Searching for black hole echoes from the LIGO-Virgo Catalog GWTC-1

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We have examined gravitational wave echo signals for nine binary black hole merger events observed by Advanced LIGO and Virgo during the first and second observation runs (O1, O2) [1–5]. Waveforms of gravitational waves of binary black hole mergers can be divided into three phases, inspiral, merger and ringdown. Ringdown phase is an important part to analyze the properties of the remnant objects. From black hole quasinormal modes in the ringdown phase, we can estimate the spin and mass of the remnant black holes [6]. And if we can also observe higher multipole modes, we can test the no-hair theorem of black holes [7]. So far, the observed ringdown phase signals are not significant enough to test the above issues, although the dominant mode is consistent with the expectation from the inspiral phase within the current detector sensitivity [8]. There is a proposal to enhance the ringdown data analysis including overtones of the quasinormal modes [9, 10].

Looking at the data succeeding to the ringdown phase, we might be able to tell whether the remnant object is a black hole or a horizonless compact object [11, 12], such as the gravastar [13] or the firewall [14]. (See Ref. [15] for details on testing exotic compact objects.) These horizonless objects are considered to be as compact as black holes with a surface located at the Planck scale outside the horizon radius due to quantum modifications. In general, the spacetime is different from black hole spacetime if the horizon does not exist. Besides the difference in the spacetime structure, the most significant difference between black holes and these horizonless objects, compact enough to possess a light ring, in the post ringdown phase is the presence of “echoes”. If the event horizon is replaced by a surface, we can expect that the merger-ringdown waveform will be reflected at the surface. Then the waveform will be partly transmitted at the angular momentum barrier and partly reflected, which will result in observable gravitational wave echoes. Abedi et al. [16] have searched for gravitational wave echoes using three binary black hole mergers observed during LIGO O1. They consider the simplest model, the horizon is replaced by a reflecting membrane at ~ Planck proper length outside the event horizon radius in Kerr spacetime. They reported echo signals at 3σ significance (0.011 in p-value) from 32 seconds data around each binary black hole event. However, Asthon et al. [17] have pointed out some problems in the analysis done by Abedi et al. and Westerweck et al. [18] have improved the background estimation using 4096 seconds data around each binary black hole event, which gave lower significance, 0.032 in p-value, than in Abedi et al. Using the same template waveform given in Abedi et al., Nielsen et al. [19] and Lo et al. [20] have also shown lower significance on echo signals evaluated by Bayes factor using Bayesian analysis, where Lo et al. have included the inspiral-merger-ringdown waveform into the template as well. Injection studies given in [18–20] have shown that echo signals with their amplitude of the first echo larger than about 15% of the merger amplitude are detectable within the current detector sensitivity.

Improvement of the evaluation of the significance is important, but we can also improve the template waveform used in Abedi et al.. If the spacetime outside the
reflecting surface is entirely Kerr spacetime, we can exactly calculate the reflection rate and the phase shift due to the reflection at the potential barrier, while the reflection rate is assumed to be a frequency-independent parameter and the overall phase shift is fixed to $\pi$ in Abedi et al. The frequency-dependent reflection rate and the phase shift at the potential barrier were calculated numerically by Nakano et al. [21] in this setup. The reflection rate also gives a frequency-dependent transmission rate, which affects the template waveform as well. In this study, we analyze the echo signals using this reflection rate that is fitted for $0.6 \leq q \leq 0.8$, where $q$ is the nondimensional Kerr parameter. Although the phase shift at the barrier can be calculated exactly, that at the reflective membrane is uncertain, i.e., model dependent. Here we leave the frequency-independent overall phase shift as a free parameter. Construction of a seed waveform is the same as Abedi et al.. Based on this template, we search for gravitational wave echo signals for binary black hole merger events observed by LIGO and Virgo during O1 and O2. And we use 4096 seconds data for each event to perform background estimation, adopting the same method done in Westerweck et al.

In this study and in the previous studies mentioned above, a perfect reflection at the membrane is assumed. However, the reflection at the membrane is also model dependent. Template waveforms for more general reflection rate at the membrane are considered in [22, 23]. A morphology-independent analysis is also proposed [24] and phenomenological templates are proposed in [25]. The validity of the constant echo interval is discussed in [26, 27]. Recent studies also provide models of gravitational wave echoes based on black hole area quantization [28], and quantum black holes [29, 30].

The paper is organized as follows. In Section 2, we explain the template waveform used in our analysis. In Section 3, we describe the method of analysis to evaluate the significance of echo signals using open LIGO data. In Section 4, we show the results of our analysis. We use p-values to evaluate the significance. And conclusions and discussions are given in Section 5.

II. TEMPLATE WAVEFORM BASED ON BLACK HOLE PERTURBATIONS

In this study, we consider a situation in which the spacetime is entirely Kerr spacetime but a reflective membrane is located at about Planck length away from the event horizon radius. In such a case, after binary black holes merge, the merger-ringdown part of waves will be partly reflected both at the membrane and the angular potential barrier of the Kerr spacetime. Every time when the waves are reflected at the potential barrier, a part of them will be transmitted through the barrier to escape to infinity, which we may observe as gravitational wave echoes. Therefore, echo waveforms are basically characterized by the reflection rates at the membrane and the potential barrier, and the time interval of echoes $\Delta t_{\text{echo}}$, which corresponds to twice the proper distance between the membrane and the potential barrier. We assume a perfect reflection at the membrane, which is the same assumption as given in [16], that is, we take into account only the reflection rate at the potential barrier.

Abedi et al. [16] assumed the reflection rate at the potential barrier is given by a frequency-independent parameter. However, if we assume the spacetime is entirely Kerr spacetime, the reflection rate can be calculated by solving the perturbation equations with appropriate boundary conditions, i.e., only outgoing waves at the spatial infinity and total reflection of waves at the membrane [21]. The reflection rate obtained from black hole perturbations depends on frequency. In this paper, we use the reflection rate $R(f)$ given in [21].

$$R(f) \approx \begin{cases} 
1 + e^{-300(x+0.27-q)} + e^{-28(x-0.125-0.6q)} & (f > 0), \\
1 + e^{-300(x+0.27-q)} + e^{-28(x-0.125-0.8q)} + e^{19(x-0.3-0.35q)} & (f < 0).
\end{cases} \quad (2.1)
$$

Here $x = 2\pi Mf$ and $q = a/M$ with black hole spin $a$ and mass $M$ in $c = G = 1$ units. Reflection rate $R(f)$ in Eq. (2.1) is a fit for numerically calculated one for $0.6 \leq q \leq 0.8$, in which the remnant spin of the binary black holes observed by LIGO and Virgo varies. The time interval between neighboring echoes $\Delta t_{\text{echo}}$ is evaluated following the formalism given in [16]

$$\Delta t_{\text{echo}} = 2 \int_{r_+ + \Delta r}^{r_+ + \Delta r} \frac{r^2 + a^2}{r^2 - 2Mr + a^2} dr, \quad (2.2)$$

where $r_{\text{max}}$ is the peak of the angular momentum barrier and $\Delta r$ is the location of the membrane away from the horizon, $r_+$. In the previous studies [16, 18–20], the frequency-independent reflection rate and $\Delta t_{\text{echo}}$ are assumed to be parameters. In our case, since both the
reflection rate and $\Delta t_{\text{echo}}$ depend on $a$ and $M$, we set $(a, M)$ as parameters instead of $[R(f), \Delta t_{\text{echo}}]$. Then the echo template waveform, including $N$ echoes, in the frequency domain $\tilde{h}(f)$ is given by

$$\tilde{h}(f) = \sqrt{1 - R^2(f)} \, h_0(f) \sum_{n=1}^{N} R(f)^{n-1} e^{-i2\pi f \Delta t_{\text{echo}} + \phi(n-1)}, \quad (2.3)$$

where $\phi(f)$ is the overall phase shift due to the reflections at the membrane and the potential barrier and $h_0(f)$ is a seed waveform in the frequency domain. Note that $\sqrt{1 - R^2(f)}$ is the transmission rate at the barrier. As for the seed waveform in the time domain, we adopt

$$h_0(t) = \frac{1}{2} \left\{ 1 + \tanh \left[ \frac{1}{2} \omega(t)(t - t_{\text{merger}} - t_0) \right] \right\} \times h_{\text{bas}}(t), \quad (2.4)$$

where $h_{\text{bas}}(t)$ is the best fit inspiral-merger-ringdown waveform for each event and $\Theta(t; t_0, \omega)$ is a cutoff function given in [16], and $t_{\text{merger}}$ is the merger time of the binary black hole. The cutoff function is determined by a cutoff parameter $t_0$ and the typical frequency around the merger time $\omega$. The cutoff parameter is also a parameter in [16] [18-20], however, since it is insensitive to the signal-to-noise ratio defined in the next section, to save the computational cost, we set $t_0$ as a constant. Following the best fit value of $t_0$ obtained in [16], we set $t_0 = -0.084 \Delta t_{\text{echo}}$ for GW150914 and $t_0 = -0.1 \Delta t_{\text{echo}}$ for the rest of the events. The phase shift at the potential barrier can be also calculated from Teukolsky equations [21]. However, since the phase shift at the membrane is highly model dependent, we take the overall phase shift $\phi(f)$ as a parameter as well. Basically, $\phi(f)$ depends on the frequency, however, we can approximate it as a linear function $\phi(f) = \phi_0 + \phi_1 f$ [21]. Then, the coefficient of the linear part $\phi_1$ can be absorbed by the parameter $\Delta t_{\text{echo}}$ and we only need to consider the zeroth order coefficient $\phi_0$. For $q = 0.7$, the frequency dependence of $\phi(f)$ due to the reflection at the barrier is shown in Fig. 1 in Ref. [21], and we can see that $\phi(f)$ only weakly depends on frequency around the quasinormal mode frequency. This fact will partly justify to replace $\phi(f)$ in Eq. (2.3) with a constant parameter $\phi_0$. We stress that one important difference from previous studies is that $h_0(t)$ here must be the complex template having the unobserved polarization mode in the imaginary part. In case we restrict the phase shift $\phi$ to 0 or $\pi$, the other polarization mode does not affect the echo signal, and hence the imaginary part of $h_0(t)$ is unnecessary.

In summary, three parameters $(a, M, \phi_0)$ are considered in the template in our analysis. Template is 32 seconds long including 30 echoes.

As an example, we show the reflection rate and the spectrum of the best fit template in our analysis and estimated average spectral densities for Hanford and Livingston detectors for GW150914 in Fig. 1. From the bottom panel of Fig. 1 we can see that amplitude of the template at a lower frequency is suppressed compared to the template given in [16]. This is because the transmission rate at the barrier is included in the template waveform Eq. (2.3). Since $R(f) \sim 1$ for $f < 200$ Hz in the case of Fig. 1, echoes at those frequencies are strongly suppressed compared with the seed template.

We further assume unknown dissipative effect so that the superradiant amplification does not occur, which is too small to affect our analysis, though.

**III. METHOD OF ANALYSIS**

**A. Analyses to search for echo signals**

We first search for the echo signals right after the binary black hole merger. We use a matched filter analysis
to evaluate the signal-to-noise ratio (SNR) $\rho$ defined as

$$\rho = (x|h) = 4\text{Re} \left( \int_{f_{\text{min}}}^{f_{\text{max}}} \frac{\tilde{x}(f)\tilde{h}^*(f)}{S_n(f)} \right),$$  \hspace{1cm} (3.1)$$

where $\tilde{x}(f)$ is the Fourier transformation of the observed data, $\tilde{h}(f)$ is the template in the frequency domain, and $S_n(f)$ is the noise power spectrum of a detector. We set $f_{\text{max}} = 4096$ Hz and $f_{\text{min}} = 40$ Hz, and we normalize the template so that $\langle h|h \rangle = 1$. We use the first 1024 seconds of the 4096 seconds data for each event to estimate the noise power spectrum using Welch’s method. Theoretically, the first echo should be at a specific time length from the merger. As described in Ref. [16], we search for the maximum value of SNR in the range

$$0.99 \leq T \equiv (t_{\text{echo}} - t_{\text{merger}})/\Delta t_{\text{echo}} \leq 1.01,$$  \hspace{1cm} (3.2)$$

where $t_{\text{echo}}$ is the starting time of the first echo. The merger time of the binary black hole merger is determined by analyzing each event by the inspiral-merger-ringdown waveform $h_{\text{tot}}(t)$, and $t_{\text{echo}}$ such that the SNR becomes maximum in the time interval Eq. (3.2) is determined by a matched filter analysis. We search for the best fit values varying three template parameters $(a, M, \phi_0)$. The search regions of $(a, M)$ are 90% credible regions estimated by the LIGO and Virgo collaborations for GW150914, GW151012, GW151226, and GW170104 and by our reanalysis using LIGO Algorithmic Library (LALInference) for the other events. Similarly, we use the best fit values for the inspiral-merger-ringdown template $h_{\text{tot}}(t)$ to construct the seed waveform as in Eq. (2.4) given by LIGO tutorial for the above four events and by our reanalysis using LALInference for the other events. Signal-to-noise ratio is also maximized for the initial phase of the template $\theta_{\text{init}}$, which can be obtained automatically by orthogonal templates for each $(a, M, \phi_0)$. This is not considered in the previous studies.

We use data of Hanford and Livingston detectors. To evaluate the network SNR, we sum the square of SNRs of respective detectors. This means that we basically perform a single detector search.

### B. Background estimation, data and software

Background estimation is necessary to evaluate the significance of the candidate obtained at the event data segment. We follow the method given in Ref. [18]. We divide the 4096 seconds data into 32-second data segments and perform the same analysis shown in the previous subsection for the all remaining data segments. Then, we count the number of the data segments which give the same or higher SNR obtained in the event data segment, and the p-value is defined as the ratio of the number of all segments.

There are two versions of LIGO open data for noise subtraction data, C01 and C02 [35]. Since the data for all observed events in O1 and O2 are given in C02 version, it may be reasonable to use only C02 data. However, 4096 seconds data of the Hanford detector is not available for GW151226. For comparison with previous works, we also use C01 for four events, GW150914, GW151012, GW151226, and GW170104. Note that for GW170809, 4096 seconds data is not available for Livingston detector, so we do not include this event. That is, we analyze nine binary black hole merger events observed in O1 and O2.

As mentioned in Ref. [18], to use 4096 seconds data for the background estimation, the data quality should be homogeneous throughout the period. It is confirmed that the variations of data quality are small for GW150914, GW151012, GW151226, and GW170104 in Ref. [18], and we confirm small variations of the noise level for the rest of five events.

Also, as mentioned in Ref. [18], a short transient noise feature is observed in the beginning of the data of GW151012. Therefore, we exclude some data located in the beginning of 4096 seconds data. The total number of the reference data segments is 127 for all events.

We use Tukey window with a parameter $\alpha = 1/8$ to cutoff the edges of time series data for all segments. Since we do not want to lose the expected echo signals by the window function, we put the merger time around 8 seconds from the beginning of 32 seconds data segment for the event segments.

We use KAGRA Algorithmic Library (KAGALI) partly to perform the analysis.

### IV. RESULTS

We summarize the results of p-values in Table I. The results are divided into two data versions, C01 and C02. Hyphen means that 4096 seconds data is not available. In general, the critical p-value is 0.05 or 0.01. In our case, if the p-value is below (above) those values, then echo signals are likely (unlikely) to present in the data. Our results show that p-values for all events and the combined p-value well exceed this critical value, that is, echo signals modeled within our framework do not exist in the data, or the amplitude of the signals are too small to be detected within the current detector sensitivity.

In our analysis, we also consider the best fit of the initial phase of the template $\theta_{\text{init}}$, which is different from the previous studies [16–18], so it might be inappropriate to compare the results directly. However, we also analyze echo signals using the same template as in Abedi et al. [16] and probably with the same condition for the analysis, the results and comparison to those given by Westerweck et al. [18] are shown in Appendix A.
ditionally analyze with this template for the O2 events, which gives similar p-value as that of O1 events. Results are shown in Appendix A.

We show the detail of the behavior of SNR in Fig. 2 for the case of the best fit parameters of GW150914 (C01) as an example. Solid, dashed, and dotted lines correspond to $\rho^2$ for combined (Hanford and Livingston), Hanford, and Livingston, respectively. We can see a peak for the combined and Livingston cases near $T \sim 1$, however, the peak of the Hanford case is located slightly outside the interval of Eq. (3.2). The figure shows that $\rho^2$ oscillates slowly against $T$ compared to Fig. 7 in Ref. [16], because we consider the best fit initial phase of the template as well.

To see the effect of including frequency-independent phase shift for the reflections as a parameter, we also analyze the case when only the phase inversion is considered for C01 data. The results are given in Appendix B. Significance becomes lower if the phase shift is not fixed except GW151226.

| Data version | Event   | C01 | C02 |
|--------------|---------|-----|-----|
| GW150914     | 0.992   | 0.984|
| GW151012     | 0.646   | 0.882|
| GW151226     | 0.276   |
| GW170104     | 0.717   | 0.677|
| GW170608     | -       | 0.488|
| GW170729     | -       | 0.575|
| GW170814     | -       | 0.472|
| GW170818     | -       | 0.976|
| GW170823     | -       | 0.315|
| total        | 0.976   | 0.921|

TABLE I. P-values for each event and total p-value. Hyphen means that 4096 seconds of data is not available.

V. CONCLUSIONS AND DISCUSSIONS

We have searched for gravitational wave echo signals for nine binary black hole merger events observed by advanced LIGO and Virgo during the first and second observation runs. We assume that the spacetime is entirely Kerr spacetime except that a reflective membrane is located near the event horizon radius. We use the template waveform given by Nakano et al. [21], in which the reflection rate and the phase shift at the potential barrier due to the angular momentum are calculated from Teukolsky equations. We assume a perfect reflection at the membrane, however, the phase shift at the membrane due to reflection is model dependent, so we assume the frequency-independent phase shift at both the membrane and the potential barrier as a parameter. Transmission rate given from the reflection rate strongly suppresses the lower frequencies contained in the seed waveform. As well as the echo parameters, we maximized the signal-to-noise ratio against the initial phase of the template. We used adjacent 4096 seconds data from open LIGO data for the background estimation, and evaluated the significance by p-values. We found no significant echo signals within our analysis. Since the method of analysis is slightly different from the analyses in the previous studies [16–18], we cannot compare our results to theirs directly, but our results suggest that the suppression of the lower frequency part in the template may affect the p-value.

As mentioned in Introduction, previous injection studies [18–20] show that if the amplitude of the first echo is larger than 15% of the peak amplitude of the binary black hole merger, echo signals can be detected by the current detector sensitivity, assuming frequency-independent reflection rate. Combined with their studies, our results suggest that the amplitude of echo signals should be much smaller than the peak amplitude of the merger even if echo signals exist.

However, p-values are much smaller when we use the template given by Abedi et al. for both O1 events and O2 events. This may imply that the waveform of Abedi et al. ‘s template is favored than that of the template in our analysis, although our assumption is physically appropriate if we assume Kerr spacetime. Signals similar to the Abedi et al. ‘s template might be produced from non Kerr spacetime or unknown exotic physics, or instrumental reactions of the detector.

The third LIGO and Virgo run started in April 2019 and about ten candidates of binary black hole mergers have been observed so far in the first two months [37]. Some of them might have higher SNR than the events observed in O1 and O2 do, which may enable us to detect echoes or to constrain their amplitude further. To
do so, it may be useful to analyze coherently the data from more than two detectors, besides improving the echo template.

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**Appendix A: Analysis using the template of Abedi et al.**

In this Appendix, we show the results using the same template given in Abedi et al. [16]. Here we fix the cutoff parameter $t_0$ as described in Sec. II, and we set $\Delta_{\text{echo}}$ and a frequency-independent reflection rate $\gamma$ as free parameters. The initial phase of the template is fixed to zero.

1. **O1 events (Reanalysis of Westerweck et al.)**

Since we follow Westerweck et al. [18] for the background estimation, it would be appropriate to compare our results with theirs. Table [1] shows the results of p-values of three O1 events. The results of Westerweck et al. are denoted as AEI. The Poisson errors of p-value for
GW151226 and GW151012 are not given in [18], so we estimate the errors from p-value and the number of segments they use. We can see that both results are almost consistent within the Poisson errors for all events. Since we use 32-second template while Westerweck et al. only show the results of 16-second template for GW170104, we do not compare the results of this event here.

| Event       | AEI [18]       | ours          |
|-------------|----------------|---------------|
| GW150914    | 0.238 ± 0.043  | 0.157 ± 0.035 |
| GW151012    | 0.063 ± 0.022  | 0.047 ± 0.019 |
| GW151226    | 0.476 ± 0.061  | 0.598 ± 0.069 |
| total       | 0.032 ± 0.016  | 0.055 ± 0.021 |

TABLE II. P-values and Poisson errors for O1 events.

2. O2 events

We also analyze for O2 events. For GW170104, we use C01 data and for the other events, we use C02 data. We set the search region of \( \Delta t_{\text{echo}} \) from the 90% credible regions of \((a, M)\) as described in Sec. III A. P-values are given in Table III. As shown in Table III, the total p-value for the six O2 events is 0.039, while when combining with O1 events shown in Table II then the p-value for nine events becomes 0.047.

| Event       | result 1 | result 2 |
|-------------|----------|----------|
| GW150914    | 0.638    | 0.992    |
| GW151012    | 0.417    | 0.646    |
| GW151226    | 0.953    | 0.276    |
| GW170104    | 0.213    | 0.717    |
| total       | 0.528    | 0.976    |

TABLE IV. P-value for each event and total p-value. Result 1 is the case when the phase shift is fixed to \( \pi \) and result 2 is the case when the total phase shift is also a parameter.

Appendix B: Effect of the phase shift due to the reflection

As mentioned in Sec. II, the phase shift at the potential barrier can be calculated numerically. However, since the phase shift at the membrane is model dependent, it is physically reasonable to assume the total phase shift as a parameter. In the previous studies [16, 18], only phase inversion at the membrane is considered. So in this section, we compare the results of two cases, when the phase shift is fixed to \( \pi \) (result 1) and when it is a free parameter (result 2), respectively, in Tab. IV. The template given in Eq. (2.3) is used. Except for GW151226, p-values become slightly larger when the phase shift due to the reflection is left as a free parameter, which we believe is a more physical condition.