Evaluation of sound velocity inside underwater acoustic materials using test panel acoustic measurements

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Abstract. Acoustic materials can be used as external coating to reduce radiated noise or target strength of immersed structures, or to improve performance of sonar systems. In order to optimise the design and the use of such coatings, it is necessary to have a good estimate of their intrinsic properties, in particular the complex sound velocity inside the materials. The type of material studied here consists in a viscoelastic slab containing a given proportion of microvoids. Determination of the complex longitudinal velocity is not an easy task, and direct methods or experimental set-ups do not give sufficient results. The method presented here consists in determining the complex longitudinal velocity by solving an inverse problem, from acoustic measurements of the reflection and transmission coefficients of a test panel in a water tank. Best results, including attenuation, are obtained by over-determining the problem, using all data available. Results are consistent to the known physical behaviour of such materials, regarding influence of frequency and hydrostatic pressure.

1. Introduction
In the scope of underwater acoustics, some problems of interest are the reduction of radiated noise or target strength of immersed structures, or to improve sonar systems performances. Regarding design or choice of relevant materials, an approximate characterisation of the coatings can be done on planar structures, as shown on figure 1a and 1b.

![Figure 1a. Acoustic decoupling.](image1.png)

![Figure 1b. Anechoism.](image2.png)
1.1. Type of material considered
On an engineering point of view, a possible solution is to realise composite materials consisting of a viscoelastic slab containing a given proportion of air in the form of micro-cavities or micro-inclusions. If $\rho_1$, $K_1$, $G_1$ are respectively the density, dynamic bulk and shear moduli of the matrix, $c$ the concentration of cavities, the composite material can be represented by an equivalent material with dynamic characteristics $\rho_{eq}$, $K_{eq}$, $G_{eq}$. As far as the cavities size is very small by comparison to the wavelength, these equivalent characteristics can be obtained using quasi-static approaches, such as Kuster-Toksöz method [1]. Under these conditions:
- density can be considered to be identical to static (physical density);
- resonance of cavities are not taken into account;
- frequency dependence of $K_{eq}$ and $G_{eq}$ appears through frequency dependence of $K_1$ and $G_1$.

![Figure 2. Representation of a composite material by an equivalent homogeneous material.](image)

1.2. Characterisation
Two main criteria are introduced: the decoupling coefficient $C_M$ and the anechoism coefficient $C_A$. It can be shown that these performance indicators are directly related to material density $\rho_{eq}$, material complex longitudinal sound velocity $c_L$, and coating thickness $h$ [2]. Once $K_{eq}$, $G_{eq}$ are known, the material is fully characterised by the density and the equivalent longitudinal and transversal sound velocities in the frequency range of interest, given by:

$$c_L = \left( \left( K_{eq} + \frac{4}{3}G_{eq} \right) \rho_{eq}^{-1} \right)^{1/2} \text{ and } c_T = \left( G_{eq} \rho_{eq}^{-1} \right)^{1/2}$$

Due to the attenuation, sound velocities are complex-valued, and can be represented, as follows, using a real part $c'_L$ and a tangent loss $\eta_L$.

$$c_L = c'_L(1 + i \eta_L)$$

That notation is used for simplicity, as propagation is best physically represented by using the complex-valued wavenumber given by $k_L = \omega c_L$.

Direct determination of $G_{eq}$ is possible using special devices such as a visco-analyser. On the other hand, direct determination of $K_{eq}$ or $c_L$ is difficult, all these methods having limitations [3], or cannot be used to evaluate the attenuation. The method proposed in this paper consists in solving an inverse problem, using acoustic measurement of the reflection and transmission coefficients of a test panel submitted to a plane wave in a water tank. Other authors have also developed similar approaches, some of them in the context of the study of acoustic metamaterials, for example [4]. However, [4]
gives explicit formulae of equivalent densities and sound velocities, that could be very sensitive to measurement errors.

The purpose of the present paper is to improve the method developed in [5], and to show some practical results. Reduction of measurement errors is obtained thanks to an over-determination of the unknown quantities, using the more data possible.

2. Presentation of the method

2.1. Test panel measurements in a water tank

As a first step, the method requires measurements of the reflection R and transmission coefficient T of a test panel in a water tank, in the frequency range of interest as shown on figure 3. It is assumed that the method can provide the phases of R and T, as well as the amplitude.

It is important to have at one’s disposal good quality data. Some problems occur with this kind of measurement, in particular diffraction effects from the edges of the test panel. To reduce these phenomena, different methods are available, for example [6] and [7].

2.2. Evaluation of the complex longitudinal sound speed using test panel data

In the present case, i.e. simple layer materials in the quasi-static regime, the problem to be solved at a given frequency consists in the determination of two scalar values, which are \( c'_L \) and \( \eta_L \), because it assumed that density \( \rho_{eq} \) has been determined using other means. Indeed, the complex reflection and transmission coefficients depend in that case only on \( c_L \), through the following expressions:

\[
T = \frac{2}{2 \cos(k_L h) + i \left( \frac{Z_{eq}}{Z_0} + \frac{Z_0}{Z_{eq}} \right) \sin(k_L h)}, \quad R = \frac{i \left( \frac{Z_{eq}}{Z_0} - \frac{Z_0}{Z_{eq}} \right) \sin(k_L h)}{2 \cos(k_L h) + i \left( \frac{Z_{eq}}{Z_0} + \frac{Z_0}{Z_{eq}} \right) \sin(k_L h)}, \tag{3}
\]

where \( Z_{eq} = \rho_{eq} c_L \) and \( Z_0 = \rho_0 c_0 \) are acoustic impedances of the material and the surrounding fluid medium (water), respectively.
The unknown values of \( c'_L \) and \( \eta_L \) are obtained by minimising numerically, in a two-dimensional space, the error between calculated and measured values of \( R \) and/or \( T \). Different minimisation criteria have been considered:

- N°1 : \(|R_{\text{calc}}| - |R_{\text{meas}}|\)
- N°2 : \(|T_{\text{calc}}| - |T_{\text{meas}}|\)
- N°3 : \(|R_{\text{calc}}| - |R_{\text{meas}}| + |T_{\text{calc}}| - |T_{\text{meas}}|\)
- N°4 : \(R_{\text{calc}} - R_{\text{meas}}\)
- N°5 : \(T_{\text{calc}} - T_{\text{meas}}\)
- N°6 : \(R_{\text{calc}} - R_{\text{meas}} + T_{\text{calc}} - T_{\text{meas}}\)

In reference [5], minimising criterion N°5 has been used, which was giving acceptable results for some materials. With the present study, we recommend criterion N°6, because it is the one that uses the more information available, i.e. both amplitude and phase of reflection and transmission coefficients. As we have four scalar data with respect to two scalar unknowns, a more robust solution is obtained, as the problem is mathematically over-determined. That result is illustrated by the following example obtained on a test anechoic sample, 50 mm thick, at 10 kHz frequency. Figures 4 to 9 give two-dimensional plots of the errors between calculus and measurement, along \( c'_L \) and \( \eta_L \), for criteria N°1 to N°6, respectively. Scale data are not shown for confidentiality reasons.

![Figure 4](image)

Figure 4. Test anechoic panel at 10 kHz – Two-dimensional plot of minimising criterion N°1.

Figure 4 shows different areas with a dark blue colour, which means that many different sets of \((c'_L, \eta_L)\) values can correspond to a minimum. This is not surprising, because here we have used a criterion base on one scalar data only, as we look for two unknown scalar values. The problem is under-determined, leading to an unacceptable result. The same problem arises for criterion N°2, shown on figure 5.
Criterion N°3, shown on figure 6, gives much better results, because it combines the two previous ones. We have here two input data for two scalar values to determine. As the ill-determination using criteria N°1 and N°2 do not coincide, we obtain here a reasonable result, circled in white. However, some other values, circled in grey, could correspond to the solution.
When both amplitude and phase of panel coefficients are used, we obtain the results shown on figure 7 and 8. As previously, we can obtain a relevant estimate of \((c', \eta_L)\), but there is a doubt between different solutions.

![Figure 7. Test anechoic panel at 10 kHz – Two-dimensional plot of minimising criterion N°4.](image)

![Figure 8. Test anechoic panel at 10 kHz – Two-dimensional plot of minimising criterion N°5.](image)
Figure 9. Test anechoic panel at 10 kHz – Two-dimensional plot of minimising criterion N°6.

Criterion N°6 on figure 9 gives the best result. There are still two possible solutions:
- one for a relatively high sound velocity and a high damping coefficient,
- an other one with lower velocity and damping.

The second solution can be discarded for different reasons:
- the minimum is weaker,
- the test material being anechoic, a strong damping coefficient is expected.

More analysis, based on numerical prediction of sound velocity of the composite, could be made to confirm the result.

3. Influence of frequency and hydrostatic pressure
This section presents more practical results on a 50 mm thick test anechoic material, in the frequency range 2 kHz – 20 kHz, and at different increasing hydrostatic pressures (P0 to P4).

3.1. Influence of frequency
Data at pressure P3 is considered here. Input information includes:
- density $\rho_{eq}$ and thickness $h$ at pressure P3,
- reflection and transmission coefficients versus frequency (amplitude and phase), shown on figures 10 and 11.
Figure 10. Test anechoic panel at pressure P3 – Level of transmission and reflection coefficients.

Figure 11. Test anechoic panel at pressure P3 – Phase of transmission and reflection coefficients.

The levels in dB on figure 10 are defined as $20 \log_{10} |T|$ and $20 \log_{10} |R|$. Scale is not shown for confidentiality reasons, the upper value in the scale being 0 dB, i.e. unit amplitude in natural value (corresponding to total transmission or total reflection). Not surprisingly, because of the relatively small thickness of the test panel compared to wavelength, and the panel being immersed with water on both sides, significant transmission is observed. Note also the material behaves like a partially soft reflector around 2.5 kHz, the phase of $R$ being close to 180°.

Applying the process discussed in section 2 for each available frequency, we obtain the evolution of complex longitudinal velocity along frequency. Results are shown on figures 12 and 13. The following remarks can be made:

- results are consistent because sound velocity increases along frequency, as expected for this kind of material,
- some spurious points appear, due to error measurements on $R$ and $T$,
- results diverge below a certain frequency, related to panel thickness,
- symmetrically, some attempts at higher frequencies did not give good results, the test panel being too opaque, acoustic measurement does not contain enough useful information.
For practical use, it is possible to fit raw data with smooth curves (dashed curves on the figures).

![Figure 12](image1.png)

**Figure 12. Test anechoic panel at pressure P3 – Estimate of sound velocity (real part).**

![Figure 13](image2.png)

**Figure 13. Test anechoic panel at pressure P3 – Estimate of sound velocity (damping).**

3.2. **Influence of hydrostatic pressure**

Same analysis is done for hydrostatic pressures P0, P1, P2 and P4. Figures 14 and 15 show fitted curves of $c'_L$ and $\eta_L$ along frequency and their evolution with increasing hydrostatic pressure. When pressure increases, there is a variation in the actual concentration of voids in the material related to the collapse of the micro-balloons. However, results show an “abnormal” behaviour for the lowest pressures P0 and P1, as we could expect a lower sound velocity and a higher damping coefficient, the void concentration being larger. As a matter of facts, that behaviour is characteristic of materials made with a viscoelastic slab containing micro-balloons, as presented in reference [8].
4. Conclusion
A method to evaluate both real part and damping of the longitudinal velocity of materials used in underwater acoustics has been presented, from test panel measurements in a water tank. Input data are material density, panel thickness and reflection and transmission coefficients of the test panel (amplitude and phase). As the problem is here mathematically over-determined, the method is less sensitive to error measurements than previous ones. It should be noted that the phase information in the data is important. Practical results have been obtained on a test anechoic panel. Dependency of longitudinal velocity along frequency and hydrostatic pressure is fully consistent with the known properties of such materials. The method suffers some frequency limitations both at low and high frequencies. However, the dimensions and thickness of the sample could be adapted to the user’s frequency range of interest.
5. References

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