Optimizing Lot Size of Flexible Job Shop Problems by Considering Expiration Aspect

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Abstract. Production planning for manufacturing companies is an aspect to meet production needs for production process. The production need of fish canning companies is uncertainty aspect and this includes in the flexible job shop system, the issues of uncertainty. The process of fulfilling the needs with consideration of expiration in the fish canning company is to optimize lot size with assumption that customer demand can be met on time without reducing consumer trust. Mathematical formulations used to meet the needs of the FJSS problem use lot-sizing techniques in making decisions. Stochastic programs can be used to obtain the desired formulation. Solution for ordering quantities and the number of quantities ordered plus expiration considerations are formulated with mixed integer linear programing.

1. Introduction

Production planning has a very important role in manufacturing industries. Therefore, the company needs to design a production plan so the costs occured in the production process can be minimized. Production costs include setup costs, processing costs and storage costs. If the company wants to produce more than one product unit processed on one machine, the costs for the setup activities at each turnover production process will be high. Conversely, if the company uses more than one machine to produce several types of products, the setup costs are low but the investment costs will be high.

In their study, Gicquel et al. (2008) explaine that if production is carried out with large batches, it can minimize the setup costs. However, as the effect, the storage costs are high. Conversely, keeping a low inventory by running production on small batches will increase the setup costs. Capacitated Lot Sizing Problem (CLSP) is developed by Gicquel et al. (2008) to determine the optimal lot size that can minimize setup costs and storage costs. This is to anticipate costs rising and reduce product storage time. A planning mechanism is needed at the production scheduling step.

Production scheduling is related to short-term production problems and this is a job shop scheduling (JSS) problem. JSS can be said as a pattern of a set of work on a set of machines. For example, there is n job(J, ..., J) with various sizes in each job that must be scheduled on each machine m (m, ..., m.). Each job is operated on each machine which requires a certain period of time for each job. Flexible job shop scheduling (FJSS) is a generalization of classic job shop problems. Each operation can be processed on a particular machine selected from limited subset of machines.
The existence of fish canning industry is a part of flexible job shop system. The manufacturing process is flexible in responding to changes in design and number of orders so that it is classified into the problem of flexible job shop scheduling (FJSS) and uncertainty issues.

Many optimization approaches to flexible problems of job shop scheduling (FJSS) have been proposed, such as exact algorithms, heuristic algorithms, and metaheuristic algorithms using priority rules. Exact algorithms such as branch-and-bound methods (Y. Tan et al, 2010), Demir and İşleyen (2013) propose Integer Programming (IP) with the objective function to minimize makespan in solving FJSS. The study includes the selection of alternative processes in determining the sequence of processes. IP formulation was also carried out by Fattahi, et al., (2007) which compared branch-and-bound with the simulated annealing method and Taboo search. However, the exact approach cannot optimally solve large-scale problems.

This research is focused on fish canning industry. Fish canning industry is a manufacturing company that produces canned fish in various sizes (large, medium and small cans). The production flow is a job shop. Available machines are used to produce various types of canned fish. Every change in the type of product size that will be produced, setup is required and setup costs will occur. As a company with a business strategy, it tries to have large quantities of raw materials and finished products in anticipation of customer needs. Because the company wants to provide maximum service to consumers, of course the availability of every item is needed. As a result, there is a buildup of inventory in each period, so the number of finished products stored is greater. The accumulation of too large quantity of finished products will also cause a high storage cost.

In this case the problem faced by the company is how the company optimizes the production lot size so the total setup costs and storage costs are minimum. Too large production lot can cause the number of products produced higher than the number of demand so the cost of production is higher. However, too small production lot can lead to unfulfillment of demand which resulting in a lack of consumer trust, product delivery delay, and penalty fees.

Canning industries producing perishable products (deteriorating or decreasing in quality after a certain time) generally has certain standards. Standardization, in this case, is the standard composition of the products produced and the raw materials used. Hsu, et al., (2010) compiles an inventory model for goods that reduce quantity and quality quickly over time until it reaches an expiry time. These types of goods include flowers, fruits, and other food products.

A fish canning industry produces a food product, where the expiration time is a problem that must be considered both the raw material and the finished product. This is because it involves the issue of food safety consumed. In addition, the raw material used also has a limited/short shelf life. Similarly, to control the quality of the products produced, the expiration of the product must be noticed because it can affect the number of demand. In order to guarantee the product received by the consumer is in good conditions, it requires consideration of product safety. Soman et al., (2004b) suggest that food producer must strive to send products to their customers by reducing product storage time to avoid expiration, decay, and shipping products at the expiration date.

The important part of this paper is to optimize lot size by considering that the raw material used is quickly damaged and the product deteriorates. The ordering of too much raw materials will decay, if not processed quickly. On the contrary, ordering little raw materials has an impact on the level of inventory of the company and incur additional costs or delays in production. Good control of raw materials is needed so the company inventory level is in optimal condition. Bramorski (2013) develops a mathematical model to control the price of products in supermarkets by paying attention to inventory levels and estimating expiration times as decision variables. Hsu et al., (2010) develop a mathematical model for inventory control for goods which experience quantity and quality reduction over time, seasonal demand levels, expiration and backorder. Chen et al., (2009) acknowledge that papers discussing production scheduling with relative expiration requirements are rare, and papers discussing simultaneous lot sizes of FJSS are even rarer. In this paper, a lot size optimization model of the FJSS problem is presented completed with consideration of expiration by using a stochastic program.
2. Developing Model

In this section, the mixed integer program model is developed on the issues described in the previous section, optimizing mathematical models by minimizing total production costs. Stochastic programs can be used from the FJSS issues by using an exact algorithm approach as an effective solution technique.

All machines can produce any product and it is assumed that each machine can be used in operations $i, k$ which is possible at time. The available capacity of each machine is limited and can vary between periods and stages. Each period is represented for one week from the planning horizon. In one period divided into several sub-periods as variable sizes.

Demand for goods depends on the next stage of production. Products can be produced in various sizes on one of the parallel machines at each stage. The level of production can vary between product and machine, but it is constant throughout the planning horizon. If switching from one product to another product, setup time is required where the machine is not productive. Cost and set up time depend on the order and can vary between machines. Set up is done when no product is being processed (setup carryover).

The model is developed with consideration that products have an expiration or shelf-life. Therefore, inventory control is needed so that there is no unused material. Therefore, the products is safe and can be stored and used. For inventory components returned when they have expired, it is necessary to note the time period of the component received. If the component reaches the end of the shelf-life and expires, the component is returned. Inventories returned are inventories that cannot be used again after the expiration date. Kallrath (2005) describes that most of data related to inventory for problems involving the shelf-life to be duplicated, by adding the shelf-life index. In addition, the amount of inventory stored as a backup, we also need to know when components must be ordered or received. Thus, the mathematical model formulation that includes the expiration factor is by adding index variables and expiration constraints in the model, as well as the withdrawal costs per unit of components returned. As a result there will be additional costs including costs for withdrawn or returned components and the holding costs.

Before the mathematical model formulation is made to make optimal decisions, it is necessary to make a model notation first. The model notation is as follows:

**Index:**
- $r$: number of items ordered
- $j$: raw material unit
- $i$: unit, product (state, item), $i \in I = \{1, 2, \ldots NI\}$
- $k$: unit, $k \in K = \{1, 2, \ldots NK\}$
- $l$: machine
- $t$: time period
- $s$: sub periode

**Parameter:**
- $I_{p0}$: Initial inventory of products
- $V_{p0}$: Initial raw material inventory
- $q_{j,t}$: Ordering raw materials $j$ at time $t$
- $d_{i,t}$: Product demand at time $t$
- $b_{j,t}$: Number of units of raw materials $j$ needed for the production process $i$
- $a_j$: Shelf-life raw material $j$
\( a_i \) Shelf-life product \( i \)
\( S_j \) Size of ordering raw material for unit \( j \)
\( l_j \) The grace period for ordering raw material for unit \( j \)
\( x_{i,j,s} \) Production time of product \( i \) on machine \( l \) in sub period \( s \)
\( r_{i,j,k}^t \) The setup time changes over the product \( i \) to product \( k \) at time \( t \)
\( f_{i,j} \) The costs of return the product \( i \) cans
\( f_j \) Disposal cost of raw material for unit \( j \)
\( c_j \) Raw material costs for unit \( j \)
\( P_i \) Production costs for product \( i \)
\( h_i \) Inventory holding costs of production for product \( i \)
\( m_j \) inventory holding costs for raw materials unit \( j \)
\( A_i \) Fixed production costs for product \( i \)
\( g_j \) Fixed costs for raw materials ordering for unit \( j \)
\( \delta_{i,k} \) Cost of setup change over from product \( i \) to product \( k \)
\( K_i \) Production capacity for product \( i \)
\( L_i \) Ordering capacity for products \( i \)
\( K_t \) The capacity of the machine \( l \) in period \( t \)
\( M_{i,j,s} \) Upper bound \( x_{i,j,s} \) or upper limit of production time of product \( i \) on machine \( l \) in sub period
\( \pi_{j,t} \) Possible raw materials of unit \( j \) at production time interval \( t \)

**Variables:**

\( x_{i,j,s} \) The number of units \( i \) produced at time \( t \)
\( I_{i,t} \) Inventory of production for product \( i \) at time \( t \)
\( Q_{j,t} \) Ordering raw material for unit \( j \) at time \( t \)
\( V_{j,s} \) Inventory of raw material unit \( j \) at time \( t \) based on the number of items ordered
\( C_{j,s} \) The use of raw materials for unit \( j \) is based on the number of items ordered at time \( t \)
\( Y_{i,t} \) Number of returns for products type \( i \)
\( Z_j \) Total disposal of raw material unit \( j \)
\( y_{i,t} \) Binary variable of ordering product \( i \) at time \( t \)
\( w_{j,t} \) Binary variables of scheduling the order of raw materials for unit \( j \) at time \( t \)
Binary variable of fixed order of raw material unit $j$ at time $t$

Binary variable if there is a change over product $i$ to product $k$ on machine $l$ in the sub period $s$

Binary variable if the process setup of machine $l$ for product $i$ products in the sub period $s$

Furthermore, developing a model formulation to present the FJSS issues is done by considering lot-size based on classic GLSP developed by Meyr (2002), Ferreira, et al. (2012) and Martinez, K.Y.P., et al. (2016). This not only considers the lot-size but also considers the expiration.

**Formulation:**

Minimize:

\[
\sum_{j=1}^{J} \sum_{t=1}^{T} g_{j} W_{j,t} + \sum_{j=1}^{J} \sum_{t=1}^{T} g_{j} W_{j,t} + \sum_{j=1}^{J} \sum_{t=1}^{T} \pi_{j,t} \left[ \sum_{j=1}^{J} \sum_{t=1}^{T} c_{j} Q_{j,t} + \sum_{j=1}^{J} \sum_{t=1}^{T} m_{j} v_{j,t} + \sum_{j=1}^{J} f_{j} z_{j} \right]
\]

Minimize:

\[
\sum_{j=1}^{J} \sum_{t=1}^{T} p_{j} X_{j,t} + \sum_{j=1}^{J} \sum_{t=1}^{T} A_{j} y_{j,t} + \sum_{l=1}^{L} \sum_{i=1}^{N} \sum_{s=1}^{S} \delta_{i,s} \alpha_{i,s,t}
\]

Minimize:

\[
\sum_{j=1}^{J} \sum_{t=1}^{T} f_{j} Y_{j} + \sum_{j=1}^{J} \sum_{t=1}^{T} \pi_{j,t} \left[ \sum_{j=1}^{J} \sum_{t=1}^{T} h_{j} l_{j,t} + \sum_{j=1}^{J} \sum_{t=1}^{T} K_{j} I_{j,t} \right]
\]

Constraints:

\[
l_{i,t} = l_{i,t-1} - X_{i,t} - d_{i,t}, \forall i \in N, \forall t \in T, \forall \sigma \in \Omega
\]

\[
\sum_{j=1}^{J} V_{j,t} = v_{i,0} + S_{j} q_{j,t} - \sum_{j=1}^{J} b_{j,t} X_{j,t}, \forall j \in J
\]

\[
\forall t \in T, \forall r \in T, t = 1
\]

\[
\sum_{j=1}^{J} V_{j,t} = \sum_{r=0}^{t-1} V_{j,t-r} + S_{j} q_{j,t} - \sum_{j=1}^{J} b_{j,t} X_{j,t}, \forall j \in J, \forall t \in T, 1 < t < a_{j}, t \leq l_{j}
\]

\[
\sum_{j=1}^{J} V_{j,t} = \sum_{r=0}^{t-1} V_{j,t-r} + S_{j} q_{j,t} - \sum_{j=1}^{J} b_{j,t} X_{j,t}, \forall j \in J, t \geq a_{j}, t \leq l_{j}
\]

\[
\sum_{j=1}^{J} V_{j,t} = \sum_{r=0}^{t-1} V_{j,t-r} + S_{j} q_{j,t} - \sum_{j=1}^{J} b_{j,t} X_{j,t}, \forall j \in J, t > l_{j}
\]

\[
\sum_{j=1}^{J} b_{j,t} X_{j,t} = \sum_{r=0}^{t} \sigma_{j,r,t}, \forall j \in J, \forall t \in T, t < a_{j}
\]
The next is obstacles to maintain the availability of raw materials

\[ V_{j,r,t} = S_j q_{j,r,t} - e_{j,r,t}, \quad \forall j \in J, \forall t \in T, \forall r \in T, r = t, t \leq l_j \]  
(9)

\[ V_{j,r,t} = S_j q_{j,r-1} - e_{j,r,t}, \quad \forall j \in J, \forall t \in T, \forall r \in T, r = t, t \leq l_j \]  
(10)

\[ V_{j,r,t} = V_{j,t-1,r} - e_{j,r,t}, \quad \forall j, t, t - r < a_j, l \leq j \leq J, 1 \leq r \leq T \]  
(11)

Constraints in the disposal of raw materials because of shelf-life exceeding

\[ Z_j = \sum_{i=1}^{T} V_{j,t,i+1-a_j}, \quad \forall j \in J \]  
(12)

For capacity of processed fish production can be written as follows

\[ Y_i = \sum_{t=1}^{T} I_{i,t} t \leq a_i, \quad \forall i \in I \]  
(13)

\[ X_{i,t} \leq K_i, \quad \forall i \in N, \forall t \in T \]  
(14)

\[ \sum_{i=1}^{N} \sum_{s \in S} x_{i,s} + \sum_{i=1}^{N} \sum_{j=1}^{N} s t_{i,j} \alpha_{i,j} \leq K'_{i,t} \]  
\[ \forall t = 1, ..., T; l = 1, ..., L \]  
(15)

\[ x_{i,s} \leq M_{i,s} \beta_{i,s} \quad \forall l = 1, ..., L; i = 1, ..., N; t = 1, ..., T; s \in S \]  
(16)

The capacity constraints of the order can be written as follows

\[ Q_{j,t} \leq L_j, \quad \forall j \in J, \forall t \in T \]  
(17)

Constraints on binary variables preparation and ordering costs can be written as follows

\[ X_{i,t} \leq M_{i,t}, \quad \forall i \in N, \forall t \in T \]  
(18)

\[ Q_{j,t} \leq M_{w,j,t}, \quad \forall j \in J, \forall t \in T \]  
(19)

\[ q_{i,t} \leq M_{w,t}, \quad \forall j \in J, \forall t \in T \]  
(20)

Constraints of negativity and binary can be written as

\[ X_{i,t}, I_{i,t}, Q_{j,t}, V_{j,t,r}, C_{j,t}, Z_j \geq 0, \forall i, j, r \]  
(21)

\[ y_{i,t}, W_{j,t}, w_{j,t} = 0 \]  
(22)

\[ M_{i,t} = \min \{ K'_{i,t} \}, \text{max}_{k: p_{ki} \neq 0} \frac{\sum_{h=1}^{T} d_{kh}}{p_{ki}} \]  
\[ \forall t = 1, ..., T; l = 1, ..., L; \]  
(23)

Constraints (4.23), (4.24) and (4.25) the relation between the setup of two successive sub-periods, thus determining changeover turn and maintaining the setup.

\[ \sum_{l=1}^{N} \alpha_{l,s} \leq \beta_{i,s} \quad \forall i = 1, ..., L; k = 1, ..., N; s = 1, ..., S \]  
(24)
$$\beta_{i(m-1)} = \sum_{j=1}^{N} \alpha_{ijs} \quad \forall \; l=1,\ldots,L; \; i=1,\ldots,N; \; s=1,\ldots,S$$  \hfill (25)

$$\sum_{j=1}^{N} \alpha_{ijs} = \beta_{il} \quad \forall \; l=1,\ldots,L; \; i=1,\ldots,N; \; s=1,\ldots,S$$  \hfill (26)

3. Algorithm Development

Completion of lot size optimization problems from FJSS with consideration of expiration in the fish canning industry is necessary to develop a settlement algorithm. The algorithm consists of two stages, namely:

Stage I:
Complete the problem in equation (1) to equation (26) with the integer relaxation conditions. If the optimal solution with an integer condition is continuously fulfilled, stop. So that an optimal solution is obtained that is feasible. If not, continue the step 1.

Step 1. Get row \(i^*\) the smallest integer infeasibility, such that \(\delta_{i^*} = \min\{f_i, 1-f_i\}\) (This choice is motivated by the desire for minimal deterioration in the objective function, and clearly corresponds to the integer basic with smallest integer infeasibility).

Step 2. Perform operations for pricing \(v_{i^*}^T = e_{i^*}^TB^{-1}\)

Step 3. Calculate\( \sigma_{ij} = v_{i^*}^T \alpha_j \) with corresponds to \(\min_j \left\{ \frac{d_j}{\alpha_{ij}} \right\} \)

Calculate the nonbasic movement \(j\) of the maximum lower bound and upper bound. Otherwise, go to next non-integer nonbasic or super basic \(j\) (if available). Finally, the \(j^*\) column will be raised lower or lowered the upper bound. If there is no variable, go to the next proceed to \(i^*\)

Step 4. Solve\( BA_j^* = \alpha_j^*\) for \(\sigma_{i^*}\)

Step 5. Perform a ratio test for basic variables by taking the limits to be feasible due to nonbasic release\(j^*\).

Step 6. Exchange base
Step 7. If line \(i^* = \{\emptyset\}\) continue to Phase 2, if there is no variable, repeat step 1 again

Stage II:
Pass 1: Move an infeasible superbasic integer with fractional steps to achieve a feasible complete integer.
Pass 2: Adjust integer superbasic. The purpose of this phase is to conduct a local neighborhood search to verify optimal locality.

4. Conclusion

This paper presents a mathematical model of lot size optimization from FJSS problems by considering expiration aspect in the fish canning industry. The model is a large scale mixed integer program. To solve this problem, the concept of the exploited superbasic variable algorithm is developed.

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