Stress waves in a cylindrical body interacting with external medium

K S Sultanov, B E Khusanov* and B B Rikhsieva
Institute of Mechanics and Seismic Stability of Structures of the Academy of Sciences of the Republic of Uzbekistan, 100187 Tashkent, Uzbekistan

* khusanov@mail.ru

Abstract. The unsteady-state axisymmetric problem of longitudinal interaction of a cylindrical body with external elastic medium is considered in the paper. The condition of Coulomb friction is accepted at the contact boundary. The solution is implemented by the finite difference method. The results are shown in graphs and analyzed. The changes in stress, velocity, and displacement of particles at some points of a cylindrical body and external medium are presented. An increase in longitudinal stresses and particle velocities in the cylinder was determined when compared to a given load due to the outflow of strain energy from the external medium. The mechanism of strain energy redistribution and friction forces formation is shown, the changes in the direction of contact forces action are revealed. A significant violation of the uniform deformability of the medium, and possible maximum value that occurs in a cylindrical body, is determined. The established dependences of tangential stresses on relative displacements qualitatively coincide with experimental data obtained during the interaction of fragments of underground pipelines with soil.

1. Introduction
The study of the dynamics of extended underground structures (pipelines) during the plane waves propagation along the structures length was mainly considered in a one-dimensional statement [1-2]. The soil taken as an external medium is considered to be undisturbed and the task is reduced to considering a pipeline with external friction. Unsteady–state problems on wave propagation in elastic rods with external Coulomb friction were considered in [1-3]. At distances close to the wave source and in some areas, depending on physical and geometric properties of underground structures and soil, ignoring the soil movement is considered unjustified [4-5]. In one-dimensional statement, this gap was filled by solving two systems of differential equations of the pipeline and soil [5], the connection between which was carried out using the interaction conditions (models) [1-3]. In these statements, it was accepted that the hypothesis of flat sections is satisfied for both the pipeline and the external medium, and the interaction conditions directly entered the equations of motion, i.e. the force acting on the interaction contact is replaced by a force uniformly distributed on the cross section of the structure and soil. The assumption of one-dimensional motion of the external medium, considered unrestricted and working not only in compression or tension, but under shear strain as well near the interaction contact, which, in turn, propagates in an unsteady-state mode at a limited velocity from the pipeline surface into soil, is doubtful. In one-dimensional problems on unsteady–state interaction of extended pipelines with soil [1-2] the stresses in the pipe sections increase manifold (from two to
several tens of times) compared to the falling load. The increase in stress is explained by the fact that the tangential stresses between the pipeline and soil under interaction turn from a passive force into an active force, which accompanies the movement.

To clarify the causes of such "phenomena", the applicability of one-dimensional calculations, and the influence of the wave amplitude on the friction force and stress state of an extended pipeline, the problem of longitudinal interaction of a cylindrical body with an external deformable medium is considered in a two-dimensional statement. The aim of the work is to study the stress wave propagation in a system “cylindrical body-external medium” under simultaneous loading of the cylinder and external medium.

2. Research methods
By virtue of axial symmetry of the problem, a cylindrical coordinate system \( r, z, \varphi \) is used, where \( r \) and \( \varphi \) are the radial and annular coordinates; \( z \) is the axis of symmetry, which coincides with the axis of cylindrical body. Let the dynamic load in the form

\[
\sigma_{zr} = \sigma_{\text{max}} H(T - t), \quad r = 0
\]

act simultaneously on the initial section of the cylinder and external medium. Before applying the load, the cylinder in question and the external medium are considered to be at rest and stress-free. The equations of the interaction dynamics of the cylindrical body and external medium, including the equations of motion, continuity and state in the form of Hooke's law, as well as the Cauchy relations relating strains and displacements, correspond to the ones given in [6–7] for axisymmetric problem. Note once again that the equations of motion, state, continuity and Cauchy relations [6–7] correspond both for a cylindrical body (for \( i=1 \)) and external medium (for \( i=2 \)). The interaction condition in the form of dry friction and equality of normal components of motion is accepted at the contact boundary of the cylinder with the external medium:

\[
\tau = \text{Sign} \left( v_{x1} (z, R_0, t) - v_{x2} (z, R_0, t) \right) \cdot f \cdot |\sigma_N|,
\]

\[
v_N = v_{N1} (z, R_0, t) = v_{N2} (z, R_0, t).
\]

Denote in (2): \( R_0 \) - the radius of cylinder; \( \sigma_N \) and \( v_N \) - normal components of stress and velocity to the contact surface. The last two parameters are defined by expressions:

\[
\sigma_N = \sigma_{zr} \sin^2 \alpha + \sigma_r \cos^2 \alpha - 2 \tau \sin \alpha \cos \alpha,
\]

\[
v_N = -v_2 \sin \alpha + v_1 \cos \alpha,
\]

where \( \alpha \) is the inclination angle of the cylinder side surface.

The tasks, i.e. the dynamics equations for a cylindrical body and external medium with boundary (1-2) and zero initial conditions are solved numerically, using the Wilkins difference scheme for axisymmetric problem [6–7].

3. Calculation results and their analysis
The initial data for the problems under consideration are taken as follows: the cylinder length is 10 m, and calculations are conducted so that the waves reflected from the end of the rod do not affect the parameters of propagating waves; the rod radius - 20 cm; the thickness of external medium is taken large enough (50 m) so that the waves reflected from the contact surface propagate without reflection at the considered time instants. For this, the region of sought solutions of external medium is divided into a non-uniform mesh: the parts adjacent to the cylinder are commensurate with the dimensions of the cylinder mesh, and then the mesh size is increasing. Physical-mechanical characteristics of the cylindrical body remained the same: initial density - 7800 kg m\(^{-3}\), elasticity modulus \( E=2.1\cdot10^8 \) MPa,
Poisson's ratio - 0.3. Two options of initial data for the external medium were taken: the first one is closer to soil media, these characteristics are as follows: initial density - 1800 kg·m$^{-3}$, longitudinal wave propagation velocity - 1000 m·s$^{-1}$ and Poisson's ratio - 0.3; for the second option - 4000 kg·m$^{-3}$, 2000 m·s$^{-1}$ and 0.3, respectively. The amplitude of acting load (1) is taken as 11.163 MPa.

Consider the results of calculations. Figure 1 shows the changes in longitudinal stresses at fixed points of the cylinder and external medium. The dotted curves correspond to $f=0.5$, the dashed curves to $f=0.1$, and the solid curves and curves 2’ and 4’ correspond to $f=0.3$. Curves 2’and 4’ are the solutions for the second option of initial data, and the remaining curves are for the first option. As seen from figure 1(a), the stress in the cylinder increases with the arrival of a propagating wave. Further, these stresses do not remain constant, but continue to increase. Such an increase in stresses was revealed in one-dimensional problems [5], where the amplitude of longitudinal stresses increased several times in comparison with the acting load. An increase in stresses also occurs in the problem under consideration. Therefore, the friction force from some moment on turns into an active force, i.e. the force accompanying the cylinder movement. Such a time comes when the propagating waves of external medium reach the point in question. Here, the velocity of external medium particles on the contact surface is ahead of the velocity of cylinder particles. However, an increase in stress, in this case, for the friction coefficient $f=0.3$ is 2-2.5 times, and with distance this increase continues. The discrepancy between these results and the results for one-dimensional solutions [5] can be explained by the second condition accepted on the cylinder-external medium contact surface (2), i.e. the radial displacement is not limited, but determined by the displacement and compressibility of external medium.

Figure 1. Dependences of the change in longitudinal stresses at fixed points of the central axis of cylindrical body (a) and the contact layer of external medium (b) in time.

Excitations of external medium begin not with the arrival of waves propagating in the external medium, but with the cylinder excitations through the reflections from the contact surface. Further, these stresses increase with the arrival of a local wave initiated from the load effect on the initial section. The change in the friction coefficient, as seen from figure 1, slightly affects the stress state of the cylinder (dotted curves), and practically does not affect the stress behavior of external medium.

Figure 2 shows the changes in longitudinal velocities of the particles of cylinder (a), external medium (b) and contact surface of external medium (c) in time. The curves numbering corresponds to the numbering in figure 1. An increase in velocity of the cylinder particles behind the wave front is observed. Here, too, the velocities of the cylinder particles increase with time. Figures 2(b) and 2(c) show that the particle velocities, therefore, the stress state of external medium along the radial sections, are not the same even for an elastic external medium. At points in external medium surrounding the cylinder, the particle velocities slightly decrease.
From figures 1 and 2 it is seen that the strain energy is redistributed during the interaction of the cylinder-external medium system, that is, the energy "outflow" from the external medium to the cylinder occurs in the sections of external medium located near the contact surface of the cylinder-external medium system. The accepted elastic model for the external medium confirms such a statement. From this, another conclusion follows that a contact layer with a limited size in radius should exist in the external medium; its existence is revealed by soil interaction with the underground structure [7].

![Figure 2](image_url)

**Figure 2.** Dependences of changes in the longitudinal velocities of particles at fixed points of the central axis of cylindrical body (a), in the section \( r=0.5 \) m (b) and on the contact surface of external medium (c) in time.

Dotted curves 1–4 in figures 2(b) and 2(c) correspond to the second option of initial data for the external medium. The velocity values of external medium particles are less (by about 4 times) than for the first option of initial data (solid curves in figures 2(b, c). The changes in external medium particles velocity show that an increase in the velocity of cylinder particles (Figure 2(a)) should not grow indefinitely. The maximum increase in velocity over time does not exceed the external medium particles velocity on the contact surface. Further increase leads to a change in direction of friction force action, and the joint movement of the cylinder-external medium system is possible.

Figure 3 shows the changes in longitudinal displacements of the cylinder particles in time; the solid curves in figure 3(a) correspond to the first option of initial data, the dotted curves to the second option, and in figure 3(b) these curves correspond to \( f=0.1 \) and 0.5. This shows the effect of mechanical characteristics of external medium and the friction coefficient on particle displacements. Changes in longitudinal displacements of external medium are shown in figure 4. Here, the dotted curves in figure 4(a) refer to the cross section \( r=0.5 \) m, and the other curves refer to the contact particle of external medium. Figure 4(b) shows that the values of the friction coefficient slightly affect (in the graph scale) the displacements of external medium particles on the contact surface, and the increase in mechanical characteristics (strain properties) significantly reduces the displacements. On the whole, the pattern of changes in the displacements of external medium particles is consistent with the changes in the particle velocity shown in figure 2(c).
Figure 5 shows the changes in tangential stresses arising on the contact surface of external medium over time. Figure 5(a) corresponds to two options of initial data of the external medium (solid curves refer to the first option, and dotted curves refer to the second option). This shows that mechanical characteristics under the action of the same external load mainly affect the time the maximum tangential stress is reached. They affect the values of the maximum tangential stresses slightly. The value of the friction coefficient significantly affects the value of shear stresses (Figure 5(b)). The dashed curves in figure 5(b) refer to \( f=0.5 \), the dotted curves to \( f=0.1 \). A change in the friction coefficient does not qualitatively affect the values of the friction force (Figure 5(b)). In quantitative terms, an increase in the friction coefficient, as one would expect, leads to an increase in the friction force on the contact surface of the cylinder-external medium system. From the above dependences of shear stress on time at fixed points, a transition from passive (positive) to active (negative) role of the friction force action for a cylindrical body is observed.

**Figure 3.** Change in longitudinal displacements of cylinder particles over time at different values of mechanical characteristics of external medium at \( f=0.3 \) (a) and friction coefficient (b)

**Figure 4.** Change in longitudinal displacements of the contact surface particles of external medium over time at different values of mechanical characteristics at \( f=0.3 \) (a) and friction coefficient (b).

The change in lateral expansion of a cylinder interacting with external medium during the wave propagation under different strain properties of external medium in time is shown in figure 6. As seen from figure 6, the lateral expansion of the cylinder, i.e. lateral strain occurs behind the front of waves propagating in the cylinder. Further, reaching the considered section of the wave in external medium, the lateral surface of the cylinder experiences significant pressure from the external body. In this case, a reverse radial displacement of the side surface occurs. However, the initial position of the cylinder radius is reached only in initial sections (curves 1), the residual displacements in the radial direction
are observed with the distance. Note that slight radial strain in the cylinder leads to a decrease in the normal stress of the cylinder. Also note that the strain properties of external medium influence the reversibility of the initial size of the cylinder radius. An increase in mechanical characteristics of external medium (dashed curves correspond to the second option of calculation) facilitates the reduction in radial displacement of the cylinder lateral surface.

![Image](image1)

**Figure 5.** The change in friction force on the contact surface over time at different values of mechanical characteristics of external medium at $f=0.3$ (a) and the friction coefficient (b).

The cross-section profile of the cylinder-external medium system at $z=1$ m is shown in figure 7. Here it is necessary to pay attention to the change in radial section of external medium, which deviates to 50% from a straight line within the radius of 5 meters. This result is necessary for calculating the longitudinal interaction of underground structures with soil in one-dimensional statement [5, 7]. In this regard, the problem statement of the underground pipeline interaction with soil, where the underground pipeline is considered in one-dimensional statement and the external medium in two-dimensional axisymmetric statement, is more real.

![Image](image2)

**Figure 6.** Change in radial displacement of the points of the cylinder lateral surface over time at $f=0.3$.

In the process of solving this problem, as well as the others problems posed in this section, the strain energy of the cylinder-external medium system was checked. The time variation of this energy is shown in figure 8. The solid curves 1–3 refer to different values of the friction coefficient of the first option, and the dotted curves correspond to the second option of calculation at $f=0.3$. As seen from this figure, the energy in the cylinder increases with time, while in the external medium, on the contrary, it decreases. This figure once again confirms the conclusion made about the energy redistribution in the...
cylinder-external medium system. Curves 0 in figures 8(a) and 8(b) are the same. They reflect the conservation of strain energy during the interaction of the cylinder–external medium system in the case of friction-free interaction on the contact surface. A slight loss of energy over time (with the accumulation of large strain energy) is a consequence of the application of external medium motion on the cylinder surface.

Figure 8. Change in strain energy of the cylinder (a) and the near-contact layer of external medium (b) over time.

In general, Figure 8, especially curves 0, show satisfactory suitability of the applied interaction calculation scheme and the reliability of the solutions obtained.

In conclusion, note that the solutions to the considered problems were obtained using the dry friction condition. The results of the dry friction problem are necessary to compare in the long term to the results obtained under more complex conditions [5] and to analyze the results. It should also be emphasized that the interaction models [5] are transformed into Coulomb friction in the second stage of interaction. This process is accelerated when the external medium is of a disturbed structure. It would be interesting to observe the dependence of the friction force on the relative displacement based on the obtained solutions to this problem. Consider the dependence of shear stress on relative displacement on the contact surface of the cylinder with the external medium (Figure 9).

Figure 9. Dependence of shear stress on relative displacement on the contact surface for different values of the friction coefficient (a) and the second calculation option for f=0.3 (b).

As seen from the obtained dependences of shear stresses on relative displacements, these dependences coincide qualitatively with the experimental data obtained at soil interaction with the fragments of underground structures, and with the result of theoretical studies using complex
interaction conditions [5]. This primarily shows that the solutions obtained using complex interaction conditions well agree with the obtained solutions.

4. Conclusion
Numerical results of the problems of unsteady-state longitudinal interaction of an extended cylindrical body with external deformable medium under simultaneous action of an external load on the cylinder and external medium were obtained. The obtained numerical solutions were given in the form of graphs. Changes in stress, velocity and particles displacement at fixed points of a cylindrical body and external medium over time were determined. The mechanism of energy redistribution and friction force formation, the changes in their direction, as well as energy redistribution on external medium-cylinder contact layer were shown. An increase in stress wave parameters in a cylindrical body, a significant violation of the uniform deformability of external medium, and the virtual maximum value that occurs in a cylindrical body, are determined.

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