Chiral symmetry and exclusive B decays in the SCET

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(Dated: March 26, 2022)

We construct a chiral formalism for processes involving both energetic hadrons and soft Goldstone bosons, which extends the application of soft-collinear effective theory to multibody B decays.

The nonfactorizable helicity amplitudes for heavy meson decays into multibody final states satisfy symmetry relations analogous to the large energy form factor relations, which are broken at leading order in $\Lambda/m_b$ by calculable factorizable terms. We use the chiral effective theory to compute the leading corrections to these symmetry relations in $B \to M_\gamma \pi\bar{\nu}$ and $B \to M_\gamma \pi^+\pi^-$ decays.

PACS numbers: 12.39.Fe, 14.20.-c, 13.60.-r

1. Introduction. The study of processes involving energetic quarks and gluons is simplified greatly by going over to an effective theory which separates the relevant energy scales. The soft-collinear effective theory (SCET) \cite{1} simplifies the proof of factorization theorems and allows a systematic treatment of power corrections. SCET has been applied to both inclusive and exclusive hard processes with energetic final state particles.

In this paper we present a combined application of the SCET with chiral perturbation theory which can be used to study exclusive processes involving both energetic light hadrons and soft pseudo Goldstone bosons and photons. The main observation is that once the dynamics of the collinear degrees of freedom has been factorized from that of the soft modes, usual chiral perturbation theory methods can be applied to the latter, unhampered by the presence of the energetic collinear particles which might have upset the momentum power counting in $p/\Lambda$. The chiral formalism has been applied previously to compute matrix elements of operators appearing in hard scattering processes, such as DIS and DVCS \cite{2,3,4}. Our paper extends these results to processes with both soft and collinear hadrons.

We focus here on exclusive B decays, which are described by three well-separated scales: hard $Q \sim m_b$, hard-collinear $\sqrt{\Lambda q}$ and the QCD scale $\Lambda \sim 500$ MeV. This requires the introduction of a sequence of effective theories QCD $\to$ SCET$_I$ $\to$ SCET$_{II}$, containing degrees of freedom of successively lower virtuality $\Lambda$. The intermediate theory SCET$_I$ contains hard-collinear quarks $\xi_n$ and gluons $A_n^\mu$ with virtuality $p_{hc}^2 \sim \Lambda Q$ and ultrasoft quarks and gluons $q, A_\mu$ with virtuality $\Lambda^2$. Finally, one matches onto SCET$_{II}$ which includes only soft $q, A_\mu$ and collinear $\xi_n, A_n^\mu$ modes with virtuality $p^2 \sim \Lambda^2$. The expansion parameter in both effective theories can be chosen as $\lambda^2 \sim \Lambda/m_b$.

In the low energy theory SCET$_{II}$ the soft and collinear modes decouple at leading order and the effective Lagrangian is simply a sum of the kinetic terms for each mode

$$\mathcal{L}^{(0)} = \mathcal{L}_{\xi}^{(0)} + \sum_q \bar{q}(i\gamma\mu - m_q)q + \mathcal{L}_{A_n}^{(0)}.$$  \hspace{1cm} (1)

The matching of an arbitrary operator $O$ onto SCET$_{II}$ can be written symbolically as \cite{5}

$$O \to T \otimes O_S \otimes O_C + O_{nf} + \cdots$$  \hspace{1cm} (2)

where the ellipses denote power suppressed contributions. The first term is a ‘factorizable’ contribution, with $O_S, O_C$ soft and collinear operators convolved with a Wilson coefficient $T$ depending on the arguments of $O_S, O_C$. $O_{nf}$ denote ‘nonfactorizable’ operators. Their precise form depends on the IR regulator adopted for SCET$_{II}$; for example, in dimensional regularization they might take the form of $T$ products of operators involving messenger modes \cite{6}.

This formalism has been used to study exclusive B decays into energetic light hadrons (e.g. $B \to \pi\ell\nu$ and $B \to K^{*}\gamma$) \cite{7,8,9}, and nonleptonic decays into 2 energetic light hadrons such as $B \to \pi\pi$ \cite{10,11}. This paper presents an extension of this formalism to describe multibody B decays to one energetic hadron plus multiple soft pions and photons. Such decays received increased attention recently \cite{12,13} due to their ability to extend the reach of existing methods for determining weak parameters.

In Sec. 2 we introduce the SCET formalism and review the derivation of the large energy symmetry relations for the $B \to M$ form factors \cite{14,15,16}. We show that similar relations exist for B decays into multibody final states containing one collinear hadron $M_n$ plus soft hadrons $X_S$, $B \to M_nX_S$. Sec. 3 develops a chiral formalism for computing the matrix elements of the soft operators in \cite{2} ($X_S|O_S|B$) with $X_S$ containing only soft Goldstone bosons. As an application we discuss in Sec. 4 the semileptonic and rare radiative decays $B \to M_n\pi_S\ell\bar{\nu}$ and $B \to M_n\pi_S\ell^+\ell^-$.

2. Symmetry relations. The most general SCET$_I$ operator appearing in the matching of SM currents $q\Gamma b$ for
$b \rightarrow u\ell \nu$ or $b \rightarrow s \gamma$ decays has the form (we neglect light quark masses, which can be included as in (12))

$$J_{\mu}^{\text{eff}} = c_1(\omega) \hat{q}_n \gamma_\mu \gamma^5 P_L b_v + [c_2(\omega)c_\mu + c_3(\omega) n_\mu] \hat{q}_n \gamma_\mu P_R b_v$$

$$+ b_{1L}(\omega_1) J_{\mu}^{(1)}(\omega_1) + b_{1R}(\omega_1) J_{\mu}^{(1)}(\omega_1) + [b_{1s}(\omega_\mu) n_\mu] J_{\mu}^{(0)}(\omega_1)$$

(3)

These are the most general operators allowed by power counting and which contain a left-handed collinear quark. We neglect $O(\lambda)$ operators of the form $\hat{q}_n P_L^I \Gamma b_0$ which do not contribute below. The relevant modes are soft quarks and gluons with momenta $k_s \sim \Lambda$ and collinear quarks and gluons moving along $n$. $n_\mu, \bar{n}_\mu$ are unit light-cone vectors satisfying $n^2 = \bar{n}^2 = 0, n \cdot \bar{n} = 2$.

The $O(\lambda)$ operators are defined as

$$J_{\mu}^{(1,1R)}(\omega_1, \omega_2) = \hat{q}_n \omega_1 \Gamma_{\mu}^{(1,1R)} \left[ \frac{1}{n \cdot \mathcal{P}} i g \mathcal{B}_{\lambda} \right] \omega_2 b_v,$$

$$J_{\mu}^{(10)}(\omega_1, \omega_2) = \hat{q}_n \omega_1 \left[ \frac{1}{n \cdot \mathcal{P}} i g \mathcal{B}_{\lambda} \right] \omega_2 P_L b_v,$$

(4)

with $\{\Gamma_{\mu}^{(1L)}, \Gamma_{\mu}^{(1L)}\} = \{\gamma_\mu \gamma^5 P_R, \gamma_\mu \gamma^5 P_R\}$. The action of the collinear derivative $i \partial_\mu$ on collinear fields is given by the momentum label operator $\mathcal{P}_\mu = \frac{1}{2} n_\mu \hat{n} \cdot \mathcal{P} + P_R^\mu$. The collinear gluon field tensor is $i g \mathcal{B}_{\mu} = W^I[\hat{n} \cdot i Dc, i D^a_{\mu}] W$. The Wilson coefficients $c_i, b_i$ depend on the Dirac structure of the QCD current $\Gamma$ and are presently known to next-to-leading order in matching [17, 18].

After matching onto SCET$_{\text{I}}$, the effective current [4] contains the factorizable operators

$$J_{\mu}^{\text{fact}} = -\frac{1}{2\omega} \int dxdzdk_z b_{1L}(x, z) J_{\mu}(x, z, k_+$$

$$\times ((qY)_k \hat{q}_n \gamma_\mu \gamma^5 P_R(Y^\dagger b_v) |\hat{q}_n \omega_1 \hat{q}_n \omega_2)$$

$$- \frac{1}{2\omega} \int dxdzd\epsilon \bar{b}_{1R}(x, z) J_{\mu}(x, z, k_+$$

$$\times ((qY)_k \hat{q}_n \gamma_\mu \gamma^5 P_R(Y^\dagger b_v) |\hat{q}_n \omega_1 \hat{q}_n \omega_2)$$

$$- \frac{1}{2\omega} \int dxdzd\epsilon \bar{b}_{1s}(x, z) \epsilon \hat{b}_{1n}(x, z, n_\mu)$$

$$\times J_{\mu}(x, z, k_+)((qY)_k \hat{q}_n \gamma_\mu \gamma^5 P_L(Y^\dagger b_v) |\hat{q}_n \omega_1 \hat{q}_n \omega_2)$$

(5)

where $J_{\perp, \parallel}$ are jet functions defined as in [11]. We denoted here $\omega_1 = x\omega, \omega_2 = -\omega(1 - x), \omega = \omega_1 - \omega_2$. This has the factorized form of Eq. (2), with the Wilson coefficient $T$ given by $b_i \otimes J_{\mu, \perp}$.

The nonfactorizable operator $O_{\text{nf}}$ in Eq. (2) arises from matching the LO SCET$_I$ operators onto SCET$_{\text{I}}$. The precise form of the latter operators is not essential for our argument, which depends only on the Dirac structure of the SCET$_I$ operators. Before proceeding to write down

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
 & constraints & parameters & # of indep. parameters \\
\hline
\hline
QCD & $H_{n}^{V,A} \times H_{m}^{V,A}$ & $H_{n}^{V,A} \times H_{m}^{V,A}$ & 7 \\
SCET & $H_{n}^{V,A} \times H_{m}^{V,A}$ & $H_{n}^{V,A} \times H_{m}^{V,A}$ & 2 + 2 \\
1-body & $H_{\perp, \parallel}^{V,A} \times O_{\text{nf}}^{(m)}$ & $\zeta_{\perp, \parallel}^{(m)}$ & 2 + 1 \\
\hline
\end{tabular}
\caption{Counting the independent hadronic parameters required for a general $B \rightarrow M_n S_X$ decays in QCD, SCET and for a 1-body hadronic state $S_X = 0$.}
\end{table}

the SCET predictions for these matrix elements, we define more precisely the kinematics of the process.

The transition $B \rightarrow M_n S_X$ induced by the current $J_\mu = \bar{q} \Gamma_\mu b$ can be parameterized in terms of helicity amplitudes defined as

$$H_\mu^{(T)}(M_n, S_X) = \langle M_n S_X | \bar{q} \Gamma_\mu \gamma^5 b | B \rangle$$

(6)

with $\epsilon_{\perp, \parallel}^{(n)}$, a set of four orthogonal unit vectors defined in the rest frame of $v$ as $\epsilon_{\perp}^{(n)} = \frac{1}{\sqrt{2}}(0, 1, \mp i, 0)$, $\epsilon_{\parallel}^{(n)} = \frac{1}{\sqrt{2}}(|q|, 0, 0, |q|)$. These definitions correspond to the choice $n = (1, 0, 0, 1)$, $\bar{n} = (1, 0, 0, -1)$.

In the language of helicity amplitudes, the most general matrix elements of the nonfactorizable operators are given in terms of the 2 parameters

$$\langle M_n S_X | \bar{q}_n \omega_\perp P_L b_v | B \rangle = 2E_M \zeta_{\perp}(E_M, S_X)$$

$$\langle M_n S_X | \bar{q}_n \omega_\parallel P_L b_v | B \rangle = 2E_M \zeta_{\parallel}(E_M, S_X)$$

(7)

$\zeta_{\perp, \parallel}(E_M, S_X)$ are complex quantities depending on the momenta, spins and flavor of the particles in the final state.

The relations Eq. (7) imply several types of SCET predictions for the nonfactorizable contributions to the helicity amplitudes. The most important one is the vanishing of the right-handed (nonfactorizable) helicity amplitudes at leading order in $1/m_b$, for any current $\Gamma$ coupling only to left chiral collinear quarks

$$H_\mu^{(0)}(\bar{B} \rightarrow M_n S_X) = 0.$$  

(8)

For decays to one-body states, this constraint leads to the well-known large energy form factor relations $m_B/m_{B^{(*)}} V(E) = (m_B + m_{B^{(*)}}) V(E)/(2E) A_1(E)$ (for $\Gamma_{V,A} = \gamma_\mu P_L$) and $T(E) = m_B/(2E) T_2(E)$ (for $\Gamma_T = i\sigma_{\mu\nu} q^\nu P_R$) [14, 15, 18]. The argument above extends this result to hadrons of arbitrary spin and multibody states $M_n S_X$.

Another prediction is a relation between the time-like and longitudinal nonfactorizable contributions to the helicity amplitudes for an arbitrary current $\Gamma$,

$$H_\mu^{(0)}(B \rightarrow M_n S_X) = \frac{c_2(v \cdot \epsilon_{\mu}^{(0)}) + c_3(n \cdot \epsilon_{\mu}^{(0)})}{c_2(v \cdot \epsilon_{\mu}^{(0)}) + c_3(n \cdot \epsilon_{\mu}^{(0)})}$$

+ $O(\Lambda_{QCD}^2/m_b^2)$

(9)
Finally, SCET predicts also the ratio of helicity amplitudes mediated by different currents, into any state $M_nX_S$ containing one energetic collinear particle, e.g.

$$\frac{H^V(A)(B \to M_nX_S)}{H^V(A)(B \to M_nX_S)} = \frac{c_2(V-A)(E_M)}{c_1(T)(E_M)} + O\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

These relations are in general broken by the factorizable contributions from Eq. (5). For a 1-body state, this is forbidden by angular momentum conservation since the collinear part of the operator can only produce a longitudinally polarized meson. However, this constraint does not apply for multibody final states $M_nX_S$ (except in channels of well defined $J^P$ quantum numbers). In particular, this means that the helicity zero Eq. (5) receives corrections at leading order in $1/m_b$. These corrections are computed in Sec. 4.

The factorizable corrections to these relations are parameterized in terms of the soft functions

$$S^{LR}(k_+,S_X) = \langle X_S|\langle qY|k_+, \gamma_\mu \gamma_\nu P_R(Y^\dagger b_\nu)|\tilde{B}\rangle$$

$$S^{LL}(k_+,S_X) = \langle X_S|\langle qY|k_+, \gamma_\mu \gamma_\nu \gamma_\lambda P_L(Y^\dagger b_\nu)|\tilde{B}\rangle$$

$$S^{00}(k_+,S_X) = \langle X_S|\langle qY|k_+, \gamma_\mu \gamma_\nu P_L(Y^\dagger b_\nu)|\tilde{B}\rangle$$

Parity invariance of the strong interactions gives one relation among these functions in channels with $X_S^{J=0}$ of well-defined spin $J$ and intrinsic parity $(-)^J$

$$S^{LL}(k_+,S_X^{J=0}) = \langle X_S^{J=0}|\langle qY|k_+, \gamma_\mu \gamma_\nu P_R(Y^\dagger b_\nu)|\tilde{B}\rangle$$

$$S^{00}(k_+,S_X^{J=0}) = \langle X_S^{J=0}|\langle qY|k_+, \gamma_\mu \gamma_\nu P_L(Y^\dagger b_\nu)|\tilde{B}\rangle$$

where $\hat{P}$ is the parity operator and $\hat{R}_\pi$ the rotation operator by $180^\circ$ around the $y$ axis.

Compared with the decays into one-body hadronic states, for which only the soft function $S^{00}$ is required, this represents an increase in the number of independent parameters. However, the total number is still less than in QCD (see Table 1), such that predictive power is retained. In the next section we construct a chiral formalism which can be used to compute these matrix elements for any state $X_S$ containing only soft pions.

3. Chiral formalism. We construct here the representation of the soft operator $O_S$ giving the soft functions in (12) in the low energy chiral theory. Since we are interested in $B$ decays, the appropriate tool is the heavy hadron chiral perturbation theory developed in Refs. 20. The main result is that the matrix elements of $O_S$ depend only on the $B$ meson light cone wave function.

The effective Lagrangian that describes the low momentum interactions of the $B$ mesons with the pseudo-Goldstone bosons $\pi, K$ and $\eta$ is invariant under chiral $SU(3)_L \times SU(3)_R$ symmetry and under heavy quark spin symmetry. This requires the introduction of the heavy quark doublet $(B, \bar{B})$ as the relevant matter field. The chiral Lagrangian for matter fields such as the $B^{(*)}$ must be written in terms of velocity dependent fields, to preserve the validity of the chiral expansion.

The chiral effective Lagrangian describing the ground state mesons containing a heavy quark $Q$ is

$$\mathcal{L} = \frac{f^2}{8} Tr \left( \partial^\mu \Sigma \partial_\mu \Sigma^T \right) + \lambda_0 Tr \left[ m_q \Sigma + m_b \Sigma^T \right]$$

$$-i Tr \hat{H}(Q)^a \nu \partial^\mu \hat{H}_a^T$$

$$+ \frac{i}{2} Tr \hat{H}(Q)^a \nu_\mu \left[ \gamma_5 \partial_\mu \xi + \gamma_\mu \gamma_5 \xi^T \right]$$

$$+ \frac{i}{2} 2 Tr \hat{H}(Q)^a H_b^T \left[ \gamma_5 \partial^\mu \xi - \gamma^\mu \gamma_5 \xi^T \right]$$

where the ellipsis denote light quark mass terms, $O(1/m_b)$ operators associated with the breaking of heavy quark spin symmetry, and terms of higher order in the derivative expansion. The pseudoscalar and vector heavy meson fields $P_{a}(Q)$ and $P_{a^T}(Q)$ form the matrix

$$H_a(Q) = \frac{1 + \hat{P}}{2} \left[ P_{a^T}(Q)\gamma_5 - P_{a}(Q)\gamma_5 \right].$$

For $Q = b$, $(P_{1^T}(b), P_{2^T}(b), P_{3^T}(b)) = (B^-, \bar{B}^+, \bar{B}_s)$, and similarly for $P_{a^T}(b)$. The field $H_a(Q)$ transforms as a 3 under flavor $SU(3)_V$.

$$H_{a^T}(Q) \to H_{a^T}(Q) U_{ba}.$$
The symmetries of the theory contain also the form of operators such as currents. For example, the left handed current \( I_\ell^a = q_a \gamma^\nu P_L Q \) in QCD can be written in the low energy chiral theory as:

\[
I_\ell^a = \frac{i\alpha}{2} \text{Tr}[\gamma^\nu P_L H_b^a(Q) \xi_{\beta \mu}^a] + \ldots,
\]

(19)

where the ellipsis denote higher dimension operators in the chiral and heavy quark expansions. The parameter \( \alpha \) is obtained by taking the vacuum to B matrix element of the current, which gives \( \alpha = f_B \sqrt{m_B} \) (we use a non-relativistic normalization for the \( [B^(*)] \) states as in 20).

In the SCET we require also the matrix elements of nonlocal operators \( O_S \), which appear in Eq. (2). To leading order in \( 1/m_b \) these operators are quark bilinears

\[
O_{L,R}^a(k_+) = \int \frac{dx}{4\pi} e^{-\frac{i}{2} k^+ x} - q^a(x_+) Y_a(x_-,0) P_{R,L} \Gamma_{\mu}(0) .
\]

(20)

Under the chiral group they transform as \( (3L,1_R) \) and \( (1_L,3_R) \). In analogy with the local current 10 we write for \( O_{L,R}^a(k_+) \) in the chiral theory

\[
O_{L}^a(k_+) = \frac{i}{4} \text{Tr}[\hat{\alpha}_L(k_+) P_L \Gamma H_b^a(Q) \xi_{\beta \mu}^a] ,
\]

(21)

\[
O_{R}^a(k_+) = \frac{i}{4} \text{Tr}[\hat{\alpha}_R(k_+) P_L \Gamma H_b^a(Q) \xi_{\beta \mu}^a] ,
\]

(22)

where the most general form for \( \hat{\alpha}_L,R(k_+) \) depends on eight unknown functions \( a_i(k_+) \)

\[
\hat{\alpha}_L,R(k_+) = a_{1L,R} + a_{2L,R} \xi_k + a_{3L,R} \xi_f + \frac{1}{2} a_{4L,R} [\xi_k, \xi_f](23)
\]

The heavy quark symmetry constraint \( H^{(Q)} \xi = -H^{(Q)} \) reduces the number of these functions to four. Taking the vacuum to B meson matrix element fixes the remaining functions as

\[
\hat{\alpha}_L(k_+) = \hat{\alpha}_R(k_+) = f_B \sqrt{m_B} [\bar{\psi}\phi_+(k_+) + \bar{\psi}\phi_-(k_+)] (24)
\]

where \( \phi_{\pm}(k_+) \) are the usual light-cone wave functions of a B meson, defined by

\[
\int \frac{dz}{4\pi} e^{-\frac{i}{2} k^+ z} \langle 0|\bar{q}(z_-) Y_n(z_-,0) b_c(0)|\bar{B}(v)\rangle = \frac{i}{4} f_B \sqrt{m_B} \left\{ \frac{1}{2} \frac{\gamma_5}{2} \right\}_{ij} \]

(25)

We find thus the remarkable result that the B meson light-cone wave functions are sufficient to fix the pion matrix elements of the nonlocal operators \( O_{L,R}^a(k_+) \).

The same result can be obtained also by considering only local operators. Let us consider the operator \( O_L^a(k_+) \) (the same results are obtained for \( O_R^a(k_+) \)). Expanding in a power series of the distance along the light cone one is led to consider the matrix elements of the operator symmetric and traceless in its indices

\[
O_{L,R}^{\mu_1 \mu_2 \cdots \mu_N}(k_+) = \bar{q}^a(-i\not{D})(\mu_1 \cdots (-i\not{D})^\mu_N) P_R \Gamma_{\mu}(0)
\]

(26)

where the sum over \( \mu \) includes the most general symmetric and traceless structures \( X \) formed from \( \gamma_\mu, \nu_\mu, g_{\mu\nu} \).

There are many such structures, but only 2 of them survive when contracted with \( n_{\mu_1} \cdots n_{\mu_N} \)

\[
X_0^{\mu_1 \mu_2 \cdots \mu_N} = (\nu_\mu \nu_{\mu_2} \cdots \nu_{\mu_N}) - (g_{\mu_1 \mu_2} - \text{terms}) (28)
\]

\[
X_1^{\mu_1 \mu_2 \cdots \mu_N} = \gamma_\mu (\nu_\mu \nu_{\mu_2} \cdots \nu_{\mu_N}) - (g_{\mu_1 \mu_2} - \text{terms}) (29)
\]

This gives the chiral representation of the projection of the operators (26) on the light-cone

\[
\bar{q}^a(-i\not{D})^N P_R \Gamma_{\mu}(0)
\]

(29)

which makes it clear that the constants \( \alpha_{N,0}, \alpha_{N,1} \) are uniquely fixed in terms of the \( B \rightarrow \) vacuum matrix elements of the operators (26). Assuming that the B light-cone wave functions are well behaved at large \( k_+ \), these matrix elements are related to the moments of \( \phi_{\pm}(k_+) \).

Specifically, one finds

\[
\alpha_{N,0} = -2f_B \sqrt{m_B} \int dk_+ (k_+) \phi_+(k_+) .
\]

(30)

In particular, for \( N = 1 \) this gives \( \alpha_{1,0} = -2f_B \sqrt{m_B} A \), which agrees with Ref. 21.

Beyond leading order in \( 1/m_b \) many more operators can be written. For example, the matrix elements of \( O_L^a \) with one insertion of the chromomagnetic term in the HQET Lagrangian \( L_m = g_b \sigma_{\mu
u} G^{\mu
u} b_c \) gives structures of the form

\[
T\{O_L^a, iL_m\} \rightarrow \text{Tr}[X^{\mu\nu} P_R \Gamma_{\mu} \frac{1}{2} \frac{\gamma_5}{2} i\sigma_{\mu\nu} H_b^a(Q) \xi_{\beta \mu}^a]
\]

(31)

with \( X^{\mu\nu} = \beta_1 [n^\mu, \gamma^\nu] + \beta_2 i\sigma^{\mu\nu} + \beta_3 [n^\mu, \gamma^\nu] + \beta_4 i\sigma^{\mu\nu} \).

The proliferation of unknown constants (see also 23) spoils the simple leading order result that knowledge of the \( B \rightarrow \) vacuum matrix element is sufficient to fix all low energy constants.

The operators in Eqs. (21), (22) (together with (24)) give the desired representation of the soft operators \( O_{L,R} \) in the chiral effective theory, and can be used to compute their matrix elements on states with a B meson and any number of pseudo Goldstone bosons.
4. **Application:** \( \bar{B} \to M_n \pi \) and \( \bar{B} \to M_n \pi \ell^+ \ell^- \). As an application we compute the factorizable corrections to the symmetry relations \([3, 10]\) for the transverse helicity amplitudes in \( \bar{B} \to M_n \pi \ell^0 \) in the region of the phase space with one energetic meson \( M_n = \pi, \rho, K^* \), etc. plus one soft pion. The factorizable contribution to the transverse helicity amplitudes for \( B \to M_n \pi \) are given by the matrix elements of Eq. \((4)\). Specifically, one has for \( M_n \) a pseudoscalar meson

\[
\begin{align*}
H^\text{fact}_+(\bar{B} \to P_n(k)X_S) &= \\
&= C \frac{1}{f_B f_{PnB}} S_R(X_S) \langle b_{1R} \phi_P \rangle
\end{align*}
\]

and for \( M_n \) a vector meson

\[
\begin{align*}
H^\text{fact}_+(\bar{B} \to V_n(k, \eta)X_S) &= \\
&= C \frac{f_B f_{V\pi B} m_B}{8n \cdot \eta v} S_R(X_S) \langle \bar{b}_{1R} \phi_V \rangle
\end{align*}
\]

\[
\begin{align*}
H^\text{fact}_-(\bar{B} \to V_n(k, \eta)X_S) &= \\
&= - \frac{1}{4} C f_B f_{V\pi B} S_L(X_S) \langle \bar{\epsilon}_n \cdot \eta^* \rangle \langle b_{1L} \phi_V \rangle
\end{align*}
\]

We used here the short notation \( \langle b_{1a} \phi^0 \rangle = \int dxdzdk_\perp b_1(x, z) J_a(x, z, k_\perp) \phi^0(x) \). The isospin factor \( C(\rho^0) = 1/\sqrt{2}, C(\rho^\pm) = 1 \). The corresponding results for the 1-body factorizable decay amplitudes are obtained from these expressions by taking \( S_R \to 0, S_L \to 1 \).

Inspection of the results \([32, 33]\) gives the following conclusions, valid to all orders in \( a_s \).

i) The null result in Eq. \((33)\) means that the symmetry relation \([10]\) for \( B \to P_n X_S \) transitions is not broken by factorizable corrections and is thus exact to leading order in \( 1/m_b \). This leads, e.g., to a relation between \( \bar{B} \to (\bar{K}_n \pi)_{h=1} \gamma^+ \gamma^- \) and \( \bar{B} \to (\pi_n \pi S)_{h=1} e^+ e^- \).

ii) The vanishing of the \( H_+ \) nonfactorizable helicity amplitudes in \( B \) decay Eq. \((3)\) is violated by the factorizable terms Eqs. \([(32), (34)]\). These terms are however calculable in chiral perturbation theory for \( X_S \) containing only soft pions. For both \( M_n = P, V \), the pion carries \( m_\pi = +1 \) angular momentum; the \( V_n \) collinear meson is emitted longitudinally polarized.

The soft functions \( S_{R,L}(p_\pi) \) in Eq. \((32), (35)\) can be computed explicitly in terms of the chiral perturbation theory diagrams in Fig. 1. We find

\[
S_R(p_\pi) = \frac{g}{f_\pi} \frac{\varepsilon_\pi \cdot p_\pi}{v \cdot p_\pi + \Delta - i\Gamma_B/2} \tag{36}
\]

\[
S_L(p_\pi) = \frac{1}{f_\pi} \left( 1 - g \frac{\varepsilon_\pi \cdot p_\pi}{v \cdot p_\pi + \Delta - i\Gamma_B/2} \right) \tag{37}
\]

with \( \Delta = m_B^2 - m_B \simeq 50 \text{ MeV} \) and \( \Gamma_B \), the width of the \( B^* \) meson. While the soft matrix elements in Eq. \((12)\) have a factorized form \( S^{(i)}(k_+X_S) = \phi_+(k_+)S_i(S_X) \), the total factorizable amplitude is not simply the product \( B \to B^* \pi \) times \( B^* \to M_n \), due to the direct graph in Fig. 1a (nonvanishing only for \( S_L \)). At threshold, the relation Eq. \((37)\) gives a soft pion theorem which fixes the soft function in \( B \to M_n \pi \) in terms of the factorizable contribution to the \( B \to M_n \) transition. Note that the \( B^* \) width in the propagator is a source of strong phases at leading order in \( 1/m_b \). These results can be extended to final states containing multiple soft pions, without introducing any new unknown hadronic parameters.

5. **Conclusions.** We presented in this paper the application of the soft-collinear effective theory to \( B \) decays into multibody final states, containing one energetic meson plus soft pseudo Goldstone bosons. The additional ingredient is the application of heavy hadron chiral perturbation theory \([20]\) to compute the matrix elements with Goldstone bosons of the nonlocal soft operators obtained after factorization. (This assumes that the only SCET operators contributing to these decays are the same as those describing \( B \to M_n \) transitions \([3]\).) Heavy quark and chiral symmetry are powerful constraints which fix all these couplings in terms of the usual \( B \) light-cone wave functions. This simplicity should be contrasted with the case of the twist-2 DIS and DVCS operators, whose matrix elements on nucleons plus soft pions require additional couplings not constrained by the nucleon structure functions \([3]\).

Some of the symmetry predictions of SCET rely on angular momentum conservation arguments which are invalidated when the final hadronic state contains more than one hadron (see Eq. \((3)\)), already at leading order in the \( 1/m_b \) expansion. The chiral formalism presented here allows the systematic computation of these effects. We point out the existence of an exact relation Eq. \((10)\) among left-handed helicity amplitudes in \( \bar{B} \to P_n X_S \) transitions induced by different \( b \to q_n \) currents.

These results extend the applicability of SCET to \( B \) decays into multibody states \( M_n X_S \) containing one energetic particle. It is interesting to note that the corrections to these predictions scale like \( \max (M/E_M, p_S/\Lambda_{\chi PT}) \), rather than \( M X/E_M \). This suggests that the range of validity of factorization in these decays might be wider than previously thought, a fact noted empirically in Refs. \([22]\) in the context of the \( B \to DX \) decays. Many more problems can be studied using the formalism described here,
e.g., the leading SU(3) violating contributions to the factorizable contributions, analogous to the effects considered in Ref. \[24].

We are grateful to Martin Savage for useful discussions. B.G. was supported in part by the DOE under Grant DE-FG03-97ER40546. D.P. was supported by the U.S. Department of Energy under cooperative research agreement DOE-FC02-94ER40818.

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