About the basic laws of non-isothermal cross-flow past of a circular cylinder with beads mix

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Abstract. The influence of mechanical impurities in the fluid on the structure of the flow near a stationary, rotating cylinder, on its hydrodynamic characteristics and the heat exchange with the stream was investigated numerically. It was shown that the presence of impurities in the liquid leads to a significant change in the pattern of the flow, to increase in the resistance and heat transfer of a stationary cylinder, reducing the lifting force. The emissivity of the rotating cylinder in a fluid flow with an admixture of substantially depends on the dimensionless velocity of the surface. If the speed is less critical, equal to 3.0, the heat is intensified. At higher values, on the contrary, the heat transfer deteriorates. The necessity of considering these circumstances when developing technologies for deposition of functional coatings is evident.

1. Introduction

For many natural phenomena, technology is characterized by the flow of bodies of various shapes, including mobile, liquid containing impurities. Often the mechanical interaction of bodies with the environment is accompanied by heat transfer. The study of patterns of occurrence of these processes is of great interest.

An overview of the research methods of these processes, the obtained results are available in the literature [1-3]. Below, in relation to the technology of deposition of functional coatings on the surface, we give the results of numerical solution of non-isothermal cross-flow past a stationary, rotating circular cylinder monodispersed mixture. To simulate the motion of the medium is used as the Lagrangian and Euler continuum approach, the flow of viscous incompressible fluid is considered to be laminar. The complete system of equations with appropriate initial and boundary conditions is integrated using the finite volume method.

2. Governing equations

In the Cartesian rectangular coordinate system $x_1, x_2$ plane for carrier environment, representing a viscous incompressible fluid is described by the equations:

$$\frac{\partial u_i}{\partial x_i} = 0,$$  

(1)
\[
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_i \partial x_j} - \frac{\bar{f}_{pi}}{\rho}, \quad i=1,2.
\]  

(2)

Here \( t \) is the time; \( u_i, u_j \) are components of the velocity vector \( \mathbf{u} \) \((i, j = 1, 2)\); \( \rho \) is the density of the fluid, \( \nu \) is kinematic viscosity coefficient of the fluid, \( f_{pi} \) is the projection on the axis \( x_i \) averaged over volume forces hydrodynamic resistance of the particles.

The equation of conservation of energy is:

\[
\frac{\partial}{\partial t}(\rho E) + \nabla \left( u (\rho E + p) \right) = \nabla (\lambda \nabla T) - \bar{Q}_p,
\]

(3)

where \( E = h - \frac{p}{\rho} + \frac{u^2}{2} \), \( h \) is the enthalpy, \( T \) is the fluid temperature, \( \lambda \) is the thermal conductivity of the environment, \( \bar{Q}_p \) is averaged by the amount of heat flux to the particles of impurities.

When considering the motion of particles of impurities, it is assumed that their concentration in the raw stream is small, the mutual influence of particle rotation is not taken into account. It is believed that particles are not deformed, have a spherical shape with the same radius \( r_p \) and average physical and mechanical properties, the material density of the particles \( \rho_p \).

In the local Cartesian coordinate system \( x\partial y \), the trajectory of the particle is described by the equations

\[
\frac{dx}{d\tau} = u_{px}, \quad \frac{dy}{d\tau} = u_{py},
\]

(4)

where \( \tau \) is the time; \( u_{px}, u_{py} \) are the projections of the velocity vector of a particle \( \mathbf{u}_p \) at the coordinate axes. Given the hydrodynamic resistance force acting from the side of the carrier medium on the particle, gravity and Archimedes the equation of motion is represented in the form

\[
\frac{d\mathbf{u}_p}{d\tau} = F_p \left( \mathbf{u} - \mathbf{u}_p \right) + \mathbf{g} \left( 1 - \frac{\rho}{\rho_p} \right),
\]

(5)

Here \( \mathbf{g} \) is the vector of gravity acceleration, \( F_p = 9C_p \Re_p \mu/12 \rho_p d_p^2, \quad d_p = 2r_p, \quad \Re_p = \frac{\rho d_p^3 |\mathbf{u}_p - \mathbf{u}|}{\mu} \)

is the Reynolds number of the particle, \( C_p = a_1 + \frac{a_2}{\Re_p} + \frac{a_3}{\Re_p^2} \) is the coefficient of hydrodynamic resistance, \( a_1, a_2, a_3 \) are constants.

According to relation (5) the power of the hydrodynamic resistance of the particles \( f_p = 0.5\pi C_p r_p^2 \rho |\mathbf{u} - \mathbf{u}_p| (|\mathbf{u} - \mathbf{u}_p|).\)

Here we find averaged over the volume of the components of the resistance forces \( \bar{f}_{pi} \) \((i=1, 2)\), in the equation (2).

Since the sizes of the particles are small, they are long enough in the fluid flow, heat exchange with the environment is weak, it is assumed that the thermal state of particles is characterized by the average in terms of temperature \( T_p \).

Provided that \( \alpha_p = \alpha_p = \text{const}, \quad T = \bar{T} = \text{const}, \quad \bar{T} \) is some time average temperature of the carrier medium

\[
T_p = \bar{T} + (T_p - \bar{T}) \exp \left( -3\alpha_p \bar{T} \tau / \tau_0 \right).
\]

(6)
Equation (6) allows us to estimate the temperature change of particles during their movement in the environment, as well as the heat exchange with the liquid medium characterized by specific surface heat flux

\[ q_p = \left( \frac{B}{r_p} \right) (T - T_p). \]

Hence, the heat flux to the particle of the impurities \( Q_p = 4\pi r_p^2 q_p \). Making this value by the volume average, we get the value \( \bar{Q}_p \) in equation (3).

If the impurity particles are thermally thin bodies, radiant heat transfer is not taken into account, the equation of thermal balance of the particles has the form

\[ m_p c_p \frac{dT_p}{d\tau} = \alpha_p A_p (T - T_p), \]

where \( m_p \) is the mass, \( c_p \) is specific heat of the material, \( A_p \) is the particle surface area.

Recorded thus equations (1) - (3) must be supplemented by the boundary and initial conditions, and (4), (5) - initial. Let’s discuss them below in relation to the specific geometry of the computational domain.

Let us choose the computational domain in the form of a rectangle the length of 0.10 m and a width of 0.04 m. We place the coordinate system origin \( x_0 \) into the center of the streamlined cylinder with radius 0.001 m in the destruction of 0.02 m from the input section, at an equal distance from the lateral boundaries.

In the input section perpendicular to the axis \( x_0 \), we accept: \( u_1 = u_{e, \infty} = \text{const} \), \( u_2 = 0 \); operating pressure \( p_0 = 0.1 \) MPa; the temperature of the carrier medium \( T = T_0 = \text{const} \).

At the outlet of the computational domain, will use a "soft" boundary conditions: \( \frac{\partial u_i}{\partial x_i} = 0 \), \( \frac{\partial u_i}{\partial x_2} = 0 \), \( \frac{\partial v_{x_2}}{\partial x_2} = 0 \), \( \frac{\partial T}{\partial x_i} = 0 \), that means the alignment of both the hydrodynamic and thermal characteristics of the flow of the carrier medium.

On the lateral boundaries we place: \( u_1 = 0 \), \( \frac{\partial u_1}{\partial x_2} = 0 \), \( \frac{\partial u_2}{\partial x_2} = 0 \), \( \frac{\partial T}{\partial x_2} = 0 \).

It is believed that on the surface of the streamlined cylinder are implemented in terms of adhesion of the carrier medium \( u = 0 \), if the body is motionless, and \( u = w \), if the body rotates. The wall temperature remains constant, equal \( T_w \). While specific heat flow is \( q = \alpha (T_w - T) \).

In the initial moment of time \( t = 0 \) the environment instantly begins to move, \( u_1 = u_{e, \infty} \), \( u_2 = 0 \), \( p = p_0 \), \( T = T_0 \).

When calculating the kinematics of particle impurities beginning of the local coordinate system \( x_0 y \) is placed at the point of touch of the particles in the carrier medium. The starting point of these is located along the input section, the axis \( x_0 \) is directed parallel to the axis \( x_1 \). At the initial moment of time \( \tau = 0 \) \( u_{x_1} = u_{e, x_1} \), \( u_{x_2} = 0 \), the temperature of the particles is \( T_p = T_{p_0} = T_0 \). In addition to the speed of the particles in place of throw, the radius, the density of the particles, the mass flow rate \( G_p \), making influence on the number of particles coming in the calculated area per unit time are given.

By using Euler continuum approach given that mass exchange between the phases does not occur, the mass conservation equation can be written as:

\[ \frac{\partial}{\partial \tau} (\rho_i \rho_j) + v_i \cdot \nabla (\rho_i \rho_j) = 0, \quad i, j = 1, 2; \]
here $\tau$ is the time, $\nabla = \hat{i} \frac{\partial}{\partial x_1} + j \frac{\partial}{\partial x_2}$, $v_i = v_{1i} + v_{2i}j$ is the velocity vector $i$-phase; $\varepsilon_1$, $\varepsilon_2$ are volume concentration of the carrier and the dispersed phase ($\varepsilon_1 + \varepsilon_2 = 1$);

$$\frac{\partial}{\partial \tau} (\varepsilon_2) + \nabla \cdot \nabla (\varepsilon_2) = 0. \quad (8)$$

The momentum equation for $i$-phase in the absence of gravitational forces has the form

$$\frac{\partial}{\partial \tau} \left( \rho_i \rho_j v_i \right) + \nabla \cdot \left( \rho_i \rho_j v_i \right) = -\varepsilon_i \nabla p_i + \nabla \cdot \Pi_i + \mathbf{R}. \quad (9)$$

Where $p_i$ is the average pressure of the $i$-phase; $\Pi_i = \varepsilon_i \mu_i \left( \nabla v_i + \left( \nabla v_i \right)^T \right) + \varepsilon_i \left( \sigma_i - \frac{2}{3} \mu_i \right) \nabla v_i \cdot \mathbf{I}$ is the stress tensor in the $i$-phase; $\mu_i$, $\sigma_i$ are shear, bulk viscosity; $\mathbf{I}$ is the unit tensor; $\mathbf{R} = \frac{(-1)^i}{V} \int \left( v_{2i} - v_{1i} \right) dV$ is the averaged interaction force between phases, $V$ is the control volume,

$$\gamma = \varepsilon_2 \rho_2 f / \delta, \quad \delta = 2 \rho_2 a^2 / 9 \mu_i, \quad f = \frac{c_d \text{Re}_2}{24},$$

$$c_d = \begin{cases} 24 \left( 1 + 0.15 \text{Re}_2^{0.667} / \text{Re}_2 \right), & \text{Re}_2 \leq 1000, \\ 0.44, & \text{Re}_2 > 1000. \end{cases}$$

$\text{Re}_2 = \frac{2 \rho_i |v_i - v_{1i}| \rho_1}{\mu_i}$ is the Reynolds number of the relative motion phases.

Equation of heat transfer in the carrier and dispersed phases

$$\frac{\partial}{\partial \tau} \left( \rho c_i T_i \right) + \nabla \cdot \left( \rho c_i T_i \right) = \nabla \cdot \left( \lambda_i \nabla T_i \right) + Q, \quad (10)$$

here $Q = \frac{(-1)^i}{V} \int 4 \pi a^2 \varepsilon_2 \beta_2 \left( T_2 - T_1 \right) dV$, $\beta_2 = \lambda_i \text{Nu}_2 / 2a$ is the heat transfer coefficient between the carrier and the dispersed phases, the Nusselt number is calculated by the formula Ranz-Marshall: $\text{Nu}_2 = 2 + 0.6 \text{Re}^{0.3} \text{Pr}^{0.3}$ ($0 \leq \text{Re}_2 \leq 200, 0 \leq \text{Pr}_2 \leq 250$).

Basic equations (7) - (10) are supplemented by the boundary and initial conditions. The input section perpendicular to the axis $0x_1$, set: $v_{1i} = v_{2i} = w$, $v_{12} = v_{22} = 0$; operating pressure $p_0 = 0.1$ MPa; temperature of the carrier and the dispersed phase is the same: $T_1 = T_2 = T_0 = \text{const}$; concentration $\varepsilon_2 = \varepsilon_0 = \text{const}$.

At the outlet of the computational domain we use the following boundary conditions:

$$\frac{\partial v_{1i}}{\partial x_1} = \frac{\partial v_{12}}{\partial x_2} = \frac{\partial p}{\partial x_1} = \frac{\partial T_i}{\partial x_1} = 0 \quad (i = 1, 2), \quad \frac{\partial \varepsilon_2}{\partial x_1} = 0.$$

On the lateral boundaries:

$$\frac{\partial v_{1i}}{\partial x_2} = \frac{\partial p}{\partial x_2} = \frac{\partial T_i}{\partial x_2} = 0, \quad v_{12} = 0 \quad (i = 1, 2), \quad \frac{\partial \varepsilon_2}{\partial x_2} = 0.$$

It is believed that on the surface of the cylinder attachment conditions implemented by the carrier and dispersed phases $v_i = v_{2i} = 0$; there is a perfect thermal contact: $T_1 = T_2 = T_w$. In this heat flux $q_i = -\lambda_i \frac{\partial T_i}{\partial n} = \alpha_i \left( T_n - T_i^* \right)$ ($i = 1, 2$), here $n$ – outward normal to the surface of the cylinder, $T_i^*$ – temperature $i$-phase away from the cylinder (average temperature), $\alpha_i$ – required local convective heat transfer coefficient.
At the initial time $\tau=0$ the fluid instantly begins to move, $v_{i1} = w$, $v_{i2} = 0$, $p = p_0$, $T_i = T_0$, $\varepsilon = \varepsilon_0$.

3. Results

It is established that in the case of a stationary cylinder at the picture wrapping his dispersed mixture have a significant impact width of the jet; the concentration, size, material density of the particles (droplets) of the impurities (figure 1).

Comparing the data shown in figure 1 a, b, convinced that if the stream of particles impurities narrow, the particles are localized outside the cylinder in distinct vortex clusters. At the same mass concentration of particles $\gamma$, the Stokes number $\text{Stk} = u_\infty \rho_p d_p^2 / 18 \mu D$ ($u_\infty$ – flow velocity, $D$ – the cylinder diameter; $\rho_p, d_p$ are the density, the particle diameter of the impurities), when the size of the streamlined body is less than the width of the jet, such localization is not observed.

If the width of the jet, the diameter of the cylinder, the mass concentration of particles are unchanged (figure 1 b, c), the increasing of the number of Stokes (density, particle size) leads again to their localization in the body, but in some different form (figure 1 c). Finally, the increase both the concentration and the number of Stokes radically changes the picture (figure 1 d). Moreover, at small Stokes numbers after the contact with the surface of the cylinder of solid particles roll down the windward side downstream, towards the vortex stagnant zone; and a high bounce off the surface, in the region behind the cylinder gets there is an extremely amount of them (figure 2).

![Figure 1. Module of the particle velocity of the impurities a) Stk=0.1, $\gamma = 23\%$; b) Stk=0.1, $\gamma = 23\%$; c) Stk=1.0, $\gamma = 23\%$; d) Stk=2.0, $\gamma = 60\%$.](image1)

![Figure 2. The position of the particle in the flow at the Reynolds number Re=100, $\beta = \rho_p / \rho = 4.6 \times 10^5$, $\gamma = 0.23$: Stk=0.1 (a); Stk=1.0 (b).](image2)
At constant other parameters, depending on the relative density $\beta$ of the volumetric, the concentration of particles $\alpha_2$ on the surface of the cylinder varies considerably (figure 3). It is seen that when $\beta > 1$ (figure 3, curve 1) the concentration of the particles of impurities takes the maximum value in the frontal point of the cylinder, then the flow direction is reduced at the point of separation of the flow reaches its minimum. On the surface of the aft portion of the cylinder, the impurity concentration of particles is high enough, varies along the surface slightly. Thus, the surface area of the cylinder $L = \frac{L}{\pi} \cdot \frac{\pi}{2}$ can be divided in three distinctive areas: frontal and stern area where a high concentration of impurities, and an intermediate region with a low concentration of impurity particles. At low relative density of the material particles of impurities $\beta < 1$ (figure 3, curve 2) their concentration along the surface remains almost unchanged.

It follows that to obtain the deposition of a uniform coating on the surface of a stationary circular cylinder we should strive to ensure that the material density of the particles (droplets) would be less than the density of the carrier medium.

It should be noted that in the case of the rotating cylinder instantaneous distribution of impurity concentration on the surface is approximately the same as on the surface of the stationary cylinder. However, due to the rotation of the cylinder, the thickness of the coating layer, a lot depends on the nature of the distribution of particles on the surface.

![Figure 3. Changes in the concentration of the impurity particles along the surface of the cylinder ($\bar{L} = \frac{l}{D}$): $1 - \beta = 21.5$, $2 - \beta = 0$.](image)

In addition to the thickness of the coating an important indicator of its quality is the continuity of the material and adhesion, characterized by the bond strength between the coating layer and the substrate. Both figures largely depend on the thermal processes occurring in the contact area of the rolling flow of the mixture and the treated body.

It is known that the addition of the gas solids improves the heat transfer of streamline bodies due to the decrease of the viscous sublayer. Indeed, if a circular cylinder is stationary, the local Nusselt number presented with increasing mass flow rate of particles of impurities increases (figure 4). In this case, as expected, the greatest heat transfer occurs from the windward side of the cylinder, in the vicinity of the separation point of the flow it is at a minimum, in the aft portion of the cylinder it increases.
The thermal interaction between heterogeneous environment with a streamlined body on the heat exchange rate, defined medium on the surface of the Nusselt number $\bar{\text{Nu}}$, is affected by the fact that near the surface concentration of impurities can increase substantially, which will change the conductivity of the mixture, the nature of the flow. From the calculations it follows that when the Reynolds number $\text{Re}=100$

$$\bar{\text{Nu}} = \bar{\text{Nu}}_0 + 1.7 \times 10^6 \frac{\gamma_0 (1-0.3/\text{Stk})}{\text{Stk} (\beta + (1-\beta)\gamma_0)},$$

where $\bar{\text{Nu}}_0$ is the average Nusselt number around the cylinder in a uniform stream.

It was also established that at a fixed Reynolds number, the change in specific heat of the material of the particles does not have significant effect on heat transfer of a cylinder. However, the thermal conductivity of the material particles of the heat exchange cylinder with the flow of the mixture varies significantly, and the stronger, the larger the Reynolds number (figure 5).

The influence of particle size on heat transfer streamlined cylinder with the environment at a fixed particle concentration impurity at the entrance to the settlement area is also largely depends on the density and thermal conductivity of the material particles. If the density of the material particles is less than or close to the density of the carrier medium, and the thermal conductivity is greater, then on the condition that the particle diameter increases, the heat transfer deteriorates.

In the case of a rotating cylinder with respect to heat transfer with two-phase flow can distinguish three characteristic change interval dimensionless speed of the surface of the cylinder $\alpha = 0.5\omega D/\nu_\infty$. 

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**Figure 4.** Local Nusselt number at $\text{Re}=100$, $\text{Pr}=0.75$, $\text{Stk}=0.6$, $\beta=4.6 \times 10^3$: mass flow rate of particles kg/s ($\bullet$), kg/s ($\Delta$).

**Figure 5.** The dependence of the average Nusselt number from Reynolds number of the cylinder for different materials of the particles: 1 – gold, 2 – aluminum; 3 – water without particles; 4 – polypropylene.
(ω is the angular speed of rotation of the cylinder): 0<α<3.0, 3.0<α<4.3, 4.3<α<4.9, (figure 6). When α<3.0 the throw of an impurity in a homogeneous viscous fluid flow several intensifies the heat exchange, the Nusselt number Nu increases by approximately 15%. A further increase in the speed of rotation of the cylinder leads to the fact that when 3.0<α<4.3 the heat transfer degrades significantly Nu reduced by almost half. Subsequent increase α (4.3<α<4.9) decrease rate of heat transfer decreases, the Nusselt number to flow with the admixture approach Nusselt number is presented for homogeneous flow.

The identified patterns are of great practical importance, because they allow to evaluate the impact on the quality of the coatings of the main indicators of the sprayed materials, enable rational selection of technological modes.

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