Backscattering in helical edge states from a magnetic impurity and Rashba disorder

Lukas Kimme,1 Bernd Rosenow,1 and Arne Brataas2
1Institut für Theoretische Physik, Universität Leipzig, D-04103 Leipzig, Germany
2Department of Physics, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway

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Transport by helical edge states of a quantum spin Hall insulator is experimentally characterized by a weakly temperature-dependent mean free path of a few microns and by reproducible conductance oscillations, challenging proposed theoretical explanations. We consider a model where edge electrons experience spatially random Rashba spin-orbit coupling and couple to a magnetic impurity with spin $S > 1/2$. In a finite bias steady state, we find for $S > 1/2$ an impurity induced resistance with a temperature dependence in agreement with experiments. Since backscattering is elastic, interference between different scatterers possibly explains conductance fluctuations.

Introduction. During the last decade, the quantum spin Hall effect (QSHE) [1–3] has become an important example of a topologically ordered state with time-reversal invariance. One of its key features is the existence of helical edge states [4] with a quantized conductance of $e^2/h$ per edge, as edge electrons are protected from elastic single particle backscattering by time-reversal symmetry [1,4,5]. Soon after the theoretical prediction [6], the QSHE was realized in HgTe/CdTe quantum wells [7], and the quantized conductance [7] as well as the demonstration of nonlocal transport [8] were crucial signatures for this first-time experimental observation. However, already in this first as well as in subsequent experiments [7–12], deviations from the quantized conductance with a weak temperature dependence were found for edges longer than approximately 1 $\mu$m. Moreover, in short samples, where the conductance is essentially quantized, small conductance fluctuations are observed as the back-gate voltage is tuned [7,8,12,13]. After the prediction [14] of the QSHE in InAs/GaSb/AlSb quantum wells, the same qualitative behavior of the conductance as in HgTe/CdTe was observed also in these devices [15–18].

A multitude of other mechanisms beyond elastic single particle backscattering have been proposed as possible explanations for the relatively short mean free path [19]: inelastic single particle [20–23] and two-particle backscattering [1,4,5,23–25] which can be caused by electron-electron or electron-phonon interactions, both of which are usually considered in combination with other time-reversal invariant perturbations; tunneling of electrons into charge puddles caused by inhomogeneous doping, giving rise to inelastic single particle backscattering [26,27]; coupling of edge electrons to a spin bath which gets dynamically polarized [28], thus effectively breaking time-reversal symmetry and giving rise to elastic backscattering in conjunction with Rashba disorder [29]; time-reversal symmetry breaking by an exciton condensate [30]; and coupling of edge electrons to a single Kondo impurity [4,31,32], to a lattice of Kondo impurities [33], to a single Kondo impurity in the presence of homogeneous Rashba spin-orbit coupling [34,35], or to several Kondo impurities with random anisotropies [36]. Although these mechanisms are very diverse, many of them have in common a pronounced temperature dependence, usually some power law $T^{\alpha}$ with positive exponent $\alpha$ for the resistance. However, only a weak temperature dependence has been observed experimentally, with the exception of a recent study using very low excitation currents [37]. In fact, in some experiments, a slight increase of the resistance is observed when the temperature is decreased [7,11,12]. With respect to the conductance fluctuations, only charge puddles [26,27] as well as coherent scattering between several magnetic impurities with large spin $S > 1$ and uniaxial single-ion anisotropy [38] were considered as possible explanations. Theories that build on scattering from local disorder are also supported by recent scanning gate microscopy experiments [39], which identified individual scattering centers.

In this Rapid Communication, we consider scattering of helical edge electrons from a magnetic impurity with spin $S > 1/2$ in combination with a spatially fluctuating Rashba spin-orbit coupling. The latter originates from a fluctuating electric field in the out-of-plane direction due to disorder in the doping layers [40–44]. From a $T$-matrix calculation accounting for the combined scattering events off these perturbations, we derive an effective additional coupling to the impurity. This coupling provides a backscattering mechanism which is enhanced by an increased polarization of the impurity with spin $S > 1/2$. The polarization of the impurity spin is determined from the steady state solution to a semiclassical scattering rate equation. We consider the linear and the nonlinear regime. Upon entering the nonlinear regime with the source drain voltage larger than temperature, the impurity gets polarized and the Rashba disorder induced effective coupling leads to an increased resistance, thus providing a possible explanation for the experimental results. We assume that the relevant Kondo temperature is exponentially suppressed and well below the temperature regime studied in our analysis. Since the dominant contribution to backscattering is elastic in our model, quantum interference between different scatterers is possible, and can give rise to conductance fluctuations as observed in Refs. [7,8,12,13].

Model. The edge states are described by

$$H_0 = \int dx \Psi_{\uparrow}(x)\sigma_\alpha^z(-i\nu\hbar\partial_x)\Psi_{\downarrow}(x),$$

(1)

where $\Psi_{\uparrow}(x)$ annihilates a right-moving electron, $\nu$ is the edge velocity, and the spin quantization axis is in the $z$ direction. A
disordered Rashba spin-orbit coupling is described by [40–43]

\[ H_R = \int dx \Psi_\sigma^\dagger(x) \mathcal{A}_\sigma(x) \Psi_\sigma(x), \]

with the correlator \( \langle a(x)a(x') \rangle_{\text{dis}} = V_0 F(x-x') \) and \( F(0) = 1 \) [44,45]. Being time-reversal invariant, \( H_R \) does not cause elastic backscattering [1]. The essential ingredient is the coupling of electrons to a local magnetic impurity with spin \( S \) via

\[ H_S = J_z \sigma^z + J_{\perp} (S^z s^z + S^- s^-) + J_{\text{aniso}} (S^z s^z). \]  

As usual, \( S^\pm = S^x \pm i S^y \) and \( s^\pm = \Psi_\sigma(0) \mathcal{A}_\sigma \Psi_\sigma(0) \) are the local spin density operators of the edge electrons. For \( J_{\text{aniso}} = 0 \), \( H_S \) describes a Heisenberg XXZ coupling, which has an axial rotation symmetry and was the subject of earlier studies [4,31–35]. In systems with axial symmetric \( H_S \), the \( z \) component of the total spin is conserved and the dc conductance is exactly quantized [32]. A finite \( J_{\text{aniso}} \) breaks the axial rotation symmetry, thus enabling persistent backscattering in the dc limit [46].

In general, a coupling to the impurity spin could also involve terms such as \( S^z s^z + S^- s^- \) and \( S^z (s^- + s^z) \) that break axial rotation symmetry. However, a microscopic analysis, based on an isotropic \( sp-d \) exchange interaction of bulk electrons with the impurity, results only in the terms in Eq. (3), at least for HgTe/CdTe quantum wells [45]. Nevertheless, as we will explain, the combined processes of impurity and Rashba disorder scattering effectively give rise to such additional terms.

Effective couplings from second-order processes. We now consider the combined scattering from the impurity and the Rashba disorder by using the \( T \) matrix \( T(e) = (H_S + H_{R}) + (H_{S} + H_{R})G(e)T \), with \( G(e) = (e - H_0)^{-1} \). The \( T \)-matrix element associated with the first term of Eqs. (3) and (2) is

\[ \langle m; -k, -\sigma | T_{Z} \rho(e) | m; k, \sigma \rangle = \frac{\sigma \epsilon_i J_z (m|S|^m)}{L \sqrt{L}} \sum_{k'} \left( \frac{(k-k') a_{-k-k'} + (k+k') a_{k-k}}{\hbar v(k-k')} \right) \]

(cf. Fig. 1). Here, \( |m; k, \sigma\rangle \) denotes a product state of the local moment in the \( S^\pm \) eigenstate \( |m\rangle \) and an electron with helical spin \( \sigma \) and momentum \( \hbar k \), \( L \) the distance between the left and right reservoir, \( \epsilon_i = \sigma \hbar v L \) the energy of the initial state \( |m; k, \sigma\rangle \), and \( a_{k} \) the Fourier components of \( a(x) \). The energy difference between the initial and intermediate state in the denominator compensates the matrix element from Rashba disorder in the numerator.

With regard to scattering rates, to be discussed below, the second-order process described in Eq. (4) can effectively be described as a first-order process resulting from the additional anisotropic coupling

\[ H_{S}^{\text{aniso}} = J_{S,R} S^z (s^z + s^-). \]  

Here, \( J_{S,R} = 2 \sqrt{\eta} J_z \) is an effective coupling constant with \( \eta = V_0 / l^2 v^2 \). Analogously to the combined process described by \( T_{Z} \), there are also second-order processes where electrons scatter from the Rashba disorder and from the impurity by either the \( J_{S} \) or the \( J_{\text{aniso}} \) term of \( H_S \) in Eq. (3). Again, these effects can be captured by considering first-order processes from the effective couplings \( J_{\perp}(S^z + S^-) s^z \) and \( J_{\text{aniso}} (S^z s^z + S^z s^- + H.c.) \), with \( J_{\perp} = \sqrt{\eta} J_z \) and \( J_{\text{aniso}} = 2 \sqrt{\eta} J_{\text{aniso}} \), respectively. \( J_{\perp} \) only renormalizes the \( J_{\text{aniso}} \) coupling already present in \( H_S \). In contrast, \( J_{\text{aniso}} \), besides renormalizing \( J_{\perp} \), opens an additional scattering channel via the \( S^z s^z + S^z s^- \) terms.

Scattering rates. Since we are interested in the effect of the impurity on dc transport, we pursue two aims: (i) We want to achieve a description of the impurity spin in a driven steady state, and (ii) we want to compute the transport scattering rates determining the impurity induced dc resistance. To achieve (i), we calculate the integrated rates \( \Gamma^\sigma m_\sigma \), which characterize the scattering of an electron from an initial state with helical spin \( \sigma \) into a final state with helical spin \( \sigma' \), as the impurity spin is flipped from \( |m\rangle \) to \( |m'|\). \( \Gamma^\sigma m_\sigma \) are obtained by weighting the rates for scattering from \( |m'; k, \sigma\rangle \) to \( |m'; k', \sigma'\rangle \) with the probability for the initial and final state to be occupied or unoccupied, respectively, and then summing over initial and final momenta. To calculate the individual rates, we employ Fermi’s golden rule for \( H^\text{eff} \), including all original and effective impurity couplings. For temperatures and voltages much smaller than the bulk excitation gap of the topological insulator, the weak momentum dependence of \( |m'; k, \sigma\rangle \) can be neglected. For forward scattering with \( \sigma' = \sigma \), we then find

\[ \Gamma^\sigma m_\sigma (\beta) = \frac{L^2}{2 \pi \hbar^3 v^2 \beta} |\langle m'; \sigma | H^\text{eff} | m; \sigma \rangle|^2. \]  

where the temperature dependence is due to the integrated occupation factors \( \int dE f_\sigma (1 - f_\sigma) = 1/\beta \). Here, \( f_\sigma \) denotes the Fermi distribution function describing the occupation of right-moving (left-moving) electrons from the left (right) reservoir. In the case of backscattering \( \sigma' = -\sigma \), the rates involve occupation factors \( f_{-\sigma} \), leading to a voltage dependence

\[ \Gamma^\sigma m_\sigma (\beta, eV) = \frac{L^2}{2 \pi \hbar^3 v^2 \beta} |\langle m'; -\sigma | H^\text{eff} | m; \sigma \rangle|^2 I^\sigma (\beta eV), \]

where

\[ I^\sigma \equiv \beta \int dE f_{-\sigma} (1 - f_{-\sigma}) = \sigma eV \frac{\epsilon_{\beta eV}^\rho}{\epsilon_{\beta eV}^\rho - 1}. \]

At low bias voltage, \( \beta eV \ll 1 \), \( I^\sigma \simeq 1 \) so that the forward and backscattering rates have the same temperature dependence.
On the other hand, when $\beta eV \gg 1$, the backscattering of right movers is linearly enhanced, while the backscattering of left movers is exponentially suppressed. For detailed results, see Table I.

**Master equation.** We describe the state of the impurity by a density matrix $\rho$ and assume that dephasing from the coupling to the electron bath is sufficiently strong, such that we can neglect coherences and consider $\rho = \sum_m P_m |m\rangle \langle m|$. In both cases, the $m$-dependent matrix elements of the ladder operators $S^\pm$ cancel. We provide an explicit expression in the Supplemental Material [45]. For the model considered here, we have $0 < \xi < 1$ and the two limiting cases have simple solutions: $\xi = 0$ implies $P_m = \delta_{m,S}$ and corresponds to a maximally polarized local moment, while for $\xi = 1$ the impurity is completely unpolarized, i.e., $P_m = 1/(2S + 1)$. Notice that $\beta eV = 0$ implies $\xi = 1$, because, without an applied transport voltage, there is no asymmetry between the rates for forward and backscattering [cf. Eqs. (6) and (7)]. The general dependence of $P_m$ on $m$ interpolates between the two limiting cases:

$$P_m = \frac{(1 - \xi) \xi^S (\frac{1}{\xi})^m}{1 + 2S + 1}.$$  

**Impurity induced resistance.** The backscattering probability $R$ can be related to a scattering rate $1/\tau$ by multiplying with the time of flight $L/v$. This compensates a dependence $1/\tau \propto 1/L$ in all scattering rates, due to the normalization of the plane wave with a factor $1/\sqrt{L}$. In this way, both $R$ and the edge conductance $G = (1 - R)v^2 / h$ are independent of the system size $L$, as expected for a single scattering site. For $R \ll 1$, $R$ equals the impurity induced resistance normalized by $h/v^2$. From the effective impurity coupling we have two important backscattering mechanisms [48], and hence obtain

$$R = \frac{L}{v} \left( \frac{1}{\tau_{\perp,R}} + \frac{1}{\tau_{\perp,L}} \right),$$

where a Fermi’s golden rule calculation yields
the magnitude of $R_0$. With increasing $\beta eV$, the backscattering of right movers via $J^2_{\min}/\beta eV J^2_{\min}$ becomes increasingly dominant relative to other scattering rates and tends to polarize the impurity, such that $\zeta \simeq 2 J^2_{\min}/\beta eV J^2_{\min}$ approaches zero in the large $\beta eV$ limit (cf. Fig. 2). With increasing polarization, the probability for the impurity spin to be in state $|S\rangle$ increases, and the probability for an individual right-moving electron to be backscattered is suppressed. This leads to the monotonic decrease of $R(\beta eV)$ shown in Fig. 2, with $R \sim J^2_{\min}/\beta eV$ in the large $\beta eV$ limit [49]. Rashba disorder, described by a finite $\eta$, has a profound effect on the impurity induced resistance. Since $J^2_{\perp} \gg J^2_{\parallel} \gg J^2_{\min}, 1/\tau_{\perp,R}$ dominates $1/\tau_{\parallel}$ already for very weak Rashba disorder with $\eta \gtrsim 4 J^2_{\min}/J^2_{\parallel}$. The magnitude of $R_0$ is then determined by $J^2_{\parallel,R}$ instead of $J^2_{\min}$, which, depending on the precise values of the couplings and $\eta$, can be a large difference. Regarding the dependence on the polarization, $1/\tau_{\perp,R}$ is qualitatively different from $1/\tau_{\parallel}$. Evaluating the sum over $m$ in Eq. (12b) for the limiting cases of perfect polarization, $P_m = \delta_m, S$, and vanishing polarization, $P_m = 1/(2S + 1)$, yields $S^2$ in the former and $S(S + 1)/3$ in the latter case, respectively. This shows that, for spin $S > 1/2$, the rate $1/\tau_{\perp,R}$ increases with increasing polarization, because $S^2 > S(S + 1)/3$. However, $1/\tau_{\perp,R}$ is independent of $\zeta$ for $S = 1/2$. Thus, in contrast to the case without Rashba disorder, $R$ is now found to monotonically increase with $\beta eV$ when $S > 1/2$.

For real samples with several impurities, the total backscattering rate is proportional to the number of impurities in the absence of localization. Based on Eq. (11), we estimate the mean free path to be $4 \mu m$ in $7.0$ nm wide HgTe quantum wells with a lattice constant $a = 0.65$ nm and $v = 4 \times 10^5$ m/s, by assuming (i) a concentration of $10^{14}$ Mn$^{2+}$ ions ($S = 5/2$) per unit cell, (ii) $\xi = 0.5$ and spatial average $\langle J^2_{\min}/\beta eV \rangle = 0.035$ [45], and (iii) $\eta \approx 3$ based on the estimate for $V_0$ from Refs. [29,44].

A plane wave $e^{i k_x x}$ with Fermi momentum $k_F$ can be backscattered elastically from impurities at $x_0 + l$ by the process Eq. (4) (see Fig. 3). The partial waves $e^{-ik_x(x-x_0-\xi l)}$ and $e^{-ik_x(x-x_0+l)}$ interfere, resulting in a contribution to the backscattering probability $\propto \cos k_F l$. Tuning $k_F$ via a back-gate voltage alternately causes constructive or destructive interference. Hence, conductance fluctuations occur, in agreement with experiment. Several impurities give rise to more complex oscillations.

Summary. We determined the dc resistance of helical edge states in the presence of Rashba disorder and a magnetic impurity with spin $S > 1/2$. As a key result, we find that combined scattering from both Rashba disorder and impurity is enhanced as the impurity becomes more polarized, giving rise to a resistance that slowly increases as the ratio of transport voltage and temperature increases, in agreement with experiments [7,11,12]. Since backscattering is elastic, quantum interference can explain the occurrence of conductance fluctuations.

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