Rayleigh waves with impedance boundary conditions in a non local micropolar thermoelastic material

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Abstract. The present paper analyzes the propagation of Rayleigh waves in a non-local micropolar thermoelastic half space with impedance boundary conditions. Dispersion equation of Rayleigh wave propagation with impedance boundary conditions is obtained and the effect of impedance and non local parameters are studied. Dispersion equation of Rayleigh waves for a micropolar thermoelastic half space with impedance boundary as well as traction free half-space is obtained as a particular case. The non-dimensional speed of Rayleigh wave is computed as a function of impedance parameters and presented graphically for a aluminum epoxy material. It is observed that non local and impedance parameters has significant effects on Rayleigh wave speed.

Keywords: Non local micropolar thermoelasticity, Impedance boundary conditions, Rayleigh waves

1. Introduction

The micropolar theory of elasticity established by Eringen’s [1] is a familiar theory of solid mechanics and being used in analyze the deformation of microstructure materials such as cellular solids, composite fibrous, granular material, polymers, bones etc. This theory takes into account the intrinsic rotational motion along with linear displacement in the materials. The motion in this theory is governed by six degrees of freedom, three of microrotation and three of translation. Nowacki [2] and Eringen [3] extended the micropolar theory by including thermal effects and presented linear theory of micropolar thermoelasticity. The micropolar theory along with generalized theory of thermoelasticity proposed by Lord and Shulman [4] and Green and Lindsay [5] can be seen in many research findings.

Surface waves are of particular significance in seismology due to their destructive nature during earthquake. Rayleigh waves are one of the surface type waves and have been explored by several researchers. Rayleigh waves in micropolar elastic materials has been explored by many researchers due to the practical applicability in the various fields such as, seismology, acoustics, aerospace and submarine structures. Eringen [6] obtained the frequency equation of Rayleigh surface waves in micropolar elastic half space along a stress free boundary. The research articles [7-9] can be seen on various problems related to propagation of Rayleigh waves in micropolar elastic material.

The wave propagation in non local micropolar elastic half space is of special interest as the non local theories is considered to cover microscopic phenomena, for example crack tip problems, high frequency vibrations and dislocations etc. Eringen [10] derived the dispersion relation for transverse plane waves in non local micropolar elastic solid. Kaliski et al [11] discussed various properties of
surface waves in non local elastic media and media with microstructure. Khurana and Tomar [12] discussed different modes Rayleigh waves in non local micropolar elastic half space.

Generally, the problems of Rayleigh waves are considered in a traction free surface. The possibilities of other type of boundary conditions are rarely consider in seismology or geophysics. However, Tiersten [13], while studying the wave propagation in an isotropic elastic solid coated with thin film used another type of boundary conditions known as impedance boundary conditions. The impedance boundary conditions prescribed on the boundary is the linear combination of the unknown function and their derivatives and stresses are assumed to be dependent upon displacement components and their derivatives. Malischewsky [14] derived secular equation for Rayleigh waves by using modified impedance boundary conditions given by Tiersten [13]. Godoy, Duran and Nedelec [15] discussed existence of surface waves in an elastic half space with impedance boundary conditions. Vinh and Hue [16] applied impedance boundary conditions to investigate Rayleigh waves in an orthotropic and monoclinic half space. Recently, Kumar et al [17] studied propagation of Rayleigh wave in a micropolar thermoelastic solid half space subjected to impedance boundary conditions.

Taking into account, the capability of non local micropolar theory to examine materials at microstructural level, in this paper the propagation Rayleigh waves in a non local micropolar thermoelastic half space with impedance boundary condition has been investigated. Secular equation for thermally insulated and isothermal surface is obtained. Some particular cases have also been discussed.

2. Basic equations

Following Eringen [3, 10, 18] the governing equations of motion for homogeneous and isotropic non local micropolar thermoelastic solid are

\[(\mu + K)\nabla^2 \tilde{\mathbf{u}} + (\lambda + \mu)\nabla(\nabla \cdot \tilde{\mathbf{u}}) + K\nabla \times \tilde{\mathbf{\phi}} - \nu \nabla T = (1 - \epsilon^2 \nabla^2)\rho \frac{\partial^2 \tilde{\mathbf{u}}}{\partial t^2}\]

\[= (\alpha + \beta + \gamma) \nabla(\nabla \cdot \tilde{\mathbf{\phi}}) - \gamma \nabla \times (\nabla \times \tilde{\mathbf{\phi}}) + K \nabla \times \tilde{\mathbf{u}} - 2K \tilde{\mathbf{\phi}} = (1 - \epsilon^2 \nabla^2) \rho \frac{\partial^2 \tilde{\mathbf{\phi}}}{\partial t^2}\]  \hspace{1cm} (2.1)

where \(\epsilon = (e_0 a)\) is the non local parameter ( \(a\) is internal characteristic length and \(e_0\) is the non local constant), \(\tilde{\mathbf{u}}\) is the displacement vector, \(\rho\) is the density of the material, \(j\) is the microinertia, \(\tilde{\mathbf{\phi}}\) is the microrotation vector, \(\lambda, \mu\) are Lame’s constants \(K, \alpha, \beta, \gamma\) are micropolar isotropic material constants. The constitutive relations are given by

\[(1 - \epsilon^2 \nabla^2)\sigma_{ij} = \lambda u_{r,r} \delta_{ij} + \mu (u_{i,j} + u_{j,i}) + K(u_{i,i} - \epsilon_{ijr} \phi_r) - \nu T \delta_{ij}\]

\[(1 - \epsilon^2 \nabla^2) m_{ij} = \alpha \phi_{r,r} \delta_{ij} + \beta \phi_{i,j} + \gamma \phi_{j,i}\]

where, \((i,j,r = 1,2,3)\), \(\sigma_{ij}\) is the stress tensor, \(m_{ij}\) is the couple stress tensor and \(\delta_{ij}\) is the Kronecker delta.

Following Lord and Shulman [4], the heat conduction equation is

\[K^* \nabla^2 T = \rho C^* \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) T + \nu T_0 \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \nabla . \tilde{\mathbf{u}}\]

\[= (1 - \epsilon^2 \nabla^2) \rho \frac{\partial^2 \phi}{\partial t^2}\]  \hspace{1cm} (2.2)

where \(K^*\) is the coefficient of thermal conductivity, \(\nu = (3\alpha + 2 \mu + K)\alpha_t\), \(C^*\) is the specific heat at constant strain, \(\alpha_t\) is the coefficient of thermal linear expansion, \(T\) is the change in temperature of the medium at any time, \(T_0\) is the reference temperature of the body and \(\tau_0\) is the thermal relaxation time

3. Formulation of the problem

A homogeneous and isotropic non local micropolar thermoelastic half space in the undeformed state is taken at uniform temperature \(T_0\). We assumed the Cartesian frame of reference having its origin on the surface \(y = 0\) and \(y\)-axis pointing vertically downward into the half space. The direction of
propagation of the waves is considered along x-axis so that all particles vibrating on a line parallel to z-axis are equally displaced. Therefore, all the field quantities will be independent of z-coordinates.

For the two-dimensional problem in x-y plane, we have
\[ \vec{\mathbf{u}} = (u, v, 0), \quad \vec{\phi} = (0, 0, \phi) \]  

(3.1)

Using (3.1), equation (2.1) and (2.2) can be written as
\[
(\lambda + 2\mu + K) \frac{\partial^2 u}{\partial x^2} + (\mu + K) \frac{\partial^2 v}{\partial y^2} + (\lambda + \mu) \frac{\partial^2 v}{\partial x \partial y} + K \frac{\partial \phi}{\partial y} - v \frac{\partial T}{\partial x} = \rho (1 - \epsilon^2 \nabla^2) \frac{\partial^2 u}{\partial t^2} \\
(\lambda + 2\mu + K) \frac{\partial^2 u}{\partial y^2} + (\mu + K) \frac{\partial^2 v}{\partial x^2} + (\lambda + \mu) \frac{\partial^2 u}{\partial y \partial x} - K \frac{\partial \phi}{\partial x} - v \frac{\partial T}{\partial y} = \rho (1 - \epsilon^2 \nabla^2) \frac{\partial^2 v}{\partial t^2} \\
\gamma \left( \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \right) + K \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) - 2K \phi = \rho j(1 - \epsilon^2 \nabla^2) \frac{\partial^2 \phi}{\partial t^2} 
\]

(3.2)

Using (3.1) in the equations (2.5) and (3.2)-(3.4), we obtained
\[
(\lambda + 2\mu + K) \nabla^2 \phi_1 - 2\mu \nabla^2 \phi_2 + \gamma \nabla^2 \phi = \rho j(1 - \epsilon^2 \nabla^2) \frac{\partial^2 \phi}{\partial t^2} \\
(\mu + K) \nabla^2 \psi_1 + K \phi = \rho (1 - \epsilon^2 \nabla^2) \frac{\partial^2 \psi_1}{\partial t^2} \\
\gamma \nabla^2 \phi - 2K \phi - K \nabla^2 \psi_1 = \rho j(1 - \epsilon^2 \nabla^2) \frac{\partial^2 \phi}{\partial t^2} \\
K^2 \nabla^2 T = \left( \frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2} \right) \left( \rho C^* T + \nu T_0 \nabla^2 \phi_1 \right) 
\]

(3.8)

4. Solution of the Problem

The surface wave solutions of the equations (3.6)-(3.9) is taken as
\[ \{\phi_1, \psi_1, T, \phi\} = \{\phi_1(y), \psi_1(y), T(y), \phi(y)\} e^{ik(x-ct)} \]  

(4.1)

where \( c \) is the phase velocity, \( k \) is the wave number, \( \omega = kc \) is the circular frequency. It is assumed that the Rayleigh surface waves possibly damped in time, propagating along x-axis with wave speed \( \text{Re}(c) = V > 0 \) and \( \text{Im}(c) \leq 0 \).

Using (4.1) in the equations (3.6)-(3.9), we get
\[ D^4 - AD^2 + B \left[ (\phi_1(y), T(y)) \right] = 0 \]  

(4.2)

\[ D^4 - A' D^2 + B' \left[ (\psi_1(y), \phi(y)) \right] = 0 \]  

(4.3)

\[ \begin{align*}
D &= \frac{d}{dy}, & A &= k^2 \left[ 2 - \frac{c^2 A_1}{A_1} \right] + k^6 c^2 c_1^2 \left( \frac{c^2}{A_1} - 2 \right), \\
B &= k^4 \left[ \frac{A_1 c^2 (1 + c_1 A_1)}{A_1 c_1^2 A_1} + \frac{c^4}{A_1 c_1} \right] + k^6 c^2 c_1^2 \left( \frac{c^2}{A_1} - 1 \right), \\
A' &= k^2 \left( 1 - \frac{c^2}{c_2^2} \right) + k^2 - \frac{k^2 c^2 \rho j}{\gamma} + \frac{2K^2}{\gamma (\mu + K)} + \frac{2k^2 c^2 \rho j e^2}{\gamma c_2^2} (c_2^2 + k^2 c^2 + k^4 c^4 e^4) \\
&\quad - \frac{2k^2 c^2 \epsilon^2}{c_2^2} \left( 1 + \frac{2K}{\gamma} \right),
\end{align*} \]  

(4.4)
\[ B' = k^2 \left( k^2 - \frac{k^2 c^2 \rho j}{\gamma} + \frac{2K}{\gamma} \right) \left( 1 - \frac{c^2}{c_j^2} \right) - \frac{k^2 K^2}{\gamma (\mu + K)} - \frac{2k^2 \epsilon^2 c^2}{\gamma c_j^2} + \frac{k^6 e^2 c^2}{c_j^2} \left( \frac{k^2 e^2 c^2 \rho j}{\gamma} + \frac{2pj}{\gamma} - \frac{pj c_j^2}{\gamma} - 1 \right), \]
\[
\frac{\lambda + 2\mu + K}{\gamma} = \frac{\mu + K}{\gamma} + i \cdot K' \cdot \sqrt{v^2 T_0}
\]

Using the radiation conditions \( \bar{\phi}_i(y), \bar{\psi}_i(y), \bar{T}(y), \bar{\phi}(y) \to 0 \) as \( y \to \infty \) on the general solutions of the equations (4.2) and (4.3) and using (4.1), we obtained

\[ \phi_1 = (B_1 e^{-b_1 y} + B_2 e^{-b_2 y}) e^{i(k(x-\alpha))} \] (4.5)
\[ \psi_1 = (B_3 e^{-b_3 y} + B_4 e^{-b_4 y}) e^{i(k(x-\alpha))} \] (4.6)
\[ T = (r_1 B_1 e^{-b_1 y} + r_2 B_2 e^{-b_2 y}) e^{i(k(x-\alpha))} \] (4.7)
\[ \phi = (r_3 B_3 e^{-b_3 y} + r_4 B_4 e^{-b_4 y}) e^{i(k(x-\alpha))} \] (4.8)

where

\[ b_1^2 + b_2^2 = A, \quad b_1^2 b_2^2 = B, \quad b_3^2 + b_4^2 = A', \quad b_3 b_4 = B' \]

\[
\begin{align*}
\begin{cases}
r_1 = \frac{k^2}{\nu} \left( \lambda + 2\mu + K \right) \left( \frac{b_1^2}{k^2} - 1 \right) + \rho c^2 (1 + \epsilon (k^2 - 1)) \quad i = 1,2 \end{cases}
\end{align*}
\] (4.9)
\[
\begin{align*}
\begin{cases}
r_j = \frac{k^2 (\mu + K)}{K} \left[ 1 - \frac{c^2}{c_j^2} \right] - \frac{b_j^2}{k^2} + \frac{\epsilon^2 c^2}{c_j^2} (b_j^3 - k^2) \quad j = 3,4
\end{cases}
\end{align*}
\] (4.10)

and \( B_1, B_2, B_3 \) and \( B_4 \) are arbitrary constants.

### 5. Boundary conditions and secular equation

Following Godoy, Duran and Nedelec [15], the impedance boundary conditions at the surface \( y = 0 \) can be written as \( \sigma_{21} + \omega Z u = 0 \). Hence for non local micropolar thermoelastic half space the impedance boundary conditions can be written as

\[
\begin{align*}
(1 - \epsilon^2 \nabla^2) \sigma_{21} + \omega Z u &= 0, \\
(1 - \epsilon^2 \nabla^2) \sigma_{22} + \omega Z v &= 0, \\
(1 - \epsilon^2 \nabla^2) m_{23} + \omega Z \phi &= 0,
\end{align*}
\] (5.1)

Where \( h \to 0 \) corresponds to thermally insulated surface and \( h \to \infty \) corresponds to isothermal surface.

Using conditions (5.1) on the surface \( y = 0 \), we get the following secular equation for the velocity of the Rayleigh waves

\[
M_1 [T_1 (L_2 N_4 - N_2 L_4) - T_2 (L_1 N_4 - N_1 L_4)] = M_2 [T_1 (L_2 N_3 - N_2 L_3) - T_2 (L_1 N_3 - N_1 L_3)]
\] (5.2)

where

\[
\begin{align*}
L_1 &= [k V_1 Z_1 - b_1 - (1 + \frac{K}{\mu}) b_1], & L_2 &= [k V_1 Z_1 - b_2 - (1 + \frac{K}{\mu}) b_2], \\
L_3 &= V_1 Z_1 b_3 - k - (1 + \frac{K}{\mu}) \left( 1 - \frac{c^2}{c_j^2} - \frac{\epsilon^2 k^2 c^2}{c_j^2} + \frac{\epsilon^2 b_j^2 c^2}{c_j^2} \right), \\
L_4 &= V_1 Z_1 b_4 - k - (1 + \frac{K}{\mu}) \left( 1 - \frac{c^2}{c_j^2} - \frac{\epsilon^2 k^2 c^2}{c_j^2} + \frac{\epsilon^2 b_j^2 c^2}{c_j^2} \right), \\
N_1 &= 2 + \frac{K}{\mu} - V_1^2 + (k^2 - 1) \epsilon^2 V_1^2 - V_1 Z_2 b_1, & N_2 &= 2 + \frac{K}{\mu} - V_1^2 + (k^2 - 1) \epsilon^2 V_1^2 - V_1 Z_2 b_2, \\
N_3 &= \left[ (2 + \frac{K}{\mu}) b_3 - k V_1 Z_2 \right], & N_4 &= \left[ (2 + \frac{K}{\mu}) b_4 - k V_1 Z_2 \right], \\
M_1 &= (k \mu V_1 Z_3 - \gamma b_3) \left( 1 - \frac{c^2}{c_j^2} - \frac{b_j^3}{k^2} + \frac{\epsilon^2 k^2 c^2}{c_j^2} + \frac{\epsilon^2 b_j^2 c^2}{c_j^2} \right), \\
M_2 &= (k \mu V_1 Z_3 - \gamma b_4) \left( 1 - \frac{c^2}{c_j^2} - \frac{b_j^3}{k^2} + \frac{\epsilon^2 k^2 c^2}{c_j^2} + \frac{\epsilon^2 b_j^2 c^2}{c_j^2} \right).
\end{align*}
\]
\[
V_i = \sqrt{\frac{\rho c_i^2}{\mu}}, \quad Z_i^* = \frac{Z_i}{\sqrt{\rho \mu}}, \quad (i = 1, 2, 3),
\]

\[
\begin{align*}
T_1 &= b_1 \left[ \left( 2 + \frac{\lambda + K}{\mu} \right) \left( \frac{b_1^2}{k^2} - 1 \right) + V_1^2 (1 + \epsilon^2 (k^2 - 1)) \right], \\
T_2 &= b_2 \left[ \left( 2 + \frac{\lambda + K}{\mu} \right) \left( \frac{b_2^2}{k^2} - 1 \right) + V_2^2 (1 + \epsilon^2 (k^2 - 1)) \right].
\end{align*}
\]

For thermally insulated surface

\[
\begin{align*}
T_1 &= \left[ \left( 2 + \frac{\lambda + K}{\mu} \right) \left( \frac{b_1^2}{k^2} - 1 \right) + V_1^2 (1 + \epsilon^2 (k^2 - 1)) \right], \\
T_2 &= \left[ \left( 2 + \frac{\lambda + K}{\mu} \right) \left( \frac{b_2^2}{k^2} - 1 \right) + V_2^2 (1 + \epsilon^2 (k^2 - 1)) \right].
\end{align*}
\]

For isothermal surface

6. Particular cases

(6.1) In the absence of nonlocal parameter \( \epsilon \), the equation (5.2) reduces to the secular equation for the phase velocity of Rayleigh waves in a micropolar thermoelastic half space with impedance boundary conditions and is same as obtained by Kumar et al [17]

(6.2) On neglecting the nonlocal and micropolar effects \( (\epsilon = K = j = 0) \) in the equation (5.2) we obtain the following equation

\[
L_3 (N_1 T_2 - N_2 T_1) - N_3 (L_1 T_2 - L_2 T_1) = 0
\]

The equation (6.1) coincides with the secular equation, obtained by author Singh [18] for Rayleigh waves in thermoelastic solid half space with impedance boundary conditions.

(6.3) If we neglect the nonlocal, impedance, micropolarity and thermal effects from the model i.e.

\[
\begin{align*}
\epsilon &= K = j = Z_1^* = Z_2^* = Z_3^* = \nu = 0, \text{ the equation (5.2) reduces to}
\end{align*}
\]

\[
\left( 2 - \frac{c_1^2}{c_2^2} \right)^2 = 4 \left( 1 - \frac{c_1^2}{c_1^*} \right) \left( 1 - \frac{c_2^2}{c_2^*} \right)
\]

where \( c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad c_2^2 = \frac{\mu}{\rho} \). This is the well-known dispersion equation for the phase velocity of Rayleigh waves in classical elastic half space.

7. Numerical results and discussions

For numerical results and discussions, we are considering aluminum epoxy material and the values of relevant physical constants are [19]

\[
\begin{align*}
\rho &= 2.19 \times 10^3 \text{kg/m}^3, \quad \lambda = 7.59 \times 10^{10} \text{N/m}^2, \quad \mu = 1.89 \times 10^{10} \text{N/m}^2, \quad K = 0.0149 \times 10^{10} \text{N/m}^2, \quad a = 0.01 \times 10^6, \quad \beta = 0.015 \times 10^6, \quad \gamma = 0.268 \times 10^6 N/m, \quad j = 0.196 \times 10^4 \text{m}^2, \quad K^* = 0.492 \times 10^5 \text{W/m} K, \quad C^* = 1.89 \times 10^{10} J/kg.K, \quad \tau_0 = 0.5 \times 10^{-10} s, T_0 = 298 K, \quad \alpha_t = 2.36 \times 10^{-6} K^{-1}
\end{align*}
\]

Under the assumption that \( c \) is a complex constant parameter with \( Re(c) = V \geq 0 \), the non-dimensional Rayleigh wave speed \( V_i = \sqrt{\frac{\rho V_i^2}{\mu}} \) is calculated by solving secular equation (5.2) using functional iteration method. The effects of nonlocal parameter and impedance parameters on the Rayleigh wave speed with respect to non-dimensional wave number \( (ka) \), where \( a \) is the internal characteristic length, under thermally insulated boundary surface have been discussed in figures (1-5). Figure (1) compares the wave speed in local and nonlocal micropolar thermoelastic half spaces under fixed values impedance parameters \( (Z_1^* = 1, Z_2^* = 0, Z_3^* = 0) \) and internal characteristic length \( a = 10^{-9} m \). As shown wave speed vanishes quickly with wave number in case of non local micropolar thermoelastic material. Figure (2) shows the effects of nonlocal parameter, for fixed internal characteristic length it shows the effect of nonlocal constant. It is noticed that as the value of nonlocal constant decreases the wave increases.

Figures (3-5) depict the influence of impedance parameters on the non-dimensional Rayleigh wave speed in a nonlocal micropolar thermoelastic half space. The impedance parameters \( Z_2^* \) and \( Z_3^* \) has significant effects on Rayleigh wave speed while it is least affected by \( Z_1^* \).
Figure 1. Variation of Non dimensional Rayleigh wave speed w.r.t. non dimensional wave number in local and non local micropolar thermoelastic half spaces.

Figure 2. Effects of non local constants on the variations non-dimensional wave speed w.r.t. non dimensional wave number.
Figure 3. Effects of impedance parameter $Z_1^*$ on Rayleigh wave speed

Figure 4. Effects of impedance parameter $Z_2^*$ on Rayleigh wave speed
8. Conclusion

In the present study, Rayleigh waves in a non local micropolar thermoelastic half space with impedance boundary conditions for thermally insulated and isothermal surface is studied. The secular equation satisfying impedance boundary conditions is obtained in the explicit form. From particular cases it is clear that on removing the non local parameter, the secular equation is in agreement with the secular equation of Rayleigh waves for micropolar thermoelastic half space with impedance boundary conditions. Further, on the removal of micropolar, impedance and thermal effects, this equation reduces to classical equation for an elastic solid as obtained by Lord Rayleigh [20]. From the numerical analysis, we can conclude that the non local parameter has significant effects on Rayleigh waves speed. Rayleigh wave speed is also affected by impedance and thermal parameters.

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