Adomian Decomposition Method for the Radiative Micropolar Nanofluid Past a Porous Channel: An Analytical Approach

Priya Mathur 1, Satyaranjan Mishra 2

1 Department of Mathematics, Poornima Institute of Engineering & Technology Jaipur, Rajasthan 302022, India; drpriyamathur21@gmail.com (P.M.);
2 Department of Mathematics, SikshaO’ Anusandhan Deemed to be University, Bhubaneswar, Odisha, 751030, India; satyaranjan_mshr@yahoo.co.in(S.R.M.);
* Correspondence: satyaranjan_mshr@yahoo.co.in (S.R.M.);
Scopus Author ID 57191609762

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Abstract: Present analysis leads to investigate the flow of conducting micropolar nanoliquid past a porous channel. The transverse magnetic field is imposed in the normal direction of the flow. The energy equation is enhanced by adopting the influence of thermal radiation. Similarity variable augmented with stream function is used to renovate PDES to ODEs. The present article’s special desirability is the approximate analytical technique for these transformed governing equations. We have employed Adomian Decomposition Method (ADM). Descriptions of various profiles for the influence of different parameters are presented through graphs. However, the computation for the rate coefficients is also obtained and deliberated through the tabular form for the influence of diverse physical parameters. A significant contribution reveals that near the lower wall, heat transfer rate enhances greater thermal radiation while impact opposes at the upper wall. A comparison between the methodology newly adopted and the earlier solutions coincided with each other and was found to be in good agreement in particular cases. However, the major findings are laid down here as higher magnetic number favors enhancing angular velocity within the channel and fluid temperature increases due to an increase in thermophoresis parameter.

Keywords: MHD flow; micropolar nanofluid; Adomian decomposition method; radiation and Brownian motion.

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1. Introduction

In the field of various branches of engineering like civil, mechanical, and electrical the flow of electrically conducting liquid via a porous channel has several interesting applications penetration from porous equipment is also significant in biological and neuroscience. Hartmann was done tremendous work concerning the effect of the homogenous magnetic field over the flow of electrically conducting fluids. Hartmann also did the experimental investigation with his colleagues. They investigated the flow of mercury for the appearance of magnetization and presented results graphically. Ashraf and his colleagues [1] studied the fluid flow of various magnetohydrodynamic flows. The steady Poiseuille flow of two immiscible incompressible micropolar fluids between two horizontal parallel plates of a channel with constant wall temperatures is studied by researchers [2,3] in terms of entropy.
generation. The authors discuss various properties of micropolar fluids with different numerical methods.

In recent years geometry of fluids attracted many researchers [4-8], they all discussed MHD effect in flow geometry. Micropolar fluid is a liquid with a microstructure. They are part of a liquid with a non-symmetric stress tensor that we would call a small liquid, and it includes, as a specialty, a well-designed Navier-Stokes model of a classic liquid that we would call a normal liquid. Physically, micropolar fluid can represent a liquid consisting of solid, randomly charged (or circular) particles suspended in a viscous area, where the transformation of liquid particles is ignored. Micropolar fluids are also very useful in many engineering applications. The theory of micropolar fluids given by Eringen [9–11] was given new dimensions to the researchers. He has been thoroughly discussed. Arimanet al. [12] and Shraf et al. [13] proposed the characteristic of polar fluid within the porous channel. They have analyzed numerically by considering the parametric behavior within a certain range. Takkar et al. [14] investigated the heat transport properties of the polar fluid also disused the behavior of fluid graphically. More work is done by many researchers later on [15–21]. A fluid containing a particle of the size nanometer, known as nanofluids, these nanoliquids have novel phenomena that are potentially useful for heat transfer applications. Cho et al. [22] were the first who declare the term nanoparticle useful. He suspended nanoparticles in the fluid and named them as base fluid. To evaluate the thermal conductivity of fluid Makinde et al. [23] construct a model with the addition of nanoparticles. The authors studied Buoyancy effects on MHD stagnation point flow and heat transfer of a nanofluid. Nanoparticle and their extensive thermal properties create space in research; later on, Ibrahim with Makinde [24] came up with another development. They have studied the flow phenomena of double stratification considering nanofluids over a vertical plate. In 2013, Khan et al. [25] investigated the cross-diffusion on the flow of heat and mass transfer with the help of The Buongiorno model. Further, computation is obtained by using effective nanofluids incorporated in the governing equations. These equations were solved by RK Fehberg method. Many researchers have done some interesting studies [26-35]. Different researchers have developed several methods to organize heat transfer and temperature augment. The literature review delivered that radiant heat transfer is instrumental in several industrial practices and useful in modern times to obtain transport energy via renewable sources. Some related studies on the topic can be seen in the open literature [36–38].

Ejtemaee and Khamehchi [39] currently did an Experimental investigation of rheological properties due to large drilling fluids applications. Boldyrev et al. [40] developed a model to Formation of microspherical particles of albumin with model drug using spray drying process. To our best knowledge to date, the flow of micropolar nanofluid to a pore channel under the influence of heat radiation has never been considered. This study is being presented to fill the gap and gain some insight into the work described. The calculations that control the flow of micropolar nanofluid are transformed into a set of different standard calculations that are later solved using a limited analysis method such as the Adomian Decomposition Method (ADM). The effect of relevant parameters on different profiles is highlighted with the help of graphs.
2. Materials and Methods

2.1. Analysis of the problem and formulation.

The two-dimensional flow of micropolar nanoliquid past a set of parallel plates situated at a distance 2h apart. It is assumed that both the walls are placed along the coordinate axes where it is defined as \( y = \pm h \) and walls of the channel are considered porous. The suction/injection velocity at the walls is consistent with magnitude \( V_0 \). Further, the consistent transverse magnetic field of strength \( B_0 \) is imposed across the flow, and because of the low magnetic Reynolds number, we have neglected the behavior of the induced magnetic field. The lower plate fluid temperature and concentration are \( T_1 \) and \( C_1 \) whereas at the upper plate; it is assumed to be \( T_2 \) and \( C_2 \) respectively (Fig.1). Assuming the above conditions, the proposed equations for the flow phenomena are:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(1)

\[
\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + (\mu + k) \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + k \frac{\partial N}{\partial y} - \sigma B^2(t) u
\]

(2)

\[
\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial y} + (\mu + k) \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + k \frac{\partial N}{\partial x}
\]

(3)

\[
\left( \frac{u \partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{k}{j} \left( 2 \alpha N \frac{\partial u}{\partial x} - v \frac{\partial v}{\partial y} \right) + \left( \frac{\mu}{j} \right) \left( \frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2} \right)
\]

(4)

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \tau \left\{ D_b \left( \frac{\partial C}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} \right) + D_t \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right] \right\}
\]

(5)

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_b \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + D_t \left[ \left( \frac{\partial^2 C}{\partial x^2} \right) + \left( \frac{\partial^2 C}{\partial y^2} \right) \right]
\]

(6)

The appropriate conditions used at the plates are
\[ v = u = 0, N = -S \frac{du}{dy}, \quad \text{at} \quad y = -h \]
\[ v = 0, u = \frac{V_0 x}{h}, N = \frac{V_0 x}{h^2} \quad \text{at} \quad y = h \]  
(7)

Where \( u \) and \( v \) stand for the \( x \)-axis and \( y \)-axis velocity component correspondingly. The microrotation of the fluid particle is described as \( N, T \), the temperature, \( C \), the fluid concentration, \( \rho \), the density, \( \mu, k \), the viscosity and the material parameter, \( p \), the pressure, the microinertia density, \( j \). \( D_B \), the Brownian diffusion, \( S \), the rotation of microelements near the channel walls. The transformation of the PDEs into ODEs are obtained by the use of the following transformations rule with the help of [14], and these are;

\[
\psi = -V_0 x f(\eta), \eta = \frac{v}{h}, N = \frac{V_0 x}{h^2} g(\eta), \theta(\eta) = \frac{T - T_2}{T_1 - T_2},
\]
\[
\phi(\eta) = \frac{C - C_2}{C_1 - C_2}, T_2 = T_1 - Ax, C_2 = C_1 - Bx
\]  
(8)

Where \( A \) and \( B \) are constants. However, the stream function is expressed as

\[
u = -\frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x}
\]  
(9)

It is obvious that, the transformations are validating the continuity equation and further, implementation of Eqs. (8) and (9) into the Eqs. (1)–(7), leads to

\[
(1 + N_1) f'' - R(f f'' - f' f'') - N_2 g'' - M^2 f'' = 0
\]  
(10)

\[
N_2 g'' + N_2 R(f' g - g' f) - N_1(2g - f') = 0
\]  
(11)

\[
\left\{1 + \frac{4}{3} R_d\right\} \theta'' + Pr R(f' \theta + f \theta') + Pr N b \phi' \theta' + Pr N t (\theta')^2 = 0
\]  
(12)

\[
\phi'' + Sc Re(f' \phi - f \phi') + \frac{N_1}{N_b} \theta'' = 0
\]  
(13)

Under the boundary condition

\[
\begin{align*}
 f(-1) &= 0, & f'(-1) &= 0, & f(1) &= 0, & f'(1) &= -1, \\
 g(-1) &= 0, & g(1) &= 1, & \theta(-1) &= 1, & \theta(1) &= 0 \\
 \phi(-1) &= 0, & N_1 \phi'(1) + N_1 \theta'(1) &= 0
\end{align*}
\]  
(14)

where \( N_1, N_3 \), the coupling parameter, \( N_2 \), the spin gradient viscosity, \( R \), the Reynolds number, \( M \), the Hartman number, \( Pr \), the Prandtl number, \( R_d \), the radiation, \( Nb \), Brownian motion, \( Nt \), the thermophoresis, \( Sc \), the Schmidt number.

\[
N_1 = \frac{k}{\mu}, N_2 = \frac{v_s}{\mu h^2}, N_3 = \frac{j}{h^2}, R_d = \frac{4T_2^3}{kk'}, Pr = \frac{v}{\alpha},
\]
\[
N_t = \left(\frac{D_T}{T_0}\right)\frac{T_2 - T_0}{\alpha}, N_b = \frac{\tau D_b C_2}{\alpha}, \quad Sc = \frac{v}{D_b}
\]  
(15)

However, the coefficient for the rate constants are

\[
Nt_s = \frac{q_s}{k_s(T_1 - T_2)} = -\theta'(-1)
\]  
(16)
\[ Sh_x = \frac{m_y}{h_k (C_1 - C_2)} = 0 \] (17)

It is the urge to describe the quantities \( q_y \) and \( m_y \) are the local parameters for heat flux and mass flux, respectively.

2.1. Solution technique ADM.

The exact analytical solutions for the set of nonlinear ODEs (10)-(13) corresponds to the boundary conditions (14) are difficult to get. Therefore, several other techniques are accessible for the solution of this set of nonlinear systems. A set of semi-analytical methods has also become fashionable in current years. An alternative approach introduced by “Adomian Decomposition Method” (ADM) is deployed. However, the specialty of the methodology is to provide an approximated analytical solution that is applicable without linearizing, perturbation, or discretization methods. The leading dimensionless ODEs, i.e., Eqns. (10)-(13) are written as:

\[ f^{iv}(\eta) = \frac{1}{1 + N_1} \left( R(f f'' - f f') + N_1 g'' + (M^2 + Kp) f^* \right) \] (18)

\[ g^{iv}(\eta) = \frac{1}{N_2} \left( -N_1 R(f g' - g f') + N_1 (2g - f^*) \right) \] (19)

\[ \theta^*(\eta) = \frac{3}{(3 + 4Rd)} \left( -Pr R(f' \theta + f \theta') - Pr Nb \phi' \theta' - Pr Nt(\theta'^*) \right) \] (20)

\[ \phi^*(\eta) = -Le Pr \left( Re(f' \phi - f \phi') + \frac{N}{N_b} \theta^* \right) \] (21)

To introduce the procedure of “Adomian Decomposition Method” (ADM), Let us assume \( L_1 = \frac{d^4}{d\eta^4} (\cdot) \) and \( L_2 = \frac{d^2}{d\eta^2} (\cdot) \) with inverse operators \( L_1^{-1}(\cdot) = \int \int \int \int d\eta d\eta d\eta d\eta \) and \( L_2^{-1}(\cdot) = \int \int d\eta d\eta \) (where \( \cdot \) is used for any function). Thus, Eqns. (15) - (17) become:

\[ f(\eta) = \frac{1}{1 + N_1} L_1^{-1} \left( R(f f'' - f f') + N_1 g'' + (M^2 + Kp) f^* \right) \] (22)

\[ g(\eta) = \frac{1}{N_2} L_2^{-1} \left( -N_1 R(f g' - g f') + N_1 (2g - f^*) \right) \] (23)

\[ \theta(\eta) = \frac{3}{(3 + 4Rd)} L_2^{-1} \left( -Pr R(f' \theta + f \theta') - Pr Nb \phi' \theta' - Pr Nt(\theta'^*) \right) \] (24)

\[ \phi(\eta) = -Le Pr L_2^{-1} \left( Re(f' \phi - f \phi') + \frac{N}{N_b} \theta^* \right) \] (25)

The functions \( f(\eta), g(\eta), \theta(\eta) \) and \( \phi(\eta) \) represented in the form of infinite series’ an expressed as:

\[ f(\eta) = \sum_{m=0}^{\infty} f_m, \quad g(\eta) = \sum_{m=0}^{\infty} g_m, \quad \theta(\eta) = \sum_{m=0}^{\infty} \theta_m, \text{ and } \phi(\eta) = \sum_{m=0}^{\infty} \phi_m \] (26)

The polynomial expression for the linear as well as nonlinear terms belonging to (22)-(25) can be presented as:
\[ \sum_{m=0}^{\infty} A_m = f f''', \sum_{m=0}^{\infty} B_m = f f'', \sum_{m=0}^{\infty} C_m = g, \sum_{m=0}^{\infty} D_m = f' \]

\[ \sum_{m=0}^{\infty} E_m = f' g, \sum_{m=0}^{\infty} F_m = f' g', \sum_{m=0}^{\infty} G_m = g, \sum_{m=0}^{\infty} H_m = f' \theta. \]

\[ \sum_{m=0}^{\infty} I_m = f' \theta', \sum_{m=0}^{\infty} J_m = \theta' \phi', \sum_{m=0}^{\infty} K_m = \theta'^2, \sum_{m=0}^{\infty} L_m = f' \phi. \]

\[ \sum_{m=0}^{\infty} M_m = f \phi', \sum_{m=0}^{\infty} N_m = \theta^* \]

(27)

From (14), invoking the boundary conditions:

\[ f(-1) = 0, \quad f'(-1) = 0, \quad f''(-1) = p_1, \quad f'''(-1) = q, \quad g(-1) = 0, \]

\[ g'(-1) = r, \quad \theta(-1) = 1, \quad \theta'(-1) = s, \quad \phi(-1) = 0, \quad \phi(-1) = t \]

(28)

The solutions of Eqns. (15) - (17) may therefore be written as

\[ f(\eta) = \frac{1}{2!} (\eta + 1)^2 p + \frac{1}{3!} (\eta + 1)^3 q + \frac{1}{1 + N_1} L_1^1 \left( R (ff'' - f f'') + N_1 g + (M^2 + Kp) f' \right) \]

(29)

\[ g(\eta) = (\eta + 1) r + \frac{1}{N_2} L_2^1 \left( -N_j R (f' g' - g f') + N_1 (2 g - f'') \right) \]

(30)

\[ \theta(\eta) = 1 + (\eta + 1) s + \frac{3}{(3 + 4 Rd)} L_2^1 \left( -Pr R (f' \theta - f \theta') - Pr Nb \phi' \theta' - Pr Nt \theta'^2 \right) \]

(31)

\[ \phi(\eta) = (\eta + 1) t - Le Pr L_2^1 \left( \text{Re} (f \phi - f \phi') + \frac{N_t}{N_b} \theta^* \right) \]

(32)

Here, the results of \( p, q, r, s, t \), the unknown functions are to be determined and the proposed guess i.e. the initial solutions along with the successive order solutions are:

\[ f_0(\eta) = \frac{1}{2!} (\eta + 1)^2 p + \frac{1}{3!} (\eta + 1)^3 q \]

(33)

\[ g_0(\eta) = (\eta + 1) r \]

(34)

\[ \theta_0(\eta) = 1 + (\eta + 1) s \]

(35)

\[ \phi_0(\eta) = (\eta + 1) t \]

(36)

and

\[ f_{m+1}(\eta) = \frac{1}{1 + N_1} L_1^1 \left( R (A_m - B_m) + N_1 C_m + (M^2 + Kp) D_m \right) \]

(37)

\[ g_{m+1}(\eta) = \frac{1}{N_2} L_2^1 \left( -N_j R (E_m - F_m) + N_1 (2 G_m - D_m) \right) \]

(38)

\[ \theta_{m+1}(\eta) = \frac{3}{(3 + 4 Rd)} L_2^1 \left( -Pr R (H_m - I_m) - Pr Nb J_m - Pr Nt K_m \right) \]

(39)

\[ \phi_{m+1}(\eta) = -Le Pr L_2^1 \left( \text{Re} (L_m - M_m) + \frac{N_t}{N_b} N_m \right) \]

(40)

For the values of \( m=0,1,2 \), Eqns. (33) - (35) in association with (22) the solutions of Eqns. (15) - (17) expressed in (23) are as follows:

\[ f(\eta) = f_0(\eta) + f_1(\eta) + f_2(\eta) + f_3(\eta) \]

\[ = \frac{1}{2!} (\eta + 1)^2 p + \frac{1}{3!} (\eta + 1)^3 q + T_5 \eta^4 + (T_1 + T_2)(\eta + 1)^5 + (T_2 + T_3)(\eta + 1)^6 \]

\[ + (T_3 + T_4)(\eta + 1)^7 + T_5 (\eta + 1)^8 + T_3 (\eta + 1)^9 + T_3 (\eta + 1)^10 + T_3 (\eta + 1)^11 \]

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https://biointerfaceresearch.com/
g(η) = g_0(η) + g_1(η) + g_2(η) + g_3(η)
= (η + 1)^{1} + T_2 \eta^2 + T_{16} \eta^3 + T_{57} \eta^4 + T_6 (η + 1)^4 + T_8 \eta^5
+ (T_7 + T_{26})(η + 1)^5 + T_{20} \eta^6 + T_{21} (η + 1)^6 + T_{22} (η + 1)^7 + T_{23} (η + 1)^8 + T_{24} (η + 1)^9

θ(η) = θ_0(η) + θ_1(η) + θ_2(η) + θ_3(η)
= 1 + (η + 1)^s + T_{25} \eta^2 + T_{13} \eta^3 + T_{26} (η + 1)^3 + T_{27} \eta^4 + (T_{27} + T_{38})(η + 1)^4 + T_{39} \eta^5
+ (T_{28} + T_{36})(η + 1)^5 + T_{34} \eta^6 + T_{37} (η + 1)^6 + T_{38} (η + 1)^7 + T_{39} (η + 1)^8 + T_{40} (η + 1)^9

ϕ(η) = ϕ_0(η) + ϕ_1(η) + ϕ_2(η) + ϕ_3(η)
= (η + 1)^t + T_{43} \eta^2 + T_{45} (η + 1)^3 + (T_{39} + T_{43})(η + 1)^4 + T_{44} (η + 1)^5
+ T_{45} (η + 1)^6 + T_{46} (η + 1)^7 + T_{47} (η + 1)^8

However, for the sake of brevity, we have not included the constants T_i', s, i = 1(1)47

3. Results and Discussion

The present investigation has considered the conducting two-dimensional flow micropolar nanofluid flow past a porous channel for the reoccurrence of thermal radiation and chemical reaction. The transverse magnetic field of the uniform field strength has been applied normally to the flow direction. An approximate analytical method such as “Adomian Decomposition method” (ADM) is deployed to handle the set of coupled nonlinear ODEs. The influence of several parameters involved in the flow phenomena is presented via graphs and tables. Moreover, the physical significance of these parameters is also described as per their contributions.

Fig. 2 presents the comparison plot of velocity profiles for various values of magnetic parameters. It is noteworthy that for both the absence/presence of magnetic parameter, the current outcome corroborate with the work of [21]. However, it is clear to see that for M=0, the velocity attains its maximum value near the lower plate and for higher values of M velocity profiles retards significantly up to the region \(-1 < η < 0.2\), and afterwards reverse effect is encountered. The reason is because the interaction of magnetic parameter produces Lorenz force, a resistive force, which has a tendency to retards the velocity profiles significantly. Moreover, in the second region due to angular momentum effect becomes opposite. Similarly, Fig. 3 deployed the comparison graph for the variation of magnetic parameter on the angular velocity profiles. Backflow occurs for various values of magnetic parameters, and more especially, in the absence of magnetic parameter, minimum velocity occurs at the middle of the channels, and then it favors in to enhance to meet the inadequate boundary conditions. Moreover the higher magnetic parameter also favors in to increase the angular velocity profiles. Influence of coupling constraint in both the presence/absence of porous matrix is presented in Fig.4. Two different variations are marked for the interaction of coupling parameters. It is interesting to observe that, point of inflection between the profiles is marked near the centerof the channel (\(η = 0.2\)), and from that particular point different variation in the profile is observed. In the first region, i.e., \(-1.0 \leq η < 0.2\), the least value of coupling parameter gives minimum velocity and profile is tilled near the middle of the domain, and further for higher values of coupling parameter the profile gives up the fluid velocity in either of the presence/absence of permeability. Moreover, in the other region, i.e., \(0.2 < η \leq 1.0\) the effect is the opposite. It is also seen that the interaction of porous medium resists the fluid motion in the first region and in the second region profile favors in to enhance the fluid velocity. Fig.5
exhibits the behavior of coupling constraint for the appearance/non-appearance of the porous matrix on the angular momentum. Oscillatory behavior is marked within the channel. Again the point of inflection is marked near \( (\eta = -0.2) \) and from that behavior is oscillatory. An increasing coupling parameter increases the angular velocity within the region \(-1.0 \leq \eta < -0.2\) for the appearance/non-appearance of the permeability of the medium; however, the impact is opposite in the region \(-0.2 < \eta \leq 1.0\). It is interesting to see that backflow occurs in the second region due to the rotational motion. Reynolds number has a significant role on the velocity, microrotation, and temperature profile in both the absence/presence of porous matrix is shown in the Figs. 6, 7, and 8, respectively. From Fig. 6, it is observed that an increase in Reynolds number diminishes the velocity profiles within the region \(-1.0 \leq \eta < 0.2\), and the opposite effect is marked in the rest of the region for both the absence/presence of porous matrix. In the clear flow, i.e., the absence of porous matrix, the maximum velocity is rendered in the first region, and the backflow occurs with minimum velocity is shown in the second region. Fig. 7 represents the behavior of Reynolds number on the angular velocity profiles, and it is observed that slight enhancement occurs near the lower plate and then sudden decreases with backflow and further increases to meet the boundary conditions. The presence of a porosity matrix enriches the flow profiles as well. Fig. 8 displays the influence of Reynolds number in both the presence/absence of porous medium on the temperature profiles. The trend of the profile seems to be linear for the low Reynolds number since at low Reynolds numbers, flows tend to be dominated by laminar for both the medium. However, for higher values of Reynolds number, the fluid temperature rises up due to the presence of magnetic number at the same time the stored energy grows, and the profile enhances. The presence of porous matrix at each level also resists the fluid temperature than the clear fluid in the clear region. Solutal distribution for various values of Reynolds number in both the absence/presence of porous matrix is presented in Fig. 9. Rapid decay in the concentration profile is marked for the higher Reynolds number in both the presence/absence of porous medium on the temperature profiles. The trend of the profile seems to be linear for the low Reynolds number since at low Reynolds numbers, flows tend to be dominated by laminar for both the medium. However, for higher values of Reynolds number, the fluid temperature rises up due to the presence of magnetic number at the same time the stored energy grows, and the profile enhances. The presence of porous matrix at each level also resists the fluid temperature than the clear fluid in the clear region. Solutal distribution for various values of Reynolds number in both the absence/presence of porous matrix is presented in Fig. 9. Rapid decay in the concentration profile is marked for the higher Reynolds number in both the presence/absence of porous medium. It shows the mass transfers from the lower plate to the upper plate to a great extent due to the presence of both the opposing forces of the magnetic and porous matrix. The effect of the spin gradient viscosity parameter on the microrotation is shown in Fig. 10. For both porous and non-porous mediums the interactions within the profiles are different, but they are nearer to the center of the channel from which the profile behaves oscillatory. As viscosity increases, the angular velocity retards within the first part of the domain, and the effect is encountered in the second region. For \( k_p = 0 \), the angular velocity attains its maximum (in magnitude) level. The impact of radiative heat on the temperature and concentration profile is described in Fig. 11 and 12, respectively. Almost linear behavior is marked for thermal radiation for which the role is insignificant earlier and later part we have produced the magnifying picture file. An increase in thermal radiation the profile retards irrespective of the appearance/non-appearance of the porous matrix. However, the concentration profiles enhance with an increase in radiation. The results of thermophoresis (\( N_t \)) on the temperature as well as concentration is depicted in Figs. 13-14. Thermophoretic expression becomes visible because of the term containing \( N_t \left( \frac{\partial \theta}{\partial \eta} \right)^2 \) from eq.(12) and \( \frac{N_t}{N_b} \frac{\partial^2 \theta}{\partial \eta^2} \) from eq.(13). Here, diffusing species is correlated to thermophoresis. From Fig. 13 it is clear to see that with an increase in thermophoresis parameter, the fluid temperature increases significantly throughout the channel. In the clear fluid, the temperature of the polar fluid enhances significantly than that of the porous medium. Moreover, from Fig. 14 few unstable
situations are found on the solutal profiles. For the low value of thermophoresis, the profile seems to be linear, whereas, for higher, it initially goes backward and then suddenly blown up from the lower plate to the upper plate. As a similar case, the backflow rate near the lower plate is maximum in the case of clear fluid. Fig. 15 depicts the characterization of Brownian motion on fluid concentration with the particular values of parameters. The mathematical expression clarifies that the composite derivative \( Nb \frac{\partial \theta}{\partial \eta} \frac{\partial \phi}{\partial \eta} \) in eq.(12) and diffusion term \( \frac{Nt}{Nb} \frac{\partial^2 \theta}{\partial \eta^2} \) in eq.(13) offers the survival of Brownian motion in the flow phenomena.

Figure 2. Comparison plot of velocity profile.

Figure 3. Comparison plot for variation of magnetic parameter on microrotation.

Figure 4. Plot of Interaction of coupling parameter.
The significant behavior of the Brownian motion shows its important characteristics on the profile that is exhibited in Fig. 15. For greater values of $Nb$ a strong decrease in the fluid concentration is marked at the channel periphery; however, the random orientation of Brownian motion with suspended nanoparticles within the traditional liquid grows up to the upper plate. Fig. 16 depicts the influence of Lewis number on the solutal concentration profiles for various physical parameters.

**Figure 5.** The plot of Interaction of coupling parameter.

**Figure 6.** Influence of Reynold Number on the velocity profile.

**Figure 7.** Influence of Reynold Number on the temperature profile.
The observation clarifies that enhanced Lewis number retards the fluid concentration, a usual phenomenon. Lewis number is the ratio of the thermal diffusion with the mass diffusion coefficients. As mass diffusion decreases, the Lewis number increases; as a result, the fluid concentration decreases. Last but not least, Table 1 presents the values of the unknowns obtained at the time of computation for the various values of pertinent parameters in both the presence/absence of porous matrix.

![Figure 8. Influence of Reynold Number on the velocity profile.](image)

![Figure 9. Influence of Reynold Number on concentration profile.](image)

![Figure 10. Influence of spin gradient viscosity on microrotation profile.](image)
Figure 11. Influence of Thermal Radiation on the temperature profile.

Figure 12. Influence of Thermal Radiation on the concentration profile.

Figure 13. Influence of Thermophoresis on the temperature profile.

Figure 14. Influence of Thermophoresis on the concentration profile.
Table 1. Values of unknowns p, q, r, s, t using ADM.

| M   | P   | q     | r     | s    | t     | N2, Kp=0 | p   | q    | r     | s     | t     |
|-----|-----|-------|-------|------|-------|----------|-----|------|-------|-------|-------|
| 1   | -1.3794 | 0.1325 | -0.4837 | 0.4386 | 0.1 | 0.8757 | -0.6484 | 1.4174 | -0.4842 | 0.4402 |
| 2   | -1.4332 | 0.1273 | -0.4843 | 0.4405 | 0.2 | 0.9151 | -0.9969 | 0.8206 | -0.4842 | 0.4401 |
| 3   | -1.6846 | 0.1146 | -0.4858 | 0.4455 | 0.3 | 0.935  | -1.1554 | 0.5597 | -0.4842 | 0.4401 |
|     | 0.8045 | 0.0995 | -0.4876 | 0.4517 | 0.4 | 0.9466 | -1.2471 | 0.4124 | -0.4842 | 0.4401 |
|     | 0.9458 | -0.9824 | 0.5181 | -0.4836 | 0.4383 | 0.3 | 0.821  | -1.4429 | 0.4692 | -0.4865 | 0.4481 |
|     | 0.9258 | -0.7689 | 0.6945 | -0.4835 | 0.4378 | 0.4 | 0.8312 | -1.5204 | 0.3466 | -0.4865 | 0.4481 |

Figure 15. The plot of Brownian motion.

Figure 16. Influence of Lewis number.

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4. Conclusions

The steady two-dimensional flow of conducting micropolar fluid through a porous channel is investigated in the present analysis. The influence of thermal radiation is also considered for the thermal enhancement of the polar fluid. Approximate analytical technique is used to solve transformed governing equations and the effects of various parameters characterizing the flow phenomena via graphs. However, the following concluding remarks are laid down here. The present result validates with the results of earlier work in particular cases.

Angular velocity enhances the higher magnetic number within the channel. An increase in Reynolds number marks an incredible rise in velocity profile; however, concentration profiles retards significantly. Thermal radiation also resists the fluid temperature, but the impact is reversed in the case of concentration distributions. Thermophoresis has a leading role in enhancing the fluid temperature, but the scenario for solutal profile is quite unstable.

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Conflicts of Interest

The authors declare no conflict of interest.

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