Managing Interference Correlation Through Random Medium Access
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Abstract—The capacity of wireless networks is fundamentally limited by interference. However, little research has focused on the interference correlation, which may greatly increase the local delay (namely the number of time slots required for a node to successfully transmit a packet). This paper focuses on the question whether increasing randomness in the MAC, specifically frequency-hopping multiple access (FHMA) and ALOHA, helps to reduce the effect of interference correlation. We derive closed-form results for the mean and variance of the local delay for the two MAC protocols and evaluate the optimal parameters that minimize the mean local delay. Based on the optimal parameters, we identify two operating regimes, the correlation-limited regime and the bandwidth-limited regime. Our results reveal that while we cannot minimize the mean local delay, two MAC protocols and evaluate the optimal parameters that effectively reduce the effect of interference correlation. We derive closed-form results for the mean and variance of the local delay for the two MAC protocols and evaluate the optimal parameters that minimize the mean local delay. Based on the optimal parameters, we identify two operating regimes, the correlation-limited regime and the bandwidth-limited regime. Our results reveal that while we cannot minimize the mean local delay, we can identify two operating regimes, the correlation-limited regime and the bandwidth-limited regime. Our results reveal that while we cannot minimize the mean local delay, two MAC protocols and evaluate the optimal parameters that effectively reduce the effect of interference correlation. We derive closed-form results for the mean and variance of the local delay for the two MAC protocols and evaluate the optimal parameters that minimize the mean local delay. Based on the optimal parameters, we identify two operating regimes, the correlation-limited regime and the bandwidth-limited regime. Our results reveal that while we cannot minimize the mean local delay, two MAC protocols and evaluate the optimal parameters that effectively reduce the effect of interference correlation. We derive closed-form results for the mean and variance of the local delay for the two MAC protocols and evaluate the optimal parameters that minimize the mean local delay. Based on the optimal parameters, we identify two operating regimes, the correlation-limited regime and the bandwidth-limited regime. Our results reveal that while we cannot minimize the mean local delay, two MAC protocols and evaluate the optimal parameters that effectively reduce the effect of interference correlation. We derive closed-form results for the mean and variance of the local delay for the two MAC protocols and evaluate the optimal parameters that minimize the mean local delay. Based on the optimal parameters, we identify two operating regimes, the correlation-limited regime and the bandwidth-limited regime. Our results reveal that while we cannot minimize the mean local delay, two MAC protocols and evaluate the optimal parameters that effectively reduce the effect of interference correlation. We derive closed-form results for the mean and variance of the local delay for the two MAC protocols and evaluate the optimal parameters that minimize the mean local delay. Based on the optimal parameters, we identify two operating regimes, the correlation-limited regime and the bandwidth-limited regime. Our results reveal that while we cannot minimize the mean local delay, two MAC protocols...
all time slots; it is a randomly variable uniquely determined by the spatial distribution of nodes. The local delay, as a random variable, in that extreme case is two-valued: either one frame (good realization of the spatial distribution of nodes) or infinite (bad realization of the spatial distribution of nodes). In this case, the transmission success events are fully correlated (one success implies success in each time slot, and vice versa), and the mean local delay is infinite.

In view of this, we consider some forms of man-made randomization by introducing MAC dynamics to reduce the interference correlation. The following analysis will be carried out in parallel under two different kinds of MAC protocols:

- **FHMA (Frequency-hopping multiple access):** FHMA is implemented by simply dividing the entire frequency band into $N$ sub-bands and letting each transmitter independently choose a sub-band uniformly randomly in each time slot. We focus on slow frequency-hopping, i.e., hopping at the time scale of a time slot, not at the time scale of a symbol. There are three benefits by splitting the entire frequency bands into sub-bands. First and foremost, it increases the uncertainty in the active pattern of interfering nodes, thereby reducing the effect of interference correlation. Second, the interference for a given transmission is also reduced because the intensity of the interfering transmitters are scaled by $\frac{1}{N}$. Third, the noise power is also scaled by $\frac{1}{N}$ since each transmission occurs in a narrow sub-band. Meanwhile, on the other side, splitting into sub-bands scales down the rate.

Since FHMA is often viewed as a spread-spectrum technique, we briefly comment on DS-CDMA. For synchronous orthogonal CDMA like those using Walsh codes, a receiver can in theory completely reject arbitrarily strong signals from interfering transmitters using different spreading sequences; thus, only those transmitters using the same spreading sequence as the desired link will cause interference. If the spreading sequence is randomly chosen for each transmission, the analysis and results of the local delay are exactly the same as that for FHMA. For asynchronous CDMA using pseudo-noise (PN) sequences, the interference comes from all transmitters and is usually approximated as Gaussian noise in the literature. The works in [8] and [9] have discussed the difference between asynchronous CDMA and FHMA in terms of outage probability and throughput. In asynchronous CDMA, although the desired signal is increased by the processing gain, the interference still comes from all transmitters. Therefore, the analysis of the local delay is similar as that for FHMA with $N = 1$, i.e., no bandwidth splitting is employed. We will show that in this case the distribution of the local delay has a heavy tail, which results in an infinite mean local delay.

- **ALOHA:** In ALOHA, if a packet is to be transmitted during a time slot, the packet will only be transmitted with a certain probability using the entire frequency band. Decreasing the transmit probability increases the uncertainty in the active pattern of interfering nodes and reduces the interference, while the noise power will not be reduced. Meanwhile, transmitting probabilistically scales down the rate.

In the last part of our work, we evaluate the mean delay-jitter tradeoff and the bounds on the tail probability of the local delay. The issue of optimizing the number of sub-bands was also considered in [21], where the optimal number of sub-bands is derived to maximize the number of concurrent transmissions. However, such outage-based framework used in [21] cannot capture the effects of correlated interference. In this work, we evaluate the mean delay-jitter tradeoff and the bounds on the tail probability of the local delay, both of which are critical issues for the system design.

Our results reveal that the means of the local delay of the two protocols, FHMA and ALOHA, coincide when the number of sub-bands $N$ in FHMA is equal to the reciprocal of the transmit probability $p$ in ALOHA (with thermal noise ignored). However, the variances of the local delay for the two protocols are drastically different: when $p = \frac{1}{N}$ and $N \to \infty$, the variance in FHMA converges to a constant which is typically small, while in ALOHA the variance scales as $\Theta(N^2)$. Moreover, we calculate bounds on the complementary cumulative distribution function (ccdf) of the local delay when no MAC dynamic is introduced. In that case, the distribution of the local delay has a heavy tail, which results in an infinite mean local delay. By employing the MAC randomness of
either FHMA or ALOHA, the cpdf of the local delay will decay fast, and the mean local delay will then be finite. This observation reveals the underlying mechanism why even such simple MAC protocols can greatly reduce the local delay.

The remaining part of this paper is organized as follows. Section II describes the network model and the MAC protocols. Section III then establishes the main analytical results of this paper, including the mean and variance of the local delay for FHMA and for ALOHA. Section IV evaluates the optimal number of sub-bands for FHMA and the optimal transmit probability for ALOHA that minimize the mean local delay. Section V presents the optimal SINR threshold that minimizes the mean local delay. Section VI presents the mean delay-jitter tradeoff and the bounds on the tail probability of the local delay, and Section VII offers the concluding remarks.

II. SYSTEM MODEL

A. Network Model

To obtain the most essential features, we consider the widely used Poisson bipolar model. In this model, the locations of the transmitters are modeled as a PPP \( \Phi = \{ x_i \} \subset \mathbb{R}^2 \) of intensity \( \lambda \). Each transmitter is associated with one receiver which is at a fixed distance \( r_0 \) to the corresponding transmitter. In the analysis, we will condition on a particular desired transmitter \( x_0 \in \Phi \), and denote by \( r_0 = |x_0| \) the distance from this transmitter to the origin where the receiver resides. Such conditioning is equivalent to adding the point \( x_0 \) to the PPP and guarantees that the link between \( x_0 \) and the origin is a typical link, in the sense that this link behaves statistically the same as all other links (see [13, Ch. 8]).

![Spatial distribution of different network entities.](image)

We assume that the time is divided into discrete slots with equal duration. Each transmission attempt occupies one time slot, and if a transmission fails in a certain time slot, a retransmission will be conducted. The local delay is defined as the number of time slots until a packet is successfully received [6, 7]. In this paper, we assume fully backlogged nodes so that whenever a node is scheduled to access the channel it always has data to transmit. The local delay is thus basically the transmission delay, but not the queueing delay.

For the propagation model, we consider the common path loss \( l(r) = \kappa r^{-\alpha} \), where \( \alpha \) is the path loss exponent and \( \kappa \) is a constant. We will further discuss the effect of bounded path loss \( l(r) = \kappa (r^\alpha + \varepsilon)^{-1} \) in the subsection [III-B].

We assume that the power fading coefficients are spatially and temporally independent with exponential distribution of unit mean (i.e., Rayleigh fading), and let \( h_{k,x} \) be the fading coefficient between transmitter \( x \) and the considered receiver located at origin \( o \) in time slot \( k \). Without loss of generality, we assume that all transmitters transmit at a normalized power level of unity. This constant power assumption is consistent with the bipolar network model, in which all link distances are identical. The thermal noise is assumed to be white Gaussian with power spectral density \( N_0 \). To simplify the notations, we introduce the normalized noise power spectral density as \( N_0 = N_r / \kappa \).

We assume that the SINR threshold model is applied. That is, for each time-frequency resource block, as long as the SINR is above a threshold \( \theta \), it can be successfully used for information transmission at spectral efficiency \( \log_2(1 + \theta) \) bits per second per Hz. We also assume that a packet of a fixed size needs exactly one time slot to be transmitted if it is allocated the entire frequency band \( W \) under SINR threshold \( \theta \) and successfully transmitted in that time slot. In that way, in the FHMA case if the entire frequency band is split into \( N \) sub-bands, a packet will need \( N \) successful time slots. Meanwhile, in the ALOHA case, each active transmission will make use of the entire frequency band; thus, only one successful time slot is needed. Notice that the local delay is measured by the number of time slots. Since different system configurations may apply different durations of time slot, we should normalize the local delay so that the actual delays of different system configurations can be compared fairly. The duration of each time slot is proportional to \( 1 / \log_2(1 + \theta) \) because the size of a packet is fixed and the spectral efficiency is proportional to \( \log_2(1 + \theta) \). Therefore, when comparing the actual delays under different SINR thresholds \( \theta \), we normalize the local delay by \( 1 / \log_2(1 + \theta) \) as the metric.

In static or moderately mobile network, the locations of the transmitters during all time slots are deemed to be correlated, resulting in the temporal interference correlation. This type of correlation decreases the successful probability for retransmissions if the first transmission attempt failed, thus increasing the local delay. In order to reduce the effect of interference correlation, we study two kinds of MAC randomness described as follows.

B. FHMA

In the FHMA case, we assume that the total frequency band \( W \) is divided into \( N \) sub-bands and each transmitter chooses a sub-band uniformly randomly, independently of the location and the time slot (i.e., memoryless both spatially and temporally). Let \( s \in S = \{ 1, 2, \cdots, N \} \) be the sub-band index, and let \( S_k(x) \in S \) denote the index of the sub-band used by node \( x \in \Phi \) in time slot \( k \). With these notations, the interference at the typical receiver located at the origin \( o \) in time slot \( k \) is given by

\[
I_k = \sum_{x \in \Phi \setminus \{x_0\}} h_{k,x} \kappa |x|^{-\alpha} \mathbf{1}(S_k(x) = S_k(x_0)),
\]
where $1(\cdot)$ is the indicator function and $|x|$ denotes the distance between $x$ and the origin $o$. Note that the exclusion of $x_0$ from the sum over the point process does not imply that $x_0 \notin \Phi$, but it ensures that when we condition on $x_0 \in \Phi$, the power received from this node is not counted as interference.

Besides reducing the interference and breaking the correlation, introducing FHMA has the additional benefit that the noise power decreases from $W \kappa N_0$ to $\frac{W \kappa N_0}{\kappa} N_0$. By taking this noise scaling into consideration, we obtain the SINR of the typical receiver in time slot $k$ as

$$\text{SINR}_k = \frac{h_{k,x_0} r_0^{-\alpha}}{W N_0 + \sum_{x \in \Phi \setminus \{x_0\}} h_{k,x} |x|^{-\alpha} 1(S_k(x) = S_k(x_0))}. \quad (2)$$

C. ALOHA

In the ALOHA case, let $\Phi_k$ be the transmitting set in time slot $k$. The interference at the typical receiver located at the origin $o$ in time slot $k$ is

$$I_k = \sum_{x \in \Phi \setminus \{x_0\}} h_{k,x} |x|^{-\alpha} 1(x \in \Phi_k). \quad (3)$$

Unlike FHMA, the noise scaling effect does not exist for ALOHA since the entire frequency band is used for each transmission. The SINR of the typical receiver in time slot $k$ is

$$\text{SINR}_k = \frac{h_{k,x_0} r_0^{-\alpha}}{W N_0 + \sum_{x \in \Phi \setminus \{x_0\}} h_{k,x} |x|^{-\alpha} 1(x \in \Phi_k)}. \quad (4)$$

III. MEAN AND VARIANCE OF THE LOCAL DELAY

In this section, we derive the mean and variance of the local delay for FHMA and for ALOHA respectively.

A. FHMA

1) Mean local delay: The following theorem gives the mean local delay in FHMA networks.

Theorem 1: In FHMA with $N$ sub-bands, the mean local delay is

$$D(N) = N \exp \left( \frac{A}{(N-1)1-\delta N^\delta} + \frac{B}{N} \right), \quad (5)$$

where $A = \lambda c_d r_0^d \delta^d C(\delta)$, $B = \delta r_0^\delta W N_0$, $\delta = d/\alpha$, $C(\delta) = 1/\text{sinc}(\delta)$, and $c_d = |b(o, 1)|$ is the volume of the $d$-dimensional unit ball.

Proof: Let $E_\Phi$ be the event that a transmission succeeds conditioned on the PPP $\Phi$. The probability for successful transmission given $\Phi$ is the same for each time slot. Our analysis below is conditioned on $\Phi$ having a point at $x_0$. This means that the probability measure of the point process is the Palm probability $P^{x_0}$ (see Ch. 8 in [13]). Correspondingly, the expectation, denoted by $E^{x_0}$, is taken with respect to the measure $P^{x_0}$. With this notation, by setting the SINR threshold to be $\theta$, we denote the probability of successful transmission

$$P^{x_0}(\epsilon_\Phi) = P^{x_0}(\text{SINR}_k > \theta | \Phi),$$

which can be evaluated as

$$P^{x_0}(\epsilon_\Phi) = P^{x_0}(\text{SINR}_k > \theta | \Phi)$$

$$= P^{x_0}(h_{k,x_0} r_0^{-\alpha} > \theta \frac{W N_0 + I_k}{N} | \Phi)$$

$$= E^{x_0}(\exp \left( -\theta r_0^\alpha \frac{W N_0 + I_k}{N} \right) | \Phi)$$

$$= E^{x_0}(\exp \left( -\theta r_0^\alpha \frac{W N_0}{N} \right. - \sum_{x \in \Phi \setminus \{x_0\}} \theta r_0^\alpha h_{k,x} |x|^{-\alpha} 1(S_k(x) = S_k(x_0)) | \Phi)$$

$$= \exp \left( -\frac{\theta r_0^\alpha W N_0}{N} \right)$$

$$\prod_{x \in \Phi \setminus \{x_0\}} E^{x_0}(\exp \left( -\theta r_0^\alpha h_{k,x} |x|^{-\alpha} 1(S_k(x) = S_k(x_0)) \right) | \Phi)$$

$$= \exp \left( -\frac{\theta r_0^\alpha W N_0}{N} \right) \prod_{x \in \Phi \setminus \{x_0\}} \left( \frac{1}{N} \right) \frac{1}{1 + \theta r_0^\alpha |x|^{-\alpha} + \frac{N-1}{N}}.$$  

(6)

In steps (a) and (b) of the derivation above, we have applied the property that the fading coefficients $h_{k,x}$ are i.i.d. random variables with exponential distribution of unit mean. The number of time slots needed until a successful time slot appears, denoted by $\Delta$, is a random variable called delay till success (DTS) [19]. Conditioned upon $\Phi$, the success events in different time slots are independent with probability $P^{x_0}(\epsilon_\Phi)$; therefore, the DTS with given $\Phi$, denoted by $\Delta_\Phi$, is a random variable with geometric distribution given by

$$P^{x_0}(\Delta_\Phi = k) = (1 - P^{x_0}(\epsilon_\Phi))^{k-1} P^{x_0}(\epsilon_\Phi). \quad (7)$$

The conditional expectation of $\Delta_\Phi$ is taken w.r.t. the fading and the MAC, given by $E^{x_0}(\Delta_\Phi) = 1/P^{x_0}(\epsilon_\Phi)$. Noticing that a packet will need $N$ successful time slots to finish transmission in FHMA, the mean local delay can be evaluated as

$$D(N) = N E^{x_0}(\Delta)$$

$$= N E^{x_0}(\epsilon_\Phi) E^{x_0}(\Delta_\Phi)$$

$$= N E^{x_0}(\epsilon_\Phi) \left( \frac{1}{P^{x_0}(\epsilon_\Phi)} \right)$$

$$= N \exp \left( -\frac{\theta r_0^\alpha W N_0}{N} \right)$$

$$\prod_{x \in \Phi \setminus \{x_0\}} \left( \frac{1}{N} \right) \frac{1}{1 + \theta r_0^\alpha |x|^{-\alpha} + \frac{N-1}{N}}.$$  

(8)
where (α) follows from [6]. By applying the probability generating functional (PGFL) of the PPP, we obtain

\[
D(N) = N \exp \left( \frac{\theta g_0^2 W N_0}{N} + \alpha \sum_{i=1}^{N} -\lambda \int_{x=1}^{\infty} \left( 1 - \frac{1}{\log(1 + \theta g_0^2 x^\alpha)} \right) dx \right)
\]

\[
= N \exp \left( \frac{\theta g_0^2 W N_0}{N} + \lambda c_d d \left( \frac{dr^{d-1}}{N_0^q r^\alpha + N - 1} \right) \right)
\]

\[
= N \exp \left( \frac{\lambda c_d d^\alpha C(\delta)}{(N - 1)^{1-\delta} N^\delta} + \theta g_0^2 W N_0 N \right),
\]

(9)

where \( \delta = d/\alpha \), \( C(\delta) \) is given by \( C(\delta) = \Gamma(1+\delta) \Gamma(1-\delta) = \frac{1}{\sin(\delta \pi)} \), and \( c_d = |b(\alpha, 1)| \) is the volume of the \( d \)-dimensional unit ball.

The result in Theorem 1 is closed-form and easy to evaluate and interpret. The value of \( A \) is determined by the interference and that of \( B \) is due to the thermal noise. From (5), we have

\[
D(N) = N \exp \left( A \left( 1 - \frac{1}{N} \right)^\delta + \frac{B}{N} \right)
\]

\[
= N \exp \left( A \left( 1 - \frac{\delta - 1}{N} + O \left( \frac{1}{N^2} \right) \right) + \frac{B}{N} \right)
\]

\[
= N + A + B + O \left( \frac{1}{N} \right).
\]

(10)

The result shows that when \( N \) is large, the mean local delay increases linearly with \( N \). Since \( D(1) \) is infinity, there exists an optimal number of sub-bands \( N_{\text{opt}} \) that minimizes the mean local delay. Inspecting \( D(N) \), we see that there are two effects by splitting the entire frequency band into \( N \) sub-bands: first, the mean local delay \( D(N) \) tends to decrease due to the reduced interference correlation; second, \( D(N) \) tends to increase since the number of time slots needed until a packet is successfully transmitted. In order to better understand the distribution of the local delay, we also derive its variance. The following theorem gives the variance of the local delay for FHMA.

**Theorem 2:** In FHMA with \( N \) sub-bands, the variance of the local delay is

\[
V(N) = N (N + 1) \exp \left( \frac{(2N - 1 - \delta) A}{N^3 (N - 1)^{2-\delta} + \frac{2B}{N}} \right) - D(N) - D^2(N).
\]

(11)

**Proof:** In order to transmit a packet in FHMA, \( N \) successful transmissions are needed. Letting \( \Delta_i \) (\( 1 \leq i \leq N \)) be the DTS of the \( i \)-th transmission, we get the local delay of a packet as \( \sum_{i=1}^{N} \Delta_i \). For \( 1 \leq i, j \leq N \) and \( i \neq j \), \( \Delta_i \) and \( \Delta_j \) are independent because the interference of the \( i \)-th transmission and that of the \( j \)-th transmission are correlated. However, if we condition on \( \Phi \), \{\( \Delta_i \)\} are i.i.d. random variables with geometric distribution given by (7). With these notations, we obtain the variance of the local delay as

\[
V(N) = \mathbb{E}[x_0] \left( \sum_{i=1}^{N} \Delta_i^2 \right) - \left( \mathbb{E}[x_0] \left( \sum_{i=1}^{N} \Delta_i \right) \right)^2
\]

\[
= \mathbb{E}[x_0] \left( \sum_{i=1}^{N} \Delta_i^2 + \sum_{i,j=1}^{N} 2\Delta_i \Delta_j - \sum_{i=1}^{N} \mathbb{E}[x_0] (\Delta_i) \right)^2
\]

\[
= \sum_{i=1}^{N} \mathbb{E}[x_0] (\Delta_i^2) + \sum_{i,j=1}^{N} 2\mathbb{E}[x_0] (\Delta_i \Delta_j) - D^2(N),
\]

(10)

Based on Theorem 1, we show how the normalized mean local delay \( \frac{D(N)}{\log_2(1 + \theta)} \) varies with \( N \) numerically. As for the parameters, we ignore the thermal noise \( (N_0 = 0) \) and set the intensity of transmitters as \( \lambda = 0.01 \text{m}^{-2} \) by default, which means that the coverage area of each transmitter is 100m² on average, reasonable for a typical deployment of WLAN. The path loss exponent is set as \( \alpha = 4 \) by default, and the distance between the receiver and the typical desired transmitter is \( r_0 = 5 \text{m} \). Let \( \theta \) be the outage threshold for SINR. The relationship between \( \frac{D(N)}{\log_2(1 + \theta)} \) and \( N \) is depicted in Fig. 2.

By changing the values of \( \alpha \) and \( \lambda \) respectively, we get the curves in Fig. 2. Comparing the curves in Fig. 2(a) with those in Fig. 2(b) and Fig. 2(c), we observe that the optimal number of sub-bands increases when \( \alpha \) decreases or when \( \lambda \) increases. This observation is consistent with the intuition: Smaller \( \alpha \) implies that the signal strength decays more slowly with distance, and larger \( \lambda \) implies that more transmitters exist in the same region, so in both cases more interference is created. Therefore, the entire frequency band should be divided into more sub-bands, namely larger \( N_{\text{opt}} \), to reduce the interference and interference correlation.

2) Variance of the local delay: The mean local delay discussed above has characterized the mean number of time slots needed until a packet is successfully transmitted. In order to better understand the distribution of the local delay, we also derive its variance. The following theorem gives the variance of the local delay for FHMA.

**Theorem 2:** In FHMA with \( N \) sub-bands, the variance of the local delay is

\[
V(N) = N (N + 1) \exp \left( \frac{(2N - 1 - \delta) A}{N^3 (N - 1)^{2-\delta} + \frac{2B}{N}} \right) - D(N) - D^2(N).
\]

(11)
Fig. 2. The normalized mean local delay $D(N)/\log_{2}(1+\theta)$ as a function of the number of sub-bands $N$, when $d = 2$, $r = 5m$, and thermal noise ignored.

where (a) follows from the definition of the mean local delay. By applying the total expectation formula, we have

$$V(N) = \sum_{i=1}^{N} E_{X_{i}}^{x_{i}} (\sum_{i=1}^{N} E_{X_{i}}^{x_{i}} (\Delta_{i}^{2} | \Phi))$$

$$+ \sum_{i=1}^{N} 2E_{X_{i}}^{x_{i}} (\sum_{i=1}^{N} E_{X_{i}}^{x_{i}} (\Delta_{i} \Delta_{j} | \Phi)) - D^{2}(N)$$

$$(b) = N E_{\Phi}^{x_{i}} \frac{2 - P_{x_{i}}(x_{0})}{(P_{x_{i}}(x_{0}))^2}$$

$$+ N (N-1) E_{\Phi}^{x_{i}} \frac{1}{(P_{x_{i}}(x_{0}))^2} - D^{2}(N)$$

$$= N (N + 1) E_{\Phi}^{x_{i}} \frac{1}{(P_{x_{i}}(x_{0}))^2}$$

$$- N E_{\Phi}^{x_{i}} \frac{1}{(P_{x_{i}}(x_{0}))^2} - D^{2}(N)$$

$$= N (N + 1) E_{\Phi}^{x_{i}} \frac{1}{(P_{x_{i}}(x_{0}))^2} - D(N) - D^{2}(N),$$

where (b) follows from the second moment of the geometrically distributed random variable.

From (6), we have

$$E_{\Phi}^{x_{i}} \frac{1}{(P_{x_{i}}(x_{0}))^2} = \exp \left( \frac{2\theta r_0^a W N_0}{N} \right)$$

$$\prod_{x \in \Phi \setminus \{x_0\}} \left( \frac{1}{N+\theta r_0^a |x| - \alpha \theta r_0^a} + \frac{N-1}{N} \right)$$

$$= \exp \left( \frac{2\theta r_0^a W N_0}{N} \right)$$

$$- \lambda \int_{\mathbb{R}^d} \left( 1 - \frac{1}{N+\theta r_0^a |x| - \alpha \theta r_0^a} + \frac{N-1}{N} \right) dx$$

$$= \exp \left( \frac{2\theta r_0^a W N_0}{N} \right)$$

$$- \lambda c d \int_{0}^{\infty} \left( 1 - \frac{\lambda N(1+\theta r_0^a r^{-\alpha})^2}{(N+(N-1)\theta r_0^a r^{-\alpha})^2} \right) r^{d-1} dr$$

$$= \exp \left( \frac{2\theta r_0^a W N_0}{N} \right) + \frac{\lambda c d \theta^\delta C(\delta)(2N - 1 - \delta)}{N^{\delta(N-1)^{2-\delta}}},$$

where (c) follows by applying the PGFL of the PPP. Plugging (13) into (12), we get the variance of the local delay as in Theorem 2.

B. ALOHA

The fundamental difference between FHMA and ALOHA is that if a packet is to be transmitted during a time slot, in FHMA the packet will be surely transmitted by randomly choosing a sub-band, while in ALOHA the packet will only be transmitted with a given probability. Similar to the analysis
of FHMA, we also assume that a packet needs exactly one time slot if it is allocated the entire frequency band \(W\) under SINR threshold \(\theta\) and successfully transmitted in that time slot. We assume that each node transmits with probability \(p\) in each time slot and if it transmits, it will make use of the entire frequency band. In that way, only one successful time slot is needed to transmit a packet, and the local delay is the DTS of one transmission, denoted by \(\Delta\).

1) Mean local delay: The following theorem gives the mean local delay for ALOHA.

**Theorem 3:** In ALOHA with transmit probability \(p\), the mean local delay is

\[
\bar{D}(p) = \frac{1}{p} \exp \left( \frac{pA}{(1-p)^{1-\alpha}} + B \right). \tag{14}
\]

**Proof:** In each time slot, a packet will be transmitted with probability \(p\) and the transmission will be successful with probability \(\mathbb{P}^x_0(SINR_k \geq \theta | \Phi)\) conditioned upon \(\Phi\). Therefore, similar to the derivation of (9), the probability for successfully transmitting a packet conditioned upon \(\Phi\) in a time slot is

\[
\mathbb{P}^x_0(\{\Phi\}) \overset{(a)}{=} \mathbb{P}^x_0(\text{SINR}_k \geq \theta | \Phi) = p\mathbb{P}^x_0(h_{k,x}r_\alpha^0 > \theta (WN_0 + I_k) | \Phi) \overset{(b)}{=} p\mathbb{P}^x_0(\exp(-\theta r_\alpha^0 (WN_0 + I_k)) | \Phi) = p\mathbb{P}^x_0(\exp(-\theta r_\alpha^0 WN_0) - \sum_{x \in \Phi \setminus \{x_0\}} \theta r_\alpha^0 h_{k,x} | x |^{-\alpha} 1(x \in \Phi_k)) | \Phi) = p\exp(-\theta r_\alpha^0 WN_0) \prod_{x \in \Phi \setminus \{x_0\}} \mathbb{P}^x_0(\exp(-\theta r_\alpha^0 h_{k,x} | x |^{-\alpha} 1(x \in \Phi_k)) | \Phi) = p\exp(-\theta r_\alpha^0 WN_0) \prod_{x \in \Phi \setminus \{x_0\}} p\mathbb{P}^x_0(\exp(-\theta r_\alpha^0 h_{k,x} | x |^{-\alpha}) | \Phi) + 1 - p) = p\exp(-\theta r_\alpha^0 WN_0) \prod_{x \in \Phi \setminus \{x_0\}} \left(\frac{p}{1 + \theta r_\alpha^0 | x |^{-\alpha}} + 1 - p\right). \tag{15}
\]

where (a) is because a transmission occurs with probability \(p\), and (b) and (c) follows because the fading coefficients \(h_{k,x}\) are i.i.d. random variables with exponential distribution of unit mean. Then, the mean local delay for ALOHA is given by

\[
\bar{D}(p) = \mathbb{E}^x_\Phi \left(\frac{1}{\mathbb{P}^x_0(\{\Phi\})}\right) = \frac{1}{p} \exp \left( \theta r_\alpha^0 WN_0 \right) - \frac{1}{\lambda} \int_{\mathbb{R}^d} \left(1 - \frac{p}{1 + \theta r_\alpha^0 | x |^{-\alpha}} + 1 - p\right) dx = \frac{1}{p} \exp \left( \frac{\lambda c_\theta r_\alpha^0 C(\delta) (2 - p - \delta p)p}{(1-p)^{1-\delta}} + \theta r_\alpha^0 WN_0 \right). \tag{16}
\]

Applying the definition of \(A\) and \(B\) in Theorem 3 we obtain the result in Theorem 4.

2) Variance of the local delay: The variance of the local delay in ALOHA is given by the following theorem.

**Theorem 4:** In ALOHA with transmit probability \(p\), the variance of the local delay is

\[
\bar{V}(p) = \frac{2}{p^2} \exp \left( \frac{(2 - p - \delta p)pA}{(1-p)^{2-\delta}} + 2B \right) - \bar{D}(p) - \bar{D}^2(p). \tag{17}
\]

**Proof:** In the ALOHA case, in order to transmit a packet, one successful transmission is needed. The variance of local delay for ALOHA is thus

\[
\bar{V}(p) = \mathbb{E}^x_\Phi \left(\frac{1}{\mathbb{P}^x_0(\{\Phi\})}\right)^2 = p^{-2} \exp \left(2\theta r_\alpha^0 WN_0\right) - \left(\frac{1}{\mathbb{P}^x_0(\{\Phi\})}\right)^{-2} - \bar{D}(p) - \bar{D}^2(p), \tag{18}
\]

where (a) follows from the total expectation formula, and (b) follows from the second moment of the geometrically distributed random variable. From (15), we have

\[
\mathbb{E}^x_\Phi \left(\frac{1}{\mathbb{P}^x_0(\{\Phi\})}\right)^2 = p^{-2} \exp \left(\frac{1}{\mathbb{P}^x_0(\{\Phi\})}\right)^2 \left(\frac{1}{\mathbb{P}^x_0(\{\Phi\})}\right)^{-2} - \mathbb{E}^x_\Phi \left(\frac{1}{\mathbb{P}^x_0(\{\Phi\})}\right)^2 - \bar{D}(p) - \bar{D}^2(p).
\]

where (c) follows from the probability generating functional (PGFL) of the PPP. Plugging (19) into (18) and applying the definition of \(A\) and \(B\) in Theorem 3 we get the variance of the local delay in Theorem 4.

C. Comparison Between FHMA and ALOHA

1) Mean local delay: The mean local delay in FHMA is given by \(D(N)\) in (5), and that in ALOHA is given by \(\bar{D}(p)\) in (14). In ALOHA, if the transmit probability \(p\) is set as \(\frac{1}{N}\), by comparing \(\bar{D}(\frac{1}{N})\) to the result of FHMA, \(D(N)\) given in (5), we observe that the only difference lies in the thermal noise term. In FHMA, the entire frequency band is divided into a number of sub-bands, thus reducing the noise power. However, with ALOHA, the noise scaling effect does not exist since the entire frequency band is used for transmission. The mean local delays of the two schemes are the same if we ignore the thermal noise term and set \(p = \frac{1}{N}\).
2) Variance of the local delay: Comparing (11) and (17), we observe that even if the noise is ignored and the transmit probability is set as \( p = \frac{1}{N} \) in ALOHA, there is still an significant difference between the variances of the two schemes when \( N > 1 \): in FHMA, a factor \( N(N+1) \) exists in the first term, while in ALOHA the factor is \( 2N^2 \). When \( N > 1 \), we have \( V(N) < V(\frac{1}{N}) \) and this illustrates that the variance of the local delay for FHMA is less than that for ALOHA (see Fig. 3). We further observe from Fig. 3 that when \( N \to \infty \), the variance for FHMA stabilizes at a typically small value, while for ALOHA, the variance increases quickly with \( N \). To understand the limiting characteristics quantitatively, we evaluate how the variances of the local delay scale with \( N \) in the following proposition.

**Proposition 1:** For FHMA with number of sub-bands \( N > 1 \), the variance of the local delay is

\[
V(N) = (2 - \delta)A + B + O\left(\frac{1}{N}\right) = \Theta(1).
\]

For ALOHA with transmit probability \( p = \frac{1}{N} < 1 \), the variance of the local delay is

\[
\tilde{V}(\frac{1}{N}) = \Theta(N^2).
\]

**Proof:** For FHMA with number of sub-bands \( N > 1 \), from (5), we have

\[
D(N) = N \exp \left( \frac{A + B}{N} + \frac{(1 - \delta)A}{N^2} + O\left(\frac{1}{N^3}\right) \right) = N + (A + B) + \frac{(A + B)^2}{2} + (1 - \delta)A + O\left(\frac{1}{N}\right).
\]

Then, \( D^2(N) \) is given by

\[
D^2(N) = N^2 + 2(A + B)N + 2(A + B)^2 + 2(1 - \delta)A + O\left(\frac{1}{N}\right).
\]

From (11), we have the variance of the local delay as

\[
V(N) = N(N + 1) \exp \left( \frac{2B}{N} + \frac{2A}{N} - \frac{3(\delta - 1)A}{N^2} \right) - D(N) - D^2(N) = N^2 + (2A + 2B + 1)N + 2(A + B) - 3(\delta - 1)A + O\left(\frac{1}{N}\right).
\]

Plugging (22) and (23) into (24), we get the result in (20). The derivations of the limiting for ALOHA is similar, and we omit the details of that proof.

**D. Finiteness of the Mean Local Delay**

To understand why the mean local delay goes to infinity when \( N \) is set to one, let us consider the expression for the mean local delay \( D = N \varphi_{\Phi} \left( \frac{1}{\varphi_{\Phi}(\xi_{0})} \right) \) in FHMA, where \( \varphi_{\Phi}(\xi_{0}) \) is a random variable with support set \((1, \infty)\) because it is the reciprocal of the successful transmit probability conditioned upon the PPP \( \Phi \). When \( N = 1 \), the expectation \( \mathbb{E}_{\Phi} \left( \frac{1}{\varphi_{\Phi}(\xi_{0})} \right) \) is infinity because the ccdf of \( \frac{1}{\varphi_{\Phi}(\xi_{0})} \) has a heavy tail. To show the heavy tail behavior, let us derive a lower bound for the ccdf of the local delay when \( N = 1 \). Ignoring the thermal noise term in (6), for \( N = 1 \) and any \( t \in (1, \infty) \), we have

\[
\mathbb{P}_{t} \left( \frac{1}{\varphi_{\Phi}(\xi_{0})} > t \right) = \mathbb{P}_{t} \left( \prod_{x \in \Phi \setminus \{x_{0}\}} (1 + \theta_{r_{0}}^{|x| - \alpha}) > t \right) > \mathbb{P}_{t} \left( 1 + \theta_{r_{0}}^{|x_{min}| - \alpha} > t \right) > \mathbb{P}_{t} \left( \theta_{r_{0}}^{|x_{min}| - \alpha} > t \right) = \mathbb{P}_{t} \left( |x_{min}| < \theta_{r_{0}}^{\frac{1}{\alpha}} t^{-\frac{1}{\alpha}} r_{0} \right),
\]

where \( x_{min} = \arg\min_{x \in \Phi \setminus \{x_{0}\}} |x| \) is the nearest interfering transmitter to the receiver. The distance between the receiver and its nearest interfering transmitter has cumulative distribution function (cdf) as \( b(r) = 1 - \exp(-c_{d} \lambda r^{d}) \). Substituting this cdf into (25) and letting \( \delta = \frac{\alpha}{\alpha} \), we get

\[
\mathbb{P}_{t} \left( \frac{1}{\varphi_{\Phi}(\xi_{0})} > t \right) > 1 - \exp \left( -c_{d} \lambda t^{-\delta} r_{0}^{d} \right) \sim \frac{c_{d} \lambda t^{-\delta} r_{0}^{d}}{t^{\delta}}, \quad t \to \infty.
\]

The function \( g(t) = 1 - \exp \left( -C_{0} t^{-\delta} \right) \), where \( C_{0} = c_{d} \lambda t^{-\delta} r_{0}^{d} \), gives a lower bound for the ccdf of the local delay when \( N = 1 \) (see Fig. 4).

By the identity of \( \mathbb{E}(X) = \int_{0}^{\infty} \mathbb{P}(X > t)dt \) for any non-negative random variable \( X \) and the inequality \( e^{-x} < 1 - x + \)
for \( x > 0 \), we have
\[
\frac{x^2}{2} \int_0^x f(x) \, dx = \frac{x^2}{2} \left( \int_0^x f(x) \, dx \right) = \frac{x^2}{2} \left( \frac{x^2}{2} \right) = \frac{x^2}{4}.
\]

In the discussion above, we have considered the unbounded path loss function \( l(r) = \kappa r^{-\alpha} \). Though the unbounded path loss function is an idealized model, it gives an effective approximation to the actual path loss and results in concise results \([22, 23]\).

In this subsection, we compare the results derived under the unbounded path loss function \( l(r) = \kappa r^{-\alpha} \) to that under the bounded path loss function \( l(r) = \kappa r^{-\alpha} + \varepsilon \), where \( \varepsilon > 0 \). The bounded path loss function is the limiting case of the bounded path loss function as \( \varepsilon \to 0 \).

Without loss of generality, we take FHMA as an example. By replacing \( l(r) = \kappa r^{-\alpha} \) with \( l(r) = \kappa r^{-\alpha} + \varepsilon \) in the derivations of Theorem 1 and Theorem 2, we obtain the mean and variance of the local delay under the assumption of bounded path loss function as follows.

\[
D_\varepsilon(N) = N \exp \left( \frac{(r_0^\alpha + \varepsilon) \theta W N_0}{N} \right) + \frac{\lambda c_\varepsilon(r_0^\alpha + \varepsilon) \theta C(\delta)}{(N\varepsilon + (N-1)\theta (r_0^\alpha + \varepsilon))^{1-\delta} N^\delta}
\]

\[
V_\varepsilon(N) = N (N+1) \exp \left( \frac{2\theta r_0^\alpha W N_0}{N} \right) + \frac{(2N\varepsilon + (2N - 1 - \delta)\theta (r_0^\alpha + \varepsilon)) \lambda c_\varepsilon (r_0^\alpha + \varepsilon) \theta C(\delta)}{(N\varepsilon + (N-1)\theta (r_0^\alpha + \varepsilon))^{2-\delta}}
\]

\[
- D_\varepsilon(N) - D_\varepsilon^2(N).
\]

When \( N = 1 \), both \( D_\varepsilon(1) \) and \( V_\varepsilon(1) \) are finite if \( \varepsilon > 0 \). Setting \( \varepsilon = 0 \) reproduces the results for the unbounded model, as expected. As can be seen, the difference between the results for the unbounded model and the bounded one decreases with increasing \( r_0 \) or decreasing \( \theta \). In order to evaluate the difference, we set \( \varepsilon \) as the typical value \( \kappa \) (i.e., the path loss becomes \( l(r) = \kappa (r^\alpha + \kappa)^{-1} \) such that the received power never exceeds the transmitted one without fading. The value of \( \kappa \) is the path loss at 1m TX-RX separation, which is rather small, typically like -30dB \([24, \text{Ch. 3}]\). Therefore, as \( \varepsilon = \kappa \to 0 \), the mean local delay for \( N = 1 \) is approximated as
\[
D_\varepsilon(1) \sim \exp \left( \theta r_0^\alpha W N_0 + \frac{\lambda c_\varepsilon r_0^\alpha \theta C(\delta)}{\varepsilon^{1-\delta}} \right), \quad \varepsilon \to 0.
\]

It is observed that \( D_\varepsilon(1) \) increases exponentially with respect to \( 1/\varepsilon^{1-\delta} \) as \( \varepsilon \to 0 \). For the realistic bounded path loss model, in which \( \varepsilon \) is rather small, the mean local delay when \( N = 1 \) is finite though extremely large. Thus, we can conclude that for the realistic bounded path loss model, when \( N > 2 \) the boundedness of the path loss has only negligible effect on the mean and the variance of the local delay; when \( N = 1 \) the mean local delay is extremely large and thus can be considered as infinity for practical purposes.

### IV. Optimal Parameters To Minimize Mean Local Delay

In this section, we analyze the optimal number of sub-bands in FHMA and optimal transmit probability in ALOHA to minimize the mean local delay. Deriving the optimal parameters is difficult, and the results may not be compact; thus, we resort to deriving tight bounds for the optimal values.

#### A. FHMA

In the derivation, we relax \( N \) to be continuous and subsequently take the actual optimal number to be a nearby integer. The following theorem gives the bounds of the optimal number of sub-bands.

**Theorem 5:** The bounds of the optimal number of sub-bands that minimizes the mean local delay are given by
\[
N_{\text{opt}} \in \left[ \lfloor t_0 \rfloor, \lfloor t_0 \rfloor + 2 \right], \quad t_0 = \frac{\lambda c_\varepsilon r_0^\alpha \theta^4 C(\delta) + \theta r_0^\alpha W N_0}{\varepsilon^{1-\delta}}.
\]

Fig. 4. Lower bound for the ccdf of the local delay, given by (26), when \( N = 1 \) in the 2-dimensional case (\( d = 2 \)). The intensity of transmitters is \( \lambda = 0.01 \text{m}^{-2} \) and the path loss exponent is \( \alpha = 4 \).
Proof: Based on the result of (5), we get the derivative of the mean local delay $D'(N)$ when $N > 1$ as follows

$$D'(N) = f(N) \exp \left( \lambda c_0 \delta \theta^C(\delta)(N-1)^{\delta-1}N^{-\delta} + \theta r_0^\alpha W N_0 - 1 \right)$$

where

$$f(N) = 1 - \frac{1}{N} \left( \lambda c_0 \delta \theta^C(\delta) \left( \frac{N}{N-1} \right)^{1-\delta} \frac{N-\delta}{N-1} + \theta r_0^\alpha W N_0 \right).$$

(33)

We observe that $f(N)$ is strictly monotonically increasing in $N$; this means that there is only one optimal value $N_{\text{opt}}$ that satisfies $D'(N_{\text{opt}}) = 0$, which is given by $f(N_{\text{opt}}) = 0$. From $f(N_{\text{opt}}) = 0$, we get

$$N_{\text{opt}} = \lambda c_0 \delta \theta^C(\delta) \left( \frac{N_{\text{opt}}}{N_{\text{opt}} - 1} \right)^{2-\delta} \frac{N_{\text{opt}} - \delta}{N_{\text{opt}}} + \theta r_0^\alpha W N_0. \quad (35)$$

Since $N_{\text{opt}}/(N_{\text{opt}} - 1) > 1$ and $0 < \delta < 1$, we have

$$N_{\text{opt}} > \lambda c_0 \delta \theta^C(\delta) \left( \frac{N_{\text{opt}}}{N_{\text{opt}} - 1} \right)^{2-\delta} \frac{N_{\text{opt}} - 1}{N_{\text{opt}}} + \theta r_0^\alpha W N_0$$

$$> \lambda c_0 \delta \theta^C(\delta) + \theta r_0^\alpha W N_0.$$ 

(36)

This gives a lower bound for the optimal value $N_{\text{opt}}$. Next we derive an upper bound for $N_{\text{opt}}$. From (35), we have

$$N_{\text{opt}} < \lambda c_0 \delta \theta^C(\delta) \left( \frac{N_{\text{opt}}}{N_{\text{opt}} - 1} \right)^{2} + \theta r_0^\alpha W N_0,$$

$$< (\lambda c_0 \delta \theta^C(\delta) + \theta r_0^\alpha W N_0) \left( \frac{N_{\text{opt}}}{N_{\text{opt}} - 1} \right)^{2}.$$ 

(37)

Then, we have

$$\frac{(N_{\text{opt}} - 1)^2}{N_{\text{opt}}} < \lambda c_0 \delta \theta^C(\delta) + \theta r_0^\alpha W N_0.$$

$$N_{\text{opt}} - 2 + \frac{1}{N_{\text{opt}}} < \lambda c_0 \delta \theta^C(\delta) + \theta r_0^\alpha W N_0.$$ 

Combining (36) and (37) and noting that $N$ is an integer, we get the bounds of $N_{\text{opt}}$. The bounds given here are rather simple and tight. If frequency splitting is not applied (the case of $N = 1$), the mean local delay will surely be infinite. To guarantee a finite mean local delay, the value of $N$ should be at least two. It is valuable to investigate for which range of parameters the optimal value $N_{\text{opt}}$ will be two. The following corollary gives such a condition.

**Corollary 1:** If the intensity of transmitters $\lambda$ satisfies the following inequality,

$$\lambda < \frac{\ln \frac{3}{2} - \frac{1}{b} \theta r_0^\alpha W N_0}{c_4 \theta^C(\delta)(2^{-\delta} - 2^{\delta-13-\delta})},$$

(38)

the optimal number of sub-bands $N_{\text{opt}}$ that minimizes the mean local delay will be two.

Proof: Since we have proved that mean local delay $D(N)$ is a function that first decreases and then increases with $N$, the condition for $N_{\text{opt}} = 2$ is $D(2) < D(3)$. By substituting the expression of mean local delay (5) into $D(2) < D(3)$, we get the condition in the corollary. Notice that the right side of the inequality in (38) may be negative, in which case the condition for $\lambda$ cannot be satisfied and then the optimal number of sub-bands will never be two.

In Fig. 5, we plot the optimal number of sub-bands $N_{\text{opt}}$ and its bounds given in (32) as a function of the path loss exponent for different $\theta$. The optimal number $N_{\text{opt}}$ is obtained by numerical calculation of the solution of the equation (35). We observe from Fig. 5 that the bounds are quite tight and give excellent approximation of the value $N_{\text{opt}}$. This figure also shows that the optimal value $N_{\text{opt}}$ decreases with increasing path loss exponent, which verifies our aforementioned discussion regarding Fig. 2. The curves show that when the path loss exponent is fixed, the optimal number $N_{\text{opt}}$ is an increasing function of the SINR threshold $\theta$, which can be also perceived from the expression (32). This is reasonable since with larger SINR threshold, the condition for successful transmission becomes harsher, and more sub-bands are needed to meet the stronger requirements. In Fig. 6, we plot the minimum value of the normalized mean local delay when the optimum number of sub-bands $N_{\text{opt}}$ is used. We observe from Fig. 6 that there are intersection points between different curves, implying that the choice of SINR threshold $\theta$ has direct impact on the mean local delay. In Section V, we will try to obtain the optimal SINR threshold.

**Fig. 5.** Optimal number of sub-bands $N_{\text{opt}}$ and its bounds $(t_0, t_0 + 2)$ as a function of the path loss exponent for varying $\alpha$.

**B. ALOHA**

Since the expressions of the mean local delays for ALOHA and for FHMA are the same when $p = 1/N$ and thermal
noise ignored, we give the optimal transmit probability in the following theorem directly and omit the proof.

**Theorem 6**: The bounds of the optimal transmit probability which minimizes the mean local delay is

\[
p_{\text{opt}} \in \left( \frac{1}{\lambda c_d r_0^2 \theta^2 C(\delta)} + 2 \frac{1}{\lambda c_d r_0^2 \theta^2 C(\delta)} \right).
\]

(39)

**V. OPTIMAL SINR THRESHOLD \( \theta \)**

In the discussion above, we have already derived tight bounds for the optimal number of sub-bands \( N_{\text{opt}} \) and the optimal transmit probability \( p_{\text{opt}} \) to minimize the mean local delay when the SINR threshold \( \theta \) is fixed. The following analysis will focus on deriving the optimal threshold \( \theta_{\text{opt}} \) or its bounds when the number of sub-bands \( N \) or the transmit probability \( p \) is fixed. However, as mentioned in Section II, the duration of each time slot is proportional to \( \frac{1}{\log_2(1+\theta)} \). In order to characterize the actual delay, we slightly modify the optimization objective as the normalized mean local delay, i.e., \( \frac{D(N)}{\log_2(1+\theta)} \) for FHMA and \( \frac{D(p)}{\log_2(1+\theta)} \) for ALOHA. In the following analysis, we consider two asymptotic regimes: the interference-limited regime and the noise-limited regime. The interference-limited regime is typically encountered in cellular radio systems like CDMA networks, where the interference dominates over the thermal noise. The noise-limited regime is appropriate if the distance between concurrent transmitters is much larger than the distance of the typical link, in which case the interference in the network is negligible.

### A. FHMA

The following theorem gives the optimal threshold \( \theta_{\text{opt}} \) and its bounds for FHMA in the interference-limited regime and noise-limited regime respectively.

**Theorem 7**: In the noise-limited regime, the optimal threshold \( \theta_{\text{opt}} \) that minimizes the normalized mean local delay

\[
\frac{D(N)}{\log_2(1+\theta)}
\]

for FHMA is given by

\[
\theta_{\text{opt}} = \exp \left( W \left( \frac{N}{r_0^2 W N_0} \right) - 1 \right),
\]

(40)

where \( W(z) \) is the Lambert \( W \) function which solves \( W(z)e^{W(z)} = z \). In the interference-limited regime, the bounds of \( \theta_{\text{opt}} \) are given by

\[
\theta_{\text{opt}} \in \left( b_0^{-1/(\delta+1)} - 1, b_0^{-1/\delta} \right),
\]

\[
b_0 = \lambda c_d r_0^2 \delta C(\delta)(N - 1)^{\delta-1} N^{-\delta}.
\]

(41)

**Proof**: The derivative of the normalized mean local delay with respect to \( \theta \) is

\[
\frac{\partial}{\partial \theta} \left( \frac{D(N)}{\log_2(1+\theta)} \right) = \frac{h(\theta) \theta^\delta - 1 N}{\log_2(1+\theta)} + \lambda c_d r_0^2 \theta^2 C(\delta)(N - 1)^{\delta-1} N^{-\delta}.
\]

(42)

where

\[
h(\theta) = \lambda c_d r_0^2 \delta C(\delta)(N - 1)^{\delta-1} N^{-\delta}
\]

(46)

Next, we prove that \( h(\theta) \) is a strictly increasing function of \( \theta \), then we show that the equation \( h(\theta) = 0 \) has a unique solution. Let \( l(\theta) = \theta^{\delta-1} (1 + \theta) \ln(1 + \theta) \) and the derivative of \( l(\theta) \) is as follows

\[
l'(\theta) = ((\delta - 1) \theta^{\delta-2} + \delta \theta^{\delta-1}) \ln(1 + \theta) + \theta^{\delta-1}
\]

\[
> ((\delta - 1) \theta^{\delta-2} + \delta \theta^{\delta-1}) \theta + \theta^{\delta-1}
\]

\[
> 0.
\]

Thus, \( l(\theta) \) is a strictly increasing function of \( \theta \). This implies that \( h(\theta) \) is also a strictly increasing function of \( \theta \). Since \( \lim_{\theta \to 0^+} h(\theta) = -\infty \) and \( \lim_{\theta \to \infty} h(\theta) = +\infty \), the equation \( h(\theta) = 0 \) has a unique solution \( \theta_{\text{opt}} \) that minimizes the mean local delay.

In the noise-limited regime, the equation \( h(\theta) = 0 \) has the form

\[
r_0^2 W N_0 N^{-1} \theta^{1-\delta} - 1 \frac{1}{\theta^{\delta-1}(1 + \theta) \ln(1 + \theta)} = 0.
\]

(43)

Solving this equation we obtain (40).

In the interference-limited regime, the noise is ignored, and \( h(\theta) = 0 \) has the form

\[
\lambda c_d r_0^2 \delta C(\delta)(N - 1)^{\delta-1} N^{-\delta}
\]

\[
- \frac{1}{\theta^{\delta-1}(1 + \theta) \ln(1 + \theta)} = 0.
\]

(45)

A closed-form solution for the above equation does not exist.

By applying the inequalities \( \frac{\theta}{1+\theta} < \ln(1+\theta) < \theta \) to (45), we get the following inequalities

\[
\frac{1}{(1+\theta)^{\delta+1}} < \frac{1}{\theta^\delta (1+\theta)}
\]

\[
< \lambda c_d r_0^2 \delta C(\delta)(N - 1)^{\delta-1} N^{-\delta} < \frac{1}{\theta^\delta}.
\]

(46)

From the above inequalities, we get the bounds in (41).
B. ALOHA

Based on the similarity between the expressions of the normalized mean local delays for ALOHA and for FHMA, we obtain the optimal threshold for ALOHA directly.

\textbf{Theorem 8}: In noise-limited regime, the optimal threshold $\theta_{\text{opt}}$ that minimizes the normalized mean local delay for ALOHA is as follows

$$\theta_{\text{opt}} = W \left( \exp \left( \frac{1}{r_0 W N_0} \right) \right) - 1.$$  \hspace{1cm} (47)

In interference-limited regime, the bounds of $\theta_{\text{opt}}$ are given by

$$\theta_{\text{opt}} \in \left( b_0 - \frac{1}{\delta + 1}, b_0 \right],$$  \hspace{1cm} (48)

where $b_0 = \lambda c d r_0^\delta C(\delta)p(1-p)^{\delta-1}$.

VI. DESIGN INSIGHTS

A. Mean Delay-Jitter Tradeoff

The jitter of delay, typically characterized by the packet delay variation, is defined in [25] and [26]. In the system design, the delay variation is an important measure that characterizes the fluctuation of delay [27]. For interactive real-time applications, e.g., VoIP, large delay variance can be a serious issue. To the best of our knowledge, the variance of local delay has not been explored in the existing work. The optimal value of $N$ that minimizes the mean local delay is often not the one that minimizes the variance; thus there is a tradeoff between the mean and the variance of the local delay. Fig. 7(a) and Fig. 7(b) visualize the relationship between the mean and the variance of the normalized local delay for FHMA and for ALOHA respectively. From Fig. 7(a) we observe that in FHMA the favorable operating point has a reasonably wide tuning range because the variance stabilizes fast as $N$ increases. In contrast, we observe from Fig. 7(b) that in ALOHA the curves turn sharply.

B. Tail Probability of the Local Delay

The tail probability is an important measure of the system performance since one may require (as a QoS constraint) that the probability that the local delay exceeds a certain threshold is less than a predefined value. Based on the mean and variance we have derived and by applying the one-tailed Chebyshev’s inequality, we obtain an upper bound for the tail probability. For example, in FHMA with $N > 1$, letting $X = \sum_{i=1}^{N} \Delta_i$ be the local delay, the tail probability is upper bounded as follows

$$P\{X > T_0\} \leq \frac{V(N)}{V(N) + (T_0 - D(N))^2}, \quad \text{for} \quad T_0 > D(N).$$  \hspace{1cm} (49)

For example, if we let the design requirement be that the probability that the local delay exceeds 10 is less than 5%. Then, when the threshold of the local delay is fixed as $T_0 = 10$, the upper bound of the tail probability given by (49) with varying $N$ is shown in Fig. 8. We observe that in order to achieve the probability 5%, the number of sub-bands $N$ for the case when $\theta = 1, 10, 100$ should be chosen larger than 15, 50, 95 respectively.

VII. CONCLUSIONS

In this work, we studied the problem of reducing the effect of interference correlation by introducing MAC dynamics. We
derived the mean and variance of the local delay and evaluated how the interference correlation can be reduced by FHMA and ALOHA. We also evaluated the optimal number of sub-bands in FHMA and the optimal transmit probability in ALOHA that minimize the mean local delay.

The results reveal that there exist two operation regimes for the network, the correlation-limited regime and bandwidth-limited regime, which are separated by the optimal number of sub-bands in FHMA and the optimal transmit probability in ALOHA. If no MAC dynamics is employed, the local delay has a heavy tail distribution which results in infinite mean local delay; meanwhile, employing FHMA and ALOHA will greatly decrease the mean local delay. By comparing the results of FHMA and ALOHA, we observed that while the mean local delays of the two protocols are the same for certain parameters, the variances are rather different. According to the results established herein, FHMA outperforms ALOHA if implementation costs like overhead are not taken into consideration; however, when considering the implementation costs, the overhead for FHMA may be much higher than ALOHA because in FHMA each transmitter should inform the corresponding receiver which sub-band to listen on.

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