Specific character of heat transfer under pulsating laminar flow in rectangular channels with different boundary conditions on the walls

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Abstract. Heat transfer in a laminar pulsating flow in rectangular channels with different ratios of the side lengths $\gamma$ was simulated numerically by the method of finite differences for two kinds of the boundary conditions on the wall: the first and the second kind. The maximum ratio of the Nusselt number to its steady-state value near the entrance to the heated portion of the channel was calculated at high amplitude of pulsations. An increase in this ratio in the regime of stabilized heat transfer is explained. The reasons for differences in the dependence of the thermal values on the regime parameters ($\gamma$, dimensionless oscillation frequency, the Prandtl number) for the two considered boundary conditions are analysed.

1. Introduction

The question on how the imposed oscillations of the flow rate affect the heat transfer in channels has been raised since the middle of the last century. Improving the heat and mass transfer with the help of such oscillations is an attractive method for the design of planar heat transfer devices composed of a system of slit channels. In spite of a laminar flow operation these heat exchangers are very effective. They are used to cool electronic elements or chemical reactors. Currently there exist some experimental data proving that in a laminar flow at high oscillation amplitudes the heat and mass transfers may increase several times with respect to their values in a steady flow. However, no substantial increase of the heat transfer at pulsating flows was yet confirmed by numerical simulations. It may be explained by the fact that all previous simulations or approximate analytical studies that the authors know of were performed for the amplitudes of the cross-section average flow rates not exceeding unity: $A < 1$. In this case the heat transfer may increase only by several percent.

A mathematical model and a method of numerical simulation of the hydrodynamics and heat transfer for the pulsating flow in rectangular channels were introduced in [1]. The proposed method allows one to simulate the heat transfer at high oscillation amplitudes. Simulation results for a rectangular channel with different aspect ratios were given in [2] for the amplitudes of the cross-section average flow rates $A = 0.25–1.5$ and the boundary conditions of the first kind. Substantial increase in the time- and perimeter-averaged Nusselt number was obtained with respect to their stationary values close to the inlet to the heated region and inside the region of thermal stabilization. For example, at high frequency oscillations with the amplitude of $A = 1.5$ the average Nusselt number may double with respect to its value in a steady flow. In the present work we compare the
computational results for the heat transfer at the oscillating flows for the boundary conditions of the first kind $T_w = \text{const}$ and of the second kind $q_w = \text{const}$.

2. Problem statement and numerical method

Momentum equation in the axial direction $x$ (1) and energy (2) equations for a developed laminar flow of an incompressible fluid with constant properties in a rectangular channel were solved. In the energy equation the fluid axial conductivity and the viscous dissipation were neglected:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right), \tag{1}$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = \alpha \left( \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right). \tag{2}$$

Here $u$ – the longitudinal velocity along the coordinate $x$, $T$ – temperature, $p$ – pressure, $t$ – time, $y$ and $z$ – transverse coordinates, measured from the channel middle axis along its width $2b$ and height $2h$ respectively; $\rho, \nu, \alpha$ are the density, kinematic viscosity and thermal diffusivity respectively.

We assumed that the flow velocity averaged over the channel cross-section is varied according to the following expression:

$$\langle u \rangle = \langle \bar{u} \rangle = [1 + A \sin(\omega t)]. \tag{3}$$

Here overline means time averaging, and $\langle \rangle$ means averaging over cross-section of the channel.

Equation (1) was solved with the no-slip boundary condition ($u = 0$) at the channel boundaries and symmetry with respect to the channel axis.

Boundary conditions for the energy equation (2) at the heated region of length $x_h$ of the channel walls were either constant temperature (boundary condition of the first kind), or constant heat flux at the wall (boundary condition of the second kind). Adiabatic wall regions of the lengths $x_0$ and $x_1$ were placed upstream and downstream of the heated region respectively. Constant temperature $T = T_0$ was assumed at the inlet of the upstream adiabatic region and at the outlet of the downstream adiabatic region. Symmetry was assumed at the axis of the channel.

The solution stabilized across the oscillation period was found iteratively, so the initial condition played no important role.

Equations (1) and (2) can be re-written in the following dimensionless form:

$$4S^2 \frac{\partial U}{\partial t_m} = P + \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial Z^2}, \tag{4}$$

$$4S_t \frac{\partial \theta}{\partial t_m} + U \frac{\partial \theta}{\partial X} = \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Z^2}. \tag{5}$$

Here $U = u / \langle \bar{u} \rangle$ and $t_m = \omega t$ – dimensionless longitudinal velocity and time respectively; $Y = y / d_h$, $Z = z / d_h$ – dimensionless coordinates; $d_h = 4h/(1+\gamma)$ – hydraulic diameter of the channel, $\gamma = h/b$ – rectangular channel aspect ratio; $P = -\frac{\partial p}{\partial X} \frac{d_h}{\rho \langle \bar{u} \rangle^2} \text{Re}$ – dimensionless pressure gradient; $\text{Re} = \langle \bar{u} \rangle d_h / \nu$ – Reynolds number for a time averaged flow; $S = d_h \sqrt{(\omega / \nu)} / 2$ – Stokes number (dimensionless oscillation frequency); $\theta$ – dimensionless temperature: $\theta = (T - T_0)/(T_w - T_0)$ for $T_w = \text{const}$, $\theta = \lambda (T - T_0)/(d_h q_w)$ for $q_w = \text{const}$, $\lambda$ – thermal conductivity; $X = x / (d_h \text{Pe})$ – dimensionless distance to the inlet of the heated section, $\text{Pe} = \text{RePr}$ – Peclet number; $S_T = (\text{Pr})^{1/2} S$ – thermal Stokes number (dimensionless oscillation frequency for the thermal quantities).
The solution of the momentum equation $U(Y,Z,t_o)$ depends on the Stokes number $S$, oscillation amplitude $A$, aspect ratio $\gamma$. The solution of the energy equation $\vartheta(X,Y,Z,t_o)$ is also affected by the thermal Stokes number $S_T$.

After solving the energy equation its solution was used for calculating the dimensionless perimeter-averaged and time-averaged Nusselt numbers. By $T_w$ = const: $<\text{Nu}> (X) = <Q_w> (X)/[1 - \vartheta_I(X)]$; and by $q_w$ = const: $<\text{Nu}> (X) = 1/[\vartheta_I(X) - \vartheta_I(X)]$; here $Q_w$ is the dimensionless heat flux density at the wall, $\vartheta_I = \int_0^{Y_0} \int_0^{Z_0} \frac{9 - u}{\langle u \rangle} dZdY$ - average bulk fluid temperature, the sign $<$ means averaging along the channel perimeter or cross-section.

Equations (4) and (5) were solved by the finite difference technique using an implicit unconditionally stable difference scheme having two levels both in time $t_o$ and with respect to the longitudinal coordinate $X$. The derivatives with respect to $Y$ or $Z$ were approximated with a central difference scheme. The dimensionless pressure gradient $P$ was calculated using the split method of L.M. Simuni [3]. The system of the finite difference equations was solved at every time layer using Gauss-Seidel iteration method. The lengths of the adiabatic regions $X_0$ and $X_1$ were adjusted in a way that they do not influence the solution in the heated region of length $X_\gamma$.

3. Results of calculations

3.1. Heat transfer for the boundary condition of the first kind

![Figure 1. Distribution of the Nusselt number along the channel length. $I - A = 0.25$; $II - A = 0.75$; $III - A = 1.5$. $a - \gamma = 0$, $b - 0.1$, $c - 0.25$, $d - 1.0$. For $II$, $I - S = 10^{-3}$, $S_T = 10^{-3}$; $2 - S = 10^{-3}$, $S_T = 7$; $3 - S = 7$, $S_T = 10^{-3}$; $4 - S = 7$, $S_T = 7$. For $III$, $I - S = 10^{-3}$, $S_T = 1.5$; $2 - S = 10^{-3}$, $S_T = 7$; $3 - S = 7$, $S_T = 1.5$; $4 - S = 7$, $S_T = 7$.](image)

The curves $<\text{Nu}>/<\text{Nu}_\infty>(X)$ in Fig. 1 have a maximum near the inlet to the heated region. This maximum increases with increasing oscillation amplitude $A$. For different ranges of the dimensionless oscillation frequencies, the maximum is found at different points and has different values (Fig. 2).

In the quasi-stationary regions ($S, S_T < 1$), this maximum is basically explained by the extension of the non-stabilized heat transfer region when the section-averaged velocity oscillates with high amplitude.

With $S < 1$ and rising $S_T$, the maximum Nusselt number decreases. This effect is more pronounced in channels with small aspect ratios. The predictions demonstrate that with an increase in the thermal Stokes number in the quasi-stationary hydrodynamic region, the temperature profiles change slightly with time and are close to the stationary profile.
With fixed $S_T$ in the high-frequency hydrodynamic regime ($S > 5$ [4]), the maximum of $\langle \bar{Nu} \rangle / \langle Nu_0 \rangle$ increases since the bulk fluid temperature increases more than the wall heat flux.

For $A > 1$, the upstream adiabatic region has a strong effect on this maximum. The reverse flow transfers hot fluid close to the inlet of the heated region, thereby increasing the bulk fluid temperature here. Due to low velocity near the wall, the temperature increases here only slightly. Therefore, the heat flux on the wall having a constant temperature also rises, but only a little. As a result, the Nusselt number increases.

![Figure 2](image)

Figure 2. Curves ($\langle \bar{Nu} \rangle / \langle Nu_0 \rangle$)$_{\text{max}}$ = const for $A = 1.5$. $a - \gamma = 0$, $b - 0.1$, $c - 0.25$, $d - 1.0$.

It is evident from Fig. 2 that, at relatively small Stokes numbers $S$, the maximum Nusselt number is independent of $S_T$, and, with increasing $S$, starting from a certain transition value of $S$, the maximum Nusselt number no longer depends on $S_T$. The smaller the aspect ratio $\gamma$, the higher is this transition value of $S$. The Stokes number below, for which the oscillating flow can be considered quasi-stationary, also increases with decreasing aspect ratio as demonstrated in [4].

In the high-frequency regime ($S > 5$), the maximum of $\langle \bar{Nu} \rangle / \langle Nu_0 \rangle$ ratio at the inlet rises considerably due to an increase in the Nusselt number along the entire channel length (curves 4 in Fig. 1). Note that end marks in the form of a point for the curve ($\langle \bar{Nu} \rangle / \langle Nu_0 \rangle$)$_{\text{max}}$ = const in Fig. 2 demonstrate that there is no maximum in the vicinity of the inlet to the heated section.

![Figure 3](image)

Figure 3. Curves ($\langle \bar{Nu} \rangle / \langle Nu_0 \rangle$)$_{\infty}$ = const for $A = 1.5$. $a - \gamma = 0$, $b - 0.1$, $c - 0.25$, $d - 1.0$.

Fig. 3 shows the ratio of the Nusselt number to its stationary value, ($\langle \bar{Nu} \rangle / \langle Nu_0 \rangle$)$_{\infty}$, far away from the heated region inlet. The above noted considerable increase in the Nusselt number under the high frequency conditions ($S > 5$), which is more pronounced in a square channel as compared with a plane channel, can be explained by a rise in the bulk fluid temperature, the value of which approaches
the wall temperature. The predicted hydrodynamic characteristics of a pulsating laminar flow in channels [4] demonstrate that, in this case, the velocity fluctuation amplitude is large, especially in the corner zones of a rectangular channel.

![Figure 4](image)

**Figure 4.** Curves \( (\text{Nu}/\text{Nu}_S)_{\max} = \text{const} \) (a) and \( (\text{Nu}/\text{Nu}_S)_{\infty} = \text{const} \) (b) for plane channel for \( A = 5 \).

The increase of the oscillation amplitude gives rise to the growth of \( \text{Nu}/\text{Nu}_S \) maximum close to the inlet to the heated region, which can be seen comparing Figs 4(a) and 1(a). However, the amplitude has practically no effect on the \( (\text{Nu}/\text{Nu}_S)_{\infty} \) ratio for the plane channel, which is seen from the comparison of Figs 4(b) and 3(a).

### 3.2. Heat transfer for the boundary condition of the second kind

As was found by calculations, the profiles of the average fluid bulk temperature and the wall temperature along the longitudinal coordinate have oscillations with amplitudes decreasing with the growth of \( S_T \) and \( X \). This effect is as well observed for the boundary condition of the first kind.

As seen from Fig 5, the profiles of \( <\text{Nu}>/ <\text{Nu}_S> \times A \) have a maximum close to the inlet to the heated region, same as for the boundary condition of the first kind, but with much smaller value. This maximum value increases with the growth of \( A \).

The main reason for the appearance of this maximum is the extension of the non-stabilized heat transfer region when the cross-section averaged flux rate oscillates with high amplitude. In contrast to the case \( T_w = \text{const} \), the upstream adiabatic region plays no important role. The reverse flow of the hot fluid towards the inlet of the heated region increases both the bulk fluid temperature and the wall temperature here. Therefore the upstream region has practically no effect on the thermal head and the Nusselt number.

![Figure 5](image)

**Figure 5.** Distribution of Nusselt number along the channel length. \( I - A = 0.25 \); \( II - A = 0.75 \); \( III - A = 1.5 \). \( a - \gamma = 0 \), \( b - 0.1 \), \( c - 0.25 \), \( d - 1.0 \). For \( I, II - I - S = 10^3 \), \( S_T = 10^3 \); \( 2 - S = 10^4 \), \( S_T = 7 \); \( 3 - S = 7 \), \( S_T = 10^3 \); \( 4 - S = 7 \), \( S_T = 7 \). For \( III - I - S = 10^3 \), \( S_T = 1.5 \); \( 2 - S = 10^3 \), \( S_T = 7 \); \( 3 - S = 7 \), \( S_T = 1.5 \); \( 4 - S = 7 \), \( S_T = 7 \).
Let us specify four characteristic zones in the $S$-$S_T$ plane with the boundaries defined by the characteristic values of $S$ and $S_T$ [1, 4]. Zone 1 ($S, S_T < 1$) is quasi-stationary, zone 4 ($S, S_T > 5$) – high-frequency, zones 2 ($S < 1, S_T > 5$) and 3 ($S > 5, S_T < 1$) are intermediate.

The above mentioned maximum of $\langle \frac{Nu}{Nu_S} \rangle$ is observed in zone 1. As seen from Fig. 6, with the growth of $S_T$ in transfer to zone 2 the maximum values increase, and with the growth of $S$ in transfer to zone 3, the maximum decreases. When $S$ is small and $S_T$ grows the temperature distribution along the channel cross-section approaches the same distribution as in a laminar flow, because the oscillation amplitude becomes small at large $S_T$.

![Figure 6](image1)

**Figure 6.** Curves $(\langle \frac{Nu}{Nu_S} \rangle)_{\text{max}} = \text{const}$ for $A = 1.5$. $a - \gamma = 0$, $b - 0.1$, $c - 0.25$, $d - 1.0$.

Same as for the condition $T_w = \text{const}$, the increase of $(\langle \frac{Nu}{Nu_S} \rangle)_{\text{max}}$ in the high-frequency regime ($S > 5$) in zones 3 and 4 is related to the increase of the Nusselt number along the whole channel length.

![Figure 7](image2)

**Figure 7.** Curves $(\langle \frac{Nu}{Nu_S} \rangle)_\infty = \text{const}$ for $A = 1.5$. $a - \gamma = 0$, $b - 0.1$, $c - 0.25$, $d - 1.0$.

The Nusselt number relative to its stationary value far away from the heated region inlet $(\langle \frac{Nu}{Nu_S} \rangle)_\infty$ is shown in Fig 7. Same as for the condition $T_w = \text{const}$, substantial increase in the Nusselt number in the high-frequency regime is explained by the growth of the average fluid bulk temperature approaching the wall temperature. However the value of $(\langle \frac{Nu}{Nu_S} \rangle)_\infty$ reaches its largest value not in the square channel ($\gamma = 1$), as for $T_w = \text{const}$, but in the slit channel (in the case $\gamma \to 0$). The reason for this is the difference in the dependence of the Nusselt number on $\gamma$ for the stationary flow in the cases $T_w = \text{const}$ and $q_w = \text{const}$ [1].
Same as for the boundary condition \( T_w = \text{const} \), the increase of the oscillation amplitude leads to the increase of the maximal of the ratio \( \frac{\overline{\text{Nu}}}{\text{Nu}_S} \) close to the inlet to the heated region, as seen from the comparison of Figs. 6(a) and 8(a). For a pulsating flow in a plane channel the oscillation amplitude does not affect this ratio at large distances form the inlet to the heated region (see Figs. 7(a), 8(b)).

![Figure 8](image)

**Figure 8.** Curves \((\overline{\text{Nu}} / \text{Nu}_S)_{\text{max}} = \text{const} \) (a) and \((\overline{\text{Nu}} / \text{Nu}_S)_\infty = \text{const} \) (b) for plane channel for \( A = 5 \).

4. Conclusion

Systematic calculations of the heat transfer by a pulsating laminar flow in rectangular channels with different aspect ratios were performed for various oscillation amplitudes, thermal and hydrodynamic Stokes numbers and under two kinds of boundary conditions: \( T_w = \text{const} \) и \( q_w = \text{const} \).

It was confirmed that period-averaged Nusselt number may substantially increase for the cross-section average velocity amplitudes larger than unity. The increase of the heat transfer takes place near the inlet to the heated region of the tube, and it is more pronounced for the condition \( T_w = \text{cons} \), than for the condition \( q_w = \text{const} \). For example, for the boundary condition of the first kind at \( A = 1.5, \gamma = 1, S = 7, S_T = 5 \) the ratio of the maximum average Nusselt number to its value in a stationary flow is \((<\overline{\text{Nu}}>/<\text{Nu}_S>)_{\text{max}} = 2.7 \) at \( X = 0.08 \). The existence of the upstream adiabatic region substantially affects the value of the indicated Nusselt number increase.

References

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