Distributed Optimal Load Frequency Control with Stochastic Wind Power Generation

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Abstract—Motivated by the inadequacy of conventional control methods for power networks with a large share of renewable generation, in this paper we study the (stochastic) passivity property of wind turbines based on the Doubly Fed Induction Generator (DFIG). Differently from the majority of the results in the literature, where renewable generation is ignored or assumed to be constant, we model wind power generation as a stochastic process, where wind speed is described by a class of stochastic differential equations. Then, we design a distributed control scheme that achieves load frequency control and economic dispatch, ensuring the stochastic stability of the controlled network.

I. INTRODUCTION

The supply-demand balance is an essential control objective in power networks. Indeed, the supply-demand mismatch leads to frequency deviations from the nominal value, which eventually may result in stability disruptions [1], [2]. For this reason, the main control objective in power networks is the so-called Load Frequency Control (LFC). Additionally, another key objective is the minimization of the generation costs, also known as economic dispatch [3]. The economic dispatch together with the LFC is called in the literature Optimal LFC (OLFC) (see for instance [3]–[6] and the references therein). However, due to the growing share of renewable generation sources in power networks, the conventional control schemes may not be adequate [7].

Different control strategies achieving LFC and OLFC have been proposed for instance in [8]–[10] and [3], [6], [11]–[14], respectively (see also the references therein). However, in all these works, only conventional power generation is taken into account.

A. Motivation and Contributions

Nowadays, renewable generation sources are widespread in power networks, leading to an inevitable increase of uncertainties affecting the overall power system and its stability, resilience and reliability. For this reason, advanced control methods that guarantee the stability of the power system also in presence of time-varying renewable sources are necessary. Indeed, due to the random and unpredictable nature of some primary energy sources such as wind, the dynamic behaviour of renewables can be usually described by stochastic processes (e.g., Ito calculus), as shown for instance in [15], [16] for wind power generation. Also, [17] proposes wind speed models based on Stochastic Differential Equations (SDEs), which can be useful in wind turbine models. Differently from [3], [6], [8]–[14] and other relevant works on the topic, in this paper we couple the wind speed model introduced in [16] with the model of wind turbines based on the Doubly Fed Induction Generator (DFIG). Then, we present a distributed passivity-based control scheme achieving OLFC and ensuring the stochastic stability of the power network.

The main contributions of this paper can be summarized as follows: (i) the OLFC problem for nonlinear power networks including the turbine-governor model of conventional generators and the model of DFIG-based wind turbines is formulated, where the wind speed is modeled by an SDE; (ii) sufficient conditions for the stochastic passivity of the open-loop system are presented, facilitating the interconnection with passive control systems; (iii) a control scheme is proposed to obtain the passivity property of the DFIG-based wind turbine; (iv) the stochastic stability of the power network controlled by the distributed control scheme proposed in [3] is proved and OLFC objective is achieved.

B. Notation

The set of real numbers is denoted by \(\mathbb{R}\). The set of positive (nonnegative) real numbers is denoted by \(\mathbb{R}_{>0}\) (\(\mathbb{R}_{\geq0}\)). Let \(0\) denote the vector of all zeros and the null matrix of suitable dimension(s), and \(\mathbf{I}_n \in \mathbb{R}^n\) denote the vector containing all ones. The \(n \times n\) identity matrix is denoted by \(\mathbf{I}_n\). Let \(A \in \mathbb{R}^{n \times n}\) be a matrix. In case \(A\) is a positive definite (positive semi-definite) matrix, we write \(A > 0\) (\(A \geq 0\)). Let \([A]\) denote the matrix \(A\) with all elements positive. The \(i\)-th element of vector \(x\) is denoted by \(x_i\). A steady-state solution to system \(\dot{x} = f(x)\), is denoted by \(\bar{x}\), i.e., \(0 = f(\bar{x})\). Let \(x \in \mathbb{R}^n, y \in \mathbb{R}^m\) be vectors, then we define \(\text{col}(x,y) := (x^\top \ y^\top)^\top \in \mathbb{R}^{n+m}\). Given a vector \(x \in \mathbb{R}^n\), \([x] \in \mathbb{R}^{n \times n}\) indicates the diagonal matrix whose diagonal entries are the components of \(x\) and \(\sin(x) := \text{col}\left(\sin(x_1), \ldots, \sin(x_n)\right)\).

II. PROBLEM FORMULATION

In this section, we introduce the nonlinear power system model together with the turbine-governor and wind turbine models. Then, two control objectives are presented: load frequency control and optimal generation (economic dispatch).
TABLE I: Symbols

| \( P_{ci} \) | Conventional power generation |
| \( P_{wu} \) | Wind power generation |
| \( P_{li} \) | Unknown constant load |
| \( \varphi_{i} \) | Voltage angle |
| \( \omega_{i} \) | Frequency deviation |
| \( V_{i} \) | Voltage |
| \( i_{ds,i} \) | \( d \) component of DFIG stator current |
| \( i_{qs,i} \) | \( q \) component of DFIG stator current |
| \( i_{dr,i} \) | \( d \) component of DFIG rotor current |
| \( i_{qr,i} \) | \( q \) component of DFIG rotor current |
| \( V_{di} \) | \( d \) component of DFIG rotor voltage |
| \( V_{qi} \) | \( q \) component of DFIG rotor voltage |
| \( V_{ti} \) | Terminal voltage of DFIG |
| \( f_{ri} \) | Rotor angular speed of DFIG |
| \( f_{bi} \) | Base speed of DFIG |
| \( \bar{v}_{i} \) | Predicted term of wind speed |
| \( \tau_{pi} \) | Moment of inertia |
| \( \tau_{ri} \) | Direct axial transient open-circuit constant |
| \( X_{di} \) | Direct synchronous reactance |
| \( X'_{di} \) | Direct synchronous transient reactance |
| \( X_{mi} \) | DFIG magnetizing reactance |
| \( X_{ri} \) | DFIG rotor reactance |
| \( X_{si} \) | DFIG Stator reactance |
| \( X_{ui} \) | Ratio between DFIG magnetizing and stator self-inductance |
| \( R_{ci} \) | DFIG Rotor resistance |
| \( R_{si} \) | DFIG Stator resistance |
| \( \psi_{i} \) | Damping constant |
| \( B \) | Susceptance |
| \( E_{f,i} \) | Exciter voltage |
| \( \tau_{ci} \) | Turbine time constant |
| \( H_{i} \) | Turbine inertia of wind turbine |
| \( T_{mi} \) | Mechanical torque of wind turbine |
| \( \lambda_{i} \) | Tip-speed ratio of wind turbine |
| \( \rho_{i} \) | Air density |
| \( \xi_{i} \) | Speed regulation coefficient |
| \( N_{i} \) | Neighboring areas of area \( i \) |
| \( A \) | Incidence matrix of power network |
| \( L_{com}^{\text{trans}} \) | Laplacian matrix of communication network |
| \( u_{ci} \) | Control input for conventional generator |
| \( u_{wi} \) | Control input for wind turbine |

where \( \omega, V : \mathbb{R}^{n} \to \mathbb{R}^{n}, P : \mathbb{R}^{n} \to \mathbb{R}^{n} \) is defined as \( P := \text{col}(P_{ci}, P_{wu}) \), with \( P_{ci} : \mathbb{R}^{n} \to \mathbb{R}^{n}, P_{wu} : \mathbb{R}^{n} \to \mathbb{R}^{n} \) denoting the vector of the power generated by conventional and wind turbine generators, respectively, \( \theta : \mathbb{R}^{n} \to \mathbb{R}^{m} \) denotes the vector of the voltage angles differences, \( \chi_{d} \in \mathbb{R}^{n \times n} \) is a diagonal matrix whose diagonal elements are defined as \( \chi_{di} := X_{di} - X'_{di} \), with \( X_{di}, X'_{di} \in \mathbb{R}^{n}, \tau_{ci}, \psi_{i}, P_{l} \in \mathbb{R}^{n \times n} \), and \( E_{f} \in \mathbb{R}^{n} \). Moreover, \( : \mathbb{R}^{n} \to \mathbb{R}^{m \times n} \) is defined as \( \Upsilon(V) := \text{diag} \{ Y_{1}, Y_{2}, ..., Y_{m} \} \), with \( Y_{k} := V_{k}^{\top}B_{ij} \), where \( k \sim \{ i, j \} \) denotes the line connecting areas \( i \) and \( j \). Furthermore, for any \( i, j \in V \), the components of \( E : \mathbb{R}^{n} \to \mathbb{R}^{n \times n} \) are defined as follows:

\[
E_{ii}(\theta) = \frac{1}{\chi_{di}} - B_{ii}, \quad \text{\( E_{ij}(\theta) = -B_{ij} \cos(\theta_{k}) = E_{ji}(\theta), \quad k \sim \{ i, j \} \in E \)}
\]

**(Remark 1: Susceptance and reactance).** According to [6, Remark 1], we notice that the reactance \( X_{di} \) of each generator \( i \in V \) is in practice generally larger than the corresponding transient reactance \( X'_{di} \). Furthermore, the self-susceptance \( B_{ii} \) is negative and satisfies \( |B_{ii}| > \sum_{j \in \mathcal{N}_{i}} |B_{ij}| \). Therefore, \( E(\theta) \) is a strictly diagonally dominant and symmetric matrix with positive elements on its diagonal, implying that \( E(\theta) \) is positive definite [18].

### B. Turbine-Governor Model for Conventional (Synchronous) Generators

In this subsection, we introduce the dynamics of the turbine-governor typically coupled with conventional (synchronous) generators. Specifically, we express the power generated by the (equivalent) synchronous generator \( i \in V_{c} \) as the output of a first-order dynamical system describing the behaviour of the turbine-governor, i.e.,

\[
\tau_{ci} \dot{P}_{ci} = -P_{ci} - \xi_{i}^{-1} \omega_{i} + u_{ci},
\]

where \( u_{ci} : \mathbb{R}_{\geq 0} \to \mathbb{R} \) is the control input and \( \tau_{ci}, \xi_{i} \in \mathbb{R}_{>0} \).

Now, we can write systems (3) compactly for all nodes \( i \in V \), as

\[
\tau_{ci} \dot{P}_{ci} = -P_{ci} - \xi_{i}^{-1} \omega_{i} + u_{ci},
\]

where \( u_{ci} : \mathbb{R}_{\geq 0} \to \mathbb{R}^{nc} \) and \( \tau_{ci}, \xi_{i} \in \mathbb{R}^{nc \times nc} \).

Now, as it is customary in the power systems literature (see for instance [3], [6], [18]), we assign to the power generated by the synchronous generator \( i \in V_{c} \), the following strictly convex linear-quadratic cost function:

\[
J_{i}^{c}(P_{ci}) = \frac{1}{2} q_{i} P_{ci}^{2} + z_{i} P_{ci} + c_{i},
\]

where \( J_{i}^{c} : \mathbb{R} \to \mathbb{R}, q_{i} \in \mathbb{R}_{\geq 0}, z_{i} \in \mathbb{R}, \) and \( c_{i} \in \mathbb{R} \) for all \( i \in V_{c} \).

### C. DFIG-Based Wind Turbine Generator Model

In this subsection, we introduce the Doubly Fed Induction Generator (DFIG) dynamics of a wind turbine generator. In the DFIG-based wind turbine generator, two back-to-back converters including a rotor side converter and a grid side converter are used. The rotor side converter controls the rotor...
current, while the grid side converter controls the DC link voltage [20], [21]. Since wind speed affects the generated power of a wind turbine, it is then important to have a realistic model of the wind speed. In our model, we consider that the wind speed at each node $i \in \mathcal{V}$ is given by the sum of a predicted constant component $\dot{v}_i$ and a stochastic component $\tilde{v}_i$. For this reason, an appropriate mathematical framework such as the Ito calculus framework is adopted to analyze the DFIG model with stochastic wind speed and to control the active power generated by the wind turbine. Before introducing the DFIG dynamics, we recall for the readers’ convenience the definition of stochastic differential equation through the Ito calculus framework [22], [23].

Definition 1: (Stochastic differential equation). A stochastic differential equation (SDE) is defined as follows:

$$dx(t) = f(x, u)dt + g(x)dβ(t),$$  

(6)

where $f(x, u) \in \mathbb{R}^N$ and $g(x) \in \mathbb{R}^{N \times M}$ are locally Lipschitz, $x(t) \in \mathbb{R}^N$ is the state vector of the stochastic process, $u(t) \in \mathbb{R}^P$ is the input of the system and $β(t) \in \mathbb{R}^M$ is the standard Brownian motion vector.

Now, according to [20], [21], the dynamics of the DFIG-based wind turbine generator $i \in \mathcal{V}_w$ are given by

$$\dot{t}_{d.si} = \frac{f_a}{K_i} \left( -R_{si}X_{si}t_{d.si} + (K_i + X_{mi}f_{ri})t_{q.si} + R_{ri}X_{mi} 
$$

$$t_{dri} + X_{mi}X_{si}f_{r.si}t_{qri} + X_{ri}V_{ti} - X_{mi}V_{dri} \right) 
$$

$$:= h_{d.si}(x_i) + b_{si}V_{dri}$$

$$\dot{t}_{q.si} = \frac{f_a}{K_i} \left( - (K_i + X_{mi}f_{ri})t_{d.si} - R_{si}X_{si}t_{q.si} 
$$

$$- X_{mi}X_{si}f_{r.si}t_{dri} - X_{mi}V_{qri} + R_{ri}X_{mi}t_{qri} \right) 
$$

$$:= h_{q.si}(x_i) + b_{si}V_{qri}$$

$$\dot{t}_{dri} = \frac{f_a}{K_i} \left( R_{si}X_{si}t_{d.si} - X_{si}X_{si}f_{r.si}t_{dri} - R_{ri}X_{si}t_{dri} 
$$

$$+ (K_i - X_{si}X_{ri}f_{r.si})t_{qri} - X_{mi}V_{ti} + X_{si}V_{dri} \right) 
$$

$$:= h_{dri}(x_i) + b_{ri}V_{dri}$$

$$\dot{t}_{qri} = \frac{f_a}{K_i} \left( X_{si}X_{si}f_{r.si}t_{d.si} + R_{si}X_{si}t_{q.si} + (X_{si}X_{ri}f_{r.si} 
$$

$$- K_i)t_{dri} - R_{ri}X_{si}t_{qri} + X_{si}V_{qri} \right) 
$$

$$:= h_{qri}(x_i) + b_{ri}V_{qri}$$

$$\dot{f}_{ri} = \frac{1}{2H_i} \left( T_{mi}(\ddot{v}_i) - X_{mi}(t_{dsi}t_{qri} - t_{dsi}t_{dri}) \right) 
$$

$$:= h_{fri}(x_i)$$

$$P_{wu} = -X_{wi}q_{ri}f_{ri} 
$$

$$:= g_i(x_i),$$  

(7)

where $t_{d.si}, t_{q.si}, t_{dri}, t_{qri}, V_{dri}, V_{qri}, f_{ri}, \ddot{v}_i, P_{wu} : \mathbb{R}_0 \rightarrow \mathbb{R}$, $x_i : \mathbb{R}_0 \rightarrow \mathbb{R}^6$ is the state vector of DFIG defined as $x_i := \text{col}(t_{d.si}, t_{q.si}, t_{dri}, t_{qri}, f_{ri}, \ddot{v}_i)$, and $b_{si}, b_{ri} \in \mathbb{R}$ are defined as $b_{si} := -\frac{f_a}{K_i} X_{mi}$ and $b_{ri} := \frac{f_a}{K_i} X_{si}$. Also, $h_{d.si}, h_{q.si}, h_{dri}, h_{qri}, h_{fri}, g_i : \mathbb{R}^6 \rightarrow \mathbb{R}$, $V_{dri}, V_{qri}, f_{ri}, X_{si}, X_{ri}, X_{mi} \in \mathbb{R}$, $R_{si}, R_{ri}, H_i \in \mathbb{R}_0$, and $K_i \in \mathbb{R}$ is defined as $K_i := X_{mi}X_{ri} - X_{ri}^2$. Moreover, $T_{mi} : \mathbb{R} \rightarrow \mathbb{R}_0$ is defined as $T_{mi}(\ddot{v}_i) := \frac{1}{2} \rho \pi r_i^2 C_Q(\lambda_i)(\ddot{v}_i + \ddot{v}_i)^2$ with $\ddot{v}_i \in \mathbb{R}$, $\lambda_i, \rho, r_i \in \mathbb{R}_0$, $C_Q : \mathbb{R}_0 \rightarrow \mathbb{R}_0$. Now, let the stochastic term of wind speed $\tilde{v}_i$ be modeled by a SDE as in [17], i.e.,

$$\tilde{v}_i = -\mu_{wi}\tilde{v}_i dt + \sigma_{wi}\tilde{v}_i d\beta, \ \forall i \in \mathcal{V}_w,$$

(8)

where $\mu_{wi}$ and $\sigma_{wi}$ are positive constant parameters. Then, we can rewrite (7) and (8) compactly for all nodes $i \in \mathcal{V}_w$ as

$$dx = (H_g(x) + B_u u_w)dt + G(x)\beta(t)$$

(9)

where $x : \mathbb{R}_0 \rightarrow \mathbb{R}^{6n_w}$ is defined as $x := \text{col}(x_{11}, ..., x_{nn_w})$, $u_w : \mathbb{R}_0 \rightarrow \mathbb{R}^{2n_u}$ with $u_{wi} : \mathbb{R}_0 \rightarrow \mathbb{R}^2$ defined as $u_{wi} := \text{col}(V_{dri}, V_{qri})$, $\beta : \mathbb{R}_0 \rightarrow \mathbb{R}^{2n_w}$ is the standard Brownian motion vector. Furthermore, $H_g : \mathbb{R}^{6n_w} \rightarrow \mathbb{R}^{2n_w}$ is defined as $H_g(x) := \text{col}(H_{g1}, ..., H_{g_{n_w}})$ with $H_{g_i}(x_i) := \text{col}(h_{d.isi}(x_i), h_{q.isi}(x_i), h_{dri}(x_i), h_{qri}(x_i), h_{fri}(x_i), -\mu_{vi})$, $G : \mathbb{R}^{6n_w} \rightarrow \mathbb{R}^{2n_w}$ is defined as $G(x) := \text{blockdiag}(G_1, ..., G_{n_w})$ with $G_i(x) := \text{diag}(0,0,0,0,0,0,0)$, $\zeta : \mathbb{R}^{6n_w} \rightarrow \mathbb{R}^{n_u}$ is defined as $\zeta(x) := \text{col}(\zeta_{11}, ..., \zeta_{nn_w})$ and $B_u \in \mathbb{R}^{6n_w \times 2n_u}$ is defined as $B_u := \text{blockdiag}(B_{u1}, ..., B_{un_u})$ with $B_{ui} := \text{col}(0, b_{si}), (0, b_{ri}), (0, b_{ri}), (0, b_{ri})$.

Now, we assign to the power generated by the wind turbine $i \in \mathcal{V}_w$, the following strictly concave linear-quadratic utility function:

$$J_{wu}^u(P_{wu}) = -\frac{1}{2} q_{wu}^2 P_{wu}^2 + z_i P_{wu} + c_i,$$

(10)

where $J_{wu}^u : \mathbb{R} \rightarrow \mathbb{R}$, $q_i \in \mathbb{R}_0$, $z_i \in \mathbb{R}$, and $c_i \in \mathbb{R}$ for all $i \in \mathcal{V}_w$. Note that $q_i$ and $z_i$ are selected in order to take into account the value of the maximum power that the wind turbine can generate given the predicted wind speed $\dot{v}_i$.

D. Control Objectives

In this subsection, we introduce and discuss the main control objectives of this work. The first objective concerns the asymptotic regulation of the frequency deviation to zero, i.e.,

Objective 1: (Load Frequency Control).

$$\lim_{t \to \infty} \omega(t) = 0,$$

(11)

Besides improving the stability of the power network by regulating the frequency deviation to zero, advanced control strategies additionally aim at reducing the costs associated with the power generated by the conventional synchronous generators and increasing the utilities associated with the power generated by the wind turbines. Therefore, we introduce the following optimization problem:

$$\min_P J(P)$$

s.t. $\sum_{i \in \mathcal{V}} \tilde{P}_i - P_{li} = 0,$$

(12)

where $J(P) = \sum_{i \in \mathcal{V}} J_{wi}^u(P_{wi}) - \sum_{i \in \mathcal{V}_w} J_{wi}^u(P_{wi}) + \frac{1}{2} P^T Q P + Z^T P + \frac{1}{n} C$ with $J_{wi}^u(P_{wi}), J_{wi}^u(P_{wi})$ given
by (5), (10), respectively. Also, $Q \in \mathbb{R}^{n \times n}$, $Z, C \in \mathbb{R}^n$ are defined as $Q := \text{diag}(q_1, \ldots, q_n, q_{n+1}, \ldots, q_{n+n})$, $Z := \text{col}(z_1, \ldots, z_n, -z_n+1, \ldots, -z_n+n)$, $C := \text{col}(c_1, \ldots, c_n, -c_n+1, \ldots, -c_n+n)$, respectively. In this regard, [6, Lemma 2], [18, Lemma 3] show that it is possible to achieve zero steady-state frequency deviation and simultaneously minimize the objective function $J(P)$ in (12) when the load $P_l$ is constant. More precisely, when the load $P_l$ is constant, the optimal value of $P$, which allows for zero steady-state frequency deviation and minimizes (at the steady-state) the objective function $J(P)$ in (12), solving the optimization problem (12), is given by:

$$P_{\text{opt}} = Q^{-1} \left( I_n Q^{-1} (P_l + Q^{-1} Z) - Z \right),$$

(13)

where $P_{\text{opt}} := \text{col}(P_{\text{opt}}, P_{\text{w}})$. This leads to the second objective, i.e., minimization of the objective function $J(P)$ in (12), which is also known in the literature as economic dispatch or optimal generation [6, 18]. Then, the second goal concerning the economic dispatch or optimal generation is defined as follows:

**Objective 2:** (Economic dispatch).

$$\lim_{t \to \infty} P(t) = P_{\text{opt}},$$

(14)

with $P_{\text{opt}}$ given by (13).

We assume now that there exists a (suitable) steady-state solution to the considered augmented power network model (1), (4) and (9).

**Assumption 1:** (Steady-state solution). There exists a constant input $(\bar{u}_c, \bar{u}_w)$ and a steady-state solution $(\bar{\theta}, \bar{\omega}, \bar{V}, \bar{P}, \bar{x})$ to (1), (4) and (9) satisfying

$$0 = A^T \bar{\omega},$$

$$0 = -\psi \bar{\omega} + \bar{P} - P_l - AT(\bar{V}) \sin(\bar{\theta})$$

$$0 = -\chi_d E(\bar{\theta}) \bar{V} + E_f$$

$$0 = -P_c - \xi \bar{\omega} + \bar{u}_c$$

$$0 = (H_g(\bar{x}) + B_u \bar{u}_w) dt + G(\bar{x}) d\beta.$$  

(15)

Additionally, (15) holds also when $\bar{\omega} = 0$ and $\bar{P} = P_{\text{opt}}$, with $P_{\text{opt}}$ given by (13).

In the next section, we present the passivity properties for the power network, turbine-governor and wind turbine. Then, we design a control scheme for regulating the frequency in the presence of stochastic wind power generation. To this end, in analogy with [6], [18], the following assumption is required:

**Assumption 2:** (Steady-state voltage angle and amplitude). The steady-state voltage $\bar{V} \in \mathbb{R}^n$ and angle difference $\bar{\theta} \in \mathbb{R}^m$ satisfy

$$\bar{\theta} \in (-\frac{\pi}{2}, \frac{\pi}{2})^m,$$

$$\chi_d E(\bar{\theta}) - \text{diag}(\bar{V})^{-1} |A| (\forall \bar{V}) \text{diag}(\sin(\bar{\theta}))$$

$$\text{diag}(\cos(\bar{\theta}))^{-1} \text{diag}(\sin(\bar{\theta})) |A| \text{diag}(\bar{V})^{-1} > 0.$$  

(16)

Note that Assumption 2 is usually verified in practice, i.e., the differences in voltage (angles) are small and the line reactances are greater than the generator reactances [6], [18].

### III. Optimal Load Frequency Control

In this section, we present the passivity properties for the power network, turbine-governor and wind turbine. Then, we use such passivity properties for designing a controller achieving Objectives 1 and 2.

#### A. Incremental Passivity of Power Network and Turbine-Governor

In this subsection, we recall from the literature the incremental passivity of the power network model introduced in Subsection II-A and the turbine-governor model introduced in Subsection II-B. In analogy with [18, Lemma 2], [3, Lemma 3], the incremental passivity of system (1) is obtained via the following lemma.

**Lemma 1:** (Incremental passivity of system (1)). Let Assumptions 1, 2 hold. System (1) is incrementally passive with respect to the storage function

$$S_1 = -1_n^T \Upsilon(\bar{V}) \cos(\bar{\theta}) + 1_n^T \Upsilon(\bar{V}) \cos(\bar{\theta}) + \frac{1}{2} V^T D V$$

$$- (\Upsilon(\bar{V}) \sin(\bar{\theta}))^T (\bar{\theta} - \bar{\omega}) - E_f d(\bar{V} - \bar{V})$$

$$\frac{1}{2} V^T D V + \frac{1}{2} (\omega - \bar{\omega})^T \tau_p (\omega - \bar{\omega}),$$  

(17)

and supply rate $(\omega - \bar{\omega})^T (P - \bar{P})$, where the steady-state solution $(\bar{\theta}, \bar{\omega}, \bar{\omega})$ satisfies (15) and $D$ is a diagonal matrix with $D_{ii} = 1 - B_{ii}(X_{ii} - X_{ii}^-)$.

**Proof:** The proof follows from combining [18, Lemma 2] and [3, Lemma 3]. Specifically, under the Assumption 2, the storage function (17) is a positive definite function and satisfies

$$S_1 = -(\chi_d E(\bar{\theta}) V - E_f d)^T \tau_p^{-1} (\chi_d E(\bar{\theta}) V - E_f d)$$

$$+ (\omega - \bar{\omega})^T (P - \bar{P}) - (\omega - \bar{\omega})^T \psi(\omega - \bar{\omega}),$$  

(18)

along the solutions to (1).

Now, we consider the following controller proposed in [3], [6] for the turbine-governor $i \in V_c$

$$\tau_{di} \bar{\delta}_i = -\delta_i + P_{ci},$$

$$u_{ci} = \bar{\delta}_i,$$  

(19)

where $\bar{\delta}_i : \mathbb{R}_0 \rightarrow \mathbb{R}$ and $\tau_{di} \in \mathbb{R}_0$. Then, in analogy with [3, Lemma 5] the incremental passivity of system (3) in closed-loop with (19) is obtained via the following lemma.

**Lemma 2:** (Incremental passivity of (3), (19)). Let Assumption 1 hold. System (3) with controller (19) is incrementally passive with respect to the storage function

$$S_{2i} = \frac{\tau_{ci}}{2} (P_{ci} - P_{\text{opt}})^2 + \frac{\tau_{di}}{2} (\delta_i - \bar{\delta}_i)^2,$$  

(20)

and supply rate $-(P_{ci} - P_{\text{opt}}) \omega_i$, where the steady-state solution $(P_{\text{opt}}, \bar{\delta}_i)$ satisfies (15) and

$$0 = -\bar{\delta}_i + P_{\text{opt}}^\text{out},$$  

(21)

with $P_{\text{opt}}^\text{out}$ given by (13).

**Proof:** See [3, Lemma 5].

■
B. Stochastic Passivity Property for DFIG-Based Wind Turbine

In this subsection, we propose a new control scheme to control the active power generated by the DFIG-based wind turbine. Then, we show that the DFIG-based wind turbine (7), (8) in closed-loop with the proposed controller is stochastically passive. Before introducing the DFIG controller, we recall for the readers’ convenience the definitions of Ito derivative and stochastic passivity through the Ito calculus framework [22], [23].

Definition 2: (Ito derivative). Consider a storage function $S(x)$, which is twice continuously differentiable. Then, $LS(x)$ denotes the Ito derivative of $S(x)$ along the SDE (6), i.e.,

$$
LS(x) = \frac{\partial S(x)}{\partial x} f(x, u) + \frac{1}{2} \text{tr} \left \{ g^T(x) \frac{\partial^2 S(x)}{\partial x^2} g(x) \right \}. \quad (22)
$$

Definition 3: (Stochastic passivity). Consider system (6) with output $y = \eta(x)$. Assume that the deterministic and stochastic terms of the SDE (6) at the equilibrium point are identically zero, i.e., $f(\bar{x}, \bar{u}) = g(\bar{x}) = 0$. Then, system (6) is said to be stochastically passive with respect to the supply rate $u^T y$ if there exists a twice continuously differentiable positive semi-definite storage function $S(x)$ satisfying

$$
LS(x) \leq u^T y, \quad \forall (x, u) \in \mathbb{R}^N \times \mathbb{R}^P. \quad (23)
$$

Now, consider the following controller for the DFIG-based wind turbine generator $i \in \mathcal{V}_w$:

$$
V_{dri} = -L_i(x_i)(\bar{K}_{1i}(x_i) + \bar{K}_{2i}(x_i) + \bar{K}_{3i}(x_i) + \bar{x}_i^T \Pi \bar{x}_i
+ \bar{x}_i^T \Pi \bar{x}_i + \bar{x}_i^T \Pi \hat{H}_{qi}(x_i)) \quad (24a)
$$

$$
V_{qri} = -L_i(x_i)(D_1(x_i)w_i + D_2(x_i)\delta_i + D_3(x_i)) \quad (24b)
$$

$$
\tau_i \delta_i \delta_i = -\delta_i + P_{wi}, \quad (24c)
$$

where

- $L_i(x_i) = \frac{X_{ri}}{X_{ri}(tds_i - \delta_i) - X_{mi}(t_dsi - tdsi)}$
- $\bar{K}_{1i}(x_i) = \frac{\rho \pi r_i^2 C_{qi}((f_{ri} - \bar{f}_{ri})v_i^2 + v_i(f_{ri} - \bar{f}_{ri})^2)}{f_{ri} - \bar{f}_{ri}}$
- $\bar{K}_{2i}(x_i) = \left(\frac{\rho \pi r_i^2 C_{qi}}{X_{ri}}\right)v_i + \omega_i(P_{wi} - P_{opt})$
- $\bar{K}_{3i}(x_i) = 2f_{ri}X_{mi}(tds_iq_{ri} - t_{qsi}tds_i)$

$$
= \left(\frac{R_{ri}X_{ri}}{X_{ri}} + \frac{R_{si}X_{mi}}{X_{si}}\right)tds_i
+ \left(\frac{R_{ri}X_{ri}}{X_{ri}} + \frac{R_{si}X_{mi}}{X_{si}}\right)t_{qri}q_{si}
$$

$$
\Pi_i = \text{diag} (R_{si}, R_{si}, R_{ri}, R_{ri}, 0, 0)
$$

$$
\hat{H}_{qi}(x_i) = \text{diag} \left(\frac{K_i}{f_{bi}X_{ri}}, \frac{K_i}{f_{bi}X_{ri}}, \frac{K_i}{f_{bi}X_{ri}}, \frac{K_i}{f_{bi}X_{ri}}, 0, 0\right)
$$

$$
D_1(x) = -X_{wi}q_{ri}f_{ri} + X_{wi}t_{qri}f_{ri}, \quad D_2(x) = (P_{wi} - P_{opt})\delta_i
$$

Note that the controller (24) requires the information of $\bar{x}_i$ and $P_{opt}$ which can be obtained by solving (15) and (13), respectively. In order to obtain the stochastic passivity of (7), (8), (24), we need to consider the following assumptions on the wind turbine and speed.

Assumption 3: (Condition on the rotational speed). The rotational speed $f_{ri}$ of the wind turbine $i \in \mathcal{V}_w$ is bounded as $|f_{ri} - \bar{f}_{ri}| < \gamma_{ri}, \gamma_{ri} \in \mathbb{R}_{>0}$.

Assumption 4: (Condition on the parameters of (8)). The wind speed parameters in (8) satisfies

$$
\mu_{wi} + \bar{f}_{ri} > \frac{\sigma_{wi}^2}{2} + v_i + \gamma_{ri}, \quad i \in \mathcal{V}_w. \quad (25)
$$

Note that Assumption 3 is true in practice, since the rotational speed of a wind turbine is limited by the mechanical characteristics of the turbine itself, which is indeed usually equipped with mechanical breaks that avoid high rotational speed. Assumption 4 is instead a sufficient technical condition to establish the stochastic passivity of the wind turbine.

Now, the stochastic passivity of DFIG-based wind turbine dynamics (7), with wind speed dynamics (8) and controller (24) is obtained via the following proposition.

Proposition 1: (Stochastic passivity of (7), (8), (24)). Let Assumptions 3 and 4 hold. System (7), (8) in closed-loop with (24) is stochastically passive with respect to the storage function

$$
S_{3i} = \frac{K_i}{2f_{bi}X_{ri}}((t_{dsi} - t_{dsi})^2 + (t_{qsi} - t_{qsi})^2)
+ (t_{dsi} - t_{dsi})^2 + (t_{qsi} - t_{qsi})^2 + 2H_{i}(f_{ri} - \bar{f}_{ri})^2
+ \rho r_i^2 C_{qi}v_i^2 + \frac{\tau_{ri}}{2}(\delta_i - \delta_i)^2, \quad (26)
$$

and supply rate $-\omega_i(P_{wi} - P_{opt})$, where the steady-state solution $(\bar{x}_i, P_{opt}, \delta_i)$ satisfies (15) and

$$
0 = -\delta_i + P_{opt}, \quad (27)
$$

with $P_{opt}$ given by (13).

Proof: The Ito derivative of the storage function (26) satisfies

$$
LS_{3i} = -R_{si}(t_{dsi} - t_{dsi})^2 - R_{si}(t_{dsi} - t_{dsi})^2
- R_{si}(t_{qsi} - t_{qsi})^2 - R_{si}(t_{dsi} - t_{dsi})^2
- (\delta_i - P_{wi})^2 - \omega_i(P_{wi} - P_{opt}) - (\delta_i - \delta_i)^2
- (f_{ri} - \bar{f}_{ri})^2 - \rho r_i^2 C_{qi}v_i^2 - \frac{\sigma_{wi}^2}{2} - v_i
$$

along the solution to (7), (8), (24). Then, we can conclude that $LS_{3i} \leq -(P_{wi} - P_{opt})^2 \omega_i$. ■

C. Closed-loop analysis

In this subsection, we show that the closed-loop system is stochastically stable, achieving Objectives 1 and 2. First, we recall the definition of (asymptotic) stochastic stability [22], [23].

Definition 4: (Asymptotic) stochastic stability. System (6) is (asymptotically) stochastically stable if a twice continuously differentiable positive definite Lyapunov function
Now, in order to achieve Objective 2, we modify controllers (19) and (24c) as follows (see [3], [6]):

\[
\begin{align*}
\tau_{\delta_i} \dot{\delta}_i &= -\delta_i + P_{ci} \\
&= -\xi_i^{-1} q_i \sum_{j \in N^\text{com}} (q_i \delta_i + z_i - (q_j \delta_j - z_j)), \forall i \in V_c \\
&= -\xi_i^{-1} q_i \sum_{j \in N^\text{com}} (q_i \delta_i + z_i - (q_j \delta_j - z_j)), \forall i \in V_w
\end{align*}
\]

where \( \tau_{\delta_i} \) is the design parameter and \( N^\text{com} \) is the set of areas communicating with area \( i \). The distributed controller (29) can be written compactly for all \( i \in V \) as

\[
\tau_{\delta} \dot{\delta} = -\delta + P - \text{blockdiag}(\xi^{-1}, I_{n_w})QL^\text{com}(Q\delta + Z),
\]

where \( \delta : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n \), \( \tau_{\delta} \in \mathbb{R}^{n \times n} \) and \( L^\text{com} \in \mathbb{R}^{n \times n} \) is the Laplacian matrix associated with a connected communication network. More precisely, the term \( Q\delta + R \) in (30) reflects the marginal cost associated with the objective function \( J(P) \) in (12) and \( L^\text{com}(Q\delta + Z) \) represents the exchange of such information among the areas of the power network. In the following theorem, we show that the closed-loop system (1), (4), (9), (24a), (24b), (30) is stochastically stable and Objectives 1 and 2 are attained.

**Theorem 1: (Closed-loop analysis).** Let Assumptions 1--4 hold. Consider system (1), (4), (9) with controller (24a), (24b), (30). Then, the solutions to the closed-loop system starting sufficiently close to \((\bar{\theta}, \bar{\omega}) = (0, V, P^{\text{opt}}, \bar{x}, \bar{\delta})\) stochastically converge to the set where \( \bar{\omega} = 0 \) and \( \bar{P} = P^{\text{opt}} \), with \( P^{\text{opt}} \) given by (13), i.e., achieving Objectives 1 and 2.

**Proof:** Following Lemmas 1, 2 and Proposition 1, we consider the storage function \( S = S_1 + S_2 + S_3 \), where \( S_1 \) is given in (17), \( S_2 = \sum_{i \in V} S_{2i} \), with \( S_{2i} \) given by (20), and \( S_3 = \sum_{i \in V_c} S_{3i} \), with \( S_{3i} \) given by (26). Now, the gradient of \( S \) is given by

\[
\nabla S = \text{col} \left( \nabla(Y) \sin(\theta) - \nabla(Y) \sin(\theta), \chi_d E(\theta) V - \bar{E}_{fd}, \right.
\]

\[
\left. \tau_p (\omega - \bar{\omega}), \tau_3 \text{blockdiag}(\xi, I_{n_w})(\delta - \bar{\delta}), \text{blockdiag}(\tau_\delta, 0_{n_w \times n_w}) (P - P^{\text{opt}}), K[f_{b1}^{-1}]X^{-1}_r \right)
\]

\[
\left( (t_{ds} - \bar{t}_{ds}), K[f_{b1}^{-1}]X^{-1}_r (t_{qs} - \bar{t}_{qs}), K[f_{b1}^{-1}]X^{-1}_r (t_{dr} - \bar{t}_{dr}), K[f_{b1}^{-1}]X^{-1}_r (t_{qr} - \bar{t}_{qr}), \right.
\]

\[
\left. 4H(f_{r} - \bar{f}_{r}), 2\rho \pi [r]^3 [C_Q] \bar{\nu} \right).
\]

We can observe from (31) that \( \nabla S \) evaluated at \((\bar{\theta}, \bar{\omega}) = (0, V, P^{\text{opt}}, \bar{x}, \bar{\delta})\) is equal to zero. Then, the Hessian matrix of \( S \) is given by

\[
\nabla^2 S = \text{blockdiag} \left( \Lambda, \tau_p, \tau_3 \text{blockdiag}(\xi, I_{n_w}), \text{blockdiag}(\tau_\delta, 0_{n_w \times n_w}), (T_m K)^\top, \right.
\]

\[
\left. (K[f_{b1}^{-1}]X^{-1}_r)^\top, (K[f_{b1}^{-1}]X^{-1}_r)^\top, (K[f_{b1}^{-1}]X^{-1}_r)^\top, 4H^\top, \right)
\]

\[
\left. (2\rho \pi [r]^3 [C_Q])^\top \right)
\]

where

\[
\Lambda = \begin{pmatrix} Y(V) \text{diag}(\cos(\theta)) & \Omega^\top \\ \Omega & \chi_d E(\theta) \end{pmatrix}
\]

with \( \Omega = (\text{diag}(V))^{-1}A[\nabla(Y) \text{diag}(\sin(\theta)) \right) \). By virtue of Assumption 2, we have \( \nabla(Y) \text{diag}(\cos(\theta)) > 0 \) for \( \theta \in (-\frac{\pi}{2}, \frac{\pi}{2}) \). Then in analogy with [18, Lemma 2] and by using the Schure complement of (33), the matrix \( \Lambda \) evaluated at \((\bar{\theta}, \bar{\omega}) = (0, V, P^{\text{opt}}, \bar{x}, \bar{\delta})\) is positive definite if and only if

\[
\chi_d E(\theta) - (\text{diag}(V))^{-1}D[\nabla(Y) \text{diag}(\sin(\theta)) \right) \text{diag}(\cos(\theta))^{-1} \text{diag}(\sin(\theta))D^\top \text{diag}(V) - 1 > 0.
\]

Thus, by virtue of Assumption 2, it can be inferred from (32)-(34) that the Hessian matrix \( \nabla^2 S \) evaluated at \((\bar{\theta}, \bar{\omega}) = (0, V, P^{\text{opt}}, \bar{x}, \bar{\delta})\) is positive definite. Consequently, the storage function \( S \) has a local minimum at \((\bar{\theta}, \bar{\omega}) = (0, V, P^{\text{opt}}, \bar{x}, \bar{\delta})\).

Now, the Ito derivative of the storage function \( S \) satisfies

\[
\dot{S} = -\left( \chi_d E(\theta)V - \bar{E}_{fd} \right)^\top \nabla^2 S \nabla^2 S \left( \chi_d E(\theta)V - \bar{E}_{fd} \right)
\]

\[
- (\omega - \bar{\omega})^\top \psi(\omega - \bar{\omega}) - (\delta - \bar{\delta})^\top \dot{\varphi}(\delta - \bar{\delta})
\]

\[
- (t_{qs} - \bar{t}_{qs})^\top R_s (t_{qs} - \bar{t}_{qs}) - (t_{ds} - \bar{t}_{ds})^\top R_s (t_{ds} - \bar{t}_{ds})
\]

\[
- (t_{dr} - \bar{t}_{dr})^\top R_r (t_{dr} - \bar{t}_{dr}) - (f_r - \bar{f}_{r})^\top \rho \pi [r]^3 [C_Q] (f_r - \bar{f}_{r})
\]

\[
- (Q_\delta + Z)^\top L^\text{com}(Q_\delta + Z) - (\delta - \bar{\delta})^\top (\delta - \bar{\delta})
\]

along the solution to (1), (4), (9), (24a), (24b), (30), where \( \dot{\theta} = \text{blockdiag}(\xi, I_{n_w}) \). Then, it follows that \( \dot{S} \leq 0 \). Thus, we can conclude that the solutions to the closed-loop system (1), (4), (9), (24a), (24b), (30) are bounded. Moreover, according to LaSalle’s invariance principle, these solutions stochastically converge to the largest invariant set contained in \( \Lambda = \{ \theta, \omega, V, P, x, \delta : \omega = 0, \chi_d E(\theta)V = \bar{E}_{fd} P = \delta, x = \bar{x}, Q_\delta + Z = Q_\delta + Z + d(t) I_{n_1} \} \), where \( d(t) : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R} \). Then, we can obtain \( \delta + d(t)Q^{\text{opt}}1_{n_1} = P^{\text{opt}} + d(t)Q^{\text{opt}}1_{n_1} = \bar{\delta} = \bar{P} \). Hence, the behavior of the power network system (1) on the set \( \Lambda \) can be described by

\[
\dot{\theta} = 0
\]

\[
0 = P^{\text{opt}} - A\nabla(Y) \sin(\theta) + d(t)Q^{\text{opt}}1_{n_1} - P_l
\]

\[
0 = -\chi_d E(\theta)V + \bar{E}_{fd}.
\]
where $\nabla'$ and $\theta'$ are constants (possibly different from $\nabla$ and $\theta$). Moreover, since $\mathbf{1}_n^\top (P_{opt} - P_l) = 0$, $\mathbf{1}_n^\top A = 0$, and $Q^{-1}$ is a positive definite diagonal matrix, we can pre-multiply the second equation of (36) by $\mathbf{1}_n^\top$ and obtain $d(t) = 0$.

Thus, we have $\delta = \bar{\delta}$ and can then infer from (21), (27) that $\bar{\omega} = 0$ and $\bar{P} = P_{opt}$ with $P_{opt}$ given by (13).

IV. Simulation Results

In this section, the simulation results show excellent performance of the proposed distributed control scheme. We consider a power network partitioned into four control areas that are interconnected as represented in [24, Fig. 1], where areas 1, 2 and 3 include conventional generation, while area 4 includes wind generation. We provide the system parameters in Table II, where the parameters are equal to [20, Table I] and [24, Table II], the nominal frequency and power base are chosen equal to $120\pi$ rad/s and 1000 MVA, respectively.

![Fig. 1: Frequency deviation in each area.](image1)

![Fig. 2: Generated power in each area.](image2)

![Fig. 3: Voltage in each area.](image3)

The system is initially at the steady-state with constant load $P_l = \text{col}(1.3, 2, 1.3, 0.5)$. Then, at the time instant $t = 5$ s the load increases to $P_l = \text{col}(1.4, 2.1, 1.4, 0.55)$ and the wind speed varies according to the stochastic differential equation (8). Fig. 1 shows that the frequency deviation in each area converges to zero after a transient time. Also, we notice from Fig. 2 that after $t = 5$ s the generated power in each area converges to the corresponding optimal value (dashed line), which has been computed according to (13) with $P_l = \text{col}(1.4, 2.1, 1.4, 0.55)$. Specifically, we observe that the additional power demand is supplied by the conventional generators while the wind turbine (Area 4) generates the maximum possible power given a certain wind speed. Moreover, we can notice from Fig. 3 that the voltages are stable.

| Parameter | Area 1 | Area 2 | Area 3 | Area 4 |
|-----------|--------|--------|--------|--------|
| $B_{ii}$ (p.u.) | -56.3 | -58.5 | -56.2 | -49.4 |
| $q_i$ (10^{-4} rad/s) | 5 | 4.5 | 5.5 | 1 |
| $\tau_v$ (s) | 6.32 | 6.63 | 7.15 | 6.46 |
| $X_{di}$ (p.u.) | 1.76 | 1.81 | 1.87 | 1.91 |
| $X_{di}'$ (p.u.) | 0.27 | 0.17 | 0.23 | 0.35 |
| $E_{f_{di}}$ (p.u.) | 3.85 | 4.43 | 3.96 | 3.88 |
| $\tau_{pi}$ (p.u.) | 3.95 | 4.71 | 5.23 | 4.17 |
| $\psi_i$ (rad) | 1.82 | 1.61 | 1.33 | 1.55 |
| $\tau_{ci}$ (s) | 7.2 | 6.8 | 8.9 | - |
| $\tau_{d_i}$ (s) | 0.2 | 0.2 | 0.2 | 0.2 |
| $\xi_i$ (Hz p.u.^{-1}) | 0.73 | 0.73 | 0.73 | - |
| $R_{di}$ (p.u.) | - | - | - | 0.031 |
| $X_{di}$ (p.u.) | - | - | - | 0.025 |
| $X_{di}'$ (p.u.) | - | - | - | 3.62 |
| $X_{mi}$ (p.u.) | - | - | - | 3.61 |
| $H_i$ (p.u.) | - | - | - | 3.6 |
| $r_i$ (m) | - | - | - | 42 |
| $\mu_{wi}$ (p.u.) | - | - | - | 17.15 |
| $\sigma_{wi}$ (p.u.) | - | - | - | 2.65 |
V. CONCLUSION

In this paper, we have considered a power network including conventional synchronous generators with turbine-governor and wind turbines based on the doubly fed induction generator, where the wind speed is described by a stochastic differential equation. Then, we have verified the (stochastic) passivity of the considered system and present a distributed control scheme that guarantees the stochastic stability of the overall system, achieving optimal load frequency control.

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