An assessment of solar-powered organic Rankine cycle systems for combined heating and power in UK domestic applications

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A domestic-scale combined solar heat and power (CSHP) system is simulated in the UK climate. The CSHP system comprises a solar collector array, an ORC engine and a hot-water cylinder. An exergy analysis, parametric study and annual performance assessment are performed. An average electrical power of 89 W plus an 86% hot water coverage are demonstrated. A total system cost as low as £2700 and a levelised cost electricity of 44 p/kW h are reported.

Performance calculations are presented for a small-scale combined solar heat and power (CSHP) system based on an Organic Rankine Cycle (ORC), in order to investigate the potential of this technology for the combined provision of heating and power for domestic use in the UK. The system consists of a solar collector array of total area equivalent to that available on the roof of a typical UK home, an ORC engine featuring a generalised positive-displacement expander and a water-cooled condenser, and a hot water storage cylinder. Preheated water from the condenser is sent to the domestic hot water cylinder, which can also receive an indirect heating contribution from the solar collector. Annual simulations of the system are performed. The electrical power output from concentrating parabolic-trough (PTC) and non-concentrating evacuated-tube (ETC) collectors of the same total array area are compared. A parametric analysis and a life-cycle cost analysis are also performed, and the annual performance of the system is evaluated according to the total electrical power output and cost per unit generating capacity. A best-case average electrical power output of 89 W (776 kW h/year) plus a hot water provision capacity equivalent to 80% of the total demand are demonstrated, for a whole system capital cost of £2700–£3900. Tracking PTCs are found to be very similar in performance to non-tracking ETCs with an average power output of 89 W (776 kW h/year) vs 80 W (701 kW h/year).

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1. Introduction

1.1. Solar heat and power in the UK

Between a quarter and 30% of the total CO₂ emissions in the United Kingdom are associated with domestic energy use [1,2]. Therefore, meeting the UK target for a reduction in CO₂ emissions of 50% by 2050 is strongly dependent on a significant contribution from dwellings, and a meaningful strategy for achieving this target should suitably address this sector. Further, in 2010 the International Energy Agency (IEA) published a roadmap [3] for emissions reduction over the next four decades, detailing the expected contributions from a range of improvements and technological developments. In this roadmap it is predicted that end-use fuel and energy efficiency will provide the largest proportion of the emissions reduction, contributing 38% of the overall target, while renewables are expected to provide a further 17%. Together, these account for more than half of the overall target and are also the areas in which the domestic sector can play a significant role.

The UK has a modest solar resource compared to countries of subtropical latitudes. The global horizontal solar irradiation received annually in London is typically between 1000 and 1100 kW h/m² [4], equivalent to approximately 120 W/m² on
average. The seasonal variability that leads to these average values is actually significant. From about April to September solar irradiance stays (over the course of an average day) well above 300 W/m² from approximately 9:00 a.m. to 5:00 p.m., peaking at about 500 W/m² at midday. The annually averaged solar irradiation is a little over half of that received in parts of southern Europe and North Africa, yet it is sufficient to reduce meaningfully the household demand for heat and power.

In recent years, the uptake of solar photovoltaic (PV) systems and solar water heating systems has increased dramatically in the UK, as noted in reports by the EPIA [5] and the Energy Saving Trust [6]. This can be largely attributed to the fall in price of these technologies and also the introduction of financial incentives by the UK government such as Feed-in Tariffs (FITs) and the Renewable Heat Incentive (RHI). In particular, demand (newly installed capacity) for solar PV panels in the UK reached 800 MW in the first six months of 2013 with 520 MW in the first quarter [7]. Cumulative PV demand now exceeds 2.5 GW, with 93% of this demand having been realised in the past two years and 52% being attributed to the domestic sector [8].

1.2. Organic Rankine cycle systems

The Organic Rankine Cycle (ORC) engine is one of a number of technologies being explored as an alternative to PV for the transformation of solar radiation into electrical energy. An ORC engine has the same operating principle as a conventional Rankine cycle equivalent, but uses an organic compound instead of water as the working fluid. Historically the cycle has been employed for the conversion of heat from low-to-medium temperature sources (e.g. geothermal, biomass combustion, process waste heat), with
The ORC technology is well established worldwide with a number of commercial systems in operation. Typical sizes range from the order of a few kW to 10 MW (rated electrical power output) for a wide range of working fluids and operating temperatures [9]. The majority of these systems incorporate turbines as the expansion device, which tend to operate at high rotational speeds and are best suited to larger scale systems with higher shaft output powers (>10 kW). Scaling down turbinomachines for smaller-scale applications presents non-trivial challenges and can be expensive and complex. Instead, positive-displacement machines may be used, which can operate at lower rotational speeds. For example, rotary screw expanders are used by some commercial manufacturers with power outputs as low as 30 kW [9]. Other non-commercial systems have used adapted scroll-compressors run in reverse as the expansion device for power outputs less than 10 kW [10,11]. Specific examples of operational solar-ORC systems include a 1 MW, concentrating solar power (CSP) plant in Arizona, USA, with a collector field area in excess of 10,000 m² [12]; a 3 kW, rural electrification project in Lesotho, Africa, with a parabolic-trough collector field of area 75 m² [10]; and an experimental 2.5 kW, solar reverse osmosis desalination system in Athens, Greece with a non-concentrating evacuated-tube collector array of area 88 m² [11].

The application of ORC technology to domestic solar power is currently without precedent in the UK. The limitations of solar availability and collector array size imply that power outputs will be lower than those of the solar-ORC systems found in the literature and mentioned above. This turns the attention towards positive-displacement expanders instead of turbinomachines as prime movers. In particular, reciprocating expanders, with their simple and rugged construction, and more advanced state of development, can be produced at low cost and with high reliability; and are able to operate with high efficiency at low power outputs, as shown by Zahoransky et al. [13].

The success of any technology aimed at the domestic market is dependent not only on efficiency, but also on affordability and reliability. The ORC components and the solar thermal collector must be matched to the system so that the highest yield of mechanical or electrical energy can be achieved for the lowest possible cost. The choice of an appropriate working fluid has a crucial role to play in maximising the net flow of heat (in fact, exergy) into the cycle, which is equal to the maximum net flow of work that can be extracted (in a reversible cycle). This net heat (and work) can be represented by the area enclosed by the cycle in a T-s diagram (see Fig. 2).

The shape of the saturation curve is a characteristic of the working fluid and determines the various processes around the thermodynamic cycle. Wet fluids (such as water) with a negative dry saturation curve, or $\partial T/\partial s < 0$, are typically considered less favourable for Rankine cycles with low-to-medium temperature heat sources. During expansion a wet fluid can undergo condensation if the two-phase region is entered, which, depending on the expander type and design, may result in significantly increased thermodynamic (exergy) losses or even damage to the expander in the case of turbinomachines. Entering the two-phase region can be avoided by superheating, although this can lead to a compromise on performance, or by selecting a dry or isentropic fluid with a positive or vertical/near-vertical saturation curve.

Increasing the pressure of the working fluid in the evaporator, thereby raising its boiling point, is one way to increase the area enclosed by the cycle. However, the temperature of the heat source (which in this work is taken as a hot fluid stream returning from a solar collector) imposes a limit on the maximum pressure in the cycle for a given working fluid, just as the temperature of the cold sink places a limit on the minimum pressure. In addition, high pressure ratios demand more heavily designed and engineered (or multiple stages of) compression and expansion components, that can lead to elevated costs. The temperatures and flow-rates of the heat source stream, the cold sink stream and the working fluid must all be considered for the best possible match with the operating pressures in the cycle.

This paper considers the isentropic fluid R245fa, which has been documented previously as being well-suited for ORC applications due to its thermodynamic properties, and also its low flammability, corrosiveness and global warming potential [14]. Only subcritical cycles are considered. Although supercritical ORCs have been shown by Chen et al. [14] to offer an improved thermal match between the heat source stream and the cycle during evaporation, with reduced irreversibility due to heat transfer across the finite temperature difference in the evaporator, the requirement for higher pressures can cause operational difficulties and safety concerns as well as increased cost of components. Furthermore, the present paper examines only a basic four-component ORC system, although it is acknowledged that variants such as the recuperative cycle can offer efficiency improvements (albeit at a cost), and as such will be investigated in future work.

1.3. Considerations for a UK domestic CHP system

The unique aspect to this particular application of solar-ORC technology is that is it is targeted for the domestic market in a geographical region that receives a relatively low yield of solar irradiation. Therefore the anticipated power output (<1 kW) in this case, which is a strong function of the regional solar and weather data, is expected to be lower than those of other solar-ORC systems discussed in the literature above. This leads to a unique combination of requirements for the system, notably: (1) low-cost solar collectors with high solar absorption efficiency under low-level and diffuse irradiation conditions, designed to deliver the highest possible outlet temperatures; (2) a small-scale expander (most likely a positive-displacement machine) that is able to operate with high efficiency under full-load and part-load conditions; (3) a working fluid and cycle design that minimise efficiency losses due to thermodynamic irreversibilities, particularly associated with heat transfer processes into and out of the cycle.

When matching the ORC to a solar heat source, the maximisation of work output from the cycle is best achieved by using solar heat at the highest available temperature, thus increasing the Carnot efficiency. However, this leads to a trade-off against the solar collector efficiency, which decreases at elevated collector temperatures due to increased thermal losses to the environment. Thus, it is expected that there is a compromise temperature at which the efficiency of the entire system will be highest, and beyond which the efficiency will fall due to the increased thermal losses [15–17].

Concentrating solar technologies such as the parabolic-trough collector (PTC) are designed to reach high (up to 400–500 °C) temperatures. Concentration ratios are typically up to ×10. The
small surface area of the absorber relative to the aperture of the
trough means that there are lower heat losses and higher efficien-
cies than for non-concentrating collectors. However, in order to
focus the reflected sunlight onto the absorber surface, the incident
beam must be parallel to the axis of the mirror to be useable. This
has two implications: (1) the collector can only use direct sunlight
and not the diffuse component that is a result of scattering by
atmospheric particulates; (2) a concentrating collector’s perfor-
mance is markedly improved if it is able to track the position of
the sun as it moves across the sky rather than remaining stationary
at a fixed orientation and elevation.

Concentrating collectors are therefore more appropriate in
locations that receive a high proportion of direct sunlight. Due to
cloudy skies in the UK, approximately 60% of annual global
irradiation received on a horizontal surface is diffuse [4]. This is
significantly higher than for example Palermo, Sicily, which has
an annual average diffuse to global irradiance ratio of 40% [4].
Non-concentrating collectors can potentially offer a higher yield
of solar energy in the UK, making use of the large proportion of
diffuse sunlight, but this is offset by the lower temperatures than
can be achieved compared to concentrating collectors. Thus the
choice of collector type is a trade-off between total yield and qual-
ity of the heat source where the deciding factor is the maximum
power from the ORC and the comparative cost.

In addition, and for similar reasons to those above, the temper-
ature of the cold reservoir should be minimised, further maximis-
ing the cycle efficiency. Heat rejection to ambient air is cheap and
simple, but dependent on weather conditions. Air temperatures
tend to be highest when solar irradiance is most abundant, mean-
ing a lower potential to reject heat. The condenser therefore needs
to be either very large in surface area or fan-assisted to achieve
high air flow-rates, which carries an associated energy require-
ment/penalty.

Mains water in the UK is typically available at around 10 °C
throughout the year, making water-cooling an attractive option
for heat rejection in a domestic setting. However, there are
associated issues of cost and wastefulness if the cooling water is
discharged without further use. In a combined power and
hot-water system, a proportion of this preheated water could be
used to top-up a domestic hot water storage cylinder in place of
cold water from the mains, thereby providing some measure of
heat and water recovery. Yet in an unvented system with no
intermediate storage capability, this would be reliant on the
demand for electricity and hot water being simultaneous. The
solar fluid may also be used to heat the hot water cylinder, as it
would in a conventional solar hot-water system. This leads to an
inevitable trade-off with electrical output from the ORC, but may
be preferable at times of high hot water demand and low
electricity demand.

In this paper we develop a techno-economic model of a
combined solar heat and power system (CSHP) using an ORC with
R245fa as the working fluid. Simulations are performed to assess
the potential and suitability of such a system for the specific case
of the UK. The system rejects heat to mains water, with the option
of recovery for domestic use when required. Concentrating and
non-concentrating collectors are compared.

2. Techno-economic model methodology

2.1. Model description

The model developed in order to assess the performance of the
UK-based CSHP system (see Fig. 1) was configured to provide
results on a daily basis, which were then used to calculate the
monthly and annual outputs of the system. All simulations were
run in MATLAB. The inputs to the model were the solar irradiance,
and the hot water and electricity demands for a 24-h period. These
inputs vary depending on the month of the year, and the electricity
demand also varies depending on whether a particular day is a
weekday or a day on the weekend. Hence, to obtain annual results
the simulation of the system was performed 24 times (12 months;
twice per month: one for a weekday and one for a weekend day),
and the values compiled to obtain monthly, and then annual,
results.

For an individual 24-h day simulation, finite time elements
were used with a default interval of 1 min such that the 24-h
period is divided into a set of 1440 inputs and outputs. This
temporal interval was chosen as it was found to be necessary in
providing a stable numerical scheme during the system
simulations (see Section 2.5). The ORC cycle analysis was
based on thermodynamic fluid data for the organic compound
R245fa that was obtained through the NIST Database [18].
Lookup tables of the key thermodynamic properties for the fluid
were used.

Three variants were tested for the ORC heat source. These are:
(1) a non-concentrating, evacuated-tube collector (ETC) array
facing due-south with a fixed inclination angle of 36° from the
horizontal (found to be the optimum angle for the highest annual
yield of solar irradiation in the UK [4]); (2) a concentrating
parabolic-trough collector (PTC) array, also fixed at the same
orientation and inclination as above; and (3) a concentrating PTC
array assumed to have perfect 2-axis solar tracking such that its
 aperture plane is orientated normal to the direction of direct
 solar irradiance at all times. It should be noted from this point
 onwards that the term “evacuated tube” will refer exclusively to
the non-concentrating collector variant (although it is acknowledged
that some concentrating parabolic-trough collectors also incorporate
an evacuated tube as insulation around the absorber pipe).

The solar collector efficiency was calculated from efficiency
curve coefficients, provided as standard by the manufacturers
(see Table 1). This is a more simplistic approach than those found
elsewhere (e.g. [10,19]), where optical and thermal efficiencies are
calculated separately, and individual energy balances are solved for
each layer or surface. The only collector geometry information
required to perform the model calculations is the total array area.
Some evacuated-tube collectors, such as the high efficiency
Sydney-tube collector modelled in this work, incorporate curved
mirror reflectors (known as compound parabolic concentrators,
CPC) behind the tubes to recover some of the irradiance that falls
between the individual elements. The mirrors are non-focusing
and thus do not require a solar-tracking device and do not affect
the collector's ability to use diffuse radiation.

Other assumptions applied to the collector array were: (1) for
simplicity the collector fluid was modelled as pressurised liquid
water rather than a water/glycol mixture; (2) solar irradiance that
is not absorbed as heat by the collector fluid is lost to the environ-
ment; and (3) the water flow-rate is divided uniformly over the
entire collector array (area). The configuration of the solar collector
fluid circuit is an active closed-loop system in which the collector
fluid first enters the ORC evaporator heat exchanger where it is
cooled by the working fluid and then flows to the heat exchanger
located in the hot water cylinder where it is cooled further. The
fluid then returns to the solar collector where it is re-heated to
begin the next cycle. A bypass was included in the circuit to allow
the fluid to recirculate in the collector until its temperature is suf-
ficiently high to evaporate the ORC working fluid.

Finally, an auxiliary heater and cold water mixing device are
included in the CSHP system model to control the final tempera-
ture of the hot water delivered for domestic consumption. It is
assumed for simplicity in the model that all hot water is required
at 60 °C.
2.2. Model equations

2.2.1. Solar collector array

The solar collector efficiency is calculated as a function of the incident solar irradiance, the mean collector temperature and the ambient air temperature as specified in European Standard EN 12975 [20]:

$$g_{sc} = \frac{c_0}{C_0} I_{sol} - c_1 \frac{T_{sc} - T_{ext}}{I_{sol}} - c_2 \left( \frac{T_{sc} - T_{ext}}{I_{sol}} \right)^2,$$

where $I_{sol}$ may be direct normal or global irradiance depending on the collector type (see Table 1).

If it is assumed that the collector cover is a glass plate with a thickness of the order of 10 mm or smaller (typically 2–4 mm) and that solar heat-flux variations have a 24-h period, the Fourier number is $>1$ and the collector can be assumed to be in a thermal steady-state ($\partial T/\partial t \to 0$). In this case, the thermal energy absorbed by the solar-collector fluid is $Q_{sol} = \eta_{sc} I_{sol} A_{sc}$. It is thus assumed that all absorbed solar energy is imparted to the mass of fluid in the collector $M_{sc}$, causing its temperature to rise. The mass of fluid held by the ETC collector is 0.42 kg/m$^2$ gross area (taken from manufacturers’ data). The fluid capacity of the PTC collector chosen for this study is not known; therefore a value of 0.45 kg/m$^2$ gross area is used, which is based on an alternative small-scale PTC collector model for which this data is available (the NEP PolyTrough 1800 [21]). The energy balance for the collector is:

$$M_{sc} c_p \frac{dT_{sc}}{dt} = Q_{sol} + m_{sc} c_p (T_{sc,in} - T_{sc}) - m_{sc} c_p T_{sc,out}.$$  \hfill (2)

The solar collector array is assumed to consist of a number of identical solar collectors connected in series in order to produce the highest possible outlet temperature from the array. For greater model accuracy it is possible to split the collector array into finite elements in order that the change in collector efficiency with temperature may be modelled along the length of the array. This is given further consideration in Section 2.5.

2.2.2. Solar fluid pump

The electrical power consumed by the solar pump, $W_{pump,sc} = \dot{m}_{sc} c_p \Delta P / (\eta_{pump,sc} P_w)$, is calculated using a fixed value of 65% for $\eta_{pump,sc}$. The pressure drop $\Delta P$ comprises pressure losses through the system pipework and components. The pipework is assumed to be of total length $L = 20$ m and diameter $D = 20$ mm. The friction factor that is related to the pressure loss is calculated according to

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Table 1

Collector efficiency curve coefficients for an evacuated-tube and a parabolic-trough collector. Reference area is the total collector area. $I_{sol}$ denotes the irradiance value used for the efficiency curve calculation in W/m$^2$ in Eq. (1). Values (with significant figures) are taken directly from the manufacturers’ data sheets.

| Type       | Model          | $c_0$  | $c_1$  | $c_2$  | Ref. | $I_{sol}$ |
|------------|----------------|--------|--------|--------|------|-----------|
| E.T.C.     | Microtherm SK-6| 0.612  | 0.54   | 0.0017 | [23] | Global    |
| P.T.C.     | PTC 1000       | 0.70   | 0.2044 | 0.001545 | [54] | DNI       |

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Fig. 1. Schematic representation of the CSHP system.
correlations based on the Reynolds number given in Incropera et al. [22]:

\[
f = 0.316 \text{ Re}^{-1/4}; \quad \text{Re} < 2 \times 10^4;
\]

\[
f = 0.184 \text{ Re}^{-1/2}; \quad \text{Re} \geq 2 \times 10^4,
\]

where \( \Delta P/L = 8 \bar{m} \rho_c D^4/\pi^2 \rho_\omega D^5 \) and Re is based on the pipe diameter.

The pressure loss through the heating coil inside the hot water cylinder is calculated according to its required length, and also assumes a \( D = 20 \text{ mm} \) internal diameter. Finally, it is important to consider the pressure-drop characteristic of the solar collector array – particularly as it is assumed that the collector modules are connected in series. No pressure drop data is available for the collector models in Table 1. Therefore, an indicative pressure drop is calculated for the evacuated-tube collector based on empirical data for a similar product (the Consolar Tubo 11, also a 6-element collector with a U-tube heat exchanger design [23]). The expression used is:

\[
\Delta P_{\text{C}} = A_{\text{C}} (21.77 \bar{m}_{\text{sc}}^2 + 3.54 \bar{m}_{\text{ac}}).
\]

Due to a lack of available information about the geometry and flow characteristics of the parabolic-trough collector, the pressure drop characteristic in Eq. (5) is used to represent both ETC and PTC collector arrays.

### 2.2.3. ORC fluid pump

The input parameters for the ORC pump calculations are the working fluid flow-rate, the cycle evaporation and condensation pressures, and the isentropic efficiency of the pump, \( \eta_{\text{pump,ORC}} \). Here, \( h_1 \) and \( h_2 \) are the specific enthalpies at the inlet and outlet of the pump respectively (refer to Fig. 2), and \( h_{\text{sc}} \) is the specific enthalpy following an isentropic pumping process that starts from the same state at the pump inlet (State 1) and ends at the same pressure at the pump outlet. State 1 is fixed from the knowledge of the ORC condensation pressure, given that the working fluid is a saturated liquid at that point. The rise in enthalpy of the working fluid through the pump is then calculated from the known isentropic efficiency. A fixed value of \( \eta_{\text{pump,ORC}} = 65\% \) was chosen, which is within the range of values used in similar studies [24–27]. The input power required by the pump is then \( W_{\text{pump,ORC}} = \eta_{\text{pump}} (h_2 - h_1)/\eta_{\text{comb}} \), with a value of \( \eta_{\text{comb}} = 90\% \) taken as the overall (mechanical and electrical) efficiency of the pump and motor [28].

#### 2.2.4. ORC evaporator

The heat addition process to working fluid in the evaporator is assumed to be an isobaric process (zero pressure loss), such that the heat source at State 3 (working fluid outlet; see Figs. 1 and 2) is equal to that at State 2. A fixed pinch temperature difference of \( \Delta T_{\text{pin}} = 5 \text{ K} \) is assumed between the heat source fluid inlet and the ORC working fluid outlet (also expander inlet) at State 3, such that the working fluid outlet temperature is calculated as:

\[
T_3 = T_{\text{hs,in}} - \Delta T_{\text{pin}}.
\]

Hence, State 3 can be fixed by the knowledge of the (imposed) evaporation pressure and the temperature at that point, from Eq. (6).

The heating stream flow-rate must be sufficient such that the temperature difference between solar-collector fluid and ORC working fluid is not less than the minimum pinch difference at any point along the length of the heat exchanger. It was stated for Eq. (6) that the minimum pinch occurs at the working fluid outlet. However, for an isothermal boiling process, a second critical pinch point can also occur at Point x (in Fig. 2), where the working fluid begins to boil (State b). At this point, it is ensured that the temperature difference between the heating stream and the working fluid stream is greater than or equal to the pinch difference, \( T_{\text{hs,x}} - T_b \geq \Delta T_{\text{pin}} \).

From Point x to the working fluid outlet from the evaporator (State 3), the increase in the specific enthalpy of the working fluid stream is:

\[
\Delta h_{b-3} = h_3 - h_b,
\]

so, by using a heat (enthalpy) balance on the two sides of the heat exchanger, the corresponding hot side (source stream) temperature at Point x is:

\[
T_{\text{hs,x}} = T_{\text{hs,in}} - \frac{\bar{m}_{\text{at}} \Delta h_{b-3}}{\bar{m}_{\text{at}} c_{\text{p,at}}}.
\]

A minimum heat source temperature is required for system operation, at which the ORC engine may undergo the cycle without the aforementioned pinch limitations in the heat exchangers being violated. In the present work the evaporation and condensation pressures, and solar collector and ORC working fluid flow-rates are set as fixed input parameters to the model. Starting from the temperature at which the working fluid is a saturated vapour at the evaporation pressure, \( T_b \), the temperature of the working fluid at the expander inlet \( T_3 \) (and also the temperature of the solar-collector fluid at the evaporator inlet that is 5 K higher, since \( T_{\text{hs,in}} = T_3 + 5 \text{ K} \) from Eq. (6)) are increased incrementally until the condition in Eq. (7) is satisfied. This heat source inlet temperature that satisfies the condition is selected as the set-point temperature \( T_{\text{hs,sp}} \) that is then used in Eq. (14) (Section 2.2.7).

#### 2.2.5. ORC expander

The enthalpy at the exit of the expander (State 4) is calculated by using a specified isentropic efficiency \( \eta_{\text{exp}} = (h_3 - h_4)/(h_3 - h_2) \), and the system electrical power produced by the expander is \( W_{\text{E,ORC}} = \eta_{\text{exp}} \bar{m}_{\text{at}} (h_3 - h_4) \), where \( h_1 \) is known (from Section 2.2.4) and \( h_{\text{at}} \) is the specific enthalpy at the expander exit for an isentropic process, and where a value of 90% is taken for the efficiency of the generator \( \eta_g \) [29]. For a low power-output system (<1 kW) such as this, positive-displacement expanders may be favoured over turbomachines (as discussed Section 1.2). Thus, for all simulations, an isentropic efficiency of 75% is used which is typical of those reported for small-scale positive-displacement
2.2.6. ORC condenser

The cooling water inlet to the condenser is set to a fixed temperature of 10 °C. It is assumed that the working fluid enters the condenser as a vapour at State 4 and leaves as a saturated liquid at State 1, which was defined in Section 2.2.3. The flow-rate of cooling water must be sufficient such that the temperature difference between working fluid and cooling water at all points along the heat exchanger does not exceed the pre-defined pinch difference. For the condenser, the critical pinch-point (Point $x$) is found to occur at the point at which the working fluid begins to condense, i.e. State $c$. At this point, the temperature difference between the working fluid and the water must be greater than or equal to the pinch difference, such that:

$$T_c - T_{cw,in} \geq \Delta T_{pin}. \quad (10)$$

From Point $x$ to the working fluid outlet of the condenser, the specific enthalpy drop in the working fluid is as follows:

$$\Delta h_{x-1} = h_c - h_1, \quad (11)$$

and the corresponding cold-side temperature at Point $x$ is:

$$T_{cw,x} = T_{cw,in} + \frac{m_{cw}\Delta h_{c-1}}{m_{fw}}. \quad (12)$$

The only unknown in Eq. (12) is $m_{cw}$.

Starting from an initial value, this parameter is adjusted incrementally until the condition in Eq. (10) is satisfied. The minimum value that satisfies the condition is then used as the cooling water mass flow-rate to calculate the annual cooling water requirement of the system. The cooling water temperature at the condenser outlet has been raised above the mains temperature. Some of this rejected flow may be used as preheated feed water to top-up the hot water cylinder (as indicated by the dashed line in Fig. 1), thereby reducing the energy consumed for domestic hot water heating. The limit for the amount of preheated water that can be determined by the hot water demand, in Eq. (19). It is assumed that any surplus cooling water is discharged to drain.

2.2.7. On/off switching and bypasses

The model uses a series of logic statements to determine whether certain parts of the system are on or off. In the solar collector circuit, a (fixed) flow of fluid through the collector $m_{sc}$ is set (by switching on the pump to its predetermined design point) only when there is availability of solar irradiance $I_{sol} > 0$, such that the pump does not operate at night without benefit:

$$m_{sc} = \begin{cases} 0 & \text{if } I_{sol} = 0; \\ m_{fw} & \text{otherwise}. \end{cases} \quad (13)$$

In the ORC working fluid circuit, a (fixed) flow through the evaporator $m_{w}$ is set (by switching on the pump to its predetermined design point), thus allowing the ORC engine to operate, only when the temperature of the solar collector fluid at the collector’s outlet $T_{sc,out}$ reaches the evaporator heat source temperature set-point $T_{hs,sp}$, as calculated from Eq. (9):

$$m_{w} = \begin{cases} 0 & \text{if } T_{sc,out} < T_{hs,sp}; \\ m_{w} & \text{otherwise}. \end{cases} \quad (14)$$

When the working fluid flow-rate is zero, the solar heat transferred to the ORC is also zero. In this case, the solar collector fluid is returned to the collector for further heating. This bypassing continues to occur until $T_{sc,out} > T_{hs,sp}$ and the ORC engine is triggered to run.

2.2.8. Domestic hot water storage cylinder

The hot water storage cylinder is assumed to consist of three fully mixed equal-volume segments which divide the cylinder along its vertical axis. Each segment is represented by a node $N$, where $N = 1$ is the bottom segment and $N = 3$ is the top one. A heat exchanger coil, through which solar collector fluid is passed, is located in the bottom segment ($N = 1$).

The cylinder equations are based on those for the Multiport Store model developed for TRNSYS [36]. A further model input is the cylinder flow-coefficient, $F_{coil}$, which defines the proportion of the solar collector return flow (0–100%, fixed) that is diverted through the cylinder’s heat exchanger coil. By default the solar collector fluid leaving the ORC evaporator is sent back to the collector to be heated again. If some of this fluid is instead diverted to the heat exchanger in the hot water cylinder there will be a reduction in the fuel demand to heat the water conventionally. However this will also decrease the collector return temperature, eventually reducing the heat available as input to the ORC when the fluid next passes around the system.

The system is set to prioritise electricity generation over water heating, therefore when the ORC engine is bypassed the hot water cylinder is also bypassed in order for the collector fluid to come up to temperature as quickly as possible. When $T_{sc,out} > T_{hs,sp}$ the ORC engine is switched on but collector fluid leaving the ORC evaporator will be sent to the hot water cylinder heating coil only if it is at a higher temperature than the hot water cylinder plus a pinch difference. No heating is delivered to the hot water cylinder if its mean temperature is above the maximum hot water storage temperature of 80 °C. This strategy can be summarised as:

$$m_{coil} = \begin{cases} F_{coil}m_{cw} & \text{if } m_{out} > 0; \\ F_{coil}m_{cw} & \text{and } \\ T_{hs,sp} > T_{hwc(N-1)} + \Delta T_{pin} & \text{and } T_{hwc(N-3)} < 80 \, ^\circ \text{C}; \\ 0 & \text{otherwise}. \end{cases} \quad (15)$$

The energy balance for the hot water cylinder is as follows:

$$M_{hwc,N}c_{p,w}\frac{dT_{hwc,N}}{dt} = Q_{dhw,N} + Q_{sl,N} + Q_{cond,N} + Q_{dump,N} - Q_{L,N}. \quad (16)$$

The amount of heat transferred in the coil from the solar fluid to the hot water cylinder is calculated from:

$$Q_{dhw,N} = \begin{cases} m_{coil}c_{p,w}(T_{coil,in} - T_{coil,out}) & \text{if } N = 1; \\ 0 & \text{otherwise}. \end{cases} \quad (17)$$

where the collector fluid leaving the coil is at the temperature:

$$T_{coil,out} = T_{hwc(N-1)} + \Delta T_{pin}. \quad (18)$$

The hot water demand profile is constructed from hourly UK data published in a study by the Energy Saving Trust [37], which has been converted from a volumetric to a mass flow-rate, $m_{hwc}$, in kg/s. Hot water extraction for domestic consumption results in a thermal energy exchange between the nodes of the hot water cylinder, as cold water is fed in at the bottom to replace the water that is drawn-off from the top:

$$Q_{0,N} = \begin{cases} m_{hwc}c_{p,w}(T_{top} - T_{hwc,N}) & \text{if } N = 1; \\ m_{hwc}c_{p,w}(T_{hwc(N-1)} - T_{hwc,N}) & \text{otherwise}. \end{cases} \quad (19)$$

The temperature of the water that re-supplies the cylinder is either at the water mains temperature (set to $T_{main} = 10 \, ^\circ \text{C}$), or at a higher temperature as a result of utilising preheated cooling water leaving the ORC condenser. In order to ensure that the flow-rate of preheated cooling water from the condenser that is
then used to re-supply the hot water cylinder is maximised within the limit imposed by the instantaneous household demand for hot water, the following strategy is applied at each time-step:

$$T_{\text{top-up}} = \begin{cases} F_{\text{ph}} T_{\text{cw, out}} + (1 - F_{\text{ph}}) T_{\text{main}} & \text{if } m_{\text{hw}} > 0; \\ T_{\text{main}} & \text{otherwise.} \end{cases}$$ (20)

where $F_{\text{ph}} = m_{\text{cw}} / m_{\text{hw}}$ is the ratio of the condenser cooling-water flow-rate to the flow-rate of domestic hot water extracted from the hot water cylinder.

The heat flux due to conduction between the layers of the store is calculated from:

$$Q_{\text{con,N}} = \frac{k_w \pi D_{\text{hwc}}}{4H_{\text{hwc,N}}} (T_{\text{hwc,N-1}} + T_{\text{hwc,N-1}} - 2T_{\text{hwc,N}}),$$ (21)

while the thermal loss through the wall of the cylinder to the ambient internal environment is:

$$Q_{L,N} = U_{\text{hwc}} A_{\text{surf}} h_{\text{cw,N}} (T_{\text{hwc,N}} - T_{\text{int}}),$$ (22)

where the indoor environmental temperature surrounding the cylinder $T_{\text{int}}$ is assumed constant at 20°C.

An upper temperature limit $T_{\text{wf,max}}$ of 500 K (227 °C) is imposed for the ORC operation, since this is the maximum temperature for which thermodynamic properties for the R245fa working fluid are available from the online NIST database [18]. Above this temperature, excess solar heat is rejected directly to the hot water cylinder, bypassing the ORC:

$$Q_{\text{dump,N}} = \begin{cases} m_{\text{cw}} c_p (T_{\text{sc,out}} - T_{\text{wf,max}} - \Delta T_{\text{pin}}) & \text{if } T_{\text{sc,out}} > T_{\text{wf,max}} + \Delta T_{\text{pin}} \text{ and } N = 1; \\ 0 & \text{otherwise.} \end{cases}$$ (23)

It is noted that the actual ORC working fluid temperatures attained in the annual simulations are far lower than this limit and that the degree of superheating is small for the average day conditions, as demonstrated in Fig. 2. The maximum temperatures reached in the cycle during the summer months in the annual simulations are not higher than 150–160 °C.

### 2.2.9. Electricity generation and demand

To assess the likely proportion of electricity demand covered by the CSHP system, electricity demand profiles were generated for each month using a model of domestic electricity use developed by the Centre for Renewable Energy Systems Technology (CREST) [38]. The CREST model generates electricity usage profiles for households based on active occupancy patterns, human activity profiles and typical appliance energy consumption data. The occupant behaviour is modified based on the month, the number of occupants, and the selection of a weekend or week day.

The auxiliary electricity consumption was calculated as the proportion of the electricity demand not met by the CSHP system. It is assumed that this proportion of the demand is met by buying the electricity from the grid:

$$W_{e,\text{aux}} = \begin{cases} W_e - W_{e,\text{ORC}} & \text{if } W_e > W_{e,\text{ORC}}; \\ 0 & \text{otherwise.} \end{cases}$$ (24)

When electricity produced by the solar-ORC system exceeds the demand, the surplus quantity is not necessarily disregarded as this can be sold back to the grid to generate income for the CSHP system owner. However, this revenue is not currently included in the annual savings calculation.

### 2.2.10. Auxiliary heating and hot water demand

For the sake of simplicity, it is assumed that all auxiliary hot-water heating takes place downstream of the hot water cylinder. The demand for auxiliary water heating is then the energy required to bring the water from the cylinder temperature to the required supply temperature, which is $T_{\text{supp}} = 60$ °C.

$$\dot{Q}_{\text{hw,aux}} = m_{\text{hw}} c_p (T_{\text{supp}} - T_{\text{hwc}}).$$ (25)

### 2.3. Exergy analysis

#### 2.3.1. Whole system exergy analysis

The equations in this section are relevant for the whole system exergy analysis, results from which are presented in Section 3.5. In this analysis, the exergy destroyed in each component of the CSHP system is tracked in order to identify the components which have the greatest capacity to limit the system’s efficiency. Exergy is the upper limit for the convertible work in cooling a stream of fluid down to a ‘dead state’ while extracting work reversibly:

$$\dot{X}_R = m_{\text{h}} [(\dot{h}_R - \dot{h}_0) - T_0 (\dot{S}_R - \dot{S}_0)].$$ (26)

where $T_0$ is the dead state temperature, taken here to be either that of the cooling medium $T_{\text{cw}}$ or of the ambient air $T_{\text{air}}$, depending on which is lowest.

In the present work a number of definitions are used which follow the exergy topological method as used by Mago et al. [39] in a study of a non-regenerative ORC system. The rate of exergy destroyed in each component $\dot{X}_{\text{dest}}$ is equal to the total exergy flowing into the component $\dot{X}_R$ minus the total exergy flowing out $\dot{X}_{\text{out}}$. Available exergy $\dot{X}_{\text{ava}}$ is the amount of exergy removed from the exergy source, which may be a fluid stream (in the case of a heat exchanger), radiation source (in the case of the solar collector) or prime-mover (in the case of a pump or compressor). Used exergy $\dot{X}_{\text{use}}$ is the amount of available exergy that is delivered as a useful output by the component. For an expander this will be shaft work whereas for a heat exchanger this will be the exergy increase experienced by the cold stream. The exergy efficiency is then defined as $\eta_{\text{ex}} = \dot{X}_{\text{use}} / \dot{X}_{\text{ava}}$. Additional indicators of interest are: (1) the degree of thermodynamic perfection $\nu = \dot{X}_{\text{ava}} / \dot{X}_{\text{ava,sys}}$; and (2) the influence coefficient $\beta = \dot{X}_{\text{ava}} / \dot{X}_{\text{ava,sys}}$, which represent the proportion of exergy that is not destroyed and the fraction of the available exergy of the entire system that is associated with a particular component, respectively.

For a solar collector, the exergy flow associated with the solar irradiance incident on its surface represents the maximum portion of the radiation energy that can be converted to useful work, and is given by [16]:

$$\dot{X}_{\text{sol}} = \left(1 - \frac{T_0}{T_{\text{sun}}} \right) \dot{I}_{\text{sol}}.$$ (27)

where $T_{\text{sun}} \approx 0.75T_{\text{sun}}$ is the equivalent temperature of the sun as an exergy source, and $T_{\text{sun}} = 5778$ K is the sun’s effective black-body temperature.

#### 2.3.2. Solar collector maximum power analysis

The equations in this section are relevant for the solar collector maximum power analysis, results from which are given in Section 3.1. Beyond accounting for the useful work lost in the various components of the whole CSHP system, exergy analysis may also be used to evaluate the theoretical maximum power that could be delivered by a solar collector operating at its optimum temperature for a given set of climatic conditions. This approach provides a useful basis for the control of collector operation through the adjustment of the mass flow-rate, and is the subject of numerous studies in the literature including [17,40,41]. It also enables the
comparison of a range of collector designs and will be used here to compare the maximum power deliverable under UK climate conditions from the concentrating and non-concentrating collectors presented in Table 1. In order to evaluate the maximum exergy flow from each solar collector, an ideal situation is considered in which the pressure loss through the collector is neglected and the fluid (assumed to be pressurised water) is returned to the solar collector inlet at the temperature of the heat rejection medium which is also the dead state/environment temperature, such that \( T_{\text{sc, in}} = T_0 \). For this case, Eq. (26) can be expressed as:

\[
X_{\text{sc, out}} = \eta_{\text{sc}} f_{\text{col}} A_{\text{sc}} - m_{\text{sc}} C_P T_0 \ln \frac{T_{\text{sc, out}}}{T_0},
\]

(28)

where \( \eta_{\text{sc}} \) is calculated according to Eq. (1) as a function of \( f_{\text{col}} \) and \( T_{\text{sc}} = T_{\text{sc, avg}} = (T_0 + T_{\text{sc, out}})/2 \). Furthermore, because \( T_{\text{sc, out}} \) is varied by adjusting the mass flow rate through the collector \( m_{\text{sc}} \), it is possible to eliminate the latter of these two variables, so that Eq. (28) becomes:

\[
X_{\text{sc, out}} = \eta_{\text{sc}} f_{\text{col}} A_{\text{sc}} \left[ 1 - \left( \frac{T_{\text{sc, out}}}{T_0} \right)^{-1} \right] \ln \frac{T_{\text{sc, out}}}{T_0}.
\]

(29)

2.4. Cost evaluation

The capital cost of the CSHP system was estimated by summing the component costs described in the sub-sections below. In this work the economic evaluation of the system was performed in the absence of any financial incentives, such as the FIT.

2.4.1. ORC evaporator and condenser

The four main components of the ORC system, i.e. the pump, expander and two heat exchangers (evaporator and condenser), represent a significant proportion of the cost of the overall system. Approximate estimates of the fraction of the overall installation costs of ORC systems, when these are used in waste heat recovery applications, indicate that these four components account for approximately 50–70% of the overall installed system cost [42,43]. In the present work, an estimate of the system cost is made by adding the individual costs of these four main components.

For heat exchangers, the main factors that influence cost are the choice of fluids and the rate of heat transfer per unit log-mean temperature difference, \( Q/\Delta T_{LM} \). This forms the basis of the C-value method for approximate costing of heat exchangers described in Hewitt et al. [44]. Although tables of C-values for heat exchangers are widely available, the range of costs and duties in these charts were found to be too high to be applicable to the small-scale system considered here. Instead, market prices were obtained for a range of small brazed-plate heat exchangers. The heat exchangers were selected for the appropriate fluids under consideration and for a range of duties, flow-rates and temperature differences. The data obtained was correlated to find an approximate relationship between cost in £ (GBP), and heat transfer per unit log-mean temperature difference \( Q/\Delta T_{LM} \) in W/K. In the model the cost curve is interpolated to return approximate costs for the evaporator and condenser based on the calculated \( Q/\Delta T_{LM} \) values.

2.4.2. ORC pump and expander

As highlighted in Section 1.2, the small-scale application may favour the use of positive-displacement expanders. Recent studies have noted that such expanders are not widely available at present as off-the-shelf items specifically for use in power generation [10,45]. Instead, equivalent data for a range of positive-displacement compressors (including piston, scroll, screw and rotary vane) was obtained. Compressor cost was tabulated as a function of volumetric flow rate and pressure ratio. A three-dimensional interpolation was then used to return an individual cost from this data using the system parameters in the MATLAB model. A similar approach was followed for the costing of the working fluid pump. Data was obtained for pumps at a range of flow duties and pressures; in particular vane pumps and diaphragm pumps, which are capable of delivering high pressures at low flow duties, were examined.

2.4.3. ORC ancillary items

As stated above, the pump, expander, evaporator and condenser form the base equipment cost. Further to these, additional ancillary ORC component costs were calculated as a percentage of the purchased equipment cost based on the cost factors in Łukawska [43]. These were piping (9%), installation (6%), instrumentation/controls (5%) and electrical equipment (4%).

2.4.4. Solar system costs

Approximate market prices for the solar system components were obtained from a brief market survey of packaged solar systems and are listed in Table 2. These are typically the market value prices for a single item, so in order to estimate the wholesale cost of components for mass-production of the system, the market prices were reduced by an approximate retail factor of 20%.

The aim of the costing exercise was to consider the electricity and hot water generating functions of the system separately, so that a total installed cost can be split into costs associated with electricity generation and costs associated with hot water provision. The reason for this will become evident in the cost analysis in Section 3.4 and the further discussion in Section 4 in which the installed cost of the system is compared with that of a PV-thermal hybrid (PVT) system (producing electricity and hot water), and a PV-only system (producing electricity but no hot water). Therefore the costs in Table 2 were divided between the power generation and hot water sub-systems. For the power generation sub-system, the full cost of the solar collector array was assigned as part of the initial investment. The hot water cylinder was not included as it is not an integral part of the power generating sub-system and was instead assigned to the hot water sub-system capital cost. The ancillary plumbing item costs and the solar thermal system installation cost were assigned as an even split between the electricity and hot water generating systems.

2.5. Model validation

2.5.1. Numerical model methodology

A numerical time-marching (TM) method was used to solve Eqs. (1)–(25) in Section 2.2 and to obtain the response of the overall CSHP system. For the purposes of model validation, the solution from this method was compared to a quasi-steady (QS) equivalent solution, which neglects the thermal inertia of the collector but not of the hot water cylinder that continues to obey the model described in Section 2.2.8. For simplicity, the ORC engine is omitted from the system model for this analysis and all of the fluid leaving the collector is passed through the hot water cylinder heating coil.

Table 2 Solar system costs.

| Item                                      | Cost     |
|-------------------------------------------|----------|
| Parabolic-trough collector                | £155/m²  |
| Evacuated-tube collector                  | £78/m²   |
| Hot water cylinder (150L volume)         | £1150    |
| Ancillary items (pump, expansion vessel, valves and pipework) | £600    |
| Solar thermal system installation cost    | £800     |

* The costs associated with solar-tracking for one of the PTC cases considered, were not included in this study.
Under the TM method, the solar collector fluid temperature exiting the hot water cylinder heating coil at time instant \( i \), \( T_{\text{collect}}(i) \), becomes the entering temperature into the collector at the next time instant \( i + 1 \), \( T_{\text{collect}}(i + 1) \). The QS method involves solving the heat fluxes and temperatures in all loops and components such that the whole system remains in steady-state at each time instant. This method effectively assumes that changes in the external/boundary conditions influencing the system are slow in comparison to the internal dynamics of the system itself. The steady-state solution then varies with time as it responds to the time-varying external/boundary conditions imposed on the system. It is expected that the QS solution should closely approximate the TM solution, providing that the time-step interval over which the TM calculations are performed is suitably small and the above QS assumption is valid. The TM and QS model results are compared in Fig. 3. The two models show excellent agreement for the predicted temperatures in the system.

As a further test, the following model parameters were adjusted: (1) the length of the time-step \( \Delta t \); and (2) the number of discrete spatial elements \( n_e \) into which the collector array is split (equivalent to splitting the array into a number of individual collector modules connected in series), while keeping the total collector area constant. In fact, the two parameters are closely linked within the set-up of the model. As the number of elements (i.e. collector modules) \( n_e \) that make up the array is increased, the mass associated with each element decreases and therefore the time-step length \( \Delta t \) required to avoid calculation instability also decreases. For this reason, the time-step and number of collector elements are varied proportionally, as can be seen in Fig. 4 where the results are shown. Based on a total available area of 15 m\(^2\) and a 1.2 m\(^2\) area for the Microtherm SK-6 ETC collector, the maximum number of in-series collector modules is 12. When the array resolution is increased beyond 9 elements, it is found that there is very little change in the system temperatures predicted by the model. For the single-element array there is a slight under-prediction of the solar collector temperature compared to the higher resolution arrays. The average difference in collector temperature between the single-element and the 9-element model is 2.7 °C. This is equal to 2.6% of the amplitude of the daily collector temperature variation, which is deemed to be an acceptable error margin. Therefore, due to the significant reduction in calculation time, the single-element model was chosen for the annual CSHP system simulations presented in this paper.

Finally, a further detail that must be considered is the effect of increasing the solar collector flow-rate on the calculation stability. It is generally observed that the calculation becomes unstable when \( n_e \Delta t \geq M_c \). For the single-element collector array model, a time-step interval of 60 s is found to be effective at providing a stable calculation output for \( n_e \leq 0.1 \text{ kg/s} \). However, at higher flow-rates the calculations become unstable and therefore the time-step interval is decreased as necessary to maintain stability.

3. Results and discussion

This section is divided into five parts. First, a concentrating and non-concentrating solar collector are compared by way of a maximum power analysis. Next, a daily performance profile of the CSHP system is examined, and this is followed by a manual sensitivity analysis that was performed to optimise the key performance parameters of the system. An annual simulation is then undertaken to determine: (1) the annual electrical power output that can be achieved from the system for a fixed 15 m\(^2\) area of collector array; (2) the cost per unit generating capacity; (3) the additional thermal output achievable and percentage of water-heating demand that can be met; (4) the system running costs, discounted payback time and levelised cost of electricity. Finally an examination of the exergy destruction in each of the system components is performed using the results from the annual simulation.

3.1. Choice of solar collector

In this section, maximum power output from the solar collector calculated in Eq. (29) is evaluated across a range of collector outlet temperatures. Fig. 5a shows a comparison of the maximum power available per unit area from the PTC and ETC collectors under steady-state direct normal irradiance (DNI) equal to 500 W/m\(^2\). In essence this figure, together with Fig. 5b, describes the compromise between the two terms on the RHS of Eq. (28). At high mass flow-rates, the temperature rise of the fluid flow through the collector is low, leading to low collector temperatures and thus high collector efficiencies (described by the first term). Nevertheless, the high mass flow-rate also causes high losses due to irreversible heat transfer (described by the second term). At the other extreme, low flow-rates lead to high temperatures and low collector efficiencies, but also reduced losses due to heat transfer, such that an optimum can be identified at some intermediate flow-rate and temperature. The PTC collector has a maximum exergy output of 96 W/m\(^2\), available at an outlet temperature of 394 °C; whereas the ETC collector has a maximum exergy output of
63 W/m², available at an outlet temperature of 284 °C. This illustrates that when a moderate amount of direct irradiance is available, the PTC collector has a higher potential to generate power. In Fig. 5b the maximum power output is calculated for a collector array of total area 15 m² (considered a representative average value for UK homes [46]) and for a London “annual-average” irradiance condition; with the PTC and ETC ability to use diffuse irradiance taken into account. It is assumed that the ETC array can use the entire (global) incident radiation (considered here for a perfect 2-axis tracking system). The maximum power output for the PTC and ETC collector arrays are 127 W and 104 W respectively. When compared to Fig. 5a it can be observed that, when the differences in useable irradiance are taken into account, the collectors are more closely matched with respect to their ideal maximum power outputs. The optimum outlet temperatures corresponding to maximum power for the PTC and ETC collectors are 159 °C and 104 °C respectively. For comparison, the maximum power output is also shown for a PTC collector at a fixed orientation and tilt angle (due South and 36 ° respectively), revealing a considerably lower power than both the PTC (tracking) and ETC (fixed) arrays. A further case may be considered in which the available solar irradiance consists entirely of the diffuse component, in which case the PTC output will be close to zero, while the ETC collector will retain a potential to generate power.

Finally, it is possible to consider a case where the maximum exergy output from each collector is achieved for the annual range of climatic conditions, by allowing the collector flow-rate to vary in order to track the optimum outlet temperature at all times. By integrating the maximum power over the entire year, the maximum annual work output is calculated, and from this a time-averaged maximum power can be found. This is found to be quite different from the maximum power at the “annual-average” climatic condition, due to non-linearity in the relationship between solar irradiance and collector efficiency. The maximum work is found to be 1685 kW h/year (192 W, average) for the ETC collector and 1707 kW h/year (195 W, average) for the PTC (tracking) collector, revealing a very closely matched ideal performance between the two collectors when considered over the annual period.

3.2. Performance of CSHP system throughout the day

Fig. 6 shows the electrical power output profiles for the CSHP system over the course of a day, for the case in which electricity production from the system is prioritised and solar fluid flow through the hot water cylinder heating coil is set to zero. The climate data used is that for an “annual average” day, which is a 24-h profile made up of annually averaged values for each hour of the day. In this figure, the CSHP system featuring a PTC solar-tracking array (Fig. 6a) is compared to the system featuring a fixed ETC collector array (Fig. 6b). The ORC evaporation pressure and system flow-rates are chosen to be equal to those that maximise the total daily work output for each collector variant (taken from Section 3.4), while obeying the full set of relations stated in Section 2.2.

The PTC system operates at a higher temperature than the ETC system and with a higher evaporation pressure in the ORC. Although this results in a higher instantaneous power production, the operation of the PTC system is more intermittent because the greater decrease in the solar collector fluid enthalpy through the ORC evaporator (with its associated increased heat input to the cycle) cannot be sustained by the collector at the temperature required. Thus for short periods the collector fluid must recirculate within the solar array, bypassing the ORC, while it recovers to the required temperature, before being sent back to the ORC evaporator. The ETC system also displays intermittent operation for a large part of the day, but is able to sustain longer periods of continuous operation during the peak solar hours. With its tracking capability the PTC system is also shown to operate across a slightly longer portion of the day due to its ability to collect irradiance when the sun is low on the horizon. However over the course of the day, the total accumulated work output for the two systems is almost identical.

In Fig. 6c the effect of providing heating to the hot water cylinder is considered for the ETC system. In the case being demonstrated here, 100% of the solar fluid flow is passed through the cylinder heating coil, downstream of the ORC evaporator, before being returned to the collector (see Fig. 1). This results in a further decrease in the enthalpy of the solar fluid stream before it is returned to the collector inlet, which in turn gives rise to the intermittency of ORC operation being more exaggerated than in Fig. 6b. The cumulative work output from the ORC is reduced considerably while the cylinder is being heated (i.e. until ~12:30 p.m.). From the time instant when the upper temperature limit for hot water storage in the cylinder (80 °C) is reached, the work output from the ORC engine increases again. Nevertheless, the early penalty in work generation means that the total cumulative work output over the course of the day is lower than in Fig. 6b.

3.3. Parametric analysis

A parametric analysis was undertaken to examine the influence of the following variables on the electrical output from the system:

1 A brief analysis of daily noon-time irradiance values for London over a whole year [Ref. 47] shows that the mean irradiance is 361 W/m² with a standard deviation of 249 W/m².
(1) cycle evaporation pressure, \( P_{\text{evap}} \); (2) solar collector fluid flow-rate, \( m_{\text{sc}} \); (3) ORC working fluid flow-rate, \( m_{\text{wT}} \); (4) cycle condensation temperature, \( T_{\text{cond}} \); and (5) fraction of solar fluid flow passed through the hot water cylinder heating coil, \( F_{\text{coil}} \). Simulations were performed for the “annual average” day and for the PTC and ETC solar array variants of the system model. In each simulation run, a single parameter was adjusted while the others were held constant.

The objective is to identify important system parameters and to understand their role in affecting the performance of the system, with a view towards maximising the total work output and determining the “maximum power” settings for the annual average condition. It follows therefore that the diverted fraction of solar collector fluid flow sent to the hot water cylinder was initially set to zero, thus prioritising power generation. The cycle condensation temperature was initially set to the lowest practically achievable value of 17 °C (allowing for 10 °C cooling water temperature, with a 5 K condenser pinch and a 2 K temperature glide between cooling water inlet and the pinch-point). Under these conditions, the PTC system (with solar tracking) was found to produce a maximum power output of 82.6 \( W_{\text{avg}} \) at an optimum working fluid evaporation pressure of 16 bar; while the ETC system was found to produce a maximum power output of 80.1 \( W_{\text{avg}} \) at an optimum evaporation pressure of 12 bar. The difference in maximum power output between the PTC and ETC collector array variants is only 3.1%. Detailed results from this analysis are shown in Fig. 7 and discussed below.

The evaporation pressure is strongly linked to the minimum heat source temperature required to operate the ORC engine, as shown in Fig. 7a and d. The optimum collector fluid outlet temperatures found for the PTC and ETC systems are 121 °C and 106 °C, respectively. The simulation results in Fig. 7d can be compared with the collector maximum power curves in Fig. 5b, noting that the optimum heat source temperature for the ETC system is very close to the optimum temperature identified for maximum power from the ETC collector (104 °C). For the PTC (tracking) system the optimum heat source temperature is considerably lower than the optimum collector outlet temperature identified in the maximum power analysis (160 °C). This may be in part due to the limitations imposed by the choice of the working fluid R245fa, whose critical temperature is 154 °C.

The work output is observed to plateau at solar collector mass flow-rates above 0.1 kg/s (see Fig. 7b) and ORC working fluid mass flow-rates above 0.01 kg/s (see Fig. 7c). The ratio of mass flow-rates in the two circuits influences the degree of superheating required to operate the ORC without violating the minimum pinch-point temperature difference in the heat exchanger. The optimal ratio of solar collector fluid mass flow-rate to ORC working fluid mass flow-rate is found to be between 8:1 and 13:1, with a minimum superheating requirement of between 8 and 9 K.

**Fig. 7.** Parametric analysis of ORC electrical power output for “annual-average” day climatic conditions. Individual system operating parameters varied as indicated, all other parameters set to maximum power settings as established in Section 3.3 unless otherwise stated.
It is noted that for the ETC chosen, the manufacturer’s maximum recommended flow rate is 0.13 kg/s, while Fig. 7b suggests that beyond 0.09 kg/s, the power consumption for the solar pump will exceed that produced by the system. It should be recalled that the solar-collector array theoretically consists of modules connected in series and therefore pressure drop is significant at high flow-rates, leading to a large power consumption by the pump. Therefore, the solar collector flow-rate has a more significant influence on the power consumption of the system than the working fluid flow-rate and evaporation pressure (which affect the ORC pump power consumption, see Fig. 7a and b) over the simulated operational range. The highest net power output (≈65–70 W_{avg} for both the PTC system and the ETC system) is found to occur at a solar collector flow-rate of ~0.03 kg/s.

Fig. 7e indicates that allowing the ORC working fluid condensation temperature \( T_{\text{cond}} \) to increase from 17°C to 35°C results in a decrease in electrical power output of 19% for the ETC system and 17% for the solar tracking PTC system. However, allowing the condensation temperature to rise also results in a lower demand for cooling water. For the ETC system the water demand decreases dramatically from 2.98 m\(^3\)/day at \( T_{\text{cond}} = 17°C \) to 0.27 m\(^3\)/day at \( T_{\text{cond}} = 35°C \). As a result, the economic benefit associated with the water savings (approximately £1100 annually at present UK water rates) far outweighs the penalty in terms of grid electricity savings. These results indicate that the energetic benefit of mains water as the ORC heat rejection medium compared to air at ambient temperatures may be economically realistic only if the water quantity is available at little or no extra cost (possibly by being shared with another domestic or industrial process).

Fig. 7f illustrates the reduction in electrical output as a result of using a fraction of the solar collector fluid to heat a domestic hot water cylinder downstream of the ORC evaporator. For the ETC system, if 100% of the fluid flow is passed through the heating coil (\( F_{\text{coil}} = 1 \)) the electrical output drops by 60% relative to the \( F_{\text{coil}} = 0 \) case. However, the system is then shown to provide water heating equivalent to 70–90% of the daily demand, depending on the whether the cylinder is designed for stratified or non-stratified charging (note that the system only provides water heating when the ORC is running). Projected over an annual period, this equates approximately to a 60 L/day (or 6 m\(^3\)/year) reduction in electricity bills (and a corresponding 150 kgCO\(_2\)/e\(^2\)) reduction on emissions saving but a 120 L/year increased saving on gas bills (and 390 kgCO\(_2\)/e\(^3\)) increase on emissions saving). It is also observed for the ETC system that values of \( F_{\text{coil}} \) greater than 0.4 produce very little further decrease in electrical output from the ORC. This is because for values of \( F_{\text{coil}} \) higher than this the maximum temperature for hot water storage (80°C) in the cylinder is reached and the cylinder is then bypassed to avoid over-heating of the hot-water store.

The total daily volume of hot-water drawn from the cylinder for domestic use is 122 L/day [37]. The total volume of mains water used for cooling in the ORC condenser (ETC system) is 2975 L/day and this water leaves the condenser at a temperature of 14.1°C. Some of this preheated water is used to top-up the hot water cylinder, but this is only permitted in the investigated configuration when there is a simultaneous draw-off of water from the cylinder. Over the course of the day the total volume of preheated water delivered to the cylinder is 23.6 L which is 19% of the total demand volume and <1% of the volume rejected from the condenser. If it were possible to preheat the entire 122 L of water delivered daily for domestic consumption to 14.1°C in the condenser (and via an intermediate storage capability) the water heating demand could potentially be reduced by a further 6.6% (0.47 kW h/day).

### 3.4 Annual simulation

In the annual simulation, the daily model was run 24 times (see Section 2.1); twice for each month (weekday and weekend day) with the appropriate electricity demand and environmental input parameters for each run. The operating pressures, solar circuit and ORC flow-rates were set to the maximum power settings determined in Section 3.3, with the solar fluid flow to the water cylinder set to zero. The results from the annual simulation, including a life-cycle cost analysis, are presented in Table 3.

The ETC system with a solar-tracking PTC collector is shown to deliver 776 kW h/year or 88.6 W\(_{\text{avg}}\) on average which, taking 3300 kW h/year (377 W\(_{\text{avg}}\) average) as the annual average household consumption [50], is equivalent to 24% of the total demand. If this figure is divided by the maximum annual work calculated for the solar collector in Section 3.1, the resulting exergy efficiency is 45%. These may be considered reasonable results, taking into account the use of fixed pressures and flow-rates, and the manual approach to system optimisation, which can be seen as a starting point for a CSHP system with improved design and operational strategies. The ETC-based system delivers 701 kW h/year (80.0 W\(_{\text{avg}}\) average). The work output from the PTC system in the annual simulation is found to be 10.7% higher than the ETC system work output. This difference is higher than the corresponding difference obtained in the “annual-average day” simulations reported in Section 3.3, which amounts to 3.2%, suggesting that the higher solar collector efficiency of the PTC system in summer months has a significant bearing on the system’s annual performance.

The large annual water volume requirement for cooling in the condenser could favour air cooling as an alternative means of heat rejection. However, there are also a number options that may be explored in order to exploit a larger proportion of the heat rejected from the ORC for preheating the domestic hot-water supply; thereby decreasing both energy demand and cooling water consumption. Thus the hot-water store is a vital component for consideration in the design and configuration of the system. A beneficial approach might be to split the water store into a high-temperature section for ORC heat-supply buffering, a low-temperature section for ORC heat-rejection, and an intermediate-temperature section for domestic hot-water supply. This could involve physical compartmentalisation of the store or exploitation of the temperature stratification gradient within a single compartment. Further use could also be made of the solar collector array in order to cool some of the stored water during the night, for later use as the heat rejection medium.

### 3.5 Exergy analysis

The results from the exergy analysis of the CSHP system are shown in Table 4 for the ETC variant, simulated over the full annual period in power generation priority mode (i.e. with the hot water cylinder bypass active). The hot water cylinder is omitted from this analysis. The thermodynamic parameters in the table are calculated according to the exergy topological method outlined in Section 2.3.1 and used, for example, by Mago et al. [39]. The proportional breakdown of exergy destruction in each component is shown in Fig. 8a. As expected, the majority of the exergy is lost in the solar collector array. This is an inevitable consequence of the conversion of solar radiation to enthalpy at a temperature that must be, for practical reasons, far lower than the apparent temperature of the sun as an exergy source.
Examining the breakdown of exergy destroyed in the ORC components only (Fig. 8b), and comparing the exergy efficiencies in Table 4 with those for the non-regenerative ORC system studied in Mago et al. [39], it is evident that: (1) the exergy efficiency of the ORC components in the present study are on the whole lower; and (2) the relative percentages of exergy destroyed are also different, with the condenser in particular exhibiting a considerably larger share of the overall exergy destruction in our study. The former of these observations may be attributable to the lower temperature operation of the system studied here, and also the lower isentropic PTC PTC ETC

| Component     | $X_{in}$ (MJ) | $X_{out}$ (MJ) | $X_{loss}$ (MJ) | $X_{use}$ (MJ) | $X_{ava}$ (MJ) | $\psi$ (%) | $\beta$ (%) | $\eta_o$ (%) |
|---------------|---------------|----------------|-----------------|--------------|----------------|------------|------------|-------------|
| Collector     | 173,000       | 119,500        | 53,500          | 3600         | 57,100         | 69.1       | 82.5       | 6.3         |
| Solar pump    | 117,200       | 115,900        | 1290            | 2730         | 4020           | 98.9       | 5.6        | 67.9        |
| ORC pump      | 104           | 73             | 31              | 63           | 94             | 69.9       | 0.1        | 66.8        |
| Evaporator    | 119,600       | 117,600        | 1960            | 4320         | 6280           | 98.4       | 8.7        | 68.7        |
| Condenser     | 1680          | 1010           | 670             | 110          | 780            | 60.2       | 1.1        | 14.5        |
| Total system  | 416,000       | 357,700        | 58,200          | 13,600       | 71,900         | 86.0       | –          | 19.0        |
| Total ORC$^a$| 125,800       | 122,300        | 3500            | 10,800       | 45,100         | 97.3       | 15.0       | 67.9        |

$^a$ Not including solar collector and solar fluid pump.

Fig. 8. Percentage of exergy loss per component in: (a) the entire system including solar collector array, and (b) the ORC system only. Note that the hot water storage cylinder is omitted because the analysis is performed for the system under power generation priority mode, with the cylinder bypassed.
efficiencies of the pump and expander which is a result of their smaller scale and positive displacement nature. The latter observation is due to the larger temperature differences between the fluids entering and exiting the condenser. The exergy destroyed in the heat exchangers can be improved by considering a regenerative cycle configuration, albeit with a likely increase in system cost due to the larger heat exchange area required. This will be considered in future work.

When the exergy analysis of the ETC collector system is compared to that for the PTC system, it is found that the total system exergy destruction is 28% lower for the PTC. The total system exergy efficiency increases from 19.0% to 25.5%. This is to be expected as a consequence of the efficient high temperature operation of the concentrating collector, which allows the ORC to operate at a higher evaporation temperature and pressure. Thus there is an improvement in the exergy efficiency, not only of the solar collector array (increasing from 6.3% for the ETC to 9.2% for the PTC), but also the ORC components. The total ORC exergy efficiency increases from 67.9% to 70.4%.

4. Further discussion and conclusions

This paper has presented a techno-economic model to investigate the potential performance and cost of a domestic-scale CSHP-ORC system featuring a positive-displacement expander, for use in the UK. The results of initial simulations based on simple component efficiency data, load profiles and operational control regimes have shown that the electrical output from the system is sensitive to the flow-rates, temperatures and working pressures in the ORC sub-system and to the design and operation of the solar collector array. Annual simulations have shown that for a fixed flow-rate system operation with 15 m$^2$ [46] of rooftop collector array, the system can produce an average power in the region of 80–90 W$_e$ (700–780 kW h$_e$/year) for an approximate total capital cost of £4400–5500, of which only £2700–£3900 can be attributed to electrical power generation and the rest to solar hot-water heating. This is equivalent to an installed total cost per unit average power generation of £55–61/W$_e$, or £34–44/W$_e$ if one considers the costs associated with electricity generation alone, and represents ~310–350 kgCO$_2$e in emissions reductions due to local power generation. In addition to this, the CSHP-ORC system model has demonstrated a potential for producing up to 86% of the required hot water for household consumption, corresponding to an additional 470 kgCO$_2$e in emissions reductions.

By comparison, a similar sized (15 m$^2$) mono-crystalline silicon (mono-Si) PV system (approx. 2.1 kW$_p$) costing around ~£7500 can be expected to have an average electrical output of ~200 W$_e$ in the UK climate, giving an installed cost of approximately ~£38/W$_e$. Interestingly, this normalised cost value is in the middle of the £34–44/W$_e$ range given above for the CSHP-ORC system. Although this PV system has a higher electrical output compared to the CSHP-ORC system, the option to provide solar hot-water does not exist because the entirety of the roof space is taken-up by the PV array that does not have a thermal output, while the total system price is significantly higher compared to the CSHP system. An improved like-for-like comparison should be made with various side-by-side configurations for a PV and solar hot-water system sharing the same roof-space (where the electrical and thermal outputs are dependent on how the roof-area is split), or a hybrid PV-Thermal (PVT) system in which the PV and solar-thermal collector components are combined in a single module. In the former case, the electrical output is compromised (due to reduced PV array-size to accommodate the solar thermal collectors) and in the latter the potential for hot-water provision is compromised (due to the collector thermal efficiency as a result of combining with the PV module), while the electrical output will be similar to a PV-only array of equivalent area. The purchase and installation cost of the side-by-side system would be in the range £9500–10,000, giving a cost per unit generating capacity of ~£73/W$_e$. 5 This figure should be directly compared to the £55–62/W$_e$ range that was given above for the CSHP-ORC system. For the PVT system the purchase and installation cost is in the region £8000–8500, with a cost per unit capacity of ~ £38/W$_e$ based on an output of ~215W$_e$. 6 This value may appear lower than the total capital cost per unit delivered power by the CSHP-ORC system (£55–61/W$_e$), however, it should be noted that the PVT alternative has a significantly reduced capacity for hot water provision, which amounts to 35% of consumption at best [53]. In both cases, the upfront cost of the purchase and installation may prove a prohibitive amount for many households. The total cost of the CSHP-ORC system, however, has the potential to be far lower because the only additional costs (relative to the solar hot-water system) are the ORC components and the additional solar-thermal collectors required to fill the remaining roof-space. Thus, there is an incentive to design the CSHP system to be available at low cost and for simple integration into an existing solar hot-water system.

Furthermore, the maximum power analysis presented in this paper has shown that there is the potential to achieve a significantly higher electrical power output from the CSHP-ORC system than has been predicted in the model simulations. Two important areas have been identified for performance improvements, which are: (1) maximising the solar collector efficiency by maintaining the optimum operating temperature; (2) allowing the system to achieve a smooth, continuous power output for better load-profile matching. The employment of variable flow-rate control has the potential to see improvements in both areas; with development of a suitable control algorithm a key requirement for implementation into the system model. In addition, the identification of a more suitable working fluid is required, particularly for the concentrating collector system, whose operational temperature has been shown to be limited by the use of the working fluid R245fa.

Acknowledgement

This work was supported by the Engineering and Physical Sciences Research Council (EPSRC), UK [grant number EP/J006041/1].

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4 Based on an approximate cost of £3500/kW$_p$ and unit size/rating of 7 m$^2$/kW$_p$ for mono-Si PV from Ref. [51]. The annual average electrical output is based on an electrical output of 825 kW, h/kW$_p$/year for PV, which is an approximate UK performance estimate for a south-facing array with a tilt-angle between 30 and 50° [51].

5 Based on a reduced array size of 10 m$^2$ to accommodate a 5 m$^2$ solar thermal array. Solar hot-water purchase and installation cost based on typical figures for domestic systems (including the hot-water cylinder) from the Energy Saving Trust [52].

6 Based on a 15 m$^2$. 2.3 kW$_e$ PV system with a total purchase and installation cost of ~£8000–8500 [53], delivering a UK-specific annual yield of 825 kW, h/kW$_p$/year [51].
