DISJET 1.2:
A Monte Carlo Program for Jet Cross Section Calculations in Deep Inelastic Scattering

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Abstract

We present a new parton level Monte Carlo program for the calculation of jet cross sections in Deep Inelastic Scattering based on Born and next-to-leading order matrix elements. Using a class of invariant jet definition schemes, the program allows for the calculation of differential distributions of jet cross sections in the basic kinematical variables (like $s, x, y, W^2, Q^2 \ldots$) as well as for total jet cross sections. Various kinematical cuts can be chosen from an input file.
1 Introduction

The start-up of the HERA electron-positron collider marked the beginning of a new aera of experiments exploring Deep Inelastic Scattering (DIS) of electrons and protons. One of the topics to be studied at HERA will be the Deep Inelastic ($\equiv$ high $Q^2$) production of multi jet events, where good event statistics are expected allowing for precision tests of QCD.

In this paper, we present a Monte Carlo program DISJET that allows for the calculation of next-to-leading order (NLO) (1 + 1),(2 + 1) and leading order (LO) (3 + 1) jet cross sections\footnote{$^1n + 1''$ denotes the remnant jet} in DIS. Using a class of invariant jet definition scheme \cite{1} the program allows for the calculation of differential distributions of jet cross sections in the basic kinematical variables as well as for the calculation of total jet cross sections.

The source code is written in FORTRAN and running ability is tested on a DEC-Station and an ALPHA-Station running the Operating Systems VMS and on a Sun-Sparc station running under UNIX. The Monte Carlo routines are using the VEGAS-package \cite{2} for numerical integration of functions depending on up to ten variables. Parton distributions are incorporated from the PDFLIB-package \cite{3}. Each ($n$ + 1) jet cross section in DISJET is calculated in fixed order QCD perturbation theory. Therefore a comparison of jet-measurements with our predicted QCD-results provides a direct tool of determining $\alpha_s$ or $\Lambda_{\overline{MS}}$.

Changes to DISJET 1.0

An error in the finite part of the $O(\alpha_s^3)$ virtual corrections to the quark initiated subprocess is corrected. An error in the labeling of the $O(\alpha_s^3)$ (3+1)-jet subprocesses [$iproc = 1001,1002,1003$] is corrected. An additional cut in the variable $z = \frac{pp_1}{pp}$, where $p_1$ is the four momentum of one of the final partons in (2+1) jet production is implemented.

2 Matrix Elements

In DISJET complete fixed order QCD matrix elements for the process

$$e^- (l) + \text{proton}(P) \rightarrow \text{proton remnant}(p_r) + \text{parton} \, 1(p_1) \ldots + \text{parton} \, n(p_n)$$

($n = 1, 2, 3$) are used. Reaction (1) proceeds via the exchange of an intermediate vector boson $V = \gamma^*, Z, W$. At present only the exchange by a virtual photon is incorporated in DISJET. We denote the $\gamma^*$-momentum by $q$, the absolute square by $Q^2$, the center of mass energy by $s$, the square of the final hadronic
mass by $W^2$ and introduce the scaling variables $x$ and $y$:

$$q = l - l'$$

$$Q^2 = -q^2 = xys > 0$$

$$s = (P + l)^2$$

$$W^2 \equiv \frac{P_f^2}{q} = (P + q)^2$$

$$x = \frac{Q^2}{2Pq} \quad (0 < x \leq 1)$$

$$y = \frac{Pq}{Pl} \quad (0 < y \leq 1)$$

At fixed $s$, only two variables in (2) are independent, since e.g.

$$xW^2 = (1 - x)Q^2, \quad Q^2 = xys.$$  

Averaging over the azimuthal angle between the parton plane and the lepton plane ($\vec{l}, \vec{l}'$) (in the ($\gamma^*\text{-initial parton})$-cms) the jet cross sections for the exchanged virtual photon factorize the following $y$ dependence [1]:

$$d\sigma[n - \text{jet}] = 2\pi\alpha^2 \frac{xyQ^2}{xyQ^2} \left\{ (1 + (1 - y)^2) d\sigma_{U+L}[n - \text{jet}] - y^2 d\sigma_{L}[n - \text{jet}] \right\}$$

(3)

In eq. (3) $\sigma_{U+L}[n - \text{jet}]$ and $\sigma_{L}[n - \text{jet}]$ denotes the cross section contributions from an unpolarized and longitudinal polarized gauge boson. These helicity cross sections are linearly related to polarization density matrix elements of the virtual boson. One has:

$$\sigma_U = \frac{1}{2}(h_{++} + h_{--}) \quad \sigma_L = h_{00} \quad \sigma_{U+L} \equiv \sigma_U + \sigma_L$$

(4)

where $h_{m'n'} = \epsilon_{\mu}(m)H^{\mu\nu}\epsilon_{\nu}(m')$, $(m, m' = +, 0, -)$ and $\epsilon_{\mu}(\pm)(\epsilon_{\mu}(0))$ are the transverse (longitudinal) polarization vectors of the $\gamma^*$ in the ($\gamma^*\text{-initial parton})$-cms. The helicity cross sections can technically be obtained by the following projections on the (partonic) hadrontensor $H^{\mu\nu}$, which is calculated in fixed order perturbation theory,

$$\hat{\sigma}_{U+L} = \left( -\frac{1}{2}\hat{g}_{\mu\nu} + \frac{3x_p}{pq}p_{\mu}p_{\nu} \right) H^{\mu\nu}[n - \text{jet}]$$

(5)

$$\hat{\sigma}_L = \frac{2x_p}{pq}p_{\mu}p_{\nu} H^{\mu\nu}[n - \text{jet}]$$

(6)

where $p$ denotes the momentum of the initial state parton and $x_p = Q^2/(2pq)$.

The available jet multiplicities in DISJET are listed below:

1) (1+1) jet $O(\alpha_s^0)$ and $O(\alpha_s^1)$
2) (2+1) jet \( O(\alpha_s^1) \)
3) (2+1) jet \( O(\alpha_s^2) \)
4) (3+1) jet \( O(\alpha_s^2) \)

The physics of 1) and 2) is extensively discussed in [4]. For numerical results see also [4, 5]. Matrix elements of the complete contributions to (2+1) jet \( O(\alpha_s^2) \) corrections are first discussed in [6]. In DISJET, a modified version of these matrix elements is implemented including the full NLO scale dependence. NLO numerical results for (2+1) jet cross sections and rates (based on DISJET) including the contributions from all helicity cross sections are presented in [4]. The (2+1) jet NLO contributions originating from the projection with the metrical tensor \(-g_{\mu\nu}\) (see eq. (5)) on the hadron tensor and a discussion of the results are given in [7]. Results for the \( O(\alpha_s^2) \) (3+1) jet cross sections are discussed in detail in [4].

The general structure of the hadronic jet cross sections within the framework of perturbative QCD is given by

\[
\frac{d\sigma^{had}[(n+1) - \text{jet}]}{d\eta} = \int d\eta \ f_a(\eta, M_f^2) \ d\hat{\sigma}^a(p = \eta P, \alpha_s(M_f^2)) \tag{7}
\]

where one sums over \( a = q, \bar{q}, g \). \( f_a(\eta, M_f^2) \) is the probability density to find a parton \( a \) with fraction \( \eta \) in the proton if the proton is probed at a scale \( M_f^2 \). \( \hat{\sigma}^a \) denotes the partonic cross section from which collinear initial state singularities have been factorized out (in the NLO calculation) at a scale \( M_f^2 \). Note that \( \hat{\sigma}^a \) depends in NLO also explicitly on the renormalization scale \( M_r \) and factorization scale \( M_f \). DISJET allows the user to choose the mass scales \( M_r \) and \( M_f \) in eq. (7) according to the following input variables \( a_x, b_x \) and \( c_x \):

\[
M_x^2 = a_x Q^2 + b_x W^2 + c_x p_T^2 \tag{8}
\]

where \( x = f, r \) (see also next section). In eq. (8) \( p_T \) denotes the transverse momentum of the two partonic jets in (2+1) jet production. The transverse momentum is defined with respect to the \( \gamma^* \) direction. \( c_x \neq 0 \) can only be chosen in the case of (2+1) jet production, otherwise \( c_x \) is set to zero. Furthermore, to avoid to small scales for perturbation theory, the renormalization and factorization scales are cut at 2 GeV, i.e.

\[
M_x^2 = \max\{2 \text{ GeV}^2, M_x^2\} \tag{9}
\]

In order to calculate the (n+1) jet cross sections we have to define what we call (n+1) jets by introducing a resolution criterion. As has been elaborated in detail in [4] energy angle cuts are not suitable for an asymmetric machine
with its strong boosts from the hadronic cms to the laboratory frame. As a jet resolution criterion we use the invariant mass cut criterion introduced in [1, 4] such that

\[
s_{ij} = (p_i + p_j)^2 \geq M^2 = \max\left\{y_{\text{cut}}M_c^2, M_0^2\right\} \quad (i, j = 1, \ldots, n, r; \quad i \neq j)
\]

(10)

where \(y_{\text{cut}}\) is the resolution parameter which should be chosen in \([0.01 \ldots 0.08]\) (see fig. 5 in [4]). \(M_c\) is a typical mass scale of the process which defines the jet definition scheme. This mass scale can be chosen by the user according to the values of \(a_c, b_c\) and a constant \(C\) [GeV\(^2\)]; (see also section 3)

\[
M_c^2 = a_cQ^2 + b_cW^2 + C
\]

(11)

The extreme choices \(a_c = 0(b_c = 0)\) and \(C = 0\) correspond to the "\(W\) scheme" ("\(Q\) - scheme") as introduced in [4]. \(M_0\) in eq. (11) is an additional fixed mass cut which we have introduced in order to clearly separate the perturbative and non-perturbative regime in the case, where \(M_c^2\) is small (i.e. \(Q^2\) and/or \(W^2\) and \(C\) small). A reasonable choice for \(M_0\) is \(M_0 = 2\) GeV [4].

### 3 Input parameters

The input parameters for DISJET are written to a file referred to as unit 4 by the local FORTRAN compiler, i.e. for004.dat on VMS and fort.4 on UNIX.

Parameters to choose are: (an example for the input file is given in the appendix)

- **srs**: real, f6.2, [GeV]
  center of mass energy \(\sqrt{(P + l)^2}\)

- **ias**: integer, i1
  choose one-loop or two-loop formula for \(\alpha_{\overline{\text{MS}}}\). The value of \(\alpha_s\) is matched at the thresholds \(q = m_q\) and the number of flavours \(n_f\) in \(\alpha_s\) is fixed by the number of flavours for which the masses are less than \(M_r\). Furthermore the numbers of quark flavours that can be pair-produced are set equal to \(n_f\) chosen in \(\alpha_s\).
  1: 1-loop-formula
  2: 2-loop-formula

- **ilambda**: \(\{0, 1\}\) integer, i1
  0: \(\Lambda^{(4)}_{\overline{\text{MS}}}\) in \(\alpha_s\) is chosen consistent to the value in the chosen parton distribution functions (see below).
1: $\Lambda_{\text{MS}}^{(4)}$ in $\alpha_s$ can be chosen by hand (=dlam), see next item.

- \textit{dlam}: real, f5.4, [GeV]
  
  only relevant in the case $\lambda = 1$, otherwise this number is changed in the main program.
  If $\lambda = 1$, $\Lambda_{\text{MS}}^{(4)} = \text{dlam}$, real, f5.4., [GeV].

- \textit{iaem}: \{0, 1\} integer, i1

  fixed or $Q^2$ dependent $\alpha_{\text{electromagnetic}}$.

  0: $\alpha = 0.00729735$ fixed
  1: $Q^2$ dependent $\alpha$

- \textit{icross}: \{0, 1\} integer, i1

  0: calculate the jet cross section as defined in (3) in [pb]
  1: only effective for iproc < 1000 (see below)

  calculates the helicity cross sections $\sigma_{U+L}(n+1)-\text{jet}$ or $\sigma_{L}(n+1)-\text{jet}$

  for fixed $x$ and $Q^2$ ($n = 1, 2$)

- \textit{x}: \([0.0001, 1]\): real, f5.4
  
  If icross=0, $x$ is changed by the integration routine.

- \textit{Q^2}: \([4 \text{GeV}^2, s]\): real, f10.2

  same as $x$

- \textit{ipola}: \{10, 11, 12\}: integer, i2

  choose helicity cross sections in the case of $(n+1)$ jet production ($n = 1, 2$).

  Depending on icross, the full cross sections (including all coefficients) or

  the helicity cross sections $\sigma_{U+L,L}$ (for fixed $x, Q^2$) are calculated.

  10: take sum of all helicity cross sections, i.e. calculates $d\sigma[n - \text{jet}]$ as

  given in (3) (only possible for icross = 0).
  11: calculates only the term $\sim d\sigma_{U+L}[n - \text{jet}]$ in (3)
  12: calculates only the term $\sim d\sigma_{L}[n - \text{jet}]$ in (3)

- \textit{iproc}: \{10, 100, 101, 102, 190, 191, 192, 1001, 1002, 1003\}: integer, i4

  characterization of the process

  10: $(1+1)$ jet $\equiv \sigma_{\text{tot}}$ in $O(\alpha_s^0)$
  100: $(2+1)$ jet: complete, i.e. sum of quark and gluon initiated processes
  101: $(2+1)$ jet: quark initiated
  102: $(2+1)$ jet: gluon initiated
190: $O(\alpha_s^4)$-contribution to $\sigma_{\text{tot}}$ in the DIS-scheme: complete
191: $O(\alpha_s^4)$-contribution to $\sigma_{\text{tot}}$ in the DIS-scheme: quark initiated
192: $O(\alpha_s^4)$-contribution to $\sigma_{\text{tot}}$ in the DIS-scheme: gluon initiated

1001: (3+1) jet: $q \rightarrow qGG$ (class D [4])
1002: (3+1) jet: $G \rightarrow Gq\bar{q}$ (class C [4])
1003: (3+1) jet $q \rightarrow qq'\bar{q}'$ (class E [4])

- **istruc**: $\{0, \ldots, 7\}$: integer, i1
  - only relevant if $iproc=100, 101, 102$
  - 0: complete NLO corrections, (note: $ias$ should be set equal to 2)
  - 1: Leading order ($O(\alpha_s)$)
  - 2: NLO virtual corrections (finite part)
  - 3: NLO final state real corrections
  - 4-7: NLO initial state real corrections:
    - 4: delta-part
    - 5: hard-part
    - 6: plus-part
    - 7: substracted-part
  (note: $ias$ should be set = 2 for $istruc=2-7$)

- $y_c$: [0.01, 0.1]: real, f5.4
  - jet resolution parameter $y_{\text{cut}}$ as defined in eq. (10)

- $M_{\text{jet}}^2$: real, f3.1, [GeV$^2$]
  - additional fixed jet cut mass as defined in eq. (10)

- $M_{\text{jet}}^2 = a_f Q^2 + b_f W^2 + c_f p_T^2$: the factorization mass in eq. (7) can be chosen by the three values $a_f, b_f$ and $c_f$ in a format f4.2 ($c_f \neq 0$ only effective for $iproc = 100, 101, 102$)

- $M_{\text{ren}}^2 = a_r Q^2 + b_r W^2 + c_r p_T^2$: the renormalization mass in eq. (7) can be chosen by the three values $a_r, b_r$ and $c_r$ in a format f4.2 ($c_r \neq 0$ only effective for $iproc = 100, 101, 102$)

- $M_{\text{jet}}^2 = a_c Q^2 + b_c W^2 + C$ [GeV$^2$]: the jet cut mass as defined in eq (10) can be chosen by the three values $a_c, b_c$ and $C$ in a format f4.2

- $p_{T,\text{min}}^2$: real, f6.2
  - minimal required transverse momentum for the jets (transverse with respect to the $\gamma^*$ direction). This entry is effective only for $iproc = 100, 101, 102$. 

7
• $z_{\text{min}}, z_{\text{max}} [0.,1.]: \text{real, f8.2}$
Cut on the invariant $z = \frac{\mathbf{p}_1}{\mathbf{p}_g}$, where $p_1$ is the four momentum of one of the final partons in (2+1) jet production. This entry is effective only for $iproc = 100, 101, 102$.

• $Nptype, Ngroup, Nset$: integer, i3,i3,i3
choose parton distribution functions according to [3].

• $iacc$: \{1, 2, 3\} integer, i1
some predefined settings for the accuracy the VEGAS-integration:
1: acc = 0.05, ncall = 20000
2: acc = 0.005, ncall = 200000
3: acc = 0.0005, ncall = 1000000

• $x_{\text{min}}, x_{\text{max}}$: [0.0001, 1] real, f8.6, Integration limits
• $y_{\text{min}}, y_{\text{max}}$: [0, 1] real, f8.6, Integration limits
• $Q^2_{\text{min}}, Q^2_{\text{max}}$: [4, s] real, f8.2, [GeV$^2$], Integration limits
• $W^2_{\text{min}}, W^2_{\text{max}}$: [5, s] real, f8.2, [GeV$^2$], Integration limits

• The next integer (format i2) determines the number of single differential cross sections to be calculated.

• the following input values are data-cards for the calculation of single differential cross sections (see the last column). The user may change the entries in the first two columns. The first (second) column defines the lower (upper) kinematical limit of the variable specified in the last column. The user may also change the single differential cross sections to be calculated by suitable changings in the subroutine dsigma.

4 Output

The output of DISJET is written to the standard output as well as to a file unit 98. The output file contains all input values specified in the unit 4 input file and all results calculated by VEGAS, i.e. the integrated cross section and the differential cross sections.

The differential cross sections will be written to file unit 98 in the following format.

• the (up to) 10 differential cross sections are written to the file one after another
• every package of data belonging to one differential cross section is preceded by the string given in the data cards of the input file

• each differential cross section is presented line by line for each requested bin in the form:
  mean value of the variable in the bin, differential cross section in this bin, number of points in this bin.

5 Summary and conclusion

We present the first version of the Monte Carlo program DISJET. The program allows to calculate NLO (1+1) and (2+1) jet cross sections and LO (3+1) jet cross sections in DIS. The contributions from all helicity cross sections are included. Note that DISJET is not an event generator like LEPTO or HERMES \[3\]. Furthermore, the freedom to choose a jet definition scheme is restricted to the schemes discussed in the text and does not include the jet definition proposed in \[9\].

Some improvements and extensions are planned for further versions of the program. We would appreciate comments and suggestions on the program to be sent to mirkes@phenom.physics.wisc.edu.

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Appendix

=============== Example for an input-file ====================
==== for004.dat under VMS ====================
==== fort.4 under UNIX ====================
=============== Example for an input-file ====================

SRS= 295.
IAS= 1
ILAMBDA= 0, DLAM= .1970
IAEM= 1
ICROSS= 0, X= .010, Q^2= 10000.00
IPOLA= 10
IPROC= 100
ISTRUC= 1
Y_C= .0200, M_0^2= 4.0 GEV^-2
M_F^2= 1.00 Q^2 + 0.00 W^2 + 0.00 PT^2
M_R^2= 1.00 Q^2 + 0.00 W^2 + 0.00 PT^2
M_C^2= 0.00 Q^2 + 1.00 W^2 + 0.00
PT2MIN= 0.
NPTYPE = 1, NGROUP = 3, NSET = 31
IACC= 1
XMIN= 0.001000 XMAX= 0.100000
YMIN= 0.040000 YMAX= 0.950000
ZP_MIN = 0.00 ZP_MAX = 1.00
Q^2_MIN= 10.00 Q^2_MAX= 87025.00
W^2_MIN= 600.00 W^2_MAX= 87025.00
10
-3., -0.1, 32,0,0,'LG(X)'
3.D-4, 1.D-1, 32,0,0,'X'
2., 10.D0, 32,0,0,'SQRT(Q^2)'
10., 1.D5, 32,0,0,'Q^2'
3., 200.D0, 32,0,0,'SQRT(W^2)'
10., 1.D5, 32,0,0,'W^2'
-2., -0.1, 32,0,0,'LG(Y)'
0.05, 0.95, 32,0,0,'Y'
0.67, 0.99, 32,0,0,'THRUST'
0., 50., 32,0,0,'PT'
0
0
