Linear modal analysis of ring shaped - elliptical plate using finite element method

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Abstract. In this paper, the wave propagation of a transversely isotropic thin ring - shaped elliptical plate made up of piezo electric material is considered. The surfaces are assumed to be traction free and coated with electrodes. The trail functions are introduced to uncouple the equations of motion and electric property. A computer program solving the finite element method (FEM) is developed and the numerical calculations are given for an array of six-noded shell elements. The non-dimensionalized natural frequencies of the elliptical plate in different wave numbers under different boundary condition for symmetric and antisymmetric modes are calculated and are plotted as dispersion curves. The proposed model is effective and simple and can be applied to problems with exact solution or a finite element approximation to the governing equations.

1. Introduction

Theoretical modeling and simulation of smart structures is an active research since the early 1990’s. Most of the investigation was done under the assumption of the time-independent material properties, which limits the applicability of the solutions obtained to certain ranges of temperature. The frequency analysis of elliptical shaped plate has many applications in various fields of science and technology such as design of structure, thermal power plants, supersonic aircraft, space shuttle and other devices. In the multi field problems, the applications of mechanical stress induce electric charge is referred as piezoelectric. These piezo electric materials are mainly used as sensors and actuators. They are applied in high voltage power resources like gas lighter and gas burners. In most of the research works, modal analysis for objects with arbitrary shapes has been performed using the numerical methods like finite element method and finite volume method. Among them, Annigeri et al [1] explained the free vibrations of clamped-clamped magneto-electro-elastic cylindrical shells using semi-analytic finite element method. Cheng et al [2] have studied the problem of linear bulking of piezo electric circular shells subjected to external pressure as well as on electric field. Heyliger and Ramirez [3] developed an efficient model for computing frequencies in laminated circular piezoelectric plates and discs by combining Discrete layer approach and Ritz method. Heyliger [4] demonstrated the fundamental behavior and the influence of piezoelectric coefficient of simply supported laminated plates. Kant [5] et al investigated the analytical solutions for the free vibration of isotropic composite and multilayered plates based on higher order refined theories. Karthikeyan et al [6] studied the damping through the imaginary parts of the complex frequencies for axisymmetric vibrations of pyrocomposite
hollow cylinder using Muller’s method. Mirsky et al [7] derived the frequency equation of flexural and longitudinal wave propagations in transversely isotropic circular cylinder. The eigen frequencies of ring shaped elliptical plate and other polygonal plates are obtained by Nagaya et al [8]. Pepper [9] explained the procedure of finite element method for various geometrical shapes. Sharma et al [10] generalized the thermo elastic wave propagation in circumferential direction of transversely isotropic curved plate using Helmholtz decomposition technique. Sunar et al [11] generalized the heat equation of thermo-piezo-magnetic smart structures using Hamilton’s principle and FEM. Selvamani et al [12] obtained the frequency equations using Fourier expansion collocation method in symmetric and anti-symmetric modes of vibration for ring shaped elliptical plate. Suganthi at el [13] discussed the computation of flexural vibrations of simply supported multilayered elliptical composite plate using Galerkin method and FEM. Zhu at el [14] discussed the analysis of sandwich TPS panel with functionally graded foam core by Galerkin method with arbitrary variation in thickness direction.

In the present study, we analyzed the free vibration behavior of elliptical structural system under different boundary conditions. The constitutive equation for piezo electric material has been considered and the governing equations reduced to weak integral form using the famous Galerkin - Finite element (semi analytic) technique. By this process, the structural problem is reduced in to standard eigen value problem. The range of resonant frequencies of a thin ring - shaped elliptical plate has been determined numerically using computational methods. The importance of eigenvalue represents the natural frequencies and the eigenvector represents the mode shapes or relative displacements are discussed for linearized multi degrees of freedom.

2. Fundamental Equations and Method of Analysis
The elliptical thin isotropic plate of Simply Supported thermo piezoelectric material is characterized by mutual coupling of mechanical field and electric field. The generalized governing equations are:

(i) The geometrical interpretation of linear stress-strain components can be expressed as:

\[ T_{rr} = c_{11} u_r + c_{12} v_\theta + c_{13} w_z + e_{31} \varphi_z \]  
\[ T_{\theta \theta} = c_{12} u_r + c_{22} v_\theta + c_{23} w_z + e_{32} \varphi_z \]  
\[ T_{zz} = c_{13} u_r + c_{23} v_\theta + c_{33} w_z + e_{33} \varphi_z \]  
\[ T_{0z} = c_{44} \left( v_z + \frac{1}{r} w_\theta \right) \frac{1}{r} e_{15} \varphi_\theta \]  
\[ T_{1z} = c_{55} (u_z + w_r) + e_{15} \varphi_r \]  
\[ T_{r \theta} = c_{66} \left( \frac{1}{r} u_\theta + v_r - \frac{v}{r} \right) \]  

(ii) The equation of equilibrium in the absence of body force given by:

\[ T_{rr,r} + \frac{1}{r} T_{r \theta, r} + T_{r z, z} + \frac{1}{r} (T_{rz} - T_{\theta \theta}) - \rho u_{tt} = f_1 \]  
\[ T_{r \theta, r} + \frac{1}{r} T_{\theta \theta, \theta} + T_{\theta z, z} + \frac{2}{r} T_{r \theta} - \rho v_{tt} = f_2 \]  
\[ T_{r z, r} + \frac{1}{r} T_{\theta z, \theta} + T_{z z, z} + \frac{1}{r} T_{r z} - \rho w_{tt} = f_3 \]  

(iii) Gauss’s law in the absence of charge density can be expressed as:

\[ \epsilon_{11} \left( \frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \varphi}{\partial \theta^2} \right) + \epsilon_{33} \frac{\partial^2 \varphi}{\partial z^2} - \epsilon_{11} \frac{\partial^2 u}{\partial r^2} - \epsilon_{12} \frac{1}{r^2} \left( \frac{\partial u}{\partial \theta} + \frac{\partial^2 v}{\partial \theta^2} \right) \]
\[ +e_{35} \frac{1}{r} \frac{\partial^2 u}{\partial \theta \partial z} - \frac{1}{r} \frac{\partial v}{\partial z} - \frac{\partial^2 v}{\partial r \partial z} - e_{13} \frac{\partial^2 w}{\partial \theta^2} = f_4, \]  

where \( T_{ij}\)-stress, \( T_{ij}\)-Strain, \( c_{ij}\)-Elastic constant, \( \rho\)-density of the material, \( e_{ij}\)-Piezoelectric constant, \( \varphi\)-Electrical potential and \( \epsilon\)-Dielectric constant respectively.

### 3. Boundary conditions

The natural boundary conditions that are assumed for elastic and electrical fields of the ring shaped elliptical plate as: along the inner and outer edges of the shell are

\[ (T_{rr}) = (T_{r\theta}) = (T_{rz}) = (\varphi) = 0 \]  

The plate is subjected to homogeneous linear isotropic with (i) simply supported (S) \{u, v, w, \( \varphi \)=0 \} and (ii) clamped condition (C) \{u, v, w, \( \varphi \}\) \( \neq 0 \)

### 4. Galerkin Approximation

In Galerkin Approach the weighting functions are chosen to be the same function as the shape functions \( (W_i)\) describes the modes of the vibration and some harmonic function of time \( t \). On integrating over the domain \( \Omega \), the equations 7 – 10 gives the Galerkin form for each of the weighting function, we have

\[ \int_{\Omega_e} N_i^u f_1 d\Omega = 0, \]  

\[ \int_{\Omega_e} N_i^v f_2 d\Omega = 0, \]  

\[ \int_{\Omega_e} N_i^w f_3 d\Omega = 0, \]  

\[ \int_{\Omega_e} N_i^\varphi f_4 d\Omega = 0, \]

where the subscripts \( e \) stands for the element and \( i \) stands for nodes of the local element.

### 5. Shape functions

The shape functions \( N_i \) of the multilayered composite cylindrical shell in the form of \( (r, \theta, z) \) which satisfies the simply supported boundary conditions are given as:

\[ U^e = \sum_{i=1}^{n} N_i^u (r, \theta, z) \Phi_i^u (t), \]

\[ \varphi^e = \sum_{i=1}^{n} N_i^\varphi (r, \theta, z) \Phi_i^\varphi (t) \]

where \( n \) is the number of nodes in the element, \( N_i^u \) and \( N_i^\varphi \) are the interpolation functions corresponds to the displacement field \( (U, V, W) \)and electric field \( \varphi \) and \( \Phi_i (t) \) are the time-dependent nodal parameter associated with the constants of unknowns at each nodes to be
determined. Substituting equation substituting the shape functions of approximations solutions of equation 16 – 17 in equations 12 – 15 and applying Galerkin method using integration by parts we can rewrite equations 12 – 15 as integrands equations. Then rearranging the like terms in the integrand equation, we get the general form as

\[
\begin{align*}
[K_{UU}] \{U\} + [M] \{\ddot{U}\} + [K_{U\Phi}] \{U\} &= 0 \quad (18) \\
[K_{\Phi U}] \{U\} - [K_{\Phi \Phi}] \{\Phi\} &= 0 \quad (19)
\end{align*}
\]

The above local element equations can be written in the global matrix form as:

\[
\begin{bmatrix}
M_{uu} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\ddot{U} \\
\ddot{\varphi}
\end{bmatrix} +
\begin{bmatrix}
K_{uu} & K_{u\varphi} \\
K_{\varphi u} & -K_{\varphi \varphi}
\end{bmatrix}
\begin{bmatrix}
U \\
\varphi
\end{bmatrix} = 0
\]

(20)

By using concatenation technique, the equations 20 yields the following second order differential equation as:

\[
[M] \{\dddot{X}\} + [K] \{X\} = 0.
\]

(21)

where \(X = \begin{bmatrix} U \\ \varphi \end{bmatrix}\) and \(\dddot{X}\) is the second order derivative w.r.t variable ‘t’. Assembling all the element contributions, the dynamic problem becomes the standard quadratic Eigen value problem as:

\[
\{[K] - \omega^2[M]\} \{X\} = 0, \text{ where } X = \exp(-i \omega t)
\]

(22)

Where \([M]\) is the Mass matrices \([K]\) is the Stiffness matrices. A Gaussian integration scheme has been adopted over the local element to obtain the mass and stiffness matrices and is listed in APPENDIX A.

6. Numerical Results and discussions

In this section, we illustrate the theoretical results obtained in the previous sections with four different boundary conditions as an example. The homogeneous isotropic ring shaped, elliptical plate made of Barium Titanate is considered. Here the traction free surface specified along the curved edges and along the top and bottom of the plate.

![Elliptical plate with inner edge abcd and the outer edge a'b'c'd'](image)

Figure 1. Elliptical plate with inner edge \(abcd\) and the outer edge \(a'b'c'd'\)

The Figure 1 presents the 3-dimensional ring shaped elliptical plate with thickness \(H\) along \(Z\)-axis. The four different boundary conditions are Case (i)SS denotes the inner edge \(abcd\) and
outer edge \( a'b'c'd' \) are both simply supported and Case (ii) SC denotes the inner edge \( abcd \) is simply supported and the outer edge \( a'b'c'd' \) is clamped and Case (iii) CS denotes the inner edge \( abcd \) is clamped and the outer edge \( a'b'c'd' \) is simply supported and Case (iv) CC denotes the inner \( abcd \) and outer edges \( a'b'c'd' \) are clamped. Here \( S- \) denotes simply supported condition and \( C \) denotes clamped condition. Also the conditions imposed only on the major axis at the nodes \( abcd \) and minor axis at the nodes \( a'b'c'd' \) and not on the curved surfaces. The physical data of the chosen materials are given as [6].

**Figure 2.** First quadrant of Six-noded quadrilateral element of Elliptical plate.

A six-noded isotropic quadrilateral element is used to model the plate structure with \( u, v, w \) and \( \phi \) as nodal degrees of freedom and the approximation functions are given as:

\[
N_i^u(r, \theta, z, t) = r^{j-1} \cos(n\theta) \sin(\lambda z) \exp(-i \omega t), \\
N_i^v(r, \theta, z, t) = r^{j-1} \cos(n\theta) \sin(\lambda z) \exp(-i \omega t), \\
N_i^w(r, \theta, z, t) = r^{j-1} \sin(n\theta) \sin(\lambda z) \exp(-i \omega t), \\
N_i^\phi(r, \theta, z, t) = r^{j-1} \sin(n\theta) \sin(\lambda z) \exp(-i \omega t),
\]

where \( j = 1, \cdots, 6 \), where \( \lambda \) is the wave number, \( \theta \) is circumferential mode and \( \omega \) is the angular frequency. Rotating the symmetric axis by 90° interchanging \((\cos n\theta)\) by \((\sin n\theta)\) we obtain a set of free vibrations which represents anti-symmetric mode and the volume of integration is replaced with \( d\Omega = rdrd\theta dz \).

### 6.1. Elliptic Cross Sectional Plate

The geometrical relations of an elliptic ring shaped plate given by Nagaya (1981b) are used for numerical calculation and are given below. The domain region of the considered problem is \( \Omega \{(r, \theta, z)\} \) where \( a = 0.4m, a' = 0.6m, b = 2m, b' = 0.4m \) and \( h_i = 0.3m \) along the thickness direction in \( z \)-axis.

The outer and inner radii of an elliptical plate are given as

\[
r_1 = a' \left( \cos^2 \theta + \left( \frac{a'}{b} \right)^2 \sin^2 \theta \right)^{\frac{1}{2}} \quad \text{and} \quad r_2 = a \left( \cos^2 \theta + \left( \frac{a}{b} \right)^2 \sin^2 \theta \right)^{\frac{1}{2}}
\]

where \( \alpha_1 = \frac{\pi}{2} - \tan^{-1} \left[ \left( \frac{b}{a} \right)^2 \tan \theta_i \right]^{-1} \) for \( \theta_i < \frac{\pi}{2} \) and \( \alpha_2 = \frac{\pi}{2} \) for \( \theta_i = \frac{\pi}{2} \).

Where \( a \) and \( a' \) are the length of inner and outer semi major axis and \( b \) and \( b' \) are the length of inner and outer semi-minor axis of an elliptical cross section. Assuming the angle \( \alpha_i \), between the normal to the segment and the reference axis.
Table 1. Normalized natural frequencies in the Elliptical plate under four different boundary conditions

| Mode | Boundary Condition | CC     | SC     | CS     | SS     |
|------|--------------------|--------|--------|--------|--------|
| 1    | 0.4012758001652    | 0.58038579450 | 0.59919591974 | 0.51245491490 |
| 2    | 0.472303789038     | 0.64831845633 | 0.68745309465 | 0.61763907386 |
| 3    | 0.53575863065      | 0.94541839  | 1.17318419967 | 0.82890784554 |
| 4    | 1.19978455481      | 1.36762431476 | 1.48472861354 | 1.35325398911 |
| 5    | 1.25773337457      | 1.55325398911 | 1.64056146663 | 1.53786613973 |

Table 2. Normalized natural frequencies in the Elliptical plate under four different boundary conditions

| Mode | Boundary Condition | CC     | SC     | CS     | SS     |
|------|--------------------|--------|--------|--------|--------|
| 1    | 0.220501670140     | 0.42458182197 | 0.4625417923 | 0.39260946188 |
| 2    | 0.397798317750     | 0.499514573615 | 0.48258147645 | 0.43763907386 |
| 3    | 0.513452285958     | 0.63805326488 | 0.69175165747 | 0.54945676256 |
| 4    | 0.925616823489     | 0.9808831779  | 0.99747167285 | 0.95731121221 |
| 5    | 1.16003781132      | 1.49205089475 | 1.5612344754 | 1.46443409235 |

By the proposed method, Matlab program was developed using finite element method and applied over the algebraic equation and hence the natural frequencies are achieved. The Eigen values are normalized as \( \omega_n = \omega \cdot a / \sqrt{c_{max}/\rho_{max}} \) where \( c_{max} \) and \( \rho_{max} \) are the maximum of material constant \( c_{ij} \) and density \( \rho \) of the sandwich plate, \( a' \) is the major axis, \( \omega \) is the natural frequency obtained from equation 22. The first five set of modes of non-dimensional frequency of the ring shaped elliptical plate made of piezo electric under four different boundary conditions have been calculated and the results are given in Table:1 and Table:2.

The dispersion curves are plotted in Fig.3, Fig.4, Fig.5 and Fig.6. It is observed that, as wave number increases the natural frequency also increase in both symmetric and anti-symmetric modes. In symmetric and anti symmetric mode, the minimum frequencies obtained in the first mode corresponding to the boundary condition CC (clamped at abcd and outer a’b’c’d’ edges) and the maximum frequencies obtained in the fifth mode corresponding to the boundary condition CS (clamped at abcd and simply supported at a’b’c’d’).
Figure 3. : Inner and Outer edges are clamped.

Figure 4. : Inner edge clamped and Outer edge simply supported.

Figure 5. : Inner edge simply supported and Outer edge is clamped.

The natural frequencies were calculated using semi analytic approach of combining Galerkin method and finite element technique. The numerical values of the normalized natural frequency for the plate made of Barium Titanate (BaTiO3) under four different cases of boundary conditions are obtained using the mentioned method and those values are given in Table:1 and Table:2 and compared with [12], it shows excellent agreement.
7. Conclusions

In this paper, Linear modal analysis of three dimensional ring shaped elliptical plate is presented to study the wave propagation under different edge conditions. Galerkin method is combined with Finite element method to solve the partial differential equation in time domain. Approximations for the mechanical displacements and electric potential are expressed as functions of the global coordinates with separate in-plane coordinate and out of-plane coordinate. By the proposed method, the normalized natural frequency of the elliptical plate made of Barium Titanate (BaTiO3) under four different boundary conditions are obtained and are given in Table:1 and Table:2. It is observed that, as wave number increases the natural frequency also increases in both symmetric and anti-symmetric modes. In symmetric and anti symmetric mode, the minimum frequencies obtained in the first mode corresponding to the boundary condition CC (clamped at abcd and outer a’b’c’d’ edges) and the maximum frequencies obtained in the fifth mode corresponding to the boundary condition CS (clamped at abcd and simply supported at a’b’c’d’). Hence, the present semi-analytic method is effective and simple and it can be applied to any linear or nonlinear modal analysis to obtain the resonant frequencies of the specified dynamic systems.

Appendix A

Stiffness Matrices [K]

\[
K_{uu} = \int_{\Omega} \left[ c_{11} \frac{\partial N_i^u}{\partial r} \frac{\partial N_j^u}{\partial r} + 2c_{12} \frac{1}{r} N_i^u \frac{\partial N_j^u}{\partial r} - c_{22} \frac{1}{r^2} N_i^u N_j^u \right. \\
+ c_{55} \frac{1}{r^2} \frac{\partial N_i^u}{\partial \theta} \frac{\partial N_j^u}{\partial \theta} + c_{55} \frac{\partial N_i^u}{\partial r} \frac{\partial N_j^u}{\partial r} \right] \, r \, dr \, d\theta \, dz
\]

\[
K_{uv} = \int_{\Omega} \left[ c_{12} \frac{1}{r} \frac{\partial N_i^u}{\partial \theta} - c_{22} \frac{1}{r^2} N_i^u \frac{\partial N_j^v}{\partial \theta} - c_{55} \frac{1}{r} \frac{\partial N_i^u}{\partial r} \frac{\partial N_j^v}{\partial r} + c_{55} \frac{\partial N_i^u}{\partial \theta} \frac{\partial N_j^v}{\partial \theta} \right] \, r \, dr \, d\theta \, dz
\]

\[
K_{uw} = \int_{\Omega} \left[ c_{31} \frac{\partial N_i^u}{\partial r} \frac{\partial N_j^w}{\partial z} + c_{35} \frac{1}{r} N_i^w \frac{\partial N_j^w}{\partial z} + c_{55} \frac{\partial N_i^w}{\partial r} \frac{\partial N_j^w}{\partial r} \right] \, r \, dr \, d\theta \, dz
\]

\[
K_{w\phi} = -\int_{\Omega} \left[ c_{15} \frac{\partial N_i^w}{\partial z} \frac{\partial N_j^\phi}{\partial r} + c_{25} \frac{1}{r} N_i^w \frac{\partial N_j^\phi}{\partial z} + c_{55} \frac{\partial N_i^w}{\partial \theta} \frac{\partial N_j^\phi}{\partial \theta} \right] \, r \, dr \, d\theta \, dz
\]
\[ K_{vv}^e = \int_{\Omega^e} \left[ c_{66} \frac{1}{r^2} N_i^v N_j^v - c_{66} \frac{2}{r} N_i^v \frac{\partial N_j^v}{\partial r} + c_{66} \frac{\partial N_i^v}{\partial r} \frac{\partial N_j^v}{\partial r} + c_{44} \frac{\partial N_i^v}{\partial r} \frac{\partial N_j^v}{\partial z} \right] r dr d\theta dz \]

\[ K_{v\varphi}^e = -\int_{\Omega^e} \left[ e_{14} \frac{\partial N_i^v}{\partial z} \frac{\partial N_j^v}{\partial \theta} - e_{14} \frac{1}{r} \frac{\partial N_i^v}{\partial z} \frac{\partial N_j^v}{\partial \theta} + e_{34} \frac{1}{r} \frac{\partial N_i^v}{\partial \theta} \frac{\partial N_j^v}{\partial z} \right] r dr d\theta dz \]

\[ K_{w\varphi}^e = \int_{\Omega^e} \left[ c_{55} \frac{\partial N_i^w}{\partial r} \frac{\partial N_j^w}{\partial r} + c_{44} \frac{1}{r^2} \frac{\partial N_i^w}{\partial \theta} \frac{\partial N_j^w}{\partial \theta} + c_{33} \frac{\partial N_i^w}{\partial \theta} \frac{\partial N_j^w}{\partial z} \right] r dr d\theta dz \]

\[ K_{\varphi\varphi}^e = -\int_{\Omega^e} \left[ e_{15} \frac{\partial N_i^\varphi}{\partial r} \frac{\partial N_j^\varphi}{\partial r} + e_{15} \frac{1}{r} \frac{\partial N_i^\varphi}{\partial r} \frac{\partial N_j^\varphi}{\partial \theta} + e_{15} \frac{1}{r^2} \frac{\partial N_i^\varphi}{\partial \theta} \frac{\partial N_j^\varphi}{\partial \theta} + e_{33} \frac{\partial N_i^\varphi}{\partial \theta} \frac{\partial N_j^\varphi}{\partial z} \right] r dr d\theta dz \]

Mass matrices \([M]\):

\[ M_{uu}^e = \rho \int_{\Omega^e} (N_i^u N_i^u) r dr d\theta dz \]

\[ M_{vv}^e = \rho \int_{\Omega^e} (N_i^v N_i^v) r dr d\theta dz \]

\[ M_{ww}^e = \rho \int_{\Omega^e} (N_i^w N_i^w) r dr d\theta dz \]

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