Electromagnetic field creation during EWPT nucleation with MSSM

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We derive the equations of motion for electroweak MSSM with a right-handed Stop, from which we derive the equations for the electromagnetic field that arises from bubble nucleation and collisions during the first order electroweak phase transition that can occur in this MSSM. Introducing an isospin ansatz we derive e.o.m. for the electrically charged W fields uncoupled from all other fields. These serve as the current for the Maxwell-like e.o.m. for the em field. The resulting electromagnetic field arising during EWPT bubble nucleation is found. This electromagnetic structure, along with that arising from bubble collisions, could seed galactic and extra-galactic magnetic fields.

I. INTRODUCTION

The origin of the large-scale galactic and extra-galactic magnetic fields which have been observed (see Ref.[1] for a review) is a long-standing problem of astrophysics. There has been a great interest in possible cosmological seeding of these magnetic fields in the early universe phase transitions: the Quantum Chromodynamics chiral phase transition (QCDPT) from the quark-gluon plasma to our hadronic universe, and the electroweak phase transition (EWPT) in which the Higgs and the other particles acquired their masses. Research on the QCDPT has explored not only seeding of the galactic and extra-galactic structure, but also possible effects in Cosmic Microwave Background Radiation (CMBR) correlations[2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. In our own work we have explored possible large scale magnetic structure which could produce observable CMBR polarization correlations[13, 14, 15] arising from bubble collisions during the QCDPT[14].

The EWPT is particularly interesting for exploring possible cosmological magnetic seeds since the electromagnetic (em) field along with the $W^\pm$ and $Z$ fields are the gauge fields of the Standard model, while in the QCDPT the em field is included in the Lagrangian through coupling to quarks. It is now believed that in the Standard model there is no first order EWPT[15], no explanation of baryogenesis, or any interesting cosmological magnetic structures created during the crossover transition. However, there has been a great deal of activity in the supersymmetric extension of the standard model[16], and with a MSSM having a Stop with a mass similar to the Higgs there can be a first order phase transition and consistency with baryogenesis[17, 18, 19].

In the present work we use the MSSM with the form of the Standard EW Model plus a right-handed Stop, which has been used by a number of authors to examine the EWPT and baryogenesis. See, e.g., Refs. [15, 20]. There have been other models for CP violation and baryogenesis, such as two-Higgs models (see Refs. [17, 18, 19] for references and discussion) and leptoquarks (see, e.g., Ref.[21] for a discussion and references). For the present exploratory work on nucleation we could use, e.g., a two-Higgs model, as the precise nature of the extension of the Standard EW Model is not needed. For future work on magnetic field generation with possible bubble collision we shall use the MSSM and treat all the equations of motion, derived below, with a right-handed Stop.

In the most detailed previous research on the study of magnetic fields arising from EWPT transitions[8, 9, 10] a model was used in which the equations of motion (e.o.m.) for the em field involved the chargeless Higgs. The EWPT bubbles were empty of the em field until the bubbles collided and overlapped. In the present work we derive the e.o.m. directly from the EW Lagrangian, using a MSSM including the right-handed Stop field for consistency with a first order phase transition. The e.o.m. which we obtain for the em field is Maxwell-like, with the current given by the charged $W^\pm$ fields, which is physically reasonable. Fermions
are not considered in the present work.

In general, the e.o.m. are complicated coupled partial differential equations, in which solutions for the Higgs and Stop fields must be found to obtain the current for the em field equation. In the present paper, however, we introduce an I-spin formulation which allows us to uncouple the e.o.m. for the $W^\pm$ fields, and carry out a study of spherically symmetric EW bubble nucleation. For pure nucleation of a bubble we solve the $W^\pm$ e.o.m. numerically, and obtain solutions for the em field using a fit to these numerical solutions for the $W^\pm$ fields. The solutions have an instanton-like form near the bubble wall, as expected.

These are the first solutions for the electromagnetic field from EWPT nucleation that have been obtained from the EW Lagrangian. Due to spherical symmetry during nucleation only electric fields are produced. These give the initial fields for bubble collisions. For the derivation of EW cosmological magnetic seeds, the more complicated collision problem must also be solved, which we shall attack in the near future, and fermion fields will be included.

In Sec. II we review the previous work on electromagnetic field creation during the EWPT. In Sec. III we derive the equations of motion with the MSSM, and in Sec. IV give the form with our I-spin ansatz. In Sec. V we discuss our method for finding solutions, and give our results. In Sec. VI we give a summary of the paper and conclusions.

II. PREVIOUS TREATMENT OF ELECTROMAGNETIC FIELD CREATION DURING A FIRST-ORDER EWPT

The most detailed derivation of the creation of magnetic fields during the EWPT is based on the Abelian Higgs model introduced by Kibble and Vilenkin[8], with the Lagrangian

$$L_{AH} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_{\mu}\Phi)^\dagger D^\mu \Phi + V(\Phi), (1)$$

where $D_{\mu} = \partial_{\mu} + ieA_{\mu}$, with $A_{\mu}, F^{\mu\nu}$ the electromagnetic 4-potential, tensor, and $\Phi$ is the complex Higgs field,

$$\Phi = \frac{1}{\sqrt{2}} \rho e^{i\theta}. (2)$$

The Higgs potential, $V(\Phi)$, has two minima, and as in Ref[8] it is assumed that there is a first order phase transition corresponding to the transition from the false to the true vacuum. In the model used in Refs[8, 9, 10] $\rho$ is assumed to be a constant corresponding to the Higgs mass. In this model, with the only electric charges that are present being associated with the gauge derivative $D_{\mu}$, the electromagnetic current is given by

$$j_\mu = ie\Phi^\dagger D_{\mu} \Phi + c.c. = -e\rho^2(\partial_\mu \Phi + A_\mu). (3)$$

Furthermore it is assumed that there is uniformity during the nucleation of the EW bubbles during the first-order phase transition, leading to nucleating bubbles empty of electromagnetic fields. Magnetic fields are created only during collisions of bubbles within the region of overlap of the bubbles.

As discussed in the Introduction, one problem with the standard model for producing magnetic seeds is that there is no first order phase transition and therefore no bubbles. With the introduction of a right-handed Stop, giving the MSSM that we use in the present paper, there can be a first order phase transition, and bubbles are produced, as assumed in the Kibble-Vilenkin model. Moreover, with this MSSM based on the standard EW model, the surfaces of the bubbles are composed, in part, from the charged gauge fields, and therefore create electromagnetic fields from bubble nucleation before collisions. We therefore find electromagnetic fields from the nucleation, before collisions. This will produce new boundary conditions for the creation of magnetic fields during the bubble collisions.

It is interesting to note that in the numerical calculations of Ref[10] the Higgs field grows at the surface of the nucleating bubble wall to resemble the instanton-like solutions that we find. Also, we should add that in our work on EWPT collisions producing magnetic fields (in progress) we have found some of the approach of Ref[9] very useful.

III. MSSM EW EQUATIONS OF MOTION WITH RIGHT-HANDED STOP

In this section we derive the equations of motion for the standard Weinberg-Salam model, in the electroweak MSSM with all partners of the standard model fields integrated out except the Stop, the partner to the top quark.

$$L_{MSSM} = L^1 + L^2 + L^3 + \text{leptonic and quark interactions} (4)$$


\[ \mathcal{L}^1 = -\frac{1}{4} W_{\mu \nu}^i W^{i \mu \nu} - \frac{1}{4} B_{\mu \nu} B^{\mu \nu} \]

\[ \mathcal{L}^2 = |(i \partial_\mu - \frac{g}{2}^* t^j \cdot W_\mu^j - \frac{g'}{2} B_\mu)|^2 - V(\Phi) \]

\[ \mathcal{L}^3 = |(i \partial_\mu - \frac{g_s}{2} \lambda^a C_{\mu a})\Phi s|^2 - V_{hs}(\Phi_s, \Phi), \]

where the pure \( C_{\mu a} \) term is omitted in \( \mathcal{L}^1 \) and

\[ \begin{align*}
W_{\mu \nu}^i &= \partial_\mu W_\nu^i - \partial_\nu W_\mu^i - g e_{ijk} W_\mu^j W_\nu^k \\
B_{\mu \nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu,
\end{align*} \]

where the \( W^i \), with \( i = (1, 2) \), are the \( W^+, W^- \) fields, \( C_{\mu a} \) is an SU(3) gauge field, \((\Phi, \Phi_s)\) are the (Higgs, right-handed Stop fields), \((\tau^i, \lambda^a)\) are the (SU(2), SU(3)) generators, and the electromagnetic and Z fields are defined as

\[ \begin{align*}
A_\mu^{em} &= \frac{1}{\sqrt{g^2 + g'^2}} (g W_3^\mu + g B_\mu) \\
Z_\mu &= \frac{1}{\sqrt{g^2 + g'^2}} (g W_3^\mu - g' B_\mu).
\end{align*} \]

The effective Higgs and Stop potentials are taken as

\[ \begin{align*}
V(\Phi) &= -\mu^2 |\Phi|^2 + \lambda |\Phi|^4 \\
V_{hs}(\Phi_s, \Phi) &= -\mu_s^2 |\Phi_s|^2 + \lambda_s |\Phi_s|^4 + \lambda_{hs} |\Phi|^2 |\Phi_s|^2.
\end{align*} \]

The various parameters are discussed in many publications. In particular we need \( g = \frac{c}{\sin \theta_W} = 0.646 \), \( g' = g \tan \theta_W = 0.343 \), and \( G = gg'/\sqrt{g^2 + g'^2} = 0.303 \).

In the picture we are using, the Higgs and Stop fields will play a dynamic role in the EW bubble nucleation and collisions, and we shall need the space-time structure of these fields rather than only the vacuum expectation value for a particular vacuum state for the complete solutions of the e.o.m. Our form for the Higgs field, \( \Phi \), is

\[ \Phi(x) = \begin{pmatrix} 0 \\ \phi(x) \end{pmatrix}. \]

and

\[ \tau \cdot W_\mu \Phi = \begin{pmatrix} (W_3^\mu - iW_\mu^2) \\ -W_3^\mu \end{pmatrix} \phi(x). \]

In the present exploratory paper treating bubble nucleation we center on the possible generation of an electromagnetic field, and the solution of all of the e.o.m. is avoided. Therefore, specific forms and solutions for the Higgs and Stop fields do not enter the equations needed for the present work. For this reason we do not choose a specific form for the right-handed Stop field, \( \Phi_s \), and for convenience write the additional MSSM gauge field as

\[ C_{\mu a} = \frac{\lambda^a}{2} C_{\mu a}. \]

We also use the definitions

\[ \phi(x) = \rho(x) e^{i\Theta(x)} \]

\[ |\phi(x)|^2 = \rho(x)^2 \]

\[ |\Phi_s(x)|^2 = \rho_s(x)^2. \]

Although we do not need specific forms for \( C_{\mu a} \) or \( \Phi_s \), we assume that a Stop condensate is formed for consistency with a first order EWPT, as in Ref. 20.

With these definitions \( \mathcal{L}^2 \) is \( (j = (1, 2, 3)) \)

\[ \begin{align*}
\mathcal{L}^2 &= \partial_\mu \phi^* \partial^\mu \phi + [i(\partial_\mu \phi^* \phi - i \phi^* \partial_\mu \phi)](-g W_3^\mu \\
&\quad -g' B_\mu) + g^2 |\phi|^4 (W_3^\mu)^2 + (g')^2 B_\mu^2 \\
&\quad - \frac{gg'}{2} W_3^\mu \cdot B \quad - V(\phi) .
\end{align*} \]

The equations of motion are obtained by minimizing the action

\[ \delta \int d^4 x [\mathcal{L}^1 + \mathcal{L}^2 + \mathcal{L}^3] = 0 , \]

i.e., we do not include the leptonic parts of \( \mathcal{L} \). The equations of motion that we obtain from the variations in \( \mathcal{L}^1 \), for \( i=(1,2) \) are

\[ \partial^2 W^i_\mu - \partial^\mu \partial_\nu W^i_\mu - g e_{ijk} W^j_\nu W^k_\mu + \frac{g^2}{2} \rho(x)^2 W^i_\nu = 0 , \]

with

\[ \begin{align*}
W^i_\nu &= \rho_i^\nu W_\nu^i + \rho_\mu^\nu \partial_\mu W_\nu^k + W^j_\mu \partial^\mu W_\nu^j W^k_\mu, \\
W^k_\mu &= \partial^\mu W_\nu^k - \partial^\nu W_\mu^k - \rho_{\delta \mu} \partial^\nu \theta_{\delta \mu} W^i_\nu \mathcal{W}^i_\nu^m.
\end{align*} \]

The e.o.m. for \( A^{em}_\mu, Z \) are

\[ \begin{align*}
\partial^2 A^{em}_\mu - \partial_\mu \partial_\nu A^{em}_\mu - \frac{gg'}{\sqrt{g^2 + g'^2}} \epsilon^{ijk} \mathcal{W}^i_\nu^j &= 0 ,
\end{align*} \]

\[ \begin{align*}
\partial^2 Z_\nu - \partial_\mu \partial_\nu Z_\mu - \frac{\rho^2 \partial^\nu \Theta}{\sqrt{g^2 + g'^2}} - \frac{g^2}{\sqrt{g^2 + g'^2}} \epsilon^{ijk} \mathcal{W}^i_\nu^j &= 0 .
\end{align*} \]
The e.o.m. for the Higgs field are

\[ \frac{1}{\rho(x)} \partial^2 \rho(x) - \mu^2 + 2 \lambda \rho(x)^2 + 2 \lambda_h \rho_h(x)^2 - H \cdot H \]

\[ - \partial_\mu \Theta \partial^\mu \Theta + \sqrt{g^2 + g'2} Z^\mu \partial_\mu \Theta = 0, \quad (18) \]

with

\[ H \cdot H \equiv (\frac{g}{2})^2 W^i \cdot W^i + (\frac{g'}{2})^2 B \cdot B - \frac{g g'}{2} W^3 \cdot B, \]

and

\[ \partial_\mu (\rho(x) \partial^\mu \Theta) - \sqrt{g^2 + g'2} \rho(x)^2 Z^\mu = 0. \quad (19) \]

The e.o.m. for the right-handed Stop is

\[ - \partial^2 \Phi_s + i g_s (\partial^\mu (C_\mu \Phi_s) + (C_\mu \partial^\mu \Phi_s)) + (g_s^2 C_\mu C_\mu + \mu_s^2 + 2 \lambda_s \rho_s^2 + \lambda_h \rho_h^2) \Phi_s = 0, \quad (20) \]

We do not give the e.o.m. for the $C_\mu$ gauge field, which is not needed in the present work.

These are exact equations of motion in our MSSM model with a right-handed stop. Note that in the absence of external currents there are trivial solutions to the e.o.m. for the gauge fields, with all $A=Z=W=0$, just as in pure electrodynamics, but the solutions with nonvanishing gauge fields are the ones of physical interest. Moreover, since the EW fields are present in the universe before EWPT bubble nucleation, from the boundary conditions, they must be nonvanishing as bubble nucleation begins. From our e.o.m. we derive the gauge fields during nucleation of the bubbles, including the electromagnetic field.

IV. I-SPIN ANSATZ AND ELECTROMAGNETIC FIELD CREATION FROM $W^\pm$

One of the most important features of the equations of motion derived directly from the EW Lagrangian is that the source current of the electromagnetic field is given by the charged gauge $W^\pm$ fields, as seen from the Maxwell-like Eq.\[10\]. Although this differs from the Kibble-Vilenkin[8], Ahonen-Enqvist[9], Copeland-Saffin-Törnkivest[10] picture, much of the underlying physics is the same. Note that our equations are charge symmetric, so there is no net electric charge produced and charge is conserved, but as explained above, individual bubbles have $W^\pm$ constituents, which produce electromagnetic fields during nucleation.

This suggests that we use an SU(2), isospin ansatz for the gauge fields. We assume in the present paper that

\[ W_\nu^j \simeq i T^j W_\nu(x) \simeq i T^j x_\nu W(x) \quad j = 1, 2, 3 \]

\[ A_{\nu m}^e \simeq i T^3 A_\nu(x) \simeq i T^3 x_\nu A(x) \]

\[ Z_\nu \simeq i T^3 Z(x) \nu \simeq i T^3 x_\nu Z(x), \quad (21) \]

with the I-spin operators defined as $\epsilon^{ijk} r j k = i r^m$. We shall see that this enables us to derive the straight-forward equations of motion for the electromagnetic field, which can be solved to a good approximation for symmetric nucleation of EW bubbles. In this section we derive the e.o.m. for spherically symmetric bubble nucleation, so that $W(x) = W(r,t)$ and $A(x) = A(r,t)$, with $x^i x_\mu = t^2 - r^2$. First note that

\[ \epsilon^{ijk} W^j_\nu = i r^i \times F[W_\nu, \partial_\nu W], \quad (22) \]

with $F$ a function of $W_\nu$ and $\partial_\nu W$ to be determined, so that the e.o.m. for $W_\nu$, Eq.\[13\], becomes

\[ \partial^\mu \partial_\nu W_\mu - \partial_\nu \partial_\mu W^\mu - g x_\nu [5 W^2 + 3 W(t \partial_t + r \partial_r) W + g s^2 W^3 - \beta s^2 \partial_r W] \]

\[ - \frac{g^2}{2} \rho^2 W_\nu = 0, \quad (23) \]

with $W_\nu = x_\nu W(r,t)$, $s^2 = t^2 - r^2$, $r = \sqrt{\sum_{j=1}^3 x^j x^j}$, $\partial_j r = x^j / r$, and $\beta = (+,-)$ for $\nu = (t,j)$. Subtracting the e.o.m. for $W_t \times x_j$ from the e.o.m. for $W_j \times x_t$ we find

\[ (\partial^2_r + \partial^2_t) W + \frac{t^2 + r^2}{rt} \partial_t \partial_r W + (\frac{3}{r} \partial_t + \frac{3}{l} \partial_l) W \]

\[ + g W(t^2 - r^2) [\frac{1}{l} \partial_t - \frac{1}{r} \partial_r] W = 0 \quad (24) \]

\[ (\partial^2_r + \partial^2_t) A + \frac{t^2 + r^2}{rt} \partial_t \partial_r A + \left( \frac{3}{r} \partial_t + \frac{3}{l} \partial_l \right) A \]

\[ + g W(t^2 - r^2) [\frac{1}{l} \partial_t - \frac{1}{r} \partial_r] A = 0. \quad (25) \]

The most significant aspect of the I-spin formulation is that we obtain e.o.m. for $W(r,t)$ and for $A(r,t)$ without contributions from the Higgs or Stop fields, because they decouple. Although the Stop and Higgs fields disappear from these equations, however, they are essential to obtain a first-order EWPT and EW bubbles, as discussed in the Introduction.
As pointed out at the beginning of this section, the current for the electromagnetic field arises entirely from the electrically charged fields/particles, $W^\pm$, as seen also in Eq. (25). Moreover, the current within our I-spin formulation is determined by a nonlinear partial equation for $W(r,t)$, without direct coupling to the Higgs or Stop fields. This will enable us to derive the electromagnetic fields from EWPT bubble nucleation as in Refs. [8, 9, 10]. In the present paper, however, we derive the electromagnetic fields produced in the EWPT via bubble nucleation before collisions, which has not been considered previously. For collisions a direction in space is singled out, so that the form $W(r,t), A(r,t)$ cannot be used. This is a topic for future work. Also, fermions contribute to the electric current, and fermion fields will be included in future work. The solution for $A^\mu$ produced during nucleation with the assumptions of the present section are found in the following section.

V. I-SPIN ANSATZ AND ELECTROMAGNETIC FIELD CREATION DURING NUCLEATION.

In the present work we make use of the gauge fields gauge conditions to reduce the partial differential equations, Eqs. (24-25), to ordinary differential equations. The philosophy is to derive the $W^\pm$ and $A^\mu$ fields as a function of time at a fixed $r$. Since from the general structure of the equations we expect at time $t$ that the bubble wall will be at $r = r_w \simeq t$, we are mainly interested in the nature of the fields near $r = r_w$. As we shall see, since the solutions are modified instanton-like in nature, the most significant region for magnetic field creation for both nucleation, and for collisions, will be at the bubble walls.

The solutions must be independent of the choice of gauge; however, the choice of gauge is important for our work. First we note that if we use the Lorentz gauge, as in our recent work on QCDPT bubble nucleation [22], the method discussed below for deriving e.o.m. for $W(r,t)$ and $A(r,t)$ does not work. It is not that the solutions are not correct, it is simply that the resulting e.o.m. is essentially the Lorentz gauge condition itself. There is no new physics.

We use the Coulomb gauge, which is consistent with spherical spatial symmetry and the forms $W(r,t)$ and $A(r,t)$:

$$
\begin{align*}
\sum_{j=1}^{3} \partial_j W_j &= \sum_{j=1}^{3} \partial_j A_j = 0 \\
\rho \partial_r W(r,t) + 3W(r,t) &= 0,
\end{align*}
$$

with solutions

$$
W(r,t) = \frac{W_r(t)}{r^3} A(r,t) = A_z(t).
$$

From Eq. (23) and the gauge condition (26) one obtains differential equations for $W^\nu(r,t)$ and $A^\nu(r,t)$ (with the notation of $W^\nu$, with $\nu = 4, \nu = j = (1,2,3)$, respectively). By combining them one finds that the Higgs and Stop fields are disconnected, and obtains an e.o.m. for the functions $W_r(t)$ and $A_z(t)$

$$
\begin{align*}
W''_r(t) - \frac{3t}{r^2} W'_r(t) + \frac{3}{r^2} W_r(t) + \frac{t^2 - r^2}{r^3} W_r(t) &= 0 \\
A''_z(t) - \frac{3t}{r^2} A'_z(t) + \frac{3}{r^2} A_z(t) + \frac{Gt^2 - r^2}{r^3} W_r(t) &= 0
\end{align*}
$$

We proceed by 1) finding initial conditions and numerical solutions to Eq. (28) for $W(t)$ for a series of $r$-values, 2) fitting a function to these values, and 3) finding the function

$$
H_r(t) = \frac{t^2 - r^2}{r^3} W_r(t) \left( \frac{1}{t^2} W'_r(t) - \frac{r}{t^2} W_r(t) \right)
$$

which is used in Eq. (29) to obtain an approximate solution for $A_z(t)$ and thereby $A(r,t)$, using Eq. (27).

In Figure 1 $W(t)$ is given for various values of $r$, and the time for the creation of the bubble wall is clearly seen. In Figure 2 similar results are shown for $A(t)$. Note that $A^\mu$ has an instanton-like behavior at the bubble wall surface in that it is infinite if $\rho \to 0$, and the effective denominator is similar to that of an instanton near the wall,

$$
A(r,t) = \frac{A_W}{(r^2 - \zeta^2)^2}.
$$

Away from the surface $A(r,t)$ becomes smaller than this instanton-like solution.
From Figure 2 one observes that the time at which one reaches the radius of the wall bubble is given approximately by

$$t \simeq 2r,$$

(32)

from which we obtain the nucleation velocity of the bubble wall.

$$v_{wall} \simeq \frac{c}{2}.$$  

(33)

This will be an important result in our future work on magnetic field generation and evolution. In our present work $A_{\mu}^{\text{em}} \sim x_{\mu}A(r,t)$, so electric fields but no magnetic fields are created. This work can provide the initial conditions for EWPT collisions in which magnetic fields are created. Preliminary work on such bubble collisions has been carried out[22].

VI. CONCLUSIONS

We have formulated the coupled equations of motion for the electroweak MSSM with a Lagrangian that adds the right-handed Stop field terms to the
Standard Model. In this model a first order EWPT can occur with satisfactory baryogenesis. By using an I-spin ansatz for spherically symmetric bubble nucleation we were able to derive a Maxwell-like equation of motion for the electromagnetic field with the current given by the electrically charged gauge fields, $W^\pm$. Moreover, by treating the $W^3$ and $W^j$ components of $W^\nu$ separately we were able to decouple the equations for the $W^\pm$ fields from the other gauge fields, and also the Higgs and Stop fields, and obtain the current for the electromagnetic field.

In the present paper we derived solutions for the electromagnetic field caused by EWPT bubble nucleation, using a Coulomb gauge condition to obtain ordinary differential equations, from which we found instanton-like solutions for the electromagnetic field in the region of the bubble wall. Although this is a very limited physical problem, it explores new physics which can arise from nucleation before collisions starting from a MSSM electroweak Lagrangian. In our future work we will also examine EWPT bubble collisions, include fermion fields, and examine possible predictions of galactic and extra-galactic magnetic structures. We stress that during nucleation EWPT bubbles generate $A^\mu_{em}$, which must be included in the electromagnetic field arising from EWPT bubble collisions, and that the length scale of the fields is that of the entire bubble, rather than random fields within the bubble, which could be of importance in seeding galactic and extra-galactic magnetic structures.

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