Euler’s computational approach for damped oscillatory solutions of RLC circuits

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Abstract. In this paper novel techniques for nonlinear frequency domain region problems with respect to linearity are examined. A notable methodology called as Modified Euler’s method and improved Euler’s method to perform nonlinear recurrence domain investigations of RLC circuit problems. Here a combination of waves approximates the driving forces to attain its damped oscillatory solutions. The two different cases were analyzed as, stability of the RLC circuit when the applied voltage is equal to zero and the forced oscillations of the RLC circuit when the applied voltage is equal to \( V_0 \sin \omega t \). To keen the strength of investigation, stability analysis and non-oscillatory behavior of the RLC circuit are explained along with periodical time.

1. Introduction

Nonlinear spatial recurrence analysis for the simulation of circuits is currently performed by various approaches like Harmonic Balance approach in the recent years. This approach deals with the nonlinear components in the field of recurrence, and the direct components by swapping with time and frequency domain via proper shift in Fourier Transform. Contrasting with conventional transient simulation with Fourier Transform functions output capacities, it has some interesting points and no valid reason to assume that underlying, a detailed assessment of inter-modulation transients to subside, specific low twisting rates of distortion levels, frequency dependent of subordinate components, etc [4,5,6].

For every Fourier components are focused on stability and efficiency along with high accuracy. Initially, the nonlinear solver converges by and by for huge dynamical problems, which is refined by unwinding system [11,12]. Furthermore, the changes ought to be exceptionally stable. Here the application zone allows extending Discrete Fourier Transform (DFT) because the calculation suits the numerical structure [9]. This further extends the usefulness of a few numerical techniques [7]. A lot of attention was received in the attainment of iterative strategies for understanding and solving nonlinear systems for circuit simulation frameworks which has been restricted when managing transient or consistent state issues [8]. Nevertheless, as late as possible, these strategies have been shown to be more effective when applied and converted to the nonlinear systems into the linear systems through Harmonic Balance [1].

Jacobi-Davidson analysis results may also be used in conduct model investigation and construction of circuit channel plan modeling [2]. For preselected information and yield factors, the poles and zeroes characterize the work on the circuit output [5]. In this case, the guarantee of poles and zeroes is done by taking care of a summed up the interest on Kirchhoff’s laws. The criteria to be interpreted or regarded essentially based on the Krichhoff’s current law [3]. RLC circuit with lower voltage can be
modeled by a system of differential algebraic equations with complementary constraints depends on time and voltages [10]. This inductor at various time shows that Modified Euler’s and Improved Euler’s methods are efficient to overcome all kinds of decays and errors [6].

This paper is organized as follows. Section II provides the linear and nonlinear hierarchical algorithm like Newton Raphson’s method and Gauss elimination structure method. Modified and improved Euler methods to arrive an optimal RLC Model are indicated in section III. Finally section IV concludes the paper.

2. Nonlinear and linear solver - Hierarchical algorithm

Nonlinear solver algorithm is processed through Newton-Raphson’s method and the linear solver is designed according with Gaussian elimination structure and tangent to the curve according to its initial values.

2.1 Nonlinear Solver – Newton Raphson’s Method:

Regularly, the structure of an electronic circuit is described by a progressive association of models with conclusive levels of active and passive components. As a rule, uncommon components of these devices (for example semiconductors) that are treated as building obstructs in the secluded structure of the circuit and these circuits have been described to characteristic the numerical calculation. Models and devices are connected in the chain of importance by their terminal unknowns to the encasing the circuit model. The single terminal circuit-level has fixed voltage from the ground hub. The conduct of the arrangement of a model is totally characterized by the limit esteems at its terminals (along with the internal sources). These sources are progressively sorted out and the numerical calculation permits for a depiction of the circuit structure. Because of the linearity of the Fourier transform, the Harmonic Balance algorithm, based on Newton-Raphson, can be planned progressively. A structure that fits is

2.2 Linear Solver - Gaussian elimination method:

Basically, the non-significant matrices with unknown terminals dealt with Gauss elimination method. Gaussian elimination method can manage a progressive structure in an exceptionally regular manner when utilizing a block or cell dividing on each level between internal unknowns and external unknowns. Each subsequent block or cell that deals with the lines and segments of the terminal unknowns is embedded in the circuit. Likewise various phases of the nonlinear solver are possible at the point when the limiting estimations of a sub-model or device are viewed as a converged. The arrangement can likewise be joined with damping or unwinding with Newton-Raphson by applying such procedures on each level in the progressive system before going further on in the progressive system. Likewise various phases of the nonlinear solver are possible at the point when the limiting estimations of a sub-model or device is viewed as a converged. The arrangement can likewise be joined with damping or unwinding with Newton-Raphson by applying such procedures on each level in the progressive system before going further on in the progressive system.

The matrices might be characterized utilizing block or cell structures of the fact that this may obliterate the diagonal value and replace it with a small negligible value or zero and choosing any one value as a pivot value (other than diagonal value) which brings about an estimation of pivot value. This estimation is improved iteratively by applying imperfect rectification on the current and voltage level before setting off to the recurrence and higher levels. This is particularly acknowledged in a Harmonic Balance investigation where the networks provide the greater impact in other relevant examinations, (for example, transient investigation). The DC-sources are autonomous and time-independent and, the AC source includes sine-waves, amplitude modulation, frequency modulation and phase modulation.
3. Modified and improved Euler method for optimal RLC Model

The modified Euler Method is the simplest case of a Taylor method, where the initial increment function is used, and the higher order terms are neglected because it’s obviously tends to zero. The method is expressed related to RLC circuit as follows:

\[ E^{i+1} = E^i + h k(E^i, t^i) \]

where \( k(E^i, t^i) = \frac{R}{L} E^i + \frac{q_0}{L} \). Here \( E \) stands as modified Euler’s coefficient across the inductor, capacitor and resistance, \( k \) be the source terms are the RLC Circuit. The same can be extended through improved Euler method as

\[ E^{i+1} = E^i + h k(E^{i+1}, t^{i+1}) \]

which is a recursive nonlinear function.

This technique is similar as describe in Euler’s method, and specifically it’s always a method of first order. An equation of this type ought to be solved iteratively and additional computational effort evaluating with previous methods. An implicit approach is obligatory with a purpose to remedy stiff issues. A preferred method for fixing the improved Euler’s method has also been evolved, the use of Newton iteration for regular differential equation related to RLC Circuit.

The numerical model (Modified Euler’s method) for analyzing the RLC Circuit is

\[
\begin{bmatrix}
1 \\
\frac{1}{RL} & -\frac{1}{LC} & \frac{1}{CR} \\
\frac{1}{LC} & \frac{1}{CR} & -\frac{1}{RL} \\
\frac{1}{CR} & \frac{1}{LR} & \frac{1}{LC}
\end{bmatrix}
\]

By applying Kirchhoff’s second rule to \([E(i)]\), the following two special cases will appear and we can solve these two cases through modified Euler’s method and improved Euler’s method.

1. Stability of the RLC circuit when the applied voltage is equal to zero.
2. Forced oscillations of the RLC circuit when the applied voltage is equal to \( V_0 \sin \omega t \).

**Modified Euler’s approach:**

Now let \( x \) be the additional voltage charge for the appearance of the stable damping factor related to the resistor \( R \). In a similar way \( y \) is the additional voltage charge for the appearance of the stable damping factor related to the inductor \( L \) and \( z \) be the additional voltage charge for the appearance of the stable damping factor related to the capacitor \( C \). These additional voltages can be determined as follows:

\[
\begin{align*}
x(t + \Delta t) &= x(t) + x'(t)\Delta t \\
y(t + \Delta t) &= y(t) + y'(t)\Delta t \\
z(t + \Delta t) &= z(t) + z'(t)\Delta t \tag{1}
\end{align*}
\]

Arbitrarily assume the forced oscillations of the applied voltage for the appearance of the damping related to the resistor is \( p \). In a similar way \( q \) is the forced oscillations of the applied voltage for the appearance of the damping related to the inductor \( L \) and \( r \) be the forced oscillations of the applied voltage for the appearance of the damping related to the capacitor \( C \).

These additional voltages can be determined as follows:

\[
\begin{align*}
p(t + \Delta t) &= x(t) + p'(t)\sin \omega_0 t \\
q(t + \Delta t) &= y(t) + q'(t)\sin \omega_1 t \\
r(t + \Delta t) &= z(t) + r'(t)\sin \omega_2 t \tag{2}
\end{align*}
\]

where \( \omega_0, \omega_1 \) and \( \omega_2 \) are the frequencies of the applied voltage and defined as,

\[
\omega_0 = \frac{1}{\sqrt{q r}} \left( 1 - \frac{p^2 r}{4 q} \right)
\]
\[
\omega_1 = \frac{1}{\sqrt{pr}} \left(1 - \frac{q^2 p}{4r}\right) \\
\omega_2 = \frac{1}{\sqrt{pq}} \left(1 - \frac{r^2 p}{4p}\right)
\]  

(3)

**Improved Euler’s approach:**

In the improved Euler’s approach equations (1) to (3) are considered the half of the portion, i.e.,

\[
\frac{1}{2}(x'(t) + x'(t + \Delta t)) = x(t + \Delta t) \\
\frac{1}{2}(y'(t) + y'(t + \Delta t)) = y(t + \Delta t) \\
\frac{1}{2}(z'(t) + z'(t + \Delta t)) = z(t + \Delta t)
\]

(4)

Arbitrarily assume the forced oscillations of the applied voltage for the appearance of the damping related to the resistor is \(p\). In a similar way \(q\) is the forced oscillations of the applied voltage for the appearance of the damping related to the inductor \(L\) and \(r\) be the forced oscillations of the applied voltage for the appearance of the damping related to the capacitor \(C\).

These additional voltages can be determined as follows:

\[
p(t + \Delta t) = x(t) + p'(t)\sin\omega_0\Delta t \\
q(t + \Delta t) = y(t) + q'(t)\sin\omega_1\Delta t \\
r(t + \Delta t) = z(t) + r'(t)\sin\omega_2\Delta t
\]

(5)

The impedance of equations (5) is given by,

\[
Z_p = \left[p^2 + (\omega_0 q - \frac{1}{\omega_0 p})^2\right]^{\frac{1}{2}} \\
Z_q = \left[q^2 + (\omega_1 r - \frac{1}{\omega_1 p})^2\right]^{\frac{1}{2}} \\
Z_r = \left[r^2 + (\omega_2 p - \frac{1}{\omega_2 q})^2\right]^{\frac{1}{2}}
\]

(6)

An equation (6) determines that how the current varies/ fluctuates with respect to time after the switch is off. So that modified Euler’s method and improved Euler’s method provides constructive solution based on applied voltage. Figure 1 shows the local and truncation Eulerian error.

**Figure 1.** Local error calculation using Euler’s point
Meanwhile the local error was identified according to Euler’s exact value and this truncated error tends to zero over the period of time. Damping factor shows that the non-oscillatory curve drawn as tangent to the solution and according to the initial values the damping values fall nearer to the local error. Figure 2 shows the global and local error based on Euler’s method.

![Global Error based on Euler's method](image1)

![Local Error based on Euler's method](image2)

**Figure 2.** Global and Local error based on Euler’s method

4. Conclusion
The applied voltage value becomes equilibrium, and then the oscillation free case occurs in RLC series circuits. If the critical impedance value becomes imaginary then $\sin\omega_0 t$ is replaced with exponential decay. This leads the over damped case and the solution becomes a exponential decay and no longer oscillates, for this reason the RLC circuit is said to be critically damped.

**References**
[1] Adeniran Adebayo O and Dare-Adeniran Olamiposi 2017 Journal of Science & Agriculture.
[2] Ali Oksasoglu and Dimitry Vavriv 1994 IEEE Transaction on Circuit and Systems-I: Fundamental Theory and Application **41**(10) pp 669-72.
[3] Charles K, Alexander and Matthew N O 2008 McGraw-Hill.
[4] Dautbegovic E, Condon M and Brennan C 2005 IEEE Trans. Microwave Theory & Techniques pp 548–55.
[5] Muthukumaran V and Ezhilmaran D 2020 International Journal of Information Technology and Web Engineering **15**(4) pp 18-36.
[6] Ganesh Gopal Deverajan, Muthukumaran V, Ching-Hsien Hsu, Marimuthu Karuppiiah, Yeh-Ching Chung and Ying-Huei Chen 2021 Transactions on Emerging Telecommunications Technologies.
[7] Meyer W S, Dommel H W 1974 IEEE Winter Power Meeting.
[8] Mohazzab J, Hussain A, Muhammad Q and Mohsin S 2008 International Journal of Computer Science and Network Security **8**(4) pp 80-90.
[9] Moosa M Al-Mazeed, Elham A, AL Juwaiser and Asma’a M Al-Mamoun 2003 IEEE ICECS pp 1292-95.
[10] Kwanamu J and Bakari J A 2016 Research, Science Domain international **7**(5) pp 1 – 9.
[11] Yang Zhi-An and Cui Yi-Hui 2005 Journal of Tangshan College **18**(4) pp 90-5.
[12] Yao Zhong-yu 2001 Journal of Guangxi University **26**(2) pp 145-9.