Quantum Corrections in

\textit{p}-adic String Theory

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Abstract

We compute loop corrections in \textit{p}-adic open string field theory. We argue that quantum effects induce a pole with \( m^2 \sim -\ln g \) for the open string field at the locally stable vacuum. We also compute the one loop effective potential and show that the potential develops an imaginary piece when the field becomes tachyonic.
The $p$-adic string turns out to be a useful model for testing Sen’s conjectures on tachyon condensation in open string field theory. The classical solutions of $p$-adic string field theory can be explicitly found. They have constant descent relations and a fluctuation spectrum that matches $p$-adic string amplitudes on D-brane backgrounds. The tachyon potential in $p$-adic string field theory also contains a classically stable vacuum where the mass of the tachyon field is pushed up to infinity, further justifying the Sen conjectures.

Of course, the Sen conjectures have been intensely studied in conventional bosonic string field theory and in the superstring analog, where the results have been nothing short of impressive. More recently, it now looks possible to find an exact verification of the Sen conjectures by expanding the string fields about the closed string vacuum.

However, the similarity between $p$-adic string theory and ordinary string field theory goes beyond the verification of the Sen conjectures. For example, $p$-adic string field theory in the $p \to 1$ limit reduces to the two derivative truncation of the tachyon effective action in boundary string field theory, suggesting that results for $p$-adic strings are applicable to boundary string field theory. Perhaps an even more compelling reason for studying $p$-adic string theory is the surprising similarity between the $p$-adic D-branes and those recently found in vacuum cubic string field theory, where the string action is expanded about the closed string vacuum. In both cases the D-branes have gaussian profiles. Furthermore, the $p$-adic string does not suffer from strong coupling problems at the local minimum, a feature shared with cubic string field theory.

Therefore, the success of the $p$-adic models in qualitatively verifying the Sen conjectures and the further similarity to ordinary string theory suggests that they might be useful for exploring other qualitative features of string field theory. In this paper we will consider quantum corrections in $p$-adic string field theory. The calculations are significantly easier than in a conventional scalar field theory due to the nature of the kinetic term. Loop calculations turn out to be gaussian integrals.

The loop calculations lead to some interesting results. In particular, we will compute the self energy correction about the stable vacuum, where we find that the classical infinite mass pole is pulled down to a finite value. We also show that the analytic continuation of the one loop effective action develops an imaginary part if the tachyon field is large enough. This occurs before the open string vacuum is reached. The presence of the imaginary piece should not come as a great surprise. This just reflects the instability of the open string vacuum — the one-loop partition function for a tachyon scalar field
formally has an IR divergence which can be continued to a finite, but imaginary piece.

One might speculate whether or not these results carry over qualitatively to conventional string field theory. If so then the conclusion would be that ordinary bosonic string theory has massive states in the closed string vacuum which are the remnants of open strings from the classical D-brane solutions. Perhaps one lesson that is applicable to vacuum cubic string field theory is that the quantum corrections about the closed string vacuum are surprisingly easy to compute. The relative simplicity of the BRST operator and the classical solutions about the closed string vacuum suggest that quantum corrections will be simpler than what one usually finds for string loop corrections.

The tachyon effective action for the $p$-adic string was derived in [2] and is given by

$$S = -\frac{1}{g^2 p - 1} \int d^p x \left[ \frac{1}{2} \phi p^{-\frac{1}{p^2}} \phi - \frac{1}{p + 1} \phi^{p+1} \right].$$

The action was derived assuming that $p$ is prime, but once this action is in place, there does not appear to be any compelling reason to keep $p$ a prime number. In fact, it was shown in [12] that the limit $p \to 1$ yields the two derivative truncation for the effective action in boundary string field theory.

The tree-level open string vacuum is at $\phi = 1$, while the locally stable solution is at $\phi = 0$. The pole position as a function of the background field $\phi$ is

$$q^2 = 2 + \frac{2(p - 1)}{\ln p} \ln \phi.$$  \hspace{1cm} (2)

Thus the solution at $\phi = 1$ has a tachyon pole, while the solution at $\phi = 0$ has its pole pushed to infinite $m^2$. Hence, in this vacuum the open string states disappear from the spectrum.

However, eq. (2) would seem to indicate that once quantum fluctuations are included, a pole with a finite $m^2$ is possible. Since the mass is infinite only when $\phi = 0$, fluctuations about the $\phi = 0$ solution could allow a state to propagate. We will explicitly verify this.

We will compute the quantum corrections by expanding about the $\phi = 0$ solution. As previously stated there is no strong coupling problem at this vacuum, so the results are trustworthy, at least at weak string coupling. For the first part of this analysis we will assume that $p$ is an odd integer. If $p$ is odd, then the potential is an even function of $\phi$ and the closed string vacuum remains at $\phi = 0$ after quantum corrections are included. Later, when we compute the one-loop effective action, we will assume that $p$ is arbitrary. If $p$ is not an odd integer, then the effective action will have a one-point function, shifting
the value of \( \phi \) at the minimum. In this case we will see that there is still a pole at finite mass squared.

The Feynman rules are easily derived from (1). The propagator with Euclidean momentum \( k \) is \( p^{-\frac{1}{2}k^2} \), while the \( p + 1 \) point vertex is given by

\[
p!g^{p-1}\left(\frac{p - 1}{p^2}\right)^{\frac{p-1}{2}}.
\] (3)

The first graph we consider is the two point diagram in Figure 1, whose existence requires that \( p \) be an odd integer. This is the lowest order correction to the two-point function and has \( \frac{p-1}{2} \) loops. The calculation of this graph is straightforward and is given by

\[
- \Sigma = p!g^{p-1}\left(\frac{p - 1}{p^2}\right)^{\frac{p-1}{2}} \frac{2^{\frac{p-1}{2}}}{(\frac{p-1}{2})!} \prod_{i=1}^{\frac{p-1}{2}} \int \frac{d^Dk_i}{(2\pi)^D} p^{-\frac{1}{2}k^2} p^{-1/2}(p-1)p^2 (2\pi \ln p)^{\frac{p-1}{2}}.
\] (4)

where \( \frac{2^{\frac{p-1}{2}}}{(\frac{p-1}{2})!} \) is a symmetry factor. Hence we find that

\[
- \Sigma = \frac{p!}{(\frac{p-1}{2})!} \left( \frac{2g^2(p - 1)}{p^2(2\pi \ln p)^{\frac{p-1}{2}}} \right)^{\frac{p-1}{2}}.
\] (5)

Figure 1: Lowest order correction to the two point function, with \( \frac{p+1}{2} \) loops.

Including this term in the effective action, we see that there is a pole when

\[
p^{\frac{1}{2}p^2} + \Sigma = 0.
\] (6)
Hence, we find a pole at

$$q^2 = \frac{p-1}{\ln p} \ln \left( \frac{2g^2(p-1)}{p^2(2\pi \ln p)^{\frac{D-2}{2}}} \right) + \frac{2}{\ln p} \ln \left( \frac{p!}{g(p-1)!} \right). \tag{7}$$

In the weak coupling limit when $g << 1$, $q^2 \sim \ln g$, so the pole is for a propagating field with a large, but finite mass squared.

We can check the robustness of this result by computing the next contribution to the two point function. The relevant diagram is shown in figure 2. This diagram has $p-1$ loops, and depends on the external momentum $q$. Using the Feynman rules and including symmetry factors, we find that the contribution $\tilde{\Sigma}(q)$ is

$$- \tilde{\Sigma}(q) = (p!)^2 g^{2(p-1)} \left( \frac{p-1}{p^2} \right)^{p-1} \frac{1}{p!} \int \prod_{i=1}^{p} \frac{d^D k_i}{(2\pi)^D} p^{-\frac{D-2}{2}} (2\pi)^D \delta^D(q + \sum k_i) \tag{8}$$

Explicitly doing one of the integrals to get rid of the $\delta$-function and then completing the squares in the gaussian integrals, we find that (8) reduces to

$$- \tilde{\Sigma}(q) = p! p^{-\frac{D-2}{2}} \left( \frac{g^2(p-1)}{p^2(2\pi \ln p)^{\frac{D-2}{2}}} \right)^{p-1} \frac{1}{p!} \frac{1}{p^2}. \tag{9}$$

Figure 2: Next order correction to the two point function, with $p-1$ loops.
Substituting the solution to $q$ in (7), we see that $\tilde{\Sigma}$ is of the order

$$\tilde{\Sigma} \sim -g^{(p-1)(2-\frac{1}{p})}.$$  \hspace{1cm} (10)

Hence, the contribution from $\tilde{\Sigma}(q)$ is suppressed by an extra factor of order $g^{(p-1)^2/p}$ as compared to $\Sigma$. Thus the solution for the pole in (7) is valid at weak coupling as long as $p \neq 1$.

We now turn to the one loop effective action. The relevant diagrams are shown in figure 3. Clearly, there are one loop contributions of the form $\phi^{n(p-1)}$. Expanding the potential to second order in the fluctuations $\tilde{\phi}$, we have

$$V(\phi) = \frac{1}{p+1} \phi^{p+1} + \phi^{p} \tilde{\phi} + \frac{p}{2} \phi^{p-1} \tilde{\phi}^2 + ...$$  \hspace{1cm} (11)

Therefore, taking into account the symmetry factors, the one loop correction to the effec-
tive potential is

$$V_{\text{loop}}(\phi) = -\sum_{N=1}^{\infty} \frac{1}{2N} (p\phi^{p-1})^N \int \frac{d^Dk}{(2\pi)^D} p^{-\frac{1}{2}}Nk^2$$

$$= -\frac{1}{2(2\pi \ln p)^D} \sum_{N=1}^{\infty} \frac{1}{N^{1+\frac{D}{2}}} (p\phi^{p-1})^N$$

$$= -\frac{1}{2(2\pi \ln p)^D} \text{PolyLog} \left( 1 + \frac{D}{2}, p\phi^{p-1} \right), \quad (12)$$

The series in (12) formally diverges if $p\phi^{p-1} > 1$. However, the result can be analytically continued, yielding a finite potential, but one with an imaginary component. The real part is combined with the tree level potential, showing that for weak coupling, $\phi$ has a small shift at the open string vacuum. Since the open string vacuum is still close to $\phi = 1$, we see that the potential here has an imaginary part. This of course arises from the tachyon mode in the open string vacuum. In particular, note that the imaginary piece develops when the value of $q^2$ in (3) changes sign. In string theory, the contribution of the tachyon to the one loop partition function, after analytic continuation, is finite but has an imaginary part.

Let us now consider the case where $p = 2$. We might expect this case to most closely mirror cubic string field theory, since the tree level action in both cases is cubic. The action is no longer invariant under $\phi \to -\phi$, so the quantum corrections will induce a one-point function and hence a shift in the local minimum. We can easily compute this shift using (12). Adding (14) to (1), we see that the new vacuum $\phi_0$ approximately satisfies

$$\phi_0 \approx \frac{1}{4(2\pi \ln 2)^{\frac{D}{2}}} g^2. \quad (13)$$

However, the contribution to $\tilde{\Sigma}(q)$ in (9) is also a one-loop contribution when $p = 2$, and because of the $q^2$ dependence in $\tilde{\Sigma}(q)$, the $\tilde{\Sigma}(q)$ term will dominate in the computation for the pole. To see this explicitly, note that the new equation for the pole is

$$2^{\frac{D}{2}}q^2 - \phi_0 - \frac{g^2}{2(4\pi \ln 2)^{\frac{D}{2}}} 2^{-\frac{1}{2}}q^2 = 0. \quad (14)$$

There is no corresponding term $\Sigma$ as in (4) because $p - 1$ is odd. Clearly, the last term on the lhs of (14) will dominate over the $\phi_0$ term and the pole is approximately at

$$q^2 = \frac{4}{3 \ln 2} \left( \ln g^2 - \ln 2 - \frac{D}{2} \ln(4\pi \ln 2) \right). \quad (15)$$
Thus, we find the same sort of $\ln g$ dependence for the pole position.

We should check that terms that contribute to next order are suppressed when evaluated at the pole. For $p = 2$, one such term has the form in figure 4. Including the

![Figure 4: Two loop contribution to the $p = 2$ two-point function.](image)

symmetry factors, we find that the contribution of this graph is

$$- \hat{\Sigma}(q) = g^4 \int \frac{d^D k_1}{(2\pi)^D} \frac{d^D k_2}{(2\pi)^D} 2^{-\frac{D}{2}} \left( k_1^2 + k_2^2 + (k_1 - k_2)^2 + (k_1 - q)^2 + (k_2 - q)^2 \right)$$

$$= \frac{g^4}{(4\pi \sqrt{2})^D} 2^{-\frac{D}{2}} q^2.$$  \hspace{1cm} (16)

Hence, this term is suppressed for the value of $q^2$ in (15).

Of course we would like to carry out the analogous computations in cubic string field theory. Here we will just speculate on the results we might find.

First, we should expect to find finite results, just as in the $p$-adic case. Ordinarily there are two types of divergences in one-loop string partition functions. The first is due to the open string tachyon, which we have previously argued can be analytically continued to a finite contribution. In the closed string vacuum there should not be any imaginary contribution since the tachyon is absent. The second type of divergence comes from the closed string zero momentum poles. Part of this divergence comes from the closed string tachyon, which can be analytically continued to a finite contribution. The other divergences are due to the graviton and dilaton, which are removed through the Fischler-Susskind mechanism. In any case, all of the closed string divergences are proportional to the coupling of the closed string fields to the D-brane which are in turn proportional to

\footnote{There has been recent work on loop corrections for boundary string field theory [17, 18, 19, 20, 21].}
the tension of the brane. In the closed string vacuum, the branes are absent, so there is nothing for the closed string tachyon, graviton and dilaton to couple to. Hence the closed string divergences should not be present in the closed string vacuum.

Second, since quantum corrections encode fluctuations about the closed string vacuum, it would seem that the mechanism is in place to produce poles from the open string fields in the closed string vacuum, perhaps with the same sort of $\ln g$ behavior. In the tree level closed string vacuum, all states with finite poles are BRST trivial. If there are to be nontrivial states, then quantum corrections must modify the form of the BRST operator.

There have been many recent works about how closed strings can appear in open string theories [22, 23, 24, 25, 26, 27, 28, 29]. One might guess that the pole described here represents a closed string state. In ordinary string theory, the closed string spectrum is independent of $g$ at weak coupling, so a pole with $\ln g$ dependence seems strange. On the other hand, in $p$-adic string theory one does not have the usual closed string states; for instance there are no infinities in the one-loop diagrams corresponding to massless closed string or off-shell tachyon exchange. So perhaps the state with $\ln g$ behavior is the unique $p$-adic closed string state.

In cubic string field theory, it is likely that if one pole appears then infinitely many will appear. Hence, it seems possible to construct an infinite tower of finite mass poles. However, it still seems as if the tower would have the masses pushed to infinity as $g$ approaches zero, making the identification of these poles with closed string states problematic. Hopefully calculations can be carried out soon to shed some light on this problem.

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