A Nonparametric Framework for Online Stochastic Matching with Correlated Arrivals

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The design of online policies for stochastic matching and revenue management settings is usually bound by the Bayesian prior that the demand process is formed by a fixed-length sequence of queries with unknown types, each drawn independently. This assumption of serial independence implies that the demand of each type, i.e., the number of queries of a given type, has low variance and is approximately Poisson-distributed. Thus, matching policies are often based on "fluid" LPs that only use the expectations of these distributions.

This paper explores alternative stochastic models for online matching that allow for higher variance demand distributions and correlated arrivals. Under a more general nonparametric framework, we propose two models, INDJP and CORR, that relax the serial independence assumption differently by combining a nonparametric distribution for the demand with standard assumptions on the arrival patterns—adversarial or random-order. In our INDJP model, the demand for each type follows an arbitrary distribution, while being mutually independent across different types. In our CORR model, the sum of demands over all types follows an arbitrary distribution, and conditional on the length of the arrival sequence, the type of each query is drawn independently. We show that our modeling framework is tractable in both settings. We find that the fluid LP relaxation based on only expected demands can be an arbitrarily bad benchmark for algorithm design. Hence, we develop tighter LP relaxations for the INDJP and CORR models that leverage the full information about the demand distribution, leading to matching algorithms that achieve constant-factor performance guarantees under adversarial and random-order arrivals.

In the context of INDJP with adversarial arrivals, we formulate an LP relaxation, called the truncated LP, that incorporates an exponential family of constraints, representing Hall’s marriage condition. For any feasible LP solution, we show that incoming queries can be routed online to resources at rates equal to those described by the LP solution. This lossless rounding enables a reduction to the single-resource prophet inequality setting. The resulting algorithm is \(1/2\)-competitive, thereby achieving the best performance ratio relative to the offline optimum. In the context of CORR, no positive constant-factor competitive ratio can be attained, even under random-order arrivals. Here, we provide an LP upper bound on the optimal online policy, called the conditional LP, by devising valid inequalities on the sample-path matching when the length of the arrival sequence is the largest. Using a reduction to prophet inequalities, our approximation algorithm matches the best-known finite-inventory performance guarantees.

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