The interaction of TeV photons from blazars with the extragalactic background light produces a relativistic beam of electron-positron pairs streaming through the intergalactic medium (IGM). The fate of the beam energy is uncertain. By means of two- and three-dimensional particle-in-cell simulations, we study the nonlinear evolution of dilute ultra-relativistic pair beams propagating through the IGM. We explore a wide range of beam Lorentz factors $\gamma_b \gg 1$ and beam-to-plasma density ratios $\alpha \ll 1$, so that our results can be extrapolated to the extreme parameters of blazar-induced beams ($\gamma_b \sim 10^6$ and $\alpha \sim 10^{-15}$, for powerful blazars). For cold beams, we show that the oblique instability governs the early stages of evolution, but its exponential growth terminates—due to self-heating of the beam in the transverse direction—when only a negligible fraction $\sim (\alpha/\gamma_b)^{1/3} \sim 10^{-7}$ of the beam energy has been transferred to the IGM plasma. Further relaxation of the beam proceeds through quasi-longitudinal modes, until the momentum dispersion in the direction of propagation saturates at $\Delta p_{b,\parallel}/\gamma_b m_\text{e}c \sim 0.2$. This corresponds to a fraction $\sim 10\%$ of the beam energy—irrespective of $\gamma_b$ or $\alpha$—being ultimately transferred to the IGM plasma (as compared to the heating efficiency of $\sim 50\%$ predicted by one-dimensional models, which cannot properly account for the transverse broadening of the beam). For the warm beams generated by TeV blazars, the development of the longitudinal relaxation is suppressed, since the initial dispersion in beam momentum is already $\Delta p_{b,\parallel}/\gamma_b m_\text{e}c \gtrsim 1$. Here, the fraction of beam energy ultimately deposited into the IGM is only $\sim \alpha \gamma_b \sim 10^{-9}$. It follows that most of the beam energy is still available to power the GeV emission produced by inverse Compton up-scattering of the cosmic microwave background by the beam pairs.

Key words: gamma rays: general – instabilities – intergalactic medium – plasmas – radiation mechanisms: non-thermal

Online-only material: color figures

1. INTRODUCTION

With the current generation of Čerenkov telescopes, hundreds of TeV sources have been discovered. By far, the extragalactic TeV sky is dominated by blazars: jets from galactic centers beaming their emission toward our line of sight. The TeV photons from distant blazars cannot travel cosmological distances, since they interact with the extragalactic background light (EBL), producing electron–positron pairs. Studies of the attenuated ~100 GeV–TeV light from distant blazars can therefore provide constraints on the strength of the EBL (e.g., Aharonian et al. 2006; Abdo et al. 2010). The produced electron-positron pairs form a relativistic beam moving in the direction of the incident TeV photons. It is usually assumed that the energy of the pair beam is lost via inverse Compton (IC) scattering off the cosmic microwave background (CMB). As a result, the TeV radiation will be reprocessed into the GeV band (Neronov & Semikoz 2009). While cooling, the pairs gyrate around the magnetic fields intergalactic medium (IGM). Depending of the field strength and length scale, the GeV emission may form an extended source, show characteristic delays with respect to the TeV flux, or be strongly suppressed. These effects make combined GeV–TeV studies a useful probe of the IGM fields (e.g., Neronov & Vovk 2010; Tavecchio et al. 2010; Dermer et al. 2011; Dolag et al. 2011; Taylor et al. 2011; Takahashi et al. 2012; Vovk et al. 2012).

Recently, it has been proposed that the destiny of the blazar-induced beams may be different. As they stream through the IGM plasma, the electron–positron pairs are expected to trigger collective plasma instabilities (as opposed to binary Coulomb collisions, that are negligible, as discussed by Miniati & Elyiv 2013). For the parameters relevant to blazar-induced beams (i.e., dilute and ultra-relativistic), the fastest growing mode is the electrostatic oblique instability (e.g., Fainberg et al. 1970; Bret et al. 2010b), whose linear growth rate can exceed the IC cooling rate by several orders of magnitude (Broderick et al. 2012). Assuming that the instability keeps growing at the linear rate until all the beam energy is deposited into the IGM, the beam energy loss will be dominated by collective beam-plasma instabilities, rather than IC cooling. In this case, the blazar TeV emission would not be reprocessed down to multi-GeV energies, thus invalidating the IGM field estimates based on the GeV–TeV flux (Broderick et al. 2012, 2013). In addition, as a result of the beam relaxation, a substantial amount of energy would be deposited into the IGM. This “volumetric heating” can have dramatic consequences for the thermal history of the IGM (Chang et al. 2012; Pfrommer et al. 2012; Puchwein et al. 2012). However, blazar-induced heating should produce an inverted temperature-density relation in the IGM, which is not supported by the observations (Rudie et al. 2012).

While plasma instabilities could, in principle, be fast enough to thermalize the pair beam, their nonlinear stages are far more complicated than what linear dispersion analysis predicts (e.g., the studies by Miniati & Elyiv 2013 and Schlickeiser...
et al. 2012b, 2013 reached opposite conclusions regarding the ultimate fate of blazar-induced beams. The nature of the fastest growing instability can change as the beam-plasma system evolves, due to the suppression of temperature-sensitive modes as the beam heats up (e.g., Bret et al. 2008). Also, beam-plasma instabilities can saturate at very small amplitudes, in particular for the extremely dilute beams produced by TeV blazars (e.g., Thode & Sudan 1975; Thode 1976).

In this work, we use first-principles particle-in-cell (PIC) simulations in two and three dimensions to study the nonlinear stages and saturation of the instabilities generated as the blazar-induced pair beams propagate through the IGM. The nonlinear effects of the beam-plasma interaction are hard to capture with analytical tools, and they require fully kinetic simulations. We explore a wide range of beam Lorentz factors \( \gamma_b \approx 1 \) and beam-to-plasma density ratios \( \alpha \ll 1 \), so that our results can be extrapolated to the extreme parameters of blazar-induced beams (\( \gamma_b \approx 10^6 \) and \( \alpha \approx 10^{-15} \), for the most powerful blazars). We find that, for ultra-relativistic dilute beams that start with a negligible thermal spread, electrostatic beam-plasma instabilities can de-pose \( \approx 10\% \) of the beam energy into the background electrons. However, if the beam is born with a significant momentum dispersion (as expected for blazar-induced beams), the fraction of energy going into IGM heating is as small as \( \approx \alpha \gamma_b \approx 10^{-6} \), for typical blazars. We conclude that most of the beam energy is still available to power the GeV emission produced by IC scattering off the CMB (leading to many photons of energy \( \approx 100 \left(E_{\gamma}/10 \text{ TeV}\right)^2 \) GeV per original TeV photon).

For a powerful blazar with TeV isotropic equivalent luminosity \( L_{\gamma} \approx 10^{45} \text{ L}_{\gamma,45} \text{ erg s}^{-1} \) (e.g., Ghisellini et al. 2010), the number density \( n_b \) of the beam pairs is set by the balance of the pair production rate with the energy loss rate (dominated by IC cooling), which gives

\[
n_b \approx \frac{2 L_{\gamma}}{4\pi D_{\gamma}^2 c} \frac{d_{IC}}{D_{\gamma}} \approx 10^{-23} K_{\text{EBL}}^{-3} L_{\gamma,45} \left( \frac{E_{\gamma}}{10 \text{ TeV}} \right) \text{ cm}^{-3},
\]

(1)

where the factor of \( d_{IC}/D_{\gamma} \approx 1 \) accounts for the rapid energy loss of the pairs due to IC.\(^5\) If the number density in the IGM is \( n_{\text{IGM}} \approx 10^{-7} \) cm\(^{-3} \) (but it may be a factor of several smaller in cosmological voids, which dominate the cosmic space at \( z \approx 0 \)), the density ratio between the streaming pairs and the background plasma is

\[
\alpha \equiv \frac{n_b}{n_{\text{IGM}}} \approx 10^{-16} K_{\text{EBL}}^{-3} L_{\gamma,45} \left( \frac{E_{\gamma}}{10 \text{ TeV}} \right) \left( \frac{n_{\text{IGM}}}{10^{-7} \text{ cm}^{-3}} \right)^{-1}.
\]

(2)

The density contrast \( \alpha \) and the beam Lorentz factor \( \gamma_b \) are the two crucial parameters determining the plasma physics of the beam-IGM interaction. For blazar-induced beams, we expect that they should vary in the range \( \gamma_b \sim 10^{-18} \)–\( 10^{-15} \) and \( \gamma_b \sim 10^6 \)–\( 10^7 \), respectively.

As discussed by Broderick et al. (2012), collective beam-plasma effects can be relevant only if many beam pairs are present within a sphere of radius equal to the wavelength of the most unstable mode. As we show below, the scale of the fastest growing modes is \( 2 \pi c/\omega_{e} \), where \( c/\omega_{e} = \sqrt{m_e e^2/4\pi e^2 n_{\text{e}}/c^2} \approx 1.5 \times 10^9 (n_{\text{e}}/10^{-7} \text{ cm}^{-3})^{-1/2} \) cm is the plasma skin depth of the IGM electrons (with number density \( n_{\text{e}} \approx n_{\text{IGM}}/2 \)). A sphere of skin-depth radius contains \( (2\pi c/\omega_{e}) n_{\text{e}} \approx 10^6 \left( \alpha/10^{-16} \right) \left( n_{\text{e}}/10^{-7} \text{ cm}^{-3} \right)^{-1/2} \) beam particles. For \( \alpha \sim 10^{-18} \)–\( 10^{-15} \), we find that collective phenomena always play a role in the evolution of blazar-induced beams.

Another important parameter is the dispersion in beam momentum at birth. Since the pair-creation cross section peaks slightly above the threshold energy, the pairs are born moderately warm, with a comoving temperature of \( k_B T_b \sim m_e c^2 \).\(^6\) Moreover, since the EBL and the blazar TeV spectra are broad, the beam energy distribution will extend over

\[\text{for } 0.1 \text{ TeV} \lesssim E_{\gamma} \lesssim 10 \text{ TeV} \text{ (e.g., Aharonian 2001).}^4\]

Each particle moves along the direction of the incident TeV photon, and it carries about half of the photon energy, so the beam Lorentz factor is \( \gamma_b \approx 10^6 \left( E_{\gamma}/10 \text{ TeV} \right)^{-1} \). Assuming that plasma instabilities in the IGM do not appreciably affect the beam propagation (an assumption that is correct a posteriori, as we demonstrate in this work), the pairs travel a distance of \( d_{IC} \approx 100 \left( E_{\gamma}/10 \text{ TeV} \right)^{-1} \) kpc before cooling by IC scattering off the CMB (leading to many photons of energy \( \approx 100 \left(E_{\gamma}/10 \text{ TeV}\right)^2 \) GeV per original TeV photon).

2. PHYSICAL PARAMETERS OF BLAZAR-DRIVEN BEAMS

In this section, we summarize the physical parameters of blazar-induced beams, including the density contrast to the IGM, the beam Lorentz factor and velocity spread. We present order-of-magnitude estimates, and we refer to Schlickeiser et al. (2012a) and Miniati & Elyiv (2013) for a more detailed analysis of the beam distribution function, and its dependence on the spectrum of the EBL and of the blazar TeV emission.

Blazar photons of energy \( E_{\gamma} \approx 10 \text{ TeV} \) travel a distance of \( D_{\gamma} \approx 80 K_{\text{EBL}} \left( E_{\gamma}/10 \text{ TeV} \right)^{-1} \) Mpc before they interact with the EBL and produce electron-positron pairs (Neronov & Semikoz 2009). Here, \( K_{\text{EBL}} \approx 1 \) accounts for uncertainties in the intensity of the EBL, with models predicting \( 0.3 \lesssim K_{\text{EBL}} \lesssim 3 \).
a wide range of Lorentz factors, so that the effective beam temperature can be as large as $k_B T_b/m_e c^2 \approx 5$–10 (Miniati & Elyiv 2013). In Section 4.2.2, we explore the role of thermal effects on the nonlinear evolution of blazar-induced beams.

3. SIMULATION SETUP

We investigate blazar-driven plasma instabilities in the IGM by means of fully kinetic PIC simulations. We employ the three-dimensional (3D) electromagnetic PIC code TRISTAN-MP (Spitkovsky 2005), which is a parallel version of the publicly available code TRISTAN (Buneman 1993), that was optimized for handling ultra-relativistic flows (see, e.g., Sironi & Spitkovsky 2011 and Sironi et al. 2013 for studies of ultra-relativistic collisionless shocks using TRISTAN-MP). We initialize a relativistic dilute pair beam that propagates along $+\hat{x}$ through an unmagnetized electron-proton plasma (with the realistic mass ratio $m_p/m_e = 1836$). The simulations are performed in the frame of the background plasma, i.e., of the IGM. No background magnetic field is assumed, so the electric and magnetic fields generated by beam-plasma instabilities will grow from noise.

To follow the beam-plasma evolution to longer times with fixed computational resources, we mainly utilize 2D computational domains in the $xy$ plane. In Section 4.1, we compare 2D and 3D runs, and we show that 2D simulations can capture most of the relevant 3D physics. In the case of 2D simulations with the beam lying in the simulation plane, only the in-plane components of the velocity, current and electric field, and only the out-of-plane component of the magnetic field are present. The simulation box is periodic in all directions, i.e., we assume that the spatial scale of the gradients (e.g., in the IGM density) is much larger than the box length. By choosing a periodic domain, we simulate the bulk of the beam-plasma system, rather than the “head” of the pair beam.

The background plasma consists of cold electrons and protons, with initial electron temperature $k_B T_e/m_e c^2 \approx 10^{-5}$. We have tested that higher temperatures of the background electrons do not significantly change the development of the relevant instabilities, as long as the electron temperature is non-relativistic, in agreement with Bret et al. (2005). The background protons are allowed to move, but we obtain similar results when the protons are treated as a static (i.e., infinitely massive) charge-neutralizing background. It follows that nonlinear Landau damping,\(^7\) which would be artificially suppressed for infinitely massive protons, does not seem to play a major role in the evolution of the beam-plasma system, for the parameters explored in our study.

The beam consists of electron-positron pairs propagating with Lorentz factor $\gamma_b$ along the $+\hat{x}$ direction. To our knowledge, our PIC simulations are the first to address the evolution of an electron–positron beam. All of the previous studies have focused on the case of an electron beam propagating through an electron–proton plasma, with the background electrons moving opposite on the case of an electron beam propagating through an electron–electron-positron beam. All of the previous studies have focused on the nonlinear evolution of blazar-induced beams.

For infinitely massive protons, does not seem to play a major role on the electromagnetic PIC code TRISTAN-MP (Spitkovsky 2005), which is a parallel version of the publicly available code TRISTAN (Buneman 1993), that was optimized for handling ultra-relativistic flows (see, e.g., Sironi & Spitkovsky 2011 and Sironi et al. 2013 for studies of ultra-relativistic collisionless shocks using TRISTAN-MP). We initialize a relativistic dilute pair beam that propagates along $+\hat{x}$ through an unmagnetized electron-proton plasma (with the realistic mass ratio $m_p/m_e = 1836$). The simulations are performed in the frame of the background plasma, i.e., of the IGM. No background magnetic field is assumed, so the electric and magnetic fields generated by beam-plasma instabilities will grow from noise.

To follow the beam-plasma evolution to longer times with fixed computational resources, we mainly utilize 2D computational domains in the $xy$ plane. In Section 4.1, we compare 2D and 3D runs, and we show that 2D simulations can capture most of the relevant 3D physics. In the case of 2D simulations with the beam lying in the simulation plane, only the in-plane components of the velocity, current and electric field, and only the out-of-plane component of the magnetic field are present. The simulation box is periodic in all directions, i.e., we assume that the spatial scale of the gradients (e.g., in the IGM density) is much larger than the box length. By choosing a periodic domain, we simulate the bulk of the beam-plasma system, rather than the “head” of the pair beam.

The background plasma consists of cold electrons and protons, with initial electron temperature $k_B T_e/m_e c^2 \approx 10^{-5}$. We have tested that higher temperatures of the background electrons do not significantly change the development of the relevant instabilities, as long as the electron temperature is non-relativistic, in agreement with Bret et al. (2005). The background protons are allowed to move, but we obtain similar results when the protons are treated as a static (i.e., infinitely massive) charge-neutralizing background. It follows that nonlinear Landau damping,\(^7\) which would be artificially suppressed for infinitely massive protons, does not seem to play a major role in the evolution of the beam-plasma system, for the parameters explored in our study.

The beam consists of electron-positron pairs propagating with Lorentz factor $\gamma_b$ along the $+\hat{x}$ direction. To our knowledge, our PIC simulations are the first to address the evolution of an electron–positron beam. All of the previous studies have focused on the case of an electron beam propagating through an electron–proton plasma, with the background electrons moving opposite on the case of an electron beam propagating through an electron–electron-positron beam. All of the previous studies have focused on the nonlinear evolution of blazar-induced beams.

The beam-to-plasma density ratio $\alpha$ and the beam Lorentz factor $\gamma_b$ expected for blazar-induced pairs streaming through the IGM (see Section 2) cannot be directly studied with PIC simulations. However, by performing dedicated experiments with a wide range of $\alpha$ and $\gamma_b$ (in the regime $\alpha \ll 1$ and $\gamma_b \gg 1$ of ultra-relativistic dilute beams), we can extrapolate the relevant physics to the extreme parameters expected in the IGM. We vary the beam Lorentz factor from $\gamma_b = 3$ up to $\gamma_b = 1000$, and the density contrast from $\alpha = 10^{-1}$ down to $\alpha = 10^{-3}$.\(^8\) For numerical convenience, the density ratio between the beam and the plasma is established by initializing the same number of beam and plasma computational particles, with the beam particles having a weight $\alpha$. We have tested that, by choosing a different weight (yet, keeping the same physical density contrast), our results do not change. In addition to studying the dependence on $\gamma_b$ and $\alpha$, we also compare the evolution of cold beams (with comoving temperature at initialization $k_B T_b/m_e c^2 \approx 10^{-4}$) with the case of warm beams, up to the limit of mildly relativistic thermal spreads $k_B T_b/m_e c^2 \approx 1$ most relevant for blazar-induced beams.

The results presented below have been extensively tested for convergence. We typically employ 50 particles per computational cell for the background plasma (25 electrons and 25 protons), and the same number for the beam particles (if each carries a weight $\alpha$). However, we have tested that our 2D results are the same when using up to 256 particles per cell, for both the beam and the plasma. We resolve the skin depth $c/\omega_e$ of the background electrons with 8 computational cells, but we have tested that our 2D results do not change when using 12 or 16 cells per skin depth.\(^9\) In 2D runs, the simulation plane is typically a square with 1024 cells ($\sim 125 c/\omega_e$) on each side, but we have checked that our results do not substantially change when employing a larger box that is 500 $c/\omega_e$ (long in the direction of beam propagation) and 250 $c/\omega_e$ wide. In 3D, we employ a box with 512 cells ($\sim 67.5 c/\omega_e$) in the transverse direction and 1024 cells ($\sim 125 c/\omega_e$) along the longitudinal direction.\(^10\) To capture the linear and nonlinear stages of the beam-plasma evolution, we follow the system up to unprecedentedly long times, in 2D up to $\omega_e t \sim 10^5$, or equivalently $\sim 1.75 \times 10^6$ timesteps, and in 3D up to $\omega_e t \sim 4 \times 10^4$ or $\sim 7 \times 10^5$ timesteps.

The number of beam particles is kept constant during the constant during the evolution of the beam-plasma system, since the photon–photon interactions that would introduce fresh electron–positron pairs are extremely rare on the timescales covered by our simulations. Also, we neglect IC cooling of the beam pairs, since it is irrelevant over the timespan of our runs. This implies that the total energy in our periodic beam-plasma system should be constant over time. However, explicit PIC codes do not conserve energy to machine precision. We track the energy conservation in our runs, and we find that at late times it is still better than 1%. This makes our estimates of the amount of beam energy transferred to the plasma electrons (of the order of $\sim 10\%$) extremely robust, for the beam parameters explored in this work.

Finally, we remark that in all PIC codes a numerical heating instability arises when cold relativistic plasma propagates for

---

\(^7\) Nonlinear Landau damping is the interaction between thermal protons in the IGM and the beat of two Langmuir waves. This would scatter the waves to higher phase velocities, where they cannot interact efficiently with the beam. As discussed by Miniati & Elyiv (2013), nonlinear Landau damping can strongly inhibit the transfer of the beam energy to the IGM plasma.

\(^8\) Beams with more extreme parameters (in particular, with $\alpha \lesssim 10^{-3}$) will take longer to evolve, and at that point the fact that explicit PIC codes do not conserve energy to machine precision (see below) might impact the reliability of our results.

\(^9\) The speed of light in the simulations is 0.45 cells/timestep, so that the temporal resolution is $\delta t = 0.05625 \omega_e^{-1}$.

\(^10\) Hereafter, “longitudinal” and “transverse” will be relative to the beam direction of motion.
large distances over the numerical grid. The instability, known as “numerical Čerenkov,” results from the coupling of electromagnetic waves with spurious beam mode aliases on the computational grid (Dieckmann et al. 2006a; Godfrey & Vay 2013; Xu et al. 2013). The numerical Čerenkov instability might artificially slow down the beam, even in the absence of physical beam-plasma instabilities. We have assessed that the results reported below arise from a physical instability (as opposed to the numerical Čerenkov mode), by comparing our beam-plasma simulations with the artificial case of a beam that propagates through the computational grid in the absence of any background plasma. The beam evolution in the two cases is dramatically different, which proves that the beam energy loss that we discuss below arises from the physical interaction of the beam with the background plasma, rather than from the numerical Čerenkov instability.

4. RESULTS

In this section, we explore the linear and nonlinear evolution of ultra-relativistic dilute pair beams by means of 2D and 3D PIC simulations. In Section 4.1, we describe the different stages of evolution of the beam-plasma system, for a representative choice of beam parameters in the regime of ultra-relativistic dilute beams ($\gamma_b = 300$, $\alpha = 10^{-2}$ and negligible beam thermal spread at initialization). In Section 4.2, we investigate the dependence of our results—in particular, of the fraction of beam energy transferred to the background electrons—on the beam-to-plasma density contrast, the beam Lorentz factor and the beam temperature at birth. The reader that is not interested in the kinetic details of the beam-plasma interaction might proceed to Section 4.3, where we extrapolate the findings of our PIC simulations to the extreme parameters of blazar-induced beams.

4.1. The Linear and Nonlinear Evolution of Ultra-relativistic Dilute Cold Beams

In this section, we follow the evolution of a cold beam with $\gamma_b = 300$ and $\alpha = 10^{-2}$. We start with the analysis of the linear phase, and then we investigate the nonlinear relaxation. We find that the exponential phase of the oblique mode (which is the fastest growing instability for dilute ultra-relativistic beams) terminates due to self-heating of the beam in the direction transverse to the beam motion. At the end of the oblique phase, only a minor fraction $\sim (\alpha/\gamma_b)^{1/3}$ of the beam energy has been deposited into the background electrons. Further evolution of the beam is governed by quasi-longitudinal modes, which operate on a timescale that is much longer (at least two orders of magnitude) than the oblique growth. At the end of the quasi-longitudinal phase, the dispersion of beam momentum in the longitudinal direction saturates at $\Delta p_{b,3}/\gamma_b m_c c \sim 0.2$, which corresponds to a fraction of $\sim 10\%$ of beam energy transferred to the plasma.

4.1.1. The Oblique Exponential Phase

The evolution of the beam-plasma system during the oblique phase is presented in panels (a)–(d) of Figure 1. The beam is set up with a small thermal spread ($k_B T_b/m_c c^2 \simeq 10^{-6}$), so that the oblique instability initially proceeds in the reactive regime, i.e., all the beam particles are in resonance with each harmonic of...
the packet of unstable modes, and the instability is the strongest. This is opposed to the kinetic regime, in which the beam velocity spread is considerable. Here, a number of unstable modes with a broad spectrum in phase velocity will be excited, with only a small number of beam particles being in resonance with each mode. This results in a slower growth, as compared to the reactive regime.

In Figure 1, we confirm that the oblique instability is the fastest growing mode for ultra-relativistic dilute beams. The reactive phase of the instability governs the evolution of the system for \( t \ll t_{\text{OBL}} \approx 600 \omega_e^{-1} \), where \( t_{\text{OBL}} \) is marked as a dash-dotted vertical blue line in panels (a) and (b). Here, \( \omega_e = \sqrt{4 \pi n_e m_e/e} \) is the plasma frequency of the background electrons. The fastest mode grows on a scale \( \sim 2 \pi c/\omega_e \), where \( c/\omega_e \) is the electron skin depth, and its wavevector is inclined at \( \sim 45^\circ \) with respect to the beam propagation. This is apparent in the 2D structure of the longitudinal electric field in Figure 1(c), as well as in the 2D plots of the transverse electric and magnetic fields (not shown). The oblique mode is also captured in 3D simulations, as shown in the 3D structure of the longitudinal electric field of Figure 2(a).

The growth rate of the oblique instability in the reactive regime is (e.g., Fainberg et al. 1970)

\[
\omega_{\text{OBL}}(k) = \frac{\sqrt{3}}{24^{1/3}} \left( \frac{2 \alpha}{\gamma_b} \right)^{1/3} \left( \frac{k^2 + k_{\parallel}^2}{k^2 + \gamma_b k_{\parallel}^2} \right)^{1/3} \omega_e, \tag{3}
\]

where the different dependence on \( k_{\perp} \) and \( k_{\parallel} \) is related to the fact that for relativistic beams the transverse inertia is much smaller than the longitudinal inertia (by a factor of \( \gamma_b^2 \)), so that the modes transverse to the beam are the easiest to be excited (for an intuitive physical description, see Nakar et al. 2011). From the pattern in Figure 1(c), we infer \( k_{\perp} \sim k_{\parallel} \sim k/\sqrt{2} \), so the growth rate of the fastest growing oblique mode will be

\[
\omega_{\text{OBL}} \sim \frac{\sqrt{3}}{24^{1/3}} \left( \frac{\alpha}{\gamma_b} \right)^{1/3} \omega_e \equiv \delta_{\text{OBL}} \omega_e, \tag{4}
\]

which nicely agrees with our results. In fact, in Figure 1(a) we show that the fraction of beam kinetic energy deposited into the background electrons (orange line), into the longitudinal (red) and transverse (blue) electric fields, and into the transverse magnetic field (green) all grow at the rate predicted by Equation (4) for \( t \lesssim 600 \omega_e^{-1} \) (dotted orange line in Figure 1(a)).

The oblique mode is quasi-electrostatic, i.e., roughly \( k \parallel E_b \) (e.g., Bret et al. 2010b). Since the angle between the wavevector and the beam is \( \gtrsim 45^\circ \), in agreement with analytical expectations (e.g., Bret et al. 2010b), it follows that the electric field component transverse to the beam is slightly larger than the longitudinal component, i.e., \( k_{\perp} \gtrsim k_{\parallel} \) implies that \( E_{\perp} \gtrsim E_{\parallel} \). This explains the small difference between the red and the blue lines in Figure 1(a). Also, since the mode is quasi-electrostatic, the magnetic component will be sub-dominant relative to the electric fields. In agreement with the analytical considerations of Lemoine & Pelletier (2010), we find that \( B_{\perp} \sim \delta_{\text{OBL}} E_{\parallel} \). Given that \( \delta_{\text{OBL}} \ll 1 \) for ultra-relativistic dilute beams, it follows that \( B_{\perp} \ll E_{\parallel} \).

For electrostatic modes in the linear phase, it is expected that the amount of kinetic energy lost by the beam should be equally distributed between electric fields and plasma heating (e.g., Thode 1976). This explains why the energy \( \epsilon_e \) of the background electrons in the exponential phase (orange line in Figure 1(a) at \( t \lesssim t_{\text{OBL}} \approx 600 \omega_e^{-1} \)) is comparable to the energy in electric fields (\( \epsilon_{E,\perp} \) and \( \epsilon_{E,\parallel} \), respectively, red and blue lines in Figure 1(a)). We have verified that the fraction of beam energy transferred to the background protons is negligible as compared to the plasma electrons, so the evolution of the protons will be ignored hereafter.

\[\text{References}\]

11 We remind that for 2D runs with the beam lying in the simulation plane, only the in-plane components of the electric field, and the out-of-plane component of the magnetic field are present.

12 In Equation (3), the factor of two that multiplies \( \alpha \) is related to our definition of \( \alpha = n_b/n_{\text{KRM}} = n_b/2 n_e \).

13 From Equation (3), one would expect that in the ultra-relativistic limit \( \gamma_b \gg 1 \), the fastest growing mode should have \( k_{\perp} \gg k_{\parallel} \) rather than \( k_{\perp} \sim k_{\parallel} \) as we find. In reality, modes with \( k_{\perp} \gg k_{\parallel} \) are efficiently suppressed due to self-heating of the beam in the transverse direction, as we show in Equation (6). It follows that the fastest growing mode lies at \( \sim 45^\circ \) of the beam direction.
Since \( E_{\perp} \sim E_{\parallel} \) during the exponential phase of the oblique mode, both the plasma and the beam are heated quasi-isotropically, so that the momentum spreads in the longitudinal and transverse directions are nearly identical (compare the red and blue lines for \( t \lesssim t_{\text{OBL}} \)) in Figure 1(b); dashed lines refer to the plasma, solid lines to the beam. The transverse momentum spread of the beam can be related to the transverse electric field \( E_{\perp} \) via the Lorentz force

\[
\frac{\Delta p_{b,\perp}}{\Delta t} \sim \delta_{\text{OBL}} \omega_e \Delta p_{b,\perp} \sim eE_{\perp},
\]

where we have assumed that the characteristic timescale is set by the oblique growth rate (i.e., \( \Delta \tau^{-1} \sim \delta_{\text{OBL}} \omega_e \)) and that the magnetic field is negligible compared to the electric force (in fact, \( B_{\perp}/E_{\perp} \sim 2 \delta_{\text{OBL}} \ll 1 \)).

Since the oblique mode is heating up the beam in the transverse direction (solid blue line in Figure 1(b)), the exponential growth of the reactive rate \( \omega_{\text{OBL}} \) will necessarily terminate, when the assumption of a cold beam required by the reactive approximation becomes invalid. It is well known that the system will transition from the reactive phase to the kinetic phase when the beam velocity dispersion \( \Delta v_b \) reaches (e.g., Fainberg et al. 1970; Bret et al. 2010b)

\[
|k \cdot \Delta v_b| \sim \omega_{\text{OBL}},
\]

namely, when the beam, due to its velocity spread, can move across one wavelength of the most unstable mode during the growth time of the reactive instability. In this case, most of the beam particles will lose resonance with the unstable mode, and the instability will transition from the reactive to the kinetic regime. For ultra-relativistic beams with isotropic momentum dispersions (in fact, Figure 1(b) shows that \( \Delta p_{b,\perp} \sim \Delta p_{b,\parallel} \)) during the reactive phase, the transverse velocity spread \( \Delta v_{b,\perp}/c \sim \Delta v_{b,\parallel}/y_b m_c \) is much larger than the longitudinal spread \( \Delta v_{b,\parallel}/c \sim (\Delta v_{b,\perp}/c)^2 + \Delta p_{b,\parallel}/y_b^3 m_c \). Since \( k_{\perp} \sim k_{\parallel} \sim \omega_e/c \), Equation (6) above reduces to

\[
\Delta P_{b,\text{OBL}} \sim \delta_{\text{OBL}} y_b m_c,
\]

where \( \Delta P_{b,\text{OBL}} \) is the expected transverse dispersion in beam momentum at the end of the reactive oblique phase. The threshold in momentum dispersion \( \Delta P_{b,\text{OBL}} \) can also be recast as a limit in beam temperature (e.g., Bret et al. 2010a). We confirm that the reactive phase of the oblique mode terminates at \( t \sim t_{\text{OBL}} \sim 600 \omega_e^{-1} \), when the beam transverse momentum reaches the threshold \( \Delta P_{b,\text{OBL}} \) in Equation (7) (which is shown as a horizontal dash-dotted blue line in Figure 1(b)).

We can now derive the expected fraction of beam kinetic energy transferred to the plasma electrons and to the electromagnetic fields at the end of the oblique reactive phase. By setting \( \Delta p_{b,\perp} = \Delta P_{b,\text{OBL}} \) in Equation (5), we find that the fraction of beam energy converted into transverse electric fields at \( t \sim t_{\text{OBL}} \) is

\[
\epsilon_{E,\perp} \equiv \frac{E_{\perp}^2}{8\pi \gamma_b n_b m_c c^2} \sim \frac{\sqrt{2} \omega_{\text{OBL}}}{32},
\]

Since the oblique mode is quasi-electrostatic (i.e., \( E_{\perp} \sim E_{\parallel} \)), it follows that \( \epsilon_{E,\perp} \sim \epsilon_{E,\parallel} \). Moreover, for electrostatic modes, the fraction \( \epsilon_e \) of the beam kinetic energy converted into plasma heating is comparable to the energy in electric fields (e.g., Thode 1976), so \( \epsilon_e \sim \epsilon_{E,\perp} \sim \epsilon_{E,\parallel} \). Finally, since \( B_{\perp} \sim 2 \delta_{\text{OBL}} E_{\perp} \), the magnetic energy fraction will be \( \epsilon_{B,\perp} \sim \sqrt{2} \omega_{\text{OBL}}/8 \). We have extensively verified that the expected scalings of the efficiency parameters \( \epsilon_e, \epsilon_{E,\perp}, \epsilon_{E,\parallel} \) and \( \epsilon_{B,\perp} \) with respect to \( \delta_{\text{OBL}} \) are in agreement with the results of our simulations, across the whole range of beam Lorentz factors and density contrasts we have explored (see the various curves in Figure 1(a) at \( t \sim t_{\text{OBL}} \), and also Section 4.2.1).

For \( t \gtrsim t_{\text{OBL}} \), the evolution of the oblique mode will proceed in the kinetic (rather than reactive) regime. The kinetic oblique mode is indeed responsible for the peak in the electric field energy observed at \( \omega_{e} t \sim 1000 \) in Figure 1(a) (red and blue lines), which produces a moderate increase in the fraction of beam energy transferred to the background electrons (orange line in Figure 1(a) at \( \omega_{e} t \sim 1000 \)). In this phase, the 2D structure of the longitudinal electric field in Figure 1(d) shows that the wavevector of the kinetic oblique mode is oriented at \( \sim 20^\circ \) relative to the beam propagation (as compared to the \( \sim 45^\circ \) angle observed during the reactive phase, see Figure 1(c)). A similar pattern is shown in the 3D plot of Figure 2(b). As expected, the increase in the transverse momentum dispersion has suppressed the modes having \( k_{\perp} \gg k_{\parallel} \), which are most sensitive to transverse temperature effects (e.g., Bret et al. 2010b).

For a beam with initial transverse velocity dispersion \( \Delta v_{b,\perp} \), the growth rate of the kinetic oblique mode for \( k_{\perp} \lesssim \omega_e/c \) is (e.g., Breizman & Ryutov 1971)

\[
\omega_k \sim \left( \frac{c}{\Delta v_{b,\perp}} \right)^2 \frac{\alpha}{y_b} \omega_e \equiv \delta_k \omega_e.
\]

At the end of the reactive oblique phase, Equation (7) prescribes that \( \Delta v_{b,\perp}/c = \Delta P_{b,\text{OBL}}/y_b m_c \sim \delta_{\text{OBL}} \), so that the growth rate of the kinetic oblique mode will be \( \omega_k \propto \delta_{\text{OBL}} \omega_e \), i.e., it will have the same scalings with \( \alpha \) and \( y_b \) as the reactive oblique mode (here, we have neglected factors of the order of unity).

We have explicitly verified that the peak in electric fields at \( \omega_{e} t \sim 1000 \) is due to the kinetic oblique mode, by performing a dedicated simulation in which at \( \omega_{e} t \sim 800 \) (i.e., shortly after the end of the reactive stage) we reset by hand the electromagnetic fields and the plasma temperature to their initial values (i.e., no seed fields and \( k_b T_{e}/m_c^2 \sim 10^{-3} \)), yet we retain the beam momentum distribution that results self-consistently from the reactive oblique phase. In this setup, we find that the fastest growing mode has the same 2D pattern as in Figure 1(d) and its growth rate scales as \( \propto \delta_{\text{OBL}} \omega_e \). This confirms that the peak in electric fields at \( \omega_{e} t \sim 1000 \) (Figure 1(a)) is indeed associated to the kinetic oblique instability.

The growth of the kinetic oblique mode terminates due to self-heating of the beam, in analogy to the reactive oblique phase. In particular, the growth in the kinetic oblique phase cannot be sustained beyond the point where, due to the self-excited electric fields, the beam momentum dispersion in the transverse direction exceeds the initial value \( \gamma_b \Delta v_{b,\perp} m_c \). At this point, the expression for the growth rate in Equation (9) becomes clearly invalid. This happens when

\[
e_{E,\perp} \sim \omega_k \gamma_b \Delta v_{b,\parallel} m_c \sim \delta_{\text{OBL}} \gamma_b \Delta v_{b,\parallel} m_c \sim 2 \delta_{\text{OBL}} E_{\perp} \sim \delta_{\text{OBL}} E_{\parallel} \sim 2 \delta_{\text{OBL}} E_{\perp},
\]

which leads to the same scaling as in Equation (8), if we take \( \Delta v_{b,\parallel}/c = \Delta P_{b,\text{OBL}}/y_b m_c \sim \delta_{\text{OBL}} \), as appropriate for the beam velocity dispersion at the end of the reactive oblique phase. In summary, apart from factors of the order of unity, the electric fields at the end of the kinetic oblique phase will
saturation at a level similar to the reactive oblique stage (compare the two peaks of electric energy in Figure 1(a) at $\omega_{e}t \sim 600$ and $\omega_{e}t \sim 1000$). It follows that the kinetic oblique instability will increase the fraction of beam kinetic energy transferred to the plasma electrons only by a factor of the order of unity, as compared to the reactive oblique mode (see the orange line in Figure 1(a), for $\omega_{e}t \gtrsim 1000$).

4.1.2. The Longitudinal Relaxation Phase

After the saturation of the oblique instability, further evolution of the beam-plasma system proceeds via quasi-longitudinal modes, as shown in the 2D plot of the longitudinal electric field in Figure 1(g), as well as in the 3D pattern of Figure 2(c). Being quasi-longitudinal, these modes are insensitive to thermal spreads in the direction perpendicular to the beam, so they can grow even after the end of the kinetic oblique phase.

The quasi-longitudinal oscillations shown in Figure 1(g) are a characteristic signature of the quasi-longitudinal relaxation of the beam (see, e.g., Grognard 1975; Lesch & Schlickeiser 1987; Schlickeiser et al. 2002; Pavan et al. 2011). In the quasi-linear relaxation, the beam generates longitudinal Langmuir waves (see the peak in $E_L$ at $\omega_{e}t \sim 10^4$ in Figure 1(e)), which scatter the beam particles and heat the background plasma (see the increase in the electron thermal energy shown by the orange line at $10^4 \lesssim \omega_{e}t \lesssim 1.5 \times 10^4$ in Figure 1(e)). Since $E_L \gg E_\perp$ (compare the red and blue curves in Figure 1(e) at $\omega_{e}t \sim 10^4$), the background electrons will be heated preferentially in the direction of motion of the beam (see the increase in $\Delta p_{e,\parallel}$ at $10^4 \lesssim \omega_{e}t \lesssim 1.5 \times 10^4$ in Figure 1(f)). For the same reason, the quasi-linear relaxation is accompanied by a substantial increase in the beam momentum spread along the direction of propagation (red solid line in Figure 1(f), showing the growth of $\Delta p_{b,\parallel}$ at $5 \times 10^3 \lesssim \omega_{e}t \lesssim 1.5 \times 10^4$). The spread in the parallel beam momentum is associated to the formation of phase space holes, that result from the trapping of beam particles by the longitudinal electric oscillations (e.g., O’Neil et al. 1971; Thode & Sudan 1975).

The quasi-linear relaxation occurs on a timescale much longer than the exponential oblique phase. Within the range of beam Lorentz factors and density contrasts probed by our simulations, we find that the characteristic relaxation time $\tau_{r}$ is at least two orders of magnitude longer than the exponential growth time of the oblique instability $\tau_{UBL} = \omega_{UBL}^{-1}$, in agreement with previous 1D simulations (Grognard 1975; Pavan et al. 2011). This emphasizes the importance of evolving our PIC simulations to sufficiently long times to capture the physics of the quasi-linear relaxation (see Section 4.2 for further details).

The quasi-linear modes broaden the beam momentum spectrum in the longitudinal direction up to the point where $\Delta p_{b,\parallel}/\gamma_b m_e c \gtrsim 0.2$ (see the red solid line in Figure 1(f), saturating at $\Delta p_{b,\parallel}/m_e c \sim 0.2 \gamma_b \sim 60$). This in agreement with the so-called Penrose’s criterion, stating that the beam will be stable to electrostatic modes only when the longitudinal dispersion in momentum approaches the initial beam Lorentz factor (e.g., Buschauer & Benford 1977). From $\Delta p_{b,\parallel}/\gamma_b m_e c \sim 0.2$, it follows that at the end of the relaxation phase, a fraction $\sim 10\%$ of the beam energy has been transferred to the background electrons (see the orange line in Figure 1(e) at $\omega_{e}t \gtrsim 2 \times 10^4$). In Section 4.2.1 we demonstrate that, irrespective of the beam Lorentz factor or the beam-to-plasma density contrast, a generic by-product of the relaxation of cold ultra-relativistic dilute beams is the conversion of $\sim 10\%$ of their energy into plasma heating.\footnote{This is smaller than the heating efficiency of $\sim 30\%$ reported by Thode & Sudan (1975) using 1D simulations. In Appendix A, we demonstrate that the transfer of beam energy to plasma electrons is indeed less efficient in 2D, as compared to the 1D case studied by Thode & Sudan (1975).}

The quasi-linear relaxation significantly affects the shape of the beam and plasma longitudinal momentum spectrum, as shown in Figure 3. As a result of the quasi-longitudinal relaxation, the plasma distribution at late times ($\omega_{e}t \gtrsim 2 \times 10^4$) develops a pronounced high-energy tail (at $3 \lesssim p_{\parallel}/m_e c \lesssim 10^2$), that bridges the main thermal peak of the plasma electrons (at $p_{\parallel}/m_e c \sim 0.5$) with the beam particles (that populate the isolated high-energy bump at $p_{\parallel}/m_e c \sim 300$ in Figure 3). This high-energy component in the background electrons, which contains a significant amount of energy, is present only in the forward direction (i.e., along the beam propagation). In the backward direction, the spectrum of plasma electrons (red dotted line in Figure 3, at $\omega_{e}t = 5 \times 10^4$) is compatible with a Maxwellian.

During the quasi-linear relaxation, the beam spectrum evolves from a quasi-monoenergetic distribution into a broad bump (from the black to the red curve at $p_{\parallel}/m_e c \sim 300$ in Figure 3). In agreement with Figure 1(f) (red solid line), most of the evolution occurs at $\omega_{e}t \lesssim 1.5 \times 10^4$, whereas the beam spectrum at longer times is remarkably steady (we have followed the system up to $\omega_{e}t \sim 1.5 \times 10^5$, finding no further signs of evolution).

We point out that the beam momentum spectrum at late times does not approach the so-called “plateau” distribution $dN/dp_{\parallel} \propto p_{\parallel}^0$ (indicated as a black dotted line in Figure 3), which is believed to be the ultimate outcome of the beam relaxation in 1D (e.g., Grognard 1975; Schlickeiser et al. 2002; Dieckmann et al. 2013). As opposed to earlier 1D claims, in...
our 2D and 3D simulations we find that the beam longitudinal relaxation leads to a momentum spectrum that is harder than the plateau distribution, yet the beam-plasma system appears stable.\textsuperscript{15} In turn, the fact that the beam spectrum is harder than $dN/dp_z \propto p_z^0$ explains why the amount of beam energy transferred to the plasma is only $\sim 10\%$ (it should be $\sim 50\%$ for a plateau distribution extending up to $\gamma_b m_e c$, see Thode & Sudan 1975).

We argue that the transverse dispersion in beam momentum, which could not be properly captured in previous 1D studies, prevents the longitudinal beam spectrum from relaxing to the plateau distribution (see Appendix B for further details). Our claim is supported by the following experiment. At the end of the kinetic oblique phase ($\omega_e t \sim 2000$), we artificially set the beam transverse dispersion to be $\Delta p_{b,\parallel}/m_e c \ll 1$ (for comparison, the self-consistent evolution in Figure 1(b) yields $\Delta p_{b,\parallel}/m_e c \sim 10$ at the end of the kinetic oblique phase, see the solid blue line). Also, in the subsequent evolution, we inhibit any growth in the transverse beam momentum. In this setup, in which any transverse dispersion effects are artificially neglected, the beam relaxation should proceed as in 1D. So, it is not surprising that, in agreement with previous 1D studies, at late times the beam relaxes to the plateau distribution (red dashed line in Figure 3). In other words, we find that the relaxation to the plateau distribution is not a general result of the multi-dimensional evolution of ultra-relativistic beams, but it only occurs when the beam transverse dispersion stays sufficiently small, so the system is quasi-1D. We now provide a qualitative explanation of this important aspect of the physics of beam-plasma interactions, deferring a more detailed analytical treatment to a future publication.

As we further discuss in Appendix B, the beam relaxation produces a plateau distribution only if $\Delta p_{b,\parallel}/m_e c \ll 1$, due to the following argument. Complete stabilization of the beam-plasma system is achieved when the longitudinal velocity spread of the ultra-relativistic beam reaches $\Delta v_{b,\parallel}/c \sim 1$ (more precisely, when the velocity spread is comparable to the beam speed). The spread in longitudinal velocity includes contributions from both the longitudinal and the transverse momentum dispersions:

$$\frac{\Delta v_{b,\parallel}}{c} \sim \frac{\Delta p_{b,\parallel}}{\gamma_b m_e c} + \left(\frac{\Delta p_{b,\perp}}{\gamma_b m_e c}\right)^2,$$

where the second term on the right hand side is absent in the case of 1D relaxation. It follows that the transverse momentum spread can appreciably modify the relaxation process only if $\Delta p_{b,\perp}/m_e c \gg \sqrt{\Delta p_{b,\parallel}/\gamma_b m_e c} \sim 1$, where we have used that $\Delta p_{b,\parallel}/\gamma_b m_e c \sim 0.2$ at the end of the relaxation phase. Then, the fact that in the case studied in Figure 1(b) the transverse beam dispersion after the oblique phase is $\Delta p_{b,\perp}/m_e c \sim 10$ explains why the beam relaxation cannot lead to a plateau distribution. In Appendix B, we provide further evidence that the relaxation to a plateau distribution requires $\Delta p_{b,\perp}/m_e c \ll 1$.

\textbf{4.1.3. The Magnetic Field Growth}

In the previous subsections, we have primarily focused on the electrostatic character of the growing modes, which determines the coupling efficiency between the beam energy and the plasma thermal energy. Here, we comment on the generation of magnetic fields associated with the evolution of the beam-plasma system.

As shown in Figure 1(a), the growth of the quasi-electrostatic oblique mode (both in the reactive and in the kinetic regime) is accompanied by a minor magnetic component. In Section 4.1.1, we have estimated that the fraction of beam kinetic energy transferred to the magnetic fields at the end of the oblique phase is $\epsilon_{\delta B,\perp} \sim \delta_{\text{BLR}}^3$, apart from factors of the order of unity. Since $\delta_{\text{BLR}} \ll 1$ for ultra-relativistic dilute beams, the magnetic fields generated by the oblique instability are generally unimportant. At the end of the relaxation phase, the beam and the plasma are highly anisotropic, with the longitudinal momentum spread much larger than the transverse one (see Figure 1(f) at $\omega_e t \sim 1.5 \times 10^4$). As a result, the system is prone to the Weibel instability (e.g., Weibel 1959; Yoon & Davidson 1987; Sakai et al. 2000; Silva et al. 2002, 2003; Jaroschek et al. 2005), which generates the transverse magnetic field pattern shown in Figure 1(h).\textsuperscript{16} As a result of the Weibel instability, the magnetic field energy increases (see the green line in Figure 1(e), at $10^4 \lesssim \omega_e t \lesssim 2 \times 10^4$), and the beam and plasma anisotropy is reduced by increasing the transverse momentum spread (see the solid and dashed blue lines at $10^4 \lesssim \omega_e t \lesssim 2 \times 10^4$ in Figure 1(f)).

The Weibel instability is predominantly magnetic, so it does not mediate any significant exchange of energy from the beam to the plasma electrons, and the heating efficiency $\epsilon_b$ does not increase beyond the value $\epsilon_b \sim 10\%$ attained at the end of the relaxation phase. However, the Weibel instability might be a promising source for the generation of magnetic fields, as discussed by Schlickeiser et al. (2012b). However, the evolution of the magnetic filaments shown in Figure 1(h) can only be captured with large-scale 3D simulations (e.g., Bret et al. 2008; Kong et al. 2009). A detailed 3D investigation of the strength and scale of the magnetic fields resulting from ultra-relativistic dilute pair beams from TeV blazars will be presented elsewhere.

\textbf{4.2. Dependence on the Beam Parameters}

In this section, we discuss the dependence of the beam-plasma evolution on the beam parameters. In Section 4.2.1, we consider the case of cold beams, and we show that the quasi-longitudinal relaxation leads to a beam momentum spread along the direction of motion $\Delta p_{b,\parallel}/\gamma_b m_e c \sim 0.2$, regardless of the beam Lorentz factor $\gamma_b$ or the beam-to-plasma density contrast $\alpha$. In turn, this implies that a fraction $\sim 10\%$ of the beam energy is converted into heat of the background plasma, irrespective of $\gamma_b$ or $\alpha$.

In Section 4.2.2, we discuss the effect of the initial beam thermal spread on the efficiency of the beam-plasma energy transfer. We find that if the initial dispersion in longitudinal momentum satisfies $\Delta p_{b,\parallel}/\gamma_b m_e c \gtrsim 0.2$ (as typically expected for blazar-induced beams), the fraction of beam energy deposited into the background plasma is much smaller than $\sim 10\%$.

\textbf{4.2.1. Cold Beams}

In Figure 4, we show how the evolution of the beam-plasma system depends on the beam Lorentz factor (that we vary from $\gamma_b = 3$ up to $\gamma_b = 1000$) and on the beam-to-plasma density contrast (from $\alpha = 3 \times 10^{-2}$ down to $\alpha = 3 \times 10^{-3}$). Most of the previous studies have focused on moderately relativistic

\textsuperscript{15} We have extensively checked that this result is numerically solid. We have confirmed our conclusions by using a larger number of computational particles per cell (up to 256), a larger 2D box (up to four times as large, in each direction), and a finer spatial resolution (up to $c/\omega_e = 16$, instead of the usual value $c/\omega_e = 8$).

\textsuperscript{16} While the quasi-linear relaxation can also be captured with 1D simulations (although in 1D one would unphysically neglect the effect of the beam transverse spread), the growth of the Weibel modes necessarily requires multi-dimensional simulations.
Figure 4. Dependence of the beam-plasma evolution on the beam Lorentz factor (from $\gamma_b = 3$ in black up to $\gamma_b = 1000$ in red, in each panel; see the legend in panels (d)–(f)) and on the beam-to-plasma density contrast ($\alpha = 3 \times 10^{-2}$ for the leftmost column, $\alpha = 10^{-2}$ for the middle column, and $\alpha = 3 \times 10^{-3}$ for the rightmost column). Panels (a)–(c): fraction of the beam kinetic energy transferred to the plasma electrons (in the inset, a zoom-in on the earliest phases of evolution). Panels (d)–(f): beam momentum dispersion in the longitudinal direction, normalized to the initial beam momentum (i.e., $\Delta p_b,\parallel/\gamma_b m_e c$). Panels (g)–(i): beam (solid) and total (dashed) momentum spectrum $p_1(dN/dp_1)$ in the longitudinal direction, at the time indicated with the dotted black lines in the upper two rows. In panels (g)–(i), the slope $dN/dp_1 \propto p_1^\gamma$ expected for the plateau distribution is shown as a dotted black line.

(A color version of this figure is available in the online journal.)

electron beams ($\gamma_b = 3$–6) with $\alpha = 10^{-1}$ (e.g., Gremillet et al. 2007; Bret et al. 2008; Kong et al. 2009). Here, we investigate the case of ultra-relativistic dilute electron-positron beams, as appropriate for blazar-induced beams.

For each choice of $\gamma_b$ and $\alpha$, we follow the beam-plasma system from the oblique phase until the quasi-linear relaxation (typically, up to $\omega_e t \sim 10^5$). We initialize a cold beam with thermal spread $k_B T_b/m_e c^2 \sim 10^{-4}$, so that the oblique instability initially proceeds in the reactive regime, for the range of $\gamma_b$ and $\alpha$ covered by our simulations. In the reactive phase, we confirm that the fastest growing mode has a wavevector oriented at $\sim 45^\circ$ to the beam direction of propagation. The growth rate is in excellent agreement with Equation (4). As described in Section 4.1.1, the exponential phase of the reactive oblique mode terminates due to self-heating of the beam in the transverse direction. At the end of the reactive phase, we find that the fractions of beam kinetic energy transferred to the plasma electrons, to the electric fields and to the magnetic fields scale, respectively, as $\epsilon_e \propto \delta_{OBL}$, $\epsilon_{E,\perp} \sim \epsilon_{E,\parallel} \propto \delta_{OBL}$ and $\epsilon_{B,\perp} \propto \delta_{OBL}^2$, as in Section 4.1.1.

The reactive oblique phase is followed by the kinetic oblique phase. We find that the characteristic wave pattern in the kinetic oblique phase (see Figure 1(d) in 2D and Figure 2(b) in 3D) appears in the evolution of all the beam-plasma systems that we present in Figure 4. In the temporal evolution of the fraction of beam energy converted into plasma heating, the kinetic oblique mode is responsible for the additional increase that is seen in the most relativistic cases ($\gamma_b \gtrsim 100$) after the end.
of the reactive oblique phase (see the insets in the top row of Figure 4). Regardless of $\gamma_b$ or $\alpha$, we find that the kinetic oblique phase deposits only a fraction $\sim \delta_{OBL}$ of the beam kinetic energy into the background electrons, i.e., comparable to the reactive oblique phase (see Section 4.1.1).

The long-term evolution of the system is controlled by the quasi-longitudinal relaxation, which operates on a timescale $\tau_k \sim 10^2 \tau_{OBL}$, where $\tau_{OBL} = \omega_{OBL}^{-1}$ is the characteristic $e$-folding time of the oblique mode. In Figure 4, the quasi-linear relaxation governs the growth in the plasma thermal energy (top row) and in the beam parallel momentum spread (middle row) occurring at $\omega_{OBL} \gtrsim 5000$. In the regime $\gamma_B \gg 1$, the quasi-linear relaxation terminates when the beam momentum spread in the longitudinal direction reaches $\Delta p_{b,\parallel}/\gamma_b m_ec \sim 0.2$, regardless of $\gamma_b$ or $\alpha$ (middle row in Figure 4). Correspondingly, the fraction of beam energy transferred to the plasma saturates at $\epsilon_e \sim 10\%$ (top row in Figure 4).

Mildly relativistic beams with moderate density contrasts deviate from such simple scalings, for the following reason. The quasi-linear relaxation will not operate if the beam dispersion at the end of the oblique phase is already $\Delta p_{b,\parallel}/\gamma_b m_ec \gtrsim 0.2$. According to Equation (7), the beam spread at the end of the oblique phase is $\Delta p_{b,\parallel} \sim \Delta p_{b,\perp} \sim \delta_{OBL} \gamma_b m_ec$, so that the quasi-linear relaxation will be suppressed if $\delta_{OBL} \sim (\alpha/\gamma_b)^{1/3} \gtrsim 0.2$, i.e., for mildly relativistic beams with moderate $\alpha$.

A similar argument explains why a plateau distribution in the longitudinal momentum spectrum (bottom row in Figure 4) is established only for relatively small $\gamma_b$. As we have argued in Section 4.1.2, a transverse spread $\Delta p_{b,\perp}/m_ec \gtrsim 1$ prevents the beam relaxation to the plateau distribution (shown as a dotted black line in the bottom row of Figure 4). At the end of the oblique phase, $\Delta p_{b,\perp}/m_ec \sim \gamma_b \delta_{OBL}$, so that the beam will relax to the plateau distribution only if $\gamma_b \delta_{OBL} \sim (\gamma_b^2 \alpha)^{1/3} \ll 1$. Clearly, this constraint is hardest to satisfy for highly relativistic beams, which explains why, at fixed $\alpha$, the momentum spectrum of more relativistic beams shows stronger deviations from the plateau distribution.

4.2.2. Hot Beams

In the previous subsection, we have assumed that the beam is born with a negligible thermal spread. Here, we discuss how the results presented above for cold beams will be modified by temperature effects. We describe separately the role of thermal temperature effects. We describe separately the role of thermal effects. We describe separately the role of thermal effects. We describe separately the role of thermal effects. We describe separately the role of thermal effects. We describe separately the role of thermal effects. We describe separately the role of thermal effects.

As we have anticipated in Section 4.1.1, the oblique instability will proceed in the kinetic (rather than reactive) regime if the initial beam dispersion in the transverse direction is $\Delta \theta_{b,\perp} \lesssim \delta_{OBL}$. The growth rate in the kinetic regime is reported in Equation (9). The plasma thermal energy grows exponentially, until the self-generated electric fields increase the transverse dispersion in beam velocity beyond the initial value $\Delta \theta_{b,\perp}$. At this point, the exponential growth at the rate in Equation (9) will necessarily terminate. From the Lorentz force applied to the beam particles, we find that this will happen when

$$eE_\perp \sim \omega_k \gamma_b \Delta \theta_{b,\perp} m_e,$$

(12)

which results in a fraction $\epsilon_{k,\perp} \sim \delta_k$ of the beam energy transferred to the electric fields at the end of the kinetic phase

$$\Delta \theta_{b,\perp} \equiv \omega_k/\omega_e$$

is defined in Equation (9)). Since the kinetic oblique mode is an electrostatic instability, the fraction of beam energy converted into heat will also be $\epsilon_e \sim \delta_k$. Moreover, due to the fact that $\omega_k \lesssim \omega_{OBL}$—they are comparable only if $\Delta \theta_{b,\perp} \lesssim \delta_{OBL}$, i.e., at the boundary between reactive and kinetic regimes—we expect the kinetic oblique mode to be less efficient in heating the plasma electrons, as compared to the reactive phase. For beams with $\gamma_b = 3 \sim 10$ and $\alpha = 10^{-3} \sim 10^{-4}$, we have indeed verified with PIC simulations (not presented here) that the exponential growth of the kinetic oblique mode terminates at smaller $\epsilon_e$ for larger values of $\Delta \theta_{b,\perp}$, in good agreement with the expected scaling $\epsilon_e \propto \Delta \theta_{b,\perp}^{-2}$ (at fixed $\gamma_b$ and $\alpha$).

Regardless of the character of the oblique mode (reactive or kinetic), the long-term evolution of the beam-plasma system is controlled by the quasi-linear relaxation. In Figure 5, we show how the relaxation phase is affected by a finite beam temperature $T_b$. For the set of beam parameters employed in Figure 5 ($\gamma_b = 300$ and $\alpha = 10^{-2}$), the oblique phase is expected to occur in the reactive regime, as long as the beam comoving temperature is non-relativistic. In fact, for all the choices of $T_b$ presented in Figure 5 (at $\gamma_b =$ units of $m_e c^2/k_b$), the early increase in the heating efficiency $\epsilon_e$ proceeds at the reactive rate $\omega_{OBL}$ (compare the curves in the inset of Figure 5(a) (with the dotted red line, that scales with the oblique growth rate). Also, the kinetic oblique instability, which is responsible for the further growth in $\epsilon_e$, at $\omega_{OBL} = 1200$ (see the inset in Figure 5(a)), does not show any dependence on temperature, in the range $10^{-6} \lesssim k_b T_b/m_e c^2 \lesssim 0.3$ explored in Figure 5.

The beam temperature has profound effects on the quasi-linear relaxation for the beam parameters employed in Figure 5. For cold beams ($k_b T_b/m_e c^2 \lesssim 10^{-4}$, yellow and red lines in Figure 5), the relaxation phase does not depend on the beam temperature. In agreement with the results presented in Section 4.2.1, the longitudinal spread in the beam momentum increases during the relaxation stage until $\Delta p_{b,\parallel}/\gamma_b m_ec \sim 0.2$, as shown in Figure 5(b). This corresponds to a fraction $\epsilon_e \sim 10\%$ of the beam kinetic energy being converted into plasma heating (Figure 5(a)). Similar conclusions hold for moderate beam temperatures ($k_b T_b/m_e c^2 \sim 3 \times 10^{-2}$, green line), whereas the quasi-linear relaxation is suppressed if the beam temperature at birth is such that the initial beam spread $\Delta p_{b,\parallel}/\gamma_b m_ec \gtrsim 0.2$ (cyan, blue, and black lines in Figure 5(b)). In this case, the quasi-linear phase does not mediate any further increase in the heating efficiency $\epsilon_e$, beyond the early oblique phase (see the black line in Figure 5(a)). In short, if the initial momentum spread is $\Delta p_{b,\parallel}/\gamma_b m_ec \sim 0.2$, the plasma heating efficiency $\epsilon_e$ stays fixed at the value $\epsilon_e \sim \delta_{OBL}$, attained at the end of the reactive oblique phase—or $\epsilon_e \sim \delta_k$, if the oblique instability proceeds in the kinetic regime.

The longitudinal momentum spectrum in Figure 5(c) clarifies why the quasi-linear relaxation is suppressed for hot beams. As we have discussed in Section 4.1.2, the relaxation is mediated

---

17 The velocity dispersion $\Delta \theta_{b,\perp}$ can be recast as a comoving beam temperature $k_b T_b/m_e c^2 \sim \gamma_b^2 \Delta \theta_{b,\perp}^2$, where we have assumed $k_b T_b/m_e c^2 \ll 1$ (i.e., non-relativistic temperatures).

18 The condition $\Delta \theta_{b,\perp} / c = 10^{-1} k_b T_b/m_e c^2 \lesssim \delta_{OBL}$, as required for the kinetic regime, together with the assumption of a quasi-monenergetic beam (i.e., with comoving beam temperature $k_b T_b/m_e c^2 \lesssim 1$), constrains $\gamma_b \delta_{OBL} \sim (\gamma_b^2 \alpha)^{1/3} \lesssim 1$, i.e., the kinetic regime can be best probed by low-$\gamma_b$ beams with $\alpha \ll 1$.

19 Since the relaxation phase is quasi-longitudinal, the same conclusions hold in 1D. We have confirmed that regardless of the nature of the fastest growing mode (oblique in 2D, longitudinal in 1D), the quasi-linear relaxation is suppressed if the beam spread at birth is such that $\Delta p_{b,\parallel}/\gamma_b m_ec \gtrsim 0.2$, both in 1D and in 2D.
slower than the wave tend to receive energy from it, and to the wave, and thus excite it, while those that are slightly faster than the wave tend to transfer energy by Langmuir waves excited by the beam. The beam particles thus damp the wave. It follows that a mode is unstable if the number of beam particles moving slightly faster than the wave exceeds that of those moving slightly slower. More precisely, the instability will be stronger for a harder slope \( d \log N/d \log p_1 \) of the longitudinal momentum spectrum at momenta \( \lesssim \gamma_b m_e c \) (this is the relevant momentum scale, since the phase velocity of the most unstable mode is slightly smaller than the beam speed). For a sharply peaked beam (i.e., with \( k_B T_b/m_e c^2 \ll 1 \)), the slope \( d \log N/d \log p_1 \) will be extremely hard, and the excitation of the Langmuir modes that mediate the relaxation process will be most efficient. As a result of the quasi-linear relaxation, the beam particles will be scattered down to lower energies, and when the beam momentum spread exceeds \( \Delta p_\parallel/\gamma_b m_e c \gtrsim 0.2 \) the slope of the momentum spectrum below the peak becomes too shallow (see the red and yellow curves in Figure 5(c) at \( p_\parallel/m_e c \lesssim 300 \), and the quasi-linear relaxation terminates. If the beam temperature at birth is too hot (i.e., such that \( \Delta p_{10.1}/\gamma_b m_e c \gtrsim 0.2 \)), the initial slope of the momentum spectrum at \( p_1 \lesssim \gamma_b m_e c \) is already too shallow to trigger efficient excitation of Langmuir waves, and the quasi-relaxation process is suppressed.

In Figure 5(c), we further support this argument by showing that in the latest stages of evolution, the shape of the beam momentum spectrum below \( \gamma_b m_e c \) (in Figure 5(c), at \( 30 \lesssim p_\parallel/m_e c \lesssim 300 \)), is nearly independent of the beam temperature at birth. For initially cold beams (yellow and red lines), the quasi-linear relaxation increases the beam momentum spread over time (see Figure 5(b)), until the beam spectrum relaxes to the broad bump shown in Figure 5(c). At this point, the beam is stable, although it has not relaxed to the so-called plateau distribution (as we explain in Section 4.1.2, the relaxation to the plateau distribution requires \( \Delta p_{10.1}/\gamma_b m_e c \ll 1 \), which is not satisfied here). For initially hot beams, the initial momentum spectrum below \( \gamma_b m_e c \) resembles the final momentum distribution of initially cold beams. This explains why hot beams starting with a longitudinal momentum spread \( \Delta p_{10.1}/\gamma_b m_e c \gtrsim 0.2 \) will not experience the quasi-linear relaxation phase.

So far, we have discussed the effect of thermal spreads on the beam relaxation, assuming that the beam spectrum at birth is a drifting Maxwellian. However, the conclusions derived above hold for more complicated beam distributions. In particular, we have performed a set of PIC simulations, assuming that the beam spectrum at birth is a power law \( dN/dp_1 \propto p_1^{-2} \) for \( p_1 \gtrsim p_{\text{min}} \gg m_e c \). For blazar environments, such a beam spectrum is expected as a result of intense IC cooling off the CMB.

If the beam power-law distribution has a negligible spread in the transverse direction (which is typically not the case for blazar-induced beams, see Section 4.3), then the oblique instability will proceed in the reactive regime. Apart from factors of the order of unity, we find that the exponential growth rate is \( \sim (\alpha m_e c/p_{\text{min}})^{1/3} \), similar to the case of mono-energetic beams. As we have discussed above, the effectiveness of the quasi-linear relaxation—which ultimately determines whether the heating efficiency \( \epsilon_e \) can reach \( \sim 10\% \)—is determined by the spread in the beam longitudinal spectrum below the peak, i.e., at \( p_1 \lesssim p_{\text{min}} \). If the beam spectrum has a sharp low-energy cutoff at \( p_{\text{min}} \), then the quasi-linear relaxation does operate, and the beam deposits \( \sim 10\% \) of its energy into the background electrons. However, if the low-energy end of the beam distribution is broader, the quasi-linear relaxation will be inhibited, and the amount of beam energy transferred to the plasma will be much smaller, in agreement with the results in Figure 5. We find that

\[ \gamma_b = 300, \quad \alpha = 10^{-2}, \]

for different beam comoving temperatures at initialization (as shown by the legend in panel (b), where the beam temperature is in units of \( m_e c^2/k_B \)). Panel (a): fraction of the beam kinetic energy deposited into the background electrons, with the inset showing the evolution at early times. Panel (b): temporal evolution of the beam longitudinal momentum spread, in units of \( \gamma_b m_e c \). Panel (c): beam (solid) and total (dashed) momentum spectra in the longitudinal direction, at the time indicated in panels (a) and (b) with the vertical black dotted line. In panel (c), the dotted oblique line shows the slope expected for a plateau distribution \( dN/dp_1 \propto p_1^0 \).

(A color version of this figure is available in the online journal.)

Figure 5. Temporal evolution of a beam-plasma system with \( \gamma_b = 300 \) and \( \alpha = 10^{-2} \), for different beam comoving temperatures at initialization (as shown by the legend in panel (b), where the beam temperature is in units of \( m_e c^2/k_B \)).
if the beam longitudinal spectrum below $p_{\text{min}}$ can be modeled as a power law $dn/dp_{\parallel} \propto p_{\parallel}^{-\gamma}$, the quasi-linear relaxation is suppressed if $s \lesssim 3$, with the case $s = 0$ corresponding to the plateau distribution.

4.3. Implications for Blazar-induced Beams

We now analyze the implications of our findings for the evolution of blazar-induced beams in the IGM. As we have discussed in Section 3, the Lorentz factors and density contrasts of blazar-driven beams ($\gamma_b \sim 10^9-10^{10}$ and $\alpha \sim 10^{-18}-10^{-15}$) cannot be directly studied with PIC simulations. However, by performing a number of experiments with a broad range of $\gamma_b \gg 1$ and $\alpha \ll 1$, we have been able to assess how the relaxation of ultra-relativistic dilute beams depends on the beam Lorentz factor and the beam-to-plasma density ratio. Our results can then be confidently extrapolated to the extreme parameters of blazar-induced beams.

We have demonstrated that the oblique instability, which governs the earliest stages of evolution of ultra-relativistic dilute beams, proceeds in the kinetic regime if the initial velocity dispersion in the direction transverse to the beam is $\Delta v_{0,\perp}/c \gtrsim \delta_{\text{OBL}}$, where $\delta_{\text{OBL}} \equiv \omega_{\text{OBL}}/\omega_e \sim (\alpha/\gamma_b)^{1/3}$ is the growth rate of the reactive oblique mode in units of the plasma frequency $\omega_e = \sqrt{4\pi e^2 n_e/m_e c^3} \sim 20 (n_e/10^{-7} \text{ cm}^{-3})^{1/2}$ rad s$^{-1}$ of the IGM electrons. Since the beam pairs are born with a mildly relativistic thermal spread in the center-of-momentum frame of the photon-photon interaction (see Section 2), the transverse velocity spread in the IGM frame will be $\Delta v_{0,\perp}/c \gtrsim 1/\gamma_b$. If follows that the oblique instability will proceed in the kinetic regime if $\gamma_b \delta_{\text{OBL}} \lesssim 1$, which is typically satisfied for blazar-induced beams ($\gamma_b \delta_{\text{OBL}} \sim 0.01-1$). The oblique instability will grow at the kinetic rate $\omega_e \sim \omega_e (c/\Delta v_{0,\perp})^{\gamma_{\alpha}/\gamma_b}$ in Equation (9), which for $\Delta v_{0,\perp}/c \sim 1/\gamma_b$, as appropriate for blazar-induced beams, reduces to $\omega_e \sim \gamma_b \alpha \omega_e$. For the parameters of blazar-induced beams, this is a factor of

$$\frac{\omega_k}{c/d_{\text{IC}}} \sim 10^5 \left(\frac{\alpha}{10^{-16}}\right) \left(\frac{n_e}{10^{-7} \text{ cm}^{-3}}\right)^{1/2}$$

larger than the IC cooling rate $c/d_{\text{IC}}$, where $d_{\text{IC}} \sim 100 (\gamma_b/10^9)^{-1}$ kpc is the IC cooling length computed in Section 2. In agreement with Broderick et al. (2012) and Schlickeiser et al. (2013), we find that the kinetic oblique instability has ample time to grow before the beam loses energy to IC emission. Furthermore, the typical lifetime of blazar activity of $\sim 10^7$ years $\gg d_{\text{IC}}/c$ is sufficient for the instability to operate.

Due to self-heating of the beam in the transverse direction (see Sections 4.1 and 4.2.2), the exponential phase of the kinetic oblique instability terminates when only a minor fraction of the beam energy has been transferred to the IGM electrons. As we have argued in Section 4.2.2, the heating efficiency of blazar-induced beams at the end of the kinetic oblique phase is only

$$\epsilon_e \sim \delta_k \equiv \frac{\omega_k}{\omega_e} \sim 10^{-5} \left(\frac{\gamma_b}{10^9}\right) \left(\frac{\alpha}{10^{-16}}\right).$$

It follows that even though the oblique instability can grow faster than the IC cooling time, its efficiency in heating the plasma electrons is extremely poor.

As we have emphasized in Section 4.1.2, a larger amount of beam energy (up to $\sim 10\%$) can be deposited into the plasma electrons by the quasi-linear relaxation phase. We find that the quasi-linear relaxation occurs on a timescale much longer than the growth time of the oblique instability, $\tau_R \gtrsim 10^7 \omega_e^{-1}$. Yet, since the oblique growth rate is much faster than the IC cooling rate, see Equation (13), the quasi-linear relaxation should have enough time to operate before the beam energy is lost to IC emission. Here, we are conservatively neglecting the possibility that $\tau_R$ might be much longer than $\sim 10^7 \omega_e^{-1}$ for more extreme beam parameters, as a result of nonlinear plasma processes—most importantly nonlinear Landau damping by the thermal ions—that reduce the strength of the electric fields available for the quasi-linear relaxation phase (Schlickeiser et al. 2012b; Miniati & Elyiv 2013).

In Section 4.2.2, we have demonstrated that the quasi-linear relaxation can occur only if the beam momentum spectrum along the longitudinal direction is sufficiently narrow. More precisely, we find that if the beam distribution peaks at $\gamma_b m_e c$ (in the case of a power-law tail, this would be the low-energy cutoff), the quasi-linear relaxation can operate only if the parallel momentum spread below the peak satisfies $\Delta p_{\text{OBL}}/\gamma_b m_e c \lesssim 0.2$. This constraint is hard to fulfill by blazar-induced beams. Since the pair-creation cross section peaks slightly above the threshold energy, the pairs are born moderately warm, with a comoving temperature $k_B T_p/m_e c^2 \sim 1$. This corresponds to a longitudinal momentum spread at birth (measured in the IGM frame) of $\Delta p_{\text{OBL}}/\gamma_b m_e c \sim 1$, which is already prohibitive for the development of the quasi-linear relaxation. Moreover, the momentum dispersion of blazar-induced beams will be even larger, since the spectrum of both the EBL and the blazar TeV emission are usually modeled as broad power laws (e.g., Miniati & Elyiv 2013 estimate that $k_B T_p/m_e c^2 \sim 5-10$ at birth).

In summary, the shape of the beam momentum distribution below the peak is essential to predict the amount of beam energy deposited into the IGM. If the beam spectrum at birth were to have $\Delta p_{\text{OBL}}/\gamma_b m_e c \lesssim 0.2$ (which is unlikely to be the case, but see Schlickeiser et al. 2012b), then the quasi-linear relaxation would transfer $\epsilon_e \sim 10\%$ of the beam energy to the IGM. Even in this optimistic case, we remark that the heating efficiency would reach at most $\sim 10\%$, so $\sim 90\%$ of the energy would still remain in the beam. At the end of the quasi-linear relaxation phase, the beam spectrum will be harder than the so-called plateau distribution, since the transverse dispersion in beam velocity does not meet the requirement $\Delta v_{0,\perp}/c \ll 1/\gamma_b$ for relaxation to the plateau spectrum. In the more realistic case $\Delta p_{\text{OBL}}/\gamma_b m_e c \gtrsim 0.2$ (e.g., as resulting from the Monte Carlo model by Miniati & Elyiv 2013), the quasi-linear relaxation will be inhibited, resulting in a lower efficiency of IGM heating— with a firm lower limit being the heating fraction at the end of the oblique phase, see Equation (14).

4.3.1. Comparison with Earlier Studies

We now compare our numerical work with earlier analytical studies of the relaxation of blazar-induced beams in the IGM. As we have emphasized in Section 3, our PIC simulations

---

20 However, jet variability may be fast enough to compete with the instability growth time. Our analysis focuses on the steady or long-term average TeV emission.

21 As we have anticipated in Section 3, nonlinear Landau damping by the thermal ions does not seem to play a major role in the evolution of the beam-plasma system, for the parameters explored in our study. Most likely, this stems from the fact that the beam energy density in our simulations is much greater than the initial thermal energy density of the background plasma, as compared to blazar-induced beams.
are the first to address the evolution of dilute ultra-relativistic electron–positron beams, as appropriate for blazar-induced beams in the IGM. All of the previous PIC studies (e.g., Dieckmann et al. 2006b; Gremillet et al. 2007; Bret et al. 2008; Kong et al. 2009) have focused on mildly relativistic electron beams.

In agreement with Broderick et al. (2012) and Schlickeiser et al. (2012a), we find that the fastest growing instability for blazar-induced beams propagating through the IGM is the oblique mode. If the pair beam were to be initially cold, the oblique instability would evolve at the reactive rate in Equation (4), with wavevector oriented at \( \sim 45^\circ \) relative to the beam (see Schlickeiser et al. 2012a). However, the initial transverse spread in beam momentum is large enough such that the oblique mode evolves at the kinetic rate in Equation (9), as argued by Broderick et al. (2012) and Miniati & Elyiv (2013). We remark that the transverse beam spread does affect the growth of the oblique mode, but it will not impact the evolution of longitudinal waves, as found by Schlickeiser et al. (2013). However, since the early evolution of blazar-induced beams is controlled by the oblique (rather than longitudinal) mode, transverse thermal effects are indeed important for the beam-plasma interaction at early times.

The evolution of blazar-induced beams at late times is governed by nonlinear plasma processes, whose efficiency depends sensitively on the assumed beam distribution. Schlickeiser et al. (2012b) and Miniati & Elyiv (2013) attempted to describe the nonlinear relaxation of blazar-induced beams, reaching opposite conclusions regarding the ultimate fate of the beam energy. Schlickeiser et al. (2012a) assumed that the beam momentum distribution can be modeled as a delta function (i.e., they approximated the beam as mono-energetic and uni-directional), and they found that the relaxation phase occurs much faster than the IC cooling losses. From this, they argued that more than 50% of the beam energy can be transferred to the IGM plasma. Miniati & Elyiv (2013) reached the opposite conclusion, when accounting for the effect of the finite transverse momentum spread of blazar-induced beams, as derived from a detailed Monte Carlo model of the electromagnetic cascade. They found that the beam energy is radiated by IC off the CMB well before the relaxation phase.

With our PIC simulations, we find that the relaxation phase occurs on a much longer timescale than the exponential oblique growth, at least by two orders of magnitude. However, it might be delayed even more for more extreme beam parameters, as suggested by both Schlickeiser et al. (2012b) and Miniati & Elyiv (2013). Even under the conservative assumption that the relaxation phase is faster than the IC cooling time, this does not imply that all of the beam energy is ultimately deposited into the IGM plasma. In short, the relaxation process being faster than the IC losses does not guarantee that it will also be efficient in heating the IGM electrons. Due to self-heating of the beam in the transverse direction, we find that the exponential phase of the kinetic oblique instability terminates when only a minor fraction of the beam energy has been transferred to the IGM plasma. At the end of the kinetic oblique phase, the heating efficiency of IGM electrons is extremely poor, with \( \epsilon_e \sim 10^{-3}(y_b/10^7)(\alpha/10^{-16}) \).

Additional transfer of energy from the beam to the plasma occurs at later times (with a delay of two or more orders of magnitude, relative to the oblique growth time), and it is mediated by the quasi-linear relaxation process. Here, the beam generates longitudinal electrostatic waves, which scatter the beam particles—thus broadening the beam momentum spectrum in the longitudinal direction—and heat the IGM electrons. The quasi-linear relaxation can operate only if the beam momentum spectrum along the longitudinal direction is sufficiently narrow. More precisely, we find that if the beam distribution peaks at \( y_b m_e c \), the quasi-linear relaxation requires the parallel momentum spread below the peak to be \( \Delta p_{\|}/y_b m_e c \lesssim 0.2 \). In this case, a fraction \( \epsilon_e \sim 10\% \) of the beam energy is transferred to the plasma (rather than \( \sim 50\%–100\% \), as assumed by Broderick et al. 2012).

The constraint \( \Delta p_{\|}/y_b m_e c \lesssim 0.2 \) on the longitudinal dispersion at birth is hard to fulfill by blazar-induced beams. Since the pair-creation cross section peaks slightly above the threshold energy, the pairs are born moderately warm, with a comoving temperature \( k_B T_b/m_e c^2 \sim 1 \). This corresponds to a longitudinal momentum spread at birth (measured in the IGM...
For Miniati & Elyiv (2013) estimate that TeV emission are usually modeled as broad power laws (e.g., larger, since the spectrum of both the EBL and the blazar TeV emission are estimated to have a power law index of -2.5). For $\Delta p_{\gamma,1}/\gamma_{e}c \sim 1$, the quasi-linear relaxation will be suppressed, which results in a much lower efficiency of IGM heating—with a firm lower limit being the heating fraction at the end of the oblique phase $\epsilon_{e} \sim \delta_{d} \ll 1$.

At the end of the relaxation phase, the beam and plasma distributions are highly anisotropic, with the longitudinal momentum spread much larger than the transverse one. As a result, the system is prone to the Weibel instability (e.g., Weibel 1959; Yoon & Davidson 1987; Sakai et al. 2000; Silva et al. 2002, 2003; Juraschek et al. 2005), which relaxes the beam and plasma anisotropy by generating transverse magnetic fields. The Weibel instability is predominantly magnetic, so it does not mediate any further exchange of energy from the beam to the plasma electrons. However, it might be a promising source for the generation of magnetic fields in the IGM, as discussed by Schlickeiser et al. (2012b). A multi-dimensional PIC investigation of the strength and scale of the magnetic fields resulting from blazar-induced beams will be presented elsewhere.

Our results have important implications for the ultimate fate of the energy of blazar-induced beams. As we have argued above, most of the beam energy is still available for IC interactions with the CMB. Therefore, the Fermi non-detection of the IC scattered GeV emission around TeV blazars can be reliably used to probe the strength of the EBL and of the IGM magnetic fields. In particular, the lower bounds on the IGM field strength derived by various authors (e.g., Neronov & Vovk 2010; Tavecchio et al. 2010) are still valid, despite the fast growth of beam-plasma instabilities in the IGM. However, when computing the expected IC signature, one should take into account that beam-plasma instabilities tend to broaden the beam distribution function. As a result, the reprocessed GeV emission will be spread over a wider frequency range (and consequently, with smaller flux), as compared to the case of mono-energetic beams.

The fraction of beam energy deposited into the IGM might have important cosmological implications, as argued by Chang et al. (2012) and Pfrommer et al. (2012). By assuming that all of the beam energy is transferred to the IGM electrons by beam-plasma instabilities, they showed that blazar heating could dominate over photo-heating in the low-redshift evolution of the IGM, by almost one order of magnitude at birth. In this case, the quasi-linear relaxation process, which mediates efficient transfer of the beam energy to the IGM electrons up to $\epsilon_{e} \sim 10\%$, will be suppressed, and the heating fraction remains $\epsilon_{e} \sim \delta_{d} \ll 1$.

1. As we have argued above, blazar-induced beams are born with a significant longitudinal spread in momentum, $\Delta p_{\gamma,1}/\gamma_{e}c \sim 1$. In this case, the quasi-linear relaxation process, which mediates efficient transfer of the beam energy to the IGM electrons up to $\epsilon_{e} \sim 10\%$, will be suppressed, and the heating fraction remains $\epsilon_{e} \sim \delta_{d} \ll 1$.

2. Even if the relaxation process were to be operating, the beam relaxation timescale might be much longer than the IC loss time, due to the competing effect of nonlinear Landau damping by the thermal protons, as argued by Miniati & Elyiv (2013; but see Schlickeiser et al. 2012b) for opposite conclusions assuming a different shape of the beam distribution function.

3. Density inhomogeneities associated with cosmic structure induce loss of resonance between the beam particles and the excited plasma oscillations, strongly inhibiting the growth of the unstable modes (Miniati & Elyiv 2013).

4. A large-scale IGM field might spread a mono-energetic uni-directional beam in the transverse and longitudinal directions, thus inhibiting the efficiency of the beam-plasma interaction. In the oblique phase, a sufficiently strong magnetic field might trigger the transition to the kinetic regime, if during the characteristic quasi-linear growth time $\tau_{k}$ the field deflects the beam such that the longitudinal dispersion in momentum reaches $\Delta p_{\gamma,1}/\gamma_{e}c \approx 0.2$. This can be recast as $\omega_{B}/\omega_{k} \geq 0.2$, where $\omega_{B} = eB/\gamma_{e}mc$ is the Larmor frequency of the beam particles in the IGM fields. The limit on the IGM field strength is

$$B \gtrsim 5 \times 10^{-15} \left( \frac{\gamma_{b}}{10^{7}} \right)^{1/3} \left( \frac{\alpha}{10^{-16}} \right)^{2/3} \left( \frac{n_{e}}{10^{-7} \, \text{cm}^{-3}} \right)^{1/2} \text{G},$$

which is generally realized for IGM fields (e.g., Neronov & Semikoz 2009).

As regards to the quasi-linear relaxation, IGM fields are expected to change the evolution of the quasi-linear process if during the characteristic quasi-linear growth time $\tau_{k}$ the field deflects the beam such that the longitudinal dispersion in momentum reaches $\Delta p_{\gamma,1}/\gamma_{e}c \approx 0.2$. This can be recast as $\alpha/\omega_{k} \approx 0.2$. In this case, the quasi-linear relaxation plays a role only if the IC cooling time is $\delta_{IC}/c \gtrsim \tau_{k}$. By setting $\delta_{IC}/c \gtrsim \tau_{k}$, we obtain a limit on the field strength that inhibits the quasi-linear relaxation

$$B \gtrsim 5 \times 10^{-14} \left( \frac{\gamma_{b}}{10^{7}} \right)^{2} \text{G},$$

which might be satisfied by IGM fields (e.g., Neronov & Semikoz 2009).

For these reasons, we conclude that blazar-induced beams are not likely to play a major role in the thermal history of the IGM.

We thank A. Bret, M. Dieckmann, and the anonymous referee for insightful comments. L.S. is supported by NASA through Einstein Postdoctoral Fellowship grant number PF1-120090 awarded by the Chandra X-ray Center, which is operated by the Smithsonian Astrophysical Observatory for NASA under contract NAS8-03060. The simulations were performed on XSEDE resources under contract No. TG-AST120010, and on NASA High-End Computing (HEC) resources through the NASA Advanced Supercomputing (NAS) Division at Ames Research Center.

APPENDIX A

1D SIMULATIONS OF RELATIVISTIC DILUTE BEAMS

In the main body of the text, we have presented our results on the relaxation of ultra-relativistic dilute beams, by employing
2D and 3D simulations. Here, we discuss how the physics of the beam-plasma evolution differs, when performing 1D simulations.

In Figure 6, we compare our 2D results (red line) with two selected 1D simulations (black and green curves), that differ in the orientation of the beam relative to the simulation box. For the black line, the beam is aligned with the simulation domain, whereas the two directions form an angle of $\theta_{\text{box}} = 45^\circ$ for the green line. Since the fastest growing oblique mode for ultra-relativistic dilute cold beams is oriented at $-45^\circ$ relative to the beam propagation (see Section 4.1.1), the 1D box at an angle $\theta_{\text{box}} = 45^\circ$ with respect to the beam should correctly capture the evolution of the oblique mode. This is confirmed by the inset in Figure 6, which shows that the exponential growth in the heating efficiency $\epsilon_e$ proceeds at the expected rate $\omega_{\text{OBL}}$ of Equation (4) both in 2D (red line) and in the 1D simulation with $\theta_{\text{box}} = 45^\circ$ (green).

On the other hand, the 1D simulation with $\theta_{\text{box}} = 45^\circ$ cannot correctly capture the relaxation phase, which is mediated by quasi-longitudinal modes. In fact, the heating efficiency $\epsilon_e$ in the 1D box with $\theta_{\text{box}} = 45^\circ$ does not significantly change after the end of the oblique phase. In contrast, as a result of the quasi-longitudinal relaxation, in 2D the heating fraction $\epsilon_e$ increases at $10^4 < \omega_{\text{TS}} < 5 \times 10^4$ up to the saturation value $\epsilon_e \sim 10\%$.

The quasi-linear relaxation, being driven by longitudinal modes, can be described in 1D with a simulation box oriented along the beam (black line in Figure 6). For a 1D box with $\theta_{\text{box}} = 0^\circ$ (black line), the quasi-linear relaxation controls the beam evolution at $2 \times 10^4 \lesssim \omega_{\text{TS}} \lesssim 3 \times 10^4$. At late times, the heating efficiency saturates at $\epsilon_e \sim 20\%$ (in agreement with Thode & Sudan 1975), which is twice as large as compared to the analogous 2D case. As anticipated in Section 4.1.2, this is related to the role of transverse spreads in the relaxation of ultra-relativistic beams. In multi-dimensions, the longitudinal velocity spread $\Delta v_{\parallel}$ required to terminate the relaxation phase can be achieved either by decelerating the beam in the longitudinal direction (with a fractional energy loss $\Delta p_{\parallel}/(\gamma_b m_e c)$), or by deflecting the beam sideways (which gives a transverse spread $\Delta p_{\perp}$, but no significant energy loss). The contribution of the two terms is presented in Equation (11). A 1D box with $\theta_{\text{box}} = 0^\circ$ can only capture beam-aligned modes, which cannot change the transverse spread $\Delta p_{\perp}$, so that the second term in Equation (11) does not contribute. It follows that the same $\Delta \gamma_{b,\parallel}$ will be attained in 1D by a larger $\Delta p_{b,\perp}/\gamma_b m_e c$ (and so, higher $\epsilon_e$), relative to its 2D counterpart. This explains why in 1D (for beam-aligned boxes) the heating fraction $\epsilon_e$ is a factor of a few larger than in 2D (compare black and red lines in Figure 6).

In summary, beam-aligned 1D simulations tend to overestimate the fraction of beam energy deposited into the background electrons by the relaxation phase. Most importantly, they cannot properly model the early evolution of the beam-plasma system, which is mediated by oblique modes. Rather, the exponential phase in 1D simulations with $\theta_{\text{box}} = 0^\circ$ will necessarily proceed at the two-stream growth rate $\omega_{\text{TS}} = \gamma_b^{2/3} \omega_{\text{OBL}}$, which is indicated as a dotted black line in Figure 6. In short, the multi-dimensional physics of the beam-plasma evolution cannot be properly captured by 1D simulations.

The difference between 1D and 2D simulations is also presented in Figure 7, where we discuss the dependence of our results on the transverse size of the computational domain, from $L_\perp = 0.125 c/\omega_e$ (1D simulation) to our standard choice $L_\perp = 125 c/\omega_e$. We also confirm that our 2D results are the same when doubling the box size in the transverse direction (compare the red lines for $L_\perp = 125 c/\omega_e$ with the yellow lines for $L_\perp = 250 c/\omega_e$). For 1D boxes aligned with the beam, the two-stream instability governs the exponential growth of $\epsilon_e$ at early times (compare the black solid and dotted lines in Figures 7(a) and (b)). The oblique mode can operate only if the transverse size of the box is $L_\perp \gtrsim 2.5 c/\omega_e$, as shown by the fact that the green line in the inset of Figure 7(a) grows at the oblique rate $\omega_{\text{OBL}}$ indicated by the dotted red line. For a box with $L_\perp = 0.625 c/\omega_e$ (blue line), the oblique phase mediates the growth of $\epsilon_e$ at early times ($\omega_{\text{TS}} \lesssim 3000$), yet at a rate smaller than $\omega_{\text{OBL}}$, whereas the exponential stage of the two-stream mode emerges at later times ($\omega_{\text{TS}} \sim 7000$) with the expected rate $\omega_{\text{TS}}$ (see the blue line in Figure 7(a) and (b)).

In agreement with Figure 6, we find that the quasi-linear relaxation in 2D proceeds in a similar way as in 1D, apart from the fact that the dispersion in longitudinal momentum at late times is smaller for larger box widths, as shown in Figure 7(b). In turn, this is related to the shape of the beam momentum distribution at the end of the quasi-linear relaxation phase. As shown in Figure 7(c), the longitudinal momentum spectrum in 1D simulations relaxes to the plateau distribution $dN/dp_\parallel \propto p_\parallel^0$ (indicated as a dotted black line in Figure 7(c)), whereas in 2D the beam spectrum below the peak stays harder than the plateau distribution. As we have argued in Section 4.1.2 (see Appendix B for further details), the difference between our 1D and 2D results is ultimately related to the transverse spread in beam momentum, which is larger in 2D than in 1D.

APPENDIX B

RELAXATION TO THE PLATEAU DISTRIBUTION

In Section 4.1.2, we have argued, based on Equation (11), that the longitudinal velocity spread $\Delta v_{\parallel}$ required to terminate the relaxation process can be sourced not only by a longitudinal spread in momentum $\Delta p_{b,\parallel}$, but also by a transverse spread...
Figure 7. Temporal evolution of the beam-plasma interaction, as a function of the box size $L_{\perp}$ in the direction transverse to the beam (instead, $L_{\parallel} = 125 \, c/\omega_{e}$ along the beam in all cases). The beam has $\gamma_b = 100$ and $\alpha = 10^{-2}$. Panel (a): temporal evolution of the heating efficiency $\epsilon_e$, with the inset showing the evolution at early times. Panel (b): temporal evolution of the beam longitudinal momentum spread, in units of $\gamma_b m_e c$. Panel (c): the solid lines show the beam momentum spectrum in the longitudinal direction, at the time indicated in panels (a) and (b) with the vertical black dotted line. For three selected cases ($L_{\perp} = 0.125 \, c/\omega_{e}$, 0.625 $c/\omega_{e}$, and 2.5 $c/\omega_{e}$), we also plot the total (beam plus plasma) momentum spectra with dashed lines. In panel (c), the dotted oblique line shows the slope expected for a plateau distribution $dN/dp_{\parallel} \propto p_{\parallel}^{0.5}$. (A color version of this figure is available in the online journal.)

$\Delta p_{b,\perp}$. In 1D, the latter is absent, whereas in 2D it is the interplay of the two that determines the shape of the beam momentum spectrum at late times. In particular, this explains why in 2D the beam does not necessarily relax to the plateau distribution, which instead is a general outcome of the beam-plasma evolution in 1D configurations.

Figure 8 clarifies the role of the transverse momentum spread for the relaxation to the plateau distribution. Using 2D simulations, we perform the following experiment. Right after the end of the oblique phase, we artificially reset the transverse momentum spread $\Delta p_{b,\perp}$ to the values indicated in the legend. Also, in the course of the subsequent evolution, we inhibit its evolution by hand. We find that if we artificially set $\Delta p_{b,\perp}/m_e c \ll 1$, the beam relaxation proceeds as in 1D, and the longitudinal momentum spectrum at late times relaxes to the plateau distribution (compare the black solid and dotted lines in Figure 8). In contrast, if $\Delta p_{b,\perp}/m_e c \gtrsim 1$ the quasi-linear relaxation does spread the beam momentum in the longitudinal direction, but not enough to approach the plateau distribution. We have verified that the threshold $\Delta p_{b,\perp}/m_e c \sim 1$ for relaxation to the plateau distribution holds irrespective of the beam Lorentz factor or the beam-to-plasma density contrast.

As a result of the growth of the oblique mode, the transverse dispersion of cold beams at the end of the oblique phase approaches $\Delta p_{b,\perp}/m_e c \sim \gamma_b \Delta \delta_{OBL}$. For the beam parameters employed in Figure 8 ($\gamma_b = 1000$ and $\alpha = 10^{-2}$), this would give $\Delta p_{b,\perp}/m_e c \sim 20$. This explains why the beam spectrum plotted as a yellow solid curve in Figure 8, which corresponds to the self-consistent evolution of the beam-plasma system (i.e., the beam transverse dispersion is not constrained by hand), does not approach the plateau distribution at late times.

REFERENCES

Abdo, A. A., Ackermann, M., Ajello, M., et al. 2010, ApJ, 723, 1082
Aharonian, F., Akhperjanian, A. G., Bazer-Bachi, A. R., et al. 2006, Natur, 440, 1018
Aharonian, F. A. 2001, in International Cosmic Ray Conference, Vol. 27, TeV Blazars and Cosmic Infrared Background Radiation, ed. R. Schlickeiser (Göttingen: Copernicus Publications), 250
