Direct effects of the resonant magnetic perturbation on turbulent transport

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Abstract
The effects of the resonant magnetic perturbations (RMPs) on the turbulent transport are analyzed in the framework of the test particle approach using a semi-analytical method. The model includes particle collisions. The influence of the RMPs on plasma confinement is determined as function turbulence parameters and of collisionality. A synergy of the turbulent transport and RMPs is found. The increase of the turbulent diffusion is much larger than the diffusion directly produced by the RMPs.

Keywords: turbulent transport, resonant magnetic perturbations, test particle approach

(Some figures may appear in colour only in the online journal)
2. The model

We consider a homogeneous and stationary turbulence represented by a stochastic potential $\phi(x, z, t)$, where $x = (x, y)$ are the coordinates in the plane perpendicular to the confining magnetic field $B_e$, (with $x$ in the radial and $y$ in the poloidal directions) and $z$ is the parallel coordinate. The trajectories of the guiding centers are solutions of

$$\frac{dx}{dt} = -\nabla \phi(x, y, z, t) + \frac{B(x, z)}{B} \eta_y(t) + \eta_z(t),$$

(1)

$$\frac{dz}{dt} = \eta_y(t),$$

(2)

where the first term is the $\mathbf{E} \times \mathbf{B}$ drift, the second term is the velocity determined by the motion along the perturbed magnetic field, $B$ is the confining magnetic field, $b(x, z)$ is the perpendicular magnetic field produced by RMP coils, $\eta_y(t)$ is the velocity determined by the motion along the perturbed magnetic field, $B$. The collisional velocities are normalized with the ion Larmor radius $\rho_i$ in the perpendicular plane and of the order of plasma major radius $R$ along the magnetic field. This leads, as shown below, to several important simplifications.

The EC of $\phi$, defined as $E(x, z, t) \equiv \langle \phi(0, 0, 0) \phi(x, z, t) \rangle_{\phi}$, is an input function in test particle studies. It is modeled, as in [12], using the results obtained in the numerical simulations for the ion temperature gradient (ITG) driven turbulence [15–17] or for the trapped electron modes (TEM) [18]. The modes with zero poloidal wave number $k_y = 0$ are stable for both types of turbulence, which leads to a special shape of the EC that has the poloidal integral equal to zero

$$E(x, z, t) = \Phi^2 \exp \left( -\frac{x^2}{2\lambda_x^2} - \frac{z^2}{2\lambda_z^2} - \frac{t}{\tau_c} \right) \partial_x \left[ \exp \left( -\frac{y^2}{2\lambda_y^2} \right) \right].$$

(3)

where $\Phi$ is the amplitude of the potential fluctuations, $\lambda_x, \lambda_z, \lambda_y$ are the correlation lengths along the three directions and $\tau_c$ is the correlation time. The derivative $\partial_x \equiv \partial / \partial y$ ensures that the poloidal integral is zero. The poloidal drift of the potential with the effective diamagnetic velocity $V_d$ is represented by $y' = y - V_d t$ in equation (3). The components of the $E \times B$ stochastic velocity $v_i = -\epsilon_{ij} \partial_j \phi(x, z, t)$ are obtained from the EC of the potential

$$E_i(x, z, t) = \left\langle \nu_i(0, 0, 0) \phi(x, z, t) \right\rangle_{\phi} = -\epsilon_{ik} \partial_k \phi(x, z, t),$$

(4)

where $\epsilon_{12} = 1, \epsilon_{21} = -1, \epsilon_{ij} = 0, \partial_1 = \partial / \partial x$, and $\partial_2 = \partial / \partial y$.

The space variation of the confining magnetic field is included in the model through a small gradient in the radial direction, $\nabla B \approx -B_0 / R e_z$. It can be approximated at the turbulent scale by

$$B(x) = B_0 \exp \left( -\frac{x}{R} \right) \approx B_0 \left( 1 - \frac{x}{R} \right).$$

(5)

The perturbation of the magnetic field $b$ is in the radial direction $b = b(x, z) e_r$. The other components can be neglected since they are smaller than the radial field and they have little influence on the radial transport. The magnetic field is represented by a stochastic function due to the fluctuating components (intrinsic topological noise and error fields [1]) that add to the deterministic component produces by the RMP coils. Moreover, the evaluation of the transport coefficients implies statistical averages that essentially are space averages over the initial conditions of the trajectories. Different values of $b$ are found along different trajectories, which enables the description of the RMP field seen by the particles as a stochastic field. The Eulerian correlation of this field is essentially determined by the configuration of the RMP coils. The correlation lengths of $b$ are at the macroscopic scale. They are much larger than the correlation lengths of the turbulence in the radial and poloidal directions $\lambda_r, \lambda_p$, which are of the order of $\rho_i$. Consequently, it is not necessary to consider the dependence of $b$ on $x$ in the study of the effects of the RMPs on turbulent transport. Only the $z$ dependence of $b$ is relevant since the parallel correlation lengths of $b$ and $\phi$ are comparable. It accounts for the non-axisymmetric structure of $b$.

Finally, the magnetic perturbation is approximated by a radial stochastic field that depends on $z (b = b(z) e_r)$ with the EC modeled by

$$C(z) \equiv \langle b(0) b(z) \rangle_b = \beta^2 \exp \left( -\frac{z^2}{2\lambda_z^2} \right),$$

(6)

where $\beta$ is the amplitude of the RMPs and $\Lambda_z$ is the correlation length. These are input parameters of the model.

The EC of the collisional velocity $\eta_y(t)$ is

$$C_\eta(t) = \left\langle \eta_y(0) \eta_y(t) \right\rangle_{\eta_y} = \lambda_i \nu \exp(-\nu |t|),$$

(7)

where $\nu$ is the frequency of collisions, $\lambda_i = \lambda_{\text{mp}} / \nu$ is the parallel diffusivity, $\lambda_{\text{mp}} = v_T / m_i$ is the mean free path, $v_T = \sqrt{T_i / m_i}$ is the thermal velocity of the ions, $m_i$ is ion mass and $\lambda_i$ is the statistical average. The ECs of the perpendicular collisional velocities $\eta_x(t), i = x, y$ are

$$C_i(t) = \left\langle \eta_x(0) \eta_x(t) \right\rangle_{\eta_i} = \lambda_i \nu \exp(-\nu |t|),$$

(8)

where $\lambda_i = \rho_i^2 \nu$.

Dimensionless variables are introduced. The perpendicular displacements, the correlation lengths $\lambda_x, \lambda_y$, and the gradient length $R$ are normalized with the ion Larmor radius $\rho_i$. The parallel displacements $z, \lambda_z$ are divided by $L_T$, the gradient length of the ion temperature $T_i$. The unit of time and of $\tau_c$ is $\tau_0 = L_T / v_T$. The collisional velocities are normalized with the amplitudes $\sqrt{\lambda_i^x \nu} = v_T$ and $\sqrt{\lambda_i^y \nu} = \rho_i \nu$. The units for the potential and for the magnetic field are $\Phi$ and $\beta$, respectively. Using the same symbols for the dimensionless variables, the equations of motion in the reference system that moves with the potential are

$$\frac{dx}{dr} = -P_0 \exp \left( \frac{x}{R} \right) \partial_x \phi(x, y, z, t) + P_0 \exp \left( \frac{y}{R} \right) b(z) \eta_x(t) + P_0 \eta_x(t)$$

(9)
\[ \frac{dy}{dt} = P_0 \exp\left(\frac{x}{R}\right) \partial_x \phi(x, y, z, t) + \nabla_x^2 \phi(t) + P_2 \eta(t) \]  

(10)

where \( \nabla_x^2 \phi \) is the diamagnetic velocity and \( R = R/\rho_i \).

All the stochastic fields in equations (9)–(11) have the amplitudes equal to one. The correlation function of the normalized collisional velocity is the same for all components \( i = x, y, z \).

Thus, this model describes the direct effect of the RMPs on turbulent transport for a stochastic potential that has the general characteristics of the ITG or TEM turbulence, taking into account particle collisions. The results apply to L-mode, since the model does not include plasma sheared rotation. The changes of the turbulent transport coefficients induced by the RMPs are determined as function of the twelve physical quantities defined in the ECs of the four stochastic functions, which are input parameters of the model. The effects of the RMPs on the turbulence are not analyzed here. The aim is to find how much is the confinement affected for different characteristics of the turbulent plasmas.

### 3. The semi-analytical solution

The change of variable from \( x \) to \( x' = x - x_b(t) \epsilon_x - \xi(t) \), where \( x_b(t) \) is the displacement produced by the RMPs and \( \xi(t) \) is the perpendicular collisional displacement, permits to define an effective velocity that includes the four stochastic functions

\[ v_{\text{eff}}^i(x', t) = -\varepsilon_i \exp\left(\frac{x' + x_b(t) + \xi(t)}{R}\right) \times \partial_x \phi(x' + x_b(t) \epsilon_x + \xi(t), z(t), t). \]  

(16)

The equation of motion in this frame is

\[ \frac{dx'}{dt} = P_0 v_{\text{eff}}(x', t) + V_d \epsilon_y. \]  

(17)

The transport formally appears in equation (17) as produced by a single stochastic velocity. This is an important simplification, which is effective if the EC of \( v_{\text{eff}}(x, t) \) can be estimated. The latter is defined as the average over all of the stochastic processes

\[ E_{\text{eff}}^i(x', t) = \left\langle \left\langle v_{\text{eff}}^i(0, 0) v_{\text{eff}}^i(x, t) \right\rangle_{\phi} \right\rangle_{\theta} \right\rangle_b. \]  

(18)

The steps for determining the semi-analytical solution of this transport problem are presented below. The statistics of the collisional motion is determined in section 3.1 The transport induced by the RMPs and the probability of the displacements \( x_b(t) \) are analyzed in section 3.2. The EC (18) of the effective velocity is calculated in section 3.3 and a short review of the DTM for determining the time dependent diffusion coefficients is presented in section 3.4.

#### 3.1. Collisional transport

The first step consists of determining the statistics of the collisional displacements obtained from equation (11) and from \( d\xi/dt = P_2 \eta(t) \), \( i = 1, 2 \), which describe the perpendicular collisional motion. All these equations are of the type \( d\xi/dt = \Theta(t) \), and they lead to Gaussian distribution of the trajectories \( \xi(t) \). We give here the results which are necessary for the following calculations. The probability of a displacement \( \zeta \) at time \( t \) is

\[ P(\zeta, t) = \langle \Delta(\zeta - \zeta(t)) \rangle = \frac{1}{\sqrt{2\pi \langle \zeta^2(t) \rangle}} \exp\left(\frac{-\zeta^2}{2 \langle \zeta^2(t) \rangle}\right). \]  

(19)

where the mean square displacement (MSD) is

\[ \langle \zeta^2(t) \rangle = 2 \int_0^t d\xi(\tau) d\tau = 2P_{\text{c}}^{-2} [P_T + \exp(-P_T) - 1] \]  

(20)

and

\[ d\xi(t) = \int_0^t C_{\xi}(\tau) d\tau = P_{\text{c}}^{-1} [1 - \exp(-P_{\text{c}})]. \]  

(21)

is the time dependent diffusion coefficient. The MSDs of the collisional displacements are \( \langle \zeta^2(t) \rangle = \langle \zeta^2(t) \rangle \) and \( \langle \xi^2(t) \rangle \).
diffusion type (magnetic field line diffusion combined with the collisional particle diffusion along field lines).

The transport can be diffusive only for $x_0(t)$ dependent stochastic magnetic fields. Any perturbation that produces the departure of the particles from the field lines leads to diffusive particle transport in this case. Such a perturbation can be the collisional perpendicular velocity, plasma rotation or even the small magnetic drifts determined by the curvature of the magnetic lines.

In the process studied here, the RMP field actually depends on the perpendicular coordinate, but its correlation length $\Lambda_\parallel$ is very large compared to that of turbulence. The other components of the motion considered in equation (1) determine the departure of the particles from the magnetic lines, and lead to diffusive transport for $x_0(t)$. Since the perpendicular collisional diffusivity is smaller than the turbulent diffusivity, the latter has the main contribution to the decorrelation of the particles from the magnetic lines. The decorrelation is produced when the turbulent MSD is larger than $\Lambda_\parallel^2$, which corresponds to a time $\tau_0 = \Lambda_\parallel^2/\chi$, where $\chi$ is the asymptotic diffusion coefficient produced by the turbulence. In these conditions, the time dependent diffusion coefficient $D_\parallel(t)$ evolves according to equation (30) for $t < \tau_0$ and it saturates at a finite value $\chi_\parallel$ for $t > \tau_0$. The transport induced by the RMPs becomes diffusive. The magnetic Kubo number $K_m = (\beta/B_0)(\Lambda_\parallel/\Lambda_\perp)$ is small and thus the asymptotic diffusion coefficient is

$$\chi_\parallel = D_\parallel(\tau_0),$$

where $D_\parallel$ is given in equation (30) (see [22], where the transport regimes in stochastic magnetic fields are analyzed using similar methods). The transport coefficient (32) determined by RMPs is very small due to the large $\tau_0$ and to the decay $D_\parallel(t)$ at large $t$.

We evaluate now the effect of the gradient of the confining magnetic field taking into account the $R$ dependent factor in equation (22), which can be written as

$$\frac{dx_0}{dt} = \exp\left(\frac{x'}{R} + x_0\right) \frac{dx_0}{dt}.\tag{33}$$

The solution in terms of $x_0(t)$ is

$$x_0(t) = -R \ln \left(1 - \frac{x_0(t)}{R} \exp\left(\frac{x'}{R}\right)\right).\tag{34}$$

The average displacement and the MSD obtained from this equation taking the linear approximation in the small parameter $1/R$ are

$$\langle x_0(t) \rangle = \frac{\langle x_0(t) \rangle}{2R} \exp\left(\frac{2x'}{R}\right) \approx \frac{\langle x_0(t) \rangle}{2R},\tag{35}$$

$$\langle x_0^2(t) \rangle_b = 2\beta^2 \Lambda_\perp \left(\Lambda_\perp^2 + \langle z^2(t) \rangle_b^{1/2}\right).\tag{36}$$

The probability of $x_0(t)$ is Gaussian in these conditions

$$P_b(x_0(t), t) \approx \frac{1}{\sqrt{2\pi \langle x_0^2(t) \rangle_b}} \exp\left(-\frac{(x_0 - x_0(t))^2}{2\langle x_0^2(t) \rangle_b}\right).\tag{37}$$

The displacements produced by the RMPs are solutions of

$$\frac{dx_0}{dt} = P_b \exp\left(\frac{x'}{R} + x_0\right) \frac{dx_0}{dt} = \eta(t).\tag{22}$$

We consider first a constant confining magnetic field $(R \to \infty)$

$$\frac{dx_0}{dt} = P_b \frac{b(z(t))}{\eta(t)} \frac{dx_0}{dt} = \eta(t).\tag{23}$$

The velocity in the right hand side of this equation is the product of two stochastic functions, the magnetic field and the collisional velocity. Its Lagrangian correlation is defined by

$$C_v \equiv \left\{ \langle b(0) b(z(t)) \rangle_b \eta(0) \eta(t) \right\}_\parallel = \left\{ \exp\left(-\frac{z^2(t)}{2\Lambda_\parallel^2}\right) \eta(0) \eta(t) \right\}_\parallel.\tag{24}$$

The correlation in equation (24) is written using a $\delta$-function to impose the condition $z(t) = z$

$$C_v = \int_\infty^\infty \int_\infty^\infty \exp\left(-\frac{z^2}{2\Lambda_\parallel^2}\right) \left\{ \delta(z - z(t)) \eta(0) \eta(t) \right\}_\parallel = \frac{1}{\pi} \int_\infty^\infty \int_\infty^\infty \exp\left(-\frac{z^2}{2\Lambda_\parallel^2} + iqz\right) \left\{ \exp(-iqz(t)) \eta(0) \eta(t) \right\}_\parallel\tag{25}$$

The average in this equation is

$$M_q \equiv \left\{ \exp(-iqz(t)) \eta(0) \eta(t) \right\}_\parallel = \frac{1}{\pi} \int_\infty^\infty \exp\left(-\frac{q^2}{2\pi^2} \int_\infty^\infty \int_\infty^\infty C_v(|r - r'|)\right).\tag{26}$$

Since $\eta$ and $z(t)$ are Gaussian functions, the average of the exponential is

$$\exp\left(-\frac{q^2}{2} \int_\infty^\infty \int_\infty^\infty C_v(|r - r'|)\right).\tag{27}$$

Straightforward calculations lead to

$$M_q = (C_v(t) - q^2 d_v^2(t)) \exp\left(-\frac{q^2}{2} \langle z^2(t) \rangle_b\right).\tag{28}$$

After calculating the integrals in equation (25), one obtains

$$C_v(t) = P_b \Lambda_\perp \left[ C_v(t) (\Lambda_\perp^2 + \langle z^2(t) \rangle_b) - d_v^2(t) \right].\tag{29}$$

The time integrals of this function give the diffusion coefficient and the MSD generated by RMPs

$$D_\parallel(t) = P_b \Lambda_\perp \left[ d_v^2(t) (\Lambda_\perp^2 + \langle z^2(t) \rangle_b) \right]^{1/2},\tag{30}$$

$$\langle x_0^2(t) \rangle_b = 2P_b \Lambda_\perp \left(\Lambda_\perp^2 + \langle z^2(t) \rangle_b^{1/2} - \Lambda_\perp\right).\tag{31}$$

One can see that the diffusion coefficient decays to zero at large $t$ as $D_\parallel(t) \sim 1/t^2$ and the MSD is $\langle x_0^2(t) \rangle_b \sim \sqrt{t}$. This is the well known subdiffusive transport [21] of the double

\[ \text{3.2. Transport induced by RMPs} \]
Thus, the gradient of the confining magnetic field generates an average displacement that is proportional to the MSD produced by the RMPs, and to the gradient. Such average displacement generates an average velocity (direct transport)

\[ V_b(t) \equiv \frac{d\langle x_b(t) \rangle}{dt} = \frac{D_x(t)}{R} \]

The velocity \( V_b \) is positive, directed towards plasma boundary. It is transitory for the subdiffusive transport, and a finite asymptotic value exists only in the presence of a process of decorrelation of the particles from the field lines. We are interested here in the nonlinear effects produced by the average displacement \( \langle x_b(t) \rangle \) on the turbulent transport.

Besides the very small direct contribution to the radial transport (32), the RMPs influence the turbulent transport through their effect on the effective drift velocity (16). Its correlation (18) depends on the magnetic displacements \( \langle x_b(t) \rangle \) and on their statistical properties. Since the EC of the stochastic potential decays in a time of the order of the decorrelation time \( \tau_c \), only the magnetic displacements for \( t \leq \tau_c \) influence the effective EC (18). Since \( \tau_c \ll \tau_b \), the MSD of the magnetic trajectories is given by equation (31) and their transport coefficient is not saturated but it has the time dependence of equation (30).

We note that the complex structure of the magnetic field lines generated by the RMPs (see for instance [1, 23]) does not influence the transport process studied here. The magnetic field lines are geometrical objects, which evidence the complex magnetic structure (the stochastic regions and the island chains) after a large number of toroidal rotations. They influence particle motion only during the time intervals when particles are tied on them. Or, the turbulence has a parallel correlation length of the order of \( R \), which means that after one toroidal rotation the particles leave the magnetic lines due to the \( \mathbf{E} \times \mathbf{B} \) drift. In other words, the particles do not ‘see’ the complex structure of the magnetic lines in the presence of turbulence, which randomly moves the particles on neighbor magnetic lines after each toroidal rotation.

### 3.3. The EC of the effective velocity

The averages over the three stochastic functions that appear in the effective velocity are calculated according to the definition (18).

The average over the stochastic potential yields the EC of the \( \mathbf{E} \times \mathbf{B} \) drift components (4)

\[ M_b \equiv \left\langle v_b^{\text{eff}}(\mathbf{0}, \mathbf{0}) v_b^{\text{eff}}(\mathbf{x}, t) \right\rangle_p = E_b\left( x + x_b(t) \right) \exp \left( \frac{x + x_b(t)}{R} \right). \]

The average over the parallel collisional velocity \( \eta_p(t) \) is obtained using the probability (19) of \( z(t) \). It applies in the case of the EC (3) only to the \( z \) dependent factor, which becomes

\[ \left\langle \exp \left( -\frac{z^2}{2\lambda_z} \right) \right\rangle_p = \int_{-\infty}^{\infty} \exp \left( -\frac{z^2}{2\lambda_z} \right) \eta_p(z, t)dz = \frac{\lambda_z}{\sqrt{\lambda_z^2 + \langle z^2(t) \rangle_p}}. \]

The average over the RMPs is calculated using the probability (37) of the magnetic displacements. This average changes only the radial factor in the EC (3) multiplied by the gradient of the confining magnetic field, which is

\[ M_b \equiv \left\langle \exp \left( -\frac{\langle x^2 + x_b(t) \rangle^2}{2\lambda_x^2} \right) \exp \left( -\frac{\langle x^2 + x_b(t) \rangle}{R} \right) \right\rangle_p \]

\[ = \int_{-\infty}^{\infty} \exp \left( -\frac{\langle x^2 + x_b(t) \rangle^2}{2\lambda_x^2} \right) \exp \left( -\frac{\langle x^2 + x_b(t) \rangle}{R} \right) P_b(x_b, t)dx_b, \]

where \( \mathbf{x}^b = \mathbf{x} + \zeta(t) \). One obtains after neglecting a small term of second order in \( 1/R \)

\[ M_b = \frac{\lambda_x}{\sqrt{\lambda_x^2 + \langle \zeta^2(t) \rangle_p}} \exp \left[ \frac{-\langle x^2 + \langle x_b(t) \rangle^2 \rangle}{2(\lambda_x^2 + \langle \zeta^2(t) \rangle_p)} \right] \]

\[ \times \exp \left( \frac{x^2 + \langle x_b(t) \rangle}{R} \right) \frac{\lambda_x^2}{\lambda_x^2 + \langle \zeta^2(t) \rangle_p} \]

The average over the perpendicular collisional displacements is similar to the average over \( x_b(t) \). Since the two components of \( \zeta(t) \) are statistically independent, the average is calculated separately for \( M_b \) and for the \( y \) dependent factor in the EC (3).

Finally, the EC of the effective velocity (18) can be written as

\[ E^{\text{eff}}_y(x, t) = -\epsilon_{ij} \epsilon_{ik} \partial_k \left[ E^{\text{eff}}(x, t) \right] \exp \left( \frac{x + \langle x_b(t) \rangle}{R^{\text{eff}}} \right). \]

where

\[ E^{\text{eff}}(x, t) = \frac{\lambda_x}{\lambda_x^2 + \lambda_y^2} \exp \left( -\frac{(x + \langle x_b(t) \rangle)^2}{2\lambda_x^2} \right) \]

\[ \times \partial_y \left[ \exp \left( -\frac{y^2}{2\lambda_y^2} \sin(k_0 y) \right) \right]. \]

\[ \lambda_x^{\text{eff}}(t) = \sqrt{\lambda_x^2 + \langle \zeta^2(t) \rangle_p}, \]

\[ \lambda_y^{\text{eff}}(t) = \sqrt{\lambda_y^2 + \langle \zeta^2(t) \rangle_p}, \]

\[ k_0^{\text{eff}} = k_0(\lambda_x^2/\lambda_y^2) \] and \( R^{\text{eff}} = R(\lambda_x^2/\lambda_y^2)^{1/2} \). The last factor in equation (42) is determined by the gradient of the confining magnetic field. The structure of \( E^{\text{eff}}_y(x, t) \) is the same as in equation (4), which relates the EC of the drift velocity to the EC of the potential. This shows that an effective potential with the EC (43) can be defined for \( \mathbf{v}^{\text{eff}}(x, t) \).

The effects of the RMPs on the effective potential can be seen by comparing equation (43) with equation (3). One can see that the modification concern the \( x \) dependence of the EC. It consists of the shift of the maximum with the average displacement generated by the RMPs, of the increase of the correlation lengths and of the decrease of the amplitude. Both collisions and the RMPs contribute to the increase of the
correlation lengths and to the time decay of the amplitude. The parallel motion eliminates the \( z \) dependent factor in (3) and leads to the time decay of the amplitude of the effective potential as \( \lambda L/\lambda_2 \). The exponential decay with \( z \) is transformed into a slow decorrelation (the decay of the amplitude as \( 1/L^3 \)). The parallel collisional velocity also determines the subdiffusive behavior of the MSD of the RMP displacements \( \chi \). This effect leads to a slow decorrelation of \( v_{\perp} \) in the radial direction, much slower than in the absence of parallel collisions when the RMP radial transport is diffusive.

The factor determined by the gradient of the toroidal magnetic field in equation (42) is modified by the RMPs, which determine a shift of the maximum and the increase of the gradient length. This means that the effect of \( R \) decreases in time.

3.4. The DTM

The multi-stochastic process that describes turbulent transport in the presence of RMPs and collisions was reduced to the problem of 2-dimensional transport in an effective velocity field that has the EC (42) with \( E_{\text{eff}}(x, t) \) given by (43).

We use the DTM [14] for determining the time dependent diffusion coefficient. This method is based on a set of deterministic trajectories, the decorrelation trajectories (DTs), which are obtained from the EC of the effective velocity. We define a set of subensembles \( S \) with given values of the stochastic functions at the origin of the trajectories \( x = 0, t = 0: \)

\[
\phi_{\text{eff}}(0,0) = \phi^0, \quad v_{\text{eff}}(0,0) = v^0.
\]

The effective velocity in each subensemble \( S \) is a Gaussian field with the average

\[
V_{\text{eff}}(x, t) = -\varepsilon_{\perp} \partial_t \phi_{\text{eff}}(x, t), \\
\Phi_{\text{eff}}(x, t) = \phi_{\text{eff}}(x, t) + \phi^0 E_{\text{eff}}(x, t)/V_{\text{eff}}^2 + v_{\text{eff}}^0 E_{\text{eff}}(x, t)/V_{\text{eff}}^2,
\]

where \( E_{\text{eff}}(x, t) = \varepsilon_{\perp} \partial_t \Phi_{\text{eff}}(x, t) \) are the correlations of the effective velocity components with the potential and \( V_{\text{eff}} = \sqrt{E_{\text{eff}}(0,0)} \) is the amplitude of the velocity. The DTs are approximate average trajectories in the subensembles obtained by solving the equation in each \( S \)

\[
\frac{dX_{\text{eff}}(t)}{dt} = P_{\text{eff}} x_{\text{eff}}^2(t, t) \exp \left( \frac{X_{\text{eff}}^2(t) + \langle x_{\text{eff}}(t) \rangle}{R_{\text{eff}}} \right) + V_{\text{eff}} \eta,
\]

The fluctuations of the trajectories are neglected in this equation. This approximation is supported by the high degree of similarity of the trajectories in a subensemble, which occurs due to the supplementary initial conditions (47), and to the small amplitude of the velocity fluctuations in a subensemble [24].

The time dependent diffusion coefficient and the average radial displacement are obtained by summing the contributions of all subensembles (see [14] for details)

\[
D_{x}(t) = \int d\phi_{\text{eff}} d\phi_{\text{eff}} P(\phi_{\text{eff}}) P(\phi_{\text{eff}}) v_{\text{eff}}^0 X_{\text{eff}}^2(t),
\]

\[
\langle x(t) \rangle = \int d\phi_{\text{eff}} d\phi_{\text{eff}} P(\phi_{\text{eff}}) P(\phi_{\text{eff}}) X_{\text{eff}}^2(t).
\]

The direct contribution of the transport produced by the RMPs (equations (30) and (32)) and by collisions (21) have to be added. Thus, the total diffusion coefficient in physical units is

\[
D_{x}(t; P_{\text{eff}}, P_{\text{rand}}, P_{\text{col}}) = \frac{\Phi}{B_0} \left( D_{x}(t) + \frac{1}{P_{\text{rand}}} D_{\text{rand}}(t) + \frac{P_{\text{rand}}^2}{P_{\text{col}}} D_{\text{col}}(t) \right),
\]

and the asymptotic diffusion coefficient is obtained taking the limit \( t \to \infty \) in this equation

\[
\chi_{x}(P_{\text{rand}}, P_{\text{rand}}, P_{\text{col}}) = \frac{\Phi}{B_0} \left( \chi_0 + \frac{1}{P_{\text{rand}}} \chi_{\text{rand}} + \frac{P_{\text{rand}}^2}{P_{\text{col}}} \chi_{\text{col}} \right)
\]

The dominant contribution is the turbulent transport \( \chi_{x} \). It depends on the whole set of parameters through the trajectories. Thus, the diffusion coefficient \( D_{x}(t) \) is obtained from equation (50) using the solutions of equation (49) for the DTs. The latter have to be numerically calculated, although they are very simple. A computer code was developed for the calculation of the decorrelation trajectories, of the running diffusion coefficient (50) and of the average displacement (51). The numerical calculations are at the microcomputer level with runs of the order of few minutes.

4. The effects of RMPs on turbulent transport

We analyze the effects of the RMPs on transport as function of turbulence parameters. The aim is to identify the direct change of the transport and possible synergy between turbulence and RMPs. The multi-stochastic process that describes the turbulent transport in the presence of RMPs depends on twelve physical parameters. This number is reduce to 10 using dimensionless variables. This large number of parameters imposes a first analysis of their ranges and of the importance of each term before quantitative evaluations.

We consider ITG or TEM turbulence that has perpendicular correlation lengths of the order of \( \rho_{\parallel} \). The values taken in the figures are \( \lambda_{\perp} = 4 \) and \( \lambda_{\parallel} = 2 \). The parallel correlation length \( \lambda_{\parallel} \) is of the order of \( 2 \pi R \), which leads to \( \lambda_{\perp}/\lambda_{\parallel} \approx 30 \) for an ITG parameter \( R/L_{\perp} \approx 5 \). The correlation length \( \lambda_{\parallel} \) of the RMPs is significantly smaller than \( \lambda_{\parallel} \) due to their non-axisymmetric nature. The value taken in the figures is \( \lambda_{\parallel} = 5 \). We have found that the variations around this fraction have weak effects on the turbulent transport. The drift velocity \( V_{\parallel} \) is of the order of the diamagnetic velocity, which leads to \( V_{\parallel} \approx 1 \). Thus, the analysis of the dependence of the transport on five of the parameters of the model is not necessary since they have narrow variation range and determine weak effects.

The normalized amplitude of the RMPs can be approximated by \( P_{\text{rand}} \approx (\beta / B_0) (a / \rho_{\parallel}) \). It depends on the amplitude of the magnetic perturbations and on plasma size parameter. Its values are of the order \( P_{\text{rand}} \approx 0.5 \) for actual experimental
conditions and will increase to maximum few units in ITER [25].

Particle collisions are strongly connected with the contribution of the RMPs in the EC of the effective velocity (43). The effects of collisions will be analyzed in both low and high collisionality regime.

We begin by a short presentation of the trapping process and of the transport regimes obtained in a turbulence with the EC of the type (3) in the absence of the RMPs (section 4.1). The effects of the RMPs on the transport coefficient and on the turbulent pinch are discussed in section 4.2.

4.1. Trajectory trapping

Particle trajectories in turbulent plasmas can have both random and quasi-coherent aspects. A typical trajectory is a random sequence of long jumps and trapping events that consists of winding on almost closed paths. Trapping introduces quasi-coherent aspects in trajectory statistics. It determines a large degree of coherence in the sense that bundles of trajectories that start from neighboring points remain close for very long time compared to the eddying time [24]. This process generates intermittent, quasi-coherent structures of trajectories similar to fluid vortices. A strong interdependence exists between ion trajectory statistics and the evolution of the drift type turbulence. Trajectory diffusion has a stabilizing effect on turbulence [26] while trajectory trapping leads to strong nonlinear effects [27]. The transport is related to the stochastic aspect of trajectories, and trajectory structures have a hindering effect. The trapped particle do not contribute to transport, but they represent a reservoir of transport. Any perturbation that liberates particles leads to increased transport and to anomalous transport regimes.

The turbulent transport in the absence of RMPs ($P_b = 0$) and collisions was studied in [12, 13] for a potential with EC of the type (3). We present here a short review of these results and of their physical image.

The process is nonlinear due to the $x$ dependence of the stochastic potential. The nonlinearity manifests as trajectory trapping or eddying due to the Hamiltonian structure of equations (9) and (10), which lead to the invariance of the Lagrangian potential for $\tau \rightarrow \infty$ and $\lambda_i \rightarrow \infty$. The trajectories remain on the contour lines of the potential, and the transport is subdiffusive in these conditions. The time variation of the potential and particle parallel motion represent decorrelation mechanisms, which lead to finite asymptotic values of the diffusion coefficient. Depending on the strength of these perturbations, represented by the decorrelation characteristic times $\tau_d$, trajectory trapping is partially or completely eliminated. The condition for the existence of trapped trajectories is $\tau_d > \tau_\alpha$, where $\tau_d \equiv \tau_\alpha(\tau_\alpha + \tau_\phi)$, $\tau_\phi = \lambda_i/\nu_l$, is the parallel decorrelation time and $\tau_\alpha = \lambda_\alpha/\nu_\alpha$ is the time of flight of the particles or the eddying time.

The average velocity $V_d$ also influences the trapping, but in a different way. It determines an average potential $xV_d$ that adds to the stochastic potential $\phi(x)$. The total potential has a strongly modified structure. Bunches of open contour lines between islands of closed lines appear for small $V_d$ ($V_d < V_\phi$).

As $V_d$ increases, the surface occupied by the islands of closed lines decreases and vanishes for $V_d > V$. Thus, the average velocity $V_d$ eliminates trajectory trapping, but not through a decorrelation mechanism.

The conditions for the existence of trajectory trapping are $\tau_d > \tau_\phi$ and $V_d < V_\phi$. In terms of the dimensionless parameters used in this paper, these conditions are

$$\tau_d \equiv \tau_\phi \frac{\lambda_i}{\nu_l} \left( \frac{\lambda_i}{\nu_l} + \frac{\lambda_\phi}{\nu_\phi} \right)_{\text{and}} P_b V_\phi > V_d. \quad (54)$$

In the absence of trapping the transport is quasilinear. At small decorrelation time ($\tau_\phi P_b < \lambda_i/\nu_l$), $\chi_s = V_d^2/\nu_l$, and it does not depend on $V_d$. When $\tau_\phi$ is large and the amplitude of the turbulence is smaller than $V_d$ ($P_b V_\phi < V_d$), the diffusion coefficient decreases with the increase of $\tau_d$ as $\chi_s = (\phi/\nu_l V_d^2)^2/\nu_l$. The decay of $\chi_s$ at large $\tau_d$ is specific to the shape (3) of the EC. For a monotonically decreasing EC, $\chi_s$ is constant at large $\tau_d$.

The nonlinear regime has a similar dependence on $\tau_d$, linear increase at small $\tau_d$, and, after a maximum that is at $\tau_d \approx \tau_\phi$, a decay as $\tau_d^\alpha$, where $\alpha > 0$. However, a different scaling of $\chi_s$ in the parameters of the turbulence is obtained, essentially due to the different position for the maximum. In these conditions, $\chi_s$ depends on the anisotropy of the turbulence and on the decay rate of the EC at large distances [13].

The physical mechanisms that determine the decay of the diffusion coefficients at large $\tau_d$ are completely different in the quasilinear and in the nonlinear regimes. The shape (3) of the EC that corresponds to roughly ordered positive and negative potential cells along $y$ produces the quasilinear decay of $\chi_s$. The decay of the nonlinear $\chi_s$ is determine by trapping, which produces the decrease of the number of particles that effectively contribute to diffusion.

Thus, the essential difference between the quasilinear and the nonlinear regimes is the existence of particle trapping or eddying in the stochastic potential. Theoretical conditions as (54) for the limits of these regimes can be deduced for the transport in different conditions. However, it is very difficult to determine experimentally the transport regime since the conditions contain not only the characteristics of the turbulence (amplitude, spectrum, correlation time), but also the decorrelation mechanisms.

4.2. RMP effects on transport

First, we analyze the time dependent diffusion coefficient (50) in order to identify the main effects of the RMPs. Typical results for $D_x(t)$ in the presence of RMPs are shown in figure 1 for both the quasilinear and the nonlinear regimes. All the decorrelation mechanisms are eliminated in these examples by taking $\tau_x \rightarrow \infty$ (static potential), $\lambda_i \rightarrow \infty$ (absence of parallel decorrelation) and $\chi_s^0 = 0$ (absence of collisional decorrelation). Without the RMPs, the transport is subdiffusive in these conditions: the diffusion coefficient decreases to zero for both quasilinear and nonlinear regimes, as shown by the dashed curves in figure 1. The RMPs produce the transition from the subdiffusive to diffusive transport. One can see in
figure 1 that the RMPs determine the saturation of $D_{\chi}(t)$ at finite values. This means that they provide a decorrelation mechanism.

The decorrelation time produced by the RMPs is of the order $10L_y/v_y$ in the examples shown in the figure 1. The parallel motion during this time corresponds to less than one toroidal rotations $10L_y/(2\pi R) \lesssim 1$, which shows that the complex structure of the perturbed magnetic surfaces does not influence the turbulent transport.

The RMPs enhance the stochastic aspects of the trajectories by destroying the quasi-coherent structures. The increase of the transport coefficient is expected in such conditions. The RMPs could produce decorrelation by the average velocity (38) or by trajectory spreading (36). The examination of the EC of the effective potential (43) shows that the average displacement $(x_0(t))$ does not modify the EC, but it only determines a shift of the EC. The RMP average velocity does not contribute to the decorrelation. It actually determines a radial drift of the stochastic potential.

Since the RMPs provide a decorrelation mechanism, their effects on the asymptotic diffusion coefficients should be understood from the analysis of the competition with the other decorrelation mechanisms.

The collisions and the magnetic perturbations determine the decay of the effective EC (43) through three factors $f_i = \bar{\lambda}/\lambda^0_i(t)$, $i = x, y, z$. $f_i$ represents the decorrelation produced by the parallel motion, $f_i$ is the effect of the perpendicular collisional decorrelation and $f_i$ includes the combined influences of RMPs and collisions. The decorrelation is dominated by the process that leads to the strongest decay of the amplitude of the EC (smallest factor $f_i$). These factors are plotted in figure 2 for low collisional regime (continuous lines) and for high collisional regime (dotted lines). One can see that the increase of the collisionality (of $P_b$) determines the increase of $f_i$ and $f_z$. This dependence is due to the parallel diffusivity $\lambda_0$, which decreases when $P_b$ increases and it leads to the decrease of $(\xi^2(t))$ and of $\langle \xi^2_0(t) \rangle$, equations (31) and (36). Moreover, the separation between the ballistic regime and the diffusive (for $z(t)$) or subdiffusive regime (for $x_0(t)$) is at $t \approx 1/P_b$, which means that the fast ballistic decorrelation is maintained only for a small time interval at large $P_b$. On the contrary, the perpendicular collisional decorrelation becomes stronger at high collisionality ($f_i$ decreases). These opposite variations can compensate in the amplitude of the effective EC.

The competition between the decorrelation mechanisms is also influenced by the values of the correlation lengths. As seen in equations (44)–(46), the MSD has to be larger than the corresponding $\bar{\lambda}$ in order to influence the transport.

The examination of the effective EC (43) shows that the RMPs are important in the decorrelation process and thus they influence the turbulent transport for small values of $P_b$. The dependence of the turbulent transport on the amplitude $P_b$ is presented in figure 3, where the asymptotic value of $\chi$ is determined by the MSD $\langle \xi^2_0(t) \rangle$ and $\chi$ decreases with $P_b$.

The dependence of the turbulent transport on the amplitude of the RMPs is presented in figure 3, where the asymptotic diffusion coefficient $\chi$ is plotted as function of $P_b$ for the quasilinear regime (dashed line) and for the nonlinear regime (continuous line). One can see that the transport in both regimes is not affected at small $P_b$, and that there is a smooth transition to a rather strong degradation of the confinement. At larger amplitudes, the tendency is reversed in both regimes, and the diffusion coefficients decrease. This effect occurs at large values of $P_b$ that are not in the range of interest.

The nonlinear dependence of $\chi$ on $P_b$ seen in figure 3 is explained by the decorrelation effect produced by the RMPs. Their contribution $f_i(t)$ is determined by the MSD $\langle \xi^2_0(t) \rangle_{b}$ that is proportional with $P_b^0$, as seen in equations (31) and (36). Thus, as $P_b$ increases, $\langle \xi^2_0(t) \rangle_{b}$ becomes larger than $\langle \xi^2(t) \rangle_{b}$ in $f_i(t)$, and $f_i(t)$ becomes smaller than the other decorrelation
For the typical values of $P_b$ in the present experiments ($P_b \lesssim 0.5$), the quasilinear regime is characterized by a stronger influence of the RMPs than the nonlinear transport (figure 3).

The dependence of the RMP effect on the collisionality is presented in figure 4 for the nonlinear ($P_b = 10$) and quasilinear ($P_b = 0.4$) transport. The high collisionality case is represented by the dashed lines and the low collisionality by the solid lines. The effect of the collisionality is rather strong in the nonlinear regime (see the curves for $P_b = 10$) and very weak in the quasilinear regime (the curves for $P_b = 0.4$ are practically superposed).

In the nonlinear case, the RMPs with small amplitude ($P_b \lesssim 0.5$) do not influence much the transport at high collisionality ($P_c = 0.1$), but they determine a significant increase of the turbulent transport at low collisionality ($P_c = 0.01$). At higher normalized amplitudes of the RMPs ($P_b \gtrsim 1$), a stronger increase of $\chi_s$ appears at high collisionality, and eventually (at $P_b \approx 2$) the collisionality does not influence the transport. It means that the perturbation of the magnetic field dominates the decorrelation process.

It is interesting to see that the quasilinear transport (the curves for $P_b = 0.4$ in figure 4) is not influenced by the collisionality. The reason is that there is a high degree of compensation of the decorrelation factors for the set of parameters considered in this example, such that the amplitude of the effective EC (43) has a weak dependence on $P_c$. The effect observed in the nonlinear transport is mainly due to the modification of the space dependence of the effective EC (increase of the correlation lengths).

The increase of the diffusion coefficient due to RMPs ($\delta \chi_s \equiv 100(\chi_s(P_b) - \chi_s(0))/(\chi_s(0))$) is shown in figure 5 as function of the turbulence amplitude $P_b$ at low collisionality $P_c = 0.01$. In this example, the limit between the quasilinear and nonlinear regime (54) is $P_b \gtrsim T_d/V_c = 4$. The normalized transport coefficient $\chi_s$ is maximum around this value, it has a fast decay towards smaller $P_b$ (in the quasilinear regime) and a weaker decay at large $P_b$ (in the nonlinear regime). However, the amplification $\delta \chi_s$ has a continuous decrease with the increase of $P_b$. The only difference between the two transport regimes is the rate of variation with $P_b$, which is larger for the nonlinear transport.

The monotonic decrease of $\delta \chi_s$ is evidence of an interesting effect. The change of turbulence amplitude due to the RMPs has an opposite effect on the confinement. Namely, if turbulence amplitude increases, then the confinement degradation is smaller than estimated at fixed amplitude. On the contrary, if the turbulence is attenuated, $\delta \chi_s$ is larger than in the case of unchanged turbulence.

The curve for $P_b = 0.5$ in figure 5 corresponds to the present experiments and the curve for $P_b = 1.5$ shows the prediction of this theory for large size plasmas that have larger values of $P_b$ (14) at the same amplitude of the magnetic field perturbation $\beta/B_0$. One can see that the increase of the diffusion is much smaller in the nonlinear regime (at large $P_b$) than in the quasilinear regime (small $R_b$). The amplification of the turbulent diffusion for $P_b = 0.5$ varies between 20–70%, with a value of the order 30% for the amplitude usually found in numerical simulations $V_c/V_\phi \approx 2$ ($P_b \approx 8$). A dramatic degradation of the confinement appears at larger $P_b$, as already suggested by figures 3 and 4. One can see in figure 5 that $\delta \chi_s$ for $P_b = 1.5$ is several times larger than in the absence or the RMPs.

The RMPs have a synergistic effect on the turbulent transport. The direct contribution of the stochastic magnetic field to the transport $\chi_s$ is very small as discussed in section 3.2, but it determines much larger variations of the turbulent transport coefficient $\delta \chi_s \gg \chi_s$ (as observed in experiments [28]). This is a nonlinear process determined by the space dependence of the stochastic potential.
It is interesting to note that, in the case of usual ECs of the potential (with monotonic decrease in space), strong effects of the small perturbations as those induced by the RMPs occur only in the nonlinear regime. The special shape (3) of the EC of the drift type turbulence leads to the extension of the synergistic effects in the quasilinear regime. The influence of RMPs is even stronger in these conditions, as seen in figure 5.

The gradient of the toroidal magnetic field generates an average velocity (38) of the trajectories \( x(t) \) determined by the RMPs. A similar effect was found in the case of turbulent plasmas [29, 30]. The turbulent pinch velocity is positive in the quasilinear regime corresponding to small \( \tau_0 \) and it becomes negative (directed inward) in the nonlinear regime (large \( \tau_0 \)) (see figure 6, the dotted line).

We determine here the influence of the RMPs on the turbulent pinch velocity. As seen in figure 6, the effect is complex and it depends on the amplitude of the RMPs, and on the decorrelation time of the turbulence. The RMPs lead to continuous decrease of the quasilinear pinch. In the nonlinear regime, a much more complicated dependence on \( P_b \) is found. A fast growth of \( \langle v_r \rangle \) appears at large \( \tau_0 \) for \( P_b \lesssim 2 \), where \( \langle v_r \rangle \) reaches positive (outward) values. The negative minimum is attenuated as \( P_b \) increases. At larger values of \( P_b \) (\( P_b \gtrsim 3 \)), the pinch becomes negative with values comparable to the turbulent pinch (\( P_b = 0 \)). However, this inward pinch occurs at values of \( P_b \) that are out of the experimental range.

The effect of the pinch velocity is determined by the dimensionless parameter \( p \equiv L_T \langle v_r \rangle / \chi_\perp \), the peaking factor. It is the estimation of the ratio of the average and diffusive displacements. The peaking factor for the direct contribution of the RMPs is small \( p_{\text{RMP}} \lesssim L_T / R \lesssim 0.2 \). The turbulent peaking factor decreases due to the RMPs, because the diffusion coefficient increases. However, it can be much larger than \( p_{\text{RMP}} \). Values of the order \( \lesssim 1 \) can be attained only for \( P_b \approx 1 \) and for large size plasmas with \( a / \rho_i \approx 1000 \). As seen in figure 6, the pinch is positive at such values, and it contributes to confinement degradation.

The direct effects of the RMPs on turbulent transport were analyzed in the framework of the test particle approach using a semi-analytical method, the DTM. The diffusion coefficient and the pinch velocity were determined as functions of the turbulence parameters and of the RMPs amplitude \( P_b \).

We underline that the effects of the RMPs on turbulence are not considered in these evaluations. One effect, which is well demonstrated and understood, is the attenuation of the modes determined by the increased diffusion, which leads to the decrease of turbulence amplitude \( \Phi \). But other processes could have opposite effects on turbulence amplitude or they can even generate a different type of turbulence. A strong interaction with the turbulence appears through the modification of the radial electric field and of plasma rotation due to the RMPs. Indeed, the experiments have found strong increase of turbulence in H-mode [3–5], but the effects are important also in the L-mode [31]. The confinement degradation by RMPs also depends on the collisionality [32].

The test particle model at the basis of the present study includes the combined action of four stochastic processes: the turbulence, the RMPs, the perpendicular and the parallel stochastic velocities introduced by collisions. Realistic models for the correlations were used. The results apply to L-mode since plasma rotation is not included in the model.

The aim of the present study is to determine the effects of the RMPs on the turbulent transport as function of the turbulence characteristics, and to identify the conditions corresponding to reduced confinement degradation.

We have shown that the effects observed in experiments (increased turbulent transport and generation of outward pinch) occur even when the turbulence is not modified. This direct influence of the RMPs on transport is produced through a decorrelation mechanism.

A synergistic effect appears. The increase of the turbulent transport in the presence of the RMPs is much larger than the direct contribution of the stochastic magnetic field.
The dependence of the diffusion coefficient on the amplitude of the RMPs is nonlinear (figure 3). A smooth threshold exists at small \( P_b \). It is determined by the condition that the characteristic time of the RMP decorrelation (a decreasing function on \( P_b \)) should be smaller than the collisional and parallel decorrelation times. At larger \( P_b \), the increase of the transport coefficients appear in both quasilinear and nonlinear regimes, with stronger effect in the first case. The values of \( P_b \) that correspond to present experiments (\( P_b \leq 0.5 \)) are near the smooth threshold seen in figure 3. In these conditions the effect of the RMPs is mixed with the influence of collisions and of parallel motion, which can hinder the confinement degradation by the RMPs. At larger \( P_b \), the effect of the RMPs is dominant and consists of a strong increase of the turbulent transport.

We have studied the effect of collisions at small and high collisionality. The results show that at high collisionality the transport is weakly affected by the RMPs, and that at low collisionality the RMPs determine significant degradation of the confinement for \( P_b \leq 0.5 \) (see figure 4). This effect appears in the nonlinear regime of the turbulent transport with large \( P_b \). The quasilinear transport does not depend on the collisionality.

The dependence of the confinement degradation \( \delta \chi_x \) on the amplitude of the turbulence \( P_b \) is a decreasing function (figure 5). This shows that the modification of turbulence amplitude by the RMPs has opposite effect on the confinement. Compared to the degradation corresponding to the turbulence amplitude in the absence of the RMPs, \( \delta \chi_x \) is smaller when \( P_b \) increases. On the contrary, a larger \( \delta \chi_x \) corresponds to the attenuation of the turbulence. This behavior is determined by trajectory trapping in the nonlinear regime and by the special shape of the EC (3) in the quasilinear transport.

The results obtained for \( P_b = 0.5 \) in the nonlinear regime (figure 5) show a degradation of the confinement of the order 30% at small collisionality. The dependence on plasma size of the normalized amplitude of the RMPs (14) leads to increased values of \( P_b \) in the case of ITER. Confinement degradation strongly increases with \( P_b \), and, as seen in figure 5, the effect is too strong even at \( P_b = 1.5 \), where the transport is amplified ten times more that at \( P_b = 0.5 \). Thus, according to our model, the results of the present experiments cannot be directly extrapolated to ITER conditions.

We have also analyzed the effect of the RMPs on the turbulent pinch velocity determined by the gradient of the toroidal magnetic field. We have shown that at large correlation times, the negative (inward) drift (dotted curve in figure 6) is reduced by the RMPs, then they generate a positive (outward) drift that is maximum for \( P_b \approx 2 \). At larger \( P_b \) the average velocity decreases and becomes again negative, but with small values of the peaking factor.

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