Nature and evolution of incommensurate charge order in manganites visualized with cryogenic scanning transmission electron microscopy

Ismail El Baggari, Benjamin H. Savitzky, Alemayehu S. Admasu, Jaewook Kim, Sang-Wook Cheong, Robert Hovden, and Lena F. Kourkoutis

*Department of Physics, Cornell University, Ithaca, NY 14853; †Rutgers Center for Emergent Materials, Rutgers University, Piscataway, NJ 08854; ‡Department of Physics and Astronomy, Rutgers University, Piscataway, NJ 08854; §School of Applied and Engineering Physics, Cornell University, Ithaca, NY 14853; and †Kavli Institute for Nanoscale Science, Cornell University, Ithaca, NY 14853

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Incommensurate charge order in hole-doped oxides is intertwined with exotic phenomena such as colossal magnetoresistance, high-temperature superconductivity, and electronic nematicity. Here, we map, at atomic resolution, the nature of incommensurate charge–lattice order in a manganite using scanning transmission electron microscopy at room temperature and cryogenic temperature (~93 K). In diffraction, the ordering wave vector changes upon cooling, a behavior typically associated with incommensurate order. However, using real space measurements, we discover that the ordered state forms lattice-locked regions over a few wavelengths interspersed with phase defects and changing periodicity. The cations undergo picometer-scale (~6 pm to 11 pm) transverse displacements, suggesting that charge–lattice coupling is strong. We further unearth phase inhomogeneity in the periodic lattice displacements at room temperature, and emergent phase coherence at ~93 K. Such local phase varia
tions govern the long-range correlations of the charge-ordered state and locally change the periodicity of the modulations, resulting in wave vector shifts in reciprocal space. These atomically resolved observations underscore the importance of lattice coupling and phase inhomogeneity, and provide a microscopic explanation for putative “incommensurate” order in hole-doped oxides.

charge order | incommensurate | manganite | electron microscopy | cryogenic STEM

Charge-ordered phases permeate the phase diagrams of strongly correlated systems such as cuprate high-temperature superconductors, colossal magnetoresistive manganites, and 2D transition-metal dichalcogenides (1–8). Charge order is a modulation of the electron density that breaks lattice translational symmetry and induces periodic lattice displacements (PLDs) via electron–lattice coupling. Bulk measurements have unearthed complex interactions between charge order and electronic phases, including direct competition with superconductivity or mediation of colossal magnetoresistance, which highlights the importance of charge order in understanding and manipulating novel phases of matter (2, 6–9).

While the precise microscopic mechanism of charge ordering remains under intense scrutiny, measurements of modulation wave vectors in various materials have established a tendency toward incommensurate order (1, 3, 5, 10–14). The presence of incommensurability coincides with the emergence of competing phases such as superconductivity and has motivated interrogation of the role of Fermi surface instabilities (10, 15–18). Scattering experiments, for instance, have measured changes in the positions of incommensurate wave vectors as a function of temperature, pressure, or doping, which is thought to reflect changes in the nesting of the Fermi surface. However, coupling to quenched impurities and attendant order parameter fluctuations profoundly alter correlation lengths and symmetry, and complicate experimental interpretation of reciprocal space behavior (14, 19, 20).

Bi$_{1−x}$Sr$_x$Ca$_y$Mn$_{2−x}$O$_{3+y}$ (BSCMO) is a model charge-ordered manganite with a high, tunable transition temperature (T$_c$) (21, 22). Dark-field transmission electron microscopy has previously visualized striped superstructures in manganites, interpreting contrast as the ordering of holes on alternating manganese sites (Mn$^{3+}$–Mn$^{4+}$) (1, 3, 10). Other experiments based on transport and electron diffraction advance that charge order corresponds to a uniform charge density wave with small valence modulations (23, 24). The debate is partly motivated by the need to reconcile incommensurate wave vectors with discrete charge ordering. Understanding striped phases in manganites is further complicated by the ubiquity of quenched disorder and nanoscale phase inhomogeneity. Optical studies on BSCMO, for instance, suggest that long-range correlations gradually develop below T$_c$, an indication that charge order is disturbed by temperature-dependent spatial inhomogeneity (25). To achieve a microscopic understanding of charge-ordered states,

**Significance**

Charge order is a modulation of the electron density and is associated with unconventional phenomena, including colossal magnetoresistance and metal–insulator transitions. Determining how the lattice responds provides insights into the nature and symmetry of the ordered state. Scanning transmission electron microscopy can measure lattice displacements with picometer precision, but its use has been limited to room-temperature phases only. Here, we demonstrate high-resolution imaging at cryogenic temperature and map the nature and evolution of charge order in a manganite. We uncover picometer-scale displacive modulations whose periodicity is strongly locked to the lattice and visualize temperature-dependent phase inhomogeneity in the modulations. These results pave the way to understanding the underlying structure of charge-ordered states and other complex phenomena.

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1 Present address: Department of Materials Science and Engineering, University of Michigan, Ann Arbor, MI 48109.
2 To whom correspondence should be addressed. Email: lena.f.kourkoutis@cornell.edu.

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atomic-scale characterization of individual degrees of freedom is necessary.

Room-temperature scanning transmission electron microscopy (STEM) enables measurements of atomic column positions with picometer precision (26) and has been used to unravel, for instance, novel ferroelectric behavior in oxides (27, 28). Recently, we have revealed transverse, PLDs associated with charge ordering at room temperature in BSCMO (T < 300 K) and visualized nanoscale inhomogeneity in the modulation field (29). Here, we demonstrate cryogenic STEM imaging with sub-angstrom resolution (~0.78 Å) and sufficient signal-to-noise ratio to visualize the charge-ordered state in BSCMO well below Tc. Earlier cryogenic STEM studies have observed ordering phenomena (30, 31); however, stage instability, limited resolution, and low signal-to-noise ratio have precluded mapping of picometer-scale lattice behavior.

To understand the local structure of incommensurate modulations, we map picometer-scale lattice distortions associated with charge ordering near and well below Tc. At both temperatures, we find that the lattice modulations tend to form locally lattice-commensurate structures interspersed with regions of disorder and changing periodicity. By extracting the phase of the lattice modulations, we uncover nanoscale temperature-dependent phase variations which govern long-range ordering. The presence of phase slips and defects locally changes the periodicity of the modulations, resulting in a smaller wave vector at room temperature. Upon cooling, the phase field becomes more homogeneous, and the wave vector increases accordingly. This observation underscores the importance of lattice locking in a manganite, and provides a visualization of the real space structure of supposed incommensurate order. More generally, cryogenic STEM paves the way for direct visualization of correlated lattice order with picometer precision over atomic and nanometer scales.

Results

Wave Vector Shift in Diffraction. BSCMO single crystals are grown using the flux method, using Bi2O3, CaCO3, SrCO3, and Mn3O4, as reported previously (29). The Bi-based manganites display robust charge ordering (Tc ≈ 320 K to 525 K for x = 0.5) compared with their rare-earth counterparts (RE1−xCa1−xMnO3, RE = La, Nd, Pr) (21). We choose a doping level in which charge ordering is nascent at room temperature to study its structure near and well below Tc, with cryogenic STEM. We measure the composition of BSCMO to be approximately x = 0.65 and y = 0.47 and find that a resistivity anomaly associated with charge ordering occurs at Tc ≈ 300 K (SI Appendix, Fig. S1). Previous reports on manganites have observed that the charge ordering wave vector, q, varies with doping, with q ≈ 1 − x reciprocal lattice units (r.l.u.) (3, 10); here, the doping level slightly deviates from x = 2/3 doping, and we expect incommensurate ordering.

Fig. 1A shows a typical, room-temperature (293 K) electron diffraction pattern of BSCMO, which we index in the Pnma space group. In addition to crystalline Bragg peaks, we observe
superlattice peaks at \( \pm q \) indicating the presence of a periodic modulation at both 293 K (Fig. 1B) and 93 K (Fig. 1C). The projected intensity of the \((-2+q,0.2)\) peak along \( a^* \) at 293 K (red) and 93 K (blue) is shown in Fig. 1D. At each temperature, the intensity profile is obtained by integrating between the tick marks in Fig. 1B and C, and is normalized by the respective integrated intensity of the 202 Bragg peak. We note a clear shift in the superlattice peak position upon cooling, from \( q = 0.318 \) r.l.u. at 293 K to \( q = 0.331 \) r.l.u. at 93 K (SI Appendix, Fig. S15). By fitting multiple satellites in the diffraction pattern, we obtain the average wave vector and its uncertainty at each temperature. The average wave vector is \( q = 0.314 \pm 0.003 \) r.l.u. at 293 K and \( q = 0.332 \pm 0.001 \) r.l.u. at 93 K. The magnitude of the wave vector shift is \( \delta q = 0.018 \pm 0.003 \) r.l.u. Temperature-dependent wave vector variations, as observed here, are typically considered an indication of incommensurate order.

In addition to a shift in the wave vector, the superlattice peak exhibits a clear increase in intensity, \( I \), and a decrease in the full-width-at-half maximum, \( \sigma_I \), at low temperature. Fitting a Lorentzian function and a linear background to the projected \( q \) peak, we find that \( I(93 \text{K})/I(293 \text{K}) \approx 1.46 \) and \( \sigma_I(93 \text{K})/\sigma_I(293 \text{K}) \approx 0.57 \) (SI Appendix, Fig. S15). The weaker \( 2q \) peak, while almost undetectable at room temperature, is relatively sharp and well defined at 93 K (Fig. 1E). These observations indicate that charge order strength and correlations increase well below Tc.

**Real Space Imaging of Lattice Order.** From previous real space measurements at room temperature, we uncovered PLDs associated with charge ordering in BSCMO (29). PLDs, \( \Delta(r) \), may be described by the order parameter

\[
\Delta(r) = \Re \{ A(r) e^{i \phi(r)} e^{i q \cdot r} \},
\]

where \( A(r) \) is the displacement vector, \( q \) is the wave vector, and \( \phi(r) \) is the phase. The modulations give rise to complex-valued satellite peaks in the Fourier transform (FT) which are given by \( S(k) \sim \sum r \exp[i(k \pm q) \cdot r] \exp[i\phi(r)] \), where \( r \) is the set of lattice positions (SI Appendix, Structure Factor of Periodic Lattice Displacements). Phase information encodes not only the particular realization of \( \Delta(r) \) but also its disorder. Since diffraction experiments probe the intensity (\( I(k) \approx |S(k)|^2 \)), they are insensitive to phase information and provide, instead, globally averaged measurements of correlation lengths, intensities, and wave vectors (6). However, the microscopic picture of incommensurate modulations and their temperature (or doping) dependence may be linked to phase information (14).

To overcome challenges associated with intensity-based techniques, we characterize \( \Delta(r) \) using phase-sensitive, real space STEM. Fig. 2A shows a high-angle annular dark-field (HAADF) STEM lattice image at 93 K; Bi/Sr/Ca columns (green) appear bright, Mn columns (red) appear faint, and O atoms are invisible. The contrast is due to the dependence of the scattering cross-section in HAADF STEM on the atomic number. The temperature of the sample (\( \sim 93 \text{K} \)) is directly read from a thermocouple near the tip of the sample rod, but the true temperature may be slightly higher. To minimize stage drift and noise, stacks of 40 fast-acquisition (0.5 ms per pixel) images are collected, registered by cross-correlation, and averaged. The data demonstrate STEM imaging near liquid nitrogen temperature (\( \sim 93 \text{K} \)) with high resolution (\( \sim 0.78 \) Å) and signal-to-noise ratio (SI Appendix, Fig. S15).
Fig. 2), which may be combined with picometer precision mapping of atomic columns.

The FT (Fig. 2A, Inset) of the lattice image exhibits sharp superlattice spots (arrows), allowing mapping of lattice modulations associated with superlattice peaks (29). We damp the amplitude of the superlattice peaks to the background level and apply an inverse FT. The result is a reference lattice image in which the targeted modulation has been removed (SI Appendix, Fig. S3). By fitting atomic columns using 2D Gaussian functions in both the original lattice and the reference lattice, we may obtain lattice shifts associated with the charge-ordered state. As previously discussed, the method accurately yields the structure of the modulation except at atomically sharp discontinuities in the modulation field (29).

Fig. 2B and D shows a STEM image at 93 K and the corresponding mapping of the lattice response, respectively. The arrows correspond to displacements of atomic columns in the original image relative to the generated reference lattice, and the color represents the angle of the displacement vector relative to q, with blue (yellow) corresponding to 90° (-90°). Thus, the low-temperature ordered state in BSCMO involves transverse, displacive modulations of both the Bi/Sr/Ca sites and the Mn sites, with amplitudes in the range of 6 pm to 11 pm. For comparison, Fig. 2C and E shows a STEM image and the corresponding PLD mapping at 293 K. The displacements appear to stack coherently along the b direction, since we do not observe intensity variations between modulated and unmodulated Mn sites (SI Appendix, Fig. S4). Due to electron channeling effects, such intensity variations are expected if the displacements stack incoherently (32). By imaging other zone axes of the crystal (SI Appendix, Figs. S4–S6), we do not observe out-of-plane displacements or modulations, which suggests that the PLDs in BSCMO occur primarily in the ac plane. These observations are consistent with neutron and X-ray powder refinements on related manganite systems, where bond length variations are observed along the modulation wave vector only (33–34). At both temperatures, the lattice displacements are visually lattice-commensurate with three-unit cell periodicity, but only over four to five PLD wavevectors.

Phase and Periodicity Variations. As shown in Wave Vector Shift in Diffraction, area-averaged diffraction yields an incommensurate wave vector at 293 K and an (almost) commensurate wave vector at 93 K. To understand the nature of incommensurate order, it is important to determine whether the periodicity from diffraction data reflects a long-range incommensurate state with uniform periodicity or some other local structures. To that end, we measure q from the FT amplitude of multiple STEM datasets with fields of view ranging from 17 nm to 40 nm. The wave vectors from local measurements span a wide range of values (SI Appendix, Phase Variations and Wavevector Shifts and Fig. S8). At room temperature, they range from \( q = 0.307 \text{ r.l.u.} \) to \( q = 0.331 \text{ r.l.u.} \) and are generally smaller than the commensurate value of \( q = 1/3 \text{ r.l.u.} \). Interestingly, the PLD even appears lattice-commensurate up to seven wavelengths in a well-ordered region at room temperature, with \( q = 0.331 \text{ r.l.u.} \) (SI Appendix, Phase Variations and Wavevector Shifts and Figs. S11 and S12). At 93 K, the wave vectors range from \( q = 0.329 \text{ r.l.u.} \) to \( q = 0.340 \text{ r.l.u.} \) and are generally larger than \( q = 1/3 \text{ r.l.u.} \). The wave vector remains smaller than the predicted value from the empirical relation \( q \approx 1 - x \approx 0.35 \text{ r.l.u.} \), which suggests that the wave vector may continue to increase upon further cooling. These observations expose a discrepancy between local (tens on nanometers) and global (micron scale) measurements of the modulation wave vector, and suggest that incommensuration reflects spatially complex textures and periodicities.

Fig. 5. Autocorrelations of the phase component and the amplitude component at 293 K (red) and 93 K (blue). Lines are guides for the eye. The slow decay of amplitude correlations at both temperatures suggests that the amplitude is not the main driver of long-range order. In contrast, the strong temperature-dependent decay of phase correlations supports that phase variations govern long-range order.
field of view, an indication of emergent phase homogeneity well below \( T_c \).

Gradients in the phase result in superlattice peak shifts in the FT amplitude, assuming variations do not average to zero (SI Appendix, Phase Variations and Wavevector Shifts and Figs. S9–S11, S13, and S14). The shift is given by \( \delta q = \langle \nabla \phi \rangle \), where \( \phi \) is the wave vector in the absence of phase variation. At room temperature, large negative phase strain results in the reduction of \( q \), as measured in diffraction or STEM FT amplitudes. Upon cooling, the compressive phase strain increases, and, consequently, the wave vector increases. The PLD mappings suggest that the modulations are often lattice-commensurate over a few wavelengths, and that, absent phase variations, we obtain \( q \approx 1/3 \) r.l.u. (Fig. 2 and SI Appendix, Phase Variations and Wavevector Shifts and Figs. S9–S12). In this picture, charge–lattice order favors lattice-locked structures, and the changes in incommensurate wave vectors reflect deformations of the phase field.

In regions where the PLD appears lattice-locked, the modulation may instead be incommensurate with a very large periodicity given by \( 1/\delta q \). Due to the limited fields of view in imaging, we cannot completely distinguish such incommensurate structure from a commensurate structure with local phase slips, especially when the incommensuration is small. However, the intertwined spatial variations of the phase and the wave vectors, as observed in our data, disfavor a long-range incommensurate state with a uniform periodicity (SI Appendix, Phase Variations and Wavevector Shifts and Figs. S9–S12). We reiterate that, due to phase variations, amplitude-based probes detect shifts in incommensurate wave vectors, which are consistent with several distinct local structures. On the other hand, phase-sensitive probes, including STEM, can measure the distribution of periodicities and textures of ordered states at the atomic scale.

Phase and Amplitude Disorder. Having revealed temperature-dependent phase disorder, we address the relative weights of amplitude and phase variations by calculating their respective autocorrelations. In Fig. 5, we observe that amplitude correlations quickly peak to \( \sim 0.9 \) at 293 K and \( \sim 0.8 \) at 93 K over accessible length scales (\( \sim 10 \) nm for reasonable statistics); their slow decay at both temperatures suggests they do not influence the correlation length of stripes. Phase correlations, on the other hand, decay rapidly at 293 K, becoming completely uncorrelated beyond \( \sim 8 \) nm. At 93 K, they remain finite and relatively significant (\( \sim 0.5 \)) beyond 10 nm. The strong temperature dependence of phase correlations suggests that the phase component is the primary driver of long-range order. The situation invites an analogy to disordered superconductors where the loss of phase coherence can cause a superconductor-to-insulator transition despite the persistence of a finite amplitude (37).

Finally, local inspection of amplitude fields at 293 K and 93 K (SI Appendix, Fig. S7) reveals an interaction between phase variation and amplitude variation. From a mean field perspective, amplitude fluctuations should be suppressed because they cost finite energy. When the phase varies slowly, the amplitude is expectedly robust and uniform. However, in regions of large phase gradients, the amplitude weakens, particularly at dislocation sites where it collapses completely (SI Appendix, Phase Variation and Amplitude Variation and Fig. S7). Phase deformations increase the elastic energy density by an amount proportional to \( |\nabla \phi(r)|^2 \), which may require amplitude suppression in regions of diverging phase gradients (38, 39). Our observations are consistent with this picture, and attest to the importance of phase fluctuations in shaping the strength and correlations of charge order.

Discussion

Based on these results, we propose that experimental observations of coincident incommensuration and emergent competing states are related to the local disordering of the phase component; phase gradients account for (i) the reduction of the correlation length, (ii) changes in the average wave vector, and (iii) the local quenching of the amplitude which may allow another order to materialize. This may explain the competition between ferromagnetism and incommensurate charge order in manganites (1, 10), or even the granular interplay between short-range incommensurate charge density waves and superconductivity in cuprates or transition metal dichalcogenides.

A successful microscopic theory for charge order, and the many exotic states it affects, should account for all relevant degrees of freedom and for the possibility of inhomogeneity. Using cryogenic STEM on a manganite, we have directly measured the lattice component and found that the cations undergo transverse displacements relative to the modulation wave vector. Models based solely on separation of charge or orbital ordering are therefore insufficient. Atomic displacements change bond distances, bond angles, and hence exchange interactions, and may be key to explaining how charge order impacts other electronic states. We have also visualized the interplay between incommensuration, phase defects, and lattice locking, reinforcing both the importance of charge–lattice coupling and the fundamental effects of nanoscale phase inhomogeneity on macroscopic behavior. We envision that mapping the lattice component using cryogenic STEM in correlated materials will elucidate structural ground states and reveal connections between atomic displacements and various symmetries of electronic and orbital order.

Materials and Methods

Sample Preparation. BSCMO single crystals are grown using the flux method, using Bi$_2$O$_3$, CaCO$_3$, SrCO$_3$, and Mn$_2$O$_3$ (29). Sample preparation for electron microscopy and energy dispersive X-ray spectroscopy (EDX) are performed on an FEI Strata 400 Focused ion Beam. From EDX, the composition is determined to be approximately \( x = 0.65 \) and \( y = 0.47 \), with negligible variations over the whole sample (size \( 0.34 \times 0.28 \) mm).

Electronic Diffraction and STEM. We perform electron diffraction and microscopy on an aberration-corrected FEI Titan Themis operating at 300 kV. Diffraction measurements across temperatures are performed in the same region of the same sample using a \( \sim \)1-μm selected area aperture. In HAADF-STEM, the convergence semiangle is 30 mrad, and the collection inner and outer angles are 68 and 340 mrad, respectively. For cryogenic experiments, we use a Gatan 636 double-tilt liquid nitrogen holder. Due to reduced stability at cryogenic temperatures, stacks of 20 to 40 fast-acquisition (0.5 μs per pixel dwell time) images are collected, registered by cross-correlation, and averaged to minimize stage drift and noise.

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