Graceful exit via monopoles in a theory with O’Raifeartaigh type supersymmetry breaking

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Abstract

We study the stability of supersymmetry breaking metastable vacua and supersymmetric vacua in the presence of solitons. The metastable vacua of supersymmetric QCD and those found elsewhere such as in models based on the SU(5) grand unified group support the existence of topological solitons. The vacua containing such topological defects can become unstable against decay into lower energy configurations. We show for a specific model that a finite region of the available parameter space of couplings becomes disallowed due to the presence of monopoles. In a manner similar to previous studies based on cosmic strings, it is shown that soliton solutions arising in supersymmetric theories can put constraints on the range of allowed values of the couplings arising in the theories. Implications for cosmology are discussed.

Keywords: topological soliton, supersymmetry breaking, metastable vacua, cosmology

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1. Introduction

Dynamical supersymmetry breaking in metastable vacua is an effective means of breaking supersymmetry which can be accommodated in various classes of models such as $\mathcal{N} = 1$ supersymmetric SU($N_c$) QCD with $N_f$ massive fundamental flavors [1, 2]. Models based on this method can have several supersymmetry breaking false vacua which have a finite lifetime based on the quantum tunneling rate to the true vacuum [3]. Metastable vacua which break supersymmetry also occur in many other models of supersymmetry breaking and mediation [4, 5, 6, 7]. Issues relating to the phenomenological implementation of these ideas have been discussed in [8, 9, 10, 11, 12, 13, 14, 15, 16, 17] and cosmological applications appear in [18, 19, 20]. A necessary condition for...
the viability of such models is that the lifetime of the metastable vacua is much larger than the age of the universe.

In a recent paper [21], we had emphasized that it is not sufficient to study the stability of supersymmetry breaking vacua in terms of their translation invariant form alone. In general, topological defects can form in the evolution of the early universe [22]. For example, the models of supersymmetry breaking described in [1, 2] break SUSY in metastable vacua which can contain cosmic strings [23]. Such defects have important consequences with regards to stability issues. The core of a cosmic string or monopole can give rise to a “seeding” of the true vacuum and render it unstable against decay into a lower energy configuration [24, 25, 26]. This is true even if the translation invariant vacuum is sufficiently stable against quantum mechanical tunneling to the true vacuum. Such a process has been studied in the context of phase transitions in Grand Unified Theories in [27]. Instabilities of domain walls in theories with compact extra dimensions are discussed in [28].

In [21] we dealt with the messenger sector of a model in which supersymmetry breaking occurs according to the gauge mediated supersymmetry breaking scenario. The seeding resulted from the presence of cosmic string solutions. As an extension of this work, we study the seeding mechanism in the presence of monopoles, and this is done for the case of direct supersymmetry breaking rather than having supersymmetry breaking being communicated from a hidden sector. In particular, we have chosen to study a non-abelian model in which both the gauge and supersymmetry breaking occur simultaneously in a O’Raifeartaigh type model through vacuum expectation values of appropriate Higgs scalars. After $SU(5)$ breaking, there are no supersymmetric vacua in this model. There are however multiple supersymmetry breaking vacua, some of which are metastable. It turns out that some of these metastable and spatially homogeneous vacuum states become disallowed in the presence of monopoles for a large range of couplings occurring in the model. In addition to the instability which can arise in supersymmetry breaking vacua, we show in the present work that this effect can also take place for two supersymmetric vacua. It is possible to study the stability numerically and also semi-analytically. Both of these approaches will be dealt with in this paper.

The classical instabilities of solitons discussed here can have a significant impact on the evolution of the early universe. In the model we deal with, there is a metastable supersymmetry breaking vacuum which is undesirable from the point of view of phenomenology. It is possible for some causally connected regions of the universe to get trapped in this false vacuum during the early stages of its evolution. If no monopoles are present, the transition to the true vacuum can only take place via quantum tunneling. This can result in a potentially inhomogeneous universe with the first-order phase transition never being completed [29, 30]. As we will show, the presence of monopoles in such metastable vacua can result in classical instabilities of such configurations and hence a graceful exit from a potentially inhomogeneous expansion.

The next section 2 discusses the classical instabilities of solitons in general. The following section 3 then briefly describes the model and its vacua, while
section 4 deals with the monopole ansatz and equations of motion. Sections 5 and 6 discuss the stability of the vacua containing monopoles from a numerical and semi-analytic approach. Implications for cosmology are presented in section 7 followed by concluding remarks in section 8.

2. Classical Instabilities of Monopoles and Vortices

We begin with brief overview of how vortex and monopole solutions can become unstable classically under certain conditions. The dissociation of $SU(5)$ monopoles which will be discussed in this paper is similar to the monopole dissociation studied in [25]. A general study of this phenomenon which applies for both vortices and monopoles is discussed in [26].

Figure 1: An example of the potential to be studied in this paper. The value of $\epsilon$ is given by the difference in energy densities of the false and true vacuum. The term $\mu$ is a mass scale which will appear in the model discussed here.

Let us consider the scalar potential for a field $\sigma_1$ as shown in figure 1. This potential is an example of the type of potential we will encounter in this study. It has a global minimum of nearly vanishing energy for which the VEV of $\sigma_1$ is close to zero. There is also a false vacuum in which the value of $\sigma_1$ is larger than that of the true vacuum. The term $\epsilon$ denotes the difference in energy densities of the false and true vacuum and it is positive in this case.

A topological soliton solution such as a cosmic string or magnetic monopole has a vanishing field strength within its core due to continuity requirements. Consequently, the field strength must increase from zero and approach the vacuum value asymptotically. Now if a monopole is present in the false vacuum of figure 1, the core of this monopole contains a region in which the field strength
corresponds to the true vacuum. Following the discussion of [26], we assume that this region is a spherical bubble of radius \( R_b \) which has a thin boundary. This is referred to as the “thin-wall” approximation. Under this approximation, the monopole consists of three regions. The inner-most region is a spherical region which corresponds to the true vacuum. This is followed by a “thin-wall” in which the field strength grows rapidly to value corresponding to the metastable vacuum. Outside this wall is a region of false vacuum in which the energy density is higher than that of the true vacuum.

A question which naturally arises is whether such a bubble of true vacuum can expand and occupy the region of false vacuum outside the core of the monopole. If the bubble expands from a radius \( R \) to \( R + dR \), the change in energy to first order in \( dR \) is \(- 4 \pi \epsilon R^2 dR\). The negative sign signifies that energy can be lost through radial expansion. Thus, the energy of the bubble is proportional to \(- \epsilon R^3\). However, an increase in radius of the bubble leads to an increase in surface area of the surrounding wall which has a positive energy density \( \sigma \). Hence, there is a term in the total bubble energy proportional to \( \sigma R^2 \). There is also a gauge field present in a topological soliton and its energy varies as \( C/R \) for some constant \( C \). Thus, the total energy of the bubble is given by

\[
E(R) = -\epsilon \frac{4\pi}{3} R^3 + 4\pi \sigma R^2 + \frac{C}{R}.
\] (1)

This energy as a function of bubble radius \( R \) is plotted in figure [2]. The profile \( E(R) \) has a local minimum for small values of \( \epsilon \). The value of \( R \) for this minimum is denoted \( R_b \) and this corresponds to a stable monopole solution. As \( \epsilon \) increases, the curvature of the local minimum becomes less positive and at a critical value \( \epsilon_0 \), it becomes zero. If \( \epsilon \) is increased beyond \( \epsilon_0 \), there is no local minimum and \( R_b \) tends to infinity. This indicates an unstable monopole configuration whose core contains a region of true vacuum which is expanding into the surrounding false vacuum.

In the supersymmetry breaking model we will consider, it turns out that value of \( \epsilon \) is governed by a parameter \( \tilde{M} \). Increasing \( \tilde{M} \) increases both the VEV of \( \sigma_1 \) and also the energy of the false vacuum (see figure [1]). Thus,

\[
\epsilon \sim \tilde{M}^4. \tag{2}
\]

As will be shown in a later section, increasing \( \tilde{M} \) beyond a certain value creates an instability in the monopole configuration which settles into the metastable vacuum.

We will be studying instability from two approaches. One is by simply looking for time-independent monopole solutions. If a monopole becomes classically unstable in the way described above, it is described by a time-dependent solution describing an expanding bubble of true vacuum. In this case, it is impossible to obtain a time-independent solution. Hence, whenever a time-independent solution cannot be obtained, that monopole configuration is understood to be unstable for those values of parameters. This will be discussed in greater detail in section [5.1]. We also follow a semi-analytic approach to the study of stability and this will be discussed in section [5.2].
Figure 2: The energy of a bubble of radius $R$. For small enough $\epsilon$, there is a minimum of $E(R)$ corresponding to a stable monopole solution with finite bubble radius. If $\epsilon$ becomes too large, there is no minimum and $R_0 \to \infty$. This corresponds to an expansion of the core resulting in an unstable monopole configuration.

3. The model and its vacua

We study a model described in [31] which enables metastable supersymmetry breaking without the use of singlet fields. This requires the use of two $SU(5)$ adjoints $\Sigma_1$ and $\Sigma_2$ whose vacuum expectation values (VEVs) are given by

$$\langle \Sigma_i \rangle = \frac{\langle \sigma_i \rangle}{\sqrt{30}} \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix}$$

(3)

where $\sigma_i$ ($i = 1, 2$) are the standard model singlets. The superpotential is

$$W = Tr \left[ \Sigma_2 \left( \mu \Sigma_1 + \lambda \Sigma_2^2 + \frac{\alpha_1}{M} \Sigma_1^3 + \frac{\alpha_2}{M} Tr(\Sigma_1^2) \Sigma_1 \right) \right]$$

$$= \frac{1}{30M} \sigma_2 \left( 30M \mu - \sqrt{30} M \lambda \sigma_1 + (7\alpha_1 + 30\alpha_2) \sigma_1^2 \right) \sigma_1$$

(4)

from which the scalar potential can be written as

$$V = \left( \mu \sigma_1 - \frac{\lambda \sigma_1^2}{\sqrt{30}} + \frac{7\alpha_1 \sigma_1^3}{30M} + \frac{\alpha_2 \sigma_1^4}{M} \right)^2$$

$$+ \frac{\sigma_1^2}{900M^2} \left( 30M \mu - 2\sqrt{30} M \lambda \sigma_1 + 3(7\alpha_1 + 30\alpha_2) \sigma_1^2 \right)^2.$$  

(5)
There are no D-Term contributions to the scalar potential since the VEVs of $\Sigma_i$ are diagonal. Defining $v_1 = \langle \sigma_1 \rangle$ and $v_2 = \langle \sigma_2 \rangle$, $SU(5)$ is broken at $v_3$ while supersymmetry is broken at $v_1$. The VEV $v_1$ can be obtained from minimizing (5) but $v_2$ is undetermined at tree order. This flat direction is lifted by nontrivial 1-loop corrections to the scalar potential which have the form $V \approx m_2^2 (\sigma_2 - v_2)^2$. Adding a term like this on the right hand side of (5) leaves the value of $v_1$ unchanged while $\langle \sigma_2 \rangle$ gets fixed at $v_2$. The units of $v_2$ are the same as those of $\sigma_2$ and can be taken to be $100\mu$ to simplify the forthcoming discussion. We thus stabilize $v_2$ in the ensuing numerical simulations by adding $(\sigma_2 - v_2)^2$ to the scalar potential (5) and hence fixing $\sigma_2$ to $v_2$ (in units of $100\mu$). An example of the scalar potential with $v_2$ and the couplings fixed is shown in figure 1.

3.1. Supersymmetry breaking vacua

The model we are considering has no supersymmetric minima unless $v_2 = 0$. When $SU(5)$ breaks with $v_2 \neq 0$, there are two supersymmetry breaking vacua which we shall denote $|V_1\rangle$ and $|V_2\rangle$. The value of $v_1$ for these minima is given by

$$v_1 = \frac{1}{3(7\alpha_1 + 30\alpha_2)} \left[ \sqrt{30} M \lambda \pm \sqrt{30} \sqrt{M^2 \lambda^2 - 21 M \mu \alpha_1 - 90 M \mu \alpha_2} \right].$$

The couplings in this model are chosen in conformity with the requirements discussed in [31]. The variables $\mu$ and $M$ in the above equation have dimensions of mass whereas $\lambda$, $\alpha_1$, and $\alpha_2$ are dimensionless. We define the dimensionless variable $\tilde{M}$ through the relation $M = \tilde{M} \mu$, and choose $\alpha_1 = \alpha_2 = 0.1$ and $\lambda = 0.5$. The resulting expression for $v_1$ becomes

$$v_1 = \frac{1}{11.1} \left[ \frac{\sqrt{30} \tilde{M}}{2} \pm \sqrt{30} \sqrt{\frac{\tilde{M}^2}{4} - 11.1 \tilde{M}} \right] \mu.$$

The value of the dimensionless variable $\tilde{M}$ is varied later in the paper to study its effect on stability of vacua containing monopoles. The $\pm$ signs in the above equation correspond to two distinct vacua, one near the origin due to almost exact cancellation and one far from the origin. The vacuum which is far from the origin is referred to as $|V_1\rangle$. Its properties for $\tilde{M} \approx 1000$ are described below:

$$|V_1\rangle : v_1 \approx 10^2 \mu \quad \langle V_1 | V | V_1\rangle \approx 10^8 \mu^4.$$

In contrast, the state $|V_2\rangle$ has the following properties:

$$|V_2\rangle : v_1 \approx \mu \quad \langle V_2 | V | V_2\rangle \approx \mu^4.$$

Both these minima have non-zero energy but the energy of $|V_2\rangle$ is much smaller than that of $|V_1\rangle$. The state $|V_2\rangle$ is thus the true vacuum for this model whereas $|V_1\rangle$ is metastable. When monopole configurations can exist in the above vacua, they shall be denoted $|V_1^{\text{monopole}}\rangle$ and $|V_2^{\text{monopole}}\rangle$ respectively.
For a given value of $v_2$, the configuration $|V_1^{\text{monopole}}\rangle$ which asymptotes to $|V_1\rangle$ at infinity must pass through the field value $v_1 \approx \mu$ corresponding to $|V_2\rangle$ near the core of the monopole. This “seeding” of the true vacuum which occurs for $|V_1^{\text{monopole}}\rangle$ inside its core can render it unstable as will be shown in section 5.1. On the other hand, such a seeding does not take place for $|V_2^{\text{monopole}}\rangle$ and it is thus expected to remain stable. This expectation is confirmed by our numerical results which will be discussed in section 5.1.

The phenomenological motivations behind the choice of the superpotential given in equation 4 is described in [31]. The two terms involving $\alpha_1$ and $\alpha_2$ are non-renormalizable terms which vanish in the limit $M \to \infty$. The desired SUSY breaking vacuum of this model turns out to be $|V_2\rangle$ and this state survives the renormalizability limit. The other vacuum $|V_1\rangle$ gets pushed further away from the origin in field space as $M$ increases and eventually becomes tachyonic for large enough $M$.

3.2. Supersymmetric vacua

This model contains two supersymmetric vacua when $v_2 = 0$. When this is the case, the value of $v_1$ for these minima is given by:

$$v_1 = \frac{1}{2(7\alpha_1 + 30\alpha_2)} \left[ \sqrt{30M\lambda} \pm \sqrt{30\sqrt{M^2\lambda^2 - 28M\mu\alpha_1 - 120M\mu\alpha_2}} \right]. \quad (10)$$

Once again, the $\pm$ signs yield one vacuum close to the origin and one far from it. The vacuum further from the origin is denoted as $|V_1^{(\text{SUSY})}\rangle$ for which the value of $v_1$ is written as $v_1^+$. The plus sign in the superscript signifies that the plus sign in equation 10 has been used. Likewise, the vacuum near the origin is referred to as $|V_2^{(\text{SUSY})}\rangle$ for which $v_1$ is given by $v_1^-$. Both these states have exactly zero energy, but since $v_1^- < v_1^+$, a monopole configuration which asymptotes into $v_1^+$ corresponding to $|V_1^{(\text{SUSY})}\rangle$ must pass through the field value $v_1^-$ corresponding to $|V_2^{(\text{SUSY})}\rangle$ near the origin. Thus, a “seeding” effect similar that of $|V_1^{\text{monopole}}\rangle$ takes place when a monopole is present in $|V_2^{(\text{SUSY})}\rangle$, rendering it unstable. This will be discussed in greater detail in section 6.

4. The monopole ansatz and equations of motion

We shall set up monopole configurations in the fields $\Sigma_1$ and $\Sigma_2$. The Lagrangian for the system can be expressed as

$$L = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2} (D_\mu \Sigma_1)^2 + \frac{1}{2} (D_\mu \Sigma_2)^2 - V(\Sigma_1, \Sigma_2) \quad (11)$$

where $a = 1, \ldots, 24$ and $F_{\mu\nu}^a$ are the gauge field strengths. The covariant derivative is expanded as

$$D_\mu \Sigma_i^a = \partial_\mu \Sigma_i^a - ie[F_\mu, \Sigma_i]^a \quad (12)$$
with \( i = 1, 2 \) and the adjoints are expressed in terms of the \( SU(5) \) generators \( T^a \) as \( \Sigma_i = \sigma_i^a T^a \). We choose a spherically symmetric ansatz for the adjoints and the gauge field:

\[
\sigma^a_1 = \frac{r^a}{er^2} G(r), \quad \sigma^a_2 = \frac{r^a}{er^2} H(r)
\]  

(13)

\[
A^a_n = \epsilon_{amn} \frac{r^m}{er^2} [1 - K(r)], \quad A^a_0 = 0.
\]  

(14)

To setup the \( SU(5) \) monopole, we take the following embedding of \( SU(2) \) in \( SU(5) \):

\[
\begin{pmatrix}
0 \\
0 \\
\tau_a \\
0
\end{pmatrix}
\]  

(15)

Here, \( \tau_a = \frac{1}{2} \sigma_a \) (\( a = 1, 2, 3 \)), and \( \sigma_a \) are the Pauli Sigma matrices. The \( 2 \times 2 \) matrices \( \tau_a \) satisfy \( [\tau_i, \tau_j] = i \epsilon_{ijk} \tau_k \). The \( SU(5) \) adjoints \( \Sigma_1 \) and \( \Sigma_2 \) given in equation (3) are aligned along the generator \( \Sigma = diag(2, 2, 2, -3, -3) \) in isospace. In order to obtain the non-trivial topology corresponding to a monopole configuration, we perform a unitary transformation on the fields \( \Sigma_1 \) and \( \Sigma_2 \) and also the gauge field \( A \) to obtain (see \([32]\))

\[
\Sigma_1 = \frac{1}{\sqrt{30}} \sum_{a=1}^{3} \begin{pmatrix}
2 \\
2 \\
-\frac{1}{2} I_2 + \frac{25}{er^2} G(5r) \tau_a r^a \\
-3
\end{pmatrix}
\]  

(16)

\[
\Sigma_2 = \frac{1}{\sqrt{30}} \sum_{a=1}^{3} \begin{pmatrix}
2 \\
2 \\
-\frac{1}{2} I_2 + \frac{25}{er^2} H(5r) \tau_a r^a \\
-3
\end{pmatrix}
\]  

(17)

\[
A_n = \sum_{i,j=1}^{3} \begin{pmatrix}
0 \\
0 \\
\frac{25}{er^2} [1 - K(5r)] \epsilon_{ijn} \tau_i r^j \\
0
\end{pmatrix}
\]  

(18)

where \( I_2 \) is the \( 2 \times 2 \) identity matrix. Using the above results in the lagrangian (\([33]\)), the Euler-Lagrange equations of motion become second-order differential equations in \( G(r) \), \( H(r) \) and \( K(r) \). Taking \( e = 1 \), they are given by

\[
r^2 K'' = K(K^2 - 1) + K(G^2 + H^2)
\]  

(19)

\[
r^2 G'' = 2G K^2 + r^4 \frac{\partial}{\partial G} (V(G, H))
\]  

(20)

\[
r^2 H'' = 2HK^2 + r^4 \frac{\partial}{\partial H} (V(G, H))
\]  

(21)
in which the function $V(G,H)$ is obtained by substituting $\sigma_1 = G/r$ and $\sigma_2 = H/r$ in the scalar potential (5). These equations of motion are ordinary differential equations involving only $r$ as a result of our spherically symmetric ansatz functions given in (13) and (14). The function $G(r)/r$ as $r \to \infty$ tends to a constant value. This means that the function $G(r)$ is proportional to $r$ and it therefore diverges at infinity. This is also true for the function $H(r)$. For the purpose of our numerical solutions for the monopole, we express equations (19) - (21) in terms of $\sigma_1$ and $\sigma_2$. The result is

$$ \frac{d^2 K}{dr^2} - \frac{1}{r^2} K(K^2 - 1) - K(\sigma_1^2 + \sigma_2^2) = 0 $$

(22)

$$ \frac{d^2 \sigma_1}{d\sigma_1^2} + \frac{2}{r} \frac{d\sigma_1}{dr} - \frac{2}{r^2} \sigma_1 K^2 - \left(\frac{\partial V}{\partial \sigma_1}\right) = 0 $$

(23)

$$ \frac{d^2 \sigma_2}{d\sigma_2^2} + \frac{2}{r} \frac{d\sigma_2}{dr} - \frac{2}{r^2} \sigma_2 K^2 - \left(\frac{\partial V}{\partial \sigma_2}\right) = 0. $$

(24)

Equations (22, 23) can be solved numerically using relaxation techniques after rescaling $\sigma_1$, $\sigma_2$ and the variable $r$ by the VEV $v_1$. A monopole configuration necessarily implies $\sigma_1 = \sigma_2 = 0$ at $r = 0$. From equation (22), the function $K(r) \to 1$ as $r \to 0$. As $r \to \infty$, the function $K(r) \to 0$ while the values of $\sigma_1$ and $\sigma_2$ are determined by solving the following set of simultaneous polynomial equations:

$$ \left(\frac{\partial V}{\partial \sigma_1}\right) = 0 $$

(25)

$$ \left(\frac{\partial V}{\partial \sigma_2}\right) = 0. $$

(26)

The initial guess for the solution is chosen to meet the boundary conditions discussed above. The value of $M$ is varied to study its effect on the stability of the vacua containing monopoles. For a given set of couplings, there are two solutions to equations (25) and (26) corresponding to the two translation invariant vacua described by (8) and (9).

5. Stability of the monopole configurations in non-supersymmetric vacua

5.1. Numerical study

The numerical methods we use are relaxation techniques implemented by discretizing the domain over which the solution is required. By dividing the interval into a sufficient number of points, we convert the differential equations into a set of simultaneous polynomial equations. The initial guess is chosen in conformity with the boundary conditions discussed at the end of section 4.

We denote by $|V_1^{\text{monopole}}\rangle$ the monopole solution which settles into $|V_1\rangle$ at infinity. Likewise, $|V_2^{\text{monopole}}\rangle$ denotes the monopole configuration which reaches $|V_2\rangle$ asymptotically. We have studied the availability of both $|V_1^{\text{monopole}}\rangle$ and
by first choosing a value of \( v_2 \) and then varying \( \tilde{M} \). An example of \( |V_1^{\text{monopole}}\rangle \) when \( \tilde{M} = 1400 \) and \( \sigma_2 = 500\mu \) is shown in figures 3 and 4. In this case, the value of \( v_1 \) is 685.3\( \mu \) and this is independent of \( \sigma_2 \). A similar solution exists for \( |V_2^{\text{monopole}}\rangle \) for which the value of \( v_1 \) is 5.5\( \mu \).

Figure 3: The functions \( f_1(r) = \sigma_1(r)/v_1 \) and \( f_2(r) = \sigma_2(r)/v_1 \) for \( |V_1^{\text{monopole}}\rangle \) with \( \tilde{M} = 1400 \) and \( v_2 = 500\mu \). The value of \( v_1 \) is 685.3\( \mu \).

Figure 4: The function \( K(r) \) for \( |V_1^{\text{monopole}}\rangle \) with \( v_2 = 500\mu \) and \( \tilde{M} = 1400 \).
As discussed in section 3, there is a “seeding” of the true vacuum within the core of $|V_{1}^{\text{monopole}}\rangle$. In the above example, the field value $\sigma_1 = v_1 = 5.5\mu$ corresponds to the true vacuum denoted by $|V_2\rangle$. The monopole configuration for $\sigma_1$ shown in figure 3 must rise from zero and pass through $\sigma_1 = 5.5\mu$ before reaching its asymptotic value of $685.3\mu$. Furthermore, the energy of the local minimum at $\sigma_1 = 685.3\mu$ increases as $\tilde{M}$ increases. Therefore, for sufficiently large values of $\tilde{M}$, we expect an instability in the configuration $|V_{1}^{\text{monopole}}\rangle$.

This expectation is confirmed by the results of our numerical simulations which are summarized in figure 5. There is a large region in the parameter space of $v_2$ and $\tilde{M}$ for which a numerical solution for $|V_{1}^{\text{monopole}}\rangle$ cannot be obtained.

![Figure 5: The allowed and disallowed regions for $|V_{1}^{\text{monopole}}\rangle$. The boundary between allowed and disallowed regions without a monopole present is defined by equation (27). When a monopole is present in $|V_{1}\rangle$, a large region of the parameter space becomes disallowed since $|V_{1}^{\text{monopole}}\rangle$ is no longer available for this region.](image)

The value of $\tilde{M}$ cannot be increased to arbitrarily high values. There is a limiting value of $\tilde{M}$ beyond which the metastable vacuum $|V_{1}\rangle$ becomes tachyonic. The condition which must be satisfied for no tachyonic states is

$$2 \left| \frac{v_2}{v_1} \right|^2 \left| \frac{F}{v_1^2} - \left( \frac{7}{30} \alpha_1 + \alpha_2 \right) \frac{v_1}{\tilde{M}} \right| \geq \left| \frac{F}{v_1^2} \right|$$

in which $F$ is defined as

$$F = v_1^2 \left[ \frac{\lambda}{\sqrt{30}} - \frac{2}{\tilde{M}} \left( \frac{7}{30} \alpha_1 + \alpha_2 \right) v_1 \right].$$

This condition puts an upper bound on possible values of $\tilde{M}$ as shown in figure 5. However, when a monopole is present in $|V_{1}\rangle$, this upper bound becomes significantly lowered.
The fact that the numerical solution for $|V_{1}^{\text{monopole}}\rangle$ cannot be obtained for a finite region of the parameter space is indicative of an inherent instability in such a configuration. This instability will be studied from a semi-analytic approach in the next subsection. The configuration $|V_{2}^{\text{monopole}}\rangle$ can be obtained for all of the parameter values which were considered. Such a configuration has no lower energy state to decay into.

5.2. Semi-Analytic approach

The numerical solutions obtained in the previous subsection are all time-independent. We can restore time-dependence in the equations of motion and study the stability of the solutions without actually solving the time-dependent equations of motion. Following the approach discussed in [24], we restore time-dependence in equation (23):

$$-\frac{d^2 \sigma_1}{dt^2} + \frac{d^2 \sigma_1}{dr^2} + \frac{2}{r} \frac{d \sigma_1}{dr} - \frac{2}{r^2} \sigma_1 K^2 - \left( \frac{\partial V}{\partial \sigma_1} \right) = 0.$$  (29)

When studying the possibility of $|V_{1}^{\text{monopole}}\rangle$ decaying into $|V_{2}^{\text{monopole}}\rangle$, it is sufficient to study the time-dependence of $\sigma_1$ alone. This is because the vacuum value of $\sigma_2$ is equal to $v_2$ for both these states. We thus treat $\sigma_2$ as a time-independent background field denoted by $\tilde{\sigma}_2(r)$ in the following discussion. The time-dependence of $\sigma_1(r, t)$ is decomposed as follows:

$$\sigma_1(r, t) = \tilde{\sigma}_1(r) + p(r) e^{i\omega t}.$$  (30)

The function $p(r) \ll \tilde{\sigma}_1(r)$ and $\tilde{\sigma}_1(r)$ is the time-independent solution to equation (23). Substituting equation (30) in (29) and linearizing in $p$, we obtain

$$\omega^2 p = - \left[ \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} \right] p + [U(r)] p.$$  (31)

This equation has the form of a one-dimensional Schrödinger equation with a potential

$$U(r)/10^4 \mu^2 = \beta \left( 147 \alpha_1^2 + 2700 \alpha_2^2 + 1260 \alpha_1 \alpha_2 \right) \left( 18 \tilde{\sigma}_2^2 \tilde{\sigma}_1^2 + 5 \tilde{\sigma}_1^4 \right)$$

$$- 20 \beta \tilde{M} \tilde{\sigma}_1 \left( 7 \alpha_1 + 30 \alpha_2 \right) \left( \sqrt{30} \lambda \tilde{\sigma}_1 - 0.18 \right)$$

$$+ 60 \beta \tilde{M}^2 \left( 2 \lambda^2 \tilde{\sigma}_2^2 + 3 \lambda^2 \tilde{\sigma}_1^2 - 0.03 \sqrt{30} \lambda \tilde{\sigma}_1 + 0.0015 \right)$$

$$+ 36 \beta \tilde{M} \tilde{\sigma}_2^2 \left( 7 \alpha_1 + 30 \alpha_2 \right) \left( \sqrt{30} \lambda \tilde{\sigma}_1 - 0.05 \right) + \frac{2 \tilde{K}^2}{r^2}$$  (32)

where $\beta = 1/450 \tilde{M}^2$ and $\tilde{\sigma}_1$, $\tilde{\sigma}_2$, and $\tilde{K}$ are time-independent dimensionless functions of $r$.

The stability of $|V_{1}^{\text{monopole}}\rangle$ depends on whether or not the frequencies of oscillation $\omega$ in equation (30) are imaginary. Real frequencies result in stable solutions whereas imaginary frequencies indicate instability. Looking for imaginary modes of oscillation or negative values of $\omega^2$ is equivalent to looking for
Figure 6: The equivalent potential for $|V_{\text{monopole}}^{1}\rangle$ with $\sigma_2 = 500\mu$. The effect of increasing $\tilde{M}$ can be seen to lower the energy of the local minimum near the origin. For large enough $\tilde{M}$, this minimum has a negative energy and the resulting bound state creates an instability in $|V_{\text{monopole}}^{1}\rangle$.

negative energy bound states of the potential given in equation (32). This potential is plotted for different values of $\tilde{M}$ with $\sigma_2 = 5$ for $|V_{\text{monopole}}^{1}\rangle$ in figure 6. Notice that there is a local minimum near the origin, and that the energy of this minimum reduces as $\tilde{M}$ increases. What this means is that $|V_{\text{monopole}}^{1}\rangle$ is only stable for small enough values of $\tilde{M}$. When this is so, there is no negative energy bound state possible. When $\tilde{M}$ is large enough, a negative energy bound state is possible and the configuration $|V_{\text{monopole}}^{1}\rangle$ is no longer stable.

6. Stability of the monopole configurations in supersymmetric vacua

For the case of the supersymmetric vacua, the monopole configuration settling into $|V_1(SUSY)\rangle$ is denoted $|V_{\text{monopole}}^{1(SUSY)}\rangle$, and likewise for $|V_{\text{monopole}}^{2(SUSY)}\rangle$. It turns out that the numerical solutions for $|V_{\text{monopole}}^{1(SUSY)}\rangle$ for values of $\tilde{M}$ larger than 200 can not be obtained. As discussed in the previous section, this failure to obtain a numerical solution is indicative of an instability of the configuration. Thus, even a supersymmetric vacuum containing a monopole can become unstable against decay into another supersymmetric vacuum.

With regards to the stability analyzed from the semi-analytic approach of the previous section, the equivalent Schrodinger potential for $|V_{\text{monopole}}^{1(SUSY)}\rangle$ can
be derived in the same manner as was done for $|V_{1}^{\text{monopole}}(\text{SUSY})\rangle$. It is given by

$$U(r)/10^{4}\mu^{2} = \gamma\tilde{\sigma}_{1}^{4}(147\alpha_{1}^{2} + 2700\alpha_{2}^{2} + 1260\alpha_{1}\alpha_{2})$$

$$+ 36\gamma\alpha_{2}\tilde{M}^{2}\left(\tilde{\sigma}_{1}^{2}\lambda^{2} - 0.01\sqrt{30}\tilde{\sigma}_{1}\lambda + 0.0005\right)$$

$$+ 4\tilde{M}\gamma\tilde{\sigma}_{1}^{2}(7\alpha_{1} + 30\alpha_{2})\left(0.18 - \sqrt{30}\lambda\tilde{\sigma}_{1}\right) + \frac{2}{r^{2}}K^{2} \quad (33)$$

where $\gamma = 1/90\tilde{M}^{2}$. This potential is plotted in figure 7 and once again, negative energy bound states indicate instability. In this case, values of $\tilde{M}$ lying near 200 represent the boundary between stable and unstable configurations.

7. Implications for cosmology

The early universe is characterized by a monotonic reduction in temperature. From calculations of thermal effective potentials [33, 34, 35], we expect a leading order temperature correction of the form $AT^{2}\sigma_{1}^{2}$ with $A > 0$ to the effective potential of $\sigma_{1}$. If we absorb this effect in the definition of $\mu_{eff}$, we have

$$\mu_{eff}^{2} = \mu^{2} + AT^{2}. \quad (34)$$

If we understand the rescaled quantity $\tilde{M}$ to be expressed in terms of $\mu_{eff}$, we see that a change in $\tilde{M}$ is equivalent to a change in $T$. Specifically, we have $\tilde{M}/\mu_{eff} = \tilde{M}$, which shows how an increase in temperature is equivalent to a reduction of $\tilde{M}$. 
In this way, the increase of $\tilde{M}$ which results in an instability in $|V_{1\text{monopole}}\rangle$ can happen naturally in the early universe with $M$ fixed and the temperature $T$ decreasing. Referring to figure 5, we see that for a fixed value of $v_2 = \langle \sigma_2 \rangle$, we may start with a high temperature period in the early universe which supports the existence of $|V_{1\text{monopole}}\rangle$. This high temperature is equivalent to low values of $\tilde{M}$. As the temperature drops, the value of $\tilde{M}$ effectively increases and at a critical temperature $T_1$, $|V_{1\text{monopole}}\rangle$ becomes unstable. Below this temperature, the solution $|V_{1}\rangle$ is no longer available and this region is referred to as “disallowed with monopole” in figure 5. As the temperature continues to drop, $\tilde{M}$ continues to rise until the state $|V_1\rangle$ becomes tachyonic at a temperature $T_2 < T_1$. For temperatures below $T_2$, the translation invariant vacuum $|V_1\rangle$ is unavailable and this is referred to as “disallowed without monopole” in figure 5.

There is hence an intermediate range of temperatures $T_2 < T < T_1$ within which $|V_1\rangle$ is available but $|V_{1\text{monopole}}\rangle$ is disallowed. When the universe passes through this temperature range, it is possible that causally connected local domains settle in the metastable state $|V_1\rangle$ instead of the true vacuum $|V_2\rangle$. Without the presence of monopoles in $|V_1\rangle$, the transition to $|V_2\rangle$ can only take place via quantum tunneling. A first-order phase transition of this type can result in an inhomogeneous universe 29, 30.

In contrast, if monopoles had formed at higher temperatures in $|V_1\rangle$, the resulting states $|V_{1\text{monopole}}\rangle$ would become unstable within this intermediate temperature range. This would result in a classical roll-over to $|V_2\rangle$ and prevent an otherwise inhomogeneous expansion. In this way, the presence of monopoles within $|V_1\rangle$ ensures a graceful exit from this false vacuum. The universe arising out of the type of SUSY breaking model discussed here is hence homogeneous like our present day universe.

8. Concluding Remarks

The results of this paper share a common theme with the results of 21. In both cases, a metastable vacuum containing a topological defect is shown to become unstable against a classical roll-over to the true vacuum. However, the exact details of this process depend on the specific model being considered. In 21, the model of gauge mediated supersymmetry breaking which we had studied permitted the existence of cosmic strings. These string configurations were responsible for the seeding of true vacuum bubbles within their cores. Furthermore, the metastable vacua which became unstable in the presence of the strings were the phenomenologically desired SUSY breaking minima. The true vacuum was an undesirable minimum in which $SU(3)_c$ was broken. In this way, the cosmic strings played a potentially disastrous role in the model.

The role of the topological soliton is quite the opposite and much more benign in the present case. Here, the $SU(5)$ grand unified group supports the existence of monopoles and as we have discussed, the metastable vacua are phenomenologically undesirable. There is hence the need for a graceful exit from the false vacuum and this is where the monopoles come in. If the universe
is trapped in a false vacuum and decays by quantum tunneling, the resulting universe can be inhomogeneous. The type of classical monopole instabilities discussed here ensure that the metastable vacuum can become unstable and undergo a “roll-over” to the global minimum. This effect hence provides a graceful exit from a potentially inhomogeneous expansion.

In general, any model based on a gauge group which supports the existence of topological solitons can undergo the process described here. Whether or not the metastable vacua containing solitons are desirable depends on the given model. Hence, the stability of a metastable vacuum against quantum mechanical tunneling is not always a sufficient condition to ensure the viability of a model containing solitons.

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