Stability of strange quark matter:  
MIT bag versus Color Dielectric Model

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Abstract

We discuss the properties of strange matter, in particular the minimum of the energy per baryon number as a function of the strangeness fraction. We utilize both the MIT bag model and the Color Dielectric Model and compare the energy per baryon with the masses of hyperons having the corresponding strangeness fraction, which are coherently calculated within both models. We also take into account the perturbative exchange of gluons. The results obtained in the two approaches allow to discuss the stability of strangelets. While the MIT bag model and the double minimum version of the Color Dielectric Model allow the existence of strangelets, the single minimum version of the Color Dielectric Model excludes this possibility.

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1 Introduction

It has been suggested long ago \cite{1} that hypermatter and/or strange quark matter can be realized in central relativistic heavy ion collisions, in which either multi–hypernuclear objects (strange hadronic matter) or strangelets (strange multiquark droplets) could be produced. The occurrence of the latter would be an unambiguous signature of the transient existence of a deconfined, strangeness rich, state of quark
gluon plasma (QGP). Of course the relevance of these objects highly depends on their stability after formation, and hence on the possibility of their detection.

An enhancement of strange particle production has been observed since the early 90’s in many experiments with relativistic heavy ion collisions [3]. Indeed a large number of $s\bar{s}$ pairs can be produced in a single central event: the antiquarks $\bar{s}$ combine with the abundantly available light quarks $u$ and $d$, thus producing antikaons, which rapidly leave the fireball region. The residual system turns out then to be a strangeness rich matter which hadronizes with a copious production of strange particles, especially $K$ mesons and $\Lambda$ hyperons: strongly enhanced yields have been by now assessed [4]. Yet this outcome cannot be considered as a reliable signature of quark–gluon plasma (QGP) formation, since kaons and hyperons can be produced in hadronic reactions as well, before the nuclear fireball reaches equilibrium [5].

The case would be different if, after formation of the deconfined plasma, this strangeness rich matter could coalesce into colorless multiquark states, the so–called strangelets. The prompt anti-kaon (and also pion) emission from the surface of the fireball could, in addition, rapidly cool the QGP, thus favouring the condensation into metastable or stable droplets of strange quark matter [1].

Long after the first definition of strangelets, by Bodmer [6] and Chin and Ker- man [7], Witten conjectured [8] that strange quark matter can be absolutely stable (namely more stable than ordinary nuclei), a result obtained within the MIT bag model with parameter $B \simeq 58$ MeV fm$^{-3}$. The main reason underlying this situation is the lowering of the Fermi energy introduced by the new degree of freedom (the $s$-quark), which in turn lowers the global minimum of the energy per baryon $(E/A_B)$ with respect to ordinary nuclear matter.

It is worth mentioning that strangelets can also be produced by coalescence of hyperons [9]: it consists of the fusion of a few hyperons and nucleons and does not imply the existence of the QGP phase: however this process can typically produce only small baryon numbers (e.g. the H dibaryon). Hence the detection of heavy strangelets ($A_B \geq 20–30$) should remain an unambiguous signature of QGP formation. For a review on the properties of strange matter and strangelets we refer the reader to the papers by Greiner and Schaffner [10, 11].

Up to now the properties and stability of strangelets have been discussed within the MIT bag model or, similarly, a Fermi gas model stabilized by the vacuum pressure $B$; the pioneering work by Fahri and Jaffe [12] also includes $\mathcal{O}(\alpha_s)$ corrections to the properties of bulk strange matter and finds the stability conditions: according to these authors heavy, slightly positively charged, strangelets could be more stable than ordinary nuclei. Greiner et al. [2] find (in pure MIT bag, without color exchange contributions) that also light strangelets (with $A_B \leq 20$) are metastable due to finite size and shell effects. A detailed calculation of strangelet properties within the MIT bag model, including shell effects and all the hadronic decay channels has been performed by J. Schaffner et al. [13]: a valley of stability clearly appears for
\( A_B = 5 \div 16 \) with charge fraction \( Z/A \) between 0 and -0.5. On the other hand, strangelets having a larger mass should be positively charged according to the results of Ref. [14].

In this paper we want to discuss the properties of strange matter, in particular the minimum of the energy per baryon number as a function of the strangeness fraction. We utilize both the MIT bag model and the Color Dielectric Model (CDM) and compare the equilibrium energy of the strange matter with the masses of hyperons having the same strangeness fraction; the latter are coherently calculated within both the MIT and CDM models. The main goal is to find out whether and to which extent the stability of strange matter and/or strangelets with respect to ordinary hadronic matter depends on the model employed to describe the confined system of quarks.

We consider homogeneous quark matter made up of \( u, \ d \) and \( s \) quarks. We assume that, during a high energy collision between heavy ions, this state of matter, if formed, can only survive for a very short time, so that it has no time to reach \( \beta \) equilibrium; hence we do not impose chemical equilibrium on the density of the strange quarks, limiting ourselves to assume that there exists in the system a strange fraction \( R_s = \rho_s/\rho, \ \rho \) being the total baryon density of quarks and \( \rho_s \) the baryonic density of strange quarks. Our aim is to find out whether this system is more or less stable than hyperons, in order to understand which state is more likely to be produced in heavy ion collisions, either hyperons and strange mesons or strangelets.

The energy per baryon number in the mean field approximation is a function of the baryon density, \( \rho \), and of \( R_s \). Thus we consider the surface which represents the energy per particle, \( E/A_B \), versus \( \rho \) and \( R_s \) and concentrate on its sections with constant \( R_s \): the resulting curves represent the energy per baryon number, at a fixed \( R_s \), as a function of \( \rho \), and in general they present a minimum. The minimal energy per baryon number can then be studied as a function of the strange fraction \( R_s \). In order to describe the system of three flavors we shall employ two different models: the MIT bag model and the CDM. We also consider the effect induced by the introduction of perturbative gluons. Electromagnetic interaction has been neglected in this paper and therefore the minimum of the energy corresponds to an equal number of \( u \) and \( d \) quarks. In the MIT bag model, \( u \) and \( d \) quarks are massless, while in the CDM they are massive: thus it will be interesting to compare the results obtained in each case, in order to find out the role of light quark masses on the bulk properties of strange quark matter. Another difference between the two models is that in the MIT bag model there is a sharp transition from the inner region to the outer region of the nucleon. In the CDM, on the contrary, both a two phase and a single phase scenario can be obtained. As we will see, these two possibilities give rather different results for the stability of strangelets.

Since we consider an infinite and homogeneous system, while strangelets are finite objects, one should be careful in drawing conclusions about the stability of
strangelets on the basis of strange matter stability; however the energy of the infinite
system appears to be a lower limit with respect to the envelop of strangelet energies
versus strangeness fraction: the latter was nicely illustrated by Schaffner et al. \[13\]
calculating the strangelet masses within the MIT bag model with shell mode filling.
We report their result in Fig. [1], which will be useful for further comparison with
our results. Here we simply recall that surface effects, which we do not consider,
would increase the energy curves of bulk matter, typically of 50-100 MeV: hence, if
hyperons should turn out to be more stable than strange matter, then this would
exclude also the stability of strangelets. If, on the contrary, strange matter is more
stable, then this provides only an indication in favour of stable strangelets.

2 Strangelets in the MIT bag model

The MIT bag model has been widely utilized in the past, both for strange matter
and for strangelets. For detailed derivations we refer the reader to the literature \[8, 12, 13, 16]\.

2.1 MIT bag model without gluons

We shall use here the simplest version of the MIT bag model, not including one
gluon exchange corrections. Therefore, we have two model parameters: the vacuum
pressure \(B\) and the strange quark mass \(m_s\). We have used different values of these
parameters, in order to discuss various possible scenarios.

Calculations of hadron spectroscopy \[10\] indicate for \(B\) a value of the order of 60
MeV/fm\(^3\). However this value is generally not adopted in quark matter calculations,
because it produces too much binding (notice that it is close to the 58 MeV/fm\(^3\) of
Witten’s conjecture); calculations of the hadronic structure functions \[17\] suggest
a value of \(B \approx 100\) MeV/fm\(^3\). Hence, besides \(B = 60\) MeV/fm\(^3\), we consider also
\(B = 100\) MeV/fm\(^3\) and an additional value, \(B = 150\) MeV/fm\(^3\). The latter does not
conform to calculations of physical quantities, but it has been indicated in the
literature \[18\] as a sensible value in a comparison between the results of MIT bag
model and lattice QCD at finite temperature.

The single flavor contribution to the energy density of the system is given by:

\[
\epsilon_f = 6 \int \frac{d^3k}{(2\pi)^3} E_f(k) \theta (k_{F_f} - k),
\]

where \(E_f(k) = \sqrt{k^2 + m_f^2}\) and \(k_{F_f}\) is the Fermi momentum of flavor \(f\). We use
natural units (\(\hbar = c = 1\)). Since \(u\) and \(d\) quarks are massless, their contribution to
the energy density is:
\[ \epsilon_{u,d} = \frac{3}{(2\pi)^2} k_{F_{u,d}}^4, \]  
while the strange quark contribution reads:
\[ \epsilon_s = \frac{3}{8\pi^2} \left[ m_s^4 \ln \left( \frac{m_s}{k_{F_s} + \sqrt{k_{F_s}^2 + m_s^2}} \right) + k_{F_s} \sqrt{k_{F_s}^2 + m_s^2 \left( 2k_{F_s}^2 + m_s^2 \right)} \right], \]  
\[ m_s \] being the mass of the quark \( s \).

The total energy density of our system turns then out to be:
\[ \epsilon_{tot} = 2\epsilon_{u,d} + \epsilon_s + B. \]  
The dependence of the above formula on \( R_s \) and \( \rho \) can be easily found by recalling that:
\[ \rho_s = R_s \rho \]  
and:
\[ k_{F_s} = \left( \frac{3\pi^2 \rho_s}{2} \right)^{1/3} \]  
\[ k_{F_{u,d}} = \left( \frac{3\pi^2 \rho}{2} (1 - R_s) \right)^{1/3}, \]  
\( \rho \) being the total baryon number density in the system (\( \rho = A_B/V \)). In the above the color degeneracy and baryon number 1/3 of the quarks have been taken into account.

From the above formulas we calculate the energy per baryon number to be:
\[ \frac{E_{tot}}{A_B} = \frac{\epsilon_{tot}}{\rho}. \]  
In Fig. 2 the results of the minimal energy per baryon (8) corresponding to \( B = 60, 100, 150 \) MeV/fm\(^3\) are shown as a function of \( R_s \). For each value of \( B \) we explore three different values of the strange mass, \( m_s = 100, 200, 300 \) MeV, and we compare these results with the experimental nucleon and hyperon masses (full circles). We have also evaluated, according to formula (3.6) of Ref. [16], the baryonic masses which are obtained within the same model employed for bulk strange matter, using the same sets of bag parameter and strange quark mass. Here we set to zero the perturbative one gluon exchange corrections, which are, instead, taken into account by De Grand \textit{et al.} [16] and which will be considered in the next subsection. For baryons, the fraction \( R_s \) is assumed to correspond to the fraction of \( s \) quarks with respect to \( u \) and \( d \) quarks in the considered hadron.
As it appears from the figure, the three lines corresponding to the different values of \( m_s \) are much lower than the experimental hyperon masses for \( B = 60 \text{ MeV/fm}^3 \) and \( B = 100 \text{ MeV/fm}^3 \), while this is not the case for \( B = 150 \text{ MeV/fm}^3 \) and \( m_s = 300 \text{ MeV} \). In this instance the experimental hyperon masses are lower than the bulk matter energy. However, if we compare the energy of strange matter with the corresponding theoretical masses of the various hyperons, we find that strange matter is always lower in energy, and thus more stable. In agreement with the results of Ref. \[13\], which are shown in Fig. \[4\], we can conclude that the MIT bag model without perturbative gluon corrections allows the existence of strangelets. It might be interesting to notice also that in the MIT bag model a minimum at a finite value of \( R_s \) is always present. The only parameter set not showing a minimum corresponds to \( B = 60 \text{ MeV/fm}^3 \) and \( m_s = 300 \text{ MeV} \).

### 2.2 MIT bag model with perturbative gluons.

We consider now the effects of the introduction of perturbative gluons in the calculation. At first order in \( \alpha_s \), two contributions to the energy can be considered, the direct and the exchange one. Since the system is globally colorless the direct term vanishes, while the exchange one gives the following contribution to the energy density of quarks of flavor \( f \) \[12\]:

\[
\epsilon_{OGE}^f = -\frac{\alpha_s}{\pi^3} m_f^4 \left\{ x_f^4 - \frac{3}{2} \left[ \ln \left( \frac{x_f + \eta_f}{\eta_f} \right) - x_f \eta_f \right]^2 + \frac{3}{2} \ln^2 \left( \frac{1}{\eta_f} \right) - 3 \ln \left( \frac{\mu}{m_f \eta_f} \right) \left[ \eta_f x_f - \ln (x_f + \eta_f) \right] \right\}.
\]

Here:

\[
x_f = \frac{k_{F_f}}{m_f},
\]

\[
\eta_f = \sqrt{1 + x_f^2}.
\]

and \( \mu \) is a renormalization scale, for which we choose the value \( \mu = 313 \text{ MeV} \), according to Ref. \[12\]. In Fig. \[3\] we show \( \epsilon_{OGE}^f \) as a function of \( k_F \) for various values of \( m \). It is evident that, for small values of \( m \), the contribution is always repulsive, while for \( m \gtrsim 200 \text{ MeV} \) there exists a range of densities for which it is attractive. Hence the effect of including perturbative gluons in the energy density of the system will crucially depend on the fraction of strange quarks, as well as on the value of

\[1\] Actually the third term in square brackets has a different sign, with respect to Ref. \[12\], in agreement with a recent re-derivation of the formula \[13\].
If the quark mass vanishes \((m_f = 0)\) eq. (9) reduces to:

\[
\epsilon_{u,d}^{OGE} = \frac{\alpha_s}{\pi^3} k_{F_{u,d}}^4.
\] (10)

Yet in the following we shall use a small nonzero mass for the \(u\) and \(d\) quarks, \(m_{u,d} = 4\) MeV. For sake of illustration, we adopted two different values for \(\alpha_s\), a small perturbative value \((\alpha_s = 0.5)\) and the canonical value which was employed by DeGrand et al. \([16]\) \((\alpha_s = 2.2)\), to reproduce the hyperon masses. The corresponding results are illustrated in Figs. 4 and 5.

By comparing Fig. 2 and 4 we can see that the gluon effect is always repulsive at low \(R_s\), while at large \(R_s\) it is repulsive for \(m_s = 100, 200\) MeV, and attractive for \(m_s = 300\) MeV. This is even more evident in Fig. 5, where the effect of gluons is stronger, due to the larger value of \(\alpha_s\): in this case the attractive effect at large \(R_s\) is so important that the curve corresponding to \(m_s = 300\) MeV becomes the lowest one, in contrast with the situation shown in Figs. 2 and 4.

From Fig. 4 we can see that, even after the inclusion of perturbative gluons, strangelets are more stable than hyperons for almost all values of the model parameters. However, when we use the stronger coupling of Fig. 5 the stability of strange matter (and hence strangelets) becomes questionable, particularly for low values of the strange mass \(m_s\). Only for \(m_s = 300\) MeV the theoretical masses of hyperons always lie above the energy of bulk matter (not so the experimental masses). It is worth keeping in mind that among all the situations illustrated here, only the upper left panel of Fig. 4 utilizes parameters close to the ones of DeGrand et al. \([16]\) \((B = 59\) MeV/fm\(^3\), \(m_s = 279\) MeV, \(\alpha_s = 2.2)\): indeed the stars \((m_s = 300\) MeV\) reproduce fairly well the experimental masses, and for this choice of parameter values the bulk strange matter turns out to be favoured.

From this analysis we can conclude (in agreement with previous findings) that, apart from rather extreme choices of the model parameters, metastable strangelets can exist in the MIT bag model.

### 3 Strangelets in the Color Dielectric Model

The Color Dielectric Model provides absolute confinement of quarks through their interaction with a scalar field \(\chi\) which represents a multi–gluon state and produces a density dependent constituent mass (see for example the review articles \([20] – [22]\)). Several versions of CDM have been employed to calculate hadronic properties \([23] – [29]\) and quark matter equation of state \([30] – [32]\).

The typical Lagrangian of the CDM is the following \([26, 31]\):

\[
\mathcal{L} = \sum_{f=u,d,s} \bar{\psi}_f i\gamma^\mu \left( \partial_\mu - ig_s \lambda^a A_\mu^a \right) \psi_f - \frac{gf\pi}{\chi} \sum_{f=u,d} \bar{\psi}_f \psi_f - m_s (\chi) \bar{\psi}_s \psi_s +
\]
where $\psi_f$ are the quark fields, $A^a_\mu$ is the (effective) gluon field, $F^a_{\mu\nu}$ its strength tensor and $\chi$ is the color dielectric field.

The $u$ and $d$ quark masses are the result of their interaction with the $\chi$–field and read:

$$m_{u,d} = \frac{g f_\pi}{\chi} ,$$

(12)

where $g$ is a parameter of the model and $f_\pi$ the pion decay constant, which is fixed to its experimental value, $f_\pi = 93$ MeV.\footnote{In some chiral invariant versions of the CDM the mass term is also coupled to the usual $(\sigma, \vec{\pi})$ fields.\cite{33,29,32}} For the strange quark mass we consider two different versions of the 3–flavors CDM, namely a scaling model, with

$$m_s = \frac{g f_\pi + \Delta m}{\chi} \equiv \frac{g' f_\pi}{\chi} ,$$

(13)

and a non–scaling model, with a constant shift of the $s$–mass with respect to the $u,d$–one:

$$m_s = \frac{g f_\pi}{\chi} + \Delta m \equiv m_{u,d} + \Delta m .$$

(14)

In the above $g'$ and/or $\Delta m$ is another parameter of the model.

Concerning the color dielectric field, there exist several versions, both for its coupling to the gluon tensor and for the potential $U(\chi)$. We adopt here both the single minimum (SM), quadratic potential:

$$U_{SM}(\chi) = \frac{1}{2} M^2 \chi^2 ,$$

(15)

which introduces the third parameter of the model, $M$ (the mass of the glueball), and the double minimum (DM), quartic potential:

$$U_{DM}(\chi) = \left( \frac{1}{2} \frac{M^2}{\chi_0^2} - \frac{3B}{\chi_0^4} \right) \chi^4 + \left( \frac{4B}{\chi_0^3} - \frac{M^2}{\chi_0^2} \right) \chi^3 + \frac{1}{2} M^2 \chi^2.$$  

(16)

The latter introduces an extra parameter, the bag pressure $B$, while the parameter $\chi_0$ is used to make the ratio $\chi/\chi_0$ dimensionless. The color–dielectric function, $\kappa(\chi)$, is usually assumed to be a quadratic or quartic function of $\chi$; we will use both options and hence we set:

$$\kappa(\chi) = \left( \frac{\chi}{\chi_0} \right)^\beta , \quad \text{with} \quad \beta = 2, 4 .$$

(17)
The field equations will be solved in the mean field approximation and without gluons: the latter are subsequently taken into account as a perturbation. For homogeneous quark matter the color dielectric field must obey the equation:

\[ \frac{dU(\chi)}{d\chi} \bigg|_{\chi = \bar{\chi}} = \frac{g f}{\chi^2} \sum_{f=u,d} \langle \bar{\psi}_f \psi_f \rangle + \frac{g (g') f}{\chi^2} \langle \bar{\psi}_s \psi_s \rangle, \]  

(18)

where the second term on the r.h.s. will contain \( g \) or \( g' \), depending on the choice of the non–scaling model \( g \) or of the scaling one \( g' \), respectively. In the above equation:

\[ \langle \bar{\psi}_f \psi_f \rangle \equiv \rho_f^F(\bar{\chi}) = 6 \int \frac{d\vec{k}}{(2\pi)^3} \frac{m_f(\bar{\chi})}{\sqrt{\vec{k}^2 + m_f^2(\bar{\chi})}} \theta(k_{Ff} - k) \]  

(19)

is the scalar density of quarks \( f \).

The unperturbed (i.e. without gluon contribution) energy density reads:

\[ \epsilon_0 = \sum_{f=u,d,s} \frac{3}{8\pi^2} \left\{ m_f^4 \ln \left( \frac{m_f}{k_{Ff} + \sqrt{k_{Ff}^2 + m_f^2}} \right) + k_{Ff} \sqrt{k_{Ff}^2 + m_f^2} \left( 2k_{Ff}^2 + m_f^2 \right) \right\} + U(\bar{\chi}), \]  

(20)

the quark masses being given by eqs. (12) and (13) [or (14)] with \( \chi = \bar{\chi} \).

Beyond \( \epsilon_0 \) we have perturbatively taken into account, to order \( \alpha_s \), the exchange of gluons, whose contribution to the energy density of an infinite, color singlet system is, in analogy with eq. (9):

\[ \epsilon_{OGE} = \frac{E_{OGE}}{V} = \sum_{f=u,d,s} \frac{6}{(2\pi)^3} \int \frac{d\vec{q}}{(2\pi)^3} V_{OGE}(\vec{q}) \theta(k_{Ff} - k) \]  

(21)

\[ = -\frac{\tilde{\alpha}_s}{\pi^3} \sum_{f=u,d,s} m_f^4 \left\{ x_f^4 - 3 \ln \left( \frac{\eta_f}{\eta_f} \right) - x_f^2 \right\}^2 + \frac{3}{2} \ln^2 \left( \frac{\rho}{\eta_f} \right) - 3 \ln \left( \frac{\rho}{m_f \eta_f} \right) \left[ x_f^2 \eta_f - \ln (x_f + \eta_f) \right] \} . \]  

(22)

Here the notations are the same as in eq. (10), but for the CDM definition of the constituent quark masses (in MFA); the effective strong coupling constant, according to the CDM Lagrangian (11), reads:

\[ \tilde{\alpha}_s = \alpha_s \left( \frac{\chi_0}{\chi} \right)^\beta \]  

(23)
and becomes very large at small densities, where $\bar{\chi} \to 0$. As already remarked above, eq. (21) only contains the exchange term of OGE, the direct one vanishing for infinite quark matter: at small baryonic densities the attractive electric contribution dominates the energy density, while the repulsive magnetic contribution becomes the dominant one at large densities.

Indeed the divergent behavior of the electric term for $\rho \to 0$ could prevent the reliability of a perturbative treatment of OGE. We have overcome this difficulty by taking into account the Debye screening of the gluon propagator in the presence of a polarized medium. This can be achieved by replacing (23) with the new effective coupling:

$$\alpha_{s}^{\text{eff}}(q) = \bar{\alpha}_s \frac{q^2}{q^2 + \frac{1}{2} \sum_{f=u,d,s} 16\bar{\alpha}_s m_f k_{F_f}^2 g(q/k_{F_f})},$$

(24)

$g(y)$ being the static limit of the polarization propagator [34]:

$$g(y) = \frac{1}{2} - \frac{1}{2y} \left( 1 - \frac{1}{4}y^2 \right) \ln \left| \frac{1 - \frac{1}{2}y}{1 + \frac{1}{2}y} \right|.$$  

(25)

Actually this expression should be utilized in $V_{\text{OGE}}(q)$ of eq. (21) and again integrated. For simplicity, since the $q$–integration is extended only up to $k_{F_f}$ and the function $g(y)$ varies at most by 9% in the range $0 \leq y \leq 1$, we have adopted $q = k_{F_f}$.

The flavor dependent effective coupling reads therefore:

$$\alpha_{s,f}^{\text{eff}} = \alpha_s \left( \frac{\chi_0}{\bar{\chi}} \right)^{\beta} \frac{k_{F_f}^2}{k_{F_f}^2 + \frac{1}{2} \sum_{f=u,d,s} 16\alpha_s (\chi_0/\bar{\chi})^\beta m_f k_{F_f}^2 g(1)}.$$  

(26)

We notice that, taking into account the dependence of $\bar{\chi}$ upon the density, this new effective coupling vanishes at small densities: $\alpha_{s,f}^{\text{eff}} \sim \rho^{\frac{1}{2}}$, while it goes to zero as $k_{F_f}^{-\beta}$ at high densities. At variance with the Debye screening in electrodynamics, which is mainly relevant at large densities, in the CDM the screening is large at small densities too, due to the divergence of $\bar{\alpha}_s$, which enhances the effect of medium polarization even at small densities. With these ingredients we shall compare our results for the minimal energy per baryon number with the hyperon masses, as they have been evaluated in the CDM. There exist in the literature two distinct calculations of this type and we shall consider both cases.

### 3.1 Stability of strangelets in the CDM: I

We consider here the work by Aoki et al. [26]: these authors solve self-consistently the mean field equations for quarks, color dielectric field and gluons, starting from a CDM Lagrangian with the Double Minimum potential [10] for the color dielectric field. Concerning the color dielectric function, Aoki et al. choose $\beta = 2$ in eq. (17).
In order to reproduce the masses of the octet and decuplet baryons (in particular hyperons) the authors of Ref. [26] employ both the scaling model and the non–scaling one. In addition, two different sets of parameters are used, whose values are dictated by two different and extreme choices for the “bag” parameter $B$: $B^{1/4} = 0$ MeV, with two degenerate vacua, and a large bag pressure, $B^{1/4} = 103.5$ MeV. The latter value of $B$ is chosen to be as large as possible, but with the requirement that the two-phase picture must hold inside hadrons. In their calculation only the strange quark mass has to be considered as a truly free parameter, the remaining ones having been fixed in a previous work on the non–strange baryons [25]. We only perform calculations with the second set of parameters, corresponding to the non-zero value of the bag pressure, since $B = 0$ does not give quark confinement at low densities.

The field equation for the color dielectric field, eq. (18), now becomes:

$$4 \left( \frac{1}{2} M^2 \chi_0^2 - \frac{3B}{\chi_0^2} \right) \bar{\chi}^3 + 3 \left( \frac{4B}{\chi_0^2} - \frac{M^2}{\chi_0^2} \right) \bar{\chi}^2 + M^2 \bar{\chi} = \frac{gf_\pi}{\chi^2} \sum_{f=u,d,s} \rho^f_s (\bar{\chi})$$

(27)

for the non-scaling model, and:

$$4 \left( \frac{1}{2} M^2 \chi_0^2 - \frac{3B}{\chi_0^2} \right) \bar{\chi}^3 + 3 \left( \frac{4B}{\chi_0^2} - \frac{M^2}{\chi_0^2} \right) \bar{\chi}^2 + M^2 \bar{\chi} = \frac{gf_\pi}{\chi^2} \left[ \rho^u_S (\bar{\chi}) + \rho^d_S (\bar{\chi}) \right] + \frac{g'f_\pi}{\chi^2} \rho^s_S (\bar{\chi})$$

(28)

for the scaling model, respectively. The parameters of Ref. [25] (with $B^{1/4} = 103.5$ MeV) appropriately converted to the notation of the present work, are the following ($\alpha_s = g_s^2/4\pi$):

$$g = 43.7 \text{ MeV} \quad M = 1177 \text{ MeV} \quad \chi_0 = 47.1 \text{ MeV} \quad \alpha_s = 2.0$$

Hence, we consider cases B and D of the work of Aoki et al.:

B: scaling model, $g' = 138.9$ MeV;

D: non–scaling model, $\Delta m = 212$ MeV.

We report in Table I the masses of a few hyperons (the ones of interest for the present work) as obtained in Ref. [26], together with their experimental values: the agreement is remarkable.

We evaluate the minimum energy per baryon number using cases B and D, both without and with the perturbative correction of the effective gluon exchange.\footnote{In the Appendix we discuss the existence of multiple solutions for field equations when the potential $U(\chi)$ has a double minimum.}
Table I. Strangeness fraction, experimental masses and theoretical masses of hyperons, according to the calculation of Ref. [26].

| Hyperon | N  | Λ  | Ξ  | Ω  |
|---------|----|----|----|----|
| $R_s$   | 0  | 0.33 | 0.67 | 1 |
| $m_{exp}$ (MeV) | 938 | 1115 | 1314 | 1672 |
| $m_B$ (MeV) | 938 | 1161 | 1346 | 1639 |
| $m_D$ (MeV) | 938 | 1113 | 1307 | 1671 |

In Fig. 6 we compare our curves with the masses calculated in Ref. [26], which always include, as already mentioned, the gluon field correction in a self-consistent approach. In addition to the theoretical masses of hyperons, we also show in the figure their experimental values (represented by full circles) which, according to Table I, are always rather close to the calculated ones.

As we can see from Fig. 6, the effect of perturbative gluons in this model is very small, due to the Debye screening, which we have included. Whether or not we take into account gluon corrections, strange matter always appears to be more stable than baryons.

Hence from the cases considered here we can conclude that this version of the CDM seems to favour strangelets as a metastable form of matter. This is due to the fact that when the DM potential is used to study hadrons, i.e. confined objects, a large contribution to the hadronic mass is given by the space fluctuations of the fields. When this version of the model is used to describe infinite quark matter, these contributions vanish due to the homogeneity of the system. For this reason, deconfined matter is favoured in this version of the model. This finding is consistent with the results of Refs. [30, 31]. It is also worth mentioning that, as shown in Fig. 6, the minimum of the energy per baryon number corresponds to $R_s = 0$, at variance with the results of the MIT bag model.

### 3.2 Stability of strangelets in the CDM: II

In this section we follow the approach of J. McGovern [27]. The model Lagrangian is still the one reported in eq. (11) with $\beta = 4$ in the color dielectric function. McGovern uses only the scaling model and the Single Minimum potential, with different values of the parameters. In this case the behavior of $\alpha_s^{eff}(\bar{x})$ is even more
divergent, for small densities, than in the case \( \beta = 2 \) previously discussed: hence the use of Debye screening in the effective strong coupling constant is mandatory.

In Ref. [27] two different sets for the model parameters are used: they allow to satisfactorily reproduce the splittings between hyperon masses, but the absolute values of the masses themselves are generally too large. The latter are reported in Table II.

In this model, the field equation for the scalar field becomes:

\[
M^2 \bar{\chi} = \frac{g f_{\pi}}{\chi^2} \left[ \rho^u_S(\bar{\chi}) + \rho^d_S(\bar{\chi}) \right] + \frac{g' f_{\pi}}{\chi^2} \rho_s(\bar{\chi}.
\]

(29)

The parameter sets used in Ref. [27] are:

Parameter set I:

\[
g = 46.7 \text{ MeV} \quad M = 2354 \text{ MeV} \quad \chi_0 = 20 \text{ MeV} \quad \alpha_s = 0.3380
\]

Parameter set II:

\[
g = 107.527 \text{ MeV} \quad M = 1000 \text{ MeV} \quad \chi_0 = 45.4 \text{ MeV} \quad \alpha_s = 0.3533
\]

We fix the ratio: \( g'/g = 1.58 \), in agreement with Ref. [27].

As in the previous section, in Fig. 7 we compare our results for the minimum energy per baryon number both with the experimental and the theoretical masses. As we can notice, also in this case the inclusion of perturbative gluons practically does not alter the quark matter energy, since here the Debye screening is overwhelming.
Strange matter turns out to be well above the experimental hyperon masses and below the theoretical ones. We must remember that in our calculations we don’t consider surface effects, which would increase the energy density of a finite system of $50 \div 100$ MeV, as we can deduce by comparing our results in the MIT bag model with Fig. 1. In this perspective, only for $R_s \approx \frac{2}{3}$ strangelets are (marginally) allowed by the present calculation and a more refined one, taking into account surface energy contributions, could clarify the situation.

We can therefore conclude that, but for $R_s \approx \frac{2}{3}$, the existence of metastable strangelets is excluded in the Single Minimum version of the CDM.

4 Conclusions

In this work we have computed the energy per baryon number of infinite quark matter having a fixed strangeness content. We have compared this quantity to the mass of hyperons having the same strangeness fraction and calculated within the same model and for the same parameter values that we adopt in our calculations.

Our analysis shows that the existence of strangelets is supported only by those models which entail a two–phase picture of hadrons, namely which maintain a false vacuum inside hadrons. This happens both in the MIT bag model, and in the Double Minimum version of the Color Dielectric Model: as we have seen in Sections 2 and 3.1, in these cases the minimum energy per baryon number versus the strangeness fraction $R_s$ turns out to be lower than the masses of hyperons with the same $R_s$.

As we have seen in Figs. 2, 4, 5, 6, the mass gap between hyperons and strangelets can be as large as 300 MeV, and the metastability of strangelets would be confirmed even by taking into account surface energy effects, typically of the order of 100 MeV, in agreement with the findings of Ref. [13].

On the contrary, the Single Minimum version of the CDM does not support the existence of strangelets: independently of the set of parameter values, the computed masses of hyperons are larger than the corresponding strange matter energy by about $50 \div 100$ MeV. Only for $R_s = \frac{2}{3}$ the gap is 150 MeV and we cannot exclude completely the presence of strangelets with this strangeness fraction.

This outcome points out that the stability of strangelets appears to depend rather crucially on the model employed: CDM supports stable strangelets only in the DM version, but not in the SM one. This fact could set serious challenges to the search for strangelets in relativistic heavy ion collisions, as a signature for the quark gluon plasma phase, out of which strangelets could be formed.

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5 Appendix

There can be multiple solutions of field equations, when for the dielectric field $\chi$ a potential having two minima is used. This possibility is known since the first calculations within the color-dielectric model \[35, 36\].

In the above quoted references, the solutions of the field equations were non-topological solitons. In the present work we are discussing plane-wave solutions for the quarks, which are considerably simpler. The solutions of the field equations for $\chi$ are graphically shown in Fig. 8, where they appear as the intersection of the dotted line with the dashed (at lower density) or continuous lines. As it appears from Fig. 8, when the density is low enough there can be three solutions. The solutions are characterized by their behavior at low density. One solution corresponds to $\chi \to 0$ as $\rho \to 0$. The other two solutions correspond to $\chi$ close to the value of relative maximum of the potential or to the relative minimum, respectively. The solution in which $\chi$ is near the relative maximum is unstable, and it corresponds to the unstable solitonic solution pointed out in Ref. \[36\].

At low density there are therefore two stable solutions. The “true” solution is clearly the one of minimal energy. As it can be seen in Fig. 8, but for very small value of the density, the solution of minimal energy is the one in which $\chi$ is close to the value of the relative minimum of the potential.

References

[1] C. Greiner, D.-H. Rischke, H. Stöcker and P. Koch, Z. Physik C 38, 283 (1988).
[2] C. Greiner, D.-H. Rischke, H. Stöcker and P. Koch, Phys. Rev. D 38, 2797 (1988).
[3] T. Abbott et al. [E-802 Collaboration], Phys. Rev. Lett. 64 (1990) 847; Phys. Rev. Lett. 66 (1991) 1567; O. Hansen, Comm. Nucl. Part. Phys. 20 (1992) 1; S. Abatzis et al. [WA85 Collaboration], Phys. Lett.B 316 (1993) 615; Nucl. Phys. A 566 (1994) 225C; J.B. Kinson et al. [WA85 Collaboration], Nucl. Phys. A 544, (1992) 321C; E. Andersen et al. [NA36 Collaboration], Phys. Lett.B 294 (1992) 127; Phys. Rev. C 46, (1992) 727; Phys. Lett.B 327 (1994) 433; M. Gazdzicki [NA35 Collaboration.], Nucl. Phys.A 566 (1994) 503C; T. Alber et al. [NA35 Collaboration.], Z. Phys. C 64 (1994) 195.
[4] S. V. Afanasev et al. [NA49 Collaboration], J. Phys. G27 (2001) 367; D. Varga [the NA49 Collaboration], hep-ex/0105035; M. Abreu et al. [NA50 Collaboration], Nucl. Phys.A 663 (2000) 721; D. Rohrich [NA49 Collaboration], Nucl. Phys.A 663 (2000) 713.
[5] R. Mattiello, H. Sorge, H. Stöcker and W. Greiner, Phys. Rev. Lett. 63, 1459 (1989).

[6] A.R. Bodmer, Phys. Rev. D 4, 1601 (1971).

[7] S.A. Chin and A.K. Kerman, Phys. Rev. Lett. 43, 1292 (1979).

[8] E. Witten, Phys. Rev. D 30, 272 (1984).

[9] C.B. Dover, Production of strange clusters in relativistic heavy ion collisions, preprint BNL–48594, 1993, presented at HIPAGS 1993.

[10] C. Greiner and J. Schaffner, Int. J. Mod. Phys. E 5, 239 (1996).

[11] C. Greiner and J. Schaffner-Bielich, To be published in 'Heavy Elements and Related New Phenomena', ed. by R.K. Gupta and W. Greiner, World Scientific Publications, nucl-th/9801062.

[12] E. Fahri and R.L. Jaffe, Phys. Rev. D30, 2379 (1984).

[13] J. Schaffner-Bielich, C. Greiner, A. Diener and H. Stocker, Phys. Rev. C 55, 3038 (1997).

[14] J. Madsen, Phys. Rev. Lett. 85, 4687 (2000).

[15] C. Greiner, J. Phys. G 25, 389 (1999) hep-ph/9809268.

[16] T. DeGrand, R.L. Jaffe, K. Johnson and J. Kiskis, Phys. Rev. D 12, 2060 (1975).

[17] F.M. Steffens, H.Holtmann, A.W. Thomas, Phys.Lett. B 358, 139 (1995)

[18] H.Satz Phys. Lett. B 113, 245 (1982)

[19] N. K. Glendenning, Compact Stars, 1997 Springer-Verlag, New York.

[20] L. Wilets, Chiral Solitons, ed. K.-F.Liu (World Scientific, Singapore, 1987) 362.

[21] M.C. Birse, Prog. Part. Nucl. Phys. 25, 1 (1990).

[22] H. Pirner, Prog. Part. Nucl. Phys. 29, 33 (1992).

[23] H. J. Pirner, G. Chanfray and O. Nachtmann, Phys. Lett. B 147, 249 (1984).

[24] L.R. Dodd, A.G. Williams and A.W. Thomas, Phys. Rev. D 35, 1040 (1987).

[25] N. Aoki and H. Hyuga, Nucl. Phys. A 505, 525 (1989).
[26] K. Nishikawa, N. Aoki and H. Hyuga, Nucl. Phys. A 534, 573 (1991).
[27] J.A. McGovern, Nucl. Phys. A 533, 553 (1991).
[28] V. Barone and A. Drago, Nucl. Phys. A 552, 479 (1993).
[29] V. Barone, A. Drago and M. Fiolhais, Phys. Lett. B 338, 433 (1994).
[30] A. Drago, M. Fiolhais and U. Tambini, Nucl. Phys. A 588, 801 (1995).
[31] V. Barone and A. Drago, J. Phys. G21, 1317 (1995).
[32] A. Drago and A. Lavagno, Phys. Lett. B 511, 229 (2001).
[33] T. Neuber, M. Fiolhais, K. Goeke and J.N. Urbano, Nucl. Phys. A 560, 909 (1993).
[34] A.L. Fetter and J.D. Walecka, Quantum Theory of Many-particle systems, McGraw–Hill (1971).
[35] L.R. Dodd, A.G. Williams and A.W. Thomas, Phys.Rev. D 35, 1040(1987).
[36] W. Broniowski, M.K. Banerjee and T.D. Cohen, MdDP-PP-87-035, ORO-5126-298, unpublished
Figure 1: The energy per baryon of all possible strangelets with $A_B < 40$, for a bag constant $B^{1/4} = 170$ MeV versus the strangeness fraction $f_s = 3R_s = |S|/A_B$. The solid line connects the masses of the nucleon and of the first lowest hyperons: it represents free baryonic matter. (Taken from Ref. [13])
Figure 2: Minimal energy per baryon number in the MIT bag model, as a function of the strangeness fraction $R_s = \rho_s/\rho$, for various choices of the values of the model parameters. The continuous line corresponds to $m_s = 100$ MeV, the dashed line to $m_s = 200$ MeV and the dotted line to $m_s = 300$ MeV. Full circles correspond to experimental masses, the other points to the masses evaluated in the model, with $m_s = 100$ MeV (open triangles), $m_s = 200$ MeV (full triangles), $m_s = 300$ MeV (stars), respectively.
Figure 3: OGE contribution to the energy density divided by \( \alpha_s \), \( \epsilon^{OGE}/\alpha_s \), as a function of \( k_F \) for different values of the quark mass.
Figure 4: Minimal energy per baryon number in the MIT bag model, including the OGE potential with $\alpha_s = 0.5$, as a function of the strangeness fraction $R_s = \rho_s/\rho$. The continuous line corresponds to $m_s = 100$ MeV, the dashed line to $m_s = 200$ MeV and the dotted line to $m_s = 300$ MeV. Full circles represent the experimental masses, the other points refer to the masses evaluated in the model, with $m_s = 100$ MeV (open triangles), $m_s = 200$ MeV (full triangles), $m_s = 300$ MeV (stars), respectively.
Figure 5: The same as in Fig. 4, but for $\alpha_s = 2.2$. 
Figure 6: Minimal energy per baryon number in the CDM, as a function of $R_s = \rho_s/\rho$, for the cases B and D with and without gluons. Full circles are the experimental hyperon masses, while triangular dots are the masses calculated in Ref. [26].
Figure 7: Minimal energy per baryon number as a function of $R_s = \rho_s/\rho$ for the Single Minimum version of the CDM. The various panels correspond to: (a) parameter set I without gluons, (b) parameter set I with gluons, (c) parameter set II without gluons and (d) parameter set II with gluons. Full circles are the experimental baryon masses, while triangular dots are the masses calculated in Ref. [27].
Figure 8: Solutions of the field equation for the scalar field, for two different values of $\rho$: the dotted line corresponds to $dU_{DM}/d\chi$, the dashed and continuous lines correspond to the r.h.s. of eq. (28), assuming $\rho = 0.01\ fm^{-3}$ (dashed line) and $\rho = 0.05\ fm^{-3}$ (continuous line), with fixed $R_s = 0.05$ in both cases.
Figure 9: Total energy per baryon number as a function of $\rho$ for $R_s = 0.5$: the solid line corresponds to the curve calculated with $\bar{\chi}$ near to the value of the relative minimum of the potential, the dashed line corresponds to $\bar{\chi}$ near to the absolute minimum of the potential.