ADAPTIVE CONTROLLABILITY OF MICROSCOPIC CHAOS GENERATED IN CHEMICAL REACTOR SYSTEM USING ANTI-SYNCHRONIZATION STRATEGY

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(Communicated by Kapil Kumar Sharma)

Abstract. In this manuscript, we design a methodology to investigate the anti-synchronization scheme in chaotic chemical reactor system using adaptive control method (ACM). Initially, an ACM has been proposed and analysed systematically for controlling the microscopic chaos found in the discussed system which is essentially described by employing Lyapunov stability theory (LST). The required asymptotic stability criterion of the state variables of the discussed system having unknown parameters is derived by designing appropriate control functions and parameter updating laws. In addition, numerical simulation results in MATLAB software are performed to illustrate the effective presentation of the considered strategy. Simulations outcomes correspond that the primal aim of chaos control in the given system have been attained computationally.

1. Introduction. Chaos can be described as an utter confusion or disorder that exists in events, due to this these events appear unpredictable and erratic. Chaos is undoubtedly an intriguing phenomenon because of its immeasurable features and surprises. Numerous interesting phenomena of extreme complexity found in nature are due to their nonlinear character. A widely known inherent characteristic of chaotic system is the extreme sensitive dependency for the initial condition- s, i.e., two nearly points in the state space would very quickly separate with the evolution of time. Consequently, prediction becomes impossible, and we end up with a random or irregular phenomenon. This establishes the beginning of chaos theory. This theory has numerous important applications in many disciplines, including physics, meteorology, computer science, environmental science, sociology, engineering, chemistry, economics, ecology, biomedical engineering, cryptography, fluid mechanics [25, 22, 35, 1, 27, 28, 6, 7, 30, 38, 39, 5, 29] etc. Subsequently, chaos theory has sought significant attention in varied research fields.

Historically, the chaos phenomenon was first glimpsed by Henri Poincare [24] at the beginning of 20th century, while examining a 3-body problem comprising of earth, moon and sun in an effort to gain stability of the solar system. More importantly, he further established that the considered 3-body problem possesses no analytical solution exhibited in terms of algebraic expression and integrals.

2010 Mathematics Subject Classification. 34K23, 34K35, 37B25, 37N35.

Key words and phrases. Adaptive control method, Chaotic system, Lyapunov stability analysis, Anti-Synchronization, Chemical reactor system.

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Lorenz [20] in 1963 reported the first 3D chaotic system while examining weather prediction model and further proposed the term ‘Butterfly Effect’ that describes the sensitive dependency for the initial conditions as a key characteristic of chaos. After 1990, there was an increasing interest among researchers and scientists in developing novel chaos controlling and synchronizing techniques to understand the complex interactions occurring among systems mostly found in nature. This major advancement was due to the pioneered work of Pecora and Carroll [23] established in 1990. More importantly, they initiated the unprecedented chaos synchronization using master-slave configuration among chaotic systems. Further, Ott et al. [31] in 1990 advocated a procedure broadly known as OGY technique of controlling chaos in nonlinear systems.

Synchronization in chaotic systems is basically a specific process wherein adjustment of two or more chaotic/hyperchaotic systems (identical or nonidentical) is made so that they depict same behaviour due to pairing to gaining stability. By now, different types of synchronization schemes are introduced in current literature like complete [32], anti [15], combination [11], hybrid [34], hybrid projective [8], combination difference [10], function projective [42], lag [13], combination-combination [12], phase [21], projective [4], modified projective [16] etc. Till date, various strategies have been proposed to visualize chaos control, for instance, active [3], adaptive [12, 10], backstepping [26], feedback [2], sliding mode [36], impulsive [14] etc. in control theory. In current literature, for an integer order nonlinear model to be chaotic, at least one Lyapunov exponent should be positive and the dimension of state equations must be at least three. Synchronization in chaotic systems utilizing adaptive control technique was first reported by Hubler [9] in the year 1989. Currently, several investigations have been discussed in the area of chaos control and its applications [19, 41, 18, 17, 40, 37, 12, 10].

Considering the above discussions and literature review in mind, our immediate goal is to investigate an anti-synchronization strategy among two identical chaotic chemical reactor systems in three dimension using ACM. Adaptive control strategy is very significant in estimating the positive parameters of master as well as slave systems. Thus, by applying this strategy, a smaller amount of information is desired to synchronize the chaotic systems. In addition, we discuss comprehensively an adaptive control law as well as an estimated parameter updating law, which is primarily based on LST.

This paper is described as: Section 2 consists of some preliminaries that have basic notations and significant terminology to be utilized here. Section 3 describes the elementary features of chaotic chemical reactor system. Section 4 investigates the ACM for stabilizing asymptotically the given chaotic system by constructing the proper controllers and parameter updating law. Section 5 deals with the numerical simulations illustrating the effectiveness and feasibility of considered anti-synchronization approach. Section 6 finally concludes the paper.

2. Preliminaries. In this section, we present few basic notations and significant terminology which are to be utilized in subsequent sections. Considering the master system and corresponding slave system as:

\[
\dot{x}_m = F_1(x_m), \\
\dot{x}_s = F_2(x_s) + v,
\]

where

\[
F_1(x_m) = \begin{pmatrix}
-a_1 x_1 \sin(x_2) + a_2 x_1 x_3 \\
a_1 x_1 \sin(x_2) - a_3 x_2 \\
a_2 x_1 x_3 + a_3 x_2 
\end{pmatrix},
\]

\[
F_2(x_s) = \begin{pmatrix}
-a_4 x_4 \sin(x_5) + a_5 x_4 x_6 \\
a_4 x_4 \sin(x_5) - a_6 x_5 \\
a_5 x_4 x_6 + a_6 x_5 
\end{pmatrix},
\]

\[
v = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
v_5 \\
v_6
\end{pmatrix}
\]

and \(a_1, a_2, a_3, a_4, a_5, a_6, v_5, v_6\) are unknown parameters.
where \( x_m = (x_{m1}, x_{m2}, \ldots, x_{mn})^T \), \( x_s = (x_{s1}, x_{s2}, \ldots, x_{sn})^T \) are the state variables of (1) and (2) respectively, \( F_1, F_2 : R^n \rightarrow R^n \) are two nonlinear continuous vector functions and \( v = (v_1, v_2, \ldots, v_n) \in R^n \) is the suitable controller to be constructed.

Defining now the anti-synchronization error as
\[
e(t) = x_s(t) + x_m(t).
\]

**Definition 2.1.** The master (1) and the slave (2) systems are said to achieve an anti-synchronization state if
\[
\lim_{t \to \infty} ||e(t)|| = \lim_{t \to \infty} ||x_s(t) + x_m(t)|| = 0,
\]
where \(||.||\) denotes matrix norm.

3. **Description of 3-D chaotic chemical reactor System.** In this section, we present in brief the chaotic chemical reactor system to be selected for anti-synchronization using ACM.

Introduced by Singh and Roy [33] in 2019, the considered chaotic chemical reactor system is described as:

\[
\begin{align*}
\dot{x}_{m1} &= a_2 x_{m1} x_{m2} - 2a_9 x_{m1}^2 + (a_9 - a_10)x_{m1} \\
\dot{x}_{m2} &= -a_1 x_{m1} x_{m2} x_{m3} + a_3 x_{m1} x_{m2} + a_5 x_{m2} x_{m3} - 2a_7 x_{m2}^2 \\
&\quad - (a_{11} - a_{12}) x_{m2} \\
\dot{x}_{m3} &= a_1 x_{m1} x_{m2} x_{m3} - a_4 x_{m1} x_{m3} - a_5 x_{m2} x_{m3} - 2a_8 x_{m2}^2 + a_{13} x_{m3},
\end{align*}
\]

where \((x_{m1}, x_{m2}, x_{m3})^T \in R^3\) is the state vector and \(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}\) and \(a_{13}\) are positive parameters. When \(a_1 = 0.88, a_2 = 10, a_3 = 29, a_4 = a_5 = 100, a_6 = 5, a_7 = 0.5, a_8 = 1.333, a_9 = a_{10} = 1000, a_{11} = 2900, a_{12} = 100\) and \(a_{13} = 10002.667\), the system (4) exhibits chaos. Further, Figure 1 depicts the phase diagram of (4) in 3-D space. In addition, Lyapunov exponents of system (4) are determined as \(L_1 = 92.298, L_2 = -5.665\) and \(L_3 = -156.091\) which show the chaotic behaviour of (4). However, the detailed analytical study and numerical results for system (4) can be found in [33].

The next section presents the anti-synchronization scheme to control chaos of (4) using adaptive control method.

4. **A simple numerical example.** Here, we study anti-synchronization strategy using ACM to derive the laws for the identification of parameters along with control functions in such a manner that all the variables \(x_{m1}, x_{m2}\) and \(x_{m3}\) approach to equilibrium points as \(t\) tends to infinity.

Conveniently, system (4) has been considered as the master system and corresponding identical slave system is defined by

\[
\begin{align*}
\dot{x}_{s1} &= a_2 x_{s1} x_{s2} - 2a_9 x_{s1}^2 + (a_9 - a_{10}) x_{s1} + v_1 \\
\dot{x}_{s2} &= -a_1 x_{s1} x_{s2} x_{s3} + a_3 x_{s1} x_{s2} + a_5 x_{s2} x_{s3} - 2a_7 x_{s2}^2 \\
&\quad - (a_{11} - a_{12}) x_{s2} + v_2 \\
\dot{x}_{s3} &= a_1 x_{s1} x_{s2} x_{s3} - a_4 x_{s1} x_{s3} - a_5 x_{s2} x_{s3} - 2a_8 x_{s2}^2 + a_{13} x_{s3} + v_3,
\end{align*}
\]

where \(v_1, v_2\) and \(v_3\) are controllers to be designed in such an adaptive manner that anti-synchronization in two identical chaotic chemical reactor systems will be achieved.
Now, we formulate adaptive controllers as:

\[ K = \begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix} \]

where the synchronized errors defined in (6) satisfy

\[ v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \]

The main objective in this section is to introduce controllers \( v_i \), \( i = 1, 2, 3 \) so that the synchronized errors defined in (6) satisfy

\[ \lim_{t \to \infty} e_i(t) = 0 \quad \text{for} \quad i = 1, 2, 3. \]

Then, the resulting error dynamics can be written as:

\[
\begin{align*}
\dot{e}_1 &= a_2(x_{s1}x_{s2} + x_{m1}x_{m2}) - 2a_6(x_{s1}^2 + x_{m1}^2) + (a_9 - a_{10})e_1 + v_1 \\
\dot{e}_2 &= -a_1(x_{s1}x_{s2}x_{s3} + x_{m1}x_{m2}x_{m3}) + a_3(x_{s1}x_{s2} + x_{m1}x_{m2}) \\
& \quad + a_5(x_{s2}x_{s3} + x_{m2}x_{m3}) - (a_{11} - a_{12})e_2 - 2a_7(x_{s2}^2 + x_{m2}^2) + v_2 \\
\dot{e}_3 &= +a_1(x_{s1}x_{s2}x_{s3} + x_{m1}x_{m2}x_{m3}) - a_4(x_{s1}x_{s3} + x_{m1}x_{m3}) \\
& \quad - a_5(x_{s2}x_{s3} + x_{m2}x_{m3}) + a_{13}e_3 - 2a_8(x_{s3}^2 + x_{m3}^2) + v_3.
\end{align*}
\]

Now, we formulate adaptive controllers as:

\[
\begin{align*}
v_1 &= -\hat{a}_2(x_{s1}x_{s2} + x_{m1}x_{m2}) + 2\hat{a}_6(x_{s1}^2 + x_{m1}^2) - (\hat{a}_9 - \hat{a}_{10})e_1 - K_1e_1 \\
v_2 &= \hat{a}_1(x_{s1}x_{s2}x_{s3} + x_{m1}x_{m2}x_{m3}) - \hat{a}_3(x_{s1}x_{s2} + x_{m1}x_{m2}) \\
& \quad - \hat{a}_5(x_{s2}x_{s3} + x_{m2}x_{m3}) + (\hat{a}_{11} - \hat{a}_{12})e_2 + 2\hat{a}_7(x_{s2}^2 + x_{m2}^2) - K_2e_2 \\
v_3 &= -\hat{a}_1(x_{s1}x_{s2}x_{s3} + x_{m1}x_{m2}x_{m3}) + \hat{a}_4(x_{s1}x_{s3} + x_{m1}x_{m3}) \\
& \quad + \hat{a}_5(x_{s2}x_{s3} + x_{m2}x_{m3}) - \hat{a}_{13}e_3 + 2\hat{a}_8(x_{s3}^2 + x_{m3}^2) - K_3e_3.
\end{align*}
\]

where \( K_1, K_2, K_3 \) are positive gaining constants.

**Figure 1.** Phase diagram of 3-D chaotic chemical reactor system in \( x_{m1} - x_{m2} - x_{m3} \) space.

Defining state errors as

\[
\begin{align*}
e_1 &= x_{s1} + x_{m1} \\
e_2 &= x_{s2} + x_{m2} \\
e_3 &= x_{s3} + x_{m3} \quad (6)
\end{align*}
\]
By putting the expressions for controllers (8) in error dynamics (7), one finds that
\[
\begin{align*}
\dot{e}_1 &= (a_2 - \hat{a}_2)(x_{s1}x_{s2} + x_{m1}x_{m2}) - 2(a_6 - \hat{a}_6)(x_{s1}^2 + x_{m1}^2) \\
&\quad + ((a_9 - \hat{a}_9) - (a_{10} - \hat{a}_{10}))e_1 - K_1e_1 \\
\dot{e}_2 &= -(a_1 - \hat{a}_1)(x_{s1}x_{s2}x_{s3} + x_{m1}x_{m2}x_{m3}) + (a_3 - \hat{a}_3)(x_{s1}x_{s2} + x_{m1}x_{m2}) \\
&\quad + (a_5 - \hat{a}_5)(x_{s1}x_{s2}x_{s3} + x_{m2}x_{m3}) - ((a_{11} - \hat{a}_{11}) - (a_{12} - \hat{a}_{12}))e_2 \\
&\quad - 2(a_7 - \hat{a}_7)(x_{s2}^2 + x_{m2}^2) - K_2e_2 \\
\dot{e}_3 &= (a_1 - \hat{a}_1)(x_{s1}x_{s2}x_{s3} + x_{m1}x_{m2}x_{m3}) - (a_4 - \hat{a}_4)(x_{s1}x_{s3} + x_{m1}x_{m3}) \\
&\quad -(a_5 - \hat{a}_5)(x_{s1}x_{s2}x_{s3} + x_{m2}x_{m3}) + (a_{13} - \hat{a}_{13})e_3 \\
&\quad - 2(a_8 - \hat{a}_8)(x_{s3}^2 + x_{m3}^2) - K_3e_3
\end{align*}
\] (9)

where $\hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4, \hat{a}_5, \hat{a}_6, \hat{a}_7, \hat{a}_8, \hat{a}_9, \hat{a}_{10}, \hat{a}_{11}, \hat{a}_{12},$ and $\hat{a}_{13}$ are estimated values of unknown parameter $a_1$, $a_2$, $a_3$, $a_4$, $a_5$, $a_6$, $a_7$, $a_8$, $a_9$, $a_{10}$, $a_{11}$, $a_{12}$ and $a_{13}$ respectively.

Describing parameter estimation error as:
\[
\begin{align*}
\hat{a}_1 &= a_1 - \hat{a}_1, \hat{a}_2 = a_2 - \hat{a}_2, \hat{a}_3 = a_3 - \hat{a}_3, \hat{a}_4 = a_4 - \hat{a}_4, \hat{a}_5 = a_5 - \hat{a}_5, \\
\hat{a}_6 &= a_6 - \hat{a}_6, \hat{a}_7 = a_7 - \hat{a}_7, \hat{a}_8 = a_8 - \hat{a}_8, \hat{a}_9 = a_9 - \hat{a}_9, \hat{a}_{10} = a_{10} - a_{10}. \\
\hat{a}_{11} &= a_{11} - \hat{a}_{11}, \hat{a}_{12} = a_{12} - \hat{a}_{12}, \hat{a}_{13} = a_{13} - \hat{a}_{13}
\end{align*}
\] (10)

Using (10), the error dynamics (9) transforms to
\[
\begin{align*}
\dot{\hat{a}}_1 &= \hat{a}_2(x_{s1}x_{s2} + x_{m1}x_{m2}) - 2\hat{a}_6(x_{s1}^2 + x_{m1}^2) + (\hat{a}_9 - \hat{a}_{10})e_1 - K_1e_1 \\
\dot{\hat{a}}_2 &= -\hat{a}_1(x_{s1}x_{s2}x_{s3} + x_{m1}x_{m2}x_{m3}) + \hat{a}_3(x_{s1}x_{s2} + x_{m1}x_{m2}) \\
&\quad + \hat{a}_5(x_{s1}x_{s2}x_{s3} + x_{m2}x_{m3}) - (\hat{a}_{11} - \hat{a}_{12})e_2 - 2\hat{a}_7(x_{s2}^2 + x_{m2}^2) - K_2e_2 \\
\dot{\hat{a}}_3 &= \hat{a}_1(x_{s1}x_{s2}x_{s3} + x_{m1}x_{m2}x_{m3}) - \hat{a}_4(x_{s1}x_{s3} + x_{m1}x_{m3}) \\
&\quad - \hat{a}_5(x_{s1}x_{s2}x_{s3} + x_{m2}x_{m3}) + \hat{a}_{13}e_3 - 2\hat{a}_8(x_{s3}^2 + x_{m3}^2) - K_3e_3
\end{align*}
\] (11)

Differentiation of parameter estimation error (10) gives
\[
\begin{align*}
\dot{\hat{a}}_1 &= -\dot{\hat{a}}_1, \dot{\hat{a}}_2 = -\dot{\hat{a}}_2, \dot{\hat{a}}_3 = -\dot{\hat{a}}_3, \dot{\hat{a}}_4 = -\dot{\hat{a}}_4, \dot{\hat{a}}_5 = -\dot{\hat{a}}_5, \\
\dot{\hat{a}}_6 &= -\dot{\hat{a}}_6, \dot{\hat{a}}_7 = -\dot{\hat{a}}_7, \dot{\hat{a}}_8 = -\dot{\hat{a}}_8, \dot{\hat{a}}_9 = -\dot{\hat{a}}_9, \dot{\hat{a}}_{10} = -\dot{\hat{a}}_{10}, \\
\dot{\hat{a}}_{11} &= -\dot{\hat{a}}_{11}, \dot{\hat{a}}_{12} = -\dot{\hat{a}}_{12}, \dot{\hat{a}}_{13} = -\dot{\hat{a}}_{13}
\end{align*}
\] (12)

Describing Lyapunov function as
\[
V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + \hat{e}_1^2 + \hat{e}_2^2 + \hat{e}_3^2 + \hat{e}_4^2 + \hat{e}_5^2 + \hat{e}_6^2 \\
+ \hat{e}_7^2 + \hat{e}_8^2 + \hat{e}_9^2 + \hat{e}_{10}^2 + \hat{e}_{11}^2 + \hat{e}_{12}^2 + \hat{e}_{13}^2)
\] (13)

which implies that $V$ is positive definite.

On differentiating the Lyapunov function $V$, we get
\[
\dot{V} = c_1\dot{e}_1 + c_2\dot{e}_2 + c_3\dot{e}_3 - \hat{a}_1\dot{\hat{a}}_1 - \hat{a}_2\dot{\hat{a}}_2 - \hat{a}_3\dot{\hat{a}}_3 - \hat{a}_4\dot{\hat{a}}_4 - \hat{a}_5\dot{\hat{a}}_5 - \hat{a}_6\dot{\hat{a}}_6 \\
- \hat{a}_7\dot{\hat{a}}_7 - \hat{a}_8\dot{\hat{a}}_8 - \hat{a}_9\dot{\hat{a}}_9 - \hat{a}_{10}\dot{\hat{a}}_{10} - \hat{a}_{11}\dot{\hat{a}}_{11} - \hat{a}_{12}\dot{\hat{a}}_{12} - \hat{a}_{13}\dot{\hat{a}}_{13}.
\] (14)
In view of (14), the parameter updating laws are defined by the rule:

\[
\begin{align*}
\dot{a}_1 &= -(x_{11}x_{22}x_{33} + x_{m1}x_{m2}x_{m3})e_2 \\
\dot{a}_2 &= (x_{11}x_{22} + x_{m1}x_{m2})e_1 + K_5 \hat{a}_1 \\
\dot{a}_3 &= (x_{11}x_{22} + x_{m1}x_{m2})e_2 + K_6 \hat{a}_3 \\
\dot{a}_4 &= -(x_{11}x_{33} + x_{m1}x_{m3})e_3 + K_7 \hat{a}_4 \\
\dot{a}_5 &= (x_{12}x_{33} + x_{m2}x_{m3})e_2 - (x_{22}x_{33} + x_{m2}x_{m3})e_3 + K_8 \hat{a}_5 \\
\dot{a}_6 &= -2(x_{11}^2 + x_{m1}^2)e_1 + K_9 \hat{a}_6 \\
\dot{a}_7 &= -2(x_{12}^2 + x_{m2}^2)e_2 + K_{10} \hat{a}_7 \\
\dot{a}_8 &= -2(x_{13}^2 + x_{m3}^2)e_3 + K_{11} \hat{a}_8 \\
\dot{a}_9 &= e_1^2 + K_{12} \hat{a}_9 \\
\dot{a}_{10} &= -e_2^2 + K_{13} \hat{a}_{10} \\
\dot{a}_{11} &= -e_3^2 + K_{14} \hat{a}_{11} \\
\dot{a}_{12} &= e_2^2 + K_{15} \hat{a}_{12} \\
\dot{a}_{13} &= e_3^2 + K_{16} \hat{a}_{13},
\end{align*}
\]

where \( K_4, K_5, K_6, K_7, K_8, K_9, K_{10}, K_{11}, K_{12}, K_{13}, K_{14}, K_{15} \) and \( K_{16} \) are positive gaining constants.

**Theorem 4.1.** The chaotic chemical reactor systems (4)-(5) are asymptotically anti-synchronized for all initial states \((x_{m1}(0), x_{m2}(0), x_{m3}(0)) \in \mathbb{R}^3\) by adaptive controller (8) and the parameter updating law (15).

**Proof.** The Lyapunov function \( V \) as defined in (13) is a positive definite function. By combining equations (11), (14) and (15), we have

\[
\dot{V} = -K_1 e_1^2 - K_2 e_2^2 - K_3 e_3^2 - K_4 \hat{a}_1^2 - K_5 \hat{a}_2^2 - K_6 \hat{a}_3 - K_7 \hat{a}_4 - K_8 \hat{a}_5 - K_9 \hat{a}_6 - K_{10} \hat{a}_7 - K_{11} \hat{a}_8 - K_{12} \hat{a}_9 - K_{13} \hat{a}_{10} - K_{14} \hat{a}_{11} - K_{15} \hat{a}_{12} - K_{16} \hat{a}_{13} < 0,
\]

which deduces that \( \dot{V} \) is negative definite.

Hence, by using Lyapunov stability theory, we conclude that the discussed anti-synchronized error \( e(t) \to 0 \) exponentially as \( t \to \infty \) for each initial conditions \( e(0) \in \mathbb{R}^3 \). This ends the proof. \( \square \)

### 5. Numerical Simulation

This section presents some simulation results for illustrating the effectiveness of the considered anti-synchronization strategy via ACM. Here, initial states of master (4) and slave systems (5) are \((200, 200, 100)\) and \((1, 2, 3)\) respectively. The control gains are selected as \( K_i = 10 \) for \( i = 1, 2, \ldots, 16 \). Further, Figure 2(A-C) show the time series of the given system. Simulation result-s are displayed in Figure 3(A-C) which exhibit anti-synchronized trajectories of systems (4) and (5). In addition, Figure 3(D) depicts the synchronization errors \((e_1, e_2, e_3) = (201, 202, 103)\) that tends to zero with \( t \) tending infinity. Moreover, Figure 3(E) indicates that the estimated values \( \hat{a}_1, \hat{a}_2, \hat{a}_3, \hat{a}_4, \hat{a}_5, \hat{a}_6, \hat{a}_7, \hat{a}_8, \hat{a}_9, \hat{a}_{10}, \hat{a}_{11}, \hat{a}_{12}, \text{and } \hat{a}_{13} \) of unknown parameters converge to their real values asymptotically with time. Therefore, the proposed anti-synchronization scheme in master and slave system using ACM is attained computationally.
5.1. A Comparative Study. In [33], the chaotic behaviour of the considered system (4) for few of the parameters using phase plane, Lyapunov exponent plots and bifurcation diagrams has been investigated. Further, nonlinear active plus integral sliding mode control is proposed and analysed for system (4) using LST. It is noticed that synchronized errors are converging to zero at $t = 0.005$ (approx.) as discussed in [33], whereas in our research work, the anti-synchronization technique is attained using ACM, in which it is noted that synchronization errors are converging to zero at $t = 0.5$ (approx.) as displayed in Figure 3(D). In addition to this, our study designs the parameter updating law which identifies the unknown parameters of the given system. This shows that our proposed anti-synchronization approach via adaptive control method is more preferred over other published work.

6. Conclusion. In this paper, we have investigated the proposed anti-synchronized scheme in identical chaotic chemical reactor systems using ACM. By designing appropriate controllers based on LST, the considered anti-synchronized approach is attained. The efficacy and accuracy of the computational results are verified in simulations by using MATLAB. Our study indicates that the discussed strategy is basic yet numerically rigorous. Furthermore, we understand that the proposed anti-synchronization strategy would be generalized by applying other control and synchronization techniques.
Figure 3. Anti-synchronization state trajectories of 3-D chaotic chemical system (A) between $x_{m1}(t) - x_{s1}(t)$, (B) between $x_{m2}(t) - x_{s2}(t)$, (C) between $x_{m3}(t) - x_{s3}(t)$, (D) synchronization error of the system, (E) Parameter estimation

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Received December 2020; 1st revision April 2021; Final revision June 2021; Early access July 2021.

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