Mathematical modeling of vertical offsets of the Earth surface during the use of the Kaluga UGS facility

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Abstract. The purpose of this paper is mathematical modeling of vertical displacements of the earth surface of the Kaluga underground gas storage. The method proposed by the authors can be used for all underground storage facilities of this type. At the moment, the technology of gas storage in reservoirs is unique and is used only in Russia and China. The development of modern methods for modeling vertical displacements of the earth surface is the most promising direction due to the transfer of gas storage technology throughout the world. Modeling will be carried out at the Kaluga UGS facility. The calculation of the real vertical offsets of the earth surface, taking into account the weight of the overlying rocks, is carried out according to the data on the values of reservoir pressure. Spatial (3D) modeling is carried out in order to determine the configuration of the subsidence surface and its dimensions in conditions when the simulated reservoir is an irregularly shaped figure. For mathematical modeling of the stress-strain state of such objects, a numerical-analytical approach is used. This approach will make it possible to assess effectively the deformation processes caused exclusively by the mode of operation of the UGS facility.

1. Introduction
The underground gas storage facilities is especially important in our country with its climatic features and the remoteness of resource sources from end consumers. Russia has a unique Unified Gas Supply System that has no analogues in the world, the UGS system of which is an integral part. Underground storage facilities allow supplying natural gas to consumers regardless of a season, temperature fluctuations and force majeure events.

It was found that the main forms of negative geodynamic consequences during the development of hydrocarbon deposits are: deep subsidence of a deposit, seismicity and activation of fault zones [1, 2]. There are two types of seismic impacts: man-made and man-made induced. Man-made do not arise without the conditions for the development of deposits, and man-made earthquakes occur only when deposits are developed in seismically active regions [3].

2. Problem statement
The importance of the underground storage system is significantly increasing under the conditions of a market economy. It is necessary to meet the increasing requirements for flexibility, efficiency, and mobility of gas supplies. At the moment, the technology of gas storage in reservoir strata is unique and is used only in two countries of the world (Russia and China). In connection with the transfer of storage technology by Russia, a number of countries are developing using this storage method. Due to this, the problem of timely observation of the processes of vertical offsets of the earth surface arises.

Oil and gas complexes are influenced by modern underground geodynamics, which has the highest in fault zones. The method of mathematical modeling will be proposed in this paper that will allow timely determination of dangerous deformations of the earth surface caused by gas injection and withdrawal using the study of Kaluga underground gas storage located on the territory of the Leninsky district of Kaluga and the Dzerzhinsky district of the Kaluga region, 10 km west of Kaluga.

3. Purpose of research

For mathematical modeling, we used a quantitative description of the processes that form extensive deformations of the earth surface of the UGS territory, which allows making an initial presentation of the physical model of the phenomena occurring in the reservoir, formulated in terms of observable and experimentally determined values. As the initial model, the idea was used that the operation of the UGS facility leads to cyclical (alternating) variations in reservoir pressure, which, in turn forms a change in the volumetric deformation of the reservoir in full compliance with Pascal's law [4, 5].

4. Research methods

During the construction of a mathematical model we used the formalism of the theory of elastic inclusions. In order to obtain the analytical expressions, a gravity-deformation analogy was used, which was based on the fact that the vertical displacements were expressed as:

\[ R = (x^2 + y^2 + z^2)^{1/2} \]  

(1)

Similar to the formula for the first derivative of the gravitational potential (vertical potential gradient) \( \Delta g \) [6].

Taking into account these analogy, it is possible to obtain analytical expressions for offsets and deformations of the free surface of an elastic half-space containing volumetric inclusions of various configurations, in which volumetric deformations change [7]. Thus, for a horizontal sheet-like inclusion of infinite length it will take the form [4]:

\[ I' = \frac{1}{2\pi} \ln \left( \frac{(x+a)^2 + d^2}{((x-a)^2 + d^2)^{1/2}} \right) - \frac{1}{2\pi} \ln \left( \frac{(x-a)^2 + d^2}{((x+a)^2 + d^2)^{1/2}} \right) - \frac{2D}{\pi} \left[ \frac{1}{\pi} \arctg \left( \frac{(x+a)(D-d)}{D} \right) - \arctg \left( \frac{(x-a)(D-d)}{D} \right) \right] \]  

(2)

In this expression \( d \) and \( D \) is the depth at the top and bottom of the reservoir where changes in reservoir pressure occur, \( a \) – half the width (length) of the formation. In order to estimate the maximum amplitude of subsidence in the center of abstinence (injection), we can take \( x = 0 \) and then the expression will be as follows:

\[ Dh = DP_{as} \left( K \left( \frac{2a\ln \left( \left( a^2 + d^2 \right) \left( a^2 + D^2 \right)^{-1} \right)}{a} \right) \right) \]  

\[ + \frac{2D}{\pi} \left[ \frac{1}{\pi} \arctg \left( \frac{a(D-d)}{D} \right) - \arctg \left( \frac{-a(D-d)}{D} \right) \right] \]  

(3)

Substituting the values for the Khadum strata \( DP_{as} = = 1.22 \text{ MPa}, K = 2.78 \cdot 10^{-4} \text{ MPa}, a=10000 \text{ m}, d=750 \text{ m}, D=820 \text{ m} \) (effective formation thickness is 70 m) we obtained [5] the total amount of subsidence in the middle part of the field (142 mm). The distribution of the subsidence calculated by the formula (2) along the profile is shown in (Figure 1). It also shows the data obtained during the field leveling in 1962 and the range of double measurement error is given [8].
Figure 1. The comparison of the measured and calculated subsidence depths of the earth surface in the Northern Stavropol Territory in 1957–1962

The obtained analytical expressions allow assessing the changes in the offsets of the earth surface for the conditions of the regime of changes in reservoir pressure during injection and withdrawal of gas at the Kaluga UGS facility. In particular, it is necessary to note that all the corresponding formulas were obtained by approximation to a non-ponderable medium. The consideration of the influence of the weight of the overlying layer leads to the need for numerical modeling, since it is impossible to obtain analytical solutions in difficult conditions [9].

In the work [10], a comparison of the offsets of the earth surface for non-ponderable and ponderable media was carried out, which was obtained within the framework of the numerical method of boundary elements. It was shown that the amplitude of vertical offsets for a ponderable medium was 15–20% higher than for a non-ponderable one. It is important to note that the singular points of the curves of the horizontal gradients of the vertical offsets of the earth surface coincide for both types of media. It is this fact that allows using an analytical model of a non-ponderable medium for the interpretation of observations, introducing an amplitude correction. Thus, the obtained estimates for the maximum vertical offset of the earth surface should be increased by 15–20%, depending on the depth of the formation from the earth surface [11].

The following reservoir dimensions were used as geometrical dimensions of the modeled object:
- formation length - 8 km;
- formation width - 2.0 km;
- effective gas-saturated thickness - 10 m;
- compressibility of the pore space - $\beta_{pore} = 1.44 \cdot 10^{-4}$/atm.

The calculation of the actual vertical offsets of the earth surface, taking into account the weight of the overlapping rocks, was carried out according to the data of the values of the injection and withdrawal pressure from the reservoir.

Within the framework of mathematical modeling, the assessments of cyclic variations in vertical offsets of the earth surface in the central part of the Kaluga UGS facility were carried out, taking into account the weight of the overlying rock stratum (Figure 2).
Figure 2. Cyclic variations in vertical offsets of the earth surface in the central part of the Kaluga UGS facility

Figure 2 shows that the values of vertical offsets do not represent significant values. Nevertheless, at some moments the relative deformations can reach significant values of $10^{-4}$ order (from the peak of injection to the peak of production). In addition, mathematical modeling of the spatial distribution of vertical offsets on the earth surface was carried out within the mining lease of the Kaluga UGS facility. Figure 3 and Figure 4 show the results of spatial modeling (3D) and the profile of the deformations of the earth surface, taking into account the weight of the underlying layers caused by gas injection. Spatial modeling (3D) was performed in order to determine the configuration of the subsidence surface and its dimensions in conditions when the simulated reservoir has an irregular shape (Figure 3). For mathematical modeling of the stress-strain state of such objects, a numerical-analytical approach was used. It is based on the principle of summing up the effects of a set of elementary finite rectangular prismatic inclusions that approach an irregularly shaped formation.

Taking into account that analytical expressions for offsets and deformations of the surface were obtained in the work [4] due to the presence of a prismatic volume inclusion in the middle space - a similar reservoir model, the numerical implementation was performed on the basis of a superposition of analytical solutions. The distribution obtained from the vertical offsets of the earth surface was also used to determine the optimal lengths of the alignment profiles, which should be calculated taking into account the limits of anomalous deformation [12].

Then, in Figure 4 we compare the offset distribution with the line of one of the projected profiles made on the basis of surface leveling. The light contour in the 3D offset image indicates the error rate in the vertical offsets for the 2nd class of alignment. This is why the endpoints that are captured on the basis of fundamental links are not only brought out of the production area, but also outside the observation error contour. In Figure 4 the surface is uplifted.
Figure 3. The distribution of vertical offsets, calculated by the hybrid model, indicating the boundaries of the GWC, mining lease and the assumed profile. On the right there is a color scale showing the most dangerous displacements in cm from blue (within normal limits) to red (requiring attention). Vertical and horizontal axis show distances in km.

Figure 4. The results of three-dimensional modeling of the deformations of the earth surface caused by gas injection, values in cm. The figure shows the scales in XYZ axes.

Figure 5 shows the results of the calculation of the distribution of vertical offsets of the earth surface of the Kaluga UGS facility with an injection of 68 atm. The boundaries of the GWC (gas-water contact) are located along the selected profile. Allotment boundaries are located along the selected profile. Vertical displacement is measured in cm.

Figure 5 shows a profile variant of the distribution of vertical offsets of the earth surface, corresponding to the projected leveling line. In this case, the vertical lines indicate the boundaries of the exploited layer along the border of the GWC and the mining lease.

It is shown that the maximum elevation of the earth surface caused by gas injection reaches 1.2 cm. This offset corresponds to a relative deformation of $5.6 \cdot 10^{-5}$. 
Figure 5. Results of modeling vertical offsets of the earth surface due to gas injection along the selected profile. The vertical axis shows the vertical offset in cm. The horizontal axis shows the distance in km.

Further, the similar studies are carried out, which correspond to the distribution of vertical offset of the earth surface during the period of gas sampling in the storage. Figure 6 shows the results of the calculation of the distribution of vertical offsets of the earth surface of the Kaluga UGS facility with 68 atm sampling. Vertical offset is presented in cm. The boundaries of lease are along the selected profile and GWC boundaries are along the selected profile.

Figure 6. The results of modeling vertical offsets of the earth surface caused by gas extraction along the selected profile. The vertical axis shows the vertical offset in cm. The horizontal axis shows the distance in km.

Figure 6 shows that the maximum subsidence of the earth surface reaches 1.84 cm. In this case, the relative deformations reach values of $6.1 \times 10^{-5}$.

5. Discussion
Analyzing the general character of the vertical offset curves, we can see that they attenuate significantly at distances greater than 0.5–1.0 km than the projection of the reservoir boundary onto the earth surface. In this case, the projected lengths of the measuring profiles should be at least 1 km from the boundary of the exploited formation.

6. Conclusion
Thus, a method for mathematical modeling of vertical offset of the earth surface during the use of UGS facilities was developed. We can state that the expected deformations of the earth surface within the territory of the Kaluga UGS facility can reach absolute offset values of 1.5–2 cm per cycle and in relative deformations this can correspond to a value of 5·6·10⁻⁵.

Moreover it is necessary to note that there are fault zones within the Kaluga UGS facility, which have been found using a set of remote sensing and geological and geophysical information. It is obvious that cyclical variations in reservoir pressure will significantly intensify the current geodeformational activity of fault zones with rates of relative deformations of the earth surface at the level of 5·7·10⁻⁵ per year, which characterizes these zones as hazardous.

It is obvious that such a level of deformation processes must be monitored creating and developing an observation system (geodynamic testing ground). The development of methods for mathematical modeling of vertical offsets of the earth surface during further studies will allow assessing the greatest efficiency of the deformation processes caused exclusively by the mode of operation of UGS facilities in reservoirs.

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