CALCULABLE UPPER LIMIT ON THE MASS
OF THE LIGHTEST HIGGS BOSON IN ANY
PERTURBATIVELY VALID SUPERSYMMETRIC THEORY

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ABSTRACT

We show that there is a calculable upper limit on the mass of the lightest Higgs boson in any supersymmetric theory that remains perturbative up to a high scale. There are no restrictions on the Higgs sector, or the gauge group or particle content. We estimate the value of the upper limit to be $m_{h^0} < 146$ GeV for $100 \text{ GeV} \lesssim M_t \lesssim 145$ GeV, from all effects except possibly additional heavy fermions beyond top (which could increase the limit by 0-20 GeV if any existed); for $M_t \gtrsim 145$ GeV the limit decreases monotonically. We expect to be able to decrease the value of the upper limit by at least a few percent by very careful analysis of the conditions. It is not normal in models for the actual mass to saturate the upper limit.
INTRODUCTION

In the minimal supersymmetric Standard Model (MSSM), with two Higgs $SU(2)$ doublet fields, the fermionic partners of the Higgs bosons must have gauge couplings. Then the supersymmetry leads to gauge couplings also for the Higgs boson self-interactions, so their masses can be expressed in terms of vacuum expectation values and gauge couplings. This is well known to lead to a tree level upper bound $^{[1]}$ $m_{h^0}^2 < M_Z^2 \cos^2 2\beta$, where $\tan \beta$ is the ratio of the vacuum expectation values of the two Higgs fields that give mass to up-type and down-type quarks, and $h^0$ is the lightest Higgs boson. It is remarkable that this upper limit is independent of the scale of supersymmetry masses and the scale of supersymmetry breaking, and holds independently of the short distance or large mass behavior of the theory.

Radiative corrections shift this limit, $^{[2]}$ adding to $m_{h^0}$ a numerically important contribution proportional to $M_t^2$ and logarithmically dependent on squark masses.

In the minimal (non-supersymmetric) Standard Model (SM) there is also an upper limit $^{[3]}$ on the Higgs boson mass $m_\varphi$ if one adds the condition that the couplings of the theory should be quantities that can be calculated perturbatively up to a scale of order $10^{16}$ GeV. Most people believe that this condition is now implied by experiment $^{[4]}$ since the gauge couplings do approximately meet at a point, or alternatively, starting with the symmetry value $\sin^2 \theta_W = 3/8$ at about $10^{16}$ GeV, the calculated value of $\sin^2 \theta_W = 0.23$ at our scale agrees with experiment in some theories to about 1% accuracy. Imposing this condition does not require a belief in any particular grand unified theory (GUT), or even in a GUT at all, but only that any acceptable theory should remain perturbative up to a scale of about $10^{16}$ GeV. Results are also insensitive to the choice of the GUT scale. A phrase is needed to describe such a condition, so we denote the requirement that any candidate theory should remain perturbative up to a scale of order $10^{16}$ GeV as “perturbative validity”, whether or not unification is assumed. Anyone who does not require that acceptable theories be perturbatively valid must maintain that the above results are accidental.

When perturbative validity is required it imposes an upper limit on the Higgs self-coupling at our scale, and thus if Higgs bosons exist it has been known for over a decade $^{[3]}$ that a SM Higgs boson cannot be heavier than about 170 GeV. The precise limit depends
on $M_t$ since the top quark Yukawa coupling enters the renormalization group equation (RGE) for the Higgs self-coupling.

Several years ago people began to explore\cite{5-9} more general supersymmetric theories, allowing additional singlets in the Higgs sector. A more fundamental theory containing the MSSM could be extended through additional Higgs multiplets (singlets, more doublets, triplets, etc.). Then new coupling terms could occur among these and the two original doublets. The effects of this sector might have destroyed the existence of a limit if the original Higgs fields were coupled to representations that got large vev’s which contributed to $m_{h^0}$. It was found\cite{5,7} that a limit still exists if any number of additional singlets and doublets are added, once the perturbative validity condition is imposed. Later additional studies were done, and recently\cite{9} the limit was also shown to exist if Higgs triplets were added whose vev’s were kept small; more precise numerical values were also provided in ref. 9. Extended models could also have a larger gauge group or larger fermion representations, or more families could exist. These will affect the numerical value of a limit, but not the existence of a limit.

The present paper extends this process further, and completes it. We allow arbitrary Higgs representations and remove the restriction that any vev’s must remain small, allowing arbitrary triplet vev’s, etc. This is crucial for a general limit since it is known\cite{10} that combinations of triplets can occur that give no contribution to the $\rho$-parameter but have large vev’s that could drive $m_{h^0}$ up. We also add numerical contributions that could affect the value of the limit in certain regions (e.g., the $b$ and $\tau$ contributions for large $\tan\beta$).

It is extraordinary that the mass of the lightest Higgs boson has an upper limit determined by weak scale parameters in a general supersymmetric theory so long as the theory remains perturbatively valid up to a high mass scale. It is also remarkable that the limit is a calculable one. It might have happened that quantities such as soft-supersymmetry breaking parameters that are bounded but unknown entered into the limit, e.g., into the equations that determine the upper limits on the self-couplings, in which case no useful numerical value could have been obtained.

In the next section we present the derivation of the limit, and then we present the numerical value of the limit. Computing the precise value of the limit is very difficult for two reasons, first because in arbitrary extended supersymmetric theories many effects feed
back on others in ways that require extensive untangling; for example, the introduction of new scalars increases the gauge coupling $\beta$-functions. Second, certain properties of mass matrices are used to obtain the upper limit, and in practice it is very hard to optimize the use of these properties. In this paper we present a conservative upper limit which we hope to improve by at least several percent later.

**DERIVATION**

We begin with a general superpotential and follow the same general line as in refs. 5, 7, 9.

$$W = A + B_a \phi^a + C_{ab} \phi^a \phi^b + D_{abc} \phi^a \phi^b \phi^c$$

where terms with sfermions are not written. From this we construct the scalar potential $V = V_F + V_D + V_{SOFT} + V_{MASS}$ in the standard way, and separate each scalar field into real and imaginary parts, $\phi^a = z^a + i z^{a+1}, a = 0, 1, 2, \ldots$. This gives

$$V = a + b_a z^a + c_{ab} z^a z^b + d_{abc} z^a z^b z^c + f_{abcd} z^a z^b z^c z^d + m^2_{(a)} z^a z^b \delta_{ab}$$

where the coefficients $a, b_a, c_{ab} \ldots$ can be expressed in terms of the superpotential coefficients and the soft-breaking coefficients. Next the minimum of the potential is obtained by calculating $\partial V/\partial z^i = 0$, and coefficients such as the $m^2_{(a)}$ are eliminated using the resulting equations. Then the positive definite mass matrix $2M^2_{ij} = \partial^2 V/\partial z^i \partial z^j$ is calculated, using a basis $Re H^o_1, Re H^o_2, Re S, Re \Sigma^o, \ldots$ if additional scalars $S, \Sigma^o, \ldots$ are present.

Finally the $2 \times 2$ submatrix corresponding to $Re H^o_1, Re H^o_2$ is examined. The important result is that it always has the form

$$M^2_{ij} = \begin{pmatrix} -J \tan \beta + K & J + L \\ J + L & -J \cot \beta + K' \end{pmatrix}.$$  

Here all of the SUSY parameters and vev’s that could grow are in $J$,

$$J = J \left( m^2_0, m^2_{1/2}, \mu, A, B, \ldots \text{ vev’s of new scalars}, g_i, v_i, \lambda_i \right),$$

while $K, K', L$ depend on the gauge couplings $g_1, g_2$, on the vev’s $v_1$ and $v_2$ that give mass to $W, Z$ (so that $v_1^2 + v_2^2 = v^2$ is fixed by $M_Z$), and on the various self-couplings $\lambda_i$. The
dependence on $\lambda_i$ means that once the $\lambda_i$ are limited by perturbative validity the functions $K, K', L$ have calculable upper limits. On the other hand, $J$ can become arbitrarily large and its value is in general not calculable.

Now we observe that $Tr M_{ij}^2$ and $Det M_{ij}^2$ both only grow as $J$ (the $J^2$ term cancels in $Det M_{ij}^2$), so one eigenvalue of $M_{ij}^2$ does not grow with $J$ (since $Tr M^2 = m_1^2 + m_2^2$, $Det M^2 = m_1^2 m_2^2$). The eigenvalues of $M_{ij}^2$ are not the actual masses. But the lowest eigenvalue of an $n \times n$ positive definite matrix is less than the smaller eigenvalue of any $2 \times 2$ submatrix (imagine the geometrical analogy of an $n$-dimensional ellipsoid and a 2-dimensional slice). Thus the bounded eigenvalue of the $2 \times 2$ submatrix is an upper limit on $m_{h^0}$. For completeness we give a few examples in Table 1 so the reader can see clearly how the above argument works.

This establishes that a limit exists. The further observation that the RGE’s for the self-couplings cannot depend on the dimensional supersymmetry masses or soft breaking parameters follows from general theorems, and has been exhibited explicitly. Thus the numerical value of the upper limit on $m_{h^0}$ is calculable in practice.

**NUMERICAL VALUE OF UPPER LIMIT**

To calculate the upper limit one can proceed as follows. The gauge sector is fixed by $M_Z$, and depends on $\tan \beta = v_2/v_1$. One then can find the maximum upper limit for any $\tan \beta$. The upper bounds on all self-couplings are calculated from their RGE’s. Again, to be conservative we do not decide on a value $\lambda_i^{\text{max}}$ above which a $\lambda_i$ is no longer considered to be perturbative, but take the value (typically a few percent larger) at which the $\lambda_i^{\text{max}}$ saturates (the meaning of this will be clear to anyone who has worked with such equations; we will describe the procedure in detail in a long paper). The $\beta$-functions for the gauge couplings $g_1$ and $g_2$ increase as more Higgs representations are added. Therefore, to calculate the upper bound we allow the $\beta$-functions to take on their maximum values such that the gauge couplings remain perturbative up to the high scale.

Then we present the results as follows. At the present time we calculate a conservative upper limit:

\[
\begin{align*}
&\text{Gauge and scalar sector,} \\
&\text{including effects of running of gauge couplings, } t, b, \tau \\
&\text{Yukawas, etc.} \\
&m_{h^0} \leq 134 \text{ GeV} \\
\end{align*}
\]

for $m_t = 135$ GeV, decreases as $m_t$ increases.
We can combine them:

\[ m_{h^0} \leq 146 \text{ GeV} \]

\( (100 \text{ GeV} \leq M_t \leq 145 \text{ GeV}; \text{ for larger } M_t \text{ the limit decreases, e.g. to } 133 \text{ GeV at } M_t = 160 \text{ GeV}) \).

To this limit must be added an amount 0-20 GeV if any new heavy fermions exist\(^{[13]}\) that get their mass by the Higgs mechanism. This number is bounded both by perturbative validity, and by precision measurements. A new fermion doublet contributes to the parameter \( S \) an amount \( \delta S = N_c/6\pi \). At 2\( \sigma \), \( S \approx 0.37 \). As precision data improves, the amount allowed from hypothetical new fermions will decrease. The heavy fermion contribution is also bounded by perturbative validity. The precision of calculations possible so far means, we think, that all of the above numbers are valid to at best \( \pm 2\text{-}3\% \). It should be understood that the effects from top (and any other new fermion) radiative correction loops in the Higgs effective potential (+12 GeV and + (0 – 20) GeV) must be added to any upper limit, either of the MSSM or any of its extensions.

We believe that a lengthy analysis which we are undertaking is likely to lower the “134 GeV” by at least a few percent, and perhaps more. Several effects enter, such as a present practical difficulty with calculating the numerical value of the lower eigenvalue of the 2 \( \times \) 2 submatrix as the parameters vary, the fact that the parameters (\( \tan \beta \), etc.) may not be able to take on simultaneously the values that give the present conservative limit, etc. Also, if it were possible to set a lower limit on \( \tan \beta \) of 2.5-3, the “134 GeV” would be lowered by over 5\%, so as the energy of LEP increases the upper limit may decrease if no signal is found.

**IMPLICATIONS**

This bound tells us that if a supersymmetric theory describes nature on the weak scale, then a light Higgs boson can be found below the limit. Conversely, if no light Higgs boson is found below the limit, no perturbative theory that requires a low energy supersymmetry to stabilize a hierarchy of scales can be correct. It is important to understand that in models the mass of the lightest Higgs boson is typically distributed from a lower value of
about 70 GeV up to the upper limit, with no strong tendency to cluster near the upper limit. Thus detecting the lightest Higgs boson is not likely to require a collider that can detect \( m_{h^o} = m_{h^o}^{\text{max}} \), though possibly it could.

In a general theory the \( ZZh^o \) coupling can change from its (one-top-loop corrected) MSSM coupling. To accompany the bound on \( m_{h^o} \) we need to calculate the minimum cross section for \( e^+e^- \to Z^{(*)} \to Z^{(*)} + h^o \) over the entire parameter space, for \( m_{h^o} = m_{h^o}^{\text{max}} \). That is extremely difficult to do because the singlets will not couple to the \( Z \); we are currently working out what can be said here. For the moment we report only that with the couplings of the radiatively corrected MSSM the cross section will not go below 0.125 pb for the maximum \( m_{h^o} \).

While we were writing this, additional papers studying particular extended supersymmetry models and tightening the limits in them have arrived.\(^{14}\)

At present the upper limit on \( m_{h^o} \) is not quite within the range where LEP could be certain to either detect \( h^o \) or to exclude the idea that supersymmetry is relevant to understanding nature near the electroweak scale, even if LEP were extended to \( \sqrt{s} \approx 240 \) GeV. But a combination of additional analysis of the limit (which we are undertaking), and improved data (which will restrict the contribution of more fermions and constrain parameters) may reduce the upper limit significantly. Whatever happens, LEP should be run to a sufficiently large energy that LEP + SSC/LHC can surely detect \( h^o \). Even if superpartners are directly detected, the supersymmetric world view will not be complete until a Higgs boson is detected.

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