Tests of Flavor Symmetry in $J/\psi$ Decays

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Abstract. We use SU(3) flavor symmetry to analyze the $PP'$, $VP$ and baryon-antibaryon decays of $J/\psi$. Both, the SU(3)-invariant and -violating contributions are considered. Particular attention is paid to the interference of the electromagnetic and strong amplitudes.

INTRODUCTION

The $J/\psi$ was discovered simultaneously in $e^+e^-$ and $p\bar{p}$ collisions in 1974 (1). Twenty years later, more than one hundred of its exclusive decay modes are reported by the Particle Data Group (2). The measurements of these decays serve several purposes (3): the study of light hadron spectroscopy, the $q\bar{q}$ content of mesons, the electromagnetic form factors at the $J/\psi$ mass, the determination of $\alpha_s$, the pattern of flavor symmetry breaking, the search for exotics ($gg$, hybrids and $4q$ states), among others.

The $J/\psi$ is only 118 MeV above the $\eta_c(1S)$, which is the lowest lying state of the charmonium family. This means that all of its decay modes, other than $J/\psi \to \eta_c\gamma$, are suppressed by the so-called OZI rule (4). This suppression occurs because the final states containing only hadrons are reached through the (a) electromagnetic (1 virtual photon) and, (b) strong (3 gluons) annihilation of $J/\psi$.

The $J/\psi$ system offers a unique opportunity to study the simultaneous effects of electromagnetic and strong interactions. On the one hand, since $\alpha_3^2(M_{J/\psi}) \approx \alpha_{em}$, this suggest that electromagnetic and strong annihilation amplitudes for $J/\psi$ hadronic decays could have similar strenghts. Moreover, the $J/\psi$ has an special status among the quarkonium vector states because it is three times heavier than the $\phi(ss)$, where non-perturbative QCD is expected to dominate the OZI suppressed decays and, on the other hand, it is three
times lighter than the $\Upsilon(bb)$, where perturbative QCD plays the main role to produce hadrons.

In this work we are concerned with tests of SU(3) flavor symmetry\footnote{Hereafter SU(3) refers to the flavor symmetry of light hadrons.} in two-body hadronic decays of $J/\psi$\footnote{Related analysis but with a different emphasis, can be found in Refs. (6-8).}. The present analysis consider $J/\psi$ decays into the $PP'$, $VP$ and $BB$ channels ($P$, $V$ and $B$ for a pseudoscalar, vector and spin-1/2 baryon, respectively). Our main focus will be on the relative size of the $1\gamma$ and $3g$ contributions and on the relative phase between them (6). This would be important in order to isolate the relevant electromagnetic form factors at the $J/\psi$ mass.

Our starting point is to realize that the $J/\psi$ is a singlet under SU(3). The main advantage of doing an SU(3) analysis of the $J/\psi$ hadronic decays relies on the fact that both, the light final state hadrons and the symmetry breaking interactions, have well defined transformation properties under SU(3). For definiteness let us introduce two important ingredients of the analysis:

(a) The relevant decay amplitudes must include the effects of SU(3) breaking. At the fundamental level, the sources of this flavor symmetry breaking are the quark-mass differences and the electromagnetic interactions\footnote{For simplicity we neglect the isospin breaking in the quark mass sector ($m_u = m_d = m$).} namely:

$$\mathcal{H}_m = \frac{1}{2} \bar{q} \{a \lambda_0 + b \lambda_8 \} q$$ \hspace{1cm} (1)

and

$$\mathcal{H}_{em} = \frac{e}{2} A_\mu \gamma_\mu \{ \lambda_3 + \frac{\lambda_8}{\sqrt{3}} \} q,$$ \hspace{1cm} (2)

where $q^T = (u, d, s)$, $A^\mu$ is the electromagnetic four-potential, $\lambda_{3,8}$ are Gell-Mann matrices, $\lambda_0 = \sqrt{2/3} I$ and $a = \sqrt{2/3}(m_s + 2m)$, $b = -2(m_s - m)/\sqrt{3}$.

(b) Following Ref. (7), it is useful to introduce a generalized charge conjugation operation $\mathcal{C}$. According to $\mathcal{C}$, $J/\psi$ decays into a pair of hadrons that belong to the same representation of SU(3) are forbidden in the SU(3) limit. Thus, the $J/\psi \to PP'$ decays are forbidden in the SU(3) limit, while $J/\psi \to VP$, $BB$ are allowed.

**TWO-PSEUDOSCALAR CHANNEL**

The measured decay modes in this channel which are reported by the Particle Data Group (2) are: $\pi^+\pi^-, K^+K^-$ and $K_LK_S$. In order to use flavor
SU(3), one introduces an octet of pseudoscalar fields $P^a$, $a = 1, \ldots, 8$. The effective Lagrangian used to describe these SU(3)-violating decays is given by (7):

$$\mathcal{L} = f_{abc} \psi^\mu P^a \partial_\mu P^b [g_M \delta^{c8} + e^{i\phi} g_E (\delta^{c3} + \delta^{c8} / \sqrt{3})]$$

where $f_{abc}$ are the SU(3) structure constants, $\psi^\mu$ describes the $J/\psi$ vector field and $g_M$, $g_E$ are the coupling constants coming from quark-mass difference and electromagnetism.

A fit to the experimental data for the $\pi^+\pi^-$, $K^+K^-$ and $K_LK_S$ decay modes (2), gives:

$$g_M = (8.5 \pm 0.6) \times 10^{-4}$$
$$g_E = (7.8 \pm 0.7) \times 10^{-4}$$
$$\phi = (90 \pm 10)^0.$$  

Thus, the electromagnetic and quark-mass difference contributions have similar strengths as expected from the naive counting of couplings ($\alpha_3^q \approx \alpha_{em}$). Observe also that there is not interference of both contributions in the decay rate ($\phi \approx \pi/2$).

**VECTOR-PSEUDOSCALAR CHANNEL**

Ten exclusive decay modes have been reported in this channel, namely (2): $\rho^0\pi^0$, $K^{*+}K^-$, $K^{*0}K^0$, $\omega\eta$, $\omega\eta'$, $\phi\eta$, $\phi\eta'$, $\rho^0\eta$, $\rho^0\eta'$ and $\omega\pi^0$. The corresponding Lorentz invariant amplitude for this decay channel is given by:

$$\mathcal{M} = g_{VP} \varepsilon_{\mu\nu\alpha\beta} \varepsilon^{\mu\eta^\nu} q^\alpha k^\beta,$$  

where $g_{VP}$ is the coupling constant, $\varepsilon^{\mu}(\eta^\mu)$ is the $J/\psi(V)$ polarization four-vector and $q(k)$ the corresponding four-momentum.

In order to provide a description based on SU(3) one needs to introduce an octet ($O^a$, $a = 1, \ldots, 8$) and a singlet ($S$) of vector and pseudoscalar fields. As usual, we define the following physical states:

$$\eta = P_8 \cos \theta_P - P_0 \sin \theta_P$$
$$\eta' = P_8 \sin \theta_P + p_0 \cos \theta_P$$
$$\omega = V_8 \sin \theta_V + V_0 \cos \theta_V$$
$$\phi = V_8 \cos \theta_V - V_0 \sin \theta_V.$$
where $\theta_P = -20^\circ$ and $\theta_V = \tan^{-1}(1/\sqrt{2})$ (2) denote the octet-singlet mixing angles.

Since the $VP$ channel is allowed by $C$ both, the SU(3)-invariant and -violating contributions are present. Following Haber and Perrier (7), the SU(3) structure of the interaction Lagrangian can be written as follows:

$$
\mathcal{L} = \psi \left\{ g_8 \delta^{ij} O_1^i O_2^j + g_1 S_1 S_2 
+ \left[ g_{M,88} d_{ab8} + g_{E,88} \left( d_{ab3} + \frac{d_{ab8}}{\sqrt{3}} \right) \right] O_1^a O_2^b 
+ \sqrt{\frac{2}{3}} \left[ g_{M,81} O_1^8 + g_{E,81} \left( O_1^3 + \frac{O_1^8}{\sqrt{3}} \right) \right] S_2 
+ (81 \rightarrow 18, 1 \leftrightarrow 2 \text{ in the previous term}) \right\}
$$

where the subindex $E(M), ij$ in the coupling constants stands for electromagnetic (mass) origin and the subindex $1(2)$ in $O, S$ is for vector (pseudoscalar) states. $d_{abc}$ are the symmetric constants of SU(3). Thus, the decay amplitudes will depend on 8 free coupling constants as well as on the electromagnetic-strong relative phase $\phi^\delta$.

If we assume nonet symmetry, we are left with only 4 free parameters because in this limit:

$$
g \equiv g_8 = g_1 
\quad g_M \equiv g_{M,88} = g_{M,81} = g_{M,18} 
\quad g_E \equiv g_{E,88} = g_{E,81} = g_{E,18}. 
$$

Thus, in order to quantify nonet symmetry breaking in these decays we introduce the ratios:

$$
r_M = \frac{g_{M,81}}{g_{M,88}}, \quad r'_M = \frac{g_{M,18}}{g_{M,88}} 
$$

and similar expressions for $r_E$ and $r'_E$.

A fit to the experimental data of Ref. (2), leads to:

$$
g_8 = (1.84 \pm 0.06) \times 10^{-3} \text{ GeV}^{-1} 
\quad g_1 = (0.98 \pm 0.05) \times 10^{-3} \text{ GeV}^{-1} 
\quad g_{M,88} = (3.84 \pm 1.57) \times 10^{-4} \text{ GeV}^{-1} 
\quad g_{E,88} = (5.46 \pm 0.56) \times 10^{-4} \text{ GeV}^{-1} 
$$

\(^\dagger\) Observe that the relative phase between $g_1$ and $g_8$ is zero because both arise from strong interactions
\[
\phi = (72 \pm 17)^0 \\
r_M = 0.48 \pm 0.28 \\
r'_M = 0.47 \pm 0.33 \\
r_E = 1.23 \pm 0.16 \\
r'_E = 1.36 \pm 0.24
\] (10)

With the above results we can predict the isospin-violating double OZI-suppressed decay \( BR(J/\psi \rightarrow \phi \pi^0) < 6.7 \times 10^{-6} \) which is consistent with the experimental upper limit \(< 6.8 \times 10^{-6} \) reported in Ref. (2).

The fit to the experimental data exhibits an interesting pattern. First, as for the \( J/\psi \rightarrow PP' \) case, the electromagnetic \( g_{E,88} \) and quark-mass \( g_{M,88} \) violations of SU(3) have similar sizes; second, nonet symmetry breaking seems to be violated by almost 50 \% and, finally, the relative phase between electromagnetic and strong contributions is close to \( \pi/2 \).

**BARYON-ANTIBARYON CHANNEL**

The Particle Data Group (2) reports measurements in five different decay modes \((p\bar{p}, n\bar{n}, \Lambda\bar{\Lambda}, \Sigma\bar{\Sigma}, \Xi\bar{\Xi})\) and an upper limit for \( \Sigma^0\bar{\Lambda} \). The decay amplitude for this decay channel can be written in terms of the ‘magnetic’ \( (G_M) \) and the ‘electric’ \( (G_E) \) form factors (9):

\[
\mathcal{M} = \varepsilon^{\mu}[G_{M}\gamma_{\mu} + \frac{2m(G_E - G_M)}{M^2 - 4m^2}(q_{B} - q_{\bar{B}})_{\mu}]v. \tag{11}
\]

In terms of these form factors, the decay rate is given by:

\[
\Gamma(J/\psi \rightarrow B\bar{B}) = \frac{G_{M}^2}{6\pi} \left\{ 1 + \frac{2m^2x^2}{M^2} \right\} p \tag{12}
\]

where \( m(M) \) denote the mass of the baryon \( (J/\psi) \) and \( p \) is the three-momentum of \( B \) in the \( J/\psi \) rest frame. The constant \( x \) is defined as \( x = G_E/G_M \).

Some theoretical arguments (10) and experimental results (11) suggest that \( 0 \leq x \leq 1 \).

The SU(3) analysis for the couplings of \( J/\psi \) to the octet of baryons gives the following (6):

\[
G_{M,E} \propto \frac{A}{2} \delta_{ab} + \epsilon^{i\phi} \left( d_{3ab} + \frac{d_{8ab}}{\sqrt{3}} \right) D + i\epsilon^{i\phi} \left( f_{3ab} + \frac{f_{8ab}}{\sqrt{3}} \right) F \\
+ \frac{d_{8ab}}{\sqrt{3}} D' + i\frac{f_{8ab}}{\sqrt{3}} F' \tag{13}
\]
where $D$ and $F$ ($D', F'$) are the symmetric and antisymmetric couplings of electromagnetic (strong) origin, and the coupling $A$ survives in the limit of exact SU(3).

Thus, we are left with seven free parameters: $A$, $D$, $F$, $D'$, $F'$, $x$ and the relative phase $\phi$. So, in order to perform a fit to the 5 experimental data, we will make a few 'reasonable' assumptions:

(i) We will set $D = 0$, because the isospin-violating decay $J/\psi \rightarrow \Sigma^0 \Lambda$ is proportional to $D$. The current upper limit on the $\Sigma^0 \Lambda$ decay mode gives a negligible value for $D$;

(ii) Since we do not know the value of $x$ we will fit the experimental data by fixing $x = 0$ or $x = 1$;

(iii) Finally we perform the fits by keeping fixed the relative phase at $\phi = 0$ and $\phi = \pi/2$.

The results of our fits for $D = 0$ and the two different values of $x$ and $\phi$ are shown in Table 1:

| Parameter | $x = 0$ | $x = 0$ | $x = 1$ | $x = 1$ |
|-----------|---------|---------|---------|---------|
|           | $\phi = \pi/2$ | $\phi = 0$ | $\phi = \pi/2$ | $\phi = 0$ |
| $A$       | 28.2± 2.1 | 29.3± 1.2 | 24.8± 1.9 | 25.9 ± 1.1 |
| $D'$      | −1.20 ± 2.5 | −0.08± 1.97 | −1.46± 2.29 | −0.38 ± 1.74 |
| $F'$      | 3.11 ± 1.88 | 1.85± 3.70 | 3.73± 1.63 | 2.46 ± 3.33 |
| $F$       | 6.09 ± 3.87 | 1.37± 2.13 | 5.64± 3.40 | 1.30 ± 1.95 |
| $\chi^2$  | $8.7 \times 10^{-3}$ | 0.20 | $1.0 \times 10^{-2}$ | 0.23 |

Although the results of the fit do not allow to draw a conclusion, we can observe the following behavior: (a) the SU(3)-violating couplings $D'$, $F'$ and $F$ are much smaller than the SU(3)-invariant coupling $A$, (b) the quality of the fit seems better in the case $\phi = \pi/2$ although they do not distinguish the cases $x = 0$, 1.

Related to point (b), we have also performed a fit (5) by keeping fixed the central values but assuming a better accuracy (3%) for the data. We obtain that only the model with no interference ($\phi = \pi/2$) survives.
CONCLUSIONS

We have performed an analysis of two-body hadronic decays of $J/\psi$ by using flavor SU(3). Our conclusions can be summarized as follows: (a) the symmetry breaking contributions of electromagnetic and quark-mass origin turn out to have similar strengths in $PP'$, $VP$ and $B\bar{B}$ channels; this is consistent with the expectations that the corresponding amplitudes are of order $\alpha_{em}$ and $\alpha_s^3(M_{J/\psi})$, respectively. (b) The interference effects between the electromagnetic and strong decay amplitudes are absent in the decay rates analysed; this is interesting because it would help to isolate the relevant electromagnetic form factors at the $M_{J/\psi}$ energy scale. Finally, a sizable breaking of nonet symmetry shows up in the $VP$ decay modes.

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