Estimation of log dead reckoning error induced by tidal currents

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Abstract. Log dead reckoning autonomous correction problem is considered. The problem is solved by using corrector, contained only one uncontrolled gyroscope and working according to single-channel inertial vertical principle. It is shown that the main dead reckoning error is currents variability which cannot be measured by log and having tidal nature mainly.

1. Introduction

A dead reckoning (DR) is one of the methods for determining vessels location. An absolute log or space navigation system is used for velocity measuring in this case. A course, which is needed for conversion of measuring velocity to navigation frame (geographic frame), is provided by inertial navigation system (INS) or stabilizer gyrocompass, which is usually constructed the INS pattern for only purpose is elimination of inertial accelerations disturbances in its orientation parameters.

A new method for eliminating inertial accelerations disturbance was presented in [1]. It was named «the method of single-channel inertial vertical (SIV) with correctable pendulum (CP)» [2]. Using the method provides the high precision correction level of DR, keeping, at the same time, the full autonomous and reducing the amount of inertial sensor elements (uncontrolled gyroscopes (UG)) to one in compare with classic INS [3]. The method consists in calculating horizontal frame and providing orientation parameters from observations of two vectors according to TRIAD algorithm [4]. The first (base) vector is the UG’s main vector, oriented parallel to the Earth equator plane. The second one is the gravity force vector, which is parallel to the vertical and calculated from accelerometers data. Orientation angles are disturbed because of this method. However, choosing the UG’s main vector as a base vector in TRIAD algorithm leads to measure angle between observed vectors (label it as $\phi'$) using only UG’s data without involvement any information, misrepresented due to inertial accelerations.

The $\phi'$ angle which is measured in SIV-plane also can be calculated with use of geographic coordinates of current location this way:

$$\cos \phi' = \cos \phi \cos \lambda_s,$$  \hspace{1cm} \hspace{1cm} (1)

where $\phi$ – local latitude; $\lambda_s = \lambda + \Omega t$ – local inertial longitude, where $\lambda$ – local longitude; $\Omega$ – the Earth’s angular velocity; $t$ – Greenwich time.

The expression (1) helps to obtain a difference measurement:
where \( \varphi_m' \) – a value of \( \varphi' \) measured in SIV loop; \( \varphi_c' \) – a value of \( \varphi' \) calculated with use of DR coordinates according to (1).

The difference measurement (2) contains DR errors \( \Delta \varphi, \Delta \lambda \). Its model can be presented within an error of inertial direction and horizontal position error as follows:

\[
z = \Delta \varphi \sin \varphi \cos \lambda_\ast + \Delta \lambda \cos \varphi \sin \lambda_\ast + \upsilon,
\]

where \( \upsilon \) – a white noise measurement error.

The measurement (3) can be used for permanent autonomous correction of DR. But at the same time despite the fact that course has an error induced by inertial accelerations mentioned above, this can be compensated in coordinates with use of information about acceleration from SIV loop and speed from an outer source. Thus the method provides DR according to CP approach [5].

The paper is about relative log DR correction problem. It is solved by use of autonomous measurement (2) provided by inertial corrector constructed according to SIV-CP approach. The DR error is caused by tidal currents. The problem is considered in both linear and non-linear statements.

Modelling results show that in terms of this kind of sea currents the problem should be solved in non-linear approach.

2. Sea current model

In all common cases a sea current model is described with Markov process [6]. A DR error estimation results were initially presented in [7]. The error was caused by currents described by Markov process. Estimation was got from measurement (2) using Kalman filter (KF) [6], [8]. Circular position error (CPE) is presented on figure 1 and calculated as:

\[
R = \sqrt{\left(\Delta \varphi R_\varphi\right)^2 + \left(\Delta \lambda R_\lambda \cos \varphi\right)^2},
\]

where \( R_\varphi, R_\lambda \) – radiuses of the Earth’s ellipsoid.

![Figure 1](image.png)

**Figure 1.** CPE is caused by sea current described with Markov process (standard deviation (±3σ) has been got from calculation results in covariance channel of KF and shown by dashed line).
Currents model described with Markov process is common and can be specified in considering with reference to certain navigation areas. The paper is considered a problem of relative log errors estimation which is induced by tidal currents since speed of constant non-periodic currents are significantly less in compare with tidal currents [9], [10] when navigation takes place in tidal seas or local to coast.

The Gulf of Ob is considered for getting a math model of currents which will be used in estimation problem statement. Total currents in Ob bay are composed of quasi-constant, tidal and wind-driven currents [11]. If speed of quasi-constant (river runoff) currents is within 0.1 – 0.2 m/s from the North to the South and wind-driven currents don’t always take place because of large ice cover then speed of tidal currents is up to 0.5 m/s. Also tidal currents have half of a day period mainly (mostly their period is close to 12 h. 25.2 m.) so it may affect their observability when using the measurement (3).

The real data has been got from automatic buoy stations deployed in the south area of Ob bay in the year 2012. Zonal and meridional flows of a current are presented on figures 2 and 3 and were published earlier in an external technical report.

Figure 2. Meridional component of sea current.
The charts presented on figures 2 and 3 show that currents have been having predominantly semidiurnal period and constant phase difference between zonal and meridional flow components during all measurement time (since 15.08.2012 till 29.09.2012) which is $\pi$.

3. Linear estimation problem statement

Based on the above, DR error model caused by currents can approximately be presented as:

$$
\Delta \phi = \frac{v_{Ny}^{(0)}}{R_\phi} + \frac{v_{Ny}^{(1)}}{R_\phi} \sin(2\Omega t + \varepsilon) + \frac{v_{Ny}^{(2)}}{R_\phi} \cos(2\Omega t + \varepsilon + \pi)
$$

$$
\Delta \lambda \cos \phi = \frac{v_{Ey}^{(0)}}{R_\lambda} + \frac{v_{Ey}^{(1)}}{R_\lambda} \sin(2\Omega t + \varepsilon)
$$

where $v_{Ny}^{(0)}, v_{Ny}^{(1)}, v_{Ny}^{(2)}$ – constant components of sea current speed; $v_{Ny}, v_{Ey}$ – constant amplitudes of meridional and zonal components of sea current speed respectively; $\varepsilon$ – meridional component init phase of sea current speed.

The model (5) is sufficient on short time – up to 6 h. of autonomous use. Variability of amplitudes $v_{Ny}, v_{Ey}$ and constant components $v_{Ny}^{(0)}, v_{Ny}^{(1)}, v_{Ny}^{(2)}$ has to take into account when time interval is more.

The unknown items in the (5) are $v_{Ny}^{(0)}, v_{Ny}^{(1)}, v_{Ny}^{(2)}, v_{Ey}^{(0)}, v_{Ey}^{(1)}$ and $\varepsilon$. An estimation problem of the parameters is non-linear for providing DR correction. However it can be rewritten in such a way:

$$
\Delta \phi = \frac{v_{Ny}^{(0)}}{R_\phi} + \frac{v_{Ny}^{(1)}}{R_\phi} \sin(2\Omega t) + \frac{v_{Ny}^{(2)}}{R_\phi} \cos(2\Omega t)
$$

$$
\Delta \lambda \cos \phi = \frac{v_{Ey}^{(0)}}{R_\lambda} - \frac{v_{Ey}^{(1)}}{R_\lambda} \sin(2\Omega t) - \frac{v_{Ey}^{(2)}}{R_\lambda} \cos(2\Omega t)
$$
where \( v_{iE}^{(1)} = v_{iE} \cos (\varepsilon) \), \( v_{iE}^{(2)} = v_{iE} \sin (\varepsilon) \) (\( i = N, E \)).

The model (6) lets us to make a linear estimation problem statement:

\[
x = \begin{bmatrix} \Delta \phi \\ \Delta \lambda \\ v_{N,T}^{(0)} \\ v_{E,T}^{(0)} \\ v_{N,T}^{(1)} \\ v_{E,T}^{(1)} \\ v_{N,E}^{(2)} \\ v_{E,E}^{(2)} \end{bmatrix}^T
\]

by the measurement (3) and solve it using KF. This statement neglects a correlation between \( v_{N,T}^{(1)}, v_{N,T}^{(2)}, v_{E,T}^{(1)}, v_{E,T}^{(2)} \) considering them to be independent values that makes a result knowingly less accurate in compare with non-linear problem statement. However it is shown in [12], [13], where the same approach was applied, that resulting accuracy decrease is usually insignificant.

CPE calculated according to (4) is presented on figure 4 as a result of KF apply.

Figure 4. CPE is caused by tidal sea current (linear problem statement) (standard deviation (±3σ) has been got from calculation results in covariance channel of KF and shown by dashed line)

Stable estimation of DR error was failed to get in case of linear statement as it follows from figure 4. It was caused by the fact that the components \( v_{N,T}^{(1)}, v_{N,T}^{(2)}, v_{E,T}^{(1)}, v_{E,T}^{(2)} \) are unobservable from the measurement (3) that can be checked by using observability criteria [14]:

\[
M_{i,i} (0, N) = \sum_{i=1}^{N} \Phi(i,i)H(i)\Phi(i,0),
\]

where \( \Phi(i,i-1) \) – a discrete state-transition model matrix; \( \Phi(i,0) = \Phi(i,i-1)\Phi(i-1,i-2)\Phi(i-2,i-3)\ldots \Phi(1,0); \) \( H(i) \) – an observation model matrix on \( i \)-step of KF loop. These components have the constant error that significantly affects an accuracy of coordinates estimations.

4. Non-linear estimation problem statement
A problem is in getting the estimation of the state vector:
\[ x = \begin{bmatrix} \Delta \phi & \Delta \lambda & v_{N_T}^{(0)} & v_{E_T}^{(0)} & v_{N_T} & v_{E_T} & \varepsilon \end{bmatrix}^T, \]

which is described with model (5), by use of the measurement (3). Such problem can be solved in optimal way by use of filters based on Rao-Blackwellization procedure for instance [15]–[17]. A key point here is that fixing the unknown parameter \( \varepsilon \) makes the estimation problem (5) linear in context of the measurement (3). Given a set of hypothesis for the parameter we can build a bank of KF. A result of each KF is a set of particle estimations of coordinates error and components of a sea current velocity. At the same time KF bank residuals let to get an approximation for likelihood function of non-linear parameter \( \varepsilon \). It can be got both an optimal Bayes estimation of \( \varepsilon \) itself and other parameters of the state vector (9) by using the likelihood function. The results of estimation are presented on figure 5 for non-linear statement.

![Figure 5](image)

**Figure 5.** CPE is caused by tidal sea current (non-linear problem statement) (standard deviation (±3\( \sigma \)) has been got from calculation results in covariance channel of KF and shown by dashed line)

The figure 5 shows that solving the estimation problem in non-linear statement is completely different and substantially (up to tens times) exceed the accuracy of estimation for linear statement by using KF. Thus it is necessary to use non-linear approach or linearized algorithms for solving the problem in case of tidal currents.

5. **Conclusion**

A relative log DR error correction problem is considered in the paper. The error is induced by tidal sea current. Linear and non-linear estimation problem statements were presented and solved with use of autonomous measurement got from inertial corrector constructed according to SIV-CP approach. Kalman filter and algorithms of non-linear filtration based on Rao-Blackwellization method were implemented. Modeling results showed that accurate result can be got only with non-linear approach because of observability absence for tidal current amplitudes and phase in case of linear statement.

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