Detection of oscillatory solutions in a vacuum diode with total electron reflection

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Abstract. Stability features of steady-state solutions for a vacuum diode with complete deceleration of electron beam is studied. A boundary line on the (inter-electrode gap, external voltage)-plane separating stable solutions from unstable ones is built up. An instability development is shown to end in a state with non-linear oscillations of the electric field but with no virtual cathode in a plasma. Existence of non-linear oscillations of the electric field in a vacuum diode with total reflection of an electron beam points out that such a diode can be a basis to create microwave generator.

1. Introduction
In order to generate strong microwave radiation, the vacuum diodes with a virtual cathode (e.g., vircators, reditrons, reflective triodes) are used [1]. In these devices, an electron flow with a velocity distribution function (VDF) being almost the mono-kinetic one enters from the emitter. When current density exceeds a certain threshold, there is a virtual cathode in a working volume which oscillations are converted into microwave radiation. In reflective triodes, an extra electrode involves a strong deceleration and reversal of the electron beam. Here, also a virtual cathode arises, but, due to extra spacing where the electrons interact with the oscillating electric field, wideband oscillations arise similar to background noise. Is it possible to increase interaction space due to reflected electrons but with no virtual cathode? To answer, we study stability features of steady-state solutions for a planar vacuum diode in which a potential difference between the electrodes is negative and so large that all emitted electrons are reflected inside an inter-electrode gap without hitting the second electrode and return towards emitter.

2. Steady state solutions
Under consideration is a vacuum diode of planar geometry with an inter-electrode gap $d$ and a potential difference $U < 0$. Non-relativistic electron flow of density $n_0$ and average velocity $v_0$ enters the inter-electrode gap from the emitter and moves with no collisions in the self-consistent electric field. We study a diode working mode with a total electron reflection from a potential barrier. It is accomplished if energy of electrons at the emitter $m v_0^2/2$ is lower than a value $|eU|$. Thus, there is a point $z_r$ in an inter-electrode space which divides this space into two regions: the plasma one ($z < z_r$), and the vacuum one ($z_r < z < d$). Steady state solutions of such a diode are featured by a monotonously decreasing potential distribution.
When transferring to dimensionless values, we take a Debye length $\lambda_D$ and energy of electrons at the emitter $W_0$ as characteristic units of length and energy, respectively, as follows:

$$\lambda_D = \left[ \frac{2e_0 W_0}{e^2 n_0} \right]^{1/2} \approx 0.3238 \cdot 10^{-2} \frac{V_0^{3/4}}{j_0^{1/2}} [\text{cm}], \quad W_0 = m v_0^2 / 2. \quad (1)$$

Here, a current density $j_0 = e n_0 v_0$ and a voltage to accelerate the electrons $V_0 = W_0 / e$ are measured in Amperes per square cm and Volts, respectively; $e$ and $m$ are the electron charge and mass; $\epsilon_0 = 8.854 \cdot 10^{-12} \text{C}^2 / \text{Nm}^2$ is the permittivity of the vacuum. The dimensionless coordinate, time, velocity, potential and electric field strength are introduced as $\zeta = z / \lambda_D$, $\tau = t \omega_0$, $u = v / \sqrt{2W_0/m}$, $\eta = e\varphi / (2W_0)$, $\epsilon = e\lambda_D / (2W_0)$, respectively; here, $\omega_0 = \left[ e^2 n_0 / (m\epsilon_0) \right]^{1/2}$ is the intrinsic frequency. Dimensionless inter-electrode length and inter-electrode voltage are denoted as $\delta = d / \lambda_D$ and $V = eU / (2W_0)$.

To take into consideration a natural spread in velocities for the electrons, their VDF at the emitter is chosen as “gate”:

$$f_e(\tau, 0, u) = \frac{1}{2\Delta} \Theta[\Delta^2 - (1 - u)^2], \quad \Delta = v / v_0 \ll 1. \quad (2)$$

Here $\Theta$ is the Heaviside step function and a parameter $\Delta$ characterises the particle velocity spread magnitude. In this case, the electrons are reflected inside a region $(\zeta_{r,1}, \zeta_{r,2})$, which boundaries are corresponded to particles with exit velocities $u_{0,min} = 1 - \Delta$ and $u_{0,max} = 1 + \Delta$.

For electron density, a formula takes place [2]:

$$n = \frac{\sqrt{2}}{\Delta} \begin{cases} \sqrt{\eta - \eta_-} - \sqrt{\eta - \eta_+}, & \zeta < \zeta_{r,1}, \\ \sqrt{\eta - \eta_+}, & \zeta_{r,1} < \zeta < \zeta_{r,2}, \\ 0, & \zeta > \zeta_{r,2}. \end{cases} \quad (3)$$

Here $\eta_{\pm} = -(1 \pm \Delta)^2$. Stationary potential distribution (PD) and the parameters of characteristic points are found from a Poisson equation

$$\frac{d^2}{d\zeta^2} \eta = n_e(\eta; \eta^-, \eta^+), \quad (4)$$

when substituting (3). Further, a relation between electric field strength in a point $\zeta_{r,2}$ and at the emitter can be determined:

$$\epsilon_r = \left[ \epsilon_0^2 - 4 \left( 1 + \Delta^2 / 3 \right) \right]^{1/2}. \quad (5)$$

From (5), one can see that a minimum value of $\epsilon_0$, for which the solutions with a total electron reflection can exist, is $\epsilon_{0,min} = 2 \left[ \left( 1 + \Delta^2 / 3 \right) \right]^{1/2}$. Relevant values of $\zeta_r$ can be found via numerical integration of the equation (4).

This work treats stability features of steady state solutions obtained. We reveal that the solutions become unstable under certain external parameters, and the non-linear oscillations develop in a plasma.

### 3. Stability features of the solutions

We consider that the electric field distribution does not change for several time-of-flight of particles through plasma layer, and it equals the stationary one, and at a moment $\tau = \tau_s$, we start to calculate the self-consistent time-dependent process. The latter develops from a small
perturbation evolving due to difference in numerical codes applied to calculate time-independent and time-dependent fields.

The VDF and electron density as well as the electric field strength are calculated consequently at every step. When density calculating, a difficulty arises due to particle flow, which is electrons reflected near the plasma layer boundary and moved against that from the emitter. Besides, a function \( n(\zeta) \) has the sharp gradients near the right boundary of reflection. So, for a true density determination, it is necessary to solve a kinetic equation for electrons and to build up their VDF. Now, no analytic solutions for the kinetic equation exist. So, we find the VDF numerically applying the high-accuracy \( E, K \)-code [3]-[5].

The code basis lies on the fact that in the case with no collisions, the VDF is kept along a trajectory of every particle: \( f(\tau, \zeta(\tau), u(\tau)) = f_0(\tau_0, u_0), \) where \( \zeta(\tau) = \zeta(\tau; u_0, \tau_0), \) \( u(\tau) = u(\tau; u_0, \tau_0), \) \( \zeta(\tau_0) = 0, \) \( \dot{\zeta}(\tau_0) = u_0, \) and \((1 - \Delta < u_0 < 1 + \Delta)\). When the VDF is calculated at a moment \( \tau = \tau' \), we know the electric field distribution for all the points of the inter-electrode gap at all past moments \( \tau \in (0, \tau^{-1}) \). An array of the field distributions \( \varepsilon(\tau_j', \zeta_k) \) are stored at the nodes of the spatial–temporal grid. When VDF calculating, the “test” particle trajectories are determined. This method feature is in that the trajectory of each particle is calculated back in time until it intersects the emitter surface. As a result, the particle velocity and its exit time are determined and the VDF value is calculated for the arrival velocity \( u \). Then, this process is repeated for a new arrival velocity \( u - \Delta u \) and so on. As a result, indeed, a dependence \( u_0(u) \) is built up. As the electron VDF at the emitter is taken as (2), we need to find the exit velocities \( u_0 \) laying in the region \((1 - \Delta, 1 + \Delta)\) to build up the required VDF. Hence, need to be determined those arrival velocities which correspond to a function \( u_0(u) \) crossing the points \( 1 - \Delta \) or \( 1 + \Delta \). A step \( \Delta u \) is taken in such a manner that the crossing points can be found with a required accuracy.

At each time step \( \tau^j \), the VDF and density of electrons having been determined, the latter is substituted into a Poisson equation:

\[
\frac{d^2}{d\zeta^2} \eta = n(\zeta), \tag{6}
\]

which is to be solved applying the boundary conditions: \( \eta(0) = 0, \eta(\delta) = V \). As a result, the distributions of a potential and electric field in a plasma are obtained. A spatial step \( \Delta \zeta_k \) is 0.002 in the plasma region. A calculation proceeds down to a point \( \zeta_r \) in which the electron density vanishes. As a result, an electric field strength \( \varepsilon_r \), a potential \( \eta_r \), and a coordinate of the point \( \zeta_r \) are determined. To the right of this point, there is no electrons and the PD is a linear function of a coordinate: \( \eta(\zeta) = \eta_r - \varepsilon_r (\zeta - \zeta_r) \).

Self-consistency of the time-dependent problem solution as a whole at each time step \( \tau^j \) is obtained via iterations. As the electric field at a moment \( \tau^j \) is known till a moment \( \tau^{j-1} \) only, at the beginning we extrapolate the field within an interval \((\tau^{j-1}, \tau^j)\). It gives an opportunity to calculate the VDF and electron density at a moment \( \tau^j \) as the first approximation. The density being known, we can recalculate the field distribution at a given moment more precisely. Then, with this new field distribution, the electron density is recalculated. As a rule, one-two iterations are well enough. To control calculations, a total current conservation is revised at each time step: for the 1D case, the total emitter and the collector current must coincide.

Study of the time-dependent process proceeds as follows: time evolution of diode parameters is determined for fixed value of the emitter field strength \( \varepsilon_0 \) and various \( \delta \) and \( V \), the values \( \delta \) and \( V \) are taken in such a manner that a plasma part of the steady state solution to be the same. For this purpose, a \( V \) value for every \( \delta \) is taken under condition: \( V = \eta_2 - \varepsilon_2 (\delta - \zeta_2) \) and \( \zeta_2, \eta_2 \) and \( \varepsilon_2 \) being a coordinate, potential, and electric field strength at the right boundary of the plasma region.
We carry out the calculations of the time-dependent processes in the diode for a series of $\varepsilon_0$ values selected from the range (2.002, 2.035). As an example, Figure 1 shows the temporal dependences of the emitter convection current for a diode with $\varepsilon_0 = 2.018$ for decreasing ($\Gamma < 0$) and increasing ($\Gamma > 0$) perturbation, respectively.

Figure 1. Temporal dependence of the emitter convection current for a diode with $\varepsilon_0 = 2.018$; $\delta = 2.0, V = -0.955$ (a), $\delta = 8.0, V = -2.568$ (b).

A high accuracy of the code gives an opportunity to determine a growth rate $\Gamma$ and a frequency $\Omega$ for inherent perturbation mode from the calculated characteristics of the time-dependent process. This values are determined numerically via the least square method. The method is essentially the functional minimization:

$$G(\Gamma, \Omega) = \sum_{i=N_1}^N \left\{ F(\tau_i) - [\alpha \cdot \cos(\Omega \tau_i) + b \cdot \sin(\Omega \tau_i)] \cdot \exp(\Gamma \tau_i) \right\}^2. \quad (7)$$

Here, $F(\tau_i)$ is an array of any time-dependent characteristic of the process. For convenience, a temporal dependence of the emitter convection current is taken as an array $F(\tau_i)$ to operate as its time average value vanishes. In Figure 2, dependences of a growth rate $\Gamma$ and a period $T = 2\pi/\Omega$ on a diode electrode length for $\varepsilon_0 = 2.018$ are shown.

The dependences $\Gamma(\delta)$ are built up for a series of $\varepsilon_0$ values from a range (2.002, 2.035). At all $\varepsilon_0 < 2.0275$, this dependence turns out to cross $\Gamma = 0$–axis, i.e. there is a gap value $\delta_{lim}(\varepsilon_0)$ above which the steady state solutions turn out to be unstable. In Figure 3(a), the dependence of a threshold value of $\delta$ on $\varepsilon_0$ is shown. The region boundary where the steady state solution turns out to be unstable has a rather fancy form. Thus, one can maintain that, under certain values of the external parameters, an instability can develop in a vacuum diode with complete deceleration of an electron beam.

4. Non-linear oscillations

Our calculations reveal that when the electrode length value exceeds the threshold one, the perturbation develops and this process terminates at the state with pronounced non-linear oscillations. An example of the emitter convection current oscillations is presented in Figure 1(b). The instability survives due to a presence of a positive capacitance feedback in the system via external circuit.
Figure 2. Growth rate $\Gamma$ (a) and oscillatory period $T$ (b) of the eigen mode via gap value $\delta$ drawn for the emitter field strength $\varepsilon_0 = 2.018$.

Figure 3. Dependencies of $\delta_{lim}$ (a) and $V_{lim}$ (b) on $\varepsilon_0$.

In Figure 4(a), a dependence of the right boundary of a plasma potential $\eta_r$ on its position $\zeta_r$ for the same diode parameters is showed. One can see that the oscillation amplitude turns out to be the rather large magnitude. A dependence of the collector electric field strength on that at the emitter for corresponding external parameters is shown in Figure 4(b).

5. Conclusion
Calculating the non-linear stage of the instability development in a plasma of a vacuum diode with a complete deceleration of an electron beam, we determined that the solutions with non-linear oscillations are implemented instead of the steady state ones. However, no virtual cathode arises. In such a manner, the development of the non-linear oscillations of the electric field in the diode can be realized. This result gives a hope to create a microwave generator based on such a diode.
Figure 4. Dependence of a potential at the right plasma boundary $\eta_r$ on its position $\zeta_r$ (a) and the collector electric field strength on that at the emitter (b) for a diode with $\varepsilon_0 = 2.018$, $\delta = 8.0$, $V = -2.568$. The bold point refers to the steady state solution.

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