One-loop corrections to multiscale effective vertices in the EFT for Multi-Regge processes in QCD

Maxim Nefedov*
Samara National Research University,
II Institute for Theoretical Physics, Hamburg University
E-mail: nefedovma@gmail.com

The computation of one-loop corrections to the $\gamma^* Q+q$ and $gR+g$ effective vertices in the framework of gauge-invariant effective theory for Multi-Regge processes in QCD is reviewed. Due to consistent implementation of the “tilted Wilson line” regularization for rapidity divergences, the gauge-invariance has been preserved at all stages of calculation independently on the rapidity regulator and cancellation of the power-like dependence on the regularization variable is traced. Only single-logarithmic rapidity divergence is left in the final result.

XXVII International Workshop on Deep Inelastic Scattering and Related Subjects
Torino, Italy, 8-12 April 2019

*Speaker.
†Work supported in part by the Foundation for the Advancement of Theoretical Physics and Mathematics BASIS, grant No. 18-1-1-30-1
1. Introduction

In the Multi-Regge Kinematics (MRK) for the $2 \rightarrow 2 + n$ partonic scattering in QCD, the final-state partons can be grouped into clusters w.r.t. their rapidity. Different clusters are highly-separated in rapidity from each-other, so that the typical $t$-channel momentum transfer is much smaller than the invariant mass of any pair of final-state clusters. At leading power in $t/s$, higher-order QCD corrections to such amplitudes are enhanced by high-energy logarithms $\log s/(-t)$. The Gauge-Invariant Effective Field Theory (EFT) for Multi-Regge processes in QCD [1, 2] has been introduced as a systematic tool for computation of asymptotics of QCD scattering amplitudes in the Multi-Regge limit in the Leading Logarithmic Approximation and beyond. The Hermitian version of this EFT [3, 4] contains all corrections, restoring the unitarity of High-Energy scattering and therefore provides a framework for studies of High-Energy QCD and gluon saturation phenomena, alternative to the Balitsky-JIMWLK or Color-Glass Condensate pictures, see Refs. [5, 6] for the recent work in this direction.

In the High-Energy EFT [1, 2], different rapidity-clusters of final-state particles are produced by different gauge-invariant subamplitudes – effective vertices. This effective vertices are connected by $t$-channel exchanges of Reggeized gluons ($R_\pm$) and Reggeized quarks ($Q_\pm$), collectively named as Reggeons – gauge-invariant degrees of freedom of the High-Energy QCD. Eventually, it should be possible to integrate-out physical quarks and gluons, order-by-order in $\alpha_s$, and formulate the high-energy limit of QCD entirely in terms of Reggeons – Reggeon Field Theory, see e.g. [6, 7, 8]. Calculation of the one-loop corrections to different effective vertices is a major task in development of this formalism.

The main technical difficulty in the Higher-Order calculations in High-Energy EFT is the appearance of Rapidity divergences in loop and phase-space integrals. These divergences arise due to the presence of “Eikonal” denominators $1/l^\pm$ in the induced vertices of interactions of Reggeons with ordinary (Yang-Mills) partons, taken together with kinematical constraints following from MRK. See Sec. 2 of Ref. [9] for the analysis of the conditions of appearance of rapidity divergences at one loop. At present, many calculations [3, 10, 11, 12] in the High-Energy EFT has been done with the use of a variant of “tilted Wilson line” regularization, where the direction vectors ($n^\pm_{\mu}$) of Wilson lines in the definition of Reggeon-parton interactions are slightly shifted from the light-cone:

$$n^\pm_{\mu} \rightarrow \tilde{n}^\pm_{\mu} = n^\pm_{\mu} + r \cdot n^\mp_{\mu}, \quad \frac{1}{l^\pm} \rightarrow \frac{1}{\tilde{l}^\pm} = \frac{1}{l^\pm + r \cdot l^\mp}, \quad (1.1)$$

where $0 < r \ll 1$ is the regularization variable. In Ref. [8] we have observed, that to keep the $Rg$-interaction gauge-invariant for $r \neq 0$ one also have to modify the usual MRK kinematic constraint, stating that four-momentum $q_1$ of $R_+\text{-Reggeon}$ has only one nonzero light-cone component $q^+_1$ and transverse momentum $q_{T1}$. The kinematic constraint for Reggeon $R_+$, consistent with gauge-invariance at $r \neq 0$ is

$$\tilde{q}^-_1 = q^-_1 + r \cdot q^+_1 = 0, \quad (1.2)$$

For Reggeized quarks, such modification is not strictly necessary, but it turns out, that many scalar integrals actually simplify in the kinematics [12], so we prefer to keep it both for Reggeized gluons and quarks.
In the present contribution we will discuss two examples of one-loop corrections to Reggeon-Particle-Particle effective vertices: $\gamma^* Q_gq$ and $gR_gg$. The first one involves an off-shell photon ($\gamma^*$), so that the vertex has two scales of virtuality: virtuality of the photon $q^2 = -Q^2 < 0$ and of the Reggeized quark $q^2 = -t_i < 0$. More details concerning this example can be found in our Ref. [9]. The second example already has been considered in Ref. [10], however in this reference part of diagrams has been cut to zero by the gauge choice for external gluons and therefore gauge-invariance of amplitude and cancellation of power-like dependence on the rapidity-regulator $r$ has not been verified. We fill this gap in the present contribution.

Our paper has the following structure: In the Sec. [8] integrals appearing in our calculation are listed and we comment on their rapidity divergences. Explicit expressions for these integrals are provided in Ref. [8]. In Sec. [8] we review the calculations for above-mentioned examples and in the Sec. [8] we summarize our conclusions.

2. One-loop rapidity-divergent integrals

It is convenient to categorize one-loop integrals appearing in our calculations according to the type of their dependence on the rapidity-regulator variable $r$. Then the simplest integrals containing only one quadratic and one or two linear propagators turn out to be the most singular ones. Integrals:

$$A_{[-]}(p) = \int \frac{[d^d l]}{(p+l)^2[l-]}, \quad A_{[-]}(p) = \int \frac{[d^d l]}{l^2[l-][l^--\bar{p}-]},$$

where $[d^d l] = (\mu^2)^d d^d l/(i\pi^{d/2} \Gamma_d)$, $d = 4 - 2\varepsilon$, $r_T = \Gamma^2(1 - \varepsilon) \Gamma(1 + \varepsilon)/\Gamma(1 - 2\varepsilon)$ and $1/[X]$ denotes the PV-prescription for the light-cone denominator: $1/[X] = (1/(X + i0) + 1/(X - i0))/2$, are related with each-other as:

$$A_{[-]}(p) = \frac{1}{\bar{p}^-} A_{[-]}(p), \quad (2.1)$$

and both are proportional to $r^{-1+\varepsilon}$.

Integrals

$$B_{[-]}(p) = \int \frac{[d^d l]}{l^2(p+l)^2[l-]}, \quad B_{[-]}(p) = \int \frac{[d^d l]}{l^2(p+l)^2[l^-][l^-+\bar{p}^-]},$$

are related as:

$$B_{[-]}(p) = \frac{2}{\bar{p}^-} B_{[-]}(p), \quad (2.2)$$

and contain the $r^\varepsilon$-dependence on the rapidity regulator for $p^2 = 0$ and also term $\propto r^{-\varepsilon}$ appears for $p^2 \neq 0$. These power-like terms come together with $1/\varepsilon^2$-factors and could lead to $\log^2 r$-dependence after expansion in $\varepsilon$, which would contradict to Reggeization of gluon and quark. Cancellation of this terms happens between different diagrams and hence is a nontrivial dynamical property of QCD.

The integral

$$B_{[+]}(p) = \int \frac{[d^d l]}{l^2(p+l)^2[l^++\bar{p}^-]},$$

where
One-loop corrections in the High-Energy EFT

Maxim Nefedov

Figure 1: Diagrams contributing to the $\gamma^* Qq$-vertex at one loop. Dashed line with an arrow – Reggeized quark.

Figure 2: Diagrams contributing to the $g_R g$-vertex at one loop. Dashed line – Reggeized gluon, dotted line – Faddeev-Popov ghost.

contributes to one-loop correction to propagators of Reggeized gluon and quark and it contains only logarithmic rapidity-divergence $\sim \log r$, related with Reggeization. Similar single-logarithmic divergence is present in a “triangle” integral:

$$C_{[-]}(-q_1^2,-q^2,q^-) = \int \frac{[d^4l]}{l^2(q_1 + l)^2(q_1 + q + l)^2l^{-1}},$$

which has been computed for the case $q^2 = 0$ in Ref. [10] and for the case $q^2 \neq 0$ in the Ref. [8]. For the $q^2 \neq 0$ case, the term $\propto r^{-\epsilon}$ also appears in the integral $C_{[-]}$. These are all scalar integrals necessary for the calculation of one-loop corrections to Particle-Particle-Reggeon effective vertices.

3. One-loop effective vertices

The set of EFT Feynman diagrams, contributing to the one-loop correction to $\gamma^* Q+q$-effective vertex is shown in the Fig. [1]. To compute them, we perform the tensor reduction procedure, similar to the standard one. However since now some integrals contain Eikonal denominators, depending on the vector $\tilde{n}_\mu$, one should include this vector to the ansatz for the tensor structure. The result [8]:

$$\Gamma_{+\mu}^{(1)}(q_1, q) = iee_q \cdot \bar{u}(q + q_1) \left[ C[\Gamma] \cdot \Gamma_{+\mu}^{(0)}(q_1, q) + C[\Delta^{(1)}] \cdot \Delta_{+\mu}^{(1)}(q_1, q) + C[\Delta^{(2)}] \cdot \Delta_{+\mu}^{(2)}(q_1, q) \right],$$

can be expressed in terms of three gauge-invariant Lorentz structures:

$$\Gamma_{+\mu}^{(0)}(q_1, q) = \gamma_{\mu} + \frac{\tilde{q} n_{\mu}}{2 q^2}, \Delta_{+\mu}^{(1)}(q_1, q) = \frac{\delta}{q^-} \left( n_{\mu} - \frac{2(q_1)_{\mu}}{q_1^-} \right), \Delta_{+\mu}^{(2)}(q_1, q) = \frac{\delta}{q^-} \left( n_{\mu} - \frac{q_{\mu}}{q^-} \right).$$
where $\Gamma^{(0)}_{+\mu}$ is the Fadin-Sherman scattering vertex and coefficients are the following:

$$C[\Gamma] = -\frac{\alpha_s C_F}{4\pi} \left\{ \frac{[(d-8)Q^2 + (d-6)t_1]B(t_1) - 2(d-7)Q^2B(Q^2)}{Q^2 - t_1} - 2\left[ (Q^2 - t_1)C(t_1, Q^2) - q_-(t_1C_{[-]}(t_1, Q^2, q_-) + (B_{[-]}(q) - B_{[-]}(q + q_1))) \right] \right\}, \hspace{1cm} (3.1)$$

$$C[\Delta^{(1)}] = -\frac{\alpha_s C_F}{4\pi} \frac{Q^2}{2(Q^2 - t_1)^2} \left\{ ((d-2)Q^2 - (d-4)t_1)B(t_1) - 2Q^2B(Q^2) \right\}, \hspace{1cm} (3.2)$$

$$C[\Delta^{(2)}] = -\frac{\alpha_s C_F}{4\pi} \frac{Q^2}{(Q^2 - t_1)^2} \left\{ ((d-6)t_1 - (d-8)Q^2)B(Q^2) + 2(t_1 - 2Q^2)B(t_1) \right\}, \hspace{1cm} (3.3)$$

were $\alpha_s = \mu^{-2\epsilon} g_s^2 r_T/(4\pi)^{1-\epsilon}$ is the dimensionless strong-coupling constant, $B(t_1)$ and $C(t_1, Q^2)$ are the usual one-loop scalar “bubble” and “triangle” integrals [13]. We observe, that integrals $A_{[-]}(q)$ appearing in the expansion of the second and third diagrams in the Fig. 1 cancel-away. Also the terms $\propto r^{\pm \epsilon}$ cancel between integrals $B_{[-]}(q)$, $B_{[-]}(q + q_1)$ and $C_{[-]}(t_1, Q^2, q_-)$ in Eq. (3.1), so that only single-logarithmic rapidity divergence is left. In Ref. [9] we have checked, that this rapidity divergence cancels in the single-Reggeon exchange contribution to the $\gamma^* + \gamma \rightarrow q + \bar{q}$-amplitude at one loop and EFT result agrees with MRK limit of one-loop QCD amplitude.

Diagrams contributing to the one-loop correction to $gR_g$-vertex with on-shell external Yang-Mills gluons $g$ with helicities $\lambda_1$ and $\lambda_2$ and momenta $q$ and $q + q_1$ are shown in the Fig. 3. This one-loop correction can be decomposed as:

$$\gamma^{abc,(1)}_{\lambda_1 + \lambda_2} = ig_s f^{abc} \cdot \epsilon^\mu(q, \lambda_1)((\epsilon^\mu(q + q_1, \lambda_2))\gamma \left[ C \left[ \gamma_{+}^{(0)} \right] \cdot \gamma_{\mu+, \nu}^{(0)} + C [\delta_\lambda] \cdot \delta_{\mu+, \nu} \right] + C [\delta_\lambda] \cdot \delta_{\mu+, \nu},$$

where the helicity-conserving (Lipatov’s) and helicity-flip Lorentz structures are:

$$\gamma_{\mu+, \nu}^{(0)} = 2q_\mu g_{\nu \mu} + 2n_\mu q_1\nu - 2n_\nu q_1\mu + \frac{n_{n\mu} n_{n\nu}}{q_-}, \hspace{0.5cm} \delta_{\mu+, \nu} = 2q_- \left[ g_{\mu \nu} + \frac{2q_\mu q_1\nu}{t_1} \right],$$

while coefficients in front of them read:

$$C \left[ \gamma_{+}^{(0)} \right] = \frac{\alpha_s C_A}{4\pi} \left[ q_- t_1 C_{[-]}(t_1, 0, q_-) - B(t_1) \right], \hspace{1cm} (3.4)$$

$$C [\delta_\lambda] = \frac{\alpha_s}{4\pi} \frac{(d-4)B(t_1)}{2(d-1)(d-2)} (2n_F - (d-2)C_A). \hspace{1cm} (3.5)$$

Eqns. (3.4) and (3.5) coincide with the results of Ref. [10], however in the calculations in this paper, the diagrams framed in the Fig. 3 where nullified by the gauge-choice for external gluons. We take them into account, and hence we can check the Slavnov-Taylor identities and trace-out the cancellation of power-like dependence on the regulator $r$. Modified kinematical constraint (2.2) guarantees the gauge-invariance of amplitude in all orders in $r$, and we observe, that contributions of integrals $A_{[-]}(q)$ and $B_{[-]}(q)$ cancel in the $O(r^1)$ and $O(r^0)$ respectively, while in higher orders in $r$ (which we eventually drop), coefficients in front of this integrals are gauge-invariant, which serves as a useful cross-check of the calculation. Cancellation of contributions of these integrals happens between different diagrams and essentially relies on relations (2.1) and (2.2), while integral $B_{[-]}(q_1) = 0$ due to the constraint (2.2). Therefore all power-like dependence on the rapidity-regulator cancels in the leading power in $r$ and we are again left with single-logarithmic rapidity divergence related with gluon Reggeization.
4. Conclusions and discussion

In the present contribution we have reviewed the structure of rapidity divergences in the one-loop integrals contributing to the one-loop corrections to Particle-Particle-Reggeon effective vertices in the gauge-invariant EFT for Multi-Regge processes in QCD [1, 2] and illustrated their application on two examples of such vertices: $\gamma^* Q + q$ and $gR + g$. The first one contains two scales of virtuality: squared transverse momentum of Reggeized quark $t_1$ and virtuality of the photon $Q^2$, and new Lorentz structure $\Delta_{\gamma\mu}^{(2)}$ appears in the answer for $Q^2 \neq 0$. Cancellation of power-like dependence on rapidity regularization parameter $r$ is observed in both cases, so that only single-logarithmic rapidity divergence is left in the end.

References

[1] L. N. Lipatov, Nucl. Phys. B 452, 369 (1995) [hep-ph/9502308].
[2] L. N. Lipatov and M. I. Vyazovsky, Nucl. Phys. B 597, 399 (2001) [hep-ph/0009340].
[3] L. N. Lipatov, Phys. Rept. 286, 131 (1997) doi:10.1016/S0370-1573(96)00045-2 [hep-ph/9610276].
[4] S. Bondarenko and M. A. Zubkov, Eur. Phys. J. C 78, no. 8, 617 (2018) doi:10.1140/epjc/s10052-018-6089-1 [hep-ph/1801.08066].
[5] M. Hentschinski, Phys. Rev. D 97, no. 11, 114027 doi:10.1103/PhysRevD.97.114027 , [hep-ph/1802.06755], (2018).
[6] S. Bondarenko and S. Pozdnyakov, Int. J. Mod. Phys. A 33, no. 35, 1850204 doi:10.1142/S0217751X18502044 , [hep-th/1806.02563], (2018).
[7] S. Bondarenko, L. Lipatov, S. Pozdnyakov and A. Prygarin, Eur. Phys. J. C 77, no. 9, 630 doi:10.1140/epjc/s10052-017-5208-8 , [hep-th/1708.05183], (2017).
[8] S. Bondarenko and S. Pozdnyakov, [hep-th/1903.11288], (2019).
[9] M. A. Nefedov, [hep-ph/1902.11030], (2019).
[10] M. Hentschinski, A. Sabio Vera, Phys. Rev. D85, 056006, doi:10.1103/PhysRevD.85.056006 , [hep-ph/1110.6741] (2012); G. Chachamis, M. Hentschinski, J. D. Madrigal Martinez, A. Sabio Vera, Phys. Rev. D87, 076009, doi:10.1016/j.physletb.2012.03.015 , [hep-ph/1212.4992] (2013).
[11] M. Nefedov, V. Saleev, Mod. Phys. Lett. A32, 1750207, doi:10.1142/S0217732317502078 , [hep-th/1709.06246], (2017).
[12] G. Chachamis, M. Hentschinski, J. D. Madrigal Martinez, A. Sabio Vera, Nucl. Phys. B876, 453âĂŞ472, doi:10.1016/j.nuclphysb.2013.08.013 , [hep-ph/1307.2591], (2013).
[13] R. K. Ellis, G. Zanderighi, JHEP 0802, 002, doi:10.1088/1126-6708/2008/02/002 , [hep-ph/0712.1851], (2008).