Squashing gravity dual of $\mathcal{N} = 6$ superconformal Chern–Simons gauge theory

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Abstract

Four-dimensional field equations are determined for perturbations of the quotient 7-sphere size and squashing parameter in 11-dimensional supergravity. The quotient 7-sphere is an $S^1$-bundle over the $\mathbb{CP}^3$ which is regarded as an $S^2$-fibration over the base $S^4$. By analyzing the AdS 4 supergravity scalar potential, the holographic supersymmetric (or nonsupersymmetric) renormalization group (RG) flow from $\mathcal{N} = 1$ (or $\mathcal{N} = 0$) $SO(5) \times U(1)$-invariant UV fixed point to $\mathcal{N} = 6$ (or $\mathcal{N} = 0$) $SU(4)_R \times U(1)$-invariant IR fixed point is obtained. The three-dimensional boundary theories are described by superconformal Chern–Simons matter theories and a dual operator corresponding to this RG flow is described.

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1. Introduction

The three-dimensional Chern–Simons matter theory with gauge group $U(N) \times U(N)$ at level $k$ which has $\mathcal{N} = 6$ superconformal symmetry was constructed recently in [1]. They describe this gauge theory as the low energy limit of $N$ M2-branes probing $\mathbb{C}^4/\mathbb{Z}_k$ singularity. At large $N$-limit, this theory is dual to the M-theory on $\text{AdS}_4 \times S^7/\mathbb{Z}_k$. In order to perform the $\mathbb{Z}_k$-quotient explicitly\(^1\), it is natural to write the 7-sphere $S^7$ metric as an $S^1$-fibration over $\mathbb{CP}^3$ [7]. When $N = 2$ with $k = 1, 2$, the theory is equivalent to Bagger–Lambert theory [8–10].

It is known in [11] that deformation from ‘round’ 7-sphere $S^7$ to ‘squashed’ one $\tilde{S}^7$ [12, 13] (see also [14, 15]) can be interpreted as renormalization group (RG) flow from UV

\(^1\) There exists a variety of orbifolds with free or nonfree actions on $S^7$ leading to different amount of supersymmetry [2]. Let us consider M2 branes at $\mathbb{C}^4/\Gamma$ singularity where the group $\Gamma$ is generated by $\text{diag}(e^{\frac{2\pi i}{a}}, e^{-\frac{2\pi i}{a}}, e^{\frac{2\pi i}{k}}, e^{-\frac{2\pi i}{k}})$ for some relatively prime integers $a$ and $k$. If $a = 1, k = 2$ we get maximal case $\mathcal{N} = 8$ with horizon manifold $\text{RP}^3$. For $a = \pm 1, k > 2$, one gets $\mathcal{N} = 6$ theory [3] where the corresponding field theory duals are present. When $a \neq \pm 1$, the theory has $\mathcal{N} = 4$ supersymmetry. Similar $\mathcal{N} = 4$ theory [4] can be obtained from the different orbifold $\mathbb{C}^2/\Gamma \times \mathbb{C}^2/\Gamma$ (see also [5]). The orbifold $\mathbb{C} \times \mathbb{C}^2/\Gamma$ provides the $\mathcal{N} = 2$ theory [6]. When $\Gamma$ is a binary dihedral group where $\mathbb{C}^2/\Gamma$ has a $D$ type singularity and we embed $\Gamma$ into $SU(2) \times SU(2)$, we get $\mathcal{N} = 5$ theory.
fixed point to IR fixed point, via AdS/CFT correspondence [16–18], by analyzing the four-dimensional effective Lagrangian from 11-dimensional supergravity solution found in [19]. The RG flow along the squashing deformation trajectory interpolates between $\mathcal{N} = 8SO(8)$-invariant conformal fixed point at the IR and $\mathcal{N} = 1$ (or $\mathcal{N} = 0$ for different orientation) $SO(5) \times SO(3)$-invariant conformal fixed point at the UV.

Then it is natural to ask what happens when we perform $\mathbb{Z}_k$-quotient [1] along the above whole RG flow [11]? At the IR fixed point, since the ‘round’ $S^7$ metric is represented by a twisted $S^3$-bundle over ‘round’ $\mathbb{CP}^3$ characterized by Fubini-Study Einstein metric [20–22], one can easily perform the $\mathbb{Z}_k$-quotient. What about at the UV fixed point? Is there any ‘squashed’ $\mathbb{CP}^3$? Nilsson and Pope [7] have realized that the ‘squashed’ $\mathbb{CP}^3$ metric can be obtained by taking the Hopf fibration of ‘round’ $S^7$ with its squashed metric from the observation by Ziller [23]. However, the explicit form for the ‘squashed’ $\mathbb{CP}^3$ metric is not presented in [7] as far as I know and it is not clear how one-parameter family of ‘squashed’ $\mathbb{CP}^3$ metric, contrary to ‘squashed’ $S^7$, arises in order to check the correct behavior of Ricci tensor in seven dimensions.

Luckily, in [24], by viewing the $\mathbb{CP}^3$ as an $S^2$-bundle over $S^4$ with the self-dual $SU(2)$ instanton, they reproduced the generic Ricci tensor for the $\mathbb{CP}^3$ [7] and the standard Fubini-Study Einstein metric (which is Kahler) arises when $\lambda^2 = 1$ where $\lambda$ is a squashing parameter while the second ‘squashed’ Einstein metric (which is nearly Kahler) arises when $\lambda^2 = \frac{1}{2}$: one-parameter family of ‘squashed’ $\mathbb{CP}^3$.

In this paper, we consider the general, one-parameter family, metric for $\mathbb{CP}^3$ described in [24], study its seven-dimensional uplift metric on an $S^1$-bundle over this $\mathbb{CP}^3$, and construct the full 11-dimensional metric with appropriate warp factors describing both breathing mode and squashing mode. By analyzing the scalar potential, the holographic supersymmetric (or nonsupersymmetric) RG flow from $\mathcal{N} = 1$ (or $\mathcal{N} = 0$) $SO(5) \times U(1)$-invariant UV fixed point to $\mathcal{N} = 6$ (or $\mathcal{N} = 0$) $SU(4) \times U(1)$-invariant IR fixed point is described.2

In section 2, we describe the round-quotient $(S^7/\mathbb{Z}_k)$ and squashed-quotient $(\widetilde{S}^7/\mathbb{Z}_k)$ 7-spheres compactification vacua in 11-dimensional supergravity. In section 3, the squashing deformation of each vacua is described by an irrelevant operator at the $\mathcal{N} = 6$ (or $\mathcal{N} = 0$) conformal fixed point and a relevant operator at the $\mathcal{N} = 1$ (or $\mathcal{N} = 0$) conformal fixed points. The RG flow is described in AdS$_4$ supergravity by a static domain wall interpolating between these two vacua. We identify the corresponding operator in the boundary conformal field theory. In appendix A, we present the details for the computations of Ricci tensor and corresponding field equations.

2. Round-quotient and squashed-quotient 7-sphere

Let us consider an 11-dimensional supergravity on AdS$_4 \times X^7$ where $X^7$ is a seven-dimensional compact Einstein manifold. When the fermion field is set equal to zero, the bosonic field equations are given by the 11-dimensional Einstein equation and Maxwell equation, as usual. In order to solve the 11-dimensional Einstein equation for the Freund–Rubin [26] form for the gauge field strength, the Ricci tensor has nonzero components for the indices of four-dimensional spacetime and the indices of seven-dimensional internal space [19]. For example, see equations (A.2). On the other hand, one also obtains the Ricci tensor from the 11-dimensional metric (A.3).

2 Recently, the $\mathcal{N} = 1$ superconformal Chern–Simons matter theory with $SO(5) \times U(1)$ global symmetry is constructed in [25] and they conjecture that this is dual to the M-theory on AdS$_4 \times \widetilde{S}^7/\mathbb{Z}_k$.2
Before we describe the 11-dimensional metric directly, we need to understand the structure of the CP^3 internal space metric first. Since we are interested in the RG flow connecting two conformal fixed points, it is necessary to obtain one-parameter family of squashed CP^3 metric which allows us to have both round and squashed CP^3's we described in the previous section. For the standard Fubini-Study metric on the round CP^3, it contains CP^2 [20–22] or CP^1 × CP^1 [21, 27] inside of CP^3. At first sight, it is natural to generalize this standard round CP^3 metric to the general one-parameter family CP^3 by putting the warp factors in front of each orthonormal basis of six-dimensional metric. However, it does not produce the correct, general Ricci tensor components found in [7] and there exists only one critical point in the scalar potential.

There is an alternative description for the squashed 7-sphere by the fact that the S^7 as an S^1-bundle over S^4 with gauge potential for the self-dual SU(2) instanton [28, 29]. Then the squashing is related to the size of the S^1-fibers relative to the base S^4. After an inverse Kaluza–Klein construction, the seven-dimensional metric consists of four-dimensional metric plus SU(2) gauge field and this metric leads to the metric [14, 15] from other approach with quaternionic projective plane. Aldazabal and Font [24] (see also [30, 31]) have constructed the family of squashed CP^3 metric by viewing the CP^3 as an S^2-bundle over S^4 with the self-dual SU(2) instanton, in the context of flux compactification on AdS_4 × CP^3. The squashing corresponds to the size of the S^2-fibers relative to the base S^4 which has the line element of standard Einstein metric on S^4. The self-dual SU(2) instanton gauge potential appears in the S^7-bundle. It turns out that the standard Fubini-Study Einstein metric (which is Kahler) occurs when λ^2 = 1 while the second squashed Einstein metric (which is nearly Kahler) occurs when λ^2 = 1/2. The isometry group corresponding to the S^4 in this case is given by SO(5). See also the relevant paper [32] which discusses about the metric on the higher dimensional case CP^2^n with the quaternionic projective space HP^n.

As noted in [7], the uplift to the seven-dimensional metric can be done from the expectation that the CP^3 solution in ten dimensions is originated from the 7-sphere S^7 solution in 11-dimensional supergravity. By writing the seven-dimensional metric on an S^1-fibration over squashed CP^3, one obtains the squashed 7-sphere S^7 metric. The gauge potential in the S^1-fibration is related to the RR 2-form flux [24]. The standard round 7-sphere metric arises when λ^2 = 1 while the squashed Einstein metric arises when λ^2 = 1/2. Note that this value is different from the one λ^2 = 1/2 in squashed CP^3. One can explicitly check that the squashed 7-sphere S^7 metric described as an S^1-bundle over S^4 with gauge potential as we mentioned before is equivalent to that on an S^7-fibration over squashed CP^3. This observation is also found in section 4 of [24].

Now a general metric that is locally the direct sum of an arbitrary four-dimensional spacetime metric and a squashed 7-sphere metric (obtained before) with an appropriate warp factor for the breathing mode [33], interpolating between round-quotient and squashed-quotient 7-spheres, may be written as

\[
\frac{ds^2}{R^2} = e^{-2u(x)}g_{\alpha\beta}(x) \, dx^\alpha dx^\beta + \frac{1}{4} e^{2e(x)+3e(x)} \left( d\theta^2 + \frac{1}{4} \sin^2 \theta (\sigma_1^2 + \sigma_2^2 + \sigma_3^2) \right) \\
+ \frac{1}{4} e^{2u(x)-4e(x)} (d\mu - \sin \phi A^\perp + \cos \phi A^\parallel)^2 \\
+ \frac{1}{4} e^{2u(x)-4e(x)} \sin^2 \mu (d\phi - \cot \mu \cos \phi A^\perp - \cot \mu \sin \phi A^\parallel)^2 \\
+ \frac{1}{4} e^{2u(x)-4e(x)} \left( \frac{d\psi}{k} - \sin \mu \cos \phi A^\perp - \sin \mu \sin \phi A^\parallel \right)^2 \\
- \cos \mu A^\parallel - \cos \mu d\phi \right)^2, \tag{2.1}
\]
where the three real 1-forms satisfy the SU(2) algebra \( d\sigma_i = -\frac{1}{2} \epsilon_{ijk} \sigma_j \wedge \sigma_k \) and the self-dual \( SU(2) \) instanton gauge potential appearing in the \( S^3 \)-bundle is as follows:

\[
A^i \equiv \cos^2 \left( \frac{\theta}{2} \right) \sigma_i, \quad i = 1, 2, 3. \tag{2.2}
\]

The \( S^4 \) that can be parametrized as \( x^5(=\theta), x^6, x^7 \) and \( x^8 \) directions has the line element of standard Einstein metric given in the second term of the first line of (2.1)\(^3\). The isometry corresponding to \( S^4 \) is given by \( SO(5) \). The \( S^2 \)-bundle given in the second and the third lines specifies the two coordinates \( (\mu, \phi) \) that play the role of \( (x^9, x^{10}) \), respectively. When \( v(x) = 0 \), the isometry corresponding to \( \mathbb{C}P^3 \) becomes reduced to \( SO(5) \) for nonzero \( v(x) \). In the last line, we do the \( \mathbb{Z}_k \)-quotient by writing

\[
\psi \rightarrow \psi/k, \quad k > 2. \tag{2.3}
\]

Note that for \( k = 1 \) and \( k = 2 \), the supersymmetry of the IR theory is enhanced to \( \mathcal{N} = 8 \) supersymmetry. The parameter \( R \) measures the overall radius of curvature and the scalar fields \( u(x) \) and \( v(x) \) parametrize ‘size’ where

\[
\text{Vol}(X^7) = \frac{1}{3k} \pi^4 e^{7u} R^7 \tag{2.4}
\]

and ‘squashing’ deformation of \( X^7 \) over four-dimensional spacetime\(^4\). The squashing is parametrized by

\[
\lambda^2(x) \equiv e^{-7u(x)}. \tag{2.5}
\]

Spontaneous compactification of M-theory to AdS\(_4 \times S^7\) before \( \mathbb{Z}_k \)-quotient is obtained from near-horizon geometry of \( N \) coincident M2-branes. Through the 7-sphere before the \( \mathbb{Z}_k \)-quotient, the M2-branes thread nonvanishing flux of 4-form field strength of the Freund–Rubin form [26]

\[
\overline{F}_{afgh} = Q' e^{-7u(x)} \tau_{afgh} = Q' e^{-21u(x)} \epsilon_{afgh}. \tag{2.6}
\]

The Page charge [19] defined by

\[
Q' = \pi^{-4} \int_{X^7} (\star \overline{F} + \overline{C} \wedge \overline{F})
\]

in the convention of [28] is related to the total number of M2-branes, \( N' \), as [1]

\[
Q' = 96 \pi^2 N' \epsilon_6.
\]

Note that the \( \int_{X^7} \overline{C} \wedge \overline{F} \) term above vanishes in the Freund–Rubin compactification. The final number of flux quanta on the \( S^7/\mathbb{Z}_k \) after the \( \mathbb{Z}_k \)-quotient (2.3) is given by [1]

\[
\hat{N} = \frac{N'}{k}.
\]

\(^3\) The variables we use in this paper can be compared with those in [24] as follows: \( \mu \leftrightarrow \theta, \phi \leftrightarrow \psi, \xi \leftrightarrow \tau, \theta \leftrightarrow \psi, \sigma_i \leftrightarrow \Sigma^i \) and \( A' \leftrightarrow A' \). By using the relations \( \Sigma_1 = \sin \phi \delta \mu + \sin \mu \cos \phi \delta \psi, \Sigma_2 = -\cos \phi \delta \mu + \sin \mu \sin \phi \delta \psi \) and \( \Sigma_3 = -d \delta \phi + \cos \mu d \psi \) with \( k = 1 \) [24], one can reexpress the three-dimensional metric parametrized by \( (\mu, \psi, \phi) \) as follows: \( \lambda^2(\Sigma_1 - \Sigma^2)^2 \). Then it is easy to see that the whole seven-dimensional metric by adding the other four-dimensional metric parametrized by \( (\theta, x^5, x^6, x^7) \) to this three-dimensional one gives rise to the usual squashed 7-sphere metric. According to the observation of [28], the squashed 7-sphere metric described above as an \( S^3 \)-bundle over \( S^4 \) is equivalent to the standard one found by [11, 14, 19] via the following identifications \( \Sigma_i \leftrightarrow \sigma_i \) and \( i \sigma_j + j \sigma_k + k \sigma_l \rightarrow V(i \omega_1 + j \omega_2 + k \omega_3) V^{-1} \) with quaternion \( V \) of unit modulus which can be parametrized by three Euler angles. Here \( i, j, k \) are imaginary quaternions.

\(^4\) The \( x \)-dependence on \( u(x) \) and \( v(x) \) only refers to the four-dimensional spacetime \( x^a \) where \( a = 1, \ldots, 4 \).
because the volume with $\mathbb{Z}_k$-quotient is smaller by a factor of $k$ (2.4) than the original volume without $\mathbb{Z}_k$-quotient. Then the Page charge can be written as

$$Q' = 96\pi^2 N' \ell^6_p = 96\pi^2 kN \ell^6_p \equiv kQ,$$

where $Q = 96\pi^2 N \ell^6_p$. (2.7)

The four-dimensional field equations from appendix A can be obtained from the effective Lagrangian

$$\mathcal{L} = \sqrt{-g} \left[ R - \frac{63}{2} (\partial u)^2 - 21 (\partial v)^2 - V(u, v) \right],$$

where the supergravity scalar potential is given by

$$V(u, v) = e^{-9u(x)}[-6 e^{3v(x)} - 48 e^{-3v(x)} + 12 e^{-10u(x)} + 2k^2 Q^2 e^{-12u(x)}],$$

which depends on $u(x)$, $v(x)$, $k$ and $Q$ with (2.7). The AdS$_4$-invariant ground state solutions correspond to setting $u = \text{const}$, $v = \text{const}$ and the four-dimensional spacetime curvature is maximally symmetric and $R^\mu_\beta = \Lambda \delta^\mu_\beta$.

The two vacua with explicit $k$-dependence can be summarized by

$$S^7/\mathbb{Z}_k: \quad u = u_1 = \frac{1}{12} \ln(3^{-2}k^2Q^2),$$

$$v = v_1 = 0, \quad (\lambda^2 = 1),$$

$$\Lambda_1 = -12 \left| \frac{kQ}{3} \right|^{3/2},$$

and

$$\tilde{S}^7/\mathbb{Z}_k: \quad u = u_2 = \frac{1}{12} \ln(3^{-4}5^{10/7}k^2Q^2),$$

$$v = v_2 = \frac{1}{7} \ln 5, \quad \left( \lambda^2 = \frac{1}{5} \right),$$

$$\Lambda_2 = -12 \times 3^{7/2}5^{-5/2} \left| \frac{kQ}{3} \right|^{3/2}.$$

The two supergravity solutions are classically stable under the changes of the size and squashing parameter (2.5) of 7-sphere by following the analysis found in [19]. Note that $u_1$ and $u_2$ depend on $k$ explicitly while $v_1$ and $v_2$ do not depend on it. Obviously, for $k = 1$, the theory becomes the original theory without $\mathbb{Z}_k$-quotient. The $S^7/\mathbb{Z}_k$ is a saddle point, corresponds to a minimum along the $v$-direction and is invariant under the $SU(4)_k \times U(1)$ isometry group while $\tilde{S}^7/\mathbb{Z}_k$ is a maximum and is invariant under $SO(5) \times U(1)$ subgroup. Since the $8_5$ of massless gravitini breaks into

$$8_5 \rightarrow 6_0 \oplus 1_2 \oplus 1_{-2}$$

under the $SU(4)_k \times U(1)$ for the left-handed orientation of the 7-sphere $S^7_k/\mathbb{Z}_k$, the near-horizon geometry preserves $N = 6$ supersymmetry [7, 34] because there are six $\mathbb{Z}_k$-quotient invariant states. On the other hand, the $8_5$ of massless gravitino breaks into

$$8_5 \rightarrow 4_{-1} \oplus 3_1$$

under the $SU(4)_k \times U(1)$ for the right-handed orientation of the 7-sphere $\tilde{S}^7_k/\mathbb{Z}_k$, the near-horizon geometry preserves no supersymmetry ($N = 0$) at all [7, 34] because there is no $\mathbb{Z}_k$-quotient invariant state. Since the left-handed squashed 7-sphere $\tilde{S}^7_1$ gives rise to a theory with $N = 1$ supersymmetry [28] (space invader scenario level crossing phenomena among massless and massive Kaluza–Klein states) and the massless gravitino is a singlet under the $U(1)$ in the squashed $\mathbb{C}P^3$ compactification, it will also have $N = 1$ supersymmetry
at $\tilde{S}^7/\mathbb{Z}_k$. For the right-handed orientation of squashed 7-sphere $\tilde{S}^7/\mathbb{Z}_k$, the near-horizon geometry preserves no supersymmetry ($N = 0$) at all.

As stressed in the introduction, by starting from the general, one-parameter family, metric for CP$^3$ described in [24], and studying its seven-dimensional uplift metric on an $S^7$-bundle over this CP$^3$, the construction of the full 11-dimensional metric (2.1) which contains the ‘squashed’ CP$^3$ with appropriate warp factors together is new. For example, the observation for the nonzero RR 2-form in the gauge potential, in appendix A, is not so obvious without knowing the full information of 11-dimensional metric.

3. (Super)conformal field theories in three dimensions

Using the results of previous section on the Kaluza–Klein spectrum under squashing deformations, an operator giving rising to a RG flow associated with the symmetry breaking $SU(4)_R \times U(1) \to SO(5) \times U(1)$ will be identified and it turns out the operator is relevant at the $\tilde{S}^7/\mathbb{Z}_k$ fixed point and irrelevant at the $S^7/\mathbb{Z}_k$ fixed point.

- SU(4)$_R \times U(1)$-invariant conformal fixed point

Let us consider the harmonic fluctuations of spacetime metric and $u(\chi)$ and $v(\chi)$ scalar fields around AdS$_3 \times S^7/\mathbb{Z}_k$. Following [19], it turns out more convenient to rewrite (2.8) in terms of the un-rescaled, M-theory metric $\bar{g}_{\mu\nu} = e^{-\gamma u} g_{\mu\nu}$ in (2.1) [11]

$$\mathcal{L} = \sqrt{-\bar{g}} e^{2u(x)} |\bar{R} - 2\bar{\Lambda}_1 - 105(\partial u)^2 - 21(\partial v)^2 - 2\bar{V}_1(u, v)|,$$

where the scalar potential is

$$\bar{V}_1(u, v) = -\bar{\Lambda}_1 \left[1 + 1/4 e^{-2u(x)-u_1)}(e^{4v(x)} + 8 e^{-3v(x)} + 2 e^{-10v(x)} + 1/4 e^{-14(u(x)-u_1)})\right].$$

in which the un-rescaled cosmological constant $\bar{\Lambda}_1 = e^{7u_1} \Lambda_1 = 1/2 e^{7u_1} V(u_1, v_1)$ is given by

$$\bar{\Lambda}_1 = -12 m_1^3 \frac{1}{\ell_p^2} = -12 \left(\frac{|kQ|}{3}\right)^{-1/3} \frac{1}{\ell_p^2} \text{ where } m_1 = \frac{1}{\tilde{\ell}_{\text{IR}}}. \quad (3.1)$$

Here $\tilde{\ell}_{\text{IR}}$ is related to $N$, $k$ and Planck scale $\ell_p$ as $\tilde{\ell}_{\text{IR}} = \ell_p (32\pi^2 kN)^{1/6}$. By rescaling the scalar fields as

$$\sqrt{210} u \equiv \bar{u}, \quad \sqrt{42} v \equiv \bar{v},$$

one obtains that the fluctuation spectrum for $\bar{\tau}$-field around the $S^7/\mathbb{Z}_k$ takes a positive value

$$M_{\bar{\tau}}^2(S^7/\mathbb{Z}_k) = \frac{g_0^2}{2\bar{V}_1} \left|\bar{\tau}\right|^2 \left|\bar{\tau}\right| = -\frac{4}{3} \bar{\Lambda}_1 \ell_p^2 = 16 m_1^2. \quad (3.2)$$

Recall that before the $\mathbb{Z}_k$-quotient, the $\bar{\tau}$-field represents squashing of $S^7$ and hence, under SO(8) isometry group, ought to correspond to 300 that is the Young tableaux $\begin{bmatrix} 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ of $SO(8)$, the lowest mode of the transverse, traceless symmetric tensor representation where the $SO(8)$ Dynkin label is given by $\{0, 2, 0, 0\}$. The spectrum of supergravity fields is simply the projection of the original spectrum on AdS$_4 \times S^7$ onto the $\mathbb{Z}_k$-invariant states [7, 3]. The branching rule of an $SO(8)$ Dynkin label $(0, 2, 0, 0)$ in terms of $\mathbb{Z}_k$-invariant $SU(4)_R \times U(1)$ Dynkin labels is given by [7, 35, 36]

$$\{0, 2, 0, 0\} \to (0, 0, 0) \oplus (1, 0, 1) \oplus (0, 2, 0) \oplus (2, 0, 2) \oplus \cdots,$$

or $SO(8)$ representation 300 decomposes into

$$300(\begin{bmatrix} 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}) \to \begin{bmatrix} 1 \\ 15 \end{bmatrix} \oplus \begin{bmatrix} 5 \\ 20 \end{bmatrix} \oplus \begin{bmatrix} 35 \\ 84 \end{bmatrix} \oplus \cdots, \quad (3.3)$$
under the $\mathbb{Z}_k$-invariant $SU(4)_R \times U(1)$. Here, the expressions for $\cdots$ have nonzero $U(1)$ charges and they are projected out by $\mathbb{Z}_k$-quotient. The singlet $\mathbf{1}$ corresponds to an overall scaling of the metric, $u(x)$. Since the $\mathbf{20}'$ (that is the Young tableau $\Young{4}{1}$ of $SU(4)_R$) is represented by a traceless symmetric matrix, the squashing with $\mathbb{Z}_k$-quotient corresponds to nonzero expectation value for the $\mathbf{20}'$. From the mass formula for $300$ [11, 37] from the eigenvalues of the Lichnerowicz operator, one gets

$$M_{20'}^2 = 16m_1^2,$$

which is equal to (3.2) and note that $m_1^2$ contains $k$-dependence from (3.1).

One concludes that, in three-dimensional conformal field theory with $\mathcal{N} = 6$ supersymmetry, the $SO(5) \times U(1)$ symmetric left-handed squashing should be an irrelevant perturbation of conformal dimension $\Delta = 4$. Note that this gives a nonsupersymmetric theory for the right-handed squashing 7-sphere orientation $S^8_k/\mathbb{Z}_k$.

$\bullet$ $SO(5) \times U(1)$-invariant conformal fixed point

Due to the skew whipping, the theory will be either left squashed quotient $\tilde{S}_5^7/\mathbb{Z}_k$ with $\mathcal{N} = 1$ supersymmetry or right squashed quotient $\tilde{S}_5^7/\mathbb{Z}_k$ with $\mathcal{N} = 0$ supersymmetry. Note that the $\mathbb{Z}_k$-quotient acts on the $U(1)$ subgroup of $SU(2)$ which is present when there is no $\mathbb{Z}_k$-quotient. The isometry of the squashed 7-sphere is $SO(5) \times SU(2)$ and this is broken to $SO(5) \times U(1)$ by the $\mathbb{Z}_k$ quotient. In terms of the un-rescaled M-theory metric, the Lagrangian (2.8) may be rewritten as

$$\mathcal{L} = \sqrt{-\bar{g}} e^{2u(x)} [\bar{R} - 2\bar{\Lambda}_2 - 105(\partial u)^2 - 21(\partial v)^2 - 2\bar{V}_2(u, v)],$$

where the scalar potential is given by

$$\bar{V}_2(u, v) = -\bar{\Lambda}_2 \left[ 1 - \frac{1}{36} e^{-2(u(x) - u_2)} (25 e^{-10(v(x) - v_2)} + 4 e^{-3(v(x) - v_2)} - 2 e^{-10(v(x) - v_2)} + \frac{3}{4} e^{-14(u(x) - u_2)}) \right],$$

and the un-rescaled cosmological constant $\bar{\Lambda}_2 = e^{7u_2} \Lambda_2 = \frac{1}{2} e^{7u_2} V(u_2, v_2)$ is given by

$$\bar{\Lambda}_2 \equiv -12m_1^2 \frac{1}{\ell_p^2} = -12 \cdot 3^{7/3} 5^{-5/3} \left( \frac{|kQ|}{3} \right)^{-1/3} \frac{1}{\ell_p^2}, \quad \text{where} \quad m_2 = \frac{1}{\ell_{UV}}.$$

Once again, the mass spectrum of the $\bar{\tau}(x)$ field is calculated straightforwardly

$$M_{\bar{\tau}}^2[S^7/\mathbb{Z}_k] \equiv \left[ \frac{\partial^2}{\partial \bar{\tau}^2} 2\bar{V}_2 \right]_{\bar{\tau}=\bar{\pi}_2, \bar{\tau}=\bar{\pi}_2} = \frac{20}{27} \bar{\Lambda}_2 \ell_p^2 = -\frac{80}{9} m_2^2. \quad (3.4)$$

The branching rule of the $\mathbb{Z}_k$-invariant $SU(4)_R$ representations appearing in the right-hand side of (3.3) in terms of $SO(5)$ representation is given by

$$\begin{align*}
1 & \rightarrow \mathbf{1}, \\
15 & \rightarrow \mathbf{5} \oplus \mathbf{10}, \\
20' & \rightarrow \mathbf{1} \oplus \mathbf{5} \oplus \mathbf{14}, \\
84 & \rightarrow \mathbf{14} \oplus \mathbf{35} \oplus \mathbf{35}.
\end{align*}$$

Similar aspect in the branching rule of $20'$ for $AdS_5 \times S^5$ compactification has been found in [38, 39] in the context of gauged supergravity. In particular, the singlet $\mathbf{1}$ in the decomposition of $20'$ corresponds to $\bar{\tau}(x)$ field we turned on. From the mass formula for the $SO(5) \times SU(2)$
representation (before $\mathbb{Z}_k$-quotient) [28, 11] from the eigenvalues of the Lichnerowicz operator, one obtains the mass squared for the singlet $1$ as follows:

$$M_1^2 = -\frac{80}{9} m_2^2,$$

and this coincides with (3.4). The perturbation that corresponds to squashing around $\tilde{S}/\mathbb{Z}_k$ has a scaling dimension either $\Delta = 4/3$ or $5/3$ and hence corresponds to a relevant operator.

We gave a nonzero expectation value to a supergravity in the $20'$ of $SU(4)_K$. Using the AdS/CFT correspondence, one identifies this perturbation with a composite operator of $\mathcal{N} = 6$ superconformal Chern–Simons matter theory with a mass term for the symmetric and traceless product between two $6'$s: $\lambda_{AB} \int d^4 x O^{AB}$ where $\lambda_{AB}$ is in the $20'$ of $\mathbb{Z}_k$-invariant $SU(4)_K$. Note that the tensor product of these leads to

$$6(\square) \times 6(\square) = 1(\square) \oplus 15(\square) \oplus 20'(\square),$$

in $SU(4)_K$ representation. Then one can construct a $6$ (that is the Young tableaux $\begin{array}{c} 2 \\ 1 \\ \end{array}$) representation by using the Clebsch–Gordan coefficient $\Gamma_{IJ}^A(A = 1, \ldots, 6)$ [40, 25] which transforms two $4$'s into $6$ of $SU(4)_K$ together with matter field $C^I$

$$\Gamma_{IJ}^A C^I$$

where $C^I(I = 1, \ldots, 4)$ are four complex scalars ($4$ under the $SU(4)_K$) transforming as $\text{SU}(4)$ with gauge group $U(N) \times U(N)$ in $\mathcal{N} = 6$ superconformal Chern–Simons gauge theory [1]. The perturbation breaking $SU(4)_K \times U(1)$ to $SO(5) \times U(1)$ is given by

$$O^{AB} = \text{Tr} \Gamma_{IJ}^A C^I \Gamma_{KL}^B C^K C^L - \frac{1}{3}\delta^{AB} \text{Tr} \Gamma_{IJ}^C C^I \Gamma_{KL}^C C^K C^L.$$

(3.5)

The singlet $1$ of this operator $O^{AB}$ which is $20'$ of $SU(4)_K$ corresponds to the supergravity field $\nu(x)$ (or $\overline{\nu}(x)$) and the conformal dimensions are given by $\Delta_{UV} = \frac{4}{3}$ (or $\frac{5}{2}$) and $\Delta_{IR} = 4$ respectively as we computed before.

The massless supermultiplets contain $(2 - \frac{1}{2})$-supermultiplet in the $SU(4)$ singlet, $(1 - \frac{1}{2})$-supermultiplet in the $10 \oplus 1$ of $SU(4)$ [7]. The full spectrum of massive supermultiplets is the subset of the multiplets on the left squashed 7-sphere that are neutral when $SU(2)$ breaks to $U(1)$. There exist $35 \oplus 30$ of $SO(5)$ massless scalars in massive supermultiplets which are the members of massive Wess–Zumino multiplets.

Since the scalar potential we are considering is almost the same as that in [11] by replacing $Q$ with $kQ$, the non-perturbative analysis for the stability given in [11], besides the perturbative analysis considered so far, appears as a static domain wall. With the equations of motion for the metric, $u$ and $v$, the four-dimensional metric ansatz satisfies the correct boundary conditions at UV and IR regions. Then the Ricci tensor and the Ricci scalar can be determined in this background. Using the asymptotics, one can check the consistency of the $SO(3)$ symmetric domain-wall configuration. One sees the rescaled $u$ field has $Q$ dependence which can be rescaled by $kQ$ at the UV and IR regions. One obtains the regular asymptote of the ratio of the scalar potential at two fixed points with monotonic rescaling of the curvature of radius. This monotonic radial behavior of the static $SO(3)$ domain-wall configuration is the holographic representation of the renormalization group flow [38, 41, 42].

- $SU(2) \times SU(2) \times U(1)$-invariant conformal fixed point

So far, we have considered a particular one-parameter RG flow between conformal fixed points of M2-brane worldvolume theory. Geometrically, the flow is induced from varying the position of M2-brane when placed near a conical singularity of an eight-dimensional manifold.
with $\text{Spin}(7)$ holonomy. In the IR limit, the conical singularity and hence squashing of M2-brane horizon are washed out completely.

At the IR fixed point with $SU(4)_R \times U(1)$ symmetry, one may flow further into another fixed points by turning on a set of relevant operators. It includes scalar operators of Dynkin label $(n, 0, 0, 0)$ with $n \geq 2$ and pseudoscalar operators of Dynkin label $(n, 0, 2, 0)$ with $n \geq 0$. Among them are 70 scalar fields $\mathbf{35}, \mathbf{35} \oplus \mathbf{35}$ of $SO(8)$ in the massless gravity supermultiplet, parametrizing the coset space $E_{7(7)}/SU(8)$ in $\mathcal{N} = 8$ gauged supergravity. Decomposing them under the $\mathbb{Z}_4$-invariant $SU(4)_R \times U(1) \subset SO(8)_R$ [7, 34],

$$\mathbf{35}_c \oplus \mathbf{35}_c \rightarrow \mathbf{15} \oplus \mathbf{15} \oplus \cdots$$

where we present only the representations with vanishing $U(1)$ charge. Turning on the two relevant operators $\mathbf{15}$'s breaks $\mathbb{Z}_4$-invariant $SU(4)_R \times U(1) \rightarrow SU(2) \times SU(2) \times U(1)$ and they have the following branching rule

$$\mathbf{15} \oplus \mathbf{15} \rightarrow (1, 1) \oplus (1, 1) \oplus \cdots$$

where we write only the representations with vanishing $U(1)$ charge. Utilizing on the known result, in [39], they have studied RG flow to a nonsupersymmetric vacuum with $SU(2) \times SU(2) \times U(1)$ global symmetry.

Moreover, in the context of three-dimensional boundary theory [43], the $SU(2) \times SU(2)$ singlet $(1, 1)$ is found from the operator $\mathbf{15}$ (which is the Young tableau $\begin{array}{c} \hline \hline \hline \hline \hline \hline \hline \hline \hline \end{array}$), appearing in the branching rule of $\mathbf{35}_c$, that corresponds to [1]

$$\mathcal{P}^I_j \sim \text{Tr} \, C^I_C^j + \cdots .$$  \hspace{1cm} (3.6)

Here $C^I_j$ ($I = 1, \ldots, 4$) are four complex scalars $\mathbf{4}$ under the $SU(4)_R$ transforming as $\mathbf{(N, N)}$ with gauge group $U(N) \times U(N)$ in $\mathcal{N} = 6$ superconformal Chern–Simons gauge theory [1]. On the other hand, the other $SU(2) \times SU(2)$ singlet $(1, 1)$ is found from the operator $\mathbf{15}$, appearing in the branching rule of $\mathbf{35}_c$, written as [43]

$$\mathcal{O}^I_j \sim \text{Tr} \, C^I_C^j C^j_C^I C^I_C^j + \cdots .$$

This is obtained from (3.6) by acting two supersymmetry transformations which are in the $\mathbf{6}$ of $SU(4)_R$.

4. Conclusions and outlook

We have constructed the full 11-dimensional metric given by (2.1) and obtained the scalar potential in (2.9) by using the Freund–Rubin ansatz (2.6) with the help of appendix A. The holographic supersymmetric (or nonsupersymmetric) RG flow from $\mathcal{N} = 1$ (or $\mathcal{N} = 0$) $SO(5) \times U(1)$-invariant UV fixed point to $\mathcal{N} = 6$ (or $\mathcal{N} = 0$) $SU(4)_R \times U(1)$-invariant IR fixed point was described. The corresponding operator in three-dimensional Chern–Simons matter theories is characterized by (3.5).

When there are gauge potential components which are proportional to totally antisymmetric torsion tensor on the 7-sphere as well as the Freund–Rubin components we have discussed so far, there is a Lorentz scalar field $w = w(x)$ providing these gauge potential components [19]. After analyzing the 11-dimensional Maxwell and Einstein equations, the effective four-dimensional scalar potential is obtained. However, this has no extremum at finite $u(x), w(x)$ if $Q'$ is positive but if $Q'$ is negative, there exists a single extremum at nonzero
$u(x)$ and $w(x) = 0$. One finds that there is no extra critical point except this single extremum and concludes that there ought to be no nontrivial RG group flow in the dual three-dimensional conformal field theory. Similar phenomena can be found in [44].

For the seven-dimensional metric, we have considered $SU(2)$-bundle over the base $S^4$ in (2.1). Also one replaces the $SU(2)$-bundle with $SO(3)$-bundle or the base $S^4$ can be replaced by either $CP^2$ or $CP^1 \times CP^1$. It is interesting to find out whether these metrics provide the nontrivial 11-dimensional solutions and whether there exist nontrivial critical points in the effective four-dimensional theory. When the $CP^1$ in [20–22] is generalized to the present form, a family of squashed $CP^3$, what happens for the 11-dimensional solutions in the gauged supergravity?

So far, it is known that there exist only for $N = 1, 2$ supersymmetric RG flows [45, 46] dual to the Chern–Simons gauge theories with mass deformation. It is an open problem whether $N = 3, 4, 5, 6, 8$ supersymmetric [2, 47] RG flows exist in the context of $N = 8$ gauged supergravity. Are there any new general $AdS_4$ vacua? Are there any new critical points in the context of $SO(5)$ gauged supergravity which might be related to $N = 5$ Chern–Simons matter theory or $SO(6)$ gauged supergravity? Are there any flows connecting $AdS_4 \times S^7/Z_k$ to $AdS_5 \times T^{1,1}$ which was suggested in [1]?

There is much progress [48–141] on the direction of [1]. It would be interesting to apply the findings of this paper to it and see whether there exist any nontrivial aspects. For example, one considers the general family of squashed $CP^3$ metric described in this paper and it is interesting to see how the squashing parameter $\lambda$ (or $v(x)$) arises in the $AdS_4 \times CP^3$ compactification.

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**Appendix A. The Ricci tensor and field equations**

The Ricci tensor [28] can be obtained from connection 1-form and curvature 2-form in the following orthonormal basis for the metric (2.1)

\[
\begin{align*}
ev_1 &= e^{-\frac{2}{7}u(x)} \sqrt{g_{11}(x)} \, dx^1, \\
ev_2 &= e^{-\frac{2}{7}u(x)} \sqrt{g_{22}(x)} \, dx^2, \\
ev_3 &= e^{-\frac{2}{7}u(x)} \sqrt{g_{33}(x)} \, dx^3, \\
ev_4 &= e^{-\frac{2}{7}u(x)} \sqrt{g_{44}(x)} \, dx^4, \\
ev_5 &= \frac{1}{2} e^{u(x)} \sqrt{g_{55}(x)} \, d\theta, \\
ev_6 &= \frac{1}{2} e^{u(x)} \sqrt{g_{66}(x)} \, d\theta, \\
ev_7 &= \frac{1}{2} e^{u(x)} \sqrt{g_{77}(x)} \, d\theta.
\end{align*}
\]

(A.1)
where the three real 1-forms are
\[\sigma_1 = \cos x^8 \, dx^6 + \sin x^8 \, sin x^6 \, dx^7,\]
\[\sigma_2 = -\sin x^8 \, dx^6 + \cos x^8 \, sin x^6 \, dx^7,\]
\[\sigma_3 = dx^8 + \cos x^6 \, dx^7,\]
and the self-dual $SU(2)$ instanton gauge potential is $A^i \equiv \cos^2 \left( \frac{\sigma_i}{2} \right)$ \sigma_i$. Intentionally, we inserted the overall factor $\frac{1}{8}$ in the seven-dimensional internal space. The RR 2-form is given by the gauge potential in $e^{11}$ through
\[d(\lambda \sin \mu \cos \phi A^1 + \lambda \sin \mu \sin \phi A^2 + \lambda \cos \mu A^3 + \lambda \cos \mu \, d\phi) = -\lambda \sin \mu \cos \phi (e^1 \wedge e^2 + e^3 \wedge e^4) - \lambda \sin \mu \sin \phi (e^1 \wedge e^3 - e^2 \wedge e^4) - \lambda \cos \mu (e^1 \wedge e^3 + e^2 \wedge e^4) - \frac{1}{\lambda} e^5 \wedge e^6,\]
where the nonzero components for $F_{13}, F_{24}, F_{14}$ and $F_{23}$ also occur, contrary to the case of [7] in which there exist only nonzero components for $F_{12}, F_{14}$ and $F_{56}$.

Starting from the gauge field strength (2.6), the 11-dimensional Einstein equation implies the following Ricci tensor components:
\[\mathcal{R}_{\beta}^{\alpha} = \frac{4}{3} Q^2 e^{-14a(x)} \delta_{\beta}^{\alpha}, \quad \mathcal{R}_{b}^{a} = \frac{2}{3} Q^2 e^{-14a(x)} \delta_{b}^{a}, \quad \mathcal{R}_{b}^{a} = \mathcal{R}_{\beta}^{\alpha} = 0. \quad (A.2)\]

Instead of using the differential forms in orthonormal basis in order to get the Ricci tensor components by hands, we compute them by using a mathematica in the basis $dx^i$ where $i = 1, 2, \ldots, 11$ and then convert them into the orthonormal basis (A.1). The metric connection can be obtained from the 11-dimensional metric and Riemann tensor can be determined from this metric connection. Finally, the Ricci tensor is defined by the contraction between this Riemann tensor and 11-dimensional metric. Furthermore, using the orthonormal basis (A.1), one obtains the following Ricci tensor components in the orthonormal basis (A.1)
\[\mathcal{R}_{\beta}^{\alpha} = e^{7a(x)} \left[ \mathcal{R}_{\beta}^{\alpha} + \frac{4}{3} \delta_{\beta}^{\alpha} u^\gamma_{(x)} u_{\gamma}(x) - \frac{2}{3} u^\alpha_{(x)} u_{\beta}(x) - 21 u^\alpha_{(x)} v_{\beta}(x) \right],\]
\[\mathcal{R}_{5} = 12 e^{-2a(x)} - 3 e^{-2a(x)} - 10 e^{-2a(x)} - e^{2a(x)} \left[ u^\alpha_{(x)} u_{\beta}(x) - \frac{1}{2} v^\alpha_{(x)} v_{\beta}(x) \right] = \mathcal{R}_{5} = \mathcal{R}_{6} = \mathcal{R}_{8},\]
\[\mathcal{R}_{9} = 2 e^{-2a(x)} - 4 e^{-2a(x)} - 10 e^{-2a(x)} - e^{2a(x)} \left[ u^\alpha_{(x)} u_{\beta}(x) - 2 v^\alpha_{(x)} v_{\beta}(x) \right] = \mathcal{R}_{10} = \mathcal{R}_{11}. \quad (A.3)\]
All these expressions are the same as that in [19]. Here $u^\alpha = g^{\alpha \beta} u_{\beta}$ where the semicolon stands for the covariant derivative which contains the metric connection, as usual. When $u(x)$ and $v(x)$ are constant, then
\[\mathcal{R}_{5} = 3 - \frac{3}{2} e^{-7e} = 3 - \frac{3}{2} \lambda^2 = \mathcal{R}_{6} = \mathcal{R}_{7} = \mathcal{R}_{8},\]
\[\mathcal{R}_{9} = e^{-7e} + \frac{1}{2} e^{7e} = \lambda^2 + \frac{1}{2 \lambda^2} = \mathcal{R}_{10} = \mathcal{R}_{11}. \quad (A.4)\]
except the overall factor $4e^{-2u-3v}$. These (A.4) are the same as that in [14, 15]. The overall factor comes from different normalization in the metric. If we substitute $e^{2u} = \frac{1}{4} e^{-3v}$ into (2.1), then this factor becomes one and the normalization for the seven-dimensional metric here becomes the standard one [14, 15]. Then the Einstein condition is satisfied by two values of $\lambda^2$. The round sphere has $\lambda^2 = 1$ while the squashed sphere has $\lambda^2 = \frac{1}{2}$.

Now substituting the last two relations of (A.3) into the first two relations of (A.2) leads to the field equations for $u(x)$ and $v(x)$

$$
\begin{align*}
    u_{;\alpha}^\alpha(x) &= \frac{5}{7} e^{-9u(x)-4v(x)} + \frac{48}{7} e^{-9u(x)-3v(x)} - \frac{12}{7} e^{-9u(x)-10v(x)} - \frac{2}{7} Q^2 e^{-21u(x)}, \\
    v_{;\alpha}^\alpha(x) &= -\frac{1}{7} e^{-9u(x)+4v(x)} + \frac{24}{7} e^{-9u(x)-3v(x)} - \frac{20}{7} e^{-9u(x)-10v(x)}.
\end{align*}
$$

(A.5)

The second equation of (A.5) implies that either $v_1 = 0$ or $v_2 = \frac{1}{4} \ln 5$ in section 2. Moreover, plugging the first equation of (A.5) into the first equation of (A.3) and equating this to the right-hand side of the first equation of (A.2) provides

$$
R^a_{\beta} = \frac{63}{2} u_{;\alpha}^\alpha(x) u_{;\beta}^\beta(x) + 21 v_{;\alpha}^\alpha(x) v_{;\beta}^\beta(x)
$$

$$
+ \delta^a_{\beta} e^{-9u(x)} [-3 e^{4v(x)} - 24 e^{-3v(x)} + 6 e^{-10v(x)} + Q^2 e^{-12u(x)}].
$$

(A.6)

Then it is easy to see that the field equations (A.5) and (A.6) are equivalent to the Euler–Lagrange equations for the effective Lagrangian (2.8). When $u(x)$ and $v(x)$ are constant, then $R^a_{\beta} = \Lambda \delta^a_{\beta} = \frac{1}{2} V \delta^a_{\beta}$.

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5 Let us recall that the Ricci tensor of the CP^3 has the following components: $R_{\alpha\beta} = 3 - \lambda^2 = R_{\alpha\beta} = R_{\alpha\beta}^0 = R_{\alpha\beta}^0$ and $\bar{R}_{\alpha\beta} = \lambda^2 + \frac{2}{3} = \bar{R}_{\alpha\beta}^0$ when we consider the six-dimensional internal space only. The standard Fubini-Study Einstein metric arises when $\lambda^2 = 1$ while the second squashed Einstein metric arises when $\lambda^2 = \frac{1}{2}$ as we mentioned in section 1.
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