Thermodynamics of AdS Schwarzschild black hole in the presence of external string cloud

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Abstract

We study the thermodynamics of AdS-Schwarzschild black hole in the presence of an external string cloud. We observe that, at any temperature, the black hole configuration is stable with non-zero entropy. We further notice that, when the value of the curvature constant equals to one, if the string cloud density has less than a critical value, within a certain range of temperature three black holes configuration exist. One of these black holes is unstable and other two are stable. Similar to the Hawking-Page phase transition, at a critical temperature, a transition between these two stable black holes takes place.

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1 Introduction

One of the crowning achievements of the Golden Age of Relativity is the discovery that the black holes exhibit thermodynamic properties. A black hole has a natural temperature associated with its surface gravity and the entropy associated with its area. These quantities follow classical laws of thermodynamics. In the semi-classical treatment, the black holes radiate and evaporate eventually. Though the Schwarzschild black hole in an asymptotically flat space-time has negative specific heat, and is thus thermodynamically unstable, the Schwarzschild black hole in an asymptotically anti-de Sitter (AdS) space possesses positive specific heat at high temperature and is therefore thermodynamically stable. In their remarkable work [1], Hawking and Page further showed that these AdS-Schwarzschild black holes acquire negative free energy relative to AdS space-time at high temperatures and exhibit a first order phase transition as one tunes the temperature. More recently, the study of black holes in AdS space-time has gained a lot of attention due to Maldacena’s discovery of the AdS/CFT conjecture [2]. Within this context, the physics of the black holes or, more precisely, the thermodynamical properties of the black hole in the bulk AdS space-time play a crucial role in triggering novel behaviour, including phase transitions, of strongly coupled dual gauge theories that reside on the boundary of the asymptotically AdS space. This line of investigations started with the work of Witten who showed that the phase transition that takes place between the thermal AdS at low temperatures and the AdS-Schwarzschild black hole at high temperatures could be realized as the confinement/deconfinement transition in the language of boundary SU($N_c$), $\mathcal{N} = 4$ SYM theory [3, 4]. Subsequently, several other extensions of this work subsequently appeared. These includes the consideration of the R-charges [5, 6], addition of the Gauss-Bonnet [7–13] corrections or the Born-Infeld [14–18] term (separately and combination [19–22] of these terms in the AdS-Schwarzschild black hole) into the action.

There has also been interest to search for the gravity dual of SU($N_c$), $\mathcal{N} = 4$ SYM theory coupled to $N_f$ massless fundamental flavors at finite temperature and baryon density [23–26]. The fundamental flavors in the dual closed string representation of SU($N_c$), $\mathcal{N} = 4$ SYM theory corresponds to adding open string sector - with one end of the string attached to the boundary of the AdS space and the body hanged into the bulk and extended up to the center of the AdS space or horizon of the black hole. In the dual gauge theory, the attached end point of the string corresponded to the quark or the anti-quark and the body of the string corresponded to the gluonic field of the dual gauge theory.

Motivated by these developments, in this paper we study the thermodynamics of
the recently developed AdS-Schwarzschild black hole in presence of an external string cloud [25]. These black holes support external matter which comprises of uniformly distributed strings, each of whose one end is stuck on the boundary. We observe that the black hole configuration is a stable one at any temperature compared to the AdS configuration. Even at zero temperature, there is a black hole with a minimum radius. The size crucially depends on the density of the string cloud. We see that the density of this cloud plays not only an important role in finding out the minimum radius of black hole, it is also an important parameter controlling the number of black holes present at any given temperature. If the cloud density is greater than a critical value, there exists only one black hole. While for the cloud density less than the critical value, and for the value of curvature constant one, within a certain range of temperature there exist three black holes. Beyond this temperature range again we have a single black hole configuration. Depending on their sizes, we call them small, medium and large black holes. Among these three holes, the small and the large come with positive specific heat, and, the remaining one has a negative specific heat. Therefore, except the medium one, the other two can be stable. We study the stability of these two black holes by analyzing their free energies as a function of the temperature and the Gibbs free energies as a function of their radii (at different temperatures). Our observation is that there is a critical temperature below which the small black hole is the stable configuration and above the critical temperature large black hole is the stable one. At the critical temperature a transition between the small and the large black holes takes place. This is similar to the Hawking-Page phase transition mentioned earlier.

We organized our paper as follows: we start by writing the action of the AdS-Schwarzschild black hole with the matter contribution coming from the infinitely long string and the corresponding black hole solution in section 2. Then we compute the thermodynamical quantities in section 3. Section 4 is devoted to the study the different phases of the black holes. Finally summarize in section 5.

2 AdS-Schwarzschild black hole in presence of external string cloud

We start this section by considering the \((n+1)\) dimensional gravitational action in presence of cosmological constant with the contribution of the external string cloud,

\[
S = \frac{1}{16\pi G_{n+1}} \int d^{n+1}x \sqrt{-g} (R - 2\Lambda) + S_m, \tag{1}
\]
here $S_m$ represents the contribution of the string cloud and can be expressed by the following way:

$$S_m = -\frac{1}{2} \sum_i T_i \int d^2 \xi \sqrt{-h} h^{\alpha \beta} \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu \nu},$$  \hspace{1cm} (2)

where $g^{\mu \nu}$ and $h^{\alpha \beta}$ are the space-time and world-sheet metric respectively with $\mu, \nu$ represents space-time directions and $\alpha, \beta$ stands for world sheet coordinates. $S_m$ is a sum over all the string contributions and $T_i$ is the tension of $i$'th string. The integration in (2) is taken over the two dimensional string coordinates. The action (1) possesses black hole solutions and the metric solution of this black hole can be written as

$$ds^2 = g_{tt}(r) dt^2 + g_{rr}(r) dr^2 + r^2 g_{ij} dx^i dx^j.$$  \hspace{1cm} (3)

Here $g_{ij}$ is the metric on the $(n-1)$ dimensional boundary and

$$g_{tt}(r) = -K - \frac{r^2}{l^2} + \frac{2m}{r^{n-2}} + \frac{2a}{(n-1) r^{n-3}} = \frac{1}{g_{rr}},$$  \hspace{1cm} (4)

where $K = 0, 1, -1$ depending on whether the $(n-1)$ dimensional boundary is flat, spherical or hyperbolic respectively, having the boundary curvature $(n-1)(n-2)K$ and volume $V_{n-1}$. The uniformly distributed string cloud density $a$ can be written as

$$a(x) = T \sum_i \delta_i^{(n-1)}(x - X_i), \quad \text{with} \ a > 0.$$  \hspace{1cm} (5)

In writing $g_{tt}(r)$, the cosmological constant is parameterized as $\Lambda = -n(n-1)/(2l^2)$. With equation (4), the metric (3) represents a black hole with singularity at $r = 0$ and the horizon is located at $g_{tt}(r) = 0$. The horizon radius, denoted by $r_+$, satisfies the equation

$$K + \frac{r_+^2}{l^2} - \frac{2m}{r_+^{n-2}} - \frac{2a}{(n-1) r_+^{n-3}} = 0.$$  \hspace{1cm} (6)

This allows us to write the integration constant $m$ in terms of horizon radius as follows

$$m = K \frac{r_+^{n-2}}{2} + \frac{(n-1) r_+^n - 2a l^2 r_+}{2(n-1)l^2}.$$  \hspace{1cm} (7)

The integration constant $m$ is related to the ADM ($M$) mass of the black hole as,

$$M = \frac{(n-1) V_{n-1} m}{8 \pi G_{n+1}}.$$  \hspace{1cm} (8)

Therefore the mass of the black hole can finally be written in the following form

$$M = \frac{(n-1) V_{n-1}}{8 \pi G_{n+1}} \left[ K \frac{r_+^{n-2}}{2} + \frac{(n-1) r_+^n - 2a l^2 r_+}{2(n-1)l^2} \right].$$  \hspace{1cm} (9)

Analysing the black hole metric solution and mass, in the next sections we discuss the thermodynamics of this type of black holes. We therefore first compute the thermodynamical quantities.
3 Thermodynamical quantities

It has been well understood that black holes behave as thermodynamic systems. The laws of black hole mechanics become similar to the usual laws of thermodynamics after appropriate identifications between the black hole parameters and the thermodynamical variables. In order to study the thermodynamics of black holes we first come across various thermodynamical quantities as calculated in the following portions: Firstly, the temperature of the black holes is found by the following standard formula;

\[ T = -\frac{1}{4\pi} \frac{dgs}{dr} \bigg|_{r=r_+} = \frac{n(n-1)r_+^{n+2} + K(n-1)(n-2)l^2r_+^n - 2al^2r_+^3}{4\pi(n-1)l^2r_+^{n+1}}. \] (10)

To find out the entropy we expect that these black holes satisfy the first law of thermodynamics. Therefore by using the first law of thermodynamics, we calculate the entropy and it takes the form as;

\[ S = \int T^{-1}dM, \] (11)

which satisfies the universal area law of the entropy,

\[ S = \frac{V_{n-1}r_+^{n-1}}{4G_{n+1}}. \] (12)

Then we compute the specific heat associated with the black holes and is found as;

\[ C = \frac{\partial M}{\partial T} = \frac{(n-1)V_{n-1}}{4G_{n+1}} \left[ \frac{n(n-1)r_+^{2n-2} + K(n-1)(n-2)l^2r_+^{2n-4} - 2al^2r_+^{n-1}}{n(n-1)r_+^{n-1} - K(n-1)(n-2)l^2r_+^{n-3} + 2(n-2)al^2} \right]. \] (13)

Free energy can be calculated by using the formula

\[ F = E - TS = \frac{V_{n-1}}{16\pi G_{n+1}} \left[ Kr_+^{n-2} - \frac{r_+^n}{l^2} - \frac{(n-2)2ar_+}{(n-1)} \right]. \] (14)

Finally we also compute the Gibbs function which is coming out in the following form,

\[ G = \frac{V_{n-1}}{16\pi l^2 G_{n+1}} \left[ (n-1)r_+^n - 4\pi l^2 Tr_+^{n-1} + K(n-1)l^2r_+^{n-2} - 2al^2r_+ \right]. \] (15)

Many interesting features of these black holes, related to local and global stabilities, can be studied from the detail analysis of the thermodynamic quantities. Here we study the thermodynamical phases of these kind of black holes in the next section.
4 Phases of black hole

In this section we consider the black holes in five dimensions \((n = 4)\) and the results can easily be extrapolated in the higher dimensions. We start the study by considering two dimensionless quantities \(\bar{a} = \frac{a}{l}\) and \(\bar{r} = \frac{l r}{l}\). In terms of these dimensionless quantities the temperature can be expressed in the following form;

\[
\bar{T} = \frac{1}{6\pi l r^2} [6\bar{r}^3 + 3K\bar{r} - \bar{a}].
\]

The behaviour of temperature with respect to \(\bar{r}\) for string cloud density \(\bar{a}\) less or greater than a critical value \(\bar{a}_c\) and \(K = 1\) are drawn in figure 1. For \(\bar{a} < \bar{a}_c\), we notice that even at zero temperature black hole exists. The size of the zero temperature black hole can be found in terms of power series of \(\bar{a}\) which takes the form

\[
\bar{r}_0 = \frac{\bar{a}}{3} + \mathcal{O}(\bar{a}^2).
\]

The size of the black hole slowly increases with temperature and at a critical temperature two more black holes nucleate. Radius of one of these two new black holes reduces and the other one increases with the temperature. Depending on the size of these three black holes we call them small, medium and large. Up to a certain value of temperature all these three black holes exist and after that small and medium size black holes merge together and vanish. Finally, only large black hole exists at high temperature.

For \(\bar{a} > \bar{a}_c\), the figure shows that at any temperature only one black hole exists. However when the size of the black hole becomes small, the associated temperature is negative. To avoid the negative temperature of the black hole, the radius of the black

![Figure 1: The plot (a) is for \(\bar{a} = 0.3 < \bar{a}_c, K = 1\) and \(l = 2\) and plot (b) is for \(\bar{a} = 0.5 > \bar{a}_c, K = 1\) and \(l = 2\).](image)
hole should be protected by a minimum size which is equal to \( r_0 \). Therefore for any value of string cloud density there will be a black hole of finite size with non-zero entropy.

To find out the \( \bar{a}_c \) we use the condition that temperature should have two extrema for two real values of radius. This gives the following relation;

\[
\bar{a}_c = \frac{1}{\sqrt{6}} \approx 0.408
\]

In order to study the stability of these black holes we study the specific heat associated with them. The specific heat by considering the above dimensionless quantities can be written as;

\[
\bar{C} = \frac{3l^3V_5}{4G_5} \left[ \frac{6\bar{r}^6 + 3K\bar{r}^4 - \bar{a}\bar{r}^3}{6\bar{r}^6 - 3K\bar{r} + 2\bar{a}} \right]
\] (18)

We study the specific heat as per the figure 2 where the specific heat is plotted as a function of \( \bar{r} \). From the figure 2, it becomes evident that for \( \bar{a} < \bar{a}_c \), the specific heat is positive for small and large sized black hole, while it is negative for the medium sized black hole. Therefore it can be expected that the black holes with positive specific heat can be stable while the black hole with negative specific heat is unstable. For \( \bar{a} > \bar{a}_c \), we notice that the specific heat monotonically increases from zero value with the increase in the radius of the black hole. So this black hole can also be stable.

We then analyse the free energy to check the stability further. The free energy in terms of dimensionless quantity \( \bar{r} \) can be rewritten as;

\[
\bar{F} = \frac{V_3l^2}{16\pi G_5} \left[ K\bar{r} - \bar{r}^3 - \frac{4}{3}\bar{a} \right]
\] (19)

The figure 3 shows that for \( \bar{a} < \bar{a}_c \), the free energy starts from zero value at \( \bar{r} = 0 \)
and increases towards the negative value with the increase of black hole radius. At a certain value of radius, free energy reaches to the minimum value and then goes to the maximum value with the increase of radius. Again it drops down to the negative region and continues to increase towards the negative value with the increase of radius. Therefore the first extrema which corresponds to small size black hole will be preferable configuration compared to the AdS configuration since its free energy is less than the latter one. However, the free energy of the small black hole is greater than the large size black hole. So the large size black hole should be more stable compared to the smaller one and there is a possibility of having a phase transition between these two black holes.

For $\bar{a} > \bar{a}_c$, again the free energy monotonically decreases with the radius. So the black hole configuration is the stable one.

Now to verify the possibility of the phase transition between these black holes we study the free energy in terms of temperature. Figure 4 represents the plot of free energy as a function of temperature. From the plot we take notice of the following scenario. For $\bar{a} > \bar{a}_c$, there is only one branch with negative free energy. Thus this branch will be stable. For $\bar{a} < \bar{a}_c$, at low temperature free energy has only one branch (I) and as the temperature is increased two new branches (II and III) with positive value appear at temperature $\bar{T}_1$. If temperature increases further free energy of both the branches continues to decrease. Branch III cuts branch I at temperature $\bar{T}_2$ and becomes more and more negative at temperature $\bar{T}_3$ where branch II meets branch I and both disappear. These three branches represent respectively small, intermediate and large black holes. Out of these three, the intermediate black hole is unstable with negative specific heat while the other two are stable with positive specific heat. Below temperature $\bar{T}_1$ only
branch I exist with the free energy less than AdS configuration. Within the range of
temperature $\tilde{T}_1$ and $\tilde{T}_2$, the free energy of the branch III is higher than the branch I. Thus
branch I should be stable configuration than the branch III. Once temperature crosses $\tilde{T}_2$
the scenario is just opposite and the branch III will be stable configuration. Therefore
at low temperature there is only small black hole and once temperature increases and
approaches towards $\tilde{T}_1$ there is a nucleation of medium and large black holes occurs.
Where as medium one is unstable. At $\tilde{T}_2$ cross over from the small black hole to the
large black hole takes place. This cross over is similar to Hawking-Page phase transition.
Above this temperature only large black hole exist. Therefore, it can be said that AdS
configuration can not be the most stable one. Only the small or the large black hole
configurations survive.

To render further support for such a scenario, we now study the Gibbs free energy.
The Gibbs free energy in terms of the dimensionless quantities can be written as follows:

$$\tilde{G} = \frac{V_3}{16\pi G_5} \left[ 3\tilde{r}^4 - 4\pi T l \tilde{r}^3 + 3K \tilde{r}^2 - 2\tilde{a} \tilde{r} \right].$$

To analyze the different phases we plot this Gibbs free energy with respect to $\tilde{r}$ for
different temperature. In figure 5(a), for $\tilde{a} < \tilde{a}_c$, we plot the free energy for three different
temperatures. There exists two black hole solutions corresponding to the temperature
$\tilde{T}_1$ but the energy of the small black hole is less than the large one. So the small one
will be stable. Similarly at temperature $\tilde{T}_2$ two black hole solutions co-exist. Finally at
Figure 5: The plot (a) is for $\bar{a} = 0.3 < \bar{a}_c$, $K = 1$ and $l = 2$ and plot (b) is for $\bar{a} = 0.45 > \bar{a}_c$, $K = 1$ and $l = 2$. The blue curve corresponds to the temperature $\bar{T}_1 = 0.208$, the green curve corresponds to $\bar{T}_2 = 0.21$ and the pink curve corresponds to $\bar{T}_3 = 0.213$ such that $\bar{T}_1 < \bar{T}_2 < \bar{T}_3$.

temperature $\bar{T}_3$, the energy of the large black hole is small compared to the small black hole and the large black hole will be stable configuration. Therefore we conclude that at low temperature small black hole will be the stable configuration and at temperature above a critical value large one will be stable and at the critical temperature there will be a Hawking page phase transition. For $\bar{a} > \bar{a}_c$, the Gibbs free energy has one minimum with negative value. Therefore we have only one stable black hole.

All the above calculations were done for curvature constant $K = 1$. For $K = 0$ and $-1$ we find that the qualitative feature of the above thermodynamical quantities are similar to the case of $K = 1$ with $\bar{a} > \bar{a}_c$. So at any temperature, there exists a single black hole phase of finite size with non-zero entropy.

## 5 Summaray

In this work we study the thermodynamics of AdS-Schwarzschild black hole in presence of external string cloud. We observe that for all values of curvature constant the black hole configuration is stable compared to the AdS configuration. However, when the value of the curvature constant equals to one and when the string cloud density is less than a critical value, within a certain range of temperature, there are three black holes, while outside this range there is only one black hole. Depending on the size these three black holes we call them as small, medium and large black holes. Among these black holes small and large one come with positive specific heats and the medium has negative one. In order
to test their stability we study the free energy and Gibbs free energy. Finally we observe that within the aforesaid temperature regime a Hawking-Page phase transition take place among the small and large black holes. Therefore, we susspect that this may lead to an instability of the bound states of a quark and anti-quark pairs in the dual gauge theory.

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