Explanation of Gravity Probe B Experimental Results using Heaviside-Maxwellian (Vector) Gravity in flat Space-time

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The Gravity Probe B (GP-B) Experiment of NASA was aimed to test the theoretical predictions of Einstein’s 1916 relativistic tensor theory of gravity in curved space-time (General Relativity (GR)) concerning the spin axis precession of a gyroscope moving in the field of a slowly rotating massive body, like the Earth. In 2011, GP-B mission reported its measured data on the precession (displacement) angles of the spin axes of the four spherical gyroscopes housed in a satellite orbiting 642 km (400 mi) above the Earth in polar orbit. The reported results are in agreement with the predictions of GR. For the first time, here we report an undergraduate level explanation of the GP-B experimental results using Heaviside-Maxwellian (vector) Gravity (HMG) in flat space-time, first formulated by Heaviside in 1893, and later considered/re-discovered by many authors. Our new explanation of the GP-B results provides a new test of HMG apart from the existing ones, which deserves the attention of researchers in the field for its simplicity and new perspective.

I. INTRODUCTION

Gravity Probe B (GP-B), launched on 20 April 2004, was a NASA physics mission to experimentally investigate Einstein’s 1916 theory of general relativity (GR) - his relativistic theory of gravity in curved space-time. GP-B used four spherical gyroscopes and a telescope, housed in a satellite orbiting 642 km (400 mi) above the Earth in a polar orbit, to measure two of its significant predictions (with unprecedented accuracy) on the precession of the spin axis of a gyroscope moving in the field of the Earth predicted by Pugh [1] in 1959 and by Schiff [2, 3], in 1960, using GR. This is called Schiff effect/precession, which was measured in a satellite (a co-moving frame) in which gyroscopes are at rest. In the context of GR, the working formula for the GP-B experiment in the polar orbit (Schematic in figure 1) was worked out by Schiff [2, 3] as

\[
\frac{ds}{d\tau} = (\Omega_{gd}^{GR} + \Omega_{fd}^{GR}) \times s
\]

where \(s\) is the spin angular momentum vector of a gyroscope in its rest frame, \(\tau\) is the time measured in gyroscope’s rest frame (i.e. the satellite), called proper time, vectors \(\Omega_{gd}^{GR}\) representing the Geodetic Effect and \(\Omega_{fd}^{GR}\) Frame-dragging effect, as they are called in the GR parlance, are given by

\[
\Omega_{gd}^{GR} = \frac{3GM}{2c^2r^3}(r \times \omega_E)
\]

\[
\Omega_{fd}^{GR} = \frac{G}{c^2r^3} \left[ \frac{3r}{r^2}(r \cdot S_E) - S_E \right]
\]

\(G\) being the gravitational constant, \(c\) is the velocity of light in vacuum, \(S_E = I\omega_E\) is the spin angular momentum of the Earth (\(I\) and \(\omega_E\) are Earth’s moment of inertia and angular velocity respectively), \(r\) is the position vector of a gyroscope from Earth’s center and \(v\) is the orbital velocity of a gyroscope. The predicted quantities \(\Omega_{gd}^{GR}\) in eq. (2) and \(\Omega_{fd}^{GR}\) in eq. (3) could be computed since each quantity in the equation eqs. (2) - (3) was known: the instantaneous orbital velocity and radius from GP-Bs on-board GPS detector; the mass, moment of inertia, and angular velocity of the Earth from geophysical and astrophysical data. As we know, GP-B experiment has confirmed both significant predictions of GR: the geodetic effect (2) to a precision of 0.3% and frame-dragging (3) to 20% [5, 6]. However, in this communication we show how one can obtain the eqs. (1)-(3) within the framework of Heaviside-Maxwellian Gravity (HMG) in flat space-time briefly described below.

II. HEAVISIDE-MAXWELLIAN GRAVITY (HMG)

The fundamental equations of HMG are analogous to the Maxwell-Lorentz equations of electromagnetism and are called gravito-Maxwell-Lorentz equations (g-MLEs). Recently Behera [7], using Galileo Newtonian Relativity (GNR) and Behera and Barik [8], using the U(1) local phage (or gauge) in-variance of Dirac’s massive field theory (of charged as well as neutral particles), have found two equivalent mathematical representations of HMG: (1) Heaviside Gravity (HG) as originally written down by Heaviside in 1893 [9-11] and (2) Maxwellian Gravity (MG) [7, 8, 12, 13]. The g-MLEs of HG and MG, which represent the same physical theory HGM, are listed in Table 1. It is to be noted that, Heaviside speculated a
The Gravity Probe B Experiment
...testing Einstein's Universe

Frame-dragging Effect
39 milliaresccond/second/year
(0.000011 degrees/year)
Guide Star
IM Pegasi (HR 8763)

The Geodetic Effect
6606 milliaresccond/second/year
(0.00018 degrees/year)

FIG. 1. Schematic of the orbit of the Gravity Probe B satellite. The Geodetic and Frame-dragging effects. NorthSouth, East-West relativistic precessions with respect to the guide star IM Pegasi for an ideal gyroscope in polar orbit around the Earth. [Figure Courtesy: C. W. F. Everitt]

gravito-Lorentz force with a wrong sign in the velocity dependent term by electromagnetic analogy, i.e. of MG-type in Table 1. But the correct form is given in Table 1, as derived Behera [7] using Schwinger’s formalism within GNR and Behera and Barik [8] using the principle of local phase (or gauge) invariance of Dirac’s massive quantum field theory.

In Table 1, in analogy with Maxwell-Lorentz equations of classical electromagnetism in SI units, we have introduced two new universal constants for vacuum:

\[
\epsilon_0 = \frac{1}{4\pi G}, \quad \mu_0 = \frac{4\pi G}{c^2} \implies c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}}
\]

where the speed of gravitational waves in vacuum \( c_0 = c \), as theoretically shown in ref. [8, 12, 13] - an important prediction of HMG which agree well with the recent (almost) simultaneous detection of the Gravitational Waves and Gamma-Rays from a Binary Neutron Star Merger events: GW170817 and GRB 170817A [14].

Gravitational mass density \( \rho_g = \rho_0 \), the positive (rest) mass density (as theoretically proved in [8, 12, 13] without sacrificing the time-tested empirical law of universality of free fall of Galileo), \( J_g = \rho_0 \mathbf{v} \) stands for the mass current density (\( \mathbf{v} \) is velocity of \( \rho_0 \)) and by electromagnetic analogy, \( \mathbf{b} \) may be called the gravito-magnetic field, the Newtonian gravitational field \( \mathbf{g} \) may be called as gravito-electric field, \( \epsilon_0 g \) and \( \mu_0 b \) may be named respectively as the gravito-electric (or gravitic) permittivity and the gravito-magnetic permeability of vacuum. The g-MLEs of MG-type have also been derived from other approaches to gravito-electromagnetic (GEM) theory using (a) Galileo-Newtonian Relativity [13], (b) special relativity [12, 13, 15, 16], (c) principle of causality [17, 18] (d) common axiomatic methods applicable to electricity and gravity [19, 20] and (e) a specific approach to linearize GR in weak field and slow motion approximation [21]. These variant of approaches that lead to HMG in its either form, establish g-MLEs as self-consistent. It is to be noted that the explanations for the (a) perihelion advance of Mercury (b) gravitational bending of light and (c) the Shapiro time delay within the vector theory of gravity exist in the literature [22, 23]. Recently Hilborn [24] following an electromagnetic analogy, calculated the wave forms of gravitational radiation emitted by orbiting binary objects that are very similar to those observed by the Laser Interferometer Gravitational-Wave Observatory (LIGO-VIRGO) gravitational wave collaboration in 2015 up to the point at which the binary merger occurs. Hilborn’s calculation produces results that have the same dependence on the masses of the orbiting objects, the orbital frequency, and the mass separation as do the results from the linear version of general relativity (GR). But the polarization, angular distributions, and overall power results of Hilborn differ from those of GR. Recognizing this and out of scientific curiosity, we ventured to adopt the MG form of HMG in Table 1 to calculate the spin axis precession in the gravito-electromagnetic (GEM) field of the spinning Earth according to MG in flat space-time by electromagnetic analogy as under.

### III. SPIN GRAVITOMAGNETIC MOMENT AND ASSOCIATED GRAVITOMAGNETIC FIELD:

For the purpose of the present paper, the relevant electromagnetic analogy is the motion of a small spherical uniformly charged sphere (corresponding to a gyroscope in GP-B Experiment) in an orbit around another huge spherical spinning charged sphere (corresponding to the huge Earth in GP-B Experiment). In electromagnetism, a spinning charged sphere creates a di-polar magnetic field \( \mathbf{B} \) around it and this field is related to its spin magnetic moment which is proportional to the spin angular momentum of the spinning object. Kramers [27], Corben [28] and Bohm [29] have provided us classical derivations of spin magnetic moment of a spinning charged particle, which precisely matches with Dirac’s quantum mechanical finding of the spin magnetic moment of the electron:

\[
\mu_s = (q/m_0)\mathbf{s} = (g_s q/2m_0) \quad \text{(in SI units)}.
\]

| TABLE I: Physically Equivalent Sets of gravito-Maxwell-Lorentz Equations (g-MLEs) representing Heaviside-Maxwellian Gravity (HMG). |
|---------------------------------------------------------------|
| \( \nabla \cdot \mathbf{g} = -4\pi G \rho_g - \frac{\mathbf{v}}{c^2} \) | \( \nabla \cdot \mathbf{g} = -4\pi G \rho_g - \frac{\mathbf{v}}{c^2} \) |
| \( \nabla \times \mathbf{b} = \mu_0 \partial \mathbf{J}_g + \frac{4\pi G}{c} \mathbf{v} \) | \( \nabla \times \mathbf{b} = \mu_0 \partial \mathbf{J}_g + \frac{4\pi G}{c} \mathbf{v} \) |
| \( \nabla \times \mathbf{g} = -\frac{\partial \mathbf{b}}{\partial t} + \nabla \times \mathbf{J}_g \) | \( \nabla \times \mathbf{g} = -\frac{\partial \mathbf{b}}{\partial t} + \nabla \times \mathbf{J}_g \) |
| \( \mathbf{b} = \mathbf{g} \times \mathbf{A}_g \) | \( \mathbf{b} = \mathbf{g} \times \mathbf{A}_g \) |
| \( \mathbf{g} = -\nabla \phi_g - \frac{\mathbf{v}}{c^2} \) | \( \mathbf{g} = -\nabla \phi_g - \frac{\mathbf{v}}{c^2} \) |
where $q = -|e|$ is the charge and $s$ is the spin angular momentum of the electron, $g_s$ is the so-called $g$-factor associated with the spin magnetic moment of a charged particle - called the gyromagnetic ratio; $g_s = 2$ for an electron as found by Dirac quantum mechanically and by others [27-29] classically. In classical electromagnetism, equation (5) also holds good for a uniformly magnetized sphere [30]. The spin magnetic moment produces a dipolar gravitomagnetic field $B(r)$ at a field point $r \neq 0$ in vacuum [30]:

$$B(r) = \frac{\mu_0}{4\pi} \left[ \frac{3\hat{r} \cdot (\mu_s - \mu_s)}{r^3} \right] \quad \text{(in SI units)},$$

where $\mu_0$ is the magnetic permeability of vacuum and $\hat{r} = r/r$. Similarly, in MG, a spinning particle/object creates a di-polar gravitomagnetic field $b$ determined by its spin gravitomagnetic moment $\mu_{gs}$ proportional to its spin angular momentum $s$. Using the classical methods of Kramers [27], Corben [28] and Bohm [29] for obtaining spin magnetic moment, in the case of MG, we found

$$\mu_{gs} = s,$$

which is true for a uniformly gravitomagnetized sphere. It is to be noted that without doing detailed calculations as in refs. [27-29], one can quickly get the result in eq. (7) from eq. (5), by replacing the electric charge $q$ with $m_0$, which represents the gravitational charge in the frame-work of MG as proved in ref. [12-13]. Further, we note that for a spin 1/2 Dirac particle Behera and Naik [10] have obtained $\mu_{gs} = s = (h/2) \sigma$. The gravitational analogue of eq. (6), in the case of MG is

$$b(r) = \frac{\mu_{0g}}{4\pi} \left[ \frac{3\hat{r} \cdot (\mu_s - \mu_s)}{r^3} \right],$$

where $\mu_{0g} = 4\pi G/c^2$ is the gravitomagnetic permeability of vacuum. The “minus” sign on the right hand side of eq. (8) is due to the fact that the source term $(j_0)$ in the gravito-Amperé of MG, viz., $\nabla \times b = -\mu_{0g} j_0$ has a minus sign before it, in contrast with the positive sign before the corresponding term $(j_e)$ in Ampère’s law $(\nabla \times B = +\mu_0 j_e)$ in electromagnetism. For the spinning Earth, $\mu_{gs} = S_E$, so the gravitomagnetic field of the Earth in the case of MG follows from eq. (8) as

$$b(r) = \frac{G}{c^2} \left[ \frac{3\hat{r} \cdot (S_E - S_E)}{r^3} \right].$$

IV. SPIN PRECESSION IN MAGNETIC FIELD AND GRAVITOMAGNETIC FIELD (NON-RELATIVISTIC THEORY):

Let the spin magnetic moment of a particle with intrinsic spin $s$, say an electron, be denoted by $\mu_s = as$, where $a$ is proportionality constant (for electron $a = q/m_0 = -|e|/m_0$). In non-relativistic classical theory of electromagnetism, if such a particle is placed in a magnetic field, its spin angular momentum $s$ and hence spin axis changes with time as

$$\frac{ds}{dt} = \mu_s \times B = (-\alpha B) \times s = \Omega_f \times s \quad (10)$$

where $B$ is the magnetic field at the position of the particle and the spin axis rotates with angular velocity

$$\Omega_f = -\alpha B.$$

Similarly, in the non-relativistic (i.e., slow motion) formulation of gravitoelectromagnetic (GEM) theory [7, 13] within domain of Newtonian physics, the gravitomagnetic analogues of eqs. (10)-(11) are

$$\frac{ds}{dt} = \mu_{gs} \times b(0) = (-b) \times s = \Omega_{f_d}^{GEM} \times s \quad (12)$$

$$\Omega_{f_d}^{GEM} = -b = \frac{G}{c^2} \left[ \frac{3\hat{r} \cdot (S_E - S_E)}{r^3} \right] = \Omega_{f_d}^{GR} \quad (13)$$

which exactly matches with eq. (3) predicted by GR, if we consider $\mu_{gs} = s$ as the gravitomagnetic moment of a gyroscope in GP-B experiment and the gravitomagnetic field $b$ of the Earth as given by eq. (9).

V. SPIN PRECESSION IN ELECTRIC FIELD AND GRAVITO-ELECTRIC FIELD (NON-RELATIVISTIC THEORY):

Suppose an electron with charge $q = -e$, rest mass $m_0$, spin magnetic moment $\mu_{es} = (q/m_0)s = -(e/m_0)s$ moves with velocity $v$ in an external electric field $E$. Then in the instantaneous rest frame of the electron it not only experiences the electric field $E$ but also a magnetic field given by (in SI units)

$$B = -\frac{v \times E}{c^2} = \frac{E \times v}{c^2} \quad (14)$$

Then the equation of motion for its spin angular momentum $s$ in its rest frame is

$$\left( \frac{ds}{dt} \right)_{\text{rest}} = \mu_{es} \times B = -\left( \frac{qE \times v}{m_0c^2} \right) \times s = \Omega_{e}^{f} \times s \quad (15)$$

and the spin axis of the electron precesses with an angular velocity

$$\Omega_{e}^{f} = -\left( \frac{qE \times v}{m_0c^2} \right) = \left( \frac{eE \times v}{m_0c^2} \right) \quad (16)$$

In the case of an atomic electron moving around a nucleus of charge $Ze$ ($Z$ being the atomic number), the field $E = (Ze\mathbf{r})/(4\pi\epsilon_0 r^3)$ and in this case eq. (10) becomes

$$\Omega_{e}^{f} = \frac{Ze^2}{4\pi \epsilon_0 m_0 c^2 r^3} (r \times v) \quad (17)$$
In the context of Maxwellian Gravity an analogous situation arises in the case of a spinning body moving in a Newtonian gravitoelectric field, just as a gyroscope moving around the Earth as in GP-B experiment. The quickest way to find the gravitational analogue of eq. (16), is to replace $qE$ by $m_qg = -(GMm_0v)/r^3$, where $m_0$ now represents the mass of the gyroscope and $g$ is the Newtonian gravito-electric field of the Earth of mass $M$; the result is

$$\Omega_g' = -b = \frac{GM}{c^2r^3} (r \times v) \quad (18)$$

which is short of

$$\Omega_{g'}^{II} = \frac{1}{2} \frac{GM}{c^2r^3} (r \times v) \quad (19)$$

to get the correct GR value in eq. (2). This deficiency is corrected from special relativistic contribution to the spin axis precession of an electron in its rest frame as calculated by Thomas [31] in 1927, which we shall consider now.

VI. SPIN PRECESSION IN ELECTRIC FIELD
AND GRAVITO-ELECTRIC FIELD
(RELATIVISTIC THEORY OF THOMAS):

For an electron with charge $q = -e$, rest mass $m_0$, spin angular momentum $s$, spin magnetic moment $\mu_s$, Thomas [31] first calculated the relativistic change in the direction of its spin axis in electron’s rest frame, which for the simple case of $\mu_s = (q/m_0)s$, is given by his eq. 4.122 in [31] (written here in SI units and in different notations):

$$\frac{d\mathbf{s}}{d\tau} = \mathbf{\Omega} \times \mathbf{s}, \quad (20)$$

where

$$\mathbf{\Omega} = -\frac{q}{m_0} \left( \mathbf{B} + \frac{1}{c^2} \frac{\gamma}{1 + \gamma} (\mathbf{E} \times \mathbf{v}) \right). \quad (21)$$

$\tau$ is the proper time, $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor, the fields $\mathbf{E}$ and $\mathbf{B}$ respectively refers to the electric and magnetic fields at the instantaneous position of the electron and $\mathbf{\Omega}$ is angular velocity of spin precession. For small velocity $|v| << c$, $\gamma \rightarrow 1$ and the equation (21) reduces to

$$\mathbf{\Omega} = -\frac{q}{m_0} \left( \mathbf{B} + \frac{1}{2c^2} (\mathbf{E} \times \mathbf{v}) \right). \quad (22)$$

If we put eq. (22) in eq. (20) we get Kramers equation (11) of 1934, in which Kramers [27] followed a simpler approach. It should be clearly noted that the 1/2 factor term in eq. (22) arises due to the relativistic $\gamma$ factor term in eq. (21), which was absent in our non-relativistic approach in the previous section. Now substituting the value of $\mathbf{B}$ from eq. (11) in eq. (22), we get

$$\Omega_{Elec} = -\frac{3}{2} \frac{m_0e^2}{m_0c^2} = \frac{3}{2} \frac{Ze^2}{4\pi\varepsilon_0c^2r^3} (r \times v), \quad (23)$$

for an atomic electron, which is the electrical analogue of geodetic precession in eq. (2) in the atomic domain.

As was done in the previous section for the gravitational situation of GP-B experiment, we now replaced $qE$ in eq. (23) by $m_qg = -(GMm_0r)/r^3$ to get the correct formula for geodetic precession according to HMG in flat spacetime as

$$\Omega_{gd}^{GEM} = -\frac{3}{2} \frac{m_0g}{m_0c^2} = \frac{3GM}{2c^2r^3} (r \times v) = \Omega_{gd}^{GR} \quad (24)$$

which is in perfect agreement with the prediction of GR in eq. (2). One can also use equation (46) of Bailey [32] to arrive at eq. (24) of the GEM frame work of HMG in flat space-time. With our results in eqs. (13) and (24), we have thus shown that the GP-B experimental results may well be explained using Heaviside-Maxwellian Gravity (HMG) in flat space-time without the requirement of Einstein’s General Relativity.

CONCLUSION

The GP-B experimental data are in close agreement with the Schiff’s formula for spin axis precession of a gyroscope in the gravitational field of the spinning Earth, which Schiff derived using Einstein’s relativistic theory of gravity in curved space-time. However, for the first time, here we report a Faraday-Maxwellian field theoretical explanation of the GP-B experimental results using Heaviside-Maxwellian (vector) Gravity (HMG) in flat space-time, first formulated by Heaviside in 1893, and later considered/re-discovered by many authors following a variant of approaches to gravito-electromagnetism.

Our new explanation of the GP-B experimental results involves an undergraduate level derivation the electrical and gravitational analogues of Schiff formula in flat space-time, which provides a new test of HMG apart from the existing ones noted in section 2. Well known physicist, C. M. Will concluded his viewpoint on GP-B results [Physics 4, 43 (2011)] by stating, “The precession of a gyroscope in the gravitational field of a rotating body had never been measured before GP-B. While the results support Einstein, this didn’t have to be the case. Physicists will never cease testing their basic theories, out of curiosity that new physics could exist beyond the “accepted” picture.”

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