A Novel Tensor RPCA Based Background Subtraction Method for Outdoor Imaging with a Low Cost Portable Radar

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ABSTRACT
Measurements acquired by low cost, low weighted, portable ultra-wide-band (UWB) radar systems are highly affected by measurement noise and clutter compared to the anechoic chamber vector network analyzer (VNA) measurements. Efficient background subtraction is vital prior to any target detection and recognition application. Low rank and sparse decomposition (LRSD) methods can be used to decompose radar data into its low rank and sparse components corresponding to target and background. In this study, instead of using robust principal component analysis (RPCA) which has high complexity due to successive singular value decomposition (SVD) operations in its iterations, we propose a tensor based method for background subtraction. Target response corresponding to an antenna location is recast into a matrix form with a size much smaller than the original data matrix. The concatenation of the responses for all antenna locations through the synthetic aperture form the data tensor which is divided using tensor RPCA (TRPCA). Experimental results show that the proposed method outperforms RPCA visually for appropriate choices of the regularization parameter with a decrease of 4–16 times in computation time.

INDEX TERMS Ultra wideband (UWB) radar, outdoor radar imaging, background subtraction, robust principal component analysis (RPCA), tensor RPCA (TRPCA)

I. INTRODUCTION
Developments in ultra wide-band (UWB) radar technology in the recent years have made it possible to develop low-dimensional and cost-effective yet high-resolution radar sensors [1]. Nowadays, UWB radar systems have found use not only in the military field, but also in civilian life due to their abilities such as high range resolution and good penetration capability. Motion estimation in the home environment, monitoring of patients’ vital signs, agricultural applications, through wall object detection and target tracking can be cited as some of these applications. Target detection, recognition and tracking in outdoor, indoor and behind obstacle environments can be made from the measurements obtained using these systems [2]–[7]. However, compared to the measurements made with vector network analyzer (VAA) in anechoic room conditions, the data obtained from these systems are more affected by noise. In particular, clutter effect, which is defined as the reflections from non-target stationary structures in the environment, is strong enough to mask the target in some cases, and reduces the performance of the methods.

While clutter can be caused by stationary objects in the observed area when viewing moving targets, it can also be caused by the signal reflected back from the wall/soil surface in through the wall/ground penetrating radar imaging. Thus many clutter removal methods proposed for ground penetrating radar (GPR) or through wall radar imaging (TWRI) can be used for background subtraction in outdoor radar imaging [8], [9].

Besides the simplest mean subtraction (MS) method, subspace based methods such as singular value decomposition (SVD), principal component analysis (PCA) or independent component analysis (ICA) have been widely used for clutter removal [10], [12], [13]. In these methods, the data collected by the radar is divided into clutter and target subspaces: wall returns in TWRI or reflections for soil surface in GPR are...
represented by the clutter component while the reflections from the target are represented by the target component. Since the clutter is much stronger than the target returns, a simple inspection of the eigenvalues of the correlation matrix of the collected data enables this separation [10].

Low rank and sparse decompositions (LRSD) is, especially robust principal component analysis (RPCA) [11], most famous one of these decompositions, have found many uses in image processing and imaging applications such as background subtraction in video, moving object detection, missing entries recovery (inpainting), denoising, anomaly detection, complex scene analysis in radar imaging, synthetic-aperture radar (SAR) interferometry, dynamic magnetic resonance imaging (MRI), clutter removal in GPR, wall mitigation in TWRI in the last decade [10]–[17]. LRSD mainly decomposes an input image into its low rank and sparse components by rank minimization with \( l_0 \) constraint for sparsity penalization. In RPCA this optimization is achieved by means of nuclear norm relaxation and \( l_1 \) norm minimization for low rank and sparse components, respectively.

In video processing background information which remains unchanged between successive frames is provided by the low rank component while the changes in the foreground are captured by the sparse component. Any outlier such as anomalies, missing pixels or noise can be thought as sparse, thus removed from the input image by recovering the low rank component.

SAR imaging assumes that all the targets are stationary in the imaging scene while the synthetic aperture is provided by the moving radar. Unfortunately this assumption does not hold for scenarios which contain moving targets besides the moving ones. Such situations, namely complex scenes, may occur frequently in real life scenarios and moving targets may cause severe artifacts in the imaging results. [15] proposes a solution to this issue. The range compressed SAR data is divided into low rank and sparse components corresponding to stationary target and background returns and moving target echoes, respectively. In [16] and [17] the authors by the use of RPCA and provide a detailed analysis on the choice of the penalization parameter \( \lambda \). Motion parameters are estimated by the sparse components provided by the RPCA decomposition of the SAR data and used to focus the moving targets in the imaging scene. To the authors knowledge these works constitute the first applications of RPCA in the radar imaging field.

RPCA has also been used in through obstacle imaging. In GPR, the data is divided into low rank and sparse components representing the clutter and target part, since the target can be considered to be sparse when comparing it to the whole image [18], [19]. Although it shows promising results, due to the successive SVD operations to update the low rank component in its iterations, RPCA has high complexity limiting its use in the field studies [20].

Since RPCA suffers from high complexity and need for successive SVD operations [21], several approaches have been proposed. Go decomposition (GoDec) [25] uses bilateral random projections (BRP) instead of SVD in its iterations. Robust non-negative matrix factorization (RNMF) [26] approximates the low rank components by matrix decomposition where the matrices are initialized randomly. The sparse component representing the target image is recovered in each iteration by subtracting the low rank component from the residual image. Both methods are SVD free but sensitive to parameter selection. An appropriate fine tuning is required to prevent the suppression of target information besides the clutter or background. Robust orthonormal subspace learning (ROSL) is a non–convex relaxation of RPCA where the nuclear norm minimization is replaced with the group sparsity of the coefficients under orthonormal subspace for the modelling of the low rank part [27]. The method does not require any choice for the rank value and is more robust to the choice of the penalization parameter value compared to RNMF.

Recently, Tensor RPCA (TRPCA) is proposed to deal with 3D data in video processing where the successive frames are cast into a tensor structure and sparse component provided by TRPCA represents foreground information or changes between successive frames [21]. Similarly in [15] multipass temporal SAR images are processed in tensor format to recover outlier free interferometric SAR (inSAR) data stacks by means of a total variation (TV) regularized robust low rank tensor decomposition method. Here the echoes due to stationary targets form the low rank part of the data matrix and those due to moving targets constitute the sparse part. Since Dynamic MRI data are acquired in both spatial and temporal dimensions, it benefits from tensor representation. The input data is either a 3D image of an organ or a 2D image which is changing with time. The reconstruction procedure is defined as a high-order low rank tensor plus sparse tensor decomposition problem [14].

In [22], C-scans are processed simultaneously to obtain the clutter free GPR data. As a different approach, tensor concept [23] is also incorporated to use frequency information besides the spatial one. The image is transformed into time-frequency domain using wavelet or contourlet transforms, where the resulting time-frequency images are recast into tensor form. A tensor version of RPCA provides low rank and sparse tensors followed by inverse transform to provides target and clutter components. Another recent work [17] is proposed to improve [24] which fails for targets that are slowly moving, or moving in a direction that is perpendicular to the direction of the SAR platform. The partially overlapped subsections of the SAR data are used to construct the tensor data whose decomposition gives better estimates for motion parameters thus better compensation results.

In this paper, we propose a new background subtraction method for outdoor imaging where the tensor concept is used to decrease the size of the raw radar data matrix, thus making the algorithm require SVD decompositions of smaller matrices in its iterations and the resulting algorithm will be faster. Each column of the raw data matrix is recast into a sub-matrix which is concatenated to form a tensor.
with much smaller size than the original raw data matrix. TRPCA is used to perform LRSD decomposition and the resulting sparse tensor is recast to the matrix form to provide the target component. Our approach for the tensor modeling of the input data is rather different than the previous works which use 3D data (temporal images) or 2D data synthetically extended to 3D with the use of time-frequency analysis or windowing or sub-aperture approach. Data were collected from moving targets in the outdoor environment with a low-cost, low-weight (≈300 gr) UWB radar, and background subtraction was performed in order to increase performance in applications such as imaging or target recognition [6].

The paper is organized as follows. Section II reviews RPCA based clutter removal for outdoor radar data. The pre-transformation step and TRPCA based LRSD decomposition are described in Section III. The radar measurements accomplished in Istanbul Technical University (ITU) campus are by a light-weight and portable radar and background removal results obtained by the proposed TRPCA based method as well as by mean subtraction, SVD, PCA and RPCA are given in Section IV. Section V concludes the paper.

II. REVIEW OF RPCA BASED BACKGROUND SUBTRACTION

Let $E^s(t, x)$ be the range compressed data, namely range time intensity (RTI) image where $t$ and $x$ denote fast and slow range, i.e. received data for a single synthetic aperture location and pulses sent along the synthetic aperture formation.

$$E^s = E^s_c + E^s_t$$

where $E^s_c \in \mathbb{R}^{N_1 \times N_2}$ and $E^s_t \in \mathbb{R}^{N_1 \times N_2}$ denote the clutter and target components, respectively [10]. It has been shown in [20] that the clutter component is much stronger than the target one and can be represented by a low rank matrix while the target which is sparse compared to the whole image can be modelled as a sparse matrix. Thus, a standard LRSD decomposition procedure with appropriate parameters can provide the target and clutter components. The conventional RPCA [28] performs this decomposition by solving the constrained optimization problem

$$\min_{L,S} \|L\|_* + \lambda \|S\|_1 \quad \text{s.t.} \quad E^s = L + S \quad (1)$$

where $\|\cdot\|_*$, $\|\cdot\|_1$, and $\lambda$ stand for the nuclear norm, L1-norm, and penalization parameter, respectively. $L \in \mathbb{R}^{N_1 \times N_2}$ and $S \in \mathbb{R}^{N_1 \times N_2}$ denote low rank and sparse components of the RPCA based decomposition. Eq. (2) can be adapted to the radar data where $E^s_c$, and $E^s_t$, correspond to clutter and target components respectively.

$$\min_{E^s_c,E^s_t} \|E^s_c\|_* + \lambda \|E^s_t\|_1 \quad \text{s.t.} \quad E^s = E^s_c + E^s_t \quad (2)$$

In the case of applications where data size is small, Principal Component Pursuit (PCP) can be executed using interior point methods which are considered as off-the-shelf tools. Different from the previous technique, this algorithm uses non-smooth convex optimization to minimize $L1$ norm and the nuclear norms. Although these methods have a high convergence rate, they can only be applied in small problems and this is a major drawback for most real-world applications [29].

The accelerated proximal gradient algorithm (APG) offers a better solution compared to the interior point methods, where the limitations on scalability was a concern. It utilizes iterative thresholding to minimize the $l_1$ norm and enhances the convergence rate by applying continuation techniques or smooth optimization [30].

APG converges much faster than any of its predecessor iterative threshold methods in solving the convex PCP. Regardless of the improved results that come along with APG, its performances deteriorates for higher size data due to high number of iterations for convergence. An Augmented Lagrange Multiplier (ALM) can be implemented to find the minimal nuclear norm yielding a higher accuracy while performing less iterations

$$\mathcal{L}(E^s_c, E^s_t, Y) = \|E^s_c\|_* + \lambda \|E^s_t\|_1 + (Y, M - E^s_c - E^s_t) + \frac{\mu}{2} \|M - E^s_c - E^s_t\|_F^2 \quad (4)$$

where $Y$ is $n \times r$ matrix with entries independently sampled from a $N(0, 1/n)$ distribution and $\lambda$, $\mu$ are the Lagrange multipliers. The solution is obtained by minimizing $\mathcal{L}$ with respect to $E^s_c$ and $E^s_t$, fixing one at a time:

$$\arg\min_x \mathcal{L}(E^s_c, E^s_t, Y) = S_{X_n}(M - E^s_c + \mu^{-1}Y) \quad (5)$$

$$\arg\min_x \mathcal{L}(E^s_c, E^s_t, Y) = D_{\mu}(M - E^s_t + \mu^{-1}Y) \quad (6)$$

considering that $S_{X_n}(1)\text{sgn}(x)\max(|x|, -\tau, 0)$ is the shrinkage operator, $D_{\mu}(x) = US\tau(\Sigma)V^*$ is the singular value thresholding operator and $X = US\Sigma V^*$ is any singular value decomposition (SVD).

The ALM method offers the best results, considering robustness and scalability of data.

III. PROPOSED METHOD

TRPCA uses an extension of RPCA by incorporating tensor concepts [21]. Eq. (2) is reformulated by replacing the matrices with the corresponding tensor representations. To simplify the expression, the superscript “s” is omitted in Eq. (3). Then, the optimization function can be expressed as

$$\min_{E^s_c,E^s_t} \|E^s_c\|_* + \lambda \|E^s_t\|_1 \quad \text{s.t.} \quad E^s = E^s_c + E^s_t \quad (7)$$

where $E^s_c$ and $E^s_t$ denote original Range time intensity (RTI), clutter component and target component tensors.
Since the input of the TRPCA method should be a tensor image, original RTI data $E_s$ which is a 2D matrix has to be converted into a tensor image $E$. Let the RTI image $E_s \in \mathbb{R}^{M \times N}$ be represented as

$$
\begin{array}{cccc}
\varepsilon_{1,1} & \cdots & \varepsilon_{1,N} \\
\varepsilon_{2,1} & \cdots & \varepsilon_{2,N} \\
\vdots & \ddots & \vdots \\
\varepsilon_{M,1} & \cdots & \varepsilon_{M,N} \\
\end{array}
$$

where $\varepsilon$ is the coefficient and $M$ and $N$ represent the indexes of the coefficients and A-scans, respectively. Then, each A-scan is converted into a 2D square matrix and concatenated to each other to construct a 3D data or a tensor $Y \in \mathbb{R}^{M \times \sqrt{M} \times N}$ by applying pre-transformation process as given in Eq. (9) where each patch corresponds to the related A-scan and it ensures to keep the structural information as patch-image.

After the pre-transformation process, tensor $Y$ can be given as input to the TRPCA method.

$$
\mathcal{L}(\varepsilon_c, \varepsilon_t, y, \mu) = ||\varepsilon_c||_s + \lambda ||\varepsilon_t||_1 + y^T(\varepsilon_c + \varepsilon_t - \mathcal{E}) + \frac{\mu}{2} ||\varepsilon_c + \varepsilon_t - \mathcal{E}||_F^2
$$

(10)

where $y$ is a Lagrange multiplier and $\mu$ is a scalar parameter. The constrained optimization problem in Eq. (7) can be solved by using alternating direction method of multipliers (ADMM) [21]

$$
\begin{align*}
\varepsilon_c^{k+1} &= \underset{\varepsilon_c}{\text{argmin}} \ ||\varepsilon_c||_s + \frac{\mu_k}{2} ||\varepsilon_c + \varepsilon_t^k - \mathcal{E} + \frac{y_k}{\mu_k}||_F^2 \\
\varepsilon_t^{k+1} &= \underset{\varepsilon_t}{\text{argmin}} \ \lambda ||\varepsilon_t||_1 + \frac{\mu_k}{2} ||\varepsilon_c^{k+1} + \varepsilon_t - \mathcal{E} + \frac{y_k}{\mu_k}||_F^2 \\
y_k &= y_k + \mu_k(\varepsilon_c^{k+1} + \varepsilon_t^{k+1} - \mathcal{E})
\end{align*}
$$

(11)

(12)

(13)

Equation (6) can be solved as

$$
\varepsilon_c^{k+1} = D_{1/\mu}(\mathcal{E} - \varepsilon_t - \frac{y_k}{\mu_k})
$$

(14)

$D_{1/\mu}$ stands for tensor-singular value thresholding (t-SVT) operator

$$
D_{1/\mu} = \mathcal{U} * S_{1/\mu} * V^* 
$$

(15)

where $\mathcal{U} \in \mathbb{R}^{N_1 \times N_2 \times N_3}$ and $V \in \mathbb{R}^{N_1 \times N_2 \times N_3}$ are orthogonal, and $S \in \mathbb{R}^{N_1 \times N_2 \times N_3}$ is an f-diagonal tensor.
Algorithm 1

**Input:**
- Radar data matrix $E^s \in \mathbb{R}^{N_1 \times N_2}$

1. Obtain image tensor $E \in \mathbb{R}^{N_1 \times N_2 \times N_3}$ via pre-transformation using (4), (5)
2. Initialize $c^0 = c^0 = y^0 = 0$, $\mu = 2$, $\epsilon = 1e-6$, $N = 1e4$
3. Main while not converged do
   - $c^{k+1} \leftarrow D_{1/\mu}(E - E^0 - \frac{y^k}{\mu})$
   - $y^{k+1} \leftarrow \text{shrink}(E - c^{k+1} - \frac{y_k}{\mu_k})$
   - $E \leftarrow \frac{E^{k+1} + E^0 + y_k}{\mu_k}$
   - $E^k \leftarrow E^{k+1}$
   - $\text{if } \max(c_1, c_2) < \epsilon \text{ or } k > N \text{ then break}$
   - $k \leftarrow k + 1$
end if
end while
4. Convert target tensor $E \in \mathbb{R}^{N_1 \times N_2 \times N_3}$ into target matrix $E_t^s \in \mathbb{R}^{N_1 \times N_2}$

**Output:**
- Target matrix $E_t^s \in \mathbb{R}^{N_1 \times N_2}$

$$S_{1/\mu} = \text{ifft}((\tilde{S} - 1/\mu)_+, [\cdot, \cdot, 3])$$

$$(\cdot)_+$$ denotes the positive-time part of $$(\cdot)$$

$S = \text{ifft}(S, [\cdot, \cdot, 3])$$ performs discrete Fourier transform (DFT) on all the tubes of $D$ (or along the 3rd dimension) the tube defines $S(i, j, \cdot): A \ast B$ denotes the product of tensors $A$ and $B$. More detailed information about t-SVD and TRPCA can be found in [21]. Target tensor $E_t$ is updated by shrinkage as

$$E_t^{k+1} = S_{1/\mu}(E - E_c^{k+1} - \frac{y_k}{\mu_k})$$

$$S_{\lambda/\mu}(\cdot) = \text{shrink}(E, \frac{\lambda}{\mu}) = \begin{cases} E - \frac{\lambda}{\mu}, & E > \frac{\lambda}{\mu} \\ 0, & |E| \leq \frac{\lambda}{\mu} \\ E + \frac{\lambda}{\mu}, & E < \frac{\lambda}{\mu} \end{cases}$$

Similar to RPCA [28], the penalization parameter $\lambda$ is generally chosen as

$$\lambda = \frac{1}{\sqrt{\text{max}(N_1, N_2) \times N_3}}$$

**FIGURE 2.** (a) Radar setup, and (b) measurement setup.

IV. EXPERIMENTS AND RESULTS

The proposed TRPCA based background subtraction method is applied to the experimental data measured by a portable low weight radar as shown in Fig. 2(a) and (b). The experimental data is measured at several different locations in the ITU campus area and is shown in Fig. 3(a)–(c) and Fig. 6(a) and (b), respectively. Fig. 3(b) shows our first measurement: three corner reflectors are arranged in “I” shape (data I with size 1200 × 350). For the second measurement corner reflectors are arranged in “L” shape (data II with size 1200 × 350) as shown in Fig. 3(c). Then, the location of the radar module is changed and located at the faculty entrance as given in the Fig. 6(a) and the measurements (data III with size 1344 × 1000) are collected from the moving objects such as people, cars, etc. The final experimental data is shown in the Fig. 6(b), the measurements (data IV with size 2208 × 200) are collected from the moving car in the empty parking lot.

For the visual results, besides the proposed method, conventional background subtraction methods MS, PCA, SVD and RPCA have been selected for comparison [10], [13], [21]. The aforementioned methods other than RPCA are parameter free. However, the $\lambda$ parameter has to be chosen in RPCA and its value is crucial for clutter/target separation. In literature, this parameter has to be chosen by the formula and fine tuning is found by grid search in the vicinity of the value computed by the formula. The same formula is also applicable for the proposed TRPCA method.

In our trials, it is observed that the fine-tuned $\lambda$ parameter instead of the one computed by the formula produces better results for our radar data measurements. Therefore, RPCA results obtained for $\lambda$ given by the formula and fine tuning are mentioned as RPCA and RPCA$_{\text{best}}$, respectively in the following results. The fine-tuned $\lambda$ is found by grid search in the vicinity of the value computed by the formula. The same formula is also applicable for the proposed TRPCA method.

As in the RPCA, the fine-tuned $\lambda$ parameter produced better results and they are given as TRPCA$_{\text{best}}$.

Time domain impulse radar PULSON 440 is used to collect the radar data. An anisotropic antenna and an horn to provide the columns of the target matrix. A pseudo-code for the proposed method is given in Algorithm [1].
antenna are used for transmission and receiving of radar pulses. The radar operates at 3.1–5.3 GHz and the pulse repetition frequency is 10 MHz. To enhance the signal-to-noise ratio (SNR) coherent pulse integration is chosen as 1024. The range is between 3–10 meters.

As the first experiment the radar was mounted on a car as shown in Fig. 3(a) and measurements from stationary corner reflectors have been collected as can be observed in Fig. 3(b)–(c). The radar targets are chosen as three corner reflectors placed one after another (“I” shape) or in “L” shape as presented in Fig. 3(b) and (c), respectively.

RTI image for the scenario (3 corner reflectors grouped in “I” shape) is shown in Fig. 3(b) and background subtraction results for MS, SVD, PCA, RPCA and the proposed TRPCA are given Fig. 4(a)–(f), respectively. The raw data is corrupted by noise and strong clutter due to the soil returns as observed at the top of the RTI image. Conventional MS and SVD methods fail to subtract the background successfully and soil returns are still visible in the RTI images. PCA has the worst performance among all, the target image is totally corrupted by vertical lines obscuring the target visibility. RPCA result presents high amount of noise for the conventional choice of λ parameter, while the target is almost lost in the proposed method result for λ parameter value. With fine tuning of the λ parameter both methods recover the targets well enough. Results of both methods have high dynamic range as seen in Fig. 4(g) and (h).

RTI image for the scenario 3 (corner reflectors grouped in “L” shape) is shown in Fig. 3(c). Raw data and background subtraction results for MS, SVD, PCA, RPCA and the proposed TRPCA are presented in Fig. 5(a)–(f), respectively. Again, the raw data is corrupted by noise and strong clutter due to the soil returns. Conventional MS fails to subtract the background successfully. Soil returns are still visible in the RTI image. PCA still presents vertical lines and target signatures are barely observed. SVD results seem to be better than the result obtained for “I” shape located targets, however dynamic range is narrow. RPCA result presents high amount of noise but an appropriate choice of λ provides background free target image with increased dynamic range. The proposed method again suppresses a large amount of target components for the conventional choice of λ however, with fine tuning of the λ parameter, the method reaches the performance of RPCA.

As a second experiment, the radar module observed in Fig. 2(a) is located at the faculty entrance and reflections from moving objects (people, cars etc.) are collected as seen in Fig. 7(a).

At the top and bottom of RTI image, reflections from static objects in the scene such as borders of the road is clearly observed as strong horizontal lines. Walking people are not clearly visible due to highly cluttered environment. The background subtraction results obtained by MS, PCA, SVD, RPCA, the proposed TRPCA are shown Fig. 7(b)–(f). Although some decrease in the clutter is observed especially at the bottom of the images For MS, SVD and PCA, the distinctions of the targets from the background is not obvious due to small dynamic range. The clutter is almost completely eliminated in the RPCA result with conventional choice of λ value. It is possible to obtain further elimination for other choices of the parameter at the expense of sum target information. Fig. 7(f) clearly demonstrates that the proposed method achieves the performance of RPCA and surpasses it with fine tuning of the regularization parameter.
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FIGURE 4. (a) Original RTI image for corner reflectors grouped in "I" shape (Data I), and background subtraction results for: (b) MS, (c) PCA, (d) SVD, (e) RPCA, (f) TRPCA, (g) RPCA_{Best}, and (h) TRPCA_{Best}.

FIGURE 5. (a) Original RTI image for corner reflectors grouped in "L" shape (Data II), and background subtraction results for: (b) MS, (c) PCA, (d) SVD, (e) RPCA, (f) TRPCA, (g) RPCA_{Best}, and (h) TRPCA_{Best}.
In the last example, the measurements are taken from a car passing in front of the radar as shown in Fig. 6(b). RTI images for raw data and background subtraction results for MS, SVD, PCA, RPCA and the proposed TRPCA are given Fig. 8(a)–(f), respectively. All the comparison methods again fail to subtract the returns of the static objects in the measurement scene. RPCA provides a higher dynamic range increasing the distinction of the target from the background however the RTI image still presents background information. Although RPCA provides clearer backgrounds with an appropriate λ parameter value, TRPCA performance does not depend on the parameter value for this experiment as seen in Fig. 8(f) and (h). We should remark that car returns are much stronger than the human target returns of the previous example. Thus, target component loss that occurred for human targets and even for corner reflectors is not encountered for this measurement.

The Error vs. number of iteration plots for the RPCA and the proposed TRPCA methods are presented in Fig. 9(a) and (b), respectively. There are two stopping conditions for both RPCA and TRPCA which can be named as minimum acceptable error value or maximum iteration number. If one of them is met, the algorithm stops. In our case, minimum acceptable error value is chosen as $1 \times 10^{-6}$ for both RPCA and proposed TRPCA. As seen in the Fig. 9(b), the TRPCA reaches the minimum error value in a very few iterations around 50. However, RPCA needs more than 1700 iterations to reach this value. As seen in the zoomed part in Figure 9(a), with the same iteration number, RPCA reaches $1 \times 10^{-3}$ error rate which is not acceptable for accurate separation. This results prove the faster convergence rate of the proposed TRPCA method compared to the RPCA.

We can conclude that the proposed TRPCA provides at least comparable or even better visual results (depending on the choice of appropriate λ value and for weak target responses) with severely less computation time compared to RPCA. To prove this, computational complexities of the methods are presented in Table 1. From Table 1 it can be observed that the TRPCA and RPCA have similar complexities for moderate input data size however, for higher input data size our proposed TRPCA has less complexity because of pre-transformation step. In addition to computational complexity comparison, all the methods are tested on the measured RTI data (Since Data I and Data II have the same size, only Data I results are presented from the first experiment) on Intel core i7 6700HQ @ 2.6GHz, 32GB DDR4-2133, Nvidia GTX950M, on a Windows 10 64-bit environment and running-time of the methods presented in Table 1 show the superiority of the proposed TRPCA over the RPCA. From the Table 2 it can be observed that TRPCA is faster than RPCA between $4–16$ times. On the other hand, both PCA and SVD have very fast running times, however their clutter removal results are not satisfactory compared to RPCA and TRPCA.

V. CONCLUSION
A new background subtraction method based on tensor concept has been proposed for outdoor target imaging. The new method decreases the size of the data matrix by recasting each column of the data matrix into matrices and forming a data tensor. TRPCA decomposition of the resulting tensor requires less computation time than the RPCA decomposition of the original data and provides at least the same performance in the visual sense. Although tensor concept and especially tensor RPCA has already been used for many imaging and image processing tasks, all these approaches either use temporal images or divide 2D inputs into subimages and recast them to tensor form. Our method does not exploit any correlation between successive echoes, only transforms them into smaller 2D images which are then recasted into tensor form. The decrease in the size results in a considerable increase in the convergence providing a background subtraction algorithm appropriate for online applications.

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FIGURE 7. (a) Original RTI image from faculty path (Data III), and background subtraction results for: (b) MS, (c) PCA, (d) SVD, (e) RPCA, (f) TRPCA, (g) RPCA_{Best}, and (h) TRPCA_{Best}.

FIGURE 8. (a) Original RTI image of the car (Data IV), and background subtraction results for: (b) MS, (c) PCA, (d) SVD, (e) RPCA, (f) TRPCA, (g) RPCA_{Best}, and (h) TRPCA_{Best}.

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such as corner reflectors, vehicles or humans in ITU campus area validate the superiority of the proposed method to the popular RPCA as well as to other conventional clutter removal/background subtraction methods. Although we provide outdoor measurements, the method can be applied to indoor imaging or behind obstacle imaging (subspace or through the wall applications) as well.

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