AN EXAMPLE CONCERNING HAMILTONIAN GROUPS OF SELF
PRODUCT, I

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Abstract. We show that \( (S^2 \times S^2, \omega_0 \oplus \lambda \omega_0) \), with \( \lambda > 1 \), is an example of symplectic manifold \( (X, \omega) \) such that the \( \pi_1 \text{Ham}(X \times X, \omega \oplus -\omega) \) contains extra elements than those from \( \pi_1 \text{Ham}(X, \omega) \times \pi_1 \text{Ham}(X, -\omega) \).

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1. Introduction

Let \( (X, \omega) \) be a compact symplectic manifold with \( \dim_{\mathbb{R}} X = 2n \) and \( \text{Ham}(X, \omega) \) the group of Hamiltonian diffeomorphisms. It’s natural to ask how \( \text{Ham}(X, \omega) \times \text{Ham}(X, -\omega) \) compares with \( \text{Ham}(X \times X, \omega \oplus -\omega) \). Firstly, there is a natural injection:

\[
m : \text{Ham}(X, \omega) \times \text{Ham}(X, -\omega) \hookrightarrow \text{Ham}(X \times X, \omega \oplus -\omega) : m(\phi, \psi) = (\phi, \psi)
\]

Secondly, since a neighbourhood of the diagonal \( \Delta \subset X \times X \) is symplectomorphic to a neighbourhood of the zero section in \( T^*X \), it is clear that the injection \( m \) can’t be surjective. On the other hand, it is not as clear how they compare homotopically. It is well known that for \( (X, \omega) = (S^2, \omega_0) \), the standard 2-sphere, the two sides of \( m \) are weakly homotopic. In this article we consider the first homotopy group, and will use \( m \) to denote the induced map on \( \pi_1 \) as well. To save notations, we use \( X \) to denote \( (X, \omega) \) and \( \overline{X} \) to denote \( (X, -\omega) \).

Seidel constructed for each \( \gamma \in \pi_1 \text{Ham}(X) \) an automorphism \( \Phi^X_\gamma \) of the quantum homology ring \( \mathbb{Q}H_*(X) \) as a module over itself. Let \( 1 = [X] \in \mathbb{Q}H_*(X) \) be the unit, then the Seidel element \( \Psi^X_\gamma = \Phi^X_\gamma(1) \in \mathbb{Q}H^X_*(X) \) is an invertible element. The map \( \Psi^X : \pi_1 \text{Ham}(X) \to \mathbb{Q}H^X_*(X) : \Psi^X(\gamma) = \Psi^X_\gamma \) is the Seidel homomorphism, where \( \mathbb{Q}H^X_*(X) \) is a group under quantum multiplication.

In this article, we consider the example \( (X, \omega) = (S^2 \times S^2, \omega_0 \oplus \lambda \omega_0) \), where \( \omega_0 \) is the standard volume form on \( S^2 \) and \( \lambda > 1 \). We prove the following statement, using explicit computation of the Seidel elements.

Theorem 1.1. \( m \) is not surjective on \( \pi_1 \) for \( (X, \omega) = (S^2 \times S^2, \omega_0 \oplus \lambda \omega_0) \) with \( \lambda > 1 \).

Remark 1.2. We note that, in fact, \( \pi_1 \text{Ham}(X, \omega) \) already has an element \( S \) which does not come from \( \pi_1 \text{Ham} \) of either of its factors. On the other hand, the factors are not symplectomorphic (after reversing one of the structures). Indeed, Gromov [1] showed that \( \text{Ham}(X, \omega_0 \oplus \omega_0) \) is weakly homotopic to \( SO(3) \times SO(3) \), which in turn is weakly homotopic to \( \text{Ham}(S^2, \omega_0) \times \text{Ham}(S^2, \omega_0) \).

Let’s start by fixing some notations. Let \( \Gamma_\omega = \pi_2(M)/\sim \) where \( \beta \sim \beta' \iff \omega(\beta - \beta') = c_1(TX)(\beta - \beta') = 0 \). As a group, the quantum homology \( \mathbb{Q}H_*(X, \omega) \cong H_*(X, \omega) \otimes \Lambda_\omega \) where \( \Lambda_\omega \) is

\footnote{Seidel’s original construction [4] gives for each choice of a reference section an automorphism as well as an element. Here, we follow McDuff [2], choosing a canonical reference section and refer to the result as the Seidel morphism and element. Both will appear in the main text.}
The quantum homology $QH$ leaves $\Gamma_{\omega}$ and define $\text{id}$ $J$ the genus 0 Gromov-Witten invariant counting the number of $J$-holomorphic rational curves in $X$ passing through representatives of the classes $a$.

Moreover, let 

$$\tau(X,\omega) = \sum_{\beta \in \Gamma_{\omega}, e \in H_*(X,\omega)} \langle a, b, c \rangle_{\beta} e^{-\beta} c$$

where $\hat{e} \in H_*(X)$ is the Poincaré dual of $e$ under the ordinary intersection product and $\langle a, b, c \rangle_{\beta}$ is the genus 0 Gromov-Witten invariant counting the number of $J$-holomorphic rational curves in $X$ passing through representatives of the classes $a$, $b$, and $\hat{e}$, representing the class $\beta$.

Next recall the effect of reversing the symplectic structure on $QH_*(X)$ and the Seidel elements. It leaves $\Gamma_{\omega}$ unchanged. Let $\tau: \pi_2(X) \to \pi_2(X): \beta \mapsto -\beta$, it induces the ring isomorphism

$$\tau: \Lambda_{\omega} \to \Lambda_{-\omega}: \sum_{\beta \in \Gamma_{\omega}} a_{\beta} e^\beta \mapsto \sum_{\beta \in \Gamma_{\omega}} a_{\beta} \tau(e^\beta) = \sum_{\beta \in \Gamma_{\omega}} (-1)^{c_1(TX)(\beta)} a_{\beta} e^{-\beta}$$

The quantum homology $QH_*(X)$ and $QH_*(\overline{X})$ are isomorphic as rings via

$$\tau: QH_*(X) \to QH_*(\overline{X}): \tau(a \otimes e^\beta) = (-1)^{\tau + c_1(TX)(\beta)} a \otimes e^{-\beta}$$

where $a \in H_*(X)$. Let $\gamma = [g] \in \pi_1 \text{Ham}(X)$ where $g \in \Omega_0 \text{Ham}(X,\omega)$ is a loop in $\text{Ham}(X)$ based at $id$ and define $\tau: \pi_1 \text{Ham}(X) \to \pi_1 \text{Ham}(\overline{X})$ by $\tau(\gamma) = [g^{-}]$, where $g^{-}(t) = g(1-t)$, then the Seidel elements are related by

$$\tau(\Psi^X_\gamma) = \overline{\Psi^X_{\tau(\gamma)}}$$

Let $(X, \omega_X)$ and $(Y, \omega_Y)$ be compact monotone symplectic manifolds, then we have the ring isomorphism extending the Künneth isomorphism for ordinary homology:

$$QH_*(X \times Y, \omega_X \oplus \omega_Y) \cong QH_*(X, \omega_X) \otimes QH_*(Y, \omega_Y)$$

For the case under consideration, although $(X, \omega) = (S^2 \times S^2, \omega_0 \oplus \lambda \omega_0)$ is not monotone, neither is $(X, -\omega)$, the manifold $(X \times X, \omega \oplus -\omega)$ can be written as a product of monotone manifolds:

$$(X \times X, \omega \oplus -\omega) = (X_1 \times X_1, \omega_1 \oplus \lambda \omega_1)$$

where $\omega_1 = \omega_0 \oplus -\omega_0$ on $X_1 = S^2 \times S^2$. Since

$$QH_*(X_1, \omega_1) \otimes QH_*(X_1, \lambda \omega_1) \cong QH_*(S^2, \omega_0) \otimes QH_*(S^2, -\omega_0) \otimes QH_*(S^2, \lambda \omega_0) \otimes QH_*(S^2, -\lambda \omega_0)$$

it follows still that

$$QH_*(X \times X, \omega \oplus -\omega) \cong QH_*(X, \omega) \otimes QH_*(X, -\omega)$$

The Hamiltonian groups are similarly related:

$$m: \text{Ham}(X, \omega_X) \times \text{Ham}(Y, \omega_Y) \to \text{Ham}(X \times Y, \omega_X \oplus \omega_Y)$$

Moreover, let $\gamma_X \in \pi_1 \text{Ham}(X, \omega_X)$ and $\gamma_Y \in \pi_1 \text{Ham}(Y, \omega_Y)$ then $\gamma_{X \times Y} := m(\gamma_X, \gamma_Y) \in \pi_1 \text{Ham}(X \times Y, \omega_X \oplus \omega_Y)$. Suppose that the ring isomorphism (1.2) holds, then the respective Seidel elements are related by

$$\Psi^X_{X \times Y}(\gamma_{X \times Y}) = \Psi^X_X(\gamma_X) \otimes \Psi^Y_Y(\gamma_Y)$$

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2. Example: \((X, \omega) = (S^2 \times S^2, \omega_0 + \lambda \omega_0)\)

Let \((S^2, \omega_0)\) be the sphere with the standard symplectic structure, \(X = (S^2 \times S^2, \omega_0 + \lambda \omega_0)\) for some \(\lambda > 0\), and \((M, \Omega) = X \times X\). Denote the factors as \(\mathbb{P}_j\) for \(j = 1, \ldots, 4\). Let

\[
(X', \omega') = \mathbb{P}_1 \times \mathbb{P}_4 \text{ and } (M', \Omega') = X' \times X',
\]

then \(M'\) and \(M\) are isomorphic symplectic manifolds, by switching the factors; while \(X'\) and \(X\) are isomorphic via an anti-symplectic involution on the second factor.

When \(\lambda \in (1, 2]\), it’s known (see for example McDuff-Tolman [3]) that \(\pi_1 \text{Ham}(X)\) is generated by 3 elements: \(r_1\) and \(r_2\) of order 2 rotating the respective factors and an element \(s\) of infinite degree. \(X\) admits another structure of \(S^2\) fibration over \(S^2\) and \(s\) defines an \(S^1\)-action on \(X\) rotating the fibers. The diagonal and the anti-diagonal are the two sections of the fibration fixed by this \(S^1\)-action, and the weight of the action on the normal bundle of the section with bigger area is \(-1\).

In order to write down the Seidel elements in \(QH_\ast(X)\) and for later convenience, we introduce a system of notations for the elements in \(H_\ast\) of the various spaces involved. The homology \(H_\ast(S^2) = \mathbb{Z} \oplus 0 \oplus \mathbb{Z}\), as graded by the degree. We write \((1) \in H_2(S^2)\) and \((0) \in H_0(S^2)\) as the respective (positive) generators (with respect to the volume form \(\omega\)). For a (positive) basis of \(H_\ast(S^2)\) with respect to the reverse form \(-\omega\), we write \((\bar{1}) := -(1) \in H_2(S^2)\) and \((\bar{0}) := -(0) \in H_0(S^2)\). The homology \(H_\ast(X)\) is then generated by \((1) \in H_4(X), (10), (01) \in H_2(X)\) and \((00) \in H_0(X)\), where, for example, \((10)\) denotes the tensor \((1) \otimes (0)\). We use similar notations for the generators of \(H_\ast(M), \) e.g. \((01\bar{0}) \in H_4(M)\).

The quantum homology \(QH_\ast(S^2)\) is determined by the fact that \((1)\) is the unit and

\[
(0) * (0) = (1)e^{-(1)}
\]

For \(QH_\ast(S^2)\), we have the corresponding \(\tau\)-version:

\[
((\bar{1})\bar{\tau}(\bar{1})) = (\bar{1})e^{-(\bar{1})} \Rightarrow (0)\bar{\tau}(0) = -(1)e^{(1)}
\]

Note that the unit in the quantum homology \(QH_\ast(X), QH_\ast(X')\) and \(QH_\ast(M)\) are respectively \((1), (1\bar{T})\) and \((11\bar{T})\). We have for example

\[
(01) * (10) = (00) \text{ and } (01\bar{0}) * (00\bar{1}) = (00\bar{1})e^{-(00\bar{1})}
\]

Using these notations, let \(r\) denote the action of \(S^1\) on \(S^2\) fixing the poles and \(\Psi_r \in QH_\ast(S^2)\) be the corresponding Seidel element, then

\[
\Psi_r^{S^2} = (0)e^{\psi(1)} \text{ and } \Psi_r^{S^2} = r(\Psi_r^{S^2}) = -(1)e^{c_1(TS^2)\psi(1)}(0)e^{-(1)} = -(0)e^{\frac{1}{2}(1)} \in QH_\ast(S^2)
\]

We write down the Seidel elements for \(R_1\) and \(R_2\):

\[
\Psi_{r_1}^X = \Psi_r^{S^2} \otimes \Psi_r^{S^2} = (01)e^{\psi(10)} \text{ and } \Psi_{r_2}^X = \Psi_1^{S^2} \otimes \Psi_1^{S^2} = (10)e^{\psi(01)}
\]

Following [3], we explicitly write down the Seidel element for \(s:\)

\[
\Psi_s^X = [(01) + (10)]e^{\frac{1}{2}(10) + h[(10) - (01)]} \text{ where } h = \frac{1}{6\lambda(\lambda - 1)}
\]

where \(\omega((10)) = 1, \omega((01)) = \lambda \text{ and } c_1((01)) = c_1((10)) = 2\). Because

\[
[(01) + (10)] \ast [(01) - (10)] = (11) \left( e^{-(10)} - e^{-(01)} \right)
\]

we see that the reversed loop \(s^-\) gives the Seidel element

\[
\Psi_{s^-}^X = (\Psi_s^X)^{-1} = [(01) - (10)]e^{\frac{1}{2}(10) - h[(10) - (01)]} \left( 1 + e^{(10) - (01)} + e^{2[(10) - (01)]} + \ldots \right)
\]

The corresponding Seidel elements in \(QH_\ast(X)\) are:

\[
\Psi_{r_1}^X = -([01])e^{-\frac{1}{2}(10)}, \Psi_{r_2}^X = -(01)e^{-\frac{1}{2}(01)} \text{ and }
\]

\[
\Psi_{r_1}^X = -([01] + (01))e^{-\frac{1}{2}(10) - h[(10) - (01)]}.
\]
Next we describe the Seidel elements in $QH_*(X')$. Those for $r'_1$ and $r'_2$ are:

$$
\Psi^X_{r'_1} = \Psi^S_0 \otimes \Psi^\infty_0 = (0\Gamma)e^{\frac{j}{4}}(10) \\
\Psi^X_{r'_2} = \Psi^S_1 \otimes \Psi^\infty_0 = -(1\Omega)e^{-\frac{j}{4}}(01).
$$

To describe the Seidel elements of infinite order, we notice that $(X', \omega')$ is symplectically identified with $(X, \omega)$ by

$$(1, c) : \mathbb{C}P^1 \times \mathbb{C}P^1 \to \mathbb{C}P^1 \times \mathbb{C}P^1,$$

where $c$ is the antipodal map. It induces on $H_*$ the isomorphism given by

$$(1, c)_* : ((00), (01), (10), (11)) \to ((00), (0\overline{\Gamma}), (10), (1\overline{\Gamma}))$$

from which can be recovered the expressions for $\Psi^X_{r'_1}$ and $\Psi^X_{r'_2}$ given above. Let $s'$ be the loop conjugate to $s$ by the map $(1, c)$ then the corresponding Seidel element is

$$\Psi^X_{s'} = [(0\overline{\Gamma}) - (1\overline{\Omega})]e^{\frac{j}{4}h(01) + h(10) + (010)} \in QH_*(X', \omega').$$

The corresponding Seidel elements in $QH_*(X')$ are:

$$
\Psi^X_{r'_i} = -(1\overline{\Omega})e^{\frac{j}{4}h(10)} - (1\overline{\Omega})e^{\frac{j}{4}h(01)} \quad \text{and} \quad
\Psi^X_{r'_j} = -(1\overline{\Omega})e^{\frac{j}{4}h(10)} - (1\overline{\Omega})e^{\frac{j}{4}h(01) - (010)}.
$$

The image of the obvious map:

$$m : \pi_1 \text{Ham}(X) \times \pi_1 \text{Ham}(X) \to \pi_1 \text{Ham}(M)$$

is generated by the image of $\{1, r_1, r_2, s\} \times \{1, \tau(r_1), \tau(r_2), \tau(s)\}$ and the corresponding Seidel elements are given by the respective tensor products. Let $m'$ be the corresponding map for $(X', \pm \omega')$:

$$m' : \pi_1 \text{Ham}(X') \times \pi_1 \text{Ham}(X') \to \pi_1 \text{Ham}(M') = \pi_1 \text{Ham}(M),$$

where the last identification is by switching the factors of $M'$. The image of $m'$ is generated by the image of $\{1, r'_1, r'_2, s'\} \times \{1, \tau(r'_1), \tau(r'_2), \tau(s')\}$. Simple algebraic observation together with the explicit description of the Seidel elements given above lead to

**Proposition 2.1.** $\text{img}(m) \neq \text{img}(m') \subset \pi_1 \text{Ham}(M, \Omega)$.

**Proof:** We first proceed as far as possible without using the exact form of the Seidel elements computed above. Let $S = m(s, 1)$, $T = m(1, \tau(s))$, $R_j = m(r_j, 1)$, $\overline{R}_j = m(1, \tau(r_j))$ for $j = 1, 2$ and the corresponding ones with $'$, be loops in $\text{Ham}(M, \Omega)$. Let $\Lambda := \Lambda_\Omega$ denote the Novikov ring for $(M, \Omega)$. It’s evident that

$$
\Psi^M_S \in \text{Span}_\Lambda((01\overline{\Omega}), (10\overline{\Gamma})), \quad \Psi^M_T \in \text{Span}_\Lambda((11\overline{\Omega}), (11\overline{\Gamma})), \quad \text{and}
$$

$$\Psi^{M'}_{S'} \in \text{Span}_\Lambda((01\overline{\Omega}), (11\overline{\Gamma})), \quad \Psi^{M'}_{T'} \in \text{Span}_\Lambda((10\overline{\Gamma}), (11\overline{\Omega})).
$$

More explicitly, we have the following

$$
\Psi^M_S = [(01\overline{\Omega}) + (10\overline{\Gamma})] e^{\frac{j}{2}(0100) + h[(0010) - (0100)]} \\
\Psi^M_T = -[(11\overline{\Omega}) + (11\overline{\Gamma})] e^{-\frac{j}{2}(0010) - h[(0010) - (0001)]} \\
\Psi^{M'}_{S'} = -[(11\overline{\Omega}) + (01\overline{\Gamma})] e^{\frac{j}{2}h[(0001) + (1000)]} \\
\Psi^{M'}_{T'} = -[(10\overline{\Gamma}) + (11\overline{\Omega})] e^{-\frac{j}{4}(0010) - h[(0010) + (0001)]}
$$

We’ll drop the superscripts such as $X$ from the notation of the Seidel elements as they can be inferred from the subscripts. The Seidel elements of loops in img($m$) are of the form

$$\sigma := \Psi^c_{R_1} \Psi^c_{R_2} \Psi^c_{\overline{R}_1} \Psi^c_{\overline{R}_2} \Psi^p_S \Psi^q_T,$$

where $\epsilon_j \in \{0, 1\}$ and $p, q \in \mathbb{Z}$. Square it we have

$$\sigma^2 = \Psi^{2p}_S \Psi^{2q}_T.$$
Suppose that $\sigma$ also lies in $\text{img}(m')$, then $\exists p', q' \in \mathbb{Z}$ so that
\begin{equation}
\sigma^2 = \Psi^{2p}_S \Psi^{2q}_{T'} = \Psi^{2p'}_{S'} \Psi^{2q'}_{T'} = \sigma'^2
\end{equation}
In the following we show that (2.3) holds iff $p = q = p' = q' = 0$.

It’s easy to see from (2.1) (also see below for the first two) that
\[\Psi^2_S \in V := \text{Span}_\Lambda((11\mathbb{1}), (00\mathbb{1})), \quad \Psi^2_T \in W := \text{Span}_\Lambda((11\mathbb{1}), (1\mathbb{1}0))\]
and $\Psi^2_{S'} \in V' := \text{Span}_\Lambda((11\mathbb{1}), (01\mathbb{1}0)), \quad \Psi^2_{T'} \in W' := \text{Span}_\Lambda((1\mathbb{1}1), (10\mathbb{1}0))$.

Notice that $V, V', W$ and $W'$ are closed under the quantum product $*$ and inverse (whenever exists).

Let us first assume that $p, q, p', q' > 0$, then $\sigma^2$ has the form:
\[(a(11\mathbb{1}) + b(00\mathbb{1}1)) * (c(1\mathbb{1}1)) = ac(11\mathbb{1}1) + ad(1\mathbb{1}00) + bc(00\mathbb{1}1) + bd(00\mathbb{1}0)\]
while $\sigma'^2$ is of the form:
\[(a'(11\mathbb{1}1) + b'(00\mathbb{1}1)) * (c'(1\mathbb{1}1)) = a'c'(11\mathbb{1}1) + a'd'(01\mathbb{1}0) + b'c'(10\mathbb{1}0) + b'd'(00\mathbb{1}0)\]
It follows that the necessary condition for (2.3) to hold is
\begin{equation}
ad = bc = a'd' = b'c' = 0 \in \Lambda
\end{equation}

Here we need the explicit form of the Seidel elements. First we have
\[\Psi^2_S = \left[2(00) + (11) \left(e^{-(10)} + e^{-(01)}\right)\right] e^{(10) + 2b(10) - (01)} \in QH_\Lambda(X).
\]
Now let $x = e^{-(10)}, y = e^{-(01)}, A = (00)$ and $B = (11)$, then for any integer $p > 0$
\[\Psi^2_p = K^p \left(A + \frac{x + y}{2} B\right)^p, \quad \text{where} \quad A^2 = Bxy, B^2 = B, AB = A \text{ and } K = 2x^{-2h-1}y^{2h}
\]
We have the explicit formula
\[\Psi^2_p = K^p \left(\sum_{i=0}^{\lfloor p/2 \rfloor} \binom{p}{i} \alpha^{p-2i} (xy)^i B + \sum_{i=0}^{\lfloor (p-1)/2 \rfloor} \binom{p-1}{2i+1} \alpha^{p-2i-1} (xy)^i A\right), \quad \text{where} \quad \alpha = \frac{x + y}{2}.
\]
Note that \[\tau(x) = e^{(10)} = x^{-1}, \quad \tau(y) = e^{(01)} = y^{-1}, \quad \tau(A) = (00) = (00) = A \quad \text{and} \quad \tau(B) = B\]
It follows that $\tau(\alpha) = (xy)^{-1} \alpha$ and $\tau(K) = 2x^{2h+1}y^{-2h} = 4K^{-1}$. Using (1.1) we get for $q > 0$
\[\Psi^q_{\tau(s)} = 4^q K^{-q} \left(\sum_{i=0}^{\lfloor q/2 \rfloor} \binom{q}{i} \alpha^{q-2i} (xy)^i B + \sum_{i=0}^{\lfloor (q-1)/2 \rfloor} \binom{q-1}{2i+1} \alpha^{q-2i-1} (xy)^{i+1-q} A\right),
\]
Since $\Psi^2_S = \Psi^2_S \otimes \Psi^2_{\tau(1)}$ and $\Psi^q_T = \Psi^q_1 \otimes \Psi^q_{\tau(s)}$, it follows that in (2.4) $ad = bc = 0 \Rightarrow p = q = 0$, i.e. $\sigma^2 = id$. Similarly $a'd' = b'c' = 0 \Rightarrow p' = q' = 0$ and $(\sigma')^2 = id$.

The other cases of the sign combinations of $p, q, p', q'$ are similar. Among $p, q, -p', -q'$, there must be 2 of the same sign. Let’s suppose $p$ and $-p'$ are of the same sign, say both $\geq 0$, then instead of (2.3) we may consider
\[\Psi^2_p \Psi^{-2p'}_{S'} = \Psi^{-2q}_{T'} \Psi^{2q'}_{T'}.
\]
Without using the details of the Seidel elements involved, we arrive at an equation similar to (2.4). Afterwards, explicit computation similar to the above gives $p = p' = 0$ and thus $\sigma^2 = (\sigma')^2 = id$.

It follows that, at least, all elements in the image of $m$ of the form $pS + qT$ with $p$ or $q \neq 0$ do not lie in the image of $m'$, and the proposition follows.

\begin{corollary}
$m$ is not surjective on $\pi_1$ for $(X, \omega) = (S^2 \times S^2, \omega_0 \oplus \lambda \omega_0)$ with $\lambda > 1$.
\end{corollary}
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