Tachyon Condensation in Noncommutative
Gauge Theory

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Abstract

We show that the decay of the $D2$-$\overline{D2}$ system with large worldvolume magnetic fields can be described in noncommutative gauge theory. Tachyon condensation in this system describes the annihilation of $D2$-$\overline{D2}$ into $D0$-branes. From the $2+1$ dimensional point of view, this is the decay of a nonabelian magnetic flux into vortices. The semiclassical approximation is valid over a long period of the decay. Our analysis allows us to clarify earlier results in the literature related to tachyon condensation and noncommutative gauge theory.

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1 Introduction

Studying the condensation of tachyons on unstable D-brane systems has led to an improved understanding of the configuration space of string theory. For instance, it has been established that D-branes can be described as solitons made of open strings [1, 2, 3, 4, 5, 6, 7, 8, 9], and that the closed string vacuum can be constructed as a nonperturbative state in open string field theory [10, 11, 12, 13, 14, 15, 16].

However, many questions remain regarding the physics around the stable vacuum. These include the nature of the spectrum of open string field theory expanded around this vacuum [17, 18, 19], the emergence of closed strings [20, 21, 22], and the potential role of strong coupling phenomena [20, 21]. Addressing these questions in open string field theory is challenging because of the infinite number of component fields which acquire expectation values, and the infinite number of higher derivative terms in the action which describe dynamics.

Recently, it was shown that considering tachyon condensation in the presence of large B-fields — or equivalently large worldvolume magnetic fields — can lead to great simplifications [22, 25, 26]. In particular, techniques of noncommutative geometry [27, 28, 29] allow an exact construction of D-branes as open string solitons, reproducing the correct tensions and fluctuation spectra [22]. In the present paper we will similarly show that the presence of large magnetic fields simplifies the description of the annihilation of D-branes via tachyon condensation. We will consider the $D2$-$\overline{D2}$ system in IIA string theory, with large but opposite sign magnetic fields on each of the branes. This system is unstable and will decay into $D0$-branes. The simplifying feature of the large magnetic field is that the decay can be described entirely within noncommutative Yang-Mills theory.

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1 This problem was solved recently [23, 24].
The Yang-Mills description of our system is provided by Matrix Theory [30]. The relation between Matrix Theory, noncommutative field theory, and tachyon condensation was noted in [29, 22] and discussed in more detail in [31]. Also, the decay of the $D\bar{D}$ system in Matrix Theory was studied in [32, 33], before the recent developments in noncommutative field theory.

Some of the questions concerning tachyon condensation in open string field theory involve understanding what happens to the open string degrees of freedom on the original unstable D-brane, what the correct variables are to describe the ground state of the system, and whether the ground state is unique [34, 35, 36, 37, 38]. These questions become most sharp in the case of decay to the closed string vacuum. In the present setup we will be left with $D0$-branes after the decay, and it is clear that the original $2 + 1$ dimensional noncommutative Yang-Mills degrees of freedom turn into the $0 + 1$ dimensional degrees of freedom describing the $D0$-branes. Furthermore, modulo gauge transformations, the classical ground states are uniquely labelled by the positions of the $D0$-branes.

The rest of this paper is organized as follows. In section 2 we discuss the $D2$-$D\bar{D}$ system in the limit of large worldvolume magnetic fields. The point is that in this limit the energy released by the decay of the system becomes small, so that rather than having to shift the entire string field we can concentrate on a small subset of modes. We then work out the spectrum of light open strings in this system, for later comparison with the noncommutative field theory. Matrix Theory provides a description of the $D2$-brane in term of $D0$-branes, as we review in section 3. As emphasized recently in [31], the $0 + 1$ dimensional Matrix Theory action for this system becomes, when rewritten using the Weyl correspondence, a $2 + 1$ dimensional noncommutative $U(1)$ gauge theory. Conversely, one can rewrite the $D2$-brane field theory as an action for a collection of $D0$-branes. It is this fact which allows us to describe the decay of the $D2$-$D\bar{D}$ system into $D0$-branes in
terms of noncommutative gauge theory, since it shows that both the initial and final states can be described as particular configurations in the same underlying action. We then confirm that the fluctuation spectrum of the gauge theory precisely matches the open string spectrum computed from free string theory. In section 4 we discuss of the decay process itself. From a $2 + 1$ dimensional point of view, this corresponds to the decay of a constant magnetic field into a gas of vortices, as pointed out in [32]. Describing the actual decay process explicitly is difficult, so we instead exhibit some paths in configuration space interpolating between the initial and final states. We also discuss an approach involving level truncation in the noncommutative gauge theory. Finally, in section 6 we conclude with some discussion.

Field theory models of tachyon condensation have been considered in [39]. The fact that $\mathcal{D}\mathcal{D}$ annihilation in the presence of fluxes can be described within field theory was discussed in [40]. The noncommutative field theory description of tachyon condensation was studied recently in the $D2 - D0$ system in [41], and in the $D5 - \overline{D5}$ system in [42].

## 2 The $D2 - \overline{D2}$ system in string theory

We are interested in describing the $D2 - \overline{D2}$ system as a bound state of $D0$-branes. A $D2$-brane constructed in this way has on its worldvolume a magnetic field whose strength is proportional to the density of $D0$-branes, as follows from the coupling

$$ S_{wz}^{D2} = -i\mu_2 \int d^3x \, C^{(1)} \wedge 2\pi\alpha'(F^+ + B), $$

(1)

where the $+$ superscript on $F$ will distinguish the field strength on the $D2$ from that on the $\overline{D2}$. Given the gauge invariant combination $F + B$ we can make an arbitrary split into an $F$ part and a $B$ part, and we will find it convenient to label the background as $B_{12} = \text{constant}$, $F = 0$, though we
of course allow $F$ to vary dynamically around this background. We describe the background field in terms of the dimensionless parameter $b$,

$$b = 2\pi\alpha' B_{12}. \tag{2}$$

We have chosen $x^1, x^2$ as the spatial worldvolume directions. The requirement that the density of $D0$ charge be positive implies $b > 0$.

The $\overline{D2}$ has an opposite sign for this coupling,

$$S_{wz}^{\overline{D2}} = +i\mu_2 \int d^3 x \, C^{(1)} \wedge 2\pi\alpha' (F^- + B). \tag{3}$$

One way to understand the sign change is to recall that a $\overline{D2}$ can be obtained from a $D2$ by a $\pi$ rotation in a $x^i - x^a$ plane, where $x^i$ is a spatial wordvolume direction and $x^a$ is a transverse direction. The implication is that in order to describe the $\overline{D2}$ as a bound state of $D0$-branes rather than $\overline{D0}$-branes we should take $F_{12}^- + B_{12} < 0$ on the $\overline{D2}$ . We accomplish this by taking a background value of $F_{12}^-$ such that

$$F_{12}^- + B_{12} = -b. \tag{4}$$

An important point is that our system thus does not correspond simply to a $D2-\overline{D2}$ in a background NS B-field — this would induce negative $D0$ charge on the $\overline{D2}$ . It was necessary to turn on the background worldvolume magnetic field in order to remedy this.

Thinking of the $D2$ as a bound state of $D0$’s, it is useful to work out the “binding” energy, in particular in the regime $b \gg 1$. This is given by the Born-Infeld term for the $D2$, which is

$$S_{BI} = -\mu_2 \int d^3 x \, \sqrt{-\det \left[ g_{ab} + 2\pi\alpha' (F^+ + B)_{ab} \right]}. \tag{5}$$

Inserting the background field $b$ and expanding for $b \gg 1$ gives

$$S_{BI} = -\mu_2 b \int d^3 x \, \sqrt{g} - \frac{\mu_2}{2b} \int d^3 x \, \sqrt{g}. \tag{6}$$
The first term represents the energy of an equivalent density of $D0$-branes, and the second term is the excess. For our purposes, the notable feature is that the latter quantity becomes small (as a density) for $b \gg 1$. Actually, it is more appropriate to express the binding energy in terms of the open string coupling and metric of [28], $G_s \sim bg_s$ and $\sqrt{G} \sim b^2 \sqrt{g}$. Then, taking $b \to \infty$ while keeping the open string quantities fixed one finds that the binding energy density scales as $1/b^2$. So in this regime, the decay of the unstable $D2 - \overline{D2}$ system releases a very small energy density. This means that we can expect to be able to describe this process in the zero slope limit, that is by a noncommutative gauge theory description. We will indeed find that this is the case.

### 2.1 Open string spectrum

We now work out the spectrum of open strings on the coincident $D2 - \overline{D2}$ system, focussing on modes which will survive in the zero slope limit. A similar analysis appears in [28]. We again take the worldvolume to lie in the $X^1, X^2$ directions, and denote transverse directions by $X^i$. It is convenient to use the complex coordinate $Z = (X^1 + iX^2)/\sqrt{2}$. As discussed above, we take

$$(F^+ + B)_{12} = b, \quad (F^- + B)_{12} = -b.$$  \hfill (7)

The spectrum of $2\bar{2}$ and $\overline{2}\bar{2}$ strings is unaffected by $b$. We now consider the $2\bar{2}$ strings. The boundary conditions on worldsheet fields are

$$\sigma = 0 : \quad \partial_\sigma Z - ib\partial_\tau Z = 0, \quad \partial_\tau X^i = 0.$$  \hfill (8)

The solution is

$$Z = \frac{1}{i} \sum_{n=\infty}^{n=-\infty} \left[ e^{i(n+\nu)(\sigma-\tau)} + e^{i\nu\pi} e^{-i(n+\nu)(\sigma+\tau)} \right] \frac{\alpha_{n+\nu}}{n+\nu} + c,$$
\[ X^i = i \sum_{n=-\infty}^{\infty} \left[ e^{in(\sigma-\tau)} - e^{-in(\sigma+\tau)} \right] \frac{\alpha_n^i}{n}, \]  
(9)

where
\[ e^{i
u\pi} = \frac{1 + ib}{1 - ib}. \]  
(10)

As \( b \) is taken from 0 to \( \infty \), \( \nu \) ranges from 0 to 1. For \( b \gg 1 \),
\[ \nu \approx 1 - \frac{2}{\pi b}. \]  
(11)

The open string spectrum is determined from
\[ \alpha' m^2 = \text{(oscillator energy)} + a. \]  
(12)

In the Ramond sector the ground state energy \( a \) vanishes as usual, \( a_R = 0 \).
In the Neveu-Schwarz sector we find
\[ a_{NS} = -\frac{1}{4} - \frac{1}{2} \left| \nu - \frac{1}{2} \right|, \]  
(13)

which for large \( b \) behaves as
\[ b \gg 1 : \quad a_{NS} \approx -\frac{1}{2} + \frac{1}{\pi b}. \]  
(14)

An important point is that the bosonic creation operator \( \alpha_{-(1-\nu)} \) raises the energy by \( 2/(\pi b) \) for large \( b \). So given any light state we can produce a tower of light states by applying \( \alpha_{-(1-\nu)} \) an arbitrary number of times.

Now we work out the spectrum, concentrating on states with \( \alpha' m^2 \sim 1/b \) or less, since these will survive the zero slope limit. We work in light cone gauge, taking \( X^1, \ldots, X^8 \) as transverse directions.

We begin with the R sector. The ground state consists of two massless, opposite chirality, 4 component SO(6) spinors. One of these is allowed by the GSO projection. The other is projected out, but we get an allowed state by acting with a worldsheet fermion. To obtain a light state we can act with
Now, given these two light states we act with $\alpha_{-(1-\nu)}$ to generate the following towers of states

$$\begin{align*}
\text{4 of } \alpha' m^2 &= \frac{2n}{\pi b}, \\
\text{4 of } \alpha' m^2 &= \frac{2}{\pi b} + \frac{2n}{\pi b}, \quad n = 0, 1, 2, \ldots.
\end{align*}$$

(15)

Next we consider the NS sector. For large $b$ the ground state has energy $a_{NS} \approx -\frac{1}{2} + \frac{1}{\pi b}$. First, we need to know whether this state is allowed by the GSO projection. When quantizing strings stretching between $D2$ and $\overline{D2}$ one takes the opposite GSO projection compared to that for $D2$-$D2$ strings. There is another consideration, which is that when we take $b$ from 0 to $b \gg 1$ the two fermionic modes $\psi_{\pm(1/2-\nu)}$ cross, shifting the fermion number of the ground state by 1. Together, this implies that for $b \gg 1$ the NS ground state is GSO projected out. Allowed light states are generated by acting on the ground state with

$$\psi_{\nu-1/2}^\dagger, \quad \psi_{-(3/2-\nu)}, \quad \text{and } \psi_{-1/2}^i.$$  

(16)

Acting on these with $\alpha_{-(1-\nu)}$ we generate the towers

$$\begin{align*}
\text{1 of } \alpha' m^2 &= -\frac{1}{\pi b} + \frac{2n}{\pi b}, \\
\text{1 of } \alpha' m^2 &= \frac{3}{\pi b} + \frac{2n}{\pi b}, \\
\text{6 of } \alpha' m^2 &= \frac{1}{\pi b} + \frac{2n}{\pi b}.
\end{align*}$$

(17)

Along with each state above, we should include its partner coming from strings beginning on $\overline{D2}$ and ending on $D2$. Note that there is then a single complex tachyon in the spectrum, signalling the instability of the $D2$-$\overline{D2}$ system. All states in the theory besides those just worked out have $\alpha' m^2 \sim O(1)$ for large $b$. 

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The zero slope limit described in [28] consists of taking
\[ \alpha' \sim \epsilon^{\frac{1}{4}} \to 0, \quad \frac{1}{b} \sim \epsilon^{\frac{1}{4}} \to 0, \]  
with \( g_{\mu\nu} \) fixed.\(^2\) In this limit, the light states we have computed remain in the spectrum while all other states are removed. We will see in the next section that the field theory describing the light states is a 2 + 1 dimensional noncommutative gauge theory with a background magnetic flux.

### 3 Matrix Theory description of \( D2\overline{D2} \)

In this section we study the description of the \( D2\overline{D2} \) system as an unstable bound state of D0-branes. The construction we use is identical to that used in Matrix Theory [30], although our interpretation is somewhat different. We will not consider the strong coupling quantum effects which lead to an eleven dimensional large \( N \) limit. Instead our considerations will be purely classical, as is appropriate in the \( g_s \to 0 \) limit at the fixed length scale \( b^{-\frac{1}{2}} \).

We will have more to say about the validity of the classical approximation in section 4.

#### 3.1 Review of \( D2 \)-branes

We begin by reviewing the construction of \( D2 \)-branes. The action describing a collection of \( N \) D0-branes is, in units with \( 2\pi \alpha' = 1 \),
\[ S = \frac{T_0}{2} \int dt \text{Tr} \left\{ D_0 X^I D_0 X^I + \frac{1}{2}[X^I, X^J]^2 - \frac{i}{2} \lambda D_0 \lambda + \frac{1}{2} \lambda \Gamma^0 \Gamma^I [X^I, \lambda] \right\} \]  
with \( I, J = 1, \cdots, 9; T_0 = \frac{\sqrt{2\pi}}{g} \) is the mass of a D0-brane; \( D_0 = \partial_t - i[A_0, \cdot] \); and \( X^I, \lambda \) are Hermitian \( N \times N \) matrices. We henceforth take the \( N \to \infty \) limit, so that the matrices become operators on an infinite dimensional space.

\(^2\)This is a coordinate transformed version of the limit presented on p. 12 of [28].
Hilbert space. Without loss of generality, it is convenient to take this Hilbert to be that of a one dimensional harmonic oscillator with a basis of energy eigenstates $|n\rangle$.

The background describing a flat, infinite $D2$-brane in the $X^1, X^2$ plane is given by $X^1 = x^1$, $X^2 = x^2$, with

$$[x^1, x^2] = i\theta.$$  \hfill (20)

It is sometimes convenient to write the solution in terms of

$$z = \frac{x^1 + ix^2}{\sqrt{2}} = \sqrt{\theta} a,$$  \hfill (21)

with $a$ the annihilation operator, $[a, a^\dagger] = 1$. To make contact with non-commutative field theory, consider a general bosonic fluctuation around this background.

$$A_0, \quad X^a = x^a + \theta \epsilon^{ab} A_b, \quad X^i,$$  \hfill (22)

$a, b = 1, 2$, and $i = 3, \cdots, 9$. Expanding out the action (19), keeping all bosonic terms, gives \cite{46, 47}

$$S = T_0 \int dt \text{Tr} \left\{ -\frac{1}{4} G^{\mu\nu} G_{\mu\nu}(F + \Phi)_{\mu\nu}(F + \Phi)_{\alpha\beta} + \frac{1}{2} G^{\mu\nu} D_\mu X^i D_\nu X^i \right\}. \hfill (23)$$

In this action Greek indices run over 0, 1, 2. We have defined the following quantities:

$$G_{\mu\nu} = \text{diag}(1, -\theta^{-2}, -\theta^{-2}),$$

$$F_{0a} = \partial_t A_a - i\theta^{-1} \epsilon_{ab}[x^b, A_0] + i[A_0, A_a],$$

$$F_{12} = i\theta^{-1}[x^a, A_a] + i[A_1, A_2],$$

$$D_a X^i = i\theta^{-1} \epsilon_{ab}[x^b, X^i] - i[A_a, X^i],$$

$$\Phi_{12} = -\theta^{-1}.$$  \hfill (24)

The 0 + 1 dimensional action (23) can be rewritten as a 2 + 1 dimensional field theory by using the Weyl correspondence (see, e.g. \cite{29}). Under this
correspondence operators on Hilbert space are replaced by functions on $\mathbb{R}^2$ according to the rules

\begin{align*}
AB &\rightarrow (A \star B)(x) \equiv e^{\frac{i}{2} \varepsilon_{a'b} \partial_a \partial_{b'}} A(x) B(x') \bigg|_{x' = x} \\
\text{Tr} &\rightarrow \frac{1}{2\pi \theta} \int d^2x.
\end{align*}

(25)

A useful formula following from the definition of the $\star$ product is

\begin{equation}
x^a \star f - f \star x^a = i\theta \varepsilon^{ab} \partial_b f
\end{equation}

(26)

for any function $f$. Using these rules, the action takes the form (23) but with the replacements

\begin{align*}
T_0 &\rightarrow T_0 \frac{1}{2\pi \theta} , \\
dt \text{Tr} &\rightarrow d^3x , \\
F_{\mu\nu} &\rightarrow \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu] , \\
D_\mu X^i &\rightarrow \partial_\mu X^i - i[A_\mu, X^i] ,
\end{align*}

(27)

and with all products replaced by $\star$ products.

The resulting action is just the noncommutative field theory describing the zero slope limit of a D2-brane in a background field $B_{12} = \theta^{-1}$, with the particular choice of $\Phi$ parameter (explained in [28]) given by $\Phi = -B$. This shows quite explicitly that a configuration of D0-branes satisfying (20) represents a D2-brane in a background $B$ field. It also demonstrates background independence [31], in that we get the same action whether we represent the configuration in terms of D0-brane quantum mechanics, or in terms of D2-brane $2 + 1$ dimensional field theory. The two descriptions correspond to expanding the same underlying actions around two different backgrounds. It is this property of background independence that will allow us to describe the decay of the $D2\overline{D2}$ system in terms of a $2 + 1$ dimensional field theory,
since it guarantees that the final state after the decay — $D0$-branes — are describable in terms of these degrees of freedom.

### 3.2 The $D2$-$\overline{D2}$ system

A $\overline{D2}$ in Matrix Theory is given by satisfying (20) with an additional minus sign,

$$[X^1, X^2] = -i\theta,$$

(28)
corresponding to reversing the orientation of the brane. The combined $D2$-$\overline{D2}$ system then corresponds to a direct sum of the individual solutions,

$$X^1 = \begin{pmatrix} x^1 & 0 \\ 0 & x^1 \end{pmatrix}, \quad X^2 = \begin{pmatrix} x^2 & 0 \\ 0 & -x^2 \end{pmatrix}.$$

(29)

Background independence implies that we can think of the $D2$-$\overline{D2}$ system as being a particular flux configuration in $U(2)$ 2 + 1 dimensional noncommutative field theory. This is easily accomplished by writing (29) as

$$X^a = x^a \otimes 1_{2\times 2} + \theta \epsilon^{ab} A_b,$$

(30)

with $A_a$ a $U(2)$ gauge field with background value

$$A_1 = \frac{2x^2}{\theta} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

(31)

Thus the decay of the $D2$-$\overline{D2}$ system is equivalent to the decay of a particular nonabelian magnetic flux in the noncommutative field theory. The background magnetic flux breaks $U(2)$ down to $U(1) \times U(1)$, which is indeed the correct gauge group for $D2$-$\overline{D2}$. As we will see shortly, the instability of the system is manifested by a tachyonic off diagonal mode.

Similarly to what we saw in the previous subsection, expanding the action (19) around the background (29) gives a field theory action of the form (23),
but now with all fields taking values in the Lie algebra of $U(2)$, and with the background magnetic flux (31).

The background gauge field (31) corresponds to a magnetic field on the $\overline{D2}$ of strength,

$$F_{12}^{-} = -\frac{2}{\theta}. \quad (32)$$

This can be compared to our discussion in section 2 once we translate the present noncommutative field strength used in this this section to the commutative field strength used there. Using the relation [28],

$$F_{12}^{\text{com}} = \frac{F_{12}^{\text{nc}}}{1 + \theta F_{12}^{\text{nc}}}, \quad (33)$$

and $\theta^{-1} = b$ we find that the commutative field strength satisfies the relation (4) up to an overall minus sign. The reason for the apparent sign discrepancy is that turning on the noncommutative gauge field (31) necessarily reverses the orientation of the worldvolume coordinates on the $\overline{D2}$, as explained in [43]. Once this is taken into account (4) is satisfied. Recall that this relation enforced the condition that the $\overline{D2}$ be composed of the same positive density of $D0$-branes as is the $D2$-brane.

### 3.3 Fluctuations around the $D2-\overline{D2}$ system

In section 2 we worked out the spectrum of light open string modes in the $D2-\overline{D2}$ system, and we noted that the spectrum should be reproduced in the zero slope limit. Checking this is a matter of computing the quadratic fluctuation spectrum of the action (19) around the background (29). This computation has already been done in the Matrix Theory literature [44, 45]. To compare, use the relations $b = c = 2\pi z^2$ and $2\pi\alpha' = 1$. (In comparing, one also needs to be careful about factors of two involving complex boson versus real boson masses). The spectra match exactly, as expected.
It is useful to identify some of the fluctuations explicitly. The eigenvectors of the fluctuation matrices are given in [44]. Writing the background (29) in complex coordinates, the tachyon corresponds to the fluctuation

\[ Z = \frac{X^1 + iX^2}{\sqrt{2}} = \sqrt{\theta} \left( \begin{array}{cc} a & 0 \\ TP_0 & a^\dagger \end{array} \right), \]  

with \([a, a^\dagger] = 1\), and \(P_0\) being the projection operator onto the harmonic oscillator ground state, \(P_0 = |0\rangle\langle 0|\). The action for \(T\) is found to be

\[ S = \frac{T_0\theta}{2} \int dt \left\{ |\dot{T}|^2 + 2\theta |T|^2 - \frac{1}{2} \theta |T|^4 \right\}, \]  

(35)

corresponding to a complex tachyon of mass \(m^2 = -2\theta\), (or \(\alpha'm^2 = -\frac{1}{\pi b}\) in the notation of section 2).

4 Decay to D0-branes

The \(D2-\overline{D2}\) system is unstable and will decay. By charge conservation, it must decay to a collection of D0-branes. The decay will be initiated by the condensation of the tachyon identified in the previous section. However, a complicating feature of the system we are considering is that tachyon condensation will not occur homogeneously over the \(D2-\overline{D2}\) worldvolume, as is seen from the following considerations. The tachyon has charge \((1, -1)\) under the unbroken \(U(1) \times U(1)\) gauge group on the \(D2-\overline{D2}\). As we have discussed, building this system up out of D0-branes means that there are background magnetic fields in each \(U(1)\), with a positive sign in the \(U(1)\) associated to the \(D2\), and a negative sign in the \(U(1)\) associated to the \(\overline{D2}\). In terms of the relative \(U(1)\), the tachyon is a charged scalar field in a background magnetic field, obeying an equation of the schematic form

\[ (\partial_\mu - iA_\mu)^2 T = V'(T). \]  

(36)
The spatial dependence of the background $A_{\mu}$ shows that the tachyon will condense inhomogeneously.

Physically, what happens is clear: the tachyon will condense into a collection of vortices, with the original constant magnetic field becoming localized in the cores. On the other hand, a vortex on the $D2\overline{D2}$ system is nothing other than a $D0$-brane [1]. So this process represents the $D2\overline{D2}$ system decaying into $D0$-branes.

While the final vortex configuration sounds rather complicated when expressed in the $2 + 1$ dimensional field theory language, it is simple when we go back to $D0$-brane quantum mechanics. If we wish to find a static configuration to which the system will decay, we can simply minimize the energy

$$H = -\frac{T_0}{2} \text{Tr} \frac{1}{2} [X^I, X^J]^2,$$

which has the obvious solution $[X^I, X^J] = 0$. We therefore expect that the $D2\overline{D2}$ system will evolve to a configuration where the matrices commute. This is a system of $D0$-branes. In the language of the BFSS Matrix Theory, we have a description of the annihilation of a membrane-antimembrane system into gravitons.

### 4.1 Validity of the classical approximation

Our analysis of the decay will be carried out entirely at the classical level. This is justified in the limit of large $b$ and small $g_s$ as follows. First of all, before the $D2\overline{D2}$ has begun to decay we can make the system arbitrarily weakly coupled at the magnetic length scale $\theta^2$ by taking $g_s$ sufficiently small. We now show that the final $D0$-brane system behaves classically for a long time. The final state consists of $D0$-branes with spacing $\Delta x \sim b^{-\frac{1}{2}} \sim \theta^{\frac{1}{2}}$ moving with characteristic velocity $v$. $v$ is determined by noting from (6) that the energy per $D0$-brane released by the decay is $E \sim \mu b^{-2} \sim \mu \theta^2$, 


where $\mu$ is the $D0$-brane mass. This implies $v \sim \theta$. Now, note that the time for the $D0$-branes to collide is $t \sim \Delta x/v \sim \theta^{-\frac{1}{2}}$ which is large in our limit. Furthermore, the Compton wavelength of the $D0$-branes, $\lambda \sim (\mu\theta)^{-1}$, can be taken to be much smaller than the inter-brane spacing $\Delta x$ by taking $g_s$ small. So in this limit there is a long time scale over which it is valid to think of the $D0$-branes as slowly moving classical particles. Large $N$ effects do not destroy this semiclassical picture because of the large size ($\sim N^{1/2}$) of the planar array of $D0$-branes. We should also note that the decay of the system occurs on a much shorter time scale, $t \sim \sqrt{\theta}$, as seen, for example, from (35). So for a long time after the decay has occurred we can treat the $D0$-branes classically; eventually though, they collide, become quantum mechanically entangled, and form bound states. We will have nothing more to say about this final stage in the evolution of our system, as it is outside the realm of the weak coupling approximation. Another way of describing this semiclassical decoupled regime is to specify that the three relevant length scales, the string length $l_s$, the magnetic length $l_m = \sqrt{\theta}$, and the eleven dimensional planck length $l_{11} = g_s^{1/3} l_s$ must obey the relation $l_{11} << l_m << l_s$.

Energetically, the final $D0$-branes can be distributed arbitrarily in the $x^1, x^2$ plane. However, given that we start with a homogeneous $D2\overline{D2}$ system it is clear that the distribution resulting from an actual decay will also be nearly homogeneous. But since such a distribution is not naturally written in terms of the diagonal matrices $X^1$ and $X^2$ we will focus on more special distributions. Our point in the following is just to exhibit some paths in configuration space along which the original $D2\overline{D2}$ system can be deformed into a zero energy state.

To analyse the decay, we shall have to consider intermediate matrix configurations where the branes have partially decayed. This is difficult to do if we are working in the limit where the matrices are infinite dimensional. It will be useful, therefore, to also consider finite dimensional representations.
of branes, and later take the size of the brane to infinity.

The problem is that it is not possible to solve the equation (20) with finite matrices. We can however find solutions of a related problem as in [30, 48, 49]. If we have matrices satisfying $[A_2, A_1] = \frac{2\pi i}{N}$, then the matrices $U = e^{iA_1}, V = e^{iA_2}$, satisfy

$$UV = \omega VU$$

(38)

where $\omega = e^{\frac{2\pi i}{N}}$.

Now although we cannot find finite dimensional representations of (20), we can find finite dimensional representations of (38). These are the clock and shift operators of rank $N$

$$U_{i,i+1} = 1, \quad i = 1, 2 \cdots N - 1$$

$$U_{N,1} = 1$$

$$V_{i,i} = \omega^{i-1}$$

(39)

and the rest of the elements are zero.

We can therefore define our finite dimensional approximation to (20) by the matrices $x_1^N, x_2^N$ satisfying

$$x_1^N = -i \ln(V), \quad x_2^N = -i \ln(U).$$

(40)

4.2 Representation 1

One natural choice of vacuum state is

$$X^1 = 0, \quad X^2 = 0$$

(41)

corresponding to all of the $D0$-branes being at the origin. More generally, we can take $X^1$ and $X^2$ to be any diagonal matrices.

One can write a sequence of classical matrix configurations connecting the initial $D2$-$\overline{D2}$ state with the final state of diagonal matrices. We will
consider an initial configuration with the $D2\overline{D2}$ represented by the rank N approximation.

\[
X^1 = \begin{pmatrix} x_1^N & 0 \\ 0 & x_1^N \end{pmatrix}, \quad X^2 = \begin{pmatrix} x_2^N & 0 \\ 0 & -x_2^N \end{pmatrix}
\]

\[
\downarrow
\]

\[
\downarrow
\]

\[
X^1 = \begin{pmatrix} x_1^{(N-k)} & 0 & 0 & 0 \\ 0 & D_1^{(k)} & 0 & 0 \\ 0 & 0 & x_1^{(N-k)} & 0 \\ 0 & 0 & 0 & D_2^{(k)} \end{pmatrix},
\]

\[
X^2 = \begin{pmatrix} x_2^{(N-k)} & 0 & 0 & 0 \\ 0 & D_3^{(k)} & 0 & 0 \\ 0 & 0 & x_2^{(N-k)} & 0 \\ 0 & 0 & 0 & D_4^{(k)} \end{pmatrix}
\]

\[
\downarrow
\]

\[
\downarrow
\]

\[
X^1 = \begin{pmatrix} D_1^N & 0 \\ 0 & D_2^N \end{pmatrix}, \quad X^2 = \begin{pmatrix} D_3^N & 0 \\ 0 & D_4^N \end{pmatrix}
\]

where the $D_i$ are diagonal matrices.

### 4.3 Representation 2

Another representation of the vacuum using commuting matrices is

\[
X^1 = \begin{pmatrix} x_1 & x_2 \\ x_2 & x_1 \end{pmatrix}, \quad X^2 = \begin{pmatrix} x_2 & x_1 \\ x_1 & x_2 \end{pmatrix}.
\]  \hspace{1cm} (42)

There is a decay process similar to the one described above

\[
X^1 = \begin{pmatrix} x_1^N & 0 \\ 0 & x_1^N \end{pmatrix}, \quad X^2 = \begin{pmatrix} x_2^N & 0 \\ 0 & -x_2^N \end{pmatrix}
\]
$X^1 = \begin{pmatrix}
  x_1^{(N-k)} & 0 & 0 & 0 \\
  0 & x_1^{(k)} & 0 & x_2^{(k)} \\
  0 & 0 & x_1^{(N-k)} & 0 \\
  0 & x_2^{(k)} & 0 & x_1^{(k)}
\end{pmatrix}$

$X^2 = \begin{pmatrix}
  x_2^{(N-k)} & 0 & 0 & 0 \\
  0 & x_2^{(k)} & 0 & x_1^{(k)} \\
  0 & 0 & x_2^{(N-k)} & 0 \\
  0 & x_1^{(k)} & 0 & x_2^{(k)}
\end{pmatrix}$

$X^1 = \begin{pmatrix}
  x_N^1 & x_N^2 \\
  x_2^N & x_1^N
\end{pmatrix}, \quad X^2 = \begin{pmatrix}
  x_N^2 & x_N^1 \\
  x_1^N & x_2^N
\end{pmatrix}$.

5 The 2d version of the decay

5.1 A puzzle

To make contact with the analysis of the previous sections, we need to take the limit of $N \to \infty$.

In this limit the decay process in the first representation looks like the process

$X^1 = \begin{pmatrix}
  x_1 & 0 \\
  0 & x_1
\end{pmatrix}, \quad X^2 = \begin{pmatrix}
  x_2 & 0 \\
  0 & -x_2
\end{pmatrix}$

$X^1 = \begin{pmatrix}
  x_1 & 0 & 0 & 0 \\
  0 & D_1^{(k)} & 0 & 0 \\
  0 & 0 & x_1 & 0 \\
  0 & 0 & 0 & D_2^{(k)}
\end{pmatrix}, \quad X^2 = \begin{pmatrix}
  x_2 & 0 & 0 & 0 \\
  0 & D_3^{(k)} & 0 & 0 \\
  0 & 0 & x_2 & 0 \\
  0 & 0 & 0 & D_4^{(k)}
\end{pmatrix}$. 
Therefore we start from a $D2\overline{D2}$ system and produce $D0$-branes; in the 2d language, this is the production of vortices on the branes. These vortices are composed purely of gauge field, and exist only in the noncommutative field theory. See [41] for a detailed discussion.

This leads us to an apparent puzzle. It would seem that the process of creating vortices on branes should require positive energy. In fact, if we consider the formation of a vortex on a single $D2$-brane, it was explicitly shown that this requires a positive energy. This is as it should be, since we know that a $D2$-brane on its own is stable.

We can belabour the point further by calculating how much the energy increase in Matrix theory. The energy of a membrane in the lightcone Hamiltonian is $M^2/N$, where $N$ is the momentum in the lightcone direction. If we remove a zero-brane, the energy becomes $M^2/(N - 1)$. Hence the energy increases by a factor $N/(N - 1)$.

To resolve this puzzle, let us consider the formation of a single vortex on a single $D2$-brane which has been regulated as we have been doing so far. Then the first guess could be that the process is described by the matrix configurations

$$X^1 = x_1^N, \quad X^2 = x_2^N$$

$$\downarrow$$

$$\downarrow$$

$$X^1 = \begin{pmatrix} x^{(N-1)}_1 & 0 \\ 0 & 0 \end{pmatrix}, \quad X^2 = \begin{pmatrix} x^{(N-1)}_2 & 0 \\ 0 & 0 \end{pmatrix}$$

(43)

where the total rank of both matrices is $N$.

However, we must now also include that the fact that membrane charge is conserved. In other words, we want $Tr[X^1, X^2]$ to be conserved. If we use the matrix configurations above, then the two matrices in the limit $N \to \infty$
do not have the same trace. The trace in the second matrix is \( \frac{N-1}{N} \) times the trace of the first.

To remedy this, we should consider the process

\[
X^1 = x_1^N, \quad X^2 = x_2^N
\]

\[
\Downarrow
\]

\[
X^1 = \sqrt{\frac{N}{N-1}} \begin{pmatrix} x_1^{(N-1)} & 0 \\ 0 & 0 \end{pmatrix}, \quad X^2 = \sqrt{\frac{N}{N-1}} \begin{pmatrix} x_2^{(N-1)} & 0 \\ 0 & 0 \end{pmatrix}.
\]

(44)

Now comparing the energies, we find that the ratio of energies of the configurations is

\[
\frac{\text{Tr}[X^1, X^2]^2_{\text{final}}}{\text{Tr}[X^1, X^2]^2_{\text{initial}}} = \frac{N}{N-1}.
\]

(45)

Hence the energy indeed increases by a factor \( \frac{N}{N-1} \), as expected.

Now in the \( D2-\overline{D2} \) system, the total membrane charge is zero. Hence the process that we wrote earlier is consistent without any extra factors. In this case, the energy of forming a vortex is indeed negative. In fact, we can calculate the ratio of energies of the intermediate configuration to the initial one to be

\[
\frac{\text{Tr}[X^1, X^2]^2_{\text{final}}}{\text{Tr}[X^1, X^2]^2_{\text{initial}}} = \frac{N-k}{N}.
\]

(46)

Hence the energy decreases, and the decay is allowed.

5.2 Decay in representation 1

Having resolved this puzzle, let us turn to the interpretation of the decay in representation 1.

The final state representing the vacuum has \( X^1 = X^2 = 0 \). From the 2d point of view, we can think of this as a fluctuation about two D2-branes
\[ X^a = x^a \otimes 1_{2 \times 2} + \theta \epsilon^{ab} A_b = 0. \] (48)

Hence
\[ A_1 = \frac{1}{\theta} x^2 \otimes 1_{2 \times 2}, \quad A_2 = -\frac{1}{\theta} x^1 \otimes 1_{2 \times 2}. \] (49)

We therefore see that in this vacuum, only the gauge fields are nonzero, and they are precisely chosen so that the covariant derivative acting on any field vanishes (as \([X^i, \cdot] = 0\)). What has happened is that the original magnetic field (31), living purely in the second \(U(1)\), has redistributed itself symmetrically in the two \(U(1)\)s via the formation of vortices. As pointed out in [36] this vanishing covariant derivative indicates that the open string degrees of freedom can no longer propagate and have become unphysical.

Furthermore, this vacuum has a classical \(U(\infty)\) symmetry. In the case of decay to nothing a vacuum state with this symmetry was proposed in [36]. By analogy, we will refer to the vacuum in representation 1 as the “GMS vacuum”.

We note, though, that more generally the vacuum in this representation corresponds to \(X^a\) diagonal, not zero. In such a vacuum \(U(\infty)\) is spontaneously broken (and in fact a gauge transformation takes this vacuum to the one discussed below). The covariant derivative is not zero, but now allows independent motion of each \(D0\)-brane. Two dimensional propagation has ceased.

Of course after very long times strong quantum fluctuations become important and the \(U(\infty)\) symmetry is restored, but these two phenomena are not linked.

As an aside let us remind the reader that the unbroken \(U(\infty)\) state of Matrix theory and the spontaneously broken \(U(\infty)\) state of an arbitrarily shaped matrix membrane (where \(U(\infty)\) is interpreted as the group of area preserving diffeomorphisms) provide an instructive analog of Witten’s ideas
about the existence an unbroken (perhaps topological) phase of gravity, as well as a more conventional spontaneously broken phase. These ideas may well be important in the search for background independent formulations of quantum gravity.

5.3 Decay in representation 2

The second representation of the decay has a final state where the off diagonal matrix is nontrivial. This implies that the tachyon is nontrivial in this vacuum. We would like to relate this vacuum to the sort of vacuum proposed by Sen, in which the tachyon condenses homogeneously while the gauge fields remain at zero. However, this is difficult to do in this context, as we have already shown that there is not expected to be a solution with a constant tachyon.

The solution instead represents, not a constant tachyon field, but rather, a configuration of tachyon vortices. These tachyon vortices each carry one unit of $D0$-brane charge, and hence this corresponds to a distribution of $D0$-branes.

It is helpful to diagonalize (42) in order to specify the $D0$-brane distribution. Working in a diagonal basis for $x_1, x_2$ is represented as $x_2 = -i\partial_{x_1}$. The eigenvectors of $X^1$ and $X^2$ are

$$\vec{v}_1 = \begin{pmatrix} f_\lambda(x_1) \\ f_\lambda(x_1) \end{pmatrix}, \quad \vec{v}_2 = \begin{pmatrix} \overline{f_\lambda(x_1)} \\ -\overline{f_\lambda(x_1)} \end{pmatrix},$$

(50)

with eigenvalues

$$X^1\vec{v}_1 = \lambda\vec{v}_1, \quad X^2\vec{v}_1 = \lambda\vec{v}_1,$$

$$X^1\vec{v}_2 = \lambda\vec{v}_2, \quad X^2\vec{v}_2 = -\lambda\vec{v}_2,$$

(51)

where we have defined

$$f_\lambda(x_1) = e^{-\frac{i}{2}(x_1-\lambda)^2}.$$  

(52)
Normalizability requires $\lambda$ to be real. The distribution of $D0$-branes thus corresponds to two diagonal lines crossing at the origin of the $X^1, X^2$ plane.

5.4 Relation between representations

We have provided two representations of the final vacuum state, and representations of the decay into these vacuum states. In the first representation only the gauge field condensed, while in the second representation the tachyon condensed as well. We now would like to understand the relation between these two representations.

In the quantum mechanical version of the decay, it is straightforward to see that these two representations are gauge equivalent. Since any pair of commuting matrices can be diagonalized, one can diagonalize the matrices (42) and obtain a diagonal matrix as in (41).

In the 2d language it is hard to see this gauge equivalence, because the $U(2)$ symmetry of the two $D2$-brane system is spontaneously broken in the $D2-D2$ system. This is a further indication that the “right” variables to describe noncommutative theories are the matrix model variables, as pointed out by Seiberg [31].

Secondly, since we are identifying representation 1 with the GMS description of the vacuum, and representation 2 with the vacuum described by Sen, we would like to argue that these two are actually the same, i.e. the GMS vacuum is gauge equivalent to the Sen vacuum.

This is not a precise statement, since we have not identified the exact vacuum corresponding to the Sen vacuum in our framework. However, we have shown that diagonal matrices are gauge equivalent to the other configurations where the tachyon is nontrivial. Furthermore, all such configurations, which have an off diagonal component turned on, can be diagonalized to a form where only the gauge field is turned on. Hence the Sen vacuum, which
must have a nonzero profile for the tachyon field, is gauge equivalent to a configuration which has a zero tachyon field, but a nontrivial gauge field.

This configuration of gauge fields is classically describable as a distribution of vortices over the plane. The Sen vacuum is expected to be gauge equivalent to a homogeneous distribution of vortices. As such, it would appear to be different from the GMS vacuum, where all the vortices are at one point. These vacua are connected by a marginal deformation, where we move the zero-branes from one configuration to the other.

Hence we can more accurately describe the GMS and Sen vacua as being on the same moduli space of vacua.

Although we have not discussed the quantum mechanics of the system so far, it is easy to see from general considerations that there will be solutions corresponding to threshold bound states in the final solution.

### 5.5 Level truncation

By turning on large magnetic fields on the worldvolumes of the $D2$ and $\overline{D2}$ and taking the zero slope limit, we have vastly reduced the number of degrees of freedom of the system. However, the problem is still sufficiently complicated that it is difficult to explicitly track the evolution of the system to its ground state. To make the problem even simpler we can employ level truncation, as is done in open string field theory [10, 11]. This corresponds to solving the equations of motion of some number of light modes, and setting all heavier modes to zero. In our case the simplest level truncation consists of retaining only the tachyon, with the action being given by (35). Minimizing the tachyon potential gives,

$$|T| = \sqrt{2}. \quad (53)$$

We now argue that this configuration corresponds, roughly, to a $D2-\overline{D2}$ with a hole around the origin. First, in operator language the tachyon
configuration is proportional to the projection operator $P_0$; in terms of functions this corresponds to a Gaussian centered at the origin [29]. To get an idea of whether the $D2$-$\overline{D2}$ has decayed in the region where the tachyon has condensed, we will introduce an extra $D0$-brane into the system to act as a probe. The point is that before $T$ condensation there is a tachyonic mode coming from strings connecting the auxiliary $D0$-brane to the $D2$-$\overline{D2}$ system, but we will see that this tachyon acquires a positive $m^2$ once $T$ condenses, indicating that the $D2$-$\overline{D2}$ system has decayed to $D0$-branes near the origin. Thus we consider in complex coordinates

$$Z = \sqrt{\theta} \begin{pmatrix} 0 & t & 0 \\ 0 & a & 0 \\ 0 & TP_0 & a^\dagger \end{pmatrix}.$$  \hfill (54)

$t$, the $D0$-$D2$ tachyon, is a row vector with a nonzero entry in its first component. The potential for $t$ is

$$V(t) = 4\theta^2(|T|^2 - 1)|t|^2.$$  \hfill (55)

This gives the desired result: $t$ is tachyonic at $T = 0$, and acquires positive $m^2$ at $|T| = \sqrt{2}$.

It would be interesting to include more modes in the level truncation. For instance at the next level we should include the $U(1) \times U(1)$ gauge fields. We expect that this will describe a $D2$-$\overline{D2}$ with an expanding hole that eventually fills the worldvolume once all modes are retained.

6 Comments

We conclude with a few comments:

1. In the case of tachyon condensation into “nothing” there is the question of what happens to the open string degrees of freedom on the original
D-brane. Expanding around the stable minimum, the spectrum must either consist of states with nonperturbatively heavy masses, or states that can be interpreted as closed strings, but how either scenario is realized is not well understood. In the present case the system decays into $D0$-branes, and it is clear that the original degrees of freedom are repackaged as strings ending on these $D0$-branes. The effect of tachyon condensation is to convert degrees of freedom which look $2+1$ dimensional into ones which look $0+1$ dimensional. It has been proposed [36] that in the case of decay to nothing the final configuration looks $0+0$ dimensional. Studying the decay to nothing seems to require dealing with the full open string field theory, rather than just the zero slope limit as was done here. In addition the scenario in [36] requires an understanding of the dynamics that produces large amounts of flux. In the current work the $D0$-branes are put in "by hand."

2. We have studied the decay of the $D\overline{D}$ system, but there are other unstable D-branes in type II string theory: $D(2p+1)$-branes in IIA, and $D(2p)$-branes in IIB. It is natural to wonder whether these branes can also be constructed out of lower dimensional branes, e.g. $D0$-branes in IIA. This seems to be complicated for the following reason. $D0$-brane charge shows up on the $D2$-brane as magnetic flux, and a similar story holds for the other $D(2p)$-branes. However on an unstable $D1$-brane, for example, $D0$-charge arises via spatial variation of the tachyon, $S_{\omega z} = \int C^{(1)} \wedge dT$. Constant $D0$ charge density then requires a linearly varying $T$, but this is not a solution to the equations of motion.
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References

[1] A. Sen, “SO(32) spinors of type I and other solitons on brane-antibrane pair,” JHEP 9809, 023 (1998) [hep-th/9808141]; Stable non-BPS bound states of BPS D-branes, JHEP 9808, 010 (1998) [hep-th/9805019]; “Tachyon condensation on the brane antibrane system,” JHEP 9808, 012 (1998) [hep-th/9805170]; “BPS D-branes on non-supersymmetric cycles,” JHEP 9812, 021 (1998) [hep-th/9812031]; ‘Descent relations among bosonic D-branes,” Int. J. Mod. Phys. A14, 4061 (1999) [hep-th/9902105].

[2] E. Witten, “D-branes and K-theory,” JHEP 9812, 019 (1998) [hep-th/9810188].

[3] P. Horava, “Type IIA D-branes, K-theory, and matrix theory,” Adv. Theor. Math. Phys. 2, 1373 (1999) [hep-th/9812135].

[4] J. A. Harvey, P. Horava and P. Kraus, “D-sphalerons and the topology of string configuration space,” JHEP 0003, 021 (2000) [hep-th/0001143].

[5] J. A. Harvey and P. Kraus, “D-branes as unstable lumps in bosonic open string field theory,” JHEP 0004, 012 (2000) [hep-th/0002117].

[6] R. de Mello Koch, A. Jevicki, M. Mihailescu and R. Tatar, “Lumps and p-branes in open string field theory,” Phys. Lett. B482, 249 (2000) [hep-th/0003031].

[7] N. Moeller, A. Sen and B. Zwiebach, “D-branes as tachyon lumps in string field theory,” JHEP 0008, 039 (2000) [hep-th/0005036].
[8] R. de Mello Koch and J. P. Rodrigues, “Lumps in level truncated open string field theory,” hep-th/0008053.

[9] N. Moeller, “Codimension two lump solutions in string field theory and tachyonic theories,” hep-th/0008101.

[10] V. A. Kostelecky and S. Samuel, “On A Nonperturbative Vacuum For The Open Bosonic String,” Nucl. Phys. B336, 263 (1990).

[11] A. Sen and B. Zwiebach, “Tachyon condensation in string field theory,” JHEP 0003, 002 (2000) [hep-th/9912249].

[12] N. Berkovits, A. Sen and B. Zwiebach, “Tachyon condensation in superstring field theory,” hep-th/0002211.

[13] N. Moeller and W. Taylor, “Level truncation and the tachyon in open bosonic string field theory,” Nucl. Phys. B583, 105 (2000) [hep-th/0002237].

[14] P. De Smet and J. Raeymaekers, “Level four approximation to the tachyon potential in superstring field theory,” JHEP 0005, 051 (2000) [hep-th/0003220]; “The tachyon potential in Witten’s superstring field theory,” JHEP 0008, 020 (2000) [hep-th/0004112].

[15] L. Rastelli and B. Zwiebach, “Tachyon potentials, star products and universality,” hep-th/0006240.

[16] V. A. Kostelecky and R. Potting, “Analytical construction of a nonperturbative vacuum for the open bosonic string,” hep-th/0008252.

[17] A. Sen and B. Zwiebach, “Large marginal deformations in string field theory,” hep-th/0007153.

[18] W. Taylor, “Mass generation from tachyon condensation for vector fields on D-branes,” JHEP 0008, 038 (2000) [hep-th/0008033].

[19] A. Iqbal and A. Naqvi, “On marginal deformations in superstring field theory,” hep-th/0008127.

[20] P. Yi, “Membranes from five-branes and fundamental strings from Dp branes,” Nucl. Phys. B550, 214 (1999) [hep-th/9901159].
[21] O. Bergman, K. Hori and P. Yi, “Confinement on the brane,” Nucl. Phys. B580, 289 (2000) [hep-th/0002223].

[22] J. A. Harvey, P. Kraus, F. Larsen and E. J. Martinec, “D-branes and strings as non-commutative solitons,” JHEP 0007, 042 (2000) [hep-th/0005031].

[23] A. A. Gerasimov and S. L. Shatashvili, “On exact tachyon potential in open string field theory,” hep-th/0009103.

[24] D. Kutasov, M. Marino and G. Moore, “Some exact results on tachyon condensation in string field theory,” hep-th/0009148.

[25] K. Dasgupta, S. Mukhi and G. Rajesh, “Noncommutative tachyons,” JHEP 0006, 022 (2000) [hep-th/0005006].

[26] E. Witten, “Noncommutative tachyons and string field theory,” hep-th/0006071.

[27] A. Connes, M. R. Douglas and A. Schwarz, “Noncommutative geometry and matrix theory: Compactification on tori,” JHEP 9802, 003 (1998) [hep-th/9711162].

[28] N. Seiberg and E. Witten, “String theory and noncommutative geometry,” JHEP 9909, 032 (1999) [hep-th/9908142].

[29] R. Gopakumar, S. Minwalla and A. Strominger, “Noncommutative solitons,” JHEP 0005, 020 (2000) [hep-th/0003160].

[30] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, “M theory as a matrix model: A conjecture,” Phys. Rev. D55, 5112 (1997) [hep-th/9610043].

[31] N. Seiberg, “A note on background independence in noncommutative gauge theories, matrix model and tachyon condensation,” JHEP 0009, 003 (2000) [hep-th/0008013].

[32] H. Awata, S. Hirano and Y. Hyakutake, “Tachyon condensation and graviton production in matrix theory,” hep-th/9902158.
[33] M. Massar and J. Troost, “The longitudinal fivebrane and tachyon condensation in matrix theory,” Nucl. Phys. B569, 417 (2000) [hep-th/9907128].

[34] J. R. David, “U(1) gauge invariance from open string field theory,” hep-th/0005085.

[35] C. Sochichiu, “Noncommutative tachyonic solitons: Interaction with gauge field,” JHEP 0008, 026 (2000) [hep-th/0007217].

[36] R. Gopakumar, S. Minwalla and A. Strominger, “Symmetry restoration and tachyon condensation in open string theory,” hep-th/0007226.

[37] A. Sen, “Some issues in non-commutative tachyon condensation,” hep-th/0009038.

[38] A. Sen, “Uniqueness of tachyonic solitons,” hep-th/0009090.

[39] B. Zwiebach, “A solvable toy model for tachyon condensation in string field theory,” hep-th/0008227; J. A. Minahan and B. Zwiebach, “Field theory models for tachyon and gauge field string dynamics,” hep-th/0008231.

[40] W. Taylor, “D-brane effective field theory from string field theory,” hep-th/001201.

[41] M. Aganagic, R. Gopakumar, S. Minwalla, and A. Strominger, “Unstable Solitons in Noncommutative Gauge Theory,” hep-th/0009142.

[42] R. Tatar, “A note on non-commutative field theory and stability of brane-antibrane systems,” hep-th/0009213.

[43] L. Cornalba, “D-brane physics and noncommutative Yang-Mills theory,” hep-th/9909081.

[44] O. Aharony and M. Berkooz, “Membrane dynamics in M(atrix) theory,” Nucl. Phys. B491, 184 (1997) [hep-th/9611215].

[45] G. Lifschytz and S. D. Mathur, “Supersymmetry and membrane interactions in M(atrix) theory,” Nucl. Phys. B507, 621 (1997) [hep-th/9612087].
[46] T. Banks, N. Seiberg and S. Shenker, “Branes from matrices,” Nucl. Phys. B490, 91 (1997) [hep-th/9612157].

[47] E. Keski-Vakkuri and P. Kraus, “Notes on branes in matrix theory,” Nucl. Phys. B510, 199 (1998) [hep-th/9706196].

[48] J. Hoppe, “Diffeomorphism Groups, Quantization And SU(Infinity),” Int. J. Mod. Phys. A4, 5235 (1989).

[49] D. B. Fairlie, P. Fletcher and C. K. Zachos, “Infinite Dimensional Algebras And A Trigonometric Basis For The Classical Lie Algebras,” J. Math. Phys. 31, 1088 (1990).