A non-minimally coupled quintom dark energy model on the warped DGP brane

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Abstract

We construct a quintom dark energy model with two non-minimally coupled scalar fields, one quintessence and the other phantom field, confined to the warped Dvali–Gabadadze–Porrati (DGP) brane. We show that this model accounts for crossing of the phantom divide line in appropriate subspaces of the model parameter space. This crossing occurs for both normal and self-accelerating branches of this DGP-inspired setup.

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1. Introduction

In the last few years, an increasing number of astronomical observations (such as data from cosmic microwave background (CMB) temperature fluctuations spectrum and Supernova type Ia redshift–distance surveys) have indicated that the current universe is almost flat and undergoing a positively accelerated phase of expansion [1]. This phenomenon is not predicted by the standard cosmology governed by the general relativity (GR) with the known matter constituents. To explain this cosmic positive acceleration, mysterious dark energy has been proposed. There are several dark energy models that can be distinguished by, for instance, their equation of state (EoS) \( \omega_{de} = \frac{P_{de}}{\rho_{de}} \) during the evolution of the universe. The cosmological constant is the simplest model of dark energy with EoS \( \omega_{de} = -1 \), but the huge fine-tuning required for its magnitude makes it unfavorable for cosmologists. It is then tempting to find alternative models of dark energy with EoS \( \omega_{de} = -1 \), but the huge fine-tuning required for its magnitude makes it unfavorable for cosmologists. An accelerated expansion can be realized by using a scalar field whose origin may be found in superstring or supergravity theories; some of these models are quintessence, k-essence, tachyonic models, dilatonic models and phantom fields [2] (see also [3]). Other alternative approaches to accommodate dark energy are modification of GR by considering additional spatial dimensions [4–6] or modified Einstein–Hilbert action [7].

In recent years, there has been a lot of interest in the extra dimensional theories by modifying the old Kaluza–Klien picture, where the extra dimensions must be sufficiently compact. These recent developments are based on the idea that ordinary matter and gauge fields could be confined to a three-dimensional (3D) world (brane), while gravity and possibly non-standard matter are free to propagate in the entire extra dimensional spacetime (the bulk). One of the most popular braneworld models is the Randall–Sundrum II setup [5]. In this model, conventional 4D gravity can be recovered at large scales (low energies) on a Minkowski braneworld embedded in a 5D anti de Sitter (AdS) bulk. On the other hand, Dvali–Gabadadze–Porrati (DGP) introduced a braneworld model in which gravity is modified at large distances rather than short distances in contrast to other popular braneworld scenarios, because of an induced 4D Ricci scalar in the action on the brane [6]. This term can be obtained by the quantum interaction between the matter confined to the brane and the bulk gravitons. The DGP braneworld scenario explains accelerated expansion of the universe in its self-accelerating branch via leakage of gravity to extra dimension without the need to introduce a dark energy component [8]. While the RS model produces ultraviolet (UV) modification to the GR, the DGP model leads to infrared (IR) modification of GR. However, by considering the effect of an induced gravity term as a quantum correction in the RS model, we have a combined model that is called ‘warped DGP braneworld’ in the literature [9]. This setup gives also a ‘self-accelerating’ phase in the brane evolution.

On the other hand, astrophysical data also indicate that \( \omega \) lies in a very narrow strip close to \(-1\). The case \( \omega = -1 \)
corresponds to the cosmological constant. For $\omega$ less than $-1$ the phantom dark energy is observed, and for $\omega$ more than $-1$ (but less than $\frac{1}{3}$) the dark energy is described by quintessence. Moreover, the analysis of the properties of dark energy from recent observational data mildly favors models of dark energy with $\omega$ crossing the $-1$ line in the near past. So, the phantom phase EoS with $\omega < -1$ is still mildly allowed by observations. In this case, the universe lives in its phantom phase, which eventually ends up with a future singularity (Big Rip). There is also a lot of evidence all around of a dynamical EoS, which has crossed the so-called phantom divide line (PDL) $\omega = -1$ recently, at the value of redshift parameter $z \approx 0.25$ [10]. Most of the dark energy models treat scalar field(s) as dark component(s) with a dynamical EoS. Currently, scalar fields play crucial roles in modern cosmology. In the inflationary scenario, they generate an exponential rate of evolution of the universe as well as density fluctuations due to vacuum energy. It appears that the presence of a non-minimal coupling (NMC) between the scalar field and gravity is also necessary. There are many lines of theoretical evidence that suggest the incorporation of an explicit NMC of the scalar field and gravity in the action [11]. The nonzero NMC arises from quantum corrections and it is required also for the renormalization of the corresponding field theory. Amazingly, it has been proven that the PDL crossing of dark energy described by a single minimally coupled scalar field with general Lagrangian is even unstable with respect to the cosmological perturbations realized on the trajectories of the zero measure [12]. This fact has motivated a lot of attempts to realize the crossing of the PDL by EoS parameter of scalar field as dark energy candidate in more complicated frameworks. One of these attempts is a hybrid model, composed of two scalar fields, quintessence and phantom, that are usually dubbed as a quintom model in the literature [13]. A quintom model was initially proposed to obtain a model of dark energy with an EoS parameter $\omega$ which satisfies $\omega > -1$ in the past and $\omega < -1$ at present. As we have emphasized, this model is mildly favored by the current observational data fitting. Thus, the quintom model is a dynamical scenario of dark energy with the property that its EoS can smoothly cross over the cosmological constant barrier $\omega = -1$. Recently, Zhang and Zhu [14] have considered a minimally coupled scalar field on the warped DGP braneworld and realized crossing of the PDL by considering two possible cases: for ordinary or canonical (quintessence) scalar field, the EoS of dark energy crosses from $\omega > -1$ to $\omega < -1$ in the normal (non-self-accelerating) branch of the DGP setup. For phantom field, the EoS of dark energy crosses from $\omega < -1$ to $\omega > -1$ in the self-accelerating branch of the DGP scenario.

With these preliminaries, in this paper we construct a quintom dark energy model with two scalar fields, one quintessence and the other phantom, non-minimally coupled to the induced gravity on the warped DGP braneworld. We study the possible realization of the PDL crossing in this setup. We show, by numerical analysis of the model parameter space, that this model accounts for crossing of the cosmological constant line in the appropriate subspaces of the model parameter space. We show that this crossing can occur in both normal and self-accelerating branches of the scenario.

2. A dark energy model on the warped DGP braneworld

2.1. Warped DGP braneworld

We start with the action of the warped DGP model as follows:

$$S = S_{\text{bulk}} + S_{\text{brane}},$$  

(1)

$$S = \int d^5 x \sqrt{-g} \left[ \frac{1}{2\kappa_5^2} \right] R + \frac{1}{2\kappa_5^2} \mathcal{L}_{\text{brane}}(g_{\alpha\beta}, \psi).$$  

(2)

Here $S_{\text{bulk}}$ is the action of the bulk, $S_{\text{brane}}$ is the action of the brane and $S$ is the total action. $X^\alpha$ with $\alpha = 0, 1, 2, 3, 5$ are coordinates in the bulk, whereas $x^\mu$ with $\mu = 0, 1, 2, 3$ are induced coordinates on the brane. $\kappa_5^2$ is a 5D gravitational constant. $(5)$ $R$ and $(5)$ $\mathcal{L}_{\text{brane}}$ are the 5D Ricci scalar and matter Lagrangian, respectively. $K^{\pm}$ is a trace of the extrinsic curvature on either side of the brane. $\mathcal{L}_{\text{brane}}(g_{\alpha\beta}, \psi)$ is the effective 4D Lagrangian. The action $S$ is actually a combination of the Randall–Sundrum II and DGP models. In other words, an induced curvature term appears on the brane in the Randall–Sundrum II model, hence the name warped DGP braneworld [9]. Now consider the brane Lagrangian given below:

$$\mathcal{L}_{\text{brane}}(g_{\alpha\beta}, \psi) = \frac{\mu^2}{2} R - \lambda + \mathcal{L}_{\text{m}},$$  

(3)

where $\mu$ is a mass parameter, $R$ is the Ricci scalar of the brane, $\lambda$ is the tension of the brane and $\mathcal{L}_{\text{m}}$ is the Lagrangian of the other matters localized on the brane. Assume that bulk contains only a cosmological constant, $(5) \Lambda$. With these choices, action (1) gives either a generalized DGP or a generalized RS II model: it gives a DGP model if $\lambda = 0$ and $(5) \Lambda = 0$, and gives an RS II model if $\mu = 0$. The generalized Friedmann equation on the brane is as follows [9]:

$$H^2 + \frac{k}{a^2} = \frac{1}{3\mu^2} \left[ \rho + \rho_0 (1 + \varepsilon A(\rho, a)) \right],$$  

(4)

where $\varepsilon = \pm 1$ corresponds to the two possible branches of solutions (two different embedding of the brane) in this warped DGP model and $A = [A_0^2 + \rho_0 (\rho - \mu^2 \frac{a^4}{\rho_0^2})]^{1/2}$ where $A_0 \equiv \left[ 1 - 2\eta \frac{\mu^2}{\rho_0} \right]^{1/2}$, $\eta \equiv \frac{6m_5^3}{\rho_0^{1/2}}$ with $0 < \eta \leq 1$ and $\rho_0 = m_5^3 + 6m_5^5/\rho_0^2$. By definition, $m_5 = \lambda^{1/4}$ and $m_5 = k_5^{2/3}$. Also, $\varepsilon_0$ is an integration constant and the corresponding term in the generalized Friedmann equation is called the dark radiation term. We neglect the dark radiation term in what follows. In this case, the generalized Friedmann equation (4) takes the following form:

$$H^2 + \frac{k}{a^2} = \frac{1}{3\mu^2} \left[ \rho + \rho_0 + \varepsilon_0 \rho \left( A_0^2 + \frac{2n_5}{\rho_0} \right)^{1/2} \right],$$  

(5)

where $\rho$ is the total energy density, including scalar fields and dust matter energy densities on the brane:

$$\rho = \rho_0 + \rho_5 + \rho_{\text{dm}}.$$  

(6)
Note that \( \rho_0 \) can be expressed in terms of the crossover distance as \( \rho_0 = \frac{6a_0}{r_c^2} \), where the crossover distance is defined as \( r_c = \kappa^2 \mu^2 \). We note that our basic equation, (5), will reduce to the case studied in [14] if we set \( \lambda_0 = 1 \), \( \eta = 1 \) and \( \xi = 0 \). As a result, the outcomes of [14] can be deduced as a special case of our more general framework; this issue will be discussed later in this paper.

### 2.2. A quintom dark energy model on the warped DGP brane

Now we consider a dark energy component consisting of two scalar fields, one quintessence and the other phantom field, both non-minimally coupled to induced gravity on the warped DGP brane. The action of this non-minimal quintom model is given by

\[
S_{\text{quint}} = \int_{\text{brane}} d^4x \sqrt{-g} \left[ -\frac{\xi}{2} R (\varphi^2 + \sigma^2) - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi 
+ \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - V_1(\varphi) - V_2(\sigma) \right],
\]

where \( \xi \) is a non-minimal coupling, \( R \) is the Ricci scalar of the brane, \( \varphi \) and \( \sigma \) are the normal (quintessence) and the phantom field, respectively, and \( V_1(\varphi) \) and \( V_2(\sigma) \) are corresponding potentials. We have assumed a conformal coupling of the scalar fields and induced gravity on the brane and these fields play the role of the quintom dark energy component on the brane. Variation of the action with respect to \( \varphi \) gives the equation of motion of the normal scalar field

\[
\dot{\varphi} + 3H \varphi + \xi \dot{R} \varphi + \frac{dV_1}{d\varphi} = 0,
\]

and variation of the action with respect to \( \sigma \) gives the equation of motion of the phantom field

\[
\dot{\sigma} + 3H \sigma - \xi \dot{R} \sigma - \frac{dV_2}{d\sigma} = 0.
\]

The energy density and pressure of this quintom field are given by the following relations

\[
\rho_{\text{quint}} = \rho_\varphi + \rho_\sigma
= \frac{1}{2} (\dot{\varphi}^2 - \dot{\sigma}^2) + V_1(\varphi) + V_2(\sigma)
+ 6\xi H (\varphi \dot{\varphi} + \sigma \dot{\sigma}) + 3\xi H^2 (\varphi^2 + \sigma^2),
\]

and

\[
p_{\text{quint}} = p_\varphi + p_\sigma
= \frac{1}{2} (\dot{\varphi}^2 - \dot{\sigma}^2) - V_1(\varphi) - V_2(\sigma)
- 2\xi \left( \dot{\varphi} (\dot{\varphi} + 2H \dot{\varphi} + \dot{\sigma} + 2\sigma H \dot{\varphi} + \dot{\sigma}^2) \right),
- \xi (2H + 3H^2) (\varphi^2 + \sigma^2),
\]

respectively. In what follows, by comparing the modified Friedmann equation in the warped DGP braneworld with the standard Friedmann equation, we deduce the EoS of the quintom field. This is reasonable since all observed features of dark energy are essentially derivable in GR [14, 15].

The standard Friedmann equation in four dimensions is written as

\[
H^2 + \frac{k}{a^2} = \frac{1}{3}\Lambda (\rho_{\text{dm}} + \rho_{\text{de}}),
\]

where \( \rho_{\text{dm}} \) is the dust matter density, while \( \rho_{\text{de}} \) is the dark energy density. Comparing this equation with equation (5), we find

\[
\rho_{\text{de}} = \rho_\varphi + \rho_\sigma + \rho_0 + \varepsilon \rho_0 \left( A_0^2 + 2\eta \frac{\rho_0}{\rho_\varphi} \right)^{1/2}.
\]

The NMC of the scalar field and Ricci curvature on the brane preserves conservation of the total scalar field energy density (see for instance [18]). For a quintom dark energy component, we have

\[
\frac{d\rho_{\text{quint}}}{dt} + 3H (\rho_{\text{quint}} + p_{\text{quint}}) = 0.
\]

Since the dust matter obeys the continuity equation and the Bianchi identity remains valid, dark energy itself satisfies the continuity equation

\[
\frac{d\rho_{\text{de}}}{dt} + 3H (\rho_{\text{de}} + p_{\text{de}}) = 0,
\]

where \( p_{\text{de}} \) denotes the pressure of the dark energy. The EoS for the dark energy can be written as follows:

\[
w_{\text{de}} = \frac{p_{\text{de}}}{\rho_{\text{de}}} = -1 + \frac{d \ln \rho_{\text{de}}}{3 \ln (1 + z)}.
\]

Using equations (14) and (16) we find

\[
\frac{d \ln \rho_{\text{de}}}{3 \ln (1 + z)} = \left[ \frac{3}{\rho_{\text{de}}} \left[ \rho_\varphi + p_\varphi + \rho_\sigma + p_\sigma
+ \varepsilon \eta \left( A_0^2 + 2\eta \frac{\rho_0}{\rho_\varphi} \right)^{-1/2}
\times (\rho_\varphi + p_\varphi + \rho_\sigma + p_\sigma + \rho_{\text{dm}}) \right] \right].
\]

There are three possible cases in this setup: if \( \left( \frac{d \ln \rho_{\text{de}}}{3 \ln (1 + z)} \right) > 0 \), we have a quintessence model; if \( \left( \frac{d \ln \rho_{\text{de}}}{3 \ln (1 + z)} \right) < 0 \) the model is phantom and if \( \left( \frac{d \ln \rho_{\text{de}}}{3 \ln (1 + z)} \right) = 0 \), the dark component is a cosmological constant. Due to explicit dependence of the scalar field energy density and pressure on the NMC, in this setup the NMC of the scalar fields and induced gravity plays a crucial role supporting or preventing PDL crossing. In this respect, the differences between the minimal and non-minimal setups will be clearer if we write the explicit dynamics of the EoS parameter. On the other hand, the effect of the warp factor that appears in the definition of \( \rho_{\text{de}} \) will be highlighted in forthcoming arguments. We choose the following exponential potential with the motivation that this type of potential can be solved exactly in the standard model:

\[
V_1(\varphi) = V_{01} \exp \left( -\lambda_1 \frac{\varphi}{\mu} \right),
\]

and

\[
V_2(\sigma) = V_{02} \exp \left( -\lambda_2 \frac{\sigma}{\mu} \right).
\]
where \( V_{01}, V_{02}, \lambda_1, A_2 \) and \( \mu \) are constants. Therefore, we have

\[
\omega = -1 + \frac{1}{\rho_{de}} \left[ (\phi^2 - \sigma^2) - 2\xi \left( -H(\psi + \sigma) \right) + H(\phi^2 + \sigma^2) + \phi \psi + \sigma \phi + \phi^2 + \sigma^2) + \phi \psi + \sigma \phi + \phi^2 + \sigma^2) + \rho_{dm}\right]
\]

\[
+ H \left( \phi^2 + \sigma^2 \right) + \phi \psi + \sigma \phi + \phi^2 + \sigma^2\right) \left( \phi^2 - \sigma^2\right) - 2\xi \left( -H(\psi + \sigma) + H(\phi^2 + \sigma^2) + \phi \psi + \sigma \phi + \phi^2 + \sigma^2) + \rho_{dm}\right]
\]

\[
\frac{1}{\rho_{de}} \left[ \frac{1}{2} (\phi^2 - \sigma^2) + \frac{1}{2} \left( \phi^2 + \sigma^2\right) + \frac{1}{2} \left( \phi^2 + \sigma^2\right) + \frac{1}{2} \left( \phi^2 + \sigma^2\right) + \rho_{dm}\right]
\]

(20)

As a comparison, in the minimal case (with \( \xi = 0 \)) and neglecting the warp effect, when we consider just a quintessence field and choosing the sign of \( \epsilon \) to be negative, two remaining terms on the right-hand side of equation (20) will have opposite signs and the EoS parameter essentially crosses the PDL [14]. However, in our non-minimal quintom model the situation is more complicated and it is not easy to determine whether or not there is crossing of the PDL just by defining \( \epsilon \) sign since NMC itself plays a crucial role in this case. We consider \( \xi \) (the NMC of the scalar fields and induced gravity) as a fine-tuning parameter in this setup. In what follows, we use some parameters like \( \Omega_{\phi} \), \( \Omega_{\phi\sigma} \) (the present value of the kinetic energy density of the quintom field over the critical density) and \( \Omega_{\rho} \) (the present value of the energy density of \( \rho_0 \) over the critical density) to calculate EoS parameter, \( \omega(z) \). If we change the values of these parameters in an appropriate manner (subject to observational constraints), the redshift at which crossing of the PDL occurs will change since it is a model-dependent quantity in this respect. We explain further this behavior of PDL crossing in our forthcoming arguments and within a numerical scheme. For these numerical calculations, we introduce a new parameter defined as \( s = -\ln(1 + z) \) and in all figures the behavior of \( \omega \) is plotted versus \( s \). In these numerical calculations, we need an explicit form of \( \psi \) (and \( \sigma \)) in terms of other quantities. These quantities can be obtained directly from their equations of motion as given in (8) and (9). For instance, calculating \( \psi \) from equation (8) and substituting the definition of other quantities lead to

\[
\dot{\psi} = \left[ -6\xi \psi \left( \psi^2 - \sigma^2 \right) - 2\xi \left( -H(\psi + \sigma\dot{\sigma}) + \psi^2 + \sigma^2\right) + \rho_{dm}\right]
\]

\[
\times \left( \frac{\mu_2}{\psi} - \frac{b}{\psi} \right) \frac{6\xi^2 \psi b}{\mu_2 - \xi(\psi^2 + \sigma^2)b}
\]

\[
+ \frac{\lambda_1}{\mu} \frac{\dot{V}_1(\psi)}{\psi \phi} - \frac{\sigma \dot{\psi}}{\phi} \frac{3\xi \psi b}{\mu_2 - \xi(\psi^2 + \sigma^2)b} - \frac{3H(\dot{\psi} + 4\xi \dot{\psi} H)}{1 + \left( \frac{\sigma}{\phi} \right) - \frac{6\xi^2 \psi b}{\mu_2 - \xi(\psi^2 + \sigma^2)b}}
\]

Table 1. Acceptable range of \( \xi \) (constrained by the age of the universe) for physical roots of \( H \) as given by equation (25) to have crossing of the PDL for the quintom model.

| \( \epsilon \)   | \( \xi \)   | Acceptable range of \( \xi \) for z = 0.25 |
|-----------------|-------------|------------------------------------------|
| Positive        | -0.217 \( < \xi < -0.088 \) | -0.126                                   |
| Negative        | No crossing | -                                       |
| Positive        | No crossing | -                                       |
| Negative        | 0.215 \( \leq \xi \leq 0.292 \) | 0.251                                   |

where we have defined

\[
x = \frac{3H(\dot{\psi} + 4\xi \dot{\psi} H)}{6\xi \psi},
\]

\[
y = \frac{3H(\dot{\psi} + 4\xi \dot{\psi} H)}{6\xi \psi},
\]

and

\[
b \equiv 1 + \epsilon \eta \left( A_2^2 + 2\eta \frac{\rho}{\rho_0} \right)^{-1/2}.
\]

On the other hand, the Friedmann equation given by (5) now takes the following complicated form:

\[
(\mu_2 + g)^2 H^4 + 2f(3\mu_2 + g)H^3 + f^2 - 2(3\mu_2 + g)g H^2 + (-2fl + \epsilon^2 \rho_0 g) f H - 2\eta \epsilon^2 \rho_0 (l - \rho_0) - \epsilon^2 \rho_0^2 A_2^2 + l^2 = 0,
\]

(25)

where

\[
g \equiv -3\xi H(\phi^2 + \sigma^2),
\]

\[
l \equiv \frac{1}{2}(\psi^2 - \sigma^2) + \frac{1}{2}(\psi^2 + \sigma^2) + \frac{1}{2}(\psi^2 + \sigma^2) + \frac{1}{2}(\psi^2 + \sigma^2) + \rho_{dm} + \rho_0,
\]

(26)

\[
f \equiv -6\xi H(\psi^2 + \sigma^2),
\]

(27)

Equation (26) is a quadratic equation in terms of \( H^2 \) and, in principle, has four roots for \( H \). We show these roots as \( h_1, h_2, h_3 \) and \( h_4 \). Two of these roots, say, \( h_1 \) and \( h_2 \), are unphysical, excluded from observational ground. The other two roots, \( h_3 \) and \( h_4 \), are physical solutions corresponding to the generalized normal branch (with \( \epsilon = -1 \)) and the self-accelerating one (with \( \epsilon = 1 \)). These roots have the capability to realize the PDL crossing. The effect of warp factor on the dynamics of these solutions will be discussed later. In table 1, we have obtained some reliable ranges of NMC to have crossing of the PDL in this setup. We have assumed the age of the universe to be 13 Gyr and with this choice the values of \( \xi \) are constrained to the ranges shown in the table. On the other hand, observational data show that crossing of the PDL occurs at redshift \( z = 0.25 \) (though a model-dependent value, but suitable for our purposes) so we have obtained the value of \( \xi \) that corresponds to this value in the last column of the table. Note that negative values of the NMC are not excluded from our analysis. In fact these negative values are theoretically interesting, corresponding to anti-gravitation (see the papers by Farahoni in [11] and references therein). The results of numerical calculations are shown in figure 1 for two branches of this DGP-inspired model and with different values of the NMC \( \xi \). In this analysis, the best ranges of values for \( \xi \) to have a reliable model in comparison with observational data are obtained. Note that in all of our numerical calculations, we
have assumed $\Omega_{ki} = 0.01$, $\Omega_{rc} = 0.01$, $\Omega_m = 0.3$ and $\lambda_1$ and $\lambda_2 = 0.5$, and also for investigating the effect of NMC we have set $A_0 = 1$, $H_0 = 1$, $\mu = 1$ and $\eta = 0.99$. Figure 1 shows the crossing of the PDL obtained in both self-accelerating and normal branches of this DGP-inspired model. However, there is a difference between crossing behavior of these solutions. Indeed, as it could be understood from the figures, for the self-accelerating branch of the model, there is crossing of the PDL (from phantom phase $\omega < -1$ to quintessence phase $\omega > -1$) only for negative values of NMC parameter. By contrast, the above result for the normal branch of the DGP-inspired model holds only for positive values of the NMC parameter, while the EoS parameter of dark energy has a transition from quintessence phase $\omega > -1$ to phantom phase $\omega < -1$.

In figure 2, we have plotted the behavior of deceleration parameter $q$ versus $s$ in both branches of our DGP-inspired scenario. We see that in the self-accelerating branch of the scenario and with negative values of the NMC, deceleration parameter vanishes at a moment in the future with redshift $z \approx -0.37$ for $\xi = -0.126$. On the other hand, in the normal branch by increasing the values of $\xi$ (which is positive in this case), the deceleration parameter vanishes at late time epochs of the universe evolution. As an important result, by incorporating the NMC we have an accelerated behavior even in the normal branch ($\epsilon = -1$) of this DGP-inspired scenario.

In figure 3, we show the behavior of the EoS parameter of dark energy with different values of the parameter $\eta$, which is related to the warp effect. It is clearly seen that for sufficiently small values of $\eta$, in both branches of the model, the EoS parameter crosses the cosmological constant line at relatively small values of redshifts. Both of the two possible crossings, that is, from phantom phase to quintessence and from quintessence phase to phantom, are possible in this scenario.

Now we discuss two special cases of our model separately: a quintessence and a phantom phase.

### 2.3. Quintessence field

In this subsection, we investigate the dynamics of a quintessence field non-minimally coupled to induced gravity on the background of a warped DGP brane. The action of the model in this case takes the following form:

$$S_q = \int_{brane} d^4x \sqrt{-g} \left[ -\frac{1}{2} \kappa R \tilde{\varphi}^2 - \frac{1}{2} \partial_\mu \tilde{\varphi} \partial^\mu \tilde{\varphi} - V_1(\varphi) \right].$$  

(29)
shows the dynamics of EoS parameter of (30) ω = −0.25, −0.2, −0.15, −0.1, −0.05, 0, 0.05, 0.1, 0.2, 0.4, 0.6, 0.8, 1.

Figure 2. (a) Deceleration parameter in DGP\textsuperscript{+1} branch of the model which vanishes for instance at $s \approx 0.45$ for $\xi = -0.217$. (b) Deceleration parameter in DGP\textsuperscript{−1} branch of the model which vanishes for instance at $s \approx 0$ for $\xi = 0.292$. This vanishing shows the transition to acceleration or deceleration phase of the cosmological dynamics. As an important consequence, we see that by incorporating the NMC, we have accelerated behavior even in the normal branch of this DGP-inspired scenario.

The energy density and pressure of this quintessence field are given by

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V_1(\phi) + 6\xi H\phi\dot{\phi} + 3\xi H^2\phi^2,$$

and

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V_1(\phi) - 2\xi(\phi\dot{\phi} + 2\phi H\dot{\phi} + \phi^2) - \xi\phi^2(2H + 3H^2).$$

(30)

The EoS parameter of this quintessence field takes the following form:

$$\omega = -1 + \frac{1}{\rho_\phi} \left[ \frac{\dot{\phi}^2}{\rho_\phi} - 2\xi \left( -H\phi\dot{\phi} + H^2\phi^2 + \phi\ddot{\phi} + \phi^2 \right) \right]$$

$$+ \left[ \dot{\phi}^2 - 2\xi \left( -H\phi\dot{\phi} + H^2\phi^2 + \phi\ddot{\phi} + \phi^2 \right) + \rho_{\text{dm}} \right]$$

$$\times \left[ \xi \eta \left( A_0^2 + 2\eta \frac{\dot{\phi}^2 + V_1(\phi) + 6\xi H\phi\dot{\phi} + 3\xi H^2\phi^2 + \rho_{\text{dm}}}{\rho_0} \right) \right]^{\frac{1}{2}}.$$  

(31)

(32)

Table 2 shows some acceptable ranges of NMC of the quintessence field and induced gravity on the warped DGP setup. These values are constrained by the age of the universe for physical roots of equation (25) (but for just one quintessence field) to have crossing of the PDL by the EoS parameter. Figure 4 shows the dynamics of EoS parameter of this scalar field on the warped DGP braneworld. As we see in the self-accelerating branch of the model (with $\varepsilon = +1$), this parameter crosses the cosmological constant line if the
Figure 4. (a) In the self-accelerating branch of the scenario and for negative values of the nonminimal coupling, \( \omega \) crosses the cosmological constant line. For instance, if \( \xi = -0.298 \) this crossing occurs at \( s = -0.22 \) or \( z = 0.25 \). (b) There is no crossing of PDL with positive values of the NMC parameter in the self-accelerating branch of the model. (c) In the normal branch of the model with negative values of the NMC, \( \omega \) never crosses the cosmological constant line. (d) In the normal branch and with positive values of the nonminimal coupling parameter, the EoS of dark energy crosses the \( \omega = -1 \) line. For instance, if \( \xi = 0.11 \) this crossing occurs at \( s = -0.22 \) or \( z = 0.25 \).

Table 2. Acceptable ranges of \( \xi \) (constrained by the age of the universe) for the physical roots of \( H \) as given by equation (25) to have crossing of the PDL by the EoS parameter of a single quintessence field.

| \( \varepsilon \) | \( \xi \) | Acceptable range of \( \xi \) | The value of \( \xi \) for \( z = 0.25 \) |
|---|---|---|---|
| +1 | Negative | \(-0.399 \leq \xi \leq -0.242\) | -0.298 |
| +1 | Positive | No crossing | |
| -1 | Negative | No crossing | |
| -1 | Positive | \(0.09 \leq \xi \leq 0.147\) | 0.11 |

NMC is negative. The crossing on the normal branch (with \( \varepsilon = -1 \)) occurs only with positive values of the NMC. In both cases, this crossing runs from phantom phase \((\omega < -1)\) to quintessence phase \((\omega > -1)\). There is no crossing of PDL in DGP\(^{+}\) branch with positive sign of \( \xi \) and in DGP\(^{-}\) branch with negative sign of \( \xi \).

Our results may be compared with the minimal case that has been investigated by Zhang and Zhu [14]. Indeed, by considering an ordinary scalar field (quintessence), they have obtained crossing of the \( \omega = 1 \) line running from \( \omega > -1 \) to \( \omega < -1 \) only in the normal branch of the DGP scenario. However, we see here that the presence of the NMC leads to crossing behavior in both branches of this DGP-inspired scenario. With a single quintessence scalar field, this crossing runs from phantom phase to quintessence phase. Figure 5 shows the deceleration parameter in two branches of the model. It is observed from the figure that an accelerated phase will occur at a sufficiently high redshift in the future. The role played by the parameter \( \eta \), which is related to the warp effect, is shown in figure 6. It should be emphasized that from this figure we see that for small values of \( \eta \), the equation of state parameter, \( \omega \), crosses the cosmological constant line at relatively small values of redshifts.

2.4. Phantom field

Now we investigate the dynamics of the EoS of a phantom field non-minimally coupled to induced gravity on the warped DGP brane. The action of the model is given by

\[
S_\sigma = \int_{\text{brane}} d^4x \sqrt{-g} \left[ -\frac{1}{2} \xi R \sigma^2 + \frac{1}{2} \partial_{\mu} \sigma \partial^{\mu} \sigma - V_2(\sigma) \right].
\]

(34)
shows that in the self-accelerating branch of this 

\[ \omega = -1 \]

\[ q = -1 \]

The energy density and pressure of this phantom field are defined as

\[ \rho_p = -\frac{1}{2} \dot{\sigma}^2 + V_2(\sigma) + 6\xi H \sigma \dot{\sigma} + 3\xi H^2 \sigma^2, \]  

\[ p_p = -\frac{1}{2} \dot{\sigma}^2 - V_2(\sigma) - 2\xi (\sigma \ddot{\sigma} + 2\sigma \dot{H} \dot{\sigma} + \dot{\sigma}^2) - 3\xi \sigma^2 (2H + 3H^2), \]

respectively. The EoS parameter of this phantom field is given by

\[
\omega = -1 + \frac{1}{\rho_c} \left[ -\dot{\sigma}^2 - 2\xi \left( -H \sigma \dot{\sigma} + H^2 \sigma^2 + \ddot{\sigma}^2 + \dot{\sigma}^2 \right) + \rho_{dm} \right] \\
+ \left[ \dot{\sigma}^2 - 2\xi \left( -H \sigma \dot{\sigma} + H^2 \sigma^2 + \ddot{\sigma}^2 + \dot{\sigma}^2 \right) + \rho_{dm} \right] \\
\times \left[ \varepsilon \eta \left( \lambda_0^2 + 2\eta \frac{1}{\rho_p} + 6\xi H \sigma \dot{\sigma} + 3\xi H^2 \sigma^2 + \rho_{dm} \right)^{-1} \right] \].

where by using the equation of motion of this phantom field as given by (9), we have

\[
\ddot{\sigma} = \left[ 3\xi \sigma \left( -\dot{\sigma}^2 - 2\xi \left( -H \sigma \dot{\sigma} + \dot{\sigma}^2 \right) + \rho_{dm} \right) \left( \frac{b}{\mu^2 - \xi \sigma^2 b} \right) + 3H(\dot{\sigma} - 4\xi \sigma H) + \frac{2\xi}{\mu^2 - \xi \sigma^2 b} V_2(\sigma) \right] \\
- \frac{1}{\mu^2 - \xi \sigma^2 b} \cdot \frac{6\xi^2 \sigma^2 b}{\mu^2 - \xi \sigma^2 b} \cdot \frac{b}{\mu^2 - \xi \sigma^2 b}
\]

(38)

Table 3 shows the acceptable range of the NMC $\xi$ to have a crossing of the cosmological line in this case.

Figure 7 shows that in the self-accelerating branch of this DGP-inspired scenario, the EoS of dark energy crosses the PDL for both positive and negative values of the NMC parameter. It is worth noting that there is very different behavior
Figure 7. (a) With negative values of the nonminimal coupling, the EoS parameter of dark energy on the self-accelerating branch of the model crosses the $\omega = -1$ line. This crossing runs from phantom to quintessence phase. (b) With positive values of the nonminimal coupling, the EoS parameter of dark energy crosses the $\omega = -1$ line in the self-accelerating branch of the model from quintessence to phantom phase and this is an important result that is supported by observational data. This crossing occurs for $\xi = 0.091$ at $z = -0.22$ or $\xi = 0.25$. (c) There is no crossing of the PDL in the normal branch of the model with negative values of the NMC. (d) There is also no crossing of the PDL in the normal branch with positive values of the NMC.

Table 3. Acceptable range of the NMC constrained by the age of the universe and physical roots of $H$ (as given by equation (25) but with just one phantom field) in order to have crossing of the PDL.

| $\varepsilon$ | $\xi$ Acceptable range of $\xi$ | The value of $\xi$ for $z = 0.25$ |
|---------------|---------------------------------|----------------------------------|
| +1 Negative   | $-0.16 \leq \xi \leq -0.126$   | -                                |
| +1 Positive   | $0.056 \leq \xi \leq 0.106$    | 0.091                            |
| -1 Negative   | No crossing                     | -                                |
| -1 Positive   | No crossing                     | -                                |

of such a crossing relative to the existing literature (for instance [14]). Whereas the EoS of dark energy crosses from below the cosmological line to above it for negative values of $\xi$, for positive values of $\xi$ this phenomenon occurs from above the cosmological constant line to below it. Figure 7 also indicates that there is no crossing in the normal branch of this setup for any sign of the NMC parameter. This result could be compared with the minimal case obtained by Zhang and Zhu [14]. In fact in their framework, for phantom field the EoS of dark energy crosses from $\omega < -1$ to $\omega > -1$ in the self-accelerating branch of DGP scenario. Here the situation differs due to the presence of the NMC and warp effect.

In figure 8, we show the dynamics of the deceleration parameter with a non-minimally coupled phantom field on the warped DGP brane. The result confirms that this parameter vanishes at sufficiently late times of the universe’s evolution.

3. Summary and conclusion

Light-curves analysis of several hundreds of type Ia supernovae, WMAP observations of the cosmic microwave background radiation and other CMB-based experiments have shown that our universe is currently in a period of positively accelerated expansion. An alternative to explain this accelerated expansion is a multi-component dark energy with at least one non-canonical phantom field. The analysis of the properties of dark energy from recent observations mildly favors models where $\omega = \frac{\xi}{\rho}$ crosses the PDL $\omega = -1$ in the near past. In this respect, construction of theoretical frameworks with potential to describe positively accelerated expansion and crossing of the PDL by the EoS parameter is an interesting challenge. In this paper, we have constructed a quintom dark energy model on a warped DGP brane. We have investigated the crossing of the PDL in this setup in
three cases: firstly, we have considered a combined scenario consisting of two scalar fields as a realization of the quintom model. This model is built on a unified treatment of an ordinary (quintessence) scalar field and a phantom field non-minimally coupled to the induced Ricci scalar on the warped DGP brane. In this case, we have realized that the cosmological constant line crossing occurs in both branches of this DGP-inspired scenario with suitable values and signs of the non-minimal coupling parameter ($\xi$). For negative values of $\xi$ (which is possible at least theoretically and corresponds to anti-gravitation), the EoS parameter of dark energy crosses the cosmological constant line ($\omega = -1$) at $s \approx -0.315$ or $z \approx 0.37$, whereas for $\eta = 0.1$, this occurs at $s \approx -0.46$ or $z \approx 0.58$.

we have obtained crossing of the cosmological constant line in both DGP ($^{(\pm)}$) branches of the model. The crossing occurs for the self-accelerating branch for negative values of $\xi$, whereas it occurs for the normal branch for positive values of $\xi$. We have compared our results with the results of a similar analysis with minimal scalar field on the ordinary DGP setup investigated by Zhang and Zhu [14]. They have considered an ordinary scalar field (quintessence) on the DGP brane and obtained a crossing of the PDL from $\omega > -1$ to $\omega < -1$ only in the normal branch of DGP scenario. Consequently, by our analysis it can be concluded that the presence of the NMC of the scalar field and induced Ricci scalar on the warped DGP brane leads to crossing behavior in both branches of this DGP-inspired scenario. Finally, we have considered a phantom scalar field non-minimally coupled to the induced Ricci scalar on the warped DGP brane. In this case, the EoS parameter of dark energy crosses the PDL only in the self-accelerating branch of the model and its behavior is sensitive to the sign of the NMC parameter. Indeed, the EoS of dark energy crosses the cosmological constant line from phantom phase to quintessence phase for negative values of $\xi$; with positive $\xi$ this phenomenon occurs reversely (from quintessence to phantom phase) and is once again in good agreement with observation. By comparing our results with the minimal case (for phantom field, the EoS of dark energy crosses from $\omega < -1$ to $\omega > -1$ in the self-accelerating branch of DGP scenario), we see that in our model this crossing behavior can occur even in the reverse direction depending on the sign of the NMC parameter.

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