SPACE TIME AS A RANDOM HEAP

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Abstract

In this paper, we demonstrate how space-time is, rather than a differentiable manifold, a Random Heap, and how this ties up with fractal dimension 2 of a Quantum Mechanical path. In this light, we can see that there is a harmonious convergence between the stochastic approach of Nelson and the de Broglie-Bohm approach. These considerations are shown to lead to the emergence of special relativity and Quantum Mechanics.

1 Introduction

Space time has generally been taken to be a differential manifold with an Euclidean (Galilean) or Minkowskian or Riemannian character. Though the Heisenberg Principle in Quantum Theory forbids arbitrarily small space time intervals, the above continuum character with space time points has been taken for granted. Indeed it has been suggested by Snyder, Lee and others[1, 2, 3, 4] that the infinities which plague Quantum Field Theory are symptomatic of the fact that space time has a granular or discrete character. This has lead to a consideration of extended particles, as against point particles of conventional theory. Wheeler’s space time foam and strings[5, 6] are in this class, with a minimum cut off at the Planck scale. This has also lead to a review of the conventional concept of a rigid background space time. More recently[4, 5, 8], it has been pointed out that it is possible to give a stochastic underpinning to space time and physical laws. This is in the spirit

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of what Wheeler calls, "Law without Law"[9].
In the light of the above remarks, we now point out that space time is fractal
and can be considered to be what may be termed a Brownian or Random
Heap. This would be at the root of Quantum behaviour.

2 The Random Heap

What distinguishes Quantum Theory from Classical Physics is the role of the
resolution of the observer or observing apparatus. Indeed as noted by Abbot
and Wise[10], in this respect the situation is similar to everywhere continuous
but non differentiable curves, the fractals of Mandelbrot[11]. This again is
tied up with the Random Walk or Brownian character of the Quantum path
as noted by Sornette and others[12]: At scales larger than the Compton
wavelength but smaller than the de Broglie wavelength, the Quantum paths
have the fractal dimension 2 of Brownian paths (cf. also Nottale,[13]). It
must be noticed that both these length scales involve the mass. It will be
seen below that this is quite meaningful.

Two important characteristics of the Compton wavelength have to be re-
emphasized (Cf.[4, 8]): On the one hand with a minimum space time cut off
at the Compton wavelength, we can recover by a simple coordinate shift the
Dirac structure for the equation of the electron, including the spin half. In
this sense the spin half, which is purely Quantum Mechanical is symptomatic
of the minimum space time cut off, as is also suggested by the Zitterbewegung
interpretation of Dirac (in terms of the Uncertainty Principle), Hestenes and
others[14, 15, 16]. The Zitterbewegung is symptomatic of the fact that by
the Heisenberg Uncertainity Principle, Physics begins only after an averaging
over the minimum space time intervals. This is also suggested by stochastic
models of Quantum Mechanics, both non relativistic and relativistic[17, 18,
19, 4].

The other aspect is that the Compton wavelength $l$ of a typical elementary
particle, the pion is given by the well known empirical "Large Number"
relation

$$ R \approx \sqrt{Nl} \quad (1) $$

where $R \sim 10^{28} \text{cm}$ is the radius of the universe and $N \sim 10^{80} \text{cm}$ is the
number of elementary particles in the universe. As pointed out,[1] [8] [1] is
also the Random Walk relation. Infact [1] and a similar equation for the
Compton time in terms of the age of the universe, viz.,

\[ T \approx \sqrt{N\tau} \]  \hspace{1cm} (2)

were the starting point for a unified scheme for physical interactions and indeed a cosmology consistent with observation. It was pointed out [20] that in the spirit of Wheeler’s travelling salesman’s ”practical man’s minimum” length that the Compton scale plays such a role, and that space time is like Richardson’s delineation of a jagged coastline with a thick brush, the thickness of the brush being comparable to the Compton scale.

Space time, rather than being a smooth continuum, is more like a fractal Brownian curve. To analyse this further, we observe that space time given by \( R \) and \( T \) of (1) and (2) represents a measure of dispersion in a normal distribution. Indeed if we have a large collection of \( N \) events (or steps) of length \( l \) or \( \tau \), forming a normal distribution, then the dispersion \( \sigma \) is given by precisely the relation (1) or (2).

The significance of this is brought out by the fact that the universe is a collection of \( N \) elementary particles, in fact typically pions of size \( l \), as seen above (Cf. ref.[21]). We consider space time not as an apriori container but as a Gaussian collection of these particles. It is a Random Heap. At this stage, we do not even need the concept of a continuum.

In this scheme the probability distribution has a width or dispersion \( \sim \frac{1}{\sqrt{N}} \) (Cf. ref.[22]), that is the fluctuation (or dispersion) in the number of particles \( \sim \sqrt{N} \). This immediately leads to equations (1) and (2). Moreover it leads to a completely consistent cosmology as pointed out earlier[23] which explains how all the so called Large Number coincidences and also Weinberg’s ”mysterious” formula relating the pion mass to the Hubble constant,

\[ m^3 = \left( \frac{Hh^2}{Gc} \right) \]  \hspace{1cm} (3)

far from being empirical, follow as a consequence of the theory, while predicting an ever expanding universe as is confirmed by latest independent observations.

It must be emphasized that equations (1) and (3) in particular bring out a holistic or Machian feature in which the large scale universe and the micro world are inextricably tied up, as against the usual differential view. This is in fact inescapable if we are to consider a Brownian Heap.
3 Stochastic Considerations

It was observed in Section 2 that the cut off length for fractal behaviour depends on the mass, via the de Broglie or Compton wavelength. The de Broglie wavelength is the non-relativistic version of the Compton wavelength. Indeed it has been shown in detail\[16, 24\] that it is the Zitterbewegung or self-interaction effects within the minimum cut off Compton wavelength that indeed give rise to the inertial mass. So the appearance of mass in the minimum cut off Compton (or de Broglie) scale is quite natural.

With this background we observe that it has been pointed out by Nottale\[13\] and others, though from a different standpoint, that the fractal nature and a stochastic underpinning are interrelated: for times less than the Compton (or de Broglie) wavelength, time runs backwards, corresponding to Nelson’s double Wiener process\[25\]. This leads to the complex wave function of Quantum Mechanics.

This is also the Zitterbewegung within the Compton scales as discussed elsewhere (Cf.ref.[16]).

In fact we have here a harmonious and meaningful convergence of Nelson’s stochastic approach and the de Broglie-Bohm approach, if we recognize the minimum space time cut offs, which again as pointed out, have a Brownian underpinning. This can be seen briefly as follows.

Let us consider the motion of a particle with position given by $x(t)$, subject to random correction given by, as in the usual theory, (Cf.[25, 8]),

$$|\Delta x| = \sqrt{\langle \Delta x^2 \rangle} \approx \nu \sqrt{\Delta t},$$

the diffusion constant $\nu$ being given by

$$\nu = \hbar/m,$$

and being related to the mean free path by

$$\nu \approx lv$$

We can then proceed to the Fokker-Planck equation.

In Nelson’s derivation, the Schrodinger wave function, exactly as in the de Broglie-Bohm approach, is decomposed as (Cf. also[26])

$$\psi = \rho e^{3i\hbar},$$

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which in both cases leads to the well known Hamilton-Jacobi type equation

$$\frac{\partial S}{\partial t} = -\frac{1}{2m} (\partial S)^2 + V + Q,$$  \hspace{1cm} (5)$$

where

$$Q = \frac{\hbar^2 \nabla^2 \sqrt{\rho}}{2m \sqrt{\rho}}$$

As pointed out by Nottale\cite{26}, the complex feature above disappears if the fractal or non-differential character is not present, (that is, the forward and backward derivates are equal): Indeed the fractal dimension 2 leads to the real coordinate becoming complex. What distinguishes Quantum Mechanics is the adhoc feature, the diffusion constant $\nu$ of (4) in Nelson’s theory and the “Quantum potential” $Q$ of (5).

However, once we recognize the minimum cut off space-time intervals, these adhoc features become quite meaningful: Equation (4) gives us the Compton wavelength and the $Q$ of the equation (5), as described in reference\cite{16} gives the inertial mass and energy of ”quantized vortices” with the same Compton scale extent.

The following will throw further insight on the foregoing considerations. Let us start with the Langevin equation in the absense of external forces,

$$m \frac{dv}{dt} = -\alpha v + F'(t)$$

where the coefficient of the frictional force is given by Stokes’s Law (cf.\cite{27})

$$\alpha = 6\pi \eta a$$

This then leads to two cases.

Case (i):
For $t$, there is a cut off time $\tau$. This is so because

$$\frac{n}{\alpha} = \frac{m}{\eta a},$$

so that, as per Stokes’s Law, as

$$\eta = \frac{mc}{a^2}$$
we get

\[ \tau \sim \frac{ma^2}{mca} = \frac{a}{c}, \]

that is \( \tau \) is the Compton time.

The expression for \( \eta \) which follows from the fact that

\[ F_x = \eta(\Delta s) \frac{dv}{dz} = m\dot{v} = \frac{a^2}{c} \dot{v}, \]

shows that the inertial mass is a type of viscosity of the background ZPF reminiscent of the work of Rueda and Haisch\[28\] and similar to the Compton wavelength mass referred to earlier (cf.also ref.[21]). To sum up case (i), for a cut off \( \tau \), the stochastic form leads us back to the minimum space time intervals \( \sim \) Compton scale.

To push these small scale considerations further, we have, using the Beckenstein radiation equation\[29\],

\[ t \equiv \tau = \frac{G^2m^3}{\hbar c^4} = \frac{m}{\eta a} = \frac{a}{c} \]

which gives

\[ a = \frac{\hbar}{mc} \quad \text{if} \quad \frac{Gm}{c^2} = a \]

In other words the Compton wavelength equals the Schwarzchild radius, which automatically gives us the Planck mass.

On the other hand if we work with \( t \geq \tau \) we get

\[ ac = \frac{2kT}{\eta a} \]

whence

\[ kT \sim mc^2, \]

which is the Hagedorn formula for Hadrons\[30\].

Thus both the Planck scale and the Compton wavelength Hadron scale considerations follow meaningfully.

Case (ii):

If there is no cut off time \( \tau \), as is known, we get back,

\[ \Delta x = \nu\sqrt{\Delta t} \]

and thence Nelson’s derivation of the non relativistic Schrodinger equation.
4 Space Time

As remarked in the previous section, the fact that forward and backward time derivatives in the double Wiener process do not cancel leads to a complex velocity (cf. [26]), $V = iU$. That is, the usual space coordinate $x$ (in one dimension for simplicity) is replaced by a coordinate like $x + ix'$, where $x'$ is a non-constant function of time. We will now show that it is possible to consistently take $x' = ct$.

Let us take the simplest choice for $x'$, viz., $x' = \lambda t$. Then the imaginary part of the complex velocity is given by $U = \lambda t$. Then we have (cf. [26]),

$$U = \nu \frac{d}{dx} (\ln \rho) = \lambda$$

where $\nu$ and $\rho$ have been defined in the equations leading to (4) and (5). We thus have, $\rho = e^{\gamma x}$, where $\gamma = \lambda / \nu$ and the quantum potential of (5) is given by

$$Q = \frac{\hbar^2}{2m} \cdot \gamma^2$$

We have already remarked in the previous section that in this stochastic formulation with Compton wavelength cut off, $Q$ turns out to be the inertial energy $mc^2$ (cf. [16]). It then follows from (6) that $\lambda = c$.

In other words it is in the above stochastic (and fractal) formulation that we see the emergence of the space time coordinates $(x, ct)$. If we now generalise to three spatial dimensions, then as is well known [31], we get the quaternion formulation with the Pauli spin matrices giving the purely Quantum Mechanical spin half of Dirac. On the other hand, the above fractal formulation with minimum space time cut offs has been shown to lead independently to the Dirac equation [4] as remarked earlier. Thus the origin of special relativity, inertial mass and the Quantum Mechanical spin half is the minimum space time cut offs. (Indeed it can be shown that a minimum space time cut off leads to Special Relativity [8, 19].

5 Comments

(i) The fact that the Quantum Mechanical wave function is complex, which indeed is one of the distinguishing features, is directly related to the above
minimum space time cut offs and fractal considerations. We must also re-
membeber that the Quantum Mechanical wave function contains as much
information of the system under consideration as possible. The values of
the time derivative of the wave function cannot therefore appear as initial
conditions.
Interestingly it can be shown that this consideration together with the re-
quirement of causality demands that the wave function be complex[^1][^2]. If
the wave function were to be real, then it is well known that we have a sta-
tionary situation in the Quantum picture. In Classical Mechanics, in contrast
the dynamics is in the second time derivative, and both a quantity like the
position and its derivative, that is velocity or momentum are required for
initial conditions. That is why in Classical theory we do not require complex
quantities.

(ii) It may be mentioned, that the holistic feature is contained in (1) and
(3). In this spirit, starting from a universal set - for example the universe of
galaxies - and considering a cascade of subsets, it is possible to see the origin
of a metric (Cf.^[3][^2] for details), rather than start with particles and consider
larger and larger sets.
(iii) The considerations of Section 3 and Section 4 show how Nelson’s stochas-
tic formulation and the de Broglie-Bohm approach converge. Both these
approaches however have been non relativistic and have had some unsatis-
factory adhoc features. For example the diffusion constant which appears in
(1) or the non local Quantum potential which appears in (2). These features
however become meaningful once we take into account the Compton scale
cut off and the Zitterbewegung effects, as described in the previous section.
It is also this stochastic or fractal feature which ensures that the underlying
theory has no hidden variables (Cf.refs.[^1][^3] and [^2][^4]).

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