Abstract: The purpose of this paper is then to theoretically prove the estimated welfare measure from such elicitation method, which encompasses a complete decision process, is more efficient than that generated from a partial choice decision process, either from single-bounded or double-bounded discrete choice format. The expenditure difference will be adopted as the interpretation of the respondent’s reply to the choice selection. The theoretical proof and empirical evidence will also be validated for the efficiency improvement under expenditure difference interpretation.

Subjects: Science; Mathematics & Statistics; Statistics & Probability

Keywords: discrete choice models; elicitation decision process; logistic distribution; standard normal distribution

1. Introduction

Even since the yes–no discrete type elicitation process was applied by Bishop and Heberlein (1979), the dichotomous choice format of contingent valuation method has been wildly used in benefit evaluation for diversified types of resources. And the discrete type contingent valuation method has even been popularized after the utility compatibility theoretical background has been developed by Hanemann (1984). This underlying theory establishment has activated the debate about the interpretation of the respondent’s reply to the choice (Cameron, 1988; Cameron & James, 1987). The duality of the utility difference and expenditure difference interpretation has been theoretically proved and empirically verified by McConnell (1990) and Wu and Hsieh (1996). This theoretical foundation also excites expanding the idea of simple yes–no dichotomous choice to a two-level discrete choice, named double-bounded dichotomous choice (Hanemann, Loomis, & Kanninen, 1991). Because more information is collected from such elicitation method, it has been theoretically and empirically shown that a relatively efficient welfare measure results.

ABOUT THE AUTHOR
Junaid Ashraf is a research student. He got an MS Statistics degree from International Islamic University, Islamabad, Pakistan. His Research Interest includes Applied Statistics, Bayesian Statistics, Bayesian Inference, and Econometrics Modelling.

PUBLIC INTEREST STATEMENT
The discrete choice followed by an open willingness to pay revelation decision process conducted in this study has a remembrance to the format of bidding game elicitation approach. The theoretical and empirical results acquired from this study have dawned upon a further search for the design of an elicitation format with optimal numbers of discrete choices followed by an open willingness to pay revelation. That is, a future study can be guided toward to construct a willingness to pay elicitation approach with a combination of discrete and continuous responses resulting a most efficient estimation and welfare measures.
Nevertheless, information obtained from double-bounded discrete choice is not as fertile as that collected from open answer format (Hammack & Brown, 1974). Furthermore, estimation of welfare measure through double-bounded discrete choice under some specification has certain complexity if it is not undoable. However, it is generally arduous for respondents to reveal the amount of willingness to pay directly for the resource that is unfamiliar to him/her. As a result, the design of an elicitation method that can incorporate both the advantage of easy response through discrete choice and full willingness to pay information from the open answer is essential.

The elicitation method designed in this study is a double-bounded discrete choice followed by a continuous open willingness to pay revelation. It is believed that respondent is benefited from the experience cumulated through a two-level choice process. Thus, final revealing amount of willingness to pay, a full information type of data, will then have a great advantage in welfare measure (Bateman, Willis, & Garrod, 1994; Drake, 1992). Furthermore, research done by Hanemann et al. (1991) has shown that more efficiency is achieved in utility difference interpretation for respondent’s reply in double-bounded dichotomous choice elicitation process. It has not been demonstrated however if similar conclusion will be retained in expenditure difference interpretation as suggested by Cameron and Quiggin (1994).

This elicitation framework is applied to the benefit evaluation of Kengting National Park in Taiwan. Data for empirical demonstration is selected from a sample of 800 households in the country. The results have shown that mean willingness to pay through a complete decision process is more efficient than those estimated both from single-bounded and from double-bounded dichotomous choices. Additionally, the efficiency of benefit estimation is also exhibited from interval estimation of mean willingness to pay.

2. Theoretical framework of benefit evaluation for National Park

2.1. Willingness to pay for the resources of the Park

Evaluation of the benefit for the resources in an existing national park can be described as that if no efforts have been made the resources will vanish with the pollution deterioration. This is the circumstance that challenges the National Park we are concerning now. Conceptually, this phenomenon can be illustrated in Figure 1. Assuming that the natural resources are lumpy and indivisible, then the willingness to pay for an individual to avoid the decrease of the resources from \( Q^0 \) to \( Q^- \) is the amount of AC (Freeman, 1993; Randall & Stoll, 1980). This is the idea of an equivalent surplus of Hicks (1943), designated as WTP\(_E\).

A utility function, \( U(Q,Y) \), is used to symbolize the satisfaction of an individual in consuming the quantity or quality of environmental goods and services, \( Q \) and values of all other goods, \( Y \). If an individual owns values of all other goods at a level of \( Y^M \), while quantity or quality of environmental goods decreases from the level of \( Q^0 \) to \( Q^- \) then the individual is willingness to pay the amount...
WTP\textsuperscript{E} to avoid the reduction of the environmental goods. In the utility function specified above, this can be described as
\[ U^-(Q^0, Y^M - WTP^E) = U^-(Q^-, Y^M) \] (1)

The amount of willingness to pay, WTP\textsuperscript{E}, for the loss or reduction of the environmental goods and services is the benefit measurement for the change of that good and services, i.e. change of Q from level Q° to Q\textsuperscript{−}. As a result, evaluation of the amount of WTP\textsuperscript{E} for the change of good and services from Q\textsuperscript{°} to 0 is reasonably justified to be the benefit measurement for the resources of a national park.

2.2. Expenditure difference interpretation for the response function

The elicitation process for willingness to pay designed in this study is a decision process revealed through a yes–no two level dichotomous choice followed by an open amount response, such as that shown in Figure 2. Individual replies to the alternatives of “yes” or “no” for maintaining the resources of the Park at current status while confronted with the first offered price S\textsubscript{Ai}. If individual i initially owns expenditure level of E\textsubscript{i}\textsuperscript{0} with a level of environmental goods at Q\textsuperscript{°}. While the level of environmental goods deteriorates or reduces to level of Q\textsuperscript{−}, E\textsubscript{i} is the amount of expenditure required to remain at the post-change utility level. These two expenditure levels can be represented as
\[ E_i^0 = H_i^0(Q^0, U_i^0(Q^0, H_i^0; Z_i)) + \epsilon_i^0 \] (2)
\[ E_i^- = H_i^- (Q^-, U_i^- (Q^-, H_i^-; Z_i)) + \epsilon_i^- \] (3)

In Equations (2) and (3), Z\textsubscript{i} is a vector of socio-demographic variables of the respondent and \epsilon_i\textsuperscript{0} and \epsilon_i\textsuperscript{−} are independent and identically distributed with zero means, respectively, and follow extreme value distribution. If the respondent consents to pay the price S\textsubscript{Ai}, it must be that the amount of S\textsubscript{Ai} is lower than or equivalent to his/her willingness to pay E\textsubscript{i}^- – E\textsubscript{i}^0 for remaining at the utility level of U\textsubscript{i}^- (Q^-, H\textsubscript{i}^0). That is,
\[ E_i^- - E_i^0 \geq A_i \] (4)

With the replacement of Equations (2) and (3), this is equivalent to
\[ H_i^- (Q^-, U_i^-; Z_i) + \epsilon_i^- - H_i^0(Q^0, U_i^0; Z_i) - \epsilon_i^0 \geq A_i \] (5)
\[ \Delta e_i^0(Q^-, Q^0, U_i^-; Z_i) + \Delta e_i^- \geq A_i \] (6)

where \Delta e_i^0 = e_i^- - \epsilon_i^0 and \Delta e_i^0(Q^-, Q^0, U_i^-; Z_i) = H_i^- (Q^-, U_i^-; Z_i) - H_i^0(Q^0, U_i^0; Z_i) . Here, the amount of willingness to pay \Delta e_i^0 + \Delta e_i^- = E_i^- - E_i^0 is further designated as Y\textsubscript{i}.

Following the first response, a higher price, S\textsubscript{Ai}^U, or a lower price, S\textsubscript{Ai}^L is then presented for further choice. If an individual’s equivalent amount of willingness to pay will not be affected while the second offered price is introduced, then the decision of acceptance of the second proffered
price is made under the comparison between \( E_i^* - E_i^0 \) and \( \$A_i^U \). That is, if an individual is a willingness to pay for the price \( \$A_i^U \) then it must be that
\[
E_i^* - E_i^0 \geq A_i^U \tag{7}
\]
Similarly, rejection of the second price is made by comparing between \( E_i^* - E_i^0 \) and \( \$A_i^L \). If the respondent is the unwillingness to pay for \( \$A_i^L \) then it must be that
\[
E_i^* - E_i^0 < A_i^L \tag{8}
\]
Cameron and Quiggin (1994) provide evidence against the use of the standard interval data model and in favor of the more general bivariate probit model. An apparent miscalculation in the estimation process overstates the poor performance of the interval data model for the application reported by Cameron and Quiggin. When the miscalculation is corrected, the data reported by Cameron and Quiggin provide results consistent with Alberini (1995) finding that interval-data models provide robust estimates of mean willingness to pay from dichotomous choice with follow-up contingent valuation questions. However, with a combination of three proffered prices, the possible choice outcome should fall in the following four ranges, i.e. \((A_i^U, \infty), (A_i^U, A_i^L), (A_i, A_i^L),\) and \((0, A_i^L)\), no matter what adjustment is adopted. Thus, we are arguing here that whether the equivalent amount of willingness to pay is altered is not important.

As a result, as long as these four alternatives are taken into account, in estimation, more efficient results will be produced with two-stage discrete choice format. Research done by Hanemann et al. (1991) has demonstrated a similar consequence in utility difference interpretation for respondent’s reply. The conclusion, however, has not been verified and illustrated in expenditure difference interpretation neither in Cameron and Quiggin studies (1994, 1998) and Haab (1998) nor in any of the studies accomplished thereafter.

3. Elicitation decision process of willingness to pay

3.1. Analysis of data collected from partial elicitation decision process

While confronted with a dichotomous choice, expenditure difference interpretation for a respondent reply not only has theoretical plausibility but also has an advantage of empirical applicability and convenience (McConnell, 1990; Wu & Hsieh, 1996). As consequence, following theoretical proof of efficiency, comparison between different stages of discrete choice and continuous willingness to pay revelation will be performed under expenditure difference interpretation.

3.1.1. Single-bounded dichotomous choice

Because the willingness to pay \( Y_i^F \) is unobservable and assuming also that respondent \( i \)'s final willingness to pay \( \Delta \varepsilon_i^S \) with the stochastic term of \( \Delta \varepsilon_i^S \) is defined to be \( Y_i^F \), an indicator \( I_i \) is thus used to represent the dichotomous choice outcome while confronted with the offered price \( \$A_i \). Let \( I_i = 1 \) designate an individual’s consent for paying \( \$A_i \) and \( I_i = 0 \) if otherwise, i.e.

\[
I_i = \begin{cases} 
1, & \text{when } Y_i^F \geq A_i \\
0, & \text{otherwise}
\end{cases} \tag{9}
\]

As a result, the probability of willingness to pay for \( \$A_i \) is
\[
\text{Prob}(I_i = 1) = \text{Prob}(Y_i^F \geq A_i)
= \text{Prob}(\Delta \varepsilon_i^S \geq A_i - \Delta \varepsilon_i^S)
= \text{Prob}
\left(\frac{\Delta \varepsilon_i^S}{\sigma_S} \geq \frac{A_i - \Delta \varepsilon_i^S}{\sigma_S}\right) \tag{10}
\]
where \( \sigma_S \) is standard deviation of \( \Delta \varepsilon_i^S \). If \( \Delta \varepsilon_i^S \) follows a symmetric probability density function with zero mean, then the probability in Equation (10) can be written as
The marginal effect of the specific explanatory variable on the willingness to pay is either consistent or decreasing as variable $g$ that coefficients, and where $\beta$ estimated to Cramer logistic distribution function.

While the respondent is presented with the first offered price $A_1$, and then followed by a higher or lower second offered price $A_1$ or $A_2$. The most commonly used cumulative distribution function $F_i(\bullet)$ is standard normal distribution and logistic distribution function.

If further assuming that there is a linear relationship between $\Delta e_i$ and $g(x_i)$, i.e. $\Delta e_i = g(x_i)\beta$ where $g(x_i) = (g(x_{i1}) g(x_{i2}) g(x_{i3}) \cdots g(x_{in}))$ and $\beta = (\beta_1 \beta_2 \beta_3 \cdots \beta_n)$. Assuming also that $g(x_i)$ is a nondecreasing quasiconcave function. Under the specification of $g(x_i) = x_i$ or $g(x_i) = \ln x_i$, the marginal effect of $x_i$ on $\Delta e_i$ is, respectively, as $\frac{\Delta e_i}{\Delta x_i} = \beta_i$ and $\frac{\partial\Delta e_i}{\partial x_i} = g(x_i)/x_i$. That is, the marginal effect of the specific explanatory variable on the willingness to pay is either consistent or decreasing as variable $x_i$ increases. This consequence will be in accordance with the assumption of diminishing marginal utility.

The likelihood function for estimating parameters $\beta$ is as follows:

$$\ln L^S = \sum_{i=1}^{N} [I_i \ln F_i + (1 - I_i) \ln(1 - F_i)]$$ (13)

where $F_i = F_i \left( \frac{g(x_i)\beta - A_i}{\sigma_S} \right)$ and the information matrix can be computed from Equation (13)

$$\text{Inf}^S(\theta) = -E[H^2] = -E \left[ \frac{\partial^2 \ln L^S(\theta)}{\partial \theta \partial \theta'} \right]$$ (14)

In Equation (14), $H^2$ is the Hessian matrix and the information matrix is computed specifically as

$$\text{Inf}^S(\beta, \sigma_S) = \begin{bmatrix} -E \left[ \frac{\partial^2 \ln L^S}{\partial \beta \partial \beta'} \right] & -E \left[ \frac{\partial^2 \ln L^S}{\partial \beta \partial \sigma_S} \right] \\ -E \left[ \frac{\partial^2 \ln L^S}{\partial \sigma_S \partial \beta} \right] & -E \left[ \frac{\partial^2 \ln L^S}{\partial \sigma_S \partial \sigma_S} \right] \end{bmatrix}$$

The mean willingness to pay and the confidence interval estimation of mean willingness to pay can be computed as those for least square estimation (Cameron, 1991)

$$E(Y^S) = \frac{\overline{g(x)}}{\hat{\beta}_S}$$ (15)

$$CI_{1-\alpha}[E(Y^S)] = \frac{\overline{g(x)}}{\hat{\beta}_S} \pm t_{\alpha/2} \sqrt{\frac{\overline{g(x)}\Omega_S g(x)}}$$ (16)

where $\overline{g(x)}$ is the vector of mean values for all explanatory variables, $\hat{\beta}_S$ is a vector of estimated coefficients, and $\Omega_S$ is the asymptotic variance–covariance matrix of $\hat{\beta}_S$. And $\Omega_S$ will be approximated to Cramer–Rao lower bound under the consistent and asymptotically efficient estimation of $\hat{\beta}_S$ (Greene, 2000). That is,

$$\Omega_S = -E \left[ \frac{\partial^2 \ln L^S}{\partial \beta \partial \beta'} \right]^{-1} = I(\hat{\beta}_S)^{-1}$$ (17)

3.1.2. Double-bounded dichotomous choice

There will be more choice combinations for data collected from a two-stage discrete choice process. While the respondent is presented with the first offered price $A_1$, and then followed by a higher or lower second offered price $A_1$ or $A_2$. There would be four different kinds of choice combinations.
under these three price levels. Assume that respondent i’s final willingness to pay \( \Delta e_i^0 \) with the stochastic term of \( \Delta e_i^0 \) is defined to be \( Y_i^0 \). Again the respondent’s willingness to pay is unobservable under such circumstance. Choice indicators \( I_{i1} \) and \( I_{i2} \) are used to represent the respondent’s first and second responses. Thus, four different choice combinations will be resulted as follows:

\[
(I_{i1}, I_{i2}) = \begin{cases} 
(1, 1), & \text{when } Y_i^0 \geq A_i \text{ and } Y_i^0 \geq A_i^U \\
(1, 0), & \text{when } Y_i^0 \geq A_i \text{ and } Y_i^0 < A_i^U \\
(0, 1), & \text{when } Y_i^0 < A_i \text{ and } Y_i^0 \geq A_i^U \\
(0, 0), & \text{when } Y_i^0 < A_i \text{ and } Y_i^0 < A_i^U 
\end{cases}
\]  \hspace{1cm} (18)

Probability for positive responses to both the first and second offered prices is computed as

\[
\text{Prob}(I_{i1} = 1, I_{i2} = 1) = \text{Prob}(Y_i^0 \geq A_i \text{ and } Y_i^0 \geq A_i^U) = \text{Prob}(Y_i^0 \geq A_i^U)
\]

\[
= \text{Prob}(\Delta e_i^0 \geq A_i^U - \Delta e_i^0)
\]

\[
= \text{Prob}
\left( \frac{\Delta e_i^0}{\sigma_0} \geq \frac{A_i^U - \Delta e_i^0}{\sigma_0} \right)
\]  \hspace{1cm} (19)

where \( \sigma_0 \) is standard deviation of \( \Delta e_i^0 \). If \( \Delta e_i^0 \) follows a symmetric probability density function with zero mean, then the above probability is rewritten as

\[
\text{Prob}(I_{i1} = 1, I_{i2} = 1) = F_i \left( \frac{\Delta e_i^0 - A_i^U}{\sigma_0} \right)
\]  \hspace{1cm} (20)

The probabilities for all other three choice combinations are, respectively as

\[
\text{Prob}(I_{i1} = 1, I_{i2} = 0) = F_i \left( \frac{\Delta e_i^0 - A_i^U}{\sigma_0} \right) - F_i \left( \frac{\Delta e_i^0 - A_i}{\sigma_0} \right)
\]  \hspace{1cm} (21)

\[
\text{Prob}(I_{i1} = 0, I_{i2} = 1) = F_i \left( \frac{\Delta e_i^0 - A_i^U}{\sigma_0} \right) - F_i \left( \frac{\Delta e_i^0 - A_i}{\sigma_0} \right)
\]  \hspace{1cm} (22)

\[
\text{Prob}(I_{i1} = 0, I_{i2} = 0) = 1 - F_i \left( \frac{\Delta e_i^0 - A_i^U}{\sigma_0} \right)
\]  \hspace{1cm} (23)

Similarly, if there is a linear relationship between \( \Delta e_i^0 \) and \( g(x_i) \), i.e. \( \Delta e_i^0 = g(x_i) \beta \) and \( g(x_i) \beta \) and \( \beta \) have the same definition as those stated above, then the likelihood function for estimation can be written as

\[
\ln L^0 = \sum_{i=1}^{N} \left[ I_{i1} I_{i2} \ln F_{i1} + I_{i1} (1 - I_{i2}) \ln (F_{i2} - F_{i1}) + (1 - I_{i1}) I_{i2} \ln (F_{i3} - F_i) + (1 - I_{i1}) (1 - I_{i2}) \ln (1 - F_i) \right]
\]  \hspace{1cm} (24)

where \( F_{i1} = F_i \left( \frac{g(x_i) \beta - A_i^U}{\sigma_0} \right), F_{i2} = F_i \left( \frac{g(x_i) \beta - A_i}{\sigma_0} \right), F_{i3} = F_i \left( \frac{g(x_i) \beta - A_i^U}{\sigma_0} \right) \) and \( 0 < F_{i1} < F_i < F_{i2} < 1 \). The information matrix is as follows:

\[
\text{Inf}^0(\beta, \sigma_0) = -E \left[ H^0 \right] = \begin{bmatrix}
-\frac{\partial^2 \ln L^0}{\partial \beta^2} & -\frac{\partial \ln L^0}{\partial \beta} \\
-\frac{\partial \ln L^0}{\partial \beta} & -\frac{\partial^2 \ln L^0}{\partial \sigma_0^2} 
\end{bmatrix}
\]

Under expenditure difference interpretation for an individual’s response, estimation of mean willingness to pay and confidence interval of mean willingness to pay is similar to the data collected from the single-bounded discrete choice.
\[ E(Y^0) = \overline{g(x)} \hat{\beta}_0 \]  
\[ \text{CI}_{1-\alpha}[E(Y^0)] = \overline{g(x)} \hat{\beta}_0 \pm t_{\alpha/2} \sqrt{\overline{g(x)} \Omega_0 \overline{g(x)}} \]

where \( \hat{\beta}_0 \) is the vector for all the estimated parameters. The asymptotic variance–covariance matrix of \( \hat{\beta}_0 \), \( \Omega_0 \) can be computed as follows and it is approximated to Cramer–Rao lower bound under the consistency and asymptotic efficiency of \( \hat{\beta}_0 \)

\[ \Omega_0 = \left[-E \frac{\partial^2 \ln L^0}{\partial \hat{\beta}_0 \partial \hat{\beta}_0^T}\right]^{-1} = I(\hat{\beta}_0)^{-1} \]  

### 3.2. Analysis of data collected from a complete elicitation decision process

Respondent gains more experiences through the first two stages of the discrete choice process; it is expected that the willingness to pay for a specific environmental good or services will be clearly identified. As a result, the final amount of revelation can be considered to be the willingness to pay from a complete decision process. Analysis of data gathered from this full process is similar to that from an open-ended elicitation method. In order to be comparable with the analyses for discrete choice, a method of maximum likelihood estimation is performed.

Assume that respondent i’s final willingness to pay \( \Delta e_i^0 \) with a stochastic term of \( \Delta e_i^0 \) is defined to be \( Y_i^0 \). The linear relationship between \( \Delta e_i^0 \) and \( g(x_i) \) exists, i.e. \( \Delta e_i^0 = g(x_i) / \hat{\beta} \), where \( g(x_i) = (g(x_1) \ g(x_2) \ g(x_3) \cdots \cdots \ g(x_K)) \) and \( \hat{\beta} = (\hat{\beta}_1 \ \hat{\beta}_2 \ \hat{\beta}_3 \cdots \cdots \ \hat{\beta}_K) \). If \( \Delta e_i^0 - N(0, \sigma_0^2) \), then \( Y_i^0 - N(g(x_i) / \hat{\beta}, \sigma_0^2) \), the probability density function for respondent i in replying \( Y_i^0 \) is

\[ f(Y_i^0) = (2\pi\sigma_0^2)^{-1/2} e^{-\frac{(Y_i^0 - g(x_i) / \hat{\beta})^2}{2\sigma_0^2}} \]  

The joint probability density function for all the respondents will be in the form of

\[ f(Y_1^0, Y_2^0, \cdots, Y_N^0) = f(Y_1^0)f(Y_2^0)\cdots f(Y_N^0) = (2\pi\sigma_0^2)^{-N/2} e^{-\sum_{i=1}^{N} (Y_i^0 - g(x_i) / \hat{\beta})^2 / 2\sigma_0^2} \]  

The likelihood function for estimation is written as

\[ \ln L^0 = -\frac{N}{2} \ln 2\pi - \frac{N}{2} \ln \sigma_0^2 - \frac{1}{2\sigma_0^2} \sum_{i=1}^{N} (Y_i^0 - g(x_i) / \hat{\beta})^2 \]

Information matrix under such specification will be in the form of the following matrix

\[ \text{Inf0}^2(\hat{\beta}, \sigma_0) = -E[H^0] = \begin{bmatrix} -E \left[ \frac{\partial^2 \ln L^0}{\partial \hat{\beta}_0 \partial \hat{\beta}_0^T} \right] & -E \left[ \frac{\partial^2 \ln L^0 / \partial \hat{\beta}_0}{\partial \sigma_0^2} \right] \\ -E \left[ \frac{\partial^2 \ln L^0 / \partial \sigma_0^2}{\partial \hat{\beta}_0} \right] & -E \left[ \frac{\partial^2 \ln L^0}{\partial \sigma_0^2 \partial \sigma_0^2} \right] \end{bmatrix} \]

And then point estimation of mean willingness to pay and confidence interval can be computed according to the formula specified as follows:

\[ E(Y^0) = \overline{g(x)} \hat{\beta}_0 \]  
\[ \text{CI}_{1-\alpha}[E(Y^0)] = \overline{g(x)} \hat{\beta}_0 \pm t_{\alpha/2} \sqrt{\overline{g(x)} \Omega_0 \overline{g(x)}} \]

where \( \hat{\beta}_0 \) is the vector of estimated coefficients. Similarly to that in single-bound dichotomous choice, variance–covariance matrix \( \Omega_0 \) will then be approximated to Cramer–Rao lower bound, i.e.

\[ \Omega_0 = \left[-E \frac{\partial^2 \ln L^0}{\partial \hat{\beta}_0 \partial \hat{\beta}_0^T}\right]^{-1} = I(\hat{\beta}_0)^{-1} \]
4. Efficiency comparison among different elicitation decision processes

4.1. Efficiency comparison between single-bounded and double-bounded discrete choice process

The maximum likelihood estimation of \( \hat{\beta}_S \) and \( \hat{\beta}_D \) will approximate the asymptotic variance-covariance matrix from either single-bounded or double-bounded discrete choice process to Cramer–Rao lower bound. And the variances for parameters estimated from single-bounded and double-bounded discrete choice are computed as follows:

\[
I(\hat{\beta}_S) = -E \left[ \frac{\partial^2 \ln L}{\partial \beta \partial \beta'} \right] = \sum_{i=1}^{n} \frac{f_i^2}{F_i} \times \frac{g(x_i)g(x_i)}{\sigma^2} \tag{34}
\]

\[
I(\hat{\beta}_D) = -E \left[ \frac{\partial^2 \ln L}{\partial \beta \partial \beta'} \right] = \sum_{i=1}^{n} \left( \frac{f_i^2}{F_i} + \frac{(f_i - f_i^*)^2}{F_i - F_i^*} + \frac{f_i - f_i^*}{F_i - F_i^*} \right) \times \frac{g(x_i)g(x_i)}{\sigma^2} \tag{35}
\]

It can be proved that the following relationship holds

\[
I(\hat{\beta}_D) - I(\hat{\beta}_S) = \begin{bmatrix}
\left( f_i^2 - f_i^* \right)^2 & \left( f_i - f_i^* \right)^2 & \left( f_i^* - f_i \right)^2 \\
\left( f_i^2 - f_i^* \right)^2 & \left( f_i - f_i^* \right)^2 & \left( f_i^* - f_i \right)^2 \\
\left( f_i^2 - f_i^* \right)^2 & \left( f_i - f_i^* \right)^2 & \left( f_i^* - f_i \right)^2 \\
\end{bmatrix} \times \frac{g(x_i)g(x_i)}{\sigma^2} \geq 0
\]

That is to say \( I(\hat{\beta}_D) - I(\hat{\beta}_S) \) is a positive semidefinite matrix, i.e. \( I(\hat{\beta}_D) \succeq I(\hat{\beta}_S) \). It is apparent that the efficiency of estimated parameters from double-bounded is higher than that from a single-bounded discrete choice process.

4.2. Efficiency comparison between discrete and continuous elicitation process

Whether respondent will benefit from the experience cumulated through a two-level discrete choice a theoretical proof for the efficiency comparison between discrete and continuous open willingness to pay response will then be performed as follows. And results are drawn for standard normal and logistic distribution, two most commonly used distributions in the elicitation method employed in the literature.

4.2.1. Efficiency comparison under standard normal distribution

Under standard normal distribution, Lemma 1 stated as follows is defined to determine the efficiency comparison between double-bounded dichotomous choice and the final open willingness to pay from a complete elicitation decision process.

Lemma 1

If \( \varphi(\cdot) \) and \( \Phi(\cdot) \), respectively, are probability density and cumulative distribution function for standard normal distribution, then

1. For any number \( a \), if \( a \in \mathbb{R} \), there exists \( a\varphi(a) < \varphi(a) \Phi(a) < 1 + a\varphi(a) - \Phi(a) \).

2. For any number \( b \), if \( b \in \mathbb{R} \), there exists \( -b\varphi(b) < \varphi(b) \Phi(b) < \varphi(b) - \Phi(b) \).

3. For any number \( a \) and \( b \), if \( a, b \in \mathbb{R} \) and \( a < b \) then there exists

\[
\frac{[\varphi(b) - \varphi(a)]^2}{\Phi(b) - \Phi(a)} < \Phi(b) - \Phi(a) + a\varphi(a) - b\varphi(b)
\]
From the asymptotic variance–covariance matrix of $\hat{\beta}_D$ and $\hat{\beta}_O$ defined in Equations (27) and (33), the difference between $I(\hat{\beta}_O)$ and $I(\hat{\beta}_D)$ thus becomes

$$I(\hat{\beta}_O) - I(\hat{\beta}_D) = \sum_{i=1}^{N} \left[ 1 - \left( \frac{(f_i^u)^2}{F_i^u} + \frac{(f_i - f_i^u)^2}{F_i - F_i^u} + \frac{(f_i^l)^2}{1 - F_i^l} \right) \right] \times \frac{g(x_i)g(x_i)^*}{\sigma^2} \tag{37}$$

According to Lemma 1, the following relations exist,

$$-af_i^u < \frac{(f_i^u)^2}{F_i^u} < F_i^u - af_i^u$$

$$af_i^u - bf_i < \frac{(f_i - f_i^u)^2}{F_i - F_i^u} < F_i - F_i^u + af_i^u - bf_i$$

$$bf_i - cf_i^u < \frac{(f_i^l)^2}{F_i^l - F_i} < F_i^l - F_i + bf_i - cf_i$$

$$cf_i^l < \frac{(f_i^l)^2}{1 - F_i^l} < 1 + cf_i^l - F_i^l$$

where $a = \frac{g(x_i)\beta - A_i}{\sigma}$, $b = \frac{g(x_i)\beta - A_i}{\sigma}$, and $c = \frac{g(x_i)\beta - A_i}{\sigma}$, and $a < b < c$. It is known that

$$0 < \frac{(f_i^u)^2}{F_i^u} + \frac{(f_i - f_i^u)^2}{F_i - F_i^u} + \frac{(f_i^l)^2}{F_i^l - F_i} + \frac{(f_i^l)^2}{1 - F_i^l} < 1 \tag{38}$$

The difference between $I(\hat{\beta}_O)$ and $I(\hat{\beta}_D)$ is proved to be a positive semidefinite matrix. The efficiency of estimated parameters from the complete elicitation decision process is higher than that from the double-bounded discrete choice, a partial elicitation decision process thus emerges.

4.2.2. Efficiency comparison under logistic distribution

Similarly, Lemma 2 is used to determine the efficiency comparison under logistic distribution through a partial and complete elicitation decision process.

**Lemma 2**

If $\varphi(\cdot)$ and $\Phi(\cdot)$, respectively, are probability density and cumulative distribution function for logistic distribution then

1. For any number $a$, if $a \in R$, there exists $\varphi(a) = \Phi(a)[1 - \Phi(a)]$.
2. For any number $a$ and $b$, if $a, b \in R$ and $a < b$ then there exists

$$\varphi(b) - \varphi(a) = [\Phi(b) - \Phi(a)][1 - \Phi(b) - \Phi(a)]$$

It is known from Lemma 2 that

$$\frac{(f_i^u)^2}{F_i^u} = F_i^u(1 - F_i^u)^2$$

$$\frac{(f_i - f_i^u)^2}{F_i - F_i^u} = (F_i - F_i^u)(1 - F_i - F_i^u)^2$$

$$\frac{(f_i^l)^2}{F_i^l - F_i} = (F_i^l - F_i)(1 - F_i^l - F_i)^2$$
\[
\frac{(f_i^L)^2}{1 - f_i^L} = (1 - f_i^O)(f_i^L)^2
\]

And it can also be shown that

\[
I(\hat{\beta}_D) = -E \left[ \frac{\partial^2 \ln L(\theta)}{\partial \theta \partial \theta} \right] = \sum_{i=1}^{N} \left( \frac{(f_i^U)^2}{f_i^O} + \frac{(f_i^L - f_i^O)^2}{f_i^O - f_i^L} + \frac{(f_i^L - f_i^O)^2}{f_i^L - f_i^O} + \frac{(f_i^L)^2}{1 - f_i^L} \right) \times \frac{g(x_i)g(x_i)^\prime}{\sigma_0^2}
\]

\[
= \sum_{i=1}^{N} \left[ \frac{f_i^O}{f_i^O - f_i^L} + (f_i^L)^2(f_i^L - f_i^O) + f_i^O - f_i^L \right] \times \frac{g(x_i)g(x_i)^\prime}{\sigma_0^2}
\]

\[
< \sum_{i=1}^{N} \frac{f_i^O}{\sigma_0^2} = I(\hat{\beta}_O)
\]

A positive semidefinite matrix for \(I(\hat{\beta}_O) - I(\hat{\beta}_D)\) is also realized. That is, the parameters estimated from the final amount of open willingness to pay are more efficient than the results estimated from the double-bounded discrete choice under logistic distribution specification.

5. Benefit evaluation of Kengting National Park

5.1. Kengting National Park

Kengting National Park, established in 1984, is the first national park in Taiwan. It is located in the southernmost part of Taiwan and belongs to the tropical-zone region. It is the only national park with unique sea-land preserve in the country. Including land and marine area, the total acreage of the Park is 33,269 hectares. Among these, the marine areas house several fancy worlds. The majestic coral coast is extraordinary by any standard. Besides, the inland geographic area includes diverse features as picturesque hills and small lakes, thickets, prairie, and even a small desert, each with its own unique beauty. The ample rainfall and diverse topography is also a good environment for sustaining the widely varied population of flora and fauna.

Individual might gain utility from the variety of resources subsisted in Kengting National Park. The satisfaction not only comes from any form of the direct and indirect use of the resources but also from the existence of such treasure assets. However, organic pollutants from surrounding industrial and agricultural wastewater, illegal fish farming and recreational activities, and pollution from the nearby nuclear power plant have deteriorated the resources in the Park in recent years. Maintaining and preserving the resources of the Park at the current level has been the task of Park administration.

5.2. Design of the questionnaire

5.2.1. Description of the resources for the Park

Most of the inland and marine areas of the Park are free of charge. It will be incomplete to evaluate the benefits of maintaining resources at the current level through entrance fee even for those who have visited the Park. Not to mention individual can acquire benefits for just knowing the existence of the resources. As a result, a method that can evaluate the use and existence values of the resources is necessary. It is not essential and technically inconceivable to evaluate use and existence value separately for those who have visited the Park. It will then be regarded as the total value for who have visited or have not visited the Park.

In order to measure the benefits of the resources in the Park, the respondent is required to evaluate the information of the resources supplied in the questionnaire. Varieties and diversity of the resources including geographic landscape, biological resources, oceanic ecology, and pre-historic remains are illustrated. Maintaining these resources at current status for not vanishing
are described to picture the overall assets in the Park. The sources of extinguishing these resources are the pollutants and devastating activities described above.

Individual is then requested to reveal his/her willingness to pay through enforced annual resource maintenance fee to keep these resources at current status. The way of describing the “change” of the resources is similar to the evaluation for establishing a new park (Kosz, 1996). In the stage of discrete choice, in addition to “yes” and “no” alternative, selection of “uncertainty” is also presented (Arrow et al., 1993). Moreover, respondent’s final open willingness to pay is followed by possible options for protest responses that no payment is exhibited.

5.2.2. Sample and determination of the offered prices
The rule of thumb recommended by Kanninen (1995) for determining offered prices is adopted in this study. The prices presented in the first and second bid are arranged from the pretest open-ended responses. The rule suggests if the prices offered for the first stage discrete choice are in the 15%–85% of the willingness to pay distribution and those for the second stage choice are in the 10%–90% then the bias will be controlled.

Under the criterion stated above, the first bid offered prices provided in the questionnaire are selected from 15%, 23%, 31%, 38%, 46%, 54%, 62%, 69%, 77%, and 85% of the pretest open willingness to pay the amount. The second offered price level is double or half of the first bid price accordingly. As a result, 10 sets of the first and the second lower and upper offered prices used in the questionnaire are 300 (150, 600), 400 (200, 800), 500 (250, 1000), 600 (300, 1200), 800 (400, 1600), 1000 (500, 2000), 1200 (600, 2400), 1500 (750, 3000), 2000 (1000, 4000), and 3000 (1500, 6000), respectively.

There are 800 households in the sample. Selection of surveyed households is composed of two stages. The first stratification is to allocate 800 households to each county by the percentage of the households of that county to the total households in the country. The second stratification is to randomly draw individual household for each county according to the total households distributed. The survey is conducted in July and August of 2000 by personal interview. A pretest was also administrated before the formal survey is performed.

6. Model specification and results analyses
Among 800 observations, 297 individuals are classified as protest respondents with various reasons. In addition, two respondents’ final willingness to pay is extremely far more than all others. These results in 501 final usable observations for further analysis. In order to estimate the model specified in Equations (13), (24), and (30), functional forms and variables for $\Delta e_i^S$, $\Delta e_i^D$, and $\Delta e_i^O$ have to be determined. A linear functional form is selected for demonstration. The explanatory variables include variable related to the visit frequency of national parks in general, Kengting National Park in specific, and various socio-demographic variables of respondents. Table 1 lists all the variables and their mean values used in estimation. And the linear specification of estimation equation is

$$
\Delta e = a_0 + a_1 \times Fre + a_2 \times Subfre + a_3 \times Sex + a_4 \times Age + a_5 \times Family + a_6 \times Edu + a_7 \times Oc_1 + a_8 \times Oc_2 + a_9 \times Oc_3 + a_{10} \times Income + a_{11} \times Green1 + a_{12} \times Green2
$$

(40)

Data collected from partial decision process, i.e. single-bounded and double-bounded elicitation method, are analyzed by probit and logit model. And data collected from the complete decision process, i.e. final open willingness to pay amount, are analyzed by maximum likelihood estimation. Results of coefficients estimation are presented in Table 2.

It is observed that except for the variable of total number of times the respondent has visited the Park for the past five years (Subfre), gender (Sex), and vocational dummy variable (Oc3)
estimated from complete decision process, the effects of explanatory variables on the willingness to pay are consistent among partial decision process and complete decision process model.

Furthermore, on average, each household is the willingness to pay 855 NT dollars per year in maintaining the park from complete decision process estimation. Mean willingness to pay estimated from probit and logit models through the single-bounded and double-bounded discrete choice decision process, respectively, is 1102 NT dollars, 1087 NT dollars, 939 NT dollars, and 884 NT dollars. Due to the similarity of normal distribution and logistic distribution, it is expected that the mean willingness to pay and 95% confidence interval of mean willingness to pay estimated from probit model resembles that estimated from logit model.

Empirical comparison of the efficiency between complete and partial elicitation decision process is performed by examining the standard deviation of mean willingness to pay and range of confidence interval estimation of mean willingness to pay via probit, logit, and final open

| Table 1. Variables used in estimation and their mean values |
|-----------------------------------------------|
| Variable name (unit) | Mean value | Standard deviation | Variable definition |
| Fre (times) | 2.287 | 6.921 | Total numbers of times the respondent has visited Kengting National Park for the past five years |
| Subfre (times) | 2.856 | 6.530 | Total numbers of times the respondent has visited all other National Parks for the past five years |
| Sex | 0.525 | 0.500 | Dummy variable 1 for male, 0 for female |
| Age (years) | 37.996 | 11.123 | Age of respondent |
| Family (persons) | 4.399 | 1.589 | Household size |
| Edu (years) | 13.273 | 3.075 | Years of education of respondent |
| Oc1 | 0.166 | 0.372 | Vocational dummy variable 1 for respondents employed by governmental sectors 0 for all others |
| Oc2 | 0.022 | 0.147 | Vocational dummy variable 1 for fishermen 0 for all others |
| Oc3 | 0.068 | 0.252 | Vocational dummy variable 1 for respondents employed by private sectors or with professional skill 0 for all others |
| Income (ten thousand NT$) | 100.710 | 66.407 | Household annual income for the year of 1999 from all sources |
| Green1 | 0.054 | 0.245 | Dummy variable 1 for respondent has been the member of non-profit environmental organization 0 otherwise |
| Green2 | 0.222 | 0.416 | Dummy variable 1 for the respondent has donated to any non-profit environmental organization 0 otherwise |
| A (n = 501) | 1129.142 | 818.333 | Average offered price for the first bid |
| AU (n = 248) | 1707.258 | 1396.217 | Average upper offered price for the second bid |
| AL (n = 253) | 699.605 | 419.254 | Average lower offered price for the second bid |
| WTP (NT$) | 855.020 | 896.640 | Average final open revealed willingness to pay |

1The exchange rate of the US dollar to NT dollar was about 1:33 in the year 1999.
The results shown in Table 3 indicate that standard deviation of mean willingness to pay and 95% confidence interval of mean willingness to pay are consistently lower resulting from open willingness to pay estimation. Additionally, the upper bound of confidence interval estimation of mean willingness to pay from complete elicitation process is about 1.19 times of its lower bound. The ranges of upper bound and lower bound of the confidence interval of mean willingness to pay resulted from partial decision process, i.e. results from single-bounded and double-bounded probit and logit model, are relatively large. The results indicate that the relative size of upper bound is about 1.45 and 1.43 times its lower bound. 

| Variable | Single-bound dichotomous choice | Double-bound dichotomous choice | Complete decision process |
|----------|---------------------------------|---------------------------------|--------------------------|
|          | Probit                          | Logit                           | Probit                   | Logit                           | Probit                   | Logit                           | Complete decision process       |
| Constant | 1338.08 (815.06)                | 1185.66 (786.91)                | 1210.57 (374.63)         | ** 958.46 (398.31)             | ** 739.69 (294.47)       | ** 712.38 (294.48)             |
| Fre      | 20.26 (17.41)                   | 19.69 (16.65)                   | 14.78 (6.26)             | *** 14.11 (6.33)              | *** 11.23 (5.48)         | *** 11.23 (5.48)              |
| Subfre   | −9.71 (15.02)                   | −8.97 (14.38)                   | −1.12 (8.12)             | −0.83 (7.33)                  | 1.19 (5.90)              |*** 1.19 (5.90)               |
| Sex      | −328.02 (226.52)                | −334.55 (217.38)                | −5.36 (105.24)           | −52.12 (98.56)               | 62.36 (80.28)            |*** 62.36 (80.28)              |
| Age      | −5.52 (10.39)                   | −3.81 (10.16)                   | −4.72 (4.76)             | −2.41 (4.44)                  | −1.76 (3.86)             |*** −1.76 (3.86)              |
| Family   | −159.51 (73.53)                 | * −149.88 (70.01)               | −46.92 (33.82)           | −30.68 (31.45)               | −17.62 (24.93)           |*** −17.62 (24.93)            |
| Edu      | −28.72 (40.78)                  | −24.38 (38.94)                  | −39.88 (17.27)           | * −32.66 (17.26)             | −21.28 (14.34)           |*** −21.28 (14.34)            |
| Oc1      | 388.75 (292.03)                 | 373.34 (278.74)                 | 285.13 (139.12)          | * 244.88 (129.92)            | 157.21 (109.78)          |*** 157.21 (109.78)           |
| Oc2      | −808.92 (879.84)                | −700.15 (664.90)                | −694.72 (660.60)         | −487.01 (552.85)             | −314.71 (262.43)         |*** −314.71 (262.43)          |
| Oc3      | 639.22 (473.01)                 | 638.85 (454.08)                 | 136.32 (230.85)          | 149.59 (198.02)              | −39.71 (152.10)          |*** −39.71 (152.10)           |
| Income   | 10.18 (2.25)                    | ** 9.95 (2.16)                  | 5.39 (0.75)              | ** 5.01 (0.71)               | ** 4.48 (0.61)           |*** 4.48 (0.61)              |
| Green1   | 898.40 (503.83)                 | 887.72 (492.24)                 | 377.92 (220.42)          | 387.89 (188.53)              | 138.05 (156.99)          |*** 138.05 (156.99)           |
| Green2   | 166.62 (249.56)                 | 162.18 (239.81)                 | 36.29 (122.67)           | 61.92 (111.15)               | 20.05 (93.22)            |** 20.05 (93.22)              |
| Sigma*   | 1668.83 (206.96)                | ** 958.01 (121.56)              | ** 962.50 (41.49)        | ** 523.20 (25.27)            | 838.98 (26.05)           |** 838.98 (26.05)             |
| Value of | Value of Likelihood Function    | 289.82 (89.24)                  | −288.94 (89.24)          | −690.29 (25.27)              | −678.96 (26.05)          |−4038.71 (26.05)             |
| Function | χ² (df.)                        | 56.03(12)                       | 56.82(12)                | 64.88(12)                    | 62.01(12)                | 65.60(12)                     |

*Numbers in parentheses are the standard deviation of estimated coefficients. Numbers with one asterisk indicate the coefficient is significantly different from zero at 5% significant level. Numbers with two asterisks indicate the coefficient is significantly different from zero at 1% significant level.

aVariable Sigma are standard deviations σ_u, σ_d, and σ_o in Equations (13), (24), and (30), respectively.
lower bound via probit and logit models from single-bounded choice process and 1.27 and 1.25 times from double-bounded choice probit and logit designs, respectively.

7. Concluding remarks
This paper has theoretically proved the estimated welfare measure from a complete elicitation method, which is a double-bounded discrete choice followed by a continuous open willingness to pay format, is more efficient than that generated from a partial choice decision process, either from the single-bounded or from double-bounded discrete choice format.

The empirical demonstration of performing a benefit evaluation of Kenting National Park of Taiwan has shown that mean willingness to pay for maintaining the park at current state via maximum likelihood estimation of the final open revelation of willingness to pay is close to sample mean as compared to those estimated from probit and logit models. The efficiency of the complete decision process is not only generated from the small standard deviation of mean willingness to pay but also from a small range of upper and lower bound of confidence interval estimation of mean willingness to pay.

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Author details
Junaid Ashraf1
E-mail: junaidashraf488@gmail.com
David Balding2
E-mail: david.balding@unimelb.edu.au
1 Statistics, International Islamic University, Islamabad, Pakistan.
2 Mathematics and Statistics, University of Melbourne, Melbourne, Australia.

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