MHD properties in the core of ITER-like hybrid scenarios

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Abstract. MHD stability is studied in the frame of ITER like hybrid scenarios, characterized by weak reversed shear, with particular attention on ideal internal kink modes and infernal mode stability. Numerical simulations for ITER like equilibria, with hollow $q$ profiles with an off axis minimum close to unity, were carried out using the 3-D equilibrium code ANIMEC, which showed the presence of a 3-D helical core [1] with the characteristics of a saturated internal kink mode. The internal kink perturbation has been investigated non-linearly using the XTOR code, in the ideal frame. A scan in the current was performed and it has been found that when the minimum of $q$ is above the unity (low currents), the helical distortion shows good agreement with the results provided by the 3-D ANIMEC simulations while for $q_{\text{min}}$ below 1 (high currents), XTOR gives a residual distortion in contrast with the ANIMEC results. Moreover infernal mode stability in hybrid scenarios has been studied analytically extending the quasi-interchange model with the inclusion of resistive and both electron and ion diamagnetic effects. This enables us to investigate kinetic effects on plasma scenarios susceptible to saturated kink structures exhibited into ANIMEC simulations.

1. Introduction
Standard tokamak operation scenarios are characterized by monotonically increasing safety factors $q$, with usually $q_0 < 1$ ($q_0$ is the value of the safety factor on the magnetic axis), full inductive plasma current and the standard high-performance operation. Another mode of operation is the advanced scenario which has weak reversed shear, produced by bib-inductive current drive and bootstrap current. A mode of operation is the hybrid scenario, which is an intermediate step between the H-mode standard scenario and an advanced scenario characterized by an extended low shear region where the safety factor $q \approx 1$. In recent hybrid experiments,
good confinement and high beta limit have been achieved [3] with stationary density profiles enclosing a large plasma volume with low magnetic shear with $q_0$ close to 1 (hybrid scenario).

As pointed out in [2, 4], these kind of discharges can be characterized by a soft magnetohydrodynamic (MHD) activity, whose most important modes can be divided as follows:

- Long lived modes - typically MAST operation regime (ideal saturated MHD instabilities infernal modes, $q_{min} > 1$ [5])
- Global oscillations - TCV (sawteeth cycles, $q_{min} > 1$)
- Snakes - JET (ideal infernal modes)

Internal kink instabilities are accessible in the regime of high $\beta$ and $q$ profiles which are close to the unity (either monotonic with $q_0 < 1$, or with low or weak reversed shear having $q_{min} \approx 1$ and $q > 1$ everywhere). Under these conditions the internal kink instability develops and 3-D helical structures appear in the plasma core, whose saturation gives rise to the above mentioned long lived modes; these saturated states have been found experimentally in the spherical tokamak MAST discharges [5] and also in "standard" tokamaks like TCV operating in the above mentioned hybrid scenario (viz. weak reversed shear) and it is conjectured that this new plasma state can be regarded as a new kind of equilibrium.

Moreover, these scenarios could be affected by infernal mode activity either ideal or resistive: in particular coupling the main kink harmonic ($m = n = 1$) to its resistive "tearing" sidebands, namely the $m' = m \pm 1$ harmonics, give rise to the infernal mode instability [6, 7], which is closely related to the quasi interchange mode [8], giving an enhanced growth rate for the internal kink mode. The resistive infernal mode has both MHD characteristics in the core region and tearing characteristics at $q = (m + 1)/n$ surface: this suggests that diamagnetic effects should be important since the tearing mode is required to rotate at the electron diamagnetic frequency at that surface [7]. Therefore an extension of the MHD model has been studied in order to include higher order corrections, viz. resistivity and plasma diamagnetism and other advanced effects including viscosity and sheared $E \times B$ toroidal rotation.

2. Ideal kink instabilities in ITER-like scenarios

Since the simplest long lived mode, in the hybrid scenarios, is the saturated internal kink, the first part of this work was devoted to the study of such MHD activity investigating the relation which occurs between magnetohydrodynamical modes and the shape of $q$ profile (i.e. density current profile). Numerical simulations for ITER like equilibria, with hollow $q$, were carried out using the 3-D equilibrium code ANIMEC [1] and the nonlinear MHD stability code XTOR [9] in the ideal frame: ANIMEC predicts the presence of 3-D helical cores in these particular configurations [1] with the characteristics of a saturated internal kink mode. XTOR simulations were performed starting from a given 2-D (i.e. axisymmetric) ITER like equilibrium ($\beta \approx 3\%, 0.96 < q_{min} < 1.03$), with the same profiles which give rise to the 3-D helical cores [1] prepared with 2-D equilibrium code CHEASE [10]. A scan in the current, ranging from 13.2 to 13.8 MA, was performed and the associated saturated state is shown in figure 1. Both ANIMEC and XTOR results predict the presence of kink-like 3-D structures, however it has been found that when the minimum of $q$ is above the unity (low currents), the helical distorsion $\delta_h$ shows good agreement with the results provided by the 3-D equilibrium simulations performed using ANIMEC and also with the analytical predictions for the nonlinearly saturated kink given by
Figure 1. Contour of constant pressure surfaces at fixed toroidal angle $\phi = 3/2\pi$ for saturated internal kink mode evaluated by the ideal stability code XTOR starting from an ANIMEC equilibrium, with toroidal plasma current (a) 13.0 MA, (b) 13.3 MA, (c) 13.4 MA, (d) 13.5 MA, (e) 13.7 MA, (f) 13.8 MA; $r_a$ denotes the plasma minor radius.

\[ \xi_h q'' = \frac{8}{71} \left( \frac{8\pi}{3} \right)^3 \Delta q \left\{ \left( \frac{1}{2} \left( \frac{\gamma^2}{\omega_A \Delta q} + 1 \right) \right) \left( \frac{\gamma^2}{\omega_A \Delta q} + 1 \right)^{1/2} + 1 \right\}^{1/2} - 1 \mid_{r=r_1} \]  

(1)

\[ \xi_h q' = \frac{13}{\pi} \frac{\gamma}{\omega_A} \mid_{r=r_1} \]  

(2)

where $\xi_h$ is the radial displacement, $\omega_A = v_A/(\sqrt{3}R_0)$ with $v_A$ the Alfvén velocity, $R_0$ the major radius, $\gamma$ is the linear growth rate of the internal kink mode, $\Delta q = |q_{\min} - 1|$ and $r_1$ is the position of the resonant $q = 1$ surface. Eq. (1) derives from the calculation of the nonlinear $m = n = 1$ saturated kink mode for nonmonotonic $q$ profiles with reversed shear having $q_{\min} \approx 1$ at $r = r_1$ [11] while equation (2) is valid for the $m = 1$ mode for monotonic safety factor profiles with the $q = 1$ surface at $r = r_1$ [12]. However, when $q_{\min} < 1$ we found that the analytical predictions almost agree with the nonlinear results, but there is a significant deviation between the 3-D equilibrium calculations and the nonlinear simulations: while the helical distortion predicted by ANIMEC decreases to 0 when $q_{\min} \approx 0.96$, XTOR gives a residual distortion in this region (as shown in figure 2) accordingly with the linear stability calculations performed with TERPSICHORE code [13]. Indeed this is to be expected because the analytic linear internal kink mode of Bussac [14] is unstable. A convenient value for the viscosity was chosen in order to avoid numerical instabilities in the core region; however, even if the growth rate for the $n = m = 1$ mode is not so sensitive to variations in the viscosity $\nu$ [15], we should take care of this parameter, since its value can affect significantly the behaviour of $n \neq m$ modes as shown in figure 3, damping potentially unstable modes.

3. Infernal mode stability

The nonlinear code XTOR-2F [9] solves an extended set of magnetohydrodynamic equations in toroidal tokamak geometry in which effects like viscosity and plasma diamagnetism are also
Figure 2. Helical distortion of the magnetic axis for an ITER like scenario as a function of the minimum of the safety factor $q$. The black line shows the ANIMEC results ($r_a \delta_h = \sqrt{R_{01}^2 + Z_{01}^2}$ where $R_{01}(Z_{01})$ is the $(m = 0, n = 1)$ Fourier component of $R(Z)$ at the magnetic axis [1]), while the blue curve refers to the nonlinear XTOR simulations ($r_a \delta_h = \sqrt{(R - R_0)^2 + (Z - Z_0)^2}$, with $(R, Z)$ being the position of the magnetic axis calculated at $\phi = 3/2\pi$ ($\phi$ is the toroidal angle) and $(R_0, Z_0)$ being the position of the magnetic axis for the axisymmetric equilibrium); The red and green lines correspond to the analytical predictions given respectively by (1) and (2).

Figure 3. Behaviour of the kinetic energy for $n = (1, ..., 9)$ modes for two different values of the viscosity ($\nu = 3 \times 10^{-6}$ (XTOR units) in left picture and $\nu = 5 \times 10^{-5}$ (XTOR units) in the right one); the other equilibrium parameters were kept the same for both simulations. We can clearly see that the $n = (4, 5)$ is strongly reduced in the linear phase. We notice also that (around 1000 alvenic times in the picture on the right) the growth rate $\gamma$ for $n > 1$ modes is given by $\gamma_{n>1} = n\gamma_{n=1}$ [16] as expected by toroidal couplings.
included. Therefore the quasi-interchange model has been analytically extended in order to retain these higher order effects. The model assumes a parabolic pressure profile with a constant subsonic toroidal rotation in the low-shear region. The analysis is split in three regions: the low-shear region, the sheared region and the resonant layer of the \( m+1 \) sideband. The equation describing the toroidal MHD modes are derived from the full set of MHD equations [17] rather than performing a minimization of the energy integral [18].

Perturbed quantities are assumed flute like having an \( \exp(-i\omega t) \) time dependence. The inverse safety factor profile is given by \( \mu = n/m_0 - \tilde{S}(r/r^*) \Lambda \). Using standard techniques [17, 19, 20, 21] the following dispersion relation in the regime of constant - \( \psi \) approximation including diamagnetic effects is derived:

\[
a\Delta' \sim S^{3/4} \left( \frac{\tilde{\omega}_s(\tilde{\omega} - \omega_s^*)}{\omega_A^2} \left( \frac{\tilde{\omega} - \omega_s^* - \omega_T^*}{\omega_A} \right)_s \right)^{3/4}
\]

where \( \tilde{\omega} \) is the Doppler shifted frequency due to toroidal rotation; the subscript \( s \) indicates that the quantity is evaluated at the resonant \( m+1 \) surface. At leading order in the shear-free region the equation for the main \( m_0 \) harmonic reads [18]:

\[
\frac{d}{dr}\left[r^3Q \frac{dX_{m_0}}{dr}\right] + r \left[(1 - m_0^2)Q - \frac{\alpha r}{2R_0} \left( \frac{1}{q^2} - 1 \right)\right] X_{m_0} + \Lambda \alpha r^{m_0+1} \int_0^r \alpha r^{1+m_0} X_{m_0} dr = 0
\]

where \( Q = \delta q^2/q^4 - \tilde{\omega}(\tilde{\omega} - \omega_s^*)(1 + 2q^2)/(m_0^2\omega_A^2) \), \( \delta q = q - m_0/n \), \( \Lambda = q^2(r^*)^{-2-2m_0}\Lambda_{\omega_{m_0,n}}^* \), \( \Lambda_{\omega_{m_0,n}}^* = [(m + 1)/2(2 + m + C)/(m - C)] \), \( C = r^* X'_1(r_\text{crit}^*)/X_+ (r^*) \) and \( \alpha = 2R_0 p_0^2 / B_0^2 \) being the usual ballooning parameter. The factor \( C \) is evaluated by means of (3) solving the sideband equations in the sheared region with the model safety factor written before. Hence the following dispersion relation for the infernal mode instability for an almost resonant \( q \approx 1 \) safety factor is derived:

\[
\frac{\tilde{\omega}(\tilde{\omega} - \omega_s^*)}{\omega_A^2} = \frac{n^2}{1 + 2q^2} \left[ \frac{(\delta q)}{q} \right]^2 - \frac{2\varepsilon_s^2 \Lambda_{\omega_{m_0,n}}^* \bar{\beta}_p^2}{(m + 1)^2(m + 2)}
\]

where \( \varepsilon_s = r^*/R_0 \) and \( \bar{\beta}_p = \bar{p} q^2 / (B_0^2 \varepsilon_s^2) \), \( \bar{p} \) denoting the plasma pressure on the magnetic axis. For a fixed \( \delta q \), the ideal MHD stability boundary is identified by \( \beta_p = \beta_\text{crit} \); however, small positive deviations in \( \delta q \) are allowed, namely \( \delta q^2 = \delta q_{\text{crit}}^2 + \Delta^2 \). Defining \( \gamma = -i\omega \), neglecting toroidal flow and diamagnetic corrections, close to marginal MHD stability the behaviour \( \gamma^{5/4}(\gamma^2 + \Delta^2) \) is recovered [7], giving the \( S^{-3/13} \) scaling for \( \Delta^2 \ll \gamma^2 \) and the \( S^{-3/5} \) behaviour for \( \Delta^2 \gg \gamma^2 \) as shown in figure (4). Using standard techniques diamagnetic corrections are introduced in the expression for \( \Delta' \) which is used to evaluate the jump of the \( m_0 + 1 \) solution at its own rational surface giving the following dispersion relation

\[
\omega (\omega - \omega_s^*) \sim 1/r_s \Delta' (\omega, \omega_s^*)
\]

which has to be solved numerically.

Neglecting diamagnetic effects but taking into account toroidal flows the following expression is derived:

\[
\left( \omega - \Omega_{E \times B}^{(s)} \right)^{5/4} \left[ \left( \omega - \Omega_{E \times B}^{(l-s)} \right)^2 - \Delta \right] \sim \beta_p^2 \sigma^{-3/4}
\]

where the symbols \( (s) \) and \( (l-s) \) indicate that the quantity is evaluated at the rational surface or in the low shear region respectively. The solution procedure is similar to the one presented
Figure 4. Scaling of the growth rates for $\Delta = 0 \ (S^{-3/13})$ and $\Delta = 0.001 \ (S^{-3/5})$ for the $m_0 = n = 7$ mode with $\delta q = 0.003$, $\varepsilon_\star = 0.18$, $\lambda = 6$ and $\hat{S} = 0.03$.

before considering three different cases with $\Delta = 0$: $\omega \ll \Omega_{E \times B}^{(s)} \ll \Omega_{E \times B}^{(l-s)}$, $\Omega_{E \times B}^{(s)} \ll \omega \ll \Omega_{E \times B}^{(l-s)}$, $\Omega_{E \times B}^{(s)} \ll \omega \simeq \Omega_{E \times B}^{(l-s)}$. The nonlinear stability code XTOR will be used in order to find the unstable roots found analytically for the different cases presented before.

4. Conclusions

A comparison between numerical simulations of three dimensional equilibria (ANIMEC) and nonlinear saturated instabilities (XTOR) in the frame of ideal MHD has been performed. When $q_{\text{min}}$ is above unity good agreement between ANIMEC and XTOR results was found in accordance with analytical predictions for saturated internal kinks [11, 14]. However nonlinear results predict a residual distorsion when $q_{\text{min}} < 1$ while in three dimensional simulations this distorsion disappears for $q_{\text{min}} \lesssim 0.96$.

Note that ANIMEC equilibrium calculations, analytic results and the nonlinear simulation carried out with XTOR neglect resistivity and this in principle allows current sheets to develop and the way in which they are handled may explain the difference observed in figure 2. However, since the $q_{\text{min}} > 1$ case is the most physically relevant, when the role of resistivity is localized at the resonant $m_0 + 1$ surface, it has been decided to drop resistive corrections in the low shear region. The quasi-interchange mode (and infernal modes) model has been extended analytically by including combined resistivity and diamagnetic effects. Moreover, recent TCV experiments [22] show that above a certain value of the plasma elongation $\kappa$ sawtooth activity is suppressed and it is replaced by continuous oscillations with frequencies around 10kHz in the direction of the electron diamagnetic drift. Thus we plan to model these experiments through examination of infernal mode stability with the extended quasi-interchange model. It is envisaged that this work will guide the non-linear numerical work to follow.

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