Topological gravity with exchange algebra

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Abstract

A topological gravity is obtained by twisting the effective (2, 0) supergravity. We show that this topological gravity has an infinite number of BRST invariant quantities with conformal weight 0. They are a tower of OSp(2, 2) multiplets and satisfy the classical exchange algebra of OSp(2, 2). We argue that these BRST invariant quantities become physical operators in the quantum theory and their correlation functions are braided according to the quantum OSp(2, 2) group. These properties of the topological effective gravity are not shared by the standard topological gravity.
It is one of the important subjects in the string theory to understand coupled systems of 2-dimensional gravity and conformal field theories (CFTs). There is much interest in the case where they are topological, i.e., topological gravity coupled with topological conformal field theories (TCFTs), since the analysis is simplified by the BRST symmetry and becomes far-reaching\cite{1,2}. Topological gravity was formulated in refs 3 and 4. In this note we give one more different formulation to discover some new properties. Namely we think of $N = 2$ supergravity coupled with a $N = 2$ CFT. The former gets dynamical through quantization, i.e., effective $N = 2$ supergravity. By twisting\cite{1,5} at this stage the total system turns into a topological effective gravity coupled with a TCFT. The novelty of this topological gravity is that there exist an infinite number of BRST invariant quantities with conformal weight 0. They are a tower of OSp(2, 2) multiplets and satisfy the classical exchange algebra of OSp(2, 2). It is natural to assume that in the quantum theory these BRST invariant quantities become physical operators. Then they define the physical ground states which are an infinite tower of OSp(2, 2) multiplets (, precisely speaking, quantum OSp(2, 2) multiplets). Moreover we may argue that correlation functions of these physical operators are braided according to the quantum OSp(2, 2) group. These ground states are purely gravitational. They contrast with the ground ring of the $c = 1$ string\cite{6}. They are rather gravitational descendants of the puncture operator\cite{1,2} in the language of the standard topological gravity.

The untwisted effective $N = 2$ supergravity can be discussed in one of the following formulations

i) light-cone gauge formulation\cite{7};

ii) geometrical formulation\cite{8};

iii) formulation by the constrained WZWN model\cite{9};

iv) conformal gauge formulation\cite{10,11}.

In ref. 12 the non-supersymmetric effective gravity was discussed in all these formulations. The consistency among them was checked at both classical and quantum levels. A similar consistency check would be done in the $N = 1$ or $N = 2$ case as well. In this note we choose the geometrical formulation. We also choose to study the effective $(2, 0)$ supergravity for simplicity. By the twisting procedure this becomes a topological gravity, which is topological in the supersymmetric sector.
We discuss that in this sector the theory is indeed such as described just above.

The supersymmetric sector of the effective (2, 0) supergravity has characteristic properties in the geometrical formulation, namely the superconformal symmetry and the exchange algebra of OSp(2, 2). They are essential for our arguments. The theory also exhibits remarkable properties in the non-supersymmetric sector like the Knizhnik-Zamolodchikov equation (simply the KZ equation,) and the Kac-Moody symmetry with OSp(2, 2). But this sector is irrelevant for our purpose in this note. So these issues will not be discussed. Suffice it to say that the appearances of the KZ equation and the Kac-Moody symmetry with OSp(2, 2) are almost evident by taking analogy to the non-supersymmetric effective gravity in the geometrical formulation\(^{12}\). Similar things have been originally shown by studying the non-supersymmetric sector of the effective (0, 0) and (1, 0) supergravities in the light-cone gauge\(^{7}\). But we would like to stress that the light-cone gauge approach does not give any proper account for the supersymmetric sector as the geometrical formulation does\(^{12}\). This is the reason why we choose the latter formulation in this note.

Let us begin by summarizing the (2, 0) superconformal group\(^{13}\). The supersymmetric sector of the (2,0) superspace is described by a real coordinate \(x\), a fermionic complex coordinate \(\theta^+\) and its complex conjugate \(\theta^-\), while the non-supersymmetric sector by a real coordinate \(t\) alone. We define the superderivatives as

\[
D_+ = \frac{\partial}{\partial \theta^+} + \theta^- \partial_x, \quad D_- = \frac{\partial}{\partial \theta^-} + \theta^+ \partial_x,
\]

which satisfy

\[
\{D_+, D_-\} = 2\partial_x, \quad \{D_\pm, D_\pm\} = 0.
\]

An element of the (2, 0) superconformal group is given by superdiffeomorphisms

\[
\begin{align*}
x & \longrightarrow f(x, \theta^+, \theta^-; t), \\
\theta^\pm & \longrightarrow \varphi^\pm(x, \theta^+, \theta^-; t), \\
t & \longrightarrow g(x, \theta^+, \theta^-; t),
\end{align*}
\]

which change the derivatives in the respective sector as

\[
D_\pm = (D_\pm \varphi^\pm) D^\varphi_+ + (D_\pm \varphi^-) D^\varphi_-,
\]

3
\[ \partial_t = (\partial_t g) \partial_g, \]

with

\[ D^g_+ = \frac{\partial}{\partial \varphi^+} + \varphi^- \frac{\partial}{\partial f}, \quad D^g_- = \frac{\partial}{\partial \varphi^-} + \varphi^+ \frac{\partial}{\partial f}, \]

\[ \partial_g = \frac{\partial}{\partial g}. \]

This imposes the following conditions on superdiffeomorphisms

\[ D_{\pm} f = \varphi^+ D_{\pm} \varphi^- + \varphi^- D_{\pm} \varphi^+, \quad (3) \]

\[ \partial_x g = \partial_{\pm} g = 0. \quad (4) \]

Since \( D_{\pm}^2 = 0 \), the superconformal conditions (3) imply that either \( D_{\pm} \varphi^\pm = 0 \) or \( D_{\pm} \varphi^\mp = 0 \). For convenience we will choose the case

\[ D_{\pm} \varphi^\mp = 0, \quad (5) \]

in this note. Then eqs (2) and (3) are reduced respectively to

\[ D_+ = (D_+ \varphi^+) D^g_+, \quad D_- = (D_- \varphi^-) D^g_- , \]

and

\[ D_+ f = \varphi^- D_+ \varphi^+, \quad D_- f = \varphi^+ D_- \varphi^- . \quad (6) \]

In terms of infinitesimal diffeomorphisms \( \delta f \) and \( \delta \varphi^\pm \) the superconformal conditions (6) are written in the forms

\[ \delta \varphi^+ = \frac{1}{D_- \varphi^-} D_- (\delta f + \varphi^+ \delta \varphi^-), \]

\[ \delta \varphi^- = \frac{1}{D_{\pm} \varphi^+} D_{\pm} (\delta f + \varphi^- \delta \varphi^+). \quad (7) \]

The \((2,0)\) superconformal transformations are obtained as solutions of the equations (4) and (7):

\[ \delta f = [\xi \partial_x + \frac{1}{2} (D_+ \xi) D_- + \frac{1}{2} (D_- \xi) D_+] f + \zeta \partial_t f, \quad (8) \]

\[ \delta \varphi^\pm = [\xi \partial_x + \frac{1}{2} (D_\mp \xi) D_{\pm}] \varphi^\pm + \zeta \partial_t \varphi^\pm , \quad (9) \]

\[ \delta g = \zeta \partial_t g. \]
Here $\xi$ and $\zeta$ are infinitesimal parameters of the $(2,0)$ superconformal transformations in the super- and non-supersymmetric sectors such as $\xi(x, \theta^+, \theta^-)$ and $\zeta(t)$. Note that $\xi$ is given by

$$\xi = \delta x + \theta^+ \delta \theta^- + \theta^- \delta \theta^+, \quad \zeta = \delta t,$$

(10)

which follows from eqs (8) and (9). A $(2,0)$ superconformal field with weight $(h, w)$ is defined as transforming by (1) according to

$$\Psi(x, \theta^+, \theta^-, t) = \Psi(f, \phi^+, \phi^-, g) \left( D_{+} \phi^+ \right)^{h-\frac{q}{2}} \left( D_{-} \phi^- \right)^{h+\frac{q}{2}} (\partial_t g)^w,$$

in which $q$ is $U(1)$ charge in the supersymmetric sector. It becomes infinitesimally

$$\delta \Psi = \{ \xi \partial_x + \frac{1}{2} (D_+ \xi) D_+ + \frac{1}{2} (D_- \xi) D_- + (h \partial_x \xi + \frac{q}{4} [D_-, D_+] \xi) + \zeta \partial_t + w(\partial_t \zeta) \} \Psi,$$

(11)

by using (10).

In the geometrical formulation the effective $(2,0)$ supergravity is formulated as a 2-dimensional field theory on the coadjoint orbit of the $(2,0)$ superconformal group$^{[14]}$. The action is given by

$$S = \frac{k}{2\pi} \int dx dt d\theta^+ d\theta^- \partial_t (\log D_+ \phi^+) \log D_- \phi^-,$$

(12)

in which $\phi^\pm$ are fermionic superfields characterized above. By the construction this action is invariant under the $(2,0)$ superconformal transformations (9). In fact we find that

$$\delta S = -\frac{1}{2\pi} \int dx dt d\theta^+ d\theta^- [\xi(x, \theta^+, \theta^-) \partial_t S + \zeta(t) \partial_x T] = 0,$$

where

$$S = k [\partial_x (\log D_+ \phi^+ - \log D_- \phi^-) + 2 \frac{\partial_x \phi^+}{D_+ \phi^+} \frac{\partial_x \phi^-}{D_- \phi^-}]$$

(13)

(N = 2 super Schwarzian derivative),

and

$$T = 2k \frac{\partial_t \phi^+}{D_+ \phi^+} \frac{\partial_t \phi^-}{D_- \phi^-}.$$
In this note we concentrate on the supersymmetric sector which exhibits a topological nature later on. In ref. 15 the Poisson brackets of $\varphi^\pm$ and $f$ with themselves were worked out in this sector, i.e., on a plane at a given time $t$. They were found as solutions to the following requirements:

i) the super Schwarzian derivative (13) is the generator of the superconformal transformations given by (8) and (9);

ii) the Poisson brackets are consistent with the superconformal conditions (6);

iii) the Jacobi identities are satisfied.

It turned out that

$$\{\varphi^+(z), \varphi^-(z')\}_{PB} = -\frac{\pi}{2k}[\theta(x-x')\{ f(z) + \varphi^+(z)( \varphi^-(z) - \varphi^-(z') ) \}$$

$$+\theta(x'-x)\{ f(z') + \varphi^-(z')( \varphi^+(z') - \varphi^+(z) ) \}],$$

$$\{f(z), \varphi^+(z')\}_{PB} = -\frac{\pi}{2k}[\theta(x-x')f(z)( \varphi^+(z) - \varphi^+(z') )$$

$$+\theta(x'-x)\{ f(z)\varphi^+(z') + f(z')( \varphi^+(z) - 2\varphi^+(z') )$$

$$+ \varphi^+(z)\varphi^+(z')\varphi^-(z') \}],$$

$$\{f(z), \varphi^-(z')\}_{PB} = -\frac{\pi}{2k}[\theta(x-x')f(z)( \varphi^-(z) - \varphi^-(z') )$$

$$+\theta(x'-x)\{ f(z)\varphi^-(z') + f(z')( \varphi^-(z) - 2\varphi^-(z') )$$

$$+ \varphi^-(z)\varphi^-(z')\varphi^+(z') \}],$$

$$\{f(z), f(z')\}_{PB} = -\frac{\pi}{2k}[\theta(x-x')\{2f(z)( f(z) - f(z') )$$

$$- f(z)( \varphi^+(z)\varphi^-(z') + \varphi^-(z)\varphi^+(z') ) \}$$

$$+\theta(x'-x)\{ 2f(z')( f(z) - f(z') )$$

$$- f(z')( \varphi^+(z)\varphi^-(z') + \varphi^-(z)\varphi^+(z') ) \}],$$

(15)

in which $z = (x, \theta^+, \theta^-)$ and the t-dependence of $\varphi^\pm(z; t)$ and $f(z; t)$ has not explicitly be written down. We have normalized the coupling constant $k$ by requiring that the last Poisson bracket tends to the one of the $(0,0)$ theory at the
non-supersymmetric limit\textsuperscript{[12]}. By using these Poisson brackets we find the superconformal transformation of the super Schwarzian derivative itself:

\[
\frac{1}{2\pi} \int dz' \xi(z') \{S(z'), S(z)\}_PB \\
= \{ [ \xi(z) \partial_x + \frac{1}{2}(D_+ \xi(z))D_- + \frac{1}{2}(D_- \xi(z))D_+ + \partial_x \xi(z) ]S(z) \\
+ \frac{k}{2}[D_+, D_-] \partial_x \xi(z),
\]

which is anomalous by the last piece. It is typical in the geometrical formulation that the superconformal anomaly already appears by a classical calculation.

The action (12) has a hidden symmetry in the non-supersymmetric sector. Consider the non-linear OSp(2, 2) transformations given by

\[
\begin{align*}
\delta f &= \epsilon^1 + \epsilon^0 f + \epsilon^{-1} f^2 \\
&+ \varphi^+ (\epsilon^+ + \epsilon^- f) + \varphi^- (\epsilon^+ - \epsilon^- f),
\end{align*}
\]

with infinitesimal parameters \( \epsilon^i = \epsilon^i(t), i = 0, \pm \frac{1}{2}, \pm 1 \). Then the transformation laws of the variables \( \varphi^\pm(z, t) \) may be found with recourse to the superconformal conditions (6)\textsuperscript{[15]}:

\[
\begin{align*}
\delta \varphi^+ &= \frac{1}{2} \epsilon^0 \varphi^+ + \epsilon^{-1} f \varphi^+ + \epsilon^+ \varphi^- + \epsilon^- (f + \varphi^+ \varphi^-) + \frac{1}{2} \epsilon \varphi^+, \\
\delta \varphi^- &= \frac{1}{2} \epsilon^0 \varphi^- + \epsilon^{-1} f \varphi^- + \epsilon^+ \varphi^+ + \epsilon^- (f - \varphi^+ \varphi^-) - \frac{1}{2} \epsilon \varphi^-,
\end{align*}
\]

in which \( \epsilon \) is a \( U(1) \) parameter of the OSp(2, 2) transformations such as \( \epsilon = \epsilon(t) = -\epsilon \). We can show that the action (12) is invariant by (17)\textsuperscript{[15]}. As for the (0, 0) effective gravity\textsuperscript{[12]}, this local symmetry may be similarly exploited to show the KZ equation and the Sugawara form of the energy-momentum tensor (14) with OSp(2, 2) in the non-supersymmetric sector. But we are not involved in the issue in this note. Therefore hereinafter it is treated simply as a global symmetry in the supersymmetric sector.

In ref. 15 it was shown that the quantities

\[
\psi = \left( \begin{array}{c} \psi_1 \\ \psi_{-\frac{1}{2}} \\ \psi_{-1} \\ \psi_0 \end{array} \right) = \frac{1}{D_+ \varphi^+} \left( \begin{array}{c} f - \varphi^+ \varphi^- \\ 1 \\ \varphi^- \end{array} \right), \quad c.c.,
\]

(18)
have remarkable properties. First of all they form the lowest dimensional representation of \( \text{OSp}(2,2) \). Namely by the transformations (16) and (17) we find

\[
\delta \psi = \left[ \sum_{i=0,\pm 1} \epsilon^i L_i + \sum_{i=\pm \frac{1}{2}} (\epsilon^i L_i^+ + \epsilon^i L_i^-) + \epsilon L \right] \psi.
\]

(19)

Here \( L_i \) (\( i = 0, \pm 1 \)), \( L_i^\pm \) (\( i = \pm \frac{1}{2} \)) and \( L \) are the generators of \( \text{OSp}(2,2) \) given by \( 3 \times 3 \) matrices. The reader may refer to ref. 15 for the explicit expressions.

Secondly the quantities (18) obey the classical exchange algebra:

\[
\left\{ \psi(z), \psi(z') \right\}_{PB} = \frac{\pi}{k^2} \left[ r^+ \theta(x-x') + r^- \theta(x'-x) \right] \psi(z) \otimes \psi(z'),
\]

(20)

by means of the Poisson brackets (15), in which the \( r \)-matrices are given by

\[
\begin{align*}
    r^+ &= L_0 \otimes L_0 - L_{-1} \otimes L_1 - \frac{1}{2} L_{-\frac{1}{2}}^- \otimes L_{-\frac{1}{2}}^+ - \frac{1}{2} L_{-\frac{1}{2}}^+ \otimes L_{-\frac{1}{2}}^- - L \otimes L, \\
    r^- &= -L_0 \otimes L_0 + L_1 \otimes L_{-1} - \frac{1}{2} L_{-\frac{1}{2}}^+ \otimes L_{-\frac{1}{2}}^- - \frac{1}{2} L_{-\frac{1}{2}}^- \otimes L_{-\frac{1}{2}}^+ + L \otimes L,
\end{align*}
\]

and satisfy the classical Yang-Baxter equation

\[
[r_{12}, r_{13}] + [r_{12}, r_{23}] + [r_{13}, r_{23}] = 0.
\]

(There are misprints in the forefactor of the classical exchange algebra and the fourth piece of \( r^\pm \) given in ref. 15.) Poisson brackets, a transformation law and an exchange algebra, such as given by (15), (19) and (20) respectively, were firstly discussed for the non-supersymmetric effective gravity in the geometrical formulation [16]. Then they were extended to the \((1,0)\) and \((2,0)\) supersymmetric cases [15,17]. But similar equations had been originally found for the Liouville theory [10,18].

We can make up higher dimensional representations of \( \text{OSp}(2,2) \) out of the lowest one \( \psi \). To this end let us introduce another set of superdiffeomorphisms \( a \) and \( \beta^\pm \), which are constrained by the same superconformal conditions as \( f \) and \( \varphi^\pm \). Define a vector such that

\[
\mathcal{X} = \begin{pmatrix} \bar{X}^\frac{1}{2} \\ \bar{X}^{-\frac{1}{2}} \\ \bar{X}^0 \end{pmatrix} = \frac{1}{D_\text{BL}} \begin{pmatrix} a - \beta^- \beta^+ \\ 1 \\ \beta^+ \end{pmatrix}.
\]
It transforms as the complex conjugate of $\psi$:

$$\delta \chi = \left[ \sum_{i=0,\pm 1} \epsilon_i L_i + \sum_{i=\pm \frac{1}{2}} (\epsilon_i^+ L_i^+ + \epsilon_i^- L_i^-) - \epsilon L \right] \chi.$$

The product

$$\chi \cdot \psi \equiv \chi_{\frac{1}{2}} \psi_{-\frac{1}{2}} - \chi_{-\frac{1}{2}} \psi_{\frac{1}{2}} + 2 \chi_0 \psi_0$$

is OSp(2, 2) invariant. By expanding the multiple product $(\chi \cdot \psi)^{2j}, j = \frac{1}{2}, 1, \frac{3}{2}, \ldots,$ we find

$$(\chi \cdot \psi)^{2j} \equiv \chi^j \cdot \psi^j = (\sum_{m=-j}^{j} (-1)^{j+m} \chi^j_{-m} \psi^j_{m} + 2 \sum_{\mu=-j+\frac{1}{2}}^{j-\frac{1}{2}} (-1)^{j+\mu - \frac{1}{2}} \chi^j_{-\mu} \psi^j_{\mu},$$

with

$$\psi^j = \left( \begin{array}{c} \psi^j_m \\ \psi^j_{\mu} \end{array} \right) = \frac{f^j}{(D_+ \phi^+)^{2j}} \left( \sqrt{\frac{(2j)!}{(j+m)!(j-m)!}} [f^m - (j + m) f^{m-1} \varphi^+ \varphi^-] \right) \sqrt{\frac{(2j)!}{(j+\mu-\frac{1}{2})!(j-\mu+\frac{1}{2})}} f^{\mu-\frac{1}{2}} \varphi^- \right), \quad (21)$$

and a similar expression for $\chi^j$ in terms of $a$ and $\beta^\pm$. It is obvious that the quantities $\psi^j$ form the $(4j+1)$-dimensional multiplets of OSp(2, 2). Examining $\delta \psi^j$ by (16) and (17) we find the OSp(2, 2) generators as $(4j+1) \times (4j+1)$ matrices.

The components of the multiplets (21) now satisfy the classical exchange algebra (20) with the $r$-matrices in the $(4j+1)$-representation.

The OSp(2, 2) multiplets $\psi^j$ have further important properties. First of all they are chiral$^{[20]}$:

$$D_+ \psi^j = 0. \quad (22)$$

We examine the transformation property by the superconformal transformations (8) and (9). It is easily shown that they are the $(2, 0)$ superconformal fields with weight $(-j, 0)$ and $U(1)$ charge $2j$ which transform according to eq. (11). By
expanding the super Schwarzian derivative (13) we find the generators of the $N = 2$ superconformal symmetry:

$$S = J + \theta^+ G_+ + \theta^- G_- + \theta^+ \theta^- T. \quad (23)$$

The global supersymmetry transformation, generated by the supercurrent $G_+$, will be of particular importance for our arguments soon later. We quickly look into the transformation property of the multiplets $\psi^j$ by this. The supercurrent $G_+$ generates a translation in the superspace such that

$$\delta \theta^+ = \text{const.}, \quad \delta \theta^- = \delta x = \delta t = 0.$$

From eq. (10) we have $\xi = \theta^- \delta \theta^+$. Then the transformation law (11) is reduced to

$$\delta \psi^j = \theta^- \delta \theta^+ \partial_x \psi^j. \quad (24)$$

for $\psi^j$. Taking into account the chirality condition we expand $\psi^j$ as

$$\psi^j = u^j + \theta^- \rho^j - \theta^+ \theta^- \partial_x u^j.$$

With this the global supersymmetry transformation (24) reads in components

$$\delta u^j = 0, \quad \delta \rho^j = -\delta \theta^+ \partial_x u^j. \quad (25)$$

Let us now twist\cite{[1,5]} the theory to get a topological gravity. The energy-momentum tensor $T$ in eq. (23) is modified by adding the $U(1)$ current $J$:

$$T \longrightarrow T + \frac{1}{2} \partial_x J.$$

Remarkably the multiplets $\psi^j$, given by eq. (21), all become chiral superconformal fields with weight $(0,0)$ with respect to this modified energy-momentum tensor. The supersymmetry current $G_+$ turns into the BRST current. As the result the global supersymmetry transformation (25) is identified with the BRST transformations. We turn to quantization of this topological theory. All the properties of $\psi^j$ hitherto found are well-based on the geometric and algebraic arguments.
Therefore it is fairly natural to suppose that they are maintained at the quantum level. Precisely speaking we assume that by quantization (i) the superconformal symmetry is took over, and (ii) the classical exchange algebra (20) becomes the operator relation

\[
\psi_{j_1m_1}(x)\psi_{j_2m_2}(y) = \theta(x - y)(R^+_{q})_{m_1m_2}^{m_1'm_2'}\psi_{j_2m_2'}(y)\psi_{j_1m_1'}(x) + \theta(y - x)(R^-_{q})_{m_1m_2}^{m_1'm_2'}\psi_{j_2m_2'}(y)\psi_{j_1m_1'}(x),
\]

(26)

in which \(R^\pm_q\) are the \(R\)-matrices of the quantum OSp(2, 2) group with the deformation parameter

\[
q = \exp\left(\frac{\pi i}{k - 1}\right).
\]

Here the coupling constant \(k\) has been shifted by the quadratic Casimir of the adjoint representation of OSp(2, 2). It is a quantum effect which may be found through analyses of the opposite sector\[^{[12]}\]. Keep in mind that in this note as well as in ref. 12 the sign of the coupling constant \(k\) is chosen to be opposite to that of refs 7. (If the multiplets have different dimensions \(j_1\) and \(j_2\), the \(R\)-matrices should be represented in a dimension which equals a common multiple of \(j_1\) and \(j_2\).) The relation (26) correctly reproduces eq. (20) at the classical limit \(k \to \infty\), since we have

\[
R^\pm_q = 1 + \frac{\pi i}{k - 1}r^\pm + O\left(\frac{1}{(k - 1)^2}\right).
\]

It is a barely possible quantum generalization of the classical algebra (20) which is consistent with the OSp (2, 2) and superconformal symmetries. It was successful to quantize the non-supersymmetric effective gravity this way\[^{[19,12]}\]. We shall see the consequences of these assumptions for the topological gravity. By the assumption (i) the multiplets \(\psi_{jm}\) are chiral primaries of the \(N = 2\) superconformal algebra at the quantum level. We have classically shown that their conformal weight is zero after twisting. In principle it would be shifted by \(\Delta\) due to quantum effects. As it was discussed in refs 19 and 21, the quantum group (assumption (ii)) and conformal symmetries (assumption (i)) interplay to fix this anomalous dimension:

\[
\Delta = -\frac{c_j}{2(k - 1)},
\]

in which \(c_j\) is the quadratic Casimir of the \((4j + 1)\)-dimensional representation. Note that the anomalous dimension \(\Delta\) tends to zero at the classical limit \(k \to \infty\). A
speciality about OSp(2, 2) is that the quadratic Casimir of the (4j + 1)-dimensional representation is vanishing. Hence the twisted multiplets ψ^j_m stay with the classical conformal weight 0 even at the quantum level. This result agrees with what is known about twisted chiral primaries by the algebraic arguments\cite{1,5,20}. The bosonic components u^j_m become physical operators since they are BRST invariant by (25). The BRST partners are given by \( \int dx \rho^j_m \) as usual\cite{5}. These physical operators define the primary states \( |u^j_m> \) and \( \int dx |\rho^j_m> \) respectively, which form an infinite tower of OSp(2, 2) multiplets (precisely speaking, quantum OSp(2, 2) multiplets). Since they have conformal weight 0, they are the BRST invariant ground states of the theory. This is reminiscent of the ground ring of the c = 1 string\cite{6}. But here the ground states are purely gravitational. We consider a correlation function of the ground primaries \( u^j_m \)

\[
< u^j_{m_1}(x_1) u^j_{m_2}(x_2) \cdots \cdot u^j_{m_N}(x_N) >.
\]

It is just a constant because conformal weight of \( u^j_m \) is zero from the quantum group argument. This can be also shown by using the BRST invariance of \( u^j_{m} \) according to the standard argument of the topological theory\cite{3}. It is worth noting here consistency between the two assumptions (i) and (ii). An interesting thing with this is that we can successively exchange the order of the ground primaries \( u^j_m \) by means of the algebra (26). Thus the correlation function is braided by the \( R \)-matrices of quantum OSp(2, 2). This braiding property can not be extended to correlation functions including the BRST partners \( \int dx \rho^j_m \). The BRST partners do not obey the exchange algebra (26), although the fermionic primaries \( \rho^j_m \) themselves do well.

In this note we have constructed chiral superconformal fields in the effective (2, 0) supergravity. They are an infinite tower of OSp(2, 2) multiplets and satisfy the classical exchange algebra with the \( r \)-matrices of the OSp(2, 2) group. By the twisting procedure the effective (2, 0) supergravity becomes a topological gravity. From those chiral superconformal fields we have found an infinite number of BRST invariant quantities with conformal weight 0 in this topological gravity, i.e., \( u^j_m \) and \( \int dx \rho^j_m \). These classical arguments are well-based on the superconformal geometry and the OSp(2, 2) group. Hence we were led to assume naturally that at
the quantum level the superconformal symmetry is maintained and the OSp(2, 2) symmetry is took over as the quantum group symmetry. The first assumption implied that the multiplets \( u^i_m \) and \( \int dx \rho^j_m \) become BRST invariant primaries at the quantum level. The second assumption was used to show that quantization does not modify the classical value of their conformal weights. Thus we have obtained an infinite number of physical operators with conformal weight 0 at the quantum level. They define the BRST invariant ground states of the theory. Furthermore the second assumption enabled us to discuss that correlation functions of the multiplets \( u^i_m \) are braided by the \( R \)-matrices of the quantum OSp(2, 2). The last property is not shared by the standard topological gravity\(^{[3,4]}\).

It is still desired to prove these assumptions rigorously. In this regard it is interesting to study the effective (2, 0) supergravity in a conformal gauge, i.e., the (2, 0) supersymmetric Liouville theory. As the (0, 0) theory\(^{[18,22]}\) it would be expected to be solvable. There would exist similar quantities to \( \psi^j \) and they could be represented in terms of free fields. It would be then possible to derive our assumptions.

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