“Spontaneous Movable Semifluxons ”- New Phenomenon arising in Nano-Electronic Superconducting System

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Abstract. Semifluxon is a very rare topological excitation. It is a spontaneous Josephson vortex carrying a half of the magnetic flux quantum $\phi_0$ created on the boundary between 0 and $\pi$ junctions. Semifluxons play an important role as qubits for quantum computing. Typically they are static and arise in a variety of Josephson 0-$\pi$ nano-junctions, in particular. The properties of semifluxons are very different from the properties of fluxon. Typically, semifluxons are pinned at the phase discontinuity point in 0-$\pi$ long Josephson junctions (LJJ) and may have two polarities carrying the flux $\pm \phi_0/2$, that was attracting to use them as qubits. In this paper, we report for a first time about a novel type of semifluxons arising in superconductors, which are movable, i.e. not pinned. We show that a spontaneous generation of movable semifluxons is created in conventional LJJ due to flux cloning in extended T-Josephson nano-junctions. We also show that there is a strong interaction between semifluxon and anti-semifluxon once they have been generated and that leads to their very short life time, i.e. to their fast annihilation and next recreation.

1. Introduction

Josephson nano-junctions are one of the most important elements in superconducting electronic circuits. Josephson vortices are stable topological excitations arising in long Josephson junctions (LJJ) subjected to magnetic field and to strong electric current pulses [1],[2]. They are also known as Sine-Gordon (SG) solitons. Magnetic flux trapped by these vortices or SG solitons is quantized with the single flux quantum $\phi_0$. The study of fluxons in Josephson nano-junctions has attracted a lot of attention because of their potential applications in superconducting electronics [3]-[6], digital logic circuitry [7]-[11] and, in general, modern quantum circuits [2]. The very long Josephson junction may be used as Josephson transmission line (JTL) through which a SG soliton can freely move along.

The standard description of fluxon dynamics in JTL is based on the (2+1)-dimensional unperturbed sine-Gordon equation. With its boundary conditions [12], this description is extremely accurate:

$$\phi_{xx} + \phi_{yy} - \phi_t - \sin(\phi) = 0$$

(1)

where $\phi(x, y, t)$ is the superconducting phase difference, and the subscripts denote partial derivatives of $x$, $y$, which are normalized to the Josephson penetration depth $\lambda_J$, and time $t$, which is normalized to the inverse of the Josephson angular plasma frequency $\omega_J$. The exact SG soliton solutions of the SG equation have been given as
\( \phi(x, y, t) = 4 \tan^{-1} \left( \exp \left( \frac{x-x_0-ut}{\sqrt{1-u^2}} \right) \right) \)  

(2)

With the SG soliton energy \( E \) as equal to

\[
E = \int_0^\infty dx \int_0^\infty dy \left[ \frac{\phi_x^2}{2} + \frac{\phi_y^2}{2} + 1 - \cos(\phi) \right] = \frac{8W}{1-u^2}  
\]

(3)

Where \( u \) is a velocity of soliton propagating in a straight two-dimensional strip of the width \( W \).

For our next consideration we assume that its initial position is located at \( x = x_0 \) for \( t = 0 \). The velocity \( u \) (normalized to the Swihart velocity \( \tilde{c} \), the maximum electromagnetic propagation velocity in the junction) may take values within the range \( 0 < u < 1 \) \([13]-[20]\).

During last years it was shown both theoretically \([21]-[22]\) and experimentally \([23]-[24]\) that there was also significant interest in vortex excitations carrying an arbitrary fraction of standard flux quantum as \( \phi = \phi_0 \kappa/2 \), where \( \kappa \) is a the value of the discontinuity \([25]\). For \( \pi \) discontinuity, Josephson vortices carry only half of the magnetic flux quantum. This half-vortex has been studied theoretically by several authors \([1],[2],[21],[25],[27],[28],[34]\). Because of containing one-half of the flux quantum, they are called “semifluxon”. In other words, a \( 2\pi \) vortex is a fluxon carrying \( \phi_0 \), while a \( \pi \) vortex is a semifluxon carrying \( \phi_0/2 \), see figure 1 \([25]\). A single semifluxon can have positive or negative polarity carrying the flux \( +\phi_0/2 \) (semifluxon) or \( -\phi_0/2 \) (anti-semifluxon), respectively \([29],[27]\). In addition, it has been conjectured that semifluxons can be utilized in superconducting memory and logic devices and accurate measurements of the Josephson penetration length of a superconductor (for details, see Refs \([22],[30]\)).

Figure 1 (color online). Comparison of fluxon and semifluxon shapes. (a) The behavior of total phase \( \phi(x) \) and of magnetic component \( \mu(x) \) only. (b) Magnetic-field profile \( \mu_x(x) \), see references \([25]\) and \([34]\), for details.

Semifluxons exist in specially designed long Josephson junctions (LJJ), which are so-called “0- \( \pi \)” LJJ, see Fig. 2. This 0- \( \pi \) junction consists of two joint parts. The 0-junction which is a conventional Josephson junction (one may literally say, “with positive critical current”) and the other second part is a \( \pi \) -junction, which is a standard junction “with a negative critical current”, see for example, the Ref. \([25]\). Semifluxons can be created at the boundaries between the 0 and \( \pi \) regions provided the Josephson junction is long enough \([31]\). The 0- \( \pi \) boundaries are sometimes called discontinuity points \([32]\).

Figure 2. 0- \( \pi \)–tunnel long Josephson junction geometry.

Semifluxons are very interesting objects which are not studied in detail, especially experimentally, because of difficulties with a fabrication of 0- \( \pi \) junctions \([29]\). Although the classical properties of
semifluxons are understood, their quantum behavior and their possible applications in the quantum domain still have to be studied [31]-[34]. The purpose of the present work is to demonstrate the existence of a new novel phenomenon for semifluxons carrying half-integer magnetic flux quantum in conventional extended Josephson junction. Whereas semifluxons and fluxons can be described by sine-Gordon-equation, their properties are very different; fluxons are topologically stable solitons with very long life time while the semifluxons created dynamically have a very short life time. We present the first evidence and observation of moving semifluxon which may arise in a manner similar to normal fluxon. In addition, we show here that semifluxon can be created and moved in extended Josephson junctions by means of flux cloning phenomenon arising in the T- shaped Josephson junctions.

2. Solitons in T- Josephson junctions

In Josephson electronic circuits, two Josephson transmission lines (JTLs) can be connected together to form a T-shaped junction. In general, T-junctions, have two different widths; a main JTL (MJTL) of width $W_0$ along the $x$-axis, and an additional JTL (AJTL) of width $W$ along the $y$-axis (see figure 4). They can be connected to form two kinds of T junctions of the different widths (see figure 3 (a) and (b)) [36]-[38]. Flux cloning is a newly discovered phenomenon that occurs as a result of vortex passing through T-shaped junctions [36],[37]. In connection area, the dynamic behavior of flux is shown that there are two different types of behaviors - either reflection from the T junction (figure 4 (a)) or flux cloning (figure 4 (b)). When the mother vortex in the MJTL approaches T-junction and its speed satisfies certain conditions, it splits to generate two vortices [36]-[38]. These behaviors are dependent on the value of velocity of fluxon, less or greater than critical velocity, which has been theoretically calculated for T-junction in figure 3 (a) [36],[37].

In T-junction, the condition of critical velocity of soliton has to satisfy the flux cloning condition:

$$u_c = \frac{\sqrt{W(W+2W_0)}}{W+W_0}$$  \hspace{1cm} (4)

Otherwise, the soliton will be reflected if the velocity ($u$) is less than $u_c$ [36]-[37]. After cloning, a new baby vortex generate in the AJTL whereas the mother vortex will still propagate continuously in MJTL. In the case $W \geq W_0$, the behavior of soliton changes because the critical velocity becomes very high close to Swihart velocity. In addition, there is a discrepancy between analytical estimation based on the soliton energy arguments[36-37] and numerical evaluation of critical velocities when $W > W_0$, see figure 5. The theoretical prediction of critical velocity can be calculated by equation (4), while the
numerical value has been evaluated when the soliton changes behavior from reflection to splitting [36], see figure 4. In this paper, we show that movable semifluxons can be excited in extended T-Josephson junctions when fluxon will be crossing T-junctions. In general, by extended T-Josephson junctions we mean that the widths of MJTL ($W_0$) and AJTL ($W$) are larger than the Josephson length ($\lambda_J$). Here, extended T-Josephson junctions are denoted to both $W_0, W_0 > \lambda_J$ and $W > W_0$.

Figure 4 (color online). Numerical simulations of the time evolution of superconducting phase difference; the yellow-green stripe corresponds to the soliton-fluxon (a) The time snapshots of the reflection of an incident fluxon-soliton propagating without cloning and (b) Cloning of the fluxon-soliton propagating through the T-junction. After the time $t=4$ due to the cloning there are created two fluxon-solitons. The color scale on a right side represents the superconducting phase difference $\phi$ [36].

Figure 5 (color online). Schematic representation of the dependence of the critical velocity on the width of the additional JTL (a) The case when $W \leq W_0$ [36]-[37]. (b) The case when $W \geq W_0$. Dots represent the data obtained in numerical calculation made within 2D geometry, solid lines are related to qualitative analytic estimations obtained from eq. (4).

In special case, when the energy of incident fluxon is very large, the T-shaped Josephson junction allows vortex excitations carrying a half of the standard flux quantum. When the width $W$ is too wide, mother vortex shows strange behavior in both MJTL and AJTL. In contrast to original prediction ($u_c$) indicating that fluxon moving with velocity high than critical will be cloned[36-37], we show here that in some cases the mother vortex is reflected from the T-junction while at the same time at the moment of the reflection some new excitations will be created and continuously propagate in MJTL. In connection area of T-junction, soliton starts to divide and to clone but it has no enough energy, ie it cannot generate a complete quantized fluxons (baby and mother vortex). Then, the fluxon-soliton will be trapped in the T-junction and semifluxons will be created. Then it becomes possible for them to
propagate in AJTL and MJTL. In MJTL, the semifluxon after creation moves and its stability looks similar of the original fluxon-soliton. In AJTL, however, a massive shape wave (tsunamis) [39],[40] will be appeared together with semifluxon which becomes unstable and fast decays into plasma waves. Because of the broad wideness of AJTL, $W$, the propagation of unstable semifluxon over a large distance requires a high energy and high velocity to move along the AJTL, which can be estimated from the equation (4). Therefore, unstable semifluxon will struggle to continue its motion due to the condition that its velocity is not large enough. Then, the unstable semifluxon, which moves along the AJTL, will be reflected and moved back to the MJTL in $x$ and -$x$ directions. In connection area of the T-junction, the trapped fluxon-soliton will be reflected as well and then starts propagating in -$x$ directions. On the other hand, in $x$ direction, unstable semifluxon starts moving and passing through the MJTL at the same time transforming as unstable anti-semifluxon. Typically unstable anti-semifluxon moves faster than stable semifluxon. Consequently, the semifluxon and anti-semifluxon are strongly attracting each other like quarkes in a high energy physics. This results typically in the fast enough annihilation between them. Hereafter, a generation of periodic stable of semifluxon and anti-semifluxon chains may be spontaneously appeared and propagate in MJTL. As a result, another and another annihilation and after consequent re-creation of such pairs will take place indicating on a complex dynamics existing between these new solitons, semifluxons and anti-semifluxons.

3. Numerical Studies of Semifluxon in T-Josephson junctions

Our numerical simulations of the flux cloning dynamics have been performed with the use of the finite element numerical analysis, in particular the technique package FEMLAB has been utilized. We have studied the time-dependent sine-Gordon-equation with boundary conditions depending on external magnetic field or a current. The dynamical behavior of Josephson vortices can be indicated by time dependent change of the value of the superconducting phase $\phi$. The solution can be presented on the figures 7 – 9. It is convenient for each value of the phase difference to put in a correspondence a color. The dark blue color represents the minimum value of the phase $\phi$ and red color represents the maximum value of the phase $\phi$. In general, semifluxons are presented as the phase difference change, $\Delta \phi = \pi$, while fluxon-solitons are presented as the phase difference change, $\Delta \phi = 2\pi$ (the intermediate color between these values represents the core of the vortex, see the color scale in figure 7). In the present paper we use the simplest boundary conditions $\phi_x(x = 0,L,y,t) = 0$ and $\phi_y(x,y = 0,W,t) = 0$ (see reference [12] for details).

In all our numerical simulations, we have used initial conditions with the following form: for the phase $\phi(x,y,t)|_{t=0} = \phi_{\text{soliton}} (x)$ and $\partial \phi(x,y,t)/\partial t|_{t=0} = -u \partial \phi_{\text{soliton}} (x)/\partial t$ for its time derivative, where $\phi_{\text{soliton}} (x) = \phi(x,y,0)$. In addition, we limit our analyses to the case when the Josephson junction widths is $W_0 = 1$ and consider many cases when $W > 1$. First, we model 0-\(\pi\)-long Josephson junction discussed in references [21-34 using the finite element methods. The results, calculated here, are similar to previous results[30], see, for example, figures 1 and 6.

**Figure 6** (color online). (a) The behavior of magnetic component $\mu(x)$ for a semi-fluxon pinned by 0-pi junction. It is stationary and does not change at any time. (b) The same, ie the behavior of semifluxon for total phase $\phi(x)$. (c) The stationary magnetic-field profile for the pinned semifluxon, $\mu_s(x)$. 

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Furthermore, we consider the case when the flux cannot be cloned, for example, when \( W = 2.2 \). The critical velocity \( u_c \) is equal to 0.94. Note, although the fluxon-soliton is moving with the speed \( u \approx 0.99 \), the flux cloning does not occur. The numerical simulations show that in this case the soliton is reflected. In the figure 7, the mother vortex starts moving with high velocity \( u \approx 0.985 \) and then it is reflected from the T-junction. Figure 7 shows that semifluxons can be created when soliton crosses T-shaped junction, i.e., at the time \( t = 5 \). After the creation, this stable semifluxon is moved in MJTL, while at the same time another semifluxon created together with a massive shape wave, see, for example, the Refs [39],[40] is moved in AJTL (see the snapshot of the semifluxon dynamics presented in the figure 7 at the time \( t = 6 \)). In addition, an unstable semifluxon in AJTL is reflected and moved in MJTL as unstable anti-semifluxon (presented on the same figure 7 at the time \( t = 8 \)). Finally at the time \( t = 12 \), semifluxon and anti-semifluxon are meet each other and annihilated. Then, spontaneous serious of movable pairs consisting of the semifluxon and anti-semifluxon are continuously generated and annihilated in MJTL. To simplify our result, we plot the phase difference \( \phi \) along \( x - \) axis; see figure 8 (a) and (b). The figure 8 (b) shows that at the time \( t = 15 \), the series of pairs consisting of the semifluxon and anti-semifluxon appear and in the next moment their annihilation event takes place.

**Figure 7** (color online). Numerical simulations of the creation of movable semifluxons. In this case no flux cloning occurs. An incident fluxon-soliton (created at \( t = 0 \)) is propagating with the velocity \( u = 0.985 \) when \( W = 2.2 \). For a comparison in this case the estimated critical velocity is \( u_c = 0.94 \)[36]. A movable semifluxon is created in MJTL after fluxon-soliton crosses the T-junction (at \( t = 5 \)). In a short time after this event, the anti-semifluxon begin to propagate in MJTL. The semifluxon and anti-semifluxon strongly interact. The color scale represents the superconducting phase difference \( \phi \). A fluxon-soliton is represented by 0 to \( 2\pi \) phase differences whereas (anti-)semifluxons are represented by phase differences as \( \Delta \phi = \pi \).
The change of the phase $\phi$ along $x$-axis. An incident fluxon is propagating with a velocity higher than the critical one [36] (a) At the time $t=0$ (solid line), the change of the phase difference of incident fluxon-soliton as usual, is equal to $2\pi$. At $t=6$ (dotted line), after fluxon has just crossed the T-junction, the change of the phase difference is equal to $\pi$ as it is needed for a semifluxon, which is located at approximately $x = 4$. It is movable semifluxon and at $t=15$, there the series of the semifluxon and anti-semifluxon pairs appears. Each semifluxon has the change of the phase difference equal to $\pi$. The semifluxon and anti-semifluxon are colliding at approximately $x = 12$ and then again are annihilated by each other.

Eventually, we consider the case when the fluxon-soliton can be cloned. In the case $W=1.7$, the critical $u_c = 0.93$[36] while the numerical evaluation gives $u_c = 0.979$. A movable semifluxon can be temporarily formed inside the T-junction only if its velocity belongs to a specific critical range. The values of such velocities should be slightly lower than the critical velocity ($u \equiv u_c$) obtained in our numerical experiments. In the figure 9, an incident fluxon-soliton propagates with the velocity $u = 0.97$. The numerical simulations show that movable semifluxons in MJTL and AJTL are created at the time $t=5$, see figure 9 (a) and (b). An annihilation of the semifluxon and a reflection of the incident fluxon are shown in figure 9 (a) at the time $t=13$. In summary, we show that there are three types of the dynamical behavior for the fluxon-solitons in extended T-Josephson junctions: 1) reflection (see figure 4 (a)), 2) flux cloning (see figure 4 (b)) and 3) a spontaneous generation of movable semifluxon and anti-semi-fluxons that arises during a reflection from the T-junction of the high energy fluxon-solitons (see figure 9 (a)).
Figure 9 (color online). Numerical simulations of spontaneous movable semifluxon generation arising during a reflection of an incident high energy fluxon-soliton propagating with the velocity $u=0.97$, $W=1.7$. The critical velocity is here equal to $u_c=0.979$. A movable semifluxon is here temporarily formed after the initial fluxon-soliton has crossed the T-junction. Then it is annihilated after a very short time. The color scale represents the superconducting phase difference $\phi$. A soliton is represented by $0$ to $2\pi$ phase differences whereas (anti-)semifluxons are represented by phase differences as $\Delta\phi = \pi$. (b) The phase difference $\phi$ along $x$ axis at $t=0$ (solid line), $t=5$ (dotted line), and $t=6$ (dashed line). A movable semifluxon has been temporarily formed after the fluxon has crossed the T-junction. For a movable semifluxon the change of the phase difference is equal to $\pi$.

4. Conclusions
We demonstrate here numerically that in conventional T-Josephson junctions it is possible the existence of movable semifluxons. In all previous studies only localised, ie unmoveable semifluxons pinned by topological defects have been discussed[29-35]. In the case of the very large extended T-shaped Josephson junctions, when $W > W_0$, our numerical calculations show clear evidence of three different interesting and important features for the semifluxon generation: (a) (anti-)semifluxons can be spontaneously appeared at the T-shaped junctions when a high energy fluxon-soliton is crossing this junction, (b) (anti-)semifluxon is not localized but it may be freely moved along the long branches of the Josephson junction, (c) there a spontaneous generation of periodic movable (anti-)semifluxon chains may occur. Also there a spontaneous series of annihilation and creation between anti- and semifluxons takes place. The finding of these semifluxon and their dynamics is very useful to understand the behavior and decay of the high elementary particles. If the fluxon is equivalent to such particle, then the semifluxon will be equivalent to a quark. Thus, the T-shaped Josephson junction may be considered as a very economic laboratory to study such a quark creation and other high energy phenomena.

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