Two-fluid model of the plasma-wall transition in the presence of warm ions

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Abstract. A one-dimensional two-fluid model of the plasma-wall transition in front of a large, negative, planar electrode is presented. The electrons are assumed to be isothermal, while for the ions the closure is made by the assumption that the ion heat flux is proportional to the negative gradient of the ion temperature. The introduction of the ion heat flow into the model changes the ion flow towards the electrode from mainly adiabatic into almost isothermal.

1. Introduction
Plasma-wall transition region is studied using a one-dimensional two-fluid model developed recently [1-3]. In this model the electrons are described by a continuity and momentum transfer equation, while the ions are described by the continuity, momentum transfer and energy transfer equation. The model is one-dimensional and steady state. For the electrons the closure is made by the assumption that the electrons are isothermal. For the ions the closure in the energy transfer equation is made by the assumption that the ion heat flux is proportional to the negative gradient of the ion temperature. Effects of variation of the heat conduction coefficient are studied in some detail.

2. Model
The basic equations of the one-dimensional two-fluid model obtained as moments of the Boltzmann equation in steady state are the following:

\[ \frac{d}{dx} (n_i u_i) = S_i, \quad \frac{d}{dx} (n_e u_e) = S_e, \]
\[ \frac{m_i n_i u_i}{dx} \frac{du_i}{dx} = -n_i e_0 \frac{d\Phi}{dx} - \frac{dp_i}{dx} + A_i - m_i u_i S_i, \]
\[ \frac{m_e n_e u_e}{dx} \frac{du_e}{dx} = n_e e_0 \frac{d\Phi}{dx} - \frac{dp_e}{dx} + A_e - m_e u_e S_e, \]
\[ \frac{1}{2} u_i \frac{dp_i}{dx} + \frac{3}{2} p_i \frac{du_i}{dx} + \frac{dq_i}{dx} = M_i - u_i A_i + \frac{1}{2} m_i u_i^2 S_i. \]

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The meaning of the symbols is the following: $x$ is the space coordinate, $m_i$ is the ion mass, $m_e$ is the electron mass, $n_i$ and $n_e$ are the respective particle densities, $u_i$ and $u_e$ are the flow velocities of ions and electrons, $e_0$ is the elementary charge, $S_i$ and $S_e$ are the source terms, discussed below, $A_i$ and $A_e$ are elastic collision terms and $M_i$ is the energy transport term. In this type of models, the terms $S_{ie}$, $A_{ie}$ and $M_i$ are assumed known functions of $x$. For simplicity, the terms $A_i$, $A_e$ and $M_i$ are neglected in this work. The potential profile $\Phi(x)$ is determined by the Poisson equation:

$$\frac{d^2 \Phi}{dx^2} = -\frac{e_0}{\varepsilon_0} \left( n_i(x) - n_e(x) \right).$$

(5)

Here $\varepsilon_0$ is the permittivity of the free space. The electron pressure $p_e$ and electron temperature $T_e$ are related by the ideal gas law $p_e = k n_e T_e$, where $k$ is the Boltzmann constant. Because the electrons are isothermal, the electron pressure gradient term in (3) can be expressed as:

$$\frac{dp_e}{dx} = k T_e \frac{dn_e}{dx}.$$  

(6)

Here $k$ is the Boltzmann constant. The ion pressure $p_i$ and ion temperature $T_i$ are related by $p_i = k n_i T_i$, but in this case the ion pressure gradient term in (2) and (4) is expressed in the following way:

$$\frac{dp_i}{dx} = k \frac{dn_i}{dx} T_i + k n_i \frac{dT_i}{dx} = \kappa T_i \frac{dn_i}{dx}.$$  

(7)

Here $\kappa$ is the polytropic function. It has been shown by Kuhn et al. [4] that the ratio between specific heats at constant pressure and at constant volume (the polytropic coefficient) in the plasma-wall transition region is not constant, but is space dependent. Thus, it is called a polytropic function instead of a polytropic coefficient. The definition of the polytropic function is read directly from (7):

$$\kappa = 1 + n_i \frac{dT_i}{dn_i} = 1 + \frac{n_i}{T_i} \frac{dT_i}{dn_i}.$$  

(8)

The ion heat flux $q_i$ is assumed to be proportional to the negative gradient of the ion temperature:

$$q_i = -K \frac{dT_i}{dx}.$$  

(9)

Here $K$ is the heat conduction coefficient.

Finally, the source terms $S_i$ and $S_e$ require some explanation. They give the difference between the number of created and annihilated charged particles of respective species per unit volume and per unit time as functions of $x$. Ionization and annihilation can have various physical mechanisms. If the main mechanism of ionization is ionizing collisions between neutrals and electrons, the number of created charged particles per unit volume and per unit time is proportional to the product $n_i n_e$, where $n_e$ is the density of neutral particles. The number of recombined ions and electrons per unit time and per unit volume is on the other hand proportional to the product $n_i n_e$. In addition, ions and electrons can be lost from the system via other mechanisms. These losses are given by the terms $\sigma_i$ and $\sigma_e$. Thus, the source terms $S_i$ and $S_e$ could be written in the following form $S_{ie} = \gamma_i n_i n_e - \gamma_e n_i n_e - \sigma_i n_e$, where $\gamma_i$ and $\gamma_e$ are the respective proportionality constants. But, in order to keep the model as simple as possible, the source terms $S_i$ and $S_e$ are written simply as:

$$S_i = S_e = \frac{n_e}{\tau}.$$  

(10)

Here $\tau$ should be understood as generalized ionization time, where creation and losses of charged particles are already taken into account.

The following variables are introduced:
\[ \lambda_D = \sqrt{\frac{e_0 k T_e}{n_0 e_0^2}}, \quad c_0 = \sqrt{\frac{k T_e}{m_i}}, \quad L = c_0 \tau, \quad \varepsilon = \frac{\lambda_D}{L}, \quad \mu = \frac{m_e}{m_i}, \quad \Psi = \frac{e_0 \Phi}{k T_e}, \] (11)

\[ N_i = \frac{n_i}{n_0}, \quad N_e = \frac{n_e}{n_0}, \quad V_i = \frac{u_i}{c_0}, \quad V_e = \frac{u_e}{c_0}, \quad \Theta = \frac{T_i}{T_e}, \quad X = x \lambda_D, \] (12)

\[ x = -\frac{d \Psi}{dX}, \quad P_i = \frac{k n_i T_i}{k_0 T_e} = N_i \Theta, \quad K_2 = K \frac{e_0 \sqrt{m_i}}{k^2 T_e \sqrt{e_0 n_0}}. \] (13)

Here \( n_0 \) is the plasma density in the region far away from the electrode, where the plasma is not perturbed by its potential. By taking into account (6), (7) and (9) - (13), the set (1) - (5) is written in the following form:

\[ \frac{d}{dX} (N V_e) = \varepsilon N_e, \quad \frac{d}{dX} (N_i V_e) = \varepsilon N_i, \] (14)

\[ N_i V_i \frac{dV_i}{dX} = -N_e \frac{d \Psi}{dX} - \frac{d}{dX} (-\varepsilon V_i N_e), \] (15)

\[ N_e V_e \frac{dV_e}{dX} = N_i \mu \frac{d \Psi}{dX} - \frac{1}{\mu} \frac{d N_i}{dX} - \varepsilon V_i N_e, \] (16)

\[ \frac{1}{2} V_i \frac{dP_i}{dX} + \frac{3}{2} P_i \frac{dV_i}{dX} - K_2 \frac{d^2 \Theta}{dX^2} = \frac{\varepsilon}{2} N_e V_i^2, \] (17)

\[ \frac{d^2 \Psi}{dX^2} = N_i (X) - N_i (X). \] (18)

**Figure 1.** Solutions of the set (14) – (18) for \( K_2 = 0, \mu = 2.724 \times 10^{-4}, \varepsilon = 10^3, \Psi(0) = 0, \) \( N_i(0) = N_e(0) = 1, V_i(0) = 0, \Theta(0) = 0.05, V_e(0) = 0.387299 \) and \( \chi(0) = 7.74594 \times 10^{-5} \).
3. Results
We present two examples of solutions of set (14) - (18). First we assume that there is no ion heat flux in the plasma, $K_2 = 0$. In this case the only plasma parameter that has to be specified is the ion mass $m_i$. The Deuterium mass $m_i = 3.34 \times 10^{-27}$ kg is selected. This yields $\mu = 2.724 \times 10^4$. In addition, $\varepsilon = 10^4$ is selected. The set (14) - (18) is integrated from $X = 0$, where the plasma is unperturbed by the electrode, towards the electrode in the positive $X$ direction. For a unique solution, seven boundary conditions defining the values of the unknown function at $X = 0$ must be specified. At $X = 0$ the plasma potential is set to zero and the plasma there is neutral, thus $\Psi(0) = 0$, $N_i(0) = N_e(0) = 1$ are a logical selection. As discussed in [2,3], the selection of the electron velocity $V_e(0)$ is rather arbitrary, so that $V_e(0) = 0$ is selected in this work. The ion temperature at $X = 0$ is selected as an independent boundary condition. In this work, $\Theta(0) = 0.05$ is selected. The ion velocity $V_i(0)$ and the electric field $\chi(0)$ are found in a self-consistent way, using the shooting method as explained in [2]. The values $V_i(0) = 0.387299$ and $\chi(0) = 7.74594 \times 10^{-5}$ are thus found. Figure 1 shows numerical solutions of the set (14) - (18).

In figure 2, the solutions of the system (14) - (18) are also presented, but with the ion heat flux being taken into account. In order to determine $K_2$, the plasma parameters have to be specified more precisely. The electron temperature and density that could correspond to the edge plasma in a small tokamak, such as COMPASS [5], are selected, or $kT_e = 5$ eV and $n_0 = 10^{19}$ m$^{-3}$. Together with $m_i = 3.34 \times 10^{-27}$ kg and $K = 1$ W/mK, this gives $K_2 = 281548$. The parameters $\mu = 2.724 \times 10^4$ and $\varepsilon = 10^4$ are the same as in figure 1. The boundary conditions are almost the same as in figure 1: $\Psi(0) = 0$, $N_i(0) = N_e(0) = 1$, $V_e(0) = 0$, $V_i(0) = 0.387298$ and $\chi(0) = 8.60663 \times 10^{-5}$. One additional boundary condition is required this time: $d\Theta/dX(0) = 0$. The corresponding numerical solutions are shown in figure 2.

![Figure 2](image-url)
At first glance, the solutions are very similar. In both cases the potential $\Psi(X)$ and the densities $N_1(X)$ and $N_2(X)$ are monotonically decreasing functions of $X$, while the electric field $\chi(X)$ and velocities $V_1(X)$ and $V_2(X)$ are increasing functions of $X$. But there is an obvious difference in the profiles of the ion temperature $\Theta(X)$, the polytropic function $\kappa(X)$ and the ion sound velocity $V_s(X)$, which is given by:

$$V_s(X) = \sqrt{\frac{1 + \kappa(X)\Theta(X)}{1 + \mu}}. \tag{19}$$

In the absence of an ion heat flux, the ion temperature decreases at first, then increases, reaches a maximum close to the sheath edge and then drops sharply in the sheath – figure 1 (d). The polytropic function - figure 1 (e) – indicates that at the entrance into the system the ion flow is adiabatic. Then ions are expanding and heating at the same time and $\kappa(X)$ becomes even negative. In the sheath, the ions are cooling adiabatically. In the presence of an ion heat flow, any ion temperature gradients are almost completely smoothed out and the ion temperature is (almost) constant – figure 2 (d). Also, the polytropic function - figure 2 (e) – confirms that the ion flow is isothermal since $\kappa(X)$ is (almost) equal to 1 everywhere in the system.

4. Conclusions
A two-fluid one-dimensional model of the plasma-wall transition in front of a planar presented earlier [1-3] is extended to include the ion heat flux in the energy exchange equation. If the heat conductivity is large, the ion flow towards the electrode becomes almost isothermal. The profiles of the potential, electric field, ion and electron density and also the ion and electron velocity remain almost unchanged when an ion heat flux term is introduced. This is valid also for the location of the singularity of set (14) – (18), which occurs when $V_2(X)$ reaches the electron thermal velocity $V_{th} = \mu^{1/2}$ (thin horizontal lines in both plots (h)) and also for the position of the sheath edge (plots (g)) in figures 1 and 2.

We conclude with the following remark. The model presented in Section 2 includes the first, the second, and in the case of ions, also the third moment of the velocity distribution function. The first moment (particle flux) is proportional to the electric current, which can be measured by, e.g., a Langmuir probe. The second and the third moment of the distribution function represent the pressure and heat flux gradients, which could be measured by force and calorimetric probes. Such advanced diagnostics of plasma sheaths are still in the early stages of development [6], but will be necessary to achieve a better understanding of the plasma-wall transition.

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