The Contribution of Pseudoscalar Mesons to Hyperfine Structure of Muonic Hydrogen

A. E. Dorokhov, N. I. Kochelev, A. P. Martynenko, F. A. Martynenko, and R. N. Faustov

Abstract—In the framework of the quasipotential method in quantum electrodynamics we calculate the contribution of pseudoscalar mesons to the interaction operator of a muon and a proton in muonic hydrogen atom. The parametrization of the transition form factor of two photons into $\pi$, $\eta$ mesons, based on the experimental data on the transition form factors and QCD asymptotics is used. Numerical estimates of the contributions to the hyperfine structure of the spectrum of the S and P levels are presented.

1. INTRODUCTION

Precise investigation of the Lamb shift and hyperfine structure of light muonic atoms is a fundamental problem for testing the Standard model and establishing the exact values of its parameters, as well as searching for effects of new physics. At present, the relevance of these studies is primarily related to experiments conducted by the collaboration CREMA (Charge Radius Experiments with Muonic Atoms) [1–4] with muonic hydrogen and deuterium by methods of laser spectroscopy. So, as a result of measuring the transition frequency $2P_{3/2} - 2S_{1/2}$ a more accurate value of the proton charge radius was found to be $r_p = 0.84184(67)$ fm, which is different from the value recommended by CODATA for $7\sigma$ [5]. The CODATA value is based on the spectroscopy of the electronic hydrogen atom and on electron-nucleon scattering. The measurement of the transition frequency $2P_{3/2} - 2S_{1/2}$ for the singlet $2S$ of the state $(\mu p)$ allowed to obtain the hyperfine splitting of the $2S$ energy level in muonic hydrogen, and also the values of the Zemach’s radius $r_Z = 1.082(37)$ fm and magnetic radius $r_M = 0.87(6)$ fm. The first measurement of three transition frequencies between energy levels $2P$ and $2S$ for muonic deuterium $(2S_{1/2} - 2P_{3/2}^{F = 1/2})$, $(2S_{1/2} - 2P_{3/2}^{F = 3/2})$, $(2S_{1/2} - 2P_{3/2}^{F = 5/2})$ allowed us to obtain in 2.7 times the more accurate value of the charge radius of the deuteron, which is also less than the value recommended by CODATA [5], by $7.5\sigma$ [4]. As a result, a situation emerges when there is an inexplicable discrepancy between the values of such fundamental parameters, like the charge radius of a proton and deuteron, obtained from electronic and muonic atoms. In the process of searching for possible solutions of the proton charge radius “puzzle” various hypotheses were formulated, including the idea of the nonuniversality of the interaction of electrons and muons with nucleons. It is possible that the inclusion in experimental studies of such muonic atoms as muonic helium $(\mu^3He)^+$, muonic tritium $(\mu t)$ with nuclei consisting of three nucleons, or other light muonic atoms will clarify the problem. In the experiments of the CREMA collaboration one very important task is posed: to obtain an order of magnitude more accurate values of the charge radii of the simplest nuclei (proton, deuteron, helion, alpha particle ...) that enter into one form or another into theoretical expressions for intervals of fine or hyperfine structure

The article is published in the original.
of the spectrum. In this case, the high sensitivity of the characteristics of the bound muon to distribution of charge density and magnetic moment of the nucleus is used. Successful realization of this program is possible only in combination with precise theoretical calculations of various energy intervals, measured experimentally. In this way, the problem of a more accurate theoretical construction of the particle interaction operator in quantum electrodynamics, the calculation of new corrections in the energy spectrum of muonic atoms acquires a special urgency.

2. GENERAL FORMALISM

To study the fine and hyperfine structure of the spectrum of the muonic hydrogen, we use a quasipotential method in quantum electrodynamics in which the bound state of a muon and a proton is described in the leading order in the fine-structure constant by the Schrödinger equation with the Coulomb potential [6–8]. The first part of the important corrections in the energy spectrum of the S- and P-states is determined by the Breit Hamiltonian [6, 7, 9] (further, the abbreviation “fs” and “hfs” is used to denote the contribution to the fine structure and hyperfine structure of the energy spectrum):

$$H_B = H_0 + \Delta V^f_s + \Delta V^h_s, \quad H_0 = \frac{p^2}{2\mu} - \frac{Z\alpha}{r},$$  \hspace{1cm}  (1)$$

$$\Delta V^f_s = -\frac{\alpha}{2m_f}\left[\frac{1}{m_f^2} + \frac{1}{m_p^2}\right] \delta(r) - \frac{\alpha}{2m_fm_pr} \left(\frac{r^2 + (rp)p}{r^3}\right)$$

$$+ \frac{Z\alpha}{2m_f^2r^3}\left[1 + \frac{2m_f}{m_p} + 2a_u \left(1 + \frac{m_f}{m_p}\right)\right] (L_s),$$  \hspace{1cm}  (2)

where \(m_f, m_p\) are the masses of muon and proton correspondingly, \(\mu\) is the proton magnetic moment, \(s_1\) and \(s_2\) are the muon and proton spins. The contribution of interactions (1)–(3) to the energy spectrum of different muonic atoms is well studied [10–17]. The operator (3) gives the main contribution of the order \(\alpha^4\) to the hyperfine structure of the energy spectrum of the muonic atom (Fermi energy). The precise calculation of the hyperfine structure, which is necessary for comparison with the experimental data, requires the consideration of various corrections.

An infinite series of perturbation theory for the particle interaction operator contains the contributions of different interactions. One such contribution due to the exchange of a pseudoscalar meson is investigated in this paper. The amplitude of this interaction is shown in Fig. 1.

The effective vertex of the interaction of the \(\pi^0\) meson (or other pseudoscalar mesons \(\eta, \eta'\)) and virtual photons can be expressed in terms of the transition form factor \(F_{\pi^0 \gamma \gamma}(k_1^2, k_2^2)\) in the form:

$$V^{\mu \nu}(k_1, k_2) = i\epsilon^{\mu \nu \alpha \beta}k_{1\alpha}k_{2\beta} \frac{\alpha}{\pi F_\pi} F_{\pi^0 \gamma \gamma}(k_1^2, k_2^2),$$  \hspace{1cm}  (4)

where \(k_1, k_2\) are four-momenta of virtual photons. The transition form factor is normalized by the condition: \(F_{\pi^0 \gamma \gamma}(0, 0) = 1\). With increasing \(k_1^2, k_2^2\), the function rapidly decreases, which ensures the ultraviolet convergence of the loop integral in the interaction amplitude. The contribution of pseudoscalar mesons to hadronic light-by-light scattering was studied earlier in the calculation of the anomalous magnetic moment of the muon and the hyperfine structure of muonium [18–24].

Let us first consider the construction of the hyperfine part of the interaction potential of particles in the case of S states. We use projection operators on the states of two particles with spin \(S = 0\) and \(S = 1\) [25]:

$$\hat{\Pi}_{S=0}[u(0)\sigma(0)]_{S=0} = \frac{1 + \frac{\epsilon^0}{2\sqrt{2}}}{2\sqrt{2}} \gamma_{S},$$

$$\hat{\Pi}_{S=1}[u(0)\sigma(0)]_{S=1} = \frac{1 + \frac{\epsilon^0}{2\sqrt{2}}}{2\sqrt{2}} \hat{\epsilon},$$  \hspace{1cm}  (5)

where \(\epsilon^\mu\) is the polarization vector of state \(\frac{3}{2}S_1\). The introduction (5) avoids the cumbersome multipli-
tion of the Dirac bispinors and immediately proceeds to calculate the trace from the factors in the numerator of the interaction amplitude:

\[ N = k_{dF}^{\mu\nu} \text{Tr} \left[ \dot{\gamma}_1 + m_1 \gamma^\nu (\dot{p}_1 - \hat{k} + m_1) \right] \times \gamma^\mu (\dot{p}_1 + m_1) \hat{\Pi} (\dot{p}_2 - m_2) \gamma_2 (\dot{q}_2 - m_2) \hat{\Pi} \]

where \( p_{1,2} \) are muon and proton four-momenta of initial state, \( q_{1,2} \) are muon and proton four-momenta of final state, \( t = p_1 - q_1 \) is the pion four momentum. For the calculation and simplification (6) the Form [26] package is used. Introducing instead of \( \mu_\alpha \), the total and relative momenta of the particles in the initial state \( \mu_1 \) and in the final state \( \mu_2 \), and also taking into account their smallness for particles in the bound state \( (\mu_1, \mu_2) \), we retain in \( N \) only the main contribution proportional to the second power of the transmitted 4-momentum \( t = p - q \):

\[ N^{\text{hfs}} = \frac{512}{3} m_p^2 m_p \left[ t^2 k_1^2 - (tk)^2 \right]. \]

Note that the index “hfs” denotes the selection of the hyperfine part in (6) using the projection operators (5).

As a result, the hyperfine part of the potential of the one-pion interaction of a muon and a proton in the S-state takes the form:

\[ \Delta V^{\text{hfs}}(p, q) = \frac{\alpha^2}{6\pi^2 m_p F_p (p - q)^2 + m_p^2} \mathcal{A}(t^2), \]

where

\[ \mathcal{A}(t^2) = \frac{2i}{\pi^2 t^2} \int d^4 k \frac{t^2 k_1^2 - (tk)^2}{k^2 (k - t)^2 (k^2 - 2kp_1)} F_{\pi\pi}(k^2, (k - t)^2). \]

The function \( \mathcal{A}(t^2) \) is characteristic for studying the imaginary and real parts of the amplitude of the decay of pseudoscalar mesons into a lepton pair [27–29]. The dispersion relation with one subtraction for \( \mathcal{A}(t^2) \) has the form:

\[ \mathcal{A}(t^2) = \mathcal{A}(0) - \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im} \mathcal{A}(s)}{s(s + t^2)}. \]

The imaginary part of \( \mathcal{A}(t^2) \), independent of the specific form of the form factor \( F_{\pi\pi}(k^2, (k - t)^2) \), is known (see [28] and Refs. there):

\[ A(0) = \sum_{n=0}^{\infty} \left( \frac{x^2}{n!} \right)^n \frac{\Gamma_{1+2n}}{\Gamma_{1+3n}\psi(n+1)} \left( 3 + 2n \right) \int_0^\infty dx G^{(n+1)}(x) \ln x \]

\[ + G^{(n)}(x = 0) \left[ 2 + (3 + 2n) \left( \ln 4x^2 - \psi(n+1) + \psi(n+1/2) - \frac{2n + 3}{(n+1)(n+2)} \right) \right], \]

where a dimensionless variable \( x = k^2/\Lambda^2 \) is introduced, \( G(x) \equiv F_{\pi\gamma\gamma}(k^2, k^2) \) and \( \psi(n) \) is the digamma function. As it was shown in [29] for the description of experimental data on transition form factors it is sufficient to use the simplest monopole parametrization

\[ G(x) = \frac{1}{1 + x}, \]

and the use of CLEO data [31] and QCD asymptotics [32] defines the parameter \( \Lambda^2 \) in the range of values

\[ \Lambda^2 = [0.448–0.549] \text{ GeV}^2. \]

With the formfactor (13), the leading logarithmic contributions can be summed as [30]

\[ A(0) = \frac{\ln \xi^2}{12 \xi^2} \times \left[ 1 + 6\xi^2 - \sqrt{1 - 4\xi^2(l + 8\xi^2)} - \frac{3}{4} + \mathcal{O}(\xi^2) \right]. \]

Thus, for an electron, the value \( A(0) \) will be equal to [29]

\[ A(0) = -21.9 \pm 0.3, \]

but for a muon

\[ A(0) = -6.1 \pm 0.3. \]
In the latter case, the power corrections to $\xi^6$ should be retained in (12), (15) for numerical estimates. It should also be noted that the effects off-shell pion are insignificant \cite{20, 33}. The maximum precise definition of the numerical value of $A(0)$ is very important for achieving high accuracy of calculation.

Going then to (8) into a coordinate representation, using the Fourier transform, we get the following single-pion exchange potential:

$$
\Delta V^{hh}(r) = \frac{\alpha^2 g_\pi^2}{6F_{\pi}m_{\pi}^2} \left\{ A(0) \left[ \delta(r) - \frac{m_{\pi}^2}{4\pi r} \right] - \frac{1}{\pi} \int_0^\infty ds I_m A(s) \right\}
$$

(18)

$$
\times \left[ \delta(r) + \frac{1}{4\pi r(s - m_{\pi}^2)} \left\{ m_{\pi}^4 e^{-m_{\pi}r} - s^2 e^{-s\sqrt{r}} \right\} \right].
$$

We preserved in (18) the contributions of both terms of the function $A(r^2)$ from (10), although numerically they can vary significantly.

Calculating the matrix elements with wave functions of 1S and 2S states, we obtain the corresponding contributions to the HFS spectrum in the form:

$$
\Delta E^{hh}(1S) = \frac{\mu_3 \alpha g_A^4}{6F_{\pi}^2 \pi^4} \left\{ A(0) \left[ \frac{4W(1 + W)}{m_{\pi}} - \frac{1}{\pi} \int_0^\infty ds I_m A(s) \right] \right\}
$$

(19)

$$
\times \left[ 1 + \frac{1}{4W^2(s - m_{\pi}^2)} \left\{ \frac{m_{\pi}^4}{(1 + m_{\pi}^2)^2} - (1 + \frac{s}{2W})^2 \right\} \right]
$$

$$
= -0.0017 \text{ meV},
$$

$$
\Delta E^{hh}(2S) = \frac{\mu_3 \alpha g_A^4}{48F_{\pi}^2 \pi^4} \left\{ A(0) \left[ \frac{W^2(8 + 11W)}{m_{\pi}^2} + \frac{8W^3}{m_{\pi}^2} + \frac{2W^3}{m_{\pi}^3} \right] \right\}
$$

(20)

$$
\times \left[ 1 + \frac{1}{(s - m_{\pi}^2)^2} \left\{ \frac{m_{\pi}^2}{2 + m_{\pi}^2} - \frac{s(2 + W^2)}{2\left(1 + \frac{W^2}{s}\right)^4} \right\} \right]
$$

$$
= -0.0002 \text{ meV},
$$

where the Goldberg–Treiman relation is used for the pion-nucleon interaction constant: $g_\pi = g_{\pi NN} = m_{\pi} g_A / F_{\pi}$ with $g_A = 1.27$, $F_{\pi} = 0.0924 \text{ GeV}$, $W = \mu \alpha$. The error in the results of (19), (20) is determined by the error in the definition of $A(0)$ from (16) and is less than 10%. Using (19), (20), one can obtain an estimate of the contribution of $\eta$ mesons. These contributions, equal to $-0.0001 \text{ meV}$ (1S), $-0.00002 \text{ meV}$ (2S), yield significantly to the contribution of the pion due to the decrease in the interaction constant $g_{NN\eta}$. The formulas (19), (20) can be used to estimate the corresponding contributions in the hyperfine structure of electron hydrogen. Thus, for the 1S-state of the hydrogen atom, we obtain $\Delta E^{hh}(1S) = -1.25 \text{ Hz}$.

The formalism of projection operators can also be used in constructing hyperfine part of the particle interaction potential for P-states, as it was proposed in \cite{34, 35} (the main contribution to the hyperfine structure of the P-levels is given by the Breit potential in the coordinate representation (3)). We shall show this in the case of the hyperfine splitting of the $2P_{1/2}$ state, taking into account only $A(0)$ from (10). We represent the wave function of the $2P$-state in the momentum representation in the tensor form

$$
\psi_{1P}(p) = (\varepsilon \cdot n_p) R_2(p),
$$

(21)

where $\varepsilon_\mu$ is the polarization vector of orbital motion, $n_p = (0, \hat{p}/\hat{p})$, $R_2(p)$ is a radial wave function in momentum representation. Using the muon bispinor in the rest frame and the polarization vector $\varepsilon_\mu$, we introduce the projection operator on the muon state with the total angular momentum $J = 1/2$:

$$
\hat{\Pi}_p^0 = \frac{i}{\sqrt{3}} \gamma_5 (\gamma_\mu - \gamma_\nu) \psi,
$$

(22)

where the introduced Dirac’s bispinor $\psi$ describes the muon state with the total angular momentum $J = 1/2$, $\nu^\mu = (1, 0, 0, 0)$. Projecting the muon-proton pair to states with the total angular momentum $F = 1/0$ by means of (5), we can represent the numerator of the muon-proton interaction amplitude (see Fig. 1) as:

$$
N_p = \frac{1}{3} k_{\mu \nu} \varepsilon^\mu \varepsilon^\nu \gamma_5 \gamma_\lambda \gamma_5 \tilde{T} \tilde{G}(\gamma_\lambda - \gamma_\nu)
$$

$$
\times \gamma_5 (\hat{q}_1 + m_1) \gamma_\mu (\hat{q}_1 - \hat{k} + m_1) \gamma_\nu (\hat{q}_1 + m_1)
$$

(23)

$$
\times \gamma_5 (\gamma_\omega - \gamma_\nu) \tilde{T} \tilde{G}(\hat{q}_2 - m_2) \gamma_\mu (\hat{q}_2 - m_2) \gamma_\nu (\gamma_\omega - \gamma_\nu)
$$

Then the potential of the hyperfine splitting of the $2P_{1/2}$ energy level can be represented in the momentum representation as follows:

$$
\Delta V^{hh}_{2P_{1/2}}(p, q) = -\frac{\alpha^2 g_A^3}{24\pi^3 F_{\pi}^2} \frac{(p^2 - q^2)}{(p - q)^2 + m_{\pi}^2} A(0).
$$

(24)

As in the previous formulas, we kept in (23) the leading contribution to the relative momenta $p, q$ pro-
portional to $\mathcal{A}(0)$. The matrix element that determines the required hyperfine splitting of the $2P_{1/2}$ level has the form:

$$\Delta E_{2P_{1/2}}^{\text{hfs}} = \int \frac{dp}{(2\pi)^{3/2}} R_{21}(p)$$

$$\times \int \frac{dq}{(2\pi)^{3/2}} R_{21}(q) \Delta V_{2P_{1/2}}^{\text{hfs}}(p, q),$$

where the radial wave function in momentum representation has the form:

$$R_{21}(p) = \frac{128}{\sqrt{3\pi}} \frac{W}{(4p^2 + W^2)^{3/2}}.$$

The expression (25) contains two typical integrals that are calculated analytically:

$$I_1 = \int \frac{dp}{(2\pi)^{3/2}} R_{21}(p) \int \frac{dq}{(2\pi)^{3/2}} R_{21}(q) \left( \frac{p + q}{q} \right) \left( \frac{p}{p - q} \right)^3,$$

$$\times \left( \frac{p + q}{q} \right)^3 + m_n^2 = \frac{2(4a + 5)}{3(a + 2)^4}, \quad a = \frac{2m_n}{W},$$

$$I_2 = \int \frac{dp}{(2\pi)^{3/2}} R_{21}(p) \int \frac{dq}{(2\pi)^{3/2}} R_{21}(q) \left( \frac{pq}{p - q} \right)^3 + m_n^2 = \frac{a(3a + 8) + 6}{2(2a + 2)^4}.$$

With the help of (27), (28) we get the following analytical formula for splitting $2P_{1/2}$ level:

$$\Delta E_{2P_{1/2}}^{\text{hfs}} = \frac{\alpha^2}{288\pi^3} \frac{g_A^2}{m_n^2} \left[ 9 + \frac{8W}{m_n} + \frac{2W^2}{m_n^2} \right] \left( 1 + \frac{W}{m_n} \right)^4 = 0.0004 \mu\text{eV}.$$

The contribution of $\eta$ meson is $8 \times 10^{-5} \mu\text{eV}$. The numerical value of the contribution in the case of the $2P_{1/2}$ level substantially decreases compared to the $2S_{1/2}$ level, since the order of the contribution itself increases. If for $2S_{1/2}$ level the order of the contribution is determined by the factor $\alpha^6$, then for $2P_{1/2}$ level it has the form $\alpha^7$. For the level $2P_{3/2}$, the further decrease in the correction value in the HFS is determined by the factor $10^7$.

3. THE POSITRONIUM EXCHANGE IN HFS OF MUONIC HYDROGEN

On the one hand, the single-pion exchange mechanism investigated in this paper gives an insignificant correction to the hyperfine splitting of the energy levels, which can not explain the “puzzle of the proton radius”. On the other hand, it can be said that this correction turned out to be “unexpectedly large” in magnitude, referring to the exotic character of the muon-proton interaction itself. In this connection it was interesting to estimate the analogous contribution that arises as a result of the positronium exchange between a muon and a proton. The amplitude of such an interaction is shown in Fig. 2. The estimation of the contributions of the hypothetical interaction with particles of mass of the order of 1 MeV both in the Lamb shift and in the HFS of the muonic hydrogen energy spectrum was discussed some time ago in [36–38] in connection with the problem of the proton charge radius.

The potential of single-positronium exchange in muonic hydrogen for the hyperfine splitting of $S$-states in the momentum representation has the form:

$$\Delta V_{\mu p}^{\text{hfs}}(t) = \frac{2\alpha^2}{3\pi^2} \sqrt{\frac{F_{Ps\gamma \gamma}^2}{F_{Ps\gamma \gamma}^2}} \mathcal{A}_{\mu}(0) \mathcal{A}_p(0) \frac{t^2}{t^2 + m_{ps}^2}(s^2 + s_2^2),$$

where for simplicity we use the approximation $\mathcal{A}_{\mu, p}(t^2) = \mathcal{A}_{\mu, p}(0)$ for the effective constants of the muon-proton interaction with positronium. Estimating the parameter $F_{Ps\gamma \gamma}(0)$ using the decay width of the positronium into two photons by the formula

$$F_{Ps\gamma \gamma}(0) = \sqrt{\frac{64\pi\Gamma(Ps \rightarrow \gamma \gamma)}{(4\pi\alpha)^2 m_{ps}^3}},$$

where $\Gamma(Ps \rightarrow \gamma \gamma)$ is the width of the positronium decay into a pair of photons, we find the contribution of this interaction to the hyperfine structure in the form:

$$\Delta E_{Ps}^{\text{hfs}}(1S) = \frac{\alpha^2}{6\pi^2 m_e^2} \mathcal{A}_{\mu}(0) \mathcal{A}_p(0) \left( 1 + \frac{m_{ps}}{W} \right).$$

Using further the expression in the Vector Dominance Model for $\mathcal{A}_{\mu, p}(0)$ (we introduce the denote...
we obtain the numerical values of the contributions to the hyperfine structure. It is convenient to represent the result of the calculation of \( \Delta E_{Ps}^{hfs}(1S) \) on the graph as a function of the cutoff parameter \( \Lambda \) (see Fig. 3). Summation over various excited states of the positronium gives an additional factor \( \sum_0^\infty 1/n^3 = 1.202 \). In the perturbative loop theoretical model, the form factor of the transition of two photons to the positronium is determined by the following tensor integral

\[
F_{\mu\nu}(k^2, k^2) = \frac{\alpha^{3/2}}{m_e \sqrt{\pi}} k^2 \left[ -L_{12} \left( \frac{2k}{\sqrt{k^2 - 4}} \right) - L_{12} \left( \frac{2k}{\sqrt{k^2 + 4}} \right) + L_{12} \left( \frac{2k}{k - \sqrt{k^2 + 4}} \right) \right],
\] (35)

where the dimensionlessness of the integral is carried out with the help of the electron mass \( m_e \). If we compare (35) and the transition factor in the Vector Dominance Model, it can be noted that the mass of positronium acts as a natural cutoff parameter. Such a form factor decreases rapidly with increasing virtuality \( k^2 \) and the magnitude of the correction \( \Delta E_{Ps}^{hfs}(1S) \) is negligible. As the cutoff parameter grows, the contribution increases logarithmically and starting with \( \Lambda \sim 1 \) GeV can already have such a value, which must be taken into account for more accurate determination of the total hyperfine splitting. An increase in the value of the cutoff parameter in the transition form factor means that the positronium production probability for large photon virtualities \( k^2 \) and \( (t-k)^2 \) remains significant.

4. CONCLUSIONS

The high precision measurement of the hyperfine splitting of the muonic hydrogen atom ground state is planned in near future (see, [40–43]). The experiment of FAMU (Fisica Atomi MUonici) collaboration [43] aims to investigate of the proton radius puzzle and determination of the Zemach radius with HFS of \( (\mu^- p)_{1S} \) and to achieve unprecedented accuracy.
\(\delta \lambda / \lambda \leq 10^{-5}\). Even higher experimental resolution for the \(\Delta E_{\text{exp}}^{(1)}\) 2 ppm is expected to obtain in [42]. Taking into account that the value of the ground state hyperfine splitting in muonic hydrogen is equal 182.725 meV [6] (see also [44]) the planned increase in the accuracy of measuring the hyperfine structure of the spectrum in muonic hydrogen will make it possible to verify various theoretical contributions of higher order, and, possibly, to reveal new terms in the particle interaction operator.

In this paper, we investigate the contribution of a pseudoscalar meson to the potential of the hyperfine interaction of the muon and the proton and into the hyperfine structure of the energy spectrum. In the framework of the quasipotential method in quantum electrodynamics and the use of the technique of projection operators on the states of two particles with a definite spin, we constructed particle interaction operators (18), (24) and obtained analytical expressions for the hyperfine splittings of the S and P energy levels (19), (20), (29). Numerical estimates of the contributions (19), (20), (29) connected with the exchange of pseudoscalar mesons are made on their basis. An important role in the numerical calculation of the studied contributions is played by the function \(\mathcal{M}(t^2)\) (10) related with the form factor of the transition of two photons to a pseudoscalar meson (4). For more accurate determination of the constant \(\mathcal{M}(0)\) in (10), we used the results of the works [20–22] in which \(\mathcal{M}(0)\) is defined in terms of the moments of the transition form factor. We also obtained numerical estimates of the contribution (32) to the hyperfine structure of the spectrum due to positronium exchange.

The obtained analytical results are in agreement with the previous calculations of this effect in the framework of chiral perturbation theory [45–47]. The numerical result for the hyperfine splitting of the 2S state \((-0.09 \pm 0.06)\) \(\mu\)eV from [47] is comparable to our value \((-0.0002)\) \(\mu\)eV, taking into account the theoretical error, and our result for HFS \(2P_{1/2}\) practically coincides with the value \(3.7 \times 10^{-4}\) \(\mu\)eV from [47]. The difference from the result of [47] for 2S-level is due to taking into account in [47] the dependence of the vertex function of the pion–nucleon interaction on the transmitted momentum.

Using the obtained result for the hyperfine interaction of a muon and a proton due to a one-pion exchange, it is possible to estimate the same contribution in the case of other light muonic atoms, for example muonic deuterium. The simplest approximation in describing the pion–deuteron interaction is that the deuteron is regarded as a state of two almost free nucleons, and the spins of the neutron and proton in the sum give the total spin \(S = 1\) of the deuteron. Consequently, it can be concluded that the contribution of the pion–neutron interaction to the hyperfine structure of muonic deuterium is the same as that of the pion–proton one, and the total contribution to the hyperfine splitting, for example, of the 2S level, is twice that, that is, has a value of \(-0.0004\) meV.

We are grateful to O. Tomalak for useful communication. The work is supported by Russian Science Foundation (grant No. RSF 15-12-10009) (A.E.D.), the Chinese Academy of Sciences visiting professorship for senior international scientists (grant no. 2013T2J0011) (N.I.K.), Russian Foundation for Basic Research (grant no. 16-02-00554) (A.P.M., F.A.M.)

REFERENCES

1. R. Pohl et al. (CREMA Collab.), “The size of the proton,” Nature 466, 213 (2010).
2. A. Antognini et al. (CREMA Collab.), “Proton structure from the measurement of 2S–2P transition frequencies of muonic hydrogen,” Science 339, 417 (2013).
3. A. Antognini, F. Kottmann, F. Biraben, P. Indelicato, F. Nez, and R. Pohl, “Theory of the 2S–2P Lamb shift and 2S hyperfine splitting in muonic hydrogen,” Ann. Phys. (N.Y.) 331, 127 (2013).
4. R. Pohl et al. (CREMA Collab.), “Laser spectroscopy of muonic deuterium,” Science 353, 669 (2016).
5. P. J. Mohr, B. N. Taylor, and D. B. Newell, “CODATA recommended values of the fundamental physical constants: 2010,” Rev. Mod. Phys. 84, 1527 (2012).
6. A. P. Martyenko and R. N. Faustov, “Hyperfine ground-state structure of muonic hydrogen,” J. Exp. Theor. Phys. 98, 39 (2004).
7. A. P. Martyenko and R. N. Faustov, “Corrections of order \((Z \alpha)^6 m_e^2/m_\mu\) in the muonium fine structure,” J. Exp. Theor. Phys. 88, 672 (1999).
8. A. A. Krutov, A. P. Martyenko, F. A. Martyenko, and O. S. Sukhorukova, “Lamb shift in muonic ions of lithium, beryllium and boron,” Phys. Rev. A 94, 062505 (2016).
9. V. B. Berestetskii, E. M. Lifshits, and L. P. Pitaevskii, Course of Theoretical Physics, Vol. 4: Quantum Electrodynamics (Nauka, Moscow, 1980; Pergamon, Oxford, 1982).
10. M. I. Eides, H. Groth, and V. A. Shelyuto, “Theory of light hydrogenic bound states,” Springer Tracts Mod. Phys. 222 (2007).
11. E. Borie, “Lamb shift in light muonic atoms: revisited,” Ann. Phys. (N.Y.) 327, 733 (2012).
12. K. Pachucki, “Theory of the Lamb shift in muonic hydrogen,” Phys. Rev. A 53, 2092 (1996).
13. U. D. Jentschura, “Lamb shift in muonic hydrogen. I. Verification and update of theoretical predictions,” Ann. Phys. (N.Y.) 326, 500 (2011).
14. S. G. Karshenboim, V. G. Ivanov, E. Yu. Korzinin, and V. A. Shelyuto, “Non-relativistic contributions in order \(\alpha^6 m_e^2\) to the Lamb shift in muonic hydrogen, deuterium and helium ion,” Phys. Rev. A 81, 060501 (2010).
15. A. P. Martynenko, “Theory of muonic hydrogen: muonic deuterium isotope shift,” J. Exp. Theor. Phys. 101, 1021 (2005).
16. A. P. Martynenko, “Hyperfine structure of s-states in muonic helium ion,” J. Exp. Theor. Phys. 106, 691 (2008).
17. A. A. Krutov and A. P. Martynenko, “Lamb shift in muonic deuterium atom,” Phys. Rev. A 84, 052514 (2011).
18. M. Knecht and A. Nyffeler, “Resonance estimates of $O(\rho^6)$ low-energy constants and QCD short distance constraints,” Eur. Phys. J. C 21, 659 (2001).
19. A. E. Dorokhov and W. Broniowski, “Pion pole light-by-light contribution to $g-2$ of the muon in a nonlocal chiral quark model,” Phys. Rev. D: Part. Fields 78, 073011 (2008).
20. A. E. Dorokhov, A. E. Radzhabov, and A. S. Zhelva-kov, “The pseudoscalar hadronic channel contribution of the light-by-light process to the muon ($g-2$), within the nonlocal chiral quark model,” Eur. Phys. J. C 71, 1702 (2011).
21. A. E. Dorokhov, A. E. Radzhabov, and A. S. Zhelvakov, “The light-by-light contribution to the muon ($g-2$) from lightest pseudoscalar and scalar mesons within nonlocal chiral quark model,” Eur. Phys. J. C 72, 2227 (2012).
22. A. E. Dorokhov, M. A. Ivanov, and S. G. Kovalenko, “Complete structure dependent analysis of the decay $P \rightarrow l^+l^-$,” Phys. Lett. B 677, 145 (2009).
23. R. N. Faustov and A. P. Martynenko, “Pseudoscalar pole terms contributions to hadronic light by light corrections to the muonium hyperfine splitting,” Phys. Lett. B 541, 135 (2002).
24. S. G. Karshenboim, V. A. Shelyuto, and A. I. Vainshtein, “Hadronic light-by-light scattering in the muonium hyperfine splitting,” Phys. Rev. D: Part. Fields 78, 065036 (2008).
25. R. N. Faustov, A. P. Martynenko, G. A. Martynenko, and V. V. Sorokin, “Radiative nonrecoil nuclear finite size corrections of order $\alpha (Z\alpha)^2$ to the hyperfine splitting of $s$-states in muonic hydrogen,” Phys. Lett. B 733, 354 (2014).
26. J. A. M. Vermaseren, “FORM,” arXiv:math-ph/0010025.
27. L. Bergstrom, “Rare decay of a pseudoscalar meson into a lepton pair: a way to detect new interactions?,” Z. Phys. C 14, 129 (1982).
28. L. Bergstrom, E. Masso, L. Amettler, and A. Bramon, “$Q^2$ duality and rare pion decays,” Phys. Lett. B 126, 117 (1983).
29. A. E. Dorokhov and M. A. Ivanov, “Rare decay $\pi_0 \rightarrow e^+e^-$: theory confronts KTeV data,” Phys. Rev. D: Part. Fields 75, 114007 (2007).
30. A. E. Dorokhov and M. A. Ivanov, “On mass corrections to the decays $P \rightarrow l^+ l^-$,” JETP Lett. 87, 531 (2008).
31. J. Gronberg et al. (CLEO), “Measurements of the meson – photon transition form-factors of light pseudoscalar mesons at large momentum transfer,” Phys. Rev. D: Part. Fields 57, 33 (1998).
32. G. P. Lepage and S. J. Brodsky, “Exclusive processes in perturbative quantum chromodynamics,” Phys. Rev. D: Part. Fields 22, 2157 (1980).
33. P. Masjuan and P. Sanchez-Puertas, “$\eta$ and $\eta'$ decays into lepton pairs,” J. High Energy Phys. 1608, 108 (2016).
34. R. N. Faustov, A. P. Martynenko, G. A. Martynenko, and V. V. Sorokin, “Pseudoscalar structure of $p$-states in muonic deuterium,” Phys. Rev. A 92, 052512 (2015).
35. A. P. Martynenko and V. V. Sorokin, “Vacuum polarization and quadrupole corrections to the hyperfine splitting of $p$-states in muonic deuterium,” J. Phys. B 50, 045001 (2017).
36. V. Barger, Ch.-W. Chiang, Wai-Yee Keung, and D. Marfatia, “Proton size anomaly,” Phys. Rev. Lett. 106, 153001 (2011).
37. D. Tucker-Smith and I. Yavin, “Muonic hydrogen and MeV forces,” Phys. Rev. D: Part. Fields 83, 101702 (2011).
38. S. G. Karshenboim, “Constraints on a long-range spin-dependent interaction from precision atomic physics,” Phys. Rev. D: Part. Fields 82, 113013 (2010).
39. T. West, “Feynman parameter and trace: programs for expressing Feynman amplitudes as integrals over Feynman parameters,” Comput. Phys. Commun. 77, 286 (1993).
40. O. Tomalak, “Forward two-photon exchange in elastic lepton–proton scattering and hyperfine splitting correction,” arXiv:1701.05514 [hep-ph].
41. R. Pohl et al. (REMA Collab.), “Laser spectroscopy of muonic hydrogen and the puzzling proton,” J. Phys. Soc. Jpn. 85, 091003 (2016).
42. Y. Ma et al., “New precision measurement for proton Zemach radius with laser spectroscopy,” Int. J. Mod. Phys. Conf. Ser. 40, 1660046 (2016).
43. A. Adamczak et al. (FAMU Collab.), “Steps towards the hyperfine splitting measurement of the muonic hydrogen ground state: pulsed muon beam and detection system characterization,” JINST 11, P05007 (2016).
44. C. Peset and A. Pineda, “Model-independent determination of the two-photon exchange contribution to hyperfine splitting in muonic hydrogen,” J. High Energy Phys. 1704, 060 (2017).
45. F. Hagelstein and V. Pascalutsa, “Proton structure in the hyperfine splitting of muonic hydrogen,” PoS(115), 077 (2016).
46. H. Q. Zhou and H. R. Pang, “One-pion-exchange effect in the energy spectrum of muonic hydrogen,” Phys. Rev. A 92, 032512 (2015).
47. N. T. Huong, E. Kou, and B. Moussallam, “Single pion contribution to the hyperfine splitting in muonic hydrogen,” Phys. Rev. D: Part. Fields 93, 114005 (2016).