A model-based method for rotor speed estimation of synchronous generators using wide area measurement system

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Abstract
In this paper a novel method is proposed to estimate the rotor speed of synchronous generators in steady state and dynamic conditions using phasor measurement unit's data. This method uses the synchronous generator model to find the relationship between generator rotor speed and frequencies at different buses of the power system. The principles of the proposed method are first presented through a simple network consisting a generator connected to a transmission line. Afterward the general formulation is developed. To demonstrate the effectiveness of our method, IEEE 9-bus and IEEE 39-bus power systems are used as the benchmark systems. Different tests are carried out under three conditions including, normal condition, presence of measurement errors and uncertainty of the power system parameters. As well, the estimation results are compared with the results of other estimation methods. The results show the high accuracy of the proposed method in different conditions.

1 | INTRODUCTION

ONLINE state estimation of synchronous generators is the fundamental part of power system analyses including power system monitoring [1], planning [2] and voltage and transient stability analyses [3–5]. More accurate estimation results in increasing the reliability of the power system and reducing the risk of cascading failures when a contingency occurs. In this regard, high sampling rate PMUs—60–120 samples/s—can be used for online state estimation purposes.

Apart from the methods that use direct measurements of the rotor angle and rotor speed via installing mechanical sensors on the generator [6, 7], due to the practical limitations, most of the presented methods use common measurements of PMU for the state estimation purposes [8–11]. Accordingly, it is required to consider a proper model for the synchronous generator to calculate state variables using the PMU's data.

Modelling a generator as a voltage source behind series impedance is a widely used approach to estimate the rotor angle [12–15]. Accordingly, the rotor angle is first estimated through a loop voltage equation. Afterward, the rotor speed is calculated via derivative of the rotor angle. In [16] based on the classical model of synchronous generators the $d$-axis transient reactance ($X_d^*$) is first estimated. Then other states and parameters are calculated using the least squares (LS) technique. Although this approach is useful to estimate the rotor angle, because of considerable error of estimation, it is not a reliable approach for analyses such as coherent generators determination, parameter estimation and transient stability prediction [17].

Another widely used state estimation method equates measured frequency at terminal bus of a generator with its rotor speed [18, 19]. Also, the rotor angle is estimated via integration of the rotor speed. Authors in [20] propose to estimate the angular speed of generators from the terminal voltage signal using the Improved Recursive Newton Type Algorithm (IRNTA). Then, based on the estimated rotor speed other states and parameters except for the inertia constant are calculated. Estimation using the terminal frequency is generally more accurate than the voltage source model; nevertheless, the estimation error is still notable. In [21], Chow proposed a method to estimate the states of generators in a radial power system. The proposed method calculates the frequency at middle of a transmission line based on the rotor speed of the equivalent generators at two sides of the line. This method is accurate but cannot be practically implemented because of its assumptions for power system configuration and PMU’s placement. In [22] the authors propose a linear relationship between the measured rotor speeds and the bus frequencies.
This relationship is used to estimate the frequencies at buses which are not equipped with PMU. This idea is extended in [23] to estimate the rotor speeds from the measured frequencies. Compatibility to different PMU’s placements and independency to network configuration, makes the method usable in practical applications. However, due to assuming a constant impedance model for the loads, it is required to update the bus susceptance matrix based on each incoming sample. Hence, estimating the rotor speed requires a large computational burden.

Beside the above-mentioned methods, some estimation techniques are proposed to improve accuracy of the estimations in presence of ambient noise and measurement errors. Kalman Filter (KF) is a well-known tool to estimate the states when the model parameters are uncertain or the noise level is considerable [24]. Due to non-linearity of the synchronous generator model, this algorithm is used in the form of Extended Kalman Filter (EKF) [25] or Unscented Kalman Filter (UKF) [26]. In [27] synchronous generator parameters including inertia constant, damping factor and mechanical input power are estimated using EKF algorithm. Similar algorithm is presented in [28] to estimate the generator states and parameters. More studies can be found in [29, 30].

The novel contribution of the paper is an approximated yet reliable and simple formula to estimate the rotor speed of generators using WAMS. The method presents a linear relationship between the rotor speeds and measured frequencies at different buses. The relationship is concluded from the synchronous generator model and a sinusoidal approximation which is proved here. As a result, it is proved that quadrature axis reactance ($X_q$) is an effective parameter in estimating the rotor speeds. As well, the method does not depend on power system configuration and is compatible with different placements of PMUs.

This paper is organized as follows. In Section 2, principles of the proposed method are explained. Section 3 extends the rotor speed estimation idea to a large power system with random distribution of PMUs. The simulation results of different rotor speed estimation methods beside the comparison results are presented in Section 4 and finally Section 5 summarizes the conclusions.

## 2 Principles of the Proposed Method

In this section, principles of the proposed method are presented. A new approximation for sinusoidal functions is first proved. Afterward, using the synchronous generator model described in Section 2, the rotor speed of a generator is estimated via measured frequencies at two buses of a simple network. General formulation for the rotor speed estimation is presented in Section 3.

### 2.1 Approximation for sinusoidal functions

In this part first, it is proved that within a short time interval the summation of multitude sinusoidal functions in which their frequencies are relatively similar, can be approximated by a sinusoidal function. Second, it is shown that there is a linear relationship between the amplitudes and frequencies of the approximated and the primary sinusoidal functions.

To prove the approximation, let us consider the summation of two sinusoidal functions as Equation (1):

$$A \cos(\omega t + \varphi) + B \cos(\omega' t + \varphi')$$

or:

$$A \cos(\omega t + \varphi) + B \cos((\omega + \Delta \omega) t + (\varphi + \Delta \varphi))$$

Where $A$ and $B$ are amplitudes of the cosine functions, $\omega$ and $\omega'$ are frequencies of them, $\varphi$ and $\varphi'$ denote phase angles of the functions and $\Delta \omega$ and $\Delta \varphi$ are differences between frequencies and phase angles of two functions respectively. Equation (1) could be rewritten as Equation (2) by replacing $(\omega t + \varphi)$ with $x$:

$$A \cos(x) + B \cos(x + \Delta x)$$

For small angles $\sin(x) \approx x$ and $\cos(x) \approx 1 - \frac{x^2}{2}$ can be approximated using the Taylor expansion as it is shown in Equation (3):

$$\cos(x) \approx 1 - \frac{x^2}{2}$$

Expanding the sine and the cosine functions in Equation (2) and replacing Equation (3) in Equation (2) yields Equation (4):

$$A \cos(x) + B \cos(x + \Delta x)$$

$$= A \left(1 - \frac{x^2}{2}\right) + B \left(1 - \frac{(x + \Delta x)^2}{2}\right)$$

$$= (A + B) \left[1 - \left(\frac{x^2}{2}\right)\right] - B.x.\Delta x - \frac{B}{2}(\Delta x)^2$$

Equation (4) could be expressed as Equation (5):

$$A + B \left[1 - \left(\frac{x^2}{2}\right)\right] - B \frac{x.\Delta x}{A + B} - \frac{B}{2(\Delta x)}(\Delta x)^2$$

For a small value of $\Delta x$, the expression $\frac{B}{2(\Delta x)}(\Delta x)^2$ is a negligible term. Therefore, we can replace this term with $\frac{B^2}{2(\Delta x)}(\Delta x)^2$. Accordingly, Equation (5) is expressed as Equation (6):

$$A + B \left[1 - \left(\frac{x^2}{2}\right)\right] - B \frac{x.\Delta x}{A + B} - \frac{B^2}{2(\Delta x)^2}(\Delta x)^2$$

$$= (A + B) \left[1 - \frac{1}{2}\left(x + \frac{B}{A + B}\Delta x\right)^2\right]$$

$$= (A + B) \left[1 - \frac{1}{2}\left(x + \frac{B}{A + B}\Delta x\right)^2\right]$$
Equation (6) could be considered as the Taylor expansion of a cosine function shown in Equation (7):

\[(A + B) \cos \left( x + \frac{B}{A + B} \Delta x \right) \quad (7)\]

Thus we can say that the summation of two sinusoidal functions shown in Equation (2) is approximately a sinusoidal function shown in Equation (7). As well, by replacing \(x\) with \((\omega t + \phi)\) a linear relationship between the frequencies of two primary functions and the resultant one is achieved:

\[
\begin{align*}
A \cos(\omega_1 t + \varphi_1) + B \cos(\omega_2 t + \varphi_2) \\
\approx (A + B) \cos(\omega_3 t + \varphi_3) \quad (8)
\end{align*}
\]

where: \(\omega_3 = \frac{A \omega_1 + B \omega_2}{A + B}\), \(\varphi_3 = \frac{A \varphi_1 + B \varphi_2}{A + B}\)

In addition, Equation (8) could be extended for more than two sinusoidal functions. The extended relation for three sinusoidal functions is proved in Equation (9):

\[
\begin{align*}
A \cos(\omega_1 t + \varphi_1) + B \cos(\omega_2 t + \varphi_2) + C \cos(\omega_3 t + \varphi_3) \\
= (A + B + C) \cos \left( \frac{A \omega_1 + B \omega_2 + C \omega_3}{A + B + C} t + \varphi_3 \right) \\
\varphi_3 = \frac{A \varphi_1 + B \varphi_2 + C \varphi_3}{A + B + C} \quad (9)
\end{align*}
\]

Finally, the approximation for \(N\) sinusoidal functions is presented in Equation (10):

\[
\sum_{k=1}^{N} A_k \cos(\omega_k t + \varphi_k) \approx \left( \sum_{k=1}^{N} A_k \right) \cos \left( \sum_{k=1}^{N} \frac{A_k \omega_k}{\sum_{k=1}^{N} A_k} t + \psi \right) \quad (10)
\]

where: \(\psi = \frac{\sum_{k=1}^{N} A_k \varphi_k}{\sum_{k=1}^{N} A_k}\)

Here, to evaluate the accuracy of the presented approximation, summation of three random sinusoidal functions including \(\cos(\omega + 0.1)\), \(1.2\cos(1.2 \omega + 0.2)\) and \(1.1\cos(1.4 \omega + 0.5)\) is compared with the approximated function results from Equation (10). This comparison is depicted in Figure 1.

Accordingly, Equation (10) is an acceptable approximation for sum of sinusoidal functions.

### 2.2 Rotor speed estimation algorithm

Consider part of a power system consisting of a generator connected to a transmission line as shown in Figure 2. In this study, the line resistance is negligible that is in accordance with the low \(R/X\) ratio in real power systems. Here \(v_1(t)\) and \(v_2(t)\) are phase voltages at both sides of the transmission line, \(i_\delta(t)\) denotes phase current flows over the line, \(L\) denotes inductance of the line and \((E < \delta)\) is internal voltage of the generator.

The transmission line phase current is calculated in Equation (11):

\[
i(t) = \int \frac{v_1(t) - v_2(t)}{L} dt \quad (11)
\]
Equation (11) could be rewritten as follows:

\[
i(t) = \int \frac{V_{M1} \cos(\theta_1) - V_{M2} \cos(\theta_2)}{L} \, dt
\]  

Where \(\theta_i = \omega_i t + \phi_i\), that \(\omega_1\) and \(\omega_2\) are frequencies at both sides of the transmission line and \(\phi_1\) and \(\phi_2\) represent the phase angles. Also \(V_{M1}\) and \(V_{M2}\) denote voltage magnitudes at both sides of the line.

Thus:

\[
i(t) = \frac{V_{M1}}{\omega_1 L} \sin(\theta_1) - \frac{V_{M2}}{\omega_2 L} \sin(\theta_2)
\]  

In this study it is assumed that, the frequency at two sides of the transmission line might be different. Replacing the Equation (13) in the synchronous generator equation described in Appendix, gives Equation (14):

\[
v_a(t) = E \cos(\delta_t - \frac{\pi}{2}) - \left(1 + \frac{L_d + L_q}{2L}\right) V_{M1} \cos(\theta_1) + \left(1 + \frac{L_d + L_q}{2L}\right) V_{M2} \cos(\theta_2)
\]

In following the sine form of \(v_a(t)\) is replaced in Equation (14) which yield Equation (15):

\[
\left(1 + \frac{L_d + L_q}{2L}\right) V_{M1} \cos(\delta_t) = E \cos(\delta_t - \frac{\pi}{2}) + \left(1 + \frac{L_d + L_q}{2L}\right) V_{M2} \cos(\theta_2)
\]

Due to the small difference between \(\theta_1\) and \(\theta_2\) as well as, voltage magnitudes at both sides of the transmission line in real power systems, the sum of the four last terms in Equation (15) is almost zero. By removing these terms, Equation (15) is expressed as Equation (16):

\[
E \cos(\delta_t - \frac{\pi}{2}) \\
\approx \left(1 + \frac{L_d + L_q}{2L}\right) V_{M1} \cos(\theta_1) - \left(1 + \frac{L_d + L_q}{2L}\right) V_{M2} \cos(\theta_2)
\]

Using Equation (16), besides the approximation introduced in part A, the relationship between rotor angle frequency \(\omega_m\) and electrical frequencies at both sides of the transmission line \((\omega_1\) and \(\omega_2\)) is obtained in Equation (17).

\[
\omega_m \approx \gamma_1 \omega_1 + \gamma_2 \omega_2
\]

where:

\[
\gamma_1 = \left(1 + \frac{L_d + L_q}{2L}\right) V_{M1}
\]

\[
\gamma_2 = -\left(1 + \frac{L_d + L_q}{2L}\right) V_{M2}
\]

Thus, the angular speed can be estimated using measured frequencies at buses 1 and 2.

The variables \(\gamma_1\) and \(\gamma_2\) could be calculated more easily by multiplying angular speed to the numerator and denominator of Equations (18) and (19). The new equations are:

\[
\gamma_1 = \left(1 + \frac{X_d + X_q}{2X_L}\right) V_{M1}
\]

\[
\gamma_2 = -\left(1 + \frac{X_d + X_q}{2X_L}\right) V_{M2}
\]

Where \(X_d\) and \(X_q\) represent the \(d\) and \(q\)-axes reactances of the generator.

Accordingly, when \(X_d\) and \(X_q\) are relatively similar, \(\gamma_1\) and \(\gamma_2\) could be calculated using Equations (22) and (23):

\[
\gamma_1 = \left(1 + \frac{X_d}{X_L}\right) V_{M1}
\]

\[
\gamma_2 = -\left(\frac{X_q}{X_L}\right) V_{M2}
\]
However, using Equations (22) and (23) instead of Equations (20) and (21) might cause a meaningful error when there is a considerable difference between \(d\) and \(q\) reactances.

Finally, it must be noted that in transient duration, steady state reactances must be replaced with the transient reactances \((x_d' , x_q')\).

3 | GENERAL FORMULATION OF THE PROPOSED METHOD

In this section, general formulation of the proposed method is presented. The start point is using the relationship between voltages and currents in power systems:

\[
I = Y_{\text{bus}} \cdot V \tag{24}
\]

Where, \(I\) and \(V\) are current injection and bus voltage matrices respectively. As well, \(Y_{\text{bus}}\) is the admittance matrix of the network. Equation (24) could be expressed as Equation (25):

\[
\begin{bmatrix}
I_{\text{WPMU}} \\
I_{\text{WOPMU}}
\end{bmatrix} =
\begin{bmatrix}
J_{11} & J_{12} \\
J_{21} & J_{22}
\end{bmatrix}
\begin{bmatrix}
V_{\text{WPMU}} \\
V_{\text{WOPMU}}
\end{bmatrix} \tag{25}
\]

Where, \(I_{\text{WPMU}}\) and \(V_{\text{WPMU}}\) stand for current and voltage of the buses equipped with PMU, and \(I_{\text{WOPMU}}\) and \(V_{\text{WOPMU}}\) represent current and voltage of the buses which PMU is not installed. In this study the electrical loads at the buses which are not equipped with PMU are modelled as constant admittance and all generator buses are equipped with PMU. Thus, buses which are not equipped with PMU are zero-injection. Accordingly, \(I_{\text{WOPMU}}\) in Equation (25) equals to zero. Therefore, Equation (25) could be rewritten based on direct relationship between current and voltage of the buses equipped with PMU:

\[
I_{\text{WPMU}} = J_{11} - J_{12}(J_{22}^{-1})J_{21}V_{\text{WPMU}}
\]

\[
= Y_{\text{bus}}^{\text{New}} V_{\text{WPMU}} \tag{26}
\]

In Equation (26) \(Y_{\text{bus}}^{\text{New}}\) is the new admittance matrix that links the voltages and currents of the buses equipped with PMU. Due to low \(R/X\) ratio in real power systems, Equation (26) could be expressed as:

\[
I_{\text{WPMU}} = B_{\text{bus}}^{\text{New}} V_{\text{WPMU}} \tag{27}
\]

According to Equation (27), the time-domain relation for injected current at bus “\(k\)” is:

\[
i_k(t) = B_{k1} V_1 \sin(\delta_1) + \cdots + B_{kn} V_n \sin(\delta_n) \tag{28}
\]

In Equation (28), “\(n\)” is number of PMUs in the power system and \(B_{ij}\) denotes the \([i,j]^{\text{th}}\) element of \(B_{\text{bus}}^{\text{New}}\) matrix.

Replacing the current calculated through Equation (28) in the synchronous generator equation, results in:

\[
v_k(t) = E \cos \left(\delta_k - \frac{\pi}{2}\right) - \left\{\left(\frac{L_d + L_q}{2}\right) (\omega_1 V_{k1} B_{k1} \cos \theta_1 + \omega_2 V_{k2} B_{k2} \cos \theta_2 + \cdots + \omega_k V_{k\delta} B_{k\delta} \cos \theta_k + \cdots + \omega_n V_{n\delta} B_{n\delta} \cos \theta_n) + \frac{3}{2} \frac{d}{dt}(i_k \cos \delta_k(t)) \right\} \tag{29}
\]

Where \(v_k(t)\) and \(i_k(t)\) are voltage and injected current at terminal bus of generator “\(k\)”. Through a slight approximation, Equation (29) is rewritten as:

\[
v_k(t) = E \cos \left(\delta_k - \frac{\pi}{2}\right) - \left(\frac{X_d + X_q}{2}\right) (V_{k1} B_{k1} \cos \theta_1 + V_{k2} B_{k2} \cos \theta_2 + \cdots + V_{k\delta} B_{k\delta} \cos \theta_k + \cdots + V_{n\delta} B_{n\delta} \cos \theta_n) \tag{30}
\]

Using Equations (10) and (30), the relationship between the generator “\(k\)” rotor speed and frequencies at different buses is calculated as:

\[
\omega_{mk} \approx \frac{\gamma_{k1} \omega_1 + \cdots + \gamma_{k\delta} \omega_n}{\gamma_{k1} + \cdots + \gamma_{k\delta}} \tag{31}
\]

where \(\omega_i\) is the measured frequency at bus “\(i\)” and \(\omega_{mk}\) is the rotor speed of generator “\(k\)” in Equation (31), \(\gamma_ks\) are calculated as:

\[
\gamma_{ki} = -\frac{X_{kd} + X_{qd}}{2} V_i |B_{ki}| \tag{32}
\]

\[
\gamma_{kk} = \left(1 + \frac{X_{kd} + X_{qd}}{2}\right) V_k |B_{kk}| \tag{33}
\]

According to Equations (32) and (33), due to dependency of \(\gamma_{ki}\) values to the bus voltages, these coefficients must be updated with new incoming samples.

The expression above defines a simple and relatively precise relation between rotor speeds and measured frequencies at different points of the power system.

4 | SIMULATION RESULTS

To evaluate performance of the proposed estimation method, the method is simulated in IEEE 9-Bus and IEEE 39-bus power systems. As well, to demonstrate the superiority of the method, the estimation results are compared with the results obtained from the terminal frequency approach and the
Frequency Divider (FD) method which is presented in [22]. Different scenarios are simulated to assess the performance of the methods in different conditions. In these studies, the power systems are simulated using DigSilent PowerFactory ver. 15.1 and the algorithms are implemented in MATLAB software. Here the calculated phasors in the simulator are used as the PMUs data. Also, the simulations were executed on a 32-bit Windows 7 operating system running on a 4 core 2.50 GHz Intel with 4 GB of RAM.

4.1 IEEE 9-bus power system

In the first study the methods are simulated in IEEE 9-Bus power system. The system is shown in Figure 3. All generators are equipped with AVR system and it is assumed that buses 1, 2, 3 and 8 are equipped with PMU. Also it is assumed that the transfer rate of the PMUs are 100 Hz.

The first study evaluates the accuracy of different methods in a normal condition. In following, impact of measurement errors and parameter uncertainties on accuracy of the estimations are investigated.

(1) Normal condition: In the first study it is assumed that the generator reactances are available and there is no error in PMUs’ data. To create a dynamic condition, an LLLG fault is applied between buses 5–7 at \( t = 5 \) s and is cleared by disconnecting the corresponding line at \( t = 5.15 \) s.

Here, measured voltages and frequencies are used to estimate the rotor speeds for three estimation methods including the conventional terminal frequency method, FD method and the proposed method. Figures 4–6 show the real and estimated values of angular speeds of the generators 1, 2 and 3 respectively.

The simulation results show the high accuracy of the proposed method in comparison with the terminal frequency approach and FD estimation method. To show the effectiveness of the proposed method, the average and maximum estimation errors during a 5 s study is calculated and the results are summarized in Table 1.

The results shown in Table 1 clearly demonstrate the high accuracy of the proposed method in comparison with two other rotor speed estimation methods. The average estimation error
TABLE 1  Average and maximum estimation errors

| Method       | Av. estimation error [%] | Max. estimation error [%] |
|--------------|--------------------------|---------------------------|
| SG1          | 0.0213                   | 0.1094                    |
| Terminal freq |                          |                           |
| FD           | 0.0370                   | 0.2168                    |
| Proposed method | 0.0056            | 0.0967                    |
| SG2          | 0.0221                   | 0.1898                    |
| Terminal freq |                          |                           |
| FD           | 0.0267                   | 0.1809                    |
| Proposed method | 0.0044            | 0.0641                    |
| SG3          | 0.0353                   | 0.2032                    |
| Terminal freq |                          |                           |
| FD           | 0.0241                   | 0.1773                    |
| Proposed method | 0.0091            | 0.0810                    |

FIGURE 7  Average error of rotor speed estimation based on TVE values

of the proposed method is considerably less than the ones resulted from the frequency approach and the FD methods.

(2) Measurement error impact: To evaluate the effectiveness of different rotor speed estimation methods in actual conditions, the estimations are performed after considering measurement errors on PMUs’ data. In this study, the result of the estimation error for each TVE value is an average of 50 tests that in each one measurement error randomly varies from zero to the TVE value. Figure 7 shows the estimation errors of different methods.

According to the Figure 7 results, the change of estimation error in the proposed method is more sensitive to the change of the measurement errors. However, the estimation error of the proposed method is still considerably less than the errors of two other methods. As shown in the figure, the estimation error is less than 0.01% in condition in which the TVE is 1% that is the maximum allowed TVE according to IEEE C37. 118. 1 Standard [31].

(3) Parameter uncertainty: In the third study the sensitivity of estimation accuracy of the three methods to the uncertainty of power system parameters is investigated. In this regard, it is assumed that the susceptance matrix is calculated with 10% error. As well, the generator reactances are estimated with 15% error. The fault type and other operational conditions are as same as the ones in the previous study.

Here, the average and maximum estimation errors are calculated and the results are summarized in Table 2.

Table 2 results show that when the power system parameters are estimated with error, accuracy of the estimations is reduced. However, the proposed method is still more accurate than the two other methods.

4.2  |  IEEE 39-bus power system

The IEEE 39-bus power system is the second test system which is used to evaluate performance of the estimation methods. The details of the system can be found in [32].

In this assessment the estimation methods are compared in terms of accuracy and computational burden. At first, it is assumed that all generator and load buses are equipped with PMUs and an LLLG fault is applied between buses 28 and 29 at $t = 21$ s. The fault is cleared after 100 ms by disconnecting the corresponding line.

Four random generators including G1, G7, G9 and G10 are selected to evaluate the performance of the methods. Among the selected generators, the G1 and G7 are far from the fault location and G9 and G10 are near it. The estimation results for three estimation methods are depicted in Figure 8.

The simulation results shown in Figure 8 clearly demonstrate the high accuracy of the proposed method. As well, it is concluded that the frequency approach is more accurate than the FD method in this power system. In Table 3 more estimation results of different methods are presented. According to the results, the proposed method could estimate the rotor speed with less than 0.025% error that is considerably lower than the estimation error of the terminal frequency (0.17%) and FD (0.24%) methods.
The next study compares the estimation time of the methods, based on the number of required operations. It is assumed that, the equivalent admittances of the loads are calculated once at the beginning of the calculation and does not update during the study.

Since the number of required operations for the proposed method depends on the number of installed PMUs, two different PMU’s placements are considered here. In the first placement, it is assumed that all generators and loads (totally 27 buses) were equipped with PMU and in the second one only the generator buses were equipped with PMU. The number of mathematical operations and the required time to estimate each rotor speed value is calculated in Tables 4 and 5. Here, the CPU speed is 10⁹ opr/sec.

According to the results of Tables 4 and 5 the computing time of the proposed method is larger than two other estimation methods. However, this time is small enough that the proposed method can be used in different applications such as transient stability studies, state estimation methods and etc.
### 5 | CONCLUSION

This paper proposed a new method for estimation of rotor speed of generators using WAMS. In this regard, we formulated a linear relationship between generators’ rotor speed and measured frequencies at different buses of the power system. This formulation is obtained from the synchronous generator model and network equations. Also, we presented a new approximation for summation of sinusoidal functions. This approximation was used to linearize the generator equations. The minimum requirement of using the proposed algorithm is installing PMU at the generator buses. Therefore, it is compatible with different PMU placements. To evaluate performance of the method, it was simulated in IEEE 9-bus and IEEE 39-bus power systems. As well, the estimation results were compared with the results of the terminal frequency approach and FD method. The results showed that the proposed method could estimate the rotor speed with less than 0.006% and 0.025% errors in IEEE-9Bus and IEEE-39Bus power systems respectively. Also, the high accuracy of the proposed method was demonstrated in the presence of the measurement error and uncertainty of the power system parameters. Finally, computational burden of the method was investigated.

### CONFLICT OF INTEREST

The authors declare no conflict of interest.

### DATA AVAILABILITY STATEMENT

Data openly available in a public repository that issues datasets with DOIs

### AUTHOR CONTRIBUTIONS

H.H.: Writing-original draft; Writing-review and editing. S.A.: Supervision

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APPENDIX A: SYNCHRONOUS GENERATOR MODEL

The electrical differential equations in Equations (A1) and (A2), as well as rotor dynamic Equations (A3) and (A4), of the synchronous generator full order model, are presented below [33]:

\[
\begin{bmatrix}
\dot{v}_a \\
\dot{v}_b \\
\dot{v}_c \\
\end{bmatrix} =
\begin{bmatrix}
-r_a & 0 & 0 \\
0 & -r_b & 0 \\
0 & 0 & -r_c \\
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b \\
i_c \\
\end{bmatrix} +
\begin{bmatrix}
\phi_a \\
\phi_b \\
\phi_c \\
\end{bmatrix} \tag{A1}
\]

\[
\begin{bmatrix}
\phi_a \\
\phi_b \\
\phi_c \\
\end{bmatrix} =
\begin{bmatrix}
I_{sia} & I_{sib} & I_{sic} & I_{siaD} & I_{sibQ} \\
I_{sia} & I_{sib} & I_{sic} & I_{siaD} & I_{sibQ} \\
I_{sia} & I_{sib} & I_{sic} & I_{siaD} & I_{sibQ} \\
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b \\
i_c \\
\end{bmatrix} \tag{A2}
\]

\[
2H\dot{\omega}_r + D\omega_r = (T_m - T_e) \tag{A3}
\]

\[
\dot{\delta} = \omega_s(\omega_r - \omega_s) \tag{A4}
\]

In Equations (A1)–(A4) \(v_{a,b,c}\) and \(i_{a,b,c}\) are phase voltages and currents at the generator terminal bus, \(\phi\) is the linkage flux of phase “A”. \(i_D\) and \(i_Q\) represent the damper currents in d–q axes. \(L_{aq}\) is the self-inductance of phase “A”, and \(L_{aj}\) is the mutual inductance between phases “A” and “J”. Also, \(\delta\) is the load angle, \(\omega_s\) is the base value of the rotor speed, \(\omega_r\) is the synchronous speed, \(\omega_s\) is the rotor speed, \(T_m\) and \(T_e\) are the mechanical and electrical torque of the generator respectively. As well, H and D denote the inertia constant and damping factor of the generator respectively. The self-inductance and mutual-inductance of phase “a” are defined in Equations (A5)–(A8):

\[
\begin{align*}
I_{sia} &= L_s + L_{mq}\cos 2\delta, \tag{A5} \\
I_{sib} &= -M_s + L_{mq}\cos(2\delta - 120), \tag{A6} \\
I_{sic} &= -M_s + L_{mq}\cos(2\delta + 120), \tag{A7} \\
I_{siaD} &= M_f \cos(\delta) \tag{A8}
\end{align*}
\]

In these equations \(\delta\) is the rotor angle of the generator in respect to a-phase axis of the stator and \(L_s, L_m, M_s\) and \(M_f\) are constant parameters.

After the sub-transient period, the damper currents could be neglected. Neglecting stator resistance and replacing the fluxes with current equations, gives the Equation (A9):

\[
v_a(t) = \frac{d}{dt}\{ -I_{siaD}i_a - I_{sibD}i_b - I_{sicD}i_c + I_{siaQ}i_{aQ} + I_{sibQ}i_{bQ} \} \tag{A9}
\]

Replacing generator reactances in Equation (A9) results in Equation (A10):

\[
v_a(t) = \frac{d}{dt}\{ -I_{siaD}i_a - I_{sibD}i_b - I_{sicD}i_c + I_{siaQ}i_{aQ} + I_{sibQ}i_{bQ} \} \tag{A10}
\]

Equation (A10) can be rewritten as Equation (A11) in symmetrical condition:

\[
v_a(t) = E\cos(\delta - \frac{\pi}{2}) - \frac{d}{dt}\{ (L_s + M_f)\frac{i_a}{2} + \frac{3}{2}i_c \cos\delta \} \tag{A11}
\]

In Equation (A11) “\(E\cos(\delta - \frac{\pi}{2})\)” is the internal voltage of the generator and is used instead of \(\frac{d}{dt}M_fi_c\cos\delta\). Equation (A11) could be rewritten as Equation (A12) by replacing the d–q reactances instead of \(L_s + M_f\). The new equation is:

\[
v_a(t) = E\cos(\delta - \frac{\pi}{2}) - \frac{d}{dt}\{ \left( \frac{L_s + L_{mq}}{2} \right) i_a + \frac{3}{2}i_c \cos\delta \} \tag{A12}
\]

where \(L_d\) and \(L_{mq}\) denote the d– and q-axes inductances of the generator respectively.

Equation (A12) defines a direct relationship between voltage and current of phase “a”. Similar equations could be written for phases “b” and “c”.

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