Quark-lepton flavored unification

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The unification of the quark and charged lepton flavor structures is revisited using the tools of Minimal Flavor Violation (MFV). Under that framework, we first identify a unique point in parameter space where flavor universality is maximally violated. Though from the point of view of the usual polynomial MFV expansions, standing close to that point would require unacceptable fine-tunings, it is automatically attained with infinite geometric MFV series. Remarkably, this peculiar point is precisely where the charged lepton Yukawa coupling can be expressed naturally in terms of those of the quarks. The most striking consequence would be that the physical electron field is dominantly the gauge lepton field of the third generation.

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1. Introduction

Not long after the advent of the Standard Model (SM), the possibility to further unify the fundamental interactions in some large but simple gauge group emerged. While this has proven very successful for the gauge sector, and is often regarded as one justification for the peculiar gauge quantum numbers of the SM fermions, these models ran into problems in the flavor sector. For example, the minimal SU(5) model has only two Yukawa couplings, and predicts $Y_d = Y^T$. Even accounting for QCD effects, this is in stark disagreement with measured quark and lepton masses. The standard way out is to introduce a third Yukawa coupling, in which case accounting for QCD effects, this is in stark disagreement with measured quark and lepton masses.

2. Could the Yukawa couplings be redundant?

To formulate the hypothesis of an intrinsic redundancy of the SM Yukawa couplings, we use the language of Minimal Flavor Violation (MFV) [2]. First, remember that gauge interactions being flavor blind, they are invariant under the flavor symmetry [3].

$$G_F = U(3)^5 = U(3)_{qL} \otimes U(3)_{qR} \otimes U(3)_{uR} \otimes U(3)_{dR} \otimes U(3)_{eR},$$  \hspace{1cm} (2.1)

with $q_L = (u,d)_{L}$, $\ell_L = (\nu,e)_{L}$. What characterizes the flavor couplings is their breaking of $G_F$. Technically, they can be treated as spurions, i.e., non-dynamical fields carrying definite $G_F$ charges: $Y_u \sim (\bar{3},3,1,1,1)$, $Y_d \sim (\bar{3},3,1,1,1)$, and $Y_e \sim (1,1,1,\bar{3},3)$.

Imagine now that at some high scale, there are only two fundamental flavor-breaking terms $Y_1$ and $Y_2$. At the low scale, $Y_{u,d,e}$ must be induced by $Y_{1,2}$, at which point they would be expressed as polynomials of the form $Y_{u,d,e} = P_{u,d,e}(Y_1,Y_2)$. Provided these polynomials are natural, i.e. involve $O(1)$ coefficients, the unknown $Y_{1,2}$ can be eliminated and one of the Yukawa coupling can be expressed as a natural polynomial in the other two. In practice, for this to work, we must choose a reduced flavor group and its two spurions. Inspired by the SU(5) model, we take:

$$G'_F = U(3)^3 = U(3)_{qL=\ell_L} \otimes U(3)_{uR} \otimes U(3)_{dR=\ell_R},$$  \hspace{1cm} (2.2)

with $Y_1 \sim Y_u \sim (\bar{3},3,1)$ and $Y_2 \sim Y_d \sim (\bar{3},1,3)$. Then, $Y_e$ is no longer fundamental but since it transforms as $Y_d$, its general expression is a polynomial in $Y_u$ and $Y_d$ of the form

$$Y_e = c_0 Y_d \cdot (1 + c_1 Y^1_u Y_u + c_2 Y^2_u Y_d + \ldots).$$  \hspace{1cm} (2.3)

Having established the $G'_F$-invariant expansion of $Y_e$, the spurions can now be frozen to their physical values. The $G'_F$ symmetry is large enough to attain the gauge basis where $Y_u \sim m_u V_{CKM}$. $Y_d \sim m_d$. Since $Y_e$ is not necessarily diagonal in that basis, our goal is to find coefficients such that $g_{R} Y_e s_{R} \sim m_e$ for some unitary $g_{RL}$. Now, of course, this is always possible since $(Y^N_u Y_u)^n$ and $(Y^N_d Y_d)^n$ for $n = 1,2,\ldots$ form an algebraic basis for $3 \times 3$ matrices. What is crucial to the whole redundancy interpretation is that these coefficients must be of $O(1)$. This is extremely constraining because the values of $Y_u$ and $Y_d$ are very peculiar, and the basis is nearly singular (the matrices...
(Y_{u,d} Y_{u,d})^a are nearly parallel to Y_{u,d} Y_{u,d}) [4]. To simplify the problem, let us keep only the first three terms in Eq. (2.3). Then, remarkably, the system does allow for a natural solution in various scenarios [1]. For example, c_{0,1,2} = 20, -7.9, 5.3 in the MSSM assuming \tan \beta = 50.

Looking back at Eq. (2.3), something special has to happen. If we write Y_e = c_0 Y_d \cdot X, the matrix X is responsible for transmuting the singular values of Y_d to that of Y_e, which are significantly different. The only way this can happen with c_{1,2} of \mathcal{O}(1) is to take advantage of the large top and bottom Yukawa couplings and ensure 1 + c_1 y_1^2 + c_2 y_2^2 \approx 0, so that the matrix X is nearly singular with X^{33} \approx 0 \ll X^{11,22} \approx 1. Interestingly, at that point, g_R Y_e g_L \sim m_e implies a reordering of the generations

\begin{equation}
\begin{pmatrix}
e_L \\
\mu_L \\
\tau_L
\end{pmatrix}_{\text{gauge}} \approx \begin{pmatrix}
0.03 & 0.98 & 0.20 \\
0.06 & 0.20 & 0.98 \\
1.00 & 0.02 & 0.07
\end{pmatrix}_{\text{phys}}.
\end{equation}

In other words, the physical e_L is actually the third generation leptonic state, partnered with (t, b)_L.

3. Geometric Minimal Flavor Violation

Unfortunately, the solution found in the previous section is intrinsically unphysical. Indeed, as shown in Fig. 1a, the singularity det X = 0 is very narrow, so the c_i have to be precisely fine-tuned to stay close to it. Adding terms to the expansion Eq. (2.3) does not change the picture, and we conclude that no finite polynomial relation between Y_u, Y_d, and Y_e will ever be truly natural.

Now, in general, there is no reason to expect the expansions to be finite. So, let us see if we can lift the fine-tuning by taking an infinite polynomial. Specifically, consider the geometric series:

\begin{equation}
Y_e(c_i) = c_0 Y_d \cdot (1 + \eta Y_u Y_u + \eta^2 (Y_u Y_u)^2 + \ldots) = c_0 Y_d \cdot (1 - \eta Y_u Y_u)^{-1}.
\end{equation}

The matrix X = (1 - \eta Y_u Y_u)^{-1} is diagonal with entries \((1 - \eta Y_{u,c,l})^{-1}\) in the basis where Y_u Y_u = diag(y_u^2, y_u^2, y_u^2). So, for \eta large enough, X \approx diag(1, 1, 0), which is precisely the structure we are after to solve Y_e = c_0 Y_d \cdot X. At this stage, it may seem we have traded the fine-tuning of X for its convergence. Indeed, the fact that \((1 - \eta y_i)^{-1} \to 0\) when \eta y_i \gg 1 requires the series to be analytically continued. Thus, for this to make sense, the resummed series must be the true MFV prediction, in which case it is its expansion which is ill-defined when \eta is too large.
To support this view, let us illustrate how imposing the usual polynomial MFV on some Lagrangian could lead to resummed geometric predictions. The idea is to view the geometric series as a Dyson series, with \( \eta Y^\dagger_u Y_u \) playing the role of some mass insertion. Consider thus adding a set of weak doublet \( X \) and weak singlet \( Z \) vector-like leptons, transforming under \( G_L^\prime \) as \( \ell_L \) and \( e_R \), respectively. Being vector like, these fields can be massive, and we assume \( M_X = M_Z = M1 \) for simplicity. In addition to that, we introduce a gauge-singlet scalar field \( h \) with flavored couplings to the leptons, on which we impose polynomial MFV. When \( H \) acquires its vacuum expectation value \( v_s \), the heavy lepton masses are shifted as \( \delta M_X = v_s (\alpha Y^\dagger_u Y_u + \beta Y^\dagger_d Y_d) \) and \( \delta M_Z = v_s e_Y Y^\dagger_d Y_d \) for some \( O(1) \) coefficients \( \alpha, \beta, \varepsilon \). Then, integrating out \( X \) and \( Z \) formally resums the infinite series shown in Fig. 1b, and the total effective lepton Yukawa coupling takes the form

\[
Y_e = \left( 1 + \frac{v_s}{M} e_Y Y^\dagger_d \right)^{-1} \gamma Y_d \left( 1 + \frac{v_s}{M} (\alpha Y^\dagger_u Y_u + \beta Y^\dagger_d Y_d) \right)^{-1},
\]

for some coefficient \( \gamma \). Imposing \( g_R Y_c g_L^\dagger \sim m_c \), infinitely many solutions exist, for example \( \gamma = 81, \alpha \equiv 1, \beta \equiv 1, \varepsilon \equiv -7, v_s / M = 2.1 \times 10^3 \). Importantly, no fine-tuning is implied, this solution is well-behaved (see Fig. 1c), and pushing the geometric series out of its radius of convergence is accomplished thanks to the theoretically-free ratio \( v_s / M \). We have thus achieved our goal of naturally generate \( Y_e \) out of \( Y_{u,d} \). The reordering observed in Eq. (2.4) not only remains but is also extended to the right-handed leptons.

4. Conclusion

MFV is known to force all New Physics flavor couplings to depart only slightly from flavor universality (but for chirality-flip insertions whenever required), with all flavor mixings tuned by the CKM matrix. Yet, within the parameter range attainable with MFV, there is a peculiar and unique point where flavor universality can in principle be totally lifted. Expansions like \( X = 1 + \alpha Y^\dagger_u Y_u \) become singular, \( \det X = 0 \), when \( \alpha \) is close to \(- \text{Tr}(Y^\dagger_u Y_u)^{-1} \), which is a natural value since the top quark Yukawa coupling is of \( O(1) \). Coincidentally, setting \( \alpha = - \text{Tr}(Y^\dagger_u Y_u)^{-1} \) is not a fine-tuning but arises automatically if the MFV series is geometric in the mathematical sense. Remarkably, this unique singularity within the MFV range is precisely the one needed to unify the quark and lepton flavor structures. Clearly, the same reasoning can be extended to GUT settings. For example, in \( SU(5) \), a successful unification could be obtained with only two fundamental Yukawa couplings. Extended to supersymmetric settings, geometric MFV structures generically predict light third generation squarks [5], and match natural SUSY-like scenarios. Implications for the neutrino sector remain to be studied, and could shed new light on the origin of their masses.

References

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