4e-condensation in a fully frustrated Josephson junction diamond chain

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Abstract

Fully frustrated one-dimensional diamond Josephson chains have been shown [B. Doucot and J. Vidal, Phys. Rev. Lett. 88, 227005 (2002)] to possess a remarkable property: The superfluid phase occurs through the condensation of pairs of Cooper pairs. By means of Monte Carlo simulations we analyze quantitatively the Insulator to 4e-Superfluid transition. We determine the location of the critical point and discuss the behaviour of the phase-phase correlators. For comparison we also present the case of a diamond chain at zero and 1/3 frustration where the standard 2e-condensation is observed.

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Josephson arrays in the quantum regime have been studied extensively [1], both experimentally and theoretically, as model systems where to investigate a variety of quantum phase transitions. The application of a magnetic field creates frustration and leads to a number of interesting physical effects. [2, 3]

Very recently, renewed interest in frustrated Josephson networks has been stimulated by the work by Vidal et al. [4] on the existence of localization in fully frustrated tight binding models with $T_d$ symmetry. Localization in this case is due the destructive interference for paths circumventing every plaquette. These clusters over which localization takes place were named Aharonov-Bohm (AB) cages. Experiments in superconducting networks have been performed and the existence of the AB cages has been confirmed through critical current measurements both in wire [4] and junction [5, 6] networks. Starting from the original paper by Vidal et al. several aspects of the AB cages both for classical [4] and quantum [11] superconducting networks have been highlighted.

The basic mechanism leading to the AB cages is also present in the (simpler) quasi-one-dimensional lattice shown in Fig.1. At fully frustration, it has been shown [12] that superconducting coherence is established throughout the system by means of 4e-condensation. The global superconducting state is due to the condensation of pairs of Cooper pairs. Predictions on the critical current of the diamond chain of Fig.1 amenable of experimental confirmation have been put forward by Protopopov and Feigelman [13, 14]. Unusual transport properties of these systems have been also predicted in semi-conducting samples [15].

In this work we present the results of our Monte Carlo simulations on the Josephson junction network with the geometry depicted in Fig.1. Our aim was to perform a detailed quantitative analysis of the phase diagram predicted in Ref. [12]. In order to have a fairly complete description of the effect of frustration in this case, we considered the stiffness and phase correlators for three values of the frustration parameter; i.e. $f = 0$, $f = 1/3$ and $f = 1/2$.

The Hamiltonian for a Josephson junction network is

$$\mathcal{H} = E_0 \sum_i n_i^2 - E_J \sum_{(i,j)} \cos(\varphi_i - \varphi_j - A_{i,j}) .$$

(1)

The first term in the Hamiltonian is due to the charging energy. Here for simplicity we consider the case in which the Coulomb interaction is on-site; see Ref. [13] for the more realistic case of long range charging interaction. The second term is the Josephson contribution. The phase of the superconducting order parameter in the i-th island is denoted by $\varphi_i$, $E_0$ is the charging energy and $E_J$ is the Josephson coupling energy. The number $n_i$ and phase $\varphi_i$ operators are canonically conjugate on each site $[n_i, e^{i\varphi_j}] = \delta_{ij} e^{i\varphi_i}$. The gauge-invariant definition of the phase in presence of an external vector potential $A$ and flux-per-plaquette $\Phi$ ($\Phi_0 = h c / 2 e$ is the flux quantum) contains the term $A_{i,j} = \frac{2\pi}{\Phi_0} \oint_{i} A \cdot dl$. All the observables are function of the frustration parameter defined as

$$f = \frac{1}{\Phi_0} \int_{\Phi} A \cdot dl = \frac{1}{2\pi} \sum_{<i,j>} A_{i,j}$$
where the line integral is performed over the elementary plaquette. Due to the periodicity of the model it is sufficient to consider values of the frustration $0 \leq f \leq 1/2$.

The Monte Carlo simulations have been performed on an effective classical $d+1$-dimensional XY-model (here $d = 1$) whose action is given by

$$S = -K \sum_{i, (kk')} \cos(\varphi_{i,k} - \varphi_{i',k'})$$
\[ -K \sum_{(ij), k} \cos(\varphi_{i,k} - \varphi_{j,k} - A_{i,j}) \]  \quad (2)

The effective dimensionless coupling is defined as $K = \sqrt{E_J/E_0}$. The index $k$ labels the extra (imaginary time) direction which takes into account the quantum fluctuations (the vector potential does not depend on the imaginary time). The first term corresponds to charging while the second is due to the Josephson coupling. The simulations where performed on $L \times L$ lattice with periodic boundary conditions (the largest lattice was $72 \times 72$). In Eq. (2) the couplings along the time and space directions have been made equal by a proper choice of the Trotter time slice [16, 17]. This choice, with no consequences on the study of the zero temperature phase transition, makes the analysis of the Monte Carlo data considerably simpler.

The expectation values of the different observables (stiffness and correlation functions) have been obtained averaging up to $10^7$ Monte Carlo configurations by using a standard Metropolis algorithm. Typically the first half of configurations, in each run, were used for thermalization.

The stiffness, related to the critical current, is used to signal the presence of the transition. It is defined through the increase of the free energy $F$ due to a phase twist $\delta$ imposed along the space direction [18]:

$$\Gamma = \frac{\partial^2 F}{\partial \delta^2}.$$

The critical point is expected to be of the Berezinskii-Kosterlitz-Thouless universality class [12, 20]. Its location can be determined using the following ansatz for the size dependence of $\Gamma(K_c)$ [19]:

$$\frac{\pi K_c}{2} \Gamma_L(K_c) = 1 + \frac{1}{2\ln(L/l_0)} \quad (3)$$

where $l_0$ is the only fit parameter. In the presence of frustration, the universality class of the transition may be different from that of the unfrustrated case. In the case of the two-dimensional fully frustrated XY-model this issue has been investigated in great detail (see Ref. 21 and refs. therein). Up to date, there is no unanimous consensus on the nature of the transition. However, in this work we suppose that the transition belongs to the BKT universality class, as suggested by Ref. 12, and determine the critical value by means of Eq. (3).

We first analyze the $f = 0$ case and extract the value of the critical coupling from the stiffness. This extrapolation has been done by performing a linear fit in logarithmic scale $\ln[\pi K_{\Gamma_L}(K) - 2]^{-1} = a(K)\ln L - \ln l_0$ and searching for the coupling value such that $a(K) = 1$. This coupling value is then identified with the critical point $K_c$. The proposed ansatz fits very well the data and the estimated value of the critical coupling is $K_c^{-1} = 1.28$ which corresponds to $(E_J/E_0)_c \sim 0.61$. Data are reported in Fig. 2.

The results of the stiffness at $f = 1/3$ and $1/2$ are shown in Fig. 3 and Fig. 4 respectively. As compared to the unfrustrated case, the critical value of the Josephson coupling required to establish superfluid coherence is slightly larger for $f = 1/3$ and further increases for the fully frustrated case $f = 1/2$. The ansatz of Eq. (3) seems to provide an accurate estimate of the transition point for $f = 0$ and $f = 1/3$. In the fully frustrated case, however, the value of $l_0 = 25$ indicates that we probably
FIG. 3: The same plots of Fig. 2 for the case of \( f = 1/3 \). The critical point is \( K_{c-1}^{-1} = 1.045 \) (with \( l_0 \sim 0.6 \)).

need larger chains in order to really enter the critical region. Another indication of this fact emerges in the upper panel of Fig. 4 where the line of slope \( 2/\pi \) crosses the data already when the stiffness is decreasing to zero. In order to put bounds to the critical point in the fully frustrated case we plot in Fig. 5 the stiffness as a function of the system size. From the raw data it is possible to bound the transition point in the range \( 0.55 \leq K_{c-1}^{-1} \leq 0.57 \).

All these results are summarized in table below:

| \( f \) | 0         | 1/3       | 1/2       |
|--------|-----------|-----------|-----------|
| \( K_{c-1}^{-1} \) | 1.28 ± 0.01 | 1.045 ± 0.005 | 0.56 ± 0.01 |
| \( (E_J/E_0)_c \) | 0.61 ± 0.01 | 0.91 ± 0.01 | 3.2 ± 0.1 |

The ratio of the obtained critical couplings for unfrustrated and fully frustrated system is \( K_{c-1/2}/K_{c-0} = 2.28±0.06 \) and not 4 as expected from the reduction by a factor 1/2 of the effective charge of the topological excitation that unbind at the critical point. This may be due to the fact that the screening of the vortices is different in the unfrustrated and fully frustred case therefore leading to a further correction in the ratio between the two critical points.

The differences in the fully frustred case manifest dramatically in the way condensation is achieved. As predicted by Douçot and Vidal [12], the destructive interference built in the diamond structure prevents Cooper pair to have (quasi-)long range order. The superfluid phase is then established via the delocalization of pairs of Cooper pairs. This is at the origin of the \( 4e \)–condensation. In order to check this point, the knowledge of the phase-phase correlators is required. Quasi-long range behaviour in a two-point correlation function of the type

\[
g_{2n}(|i-j|) = \langle \cos n(\varphi_i - \varphi_j) \rangle
g_{4}(|i-j|) = \langle \cos 2(\varphi_i - \varphi_j) \rangle
g_{2}(|i-j|) = \langle \cos 4(\varphi_i - \varphi_j) \rangle
\]

signals the existence of condensation of \( 2n \) charged objects. In Fig. 6 we discuss their properties. In the upper panels, we consider the phase-phase correlator \( g_2 \) for two different couplings deep in the superfluid and Mott insulating phases respectively. What is evident from the figure is that, despite the fact that the system is phase coherent, phase correlations decay very fast almost independently from the value of \( K \). As explained in [12], this behaviour should be ascribed to the existence of the Aharonov-Bohm cages. Even if hopping of single Cooper pairs is forbidden because of quantum interference, correlated hopping of two pairs does not suffer the same destructive interference. In the lower panels of the same figure, the space dependence of the correlator \( g_4 \) is plotted for the same coupling as upwards. The different behaviour between the Mott and the superfluid phase is now evident. The correlator decays exponentially only
for $K^{-1} = 1.2 > K_c$ (right side): in the other panel, differently from $g_2$, the decay is power-law like. For comparison we report also simulations of the phase correlators for the case $f = 1/3$. In this case the “standard” condensation of Cooper pair is observed as witnessed by the behaviour of $g_2$ shown in Fig. 7.

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