Symmetry Tests of the Electroweak Interaction from Muon Capture on $^3$He

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Abstract

Precision measurements in muon capture on $^3$He to the triton channel provide for interesting tests of the charged electroweak interaction, whether within the Standard Model or beyond it. Based on the statistical capture rate and the triton asymmetry, both the object of recent and on-going experiments using polarised $^3$He targets, examples of such tests are presented, and shown to lead to stringent complementary constraints for which further dedicated experimental as well as theoretical efforts are required.

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1 Introduction

Ever since the discovery of parity violation in the weak interactions, muon capture on nuclei has remained a topic of wide ranging interest[1]. Given the presently available high intensity muon beams as well as new experimental techniques and particle detectors, enabling new measurements of much improved precision, this interest has been revived in recent years[2].

Essentially two types of observables are presently available in the case of muon capture on \(^3\)He to the triton channel, \(\mu^- + \(^3\)He \rightarrow \nu_\mu + \(^3\)H\), namely the statistical muon capture rate \(\lambda_{\text{stat}}\) and the triton asymmetry \(A_v\) for a polarised initial \(\mu^- - \(^3\)He\) state. The latter measurement is in progress, and only a preliminary result is available so far[3]. In the former case however, a recent high precision measurement has reached the following value[4], precise to 0.3%,

\[
\lambda_{\text{stat}}^{\exp} = 1496 \pm 3 \text{ (stat.)} \pm 3 \text{ (syst.)} \text{ s}^{-1} = 1496 \pm 4 \text{ s}^{-1} \quad . \quad (1)
\]

This result, which confirms earlier experiments of much less statistics[5], is in beautiful agreement with the latest and most precise theoretical analysis[6] whose prediction is \(\lambda_{\text{stat}}^{\text{theor}} = 1497 \pm 21 \text{ s}^{-1}\).

Given such precision measurements and prospects for new ones using polarised \(^3\)He targets, it thus seems timely to consider the possible physics information which may be gathered from such experiments. In the next section, this issue is first addressed within the Standard Model (SM) for the electroweak interactions. The following section considers the sensitivity of the aforementioned observables to possible new interactions which may be lurking behind the horizon of the Standard Model. In both cases, the complementarity of different types of measurements is most apparent, thus calling for new experiments aiming for the best precisions attainable, as well as for a renewed theoretical effort to improve on the precision of different inputs.

Even though the present analysis is developed in the context of muon capture on \(^3\)He, a similar discussion applies equally well in the case of muon capture on hydrogen. Indeed, both cases being that of muon capture on a spin \(\frac{1}{2}\) isospin doublet, the relevant expressions remain identical; only numerical values of form factors differ from one case to the other. In fact, the most general situation possible is being analysed[7]—including polarised muons and \(^3\)He nuclei, both within the Standard Model and beyond it—in order to ascertain the interest of measurements other than the statistical capture rate, be it the triton asymmetry or new experiments altogether such as the triton polarisation or tensor analysing power. Some of the present conclusions are based on preliminary results of that work in progress.
2 Muon Capture in the Standard Model

Within the SM, the charged current interaction possesses the \((V-A)\) left-handed chirality structure in both the muonic and hadronic vertices. The effective interaction amplitude is of the form \[ \frac{q^2}{8M^2} V_{ud} J_{\text{lept}}^\mu \mathcal{J}_{\text{hadr}} \], with \(g\) being the usual \(SU(2)_L\) gauge coupling constant, \(M\) the \(W^\pm\) gauge boson mass, and \(V_{ud}\) the Cabibbo-Kobayashi-Maskawa flavour mixing matrix element. The leptonic current is \(J_{\text{lept}}^\mu = \bar{\mu} \gamma^\mu (1-\gamma_5) \nu_\mu\), the neutrino being massless without flavour mixing. The hadronic current is of the form \(J_{\text{hadr}}^\mu = V_{\text{hadr}}^\mu - A_{\text{hadr}}^\mu\), with the vector and axial contributions given by the general Lorentz covariant parametrisation in momentum space,

\[
V_{\text{hadr}}^\mu = \bar{\psi}_2 \left[ F_V \gamma^\mu + i F_M \sigma^\mu\nu \frac{q^\nu}{2M} + F_S \frac{q^\mu}{2M} \right] \psi_1 , \\
A_{\text{hadr}}^\mu = \bar{\psi}_2 \left[ F_A \gamma^\mu \gamma_5 + F_P \gamma_5 \frac{q^\mu}{2M} + i F_T \sigma^\mu\nu \frac{q^\nu}{2M} \right] \psi_1 . \tag{2}
\]

Here, \(\psi_1\) and \(\psi_2\) are Dirac spinors for on mass-shell initial and final spin \(\frac{1}{2}\) nuclei of energy-momenta \(p_1\) and \(p_2\), respectively, \(M\) is a mass scale chosen to be their average mass value, and \(q^\mu = p_2^\mu - p_1^\mu\) is the energy-momentum transfer of the process. The quantities \(F_V\), \(F_M\), \(F_S\), \(F_A\), \(F_P\) and \(F_T\) are phenomenological \(q^2\)-dependent nuclear form factors, real under complex conjugation for time reversal invariant interactions, whose values may be determined using experimental data and the hadronic symmetries of CVC and PCAC\([\text{1}]\). In the limit of exact isospin symmetry and charge conjugation invariance, the induced second-class form factors \(F_S\) and \(F_T\) vanish identically; by analogy with the situation for the nucleon\([\text{8}]\), their normalised values \(|F_S/F_V|\) and \(|F_T/F_A|\) should thus not exceed, say, 0.02. The vector form factors \(F_V\) and \(F_M\) are rather well established on the basis of CVC, taking the following values\([\text{6}]\) at the relevant invariant momentum transfer \(q_0^2 = -0.954 m_\mu^2\): \(F_V(q_0^2) = 0.834 \pm 0.011\) and \(F_M(q_0^2) = -13.969 \pm 0.052\). The axial form factor \(F_A(q^2 = 0)\) is well determined from the \(\beta\)-decay rate of \(^3\)H, but its extrapolation to \(q_0^2\) requires some model dependent assumptions. In Ref.\([\text{6}]\), two uncertainties are considered for that extrapolation, the larger of which accounts generously for meson exchange contributions possibly overlooked in the smaller one, even though the latter should already be rather reliable since it uses the \(q^2\)-dependence of \(F_A\) inferred from the impulse approximation\([\text{7}]\). Correspondingly, the value

\[\text{In the case of muon capture on hydrogen, the corresponding form factors are usually denoted by } g_V, g_M, g_S, g_A, g_P \text{ and } g_T, \text{ respectively. The values of } |g_S/g_V| \text{ and } |g_T/g_A| \text{ should not exceed, say, 0.02, on account of isospin symmetry breaking}\([\text{8}]\).\]

\[\text{A precise calculation of the capture rate is also available in the impulse approximation including meson exchange currents}\([\text{9}]\), which, on basis of the experimental result (1), enables a test\([\text{9,4}]\) of the QCD-corrected PCAC prediction\([\text{10}]\) for \(g_P\), the precision of the latter being better than 3\%.\]
for $F_A$ is $F_A(q_0^2) = -1.052 \pm 0.005$ (or $\pm 0.010$). Finally, the value for the induced pseudoscalar form factor $F_P$ may be related to that of $F_A$ on the basis of PCAC—a consequence of the spontaneous breaking of the approximate chiral symmetries of QCD—, through $F_P^{\text{PCAC}}(q^2) = 4MF_A(q^2)/(m_\pi^2 - q^2)$. Here, a correction—not exceeding a few percent[11]—due a possibly different $q^2$-dependence of the $\pi^-\text{He}^3\text{H}$ coupling constant and of $F_A$ is ignored.

Given the above parametrisation of the muon capture amplitude, it is possible to compute different observables[12,13,6], in particular the statistical capture rate $\lambda_{\text{stat}}$ and the triton asymmetry $A_v$. In the limit that both $F_S$ and $F_T$ vanish, and with the values quoted above for the other form factors, Ref.[6] obtains

$$\lambda_{\text{theor}} = 1497 \pm 12 \text{ (or } \pm 21) \text{ s}^{-1} \text{ and } A_{v \text{theor}} = 0.524 \pm 0.006 \text{ (or } \pm 0.006),$$

where the first (resp. second) error indicated in each case corresponds to the smallest (resp. largest) uncertainty of $\pm 0.005$ (resp. $\pm 0.010$) on $F_A$ (this notation is used throughout). Note how $A_v$ is much less sensitive to $F_A$ than is $\lambda_{\text{stat}}$. Indeed, the normalised variations $F_A/O dO/dF_A$, with $(O = \lambda_{\text{stat}}, A_v)$, are 1.521 and $-0.134$, respectively. The situation is quite the opposite with respect to $F_P$, since the normalised variations are then[6] $-0.116$ and $-0.377$, respectively. Consequently, a precision measurement of $A_v$ provides a determination of $F_P$ not much sensitive to the uncertainty on $F_A$, in contradistinction to $\lambda_{\text{stat}}$. Moreover, given the values for $F_V$ and $F_M$ and assuming no second-class contributions $F_S = 0 = F_T$, a combined analysis using these two observables would lead to a purely experimental determination both of $F_A$ and of $F_P$, independent of the $^3\text{H} \beta$-decay rate and of assumptions on the $q^2$-dependence of $F_A$.

To numerically assess the situation when $F_S = 0 = F_T$, let us consider the experimental result $\lambda_{\text{stat}}$ in (1), as well as a hypothetical value for $A_v$ equal to its prediction $A_{v \text{theor}} = 0.524$ with a relative precision of 5% and 1%. From $\lambda_{\text{stat}}$, one obtains[4],

$$F_P^{\text{norm}} \equiv \frac{m_\mu}{2M}F_P(q_0^2) = -20.80 \pm 1.57 \text{ (or } \pm 2.74), \tag{3}$$

to be compared to the PCAC prediction of $-20.72 \pm 0.10$ (or $\pm 0.20$). This $\pm 7.6\%$ (or $\pm 13.2\%$) result is thus a beautiful confirmation of PCAC in contradistinction with the conclusion reached recently from a radiative muon capture experiment on hydrogen[17] which finds a value for $g_P$ 1.5 times larger

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3 Refs.[12,13,6] ignore the induced second-class contributions, while Ref.[14] includes them but only numerically. Ref.[7] will provide explicit analytic expressions in the most general case possible.

4 In turn, this test of PCAC allows both for a somewhat improved[9,4] precision of $g_P = (1.05 \pm 0.19)g_P^{\text{PCAC}}$ on the present combined 22% uncertainty on $g_P$[15] from all muon capture measurements on hydrogen, and for a precise determination of the pion-nucleus coupling constant[16].
than the PCAC prediction[10]. On the other hand, a 5% (resp. 1%) precision on \( A_v \) would lead to uncertainties on \( F_P^{\text{norm}} \) of \( \pm 2.81 \) (resp. \( \pm 0.80 \)), irrespective of whether the error on \( F_A \) is \( \pm 0.005 \) or \( \pm 0.010 \). Therefore, in order to improve the precision on \( F_P^{\text{norm}} \) reached using (1), the precision aim of a triton asymmetry measurement should at least be on the order of 1% to 2%, a real challenge indeed. It may also be shown that for a combined analysis using both \( \lambda_{\text{stat}} \) and \( A_v \), and when accounting for the errors both on \( \lambda_{\text{stat}}^{\exp} \) and \( A_v^{\exp} \), and on \( F_V \) and \( F_M \), a 1% measurement of \( A_v \) would lead to a determination\(^5\) of \( F_A \) with an uncertainty close to the value of \( \pm 0.005 \). As a matter of fact, when the errors on \( F_V \), \( F_M \) and \( F_A \) are not included but only those on the experimental values \( \lambda_{\text{stat}}^{\exp} \) and \( A_v^{\exp} \), the present uncertainty on \( F_P^{\text{norm}} \) as determined from (1) is reduced to \( \pm 0.48 \), while using the value for \( A_v \) precise to 5% (resp. 1%) brings this uncertainty down to \( \pm 2.75 \) (resp. \( \pm 0.55 \)). This establishes that on the theoretical front as well, an improvement on the determination of the form factors \( F_V \), \( F_M \) and \( F_A \) is clearly also called for, especially in the case of \( F_A \) whose error presently dominates the uncertainty on \( F_P^{\text{norm}} \) given above.

A similar analysis is possible[18] for the second-class form factors \( F_S \) and \( F_T \), when assuming that either one of these vanishes, while the remaining form factors take the values listed above, with \( F_P \) given by the PCAC relation quoted previously. An analysis using both \( \lambda_{\text{stat}} \) and \( A_v \) could also determine \( F_S \) and \( F_T \) from a combined fit. Here again, the sensitivity of \( A_v \) to \( F_S \) and \( F_T \) is much larger than that of \( \lambda_{\text{stat}} \), since with \( (\mathcal{O} = \lambda_{\text{stat}}, A_v) \), one has \( 1/\mathcal{O} d\mathcal{O}/dF_S = (0.007, 0.017) \) and \( 1/\mathcal{O} d\mathcal{O}/dF_T = (-0.006, -0.019) \). The experimental result (1) then implies,

\[
F_S = -0.062 \pm 1.18 \text{ (or } \pm 2.02) \quad , \quad F_T = 0.075 \pm 1.43 \text{ (or } \pm 2.45) \ , \quad (4)
\]

while these uncertainties reduce to \( \pm 0.38 \) and \( \pm 0.46 \), respectively, when only the experimental error on \( \lambda_{\text{stat}}^{\exp} \) is included. These results thus provide some improvement on the uncertainties obtained in Ref.[18] in the impulse approximation for the nucleon form factors \( g_S \) and \( g_T \), but with central values much closer to expectations. Nevertheless, they also call for a theoretical improvement on the uncertainties of \( F_V \), \( F_M \) and especially of \( F_A \), for the same reasons as above.

Similarly, a measurement of \( A_v \) precise to 5% (resp. 1%) at \( A_v^{\text{theor}} = 0.524 \) would imply uncertainties on \( F_S \) and \( F_T \) of \( \pm 3.0 \) (resp. \( \pm 0.9 \)) and \( \pm 2.8 \) (resp. \( \pm 0.8 \)), respectively, whether the error on \( F_A \) is \( \pm 0.005 \) or \( \pm 0.010 \). When only the experimental uncertainty on \( A_v \) is included, these numbers reduce to \( \pm 2.9 \) (resp. \( \pm 0.58 \)) and \( \pm 2.7 \) (resp. \( \pm 0.54 \)), respectively. The situation is thus comparable to that for \( F_P \). When accounting for the present errors on \( F_V \), \( F_M \) and \( F_A \), a 1% measurement of \( A_v \) would improve by about a factor two the present

\(^5\) The precision on \( F_P \) is then set by that on \( A_v \), namely \( \pm 0.80 \).
uncertainties both on $F_S$ and $F_T$ established in (4) on basis of the experimental result (1). Such results would also be comparable or would improve some of the limits presently available on these contributions from $\beta$-decay processes[19].

3 Muon Capture Beyond the Standard Model

Assuming the given values for the hadronic form factors$^6$, precision measurements of $\lambda_{\text{stat}}$ and $A_e$ may also be used to set limits for physics beyond the SM. At the $(u,d)$ quark level, such new interactions may effectively be represented by the amplitude,

$$\frac{4g^2}{8\pi M^2} V_{ud} \sum_{\eta_1, \eta_2 = \pm, \pm} \left[ (h^{V}_{\eta_1 \eta_2})* \bar{\nu}_\mu \gamma^\mu P_{\eta_1} \mu \bar{d} \gamma^\mu P_{\eta_2} u + \right. $$

$$+ (h^{S}_{\eta_1 \eta_2})* \bar{\nu}_\mu P_{\eta_1} \mu \bar{d} P_{\eta_2} u + \left( h^{T}_{\eta_1 \eta_2})* \bar{\nu}_\mu \sigma^{\mu\nu} P_{\eta_1} \mu \bar{d} \sigma^{\mu\nu} P_{\eta_2} u \right], \quad (5)$$

where $P_{\pm} = (1 \pm \gamma_5)/2$ are the chirality projectors, and $g$, $M$ and $V_{ud}$ are arbitrary parameters which in the limit of the SM reduce to the usual ones introduced in Sect.2. Finally, the coefficients $h^{V,S,T}_{\pm\pm}$ are arbitrary complex coefficients associated to vector, scalar and tensor interactions in the charge exchange form, with $\eta_1$ (resp. $\eta_2$) being the muon (resp. $u$ quark) chirality which equals the neutrino (resp. $d$ quark) chirality for vector interactions and is opposite to it for scalar and tensor interactions. Finally, without loss of generality, one may set $h^T_{\pm-} = 0 = h^T_{T+}$. The SM amplitude is recovered in the limit that all coefficients $h^{S,V,T}_{\pm\pm}$ vanish except for $h^V_{-+} = +1$.

Like-wise, effective interactions for $\beta$- and $\mu$-decay may be parametrised in terms of analogous coefficients $f^{V,S,T}_{\pm\pm}$ and $g^{V,S,T}_{\pm\pm}$, respectively, with the electron playing the rôle of the muon in the two cases, and the muon playing that of the $d$ quark in the latter case$^7$. Consequently, the relation between the coupling constant $g^2/8M^2$ and Fermi’s constant $G_F$, as well as the mixing parameter $V_{ud}$, determined from the muon decay rate and $0^+ - 0^+$ superallowed $\beta$-decay rates, respectively, are modified accordingly, leading to a computable correction factor for the muon capture rate. Note that the amplitude in (5) also determines the $\pi \rightarrow \mu \nu_\mu$ decay rate, as well as the $\pi \rightarrow e \nu_e$ decay rate in terms of the coefficients $f^{S,V,T}_{\pm\pm}$. In these two cases however, only the axial and

$^6$ In this first order analysis, the possible second-class contributions of $F_S$ and $F_T$ are both set equal to zero, an approximation which, in view of the results of Sect.2, is already quite satisfactory. Realistic values for $F_S$ and $F_T$ would mostly affect only the uncertainties on the constraints derived in the present section, and then by a small amount at best.

$^7$ In the case of muon decay, this parametrisation is now standard[20].
constraint, Here, the experimental branching ratio \(R_{\pi}^{\text{exp}} = \Gamma(\pi^+ \to e^+\nu_e(\gamma))/\Gamma(\pi^+ \to \mu^+\nu_\mu(\gamma))\) corrected for radiative corrections such that \(^{[21]}\) \(R_{\pi}^{\text{exp}}/R_{\pi}^{\text{SM}} = 0.9960 \pm 0.0033\) implies the constraint,

\[
\frac{|(h^V_+ - h^V_-) + C_\mu (h^S_+ - h^S_-)|^2 + |(h^V_+ - h^V_-) + C_\mu (h^S_+ - h^S_-)|^2}{|(f^V_+ - f^V_-) + C_\mu (f^S_+ - f^S_-)|^2 + |(f^V_+ - f^V_-) + C_\mu (f^S_+ - f^S_-)|^2} = 1.0040 \pm 0.0033 ,
\]

with \(C_\mu = m^2_\pi/(m_\mu(m_u + m_d)) \simeq 12.29\) and \(C_e = m^2_\pi/(m_e(m_u + m_d)) \simeq 2541\). Note that it is only when the interaction is a purely left-handed vector one that this constraint provides a \(e - \mu\) universality test of the SM, namely \(|h^V_-/f^V_-|^2 = 1.0040 \pm 0.0033\).

In the case of muon capture on \(^3\)He, let us also first consider this situation, namely when all coefficients \(h^S_{\pm \pm}\) and \(f^S_{\pm \pm}\) vanish except for \(h^V_-\) and \(f^V_-\) which are left unspecified. The experimental result (1) for the capture rate then implies the constraint,

\[
|h^V_-/f^V_-|^2 = 0.9996 \pm 0.0083 \text{ (or } \pm 0.0142\text{) ,}
\]

the uncertainty on this constraint reducing to \(\pm 0.0027\) when only the experimental error on \(\lambda_{\text{stat}}\) is included. Therefore, under these much restricted assumptions, and because of the uncertainties on the form factors—mostly that on \(F_A^-\), this \(e - \mu\) universality test does not improve on the present limit from pion decay. Nevertheless, once the parameter space is enlarged, the two types of observables lead to complementary constraints on the effective coupling coefficients \(h^S_{\pm \pm}\) and \(f^S_{\pm \pm}\).

General left-right symmetric models (LRSM)\(^{[22]}\) whose gauge symmetry group \(SU(2)_L \times SU(2)_R \times U(1)_{B-L}\) provides an attractive gauge extension of the SM, are an interesting context for the present discussion. Let us introduce the notations,

\[
r = \frac{g_R}{g_L} , \quad \delta = \frac{M_2^2}{M_1^2} , \quad t = \tan \zeta ,
\]

\[
v_{ud} = \frac{V^{R}_{ud}}{V^{L}_{ud}} , \quad v_u = |v_{ud}|^2 , \quad v_\ell = \sum_i |U^{R,L}_{\ell i}|^2 , \quad \ell = e, \mu .
\]

Here, \(g_R\) and \(g_L\) are the gauge coupling constants of the chiral \(SU(2)_{R,L}\) gauge groups, \(M_1\) and \(M_2\) are the masses of the physical charged gauge bosons \(W^\pm_1\) and \(W^\pm_2\)—the former being the lightest—\(\zeta\) is their mixing angle in terms of the \(W^\pm_{L,R}\) gauge bosons, and \(V^{R,L}_{ud}\), \(U^{R,L}_{\ell i}\) are Cabibbo-Kobayashi-Maskawa flavour mixing matrix elements in each chiral sector, both in the
(u, d) quark sector and in the lepton sector, the latter coupling the charged lepton of flavour $\ell$ to the mass eigenstate neutrino $\nu_i$. The prime "′" on the summation symbols in the lepton sector indicates that only those neutrinos whose mass is sufficiently small in order to be produced in a given process are included, while numerically in this note their mass is then also neglected in the kinematics of that process. Finally, in the mixing of the $W_{R,L}^\pm$ gauge bosons leading to the physical charged bosons $W_{1,2}^\pm$, in general there may also appear a CP violating phase $\omega$. By definition, so-called manifest left-right symmetric models (MLRSM) are such that $r = 1$, $v_{u d} = 1$, $v_\mu = 1 = v_e$ and $\omega = 0$.

Within LRSM, and when the much suppressed charged Higgs contributions are ignored, only effective vector interactions are induced, leading in general to non vanishing coefficients $h_{V\pm}^Y$ and $f_{V\pm}^Y$. First, let us assume that the neutrinos produced in $\mu^\rightarrow^3$He capture and in $0^+ \rightarrow 0^+$ superallowed $\beta$-decays are such that $v_\mu = v_e$, and let us also take it for granted that $(\tan\zeta)$ and $\delta$ are sufficiently small to warrant an expansion in these two parameters. Under these circumstances, the muon capture rate $\lambda_{stat}$ is essentially independent of the mass $M_2$ of the heavier charged gauge boson, but is rather sensitive to the mixing angle $\zeta$, while the situation is reversed for the triton asymmetry $A_V$. Given these assumptions, the experimental value (1) for the capture rate then leads to the result,

$$\frac{g_R}{g_L} \text{Re} \left( e^{i\omega} v_{u d} \right) \tan \zeta = -0.00016 \pm 0.00295 \text{ (or } \pm 0.00506 \text{)} \quad (9)$$

with the uncertainty reducing to $\pm 0.00095$ when only the experimental error on $\lambda_{stat}^{exp}$ is included. This new constraint on the mixing angle $\zeta$, which applies in general LRSM and thus includes a dependence on $r$ and $e^{i\omega} v_{u d}$, is among the very best available in the context of MLRSM for which all other such constraints have been obtained from muon decay, $\beta$-decay and other semileptonic processes[24–26].

Given that the mixing angle $\zeta$ must be small, let us now relax the condition $v_\mu = v_e$ but take the limit $\zeta = 0$. Corresponding to the universality limit (7), the experimental result (1) then implies the constraint

$$(v_\mu - v_e) v_u r^4 \delta^2 = 0.0004 \pm 0.0083 \text{ (or } \pm 0.0142 \text{)} \quad (10)$$

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8 This is also the case for the hyperfine muon capture rates.
9 Unless $v_{u d}$ vanishes altogether, which would make the collider limits[27] on $M_2$ void of content.
10 Here, $v_\mu$ includes the sum over all neutrinos produced in muon capture, while $v_e$ includes all neutrinos produced in $0^+ \rightarrow 0^+$ superallowed $\beta$-decays.
the uncertainty reducing to ±0.0027 when only the experimental error on $\lambda_{\text{stat}}$ is included. From pion decay, the analogous constraint is $(v_\mu^{(\pi)} - v_\mu^{(\pi)}) v_\nu r^4 \delta^2 = 0.0040 \pm 0.0033$, where $v_\mu^{(\pi)}$ and $v_\mu^{(\pi)}$ include summations over all neutrinos produced in the muon and electron branches of pion decay, respectively.

Finally, $A_v$ being essentially insensitive to $\zeta$, in the approximation that $\zeta = 0$ one obtains\[28],

\[ (A_v^{\text{LRSM}})_{\zeta=0} = \frac{1 - v_\mu v_u r^4 \delta^2}{1 + v_\mu v_u r^4 \delta^2} A_v^{\text{SM}}. \tag{11} \]

Therefore, a 1% measurement of $A_v$ provides an upper bound of 0.005 on $|v_\mu v_u r^4 \delta^2|$, which in the case of MLRSM corresponds to a lower bound on $M_2$ of 300 GeV/c$^2$ only.

It is also interesting to consider limits that the experimental result (1) may set on scalar and tensor interactions, characterised by the coefficients $h_{S,T}^{\pm\pm}$, which typically arise in theories with leptoquarks. Mostly for illustrative purposes, only interactions coupling to left-handed neutrinos are included here, motivated by the idea that right-handed ones may be too heavy to be produced in muon capture and $\beta$-decay. This corresponds to $h_{V,T}^{\pm\pm} = h_{S,T}^{\pm\pm} = 0$, and similarly for the $f_{S,V,T}^{\pm\pm}$ coefficients. Moreover, for simplicity, one also assumes that $|h_{V,T}^{\pm\pm} + h_{V,T}^{\pm\pm}|^2 \approx |f_{V,T}^{\pm\pm} + f_{V,T}^{\pm\pm}|^2$. Associated to such interactions, one must also consider the corresponding hadronic matrix elements of the $(u,d)$ scalar, pseudoscalar and tensor nuclear form factors $G_S$, $G_P$ and $G_T$, respectively, where in contradistinction to (2), induced recoil corrections are not included for the obvious reason that they would be even further suppressed in comparison to the genuine scalar, pseudoscalar and tensor interactions.

First, consider the case when the only such additional interactions are restricted to be the scalar ones, namely when $h_{V,T}^{\pm\pm} = 0$, and assume then also that\[11] $|G_S^{\pm\pm} f_{V,T}^{\pm\pm} + f_{V,T}^{\pm\pm}|^2 << |F_{V,T}^{\pm\pm} f_{V,T}^{\pm\pm} + f_{V,T}^{\pm\pm}|^2$, where $G_S^{\pm\pm}$ and $F_{V,T}^{\pm\pm}$ are the hadronic matrix elements in the case of $0^+ - 0^+$ superallowed $\beta$-decays for the scalar and vector $(u,d)$ quark currents, respectively. Then, when either one of the pseudoscalar or scalar combinations $(h_{S,T}^{\pm\pm} - h_{S,T}^{\pm\pm})$ and $(h_{S,T}^{\pm\pm} + h_{S,T}^{\pm\pm})$ vanishes, the experimental value (1) leads to the contraints, respectively,

\[
\begin{align*}
\frac{h_{S,T}^{\pm\pm} + h_{S,T}^{\pm\pm}}{h_{V,T}^{\pm\pm} + h_{V,T}^{\pm\pm}} G_S &= -0.0012 \pm 0.0221 \text{ (or } \pm 0.0380) \text{ ,} \\
\frac{h_{S,T}^{\pm\pm} - h_{S,T}^{\pm\pm}}{h_{V,T}^{\pm\pm} + h_{V,T}^{\pm\pm}} G_P &= -0.08 \pm 1.49 \text{ (or } \pm 2.56) .
\end{align*}
\tag{12}
\]

\[11\] This condition is certainly satisfied to a very good approximation.
When only the experimental error on $\lambda_{\text{exp}}^{\text{stat}}$ is included, these uncertainties reduce to $\pm 0.0071$ and $\pm 0.480$, respectively. In fact\cite{7}, the result involving $G_S$ is the one obtained in (4) for $m_\mu F_S / (2M)$, while the precision on the result involving $G_P$ is that obtained for $F_P^{\text{form}}$ in (3). Clearly, these limits are complementary to those which, under the same approximations, may be obtained from the pion decay branching ratio (6) for the pseudoscalar couplings $(f_{++}^s - f_{--}^s)$ and $(h_{++}^s - h_{--}^s)$, given the rather large ratio $C_e / C_\mu \simeq 207$. In this respect, the stringent limit (12) for the scalar coupling $(h_{++}^s + h_{--}^s)$ is quite interesting.

Still under the same chirality assumptions as stated above in relation to left-handed neutrinos, consider now the situation when only new tensor interactions are included, namely when $h_{+-}^T = 0 = h_{-+}^T$. In that case, (1) provides the constraint,

$$h_{+-}^T G_T = -0.00008 \pm 0.00143 \text{ (or } \pm 0.00245\text{) ,} \quad (13)$$

the uncertainty reducing to $\pm 0.000460$ when only the experimental error on $\lambda_{\text{exp}}^{\text{stat}}$ is included. This result thus determines from the muon capture rate on $^3\text{He}$ a most stringent upper bound on tensor interactions coupling to left-handed neutrinos only.

Limits that may be set similarly on scalar and tensor interactions from a 1\% measurement of the triton asymmetry $A_v$ will be considered elsewhere\cite{7}. In view of the results obtained in that case in Sect.2 for the second-class induced form factors $F_S$ and $F_T$, one may expect quite stringent constraints to apply as well.

### 4 Conclusions

As this contribution illustrates, precision measurements in muon capture on $^3\text{He}$ to the triton channel provide for stringent tests of the charged electroweak interaction, both within the Standard Model and for new physics beyond it. The recent precision result\cite{4} for the statistical capture rate enables a series of such tests, some new, the others improving on previous limits, which are complementary to tests that precision measurements of other observables using polarised $^3\text{He}$ targets would make possible. A 1\% measurement of the triton asymmetry\cite{3} is certainly such an example offering much interest, but the potential of other possibilities should also be assessed\cite{7}. Making the most out of muon capture on polarised $^3\text{He}$ calls both for much dedicated experimental efforts and ingenuity, as well as for a renewed theoretical evaluation of the inputs involved in the description of that process.
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