How light the lepton flavor changing gauge bosons can be?

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Spontaneous breaking of inter-family (horizontal) gauge symmetries can be at the origin of the mass hierarchy between the fermion families. The corresponding gauge bosons have flavor-nondiagonal couplings which generically induce the flavour changing phenomena, and this puts strong lower limits on the flavor symmetry breaking scales. However, in the special choices of chiral horizontal symmetries the flavor changing effects can be naturally suppressed. For the sake of demonstration, we consider the case of leptonic gauge symmetry \(SU(3)\), acting between right-handed leptons and show that the respective gauge bosons can have mass in the TeV range, without contradicting the existing experimental limits.

1. The replication of fermion families is one of the main puzzles of particle physics. Three fermion families are in identical representations of the Standard Model (SM) gauge symmetry \(SU(3) \times SU(2) \times U(1)\). Its electroweak (EW) part \(SU(2) \times U(1)\) is chiral with respect to fermion multiplets: the left-handed (LH) leptons and quarks, \(\ell_L, u_R, d_R\), and the right-handed (RH) ones \(e_R, u_L, d_L\) transform as weak isodoublets while the right-handed (RH) ones \(e_R, u_L, d_L\) as isosinglets, \(i = 1, 2, 3\) being the family index. The chiral fermion content of SM has a remarkable feature that the fermion masses emerge only after spontaneous breaking of \(SU(2) \times U(1)\) by the vacuum expectation value (VEV) \(\langle \phi^0 \rangle = v_w = 174\text{ GeV}\) of the Higgs doublet \(\phi\), via the Yukawa couplings

\[
Y_{ij}^e \phi_L^i e_R^j + Y_{ij}^d \phi_L^i d_R^j + Y_{ij}^u \phi_L^i u_R^j + \text{h.c.},
\]

where \(Y_{e,u,d}\) are the Yukawa constant matrices, and \(\phi = i\sigma^2 \phi^*\). The fermion mass matrices \(M_f = Y_f v_w\), \(f = e, u, d\), can be diagonalized via bi-unitary transformations \(V_{L,f}^\dagger M_f V_{R,f} = M_f^{\text{diag}}\). The masses of leptons \(m_e, m_u, m_d\) and quarks \(m_u, m_d, \ldots\) are the eigenvalues of these mass matrices. The “right” matrices \(V_{R,f}\) have no physical meaning in the SM context while the “left” ones \(V_{L,f}\) determine the mixing matrices in charged currents coupled to weak \(W^\pm\) bosons, namely \(V_{CKM} = V_{L,e} V_{L,d}\) for quarks. However, no flavor mixing emerges in neutral currents coupled to \(Z\) boson and Higgs boson. In this way, the SM exhibits a remarkable feature of natural suppression of flavor-changing neutral currents (FCNC) \[1\]: all FCNC phenomena are suppressed at tree level and emerge exclusively from radiative corrections. At present, the majority of experimental data on flavor changing and CP violating processes are in good agreement with the SM predictions. There are a few anomalies, not definitively confirmed yet, which could point towards new physics beyond the Standard Model (BSM).

In a sense, the SM is technically natural since it can tolerate any Yukawa matrices \(Y_{ij}^f\), but it tells nothing about their structures which remain arbitrary. So the origin of the fermion mass hierarchy and their weak mixing pattern remains a mystery.

2. The key for understanding the fermion mass and mixing pattern may lie in symmetry principles. One can assign, e.g. different charges of abelian flavor symmetry \(U(1)\) to different fermion species \[2\], or one can introduce non-abelian horizontal gauge symmetries \(G_H\), e.g. \(SU(3)_H\) \[3\] with the flavor gauge fields dynamically marking the family indices. Such a gauge theory of flavor can be considered as quantum flavordynamics, provided that it is built in a consistent way and sheds some more light on the origin of the fermion mass hierarchy.

Namely, one can envisage that the form of the Yukawa matrices in \[1\] is related to the VEV structures of horizontal scalar fields (known also as flavons) which spontaneously break \(G_H\), and the fermion mass hierarchy emerges from the hierarchy between the scales of this breaking. In Ref. \[4\] this conjecture was coined as hypothesis of horizontal hierarchies (HHH). It implies that the fermion masses cannot be induced without breaking \(G_H\) so that it cannot be a vector-like symmetry, but it should have a chiral character transforming the LH and RH particle species in different representations. In such a picture the fermion Yukawa couplings should emerge from the higher order “projective” operators containing flavon scalars which transfer the VEV pattern of flavons to the structure of the Yukawa matrices \(Y_f\). In the UV-complete pictures such operators can be induced via renormalizable interactions after integrating out some extra heavy fields, scalars \[3\] or vector-like fermions \[4\]. In the context of supersymmetry, such horizontal symmetries can lead to interesting relations between the mass spectra of fermions and their superpartners and naturally realize the minimal flavor violation scenarios \[3\] \[7\].

Discovery of the flavor gauge bosons of flavor and/or related FCNC effects would point towards new BSM physics of flavor. However, a direct discovery at future accelerators can be realistic only if the scale of \(G_H\) symmetry breaking is rather low, in the range of few TeV. Therefore, the following questions arise: (i) for which choice of symmetry group \(G_H\) one can realize the
HHH paradigm, relating the fermion mass hierarchy to its breaking pattern, and (ii) which is the minimal scale of $G_H$ symmetry allowed by present experimental limits, and namely, can this scale be low enough to have $G_H$ flavor bosons within the potential experimental reach?

3. In the limit of vanishing Yukawa couplings, $Y_f \to 0$, the SM acquires a maximal global chiral symmetry

$$U(3)_e \times U(3)_c \times U(3)_Q \times U(3)_u \times U(3)_d$$

(2)

under which fermion species transform as triplets of independent $U(3)$ groups respectively as $\ell_L \sim 3_\ell, e_R \sim 3_e$, etc. The Yukawa couplings can be induced by the VEVs of flavons in mixed representations of these symmetry groups. One can consider the higher order operators such as e.g. for leptons

$$X_e \frac{\phi \bar{\ell}_L e_R}{M} + h.c.$$  \hfill (3)

where $X_e \sim (3_e, 3_e)$ is a flavon in mixed representation of $U(3)_e \times U(3)_c$ which can be also viewed as composite tensor product of scalars in fundamental representations of $U(3)_\ell$ and $U(3)_c$.

In the SM extensions the maximal flavor symmetry reduces to a smaller symmetry. E.g. in the context of $SU(5)$ grand unified theory (GUT) which unifies $\ell_L$ and $d^c_L$ fragments of each family in 5-plets and $e^c_L, u^c_L$ and $Q_L$ fragments in 10-plets ($\psi^c_L = C \bar{\psi}_R^T, C$ is a charge conjugation matrix), the maximal symmetry reduces to two factors $U(3)_\ell \times U(3)_c$:

$$5_L = (\ell, d^c)_L \sim (3_\ell, 1), \ 10_L = (e^c, u^c, Q)_L \sim (1, 3_e)$$  \hfill (4)

In the context of $SO(10)$ GUT all fermions of one family including the RH neutrino $N_R$ reside in the spinor multiplet $16_L = (5 + 10 + 1)_L$. Hence, there can be only one chiral symmetry $U(3)_H$ between three families of 16-plets, with all LH fermions $\ell_L, Q_L$ transforming as triplets and the RH ones $N_R, e_R, u_R, d_R$ as anti-triplets, in the spirit of chiral horizontal $SU(3)_H$ of Refs. [3–5]. For predictive models based on $SO(10) \times SU(3)_H$ see e.g. in Refs. [5].

It is tempting to consider some part of the maximal flavor symmetry, or its GUT-restricted versions, as a gauge symmetry $G_H$. Gauging of chiral $U(1)$ factors is difficult since they are anomalous with respect to the SM. Therefore, we consider a situation in which only some of non-abelian $SU(3)$ parts in (4) are gauged. In particular, in this paper we concentrate on the lepton sector and discuss a simple model with a gauge symmetry $G_H = SU(3)_c$ transforming the RH leptons as a triplet $e_R = (e_1, e_2, e_3)_R$, while the LH leptons $\ell_L = (\ell_1, \ell_2, \ell_3)$ have no symmetry and $i = 1, 2, 3$ is just a family number. We show that the lepton mass hierarchy $m_\tau \gg m_\mu \gg m_e$ can be directly related to the hierarchy of $U(3)_c$ symmetry breaking scales. As for the lepton flavor violating (LFV) phenomena induced by $SU(3)_c$ gauge bosons, we show that they are strongly suppressed since the intermediate $SU(2)_L$ subgroup acts as an approximate custodial symmetry. The respective scale is allowed to be as low as 2 TeV, without contradicting the present experimental limits on the LFV processes.

4. The LH and RH lepton fields of our model are in the following representations:

$$\ell_{Li} = \left( \frac{\nu_i}{\ell_i} \right) \sim (2, -1, 1), \ e_{Ra} \sim (1, -2, 3_e)$$  \hfill (5)

where we explicitly indicate the multiplet content with respect to the EW $SU(2) \times U(1)$ and horizontal $SU(3)_c$. This set of fermions is not anomaly free. The ways of the anomaly cancellations will be discussed in next sections.

We assume that there is only one Higgs doublet $\phi$ with the standard Higgs potential $V(\phi) = \lambda(|\phi|^2 - \nu^2)^2$. However, the Yukawa couplings of $\phi$ with the fermions $\ell_L$ and $e_{Ra}$ are forbidden by $SU(3)_c$ symmetry. So, for generating the lepton masses this symmetry should be broken.

For breaking $SU(3)_c$ we introduce three flavon fields $\xi^c_n, n = 1, 2, 3$, each transforming as $SU(3)_c$ (anti)triplet. The charged lepton masses then can emerge from the gauge invariant dimension 5 operators

$$\sum_n \frac{g_{ln} \xi^c_n}{M} \phi \bar{\ell}_{Li} e_{Ra} + h.c.$$  \hfill (6)

where $g_{ln}$ are order one constants (see upper diagram of Fig. 1). For having an UV-complete theory, one can consider these operators as induced form the renormalizable terms via seesaw-like mechanism. E.g. one can integrate out from the following Yukawa Lagrangian

$$h \phi \overline{L_\alpha} e_{Ra} + M \overline{R_\alpha} L_\alpha + \sum_n g_{ln} \xi^c_n \overline{L_\alpha} R_\alpha + h.c.,$$  \hfill (7)

the extra heavy vector-like lepton doublets

$$L_\alpha, R_\alpha = \left( \frac{N_\alpha}{E_\alpha} \right)_{L,R} \sim (2, -1, 3_e)$$  \hfill (8)

with a large Dirac mass $M$ (see Fig. 1 lower diagram). Operator (8) has a global symmetry $U(3)_c = SU(3)_c \times U(1)_c$, where the abelian part $U(1)_c$ is related to the

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1. However, there are models in which string-inspired anomalous gauge symmetry $U(1)_A$ is used as a flavor symmetry [9].

2. Alternatively, one could say that also $SU(3)_c$ is a flavor symmetry but broken at some higher scales. More complete model with $SU(3)_c \times SU(3)_c$ symmetry will be discussed elsewhere [10].

3. Also the vector-like $SU(2)$ acting on both LH and RH fermion species has custodial properties [11]. However, it allows degenerate mass spectrum which makes problematic the naturalness of inter-family mass hierarchy.

4. For comparison, the naive lower limit on the scale of flavor changing bosons is over 100 TeV [12]. In the models this scale was assumed to be close to the GUT scale, and in any case larger than a PeV, for avoiding the excessive FCNC. For an exception, see Ref. [13].
where we denote $\phi \xi$ and $\xi_n$. In order to generate non-zero masses of all three leptons $e, \mu, \tau$, this global symmetry must be fully broken. This means that all three flavons $\xi_n$ should have the non-zero VEVs with disoriented directions. In other words, the VEVs $\langle \xi_n \rangle$ becomes diagonal, $\langle \xi_n \rangle = v_n \delta_{n}^{\alpha}$, or explicitly

$$\langle \xi_1 \rangle = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle \xi_2 \rangle = \begin{pmatrix} 0 \\ v_2 \\ 0 \end{pmatrix}, \quad \langle \xi_3 \rangle = \begin{pmatrix} 0 \\ 0 \\ v_3 \end{pmatrix}, \quad (9)$$

ordered as $v_3 > v_2 > v_1$ reflecting the steps of the global symmetry breaking $U(3)_e \rightarrow U(2)_e \rightarrow U(1)_e \rightarrow$ nothing. By substituting these VEVs in operator (6), it reduces to the SM Yukawa couplings

$$Y^{\alpha \beta}_{e} \theta \bar{U}_L \ell \phi \xi + \text{h.c.}, \quad Y^{\alpha}_{e} = \sum_{n} \frac{g_{\alpha n}(\xi_n)}{M} = g_{\alpha n} \frac{v_n}{M}. \quad (10)$$

Without loss of generality, $\ell_L$ states can be turned to the basis in which matrix $g_{\alpha n}$ has a triangular form and the diagonal elements $g_{33}, g_{22}, g_{11}$ are real. Then

$$Y_e = \frac{1}{M} \begin{pmatrix} g_{11} v_1 & 0 & 0 \\ g_{21} v_1 & g_{22} v_2 & 0 \\ g_{31} v_1 & g_{32} v_2 & g_{33} v_3 \end{pmatrix} = \frac{v_3}{M} \begin{pmatrix} g_{11} \epsilon & 0 & 0 \\ g_{21} \epsilon & g_{22} \epsilon & 0 \\ g_{31} \epsilon & g_{32} \epsilon & g_{33} \epsilon \end{pmatrix} \quad (11)$$

where we denote $v_2/v_3 = \epsilon$ and $v_1/v_2 = \bar{\epsilon}$. The Yukawa matrix $Y_e$ (and the mass matrix $M_e = Y_e v_w$) can be diagonalized via bi-unitary transformation

$$Y_e \rightarrow V_L^{\dagger} Y_e V_R = \text{diag}(y_e, y_\mu, y_\tau). \quad (12)$$

Hence, the lepton mass hierarchy $m_\tau : m_\mu : m_e$ corresponds to the hierarchy between the scales $v_3 : v_2 : v_1$. Namely, neglecting the small $\sim \epsilon^2$ corrections, we have

$$m_\tau = \frac{g_{33} v_3}{M} v_w, \quad m_\mu = \frac{g_{22} v_2}{M} v_w, \quad m_e = \frac{g_{11} v_1}{M} v_w. \quad (13)$$

Let us discuss whether such a hierarchy of the VEVs can be natural. Since three flavons have identical quantum numbers, their scalar potential has a generic form

$$V(\xi) = \lambda_n \left( |\xi_n|^2 - \frac{\mu_n^2}{2\lambda_n} \right)^2 + \lambda_{klnm} \xi_k^\dagger \xi_l^\dagger \xi_m^\dagger \xi_n^\dagger + (\mu \xi_2 \xi_3 + \text{h.c.}) \quad (14)$$

We assume that all constants $\lambda$ are say in the range $\lambda \sim 0.1 \div 1$, and the mass terms $\mu_n^2$, positive or negative, are of the same order, say within $1 \div 10$ TeV. The last (trilinear) coupling $\mu_{\alpha \beta \gamma} \xi_{\alpha}^\dagger \xi_{\beta}^\dagger \xi_{\gamma}^\dagger$ has a dimensional constant $\mu$ which is however allowed (by ‘t Hooft’s naturalness principle) to be arbitrarily small since in the limit $\mu \rightarrow 0$ the Lagrangian acquires global $U(1)_e$ symmetry respected also by the Yukawa terms [4]. In fact, this latter coupling softly breaks $U(1)_e$ and thus reduces the global symmetry $U(3)_e$ to $SU(3)_e$.

For full breaking of gauge $SU(3)_e$ symmetry, just two flavons with non-aligned VEVs are sufficient. An order of magnitude hierarchy between the scales $v_2$ and $v_3$, $v_2/v_3 \sim m_\mu/m_\tau$, can emerge due to some moderate conspiracy of parameters admitting a natural “spread”, say within an order of magnitude, between the mass terms and coupling constants of $\xi_2$ and $\xi_3$ in (14). But large hierarchy $v_1/v_3 \sim m_\mu/m_e$ at first sight requires a strong fine tuning. However, in fact small $v_1$ can be obtained naturally considering the following situation. In the limit $\mu \rightarrow 0$ the VEV matrix $\langle \xi_n \rangle$ has rank 2, so that only two flavons $\xi_2$ and $\xi_3$ get the VEVs $v_n = \mu_n/\sqrt{2\lambda_n}$, $n = 2, 3$, oriented as in (6), because of their negative mass$^2$ terms in (14). As for third flavon $\xi_1$, it has a positive mass$^2$ term, i.e. $\mu_1^2 < 0$, and in the limit $\mu = 0$ it remains VEV-less. However, for $\mu \neq 0$, the last term in (14) explicitly breaks global $U(1)_e$ symmetry and induces non-zero VEV $\langle \xi_1 \rangle$, $v_1 = \mu v_2/\mu_1^2$. Thus, taking e.g. $\mu \sim v_3$, we have $v_1/v_3 \sim \mu/v_3$, and $\mu < v_2$ would suffice for having $v_1$ in the needed range.

The unitary matrix $V_R$ in (12) connecting the initial flavor basis of the RH leptons to the mass basis,

$$\left( \begin{array}{c} e_1 \\ e_2 \\ e_3 \end{array} \right)_R = V_R \left( \begin{array}{c} e \\ \mu \\ \tau \end{array} \right) = \left( \begin{array}{ccc} V_{1e} & V_{1\mu} & V_{1\tau} \\ V_{2e} & V_{2\mu} & V_{2\tau} \\ V_{3e} & V_{3\mu} & V_{3\tau} \end{array} \right) \left( \begin{array}{c} e \\ \mu \\ \tau \end{array} \right)_R \quad (15)$$

has no physical meaning for the EW interactions, but it is meaningful for the LFV interactions mediated by the gauge bosons of $SU(3)_e$. Since the mixing angles in $V_R$ are small, we have (modulo $\epsilon^2$ corrections)
where $\mathcal{M}$ is a new scale which in the context of seesaw mechanism can be related to the Majorana masses of RH neutrinos. In our scenario the states $\ell_L$, $\ell_R$ are not distinguished by any symmetry and the matrix $Y_{\nu}^{\mu}$ is a generic non-diagonal matrix, supposedly with all elements of the same order. Thus, the unitary matrix $V_R$ which diagonalizes it, $V_R^\dagger Y_\nu V_R = Y_{\nu}^{\mu \nu}$ contains large rotations and the neutrino mixing angles are expected to be large.

5. Gauge bosons $F_{\alpha}^\mu$ of $SU(3)_c$, associated to the Gell-Mann matrices $\lambda_\alpha$, $\alpha = 1, 2, ..., 8$, interact as $g F_{\alpha}^\mu J_{\alpha \mu}$ with the respective currents $J_{\alpha \mu}$, where $g$ is the $SU(3)_c$ gauge coupling and $e_R = (e_1, e_2, e_3)^T_R$ denotes the triplet of the RH leptons. Clearly, these currents are generically FCNC; e.g., those related to non-diagonal generators $\lambda_3, \lambda_2$ transform $e_1$ into $e_2$, etc. Nevertheless, as we shall see below, the processes mediated by flavor bosons exhibit no LFV in the basis of eigenstates $e_{R1}$, $e_{R2}$, $e_{R3}$ of flavor diagonal generators $\lambda_3$ and $\lambda_6$.

At low energies the flavor bosons induce four-fermion (current $\times$ current) interactions:

$$L_{\text{eff}} = -g^2 \frac{1}{2} M_{2 \beta} (2M_{2 \beta})^{-1} J_{\beta \mu}$$

where $M_{2 \beta}$ is the (symmetric) mass matrix of gauge bosons $F_{\alpha}^\mu$. In the flavon VEV basis [4] this matrix is essentially diagonal apart of a non-diagonal $2 \times 2$ block related to $F_3 - F_8$ mixing. Namely, for the masses $\nu$ of gauge bosons $F_{4,5}^\mu$, $F_{6,7}^\mu$ and $F_{1,2}^\mu$ we have respectively

$$M_{4,5}^2 = \frac{g^2}{2} (v_3^2 + v_1^2), \quad M_{6,7}^2 = \frac{g^2}{2} (v_3^2 + v_2^2),$$

$$M_{1,2}^2 = \frac{g^2}{2} (v_2^2 + v_1^2),$$

while for the mass matrix of $F_3^\mu - F_8^\mu$ system we get

$$M_{38}^2 = \frac{g^2}{2} \left( \frac{v_2^2 + v_1^2}{\sqrt{3}}, \frac{1}{\sqrt{3}} (v_1^2 - v_2^2), \frac{1}{3} (4v_3^2 + v_2^2) \right).$$

(20)

Obviously, the factor $g^2$ in operators [18] cancels and their strength is determined solely by $SU(3)_c$ symmetry breaking scales $v_2$ and $v_3$. In the following we neglect a small contribution $\nu_1^2/\nu_2^2 = \frac{\epsilon^2}{2} \ll 1$ in the gauge boson mass terms and in respective effective operators. Then

$$g^2 (2M_{38}^2)^{-1} = \frac{1}{v_2^2} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + \frac{1}{4v_3^2} \begin{pmatrix} 1 & \sqrt{3} \\ \sqrt{3} & 3 \end{pmatrix}$$

(21)

Hence, from operators [18] one can single out the one cut off by the scale $v_2$:

$$L_2 = -\frac{1}{v_2^2} \sum_{a=1}^3 (J_{\alpha a}^\mu)^2 = -\frac{1}{4v_2^2} \sum_{a=1}^3 (\bar{e}_R \lambda_{a \gamma} e_R)^2$$

(22)

which involves only $e_{R1}$ and $e_{R2}$ states. Using Fierz identities for $\lambda_1, \lambda_2, \lambda_3$ which in fact are the Pauli matrices, this operator can be rewritten as

$$L_2 = -\frac{1}{v_2^2} (J_{\lambda 0}^\mu)^2 = -\frac{1}{4v_2^2} (\bar{e}_R \lambda_{0 \gamma} e_R)^2$$

(23)

where we denote $\lambda_0 = \text{diag}(1, 1, 0)$. The remaining operators in [18] are related to the scale $v_3$:

$$L_3 = -\frac{1}{v_3^2} \left[ (J_{\lambda 0}^\mu)^2 + (J_{\lambda 0}^\mu)^2 + (J_{\lambda 0}^\mu)^2 \right]$$

$$-\frac{1}{4v_3^2} \left[ (J_{\lambda 0}^\mu)^2 + (J_{\lambda 0}^\mu)^2 \right]$$

(24)

where the current $J_{\lambda 0}^\mu = \frac{1}{2} J_0^\mu + \sqrt{3} J_3^\mu$ has a form $\frac{1}{2} \bar{e}_R \lambda_{3 \gamma} e_R$ with $\lambda_3 = \text{diag}(1, 0, -1)$. Hence, $\lambda_3, \lambda_4$ and $\lambda_5$ form a $SU(2)$ subalgebra of $SU(3)_c$, and using the Fierz identities for these matrices one can rewrite [24] as

$$L_3 = -\frac{1}{v_3^2} \left[ (J_{\lambda 0}^\mu)^2 + (J_{\lambda 0}^\mu + iJ_{\lambda 0}^\mu)(J_{\lambda 0}^\mu - iJ_{\lambda 0}^\mu) \right]$$

$$-\frac{\epsilon^2}{4v_3^2} \left[ (\bar{e}_R \lambda_{0 \gamma} e_R)^2 + 4(\bar{e}_R \lambda_{3 \gamma} e_R)(\bar{e}_R \lambda_{5 \gamma} e_R) \right]$$

(25)

where $\lambda_0 = \text{diag}(1, 0, 1)$. So, operators $L_2$ and $L_3$ do not induce any LFV transition between $e_{R1}, e_{R2}, e_{R3}$ states.

In basis of mass eigenstates $(e, \mu, \tau)$ all currents involved in operators [23] and [25], including those related to $\lambda_0$ and $\lambda_0$, should be rotated with the matrix $V_R$ [15], or in explicit form

$$J_{\alpha \mu} = (\bar{e}, \mu, \tau) R_{\alpha \gamma} \frac{V_R^\dagger \lambda_{\alpha} V_R}{2} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_R$$

(26)

In particular, in $(e, \mu, \tau)$ basis $\lambda_0$ in operator [23] is deformed to $\lambda_V = V_R^\dagger \lambda_0 V_R$, or explicitly

$$\lambda_V = \begin{pmatrix} 1 - |V_{3e}|^2 & -V_{3e}^* V_{3\mu} & -V_{3e}^* V_{3\tau} \\ -V_{3e} V_{3\mu}^* & 1 - |V_{3\mu}|^2 & -V_{3e}^* V_{3\tau} \\ -V_{3e} V_{3\tau}^* & -V_{3e}^* V_{3\tau} & |V_{1\tau}|^2 + |V_{2\tau}|^2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & \sim \epsilon \tilde{e}^2 & \sim \epsilon \tilde{e} \\ \sim \epsilon \tilde{e}^2 & 1 & \sim \epsilon \\ \sim \epsilon \tilde{e} & \sim \epsilon & \sim \epsilon^2 \end{pmatrix}.$$
and \( \lambda_{\nu} = V_{\tilde{R} \tilde{R}}^2 \tilde{\lambda}_{\nu} R_{\mu} \):

\[
\tilde{\lambda}_{\nu} = \left( \begin{array}{cccc}
1 & -|V_{2e}|^2 & V_{2e}^* V_{3\mu} & -V_{2e}^* V_{3\tau} \\
-\bar{V}_{2e} V_{2e}^* & |V_{1\mu}|^2 + |V_{3\mu}|^2 & -V_{2e}^* V_{2\tau} & -V_{2e} V_{2\tau} \\
-\bar{V}_{2e} V_{2e}^* & -V_{2e}^* V_{2\tau} & 1 - |V_{2\tau}|^2 & V_{2e}^* V_{3\tau} \\
-\bar{V}_{2e} V_{2e}^* & -V_{2e} V_{2\tau} & V_{2e}^* V_{3\tau} & 1 - |V_{2\tau}|^2
\end{array} \right)
\]

(28)

where we indicate the order of magnitude for the LFV entries in terms of the small parameters \( \epsilon, \tilde{\epsilon} \).

6. Let us question now how small the scale \( v_2 \) can be without contradicting to existing experimental limits. The leading terms in (23) give rise to flavor conserving operators

\[- \frac{1}{4v_2^2} (\bar{\nu}_e \gamma^\nu e_R)^2 - \frac{1}{2v_2^2} (\bar{\nu}_e \gamma^\nu e_R)(\bar{\nu}_R \gamma^\nu \mu_R) \]

(29)

constrained by the compositeness limits \( \Lambda_{\tilde{R} \tilde{R}} (eeee) > 10.2 \text{ TeV} \) and \( \Lambda_{\tilde{R} \tilde{R}} (ee \mu \mu) > 9.1 \text{ TeV} \) as reported respectively in Refs. [15] and [16]. Translating the formal definitions of compositeness scales to the scale \( v_2 \), we get

\[ v_2 = (8\pi)^{-1/2} \Lambda_{\tilde{R} \tilde{R}} (eeee) > 2.0 \text{ TeV} \]

\[ v_2 = (8\pi)^{-1/2} \Lambda_{\tilde{R} \tilde{R}} (ee \mu \mu) > 1.8 \text{ TeV} \]

(30)

Hence, these limits allow the scale of \( SU(2)_c \) symmetry to be as small as \( v_2 = 2 \text{ TeV} \) or so. As for the flavor-changing phenomena induced by operators (23) and (25), they are suppressed by small parameters \( \epsilon, \tilde{\epsilon} \) and agree with severe experimental limits on the LFV even for such a low scale of flavor symmetry.

E.g. both \( L_2 \) and \( L_3 \) terms contribute the following operator which induces the LFV decay \( \mu \rightarrow e\tilde{e} \):

\[
\frac{4 G_{\mu ee}}{\sqrt{2}} (\bar{\nu}_e \gamma^\nu \mu_R) (\bar{\nu}_R \gamma^\nu e_R),
\]

\[
\frac{4 G_{\mu ee}}{\sqrt{2}} = \frac{1}{2v_2^2} V_{3\mu}^* V_{3\mu} + \frac{1}{2v_2^2} V_{2\mu}^* V_{2\mu} \sim \frac{\epsilon^2 \tilde{\epsilon}}{2v_2^2}.
\]

(31)

Its amplitude can be normalized to the amplitude of the muon standard decay due to the weak interactions

\[- \frac{4 h}{\sqrt{2}} (\bar{\nu}_e \gamma^\nu e_R) (\bar{\nu}_\mu \gamma^\nu \mu_L), \]

(32)

where \( 4h/\sqrt{2} = 1/v_w^2 \), \( v_w = 174 \text{ GeV} \). Hence, we get

\[
\Gamma(\mu \rightarrow e\tilde{e}) = \frac{1}{2} \left| \frac{4 G_{\mu ee}}{G_F} \right|^2 \sim \frac{\epsilon^2 \tilde{\epsilon}^2}{8} \left( \frac{v_w}{v_2} \right)^4.
\]

(33)

Hence, \( v_2 > 2 \text{ TeV} \) and \( \epsilon, \tilde{\epsilon} \lesssim 1/20 \) or so, this branching ratio is compatible with the existing experimental limit \( Br(\mu \rightarrow 3e)_{\exp} < 10^{-12} \) [17].

For \( \tau \) lepton decay modes as \( \tau \rightarrow \mu e\tilde{e} \) and \( \tau \rightarrow 3\mu \) leading contributions arise from operator (23). From (27) we get the relevant constants as \( 4G_{\tau ee} \sqrt{2} = 4G_{\tau \mu \mu} \sqrt{2} = V_{3\mu}^* V_{3\tau}/2v_2^2 \). Hence, the widths of these decays are suppressed by a factor \( \sim \epsilon^2/v_2^2 \), and are compatible with the experimental limits [17]. The summary of predicted branching ratios of relevant LFV processes compared with experimental limits is given in Table 1 with the parameters \( \epsilon, \tilde{\epsilon} \) normalized to a benchmark value 1/20.

Yet another LFV phenomena to be considered is muonium-antimuonium conversion \( M(\mu e) \rightarrow \bar{M}(\bar{\mu}e) \) [18]. The relevant operator emerging from (25) reads

\[- \frac{4 G_{M\bar{M}}}{\sqrt{2}} (\bar{\mu}_R \gamma^\nu e_R)^2, \quad G_{M\bar{M}} = \frac{\epsilon^2 (V_{2e} V_{2\mu})^2}{8\sqrt{2}v_2^2} \]

(34)

Thus the amplitude of \( M - \bar{M} \) transition is doubly suppressed, by a factor \( \sim \epsilon^2 \tilde{\epsilon}^2 < 10^{-5} \) or so, and is much below the experimental limit \( |G_{M\bar{M}}/G_F| < 3 \times 10^{-6} \) [19].

One loop contribution of flavor bosons to the electron magnetic moment has no suppression by mixing angles in \( V_{e\mu} \) (the electric dipole gets no contribution at one loop). By computing parameter \( a_e = \frac{\mu_e}{2(g_e - 2)} \) with formulas in Ref. [20], we get:

\[
a_e = - \frac{m_e^2}{8\pi^2 v_2^2} = -8.3 \times 10^{-16} \left( \frac{2 \text{ TeV}}{v_2} \right)^2 \]

(35)

which is about 3 orders of magnitude smaller than the present difference between the experimental [17] and theoretical [21] determinations of the electron anomalous magnetic moment, \( a_e^{\exp} - a_e^{\text{SM}} = (-7.0 \pm 3.5) \times 10^{-13} \). Similarly, the contribution for muon anomalous magnetic moment obtained by substituting \( m_\mu \rightarrow m_\mu \) in (35), \( a_\mu = -3.5 \times 10^{-11} (2 \text{ TeV}/v_2)^2 \), is two orders of magnitude below the existing discrepancy \( a_\mu^{\exp} - a_\mu^{\text{SM}} = (2.7 \pm 0.8) \times 10^{-6} \) [17]. So, these contributions are irrelevant for both electron and muon.

Let us remark that potentially also flavons can mediate the LFV processes. From the effective operators (9), after substituting the VEV \( \langle \phi \rangle = v_w \) we obtain for the lepton Yukawa couplings with the flavon fields \( \xi_n \):

\[
h_{in} \xi_n \frac{\alpha}{v_L} e_R \ln, \quad h_{in} = \frac{g_{in} v_w}{M} \]

(36)

which are generically flavor-changing. For example, in the basis [11] the Higgs mode of the flavon \( \xi_2 \) which is presumably the lightest, with the mass \( \mu_2 \sim v_2 \), induces the following effective operator:

\[- \frac{h_{32} h_{22}}{M^4} \frac{(\tau \mu)}{(\bar{\tau} \bar{\mu})}, \quad \frac{h_{32} h_{22}}{M^4} \sim \frac{m^2}{v_2^4} \]

(37)

where we have taken into account the relations [13]. Thus, for \( v_2 > 2 \text{ TeV} \), the width of \( \tau \rightarrow 3\mu \) decay induced by this operator is more than 12 orders of magnitude below the experimental limit. The width of \( \mu \rightarrow 3e \) decay induced by analogous operator mediated by flavon \( \xi_1 \) is also suppressed by many orders of magnitude.

7. Let us remark that for promoting the chiral non-abelian factors in (2) as \( SU(3)_c \), etc. a s gauge symmetries, one has to take care of anomaly cancellations. For
In this picture, the parity can be understood as a disconnected symmetry of exchange between ordinary and mirror particles as it was suggested. As for the horizontal gauge factors, they in fact all assume that horizontal symmetries are common symmetries between ordinary and mirror leptons. Hence, we have

\[ \sum_n \frac{g_{\nu R}^n c_n}{M} (\phi \ell_L R_{\nu A} + \phi' \ell_R R_{\nu L}) + \text{h.c.} \]  

Then, if mirror symmetry is exact, i.e. \( \langle \phi' \rangle = \langle \phi \rangle = v_w \), the ordinary and mirror leptons should have identical mass spectra. As for neutrinos, now besides the operator generating the neutrino Majorana masses, we should have its mirror copy generating Majorana masses of mirror neutrinos, and also a mixed operator between ordinary \( \ell \) and mirror \( \ell' \) leptons

\[ \frac{Y_{\nu}^{ij}}{M} (\phi \ell_L C_{i} \ell_{R} + \phi' \ell'_R C_{i} \ell'_{R}) + \frac{\tilde{Y}_{\nu}^{ij}}{M} \phi' \ell_L C_{i} \ell'_{R} + \text{h.c.} \]  

The last operator mixes ordinary (active) and mirror (sterile) neutrinos, and also can play a key role in co-leptogenesis scenario which can generate baryon asymmetries in both ordinary and mirror sectors. Interestingly, if lepton numbers (or better \( B - \ell \)) are conserved in each sector, then these operators are forbidden and all neutrinos remain massless. However, if the combination \( B - L \) is conserved, then the last operator is allowed. In this case the neutrinos will be Dirac particles having masses \( \sim v^2_{\nu}/M \), with their LH components living in ordinary world and the RH components living in mirror world.
For its cancellation, new fermion species should be introduced in the proper representations of the SM and $SU(3)_c$. There are several ways of doing this. Let us consider one of possibilities by introducing in our sector, in addition to the regular leptons $[5]$, the new lepton species in representations

$$\mathcal{E}_{La} \sim (1, -2, 3; X), \quad \mathcal{E}_{Ri} \sim (1, -2, 1; X),$$  

(43)

and, for mirror parity, analogous species in mirror sector:

$$\mathcal{E}'_{Ro} \sim (1, -2', 3; X), \quad \mathcal{E}'_{Li} \sim (1, -2', 1; X),$$  

(44)

where $\alpha = 1, 2, 3$ is a gauge $SU(3)_c$ index and $i = 1, 2, 3$ is just for numbering three species. We assign to these fermions a new charge $X$ of additional gauge symmetry $U(1)_X$ while ordinary leptons have no $X$-charges. This additional charge is introduced in order to forbid the mixing of new fermions $[43]$ with ordinary leptons $[5]$ due to the mass term $M_{\mathcal{E}_{La}} e_{Ro}$ and the Yukawa terms $\mathcal{E}'_{Li} e_{Rj} \phi$ which would ruin the flavor structure induced by the operator $[6]$. It is easy to check that by introducing extra fermions $[43]$ and by taking the mixed triangle anomalies including $U(1) \times SU(3)_c \times U(1)_X \times SU(3)'_c, U(1) \times U(1)'_X$ and $U(1)_X \times U(1)'_X$ are all cancelled.

The new fermions get masses from couplings with flavons $\xi_n$:

$$m_{\mathcal{E}_{Ro}} \sim v_n \xi_{Ro} e_{Ro} + m_{\mathcal{E}_{Li}} \xi_{Li} e_{Li} + h.c.$$  

(45)

where $v_n$ are order 1 Yukawa constants. Therefore, their mass spectrum should reflect the hierarchy $v_3 : v_2 : v_1 \sim 1 : \varepsilon : \varepsilon \varepsilon$. In particular, if $v_2$ is the TeV range, then $v_1$ should be in the range of 100 GeV and thus the lightest of new leptons will have a mass of this order. In addition, if $U(1)_X$ symmetry is unbroken, then the lightest of these states should be stable (the heavier ones will decay in to lighter one via $SU(3)_c$ flavor boson mediated operators). Interestingly, the LEP direct experimental lower limit on the mass of new charged leptons is 102.6 GeV $[27]$. Such heavy leptons can be within the reach of new $e^+e^-$ machines as ILC/CLIC or CEPC/FCC-ee. If $U(1)_X$ symmetry is spontaneously broken, then the mixing of $e, \mu, \tau$ with new leptons can be allowed and thus the latter can be rendered unstable.

Let us turn to flavor gauge bosons of $SU(3)_c$ which now interact with both normal leptons mirror leptons. Now their exchange should create mixed effective operators involving both ordinary and mirror leptons. In particular, the bosons $\mathcal{F}_{1,2,3}$ mediate the following operators involving only first two families of $\ldots$

$$1 \sum_{a=1}^{3} J_\mu J_{\mu} = \frac{1}{4v^2} \sum_{a=1}^{3} (\mathcal{E}_R \lambda_\alpha \gamma^\mu \mathcal{E}_R)(\mathcal{E}_L \lambda_\alpha \gamma^\mu \mathcal{E}_L)$$  

(46)

Thus, this operator induces muonium - mirror muonium conversion $M(\mu e) \rightarrow M'(\mu' e')$ with $G_{M'M'/GF} = (v_\mu/v_2)^2 = 7.6 \times 10^{-3}(2 \text{TeV}/v_2)^2$. In difference from the muonium-antimuonium conversion $[33]$, here is no suppression by small mixing angles. The present limit on the muonium disappearance reads $Br(M \rightarrow \text{invisible}) < 5.7 \times 10^{-6}$ $[29]$ which is respected for $v_2 \sim 400 \text{ GeV}$ or so, however this limit can be improved by several orders of magnitude as discussed in Ref. $[29]$. Analogously, this operator should induce positronium conversion into mirror positronium $[30]$, but for $v_2 > 2 \text{ TeV}$ the positronium disappearance rate is much below the present experimental limit $Br(\Omega_{P_s} \rightarrow \text{invisible}) < 6 \times 10^{-4}$ $[31]$.

8. In this paper we discussed phenomenological implications of horizontal gauge symmetry $SU(3)_c$ acting only in lepton sector, between three families of right-handed leptons. The lepton mass hierarchy $m_e \gg m_\mu \gg m_\tau$ can be related to the hierarchy of the symmetry breaking scales $v_3 \gg v_2 \gg v_1$. We have shown that the LFV effects induced by flavor changing gauge bosons are strongly suppressed due to custodial properties of $SU(2)_c \subset SU(3)_c$ symmetry and respective scale can be as small $v_2 = 2 \text{ TeV}$, which limit is in fact set from the compositeness limits on the flavor-conserving operators while the limits obtained from the LFV processes itself are weaker. Taken into account that the flavor gauge constant $g$ of horizontal $SU(3)_c$ can be less than 1, then masses of the $SU(2)_c$ gauge bosons $M_{Q,2,3} \approx (g/\sqrt{2})v_2$ can be as small as 1 TeV or even smaller, and thus can be accessible at new electron-positron machines.

Analogously to $SU(3)_c$, all $SU(3)_{Q}$ factors in $[35]$ can be rendered anomaly along the lines discussed in previous section, and thus they also can be gauge symmetries $[7]$. The quark mass hierarchy can be related with hierarchies in breaking of $SU(3)_{Q} \times SU(3)_d \times SU(3)_u$ gauge factors, i.e. with the ratios $\varepsilon_d = v_d^2/v_3^2$ and $\varepsilon_u = v_u^2/v_3^2$ for between the VEVs of $SU(3)_d$ triplet flavons, and the same for $SU(3)_u$ and $SU(3)_Q$. In this way, the hierarchy of down quark masses will go parametrically as $1 : \varepsilon_d : \varepsilon_u : \varepsilon_d \varepsilon_u$. The quark flavor violating processes mediated by gauge bosons of $SU(3)_d$ and $SU(3)_u$ will be suppressed due to custodial symmetry in the same way as the LFV processes mediated by $SU(3)_c$ bosons. In particular, the operator $(\overline{\mathcal{E}}_R \gamma^\mu e_R)^2$ which introduces $K^0 \rightarrow K^0$ oscillation (analogously as leptonic operator $[34]$) induces $M \rightarrow \overline{M}$ conversion will be suppressed by a factor $\sim \varepsilon_d^2 \varepsilon_u^2 \ll 1$. This can allow to quark flavor changing gauge bosons to be in the range of few TeV, in fact limited only by the quark compositeness bounds. Interestingly, the flavor bosons of $SU(3)_L$ and $SU(3)_Q$ can give also anomalous contributions imitating the charged current × current operators of the SM, and so they will have interference with the latter. E.g. $SU(3)_L$ bosons induce operator $(\overline{\mathcal{E}}_R \gamma_{\mu} H_L)(\overline{\mathcal{E}}_R \gamma^\mu \nu e)$ which is nothing but the Fierz-transformed SM operator $[32]$ responsible for the muon decay. Analogously, $SU(3)_Q$ bosons should

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5 Moreover, also some anomaly free combinations of $U(1)$ factors in $[2]$ can be promoted as gauge symmetries, as e.g. common $U(1)_{B-L}$ acting between ordinary and mirror sectors $[28]$. 
induce e.g. operator \((\pi \gamma_\rho c_L)(\bar{\pi} \gamma^\rho d_L)\) which also interferes with the charged current operators in the SM. So, presence of such operators can have impact for the unitarity tests of the CKM mixing of quarks. Detailed analysis of these issues will be given elsewhere [10].

As far as the presence of mirror sector is concerned, mirror matters a viable candidate for light dark matter dominantly consisting of mirror helium and hydrogen atoms [32]. The flavor gauge bosons related to both quarks and mirror quarks appear as messengers between two sectors and can give an interesting portal for direct detection via mirror matter scattering off normal nuclei in detectors [33]. The flavor gauge bosons related to both symmetric version the respective flavor gauginos, complemented by R-parity breaking, can induce the mixing between the neutral ordinary and mirror nuclei in detectors [34].

In this paper we demonstrated that in the TeV range there may exist other sorts of new physics related to the fermion flavor which can be revealed in future experiments at the energy and precision frontiers. In particular, the lepton-flavor changing gauge bosons can be as light as a TeV, or even lighter, since the LFV processes are strongly suppressed by custodial symmetry reasons. Nevertheless, some of these LFV processes, as e.g. \(\tau \rightarrow 3\eta\), can have widths close to present experimental limits and can be within the reach of future high precision experiments.

The preliminary version of this work was presented by B.B. at the European Physical Society Conference on High Energy Physics EPS-HEP 2017 [35].

Oscillation between ordinary and mirror neutral particles are effective if they are degenerate in mass, i.e. mirror parity is unbroken and the weak scales \(\langle \phi \rangle = v_w\) and \(\langle \phi' \rangle = v'_w\) are exactly equal in two sectors, \(v'_w = v_w\). However, the cancellation of horizontal anomalies between two sectors does not require that mirror parity is unbroken, and in fact one can consider models where it is spontaneously broken, e.g. \(v'_w > v_w\), with interesting implications for mirror dark matter properties and sterile mirror neutrinos [36] and axion physics [37]. In particular, the mirror twin Higgs mechanism for solving the little hierarchy problem, in supersymmetric [38] or non-supersymmetric [39] versions, needs \(v'_w\) in the TeV range, and after all, a scale of few TeV is of interest as a realistic scale of consistent supersymmetric models [40].

In this paper we demonstrated that in the TeV range there may exist other sorts of new physics related to the fermion flavor which can be revealed in future experiments at the energy and precision frontiers. In particular, the lepton-flavor changing gauge bosons can be as light as a TeV, or even lighter, since the LFV processes are strongly suppressed by custodial symmetry reasons. Nevertheless, some of these LFV processes, as e.g. \(\tau \rightarrow 3\eta\), can have widths close to present experimental limits and can be within the reach of future high precision experiments.

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