We investigate the implications of $R$-Parity violation (RPV) for quark flavour violation both by constraining the sneutrino masses to be positive and by studying the processes $b \to s\gamma$ and $K^0 - \bar{K}^0$ mixing. In the latter there are two major contributions, one from “direct” one loop diagrams involving RPV couplings, and one from the “indirect” contributions generated by the renormalisation group. We compare the effects and discuss the implications of our results.
1 Introduction

One of the most promising candidates for physics beyond the so-called Standard Model (SM) is that of supersymmetry (SUSY) \[1\]. SUSY has the highly attractive properties of giving a natural explanation to the hierarchy problem of how it is possible to have a low energy theory containing light scalars (the Higgs) when the ultimate theory must include states with masses of order the Planck mass. This allows the ultimate hope of constructing a theory of gauge unification and fermion (and superpartner) masses defined at a scale near the Planck mass yet whose structure and parameters may be deduced from physics at accessible scales.

In this paper we shall be concerned with the implications of a particular possible feature of SUSY, namely that of \(R\)-Parity violation (RPV) \[2, 3, 4\]. \(R\)-Parity is a \(Z_2\) symmetry of both the SM and its minimal SUSY extension, the MSSM, under which all of the SM particles have charge 0, while all their SUSY partners have charge 1. Its implications include the stability of the lightest supersymmetric particle (LSP), and hence the typical SUSY collider signatures of missing \(E_T\) and the existence of a source of dark matter. Its violation changes both the implied cosmology and the expected collider signatures, allowing such effects as LSPs decaying inside the detector and leptoquarks \[3\]. In addition to these, further constraints on RPV can be derived by considering experimental limits on rare decays \[5, 6, 7\], and by the demands of proton stability. In practice it is usual to evade the problems of proton decay by considering either baryon number or lepton number violation but not both simultaneously.

\(R\)-Parity is violated by the superpotential and soft potential

\[
W = h_u^{ij} Q_i H_2 u_j + h_d^{ij} Q_i H_1 d_j + h_e^{ij} L_i H_1 e_j \\
+ \frac{1}{2} \lambda_{ijk} L_i L_j e_k + \lambda'_{ijk} L_i Q_j d_k + \frac{1}{2} \lambda''_{ijk} u_i d_j d_k \\
+ \mu_4 H_1 H_2 + \mu_5 L_i H_2 \\
V_{\text{soft}} = \eta_u^{ij} Q_i H_2 u_j + \eta_d^{ij} Q_i H_1 d_j + \eta_e^{ij} L_i H_1 e_j + \text{h.c.}
\]
\[ + \frac{1}{2} C'_{ijk} L_i L_j e_k + C''_{ijk} L_i Q_j d_k + \frac{1}{2} C''_{ijk} \bar{u}_i d_j d_k \text{ h.c.} \]
\[ + \frac{1}{2} M_0 \lambda_a \lambda_a + \sum_{a,b} m^2_{ab} \varphi_a \bar{\varphi}_b \]
\[ + D_i H_1 H_2 + D_i L_i H_2 + \text{h.c.} \]

From the point of view of deriving constraints on the $R$-Parity violating couplings in the model, the most extensively studied couplings are the dimensionless couplings $\lambda$, $\lambda'$, and $\lambda''$, which directly generate many effects which can be experimentally limited. The extra soft terms by definition mostly couple only heavy SUSY particles and hence are relevant mostly because of their impact on the Renormalisation Group Equations (RGEs), although they can have significant effects on the neutrino-neutralino and Higgs boson-sneutrino sectors [4, 9, 10].

In SUSY models, flavour-changing effects may be caused by the existence of off-diagonal terms in the sfermion mass matrices in the basis in which the fermion masses are diagonal, in which there are no tree level SM flavour-changing neutral currents (FCNC). Such flavour-violating soft masses can be generated either from the high energy theory such a GUT directly, or else through the RGEs by couplings which violate flavour symmetries, such as Yukawa couplings mediated by the CKM matrix and both possibilities have been studied extensively [1, 2]. However, the inclusion of the RPV couplings in the RGEs allows many extra violations of quark and lepton flavour.

In a previous analysis [10], we presented the renormalisation group equations (RGEs) for the couplings of the full $R$-Parity violating sector of the model, and investigated the implications of typical scenarios at the GUT scale for the generation of neutrino masses and the decay $\mu \to e\gamma$. Contributions can conveniently be split into two categories which we term “direct” (where the flavour violation occurs directly through $R$-Parity violating vertices in the diagrams) and “indirect” (where $R$-Parity violation induces flavour violation through the RGEs on the soft masses). We found
that the indirect effects were typically large, often orders of magnitude larger than
the direct ones. However, the extremely complicated dependence on the spectrum
and the existence of many possible cancellations in the amplitude render the process
of deriving bounds impossible except on an order of magnitude basis.

The intention of this paper is to extend the work of reference \[10\] to consider
quark flavour violation (QFV) effects (for recent work on the subject see ref. \[13\]).
Following this introduction, we give a short discussion on a new bound from requiring
the sneutrino masses to be above their experimental limits in section 2, while section
3 is devoted to the effects of RPV on $b \rightarrow s\gamma$, and section 4 to the $K^0$ – $\bar{K}^0$ mixing
term $\Delta m_K$. Many useful formulae for our RGEs and definitions are relegated to
the appendices, while section 5 contains our conclusions.

\section{Bounds from Sneutrino Masses}

We now comment on a new simple bound on certain of the Yukawa couplings, which
is associated with our assumptions of unification of couplings at a high scale. If we
add a new Yukawa coupling associated with the sneutrinos (here $\lambda'$), the mass of the
sneutrino is driven down by its effect on the RGEs, and so increasing the value of
the coupling will ultimately drive the mass of the sneutrino below the experimental
limit. The low energy values of the lepton masses and the corresponding soft masses
are given by

\begin{align}
    m_{L_i}^2 &= m_0^2 + 0.51 M_{1/2}^2 - \sum_{j,k} \lambda'^2_{ijk} (M_{GUT}) [13 m_0^2 + 49 M_{1/2}^2 - 1.5 M_{1/2} A_0 - 12 A_0^2] \\
    m_{\tilde{\nu}_i}^2 &= m_{L_i}^2 + \frac{1}{2} M_Z^2 \cos 2\beta
\end{align}

(2.1)

where we have assumed universal masses at the unification scale and solved the RGEs
numerically. The experimental limit on sneutrinos is 37 GeV \[14\] (assuming that the
sneutrinos are not degenerate), so that we conclude that the Yukawa couplings $\lambda'$
must be bounded by

\[
\sum_{jk} \lambda'^2_{jk}(M_{\text{GUT}}) < \frac{m_0^2 + 0.51 M_{1/2}^2 + \frac{1}{2} M_Z^2 \cos 2\beta - (37 \text{GeV})^2}{13m_0^2 + 49 M_{1/2}^2 - 1.5 M_{1/2} A_0 - 12 A_0^2} \tag{2.2}
\]

Given that masses for squarks and sleptons as low as 100 GeV are barely tenable in a unified framework with present collider limits, this constraint is the tightest available on certain \(\lambda'\). For example, if we set \(A_0 \simeq 0\), \(M_{1/2} = m_0 = 200\) GeV and \(\tan \beta = 10\) then this gives a bound on all of the \(\lambda'\) of 0.15, while if two of the \(\lambda'\) are equal then they must be less than 0.11, bounds which become only very slightly weaker with increasing soft masses, while the corresponding numbers with \(m_0\) and \(M_{1/2}\) both 100 GeV are 0.12 and 0.09. Note that these couplings are at the GUT scale, and the electroweak scale values are then around three times larger, while there is an error of at least 10 to 20% from the uncertainty in the value of the strong coupling.

Although we have neglected the effect of the tau Yukawa coupling, this in fact only makes the bound for the third generation rather tighter by reducing the sneutrino mass for that generation still further. We can also derive bounds on the \(\lambda\) and \(\lambda''\) couplings from similar arguments, but the results are not very restrictive.

3 \(b \to s\gamma\)

3.1 Contributions

The rare decay process \(b \to s\gamma\) has a branching rate which can be deduced from the decay \(B \to K^*\gamma\) which has been measured \cite{13} to obtain a value

\[
B(b \to s\gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4} \tag{3.1}
\]

consistent with the standard model result, and hence a 95% confidence level limit of

\[
1.0 \times 10^{-4} < B(b \to s\gamma) < 4.2 \times 10^{-4} \tag{3.2}
\]
This gives a very strong test of new physics such as supersymmetry, since SUSY models with light spectra can give large contributions to this process through charged Higgs and chargino diagrams [10].

The total branching ratio for the process $b \to s\gamma$ can be written (in units of the BR for the semileptonic $b$ decay) as:

$$\frac{B(b \to s\gamma)}{B(b \to ce\nu)} = \frac{3\pi\alpha}{G_F^2|K_{cb}|^2I(z)} \left(|\bar{A}_{LR}|^2 + |\bar{A}_{RL}|^2\right) F$$

(3.3)

where $G_F$ is the Fermi constant, $z = m_c/m_b$, $I(z) = 1 - 8z^2 + 8z^6 - 24z^4\log(z)$ is the phase space factor, $K_{ij}$ will be the different CKM matrix elements, $\bar{A}_{LR}, \bar{A}_{RL}$ are the total amplitudes to the LR and RL transitions respectively, and

$$F \sim \left(1 - \frac{8\alpha_s(m_b)}{3\pi}\right) \frac{1}{\kappa(z)}$$

(3.4)

contains NLO effects ($\kappa(z)$ being the NLO correction to the semileptonic decay). Here

$$\bar{A}_i = \eta^{16/23} \bar{A}_i^\gamma + \frac{8}{3} (\eta^{14/23} - \eta^{16/23}) \bar{A}_i^g + C\bar{A}_i^0$$

(3.5)

with $i = LR, RL$, $\eta = \alpha_s(M_W)/\alpha_s(m_b)$ and the different terms are as follows: $\bar{A}_i^{\gamma,g}_{LR,RL}$ are the coefficients of the effective operators for the $bs\gamma$ and $bsg$ interactions

$$C_i^{\gamma}_{LR,RL} = \frac{e}{4\pi} m_b (\bar{s}\sigma^{\mu\nu}P_{RL}b) F_{\mu\nu}$$

$$C_i^g_{LR,RL} = \frac{g_3}{4\pi} m_b (\bar{s}i\sigma^{\mu\nu}P_{RL}b_J) G^{a}_{\mu\nu} T^{aj}$$

(3.6)

$\bar{A}_i^0 = -\alpha_W K_{L_s}^* K_{tb}/M_W^2$ is from the coefficient of an operator

$$C_i^0 = [\bar{s}P_L c][\bar{c}P_L b]$$

(3.7)

and $C$ stands for the leading logarithmic QCD corrections (for a complete list of references see [17]).

The LR amplitudes can be divided into an $R$-Parity conserving part plus an RPV one; the former has been calculated in ref. [18], and corresponds mainly to the contributions coming from the SM diagram plus those with top quark and charged Higgs,
and stops/scharms and charginos running in the loop, plus smaller contributions from loops with neutralinos or gluinos and d–type squarks, which are generated due to QFV explicitly through the CKM matrices. Their expressions are

\[ \tilde{A}_{\varphi} = \frac{\alpha_W}{2} K_{ts} K_{tb} \left( \frac{m_t^2}{M_W^2} f^{(1)}_{\gamma,g} \left( \frac{m_t^2}{M_W^2} \right) + \frac{1}{\tan^2 \beta} f^{(2)}_{\gamma,g} \left( \frac{m_t^2}{m_{H^-}^2} \right) \right) \]

\[ \tilde{A}_{\chi} = -\alpha_W K_{ts} K_{tb} \sum \left( \frac{1}{m_{\tilde{t}_i}^2} V_{ij} T_{k1} - \frac{m_i V_{j2} T_{k2}}{\sqrt{2} M_W \sin \beta} \right)^2 f^{(3)}_{\gamma,g} \left( \frac{M_{\tilde{x}_j}^2}{m_{\tilde{t}_i}^2} \right) \]

\[ \tilde{A}_{\chi} = -\frac{U_{j2}}{\sqrt{2} \cos \beta} \frac{M_{\tilde{x}_j}}{M_W} \left( \frac{1}{m_{\tilde{t}_i}^2} V_{ij} T_{k1} - \frac{m_i V_{j2} T_{k2}}{\sqrt{2} M_W \sin \beta} \right) T_{k1} f^{(4)}_{\gamma,g} \left( \frac{M_{\tilde{x}_j}^2}{m_{\tilde{t}_i}^2} \right) \}

where our notation is as in ref. [10]. T, B are the orthogonal matrices that diagonalise the stop and sbottom mass matrices respectively through \( T M_{\tilde{t} \text{ - weak}} T^\dagger = M_{\tilde{t} \text{ - diag}} \)

where \( T_{11} = T_{22} = \cos \theta_t, T_{12} = -T_{21} = \sin \theta_t \), so that we may write the mass eigenstates \( |\tilde{t}^{(1)}\rangle > \) and \( |\tilde{t}^{(2)}\rangle > \) as \( \cos \theta_t |\tilde{t}_L > + \sin \theta_t |\tilde{t}_R > \) and \( -\sin \theta_t |\tilde{t}_L > + \cos \theta_t |\tilde{t}_R > \) respectively, and similarly for other flavours. For the first and second generations with small left-right mixing we shall take \( \cos \theta_i = 1 \), so that for example \( \tilde{d}_i^{(1)} = \tilde{d}_L \) and \( \tilde{d}_i^{(2)} = \tilde{c}_R \). For simplicity, we shall use the notation \( \tilde{s}_1 \) rather than \( \tilde{d}_2^{(1)} \), except where confusion might arise. The different functions \( f^{(i)}_{\gamma,g} \) are defined in Appendix C.

We generate new contributions to both the LR and RL amplitudes from \( R \)-parity violating couplings (\( \lambda' \) and \( \lambda'' \)) directly and also indirectly from the induced QFV soft terms. Therefore we can write

\[ \tilde{A}_{LR}^{RPV} = \tilde{A}_{LR}^\lambda + \tilde{A}_{LR}^{m_{\chi^-}^0} + \tilde{A}_{LR}^{m_{\chi^-}} + \tilde{A}_{LR}^{m_\phi} . \]
The direct amplitude is given by

\[ A_{LR}^{\gamma} = Q_d A_{LR}^{\gamma'} = -Q_d \sum_{i,j=1}^{3} \frac{\Delta m_{\tilde{\chi}_i}^2}{4\pi} \left( \frac{1}{12} \left[ \frac{\sin^2 \theta_{d_{ij}}}{m_{d_{j(1)}}^2} + \frac{\cos^2 \theta_{d_{ij}}}{m_{d_{j(2)}}^2} \right] - \frac{1}{m_{\tilde{\nu}_i}^2} F_1(x_{ji}) \right), \]

(3.12)

where \( \tilde{\chi}' \) are the different RPV couplings in the fermion mass eigenstate basis which, in this case, is related to the weak one by:

\[ \tilde{\chi}'_{ijk} = \lambda'_{imk} K_{mj}, \]

(3.13)

where a sum over \( m \) is understood. Note that this still leaves the possibility of generating effects even with only one non-zero RPV coupling in the weak basis [19]. Here \( x_{ji} = m_{d_{ji}}^2/m_{\tilde{\nu}_i}^2 \).

The other amplitudes are:

\[ A_{LR}^{\gamma_m \Delta m_{\chi}} = -\alpha_W K_{e_b} K_{tb} \sum_{n=1}^{2} \frac{\Delta m_{\tilde{\chi}_i}^2}{m_{e_n}^2 - m_{e_1}^2} \sum_{j=1}^{2} V_{j_1} \left\{ \left( V_{j_1}^{*} T_{n_1} - \frac{m_{t} V_{j_2}^{*} T_{n_2}}{\sqrt{2} M_W \sin \beta} \right) \right\} \]

\[ \times \left( \frac{f_{\gamma_1}^{(3)}(x_{jn})}{m_{e_n}^2} - \frac{f_{\gamma_1}^{(3)}(x_{j1})}{m_{e_1}^2} \right) - \frac{U_{j_2} T_{n_1}^*}{\sqrt{2} \cos \beta} M_{\tilde{\chi}_i} \left( \frac{f_{\gamma_1}^{(4)}(x_{jn})}{m_{e_n}^2} - \frac{f_{\gamma_1}^{(4)}(x_{j1})}{m_{e_1}^2} \right) \right\} \]

(3.14)

\[ A_{LR}^{\gamma_m \Delta m_{\chi'}} = Q_d A_{LR}^{\gamma_m \Delta m_{\chi'}} = -2\alpha_W Q_d \sum_{n=1}^{2} \frac{\Delta m_{\tilde{\chi}_i}^2}{m_{b_{2n}}^2 - m_{b_{21}}^2} \sum_{j=1}^{4} \left\{ \right\} \]

\[ |s_W Q_d N_{j1}' - \frac{1}{c_W} (1/2 + Q_d s_W^2) N_{j2}'|^2 B_{n_1}^* \left[ \frac{F_2(x_{jn})}{m_{b_{2n}}^2} - \frac{F_2(x_{j1})}{m_{b_{21}}^2} \right] \]

\[ - \left( s_W Q_d N_{j1}' - \frac{1}{c_W} (1/2 + Q_d s_W^2) N_{j2}' \right) \left[ \frac{F_1(x_{jn})}{m_{b_{2n}}^2} - \frac{F_1(x_{j1})}{m_{b_{21}}^2} \right] B_{n_2}^* \]

\[ - \frac{m_b}{2M_W \cos \beta} N_{j3} B_{n_1}^* \left[ \frac{M_{\tilde{\chi}_i}^2}{m_{b_{2n}}^2} - \frac{F_1(x_{jn})}{m_{b_{2n}}^2} \right] \]

(3.15)

\[ A_{LR}^{\gamma_{m_b}} = -2\alpha_s Q_d C(R) \sum_{n=1}^{2} \frac{\Delta m_{\tilde{\chi}_i}^2}{m_{b_{2n}}^2 - m_{b_{21}}^2} \]

\[ \times \left[ B_{n_1}^* \left( \frac{F_2(x_{gn})}{m_{b_{2n}}^2} - \frac{F_2(x_{g1})}{m_{b_{21}}^2} \right) - B_{n_2}^* \frac{M_{\tilde{\chi}_i}^2}{m_{b_{2n}}^2} \left( \frac{F_4(x_{gn})}{m_{b_{2n}}^2} - \frac{F_4(x_{g1})}{m_{b_{21}}^2} \right) \right] \]

(3.16)

\[ A_{LR}^{\gamma_{m_b'}} = -\alpha_s \sum_{n=1}^{2} \frac{\Delta m_{\tilde{\chi}_i}^2}{m_{b_{2n}}^2 - m_{b_{21}}^2} \left\{ -B_{n_1}^* C(G) \left( \frac{F_1(x_{gn})}{m_{b_{2n}}^2} - \frac{F_1(x_{g1})}{m_{b_{21}}^2} \right) \right\} \]

\[ + B_{n_1}^* (2C(R) - C(G)) \left( \frac{F_2(x_{gn})}{m_{b_{2n}}^2} - \frac{F_2(x_{g1})}{m_{b_{21}}^2} \right) \]
\[ + B_{n2}^* \frac{M_g}{m_b} C(G) \left( \frac{F_3(x_{g1}^b)}{m_{b_{1_1}}} - \frac{F_3(x_{g1}^s)}{m_{s_{1_1}}} \right) \]
\[ - B_{n2}^* \frac{M_g}{m_b} (2C(R) - C(G)) \left( \frac{F_4(x_{g1}^b)}{m_{b_{1_1}}} - \frac{F_4(x_{g1}^s)}{m_{s_{1_1}}} \right) \} , \tag{3.17} \]

where \( \alpha_s \) is the strong gauge coupling constant. Also,

\[ x_{j1}^c = \frac{M_{n_{i_1}}}{m_{E_i}} , \quad x_{j1}^s = \frac{M_{n_{i_1}}}{m_{E_i}} , \quad x_{g1}^s = \frac{M_{n_{i_1}}}{m_{E_i}} \tag{3.18} \]
\[ x_{jn}^c = \frac{M_{n_{i_{j 1}}}}{m_{E_i}} , n = 1, 2 \quad x_{jn}^b = \frac{M_{n_{i_{j 1}}}}{m_{E_i}} , n = 1, 2 \quad x_{g1}^b = \frac{M_{n_{i_{j 1}}}}{m_{E_i}} \tag{3.19} \]

and the QCD factors are \( C(R) = 4/3 \), \( C(G) = 3 \).

Now we turn to the RPV contributions to \( \tilde{A}_{RL} \). These are given by:

\[ \tilde{A}_{RL}^{RPV} = \tilde{A}_{RL}^\gamma + \tilde{A}_{RL}^\lambda + \tilde{A}_{RL}^{\Delta m_{\chi^0}} + \tilde{A}_{RL}^{\Delta m_{\gamma}} , \tag{3.20} \]

with:

\[ \tilde{A}_{RL}^\gamma^\lambda^{\rho \lambda} = - \sum_{i,j=1}^3 \left[ \tilde{\lambda}_{ij2} \tilde{\lambda}_{ij3} \frac{4\pi}{Q_d} \left( \frac{12}{m_{d_{i_1}}^2} \right) - \frac{12}{m_{d_{i_1}}^2} \right] F_1(x_{ji}) \]
\[ + \frac{\lambda_{ij2} \lambda_{ij3}}{4\pi} \left( - \frac{\cos^2 \theta_{w} f_{(1)}^{\gamma g} \left( \frac{m_{u_{i_1}}^2}{m_{e_{j_1}}^2} \right) + \sin^2 \theta_{w} f_{(1)}^{\gamma g} \left( \frac{m_{u_{i_1}}^2}{m_{e_{j_1}}^2} \right) - \frac{\cos^2 \theta_{w} f_{(3)}^{\gamma g} \left( \frac{m_{u_{i_1}}^2}{m_{e_{j_1}}^2} \right) + \sin^2 \theta_{w} f_{(3)}^{\gamma g} \left( \frac{m_{u_{i_1}}^2}{m_{e_{j_1}}^2} \right) \right) \right] \tag{3.21} \]

where both \( \tilde{\lambda} \) and \( x_{ji} \) are defined after eq. (3.12).

\[ \tilde{A}_{RL}^\gamma^\lambda^{\rho \lambda} = -2 \sum_{i,j=1}^3 \frac{\tilde{\lambda}_{ij2} \tilde{\lambda}_{ij3}}{4\pi} \left[ \frac{\sin^2 \theta_{w} f_{(5)}^{\gamma g} \left( \frac{m_{u_{i_1}}^2}{m_{d_{i_1}}^2} \right) + \cos^2 \theta_{w} f_{(5)}^{\gamma g} \left( \frac{m_{u_{i_1}}^2}{m_{d_{i_1}}^2} \right) \right] \]
\[ + \frac{\sin^2 \theta_{w} f_{(6)}^{\gamma g} \left( \frac{m_{u_{i_1}}^2}{m_{d_{i_1}}^2} \right) + \cos^2 \theta_{w} f_{(6)}^{\gamma g} \left( \frac{m_{u_{i_1}}^2}{m_{d_{i_1}}^2} \right) \right] \] \tag{3.22}

\[ \tilde{A}_{RL}^{\Delta m_{\chi^0}} = Q_d \tilde{A}_{RL}^{\Delta m_{\chi^0}} = -2 \alpha_W Q_d \sum_{n=1}^2 \frac{\Delta m_{\chi^0}}{m_{b_{n_1}}^2 - m_{s_{j_1}}^2} \sum_{j=1}^4 \left( s_W Q_d N'_{j_1} - \frac{s_W^2}{c_W} Q_d N'_{j_2} \right)^2 \]
\[ \times B_{n2}^* \left[ \frac{F_2(x_{g1}^b)}{m_{b_{n_1}}^2} - \frac{F_2(x_{g1}^s)}{m_{s_{j_1}}^2} \right] - \left( s_W Q_d N'_{j_1} - \frac{s_W^2}{c_W} Q_d N'_{j_2} \right) \]
\[ \times \left( s_W Q_d N^*_{j1} - \frac{1}{c_W} (1/2 + Q_d s^2_W) N^*_{j2} \right) B_{n1} + \frac{m_b}{2 M_W \cos \beta} N^*_{j3} B_{n2} \]

\[ \times \frac{M_{\chi^0}}{m_b} \left( \frac{F_4(x_{jn})}{m^2_{b_n}} - \frac{F_4(x_{j2})}{m^2_{s_2}} \right) \right\} \right] \] (3.23)

\[ \tilde{A}^\gamma_{RL} = -2 \alpha_s Q_d C(R) \sum_{n=1}^{2} \frac{\Delta m^2_{s_2b_n}}{m^2_{b_n} - m^2_{s_2}} \times \left[ B_{n2}^* \left( \frac{F_2(x_{gn}^b)}{m^2_{b_n}} - \frac{F_2(x_{g2}^s)}{m^2_{s_2}} \right) - B_{n1}^* \frac{M_{\tilde{g}}}{m_b} \left( \frac{F_4(x_{gn}^b)}{m^2_{b_n}} - \frac{F_4(x_{g2}^s)}{m^2_{s_2}} \right) \right] \] (3.24)

\[ \tilde{A}^g_{RL} = -\alpha_s \sum_{n=1}^{2} \frac{\Delta m^2_{s_2b_n}}{m^2_{b_n} - m^2_{s_2}} \left\{ -B_{n2}^* C(G) \left( \frac{F_1(x_{gn}^b)}{m^2_{b_n}} - \frac{F_1(x_{g2}^s)}{m^2_{s_2}} \right) + B_{n2}^* (2C(R) - C(G)) \left( \frac{F_2(x_{gn}^b)}{m^2_{b_n}} - \frac{F_2(x_{g2}^s)}{m^2_{s_2}} \right) \right. \]

\[ \left. + B_{n1}^* \frac{M_{\tilde{g}}}{m_b} C(G) \left( \frac{F_3(x_{gn}^b)}{m^2_{b_n}} - \frac{F_3(x_{g2}^s)}{m^2_{s_2}} \right) - B_{n1}^* \frac{M_{\tilde{g}}}{m_b} (2C(R) - C(G)) \left( \frac{F_4(x_{gn}^b)}{m^2_{b_n}} - \frac{F_4(x_{g2}^s)}{m^2_{s_2}} \right) \right\} \] (3.25)

### 3.2 Analytical Discussion

In general the results for the implications of the MSSM for \( b \to s \gamma \) are well known \textsuperscript{[16]}. The SM contribution to the branching rate is large and in good agreement with the data at around \( 3 \times 10^{-4} \). The MSSM charged Higgs contribution is always of the same sign as that from the \( W \) and can easily push the total over the experimental limit, hence giving quite a tight constraint on the charged Higgs mass or equivalently \( m_A \). In practice, however, the situation changes when the full spectrum is considered, because the SUSY partners can give contributions to the amplitude of both signs with the dominant contribution being that of the chargino which can be of opposite sign to that of the SM and charged Higgs \textsuperscript{[18]}. There are thus two separate scenarios according to the sign of \( \mu \). For \( \mu_4 < 0 \), the structure of the chargino mass matrix and mixings makes its contribution comparable in size to the other two with the opposite sign giving rise to a strong cancellation, and hence the constraints on SUSY parameters
are rather weak. For $\mu_4 > 0$ the chargino amplitude is not so big and moreover its sign is not always opposed to that of SM and charged Higgs, and therefore cannot cancel them off completely so we must have a fairly heavy charged Higgs, which given our unification and radiative electroweak symmetry breaking assumptions implies fairly large soft masses.

If we now turn to the implications of $R$-parity violating couplings, there are three distinct possibilities. The first of these is where the product $\lambda'_{ij2}\lambda'_{ij3}$ is non-zero. Here the main effects will be to change $\tilde{A}_{LR}$, since mixing is generated in left handed superfields only. This will give an additional term which is added directly to the amplitude from the usual MSSM terms, but may be of opposite sign and hence less restricted. By contrast, the cases of non-zero $\lambda'_{ij2}\lambda'_{ij3}$ or $\lambda''_{ij2}\lambda''_{ij3}$ generate contributions to $\tilde{A}_{RL}$ (negligible in the MSSM), which cannot therefore have interference with the MSSM effects but which are only added to them in quadrature.

Beginning with the direct $R$-Parity violating contributions, the SM contribution to $\tilde{A}_{LR}$ is $-6.7 \times 10^{-8}$GeV$^{-2}$, with an experimental limit on $\sqrt{(\tilde{A}_{LR})^2 + (\tilde{A}_{RL})^2}$ derived from equation (3.2) of 4.1 to 8.5$x10^{-8}$GeV$^{-2}$. Since the QCD corrections typically give a contribution to $\tilde{A}_i$ of around 0.65$\tilde{A}_i$, we shall consider what values of the couplings give a contribution of $-2 \times 10^{-8}$GeV$^{-2}$ to $\tilde{A}_{LR}$ or $7 \times 10^{-8}$GeV$^{-2}$ to $\tilde{A}_{RL}$ (recall that the sign of the former matters, since it is being added to a negative SM contribution, while the latter is added in quadrature). It is then straightforward to derive the bounds

$$\tilde{\lambda}'_{ij2}\tilde{\lambda}'_{ij3}(M_Z) \lesssim 0.09 \left[ 2 \left( \frac{100\text{GeV}}{m_{\tilde{\nu}_i}} \right)^2 - \left( \frac{100\text{GeV}}{m_{\tilde{d}_R}} \right)^2 \right]^{-1}$$  (3.26)

$$|\lambda'_{ij2}\lambda'_{ij3}(M_Z)| \lesssim 0.035 \left[ \left( \frac{100\text{GeV}}{m_{\tilde{\nu}_i}} \right)^2 - \left( \frac{100\text{GeV}}{m_{\tilde{d}_L}} \right)^2 \right]^{-1}$$  (3.27)

$$|\lambda''_{ij2}\lambda''_{ij3}(M_Z)| \lesssim 0.16 \left( \frac{m_{\tilde{q}_R}}{100\text{GeV}} \right)^2$$  (3.28)

where we have neglected the distinction between $\tilde{\lambda}'$ and $\lambda'$ in deriving the second of
these.

The indirect contributions are far more complicated, but it is instructive to consider their size in the case where we include only the diagrams with a helicity flip on the gaugino line (and hence an overall factor of the gaugino mass), set all sparticle masses approximately degenerate at $\tilde{m}$ and ignore the various mixing factors. This then gives an approximate contribution for each chargino of

$$\tilde{A}_{LR}^{\gamma \Delta m_{\chi^{-}}} \sim \lambda'_{i2j} \lambda'_{i3j} (M_{\text{GUT}}) \frac{10^{-7} \text{GeV}^{-2}}{\cos \beta} \left( \frac{100 \text{GeV}}{\tilde{m}} \right)^2 \frac{M_{\chi^{-}}}{M_W}$$

(3.29)

where we have used equation (B.1) and have evaluated the functions with all masses equal. Since the product of two of these couplings at the GUT scale is an order of magnitude smaller than at $M_Z$ [10], for $\lambda'_{i2j} \lambda'_{i3j}$ we expect a significant chargino contribution relative to the direct contributions for large $\tan \beta$ unless the gaugino-higgsino mixing is small. A simple way of understanding how the chargino contribution must be relevant is to consider this in terms of results from the MSSM. The chargino contribution here is driven by the CKM matrix element $K_{23} \simeq 0.04$, while the $R$-Parity Violating contributions are driven by a similar mixing $\Delta m^2/m_q^2$, which is roughly equal to the product of $\lambda'_{i2j} \lambda'_{i3j} (M_{\text{GUT}})$ and hence will be similarly important when this product of couplings at the GUT scale is around $10^{-2}$.

The gluino contribution is simpler, and neglecting mixing between sbottom squarks and the mass difference between squarks and gluinos we find a bound on each of the pairs of couplings

$$\lambda'_{i2j} \lambda'_{i3j} (M_Z) \lesssim 0.003 \left( \frac{\tilde{m}}{100 \text{GeV}} \right)^2$$

(3.30)

$$\lambda'_{i2j} \lambda'_{i3j} (M_Z) \lesssim 0.006 \left( \frac{\tilde{m}}{100 \text{GeV}} \right)^2$$

(3.31)

$$\lambda''_{i2j} \lambda''_{i3j} (M_Z) \lesssim 0.006 \left( \frac{\tilde{m}}{100 \text{GeV}} \right)^2$$

(3.32)

where the bound is in fact on the sum of these indirect terms plus the direct contributions quoted above. Unfortunately, we might expect the suppression from the
mixing in the sbottom sector to weaken these by an order of magnitude or so. We can conclude however that it is likely that the $RL$ contributions are dominated by the gluino, especially in the case of non-zero $\lambda''_{ij2}\lambda''_{ij3}$, while for $\lambda'_{ij2}\lambda'_{ij3}$ the chargino is likely to dominate, results which are confirmed by our numerical studies.

### 3.3 Numerical Results

Given the discussion above, we are now in a position to discuss the effects of setting these products non-zero with a realistic spectrum from universality at the GUT scale. The products of couplings which we are considering have not been constrained before, but we find $\lambda'_{ij2}\lambda'_{ij3} < 0.06(100\text{GeV}/\tilde{m})^2$ using the bounds on each coupling independently from $K^+ \text{ decays}$. Other such bounds can always be evaded by selecting the indices in the products appropriately; for example the tight bounds on $\lambda''_{ijk}$ from neutron anti-neutron oscillation only apply if $i = 1$.

In Figure 1, we show a simple example of mixing involving the product $\lambda'_{ij2}\lambda'_{ij3}$, where we choose both couplings 0.05 at the GUT scale, hence making the product around 0.023 at low energy, and with other parameters $\tan\beta = 10$, $\mu_4 < 0$, $m_0 = 100$ GeV, $A_0 = 0$, and $M_{1/2}$ varying. We see that in fact here the gluino contribution is dominated by the direct one, and this in turn is dominated by the chargino contribution. However, even at very low values of the mass spectrum (for $M_{1/2} \lesssim 110$ GeV the chargino mass is actually below the experimental limit) and even though we have taken the product of couplings to be close to the limit of equation (2.2), the contributions are still too small to greatly affect the total. Hence we conclude that bounds on this product are really not usefully found from $b \to s\gamma$ decays.

Similarly, for $\lambda'_{ij2}\lambda'_{ij3}$ we expect that the product at the GUT scale will be $\lesssim 0.01$ from equation (2.2), which in turn leads to the product at the weak scale being around $\lesssim 0.1$, which can only give a useful bound if the spectrum is extremely light, and in
practice it again appears that constructing a spectrum so light as to have interesting
consequences for $b \rightarrow s\gamma$ through $R$-Parity Violation requires one or other of the
masses to become lighter than its experimental bound.

For the case of $\lambda''_{ij2} \lambda''_{ij3}$ we do not have such tight bounds from the sneutrino mass
and can increase the coupling to quite a large value. In Figure 2, we show a plot with
the same input parameters as in Figure 1, but with $\lambda''_{ij2} \lambda''_{ij3}(M_{GUT}) = 0.01$. Here
again the effects on $b \rightarrow s\gamma$ are only really beginning to be significant for a very
light spectrum. However, it is interesting that the contribution is very much larger
than might be expected from equation (3.28), since the gluino term is overwhelmingly
dominant.

In conclusion, while the $R$–Parity violating couplings give a contribution to $b \rightarrow s\gamma$
which is enhanced typically by an order of magnitude when the indirect contributions
are included, and although this in principle gives a tighter bound than others in the
literature, in fact these constraints are very weak in the context of universal mass
spectrum at the GUT scale. This is because, for example, it is virtually impossible
to arrange that squarks should have masses as light as 100 GeV without violating
one or another of the experimental limits. In the case of non-zero $\lambda'$ couplings the
bounds from $b \rightarrow s\gamma$ are weaker than those from the sneutrino mass limits given in
equation (2.2). For the $\lambda''$ couplings, the indirect gluino contribution is far larger
than the direct contribution, and we find that for a light spectrum with $m_0$ and $M_{1/2}$
of order 100 GeV, the product of $\lambda''_{ij2} \lambda''_{ij3}(M_Z)$ should be $\approx 0.2$. This bound scales
very roughly as $M_{1/2}$, but with some complicated dependence on the sbottom squark
mixing as well as on the mass spectrum.

4 $K^0 - \bar{K}^0$ mixing
4.1 SUSY contributions

The $K^0 - \bar{K}^0$ sector has long been a probe of physics beyond the standard model, starting with the original motivation of the study of CP violation and recently also as a way of investigating flavour changing neutral currents in SUSY. These occur even in the absence of $R$-Parity violation, since in the MSSM (and SM) quark flavour is not conserved. Hence we shall find that the RPV contribution is complementary to the already existing CKM contribution. We shall ask the question of how much it is possible to constrain RPV couplings through their impact on $\Delta m_K$. Here we shall consider the relative sizes of three effects, the MSSM $\Delta m^2$ contribution and the direct and indirect RPV contributions. The first has already been investigated [11, 12], as has the last [3, 8, 19, 21, 22], but the question of whether the simple direct effects are in fact dominant has never been studied.

The upper bound on the $K_L - K_S$ mass difference has been measured [14] as

$$\Delta m_K = (3.491 \pm 0.009) \times 10^{-15}\text{GeV} \quad (4.1)$$

$K^0 - \bar{K}^0$ mixing is generated by the effective Lagrangian

$$\Delta \mathcal{L}^{s=2} = c_{LL}[\bar{d}_i P_L s^i][\bar{d}_j P_L s^j] + c'_{LL}[\bar{d}_i P_L s^i][\bar{d}_j P_L s^j] + c_{RR}[\bar{d}_i P_R s^i][\bar{d}_j P_R s^j] + c'_{RR}[\bar{d}_i P_R s^i][\bar{d}_j P_R s^j] + c_{LR}[\bar{d}_i P_L s^i][\bar{d}_j P_R s^j] + c'_{LR}[\bar{d}_i P_L s^i][\bar{d}_j P_R s^j] + \frac{m^2_K}{(m_d + m_s)^2} \gamma^\mu P_L s^i][\bar{d}_j \gamma^\mu P_R s^j] \quad (4.2)$$

giving a contribution to the $K_L - K_S$ mass difference

$$\Delta m_K = \frac{1}{12} f_K^2 m_K \left( c_{LR} + 3 c'_{LR} + 8 d_{LL} + 8 d_{RR} \right. \left. \frac{m^2_K}{(m_d + m_s)^2} \left( 6 c_{LR} + 2 c'_{LR} + 5 c_{LL} - c'_{LL} + 5 c_{RR} - c'_{RR} \right) \right) \quad (4.3)$$

Here the terms involving both left and right handed fields in the effective lagrangian will give a larger contribution to $\Delta m_K$ than those involving purely left or right handed
fields because of the factor of $m_R^2/m_0^2$. We shall not try to include QCD corrections to these formulae since these are unlikely to be more significant than the errors implicit in, for example, our choice of $\alpha_3$, $m_s$, and SUSY parameters.

Contributions to the parameters in the effective Lagrangian come from a variety of different classes of diagram, some of which are shown in Figure 3. The standard model contribution from Figure 3A is

$$d_{LL}^{SM} = \sum_{i,j} \frac{g^4}{64\pi^2} K_{i1} K_{j2}^* K_{j1}^* K_{i2} I_4(m_{u_i}^2, m_{u_j}^2, M_W^2, M_W^2)$$

$$\simeq \frac{g^4}{128\pi^2} |K_{cd}|^2 \frac{m_c^2}{M_W^2}$$

(4.4)

where for the second line we only consider the charm and up quark contributions.

Apart from the standard model contributions, there are contributions from direct $R$-Parity violating diagrams which have been calculated previously [3, 19, 21, 22]. The only tree level diagram is that with the interchange of a sneutrino and two $\tilde{\chi}$ couplings shown in Figure 3b

$$c_{LR}^{TL} = -\sum_i \frac{\tilde{\chi}_{i21} \tilde{\chi}_{i12}^*}{m_{\tilde{\nu}_i}}$$

(4.5)

where our notation is as in the previous section and our earlier work [10].

The $\chi$ box diagrams such as Figure 3c give a contribution

$$d_{LL}^{\chi} = \sum_{i,j,k,m} \frac{1}{64\pi^2} \tilde{\chi}_{i1k} \tilde{\chi}_{j2k}^* \tilde{\chi}_{j1m}^* \tilde{\chi}_{i2m} I_4(m_{\nu_i}^2, m_{\nu_j}^2, m_{d_k}^2, m_{d_m}^2)$$

$$+ \sum_{i,j,k,m} \frac{1}{64\pi^2} \tilde{\chi}_{i1k} \tilde{\chi}_{j2k}^* \tilde{\chi}_{j1m}^* \tilde{\chi}_{i2m} I_4(m_{\nu_i}^2, m_{\nu_j}^2, m_{d_k}^2, m_{d_m}^2)$$

(4.6)

$$d_{RR}^{\chi} = \sum_{i,j,k,m} \frac{1}{64\pi^2} \tilde{\chi}_{i1k} \tilde{\chi}_{j2k}^* \tilde{\chi}_{j1m} \tilde{\chi}_{i2m} I_4(m_{\nu_i}^2, m_{\nu_j}^2, m_{d_k}^2, m_{d_m}^2)$$

$$+ \sum_{i,j,k,m} \frac{1}{64\pi^2} \tilde{\chi}_{i1k} \tilde{\chi}_{j2k}^* \tilde{\chi}_{j1m} \tilde{\chi}_{i2m} I_4(m_{\nu_i}^2, m_{\nu_j}^2, m_{d_k}^2, m_{d_m}^2)$$

(4.7)
\[ c'_{LR} \lambda' = - \sum_{i,j,k,m} \frac{1}{32 \pi^2} \tilde{X}_{i1k} \tilde{X}^{*}_{j2k} \tilde{X}_{im1} \tilde{X}^{*}_{jim2} \mathcal{I}_4(m_{\tilde{p}_i}^2, m_{\tilde{p}_j}^2, m_{\tilde{d}_k}^2, m_{\tilde{d}_m}^2) \]
\[ - \sum_{i,j,k,m} \frac{1}{32 \pi^2} \tilde{X}_{i1k} \tilde{X}^{*}_{j2k} \tilde{X}_{im1} \tilde{X}^{*}_{jim2} \mathcal{I}_4(m_{\tilde{p}_i}^2, m_{\tilde{p}_j}^2, m_{\tilde{d}_k}^2, m_{\tilde{d}_m}^2) \] (4.8)

while \( \lambda'' \) box diagrams such as Figure 3f give

\[ d_{RR} \lambda'' = \sum_{i,j} \frac{1}{32 \pi^2} \lambda''_{i13} \lambda''_{j23} \lambda''_{j13} \lambda''_{j23} \left[ \mathcal{I}_4(m_{\tilde{b}_{bR}}^2, m_{\tilde{b}_{bR}}^2, m_{\tilde{u}_i}^2, m_{\tilde{u}_j}^2) \right]^2 \] (4.9)

In addition we have diagrams such as those Figure 3e with one internal \( W \) boson and one internal squark line, where a helicity flip is needed on the internal fermion line, forcing it to be a top or charm. Hence we have

\[ c_{LR} \lambda'' \bar{W} = - \sum_{i,j} \frac{\alpha}{4 \pi \sin^2 \theta_W} \lambda''_{i13} \lambda''_{j23} \lambda''_{j13} \lambda''_{j23} \left[ \mathcal{I}_4(m_{\tilde{b}_{bR}}^2, m_{\tilde{b}_{bR}}^2, m_{\tilde{u}_i}^2, m_{\tilde{u}_j}^2) \right] \] (4.10)

We neglect similar diagrams with \( \lambda' \) vertices, since these are always dominated by the tree level contribution.

Although they have not been considered in the context of \( R \)-Parity, the diagrams involving mass insertions on squark lines have been analysed in the case where the off-diagonal mass terms are generated by the CKM matrix and by boundary conditions at the unification scale [11, 12, 18]. Most of these analyses include only gluino mediated box diagrams, such as Figure 3f:

\[ d_{LL} \bar{g} = \left( \Delta m_{\tilde{s}_L d_L}^2 \right)^2 \alpha_3^2 \left[ \frac{11}{18} \mathcal{I}_4(m_{\tilde{d}_L}^2, m_{\tilde{d}_L}^2, M_g^2, M_g^2) \right] \] (4.11)

\[ d_{RR} \bar{g} = \left( \Delta m_{\tilde{s}_R d_R}^2 \right)^2 \alpha_3^2 \left[ \frac{11}{18} \mathcal{I}_4(m_{\tilde{d}_R}^2, m_{\tilde{d}_R}^2, M_g^2, M_g^2) \right] \] (4.12)

\[ c_{LR} \bar{g} = \left( \Delta m_{\tilde{s}_L d_L}^2 \right) \left( \Delta m_{\tilde{s}_R d_R}^2 \right) \alpha_3^2 \left[ -\frac{2}{3} \mathcal{I}_4(m_{\tilde{d}_L}^2, m_{\tilde{d}_R}^2, M_g^2, M_g^2) \right] \]
where we have assumed that $m_{\tilde{d}_L} \simeq m_{\tilde{s}_L}$. These results and those for the resulting contribution to $\Delta m_K$ in equation (4.3) agree with those presented in Gabbiani et al [12], where the disagreements between these results and others in references [11, 12] are discussed.

The chargino mediated box diagrams give

$$d_{LL} \chi^\pm = \frac{(\Delta m_{\tilde{c}_{L\tilde{u}}})^2}{4 \sin^4 \theta_w} \sum_{i,j} |V_{ti}|^2 |V_{tj}|^2 I''_4(m_{\tilde{u}_L}, m_{\tilde{u}_L}, M^2_{\tilde{c}, \tilde{c}}, M^2_{\tilde{c}, \tilde{c}}) (4.19)$$

while from neutralino mediated box diagrams we have

$$d_{LL} \chi^0 = \sum_{ij} \frac{1}{16 \pi^2} (\Delta m_{\tilde{c}_{L\tilde{d}_L}})^2 \left[ |A_{d_{LL}}|^2 |A_{d_{LL}}|^2 I''_4(m_{\tilde{d}_L}, m_{\tilde{d}_L}, M^2_{\tilde{c}, \tilde{c}}, M^2_{\tilde{c}, \tilde{c}}) - (A_{d_{LL}})^2 (A_{d_{LL}}^*)^2 M_{\tilde{c}^0_i} M_{\tilde{c}^0_j} J''_4(m_{\tilde{d}_L}, m_{\tilde{d}_L}, M^2_{\tilde{c}^0_i}, M^2_{\tilde{c}^0_j}) \right] (4.20)$$

$$d_{RR} \chi^0 = \sum_{ij} \frac{1}{16 \pi^2} (\Delta m_{\tilde{b}_{R\tilde{d}_R}})^2 \left[ |A_{d_{RR}}|^2 |A_{d_{RR}}|^2 I''_4(m_{\tilde{d}_R}, m_{\tilde{d}_R}, M^2_{\tilde{c}, \tilde{c}}, M^2_{\tilde{c}, \tilde{c}}) - (A_{d_{RR}})^2 (A_{d_{RR}}^*)^2 M_{\tilde{c}^0_i} M_{\tilde{c}^0_j} J''_4(m_{\tilde{d}_R}, m_{\tilde{d}_R}, M^2_{\tilde{c}^0_i}, M^2_{\tilde{c}^0_j}) \right] (4.21)$$

$$c_{LR} \chi^0 = - \sum_{ij} \frac{A_{d_{LL}} A_{d_{RR}} A_{d_{LJ}} A_{d_{RJ}}}{4 \pi^2} \left( \Delta m_{\tilde{c}_{L\tilde{d}_L}}^2 \right) \left( \Delta m_{\tilde{b}_{R\tilde{d}_R}}^2 \right) \times M_{\tilde{c}^0_i} M_{\tilde{c}^0_j} J''_4(m_{\tilde{d}_L}, m_{\tilde{d}_L}, M^2_{\tilde{c}^0_i}, M^2_{\tilde{c}^0_j}) (4.22)$$

$$c'_{LR} \chi^0 = \sum_{ij} \frac{A_{d_{LL}} A_{d_{RR}} A_{d_{LJ}} A_{d_{RJ}}^*}{4 \pi^2} \left( \Delta m_{\tilde{c}_{L\tilde{d}_L}}^2 \right) \left( \Delta m_{\tilde{b}_{R\tilde{d}_R}}^2 \right) (4.22)$$
and from those with one gluino and one neutralino:

\[ d_{LL} \tilde{\chi}^0 \tilde{g} = - \sum_i \frac{\alpha_3}{6\pi} \left( \Delta m^2_{s, d_L} \right)^2 \left[ |A_{dLi}|^2 I''_4\left( m^2_{\tilde{d}_R}, m^2_{d_L}, M^2_{\tilde{\chi}_i}, M^2_{\tilde{\chi}'_j} \right) 
+ \frac{1}{2}(A^*_{dLi} + A_{dLi}) M_{\tilde{\chi}_i} M_{\tilde{\chi}'_j} J''_4\left( m^2_{d_L}, m^2_{\tilde{d}_R}, M^2_{\tilde{\chi}_i}, M^2_{\tilde{\chi}'_j} \right) \right] \]  

(4.24)

\[ d_{RR} \tilde{\chi}^0 \tilde{g} = - \sum_i \frac{\alpha_3}{6\pi} \left( \Delta m^2_{s, d_R} \right)^2 \left[ |A_{dRi}|^2 I''_4\left( m^2_{d_R}, m^2_{\tilde{d}_L}, M^2_{\tilde{\chi}_i}, M^2_{\tilde{\chi}'_j} \right) 
+ \frac{1}{2}(A^*_{dRi} + A_{dRi}) M_{\tilde{\chi}_i} M_{\tilde{\chi}'_j} J''_4\left( m^2_{d_R}, m^2_{\tilde{d}_L}, M^2_{\tilde{\chi}_i}, M^2_{\tilde{\chi}'_j} \right) \right] \]  

(4.25)

\[ c_{LR} \tilde{\chi}^0 \tilde{g} = - \sum_i \frac{\alpha_3}{2\pi} \left( \Delta m^2_{s, d_L} \right)^2 \left( \Delta m^2_{s, d_R} \right)^2 \left[ (A_{dRi} A_{dLi} + A^*_{dRi} A^*_{dLi}) \right. 
\left. \times I''_4\left( m^2_{d_L}, m^2_{d_R}, M^2_{\tilde{\chi}_i}, M^2_{\tilde{\chi}'_j} \right) 
+ (A_{dRi} A^*_{dLi} + A^*_{dRi} A_{dLi}) M_{\tilde{\chi}_i} M_{\tilde{\chi}'_j} J''_4\left( m^2_{d_L}, m^2_{d_R}, M^2_{\tilde{\chi}_i}, M^2_{\tilde{\chi}'_j} \right) \right] \]  

(4.26)

\[ c'_{LR} \tilde{\chi}^0 \tilde{g} = \sum_i \frac{\alpha_3}{6\pi} \left( \Delta m^2_{s, d_L} \right)^2 \left( \Delta m^2_{s, d_R} \right)^2 \left[ (A_{dRi} A_{dLi} + A^*_{dRi} A^*_{dLi}) \right. 
\left. \times I''_4\left( m^2_{d_L}, m^2_{d_R}, M^2_{\tilde{\chi}_i}, M^2_{\tilde{\chi}'_j} \right) 
+ (A_{dRi} A^*_{dLi} + A^*_{dRi} A_{dLi}) M_{\tilde{\chi}_i} M_{\tilde{\chi}'_j} J''_4\left( m^2_{d_L}, m^2_{d_R}, M^2_{\tilde{\chi}_i}, M^2_{\tilde{\chi}'_j} \right) \right] \]  

(4.27)

Here we use

\[ A_{dLj} = e Q_d N''_{j1} - \frac{g}{\cos \theta_W} \left( \frac{1}{2} + Q_d \sin^2 \theta_W \right) N''_{j2} \]  

\[ A_{dRj} = -e Q_d N'_{j1} + \frac{g Q_d \sin^2 \theta_W}{\cos \theta_W} N_{j2} \]  

(4.28)

and we note that

\[ \Delta m^2_{s, d_L} = m^2_{Q_i Q_2} \]  

\[ \Delta m^2_{s, d_R} = m^2_{d_1 d_2} \]  

(4.29)

\[ \left( \Delta m^2_{s, d_L} \right)^2 = \nu^2_1 \left( \eta^d_{21} \right)^2 + \nu^2_2 \eta^d_{21} \left( \eta^d_{11} m^2_{Q_1 Q_2} \frac{\delta}{\delta m^2_{d_L}} + \eta^d_{22} m^2_{d_1 d_2} \frac{\delta}{\delta m^2_{d_R}} \right) + \ldots \]  

\[ \left( \Delta m^2_{s, d_R} \right)^2 = \nu^2_1 \left( \eta^d_{12} \right)^2 + \nu^2_2 \eta^d_{12} \left( \eta^d_{21} m^2_{Q_1 Q_2} \frac{\delta}{\delta m^2_{d_L}} + \eta^d_{11} m^2_{d_1 d_2} \frac{\delta}{\delta m^2_{d_R}} \right) + \ldots \]  

\[ \Delta m^2_{s, d_L} \Delta m^2_{s, d_R} = \nu^2_1 \left( \eta^d_{21} \eta^d_{12} + \frac{1}{2} \eta^d_{21} \right) \left( \eta^d_{22} m^2_{Q_1 Q_2} \frac{\delta}{\delta m^2_{d_L}} + \eta^d_{11} m^2_{d_1 d_2} \frac{\delta}{\delta m^2_{d_R}} \right) + \ldots \]  

\[ \left( \frac{1}{2} \eta^d_{21} \right) \left( \eta^d_{22} m^2_{Q_1 Q_2} \frac{\delta}{\delta m^2_{d_L}} + \eta^d_{11} m^2_{d_1 d_2} \frac{\delta}{\delta m^2_{d_R}} \right) + \ldots \]  

\[ \times I''_4\left( m^2_{d_L}, m^2_{d_R}, M^2_{\tilde{\chi}_i}, M^2_{\tilde{\chi}'_j} \right) \]  

(4.23)
where the ellipsis stands for higher order contributions, the derivatives are assumed to act only of one of the arguments of the appropriate $I''_4$, and we have only included terms proportional to $\Delta m_{sL\bar{d}R}^2$ and $\Delta m_{sR\bar{d}L}^2$ in the dominant gluino contributions, not in the neutralino contributions where such terms also appear but are smaller.

### 4.2 Analytical Discussion

We begin by noting that the standard model term from equation (4.4) gives a contribution to $\Delta m_K$ of around $2 \times 10^{-15}$ GeV, which given the large theoretical errors in the input parameters to the calculation is in reasonable agreement. Hence we shall here require that the new contributions from SUSY and RPV do not destroy this agreement by being larger than the experimental limit themselves. However, it is important to note that there are unknown relative signs between the various contributions, and hence that there can be cancellations.

The direct contributions have been discussed in refs. [3, 8, 19, 21, 22]. The most stringent bound on couplings is that from the tree level diagram of Figure 3b which leads to a constraint [8, 22]

$$\tilde{\lambda}^i_{12} \tilde{\lambda}^i_{21}(M_Z) \equiv \lambda^i_{12} \lambda^i_{21}(M_Z) K_{j1} K_{k2} \lesssim 1.3 \times 10^{-7} \left( \frac{\tilde{m}_{\tilde{\nu}_i}}{500 \text{GeV}} \right)^2$$

The box diagrams of Figures 3c and 3d and the competing diagram with an internal W line of Figure 3e lead to bounds on products of two couplings of order $10^{-2}$ to $10^{-3}$ [3, 21] for a very light spectrum. Ref. [19] was mostly concerned with the case where, in the weak basis, only one $R$-Parity violating coupling is non-zero. This leads to bounds of order 0.1 on $\lambda^i_{1ik}$ and $\lambda^i_{12k}$ for a very light spectrum, but scaling more weakly with masses.

We now consider how large the expected contributions from the main indirect processes are, beginning with non-zero $\lambda''$, by deriving approximate bounds from each of the most important direct and indirect contributions in turn. The term which
we shall bound will be $\lambda''^2 \equiv \lambda''^\prime_{31} \lambda''^\prime_{32}$, and for purposes of the discussion in this section we shall assume that all superpartners are degenerate at $\tilde{m} \simeq 3M_{1/2}$.

The main indirect contributions are the gluino mediated box diagrams, which arise because $\Delta m^2_{\tilde{s}_L \tilde{d}_R}$ is non-zero. Using the various functions in Appendix C and Eqs. (B.1) and (1.26) we can find $d^\tilde{g}_{R \tilde{R}}$ and hence derive an approximate bound of

$$\lambda''^2(M_Z) \lesssim 0.07 \left( \frac{\tilde{m}}{500 \text{GeV}} \right)$$  (4.31)

Although other indirect contributions exist, this is the simplest and is sometimes the largest. However, the various contributions from $\Delta m^2_{\tilde{s}_L \tilde{d}_R}$ and $\Delta m^2_{\tilde{s}_R \tilde{d}_L}$ often in practice give comparably large effects, and so equation (4.31) should be treated with caution.

The direct contributions are those from the $\lambda''$ box diagram and the $\lambda'' - W$ diagram [3, 21]. The former gives a bound of

$$\lambda''^2(M_Z) \lesssim 0.01 \left( \frac{\tilde{m}}{500 \text{GeV}} \right)$$  (4.32)

which may be rather inaccurate if the first index of the non-zero $\lambda''$ is 3. The second gives

$$\lambda''^\prime_{213} \lambda''^\prime_{223}(M_Z) \lesssim 0.05 \left( \frac{\tilde{m}}{500 \text{GeV}} \right)^2$$  (4.33)

$$\lambda''^\prime_{213} \lambda''^\prime_{323}(M_Z) \lesssim 0.1 \left( \frac{\tilde{m}}{500 \text{GeV}} \right)^2$$  (4.34)

$$\lambda''^\prime_{313} \lambda''^\prime_{223}(M_Z) \lesssim 0.2 \left( \frac{\tilde{m}}{500 \text{GeV}} \right)^2$$  (4.35)

$$\lambda''^\prime_{313} \lambda''^\prime_{323}(M_Z) \lesssim 0.1 \left( \frac{\tilde{m}}{500 \text{GeV}} \right)^2$$  (4.36)

respectively, where the scaling is very approximate for the cases involving stop squarks.

These numbers are in reasonable agreement with those in reference [21] given the uncertainties in (for example) $m_s$.

We conclude that although the indirect contribution appears rather smaller, none of the contributions to $\Delta m_K$ is obviously negligible in the region of interest, and so we
must turn to a numerical analysis, noting that we expect \( \lambda''(M_{GUT}) \sim 10^{-3} \) to give a result comparable with the experimental limit for masses of order a few hundred GeV.

We now turn to the case of non-zero \( \lambda' \). Here the tree level diagram is inevitably completely dominant where it exists [8, 22], and even with only one non-zero coupling in the weak basis there can be measurable effects [13]. Note that here it is possible to arrange the couplings so that the indirect contributions are zero but the direct contribution is not, for example if we arrange for non-zero \( \lambda'_{i13}, \lambda'_{223}, \lambda'_{212}, \) and \( \lambda'_{122} \). However, such scenarios seem contrived given the need to avoid the extremely tight bounds from the tree level term while simultaneously having non-negligible values for four couplings, requiring remarkable cancellations in \( \lambda'_{i12}\lambda'_{i21} \).

The only scenario which we shall consider is thus non-zero \( \lambda'_{i13} \) and \( \lambda'_{i23} \). For this case the direct contribution lead to a constraint which is similar to that of Eq. (4.31),

\[
\lambda'^2(M_Z) \lesssim 0.07 \left( \frac{\tilde{m}}{500\text{GeV}} \right) \quad (4.37)
\]

where again this result is very approximate given the number of different contributions which can be relevant. There is no \( \lambda' - W \) diagram by construction since it only exists when the tree level diagram exists, while the direct contribution from the box diagram then gives

\[
\lambda'^2(M_Z) \lesssim 8 \times 10^{-3} \left( \frac{\tilde{m}}{500\text{GeV}} \right) \quad (4.38)
\]

Since these results are in general only reliable as order of magnitude estimates, we use a numerical study to find out which contributions are most significant.

### 4.3 Numerical Results

The results for \( \Delta m_K \) are fairly straightforward. We find that for each scenario which we have considered the indirect contributions are in fact completely dominated by the direct ones, with the gluino term being between one and two orders of magnitude
too small to compete with the direct box diagrams. We illustrate this in Figure 4, which shows a typical situation. Here we have set $\lambda''_{213}(M_{GUT}) = \lambda''_{223}(M_{GUT}) = 0.02$, $M_{1/2} = 100$ GeV, $A_0 = 0$, $\tan \beta = 10$, $\mu_4 < 0$, and show the relevant contributions.

Apart from the fact that the direct contributions are dominant here, we note that the contribution from diagrams with one neutralino and one gluino line is comparable in magnitude to that from gluinos alone. This clearly will be significant for models involving mass insertions from a GUT theory. In general we find that for flavour violation in the left handed sector the contributions from charginos and mixed gluino-neutralino diagrams are up to half that of the gluino, while for right handed flavour violation the chargino contribution is negligible but the mixed neutralino-gluino contribution is similar.

In summary then, we find that in practice this is the only process which has been studied where the indirect contributions are generically overwhelmed by the direct one, and hence we can essentially simply use the results quoted above in equations (4.32) and (4.38) to bound the couplings, thus confirming the results of references [8, 19, 21, 22]. Why this should be so is unclear, since the formulae for the contributions to $\Delta m_K$ are so complicated, but it appears the values of the various four point functions are simply such as to suppress the indirect contributions, while for the three point functions relevant for $\mu \to e\gamma$ and $b \to s\gamma$ the indirect contributions are enhanced. We also find that different parts in the gluino contribution are of different signs and comparable magnitude, and so tend to partially cancel.

Finally, we note that the qualitative nature of this result is such that the direct contribution will also dominate in $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$ mixing.
5 Conclusion

We now conclude with a brief summary of our results. We have extended our analysis of the RGEs for $R$-Parity violating supersymmetry to include the effects of the CKM matrix, and have studied two well-known flavour changing processes, including both the direct contributions, with $R$-Parity violating couplings at the vertices of diagrams, and the indirect ones, where flavour violation arises through soft masses generated by the Renormalisation Group Equations.

We first showed that constraints on $\lambda'$ can be derived by demanding that the sneutrino masses not be driven below their experimental limits, which gives a bound of around 0.3 on the low energy values of the $\lambda'$ couplings, with the particularly interesting feature that it is extremely insensitive to the sparticle spectrum and does not disappear in the limit where the masses go to infinity. Given that the sparticle spectrum is now known to be heavy (with many limits well above 100 GeV if we assume unification) this means that this bound is one of the tightest existing in such a scenario.

For the case of $b \rightarrow s\gamma$ we find that the indirect contributions caused by the flavour violation are dominant, and enhance the amplitude contribution by up to an order of magnitude, allowing new constraints on both $\lambda''$ and $\lambda'$. However, these are rather weak, and the $\lambda'$ constraints are weaker than those from sneutrino masses derived earlier in this paper.

With regard to $K^0 - \bar{K}^0$ mixing, we have included all the indirect contributions including those from gluino, neutralino, chargino, and mixed neutralino-gluino diagrams. We find that here the chargino and mixed diagrams are sometimes comparable in size to those from gluinos, with consequences for the study of flavour changing from GUT theories. However, in the context of $R$-Parity violation, we find that the indirect contributions are consistently around an order of magnitude or more less than the
direct ones, so that for this process and for the similar ones of $B^0 - \bar{B}^0$ and $D^0 - \bar{D}^0$ mixing we cannot improve on results in the literature

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A Renormalisation Group Equations

We now explain how to derive the RGEs which we shall use. Full RGEs for our model are given in reference [10], but there are two slight complications which arise when considering quark flavour violation which do not occur in the lepton flavour violating processes considered there. The first of these is that the MSSM (and the SM) contain QFV explicitly through the CKM matrices and through the MSSM soft trilinears. The second is that the RGEs of [10] are presented in a weak basis while we work in a fermion mass eigenstate basis.

The inclusion of the CKM matrix is straightforward, since defining

$$\Lambda_d^{ij} = \delta_{ij} h_d^j$$

with $h_d^j$ one of the eigenvalues of the Yukawa matrix $h_d^{ij}$, and similarly for $\Lambda_u$, $\Lambda_e$, we can simply put

$$\lambda'_{4ij} = -h_{ij}^d = -K^* \Lambda_d = -\Lambda_d - k\Lambda_d$$

where $K$ is the CKM matrix, which we shall henceforth consider to be real, defining $K = 1 + k$ so that $k$ is anti-symmetric to first order in $k$. A full description of our other notation is contained in reference [10]. This leads to off-diagonal terms in the
running of the soft squark masses

\[
16\pi^2 \frac{dm^2_{Q_iQ_j}}{dt} \simeq k_{ij}(h_j^2 - h_i^2)(m^2_{Q_i} + m^2_{Q_j} + 2m^2_{H_1}) + 2k_{ij}(h_j^2 m^2_{d_i} - h_i^2 m^2_{d_i}) + 2\eta^u_{ik}\eta^u_{kj} + 2\eta^d_{ik}\eta^d_{kj}
\]

which are in addition to the \(R\)-Parity violating contributions. Here \(\eta^d\) is given at the unification scale by \(\eta^d = h^d A_0 = K^* \Lambda^d A_0\), but in general need not run in such a way as to preserve this relationship, and we have assumed that the diagonal terms in the mass matrix \(m^2_x := m^2_{xx}\) are very much larger than the off-diagonal ones. Including the down and lepton soft trilinears in the equations given in [10] can be done by treating \(H_1\) as \(L_4\) as noted there.

Including the full matrix of \(\eta^u_{ij}\) is straightforward. We give here the full RGE

\[
16\pi^2 \frac{d\eta^u_{ij}}{dt} = \eta^u_{ij}(3h^u_k + 5h^u_i + 4h^u_j) + 6\delta_{ij}h^u_i h^u_k \eta^u_{kk} + 4h^u_i \lambda''_{mn} \lambda''_{jmn} + 2\eta^u_{ik} \lambda''_{mkn} \lambda'_{min} + 2h^u_i \lambda'_{mjn} C'''_{min} - \eta^u_{ij} \left(\frac{13}{9} g_1^2 + 3g_2^2 + \frac{16}{3} g_3^2\right) + 2\delta_{ij}h^u_k \left(\frac{13}{9} g_1^2 M_1 + 3g_2^2 M_2 + \frac{16}{3} g_3^2 M_3\right)
\]

where we have included the down and lepton Yukawa couplings and trilinear soft terms only implicitly (i.e. through the \(\lambda\), \(\lambda'\), \(C\), and \(C''\)). We also must then include a term in the RGE for \(C''_{ijk}\) of \(2\eta^u_k h^u_i \lambda'_{ik} + h^u_i C''_{ijk}\) and one in the RGE for \(C'''_{ijk}\) of \(4\eta^u_k h^u_i \lambda''_{ij} + 2h^u_i C''''_{ijk}\). Inclusion of the remaining Yukawa couplings is trivial.

The second and rather more complicated issue is that of the difference between the weak and mass bases. We have three Yukawa matrices \(h^u\), \(h^d\), \(h^e\), and it is conventional to choose a basis such that

\[
h^u = \Lambda^u \quad h^d = K\Lambda^d \quad h^e = \Lambda^e
\]

where \(K\) is now being taken real. Such a choice is also a weak eigenstate basis. However, these relations are not stable under the RGEs with either a non-trivial
CKM matrix or $R$-Parity violating couplings. Hence we will find in general that all of
the Yukawa matrices will have non-zero off-diagonal elements, greatly complicating
our analysis. The simplest way of dealing with this is to define a basis where the field
rotations to impose equation (A.3) are performed in a scale-dependent way. Then
equation (A.3) will always be satisfied, but the RGEs will differ slightly.

If we define

$$\Lambda^u = U h^u V^T_u$$  \hspace{1cm} (A.6)

where $U$, $V$ are orthogonal matrices with explicit dependence on the renormalisation
scale, then

$$\dot{\Lambda}^u = \dot{U} h^u V^T_u + U \dot{h}^u V^T_u + U h^u \dot{V}^T_u$$  \hspace{1cm} (A.7)

where the dots indicate $16\pi^2 d/dt$ and defining $u = \dot{U} U^T$ and $v^u = \dot{V} u V^T$ we find that

$$\dot{\Lambda}^u = u \Lambda^u - \Lambda^u v^u + \gamma Q \Lambda^u + \Lambda^u (\gamma^u + \gamma^{H_2})$$  \hspace{1cm} (A.8)

where the anomalous dimension matrices $\gamma$ are solved in the basis where equa-
tion (A.3) is satisfied. Hence we can solve for $u$ and $v^u$ in terms of the Yukawa
couplings and anomalous dimensions. The situation for the down-type Yukawas is
slightly more complicated. We choose to define

$$\Lambda^d = K^T (W h^d V^T_d)$$  \hspace{1cm} (A.9)

where $K$ is the CKM matrix and $W$ and $V_d$ are unitary as before. Hence we may
calculate $w$ and $v^d$ defined by $w = \dot{W} W^T$ and $v^d = \dot{V}_d V^T_d$. For the leptonic case we
define $u^e$ and $v^e$ exactly as for up-type quarks.

Imposing that the $\Lambda$ remain diagonal we find

$$u_{ij} = -\gamma_{ij}^Q \left( \frac{h_{ij}^2 + h_{ii}^2 - h_{ij}^2}{h_{ii}^2 - h_{ij}^2} \right) - \gamma_{ij}^u \left( \frac{2h_{ii}^u h_{ij}^u}{h_{ii}^2 - h_{ij}^2} \right)$$

$$v_{ij}^u = -\gamma_{ij}^Q \left( \frac{2h_{ii}^u h_{ij}^u}{h_{ij}^2 - h_{ii}^2} \right) - \gamma_{ij}^u \left( \frac{h_{ij}^2 + h_{ii}^2}{h_{ii}^2 - h_{ij}^2} \right)$$
Here we use $\tilde{w} = K^T w K$ and $\tilde{\gamma} = K^T \gamma K$.

Explicitly expanding taking the masses to be very strongly ordered with $h_1 \ll h_2 \ll h_3$, we find that for $i > j$

\begin{align}
\tilde{w}_{ij} &= -\gamma_{ij}^Q \left( \frac{h^d_j + h^d_i}{h^d_j - h^d_i} \right) - \gamma_{ij}^d \left( \frac{2h^d_i h^d_j}{h^d_j - h^d_i} \right) \\
v^d_{ij} &= -\gamma_{ij}^Q \left( \frac{2h^d_i h^d_j}{h^d_j - h^d_i} \right) - \gamma_{ij}^d \left( \frac{h^d_j + h^d_i}{h^d_j - h^d_i} \right) \\
u^e_{ij} &= -\gamma_{ij}^L \left( \frac{h^e_j + h^e_i}{h^e_j - h^e_i} \right) - \gamma_{ij}^e \left( \frac{2h^e_i h^e_j}{h^e_j - h^e_i} \right) \\
u^c_{ij} &= -\gamma_{ij}^L \left( \frac{2h^e_i h^e_j}{h^e_j - h^e_i} \right) - \gamma_{ij}^e \left( \frac{h^e_j + h^e_i}{h^e_j - h^e_i} \right)
\end{align}

(A.10)

Hence we can simply use the RGEs in reference [10], but adding the explicit CKM contributions and including the conversion of basis by, for example

\begin{equation}
16\pi^2 \frac{d m^2_{Q_i Q_j}}{dt} \quad \text{mass basis} \quad = \quad 16\pi^2 \frac{d m^2_{Q_i Q_j}}{dt} \quad \text{weak basis} \quad + \sum_k (u_{ik} m^2_{Q_k Q_j} + u_{jk} m^2_{Q_i Q_k}) \quad (A.12)
\end{equation}

This gives the fermion mass basis RGEs for the off-diagonal terms in the soft masses

\begin{equation}
16\pi^2 \frac{d m^2_{Q_i Q_j}}{dt} = \sum_{mn} \left( \lambda'_{mni} \lambda'_{mnj} (m^2_{Q_i} + m^2_{Q_j}) + 2m^2_{u_n} + 2m^2_{d_m} + 2C'_{min}C'_{mnj} \right)
\end{equation}

27
\[ 16\pi^2 \frac{d m_{u_{ij}}^2}{d t} = \sum_{mn} \left( \lambda''_{immn} \lambda''_{jmn} (m_{u_i}^2 + m_{u_j}^2 + 4m_{u_n}^2) + 2C''_{immn} C''_{jmn} \right) + 4\eta_{ik}^u \eta_{jk}^u + v_{ij}^u (m_{u_i}^2 - m_{u_j}^2) \] (A.13)

\[ 16\pi^2 \frac{d m_{d_{ij}}^2}{d t} = \sum_{mn} \left( 2\lambda''_{immn} \lambda''_{jmn} (m_{d_i}^2 + m_{d_j}^2 + 2m_{Q_n} + 2m_{L_m}) + 4C''_{immn} C''_{jmn} \right) + 4\eta_{ik}^d \eta_{jk}^d + v_{ij}^d (m_{d_i}^2 - m_{d_j}^2) \] (A.14)

\[ 16\pi^2 \frac{d m_{L_{ij}}^2}{d t} = \sum_{mn} \left( \lambda_{immn} \lambda_{jmn} (m_{L_i}^2 + m_{L_j}^2 + 2m_{e_n} + 2m_{L_m}) + 2C_{immn} C_{jmjn} \right) + 3\lambda_{immn} \lambda_{jmn} (m_{L_i}^2 + m_{L_j}^2 + 2m_{Q_n} + 2m_{Q_m}) + 6C''_{immn} C''_{jmjn} \right) + 2\eta_{ik}^e \eta_{jk}^e + w_{ij}^e (m_{L_i}^2 - m_{L_j}^2) \] (A.15)

\[ 16\pi^2 \frac{d m_{e_{ij}}^2}{d t} = \sum_{mn} \left( \lambda_{mmn} \lambda_{mnj} (m_{e_i}^2 + m_{e_j}^2 + 4m_{L_n}^2) + 2C_{mmn} C_{mnj} \right) + 4\eta_{ik}^e \eta_{jk}^e + v_{ij}^e (m_{e_i}^2 - m_{e_j}^2) \] (A.16)

It is clear from these equations that when we consider the first and second generations, where the soft masses are nearly degenerate, the difference in basis is totally irrelevant, but for cases involving the third generation which is typically rather lighter the effects may be non-negligible.

The RGE for the CKM matrix is then

\[ 16\pi^2 \frac{d k_{ij}}{d t} = u_{ij} - w_{ij} \] (A.18)

### B Approximate Solutions of RGEs

For our discussion of the indirect contributions to the various processes, it will now be helpful to present some approximate formulae for the various off-diagonal mass insertions. For convenience we will present here formulae for all the elements, not merely those involved in \( b \to s \gamma \). If we begin by switching off the CKM matrix we
find

\[ m^2_{Q_i Q_j}(M_Z) = -\sum_{m,n} \lambda'_{mn} \lambda'_{mjn}(M_{GUT})(5m_0^2 + 15M_{1/2}^2 - 5A_0 M_{1/2} + A_0^2) \]

\[ m^2_{d_i d_j}(M_Z) = -\sum_{m,n} \lambda'_{mn} \lambda'_{mnj}(M_{GUT})(10m_0^2 + 30M_{1/2}^2 - 10A_0 M_{1/2} + 2A_0^2) \]

\[ -\sum_{m,n} \lambda''_{mn} \lambda''_{mnj}(M_{GUT})(10m_0^2 + 50M_{1/2}^2 - 20A_0 M_{1/2} + 5A_0^2) \]

\[ m^2_{u_i u_j}(M_Z) = -\sum_{m,n} \lambda''_{mn} \lambda''_{jmn}(M_{GUT})(10m_0^2 + 50M_{1/2}^2 - 20A_0 M_{1/2} + 5A_0^2) \]

(B.1)

while for the \( \Delta m^2_{LR} \) and \( \Delta m^2_{RL} \) insertions we find

\[ \eta_{d_12}^d \nu_1 = m_s \sum_{mn} \lambda'_{m1n} \lambda'_{m2n}(M_{GUT})(3M_{1/2} - 1.5A_0) \]

\[ \eta_{d_21}^d \nu_1 = m_s \sum_{mn} \lambda'_{mn1} \lambda'_{mn2}(M_{GUT})(9M_{1/2} - 4.5A_0) \]

\[ + m_s \sum_{mn} \lambda''_{mn1} \lambda''_{mn2}(M_{GUT})(9M_{1/2} - 3.5A_0) \]

(B.2)

where \( m_s \) is the running strange mass at \( M_Z \). Since all these formulae depend quite strongly on \( \alpha_3 \) they should not be trusted to better than about a factor of two. Note that the \( \Delta m^2_{LR} \) and \( \Delta m^2_{RL} \) insertions are very much smaller than the others because they are proportional to a soft mass times a strange quark mass rather than two soft masses.

Including the CKM matrix is straightforward, and leads extra contributions to those given above of

\[ m^2_{Q_i Q_j}(M_Z) = -\frac{10^{-7}}{\cos^2 \beta}(3m_0^2 + 10M_{1/2}^2 - 4A_0 M_{1/2} + A_0^2) \]

\[ m^2_{Q_i Q_j}(M_Z) = -\frac{10^{-6}}{\cos^2 \beta}(2m_0^2 + 8M_{1/2}^2 - 3A_0 M_{1/2} + 0.8A_0^2) \]

\[ m^2_{Q_i Q_j}(M_Z) = -\frac{10^{-5}}{\cos^2 \beta}(2m_0^2 + 8M_{1/2}^2 - 3A_0 M_{1/2} + 0.8A_0^2) \]

\[ \eta_{12}^d \nu_1 = m_s(-0.8M_{1/2} + 0.2A_0) \]

\[ \eta_{21}^d \nu_1 \ll \eta_{12}^d \nu_1 \]  

(B.3)

although here the errors are larger, owing to the dependence on poorly known Yukawa couplings. Although we have only shown the contributions to \( m^2_{Q_i Q_j} \), in fact there
are also contributions to $m_{d_d}^2$ and $m_{udu_3}^2$ in particular which are much smaller but non-zero.

### C Definition of Functions

The expressions for the amplitudes in $b \to s \gamma$ and $K^0\bar{K}^0$ mixing presented earlier employ a number of functions, many of which are contained in [10], and the remainder of which are given explicitly here. Functions occurring in $b \to s \gamma$ are

\[
\begin{align*}
  f_{\gamma}^{(1)}(x) &= Q_u F_1(x) + F_2(x) \\
  f_{g}^{(1)} &= F_1(x) \\
  f_{\gamma}^{(2)}(x) &= Q_u F_3(x) + F_4(x) \\
  f_{g}^{(2)} &= F_3(x) \\
  f_{\gamma}^{(3)}(x) &= F_1(x) + Q_u F_2(x) \\
  f_{g}^{(3)} &= F_2(x) \\
  f_{\gamma}^{(4)}(x) &= F_3(x) + Q_u F_4(x) \\
  f_{g}^{(4)} &= F_4(x) \\
  f_{\gamma}^{(5)}(x) &= -Q_u F_1(x) + Q_d F_2(x) \\
  f_{g}^{(5)} &= \frac{1}{2} F_1(x) - \frac{1}{2} F_2(x) \\
  f_{\gamma}^{(6)}(x) &= -Q_d F_1(x) + Q_u F_2(x) \\
  f_{g}^{(6)}(x) &= f_{g}^{(5)}(x)
\end{align*}
\]  

(C.1)

In addition, for the $\Delta m_K$ calculation the functions $I_4$ and $J_4$ used in the text are given by

\[
\begin{align*}
  I_4(m_1^2, m_2^2, m_3^2, m_4^2) &= \int_0^1 dx dy dz \frac{z(1-z)}{D_4} \\
  J_4(m_1^2, m_2^2, m_3^2, m_4^2) &= \int_0^1 dx dy dz \frac{z(1-z)}{D_4^2}
\end{align*}
\]  

(C.2)

where we have defined

\[
D_4(m_1^2, m_2^2, m_3^2, m_4^2) = [xm_1^2 + (1-x)m_2^2]z + [ym_3^2 + (1-y)m_4^2](1-z)  
\]  

(C.3)
Note that $I_4$ and $J_4$ are totally symmetric about interchange of arguments, and can be simplified in certain limits as follows.

\[
I_4(m^2, m^2, M^2, M^2) = \frac{1}{M^2} F_4(x) \tag{C.4}
\]

\[
I_4(m_1^2, m_2^2, M^2, 0) = \frac{1}{M^2} \left[ \frac{(1-x_1)x_2 \ln x_2 - (1-x_2)x_1 \ln x_1}{2(x_1-1)(x_1-x_2)(x_2-1)} \right] \tag{C.5}
\]

\[
I_4(m_1^2, m_2^2, M^2, M^2) = \frac{1}{M^2} \left[ \frac{1}{2(1-x_1)(1-x_2)} \right.
+ \frac{1}{x_2-x_1} \left( \frac{x_2^2 \ln(x_2)}{2(1-x_2)^2} - \frac{x_1^2 \ln(x_1)}{2(1-x_1)^2} \right) \right] \tag{C.6}
\]

\[
I_4(m^2, M^2, 0, 0) = \frac{1}{M^2} \left[ \frac{\ln x}{2(x-1)} \right] \tag{C.7}
\]

\[
I_4(m^2, m^2, M^2, 0) = \frac{1}{2m^2} \left[ \frac{1-y + y \ln(y)}{(1-y)^2} \right] \tag{C.8}
\]

\[
I_4''(m^2, m^2, M^2, M^2) = \frac{1}{m^8} G(y) = -\frac{1}{2m^6} \tilde{f}_6(x) \tag{C.9}
\]

\[
J_4(m^2, m^2, M^2, M^2) = \frac{1}{m^4(M_1^2 - M_2^2)} \left[ y_1 F_1(y_1) - y_2 F_1(y_2) \right] \tag{C.10}
\]

\[
J_4''(m_1^2, m_2^2, M^2, M^2) = \frac{1}{M^6} \left[ \frac{x_1 + x_2 + x_1^2 + x_2^2 - 6x_1x_2 + x_1^2x_2 + x_1x_2^2}{2(x_1-1)^2(x_2-1)^2(x_1-x_2)^2} \right.
+ \left( \frac{x_1x_2 - x_1^2}{x_1-1} \right) \ln(x_1) - \left( \frac{x_1x_2 - x_2^2}{x_2-1} \right) \ln(x_2) \right] \tag{C.11}
\]

\[
J_4(m_1^2, m_2^2, M^2, M^2) = \frac{1}{M^4} \left[ \frac{-1}{(1-x_1)(1-x_2)} \right.
+ \left( \frac{x_1 \ln(x_1)}{1-x_1} - \frac{x_2 \ln(x_2)}{1-x_2} \right) \right] \tag{C.12}
\]

\[
J_4(0, m_1^2, m_2^2, M^2) = \frac{1}{M^4} \left[ \frac{(x_2-1) \ln(x_1) - (x_1-1) \ln(x_2)}{(x_2-1)(x_2-x_1)(x_1-1)} \right] \tag{C.13}
\]

\[
J_4''(m^2, m^2, M^2, M^2) = \frac{2}{m^4 M^4} F(x) = \frac{1}{m^8} x f_6(x) \tag{C.14}
\]

\[
J_4''(m_1^2, m_2^2, M^2, M^2) = \frac{1}{M^8} \left[ \frac{-2(1-x_1 - x_2 + x_1^2 + x_2^2 - x_1x_2)}{(x_1-1)^2(x_2-1)^2(x_1-x_2)^2} \right.
+ \left( \frac{x_1x_2 - x_1^2}{x_1-2} \right) \ln(x_1) \right.
- \left( \frac{x_1x_2 - x_2^2}{x_2-1} \right) \ln(x_2) \right]
\right]
\tag{C.15}
\]

\[
J_4''(m_1^2, m_2^2, M_1^2, M_2^2) = \frac{1}{m_2^8} \left[ \frac{y_2 \ln(y_2)}{(y_2-1)^2(y_2-y_2)} - \frac{y_1 \ln(y_1)}{(y_1-1)^4(y_1-y_2)} \right.
+ \left( \frac{y_2}{6(y-1)^3(y_2-1)^3} \right) \ln(y_2) - \left( \frac{y_1}{6(y_1-1)^3(y_2-1)^3} \right) \ln(y_1) \right]
\tag{C.16}
\]

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\[-2(y_1^2 + y_2^2 - 5y_1 y_2) - 5(y_1 + y_2)y_1 y_2 + y_1^2 y_2^2)\]\]

Here \(x = m^2/M^2\), \(x_i = m_i^2/M^2\), \(y = M^2/m^2\) and \(y_i = M_i^2/m^2\), and

\[
\frac{\partial^2}{\partial m_1^2 \partial m_2^2} I'_4(m_1^2, m_2^2, m_3^2, m_4^2) = \frac{\partial^2}{\partial m_1^2 \partial m_2^2} J'_4(m_1^2, m_2^2, m_3^2, m_4^2)
\]

\[(C.17)\]

We give \(f_6\) and \(\tilde{f}_6\) defined in reference [1] for convenience when comparing our results with other authors.

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D Figure Captions

Figure 1
Absolute values for various contributions to the $b \rightarrow s\gamma$ amplitude $\tilde{A}_{LR}$. Parameters are $\lambda'_{121}(M_{GUT}) = \lambda'_{131}(M_{GUT}) = 0.05$, $m_0 = 100$ GeV, $A_0 = 0$, $\tan \beta = 10$, $\mu_4 < 0$. Contributions from Higgs (dotted), SM (horizontal dotted), neutralino (double dashes), chargino (dashes for the MSSM part with triple dashes for the remainder), gluino (quadruple dashes), direct RPV diagrams (dot dashed) and total (solid) are shown. The experimental upper and lower limits are shown as horizontal solid lines.

Figure 2
Absolute values for various contributions to the $b \rightarrow s\gamma$ amplitude $\tilde{A}_{RL}$. Parameters are $\lambda''_{112}(M_{GUT}) = \lambda''_{113}(M_{GUT}) = 0.1$, $m_0 = 100$ GeV, $A_0 = 0$, $\tan \beta = 10$, $\mu_4 < 0$. Contributions from neutralino (double dashes), gluino (quadruple dashes), direct RPV diagrams (dot dashed) and total (solid) are shown. The experimental upper limit shown varies with $M_{1/2}$ because the MSSM part of $\tilde{A}_{LR}$ does.

Figure 3
Some of the diagrams contributing to $K^0 - \bar{K}^0$ mixing.

Figure 4
Absolute values of various contributions to $\Delta m_K$. Parameters are $\lambda''_{213}(M_{GUT}) = \lambda''_{223}(M_{GUT}) = 0.02$, $M_{1/2} = 100$ GeV, $A_0 = 0$, $\tan \beta = 10$, $\mu_4 < 0$. Contributions from neutralino (double dashes), gluino (quadruple dashes), mixed gluino-neutralino (dashed), direct RPV diagrams with and without $W$ lines (dotted and dot dashed respectively) and total (solid) are shown, together with the experimental upper limit (horizontal solid line).
Figure 1:

Amplitude for $b \rightarrow s \gamma$ (GeV$^{-2}$) vs $M_{1/2}$ (GeV)

The graph shows the amplitude for the decay $b \rightarrow s \gamma$ in units of GeV$^{-2}$ as a function of $M_{1/2}$ in GeV. The lines represent different theoretical models, with each line corresponding to a specific parameter set. The scale on the y-axis ranges from $10^{-10}$ to $10^{0}$ and the scale on the x-axis ranges from 100 to 500 GeV. The graph includes a log-log scale for both axes, indicating a power-law dependence of the amplitude on $M_{1/2}$. The exact nature of the dependence is not specified in the text, but it is clear that the amplitude decreases with increasing $M_{1/2}$.
Figure 2:

Amplitude for $b \to s \gamma$ (GeV$^2$)

$M_{1/2}$ (GeV)

Experimental limit
Figure 3:
Figure 4: