Determination of the CKM matrix by semileptonic rare B-decays *

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Abstract

The possibility to test the unitarity triangle of CKM matrix is examined by using the semileptonic rare B decays, \( B \to X_q \ell^+ \ell^- \) with \( q = d, s \) and \( \ell \)'s are leptons. The discussion is emphasized on the CP asymmetry in the framework of standard and beyond the standard models. For the beyond standard model, a minimal extension of the standard model containing an additional isosinglet charge (-1/3) charge quark, which leads to a deviation from CKM unitarity, is considered. As the results, in the standard model the CP asymmetry is visible only for \( B \to X_q \ell^+ \ell^- \), while in the latter model it is sizeable even for \( B \to X_s \ell^+ \ell^- \).

1 Introduction

After the measurement of the radiative \( B \to X_s \gamma \) decay, the next target for the experiments is other higher order decays, e.g. \( B \to X_s \ell^+ \ell^- \). The CLEO Collaboration's study indicates to be able to measure few events number of this mode in near future [1].

Theoretically, the decays are very attractive and should be good probes to test the standard model as well as open a window to the theory beyond it. In general the rare decays, that means the decays which occur in one-loop level or more, are very sensitive to the new contributions. Here according to the unitarity triangle of CKM matrix, the new physics can be divided into two types, 1) the new physics which conserve the unitarity, and 2) the new physics which violate it. In this proceeding, I consider only the standard model and the latter type of new physics that can be realized by introducing an additional isosinglet charge (-1/3) charge quark.

As the measurement that must be examined in the experiments, the interest is focused on the CP asymmetry. Remark that, in the decay which is governed by single operator

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like $B \to X_s\gamma$, the CP asymmetry is tiny as indicated, for example, in some previous papers \[2\]. In another word, $B \to X_s\gamma$ decay is not useful to extract the CP violation. Then one needs to consider more promising modes. The best candidate is the semileptonic $B \to X_q\ell^+\ell^-$ decays with $q = d, s$.

The reason is very clear, that is $B \to X_q\ell^+\ell^-$ decay contains minimally three operators in its effective hamiltonian, that is

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{tb}^{\ast} V_{ts} \left\{ C_9^{\text{eff}} \left[ \bar{q} \gamma_{\mu} L b \right] \left[ \bar{\ell} \gamma^{\mu} \ell \right] + C_{10}^{\text{eff}} \left[ \bar{q} \gamma_{\mu} L b \right] \left[ \bar{\ell} \gamma^{\mu} \gamma_5 \ell \right] - 2C_7^{\text{eff}} \left[ \bar{q} i \sigma_{\mu\nu} \bar{q}'^\nu \left( R + m_q L \right) b \right] \left[ \bar{\ell} \gamma^{\mu} \ell \right] \right\}. \quad (1)$$

Here, $C_i^{\text{eff}}$ denotes the Wilson coefficient for each operator, $q'^\mu$ is four-momentum of the dilepton and $s = q^2$. Therefore, the CP violation will be occured if there is different phases in, at least, one of the Wilson coefficients.

## 2 Conserved CKM unitarity case

Now, I am not going to give a specific model of new physics that conserves the unitarity of CKM matrix, but discuss only the case in the framework of standard model.

In the standard model, some different phases are possibly appearing in the long-distance contributions of the decay. The reason is because $m_b$ is heavy enough to generate on-shell states of uponium and/or charmonium, i.e. $u^i\bar{u}^i$ with $u^i = u, c$. These $u^i\bar{u}^i$ loops are generated from the $b \to q u^i \bar{u}^i$ processes that is governed by the following operators,

$$O_1 = \left( \bar{q}_\alpha \gamma_{\mu} L b_\alpha \right) \sum_{i=1,2} \left( \bar{u}^i_\beta \gamma^{\mu} L u^i_\beta \right), \quad (2)$$

$$O_2 = \left( \bar{q}_\alpha \gamma_{\mu} L b_\beta \right) \sum_{i=1,2} \left( \bar{u}^i_\beta \gamma^{\mu} L u^i_\alpha \right), \quad (3)$$

after doing Fierz transformation. Here, $u^1 = u$, $u^2 = c$ and the lower suffixes denote the color. For $q = s$, $u\bar{u}$ loop contribution can be ignored \[4\], while for $q = d$ the situation is quite different, since \[3\]

$$q = s : |V^*_{us} V_{ub}| \sim \begin{cases} O(\lambda^2), & u^i = u \\ O(1), & u^i = c \end{cases}$$
Figure 1: The CKM unitarity triangle on the $\rho - \eta$ plane.

$$q = d : \left| \frac{V_{u_d}^* V_{u_b}}{V_{td}^* V_{tb}} \right| \sim \begin{cases} O(1), & u^i = u \\ O(1), & u^i = c \end{cases}$$

if we normalize the amplitude with $V_{tq}^* V_{tb}$ as usual. $\lambda$ is a parameter in Wolfenstein parametrization of CKM matrix [7] and the world average is $\lambda \sim 0.22$ [8].

Further, if the triangle is parametrized as Fig. 1, then one finds

$$\frac{V_{u_q}^* V_{ub}}{V_{tq}^* V_{tb}} \sim \begin{cases} r \left( e^{-i\gamma} - r \right), & q = d \\ \lambda^2 r e^{-i\gamma}, & q = s \quad (\lambda^2 \sim 5\%) \end{cases}$$

From the result, it is obvious that for $q = d$ the uponium ($u\bar{u}$) is large enough to induce the CP violation in the decay. On the other hand, for $q = s$ the CP violation will be suppressed by a factor of $\lambda^2$. These long-distance effects lead the Wilson coefficients to be [3],

$$C_{7 \text{eff}} = C_{7 \text{NLO}}, \quad C_{9 \text{eff}} = C_{9 \text{NLO}} + \frac{V_{u_q}^* V_{ub}}{V_{tq}^* V_{tb}} C_{9 \text{CP}} \quad C_{10 \text{eff}} = C_{10},$$

where $C_{7 \text{eff}}$ and $C_{9 \text{eff}}$ have been calculated up to next-to-leading order [5] [6], while $C_{10 \text{eff}}$ obtains no correction at all. The $C_{9 \text{CP}}$ is the CP violation factor and given as,

$$C_{9 \text{CP}} = (3 C_1 + C_2 + 3 C_3 + C_4 + 3 C_5 + C_6)$$

$$\times \left[ g(m_c, \hat{s}) - g(m_u, \hat{s}) \right] - \frac{16\pi^2}{9} \left( \sum_{V=\psi,\ldots} F_V(\hat{s}) - \sum_{V=\rho,\omega} F_V(\hat{s}) \right).$$
The expressions for the continuum and resonances contribution can be seen in reference [3].

3 Violated CKM unitarity case

For the case when the CKM unitarity is violated, I adopt a definite extension of the standard model containing an additional isosinglet charge (-1/3) charge quark [9].

In this model, the gauge sector are modified like below,

\[ \mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} V_{is} \bar{u}_i \gamma^\mu L d_\mu W^{\pm} + \text{h.c.} , \]  
\[ \mathcal{L}_Z = \frac{g}{2 \cos \theta_W} d_\alpha \gamma^\mu \left[ \left( \frac{2}{3} \sin^2 \theta_W \delta_{\alpha\beta} - z_{\alpha\beta} \right) L + \frac{2}{3} \sin^2 \theta_W \delta_{\alpha\beta} R \right] d_\beta Z_\mu , \]  

where \( z_{\alpha\beta} = \sum_{i=1}^{3} U^d_{\alpha i} U^{d^*_\beta} = \delta_{\alpha\beta} - U^d_{\alpha4} U^{d^*_4} \) and the new 3 \times 4 CKM matrix satisfies the relation \( \sum V^*_{iq} V_{tb} = z_{qb} \) that shows the violated unitarity for \( z_{qb} \neq 0 \).

As long as one considers only the tree-level contributions of new physics, in this case is the Z–exchange tree-level diagram, the Wilson coefficients would read [10],

\[ C_7^{\text{eff}} \rightarrow C_7^{\text{eff}} \]  
\[ C_9^{\text{eff}} \rightarrow C_9^{\text{eff}} + \left( 1 + \frac{\alpha_s(\mu)}{\pi} \omega(\hat{s}) \right) \frac{\pi}{\alpha} \frac{z_{qb}}{V^*_{tq} V_{tb}} \left( 4 \sin^2 \theta_W - 1 \right) \]  
\[ C_{10}^{\text{eff}} \rightarrow C_{10}^{\text{eff}} + \frac{\pi}{\alpha} \frac{z_{qb}}{V^*_{tq} V_{tb}} \]

The problem is then how to determine the size of the new mixings \( z_{qb}/V^*_{tq} V_{tb} \). Fortunately, in principle the upper bounds for the mixings can be extracted from the experimentally known \( B \rightarrow X_s \gamma \) decay for \( q = s \) and from \( B^0_d - \bar{B}^0_d \) mixing for \( q = d \) [10].

4 Summary

By including the long-distance contributions in the \( B \rightarrow X_q \ell^+ \ell^- \) decays, there will be additional phases in the amplitude. These should induce the CP asymmetry that is defined as,

\[ \tilde{A}_{CP} = \frac{d\mathcal{B}/d\hat{s} - d\tilde{\mathcal{B}}/d\hat{s}}{d\mathcal{B}/d\hat{s} + d\tilde{\mathcal{B}}/d\hat{s}} = \frac{-2 dA_{CP}/d\hat{s}}{d\mathcal{B}/d\hat{s} + 2 dA_{CP}/d\hat{s}} , \]  

with \( \mathcal{B} = \text{Br}(B \rightarrow X_q \ell^+ \ell^-) \) and \( \tilde{\mathcal{B}} \) is its complex conjugate.
Lastly, as the results in both cases I found:

1. In the standard model where the CKM unitarity is conserved, $\bar{A}_{\text{CP}}(B \rightarrow X_s \ell^+ \ell^-) \sim \text{few} \times 10^{-3}$ and $\bar{A}_{\text{CP}}(B \rightarrow X_d \ell^+ \ell^-) \sim 5\%$.

2. In the beyond standard model where the CKM unitarity is violated, $\bar{A}_{\text{CP}}(B \rightarrow X_s \ell^+ \ell^-) \sim 2\%$ and $\bar{A}_{\text{CP}}(B \rightarrow X_d \ell^+ \ell^-) \sim 5\%$, by including only the $Z-$exchange tree diagram.

Therefore, one can say that once the CP asymmetry in $B \rightarrow X_s \ell^+ \ell^-$ is detected, it will be a strong proof of the new physics beyond the standard model.

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