Comparison of isothermal and isoflux $g$-functions for borehole-heat-exchanger fields

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Abstract. A new numerical approach to determine the $g$-function of a borehole field with the boundary condition of uniform temperature and time-constant mean heat flux at the surface of the boreholes is presented. The method is employed to compare the $g$-functions obtained by this boundary condition with those obtained by the usual condition of uniform and constant heat flux, for a single borehole and for a field of six boreholes placed in two lines. Boreholes with length 150 m, diameter 15 cm, buried depth 1.5 m and mutual distance 7.5 m, for the field, are considered. The results show that the difference between the two kinds of $g$-functions is less than 1.5 % for a single borehole, while it reaches 8.7 % for the borehole field, at high values of time. Finally, we show that the superposition of the effects of the single boreholes yields correctly the $g$-function of the field in the case of uniform heat flux, but overestimates the $g$-function in the case of uniform temperature and constant mean heat flux.

1. Introduction

Ground-source heat pumps (GSHPs) are becoming a rather diffuse technology for building heating and cooling, especially in countries with a cold climate, where air source heat pumps are not very efficient. Self et al. [1] compared GSHPs and conventional heating systems in terms of costs, CO$_2$ emissions and other parameters. They concluded that GSHPs are not only the most efficient and the most eco-friendly system, but also the most convenient system for building heating in Canada. The most widely used GSHPs are ground-coupled heat pumps (GCHPs), that commonly employ fields of vertical closed-loop ground heat exchangers, often called borehole heat exchangers (BHEs).

The design of BHE fields is usually performed by employing dimensionless thermal response factors, called $g$-functions, which are often embedded in programs for the simulation of BHE fields, such as EED [2], GHLEPRO [3], and EnergyPlus [4].

The $g$-function of a BHE field is the time-dependent mean dimensionless temperature of the surface of the BHEs produced by a time-constant mean heat flux per unit length applied at that surface. For a single BHE, $g$-functions are usually evaluated not only on the BHE surface, where the heat flux is applied, but also on cylindrical surfaces placed at different radial distances from the BHE axis. In fact, the $g$-function for a BHE field can be often calculated by the spatial superposition of effects in space.

$G$-functions are usually evaluated by employing the boundary condition of uniform and constant heat flux per unit length at the surface between BHEs and ground. The simplest scheme adopted is the Finite Line-Source (FLS) scheme, where each BHE is considered as a line-source with finite length. The analytical solution of the temperature field under this scheme was determined by Claesson and Eskilson [5] and by Zeng et al. [6]. Since the solution has the form of an integral, the temperature
averaged along the BHE length has the form of a double integral. Simpler expressions of this
temperature, in the form of a single integral, were developed by Lamarche and Beauchamp [7]
and by Bandos et al. [8]. An extended solution, valid also for BHEs whose top is buried under the ground
surface, was determined by Claesson and Javed [9]. The FLS scheme at uniform and constant heat flux
allows determining the $g$-function for a BHE field by the superposition of the effects of single BHEs.

Another simple scheme is the Finite Cylindrical-Source (FCS) scheme, where each BHE is
considered as a cylinder with a finite length that delivers a uniform and constant power per unit length
to the surrounding solid medium. This scheme is suitable for numerical evaluations of the $g$-functions,
and was employed, for instance, by Zanchini and Lazzari [10, 11] and by Monzó el al. [12]. It allows
employing the superposition of the effects of the single BHEs.

A different FCS scheme for the numerical calculation of the $g$-functions was employed by
Eskilson [13]. He imposed a uniform temperature and a time-constant mean heat flux per unit length at
the surface between the BHE field and the ground, and evaluated the $g$-functions for many BHE fields
with this boundary condition. The results were obtained by finite-difference numerical simulations and
were reported in graphical form. Lamarche and Beauchamp [7] and Fossa et al. [14] compared
$g$-functions obtained by the FLS model and the superposition of the effects of single BHEs with the
$g$-functions obtained by Eskilson [13], for some BHE fields. They found that the FLS scheme slightly
overestimates the $g$-functions with respect to the boundary condition employed by Eskilson.

Eskilson’s boundary condition is not easily applied, since the time-dependent heat flux distribution
that yields a uniform surface temperature and a time-constant mean heat flux is unknown. Moreover,
this boundary condition does not allow applying the superposition of the effects of single BHEs.
Nevertheless, Eskilson’s boundary condition has been employed in recent scientific papers, because it
is closer to the conditions that occur in practical applications.

Cimmino and Bernier [15] developed a semi-analytical method, based on the FLS solution, to
determine $g$-functions with Eskilson’s boundary condition. In this method, BHEs are divided into
segments releasing different time-dependent heat fluxes per unit length, such that the surface
temperature of the BHE field is uniform and the heat flux averaged on the surface of the BHE field is
constant. An improvement of the method of Ref. [15], that takes into account the effects of the BHE
thermal resistance, was developed by Cimmino [16]. A technique to reduce the computation time in
the application of the method of Ref. [15], by reducing the number of numerical computations of the
FLS solutions, was presented in Ref. [17]. Lamarche [18] proposed a modification of the method of
Ref. [15], which allows reducing the number of segments for each borehole.

Priaorone and Fossa [19] presented a numerical method to determine $g$-functions with Eskilson’s
boundary condition for a single BHE. To obtain a uniform surface temperature, they imposed a
uniform and constant heat flux at the inner surface of a superconductive layer adjacent to the BHE
surface. Monzó et al. [20] proposed a more general numerical method to evaluate $g$-functions with
Eskilson’s boundary condition, valid for any BHE field. In this method, the BHEs are supposed to be
filled with a superconductive material that also interconnects them. The authors compared their
$g$-function for a $3 \times 2$ BHE field with that obtained in Ref. [15], and found a good agreement.

In this paper, we present an alternative numerical method to obtain the $g$-function of a BHE field
with Eskilson’s boundary condition, that we will call isothermal $g$-function. The method is based on a
recursive procedure that starts by imposing, as a first-trial isothermal boundary condition, the isoflux
$g$-function, namely the uniform time-dependent surface temperature determined by applying the usual
boundary condition of uniform and constant wall heat flux. The time averaged mean heat flux per unit
length obtained is then used to determine a correction factor that yields the first-trial isothermal
$g$-function. The latter can be used as isothermal boundary condition in another iteration. By employing
this method, we analyze the difference between isothermal and isoflux $g$-functions for a single BHE
and for a $3 \times 2$ BHE field, with the top of BHEs placed at a typical buried depth employed in Italy,
namely 1.5 m. Moreover, we show that the superposition of the effects of the single BHEs cannot be
applied to determine the isothermal $g$-function of a BHE field.
2. Numerical method
The g-functions were evaluated by accurate numerical simulations performed through the software COMSOL Multiphysics. Each borehole was schematized as a finite cylindrical source, buried at some distance from the ground surface. The ground was considered as a semi-infinite solid medium, with constant thermophysical properties. A single borehole and a 3 × 2 BHE field were considered, with the following geometric characteristics: BHE diameter $D = 15$ cm, BHE length $L = 150$ m, buried depth $B_d = 1.5$ m, spacing between adjoining BHEs in the 3 × 2 field $S = 7.5$ m.

The simulations of the single borehole were performed by employing the 2D axis-symmetric domain illustrated in figure 1. The selected domain is a cylinder coaxial with the borehole, with a radius equal to 2500 BHE diameters and a height equal to 2510 BHE diameters. The axis of the vertical coordinate, $z$, is directed downwards, with $z = 0$ at the BHE top.

![Figure 1](image1.jpg)

**Figure 1.** Computational domain and mesh for a single borehole (left) and zoom on the borehole top (right).

![Figure 2](image2.jpg)

**Figure 2.** Sketch of the 3 × 2 BHE field (left), and meshed computational domain (right).
Figure 2 illustrates a sketch of the $3 \times 2$ BHE field and the 3D computational domain employed for the simulations. Thanks to the double symmetry of the $3 \times 2$ field, only a quarter of the field was simulated, by adopting as computational domain a quarter of cylinder that contains one borehole and a half. The cylinder has radius 2500 BHE diameters and height 2510 BHE diameters; the $z$-axis is directed downwards, with $z = 0$ at the BHE top.

The differential equation of transient heat conduction is

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T,$$

where $T$ is temperature, $t$ is time and $\alpha$ is the ground thermal diffusivity.

Both in the computational domain of the single borehole and in that of the $3 \times 2$ BHE field, the outer vertical surface and the bottom horizontal surface of the domain were set as adiabatic. The upper horizontal surface (ground surface) was considered as isothermal, with a constant temperature equal to the undisturbed ground temperature, $T_g$. The BHEs top and bottom were set as adiabatic. At the vertical surface between BHEs and ground, either a uniform and time-constant heat flux, or a uniform and time-dependent temperature were imposed, to evaluate the isoflux or the isothermal $g$-function, respectively. At the initial time instant, a uniform temperature equal to $T_g$ was set in the whole domain.

The isoflux $g$-function, $g_{s\pi}$, was evaluated by imposing a uniform and time-constant heat flux per unit area, $q$, at the BHE-ground interface:

$$q = -k (\nabla T \cdot \mathbf{n})|_{\text{BHE-ground}} = \frac{q_i}{\pi D},$$

where $k$ is the ground thermal conductivity, $\mathbf{n}$ is the outward unit normal and $q_i$ is the linear heat flux applied at the BHE wall.

Results were reported in dimensionless form by employing the following dimensionless quantities

$$x^* = \frac{x}{D}, \quad y^* = \frac{y}{D}, \quad z^* = \frac{z}{D}, \quad r^* = \frac{r}{D}, \quad \nabla^* = D \nabla, \quad t^* = \frac{\alpha t}{D^2}, \quad T^* = k \frac{T - T_g}{q_i},$$

where $(x, y, z)$ are the coordinates of the domain for the $3 \times 2$ BHE field, $(r, z)$ are the coordinates of the domain for the single borehole, and $\nabla$ is the nabla vector differential operator. In dimensionless form, the boundary condition (2) becomes

$$q^* = - (\nabla T^* \cdot \mathbf{n})|_{\text{BHE-ground}} = 1/\pi,$$

and the corresponding dimensionless heat flux per unit length is

$$q^*_i = \pi q^* = 1.$$

The dimensionless size of the computational domain for the single BHE was $0 \leq r^* \leq 2500$, $-10 \leq z^* \leq 2500$, and that for the $3 \times 2$ field was $0 \leq x^* \leq 2500$, $0 \leq y^* \leq 2500$, $-10 \leq z^* \leq 2500$. The dimensionless geometric parameters of the boreholes were $L^* = L/D = 1000$, $B_d^* = B_d/D = 10$, $S^* = S/D = 50$ (for the $3 \times 2$ field).

An unstructured mesh with 440 109 triangular elements was employed for the simulation of the single BHE (see figure 1), and an unstructured mesh with 1 350 358 tetrahedral elements was adopted for the simulation of the $3 \times 2$ BHE field (see figure 2). In both cases, finer elements were employed in proximity of the boreholes and coarser elements towards the external boundaries of the domain.

The simulation-time interval was $10^6 \leq t^* \leq 10^8$, namely $-6 \leq \log_{10}(t^*) \leq 6$, divided in time steps of $\log_{10}(t^*) = 0.05$. With a borehole diameter of 15 cm and a ground diffusivity of $10^6$ m$^2$/s, the simulation-time interval is larger than 713 years.

Both for the single BHE and for the field, the simulation with uniform and time-constant heat flux per unit area at the BHEs surface yielded the isoflux $g$-function, $g_{s\pi}$, which was then used as
time-dependent isothermal boundary condition at the surface between the BHE field and the ground in the second simulation. The inverse of the time averaged mean dimensionless heat flux per unit length at the surface between the BHE field and the ground was used as a correction factor of \( g_{ni} \), to determine the first-trial isothermal \( g\)-function, \( g_{T1} \). Then, \( g_{T1} \) was employed as isothermal boundary condition at the surface of the boreholes in the third simulation, to determine a new mean dimensionless heat flux per unit length at the surface between BHEs and ground, which yielded the correction factor for \( g_{T1} \), and thus the second-trial isothermal \( g\)-function, \( g_{T2} \), and so on. The accuracy of the isothermal \( g\)-function obtained was finally checked by performing an additional simulation with this \( g\)-function as boundary condition, and comparing the mean time-dependent dimensionless heat flux per unit length at the BHEs surface to the desired value, 1.

In order to ensure that results are independent of the domain size, the simulation for the isoflux \( g\)-function of the single BHE and that for the isoflux \( g\)-function of the \( 3 \times 2 \) BHE field were repeated, by replacing the adiabatic boundary condition at the outer vertical surface and at the bottom horizontal surface of the domain with the isothermal boundary condition \( T^* = 0 \). The maximum relative error was 0.03 % for the single BHE, and 0.06 % for the \( 3 \times 2 \) field, both occurring at the last time instant.

In order to ensure that results are independent of the selected mesh, the simulation for the isoflux \( g\)-function with the adiabatic boundary condition was repeated by employing a different mesh. A mesh with 315 877 elements was used for the single BHE, and a mesh with 2 980 542 elements was used for the \( 3 \times 2 \) BHE field. In each case, the relative discrepancy between the results obtained by different meshes decreases with time. For the single borehole, it is equal to 0.94 % at \( \log_{10}(t^*) = 1 \) and to 0.32 % at the last time instant; for the \( 3 \times 2 \) BHE field, it is equal to 0.17 % at \( \log_{10}(t^*) = 1 \) and to 0.11 % at the last time instant.

3. Results

The evaluation of isothermal \( g\)-functions requires as boundary condition at the surface between BHE field and ground both a uniform temperature and a time-constant surface averaged dimensionless linear heat flux, \( \bar{q}_i^* \), equal to 1. By employing the recursive procedure described in Section 2, for each iteration a uniform time-dependent temperature at the surface of the BHE field was imposed as a boundary condition, and the corresponding time evolution of \( \bar{q}_i^* \) was determined.

![Figure 3](image-url)

**Figure 3.** Plots of \( \bar{q}_i^* \) versus dimensionless time, for the \( g\)-functions \( g_{T1}, g_{T2}, g_{T3} \), single BHE.

For the single borehole, figure 3 illustrates the time evolution of \( \bar{q}_i^* \) yielded by the isothermal \( g\)-functions obtained through the recursive procedure. In particular, \( \bar{q}_{n1}^* \) is the surface averaged
dimensionless linear heat flux produced by the first-trial isothermal \( g \)-function, \( q_{11} \), \( q_{12}^* \) is that produced by the second-trial isothermal \( g \)-function, \( q_{22} \), and \( q_{13}^* \) is that produced by the last isothermal \( g \)-function, \( q_{33} \). The figure shows the convergence of \( q_i \) to the constant value 1.

The values of the isothermal \( g \)-functions for the single BHE obtained by each iteration are reported in table 1, for selected time instants, together with the percent difference between the values of the second and third \( g \)-function. The maximum value of this percent difference, in the range \( 0 \leq \log_{10}(t^r) \leq 6 \), is 0.37 \%. The maximum absolute difference between the values of the last two \( g \)-functions is 0.0008 and occurs at \( \log_{10}(t^r) = 0.25 \). Even the difference between the first and the third \( g \)-function is rather small: the maximum relative difference, in the range \( 0 \leq \log_{10}(t^r) \leq 6 \), is 0.56 \% and occurs at \( \log_{10}(t^r) = 0 \). Consequently, even the first \( g \)-function, obtained by two simulations, is sufficiently accurate for technical purposes.

| \( \log_{10}(t^r) \) | \( g_{r1} \) | \( g_{r2} \) | \( g_{r3} \) | \( \frac{100 \times (g_{r2} - g_{r3})}{g_{r3}} \) |
|---------------------|------------|------------|------------|-------------------|
| 0                   | 0.1933     | 0.1951     | 0.1944     | 0.37              |
| 1                   | 0.3560     | 0.3564     | 0.3559     | 0.15              |
| 2                   | 0.5335     | 0.5335     | 0.5333     | 0.03              |
| 3                   | 0.7107     | 0.7105     | 0.7104     | 0.02              |
| 4                   | 0.8747     | 0.8739     | 0.8737     | 0.03              |
| 5                   | 0.9965     | 0.9947     | 0.9943     | 0.03              |
| 6                   | 1.0348     | 1.0334     | 1.0337     | -0.03             |

In the following, the isothermal \( g \)-function obtained by the last iteration will be called simply isothermal \( g \)-function.

A comparison between the isoflux and the isothermal \( g \)-function for the single BHE is illustrated in figure 4. For \( \log_{10}(t^r) < 3.5 \), the isothermal \( g \)-function, \( g_{r} \), is graphically indistinguishable from the isoflux \( g \)-function, \( g_{q} \). At the highest values of the dimensionless time, \( g_{q} \) is slightly higher than \( g_{r} \), and the maximum relative difference, which occurs at the last time instant, is 1.4 \%.

**Figure 4.** Isoflux \( g \)-function, \( g_{q} \), and isothermal \( g \)-function, \( g_{r} \), single BHE.
In figure 5, a comparison is made between the isoflux and the isothermal $g$-functions evaluated on cylindrical surfaces placed at different radial distances from the single-BHE axis. The isoflux and isothermal $g$-functions evaluated at 50, 100 and 400 diameters from the single-BHE axis are reported in the figure for the time interval $2 \leq \log_{10}(t^*) \leq 6$. For $\log_{10}(t^*) < 2$, all these $g$-functions are practically equal to 0. At low values of the dimensionless time, the isothermal $g$-functions are graphically indistinguishable from the isoflux $g$-functions evaluated at the same radial distance. At high values of the dimensionless time, the isothermal $g$-functions are slightly lower than the isoflux $g$-functions evaluated at the same radial distance. The absolute discrepancy between the isoflux and the isothermal $g$-function decreases with the distance from the BHE axis. In fact, the absolute difference between the two $g$-functions at the last time instant is 0.01 at a radial distance of 50 diameters (see black and red solid lines in figure 5), whereas it is equal to 0.003 at 400 diameters (see black and red dashed lines in the figure). The relative difference between the isoflux and the isothermal $g$-function is nearly independent of the distance from the BHE axis. At the last time instant it ranges from 3.15% (at 50 diameters) to 3.82% (at 400 diameters).

![Figure 5. Isoflux $g$-functions, $g_{\theta}$, and isothermal $g$-functions, $g_{\tau}$, at 50, 100 and 400 diameters from the single-BHE axis.](image)

The isoflux and isothermal $g$-functions were evaluated also for the $3 \times 2$ BHE field, by employing the recursive procedure described in Section 2. Five iterations were performed, and the convergence of the mean dimensionless linear heat flux on the BHE-field surface was checked.

Plots of the isoflux $g$-function, $g_{\theta}$, and of the isothermal $g$-functions $g_{\tau1}$, $g_{\tau2}$, $g_{\tau3}$, $g_{\tau4}$, $g_{\tau5}$ are illustrated in figure 6, for the time interval $0 \leq \log_{10}(t^*) \leq 6$.

For $\log_{10}(t^*) < 4$, the isothermal $g$-functions are graphically indistinguishable from each other, whereas slight differences are visible at the highest values of the dimensionless time, as highlighted by the zoom on the graph of figure 6, which refers to the time interval $5.7 \leq \log_{10}(t^*) \leq 6$. The maximum relative difference between the values of the last two isothermal $g$-functions is 0.057% and occurs at $\log_{10}(t^*) = 5.60$. Moreover, the maximum absolute difference between the values of the last two isothermal $g$-functions is 0.0013 and occurs at $\log_{10}(t^*) = 5.65$. The maximum relative difference between the first and the last isothermal $g$-function, occurring at $\log_{10}(t^*) = 5$, is 1.36%, which confirms that even the first isothermal $g$-function is sufficiently accurate for technical purposes.

From figure 6, one can note that the isoflux $g$-function, $g_{\theta}$, is graphically indistinguishable from the isothermal $g$-functions for $\log_{10}(t^*) < 3.5$. At the highest values of the dimensionless time, on the contrary, $g_{\theta}$ is significantly higher than $g_{\tau5}$, with a maximum relative difference of 8.67%, occurring at the last time instant. As a consequence, adopting an isoflux boundary condition at the BHE-field...
surface, instead of an isothermal boundary condition, yields a significant overestimation of the $g$-function of the field at high times. In the following, the isothermal $g$-function obtained by the last iteration will be called simply isothermal $g$-function and will be denoted by $g_T$.

![Figure 6](image)

**Figure 6.** Isoflux $g$-function, $g_q$, and isothermal $g$-functions $g_{T1}$, $g_{T2}$, $g_{T3}$, $g_{T4}$, $g_{T5}$, $3 \times 2$ BHE field.

The difference between $g_q$ and $g_T$ is because employing an isoflux boundary condition at the BHE-field interface yields a non-uniform temperature distribution at the BHE surface. Figure 7 illustrates the vertical distribution of the dimensionless surface temperature, averaged on the circumference of the central BHEs, obtained at selected time instants by employing the isoflux boundary condition. For $\log_{10}(t^*) = 0$, the temperature distribution is almost uniform. For $\log_{10}(t^*) = 3$, the temperature distribution is still uniform, except close to the BHE top ($z^* = 0$) and bottom ($z^* = 1000$), where the temperature decreases. For $\log_{10}(t^*) = 6$ (i.e. at the last time instant), the temperature distribution has an inverted-U profile, with values at the borehole top lower than those at the bottom.

![Figure 7](image)

**Figure 7.** Dimensionless surface temperature of the central BHEs, versus $z^*$, isoflux boundary condition, $3 \times 2$ BHE field.
To check the applicability of the superposition of the effects of single BHEs, this method was applied to evaluate both the isoflux g-function, \( g_{\text{qse}} \), and the isothermal g-function, \( g_{\text{Tse}} \), of the 3 \times 2 BHE field.

Figure 8 illustrates a comparison between \( g_{\text{qse}} \) and \( g_q \), and between \( g_{\text{Tse}} \) and \( g_T \). While \( g_{\text{qse}} \) is practically coincident with \( g_q \) (blue and green lines in figure 8 are overlying), \( g_{\text{Tse}} \) is sharply greater than \( g_T \) for high values of time (compare red and purple lines in the figure). The maximum relative difference between \( g_{\text{Tse}} \) and \( g_T \), which occurs at the last time instant, is 6.0 %.

The results confirm the applicability of the superposition of the effects of single BHEs to evaluate the isoflux g-function of a BHE field. On the opposite, employing the superposition of the effects of single BHEs to evaluate the isothermal g-function of the field yields a significant overestimation of the g-function at high values of time.

### Figure 8

Isoflux and isothermal g-functions by superposition of the effects \( (g_{\text{qse}}, g_{\text{Tse}}) \), and isoflux and isothermal g-functions by 3D simulations \( (g_q, g_T) \), 3 \times 2 BHE field.

4. Conclusions

We have presented a new numerical approach to determine the isothermal g-function of a borehole-heat-exchanger (BHE) field, i.e. the g-function yielded by the boundary condition of uniform temperature and time-constant mean heat flux at the surface between BHEs and ground. By this method, we have analyzed the difference between the isothermal g-function and the isoflux one, namely that yielded by the boundary condition of uniform and constant heat flux at the interface BHEs-ground. Moreover, we have checked the applicability of the superposition of the effects of single BHEs to determine the g-function of the field. The analysis has been performed for a single BHE and for a 3 \times 2 BHE field, for BHEs with length 150 m, diameter 15 cm, top placed at 1.5 m under the ground surface, and spacing from each other 7.5 m in the case of the field. The following results have been found.

The difference between the isoflux and the isothermal g-function is positive and becomes appreciable only for values of the dimensionless time defined here greater than \( 10^{3.5} \), i.e. about 2.3 years for BHEs with diameter 15 cm placed in a ground with thermal diffusivity \( 10^{-6} \text{ m}^2/\text{s} \). For a dimensionless time equal to \( 10^6 \), i.e. about 713 year in the conditions stated above, the difference is about 1.4 % for a single BHE, and about 8.7 % for the 3 \times 2 BHE field considered here.

The superposition of the effects of the single BHEs yields correctly the isoflux g-function of the field; on the contrary, it cannot be applied to determine the isothermal g-function of a BHE field.
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