\) comparable to its ‘typical’ value, \(|a_0| \sim 0.2\), due to the strong sensitivity of \(a_0\) to these parameters.

One way of understanding such a strong sensitivity is to neglect the threshold corrections to \(a_0\) and to express the dependence of \(a_0\) on the EW parameters through an integral involving the top Yukawa coupling renormalized at energies \(\mu\) higher than the EW scale: \(a_0 \approx 1 - \frac{3}{2} \rho\), where

\[
\rho \equiv 1 - \text{exp} \left( - 6 \int_{\ln m_0}^{\ln M_{\text{GUT}}} \lambda_t^2(\mu) \frac{d\ln \mu}{8\pi} \right). 
\]

While \(M_t\) and \(\alpha_3(M_Z)\) are known with few \% uncertainty, there is a larger uncertainty on \(\lambda_t(\mu)\): unknown sparticle threshold corrections affect the value of \(\lambda_t\) just above the SUSY breaking scale; the running up to higher scales depends on the gauge couplings (also affected by unknown sparticle threshold corrections) amplifying the uncertainties in \(\lambda_t\). Of course, by solving the RGE equation for \(\lambda_t\), \(\rho\) can be written in terms of the value of \(\lambda_t\) renormalized at any scale between \(m_0\) and \(M_{\text{GUT}}\): for example

\[
\rho = \frac{\lambda_t^2(m_0)}{E/6F} = \frac{1}{1 + [6F\lambda_t^2(M_{\text{GUT}})]^{-1}}
\]

where \(E\) and \(F\) are functions of the gauge couplings \(g_i\) defined as

\[
E_\mu(\lambda_t) \equiv \frac{\alpha_3(M_{\text{GUT}})}{\alpha_i(\mu)}, \quad E(\mu) \equiv \frac{1}{f_1^{1/99}} f_2 f_3^{16/9}
\]

\(E \equiv E(m_0) \approx 11\). and

\[
F \equiv \int_{\ln m_0}^{\ln M_{\text{GUT}}} E(\mu) \frac{d\ln \mu}{8\pi} \approx 1.5
\]

Writing \(a_0\) in terms of \(\lambda_t(m_0)\) we can estimate the uncertainty on \(a_0\) due to the uncertainty on the couplings as

\[\delta a_0 \approx -2.5 \delta \lambda_t(m_0) + 2.2 \delta g_3(m_0) + \cdots.\]

\[\alpha_3(M_{\text{GUT}}) \equiv \frac{\alpha_3(M_Z)}{\alpha_i(\mu)} \]  

†We have varied the range because knowing the allowed range the top quark Yukawa coupling renormalized at the unification scale as function of \(\tan \beta\) is also interesting for lepton-flavour violating signals of supersymmetric unification.‡ Fig. 1b shows in a less direct but more precise way than fig. 1a that there is no evidence for a very small value of \(a_0\).

‡If this conclusion were not true, any supersymmetric model with very heavy sparticles could be made ‘natural’ provided that the soft terms depend on unmeasured couplings. Even the quantum corrections to the higgs mass in the non-supersymmetric SM could be made ‘naturally’ vanishing by choosing an experimentally allowed appropriate value of the SM quartic higgs coupling.
The FT parameters quantify how sensitive is $M_Z$ with respect to variations of the parameters. Sensitivity and naturalness are however two different things\footnote{This is the minimal uncertainty that would be obtained if $M_t$ were known with negligible error; including the present error on $M_t$ would make $\lambda_t(M_{\text{GUT}})$ more uncertain, see fig. 3 strengthening our conclusions.}. Nevertheless, 1/FT, if much smaller than one and if divided by the ‘total allowed parameter space’, gives a rough measure of the percentage of the allowed parameter space where a certain cancellation happens\footnote{This is the minimal uncertainty that would be obtained if $M_t$ were known with negligible error; including the present error on $M_t$ would make $\lambda_t(M_{\text{GUT}})$ more uncertain, see fig. 3 strengthening our conclusions.}. In absence of a theoretical justification, very strong cancellations happen only in very small corners of the parameter space and are consequently very unlikely. To estimate how unlikely are the cancellations that allow a large $m_0$, we must therefore include the FT with respect to each relevant parameter $\varphi$ and normalize it with respect to their experimentally allowed range $\Delta \varphi$\footnote{This is the minimal uncertainty that would be obtained if $M_t$ were known with negligible error; including the present error on $M_t$ would make $\lambda_t(M_{\text{GUT}})$ more uncertain, see fig. 3 strengthening our conclusions.}. More precisely, we will compute

$$\Delta(\varphi) \simeq \left| \Delta \varphi \frac{\partial M_Z^2}{\partial \varphi} \right| \text{ instead of } \text{FT}(\varphi) \simeq \left| \frac{\varphi}{M_Z^2} \frac{\partial M_Z^2}{\partial \varphi} \right|$$

for each parameter $\varphi$. We will then combine different FTs in the ‘usual’ way:

$$\Delta = \max_{\varphi} [\Delta(\varphi)]$$

although, since we want to estimate the probability that two different and almost independent cancellations could occur, it would be safe to multiply the FT parameters relative to the two cancellations, obtaining stronger bounds.

While $m_0$ was the only parameter considered in \footnote{This is the minimal uncertainty that would be obtained if $M_t$ were known with negligible error; including the present error on $M_t$ would make $\lambda_t(M_{\text{GUT}})$ more uncertain, see fig. 3 strengthening our conclusions.}, we consider in addition the FTs with respect to variations of $M_t, \alpha_t(M_Z), \ldots$ in their experimental ranges. At present, $\Delta(M_t)$ is actually the only relevant FT besides $\Delta(m_0)$. We assume that the uncertainty on $m_0^2$ is comparable to $m_0^2$ (so that $\Delta(m_0^2) \approx \text{FT}(m_0^2)$), and we (optimistically) assume a $\Delta \varphi = 0.1$ total uncertainty range on $\varphi = \lambda_t(M_{\text{GUT}})$. As observed above, one can exploit the relation between $M_t$ and $\lambda_t(M_{\text{GUT}})$ and use $\Delta(\lambda_t(M_{\text{GUT}}))$ instead of $\Delta(M_t)$. The two possibilities are equivalent in the limit in which the uncertainty on $M_t$ is the dominant one. If the error on $M_t$ will be reduced down to a negligible level, $\Delta(\lambda_t(M_{\text{GUT}}))$ will still take into account (some of) the FT associated to, e.g., $\alpha_t$ so that our conclusions will still hold.

Let us discuss analytically the magnitude of $\Delta(\lambda_t(M_{\text{GUT}}))$. If the experimental measure of $M_t$ implied that $|\alpha_t| \ll 1$, our FT-like parameter would consider as natural very large values of $m_0$. The variation of $\alpha_t$ with $\lambda_t(M_{\text{GUT}})$ is however sufficiently strong to disfavour such a possibility:

$$\text{FT}(\lambda_t(M_{\text{GUT}})) = \frac{36F \cdot \lambda_t^2(M_{\text{GUT}})}{[1 + 16F \cdot \lambda_t^2(M_{\text{GUT}})]^2} \frac{m_0^2}{M_Z^2}$$

where $F$ has been defined in \footnote{This is the minimal uncertainty that would be obtained if $M_t$ were known with negligible error; including the present error on $M_t$ would make $\lambda_t(M_{\text{GUT}})$ more uncertain, see fig. 3 strengthening our conclusions.}. For $\lambda_t(M_{\text{GUT}})$ close to the value where the cancellation in $a_0$ takes place,

$$\Delta(\lambda_t(M_{\text{GUT}})) \approx \frac{0.1}{\lambda_t(M_{\text{GUT}})} \text{FT}(\lambda_t(M_{\text{GUT}})) \approx 0.25 \frac{m_0^2}{M_Z^2}$$

The effect of taking $\Delta(\lambda_t(M_{\text{GUT}}))$ into account is shown in fig. 3. We have assumed fixed values for the gaugino masses and for the gauge couplings. For heavy $m_0$ there is a small portion of parameter space (limited by the dashed lines) where $\Delta(m_0^2) < 10,30$. As explained, the smallness of this region means that there is a significant FT with respect to some other parameter. In fact this regions disappears when $\Delta(\lambda_t(M_{\text{GUT}}))$ (solid line) is taken into account.

In conclusion, very heavy values of $m_0$ require an unnatural FT of the relevant parameters inside their present experimental range. We have used the FT-like parameter introduced in \footnote{This is the minimal uncertainty that would be obtained if $M_t$ were known with negligible error; including the present error on $M_t$ would make $\lambda_t(M_{\text{GUT}})$ more uncertain, see fig. 3 strengthening our conclusions.} and repeated the computation in appendix $A$ using the more accurate technique presented in \footnote{This is the minimal uncertainty that would be obtained if $M_t$ were known with negligible error; including the present error on $M_t$ would make $\lambda_t(M_{\text{GUT}})$ more uncertain, see fig. 3 strengthening our conclusions.}. With respect to this problem both criteria are less restrictive than a ‘naive’ complete FT analysis. In both cases the result is that too large values of $m_0$ are unnatural, as it can simply be seen by inserting a typical value of $|a_0|$ and the preferred confidence level on unlikely cancellations (for example $\text{FT}_{\text{lim}} \lesssim 1/10\%$) in the naive bound $m_0^2 \lesssim \text{FT}_{\text{lim}} M_Z^2/|2a_0|$. Since $a_0$ is typically small, eq. (3), one obtains the usual weak naturalness bound on $m_0$, well above all present accelerator bounds, but not above 1 TeV. Due to the smallness of $|a_0|$, the naturalness upper bound on $m_0$ has almost no impact on the ‘naturalness status’ of mSUGRA models, as discussed in appendix $A$.

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A Naturalness bound on $m_0$

As said naturalness disfavours heavy $m_0$ because very strong cancellations (either between different soft terms, or between the tree level $m_0^2$ term and the radiative corrections to it) are needed in order to accommodate very large values of $m_0$. Setting a naturalness upper bound on $m_0$ amounts to estimate how unlikely is the required cancellation in the light of our experimental and theoretical knowledge.

If we assign to the parameter space an arbitrary probability distribution function (pdf) we can compute the probability of any event, for example of the required cancellation. The pdf is however totally arbitrary in absence of experimental data. This same assumption (the choice of an arbitrary pdf, called ‘Bayes prior’ in statistical inference) is the crucial ingredient that allows to convert experimental data into measured ranges of fundamental parameters, like the top mass. Starting from an arbitrary pdf and using simple properties of probability, it is possible to follow how experimental data modify the probability of different values. When

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Figure 2: Naturalness upper bounds on $m_0$ as a function of $M_t$ ($\Delta < 10,30$) before (dashed lines) and after (continuous lines) having properly taken into account the uncertainty on the relevant parameters.

In conclusion, very heavy values of $m_0$ require an unnatural FT of the relevant parameters inside their present experimental range. We have used the FT-like parameter introduced in \footnote{This is the minimal uncertainty that would be obtained if $M_t$ were known with negligible error; including the present error on $M_t$ would make $\lambda_t(M_{\text{GUT}})$ more uncertain, see fig. 3 strengthening our conclusions.} and repeated the computation in appendix $A$ using the more accurate technique presented in \footnote{This is the minimal uncertainty that would be obtained if $M_t$ were known with negligible error; including the present error on $M_t$ would make $\lambda_t(M_{\text{GUT}})$ more uncertain, see fig. 3 strengthening our conclusions.}. With respect to this problem both criteria are less restrictive than a ‘naive’ complete FT analysis. In both cases the result is that too large values of $m_0$ are unnatural, as it can simply be seen by inserting a typical value of $|a_0|$ and the preferred confidence level on unlikely cancellations (for example $\text{FT}_{\text{lim}} \lesssim 1/10\%$) in the naive bound $m_0^2 \lesssim \text{FT}_{\text{lim}} M_Z^2/|2a_0|$. Since $a_0$ is typically small, eq. (3), one obtains the usual weak naturalness bound on $m_0$, well above all present accelerator bounds, but not above 1 TeV. Due to the smallness of $|a_0|$, the naturalness upper bound on $m_0$ has almost no impact on the ‘naturalness status’ of mSUGRA models, as discussed in appendix $A$.

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experimental information is sufficiently strong, the final pdf
does not depend on the arbitrary pdf needed to start with.
This is why we can today assume that the pole top mass is
distributed according to a $175 \pm 5$ gaussian.
Since the soft terms are totally unknown we assume some
broad pdf for them. Our results have only a mild dependence
on the pdf, unless some crazy pdf is chosen. Since $M_Z$
(that is one combination of soft terms) has been already
measured with a practically infinite precision, it is simpler
to take this experimental constraint into account with the pro-
cedure used in $\Box$ we assume a probability distribution for the
dimensionless ratios of the soft terms, and compute the
overall scale of soft terms from the EWSB condition. Since
in this way we never specify how heavy are the sparticles,
the connection of this procedure with naturalness is quite
transparent.
Sampling all parameters, like $M_t$ and $m_0/m_{1/2}$, according
to their assumed pdf, we estimated $\Box$ that only in $p \sim
5\%$ of the cases a cancellation in the EWSB conditions gen-
erates sparticle masses above all experimental bounds in
mSUGRA. In order to set upper bounds on $m_0$ we repeat the
analysis in $\Box$, but without averaging $p$ over the dis-
tribution of $m_0/m_{1/2}$: we here compute $p$ as function of
$m_0/m_{1/2}$ at fixed $\tan \beta = 10$. We find that $p(m_0/m_{1/2})$
has a maximum at $m_0 \sim 3m_{1/2}$, decreases when $m_0 \ll m_{1/2}$
(because too small values of $m_0$ give light right-handed slep-
tons) and becomes negligibly small when $m_0 \gg m_{1/2}$ (more
precisely when $m_0 \gtrsim 3M_3$). We again conclude that values of
$m_0$ significantly above 1 TeV require very unlikely cancel-
lations in the EWSB condition. A certain minimal amount of
cancellation is however required even for $m_0$ below 1 TeV
in order to accommodate experimental bounds, as recalled in appendix C.

B Heavy $m_0$ and the naturalness problem
The $Z$ mass is given, as function of the soft terms, by a poten-
tial minimization condition that in mSUGRA with vanishing
$A_0$ and large $\tan \beta \approx 10$ can be approximated as

$$M_Z^2 = -2(a_0m_0^2 + a_{1/2}m_{1/2}^2 + \mu^2).$$

One important success of supersymmetry is the prediction
that RGE effects typically induce negative $a_i$ coefficients,
thus establishing a direct link between SUSY-breaking and
EW-breaking. This nice feature is however due to $\lambda_1$ and
$g_3$ interactions: SUSY breaking most naturally induce a non
vanishing $Z$-boson mass comparable to the gluino and top-
squark masses, that are typically heavier than the other non
coloured sparticles. On the contrary experiments now tell
that the $Z$ boson is lighter than (almost) all sparticles. This
kind of naturalness problem manifests itself in eq. $\Box$ if the
bounds on sparticle masses imply that the single contribu-
tions to $M_Z^2$ are much larger than $M_Z^2$ itself. What happens
is that the $M_Z^2$ contribution gives no problems, while the $m_{1/2}\ddot{}$
term gives an unpleasantly large contribution $\Box$ to $M_Z^2$, that
can be canceled by the $\mu^2$ term.

We could also study $p$ as function of $m_0/M_Z$. However $m_0 \gg
M_Z$ is possible either because $|a_0| \ll 1$, or due to a cancella-
tion between different soft terms. We study $p(m_0/m_{1/2})$ rather
than $p(m_0/M_Z)$ because we here want to concentrate our attention
on the first possibility. Bounds on $m_0/M_Z$ have a more direct
impact on phenomenology. Bounds on $m_0/m_{1/2}$ have a more
direct impact on theoretical attempts of predicting $m_0/m_{1/2}$.

The $m_Z^2$ contribution does not pose naturalness problems
because the experimental bound on $m_0$ is weak ($m_0$ could
even be zero), and because the coefficient $a_0$ is typically
small, $|a_0| < 1/3$. The particular structure of the SUSY
RGE protects the $m_Z^2$ contribution from QCD corrections,
that instead affect the $m_{1/2}$ contribution. This well known
fact can be easily understood with the techniques of $\Box$.

The $m_{1/2}$ term is problematic because it has a large co-
efficient $a_{1/2} \sim -(3/5)/2$ and because LEP and Tevatron
experiments provide significant lower bounds on $m_{1/2}$. The
$m_{1/2}$ contribution to $M_Z^2$ is approximately given by

$$M_Z^2 \over (91 \text{GeV})^2 \approx (5/11)(M_3/290 \text{GeV})^2 + \cdots$$

where $M_3 \approx 2.5m_{1/2}$ is renormalized at $Q = 500 \text{GeV}$ and
lower values in the given range can be obtained for higher
$\tan \beta$ and lower $\lambda_1(M_{\text{GUT}})$. The LEP limit on the
chargino mass gives rise, due to our assumption of gaugino mass
unification, to a strong but indirect bound on the gluino
mass, $M_3 \gtrsim 290 \text{GeV}$. Abandoning gaugino mass unification
only the Tevatron direct bound on the gluino mass applies
($M_3 \gtrsim 180 \pm 280 \text{GeV}$, depending on the squark spectrum)
so that the situation can be partially improved $\Box$ $\Box$. The
value of $m_0$ has only a small indirect impact on the natural-
ness problem: since $\tan \beta$ is determined by minimizing the
potential, a moderately large $m_0$ allows to naturally obtain
the moderately large values of $\tan \beta \approx 10$ for which the $m_{1/2}$
problem is minimized $\Box$.

References
[1] The top averaging group and CDF and D0 collaborations (L.
Demortier et al.). FERMILAB-TM-2084, sep. 1999.
[2] J.L. Feng and T. Moroi, hep-ph/9907315, J.L. Feng,
K.T. Matchev and T. Moroi, hep-ph/9908309 and hep-
ph/991047.
[3] R. Barbieri and G.F. Giudice, Nucl. Phys. B306 (1988) 63.
[4] See e.g. P.H. Chankowski, J. Ellis and S. Pokorski, Phys.
Lett. B423 (1998) 327 (hep-ph/9712234).
[5] L. Giusti et al, Nucl. Phys. B550 (1999) 3 (hep-ph/9811386).
[6] L. Ibáñez and C. Lopez, Nucl. Phys. B233 (1984) 511; L.
Ibáñez, C. Lopez and C. Muñoz, Nucl. Phys. B256 (1985)
218; A. Bouquet, J. Kaplan and C.A. Savoy, Nucl. Phys.
B262 (1985) 299.
[7] R. Barbieri and L.J. Hall, Phys. Lett. B338 (1994) 212.
[8] The use of FT to quantify naturalness has been criticized
in B. de Carlos and J.A. Casas, Phys.
Lett. B309 (1993) 320. In order to avoid such criticisms an improved measure of
naturalness (FT divided by an ‘average FT’) has been
proposed in G.W. Anderson and D.J. Castaño,
Phys. Lett. B347 (1995) 308, Phys.
Rev. D52 (1995) 1995. In limiting cases of practical interest FT can be used to obtain a cor-
rect rough estimate of how unlikely is a numerical accident,
see [8]. In more realistic cases this probability can be more
accurately estimated as discussed in [3] and in A. Strumia,
hep-ph/9904247.
[9] P. Chalabi and A. Strumia, Nucl. Phys. B494 (1997) 41
hep-ph/9611204.
[10] C. Giudice and R. Rattazzi, Nucl. Phys. B511 (1998) 25
hep-ph/9706546.
[11] D. Wright, hep-ph/9801447, G.L. Kane and S.F. King, hep-
ph/9810374. See also K. Agashe and M. Graesser, Nucl.
Phys. B507 (1997) 3 (hep-ph/9704204).
[12] P.H. Chankowski, J. Ellis, M. Olechowski and S. Pokorski,
hep-ph/9808273.