Text S3: Integrating binocular term over frequency and orientation

The binocular term in the response of a single energy-model complex cell is

\[ B = 2(S_{L1}S_{R1} + S_{L2}S_{R2}) \]

\[ = 2 \int dx \int dy \int dx' dy'I_R(x, y)I_R(x', y') \exp \left( -\frac{(x-x_R)^2 + (y-y_R)^2}{2\sigma^2} \right) \right] \]

\[ \left[ \cos(k_x x + k_y y + \phi_L) \cos(k_x' x + k_y' y + \phi_R) + \sin(k_x x + k_y y + \phi_L) \sin(k_x' x + k_y' y + \phi_R) \right] \]

This cell is tuned to a spatial frequency and orientation specified by the wavenumbers \( k_x \) and \( k_y \) and has receptive fields centered at \((x_L, y_L)\) and \((x_R, y_R)\), with phases \( \phi_L \) and \( \phi_R \) respectively. We now compute the total response of many such cells tuned to many spatial frequencies and orientations, but all with the same receptive field centers and phases:

\[ B_{int} = \int Bdk, dk, \]

\[ = 2 \int dx \int dy \int dx' dy'I_R(x, y)I_R(x', y') \exp \left( -\frac{(x-x_R)^2 + (y-y_R)^2}{2\sigma^2} \right) \right] \]

\[ \left[ \cos(k_x x + k_y y + \phi_L) \cos(k_x' x + k_y' y + \phi_R) + \sin(k_x x + k_y y + \phi_L) \sin(k_x' x + k_y' y + \phi_R) \right] \]

Doing the innermost integral first, we obtain

\[ \int dk, dk, \left[ \cos(k_x x + k_y y + \phi_L) \cos(k_x' x + k_y' y + \phi_R) + \sin(k_x x + k_y y + \phi_L) \sin(k_x' x + k_y' y + \phi_R) \right] \]

\[ = \frac{1}{4} \int dk, dk, \left[ \exp i(k_x x + k_y y + \phi_L) + \exp -i(k_x x + k_y y + \phi_L) \right] \exp i(k_x' x + k_y' y + \phi_R) + \exp -i(k_x' x + k_y' y + \phi_R) \]

\[ - \exp i(k_x x + k_y y + \phi_L) \exp -i(k_x' x + k_y' y + \phi_R) + \exp i(k_x' x + k_y' y + \phi_R) \exp -i(k_x x + k_y y + \phi_L) \]

\[ + \exp i(k_x x + k_y y + \phi_L) \exp -i(k_x x + k_y y + \phi_L) \exp i(k_x' x + k_y' y + \phi_R) \exp -i(k_x' x + k_y' y + \phi_R) \]

\[ = \frac{1}{2} \int dk, dk, \left[ \exp i(k_x x + k_y y + \phi_L) - \exp i(k_x x + k_y y + \phi_L) \right] \exp i(k_x' x + k_y' y + \phi_R) \exp i(k_x' x + k_y' y + \phi_R) \]

\[ = \frac{1}{2} \exp i(\phi_R - \phi_L) \int dk, dk, \left[ \exp i(k_x x + k_y y + \phi_L) - \exp i(k_x x + k_y y + \phi_L) \right] \exp i(k_x x + k_y y + \phi_L) \exp i(k_x x + k_y y + \phi_L) \]

\[ = \frac{1}{2} \exp i(\phi_R - \phi_L) \int dk, dk, \left[ \exp i(k_x x + k_y y + \phi_L) + \exp i(k_x x + k_y y + \phi_L) \right] \exp i(k_x x + k_y y + \phi_L) \exp i(k_x x + k_y y + \phi_L) \]

\[ = \frac{1}{2} \left( e^{i\phi_R + i\phi_L} - e^{-i\phi_R - i\phi_L} \right) \delta(x - x') \delta(y - y') = \cos(\Delta\phi) \delta(x - x') \delta(y - y') \]

where \( \Delta\phi = \phi_R - \phi_L \) is the phase disparity of the cells. Using this result in the equation for the integral of \( B \) gives us:

\[ B_{int} = \int Bdk, dk, = 2 \cos(\Delta\phi) \int dx \int dy \exp \left( -\frac{(x-x_R)^2 + (y-y_R)^2}{2\sigma^2} \right) I_R(x, y) \exp \left( -\frac{(x-x_R)^2 + (y-y_R)^2}{2\sigma^2} \right) I_R(x, y) \]