Characterizing High-Energy \( pp \) Collisions at the Large Hadron Collider using Thermal Properties

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High-multiplicity \( pp \) collisions at the Large Hadron Collider energies have created a new domain of research to look for possible formation of Quark-Gluon Plasma in these events. In this work, we have estimated various thermal properties of the matter formed in high-energy \( pp \) collisions, like mean free path \((\Lambda)\), isobaric expansivity \((\alpha)\), thermal pressure \((\frac{\partial p}{\partial T})\), heat capacity \((C_V)\) and the degrees of freedom \((\epsilon/T^4)\) using a thermodynamically consistent Tsallis distribution function, which happens to describe the identified particle spectra very well. Particle species dependent mean free path and isobaric expansivity are studied as a function of final state charged particle multiplicity for \( pp \) collisions at \( \sqrt{s} = 7 \) TeV. The findings are confronted to the theoretical expectations. The role of degree of non-equilibrium, baryochemical potential and temperature on these thermal properties are studied.

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I. INTRODUCTION

One of the many astounding revelations of 20th century physics was the relatively new state of matter called Quark-Gluon Plasma (QGP), which is expected to be formed in ultra-relativistic high-energy collisions. The temperature in such collisions reaches up to a few hundred MeV. To have a better grasp, the temperature in the core of the Sun is around \( 10^5 \) times less than that is produced in high-energy collisions at Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC).

At such high temperatures, the normal baryonic matter with hadrons as their degrees of freedom goes through first order/cross-over phase transition to produce QGP, which has quarks and gluons as its degrees of freedom. Earlier, the general understanding was that only high-energy heavy-ion collisions can produce QGP, while \( pp \) collisions were taken as baseline measurements to understand medium formation in nuclear collisions. However, recent advances in high-energy \( pp \) collisions have shown us that there is a possibility of formation of QGP droplets, hopefully in high-multiplicity \( pp \) collisions in view of the observed heavy-ion-like signatures in such collisions 1,2. Perturbative QCD (pQCD) based theoretical models like PYTHIA8 in their advanced tunes like multipartonic interactions (MPI), color reconnection (CR) and rope hadronization, have been successful in explaining some of the features in small collision systems 3.

These include – color reconnection brings out flow-like features in small systems 4, which is in principle a macroscopic hydrodynamic feature with a possible microscopic thermalization features, rope hadronization explains the enhancement of multi-strange particles with respect to pions 5. Although there is no inbuilt thermalization in PYTHIA8, it is successful in mimicking the macroscopic flow features in \( pp \) collisions. In contrast to PYTHIA8, EPOS model also describes the heavy-ion like signatures in small collision systems 6. It includes event-by-event 3+1D hydrodynamic evolution of the system produced in high-energy collisions 6–8. The description of hard sector for PYTHIA8 and EPOS are similar, however, the key difference in the EPOS model is that it includes a hydrodynamical description of the underlying mechanisms for generating the collectivity instead of the color reconnection mechanism in PYTHIA8. So, unlike PYTHIA8, EPOS includes the thermalization of core (bulk) part of the system.

In view of these, there is a need to have a deeper understanding of high-multiplicity \( pp \) events to have a better understanding of the produced systems, possible associated thermodynamics, particle production dynamics and freeze-out. There have been attempts to study these events using event topology, which separates jetty (pQCD based hard scattering events) from isotropic (dominated by soft processes) events 9,13. Using the temperature, \( T \) and Tsallis non-equilibrium parameter, \( q \), extracted from the \( p_T \) spectra various thermodynamical quantities are estimated in order to characterize the system created in \( pp \) 15 and heavy-ion collisions 10.

Going in line with these explorations from various fronts, in this work, let’s make an attempt to understand some of the thermodynamic features of \( pp \) collisions using experimentally motivated Tsallis non-extensive distribution, which describes the particle spectra. In this work, we consider a cosmological expansion scenario, where the fireball produced in high energy hadronic and nuclear collisions cools down during its spacetime expansion, following a temperature profile with time. Final state particles decouple from the system following a differential freeze-out scenario – heavier particles coming out of the system early in time, which corresponds to higher decoupling temperatures.

To understand the behavior of the matter formed in high-energy collisions at ultra-relativistic energies, we

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need to have an overall idea about its thermodynamical properties. Mean free path of a system gives the average distance which a particle has to travel before it collides with another particle in the system. It can give us idea about the state of the system under consideration. One of the important thermodynamic properties of a system to study is isobaric expansivity ($\alpha$). The tendency of matter to change its shape, area or volume is known as thermal expansion of that matter. The relative expansion per change in temperature at constant pressure is called the matter’s coefficient of thermal expansion or isobaric expansivity $[17]$. The study of isobaric expansivity of the matter formed in high-energy collisions tells us about the key features in such systems. Thermal pressure ($\frac{\partial P}{\partial T}$) is nothing but the ratio of isobaric expansivity and isothermal compressibility. It tells us how much does the pressure of the system increase with the increase in temperature at constant volume. The heat capacity of the system is the amount of heat energy that is required to raise the temperature of the system by one unit. It is seen to give a thermodynamically consistent Tsallis distribution function, which is given as $[25]$, 

$$f(E, q, T, \mu) = \frac{1}{[1 + (q - 1) \frac{E - \mu}{T}]^{\frac{q}{q-1}}},$$

where $T$ and $\mu$ are the temperature and the chemical potential, respectively. Here $E = \sqrt{p^2 + m^2}$, is the energy with $p$ being the momentum and $m$ being the mass of the particle under study. The thermodynamical quantities in non-extensive statistics are given by,

$$n = g \int \frac{d^3p}{(2\pi)^3} \left[1 + (q - 1) \frac{E - \mu}{T}\right]^{\frac{1}{q-1}},$$

$$\epsilon = g \int \frac{d^3p}{(2\pi)^3} E \left[1 + (q - 1) \frac{E - \mu}{T}\right]^{\frac{1}{q-1}},$$

$$P = g \int \frac{d^3p}{(2\pi)^3} \left[1 + (q - 1) \frac{E - \mu}{T}\right]^{\frac{1}{q-1}}.$$

As the identified particle $p_T$ spectra for $pp$ collisions are well described by Tsallis non-extensive distribution function, we estimate various thermal properties of the produced fireball by using a thermodynamically consistent Tsallis distribution function, which is given as $[26]$,

$$\frac{\partial E}{\partial S}|_{N,V}.$$

Further, the variable $T$, used in the above expressions, obeys the thermodynamic relation: $T = \frac{\partial E}{\partial S}|_{N,V}$. The paper is organized as follows. Section II gives a brief account of the formulations that we have used in this work. In section III we discuss our findings and the results, and in section IV we have summarized our work.
and hence, could be termed as the temperature even though the system follows Tsallis non-extensive statistics.

Going ahead with the above prescription, in view of thermodynamic consistency and Tsallis non-extensive statistics successfully describing final state particle spectra in the experiments, the expressions for the discussed thermodynamic observables could be written as follows. The general expression of mean free path is given by,

\[ \Lambda = \frac{1}{n \sigma}. \]  

Using Eq.2 the expression for mean free path becomes:

\[ \Lambda = \frac{1}{\sigma g \int \frac{d^3 p}{(2\pi)^3} [1 + (q - 1) \frac{E - \mu}{T}] \frac{1}{\pi^4}} \]  

where \( \sigma \) is the hadron scattering cross-section and can be taken as 11.3 mb considering a hard core hadron radius, \( r_h = 0.3 \text{ fm} \) and using \( \sigma = 4\pi r_h^2 \) [26][27]. The choice of an average value of hard core hadron radius, \( r_h = 0.3 \text{ fm} \) seems to be a good choice both for the estimation of viscosity of the system [26], while giving proper estimation of hadron yields in a statistical hadron gas model [27]. A change in the hard core hadron radius in the calculation of hadron scattering cross section will induce a change in the absolute values of the mean free path. However, this will have no impact on the trend of mean free path of the particles, when studied as a function of temperature, final state multiplicity, \( q \)-parameter or the baryochemical potential of a system.

From thermodynamics, the expression of isobaric expansivity is given as [17],

\[ \alpha = \frac{1}{V} \frac{\partial V}{\partial p}. \]  

It is very difficult to work with volume explicitly in high-energy collisions as the exact freeze-out volume is not known for identified charged particles. To avoid this particular problem, we modify Eq.11 and write it in terms of particle number density as:

\[ \alpha = \frac{1}{n} \frac{\partial n}{\partial T}. \]  

where \( n \) is the number density of the system. After substituting \( n \) and simplifying, the above equation can be written as:

\[ \alpha = -\int g [1 + (q - 1) \frac{E - \mu}{T}] \frac{1}{\pi^4} \frac{d^3 p}{m^3} \]  

The thermal pressure is given by,

\[ \left( \frac{\partial P}{\partial T} \right)_V = g \int \frac{d^3 p}{(2\pi)^3} \frac{gq^2}{3E} \left[1 + (q - 1) \frac{E - \mu}{T} \right] \frac{1}{\pi^4} \frac{d^3 p}{m^3}. \]

The heat capacity or the specific heat at constant volume is defined as,

\[ C_V = \frac{\partial e}{\partial T}, \]  

and in the present formalism, this could be written as:

\[ C_V = g \int \frac{d^3 p}{(2\pi)^3} qE \left[1 + (q - 1) \frac{E - \mu}{T} \right] \frac{1}{\pi^4} \frac{d^3 p}{m^3}. \]  

In our work, we have dealt with mostly the pion gas because of its dominance in the final state created in high-energy collisions. In our calculations, we have taken the degeneracy factor, \( g = 2 \), as we compare our results with experimental data, where only charged pions are considered. In LHC pp collisions the baryochemical potential is assumed to be zero, because of the baryon-antibaryon symmetry. So in our calculations we have taken \( \mu = 0 \), except when we are studying the effect of chemical potential on various thermal quantities.

### III. RESULTS AND DISCUSSION

To calculate the mean free path, we have used Eq.10. In figure 1 we have plotted the mean free path (\( \Lambda \)) of a pion gas system as a function of temperature for different values of the non-equilibrium parameter, \( q \). We observe that for low temperature, the mean free path of the system is very high and it suddenly decreases becoming minimum at high temperature region, which usually corresponds to freeze-out temperatures in high energy hadronic and nuclear collisions [28]. We also notice that the mean free path is highly sensitive to the \( q \)-parameter in the low temperature regime, whereas the dependency decreases at higher temperatures becoming almost negligible. These trends are expected because, at low temperature the number density in the system is less, which results in a high \( \Lambda \). As the temperature increases, the number density also increases drastically between a short temperature range before becoming almost constant at very high temperatures. For a system temperature, \( T \geq (100-120) \text{ MeV} \), the mean free path of the system becomes almost insensitive to the \( q \)-parameter. Below this threshold temperature, which usually correspond to kinetic freeze-out temperature(s), at a given temperature, the mean free path decreases, when the system slowly goes away from equilibrium. For an expanding fireball, initially the system volume is low and hence for a given number of particles the mean free path is smaller. The constituents of the system go through mutual collisions as a part of the process of equilibration, while the system is still expanding resulting in higher mean free path.

Figure 2 shows the mean free path of a pion gas as a function of temperature for different values of baryochemical potentials. Here we have taken the non-extensive parameter, \( q=1.001 \), which corresponds to a near equilibrium state. We observe that when \( \mu_B = 0 \),.
The hardening of the final state multiplicity has been taken from the ALICE experiment [1]. The multiplicity of identified particles is a function of final state event multiplicity, which is shown in Fig. 3. The definition of event class through the baryochemical potential of the system higher. This makes the mean free path to decrease with temperature irrespectively of the trend of the mean free path of the system mimics the trend which we get from Fig. 1. However, as we go on increasing the value of \( \mu_B \), which happens when one moves down in collision energy, the trend becomes flatter and flatter at low temperature regime. In addition, one observes a change in the curvature from concave upwards to a convex structure, towards lower collision energies (higher \( \mu_B \)). The dependence on baryochemical potential on the mean free path is clearly visible in the low temperature regime. There is an universality in the decreasing nature of the mean free path as a function of temperature for various \( \mu_B \) values. This happens for temperatures higher than the expected freeze-out temperatures. Usually one expects to produce more number of particles at higher system temperatures, making the number density in the system higher. This makes the mean free path to decrease with temperature irrespective of the baryochemical potential of the system. This is clearly evident from Fig. 2.

Beyond these studies on the dependencies of mean free path on \( \mu_B, T \) and \( q \)-parameter of the system for a pion gas, one must be curious to look into the behaviour of mean free path as a function of final state event multiplicity for \( pp \) collisions at LHC energies for different identified particles. In order to do that, we have taken the multiplicity dependent identified particle \( p_T \)-spectra for \( pp \) collisions at \( \sqrt{s} = 7 \) TeV and using the discussed Tsallis non-extensive statistics, we have obtained the \( T \) and \( q \)-parameters [31]. Taking these as inputs, we estimate \( \Lambda \) as a function of final state event multiplicity, which is shown in Fig. 3. The definition of event class through final state multiplicity has been taken from the ALICE experiment [1]. The hardening of \( p_T \)-spectra with event multiplicity for \( pp \) collisions at \( \sqrt{s} = 7 \) TeV has been seen recently [1]. This points to an increase in system temperature with event multiplicity. Hence, in Fig. 3 the \( x \)-axis becomes a replica of temperature, although the explicit functionality with event multiplicity is not known. In view of this, the experimental behaviour of particle mean free path assuming a common cross-section for all particle species [26] as a function of temperature agrees with the theoretical expectations for \( q \sim 1.001 \) and \( \mu_B = 0 \) as shown in Figs. 1 and 2. After a threshold of \( \langle dN_{ch}/d\eta \rangle \sim (10-15) \), the mean free path becomes independent of particle species and for higher event multiplicities, it attains a lower asymptotic value between (1-10) fm.

We have used Eq. 13 to calculate the isobaric expansivity of a pion gas. Fig. 3 shows the isobaric expansivity (\( \alpha \)) as a function of temperature for different values of
the non-extensive parameter $q$. We observe that for lower temperatures, the system shows lower $\alpha$ values, and as the temperature increases slowly, $\alpha$ increases rapidly and the trend becomes almost flat for higher values of temperature, $T \sim 180$ MeV. We also see that the isobaric expansivity is highly dependent on $q$ at low temperature, which is not the case for temperatures higher than the freeze-out temperatures of high energy collisions. For $q \to 1$, which is for an equilibrated system, $\alpha$ is the lowest at any given temperature. The value of $\alpha$ increases as $q$ increases, i.e., as the system stays away from equilibrium. The expansivity of the system is thus found to be correlated with the degree of non-equilibrium of the system. This indicates that at lower temperatures for an equilibrated system, the isobaric expansivity is lower than that of a non-equilibrated system at the same temperature. However, as the temperature goes on increasing, the $q$-dependency becomes negligible after a certain temperature.

![Graph](image)

**FIG. 4:** (Color online) Isobaric expansivity as a function of temperature for different values of non-extensive parameter $q$.

The isobaric expansivity of pion gas as a function of temperature for different values of baryochemical potential ($\mu_B$) is shown in Fig. 5. Here we have taken the non-extensive parameter $q = 1$ and estimated $\alpha$ for different $\mu_B$ values. For $\mu_B = 0$ (the case at the LHC energies), we get the similar trend as we have got for the case of $q = 1.001$ shown in Fig. 4. Interestingly, when we increase the baryochemical potential to 200 MeV, the trend becomes a little flatter at the low temperature region. As we further decrease the $\mu_B$ values, we see the trend becoming flatter and flatter (making it almost independent of temperature), and for $\mu_B$ more than 300 MeV, we observe a completely opposite trend of isobaric expansivity at low temperature region – convex upwards trend becomes flat and then concave upwards. The reason for negative expansivity can be because of a number of physical processes, such as transverse vibrational modes, rigid unit modes and phase transitions [29]. However, in our case, further studies are necessary to understand the underlying dynamics.

![Graph](image)

**FIG. 5:** (Color online) Isobaric expansivity as a function of temperature for different values of baryochemical potential $\mu_B$ (in GeV).

Figure 6 shows the isobaric expansivity of the hadron gas system formed in LHC $pp$ collisions at $\sqrt{s} = 7$ TeV as a function of final state charged particle multiplicity, which is an event classifier in $pp$ collisions. We have taken the temperature and $q$ values from the Tsallis fit to the $p_T$ spectra of the identified particles produced in LHC $pp$ collisions at $\sqrt{s} = 7$ TeV [30] to estimate the isobaric expansivity of the system for different identified particles [31]. At the LHC energies, the baryochemical potential of the system is almost zero and for our studies, we use $\mu = 0$ in the calculations. We observe an increasing trend in the expansivity of the system as the final state charged particle multiplicity increases. For lower $\langle dN_{ch}/d\eta \rangle$, the expansivity of identified particles are different and mostly follow a mass ordering, with massive particle having lower $\alpha$ than lighter particles. This is a consequence of the fact that in the hadronic phase, the number density of the lighter particles are higher than that of the massive particles. When we increase the temperature, the number density of all the particles increases. The given temperature can be translated into the kinetic energy of the particles in the system. Taking the solid state and gaseous phase analogy, for a denser system, the increase in temperature makes a small increase in the volume of the system, whereas for a less dense system, the change is very high. However, after a certain charged particle multiplicity ($\langle dN_{ch}/d\eta \rangle = 10$), we observe that all the hadronic species in the system have almost the same isobaric expansivity. This suggests a change in the dynamics of the system at higher $\langle dN_{ch}/d\eta \rangle$ [32].

It is a well-known fact that water has a negative expansivity from $0^0$ C to $4^0$ C. However, with the increase in
temperature it gains positive expansivity. At $30^0C$, water has the isobaric expansivity of about $3.515 \times 10^3$ GeV$^{-1}$ [33]. This comparison is made, to have a physical feeling of the system produced in high-energy collisions.

We observe that for low temperatures, the specific heat of the system is low and it increases gradually as the temperature increases. We also observe that for $q=1.001$, i.e. when the system is near equilibrium, the specific heat of the system is the lowest. This is expected, as for an equilibrated system the degree of change of internal energy of the system is less. For higher $q$ values, $C_V$ becomes higher at any given temperature. From Fig. 9 we observe that the baryochemical potential plays a big role in the outcome of $C_V$. For $\mu_B = 0$, the $C_V$ of the system changes very little with system temperature, but as the baryochemical potential increases (corresponds to lower collision energies), the specific heat of the produced system increases, which is more evident at higher temperatures. For $\mu_B \geq 300$ MeV, we observe that the specific heat of the system is negative up to certain temperatures. In general, most of the physical systems exhibit a positive heat capacity. But there are some systems which have negative heat capacities, which are said to be inhomogeneous in nature and they are not thermodynamically equilibrated [34]. This could mean that even if we have taken $q = 1.001$, which is the case of a system near equilibrium, for higher $\mu_B$, the produced system seems to behave like a non-equilibrated one.

We use Eq. 14 to estimate $\frac{\partial P}{\partial T}$, which is the thermal pressure of the produced system. Fig. 7 shows the variation of $\frac{\partial P}{\partial T}$ as a function of temperature for different values of the non-extensive parameter $q$. We observe that for low temperatures, the thermal pressure is lower for the system, but as the temperature increases, the thermal pressure also increases. We also see that when the system is near equilibrium, $\frac{\partial P}{\partial T}$ is the lowest. For higher $q$ values, $\frac{\partial P}{\partial T}$ increases, meaning, as the degree of non-equilibrium of the system increases, the thermal pressure also increases at any given temperature. Hence, the thermal pressure is responsible for the degree of non-equilibration of the system. In addition, at lower temperatures, the number of particles in the system would be less. With a small change in temperature, thus the change in pressure of the system would be less. But at higher temperature, the number of particles in the system would be much higher, which contributes to a higher thermal pressure.

We have calculated the specific heat at constant volume for a system with pions as the constituents using Eq. 16. Fig. 8 shows the variation of $C_V$ as a function of temperature. We see that for low temperatures, the specific heat of the system is low and it increases gradually as the temperature increases. We also observe that for $q=1.001$, i.e. when the system is near equilibrium, the specific heat of the system is the lowest. This is expected, as for an equilibrated system the degree of change of internal energy of the system is less. For higher $q$ values, $C_V$ becomes higher at any given temperature. From Fig. 8, we observe that the baryochemical potential plays a big role in the outcome of $C_V$. For $\mu_B = 0$, the $C_V$ of the system changes very little with system temperature, but as the baryochemical potential increases (corresponds to lower collision energies), the specific heat of the produced system increases, which is more evident at higher temperatures. For $\mu_B \geq 300$ MeV, we observe that the specific heat of the system is negative up to certain temperatures. In general, most of the physical systems exhibit a positive heat capacity. But there are some systems which have negative heat capacities, which are said to be inhomogeneous in nature and they are not thermodynamically equilibrated [34]. This could mean that even if we have taken $q = 1.001$, which is the case of a system near equilibrium, for higher $\mu_B$, the produced system seems to behave like a non-equilibrated one.

It would be interesting to estimate the effective degrees of freedom of a pion gas as a function of temperature for different $q$-values. We use Eq. 3 to estimate $\epsilon/T^4$,
of q is a sudden jump in the degrees of freedom for all values observe that at certain temperature, (T,q) the mean free path becomes independent of particle species. For higher event multiplicities, the mean free path seems to attain a lower asymptotic value towards higher temperatures.

The observations of heavy-ion-like features in high-multiplicity pp collisions at the Large Hadron Collider energies and the corresponding explanations using different tunes in pQCD inspired models like PYTHIA8, have warranted a deeper look into these events. A systematic study of various observables as a function of final state charged particle multiplicity to look into a possible new domain of particle production has been one of the new studies at the LHC. Transverse momentum spectra of various identified particles are seen to be well explained by a thermodynamically consistent Tsallis distribution function, which has been very successful in bringing out information about the produced system. In this work, we have estimated various thermal properties of the matter formed in high-energy pp collisions, like mean free path (Λ), isobaric expansivity (α), thermal pressure (∂P/∂T), heat capacity (C_v) and the degrees of freedom (ε/T⁴) using a thermodynamically consistent Tsallis distribution function. The role of degree of non-equilibrium (q), baryochemical potential and temperature on these thermal properties are studied. Some of these observables like mean free path and isobaric expansivity are studied as a function of final state charged particle multiplicity for pp collisions at √s = 7 TeV and the findings are confronted to the theoretical expectations. The important results are summarized below.

1. The mean free path is found to be highly sensitive to the equilibration parameter, q at the lower temperatures. In the domain beyond the kinetic freeze-out temperature(s), it is found to be almost independent of the q-parameter. The impact of baryochemical potential on the system mean free path is found to be very interesting. For a near equilibrium pion gas at lower temperatures, the shape of the (Λ − T)-spectrum changes from convex upwards to convex downwards as one moves from a baryon-rich environment to a baryon-free environment. The high temperature behaviour is somewhat different – for all values of baryochemical potential, the spectra show a decreasing trend with temperature.

2. The mean free path is then studied for identified particles in pp collisions at the LHC for √s = 7 TeV with the (T,q) inputs from the experimental spectra. \(dN_{ch}/dη\) is found to be a threshold in the final state event multiplicity, after which the mean free path becomes independent of particle species. For higher event multiplicities, the mean free path seems to attain a lower asymptotic value between (1-10) fm.

3. We observe a negative isobaric expansivity of the pion gas system produced in high-energy collisions. The isobaric expansivity is sensitive to the q parameter at lower temperatures and the q depen-
dency becomes minuscule in the higher temperature regime. However, for a baryon-rich environment ($\mu_B \geq 300$ MeV), we observe positive isobaric expansivity for lower temperatures which slowly becomes negative in the higher temperature region.

4. We have studied the isobaric expansivity of the identified particles produced in $pp$ collisions at the LHC for $\sqrt{s} = 7$ TeV. We observe a threshold charged particle multiplicity ($\langle dN_{ch}/d\eta\rangle > 10$) after which the isobaric expansivity for the identified particles almost converges to the same value, which may indicate a change in the dynamics of the system.

5. The thermal pressure is found to be weakly dependent on the $q$ parameter at lower temperatures, but the dependency becomes stronger with the increase of temperature. As expected, at lower temperatures, the thermal pressure in the system is lower, which goes up, as the temperature of the system increases. The thermal pressure is found to be positively correlated with the degree of non-equilibrium of a physical system.

6. The specific heat at constant volume is weakly dependent on the equilibration parameter $q$ at lower temperatures and highly dependent on $q$ in the higher temperature region. As the temperature increases, we observe an increase in the specific heat of the system. We have also studied the effect of baryochemical potential on the specific heat and observed that for a baryon-rich environment, the specific heat becomes negative at lower temperatures. But at higher temperatures, the specific heat gradually becomes positive for all baryochemical potentials.

7. The degrees of freedom is found to be highly sensitive to the parameter $q$. At all temperatures, we observe that the system which is away from the equilibrium has higher degrees of freedom than a system which is near the equilibrium.

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