THE BRIGHTEST POINT X-RAY SOURCES IN ELLIPTICAL GALAXIES AND THE MASS SPECTRUM OF ACCRETING BLACK HOLES

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ABSTRACT

We propose that the shape of the upper-end X-ray luminosity function observed in elliptical galaxies for point sources is a footprint of the black-hole (BH) mass spectrum among old X-ray transients formed in the galaxies. We show that this underlying BH mass spectrum is modified by a weighting factor that is related to the transient duty cycle and it generally depends on the BH mass and XRB donor type (main-sequence, red-giant, or white-dwarf donors). A duty cycle that depends on the binary mass-transfer rate relative to the critical rate for transient behavior is most probably favored. We also find that the derived BH mass spectrum slope depends on the strength of angular momentum loss due to magnetic braking for main-sequence donors. More specifically, we find that, for “standard” magnetic braking, BH XRBs with red-giant donors dominate the upper-end XLF; for weaker magnetic braking prescriptions main-sequence donors are found to be dominant. In both cases the BH mass spectrum has a differential slope of $\sim 2.5$ and an upper BH mass cut-off at $\sim 20 M_\odot$ is needed to understand the very brightest of the BH XRBs in elliptical galaxies. We expect that our result will help to constrain binary population synthesis models and the adopted relations between black holes and the masses of their progenitors.

Subject headings: galaxies: elliptical – methods: statistical – X-rays: binaries

1. INTRODUCTION

\textit{Chandra} has revolutionized the study of point X-ray sources in the nearby Universe. The majority of these are interpreted to be X-ray binaries (XRBs; for a general review on \textit{Chandra} performance see Weisskopf et al. 2003, for extragalactic X-ray binaries see, e.g., \textbf{Kim & Fabbiando 2004} \textbf{Jordán et al. 2004}). Elliptical galaxies out to the Virgo cluster have now been studied and have surprised us with the large number (typically $\sim 100$ per galaxy) of detectable point X-ray sources down to X-ray luminosities of about $10^{37} \text{ergs s}^{-1}$. Two main population characteristics have attracted considerable attention so far: (i) the X-ray luminosity function (hereafter XLF) that may or may not exhibit a break at about $4 - 5 \times 10^{38} \text{ergs s}^{-1}$ (for a recent update see \textbf{Kim & Fabbiando 2004}), (ii) the high fraction of sources coincident with identified globular clusters (GCs) in ellipticals.

The shape of the XLF has been debated since the first observations of ellipticals were reported. \textbf{Sarazin et al. 2003} identified a shape that required two power laws with a “break” or a “knee” at $\sim 3.2 \times 10^{38} \text{ergs s}^{-1}$. \textbf{Kim & Fabbiando 2003} argued that the break may result from biases affecting the detection threshold of the data. In the following few years longer exposures became possible and more and more ellipticals were added in the observed sample with low enough sensitivity. The current situation is probably best summarized in \textbf{Kim & Fabbiando 2004}. They analyzed a large sample of elliptical galaxies with varying sizes of point source samples, and they concluded that: although XLFs of individual galaxies do not require a broken-power-law fit, the combined sample of sources from all the galaxies considered shows a statistically significant requirement for two power laws and a break at $5 \pm 1.6 \times 10^{38} \text{ergs s}^{-1}$. They found the best-fit slope of the lower end of the XLF to be $\beta = 1.8 \pm 0.2$ and the best-fit slope of the upper end to be $\beta = 2.8 \pm 0.6$. It is important to note that the break location is consistent with the Eddington luminosity for a $1.9 \pm 0.6 M_\odot$ neutron star (NS) accreting helium-rich material (for hydrogen rich donor this value is as large as $3.2 \pm 1 M_\odot$). In what follows we consider the results of the \textbf{Kim & Fabbiando 2004} study as representing our current observational understanding of the XLF in ellipticals. We address the question of the interpretation of this understanding and what it implies about the properties of the sources contributing to the observed XLFs.

Large fractions ($20\% - 70\%$) of the point sources in ellipticals have been reported to be associated with globular clusters (see, e.g., \textbf{Sarazin et al. 2003} and references therein). These high fractions have led to the suggestion that all point X-ray sources seen currently in ellipticals have been formed through stellar interactions and that sources that are not associated with GCs have originated in GCs and have either been ejected or the parent GCs have been destroyed by the galaxian tidal forces. As much interesting as this suggestion is, it raises the question: why would the field stellar population of ellipticals not lead to XRB formation as it has occurred in the Milky Way, for example? One could speculate that the field population is just too old and the XRBs that were formed at some point have completed their X-ray emitting life. However, such a speculation is inconsistent with the expectation that XRBs with low mass donors can live for several Gyrs as the donors lose mass and the binaries enter a transient phase. Such systems would become detectable as bright X-ray sources during the disk outbursts \textbf{Pablo & Bildsten 2002}. Moreover, it has recently been pointed out that the high rate of source coincidence with GCs actually appears consistent with some of the sources having been formed in the field \textbf{Juciet 2007}, although
the result may sensitively depend on the definition of GC concentration in ellipticals. Most importantly for the present study, of the bright sources above the XLF break only a very small fraction is associated with GCs (e.g., only one source in the Virgo cluster sample as reported by Jordán et al. 2004).

In a recent Letter Bildsten & Deloye (2004 hereafter DB) have suggested an explanation that couples the two population characteristics: the XLF shape and the source coincidence with GCs. They identify the point X-ray sources as ultra-compact binaries (UCBs) that form predominantly in GCs. They are neutron stars accreting from low-mass He or C/O white dwarfs and they contribute to the ellipticals XLF early in their lifetime when they are still bright. Their association with GCs is important in replenishing the population through tidal interactions and allowing a significant number of sources in this bright phase, even though there is no ongoing star formation in ellipticals nor in GCs. DB estimate the number of sources observed and conclude that the model XLF slope is in agreement with the slope below the break as derived by Kim & Fabbiano (2004). The association of the XLF break with the NS Eddington limit for He-rich accretion is also consistent with this interpretation. However, the upper end of the XLF (at luminosities in excess of the break location) are not easy to interpret. Deloye & Bildsten suggest that some of the NS ultra-compact sources can reach super-Eddington luminosities. However luminosities in excess of $10^{39}\text{erg s}^{-1}$ are very difficult to explain with NS accretors. Therefore the origin of the upper-end slope is not naturally connected to NS UCBs formed in GCs.

In this paper we address the question of the upper-end XLF slope and its origin. We consider the previously made suggestion Sarazin et al. 2000 that the XLF above the break at $5 \times 10^{38}\text{erg s}^{-1}$ is populated by XRBs with black hole (BH) accretors. Given the difficulty of GCs harboring a significant number of BH-XRBs (see Kalogera et al. 2004 and references therein), we suggest that the vast majority of these BH-XRBs are part of the galactic-field stellar population in ellipticals. Most of donors in these binaries are of low-enough mass that the XRBs are expected to be transient and therefore they populate the XLF only during disk outbursts when they typically emit at the Eddington luminosity for their BH mass. We further suggest that the slope of the upper XLF is a footprint of the BH mass spectrum in the BH XRBs under consideration. We present analytical derivations that demonstrate this link and allow us to infer the underlying BH mass spectrum consistent with the current upper-end XLF slope (Kim & Fabbiano 2004). We also show that given the current observations it is possible to constrain the strength of magnetic braking acting in these XRBs, the type of BH donors, as well as the transient duty cycle. This analysis is presented in §2 and 3. We conclude with a discussion of our results and possible connections to population synthesis calculations (§4).

2. BLACK HOLE X-RAY BINARIES IN ELLIPTICALS

We consider XRBs that could possibly populate the part of the observed XLF above the reported break at $4 - 6 \times 10^{38}\text{erg s}^{-1}$, and therefore we focus on BH accretors (masses in excess of $2 - 3M_\odot$). Given the current estimates for the ages of stellar populations in ellipticals (in their majority 8 to 12 Gyr, although some estimates are slightly shorter than 5 Gyr; see Ryden et al. 2001 Temi et al. 2007), we expect that donor masses are lower than $\sim 1 - 1.5M_\odot$. Given these mass ratios and the properties of similar observed systems in the Milky Way (i.e., soft X-ray transients), these BH XRBs are expected to be transient X-ray sources (McClintock & Remillard 2003; for review of BH X-ray binaries in the Milky Way, see), where mass transfer is driven by the Roche-lobe filling donor. Given the above upper limit on the donor mass for ellipticals, we expect that there will be three different types of low-mass donors: (i) Main Sequence (MS) stars, (ii) Evolved or Red Giant Branch (RG) stars, and (iii) White Dwarf (WD) donors. Each of these sub-populations of BH XRBs will have different typical mass-transfer rates and binary property distributions, and therefore we examine them separately in our analysis that follows.

2.1. TRANSIENT X-RAY SOURCES

We adopt the current understanding for the origin of transient behavior in XRBs (for a recent review, see King 2005). To identify transient systems in our modeling we consider the typical mass-transfer (MT) rate associated with each type of XRB donor ($\dot{M}$) and compare it to the critical MT rates for transient behavior ($\dot{M}_{\text{crit}}$): if $\dot{M} < \dot{M}_{\text{crit}}$, then the accretion disk is expected to be thermally unstable and the binary system is assumed to be a transient X-ray source. For hydrogen-rich donors, we adopt the $\dot{M}_{\text{crit}}$ value derived by Dubus et al. (1999), and for helium or carbon-oxygen donors, we adopt the value derived by Menou et al. (2002).

To account for the contribution of transient sources in the XLF among any persistent sources, assumptions about the XRB luminosity at outburst and the transient duty cycle need to be made.

When a XRB is identified as transient, we assume that during the disk outburst the X-ray luminosity $L_X$ is equal to the Eddington luminosity $L_{\text{Edd}}$ associated to the BH accretor:

$$L_{\text{Edd}} = \frac{4\pi c G M_{\text{BH}}}{\kappa} = 5 \times 10^{37} \frac{m_{\text{BH}}}{\kappa} \text{[ergs s}^{-1}] \quad (1)$$

where $m_{\text{BH}}$ is the accretor mass in $M_\odot$ and $\kappa$ is the opacity of the accreting material. We adopt electron scattering opacities equal to 0.32 and 0.19 for hydrogen and helium or carbon-oxygen rich material, respectively. We note that of the 15 confirmed transient BH XRBs in our Galaxy, 3 appear to reach possibly super-Eddington luminosities (McClintock & Remillard 2003) at outburst (although distance estimate uncertainties cannot be ignored). Two of them, V4641 Sgr and 4U 1543-47, have early-type donors. Such donors are not present in ellipticals when they are still bright. Their association with GCs is important in replenishing the population through tidal interactions and allowing a significant number of sources in this bright phase, even though there is no ongoing star formation in ellipticals nor in GCs. DB estimate the number of sources observed and conclude that the model XLF slope is in agreement with the slope below the break as derived by Kim & Fabbiano (2004). The association of the XLF break with the NS Eddington limit for He-rich accretion is also consistent with this interpretation. However, the upper end of the XLF (at luminosities in excess of the break location) are not easy to interpret. Deloye & Bildsten suggest that some of the NS ultra-compact sources can reach super-Eddington luminosities. However luminosities in excess of $10^{39}\text{erg s}^{-1}$ are very difficult to explain with NS accretors. Therefore the origin of the upper-end slope is not naturally connected to NS UCBs formed in GCs.
At present there are no strong constraints on the duty cycles either from observations or from theoretical considerations. Among known Galactic X-ray transients, a typical duty cycle of a few \% is favored for hydrogen donors (Tanaka & Shibazaki 1990). To our knowledge, there are no data on duty cycles for transients with a WD companion. In what follows we investigate how plausible duty cycle assumptions affect the upper-end XLF shape. In particular, we consider two specific cases: one of constant duty cycle equal to $\eta = 0.01$; another of a variable (dependent on MT rates) duty cycle equal to

$$\eta = 0.1 \frac{M_d}{M_{\text{crit}}}.$$  

We also show that the latter prescription for the duty cycle leads to very small duty cycle values for WD donors (see §2.3).

### 2.2. Main Sequence Donors

Mass transfer in BH XRBs with hydrogen-rich, low-mass MS donors still on the Main Sequence is expected to be driven by angular momentum losses due to magnetic braking (MB) and possibly gravitational radiation (GR). In the case of conservative mass transfer (Verbunt 1993)

$$\frac{\dot{J}_{\text{orb}}}{\dot{J}_{\text{mb}}} = \frac{M_d}{M_d} \left( \frac{5}{6} + \frac{n - 2}{M_d} \right),$$  

where $n$ is the radius-semi-major exponent for the donor. For a low-mass MS star:

$$r_d \simeq m_d,$$

where $r_d = R_d/R_\odot$ and $m_d = M_d/M_\odot$ are the donor stellar radius and mass in solar units. Therefore $n \equiv d \ln R_d/d \ln M_d$ is equal to $\simeq 1$ for MS donors.

According to general relativity the rate of angular momentum loss due to GR is given by:

$$\frac{\dot{J}_{\text{orb}}}{\dot{J}_{\text{orb}}} = -\frac{32G^2}{5c^5} \frac{M_d (M_d + M_d)}{A^4}$$

$$= -2.6 \times 10^{-17} s^{-1} \frac{m_d (m_d + m_d)}{a^4},$$  

where $a$ is the orbital semi-major axis in units of solar radius. For a mass ratio $q \equiv M_d/M_{\text{BH}} < 0.8$ (Paczynski 1971) and using the mass-radius relation eq. (4):

$$a = \frac{1}{0.46} \frac{m_{d}^{2/3}}{(m_b + m_d)^{1/3}}.$$

We consider two derivations of the angular momentum loss rate due to magnetic braking: (i) the Skumanich law based on the empirical relation for slowly rotating stars adopted from Rappaport et al. 1983 (RVJ), and (ii) the revised law based on X-ray observations of faster rotating dwarfs adopted from Ivanova & Taam 2003 (IT):

$$\begin{align*}
\dot{J}_{\text{mb}}^{\text{RVJ}} &= -3.8 \times 10^{-30} M_d R_d^4 (R_d / R_\odot)^2 \Omega^3 \\
\dot{J}_{\text{mb}}^{\text{IT}} &= -6 \times 10^{-30} (R_d / R_\odot)^4 \left( \frac{\Omega_4}{15} \right) \left( \frac{\Omega_4}{15} \right),
\end{align*}$$

where $\Omega$ is the stellar angular velocity which is equal to the binary orbital velocity assuming the star is in full synchronization with the binary orbit, $\Omega_\odot = 5 \times 10^{-6}$ s$^{-1}$ is the Sun’s angular velocity, and $\Omega_\odot = 10\Omega_\odot$. Using Kepler’s law the above are re-written as:

$$\begin{align*}
\dot{J}_{\text{mb}}^{\text{RVJ}} &= -7.2 \times 10^{-15} \frac{m_d^4 (m_d + m_d)^2}{m_{\text{BH}} a^5}, \\
\dot{J}_{\text{mb}}^{\text{IT}} &= -2.7 \times 10^{-17} \frac{m_d^3 (m_d + m_d)^{1.15}}{m_{\text{BH}} a^5}. 
\end{align*}$$

In XRBs with BH masses $\geq 3M_\odot$ and MS donor masses $\leq 1.0M_\odot$, it is $\dot{J}_{\text{orb}}^{\text{RVJ}}$ and $\dot{J}_{\text{orb}}^{\text{IT}}$. The lifetime of the MS-BH XRBs is much longer in the latter case.

The critical MT rate, below which the accretion disk becomes unstable for irradiated disks (Dubus et al. 1999) is:

$$\dot{M}_{\text{crit}} = -1.5 \times 10^{15} m_{-0.4} \left( \frac{R_{\text{disk}}}{10^{10}} \right)^{2.1} [\text{g s}^{-1}].$$

From eq. (3), (10) and (11) it can be shown numerically, that for the RVJ MB law and for a given donor mass, there is a BH mass $M_{\text{PT}}$ of $\sim 5 M_\odot$ that separates BH-MS systems into persistent ($M < M_{\text{PT}}$) and transient ($M > M_{\text{PT}}$). From more detailed binary evolutionary calculations using the stellar evolution and MT code described in Ivanova & Taam 2003, we find that this boundary is about $10 M_\odot$ (see Fig. 4). For BHs less massive than this critical mass, the MT rates driven due to the RVJ type of MB turn out to be $0.01 - 0.25$ of the Eddington rate. The maximum resultant persistent X-ray luminosity is $\lesssim 10^{38} \text{erg s}^{-1}$, i.e., below the bright $L_X$ range we consider here. On the other hand, the Eddington luminosity that corresponds to the critical BH mass for transient behavior is $\sim 1.5 \times 10^{39} \text{erg s}^{-1}$. This is comparable to the highest luminosity seen currently in XLFs of ellipticals (Kim & Fabbiano 2004), and therefore these systems cannot contribute significantly to the observed XLFs.

For the RVJ MB prescription, we also find that all BH-MS systems become transient once a donor is less massive than $0.15 - 0.3 M_\odot$. This low mass donor is out of thermal equilibrium and is significantly expanded ($\simeq 3x$) compared to a normal MS star of the same mass. In the case of the MT dependent duty-cycle $\eta$ is about a few \%.

We note, however, that applicability of MB for these stars is very questionable, as it is generally accepted that MB does not operate in fully convective stars, which are less massive than $0.35 M_\odot$. Such “eroded” low-mass MS donors do not become fully convective like stars of the same mass with no prior MT evolution, as they are not in thermal equilibrium. However our detailed MT calculations show (Fig. 4) that the radiative core is extremely small and therefore applying angular momentum loss due to MB is not reasonable. We therefore exclude the possibility that BH-MS binaries evolving according to the RVJ MB prescription can contribute to XLFs of ellipticals. We note that if MB is interrupted, then eventually GR will drive the mass transfer and therefore transfer rates and duty cycles will be somewhat different; however here we concentrate on mass transfer driven by MB.

In the case of the IT MB prescription, BH-MS systems are transient for all BHs masses $M_{\text{BH}} > 3M_\odot$, and for all low-mass MS donors. The reason is that the IT MB is weaker; consequently the donors are mildly out of thermal equilibrium and the mass transfer rates are lower...
MT evolutionary simulations we find that
line shows the donor mass evolution.

\begin{equation}
\dot{M}_{\text{crit}} \approx (m_{\text{BH}} + m_d)0.465 \ [\text{g s}^{-1}] .
\end{equation}

\textbf{2.3. Red Giant Donors}

For orbital periods more than about a day, MT occurs when the low-mass donor is a subgiant or a giant. The driving force is the nuclear expansion of the donor, and a simple analytic prescription for the MT is (Webbink, Rappaport, Savonije 1983; Ritter 1999; see also King 2005):

\begin{equation}
\dot{M}_{\text{rg}} = -3.4 \times 10^{15} \alpha^{1.4} \frac{m_d^{1.47}}{(m_{\text{BH}} + m_d)^{0.465}} \ [\text{g s}^{-1}] .
\end{equation}

King et al. (1997) have shown that such wider XRBs are transient (based on the criterion \cite{11}), regardless of the BH mass.

\textbf{2.4. White Dwarf Donors}

A typical WD mass-radius relation is (see, e.g., Rappaport et al. 1987):

\begin{equation}
r_d = 0.0128 m_d^{-1/3}
\end{equation}

Using (13) and assuming that the mass of WD is much smaller than a BH mass, we can show that

\begin{equation}
a = 0.0278 m_d^{-1/3} \left( \frac{m_d}{m_{\text{BH}}} \right)^{-1/3}
\end{equation}

We consider again conservative mass transfer \cite{3} but without any MB losses, adopting $n = -1/3$ and assuming that $m_d \ll m_{\text{BH}}$. Then

\begin{equation}
\dot{M}_d \approx -\frac{32 G^3 M_{\text{BH}}^2 M_d}{3.5 c^5 A^4} = -7.9 \times 10^{16} \frac{m_{\text{BH}} M_d^2}{a^4} \ [\text{g s}^{-1}]
\end{equation}

We substitute here eq. (13) and then have

\begin{equation}
\dot{m}_d = -2 \times 10^{-3} m_d^{-3/4} m_{\text{BH}}^{-1/4} M_\odot \text{yr}^{-1}
\end{equation}

The critical MT rate for He-rich donors Menou et al. (2002):

\begin{equation}
\dot{M}_{\text{crit}} = -5.9 \times 10^{16} m_{\text{BH}}^{-0.87} \left( \frac{R_{\text{disk}}}{10^{10}} \right)^{2.62} \ [\text{g s}^{-1}]
\end{equation}

For a large mass ratio $q_{\text{BH}} = m_{\text{BH}}/m_d$ the Roche lobe of the accretor is $r_{\text{RL}} \simeq 0.7 a$ and

\begin{equation}
r_{\text{disk}} \simeq 2/3 r_{\text{RL}} = 0.013 m_d^{-2/3} m_{\text{BH}}^{1/3}
\end{equation}

\begin{equation}
\dot{m}_{\text{crit}} = -1.7 \times 10^{-12} m_d^{-1.74} M_\odot \text{yr}^{-1}
\end{equation}

BH-WD binaries will be transient if

\begin{equation}
\frac{\dot{m}_d}{\dot{m}_{\text{crit}}} = 1.2 \times 10^{0.64} m_d^{-0.4} m_{\text{BH}}^2 \leq 1
\end{equation}

Therefore the maximum donor mass that leads to transient behavior in BH-WD binaries is:

\begin{equation}
m_{\text{tr}} = 0.038 m_{\text{BH}}^{-0.1}
\end{equation}

The time interval in Gyr needed for the WD donor mass to evolve from $m_d^{-11/3}(T_1)$ to $m_d^{-11/3}(T_2)$ is (using eq. 10):

\begin{equation}
m_{\text{tr}} = \frac{0.038}{m_{\text{BH}}^{-0.1}}
\end{equation}

It is rather unlikely that persistent sources with a BH accretor and a hydrogen-rich, low-mass MS donor populate at any significant fraction the upper-end XLF of ellipticals.

compared to the RVJ MB case. Using again detailed MT evolutionary simulations we find that $\dot{M}/\dot{M}_{\text{crit}} \simeq 0.25 \pm 0.15$ (see Fig. 2). Therefore, in the case of the MT dependent duty-cycle $\eta$ is again about a few %. In both cases (RVJ MB and IT MB), the value of the DC is consistent with observations for BHs of different masses, though the transiency occurs at very different donor masses.

We conclude that, regardless of the specific MB law,
\[
T_2 - T_1 = \frac{3}{22} \times 10^{-6} m_{BH}^{-2/3} \times \\
\left( m_d(T_2)^{-11/3} - m_d(T_1)^{-11/3} \right)
\]  
(22)

Consequently and using eq. (22), we can find that the time a BH-WD system spends in the persistent state is \( t_{\text{pers}} \approx 20 \times 10^6 m_{BH}^{0.3} \) yr, i.e., it very weakly depends on the accretor mass. We note that through this persistent phase the MT rate will be comparable or higher to the Eddington limit only for a very short time fraction, \( t_{\text{eddd}} \approx 2 \times 10^6 m_{BH}^{-0.9} \) yr \(^{-1}\).

The evolution of BH-WD systems with C/O WD companions is rather similar. The critical MT rate (using Menou et al. 2002) is

\[
\dot{m}_{\text{crit}} = -9.4 \times 10^{-13} m_d^{-1.47}[M_\odot \text{yr}^{-1}]
\]  
(23)

and

\[
\dot{m}_{\text{tr}} = 0.03 m_{BH}^{-0.1}.
\]  
(24)

The time that a BH-WD system with a C/O rich donor spends in the persistent state is also not very long, \( t_{\text{pers}} \approx 50 \times 10^6 m_{BH}^{-0.3} \) yr.

We conclude that BH-WD binaries that contribute to the current upper-end XLFs of ellipticals are expected to be transient sources\(^1\).

3. BLACK HOLE MASS SPECTRUM OF TRANSIENTS IN OUTBURST

In the previous section we have shown that the upper-end XLF of ellipticals is dominated by transient BH XRBs possibly with a variety of donors: MS (for the case of the IT MB prescription) and RG stars, and WD donors with masses lower than \( \sim 0.035 M_\odot \). All these systems contribute to the XLF only when in outburst, when their \( L_X \approx L_{\text{Edd}} \propto M_{BH} \). Consequently the slope of the upper-end XLF is a footprint of the BH mass distribution of accretors in the \textit{contributing} BH XRBs. These contributing BH XRBs are just a sub-set (those in outburst) of the true population of BH XRBs in ellipticals determined by the duty cycle of BH transients binaries. For the general case of a transient duty cycle that is dependent on the BH accretor mass, the \textit{differential} XLF \( n(L)_\text{obs} \) and the underlying BH mass distribution in XRBs \( n(m_{BH}) \) are connected by:

\[
n(L)_\text{obs} = n(m_{BH}) \times W(m_{BH}),
\]  
(25)

where \( W(m_{BH}) \) is a \textit{weighting} factor related to the dependence of the transient duty cycle on \( m_{BH} \).

The observed slope of the \textit{differential} upper-end XLF is \( \alpha_d = 2.8 \pm 0.6 \): \( n(L)_\text{obs} \propto L_X^{-\alpha_d} \) (the slope of the cumulative upper-end XLF reported by Kim & Fabiano 2004 is \( \alpha_c = 1.8 \pm 0.6 \)). Assuming that \( n(m_{BH}) \propto m_{BH}^{-\beta} \), and \( W(m_{BH}) \propto m_{BH}^{\gamma} \), the slope characterizing the underlying BH mass distribution in XRBs is:

\[
\beta = \alpha_d - \gamma.
\]  
(26)

For the standard assumption of a constant duty cycle, \( \beta = \alpha_d = 2.8 \pm 0.6 \). In the following subsections we derive \( W(m_{BH}) \) and \( \gamma \) for all three types of donors in the case of a duty cycle dependent on MT and the \( m_{BH} \) (see also eq2).

3.1. Main Sequence Donors

In what follows we estimate the typical duty cycle for BH-MS transients averaged over the possible distribution of donor masses. In the case of the IT MB prescription, the angular momentum loss rate due to GR is comparable or even more important than MB for all BH masses above 3 \( M_\odot \) and donor masses \( \lesssim 1.0 - 1.2 M_\odot \). The MT timescale during the transient phase is longer than the donor’s thermal timescale when the donor is \( \gtrsim 0.25 M_\odot \). For this range the donor is in thermal equilibrium and the approximation for the mass-radius dependence eq. (4) can be used. Using eq. (3), (4) and (21), we find:

\[
\dot{m}_{\text{crit}} \approx 0.054 m_{BH}^{0.4} q_{BH}^{-2/3} \log(1 + 0.3 q_{BH}^{-1/3} + 0.6)^{2.1} m_{d}^{-2}(1 + q_{BH}^4/(4/3 - 1/q_{BH}^4)),
\]  
(27)

where \( q_{BH} = m_{BH}/m_d \) is the mass ratio. For \( q_{BH} \gg 1 \) we have

\[
\dot{m}_{\text{crit}} \approx 0.014 m_{BH}^{0.4} m_d^{-2}
\]  
(28)

At smaller donor masses, the donor is out of the thermal equilibrium and its radius is about twice bigger than predicted by eq. (4). We find:

\[
\dot{m}_{\text{crit}} \approx 0.0004 m_{BH}^{0.4} m_d^{-2}
\]  
(29)

As discussed in § 2.2 for IT MB, a BH-MS system is transient throughout the MT phase. The MT rates are well below the Eddington limit for the BH mass and therefore we assumed that MT is fully conservative; i.e., \( M_d + M_{BH} = M_{\text{tot}} \) is constant with time. Assuming a \textit{flat} current mass distribution for donors (\( \partial N/\partial m_d = \text{const} \)) and integrating eq. (28) for \( m_d \) from \( \sim 0.25 \) to \( 1 M_\odot \), we find that at present the probability that a system with a BH accretor of \( m_{BH} \) is in outburst and therefore contributes to the upper-end XLF is:

\[
W(m_{BH}) = \int_{0.25}^{m_{BH}} \frac{\partial N}{\partial m_d} \, dm_d \approx 0.05 m_{BH}^{0.4}
\]  
(30)

We note that this result is valid only for large mass ratio \( q_{BH} \). The contribution of BH-MS system when the donors have masses \( \lesssim 0.25 M_\odot \) (systems where the donor is out of the thermal equilibrium) is less significant. We also find that \( W(m_{BH}) \) is a very weak function of the maximum donor mass (which is the mass of the turn-off MS star in the ellipticals), and therefore it is not sensitive to the elliptical age.

We have further examined the effect of mass accretion on the BH mass spectrum for a more general case of BH-MS mass ratios. With a simple Monte Carlo code, we tested the behavior of \( W(m_{BH}) \) assuming a flat birth rate and flat mass distribution for donors at the beginning of

\(^1\) Although MT is non-conservative during the super-Eddington accretion, and eq. (5) formally should not be applied, this result is well consistent with the detailed calculations that take into account non-conservative MT.

\(^2\) This would not be true if BH-WD binaries continuously formed, but this is not possible in the galactic field of ellipticals and is not even expected in globular clusters, since BHs tend to dynamically separate from the rest of the cluster and eject one another (Kulkarni et al. 1992; Sigurdsson & Hernquist 1993; Watters et al. 2000).
We can then calculate the probability for a BH-WD system to become a transient BH-WD system within a short interval of elliptical ages \(T_{\text{start}}\) to \(T_{\text{fin}}\) (in Gyrs) several Gyrs ago when star formation was still occurring in the elliptical; and (ii) \(T - T_{\text{fin}} > t_{\text{pers}}, \) i.e., the binary is a transient at time \(T\). The latter assumption is well justified given the short duration of the persistent phase (see §2.4). We further adopt a constant BH-WD formation rate between \(T_{\text{start}}\) and \(T_{\text{fin}}, \) i.e., \(\frac{\partial m}{\partial t} = \text{const.}\) The probability then is expressed by the duty-cycle weighting factor at \(T\) for a given BH mass:

\[
W(T; m_{\text{BH}}) = \frac{\int_{m_{\text{crit}}}^{m_{\text{BH}}} \eta \frac{\partial N}{\partial m} dm}{\int_{m_{\text{crit}}}^{m_{\text{BH}}} \frac{\partial N}{\partial m} dm} = 0.1 \frac{\int_{m_{\text{crit}}}^{m_{\text{BH}}} \frac{\partial N}{\partial m} \frac{\partial t}{\partial m} dm}{\int_{m_{1}}^{m_{2}} \frac{\partial N}{\partial m} dm}
\]

Here \(m_{1} = m(T - T_{\text{start}}; m_{\text{BH}}) = m(t_{1}; m_{\text{BH}})\) and \(m_{2} = m(T - T_{\text{fin}}; m_{\text{BH}}) = m(t_{2}; m_{\text{BH}})\) and \(m_{\text{crit}} > m_{1} > m_{2} > m_{1} \). Then, using eqs. (10), (11) and (23), we obtain:

\[
W(T; m_{\text{BH}}) = 1.6 \times 10^{-4} \frac{m_{\text{BH}}^{-0.5} t_{1}^{-7/4}}{1 - (t_{2}/t_{1})^{-3/4}}
\]

Therefore for WD donors \(\gamma = 0.5\).

4. ACCRETING BLACK HOLE MASS SPECTRUM

So far we have derived the dependence of the duty-cycle weighting factor \(W\) (eq. 25) on the accreting BH mass for the different types of BH donors. In order to make progress and derive constraints on the slope \(\beta\) of the underlying accreting BH mass spectrum we need to examine which of the possible donor populations dominate the observed upper-end XLF under what conditions. The answer to this question requires large-scale population synthesis models that are not part of the scope of this paper. However, as is shown below, we can use a number of different arguments and pieces of evidence to derive tentative constraints.

For RG donors we find that in the case of a MT-dependent duty cycle \(\eta\) is about an order of magnitude smaller than for MS donors (see eq. 21). We also note that among known BH X-ray transients in our Galaxy, the ratio of transient BH-RG systems to transient BH-MS systems is about 1:2 (see Table 4.1 in McClintock & Remillard 2006). Consequently we conclude that BH-RG transients cannot be important contributors to the upper-end XLF of elliptical galaxies for the case of the MT-dependent duty cycle as defined by \(\eta\). Only in the case of the constant, MT-independent (and therefore donor and BH-mass independent) duty cycle we expect BH-RG transients to be a significant population of the observed upper-end XLF.

Let us consider the case when the number of transient BH-RG systems exceeds the number of transient BH-MS system in a way that their contribution becomes comparable at some age of the elliptical. We also assume that \(\beta_{0}\) should be the same for both populations. In this case, first of all, the resultant combined XLF will be flatter than the XLF provided by only BH-MS contributors.
Secondly, as the contribution of BH-RG system decreases with elliptical age (see eq. 29), the XLF becomes steeper, evolving towards the slope characteristic for BH-MS binaries. It is possible that this is the kind of behavior that we observe in XLFs of ellipticals (see Fig. 3): we note that younger ellipticals possibly have flatter XLFs, although uncertainties are significant.

For WD donors we find $W(m) \propto t_1^{-7/4}$ (see eq. 29), implying that the probability of each BH-WD transient contributing to the observed XLF decreases with the age of the elliptical galaxy. Let us consider an elliptical where the formation of BH-WD systems has ended a few Gyr ago. We also consider that BH-MS systems are transient and have $W(M)$ according eq. 30. In order for BH-WD binaries to contribute significantly to the observed XLF they must form at a rate such that more than ~ 8,000 BH-WDs form for each BH-MS at a given BH accretor mass. According to binary population synthesis models for the Milky Way published so far, the number of formed BH-WD LMXBs exceeds the number of BH-MS LMXBs by at most a factor of 100 (Hurley et al. 2002). So, if the ratio of BH-MS binaries to BH-RG binaries is the same as in Milky Way, BH-WD XRBs will not be a significant contributor to the XLF. Based again on the discrepancy between the duty cycles for WD and RG donors (smaller for WDs by a factor of ~ 500), expect that BH-RG transients dominate over BH-WD transients too.

For the case of a MT-dependent duty cycle (expressed by $\eta$ in eq. 2) we conclude that: (i) if MT describes the angular momentum loss best, then only BH-MS transients significantly contribute to the XLFs of elliptical galaxies; consequently $\beta = 2.5 \pm 0.6$; (ii) if instead R VJ MB is a better prescription, then the XLF is dominated by BH-RG binaries and $\beta = 2.8 \pm 0.6$.

For the case of a constant duty cycle independent of the donor type, it is clear that the XRB type with the highest formation rate should dominate the XLF. According to formation rates calculated by Hurley et al. (2002), BH-WD binaries form at a rate about 100 times higher than BH-MS and BH-RG binaries. Consequently, WDs would be expected to dominate the transient population and this is certainly not true for the Milky Way. Therefore we conclude that the assumption of a constant, MT-independent is most probably not realistic.

Overall, we conclude that MS or RG donors dominate, depending on whether the IT or RVJ MB prescription is more realistic. Consequently, the slope of the accreting BH mass spectrum is $\beta = 2.5 \pm 0.6$ ($\beta_0 = 2.3 \pm 0.6$) or $\beta = 2.8 \pm 0.6$, respectively.

Next we consider the fact that the upper-end XLF of ellipticals is not a perfect power-law up to arbitrarily high $L_X$ values; instead there is a usually smooth cutoff behavior that limits the maximum $L_X$ observed at $\approx 2 \times 10^{39} \text{erg s}^{-1}$. In Fig. 4 we show the cumulative XLF associated with a model population of BH-MS binaries with a BH mass spectrum with a differential slope of $\beta = 2.3$ and with an imposed upper limit of 15 $M_\odot$ on the maximum BH mass present in the XRB population. We obtain a model XLF that behaves very similarly to observed XLFs (dash-dotted line). Clearly this is just to show the importance of the qualitative effect of a BH mass cut-off on the cumulative XLF.

5. DISCUSSION

We consider the upper-end XLF of ellipticals (above the reported break at $\approx 4 - 6 \times 10^{38} \text{erg s}^{-1}$) and suggest that it is populated by BH X-ray transients at outburst emitting approximately at the Eddington limit. We argue that the upper-end XLF slope is a footprint of the underlying accreting BH mass spectrum modified by a weighting function related to the transient duty cycle. We show that this weighting factor is generally dependent on the BH mass and the derived power-law dependence is different for each of the possible BH donor types: MS, RG, and WD.

Based on our analysis and prior population synthesis results we conclude that a constant transient duty cycle independent of the donor type can be excluded. Instead a duty cycle that depends on the ratio of the binary mass transfer rate to the critical rate for transient behavior (see eq. 2) appears consistent with current observations. In this case we conclude that the XLF is formed by an underlying spectrum of accreting black holes. The BH X-ray transients have a dominant donor type and an accreting BH mass spectrum slope $\beta$ that depend on the strength of MB angular momentum loss: (i) for the IT MB prescription, only BH-MS transients significantly contribute to the upper-end XLF and $\beta = 2.5 \pm 0.6$ ($\beta_0 = 2.3 \pm 0.6$); (ii) for the R VJ MB prescription, the XLF is dominated by BH-RG binaries and $\beta = 2.8 \pm 0.6$. We note that the results do depend on our conclusions about which donor-type population dominates based on currently published population synthesis models (Hurley et al. 2002).

We expect that our results can be used to reveal more information about the formation of BH XRBs in elliptical galaxies. More specifically, they can be used to constrain the physical connection between massive stars in XRB progenitors and the resultant BH masses. Current simulations assume either an artificially constant mass for BHs formed (usually at $\approx 10 M_\odot$), or a constant mass fraction of the progenitor leading to the remnant objects, or a remnant mass relation consistent with core-collapse simulations. The constraints on the accreting BH mass spectrum derived here could contribute to our understanding of core collapse, and the connection of BH masses to their progenitor masses.

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