Superconformal Transformation Properties of the Supercurrent II: Abelian Gauge Theories

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Abstract

We derive the superconformal transformation properties of the supercurrent for $N = 1$ supersymmetric QED in four dimensions within the superfield formalism. Superconformal Ward identities for Green functions involving insertions of the supercurrent are conveniently derived by coupling the supercurrent to the appropriate prepotential of a classical curved superspace background, and by combining superdiffeomorphisms and super Weyl transformations. We determine all superconformal anomalies of SQED on curved superspace within an all-order perturbative approach and derive a local Callan-Symanzik equation. Particular importance is given to the issue of gauge invariance.

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1 Introduction

Conformal symmetry and superconformal symmetry in particular have attracted much attention within four-dimensional quantum field theory for a long period of time as well as very recently. One of the numerous interesting aspects of conformal symmetry is its interplay with renormalisation. For quantised field theories, conformal symmetry is broken by anomalies in general. The exact determination of these conformal anomalies provides detailed information about the renormalisation behaviour of a given quantum field theory. In particular it is of interest to study the superconformal transformation properties of the supercurrent, an axial vector superfield from which all currents of the superconformal group may be obtained by means of a moment construction. The exact knowledge of this transformation behaviour is expected to be of relevance for instance for a potential proof of the Zamolochikov C theorem in four dimensions. In this context it is essential to study gauge theories, since for several examples of $\mathcal{N} = 1$ supersymmetric non-abelian gauge theories it has been shown that the coefficient of the topological Euler density is larger in the UV than in the IR limit, in agreement with the conjectured extension of the C theorem [1].

Here we consider field theories away from fixed points, where anomalies involving the dynamical fields may be present. In a recent series of papers [2, 3, 4] we have determined the superconformal transformation behaviour of the supercurrent for the massless Wess-Zumino model. For deriving superconformal Ward identities for correlation functions involving the supercurrent we have coupled the supercurrent to the appropriate prepotential of a classical curved superspace background. On curved superspace, superconformal transformations are given by the combination of superdiffeomorphisms and super Weyl transformations. For theories with a conserved energy-momentum tensor, superdiffeomorphisms are anomaly-free, such that the breakdown of conformal symmetry upon quantisation is entirely determined by the anomalies of Weyl symmetry. These manifest themselves in anomalous contributions to the Weyl symmetry Ward identity. In [2] we have determined all dynamical anomalies in this Ward identity. Since all dynamical anomalies are given by terms which involve the usual $\beta, \gamma$ functions as their coefficients, it may be interpreted as a local Callan-Symanzik (CS) equation. The integral of this equation agrees with the well-known global CS equation. Furthermore there are purely geometrical superconformal anomalies which involve only the external classical supergravity fields [3]. These lead to purely local contributions to Ward identities for Green functions with multiple insertions of the supercurrent. Such Ward identities have been discussed in [4], paying careful attention to the renormalisation behaviour of Green functions with multiple insertions of composite operators. All these results are valid in a perturbative approach to all orders in $\hbar$.

In the present paper we extend this analysis to the more involved case of $\mathcal{N} = 1$ supersymmetric QED with massless matter and massive vector fields. Superconformal Ward identities are again derived by considering a classical curved space background [4].
However in this case the compatibility of superconformal transformations and of gauge transformations has to be ensured. For renormalisation we use the BPHZ approach, and we follow the procedure developed in [6, 7] for abelian gauge theories on flat space, i.e. for the superconformal transformations properties of Green functions without insertions of the supercurrent.

For defining a covariant gauge field propagator, we fix the gauge explicitly, such that gauge invariance is broken. The local Callan-Symanzik equation and the conformal transformation properties of the supercurrent may then be derived in close analogy to the Wess-Zumino model. Since the gauge fixing terms are not conformally invariant, they contribute to the final results. However we are able to show that these terms are unphysical in the sense that they do not contribute when acting on states in the physical Hilbert space. As far as the superconformal transformation of the supercurrent is concerned, we show that some of the gauge non-invariant terms cancel with the superconformal transformations of the physical states themselves.

This supersymmetric renormalisation scheme appears to be well-suited for deriving the transformation properties of composite operators to all orders in $\hbar$, and extends in a natural way to the classical curved space background. A different approach to studying supersymmetric composite operators which keeps gauge invariance manifest has been discussed in [8] on flat space. This has also been applied to anomalies at the one-loop level.

This paper is organised as follows. We begin by discussing the superconformal and gauge transformation properties of the dynamical and the background fields in section 2. Using these results we discuss the superconformal and gauge transformation properties of SQED on curved superspace in section 3. In particular we derive a local CS equation which characterises all dynamical superconformal anomalies in terms of the usual $\beta$ and $\gamma$ functions. With the help of this equation we derive the superconformal transformation properties of the supercurrent on flat space. In section 4 we address the issue of gauge invariance. Section 5 contains some concluding remarks.

2 Symmetry transformations of the fields

2.1 Superconformal Transformations

We consider curved superspace in an approach to supergravity characterised by an axial vector superfield $H$, \[ H = H^a \partial_a \] \footnote{To lowest order in the supergravity prepotentials, the geometrical method for deriving such Ward identities has been discussed in [8].}
and a chiral compensator $\phi \equiv \exp(J)$ satisfying

$$D_\alpha \phi = 0, \quad D_\alpha \bar{\phi} = 0,$$

where

$$D_A \equiv (\partial_a, D_\alpha, \bar{D}^\dot{\alpha})$$

are the well known partial and flat space supersymmetry derivatives respectively, which span the tangent space. This approach to supergravity is presented in [2], [3], [4]. Further details may be found in the textbooks [9], [10]. This applies also to the definition the supercovariant derivatives, denoted by

$$\mathcal{D}_A \equiv (D_a, D_\alpha, \bar{D}^\dot{\alpha}),$$

which are the covariant derivatives associated to the group of superdiffeomorphisms on curved superspace and which depend on $H$. Moreover it should be noted that throughout this paper we use the curved space chiral representation which is analogous to the flat space chiral representation in which all fields $\Phi$ are replaced by $\Phi = e^{i\theta^a \theta_\alpha \bar{\phi}}$. Complex conjugation of $\Phi$ in the chiral representation yields $\bar{\Phi}$ in the antichiral representation. On curved space, the chiral representation expression for the conjugate of $\Phi$ is given by $e^{2iH\bar{\Phi}}$. This is of particular importance for the gauge superfield.

The superdiffeomorphisms are given by a complex superfield $\Lambda$ and its conjugate $\bar{\Lambda}$,

$$\Lambda = \Lambda^a \partial_a + \Lambda^\alpha D_\alpha + \Lambda_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} + \Lambda^{\alpha\dot{\beta}} M_{\alpha\beta} + \Lambda^{\dot{\alpha}\dot{\beta}} M_{\dot{\alpha}\dot{\beta}},$$

$$\bar{\Lambda} = \bar{\Lambda}^a \partial_a + \bar{\Lambda}^\alpha D_\alpha + \bar{\Lambda}_{\dot{\alpha}} \bar{D}^{\dot{\alpha}} + \bar{\Lambda}^{\alpha\dot{\beta}} M_{\alpha\beta} + \bar{\Lambda}^{\dot{\alpha}\dot{\beta}} M_{\dot{\alpha}\dot{\beta}},$$

(2.5)

where $\Lambda$ is in the chiral and $\bar{\Lambda}$ in the antichiral representation, and $M, \bar{M}$ are the generators of infinitesimal Lorentz transformations. Covariance constraints imply that the diffeomorphism components may be written as

$$\Lambda^{\alpha\dot{\alpha}} = i\bar{D}^{\dot{\alpha}} \Omega^\alpha, \quad \Lambda^{a} = \frac{1}{4} \bar{D}^2 \Omega^a, \quad \Lambda_{\dot{\alpha}} = e^{2iH} \bar{\Lambda}_{\dot{\alpha}}, \quad \Lambda_{\alpha\beta} = e^{2iH} \bar{\Lambda}_{\alpha\beta},$$

(2.6)

in terms of an unconstrained superfield $\Omega$, with corresponding expressions for $\bar{\Lambda}$. The diffeomorphism transformation properties of the supergravity prepotentials are given by

$$e^{2iH} \rightarrow e^{\Lambda} e^{2iH} e^{-\bar{\Lambda}},$$

(2.7)

for the real axial vector prepotential $H$ and by

$$\phi^3 \rightarrow \phi^3 e^{\bar{\Lambda} c}, \quad \Lambda_c = \Lambda^a \partial_a + \Lambda^\alpha D_\alpha$$

(2.8)

for the chiral compensator, both in the chiral representation.

\footnote{In our conventions the metric has diagonal elements $(+1, -1, -1, -1)$. The covariant derivatives satisfy $\{D_a, \bar{D}_a\} = 2i\sigma^a_{\alpha\dot{\alpha}} \partial_a$.}
The super Weyl transformations are given by superfields \( \sigma, \bar{\sigma} \) which satisfy \( \bar{D}_\alpha \sigma = 0, \ D_\alpha \bar{\sigma} = 0 \). For the Weyl transformation properties of the prepotentials we have

\[
H \rightarrow H, \quad \phi \rightarrow e^\sigma \phi, \quad \bar{\phi} \rightarrow e^{\bar{\sigma}} \bar{\phi}.
\] (2.9)

It is crucial to note that \( H \) is Weyl invariant. Superconformal transformations satisfy the additional relations

\[
\Lambda = \bar{\Lambda} \Rightarrow \bar{D}_\dot{\alpha} \Omega^\alpha = D^\alpha \bar{\Omega}^\dot{\alpha}, \quad \sigma = -\frac{1}{12} D^2 D^\alpha \Omega_\alpha.
\] (2.10)

For the dynamical fields of supersymmetric QED we have two chiral matter fields \( A^+, A^- \) and their chiral partners, which under parity transform as

\[
\mathcal{P} A^+_+ = \bar{A}^-, \quad \mathcal{P} A^-_+ = \bar{A}^-_+.
\] (2.11)

Their diffeomorphism and Weyl transformation properties are given by

\[
A^\pm_+ \rightarrow e^\Lambda A^\pm_+, \quad A^\pm_- \rightarrow e^{-\sigma} A^\pm_+.
\] (2.12)

The spinorial components \( \psi^\alpha_+, \psi^\alpha_- \) of \( A^+_, A^- \) form a Dirac spinor

\[
\Psi_D = \begin{pmatrix} \psi^\alpha_+ \\ \bar{\psi}^\alpha_- \end{pmatrix},
\] (2.13)

such that a theory involving \( A^+_, A^- \) describes the coupling of a Dirac particle to the electromagnetic field. The appropriate gauge superfield \( \Phi \) is dimensionless and real. Under diffeomorphisms

\[
\Phi \rightarrow e^\Lambda \Phi.
\] (2.14)

Since its canonical dimension is zero, \( \Phi \) is Weyl invariant in the classical case discussed here.

It should be noted that in the chiral representation used above, the reality condition on \( \Phi \) reads

\[
\Phi = e^{2iH} \bar{\Phi}.
\] (2.15)

In order to avoid such \( H \) dependent constraint on \( \Phi \), it is more convenient to use the real representation in which

\[
\bar{\Phi} = e^{-iH} \Phi,
\] (2.16)

such that \( \bar{\Phi} = \bar{\Phi} \). By explicit calculation we find that in the real representation, to lowest order in \( H \), the gauge field has the diffeomorphism transformation property

\[
\bar{\Phi} \rightarrow e^{(1/2(\Lambda^a + \bar{\Lambda}^\dot{a}) \partial_\alpha + \Lambda^a D_\alpha + \bar{\Lambda}_\dot{a} \bar{D}^\dot{a})} \bar{\Phi}.
\] (2.17)
2.2 Gauge transformations

The gauge transformations of the matter fields are given by

\[ \delta_\lambda A_+ (z) = -ig\lambda (z)A_+ (z), \quad \delta_\lambda A_- (z) = +ig\lambda (z)A_- (z), \quad \bar{D}^\alpha \lambda = 0, \]  

(2.18)

with \( g \) the charge and \( \lambda \) chiral. For the gauge field we have

\[ \delta_\lambda \Phi = i \left( e^{-iH} \lambda - e^{iH} \bar{\lambda} \right). \]  

(2.19)

Again it is not convenient to consider such \( H \) dependent gauge transformation. Therefore we use the ‘gauge flat’ representation of \([10]\),

\[ \bar{\Phi} = \left( \cosh(-iH) + \frac{\sinh(-iH)}{-iH} \circ \frac{1}{2} H^\alpha \bar{\alpha} [D_\alpha, \bar{D}_\bar{\alpha}] \right) \Phi \]  

(2.20)

for which

\[ \delta_\lambda \bar{\Phi} = i (\lambda - \bar{\lambda}). \]  

(2.21)

\( \bar{\Phi} \) is real, \( \bar{\Phi} = \bar{\bar{\Phi}} \).

Moreover the supergravity fields are gauge invariant. The infinitesimal forms for all field transformations discussed may be found in appendix \([A.1]\). These are relevant for deriving Ward identities.

3 Transformation properties of the supercurrent

Our aim is to derive superconformal Ward identities for Green functions involving insertions of the supercurrent for supersymmetric quantum electrodynamics (SQED). For renormalisation we use the BPHZ approach \([11]\), in which integrands whose integrals are potentially divergent are expanded into power series in external momenta. Then terms of this series are subtracted such that the integrals over the remaining terms are well-defined finite expressions.

We determine the dynamical anomalies present in the Weyl symmetry Ward identity to all orders in perturbation theory. The purely geometrical anomalies depending on the classical background supergravity fields only are the same as discussed in \([3]\), save for the coefficients which are model dependent.

The transformation properties of the supercurrent \( V_a \) are obtained by virtue of the action principle, according to which the variation of the vertex functional with respect to the external field \( H_a \) yields an insertion of the supercurrent,

\[ [V_a] \cdot \Gamma = 8 \frac{\delta}{\delta H_a} \Gamma. \]  

(3.1)
As shown in [2, 4], superconformal Ward identities are conveniently obtained by combining superdiffeomorphism and super Weyl transformations on curved superspace. The diffeomorphism invariance of SQED is preserved by the renormalisation scheme to all orders in perturbation theory, which ensures energy-momentum conservation. The anomalies of superconformal symmetry are thus given by the anomalies of super Weyl symmetry.

The use of the field \( \hat{\Phi} \) as defined implicitly in (2.20) is vital for ensuring that - discarding gauge fixing terms - \( V_a \) as given by (1.11) is gauge invariant. Furthermore the use of \( \hat{\Phi} \) ensures that at the classical level where no anomalies are present, \( V_a \) is a conformally covariant, i.e. quasi-primary field. This is due to the fact that the Ward operator

\[
W(\hat{\Phi}) \equiv \int d^8 z \Omega_{\text{conf}}^\alpha w_{\alpha}^{(A)}(\hat{\Phi}) + c.c.
\]

with \( w_{\alpha}^{(A)}(\hat{\Phi}) \) as in appendix A.2 and with \( \Omega_{\text{conf}}^\alpha \) satisfying (2.10) has no \( H \) dependent contribution to first order in \( H \).

### 3.1 SQED on curved superspace

We discuss the quantised theory within perturbation theory to all orders in \( \hbar \). A basis of diffeomorphism invariant field monomials involving the chiral matter superfields \( A_+, A_- \), as well as the real abelian gauge superfield field \( \Phi \), is given by

\[
\begin{align*}
I_{\Phi} &= \int d^6 z \phi^3 F^\alpha F_\alpha, \\
I_{gl} &= \int d^8 z E^{-1}(\bar{D}^2 + R)\Phi(\bar{D}^2 + \bar{R})\Phi, \\
I_g &= \int d^8 z E^{-1} (\bar{A}_+ e^{\Phi} A_+ + \bar{A}_- e^{-\Phi} A_-), \\
I_\xi &= \int d^6 z \phi^3 R A_+ A_- , \\
I_M &= \int d^8 z E^{-1} \Phi^2, \\
I_m &= \int d^6 z \phi^3 A_+ A_- ,
\end{align*}
\]

where \( F_\alpha = (\bar{D}^2 + R)D_\alpha \Phi \) is the field strength, with \( R \) the chiral supersymmetric curvature scalar. \( E^{-1} \) is the general curved superspace integration measure and \( \phi^3 \), the cube of the chiral compensator, is the integration measure for chiral subspace. \( (\bar{D}^2 + R) \) and \( (D^2 + \bar{R}) \) are chiral and anti-chiral projection operators respectively. Furthermore there is also a basis for local chiral field monomials leading to (3.3) upon integration, which is given by

\[
\begin{align*}
\mathcal{L}_{\Phi} &= \phi^3 F^\alpha F_\alpha , \\
\mathcal{L}_{gl} &= \phi^3 (\bar{D}^2 + R) (\Phi(\bar{D}^2 + R)(\bar{D}^2 + \bar{R})\Phi) , \\
\mathcal{L}_g &= \phi^3 (\bar{D}^2 + R) (\bar{A}_+ e^{\Phi} A_+ + \bar{A}_- e^{-\Phi} A_-) , \\
\mathcal{L}_\xi &= \phi^3 R A_+ A_- , \\
\mathcal{L}_M &= \phi^3 (\bar{D}^2 + R)\Phi^2 ,
\end{align*}
\]

\[
\begin{align*}
\mathcal{L}_m &= \phi^3 A_+ A_- ,
\end{align*}
\]

(3.4)
With the basis (3.3) the effective action (in the sense of Zimmermann) for SQED is given by
\[ \Gamma_{\text{eff}} = -\frac{1}{128} z_\Phi I_\Phi + \frac{1}{16} z_g I_g + \frac{1}{8} \hat{\xi} (I_\xi + \bar{I}_\xi) + \frac{1}{4} m(s - 1) (I_m + \bar{I}_m) + \frac{1}{16} \hat{M}^2 I_M - \frac{1}{128} \alpha I_{gf}. \] (3.5)

Here the coefficients \( z_\Phi, z_g, \hat{\xi} \) and \( \hat{M}^2 \) are power series in \( \hbar \). \( I_\Phi \) and \( I_g \) are the curved superspace generalisations of the usual terms contributing to SQED on flat space. \( I_\xi \) describes the coupling of the curved space background to the matter fields. The factor \( (s - 1) \) in front of the matter field mass terms is an auxiliary parameter participating in the subtractions, which is necessary for consistent renormalisation of massless theories, as described below. It may be set to \( s = 1 \) at the very end of the renormalisation procedure.

We also include a mass term \( \hat{M}^2 I_M \) for the gauge field. This ensures in particular that there is no infrared problem. Finally for a consistent perturbative approach which requires a well-defined covariant gauge field propagator, we fix the gauge by including the gauge fixing term \( I_{gf} \) with \( \alpha \) the gauge parameter. Therefore the gauge Ward identity has the form
\[ w^{(\lambda)}(A, \hat{\Phi}) \Gamma = i \phi^3 (\bar{D}^2 + R) \left\{ -\frac{1}{128\alpha} R(\bar{D}^2 + R) \Phi - \frac{1}{128\alpha} (\bar{D}^2 + R)(\bar{D}^2 + R) \Phi + \frac{1}{4} \hat{M}^2 \Phi \right\}, \] (3.6)
where the local gauge Ward operator is given by
\[ w^{(\lambda)}(A, \hat{\Phi}) = ig \left( A_- \frac{\delta}{\delta A_-} - A_+ \frac{\delta}{\delta A_+} \right) + i \bar{D}^2 \frac{\delta}{\delta \Phi}. \] (3.7)

### 3.2 Dynamical anomalies

We derive a local Callan-Symanzik equation in close analogy to the procedure followed for the Wess-Zumino model in [2]. According to the discussion of the Weyl transformation properties of both dynamical and geometrical fields in section 2, the local chiral Weyl symmetry Ward operator is given by
\[ w^{(\sigma)}(A, J) = \frac{\delta}{\delta J} - A_+ \frac{\delta}{\delta A_+} - A_- \frac{\delta}{\delta A_-}, \] (3.8)
where \( J \) is given by \( \phi \equiv \exp(J) \) with \( \phi \) the chiral compensator. Applying this operator to the vertex functional \( \Gamma \) as given by (3.3) we obtain
\[ w^{(\sigma)}(A, J) \Gamma = -\frac{3}{2} [S]_3^3 \cdot \Gamma, \] (3.9)
\[ S = \frac{1}{16} \hat{M}^2 L_M + \frac{1}{4} m(s - 1) L_m + \frac{1}{8} \hat{\xi} (L'_\xi - L_\xi) + \frac{1}{128\alpha} (L'_{gf} - L_{gf}), \] (3.10)
where the square brackets denote an insertion of local composite operators, and \( L_M, L_m, L_\xi, L_{gf} \) are defined in (3.4). The indices \( \frac{3}{3} \) stand for the UV and IR subtraction degrees.
We see that in addition to the mass terms, the matter-background coupling and the gauge fixing terms are not Weyl invariant. For exploring the consequences of this Ward identity further, we use appropriate Zimmermann identities [12, 7]. These identities express hard insertions in terms of the corresponding soft insertion plus a basis of hard local field monomials with the appropriate symmetries and quantum numbers. Here the relevant identities are, using the basis (3.4),

\[ \left[ m(s-1)\mathcal{L}_m \right]_3^3 \cdot \Gamma = u_m m(s-1) \left[ \mathcal{L}_m \right]_2^2 \cdot \Gamma \\
+ [ u_\phi \mathcal{L}_\phi + u_g \mathcal{L}_g + u_\xi \mathcal{L}_\xi + u'_\xi \mathcal{L}'_\xi ]_3^3 \cdot \Gamma, \quad (3.11) \]

\[ \left[ \hat{M}^2 \mathcal{L}_M \right]_3^3 \cdot \Gamma = v_M \hat{M}^2 \left[ \mathcal{L}_M \right]_1^1 \cdot \Gamma \\
+ [ v_g \mathcal{L}_g + v_\xi (\mathcal{L}_\xi + \mathcal{L}'_\xi) ]_3^3 \cdot \Gamma \quad (3.12) \]

for the mass terms as well as

\[ \left[ \mathcal{L}'_g + \mathcal{L}_g \right]_3^3 \cdot \Gamma = \left[ \mathcal{L}'_g + \mathcal{L}_g \right]^1_1 \cdot \Gamma + [ 2r_g \mathcal{L}_g + 2r_\xi (\mathcal{L}_\xi + \mathcal{L}'_\xi) ]_3^3 \cdot \Gamma, \quad (3.13) \]

\[ \left[ \mathcal{L}'_g - \mathcal{L}_g \right]_3^3 \cdot \Gamma = \left[ \mathcal{L}'_g - \mathcal{L}_g \right]^1_1 \cdot \Gamma + t \left[ \mathcal{L}'_g - \mathcal{L}_g \right]_3^3 \cdot \Gamma, \quad (3.14) \]

\[ [I_g]_4^4 \cdot \Gamma = [I_g]_2^2 \cdot \Gamma + [ r_g I_g + r_\xi (I_\xi + \bar{I}_\xi) ]_4^4 \cdot \Gamma \quad (3.15) \]

for the gauge fixing terms. The coefficients \( u, v, r, t \) are power series in the couplings and in \( \hbar \). By integrating the identity (3.14) over chiral superspace, it may be shown that \( t = 0 \).

In order to eliminate the symmetry breaking terms \( \mathcal{L}_\xi \) of matter-background coupling type from the Ward identity (3.9), we impose \( R \) invariance

\[ W^R \Gamma = 0 \bigg|_{s=1}. \quad (3.16) \]

Here the \( R \) symmetry Ward operator is obtained from the superconformal Ward operator combining diffeomorphisms and Weyl transformations, by using

\[ \Omega^\alpha(R) = -i\theta^\alpha \bar{\theta}^2 r, \quad \sigma^{(R)} = \frac{2}{3} i r, \quad (3.17) \]

for the parameter \( \Omega^\alpha \) of local superconformal transformations (2.10), as appropriate for \( R \) symmetry transformations. This gives

\[ W^R \Gamma = \int d^8 z \left( \Omega^\alpha(R) w_\alpha - \bar{\Omega}^{\dot{\alpha}}(R) \bar{w}^{\dot{\alpha}} \right) \Gamma, \quad (3.18) \]

\[ w_\alpha \equiv w_\alpha^{(A)}(H,J,\Phi,A_{\pm}) + \frac{1}{12} D_\alpha w^{(\sigma)}(A_{\pm},J), \quad (3.19) \]

using the Ward operators of appendix A.2.
The imposition of $R$ invariance implies

$$\hat{\xi} = u\xi - u'\xi, \quad (3.20)$$

which fixes the coupling $\hat{\xi}$ uniquely as a function of the coupling $g$ and thus amounts to a reduction of couplings \[13\].

The Ward identity \[3.9\] leads to a Callan-Symanzik (CS) equation since from dimensional analysis we have

$$\mu \partial_\mu \Gamma_{\text{eff}} = -iW^D\Gamma_{\text{eff}}, \quad W^D = \int d^6z w^{(\sigma)} + \int d^6\bar{z} \bar{w}^{(\sigma)}, \quad (3.21)$$

where $\mu$ stands for all mass parameters of the theory. Thus using the Zimmermann identities \[3.11\] and \[3.12\] we obtain the CS equation

$$C \Gamma = \frac{1}{4} (1 - 2\gamma_A) m(s - 1) [I_m + \hat{I}_m]_3 \cdot \Gamma$$

$$+ \left( \frac{1}{2} (1 - \beta) \hat{M}^2 + \frac{1}{16} \beta (g \partial_g - 4\alpha \partial_\alpha) \hat{M}^2 \right) [I_{\hat{M}}]_2 \cdot \Gamma,$$  

$$C \equiv \mu \partial_\mu + \beta (g \partial_g - N_\Phi - 2\alpha \partial_\alpha) - \gamma_A N_A, \quad (3.22)$$

which holds subject to the coefficients satisfying

$$\left( 1 - \beta - 2\beta \alpha \frac{\partial_\alpha M^2}{M^2} + \frac{1}{2} \beta \frac{\partial_g M^2}{M^2} \right) v_g + 4(1 - 2\gamma_A) u_g + \left( \frac{1}{2} \beta g \partial_g - \beta \alpha \partial_\alpha - \gamma_A \right) z_g = 0,$$  

$$\frac{1}{64} (\beta + \beta \alpha \partial_\alpha - \frac{1}{2} \beta g \partial_g) z\Phi + \frac{1}{2} (1 - 2\gamma_A) u\Phi = 0.$$  

(3.25)

These relations define the functions $\beta$ and $\gamma_A$, which are the usual $\beta$ and $\gamma$ functions, noting that

$$\beta^g = \beta \cdot g, \quad \beta \equiv \gamma_\Phi. \quad (3.26)$$

$N_\Phi$ and $N_A$, the operators counting fields in $C$, are given by

$$N_\Phi = \int d^6z w^{(\sigma)}(\hat{\Phi}) + c.c., \quad N_A = \sum_\pm \int d^6z A_\pm \frac{\delta}{\delta A_\pm} + \int d^6\bar{z} \bar{A}_\pm \frac{\delta}{\delta \bar{A}_\pm}, \quad (3.27)$$

where $w^{(\sigma)}(\hat{\Phi})$ is a power series in $H^{\alpha\dot{\alpha}}$ as given in appendix \[A.2\]. To lowest order it is given by

$$w^{(\sigma)}(\hat{\Phi}) \Gamma = D^2 \left( \hat{\Phi} \frac{\delta \Gamma}{\delta \Phi} \right). \quad (3.28)$$

Classically, $\hat{\Phi}$ is Weyl invariant. However in the quantised theory it acquires an anomalous Weyl weight $\beta$.  

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For the proof of the CS equation (3.22) to all orders we make use of the consistency condition

$$[W^R, \mathcal{C}] = 0$$  \hspace{1cm} (3.29)$$

for the operators defined in (3.18) and (3.23), proceeding by induction in orders of $\hbar$. In particular for the matter-background coupling $I_\xi + \bar{I}_\xi$, (3.29) ensures that subject to (3.20)

$$2(1 - \beta)v_\xi + 4(1 - 2\gamma A)(u_\xi + u'_\xi) + 2(\beta g \partial g - 2\gamma A)\hat{\xi} + \beta g \frac{\partial_\alpha M^2}{M^2} v_\xi - 4\beta \alpha \partial_\alpha \hat{\xi} = 0, \hspace{1cm} (3.30)$$

such that there are no $I_\xi + \bar{I}_\xi$ contributions to the r.h.s. of the CS equation (3.22).

### 3.3 Gauge parameter dependence

The CS operator $\mathcal{C}$ given by (3.23) involves $\partial_\alpha$, the derivative with respect to the gauge parameter $\alpha$. When acting on the vertex functional $\Gamma$ this derivative yields

$$\partial_\alpha \Gamma = \left[\partial_\alpha \Gamma_{\text{eff}}\right]^4_4 \cdot \Gamma$$

$$= \left[-\frac{1}{128} \partial_\alpha z_\Phi I_\Phi + \frac{1}{16} \partial_\alpha \hat{\omega} \hat{M}^2 I_M + \frac{1}{128\alpha^2} I_{gf} + \frac{1}{16} \partial_\alpha z_g I_g + \frac{1}{8} \partial_\alpha \hat{\xi} (I_\xi + \bar{I}_\xi)\right]^4_4 \cdot \Gamma. \hspace{1cm} (3.31)$$

This hard symmetry breaking term may be simplified using the Zimmermann identity (3.15). Moreover we are free to choose the normalisation conditions

$$\partial_\alpha z_\Phi = 0, \quad \partial_\alpha \hat{M}^2 = 0, \quad 8\partial_\alpha z_g + \frac{1}{\alpha^2} r_g = 0, \hspace{1cm} (3.32)$$

such that (3.31) reduces to

$$\partial_\alpha \Gamma = \frac{1}{128\alpha^2} [I_{gf}]^2_2 \cdot \Gamma + (\frac{1}{8} \partial_\alpha \hat{\xi} - \frac{1}{128\alpha^2} r_\xi) [I_\xi + \bar{I}_\xi]^4_4 \cdot \Gamma. \hspace{1cm} (3.33)$$

The coupling $\hat{\xi}$ - and thus its $\alpha$ dependence - have already been fixed by imposing $R$ invariance in (3.16). By making use of the consistency condition

$$[W^R, \partial_\alpha] = 0$$  \hspace{1cm} (3.34)$$

we may show that

$$\frac{1}{8} \partial_\alpha \hat{\xi} + \frac{1}{128\alpha^2} r_\xi = 0, \hspace{1cm} (3.35)$$

such that

$$\partial_\alpha \Gamma = \frac{1}{128\alpha^2} [I_{gf}]^2_2 \cdot \Gamma. \hspace{1cm} (3.36)$$

When reducing to flat space, the gauge fixing term on the r.h.s. vanishes when acting on states of the physical Hilbert space, as was shown in [8, 9]. This is essentially due to its reduced subtraction degree. Thus the physical degrees of freedom in SQED on flat space
are gauge parameter independent. Here it is straightforward to extend this argument to Green functions involving an insertion of the supercurrent by making use of (3.31) and varying (3.36) with respect to the superpotential $H_{\alpha\dot{\alpha}}$, which gives, when restricting to flat superspace,

$$\partial_\alpha [V_{\alpha\dot{\alpha}}]_3^3 \cdot \Gamma \bigg|_{H=0} = [V_{\alpha\dot{\alpha}}]_0^0 \cdot \Gamma + \frac{1}{128\alpha} [V_{\alpha\dot{\alpha}}]_3^3 \cdot [I_{gf}]_2^2 \cdot \Gamma \bigg|_{H=0},$$

(3.37)

with $V_{\alpha\dot{\alpha}}$ the gauge non-invariant part of the supercurrent. Each of the terms on the r.h.s. have been shown to be unphysical on flat space, as discussed in [7]. Thus when acting on states of the physical Hilbert space, the supercurrent insertion is gauge parameter independent.

### 3.4 Local Callan-Symanzik equation

Just as for the Wess-Zumino model [4], we derive a *local* CS equation, i.e. we express the anomalies present in the local chiral Weyl symmetry Ward identity (3.9) in such a way that upon integration and combination with the corresponding antichiral equation we recover the *global* CS equation (3.22). For this purpose we introduce the chiral effective Lagrangian

$$\mathcal{L}_{\text{eff}} \equiv -\frac{1}{256} z_0 \Phi \mathcal{L}_\Phi + \frac{1}{32} z_0 g \mathcal{L}_g + \frac{1}{4} m(s-1) \mathcal{L}_m + \frac{1}{32} \hat{M}^2 \mathcal{L}_M + \frac{1}{8} (\hat{\xi} - \hat{\epsilon}) \mathcal{L}_\xi + \frac{1}{8} \hat{\epsilon} \mathcal{L}'_\xi,$$

(3.39)

using the basis (3.4), such that

$$\Gamma_{\text{eff}} = \int d^6 z \mathcal{L}_{\text{eff}} + \int d^6 \bar{z} \bar{\mathcal{L}}_{\text{eff}}$$

(3.40)

with $\Gamma_{\text{eff}}$ as given by (3.3). $\hat{\epsilon}$ is an additional coupling parameter which does not contribute to $\Gamma_{\text{eff}}$ due to

$$\int d^6 z (\mathcal{L}_\xi - \mathcal{L}'_\xi) + c.c. = 0.$$

(3.41)

Then from the Ward identity (3.9) and the relations (3.24), (3.25) and (3.30) we obtain the local CS equation

$$w^{(\sigma,\gamma)} \Gamma = -\beta g [\partial_\gamma \mathcal{L}_{\text{eff}}]_3^3 \cdot \Gamma$$

$$- \frac{1}{128\alpha} [\mathcal{L}_g' - \mathcal{L}_g]_1^1 \cdot \Gamma + \frac{1}{256\alpha} \beta [\mathcal{L}_g' + \mathcal{L}_g^\prime]_1^1 \cdot \Gamma$$

$$+ \frac{1}{16} v_M (1 - \beta + \frac{1}{2} \beta g \partial_\gamma \hat{M}^2 \mathcal{L}_M) \cdot \Gamma$$

$$+ \frac{1}{4} (1 - 2 \gamma_A) u_m m(s-1) [\mathcal{L}_m]_2^2 \cdot \Gamma,$$

(3.42)

$$w^{(\sigma,\gamma)} \Gamma \equiv \left( w^{(\sigma)} \Gamma - \gamma_A A_\pm \frac{\delta \Gamma}{\delta A_\pm} - \frac{1}{2} \beta w^{(\sigma)} (\hat{\Phi}) \Gamma \right),$$

(3.43)
which holds subject to the additional condition
\[ \beta g \partial_g \hat{\epsilon} - \frac{1}{2} \beta g \partial_g \hat{\xi} + 3\gamma_A \hat{\xi} = 0 \] (3.44)
for the coupling \( \hat{\epsilon} \). In (3.43), \( w^{(\sigma)}(\hat{\Phi}) \) is given by (3.27) and (3.28). The local CS equation (3.42), which is chiral, characterises all dynamical anomalies of superconformal symmetry. The global CS equation (3.22) is recovered by integrating (3.42) and adding the complex conjugate, where, as far as the gauge fixing terms are concerned, the agreement is due to (3.36). From the local CS equation (3.42) we see that the breakdown of superconformal invariance manifests itself in anomalous dimensions for the fields and in an insertion of \( \mathcal{L}_{\text{eff}} \) with the usual \( \beta \) function as coefficient. Moreover the mass terms break conformal symmetry as expected. In (3.42) we may now set \( s = 1 \). Furthermore there are gauge fixing terms breaking superconformal symmetry. However these do not contribute when acting on states of the physical Hilbert space, as is discussed in section 4 below.

### 3.5 Transformation properties of the supercurrent

By combining the local Callan-Symanzik equation (3.42) with the local Ward identity expressing diffeomorphism invariance, we obtain the superconformal transformation properties of the supercurrent. In agreement with the discussion of section 2 and using the local Ward operators listed in appendix A.2 and in (3.43), we have, restricting the transformation parameters \( \Omega^\alpha \) and \( \sigma \) to be of the superconformal form (2.10),

\[ \int \! d^8z \Omega^\alpha \left( w^{(\Lambda)}_\alpha + \frac{1}{12} D_\alpha w^{(\sigma, \gamma)} \right) \Gamma + \text{c.c.} = \int \! d^6z \sigma[S^{(\gamma)}] \cdot \Gamma + \text{c.c.,} \] (3.45)

where \( S^{(\gamma)} \) stands for the r.h.s. of the local CS equation (3.42). From this superconformal Ward identity we obtain the superconformal transformation properties of the supercurrent insertion by varying (3.45) with respect to \( H_{a\dot{\alpha}} \) and subsequently restricting to flat superspace, which yields

\[
\begin{align*}
&\delta A^\Lambda_{\dot{\alpha}} W^{(\gamma_{a\dot{\alpha}}, \beta)}(\Omega_{\text{conf}})[V_{a\dot{\alpha}}(z)] \cdot \Gamma \\
&= [\delta V_{a\dot{\alpha}}(z)] \cdot \Gamma \\
&+ \int \! d^6z' \sigma(z') \left\{ -\beta g \partial_g \mathcal{L}_{\text{eff}}(z') \cdot V_{a\dot{\alpha}}(z) \right\} \cdot \Gamma + \text{c.c.} \\
&- \frac{1}{16\alpha} \frac{\delta}{\delta H^{a\dot{\alpha}}(z)} \left[ \int \! d^6z' \sigma(z') \left( (\mathcal{L}^\prime_{\text{gt}} - \mathcal{L}_{\text{gt}}) - \frac{1}{2} \beta (\mathcal{L}_{\text{gt}} + \mathcal{L}^\prime_{\text{gt}}) \right) + \text{c.c.} \right]^2 \cdot \Gamma \\
&+ \frac{1}{16\alpha M} \left( 1 - \beta (1 + \frac{1}{2} g \partial_g) \right) \int \! d^6z' \sigma(z') \mathcal{M}^2 \frac{\delta}{\delta H^{a\dot{\alpha}}(z)} \left[ \int \! d^6z' \sigma(z') \mathcal{L}_M(z') + \text{c.c.} \right]^2 \cdot \Gamma \\
&- \frac{1}{2} \beta \left( D_\alpha \sigma \bar{D}_\dot{\alpha} \hat{\Phi} - \bar{D}_\dot{\alpha} \sigma D_\alpha \hat{\Phi} \right) \frac{\delta \Gamma}{\delta \hat{\Phi}},
\end{align*}
\] (3.46)
where we have set $s = 1$. Here the flat space superconformal Ward operator is given by

$$A^\Phi_{\Lambda\sigma} W^{(\gamma\lambda, \beta)}(\Omega_{\text{conf}}) = \int d^8 z \Omega_{\text{conf}}^\Lambda \left( w^{(\gamma\lambda)}_\alpha (A_\pm) + w^{(\beta)}_\alpha (\hat{\Phi}) \right) + \text{c.c.}, \quad (3.47)$$

$$w^{(\gamma\lambda)}_\alpha (A_\pm) = \frac{1}{4} D_\alpha A_\pm \frac{\delta}{\delta A_\pm} - \frac{1}{12} (1 + \gamma A) D_\alpha \left( A_\pm \frac{\delta}{\delta A_\pm} \right),$$

$$w^{(\beta)}_\alpha (\hat{\Phi}) = \frac{1}{2} \tilde{D}^{\alpha} \left( \tilde{D}_\alpha \hat{\Phi} \frac{\delta}{\delta \hat{\Phi}} \right) + \frac{i}{4} \tilde{D}^2 \left( D_\alpha \hat{\Phi} \frac{\delta}{\delta \hat{\Phi}} \right) - \frac{1}{12} \beta D_\alpha \tilde{D}^2 \left( \hat{\Phi} \frac{\delta}{\delta \hat{\Phi}} \right),$$

where the superconformal transformation parameter $\Omega_{\text{conf}}$ satisfies the condition (2.10) and where $w^{(\gamma\lambda)}_\alpha (A_\pm)$ and $w^{(\beta)}_\alpha (\hat{\Phi})$ are flat space superconformal Ward operators. These are obtained from the flat space restriction of the combined diffeomorphism and Weyl operators. Moreover the classical superconformal transformation of the supercurrent is given by

$$\delta V_{a\dot{a}} (z) = (\Lambda - \frac{2}{3} (\sigma + \bar{\sigma})) V_{a\dot{a}} (z)$$

$$= \{ D_\beta, \tilde{D}_\bar{\beta} \} \left( \tilde{D}^{\beta} \Omega V_{a\dot{a}} \right) + \{ D_\alpha, \tilde{D}_\bar{\alpha} \} \left( \tilde{D}^\beta \hat{\Phi} \right) + D_\beta \left( \tilde{D}^2 \Omega V_{a\dot{a}} \right) + \text{c.c.}. \quad (3.48)$$

For the classical theory without gauge fixing, the supercurrent $V_{a\dot{a}}$ is conformally covariant, i.e., a quasi-primary field. Furthermore in (3.46), for any insertion $[S]$ the curly brackets denote

$$\{ V_{a\dot{a}} (z) \cdot S(z') \} \cdot \Gamma \equiv 8 \frac{\delta}{\delta H^{a\dot{a}}(z)} [S(z')] \cdot \Gamma = [V_{a\dot{a}} (z)] \cdot [S(z')] \cdot \Gamma + 8 \left[ \frac{\delta S(z')}{\delta H^{a\dot{a}}(z)} \right] \cdot \Gamma. \quad (3.49)$$

Finally the last term on the r.h.s. of (3.46) is due to the $H$ dependence of $\hat{\Phi}$ as given by (2.20), which leads to an anomalous conformal transformation of $\hat{\Phi}$ in the quantised theory. This reflects the fact that after quantisation, the gauge field $\hat{\Phi}$ is no longer quasi-primary.

## 4 Gauge invariance

The superconformal transformation of the supercurrent as given by (3.46) contains terms involving the gauge fixing and thus is seemingly not gauge invariant. However this is only apparent. In this section we show that when acting on elements $|p\rangle$ of the Hilbert space of physical states, with infinitesimal superconformal transformation $|\delta p\rangle$, then

$$\Delta (\langle p'| V_{a\dot{a}} |p \rangle) = \langle p'| (\Delta V_{a\dot{a}}) |p \rangle + \langle \delta p'| V_{a\dot{a}} |p \rangle + \langle p'| V_{a\dot{a}} |\delta p \rangle$$

is gauge invariant. $\Delta$ is the superconformal transformation corresponding to $A^\Phi_{\Lambda\sigma} W^{(\gamma\lambda, \beta)}(\Omega)$ acting on $[V_{a\dot{a}}(z)] \cdot \Gamma$ in (3.46). For showing gauge invariance we note that the physical states satisfy

$$(D^2 \Phi)^{-} |p\rangle = (\tilde{D}^2 \Phi)^{-} |p\rangle = 0, \quad \langle p'| (D^2 \Phi)^{+} = \langle p'| (\tilde{D}^2 \Phi)^{+} = 0,$$
Furthermore using (4.2) we have

\[ 0 = \delta(\langle p'|D^2\Phi|p\rangle) = (1 - \frac{1}{2}\beta)\langle p'|D^2(\bar{\sigma}\Phi)|p\rangle + \langle \delta p'|D^2\Phi|p\rangle + \langle p'|D^2\Phi|\delta p\rangle, \]

\[ 0 = \delta(\langle p'|D^2\Phi|p\rangle) = (1 - \frac{1}{2}\beta)\langle p'|D^2(\sigma\Phi)|p\rangle + \langle \delta p'|D^2\Phi|p\rangle + \langle p'|D^2\Phi|\delta p\rangle. \]  

(4.3)

Furthermore using (4.2) we have

\[ \langle p'|D^2(\bar{\sigma}\Phi)|p\rangle = \langle p'|2\bar{D}_\alpha\bar{\sigma}\bar{D}^\alpha\Phi|p\rangle. \]  

(4.4)

Using these relations we find for the gauge non-invariant part \( V^g_{\alpha\bar{\alpha}} \) of the supercurrent defined in (3.38), when acting on the transformation of the physical states \( |\delta p\rangle \),

\[ \langle \delta p'|V^g_{\alpha\bar{\alpha}}|p\rangle + \langle p'|V^g_{\alpha\bar{\alpha}}|\delta p\rangle = (1 - \frac{1}{2}\beta)\langle p'| (T_{\alpha\bar{\alpha}} + c.c.) |p\rangle, \]  

(4.5)

with

\[ T_{\alpha\bar{\alpha}} \equiv \frac{1}{24\alpha} \left([D_\alpha, \bar{D}_{\bar{\alpha}}]|D^2(\bar{\sigma}\sigma D_\beta\Phi) - \bar{\Phi}\bar{D}_{\bar{\alpha}}D_\alpha\bar{D}^2(\bar{\sigma}\sigma D_\beta\Phi) + \bar{D}_{\bar{\alpha}}\Phi D_\alpha\bar{D}^2(\bar{\sigma}\sigma D_\beta\Phi)\right). \]  

(4.6)

Furthermore by explicit calculation we find for the contribution of the gauge fixing terms to \( \langle p'|\Delta V_{\alpha\bar{\alpha}}|p\rangle \) in (4.1) as given by (3.46)

\[ \langle p'|\Delta V_{\alpha\bar{\alpha}}|gf|p\rangle \equiv -\frac{1}{16\alpha}\langle p'|\frac{\delta}{\delta H_{\alpha\bar{\alpha}}(z)} \left( \int d^6z' \sigma(z') \left( (\mathcal{L}^g_{gf} - \mathcal{L}^g_{gf}) - \frac{1}{2}\beta(\mathcal{L}^g_{gf} + \mathcal{L}^g_{gf}) \right) + c.c. \right) |p\rangle \]

\[ = -(1 - \frac{1}{2}\beta)\langle p'| (T_{\alpha\bar{\alpha}} + c.c.) |p\rangle, \]  

(4.7)

with \( T_{\alpha\bar{\alpha}} \) as in (4.3). Therefore the contributions of (4.3) and (4.4) to (4.1) cancel each other,

\[ \langle p'|\Delta V_{\alpha\bar{\alpha}}|gf|p\rangle + \langle \delta p'|V^g_{\alpha\bar{\alpha}}|p\rangle + \langle p'|V^g_{\alpha\bar{\alpha}}|\delta p\rangle = 0. \]  

(4.8)

Thus the conformal transformation of the gauge fixing contribution to the supercurrent is compensated by the conformal transformation of the physical states. Furthermore we show that the remaining contributions to (3.46) vanish between physical states. Using the results of [7] we have

\[ \langle p'|\delta V^g_{\alpha\bar{\alpha}}|p\rangle \equiv \langle p'| \left( (\Lambda - \frac{3}{2}(\sigma + \bar{\sigma})) V^g_{\alpha\bar{\alpha}} \right) |p\rangle = 0 \]  

(4.9)

for the contribution of the gauge fixing term to the classical superconformal transformation of the supercurrent. Furthermore the gauge field contact terms in (3.46) may be shown to be gauge invariant by making use of the Zimmermann identity (3.13) once more. Since for superconformal transformations \( \partial_\sigma = \partial_\bar{\sigma} \), we have for the last term in (3.46)

\[ (D_\alpha\sigma\bar{D}_{\bar{\alpha}}\Phi - \bar{D}_{\bar{\alpha}}\bar{\sigma}D_\alpha\Phi) \frac{\delta \Gamma}{\delta \Phi} = \bar{D}_{\bar{\alpha}} \left( D_\alpha\sigma\Phi \frac{\delta \Gamma}{\delta \Phi} \right) - D_\alpha \left( \bar{D}_{\bar{\alpha}}\bar{\sigma}\Phi \frac{\delta \Gamma}{\delta \Phi} \right). \]  

(4.10)
The gauge fixing terms contributing to this expression are given by

\[
\Phi \frac{\delta \Gamma}{\delta \Phi} \bigg|_{gf} = -\frac{1}{128\alpha} \left[ \Phi D^2 \bar{D}^2 \Phi + \Phi \bar{D}^2 D^2 \Phi \right]_2^2 \cdot \Gamma. \tag{4.11}
\]

Without chiral projection, the Zimmermann identity (3.13) reads

\[
\left[ \Phi D^2 \bar{D}^2 \Phi + \Phi \bar{D}^2 D^2 \Phi \right]_2^2 \cdot \Gamma = \left[ \Phi D^2 \bar{D}^2 \Phi + \Phi \bar{D}^2 D^2 \Phi \right]_0^0 \cdot \Gamma \quad \tag{4.12}
\]

\[
[\Gamma_{g}] = [\Gamma_{g}] \quad \text{on flat space. Thus the gauge fixing terms contributing to (4.11) are not subtracted and therefore vanish between physical states by virtue of (4.2). The remaining terms in (4.12) are gauge invariant. - This completes our proof that (4.1) with } \Delta = 0 \text{ is gauge invariant for physical states.}
\]

Similarly we may show that the local CS equation (3.42) is gauge invariant when sandwiched between physical states. For this we generalise (4.12) to curved superspace. The operators \( D^2 \Phi, \bar{D}^2 \Phi \) are solutions of the flat space chiral equations of motion

\[
\frac{1}{16} \bar{D}^2 D^2 A - m^2 A = 0, \quad \frac{1}{16} D^2 \bar{D}^2 \bar{A} - m^2 \bar{A} = 0, \tag{4.13}
\]

with \( A \equiv \bar{D}^2 \Phi, \bar{A} \equiv D^2 \Phi \), such that \( D^2 \Phi, \bar{D}^2 \Phi \) are free fields. The curved space generalisation of (4.13) is obtained by varying the action

\[
\Gamma_2 = -\frac{1}{128\alpha} \int d^8z \, E^{-1} \Phi (\bar{D}^2 + R)(D^2 + \bar{R})\Phi + \frac{1}{16} M^2 \int d^8z \, E^{-1} \Phi^2 \\
- \frac{1}{64\alpha} \int d^8z \, E^{-1} (\bar{R} \Phi (\bar{D}^2 + R)\Phi + \bar{R} \Phi (D^2 + \bar{R})\Phi) \tag{4.14}
\]

given in the chiral representation. This action consists of the gauge fixing and mass terms as well as of an additional term coupling the gauge field to the supergravity background. For

\[
A \equiv (\bar{D}^2 + R)\Phi, \quad \bar{A} \equiv (D^2 + \bar{R})\Phi \tag{4.15}
\]

we obtain

\[
\frac{1}{16}(\bar{D}^2 + R) ((\bar{D}^2 + R)\bar{A} + (\bar{D}^2 + R)A) \\
+ \frac{1}{16}(D^2 + \bar{R})(RA + \bar{R}A) - m^2 A = 0, \\
\frac{1}{16}(D^2 + \bar{R}) ((D^2 + \bar{R})A + (D^2 + \bar{R})\bar{A}) \\
+ \frac{1}{16}(\bar{D}^2 + \bar{R})(RA + \bar{R}A) - m^2 \bar{A} = 0, \tag{4.16}
\]

with \( m = \sqrt{\alpha} M \), which is the equation of motion for chiral fields with minimal coupling to supergravity, \( \xi = \frac{1}{4} \) for \( \xi \) as in (3.3). Thus (4.15) is a solution of the free chiral equation of motion on curved superspace, such that the physical Hilbert space is given by

\[
\langle \Phi^- | \langle \bar{D}^2 + R ) \Phi | p \rangle = \langle \Phi^- | \langle D^2 + \bar{R} ) \Phi | p \rangle = 0, \tag{4.17}
\]

\[
\langle p'| \langle \bar{D}^2 + R ) \Phi | p \rangle = \langle p'| \langle D^2 + \bar{R} ) \Phi | p \rangle = 0,
\]

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in analogy to (4.2). Using this it is immediately obvious that the local CS equation (3.42) is gauge invariant between physical states. The last term in (4.14) has not been taken into account in the discussion of the local CS equation in section 3.4. However using (4.17) these terms may be easily seen to vanish between physical states.

5 Conclusion

From (3.46) it is straightforward to obtain the superconformal transformation properties for connected Green functions with an insertion of the supercurrent. We find

\[ \delta \langle V_{a\dot{a}}(z) X \rangle = \int d^6 z' \sigma(z') \langle \beta g \partial_g \mathcal{L}_{\text{eff}}(z') \cdot V_{a\dot{a}}(z) X \rangle + \text{c.c.} + \text{soft terms}, \]

where

\[ X = A_\pm(z_1) \ldots A_\pm(z_n) \bar{A}_\pm(z_{n+1}) \ldots \bar{A}_\pm(z_m). \]

Since the connected Green functions are defined according to the Gell-Mann-Low formula, the discussion of section 4 applies and (5.1) is gauge invariant. The superconformal transformation \( \delta \) is defined by

\[ \delta \langle V_{a\dot{a}}(z) X \rangle = \langle \delta V_{a\dot{a}}(z) X \rangle \]

\[ + \sum_{k=1}^{n} \langle V_{a\dot{a}}(z) A_\pm(z_1) \ldots \delta A(z_k) \ldots A_\pm(z_n) \bar{A}_\pm(z'_1) \ldots \bar{A}_\pm(z'_m) \rangle \]

\[ + \sum_{k=1}^{m} \langle V_{a\dot{a}}(z) A_\pm(z_1) \ldots \delta \bar{A}_\pm(z'_1) \ldots \bar{A}_\pm(z'_m) \rangle, \]

with \( \delta V_{a\dot{a}}(z) \) the classical conformal transformation given by (3.48), and

\[ \delta A_\pm = (\Lambda - (1 + \gamma_A) \sigma) A_\pm, \quad \delta \bar{A}_\pm = (\bar{\Lambda} - (1 + \gamma_{\bar{A}}) \bar{\sigma}) \bar{A}_\pm. \]

We consider matter fields only in (5.2) for simplicity. For gauge fields \( \hat{\Phi} \) there are additional terms arising from the last term in (3.46).

In this paper we have determined a suitable field structure for discussing both conformal transformations and gauge invariance, from which we have obtained Ward operators which ensure conformal covariance of the supercurrent. Our result (5.1) states that there is a gauge invariant expression for the superconformal transformation properties of Green functions with a supercurrent insertion which is well-defined to all orders in perturbation theory, including the soft mass breaking terms. Discarding these we see that \( \langle V_{a\dot{a}} \cdot X \rangle \) is conformally invariant when the \( \beta \) function vanishes (which for SQED considered here applies just to the free theory). For superconformal theories as relevant to RG fixed
points, correlation functions involving the supercurrent have been constructed explicitly in [14] using the symmetry constraints.

So far we have considered one insertion of the supercurrent only. Ward identities involving Green functions with multiple insertions of the supercurrent, \( \langle V\alpha\bar{\alpha}(z_1) \cdot V\beta\bar{\beta}(z_2) \cdot \cdots \cdot V\omega\bar{\omega}(z_n) \cdot X \rangle \), may in principle be obtained by varying (3.47) an appropriate number of times with respect to the supergravity prepotential \( H \). However in this case the proof of gauge invariance in analogy to the calculation performed in section 4 gets very tedious since higher orders in \( H \) have to be considered explicitly. In this case a BRS approach seems more appropriate, which applies of course also to a possible generalisation to non-abelian gauge theories.

As a final remark we discuss the consequences of \( R \) invariance. We write the \( R \) symmetry operator defined in (3.18) in the form
\[
W^R \Gamma \equiv i \int d^4x \ w\Gamma.
\]
(5.5)

For the subsequent discussion, it is important to remember that \( w \) contains both diffeomorphisms and Weyl transformations. \( R \) invariance implies
\[
W^R \Gamma = 0 \Rightarrow w\Gamma = \text{total divergence}.
\]
(5.6)

Calculating \( w\Gamma \) as given by (5.5) we find, when restricting to flat space,
\[
\langle \partial^a V_a \cdot X \rangle = i \langle (D^2 S - \bar{D}^2 \bar{S}) \cdot X \rangle + \sum_j f_R(z - z_j) \langle X' \rangle,
\]
(5.7)

where \( S \) is the Weyl symmetry breaking term given by (3.9). Evaluating this term with the help of the Zimmermann identities (3.11), (3.12) we find
\[
i \langle (D^2 S - \bar{D}^2 \bar{S}) \cdot X \rangle = 8i \partial^a (2\partial_a + i(D\sigma \bar{D})_a) \langle B \cdot X \rangle,
\]
(5.8)

\[
B = B_5 + B_\Phi,
\]
(5.9)

\[
B_5 = -\frac{1}{24} (4u_g + v_g) \left( \bar{A}_+ e^{g\Phi} A_+ + \bar{A}_- e^{-g\Phi} A_- \right) - \frac{1}{24} (2u_\xi + 2u'_\xi + v_\xi) (\bar{A}_+ A_- + A_+ A_-) - \frac{1}{24} \bar{M}^2 \Phi^2,
\]
(5.10)

\[
B_\Phi = -\frac{1}{6} u_\Phi \left( D^a \bar{\Phi} \bar{D}^2 D_\bar{\alpha} \bar{\Phi} + \bar{D}_a \bar{\Phi} D^2 \bar{D}_\bar{\alpha} \bar{\Phi} + \bar{\Phi} D^a \bar{D}^2 D_\bar{\alpha} \bar{\Phi} \right).
\]
(5.11)

\( V_a \) is the supercurrent of SQED as given by (3.1) with the action (3.5). \( B_\Phi \) arises from the gauge field term \( D^2 F^a F_a - \bar{D}^2 \bar{F}_a \bar{F}_\bar{a} \). The Zimmermann coefficients \( u_g, u_\xi, u'_\xi, u_\Phi, v_g, v_\xi \) are related to the \( \beta \) and \( \gamma \) functions by virtue of (3.24), (3.25) and (3.30). The terms involving \( f^R(z - z_j) \) in (5.7) are contact terms arising from varying \( w\Gamma \) as given by (5.5) with respect to the matter fields in order to obtain \( X \) as in (5.2). It is a specific feature

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\( ^6 \)For a complementary approach using the component formalism see [15].
of the Weyl anomalies present in SQED with massless matter fields that they may be written as total divergences in the $R$ symmetry Ward identity. This is due to the fact that they are of the form $S = \bar{D}^2 B$.

As follows from the discussion of [7], the supercurrent $V_a$ in (5.7) as given by (3.1) has the component decomposition

$$V_a(z) = R_a(x) - i\theta^a(Q_a\alpha(x) - \frac{1}{3}(\sigma_a\sigma_b)\beta^a Q^b\beta) + i\bar{\theta}_{\dot{a}}(\bar{Q}_{\dot{a}}\hat{\alpha}(x) - \frac{1}{3}(\sigma_a\sigma_b)\hat{\alpha}_\beta\bar{Q}^b\beta)$$

$$- 4(\theta\sigma\bar{\theta})^a_{\dot{a}}(T_{ab}(x) - \frac{1}{3}\eta_{ab}T_{c}^c(x) + \frac{1}{4}\varepsilon_{abcd}\partial^d R^c) + \cdots ,$$

with the $R$ current, the supersymmetry currents $Q_{\alpha a}$, $\bar{Q}_{\dot{a}}\hat{\alpha}$ and the energy momentum tensor $T_{ab}$. Furthermore we may define a current

$$V^B_a = V_{\alpha a} + 4[D_{\alpha}, \bar{D}_{\dot{a}}]B ,$$

with $B$ as in (5.9), whose divergence is anomaly-free,

$$\langle \partial^a V^B_a \cdot X \rangle = \sum_j f^R(z - z_j)(X') .$$

This is referred to as changing from so-called ‘S’ to ‘B’ breaking in [7] (see also [2]). The current (5.13) has the component decomposition

$$V^B_a(z) = R_a(x) + \theta^a Q_{\alpha a}(x) + \bar{\theta}_{\dot{a}}\bar{Q}_{\dot{a}}\hat{\alpha}(x) + (\theta\sigma\bar{\theta})^a_{\dot{a}}T_{ab}(x) + \cdots .$$

We note that $V^B_a$ is not gauge invariant since $B_\phi$ is not gauge invariant.

Finally we discuss a current whose divergence agrees with the chiral anomaly. The current

$$V'_{\alpha a} = V_{\alpha a} + K_{\alpha a},$$

$$K_{\alpha a} = 4[D_{\alpha}, \bar{D}_{\dot{a}}]B_5$$

$$= -\frac{1}{3}[D_{\alpha}, \bar{D}_{\dot{a}}]\left((2u_g + \frac{1}{2}v_g)(\bar{A}_+ e^{\phi} A_+ + \bar{A}_- e^{-\phi} A_-)
+ (u_\xi + u'_\xi + \frac{1}{2}v_\xi)(A_+ A_- + \bar{A}_+ \bar{A}_-) + \frac{1}{2}M^2 \tilde{\Phi}^2 \right) ,$$

which coincides with $V^B_a - 4[D_{\alpha}, \bar{D}_{\dot{a}}]B_\phi$, satisfies the Ward identity

$$\langle \partial^a V'_a \cdot X \rangle = -\frac{1}{6} i u_\phi \langle (D^2 F_\alpha - \bar{D}^2 \bar{F}_{\dot{a}}\hat{F})^a \cdot X \rangle$$

$$+ \sum_j f^R(z - z_j)(X') .$$

$V'_a$ is gauge invariant except for the soft mass term. Its significance arises from the fact that the coefficient $u_\phi$ of its anomaly has no corrections beyond one loop [13]. This same coefficient appears in the anomaly of the so-called Konishi current [17], which is
the current associated with $R$ symmetry on flat space without coupling to supergravity, where the chiral local $R$ symmetry Ward operator is given by

$$w_5 \equiv A_+ \frac{\delta}{\delta A_+} + A_- \frac{\delta}{\delta A_-}. \quad (5.19)$$

Therefore the current $V'_a$ as given by (5.16) satisfies an Adler-Bardeen theorem in the sense that its gauge anomaly has no quantum corrections beyond one loop. It has to be noted however that the higher $\theta$ components of $V'_a$ neither correspond to the supersymmetry currents nor to the energy-momentum tensor.

(5.18) shows that although the chiral anomaly and the superconformal trace anomaly as expressed by the divergence of the supercurrent are related, the trace anomaly and thus the $\beta$ functions may acquire higher order corrections while the chiral anomaly is one-loop. This issue is discussed from a different point of view in [18]. The discussion here shows how the geometrical structure of the supergravity background relates the two anomalies. Effectively the Konishi anomaly appears as a term breaking Weyl invariance, whereas the supercurrent is present in the diffeomorphism Ward identity since it is coupled to the prepotential $H^{\beta\beta}$. Thus in the superconformal Ward identity which relates Weyl symmetry and diffeomorphisms both currents are present.
A Appendix

A.1 Infinitesimal Transformations

Here we list the infinitesimal symmetry transformations for the symmetries discussed in section 2. For the supergravity prepotential $H^{\alpha\dot{\alpha}}$ and the gauge field $\hat{\Phi}$, these are power series in $H$.

Diffeomorphisms
\begin{align}
\delta_\Omega A_\pm &= \frac{1}{4} \bar{D}^2 (\Omega^\alpha D_\alpha A_\pm) \\
\delta_\Omega H^{\alpha\dot{\alpha}} &= \frac{1}{2} D^\alpha \Omega^\alpha \\
&\quad + \frac{1}{4} \bar{D}^\beta \Omega^\beta \{D_\beta, \bar{D}_\dot{\beta}\} H^{\alpha\dot{\alpha}} - \frac{1}{4} H^{\beta\dot{\beta}} \{D_\beta, \bar{D}_\dot{\beta}\} \bar{D}^\alpha \Omega^\alpha + \frac{1}{4} \bar{D}^2 \Omega^\beta D_\beta H^{\alpha\dot{\alpha}} \\
&\quad + O(H^2) \\
\delta_\Lambda J &= \frac{1}{4} D^2 (\Omega^\alpha D_\alpha J) + \frac{1}{12} \bar{D}^2 D^\alpha \Omega^\alpha \\
\delta_\Lambda \hat{\Phi} &= \delta_\Lambda^{(0)} \hat{\Phi} + \delta_\Lambda^{(1)} \hat{\Phi} + O(H^2) \\
\delta_\Lambda^{(0)} \hat{\Phi} &= \frac{1}{2} D^\alpha \Omega^\alpha D_\alpha \hat{\Phi} + \frac{1}{4} \bar{D}^2 \Omega^\alpha D_\alpha \hat{\Phi} + c.c. \\
\delta_\Lambda^{(1)} \hat{\Phi} &= \frac{1}{8} H^{\alpha\dot{\alpha}} \bar{D}^\beta \Omega^\beta \left( [D_\alpha, D_\dot{\alpha}] [D_\beta, \bar{D}_\dot{\beta}] - \{D_\alpha, D_\dot{\alpha}\} \{D_\beta, \bar{D}_\dot{\beta}\} \right) \hat{\Phi} \\
&\quad - \frac{1}{8} H^{\alpha\dot{\alpha}} \bar{D}^2 \Omega^\alpha D^2 \hat{\Phi} + \frac{1}{4} H^{\alpha\dot{\alpha}} D_\alpha \bar{D}_\dot{\alpha} \bar{D}^2 D_\beta \hat{\Phi} + c.c. \\
\end{align}

Weyl Transformations
\begin{align}
\delta_\sigma A_\pm &= -\sigma A_\pm \\
\delta_\sigma H^{\alpha\dot{\alpha}} &= 0 \\
\delta_\sigma J &= \sigma \\
\delta_\sigma \hat{\Phi} &= \delta_\sigma^{(0)} \hat{\Phi} + \delta_\sigma^{(1)} \hat{\Phi} + O(H^2) \\
\delta_\sigma^{(0)} \hat{\Phi} &= (\sigma + \bar{\sigma}) \hat{\Phi} \\
\delta_\sigma^{(1)} \hat{\Phi} &= -\frac{1}{2} H^{\alpha\dot{\alpha}} \{D_\alpha, \bar{D}_\dot{\alpha}\} (\sigma - \bar{\sigma}) \hat{\Phi} + \frac{1}{2} (\sigma + \bar{\sigma}) H^{\alpha\dot{\alpha}} [D_\alpha, \bar{D}_\dot{\alpha}] \hat{\Phi} \\
&\quad - \frac{1}{2} H^{\alpha\dot{\alpha}} [D_\alpha, \bar{D}_\dot{\alpha}] \left( (\sigma + \bar{\sigma}) \hat{\Phi} \right) \\
\end{align}

Gauge Transformations
\begin{align}
\delta_\lambda \hat{\Phi} &= i(\lambda - \bar{\lambda}) \\
\delta_\lambda A_\pm &= \mp ig A_\pm \\
\end{align}
A.2 Local Ward operators

Here we list the local Ward operators corresponding to the infinitesimal transformations of [A.1].

Diffeomorphisms

\[ w_{\alpha}^{(A)}(A_\pm) = \frac{1}{4} D_\alpha A_\pm \frac{\delta}{\delta A_\pm}, \] (A.2.13)

\[ w_{\alpha}^{(A)}(J) = \frac{1}{4} D_\alpha J \frac{\delta}{\delta J} - \frac{1}{12} D_\alpha \frac{\delta}{\delta J}. \] (A.2.14)

\[ w_{\alpha}^{(A)}(H) = w_{\alpha}^{(0)}(H) + w_{\alpha}^{(1)}(H) + w_{\alpha}^{(2)}(H) + O(H^3), \] (A.2.15)

\[ w_{\alpha}^{(0)}(H) = \frac{1}{2} \bar{D}^{\alpha} \frac{\delta}{\delta H^{\alpha\bar{\alpha}}}, \] (A.2.16)

\[ w_{\alpha}^{(1)}(H) = \frac{1}{4} \bar{D}^{\alpha} \left( \{ D_\alpha, \bar{D}_\alpha \} H^{\beta\bar{\beta}} \frac{\delta}{\delta H^{\beta\bar{\beta}}} \right) + \frac{1}{4} \{ D_\beta, \bar{D}_\bar{\beta} \} \bar{D}^{\alpha} \left( H^{\beta\bar{\beta}} \frac{\delta}{\delta H^{\alpha\bar{\alpha}}} \right) \]
\[ + \frac{1}{4} \bar{D}^2 \left( D_\alpha H^{\beta\bar{\beta}} \frac{\delta}{\delta H^{\beta\bar{\beta}}} \right), \] (A.2.17)

\[ w_{\alpha}^{(A)}(\hat{\Phi}) = w_{\alpha}^{(A)}(\hat{\Phi})^{(0)} + w_{\alpha}^{(A)}(\hat{\Phi})^{(1)} + O(H^2) \] (A.2.18)

\[ w_{\alpha}^{(A)}(\hat{\Phi})^{(0)} = \frac{1}{2} \bar{D}^{\alpha} \left( \bar{D}_{\alpha} D_\alpha \hat{\Phi} \frac{\delta}{\delta \hat{\Phi}} \right) + \frac{1}{4} \bar{D}^2 \left( D_\alpha \hat{\Phi} \frac{\delta}{\delta \hat{\Phi}} \right), \] (A.2.19)

\[ w_{\alpha}^{(A)}(\hat{\Phi})^{(1)} = \frac{1}{8} D^{\alpha} \left( H^{\beta\bar{\beta}} \left[ D_\beta, D_{\bar{\beta}} \right] [D_\alpha, D_{\alpha}] \hat{\Phi} \frac{\delta}{\delta \hat{\Phi}} - H^{\beta\bar{\beta}} \left\{ D_\beta, D_{\bar{\beta}} \right\} [D_\alpha, D_{\alpha}] \hat{\Phi} \frac{\delta}{\delta \hat{\Phi}} \right) \]
\[ - \frac{1}{8} \bar{D}^2 \left( H_{\alpha\bar{\alpha}} D^2 \bar{D}^{\alpha} \hat{\Phi} \frac{\delta}{\delta \hat{\Phi}} \right) - \frac{1}{4} \bar{D}_\beta D_\beta \left( H^{\beta\bar{\beta}} \bar{D}^2 D_{\alpha} \hat{\Phi} \frac{\delta}{\delta \hat{\Phi}} \right). \] (A.2.20)
Weyl Transformations

\[ w^{(\sigma)}(A_\pm) = -A_\pm \frac{\delta}{\delta A_\pm} \quad \text{(A.2.21)} \]

\[ w^{(\sigma)}(J) = \frac{\delta}{\delta J} \quad \text{(A.2.22)} \]

\[ w^{(\sigma)}(H) = 0 \quad \text{(A.2.23)} \]

\[ w^{(\sigma)}(\Phi) = w^{(\sigma)}(\Phi)^{(0)} + w^{(\sigma)}(\Phi)^{(1)} + O(H^2) \quad \text{(A.2.24)} \]

\[ w^{(\sigma)}(\Phi)^{(0)} = \bar{D}^2 \left( \frac{\delta}{\delta \Phi} \right) \quad \text{(A.2.25)} \]

\[ w^{(\sigma)}(\Phi)^{(1)} = \frac{1}{2} \bar{D}^2 \{ D_\alpha, \bar{D}_{\dot{\alpha}} \} \left( H^{\alpha\dot{\alpha}} \frac{\delta}{\delta \Phi} \right) + \frac{1}{2} \bar{D}^2 \left( H^{\alpha\dot{\alpha}} [ D_\alpha, \bar{D}_{\dot{\alpha}} ] \frac{\delta}{\delta \Phi} \right) \]

\[ - \frac{1}{2} \bar{D}^2 \left( \bar{D}_\alpha \bar{D}_{\dot{\alpha}} \right) \left( H^{\alpha\dot{\alpha}} \frac{\delta}{\delta \Phi} \right) \quad \text{(A.2.26)} \]

Gauge Transformations

\[ w^{(\lambda)}(A) = i g \left( A_- \frac{\delta}{\delta A_-} - A_+ \frac{\delta}{\delta A_+} \right) \quad \text{(A.2.27)} \]

\[ w^{(\lambda)}(J) = 0 \quad \text{(A.2.28)} \]

\[ w^{(\lambda)}(H) = 0 \quad \text{(A.2.29)} \]

\[ w^{(\lambda)}(\Phi) = i \bar{D}^2 \frac{\delta}{\delta \Phi} \quad \text{(A.2.30)} \]
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