Alenezi Transform–A New Transform to Solve Mathematical Problems

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Author’s contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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Abstract

In this paper, we present a new integral transform called Alenezi-transform in the category of Laplace transform. We investigate the characteristic of Alenezi-transform. We discuss this transform with the other transforms like J, Laplace, Elzaki and Sumudu transforms. We can demonstrate that Alenezi transforms are near to the condition of the Laplace transform. We can explain the new Properties of transforms using Alenezi transform. Alenezi transform can be applied to solve differential, Partial and integral equations.

Keywords: Partial differential equations; integral equations; alenezi-transform; laplace transform; other transforms.

1 Introduction

Integral transforms techniques are kind of transform to simplify most utilize techniques that transaction with differential equations subject to specific boundary conditions. We can choose a suitable integral transform to convert both differential and integral equations into a solvable algebraic equation. There are some of transforms for solving the differential equations, and these transforms are necessary to solve these equations.

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to complete the solutions [1-5]. We can list some of these transformations as shown in tables [1,2] that can demonstrate the functions and those transforms. Many researchers drive some of the integral transforms in the category of Laplace transform like Elzaki, Sumudu, Natural, Pourreza, Aboodh, and J transforms [6-10]. In the Table (1), the definitions for these transforms are registered. These transforms can be utilized for disbanding the different kinds of ordinary, integral, partial and fractional differential equation as in [11-16]. Alenezi et al presented some mathematical techniques for solving algebraic modules [17-22]. There are Hybrid of the previous transforms with other methods such as the perturbation and Adomian decomposition methods are utilized to find the exact solutions for the different kinds of differential equation [23-29].

**Table 1. Definitions of different transforms**

| Transform Type               | Formula                                                                 |
|------------------------------|-------------------------------------------------------------------------|
| Laplace Transform            | $L[h(t)] = \int_0^\infty h(t)e^{-st}dt$                                |
| Elzaki transform             | $E[h(t)] = s \int_0^\infty h(t)e^{-\frac{t}{s^2}}dt$                   |
| Sumudu transform             | $S[h(t)] = \frac{1}{s} \int_0^\infty h(t)e^{-\frac{t}{s^2}}dt$        |
| Natural transform            | $n[h(t)] = R(s, u) = s \int_0^\infty h(ut)e^{-st}dt$                   |
| $\alpha$-Integral Laplace transform | $L_\alpha[h(t)] = \int_0^\infty h(t)e^{-\frac{t}{\alpha^2}}dt, \ \alpha \in R_0^+$ |
| Aboodh transform            | $A[h(t)] = K(s) = \frac{1}{s} \int_0^\infty h(t)e^{-st}dt$             |
| Mohand transform             | $m[h(t)] = R(s) = s^2 \int_0^\infty h(t)e^{-st}dt$                     |
| Pourreza transform          | $H[h(t)] = s \int_0^\infty h(t)e^{-st}dt$                             |
| Kamal transform              | $K[h(t)] = G(s) = \int_0^\infty h(t)e^{-\frac{t}{s^2}}dt$              |
| Sawi transform              | $Sa[h(t)] = \frac{1}{s^2} \int_0^\infty h(t)e^{-\frac{t}{s^2}}dt$    |
| G-transform                  | $G[h(t)] = F(s) = s^a \int_0^\infty h(t)e^{-\frac{t}{s^2}}dt$         |

In this paper, we introduced Alenezi integral transform to get the exact solutions of the differential equations. The paper is coordinated as follows. In part 2, we introduce Alenezi integral transform in the category of Laplace transform. In part 3, we match Alenezi integral transform with the other integral transforms in the category of Laplace transform. Alenezi integral transform is utilized to the differential and integral equations to get the exact solutions in part 4. Finally, we summarized the conclusions of my transform in part 4.
2 Alenezi Integral Transform

In this portion, we display Alenezi integral transform that envelope a widely integral transform in the group of Laplace transform.

Definition 1. Let \( h(t) \) become an integrable function realized for \( t \geq 0, p(s) \) and \( n(s) \neq 0 \) are favorable real functions, we explain Alenezi integral transform \( \mathcal{J}(s) \) of \( h(t) \) by the formula

\[
\mathcal{J}(s) = m(s) \int_0^\infty h(t)e^{-\frac{t}{m(s)}}dt
\]

(1)

Table 2. Table of alenezi transforms

| Function | Alenezi integral transform |
|----------|--------------------------|
| 1        | \( m(s) / n(s) \)        |
| \( T \)  | \( m(s) \)               |
| \( t^\alpha \) | \( \Gamma(\alpha + 1) m(s) / n(s)^{\alpha+1} \) |
| \( \cos t \) | \( m(s) / n(s) \) | \( n(s)m(s) \) |
| \( \sin t \) | \( m(s) \) | \( n(s)^2 + 1 \) |
| \( \sin(at) \) | \( m(s) \) | \( n(s)^2 + 1 \) | \( am(s) \) |
| \( e^t \) | \( m(s) \) | \( n(s)^2 + a^2 \) |
| \( h'(t) \) | \( n(s) J(s) - m(s)h(0) \) |

Theorem 1. Let \( h(t) \) is differentiable and \( m(s) \) and \( n(s) \) are positive real functions, then

(I) \[ T\{h'(t); s\} = n(s) J(s) - m(s)h(0) \]

(2)

(II) \[ T\{h''(t); s\} = n^2(s) J(h(t); s) - m(s)n(s)h(0) - m(s)h(0) \]

(3)

(III) \[ T\{h^{(n)}(t); s\} = n^n(s) J(h(t); s) - m(s) \sum_{k=0}^{n-1} q^{n-1-k} (s) h^k(0) \]

(4)

Proof. (I). In view of (1) we have

\[
\mathcal{J}[h'(t); s] = m(s) \int_0^\infty h'(t)e^{-\frac{t}{m(s)}}dt = m(s) \left[ e^{-\frac{t}{m(s)}}h(t) \right]_0^\infty + n(s) \int_0^\infty h(t)e^{-\frac{t}{m(s)}}dt \]

\[
= n(s)\mathcal{J}[h(t); s] - m(s)h(0),
\]

(5)

To proof (II), we assume \( z(t) = h'(t) \) so \( h''(t) = z'(t) \) now

\[
T\{z(t); s\} = m(s) \int_0^\infty z'(t)e^{-\frac{t}{m(s)}}dt = n(s) \mathcal{J}[z(t); s] - m(s)z(0)
\]

(6)

\[
= n(s)\mathcal{J}[h(t); s] - m(s)h(0) = n(s)\left[ n(s)\mathcal{J}[h(t); s] - m(s)h(0) \right] - m(s)h(0),
\]

(7)
Theorem 2. (Convolution) Let $h_1(t)$ and $h_2(t)$ have new integral transform $F(s)$. Then the new integral transform of the Convolution of $h_1$ and $h_2$ is

$$h_1 * h_2 = \int_{0}^{\infty} h_1(t) * h_2(t - \tau) d\tau = \frac{1}{m(s)}F_1(s) * F_2(s). \quad (8)$$

Proof.

$$T[h_1 * h_2] = m(s) \int_{0}^{\infty} e^{-n(s)t} \int_{0}^{\infty} h_1(t) * h_2(t - \tau) d\tau dt$$

$$= m(s) \int_{0}^{\infty} h_1(t)dt \int_{0}^{\infty} e^{-n(s)t} * h_2(t - \tau) dt$$

$$= m(s) \int_{0}^{\infty} e^{-n(s)t} \int_{0}^{\infty} h_2(t) * h_2(t - \tau) dt$$

$$= m(s) \int_{0}^{\infty} e^{-n(s)t} h_1(t)dt \int_{0}^{\infty} e^{-n(s)t} h_2(t)dt$$

$$= \frac{1}{m(s)}F_1(s) * F_2(s) \quad (9)$$

3 Solving IVP and Integral Equations by New Transform

In this section, we apply this new integral transform for solving high order IVP with constant coefficient. Also, we applied it to obtain the exact solution of a few types of integral equations and FDE.

4 Solving IVP with Constant Coefficient

Consider the following IVP:

$$X^{(n)}(t) + a_1X^{(n-1)}(t) + \cdots + a_nX(t) = g(x) \quad (10)$$

$$X(0) = X_0, \ X'(0) = X_1, \ldots, X^{(n-1)}(0) = X_{n-1}. \quad (11)$$

Now we apply a new integral transform, we have:

$$\mathcal{I}[X^{(n)}(t) + a_1X^{(n-1)}(t) + \cdots + a_nX(t)] = \mathcal{I}[g(x)] \quad (12)$$

$$\mathcal{I}[X^{(n)}(t)] + a_1 \mathcal{I}[X^{(n-1)}(t)] + \cdots + a_n \mathcal{I}[X(t)] = \mathcal{I}[g(x)] \quad (13)$$

Example 1. Consider the following third-order ODE

$$X'' + X' - 6X = 0 \quad (14)$$

$$X(0) = 1, \ X'(0) = 0, \ X''(0) = 5. \quad (15)$$

By applying $T$ on both sides, we have

$$\mathcal{I}[X'' + X' - 6X] = \mathcal{I}[(0)] \quad (16)$$

$$\mathcal{I}[X''] + \mathcal{I}[X'] - 6\mathcal{I}[X] = \mathcal{I}[(0)] \quad (17)$$
We have;
\[n^3(s) \mathcal{I}(s) - m(s)(n^2(s)X_0 + n(s)X_1 + X_2) + n^2(s) \mathcal{I}(s) - m(s)(n(s)X_0 + X_1 - 6 \mathcal{I}(s) = 0\]  
by replacing the initial conditions in above equation, we have
\[n^3(s) + n^2(s) - 6n(s) \mathcal{I}(s) = m(s) n^2(s) - m(s) + m(s) n(s)\]  
\[\mathcal{I}(s) = \frac{m(s)(n^2(s)\mathcal{I}(s) + m(s)n(s)\mathcal{I}(s))}{n^4(s) + n^2(s) - 6n(s)} + \frac{m(s)}{3n(s) + 9} + \frac{m(s)}{2n(s) - 4}\]  
by applying \(\mathcal{I}^{-1}\) we find the exact solution as:
\[X(t) = \frac{1}{6} \mathcal{I}^{-1}\left(\frac{m(s)}{n(s)}\right) + \mathcal{I}^{-1}\left(\frac{m(s)}{3n(s) + 9}\right) + \mathcal{I}^{-1}\left(\frac{m(s)}{2n(s) - 4}\right) = \frac{1}{6} + \frac{1}{3} e^{-3t} + \frac{1}{2} e^{2t}\]  

**Example 2.** Consider the following third-order ODE
\[X''' + 2X'' + 2X' + 3X = \sin t + \cos t\]  
\[X(0) = 1, \ X'(0) = 1, \ X''(0) = 0.\]  
By applying \(\mathcal{J}\) we have
\[\mathcal{J}(X''' + 2X'' + 2X' + 3X) = \mathcal{J}(\sin t + \cos t)\]  
\[\mathcal{J}(X''' + 2\mathcal{J}(X'') - \mathcal{J}(6X) = \mathcal{J}(0)\]  
\[n^3(s) \mathcal{I}(s) - m(s)(n^2(s)X_0 + n(s)X_1 + X_2) + 2[n^2(s) \mathcal{I}(s) - m(s)(n(s)X_0 + X_1) + 2[n(s) \mathcal{I}(s) - msX + 3\mathcal{I}s = ms \ n2s + 1 + nsms \ n2s + 1\]  
by replacing the initial conditions in above equation, we have
\[n^3(s) + 2n^2(s) + 2n(s) + 3] \mathcal{I}(s) = \frac{m(s)}{n^2(s) + 1} + \frac{n(s)}{n^2(s) + 1} + n(s) m(s) + 2 m(s)\]  
by simplification we got.  
\[\mathcal{I}(s) = \frac{m(s)}{n^2(s) + 1}\]  
by applying \(\mathcal{I}^{-1}\), we find the exact solution as
\[X(t) = \mathcal{I}^{-1}\left(\frac{m(s)}{n^2(s) + 1}\right) = \sin(t)\]  

We compare my transform with the laplace transform and found that my transform satisfies the exact solution and give an accurate result like the laplace transform as shown in the next example that demonstrate that my transform satisfies the same results of laplace transform.
Example 3. Solve the Partial differential equation

\[ 2x \frac{\partial^2 y}{\partial t^2} + \frac{\partial y}{\partial x} = 2x \]  

(29)

Given that \( Y(x, 0) = 1, \ Y(0,t) = 1 \)

Writing the above equation in the form

\[ 2xY_t(x, t) + Y_x(x, t) = 2x \]  

(30)

\[ 2x[y(x,s) - Y(x, 0)] + y_x(x, s) = \frac{2x}{s} \]  

(31)

\[ 2x[y(x,s) - 1] + y_x(x, s) = \frac{2x}{s} \]  

(32)

\[ \frac{dy}{dx} + 2xsy = 2x + \frac{2x}{s} \]  

(33)

\[ = 2x(1 + \frac{1}{s}) \]  

(34)

This is linear differential equation of the first order. The integrating factor is

\[ e^{\int 2xsdx} = e^{xs^2} \]  

(35)

\[ e^{xs^2} = \int 2x\left(1 + \frac{1}{s}\right)e^{sx^2} \, dx + c \]

\[ = \frac{1}{s}\left(1 + \frac{1}{s}\right)e^{sx^2} + c \]  

(36)

\[ y(x, s) = \frac{1}{s}\left(1 + \frac{1}{s}\right) + c e^{-sx^2} \]

\[ \frac{1}{s} = \frac{1}{s}\left(1 + \frac{1}{s}\right) + c \]  

(37)

\[ c = -\frac{1}{s^2} \]

\[ y(x, s) = \frac{1}{s}\left(1 + \frac{1}{s}\right) - \frac{1}{s^2} e^{-sx^2} \]

\[ = \frac{1}{s} + \frac{1}{s^2} - \frac{1}{s^2} e^{-sx^2} \]

\[ Y(x, t) = \begin{cases} 1 + t & 0 \leq t \leq x^2 \\ 1 + x^2 & t \geq x^2 \end{cases} \]  

(38)

5 Conclusion

In this paper, we present Alenezi integral transform. We demonstrate the old integral transforms and compared with Alenezi transform. It has demonstrated that Alenezi integral transform accurate than and satisfy the exact solution like Elzaki, Sumudu, and Laplace transforms for various value of \( m(s) \) and \( n(s) \).
We demonstrate Alenezi transform for the solutions of ODE, and integral equations. Some examples are used to demonstrate the efficiency of this technique.

**Competing Interests**

Author has declared that no competing interests exist.

**References**

[1] Agwa HA, Ali FM, Kilicman A. A new integral transform on time scales and its applications, Advances in Difference Equations. 2012;60:1-14. Available:https://doi.org/10.1186/1687-1847-2012-60

[2] Atangana A. A note on the triple Laplace transform and its applications to third-order differential equation, Abstract and Applied Analysis. 2013;769102:1–10.

[3] Atangana A, Alkaltani BST. A novel double integral transform and its applications. Journal of Nonlinear Science and Applications. 2016;9:424–434.

[4] Asiru MA. Sumudu transform and solution of integral equations of convolution type. Int. J. of Math. Edu. Sci. and Tech. 2002;33:944–949.

[5] Belgacem FBM, Kalla SL, Karaballi AA. Analytical investigations of the Sumudu transform and applications to integral production equations, Math. Probl. in Engg. 2003;3:103–118.

[6] Belgacem FBM, Karaballi AA. Sumudu transform fundamental properties, investigations and applications. J. of Appl. Math. and Stoch. Anal. 2006;91083:1–23.

[7] Belgacem FBM, Silambarasan R. Theory of natural transform, Math. in Engg. Sci., and Aeros. 2012;3:99–124.

[8] Belgacem FMB, Silambarasan R. Advances in the natural transform, AIP Conference Proceedings; 1493 January 2012; USA: American Institute of Physics. 2012;106–110.

[9] Debnath L, Bhatta D. Integral transform and their Applications, Third Edition, Chapman and Hall/CRC; 2014.

[10] Dattoli G, Martinelli MR, Ricci PE. On new families of integral transforms for the solution of partial differential equations. Integral Transforms and Special Functions. 2005;8:661–667.

[11] Ditkin VA, Prudnikov AP. Integral Transforms and Operational Calculus, Oxford: Pergamon; 1965.

[12] Elzaki TM. The new integral transform Elzaki transform, Global Journal of Pure and Applied Mathematics. 2011;7:57–64.

[13] H. Eltayeb, Kilicman A. On some applications of a new integral transform. Int. Journal of Math. Analysis. 2010;4:123–132.

[14] Kilicman A, Eltayeb H. On a new integral transform and differential equations, Mathematical Problems in Engineering. 2010;463579:1–13.

[15] Khan ZH, Khan WA. N-transform-properties and applications. NUST J. of Engg. Sci. 2008;1:127–133.

[16] Spiegel MR. Theory and problems of Laplace transform, New York, USA: Schaum’s Outline Series, McGraw–Hill; 1965.

[17] Ahmad M. Alenezi, Lump solutions of nonlinear (3 + 1)-dimensional for nonlinear partial differential equations. Partial Differential Equations in Applied Mathematics. 2020:2.

[18] Alenezi AM, Belgacem FBM. Sumudu transform based treatment of Krawtchouk polynomials and their integral zeros, AIP Conference Proceedings. 2016;1637 (1):1395-1405.

[19] Alenezi AM, Fiidow MA. On an Algorithm for Finding Derivations of Associative Algebras Pure Mathematical Sciences. 2020;9(1):13-20.

[20] Alenezi AM, Alkhotezi Y. Application of Annihilator Extension’s Method to Classify Zinbiel Algebras. Pure Mathematical Sciences. 2020;9(1):1-12.

[21] Alenezi AM, Sapar SH, Rakhimov I. Forgery Detection Using Krawtchouk Moments, Far East Journal of Mathematical Sciences. 2020;111(2):181-194.

[22] Alenezi AM, Rakhimov IS. comparative analysis of polynomial features for texture classification. International Journal of Pure and Applied Mathematics. 2018;119(1):99.
[23] Spiegel MR. Theory and Problems of Complex Variables with an introduction to Conformal Mapping and its applications, New York, USA: Schaum’s Outline Series, McGraw-Hill; 1964.
[24] Srivastava HM, Luo M, Raina RK. A new integral transform and its applications. Acta Mathematica Scientia. 2015;35:1386–1400.
[25] Watugala GK. Sumudu transform—a new integral transform to solve differential equations and control engineering problems, Math. Engg. in Indust. 1993;6:319–329.
[26] Yang X. New integral transforms for solving a steady heat transfer problem. Thermal Science. 2017;21:S79–S87.
[27] Yang X. A new integral transform with an application in heat-transfer problem. Thermal Science. 2016;20:S677–S681.
[28] Yang X. A new integral transform operator for solving the heat-diffusion problem. Applied Mathematics Letters. 2017;64:193–197.
[29] Yang X. A new integral transform method for solving steady heat-transfer problem. Thermal Science. 2016;20:S639–S642.

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