MATHEMATICAL MODELING OF INTERACTION OF SPHERICAL MILL BODIES WITH ELEMENTARY AREAS OF MILL DRUM SURFACE

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Abstract: The purpose of the work is a mathematical modelling of interaction of spherical mill bodies with elementary areas of a mill drum surface for creation of foundations of control facilities of technological aggregate overload.

Methods of the ball mill theory for obtaining efficient characteristics of fineness of ball load, provision of their invariability in the operation, determination of peculiarities of stratification of mill bodies by fineness along the drum and in separate areas of approximately equal fineness, modelling methods for determination of fineness characteristics of ball load, analysis methods for establishment of motion conditions of balls along an inclined heel, methods of the ball mill theory, probability theory and mathematical statistics, when grounding impact frequency of balls of identical size with elementary area on the mill drum internal surface, are used.

Academic novelty consists in foremost grounding of probability to implement identification tool of ball mill overload with ore by energetic efficiency of material destruction directly into the technological apparatus drum.

Practical significance of the conducted investigations is high, since the obtained results allow to develop tools of ball mill ore overload of new type. Thus, technological aggregations can be fully operational without fear of emergency mode.

It is shown that one can select the best ball load composition and maintain it in service for separate technological ore type and its definite fineness. Along the drum, balls are located in separate areas by fineness, the smallest ones are near the charging nose. The smallest balls of almost identical size are situated in separate areas of approximately equal fineness due to segregation near the lining. This creates an external layer that moves during rotation of the drum and accomplishes a three-phase trajectory. Five impacts of a ball of 50 mm in diameter with elementary area of 30 mm in diameter occur for four minutes. One impact occurs, provided that its diameter is decreased by 15 mm. It allows to develop innovative means for identifying ball mill ore overload. Keywords: fixed area, uniform-sized balls, motion conditions, impact, overload.

Introduction. The raw component of ferrous metallurgy in Ukraine is reached by concentration of poor iron ores. That requires their grinding until magnetic iron elements are discovered and extracted into industrial product, and thereafter into magnetite concentrate. Despite a huge ferrous metallurgy output in Ukraine, significant overconsumption of electric power and metal in the form of balls and lining takes place in the ore processing. Automation of these technological processes is one of efficient ways to reduce overconsumption. Considering excessive power consumption in ore processing and decrease in competitiveness of ferrous metallurgy production, high priority measures must be taken to improve this situation. Therefore, the article intends to tackle one of these urgent problems. Research relevance is based on the state documents on reduction of power and material consumption in industry, in particular, in ferrous ore mining, and inclusion of these tasks into science topics of the Central Ukrainian National Technical University.

Ore beneficiation automation has been carried out at Ukrainian concentration plants for a long time, as well as inside and outside of the countries of the former USSR. [1, 2, 3, 4, 5, 6]. However, automation efficiency results have fallen below expectations. Therefore, present investigations aim to improve ore beneficiation automation [7, 8, 9, 10, 11, 12]. Attention is paid to find solutions of key tasks. Thus, in the work [13] robust automated control of closed crushing cycle on basis of -norm is offered; the work [14] is dedicated to formation of adaptive control of iron ore raw material crushing in terms of uncertainty of object characteristics; in the work [15], it is shown that automation of first crushing, classification and magnetic separation is a real way to improve iron ore concentration efficiency. It is indicated that development of primary control measures of technological parameters is left off and a set of such measures must be created and improved [10]. The main problem of delay in ore beneficiation technological processes is absence of reliable control measures of required precision or their high price [16]. Such control measures include a ball mill ore overload sensor. In spite of attempts of their development, they are not used at concentration plants. As a result of many defects [17], well-known sound-measuring devices fail to identify overload more or less precisely. In
the work [10], it is indicated that this problem has to be solved by search and grounding of automatic control parameters and directly characterize energetic efficiency of material destruction process in the mill drum. In order to develop such measures, first of all, interaction of spherical mill bodies with elementary areas of the mill drum should be investigated. This article aims at mathematical modelling of interaction of spherical mill bodies with elementary areas of mill drum surfaces for creation of foundations for development of technological aggregate overload control measures based upon material destruction energy.

**Materials and Methods.** For continuous crushing of output ore at pre concentration plants the ball mills are used that consume a large quantity of power, balls and lining. Their overconsumption results in high damages. That is why conditions must be created for the technological aggregate, in which it operates with maximum efficiency.

In ore crushing in ball mills both balls and lining are worn out. As balls are worn out faster, they are added into the mill during operation. It is found out in practice that wear of balls is proportional to useful energy consumed by the technological aggregate. Wear of mill bodies is determined by many factors including material, production method, sizes, mass, diameter of balls and drum rotation speed, crushing characteristic of the material, lining type, pulp density etc. In the mill, a metal ball operates with loads significantly lower than those capable to destroy it. Under such operating conditions metal loss takes place mostly due to rubbing of the ball surface, as a result its diameter decreases. If the mill is loaded with balls of identical diameter at first and mill bodies of the same size are added in service, in a certain period of time, when initial crushing load decreases, a mixture of balls of various diameters in the technological aggregate is created, whose composition is determined by their wearing regularity [18]. Further, composition of mill bodies mixture remains stable, i.e. the steady-state ball load is established in the mill, in which coarse balls prevail. Such ball load characteristic does not provide optimal indices by productivity and efficiency because of lack of small and mid-size mill bodies providing larger surface per volume unit [18]. Efficient operation of the ball mill requires ball load to contain not only sufficient number of coarse balls for crushing relatively large ore pieces, but both mid-size and small mill bodies for attrition of little grains. For every fineness of the material having particular grind ability, fineness characteristic of ball mixture can be selected for providing the highest productivity of the mill. Fineness characteristic of ball load providing the highest productivity has to be selected experimentally. Nevertheless, some particular recommendations for provision of such ball loads have been accumulated as well. To maintain the required ratio of large, mid-size and small balls in the ball load rational additional loading is used that means supply of a portion of mill bodies of various sizes inside the mill. Developed graphical and analytical methods permit to determine composition of balls in the additional loading portion [19]. Such balls portion with determined composition by sizes is usually supplied into the technological aggregate once per day that does not fully meet conditions of providing ball load fineness characteristics, disturbs operation of the aggregate and changes conditions of applying methods of determination of charging the mill with ore. As a result, possibility of another approach to provision of desirable composition of ball load of the mill in its operation has been examined [20].

In modelling ball load characteristics, it is established that one can get ball media for different ore fineness that will provide the most efficient crushing of bulk material. For finer ore initial ball load: 90 mm – 32%, 75 mm – 27%, 65 mm – 23%, 50 mm – 18%. For coarser material initial ball loads are as follows: 100 mm – 32%, 75 mm – 35%, 65 mm – 18%, 50 mm – 15%. Consequently, by changing the primary composition of mill medium one can affect its fineness characteristic in the steady-state mode, providing the best operating efficiency of the mill. Steady-state fineness value of ball will not be changed during operation of the mill. However, the problem of its efficient obtaining arises. If the initial set of balls of four sizes is loaded, optimal composition of ball load is created only for a definite period of time, when the first ball is completely worn off. Calculations show that ball load in the mill with steady-state sizes is
established in 1.7 months of continuous operation. Operating mode of the ball mill does not conform to optimal conditions, charging identification process of the mill with ore is disturbed and duration of operation in the optimal mode does not exceed six months. Therefore, it is expedient to perform initial charge of the mill under steady-state structure of mill bodies, which composition meets the obtained fineness characteristic. At the same time the steady-state operating mode is started at once, which has to be maintained.

At concentration plants balls are usually added into the mill once per day. Parameters of technological processes and, consequently, in ball mills have to be maintained with deviation not exceeding ±3.0% [21]. If cast iron balls are used, in ore middle crushing down to 0.15 mm their wear is 1.25 kg per ore ton. With mill productivity of 180 t/h the daily norm of additional loading of mill medium is 4.22% that does not meet technological process requirements. Absence of 0…4.22% of balls for one day reduces technological aggregate productivity that makes up abovementioned consumption in ore beneficiation. Therefore, balls must be added not once a day, but more frequently, for example, in one hour of operation or crushing of certain volume of the material.

Considering that operating efficiency of balls of various sizes is almost identical, let us determine composition of the balls portion that has to be added after crushing 160 t of ore with loss of 1 kg of mill bodies per ton of the material and loading of 134.4 t of steel mill medium into the mill that meets the technological aggregate highest productivity. Let us record calculation data into Table 1.

Table 1. Data on additional loading of the mill with balls after crushing 160 t of ore

| Diameter of balls of initial size, mm | 90   | 75   | 65   | 50   |
|-------------------------------------|------|------|------|------|
| Quantity of balls of initial size that has to be added after crushing 160 t of ore, pcs. | 17   | 25   | 33   | 57   |

Analogously it is possible to examine crushing conditions of other types of ore in mills with certain ball media. Proceeding from data of Table 1, it is clear that it is easy to load the mill with additional balls. For ore crushing conditions, which meet data of Table 1 after processing 160 t of the material (approximately for an hour) 17 balls of 90 mm in diameter, 25 balls of 75 mm in diameter, 33 balls of 65 mm in diameter and 57 balls of 50 mm in diameter are added inside the mill. It can be done by four-channel dosing units acting under principles stated above in [22].

Considering longitudinal segregation of mill bodies ideal, one can determine location of balls along the drum. The drum of mill MShR-40-50 is 5 m long [21]. With total ball load 134.4 t, specific load is 26.88 t/m or 0.2688 t/cm permitting to determine mill medium areas of the ball mill by fineness with determined masses of balls of various sizes involved. Calculation data are summarized in Table 2.

Table 2. Calculation data

| Diameter of balls of initial size, mm | 90   | 75   | 65   | 50   |
|-------------------------------------|------|------|------|------|
| Speed is higher compared to fine bodies. Consequently, during rotation of the drum a small ball rises higher and gets into the external layer of ball load. Larger ball has lower speed, rises to lower height and during separation from the lining remains in one of internal layers. As a result, large balls remain in the middle part of the load and do not mix with small balls in whole mass of the load [23].

Behaviour of balls inside the ball mill drum is important as well. The mill medium close to the drum charging end is on higher mark comparing to the discharging one. It results into rolling of mill bodies along the inclination. Fine balls easily fall among coarser ones and do not roll down far. The coarser are the balls, the farther they roll down. Longitudinal segregation of mill bodies occurs. As a result of segregation finer balls concentrate close to the discharging end that decreases their operating efficiency.
From data of Table 2 it is seen that in the ball mill along the drum mill medium areas with approximately identical fineness of balls are created. The widest areas conform to balls of 40 mm, 50 mm, 60 mm in diameter. These areas of approximately equal length, in them balls fineness is within the limits of 35...45 mm, 45...55 mm, 55...65 mm. These areas are located in the ball mill middle part. This conclusion is confirmed experimentally by means of actual mills, because a sound-ranging signal changes its characteristics depending on installation place of the receiver. It can be explained only by effect of balls of different sizes in a definite area along the mill drum and crushing of ore particles of different fineness.

In ball mills revolving with relatively low speed an explicit stratification of mill bodies in drum cross sections is observed. Under these conditions fine balls get into external rows and when they reach the lining, they seem to cover it with a layer, preventing access to coarse mill bodies. Coarse balls are concentrated inside the load. Layout of balls along the circular trajectory is explained by the fact that center of mass of a small mill body can be closer to the mill drum than center of mass of a large ball. As a result, rotation radius of small balls is larger and it means that speed is higher compared to fine bodies. Consequently, during rotation of the drum a small ball rises higher and gets into the external layer of ball load. Larger ball has lower speed, rises to lower height and during separation from the lining remains in one of internal layers. As a result, large balls remain in the middle part of the load and do not mix with small balls in whole mass of the load.

During additional loading coarse balls are gradually displaced to the mill discharging nose. Taking into account effect of balls' stratification in a definite cross section, one can assert that in the ball load external layer there will be finer mill bodies from the available narrow range of variation of their size. Thus, balls of almost identical size will be concentrated in the definite drum cross section in the external layer. In displacement of coarse balls along the drum they will not get into the external layer.

In the concentration branch waterfall operating mode of ball mills is mainly used. In the waterfall mode when balls and ore are lifted to a certain height, they are separated from the drum surface and further move in a free flight along parabolic trajectories. Crushing is carried out mainly by impact and partially by rubbing and suppression. Balls in the external layer pressed against the lining, if being separated, move along the parabolic trajectory and start circular motion after impact with the drum internal wall. Motion speed of the ball before impact equals [19]

$$U = 2\pi n_p R \sqrt{1 + 8\sin^2(\arccos \psi^2)}, \quad (1)$$

where $\psi$ - is share of revolution speed of the ball mill drum from the critical $n_p; R$ - is internal radius of the mill drum.

Depending on (1) parameters $n_p$ and $\psi$ are constant and internal radius of mill drum varies because of wear of the lining. For this reason, speed of balls in the external layer at the moment of impact with the lining in the waterfall operating mode in service is not a constant value.

In the waterfall operating mode of the ball mill the number of full motion cycles of the ball mill exceeds the revolutions number of the
technological aggregate drum for a certain period of time owing to motion of balls along the parabolic trajectory. Such rise is estimated by cycles number of the ball per duration of one revolution of the mill drum that is determined by the following dependence [19]

\[
U = \frac{t_n}{T_u} = \frac{\pi}{(\pi - 2\alpha\pi\sin2\alpha)},
\]

where \(t_n\) – is duration of one revolution of the mill drum; \(T_u\) – is duration of one cycle of ball motion; \(\alpha\) – is angle of separation of the ball from the lining surface.

As it is seen from (2), the cycles number of the ball depends from the angle of separation \(\alpha\). This index is not equal for different layers of balls with constant rotation speed of the ball mill. Taking into account that \(\cos \alpha = \psi^2\), drum rotation speed \(n = \psi n_u\), and critical rotation speed of the mill drum depends on its internal diameter, the cycles number of the ball varies to some extent with wearing of the lining. In spite of this the cycles number exceeds the number of drum revolutions.

Revolution frequencies of drums of ball mills mainly make up 68...82% from the critical one. At the same time N.P. Neronov established that theoretically the external layer of balls can transfer to trajectories, which correspond to purely waterfall operating mode, only at 84% or higher revolution speed from the critical one [24]. It indicates that two-phase cycle of balls motion under the classical Davis theory is almost infeasible in practice. Unreality of two-phase cycle of balls motion in the mill in operating modes of industrial aggregates is also shown in the work [25]. Experimental investigations simultaneously with the theoretical ones revealed that distribution of ball load in the ball mill really differs from the one offered by the classical theory [26]. It is established that with parameters of mechanical modes used in industry the availability of the heel area is distinctive, where intensive crushing by rubbing and impact takes place. At the same time in the drum cross section a sluggish core is created, around which the remaining balls keep circulating along closed trajectories. Investigations conducted at the beginning of the new century [27] confirmed that in operation of ball mills in the modes approximated to industrial conditions the two-phase motion cycle of balls is not realized. In practice the contour of ball load external layer is three-phase (Fig.1).

\[ y = \frac{e}{\cos \alpha} - (x - R\sin \alpha - R \cos \alpha). \]

Consequently, before impact of the ball and lining mill bodies of the external layer move along the inclined straight line at a distance that equals length of section \(BC\). When balls fall down the heel, they are being braked, further mill bodies move along the external contour of the heel (straight line \(BC\)) at the speed much lower than in motion along the parabolic trajectory. Nevertheless, even when balls move over the heel surface, on impact an efficient destruction of ore pieces occurs, taking into account that mill bodies influence upon grains of the material crushed with efforts exceeding the optimal value conditioned by their strength characteristics by several times [28].
In the three-phase loading motion mode the eccentricity value $e$ of absolute trajectory of balls of the heel external contour and the value of speed of mill bodies depend not only on mechanical mode parameters, but on type of plates lined. Besides, level of pulp above the heel surface, solid-to-liquid ratio, material fineness and other factors influence motion speed of balls along the inclined surface. Nevertheless, for the foregoing reasons it follows that in spite of the fact that motion speed of balls in the external layer during impact with the lining changes under the influence of many factors, this value remains almost the same under definite operating conditions of the ball mill. According to S.F. Shynkarenko [29], decrease in balls falling down speed in the pulp hardly reduces revolution of ball loading as well that permits to apply the dependence for such conditions (2).

The pulp volume in the mill drum significantly affects its productivity. Application of a ball mill of new type with low level of pulp draining allows to get rid of drawbacks of conventional technological aggregates. Furthermore, new ball mills have many advantages before the existing ore crushing aggregates [29]. In these mills the volume of pulp available into the drum is optimized and does not change in service. It ensures coating of a bedding layer of balls along the whole drum length [29]. Therefore, in the concentrating branch it is more beneficial to use ball mills of new type, especially, taking into account that conventional technological aggregates can be easily modified in the new ones.

In the mill drum the ball load interacts with pulp and large ore pieces. In dynamics total pulp volume available into the drum consists of volume of the flooding zone of its lower part with the level equal to the mill draining level and volume of pulp lifted together with ball load. For instance, for mills with central discharging with filling factor of volume with ball load 0.4 and revolution frequency 0.8 from the critical one the mill drain above the drum bottom point makes up 0.38 of its internal diameter $D$ [29]. Average height of draining layer in the mill nose is 10 cm. One determines immersion depth of a ball from the external layer in pulp while falling down. This parameter is the best for ball mills of new type with low draining level, where pulp volume in the drum is optimized and remains constant and provides coating of the layer of bedding balls along the whole technological aggregate.

Bedding layer height in usual operating modes on the average makes up 0.1...0.2$D$ of the mill and the nose diameter must be within the limits of 0.6...0.8$D$ [30]. Analysis of data of dependences shows that within the indicated limits of variation of design and technological parameter it is necessary to select the best ratios for every mill diameter. Then in service the pulp volume into the drum remains almost optimal. The pulp level above the bedding layer of mill bodies mill remains constant in definite technological situations as well.

Energetic efficiency of material destruction process in the mill drum can be registered at impact of a ball moving along the inclined heel and lining internal surface, but at that one has to investigate interaction of uniform-sized mill bodies moving with almost invariable speed under constant technological conditions with elementary areas fixed on the internal surface of the drum.

Interaction of the elementary area with balls fixed on the drum internal surface when the drum rotates is a casual process. Revolution speed of the drum equals [19] $42.3
\begin{equation}
    n = \psi n_{kp} = \psi \frac{42.3}{\sqrt{D}}, \text{rev/min}
\end{equation}
\] where $n_{kp}$ - is critical speed of drum revolution that depends on drum internal diameter $D$ and equals $42.3\sqrt{D}$.

With homogeneous layout of balls in the drum section in the point of separation of mill bodies from the lining the following number of balls is transferred to parabolic trajectory per time unit
\begin{equation}
    n_{dl} = \frac{\pi n D}{60 d_i}, \text{balls/sec}
\end{equation}
where $d_i$ – is diameter of a ball.

In industrial conditions ball mills operate at value $\psi=0.7...0.8$ that conforms to three-phase motion of external layer balls [27]. Taking into account that with three-phase contour of mill load external layer duration of the cycle of this motion does not vary [29], at the end of the previous cycle and at the beginning of the new one during rotation balls are arranged along the circle of cross section in the sequence of their arrival to the last section of the trajectory.
The fixed elementary area is installed with its central part in a definite middle cross section of the ball mill drum. According to motion trajectories balls are arranged randomly, but they cannot shift freely, taking into account close location of adjacent mill bodies. When the drum rotates balls “bombard” the lining in the lower zone. Under such conditions coordinates of impacts of balls in the defined zone conform to the law of uniform density [31], if deviation along the drum of mill is considered. Analysis shows that deviations can take place within the limits of ball diameter $d_k$, and the selected zone will be restricted by its diameter $d_P$ (Fig.1). Probability of entry of the ball, which coordinates are distributed under the law of uniform density, within the zone of size $d_P$ being a part of the total zone is the area hatched in Fig.2. This probability equals [31]

$$P_T = d_n / d_k$$

(6)

Using ball fluxes probability $P_T$ can be given as

$$P_T = \frac{n_{km}}{n_{vk}}$$

(7)

where $n_{km}$ - is productivity of a ball flux acting in the selected zone.

From equations (5), (6) and (7) let us determine productivity of the ball flux acting in the selected zone

$$n_{km} = \frac{\pi n D}{60} \cdot \frac{d_n}{d_k}.$$  

(8)

Balls bombarding the selected zone of $d_n$ in diameter will inflict blows in a certain distance from one another along the arch of the drum circle. Mathematical expectation of the distance between points of blows inflicted by balls can be defined as ratio $m_s = \nu_{km}$, where $\nu$ is a linear motion speed of points of the lining during rotation of the drum. It can be defined as

$$\nu = \frac{\pi n D}{60}.$$

(9)

Taking into account (8) and (9), mathematical expectation of the distance between points of blows inflicted by balls will equal

$$m_s = \frac{d_s^2}{d_n^2}.$$  

(10)

All these balls will act covering the space width conforming the selected zone of $d_n$ in diameter. Every section (Fig.3, a) on the length of the drum section circle (Fig.3, b) can be represented as the one consisting of a rectangular having length $m_s$, width $d_n$ and a circle of the selected zone of $d_n$ in diameter located into it (Fig.3). This area is shown conditionally, because a converter really exists only on one such section (fig.3b). Nevertheless, this presentation is fair, taking into account that at the moment of passing the selected circular zone of $d_n$ in diameter in the field, where balls move over a straight line, precisely this interaction of the system “selected zone – ball” will arise. In this case probability of entry of the ball into the selected elementary area can be defined as a ratio of measured values [32]. Probability of entry of the ball into the selected elementary area equals a ratio of its area to the whole field, i.e.

$$P_{nm} = \frac{\pi \cdot \frac{d_n}{4}}{m_s}.$$  

(11)

Taking into account (10), the expression (11) is given in the form as follows

$$P_{nm} = \frac{\pi \cdot \frac{d_n^2}{4}}{d_k^2}.$$  

(12)

When the selected elementary area enters in the action zone of balls moving along an inclined straight line towards the lining, probability of entry of the ball into it will be determined by the expression (12).

To provide impact of the ball with the selected elementary area the latter must get in the ball mill zone, where mill bodies of external layer move along the inclined straight line towards the lining. Probability of entry of the selected elementary area in the action zone of balls is defined analogously [32] as a ratio of area of the
elementary area to total area of the lining that equals length of the drum circle multiplied by elementary area diameter (Fig.3, b).

\[ P_{rr} = 0.25 \cdot \frac{d_n}{D} \]  \hspace{1cm} (13)

Figure 3. Conditional presentation of the lining area being “bombarded” by balls during rotation of the drum at the section of selected zone of \( d_n \) in diameter:
a – elementary area; b – full area

Entry of the elementary area in the action zone of balls and entry of the ball into it are independent events. At the same time both of them determine probability of impact of the ball and elementary area. As these events are independent, probability of entry of the ball into the elementary area is determined by the product of obtained probabilities [33]

\[ P_{rr} = P_{rri} \cdot P_{rr} = \frac{\pi}{16D} \cdot \frac{d_n^3}{d^3_k}. \]  \hspace{1cm} (14)

Analysis of the dependence (14) shows that probability \( P_{rr} \) is sufficiently small. Besides, it can be approximately expressed as a ratio of number of entries of balls into the elementary area \( N_r \) to number of tests \( N_n \), i.e.

\[ P_{rr} = \frac{N_r}{N_n} \]  \hspace{1cm} (15)

In this case, number of tests can be expressed as

\[ N_n = n_{atk} \cdot T = \frac{\pi n d_n D}{60} \cdot T, \]  \hspace{1cm} (16)

where \( T \) – is an arbitrary period of time.

Considering (16), the expression (15) can be given in the following form:

\[ P_{rr} = \frac{N_r}{\pi n d_n D T/60 d^2_k}. \]  \hspace{1cm} (17)

Probability of a casual event is the number that objectively characterizes probability of its occurrence under this complex of conditions. It is related to the expression (14), because in theory it is obtained objectively. One cannot say that about the expression (15), taking into account that it is a relative frequency of this event with distribution of probabilities, which center is the probability \( P_{rr} \) (14). From the theory of probabilities it is clear that in case of \( N_r \) –fold repetition of the test, if the casual event has occurred \( N_r \) times, ratio \( N_r/N_n \) will be a partial, experimental value of the relative frequency. With sufficiently high \( N_n \) one can be sure that the approximate parity is fulfilled

\[ N_r/N_n = P_{rr}, \]  \hspace{1cm} (18)

with any pre-set accuracy [33]. In practice it is shown in the fact that value \( N_r/N_n \) of the relative frequency is stable. With definite values of \( N_n \) accuracy of the approximate parity (18) requires estimation [33].

Let us estimate the probability \( P_{rr} \) being investigated. As the number \( P_{rr} \) (14) is the center of distribution of probability of the ball entry into the elementary area, let us take it as the experimentally obtained value. Using (14) let us determine value of \( P_{rr} \) for conditions of the ball mill with \( D=4.0 \text{ m}, d_k = 50 \text{ mm}, d_n=0.6d_k=30 \text{ mm} \). It will equal \( P_{rr}=0.0003 \). We will determine reliable limits with probability level that equals \( P=0.997 \). For \( P=0.997 \) parameter \( t_{0.997}=3 \) [34]. In order to determine the reliable intervals of unknown probability the most widespread approach described in [34] will be used. It guarantees satisfactory results with \( N_q P_{rr} > 9 \), where \( q=1-P_{rr} \). The case with \( N_q=18000 \), when \( N_qP_{rr} = 9.54 \) meets this condition.

In this method the limit probabilities have the following values [34]

\[ P_1(P_{rr}, N_q) = \frac{2N_q P_{rr} + t_p^2 - t_p \sqrt{D_1}}{2(N_q + t_p^2)}, \]  \hspace{1cm} (19)

\[ P_2(P_{rr}, N_q) = \frac{2N_q P_{rr} + t_p^2 + t_p \sqrt{D_1}}{2(N_q + t_p^2)}, \]  \hspace{1cm} (20)

where \( D_1=4N_q P_{rr}(1-P_{rr})+t_p^2 \).
With acceptable number of tests $D_I=47.14$, and $P(P_{MT},N_0) = 0.00021$, $P(P_{MT},N_0) = 0.00135$.

Consequently, in this case the interval $0.00021 < P_{MT} < 0.00135$ conforms to the reliable probability $P=0.997$.

The obtained reliable interval shows that from 10000 tests there will be at least 2 and at most 13 entries of the ball into the elementary area. Proceeding from the fact that in theory the probability $P_{MT}$ is calculated precisely, one shall take into account that this is not its estimation and the expression for calculation of entries number of the ball into the elementary area can be determined. Considering the parity (14) and (16), (18) the entries number will be equal

$$N_n = \pi n d_i^2 \frac{d_o^2}{d_r^2} T,$$

(21)

where $n$ – is rotation speed of the mill drum, rev/min; $T$ – is an arbitrary time interval, s.

In the equation (21) the arbitrary time interval $T$ has to be taken so that to ensure sufficiently large number of tests. It can be determined proceeding from the following assumptions. With 16.5 rev/min the ball mill drum with $D=4$ m, $d_i=30$ mm, $d_o=50$ mm the ball flux productivity acting on the zone of the leg converter end, $n_{\text{nom}} =41.448$ balls/s. The tests number can be taken approximately 10000. Then the temporary interval can be taken $T=240$ c. For this interval performs 66 revolutions and the entries number of the ball into the elementary area will equal $N_n=5$. Such temporary interval can be taken for processing converter signals, because the aggregate is inertial and any serious changes cannot take place in it for 4 minutes.

Besides, equation (21) allows to find the temporary interval for interaction of balls with much smaller elementary areas. With elementary area diameter $d_o=15$ mm and acceptable remaining parameters for 960 s (16 min) a single entry of the ball into it is guaranteed. These signals can be averaged for the longer period of time, taking into account that the mode in the ball mill does not change quickly.

Conducted modelling confirms that probability of determination charging and discharging of the ball mill with ore by energetic efficiency of the material destruction process in the ball mill drum.

Results. It is shown that for every type of ore and its fineness the fineness characteristic of ball mixture can be selected that provides the highest productivity of the technological aggregate. In operation such ball composition can be maintained constant, if the calculated number of balls of acceptable dimensions is added through the established volume of treated ore. In operation the ball load represented by balls of certain dimensions is arranged along the drum, by their fineness with sections of certain length with approximately uniform-sized mill bodies. In the ore charging zone the finest balls and at the end of the drum the coarsest balls are located. In separate sections the finest mill bodies are located near the lining, thus creating an external layer of almost uniform-sized balls. In industrial conditions mills operate in three-phase mode of balls motion. The ball mill with low draining level is the most efficient. Besides, almost invariable pulp layer is marinated into it. Balls turnover in three-phase motion hardly changes and number of cycles of their displacement remains higher than drum revolutions number. Comparably stable conditions of balls motion along the inclined heel and practical invariability of their mass permit to realize possibility to determine ore load and overload of the mill by energetic efficiency of material destruction process in the technological aggregate drum. It will be proven that there will be 5 impacts of balls of 50 mm in diameter with the elementary area fixed on internal surface the mill drum having 30 mm in diameter for 4 minutes. If diameter of the elementary area is decreased down to 15 mm, there will be one impact in 16 minutes of operation. It permits to implement the new approach to determination of ore overload of ball mills. Obtained results are trustworthy, because investigated processes in ball mills are based upon acknowledgement of theoretical statements, industrial experiments and practical operation of technological aggregates and interaction of mill bodies with elementary areas is supported by checked theoretical provisions of theory of probabilities and mathematical statistics.

Conclusions. Consequently, proceeding from carried out mathematical modelling of interaction of spherical mill bodies with elementary areas of the mill drum internal surface...
the following is found out. For every technological type of ore and its fineness the best ball load and its fineness characteristics can be selected and continuously supported. In the ball mill zones with approximately uniform-sized balls are created, which superficial layer consists of almost uniform-sized ones. In ball mills of new type with low draining level almost invariable conditions of motion of uniform-sized balls are created. It is established that impact frequency of balls with selected elementary area is sufficient for implementation of technical identification tools of ore overload of ball mills by energetic efficiency of material destruction.

Development of approaches of ore overload of ball mills by energetic efficiency of material destruction is perspective for further investigations.

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