Lepton-rich cold quark matter

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We explore protoneutron star matter in the framework of cold and dense QCD using the state-of-the-art perturbative equation of state including neutrinos fixed by a lepton fraction that is appropriate for this environment. Furthermore, we calculate the modifications in the lepton-rich equation of state showing that stable strange quark matter has a more restricted parameter space.

PACS numbers: 25.75.Nq, 11.10.Wx, 12.39.Fe, 64.60.Q-
bounce stage of core collapse supernovae, one has to consider an EoS that is rich in leptons. In particular, one has to include trapped neutrinos in the pQCD framework. In the following, we set a widely accepted value for lepton fraction in PNS matter, i.e., $Y_L = 0.4$ [23,24], which forces us to introduce a chemical potential for the neutrinos, $\mu_\nu$.

As usual, we define the total quark number density as

$$n = n_u(\mu_u, X) + n_d(\mu_d, X) + n_s(\mu_s, X), \quad (1)$$

where each quark density depends on its respective chemical potential and on the renormalization scale $X$. The baryon number density is defined as $n_B = n/3$. Since we aim to study the properties of compact stars, one has to impose charge neutrality and lepton fraction conservation. Doing it locally:

$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s = n_e, \quad (2)$$

$$\frac{n_e + n_\nu}{n_B} = Y_L = 0.4, \quad (3)$$

while the weak interaction equilibrium conditions imply

$$\mu_d + \mu_\nu = \mu_u + \mu_e, \quad (4)$$

$$\mu_d = \mu_s \equiv \mu. \quad (5)$$

Here, $\mu_u$, $\mu_d$, $\mu_s$, $\mu_e$ and $\mu_\nu$ are the chemical potentials of the up, down and strange quarks, the electron and the electron neutrino. The latter are introduced as degenerate Fermi gas contributions.

Using the constraints above, we can write all quark and lepton chemical potentials in terms of the strange quark chemical potential, $\mu_s \equiv \mu$, only. Following Kurkela et al. [7], we use the quark and lepton number densities as the building blocks from which one can construct the thermodynamic variables, in our case the total pressure, demanding thermodynamic consistency at each step of the calculation and preserving terms up to $O(\alpha_s^2)$.

When some quark density flavor $n_f(\mu_f, X)$ becomes negative below a given chemical potential flavor, $\mu_f < \mu_f^0(X)$, we set it to $n_f \equiv 0$. Integrating the number densities from their minimal value $\mu_f^0(X)$ to some arbitrary strange quark chemical potential $\mu$ and taking into account Eqs. (2)–(5), we obtain the total pressure for lepton-rich quark matter as follows:

$$P(\mu, X) = \int^{\mu}_{\mu_0(X)} d\bar{\mu} \left[ n_u \left( 1 + \frac{d\mu_e}{d\mu_s} - \frac{d\mu_s}{d\mu_e} \right)^2 \right.$$  

$$+ n_d + n_e \frac{d\mu_s}{d\mu_e} \frac{d\mu_s}{d\mu_e} \left. \right] . \quad (6)$$

We express the pressure as a function of the baryon chemical potential, $P = P(\mu_B)$, where $\mu_B = \mu_u + \mu_d + \mu_e$. In Fig. 2 one can see how the cold quark matter EoS (KRV) is modified by the presence of trapped neutrinos ($Y_L$) for different values of the renormalization scale $X$. Notice that, at high $\mu_B$, since $m_s(X)$ tends to be constant [7,19] and $\alpha_s(X)$ is non-zero (unless we are at asymptotically high densities [7]), the total pressure of quarks and leptons will increase faster, in contrast to the lepton-poor case. However, at low $\mu_B$, the lepton-poor total pressure appears to be higher (see Ref. [10] for a discussion). To clarify this issue, we show, in Fig. 3 the total pressure of quarks and leptons as a function of $n_B$.

**IV. LEPTON-RICH STABLE STRANGE QUARK MATTER**

Bodmer [25] and Witten [26] investigated in different contexts a system formed by massless up, down and
strange quarks. If they have at zero pressure a energy per baryon
\[ E/A \leq 0.93 \text{GeV}, \]
i.e., lower than the most stable nuclei Fe\textsuperscript{56}, one would find configurations of absolutely \textit{stable strange quark matter} (SQM) as the true ground state of hadronic matter in the vacuum.

We explore the criterion above using the thermodynamic potential discussed in Sec. III. To do that, we use the Hugenholtz-Van Hove theorem \cite{27} generalized to a system with many components \cite{28}. It requires only the quark and lepton densities and chemical potentials as input, giving the following energy per baryon:
\[ \frac{E(\mu_s, X)}{A} = \frac{n_u}{n_B} (\mu_\nu - \mu_e) + 3\mu_s, \]
where we implicitly assumed that all the quantities on the rhs of the equation above are functions of the strange chemical potential \( \mu_s \) and renormalization scale \( X \).

Constraining the values of \( X \) such that \( \mu_s \) and \( n_s \) are not zero and satisfy Eq. (7) we obtain, for the cold case, \( X \in [2.95, 4] \), and for the lepton-rich case \( X \in [3.45, 4] \), as can be seen in Fig. 3. Even if the parameter space of \( X \) is not radically modified when trapped neutrinos are included, one can notice from Fig. 3 that the band for \( X \) tends to shrink to \( \mu_B \in [0.86, 0.88] \text{GeV} \) for vanishing pressure (as compared to \( \mu_B \in [0.803, 0.93] \text{GeV} \) in the cold case). Figure 3 also indicates that lepton-rich strange quark matter becomes essentially \textit{independent} of the renormalization scale \( X \).

One can infer from this that the presence of leptons fixed to some non-trivial value makes the SQM hypothesis less favorable, i.e., the stability windows of critical densities with vanishing pressure is narrower. A similar behavior was observed in Ref. \cite{29}, where the authors also included finite-temperature contributions in different quark models. This leaves us with \( X \in [1, 3.44] \) for lepton-rich quark matter having as ground state hadronic matter in the vacuum.

\section{V. SUMMARY}

In this work we have explored protoneutron star matter, i.e., stellar matter formed after a core-collapse supernova explosion which after a minute produces a regular neutron star, using the state-of-the-art perturbative equation of state for cold and dense QCD matter in the presence of a fixed lepton fraction in which both electrons and neutrinos are included. Finite-temperature effects on the equation of state can be neglected since they have a minor effect in the PNS scenario we have in mind where densities are high enough compared to thermal effects. Even if the presence of neutrinos does not modify appreciably the EoS at low densities compared with other effective models for lepton-rich quark matter, for some values of the renormalization scale \( X \) their presence significantly increases the pressure as one goes to higher densities, still within the region relevant for the physics of PNS, making the EoS stiffer.

Modifications in the equation of state due to the presence of trapped neutrinos were investigated showing that the stable strange quark matter hypothesis is less favorable in this environment, i.e., the parameter space for the formation of strange quark matter with neutrinos decreases considerably.
ACKNOWLEDGMENTS

This work was supported by CNPq and FAPERJ, being also part of the project INCT-FNA Process No. 464898/2014-5.

[1] N. K. Glendenning, Compact Stars – Nuclear Physics, Particle Physics and General Relativity, (Springer, New York, 2000).
[2] B. P. Abbott et al. (LIGO Scientific and Virgo Collaborations), Phys. Rev. Lett. 119, 161101 (2017).
[3] N. Andersson, V. Ferrari, D. I. Jones, K. D. Kokkotas, B. Krishnan, J. S. Read, L. Rezzolla and B. Zink, Gen. Relativ. Gravit. 43, 409 (2011).
[4] P. de Forcrand, Proc. Sci. LAT 2009 (2009) 010.
[5] J. I. Kapusta and C. Gale, Finite-Temperature Field Theory: Principles and Applications (Cambridge University Press, Cambridge, England, 2006).
[6] M. Laine and A. Vuorinen, Lect. Notes Phys. 925, pp.1 (2016).
[7] A. Kurkela, P. Romatschke and A. Vuorinen, Phys. Rev. D 81, 105021 (2010).
[8] E. S. Fraga, A. Kurkela and A. Vuorinen, Astrophys. J. 781, L25 (2014).
[9] E. S. Fraga, A. Kurkela and A. Vuorinen, Eur. Phys. J. A 52, 49 (2016).
[10] J. C. Jiménez and E. S. Fraga, Phys. Rev. D 97, 094023 (2018) [arXiv:1712.04773 [hep-ph]].
[11] B. A. Freedman and L. D. McLerran, Phys. Rev. D 16, 1169 (1977).
[12] V. Baluni, Phys. Rev. D 17, 2092 (1978).
[13] T. Toimela, Int. J. Theor. Phys. 24, 901 (1985) ; 26, 1021 (E) (1987).
[14] E. S. Fraga, R. D. Pisarski and J. Schaffner-Bielich, Phys. Rev. D 63, 121702 (2001).
[15] J. P. Blaizot, E. Iancu and A. Rebhan, Phys. Rev. D 63, 065003 (2001).
[16] B. Freedman and L. D. McLerran, Phys. Rev. D 17, 1109 (1978).
[17] E. Farhi and R. L. Jaffe, Phys. Rev. D 30, 2379 (1984).
[18] N. Itoh, Prog. Theor. Phys. 44, 291 (1970).
[19] E. S. Fraga and P. Romatschke, Phys. Rev. D 71, 105014 (2005).
[20] A. Kurkela, P. Romatschke, A. Vuorinen and B. Wu, arXiv:1006.4062 [astro-ph.HE].
[21] S. Aoki et al., Eur. Phys. J. C 77, 112 (2017).
[22] A. Bazavov, N. Brambilla, X. Garcia i Tormo, P. Petreczky, J. Soto and A. Vairo, Phys. Rev. D 90, 074038 (2014).
[23] J. A. Pons, S. Reddy, M. Prakash, J. M. Lattimer and J. A. Miralles, Astrophys. J. 513, 780 (1999).
[24] A. Burrows and J. M. Lattimer, Astrophys. J. 307, 178 (1986).
[25] A. R. Bodmer, Phys. Rev. D 4, 1601 (1971).
[26] E. Witten, Phys. Rev. D 30, 272 (1984).
[27] N. M. Hugenholtz and L. van Hove, Physica 24, 363 (1958).
[28] R.C.Nayak and S. Pattnaik, DAE Symp. Nucl. Phys. 56, 236 (2011).
[29] V. Dexheimer, J. R. Torres and D. P. Menezes, Eur. Phys. J. C 73, 2569 (2013).