Scaling Law for Spin Deployment of Large-Membrane Structures Acceptable for Geometrical Mismatch

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There have been several advanced space missions using large gossamer space structures proposed recently, such as a spinning solar sail. However, only certain missions have been implemented, primarily because the on-orbit dynamic behavior of such structures cannot be precisely predicted through ground tests. To address this limitation, a scale law between a small- and a full-scale model of large membrane structures is proposed herein. In addition, small or reduced-order models for numerical simulations are proposed to reduce computation costs. However, the development of a small model, which has to be completely identical (geometrically) to the full-scale model, is challenging. Thus, a scale law between the small- and full-scale models of a large gossamer space structure, which is geometrically non-similar to the full-scale model, is also proposed. The scale law is also applied to the spin deployment motion of thin membrane and verified using numerical simulations.

Key Words: Scale Law, Membrane Structures, Mechanical Property, Solar-Power Sail

Nomenclature

- $A$: surface area or cross-sectional area
- $C$: elastic tensor
- $CD$: drag coefficient
- $E$: Young’s modulus
- $F$: external force
- $g$: gravity acceleration
- $G$: metric tensor of membrane
- $h$: thickness
- $L_f$: length of fold line or equivalent total length of fold line
- $n$: normal vector of membrane, $n = \hat{\xi}_1 \times \hat{\xi}_2 / |\hat{\xi}_1 \times \hat{\xi}_2|$
- $N_f$: fold number of membrane in undeployed state
- $R$: curvature of crease
- $t$: time
- $T_k$: kinetic energy
- $T$: characteristic time
- $V_e$: potential energy
- $\delta W$: virtual energy
- $\delta x$: virtual displacement
- $X$: characteristic length
- $y$: distance from the center line of membrane
- $\gamma$: damping factor
- $e$: Green–Lagrange strain vector
- $\theta_{ref}$: crease angle of membrane
- $v$: Poisson’s ratio of membrane
- $\hat{\xi}_3$: base vector of local coordinate

- $TT$: energy of entire system
- $\rho$: density
- $\sigma$: 2nd Piola-Kirchhoff stress vector
- $\omega$: spin rate of satellite body
- $\square$: non-dimensional parameter
- $\Delta \square$: increment at each time step
- $\square^*$: differential by time
- $\square^*$: scale-law parameter
- $\square^\circ$: normalized scale-law parameter
- $\square$: differential by non-dimensional time

Subscripts and superscripts

- No script, $m$: parameter of membrane
- $air$: parameter of air in experimental space
- $c$: parameter of cable (center tether, tip tether)
- $e$: converged value of parameter
- $f$: parameter of fold
- $L$: parameter of full-scale model
- $mod$: parameter when modifying fold length
- $o$: initial value of parameter
- $org$: parameter when not modifying fold length
- $s$: parameter of small model
- $sb$: parameter of spring back in fold line
- $tip$: parameter of tip mass
- $rigid$: parameter of satellite body
- $rot$: parameter of rotation motion

1. Introduction

Gossamer space structures are large and ultra-lightweight deployable structures that use membranes and cables. Several advanced space missions featuring large gossamer space structures have been proposed in recent years.$^{1)}$ Figure 1 illustrates these structures, namely a large solar-power sail$^{2)}$ (Fig. 1 left) and a starshade$^{3)}$ (Fig. 1 right).
However, only a handful of these missions have been implemented to date because the dynamic behavior of these systems, including their deployment, cannot be appropriately predicted during ground tests (i.e., in an atmosphere under gravity). Thus, numerical prediction of the deployment is vital for implementing large gossamer space structures.

In particular, the numerical analysis of the cable–membrane structure has a detailed history since the 1970s, which has been applied to the design and development of gossamer space structures. The world’s first solar sail—“IKAROS”—achieved the deployment of a 20 m-class thin membrane in 2010 (Fig. 2 left). The deployment motion of the thin membrane of IKAROS was predicted using two kinds of in-house numerical analysis software based on the finite element method (FEM) and multiparticle method. Notably, the predicted motion corresponded well with the flight data (Fig. 2 right).

Moreover, an appropriate ground test for the deployment of larger gossamer space structures is required before launch. However, ground deployment testing of a full-scale model is impossible because the effect of gravity cannot be neglected for such a large structure.

Thus, the deployment motion of a full-scale model is required to be predicted using a small model that is available for ground testing. This requires a scaling law for the deployment motion. Consequently, several scale laws have been proposed thus far. The motion of two models of varying sizes are similar if the scale law holds true between them. However, the detailed miniaturization of large deployable structures and fabrication of a small model that is geometrically similar to the full-scale model is challenging owing to limitations of the small model in terms of folding the membrane with a lesser width. For instance, the storage of a small membrane with numerous folds into a low volume is difficult. Therefore, the full-scale model must be simplified during fabrication of the small model, regardless of the geometrical mismatch with the full-scale model. In addition, simplification of the model reduces the computation time of the numerical simulation.

This paper proposes a scale law between the full-scale model of a large gossamer space structure and its geometrically non-similar small model. The scale law is also applied to the spin-deployment motion of a thin membrane and subsequently verified through numerical simulations. Applications of the proposed method to the spin-deployment of a z-fold membrane are assumed, such as that of a thin solar panel and that of the IKAROS 20 m-class solar power sail, are shown as the numerical examples.

2. Scale-Law Parameters

This section proposes a scale law and derives the scale-law parameters for the spin-deployment motion of the IKAROS solar sail. In addition, the scale-law parameters were matched between the small and full-scale models, regardless of the geometric similarity between the small and full-scale models. Note that “small model” indicates a geometrically non-similar model, unless stated otherwise.

2.1. Concept of scale law

The scale law proposed in this section is based on conventional scale laws that is, the small model is designed such that the ratio between each energy component is equal to that of the full-scale model. Let us express the total mechanical energy \( \Pi \) as the summation of the kinetic energy components \( T_i \) and potential energy components \( V_j \).

\[
\Pi = \sum_{i=1}^{M} T_i + \sum_{j=1}^{N} V_j, \tag{1}
\]

where \( M \) and \( N \) are the number of components of kinetic energy and potential energy, respectively. Equation (1) can then be rewritten as

\[
\Pi = \sum_{i=1}^{M} T_i^* \tilde{T}_i + \sum_{j=1}^{N} V_j^* \tilde{V}_j, \tag{2}
\]

where \( T_i^* \) and \( V_j^* \) are dimensional constants, while \( \tilde{T}_i \) and \( \tilde{V}_j \) are nondimensional time-varying quantities. The normalization of \( T_i^* \) and \( V_j^* \) with respect to \( T_i^* \) reduces the parameters for correspondence between the small and full-scale models. Therefore, Eq. (2) can then be rewritten as

\[
\Pi = T_1 \left( \tilde{T}_1 + \sum_{i=2}^{M} \frac{T_i^*}{T_1} \tilde{T}_i + \sum_{j=1}^{N} \frac{V_j^*}{T_1} \tilde{V}_j \right), \tag{3}
\]

The dimensional factor using normalization can be any parameter of the kinetic or potential energy. Moreover, the motion of both of the models is similar if \( \tilde{T}_i \) and \( \tilde{V}_j \) are identical to those of the full-scale model. In particular, \( \tilde{T}_i \) and \( \tilde{V}_j \) are identical in almost all cases. Therefore, the design considerations of the small model should ensure that its \( \tilde{T}_i \) and \( \tilde{V}_j \) are identical to those of the full-scale model. The parameters \( T_i^* \) and \( V_j^* \) are termed as “scale-law parameters,” and \( \tilde{T}_i \) and \( \tilde{V}_j \) are termed as “norm-
2.2. Scale-law parameters of IKAROS

The scale-law parameters for the spin-deployment motion of IKAROS are derived herein.

2.2.1. Energy components of IKAROS

The small IKAROS model for simulation of the spin-deployment motion is illustrated in Fig. 3, where the devices attached to the sail membrane of IKAROS are ignored in the small model, and the number of folds varies from that of IKAROS.

In addition, considering the configuration of the ground test, gravity was assumed to act in the negative direction of the z-axis of the coordinate system, as depicted in Fig. 4.

The energy components and non-conservative disturbances for the scale-law parameters are stated as follows:

1) Kinetic energy of membrane
2) Strain energy of membrane
3) Spring back energy in fold line
4) Kinetic energy of tether
5) Strain energy of tether
6) Kinetic energy of tip mass
7) Kinetic energy of satellite body
8) Gravitational potential energy
9) Aerodynamic drag
10) Structural damping (membrane and cable)

Furthermore, the unique parameter pertaining to the spin-deployment motion is as follows:

11) Initial spin rate

These parameters are defined in the following subsections.

2.2.2. Kinetic energy of membrane ($T_m$)

The kinetic energy of an infinitesimal membrane element $T_m$ can be expressed as

$$T_m = \int 1 2 \rho \dot{\mathbf{r}} \cdot \mathbf{\ddot{r}} \, dA$$

where $\rho$ and $h$ are the density and thickness of the membrane, respectively. In addition, $dA$ is the infinitesimal area of the membrane as shown in Fig. 5.

Therefore, the infinitesimal area of membrane $dA$ can be expressed as

$$dA = \frac{\partial x}{\partial \xi_1} \times \frac{\partial x}{\partial \xi_2} \, d\xi_1 \, d\xi_2 = X^2 \frac{\partial x}{\partial \xi_1} \times \frac{\partial x}{\partial \xi_2} \, d\xi_1 \, d\xi_2$$

where $X$ and $T$ represent the characteristic length and time for normalizing the design parameters, respectively, and they satisfy the following:

$$x = X \bar{x}, \quad t = T \bar{t}.$$  

Using Eq. (6), differentiation of the position vector with respect to time $\dot{x}$ can be rewritten as

$$\dot{x} = \frac{\partial x}{\partial t} = X \frac{\partial \bar{x}}{\partial \bar{t}} \equiv X \frac{T}{T} \dot{\bar{x}},$$

where $\dot{\bar{x}}$ represents the differentiation of the non-dimensional position vector with respect to non-dimensional time. $X$ and $T$ of the full-scale model are defined arbitrarily by the designer. For the spin-deployment of IKAROS, $X$ is defined as the side length of the square membrane herein. Furthermore, $T$ of the small model is defined by matching a scale-law parameter, because the ratio of the characteristic time between the full-scale and small models represents the ratio of deployment time for each model. The evaluation of $T$ is described in Section 3. Substituting Eq. (5) into Eq. (4), the kinetic energy of an infinitesimal membrane element can be rewritten as

$$T_m = \rho h \int \frac{1}{2} \dot{\bar{x}} \cdot \mathbf{\ddot{\bar{x}}} | \mathbf{\dot{G}} | d\xi_1 \, d\xi_2$$

Thus, the dimensionless factor $\tilde{T}_m$ expressed in Eq. (8) can be obtained as

$$\tilde{T}_m = \frac{\rho h X^4}{T^2} \int \frac{1}{2} \dot{\bar{x}} \cdot \mathbf{\ddot{\bar{x}}} | \mathbf{\dot{G}} | d\xi_1 \, d\xi_2,$$

and the scale-law parameter for the kinetic energy of membrane $\tilde{T}_m$, which is the dimensional factor in Eq. (8), is obtained as

$$\tilde{T}_m = \int \frac{1}{2} \dot{\bar{x}} \cdot \mathbf{\ddot{\bar{x}}} | \mathbf{\dot{G}} | d\xi_1 \, d\xi_2.$$

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2.2.3. Strain energy of membrane ($V_m^*$)

Let us assume that the membrane material is isotropic. Thus, the constitutive law can be expressed as

$$
\sigma = C \epsilon
$$

(11)

$$
C = \begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{xy}
\end{bmatrix}, \quad
\epsilon = \begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
2\gamma_{xy}
\end{bmatrix},
$$

(12)

$$
C = \frac{E}{1 - \nu^2} \begin{bmatrix}
1 & \nu & 0 \\
\nu & 1 & 0 \\
0 & 0 & 1 - \nu
\end{bmatrix} \equiv E\bar{C}.
$$

The strain energy of a membrane element $V_m$ is given as

$$
V_m = \int \frac{1}{2} \epsilon^T \bar{C} \epsilon \, dA
\approx \int \frac{1}{2} \epsilon^T \bar{C} \epsilon \, |\Omega| \, dl
$$

(13)

$$
= E h X^2 \int \frac{1}{2} \epsilon^T \bar{C} \epsilon \, |\Omega| \, dl
$$

Thus, the dimensionless factor $\bar{V}_m$ in Eq. (13) is written as

$$
\bar{V}_m = \int \frac{1}{2} \epsilon^T \bar{C} \epsilon \, |\Omega| \, dl.
$$

(14)

and the scale-law parameter for the strain energy of membrane $V_m^*$ is obtained as

$$
V_m^* = E h X^2.
$$

(15)

2.2.4. Spring-back energy in fold line ($V_{sb}^*$)

The strain energy of the spring-back at the crest of length $L_f$ can be estimated using the curved beam theory. Let us consider the crease in the self-equilibrium state as a curved beam with radius $R$, thickness $h$, length $L_f$, and inner angle $\theta_{ref}$, as depicted in Fig. 6.

If the inner angle of the crease changes to $\theta$ upon bending as in Fig. 6, the strain energy of the spring-back $V_{sb}$ can be expressed as

$$
V_{sb} = \frac{E L_f}{2\theta_{ref}} \left[ R h - \frac{h^2}{\log \left( \frac{R + h/2}{R - h/2} \right)} \right] (\theta - \theta_{ref})^2.
$$

(16)

Next, we assume that the membrane is completely folded to form a crease, as illustrated in Fig. 7. Then, the radius of curvature of the crease in the self-equilibrium state can be written as

$$
R = \frac{h \pi}{2 \theta_{ref}}.
$$

(17)

Substituting Eq. (17) into Eq. (16) yields

$$
V_{sb} = \frac{E L_f h^2}{2\theta_{ref}^3} \left[ \pi \log \left( \frac{\pi + \theta_{ref}}{\pi - \theta_{ref}} \right) \right] (\theta - \theta_{ref})^2,
$$

(18)

which indicates that the rotational spring constant per unit fold length, denoted as $k_f$, can be expressed as

$$
k_f = \frac{E h^2}{\theta_{ref}^3} \left[ \frac{\pi}{2\theta_{ref}^2} \log \left( \frac{\pi + \theta_{ref}}{\pi - \theta_{ref}} \right) \right].
$$

(19)

In the numerical simulation, the spring-back effect is considered by implementing the rotational spring element with the potential energy as

$$
V_{sb} = k_f L_f (\theta - \theta_{ref})^2.
$$

(20)

If the self-equilibrium angle $\theta_{ref}$ of the small model is equal to that of the full-scale model, the scale-law parameter can be stated as $k_f L_f$ (or $EL_f h^2$). However, the corresponding $\theta_{ref}$ of the small model with that of the full-scale model is challenging because the folding methods for controlling the crease angle are not yet established and the values of $\theta_{ref}$ are uncertain when the membrane is folded. Consequently, the scale law does not hold true in such cases. Therefore, the authors propose an approximated scale law that holds true only in a certain case $\theta = \theta_{ref}$. Thus, $\theta_{sb}$ of the small model must be the same as that of the full-scale model. With this approximation, the scale-law parameter can be stated as

$$
V_{sb} = k_f L_f (\theta_{sb} - \theta_{ref})^2,
$$

(21)

and the strain energy $V_{sb}$ is written as

$$
V_{sb} = V_{sb}^* V_{sb},
$$

(22)

where

$$
\bar{V}_{sb} = \frac{(\theta - \theta_{ref})^2}{2(\theta_{sb} - \theta_{ref})^2}.
$$

(23)

If every crease in the membrane is formed in the same way, which was true in the case of IKAROS, $L_f$ in Eq. (21) denotes the total length of the fold, as represented by the red lines in Fig. 8.
As described in detail in Section 3, $L_f$ of the small model can be determined in a few ways, which depends on the fold angle $\theta_{db}$. Thus, the fold angle $\theta_{db}$ affects the error of the scale law. In this study, the authors determined that

$$\theta_{db} = \pi,$$  (24)

that is, the scale law holds at the folded state when the effect of the spring-back energy is largest during deployment.

2.2.5. Kinetic energy of tether ($T_{\text{c}}^*$)

The kinetic energy of a cable element $T_c$ can be written as follows.

$$T_c = \int \frac{1}{2} \beta_e \cdot \dot{\beta}_e L_c \, d\xi_e = \frac{\rho_c A_c X^3}{T^2} \int \frac{1}{2} \dot{\beta}_e \cdot \dot{\beta}_e L_c \, d\xi_e$$  (25)

Thus, the dimensionless factor $\tilde{T}_c$ is obtained as

$$\tilde{T}_c = \int \frac{1}{2} \dot{x}_e \cdot \dot{x}_e L_c \, d\xi_e,$$  (26)

and the scale-law parameter for the kinetic energy of tether $T_{\text{c}}^*$ is obtained as

$$T_{\text{c}}^* = \frac{\rho_c A_c X^3}{T^2}.$$  (27)

2.2.6. Strain energy of tether ($V_{\text{c}}^*$)

The strain energy of a cable element $V_c$ can be expressed as

$$V_c = \int \frac{1}{2} E_c \varepsilon_e^2 L_c \, d\xi_e = E_c A_c X \int \frac{1}{2} \varepsilon_e^2 L_c \, d\xi_e.$$  (28)

Thus, the non-dimensional factor $\tilde{V}_c$ is derived as

$$\tilde{V}_c = \int \frac{1}{2} \varepsilon_e^2 L_c \, d\xi_e,$$  (29)

and the scale-law parameter describing the strain energy of tether $V_{\text{c}}^*$ is obtained as

$$V_{\text{c}}^* = E_c A_c X.$$  (30)

2.2.7. Kinetic energy of tip mass ($T_{\text{tip}}^*$)

The kinetic energy of a tip mass (mass point) $T_{\text{tip}}$ can be expressed as

$$T_{\text{tip}} = \frac{1}{2} m_{\text{tip}} \dot{\mathbf{x}} \cdot \dot{\mathbf{x}} = \frac{m_{\text{tip}} X^2}{T^2} \frac{1}{2} \dot{x} \cdot \dot{x}.$$  (31)

Subsequently, the dimensionless factor $\tilde{T}_{\text{tip}}$ is obtained as

$$\tilde{T}_{\text{tip}} = \frac{1}{2} \dot{x} \cdot \dot{x},$$  (32)

and the scale-law parameter for the kinetic energy of a tip mass $T_{\text{tip}}^*$ is obtained as

$$T_{\text{tip}}^* = \frac{m_{\text{tip}} X^2}{T^2}.$$  (33)

2.2.8. Kinetic energy of satellite body ($T_{\text{rigid}}^*$)

The kinetic energy associated with the rotational motion of a satellite body (rigid body) $T_{\text{rigid}}$ can be expressed as

$$T_{\text{rigid}} = \frac{1}{2} I_{\text{rigid}} \dot{\phi}_{\text{rot}}^2 = \frac{I_{\text{rigid}}}{T^2} \frac{1}{2} \dot{\phi}_{\text{rot}}^2.$$  (34)

Thus, the dimensionless factor $\tilde{T}_{\text{rigid}}$ is derived as

$$\tilde{T}_{\text{rigid}} = \frac{1}{2} \dot{\phi}_{\text{rot}}^2,$$  (35)

and the scale-law parameter for the kinetic energy of the rotational motion of the satellite body $T_{\text{rigid}}^*$ is obtained as

$$T_{\text{rigid}}^* = \frac{I_{\text{rigid}}}{T^2}.$$  (36)

2.2.9. Gravitational potential ($V_g^*$)

The gravitational potential energy $V_g$ in the coordinate system illustrated in Fig. 4 can be written as

$$V_g = -\int \left( \sum_i \phi \delta \mathbf{x} \right)^T \rho x \mathbf{g} |d\xi_1| d\xi_2$$

$$= -\rho g X^3 \int i_1 \sum_i \phi \delta \mathbf{x} |d\xi_1| d\xi_2.$$  (37)

Thus, the dimensionless factor $\tilde{V}_g$ expressed in the above equation is derived as

$$\tilde{V}_g = -\int i_1 \sum_i \phi \delta \mathbf{x} |G| |d\xi_1| d\xi_2,$$  (38)

and the scale-law parameter for the gravitational potential energy $V_g^*$ is obtained as

$$V_g^* = \rho g X^3 g.$$  (39)

2.2.10. Aerodynamic drag ($\delta W_{\text{air}}$)

The external virtual work due to the aerodynamic drag can be expressed as

$$\delta W_{\text{air}} = -\int \int \frac{1}{2} \rho_{\text{air}} C_D |\dot{\mathbf{x}}|^2 n \cdot \delta \mathbf{x} |G| |d\xi_1| d\xi_2$$

$$= -\frac{\rho_{\text{air}} X^4}{T^2} \int \int \frac{1}{2} C_D |\dot{\mathbf{x}}|^2 n \cdot \delta \mathbf{x} |G| |d\xi_1| d\xi_2,$$  (40)

where $\delta \mathbf{x}$ represents the virtual displacement. Thus, the dimensionless factor $\delta \tilde{W}_{\text{air}}$ in Eq. (40) is obtained as follows:

$$\delta W_{\text{air}} = -\int \int \frac{1}{2} C_D |\dot{\mathbf{x}}|^2 n \cdot \delta \mathbf{x} |G| |d\xi_1| d\xi_2,$$  (41)

and the scale-law parameter of air resistance $\delta W_{\text{air}}^*$ is obtained as
2.2.11. Structural damping ($V_{mc}^*$)

The amount of energy dissipation in the stress proportional damping of membrane $V_{mc}^*$ can be expressed as

$$V_{mc}^* = -\int \varepsilon_p \cdot \gamma C \varepsilon_p \, dV,$$

where $\varepsilon_p$ denotes the principal strain, and $\gamma$ represents the damping factor. Thus, Eq. (43) can be rewritten as

$$V_{mc}^* = \frac{E h X^2 \gamma}{T} \int \left( -\varepsilon_p \cdot C \varepsilon_p \right) |G| d\xi_1 d\xi_2.$$

Thus, the dimensionless factor $\bar{V}_{mc}^*$ in Eq. (44) is obtained as

$$\bar{V}_{mc}^* = \frac{\int \left( -\varepsilon_p \cdot C \varepsilon_p \right) |G| d\xi_1 d\xi_2}{\int \left( -\varepsilon_p \cdot C \varepsilon_p \right) dV},$$

and the scale-law parameter for the damping of membrane $V_{mc}^*$ is obtained as

$$V_{mc}^* = \frac{E h X^2 \gamma}{T}.$$

Similarly, the scale-law parameter for the damping of cable $V_{mc}^*$ is obtained as

$$V_{mc}^* = \frac{E_c A_c X Y_c}{T},$$

where $Y_c$ represents the damping factor of the cable.

2.2.12. Initial spin rate ($\phi_{rot}^*$)

The primary function of this scale-law parameter is to facilitate exact correspondence of the rotation angle and not the energy ratio. The rotation angle $\phi_{rot}$ at the initial spin rate $\omega_0$ can be expressed as

$$\phi_{rot} = \omega_0 t_0 T.$$

Thus, the dimensionless factor $\bar{\phi}_{rot}$ is obtained as

$$\bar{\phi}_{rot} = \bar{t},$$

and the scale-law parameter of the initial spin rate $\phi_{rot}^*$ is described as

$$\phi_{rot}^* = \omega_0 T.$$

The influence of the variation in the spin rate during deployment must be considered for satisfying the scale law, which is discussed in Section 5.

2.3. Modification of scale-law parameters

The scale-law parameters derived in the previous subsections are summarized in Table 1. The parameters involving the gravitational potential and aerodynamic drag can be neglected if the scale law is established between the small model and IKAROS, because the gravity acting on IKAROS is negligible and the aerodynamic drag does not act on IKAROS while it may act on the small model during ground testing.

In this study, the scale-law parameter for the kinetic energy of the membrane $T_{m}^*$ is selected as the scale-law parameter $T_{m}^*$ expressed in Eq. (3) for normalization. Consequently, nine normalized scale-law parameters are obtained for coordinating the full-scale and small models, as listed in Table 2.

3. Procedure of Determining Design Parameters of Small Model from Scale-Law Parameters

As listed in Table 2, the normalized scale-law parameters of the small model must correspond to those of the full-scale model in order to an accurate predict on-orbit behavior of the large-membrane structure using numerical simulations/ground testing of the small model, which is geometrically non-similar to the full-scale model.

The following list describes the procedure for determining the design parameters of the small model, assuming that the design or $X$ parameters of the full-scale model are already known. Note that the superscript $s$ (i.e., indicating values of the small model) is omitted in the list for simplicity.

1) Determination of the characteristic length $X$ (i.e., side length of the square membrane) from the experimental space (e.g., size of the vacuum chamber).

2) Determination of the fold number $N_f$: Preferably, the fold number of both of the models should be equal. However, the generation of several folds in the small-scale model such as that in the full-scale model is often challenging; that is, $N_f$ of the small-scale model is less than that of the full-scale model because the small-scale model poses limitations in holding a membrane with a smaller width. Moreover, an unequal fold number results in a geometrical mismatch between

| Table 1. Scale-law parameters. |
|-----------------------------|-----------------|
| Item | Parameter |
|---|---|
| Kinetic energy of membrane $T_m^*$ | $\rho h X^2 T^2$ |
| Strain energy of membrane $V_{mc}^*$ | $E h X^2$ |
| Spring-back energy in fold line $V_{fb}^*$ | $k_1 L (\beta_{ref} - \beta_{ref})^2$ |
| Kinetic energy of cable $T_c^*$ | $\rho A_c X^2 T^2$ |
| Strain energy of cable $V_{sc}^*$ | $E A_c T / \rho h X^2$ |
| Kinetic energy of tip mass $T_{tip}^*$ | $m h X^2 T^2$ |
| Kinetic energy of satellite body $T_{sat}^*$ | $I_{ref} h X^2 / \rho h X^2$ |
| Gravitation potential $V_g$ | $-g h X^2$ |
| Aerodynamic drag $\delta W_{air}^t$ | $C_d h X^2$ |
| Damping of membrane $V_{mc}^*$ | $E h X^2 / \rho h X^2$ |
| Damping of cable $V_{mc}^*$ | $E_c A_c X Y_c / \rho h X^2$ |
| Initial spin rate $\phi_{rot}^*$ | $\phi_0 T$ |

| Table 2. Scale-law parameters (modified). |
|-----------------------------|-----------------|
| Item | Parameter |
|---|---|
| Kinetic energy of membrane $T_m^*$ | $E h X^2 / \rho h X^2$ |
| Strain energy of membrane $V_{mc}^*$ | $k_1 L (\beta_{ref} - \beta_{ref})^2 / \rho h X^2$ |
| Spring-back energy in fold line $V_{fb}^*$ | $E A_c T / \rho h X^2$ |
| Kinetic energy of cable $T_c^*$ | $m h X^2 T^2$ |
| Strain energy of cable $V_{sc}^*$ | $I_{ref} h X^2 / \rho h X^2$ |
| Kinetic energy of tip mass $T_{tip}^*$ | $-g h X^2$ |
| Kinetic energy of satellite body $T_{sat}^*$ | $C_d h X^2$ |
| Gravitation potential $V_g$ | $\rho h X^2$ |
| Aerodynamic drag $\delta W_{air}^t$ | $D h X^2$ |
| Damping of membrane $V_{mc}^*$ | $E h X^2 / \rho h X^2$ |
| Damping of cable $V_{mc}^*$ | $E_c A_c X Y_c / \rho h X^2$ |
| Initial spin rate $\phi_{rot}^*$ | $\phi_0 T$ |
the two models, thus necessitating modification of the model, as described in Section 4.

3) Selection of the membrane material for determining the physical properties of the membrane, such as $E$, $\rho$, $v$, $h$, and $\theta_{\text{ref}}$.

4) Determination of the characteristic time $T$ by matching the scale-law parameters for the strain energy of the membrane of the small model, $V_m^{\psi}$, with that of the full-scale model, $V_m^{\psi}$,

$$T = \sqrt[4]{\frac{V_m^{\psi}}{\rho X^2}}. \quad (51)$$

5) Determination of the mass of tip $m_{\text{tip}}$ by matching $T_{\text{tip}}^{\psi}$ with $T_{\text{tip}}^{\psi}$,

$$m_{\text{tip}} = \frac{\rho L_{\text{tip}}}{\rho X^2}. \quad (52)$$

Determination of the moment of inertia of the satellite body $I_{\text{rigid}}$ by matching $T_{\text{rigid}}^{\psi}$ with $T_{\text{rigid}}^{\psi}$,

$$I_{\text{rigid}} = \frac{V_m^{\psi}}{\rho X^2}. \quad (53)$$

Determination of the initial spin rate $\omega_0$ by matching $\omega_{\text{rot}}^{\psi}$ with $\omega_{\text{rot}}^{\psi}$,

$$\omega_0 = \frac{\rho L_{\text{tip}}}{\rho X^2}. \quad (54)$$

6) Selection of the tether material for evaluating the Young’s modulus $E$, and subsequently determining the physical properties of each tether $\rho_w$, $A_w$ by matching $T_{\text{teth}}^{\psi}$ with $T_{\text{teth}}^{\psi}$ and $V_v^{\psi}$ with $V_v^{\psi}$, respectively,

$$\rho_w = \frac{V_v^{\psi}}{A_w} \quad (55)$$

$$A_w = \frac{V_v^{\psi}}{E X^2}. \quad (56)$$

7) Determination of the damping factor of the membrane and tether $\gamma$ by matching $V_v^{\text{diss}}$ with $V_v^{\psi}$ and $V_v^{\psi}$ with $V_v^{\psi}$, respectively,

$$\gamma = \frac{V_v^{\psi}}{T}. \quad (57)$$

$$\gamma = \frac{V_v^{\psi}}{T}. \quad (58)$$

In this case, the damping factor of the membrane and cable can be determined using Eqs. (57) and (58), because the scale-law parameters of the strain energy of the membrane and cable–$V_m^{\psi}$ and $V_{\text{teth}}^{\psi}$, and $V_v^{\psi}$ and $V_v^{\psi}$–have already been matched to each other in the steps 4) and 6), respectively.

8) Determination of the total length of the fold line $L_f$ or modification of the rotational spring constant $k_f$: The scale-law parameter in Eq. (21) depends on $\theta_{\text{tip}}$, $\theta_{\text{org}}$, $k_f$, and $L_f$. $\theta_{\text{org}}$ should be determined as the angle that generates the most significant effects on the spring-back energy; therefore, the authors set $\theta_{\text{org}}$ as 0. Crease angle of membrane of small model $\theta_{\text{ref}}$ is better to be the same as that of full-scale model $\theta_{\text{ref}}$. However, as explained in Section 2.2.4, it is difficult to make $\theta_{\text{ref}}$ coincide with $\theta_{\text{ref}}$ if the material properties of the membrane of the small model are not the same as those of the full-scale model. $k_f$ is determined using Eq. (19) if the material property of the membrane and $\theta_{\text{ref}}$ are given. $L_f$ is determined from the geometry of the small model if the membrane size and the fold number $N_f$ are given. Therefore, some modification is necessary. In this study, the following two cases are considered:

Case 1: $k_f$ is modified in the numerical simulation: Eq. (19) is ignored and $k_f$ is determined so that the non-dimensional scale-law parameter $V_{\text{sb}}$ of the small model coincides with that of the full-scale model,

$$k_f = k_f^{\text{mod}} = \frac{\rho L_{\text{ref}}}{L_f (\theta_{\text{org}} - \theta_{\text{ref}})^2 T^2} V_{\text{sb}}^{\psi}. \quad (59)$$

Case 2: The scale-law parameter $V_{\text{sb}}$ is neglected, $L_f = L_f^{\text{org}}$, $k_f = k_f^{\text{org}}$, and $V_{\text{sb}}$ does not match $V_{\text{sb}}^{\psi}$. It should be noted that Case 1 is not practically applicable for use during ground tests because it is difficult to fabricate the crease satisfying $k_f = k_f^{\text{mod}}$. In this case, $L_f$ must be modified to

$$L_f = L_f^{\text{mod}} = \frac{\rho L_{\text{ref}}}{k_f (\theta_{\text{org}} - \theta_{\text{ref}})^2 T^2} V_{\text{sb}}^{\psi}. \quad (60)$$

If $L_f^{\text{mod}}$ is smaller than the original fold length that is geometrically determined and denoted by $L_f^{\text{org}}$, $L_f$ can be modified by cutting the fold line partially to make the residual fold length equal to $L_f^{\text{mod}}$, as illustrated in Fig. 9. If $L_f^{\text{mod}}$ is longer than $L_f^{\text{org}}$, then $\theta_{\text{org}}$ must be altered by remaking the fold line.

10) The additional design parameters of the small model, the length of each tether, and the diameter of the satellite body are determined based on geometry.

4. Modification of Spin Rate in a Geometrically Non-Similar Model

The proposed scale law has a critical problem when trying to predict the spin deployment motion.

Generally, it is difficult to fold a membrane with a small fold width, so a small membrane model with the same number of folds as that of the full-scale model is much more difficult to fold than the full-scale model, as mentioned in the previous section. Therefore, the fold number $N_f$ of the small model may be different from that of the full-scale model.

In the case of spin deployment of a thin membrane such as a solar sail, the variation of $N_f$ is reflected as the variation of moment of inertia in the initial (undeployed) state (Fig. 10).

According to the conservation law of angular momentum,
Eq. (61) holds.

$$I_{e}^{t} \omega_{e}^{t} = I_{o}^{t} \omega_{o}^{t}, \quad I_{e}^{f} \omega_{e}^{f} = I_{o}^{f} \omega_{o}^{f}, \quad (61)$$

where the subscripts $o$ and $e$ represent the initial and final values, respectively. If the small model presents similar motion in comparison to that of the full-scale model, Eq. (62) must hold true as well.

$$\frac{\omega_{e}^{f}}{\omega_{o}^{f}} = \frac{\omega_{e}^{t}}{\omega_{o}^{t}} \quad (62)$$

However, Eq. (62) holds only if the following equation is true:

$$I_{e}^{f} \omega_{e}^{f} = I_{o}^{f} \omega_{o}^{f}. \quad (63)$$

This equation does not hold if $N_f$ of the small model is not equal to that of the full-scale model. In such a case, after the motion data of the small model are obtained from numerical simulations or experiments, the data must be modified to predict the motion of the full-scale model. In this paper, the authors propose to modify the spin rate of the small model $\omega_{e}^{f}(\vec{t})$ during deployment, as shown by using $\omega_{e}^{mod}(\vec{t})$ in Eq. (64).

$$\frac{\omega_{e}^{mod}(\vec{t})}{\omega_{o}^{f}} = 1 - \frac{\omega_{e}^{L} - \omega_{e}^{C}}{\omega_{o}^{L} - \omega_{o}^{C}} - \frac{\omega_{e}^{L} - \omega_{e}^{f}(\vec{t})}{\omega_{o}^{L} - \omega_{o}^{C}} \quad (64)$$

Using this modification, Eq. (62) holds true, regardless of the validity of Eq. (63).

5. Numerical Example

Two numerical examples are shown in this section to evaluate the proposed scale law. The first one is the spin deployment of simply z-folded panels, and the second one is the spin deployment of a membrane structure and comparison with the flight data of IKAROS.

5.1. Deployment of solar panel

Let us consider a spinning rigid body with two z-folded panels. The panels deploy as in Fig. 11. Figure 12 illustrates the coordinate system and the parameters. The characteristic length $X$ of this model is defined as the total length of the panel as shown in Fig. 11.

In this example, two small models are considered. One of them, model 1, is geometrically similar to the full-scale model, whereas the other model, “model 2,” is geometrically non-similar to the full-scale model; that is, the fold number of model 2 is less than that of the full-scale model and model 1. The design parameters of the small models are determined as shown in Section 3. The specifications of the full-scale model and each small model used in the numerical analysis are listed in Table 3.

Table 3. Specifications of full-scale and small models (deployment of z-folded solar panels).

| Item | Full-scale model | Small model |
|------|------------------|-------------|
| Characteristic parameter | | |
| Characteristic time $T$ [s] | 6.0 | 0.5 |
| Characteristic length $X$ [m] | 12.00 | 1.00 |
| Satellite body | | |
| Young’s modulus $E$ [GPa] | 3.00 | 300 |
| Density $\rho$ [kg/m$^3$] | 1400 | 1400 |
| Fold number $N_f$ [-] | 24.5 | model 1: 24.5 |
| | | model 2: 5.5 |
| Length of each panel $L_{x}$ [m] | 0.490 | model 1: 0.041 |
| | | model 2: 0.182 |
| Width $L_{y}$ [m] | 4.8 | 0.4 |
| Thickness $h$ [µm] | 50.0 | 50.0 |
| Poisson’s ratio $v$ [-] | 0.3 | 0.3 |
| Damping factor $\gamma$ [s] | $12.0 \times 10^{-6}$ | $1.00 \times 10^{-6}$ |
model 2 is less than that of model 1, geometrically similar model. Thus, using a small model that is not geometrically similar can reduce computational costs by about 97% compared to the full-scale model in this example.

The spin rate during the deployment obtained using the full-scale FE model and the geometrically similar FE model based on the proposed scale law are described in Fig. 13, and it is shown that they agree well with each other. Thus, the proposed scale law is effective if the small model is geometrically similar to the full-scale model.

The spin rates during deployment obtained using the full-scale FE model and small FE model that is geometrically non-similar to the full-scale model are shown in Fig. 14, where Fig. 14(b) presents the spin rate modified using Eq. (64) and Fig. 14(a) presents the results without modification. Figure 15 shows the spin rate of $0.4 \leq t \leq 1.0$. There is a difference between the modified and non-modified results, but the difference is small. Table 5 shows the moment of inertia of each model in undeployed and deployed states. The ratio of the moment of inertia in Eq. (63) is almost the same for all models. The difference between model 1 and model 2 is only an order of $10^{-3}$. So, the effect of the modification is quite small.

Figures 16 and 17 illustrate the shape of the membrane during the deployment of model 1 (geometrically similar small model) and the full-scale model, respectively. The deployment behavior of the small model agrees well with that of the full-scale model.

5.2. Spin-deployment of IKAROS

In this subsection, the FE analysis results of the small model obtained using the proposed scale law are compared with the flight data of IKAROS to evaluate the proposed scale law. The results are compared with the full-scale FE analysis results of IKAROS as well.12,21

The specifications of the small and full-scale models used in the numerical analysis are listed in Table 6. The rotational spring constant of the crease unit length of the full-scale FE finite analysis is assumed to be zero because the spring-back effect is negligible for the deployment of a large membrane.22

The spin rate during the deployment obtained using the full-scale FE model of IKAROS and the small-scale FE model obtained using the proposed scale law are plotted in Fig. 18. The rotational spring constant of the crease $k_f$ of the small model is modified using Eq. (59). Figure 18(b)
Table 5. The ratio of the moment of inertia in undeployed and deployed states of the full-scale model and two small models (deployment of z-folded solar panel).

| Item                                  | Full-scale model | Small model (model 1) | Small model (model 2) |
|---------------------------------------|------------------|-----------------------|-----------------------|
| Moment of inertia in undeployed state | 2.506 × 10^7 kg m^2 | 0.1212 kg m^2 | 0.1209 kg m^2 |
| Moment of inertia in deployed state   | 3.357 × 10^7 kg m^2 | 0.1620 kg m^2 | 0.1620 kg m^2 |
| Ratio of moment of inertia           | 1.3396            | 1.3396                | 1.3365                |

Fig. 17. Shape of the membrane during deployment of the full-scale model.

Table 6. Specification of full-scale and small models (spin-deployment of IKAROS).

| Item                                  | Full-scale   | Small model |
|---------------------------------------|--------------|-------------|
| Characteristic time T [s]             | 5.000        | 0.319       |
| Characteristic length X [m]           | 13.56        | 1.000       |
| Satellite body                        |              |             |
| Mass M_{total} [kg]                   | 291          | 1.30        |
| Inertia of z-axis I_{z-axis} [kg m^2]  | 66.5         | 3.30 × 10^7 |
| Initial spin rate \(\omega_0\) [rpm]  | 5.147        | 80.74       |
| Diameter \(\phi_{total}\) [mm]       | 1580         | 116.5       |
| Tip mass                              |              |             |
| Mass per tip mass \(m_t\) [kg]        | 5.0 × 10^{-1} | 4.6 × 10^{-1} |
| Membrane                              |              |             |
| Young’s modulus \(E\) [GPa]           | 3.0          | 4.1         |
| Density \(\rho\) [kg/m^3]             | 1420         | 1450        |
| Thickness \(h\) [μm]                  | 7.5          | 12.5        |
| Poisson’s ratio \(\nu\) [—]           | 0.3          | 0.3         |
| Damping factor \(\gamma\) [s]        | 7.80 × 10^{-6} | 4.97 × 10^{-6} |
| Fold number \(N_f\) [—]               | 18.5         | 7.5         |
| Spring constant of crease unit length \(k_{nf}\) [N] | 0.40 × 10^{-4} | 4.81 × 10^{-4} |
| Crease angle \(\theta_{cre}\) [deg]   | 23.80        | 16.97       |
| Equivalent total length of fold line \(L_f\) [m] | 593.28 Case 1: 0.90 | Case 2: 16.6 |
| Tether                                |              |             |
| Young’s modulus \(E_c\) [GPa]         | 11           | 100         |
| Density \(\rho_c\) [kg/m^3]           | 1813         | 1355        |
| Cross-sectional area \(A_c\) [m^2]    | 6.70 × 10^{-6} | 1.12 × 10^{-5} |
| Damping factor \(\gamma_c\) [s]      | 1.00 × 10^{-6} | 12.7 × 10^{-6} |

Fig. 16. Shape of the membrane during deployment of the small model (geometrically non-similar model, model 2).

Fig. 19(a). Shape of the membrane during deployment of the small model (geometrically non-similar model, model 2).

Fig. 22 and Fig. 23. of IKAROS are presented in Fig. 22 and Fig. 23. \(k_f\) was modified in Fig. 22 but not in Fig. 23. In the on-orbit deployment of the sail membrane of IKAROS, slight sticking occurred at the beginning of deployment (i.e., \(0 \leq i \leq 6\)). Therefore, the non-dimensional time \(i\) of the numerical result pertaining to the small-scale model was shifted by an equal amount as that of the full-scale FEM, as shown in Fig. 2.
By modifying the $k_f$, the numerical result of the small model agreed well with that of the flight data. On the other hand, there is a large difference between the flight data and model 2, non-modified model, because $k_f$ is much larger than that of model 1, modified model.

6. Conclusion

This paper proposed a scale law for the spin-deployment motion of membrane structures to predict the motion of a larger model based on that of a smaller model. A procedure for the development of a small model satisfying the scale law was also presented. The scale law was based on the stored mechanical energy, and it was usable even if the smaller
model was geometrically non-similar to the larger model (e.g., even if the number of folds of the membranes did not match). However, if the smaller model is geometrically non-similar, that is, there is a geometrical mismatch between the smaller and larger model, its spin rate differs from that of the large model. Therefore, a method for modifying the spin rate of the smaller model was introduced. In addition, a modification method was proposed for the rotational spring constant of the crease of the membrane in the case of a geometrical mismatch.

The scale law and modification methods were successfully verified by comparing the numerical results of the small model, flight data of IKAROS, and numerical results of the full-scale IKAROS model.

The authors plan to expand the scale law to enable minimizing the effect of gravity and verify it through a ground experiment using a vacuum chamber. The results will be presented in the near future.

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