Sliding Singlet Mechanism Revisited

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Abstract

We show that the unification of the doublet Higgs in the standard model (SM) and the Higgs to break the grand unified theory (GUT) group stabilizes the sliding singlet mechanism which can solve the doublet-triplet (DT) splitting problem. And we generalize this attractive mechanism to apply it to many unified scenarios. In this paper, we try to build various concrete $E_6$ unified models by using the generalized sliding singlet mechanism.
1 Introduction

The well-known success of the gauge coupling unification in the minimal supersymmetric standard model (MSSM) likely supports the attractive idea of supersymmetric grand unified theory (SUSY-GUT). On the other hand, we know there are some obstacles in constructing a realistic SUSY-GUT. One of the biggest problems is the so-called DT splitting problem. Generically in SUSY-GUTs, there are color triplet partners of the MSSM Higgs, and the nucleon decay via dimension five operators becomes too rapid. In order to suppress this proton decay, the color triplet partners must have very large mass ($\gg M_{\text{GUT}} \sim 10^{16}\text{GeV}$), in contrast to the doublet Higgs whose mass has to be of order the weak scale $M_W \sim 10^{2}\text{GeV}$. Some ideas to solve this problem have been proposed: the sliding-singlet mechanism\cite{1, 2, 3, 4}, the missing partner mechanism\cite{5, 6}, the Dimopoulos-Wilczek (DW) mechanism\cite{7}, the GIFT mechanism\cite{8}, and via orbifold boundary condition\cite{9}.

Among these ideas, the first mechanism is the smartest solution which realizes the DT splitting dynamically. Although it was shown that the originally proposed $SU(5)$ model cannot act effectively if SUSY breaking effect is considered\cite{10}, some authors have proposed $SU(6)$ extensions in which this mechanism acts without destabilization due to SUSY breaking\cite{2, 3, 4}. In this paper, we abstract the essence of this sliding singlet mechanism in $SU(6)$ models and generalize it to apply to many other unified theories. Actually in $E_6$ unification it is found that for many directions of VEV of the adjoint Higgs this mechanism may act. Corresponding to these breaking patterns, we construct some $E_6$ Higgs sectors in which the DT splitting problem is indeed solved through this mechanism. Several concrete models are propose in the context of the SUSY-GUT in which an anomalous $U(1)_A$ gauge symmetry\cite{11}, whose anomaly is cancelled by the Green-Schwarz mechanism\cite{12}, plays an important role\cite{13, 14, 15, 16, 17, 18, 20} in solving various problems in SUSY scenario. And we examine whether the already proposed realistic quark and lepton sector\cite{14} is compatible with such a Higgs sector or not. Note that this $E_6$ group is interesting as a unified group, in the sense that the SUSY flavor problem can be solved in $E_6$ SUSY-GUT with anomalous $U(1)_A$ and non-abelian horizontal gauge symmetry\cite{15}.

In section 2, we briefly review the sliding singlet mechanism in the context of $SU(5)$ and $SU(6)$. In section 3, we generalize this mechanism to the general gauge group. In section 4, we construct some concrete Higgs sectors.

2 The Sliding Singlet Mechanism

In this section, We review the present status of the sliding singlet mechanism. For this purpose, we sometimes omit details, which are described in each references.

2.1 $SU(5)$

The sliding singlet mechanism was originally proposed in the context of $SU(5)$\cite{1}, in which the following terms are allowed in the superpotential;

$$W_{\text{ss}} = \bar{H}(A + Z)H.$$

(2.1)
Here, the adjoint Higgs $A(24)$ is assumed to have the VEV $\langle A \rangle = \text{diag}(2, 2, 2, -3, -3) v$ which breaks $SU(5)$ into $SU(3)_C \times SU(2)_L \times U(1)_Y$ ($G_{SM}$), and the (anti)fundamental Higgs $H(5)$ and $\bar{H}(\bar{5})$ contain the MSSM doublet Higgs, $H_u$ and $H_d$, respectively. Since the doublet Higgs have non-vanishing VEVs $\langle H_u \rangle$ and $\langle H_d \rangle$ to break $SU(2)_L \times U(1)_Y$ into $U(1)_{\text{EM}}$, the minimization of the potential,

$$V_{\text{SUSY}} = |F_H|^2 + |F_\bar{H}|^2 = \left( |\langle H \rangle|^2 + |\langle \bar{H} \rangle|^2 \right) | -3v + \langle Z \rangle |^2,$$

leads to the vanishing doublet Higgs mass $\mu = (\langle A \rangle + \langle Z \rangle)_{2} = -3v + \langle Z \rangle = 0$ by sliding the VEV of the singlet Higgs $Z(1)$\footnote{Here, the contributions to the potential from $F_A$ and $F_Z$ are neglected, because they are of order $(\langle H \bar{H} \rangle)^2$. In this sense, the doublet Higgs mass $\mu$ does not vanish exactly but may become of order SUSY breaking scale $M_{\text{SB}}$.}. For these VEVs, $\langle A \rangle + \langle Z \rangle = \text{diag}(5, 5, 5, 0, 0)v$, the color triplet partners of doublet Higgs have a large mass $5v \sim 10^{16}\text{GeV}$.

Unfortunately, it is known that if SUSY breaking is taken into account, this DT splitting is failed. For example, the soft SUSY breaking mass term $\sqrt{\langle Z \rangle}$ leads to the vanishing doublet Higgs mass $\mu$. For these VEVs, $\langle A \rangle + \langle Z \rangle = \text{diag}(5, 5, 5, 0, 0)v$, the color triplet partners of doublet Higgs have a large mass $5v \sim 10^{16}\text{GeV}$.

It is obvious that this problem can be solved if we take the large VEV from that in the SUSY limit by a large amount. This is caused by the fact that the terms $|F_H|^2 + |F_\bar{H}|^2$ give only a mass of order $\langle H \rangle$ to $Z$, which are the same order as (or smaller than) the SUSY breaking contribution. Since this mass parameterizes the stability of $\langle Z \rangle$ against other contributions to the potential, e.g. SUSY breaking effects $\tilde{m}^2 |Z|^2$, soft terms of order $M_{\text{SB}}$ easily shift the VEV from that in the SUSY limit by a large amount.

It is obvious that this problem can be solved if we take the large VEVs $\langle H \rangle$ and $\langle \bar{H} \rangle$ (larger than $\sqrt{M_{\text{SB}} M_{\text{GUT}}}$) which give larger mass to $Z$ and stabilize the VEV of $Z$ against SUSY breaking effects. Of course, it is not consistent with the experiments to take such large VEVs for SM doublet Higgs. But for other Higgs, for example, that breaks a larger gauge group into the SM gauge group, we can take such large VEV. This is an essential idea of Sen\footnote{The soft term $\tilde{m} Z F_Z$ also destabilizes the sliding singlet mechanism because this term alters the contribution of $F_Z$ to the scalar potential as $|\langle H \bar{H} + \tilde{m} Z F_Z |^2$, that is the order of $M_{\text{SB}}^2 M_{\text{GUT}}^2 (\gg M_{\text{SB}}^4)$ if $\langle H \rangle$ is order of the weak scale. Such a term is induced by loop effects through the coupling between $Z$ and the color triplet Higgs. Therefore, even if the terms $\tilde{m} Z F_Z$ and $\tilde{m}^2 |Z|^2$ are absent at the tree level, this problem cannot be avoided.}.

### 2.2 \textit{SU(6)}

$SU(6)$ is the simplest candidate for the above purpose, and some authors examine the possibility\footnote{$A(35)$ has a VEV $\langle A \rangle = \text{diag}(1, 1, 1, -1, -1, -1) v$ which breaks $SU(6)$ into $SU(3)_C \times SU(3)_L \times U(1)_{Y}$ and $H(6)$ ($\bar{H}(\bar{6})$) denotes (anti)fundamental Higgs which has a VEV in $SU(5)$ singlet component. Note that the simplest extension, in which one pair of (anti)fundamental Higgs is introduced, cannot act effectively. This is because

- the $F$-term of $Z$ gives a contribution of $O(M_{\text{GUT}}^4)$ to the scalar potential.}. The relevant part of the superpotential is written in a similar form as $\text{[2.1]}$, where $A(35)$ has a VEV $\langle A \rangle = \text{diag}(1, 1, 1, -1, -1, -1) v$ which breaks $SU(6)$ into $SU(3)_C \times SU(3)_L \times U(1)_{Y}$ and $H(6)$ ($\bar{H}(\bar{6})$) denotes (anti)fundamental Higgs which has a VEV in $SU(5)$ singlet component. Note that the simplest extension, in which one pair of (anti)fundamental Higgs is introduced, cannot act effectively. This is because
• the term $\bar{H}A H$ gives a contribution to the $F$-flatness condition of $A$ which destabilizes the required form of VEV of $A$, if $\langle A \rangle$ is determined from $F_A$.

• this term also gives mass terms of $5$ and $\bar{5}$ of $SU(5)$, $\langle H \rangle AH$ and $\bar{H} A \langle H \rangle$.

• one pair of doublet of (anti)fundamental Higgs is the Nambu-Goldstone (NG) mode and unphysical.

The first two and the last one are resolved if one of the pair of (anti)fundamental Higgs has vanishing VEV. Thus, (2.1) is altered as

$$W_{SS} = \bar{H}'(A + Z)H + \bar{H}(A + Z)H',$$  \hspace{1cm} (2.3)

where primed fields have vanishing VEVs. Because of the third reason, at least one more pair of (anti)fundamental Higgs is required.

For example in Ref.[3], four pairs are introduced and the relevant part of the super-potential is given as

$$W = W(A) + W(\bar{H}_i, H_i)$$

$$+ \sum_{i=1,2} a_i H'_i(A + Z_i)H_i + \sum_{i=1,2} \bar{a}_i \bar{H}_i(A + \bar{Z}_i)H'_i,$$  \hspace{1cm} (2.4)

where $a_i$ and $\bar{a}_i$ are coupling constants and $W(A)$ and $W(\bar{H}_i, H_i)$ are some sets of terms which give the desired VEV (as one of discrete vacua) to $A$ and $\bar{H}_i, H_i$ respectively. This gives following mass matrix of $5 \times 5$ of $SU(5)$:

$$M_I = \begin{pmatrix}
I_A & I_{H'_1} & I_{H'_2} & I_{H_1} & I_{H_2} \\
\bar{I}_{A} & M_I & a_1 \langle \bar{H}_1 \rangle & a_2 \langle \bar{H}_2 \rangle & 0 & 0 \\
I_{H'_1} & a_1 \langle H_1 \rangle & 0 & 0 & 2\alpha_1 a_1 v & 0 \\
I_{H'_2} & a_2 \langle H_2 \rangle & 0 & 0 & 0 & 2\alpha_1 a_2 v \\
\bar{I}_{H_1} & 0 & 2\alpha_1 \bar{a}_1 v & 0 & c|\langle H_2 \rangle|^2 & -c \langle H_1 H_2^* \rangle \\
\bar{I}_{H_2} & 0 & 0 & 2\alpha_1 \bar{a}_2 v & -c \langle H_1^* H_2 \rangle & c|\langle H_1 \rangle|^2
\end{pmatrix}.$$  \hspace{1cm} (2.5)

Here $\alpha_2 = 0$ and $\alpha_3 = 1$, which are realized by the sliding singlet mechanism, and $M_3 = 0$ because $3_A$ and $\bar{3}_A$ are NG modes by breaking $SU(6) \rightarrow SU(3)_C \times SU(3)_L \times U(1)$. $M_2$ and $c$ are determined from $W(A)$ and $W(\bar{H}_i, H_i)$, respectively. From this mass matrix, it can be found that there are two massless modes for $I = 2$ and one for $I = 3$. Since one pair of $5$ and $\bar{5}$ of $SU(5)$ are absorbed by the Higgs mechanism, only one pair of doublets remains massless and therefore the DT splitting is realized. In this model, the massless modes come from a linear combination of the primed fields.

Note that, due to the large VEVs of $H_i$ and $\bar{H}_i$, this hierarchy is stable against the SUSY breaking corrections, which means that all the elements of the mass matrix have corrections due to SUSY breaking at most $O(M_{SB})$.

Another example was proposed in Ref.[4] in the context of $SU(6) \times SU(2)$. The relevant part is similar as previous model except the absence of the indices $i$ of the singlet Higgs. The indices of the (anti)fundamental Higgs are understood as those of the
symmetry $SU(2)$. This symmetry guarantees that the doublet components of $H_2$ and $\bar{H}_2$ are massless even if they do not have non-vanishing VEVs. This means that the doublets are not NG modes and therefore physical modes. In this model, the SM doublet Higgs comes from the unprimed fields which have non-vanishing VEVs, in contrast to the previous model.\footnote{In order to give masses to the primed fields, some additional terms, e.g. $\bar{H}'_i H'_i$, are needed.}

The author of Ref.[4] mentions that this is because the symmetry $SU(2)$ relates the doublets to NG modes in $H_1$ and $\bar{H}_1$. To be more precise, the doublets and the NG modes belong to a single multiplet of the $SU(2)$ symmetry to which the mass parameter $(\langle A \rangle + \langle Z \rangle)$ respect. However, in the spirit of the sliding singlet, it may be more appropriate to say that the mass parameter $(\langle A \rangle + \langle Z \rangle)$ gives the same value for the doublets as for the $SU(5)$ singlet components of $H_1$ and $\bar{H}_1$, which must vanish due to the non-vanishing VEVs. This observation makes it possible to apply the sliding singlet mechanism in more general case.

3 Generalization

Now, we examine how we can generalize the sliding singlet mechanism.

The essential idea of the sliding singlet mechanism is following.

- If the mass parameter of a certain component is guaranteed to be the same value as that of the other component which has a non-vanishing VEV and the later vanishes dynamically due to the VEV, the former also vanishes. And if the non-vanishing VEVs are sufficiently large, the mass hierarchy is stable against possible SUSY breaking effects.

It is the case that a doublet component and a singlet component with non-vanishing VEV belong to a single multiplet of the symmetry to which the mass terms respect, e.g. $SU(3)_C \times SU(3)_L \times U(1)$ for the previous $SU(6)$ example in Ref.[3]. In this case, the DT splitting problem can be solved.

Moreover, if the mass parameter depends only on the VEVs of adjoint Higgs and singlet Higgs, the above condition for the sliding singlet mechanism to act can be easily examined. This is because the mass parameter for each component is determined by each quantum number of $U(1)$ which are fixed by the non-vanishing VEV of the adjoint Higgs. Therefore, if the charge of the doublet Higgs component is the same as that of the singlet component which has non-vanishing VEV, then the massless doublet Higgs can be realized by the sliding singlet mechanism. This perspective holds for any gauge group, even if the mass term involves non-renormalizable terms.

It is obviously important to know the charges for the SM singlet and the doublet Higgs under the $U(1)$ generator which are fixed by the non-vanishing VEV of the adjoint Higgs. If a GUT group $G$ includes $SU(3)_C \times SU(2)_L \times Y \times U(1)^r$ as a subgroup, where $r$ is the rank of $G$, the $U(1)$ generator must be a linear combination of $r - 3$ $U(1)$ generators. Therefore, it is helpful to know the charges of these $U(1)$ generators in order to classify
the models in which sliding singlet mechanism acts effectively. We present the charges for $SU(6)$ unification group in Table 1 and for $E_6$ unification group in Table 2.

| $SU(6)$ | 6 | 35 |
|---------|---|----|
| $SU(5)$ | 5 | 1 | 24 | 5 |
| $G_{SM}$ | $D, \bar{L}$ | $N$ | $X$ | $D, \bar{L}$ |
| $V_6$ | 1 | $-5$ | 0 | $-6$ |
| $6Y$ | $-2, 3$ | 0 | $-5$ | $2, -3$ |
| $V_6 - 12Y$ | $5, -5$ | $-5$ | 10 | $-10, 0$ |

Table 1: The charges of the relevant $U(1)$ for $SU(6)$ model. Here, $V_6$ is defined by the relation $SU(6) \supset SU(5) \times U(1)_{V_6}$ and we omit representations $G$ and $W$ which have trivial charges and 5 in 35 which is a conjugated field of 5 in 35.

From Table 1 we can see that there is one possible breaking pattern, $V_6 - 12Y$ direction, i.e. $\langle A \rangle \propto \text{diag}(1, 1, 1, -1, -1, -1)$, for the sliding singlet mechanism to act in the mass terms of 6 and 6. In other words, only for the $U(1)$ generated by the generator of this direction, the singlet and doublet of $G_{SM}$ have the same charge.

| $E_6$ | 27 | 78 |
|-------|----|----|
| $SO(10)$ | 16 | 10 | 1 |
| $SU(5)$ | 10 | $\bar{5}$ | 1 | 5 | 5 | 1 | 24 | 10 | 10 | $\bar{5}$ | 1 |
| $G_{SM}$ | $Q, U, E$ | $D, \bar{L}$ | $N$ | $D, \bar{L}$ | $N$ | $X$ | $Q, U, E$ | $D, \bar{L}$ | $N$ |
| $V'$ | 1 | $-2$ | 4 | 0 | $-3$ |
| $V$ | 1 | $-3$ | 5 | $-2$ | 2 | 0 | 0 | $-4$ | 1 | $-3$ | 5 |
| $6Y$ | $-1, -4, 6$ | $2, -3$ | 0 | $-2, 3$ | $2, -3$ | 0 | $-5$ | $1, -4, 6$ | $1, -4, 6$ | $2, -3$ | 0 |
| I: $\frac{(0,3,12)}{5}$ | $1, -1, 3$ | $-1, -3, 3$ | $3, -2$ | $0, 2$ | $0, 0$ | $0, -2, -4, 0$ | $1, -1, 3$ | $-1, -3, 3$ | $1, -1, 3$ | $-1, -3, 3$ | 3 |
| II: $\frac{(5, -48)}{20}$ | $0, 2, -2$ | $-1, 1$ | 1 | $0, -2$ | $-1, 1$ | 1 | $2, -1, 1, -3$ | $-1, -3, -2, 0$ | 0 |
| III: $\frac{(5, -924)}{20}$ | $0, -1, 1$ | 2 | 1 | $-2, 0$ | $1, -1, -2$ | 1 | $-1, 2, 3$ | $-1, -2, 0$ | 1, 0 | $-3$ |
| IV: $\frac{(5, 3, 72)}{20}$ | $1, -2, 4$ | 1 | $-2$ | 1 | $-2, 1$ | $-1, -2$ | 1 | $-3$ | $0, -3, 3$ | $0, -3, 3$ | 0, $-3$ |

Table 2: The charges of the relevant $U(1)$ for $E_6$ model. Here, we omit representations $\bar{10}$ in 45 which is a conjugated field of 10 in 45, and the fields omitted in Table 1. In the first column of the last four rows, $\frac{(a, b, c)}{d}$ denotes the linear combination $\frac{a}{d}V' + \frac{b}{d}V + \frac{c}{d}Y$, where $V'$ and $V$ are defined by the relations $E_6 \supset SO(10) \times U(1)_{V'} \supset SU(5) \times U(1)_{V} \times U(1)_{V'}$.

On the other hand, as seen in Table 2 there are many possible breaking patterns for the generalized sliding singlet mechanism in $E_6$ models. Actually there are infinite patterns.\footnote{We denote each representation of $G_{SM}$, $(3, 2)$, $(\bar{3}, 1)$, $(3, 1)$, $(1, 2)$, $(1, 1)_1$, $(1, 1)_0$, $(3, 2)_0$, $(8, 1)_0$, and $(1, 3)_0$ as $Q, U, D, L, E, N, X, G,$ and $W$, respectively, and the conjugate representation of a representation $R$ by $\bar{R}$.}
possibilities because there is only one relation, namely, the charge of a singlet is equal to that of a doublet, and three independent $U(1)'s$ in $E_6$ unified models. If we impose two relations, the $U(1)$ can be fixed except for the normalization. For example, we require that the charge of a singlet is equal to the charges of two doublets. Because there are two singlets and three doublets of SM gauge group in a single field $27$ of $E_6$, there are several solutions as in Table 2. Essentially there are three solutions (Model I, II, and III in Table 2). The Model I gives the DW type of VEV which breaks $E_6$ into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_{V'}$, and the models in Ref.[16] are classified as this case. The breaking pattern of Model II is that $E_6$ is broken into $SU(3)_C \times SU(3)_L \times U(1) \times SU(2)_E$, where $SU(2)_E$ rotates two $\bar{5}$s and 1s of $SU(5)$ in 27 of $E_6$ as the doublets. This produces similar models as in Ref.[14] if $SU(2)_E$ is identified as the symmetry $SU(2)$ in Ref.[14]. The Model III preserves $SU(3)_C \times SU(3)_L \times SU(2)_{RE} \times U(1)$, where $SU(2)_{RE}$ denotes the $SU(2)$ sub-group of $SU(3)_R$ under which the component of 3 which belongs to the doublet of both $SU(2)_R$ and $SU(2)_E$ is singlet. If we require that $SU(2)_E$ symmetry is not broken by the VEV of the adjoint Higgs and the charge of a singlet is equal to the charge of a doublet, then in addition to Model II, Model IV is satisfied with the requirements. In the Model IV, $E_6$ is broken into $SO(10)_F \times U(1)$.

From the above observation, models in which the DT splitting is realized through the sliding singlet mechanism may be constructed for each breaking pattern, because some doublet components $L$ and/or $\bar{L}$ in 27 have the same charge as some singlet components $N$. However, as illustrated in the previous section, this is not a sufficient condition. Actually we have to take care of the following points in constructing models:

- the massless mode may be an unphysical NG mode.
- other part of superpotential may give a large mass to the would-be massless mode.
- an effort to avoid the above two obstructions may yield unwanted additional massless modes.

In the next section, we try to construct several models in which DT splitting is realized.

4 Models

Here we construct concrete models corresponding to the breaking patterns listed in Table 2 in the context of anomalous $U(1)_A$ gauge symmetry. Since, in GUTs with anomalous $U(1)_A$ gauge symmetry with generic interaction, positively charged fields have vanishing VEVs,[14][17][18], they naturally play the role of the primed fields.

4.1 Model I: $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_{V'}$

Recently we proposed a $E_6$ unified scenario in which the DT splitting problem is solved by the DW mechanism[16]. In Ref.[16], we emphasized that the sliding singlet mechanism is also workable in the model but focused on the DW mechanism. However, according to the new perspective proposed in the previous section, the DT splitting in the model can be understood only by using the generalized sliding singlet mechanism.
We consider the Higgs sector defined by Table 3, where lowercase letters denote the anomalous $U(1)_A$ charge, and $\Theta$ is the Froggatt-Nielsen field \cite{19}. From this table, we can lead the relevant superpotential to determine VEVs:

\[ W = W_{A'} + W_{\Phi'} + W_{C'}, \quad (4.1) \]

where

\[
W_{A'} = A'(A + A^3 + A^4 + A^5) \quad (4.2)
\]

\[
W_{\Phi'} = \Phi'(1 + A + Z_i + A^2 + AZ_i + Z_i^2)\Phi \quad (4.3)
\]

\[
W_{C'} = \bar{C}(1 + A + Z_i + \cdots + (\bar{C}\Phi)^2)C'. \quad (4.4)
\]

Combining with the $D$-flatness conditions, we can see the following VEVs

\[
\langle 16_C \rangle \sim \langle 1_A \rangle \sim \langle 1_F \rangle \sim \lambda^{-a} \lambda^r \quad (4.5)
\]

\[
\langle 1_C \rangle \sim \langle 16_F \rangle \sim \lambda^{-a} \lambda^{2r} \quad (4.6)
\]

\[
\langle 45_A \rangle \sim \lambda^{-a} \quad (4.7)
\]

Other VEVs = 0 \quad (4.8)

are indeed one of vacua of this model. Here, $r = \frac{2a-\phi-\bar{\phi}}{3}$, $\lambda$ is the ratio of the VEV of $\Theta$ to the cutoff scale $\Lambda$, and the diagonal part of the VEV of $A$ points to the direction $V + 4Y$, $\langle 45_A \rangle = \tau_2 \times \text{diag}(1,1,1,0,0)v$ (DW form), which breaks $E_6$ into $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_{U'} (\equiv G_A)$. Here, $\tau_2$ denotes the second Pauli matrix.

For these VEVs, the sliding singlet mechanism acts in $W_{\Phi'}$ as mentioned in the previous section. One of the $F$-flatness conditions from $W_{\Phi'}$,

\[ F_{1_{\Phi'}} = (1 + A + Z_i + A^2 + AZ_i + Z_i^2)1_{\Phi} = 0, \quad (4.9) \]

makes the mass terms of doublet components in $\Phi' \times 10_{\Phi}$ of $SO(10)$ vanishing. This is because they have the same charges as $1_{\Phi}$ (which happen to be zero\footnote{See Ref.\cite{16} and its references\cite{14, 17, 18}.}) of $U(1)$ which determined by $\langle A \rangle$, \textit{i.e.} $U(1)_{B-L}$, and therefore the mass parameters become the same\footnote{Because of the zero, the DW mechanism can act in $SO(10)$ model if only $10 \cdot 45 \cdot 10$ contributes the doublet mass terms. In order to forbid the dangerous terms, \textit{e.g.} $10 \cdot 10$, the additional symmetry, \textit{e.g.} $Z_2$ parity, is required in $SO(10)$ model. On the other hand, in $E_6$ model, the prohibition can be realized dynamically.}.
value as for $1_{\Phi'} \times 1_{\Phi}$, which vanishes due to Eq. (4.9). Note that because $16_A$ has non-vanishing VEV, we have to examine carefully the other $F$-flatness conditions of $16_{\Phi'}$, in which the VEV $\langle 16_A \rangle$ appears. As we pointed out in Ref. [16], because of the $E_6$ group theoretical reason and the $F$-flatness condition (4.9), the $F$-term is factorized as
\[ F_{16_{\Phi'}} = (1 + Z_i + A)(45_A 16_{\Phi} + 16_A 1_{\Phi}) = 0 \] (4.10)
in this vacuum. Therefore, in this model, the sliding singlet mechanism acts for the singlets in $16_{\Phi}$ and $16_A$, namely, the factor $(1 + Z_i + A)$ in Eq. (4.10) vanishes by sliding the singlet VEV $\langle Z_i \rangle$. Actually, in the model in Table 3, two $E$ fields of $16_{\Phi}$ and $16_A$, which have the same $U(1)$ charges as $N$ in $16_{\Phi}$ and $16_A$, respectively, become massless, though they are absorbed by the Higgs mechanism. In order to confirm that only one pair of doublet is massless, we have to check all mass matrices explicitly. Straightforward calculation shows it, but we would like to skip it here. And we just mension that we have constructed a model which solves the DT splitting problem by using the generalized sliding singlet mechanism. We stress that, in this model, the massless Higgs doublets come from a single multiplet $\Phi(27)$. To be more precise, they come from $10_{\Phi}$ which is not related to any NG modes by the remaining symmetry $G_A$. This situation is quite different from the $SU(6)$ cases in which by the sliding singlet mechanism a pair of doublets becomes massless but is absorbed by the Higgs mechanism. Note that in mass terms generated from $W_C'$, generally the sliding singlet mechanism does not act, since the VEV of $\bar{C}\Phi$ respects only $G_{SM}$.

And, as mentioned in Ref. [16], this Higgs sector is compatible with the matter sector proposed in Ref. [14]. This is because the main modes of doublet Higgs come from a fundamental representation field $\Phi(27)$ and not from an anti-fundamental fields $\bar{C}(27)$. This fact results in comparatively large Yukawa couplings which are important to realize large top Yukawa coupling and to avoid too small $\tan \beta$.

### 4.2 Model II: $SU(3)_C \times SU(3)_L \times U(1) \times SU(2)_E$

This breaking pattern is similar as that of the $SU(6)$ models in §2.2 and therefore the reason that the massless doublet Higgs appear can be understood in the similar way.

We consider the Higgs sector defined by Table 3. Though we adopt the same anomalous $U(1)_A$ charges as in Table 3, we take different number of the singlet Higgs fields. The difference between the previous vacuum and this vacuum is essentially that in this vacuum, the diagonal part of the VEV of $A$, $\langle (45 + 1)_A \rangle$, points to the direction $5V' + 3V - 48Y$, which breaks $E_6$ into $SU(3)_C \times SU(3)_L \times U(1) \times SU(2)_E$. For this vacuum, possible candidates for the NG mode in the representation $L (\bar{L})$ are from the fields $\Phi (\bar{C})$ and not from $A$. Therefore, at least one of the two $L$ of $\Phi$ is the NG mode, namely becomes massless. On the other hand, the two $L$ of $\Phi$ are doublet under the the symmetry $SU(2)_E$, that leads to the both of the $L$ must be massless. One of the two $L$ is absorbed by the Higgs mechanism, but the other $L$ becomes a physical massless mode.

On the other hand, according to the new perspective proposed in the previous section, this can be understood in the different way. Namely, the vanishing mass term is caused by

\[ \text{On the other hand, because the mass term of } \bar{C} \text{ does not respect } SU(2)_E \text{ due to the term } \bar{C}(\bar{C}\Phi)^2C', \text{ one linear combination of the two } \bar{L} \text{ of } C \text{ have a non-vanishing mass parameter.} \]
the $F$-flatness condition (119). This condition makes the two mass terms for $L_{10\Phi} \times L_{10\Phi}$ and $\bar{L}_{10\Phi}^{\dagger} \times L_{16\Phi}$ also vanishing, due to their same charges as $1_\Phi$. Because $16_A$ has non-vanishing VEV, we have to take care of the $F$-flatness condition of $16_A$.

$$F_{10\Phi} = (1 + A + Z_i + A^2 + AZ_i + Z_i^2)16_\Phi + (1 + Z_i + A)16_A1_\Phi = 0. \quad (4.11)$$

Note that in this breaking pattern, this $F$-term is not factorized. (This is because the charge of $1_\Phi$ does not vanish in this vacuum, but that vanishes in the previous vacuum.) The first term in Eq. (4.11) vanishes by the sliding singlet mechanism because $16_\Phi$ has the same $U(1)$ charge as $1_\Phi$. Therefore the second term must vanish by itself and the VEV of a singlet field again slides to satisfy this relation. The coefficient $(1 + Z_i + A)$ of the second term is equivalent to that of the term $\bar{L}_{10\Phi}^{\dagger} \langle 16_A \rangle L_{16\Phi}$, because $16_A$ has vanishing $U(1)$ charge and $L_{16\Phi}$ has the same $U(1)$ charge as $1_\Phi$. Because of these effects, the two $L_\Phi$ become massless. One of the two $L_\Phi$ are absorbed by the Higgs mechanism and the other becomes a physical massless doublet Higgs. Because the other part does not contribute to the mass of $\Phi$ and it can be checked straightforwardly that there are no additional massless modes, the DT splitting is realized.

In this model, the massless $L$ comes from $\Phi$ and the massless $\bar{L}$ comes from a certain linear combination, the main mode of which comes from a primed field. This is because positively charged fields (primed fields) have smaller couplings and therefore smaller masses than negatively charged fields (unprimed fields). Unfortunately we do not know a realistic quark and lepton sector compatible with this model, in contrast to the model in the previous subsection. The biggest difference is that the main component of the $\bar{L}$ Higgs comes from the primed fields and not from unprimed fields in this model. And therefore, the top Yukawa coupling is suppressed because the Higgs $\bar{L}$ has positive charge. It is the essential reason for this suppression that the sliding singlet mechanism does not act for $\bar{L}$ in 27. To avoid this situation, it is important to take the same $U(1)$ charge of $\bar{L}$ in 27 as that of 1 in 27.

In the following, we examine models in which $\bar{L}$ in 27 has the same charge as 1 in 27.

### 4.3 Model III: $SU(3)_C \times SU(3)_L \times SU(2)_{RE} \times U(1)$

This is the last breaking pattern in which the sliding singlet mechanism acts for two doublet components in 27. Because one of them is $\bar{L}$, large top Yukawa coupling can be expected to be realized. Here we consider the same Higgs sector as in §4.1 and §4.2 defined by Table 3 except for the number of singlet fields. The relevant superpotential is given by (111)-(124). The essential difference is that the VEV of the diagonal component of $\Phi$ points to the direction $5V' - 9V + 24Y$.

Because the contribution to mass of $\Phi$ comes only from $W_{16}$, we concentrate on this interaction. The component fields which have the same $U(1)$ charges as $1_\Phi$ are $L_{10}$, $L_{16}$, and $E_{16}$, and they are massless by the sliding singlet mechanism, though the component fields $L_{16}$ and $E_{16}$ are absorbed by the Higgs mechanism. The component field which has the same $U(1)$ charge as $16_\Phi$ is $L_{10}$, so we have to examine whether the sliding singlet mechanism acts for $16_\Phi$ in this model. In the $F$-flatness condition

$$F_{10\Phi}^{\dagger} = (1 + A + Z_i + A^2 + AZ_i + Z_i^2)16_\Phi + (1 + A + Z_i)16_A1_\Phi = 0, \quad (4.12)$$
the coefficient of the first term does not vanish because the component $16_\Phi$ has the different charge from $1_\Phi$. Therefore, the above $F$-flatness condition means just that the sum of the two terms must vanish, namely, the sliding singlet mechanism does not act for the component $16_\Phi$. The remaining task is to check that there are no additional massless modes, which can be done straightforwardly, and we skip it here. In this model, the massless Higgs $\bar{L}$ comes from $10_\Phi$ and the main mode of massless Higgs $L$ comes from a primed field.

Unfortunately we have not found a realistic quark and lepton sector compatible with this model, in contrast to the model in §4.1. This is because, in the context of anomalous $U(1)_A$, primed fields, which have positive anomalous $U(1)_A$ charges, tend to have suppressed coupling constant, and therefore the bottom and $\tau$ Yukawa couplings become too small. In order to avoid this, the massless doublet Higgs should belong to unprimed fields. However, it is difficult to construct such a model without extra massless modes, in this breaking pattern. One of the reason is that the sliding singlet mechanism acts for the $L$ mode in $16_\Phi$ which is absorbed by the Higgs mechanism. To avoid this situation, we can change the direction of $U(1)$ so that the sliding singlet mechanism acts to $L$ and $\bar{L}$ modes in $10_\Phi$, but this vacuum is nothing but that of the DW type. It is another possibility that the massless Higgs $L$ comes from $16_{\bar{C}}$. To realize this situation, we can apply the sliding singlet to the $\bar{C}$ field. However, in that situation, there appear several undesired massless modes, which spoils the success of the gauge coupling unification.

4.4 Model IV: $SO(10)_F \times U(1)$

For this breaking pattern, even if the sliding singlet mechanism acts well, the DT splitting can not be realized, although the doublet Higgs indeed become massless. This is because the triplet Higgs in $16 \subset 27$ also become massless. In terms of $SO(10)_F \times U(1)$, it is possible to give a large mass to the triplet Higgs while the doublet remains massless, through the missing partner mechanism similar as $SU(5)_F \times U(1)$ case. However, it is difficult to embed this $SO(10)_F \times U(1)$ model into an $E_6$ model. This topic is discussed in detail in Ref. [20].

5 Summary and Discussion

In this paper, we extracted the essence of the sliding singlet mechanism in which SUSY breaking effect does not spoil the doublet-triplet splitting. And we generalized the sliding singlet mechanism. The essential point in this mechanism is that the sliding singlet mechanism makes the doublet components massless which have the same $U(1)$ charges as the SM singlet components which have non-vanishing VEVs. By choosing the $U(1)$ which is determined by the VEV of the adjoint Higgs field, we can build various GUTs in which the generalized sliding singlet mechanism acts. In this paper, we examined various $E_6$ GUTs by using anomalous $U(1)_A$ gauge symmetry, by which we can construct GUT models with generic interactions (including even higher dimensional interactions). Since

8Conversely, components with different charges from the SM singlet component become massive, that has been used to make pseudo NG modes massive in the literature [21].
in $E_6$ group there are three $U(1)$ which commute with the SM gauge group, we can select two of three doublets in the fundamental representation $27$ to become massless by the sliding singlet mechanism, though one of the three is enough to realize the DT splitting. Among the various GUTs we examined in this paper, it is the most promising model in which the $L$ and $\bar{L}$ components in $10$ of $SO(10)$ in $27$ of $E_6$ become massless by the sliding singlet mechanism. This vacuum is nothing but the Dimopoulos-Wilczek type vacuum. To make the $\bar{L}$ component in $27$ massless by the sliding singlet mechanism is important to realize large top Yukawa coupling. For this purpose, it is enough that $\bar{L}$ in $27$ becomes massless by the sliding singlet mechanism, namely, the charge of $\bar{L}$ is taken to be the same as that of the SM singlet in $27$. The concrete condition is that $(a, b, c) = (\frac{1}{4}, -\frac{3}{4} + \frac{3}{2}c, c)$ in the notation in Table 2. Because $L$ component in $16$ is absorbed by the Higgs mechanism, it is better that the other $L$ component in $10$ becomes massless independently by some mechanism. Of course, to realize DT splitting, it is enough that $L$ component Higgs is guaranteed to be massless, because there must be its massless partner $\bar{L}$. However, in that case, the main component of the partner $L$ tends to come from positively anomalous $U(1)_A$ charged field (primed field), because positively charged fields have smaller masses than negatively charged fields. And positively charged Higgs $L$ leads to small down-type quark Yukawa couplings and therefore too small $\tan \beta$.

The generalized sliding singlet mechanism has opened the new possibility to build various GUT models in which DT splitting is realized. The DW type vacuum is the most promising vacuum in $E_6$ GUTs even in the sense of the sliding singlet mechanism, though the other possibilities may become also interesting. We hope that such an observation gives us a key leading to the real GUT which describes our world.

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