A Nonlinear Weighted Anisotropic Total Variation Regularization for Electrical Impedance Tomography

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Abstract—This article proposes a nonlinear weighted anisotropic total variation (NWATV) regularization technique for electrical impedance tomography (EIT). The key idea is to incorporate the internal inhomogeneity information (e.g., edges of the detected objects) into the EIT reconstruction process, aiming to preserve the conductivity profiles (to be detected). We study the NWATV image reconstruction using a novel soft thresholding-based reformulation included in the alternating direction method of multipliers (ADMM). To evaluate the proposed approach, numerical simulations and human EIT lung imaging are carried out. It is demonstrated that the properties of the internal inhomogeneity are well-preserved and improved with the proposed regularization approach, in comparison to traditional total variation (TV) and recently proposed fidelity embedded regularization (FER) approaches. Owing to the simplicity of the proposed method, the computational cost is significantly decreased compared with the well-established primal-dual algorithm. Precisely, with the proposed algorithm, we are able to alleviate the staircase effect arising in TV regularization and improve the reconstruction accuracy for FER does. To achieve similar accuracy as TV does, the computational times are reduced from 2.311 to 0.629 and 1.733 to 0.428 s in 2-D and 3-D simulations, respectively, and the computational time is reduced from greater than 2 s to less than 0.2 s in the human experiment. Meanwhile, it was found that the proposed regularization method is quite robust to the measurement noise, which is one of the main uncertainties in EIT.

Index Terms—Anisotropic total variation (TV), electrical impedance tomography (EIT), lung imaging, nonlinear weighted, regularization.

I. INTRODUCTION

ELECTRICAL impedance tomography (EIT) aims to reconstruct the (change in) conductivity distribution inside objects by injecting a current and measuring the voltage responses through pairs of surface electrodes mounted on the object. EIT has the advantages of being noninvasive, portable, low cost, capable of high temporal resolution, long duration and continuously monitoring, and much more [1], [2], [3]. These advantages make EIT useful for bedside medical apparatus in clinical applications. For this, EIT was commercialized and introduced in medical applications since the 1980s [4]. However, EIT has not yet been widely used in routine clinical applications due to the fact that it is a diffusive modality.

The existing reconstruction algorithms include the back-projection method [4], the D-bar method [5], [6], [7], Newton’s one-step error reconstructor (NOSE) [8], the method based on deep learning [9], etc. To date, many groups published their EIT database, e.g., Finnish Inverse Problems Society for phantom experiment [10] and Michael and Thomas’ [11] group for human experiment. Since linearization does not cause an error in shape reconstruction [12], the EIT reconstruction process is commonly recasted into a (least-square-based) data-fitting inverse problem between the boundary measurement and the computational data. To deal with the ill-posedness, regularization techniques are widely added to the data-fitting to attract the solution satisfying application-driven constraints. Depending on the form of different constraints, the regularization methods can be roughly classified into three categories: projection-based regularization, a priori conductivity-based penalization, and learning-based regularization.

For projection-based regularization, typical examples are the truncated singular value decomposition (tSVD) method [13], [14] and principle component analysis (PCA) method [15]. Even though these methods are capable of providing stable reconstructions, they generally produce ringing artifacts due to the dropout of certain frequency components [16].

In the case of penalty-based regularization, typical examples include Tikhonov regularization [8], [15] and its multiplicative form [17], monotonicity-based regularization [18],
factorization-based regularization [16], and sparsity-based regularization [19], [20], [21]. These methods are able to provide stable reconstructions at the cost of blurring the edges of internal inhomogeneities. The total-variation (TV)-based regularization [22], [23] and its variants [24], [25], [26] have the advantage of preserving the discontinuities of the internal structures especially for dealing with the cases of piecewise constant conductivity distributions. Since TV regularization is nondifferentiable, the so-called primal-dual algorithm [23] is usually used to deal with the nondifferentiability. However, this method needs to handle two optimization problems and hence, it is time-consuming [27] and could produce pseudoedges and lead to staircase effects in the reconstruction. Meanwhile, the split Bregman method and the first-order TV regularization [28] are also used at the cost of decreasing the effect of edge preserving [29].

Lee et al. [30] proposed a so-called fidelity embedded regularization (FER) technique. Using this method, high-quality geometries of the internal inhomogeneities can be obtained. However, since the regularization in the method does not depend on the internal structures, the accuracy of values of the reconstructed conductivity, which could be useful information for clinical use, cannot be guaranteed. In actuality, numerical simulations show that when the regularization parameter was set to be infinity, the estimated conductivity is usually far away from the true value (see Section V for details).

Recently, regularization based on manifold learning [31] has been published using the results in [30] as the training data. Since EIT has not been widely used in clinical applications, it is difficult to obtain enough training data to improve the performance continuously.

In this work, we proposed a nonlinear weighted anisotropic TV regularization approach for the EIT reconstruction problem. In comparison to the well-established isotropic TV regularization methods, the nonlinear weighted anisotropic regularization takes use of a nonlinear weight related to the conductivity distribution in the internal structures. Moreover, a soft thresholding formula is derived in the alternating direction method of multipliers (ADMM) [32]-type algorithm to accelerate the reconstruction. Compared with the anisotropic TV regularization method [23], [28], the proposed regularization approach uses nonlinear weight to pull back the internal edges from possible distortions along the coordinate axes [33] and provides more accurate EIT reconstructions. To contextualize the proposed regularization among contemporaries, Table I provides a comparison of the pros and cons associated with the existing EIT regularization methods in the field of EIT. To validate the proposed method, we conduct 2-D and 3-D numerical simulations and a human EIT lung imaging experiment. These experiments are carried out to illustrate the main advantages of the proposed nonlinear weighted anisotropic TV (NWATV)-based method with respect to the existing TV, first-order TV, and FER regularization methods.

The remaining sections of this article are organized as follows. In Section II, we review the forward and inverse problems in EIT. Then we describe the proposed nonlinear weighted anisotropic TV regularization approach in Section III and provide the reconstruction algorithm in Section IV. Next, we provide the numerical and human lung experiments in Section V. Finally, we provide a discussion in Section VI and conclude the article in Section VII.

II. Forward and Inverse Problems in EIT

Let $\Omega \subset \mathbb{R}^n$ ($n = 2, 3$) be the imaged object with a smooth boundary. Let $\gamma = \sigma + i\omega \epsilon$ represent the admittivity distribution of the region $\Omega$. Here, the conductivity $\sigma$ and permittivity $\epsilon$ depend on position $r = (x, y, z)$ and angular frequency $\omega$.

For an $E$-channel EIT system, $E$ electrodes $E_1, E_2, \ldots, E_E$ are attached on $\partial \Omega$, the boundary of $\Omega$. We inject a series of time-harmonic currents with magnitude $I$ mA following, e.g., the neighboring protocol. Under such a protocol, we sequentially inject several currents using pairs of electrodes $(E_j, E_{j+1})$ for $j = 1, \ldots, E$, where we assign $E + 1$ to be 1. The induced electric potential $u^j$ is governed by the following elliptic partial differential equation (PDE) with mixed boundary conditions:

$$
\begin{align*}
\nabla \cdot (\sigma \nabla u^j) &= 0, \quad \text{in } \Omega \\
\sigma \nabla u^j \cdot n &= 0, \quad \text{on } \partial \Omega \setminus \bigcup_{i=1}^{E} E_i \\
\int_{E_i} \sigma \frac{\partial u^j}{\partial n} dS &= I - \int_{E_{i+1}} \sigma \frac{\partial u^j}{\partial n} dS \\
u^j + z_i \sigma \frac{\partial u^j}{\partial n}_{|E_i} &= U^j, \quad i, j = 1, 2, \ldots, E \\
\int_{E_i} \sigma \frac{\partial u^j}{\partial n} dS &= 0, \quad k \in \{1, 2, \ldots, E\} \setminus \{j, j + 1\}.
\end{align*}
$$

Here, $z_i$ is the contact impedance between the electrode and $\partial \Omega$, and $U^j$ is the voltage potential on $E_i$ caused by the injection currents using the electrode pair $(E_j, E_{j+1})$. Using an EIT measurement device, the quantity $V^j_i[\sigma] = U^j_i[\sigma] - U^j_{i+1}[\sigma]$ is measurable. Note that $z_i$ is unknown; however, for $i \in \{1, 2, \ldots, E\} \setminus \{j - 1, j, j + 1\}$, the contact impedance can be neglected [30] and hence $V^j_i \approx u^j_i|_{E_i} - u^j_i|_{E_{i+1}}$.

To reconstruct $\sigma$, EIT uses the following reciprocity principle:

$$
V^j_i[\sigma] \approx u^j_i|_{E_i} - u^j_i|_{E_{i+1}} = \frac{1}{T} \int_{\Omega} \sigma(r) \nabla u^j_i(r) \cdot \nabla u^j_i(r) dr.
$$

EIT uses $E \times (E - 3)$ data $V_i^j$ and (2) to reconstruct the unknown quantity $\sigma$. However, the above problem is nonlinear and ill-posed. Linearization is usually applied to deal with the nonlinearity and regularization is used to handle the ill-posedness.

To be precise, we assume that $\sigma$ is a perturbation of a constant $\sigma_0$, that is, $\sigma(r) = \sigma_0 + \delta \sigma(r)$ for $r \in \Omega$. Here, $\delta \sigma$ has compact support in $\Omega$. Then $\delta \sigma$ approximately satisfies the following integral equation:

$$
\int_{\Omega} \delta \sigma(r) \nabla u^j_0(r) \cdot \nabla u^j_0(r) dr = I \delta V^j_i[\sigma]
$$

where $\delta V^j_i[\sigma] = V^j_i[\sigma] - V^j_i[\sigma_0]$ and $u^j_0$ is the solution of (1) with $\sigma$ replaced by $\sigma_0$.

Discretize the domain $\Omega$ as $\Omega = \bigcup_{k=1}^{N} T_k$, where $T_k$ is the triangular element, and then $\sigma|_{T_k}$ can be considered as a constant $\sigma_k$. $\sigma$ can be approximated by the vector...
### Table I

**Comparisons of the Pros and Cons of Different Regularization Schemes in EIT**

| Categories          | Name of regularizer                                      | Pros                                                                 | Cons                                                                 |
|---------------------|-----------------------------------------------------------|----------------------------------------------------------------------|----------------------------------------------------------------------|
| **Projection method** | GSV method \[13], \[14\]                                 | Singular value of $S >$ a threshold                                  | Easy to implement.                                                   |
|                     | Principle component analysis (PCA) \[15\]                | $\sigma$ lies in the solution space $S_{\omega}$                     | Apply geometrical information from MRI and CT etc.                  |
|                     | Tikhonov \[8], \[15\]                                    | $\|\sigma - \sigma_0\|_{L^2(\Omega)}$                                | The optimization problem is convex and differentiable.              |
|                     | Multiplicative Tikhonov regularization \[17\]           | $\left| \frac{\nabla \sigma^2 + \delta_{n-1}^2}{|\nabla \sigma^{n-1}|^2 + \delta_{n-1}^2} \right|_{L^2(\Omega)}$ | Reconstruction results are independent of the regularization parameter choice. |
|                     | Total variational (TV) regularization \[23\]       | $\int_\Omega |\nabla \sigma|$                                          | Can preserve the edge of internal inhomogeneities                     |
| **a priori conductivity-based penalty** | TGV \[21], \[34\]                                    | $TGV^0(\sigma)$                                                      | Efficiency in removing the staircase effect when using TV for piecewise linear conductivity distribution. |
|                     | First-order TV regularization \[28], \[33\]           | $\|\nabla \sigma\|_{L^1(\Omega)}$                                  | The imaging speed is high                                            |
|                     | Monotonicity based method \[18\]                       | The sign of $\frac{\partial \sigma}{\partial t}$ coincides with the breathing process. | No confusion between breathing and expiration.                       |
|                     | Factorization based method \[16\]                      | Highlight the right singular vector associate with high singular values. | Alleviated Gibbs ringing artifacts                                   |
|                     | Wavelet frame TGV \[26\]                                | TGV $+ \|\lambda \cdot W\sigma_{recon}\|_0$                         | Using the $L^0$ norm of the wavelet frame to sharpen the TGV reconstructed conductivity |
|                     | Fidelity-embedded regularization \[30\]                | $\sqrt{\sum_{i} (S_h, S_i)}$                                      | Reconstruction results are independent of the regularization parameter choice and fast imaging due to only a direct algebraic inversion is needed to compute. |
| **Learning method**   | Manifold learning method \[31], \[35\], \[36\]          | $\sigma \in$ a manifold $\mathcal{M}$                                | Imaging quality is higher than the model-based regularizer.          |
|                     | Proposed method                                           | $\left| \frac{\nabla \sigma^2}{|\nabla \sigma|^2} \right|_{L^1(\Omega)}$ | Improved preservation of the internal inhomogeneity edges; the computational time is significantly decreased; robust to the additional Gaussian random noise. |

Reconstruction results depend on an artificially chosen regularization parameter. Inclusions are blurred.

A weighted $L^2$ regularizer could still blur the image. In each step, numerical integral and differentiation are needed to calculate. Moreover, Gauss-Newton method with step size using line search method is used to solve a non-quadratic cost functional which is time consuming. Staircase effect and pseudo edge exists. Reconstructing is slow and resolution is high when the prime-dual algorithm is used while resolution is low and reconstructions is fast when the Split-Bregman method is used to minimized the TV constrained optimal problem. In some imaging cases such as human lung imaging, piecewise linear is not a reasonable assumption. Balancing efficiency and quality is difficult as that in TV regularization. The capacity of edge preserving is lower than TV. Distort the inclusion boundaries along the coordinate axes. Still a kind of $L^2$ regularization which could blur the internal edges. Heuristical argument without strict mathematical theory. Primal-dual method is used for the TGV minimization, reconstruction is time consuming. Postprocessing the TGV reconstructions needs more time. The accuracy of the reconstruction can not be guaranteed due to the lack of mathematical background.
\(\sigma := (\sigma_1, \sigma_2, \ldots, \sigma_N)^T\). Then we convert (3) into the following system:

\[
S\delta\sigma = \partial V
\]

where \(S\) is an \(E(E - 3) \times N\) sensitivity matrix whose element \(S_{pq} = (1/n) \int_{\Omega} \nabla u_{pq}^m(r) \cdot \nabla u_{pq}^n(r) dr\), where \(m = [(p - 1)/(N - 3)] + 1\), \(n = p - 12[(p - 1)/(N - 3)] + 2\), \(\delta V \in \mathbb{R}^{E(E - 3)}\) whose \(q\)th element is \(V_q = \delta V_{pq}\), and the operator \(\lfloor \cdot \rfloor\) represents the largest integer less than \(\cdot\).

### III. NONLINEAR WEIGHTED ANISOTROPIC TV REGULARIZATION

To solve (4), we reformulate it to the following least-squares problem:

\[
\delta\sigma^* = \arg \min_{\delta\sigma} \|S\delta\sigma - \partial V\|_I^2
\]

where \(\|\cdot\|_I\) denotes the Euclidean norm in \(\mathbb{R}^{E(E - 3)}\).

Let \(s_l\) denote the \(l\)th column vector of \(S\) for \(l = 1, 2, \ldots, N\). The condition between the \(l\) and \(k\) column vectors \(((s_l/|s_l|), (s_k/|s_k|))\) decreases rapidly as the distance between \(T_l\) and \(T_k\) increases [30]; this makes the condition number of the matrix \(S^T S\) to be approximately infinity. Hence, the minimization problem (5) is unstable against the measurement errors and noises in \(\partial V\). We use the following minimization problem to approximate (5):

\[
\delta\sigma^*_l = \arg \min_{\delta\sigma} \left\{ \frac{1}{2} \|S\delta\sigma - \partial V\|_I^2 + \lambda \text{Reg}(\delta\sigma) \right\}.
\]

Here, instead of minimizing the term \(\|S\delta\sigma - \partial V\|_I^2\), we also minimize the other regularization term \(\text{Reg}(\delta\sigma)\), where \(\lambda\) is a parameter balancing the fidelity term \(\|S\delta\sigma - \partial V\|_I^2\) and the regularization term. The choice of regularization depends on the a priori information of the conductivity.

Following, for ease of explanation we write \(\Omega = \bigcup_{i=1}^{n_f} D_i \cup (\Omega \setminus \bigcup_{i=1}^{n_f} D_i)\), where \(D_i (i = 1, 2, \ldots, n_f)\) represents \(n_f\) internal structures with \(n_f\) being a positive integer. Note that \(\zeta(r) := 1/|\nabla \sigma(r)|^2\) can be considered as edges of indicators of conductivity inhomogeneities. To be precise, as \(r \to \partial D_i\), \(\zeta(r) \to 0\) and \(\zeta(r) \approx +\infty\) for \(r\) away from \(\partial D_i\) (\(i = 1, 2, \ldots, n_f\)). Moreover, \(|\nabla \sigma|\) is relatively sparse in lung imaging due to the piecewise constant structure of \(\sigma\) at a given time. Based on the above observations, we construct the nonlinear weighted regularized term \(\|\nabla \sigma/|\nabla \sigma|^2\|_{L^1(\Omega)}\).

Hence, (6) is changed to

\[
\delta\sigma^*_l = \arg \min_{\delta\sigma} \left\{ \frac{1}{2} \|S\delta\sigma - \partial V\|_I^2 + \lambda \|p \cdot D(\delta\sigma)\|_I \right\}.
\]

Here, \(p = (\zeta(\delta\sigma), \zeta(\delta\sigma)) \in \mathbb{R}^{2N}\), \(D(\delta\sigma) = (D_1 \delta\sigma; D_2 \delta\sigma) \in \mathbb{R}^{2N}\), where \(D_1\) and \(D_2\) are, respectively, the first-order difference operators along the \(x\)- and \(y\)-directions.

Given the fact that the minimization problem (7) is non-differentiable, we adopt the well-established ADMM [32] to solve it. ADMM has been widely used in EIT, e.g., [28], [37], [38]. To be precise, in the proposed scheme, given the initial guesses \((z_0, p_0, y_0), \delta\sigma\) are updated via the following schemes:

\[
\delta\sigma_{n+1} = \arg \min_{\delta\sigma} \mathcal{L}_p(\delta\sigma, z_n, p_n; y_n)
\]

\[
p_{n+1} = (\zeta(\delta\sigma_{n+1}); \zeta(\delta\sigma_{n+1}))
\]

\[
y_{n+1} = y_n + \rho(D\delta\sigma_{n+1} - z_{n+1}).
\]

Here, \(z \in \mathbb{R}^{2N}\) is an auxiliary variable, and \(\mathcal{L}_p\) is the augmented Lagrangian functional defined as

\[
\mathcal{L}_p(\delta\sigma, z, p; y) := \frac{1}{2} \|S\delta\sigma - \partial V\|_I^2 + \lambda \|p \cdot z\|_I + \frac{\rho}{2} \|D\delta\sigma - z\|_I^2 + y^T (D\delta\sigma - z)
\]

where \(y\) is the Lagrangian multiplier, and \(\rho\) is a scalar penalty parameter.

Meanwhile, from (8a) and (8b), we can derive

\[
\delta\sigma_{n+1} = \left( \frac{1}{\rho} S^T S + D^T D \right)^{-1} \left( \frac{1}{\rho} S^T \partial V + D^T z_n - D^T Y_n \right)
\]

and

\[
z_{n+1} = h_{\rho}(\arg \min_{\delta\sigma} (D\delta\sigma_{n+1}) + y_{n+1}[k]).
\]

Here, \(y_{n+1}[k]\) and \(z_{n+1}[k]\) are, respectively, the \(k\)th element of \(y_n\) and \(z_{n+1}\) for \(k = 1, 2, \ldots, 2N\), and \(h_{\rho}(\cdot)\) is the soft thresholding operator defined as [39]

\[
h_{\rho}(\cdot) = \begin{cases} 
- \text{sgn}(\cdot), & |\cdot| > \rho \\
0, & \text{otherwise}
\end{cases}
\]

where \(\text{sgn}\) is the sign function. We provide a proof of (11) in the Appendix.

### IV. RECONSTRUCTION ALGORITHM

In this section, we summarize the reconstruction algorithm using the proposed nonlinear weighted anisotropic TV regularization. Note that in the human experiment there is unavoidable noise and artifacts in the measured data, and we first preprocess the data as that in [30] to reduce the artifacts caused by rib cage movement [40].

#### A. Data Preprocessing and Modeling Error Blocking

Instead of using the direct measurement \(\partial V\), we use the preprocessed data \(\partial V^c\). Here

\[
\partial V^c = \partial V - S_{bdy}(S_{bdy}^T S_{bdy} + \lambda_0 I)^{-1} S_{bdy}^T \partial V
\]

where \(S_{bdy}\) is a submatrix of \(S\) including the columns related with the boundary elements, \(\lambda_0\) is a regularization parameter, and \(I\) is the identity matrix.

Because the reconstruction problem is iteratively solved, the modeling error could propagate in the forward problem, especially the estimated conductivity near the electrodes during the iterations. The modeling error propagation could heavily deteriorate the accuracy of the reconstructed images. To this end, we assume that the conductivity near the electrodes is invariant with respect to patients breathing and it only changes in the domain \(\Omega\), where \(\Omega \subset \subset \Omega\) represents the domain including two lungs. For each iteration, we modify \(\delta\sigma_{n+1}\) by

\[
\delta\sigma_{n+1} = \delta\sigma_{n+1} \chi_{\Omega}
\]
where \( \chi_{\tilde{\Omega}} \) is the characteristic function of the domain \( \tilde{\Omega} \). Subsequently, we will nominate modified nonlinear weighted anisotropic TV (MNWATV) as the proposed regularization using the preprocessed data.

B. Pseudocode for the Proposed Method

We summarize the above procedures as the following algorithm in the form of pseudocode.

**Algorithm 1 Proposed Algorithm**

**Input:** Measured data \( V \), the maximum iteration number \( M \), the parameters \( \lambda, \rho \), tolerance \( tol \), and the sensitivity matrix \( S \)

**Output:** conductivity distribution \( \sigma \)

1. Initialize: \( y_0 = 0, z_0 = 0, p_0 = 1, n = 1 \).
2. while \( n < M \) do
3. Update \( \sigma_{n+1} \) using (10) and set \( \sigma_{n+1} \leftarrow \sigma_{n+1} \chi_{\tilde{\Omega}} \).
4. Update \( z_{n+1} \) using (11).
5. Update \( p_{n+1} \) using (8c).
6. Update \( y_{n+1} \) using (8d).
7. if \( ||\sigma_{n+1} - \sigma_n|| < tol \) then
8. break
9. end if
10. end while

V. NUMERICAL AND HUMAN EXPERIMENTS

To test the performance of the proposed algorithm, we carried out numerical examples and human EIT lung imaging experiment.

A. Experimental Setup

In the experiments, we attach 16 electrodes to the boundary of the subject as uniform as possible. In all the experiments, we inject currents with an amplitude of 1 mA and measure the voltages between adjacent pairs of electrodes. For each frame, the voltages \( V = (V[p]) \) with \( 16 \times 13 = 208 \) elements are measured through the attached electrodes avoiding the driving electrodes. Here, \( V[p] = V_i^j \) for \( j = [(p - 1)/13] + 1, i = p - 13[(p - 1)/13] + j, j = 1, 2, \ldots, 16 \) and \( p = 1, 2, \ldots, 208 \). To avoid having 0 in the denominator, in all the experiments we approximate \( \zeta(\delta\sigma_{n+1}) \) by \( 1/(\|D\sigma_{n+1}\|^2 + \delta) \) for a small \( \delta > 0 \).

The results of the experiments are compared with methods using TV, first-order TV, and FER regularizers. For comparison, in the numerical experiments, we computed the relative error (RE) and the peak signal-to-noise ratio (PSNR) to evaluate the accuracy of reconstructed images. At the \( n \)th step, RE(\( n \)) is defined as

\[
RE(n) = \frac{||\sigma_n - \sigma^*||_2}{||\sigma^*||_2} \tag{14}
\]

and PSNR(\( n \)) is defined as

\[
PSNR(n) = 10 \log_{10} \frac{\max(\sigma_n \odot \sigma_n)}{\text{MSE}(n)}. \tag{15}
\]

Here, \( \cdot \odot \cdot \) represents the componentwise multiplication. \( \sigma^* \) represents the true conductivity and MSE(\( n \)) is the mean square error between \( \sigma^* \) and \( \sigma_n \).

Since we do not know the true conductivity in the human lung imaging experiment, we define RE(\( n \)) and PSNR(\( n \)), \( \sigma^* \) was replaced by \( \sigma_{TV} \), which is the reconstructed conductivity using TV regularization. In addition, we also compare the CPU time in the reconstruction process. The reconstructions are carried out using MATLAB 2016a (The MathWorks, Inc., Natick, MA, USA) with EIDORS [41] on a workstation with 2 GHz Intel\(^1\) Xeon\(^1\) Gold 6138 CPU, 256-GB memory, and Windows 10 operating system.

To decrease the computational cost, we use the point electrodes at the center of the true electrodes. Hanke et al. [42] provide a theoretical result toward the reasonability of such an approximation. We present the reconstruction results in the image with \( 256 \times 256 \) pixels. Moreover, in all the experiments, we set \( tol = 10^{-5} \).

B. 2-D Numerical Experiments

We construct a disk with a radius of 0.1 m centered at the original point (0, 0). We put two ellipses inside the disk to simulate the lungs. The center of the constructed ellipses is, respectively, (0.04, −0.01) and (−0.04, −0.01). The size of the ellipses varies for ten different models (see the first row of Fig. 2). For the \( k \)th model \( (k = 1, 2, \ldots, 10) \), the major and minor semi-axes of the left ellipse are, respectively, \( a_i = (0.012 + 0.001k, 0.024 + 0.002k) \), \( b_i = (0.012 - 0.001k, 0.006 + 0.0005k) \), while the values of the right ellipse are, respectively, \( a_i = (0.012 - 0.001k, 0.024 + 0.002k) \) and \( b_i = (0.012 + 0.001k, 0.006 + 0.0005k) \). We set the conductivity of background and inclusion to be 1.0 and 1.1 S/m, respectively. One of the ten models is shown in Fig. 1(a). In the inversion, a finite element mesh with 1024 triangular elements and 545 nodes as shown in Fig. 1(b) is used.

We first solve the 2-D forward problem (1) to obtain the boundary voltage data. The data for the seventh model are depicted in Fig. 1(c). To simulate inherent noise of EIT systems in reality, we added the Gaussian noise to the simulated data. The selected noise level corresponds to the signal-to-noise ratio (SNR) SNR = 50 dB, which represents well the noise level of modern EIT systems [43]. Using

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these noisy data, we reconstruct the conductivity images using the proposed algorithm and three existing methods. We set $\lambda = 5 \times 10^{-13}$, $\rho = 1 \times 10^{-10}$, $\delta = 0.01$, and $M = 20$ for the proposed method. We explain the selected parameters in Section VI. For the FER method, we only consider the case when the regularization parameter is set to be $\infty$. The reference conductivity $\sigma_0$ is set to be background conductivity $1$. The parameters of the other regularization methods are empirically chosen. The results are shown in Fig. 2.

Fig. 3(a)–(c) illustrates the profiles along the solid, dash, and dot dash lines, respectively, shown in the seventh model in Fig. 2. Fig. 4(a) and (b) shows the behavior of $\text{RE}(n)$ and $\text{PSNR}(n)$ as $n$ increases for each regularizer except FER since it is a direct reconstruction method. In Table II, we compare the computational time for the reconstructions using the aforementioned regularizers.

From comparisons, as shown in Figs. 2 and 3, the NWATV reconstructions are more accurate than FER does. As shown in Fig. 3, the staircase effect in TV regularization is alleviated. Moreover, due to the first-order approximation of the anisotropic TV norms, the first-order TV regularizer is at the risk of merging the two inhomogeneities when the distance between them is small. However, the NWATV has more advantage of edge preserving than the first-order TV does especially when the distance between the two anomalies decreases.

For Fig. 4(a) and (b), even though the blue line decreases rapidly, it will take more time for each iteration step. Even though the RE value of TV reconstruction at the tenth step achieves its minimum value, its performance is worse than that one corresponding to the 20th step. Indeed, the TV-based image has more staircase effects in the tenth step. One of the
of each colorful line is shown in the subfigure.

We attach16 circular electrodes along the thorax model in EIDORS [41]. The geometry of the model C. 3-D Numerical Experiment

accuracy as that for the TV does.

The human lung EIT experiment is approved by the ethics committee of the science and technology division, Shandong Normal University. In this experiment, we attach 16 electrodes [44] around the exterior of the object’s torso as shown in Fig. 9(a). We choose the disposable Ag/AgCl electrocardiogram (ECG) electrodes with the size of 20 × 27 mm. We use the 3-D scanner [45] to scan the body with electrodes and obtain the geometry and the positions of the electrodes as accurate as possible. The geometry obtained is shown in Fig. 9(b). To obtain the position of the electrodes, we use point electrode approximations which lie on the center of the true electrodes [42]. Then we discretize the imaging slice with point electrodes into 614 nodes and 1132 triangles. The discretization result is shown in Fig. 9(c).

The human EIT data were collected with the Sciospec 16-channel EIT system [46]. The frequency of the injection current is 10 kHz and the speed of the data acquisition is 30 frames/s. The data for \( t_3 \) are depicted in Fig. 9(c). Using the data obtained from the EIT scanner, we reconstruct ten frames of the human lung images. The time step for each frame is around 0.66 s (\( t_n - t_{n-1} \approx 0.66 \) s for \( n = 1, 2, \ldots, 10 \)).
During reconstruction, we assign the reference conductivity \( \sigma_0 = 0.0107 \) S/m [41] as the conductivity of the lungs at the state of the end expiration and use this \( \sigma_0 \) to obtain the sensitivity matrix \( S \) in (4). In Fig. 10 we compare the reconstructions using TV, FER, first-order TV, NWATV, and modified NWATV regularizer (MNWATV). In this experiment, the regularization parameters for MNWATV regularizer are set to be \( \lambda = 1 \times 10^{-14}, \rho = 1.25 \times 10^{-5}, \delta = 7 \times 10^{-8}, M = 4, \) and \( \lambda_b = 1 \times 10^{-7} \). Again for the regularizers of TV and first-order TV, the regularization parameters are set to be as optimal as we can.

Due to the noise in the measured data, to block the error propagation in the iteration process, in each step we modify the reconstruction result by (13). The results of the reconstructions using each regularizer are shown in Fig. 10. Especially the results of the modified reconstructions are shown in the last row of Fig. 10. Table III illustrates the behavior of \( \text{RE}(n) \) and \( \text{PSNR}(n) \) for the frames \( t_3, t_6, \) and \( t_{10} \). In Table IV, we provide the computational time when using different regularizers.

As we can see through the use of the proposed method, we obtain a similar reconstruction as observed in TV images and a more accurate image than FER. Precisely, with the same gray scale, FER has the lowest accuracy; the first-order TV
Fig. 10. Ten frames of human lung images using different regularizers. The first to third rows illustrate the reconstruction results using FER, TV, and first-order TV regularizers, while the fourth and fifth rows illustrate the results using NWATV and MNWATV regularizers, respectively, for \( t = t_1, \ldots, t_{10} \).

| \( t \) | FER | TV | First-order TV | NWATV | MNWATV |
|---|---|---|---|---|---|
| \( t_1 \) | 0.0107 | 0.0039 | 0.0021 | 0.0017 | 0.0021 |
| \( t_2 \) | 0.0089 | 0.0037 | 0.0022 | 0.0017 | 0.0022 |
| \( t_3 \) | 0.0081 | 0.0034 | 0.0017 | 0.0016 | 0.0017 |
| \( t_4 \) | 19.13 | 27.83 | 33.70 | 33.70 | 33.70 |
| \( t_5 \) | 20.63 | 28.19 | 35.91 | 35.91 | 35.91 |
| \( t_6 \) | 21.36 | 28.71 | 35.00 | 35.00 | 35.00 |

Table III

Behaviors of \( \bar{\text{RE}}(n) \) and PSNR(\( n \)) for Frames \( t_3, t_6, \) and \( t_{10} \)

Fig. 11. Changes in (a) \( \text{RE} \) and (b) PSNR with respect to \( \lambda/\rho \) and \( \delta \) in the seventh image of 2-D numerical simulation.

Table IV

Comparison of the Five Methods in Terms of Computational Time for Human Experiments

| \( \lambda/\rho \) | \( \delta \) | \( t_1 \) | \( t_2 \) | \( t_3 \) | \( t_4 \) | \( t_5 \) | \( t_6 \) | \( t_7 \) | \( t_8 \) | \( t_9 \) | \( t_{10} \) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 1.0 | 2.055 | 0.020 | 0.116 | 0.109 | 0.153 | 2.131 | 0.017 | 0.104 | 0.105 | 0.140 |
| 1.0 | 0.8 | 2.056 | 0.019 | 0.119 | 0.108 | 0.169 | 2.071 | 0.018 | 0.109 | 0.113 | 0.163 |
| 1.0 | 0.6 | 2.081 | 0.021 | 0.116 | 0.112 | 0.149 | 2.088 | 0.022 | 0.107 | 0.129 | 0.158 |
| 1.0 | 0.4 | 2.140 | 0.020 | 0.105 | 0.122 | 0.199 | 2.136 | 0.019 | 0.110 | 0.109 | 0.176 |
| 1.0 | 0.2 | 2.157 | 0.02 | 0.107 | 0.108 | 0.135 | 2.161 | 0.019 | 0.112 | 0.1149 | 0.135 |

Method tends to blur the heart especially for the frames corresponding to \( t_3, t_6, t_9, t_{10} \); NWATV regularization provides the closest reconstruction results with the TV regularization method with the primal-dual algorithm. Moreover, the computational cost is acceptable in clinical applications since it produces up to 6 frames/s, which is much higher than the typical respiratory rate.

VI. DISCUSSION: REGULARIZATION PARAMETER SELECTION AND ROBUSTNESS STUDIES

TV regularization has the advantage of preserving intermediate discontinuities [38] especially for a piecewise constant images and definitely is promising in EIT clinical applications. Dobson and Santos [47] introduced the TV regularization in EIT, and Borsic et al. [23] introduced a primal-dual interior point method for the minimizing process. However, this method is time-consuming, and consequently it is difficult to apply in real-time applications. The proposed nonlinear weighted anisotropic TV regularization method reduces the computational by using a weighted anisotropic TV regularizer. Moreover, this method is promising in producing a better result than using only traditional TV (isotropic TV) or anisotropic TV regularization. This is because the weight \( 1/|\nabla \sigma|^2 \) could avoid possible pseudoedges in isotropic TV or distortion along the coordinates axes in anisotropic TV regularizations [33].

A. Parameter Selection

For regularization-based reconstructions, the choices of regularization parameter(s) are always critical. In this article, the choices of \( \rho \) and \( \delta \) depend on the mesh size used in the process of inversion. \( \rho \) has been chosen in such a way that \( (1/\rho)S^T S \) and \( D^T D \) in the similar order so that \( (10) \) makes sense. \( \delta \) is chosen in the similar way as \( \rho \) such that \( \delta \) is in the similar order as \( |D\sigma| \). This explains why the parameters \( \rho \) and \( \delta \) are
Fig. 12. Reconstructions using the noisy data with SNR = 10, 20, 30, 40, 60, and ∞ dB, respectively, in the 2-D model as that in Fig. 2.

Fig. 13. (a)–(f) Reconstruction result of the 3-D numerical experiment using the noisy data with SNR = 10, 20, 30, 40, 60, and ∞ dB, respectively.

chosen so small. From (11), only the ratio \( \lambda/\rho \) is meaningful. Hence, the parameter \( \lambda \) has to be chosen according to the selection of \( \rho \). Fig. 11(a) and (b) illustrates the change in RE and PSNR with respect to \( \delta \) and \( \lambda/\rho \), respectively. As can be seen from this figure, for a fixed \( \lambda/\rho \), RE and PSNR are almost invariant with respect to \( \delta \). Moreover, there is optimal choice of \( \lambda/\rho \) between 0 and 0.01 to minimize RE and maximize PSNR. For given \( \lambda/\rho \) close to 0, the values of RE and PSNR are almost invariant with the parameter \( \delta \).

B. Robustness Against Noise

In this section, we study the robustness of the proposed NWATV regularization method with respect to the additive noise. The Gaussian noise with SNR = 10, 20, 30, 40, 60, and ∞ dB is, respectively, added to the measurements and the corresponding values of RE(\( n \)) and PSNR(\( n \)) are computed. Here, parameters \( n = 20 \) and 10 are set for the seventh model of the 2-D experiment and the 3-D numerical experiment, respectively. We use the same regularization

| Experiments | 7th model of 2D cases | 3D cases |
|-------------|----------------------|----------|
| SNR         | RE       | PSNR     | RE       | PSNR     |
| 10          | 0.040931 | 20.82942 | 0.085788 | 16.06117 |
| 20          | 0.03735  | 23.13503 | 0.086654 | 16.46238 |
| 30          | 0.03632  | 24.16227 | 0.087661 | 17.1945  |
| 40          | 0.036267 | 24.23114 | 0.067352 | 17.34165 |
| 50          | 0.0362   | 24.26316 | 0.067634 | 17.60159 |
| 60          | 0.036186 | 24.28306 | 0.067591 | 17.606   |
| ∞           | 0.036184 | 24.28693 | 0.067577 | 17.6093  |

Fig. 14. (a) and (b) RE and PNSR, respectively, as functions of SNR for the 20th step iteration of the seventh model in 2-D numerical experiment.
parameter as that in Section V. The reconstruction results are, respectively, depicted in Figs. 12 and 13 for the 2-D and 3-D cases. The corresponding values of RE(n) and PSNR(n) are shown in Table V. RE and PSNR as functions of SNR are, respectively, depicted in Figs. 14 and 15 for the 2-D and 3-D numerical experiments.

From the above results, even for signals with a low SNR (e.g., SNR = 20), we can still obtain a satisfied reconstruction result. Hence, we conclude that the proposed regularization method is quite stable against the added Gaussian random noise. This is an expected result, since the current work is based on the context of difference imaging, which is tolerant to noise to some extent [48].

VII. CONCLUSION AND FUTURE WORK

In this article, we propose a nonlinear weighted anisotropic TV regularization method in EIT to reduce the computational time and improve the capability of edge preservation in comparison to TV regularization-based imaging. To validate the advantages of the proposed regularization method corroborating with the well-established FER, TV, and first-order TV regularizers, we carried out 2-D, 3-D simulations and human EIT lung imaging. In the testing campaign, it was shown that the proposed method reduces the computational time significantly while providing the reconstruction images comparable to the traditional TV method with the primal-dual method. Indeed, using the proposed NWATV regularization, we can alleviate the staircase effect arising in TV regularization and improve the reconstruction accuracy for FER. Meanwhile, the computational times are reduced from 2.311 to 0.629 and 1.733 to 0.428 s in 2-D and 3-D simulations, respectively, and the computational time is reduced from more than 2 s to less than 0.2 s in human experiment. Moreover, NWATV regularization is quite robust to measurement noise.

Future studies should cover a strict mathematical theory on the convergence analysis of the iteration process. Moreover, we need to study the stableness and the stopping criteria for the real applications. For clinical applications, a potential consideration is to add low-rank constraints of the sequential images [19] in the reconstruction.

APPENDIX

In this section, we provide the proofs of (11).

A. Proof of (11)

Direct calculation yields that

\[ z_{n+1} = \arg \min_{z} \lambda \| p_n \cdot z \|_{l_1} + \frac{\rho}{2} \| D \delta \sigma_{n+1} - z \|_{l_1} + y_n^T (D \delta \sigma_{n+1} - z) \]

\[ = \arg \min_{z} \| D \delta \sigma_{n+1} - z \|_{l_1}^2 + \frac{\rho}{2} y_n^T (D \delta \sigma_{n+1} - z) + \frac{\lambda}{\rho} \| p_n \cdot z \|_{l_1} \]

\[ = \arg \min_{z} \left\| z - \left( D \delta \sigma_{n+1} + \frac{y_n}{\rho} \right) \right\|_{l_1}^2 + \frac{2 \lambda}{\rho} \| p_n \cdot z \|_{l_1} \]

where the last equality comes from the fact that \( \| y_n / \rho \|_{l_1}^2 \) is independent of \( z \). From the definition of \( l_p \) \( (p = 1, 2) \) norm, we obtain that

\[ z_{n+1} = \arg \min_{z} \sum_{k=1}^{N} F_{p, \lambda}(z[k]) \]

where \( F_{p, \lambda}(z[k]) \) is defined as follows:

\[ F_{p, \lambda}(z[k]) = \left\| z[k] - \left( D \delta \sigma_{n+1}[k] + \frac{y_n[k]}{\rho} \right) \right\|_{l_1}^2 + \frac{2 \lambda}{\rho} \| p_n[k] \cdot z[k] \|_{l_1} \].

Hence

\[ z_{n+1}[k] = \arg \min_{z[k]} F_{p, \lambda}(z[k]). \quad (16) \]

Next, we will calculate the minimizer of \( F_{p, \lambda}(z[k]), z_{n+1}[k] \) explicitly. Indeed, from the first-order optimal condition, we obtain

\[ 0 = \frac{1}{2} \frac{\partial F_{p, \lambda}(z_{n+1}[k])}{\partial z} \bigg|_{z = z_{n+1}[k]} \]

\[ = \left( D \delta \sigma_{n+1}[k] + \frac{y_n[k]}{\rho} \right) + \frac{\lambda p[k]}{\rho} \text{sgn}(z_{n+1}[k]). \]

Hence, \( z_{n+1}[k] \) satisfies the following identity:

\[ z_{n+1}[k] = D \delta \sigma_{n+1}[k] + \frac{y_n[k]}{\rho} - \frac{\lambda p[k]}{\rho} \text{sgn}(z_{n+1}[k]). \quad (17) \]

From (17) and the fact that \( \lambda p[k] / \rho \geq 0 \) for \( k = 1, 2, \ldots, N \), it is obvious that

\[ \text{sgn}(z_{n+1}[k]) = \text{sgn}(D \delta \sigma_{n+1}[k] + y_n[k]/\rho). \quad (18) \]

Note that from the basics of calculus, the candidates of \( z_{n+1}[k] \) are either the stationary point or the nondifferentiable point of \( F_{p, \lambda} \). Hence,

1) For the case \( |D \delta \sigma_{n+1}[k] + y_n[k]/\rho| > ((\lambda p[k]) / \rho) \).

From (12), (17), and (18) we obtain

\[ z_{n+1}[k] = (\mathcal{I} - \lambda p[k] / \rho \cdot \text{sgn})[D \delta \sigma_{n+1}[k] + y_n[k]/\rho] \]
or $z_{n+1}[k] = 0$, where $I$ is the identity map. Since

$$F_{p,i} = \frac{\lambda}{\rho^2} \left( \frac{1}{\rho} p_n[k] \right)^2 + \frac{2\lambda}{\rho} p_n[k]$$

we have

$$\frac{1}{\rho} p_n[k] = \frac{\lambda}{\rho} p_n[k]$$

or $p_n[k] = 0$. This contradicts with the relation (18). Hence, the only minimizer of (16) is $z_{n+1}[k] = 0$.

Finally, we obtain (11).

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