Constituent quark models and pentaquark baryons

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Abstract. We discuss certain general features of the pentaquark picture for the \( \theta \), its \( \bar{10}_F \) partner, \( \Xi_{3/2} \), and possible heavy quark analogues. Models employing spin-dependent interactions based on either effective Goldstone boson exchange or effective color magnetic exchange are also used to shed light on possible corrections to the Jaffe-Wilczek and Karliner-Lipkin scenarios. Some model-dependent features of the pentaquark picture (splitting patterns and relative decay couplings) are also discussed in the context of these models.

1. Introduction
As a basis for the discussion which follows, I adopt the following version of the current “experimental situation” with regard to reported pentaquark resonances:

- A significant number of medium-energy experiments report positive evidence for the \( \theta \). Its existence is thus plausible. If these experiments are correct, the \( \theta \) mass and width are \( m \sim 1540 \text{ MeV} \) and \( \Gamma \sim \mathcal{O}(1) \text{ MeV} \). See Ref. [1] for a critical assessment of both positive and negative experimental searches, as well as references to the experimental literature.
- The NA49 exotic \( I = 3/2, \Xi (\Xi_{3/2}) \) signal with \( m \sim 1860 \text{ MeV} \) and narrow width [2] has not been confirmed [3], and is hard to reconcile with high statistics \( \Xi \) production experiments [4]. Though a \( \Xi_{3/2} \) partner is unavoidable if the \( \theta \) lies in a \( \bar{10}_F \), in view of the experimental situation, no assumption will be made about its mass.
- The H1 report of an anticharmed baryon at 3099 MeV [5] is also unconfirmed [6], so no assumptions will be made about the masses of any possible heavy pentaquark states.

A low-lying, relatively narrow (\( \mathcal{O}(10 \text{ MeV}) \)), \( \bar{10}_F \), \( J^P = 1/2^+ \), \( S = +1 \) exotic baryon is natural in the chiral soliton model (CSM) [7, 8]. In the quark model, the ground state of the exotic system would have negative parity (\( P = - \)) if all five particles were in a symmetric relative s-wave configuration. Whether such a configuration represents the true model ground state, however, is a dynamical question. Exciting one of the four light (\( \ell = u, d \)) quarks into a p-orbit, reducing the \( \ell^4 \) spatial symmetry from \( [4]^L \) to \( [31]^L \), allows spin-flavor-color configurations with significantly increased hyperfine attraction which were previously Pauli-forbidden. If the increased attraction is greater than the p-wave excitation cost, the ground state will have \( P = + \). A dynamical calculation is needed to determine which of the two effects wins out.

Two proposals of possible \( P = + \) quark model configurations for the \( \theta \) have received considerable attention, and are also useful for framing the discussions below. The Jaffe-Wilczek (JW) ansatz for the \( \theta \) [9] involves two \( I = J = 0, C = 3 \) diquark pairs, coupled to color \( C = 3 \), with only confinement forces between the diquarks and \( s \). Bose statistics for the diquarks imply a relative p-wave between them, hence \( P_\theta = + \). The Karliner-Lipkin (KL) scenario for the \( \theta \) [10]
involves one JW diquark and one $ud\bar{s}$ triquark. In the triquark, the spin and color of a JW diquark have been flipped and anti-aligned with those of the $\bar{s}$, producing triquark quantum numbers $I = 0, J = 1/2, C = 3$. The ansatz is motivated by the color-magnetic-exchange model, in which the hyperfine attraction is stronger for the KL than for the JW correlation. A relative diquark-triquark p-wave will yield $P_0 = +$. In both scenarios, the relative intercluster p-wave is assumed to make cross-cluster interaction and antisymmetrization (AS'n) effects negligible.

The “GB model” is a model with spin-dependent interactions generated by effective Goldstone boson exchange. The spin-dependent part of the Hamiltonian is

$$H_{GB} \sim - \sum_{i<j} \vec{\sigma}_i \cdot \vec{\sigma}_j \frac{\lambda_{F,i}^a \lambda_{F,j}^a f^a(\vec{r}_{ij})}{m_i m_j}$$

(1)

with $\vec{\sigma}_i$ the Pauli spin matrices, $m_i$ the constituent quark mass, $\lambda_{F,i}^a$ the $SU(3)_F$ Gell-Mann matrices, and $f^a(\vec{r})$ as given in Ref. [11]. The JW diquark is by far the most attractive $qq$ correlation for $H_{GB}$. If the sum in Eq. (1) runs only over quark labels [11], the model also matches the JW assumption for the $\bar{s}\ell$ interactions. Assuming an orbital excitation cost similar to that in the ordinary baryon sector of the model, the gain in hyperfine energy in the $[31]_L$ configuration leads to a $P = +$ ground state, one which should be dominated by the JW correlation. The model thus provides a dynamical setting for studying (i) corrections to the strict JW scenario (e.g., cross-cluster AS'n/interaction effects, configuration mixing) and (ii) the pattern of excitations above the JW-correlation-dominated ground state.

The “CM model” is a model with spin-dependent interactions generated by effective color magnetic exchange. The spin-dependent part of the Hamiltonian is

$$H_{CM} \sim - \sum_{i<j=1,\ldots,5} \vec{\sigma}_i \cdot \vec{\sigma}_j F_i^a F_j^a \frac{f_{CM}(\vec{r}_{ij})}{m_i m_j}$$

(2)

with $\vec{\sigma}_i$ and $m_i$ as above, $F_i^a = \frac{\lambda_i^a}{2} (-\lambda_i^a/2)$ for quarks (antiquarks) the $SU(3)_c$ Gell-Mann matrices and $f_{CM}(\vec{r}_{ij})$ a smeared delta function. The JW diquark is again the most attractive $qq$ correlation, but, as noted above, the more complicated KL configuration lies lower than the JW configuration. The CM model thus provides a dynamical setting for studying (i) corrections to the KL scenario and (ii) the expected excitation pattern above the CM ground state. An important correction results from the fact that the same $\bar{s}\ell$ interactions responsible for lowering the KL triquark hyperfine energy also mix the JW and KL correlations, producing a configuration significantly lower than either the JW or KL correlations separately. This configuration turns out to be a roughly equal admixture of the two correlations [12].

2. Results

Space constraints prevent discussion of all points covered in the conference talk. (See the conference website and Refs. [12] [13] [14] for more details; Ref. [14], in particular, gives an explicit definition of the dimensionless form used in quoting results below.)

(a) The $\theta$ mass and pentaquark widths in the quark model: Models such as the GB and CM models, with parameters fixed from the ordinary baryon sector, cannot predict exotic pentaquark masses because of the absence of terms representing the response of the non-perturbative vacuum to the hadronic constituents. This response will differ depending on the number of constituents. This effect will be absorbed by the additive constant used to set the overall scale in each sector; the constant should thus be different for ordinary and pentaquark baryons, just as it is for mesons and baryons. In the bag model, where some modelling of vacuum response does occur, the difference in 1-body energies between a single $q^6$ bag and two isolated $q^3$ bags is an extremely sensitive function of the $B^{1/4}$ [15], suggesting it is impossible to anticipate in advance the size,
or even sign, of this effect. Thus, the fact that quark model calculations for \( m_\theta \) using parameter values fixed from the ordinary baryon spectrum yield masses 100 – 300 MeV or so too high does not rule out the validity of a pentaquark picture for such states. Other observables (widths, splittings, relative decay couplings) must be used to test the models.

If the \( \theta \) is well described as a pentaquark, the initial and final states in \( \theta \rightarrow NK \) have the same number of constituents. The decay thus proceeds by rearrangement, or “fall-apart”, and should have a width \( \sim \hat{\Gamma}S \), where \( \hat{\Gamma} \) is the width for a \( KN \) potential scattering resonance with mass \( m_\theta \) in an attractive potential of roughly hadronic size, and \( S \) is a spectroscopic, or squared-overlap, factor \([12]\). A single-scale s-wave attractive potential too weak to bind does not produce resonant behavior \([9]\); hence a \( J^P = 1/2^- \) assignment for the \( \theta \) is impossible. For a p-wave (d-wave) decay, \( \hat{\Gamma} \sim 200 \) (respectively, 20) MeV. The spin-flavor-color (JFC) part of the overlap suppression is 1/24 for the JW correlation and \( \sim 1/25 \) for the optimized JW/KL combination in the CM model \([12]\). With additional suppression from the spatial overlap, \( \theta \) widths \( \sim \) a few to several MeV are expected for \( P = + \) ground states involving such correlations. As for the CSM, some fine tuning is needed to reduce this to \( \sim 1 \) MeV or less. (A very narrow \( \theta \) is natural for a \( J^P = 3/2^- \) assignment, where a d-wave \( NK \) decay is required. However, at least in the GB and CM models, such a ground state assignment is impossible \([12]\).)

The JFC “overlap” suppression effect is rather general in the quark model approach. In the soft-\(K\)/PCAC approximation, the \( \theta \rightarrow nK^+ \) coupling is \( \propto \langle n|\bar{s}u|\theta \rangle \). Three of the light quarks in the \( \theta \) must project onto the \( n \) and the remaining constituent \( \ell \) and \( \bar{s} \) be annihilated by the pseudoscalar density with \( K^+ \) quantum numbers. The JFC factors are the same as would occur in a \( \theta \) to \( NK \) overlap calculation using a constituent \( \bar{s}u \) picture for the \( K^+ \). A model calculation \([16]\) confirms the reduction of \( \hat{\Gamma}_\theta \) to \( \sim 10 \) MeV for the JW correlation, before inclusion of spatial overlap effects, and can be interpreted as a model-dependent version of the soft-\(K\)/PCAC estimate. As stressed in Ref. \([17]\), for \( P = + \) pentaquarks with p-wave fall-apart decays, decay coupling ratios should be given by the corresponding overlap ratios squared.

(b) Corrections to the JW and KL scenarios in the GB and CM models: For the GB model, \( \langle H_{GB} \rangle \) in the strict JW limit is \(-16\). The lowest-lying \( P = + \) state has JW quantum numbers (which, up to a \( J^P = 3/2^+ \) spin-orbit partner, are also the CSM ground state quantum numbers). Cross-cluster interaction/AS’n and configuration mixing lower the actual ground state expectation to \(-21.9 \pm 1.3\) \([12]\). The JW approximation is reasonable, but not exact. The ground state expectation is, however, the same in the \( \theta \) and heavy \( Q\ell^4 \) \((Q = c, b)\) sectors, as in the JW scenario. With the JW estimate for the 1-body contribution to \( m_{\theta_{c,b}} - m_\theta \) \([9]\), the JW prediction of strong interaction stable \( \theta_{c,b} \) is thus preserved.

For the CM model, in the \( \theta \) sector, the optimized combination of JW and KL correlations provides a good approximation to the lowest \( P = + \) eigenvalue \([12]\), which again occurs for the channel with CSM quantum numbers. In the heavy sector, the \( Q\ell \) interactions are strongly suppressed by the \( 1/m_Q \) factor in \( H_{CM} \). This (i) decreases the KL triquark hyperfine attraction, making the KL diquark-triquark configuration higher in energy than the JW correlation for both \( Q = c, b \), and (ii) strongly suppresses KL-JW mixing. The heavy pentaquark \( P = + \) ground state should thus be dominated by the JW correlation. The JW expectation is \(-4\), c.f. the actual ground state expectation \(-3.48 \pm 0.04\) \([13]\). The JW approximation is again reasonable, but not exact. The KL scenario for the heavy quark analogues \([18]\) assumes the same triquark-diquark clustering as for the \( \theta \) persists in the heavy system. This assumption is not compatible with the CM model effective interactions, which motivated the original triquark-diquark proposal. The KL heavy sector ansatz thus leads to an overestimate of the \( \theta_{c,b} \) masses in the CM model. Modified estimates put the \( \theta_{c,b} \) close to the \( ND, NB \) strong decay thresholds \([14]\).

(c) Splittings and relative decay couplings in the light and heavy \( P = + Q\ell^4 \) sectors: Though one cannot predict absolute pentaquark masses in the quark model approach, splittings are independent of any additive constant and hence are predictable. Predictions for a given \( Q\ell^4 \)
sector (with $Q = s, c, b$ fixed) are particularly immune to uncertainties in treating 1-body energies, and are on the same footing as analogous predictions for the ordinary baryon sector. The relative ordering of states is fixed by the JFC structure of the effective operators, with an overall scale, dependent on the size of the pentaquark states. Ratios of overlap factors relevant to the decays to $BM$, with $B$ the relevant ground state baryon and $M$ the relevant vector or pseudoscalar meson, are also well-determined [13].

For the GB model, the first excitation above the $P = +, Q\ell^4$ ground state, for all of $Q = s, c, b$, is a degenerate pair with $(I, J_q) = (1, 1/2), (1, 3/2)$, where $J_q$ is the total intrinsic spin. The excitation energy is $\sim 130 - 170$ MeV. No other states are expected within $\sim m_\Delta - m_N$ of the ground state. For the $(1, 1/2)$ state, the combination of phase space and squared-overlaps enhances the width to $NK$ by a factor of at most $\sim 12$ c.f. the $\theta$, making this prediction potentially problematic. A similar, potentially problematic, state with $(F, I, J^P) = (27, 1, 3/2^+)$, mass $\sim 60$ MeV above the $\theta$, and relatively narrow $\sim 66$ MeV $NK$ width, is also predicted in the CSM [8]. For the CM model, in the $\ell^4s$ sector, a single low-lying $(I, J_q) = (1, 1/2)$ excitation is predicted $\sim 80$ MeV above the $\theta$. The next excitation lies $\sim 160$ MeV higher. Phase space and the overlap factor for $(1, 1/2)$ state’s decay to $NK$ are again such that a relatively narrow width is expected. Even more problematic are the predictions in the heavy sector, where five low-lying excitations are expected, with $(I, J_q) = (0, 1/2), (1, 1/2), (0, 3/2), (1, 3/2), (1, 1/2)$, the first $\sim 80 - 90$ MeV above the ground state, and the last $\sim 130 - 150$ MeV above the ground state [14]. Such a dense, low-lying excitation spectrum seems unlikely, but would be a striking prediction of the CM model, if confirmed. None of the overlap ratios are such as to allow any of these low-lying states to be experimentally undetectable in 2-body decay modes, provided at least one of either them, or the ground state, is experimentally accessible [14].

For the GB model, the JW dominance of the ground state and decrease in JW diquark hyperfine energy under $d \leftrightarrow s$ imply a reduction in $\Xi_{3/2}$ hyperfine energy c.f. the $\theta$. An estimate of the splitting, using only the $[4]_{F, I}$ component in the $\Xi_{3/2}$ wavefunction, yields $m_{\Xi_{3/2}} = 1962$ MeV [19]. Allowing other wavefunction components (known to be small for the $\theta$, but expected to be more important for the $\Xi_{3/2}$) will lower this value. In the KL scenario, the $\theta$ and $\Xi_{3/2}$ have almost identical ($H_{CM}$) values, leading to $m_{\Xi_{3/2}} \sim 1720$ MeV [10]. The lowest hyperfine eigenvalues in the $\theta$ and $\Xi_{3/2}$ channels turn out to share this near equality, even though the ground state is not well approximated by the KL correlation. The CM model thus produces a significantly smaller $\Xi_{3/2} - \theta$ splitting than does the GB model.

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