The $B \to \pi K$ Puzzle and Supersymmetry

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(February 14, 2008)

Abstract

At present, there are discrepancies between the measurements of several observables in $B \to \pi K$ decays and the predictions of the standard model (the “$B \to \pi K$ puzzle”). Although the effect is not yet statistically significant – it is at the level of $\gtrsim 3\sigma$ – it does hint at the presence of new physics. In this paper, we explore whether supersymmetry (SUSY) can explain the $B \to \pi K$ puzzle. In particular, we consider the SUSY model of Grossman, Neubert and Kagan (GNK). We find that it is extremely unlikely that GNK explains the $B \to \pi K$ data. We also find a similar conclusion in many other models of SUSY. And there are serious criticisms of the two SUSY models that do reproduce the $B \to \pi K$ data. If the $B \to \pi K$ puzzle remains, it could pose a problem for SUSY models.
Over the past several years, measurements have been made of a number of observables in the decays of $B$ mesons which are in disagreement with the predictions of the standard model (SM): e.g. indirect CP asymmetries in penguin-dominated $B$ decays [1], triple-product correlations in $B \to \phi K^*$ [2], polarizations in $B \to V_1 V_2$ decays ($V_i$ is a vector meson) [3], etc. None of these discrepancies is statistically significant, so that these disagreements only point to a hint of physics beyond the SM. Still, if these hints are taken together, the statistical significance increases. Furthermore, they are intriguing since they all point to new physics (NP) in $\bar{b} \to \bar{s}$ transitions.

Arguably, the most stringent discrepancy appears in $B \to \pi K$ decays. Briefly, the effect goes as follows. There are four $B \to \pi K$ decays: $B^+ \to \pi^+ K^0$ (designated as $+0$ below), $B^+ \to \pi^0 K^+$ ($0+$), $B^0 \to \pi^- K^+$ ($-+$) and $B^0 \to \pi^0 K^0$ (00). In terms of diagrams [4], the amplitudes are given by

$$
A^{+0} = -P',
$$
$$
\sqrt{2}A^{0+} = P' - T' e^{i\gamma} - C' e^{i\gamma} - P'_{\text{EW}},
$$
$$
A^{-+} = P' - T' e^{i\gamma},
$$
$$
\sqrt{2}A^{00} = -P' - P'_{\text{EW}} - C' e^{i\gamma}.
$$

(1)

In the above, we have neglected small diagrams and written the amplitudes in terms of the color-favored and color-suppressed tree amplitudes $T'$ and $C'$, the $t$-quark-dominated gluonic penguin amplitude $P'$, and the color-favored electroweak penguin amplitude $P'_{\text{EW}}$. (The primes on the amplitudes indicate $\bar{b} \to \bar{s}$ transitions.) In addition, we have explicitly written the weak-phase dependence (including the minus sign from $V^*_{tb} V_{ts} [P']$), while the diagrams contain strong phases. (The phase information in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix is conventionally parametrized in terms of the unitarity triangle, in which the interior (CP-violating) angles are known as $\alpha$, $\beta$ and $\gamma$ [5].) The amplitudes for the CP-conjugate processes can be obtained from the above by changing the sign of the weak phase ($\gamma$). Note that these diagrams include the magnitudes of their associated CKM matrix elements.

The diagram $P'_{\text{EW}}$ is not independent. To a good approximation, it can be related to $T'$ and $C'$ using flavor SU(3) symmetry [6]:

$$
P'_{\text{EW}} = \frac{3c_9 + c_{10}}{4c_1 + c_2} R(T' + C') + \frac{3c_9 - c_{10}}{4c_1 - c_2} R(T' - C') .
$$

(2)

Here, the $c_i$ are Wilson coefficients [7] and $R \equiv |(V^*_{tb} V_{ts})/(V^*_{ub} V_{us})|$. $\bar{\lambda}$ is the prediction of NLO pQCD [8], and $|\lambda| \sim 0.3$ is the maximal SCET (QCDf) prediction [9, 10].

Now, in Ref. [4], the relative sizes of the diagrams were estimated to be roughly

$$
1 : |P'| , \quad \mathcal{O}(\bar{\lambda}) : |T'| , \quad |P'_{\text{EW}}| , \quad \mathcal{O}(\bar{\lambda}^2) : |C'| .
$$

(3)

where $\bar{\lambda} \sim 0.2$. With this estimate, the diagram $C'$ should also be neglected in the $B \to \pi K$ amplitudes above [Eq. (1)]. Note that the smallness of $|C'|$ is verified by more robust hadronic computations: $|C'/T'| \sim 0.3$ is the prediction of NLO pQCD [8], and $|C'/T'| \sim 0.6$ is the maximal SCET (QCDf) prediction [9, 10].
There are nine measurements that have been made of $B \to \pi K$ decays: the four branching ratios, the four direct CP asymmetries $A_{ij}^{\ell}$ ($ij = +0, 0+, -+, 00$), and the mixing-induced CP asymmetry $S_{CP}^{ij}$, in $B_d^0 \to \pi^0 K^0$ [11]. With this data and the expressions for the $B \to \pi K$ amplitudes, one can perform a fit [12]. In the first fit, $C'$ was neglected in the $B \to \pi K$ amplitudes. A very poor fit was found: $\chi^2_{\text{min}}/d.o.f. = 25.0/5 (1.4 \times 10^{-4})$. (The number in parentheses indicates the quality of the fit, and depends on $\chi^2_{\text{min}}$ and d.o.f. individually. It shows the percentage of the parameter space which has a worse $\chi^2_{\text{min}}$. 50% or more is a very good fit; fits which are substantially less than 50% are poorer. $1.4 \times 10^{-4}$ corresponds to a 3-4$\sigma$ discrepancy with the SM.) This result has led some authors to posit the existence of a “$B \to \pi K$ puzzle” [13].

In the second fit, $C'$ was kept and the full amplitudes of Eq. (1) used. In this case, a good fit was found: $\chi^2_{\text{min}}/d.o.f. = 1.0/3 (80\%)$. This has led some people to argue that there is in fact no $B \to \pi K$ puzzle (for example, see Ref. [14]). However, $|C'/T'| = 1.6 \pm 0.3$ is required here. This is much larger then the theoretical estimates described above. If one takes this theoretical input seriously – as we do here – this shows explicitly that the $B \to \pi K$ puzzle is still present, at $\gtrsim 3\sigma$ level.

The question now is: what type of new physics can explain the $B \to \pi K$ puzzle? All NP operators in $b \to s \bar{q}q$ transitions take the form $\mathcal{O}_{NP}^{ij} \sim s\Gamma_i b \bar{q}\Gamma_j q$ ($q = u, d, s, c$), where the $\Gamma_{ij}$ represent Lorentz structures, and color indices are suppressed. These operators contribute to the decay $B \to \pi K$ through the matrix elements $\langle \pi K | \mathcal{O}_{NP}^{ij} | B \rangle$. Each matrix element has its own NP weak and strong phase. Now, it has been argued that all NP strong phases are negligible [15]. In this case one can combine all NP matrix elements of $B \to \pi K$ into a single NP amplitude, with a single weak phase:

$$\sum \langle \pi K | \mathcal{O}_{NP}^{ij} | B \rangle = A^q e^{i\Phi_q}. \tag{4}$$

$B \to \pi K$ decays involve only NP parameters related to the quarks $u$ and $d$. These operators come in two classes, differing in their color structure: $s\bar{q}\Gamma_i b \bar{q}\Gamma_j q$ and $s\bar{q}\Gamma_i b \bar{q}\Gamma_j q$ ($q = u, d$). The matrix elements of these operators can be combined into single NP amplitudes, denoted $A^{i,q} e^{i\Phi_q^{i}}$ and $A^{IC,q} e^{i\Phi_q^{IC}}$, respectively [16]. Here, $\Phi_q^{i}$ and $\Phi_q^{IC}$ are the NP weak phases; the strong phases are zero. Each of these contributes differently to the various $B \to \pi K$ decays. In general, $A^{i,q} \neq A^{IC,q}$ and $\Phi_q^{i} \neq \Phi_q^{IC}$. Note that, despite the “color-suppressed” index $C$, the matrix elements $A^{IC,q} e^{i\Phi_q^{IC}}$ are not necessarily smaller than the $A^{i,q} e^{i\Phi_q^{i}}$.

The $B \to \pi K$ amplitudes can now be written in terms of the SM amplitudes to $O(\lambda)$ [$P_{EW}$ and $T'$ are related as in Eq. (2)], along with the NP matrix elements [16]:

$$A^{+0} = -P' + A^{IC,d} e^{i\Phi_q^{IC}}, \tag{5}$$

$$\sqrt{2}A^{0+} = P' - T' e^{i\gamma} + P_{EW}^\prime + A'^{comb} e^{i\Phi'} - A^{IC,u} e^{i\Phi_q^{IC}};$$

$$A^{-+} = P' - T' e^{i\gamma} - A^{IC,u} e^{i\Phi_q^{IC}},$$

$$\sqrt{2}A^{00} = -P' + P_{EW}^\prime + A'^{comb} e^{i\Phi'} + A^{IC,d} e^{i\Phi_q^{IC}},$$

where $A'^{comb} e^{i\Phi'} \equiv -A'^{u} e^{i\Phi'} + A'^{d} e^{i\Phi_d'}$. 

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In 1999, Grossman, Neubert and Kagan (GNK) proposed a new version of supersymmetry (SUSY) [17]. This model was promising for NP contributions to $B \to \pi K$ decays because it incorporates a new CP phase, and because it breaks isospin. In this paper we explore whether the GNK SUSY model can in fact explain the $B \to \pi K$ puzzle, i.e. whether it gives the appropriate contributions to $A'_{\text{comb}} e^{i\Phi'}$, $A'_{\text{u}} e^{i\Phi'_d}$ and $A'_{\text{d}} e^{i\Phi'_d}$.

We begin with a review of the GNK SUSY model, emphasizing those points which are important to our calculation. In R-parity-conserving SUSY models, the largest contributions to flavor-changing neutral current (FCNC) processes potentially come from the gluino-exchange SUSY box or penguin diagrams. The chargino and neutralino contributions are parametrically suppressed due to their small gauge couplings. The source of the gluino-mediated FCNC is the off-diagonal components in the scalar mass matrix in the basis where the quark mass matrices are diagonalized (super-CKM basis). Since we are interested only in the $\bar{b} \to \bar{s}$ transition, we consider only the down-type scalar mass matrix.

However, a generic form of scalar mass matrices is not acceptable because it leads to too-large contributions to FCNC processes (SUSY FCNC problem) and/or to the electric dipole moments of the neutron and electron (SUSY CP problem). To evade these problems, people usually assume that SUSY is broken in a hidden sector and mediated to the observable sector by some flavor-blind interactions, such as gravity or gauge interactions. Then the squark mass matrices are diagonal matrices at a high-energy scale. The off-diagonal components in the squark mass matrices are generated by renormalization group (RG) running. In these popular models, such as minimal supergravity (mSUGRA) [18], anomaly-mediated SUSY breaking (AMSB) [19] or gauge-mediated SUSY breaking (GMSB) [20] models, the SUSY FCNC/CP problems are solved because the RG-generated off-diagonal terms are typically very small and they do not include new sources of CP violation. On the other hand, as a consequence, they also cannot explain any possible deviation in the CP asymmetries in $B$ decays.

The GNK model assumes the following form of squark mass-squared matrices:

$$M^2_{d,LL(RR)} = \begin{pmatrix}
\tilde{m}_{L(R)11}^2 & 0 & 0 \\
0 & \tilde{m}_{L(R)22}^2 & \tilde{m}_{L(R)23}^2 \\
0 & \tilde{m}_{L(R)32}^2 & \tilde{m}_{L(R)33}^2
\end{pmatrix}, \quad M^2_{d,LR(RL)} \equiv 0_{3 \times 3}, \quad (6)$$

where off-diagonal components can be as large as the diagonal components. Although Eq. (6) is not supported by the above-mentioned popular SUSY-breaking models, it is well-motivated in SUSY GUT theories, where neutrinos are in the same supermultiplet as down quarks [21]. The zeroes in the above mass matrix are justified by the fact that the experimental results for $K^0-\bar{K}^0$ mixing, $B^0_d-\bar{B}^0_d$ mixing and $B \to X_s \gamma$ are in good agreement with the SM predictions. In general they can get small non-zero values, but they do not affect our results much as long as we do not consider the very large $\tan \beta$ region [22]. In our analysis below, we consider two scenarios: (i) only $LL$ mixing is present (i.e. $M^2_{d,RR}$ is diagonal), and (ii) both $LL$ and $RR$ mixing are present.
The mass matrix $M^2_{d,LL}$ is diagonalized by

$$
\Gamma_L M^2_{d,LL} \Gamma_L^\dagger = \text{diag}(m^2_{dL}, m^2_{sL}, m^2_{bL}),
$$

(7)

with

$$
\Gamma_L = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta_L & \sin \theta_L e^{i\delta_L} \\
0 & -\sin \theta_L e^{-i\delta_L} & \cos \theta_L
\end{pmatrix}.
$$

(8)

Similarly, the exchange $L \leftrightarrow R$ in (8) gives $\Gamma_R$. We restrict to $-\pi/4 < \theta_{L(R)} < \pi/4$ ($\theta_R = 0$ if $RR$ mixing is absent) and $-\pi < \delta_{L(R)} < \pi$.

The form given in Eq. (6) is not sufficient to give large SUSY contributions to $P_{EW}$. (Actually it is known that the gluino contribution to the $Z$-penguin is small [23].) In Ref. [17], the authors assumed that there is a significant mass splitting between the right-handed up and down squarks. Then the gluino box diagrams become the main source of the isospin breaking, and the scale $\alpha_s^2/m_{SUSY}^2$ (SUSY contribution) is comparable with $\alpha/M^2_W$ (SM contribution).

We now turn to a review of the new-physics amplitudes $A_{t,comb}^q e^{i\Phi'_q}$, $A_{t,c} e^{i\Phi'_s}$ and $A_{t,c}^c e^{i\Phi'_q}$. These same NP amplitudes also contribute to $B^0_s \rightarrow K^+K^-$ and $B^0_s \rightarrow K^0\bar{K}^0$, and have been calculated within GNK SUSY in Ref. [24]. We closely follow this reference in our analysis, and use its treatment of the NP SUSY amplitudes. The color-allowed and color-suppressed NP amplitudes are given by

$$
A_{t,comb}^q e^{i\Phi'_q} = \frac{G_F}{\sqrt{2}} \left[ (c^q_1 + \frac{1}{3} c^q_2) - (c^q_3 + \frac{1}{3} c^q_4) - \chi_\pi \left( \frac{1}{3} c^q_3 + c^q_6 \right) \right] A_{K^\pi},
$$

$$
A_{t,c}^c e^{i\Phi'_q} = \frac{G_F}{\sqrt{2}} \left[ -\chi_K \left( \frac{1}{3} c^q_1 + c^q_3 \right) - \left( \frac{1}{3} c^q_1 + c^q_3 \right) + \left( c^q_3 + \frac{1}{3} c^q_6 \right) \right. \\
\left. - \lambda_t \frac{2\alpha_s}{3\pi} c_{8g} \left( 1 + \frac{\chi_K}{3} \right) \right] A_{s,K},
$$

(9)

where $q = u, d$. (Note: in Ref. [24], $A_{t,c} e^{i\Phi'_q}$ and $A_{t,q} e^{i\Phi'_k}$ are switched.) In the above, $\lambda_t = V_{tb}^* V_{ts}$ and

$$
\begin{align*}
\bar{c}^q_i &= c_i - \bar{c}_i, \\
\bar{c}^{eff}_{8g} &= c_{8g} + \frac{1}{3} c^q_1, \\
A_{s,K} &= i(m^2_{b} - m^2_{\pi}) P^{B-K}_0 f_K, \\
A_{K^\pi} &= i(m^2_{b} - m^2_{\pi}) P^{B-K}_0 f_\pi, \\
\chi_K(\mu) &= \frac{2m^2_{\pi}}{m_b(\mu)(m_q(\mu) + m_s(\mu))}, \\
\chi_\pi(\mu) &= \frac{2m^2_{\pi}}{m_b(\mu)(m_u(\mu) + m_d(\mu))}.
\end{align*}
$$

(10)
where the $c$’s and $\tilde{c}$’s are Wilson coefficients of the effective operator in the GNK basis, $m_q$ is the averaged mass of up and down quarks, and naive factorization has been used for the hadronic matrix elements $A_{\pi K}$ and $A_{K\pi}$. Also, $c_{8g} = -\lambda_i C_{8g}$, where $C_{8g}^{\text{eff}} = C_{8g} + C_5$ in the standard basis.

When only mixing between components 2 and 3 of the down-squark mixing matrices is allowed, the Wilson coefficients are given by

\[
\begin{align*}
    c_1^q &= \frac{\alpha_s^2}{4\sqrt{2}G_F m_g^2} \left[ \frac{1}{18} F(x_{bL\bar{g}}, x_{\bar{q}R\bar{g}}) - \frac{5}{18} G(x_{bL\bar{g}}, x_{\bar{q}R\bar{g}}) + \frac{1}{2} A(x_{bL\bar{g}}) + \frac{2}{9} B(x_{bL\bar{g}}) \right] \\
    c_2^q &= \frac{\alpha_s^2}{4\sqrt{2}G_F m_g^2} \left[ \frac{7}{6} F(x_{bL\bar{g}}, x_{\bar{q}R\bar{g}}) + \frac{1}{6} G(x_{bL\bar{g}}, x_{\bar{q}R\bar{g}}) - \frac{3}{2} A(x_{bL\bar{g}}) - \frac{2}{3} B(x_{bL\bar{g}}) \right] \\
    c_3^q &= \frac{\alpha_s^2}{4\sqrt{2}G_F m_g^2} \left[ \frac{5}{9} F(x_{bL\bar{g}}, x_{\bar{q}R\bar{g}}) + \frac{1}{36} G(x_{bL\bar{g}}, x_{\bar{q}R\bar{g}}) + \frac{1}{2} A(x_{bL\bar{g}}) + \frac{2}{9} B(x_{bL\bar{g}}) \right] \\
    c_4^q &= \frac{\alpha_s^2}{4\sqrt{2}G_F m_g^2} \left[ \frac{1}{3} F(x_{bL\bar{g}}, x_{\bar{q}R\bar{g}}) + \frac{7}{12} G(x_{bL\bar{g}}, x_{\bar{q}R\bar{g}}) - \frac{3}{2} A(x_{bL\bar{g}}) - \frac{2}{3} B(x_{bL\bar{g}}) \right] \\
    c_5^q &= c_6^q = 0 ,
\end{align*}
\]

where $x_{ab} = m_a^2/m_b^2$. Wilson coefficients with inverse chirality $\tilde{c}$’s have exactly the same form, with the replacement $L \leftrightarrow R$. Loop integrals are given by

\[
\begin{align*}
    F(x, y) &= -\frac{x \ln x}{(x-y)(x-1)^2} - \frac{y \ln y}{(y-x)(y-1)^2} - \frac{1}{(x-1)(y-1)} , \\
    G(x, y) &= \frac{x^2 \ln x}{(x-y)(x-1)^2} + \frac{y^2 \ln y}{(y-x)(y-1)^2} + \frac{1}{(x-1)(y-1)} , \\
    A(x) &= \frac{1}{2(1-x)} + \frac{(1+2x) \ln x}{6(1-x)^2} , \\
    B(x) &= -\frac{11 - 7x + 2x^2}{18(1-x)^3} - \frac{\ln x}{3(1-x)^4} .
\end{align*}
\]

Finally, for the chromomagnetic penguin, we have

\[
\lambda_i \frac{2\alpha_s}{3\pi} C_{8g}^{\text{eff}} = \frac{8 \alpha_s^2}{3} \frac{e^{i\delta_L}}{4\sqrt{2}G_F m_g^2} \left[ f_{8}^{\text{SUSY}}(x_{bL\bar{g}}) - (b_L \leftrightarrow s_L) \right] ,
\]

where

\[
f_{8}^{\text{SUSY}}(x) = \frac{-11 + 51x - 21x^2 - 19x^3 - 6x(1 - 9x) \log x}{72(x - 1)^4} .
\]
The equations presented above allow one to calculate Wilson coefficients at the SUSY scale, taken to be $m_t$. They then need to be renormalized to the scale $\mu = m_b$. The renormalization procedure described in Ref. [24] is used. This then gives the three NP SUSY amplitudes $A_{\text{comb}}^{\mu}e^{i\Phi}$, $A_{\text{C},u}^{\mu}e^{i\Phi_u}$ and $A_{\text{C},d}^{\mu}e^{i\Phi_d}$ at scale $m_b$.

We can now see if GNK can explain the $B \to \pi K$ puzzle. In Ref. [12], fits were done with NP. The value of $\gamma$ was taken from independent measurements. (The value of $\gamma$ is the same as in the SM even in the presence of NP [25].) However, if all NP amplitudes are kept, there are more theoretical parameters (10) than measurements (9), and a fit cannot be done. For this reason, a single NP amplitude was assumed to dominate. Four possibilities were considered: (i) only $A_{\text{comb}}^{\mu} \neq 0$, (ii) only $A_{\text{C},u}^{\mu} \neq 0$, (iii) only $A_{\text{C},d}^{\mu} \neq 0$, (iv) $A_{\text{C},u}^{\mu}e^{i\Phi_u} = A_{\text{C},d}^{\mu}e^{i\Phi_d}$, $A_{\text{comb}}^{\mu} = 0$ (isospin-conserving NP). A very good fit was found only if the NP is in the form of $A_{\text{comb}}^{\mu}e^{i\Phi'}$ (i.e. the SM electroweak-penguin amplitude). It is therefore often said that any NP invoked to explain the $B \to \pi K$ puzzle must contribute mainly to $A_{\text{comb}}^{\mu}$ and little to $A_{\text{C},u}^{\mu}$ and $A_{\text{C},d}^{\mu}$. (However, it should be noted that the fit with only $A_{\text{C},u}^{\mu} \neq 0$ is not bad.) On the other hand, the GNK SUSY model gives nonzero values to all three NP amplitudes, and so the results of Ref. [12] do not hold. Another procedure must be used.

Our analysis proceeds as follows. The three NP SUSY amplitudes depend on a number of theoretical inputs. We generate these randomly in the following ranges:

- $300 \leq m_\tilde{g} \leq 2000$ GeV,
- $100 \leq m_\tilde{q} \leq 2000$ GeV,
- $-\pi/4 < \theta_{L,R} < \pi/4$,
- $-\pi < \delta_{L,R} < \pi$,
- $\gamma = 67.6^{+2.8}_{-4.5}$ [26],
- $m_u, m_d$ (2 GeV) = 2.5 to 5.5 MeV [5],
- $m_s$ (2 GeV) = 0.095 ± 0.025 GeV [5],
- $F_{B\to K}(q^2 = 0) = 0.34 \pm 0.05$ [27],
- $F_{B\to \pi}(q^2 = 0) = 0.28 \pm 0.05$ [27].

Note that we have taken $m_{\tilde{u}_L} = m_{\tilde{d}_L}$ following $SU(2)_L$ symmetry. The weak phase $\gamma$ is allowed to vary in the ±2σ range. For the other (theoretical) quantities for which an error is given, we take the range as ±1σ. With these values, $A_{\text{comb}}^{\mu}e^{i\Phi}$, $A_{\text{C},u}^{\mu}e^{i\Phi_u}$ and $A_{\text{C},d}^{\mu}e^{i\Phi_d}$ are generated.

Given the knowledge of the three NP amplitudes and $\gamma$, the $B \to \pi K$ amplitudes [Eq. (5)] and observables depend only on the two SM diagrams $P'$ and $T'$ (magnitudes and strong phases; $P'_{EW}$ is related to $T'$). We can therefore do a fit to see how well the
Table 1: Branching ratios, direct CP asymmetries $A_{CP}$, and mixing-induced CP asymmetry $S_{CP}$ (if applicable) for the four $B \rightarrow \pi K$ decay modes. The data is taken from Refs. [1] and [11].

| Mode          | BR[10^{-6}] | $A_{CP}$                  | $S_{CP}$ |
|---------------|-------------|---------------------------|----------|
| $B^+ \rightarrow \pi^+ K^0$ | 23.1 ± 1.0  | 0.009 ± 0.025             |          |
| $B^+ \rightarrow \pi^0 K^+$  | 12.9 ± 0.6  | 0.050 ± 0.025             |          |
| $B^0_d \rightarrow \pi^- K^+$ | 19.4 ± 0.6  | −0.097 ± 0.012            |          |
| $B^0_d \rightarrow \pi^0 K^0$ | 9.9 ± 0.6   | −0.14 ± 0.11              | 0.38 ± 0.19 |

$B \rightarrow \pi K$ data is reproduced. If the $\chi^2_{min}$ is acceptable, then we can conclude that the GNK SUSY model explains the $B \rightarrow \pi K$ puzzle. If not, then it does not.

In order to establish what constitutes an “acceptable” fit, we take our cue from ordinary observables. There, the $2\sigma$ limit implies that 4.55% of the points of a Gaussian distribution lie outside this interval. In this spirit, we assume that the $\chi^2_{min}$ is acceptable if the percentage of the parameter space which has a worse $\chi^2_{min}$ is 4.55%, i.e. $\chi^2_{min}$ is taken to be < 11.31. (Note: in practice, there is no relation between Gaussian and $\chi^2_{min}$ distributions. We use the information from the Gaussian distribution only as a guide.)

Before presenting the conclusions of this analysis, we must consider other constraints. There are many constraints on SUSY models – electroweak precision tests, $\Delta m_d$, $\Delta m_K$, $b \rightarrow s(d)\gamma$, etc. However, by far the most stringent is that coming from $B^0_s$-$\bar{B}^0_s$ mixing. This is discussed in detail in Ref. [28], and we closely follow the analysis presented here. We find that $|\Delta m_s/\Delta m_s^{SM}| = 0.788 \pm 0.195$. This limits the SUSY contribution to $B^0_s$-$\bar{B}^0_s$ mixing. Using the expression given in Ref. [28], we compute the GNK SUSY contribution to $|\Delta m_s|$. To do so, three more theoretical parameters are needed, and we generate them randomly:

- $B_1 = 0.86^{+0.05}_{-0.04}$ [28],
- $B_4 = 1.17^{+0.05}_{-0.07}$ [28],
- $B_5 = 1.94^{+0.23}_{-0.08}$ [28].

For each set of theoretical parameters generated, we check whether the constraint is satisfied (within ±2σ).

The parameter space of GNK SUSY models is enormous – there are 12 SUSY parameters alone. In order to do our best to adequately sample this parameter space, 500,000 sets of theoretical parameters were generated. For each set, we checked whether the $B \rightarrow \pi K$ and the $B^0_s$-$\bar{B}^0_s$ mixing data were reproduced. The results are shown in Table 2, for the cases where (i) only $LL$ mixing is allowed, and (ii) both $LL$ and $RR$ mixings are allowed. From this Table we see that the case with only $LL$ mixing is preferred by the $B^0_s$-$\bar{B}^0_s$ mixing data. However, neither mixing scenario can explain the
$\chi^2_{\text{min}} < 11.31$  $\Delta m_s$ both  
74 414357 15 

$\chi^2_{\text{min}} < 11.31$  $\Delta m_s$ both  
102 92844 1

Table 2: The number of points (out of 500000) which satisfy $\chi^2_{\text{min}}(B \to \pi K) < 11.31$, the $\Delta m_s$ constraint within $\pm 2\sigma$, and both constraints. In the left table, only LL mixing is allowed, while in the right table, both LL and RR mixings are allowed.

$B \to \pi K$ puzzle – in both cases, the $B \to \pi K$ data is reproduced only in a tiny region of parameter space. The $B_s^0 - \bar{B}_s^0$ mixing constraint reduces this (already small) region. We therefore conclude that it is very unlikely that the GNK SUSY model obeys the constraints from $B \to \pi K$ decays, and virtually impossible that it reproduces the data from both $B \to \pi K$ and $B_s^0 - \bar{B}_s^0$ mixing.

In Fig. 1, we present the SUSY contributions to several $B \to \pi K$ observables. This helps identify which measurements lead to the large $\chi^2_{\text{min}}$ for each of the 500,000 GNK sets of parameters. In particular, we show $R_c$ vs. $R_n$, where

$$
R_c \equiv 2 \frac{\text{BR}(B^+ \to \pi^0 K^+) + \text{BR}(B^- \to \pi^0 K^-)}{\text{BR}(B^+ \to \pi^+ K^0) + \text{BR}(B^- \to \pi^- K^0)} ,
$$

$$
R_n \equiv \frac{1}{2} \frac{\text{BR}(B_s^0 \to \pi^- K^+) + \text{BR}(\bar{B}_s^0 \to \pi^+ K^-)}{\text{BR}(B_s^0 \to \pi^0 K^0) + \text{BR}(\bar{B}_s^0 \to \pi^0 K^0)} , \quad (15)
$$

$A^{00}_{CP}$ vs. $S^{00}_{CP}$, and $A'^{0+}_{CP}$ vs. $A'^{-+}_{CP}$. All are scatter plots, showing the contribution of the GNK SUSY model to the various observables. As can be seen from this Figure, GNK has little difficulty in reproducing the combined $R_c$ and $R_n$ quantities. However, the SM can do this alone, showing that there is no discrepancy with the SM for $R_c$ and $R_n$. GNK can also explain the $A^{00}_{CP}$ and $S^{00}_{CP}$ observables. Note that the SM alone has difficulty with these measurements. On the other hand, it is almost impossible for GNK to simultaneously reproduce $A'^{0+}_{CP}$ and $A'^{-+}_{CP}$. This shows explicitly that it is the direct CP asymmetry measurements which are most problematic.

There are several reasons that the GNK SUSY model cannot explain the $B \to \pi K$ puzzle. First, for much of the parameter space, all three NP amplitudes are small. Thus, despite the presence of SUSY, the $B \to \pi K$ system is basically described by the SM. However, we saw that the SM has a very poor fit in explaining the $B \to \pi K$ observables, and so the same is true here. Second, the $B \to \pi K$ measurements suggest that there is NP in the $P'_{EW}$ diagram ($A'^{\text{comb}}$). However, as indicated earlier, SUSY does not contribute significantly to $P'_{EW}$. As a result, it is very difficult for SUSY to explain the $B \to \pi K$ puzzle, and the GNK SUSY fits are generally poor. Third, we saw in the fits in which a single NP amplitude was assumed to dominate that the fit with $A^{0+}_{CP} \neq 0$ was not bad. However, GNK generally does not generate only a large $A^{0+}_{CP}$ – a large $A^{0+}_{CP}$ is also usually found. Again, this leads to a poor fit. All of these can be seen in Fig. 2, which shows the plots of $A^{0+}_{CP}$ vs. $A'^{\text{comb}}$ and $A^{0+}_{CP}$ vs. $A'^{-+}_{CP}$. The bottom line is that it requires a very precise pattern of SUSY parameters to explain
Figure 1: Scatter plots of $R_c$ vs. $R_n$ (top), $A_{CP}^{00}$ vs. $S_{CP}^{00}$ (middle), and $A_{CP}^{0+}$ vs. $A_{CP}^{-+}$ (bottom), for LL mixing only (left), and LL and RR mixing (right). Horizontal and vertical lines represent experimental values within 1σ and 2σ. Plots include all 500,000 GNK sets of parameters. Red points (dark grey in black and white) indicate only the SM piece of the SM + SUSY contribution. For $A_{CP}^{00}$ vs. $S_{CP}^{00}$, the SM piece is a single dot because there is no direct CP violation when SUSY is not added.
Figure 2: Scatter plots of $A^{c,u}_t$ vs. $A^{c,\text{comb}}_t$ (top) and $A^{c,u}_t$ vs. $A^{c,d}_t$ (bottom), for LL mixing only (left), and LL and RR mixing (right). Plots include all 500,000 GNK sets of parameters.

the $B \to \pi K$ puzzle, and this is not found in most of the GNK SUSY parameter space.

Of the very few points which satisfy both constraints, the great majority correspond to a large $A^{c,u}_t$ and a small $A^{c,d}_t$ and $A^{c,\text{comb}}_t$. Also, all the points with $\chi^2_{\text{min}}(B \to \pi K) < 11.31$ have a gluino mass less than 1.3 TeV. This is the only direct constraint on the SUSY parameters.

As we have seen, it is extremely unlikely that the GNK SUSY model explains the $B \to \pi K$ puzzle. As noted earlier, there are other popular SUSY models: mSUGRA \cite{18}, AMSB \cite{19}, GMSB \cite{20}, etc. However they all automatically solve the SUSY FCNC/CP problems by not allowing any CP-violating phases. So these models cannot explain the $B \to \pi K$ data either.

There are two SUSY models which do reproduce the $B \to \pi K$ data. They have (i) a large chargino contribution which allows large $(2,3)$ mass terms in the up-squark sector \cite{29}, or (ii) R-parity violation \cite{30}. However, these two models have their own problems. The one with chargino contributions seems to be fine-tuned. It is not natural, i.e. it is hard to find a more microscopic theory which generates only $(2,3)$ up-squark mass components in the $LL$ or $RR$ sector. And the R-parity-violating model lacks the beauty of SUSY, e.g. it does not have dark-matter candidates. We therefore conclude that if the $B \to \pi K$ puzzle persists, SUSY models could have some difficulty.
To summarize, the supersymmetry (SUSY) model of Grossman, Neubert and Kagan (GNK) [17] has great difficulty in explaining the $B \to \pi K$ puzzle. The $B \to \pi K$ data can be reproduced in the GNK model, but only in a tiny region of parameter space. Other SUSY models, such as those with minimal supergravity [18], anomaly-mediated SUSY breaking [19] or gauge-mediated SUSY breaking [20], fare no better, as they do not allow any new CP-violating phases. There are two SUSY models which do reproduce the $B \to \pi K$ data [29, 30]. However, these models are either fine-tuned or lack some elements of ordinary SUSY theories. The $B \to \pi K$ puzzle is still only a $\gtrsim 3\sigma$ effect, and so cannot be considered statistically significant. However, if this discrepancy with the SM remains in the years to come, it could pose a problem for SUSY models.

Acknowledgments:
This work is financially supported by NSERC of Canada (MI and DL) and by the Korea Research Foundation Grant funded by the Korean Government (MOEHRD) No. KRF-2007-359-C00009 (SB).

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