Radiative Heat Transfer on Optically Thick Fluid Past an Oscillating Vertical Plate with Variable Temperature

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Author’s contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

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ABSTRACT

A theoretical study of radiation heat transfer with reference to an optically thick fluid past an oscillating vertical flat plate with variable temperature in the presence of convection and radiation has been presented. The fluid is considered to be a gray, absorbing-emitting radiation but non-scattering medium. The Rosseland flux approximation plays an important role in determining the effect of radiation heat transfer contribution. This problem is an improvement of Stoke’s first and second problem to justify the physical significance on this problem. This problem is solved by employing Laplace transform method. Numerical results of velocity and temperature distributions are depicted graphically. Also, numerical results of frictional shearing stress and critical Grashof number are presented in tables.

Keywords: Thermal radiation; gray gas flow; Grashof number; Rosseland model; radiative heat-flux.

NOMENCLATURE

\( c_p \) : Specific heat at constant pressure
\( g \) : Gravity acceleration

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Grashof number

Gr = \frac{\beta \rho (T_w^2 - T_\infty^2)}{\nu_0}

: Grashof number

\( i = \sqrt{-1} \)

: Complex unity

k

: Thermal conductivity

k^*

: Rosseland mean absorption coefficient

k = \frac{16\pi T}{\lambda}

: Radiation parameter

Pr = \frac{\nu}{\alpha}

: Prandtl number

q_r

: The radiative heat flux

t'

: Non-dimensional time

T

: Non-dimensional fluid temperature

u_0

: Characteristic velocity

u'

: Fluid velocity

u

: Non-dimensional fluid velocity

x', y'

: Co-ordinate axes

y

: Non-dimensional distance

GREEK SYMBOLS

\beta

: Thermal expansion coefficient

\nu

: Kinematic coefficient of viscosity

\sigma

: Stefan-Boltzman constant

\rho

: Density

\omega'

: Frequency of oscillations

\frac{\omega' \nu}{\alpha}

: Non-dimensional frequency of oscillations

\omega t

: Phase angle

\tau

: Shear stress

SUBSCRIPTS

W

: Plate surface

\infty

: Away from the plate

1. INTRODUCTION

Radiation heat transfer of an optically dense medium is subjected to a large optical thickness as defined just as with molecular conductivity the transfer of radiant energy in a medium to compare with diffusion transfer. Here, the interphoton collision becomes predominant. For large optical thickness (optically dense medium), a gray body radiation depends on the basis of diffusion concept of radiation heat transfer. This leads to Rosseland approximation for an optically thick medium. The study of radiation heat transfer of an optically thick fluid has received wide attention to many researchers in the field of engineering and space physics. The importance of a study of thermal radiation effect takes place in a numerous applications of condensed fuel combustion, solar energy collectors, heat exchangers, glass and ceramics manufacture,
rocket propulsion system and high temperature physics. The purpose of present investigation is to deal with radiative heat transfer of an optically dense medium with temperature variation along an infinite oscillating vertical flat plate with reference to gravity driven radiation-convection flow. The importance of a study of such fluid flow problem by employing Rosseland diffusion flux model takes place of a gray gas flow by which the fluid is considered to be gray, absorbing-emitting radiation but non-scattering medium. This problem is an improvement of Stoke’s first and second problem to generate gravity driven radiation-convection flow. This mathematical study does not seem to appeared in the literature. The importance of a study of this problem lies in its application of fluid engineering, space craft propulsion system and high temperature physics.

2. FORMULATION OF THE PROBLEM AND ITS SOLUTION

Consider an unsteady flow of a viscous incompressible fluid occupying a semi-infinite region of space bounded by an infinite vertical flat plate with variable temperature moving with uniform velocity \( u_0 \) which varies harmonically with time in the presence of convection and radiation. The fluid is considered to be a gray, absorbing-emitting radiation but non-scattering
medium. To choose cartesian co-ordinate system in such a way that $x'$-axis is taken along the plate and $y'$-axis is normal to it. It is considered that all fluid properties are constant except the influence of density variation in the body force term. Initially, the plate and fluid are at same temperature in a stationary condition. At time $t' > 0$, the plate is given an impulsive motion in the vertical direction against the gravitational field with the constant velocity $u_0$, which varies harmonically with time and the plate temperature is made to rise linearly with time. Since the plate is infinite along $x'$-direction, all physical quantities are functions of $y'$ and $t'$ only.

Under the Boussinesq approximation, the flow is governed by the following equation:

$$\frac{\partial u'}{\partial t'} + v \frac{\partial^2 u'}{\partial y'^2} + g \beta (T' - T_o) = 0$$

(1)

The energy equation becomes:

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q_u}{\partial y'}$$

(2)

where $u'$, $t'$, $v$, $g$, $\beta$, $T'$, $T_o$, $k$, $c_p$, $\rho$ and $q_u$ are respectively, the velocity component along the plate, the time, the kinematic coefficient of viscosity, the gravitational acceleration, the coefficient of thermal expansion, the temperature of the fluid, the temperature of the fluid far away from the plate, the thermal conductivity, the specific heat at constant pressure, the density of the fluid and the radiative heat-flux.

The boundary conditions are

$$u' = 0, T' = T_w' \quad \text{for all } y', t' \leq 0$$

$$t' > 0: u' = u_0 \cos \omega t', T' = T_w' + (T_o' - T_w') A t' \quad \text{at } y' = 0$$

$$u' \rightarrow 0, T' \rightarrow T_o' \quad \text{as } y' \rightarrow \infty$$

(3)

where $A = u_0^2 / \nu$, $T_w'$ is the temperature at the plate, $u_0$ is the velocity of the plate and $\omega'$ is the frequency of oscillations.

Introducing dimensionless quantities reads:

$$u = \frac{u'}{u_0}, \quad y = \frac{y'}{u_0}, \quad t = \frac{t'}{u_0^2}, \quad \omega = \frac{\omega' \nu}{u_0}$$

$$T = \frac{T' - T_o'}{T_o'}$$

$$Pr = \frac{\nu}{a}$$

$$Gr = \frac{g \beta (T_o' - T_w')}{u_0^3}$$

(4)

Equation (1) together with the dimensionless quantities (4) transform into

$$\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial y^2} = Gr T$$

(5)

The radiation flux vector can be found from Isachenko et al. [40] and its formula is derived on the basis of the diffusion concept of radiation heat transfer in the following way:

$$q_u = -\frac{4\sigma}{3k} \frac{\partial T'^4}{\partial y'}$$

(6)

where $\sigma$ and $k'$ are respectively, the Stefan-Boltzmann constant and the spectrul mean absorption coefficient of the medium.

It is assumed that the temperature differences within the flow are sufficiently small such that $T'^4$ may be regarded as a linear function of temperature. It can be established by expanding $T'^4$ i.e. a Taylor series about $T_o'$ and neglecting higher order term. Therefore $T'^4$ can be expressed in the following way

$$T'^4 = 4T_o'^3 T' - 3T_o'^4$$

(7)

Using equations (6) and (7), equation (2) takes the form

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma T_o'^3}{3k^2} \frac{1}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2}$$

(8)

Using dimensionless quantities (4) , equation (8) can be written in a dimensionless form reads

$$\left(1 + k_i\right) \frac{\partial^2 T}{\partial y^2} - Pr \frac{\partial T}{\partial t} = 0$$

(9)

where $k_i = \frac{16\sigma T_o'^3}{3k^2}$ is the radiation parameter.

The dimensionless boundary conditions turn into

$$u = 0, T = 0 \quad \text{for all } y, t \leq 0$$

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By applying temperature boundary conditions given by (13), equation (12) becomes

\[ T^* = \frac{1}{s^2} e^{-\frac{Pr}{1+k_1} y} \]  

(15)

Using equation (15), equation (11) gives

\[ s u^* = \frac{\partial^2 u^*}{\partial y^2} + \frac{Gr}{s^2} e^{-\frac{Pr}{1+k_1} y} \]  

(16)

By applying Laplace inversion method, the equation (14) gives

\[ u(\eta, \tau) = \frac{1}{4} \left[ e^{-(\sqrt{1-\omega} - i\omega) \sqrt{2\sqrt{\tau}} - i\omega}} + e^{-(\sqrt{1+\omega} + i\omega) \sqrt{2\sqrt{\tau}} - i\omega}} \right] \]

\[ + e^{(\sqrt{1+\omega} + i\omega) \sqrt{2\sqrt{\tau}} + i\omega}} + e^{-(\sqrt{1-\omega} - i\omega) \sqrt{2\sqrt{\tau}} - i\omega}} \left[ 1 - \frac{1}{2} \left( \frac{t^2}{2} + \frac{y^2}{2} \right) \right] \]

\[ + \frac{Gr}{\alpha - 1} \left[ \frac{1}{2} \left( t^2 + y^2 \right) \right] \left[ e^{\sqrt{t^2} \frac{y^2}{2\sqrt{\tau}} - i\omega}} \right] - \frac{Gr}{\alpha - 1} \left[ \frac{1}{2} \left( t^2 + \alpha y^2 \right) \right] \left[ e^{\sqrt{t^2} \frac{y^2}{2\sqrt{\tau}} - i\omega}} \right] \]

\[ T(\eta, \tau) = \left( t + \frac{y^2}{2} \right) \frac{1}{\alpha} e^{\sqrt{t^2} \frac{y^2}{2\sqrt{\tau}} - i\omega}} \left[ e^{\sqrt{t^2} \frac{y^2}{2\sqrt{\tau}} - i\omega}} \right] \]  

(18)

where \( \omega \) and \( \omega t \) are, respectively, the dimensionless frequency of oscillations and phase angle and \( \alpha = \frac{Pr}{1+k_1} \).
Particular case of interest:

Non-oscillatory case by putting $\omega = 0$ and $\omega t = 0$. The dimensionless boundary conditions (10) turns into:

$$u = 0, \; T = 0 \quad \text{for all} \quad y, \; t \leq 0 \; t > 0: \; u = 1, \; T = t \quad \text{at} \quad y = 0 \tag{20}$$

$$u \to 0, \; T \to 0 \quad \text{as} \quad y \to \infty$$

By applying Laplace transform subject to equations (11) and (12) together with the boundary condition (20) the velocity and temperature distribution can be obtained by the help of Laplace Inversion method such as

$$u(y, t) = \text{erfc} \left( \frac{y}{2 \sqrt{t}} \right) + \frac{\text{Gr}}{\alpha - 1} \left[ \frac{1}{2} \left( t^2 + y^2 \right) + \frac{y^4}{24} \right] \text{erfc} \left( \frac{y}{2 \sqrt{t}} \right) - y \sqrt{t} \left( \frac{5}{6} + \frac{y^2}{12} \right) e^{-\frac{y^2}{4t}}$$

$$- \frac{\text{Gr}}{\alpha - 1} \left[ \frac{1}{2} \left( t^2 + \alpha y^2 \right) + \frac{\alpha^2 y^4}{24} \right] \text{erfc} \left( \frac{y \sqrt{\alpha}}{2 \sqrt{t}} \right) - y \sqrt{\alpha} \sqrt{t} \left( \frac{5}{6} + \frac{y \alpha}{12} \right) e^{-\frac{y \alpha}{4t}} \tag{21a}$$

$$T(y, t) = \left( t + \frac{y^2}{2} \alpha \right) \text{erfc} \left( \frac{y \sqrt{\alpha}}{2 \sqrt{t}} \right) - y \sqrt{\alpha} \sqrt{t} \left( \frac{5}{6} + \frac{y \alpha}{12} \right) e^{-\frac{y \alpha}{4t}} \tag{21b}$$

Shear stress at the plate $y = 0$ for $\omega = 0$ and $\omega t = 0$ takes the form

$$\tau_s = \left. \frac{du}{dy} \right|_{y=0} = -\frac{1}{\sqrt{\pi t}} - \frac{4 \text{Gr}}{3 (\alpha - 1) \sqrt{\pi}} + \frac{1}{2} \sqrt{\frac{t}{\pi}} \frac{\text{Gr}}{\alpha - 1} \left( 1 - \frac{5 \sqrt{\alpha}}{3} \right) \tag{22}$$

In the absence of Grashof number ($\text{Gr} = 0$), the velocity distribution $u(y, t)$ given by equation (21a) turns into

$$u(y, t) = \text{erfc} \left( \frac{y}{2 \sqrt{t}} \right) \tag{23}$$

Equation (23) gives the velocity of Stoke’s first problem reads:

$$u(y, t) = \text{erfc} \left( \frac{y}{2 \sqrt{t}} \right) + 1 - \text{erf} \left( \frac{y}{2 \sqrt{t}} \right)$$

The temperature distribution (19) gives the Nusselt number $Nu$ at the plate:

$$Nu = -\left. \frac{dT}{dy} \right|_{y=0} = 2 \sqrt{\frac{t}{\pi}} \frac{Pr}{1 + k_t} \tag{24}$$

3. RESULTS AND DISCUSSION

The graphical discussions in relevance to the physical interpretation has been made with arbitrary values of radiation parameter ($k_t$), Grashof number ($\text{Gr}$), Prandtl number ($\text{Pr}$), frequency parameter ($\omega$), phase angle ($\omega t$) and time ($t$) in Figs. 1 to 7. In Fig. 1, it is evident that the buoyancy force ($\text{Gr}$) on velocity field...
leads to increase the flow behavior with increase in Grashof number $Gr$. As the buoyancy effects become relatively large due to increasing value of $Gr$, the fluid velocity increases, reaching its peak value near the plate surface and then decreases monotonically to the zero-free stream value satisfying the far field condition. In the case of higher buoyancy it is important to note that there is no flow reversal on velocity field and the maximum peak of the profile occurs at the plate $y = 0$ while the peak of the profile decreases steadily near the plate surface. This situation happens in the case of an oscillating plate so that the flow velocity is characterised by the higher buoyancy to increase the fluid velocity with an increase in $Gr$. It is noticed from Fig. 2 that the fluid velocity decreases with an increase of phase angle $(\omega t)$. This situation reveals that the phase angle $(\omega t)$ leads to fall the flow velocity on increasing $\omega t$ with reference to impulsive onset into motion. There arises a phase lag on molecular diffusion region with interphoton collision. Fig. 3 shows that, for buoyancy added flow $(Gr > 0)$, the velocity increases with increase in time $(t)$. The maximum peak of the velocity profile occurs adjacent to the plate whereas the peak of the profile quickly decreases on the plate surface. Since the plate oscillates harmonically with time, the velocity profiles are skewed near the plate surface. The skewness is characterised by the impulsive movement of the plate with time variation at the plate. It is observed from Fig. 4 that in the absence of oscillation $(\omega = 0$ and $\omega t = 0)$ and the buoyancy force $(Gr = 0)$, this represents Stoke’s flow with reference to the velocity distribution

\[ u(y,t) = \text{erfc} \left( \frac{y}{2\sqrt{t}} \right) - \text{erfc} \left( - \frac{y}{2\sqrt{t}} \right) \]

This situation reveals that the velocity increases with increase in time $t$. Fig. 5 shows that the temperature field $(T)$ increases with an increase in radiation parameter $(k_i)$. Larger values of radiation parameter $(k_i)$ exert its influence on Rosseland approximation in the determination of an increased dominance of thermal radiation over conduction. As such thermal radiation supplements the thermal diffusion and increases the local radiative flux of the regime since the local radiative diffusion flux model adds radiation conductivity to the conventional thermal conductivity. As a result, the fluid temperature and velocity in the fluid regime of flow are increased. Fig. 6 demonstrates that with the increase in Prandtl number $(Pr)$ the temperature field $(T)$ decreases near the plate. This is true since; in general, fluid with low Prandtl number has higher thermal conductivity. The higher thermal conductivity means fluid has affinity for heat and so low Prandtl fluid attains comparatively higher temperature. The effect of Prandtl number plays a significant role on diffusion concept of flow medium. If Prandtl number is greater than one $(Pr > 1)$ the diffusivity of the flow medium tends to ionization of the flow. In a highly ionized fluid $Pr > 1$, the effect of Prandtl number $(Pr = 0.72)$ for air transformed into ionized state to water. Fig. 7 reveals that with an increase in $t$, there is a strong acceleration in the flow. It is stated that the temperature field $(T)$ increases with an increase in time $(t)$. Thus, time variation at the plate gives rise to increase in temperature with an increase in time $(t)$.

**Frictional shear stress at the plate $y = 0$:**

Frictional shearing stress at the plate can be obtained from

\[ \left. \frac{du}{dy} \right|_{y=0} = 0 \]

with reference to the solution $u(y,t)$ of (18)

\[
\tau_s = \left. \frac{du}{dy} \right|_{y=0}
= \frac{1}{4} \left[ -\frac{2}{\sqrt{\pi t}} + \sqrt{-i\omega} e^{-i\omega} \left\{ \text{erfc} \left( \sqrt{i\omega t} \right) - \text{erfc} \left( -\sqrt{i\omega t} \right) \right\} 
- \frac{2}{\sqrt{\pi t}} + \sqrt{i\omega} e^{i\omega} \left\{ \text{erfc} \left( \sqrt{i\omega t} \right) - \text{erfc} \left( -\sqrt{i\omega t} \right) \right\} \right] + \frac{5}{6} \frac{Gr}{\alpha - 1} t \frac{1}{\pi} \left( \sqrt{\alpha} - 1 \right)
\]

\[ (25) \]
Fig. 1. Velocity profiles for large Grashof number $Gr$

Fig. 2. Velocity profiles for increasing $\omega t$
Fig. 3. Velocity profiles for increasing time $t$

Fig. 4. Velocity profiles for increasing time $t$
Fig. 5. Temperature profiles for increasing $k_1$

Fig. 6. Temperature profiles for increasing $Pr$
Critical Grashof Number:

Critical Grashof Number can be obtained by putting \( \frac{du}{dy} \bigg|_{y=0} = 0 \) in equation (25) reads:

\[
Gr = \frac{1}{A\sqrt{\pi t}} \left[ \frac{1}{4A} \left[ \sqrt{-i\omega} e^{-i\omega t} \left\{ \text{erfc} \left( \sqrt{-i\omega t} \right) - \text{erfc} \left( -\sqrt{-i\omega t} \right) \right\} + \sqrt{i\omega} e^{i\omega t} \left\{ \text{erfc} \left( \sqrt{i\omega t} \right) - \text{erfc} \left( -\sqrt{i\omega t} \right) \right\} \right] \right]
\]  
(26)

where \( A = \frac{5}{6} \frac{1}{\alpha - 1} t \frac{t}{\pi} \left( \sqrt{\alpha - 1} \right) \)

Nusselt number \( Nu \) at the plate becomes:

\[
2 \sqrt{\frac{\alpha t}{\pi}}
\]  
(27)

Numerical results of shear stress and critical Grashof number are presented in tables. Table 1 shows that the frictional shear stress at the plate increases with an increase in either radiation parameter \( (k_i) \) or time variation \( (\eta) \). Also, there exists a seperation at the plate for \( t > 0.2 \). Also, frictional drag increases to impede thermal diffusion at the plate surface. Table 2 demonstrates that the frictional shear stress decrease in magnitude with increase in either Grashof number \( (Gr) \) or radiation parameter \( (k_i) \). This implicates the situation of drag reducing effect to produce stronger thermal diffusion at the plate surface. It is noticed from Table 3 that the frictional shear stress increases with an increase in either phase angle \( (\omega t) \) or time \( (\eta) \). It is interesting to note that there exists separation when \( t > 0.4 \). Since phase angle rotates about the time variation at the plate, the frictional drag is increased to show the influence of thermal radiation at the plate surface. It is evident from Table 4 that there arises a destabilizing influence on the flow field on increasing radiation parameter \( (k_i) \). Table 5 indicates that the Critical Grashof number \( (Gr_{crit}) \) decreases with increase in either phase angle \( (\omega t) \) or time \( (\eta) \) to show the influence of destabilizing effect on the flow field. It is noticed from Tables 4 and 5 that no flow reversal occurs at the plate surface.
Table 1. Shear stress at the plate $\tau_x$ for $Pr = 0.72$, $Gr = 10$, $\omega = 0.2$ and $\omega t = \frac{\pi}{4}$

| $t/k_1$ | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 |
|---------|-----|-----|-----|-----|-----|
| 0.2 | −0.78045 | −0.76103 | −0.76802 | −0.73844 | −0.73095 |
| 0.4 | 0.06960 | 0.12454 | 0.16132 | 0.18844 | 0.20961 |
| 0.6 | 0.85560 | 0.95652 | 1.02411 | 1.07392 | 1.11282 |
| 0.8 | 1.69011 | 1.84549 | 1.94954 | 2.02623 | 2.08612 |
| 1.0 | 2.59257 | 2.80973 | 2.95515 | 3.06233 | 3.14602 |

Table 2. Shear stress at the plate $\tau_x$ for $Pr = 0.72$, $\omega = 0.2$, $\omega t = \frac{\pi}{4}$ and $t = 0.2$

| Gr/$k_1$ | 5.0 | 10.0 | 15.0 | 20.0 | 25.0 |
|---------|-----|-----|-----|-----|-----|
| 1.0 | −0.91187 | −0.78045 | −0.64905 | −0.51763 | −0.38621 |
| 2.0 | −0.90216 | −0.76103 | −0.61991 | −0.47878 | −0.33766 |
| 3.0 | −0.89565 | −0.74802 | −0.60040 | −0.45277 | −0.30514 |
| 4.0 | −0.89086 | −0.73844 | −0.58602 | −0.43360 | −0.28117 |
| 5.0 | −0.88712 | −0.73095 | −0.57479 | −0.41862 | −0.26246 |

Table 3. Shear stress at the plate $\tau_x$ for $Pr = 0.72$, $\omega = 0.2$, $Gr = 10$ and $k_1 = 1$

| $t/\omega t$ | 0.0 | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ |
|-------------|-----|--------|--------|--------|--------|
| 0.2 | −0.99874 | −0.78045 | −0.78045 | −0.68115 | −0.50544 |
| 0.4 | −0.14867 | −0.76103 | 0.06960 | 0.16890 | 0.34462 |
| 0.6 | 0.63731 | 0.76096 | 0.85560 | 0.95490 | 1.13062 |
| 0.8 | 1.47182 | 1.59547 | 1.69011 | 1.78941 | 1.96512 |
| 1.0 | 2.37429 | 2.49794 | 2.59257 | 2.69188 | 2.86759 |

Table 4. Critical Grashof number $Gr_{crit}$ for $Pr = 0.72$, $\omega = 0.2$ and $\omega t = \frac{\pi}{4}$

| $t/k_1$ | 1.0 | 2.0 | 3.0 | 4.0 | 5.0 |
|---------|-----|-----|-----|-----|-----|
| 0.2 | 39.69485 | 36.96329 | 35.33497 | 34.22374 | 33.40347 |
| 0.4 | 9.06368 | 8.43998 | 8.06817 | 7.81444 | 7.62715 |
| 0.6 | 3.73500 | 3.47798 | 3.32477 | 3.22194 | 3.14303 |
| 0.8 | 1.96185 | 1.82685 | 1.74637 | 1.69145 | 1.65091 |
| 1.0 | 1.17716 | 1.09615 | 1.04787 | 1.01491 | 0.99059 |

Table 5. Critical Grashof number $Gr_{crit}$ for $Pr = 0.72$, $k_1 = 1.0$ and $\omega = 0.2$

| $t/\omega t$ | $\theta^0$ | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ |
|-------------|-----------|--------|--------|--------|--------|
| 0.2 | 48.00000 | 43.29557 | 39.69485 | 35.91655 | 33.40347 |
| 0.4 | 12.00000 | 10.33673 | 9.06368 | 7.72789 | 5.36414 |
| 0.6 | 5.33333 | 4.42796 | 3.73500 | 3.00789 | 1.72123 |
| 0.8 | 3.00000 | 2.41194 | 1.96185 | 1.48958 | 0.65387 |
| 1.0 | 1.92000 | 1.49922 | 1.17716 | 0.83923 | 0.24124 |

4. CONCLUSION

Radiation heat transfer aspect on transient gray gas flow of an optically thick fluid past an oscillating vertical flat plate with variable temperature in the presence of thermal radiation has been presented. This problem is an improvement of Stoke’s first and second problem.
with Rosseland radiation – conduction parameter. The present problem deals with optically dense medium with a decisive importance to black body radiation. The governing equations have been solved by using Laplace transform method. The velocity profiles are influenced by the Rossel and radiation – conduction parameter. The fluid velocity greatly increases for increasing values of Grash of number while the fluid velocity is accelerated when time progresses. The frictional shear stress at the plate is reduced in magnitude with increase in Grash of number or radiation parameter while it is enhanced for increasing values of phase angle. It is stated that the critical Grashof number increases for increasing values of phase angle or time whereas it decreases for increasing values of radiation parameter.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

1. Chen S L, Ma H K, Chen DY. Radiation blockage by the interaction of thermal radiation with conduction and convection in the combustion of the condensed fuel, Int. Commun. Heat and Mass Transfer. 1993;20:145-157.
2. Reddy KS, Kumar NS. Combined laminar natural convection and surface radiation heat transfer in a modified cavity receiver of solar parabolic dish, Int. J. Thermal Sci. 2008;47:1647-1657.
3. Gandjalikhan Nassab SAG, Maramisaran M. Transient numerical analysis of a multi-layered porous heat exchanger including gas radiation effects. Int J Therm Sci. 2009;48(8):1586-95.
4. Obidina SP, Kiseleva MN. Production of glass with high radiation absorption in the range 0.9-1.2 μm. Glass Ceram. 1980;37(8):376-8.
5. Saladino AJ, Farmer RC. Radiation/Convection coupling in rocket motor and plume analysis [final report]. Huntsville, AL: Seca Inc; 1993.
6. Gedda H, Powell J, Wahlström G, Li W-B, Engström H, Magnusson C. Energy redistribution during CO₂ laser cladding. J Laser Appl. 2002;14(2):78-82.
7. Siegel R, Howell JR. Thermal radiation Heat Transfer. student ed. McGraw-Hill; 1972.
8. Davies TW. The cooling of a plate by combined thermal radiation and convection. Int Commun Heat Mass Transf. 1985;12(4):405-15.
9. Ali MM, Chen TS, Armaly BF. Natural convection radiation interaction in boundary layer flow over horizontal surface. AIAA J. 1984;22(12):1797-803.
10. Chamkha AJ, Takhar HS, Beg OA. Radiative free convective non-Newtonian fluid flow past a wedge embedded in a porous medium. Int J Fluid Mech Res. 2004;31(2):1-15.
11. Bestman AR. Compressibility effect on laminar convection to a radiating gas past a vertical plate. Z Angew Math Phys. 1985;36(5):767-74.
12. Campo A, Schuler C. Thermal radiation and laminar forced convection in a gas pipe flow. Heat Mass Transf. 1988;22(5):251-7.
13. Yih KA. Radiation effect on natural convection over a vertical cylinder embedded in a porous media, Int. Commun. Heat Mass Transf. 1999; 26:259-67.
14. Hossain MA, Khanafar K, Vafai K. The effect of radiation on free convection flow of fluid with variable viscosity from a porous vertical plate. Int J Therm Sci. 2001;40(2):115-24.
15. Jaiswal BS, Soundalgekar VM. Oscillating plate temperature effects on a flow past an infinite vertical porous late with constant suction and embedded in a porous medium. Heat Mass Transf. 2001;37(2-3):125-31.
16. Muthucumaraswamy R, Ganesan P. Radiation effects on flow past an impulsively started infinite vertical plate with variable temperature. Int J Appl Mech Eng. 2003;8(1):125-9.
17. Makinde OD. Free convection flow with thermal radiation and mass transfer past a moving vertical porous plate. Int Commun Heat Mass Transf. 2005;32(10):1411-9.
18. Kumar AGV, Verma SVK. Thermal radiation and mass transfer effects on MHD flow past a vertical oscillating plate
with variable temperature effects variable mass diffusion. Int J Eng. 2011;3:493-9.
19. Muthucumaraswamy R, Saravanan B. Numerical solution of unsteady radiative flow past an oscillating semi-infinite vertical plate with uniform mass flux. Comp Methods Sci Technol. 2013;19(1):23-31.
20. Ghosh SK, Das S, Jana RN. Transient MHD free convection flow of an optically thick gray gas past a moving vertical plate in the presence of thermal radiation and mass diffusion. J Appl Fluid Mech. 2015;8:65-73.
21. Ahmed S, Batin A, Chamkha AJ. Numerical/Laplace transform analysis for MHD radiating heat/mass transport in a Darcian porous regime bounded by an oscillating vertical surface. Alex Eng J. 2015;54(1):45-54.
22. Kumar GS. Thermal radiation on oscillatory flow past a moving vertical plate in a time varying gravity field, Global journal of engineering and technology advances. 2020;2(1):001-7.
23. Biswas R, Hossain MS, Afikuzzaman M, Ahmmed SF. Computational treatment of MHD Maxwell nanofluid flow across a stretching sheet considering higher-order chemical reaction and thermal radiation. J Comp Math Data Sci. 2022;4. doi: 10.1016/j.mbs.2016.11.009, PMID 27840282.
24. Biswas R, Ahmmed SF. Effects of Hall current and chemical reaction on MHD unsteady heat and mass transfer of Casson nanofluid flow through a vertical plate. J Heat Transf. 2018;140(9): 140/092402-1.
25. Biswas R, Afikuzzaman M, Mondal, Ahmmed SF. MHD free convection and heat transfer flow through a vertical porous plate in the presence of chemical reaction. Front Heat Mass Transf (FHMT). 2018;11:13.
26. Biswas R, Mondal M, Islam A. A steady MHD natural convection heat transfer fluid flow through a vertical surface in the existence of Hall current and radiation. Instrum Mesure Métrologie. 2019;2:331-56.
27. Ahmmed SF, Biswas R. Effects of radiation and chemical reaction on MHD unsteady heat and mass transfer of nanofluid flow through a vertical plate. Model Meas Control B. 2018;87(4):213-20.
28. Gazi MA, Suma UK, Katun M, Taibur Rahaman Md, Ahmmed SF, Biswas R. A numerical evolution of MHD Maxwell fluid flow through an isothermal radiated stretching sheet with higher order chemical reaction. Asian J Pure Appl Math. 2022;4(3):329-53.
29. Ghosh SK. A new investigation on resonance of the Sun with a decisive importance to magnetohydrodynamic (MHD) pure shear flow permeated by an oblique magnetic field. J Energy Res Rev. 2022;12(2):14-25.
30. Ghosh SK. Temperature dependence on transient hydrodynamic gravity driven flow down an inclined plane. J Energy Res Rev. 2022;12(3):16-25.
31. Fakour M, Ganji DD, Abbasi M. Scrutiny of underdeveloped nanofluid MHD flow and heat conduction in a channel with porous walls. Int J Stud Therm Eng. 2014;4: 202-14.
32. Fakour M, Vahabzadeh A, Ganji DD, Hatami M. Analytical study of micropolar fluid flow and heat transfer in a channel with permeable walls. J Mol Liq. 2015;204:198-204.
33. Fakour M, Vahabzadeh A, Ganji DD. Study of heat transfer and flow of nanofluid in permeable channel in the presence of magnetic field. Propul Power Res. 2015;4(1):50-62.
34. Rahbari A, Fakour M, Hamzehnezhad A, Vakilabadi MA, Ganji DD. Heat transfer and fluid flow of blood with nanoparticles through porous vessels in a magnetic field: A quasi-one dimensional analytical approach. Math Biosci. 2017;283:38-47, DOI: 10.1016/j.mbs.2016.11.009, PMID 27840282.
35. Fakour M, Ganji DD, Khalili A, Bakhshi A. Heat transfer in nanofluid MHD flow in a channel with Permeable walls. Heat Transf Res. 2017;48(3):221-38.
36. Fakour M, Rahbari A, Khodabandelh E, Ganji DD. Nanofluid thin film flow and heat transfer over an unsteady stretching elastic sheet by LSM. J Mech Sci Technol. 2018;32(1):177-83.
37. Damala CK, Bhumarapu V, Makinde OD. Radiative MHD Walter’s liquid-B flow past a semi-infinite vertical plate in the presence of viscous dissipation with a heat source. Eng Trans. 2021;69(4):373-401.
38. Chenna Kesavaiah D, Govinda Chowdary P, Rami Reddy G, Nookala V. Radiation, radiation absorption, chemical reaction and hall effects on unsteady flow past an isothermal vertical plate in a rotating fluid with variable mass diffusion with heat.
source. Neuro Quantology. 2022; 20(11):800-15.
39. Chenna Kesavaiah D, Govinda Chowdary P, Ahmed A, Devika B. Radiation and mass transfer effects on MHD mixed convective flow from a vertical surface with heat source and chemical reaction. Neuro Quantology. 2022;20(11):821-35.
40. Isachenko VP, Osipova VA, Sukomel AS. Heat transfer. Moscow, USSR: Mir Publishers; 1980.

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