Joint Successive Cancellation Decoding of Polar Codes over Intersymbol Interference Channels

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Abstract—Polar codes are a class of capacity-achieving codes for the binary-input discrete memoryless channels (B-DMCs). However, when applied in channels with intersymbol interference (ISI), the codes may perform poorly with BCJR equalization and conventional decoding methods. To deal with the ISI problem, in this paper a new joint successive cancellation (SC) decoding algorithm is proposed for polar codes in ISI channels, which combines the equalization and conventional decoding. The initialization information of the decoding method is the likelihood functions of ISI codeword symbols rather than the codeword symbols. The decoding adopts recursion formulas like conventional SC decoding and is without iterations. This is in contrast to the conventional iterative algorithm which performs iterations between the equalizer and decoder. In addition, the proposed SC trellis decoding can be easily extended to list decoding which can further improve the performance. Simulation shows that the proposed scheme significantly outperforms the conventional decoding schemes in ISI channels.

I. INTRODUCTION

Polar codes were proposed by Arikan as a class of capacity-achieving codes for the binary-input discrete memoryless channels (B-DMCs) [1]. The codes perform well in many situations, such as source coding [2], physical-layer security [3], multiple access channel [4], flash memory [5], and cooperative relaying [6]. However, there is often intersymbol interference (ISI), which leads to memory channels, in wireless communications, storage and other communication systems. The memory property of the ISI channels increase the error propagation phenomenon occurring in the successive cancelation (SC) decoding of polar codes.

To deal with the ISI problem, interleaving and turbo equalization can be performed [7]. However, the iteration decoding in the turbo equalization structures may obstruct the selection of the optimal frozen set of polar codes, which limits the performance.

In this paper, we propose a joint decoding method, called SC trellis decoding, to decode the polar codes for ISI channels. In contrast to the conventional iterative algorithm which performs iterations between the equalizer and decoder, the decoder combines the equalization and conventional decoding and is performed using recursion formulas like conventional SC decoding without iterations. Only the conventional bit-reversal interleaving is exploited and the selection of the frozen set can be obtained by Monte Carlo approach [1]. The proposed scheme significantly outperforms the conventional decoding schemes in ISI channels. In addition, the proposed SC trellis decoding can be easily extended to list decoding, which can further improve the performance. The decision functions of the method have the same form as the conventional SC decoding. For information vector \( u_0^{N-1} = (u_0, u_2, \ldots, u_{N-1}) \), \( 0 \leq i \leq N - 1 \) and received vector \( r_0^{N-1} \), we compute the probability \( P(r_0^{N-1}, u_i^{N-1} | u_i = 0) \) and \( P(r_0^{N-1}, \hat{u}_i^{N-1} | u_i = 1) \). Compared with the conventional SC decoding, the SC trellis decoding introduce the trellis state variables in the recursive calculations.

The rest of this paper is organized as follow. Section II briefly presents some preliminaries of polar codes. The proposed SC trellis decoding algorithm is introduced in Section III. Simulation results are discussed in Section IV.

II. PRELIMINARIES

A. Polar Codes

The generator matrices of polar codes with lengths \( N = 2^n, n = 1, 2, \ldots, \) can be written as \( G_N = B_N G_2 \otimes^n \), where \( G_2 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \otimes^n \) is the Kronecker power and \( B_N \) is the bit-reversal interleaving [1]. Due to the polarization property of polar codes, the channel will be polarized into a set of good noiseless and poor noisy sub-channels, as the block length of codewords approaches infinite. The good noiseless sub-channels transmit the information bits while the poor noisy sub-channels transmit the frozen bits, which are known to the receiver. The set of the index of the frozen bits is denoted by \( F \), which can be chosen by Monte Carlo approach or density evolution [8] in B-DMCs. In this paper, the Monte Carlo approach is exploited for ISI channels.

Let \( u_{N-1}^0 = (u_0, u_1, \ldots, u_{N-1}) \) denote the information vector, where \( N \) is the code length, \( u_i \in \mathcal{U} = \{0, 1\} \). Let \( c_{N-1} = (c_0, c_1, \ldots, c_{N-1}) = u_0^{N-1} G_N \) denote the codeword vector. \( r_{N-1}^0 = (r_0, r_1, \ldots, r_{N-1}) \) denotes the received vector and \( \hat{u}_i \) represents the decoded bit of \( u_i \). The likelihood function of the \( i \)th information bit \( u_i \) is

\[
P \left( r_0^{N-1}, \hat{u}_i^{N-1} | u_i \right) = \sum_{u_{i+1}^{N-1} \in \mathcal{U}^{N-1}} \frac{1}{2^{N-1}} P \left( r_0^{N-1}, \hat{u}_i^{N-1} | u_i^{N-1} \right).
\]
decoded recursively. The encoder structure of intermediate codeword bit of \( j \)th codeword symbol \( u \) is represented by \( \pi \) of \( u \) under the constraint of \( (j_1,j_2) \)-th phase in the layer \( m \). Note that, \( \pi(\cdot) = \pi^{-1}(\cdot) \) denotes the bit-reversal interleaving.

Let \( \rho^{(m)}_{(j_1,j_2),m}(\cdot) \) denote the sum of the likelihood functions of codeword symbols \( x_{(j_1,2^{m-1})} \) under the constraint intermediate codeword bit \( v_{(j_1,j_2)}^{(m)} = b \) and fixed \( u_i \), \( i \)th bit, \( \tilde{u}_{i-1} \), i.e.,

\[
\rho^{(m)}_{(j_1,j_2),m}(\tilde{u}_{i-1}, u_i) = \sum_{v_{(j_1,j_2)}^{(m)}(\cdot)} \mathbb{P}(v_{(j_1,2^{m-1})} | x_{(j_1,2^{m-1})} = b, u_i, \tilde{u}_{i-1})
\]

where \( v_{(j_1,j_2)}^{(m)}(\cdot) \) represents the condition \( u_0^{N-1} \in \mathcal{U}^{N-i} \) with fixed \( u_i \), \( i \)th bit, \( \tilde{u}_{i-1}^{N-1} \) and under the constraint \( v_{(j_1,j_2)}^{(m)} = b \). Note that, given an information vector \( u_0^{N-1} \), all the intermediate codeword bit can be obtained uniquely, so are the codeword symbols \( x_i \). The summation \( \rho^{(m)}_{(j_1,j_2),m}(\cdot) \) is under the constraint \( v_{(j_1,j_2)}^{(m)} = b \), which means that given \( \tilde{u}_{i-1} \) and \( u_i \), all the information bits \( u_i^{N-1} \) in the summation satisfy that the intermediate codeword bit \( v_{(j_1,j_2)}^{(m)} = b \) according to the encoding rule.

Then the likelihood function of \( u_i = v_i^{n-1} \) can be rewritten by

\[
P(r_0^{N-1}, u_0^{N-1} | u_i = b) = \frac{1}{2^{N-1}} \sum_{(n-1)(i),b} v_{(n-1)(i),b}^{(n-1)}(\cdot)
\]

\[
= \frac{1}{2^{N-1}} \rho^{(m)}_{(n-1)(i),b}(u_0^{N-1}, u_i).
\]

Because of the good structures of polar codes, \( \rho^{(m)}_{(j_1,j_2),m}(\cdot) \) can be calculated recursively like the conventional likelihood ratio calculation in [1].

If \( j_2 < 2^{m-1} \), we have

\[
\rho^{(m)}_{(j_1,j_2),b}(\cdot) = \rho^{(m)}_{(j_1,j_2),b}(\cdot)
\]

\[
= \rho^{(m-1)}_{(2^{j_1}+j_2),m-1}(\cdot) \rho^{(m-1)}_{(0,0),b}(\cdot)
\]

\[
+ \rho^{(m-1)}_{(2^{j_1}+j_2),m-1}(\cdot) \rho^{(m-1)}_{(0,1),b}(\cdot)
\]

\[
= P(r_0|c_0 = 0) P(r_1|c_1 = 1 - b)
\]

\[
+ P(r_0|c_0 = 1) P(r_1|c_1 = 1 - b)
\]

\[
\rho^{(m-1)}_{(1,0),b}(\cdot)
\]

\[
= \rho^{(m-1)}_{(1,0),b}(\cdot)
\]

\[
= P(r_0|c_0 = 0) P(r_1|c_1 = 1 - b)
\]

\[
+ P(r_0|c_0 = 1) P(r_1|c_1 = 1 - b)
\]

\[
C. ISI Channels
\]

Let \( x_0^{N-1} = (x_0, x_1, \ldots, x_{N-1}) \in \mathcal{X}^N = \{1,-1\}^N \), \( x_i = (-1)^c \), and \( r_0^{N-1} = (r_0, r_1, \ldots, r_{N-1}) \) represent the codeword symbol and received vectors, respectively. The \( i \)th
received symbol $r_i$ can then be written as

$$r_i = y_i + n_i = \sum_{j=0}^{D} h_j x_{i-j} + n_i,$$

where $D$ is the memory length of ISI, $n_i$ is the Gauss noise, and $y_i \in \mathcal{Y}$ is an intermediate variable, called ISI codeword symbol in this paper. $h_i$ is the tap coefficient of the channel. When $D = 0$, the channel becomes the binary-input additive white Gaussian noise (BI-AWGN) channel.

The ISI channels can also be represented by trellis diagrams. Let $s_i = x_{i-D} \in \mathcal{S}$ denote the trellis state. Then in each stage there are $2^D$ states in the trellis. For example, in dicode channel $h_0 = 1, h_1 = -1$ and there are two states in each stage. The trellis diagram of the dicode channel is shown in Fig. 2.

**III. SC TRELLEIS DECODING**

When performing soft decoding, estimations of the code-word symbols $x_i$ are required for initialization. In B-DMCs, the likelihood function of each codeword symbol $P(r_i|x_i)$ is independent. On the other hand, in ISI channels, the estimations are usually obtained by equalization, and these estimations of the codeword symbols are not independent any more due to the channel memory.

To overcome the problem, in this section, we propose SC trellis decoding method for polar codes in ISI channels. We use likelihood functions of the ISI codeword symbol $y_i$ as the initialization information and combine the processes of equalization and decoding. This method is calculated recursively and without iteration.

**A. Initialization Information**

In this subsection, we will show how to use the independent likelihood functions of $y_i$ as initialization information.

If $D = 0$, the ISI channel becomes BI-AWGN channel. Let $h_0 = 1$, then $r_i = y_i + n_i = h_0 x_i + n_i = (-1)^c_i + n_i$. The likelihood functions of the ISI codeword symbols $y_i$ are equal to the likelihood functions of the codeword symbols $x_i$, $P(r_i|y_i) = P(r_i|x_i)$, due to the bijective mapping between set $\mathcal{X}$ and $\mathcal{Y}$, i.e. $y_i = x_i = (-1)^c_i$.

However, if $D \neq 0$, although the bijective mapping between $\mathcal{U}$ and $\mathcal{X}$ is also satisfied ($x_i = (-1)^c_i$), the bijective mapping between each codeword symbol $x_i \in \mathcal{X}$ and symbol $y_i \in \mathcal{Y}$ is not satisfied any more. For example, in the dicode channel, all the mapping between $x_i$ and $y_i$ are showed in Table I. Note that the same input symbols $x_i$ leads different $y_i$, such as \( x_{i+1} = (1, 1) \), $y_{i+1}$ can be $(0, 0)$ or $(2, 0)$. However, if the state $s_i = x_{i-D} \in \mathcal{S}$ is fixed, we can obtain the bijective mapping between each codeword symbol $x_i \in \mathcal{X}$ and symbol $y_i \in \mathcal{Y}$ again, i.e., there is bijective mapping between $\mathcal{X} \times \mathcal{S}$ and $\mathcal{Y}$.

We consider the likelihood functions with state $s_i \in \mathcal{S}$. Then we can obtain that $P(r_i|s_i, x_i) = P(r_i|y_i)$. Because of the bijective mapping, instead of using the likelihood functions of the codeword symbol $x_i$, we can use the likelihood functions of the codeword symbol $x_i$ and state $s_i$, i.e. the likelihood functions of the ISI codeword symbols $y_i$, as the initialization information. Fortunately, the ISI channels only have finite trellis states. For each $i$, we can calculate the likelihood functions of $y_i$ based on all the possible values of $x_i$ and $s_i$.

**B. Recursive Formulas**

Note that, the likelihood functions of ISI codeword symbols $P(r_i|y_i)$ are independent, but unlike decoding polar codes in B-DMCs, we cannot directly use the recursive formulas (5) and (6) due to correlation between $x_{0}^{N-1}$ and $r_{0}^{N-1}$.

In ISI channels, in order to corresponding to the likelihood functions of $y_i$, the initialization information of the decoding, we consider the probability $P(r_{0}^{N-1}, \hat{u}_{0}^{N-1}, s_0|u_0 = b)$ instead of $P(r_{0}^{N-1}, \hat{u}_{0}^{N-1}|u_0 = b)$. Here, $s_0$ denotes the initial state of the ISI channel. Usually, we can assume $s_0 = 0$ in the presence of zero padding. Intuitively, we can also decode the codes recursively like (5) and (6). Note that, we have

\[
P\left(r_{0}^{N-1}|y_{0}^{N-1}\right) = \prod_{i=0}^{N-1} P\left(r_i|y_i\right) = \prod_{i=0}^{N-1} P\left(r_i|s_i, x_i\right) = \prod_{i=0}^{N-1} P\left(r_i|s_i, x_i, s_{i+1}\right) = P\left(r_{0}^{N-1}|s_0, x_{0}^{N-1}, s_\mathcal{T}\right) \times P\left(y_{0}^{N-1}|s_\mathcal{T}, x_{\mathcal{T}}^{N-1}, s_\mathcal{T}\right).
\]

(10)

The difference between using the likelihood functions of $y_i$ and $x_i$ is that in addition to $x_i$, a state value $s_i$ is given. Note

![The trellis diagram of dicode channel](image_url)
that, if $s_i$ and $x_i$ are fixed, we can obtain the end state $s_{i+1}$ uniquely, so we rewrite $P(r_{i} | s_{i}, x_{i})$ by $P(r_{i} | s_{i}, x_{i}, s_{i+1})$. We can divide the vector $x_{k}^{i}$ into two parts $x_{k}^{i}$ and $x_{k+1}^{i}$. If the end state $s_{k+1}$ of the first part is the same with the initial state of the second part, we have

$$
P \left( r_{k}^{i} | s_{i}, x_{j}^{j}, s_{j+1} \right) = P \left( r_{k}^{i} | s_{i}, x_{k}^{i}, s_{k+1} \right) P \left( r_{k+1}^{i} | s_{k+1}, x_{j}^{j+k+1}, s_{j+1} \right), \tag{11}
$$

where $i < k < j$. For example, in a decode channel, we have

$$
P \left( r_{i} | r_{i+1} | y_{i} = 0, y_{i+1} = -2 \right) = P \left( r_{i} | s_{i} = 0, x_{i} = 0, s_{i+1} = 0 \right) \cdot P \left( r_{i+1} | s_{i+1} = 0, x_{i+1} = 1, s_{i+2} = 1 \right). \tag{12}
$$

Let $\rho_{m}^{(m)}(j_1, j_2, b)$ denote the summation of the likelihood functions of ISI codeword symbols $y_{j_1}(j_1,0)_{m}$ with fixed $u_i$, decoded $\hat{u}_{i-1}$, intermediate codeword bit $u_{(j_1, j_2)} = b$, initial state $s_{j_1}(j_1,0)_{m}$, end state $s_{j_2}(j_2,0)_{m}$, i.e.,

$$
\rho_{m}^{(m)}(j_1, j_2, b) \left( \hat{u}_{i-1}, u_{i}, s_{j_1}(j_1,0)_{m}, s_{j_2}(j_2,0)_{m} \right) = \sum_{u_{(j_1, j_2)}} P \left( r_{j_1, j_2}^{(m)} \mid s_{j_1}(j_1,0)_{m}, s_{j_2}(j_2,0)_{m} \right). \tag{13}
$$

Like (14), we have

$$
P \left( r_{0}^{N-1}, u_{i-1}^{i-1}, s_{0}, s_{N} \mid u_{i} = b \right) = \frac{1}{2^{N-1}} \rho_{m}^{(m)}(0, i, b) \left( u_{0}^{i-1}, u_{i}, s_{0}, s_{N} \right). \tag{14}
$$

For the summation $\rho_{m}^{(m)}(j_1, j_2, b) = \left( \hat{u}_{i-1}, u_{i}, s_{j_1}(j_1,0)_{m}, s_{j_2}(j_2,0)_{m} \right)$, we can group the terms by middle states $s_{j_2}(j_2,0)_{m}$, i.e.,

$$
\rho_{m}^{(m)}(j_1, j_2, b) \left( \hat{u}_{i-1}, u_{i}, s_{j_1}(j_1,0)_{m}, s_{j_2}(j_2,0)_{m} \right) = \sum_{s_{j_2}(j_2,0)_{m}} \rho_{m}^{(m)}(j_1, j_2, b) \left( \hat{u}_{i-1}, u_{i}, s_{j_1}(j_1,0)_{m}, s_{j_2}(j_2,0)_{m} \right). \tag{15}
$$

Using (14), (15) and the recursive formulas (5) and (6), we can obtain the new recursive formulas.

If $j_2 < 2^{m-1}$, we have

$$
\rho_{m}^{(m)}(j_1, j_2, b) \left( \cdot \right) = \sum_{s_{j_2}(j_2,2^{m-1})_{m}} \left( \left( \begin{array}{c}
m-1 \\hline
2ji - 1 \end{array} \right) \rho_{m}^{(m-1)}(2ji - 1, j_2, 2^{m-1} b) \left( \cdot \right) + \rho_{m}^{(m-1)}(2ji - 1, j_2, 2^{m-1} b, 1) \left( \cdot \right) \right). \tag{16}
$$

else

$$
\rho_{m}^{(m)}(j_1, j_2, b) \left( \cdot \right) = \sum_{s_{j_2}(j_2,2^{m-1})_{m}} \left( \left( \begin{array}{c}
m-1 \\hline
2ji - 1 \end{array} \right) \rho_{m}^{(m-1)}(2ji - 1, j_2, 2^{m-1} b, 1) \left( \cdot \right) \right). \tag{17}
$$

(16) and (17) is the new recursive formulas instead of (5) and (6). Introducing the middle states, the probability of the information $u_i$ can be calculated over specific states and recursively like conventional SC decoding. The initialization information of the recursive calculations is the likelihood functions of ISI codeword symbol $y_i$, i.e., $\rho_{0}^{(0)}(i, b) = P \left( r_{i} | s_{i}, x_{i}, s_{i+1} \right) = P \left( r_{i} | y_{i} \right)$.

The SC trellis decoding criterion for ISI channel is the same with (2), i.e., we need to compare $P \left( r_{0}^{N-1}, \hat{u}_{0}^{i-1} | u_{i} = b \right)$ where $b \in U = \{0, 1\}$. So after calculate the probability $P \left( r_{0}^{N-1}, \hat{u}_{0}^{i-1}, s_{0}, s_{N} \mid u_{i} = b \right)$ by (14) and the new recursive formulas, we need to merge the results. We assume the initial states are 0, so the probability can be rewritten as $P \left( r_{0}^{N-1}, \hat{u}_{0}^{i-1}, s_{0} = 0 \mid u_{i} = b \right), b \in U$, which can be calculated by

$$
P \left( r_{0}^{N-1}, \hat{u}_{0}^{i-1}, s_{0} = 0 \mid u_{i} = b \right) = \sum_{s_{N}} P \left( r_{0}^{N-1}, \hat{u}_{0}^{i-1}, s_{0} = 0, s_{N} \mid u_{i} = b \right) \tag{18}
$$

C. List Decoding

SC decoding can be improved by list decoding [9]. The proposed method can also be improved by list decoding. We calculate the probability of sequences $P \left( r_{0}^{N-1} \mid u_{0}^{i-1} = b_{0}^{i-1} \right), b_{i} \in U$, and keep the best L paths. Note that, we have the following relationship.

$$
P \left( r_{0}^{N-1} \mid u_{0}^{i-1} = b_{0}^{i-1} \right) = 2^{i} P \left( r_{0}^{N-1} \mid u_{i-1}^{i-1} = b_{0}^{i-1} \right) \left( u_{i} = b_{1} \right). \tag{19}
$$

For each $0 \leq i \leq N - 1$, we keep the best L paths, who have the maximum values of the probability $P \left( r_{0}^{N-1}, u_{i}^{i-1} = b_{0}^{i-1} \right)$. For $1 \leq l \leq L$, there is an estimation of $u_{0}^{i-1}$, denoted by $(b_{0}^{1}, b_{1}^{1}, \ldots, b_{l}^{1})$. And for $i + 1$, we calculate the new 2L paths using the estimation $(b_{0}^{1}, b_{1}^{1}, \ldots, b_{l}^{1})$ of the last L paths and keep the best L paths.

Although the SC trellis decoding requests the probability with trellis states, the decision functions $P \left( r_{0}^{N-1}, u_{i}^{i-1}, s_{0} \mid u_{i} \right)$ are the same with the probability $P \left( r_{0}^{N-1}, u_{i}^{i-1} \mid u_{i} \right)$ if $s_{0}$ is fixed. It is straightforward to change SC trellis decoding to list decoding. We show the performance in Section IV.

IV. Simulation

In this section, we compare the performance of the proposed decoding method with the turbo equalization structures of polar codes. In the turbo equalization structures, first, BCJR equalization is performed, then it passes soft estimates to the
decoder after deinterleaving. If the decoder is belief propagation (BP) decoder, it operates for \( I \) iterations. The resulting soft estimates are passed back to equalizer after interleaving. This forms one turbo iteration, and is repeated \( I_0 \) times. We also compare the performance with LDPC codes decoded with the turbo equalization structure using BP decoding \[10\]. The LDPC codes are with column weight 3. We choose EPR4 channel to show the performance. The channel is modelled by

\[
r_i = y_i + n_i = x_i + x_{i-1} - x_{i-2} - x_{i-3} + n_i, \quad (20)
\]

Fig. 3 shows the performance of codes with code lengths \( N = 1024 \) and code rates \( R = 1/2 \). The proposed SC trellis decoding method is compared with the turbo equalization structure using SC decoding. We can find that the proposed method significantly outperforms the turbo equalization structure and can obtain 1 dB gain at BER \( 10^{-4} \). Note that, the bit error rate (BER) of turbo equalization structure is close to the frame error rate (FER) due to the critical error propagation.

We show another comparison in Fig. 4. The code lengths are all \( N = 256 \), and code rates are \( R = 1/2 \). In this simulation, list decoding using the proposed method is compared with the LDPC code and the proposed SC trellis decoding method is compared with the turbo equalization structure using BP decoding \[11\]. The performance of the list decoding with 4 list paths obtain 1 dB gain at BER \( 10^{-4} \) as compared to the LDPC code and the SC trellis decoding method obtain more than 0.5 dB gain at BER \( 10^{-4} \) as compared to the turbo equalization structure using BP decoding.

Furthermore, the performance of the proposed method will be enhanced if a more suitable frozen set is selected, especially at high \( E_b/N_0 \) regions.

V. CONCLUSION

In this paper, we proposed SC trellis decoding to deal with the performance degradation of polar codes in ISI channels. We show the recursive formulas of the SC trellis decoding by introducing the trellis states. Using the recursive formulas, we can decode the polar codes in ISI channels like decoding it in B-DMCs without iterations. The error propagation can be reduced by the method. Furthermore, the proposed decoding method can be easily extended to list decoding, which gives an outstanding performance. The simulation shows that the decoding method perform well and significantly outperforms the conventional decoding schemes.

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