Consistent Histories and Quantum Delayed Choice

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Abstract

John Wheeler devised a gedanken experiment in which a piece of apparatus can be altered just before the arrival of particle, and this “delayed choice” can, seemingly, alter the quantum state of the particle at a much earlier time, long before the choice is made. A slightly different gedanken experiment, which exhibits the same conceptual difficulty, is analyzed using the techniques of consistent history quantum theory. The idea that the future influences the past disappears when proper account is taken of the diversity of possible quantum descriptions of the world, and their mutual compatibility or incompatibility.

John Wheeler proposed a delayed-choice experiment in which the second beam splitter of a Mach-Zehnder interferometer is or is not removed at the very last instant before a photon reaches it and passes out of the interferometer [1]. A modified version which exhibits essentially the same paradox is shown in Fig. 1. A photon enters the interferometer through channel $a$, and two detectors in the $c$ and $d$ arms are either (a) left in position, or (b) removed just before the photon reaches (one of) them, so that it is free to proceed on to the second beam splitter. To be sure, physically moving detectors out of the way at the last instant is a theorist’s fantasy, but clever experimentalists can do essentially the same thing using Pockels cells and polarizers [2, 3].

The paradox arises in the following way. Suppose detectors $C$ and $D$ are present, Fig. 1(a), and one of them, say $C$, detects the photon. Then it seems plausible that at an earlier time the photon was in the $c$ arm of the interferometer. There are various ways in which this idea could be supported experimentally. For example, by seeing what happens when one removes mirror $M_c$, or mirror $M_d$, or inserts an absorber in the $c$ or $d$ arm of the interferometer upstream from the detectors, or lengthens the path in one arm while timing when the photon reaches the detector. These considerations make it plausible that if one of the detectors detects the photon, the latter was, at an earlier time, in the corresponding arm of the interferometer.

On the other hand, if $C$ and $D$ are out of the way, Fig. 1(b), removed at the very last instant, the second beamsplitter and detectors $E$ and $F$ which follow it can be used to test
whether the photon was in a coherent superposition state between the two arms while inside the interferometer. To be specific, suppose the action of the first beam splitter is described by the unitary transformation

\[ |a⟩ \mapsto |s⟩ = \left( |c⟩ + |d⟩ \right)/\sqrt{2}, \]  

(1)

where \( |a⟩ \) is a wave packet in the entrance channel of the interferometer, and \( |c⟩ \) and \( |d⟩ \) are wave packets in the \( c \) and \( d \) arms, respectively. Similarly, assume a unitary transformation

\[ |c⟩ \mapsto |u⟩ = \left( |e⟩ + |f⟩ \right)/\sqrt{2}, \quad |d⟩ \mapsto |v⟩ = \left( -|e⟩ + |f⟩ \right)/\sqrt{2} \]  

(2)

at the second beamsplitter. As a consequence,

\[ |a⟩ \mapsto |s⟩ \mapsto |f⟩, \]  

(3)

so that a photon which enters the interferometer in channel \( a \) will emerge in channel \( f \), to be detected by detector \( F \) and not by detector \( E \). If the experiment is repeated many times, and if every time \( C \) and \( D \) are pushed out of the way the photon is later detected by \( F \) rather than \( E \), then it is plausible that inside the interferometer the photon was in the coherent superposition state \( |s⟩ \) with a definite phase relationship between the wave packets in the two arms, rather than entirely in the \( c \) arm or in the \( d \) arm. Indeed, it is hard to see how else one could explain the outcome.

But now we have a paradox. For it the detectors \( C \) and \( D \) are left in place, the photon was (plausibly) in either the \( c \) or the \( d \) arm, that is, in the arm in which it will later be detected, ever since it left the first beamsplitter. On the other hand, if \( C \) and \( D \) are removed at the very last moment, then, in order that it be later detected by \( F \) and not by \( E \), the photon needs to have been in the superposition \( |s⟩ \), and this must have been the case ever since it left the first beam splitter. Consequently, the photon when it passed the first beamsplitter had to “know” whether the detectors \( C \) and \( D \) would or would not be present later on.

Wheeler used this paradox to argue, in effect, that the process by which the photon moves through the interferometer cannot be described or thought about in a coherent way. It is, to use his words, a “great smoky dragon”: one cannot say anything at all about what is going on before the dragon “bites” one of the detectors, thus bringing the experiment to a close by an irreversible act of measurement. Or, if I may use a different metaphor, a quantum system is like a “black box”, and one should not try and figure out what is going on inside the box.

The whole field of quantum foundations is littered with these black boxes, and physicists foolish enough to open them have often been stung by the paradoxes (or burned by the dragons) which then emerge. Although there are important differences, Wheeler’s paradox, in which the future seems to influence the past, bears a certain resemblance to the Elitzur-Vaidman paradox of “interaction free measurement” [1], a delayed-choice version of which is obtained by discarding one of the counters (say \( D \)) in Fig. 1(a), and Hardy’s paradox [2], in which there seems to be a mysterious long-range influence in violation of relativity theory.

The consistent history formalism [4-8] is ideally suited to addressing problems of this sort, because it allows one to discuss what is going on in a closed quantum mechanical system in a realistic way, without running into logical inconsistencies. One can, so-to-speak, open the black box and disarm the paradox without being stung (or burned). Omnès’ book
[10] discusses a modified form of Wheeler’s paradox from the consistent history perspective. While it is technically sound (once some misprints have been corrected [11]), this treatment is not entirely satisfactory, because it relies on a distinction between “true” and “reliable” which both Omnès [12] and his critics [13, 14] agree is flawed. Consequently, it is worthwhile exploring Wheeler’s paradox once again in the light of a recent restatement of the principles of consistent history reasoning found in [15], and augmented in [16], which (I believe) does not suffer from the problem just referred to. (Incidentally, [16] is also a response to certain criticisms of the consistent history approach found in [13].)

As usual, when employing consistent histories the entire system, including detectors, must be treated quantum mechanically. Thus for the situation in Fig. 1, let \(|C\rangle\) indicate the “ready” state of detector \(C\) before a photon has arrived, \(|C^*\rangle\) the corresponding state in which it has detected a photon, and let the unitary transformation representing detection be given by

\[
|c\rangle|C\rangle \mapsto |C^*\rangle. \tag{4}
\]

A similar notation is employed for the other detectors. One should think of \(|C\rangle\) and \(|C^*\rangle\) as macroscopically distinct states; e.g., in the quaint but picturesque language of quantum foundations, there is a very visible pointer attached to the detector, which (say) points down for \(|C\rangle\) and up for \(|C^*\rangle\).

A consistent history analysis of the situation in Fig. 1(a) proceeds by introducing a consistent family of possible histories. There are many different ways to choose a family; here is one of them:

\[
|a\rangle|CD\rangle \rightarrow \begin{cases} |c\rangle|CD\rangle \rightarrow |C^*D\rangle, \\ |d\rangle|CD\rangle \rightarrow |CD^*\rangle, \end{cases} \tag{5}
\]

where the notation is to be interpreted in the following way. Time progresses from left to right, starting at \(t_0\) when the photon is in a wave packet \(|a\rangle\), that is, about to enter the interferometer, and detectors \(C\) and \(D\) are ready. At the next time, \(t_1\), the photon is either in a wave packet \(|c\rangle\) in the \(c\) arm, or else a wave packet \(|d\rangle\) in the \(d\) arm. If it is in \(|c\rangle\) at \(t_1\), then at a later time \(t_2\) it will have triggered the \(C\) detector, changing it from \(|C\rangle\) to \(|C^*\rangle\), while detector \(D\) remains in its untriggered (“ready”) state. Similarly, in the second history in (5), the photon is in \(|d\rangle\) at \(t_1\), and has triggered \(D\), and not \(C\), at time \(t_2\). The representation (5) is somewhat abbreviated in that the beam splitters and the detectors \(E\) and \(F\), which play only a passive role, are not shown explicitly. If we put all of these into a single initial state \(|\Psi\rangle\), then, in the notation of [15], family (5) contains the two histories,

\[
|c\rangle|CD\rangle|C^*\rangle|D\rangle, \quad |d\rangle|CD\rangle|C\rangle|D^*\rangle. \tag{6}
\]

But we will not use (6) in the subsequent discussion, so the reader unfamiliar with this notation can ignore it and simply refer to (5).

It is important to notice that the two histories in (5), even though they start with the same initial state, are mutually exclusive alternatives (a point which is, perhaps, a bit clearer in the notation of [15]): if the photon is in the \(c\) arm at time \(t_1\), it is definitely not in the \(d\) arm, and it will later trigger detector \(C\), and not trigger detector \(D\). By applying an appropriate probability calculus, as explained in [15], to (5), one can conclude that in the

\footnote{More realistic models of measurement can be employed in the consistent history formalism; for an example, see Sec. VI C of [15].}
case in which the photon was detected by detector $C$, it definitely (that is, with conditional probability equal to one) was in the $c$ arm at an earlier time, just as a naive physicist would tend to believe (at least when not being chased by a smoky dragon!). Thus in this particular instance, consistent histories supports the usual intuition employed for designing experimental apparatus. We have opened the black box, and nothing has gone wrong—at least, not yet.

But how are we going to handle the situation in Fig. 1(b)? Again, it is necessary to choose a consistent family, and one which will work very well for this purpose is:

$$|a⟩|EF⟩ → |s⟩|EF⟩ → \begin{cases} |e⟩|EF⟩ → |E^*F⟩, \\ |f⟩|EF⟩ → |EF^*⟩, \end{cases}$$

(7)

where there are now four different times: the initial time $t_0$, $t_1$ when the photon is in the superposition state $|s⟩$ (defined in (1)) inside the interferometer, $t_2$ when it has passed through the second beam splitter to emerge in either the $e$ or $f$ channel, and $t_3$ when it has been detected by either $E$ or $F$. In fact, with the unitary transformations indicated in (1) and (2), the photon will surely emerge in $|f⟩$, see (3). As a consequence, the upper history in (7) has probability 0, so it will never occur, while the lower history, ending with $F^*$, will occur with probability 1. (Actually, we omitted from (5) a certain number of histories whose presence is needed for a consistent formalism according to the rules of [15], but which occur with probability zero, because they are dynamically impossible, so we could also have omitted the upper history in (7).) By considering the family (7) we can conclude from the fact that the photon was detected by $F$, or simply on the basis of the initial state, that the photon was certainly (probability 1) in the coherent superposition $|s⟩$ while inside the interferometer. So once again we have opened up the black box, and found a physically reasonable result.

But something looks suspicious. For the case in which detectors $C$ and $D$ remain in place, we used (5), in which the photon is definitely in the $c$ or $d$ arm after it passes through the first beam splitter, whereas for the situation in which both $C$ and $D$ are removed at the last moment, we used (7), in which the photon is in a coherent superposition all the time it is inside the interferometer. Isn’t this precisely Wheeler’s paradox? No, because the choice of family is one made by the physicist in constructing a (possible) description of the quantum system; it is not something forced upon him either by a law of nature or by later events in the system under consideration. To see that this is so, let us examine what happens if, in place of (7), we use a consistent family of histories in which the photon is in a definite arm, $c$ or $d$, while it is inside the interferometer:

$$|a⟩|EF⟩ → \begin{cases} |c⟩|EF⟩ → |u⟩|EF⟩ → |U⟩, \\ |d⟩|EF⟩ → |v⟩|EF⟩ → |V⟩. \end{cases}$$

(8)

Here the four successive times are the same as in (7), $|u⟩$ and $|v⟩$ were defined in (2), and

$$|U⟩ = (|E^*F⟩ + |EF^*⟩)/\sqrt{2}, \quad |V⟩ = (-|E^*F⟩ + |EF^*⟩)/\sqrt{2}$$

(9)

are macroscopic quantum superposition (MQS) or Schrödinger cat states, since they are coherent superpositions of macroscopically distinct situations.

Both families (7) and (8) apply to the same dynamical situation (detectors $C$ and $D$ out of the way, Fig. 1(b)), and from the point of view of consistent histories, either is equally
valid as a description of the quantum world. To be sure, (8) involves MQS states at the final time \( t_3 \), and we shall say more about this later. However, it is just as significant that at time \( t_1 \), that is, while the photon is inside the interferometer, (7) assigns to it a state \( |s\rangle \), and (8) one of the two states \( |c\rangle \) or \( |d\rangle \). Surely these cannot both be right. Is it (7) or is it (8) which tells us what the photon is really doing inside the interferometer?

The question just posed has no answer, and it is important to understand why that is so, as it goes to the very heart of what distinguishes quantum theory from classical physics. Consider the simplest of all quantum systems, a spin half particle with a two-dimensional Hilbert space, where each ray (i.e., one-dimensional subspace) corresponds to a spin angular momentum of \( +1/2 \) (in units of \( \hbar \)) in a particular direction. In standard quantum mechanics (no hidden variables) the statement “\( S_x = 1/2 \)” makes sense, for it corresponds to something definite in the Hilbert space, a particular ray (i.e., a particular ket, up to multiplication by an arbitrary complex number), and so, of course, does “\( S_z = 1/2 \)”.

On the other hand, “\( S_x = 1/2 \) AND \( S_z = 1/2 \)” does not make sense, for there is no ray which corresponds to it. But if connecting two propositions with AND makes no sense, connecting them with OR is no better—at least this is the consistent histories perspective—and thus the question “is the spin in the +z direction or is it in the +x direction?” is meaningless (in the sense that quantum theory ascribes it no meaning). And, by analogy, the question “Was the photon in \( |s\rangle \) or in \( |c\rangle \) at \( t_1 \)” makes no sense. In the technical terminology of consistent histories, the families (7) and (8) are incompatible: descriptions based upon one cannot be combined with those based upon the other, nor does it make sense to ask “which is right?”.

For further details, see [16].

To be sure, Wheeler’s “great smoky dragon” is also a way of saying that certain things cannot be discussed in quantum theory, certain things do not make sense. The difference is that in the consistent history approach the rules as to what makes sense and what does not are precise, and based upon the mathematical structure of quantum theory itself. They do not take the form of a blanket prohibition: “Don’t talk about microscopic systems, because that is dangerous.” Instead, they are of the form: “This makes sense (if you use a Hilbert space) but that doesn’t make sense,” or: “This family satisfies the consistency conditions, so probabilities can be assigned, whereas that other family does not satisfy the consistency conditions.” The basic point is that if one employs, as in standard quantum mechanics, a Hilbert space and unitary time transformations to produce descriptions of a quantum system, the logical rules for interpreting these descriptions should be compatible with the underlying mathematics. Failure to pay attention to this requirement is the source of many paradoxes, superluminal influences, and other quantum ghosts.

We have seen that the absence of detectors \( C \) and \( D \) does not force one to adopt a coherent superposition state \( |s\rangle \) for the photon while it is inside the interferometer; (8) can be used rather than (7). Similarly, when \( C \) and \( D \) are present, one does not have to employ a family, such as (4), in which the photon is in one arm or the other. Here is a perfectly good alternative:

\[
|a\rangle|CD\rangle \rightarrow |s\rangle|CD\rangle \rightarrow |S\rangle,
\]

\[\text{Since every ray corresponds to the spin being in a particular direction, there are none left over to represent two different directions joined by AND. For a fuller treatment of this point, see [16], Sec. IV A.}\]
where
\[ |S\rangle = (|C^*D\rangle + |CD^*\rangle)/\sqrt{2} \] (11)
is another MQS state; as before, various histories with zero probability belonging to this family are not shown explicitly. Once again, there is no “law of nature” which singles out (11) in contrast to (10) as the “correct” consistent family; from the perspective of fundamental quantum theory, both are equally correct, both provide perfectly good descriptions of the quantum world. However, they are incompatible (remember \(S_x\) and \(S_z\)!), so that only one, not both, can be used to describe a given situation.

But if the consistent histories approach allows many different consistent families, and gives no procedure for specifying which is the right one, how can it be a rational scientific theory of the world? Examining the different consistent families introduced above, one sees that certain families can address certain questions, and other families can address other questions. Suppose, for example, one wants to know whether it is detector \(E\) or detector \(F\) which detects the photon in Fig. 1(b). While (5) and (8) are equally good consistent families from the perspective of fundamental quantum theory, the question of “which detector” can be addressed using (5), but not (8). The reason is that the MQS states \(|U\rangle\) and \(|V\rangle\) in (8) are incompatible (in the quantum sense) with a discussion of whether detector \(E\) has or has not detected a photon; think of \(|U\rangle\) as something like \(S_x = 1/2\) and \(|EF^*\rangle\) as something like \(S_z = 1/2\). Similarly, if we are interested in whether \(C\) or \(D\) detected the particle in Fig. 1(a), we can use (3), but we cannot use (10). However, (3) is not the only possibility for discussing “\(C\) or \(D\)?”; the consistent family
\[ |a\rangle|CD\rangle \rightarrow |s\rangle|CD\rangle \rightarrow \begin{cases} \left(\begin{array}{c} |C^*D\rangle \\ |CD^*\rangle \end{array}\right), 
\end{cases} \] (12)
will work equally well. An important difference between (3) and (12), however, is that the former allows us to address the question “in which arm of the detector was the photon before it was detected?”, whereas the latter does not: \(|c\rangle\) and \(|d\rangle\) are incompatible with \(|s\rangle\) at the time \(t_1\).

Consequently, one sees that the multiplicity of quantum descriptions, that is, consistent families, allowed by the consistent history approach is sharply narrowed as soon as we focus on particular questions of physical interest. Some families can be used to address certain questions, other families are needed to address other questions. In addition, when more than one consistent family which can be used to address a particular physical question—e.g., (3) and (12) can both be used to answer “\(C\) or \(D\)?”—the answers (in terms of probabilities) given by the different families are the same; see Sec. IV of [15].

The various consistent families introduced above show that opening the black box of quantum theory does not lead to the conclusion that the future influences the past, for we can adopt either a coherent superposition description, \(|s\rangle\), or the one-arm-or-the-other description using \(|c\rangle\) and \(|d\rangle\) for the photon inside the interferometer, whether or not the detectors \(C\) and \(D\) will later be present. However, the presence or absence of the \(C\) and \(D\) detectors does have an important dynamical effect: compare (3) with (10). In particular, MQS states, \(|U\rangle\) and \(|V\rangle\), appear in (8) but not in (3). Physicists tend to find MQS states

\footnote{In order to keep the notation from becoming unwieldy, we have omitted detectors \(E\) and \(F\) from (3), and \(C\) and \(D\) from (8); putting them in causes no problem, and makes the expressions look more similar.}
embarrassing, and one can sympathize with a certain reluctance to use (8), and a corresponding preference for (7). However, this is not a matter of “future influencing past”, at least as a physical effect. It is more like what happens when an author is writing a novel and adjusts certain events near the beginning so that the last chapter turns out the way he wants. Note that the consistent history approach does not “rule out” MQS states; indeed, they have to be present in the Hilbert space of standard quantum theory precisely because it is a linear vector space. However, they do not occur in every consistent family, and for this reason the consistent history approach is not troubled by the infamous “measurement problem” which has given rise to a great deal of work, and an enormous amount of confusion, in the field of quantum foundations; see the valuable critique by John Bell in [17].

The analysis given above is incomplete in one respect: we have treated the situations in Fig. 1(a) and (b) separately, and have not tried to put the whole story together by, for example, introducing a quantum coin and an associated servo mechanism which pushes $C$ and $D$ out of the way at the last instant if the coin turns up heads, or leaves it in place if the result is tails. (While this may sound like another theorist’s fantasy, the experimentalists already know how to do what is essentially the same thing [2].) Putting the whole story together does not yield any new insights into the question of whether the future influences the past, which I believe can be answered (in the negative) on the basis of the considerations given above. However, it does open up additional possibilities for consistent families, including the (analog of the) one discussed by Omnès [18], and allows one to pose counterfactual questions such as the following: Suppose that $C$ and $D$ were left in place. What would have happened if the quantum coin had turned up the other way, and $C$ and $D$ had been removed before the photon arrived? Discussing this interesting question goes beyond the scope of the present paper.

In conclusion, the consistent histories approach provides an analysis of the situation in Fig. 1 in terms of what the photon was doing inside the interferometer, without running into a paradox in which the future influences the past. The key idea is to use consistent histories in order to have a logical structure for quantum theory which is consistent with the mathematics, and which can be used to discuss the time development of a quantum system without running into paradoxes.

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Figure 1: Mach-Zehnder interferometer with (a) detectors in arms $c$ and $d$; (b) detectors removed at the last moment.