Physics potential for the measurement of $\sigma(H\nu\bar{\nu}) \times BR(H \rightarrow \mu^+\mu^-)$ at the 1.4 TeV CLIC collider

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Abstract The future Compact Linear Collider (CLIC) offers a possibility for a rich precision physics programme, in particular in the Higgs sector through the energy staging. This is the first paper addressing the measurement of the Standard Model Higgs boson decay into two muons at 1.4 TeV CLIC. With respect to similar studies at future linear colliders, this paper includes several novel contributions to the statistical uncertainty of the measurement. The later includes the Equivalent Photon Approximation and realistic forward electron tagging based on energy deposition maps in the forward calorimeters, as well as several processes with the Beamstrahlung photons that result in irreducible contribution to the signal. In addition, coincidence of the Bhabha scattering with the signal and background processes is considered, altering the signal selection efficiency. The study is performed using a fully simulated CLIC_ILD detector model. It is shown that the branching ratio for the Higgs decay into a pair of muons BR($H \rightarrow \mu^+\mu^-$) times the Higgs production cross-section in $WW$-fusion $\sigma(H\nu\bar{\nu})$ can be measured with 38% statistical accuracy at $\sqrt{s} = 1.4$ TeV, assuming an integrated luminosity of 1.5 ab$^{-1}$ with unpolarised beams. If 80% electron beam polarisation is considered, the statistical uncertainty of the measurement is reduced to 25%. Systematic uncertainties are negligible in comparison to the statistical uncertainty.

1 Introduction

Measurements of Higgs branching ratios, and consequently Higgs couplings, provide a strong test of the Standard Model (SM) and possible physics beyond. Models that could possibly extend the SM Higgs sector (Two Higgs Doublet model, Little Higgs models or Compositeness models) will require Higgs couplings to electroweak bosons and Higgs-fermion Yukawa couplings (coupling-mass linearity) to deviate from the SM predictions

The Compact Linear Collider (CLIC) represents an excellent environment to study properties of the Higgs boson, including its couplings, with a very high precision. Measurements of rare $H \rightarrow \mu^+\mu^-$ decays are particularly challenging because of the very low branching ratio of $2 \times 10^{-4}$ predicted in the SM for a Higgs mass of 126 GeV. Current results indicate that the LHC was not able to access Higgs coupling to muons ($g_{H\mu\mu}$), based on the runs at 7 TeV and 8 TeV centre-of-mass (CM) energies. Projections for the HL-LHC, assuming 300 fb$^{-1}$ and 3 ab$^{-1}$ of data, predict uncertainties of 23% and 8% respectively for the $g_{H\mu\mu}$ coupling. In order to provide the best physics reach in the shortest time and for an optimal cost, the operation of the CLIC accelerator is foreseen in energy stages of 350 GeV, 1.4 TeV and 3 TeV. At 1.4 TeV and 3 TeV, sufficiently large Higgs boson samples can be produced to allow studies of rare Higgs decays. A sample of $3.7 \times 10^3$ Higgs bosons can be produced at 1.4 TeV CM energy, for an integrated luminosity of 1.5 ab$^{-1}$ with unpolarised beams. With the expected instantaneous luminosity of $3.2 \times 10^{34}$ cm$^{-2}$s$^{-1}$ this can be achieved in approximately five years of detector operation, with 200 running days per year and an effective up-time of 50%. The signal sample size will be doubled at 3 TeV CM energy due to rising cross-section for $WW$-fusion.

A similar study has been performed at 3 TeV CM energy. Compared to the study at 3 TeV, several challenges for the measurement of $H \rightarrow \mu^+\mu^-$ at CLIC are discussed for the first time in this paper. First, background processes with photons in the initial state simulated using both the expected Beamstrahlung spectrum at CLIC and the Equivalent Photon Approximation (EPA), were considered. Forward
electron tagging (Section 5) leads to a rejection of 48% and 42% of the $e^-e^+ \rightarrow e^-e^+\mu^+\mu^-$ and $e^+\gamma \rightarrow e^+\mu^+\mu^-$ background events, respectively. The impact of Bhabha scattering events on the rejection of events with forward electrons is investigated.

The paper is organised as follows. In Section 2 the simulation tools used for the analysis are listed and in Section 3 the CLIC_ILD detector model is briefly described. Signal and background processes and event samples are discussed in Section 4. Tagging of background high-energy electrons is described in Section 5. Event preselection and the final selection based on a Multivariate Analysis (MVA) approach are described in Section 6. The di-muon invariant mass fit is performed using Mokka [18] based on Geant4 [19]. Background processes and event samples are discussed in Section 7. Additional silicon trackers of the CLIC_ILD detector can be found in [25].

The ILD detector concept [24] is modified for CLIC according to the specific experimental conditions at higher energies [3]. The subsystems of particular relevance for the presented analysis are discussed here. A complete description of the CLIC_ILD detector can be found in [25].

The main tracking device of CLIC_ILD is the Time Projection Chamber (TPC) providing a point resolution in the $r\phi$ plane better than 100 $\mu$m, for charged particles in the detector angular acceptance [3]. Additional silicon trackers cover polar angles down to 7°. They have a single point resolution of 7 $\mu$m, and together with the TPC improve the tracking accuracy in the $r\phi$ plane. In order to provide precision tracking and vertexing closer to the beam-pipe, a Vertex Detector capable of an impact parameter resolution of 3 $\mu$m [26] is foreseen. Calorimetry at CLIC is based on fine-grained sandwich calorimeters optimized for particle-flow analysis (PFA). PFA is based on reconstruction of four-vectors of visible particles, combining the information from precise tracking with highly granular calorimetry. The detector comprises a central solenoid magnet, with a field of 4 T. High muon reconstruction efficiency of 99%, for muons above 7.5 GeV, is achieved by combining information from the central tracker (TPC plus silicon tracker) with information provided by the iron yoke instrumented with the 9 layers of Resistive Plate Chamber detectors.

In principle, hadrons produced in the interaction of the beam-induced photons affect the TPC occupancy and consequently the muon reconstruction efficiency. However, in the studied sample of muons from $H \rightarrow \mu^+\mu^-$ decays, muon reconstruction efficiency is above 99% in the barrel region, in the presence of $\gamma\gamma \rightarrow$ hadrons.

The average muon transverse momentum resolution for the signal sample is $\Delta(1/p_T) = 3.3 \times 10^{-5}$ GeV$^{-1}$ in the barrel region. The impact of transverse momentum resolution on the statistical uncertainty of $\sigma(H\nu\nu) \times BR(H \rightarrow \mu^+\mu^-)$ measurement is discussed in Section 6.

In the very forward region of the CLIC_ILD detector, below $\theta=8^\circ$, no tracking information is available. The region between 0.6° and 6.3° is instrumented with the two silicon-tungsten sampling calorimeters, LumiCal and BeamCal [27], for the luminosity measurement, beam-parameter control, as well as for the tagging of high-energy electrons escaping the main detector at low angles. Together with the very forward segments of the electromagnetic calorimeter covering the polar-angle region between 6.3° and 8°, it is possible to suppress the four-fermion SM background with the characteristic low-angle electron signature. The simulation of the very-forward electron tagging is described in Section 5.

2 Simulation and analysis tools

Higgs production through WW-fusion is simulated in Whizard 1.95 [13, 14] including a realistic CLIC beam spectrum and initial state radiation. The generator Pythia 6.4 [15] is used to simulate the Higgs decay into two muons. The CLIC luminosity spectrum and beam-induced processes are obtained by GuineaPig 1.4.4 [16]. Background events are also generated with Whizard using Pythia 6.4 to simulate the hadronization and fragmentation processes. Simulation of tau decays is done by Tauola [17]. The CLIC_ILD detector simulation is performed using Mokka [18] based on Geant4 [19]. Before digitisation of the detector signals, pile-up from $\gamma\gamma \rightarrow$ hadrons interactions is overlaid on the physics events. The particle flow algorithm, PandoraPFA [20, 21] is employed in reconstruction of the final-state particles within the Marlin reconstruction framework [22]. The TMVA package [23] is used to separate signal from background by MVA of signal and background kinematic properties.

3 The CLIC_ILD detector model

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4 Event samples

At $\sqrt{s} = 1.4$ TeV the SM Higgs boson is predominantly produced via WW-fusion (Figure 1). The effective cross-section for Higgs production in WW-fusion is 244 fb without beam polarization. The Higgs production cross-section above 1 TeV can be measured with a statistical precision better than 1% as shown in [1]. The $e^+e^- \rightarrow H\nu\nu, H \rightarrow \mu^+\mu^-$ signal statistics are expected to be small (of the order of a few tens of events) because of the small branching fraction for this particular decay.
crossings in the simulation, before the digitisation and event hadrons are included in the analysis by overlaying 60 bunch crossings 

In addition to the processes listed in Table 1, we have considered s-channel $e^+e^-\rightarrow \mu^+\mu^-$ production, as well as several processes with tau pair in the final state $e^+e^-\rightarrow \tau^+\tau^-$, $e^+e^-\rightarrow \nu_\tau\tau^+\tau^-$, $e^+e^-\rightarrow e^+e^-\tau^+\tau^-$. Tau decays become relevant if both taus decay into two muons which happens in $\sim$3% of cases. However, the invariant mass of the di-muon system will not match the Higgs mass window considered in this analysis (see Section 6.1). The same holds for $e^+e^-\rightarrow \mu^+\mu^-$ production.

In addition, processes with quasi-real photons in the initial state are simulated using the EPA. In this analysis, such events are grouped together with the processes involving Beamstrahlung photons. In this way, processes with roughly similar kinematic characteristics are grouped together, as shown in Table 1. The notation $e^+\gamma$ represents the sum of cross-sections for the processes with either $e^-\gamma$ or $e^+\gamma$ in the initial state.

At $\sqrt{s}=1.4$ TeV, the Higgs boson is also produced via ZZ-fusion, with a cross-section of about 10% of the Higgs production cross-section in WW-fusion. However, on a test sample of 300 ZZ-fusion events followed by the Higgs decay to a pair of muons, not a single event passed the selection described in Sections 6.1 and 6.2. This implies an efficiency smaller than 1.2% (95% CL) for this channel equivalent to less than 0.1 events passing the final selection. Therefore, the Higgs production through ZZ-fusion is not considered relevant for this analysis.

Photons, dominantly emitted by Beamstrahlung, produce incoherent pairs deposited mainly in the low-angle calorimeters. On average, 1.3 two-photon interactions producing hadronic final states occur per bunch crossing which may affect the muon reconstruction in the tracking detectors. These hadrons are included in the analysis by overlaying 60 bunch crossings in the simulation, before the digitisation and event reconstruction phase. These events, as well as other physics events, are passed through the full detector simulation [30].

### Table 1 List of considered processes with corresponding cross-sections

| Process | $\sigma (fb)$ |
|---------|--------------|
| $e^+e^-\rightarrow H\nu\nu, H\rightarrow \mu^+\mu^-$ | 0.0522 |
| $e^+e^-\rightarrow \nu\nu\mu^+\mu^-$ | 129 |
| $e^+e^-\rightarrow e^+e^-\mu^+\mu^-$ | 24.5* |
| $e^+\gamma\rightarrow e^+\nu_\mu\nu_\mu$ | 1098* |
| $\gamma\gamma\rightarrow \nu_\mu\nu_\mu$ | 30 |
| $e^+e^-\rightarrow e^-\nu_\mu\nu_\mu$ | 162 |
| $e^+e^-\rightarrow e^-\nu_\mu\nu_\mu$ | 1.6 |

### 5 Tagging of EM showers in the very forward region

In the polar angle region below $\theta=8^\circ$, tracking information and hadronic calorimetry are not available. The four-fermion background $e^+e^-\rightarrow e^+e^-\mu^+\mu^-$ of multiperipheral type and similar processes like $e^+\gamma\rightarrow e^+\mu^+\mu^-$ can fake the missing energy signature of the signal if the final state electrons (spectators), emitted at the polar angles smaller than $\theta=8^\circ$, escape undetected.

Electron detection in the very forward region involves the reconstruction of electromagnetic showers in the presence of intense beam-induced background depositing in the very forward calorimeters a large number of low-energy particles, mostly incoherent pairs from Beamstrahlung [31]. This deposition amounts to several hundred thousand of $e^+e^-$ pairs per bunch crossing [32].

Furthermore, Bhabha events where one or both electrons are detected in the very forward calorimeters may occur in coincidence with either signal or background, even within the 10 ns time stamp. Tagging of such Bhabha electrons will result in the rejection of signal (or background). In order to prevent significant loss of signal statistics, the electron tagging was optimized to identify showers with energy higher than 200 GeV and a polar angle above $1.7^\circ$ only. Under these requirements, the loss of the number of signal events due to tagging of Bhabha electrons amounts to 7%. Out of these 7%, in slightly more than a half of events one electron is added to the final state and, in the remainder two

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1Simulation with the reconstruction algorithm from Ref. [33] shows that assuming these cuts, the reconstruction efficiency, in BeamCal and LumiCal is above 98%, with a negligible fake rate.
Bhabha electrons are added. Table 2 shows rejection rates for signal and background obtained by the forward electron tagging due to Bhabha pile-up.

In conclusion, very forward tagging of high-energy electrons serves to half the fraction of background with spectator electrons, with a moderate loss of signal of 7% in the presence of Bhabha coincidence.

Table 2 Selection rates for signal and background by the forward electron tagging.

| Process | Rejection rate |
|---------|----------------|
| $e^+e^-\rightarrow e^+e^-\mu^+\mu^-$ | 48% |
| $e^+\gamma\rightarrow e^+\mu^+\mu^-$ | 42% |
| Signal | 7% |

6 Event selection

The event selection is done in two steps. First, a preselection is performed aiming to suppress background originating from beamstrahlung as well as the processes with spectator electrons described in Section 5. The final event selection uses a multivariate classifier based on boosted decision trees (BDT) to suppress remain background processes on the basis of their kinematic properties.

6.1 Preselection

In order to suppress the impact of the beam-induced background, only reconstructed particles with transverse momenta $p_T > 5$ GeV are used in the analysis. Furthermore, the preselection of events was made by requiring a reconstruction of exactly two muons in the event, with an invariant mass of the di-muon system in the window centered around the Higgs mass 105-145 GeV. In addition, the absence of tagged electrons with energy above 200 GeV and polar angle above 1.7° is required in order to suppress background with spectator electrons emitted in the very forward region of the detector.

6.2 MVA selection

As a second step in the event selection, MVA techniques are used based on the BDT classifier implemented in the TMVA package. From the signal sample, quarter of all events are reserved for TMVA training, as well as 0.5 ab$^{-1}$ of each background. The following observables were used for the classification of events, similar to the CLIC study at $\sqrt{s} = 3$ TeV [10]:

- visible energy of the event excluding the energy of the di-muon system, $E_{vis}$,
- transverse momentum of the di-muon system, $p_T (\mu\mu)$,
- scalar sum of the transverse momenta of the two selected muons, $p_T (\mu_1) + p_T (\mu_2)$,
- boost of the di-muon system, $\beta_{\mu\mu} = |p_{\mu\mu}|/E_{\mu\mu}$,
- polar angle of the di-muon system, $\theta_{\mu\mu}$,
- cosine of the helicity angle, $cos \theta^*$.

The process $e^+e^-\rightarrow \nu\bar{\nu}\mu^+\mu^-$, with the same final state as the signal, represents an irreducible background and can not be substantially suppressed before the invariant mass fit of the di-muon system. The processes $\gamma\gamma \rightarrow \nu\bar{\nu}\mu^+\mu^-$ has a similar final state, but a different CM energy distribution in the initial state, since it involves Beamstrahlung or EPA photons rather than initial electrons. This leads to a different distribution of the boost of the di-muon system, allowing separation from the signal to some extent. The processes $e^+e^-\rightarrow \nu\bar{\nu}\mu^+\mu^-$ and $\gamma\gamma \rightarrow \nu\bar{\nu}\mu^+\mu^-$, have slightly different distributions of the helicity angle from the signal. All processes with one or two spectator electrons show significant differences from the signal, primarily in the distribution of the visible energy (Figure 2). These processes are also effectively suppressed by the $p_T (\mu_1) + p_T (\mu_2)$ observable. In addition, for the $e^+e^-\rightarrow e^+e^-\mu^+\mu^-$ process, the distribution of $p_T (\mu\mu)$ exhibits a peak at lower values than the signal (Figure 3). This peak corresponds to events in which the di-muon system recoils against electron spectators or outgoing photons that are emitted below the angular cut of the very forward EM-shower tagging. The above is illustrated in Figure 3 showing the $p_T$ distributions for representative background processes.

![Fig. 2](https://example.com/fig.png)
7 Di-muon invariant mass fit

The quantity $\sigma(Hv\bar{v}) \times BR(H \rightarrow \mu^{+}\mu^{-})$ is determined from the equation:

$$\sigma(Hv\bar{v}) \times BR(H \rightarrow \mu^{+}\mu^{-}) = \frac{N_{s}}{L \cdot \varepsilon_{s}}$$

(1)

where $L$ stands for the integrated luminosity and $\varepsilon_{s}$ is the total counting efficiency for the signal, including the reconstruction, preselection and MVA selection. In the experiment, the number of signal events $N_{s}$ will be determined by fitting the di-muon invariant mass distribution with a function $f(m_{\mu\mu})$:

$$f(m_{\mu\mu}) = N_{s}f_{s}(m_{\mu\mu}) + N_{b}f_{b}(m_{\mu\mu})$$

(2)

where $f_{s,b}$ are probability density functions (PDF) used to describe the signal and the sum of all background processes, and $N_{s}$ and $N_{b}$ are the respective numbers of signal and background events in the fitting mass window. In this analysis, an unbinned likelihood fit, with all parameters of $f_{s,b}(m_{\mu\mu})$ fixed, is performed on simulated signal and background samples. $N_{s}$ and $N_{b}$ are left as free parameters determined from the fit. The way the signal and background PDFs are obtained is discussed in Section 7.1.

In order to estimate the statistical uncertainty of the signal count, 5000 toy Monte Carlo (MC) experiments are performed, where pseudo-data are obtained by randomly picking the signal $m_{\mu\mu}$ values from the fully simulated signal sample, while background $m_{\mu\mu}$ values are randomly generated from the total background PDF $f_{b}(m_{\mu\mu})$. The size of the signal sample $N'_{s}$ and sample sizes $N'_{b}$ of individual backgrounds considered, are obtained from the Poisson distribution for the integrated luminosity of 1.5 ab$^{-1}$, taking into consideration corresponding cross-sections $\sigma$ and the selection efficiencies $\varepsilon_{s}$, $\varepsilon_{b}$.

$$\langle N'_{s} \rangle = L \cdot \sigma_{s} \cdot \varepsilon_{s}, \quad \langle N'_{b} \rangle = L \cdot \sigma_{b} \cdot \varepsilon_{b}$$

where $i$ is indexing the different background processes listed in Table I.

For each toy MC experiment, the $m_{\mu\mu}$ distribution is fitted by the function $f(m_{\mu\mu})$ given in Eq.2, and the standard deviation of the resulting distribution of $N_{s}$ over all toy MC experiments is taken as the estimate of the statistical uncertainty of the $\sigma(Hv\bar{v}) \times BR(H \rightarrow \mu^{+}\mu^{-})$ measurement.

As will be discussed in Section 9.1, the di-muon invariant mass distribution is sensitive to the detector $p_{T}$ resolution, while the Higgs width $\Gamma_{H}$ can be considered negligible in comparison to the detector energy resolution.

7.1 Signal and background PDFs

Fully simulated samples of signal and background (Table I) are fitted to extract the PDFs. The sizes of the samples vary from several tens of thousands of events for the signal, up to a few million of events for various background processes.

The signal PDF was defined as a linear combination of a Gaussian function with exponential tails, $f_{exp}$, and a Gaussian function with tails that asymptotically approach con-
Fig. 4 Stacked histograms of the BDT output variable for signal and background processes (a); Significance as a function of the BDT cut value (b).

Fig. 5 Stacked histograms of the di-muon invariant mass distributions with preselection only (a) and after MVA selection (b). The distributions are normalised to the same integrated luminosity of 1.5 ab\(^{-1}\). The same legend applies to (a) and (b).

The parameters of Eq. 3 are determined by fitting the di-muon invariant mass distribution for the signal (Figure 6).

The total background PDF is defined as a linear combination of a constant and exponential term:

\[
 f_b = p_0 \cdot e^{p_2(m_{\mu\mu} - m_H)} + (1 - p_1) \]  

\[ f_c = f_{flat} + C \cdot f_{exp} \]  

where

\[
 f_{flat} = \begin{cases} 
 e^{- \frac{(m_{\mu\mu} - m_H)^2}{2 \sigma^2}} & m_{\mu\mu} < m_H \\
 e^{- \frac{(m_{\mu\mu} - m_H)^2}{2 \sigma^2}} & m_{\mu\mu} > m_H 
\end{cases} \]  

and

\[
 f_{exp} = \begin{cases} 
 e^{- \frac{(m_{\mu\mu} - m_H)^2}{2 \sigma^2}} & m_{\mu\mu} < m_H \\
 e^{- \frac{(m_{\mu\mu} - m_H)^2}{2 \sigma^2}} & m_{\mu\mu} > m_H 
\end{cases} \]  

The di-muon invariant mass fit of the total background is shown in Figure 7, together with the fit results for the free parameters in Eq. 4. As the normalisation to the common integrated luminosity requires different normalisation coefficients for different processes, binned data were used.
to combine the background processes in a straightforward manner and a binned $\chi^2$ fit was performed. The $\chi^2/N_{df}$ of the background fit was 62/61.

Fig. 6 Distribution of the di-muon invariant mass $m_{\mu\mu}$, of the signal after MVA selection, used for the PDF determination. The distribution is fitted with the function $f_i$ given in Eq. 4.

![Figure 6](image)

7.2 Distribution of the signal count

The overall function $f(m_{\mu\mu})$ (Eq. 2) is fitted to the pseudodata of each toy MC experiment using the unbinned likelihood fit. An example of a toy MC fit is given in Figure 8.

The standard deviation of the resulting signal count distribution in 5000 repeated toy MC experiments corresponds to the statistical uncertainty of the measurement and is 38%. (Figure 9). According to Eq. 1 it translates into the statistical uncertainty of the measurement, having in mind that the total uncertainty of the integrated luminosity can be determined at the permille level.

The statistical uncertainty of the signal counting is dominated by contributions from the limited signal statistics and from a presence of irreducible backgrounds. To estimate the significance of the signal against the null-hypothesis, another set of 5000 toy MC experiments was performed with zero signal count, and $f(m_{\mu\mu})$ (Eq. 2) was fitted to the pseudodata. The resulting $N_s$ distribution was centered on zero with a standard deviation of 5.4. Thus, in the case where the SM expected number of 19 signal events are found in an experiment, the corresponding signal significance would be 3.7 $\sigma$.

Because $\sigma(H\nu\bar{\nu}) \times BR(H \to \mu^+\mu^-) \propto g_{HWW}^2 g_{H\mu\mu}^2$ and having access to $\Gamma_H$ and $g_{HWW}$ from other measurements, extraction of $g_{H\mu\mu}$ is possible solely from the measurement presented here. An example of a minimal set of measurements giving a model-independent access to $\Gamma_H$ and $g_{HWW}$ is the following: the $H \to bb$ measurements at both 350 GeV and 1.4 TeV give access to the ratio $g_{HWW}$, the recoil mass measurement at 350 GeV CM energy gives access to $g_{HZZ}$, and the $H \to W^+W^-$ measurement at 1.4 TeV gives access to the ratio $g_{HWW}^2/\Gamma_H$. The contributions of these measurements towards the final $\Delta g_{H\mu\mu}$ is negligible at the second significant digit.
uncertainty of the $H \rightarrow \mu^+ \mu^-$ branching ratio measurement at the systematic level. It has been shown that at 3 TeV CLIC [35], where the impact of the beam-induced processes is the most severe, the luminosity above 75% of the nominal CM energy can be determined at the permille level, using low-angle Bhabha scattering. Below 75% of the nominal CM energy, the luminosity spectrum can be measured with a precision of a few percent using wide-angle Bhabha scattering [36]. About 17% of all Higgs production events occur at a CM energy below 75% of the nominal CM energy. Having in mind the intrinsic statistical limitations of the signal sample, this source of systematic uncertainty can be considered negligible.

On the detector side, an important systematic effect is the uncertainty on the transverse momentum resolution, because it directly influences the expected shape of the signal $m_{\mu\mu}$ distribution. The sensitivity of the signal count to the accuracy of the knowledge of the $p_T$ resolution $\sigma_{p_T}$ has been studied by performing the analysis with an artificially introduced uncertainty of an exaggerated magnitude on the assumed $p_T$ resolution used to extract the signal PDF. Results of the relative shift in signal counts w.r.t. the relative shift of $\sigma_{p_T}$ are shown in Figure 10. The relative bias in signal counting per one percent change of $\sigma_{p_T}$ is 0.35%.

The uncertainty of the muon identification efficiency will directly influence the signal selection efficiency. In addition, the uncertainty of the muon polar angle resolution impacts the $m_{\mu\mu}$ reconstruction. Based on the results of the LEP experiments [37], it can be assumed that these detector related uncertainties are below a percent.

The systematic uncertainty of the signal count caused by the fit with $f_{m_{\mu\mu}}$ defined in Eq.3 was found to be about 1% which is small compared to the statistical error.

Because of the forward EM shower tagging, about 7% of all events are rejected by coincident detection of Bhabha events. This fraction must be precisely calculated taking into account Bhabha event distributions, beam-beam effects, as well as the dependence of the tagging efficiency on energy and angle of the incident electrons and photons. This is work in progress [38][40], but the uncertainty of this effect is also expected to be negligible compared to the statistical uncertainty of the measurement.

8 Impact of electron polarization

If 80% left-handed polarisation of the electron beam is assumed during the entire operation time at 1.4 TeV, the Higgs production cross-section through $WW$-fusion would be enhanced by a factor 1.8 [4]. The most important background contribution after the MVA selection, the $e^+e^- \rightarrow \nu_{\mu} \bar{\nu}_{\mu} \mu^+ \mu^-$ process, is enhanced by the same factor because it is also mediated by $W$ bosons which have only left-handed interactions. The process $e^+ \gamma \rightarrow e^+ \mu^+ \mu^-$ is enhanced by a factor 1.32, while cross-sections for other background processes are not significantly changed w.r.t. the unpolarised case. The overall selection efficiency of the signal is 30%, because the classifier cut position is moved to a lower value which consequently leads to a higher signal efficiency. The final statistical uncertainty of the $\sigma(H\nu\nu) \times BR(H \rightarrow \mu^+ \mu^-)$ measurement is 25%. The corresponding uncertainty of $\sigma_{\mu\mu}$ is 13%. A summary of the results of the presented analysis is given in Table 3. It is important to note that all kinematic variables are unaffected by the beam polarization.

9 Systematic uncertainties

From Eq.1 it is clear that uncertainties of the integrated luminosity and muon identification efficiency influence the uncertainty of the $H \rightarrow \mu^+ \mu^-$ branching ratio measurement at the systematic level. It has been shown that at 3 TeV CLIC [35], where the impact of the beam-induced processes is the most severe, the luminosity above 75% of the nominal CM energy can be determined at the permille level, using low-angle Bhabha scattering. Below 75% of the nominal CM energy, the luminosity spectrum can be measured with a precision of a few percent using wide-angle Bhabha scattering [36]. About 17% of all Higgs production events occur at a CM energy below 75% of the nominal CM energy. Having in mind the intrinsic statistical limitations of the signal sample, this source of systematic uncertainty can be considered negligible.

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Table 3 Summary of the $\sigma(H\nu\nu) \times BR(H \rightarrow \mu^+ \mu^-)$ measurement at 1.4 TeV CLIC with unpolarised and 80% polarized electron beams. All uncertainties are statistical.

|                  | Unpolarised | Polarised (80%, 0%) |
|------------------|-------------|---------------------|
| $N_e$            | 19.3 ± 0.1  | 35 ± 9              |
| $\epsilon_{\mu}$| 24%         | 25%                 |
| $\delta(\sigma(H\nu\nu) \times BR(H \rightarrow \mu^+ \mu^-))$ | 38% | 25% |
| $\delta(\sigma(H\nu\nu)/\sigma(H\nu\nu))$ | 19% | 13% |

The dominant contribution to the $\sigma_{\mu\mu}$ coupling uncertainty is the statistical uncertainty of the measurement presented here. Systematic uncertainties affect the total uncertainty of $\sigma_{\mu\mu}$ determination only at the third significant digit, and thus can be neglected (Section 9). Under these assumptions, the relative uncertainty of $\sigma_{\mu\mu}$ is approximated to be 19%.
9.1 Benefit of a improved $p_T$ resolution

To estimate the benefit of a better $p_T$ resolution, the analysis was repeated by substituting the muon four-momenta reconstructed in the full simulation of the signal by a parametrisation of the momentum resolution for several different values of the detector resolution. Figure 11 displays the approximate dependence of the statistical uncertainty of the measurement on the average transverse momentum resolution in the whole detector. Even a large improvement of the muon momentum resolution would result in only a moderate improvement of the statistical uncertainty of the measured product of the Higgs production cross-section and the branching ratio for the $H \rightarrow \mu^+ \mu^-$ decay.

10 Conclusions

It has been shown that the measurement of the cross-section times the branching ratio for the SM Higgs decay into two muons can be performed with a relative statistical uncertainty of 38% at 1.4 TeV CLIC, assuming 1.5 ab$^{-1}$ integrated luminosity with unpolarised beams. The result is dominated by the limited signal statistics and the irreducible background. The systematic uncertainties are negligible in comparison to the statistical one. This translates into a relative uncertainty of the coupling of Higgs to muons $g_{H\mu\mu}$ of approximately 19%. If the same integrated luminosity is collected with 80% left-handed polarisation for the electrons, the relative statistical uncertainty improves to 25% and 13% for $\sigma(H\nu\bar\nu) \times BR(H \rightarrow \mu^+ \mu^-)$ and $g_{H\mu\mu}$, respectively.

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