Vertical and radial metallicity gradients in high latitude galactic fields with SDSS

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Abstract

We used the \textit{ugr} magnitudes of 1,437,467 F-G type main-sequence stars with metal abundance $-2 \leq \text{[Fe/H]} \leq +0.2$ dex and estimated radial and vertical metallicity gradients for high Galactic-latitude fields, $50^\circ < b \leq 90^\circ$ and $0^\circ < l \leq 360^\circ$, of the Milky Way Galaxy. The radial metallicity gradient $d(\text{[Fe/H]})/dR = -0.042 \pm 0.011$ dex kpc$^{-1}$ estimated for the stars with $1.31 < z \leq 1.74$ kpc is attributed to the thin-disc population. While, the radial gradients evaluated for stars at higher vertical distances are close to zero indicating that the thick disc and halo have not undergone a radial collapse phase at least at high Galactic latitudes. The vertical metallicity gradients estimated for stars with three different Galactic latitudes, $50^\circ < b \leq 65^\circ$, $65^\circ < b \leq 80^\circ$ and $80^\circ < b \leq 90^\circ$ do not show a strong indication for Galactic latitude dependence of our gradients. The thin disc, $0.5 < z \leq 2$ kpc, with a vertical metallicity gradient $d(\text{[Fe/H]})/dz = -0.308 \pm 0.018$ dex kpc$^{-1}$, is dominant only in galactocentric distance interval $6 < R \leq 10$ kpc, while the thick disc ($2 < z \leq 5$ kpc) could be observed in the intervals $6 < R \leq 10$ and $10 < R \leq 15$ kpc with compatible vertical metallicity gradients, i.e. $d(\text{[Fe/H]})/dz = -0.164 \pm 0.014$ dex kpc$^{-1}$ and $d(\text{[Fe/H]})/dz = -0.172 \pm 0.016$ dex kpc$^{-1}$. Five vertical metallicity gradients are estimated for the halo ($z > 5$ kpc) in three galactocentric distance intervals, $6 < R \leq 10$, $10 < R \leq 15$ and $15 < R \leq 30$ kpc.

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$R \leq 15$ and $15 < R \leq 20$ kpc. The first one corresponding to the interval $6 < R \leq 10$ kpc is equal to $d[\text{Fe}/\text{H}]/dz = -0.023 \pm 0.006$ dex kpc$^{-1}$, while the others at larger galactocentric distances are close to zero. We derived synthetic vertical metallicity gradients for 2,230,167 stars and compared them with the observed ones. There is a good agreement between the two sets of vertical metallicity gradients for the thin disc, while they are different for the thick disc. For the halo, the conspicuous difference corresponds to the galactocentric distance interval $6 < R \leq 10$ kpc, while they are compatible at higher galactocentric distance intervals.

**Keywords:** Galaxy: disc Galaxy: halo stars: abundances stars: distances

1. **Introduction**

Metallicity is an important tool used for understanding the formation and evolution of our Galaxy. [Eggen, Lynden-Bell & Sandage (1962)] proposed the first model for the formation and evolution of the Milky Way. In this model, the first stars are formed from the metal-poor protogalactic cloud. The heavier elements produced by massive stars are spread to the interstellar medium with supernova explosions and the second generation of stars formed from relatively metal-rich molecular clouds, which will proceed to collapse of the matter to the Galactic plane and eventually become the disc component. According to the model, this collapse process happened in a short time scale (a few of $10^8$ yr) and this might cause a metallicity gradient in the Galaxy (cf. [Andrievsky et al., 2002; Karaali, Bilir & Hamzaoglu, 2004; Bilir, Karaali & Gilmore, 2006]). Over time, understanding that the halo global clusters have different ages and different chemical abundances has led [Searle & Zinn (1978)] to suggest an alternative model. In this model, the lack of a metallicity gradient in the halo indicates that the Milky Way was formed by the accretion of clouds with different chemical composition over a long time scale (several of $10^9$ yr).

Vertical metallicity gradients in the Milky Way are deeper than the radial ones, and their numerical values depend on the observed object itself as well as on its position. The numerical values in [Ak et al. (2007a)], $d[\text{Fe}/\text{H}]/dz = -0.38 \pm 0.06$ dex kpc$^{-1}$, and in [Huang et al. (2015)], $d[\text{Fe}/\text{H}]/dz = -0.052 \pm 0.010$ dex kpc$^{-1}$ can be given as two extreme values for the vertical metallicity gradients. The first one was estimated for G spectral type stars in the distance interval $3 \leq z < 5$ kpc while the second one belongs to the red clump.
(RC) stars observed in the distance intervals $|z| \leq 1.9$ kpc and $9 < R \leq 10$ kpc. The deepest radial metallicity gradient is the one in Frinchaboy et al. (2013), i.e. $d[\text{Fe/H}]/dR = -0.20 \pm 0.08$ dex kpc$^{-1}$, estimated for open clusters with galactocentric distance $7.9 < R < 10$ kpc. The radial metallicity gradients in Xiang et al. (2015), $d[\text{Fe/H}]/dR = -0.100 \pm 0.003$ dex kpc$^{-1}$, and in Önal Taș et al. (2016), $d[\text{Fe/H}]/dR = -0.047 \pm 0.003$ dex kpc$^{-1}$ represent the intermediate and relatively low radial metallicity gradients in the literature. The first gradient was estimated for the turnoff stars with distance $|z| \leq 0.1$ kpc and age $2 < t < 16$ Gyr while the second one belongs to RC stars with $|z| \leq 0.5$ kpc. A large reference table related to the metallicity gradients can be found in Önal Taş et al. (2016). Positive metallicity gradients which are usually estimated for stars at relatively large vertical and radial distances (cf. Cheng et al. 2012) confirm the argument that some of the components of our Galaxy are formed from merger and aggregation of numerous fragments (Searle & Zinn, 1978; Freeman & Bland-Hawthorn, 2002).

Metallicity of a star can be determined spectroscopically or photometrically. The first procedure requires high resolution which is available for nearby dwarf and giant stars at different distances. While the second one can be applied to distant stars as well. The distance of a star relative to the Sun is necessary for metallicity gradient estimation which can be determined by using trigonometric or photometric parallaxes. Trigonometric parallax can be estimated only for nearby stars and one of the main sources is the Hipparcos catalogue (ESA, 1997; van Leeuwen, 2007). Another source is the Tycho-Gaia Astrometric Solution (TGAS) catalogue of Michalik, Lindegren & Hobbs (2015) which covers the trigonometric parallaxes (and proper motions) of about $2.5 \times 10^6$ stars supplied from the Gaia Data Release 1 (Lindegren et al. 2016). Photometric parallax of a star can be provided by a combination of its apparent and absolute magnitude (cf. Bilir, Karaali & Tunçel, 2005; Bilir et al. 2008a, 2009), where absolute magnitude needs also to be determined. The procedure usually used for this purpose is based on its offset from the fiducial sequence of a cluster such as Hyades (Laird et al., 1988; Nissen & Schuster, 1991; Karaali et al., 2003a,b, 2005, 2011; Karatas & Schuster, 2006). Alternative methods can be found in Phleps et al. (2000), Chen et al. (2001), Siegel et al. (2002) and Ivezić et al. (2008).

In Tunçel Güçtekin et al. (2016, hereafter Paper I), we calibrated the iron abundance $[\text{Fe/H}]$ and absolute magnitude offset $\Delta M_V$ in terms of the ul-
traviolet (UV) excess $\delta_{0.6}(U - B)$, while in Tuncel Güctekin et al. (2017, hereafter Paper II) we carried out similar calibrations for the ugr photometric data of the Sloan Digital Sky Survey (SDSS, York et al., 2000). In this study we aim to test the mentioned calibrations in Paper II on stars in a large, $5,280 \text{ deg}^2$, high Galactic latitude, $b > 50^\circ$, star field. Thus, we should obtain vertical and radial metallicity gradients, $d[\text{Fe}/\text{H}]/dz$ and $d[\text{Fe}/\text{H}]/dR$ respectively, in this direction of our Galaxy and compare them with the counterparts. The paper is organized as follows: the data are presented in Section 2, estimations of the metallicity gradients are given in Section 3, and finally Section 4 is devoted to the summary and discussion.

2. Data

2.1. The selection of the F-G type main-sequence stars

The de-reddened ugr magnitudes used in this study are provided from the recent survey DR12 of SDSS III (Alam et al., 2015). There are 26,719,324 objects with de-reddened ugriz magnitudes in an area of $5,280 \text{ deg}^2$ centred on the north Galactic pole. The ugr magnitudes and their errors are taken from the SQL webpage of SDSS\footnote{https://skyserver.sdss.org/dr12/en/tools/search/sql.aspx}. We applied a series of constraints, as explained in the following, and obtained a sample of F-G spectral type dwarfs suitable to estimate vertical and radial metallicity gradients. We applied the procedure of Chen et al. (2001), i.e. $g_0 \leq 23$ and $(u - g)_0 > 0.5$ mag, by doing so the extragalactic objects such as quasars are omitted. Thus, the original set of objects is reduced to 16,281,265 stars. We noticed that there are large scattered objects in the $(g - r)_0 \times (r - i)_0$ two-colour diagram (Fig. 1). Hence, we adopted the following equation of Juric et al. (2008) and limited the number of stars to 15,456,605 which lie within $\pm 2\sigma$ of this equation:

$$
(g - r)_0 = 1.39(1 - \exp[-4.9(r - i)_0^3 - 2.45(r - i)_0^2 - 1.68(r - i)_0 - 0.050]).
$$

The next step concerns the giants in this field which are identified by the following set of equations of Helmi et al. (2003, see also Bilir et al. (2008b)):
1.1 \leq (u - g)_0 \leq 2, \\
-0.1 < P_1 < 0.6, \\
|s| > m_s + 0.05.

where $P_1$, $s$ and $m_s$ are defined as follows:

\begin{align*}
P_1 &= 0.910(u - g)_0 + 0.415(g - r)_0 - 1.28, \\
s &= -0.249u_0 + 0.794g_0 - 0.555r_0 + 0.240, \\
m_s &= 0.12.
\end{align*}

This constraint revealed 7,613 giants which reduced our set of stars to 15,448,992 dwarfs (Fig. 2). The colour range of the F-G spectral type main-sequence stars, 0.15 < (g - r)$_0$ < 0.50 mag, further decreased the number of dwarfs down to 4,402,895. The additional constraint on the apparent magnitude, $g_0 \leq 20$, to exclude stars with relatively large errors reduced the number of dwarfs to 1,895,264. The brighter limit for the apparent magnitude is $g_0 = 14$ mag. Finally, we omitted the dwarfs whose metallicities
lie out of the metallicity range of our calibration obtained in Paper II, i.e. $-2 \leq [\text{Fe/H}] \leq 0.2$ dex. Thus, the final F-G spectral type dwarf sample consists of 1,437,467 stars. The magnitude and colour errors for these stars are as follows: $g_{\text{err}} = 0.006$, $(u-g)_{\text{err}} = 0.022$, $(g-r)_{\text{err}} = 0.008$, and $(r-i)_{\text{err}} = 0.008$ mag.

2.2. Metallicities, absolute magnitudes and distances

The $[\text{Fe/H}]$ metallicities and $M_g$ absolute magnitudes for the (final) sample stars are estimated by using the calibrations obtained in Paper II. For the metallicity we used the following equation:

$$[\text{Fe/H}] = 0.105(0.010) - 5.633(0.521) \times \delta_{0.41} + 2.984(1.895) \times \delta_{0.41}^3 - 27.209(16.359) \times \delta_{0.41}^3,$$

where $\delta$ is the ultra-violet excess of a star relative to a Hyades star with the same colour reduced to $(g-r)_0 = 0.41$ mag. While for the absolute magnitude, we estimated an offset $\Delta M_g$ for a given star by the following equation and added it to the absolute magnitude of a Hyades star with the same $(g-r)_0$ colour:

$$\Delta M_g = -0.133(0.031) + 5.169(0.965) \times \delta_{0.41}.$$
\(-3.623(0.978) \times \delta_{0.41}^2 + 21.497(7.799) \times \delta_{0.41}^3.\)

\((M_g)_s = \Delta M_g + (M_g)_H.\)

where \((M_g)_s\) and \((M_g)_H\) correspond to the absolute magnitudes of the star in question and that of the Hyades star with the same \((g - r)_0\) colour.

The distance \(d\) of a star relative to the Sun is estimated by means of the Pogson formula, \(g_0 - M_g = 5 \log d - 5\), while its distance to the Galactic plane, \(z\), is evaluated via the equation \(z = d \times \sin b\) where \(b\) is the Galactic latitude of the star in question. Finally, radial distances \((R)\) of the sample stars are provided by the following equation:

\[ R = \sqrt{R_0^2 + d^2 \cos^2 b - 2R_0 d \cos b \cos l + \frac{d^2 \sin^2 b}{\eta^2}}. \]  

(6)

where \(l\) and \(b\) are the Galactic longitude and latitude of the related star, respectively, and where \(R_0 = 8\) kpc is the distance to the Galactic centre [Majewski, 1993], and \(\eta\) is the oblate parameter of the halo whose numerical value is usually adopted as \(\eta = 0.85\) [Carney et al., 1996]. We used the same equation for the disc stars to avoid any discontinuity in the metallicity distribution versus distances (see Section 3). However we should note that the differences of the radial distances estimated for a spherical and oblate disc galaxy is less than 0.06 kpc.

3. Metallicity gradients

3.1. Radial metallicity gradients

We estimated radial metallicity gradients for 14 sub-samples defined by distances relative to the Galactic mid-plane, as explained in the following. First, we separated the sample stars into 18 Galactic longitude intervals with equal scale, i.e. \(0^\circ < l \leq 20^\circ, 20^\circ < l \leq 40^\circ, \ldots, 320^\circ < l \leq 340^\circ, 340^\circ < l \leq 360^\circ\) and re-separated each of these intervals into 14 sub-samples with distances relative to the Sun, \(0.5 < d \leq 1, 1 < d \leq 1.5, 1.5 < d \leq 2, 2 < d \leq 3, 3 < d \leq 4, 4 < d \leq 5, 5 < d \leq 6, 6 < d \leq 7, 7 < d \leq 8, 8 < d \leq 9, 9 < d \leq 10, 10 < d \leq 12, 12 < d \leq 14\) and \(14 < d \leq 16\) kpc. We performed metallicity histograms for each of these sub-samples and determined their modes. Also, we estimated the median values of the radial \((\bar{R})\) and vertical
(\tilde{z}) distances for the stars in these sub-samples. We present the metallicity histogram for the Galactic longitude interval \(160^\circ < l \leq 180^\circ\) in Fig. 3 and the corresponding derived data just mentioned in Table 1 as an example. Second, we plotted the metallicity modes determined for each of the 14 sub-samples against radial distances (\(\tilde{R}\)) and estimated the radial metallicity gradient for each of the mentioned sub-samples. The results are given in Fig. 4. The radial metallicity gradients in three vertical distance intervals corresponding to short distances, i.e. \(0.44 < z \leq 0.87\), \(0.87 < z \leq 1.31\) and \(1.31 < z \leq 1.74\) kpc, are \(d[\text{Fe}/\text{H}]/dR = -0.010 \pm 0.022\), \(d[\text{Fe}/\text{H}]/dR = -0.028 \pm 0.013\), and \(d[\text{Fe}/\text{H}]/dR = -0.042 \pm 0.010\) dex kpc\(^{-1}\), while for distances larger than \(z = 1.74\) kpc three of the radial metallicity gradients are rather small negative numbers and eight of them are positive (Fig. 5). The \(z\)-intervals just mentioned correspond to the intervals \(0.5 < d \leq 1\), \(1 < d \leq 1.5\), \(1.5 < d \leq 2\) kpc, and the \(z\) values in Fig. 5 are taken from the fifth column of Table 1 which correspond to the \(d\)-values in the first column of the same table. The spatial distribution of the sample stars are given in the \(z-R\) plane in three panels in Fig. 6 with colour-coded for number of stars (\(N\)), the distance relative to the Sun (\(d\)) and the metallicity ([Fe/H]).

### 3.2. Vertical metallicities

The vertical metallicity gradients are estimated for thin disc, thick disc, and halo components of the Galaxy separately. We adopted the galactocentric distance ranges \(6 < R < 10\), \(10 < R < 15\), and \(15 < R < 20\) kpc and carried out this analysis for each range for stars with \(0.5 < z \leq 2\) (thin disc), \(2 < z \leq 5\) (thick disc), and \(z > 5\) (halo) kpc, by using two procedures explained in the following. In the first procedure, the stars are separated into a series of sub-samples by using the three Galactic latitude and four Galactic longitude intervals: \(50^\circ < b \leq 65^\circ\), \(65^\circ < b \leq 80^\circ\), \(80^\circ < b \leq 90^\circ\) and \(0^\circ < l \leq 90^\circ\), \(90^\circ < l \leq 180^\circ\), \(180^\circ < l \leq 270^\circ\), \(270^\circ < l \leq 360^\circ\). The halo stars with \(6 < R \leq 10\) kpc could be observed only in the vertical distance interval \(5 < z \leq 8\) kpc, while those with \(10 < R \leq 15\) kpc and \(15 < R \leq 20\) kpc could be separated into two sub-samples, i.e. \(5 < z \leq 10\), \(10 < z \leq 12\) kpc, and \(7 < z \leq 10\), \(10 < z \leq 15\) kpc respectively. The results are tabulated in Table 2. One can see that vertical metallicity gradients could not be estimated for the halo stars in the second and third quadrants for three Galactic latitudes, corresponding to the interval \(6 < R \leq 10\) kpc, due to the insufficient number of stars. A similar case holds for the halo stars with Galactic latitude \(50^\circ < b \leq 65^\circ\) and, galactocentric distance \(10 < R \leq 15\).
Figure 3: Metallicity histograms for stars with Galactic longitudes $160^\circ < \ell < 180^\circ$. The thin line denotes the Gaussian curve whose mode is fitted to the one of the metallicity distribution. The Gaussian curve deviates gradually from the metallicity distribution at distances larger than $d = 4$ kpc.
Figure 4: Metallicity distribution with respect to the radial distance for stars in 14 sub-samples defined in the text and given in Table 1. Each panel covers the metallicities of stars in 18 Galactic longitude intervals defined in the text. The corresponding metallicity gradients are indicated in each panel. The blue line corresponds to the metallicity calibration in terms of radial distance, while the red bar denotes the metallicity uncertainties.
Table 1: Data for the stars with Galactic longitudes $160^\circ < l \leq 180^\circ$. The columns give: $(d_1 - d_2)$ distances relative to the Sun, $(z_1 - z_2)$ vertical distances, $(N)$ number of stars, $(\bar{R})$ and $(\bar{z})$ median radial and vertical distances, respectively, and ([Fe/H]) metallicity modes with their uncertainties.

| $d_1 - d_2$ (kpc) | $z_1 - z_2$ (kpc) | $N$ | $\bar{R}$ (kpc) | $\bar{z}$ (kpc) | [Fe/H] (dex) |
|-------------------|-------------------|-----|----------------|----------------|------------|
| 0.5-1             | 0.44-0.87         | 3420| 8.43           | 0.74           | $-0.45\pm0.01$ |
| 1-1.5             | 0.87-1.31         | 7094| 8.67           | 1.09           | $-0.61\pm0.00$ |
| 1.5-2             | 1.31-1.74         | 6019| 8.99           | 1.51           | $-0.71\pm0.00$ |
| 2-3               | 1.74-2.62         | 8866| 9.46           | 2.15           | $-0.86\pm0.01$ |
| 3-4               | 2.62-3.49         | 5890| 10.23          | 3.03           | $-1.00\pm0.01$ |
| 4-5               | 3.49-4.36         | 4404| 11.06          | 3.92           | $-1.40\pm0.01$ |
| 5-6               | 4.36-5.23         | 3679| 11.91          | 4.85           | $-1.50\pm0.01$ |
| 6-7               | 5.23-6.10         | 3143| 12.86          | 5.72           | $-1.50\pm0.01$ |
| 7-8               | 6.10-6.98         | 2623| 13.78          | 6.64           | $-1.50\pm0.01$ |
| 8-9               | 6.98-7.85         | 2027| 14.76          | 7.49           | $-1.60\pm0.01$ |
| 9-10              | 7.85-8.72         | 1525| 15.71          | 8.47           | $-1.60\pm0.02$ |
| 10-12             | 8.72-10.46        | 1745| 17.04          | 9.67           | $-1.47\pm0.01$ |
| 12-14             | 10.46-12.21       | 665 | 19.06          | 11.60          | $-1.55\pm0.02$ |
| 14-16             | 12.21-13.95       | 280 | 21.23          | 13.25          | $-1.50\pm0.05$ |

Figure 5: Distribution of the radial metallicity gradients relative to vertical distance, for stars in the 14 sub-samples.
Figure 6: Distribution of the sample stars in the $z - R$ plane in three panels, colour-coded for the number of stars (a), the distance relative to the Sun (b), and the metallicity (c).
kpc, again, in the second and third quadrants. Also, vertical metallicity gradients could not be estimated for the thick-disc stars ($2 < z \leq 5$ kpc) with $80^\circ < b \leq 90^\circ$ and $10 < R \leq 15$ kpc in the first and fourth quadrants, due to the reason just cited. The final sub-samples for which vertical metallicity gradients could not be estimated consist of halo stars with Galactic latitudes $65^\circ < b \leq 80^\circ$ and $80^\circ < b \leq 90^\circ$, and galactocentric distances $15 < R \leq 20$ kpc in the first and fourth quadrants.

The vertical metallicity gradient could be estimated for the thin disc stars only in the galactocentric distance interval $6 < R \leq 10$ kpc. There is no systematic difference between the gradients estimated for the thin disc for different sub-samples. Correspondingly the metallicity gradients estimated for the thin disc are neither Galactic latitude nor Galactic longitude dependent. The vertical metallicity gradient $d[\text{Fe}/\text{H}]/dz = -0.128 \pm 0.072$ dex kpc$^{-1}$ for the thick disc in the galactocentric distance interval $6 < R \leq 10$ kpc is a bit different than the counterparts. The uncertainty of this metallicity gradient is also (absolutely) larger than the counterparts. However, the cited difference can be explained by fewer number of stars used for estimation of the metallicity gradient in question. The different vertical metallicity gradients for the thick disc stars with $10 < R \leq 15$ kpc, such as the one of $d[\text{Fe}/\text{H}]/dz = -0.018 \pm 0.080$ dex kpc$^{-1}$, are due to the fewer number of stars considered in the gradient determination. One can see a small difference between the vertical metallicity gradients, $d[\text{Fe}/\text{H}]/dz = -0.156 \pm 0.038$ and $d[\text{Fe}/\text{H}]/dz = -0.131 \pm 0.038$ dex kpc$^{-1}$, in the second and third quadrants, estimated for the thick-disc stars with $10 < R \leq 15$ kpc and $65^\circ < b \leq 80^\circ$. The first one is compatible with the counterparts in the interval $6 < R \leq 10$ kpc. Hence, it seems that the difference in question originates from the positions of the stars in the third quadrant. The vertical metallicity gradients estimated for the halo stars with different galactocentric distance -and Galactic coordinate- intervals are compatible, with some exceptions however where only a small number of stars are used in the estimate.

In the second procedure, the Galactic latitudes and longitudes are omitted in the definition of the sub-samples. The results are given in the third column of Table 3 and in Fig. 7. The vertical metallicity gradient for the thin disc, $0.5 < z \leq 2$ kpc, is high in the interval $6 < R \leq 10$ kpc, namely $d[\text{Fe}/\text{H}]/dz = -0.308 \pm 0.018$ dex kpc$^{-1}$, while it could not be estimated at higher galactocentric distances due to the insufficient number of stars. The vertical metallicity gradients for the thick disc, $2 < z \leq 5$ kpc, estimated for the intervals $6 < R \leq 10$ and $10 < R \leq 15$ kpc are compatible,
$d[\text{Fe}/\text{H}]/dz = -0.164 \pm 0.014$ dex kpc$^{-1}$ and $d[\text{Fe}/\text{H}]/dz = -0.172 \pm 0.016$ dex kpc$^{-1}$ respectively, while it could not be carried out at distances larger than $R = 15$ kpc. There is a small vertical metallicity gradient for halo stars, $5 < z \leq 8$ kpc, i.e. $d[\text{Fe}/\text{H}]/dz = -0.023 \pm 0.006$ dex kpc$^{-1}$ only in the galactocentric distance interval $6 < R \leq 10$ kpc, while it is close to zero at larger radial distances.

As expected, the vertical metallicity gradient in our study is highest for short vertical distances (for thin-disc stars), while, gradually it becomes lower at higher distances and diminishes at distances larger than $z = 5$ kpc where halo stars dominate. This result can be confirmed by all studies in the literature. The metallicity gradient $d[\text{Fe}/\text{H}]/dz = -0.308 \pm 0.018$ dex kpc$^{-1}$ estimated for the interval $0.5 < z \leq 2$ kpc in our study is similar to the value, $d[\text{Fe}/\text{H}]/dz = -0.320 \pm 0.010$ dex kpc$^{-1}$, obtained in [Yaz & Karaali (2010)] estimated for G type main-sequence stars with $z < 2.5$ kpc, and likewise our gradient $d[\text{Fe}/\text{H}]/dz = -0.164 \pm 0.014$ dex kpc$^{-1}$ corresponding to the interval $2 < z \leq 5$ kpc confirms the metallicity gradient in [Kordopatis et al. (2011)], $d[\text{Fe}/\text{H}]/dz = -0.140 \pm 0.050$ dex kpc$^{-1}$, estimated for the FGK main-sequence stars with $1 \leq z \leq 4$ kpc (see Section 4 for further discussion).

We estimated the uncertainties of the metallicity gradients by a procedure the following. We adopted the uncertainties of the metallicity and absolute magnitude cited in Paper II, i.e. $\Delta[\text{Fe}/\text{H}] = \pm 0.137$ dex and $\Delta M_g = \pm 0.18$ mag, and produced 1,000 catalogues which cover $1.44 \times 10^6$ stars with their metallicities plus uncertainties with range $-0.137 \leq \Delta[\text{Fe}/\text{H}] \leq +0.137$ dex, and distances plus the uncertainties estimated via their absolute magnitude uncertainties, $-0.18 \leq \Delta M_g \leq +0.18$ mag. Then, we estimated the metallicity gradients and the corresponding uncertainties for each catalogue. It turned out that the maximum uncertainty for the metallicity gradient is $\varepsilon_{[\text{Fe}/\text{H}]} = \pm 0.03$ dex kpc$^{-1}$. Actually, the uncertainties for the metallicity gradients in Table 2 and Fig. 4 are less than this value. The uncertainties of the distance are estimated by means of the uncertainties of the apparent magnitude $g$, $-0.025 \leq g \leq 0.025$, and the uncertainties of the absolute magnitude $M_g$, $-0.18 \leq M_g \leq +0.18$. The relative distance uncertainty is as follows: $\%8.6 \leq \Delta d/d \leq \%10.2$.

The uncertainties claimed in Section 3 are the combination of probable and systematic errors estimated as explained in the following. We fitted the $[\text{Fe}/\text{H}]$ metallicities to the corresponding $z$ distances for each sub-sample of stars defined in the tables: $[\text{Fe}/\text{H}] = a_0 z + b_0$. The uncertainty of the inclination for such a linear fitting ($\varepsilon_1$) is the probable error for the metallicities
Figure 7: Vertical metallicity gradients for three galactocentric distance intervals, (a) $6 < R \leq 10$, (b) $10 < R \leq 15$, and (c) $15 < R \leq 20$ kpc separated into different sub-intervals defined by vertical $z$ distances.
in that sub-sample. The uncertainty for the metallicity of a single star in our study is estimated as $\epsilon_2 = \pm 0.03$ dex, while for the distance ($\epsilon_3$) it lies in the interval $0.86 \leq \Delta d/d \leq 10.2$. We reduced the distances $d$ to the $z$ ones, combined them with the corresponding uncertainties and fitted them to the metallicities which are also the combination of the original values and their uncertainties: $[\text{Fe/H}] \pm \epsilon_2 = a_1(Z \pm \epsilon_3) + b_1$. Then, we adopted the difference between two inclinations, $\epsilon_4 = |a_0 - a_1|$, as the systematic error of the metallicity gradients for the sub-sample in question. Then, the final uncertainty is $(\sqrt{\epsilon_1^2 + \epsilon_4^2})$.

4. Summary and Discussion

We applied the metallicity and absolute magnitude calibrations in terms of the UV excess, $\delta_{0.41}$, presented in Paper II, and estimated vertical and radial metallicity gradients for 1,437,467 F-G type main-sequence stars with metallicities $-2 \leq [\text{Fe/H}] \leq 0.2$ dex, provided from the recent survey DR12 of SDSS III (Alam et al., 2015). The radial metallicity gradients are estimated for 14 distance intervals $0.44 < z \leq 0.87$, $0.87 < z \leq 1.31$, ..., $12.21 < z \leq 13.95$ kpc. The evaluation of the gradients is carried out by fitting the mean metallicities of the stars in 18 Galactic longitude intervals, $0^\circ < l \leq 20^\circ$, $20^\circ < l \leq 40^\circ$, ..., $340^\circ < l \leq 360^\circ$ to the corresponding median radial distances, $\tilde{R}$. The inclination of each fit is adopted as the metallicity gradient for the star in question.

The radial metallicity gradients for two intervals with short distances, $0.44 < z \leq 0.87$, $0.87 < z \leq 1.31$ kpc, are less than expected: $d[\text{Fe/H}]/dR = -0.011 \pm 0.022$ dex kpc$^{-1}$ and $d[\text{Fe/H}]/dR = -0.029 \pm 0.013$ dex kpc$^{-1}$, respectively. This is due to the short radial range of the corresponding stars: $7.62 < R < 8.42$ and $7.50 < R < 8.65$ kpc. The radial metallicity gradient $d[\text{Fe/H}]/dR = -0.041 \pm 0.011$ dex kpc$^{-1}$ in the interval $1.31 < z \leq 1.74$ kpc is close to the ones in Boeche et al. (2013): $d[\text{Fe/H}]/dR = -0.059 \pm 0.002$ dex kpc$^{-1}$, and Recio-Blanco et al. (2014): $d[\text{Fe/H}]/dR = -0.058 \pm 0.008$ dex kpc$^{-1}$, where the first gradient is estimated for the F-G main-sequence stars with $4.5 < R < 9.5$ kpc, while the second one is carried out for the FGK main-sequence thin-disc stars. It is interesting that the radial metallicity gradient in Önal Taş et al. (2016), estimated for RC stars at shorter distances $|z| < 0.5$ kpc, $d[\text{Fe/H}]/dR = -0.047 \pm 0.003$ dex kpc$^{-1}$ is almost the same as our gradient $d[\text{Fe/H}]/dR = -0.041 \pm 0.011$ dex kpc$^{-1}$. The radial metallicity gradients for distances larger than $z = 1.74$ kpc are positive or negative.
Table 2: Vertical metallicity gradients for three galactocentric distance intervals, (a) 6 < R ≤ 10, (b) 10 < R ≤ 15, and (c) 15 < R ≤ 20, and 96 sub-samples, as defined in the text.

| z (kpc) | 50° < b < 65° | 65° < b < 80° | 80° < b < 90° |
|--------|---------------|---------------|---------------|
| 6 < R ≤ 10 kpc | | | |
| 0.5 < z ≤ 2 | 0 < l ≤ 90 | N [Fe/H] | d[Fe/H]/dz (dex) | N [Fe/H] | d[Fe/H]/dz (dex) | N [Fe/H] | d[Fe/H]/dz (dex) |
| 0 < l ≤ 180 | 10423 | -0.70 | -0.296±0.015 | 23504 | -0.70 | -0.292±0.020 | 38337 | -0.73 | -0.312±0.023 |
| 180 < l ≤ 270 | 64145 | -0.70 | -0.317±0.016 | 24679 | -0.72 | -0.328±0.019 | 4532 | -0.75 | -0.293±0.040 |
| 270 < l ≤ 360 | 71916 | -0.68 | -0.317±0.025 | 33661 | -0.69 | -0.311±0.022 | 4720 | -0.76 | -0.367±0.055 |
| 2 < z ≤ 5 | 0 < l ≤ 90 | 122960 | -1.12 | -0.166±0.016 | 44165 | -1.13 | -0.162±0.011 | 7014 | -1.13 | -0.174±0.016 |
| 0 < l ≤ 180 | 26706 | -0.98 | -0.128±0.072 | 20450 | -1.05 | -0.151±0.033 | 5621 | -1.07 | -0.171±0.017 |
| 180 < l ≤ 270 | 31401 | -1.03 | -0.150±0.035 | 23535 | -1.06 | -0.169±0.011 | 5705 | -1.15 | -0.180±0.015 |
| 270 < l ≤ 360 | 90278 | -1.11 | -0.160±0.014 | 49471 | -1.09 | -0.171±0.011 | 7127 | -1.15 | -0.158±0.021 |
| 5 < z ≤ 8 | 0 < l ≤ 90 | 34457 | -1.37 | -0.022±0.007 | 10265 | -1.37 | -0.016±0.022 | 746 | -1.37 | -0.032±0.080 |
| 0 < l ≤ 180 | — | — | — | — | — | — | 5 | — | — |
| 180 < l ≤ 270 | — | — | — | — | — | — | 1 | — | — |
| 270 < l ≤ 360 | 28982 | -1.34 | -0.027±0.006 | 12471 | -1.35 | -0.030±0.014 | 825 | -1.39 | -0.044±0.085 |
| 10 < R ≤ 15 kpc | | | | |
| 50° < b < 65° | 65° < b < 80° | 80° < b < 90° |
| 2 < z ≤ 5 | 0 < l ≤ 90 | 542 | -1.30 | -0.018±0.080 | 19 | — | — |
| 0 < l ≤ 180 | 32048 | -1.21 | -0.171±0.014 | 6937 | -1.31 | -0.156±0.038 | 461 | -1.33 | -0.136±0.058 |
| 180 < l ≤ 270 | 33360 | -1.24 | -0.167±0.017 | 8065 | -1.32 | -0.131±0.038 | 464 | -1.37 | -0.196±0.077 |
| 270 < l ≤ 360 | 374 | -1.33 | +0.021±0.084 | 29 | — | — |
| 5 < z ≤ 10 | 0 < l ≤ 90 | 24897 | -1.38 | -0.014±0.005 | 18487 | -1.40 | -0.010±0.004 | 4283 | -1.41 | -0.016±0.008 |
| 0 < l ≤ 180 | 22253 | -1.36 | -0.028±0.025 | 17445 | -1.38 | -0.011±0.006 | 4643 | -1.39 | -0.002±0.004 |
| 180 < l ≤ 270 | 29162 | -1.37 | +0.010±0.013 | 20756 | -1.38 | -0.007±0.005 | 4480 | -1.41 | +0.003±0.012 |
| 270 < l ≤ 360 | 21172 | -1.38 | -0.000±0.005 | 22013 | -1.37 | -0.005±0.006 | 4416 | -1.40 | -0.003±0.007 |
| 10 < z ≤ 12 | 0 < l ≤ 90 | 2888 | -1.40 | -0.034±0.041 | 2814 | -1.41 | +0.016±0.042 | 487 | -1.38 | -0.039±0.076 |
| 0 < l ≤ 180 | — | — | — | 77 | -1.38 | -0.004±0.150 | 149 | -1.36 | -0.005±0.031 |
| 180 < l ≤ 270 | — | — | — | 82 | -1.37 | -0.003±0.032 | 133 | -1.45 | -0.002±0.081 |
| 270 < l ≤ 360 | 2936 | -1.38 | +0.013±0.028 | 3339 | -1.38 | -0.020±0.013 | 517 | -1.43 | -0.047±0.052 |
| 15 < R ≤ 20 kpc | | | |
| 50° < b < 65° | 65° < b < 80° | 80° < b < 90° |
| 7 < z ≤ 10 | 0 < l ≤ 90 | 110 | -1.30 | -0.003±0.096 | — | — | — |
| 0 < l ≤ 180 | 6863 | -1.37 | +0.012±0.024 | 1422 | -1.39 | +0.060±0.068 | 11 | — | — |
| 180 < l ≤ 270 | 8944 | -1.38 | +0.010±0.016 | 1821 | -1.39 | +0.053±0.040 | 7 | — | — |
| 270 < l ≤ 360 | 80 | -1.26 | +0.002±0.086 | — | — | — |
| 10 < z ≤ 15 | 0 < l ≤ 90 | 1925 | -1.31 | +0.052±0.071 | 2101 | -1.40 | +0.014±0.013 | 536 | -1.35 | -0.007±0.028 |
| 0 < l ≤ 180 | 1977 | -1.35 | -0.013±0.013 | 2957 | -1.39 | +0.020±0.013 | 898 | -1.36 | +0.001±0.021 |
| 180 < l ≤ 270 | 3082 | -1.37 | -0.015±0.015 | 3983 | -1.40 | +0.011±0.015 | 850 | -1.41 | +0.023±0.022 |
| 270 < l ≤ 360 | 1946 | -1.36 | +0.020±0.029 | 2577 | -1.37 | -0.000±0.009 | 602 | -1.42 | -0.023±0.026 |

Numbers close zero, which confirm previous findings in the literature. The study of Önal Tas et al. (2016) is an example: d[Fe/H]/dR = -0.001 ± 0.003 and d[Fe/H]/dR = +0.015 ± 0.008 dex kpc⁻¹ were estimated for the intervals 0.5 < |z| ≤ 1 and 1 < |z| ≤ 3 kpc, respectively.
Table 3: Vertical metallicity gradients estimated by using the observed (third column) and synthetic (fourth column, *Galaxia*) data.

| $R$ interval (kpc) | $z$ interval (kpc) | $d[\text{Fe}/\text{H}]/dz$ (dex kpc$^{-1}$) | $d[\text{Fe}/\text{H}]/dz$ (dex kpc$^{-1}$) |
|-------------------|-------------------|---------------------------------|---------------------------------|
| $6 < R \leq 10$   | $0.5 < z \leq 2$  | $-0.308 \pm 0.018$              | $-0.333 \pm 0.036$              |
|                   | $2 < z \leq 5$    | $-0.164 \pm 0.014$              | $-0.108 \pm 0.006$              |
|                   | $5 < z \leq 8$    | $-0.023 \pm 0.006$              | $-0.143 \pm 0.014$              |
| $10 < R \leq 15$  | $2 < z \leq 5$    | $-0.172 \pm 0.016$              | $-0.233 \pm 0.015$              |
|                   | $5 < z \leq 10$   | $-0.007 \pm 0.003$              | $-0.034 \pm 0.005$              |
|                   | $10 < z \leq 12$  | $-0.002 \pm 0.007$              | $+0.006 \pm 0.008$              |
| $15 < R \leq 20$  | $7 < z \leq 10$   | $+0.008 \pm 0.014$              | $-0.011 \pm 0.007$              |
|                   | $10 < z \leq 15$  | $+0.002 \pm 0.004$              | $-0.012 \pm 0.003$              |

We adopted the galactocentric distance ranges $6 < R \leq 10$, $10 < R \leq 15$ and $15 < R \leq 20$ kpc and estimated vertical metallicity gradients for stars with $0.5 < z \leq 2$ (thin disc), $2 < z \leq 5$ (thick disc), and $z > 5$ (halo) kpc by using two procedures. In the first procedure the stars are separated into a series of sub-samples by using three Galactic latitude and four Galactic longitude intervals: $50^\circ < b \leq 65^\circ$, $65^\circ < b \leq 80^\circ$, $80^\circ < b \leq 90^\circ$ and $0^\circ < l \leq 90^\circ$, $90^\circ < l \leq 180^\circ$, $180^\circ < l \leq 270^\circ$, $270^\circ < l \leq 360^\circ$. The halo stars with $6 < R \leq 10$ kpc could be observed only in the vertical distance interval $5 < z \leq 8$, kpc, while those with $10 < R \leq 15$ kpc and $15 < R \leq 20$ kpc could be separated into two sub-samples, i.e. $5 < z \leq 10$, $10 < z \leq 12$ kpc, and $7 < z \leq 10$, $10 < z \leq 15$ kpc respectively. Vertical metallicity gradients could not be estimated for the thick disc stars with $10 < R \leq 15$ kpc and $80^\circ < b \leq 90^\circ$ in the first and fourth quadrants due to a insufficient number of stars in these sub-samples. A similar case holds for the following halo stars: a) stars with $6 < R \leq 10$ kpc and $50^\circ < b \leq 90^\circ$ in the second and third quadrants, b) stars with $10 < R \leq 15$ kpc, $50^\circ < b \leq 65^\circ$ and $10 < z \leq 12$ kpc in the second and third quadrants, and c) stars with $15 < R \leq 20$ kpc, $65^\circ < b \leq 90^\circ$ and $7 < z \leq 10$ kpc in the first and fourth quadrants.

The vertical metallicity gradient could be estimated for the thin-disc stars only in the galactocentric distance interval $6 < R \leq 10$ kpc. There is no any systematic difference between the gradients estimated for the thin disc for different sub-samples. The thick disc is dominant in two galactocentric
distance intervals, \(6 < R \leq 10\) and \(10 < R \leq 15\) kpc, and the gradients estimated for different sub-samples are compatible with two exceptions. The first one is related with less number of stars, such as \(d[\text{Fe}/\text{H}]/dz = -0.018 \pm 0.080\) dex kpc\(^{-1}\) in the sub-sample defined by \(10 < R \leq 15\) kpc, \(50^\circ < b < 65^\circ\) and \(0^\circ < l \leq 90^\circ\). While the second one, i.e. \(d[\text{Fe}/\text{H}]/dz = -0.131 \pm 0.038\) dex kpc\(^{-1}\) estimated for stars with \(10 < R \leq 15\) kpc, \(65^\circ < b < 80^\circ\) and \(180^\circ < l \leq 270^\circ\) could not be explained. The vertical metallicity gradients estimated for the halo stars with different galactocentric distance- and Galactic coordinate- intervals are compatible, with a few exceptions where a small number of stars are used in the study.

The second set of vertical metallicity gradients is free of Galactic latitude and longitude. The high gradients cover the short vertical -and radial- distances, while they decrease gradually with increasing vertical distances and become almost zero at large distances. The highest vertical metallicity gradient \(d[\text{Fe}/\text{H}]/dz = -0.308 \pm 0.018\) dex kpc\(^{-1}\) estimated for the thin-disc stars in our study is close to the value \(d[\text{Fe}/\text{H}]/dz = -0.305 \pm 0.011\) dex kpc\(^{-1}\) given in Hayden et al. (2014) as well as \(d[\text{Fe}/\text{H}]/dz = -0.290 \pm 0.060\) dex kpc\(^{-1}\) in Marsakov & Borkova (2006) estimated for red-giant stars with \(0 < |z| \leq 2\) kpc and thin disc F-G type stars, respectively. The two vertical metallicity gradients estimated for the thick-disc stars for the intervals \(6 < R \leq 10\) and \(10 < R \leq 15\) kpc in our study, \(d[\text{Fe}/\text{H}]/dz = -0.164 \pm 0.014\) dex kpc\(^{-1}\) and \(d[\text{Fe}/\text{H}]/dz = -0.172 \pm 0.016\) dex kpc\(^{-1}\) are compatible with the vertical metallicity gradients in Ak et al. (2007b) and Kordopatis et al. (2011), i.e., \(d[\text{Fe}/\text{H}]/dz = -0.160 \pm 0.020\) dex kpc\(^{-1}\) and \(d[\text{Fe}/\text{H}]/dz = -0.140 \pm 0.050\) dex kpc\(^{-1}\) which were estimated for G type main-sequence and FGK main-sequence stars, respectively. However, the distance ranges covered by the sample stars in the cited studies are different, \(z < 3\) kpc (north) and \(1 \leq z \leq 4\) kpc, respectively. Vertical metallicity gradients for stars at distances larger than \(z = 5\) kpc, where the halo component dominates, is almost zero. A case which is valid in the studies appeared in the literature.

In summary, the vertical metallicity gradients are high at short vertical distances, while they become lower at higher vertical distances. Also, the vertical metallicity gradients of stars of different populations (dwarfs, giants, etc.) may be compatible, though their distance ranges may be different. Some vertical metallicity gradients that appeared in the literature and are cited in our study, were attributed to the three main populations of our Galaxy, i.e. thin and thick discs and halo. The highest vertical metallicities belong to the thin-disc stars very close to the Galactic plane, and the smallest
ones (absolutely small numbers) were obtained at large vertical distances, i.e. 
$z > 5$ kpc, and were attributed to halo stars. While the vertical metallicity 
gradients determined for the thick-disc stars, that lie between those of the 
thin-disc and of the halo. However, the vertical metallicity gradients and the 
distance ranges of the corresponding stars cited by different researchers for 
a given population may be different. In our study, the metallicity gradient 
$\frac{d[\text{Fe}/\text{H}]}{dz} = -0.308 \pm 0.018 \text{ dex kpc}^{-1}$, estimated for the stars with $0.5 < z \leq 2$ kpc can be attributed to the thin-disc stars. While the two values, 
$\frac{d[\text{Fe}/\text{H}]}{dz} = -0.164 \pm 0.014$ and $\frac{d[\text{Fe}/\text{H}]}{dz} = -0.172 \pm 0.016 \text{ dex kpc}^{-1}$ 
estimated for the stars with $2 < z \leq 5$ kpc correspond to the thick-disc 
stars. In contrast, the metallicity gradients $-0.023 \leq \frac{d[\text{Fe}/\text{H}]}{dz} \leq 0.008$ 
dex kpc$^{-1}$ estimated for stars with $z > 5$ kpc belong to the halo stars.

The three radial metallicity gradients estimated for stars with $0.44 < z \leq 
0.87$, $0.87 < z \leq 1.31$ and $1.31 < z \leq 1.74$ kpc can be attributed to thin-disc 
stars. The lack of a radial metallicity gradient for stars with $z > 1.74$ kpc 
indicates that the thick disc has not undergone a radial collapse phase as 
observed in recent spectroscopic surveys (Cheng et al., 2012; Anders et al., 
2014; Hayden et al., 2014; Recio-Blanco et al., 2014).

Our final comparison is carried out between the vertical metallicity gradients 
estimated in our study and those produced by Galaxia (Sharma et al., 2011). 
Galaxia is a C++ code to generate a synthetic structure of the Milky 
Way for different sky surveys. Given one or more colour-magnitude diagrams, 
a survey size and geometry, the code returns a catalogue of stars in accordance 
with a given model of our Galaxy. In our case, the synthetic stars are 
generated for a field with size $5.280 \text{ deg}^2$, centered at the north Galactic pole. 
We applied the constraints presented in Section 2.1 and obtained 2,230,167 
synthetic stars for our purpose. As we could not detect a strong indication 
for the dependence of our vertical metallicity gradients on the Galactic 
latitude and longitude, we restricted our comparison on the sub-samples 
defined by the galactocentric distance ranges $6 < R \leq 10$, $10 < R \leq 15$ and 
$15 < R \leq 20$ kpc. As in the observational data, the gradients are carried 
out for the thin disc ($0.5 < z \leq 2$ kpc), thick disc ($2 < z \leq 5$ kpc), and 
halo ($z > 5$ kpc). We produced synthetic $[\text{Fe}/\text{H}]$ metallicities and vertical 
$z$-distances for 2,230,167 F-G type main-sequence stars with Galactic coordinates 
$50^\circ < b < 90^\circ$ and $0^\circ < l \leq 360^\circ$ for our purpose. The results are 
given in the last column of Table 3 and in Fig. 8. The distribution of the 
(synthetic) stars corresponding to the lines in Table 3 from top to bottom is 
as follows: 788,679; 852,941; 142,718; 113,438; 238,559; 23,898; 24,940; and
There is a good agreement between the observed and synthetic vertical metallicity gradients for three galactocentric distance intervals in the *Galaxia* sample, (a) $6 < R \leq 10$, (b) $10 < R \leq 15$, and (c) $15 < R \leq 20$ kpc. Each interval is separated into sub-intervals of $z$-distance.
metallicity gradients for the thin disc component of our Galaxy (first line in Table 3). However, one can not confirm the same agreement for the thick disc, neither with stars with $6 < R \leq 10$ kpc (second line) nor with the ones with $10 < R \leq 15$ kpc (fourth line). It is interesting that the mentioned two observed gradients, i.e. $d[\text{Fe/H}]/dz = -0.164 \pm 0.014$ dex kpc$^{-1}$ and $d[\text{Fe/H}]/dz = -0.172 \pm 0.016$ dex kpc$^{-1}$, are compatible while the synthetic counterparts are different, i.e. $d[\text{Fe/H}]/dz = -0.108 \pm 0.006$ dex kpc$^{-1}$ and $d[\text{Fe/H}]/dz = -0.233 \pm 0.015$ dex kpc$^{-1}$. Hence, we can argue that the (observed) vertical metallicity gradients estimated for the thick disc are more reliable than the synthetic ones. For the halo, we could detect only a difference between the observed and synthetic vertical metallicity gradients, i.e. $d[\text{Fe/H}]/dz = -0.023 \pm 0.006$ dex kpc$^{-1}$, $d[\text{Fe/H}]/dz = -0.143 \pm 0.014$ dex kpc$^{-1}$ (third line) which corresponds to the stars with $6 < R \leq 10$ kpc. While they are compatible in four vertical distance intervals (fifth, sixth, seventh and eight lines) at larger galactocentric distances. Finally, we should note that the observed vertical metallicity gradients estimated in our study are in agreement with those in the literature. Hence, some differences between the observed and synthetic vertical metallicity gradients just cited may originate from the “input data” of Galaxia.

We investigated the probable impact of the streams and substructures on our results. The spatial distribution of our sample stars in Galactic coordinates is $50^\circ < b \leq 90^\circ$ and $0^\circ < l \leq 360^\circ$. Also, they are apparently brighter than $g = 20$ mag and at distances $d \leq 16$ kpc relative to the Sun. Then, there are three candidate streams, Sagittarius, Hercules-Aquila and Virgo Overdensity, which would impact our results. The spatial distributions in Galactic coordinates, their lengths and the distances of these streams are tabulated in Table 4. The distance of the Sagittarius stream cited in Ibata et al. (2001), $d = 46$ kpc, is out of the distance range in our study. Hence, one can not expect any impact on our results. The impact of the same stream in Belokurov et al. (2006) is limited with distance $d = 15$ kpc and Galactic latitude $b = 50^\circ$. However, the Galactic longitude interval for this stream is $180^\circ < l \leq 230^\circ$, and one can not detect any mean metallicity different than its counterparts in any third quadrant in Table 2. Hence, we can say that there is no any impact of the stream just claimed on our results. The spatial distributions of the streams Sagittarius, Hercules-Aquila, and Virgo Overdensity in Koposov et al. (2012), Simion et al. (2014), and Carlin et al. (2012), respectively, and the stars in our study have limited overlap in spatial distribution in Galactic coordinates and distance. There-
fore the metallicity gradients are not contaminated by those of the stars in
the cited streams. Actually, the mean metallicity, \( \langle [\text{Fe}/\text{H}] \rangle \), for stars in our
study with Galactic longitude corresponding to the stream in question is
compatible with those estimated for stars in different quadrants for a given
vertical distance interval. This is a strong indication for our argument. As
an example, we compare the mean metallicities for stars with \( 10 < R \leq 15 \)
kpc, \( 2 < z \leq 5 \) kpc and \( 50^\circ < b \leq 65^\circ \) in Table 2. The mean metal-
licities \( [\text{Fe}/\text{H}] = -1.24 \) dex and \( [\text{Fe}/\text{H}] = -1.33 \) dex in the Galactic lon-
gitude intervals \( 180^\circ < l \leq 270^\circ \) and \( 270^\circ < l \leq 360^\circ \) which cover the
Galactic longitude distribution \( 180^\circ < l \leq 350^\circ \) of the Sagittarius stream in
Koposov et al. (2012), are compatible with those in the Galactic longitude
intervals \( 0^\circ < l \leq 90^\circ \) and \( 90^\circ < l \leq 180^\circ \), i.e. \( [\text{Fe}/\text{H}] = -1.30 \) dex and
\( [\text{Fe}/\text{H}] = -1.24 \) dex.

We could detect 4,819 main sequence stars with \( S/N \geq 50 \) and \( 15 < g < 17 \) mag for
which SDSS spectroscopic metallicities are available in the SEGUE survey, and compared these metallicities with those determined pho-
tometrically in our study. The result is given in Fig. 9. The mean and
the standard deviation of the differences between the two sets of metal-
icities are \( \langle \Delta[\text{Fe}/\text{H}] \rangle = 0.13 \) and \( \sigma = 0.33 \) dex respectively. The large dis-
persion is due to the medium-resolution of the spectra (Lee et al., 2008a,b;
Allende Prieto et al., 2008).

We used the oblate parameter \( \eta = 0.85 \) in our calculations. However,
we tested three more values, \( \eta = 0.9, 0.95 \) and \( 1 \), just to see the difference.
The ranges of the metallicity gradients for the sub-intervals \( 5 < Z \leq 8, \)
\( 5 < Z \leq 10, 10 < Z \leq 12, 7 < Z \leq 10 \) and \( 10 < Z \leq 15 \) kpc for different
three radial distance intervals are \( d[\text{Fe}/\text{H}]/dz : [-0.023, -0.017], [-0.007, -0.006], [-0.002, 0.000], [+0.009, +0.005] \) and
\([+0.002, +0.013]\) dex kpc\(^{-1}\), respectively. Then, we can say that a different oblate parameter would not
change our results.

Improving the new metallicity and absolute magnitude calibrations by us-
ing the ultraviolet-excesses of stars provided from the Gaia era, and their
applications to the photometric data observed in the deep sky survey pro-
grams will be an important contributor for understanding of the structure,
formation and evolution of the Galaxy, as well as for testing the Galactic
models.
Table 4: Stellar streams in the star field used in this study. The columns give: the name of the stream, the Galactic coordinates, the extension of the stream, distance relative to the Sun, and reference.

| Designation       | $l$   | $b$   | $L$  | $d$  | Reference                  |
|-------------------|------|------|-----|-----|---------------------------|
| Sagittarius       | 350  | 50   | 50  | 46  | Ibata et al. (2001)       |
| Sagittarius       | 180  | < $l$ < 230 | 50 | 45  | 15 | Belokurov et al. (2006)   |
| Sagittarius       | 180  | < $l$ < 350 | < $b$ < 70 | — | 10-60 | Koposov et al. (2012)     |
| Hercules-Aquila   | < $l$ < 30 | 50   | —   | 1-6 | 10-20 | Simion et al. (2014)      |
| Virgo Overdensity | 265  | < $l$ < 283 | < $b$ < 68 | — | 14 | Carlin et al. (2012)      |

Figure 9: Comparison of the metal abundances for 4,189 stars estimated by the photometric and spectroscopic procedures.
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