The Cyclotron Spin-Flip Mode as the Lowest-Energy Excitation of Unpolarized Integer Quantum Hall States

L.V. Kulik, a,b I.V. Kukushkin, a,b S. Dickmann, a V.E. Kirpichev, a,b A.B. Van’kov, a A.L. Parakhonsky, a J.H. Smet, b K. von Klitzing, b W. Wegscheider c,d

a Institute for Solid State Physics, Russian Academy of Sciences, Chernogolovka, 142432 Russia
b Max-Planck-Institut für Festkörperforschung, Heisenbergstraße 1, 70569 Stuttgart, Germany
c Walter Schottky Institut, Technische Universität München, 85748 Garching, Germany
d Institut für Experimentelle und Angewandte Physik, Universität Regensburg, 93040 Regensburg, Germany

The cyclotron spin-flip modes of spin unpolarized integer quantum Hall states (ν = 2, 4, 6) have been studied with inelastic light scattering. The energy of these modes is significantly smaller compared to the bare cyclotron gap. Second order exchange corrections are held responsible for a negative energy contribution and render these modes the lowest energy excitations of unpolarized integer quantum Hall states.

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According to Kohn’s theorem [1], homogenous electromagnetic radiation incident on a translationally invariant electronic system can only couple to the center-of-mass coordinate. Such radiation is unable to excite internal degrees of freedom associated with the Coulomb interaction. As a result, physical phenomena originating from electron-electron interactions leave the cyclotron resonance unaffected. Hence, spin-unperturbed magneto-plasmons excited under these conditions have an energy equal to the bare cyclotron energy, irrespective of existing electron-electron correlations [2]. A similar statement also holds for spin-excitons, intra-Landau level spin-flip excitations. In a system with rotational invariance in spin space, Larmor’s theorem [3] dictates that Coulomb interactions do not contribute to the energy of zero-momentum spin-excitons. In contrast to these magneto-plasma and spin-exciton excitations, there exist no symmetry arguments which restrict the energy of the combined zero momentum cyclotron spin-flip mode (CSFM). It is well established that the cyclotron-spin-flip mode excited from spin-polarized ground states acquires considerable exchange energy even for zero momentum [4, 5, 6]. The energy of this mode may thus serve as a unique probe of many-body interactions in the electronic system.

Hitherto, it has not been considered that there is also an exchange contribution to the energy of the zero momentum cyclotron-spin-flip modes of unpolarized quantum Hall ground states at even integer fillings (ν = 2, 4, 6, ...). First order perturbation calculations in the ratio r_c = E_C/hω_c explicitly predicted a zero exchange contribution to the total energy of this combined mode of unpolarized quantum Hall ground states [2] (E_C is the characteristic Coulomb energy scale and hω_c the cyclotron energy). Here, we experimentally demonstrate however that the energy of these modes is considerably reduced compared with the bare cyclotron gap. We corroborate with theoretical considerations that the negative energy contribution arises from second order Coulomb corrections and so was not captured by previous first order perturbation calculations.

Two high-quality heterostructures were studied. Each consisted of a single-side modulation doped 30 nm AlGaAs/GaAs quantum well (QW) with an electron density between 1 and 1.2 · 10^{11} cm^{-2} and a mobility of 5−7 · 10^{6} cm^{2}/Vs. The density n_e was tuned continuously via the opto-depletion effect and was measured with luminescence [7]. Inelastic light scattering (ILS) spectra were recorded at 1.5 K in the back-scattering geometry in a split-coil cryostat. Three optical fibers were utilized. One fiber transmitted a dye laser pump beam, tuned above the fundamental gap of the QW. The remaining fibers collected the scattered light and guided it out of the cryostat. The angles between the sample surface, pump beam fiber and collecting fibers define the in-plane momentum transferred to the electron system via inelastic light scattering. The collecting fibers selected excitations with in-plane momenta of 0.4 and 1.0 · 10^5 cm^{-1}. The scattered light was dispersed in a triple grating monochromator and detected with a CCD camera.

Fig. 1 shows typical ILS spectra of inter Landau-level (LL) excitations as well as the magnetic field (B) dependence of the energy of the various lines in these spectra in a sample with a density of 1.2 · 10^{11} cm^{-2}. The experimental configuration selected excitations with an in-plane momentum of q = 1.0 · 10^5 cm^{-1}. The polarization selection rules allowed to identify that lines at low B (<1 T) correspond to charge density excitations. The principal magneto-plasmon mode as well as a Bernstein mode (B_1) are observed in the geometry where the incident and scattered photons have parallel polarization vectors. At non-zero B the magneto-plasmon mode has a strong linear dispersion in the long wavelength limit and at B = 0 its energy equals the plasma energy for momentum q. In contrast, the Bernstein mode is nearly dispersionless. Both modes couple through many body
Coulomb interactions near $\sim 0.8$ T. At large $B$, their energies converge asymptotically to the cyclotron energy and twice the cyclotron energy, respectively [1].

Of main interest here is the appearance of a triplet ILS resonance when the system is in the $\nu = 2$ spin-unpolarized quantum Hall state (bottom, right inset Fig. 2). Near $B \approx 2.4$ T, the central line of the triplet is clearly resolved, but the side lines only appear as shoulders. The splitting between the features corresponds approximately to the electron Zeeman energy $E_Z$ in GaAs, so they are attributed to the three cyclotron spin-flip modes with different spin projections along the $B$-field axis ($S_z = -1, 0, 1$). The shoulder structures are assigned to the cyclotron spin-flip modes with $S_z = -1$ and 1, and the central line ($S_z = 0$) is associated with a cyclotron spin-wave, i.e. out-of-phase oscillations of the two spin subsystems of the Landau levels with orbital index 0 and 1 [10]. This identification of the triplet is confirmed by measurements in tilted fields. ILS spectra in tilted fields are plotted in the left inset to Fig. 2. The triplets are much better resolved due to the larger total fields $B_{\text{tot}}$. Well-separated peaks appear and the spin splitting can be directly measured. The Zeeman effect in essence is a three dimensional phenomenon and so enters as a function of $B_{\text{tot}}$. The data points fit well to a $g$-factor $g_{\text{CSFM}} = -0.4$ (solid line). The dashed line corresponds to $g_{\text{GaAs}, \mu_B B}$, where $g_{\text{GaAs}} = -0.44$ is the effective $g$-factor of bulk GaAs. A significant reduction of the $g$-factor is not uncommon in AlGaAs-heterostructures and has been accounted for by bandstructure non-parabolicity, confinement and wavefunction penetration effects [11].

In the inset of Fig. 3 we compare ILS spectra measured at $\nu = 2$ for two different values of in-plane momenta: 0.4 and 1.0 $10^5$ cm$^{-1}$. In agreement with existing theories [2, 12], the CSFM energy does not show any appreciable dispersion at momentum values accessible with ILS techniques. Therefore, the CSFM line is regarded as the energy of the cyclotron spin-flip mode when $q \rightarrow 0$. The key experimental finding is a downward shift of the energy of this mode with $-0.35$ meV as compared to the bare cyclotron energy. This shift exceeds by far the single electron Zeeman energy in GaAs at this magnetic field (0.08 meV) and we therefore assert it is strongly influenced by exchange interactions.

The $B$-dependence of the energy of the cyclotron spin-flip mode for fixed filling $\nu = 2$ is plotted in Fig. 3. The slope is identical to the bare cyclotron energy line in GaAs. Hence, the dependence of the cyclotron spin-flip mode on $B$ takes on the functional form $\hbar \omega_{\text{CSFM}} = \hbar \omega_c + \Delta E_{\text{SF}}$ over a rather broad magnetic field interval: $0.6 T < B < 2.7 T$. Here $\Delta E_{\text{SF}}$ is the $B$-independent downward shift of approximately $-0.35$ meV. It is worthwhile to note that a dimensional analysis of second order Coulomb corrections to the energies of inter-Landau levels would yield a similar dependence on $B$: $E = \hbar \omega_c + \Delta E_{\text{SF}}$, where $\Delta E_{\text{SF}} \sim \hbar \omega_c r^2_c$. Indeed, if $E_C = \alpha e^2/\epsilon l_B$ then $\Delta E_{\text{SF}}$ is independent of the field. The renormalization factor $\alpha$ is determined by the size-quantized wave function of electrons confined in the QW. In the ideal 2D case $\alpha = 1$. However, the larger the width of the 2D electron system (2DES), the smaller $\alpha$ becomes and thereby reducing $r_c$. This is certainly relevant for the width of our quantum well.

An analytical calculation of the second order correction to the CSFM energy is performed in terms of small $r_c$. The theory is based on the following general features of the system. The state of the system is described by the exact quantum numbers $S_z, S_y$ and $q$ and by the 'good' quantum number $\delta n$ characterizing the excitation kinetic energy $\hbar \omega_c \delta n$ (good but not exact due to LL-mixing). The relevant excitations with $q=0$ and $\delta n = 1$ may be presented in the form $\hat{K}^{\uparrow}_{S,S_z} |0\rangle$, where $|0\rangle$ is the ground state and $\hat{K}^{\uparrow}_{S,S_z}$ are "raising" operators: $\hat{K}^{\uparrow}_{0} = \sum_n \epsilon_n \hat{c}_n^\dagger \hat{c}_n$ and $\hat{K}^{\uparrow}_{1} = \sum_n \epsilon_n \hat{c}_n^\dagger \hat{c}_{n+1}$. All three commutators with the kinetic-energy operator $\hat{H}_1$ are $[\hat{H}_1, \hat{K}^{\uparrow}_{S,S_z}] = \hbar \omega_c \delta n |0\rangle$.

The total Hamiltonian is $\hat{H}_{\text{tot}} = \hat{H}_1 + \hat{H}_{\text{int}}$, where $\hat{H}_{\text{int}}$ is the exact Coulomb-interaction Hamiltonian. If $|0\rangle$ is unpolarized, we have $2^S \hat{S}^z \hat{K}^{\uparrow}_{S,S_z} |0\rangle = S(S+1) \hat{K}^{\uparrow}_{S,S_z} |0\rangle$ and $\hat{S}_z \hat{K}^{\uparrow}_{S,S_z} |0\rangle = S \hat{K}^{\uparrow}_{S,S_z} |0\rangle$. Moreover, the identity $\hat{H}_{\text{int}} \hat{K}^{\uparrow}_{S,S_z} |0\rangle = 0$ holds ($0\rangle$, to describe the zero'th order ground state). It implies that first-order Coulomb corrections vanish for both the spin unperturbed or singlet magnetoplasmon (where $S = 0$) and the combined CSFM triplet ($S = 1$). At the same time, $[\hat{H}_{\text{int}}, \hat{K}^{\uparrow}_{0}] = 0$ but $[\hat{H}_{\text{int}}, \hat{K}^{\uparrow}_{1}] \neq 0$. Hence, whereas the magnetoplasmon has no exchange energy correction calculated to any order in $r_c$, the combined modes should have second and higher order exchange corrections.

The second-order correction is based on the Excitonic Representation (ER) technique [13, 14, 15]. It utilizes exciton states $\hat{Q}^{\dagger}_{ab|q}\langle 0\rangle$ as a basis set, instead of single-electron states of a degenerated LL. The exciton creation operator is defined as [13, 14, 15]

$$\hat{Q}^{\dagger}_{ab|q} = N^{-1/2}_\phi \sum_{p,q} e^{-iq \cdot q c^\dagger_{p+q} b^\dagger_{p+q}} / \sqrt{2^{a+b}}. \tag{1}$$

Here, $N_\phi = A/2 \pi g^2 B$ stands for the number of magnetic flux quanta and $q = (q_x, q_y)$ is given in units of $1/l_B$. The binary indices $a$ and $b$ denote both the LL number and the spin index: $a, b = (n_{a,b}, \sigma_{a,b})$. All three CSFM states have certainly the same exchange energy,


Ref. [2] in which a non-zero energy shift was predicted for the roton minimum at large wave-vector. This assignment can easily be performed. It yields \( \Delta E \) depending on the non-zero thickness of the 2DES. The effective thickness parameter is chosen, then \( |\langle q| V| - \rangle_q |^2 \) is a form factor capturing the Coulomb interaction. A second order perturbation theory of the Coulomb interaction explains the experimental results.

In conclusion, the inelastic light scattering response from the combined cyclotron spin-flip modes of unpolarized quantum Hall states at \( q = 0 \) has been studied. A negative energy term was found to decrease their energy and was attributed to many-body Coulomb exchange interaction. A second order perturbation theory of the Coulomb interaction explains the experimental results.

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FIG. 1: $B$-dependence of ILS line energies. Triangles mark charge density excitations, circles the CSFM. The dashed lines correspond to $\hbar \omega_c$ and $2\hbar \omega_c$. Top, left inset: ILS spectra at $B=0$ and 1 T. Bottom, right inset: ILS spectrum at 2.4 T.
FIG. 2: Electron Zeeman energy $E_Z$. The dashed line plots the expected Zeeman energy when taking $g_{GaAs} = -0.44$. The solid line is a linear fit to the data for $g_{QW} = -0.4$. The inset depicts ILS spectra at a constant perpendicular field of 2T but two different values of the total field. The right inset schematically illustrates the spin-triplet ($S = 1, S_z = -1, 0$ and 1) cyclotron excitations for $\nu = 2$.

FIG. 3: The CSFM energy for the spin unpolarized $\nu = 2$ quantum Hall state versus perpendicular field. The dashed line gives the cyclotron energy. The upper inset displays ILS spectra of the CSFM for two different in-plane momenta.
FIG. 4: The CSFM exchange shift calculated from Eq. (3) with the modified Coulomb interaction $V(q) = q^{-1}e^2w^2erfc(qw)$. Its absolute value at $w = 0$ equals $(1 - \ln 2)\text{Ry}$. 

FIG. 5: ILS spectrum of the cyclotron spin-flip mode at the $\nu = 4$ unpolarized quantum Hall state at the indicated values for $B_L$ and $B_{tot}$. 

$h\omega_{\text{CSFM}} = h\omega_c + \Delta E_{\text{SF}}$

$\Delta E_{\text{SF}} = -0.17 \text{ meV}$

$\nu = 4$

$B_{tot} = 5.7 \text{T}$

$B_L = 1.0 \text{T}$

Int. (arb. units)

$\omega_c$

Raman Shift (meV)