1. Introduction

The purpose of this paper is to compute the global optimum values for the location of outlets, height of APB and shroud immersion depth which can yield the maximum value for mean residence time and the ratio of mix to dead volume of a given tundish, which are regarded as the most important fluid dynamical parameters to indicate the level of mixing in it. The earlier work of Jha et al. indicates that there exists optimums in all the three parameters i.e. the position of the outlets, height of APB (at optimum location of outlets) and shroud immersion depth (at optimum height of APB). Naturally one becomes more curious to know whether the optimums obtained in their study were unique or just a local optimum? If they had got only local optimum values then there may exist global optimum values for the above parameters at which the ratio of mixed to dead volume and mean residence time may be the highest for a given tundish. This has been the main motivating factor behind the present work where we intend to compute the ratio of mixed to dead volume and mean residence time for all possible combinations of outlet positions, height of APB and shroud immersion depth to explore the global optimums for these parameters such that one can obtain highest possible mixing in a given tundish or at least can use the present methodology to design a tundish or modify an existing tundish for best possible mixing.

The details about the geometry of the six strand billet caster tundish, the boundary conditions imposed on it as well as the mathematical formulations and assumptions remain the same as has been discussed by Jha et al.

2. Computation of Mixed and Dead Volume

The flow field in the six strand billet caster tundish has been obtained by solving the Navier Stokes equations numerically by using a k–ε model. The tracer dispersion in the tundish was computed from the solution of the species conservation equation, by injecting some dye into the inlet for a very short time. From the tracer dispersion curve the mixed volume and the dead volume were computed according to the Eqs. (4), (5) and (6).

Theoretical residence time:
\[ \tau = \frac{\text{Volume of tundish}}{\text{Volumetric flow rate}} \] ....(1)

Actual residence time:
\[ t_i = \frac{\sum C_{av}t_i}{\sum C_{av}} \quad i=1,2,3 \quad \text{(for the three outlets)} \] ....(2)

Average break through time:
\[ t_p = \text{First appearance of tracer at the exits (time is averaged because of multi exits)} \] ....(3)

Fraction of dead volume:
\[ \frac{V_d}{V} = 1 - t_p/\tau \] ....(4)

Fraction of plug volume:
\[ \frac{V_p}{V} = t_p/\tau \] ....(5)

Fraction of mixed volume:
\[ \frac{V_m}{V} = 1 - \frac{V_p}{V} - \frac{V_d}{V} \] ....(6)

3. Results and Discussions

Variation of the ratio of mixed to dead volume and mean residence time as a function of the outlet positions, height of APB and shroud immersion depth will be discussed here.

3.1. Variation of $V_m/V_d$ and Mean Residence Time with Height of APB for Various Outlet Positions

From the concentration time distribution curve the mixed and dead volume can be computed according to Eqs. (6) and (4) respectively and the actual mean residence time according to Eq. (2). A plot of $V_m/V_d$ is made against the height of APB keeping the shroud immersion depth at zero for various positions of the outlets in Fig. 1(a). It can be seen from Fig. 1(a) that $V_m/V_d$ is the highest when the outlets are at position-3 (closer to the inlet plane) and the height of APB is at 90 mm. It should be noted that when the height of APB increases beyond 90 mm at position-3, $V_m/V_d$ decreases signifying deterioration in mixing. When the height of APB increases, the rising plume from the APB comes closer to the top free surface of the tundish and then the plume disperses on the top free surface on to a much greater surface area which fails to induce global recirculation in the tundish causing mixing to be less. So an optimum height is found to induce better recirculation in the tundish and cause better mixing. However, this should be noted that the optimum APB height varies from outlet positions-1 to 3 which can be seen in Fig.1(a) as well as in Figs. 1(b), 1(c) and 1(d). Figure 2(a) shows the variation of mean residence time with the height of the APB when the outlets are placed at positions-1, 2 and 3. It can be seen

![Fig. 1(a). Variation of the ratio of mix to dead volume as a function of APB height for different outlet positions.](image)
from the figure that the mean residence time is the highest for the outlets at position-3 when the APB height is 90 mm. An increase in APB height at position-3 spoils overall recirculation in the tundish and hence the mixing time becomes less signifying more plug flow towards the outlets.

3.2. Variation of $V_m/V_d$ and Mean Residence Time with Shroud Immersion Depth as a Function of Height of APB at Different Outlet Positions

3.2.1. Outlets at Position-1

Figure 1(b) shows the variation of $V_m/V_d$ with shroud immersion depth for different APB heights at outlet position-1.

Fig. 1(b). Variation of the ratio of mix to dead volume as a function of shroud immersion depth for different APB heights at outlet position-1.

Fig. 1(c). Variation of the ratio of mix to dead volume as a function of shroud immersion depth for different APB heights at outlet position-2.

Fig. 1(d). Variation of the ratio of mix to dead volume as a function of shroud immersion depth for different APB heights at outlet position-3.

Fig. 2(a). Variation of the mean residence time as a function of APB height for different positions.

Fig. 2(b). Variation of the mean residence time as a function of shroud immersion depth for different APB heights at outlet position-1.

Fig. 2(c). Variation of the mean residence time as a function of shroud immersion depth for different APB heights at outlet position-2.
be seen that when the shroud immersion depth increases, \( V_m/V_d \) decreases for APBs having height of 90 and 210 mm while it increases for APB height of 340 mm. However, when the APB height is 340 mm, a peak in \( V_m/V_d \) is attained at a shroud immersion depth of 200 mm after which \( V_m/V_d \) falls with the increase of shroud immersion depth. This signifies that immersion depth of low magnitude does not improve mixing and an optimum immersion is required to induce global recirculation in the tundish. For the case under study an APB height of 90 mm with an immersion depth of 300 mm gives highest value for \( V_m/V_d \) when the outlets are at position-1. An elaborate fluid dynamic analysis of the mixing phenomenon due to an increase in the shroud immersion depth can be found in Jha et al.\(^2\) and we do not discuss this again just to save space. Figure 2(b) shows the mean residence time against the variation of shroud immersion depth while the height of the APB varies from 90 to 340 mm at position-1. This has similar trend like that of the variation of \( V_m/V_d \) but the mean residence time is the highest for an APB height of 340 mm at a shroud immersion depth of 200 mm. Normally higher mean residence time signifies better mixing and higher \( V_m \) but when it is compared with the quantity \( V_m/V_d \) then the one to one correspondence is little smeared.

3.2.2. Outlets at Position-2

Figure 1(c) shows the variation of \( V_m/V_d \) with shroud immersion depth while the height of the APB is varying from 0 to 340 mm and Fig. 2(c) shows the corresponding variation in the mean residence time when the outlets are placed at position-2. At this position of the outlet there is an optimum shroud immersion depth and optimum height of the APB which gives highest ratio of \( V_m/V_d \) and highest mean residence time. An APB height of 210 mm and shroud immersion depth of 300 mm gives highest value for \( V_m/V_d \) and mean residence time compared to all other APB heights and shroud immersion depth considered for outlets at position-2.

3.2.3. Outlets at Position-3

Figure 1(d) shows the variation of \( V_m/V_d \) with shroud immersion depth while the height of the APB varies from 90 to 340 mm and Fig. 2(d) shows the mean residence time in the tundish for the same case when the outlets are at position-3. From Fig. 1(d) it can be seen that \( V_m/V_d \) is having the highest value when the shroud immersion depth is zero and the height of the APB is 90 mm (this is the global highest for \( V_m/V_d \)). When the shroud immersion depth increases at position -3, \( V_m/V_d \) first decreases for the APB height of 90 and 210 mm but for the case of 340 mm APB height, \( V_m/V_d \) increases with the increases of shroud immersion depth. The trend of the mean residence time is exactly similar to the variation of \( V_m/V_d \) and this can be seen from Fig. 2(d). Further increase in the shroud immersion depth beyond 100 mm causes \( V_m/V_d \) to increase further and attain a peak at about a shroud immersion depth of 200 mm for all the cases of APB height. Still further increase in shroud immersion depth beyond 200 mm causes \( V_m/V_d \) to fall for all the heights of APB. Increase beyond a depth of 300 mm in shroud immersion causes very little change in \( V_m/V_d \) for APB heights of 210 and 340 mm. But for the case of APB height of 90 mm there is a significant increase in \( V_m/V_d \).

3.3. Local and Global Optimum

In the earlier work of Jha et al.\(^1\) it was found that highest value of \( V_m/V_d \) occurred when outlets were placed at position-2 with an APB height of 210 mm and a shroud immersion depth of 300 mm. In fact the same conclusion can be derived from the present results when one fixes the outlets at position-2 because the above combination gives the highest value for \( V_m/V_d \) (Fig. 1(c)). But this should be noted that such a highest for \( V_m/V_d \) is a local maximum not a global maximum. The global maximum for \( V_m/V_d \) occurs when the outlets are at position-3 with an APB of 90 mm and a shroud immersion depth of 0 mm (Fig. 1(d)). Jha et al. in their earlier work got the local maximum because after getting the optimum outlet position they moved to get the optimum APB height and subsequently the optimum shroud immersion depth without exploring the global optimum for any of the above three parameters. However, it should be mentioned that, the only way to obtain a global optimum is to explore all possible combinations, which has been done in the present computation. It should be noted that the local and global optimum values for APB height and shroud immersion depth which give highest value for \( V_m/V_d \) and the mean residence time would not have been clear without the present computation.

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