Tachyon Condensation and Universal Solutions in String Field Theory

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Abstract

We investigate a non-perturbative vacuum in open string field theory expanded around the analytic classical solution which has been found in the universal Fock space generated by matter Virasoro generators and ghost oscillators. We carry out level-truncation analyses up to level (6, 18) in the theory around one-parameter families of the solution. We observe that the absolute value of the vacuum energy cancels the D-brane tension as the approximation level is increased, but this non-perturbative vacuum disappears at the boundary of the parameter space. These results provide strong evidence for the conjecture that, although the universal solutions are pure gauge in almost all the parameter space, they are regarded as the tachyon vacuum solution at the boundary.

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1 Introduction

String field theory is a promising approach to investigate non-perturbative aspects of string theory. As conjectured by Sen [1, 2], we can describe the annihilation process of D-branes by using the condensation of the tachyon in open string field theory, in which there is a stable vacuum [3] and the energy difference between the stable and unstable vacua is in precise agreement with a D-brane tension [4, 5, 6]. Since these discoveries much progress has been made in string field theory, particularly in the formulation of vacuum string field theory [7], where some exact results are obtained [8]. However, despite these developments, we have not yet found the analytic classical solution, which is eagerly awaited, corresponding to the tachyon vacuum in open string field theory.

While there are some formal attempts to construct analytic solutions in string field theory [9, 10, 11, 12], strange problems often arise from such formal solutions. For example, if $Q_L$ stands for the left-half integration of the BRS current and $I$ is the identity string field, $Q_L I$ is a formal solution in purely cubic string field theory [13]. The equation of motion is given by $\Psi^* \Psi = 0$ and then $-Q_L I$ is also a solution, around which the kinetic operator becomes $-Q_B$ and the theory should describe open strings with a negative tension. Therefore ordinary D-branes and negative tension branes are realized with the same energy density. We find another example in the context of vacuum string field theory as pointed out in [7]. From similar discussions, it follows that the solutions leading to pure ghost kinetic operators provide the same energy density as the perturbative vacuum. Usually, these unreliable results are caused by midpoint singularities in a half string formulation. In the former example, though the operator $Q_L^2$ appears in solving the equation of motion, this operator itself is ill-defined due to a midpoint singularity, as pointed out for example in [10]. We find the same singularity in the latter case.

Fortunately, the analytic solutions found in [9, 10] escape from all these problems related to the midpoint singularity. In addition they have many remarkable features: The solutions can be expressed by states in the universal Fock space which is spanned by matter Virasoro generators and ghost oscillators acting on the $SL(2, R)$ invariant vacuum (so we call them universal solutions). This universality is necessary for the solution corresponding to the tachyon vacuum [11]. Secondly, open string excitations disappear after the string field condensation to a specific class of the universal solutions [14]. This property is also indispensable for the tachyon vacuum solution. Consequently, we naturally expect that a certain kind of the universal solutions is regarded as the tachyon vacuum solution.
Even if the universal solutions are irrelevant to the tachyon vacuum, it should be emphasized that they have intimate relations to gauge symmetry in string field theory, which is an underlying principle in the theory and which is much larger symmetry than existing in the low energy effective theory. We can construct the universal solutions with a parameter. They are pure gauge solutions in almost all the region of the parameter, but they become non-trivial solutions at the boundary of the parameter space. Hence, the non-trivial universal solutions can be regarded as a kind of singular gauge transformations from the perturbative vacuum [10]. Moreover, the gauge symmetry and the annihilation mechanism of open strings are inseparable. In the theory around the non-trivial solution, the kinetic operator is given by the modified BRS charge which has the cohomology with different ghost numbers from the original cohomology. Therefore all of on-shell states are reduced to gauge degrees of freedom in the gauge unfixed theory and then open string excitations disappear after the condensation [14].

Although the tachyon vacuum solution has already been obtained approximately in the level truncated theory, we can not so simply compared the universal solutions with the level truncated solution, because the gauges of these solutions differ. The energy density of the universal solutions, then, should be calculated in order to clarify the relation between these solutions. However, if we try to calculate the potential height directly, there are some technical problems which are remained to be resolved. To avoid these we should adopt other approaches at present.

The purpose of this paper is to apply level truncation scheme to investigate the theory around the universal solutions, and to determine the potential height of the solutions indirectly. If a one-parameter family of the solutions can be interpreted as explained above, the theory around the solutions should describe the perturbative vacuum in the almost region of the parameter and the tachyon vacuum at the boundary. Hence, we should observe the situation that in the theory for the almost parameter, there is a non-perturbative vacuum which gives the same energy density as the D-brane tension, but the non-trivial vacuum disappears at the endpoint.

In Section 2 we review the universal solutions in string field theory with some new results, and we explain the difficulty of calculating the potential height. In Section 3 we analyze the non-perturbative vacuum in the theory around the universal solutions up to level (6,18). Our results strongly suggest that the non-trivial universal solution corresponds to the tachyon vacuum. In Section 4, we give summary and discussions.
2 Classical solutions and potential heights

2.1 universal solutions

The action of cubic open string field theory is given by [13]

\[ S = -\frac{1}{g^2} \int \left( \frac{1}{2} \Psi^* Q_B \Psi + \frac{1}{3} \Psi^* \Psi^* \Psi \right). \]  

(2.1)

By variation of the action, we find the classical equation of motion

\[ Q_B \Psi + \Psi^* \Psi = 0. \]  

(2.2)

One of the analytic solutions with translational invariance has been found as [10]

\[ \Psi_0 = Q_L(e^h - 1) I - C_L((\partial h)^2 e^h) I, \]  

(2.3)

where \( I \) denotes the identity string field. The operators \( Q_L \) and \( C_L \) are defined by

\[ Q_L(f) = \int_{C_{\text{left}}} \frac{dw}{2\pi i} f(w) J_B(w), \quad C_L(f) = \int_{C_{\text{left}}} \frac{dw}{2\pi i} f(w) c(w), \]  

(2.4)

where \( J_B(w) \) and \( c(w) \) are the BRS current and the ghost field, respectively, and \( C_{\text{left}} \) stands for the contour along the left-half of strings. The function \( h(w) \) satisfies \( h(-1/w) = h(w) \) and \( h(\pm i) = 0 \).

The solution (2.3) obeys the equation of motion (2.2) in the following. The anti-commutation relations of \( Q_L \) and \( C_L \) are given by

\[ \{Q_L(e^h - 1), Q_L(e^h - 1)\} = 2\{Q_B, C_L((\partial h)^2 e^h)\}, \]  

\[ \{Q_L(e^h - 1), C_L((\partial h)^2 e^h)\} = \{Q_B, C_L((\partial h)^2(e^{2h} - e^h))\}, \]  

(2.5)

and others are zero. We define similar operators \( Q_R(f) \) and \( C_R(f) \) by replacing the contour \( C_{\text{left}} \) in (2.4) to \( C_{\text{right}} \) corresponding to the right half of strings. Then, for the star product, these operators satisfy

\[ (Q_R(e^h - 1) A) \star B = -(-1)^{|A|} A \star (Q_L(e^h - 1) B), \]  

\[ (C_R((\partial h)^2 e^h) A) \star B = -(-1)^{|A|} A \star (C_L((\partial h)^2 e^h) B). \]  

(2.6)

Through conservation of the BRS current and the ghost field on the identity string field, we find that

\[ (Q_L(e^h - 1) + Q_R(e^h - 1)) I = 0, \quad (C_L((\partial h)^2 e^h) + C_R((\partial h)^2 e^h)) I = 0. \]  

(2.7)
From (2.5), (2.6) and (2.7), it follows that

\[
Q_B \Psi_0 = -\{Q_B, C_L((\partial h)^2e^h)\}I, \\
\Psi_0 \ast \Psi_0 = (Q_L(e^h - 1) - C_L((\partial h)^2e^h))^2 I = \{Q_B, C_L((\partial h)^2e^h)\}I. \tag{2.8}
\]

As a result, the equation of (2.3) is a classical solution in the string field theory.

Though the function \(h(w)\) must cancel the midpoint singularity of the ghost field on \(I\) to make the operator \(C_L I\) well-defined, this cancellation actually occurs due to the previous two conditions for \(h(w)\). Around the midpoint \(w_0 = \pm i\), the ghost field behaves as \([9, 10, 16]\)

\[
c(w)I \sim \frac{1}{w - w_0} \left(-c_0 + \frac{w_0}{2}(c_1 - c_{-1})\right) I + O((w - w_0)^0), \tag{2.9}
\]

and then its singularity is a pole at the midpoint. If the function \(h(w)\) is analytic around \(w_0\), \(h(w)\) can be expanded as \(h(w) = h''(w_0)(w - w_0)^2 + \cdots\) because \(h'(w) = h'(-1/w)/w^2\). Therefore, \((\partial h)^2e^h c(w)\) becomes regular at the midpoint and then the operator \(C_L\) is well-defined on the identity string field.

In the following, let us consider the solution generated by the function

\[
h_a(w) = \log \left(1 + \frac{a}{2} \left(w + \frac{1}{w}\right)^2\right), \tag{2.10}
\]

and we parameterize the solution by a real number \(a\). The function is rewritten as \(h_a(\sigma) = \log(1 + 2a \cos^2 \sigma)\) on the unit circle \(w = \exp(i\sigma)\), and the parameter \(a\) is larger than \(-1/2\) accordingly. In the region \(a \geq -1/2\), the function \(h_a(\sigma)\) can be expanded by the Fourier series

\[
h_a(\sigma) = -\log(1 - Z(a))^2 - 2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n} Z(a)^n \cos(2n\sigma), \tag{2.11}
\]

where \(Z(a) = (1 + a - \sqrt{1 + 4a})/a\) and \(-1 \leq Z(a) < 1\) \((-1/2 \leq a < \infty)\).

Substituting the function into the form (2.3) and expanding it by oscillators, we find the solution up to level two

\[
|\Psi_0(a)\rangle = J_1(a) c_1 |0\rangle + \left(\frac{8a}{3\pi} + J_1(a)\right) L^X_2 c_1 |0\rangle \\
+ \left(\frac{8a}{\pi} + J_2(a)\right) c_{-1} |0\rangle + \left(\frac{8a}{3\pi} + 2J_1(a)\right) c_0 b_{-2} c_1 |0\rangle + \cdots, \tag{2.12}
\]

where \(L^X_n\) denote matter Virasoro generators and \(J_1(a)\) and \(J_2(a)\) are given by

\[
J_1(a) = -\int_{-\pi/2}^{\pi/2} \frac{d\sigma}{2\pi} h_a'(\sigma)^2 e^{h_a(\sigma)} \frac{1}{2 \cos \sigma}, \\
J_2(a) = -\int_{-\pi/2}^{\pi/2} \frac{d\sigma}{2\pi} h_a'(\sigma)^2 e^{h_a(\sigma)} \frac{1 + 2 \cos(2\sigma)}{2 \cos \sigma}. \tag{2.13}
\]
Using the Fourier series (2.11), we can carry out these integrations. The results of the calculations are, for \( a \geq 0 \)
\[
J_1(a) = -\frac{4a}{\pi} \left\{ 1 - \frac{1}{2} \left( \sqrt{Z(a)} + \frac{1}{\sqrt{Z(a)}} \right) \log \frac{1 + \sqrt{Z(a)}}{1 - \sqrt{Z(a)}} \right\}, \tag{2.14}
\]
\[
J_2(a) = \frac{4a}{\pi} \left\{ \frac{1}{3} + Z(a) + \frac{1}{Z(a)} - \frac{1}{2} \left( Z(a) \sqrt{Z(a)} + \frac{1}{Z(a) \sqrt{Z(a)}} \right) \log \frac{1 + \sqrt{Z(a)}}{1 - \sqrt{Z(a)}} \right\}, \tag{2.15}
\]
and for \(-1/2 \leq a < 0\),
\[
J_1(a) = -\frac{4a}{\pi} \left\{ 1 + \left( \sqrt{-Z(a)} - \frac{1}{\sqrt{-Z(a)}} \right) \arctan \sqrt{-Z(a)} \right\}, \tag{2.16}
\]
\[
J_2(a) = \frac{4a}{\pi} \left\{ \frac{1}{3} + Z(a) + \frac{1}{Z(a)} + \left( Z(a) \sqrt{-Z(a)} - \frac{1}{Z(a) \sqrt{-Z(a)}} \right) \arctan \sqrt{-Z(a)} \right\}. \tag{2.17}
\]

It is interesting to note that the solution has a well-defined Fock space expression and the coefficients of its component states have no divergence. This situation is different from the case of the dilaton condensation in light-cone type string field theories [17, 18]. For instance, the functions \( J_1(a) \) and \( J_2(a) \) have finite values as depicted in Fig. 1 and then the coefficients become finite up to level two. Moreover, the Fock space used in the solution can be spanned by the universal basis, because the solution is made of the BRS current, the ghost field and the identity string field. This universality is indispensable for the tachyon solution [1]. Finally, we indicate that the solution is outside Siegel gauge since it contains states proportional to the ghost zero mode \( c_0 \).

### 2.2 physical interpretation

To consider physical meaning of the solution, we expand the string field \( \Psi \) by the solution \( \Psi_0 \) and the quantum fluctuation \( \Phi \) as
\[
\Psi = \Psi_0(a) + \Phi. \tag{2.18}
\]
Substituting this form into (2.1), the action becomes
\[
S[\Psi] = S[\Psi_0(a)] - \frac{1}{g^2} \int \left( \frac{1}{2} \Phi * Q_B'(a) \Phi + \frac{1}{3} \Phi * \Phi * \Phi \right), \tag{2.19}
\]
where the modified BRS charge \( Q_B' \) is given by
\[
Q_B'(a) = Q(e^{ha}) - C((\partial h_a)^2 e^{ha}), \tag{2.20}
\]
5
and $Q(f)$ and $C(g)$ are defined by $Q_L(f) + Q_R(f)$ and $C_L(g) + C_R(g)$, respectively. The first term in the action corresponds to the vacuum energy at the solution, and the second term represents the action for the quantum fluctuation.

We can show that the solution for $a > -1/2$ is expressed by a gauge transformation of the trivial vacuum as

$$\Psi_0(a) = \exp(K_L(h_a)I) \ast Q_B \exp(-K_L(h_a)I), \quad (2.21)$$

where $\exp A$ for a string field $A$ is defined by its series as $\exp A = I + A + A \ast A/2! + \cdots$, and the operator $K_L$ is defined by

$$K_L(f) = \int_{C_{\text{ret}}} \frac{dw}{2\pi i} f(w) \left( J_{gh}(w) - \frac{3}{2} w^{-1} \right). \quad (2.22)$$

Consequently, we naturally expect that the theory around the solution for $a > -1/2$ describes the physics on the perturbative vacuum. Indeed, we can transform the action for the fluctuation $\Phi$ into the original action through the string field redefinition

$$\Phi' = e^{K(h_a)}\Phi, \quad (2.23)$$

where $K(f) = K_L(f) + K_R(f)$, and $K_R(f)$ is the counterpart of $K_L(f)$ related to right strings. The equivalence of these actions is based on the fact that the original and modified BRS charges are connected by the similarity transformation

$$Q_B'(a) = e^{K(h_a)}Q_B e^{-K(h_a)}. \quad (2.24)$$
However, since the operators $e^{K_L}$ and $e^K$ becomes ill-defined at $a = -1/2$, the solution can not be represented by the pure gauge, and the theory around it can not be connected to the original theory through the string field redefinition. Consequently, the solution at $a = -1/2$ represents a non-trivial vacuum, while the solution is a pure gauge for $a > -1/2$. For example, the operator $e^K$ is written by the normal ordered form
\begin{equation}
\begin{align*}
e^{K(h_a)} &= \left(1 - Z(a)^2\right)^{-1} \exp(-\tilde{q}_0 \log(1 - Z(a))^2) \\
&\quad \times \exp\left(-\sum_{n=1}^{\infty} \frac{(-1)^n}{n} q_{-2n} Z(a)^n\right) \exp\left(-\sum_{n=1}^{\infty} \frac{(-1)^n}{n} q_{2n} Z(a)^n\right),
\end{align*}
\end{equation}
where $q_n$ denote the oscillators of the ghost number current and they are written by the ghost oscillators as
\begin{equation}
\begin{align*}
\tilde{q}_0 &= \frac{1}{2}(c_0 b_0 - b_0 c_0) + \sum_{n=1}^{\infty} (c_{-n} b_n - b_{-n} c_n), \\
q_n &= \sum_{m=-\infty}^{\infty} c_{n-m} b_m \quad (n \neq 0).
\end{align*}
\end{equation}
If we take $a = -1/2$, the first factor in (2.25) diverges because $Z(-1/2) = -1$. In order to find the singularity more explicitly, we write the string field by the oscillator expression as
\begin{equation}
|\Psi\rangle = \phi(x) c_1 |0\rangle + \cdots + \beta(x) c_{-1} |0\rangle + \gamma(x) b_{-2} c_0 c_1 |0\rangle + \cdots.
\end{equation}
Using the normal ordered expression (2.25), we find that, through the redefinition of (2.23), the lowest level component field $\phi(x)$ is transformed as
\begin{equation}
\phi'(x) = \frac{1}{1 + Z(a)} \phi(x) + \frac{Z(a)}{1 + Z(a)} (-\beta(x) + 2\gamma(x)) + \cdots,
\end{equation}
where the abbreviation denotes the contribution from the higher level component fields. Thus, by the string field redefinition, the component fields are transformed into the linear combination of an infinite number of fields. However, its coefficients diverge at $a = -1/2$ and then this redefinition is ill-defined. Similarly, the operator $e^{K_L}$ has a singularity at $a = -1/2$.

To find the physical meaning of the solution at $a = -1/2$ from a different viewpoint, we can determine the cohomology of the new BRS charge and the perturbative spectrum around the solution. As in ref. [14], the new cohomology appears only in the sector with different ghost numbers from the original cohomology. Consequently, in the gauge unfixed theory around the solution, we can solve the equation of motion $Q_B'(-1/2)\Phi = 0$ as $\Phi = Q_B'(-1/2)\phi$. Since the theory is invariant under the gauge transformation $\delta \Phi = Q_B'(-1/2)\delta \Lambda$, all of the on-shell physical states become gauge degrees of freedom in the theory perturbatively.
Hence, we find that the universal solution at \( a = -\frac{1}{2} \) represents a non-trivial vacuum, where there is no physical excitation perturbatively. This is the very feature required for the tachyon vacuum. As discussed above, the solution satisfies universality. Putting these observations together, we expect that the universal solution corresponds to the tachyon vacuum itself.

### 2.3 potential heights

Formally, we can find that the potential height \( -S[\Psi_0(a)] \) is zero for \( a > -\frac{1}{2} \). Indeed, the derivative of \( S[\Psi(a)] \) with respect to \( a \) is given by

\[
\frac{d}{da} S[\Psi_0(a)] = -\frac{1}{g^2} \int (Q_B \Psi_0(a) + \Psi_0(a) \ast \Psi_0(a)) \ast \frac{d\Psi_0(a)}{da} = 0,
\]

where we have used the equation of motions for the last equality [19]. Since \( h_{a=0} = \partial h_{a=0} = 0 \) and then \( S[\Psi_0(a = 0)] = 0 \), it turns out that \( S[\Psi_0(a)] \) is equal to zero. This zero potential height can be shown only for \( a > -\frac{1}{2} \), because the solution is ill-defined for \( a < -\frac{1}{2} \) and it is undifferentiable at \( a = -\frac{1}{2} \). This formal discussion is consistent with the expectation that the universal solution is a pure gauge for \( a > -\frac{1}{2} \).

However, we cannot calculate the potential height more explicitly beyond the formal evaluation. Substituting the solution (2.3) into the action (2.1), we find that

\[
S[\Psi_0(a)] = -\frac{1}{6g^2} \langle I | C_L((\partial h_a)^2 e^{h_a}) Q_B C_L((\partial h_a)^2 e^{h_a}) | I \rangle
\]

\[
= -\frac{1}{6g^2} \int_{\text{conf}} dw \int_{\text{conf}} dw' \frac{dw}{2\pi i} \frac{dw'}{2\pi i}
\]

\[
\times (\partial h_a(w))^2 e^{h_a(w)} (\partial h_a(w'))^2 e^{h_a(w')} \langle I | c(w) \partial c(w') | I \rangle,
\]

where we have used \( Q_B I = 0 \) and \( \{Q_B, c\} = c \partial c \). The identity string field is written by the tensor product of the matter and ghost sectors and the matter sector of the identity string field is given by [20]

\[
| I^X \rangle = \exp \left( -\sum_{n=1}^{\infty} \frac{(-1)^n}{2n} \alpha_{-n} \cdot \alpha_{-n} \right) | 0 \rangle.
\]

Then, \( \langle I | c \partial c | I \rangle \) is an indefinite quantity for any \( a \) due to the infinite determinant factor of the matter sector. Thus, it is difficult to evaluate the potential height because we have not yet known how it should be regularized.

*More precisely, the potential \( V \) is divided by the D-brane volume \( V_D \) as \( V = -S[\Psi_0]/V_D \).

†We are still suffering from the disastrous divergence even if we use usual regularization with the insertion of \( e^{-\epsilon L_0} [21] \).
Instead of the exact calculation, we try to use level truncated solutions to evaluate the potential height. For \( a = -1/2 \), the solution (2.12) becomes

\[
|\Psi_0(-1/2)\rangle = \frac{2}{\pi} c_1 |0\rangle + \frac{2}{3\pi} L_{-2}^c c_1 |0\rangle - \frac{2}{3\pi} c_{-1} |0\rangle + \frac{8}{3\pi} c_0 b_{-2} c_1 |0\rangle + \cdots. \tag{2.32}
\]

At level zero, the truncated solution is \( 2/\pi \times c_1 |0\rangle \) and so the function \( f(T) \) defined in ref. [4] takes the value

\[
2\pi^2 \left( -\frac{1}{2} \left( \frac{2}{\pi} \right)^2 + \frac{1}{3} \left( \frac{2}{\pi} \right)^3 \right) \simeq -0.279. \tag{2.33}
\]

This provides 28% of the D-brane tension. Furthermore, we calculate the function \( f(T) \) at level two and, then, it takes the value \( \sim 85 \), which is far from a stationary point at the level two potential. For general \( a \), we are faced with such terrible behavior. Although this result discourages us to perform further calculations, this is a natural result because our solution is not a solution in the level truncated theory.

Hence, we can not calculate the vacuum energy of the universal solutions at present, in order to compare it with the D-brane tension.

3 Level truncation in string field theory with the modified kinetic term

In this section we explore another possibility of clarifying the relation between the universal solutions and the tachyon vacuum. Instead of the direct calculation of the potential height, we analyze the non-perturbative vacuum in the theory expanded around the universal solutions. By using the level truncation scheme in Siegel gauge, we can find the non-perturbative vacuum without any difficulty, and moreover we come across the impressive result which supports our conjecture for the universal solutions.

3.1 conjectures and setup

As in (2.19), the action for the fluctuation \( \Phi \) is written by

\[
S[\Phi] = -\frac{1}{g^2} \int \left( \frac{1}{2} \Phi \ast Q'_B(a) \Phi + \frac{1}{3} \Phi \ast \Phi \ast \Phi \right), \tag{3.1}
\]

where the modified BRS charge is given by (2.20). Substituting (2.11) into (2.20) and performing the \( w \) integration, we obtain the oscillator expressions of the new BRS charge

\[
Q'_B(a) = (1 + a) Q_B + \frac{a}{2} (Q_2 + Q_{-2}) + 4aZ(a) c_0 - 2aZ(a)^2 (c_2 + c_{-2})
- 2a(1 - Z(a)^2) \sum_{n=2}^{\infty} (-1)^n Z(a)^{n-1} (c_{2n} + c_{-2n}). \tag{3.2}
\]
where we expand the BRS current as \( J_B(w) = \sum_n Q_n w^{-n-1} \). The details of the calculation are presented in Appendix A.

Under the Siegel gauge condition \( b_0 \Phi = 0 \), the quadratic term in the action becomes

\[
-\frac{1}{g^2} \int \frac{1}{2} \Phi * Q'_B(a) \Phi = -\frac{1}{g^2} \int \frac{1}{2} \Phi * c_0 L(a) \Phi,
\]

where \( L(a) = \{Q'_B(a), b_0\} \). Using the anti-commutation relation \[14\]

\[
\{Q_m, b_n\} = L_m + n + \frac{3}{2} m(m - 1) \delta_{m+n,0},
\]

we can calculate the operator \( L(a) \) as

\[
L(a) = (1 + a)L_0 + \frac{a}{2}(L_2 + L_{-2}) + a(q_2 - q_{-2}) + 4aZ(a).
\]

Therefore, according to the notations of \[11, 4\], the potential in the new string field theory is given by the ‘modified’ universal function

\[
f_a(\Phi) = 2\pi^2 \left( \frac{1}{2} \langle \Phi, c_0 L(a) \Phi \rangle + \frac{1}{3} \langle \Phi, \Phi * \Phi \rangle \right).
\]

As seen in the previous section, we expect that the solutions \( \Psi_0(a > -1/2) \) and \( \Psi_0(a = -1/2) \) are regarded as a pure gauge and the tachyon vacuum, respectively. Consequently, in the case of \( a > -1/2 \), the action of \[3.1\] describes the perturbative vacuum, and then, in the potential, there is the stationary point which corresponds to the tachyon vacuum. On the other hand, the stationary point must vanish at \( a = -1/2 \), because the theory has already stayed on the non-perturbative vacuum. Hence, due to our conjectures, the modified universal function at the stationary point \( \Phi_0 \) must satisfy

\[
f_a(\Phi_0) = \begin{cases} 
0 & (a = -1/2) \\
-1 & (a > -1/2).
\end{cases}
\]

Let us consider a level truncated expression of the modified universal function. The new action is invariant under the twist transformation \( \sigma \rightarrow \pi - \sigma \) \[4\]. Due to this symmetry, we have only to look for a stationary point where \( \Phi_0 \) contains even level states as well as the case of the original level truncated analysis. In general, we can write a scalar string field \( |\Phi\rangle \) by the tensor product of the matter and ghost states as

\[
|\Phi\rangle = \sum_{i=0}^{\infty} \psi_i |s_i\rangle, \quad |s_i\rangle = |\eta_{m(i)}\rangle \otimes |\chi_{g(i)}\rangle.
\]

Our notations for the decomposition of states are almost same as in Ref. \[5\]. Up to level 6, the matter and ghost states, \( |\eta_i\rangle \) and \( |\chi_i\rangle \), are given in Appendix B and the decomposition of states is in Appendix C.
Using the component fields $\psi_i$ with zero momentum, we can express the modified universal function as follows,

$$f_a(\psi) = 2\pi^2 \left( \frac{1}{2} \sum_{ij} d_{ij}(a) \psi_i \psi_j + \frac{1}{3} \sum_{ijk} t_{ijk} \psi_i \psi_j \psi_k \right).$$

(3.9)

The cubic coefficients $t_{ijk}$ does not change from the previous analysis in Ref. [2]. The quadratic coefficients are calculated as

$$d_{ij}(a) = \left\{ (1 + a) (\text{level}(i) - 1) A_{ij}^{\text{mat}} A_{ij}^{gh} + 4a Z(a) \right\} A_{ij}^{\text{mat}} A_{ij}^{gh}$$

$$+ a B_{ij}^{\text{mat}} A_{ij}^{gh} + a A_{ij}^{\text{mat}} B_{ij}^{gh},$$

(3.10)

where level$(i)$ denotes the level of the state $|s_i\rangle$, and $A_{ij}^{\text{mat(gh)}}$ and $B_{ij}^{\text{mat(gh)}}$ are defined by

$$A_{ij}^{\text{mat}} = \langle \eta_m(i) | \eta_m(j) \rangle,$$

(3.11)

$$A_{ij}^{gh} = \langle \chi_g(i) | c_0 | \chi_g(j) \rangle,$$

(3.12)

$$B_{ij}^{\text{mat}} = \langle \eta_m(i) | L_{-2}^{\text{mat}} | \eta_m(j) \rangle,$$

(3.13)

$$B_{ij}^{gh} = \langle \chi_g(i) | c_0 (L_{-2}^{gh} - q_{-2}) | \chi_g(j) \rangle.$$

(3.14)

Here, $L_{n}^{\text{mat}}$ and $L_{n}^{gh}$ are the matter and ghost parts of the total Virasoro generators $L_n$. In this expression, $L_2$ and $q_2$ are converted to $L_{-2}$ and $-q_{-2}$ by the hermitian conjugation. A list of the coefficients $A_{ij}^{\text{mat(gh)}}$ up to level 6 is given in Appendix B. Up to level 6, the coefficients $B_{ij}^{\text{mat(gh)}}$ can be calculated by using $A_{ij}^{\text{mat(gh)}}$ through the following equations,

$$L_{-2}^{\text{mat}} | \eta_0 \rangle = \frac{1}{2} | \eta_1 \rangle,$$

$$L_{-2}^{\text{mat}} | \eta_1 \rangle = 2 | \eta_3 \rangle + \frac{1}{2} | \eta_5 \rangle,$$

$$L_{-2}^{\text{mat}} | \eta_3 \rangle = 3 | \eta_5 \rangle + | \eta_8 \rangle + \frac{1}{2} | \eta_9 \rangle,$$

$$L_{-2}^{\text{mat}} | \eta_4 \rangle = 4 | \eta_7 \rangle + | \eta_{10} \rangle,$$

$$L_{-2}^{\text{mat}} | \eta_5 \rangle = 4 | \eta_9 \rangle + \frac{1}{2} | \eta_{12} \rangle,$$

$$\left( L_{-2}^{gh} - 2q_{-2} \right) | \chi_0 \rangle = | \chi_1 \rangle,$$

$$\left( L_{-2}^{gh} - 2q_{-2} \right) | \chi_1 \rangle = 3 | \chi_4 \rangle + | \chi_6 \rangle,$$

$$\left( L_{-2}^{gh} - 2q_{-2} \right) | \chi_4 \rangle = 5 | \chi_7 \rangle + | \chi_9 \rangle,$$

$$\left( L_{-2}^{gh} - 2q_{-2} \right) | \chi_5 \rangle = 4 | \chi_8 \rangle + 2 | \chi_{10} \rangle + | \chi_{12} \rangle,$$

$$\left( L_{-2}^{gh} - 2q_{-2} \right) | \chi_6 \rangle = 3 | \chi_9 \rangle + 3 | \chi_{11} \rangle.$$

(3.15)

We set the level of ground states as level$(\eta_0) = 0$ and level$(\chi_0) = 0$.
3.2 level zero analysis

At level (0, 0) approximation, the component field is \( t c_1 |0\) and then the modified universal function is

\[
f_a(t) = 2\pi^2 \left( -\frac{1}{2} \lambda(a) t^2 + \frac{1}{3} K^3 t^3 \right),
\]

where \( \lambda(a) = 4\sqrt{1 + 2a} - 3(1 + a) \) and \( K = 3\sqrt{3}/4 \). It is easy to see that \( \lambda(a) \) has two roots

\[
a^+ = \frac{7 + 4\sqrt{7}}{9} = 1.954 \cdots, \quad a^- = \frac{7 - 4\sqrt{7}}{9} = -0.398 \cdots,
\]

then

\[
\text{if } a^- < a < a^+, \quad \lambda(a) > 0,
\]

\[
\text{if } -1/2 \leq a < a^- \text{ or } a > a^+, \quad \lambda(a) < 0.
\]

Therefore, \( f_a(t) \) has a local minimum at

\[
t_0 = \begin{cases} 
K^{-3} \lambda(a) & (a^- \leq a \leq a^+) \\
0 & (-1/2 \leq a < a^- \text{ or } a > a^+),
\end{cases}
\]

and \( f_a(t) \) at this minimum takes the value

\[
f_a(t_0) = \begin{cases} 
-\frac{\pi^2}{3K^6} \lambda(a)^3 & (a^- \leq a \leq a^+) \\
0 & (-1/2 \leq a < a^- \text{ or } a > a^+).
\end{cases}
\]

The \( a \) dependence of this value is depicted in Fig. 2. Though this behavior is quite different from the expectation that \( a \) does not affect the potential minimum for \( a > -1/2 \) as in (3.7), this \( a \) dependence is introduced merely by the level truncation approximation.

The value \(-0.684 \cdots \) at \( a = 0 \) equals to the minimum derived from the previous level truncation analysis \[4\], because the kinetic operator \( L(a) \) becomes \( L_0 \) at \( a = 0 \) and the modified universal function agrees with the universal function. This agreement is realized for any level analysis as seen in (3.5). In addition, it should be noticed that the minimum is exactly zero at \( a = -1/2 \).

3.3 higher level analysis

We apply the iterative approximation algorithm used by Moeller and Taylor \[5\] to higher level calculations. However, there is a slight change in the procedure to find the stable vacuum.
According to Ref. [6], the solution which minimizes the potential of (3.9) has many branches, and which branch should be chosen is determined by the condition that the solution becomes the level zero stable vacuum if higher level fields are turned off. As a result, the branch depends on the sign of the quadratic coefficients $d_{ij}(a)$. As an example, let us see the tachyon field $\psi_0 = t$. Since the coefficient $d_{00}(a)$ equals to $-\lambda(a)$ for any level analysis, the tachyon field which minimizes the potential can be expressed by other fields in the following,

$$
 t = \begin{cases} 
 -\beta + \sqrt{\beta^2 - 4\alpha\gamma} & (a^- \leq a \leq a^+) \\
 -\beta - \sqrt{\beta^2 - 4\alpha\gamma} & (-1/2 \leq a < a^-, \ a > a^+) 
\end{cases}
$$

(3.21)

Here, $\alpha$, $\beta$ and $\gamma$ are given by

$$
\alpha = t_{000}, \\
\beta = -\lambda(a) + \sum_{i=1}^{N} t_{00i}\psi_i, \\
\gamma = \sum_{i=1}^{N} d_{0i}(a)\psi_i + \sum_{i,j=1}^{N} t_{0ij}\psi_i\psi_j,
$$

(3.22) (3.23)

where $N$ is the number of truncated fields. Thus, the branch is determined depending on the value of $a$ in our analysis.

Let us consider the level two approximation. The level two field is given by

$$
\Phi^{(2)} = t |s_0\rangle + \psi_1 |s_1\rangle + \psi_2 |s_2\rangle
$$

(3.24)

$$
= t c_1 |0\rangle + \psi_1 (\alpha_{-1} \cdot \alpha_{-1}) c_1 |0\rangle - \psi_2 c_{-1} |0\rangle.
$$

(3.25)
From (3.10) and (3.15), we find the quadratic term of the modified universal function as
\[
2\pi^2 \left( -\frac{1}{2} \lambda(a) t^2 + 26 (1 + a + 4aZ(a)) \psi_1^2 - \frac{1}{2} (1 + a + 4aZ(a)) \psi_2^2 \right. \\
\left. + 13at \psi_1 + \frac{1}{2}at \psi_2 \right) .
\] (3.26)

The potential minimum is depicted in Fig. 3. We observe that (2,4) and (2,6) approximations lead to almost same results. At \( a = -1/2 \), the vacuum expectation values of component fields and the potential minimum are equal to zero as used (0,0) approximation. The vacuum energy are varying slowly in the neighborhood of \( a = 0 \), in which the potential height provide 96% of the D-brane tension. For our conjectures, it is a desirable fact that the potential minimum changes slowly along the expected vacuum energy. Thus, even at this level, we can expect that the universal solution is a pure gauge at least near \( a = 0 \), as our conjecture.

![Figure 3: The potential minimum in level two truncations.](image)

Let us progress to the higher level approximation. The potential minimum using level four and six approximations is depicted in Fig. 4. Like level zero and two cases, the results of the level \((L, 3L)\) calculation slightly change with the level \((L, 2L)\). In the cases of both of level four and six, the potential minimum displays a flat region along the expected vacuum energy. The potential heights and the vacuum expectation values become zero at \( a = -1/2 \) as before. This behavior can be seen in detail in Fig. 5 which magnifies the area near \( a = -1/2 \).

In these analyses, it is a remarkable fact that the higher the approximation level is increased, the wider the flat region grows. Moreover, the potential value in the flat region approaches the expected value increasingly as the level is raised. At level six, the vacuum
energy becomes almost $-1$ in the region from $-0.2$ to $1$. In Fig. 6 we pick out the values of the potential height for several points of $a$.\textsuperscript{5} All of the values are about 99% of the D-brane tension at level six. For our conjecture, the most important result is that the stable vacuum disappears at $a = -1/2$ in every level analysis. These potential behavior to the parameter $a$ suggests that, if the approximation level approaches infinity, the potential minimum takes the value of $-1$ for $a > -1/2$, but it remains being zero at $a = -1/2$. Hence, these results lead us to believe that the conjecture for the universal solutions, which is expressed by (3.7), should be true, and then the universal solution at $a = -1/2$ corresponds to the tachyon condensation conjectured by Sen.

\textsuperscript{5}Of course, our values at $a = 0$ agree with previous results in refs. [4, 5, 6]. But only the value at level (6,12) does not coincide with a result in [5].
Figure 5: The enlarged graph of the potential minimum around $a = -1/2$.

| level   | $V/T_{25}$          |
|---------|---------------------|
|         | $a = -0.2$ | $a = -0.1$ | $a = 0.0$ | $a = 0.1$ | $a = 0.2$ |
| (0,0)   | -0.233203 | -0.462912 | -0.684616 | -0.866692 | -0.995360 |
| (2,4)   | -0.777067 | -0.908062 | -0.948553 | -0.955031 | -0.956909 |
| (2,6)   | -0.854866 | -0.944975 | -0.959377 | -0.955239 | -0.955100 |
| (4,8)   | -0.965369 | -0.986459 | -0.986403 | -0.985358 | -0.985329 |
| (4,12)  | -0.988826 | -0.990313 | -0.987822 | -0.986499 | -0.985300 |
| (6,12)  | -0.993496 | -0.995449 | -0.994773 | -0.994077 | -0.993590 |
| (6,18)  | -0.996274 | -0.996056 | -0.995177 | -0.994346 | -0.993715 |

Figure 6: Vacuum energy in level truncation scheme for several points of $a$.

### 3.4 other universal solutions

We can provide other universal solutions by choosing the function in (2.3) as

$$ h_l^l(w) = \log \left( 1 - \frac{a}{2} (-1)^l \left( w^l - (-1)^l w^{-l} \right)^2 \right), \quad (3.28) $$

where $l = 1, 2, 3 \cdots$ [14]. The case of $l = 1$ corresponds to the previous example. The action around the solution has the modified BRS charge

$$ Q_B^l(a) = (1 + a) Q_B + (-1)^l \frac{a}{2} (Q_{2l} + Q_{-2l}) + 4a l^2 Z(a) c_0 + (-1)^l a l^2 Z(a)^2 (c_{2l} + c_{-2l}) $$

$$ -2a l^2 (1 - Z(a)^2) \sum_{n=2}^{\infty} (-1)^{nl} Z(a)^{n-1} (c_{2nl} + c_{-2nl}). \quad (3.29) $$

The kinetic operator in Siegel gauge is given by

$$ L^l(a) = \{ Q^l(a), b_0 \} \quad (3.30) $$

16
\[
= (1 + a)L_0 - (-1)^l \frac{a}{2} (L_{2l} + L_{-2l}) - (-1)^l a (q_{2l} - q_{-2l}) + 4al^2 Z(a). \quad (3.31)
\]

At level zero, the modified universal function becomes
\[
f_l(t) = 2\pi^2 \left( -\frac{1}{2} \lambda_l(a) t^2 + \frac{1}{3} K^3 t^3 \right),
\]
\[
\lambda_l(a) = 4l^2 \sqrt{1 + 2a} - (4l^2 - 1)(1 + a).
\quad (3.32)
\]

At the local minimum it takes the value
\[
f_l(t_0) = \begin{cases} 
-\frac{\pi^2}{3K^6} \lambda_l(a)^3 & (a^- \leq a \leq a^+) \\
0 & (-1/2 \leq a < a^- \text{ or } a > a^+) \end{cases}
\quad (3.33)
\]
where the branch points \( a_i^\pm \) are given by
\[
a_i^\pm = \frac{8l^2 - 1 \pm 4l^2 \sqrt{8l^4 - 1}}{(4l^2 - 1)^2}. \quad (3.34)
\]

Let us consider the case of \( l = 2 \). The kinetic operator is given by
\[
L^2(a) = (1 + a)L_0 - \frac{a}{2}(L_4 + L_{-4}) - 2a(q_4 + q_{-4}) + 16a Z(a). \quad (3.35)
\]
Similarly to the case of \( l = 1 \), the quadratic terms up to level six can be calculated by the following equations,
\[
L_{-4}^{\text{mat}} |\eta_6\rangle = |\eta_3\rangle + \frac{1}{2} |\eta_4\rangle,
\]
\[
L_{-4}^{\text{mat}} |\eta_1\rangle = |\eta_9\rangle + |\eta_{10}\rangle + \frac{1}{2} |\eta_4\rangle,
\]
\[
(L_{-4}^{\text{gh}} - 4q_{-4}) |\chi_0\rangle = -3 |\chi_4\rangle - 2 |\chi_5\rangle - |\chi_6\rangle,
\]
\[
(L_{-4}^{\text{gh}} - 4q_{-4}) |\chi_1\rangle = 5 |\chi_7\rangle + |\chi_{11}\rangle + 2 |\chi_{12}\rangle. \quad (3.36)
\]

The branch points are
\[
a_2^- = -0.258 \ldots, \quad a_2^+ = 0.533 \ldots. \quad (3.37)
\]

The results of numerical analysis up to level six are depicted in Fig. 7. At \( a = -1/2 \), the potential minimum remains being zero for any approximation level. From level four, the flat region begins to spread along the expected vacuum energy. As a different feature from the previous case, we observe that, if the level is raised, the shape of the curve changes with the period of level two, in particular near the point that the potential minimum approaches to zero. On the whole we can see that the curve tends to approach the step function \( (3.7) \) as increasing the level. However, it seems to converge more slowly than the case of \( l = 1 \).
Summary and discussions

We have studied a non-perturbative vacuum in open string field theory around the universal solutions by using a level truncation scheme. We have observed that, as increasing the approximation level, the potential minimum gradually approaches to the step function which equals to the negative D-brane tension for almost parameters and becomes zero at the boundary. These results strongly support our conjecture that the universal solution are pure gauge for the almost parameter region, but the non-trivial solution at the boundary corresponds to the tachyon vacuum solution.

Though this physical interpretation for the universal solutions is plausible, we can construct them as many as functions used in eq. (2.3). At least, the non-trivial solutions are countable by the natural number as in eq. (3.28). On the other hand the tachyon vacuum should be unique.
in string theory. In order for them to be consistent, these universal solutions must be changed by gauge transformations. Though this gauge equivalence remains to be proved, this may be probably possible by the gauge transformation related to the operators $K_n = L_n - (-1)^n L_{-n}$ as discussed in \cite{14}. In addition, the equivalence between our solutions in this paper is consistent with the conjecture proposed by Drukker \cite{22, 23} which concerns the order of zero of the function with the number of D-branes.

Finally, two problems at least are remained to be solved in order to prove the correspondence between the universal solutions and the tachyon vacuum. First, we should calculate the potential height of the universal solutions exactly instead of these numerical analysis. Secondly, we should find closed strings in the theory around the solutions. In \cite{23} Drukker discussed how closed strings should be appeared in the theory, but we should show them more explicitly, for example as closed string poles in amplitudes.

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Appendix

A Mode expansion of the modified BRS charge

The modified BRS charge $Q'_B(a)$ is defined in (2.20) and the function $h_a(w)$ is given by (2.10). We can easily evaluate the first term of (2.20) as

$$Q(e^{h_a}) = Q\left(1 + \frac{a}{2}(w + \frac{1}{w})^2\right) = Q\left(1 + a + \frac{a}{2}(w^2 + w^{-2})\right) = (1 + a)Q_B + \frac{a}{2}(Q_2 + Q_{-2}). \quad (A.1)$$

From (2.10), we can find

$$\left(\partial h_a(w)\right)^2 e^{h_a(w)} = -a w^{-1} \partial h_a(w) (w^2 - w^{-2}). \quad (A.2)$$

If we differentiate the Fourier series of (2.11), we get

$$\partial h_a(w) = -4i w^{-1} \sum_{n=1}^{\infty} (-1)^n Z(a)^n \sin 2n\sigma, \quad (A.3)$$

where $w = \exp(i\sigma)$. From these equations, it follows that

$$\left(\partial h_a(w)\right)^2 e^{h_a(w)} = 8a \sum_{n=1}^{\infty} (-1)^n Z(a)^n \sin 2n\sigma \sin 2\sigma$$

$$= 8a w^{-2} \left(-Z(a) \sin^2 2\sigma + \sum_{n=2}^{\infty} (-1)^n Z(a)^n \sin 2n\sigma \sin 2\sigma\right). \quad (A.4)$$

Substituting (A.4) into (2.20), we can evaluate the second term of (2.20) as

$$C((\partial h_a)^2 e^{h_a}) = -8a Z(a) \sum_{n=-\infty}^{\infty} c_n \int_{-\pi}^{\pi} \frac{d\sigma}{2\pi} e^{-i n \sigma} \sin^2 2\sigma$$

$$+ 8a \sum_{n=-\infty}^{\infty} c_n \sum_{m=2}^{\infty} (-1)^m Z(a)^m \int_{-\pi}^{\pi} \frac{d\sigma}{2\pi} e^{-i n \sigma} \sin 2m\sigma \sin 2\sigma. \quad (A.5)$$

We can calculate the integrations in this equation and the results are

$$\int_{-\pi}^{\pi} \frac{d\sigma}{2\pi} e^{-i n \sigma} \sin^2 2\sigma = \begin{cases} 
\frac{1}{2} & (n = 0) \\
-\frac{1}{4} & (n = \pm 4) \\
0 & \text{(otherwise),} 
\end{cases} \quad (A.6)$$
\[
\int_{-\pi}^{\pi} \frac{d\sigma}{2\pi} e^{-i n \sigma} \sin 2m \sigma \sin 2\sigma = \begin{cases} 
\frac{1}{4} & (n = \pm 2(m - 1)) \\
-\frac{1}{4} & (n = \pm 2(m + 1)) \\
0 & \text{(otherwise)}.
\end{cases}
\] (A.7)

Substituting (A.6) and (A.7) into (A.5), we find that

\[
C((\partial_h a)^2 e^{h_a}) = -8aZ(a) \left( \frac{1}{2} c_0 - \frac{1}{4} (c_4 + c_{-4}) \right) \\
+ 8a \sum_{m=2}^{\infty} (-1)^m Z(a)^m \left( \frac{1}{4} (c_{2(m-1)} + c_{-2(m-1)}) \\
- \frac{1}{4} (c_{2(m+1)} + c_{-2(m+1)}) \right) \\
= -4aZ(a)c_0 + 2aZ(a)^2 (c_2 + c_{-2}) \\
+ 2a(1 - Z(a)^2) \sum_{n=2}^{\infty} (-1)^n Z(a)^{n-1} (c_{2n} + c_{-2n}).
\] (A.8)

Finally, from (A.1) and (A.8), we obtain the oscillator expression of the modified BRS charge as in (3.2).
## B Table of matter and ghost states at levels $\leq 6$

The following table describe the matter and ghost states which contribute to scalar fields at levels $\leq 6$. The inner products $A_{ij}^{\text{mat}}$ and $A_{ij}^{\text{gh}}$ are defined by eqs. (3.11) and (3.12).

| state | inner products |
|-------|----------------|
| $|\eta_0\rangle$ | $|0\rangle$ | $A_{00}^{\text{mat}} = 1$ |
| $|\eta_1\rangle$ | $(\alpha_{-1} \cdot \alpha_{-1}) |0\rangle$ | $A_{11}^{\text{mat}} = 52$, $A_{22}^{\text{mat}} = 52$ |
| $|\eta_2\rangle$ | $(\alpha_{-1} \cdot \alpha_{-2}) |0\rangle$ | $A_{33}^{\text{mat}} = 78$, $A_{44}^{\text{mat}} = 208$ |
| $|\eta_3\rangle$ | $(\alpha_{-1} \cdot \alpha_{-3}) |0\rangle$ | $A_{55}^{\text{mat}} = 5824$, $A_{66}^{\text{mat}} = 130$, $A_{77}^{\text{mat}} = 208$, $A_{88}^{\text{mat}} = 468$ |
| $|\eta_4\rangle$ | $(\alpha_{-2} \cdot \alpha_{-2}) |0\rangle$ | $A_{99}^{\text{mat}} = 4368$, $A_{1010}^{\text{mat}} = 10816$, $A_{1011}^{\text{mat}} = 416$, $A_{1111}^{\text{mat}} = 5616$, $A_{1110}^{\text{mat}} = 416$ |
| $|\eta_5\rangle$ | $(\alpha_{-1} \cdot \alpha_{-1}) (\alpha_{-3} \cdot \alpha_{-1}) |0\rangle$ | $A_{1121}^{\text{mat}} = 1048320$ |
| $|\chi_0\rangle$ | $|1\rangle$ | $A_{00}^{\text{gh}} = 1$ |
| $|\chi_1\rangle$ | $b_{-1}c_{-1} |1\rangle$ | $A_{11}^{\text{gh}} = -1$, $A_{22}^{\text{gh}} = -1$ |
| $|\chi_2\rangle$ | $b_{-1}c_{-2} |1\rangle$ | $A_{33}^{\text{gh}} = -1$, $A_{44}^{\text{gh}} = -1$ |
| $|\chi_3\rangle$ | $b_{-2}c_{-1} |1\rangle$ | $A_{55}^{\text{gh}} = -1$, $A_{66}^{\text{gh}} = -1$ |
| $|\chi_4\rangle$ | $b_{-1}c_{-3} |1\rangle$ | $A_{77}^{\text{gh}} = -1$, $A_{88}^{\text{gh}} = -1$ |
| $|\chi_5\rangle$ | $b_{-2}c_{-2} |1\rangle$ | $A_{99}^{\text{gh}} = -1$, $A_{1010}^{\text{gh}} = -1$ |
| $|\chi_6\rangle$ | $b_{-3}c_{-1} |1\rangle$ | $A_{1111}^{\text{gh}} = -1$, $A_{1110}^{\text{gh}} = -1$ |
| $|\chi_7\rangle$ | $b_{-1}c_{-5} |1\rangle$ | $A_{1212}^{\text{gh}} = 1$ |
| $|\chi_8\rangle$ | $b_{-2}c_{-4} |1\rangle$ | $A_{13}^{\text{gh}} = 4368$, $A_{14}^{\text{gh}} = 468$ |
| $|\chi_9\rangle$ | $b_{-3}c_{-3} |1\rangle$ | $A_{15}^{\text{gh}} = 130$, $A_{16}^{\text{gh}} = 5824$ |
| $|\chi_{10}\rangle$ | $b_{-4}c_{-2} |1\rangle$ | $A_{17}^{\text{gh}} = 208$, $A_{18}^{\text{gh}} = 416$ |
| $|\chi_{11}\rangle$ | $b_{-5}c_{-1} |1\rangle$ | $A_{19}^{\text{gh}} = 5616$, $A_{20}^{\text{gh}} = 10816$ |
| $|\chi_{12}\rangle$ | $b_{-2}b_{-1}c_{-2}c_{-1} |1\rangle$ | $A_{21}^{\text{gh}} = 416$, $A_{22}^{\text{gh}} = 1048320$ |
C Table of scalar states at levels \( \leq 6 \)

The following table lists scalar states at levels \( \leq 6 \). \( \langle \psi_i \rangle_{(6,18)} \) denote vacuum expectation values of the scalar states \( \psi_i \) in level truncation calculations at \( (6,18) \).

| \( \psi_i \) | state | \( a = -0.50 \) | \( a = -0.40 \) | \( a = -0.25 \) | \( a = 0.00 \) | \( a = 0.50 \) | \( a = 1.00 \) |
|---|---|---|---|---|---|---|---|
| \( \psi_0 \) | \( \eta_0 \otimes \chi_0 \) | 0.0000 | 0.49637 | 0.56739 | 0.54793 | 0.48059 | 0.42623 |
| \( \psi_1 \) | \( \eta_1 \otimes \chi_0 \) | 0.00000 | 0.07446 | 0.05809 | 0.02857 | 0.00038 | -0.01221 |
| \( \psi_2 \) | \( \eta_0 \otimes \chi_1 \) | 0.00000 | -0.31083 | -0.30955 | -0.21181 | -0.10466 | -0.04878 |
| \( \psi_3 \) | \( \eta_3 \otimes \chi_0 \) | 0.00000 | 0.01000 | -0.00008 | -0.00573 | -0.00524 | -0.00271 |
| \( \psi_4 \) | \( \eta_4 \otimes \chi_0 \) | 0.00000 | -0.00351 | -0.00355 | -0.00255 | -0.00152 | -0.00100 |
| \( \psi_5 \) | \( \eta_5 \otimes \chi_0 \) | 0.00000 | 0.00426 | 0.00176 | -0.0016 | -0.00055 | -0.00018 |
| \( \psi_6 \) | \( \eta_1 \otimes \chi_1 \) | 0.00000 | -0.02664 | -0.01077 | 0.00370 | 0.00817 | 0.00629 |
| \( \psi_7 \) | \( \eta_0 \otimes \chi_4 \) | 0.00000 | -0.05489 | 0.00881 | 0.05739 | 0.06430 | 0.04973 |
| \( \psi_8 \) | \( \eta_0 \otimes \chi_5 \) | 0.00000 | 0.04502 | 0.04922 | 0.03406 | 0.01876 | 0.01125 |
| \( \psi_9 \) | \( \eta_0 \otimes \chi_6 \) | 0.00000 | -0.01830 | 0.00294 | 0.01913 | 0.02143 | 0.01658 |
| \( \psi_{10} \) | \( \eta_0 \otimes \chi_0 \) | 0.00000 | 0.00273 | 0.00178 | 0.00175 | 0.00180 | 0.00138 |
| \( \psi_{11} \) | \( \eta_0 \otimes \chi_0 \) | 0.00000 | 0.00049 | 0.00128 | 0.00146 | 0.00108 | 0.00075 |
| \( \psi_{12} \) | \( \eta_0 \otimes \chi_6 \) | 0.00000 | 0.00118 | 0.00084 | 0.00072 | 0.00066 | 0.00049 |
| \( \psi_{13} \) | \( \eta_0 \otimes \chi_0 \) | 0.00000 | 0.00111 | 0.00025 | 0.00014 | 0.00036 | 0.00031 |
| \( \psi_{14} \) | \( \eta_0 \otimes \chi_0 \) | 0.00000 | -0.00015 | -0.00003 | 0.00008 | 0.00009 | 0.00007 |
| \( \psi_{15} \) | \( \eta_1 \otimes \chi_0 \) | 0.00000 | 0.00002 | 0.00001 | 0.00001 | 0.00001 | 0.00000 |
| \( \psi_{16} \) | \( \eta_2 \otimes \chi_0 \) | 0.00000 | 0.00012 | 0.00003 | 0.00000 | 0.00001 | 0.00001 |
| \( \psi_{17} \) | \( \eta_3 \otimes \chi_0 \) | 0.00000 | -0.00501 | -0.00232 | -0.00132 | -0.00167 | -0.0015 |
| \( \psi_{18} \) | \( \eta_4 \otimes \chi_1 \) | 0.00000 | -0.00059 | -0.00074 | -0.00058 | -0.00034 | -0.00022 |
| \( \psi_{19} \) | \( \eta_5 \otimes \chi_1 \) | 0.00000 | -0.00096 | -0.00021 | -0.00004 | -0.00025 | -0.00027 |
| \( \psi_{20} \) | \( \eta_2 \otimes \chi_2 \) | 0.00000 | -0.0012 | -0.00008 | -0.00006 | -0.00005 | -0.00003 |
| \( \psi_{21} \) | \( \eta_3 \otimes \chi_3 \) | 0.00000 | -0.00006 | -0.00004 | -0.00003 | -0.00003 | -0.00002 |
| \( \psi_{22} \) | \( \eta_4 \otimes \chi_4 \) | 0.00000 | -0.00506 | -0.00131 | -0.00168 | -0.00376 | -0.00371 |
| \( \psi_{23} \) | \( \eta_5 \otimes \chi_5 \) | 0.00000 | 0.00126 | -0.00038 | -0.00142 | -0.00123 | -0.00084 |
| \( \psi_{24} \) | \( \eta_6 \otimes \chi_6 \) | 0.00000 | -0.00169 | -0.00044 | -0.00056 | -0.00125 | -0.00124 |
| \( \psi_{25} \) | \( \eta_7 \otimes \chi_7 \) | 0.00000 | -0.04011 | -0.03204 | -0.03001 | -0.03071 | -0.02579 |
| \( \psi_{26} \) | \( \eta_8 \otimes \chi_8 \) | 0.00000 | -0.00536 | -0.01649 | -0.01875 | -0.01288 | -0.00819 |
| \( \psi_{27} \) | \( \eta_9 \otimes \chi_9 \) | 0.00000 | -0.01448 | -0.01167 | -0.01142 | -0.01206 | -0.01018 |
| \( \psi_{28} \) | \( \eta_{10} \otimes \chi_{10} \) | 0.00000 | -0.00268 | -0.00824 | -0.00938 | -0.00644 | -0.00409 |
| \( \psi_{29} \) | \( \eta_{11} \otimes \chi_{11} \) | 0.00000 | -0.00802 | -0.00641 | -0.00600 | -0.00614 | -0.00516 |
| \( \psi_{30} \) | \( \eta_{12} \otimes \chi_{12} \) | 0.00000 | -0.00765 | -0.01029 | -0.00773 | -0.00413 | -0.00243 |
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