Hierarchy analysis for monitoring the uniformity of measurements system

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Abstract. An approach to monitoring the state of the system for ensuring the uniformity of measurements and forecasting its state, based on the method of hierarchy analysis, is proposed. A decomposition of the system for ensuring the uniformity of measurements is proposed, the further processing of the judgments sequence for decision-making based on paired comparisons, is discussed. The processing of experts judgments in order to obtain a generalized judgment is described.

1. Introduction
One of the main priorities in developing the national system for ensuring the uniformity of measurements is the creation of a mechanism for forecasting the needs of the economy and society in measurements, while the lack of such a mechanism should be attributed to systemic problems.

Monitoring and analysis of trends in priority areas of the economy should provide information about the objects and types of measurements, the required accuracy characteristics.

The development of such forecasts is a complex task that requires appropriate support. To solve this problem the All-Russian Research Institute of Metrology Service in 2019 established the Center for Monitoring and Forecasting. The task of the Center is to develop methods to assess the state of ensuring the uniformity of measurements, to forecast the needs in measurements, to assess the impact of metrology on quality of life and the economy nationwide.

The unity of measurements in the country is reached as a result of functioning of the system of maintenance of uniformity of measurements which is formed by set of legislative and other normative legal acts, normative-technical and methodical documents in the field, participants of system of maintenance of uniformity of measurements, the reference base and park of measuring instruments. Thus, the system for ensuring the uniformity of measurements is a complex ergatic system, and the task of monitoring its state is actual.

The state of the system is assumed as a set of quantitative and qualitative parameters (indicators) characterizing: the structure of the system for ensuring the uniformity of changes; the number of personnel as a whole and by categories; technical equipment of measuring instruments; control, training subsystems, infrastructure of the system for ensuring the uniformity of measurements, scientific support subsystem and regulatory legal basis (figure 1).
Figure 1. Block diagram of indicators of the state of the system for ensuring the uniformity of measurements.

The quantitative and qualitative composition of the system for ensuring the uniformity of measurements is understood as a set of different for its purpose organizations engaged in activities to ensure the uniformity of measurements.

The structure of the system for ensuring the uniformity of measurements is assumed as a set of hierarchically distributed control bodies for ensuring the uniformity of measurements and the components included in them with the purpose of more effective performance of tasks assigned to them on ensuring the uniformity of measurements.

Evaluation of the state of the system for ensuring the uniformity of measurements is to assess the current state of the system as a whole and its subsystems, the main problem issue is to take into account the integrity of the system, the interaction of subsystems, as well as the presence of indicators, the description of which is possible only at the verbal level.

Thus, the methodological apparatus to assess the state of the system of ensuring the uniformity of measurements, as a complex ergonomic system, should provide an account of interaction between subsystems and the possibility of converting the verbal description of indicators into their numerical values. Nowadays there are numerous methods to solve problems of this type [1, 2]. One of the most effective and relatively simple methods that allow to implement these requirements is the analysis of hierarchies [3–5].

The method of analysis of hierarchies is a systematic procedure for hierarchical representation of elements that determine the essence of a problem. The essence of the method consists in decomposition of the problem into simpler components with further processing of the sequence of judgments of the decision maker by pair comparison. As a result, a relative degree (intensity) of interaction of elements in the hierarchy can be obtained. The method of hierarchy analysis includes procedures for synthesizing multiple judgments, obtaining priority criteria and finding alternative solutions [3, 4].

The main stages of the hierarchy analysis method are the following:
1. Building a qualitative model of the problem in the form of a hierarchy that includes a goal, alternatives to achieve the goal and criteria for assessing the quality of alternatives;
2. Determining the priorities of all elements of the hierarchy using the paired comparison method;
3. Synthesis of global alternative priorities by linear convolution of priorities of elements in the hierarchy;
4. Checking of judgments for consistency.
5. Decision making on the basis of the obtained results.

Let's consider the application of the method of hierarchies analysis to assess the state of the system of ensuring the uniformity of measurements.

The first stage of the method of hierarchy analysis is decomposition of the task of estimating. Thus, the general objective is to evaluate the state of ensuring the uniformity of measurements; it is on the first (highest) level. At the next level, there are goals of ensuring the uniformity of measurements, at
the third level, the methods of state regulation in the field of ensuring the uniformity of measurements, and at the fourth level, the subsystems of the system for ensuring the uniformity of measurements (figure 2).

![Hierarchical model for assessing the state of the system for ensuring the uniformity of measurements](image)

**Figure 2.** Hierarchical model for assessing the state of the system for ensuring the uniformity of measurements.

In this hierarchy, each element of the level, except for the highest one, depends on all below located elements. The next step establishes the priorities of the criteria, and the evaluation of each of the alternatives by criteria in order to identify the most important of them. The elements are compared in pairs using subjective judgments, numerically evaluated on a scale, with subsequent solving the problem of finding the weight (intensity) of each of the criteria.

To compare the relative importance of the criteria at the second level with respect to the common goal, a matrix of paired comparison of each of the second level alternatives with respect to the first level goal is constructed at the first level. Similar matrices are constructed for paired comparison of each of the alternatives on the third and fourth levels in relation to the criteria of the second and third levels respectively.

The matrix compiled for comparison of relative importance of criteria at the second level for the hierarchy presented in figure 2 is given in table 1.

**Table 1.** Assessment of the state of the system for ensuring the uniformity of measurements: structure of a matrix of pairwise comparisons.

| The state of the system | Uniformity of measurements | The required accuracy of measurements | Completeness of measurements |
|-------------------------|----------------------------|--------------------------------------|------------------------------|
| Uniformity of measurements |                           |                                      |                              |
| The required accuracy of measurements |                     |                                      |                              |
| Completeness of measurements |                     |                                      |                              |

A structure of matrix of paired comparisons for the Unity of Measurement criteria of the 3rd level of the hierarchy is presented in Table 2, such matrices are compiled for each criterion of the 2nd level, i.e. 3 matrices.
Table 2. Example of a matrix of relative importance for the element of unity of measurements: Matrix of paired comparisons for level 3.

| Unity of measurements | Type Approval | Verification of measuring instruments | Metrological examination | Federal State Metrological Supervision | Attestation of measurement methods | Accreditation |
|-----------------------|---------------|----------------------------------------|--------------------------|----------------------------------------|------------------------------------|---------------|
|                       |               |                                        |                          |                                        |                                    |               |

Similarly, matrices of pairwise comparisons are formed for each level. The cells of the given matrices are filled with estimates or judgments about the relative importance of the compared individual subsystems (items) in relation to the goal or criterion indicated at the top. In this case, the matrix cells are planned to be filled with estimates obtained as a result of subjective, but thoughtful judgments of a group of experts who solve this problem. In this case, it is advisable to use the scale for measuring such judgments shown in table 3. The effectiveness of this scale is proved by a large number of applications.

Table 3. Relative importance Scale.

| Intensity of relative importance | Definition                                      | Explanations                                                   |
|---------------------------------|-------------------------------------------------|---------------------------------------------------------------|
| 1                               | Of equal importance                             | Equal contribution of two activities to the goal               |
| 3                               | Moderate superiority                           | Experience and judgment give an easy superiority to one activity over another |
| 5                               | Significant or strong superiority              | Experience and judgment give an considerable superiority to one activity over another |
| 7                               | A significant advantage                        | One type of activity is given such a strong superiority that it becomes significant |
| 9                               | Very strong superiority                        | The evidence of the superiority of one type of activity over another is most strongly confirmed |
| 2, 4, 6, 8                      | Intermediate decisions between two adjacent judgments | Apply in a compromise case                                    |

Inverse values of the above numbers

Let us consider the basic steps of data processing. Obtained in the course of paired comparisons a matrix of paired comparisons of dimension $n \times n$:
Calculating the vector of local priorities \( X = (x_1, x_2, \ldots, x_n) \):

\[
x_i = \frac{\sqrt{\prod_{j=1}^{n} w_j \times w_j \times \ldots \times w_j}}{\sum_{j=1}^{n} \sqrt{w_j \times w_j \times \ldots \times w_j}}.
\]  

(2)

The next step is to calculate the own value \( \lambda_{\text{max}} \) of matrix \( W \), the IP consistency index and the OS consistency ratio. To control the consistency, the following indicators are calculated:

\[
Y = \left( \sum_{j=1}^{n} w_j \sum_{j=1}^{n} w_j \sum_{j=1}^{n} w_j \ldots \sum_{j=1}^{n} w_j \right); \\
\lambda_{\text{max}} = X \cdot Y^T; \\
IP = \frac{\lambda_{\text{max}} - n}{n - 1}; \\
OS = \frac{IP}{IP_r},
\]

(3) (4) (5) (6)

where \( IP_r \) – the normalized consistency index which depends on matrix dimension \( n \) (table 4).

**Table 4.** Standard consistency index.

| Matrix size | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|-------------|----|----|----|----|----|----|----|----|----|----|
| \( IP_r \)  | 0  | 0  | 0.58 | 0.9 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.49 |

The \( OS \) value should not exceed 10%. If the \( OS \) goes beyond these limits, it is necessary to investigate the problem in more detail and check the judgments.

An example of formation of initial data by the expert for the Level 2 is presented in Table 5.

**Table 5.** Evaluation of the Uniformity of Measurement System Status: Matrix of paired comparisons for Level 2.

| State of the Uniformity of Measurement System | Unity of measurements | Required measurement accuracy | Completeness of measurements |
|---------------------------------------------|-----------------------|------------------------------|-----------------------------|
| Unity of measurements                       | 1.00                  | 3.00                         | 5.00                        |
| Required measurement accuracy                | 0.33                  | 1.00                         | 5.00                        |
| Completeness of measurements                 | 0.20                  | 0.20                         | 1.00                        |
As a result of processing the matrix of paired comparisons in accordance with the above-described formulas, we obtain indicators characterizing the degree of influence of the second level elements on the highest level element. An example of the impact assessment results calculated using formula 2 is shown in Figure 3.

Formulas 3–6 calculate the ratio of consistency for the matrix of judgments $OS = 12\%$, which exceeds 10\% and indicates that the judgment in the example needs to be reviewed.

![Figure 3. Indicators characterizing the influence of level 2 elements on the Uniformity of Measurement System Status.](image)

An example of filling the matrix of paired comparisons of the third level elements for the second level target «Unity of measurements» is presented in Table 6.

| Unity of measurements | Type Approval | Verification of measuring instruments | Metrological examination | Federal State Metrological Supervision | Attestation of measurement methods | Accreditation |
|-----------------------|---------------|---------------------------------------|--------------------------|---------------------------------------|----------------------------------|---------------|
| Type Approval         | 1.00          | 7.00                                  | 5.00                     | 5.00                                  | 3.00                             | 5.00          |
| Verification of measuring instruments | 0.14 | 1.00 | 7.00 | 5.00 | 5.00 | 5.00 |
| Metrological examination | 0.20 | 0.14 | 1.00 | 5.00 | 3.00 | 7.00 |
| Federal State Metrological Supervision | 0.20 | 0.20 | 0.20 | 1.00 | 3.00 | 0.33 |
| Attestation of measurement methods | 0.33 | 0.20 | 0.33 | 0.33 | 1.00 | 5.00 |
| Accreditation         | 0.20          | 0.20                                  | 0.14                     | 3.00                                  | 0.20                             | 1.00          |

An example of the influence indicator (2) for the second level target «Unity of measurements» is presented in Figure 4.

![Figure 4. Indicators characterizing the influence of level 3 elements on the Level 2 target «Unity of measurements».](image)
These calculations are performed for each level of the hierarchy, then a global priority vector is calculated. The resulting own vectors (one own value of each criterion) are weighed with the corresponding component of the own vector of the combined criterion, and the results are summed up and normalized.

Thus, application of the method of hierarchy analysis when assessing the state of the system for ensuring uniformity of measurement allows to determine weight (intensity) of influence of this or that subsystem on the system as a whole, which is necessary when determining priority directions of improvement of the metrological system.

References
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