Conceptions and pedagogical practices on the mathematical processes of teachers

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Abstract. The objective of this research work is to establish the disciplinary, didactic and pedagogical conceptions of teachers in the area of mathematics in basic education against the formative process by competences and mathematical processes. The relationship between the standards developed in the institution against those stipulated by the “Ministerio de Educación Nacional” of Colombia was analyzed. This research was developed considering a hermeneutical historical paradigm, guided by the integrated qualitative approach, from an ethnographic design and therefore microethnography as a method. The research contributes from the epistemological, pedagogical and didactic categories, towards the achievement of evidence related to the classroom practices of teachers where contextualized problematic situations do not prevail so that the classes of the students are routine, not very impactful and the environment or Space becomes unmanageable, they do not have an adequate use of educational texts, and the use of educational material such as technological tools is not evident.

1. Introduction
Education must move away from typical traditional teaching methods to overcome reductivism visions that do not motivate innovation in pedagogical practices. Within the framework of the commitment to the quality of education and specifically in the necessary processes of training for competencies for an active life, it is necessary to recognize the state in which the pedagogical practices of teachers are found, which requires a deeper analysis of the epistemological, pedagogical, and didactic aspects involved in the formative experience of their area of knowledge.

Pedagogical practices, in any area of knowledge, must address three essential areas inherent to the educational process; in this regard, it is necessary to emphasize that it is based on the knowledge (epistemological dimension) that the teacher possesses in relation to the area of mastery, practices with emphasis on learning approaches (pedagogical dimension), where teachers value student performance and the development of strategies (didactic dimension) to achieve correspondence between teaching and learning processes.

In this scenario, pedagogical practices in mathematics do not escape, given that these practices, in addition to a formative value, lead to a cultural and social value. The process of teaching and learning in the field of mathematics has been throughout history a routine that has shown very little interest in students, which is why most of the time is pointed out as the subject with the greatest reproach, leading in some cases to be part of the reason for the loss of the school year in students; at the same time, the procedures that are maintained in pedagogical practice due to technological progress must be increasingly innovative, allowing them to be effective and enriching for the education of students.
This research focuses on the description of the real behaviors of teachers in the area of mathematics, in their educational activity based on them, infer the conceptions and pedagogical practices used from this discipline of knowledge and from the perspective of pedagogy give response to the problem posed. It is also pertinent to analyses the impact of the development of these pedagogical strategies, in order to identify what benefit is being generated for the student and, at the same time, to interpret what factors are failing and what actions should be taken to re-establish these strategies.

At the academic level, the “Ministerio de Educación Nacional de Colombia (MinEducación)” should analyses the relationship between disciplinary and didactic knowledge as opposed to curricular standards. [1]. Similarly, the pedagogical component requires reflection on the strengths and weaknesses of their work in the classroom, through the experience acquired in teaching, the essential foundation of institutional pedagogical practices, where the epistemological conception of them is manifested.

Concepts of: Pedagogical Practice: [2-5]. According to Zuluaga, pedagogical practice names the processes of institutionalization of pedagogical knowledge, that is, its functioning in educational institutions. But it also includes the forms of enunciation and circulation of the knowledge taught in such institutions [2]; Conceptions and thinking of the teacher as a mediating variable of the classroom system, especially in mathematics: [6-14]. According to Porlán and Martin, the teacher's thinking should be organized according to knowledge schemes, which include personal beliefs and theories, as well as strategies and procedures for the correct planning, intervention and evaluation of the results obtained in the classroom. [7]; mathematics didactics [15-17]. For D’Amore didactics is the part of the educational sciences that aims to study teaching and learning processes regardless of the discipline in question but considering the institutional relationship [17]; epistemology of mathematics [18], where mathematical knowledge refers to the epistemological, from the evidence of the mathematical foundation. And finally, the five general processes of mathematical activity [19] which are to formulate and solve problems; model processes and phenomena of reality; communicate; reason and formulate compare and exercise procedures and algorithms.

Within the antecedents that served as a frame of reference, at the international level we have the works of [20-24]; similarly, at the national level are the contributions [25-28].

In view of the above, it is necessary to establish the disciplinary, didactic and pedagogical conceptions of teachers in the area of mathematics in basic education as opposed to the training process by competencies.

2. Method

2.1. Design
The interpretative or hermeneutic paradigm, which seeks to understand, interpret, and analyze the pedagogical practices of teachers in the area of mathematics.

2.2. Approach
This research is framed within the qualitative approach [29], which is reflected in the possibility of characterizing pedagogical practices, in order to understand their impact on reality, these orientations are the starting point for defining key elements within the characterization of the pedagogical practices of mathematics teachers as opposed to competency-based training.

2.3. Qualitative research design
The ethnographic design is taken, which is applied in the formation of an image of the group that conforms a classroom that allows to understand the own characteristics of its conduct and to be able to compare and to relate the practices of the teacher, since it seeks to describe the traits and characters to create a realistic image that reflects the practices exerted by the teachers of the area of mathematics. The method chosen is micro ethnography and allows us to study the cultural processes and phenomena that occur within schools and that are related to the daily practices of teachers.
2.4. Scenario
The setting where the research was developed to achieve the objective, was the public educational institution located in the Municipality of Villa del Rosario, San José de Cúcuta Metropolitan Area, Norte de Santander, Colombia.

2.5. Key informants
It was considered that the key informants were five teachers and their pedagogical practices in the area of mathematics taught in the different grades of the educational institution in question, taken as units of study, from which the data were observed, described and originated. It should be noted that the key informants were approached from the beginning of the research in order to provide information without interference from other aspects that at some point did not make them feel comfortable, observations were made. In addition, the participation of the study informants was carried out by means of an informed consent for the application of the interview, in this way, for the systematization of the information gathered from the study, codes DC1, DC2, DC3 and DC4 were assigned to refer to teacher 1, teacher 2, teacher 3 and teacher 4 respectively.

2.6. Data collection techniques
In order to establish the disciplinary, didactic and pedagogical conceptions of mathematics teachers as opposed to the training process by competencies, the technique of in-depth interviews was applied. The interview made use of the script of questions, constructed based on the deductive categories of the investigation (epistemology, pedagogy and didactics).

2.7. Procedure for the analysis of information
Content analysis was used. The categorization process starts from the revision of the live text that led to the identification of units of analysis, which were reduced to inductive categories, and these in turn were grouped according to the common characteristics they shared, thus generating axial categories. In addition to the above, the triangulation is presented, which allowed the confrontation of testimonies where it was possible to corroborate the situation required to respond to the proposed objectives. Data triangulation will be used, which occurs when there is agreement or discrepancy between these sources. Next, the contrast was made, which consisted of relating and comparing the results with those studies presented in the theoretical frame of reference and will allow better explaining what the research means. This stage allows to reformulate, restructure, extend or correct the previous theoretical constructions, achieving with it a significant advance in the area. It is necessary to clarify that during the information systematization process a codification was used that refers to the teachers who were focused for the respective interviews in the institution: ED1, ED2, ED3 and ED4 to refer to teacher interview 1, teacher interview 2, teacher interview 3 and teacher interview 4 respectively.

3. Result

3.1. Epistemology
In relation to the deductive category of epistemology, seven axial categories emerged among which are: mathematically competent, vision of mathematics from the axiomatic nature, mathematics as a discipline in culturally mediated development, mathematics from the development of comprehension processes, mathematics transversally, contextualized curricular adaptation and didactic resources, and three emerging axial categories such as science as a tool, structured and verifiable knowledge, and science as an explanation of reality. Table 1 shows in detail the recurrence of axial and inductive categories that emerged in this research from the deductive category of epistemology.

Table 2 shows three emerging categories among which is science as a tool, structured and verifiable knowledge and science as an explanation of reality.
Table 1. Axial and inductive categories evidenced from the deductive category of epistemology.

| Epistemology category | Axial category | Recurrence of axial categories |
|-----------------------|----------------|-------------------------------|
| Mathematically competent | (25)            |                               |
| Vision of mathematics from the axiomatic nature | (8)             |                               |
| Mathematics as a developmental discipline culturally mediated | (3)             |                               |
| Mathematics from the development of comprehension processes | (3)             |                               |
| Transversely of mathematics | (4)             |                               |
| Contextualized curricular adaptation | (4)             |                               |
| Didactic resources | (1)             |                               |

Table 2. Emerging and inductive axial categories evidenced from the deductive category of epistemology together with its recurrences.

| Epistemology category | Emerging axial categories | Recurrence of emerging axial categories |
|-----------------------|---------------------------|----------------------------------------|
| Science as a tool | (4)                        |                                       |
| Structured and verifiable knowledge | (1)                  |                                       |
| Science as an explanation of reality | (1)                    |                                       |

3.2. Pedagogy

Pedagogy, framed as a theoretical and organized reconstruction for the fulfillment of certain educational purposes, in contribution to society having the vision of the citizen who wants to train, where the teacher is the main factor to these ends, starting from his practices, purposes and commitments in his profession as a pedagogue. From the point of view of mathematics, there is a great responsibility and commitment for those who teach them, since these are reflected in all areas and in the evolution of the human being in the learning and acquisition of their competences. The emerging axial and axial categories were determined based on interviews conducted under the teacher's own mathematical conceptions based on his or her experience and knowledge. Table 3 shows the axial and emerging axial categories in relation to the deductive category of pedagogy that emerged in the interview together with their recurrences.

However, it is important to highlight the emerging categories that emerged from the analysis of the deductive category of pedagogy where it is found difficulty in the processes of understanding mathematics, education mediated by information and communication technologies (ICT), lack of project research, lack of updating of pedagogical and didactic knowledge. Table 4 shows the recurrence of the emerging axial categories and their relationship with the inductive categories.

Table 3. Axial and inductive categories evidenced from the deductive category of pedagogy.

| Pedagogy category | Axial category | Recurrence of axial categories |
|-------------------|----------------|-------------------------------|
| Mathematically competent | (15)           |                               |
| Vision of mathematics from the axiomatic nature | (1)             |                               |
| Mathematics as a developmental discipline culturally mediated | (2)             |                               |
| Mathematics from the development of comprehension processes | (6)             |                               |
| Transversely of mathematics | (5)             |                               |
| Contextualized curricular adaptation | (4)             |                               |

Table 4. Emerging and inductive axial categories evidenced from the deductive category of pedagogy.

| Pedagogy category | Emerging axial categories | Recurrence of emerging axial categories |
|-------------------|---------------------------|----------------------------------------|
| Difficulty of mathematical comprehension processes | (1)                        |                                       |
| Education mediated by ICTs | (3)                 |                                       |
| Failure to investigate | (1)                      |                                       |
| Lack of updating of pedagogical and didactic knowledge | (1)                      |                                       |
3.3. Didactic

Didactics as the main precursor in the study of teaching and learning methods, which takes into account how to transmit and how to acquire different mathematical knowledge through methods and techniques applied in order to improve teaching, where one of the main purposes is the search for the acquisition of knowledge in a meaningful way. Within the analysis of teachers' conceptions, these are derived from their professional vision and tasks within their teaching practices. In the axial categories there is competent mathematics, mathematics from the axiomatic nature, mathematics from the development of comprehension processes, teaching from situations (Brousseau), cooperative learning, formative evaluation and didactic resource, and from the emerging axials there is context influence in the teaching processes, teaching mathematics through play, summative evaluation. In the following, Table 5 details the emerging axial and axial categories taken from the inductive ones together with the recurrences, for a better understanding.

Table 5. Axial and inductive categories evidenced from the deductive category of didactics.

| Didactic category - axial category | Recurrence of axial categories |
|-----------------------------------|-------------------------------|
| Didactic resources                | (9)                           |
| Mathematics from the development of comprehension processes | (6) |
| Cooperative learning              | (5)                           |
| Teaching from situations [15]     | (4)                           |
| mathematically competent          | (3)                           |
| Formative evaluation              | (3)                           |
| View of mathematics from the axiomatic nature | (1) |

Table 6 presents the recurrence of emerging categories and their relationship with inductive categories from the didactic category.

Table 6. Emerging and inductive axial categories evidenced from the deductive category of didactics.

| Category deductive didactic - emerging axial categories | Recurrence of emerging axial categories |
|---------------------------------------------------------|----------------------------------------|
| Influence of context on teaching processes              | (4)                                    |
| Teaching mathematics through play                       | (3)                                    |
| Summative evaluation                                   | (1)                                    |

4. Discussion

4.1. Epistemology

From this category, different axial categories become evident, such as being mathematically competent, which reflects the need to construct significant knowledge so that subjects develop in an adequate manner in social contexts. It is also necessary to emphasize mathematics from the development of comprehension processes, where the development of subjects' thinking is evident, in this case, the situation is repetitive, since teachers use methods inherent to such a situation, in regard to Sierpinska and Lerman point out that the writings of Lakatos of the seventies, on the nature of mathematics, establishes the idea that Euclidean epistemology framed the rationalist thought that for more than two millennia proposed that mathematical knowledge was deduced from a small number of axiomatic propositions [30]. Within this analysis, in the epistemology category, it was also possible to detect that science is present as a tool, where phenomena are formulated that orient an adequate pedagogical practice to the basic standards, which allows the formation of a structured and verifiable knowledge, given its scientific nature, which allows the adoption of science as an explanation of reality, so as to refer to teaching work in a way associated with science, that is, where the discipline attend all the scientific elements associated with the development of student thinking, in this respect, Godino, mathematics, is a science that serves as a tool for human development [18].
4.2. Pedagogy
In relation to pedagogy, it was possible to determine that within the axial categories, they allow evidencing the fact of the formation of a mathematically competent subject, which means that the students have the necessary mechanisms for the development of significant knowledge. In this regard, the “Ministerio de Educación Nacional de Colombia” defines that in order to be mathematically competent it is necessary to be skilful, effective and efficient in the development of each one of these general processes, in which each student goes through different levels of competence, in this case, the construction and development of the skills and abilities of the students is assumed [19]. In addition to this, it was possible to establish as emerging categories in this case, the difficulty of the processes of understanding mathematics, given the complexity of the area, likewise, is presented as an element that has not been widely valued, however, Guzman argues that understanding mathematics is one of the skills that has the thinking of human beings, specifically logical thinking, in this sense, understanding mathematics, is manifested in terms of situations inherent to human development and as such, logical thinking [31]. This development of mathematical logical thinking is assumed from the education mediated by ICT, which is an innovative element associated with the teaching of mathematics.

4.3. Didactics
As for didactics, it is evident from the axial categories, mention is made of the use of didactic resources; in this sense D'Amore in a didactic situation, proposes as a model the game where the teacher encourages the student to return an a-didactic situation that provokes in him an interaction as independent and fruitful as possible [17]. So that the adoption of didactic resources is reflected in reality itself, and the main responsible for their use is the teacher, they can promote cooperative learning, which reflects the collaboration between peers and the teacher, to achieve the realization of actions inherent to human development, from the understanding that learning is a heuristic element, where global phenomena, typical of reality, are integrated, in which student interactions are promoted. It is evident that the influence of context in teaching processes, in educational institutions, it is common to observe how subjects assume the same imminently since it is assumed from the context, which serves as the basis for the adoption of the teaching process, that is to say, it is a question of students from the community being able to put into practice the development of knowledge aimed at strengthening their own cognitive structure, in this respect Brousseau emphasizes that learning is more effective when relations are established with their context [15].

5. Conclusions
The pedagogical practices of teachers as seen from institutional documents (area plan - classroom) make presence of humanism and a tendency towards constructivism although it can be evidenced in class observations made to focused teachers that their practices are given by a traditional teaching, since sometimes it is still the teacher who gives the concept, the process and arrives at some conclusions of the issues seen, by which it is not promoted that the students have an integral formation. It is important to clarify that some moments try to construct knowledge together with the students in such a way that their processes are active and participative, but they are not continuous, since there is a lack of knowledge of this methodology on the part of the teacher.

For what has been said before, the vision of mathematics is relevant from the axiomatic nature since the students learn the concepts, solve the operations and give a result according to the property needed to carry it out. We can appreciate that there is no evidence within these practices of transversely and interdisciplinarity. If we look at the didactic resources used in the pedagogical practices of teachers, we observe that educational texts do not have an adequate use, educational material such as technological tools is not evident since teachers state that they do not have the appropriation for their manipulation, there are not sufficient quantities for all students, or on the contrary the tools are defective and the spaces do not favor learning. Also, within this process, the summative evaluation that is the written tests provided to the students, this type of tests allows mechanically evaluating the knowledge acquired by
the students in accordance with the topics exposed during a period and which allows measuring the student's learning.

It is observed that in most of the classroom practices of the teachers, contextualized problematic situations do not prevail, which makes the students' classes routine, with little impact and the environment or space becomes unmanageable. This makes it possible to deduce that teachers require an adequate appropriation of pedagogical methodologies and the use of didactic resources in order to be in accordance with current trends and to improve classroom practices.

Finally, this research offers educational institutions the opportunity to identify strengths, weaknesses, and possible improvements in the characterization of classroom practices so that principals and teachers become aware of the need to redefine, restructure, and reorganize institutional documents in order to fulfill the objective of transforming the educational quality of the institution and to be competent in current education.

References

[1] Ministerio de Educación Nacional (MEN) 2003 Estándares básicos de competencias en matemáticas (Colombia: Ministerio de Educación Nacional)
[2] Zuluaga O 1999 Pedagogía e historia. La historicidad de la pedagogía, la enseñanza, un objeto de saber (Bogotá: Siglo del Hombre Editores)
[3] Martínez A 1990 Teoría Pedagógica, una mirada arqueológica de la pedagogía Pedagogía y Saberes 1 8
[4] Barragán D, et al. 2012 Practica pedagógica perspectivas teóricas inteligencias (Bogotá: Ecoe Ediciones)
[5] De Longhi A, Bermudez G 2010 La comunicación en el aula (Cordoba: Editorial FCEF y N- UNC)
[6] Pérez A 1985 Paradigmas contemporáneos de investigación didáctica la enseñanza: Su teoría y su práctica ed Gimeno J (Madrid: Ediciones Akal) p 95
[7] Porlán R, Martín J 2004 El diario del profesor. Un recurso para la investigación en el aula (Sevilla: Diada)
[8] Gascón J 1998 Evolución de la didáctica de las matemáticas como disciplina científica Recherches en Didactique des Mathématiques 18 7
[9] Hamachek D 1987 Encounters with the self (New York: Holt, Rinehart and Winston)
[10] Barragan A 2012 Psicología positiva y humanismo: premisas básicas y coincidencias en los conceptos Revista Electrónica e Psicología 15 1512
[11] Rodríguez M 2008 La teoría del aprendizaje significativo en la perspectiva de la psicología cognitiva (Barcelona: Ediciones Octaedro)
[12] Sílvio J 2000 La virtualización de la universidad: ¿Cómo transformar la educación superior con la tecnología? (Caracas: UNESCO)
[13] Gómez I 2000 Matemática emocional: Los afectos en el aprendizaje matemático (Madrid: Narcea)
[14] Arcavi A and Nurit H 2007 Computer mediated learning an example of an approach International Journal of Computers for Mathematical Learning 5 25
[15] Brousseau G 2000 Educación y didáctica de las matemáticas Educación Matemática 12 5
[16] G Brousseau 1986 Fondements et méthodes de la didactiques desmathématique Recherches en Didactique des Mathématiques 7 33
[17] D’Amore B 2011 Didáctica de la matemática (Bogotá: Editorial Magisterio)
[18] Moreno C, García M 2009 La epistemología matemática y los enfoques del aprendizaje en la movilidad del pensamiento instruccional del profesor Investigación y Postgrado 24 218
[19] Godino J 2002 Perspectiva ontosemiótica de la competencia y comprensión matemática (Bologna: XVI Convenio Nazionale. Incontrì con la Matemática)
[20] Lebrija A Flores R, Trejos M 2010 El papel del maestro, el papel del alumno: un estudio sobre las creencias e implicaciones en la docencia de los profesores de matemáticas en Panamá Educación Matemática 22 31
[21] Vilora N, Godoy G 2010 Planificación de estrategias didácticas para el mejoramiento de las competencias matemáticas de sexto grado Investigación y Postgrado 25 95
[22] Pérez Y and Ramírez R 2011 Estrategias de enseñanza de la resolución de problemas matemáticos: Fundamentos teóricos y metodológicos Revista de Investigación 35 169
[23] Alpizar M 2014 Actitudes del docente de matemáticas de enseñanza secundaria (ESO y Bachillerato) en la relación docente–estudiante: Un estudio mediante el grupo de discusión, sobre metaconsciencia actitudinal de los docentes de matemáticas de ESO-Bachillerato en su práctica docente (Bellaterra: Universitat Autònoma de Barcelona)
[24] Varón V, Otalora Y 2012 Estrategias de intervención con maestros centradas en la construcción de espacios educativos significativos para el desarrollo de competencias matemáticas *Avances en Psicología Latinoamericana* **30** 93

[25] Zambrano J 2013 Análisis multinivel del rendimiento escolar en matemáticas para cuarto grado de educación básica primaria en Colombia *Sociedad y Economía* **25** 205

[26] Martínez F, Mosquera D, Ordoñez M, Jiménez C 2014 *Prácticas pedagógicas matemáticas en atención a la diversidad: el imaginario del docente* (Colombia: Universidad de Manizalez)

[27] Ramírez C 2015 *La formación por competencias a partir de las practicas pedagógicas desarrolladas por los docentes en el colegio Santos Apóstoles* (Colombia: Universidad Francisco de Paula Santander)

[28] Martínez M 2007 *La investigación cualitativa etnográfica en educación* (Mexico: Trillas)

[29] Sierpinska A, Lerman S 1996 *Epistemologies of mathematics and of mathematics education* *international handbook of mathematics education* ed Bishop A J (Dordrecht: Springer) **22** 827

[30] Guzmán M 1995 *Para pensar mejor: desarrollo de la creatividad en los procesos matemáticos* (Madrid: Pirámide)

[31] Ministerio de Educación Nacional 1998 *Lineamientos curriculares: Matemáticas* (Bogotá: Ministerio de Educación Nacional)