Exp-function method for solving the Burgers-Fisher equation with variable coefficients

Bo-Kui Chen\(^1\), Yang Li\(^1\), Han-Lin Chen\(^2\), and Bing-Hong Wang\(^1\), \(^3\)

\(^1\)Department of Modern Physics, University of Science and Technology of China, Hefei 230026, China
\(^2\)School of Science, Southwest University of Science and Technology, Mianyang 621010, China
\(^3\)The Research Center for Complex System Science, University of Shanghai for Science and Technology and Shanghai Academy of System Science, Shanghai 200093 China

In this paper, the exp-function method with the aid of symbolic computational system is used to obtain generalized travelling wave solutions of a Burgers-Fisher equation with variable coefficients. It is shown that the exp-function method, with the help of symbolic computation, provides a straightforward and powerful mathematical tool to solve the nonlinear evolution equation with variable coefficients in mathematical physics.

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I. INTRODUCTION

The investigation of exact solutions of nonlinear evolution equations (NLEEs) plays an important role in the study of nonlinear physics phenomena. The importance of obtaining the exact solutions of these nonlinear equations, if available, will facilitate the verification of numerical solvers and aids in the stability analysis of solutions. In the past several decades, many effective methods for obtaining exact solutions of NLEEs have been presented, such as the tanh-function method [1–3], extended tanh method [4, 5], F-expansion method [6, 7], sine-cosine method [8–10], Jacobian elliptic function method [11–14], homotopy perturbation method [15–18], variational iteration method [19, 20] and Adomian method [21–23] and so on.

Recently, He and Wu [24] proposed a straightforward and concise method, called Exp-function method, to obtain generalized solitary solutions and periodic solutions. Applications of the Exp-function method can be found in [25–27] for solving nonlinear evolution equations arising in mathematical physics. The solution procedure of this method, with the aid of Maple, is of utter simplicity and this method can easily extended to other kinds of nonlinear evolution equations.

The present Letter is motivated by the desire to extend the exp-function method to the general types of Burgers-Fisher equation with variable coefficients, which reads:

\[ u_t - u_{xx} + \alpha(t) u u_x = \beta(t) u (1 - u), \]  

(1)

where \( \alpha(t) \) and \( \beta(t) \) are arbitrary functions of \( t \).

The Burgers-Fisher equation has a wide range of applications in plasma physics, fluid physics, capillary-gravity waves, nonlinear optics and chemical physics. When \( \alpha(t) = 0 \), \( \beta(t) \) is an arbitrary constant, Eq. (1) turns to Fisher equation,

\[ u_t - u_{xx} = \beta u (1 - u), \]  

(2)

Kolmogorov, Petrovskii, and Piskunov studied this equation in [28]. They showed if initial datum satisfies some conditions then the solution of Eq. (2) approaches a travelling wave of speed \( C_0 = 2 \). Exact solution of Eq. (2) was found by Ablowitz and Zeppetella in [29] at \( C_0 = \pm \frac{2}{\sqrt{3}} \). When \( \beta(t) = 0 \), \( \alpha(t) \) is an arbitrary constant, Eq. (1) turns to Burgers equation,

\[ u_t - u_{xx} + \alpha u u_x = 0, \]  

(3)

which is used to describe the spread of sound wave in the medium with viscosity and heat exchange if we do not consider the medium’s frequently dispersive character and the slack comfort process, at the same time, the Burgers equations with variable coefficient can be used to describe the cylinder and spherical wave in these questions such as overfall, traffic flow model and so on.

Therefore, there are important theoretic and factual value for us to look for the exact solutions of Eq. (1).

II. THE EXP-FUNCTION METHOD

We now present briefly the main steps of the Exp-function method that will be applied. A traveling wave transformation \( u = u(\xi) \), \( \xi = k x + w t \) converts a partial differential equation

\[ \Psi(u, u_\xi, u_x, u_{xx}, u_{tt}, \cdots) = 0, \]  

(4)

\*Electronic address: chenssx@mail.ustc.edu.cn
where prime denotes the differential with respect to \( t \), and the linear term of lowest order in Eq. (8) with the lowest order nonlinear term, respectively. By simple calculation, we have

\[
\Phi(u, wu', ku', k^2 u'', w^2 u'', kwu'', \cdots) = 0, \tag{5}
\]

The Exp-function method is based on the assumption that traveling wave solutions can be expressed in the form

\[
u(\xi) = \sum_{n=-c}^d a_n \exp(n\xi) \sum_{m=-p}^c b_m \exp(m\xi), \tag{6}
\]

where \( c, d, p \) and \( q \) are positive integers which are unknown to be further determined, \( a_n \) and \( b_m \) are unknown constants. To determine the values of \( c \) and \( p \), and the value of \( d \) and \( q \), we balance the linear term of highest order in Eq. (8) with the lowest order nonlinear term, respectively.

### III. APPLICATION TO THE BURGERS-FISHER EQUATION

In order to obtain the solution of Eq. (1), we consider the transformation

\[
u = u(\xi), \quad \xi = kx + \int \tau(t) dt, \tag{7}
\]

where \( k \) is a constant, \( \tau(t) \) is an integrable function of \( t \) to be determined later, then Eq. (1) becomes an ordinary differential equation, which reads

\[
\tau(t)u' + k\alpha(t) uu' - k^2 u'' - \beta(t) u(1 - u) = 0, \tag{8}
\]

where prime denotes the differential with respect to \( \xi \).

According to the Exp-function method, we assume that the solution of Eq. (8) can be expressed in the form

\[
u(\xi) = \sum_{n=-c}^d a_n \exp(n\xi) \sum_{m=-p}^c b_m \exp(m\xi) = \frac{a_{-c} \exp(-c\xi) + \cdots + a_d \exp(d\xi)}{b_{-p} \exp(-p\xi) + \cdots + b_q \exp(q\xi)}, \tag{9}
\]

where \( c, d, p \) and \( q \) are unknown constants.

In order to determine values of \( c \) and \( p \), we balance the linear term of highest order in Eq. (8) with the highest order nonlinear term, and the linear term of lowest order in Eq. (8) with the lowest order nonlinear term, respectively. By simple calculation, we have

\[
u''(\xi) = \frac{h_1 \exp[(d + 3q)\xi] + \cdots}{h_2 \exp(4q\xi) + \cdots}, \tag{10}
\]

and

\[
u(\xi)u'(\xi) = \frac{h_3 \exp(2d + q)\xi + \cdots}{h_4 \exp(3q\xi) + \cdots} = \frac{h_3 \exp(2d + 2q)\xi + \cdots}{h_4 \exp(4q\xi) + \cdots}, \tag{11}
\]

where \( h_i \) are determined coefficients only for simplicity. Balancing highest order of Exp-function in Eq. (10) and (11), we have

\[
d + 3q = 2d + 2q, \quad d = q. \tag{12}
\]

Similarly to determine values of \( c \) and \( p \), we balance the linear term of lowest order in Eq. (8)

\[
u''(\xi) = \frac{\cdots + s_1 \exp[-(c + 3p)\xi]}{\cdots + s_2 \exp(-4p\xi)}, \tag{13}
\]

and

\[
u(\xi)u'(\xi) = \frac{s_3 \exp[-(2c + p)\xi] + \cdots}{\cdots + s_4 \exp(-3p\xi)} = \frac{\cdots + s_3 \exp[-(2c + 2p)\xi]}{\cdots + s_4 \exp(-4p\xi)}, \tag{14}
\]

where \( s_i \) are determined coefficients only for simplicity. Balancing highest order of Exp-function in Eq. (13) and (14), we have

\[
c + 3p = 2c + 2p, \quad c = p. \tag{15}
\]
we can freely choose the values of \(c\) and \(d\), but the final solution does not strongly depends upon the choice of values of \(c\) and \(d\)[26]. For simplicity, we set \(b_1 = 1\), \(p = c = 1\) and \(d = q = 1\) Eq. (9) becomes

\[
u(\xi) = \frac{a_1 \exp(\xi) + a_0 + a_{-1} \exp(-\xi)}{\exp(\xi) + b_0 + b_{-1} \exp(-\xi)},
\]

(16)

Substituting Eq. (16) into Eq. (8), and with the help of Maple, we have

\[
\frac{1}{A} [C_3 \exp(3\xi) + C_2 \exp(2\xi) + C_1 \exp(\xi) + C_0 + C_{-1} \exp(-\xi) + C_{-2} \exp(-2\xi) + C_{-3} \exp(-3\xi)] = 0,
\]

(17)

where

\[
A = (\exp(\xi) + b_0 + b_{-1} \exp(-\xi))^3;
\]

\[
C_3 = -a_1 \beta(t) + a_1 \beta(t); \\
C_2 = -2a_1 b_0 \beta(t) - a_1 a_0 \alpha(t) + a_1 \beta(t) + a_1 b_0 \tau(t) + ka_0 b_0 \alpha(t) - a_0 \tau(t) - a_0 \beta(t) + k^2 a_1 b_0 - k^2 a_0 + 2a_1 a_0 \beta(t); \\
C_1 = a_2 \beta(t) - 2a_1 \alpha(t) - a_0 b_0 \tau(t) - 2a_0 a_0 b_0 \alpha(t) - k^2 a_1 b_0^2 - ka_0 a_0 \alpha(t) + 2ka_0 b_{-1} \alpha(t) - a_{-1} \tau(t) - a_{-1} \beta(t) - 2a_{-1} \tau(t); \\
+ a_1 b_0^2 \tau(t) - 2a_0 b_0 \alpha(t) + 2a_1 a_{-1} \beta(t) + k^2 a_0 b_0 - a_1 b_0^2 \beta(t) - 4k^2 a_{-1} + 2a_{-1} \tau(t) + 4k^2 a_{-1} b_{-1} \\
- 2a_{-1} b_{-1} \beta(t) + a_1 b_0^2 \beta(t) + k a_0 a_{-1} \alpha(t); \\
C_0 = 6ka_0 b_{-1} + 2a_0 a_{-1} \beta(t) - 2a_1 b_0 b_{-1} \beta(t) - 2a_{-1} b_0 \beta(t) - 3a_{-1} b_0 \tau(t) - 3k^2 a_{-1} b_0 + 3a_1 b_0 a_{-1} \tau(t) - 3k^2 a_1 b_0 b_{-1}; \\
+ 2a_1 a_0 b_0 \beta(t) + 3ka_0 a_{-1} \alpha(t) + 3ka_1 a_0 b_{-1} \alpha(t) - 2a_0 b_{-1} \beta(t) - a_0 b_{-1} \beta(t) + a_0 b_{-1}; \\
C_{-1} = 2ka_2 \alpha(t) - a_0 b_{-1} \beta(t) + a_2 \beta(t) + a_0 b_{-1} \beta(t) + 2ka_0 a_{-1} \alpha(t) + 2a_1 b_{-1} \tau(t) - a_{-1} b_{-1} \tau(t) + 2a_{-1} b_0 \beta(t) \\
- a_{-1} b_{-1} \beta(t) + 2ka_0 b_{-1} + 2a_{-1} b_{-1} \beta(t) - 2a_0 b_{-1} \beta(t) - 4ka_1 b_{-1}^2 + 4k^2 a_{-1} + 2a_{-1} \tau(t) - 2a_{-1} \beta(t) \\
- ka_0 a_{-1} \alpha(t) - 2a_{-1} b_{-1} \tau(t) - k^2 a_{-1} b_{-1} + k a_0 b_{-1} \alpha(t); \\
C_{-2} = 2a_{-1} b_0 b_{-1} \beta(t) - 2a_{-1} b_0 \beta(t) - 2a_{-1} a_{-1} \beta(t) - 2a_{-1} b_0 \beta(t) - 4k^2 a_{-1} b_{-1} + a_0 a_{-1} \beta(t) \phi \left(a_{-1}, b_{-1} \right); \\
+ k a_0 a_{-1} \alpha(t) + a_{-1} b_{-1} \alpha(t) + 2a_{-1} a_{-1} \beta(t) + 2a_{-1} b_{-1} \beta(t); \\
C_{-3} = a_2 b_0 \alpha(t) - a_{-1} b_{-1} \beta(t) + a_{-1} b_{-1} \beta(t). \\
\]

Equating to zero the coefficients of all powers of \(\exp(a\xi)\) yields a set of algebraic equations for \(a_1, a_0, a_{-1}, b_{-1}, b_0, b_1, k, \tau(t), \alpha(t), \beta(t)\). Solving the system of algebraic equations with the aid of Maple, we obtain:

**case1.**

\[
\tau(t) = -k^2 - \beta(t), \quad \alpha(t) = \frac{\beta(t)}{k}, \quad a_1 = 0, \quad a_0 = a_0, \quad a_{-1} = a_0 b_0, \quad b_0 = b_0, \quad b_{-1} = 0.
\]

(18)

**case2.**

\[
\tau(t) = 2k^2 + \frac{1}{2} \beta(t), \quad \alpha(t) = -4k, \quad a_1 = 1, \quad a_0 = 0, \quad a_{-1} = 0, \quad b_0 = 0, \quad b_{-1} = b_1.
\]

(19)

**case3.**

\[
\tau(t) = -k^2 - \beta(t), \quad \alpha(t) = 2k, \quad a_1 = 0, \quad a_0 = \frac{b_0 + \sqrt{b_0^2 - 4b_{-1}}}{2}, \quad a_{-1} = b_{-1}, \quad b_0 = b_0, \quad b_{-1} = b_{-1}.
\]

(20)

**case4.**

\[
\tau(t) = k^2 + \beta(t), \quad \alpha(t) = -2k, \quad a_1 = 1, \quad a_0 = \frac{b_0 - \sqrt{b_0^2 - 4b_{-1}}}{2}, \quad a_{-1} = 0, \quad b_0 = b_0, \quad b_{-1} = b_{-1}.
\]

(21)

**case5.**

\[
\tau(t) = -k^2, \quad \alpha(t) = \frac{\beta(t)}{k}, \quad a_1 = 1, \quad a_0 = a_0, \quad a_{-1} = -b_0^2 + a_0 b_0, \quad b_0 = b_0, \quad b_{-1} = 0.
\]

(22)

**case6.**

\[
\tau(t) = \frac{k^2 \left(12 \sqrt{2b_{-1}} a_0^2 + 7 \sqrt{2b_{-1}} + 40 \sqrt{2b_{-1}} a_{-1}^2 + 45a_{-1}^2 b_{-1} + 2a_0^2 + 37a_0 \right)}{10 \sqrt{2a_0^2 b_{-1}^2 + \sqrt{2b_{-1}} + 6 \sqrt{2b_{-1}} a_0^2 + 2a_0^2 + 7a_{-1}^2 + 15a_{-1}^2 b_{-1}}}, \quad \alpha(t) = -2k, \quad b_{-1} = b_{-1}.
\]

(23)
\[
\beta(t) = \frac{6k^2(2a_0b_1^2 + \sqrt{2} + \sqrt{2b-1}a_0^3 + 3a_0)}{4\sqrt{2b-1}a_0^3 + 3\sqrt{2a_0b_1^2 + 7a_0^2b_1 - 2a_0^2 + b_1^2}}, \quad a_1 = 1, \quad a_0 = a_0, \quad a_- = 0, \quad b_0 = -\sqrt{2b-1}. \tag{24}
\]

Substituting Eqs. (18)-(24) into Eq. (16) yields
\[
u_1(x, t) = \frac{a_0 + ab_0 \exp [-kx + \int k^2 + \beta(t) dt]}{\exp [kx - \int k^2 + \beta(t) dt] + b_0}, \tag{25}
\]
\[
u_2(x, t) = \frac{\exp [kx + \int 2k^2 + \frac{\beta(t)}{2} dt]}{\exp [kx - \int 2k^2 + \frac{\beta(t)}{2} dt] + b_- \exp [-kx - \int 2k^2 + \frac{\beta(t)}{2} dt]}, \tag{26}
\]
\[
u_3(x, t) = \frac{\frac{b_0 + \sqrt{b_0^2 - 4b_-}}{2} + b_- \exp [-kx + \int k^2 + \beta(t) dt]}{\exp [kx - \int k^2 + \beta(t) dt] + b_0 + b_- \exp [-kx + \int k^2 + \beta(t) dt]}, \tag{27}
\]
\[
u_4(x, t) = \frac{\exp [kx + \int k^2 + \beta(t) dt]}{\exp [kx - \int k^2 + \beta(t) dt] + b_0 + b_- \exp [-kx - \int k^2 + \beta(t) dt]}, \tag{28}
\]
\[
u_5(x, t) = \frac{\exp(kx - k^2t) + a_0 + (-b_0^2 + a_0b_0) \exp(-kx + k^2t)}{\exp(\nu k^2 - k^2t) + b_0}, \tag{29}
\]
\[
u_6(x, t) = \frac{\exp(kx + \int \tau(t) dt) + a_0}{\exp(kx + \int \tau(t) dt) - \sqrt{2b_- + b_- \exp(-kx - \int \tau(t) dt)}}, \tag{30}
\]
where \(\tau(t) = \frac{\sqrt{2a_0b_1^2 + 7a_0^2b_1 - 2a_0^2 + b_1^2} + 2a_0^2 + 7a_0b_1 + 15a_0b_1^2 + 4a_0b_1^2 + 12a_0b_1^2 + 45a_0^2b_- + 2a_0^2 + 37a_0}{2a_0b_1^2 + 7a_0^2b_- + 2a_0^2 + 7a_0b_1 + 15a_0b_1^2 + 4a_0b_1^2 + 12a_0b_1^2 + 45a_0^2b_- + 2a_0^2 + 37a_0} \). As we will show later, these solutions have included all the solutions obtained in [30] using the first integral method.

**IV. SOME DISCUSSIONS ABOUT THE SOLUTIONS**

(1) If we take \(b_0 = 0\) in Eq. (25), we have
\[
u_{11}(x, t) = a_0 \exp \left[-kx + \int k^2 + \beta(t) dt\right]. \tag{31}
\]

(2) If we take \(c(t) = c\) is a constant, \(b_- = 1\) and \(\beta(t) = \frac{2ac-a^2}{4}\) in Eq. (26), where \(c\) is a arbitrary constant. Then we have,
\[
u_{21}(x, t) = \frac{1}{2} - \frac{1}{2} \tanh \left[\frac{a}{4}(x - ct)\right]. \tag{32}
\]

This is the solution (36) obtained in [30]. Other solutions in [30] also included in our solution (25) - (30).

(3) If we take \(b_0 = 4, b_- = 1\) and \(k = 1\) in Eq. (27), we have,
\[
u_{31}(x, t) = \frac{-1 + (2 + 2\sqrt{3}) \csc \left[x - \int 1 + \beta(t) dt\right] + \coth \left[x - \int 1 + \beta(t) dt\right]}{4 \csc \left[x - \int 1 + \beta(t) dt\right] + 2 \coth \left[x - \int 1 + \beta(t) dt\right]}. \tag{33}
\]
FIG. 1: (a) Solution of Eq. (31) with $a_0 = 1$, $k = 1$ and $\beta(t) = t$. (b) Solution of Eq. (32) with $a = 4$ and $c = 5$.

(4) If we take $b_0 = 0$, $b_{-1} = -5$ and $k = 1$ in Eq. (28), we get,

$$u_{41}(x, t) = \cosh \left[ x + \int 1 + \beta(t) dt \right] + \sinh \left[ x + \int 1 + \beta(t) dt \right] - \frac{\sqrt{5}}{4} \cosh \left[ x + \int 1 + \beta(t) dt \right] + 6 \sinh \left[ x + \int 1 + \beta(t) dt \right].$$

(34)

(5) If we take $b_0 = 2$ and $a_0 = \frac{3}{2}$, we have

$$u_{51} = \frac{2 \tanh(kx - k^2 t) + \frac{3}{2} \sec(kx - k^2 t)}{1 + \tanh(kx - k^2 t) + 2 \sec(kx - k^2 t)}.$$  

(35)

(6) If we take $a_0 = 0$ in Eq. (30), we have

$$u_{61}(x, t) = \frac{\exp(kx + 7k^2 t)}{\exp(kx + 7k^2 t) - \sqrt{2b_{-1}} + b_{-1} \exp(-kx - 7k^2 t)}.$$  

(36)

These solutions for local area structure are shown in Figs. 1-3.

V. CONCLUSION

The Burgers-Fisher equation with variable coefficients is investigated by Exp-function method. The generalized Travelling wave solutions of this equation are obtained with the help of symbolic computation. From these results, we can see that the Exp-function method is one of the most effective methods to obtain exact solutions.

Finally, it is worthwhile to mention that the Exp-function method can be also extended to other nonlinear evolution equations with variable coefficients, such as the mKdV equation, the (3+1)-dimensional Burgers equation, the generalized Zakharov-Kuznetsov equation and so on. The Exp-function method is a promising and powerful new method for nonlinear evolution equations. This is our task in future work.
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