ABSTRACT

We describe a numerical study to quantify the influence of tool-eccentricity on wireline (WL) and logging-while-drilling (LWD) sonic logging measurements. Simulations are performed with a height-polynomial-adaptive (hp) Fourier finite-element method that delivers highly accurate solutions of linear visco-elasto-acoustic problems in the frequency domain. The analysis focuses on WL instruments equipped with monopole or dipole sources and LWD instruments with monopole excitation. Analysis of the main propagation modes obtained from frequency dispersion curves indicates that the additional high-order modes arising as a result of borehole-eccentricity interfere with the main modes (i.e., Stoneley, pseudo-Rayleigh and flexural). This often modifies (decreases) the estimation of shear and compressional formation velocities, which should be corrected (increased) to account for borehole-eccentricity effects. Undesired interferences between different modes can occur at different frequencies depending upon the properties of the formation and fluid annulus size, which may difficult the estimation of the formation velocities.

Key words: Eccentred, Wireline, Borehole.

INTRODUCTION

Sonic measurements are routinely used in the geophysical description of hydrocarbon reservoirs (Tang and Cheng 2004). They provide important information about elastic and petrophysical properties and are widely used in combination with surface seismic amplitude measurements. Acoustic logging measurements are also commonly acquired during drilling and production to quantify mechanical properties of rock formations. They provide quantitative estimates of shear and compressional velocities of rock formations in the proximity of a borehole (Bassiouni 1994) and are used for the diagnosis and quantification of in-situ rock stress, presence of fractures and elastic anisotropy.

There exist two types of sonic logging instruments: wireline (WL) and logging-while-drilling (LWD). LWD tools have a larger diameter than WL tools (Jackson and Heysse 1994), thereby comprising a significant volume of the borehole and leaving only a small fluid-filled annulus between the tool and the borehole wall. Likewise, they contain an inner fluid filled channel that poses significant challenges to the computer-aided simulation of full-waveform sonic measurements.

For both LWD and WL tools, the main physical principles behind sonic measurements are fairly well-understood in the case of a logging instrument located at the centre of a vertical borehole (axis of symmetry) penetrating horizontal isotropic layers (e.g., Hsu and Sinha 1998; Sinha, Simsek and...
Asvadurov 2009). However, in the presence of anisotropic formations, deviated wells and/or borehole-eccentred logging instruments, proper interpretation of sonic measurements is substantially more challenging. The aim of this paper is to study numerically the influence of borehole eccentricity on sonic measurements acquired in vertical wells penetrating isotropic layered formations.

Most LWD logging instruments employ stabilizers to place the logging instrument at the centre of the borehole. Similarly, WL instruments are equipped with centralizers. Nevertheless, the location of the tool is often borehole-eccentric in actual logging conditions. In the case of a WL tool, one often encounters such a situation when logging in deviated wells, since the tool tends to descend toward the lower side of the borehole. The stabilization of an LWD tool is also often problematic (mainly due to variations on the borehole trajectory during directional drilling) and one usually works in off-centred regimes (Market and Canady 2007). In this paper, we show that sonic measurements can be highly affected by the distance from the centre of the tool to the borehole axis (eccentricity distance), regardless of the tool type. Thus, simulation of borehole-eccentred tools is imperative to quantify and interpret acoustic measurements acquired under these adverse but common conditions.

Numerical studies in this area include the pioneering results obtained by (Roever, Rosenbaum and Vining 1974; Willis, Toksz and Cheng 1983), where analytical methods were applied to investigate the influence of an off-centred point source in an open borehole. This approach was extended to dipole sources (Leslie and Randall 1990; Schmitt 1993), cylindrical tool modelling (Zhang, Wang and Dong 1996) and quadrupole sources (Byun and Toksö 2006). The finite-difference time-domain method was also applied to study borehole eccentricity effects on LWD sonic tools (Huang 2003; Wang and Tang 2003). Lately, the finite-element has been used to investigate the influence of LWD-tool eccentricity on guided wave dispersion properties in a borehole penetrating homogeneous formations (Zheng, Huang and Toksö 2004; Cheng and Toksö 2004). In the former paper, it was recognized that when the tool is eccentred, the Stoneley mode exhibits a lower phase velocity and dispersion curves related to dipole and quadrupole modes split into two curves with different phase velocities. In the latter paper, the problem was solved in the frequency domain combined with a wavenumber-domain (in the axial direction) formulation on the plane perpendicular to the axis of symmetry, thereby restricting the analysis to cases with no variations of material properties in the z-axis.

In this paper, we make use of a high-accuracy hp-adaptive Fourier finite-element method to perform a detailed numerical study of the influence of borehole-eccentred tools in full-wave sonic measurements for both WL and LWD instruments. In contrast to previously existing works, our method is flexible in the sense that it enables simulations of different formations (isotropic, vertically transversely anisotropic, with attenuation, vertical and/or horizontal layers, fractures, etc.) and sonic borehole-eccentred logging instruments, including WL and LWD tools in vertical wells and, at the same time, it guarantees a high-accuracy solution in all cases. Results presented here are a continuation of those presented in Pardo et al. (2011), where they described in detail and verified the hp Fourier finite-element solution method and confirmed its high-numerical accuracy by showing a perfect match between numerical results and existing analytical solutions. The work presented here invokes the hp Fourier finite-element method for analysing and quantifying the sensitivity of sonic logging measurements to tool eccentricity. We also show that a frequency-domain based method could be used to efficiently interpret measurements acquired with borehole-eccentred sonic instruments.

The remaining components of the paper introduce the simulation method and provide additional verification results to subsequently study in detail the sensitivity of sonic measurements to eccentricity distance in different borehole conditions involving both WL and LWD tools.

METHOD

The hp Fourier finite-element method is intended to solve the following linear time-harmonic coupled visco-elastic-acoustic problem expressed in its variational (weak) form in terms of the fluid pressure \( p \) and the solid displacement \( u \) as:

\[
\text{Find } (p, u) \in H^1(\Omega_A) \times H^1(\Omega_E) \\
\text{such that for all } \varphi \in H_0^1(\Omega_A) \text{ and all } \psi \in H_0^1(\Omega_E) \\
\langle \nabla \varphi, \nabla \psi \rangle_{L^2(\Omega_A)} + k_f^2 \langle \varphi, \psi \rangle_{L^2(\Omega_A)} - \rho_f \omega^2 \langle \varphi, n_f \cdot u \rangle_{L^2(\Gamma_A)} = 0, \\
\langle \kappa(v), Ce(u) \rangle_{L^2(\Omega_E)} - \rho_c \omega^2 \langle v, u \rangle_{L^2(\Omega_E)} + \langle n_c \cdot v, p \rangle_{L^2(\Gamma_E)} = 0,
\]

where \( q \) and \( \varphi \) are test functions, \( \Omega_A \) and \( \Omega_E \) are the acoustic and elastic parts of the domain, respectively, \( \Gamma_A \) is the interface between the acoustic and elastic parts, \( \Gamma_E \) is part of the acoustic boundary where Neumann loading is imposed, \( k_f \) is the wavenumber in a fluid, \( \omega \) is the angular frequency, \( n_f \) and \( n_c \) are the unit normal (outward) vectors with respect to
the fluid and solid, respectively, \( \epsilon \) denotes the strain (symmetric part of the displacement gradient), \( \mathbf{C} \) is the fourth-order (visco-)elastic stiffness tensor, \( \rho_s \) and \( \rho_f \) are the solid and fluid mass densities, respectively, \( f_A \) is the acoustic excitation, and \( H^1(\Omega) \) and \( H^1(\Omega) \) denote the scalar and vector-valued proper Sobolev spaces, respectively, for finite-element computations (in this case, the set of \( L^2 \) functions whose gradients are also \( L^2 \) functions). Two boundary integrals defined on \( \Gamma_j \) express a weak coupling between the acoustic and elastic domains.

When a logging instrument is borehole-centric, the axial symmetry of the problem is lost. Therefore, the problem cannot be simply reduced to a two-dimensional domain. In view of this challenge, our simulation method incorporates a change of coordinates intended to minimize the computational cost of the overall algorithm. Instead of using a traditional cylindrical system of coordinates everywhere in the computational domain, we select a new parametrization in which: (a) the innermost subdomain containing the logging instrument is parametrized with a cylindrical system of coordinates whose centre is that of the mandrel, (b) the outermost subdomain containing the formation is parametrized with a cylindrical system of coordinates whose centre is that of the borehole and (c) the remaining part of the computational domain (i.e., the space occupied by the borehole without the mandrel) is parametrized with any system of coordinates such that the change of coordinates is globally continuous and therefore, it can be employed for finite-element computations. An example of such a parametrization is described in Fig. 1 (top panel). This change of coordinates is designed to parametrize the geometry of a borehole-centric logging instrument in such a way that all material properties are constant along the new quasi-azimuthal direction. For details, we refer to Pardo et al. (2011).

Since material properties of the medium are constant along the newly created quasi-azimuthal direction, the solution is expected to be smooth and a high-order method along the direction where materials are invariant should provide highly accurate solutions. Thus, we perform a Fourier series expansion along the quasi-azimuthal direction. Moreover, the resulting formulation in terms of Fourier modes is only weakly coupled within the domain occupied by the formation as well as by the mandrel, which significantly reduces the computational cost with respect to traditional formulations based on cylindrical coordinates. The remaining two spatial (meridian) components are discretized with a multi-physics two-dimensional self-adaptive \( h p \)-finite-element method (Matuszyk, Demkowicz and Torres-Verdin 2011, 2012), where \( h \) indicates the element size and \( p \) the polynomial order of approximation, both varying locally throughout the computational grid. Given an initial grid that is designed in such a way that it roughly resolves the waves in the domain using the estimation for the dispersion error, as given in Theorem 3.3 in Ainsworth (2004), the self-adaptive strategy automatically performs optimal local \( hp \)-refinements by either dividing the element size or increasing its polynomial order of approximation, cf. Demkowicz (2006) and Pardo et al. (2007a). The choice between \( h \) and \( p \)-refinement is based on minimizing the projection error, as detailed in Demkowicz (2006). The resulting method provides exponential convergence in terms of the error versus the problem size, as proved in Babuška and Guo (1996). From a practical point of view, this implies a superior accuracy with respect to other methods. Notice that no other finite-element or finite-difference method provides exponential convergence for the type of problems considered in this paper (Babuška and Guo 1996). The suitability of this \( hp \)-finite-element method in various borehole logging applications has been shown in Nam, Pardo and Torres-Verdin (2008), Pardo, Torres-Verdin and Demkowicz (2007b), Pardo, Torres-Verdin and Zhang (2008) and Pardo et al. (2006).

The method also incorporates a Perfectly Matched Layer (PML) for truncation of the computational domain, as first proposed by Bérenger (1994). Our understanding and implementation of the PML is consistent with the one provided by Chew and Weedon (1994), which is based on a change of coordinates of the governing equations into the complex plane within the PML layer (Matuszyk et al. 2012; Matuszyk and Demkowicz 2012). The coordinates are transformed according to the formula

\[
x_{ij} := X_j(x_j, k), \quad \frac{\partial}{\partial x_j} := \frac{1}{X_j} \frac{\partial}{\partial x_j} \quad \text{where} \quad X_j = \frac{\partial X_j}{\partial x_j},
\]

where \( x_j \) and \( X_j \) represent unstretched and stretched coordinates, respectively, \( k = 2\pi/\lambda_{\text{min}} \) (\( \lambda_{\text{min}} \) is the shortest anticipated wavelength for a given frequency and all the material properties) and

\[
X_j = \frac{2p^p}{k^q} \frac{\xi^p}{|\xi|^q} (1 - i) + x_j, \quad \xi(x_j) = \begin{cases} \frac{x_j^p - x_j}{\delta_j} & x_j < x_j^p \\ \frac{x_j^p - x_j}{\delta_j} & x_j > x_j^p \\ 0 & \text{otherwise.} \end{cases}
\]

The coordinates \( x_j^p \) and \( x_j^p \) define the PML region for the coordinate \( x_j \) and \( \delta_j \) denotes its thickness. In the presented examples, we select based on numerical experimentation \( p = 6, m = 3, a(\xi) = \xi \). The thickness of the PML layer is computed...
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for each frequency separately according to the formula

\[ b = \min(2\lambda_{\text{min}}, 0.1 \text{diam}(\Omega)) \]

Finally, homogeneous Dirichlet boundary conditions are prescribed at the outermost boundary of the PML layer.

Numerical results are post-processed using a modified matrix-pencil based inversion procedure (Ekström 1996) to obtain dispersion curves directly from frequency-domain data. This inversion algorithm fails to provide a unique solution when the Nyquist frequency sampling theorem is not satisfied, which occurs at high frequencies. This is not an algorithmic problem but rather a fundamental physical problem, since at high frequencies different formations may provide the exact same acoustic pressures at the receivers. In order to overcome this problem and determine the correct dispersion curves, it is necessary to include two receivers very close to each other. More precisely, the proximity between the receivers should be inversely proportional to the frequency to be resolved. This physical observation seems to suggest that the sonic logging instrument should contain non-equally spaced receivers, which is confirmed with our numerical experiments. We have included additional receivers in our numerical simulations in order to obtain a unique set of dispersion curves for each case.

For illustration purposes, time-domain results (waveforms) obtained from frequency-domain simulations via inverse Fourier transform are also displayed. We use as the acoustic ring source a Ricker wavelet centred at \( t = 0 \). The central frequency is 8 kHz for monopole excitation and 3 kHz for dipole excitation.

**VERIFICATION**

We describe verification results that are complementary to those performed in Pardo et al. (2011).

First, we consider two different changes of coordinates for our method, as illustrated in Fig. 1 (top panel). The
inner-dotted line denotes the outer-boundary of the cylindrical system of coordinates within the tool and the outer-dotted line represents the inner-boundary of the cylindrical system of coordinates within the formation. Both parametrizations correspond to the same physical problem and, therefore, solutions associated to both changes of coordinates should coincide to an infinite number of Fourier modes. However, discrete solutions with a finite number of Fourier modes will be quite different. Indeed, having two different parametrizations is the same as considering two different computational methods, both converging towards the exact solution. The fact that both parametrizations provide a good agreement constitutes a strong indication of the correctness of the method, since it implies that two different methods provide the same exact solution.

Figure 1 (bottom panel) describes the convergence history of both parametrizations with respect to a reference solution (obtained with a third parametrization) for a model with WL and LWD logging instruments. Curves corresponding to different parametrizations exhibit a fast convergence as we increase the number of Fourier modes, which indicates that the software is error free.

Additionally, in the section ‘Numerical Results’ we also compare results obtained with our finite-element software with those corresponding to the analytical solutions for the centred homogeneous isotropic case (see Fig. 5a,b, Fig. 9a,b, Figs 13 and 16). Results exhibit in all cases a perfect agreement between the analytical and numerical solutions.

### Table 1 Material parameters assumed for borehole sonic logging with a borehole-eccentred tool.

| Nr | Medium | $V$ [m/s] | $S$ [μs/m] | $Q$ | $\rho$ [kg/m$^3$] |
|----|--------|-----------|------------|-----|------------------|
| 1  | tool   | $P_1$     | 5862       | 170.59 | 100              | 5900          |
|    |        | $S_1$     | 2519       | 396.98  | 50               |               |
| 2  | fluid  | $P_2$     | 1524       | 656.17  | $\infty$        | 1100          |
|    |        | $S_1$     | 2621       | 381.53  | 1000             |               |
| 3  | fast   | $P_3$     | 4354       | 229.67  | 1000             | 2300          |
|    |        | $S_3$     | 1269       | 788.02  | 1000             |               |
| 4  | slow   | $P_4$     | 2540       | 393.70  | 1000             | 2100          |
|    |        | $S_4$     | 1269       | 788.02  | 1000             |               |

### NUMERICAL RESULTS

In this section we present numerical simulations of sonic measurements acquired with borehole-eccentred logging instruments. In order to study the effects of borehole-eccentricity, we shall consider two different homogeneous formations: slow and fast. Table 1 describes the elastic properties of our material data. To facilitate the interpretation of the numerical results, each mode shown in the figures is identified by the corresponding physical mode (e.g., Stoneley, flexural, tool, etc.).

#### Wireline logging

First, we consider the WL logging instrument described in Fig. 2(a), which is composed of a mandrel with a radius equal to 4.5 cm, a transmitter and an array of 13
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Figure 3 Dispersion curves (on the left) and mode amplitudes (on the right) for a WL monopole logging in a fast formation. (a,b) No tool eccentricity, (c,d) 2 cm (31%) eccentric tool and (e,f) 5 cm (77%) eccentric tool.

equidistant receivers, spaced 0.1524 m apart. We emphasize that some modern logging instruments are designed to minimize the interference between the mandrel and the recorded measurements by incorporating slotted sleeves that significantly delay the arrival of mandrel-associated modes. However, following Hsu and Sinha (1998), in this paper we model the mandrel as a visco-elastic tube, since one of the objectives of this work is to investigate the effects of borehole-eccentred mandrels on the recorded measurements.

Monopole, fast formation: Fig. 3 shows the dispersion curves and, side-by-side, the corresponding mode amplitudes obtained for WL monopole logging in a fast formation with different tool eccentricities (in the range 0–5 cm). The first pair of plots, Fig. 3(a,b), display the reference case corresponding to the centred tool. We can observe here classical modes excited in the fast formation: Stoneley (St), pseudo-Rayleigh (p-R) and formation compressional mode (P). For a comprehensive exposition of sonic modes corresponding to borehole-centred measurements, we refer to Paillet and Cheng (1991) and Tang and Cheng (2004). For the case of borehole-eccentred measurements, we observe new higher-order modes that become stronger in amplitude as the distance between the tool and borehole centres increases (Fig. 3c,d for 2 cm eccentricity (31%) and Fig. 3e,f for 5 cm eccentricity (77%)). They are the
so-called dipole modes: flexural mode (Fl) and tool mode (T). The former one begins at a cut-off frequency with a slowness equal to the shear-slowness of the formation and it approaches asymptotically the Scholte wave velocity for high frequencies. The tool dipole mode perturbs locally the Stoneley (at 4 kHz), flexural mode (at 7–8 kHz) and pseudo-Rayleigh (at 13 kHz) modes due to interferences produced by similar wavenumbers co-existing at a certain range of frequencies. These interferences prevent proper recovering of the exact modes and result in the presence of two or more branches of the coupled mode (e.g., T, Fl and p-R modes in Fig. 3c). Such a perturbation can also be observed in the rapid decrease of the mode amplitudes. The Stoneley mode remains nearly intact independently of the borehole-eccentricity distance, especially in the low-frequency regime. The pseudo-Rayleigh mode is more sensitive to the logging tool position. For smaller borehole-eccentred distances, only the high-frequency spectrum of the mode is perturbed, leaving the cut-off frequency region intact. The greater the distance between the borehole and mandrel centres, the larger the amplitudes of the higher-order modes (see Fig. 3e,f). These higher-order modes interfere with the p-R mode, perturbing significantly the p-R curve. Moreover, the presence of the excited dipole tool mode at higher frequencies produces additional pollution on the p-R mode. Therefore, estimation of the formation shear-slowness based on processing the pseudo-Rayleigh mode is very challenging for borehole-eccentred measurements and needs special treatment.

In the presented case, the effect of contamination of the monopole modes by the excited higher-order modes can also be observed in the waveforms (see Fig. 4a,c,e). For the largest tool eccentricity distance, we observe in Fig. 4(e) the presence of the tool dipole mode following the formation compressional mode $P_3$ and the dipole formation mode preceding the Stoneley mode. However, the arrivals of the compressional mode of the formation and low-frequency component of the Stoneley mode remain unaffected.

A summary comparison of the dispersive modes is presented in Fig. 5(a), where we also included analytical dispersion curves corresponding to borehole-centred measurements to facilitate the interpretation of the results. Here, it can again be observed that the Stoneley mode is almost insensitive to the borehole-eccentricity distance. On the other hand, the pseudo-Rayleigh mode is significantly influenced by the borehole-eccentricity distance at higher frequencies but close
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Figure 5 Dispersion curves for simulations of a WL tool equipped with: (a) a monopole and (b) a dipole source in a fast formation. Eccentricity effects on Stoneley (St), pseudo-Rayleigh (p-R) and flexural (Fl) modes. Different colours correspond to various distances from the centre of the tool to the centre of the borehole. Analytical dispersion curves for the centred case are displayed by red solid (for monopole), red dashed (for dipole) and red dotted lines (for quadrupole).

to the cut-off frequency gives a correct estimation of the shear-slowness of the formation.

Dipole, fast formation: Fig. 6 shows the dispersion curves and the corresponding mode amplitudes obtained for WL dipole logging in a fast formation with different tool eccentricities. Corresponding waveforms are displayed in Fig. 4(b,d,f). Plots 6(a,b) show the results for the centred tool. We can observe here three modes: the first flexural mode (branches Fl1 and Fl2), the tool mode (branches T1 and T2) and at higher frequencies, the second flexural mode (in red). Crossing of the two former modes in the frequency-slowness domain results in two new hybridized modes, namely T1-Fl2 and Fl1-T2 (compare analytical dispersion curves in Fig. 5). However, it does not affect the low-frequency limits of both flexural modes, which indicate the shear of the formation. For borehole-eccentred measurements, the excited quadrupole screw mode (Sc1 and Sc2), as well as other modes, strongly interfere with the dipole tool mode (see Fig. 5b). However, the latter one interferes with other modes at higher frequencies. Therefore, most of the higher-order modes (flexural and quadrupole) are not perturbed around their cut-off frequencies, which facilitates a reliable estimation of the shear-slowness of the fast formation. For the largest borehole-eccentred distance of the logging instrument, we also observe a split of the formation flexural mode into two modes, as seen in Zheng et al. (2004).

Monopole, slow formation: Fig. 7 shows the dispersion curves and the corresponding mode amplitudes obtained for WL monopole logging in a slow formation with different tool eccentricities. Plots 7(a,b) show the results for the centred tool. We can observe the classical modes excited in the slow formation: Stoneley (St) mode, which has the largest amplitude at low frequencies; formation compressional (P); and tool compressional mode (T1), which has the largest amplitude at higher frequencies. Similarly to the previous case, tool-eccentricity causes excitation of the dipole tool and higher-order modes in the formation. The higher-order modes excited on the formation are generally weak but the dipole tool mode becomes stronger than that for the fast formation. Moreover, the frequency band where interferences between tool and formation flexural modes and the Stoneley mode appear, is shifted toward lower frequencies (in comparison to the fast formation case). This results in a split of the Stoneley mode into two branches (St1 and St2). Therefore, (see Fig. 9a), the Stoneley mode can be used for slowness estimation only for small tool-eccentricities. Moreover, this effect becomes even more noticeable for very slow formations, since the dispersion curve corresponding to the Stoneley mode is slower and the interference with the tool dipole mode appears at lower frequencies. On the other hand, in a slow formation, the flexural mode seems to be useless in the estimation of the formation slowness due to its interference with the tool mode. The excited quadrupole is very weak, hence also not suitable for assessing formation shear-slowness. In the waveforms plotted in Fig. 8(a,c,e) we observe the arrival of the strong dipole tool mode (P1), as well as the presence of the excited formation flexural mode preceding the Stoneley mode for the largest tool eccentric distance.

Dipole, slow formation: Fig. 10 shows the dispersion curves and the corresponding mode amplitudes and waveforms

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obtained for WL dipole logging in a slow formation with different tool eccentricities. Plots 10(a,b) show the results for the centred tool. Similarly to the previous case, here we observe two coupled modes: the first flexural mode (Fl1 and Fl2) and dipole tool mode (T1 and T2). However, in the considered case, the flexural mode is slower than that for a fast formation and its cut-off frequency shifts toward very low frequencies as the formation shear-slowness increases. Additionally, at lower frequencies, the dipole tool mode becomes dominant. Due to the shape of the dispersion curve related to the dipole tool mode, the deleterious mode interference occurs around the flexural cut-off frequency, which hampers estimation of the formation shear-slowness. The effect of tool eccentricity perturbs the formation flexural mode due to excitation of the Stoneley mode. The mixing of different modes is revealed also in amplitude plots through fast changes of mode amplitude at nearby frequencies (see Fig. 10f). This destructing interference phenomenon can also be observed in Fig. 8(b,d,f) where waveforms are plotted, as well as in Fig. 9(b), which summarizes the selected dipole modes. Recovering the shear-slowness of the formation from the dispersion curves becomes a much more challenging task when we consider borehole-eccentred
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Figure 7 Dispersion curves (on the left) and mode amplitudes (on the right) for a WL monopole logging in a slow formation. (a,b) No tool eccentricity, (c,d) 2 cm (31%) eccentred tool and (e,f) 5 cm (77%) eccentred tool.

measurements due to interferences and the shift produced on the first flexural mode.

Logging-while-drilling

Now, we consider the LWD instrument described in Fig. 2(b), which is modelled by a visco-elastic tube with an inner section filled with drilling fluid, a transmitter and 13 equidistant receivers, spaced 0.1524 m apart. We assume the same formation, tool and borehole fluid parameters as those used for simulation of WL sonic measurements. We consider only a monopole excitation, since dipole sources are typically not employed in combination with LWD tools due to difficult separation of the dipole collar mode and formation flexural mode, despite the existing attempts to use dipole sources and measure excited hexapole mode (Chi et al. 2005).

We first consider the case of a fast formation. Figures 11 and 12 display the dispersion curves, the corresponding mode amplitudes and waveforms obtained for the monopole excitation, for the borehole-centred and 1 cm (41%) borehole-eccentred cases. The borehole-eccentred case contains all modes corresponding to the borehole-centred case (Stoneley mode (St), 1st
Figure 8: Waveforms for a WL logging instrument equipped with a monopole (on the left) and dipole (on the right) source in a slow formation. Tool eccentricity: (a,b) 0 cm, (c,d) 2 cm (31%) and (e,f) 5 cm (77%). Arrivals of the modes are marked with symbols defined in Table 1.

Figure 9: Dispersion curves for simulations of a WL tool equipped with: (a) monopole and (b) dipole source in a slow formation. Eccentricity effects on Stoneley (St) and flexural (Fl) modes. Different colours correspond to various distances from the centre of the tool to the centre of the borehole. Analytical dispersion curves for the centred case are displayed by red solid (for monopole), red dashed (for dipole) and red dotted lines (for quadrupole).

We note that the Stoneley mode is affected at low frequencies by the excited dipole tool mode and at high frequencies by the flexural mode. An increase of the borehole eccentric distance of the tool produces a shift on the Stoneley mode, which becomes slightly slower than for the centred tool case, as described in Fig. 13. However, at the low-frequency limit, the Stoneley mode remains invariant, since the borehole-eccentric distance is small and the Stoneley mode is slower than the dipole tool mode. In the considered case, the pseudo-Rayleigh mode (p-R) and two fast dispersive LWD mandrel modes (T1 and T2) plus additional dipole modes: tool mode (T3 and T4) and flexural mode (Fl1 and Fl2). We observe that the Stoneley mode is affected at low frequencies by the excited dipole tool mode and at high frequencies by the flexural mode. An increase of the borehole eccentric distance of the tool produces a shift on the Stoneley mode, which becomes slightly slower than for the centred tool case, as described in Fig. 13. However, at the low-frequency limit, the Stoneley mode remains invariant, since the borehole-eccentric distance is small and the Stoneley mode is slower than the dipole tool mode. In the considered case, the pseudo-Rayleigh mode is only affected by the excited higher-order modes at higher frequencies; the dipole tool mode has a negligible influence and, thus, the pseudo-Rayleigh mode can be used to estimate formation shear-slowness. Additionally, we observe that as the formation shear-slowness increases, we could expect a stronger interference of the dipole tool mode with the pseudo-Rayleigh mode, which can hamper slowness estimation. Thus, the shear-slowness of the fast formation can be properly recovered, also for the case of borehole-eccentric LWD measurements. Notice that interferences between different modes may occur at different frequencies depending upon the properties of the formation, which may difficult the estimation of the formation velocities.
A similar situation occurs in the case of a LWD tool in a slow formation (see Figs 14–16). In Fig. 14, we observe that collar and formation dipole modes are excited when the tool is borehole-eccentred. The collar mode perturbs the Stonely mode at low frequencies. On the other hand, the excited formation flexural mode is too weak to reliably estimate formation shear-slowness. Therefore, LWD monopole logging measurements are not appropriate to assess the shear velocity of the formation.

**CONCLUSIONS**

We have employed simulation results based on a highly-accurate $hp$-finite-element code to quantify the influence of tool eccentricity in wire-line and logging-while-drilling acoustic measurements. Results obtained for the considered cases indicate that excited higher-order (dipole and quadrupole) modes appearing as a result of tool eccentricity interfere with the ones corresponding to the case of borehole-centred
Figure 11 Dispersion curves for an LWD tool in a fast formation. a) The logging instrument is located at the axis of symmetry. b) The logging instrument is 1 cm (41%) eccentric.

Figure 12 Dispersion curves for an LWD tool in a fast formation. a) The logging instrument is located at the axis of symmetry. b) The logging instrument is 1 cm (41%) eccentric.

measurements. These interferences occurring at particular frequency regimes, compromise in many cases the proper estimation of the formation shear velocity.

In WL logging, the Stoneley mode (for the monopole source) and the flexural mode (for dipole excitation) are marginally affected by the eccentricity distance. The presence of an additional dipole tool mode for monopole logging influences

Figure 13 Dispersion curves for an LWD tool in a fast formation. Different colours correspond to various distances from the centre of the tool to the centre of the borehole. Analytical dispersion curves for the centred case are displayed by red solid (for monopole) and red dashed lines (for dipole).
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Figure 14 Dispersion curves for an LWD tool in a slow formation. a) The logging instrument is located at the axis of symmetry. b) The logging instrument is 1 cm (41%) eccentred.

Figure 15 Dispersion curves for an LWD tool in a slow formation. a) The logging instrument is located at the axis of symmetry. b) The logging instrument is 1 cm (41%) eccentred.

Figure 16 Dispersion curves for an LWD tool in a slow formation. Different colours correspond to various distances from the centre of the tool to the centre of the borehole. Analytical dispersion curves for the centred case are displayed by red solid (for monopole) and red dashed lines (for dipole).
case of slow formations, the dipole tool mode typically difficulties reliable estimation of the formation shear velocity. Tool eccentricity perturbs modes under interest, which additionally makes this problem more challenging.

In the case of monopole LWD logging, the effects due to a borehole-eccentred position of the tool are not as prominent as those for the WL tool and they influence mainly the Stoneley mode, which becomes slightly slower with respect to that of a centred tool. In fast formations, the pseudo-Rayleigh mode can still be used to estimate formation shear-slowness. For slow formations, monopole LWD is not adequate to estimate shear-slowness.

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