Soil stiffness identification using fuzzy logic based on seismic tomography and its relationship with dynamic elastic moduli

L Yufajjiru¹, M Maryadi¹,²*

¹Undergraduate Program of Geophysics, Faculty of Mathematics and Natural Sciences, Universitas Indonesia, Depok 16424, Indonesia
³Department of Earth Resources Engineering, Graduate School of Engineering, Kyushu University, Fukuoka, 819-0395, Japan.
*E-mail: maryadi@sci.ui.ac.id

Abstract. Either P-wave or S-wave tomography has been used to interpret soil stiffness zone to extend subsurface conditions’ visibility. The dynamic elasticity moduli have been calculated to connect both seismic wave velocities to geotechnical rationale. However, the interpretation has to be more integrated and less subjective. Fuzzy logic has been applied for both velocities fusion to generate geotechnical cross-sections to diminish the subjectivity. Nevertheless, this is a new application, and it is not known whether this result has a relation to the dynamic elasticity moduli or not. Therefore, this research uses Fuzzy Logic to identify the soil stiffness zone and its relationship to the dynamic elasticity moduli. Meta-hypotheses are produced by possibility function in the early phase of the fusion. The possibility function is a fuzzy set that describes each seismic wave velocities to a geotechnical rationale in a possibility scale. Meta-hypotheses fused into hypotheses: the possibility of solid-state behavior and plastic-state behavior. It is found that solid-state behavior is analog to Bulk modulus, and the plastic state is analog to the elastic modulus. Both hypotheses’ extreme values are highlighting the target based on the geotechnical rationale from the possibility function.

1. Introduction
Soil investigation has been being used in the early phase of civil works. It needs plenty of either a standard penetration test (SPT) or the other penetration test points in an extensive area. These kinds of jobs are costly. We need to map at a larger scale to makes penetration tests are more targeted.

Seismic refraction is the most common geophysical tool to maps the P-wave velocity in the geotechnical assessment. However, this methodology is not directly indicated the soil stiffness. The S-wave velocity is meant to be the best indicator seismically [5]. The shear modulus is directly linked to the material stiffness. Multichannel Analysis Surface Wave (MASW) has been used to maps the S-wave velocity by using the Rayleigh wave dispersion.

The dynamic elasticity moduli have been calculated to interpret both seismic wave velocities for the geotechnical rationale. These moduli required a density model to produce. However, the interpretation has to be more integrated and less subjective. Fuzzy logic has been applied to diminish the subjectivity by fusing both seismic wave velocities. The first application has shown that it generates a geotechnical cross-section without creating any initial model [2]. Nevertheless, it is not known whether this result has a relation to the dynamic elasticity moduli or not. Therefore, this research uses Fuzzy Logic to identify soil stiffness zone from geotechnical cross-section and find its
relationship to the dynamic elasticity moduli. A complete workflow of this study is described in figure 1.

Figure 1. Workflow diagram.

2. Seismic Refraction
Seismic refraction is an active seismic method that used critically refracted waves for analysis. Head wave is a wave that returns to the surface when a seismic wave is being critically refracted. The critical angle is an incoming angle that made the refraction angle reach 90° when the wave hits a new medium with speed more significant than the previous medium. The medium which has a lower velocity or slightly different than a previous medium is not detected or called a blind zone [6]. The P-wave velocity tomography is generated by the general reciprocal method (GRM). The GRM method has excellent lateral difference detection, which good for geotechnical investigation [6].

3. Multichannel Analysis Surface Wave
Multichannel analysis surface wave is a methodology that generates S-wave velocity from surface waves, particularly the Rayleigh wave. The Rayleigh wave has the most considerable energy recorded in seismic traces. It could 67% for Rayleigh wave, 23% for S-wave, and 7% for P-wave [5]. Thus Rayleigh wave could serve as an excellent signal to noise ratio. Another surface wave velocity could invert to S-wave velocity. The Love wave velocity has equal to the S-wave velocity, and the Rayleigh wave velocity is about 91% of the S-wave velocity. However, the Love wave velocity characteristic made it does not always have been recorded in vertical geophones, and the Rayleigh wave has been
consistently appearing in any seismic recordings as a ground roll. Thus still, the Rayleigh wave is favorable to obtains S-wave velocity in this research.

The Rayleigh wave dispersion analysis has produced the S-wave velocity. The Rayleigh wave is dispersed in a non-homogeneous half-space. Three parameters defined the dispersion property, such as S-wave velocity, Poisson's ratio, and density. The S-wave velocity is the most affecting the dispersion up to 98% dependency [5]. The theoretical dispersion curve models the S-wave velocity over a thousand iteration that matches the extracted dispersion curve.

4. The Dynamic Elastic Moduli

The dynamic elastic moduli consist of an elastic modulus, Bulk modulus, Shear modulus, and Poisson's ratio derived from wave velocity.

\[
\sigma = \frac{v_p^2 - 2v_s^2}{2(v_p^2 - v_s^2)} \\
G_{\text{max}} = \rho \times V_s^2 \\
E = G \left( \frac{3v_p^2 - 4v_s^2}{v_p^2} \right) \\
K = \rho \left( V_p^2 - \frac{4}{3} V_s^2 \right)
\]

\(\sigma\) = The Poisson’s ratio,
\(G_{\text{max}}\) = Maximum shear modulus,
\(E\) = Elastic modulus,
\(K\) = Bulk modulus,
\(v_p\) = P-wave velocity, and
\(v_p\) = S-wave velocity.

5. Fuzzy Set

The set is a collection that defining an object. Thus every object could be differentiated from each other from the set. A classical set has a crisp boundary. This classical set could not recognize an object if we have a lack of information. It does not seem enough to satisfy the object definition. This lack of information condition made a "fuzzy" condition.

A fuzzy set defines how "fuzzy" the object is. An example is when we talk about how "tall" is someone, so we need a reference for the tallness. This tallness does not have an absolute value. Thus it is defined by a degree of tallness from the reference. A fuzzy set defines an object through the fuzzy membership. There are many fuzzy membership functions. In this research, the trapezoidal function is used, as shown in Equation 5.

\[
\mu_x(x) = \begin{cases} 
\frac{x-a}{m-a}, & a < c \leq m \\
1, & m < x \leq n \\
\frac{b-x}{b-m}, & n < x \leq b 
\end{cases}
\]

\(x\) = The condition's universe,
\(\mu_x\) = Fuzzy set in x universe,
\(a\) = The first boundary of the lowest value of the degree,
\(m\) = The first boundary of the highest value of the degree,
\(n\) = The end boundary of the highest value of the degree,
\(b\) = And the end boundary of the lowest value of the degree.
6. Fuzzy Operator

6.1. And/Intersection

\[(A \cap B)(x) = \min (A(x), B(x))\]  \hspace{1cm} (6)

The standard intersection has shown in Equation 6. The \((A \cap B)(x)\) is the notation of the intersection of set A and set B in the x universe. The illustration of an intersection is shown in figure 2a. There are the non-standard intersections, as described in Equation 7 – Equation 9.

- **Algebraic Product**: \(I(a, b) = a \cdot b\)  \hspace{1cm} (7)
- **Bounded Difference**: \(I(a, b) = \max (0, a + b - 1)\)  \hspace{1cm} (8)
- **Drastic Intersection**: \(I_{min}(a, b) = \begin{cases} a & \text{when } b = 1 \\ b & \text{when } a = 1 \\ 0 & \text{others} \end{cases}\)  \hspace{1cm} (9)

6.2. Complement

Complement is an inverse of an element in a set. The complement operation is shown in Equation 10. The illustration of complement is shown in figure 2b.

\[\bar{A}(x) = (1 - A(x))\]  \hspace{1cm} (10)

![Figure 2](image-url)  \hspace{1cm} (a) Intersection of fuzzy sets and (b) operation of complement.

7. Fusion Operator

\[\pi(x) = \pi_1(x) \otimes \pi_2(x) = \frac{\pi_1(x) \land \pi_2(x)}{\sup(\pi_1(x) \land \pi_2(x))}\]  \hspace{1cm} (11)

A fusion operator is an operator that fusing several pieces of information into new information.

- \(\pi(x)\) = A result set,
- \(\pi_1(x)\) = First set,
- \(\pi_2(x)\) = Second set,
- \(\otimes\) = The fusion operator,
- \(\land\) = The intersection operator,
\( \text{sup} \quad = \text{And supremum.} \)

Supremum is the maximum of the list of intersection sets, which is used for normalizing the results. The maximum value in the set turns into ”1” and references the other value.

8. Meta-hypotheses and Hypotheses

Meta-hypotheses define each velocity tomography to the geotechnical definition. We define that when the velocity below 500 m/s, the soil has been cracked. When it exceeds 1500 m/s, the soil has not been cracked. We referenced these definitions as a possibility of cracked soil, as shown in figure 3a. The geotechnical definition for S-wave velocity is when it goes below 500 m/s, the soil has been sheared, and when it surpasses 1000 m/s, the soil has not been sheared and stated as a possibility of sheared soil as shown in figure 3b. These meta-hypotheses posed as a first step calculation from physical value-form into fuzzy logic operation. It is behaved as a translator from seismic wave velocity into physical reason in geotechnical properties.

The hypotheses are the form of physical inference of our fusion through the formula. The formulas of those hypotheses are a primary key in this fusion process. It drives away the inputs into a new form of tomography corresponding to the formula’s physical rationale. The hypotheses serve a new form of tomography from the fusion of several geophysical data. In this case, we are using P-wave velocity and S-wave velocity tomography. It combines those meta-hypotheses using the following general equation [3]:

\[
\pi = \pi_1 \otimes \pi_2 \quad (12)
\]

\( \pi \quad = \text{hypothesis} \)
\( \pi_1 \quad = \text{First meta-hypothesis} \)
\( \pi_2 \quad = \text{Second complement meta-hypothesis} \)
\( \otimes \quad = \text{Fusion operator} \)

![Figure 3](image)

**Figure 3.** Fuzzy set for possibility of (a) cracked soil and (b) sheared soil meta-hypothesis.

8.1. Solid State Behavior Hypothesis

Solid-state behavior is a hypothesis in this research that delivers a geotechnical characteristic that the rocks are vulnerable to the compressional force. When the soil is likely cracked (\( \pi_1 \)) or not likely sheared (\( \pi_2 \)), then the soil is likely vulnerable to compressional force. That condition’s possibility is inherited from the minimum value between the possibility of being cracked or not being sheared. When the soil is not cracked, or soil is sheared, the solid-state behavior possibility is zero. In other words, when neither the soil is not cracked nor sheared, the soil is in solid-state behavior.
8.2. Plastic State Behavior Hypothesis

Plastic-state behavior is a hypothesis in this research that delivers a geotechnical characteristic that the rocks are vulnerable to the shear force. When the soil is likely sheared ($\tau_1$) or not likely cracked ($\tau_2$), then the soil is likely vulnerable to shear force. That condition's possibility is inherited from the minimum value between the possibility of being sheared or not being cracked. When the soil is not sheared, or the soil is cracked, the possibility of plastic-state behavior is zero. In other words, when neither the soil is not sheared nor cracked, the soil is in plastic-state behavior.

9. Geological Overview and Methodology

The geology of the research area is a sedimentary environment. The research area's surface is covered by alluvium fans, which has poorly sorted grains presumed that produced by landslides, as seen in figure 4a. It is shown that at the east of the area is covered by alluvium and has an active meandering river from southwest to northeast. The alluvium is considered to be a past deposit from the active river. It is shown that the seismic line lies along 67.5 m in the west-east direction and crossing the two cone penetration test points in figure 4b.

The seismic recording was done using the seismic refraction method and the MASW method. The 24 geophones had planted along 67.5m, 2.5m between each geophone, 10m offset, and 14 shots from the west side with a 5m increment for the seismic refraction method. The distance between each geophone and the offset is the same. However, it only uses 16 actives geophones and moves by one geophone from the first configuration sequentially.

The P-wave velocity tomography was obtained from Easyrefract software, which uses the general reciprocal method for time travel interpretation. The S-wave velocity tomography was obtained from EasyMASW, which uses the phase-shift method for the dispersion curve transformation. The dynamic elastic modeling and fuzzy logic fusion was performed using Python 3.7, employing Numpy and Scikit-Fuzzy modules.

10. Results and Discussion

The P-wave velocity has shown that there is a possibility that the research area has a channel-fill with the 10m – 20m thick at 32.5m, as shown in 5a. This is agreed with geological interpretation in figure 4a which shows that the current river moves from west-east direction in the past and changes to east-west in the current era [4]. The bedrocks lie in 20 m – 23 m depth and >25 m depth, as shown by S-wave velocity and density model in figure 5b and 5c. This finding accords with the previous CPT investigation as shown in figure 5d.
Figure 5. Tomography of (a) P-wave velocity, (b) S-wave velocity, and (c) density model, and integrated interpretation of S-wave velocity tomography and Cone Penetration Test (CPT) data.

The dynamic Bulk modulus illustrates the subsurface in more detail, as shown in figure 6. The Bulk modulus physically explains the compression of the rocks, which relies more on P-wave velocity. The rocks are more fragile to the pressure crack, or the P-wave tomography could not detect the layers in that depth. Thus there are no velocity changes in this depth that made this situation.

Figure 6. Subsurface dynamic elastic moduli: (a) Poisson’s ratio, (b) Young’s modulus, (c) bulk modulus, and (d) shear modulus.
The dynamic elastic moduli unit in the GPa should not make sense in term of the stiffness of the soil, this because the elastic moduli are dynamic and should be corrected with static testing data. Unfortunately, those static data are not in the range of the tomography zone. The anomaly in dynamic Poisson's Ratio could be possible by the blind zone. The ambiguity of these results could be minimized by doing fusion with Fuzzy Logic fusion.

![Figure 7. Results of fuzzy logic fusion: (a) possibility of solid-state behavior, and (b) possibility of plastic-state behavior.](image)

It is shown from figure 7a and 7b that below 18m - 24m and > 26m depths are hard layers, while 0m - 10m are layers that can be deformed by stress or cracking forces. The degree of cracking at a depth of 0m - 5m not greater than in depth of 5m - 10m. This finding coincides with anomaly A at the Poisson's ratio, shown earlier in figure 6a. The seam is thought to be an unconsolidated sand layer. At a distance of 32.5m and a depth of 20m, the object is shown to have a Poisson's ratio value of 0.36, and a high value of cracked soil possibility as illustrated in figure 7. The object also has a similar shape between the Poisson's cross-section and the possibility of solid-state behavior. The possibility of the plastic state behavior has an anomalous geometric shape, analogous to Young's modulus of elasticity and dynamic shear modulus. Physically, the cross-section has the same substance, namely Young's modulus, and shear modulus, which describe the resistance of rock or soil to shear forces.

**11. Conclusions**

The combined result of the P wave velocity and S wave velocity with the concept of fuzzy logic provides ease of interpretation because it is following the analytical calculation of dynamic modulus of elasticity and is more comfortable and more intuitive to use. However, this study's results are subject to improvement because there could be other influential data that need to be considered, e.g., resistivity, and each data's likelihood is not accounted yet.

**Acknowledgment**

This work was financially supported by Universitas Indonesia through PUTI Saintekes Program No. KB-4883/UN2.RST/HKP.05.00/2020. We acknowledge the Division of Facility Management and Maintenance of Universitas Indonesia to cooperate and support this study.
References

[1] Ekinci B 2012 The Determination of The Dynamic Elastic Moduli by MASW and Seismic Refraction Method 12th International Multidisciplinary Scientific GeoConference and EXPO - Modern Management of Mine Producing, Geology and Environmental Protection SGEM 649–54

[2] Grandjean G, Malet J-P and Bitri A 2007 Geophysical data fusion by fuzzy logic for imaging the mechanical behaviour of mudslides Bulletin de la Société Géologique de France 178 127–36

[3] Hajian A and Styles P 2019 Application of Soft Computing and Intelligent Methods in Geophysics (Springer) p 533

[4] Nichols G 2009 Sedimentology and Stratigraphy (John Wiley & Sons Ltd) p 432

[5] Park C B, Richter J, Rodriguez R and Cirone A 2018 MASW applications for road construction and maintenance The Leading Edge 27 724–30

[6] Sheriff R E and Geldart L P 1995 Exploration Seismology (Cambridge: Cambridge University Press) p 592

[7] Sukardi T 1992 Peta Geologi Lembar Jakarta (Bandung: Pusat Penelitian dan Pengembangan Geologi)