Dynamical generation of flavor vacuum and Lorentz invariance

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Abstract. In this paper we review dynamical generation of field mixing after chiral symmetry breaking. We also study the explicit form of Lorentz boosts transformations of flavor states in a two-flavor scalar model with field mixing. We find that Lorentz symmetry is spontaneously broken on flavor vacuum because of its dynamically generated condensate structure.

1. Introduction

Lorentz invariance of flavor mixing and oscillations has been considered in modern theoretical physics literature [1, 2, 3, 4]. Within the context of neutrino oscillations in quantum field theory (QFT) [5, 6, 7, 8, 9, 10, 11], deformations of the Lorentz dispersion relations were considered in Ref. [2, 3]. This result was achieved implementing the formalism of Ref. [12, 13]. Similar deformations were also found in string theory [14], which predicts deformations of the Lorentz dispersion relation of the order of some power of $E/M_P$, where $E$ is the energy of a particle and $M_P$ is the Planck mass. In that context, the scattering between open strings and $D0$-branes is described by an effective action and field mixing is dynamically generated via flavor vacuum condensates [15, 16, 17, 18, 19, 20]. This mechanism was generalized in Refs. [21, 22], with algebraic methods.

Standard Model extension (SME) [23] also derives from effective string theory scenarios. In that context, Lorentz violating terms were added to the Standard Model Lagrangian, looking at possible phenomenological consequences, e.g., for neutrino oscillations [24] and for other physical systems [25, 26, 27]. Bounds on SME can be also put in connection [28] with the generalized uncertainty principle [29, 30, 31, 32, 33].

In this paper we briefly review the dynamical mechanism of flavor vacuum condensate and field mixing generation presented in Ref. [21]. As a consequence, Lorentz symmetry is spontaneously broken. We here limit our study to the case of Lorentz boosts in a simple effective model of two flavor scalar fields with mixing [34, 35]. We explicitly show that this model presents a spontaneous symmetry breaking (SSB) of Lorentz symmetry, which is an exact symmetries of the Lagrangian. To this end, the transformation properties of flavor creation and annihilation

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operators are explicitly derived. It is also shown that QFT flavor oscillation formula is Lorentz invariant.

The paper is organized as it follows: In Section 2 we review the mechanism of dynamical mixing generation described in Ref. [21]. In Section 3 quantization of flavor (scalar) fields is reviewed [34, 35]. Lorentz boosts in the flavor representation are studied in Section 4. Finally, in Section 5, conclusions and perspectives are presented.

2. Dynamical field mixing generation

Following Ref. [21], let us take a Lagrangian density $\mathcal{L}$ which is invariant under the global chiral-flavor group $G = SU(2)_L \times SU(2)_R \times U(1)_V$. We here introduce the fermion flavor doublet $\psi = \left[ \tilde{\psi}_1 \ \tilde{\psi}_2 \right]$. (1)

The action of a generic chiral-group transformation $g$ on $\psi$ is given by [36]

$$\psi' = g \psi = \exp \left[ i \left( \phi + \omega \cdot \frac{\sigma}{2} + \omega_5 \cdot \gamma_5 \right) \right] \psi.$$ (2)

Here $\sigma_j, j = 1, 2, 3$ are the Pauli matrices and $\phi, \omega, \omega_5$ are real-valued transformation parameters of $G$. From Noether’s theorem it follows that the vector and axial currents

$$J^\mu = \overline{\psi} \gamma^\mu \psi, \quad J^\mu = \overline{\psi} \gamma^\mu \frac{\sigma}{2} \psi, \quad J_5^\mu = \overline{\psi} \gamma^\mu \gamma_5 \frac{\sigma}{2} \psi,$$ (3)

and the ensuing charges

$$Q = \int d^3 x \, \psi^\dagger \psi, \quad Q = \int d^3 x \, \psi^\dagger \frac{\sigma}{2} \psi, \quad Q_5 = \int d^3 x \, \psi^\dagger \gamma_5 \frac{\sigma}{2} \psi,$$ (4)

are conserved quantities. The Lie algebra of the chiral-flavor group $G$ is thus recovered:

$$[Q_i, Q_j] = i \varepsilon_{ijk} Q_k, \quad [Q_i, Q_5, j] = i \varepsilon_{ijk} Q_{5,k}, \quad [Q_{5,i}, Q_{5,j}] = i \varepsilon_{ijk} Q_k, \quad [Q, Q_{5,j}] = [Q, Q_5] = 0.$$ (5)

Here $i, j, k = 1, 2, 3$ and $\varepsilon_{ijk}$ is the Levi-Civita pseudo-tensor.

SSB is characterized by the existence of some local operator(s) $\phi(x)$ [36] so that

$$\langle [N_i, \phi(0)] \rangle = \langle \varphi_i(0) \rangle \equiv v_i \neq 0,$$ (6)

where $\langle . \rangle \equiv \langle \Omega | . | \Omega \rangle$ and $| \Omega \rangle$ is the vacuum state. Here $v_i$ are the order parameters and $N_i$ represent group generators from the quotient space $G/H$, with $H$ being the vacuum stability group. In our case $N_i$ will be identified with $Q$ and $Q_5$ according to the SSB scheme discussed.

By analogy with quark condensation in QCD [36], we will limit our considerations to order parameters that are condensates of fermion-antifermion pairs. In order to simplify the notation, we introduce the following composite operators

$$\Phi_k = \overline{\psi} \sigma_k \psi, \quad \Phi_5 = \overline{\psi} \sigma_5 \gamma_5 \psi, \quad \sigma_0 \equiv 1,$$ (7)

with $k = 0, 1, 2, 3$. For simplicity we now assume $(\Phi_5^5) = 0$.

Let us now consider three specific SSB schemes $G \to H$:
i) SSB sequence corresponding to a single mass generation is [36, 37]

\[ SU(2)_L \times SU(2)_R \times U(1)_V \longrightarrow U(2)_V . \] (8)

The broken-phase symmetry (which corresponds to dynamically generated mass matrix \( M = m_0 \mathbb{I} \)) is characterized by the order parameter

\[ \langle \Phi_0 \rangle = v_0 \neq 0 , \quad \langle \Phi_k \rangle = 0 , \quad k = 1, 2, 3 . \] (9)

One can easily check that this is invariant under the residual symmetry group \( H = U(2)_V \) but not under the full chiral group \( G \).

ii) As a second case we consider the SSB scheme

\[ SU(2)_L \times SU(2)_R \times U(1)_V \longrightarrow U(1)_V \times U(1)_V^3 , \] (10)

which is responsible for the dynamical generation of different masses \( m_1, m_2 \). In this case the order parameters take the form

\[ \langle \Phi_0 \rangle = v_0 \neq 0 , \quad \langle \Phi_3 \rangle = v_3 \neq 0 . \] (11)

iii) Finally, we consider the SSB pattern

\[ SU(2)_L \times SU(2)_R \times U(1)_V \longrightarrow U(1)_V \times U(1)_V^3 \longrightarrow U(1)_V , \] (12)

which is responsible for the dynamical generation of field mixing.

We now define

\[ \Phi_{k,m} = \overline{\psi}_{k,\sigma} \psi , \quad k = 1, 2, 3 , \] (13)

where \( m \) indicates that \( \psi \) is now a doublet of fields \( \psi = [\psi_1, \psi_2]^T \) in the mass basis. The SSB condition now reads

\[ \langle \Phi_{1,m} \rangle \equiv v_{1,m} \neq 0 . \] (14)

Therefore, we understand that a necessary condition for dynamical generation of field mixing within chiral symmetric systems, is the presence of pairs in the vacuum which mix fermions and antifermions with different masses [21]:

\[ \langle \overline{\psi}_i(x) \psi_j(x) \rangle \neq 0 , \quad i \neq j . \] (15)

In other words, field mixing generation requires mixing at the level of the vacuum condensate structure. This statement is consistent with similar results obtained in the QFT treatment of neutrino oscillations. There, a flavor vacuum was defined, which has a non-trivial condensate structure, where neutrinos and antineutrinos with different masses are mixed [5, 6, 7, 8, 9, 10, 11]. Moreover, the present result agrees with Ref. [15, 16, 17, 18], where the condensate structure was dynamically generated by string-brane scattering. We remark that the above achievement is almost model independent (the only strong hypothesis made was the global chiral symmetry), and it is non-perturbative.

We can see the strong analogy with neutrino physics [6, 7, 8, 9, 10, 11], by looking at mean-field approximation [21]. In that case, the vacuum responsible for (14) is the aforementioned flavor vacuum

\[ |0\rangle_{e,\mu} = \prod_k \prod_r \left[ (1 - \sin^2 \theta V^2_k) - \eta^r \sin \theta \cos \theta V_k (\alpha^r_{k,1} \beta^r_{k,2} - \alpha^r_{k,2} \beta^r_{k,1}) \right. \]

\[ \left. + \eta^r \sin^2 \theta V_k U_k (\alpha^r_{k,1} \beta^r_{k,1} - \alpha^r_{k,2} \beta^r_{k,2}) + \sin^2 \theta V^2_k \alpha^r_{k,1} \beta^r_{k,1} \right] |0\rangle_{1,2} , \] (16)
The Lagrange density (22) describes two free scalar fields with masses
where \( \alpha_{k,j}^\dagger, \beta_{k,j}^\dagger, \alpha_{-k,j}^\dagger, \beta_{-k,j}^\dagger \) are the ladder operators, and

\[
U_k = A_k \left( 1 + \frac{|k|^2}{(\omega_{k,1} + m_1)(\omega_{k,2} + m_2)} \right), \quad V_k = A_k \left( \frac{|k|}{\omega_{k,1} + m_1} - \frac{|k|}{\omega_{k,2} + m_2} \right),
\]

with \( A_k = \sqrt{\frac{(\omega_{k,1} + m_1)(\omega_{k,2} + m_2)}{4\omega_{k,1}\omega_{k,2}}} \).

In this case [21]:

\[
v_{1,m} = 2 \sin 2\theta \int \frac{d^3k}{(\omega_{k,2} - \omega_{k,1})}. \]

Let us remark that QFT approach to flavor oscillations can be equally well applied to the study of the boson mixing [35]. Therefore, just for technical simplicity, we here study the properties of flavor vacuum in this simplified framework. However, it is implicit that the origin of such condensate and the consequent Lorentz SSB have to be retraced in a dynamical generation mechanism similar to the one just described.

3. Scalar field mixing

Let us consider the effective Lagrange density

\[
\mathcal{L}(x) = \partial^\mu \varphi^\dagger_1(x) \partial_\mu \varphi_1(x) - \varphi^\dagger_1(x) M^2 \varphi_1(x),
\]

where

\[
\varphi_1(x) = \begin{bmatrix} \varphi_A(x) \\ \varphi_B(x) \end{bmatrix}, \quad M^2 = \begin{bmatrix} m_A^2 & m_{AB}^2 \\ m_{AB}^2 & m_B^2 \end{bmatrix},
\]

which describes the dynamics of two linearly coupled (mixed) scalar fields. Here we use the name flavor fields, in analogy with the case of quark and neutrino physics. The Lagrange density (20) can be diagonalized with the transformation:

\[
\varphi_1(x) = U \varphi_m(x), \quad U = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix},
\]

where \( \tan 2\theta = 2 m_{AB}^2 / (m_B^2 - m_A^2) \). Therefore, \( \mathcal{L} \) becomes

\[
\mathcal{L}(x) = \partial^\mu \varphi^\dagger_m(x) \partial_\mu \varphi_m(x) - \varphi^\dagger_m(x) M_d^2 \varphi_m(x),
\]

where

\[
\varphi_m(x) = \begin{bmatrix} \varphi_1(x) \\ \varphi_2(x) \end{bmatrix}, \quad M_d^2 = \begin{bmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{bmatrix}.
\]

The Lagrange density (22) describes two free scalar fields with masses \( m_1 \) and \( m_2 \). These can be Fourier expanded as:

\[
\varphi_j(x) = \int \frac{d^3k}{2\omega_{k,j}(2\pi)^3} \left[ a_{k,j} e^{-ik\cdot x} + b_{k,j}^\dagger e^{ik\cdot x} \right] e^{ik\cdot x}, \quad j = 1, 2
\]

where ladder operators satisfy the Lorentz invariant commutation relations:

\[
\left[ a_{k,i}, a_{p,j}^\dagger \right] = \left[ b_{k,i}, b_{p,j}^\dagger \right] = 2\omega_{k,i} (2\pi)^3 \delta(k - p) \delta_{ij}.
\]
The mass vacuum is defined by the relations:

\[ a_{k,j}|0\rangle_{1,2} = b_{k,j}|0\rangle_{1,2} = 0. \] (25)

We now expand flavor fields in a similar way:

\[ \varphi_\sigma(x) = \int \frac{d^3k}{2\omega_{k,\sigma}(2\pi)^3} \left[ a_{k,\sigma}(t) e^{-i\omega_{k,\sigma}t} + b_{k,\sigma}^\dagger(t) e^{i\omega_{k,\sigma}t} \right] e^{ik\cdot x}, \quad \sigma = A, B \] (26)

with \( \omega_{k,\sigma} = \sqrt{|k|^2 + \mu_\sigma^2} \) and \( \mu_\sigma \) are mass parameters which have to be specified. From mixing transformation (21), it follows:

\[ a_{k,A}(t) = \int d^3x e^{i(\omega_{k,A}t-k\cdot x)} i \frac{\partial}{\partial t} \left( \cos \theta \varphi_1(x) + \sin \theta \varphi_2(x) \right), \] (27)

and similar expressions work for the other operators. Explicitly:

\[
\begin{bmatrix}
  a_{k,A} \\
  b_{-k,A}^\dagger \\
  a_{k,B} \\
  b_{-k,B}^\dagger
\end{bmatrix} =
\begin{bmatrix}
  c_\theta \rho^{k}_{A1} & c_\theta \lambda^{k}_{A1} & s_\theta \rho^{k}_{A2} & s_\theta \lambda^{k}_{A2} \\
  \overline{c_\theta \rho^{k}_{A1}} & \overline{c_\theta \lambda^{k}_{A1}} & \overline{s_\theta \rho^{k}_{A2}} & \overline{s_\theta \lambda^{k}_{A2}} \\
  -s_\theta \rho^{k}_{B1} & -s_\theta \lambda^{k}_{B1} & c_\theta \rho^{k}_{B2} & c_\theta \lambda^{k}_{B2} \\
  \overline{-s_\theta \rho^{k}_{B1}} & \overline{-s_\theta \lambda^{k}_{B1}} & \overline{c_\theta \rho^{k}_{B2}} & \overline{c_\theta \lambda^{k}_{B2}}
\end{bmatrix}
\begin{bmatrix}
  a_{k,1} \\
  b_{-k,1}^\dagger \\
  a_{k,2} \\
  b_{-k,2}^\dagger
\end{bmatrix},
\] (28)

where \( c_\theta \equiv \cos \theta, \ s_\theta \equiv \sin \theta \),

\[
\rho^{k}_{\sigma j} = |\rho_{\sigma j}| e^{i(\omega_{k,\sigma} - \omega_{k,j})t}, \quad \lambda^{k}_{\sigma j} = |\lambda_{\sigma j}| e^{i(\omega_{k,\sigma} + \omega_{k,j})t}, \quad (\sigma, j) = (A, 1), (B, 2), \] (29)

and

\[
|\rho^{k}_{\sigma j}| = \frac{1}{2} \left( \frac{\omega_{k,\sigma}}{\omega_{k,j}} + 1 \right), \quad |\lambda^{k}_{\sigma j}| = \frac{1}{2} \left( \frac{\omega_{k,\sigma}}{\omega_{k,j}} - 1 \right). \] (30)

One can verify that Eq.(28) is a canonical transformation:

\[
\begin{bmatrix}
  a_{k,\sigma}(t), a_{p,\rho}^\dagger(t)
\end{bmatrix} =
\begin{bmatrix}
  b_{k,\sigma}(t), b_{p,\rho}^\dagger(t)
\end{bmatrix} = 2\omega_{k,\sigma} (2\pi)^3 \delta(k - p) \delta_{\sigma \rho}.
\] (31)

As it was previously mentioned, mass parameters \( \mu_\sigma \) have to be specified [6, 7, 8, 9, 10, 11, 39]: as in QFT in curved spacetime [40], unitarily inequivalent representations of canonical commutation relation are connected by improper Bogolyubov transformations. In Ref. [41], it was shown that different mass parameters \( \mu_\sigma \) correspond to different forms of the Casimir force between two plates.

**Flavor vacuum** is defined, at a fixed time \( t \), as

\[ a_{k,\sigma}(t) |0(t)\rangle_{A,B} = b_{k,\sigma}(t) |0(t)\rangle_{A,B} = 0. \] (32)

This is characterized by a boson-condensate structure, similar to the one shown in Eq. (16). In the case \( \mu_A = m_1, \mu_B = m_2 \) one has [35]:

\[ A,B |0(t)\rangle |a_{k,j}^\dagger a_{k,j}|0(0)\rangle_{A,B} = A,B |0(t)\rangle |b_{-k,j}^\dagger b_{-k,j}|0(0)\rangle_{A,B} = \sin^2 \theta |V_k|^2 2 \omega_{k,j} (2\pi)^3, \] (33)

where \( |V_k| = |\lambda_k|_{\mu_A = m_1, \mu_B = m_2} \), analogous to the function encountered in Eq.(17). This structure is responsible for the Lorentz symmetry breaking discussed below.

\(^1\) Here the time dependence of creation and annihilation operators indicates that flavor fields are interacting fields. Actually, this interacting model can be solved exactly, without perturbation expansion.
Flavor states are defined as excitations of the flavor vacuum:

$$|a_{k,\sigma}(t)\rangle \equiv a_{k,\sigma}^\dagger(t)|0(t)\rangle_{A,B}.$$  (34)

These are eigenstates of the flavor charges

$$Q_{\sigma}(t) = i \int d^3x : \varphi_{\sigma}^\dagger(x) \partial_0 \varphi_{\sigma}(x) :$$

$$= \int \frac{d^3k}{2\omega_{k,\sigma}(2\pi)^3} \left( a_{k,\sigma}^\dagger(t) a_{k,\sigma}(t) - b_{k,\sigma}^\dagger(t) b_{k,\sigma}(t) \right),$$  (35)

at fixed time $t$

$$Q_{\sigma}(t)|a_{k,\sigma}(t)\rangle = |a_{k,\sigma}(t)\rangle, \quad \sigma = A, B.$$  (36)

4. Flavor vacuum and Lorentz invariance

Let us now analyze the Lorentz boost transformation properties of flavor operators. The generator of a boost along the $l$-th axis is:

$$K_l = \int d^3x \left( x^0 T^{0l} - x^l T^{00} \right).$$  (37)

This can also be written as

$$K_l = x^0 P^l - \int d^3x x^l \mathcal{H}.$$  (38)

In our case:

$$K_l = \int d^3x \left( \pi^l_\sigma(x) \partial^l \varphi_f(x) + \partial^l \varphi_f^\dagger(x) \pi_f(x) \right)$$

$$- \int d^3x x^l \left( \pi^l_\sigma(x) \pi_f(x) + \nabla \varphi_f^\dagger(x) \cdot \nabla \varphi_f(x) + \varphi_f^\dagger(x) M^2 \varphi_f(x) \right),$$  (39)

A generic boost can be thus expressed as:

$$U(L) = \exp \left( -i \sum_{l=1}^{3} \xi_l^l K_l \right),$$  (40)

where $L(\xi)$ indicates the Lorentz transformation:

$$x^\mu \rightarrow x'^\mu = L^\mu_\nu(\xi) x^\nu.$$  (41)

Then, we get

$$U(L) \varphi_{\sigma}(x) U^{-1}(L) = \varphi_{\sigma}(x'),$$  (42)

i.e. $\varphi_{\sigma}$ behaves as scalar under Lorentz boosts. From Eq.(26) we get:

$$\varphi_{\sigma}(x') = \int \frac{d^3k}{2\omega_{k,\sigma}(2\pi)^3} \left[ a_{k,\sigma}(t') e^{-ikx'} + b_{k,\sigma}^\dagger(t') e^{ikx'} \right].$$  (43)

Here and in the following we formally use the notation $L_k$ to indicate $L_j^\mu k^\mu$ ($j=1,2,3$), respectively. This equation should be actually written in the form:

$$\varphi_{\sigma}(x') = \int \frac{d^4k}{(2\pi)^4} (2\pi)^3 \delta^4(k^2 - \mu_\sigma^2) \theta(k_0) \left[ a_{k,\sigma}(t') e^{-ikx'} + b_{k,\sigma}^\dagger(t') e^{ikx'} \right].$$  (44)
Performing the change of variables $k \rightarrow k' = L^{-1} k$ [38], we get:

$$\varphi_{\sigma}(x') = \int \frac{d^4k}{(2\pi)^4} (2\pi) \delta^4(k^2 - \mu^2) \theta(k_0) \left[ a_{Lk,\sigma}(t') e^{-ikx} + b_{Lk,\sigma}^\dagger(t') e^{ikx} \right].$$  \hspace{1cm} (45)

By integrating once more on $k_0$ we find:

$$\varphi_{\sigma}(x') = \int \frac{d^3k}{2\omega_{k,\sigma}(2\pi)^3} \left[ a_{Lk,\sigma}(t') e^{-ikx} + b_{Lk,\sigma}^\dagger(t') e^{ikx} \right].$$  \hspace{1cm} (46)

Therefore we get

$$U(L) a_{k,\sigma}(t) U^{-1}(L) = a_{Lk,\sigma}(t'),$$  \hspace{1cm} (47)

$$U(L) b_{k,\sigma}(t) U^{-1}(L) = b_{Lk,\sigma}(t').$$  \hspace{1cm} (48)

We see immediately that flavor vacuum is not left unchanged by a Lorentz boost, i.e. $U(L)|0(t)\rangle_{A,B} \neq |0(t)\rangle_{A,B}$. However, we show that flavor oscillations can be equivalently described in every Lorentz frame.

Let us consider a flavor wavepacket:

$$|a_{\sigma}(x_0)\rangle \equiv \int \frac{d^3k}{2\omega_{k,\sigma}(2\pi)^3} e^{-ikx_0} f(k) a_{k,\sigma}(t_0)|0(t_0)\rangle_{A,B}.$$  \hspace{1cm} (49)

We suppose that the momentum space distribution $f(k)$ were Lorentz invariant. Therefore:

$$|a'_{\sigma}(x_0)\rangle \equiv U(L)|a_{\sigma}(x_0)\rangle = |a_{\sigma}(x'_0)\rangle,$$  \hspace{1cm} (50)

as one can derive from Eqs.(47),(48). Covariant oscillation formula should be written as:

$$\mathcal{J}_{\sigma\rightarrow \rho}^\mu(x - x_0) = \langle a_{\sigma}(x_0)|J^\mu_{\rho}(x)|a_{\sigma}(x_0)\rangle,$$  \hspace{1cm} (51)

where $J^\mu_{\rho}(x)$ are the flavor currents defined as [35]

$$J^\mu_{\rho}(x) \equiv \partial^\mu \varphi_{\rho}(x).$$  \hspace{1cm} (52)

In the primed Lorentz frame Eq.(51) reads

$$\langle a'_{\sigma}(x_0)|J^\mu_{\rho}(x')|a'_{\sigma}(x_0)\rangle = \langle a_{\sigma}(x_0)|J^\mu_{\rho}(x')|a_{\sigma}(x'_0)\rangle = \mathcal{J}_{\sigma\rightarrow \rho}^\mu(x' - x'_0).$$  \hspace{1cm} (53)

Therefore, flavor oscillation formula in the primed Lorentz frame is the same as in the unprimed one.

5. Conclusions

In this paper, we reviewed basic facts on dynamical generation of field mixing via flavor vacuum condensate, as described in Ref. [21]. Then, we have studied the properties of flavor creation and annihilation operators under Lorentz boost transformations, in an effective model describing a flavor scalar doublet with mixing. We have proved that the Lagrangian under consideration is Lorentz invariant, while flavor vacuum is not. This SSB can be driven by a mechanism analogous to the one described in Section 2, in the fermionic case. However, we also proved that flavor oscillation formula Eq. (51) is Lorentz invariant and the flavor oscillations can be equivalently described in every Lorentz frame.
As it was mentioned in the introduction, in Refs. [23, 24], explicit Lorentz violating terms were added to the Standard Model Lagrangian, in order to analyze, at an effective level, signal of Lorentz SSB. An interesting perspective would be to find a possible connection of these researches with the present approach.

The problem of the interplay of Poincaré transformations and flavor states was recently treated in Ref. [42], where Standard Model was generalized in an appropriate way. It is then evident that the analysis of vacuum structure in connection with dynamical mixing generation [15, 16, 17, 18, 19, 20, 21, 22] could be important to understand new physics beyond Standard Model.

Acknowledgments

P.J. was supported by the Czech Science Foundation Grant No. 17-33812L.

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