Contact problem for a solid indenter and a viscoelastic half-space described by the spectrum of relaxation and retardation times

To cite this article: F I Stepanov 2018 J. Phys.: Conf. Ser. 991 012076

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Contact problem for a solid indenter and a viscoelastic half-space described by the spectrum of relaxation and retardation times

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Abstract. The mechanical properties of a material which is modeled by an exponential creep kernel characterized by a spectrum of relaxation and retardation times are studied. The research is carried out considering a contact problem for a solid indenter sliding over a viscoelastic half-space. The contact pressure, indentation depth of the indenter, and the deformation component of the friction coefficient are analyzed with respect to the case of half-space material modeled by single relaxation and retardation times.

1. Introduction

3D contact problems for a viscoelastic half-space interacting with hard bodies has been considered by many researchers in different statements of the problem [1–8]. Many of the problems are solved by the boundary element method (BEM) that allows one to consider any type of material model including a viscoelastic half-space described by a spectrum of relaxation and retardation times. Nevertheless, no analysis has still been made for the mechanical properties of a viscoelastic half-space described by a spectrum of relaxation times. The purpose of this research is to study the contact parameters of a solid indenter sliding over a viscoelastic half-space and to compare them with the case where the half-space is described by a single relaxation and retardation time.

2. Statement of the problem

We consider a solid indenter sliding over a viscoelastic half-space with a constant velocity $V$. The indenter is loaded by a vertical force $Q$ and a tangential force $T$ which ensure constant moving (figure 1). The material model is specified by the Volterra integral operator that defines
the relationship between strains \((\gamma(t), e(t))\) and stresses \((\tau(t), \sigma(t))\) in the form

\[
\gamma(t) = \frac{1}{G} \tau(t) + \frac{1}{G} \int_{-\infty}^{t} \tau(t)K(t-\tau)d\tau,
\]

\[
e_x(t) = \frac{1}{E} \{\sigma_x(t) - \nu[\sigma_y(t) + \sigma_z(t)]\} + \frac{1}{E} \int_{-\infty}^{t} \{\sigma_x(t) - \nu[\sigma_y(t) + \sigma_z(t)]\}K(t-\tau)d\tau,
\]

\[
e_y(t) = \frac{1}{E} \{\sigma_y(t) - \nu[\sigma_x(t) + \sigma_z(t)]\} + \frac{1}{E} \int_{-\infty}^{t} \{\sigma_y(t) - \nu[\sigma_x(t) + \sigma_z(t)]\}K(t-\tau)d\tau,
\]

\[
e_z(t) = \frac{1}{E} \{\sigma_z(t) - \nu[\sigma_x(t) + \sigma_y(t)]\} + \frac{1}{E} \int_{-\infty}^{t} \{\sigma_z(t) - \nu[\sigma_x(t) + \sigma_y(t)]\}K(t-\tau)d\tau,
\]

\[
K(t') = \sum_i k_i \exp \left( -\frac{t'}{\lambda_i} \right),
\]

where \(G\) and \(E\) are the shear modulus and the modulus of elasticity, \(\nu\) is the Poisson ratio, \(\lambda_i\) and \(1/k_i\) are the retardation and relaxation times, and \(K(t')\) is the creep kernel.

The following contact conditions are met:

\[
\begin{align*}
z &= 0: \quad \tau_{xz}(x,y) = 0, \quad \tau_{yz} = 0, \quad w(x,y) = f(x,y) + D, \quad (x,y) \in \Omega, \\
\sigma_z &= 0, \quad \tau_{xz} = 0, \quad \tau_{yz} = 0, \quad (x,y) \notin \Omega, \\
-\infty < x < +\infty, \quad -\infty < y < +\infty.
\end{align*}
\]

Here \(D\) is the indentation depth, \(w(x,y)\) is the surface vertical displacement, and \(\Omega\) is the unknown contact region to be determined by solving the contact problem. The indenter shape is described by the function

\[
f(x,y) = \frac{x^2 + y^2}{2r},
\]

where \(r\) is the indenter radius. The equilibrium condition is considered in the form

\[
Q = \iint_{\Omega} p(x,y) \, dx \, dy.
\]
The relation between the contact pressure and the surface displacement is given by the equation [8]

\[ w(x, y, 0) = \frac{1 - 2\nu}{4\pi G} \int_{\Omega} \tau_{xz}(\xi, \eta) \left[ \frac{x - x}{R^2} + \frac{1}{V} \sum_{i=1}^{n} k_i I_1 \left( \frac{\xi - x}{\lambda_i V}, \frac{\eta - y}{\lambda_i V} \right) \right] d\xi d\eta - \frac{1 - \nu}{2\pi G} \int_{\Omega} q(\xi, \eta) \left[ \frac{1}{R} + \frac{1}{V} \sum_{i=1}^{n} k_i I_2 \left( \frac{\xi - x}{\lambda_i V}, \frac{\eta - y}{\lambda_i V} \right) \right] d\xi d\eta, \quad (5) \]

The contact problem is solved by the BEM. The solution method is presented explicitly in [6, 7]. An arbitrary area containing the contact area is a priori divided into square elements of constant size. The contact problem is solved again. The iteration procedure stops when the solution contains no elements with negative pressure. The deformation component of the friction coefficient arises due to asymmetric distribution of the contact pressure and can be obtained after solving the contact problem. It is determined by the equation:

\[ \mu' = \frac{\iint_{R} xp(x, y) \, dx \, dy}{\iint_{R} p(x, y) \, dx \, dy}. \quad (7) \]

### 3. Calculation results

The numerical results are used to analyze the contact pressure distribution, the indentation depth, and the deformation component of the frictional force in the case where the material model is determined by three relaxation and retardation times. The following dimensionless parameters where used in the calculations:

\[ (x', y', \xi', \eta') = \frac{1}{r} (x, y, \xi, \eta), \quad c = \lambda_i k_i = \frac{G}{G_i}, \quad A_i = \frac{r}{\lambda_i V}, \quad B_i = k_i \frac{r}{V}, \quad Q' = \frac{Q}{G_i r^2}, \quad p'(x, y) = \frac{p(x, y)}{G_i}, \quad V' = \frac{V}{r}. \quad (8) \]

Figure 2 illustrates the normal displacement of the surface due to the impact of a moving load distributed inside the square element with center at (0,0). The area of the square element is \( S' = 1/900 \), the pressure inside the element is constant (\( p' = 1.0 \)). The lines in the graph represent the surface displacement considering \( y' = 0 \) for three different relaxation times and for the spectrum consisting of the above-mentioned relaxation times. Figure 2 clearly presents the effect of retardation (\( \lambda \)) and relaxation times (1/\( k \)). The less the retardation time (\( \lambda \)), the greater the surface displacement that occurs due to the moving load. In the case of a smaller value of the relaxation time (1/\( k \)), the surface displacement decreases more intensively after passing of the distributed load. Line 4 corresponds to the spectrum of relaxation times and shows the...
Figure 2. Normal displacement of the surface. Calculations for following parameters: $A = 5$, $P' = 1.0$, $\nu = 0.3$, $V' = 3$; $\lambda = 0.005$, $k = 1000$ for line 1; $\lambda = 0.001$, $k = 5000$ for line 2; $\lambda = 0.0002$, $k = 25000$ for line 3; $\lambda_{1,2,3} = 0.0002, 0.001, 0.005$, $k_{1,2,3} = 25000, 5000, 1000$ for line 4.

The greatest surface displacement due to the moving load. The surface displacement decreases after passing of the distributed load more gradually than in the case of a single relaxation time of the smallest value.

The dependence of the mechanical component of the friction coefficient on the sliding velocity was studied. The parameter $c = 5$ which is the relation between the instant and longitudinal shear moduli is assumed to be constant. The calculations were carried out for the materials described by a creep kernel with a single relaxation time (figure 3, lines 1–3) and for the spectrum of relaxation times (figure 3, line 4). The maximum value of the mechanical component of the friction coefficient does not depend on the relaxation time, though it appears at different sliding velocities relevant to the relaxation time (lines 1–3). In the case of a greater relaxation time, the maximum of the friction coefficient appears at a smaller sliding velocity. The friction coefficient attains its maximum and then decreases more gradually in the case of relatively small values of the relaxation time.

When the material is characterized by the spectrum of relaxation times, the value of the mechanical component of the friction coefficient is greater than that in the case of a material characterized by a single relaxation time.

The dependence of the indentation depth on the sliding velocity is presented in figure 4. The calculations were made using the same parameters as in figure 3. Lines 1–3 present the indentation of the indenter into the half-space described by a single relaxation time. Line 4 corresponds to the spectrum of relaxation times. The indentation of a body is the smallest in the case of a material characterized by the greatest relaxation time. In the case of the spectrum of relaxation times, the indentation is the greatest of all the cases under study. The indentation depth decreases more rapidly as the sliding velocity increases in the case of a greatest value of
Figure 3. Mechanical component of friction. Calculations for following parameters: $A = 5$, $Q' = 0.06$, $\nu = 0.3$; $\lambda = 0.005$, $k = 1000$ for line 1; $\lambda = 0.001$, $k = 5000$ for line 2; $\lambda = 0.0002$, $k = 25000$ for line 3; $\lambda_{1,2,3} = 0.0002, 0.001, 0.005$, $k_{1,2,3} = 25000, 5000, 1000$ for line 4.

Figure 4. Indentation depth. Calculations for following parameters: $A = 5$, $Q' = 0.06$, $\nu = 0.3$; $\lambda = 0.005$, $k = 1000$ for line 1; $\lambda = 0.001$, $k = 5000$ for line 2; $\lambda = 0.0002$, $k = 25000$ for line 3; $\lambda_{1,2,3} = 0.0002, 0.001, 0.005$, $k_{1,2,3} = 25000, 5000, 1000$ for line 4.

the relaxation time.
Conclusions
The mechanical properties of viscoelastic half-space modeled by different relaxation times and the spectrum being a combination of the above-mentioned relaxation times have been studied. The calculations were carried out for the surface displacement due to the impact of a distributed load moving over a viscoelastic half-space and for a solid indenter sliding with a constant velocity over the viscoelastic half-space. The performed analysis allowed making the following conclusions:

• under the assumption that the relation between the instant and longitudinal shear moduli is constant, the maximum of the mechanical component of the friction coefficient is the same for different relaxation times, it occurs at different sliding velocities (the smaller the relaxation time, the greater the velocity);
• in the case where the spectrum of relaxation times is considered, the maximum value of the mechanical component of the friction coefficient is greater than that in the cases of a single relaxation time;
• the mechanical component of the friction coefficient decreases more rapidly with velocity when the value of the relaxation time is greater;
• the indentation depth is greater in the case of a smaller value of the relaxation time and it is the greatest in the case of a relaxation time spectrum;
• the indentation depth decreases more rapidly with the velocity in the case of a greater relaxation time.

Acknowledgments
This work was supported by the Russian Science Foundation (grant No. 14-29-00198).

References
This work was supported by the Russian Science Foundation (grant No. 14-29-00198).

[1] Flom D G and Bueche A M 1959 Theory of rolling friction for spheres J. Appl. Phys. 30 (11) 1725–30
[2] Panek C and Kalker J J 1977 A solution for the narrow rectangular punch J. Elasticity 7 (2) 213–8
[3] Panek C and Kalker J J 1980 Three-dimensional contact of a rigid roller traversing a viscoelastic half-space IMA J. Appl. Math. 26 (2) 299–313
[4] Kusche S 2016 Frictional force between a rotationally symmetric indenter and a viscoelastic half-space ZAMM 97 (2) 226–39
[5] Stepanov F I and Torskaya E V 2016 Study of stress state of viscoelastic half-space in sliding contact with smooth indenter J. Frict. Wear 37 (2) 101–6
[6] Goryacheva I G, Stepanov F I, and Torskaya E V 2015 Sliding of a smooth indenter over a viscoelastic half-space when there is friction J. Appl. Math. Mech. 79 (6) 596–603
[7] Goryacheva I G, Stepanov F I, and Torskaya E V 2016 Effect of friction in sliding contact of a sphere over a viscoelastic half-space In Computational Methods in Applied Sciences vol 40 Mathematical Modeling and Optimization of Complex Structures (Springer) pp 93–103
[8] Alexandrov V M and Goryacheva I G 2005 Mixed problems of mechanics of deformable solid In Mixed Problems in Solid Mechanics: Proc. of V Russian Conf. with Int. Participation ed N F Morozov (Saratov: Izdat. Sarat. Univ.) pp 23–5 [in Russian]