Hubble tension and Reheating: Hybrid Inflation Implications

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(Dated: August 5, 2022)

We investigate a new possible solution to the Hubble constant tension, we propose a simple resolution to the problem assuming that a first-order phase transition related to $H_0$ transition occurred in the early Universe. The early evolution of the Universe is a result of hybrid inflation which has lasted for a specific period until symmetry breaking takes place. Fitting our model to measurements from Planck and SH0ES data provides a key explanation of discrepancies of $H_0$ measurements. The quantum fluctuations calculated in this model have significant results on the reheating parameters $N_{re}$ and $T_{re}$. Therefore, new constraints must be taken into consideration to fit these parameters to recent results.

I. INTRODUCTION

The Hubble parameter $H_0$ calculated using CMB radiation observations, puts the $ΛCDM$ model in a crisis since it does not agree with the results obtained by directly measuring the expansion rate today using supernovae redshift measurements. The Planck satellite which offers the most accurate observations of temperature fluctuations, polarization, and lensing in CMB radiation, considers that the $ΛCDM$ model predicts the current value of the expansion rate today to be $H_0 = 67.36 \pm 0.54 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [1]. On other hand, the expansion rate measured from Cepheids-calibrated supernovae by the SH0ES team [2], was found to be $H_0 = 74.03 \pm 1.42 \text{ km s}^{-1} \text{ Mpc}^{-1}$, several other Hubble rate measurements including H0LiCoW [3] also show substantial discrepancies. A popular conclusion among astronomers is that additional physics outside of the $ΛCDM$ model is needed to resolve the tension [1,2], but no consensus has yet been reached on what those effects may be. While it is important to continue to look for possible systematic impacts, the possibility of an extra component of dark energy in the early Universe could be the reason for the change in the measurement of $H_0$. According to [3,9] an early scenario could occur thanks to an extra dynamical scalar field, which behaves like dark energy until a crucial moment, at which point its energy density quickly decays.

Reheating is the phase at which the transition from inflation to the radiation era occurs, during this step the production of relativistic particles take place thanks to the inflaton field decay, reheating could either involve a perturbative decay of the inflaton field at the end of inflation [10], or include a scenario of parametric resonance we call preheating [11]. The physics of reheating is usually studied independently on dark energy effects. However, according to [17] inflationary quantum fluctuations could be the source behind the expansion of the Universe described through dark energy, following this results it is possible to constrain reheating parameters $N_{re}$ and $T_{re}$ according to recent data, considering that hybrid inflation behaves like dark energy until the symmetry breaking occurs.

In the present paper, we consider hybrid inflation as a solution to the discrepancies of the expansion rate measured from deferents sources. Hybrid inflation models are described usually by at least two scalar fields, we will study a model with $\phi$ and $\sigma$ fields, which looks like a hybrid of chaotic inflation with $V(\phi) = m^2 \phi^2$ and a non-inflationary potential with spontaneous symmetry breaking given by $V(\sigma) = (1/4\lambda) (M^2 - \lambda \sigma^2)^2$ [12]. In this model, inflation ends by a first-order phase transition, in this direction we will investigate the possibility that the phase transition that occurred in the early universe before recombination, which could be responsible for the transition of $H_0$ from its initial value to its current value. Finally, we extract information about reheating in terms of the e-folds numbers $N_{re}$ and the reheating temperature $T_{re}$ and put new constraints on this phase studying the effects of the hybrid inflation parameters on reheating parameters.

Our paper is organized as follows, in the next section, we first recall the basic notions of the inflationary quantum fluctuations and their effects on initial measurements of $H_0$. In sect. [13] we discuss the mechanism of the symmetry breaking that could explain the expansion rate transition. In sect. [14] we investigate the constraints on hybrid inflation parameters from the early evolution of the universe. In sect. [15] we study the reheating phase, and we focus on the duration $N_{re}$ and temperature $T_{re}$ constraints according to recent observations.
II. INITIAL MEASUREMENT FROM QUANTUM FLUCTUATIONS

According to the hybrid inflation model proposed in [13], which is given as

\[ V(\phi, \sigma) = \frac{1}{4\lambda} (M^2 - \lambda \sigma^2)^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{2} g^2 \phi^2 \sigma^2, \]  

(1)

during the slow-roll of the inflaton field, \( \phi \) moves towards the critical value \( \phi_c = M/g \), and the field fluctuate at the minimum of the potential \( \sigma = 0 \), that takes the form \( V(\phi) = M^4/4\lambda + m^2 \phi^2/2 \). For \( \phi > \phi_c \), \( \sigma \) has positive effective mass squared. Inflation occurs while \( \phi \) fluctuates at the minimum of the potential \( \frac{V}{\phi} \).

Inflation ends at \( \phi = \phi_c \), the energy density became dominated by the false vacuum contribution. When \( \phi < \phi_c \), \( \sigma \) acquires a tachyonic effective mass and the fields roll towards the true minimum at \( \phi = 0 \).

Considering that \( \phi > \phi_c = M/g \), a model was considered in [21] where they described a rapid rolling of \( \sigma \) field to realize the waterfall mechanism which makes inflation ends in a different way, taking into account that the mass \( m \) of the scalar field \( \phi \) is in the order of the Hubble scale \( m \sim H_0 \sim 1.4 \times 10^{-33} eV \). The massless scalar field will acquire quantum fluctuations during inflation since the parameter \( H_I \) satisfies the condition \( H_I > H_0 \). We decompose the classical field \( \phi \) into homogeneous mode and linear perturbations as

\[ \phi(x, t) = \varphi(t) + \delta\phi(x, t) \]

(2)

we then make Fourier transform of the linear perturbations \( \delta\phi(x, t) \) as [13, 16]

\[ |\delta\phi_k|^2 \approx \frac{H_I^2}{2k^3} \left( \frac{k}{aH_I} \right)^{2n^2/(3H_I^2)} \],

(3)

knowing that inflation start at \( a_i \), variations in the real space field caused by super-Hubble fluctuations for \( N = \ln (a_c/a_i) \) inflation e-folds are expressed as [17]:

\[ \langle \delta\phi^2 \rangle \approx \int_{a_i H_i}^{a H_i} \frac{dk}{(2\pi)^3} |\delta\phi_k|^2 = \frac{3H_I^4}{8\pi^2 m^2} \left[ 1 - \exp \left( \frac{-2m^2}{3H_I^2} N \right) \right] \],

(4)

the exponential term which represents the evolution of the homogeneous mode must be suppressed compared to the field fluctuations [17], for that reason inflation lasts long enough for the exponential term to cancel, which will lead to \( \langle \delta\phi^2 \rangle \approx 3H_I^4/8\pi^2 m^2 \). Before phase transition, the energy density in the first phase supported by quantum fluctuations of the inflationary potential \( V(\phi) \) is roughly

\[ V(\phi) \approx \frac{1}{2} m^2 \langle \delta\phi^2 \rangle \approx \frac{3H_I^4}{16\pi^2}, \]

(5)

from Eq. (5) and the density parameter, one should express the parameter \( H_I \) as

\[ H_I \approx (\Omega_\Lambda^{1/4} - 4\pi H_0 M_p, \]

(6)

here, \( M_p \) is the Planck mass, and \( \Omega_\Lambda \) is the density parameter associated with a cosmological constant [1]. Using the current data from Planck, one gets the results in table (I).

![FIG. 1. Variation of \( H_I \) as a function of \( H_0 \). The dark energy density is considered to be \( \Omega_\Lambda = 0.68 \).](image)

From the estimations in table and the massless inflaton field \( m \ll H_I \), one would consider an extremely large number of e-folds as a requirement from Eq. (1). As a consequence, the homogeneous mode of the scalar field is therefore considerably reduced, such that the long wave fluctuation \( \langle \delta\phi \rangle \) determines the value of the classical field.

In Fig. (1) the variation of \( H_I \) as a function of the Hubble rate \( H_0 \) is presented, the vertical light blue, pink and green regions represents Planck’s bounds on \( H_0 \). We observe that \( H_I \) is an increasing function that shows good compatibility with observation when \( H_I > 5.99 \times 10^{-3} \). However, this parameter must obey the condition \( H_I < 6.05 \times 10^{-5} \) in order to reproduce valid values of \( H_0 \) according to observations.

III. SECONDARY MEASUREMENT FROM PHASE TRANSITION

At the moment when the inflaton field \( \phi \) becomes smaller than \( \phi_c = M/g \), symmetry breaking occurs, then the phase transition caused by the \( \sigma \) field fluctuations
TABLE I. The Hubble constant associated with inflation considering parameter 68% intervals for the base-$\Lambda$CDM model from Planck CMB, in combination with CMB lensing and BAO.

| $H_0$ [km s$^{-1}$ Mpc$^{-1}$] | $H_I$ [eV] |
|-------------------------------|-------------|
| TT,TE,EE+lowE                 | 67.27 ± 0.60 | 67.36 ± 0.54 | 67.66 ± 0.42 |
| TT,TE,EE+lowE+lensing        | 6.017 × 10$^{-3}$ | 6.021 × 10$^{-3}$ | 6.034 × 10$^{-3}$ |

ends inflation. When $m^2\phi^2 < m^2\phi_c^2 = m^2M^2/g^2$, inflation ends due to the waterfall mechanism. Thus, any quantum fluctuation will not be produced in this phase since $\sigma \rightarrow M/\sqrt{\lambda}$ and $\phi \rightarrow 0$, this will cause an instantaneous transition and as in the original hybrid inflation model, inflation will come to an abrupt end [18,19]. The potential energy at the transition is given as [21]

$$V_T \simeq \frac{M^4}{4\lambda} = \frac{H_I^4}{4},$$

(7)

to explain the expansion rate transition one needs

$$H_T \simeq \left(\Omega_\Lambda\right)^{1/4} \sqrt{12 H_0 M_p}.$$  

(8)

The present data from $SH0ES$ team [2], provides the results in table I.

![Graph](image)

FIG. 2. Variation of $H_T$ as a function of $H_0$. The observational data were taken from $SH0ES$ team.

Fig. 2 represents the variation of $H_T$ according to $H_0$, the value of the Hubble rate is restricted between $71.85$ km s$^{-1}$ Mpc$^{-1} < H_0 < 75.92$ km s$^{-1}$ Mpc$^{-1}$ from observations. We observe that the parameter $H_T$ should be bounded as $6.08 \times 10^{-3}$ eV $< H_T < 6.24 \times 10^{-3}$ eV

in order to obtain compatible values from the $SH0ES$ team measurements of $H_0$.

When we consider $M > H_I$, we can automatically conclude that the parameter $\lambda$ is bounded as

$$\lambda > \frac{H_I^4}{H_T^4}.$$  

(9)

IV. CONSTRAINTS FROM EARLY EVOLUTION

Since inflation lasted for an extremely long period according to the above analysis, the early evolution could therefore be a natural consequence of cosmic inflation. The stage of inflation is defined at large $\phi$, where the effective potential $V(\sigma, \phi)$ is at the local minimum $\sigma = 0$, therefore, $V(0, \phi) = M^4/4\lambda + m^2\phi^2/2$.

When we assume $m \ll H_I$, the condition from the lower bound on the inflationary quantum fluctuation $\langle \delta \phi^2 \rangle = 3H_I^2 / 8\pi^2 m^2$ can be given as

$$\langle \delta \phi^2 \rangle \gg \frac{3H_I^2}{8\pi^2}.$$  

(10)

The amplitude of density perturbations produced in the model $V \sim m^2\phi^2/2$ can be estimated as [1,21]

$$P_{R}^{1/2} = \frac{16\sqrt{6\pi}}{5} \frac{V^{3/2}}{M_p^3} \frac{\partial V}{\partial \phi} \sim \frac{2\sqrt{6\pi}}{5} m \frac{\phi^2}{M_p^3},$$  

(11)

$$P_R \simeq A_s = 2.196^{+0.051}_{-0.06} \times 10^{-9}$$  

(12)

in this case, the amplitude depends on the field $\phi > \phi_c$, as a consequence the constant $g$ and the masses $M,m$ should satisfy the condition

$$\frac{m}{M_p^3 \sqrt{g^2}} < 2.69 \times 10^{-5},$$  

(13)

the constant $g$ with the bare masses could practically have any value, as long as they satisfy the constraint above.

V. REHEATING CONSTRAINTS FROM EARLY EVOLUTION

Reheating is a phase that occurs in the early universe, at which, the creation of elementary particles and their decay takes place. In fact, it has been proposed in [27,29] that during the first stage of reheating (preheating), phenomena like primordial gravitational waves can be produced and the density spectrum of gravitational waves produced during this stage can satisfy constraints from the latest $Planck$’s data. Knowing that
preheating is characterized by parametric resonance, the created particles can only decay and thermalize with a final temperature called reheating temperature in the last stage of reheating, the decay rate of the inflaton oscillations parametrized with \( \Gamma \) \[26, 30\], is added as a friction term to the inflaton equation of motion (EoM) \( \dot{\phi} + 3H\phi + V(\phi) = 0 \), the decay rate describing the energy transferred to new particles is given by the formula \( \Gamma = \Gamma (\phi \rightarrow \chi \chi) + \Gamma (\phi \rightarrow \psi \psi) \). In our model reheating must occur considering the condition \( \Gamma \sim m^2\phi^2/2 \). We can extract information about reheating in terms of the e-folds numbers \( N_{\text{re}} \) and the reheating temperature \( T_{\text{re}} \) considering the time observed CMB modes crossed beyond the Hubble radius till the present time \[22, 28\]

\[
N_{\text{pre}} = \left[ 61.6 - \frac{1}{4} \ln \left( \frac{V_{\text{end}}}{H_k^2} \right) - N \right] \quad (14)
\]

\[
T_{\text{re}} = \left[ \left( \frac{43}{11g} \right)^{\frac{2}{3}} \frac{a_0 T_0}{k} H_k e^{-N} \left( \frac{3^2 \cdot 5 V_{\text{end}}}{\pi^2g} \right)^{-\pi^2/18} \right]^{\frac{2}{11+g} \sigma}, \quad (15)
\]

next, we express the two inflationary parameters \( N \) and \( V_{\text{end}} \) as a function of the scalar spectral index \[22\]

\[
N = \frac{2}{1 - n_s}, \quad (16)
\]

\[
V_{\text{end}} = 6\pi^2 M_p^4 A_s \left( \frac{1 - n_s}{2} \right)^2, \quad (17)
\]

the parameter \( H_k \) can be written in terms of the chaotic potential \[24\]

\[
H_k = 2\pi \sqrt{A_s M_p \left( \frac{V''}{V} \right)^2}, \quad (18)
\]

after calculation we make the transition \( \phi^2 \rightarrow \langle \delta \phi^2 \rangle \), knowing that \( \langle \delta \phi^2 \rangle = 3 H_{\text{inf}}^4/8\pi^2 m^2 \)

\[
H_k = \frac{\pi^2 m^2}{H_T^2} \sqrt{\frac{128}{3}} A_s, \quad (19)
\]

Fig. \[23\] presented the variation of reheating parameters \( N_{\text{re}} \) and \( T_{\text{re}} \) as a function of the spectral index \( n_s \), the equation of state (EoS) parameter \( \omega \) takes different values each with a specific color, the EoS varies from \(-1/3\) to \(1/4\), knowing that \( \omega = 1/3 \) cannot present any predictions for \( N_{\text{re}} \) and \( T_{\text{re}} \). The blue vertical regions show the values of the scalar spectral index released by Planck’s data, and the horizontal regions for the reheating temperature plot, represent the temperatures below the electroweak scale \((T < 100 GeV)\) and below the big bang nucleosynthesis scale \((T < 10 MeV)\) respectively. Note that for the reheating duration \( N_{\text{re}} \), we define the instantaneous reheating by the limit \( N_{\text{re}} \to 0 \) at the point where all the lines converge, our chaotic potential of inflation shows compatibility with observations for all different values of \( \omega \), in addition to that the case of \( \omega = 1/4 \) gives good consistency even for extremely higher e-folds number. On the other hand for the case of the reheating temperature \( T_{\text{re}} \), An instantaneous reheating leads to the maximum temperature \( T_{\text{re}} \sim 10^{16}\text{GeV} \). Our model chooses to be compatible for all cases of \( \omega \) especially for \( \omega = 1/4 \) that can reach the lowest possible values of the reheating temperature \( T_{\text{re,min}} > 4\text{MeV} \)[25].

At the end of hybrid inflation, the behavior of the fields could describe the explosive preheating with a production of \( \phi \) and \( \sigma \) particles \[12\]. While in \[28\] the behavior of particle production was studied considering an extra field \( \chi \) coupled to both \( \phi \) and \( \sigma \) fields. Particle production in \( \chi \) field that interacts with the fields \( \phi \) and \( \sigma \) require amplified fluctuations which allows an explosive production of \( \chi \)-particles, this means we are in the broad parametric resonance region, the expansion of the universe plays an important role in ending the parametric resonance regime, particle production will be ended by the redshifted modes that will fall out of the resonance band because of the expansion.

### TABLE II. The Hubble constant associated with the transition considering different measurements of the best Estimates of \( H_0 \). Including Systematics from SH0ES team.

| \( H_0 \) \( [km \ s^{-1} \ Mpc^{-1}] \) | LMC + NGC 4258 | LMC + MW | NGC 4258 + MW |
|---------------------------------|-----------------|----------|----------------|
| \( H_T \) \( [eV] \)            | 73.40 ± 1.55    | 73.94 ± 1.58 | 74.47 ± 1.45 |
| \( 6.142 \times 10^{-3} \)      | 6.164 × 10^{-3} | 6.186 × 10^{-3} |                |

### VI. CONCLUSION

In this work we considered hybrid inflation as a solution to the Hubble tension, we have discussed the possibility that the transition of Hubble rate parameter is a result of the phase transition which happened after inflation that has lasted for a certain e-folds number. We have derived two parameters \( H_I \) and \( H_T \) that take different values which could explain the discrepancy of the Hubble rate parameter measurements. From the Hybrid potential \( V(\phi, \sigma) \), the coupling constants and the bare masses of the fields must be constrained to satisfy the model we have proposed in the present paper. Although the
FIG. 3. Variation of $N_{re}$ and $T_{re}$ as a function of the spectral index $n_s$. The observational data were taken from Planck’s results.

The physics of reheating is studied independently on dark energy effects, in this work we consider that dark energy is a result of inflationary quantum fluctuations, this provides the possibility to give new constraints on the reheating parameters $N_{re}$ and $T_{re}$ according to recent data.

[1] Aghanim, N. et al., (2020), Planck 2018 results-VI. Cosmological parameters, Astron. Astrophys., 641, A6.
[2] Riess, Adam G., et al., (2019), Large Magellanic Cloud Cepheid standards provide a 1% foundation for the determination of the Hubble constant and stronger evidence for physics beyond $\Lambda$-CDM, Astrophys. J. 876.1, 85.
[3] Chen, Geoff CF, et al., (2019), A SHARP view of H0LiCOW: H 0 from three time-delay gravitational lens systems with adaptive optics imaging, Mon. Notices Royal Astron. Soc. 490.2, 1743-1773.
[4] Knox, Lloyd, and Marius Millea., (2020), Hubble constant hunter’s guide, Phys. Rev. D, 101.4, 043533.
[5] Vagnozzi, S. (2020). New physics in light of the $H_0$ tension: an alternative view. Physical Review D, 102(2), 023518.
[6] Lin, Meng-Xiang, et al., (2019), Acoustic dark energy: potential conversion of the Hubble tension, Phys. Rev. D, 100.6, 063542.
[7] Alexander, S., and E. McDonough., arXiv 2019, Axion-Dilaton Destabilization and the Hubble Tension, arXiv preprint arXiv:1904.08912.
[8] Ivanov, Mikhail M., et al. (2020), Constraining early dark energy with large-scale structure, Phys. Rev. D, 102.10, 103502.
[9] Vagnozzi, S. (2021). Consistency tests of $\Lambda$CDM from the early integrated Sachs-Wolfe effect: Implications for early-time new physics and the Hubble tension. Physical Review D, 104(6), 063524.
[10] Abbott, Laurence F., Edward Farhi, and Mark B. Wise, (1982), Particle production in the new inflationary cosmology, Phys. Lett. B, 117.1-2, 29-33.
[11] Kofman, Lev., 1998, Preheating after inflation, COSMO-97, 312-321.
[12] Garcia-Bellido, Juan, and Andrei Linde., (1998), Preheating in hybrid inflation, Phys. Rev. D, 57.10, 6075.
[13] Linde, Andrei., (1991), Axions in inflationary cosmology, Phys. Lett. B, 259.1-2, 38-47.
[14] Dufaux, Jean-Francois, et al., (2009), Gravity waves from tachyonic preheating after hybrid inflation, J. Cosmol. Astropart. Phys. 2009.03, 001.
[15] Starobinsky, Alexei A., (1980), A new type of isotropic cosmological models without singularity, Phys. Lett. B, 91.1, 99-102.
[16] Mukhanov, Viatcheslav F., and G. V. Chibisov., (1981), Quantum fluctuations and a nonsingular universe, ŽETF, Pisemared., 33, 549-553.
[17] Ringeval, Christophe, et al., (2010), Dark energy from primordial inflationary quantum fluctuations, Phys. Rev.
[18] Linde, Andrei., (1991), *Axions in inflationary cosmology*, Phys. Lett. B, 259.1-2, 38-47.
[19] Copeland, Edmund J., et al., (1994), *False vacuum inflation with Einstein gravity*, Phys. Rev. D, 49.12, 6410.
[20] Garcia-Bellido, Juan, and David Wands., (1996), *Spectrum of curvature perturbations from hybrid inflation*, Phys. Rev. D, 54.12, 7181.
[21] Linde, Andrei., (1994), *Hybrid inflation*, Phys. Rev. D, 49.2, 748.
[22] Nautiyal, Akhilesh., (2018), *Reheating constraints on tachyon inflation*, Phys. Rev. D, 98.10, 103531.
[23] Asadi, Kosar, and Kourosh Nozari., (2019), *Reheating constraints on a two-field inflationary model* Nucl. Phys. B., 949, 114827.
[24] Sakhi, Z., El Bourakadi, K., et al., (2020), *Effect of brane tension on reheating parameters in small field inflation according to Planck-2018 data*, Int J Mod Phys A, 2050191.
[25] Hannestad, S. (2004). *What is the lowest possible reheating temperature?*, Physical Review D, 70(4), 043506.
[26] Kofman, L., Linde, A., & Starobinsky, A. A. (1997). *Towards the theory of reheating after inflation*, Physical Review D, 56(6), 3258.
[27] El Bourakadi, K., Ferricha-Alami, M., Filali, H., Sakhi, Z., & Bennai, M. (2021). *Gravitational waves from preheating in Gauss–Bonnet inflation.*, The European Physical Journal C, 81(12), 1-8.
[28] El Bourakadi, K., et al. (2021), *Preheating and reheating constraints in supersymmetric braneworld inflation*, Eur. Phys. J. Plus, 136.8, 1-19.
[29] El Bourakadi, K., Sakhi, Z., & Bennai, M. (2022). *Preheating constraints in α-attractor inflation and Gravitational Waves production*. International Journal of Modern Physics A.
[30] Ferricha-Alami, M., et al. (2017). *Mutated hybrid inflation on brane and reheating temperature*. Eur. Phys. J. Plus, 132(7), 1-10.