Lack of collective motion in granular gases of rotators

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Abstract

The dynamics of gases made of particles interacting dissipatively—known as granular gases—can be fully described by the translational and rotational motion of the individual particles; however, most of the results in the field refer to the limit of smooth particles, which implies that the rotational degrees of freedom are suppressed. Here we investigate the opposite limit: we consider a granular gas where the translational degrees of freedom are suppressed, and the key degrees of freedom are rotational. Our results indicate that for many-particle systems of pure rotators collective effects almost completely suppressed. This is in a sharp contrast to granular gases of smooth particles and other conventional matter where the translational degrees of freedom dominate the kinetics.

1. Introduction

The dilute limit of granular systems—granular gases—has been intensively studied both experimentally and theoretically, mainly because of their rich phenomenology. Unlike dense granular matter, in granular gases the particles interact via instantaneous collisions rather than long-lasting contacts. Therefore, the dynamics of the particles is characterized by ballistic motion interrupted by collisions of short duration. In the limit of instantaneous collisions corresponding to perfectly hard and rigid particles, the particles undergo exclusively binary interactions. By adding the assumption of molecular chaos, granular gases can be described by the kinetic theory on the level of the Boltzmann equation. As a characteristic for granular systems, particle–particle interactions are dissipative, and coefficients of restitution quantify the loss of kinetic energy due to collisions. While a general theoretical description of granular systems is still missing, for the case of granular gases, we can apply the tools of the kinetic theory of molecular gases with appropriate modifications, see e.g. [1–3]. The kinetic theory of granular gases has been successfully applied to describe rapid granular flows in many physical situations; see [4] and many references therein.

Albeit by definition, granular gases are similar to molecular gases, except for the loss of kinetic energy due to collisions. There are several exciting features of granular gases that do not have equivalents in the physics of molecular gases. Examples are instabilities of the homogeneous state [5, 6], anomalous diffusion [7], violation of energy equipartition [8, 9], and even correlations between the translational and rotational velocities [10, 11]. While these phenomena apply to the force-free case, that is, in the absence of an external supply of energy, there are also several exciting effects in driven granular gases, e.g., in the presence of gravity. Here we observe giant fluctuations [12], floating cluster instabilities (also called granular Leidenfrost effect) [13, 14], several types of Maxwell daemon effects [15–18], self-organized shocks [19, 20], and others.
Figure 1. Interacting vibrots. (a) Each of the particles is caged in its cavity where it can freely rotate between collisions due to the vibration of the plate underneath. The translational motion is strongly constrained by the cavity. (b) Detailed view of the vibrot explaining the parameters of the system $d_c$ (the maximal diameter of the vibrot), $d_{eff}$ (the effective diameter, defined in section 'kinetic theory'), $r_h$ (the radius of the cavity) and $r_l$ (the distance from the vibrot center to a leg).

Similar to molecular gases, the most fundamental characteristic of a granular gas is its velocity distribution function. The important hydrodynamic fields of density, pressure, flow velocity, temperature, etc, are related to the velocity distribution function’s low moments. Unlike molecular gases where the velocity distribution in equilibrium obeys a Maxwell distribution, granular gases reveal significant non-Maxwellian behavior: for velocities close to the thermal velocity, $v/v_T \sim 1$, the deviations of the distribution function from the Maxwell distribution can be characterized by low order coefficients of a Sonine expansion [1, 21, 22]. A more fundamental deviation can be found for the high energy tail, $v/v_T \gg 1$, where the distribution function shows an overpopulated tail, $f(v) \sim \exp(-v^\alpha)$, where $\alpha = 1$ for the force-free homogeneous cooling state [19] and $\alpha = 3/2$ for the case of a homogeneously heated system [23].

The kinetic theory of granular gases is well developed as far as the limit of smooth spheres is concerned, and most of the published results mentioned above refer to systems of smooth spheres or disks. In the smooth (frictionless) limit, the particles’ rotational velocities do not change due to collisions, so the rotational degrees of freedom are not considered. Only a small fraction of the literature on granular gas theory deals with frictional particles, e.g. [11, 24]. The limit of smooth particles is frequently considered since the kinetic theory of frictional particles is much more involved, and even simple results require either strong (and questionable) assumptions on the coupling between rotation and translation and/or very extensive calculations, e.g. [10, 11, 24, 25]. Nevertheless, we wish to point out that the limit of mathematically smooth particles is problematic since the particles cannot transfer torque between one another, so the rotational and translational degrees of freedom decouple. Consequently, any initial distribution of the rotational velocity is stable, i.e., there is no thermalization concerning the rotational motion. Note that similar artifacts for the translational velocity of a one-dimensional granular gas led to the conclusion of a ‘breakdown of hydrodynamics’ [26].

Here we study the opposite limit of a granular gas: we consider a gas of particles that rotate and exchange angular momentum while the translational degrees of freedom are suppressed. Our experimental system is comprised of interacting rotators, called vibrots [27–32], whose spatial positions are fixed on a lattice. We analyze the impact of particle interactions on the system’s overall dynamics and the role of collective modes. Surprisingly, we find the system’s physical behavior rather independent of the topology of the lattice and—in sharp contrast to granular gases of smooth particles—the collective motion is unimportant for the overall dynamics of the system. Instead, the dynamics is mainly determined by random uncorrelated collisions between particles and their neighbors. Based on our experimental findings, we develop a kinetic theory for granular gases in the limit of pure rotators and find good agreement with the experimental data. Moreover, both experiment and theory show that even a significant increase in the system’s density does not noticeably enhance collective effects. Again, this is in sharp contrast with systems dominated by translational motion, where increasing density results in a strongly correlated motion of the particles.

2. Experimental setup

We studied a two dimensional gas of active granular particles sketched in figure 1(a).

These particles named vibrots [27] and sketched in figure 1(b) are disks resting on circularly aligned tilted elastic legs. An additional cylinder is attached below the disk in order to keep the center of mass low. When placed on a vertically vibrating table, these particles transfer vibrational energy into rotation through
Table 1. Characteristics of the regular lattice templates used in the experiments. Both the lattice type and lattice constant determine the intensity of the interaction between the particles.

| Lattice type | #neighbors | Lattice constant |
|--------------|------------|-----------------|
| Linear       | $n = 2$    | $a = \{18, 19, 20, 21, 22\}$ mm |
| Triangular   | $n = 3$    | $a = \{18, 19, 20, 21, 22\}$ mm |
| Rectangular  | $n = 4$    | $a = \{18, 19, 20, 21, 22\}$ mm |
| Hexagonal    | $n = 6$    | $a = \{18, 19, 20, 21, 22\}$ mm |

an interplay of friction, inertial forces, and inelastic interaction between the particle and the vibrating table [28, 29]. The motion of the particles depends on the vibration parameters and characteristics of the vibrot such as number and inclination of legs, total mass, and others [28, 30]. It was shown that a gas of such particles has very similar properties to a homogeneously heated granular gas [31]. In particular, it was shown that the velocity distribution function shows the same overpopulated high-energy tail as for granular gases, up to high precision. Like granular gases, the diffusion of such gases deviates from simple gas diffusion due to the finite inertia of the particles [32]. This is different from other systems of active particles, such as [33–37].

In the experiments, we used identical vibrots with a 2 mm-thickness toothed wheel head of maximum diameter $d_t = 21$ mm and teeth depth of 3 mm, resting on seven legs tilted by 18°, see figure 1. The teeth are designed to enhance the transfer of angular momentum when particles collide. Another cylinder of diameter 8 mm and height 6 mm is attached underneath the toothed wheel to lower the particle’s center of mass (these cylinders are seen at the bottom of each toothed wheel in figure 1(a), and on top of the toothed wheel in the lower panel of figure 1(b).

A large number of vibrots were produced by additive manufacturing using a formlabs 3D-printer. As printing material, grey resin consisting of a mixture of urethane dimethacrylate and methacrylate monomer(s) was used [38]. The resulting 3D-printed particles have density, $\rho = 1.08$ g cm$^{-3}$ and Young modulus $Y = 2.8$ GPa. A white cross marks the particles on the top to determine the positions and orientations using image processing, see figure 1.

We placed the particles within the cavities of different templates. Such templates were attached to a vibrating table, consisting of a smooth circular acrylic plate of diameter 300 mm and width 30 mm. The vibrating table was attached to an electromagnetic shaker, and the whole setup was leveled horizontally at an accuracy of $1 \times 10^{-3}$ degrees. The vibration frequency and amplitude were set to $f = 80$ Hz and $A_v = 0.155$ mm (peak-to-peak), respectively. We recorded the motion of the particles using a high-speed camera at 80 frames/s. Using standard image recognition methods, we tracked the particles’ positions and orientations with sub-pixel accuracy and subsequently computed their rotational velocities.

The translational motion of the particles on the table is suppressed by means of a template rigidly attached to the vibrating table 1 (a). The template is textured by a regular lattice of circular cavities forming individual cages to the particles from where they cannot escape. The type of the lattice and the lattice constant, $a$, determine the number of next neighbors of each particle, $n$, and the distance between next neighbors, respectively. Both lattice type and lattice constant determine the intensity of interaction between the particles. The radius of the cavities was chosen $r_b = 8.5$ mm, allowing a translational motion of the active particles of $r_t - r_b = 1.5$ mm in every radial direction measured from the center of the cavity. Here, $r_t = 7.0$ mm represents the distance from the center of the vibrot to the most extreme point of a leg.

3. Experimental results

3.1. Statistical properties of rotation motion

We study the angular velocity distribution function, $f(\omega)$ (normalized as $\int f(\omega)d\omega = 1$) and in particular its first and second moments, that is, the average angular velocity, $\overline{\omega}$, and the variance, $\text{var(}\omega)$, as a function of the number of next neighbors, $n$, and the lattice constant of the grid of cavities, $a$. Before investigating the collective properties of the granular gas, we carefully checked the one-particle distribution function of isolated vibrots. In spite of the fact that we obtain that the mean rotational velocity is slightly smaller than for the collision-free rotors studied by [31], we corroborate the asymmetric Gaussian shape of $f(\omega)$ shown in figure 2(b) in reference [31]. In summary, collisions with the wall of the cavity only reduce the mean rotational velocity while conserving the direction of rotation as well as the shape of the distribution. In our study we explore mostly the systems where the inter-particle collisions dominate and collisions with the walls play a minor role.

Possible values of $n$ for a two-dimensional regular and homogeneous lattice are $n \in \{2, 3, 4, 6\}$, where $n = 4$ requires a rectangular lattice and $n \in \{3, 6\}$ a hexagonal lattice. For $n = 2$, both lattice types are
Figure 2. Rotational granular gas lattices. By appropriate choice of the arrangement of the cavities, regular lattices of different types can be obtained where the particles have a different number of neighbors. (a) Two neighbors; (b) three neighbors; (c) four neighbors; (d) six neighbors. Also, the lattice constant, a can be chosen.

possible. Figure 2 shows snapshots of gases for different lattices, and table 1 summarises the characteristics of the lattices used.

For each finite system, there are particles close to the boundary having less than n neighbors. Such particles have been excluded from the subsequent analysis. Obviously, the results must depend on the number of elements of the gas, N, since the rotation of a particle is affected by the rotational motion of its neighbors. As an example, figure 3(a) shows the average rotational velocity as a function of the number of gas molecules for the case n = 2. Here, the particles form a chain, such that each molecule has two neighbors, except the first and the last ones, see figure 2(a).

For all values of the lattice constant, 18 mm ≤ a ≤ 2 mm, we find that the fluctuations of the average rotational velocity are small if the systems consist of at least about 15 particles. That is, the properties of larger systems (N > 15) are determined by the bulk part and the boundary effects are not important, figure 3(a). This is an interesting property of the system – due to the lack of collective behavior, as it is shown below, even a relatively small system demonstrates the size independence of its properties.

We also see that ω depends strongly on the value of the lattice constant, i.e., on the average distance of next neighbors. While for a = 18 mm we find ω ≈ 0, for a = 22 mm, the value of ω approaches the isolated particles’ value. Figure 4 shows the function ω(a) obtained from experiments.

Similar to the average value ω, the entire distribution function is invariant with respect to the system size, provided we have more than about 15 particles. Figure 3(b) shows the experimentally measured distribution function of the rotational motion, f(ω) for different system sizes N. Except for the case N = 1 where the system consists only of one particle with a complete neighborhood, all other distribution functions are rather similar. Full convergence is reached for N ≳ 15, evidenced by the fact that the curves for N = 15 and N = 28 coincide, up to small fluctuations. The dashed line in figure 3(b) represents an asymmetric Gaussian fit for N = 28, showing overpopulated high-energy tails of the distributions, which differs from the behavior of single isolated particles, see figure 2(b) in reference [31].

Figure 4 (black dots) shows the experimentally measured values ω(a), corresponding to n = 2 and N = 28. The dashed line represents the theoretical prediction discussed in section kinetic theory based on the simplified Boltzmann equation (5). From the good agreement between the experimental data and the theory, we conclude that the kinetic theory captures the problem’s physics with good reliability. We will discuss this statement in more detail below.

Figure 4 reveals a sensitive dependence of the average rotational velocity on the lattice constant, and also on the number of next neighbors (see the inset). Increasing the number of neighbors increases the probability of collisions, which lowers the system’s temperature due to their dissipative nature.
Figure 3. Rotational velocities for a gas consisting of a line of vibrots. (a) The average rotational velocity $\bar{\omega}$ as a function of the number of gas molecules, $N$ for the case $n = 2$. In the experiment, we used $N + 2$ particles, but only $N$ of them have two neighbors. The graph is shown for different values of the lattice constant $a \in \{18, \ldots, 22\}$ mm. In all cases, for $N \geq 15$, fluctuations of $\bar{\omega}$ are small. (b) Experimental result for the probability distribution function of the rotational velocity, $f(\omega)$ for the case $n = 2$ and $a = 18$ mm for different system size, $N \in \{1, \ldots, 28\}$. The gas consisted of $N + 2$ particles; the first and last are disregarded for the data analysis. The dashed line represents an asymmetric Gaussian fit for $N = 28$.

Consequently, the mean rotational velocity decreases. Therefore, when the strength of the interaction decreases i.e., the number of next neighbors decreases, $n = 2$, and the lattice constant increases, $a = 22$ mm, the characteristics of a single isolated particle, $f(\omega)$, $\bar{\omega}$ and $\text{var}(\omega)$, must be recovered. For this case, we obtain $\bar{\omega} \approx -11.8$ rad s$^{-1}$, which is close to the value for isolated vibrots, $\omega_0 = -11.9$ rad s$^{-1}$.

Figure 4 also shows the dependence of the variance of $\omega$ on the lattice constant for $n = 2$. It is represented by the error bars in the inset and also by the color graphs showing the velocity distribution function, $f(\omega)$. Here, the vertical extension of the color graphs stands for the variance. For different neighborhood relation, $n \in \{2, 3, 4, 6\}$, figure 5 shows the variance, $\text{var}(\omega)$, over the lattice constant, $a$. Concerning the variance, both extremes, $a \to 0$ and $a \to \infty$, are equivalent. The former case that may be called frustrated or jammed-- using the familiar term in the field of granular matter [39–41]-- corresponds to a dense system with very low temperature. In this case, the average rotational velocity ceases, $\bar{\omega} \approx 0$.

Here, essentially all vibrots are entangled, such that any significant motion of one vibrot would require a collective motion of all others, which is frustrated. Consequently, the distribution function, $f(\omega)$, is sharply peaked around zero and reveals a small variance. The latter extreme is analogous to a rarified Knudsen gas: in a Knudsen gas, molecules collide more frequently with the walls of a vessel than with themselves. In our system, the particle–particle collisions are rarer than collisions with the walls of the cavities. These results show that both the mean and the variance of the rotational velocity coincide with the ones of isolated rotators, which represent an extremely rarified gas. That is, in this limit, only collisions with the walls of the...
Figure 4. Rotational velocities for a gas consisting in a line of vibrots. Average rotational velocity, $\omega$, as a function of the lattice constant, $a$, for the case $n = 2$ and $N = 28$. Black dots show the experimental time-averaged data. The color graph around each data point shows the distribution function, $f(\omega)$, for discussion see the text. The dashed line shows $\omega(a)$ as given by equations (11) and (13) for $\gamma = 0.1 \text{ s}^{-1}$, $\Gamma = 0.6 \text{ s}^{-1}$, and $\omega_0 = -11.9 \text{ rad s}^{-1}$. Inset: average rotation velocity as a function of the lattice constant for different number of next neighbors, $n \in \{3, 4, 6\}$.

Figure 5. Variance of the rotational velocity, $\text{var}(\omega)$, as a function of the lattice constant, $a$, for $n \in \{2, 3, 4, 6\}$. The inset shows the probability distribution function of the rotational velocity, $f(\omega)$, for the same values of $n$ and $a = 21 \text{ mm}$, which corresponds to the maximum variance; see the main figure. The dashed lines show Gaussian fits.

cavity are relevant for the dynamics of the system. Hence, the distribution function is sharply peaked around the mean value, which is determined by the interplay of the driving torque and friction. Albeit both limits correspond to small variance, the mechanisms and, thus, the numerical values of the variance are different: in the jammed case, the small variance is a consequence of the inter-particle collisions and entanglement, while in the Knudsen regime, it arises from the random torques acting on isolated vibrots. This difference can be seen in figure 5 by comparing $\text{var}(a = 18 \text{ mm})$ and $\text{var}(a = 22 \text{ mm})$.

The maximum variance is achieved for some intermediate value $a'$. While for small $a$, entanglement between the particles impedes independent motion enforcing small variance, for large values of $a$ the
variance is only due to the fluctuating driving; otherwise, the particles are almost independent. For intermediate values, \( a \approx a' \), frequently occurring collisions lead to substantial changes of the rotational velocities, implying large variance. Indeed, for bouncing collisions, the change of the rotational velocity, \( \Delta \omega \propto \omega \) (see theoretical model below), so the variance increases with the average rotational velocity \( |\omega| \). In turn, it increases with the lattice constant \( a \), see figure 4. This mechanism does not apply to collision-free systems. That is why the variance decreases for very large \( a \). Consequently, we can understand the plot shown in figure 5 from plausible arguments.

The inset in figure 5 shows the probability distribution function, \( f(\omega) \) for \( a = 21 \) mm, and other numbers of next neighbors, \( n \in \{2, 3, 4, 6\} \), together with Gaussian fits. The remarkable agreement between the experimental data and the fits supports the hypothesis of molecular chaos, discussed in the next section. Note that both \( \Sigma \) and \( \sigma(\omega) \) do not significantly depend on \( n \). In systems dominated by the translational degrees of freedom, like liquids or crystals, the collective motion arises from a correlation between particles and their local environment \([42, 43]\). The latter is completely determined by the density of particles, associated with the number of next neighbors \( n \) and the distance to the next neighbors. In our system, however, figures 4 and 5, do not show significant changes in the system’s dynamics for different numbers of next neighbors, as expected for systems dominated by translational motion. From this observation, we conclude that the collective motion, which should be different for different lattices, does not play a major role in the system’s dynamics. Instead, the dynamics is governed by random uncorrelated collisions, again supporting the assumption of molecular chaos.

3.2. Characteristics of collective motion

As have been stated above, the explored statistical properties of the rotational motion do not manifest any collective behavior in fluids with a pure rotation. This is in a sharp contrast with ordinary fluids, which display collective modes. To analyze the collective behavior for translational motion, the van Hove functions are exploited \([42, 43]\). To analyze the collective effects for orientational degrees of freedom, Landau-De Gennes order parameter for liquid crystals has been introduced \([44]\), which is associated with the local director of the system. For two-dimensional nematics, De Gennes proposed another characteristic which seems to be more relevant here \([45]\). Let us define \( \theta(\mathbf{r}) \), where \( \theta \in [-\pi, \pi] \), denotes an angle between the laboratory axis \( OX \) and the vector specifying the orientation of the vibrot, located at point \( \mathbf{r} \). Then, the following function characterizes the orientational correlations:

\[
\Phi(\mathbf{r}_{12}, t) = \langle (\theta(\mathbf{r}_1, 0) - \theta(\mathbf{r}_2, t))^2 \rangle. \tag{1}
\]

If the system is spatially uniform and isotropic, \( \langle \theta(\mathbf{r}, t) \rangle = 0 \), and lacks the orientational correlations, the function \( \Phi(\mathbf{r}_{12}, t) \) reads

\[
\Phi(\mathbf{r}_{12}, t) = \langle \theta^2(\mathbf{r}_1, 0) \rangle + \langle \theta^2(\mathbf{r}_2, t) \rangle - 2\langle \theta(\mathbf{r}_1, 0)\theta(\mathbf{r}_2, t) \rangle \\
= 2\langle \theta^2 \rangle - \frac{2}{3}\pi^2, \tag{2}
\]

which corresponds to the uniform angular distribution \( 1/(2\pi) \) in the interval \( [-\pi, \pi] \).

Additionally in our system it is worth to study the angular velocity correlation function:

\[
\Phi_v(\mathbf{r}_{12}, t) = \langle (\omega(\mathbf{r}_1, 0) - \overline{\omega}) (\omega(\mathbf{r}_2, t) - \overline{\omega}) \rangle, \tag{3}
\]

where \( \overline{\omega} \) is the average angular velocity. For vanishing correlations the above function is trivial, i.e. \( \Phi_v(\mathbf{r}_{12}, t) = 0 \).

For the addressed lattice systems the functions \( \Phi(\mathbf{r}_{12}, t) \) and \( \Phi_v(\mathbf{r}_{12}, t) \) are defined only for \( \mathbf{r}_{12} \) compatible with the distances between the lattice sites. Figures 6 and 7 illustrate the behavior of \( \Phi(\mathbf{r}_{12}, t) \) and \( \Phi_v(\mathbf{r}_{12}, t) \) for \( n = 6 \) and \( a = 20 \) mm and \( a = 19 \) mm, respectively. As it may be seen from the figures, correlations in the rotational motion are very small; comparable with the accuracy of the data.

4. Kinetic theory

For the theoretical description of the system, we take into account that each vibrot is caged in its cavity. Its position inside the cavity is uniformly distributed within the area accessible to its center. For an ideal system with totally pure rotational motion the translational velocity distribution function \( f(v) \) would be a delta function centered at zero, \( v = 0 \). In our system the residual translational motion does exist, but its impact on the dynamics of the system is negligible. The corresponding distribution function \( f(v) \) is sharply peaked near zero, with the width, significantly smaller than that of the rotational velocity distribution function, see
Figure 6. Orientational correlation function, $\Phi(r_{12}, t)$, defined by equation (1) for $n = 6$ and $a = 20$ mm. For uncorrelated rotational motion, $\Phi(r_{12}, t) = \frac{1}{3} \pi^2$ (shown by the dashed line).

Figure 7. Angular velocity correlation function, $\Phi_\omega(r_{12}, t)$, equations (2) and (3), for $n = 6$ and $a = 19$. $r_{12} = a$ (blue), $r_{12} = 2a$ (yellow) and $r_{12} = 3a$ (green). No noticeable correlations are observed.

Figure 9. The detailed analysis presented in the appendix (subsection Fokker–Planck–Boltzmann equation) demonstrates that the translational motion of a vibrot can be disregarded, so that one can focus on the rotational motion only. Hence we start with the Langevin equation for the angular velocity which fully describes the dynamics.

$$\frac{d\omega}{dt} = M + \xi(t) - \gamma' \omega + M_{col}(t),$$

where $I$ is the moment of inertia of a vibrot. The motion is driven by a constant torque, $M$, due to the vibration of the table and Gaussian white noise, $\xi(t)$, with zero mean and intensity $\Gamma'$, $\langle \xi(t) \rangle = 0$, $\langle \xi(t)\xi(t') \rangle = \Gamma' \delta(t - t')$. The frictional force $-\gamma' \omega$, damps the rotation of the vibrots, where $\gamma'$ is the friction coefficient. The interaction between the considered vibrot and its neighbors is described by the torque $M_{col}(t)$. The corresponding Fokker–Planck–Boltzmann equation [1, 42] for the rotational velocity distribution function, $f(\omega, t)$, reads (see appendix (subsection Fokker–Planck–Boltzmann equation) for more detail):

$$\frac{\partial f}{\partial t} + \frac{M}{I} \frac{\partial f}{\partial \omega} = \gamma' \frac{\partial \omega f}{\partial \omega} + \frac{\Gamma'}{2} \frac{\partial^2 f}{\partial \omega^2} + L_{col}(f),$$

(5)
where $\gamma = \gamma'/I$ and $\Gamma = \Gamma'/I^2$. The collision integral $I_{col}(f, f)$ describes the instantaneous change of the rotational velocity of a vibro due to a collision with one of its neighbors. It can be written as a sum of a gain and a loss term [1]:

$$I_{col} = p_n(a) \left[ f(-\omega) \int f(\omega')/(\omega' - \omega) d\omega' - f(\omega) \int f(\omega')/(\omega + \omega') d\omega' \right],$$

(6)

where $p_n(a)$ is the probability that the vibro is located at positions that lead to a collision. We assume that this probability is independent of the angular orientation of the vibro, but it is a function of the lattice constant, $a$, and the number of next neighbors, $n$, given by the topology of the lattice. Similar to the collision integral of the ordinary Boltzmann equation for granular particles [1, 42], (6) is based on the molecular chaos hypothesis and consists of gain and loss terms. These terms are, however, slightly different in detail: the gain term accounts for the collision of vibros at rotational velocity $-\omega$ and $\omega'$, resulting in the post-collision velocities $\omega$ and $-\omega'$, according to our collision model: colliding vibros exchange their rotational velocities, similar to the exchange of linear velocities of particles colliding on a line [26]. Such collisions increase the number of particles with rotational velocity $\omega$. The collision frequency of two neighboring rotators with angular velocities $\omega_1$ and $\omega_2$ is proportional to $(\omega_1 + \omega_2)$, motivated by the same arguments as the collision frequency in the ordinary Boltzmann equation, based on the collision cylinder. The (second) loss term in (6) accounts for collisions of vibros spinning with rotational velocities $\omega$ and $-\omega'$, that decrease the number of vibros with $\omega$. The (first) gain term accounts for the collisions of vibros with $\omega$ and $-\omega'$. The collision integral can be written in the form

$$I_{col} \approx p_n(a) \left[ (\bar{\omega} - \omega) f(-\omega) - (\bar{\omega} + \omega) f(\omega) \right],$$

(7)

with the average rotational velocity $\bar{\omega} \equiv \int f(\omega) \omega d\omega$ and normalization, $\int f(\omega) d\omega = 1$.

We consider the stationary state distribution $f(\omega)$, where $\partial f/\partial t = 0$, and neglect the gain term in the collision integral. This approximation is justified since for characteristic velocities, $\omega \sim \bar{\omega}$, our experiments show that $f(-\bar{\omega}) \ll f(\omega)$, see color graphs in figure 4. The same approximation was used in reference [19] to derive the overpopulated tail of the distribution function for the translational velocities in the (quasi-) stationary state. In this approximation, we obtain for the stationary state

$$\frac{\Gamma}{2} \frac{\partial^2 f}{\partial \omega^2} + \frac{\partial f}{\partial \omega} - \frac{M}{T} \frac{\partial f}{\partial \omega} - p_n(a) f(\omega)(\bar{\omega} + \omega) = 0.$$  

(8)

As an interesting detail, the analysis of (8) in the limit $|\omega| \to \infty$ reveals a biased over/under-population of the high-energy tails of $f(\omega)$, whose inclination depends on the lattice constant, $a$. The details of this analysis are given in the appendix (subsection Solution of the Fokker–Planck–Boltzmann equation in the stationary state).

To analyze the dependence of the average rotational velocity on the lattice constant, $\bar{\omega}(a)$, we multiply (8) by $\omega$ and $\omega^2$ and integrate over $\omega$:

$$\omega_0 = \bar{\omega} + \frac{\gamma}{\gamma'} \left( \omega^2 + \bar{\omega}^2 \right)$$

(9)

$$\omega_0 \bar{\omega} = \bar{\omega}^2 + \frac{p_n(a)}{2\gamma} \left( \omega^2 \bar{\omega}^2 + \bar{\omega}^4 \right) - \frac{\Gamma}{2\gamma},$$

(10)

where $\omega_0 \equiv M/I\gamma$. To close this set of equations, we need an equation for $\bar{\omega}^2$, which, however, would contain even higher powers, resulting in an infinite hierarchy of equations. To obtain an approximate solution, we compute $\omega^2$ from equations (9) and (10) with $p_n(a) = 0$, and substitute the solution, $\bar{\omega}^2 = \omega_0^2 + \Gamma/(2\gamma)$, into (9):

$$\bar{\omega} = \frac{\gamma}{4p_n(a)} \left[ \sqrt{1 + 8\omega_0} + \frac{p_n(a)}{\gamma} - \frac{4\Gamma}{\gamma} \left( \frac{p_n(a)}{\gamma} \right)^2 - 1 \right].$$

(11)

As expected, for vanishing collision frequency, $\lim_{p_n(a) \to 0} \bar{\omega} = \omega_0$.

To evaluate (11), we still need the probability $p_n(a)$. We assume that it does not depend on the rotational velocities of adjacent vibros but only on their relative location: two vibros might collide if the distance between their centers is less than $d$, see figure 1(b). Thus, the probability, $p_n(a)$, of a collision of a vibro with any of its neighbors is given by

$$p_n(a) = \sum_{i=1}^{n} \frac{1}{(\pi R^2)^2} \int_{\Omega_0} d\vec{r}_0 \int_{\Omega_i} d\vec{r}_i \Theta \left( d_i - |\vec{r}_i - \vec{r}_0| \right).$$

(12)
Figure 8. Sketch for the computation of the collision probability, \( p_2(a) \). For clarity we have strongly decreased the size of the vibrots and exaggerated the size of the cavities. Therefore, in the sketch \( 2\varphi_0 < \pi \), although for the most realistic cases \( 2\varphi_0 > \pi \). The effective parameter \( d_{\text{eff}} \) accounts for the actual difference between the area of the simplified and true manifold where collisions happen.

Figure 9. Translation and rotational velocity distribution functions for \( n = 2 \) and \( a = \{18, 19, 20, 21, 22\} \) mm. Similar behaviors are obtained for \( n = \{3, 4, 6\} \).

Here we integrate over all possible locations of a vibrot, \( \vec{r}_0 \) residing in the central cavity \( \Omega_0 \) of radius \( R \) (see figure 8), and over all possible locations of vibrots \( \vec{r}_i, i = 1, \ldots, n \), residing in neighboring cavities, \( \Omega_i \), with the same radii. The factor \( d\vec{r}_i/(\pi R^2) \) describes the probability of a vibrot’s center located in a small area \( d\vec{r}_i \), and we assume that this probability is uniform within a cavity. Thus, the integrals in (12) evaluate the total area (divided by \( (\pi R^2)^2 \)) within the central cavity and one of the neighboring cavities, where the inter-center distance between two vibrots is less than \( d_{\text{eff}} \). The summation is performed over all \( n \) next neighbors. The center of cavity \( \Omega_i \) is shifted against the center of the central cavity \( \Omega_0 \) by the lattice vector \( \vec{a}_i \). The unit step function, \( \Theta(x) \), in the integrand of (12) discriminates relative positions of the vibrots that lead to a collision with the central one from those which would not collide. The integrals in (12) resemble the virial integrals in fluid cell models [43] and are rather cumbersome. In the appendix (subsection evaluation of the collision probability \( p_2(a) \)), we provide the analytical solution, but here we follow a different approach.

Consider two symmetrically located segments in neighboring cavities as sketched in figure 8. The dashed chords are perpendicular to the line connecting the centers of the cavities. The distance \( d_{\text{eff}} \) approximates \( d_{\text{v}} \), see figure 1. If the centers of the vibrots reside within the adjacent segments, they will collide for most of their positions inside the segments. The deviation of the area of the simplified manifold in figure 8 from the exact area may be corrected by the effective parameter \( d_{\text{eff}} \). With simple geometry, we obtain from (12)

\[
\begin{align*}
  p_2(a) &= 2q^2 = 2 \left[ \frac{1}{2\pi} \left( 2\varphi_0 - \sin 2\varphi_0 \right) \right]^2 \\
  \varphi_0 &= \pi - \arccos \left[ \frac{d_{\text{eff}} - a}{2R} \right].
\end{align*}
\]

The quantity \( q \) allows us to understand (13) easily. It describes the probability that one of the vibrots is located in a position where a collision with another vibrot is possible. Therefore, \( q^2 \) characterizes the probability that two vibrots are located in such a region, which guarantees a collision. The factor 2 arises from the fact that the vibrot has two neighbors.
The obtained theoretical results, (11) and (13), can be directly compared to the experimental observations: the dashed line in figure 4 shows the average rotational velocity predicted by the theory as a function of the lattice constant, $\Xi(a)$, for $\gamma = 0.1 \text{ s}^{-1}$, $\Gamma = 0.6 \text{ s}^{-3}$, and $\omega_0 = -11.9 \text{ rad s}^{-1}$. Here we use $d_{\text{eff}}/R = 7.6$, estimated from the experimental data, while the lattice constant ranges from $a = 22 \text{ mm}$, to $a = 18 \text{ mm}$. Note that $d_{\text{eff}}$ is an effective parameter, which accounts for the complicated shape of the manifold, where the vibrot center may be located to guarantee a collision. The good agreement between the theoretical and experimental data supports the assumption of molecular chaos made in the derivation.

5. Conclusion

While the limiting case of a granular gas with suppressed rotational degrees of freedom is widely discussed in the literature, here we address the opposite limit: a gas of particles with suppressed translational degrees of freedom. To that end, we studied systems of active rotators constrained to move within small circular cavities and, thus, constitute lattices of different topologies.

Surprisingly, we find that the lattice’s topology does not play a major rôle in the dynamics of the system. Instead, the dynamics of the system is mainly governed by the lattice constant. By varying it, we can significantly change the system’s dynamical properties, ranging from a Knudsen rotation gas regime corresponding to an extremely dilute hard sphere gas, to the regime of frustrated rotation corresponding to the jammed state of a hard sphere gas at high density. In between these extremes, there is a regime corresponding to an ordinary gas at intermediate density. The Boltzmann equation can describe the kinetics of ordinary gases at low to intermediate density with the assumption of molecular chaos. Here we showed that a rather similar approach, also relying on the (modified) Boltzmann equation and on the assumption of molecular chaos, leads to theoretical predictions in good agreement with experiments.

Albeit the mathematical tools for the description of ordinary gases can also be applied to gases of rotators, we have to acknowledge that both systems are fundamentally different. In a system with suppressed rotational degrees of freedom (smooth limit), $f(\vec{r})$ evolves independently of $f(\vec{\omega})$, which is invariant and arbitrary. The opposite situation where $f(\vec{\omega})$ evolves independently of an invariant and arbitrary $f(\vec{r})$ is not possible. Likewise, the statistical properties of rotator gases are different from those of ordinary gases. In particular, both experimental and theoretical results support the finding that the collective effects are rather weak and the collective motion is not important for the dynamics of rotator gases, which is in strong contrast to ordinary fluids. Surprisingly, this is true even for dense systems. Finally, we observe an interesting fact: due to the lack of collective behavior, even relatively small systems already demonstrate the size independence of their properties.

We believe that the ensemble of rotators introduced here may be a prototype of a wider class of statistical systems whose behavior is dominated by the rotational degrees of freedom.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

Appendix A

A.1. Fokker–Planck–Boltzmann equation

Generally, the translational and rotational motion of a vibrot in a cavity are described by the Langevin equations:

$$\frac{d\omega}{dt} = M + \xi(t) - \gamma' \omega + M_{\text{col}}(t)$$

$$m \frac{dv}{dt} = \xi^T(t) - \gamma'' v + F_{\text{wall}}(t),$$

where $\omega$ and $v$ are the rotational and translational velocities, respectively, $M$ and $M_{\text{col}}(t)$ are the forces due to the cavity and collisions, $\xi(t)$ and $\xi^T(t)$ are the translational and rotational noise, and $\gamma'$ and $\gamma''$ are the rotational and translational friction coefficients.
where $m$ and $I$ are the mass and moment of inertia of a vibrot and $\omega$ and $v$ are its angular velocity and linear velocity. The rotation motion is driven by a constant torque, $M$, due to the vibration of the table and Gaussian white noise, $\xi(t)$, with zero mean and intensity $\Gamma$, i.e., $\langle \xi(t) \rangle = 0$, $\langle \xi(t) \xi(t') \rangle = \Gamma \delta(t-t')$. The friction force, $-\gamma \omega$, damps the rotation of the vibrot, where $\gamma$ is the friction coefficient. The interaction between the considered vibrot and its neighbors is described by the torque $M_{\text{coll}}(t)$.

Similarly, the translational motion is driven by the Gaussian white noise $\xi_1(t)$, with zero mean and intensity $\Gamma$, that is, $\langle \xi_1(t) \rangle = 0$, $\langle \xi_1(t) \xi_1(t') \rangle = \Gamma \delta(t-t')$, where $i,j = x,y$. The friction force for the translational motion, $-\gamma_v v$, depends on the translational friction coefficient $\gamma_v$. Finally, $F_{\text{wall}}(t)$ describes the collisions of a vibrot with the walls of the cavity. As we observe in our experiments, the role of such collisions is negligible, as compared to the collisions between the vibrots themselves. Therefore, we neglect the term $F_{\text{wall}}(t)$.

Applying the standard techniques to obtain the Fokker–Planck equation from the Langevin equation, see e.g. [1, 42], we arrive at the following Fokker–Planck–Boltzmann equation for the rotational and translational velocity distribution function, $f_\psi(\omega_1, v_1, t)$:

$$
\frac{\partial f_\psi}{\partial t} + \frac{M}{I} \frac{\partial f_\psi}{\partial \omega_1} = \gamma \frac{\partial f_\psi}{\partial \omega_1} + \frac{\partial f_\psi}{\partial \omega_1} + \frac{\partial f_\psi}{\partial \omega_1} + \frac{\partial f_\psi}{\partial \omega_1} + \frac{\partial f_\psi}{\partial \omega_1} + \frac{\partial f_\psi}{\partial \omega_1} + I_{\text{coll}},
$$

(16)

where $\gamma = \gamma / I$, $\gamma_1 = \gamma_1 / m$, $\Gamma = \Gamma_1 / I$ and $\Gamma_1 = \Gamma_1 / m^2$. The collision integral describes the instantaneous change of the rotational and translational velocities of a vibrot due to a collision with one of its neighbors. It has a common structure of a difference of the gain and loss terms, $[1, 42]$,

$$
I_{\text{coll}} = \int d\omega_2 d\nu_2 Q'_{\psi} f_\psi(\omega_1, v_1, t) f_\psi(\omega_1, \nu_2, t) - \int d\omega_2 d\nu_2 Q'_{\psi} f_\psi(\omega_1, v_1, t) f_\psi(\omega_1, v_2, t)
$$

(17)

and describes the direct collision of a pair of particles with angular and linear velocities $\omega_1, v_1$ and $\omega_2, v_2$ (the loss term) and the inverse collision of the pair with $\omega'_1$, $v'_1$ and $\omega'_2$, $v'_2$ (the gain term), which results in $\omega_1, v_1$ and $\omega_2, v_2$; the inverse angular and linear velocities are functions of the direct ones. The factors $Q$ and $Q'$ are associated with the volume of the collision cylinder for the direct and inverse collision, that is, they quantify the collision frequency $[1, 42]$. In the literature, however, another form is commonly used for the gain term of the collision integral for the function $f_\psi(\omega, v, t)$ [46–51]. For the case of interest, the translational motion is frustrated, which is also clearly visible in our experiments. Indeed, the velocity distribution function, $f(\nu, t) = \int d\omega f_\psi(\omega, \nu, t)$ is narrowly peaked around $\nu = 0$, see figure 9.

In the current context it implies that neither collision cylinder nor the collision outcome noticeably depend on the linear velocities of the colliding vibrots. These depend mainly on the angular velocities and particles location.

Therefore we can write for the factor $Q$ and $Q'$, which gives the number of collisions per unit time:

$$
Q \simeq p_\psi(a)(\omega_1 + \omega_2), \quad Q' \simeq p_\psi(a)(\omega'_1 + \omega'_2).
$$

(18)

Here $p_\psi(a)$ is the probability that two vibrots location allows the collision of their blades, which depends on the lattice size $a$ and $n$ – the number of nearest neighbors. $(\omega_1 + \omega_2)$ give the rate at which the rotating blades approach. An impact of vibrots is a complicated process, therefore we apply here a simplified collision model, motivated by our experimental observations: vibrots exchange their one-dimensional rotational velocities (up to a sign), similar as two colliding particles on a line exchange their linear velocities. That is, the after-collision angular velocities $\omega'_1$, $\omega'_2$ read in terms of the pre-collision ones, $\omega_1$, $\omega_2$, as

$$
\omega'_1 = -\omega_2, \quad \omega'_2 = -\omega_1,
$$

and similar for the inverse collision:

$$
\omega''_1 = -\omega_2, \quad \omega''_2 = -\omega_1.
$$

(19)

This corresponds to an approximation of an elastic collision; similar we assume that the change of the linear velocities also corresponds to an elastic collision. A more general consideration with different restitution coefficients is also possible, but we leave this for future studies.

We will consider the angular velocity distribution function, defined as

$$
f(\omega, t) = \int f_{\psi,\nu}(\omega, \nu, t) d\nu.
$$

(20)
Integrating then equation (16) w.r.t. linear velocity $v_1$, we observe that the terms containing the velocity derivatives vanish, the loss term may be recast into the form,

$$-f(\omega, t) \int d\omega' f(\omega')(\omega' + \omega') p_n(a),$$

where we use equations (18)–(20) and change the notations, $(\omega_1, \omega_2) \rightarrow (\omega, \omega')$. To transform the gain term we also use equations (18)–(20), but additionally, the transformation Jacobian from variables $(v_1, v_2)$ to $(v_1', v_2')$, which is equal to unity for the elastic collision. Hence, changing again the notations, $(\omega_1, \omega_2) \rightarrow (\omega, \omega')$, we obtain for the gain term,

$$\int d\omega' f(-\omega, t)f(\omega')(\omega' - \omega) p_n(a).$$

With equations (21) and (22), we arrive at the Boltzmann equation (5) with the collision integral given by equation (6).

### A.2. Solution of the Fokker–Planck–Boltzmann equation in the stationary state

Let us analyze the limit of vanishing vibrot–vibrot interactions, $p_n(a) = 0$, for the stationary state. From (8), we obtain

$$f(\omega) = \sqrt{\frac{\gamma}{\pi T}} e^{-\frac{\gamma (\omega - \omega_0)^2}{\omega^0}} \text{ with } \omega_0 \equiv \frac{M}{\Gamma},$$

where $\omega_0$ is the average rotational velocity of an isolated vibrot.

To solve (8), including the vibrot–vibrot interaction, we substitute the ansatz

$$f(\omega) \sim e^{-\frac{\gamma (\omega - \omega_0)^2}{\omega^0} + \beta \omega}$$

into (24) and obtain the condition

$$\omega \left[ 2\frac{\gamma}{\Gamma} - \beta - \frac{p_n(a)}{\gamma} \right] + \left[ \beta^2 - \omega_0 \beta + \frac{p_n(a)}{\gamma} \right] = 0. \tag{25}$$

To analyze the high-energy tails of the distribution, $|\omega| \rightarrow \infty$, we can neglect the second term in (25) compared to the first term and solve for $\beta$. This yields

$$f(\omega) \sim e^{-\frac{\gamma (\omega - \omega_0)^2}{\omega^0} + \beta \omega}, \text{ for } |\omega| \rightarrow \infty$$

$$\beta = \frac{2\omega_0 \gamma}{\Gamma} - \frac{2 p_n(a)}{\gamma} \tag{26}$$

For vanishing vibrot–vibrot interaction, $\omega_0 = \omega$ and $p_n(a) = 0$ we recover (23). Equation (26) has a clear physical interpretation: it shows that the main part of the rotational velocity distribution obeys a Maxwell distribution with the average rotational velocity $\omega$, in agreement with the experimental observation shown in figure 5.

At the same time, (26) predicts a biased over/under-population of the high-energy tails of $f(\omega)$, depending on the sign of $\beta$, which, in turn, is determined by the lattice constant, $a$. For instance, for $\beta > 0$, one observes overpopulation of the tail for positive $\omega$, with regard to the Gaussian distribution, (23), and underpopulation for negative $\omega$. The opposite applies for $\beta < 0$. In the experiments, we could not unambiguously confirm this behavior of the tails of $f(\omega)$ due to the lack of data for a wide enough interval of $\omega$.

### A.3. Evaluation of the collision probability $p_2(a)$

For the evaluation of the collision probability given by (12), we consider the case $n = 2$. Generalization is straightforward. Then from (12) we obtain by simple but extensive algebra

$$p_2(a) = \frac{2}{\pi^2} \int_0^1 dx \int_0^x d\varphi' \Phi(2\phi_1) + \tilde{\Phi}(2\phi_2), \tag{28}$$
where

\[
\Phi(x) \equiv x - \sin x
\]

\[
\varphi' \equiv \arccos\left[\frac{x^2 + \bar{a}^2 - (2\bar{d} + 1)^2}{2\bar{ax}}\right]
\]

\[
\varphi_1 \equiv \arccos\left[\frac{1 + x^2 + \bar{a}^2 - 2x\bar{a} \cos \varphi - \bar{d}^2}{2\sqrt{x^2 + \bar{a}^2 - 2x\bar{a} \cos \varphi}}\right]
\]

\[
\varphi_2 \equiv \arccos\left[\frac{x^2 + \bar{a}^2 - 2x\bar{a} \cos \varphi + \bar{d}^2 - 1}{2d\sqrt{x^2 + \bar{a}^2 - 2x\bar{a} \cos \varphi}}\right]
\]

with \(\bar{a} \equiv a/R\) and \(\bar{d} \equiv d_0/R\).

The obtained expression for \(p_2(a)\), though exact, is complicated and not practical: the parameter \(d_0 = d_0/R\) is essentially an effective parameter. Indeed, the value of \(R \equiv \rho_0 - \rho_1\) (see figure 1(b)) characterising the accessible area in a cavity, corresponds to the distance from the center of a vibrot to its legs. The value of \(p_2(a)\) is associated with the area of the complicated manifold specified by the conditions of (12). In section kinetic theory, we approximate \(p_2(a)\) through a much simpler manifold.

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