Generalization of the Keller–Osserman theorem for higher order differential inequalities

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Abstract

We obtain exact conditions guaranteeing that any global weak solution of the differential inequality

\[
\sum_{|\alpha|=m} \partial^\alpha a_\alpha (x,u) \geq g(|u|) \quad \text{in } \mathbb{R}^n
\]

is trivial, where \(m, n \geq 1\) are integers and \(a_\alpha\) and \(g\) are some functions. These conditions generalize the well-known Keller–Osserman condition.

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1. Introduction

We study the differential inequality

\[
\sum_{|\alpha|=m} \partial^\alpha a_\alpha (x,u) \geq g(|u|) \quad \text{in } \mathbb{R}^n,
\]

where \(m, n \geq 1\) are integers and \(a_\alpha\) are Caratheodory functions such that

\(|a_\alpha(x, \zeta)| \leq A|\zeta|, \quad |\alpha| = m.\)

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with some constant $A > 0$ for almost all $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ and for all $\zeta \in \mathbb{R}$. By $\alpha = (\alpha_1, \ldots, \alpha_n)$ we mean a multi-index with $|\alpha| = \alpha_1 + \ldots + \alpha_n$ and $\partial^\alpha = \partial^{\alpha_1}_{x_1} \cdots \partial^{\alpha_n}_{x_n}$. It is also assumed that $g$ is a non-decreasing convex function on the interval $[0, \infty)$ and, moreover, $g(\zeta) > 0$ for all $\zeta > 0$.

Let us denote by $B_r^x$ the open ball in $\mathbb{R}^n$ of radius $r > 0$ and center at $x$. In the case of $x = 0$, we write $B_r$ instead of $B_0^r$.

A function $u \in L_{1,\text{loc}}(\mathbb{R}^n)$ is called a global weak solution of (1.1) if $g(|u|) \in L_{1,\text{loc}}(\mathbb{R}^n)$ and

$$
\int_{\mathbb{R}^n} \sum_{|\alpha| = m} (-1)^{|\alpha|} a_\alpha(x, u) \partial^\alpha \varphi \, dx \geq \int_{\mathbb{R}^n} g(|u|) \varphi \, dx \tag{1.2}
$$

for any non-negative function $\varphi \in C_0^\infty(\mathbb{R}^n)$.

In their pioneering works [1, 2], Keller and Osserman proved that, under the condition

$$
\int_{1}^{\infty} \left( \int_{1}^{\xi} g(\xi) \, d\xi \right)^{-1/2} \, d\xi < \infty, \tag{1.3}
$$

the elliptic inequality

$$
\Delta u \geq g(u) \quad \text{in} \quad \mathbb{R}^n \tag{1.4}
$$

has no positive global solutions. Since then, a lot of papers appeared on the absence of solutions for various differential equations and inequalities [3–21]. In so doing, the most studied case is the case of the Emden–Fowler nonlinearity $g(t) = t^\lambda$. For this nonlinearity, condition (1.3) takes the form $\lambda > 1$. This obviously differs from the condition

$$
1 < \lambda < \frac{n}{n - 2}
$$

given in [3–5] which, for $n \geq 3$, guarantees the absence of positive global solutions for the so-called noncoercive inequalities

$$
-\Delta u \geq u^\lambda \quad \text{in} \quad \mathbb{R}^n.
$$

Thus, conditions for the absence of global solutions or, in other words, blow-up conditions depend on the sign before the Laplace operator.

For the Emden–Fowler nonlinearity, it was managed to obtain blow-up conditions for solutions of higher order differential inequalities [5–7] while, in the case of general nonlinearity, all studies were limited to inequalities of the second order [8–14]. For these inequalities, in particular, it is possible to get rid of the function $g$ convexity requirement. Moreover, for second order differential inequalities, there is no need to require the function $g$ to be non-decreasing [8]. This was achieved thanks to the maximum principle which, unfortunately, is not valid for higher order inequalities.

In our paper, we obtain exact conditions guaranteeing that any global weak solution of (1.1) is trivial or, in other words, equal to zero almost everywhere in $\mathbb{R}^n$. For non-negative solutions of (1.4), these conditions are equivalent to (1.3). In so doing, they are also exact in the noncoercive case (see example 2.1).

Note that a priori we do not impose any ellipticity conditions on the differential operator in the left-hand side of (1.1). In addition, it does not matter to us what sign a solution of (1.1) has. Therefore, our results can be applied to a wide class of differential inequalities.
2. Main results

**Theorem 2.1.** Let
\[
\int_{1}^{\infty} g^{-1/m}(\zeta)\zeta^{1/m-1} \, d\zeta < \infty \tag{2.1}
\]
and
\[
\lim_{t \to 0+} \inf G^{n-m}(t) t < \infty, \tag{2.2}
\]
where
\[
G(t) = \int_{t}^{\infty} g^{-1/m}(\zeta)\zeta^{1/m-1} \, d\zeta. \tag{2.3}
\]
Then any global weak solution of (1.1) is trivial.

**Theorem 2.2.** Let
\[
\int_{0}^{\infty} g^{-1/m}(\zeta)\zeta^{1/m-1} \, d\zeta < \infty \tag{2.4}
\]
Then (1.1) has no global weak solutions.

**Theorem 2.3.** Let (2.1) be valid, then any global weak solution of (1.1) satisfies the estimate
\[
\frac{1}{r^m} \int_{B_r} |u| \, dx \leq CG^{-1}(kr) \tag{2.5}
\]
for all \( r > 0 \), where \( G^{-1} \) is the inverse function to (2.3) and the constants \( C > 0 \) and \( k > 0 \) depend only on \( A, m, \) and \( n \).

The proof of theorems 2.1–2.3 is given in section 3.

**Remark 2.1.** If (2.1) holds and inequality (1.1) has a global weak solution, then in accordance with theorem 2.2 we obviously have
\[
\lim_{t \to 0+} G(t) = \infty. \tag{2.6}
\]
Thus, the right-hand side of (2.5) is defined for all \( r > 0 \). Since \( g \) is a a non-decreasing convex function on \([0, \infty)\), condition (2.4) implies that \( g(0) > 0 \).

**Theorem 2.4.** Let (2.1) be valid, then
\[
\lim_{r \to \infty} \frac{1}{r^m} \int_{B_r} |u| \, dx = 0 \tag{2.7}
\]
for any global weak solution of inequality (1.1).
Theorem 2.5. Let (2.1) be valid and, moreover, \( m \geq n \). Then any global weak solution of (1.1) is trivial.

Proof of theorems 2.4 and 2.5. Since \( G^{-1}(r) \to 0 \) as \( r \to \infty \), relation (2.7) readily follows from estimate (2.5) of theorem 2.3. In turn, to prove theorem 2.5, it is sufficient to use theorem 2.1.

Remark 2.2. In the case of \( m = 2 \), condition (2.1) takes the form

\[
\int_1^\infty (g(\zeta)\zeta)^{-1/2} \, d\zeta < \infty. \tag{2.8}
\]

It does not present any particular problem to verify that (2.8) is equivalent to the well-known Keller–Osserman condition (1.3). Really, taking into account the fact that \( g \) is a non-decreasing positive function on the interval \((0, \infty)\), we obtain

\[
\int_1^\zeta g(\xi) \, d\xi \geq \int_{\zeta/2}^\zeta g(\xi) \, d\xi \geq \frac{\zeta}{2}g\left(\frac{\zeta}{2}\right), \quad \zeta \geq 2.
\]

Hence, (2.8) implies (1.3). At the same time,

\[
\int_1^\zeta g(\xi) \, d\xi \leq \zeta g(\zeta), \quad \zeta \geq 1;
\]

therefore, (2.8) follows from (1.3).

Corollary 2.1 (Keller–Osserman). Suppose that (2.8) is valid, then any non-negative global weak solution of (1.4) is trivial.

Proof. Let \( u \) be a non-negative global weak solution of (1.4). By the submean-value property, we have

\[
u(x) \leq \frac{1}{|B_r|} \int_{B_r} u \, dy
\]

for all \( r > 0 \) and for almost all \( x \in \mathbb{R}^n \), where \(|B_r|\) is the \( n \)-dimensional volume of the ball \( B_r \), whence in accordance with theorem 2.4 it follows that \( u = 0 \) almost everywhere in \( \mathbb{R}^n \).

Example 2.1. Consider the inequality

\[
\sum_{|\alpha|=m} \partial^\alpha a_\alpha(x,u) \geq c_0 |u|^\lambda \quad \text{in} \ \mathbb{R}^n, \quad c_0 = \text{const} > 0. \tag{2.9}
\]

By theorem 2.1, the conditions

\[
\lambda > 1 \quad \text{and} \quad \lambda(n-m) \leq n
\]

imply that any global weak solution of (2.9) is trivial. It can be shown that these conditions are the best possible [5, 6].
Example 2.2. Let us examine the critical exponent $\lambda = 1$ in the right-hand side of (2.9). Namely, consider the inequality
\[ \sum_{|\alpha| = m} \partial^\alpha a_\alpha (x, u) \geq c_0 |u| \ln^{\nu} (2 + |u|) \quad \text{in } \mathbb{R}^n, \quad c_0 = \text{const} > 0, \]
(2.10)

By theorem 2.1, if $\nu > m$,

then any global weak solution of (2.10) is trivial. At the same time, for all positive even integers $m$ and real numbers $\nu \leq m$ and $c_0 > 0$ the inequality
\[ \Delta^{m/2} u \geq c_0 |u| \ln^{\nu} (2 + |u|) \quad \text{in } \mathbb{R}^n \]
has a positive infinitely smooth global solution. As such a solution, one can take
\[ u(x) = e^{k (1 + |x|^{2})^{1/2}}, \]
where $k > 0$ is a sufficiently large real number. Thus, condition (2.11) is also the best possible.

3. Proof of theorems 2.1–2.3

In this section, by $C$ we denote various positive constants that can depend only on $A$, $m$, and $n$.

Lemma 3.1. Let $u$ be a global weak solution of (1.1), then
\[ \int_{B_{r_2} \setminus B_{r_1}} |u| \, dx \geq C (r_2 - r_1)^m \int_{B_{r_1}} g(|u|) \, dx \]
for all real numbers $0 < r_1 < r_2$ such that $r_2 \leq 2r_1$.

Proof. Take a non-negative function $\varphi_0 \in C^\infty (\mathbb{R})$ satisfying the conditions
\[ \varphi_0 |_{(-\infty, 0]} = 0 \quad \text{and} \quad \varphi_0 |_{[1, \infty)} = 1. \]

Putting
\[ \varphi(x) = \varphi_0 \left( \frac{r_2 - |x|}{r_2 - r_1} \right) \]
as a test function in (1.2), we obtain
\[ \int_{\mathbb{R}^n} \sum_{|\alpha| = m} (-1)^m a_\alpha (x, u) \partial^\alpha \varphi_0 \left( \frac{r_2 - |x|}{r_2 - r_1} \right) \, dx \geq \int_{\mathbb{R}^n} g(|u|) \varphi_0 \left( \frac{r_2 - |x|}{r_2 - r_1} \right) \, dx. \]

Combining this with the estimates
\[ \left| \int_{\mathbb{R}^n} \sum_{|\alpha| = m} (-1)^m a_\alpha (x, u) \partial^\alpha \varphi_0 \left( \frac{r_2 - |x|}{r_2 - r_1} \right) \, dx \right| \leq \frac{C}{(r_2 - r_1)^m} \int_{B_{r_2} \setminus B_{r_1}} |u| \, dx \]

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and

\[ \int_{B_{r_1}} g(|u|) \, dx \leq \int_{B_{r_2}} g(|u|) \varphi_0 \left( \frac{r_2 - |x|}{r_2 - r_1} \right) \, dx, \]

we complete the proof. \qed

**Lemma 3.2.** Let \( u \) be a global weak solution of \((1.1)\) and \( r_1 < r_2 < 2r \) be positive real numbers. Then

\[ J_r(r_2) - J_r(r_1) \geq C(r_2 - r_1)^m g(J_r(r_1)), \]

where

\[ J_r(\rho) = \frac{1}{|B_2|} \int_{B_\rho} |u| \, dx. \quad (3.1) \]

**Proof.** By lemma 3.1, we have

\[ J_r(r_2) - J_r(r_1) \geq C(r_2 - r_1)^m \int_{B_{r_1}} g(|u|) \, dx \geq \frac{C(r_2 - r_1)^m}{2^n|B_{r_1}|} \int_{B_{r_1}} g(|u|) \, dx. \]

Thus, to complete the proof it remains to note that

\[ \frac{1}{|B_{r_1}|} \int_{B_{r_1}} g(|u|) \, dx \geq g \left( \frac{1}{|B_{r_1}|} \int_{B_{r_1}} |u| \, dx \right) \geq g(J_r(r_1)) \]

since \( g \) is a non-decreasing convex function. \qed

**Lemma 3.3.** Let \( u \) be a global weak solution of \((1.1)\) and \( r > 0 \) be a real number such that

\[ \int_{B_r} |u| \, dx > 0. \quad (3.2) \]

Then at least one of the following two inequalities is valid:

\[ \int_{J_r(r)}^{J_r(2r)} \frac{d\zeta}{g(\zeta/2)} \geq Cr^m, \quad (3.3) \]

\[ \int_{J_r(r)}^{J_r(2r)} g^{-1/m}(\zeta/2)\zeta^{1/m-1} \, d\zeta \geq Cr, \quad (3.4) \]

where the function \( J_r \) is defined by \((3.1)\).

**Proof.** Consider a finite sequence of real numbers \( \{r_i\}_{i=0}^l \) constructed as follows. We take \( r_0 = r \). Assume further that \( r_i \) is already known. If \( r_i \geq 3r/2 \), then we put \( l = i \) and stop; otherwise we take

\[ r_{i+1} = \sup \{ \rho \in [r_i, 2r] : J_{r}(\rho) \leq 2J_{r}(r_i) \}. \]
Since $J(r_0) > 0$ and $u \in L^{1, \infty}(\mathbb{R}^n)$, this procedure must terminate at a finite step. In so doing, we obviously have either
\begin{equation}
    r_i = 2r \quad \text{and} \quad J_i(r_i) \leq 2J_i(r_{i-1}) \tag{3.5}
\end{equation}
or
\begin{equation}
    J_i(r_{i+1}) = 2J_i(r_i), \quad i = 0, \ldots, l - 1. \tag{3.6}
\end{equation}

At first, let (3.5) hold. By lemma 3.2, we obtain
\[
    J_i(r_{i+1}) - J_i(r_i) \quad \frac{g(J_i(r_i))}{g(J_i(r_{i-1}))} \leq C(r_i - r_{i-1})^m.
\]

Since
\[
    \int_{J_i(r_{i-1})}^{J_i(r_i)} \frac{d\zeta}{g(\zeta/2)} \geq \frac{J_i(r_{i+1}) - J_i(r_i)}{g(J_i(r_{i-1}))}
\]
and $r_i - r_{i-1} \geq r/2$, this yields (3.3).

Now, let (3.6) be valid. Lemma 3.2 implies that
\[
    \left( \frac{J_i(r_{i+1}) - J_i(r_i)}{g(J_i(r_i))} \right)^{1/m} \geq C(r_{i+1} - r_i), \quad i = 0, \ldots, l - 1.
\]

Combining this with the inequalities
\[
    \int_{J_i(r_i)}^{J_i(r_{i+1})} \frac{g^{-1/m}(\zeta/2)\zeta^{1/m-1}d\zeta}{g(\zeta/2)} \geq C \left( \frac{J_i(r_{i+1}) - J_i(r_i)}{g(J_i(r_i))} \right)^{1/m}, \quad i = 0, \ldots, l - 1.
\]
we have
\[
    \int_{J_i(r_i)}^{J_i(r_{i+1})} g^{-1/m}(\zeta/2)\zeta^{1/m-1}d\zeta \geq C(r_{i+1} - r_i), \quad i = 0, \ldots, l - 1.
\]

Finally, summing the last formula over all $i = 0, \ldots, l - 1$, we conclude that
\[
    \int_{J_i(r_0)}^{J_i(r_l)} g^{-1/m}(\zeta/2)\zeta^{1/m-1}d\zeta \geq C(r_l - r_0).
\]

This implies (3.4).
\[
\square
\]

We need the following assertion proved in [21, lemma 2.3].

**Lemma 3.4.** Let $\psi : (0, \infty) \to (0, \infty)$ and $\gamma : (0, \infty) \to (0, \infty)$ be measurable functions satisfying the condition
\[
    \gamma(\zeta) \leq \text{ess inf}_{(\zeta/0, \theta \zeta)} \psi
\]
with some real number $\theta > 1$ for almost all $\zeta \in (0, \infty)$. Also assume that $0 < \alpha \leq 1, M_1 > 0, M_2 > 0,$ and $\nu > 1$ are some real numbers with $M_2 \geq \nu M_1$. Then
\[
\left( \frac{1}{\alpha} \int_{M_2}^{M_1} \gamma^{-\alpha}(\zeta)\zeta^{\alpha-1} \, d\zeta \right)^{1/\alpha} \geq A \int_{M_1}^{M_2} \frac{d\zeta}{\psi(\zeta)},
\]
where the constant \( A > 0 \) depends only on \( \alpha, \nu, \) and \( \theta. \)

**Lemma 3.5.** Under the hypotheses of lemma 3.3, let (2.1) be valid. Then
\[
\int_{J_r(\rho)} g^{-1/m}(\zeta/4)\zeta^{1/m-1} \, d\zeta \geq C r,
\]
where the function \( J_r \) is defined by (3.1).

**Proof.** In view of lemma 3.3, at least one of inequalities (3.3) and (3.4) holds. In the case where (3.4) holds, we obviously have
\[
\int_{J_r(\rho)} g^{-1/m}(\zeta/2)\zeta^{1/m-1} \, d\zeta \geq C r,
\]
whence (3.7) follows at once. Now, let (3.3) be valid. Lemma 3.4 yields
\[
\left( \int_{J_r(\rho)} g^{-1/m}(\zeta/4)\zeta^{1/m-1} \, d\zeta \right)^m \geq C \int_{J_r(\rho)} \frac{d\zeta}{g(\zeta/2)}.
\]
Combining this with (3.3), we again obtain (3.7).

**Proof of theorem 2.2.** Assume that (2.4) holds and, moreover, \( u \) is a global weak solution of (1.1). Since \( g \) is a a non-decreasing convex function, we have \( g(0) > 0 \). This means that for all \( r > 0 \) inequality (3.2) is valid. Really, if \( g(0) > 0 \), then in accordance with (1.2) a global weak solution of (1.1) can not vanish on a non-empty open set. Therefore, in view of lemma 3.5, for all \( r > 0 \) estimate (3.7) is valid. Thus, passing in (3.7) to the limit as \( r \to \infty \), we arrive at a contradiction.

**Proof of theorem 2.3.** Let \( r > 0 \) be a real number and \( u \) be a global weak solution of (1.1). If \( u = 0 \) almost everywhere in \( B_r \), then (2.5) is obvious; otherwise (3.2) holds and estimate (2.5) follows from inequality (3.7) of lemma 3.5.

**Proof of theorem 2.1.** Let \( u \) be a global weak solution of (1.1). In view of theorem 2.2, relation (2.6) is valid. Thus, we have \( g(0) = 0 \) and \( G^{-1}(r) \to 0 \) as \( r \to \infty \). In so doing, \( G \) is an one-to-one continuous map of the open interval \((0, \infty)\) onto itself and \( g \) is an one-to-one continuous map of the closed interval \([0, \infty)\) onto itself.

Lemma 3.1 with \( r_1 = r/2 \) and \( r_2 = r \) yields
\[
\frac{1}{r^n} \int_{B_r \setminus B_{r/2}} |u| \, dx \geq C \int_{B_{r/2}} g(|u|) \, dx
\]
(3.8)
for all real numbers \( r > 0 \). By theorem 2.3, this implies the estimate
\[
\int_{B_r/2} g(|u|) \, dx \leq C r^{n-m} G^{-1}(kr)
\] (3.9)
for all real numbers \( r > 0 \). In the case of \( n \leq m \), passing in (3.9) to the limit as \( r \to \infty \), we obviously obtain \( u = 0 \) almost everywhere in \( \mathbb{R}^n \). Consequently, we can further assume that \( n > m \).

Condition (2.2) is equivalent to
\[
\lim \inf_{r \to \infty} r^{n-m} G^{-1}(r) < \infty,
\]
whence in accordance with (3.9) it follows that
\[
\int_{\mathbb{R}^n} g(|u|) \, dx < \infty;
\]
therefore,
\[
\int_{B_r \setminus B_r/2} g(|u|) \, dx \to 0 \quad \text{as} \quad r \to \infty. \tag{3.10}
\]
Since \( g \) is a convex function, we have
\[
\frac{1}{\operatorname{mes}(B_r \setminus B_r/2)} \int_{B_r \setminus B_r/2} g(|u|) \, dx \geq g \left( \frac{1}{\operatorname{mes}(B_r \setminus B_r/2)} \int_{B_r \setminus B_r/2} |u| \, dx \right)
\]
or, in other words,
\[
\operatorname{mes}(B_r \setminus B_r/2) g^{-1} \left( \frac{1}{\operatorname{mes}(B_r \setminus B_r/2)} \int_{B_r \setminus B_r/2} g(|u|) \, dx \right) \geq \int_{B_r \setminus B_r/2} |u| \, dx
\]
for all real numbers \( r > 0 \), where \( g^{-1} \) is the inverse function to \( g \). By (3.8), this implies the inequality
\[
\frac{\operatorname{mes}(B_r \setminus B_r/2)}{r^m} g^{-1} \left( \frac{1}{\operatorname{mes}(B_r \setminus B_r/2)} \int_{B_r \setminus B_r/2} g(|u|) \, dx \right) \geq C \int_{B_r/2} g(|u|) \, dx
\]
for all real numbers \( r > 0 \), whence it follows that
\[
\left( \int_{B_r \setminus B_r/2} g(|u|) \, dx \right)^{n-m} g^{n-m}(f(r)) f^n(r) \geq C \left( \int_{B_r/2} g(|u|) \, dx \right)^n \tag{3.11}
\]
for all real numbers \( r > 0 \), where
\[
f(r) = g^{-1} \left( \frac{1}{\operatorname{mes}(B_r \setminus B_r/2)} \int_{B_r \setminus B_r/2} g(|u|) \, dx \right).
\]
Let us note that \( f \) is a continuous function and, moreover, \( f(r) \to 0 \) as \( r \to \infty \). In so doing, since
\[
G(t) \geq \int_t^{2t} g^{-1/m}(\zeta)\zeta^{1/1-m-1} d\zeta \geq 2^{1/m-1} g^{-1/m}(2t)t^{1/m}
\]
for all \( t > 0 \), condition (2.2) implies the relation
\[
\liminf_{r \to 0^+} g^{m-n}(r) r^n < \infty
\]
from which it follows that
\[
\liminf_{r \to \infty} g^{m-n}(f(r)) f^n(r) < \infty. \quad (3.12)
\]

Taking into account (3.11), we obtain
\[
\liminf_{r \to \infty} \left( \int_{B_r} g(|u|) \, dx \right)^{n-m} \g^{m-n}(f(r)) f^n(r) \geq C \left( \int_{\mathbb{R}^n} g(|u|) \, dx \right)^n.
\]
In view of (3.10) and (3.12), the limit in the left-hand side of the last expression is equal to zero. Thus, \( u = 0 \) almost everywhere in \( \mathbb{R}^n \).

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