Gravitational lens time delays for distant supernovae: break the degeneracy between radial mass profiles and the Hubble constant

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ABSTRACT

An attempt to measure the Hubble constant with gravitational lens time delays is often limited by the strong degeneracy between radial mass profiles of lens galaxies and the Hubble constant. We show that strong gravitational lensing of type Ia supernovae breaks this degeneracy; the standard candle nature of type Ia supernova luminosity function allows one to measure the magnification factor directly, and this information is essential to constrain radial mass profiles and the Hubble constant separately. Our numerical simulation demonstrates that the Hubble constant can be determined with \(\sim 5\%\) accuracy from only several lens events if magnification factors are used as constraints. Therefore, distant supernova survey is a promising way to measure the global Hubble constant independently with the local estimates.

Key words: cosmology: theory — distance scale — galaxies: structure — gravitational lensing — supernovae: general

1 INTRODUCTION

Gravitational lensing is known to be a powerful tool to determine the Hubble constant \(H_0\) directly, without using the distance ladder (Refsdal 1964). Derived values of \(H_0\) show good agreement among known several lens systems, once the radial mass profiles of lens galaxies are fixed (Koopmans & Fassnacht 1999; Kochanek 2002a). If the lens galaxies are assumed to have the singular isothermal mass distribution, analysis of five gravitational lens systems indicates that the value of \(H_0\) is \(H_0 \sim 50\text{km s}^{-1}\text{Mpc}^{-1}\) (Kochanek 2002a) and hence too low to be consistent with the local measurement \(H_0 \sim 70\text{km s}^{-1}\text{Mpc}^{-1}\) (Freedman et al. 2001). This discrepancy may be ascribe to inhomogeneity in the universe (Tomita 2000a), and therefore it is important to study the global Hubble constant independently with the local measurement which relies on the distance ladder.

The main limitation of this technique is that there is a strong degeneracy between radial mass profiles and \(H_0\) (Wambsganss & Paczynski 1994; Keeton & Kochanek 1997; Koopmans & Fassnacht 1999; Witt, Mao, & Keeton 2000; Tada & Futamase 2000; Williams & Saha 2000; Chiba & Takahashi 2002; Wucknitz 2002; Kochanek 2002a; Zhao & Qin 2003). Therefore, unless we specify the radial mass distribution in the lens object, we hardly constrain the value of \(H_0\). The strong dependence of differential time delays on the radial mass distribution, on the other hand, indicates that statistics of time delays provide a powerful probe of density profiles (Oguri et al. 2002). This degeneracy can be broken if the Einstein ring images of host galaxies are observed (Kochanek, Keeton, & McLeod 2001), but the observation of host galaxies is often very difficult because of the large brightness contrast between quasars and host galaxies. Other way to break this degeneracy comes from the central core images (Rusin & Ma 2001; Evans & Hunter 2002; Keeton 2003). The lack of central core images, however, places only the lower limit of the mass concentration, and this corresponds to the lower limit of \(H_0\). Information of stellar kinematics and the mass-to-light ratio also allows one to constrain the radial mass profile of the lens galaxy and to break the degeneracy in the Hubble constant, although the measurement of velocity dispersions for distant galaxies is very difficult and involves large uncertainties. Treu & Koopmans (2002), for instance, concluded that the mass density profile of the lens PG1115+080 is steeper than the singular isothermal mass distribution, and obtained a value of the Hubble constant \(H_0 \sim 60\text{km s}^{-1}\text{Mpc}^{-1}\) which is marginally consistent with the local measurement.

In this Letter, we consider strong lensing of distant supernovae (SNe) which is expected to be observed by future observational plans (Kolatt & Bartelmann 1998; Wang 2000).
In this section, we estimate how accurately we can constrain radial mass profiles and the Hubble constant separately, on the basis of simple analytic consideration. Following Wucknitz (2002), we consider the degeneracy between radial mass profiles and \( H_0 \). First we provide the reason for this degeneracy breaking analytically, and next we numerically demonstrate the importance of magnification factors.

In what follows, we adopt a lambda-dominated universe with cosmological constant \( \Lambda \) and Hubble constant \( h \).

### 2 ANALYTIC CONSIDERATION

First start from the lens equation:

\[
y = x_i - \nabla \psi(x_i),
\]

where \( y \) and \( x_i \) are the positions of the source and images, and \( \psi(x_i) \) denotes the lens potential (see Schneider, Ehlers, & Falco 1992). If we multiply the lens equation with \( (1 - \kappa) \), then the lens equation can be rewritten as

\[
(1 - \kappa)y = x_i - \nabla \left( (1 - \kappa)\psi(x_i) + \kappa \frac{x_i^2}{2} \right).
\]

Therefore, the image position \( x_i \) is never changed if we transform unobservable quantities as \( y \rightarrow (1 - \kappa)y \) and \( \psi(x_i) \rightarrow (1 - \kappa)\psi(x_i) + \kappa(x_i^2/2) \). This is the mass-sheet degeneracy. Since the time delay is calculated from

\[
h\Delta t_i \propto \frac{(y - x_i)^2}{2} - \psi(x_i),
\]

this transform also changes the estimation of the Hubble constant, \( h \rightarrow (1 - \kappa)h \) if \( \Delta t_{ij} \) is fixed to the observed value. Therefore, the Hubble constant \( h \) cannot be uniquely determined from information of \( \{x_i\} \) and \( \{\Delta t_{ij}\} = \{\Delta t_i - \Delta t_j\} \). In the usual quasar lensing, additional information is supplied by the flux ratio. The flux ratio is simply derived from the ratio of magnification factors \( \mu_i \):

\[
\mu_i = \left| \frac{\partial y}{\partial x_i} \right|^{-1}.
\]

From this expression, it is found that \( \mu_i \) is transformed as \( \mu_i \rightarrow (1 - \kappa)^{-2}\mu_i \). Hence the flux ratio \( r_{ij} = \mu_i/\mu_j \) is also never changed. This means that information of \( r_{ij} \) cannot break this degeneracy. The degeneracy between radial mass profiles, \( \psi \propto r^3 \), and the Hubble constant \( h \) can be interpreted in this context because of the simple relation

\[
1 - \kappa = 2 - \beta \quad \text{(Wucknitz 2002)}.
\]

This yields a general scaling law,

\[
h \propto 2 - \beta, \quad (5)
\]

without changing observable values such as \( x_i, \Delta t_{ij}, \) and \( r_{ij} \).

In the case of SN Ia lensing, the situation changes drastically. “Standard candle” nature of SNe Ia (Phillips 1993; Riess, Press, & Kirshner 1996) allows one to observe magnification factors \( \mu_i \) directly. The magnification factor has the strong dependence on the radial mass profile (see also Wambsganss & Paczynski 1994, Kawano et al., in preparation),

\[
\mu_i \propto (2 - \beta)^{-2},
\]

thus we can break the degeneracy by the measurement of magnification factors. We note that Kolatt & Bartelmann (1998) also proposed this method to break the mass-sheet degeneracy in galaxy clusters and reconstruct galaxy cluster mass. Although \( \mu_i \) should have significant error which arises from the intrinsic dispersion in SNe Ia peak luminosities as well as substructure in the lens galaxy (Mao & Schneider 1998), its effect on \( h \) estimation is not so severe because equations (6) and (6) implies that the resulting error of \( h \) is a half of that of \( \mu_i \).

### 3 SIMULATED RESULT

To illustrate how accurately we can determine \( \beta \) and \( h \), we show the results of our simulation. First, we assume the lens galaxy is well characterized by the following lens potential (Kawano et al., in preparation):

\[
\psi(x) = r^3 (a_0 + a_2 \cos 2\theta + a_3 \cos 3\theta + b_2 \sin 2\theta + b_3 \sin 3\theta), \quad (7)
\]

where \( \theta = (r, \theta) \) is the position in the polar coordinate. We note that best-fit values of \( h \) and \( \mu \) in the analysis of quasar PG1115+080 are quite insensitive to the choice of the angular part of the lens potential (Kochanek 2002). Kawano et al., in preparation, thus it is sufficient to analyze this lens potential only. We also take account of the effect of external shear as

\[
\psi_{\text{shear}}(x) = \frac{1}{2} \gamma r^2 \cos(2\theta - \theta_{\gamma}). \quad (8)
\]

We randomly put a source, generate quadruple images, and calculate differential time delays and magnification factors, assuming following parameters: \( \beta = 1.0, h = 0.5, a_0 = 0.5, a_2 = b_2 = 0.01, a_3 = b_3 = 0.001, \gamma_1 = \gamma \cos 2\theta_{\gamma} = 0.1, \) and \( \gamma_2 = \gamma \sin 2\theta_{\gamma} = -0.01 \). We also determine \( a_i \) and \( b_i \) values when \( x \) is in units of arcsec. Source and lens are placed at \( z_s = 1.5 \) and \( z_l = 0.5 \), respectively. Generated images have separations on the order of 1”. For the observable quantities, such as image positions and time delays, the Gaussian noise is added. We assume following dispersions:

\[
\sigma_x = 0.01” \quad , \quad \sigma_{\Delta t} = 0.05 \text{[day]}, \quad \sigma_{\alpha x_{ij}} = 0.09, \quad \sigma_{\alpha_{\Delta t}} = 0.01” \quad , \quad \sigma_{\alpha_{\Delta t}} = 0.05 \text{[day]}, \quad \sigma_{\alpha_{\Delta t}} = 0.09.
\]

\[\text{http://snap.lbl.gov/}\]
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Figure 1. Constraints on the radial mass profile $\beta$ (eq. [7]) and the Hubble constant $h$. The contours of $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$ in the $\beta$-$h$ plane are calculated from one quadruple lens event. Crosses indicate the assumed value in generating observable quantities; $\left(\beta, h\right) = (1.0, 0.5)$. Upper panel: image positions $x_i$, differential time delays $\Delta t_{ij}$, and magnification ratios $r_{ij}$ are used to calculate $\chi^2$. Lower panel: instead of $r_{ij}$, magnification factors $\mu_i$ are used to calculate $\chi^2$. Dotted lines are same as solid lines, but in this case additional non-Gaussian errors due to microlensing are also included.

$\sigma_{\log \mu_i} = 0.12$. \hfill (12)

The precisions in positions and time delays are consistent with 0.1 pixel of the instrument and estimated accuracy for SNe Ia lightcurves in SNAP survey, respectively (Goobar et al. 2002). The dispersion of magnification ratio, which roughly corresponds to $\sim 20\%$ fractional error, is a fiducial error often assumed in $\chi^2$ minimization (e.g., Kochanek 2002a). We assume that the dispersion of the magnification factor is somewhat larger than this, roughly corresponds to $\sim 30\%$ fractional error, because not only substructure in the lens galaxy (Mao & Schneider 1998) but also the intrinsic dispersion of SNe Ia peak magnitudes contribute to $\sigma_{\log \mu_i}$. Other possible source of the dispersion is dust extinction in the lens galaxy. However, the effect of dust extinction can be corrected from the observed reddening because of knowledge of an SN Ia’s intrinsic color (e.g., Riess et al. 1996).

After the virtual “observational data” is generated, we perform $\chi^2$ minimization using the same lens model. At that time, we fix values of $\beta$ and $h$, and optimize the other parameters such as $a_i$, $b_i$, $\gamma_i$, and the source position. Faint core images which may appear when $\beta > 1$ are always neglected. In calculating $\chi^2$, we consider following two cases: (1) Only the magnification ratio $r$ is measured. This case corresponds to traditional quasar lensing. (2) The magnification factor is directly measured. This is the case of SN Ia lensing we are interested in. For each case, we calculate the contour of $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$ in the $\beta$-$h$ plane. Figure 1 plots constraints on $\beta$ and $h$ from one quadruple lens event. This figure clearly shows that in the case of SN Ia lensing $\beta$ and $h$ are well constrained separately. It is surprising that the Hubble constant is determined with $\sim 10\%$ accuracy (68% confidence) from only one lens system. On the other hand, when magnification factors are not used, $\beta$ and $h$ are poorly determined; they show the strong degeneracy $h \propto 2 - \beta$. We note that in practice constraints from quasar lensing may be worse than our result using magnification ratios, because the error of time delays is usually much larger than our assumption (eq. [10]). Figure 2 shows constraints from five quadruple lens events. In generating observables for each event, the position of the source is changed while the lens model is always fixed. In this figure, the Hubble constant $h$ is determined with $\sim 5\%$ accuracy (68% confidence) when magnification factors are used, while the accuracy is still $\sim 20\%$ (68% confidence) when magnification ratios are used. We also examine the case that lens galaxies have different values of $\beta$, and the result is shown in Figure 3. In this plot, we assume that five lens systems have different radial mass profiles; $\beta = 0.8, 0.9, 1.0, 1.1,$ and 1.2, respectively. This figure clearly indicates that the magnification factor is quite useful to constrain the Hubble constant even if the scatter of $\beta$ is taken into account. The contour is slightly

Figure 2. Same as Figure 1, but from five quadruple lens events.
elongated along the $\beta$-direction by the scatter of $\beta$, but the accuracy of the Hubble constant determination is almost same as that in Figure 2. On the other hand, the strong degeneracy still remains when only the magnification ratio is used as constraints. Therefore, we conclude that magnification factors which are observed in SN Ia lensing provide indeed important information to break the $\beta$-h degeneracy.

The most important effect we neglect here is the effect of microlensing by stars in the lens galaxy. The magnification probability distribution of microlensing has a long tail because the amount of microlensing fluctuation depends on the fraction of the stellar component which is still unclear (e.g., Schechter & Wambsganss 2002). Therefore, in this paper we show the effect of microlensing by simply adding non-Gaussian errors. We assume the following log-normal probability distribution:

$$P(\log \mu_{ML}) = \frac{\exp \left\{ -\frac{[\ln(\log \mu_{ML} + m)]^2}{2s^2} \right\}}{\sqrt{2\pi s^2} (\log \mu_{ML} + m)};$$

(13)

where $\mu_{ML}$ indicates the additional magnification due to microlensing. The parameters $m = 1$ and $s = 0.2$ are chosen so as to reproduce the typical fraction which suffers from microlensing variability; in our model the fraction magnified by $\log \mu_{ML} > 0.4$ (0.2) is $\sim 5\%$ ($\sim 20\%$) respectively, and this fraction is roughly consistent with the previous estimations (Koopmans & Wambsganss 2001; Wyithe & Turner 2002). The probability distribution of microlensing should depend on convergence and shear at images and therefore each image may have different probability distributions. We neglect this effect because we now consider quadruple lens systems in which images have approximately the same separations from the lens center and are likely to have similar convergence and shear. Constraints on $\beta$ and $h$ including the effect of microlensing are shown in all figures (dotted lines). These figures indicate that the effect of microlensing is fairly small. The magnification by microlensing systematically moves contours mainly to the lower $h$, but the deviations are sufficiently small and our assumed model ($\beta = 1$ and $h = 0.5$) lies still within contours. Our conclusion is therefore that the effect of microlensing is not so severe.

4 SUMMARY AND DISCUSSION

We have shown that the strong degeneracy between radial mass profiles $\beta$ and the Hubble constant $h$ can be broken if we use magnification factors as constraints. This means that SN Ia lensing has the great advantage over traditional quasar lensing. We have found that in the case of SN Ia lensing the Hubble constant is constrained quite accurately, with $\sim 5\%$ accuracy from only several lens events. We have found also that both the scatter of radial mass profiles and microlensing do not affect our result so much. In contrast to this, quasar lensing can poorly constrain the Hubble constant, even if the same accuracy of time delay measurements is assumed. Of course, this method can be applicable to any distant astronomical objects which have quite narrow luminosity function. The limitation of our method is that it probes only the local slope of the mass profile. Therefore, the value of the Hubble constant derived from our method may be different from the true value if the radial mass profile is significantly different from a power-law.

Although SN lensing has not ever been observed, lensed SN will be found in the future observations such as SNAP. Then how many lensed SNe are expected to be observed in the future observational plans? SNAP survey, for instance, can catch $2000 \sim 3000$ Type Ia SNe at $z \leq 1.7$ per year. Since the lensing probability at $z \sim 1.5$ is $\sim 10^{-3}$, we observe at least a few lensed SNe per year (Holz 2001; Oguri et al. 2003). Time delays between images are always observed in the case of SN lensing because SNe are transient phenomena. Hence all lensed SNe can be used to constrain the Hubble constant. Therefore, a few year’s observation by SNAP is enough to derive the accurate value of the Hubble constant. Note that our method can constrain the Hubble constant accurately from even one lens system. On the other hand, large-scale surveys, such as two degree Field system (2dF) and Sloan Digital Sky Survey (SDSS), will also find more than one hundred of quasar lenses. The number of lens systems for which time delays are measured, however, should be much smaller than this because of the difficulty in measuring time delays. Moreover, additional observations, such as host galaxies of quasars or velocity dispersions of the lens galaxies, are needed to break the $\beta$-h degeneracy.

Therefore we conclude that our method is practically applicable and indeed have advantages compared with methods using quasar lenses.
In our simulation, we considered only quadruple lens systems which have larger numbers of constraints. In practice, SNAP-like survey will catch double lenses much more than quadruple lenses because the magnification bias is neglected at least for low-z SNe ($z \leq 1.7$ in the case of SNAP survey). Time delay bias (Oguri et al. 2003) favors quadruple lenses, but the time delay bias is not so significant at typical lens separation $\theta \sim 1''$. We believe, however, our results are also applicable to double lenses, because magnification factors are sensitive to radial mass profiles even when the source is far from the center of the lens and thus likely to be the double lens (see Oguri et al. 2002).

Finally, we comment on the measurement of the magnification factor $\mu$. Consider the situation that only lensed SNe are observed and the absolute magnitude of (unlensed) SNe, which is estimated using the local Hubble constant, is known. In this case, the flux of unlensed SNe at the same redshift, expected from the absolute magnitude, scales as $f_{\text{unlensed}} \propto h^2$ if the assumption that the local Hubble constant is the same as the global Hubble constant (denoted by $h$) is relaxed. Since we observe $f_{\text{lensed}}$ directly, the magnification factor should scale as $\mu = f_{\text{lensed}}/f_{\text{unlensed}} \propto h^{-2}$. This scaling is exactly same as that derived in Eqs. (2), which means that the $\beta$-$h$ degeneracy is never broken in this situation. But if the redshift of lensed SN is not so larger, huge numbers of unlensed SNe which have the similar redshift should be also observed. In this case, $\mu$ can be simply estimated from the magnitude difference between lensed and unlensed SNe independently with $h$. The only assumption needed to estimate $\mu$ is therefore that SN Ia is an excellent standard candle. We emphasize that several important uncertainties in the SN survey, such as cosmological parameters, possible evolution of intrinsic luminosities, dust extinction outside the lens galaxy, and the Cepheid calibration, do not affect the measurement of $\mu$.

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