SUSY–QCD corrections to squark production and decays in $e^+e^-$ annihilation

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Abstract

We discuss the supersymmetric $O(\alpha_s)$ QCD corrections to $e^+e^- \to \tilde{q}_i \bar{\tilde{q}}_j$ ($i, j = 1, 2$) and to $\tilde{q}_i \to q' \tilde{\chi}^\pm_j$, $q \tilde{\chi}_0_k$ ($i, j = 1, 2$; $k = 1 \ldots 4$) within the Minimal Supersymmetric Standard Model. In particular we consider the squarks of the third generation $\tilde{t}_i$ and $\tilde{b}_i$ including the left–right mixing. In the on–shell scheme also the mixing angle has to be renormalized. We use dimensional reduction (which preserves supersymmetry) and compare it with the conventional dimensional regularization. A detailed numerical analysis is also presented.

1 Introduction

In supersymmetry (SUSY) one has two types of scalar quarks (squarks), $\tilde{q}_L$ and $\tilde{q}_R$, corresponding to the left and right helicity states of a quark. $\tilde{q}_L$ and $\tilde{q}_R$, however, mix due to the Yukawa coupling to the Higgs bosons, which is proportional to the mass of the quark. One therefore expects large mixing in the case of the stop quarks so that one mass eigenstate ($m_{\tilde{t}_1}$) might be rather light and even reachable at present colliders. The sbottoms $\tilde{b}_L$, $\tilde{b}_R$ may also strongly mix for large $\tan \beta$. The mass matrix in the basis ($\tilde{q}_L$, $\tilde{q}_R$) is given by:

$$M^2 = \begin{pmatrix} m_{\tilde{Q}}^2 + m_{\tilde{q}}^2 + D_1 & m_q \left( A_q - \mu \left\{ \frac{\cot \beta}{\tan \beta} \right\} \right) \\ m_q \left( A_q - \mu \left\{ \frac{\cot \beta}{\tan \beta} \right\} \right) & m_{\tilde{U}, \tilde{D}}^2 + m_{\tilde{q}}^2 + D_2 \end{pmatrix}. \quad (1)$$

Here $m_{\tilde{Q}}$, $m_{\tilde{G}}$, $m_{\tilde{D}}$, and $A_q$ are SUSY soft–breaking parameters, $\mu$ is the Higgsino mass parameter, and $\tan \beta = \frac{v_2}{v_1}$. $D_1$ and $D_2$ are the $D$ terms: $D_1 = m_{\tilde{q}}^2 \cos 2\beta (I_q^{3L} - e_\tilde{q} \sin^2 \theta_W)$, $D_2 = m_{\tilde{q}}^2 \cos 2\beta e_\tilde{q} \sin^2 \theta_W$, with $I_q^{3L}$ the third component of the weak isospin of $q$. In the off–diagonal elements of Eq. (1) $\cot \beta$ enters in the case of the stops and $\tan \beta$ in that of the sbottoms. Diagonalizing the matrix one gets the mass eigenstates $\tilde{q}_1 = \tilde{q}_L \cos \theta_{\tilde{q}} + \tilde{q}_R \sin \theta_{\tilde{q}}$, $\tilde{q}_2 = -\tilde{q}_L \sin \theta_{\tilde{q}} + \tilde{q}_R \cos \theta_{\tilde{q}}$ with the masses $m_{\tilde{q}_1}$, $m_{\tilde{q}_2}$ (with $m_{\tilde{q}_1} < m_{\tilde{q}_2}$) and the mixing angle $\theta_{\tilde{q}}$.

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Conventional QCD corrections to squark pair production $e^+ e^- \rightarrow \tilde{q}_i \tilde{q}_j$ ($i, j = 1, 2$) can be very large. The SUSY–QCD corrections including squark and gluino exchange will be discussed here following closely ref. These corrections were also treated in. The new feature in the calculation of the SUSY–QCD corrections is that in the on–shell scheme a suitable renormalization condition has to be found for the mixing angle $\theta_{\tilde{q}}$ because the tree–level amplitude explicitly depends on it. We will explain this in detail below.

The SUSY–QCD corrections to the squark decays into chargino or neutralino, $\tilde{q}_i \rightarrow q \tilde{\chi}^\pm_j$, $q \tilde{\chi}^0_k$ ($i, j = 1, 2; k = 1 \ldots 4$), have been calculated in and will also be discussed in the following. Here the dependence on the nature of the charginos/neutralinos (gaugino–like or higgsino–like) is particularly interesting.

We work in the on–shell scheme and use dimensional reduction (DR) to regularize the integrals, which is necessary to preserve supersymmetry (at least up to two loops). We will comment on the differences between this and the dimensional regularization scheme used in the Standard Model.

## 2 The Production Process $e^+ e^- \rightarrow \tilde{q}_i \tilde{q}_j$

The cross section at tree level is given by:

$$\sigma^0 (e^+ e^- \rightarrow \tilde{q}_i \tilde{q}_j) = \frac{\pi a^2}{s} \lambda_{ij}^{3/2} \left[ e_q^2 \delta_{ij} - T_{\gamma Z} c_q a_{ij} \delta_{ij} + T_{ZZ} a_{ij}^2 \right]$$  \hspace{1cm} (2)

with

$$T_{\gamma Z} = \frac{v_e}{8 s_W c_W} \frac{s(s - m_Z^2)}{2 s_W c_W ((s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2)}$$  \hspace{1cm} (3)

$$T_{ZZ} = \frac{a_{ij}^2 + v_e^2}{256 s_W^4 c_W^4} \frac{s^2}{(s - m_Z^2)^2 + \Gamma_Z^2 m_Z^2}.$$  \hspace{1cm} (4)

Here $\lambda_{ij} = (1 - \mu_{i,j}^2 - \mu_{j,i}^2)^2 - 4 \mu_{i,j}^2 \mu_{j,i}^2$ with $\mu_{i,j}^2 = m_{\tilde{q}_{i,j}}^2 / s$. $e_q$ is the charge of the squarks (in units of $e$), $v_e = -1 + 4 s_W^2$, $c_e = -1$, $s_W = \sin \theta_W$, $c_W = \cos \theta_W$. $a_{ij}$ are the relevant parts of the couplings $Z \tilde{q}_j \tilde{q}_i^*$:

$$a_{11} = 4 (f_q^R \cos^2 \theta_{\tilde{q}} - s_W^2 e_q), \quad a_{22} = 4 (f_q^R \sin^2 \theta_{\tilde{q}} - s_W^2 e_q),$$

$$a_{12} = a_{21} = -2 f_q^R \sin 2 \theta_{\tilde{q}}.$$  \hspace{1cm} (5)

The SUSY–QCD corrections in $O(\alpha_s)$ consist of the conventional QCD corrections due to gluon exchange and real gluon radiaton, as well as of the corrections due to the exchange of a gluino and squarks, see Fig. 1. Our input parameters are the physical masses $m_{\tilde{q}_1}, m_{\tilde{q}_2}, m_{\tilde{q}}, m_{\tilde{\chi}}$, and the mixing angle $\theta_{\tilde{q}}$. We use the on–shell scheme where the masses are fixed by the respective poles of the propagators.
In renormalizing the lagrangian we follow the usual procedure:

\[ \mathcal{L}_0 = \mathcal{L} + \delta \mathcal{L} \]  

with

\[ \mathcal{L} = -e e_q \delta_{ij} A^\mu \tilde{\bar{q}}_i \gamma_\mu (i\partial_\mu) \tilde{q}_j - \frac{e}{4 s_W c_W} a_{ij} Z^\mu \tilde{\bar{q}}_i \gamma_\mu (i\partial_\mu) \tilde{q}_j, \]  

\[ \mathcal{L}_0, \text{ the bare lagrangian, has the same form with the bare quantities:} \]

\[ e_0^q \delta_{ij} = e_0 \delta_{ij} + (\delta e_0)_{ij}, \]  

\[ a_0^i_j = a_{ij} + \delta a_{ij}, \]  

\[ \tilde{q}_{i}^{\ast 0} = (1 + \frac{1}{2} \delta Z_{ii}) \tilde{q}_{i}^{\ast} + \delta Z_{ij'} \tilde{q}_{j'}^{\ast}, \quad i \neq i', \]  

\[ \tilde{q}_{j}^{0} = (1 + \frac{1}{2} \delta Z_{jj'}) \tilde{q}_{j} + \delta Z_{jj} \tilde{q}_{j}, \quad j \neq j'. \]  

Notice that because of \( \theta_0^q = \theta_{\tilde{q}} + \delta \theta_{\tilde{q}}, \delta a_{ij} \) is a function of \( \delta \theta_{\tilde{q}}. \) The total correction in \( \mathcal{O}(\alpha_s) \) can be written as:

\[ \Delta a_{ij} = \delta a_{ij}^{(v)} + \delta a_{ij}^{(w)} + \delta a_{ij}^{(\tilde{q})}, \]  

\[ (\Delta e_0^q)_{ij} = (\delta e_0^q)_{ij}^{(v)} + (\delta e_0^q)_{ij}^{(w)}, \]
where \((v)\) denotes the vertex corrections and \((w)\) the wave–function corrections. The contributions come from gluon, gluino, and squark exchange. \(\delta a_{ij}^{(g)}\) is due to the shift from the bare to the on–shell couplings. As already mentioned, we use dimensional reduction instead of dimensional regularization. Up to first order this is achieved technically by taking \(D = (4 − r)\) with \(r \rightarrow 0\) (see section 4). In the case of \(e^+ e^− \rightarrow \tilde{q}_i \tilde{q}_j\) there is, however, no difference between the two schemes as will be explained later.

Let us first discuss the vertex corrections \(\delta e_{ij}^{(v)}\) and \(\delta e_{ij}^{(w)}\) coming from the exchange of SUSY particles. The gluino contribution due to the graph in Fig. 1a is given by:

\[
\delta e_{ij}^{(v)(q)} = \frac{2 \alpha_s}{3 \pi} \left\{ \gamma_{mq}v_qS_{ij}^q \left( 2C_{ij}^+ + C_{ij}^0 \right) + v_q\delta_{ij}\left[ (2m_q^2 + 2m_q^2 + m_{\tilde{q}_i}^2 + m_{\tilde{q}_j}^2)C_{ij}^+ + 2m_q^2C_{ij}^0 + B^0(s, m_q^2, m_{\tilde{q}_i}^2) \right] + a_qA_{ij}^q \left[ (2m_q^2 - 2m_q^2 + m_{\tilde{q}_i}^2 + m_{\tilde{q}_j}^2)C_{ij}^+ + (m_{\tilde{q}_i}^2 - m_{\tilde{q}_j}^2)C_{ij}^- + 2m_q^2C_{ij}^0 + B^0(s, m_q^2, m_{\tilde{q}_i}^2) \right] \right\} \]
\[(14)\]

and

\[
\delta e_{ij}^{(w)(q)} = \frac{2 \alpha_s}{3 \pi} e_q \left\{ \gamma_{mq}v_qS_{ij}^q \left( 2C_{ij}^+ + C_{ij}^0 \right) + \delta_{ij}\left[ (2m_q^2 + 2m_q^2 + m_{\tilde{q}_i}^2 + m_{\tilde{q}_j}^2)C_{ij}^+ + 2m_q^2C_{ij}^0 + B^0(s, m_q^2, m_{\tilde{q}_i}^2) \right] \right\} \]
\[(15)\]

with \(v_q = 2I_{q}^{3L} - 4s_W e_q, a_q = 2I_{q}^{3L}, S_{i1}^q = -\sin 2\theta_{\tilde{q}} = -S_{22}^q = A_{i1}^q = A_{21}^q, \) and \(S_{12}^q = S_{21}^q = -\cos 2\theta_{\tilde{q}} = -A_{11}^q = A_{12}^q.\) The functions \(C_{ij}^+\) are defined by

\[
C^+ = \frac{C^1 + C^2}{2}, \quad C^- = \frac{C^1 - C^2}{2}. \quad (16)
\]

\(B^0\) and \(C_{ij}^{0,1,2}\) are the usual two– and three–point functions as given, for instance, in \(\text{[1]}\). The arguments of all \(C\)–functions are \((m_{\tilde{q}_i}^2, s, m_{\tilde{q}_j}^2, m_{\tilde{q}_j}^2, m_{\tilde{q}_i}^2).\)

The squark exchange graph Fig. 1c is proportional to the four–momentum of \(Z^0\), and therefore does not contribute to the physical matrix element. The wave–function corrections (Figs. 1b,d) can be written as, using Eqs. (3) to (4) \((i \neq i', j \neq j'):\)

\[
\delta e_{ij}^{(w)} = \frac{1}{2}(\delta Z_{ii} + \delta Z_{jj})a_{ij} + \delta Z_{i'j} a_{ij} + \delta Z_{jj} a_{ij} + \delta Z_{ij} a_{ij}'. \quad (17)
\]

An analogous formula holds for \(\delta e_{ij}^{(w)}\) with \(a_{ij} \rightarrow e_q \delta_{ij}\). One obtains from Fig. 1b:

\[
\delta e_{ij}^{(w)(g)} = -\text{Re} \left\{ \frac{1}{2} \left[ \sum_{ii}^{(g)}(m_{\tilde{q}_i}^2) + \sum_{jj}^{(g)}(m_{\tilde{q}_j}^2) \right] a_{ij} \right\} \]

\[(18)\]

where \(\Sigma_{ij}^{(g)}(m_{\tilde{q}_j}^2)\) is given by

\[
\Sigma_{ij}^{(g)}(m_{\tilde{q}_j}^2) = 2 \alpha_s \frac{m_{\tilde{q}_j}^2}{\pi} \left\{ \gamma_{mq}v_qS_{ij}^q \left( 2C_{ij}^+ + C_{ij}^0 \right) + \delta_{ij}\left[ (2m_q^2 + 2m_q^2 + m_{\tilde{q}_i}^2 + m_{\tilde{q}_j}^2)C_{ij}^+ + 2m_q^2C_{ij}^0 + B^0(s, m_q^2, m_{\tilde{q}_i}^2) \right] \right\} \]
\[(19)\]

The arguments of all \(C\)–functions are \((m_{\tilde{q}_i}^2, s, m_{\tilde{q}_j}^2, m_{\tilde{q}_j}^2, m_{\tilde{q}_i}^2).\)
\[ \frac{\Sigma^{(\bar{\theta})}_{ij}(m_{\tilde{q}_i}^2)}{m_{\tilde{q}_i}^2 - m_{\tilde{q}_j}^2} a_{i'j} + \frac{\Sigma^{(\bar{\theta})}_{jj}(m_{\tilde{q}_j}^2)}{m_{\tilde{q}_j}^2 - m_{\tilde{q}_i}^2} a_{i'j'} \} \] (18)

and

\[ \delta(e_q)_{ij}^{(\bar{\theta})} = -e_q \text{Re} \left\{ \Sigma^{(\bar{\theta})}_{ii}(m_{\tilde{q}_i}^2) \right\} , \] (19)

\[ \delta(e_q)_{12}^{(\bar{\theta})} = \frac{e_q}{m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2} \text{Re} \left\{ \Sigma^{(\bar{\theta})}_{21}(m_{\tilde{q}_2}^2) - \Sigma^{(\bar{\theta})}_{12}(m_{\tilde{q}_1}^2) \right\} , \] (20)

where \( \Sigma^{(\bar{\theta})}_{ij}(m^2) \) are self-energies and \( \Sigma'_{ii}(m_{\tilde{q}_i}^2) = \partial \Sigma^{(\bar{\theta})}_{ii}(p^2) / \partial p^2 \bigg|_{p^2=m^2} \).

Notice that \( \delta(e_q)_{ij}^{(\bar{\theta})} = 0 \) because the contributions with the squark loop attached at either external squark line in Fig. 1d cancel each other.

The wave-function correction \( \delta a^{(\bar{\theta})}_{ij} \) due to Fig. 1d plays an important role in the renormalization of the squark mixing angle \( \bar{\theta}_{\tilde{q}} \).

### 2.1 Renormalization of the Mixing Angle \( \bar{\theta}_{\tilde{q}} \)

The total correction \( \Delta a_{ij} \), Eq. (12), using Eq. (17) can be written as \( i \neq i', j \neq j' \)

\[ \Delta a_{ij} = \delta a_{ij}^{(v)} + \frac{1}{2} (\delta Z_{ii} + \delta Z_{jj}) a_{ij} + \delta Z_{ij} a_{i'j} + \delta Z_{j'i} a_{i'j'} + \delta a_{ij}^{(\bar{\theta})} . \] (21)

Notice that the first part of the right-hand side, \( \delta a_{ij}^{(v)} + \frac{1}{2} (\delta Z_{ii} + \delta Z_{jj}) a_{ij} \), is already free of ultra-violet divergencies. Hence, the second part of Eq. (21) has to be finite, too. We therefore may require for \( i = 1 \) and \( j = 2 \)

\[ \delta a_{12}^{(\bar{\theta})} = (a_{22} - a_{11}) \delta \theta_{\tilde{q}} = - (\delta Z_{21} a_{22} + \delta Z_{12} a_{11}) . \] (22)

One can easily see that \( \Delta a_{ij} \) is then also finite for all \( i, j \). The condition, Eq. (22), means that the non-diagonal self-energy graphs Fig. 1b and 1d cancel the counterterm \( \delta a_{12}^{(\bar{\theta})} \) in Eq. (21). Notice also that the total squark contribution \( \Delta a_{ij}^{(\bar{\theta})} \) is zero. Other authors used the same basic idea but took, for instance, the condition analogous to Eq. (22) for \( \delta a_{11}^{(\bar{\theta})} \) or \( \delta a_{22}^{(\bar{\theta})} \), see 4, or a similar condition valid at a point \( Q^2 \), see ref. 9. The differences between these schemes are, however, numerically very small.

### 2.2 Total QCD Correction in \( \mathcal{O}(\alpha_s) \)

The total QCD correction \( \Delta \sigma \) to the cross section is

\[ \Delta \sigma = \Delta \sigma^{(g)} + \Delta \sigma^{(\bar{\theta})} , \] (23)
as $\Delta\sigma^{(\tilde{g})} = 0$ in our renormalization scheme of the squark mixing angle. The gluon contribution factorizes:

$$\sigma^{(g)} = \sigma^0 \left[ \frac{4 \alpha_s}{3 \pi} \Delta_{ij} \right],$$

where $\Delta_{ij}$ is given in ref. 2. The total gluino contribution is given by:

$$\Delta\sigma^{(\tilde{g})} = \frac{\pi \alpha_s^2}{8} \lambda_{ij}^{3/2} \left\{ 2 c_q (\Delta c_q(i))_{ij} + 2 T_{zz} a_{ij} \Delta a_{ij}^{(\tilde{g})} \right\}$$

$$- T_{\gamma Z} [c_q \delta_{ij} \Delta a_{ij} + (\Delta c_q)_{ij} a_{ij}] \right\}$$

with

$$\Delta a_{ij}^{(\tilde{g})} = \delta a_{ij}^{(v, \tilde{g})} - \text{Re} \left\{ \frac{1}{2} \left[ \Sigma_{ii} (m_{\tilde{q}}^2) + \Sigma_{jj} (m_{\tilde{q}}^2) \right] a_{ij} + \frac{4 \alpha_s}{3 \pi} \frac{m_{\tilde{q}} m_{\tilde{q}}}{m_{\tilde{q}} - m_{\tilde{q}}} \delta_{ij} \right\}$$

$$\cdot \left[ B^0 (m_{\tilde{q}}^2, m_{\tilde{g}}^2, m_{\tilde{q}}^2) \left[ (-1)^{i+1} 2 a_{ii'} \cos 2\theta_{\tilde{q}} - a_{i'i'} \sin 2\theta_{\tilde{q}} \right] \right.$$  

$$\left. + B^0 (m_{\tilde{l}}^2, m_{\tilde{g}}^2, m_{\tilde{l}}^2) a_{ii} \sin 2\theta_{\tilde{q}} \right\}$$

(26)

2.3 Discussion

First, we have calculated the SUSY–QCD corrections to the cross section of $e^+ e^- \rightarrow \tilde{t}_1 \tilde{t}_1$ in the LEP energy range $\sqrt{s} \leq 200$ GeV. We have found that, whereas the conventional QCD correction may be rather large, the gluino correction is only about 1% of the tree–level cross section, quite independent of $m_{\tilde{t}_1}$. The correction due to gluino exchange is, however, not negligible ($2$–$8\%$) in the energy range of a linear $e^+ e^-$ collider ($\sqrt{s} = 500$–$2000$ GeV).

The $\sqrt{s}$–dependence of the SUSY–QCD corrections to the cross section $\sigma(e^+ e^- \rightarrow \tilde{t}_1 \tilde{t}_1)$ is shown in Fig. 2 for $m_{\tilde{t}_1} = 100$ GeV, $m_{\tilde{t}_2} = 400$ GeV, $m_{\tilde{q}} = 300$ GeV, and $\cos \theta_{\tilde{t}} = 1/\sqrt{2}$. The peak at $\sqrt{s} = 350$ GeV is due to the $t\bar{t}$ threshold. In Fig. 3 we show the $\cos \theta_{\tilde{t}}$ dependence of the corrections for this process at $\sqrt{s} = 500$ GeV for the same masses of the stops and the gluino as in Fig. 2. Whereas the gluon correction has the same behaviour in $\theta_{\tilde{t}}$ as the tree–level cross section, the gluino correction is different. In Figs. 4 and 5 we exhibit the corrections to $\sigma(e^+ e^- \rightarrow \tilde{t}_1 \tilde{t}_2)$ and $\sigma(e^+ e^- \rightarrow \tilde{t}_2 \tilde{t}_3)$, respectively, at $\sqrt{s} = 2$ TeV for $m_{\tilde{t}_1} = 400$ GeV, $m_{\tilde{t}_2} = 800$ GeV, $m_{\tilde{q}} = 600$ GeV. The gluino contributions can go up to about $-10\%$. Fig. 6 shows the dependence on the gluino mass. It is interesting to notice that the gluino correction decreases very slowly with the gluino mass.
Figure 2: SUSY–QCD corrections \( \delta \sigma^g / \sigma^{\text{tree}} \) and \( \delta \sigma^g / \sigma^{\text{tree}} \) for \( e^+ e^- \rightarrow \tilde{t}_1 \tilde{t}_1 \) as a function of \( \sqrt{s} \) for \( \cos \theta_\tilde{t} = 1/\sqrt{2} \), \( m_{\tilde{t}_1} = 100 \) GeV, \( m_{\tilde{t}_2} = 400 \) GeV, and \( m_\tilde{g} = 300 \) GeV.

Figure 3: SUSY–QCD corrections \( \delta \sigma^g / \sigma^{\text{tree}} \) and \( \delta \sigma^g / \sigma^{\text{tree}} \) for \( e^+ e^- \rightarrow \tilde{t}_1 \bar{\tilde{t}}_1 \) as a function of \( \cos \theta_\tilde{t} \) for \( \sqrt{s} = 500 \) GeV, \( m_{\tilde{t}_1} = 100 \) GeV, \( m_{\tilde{t}_2} = 400 \) GeV, and \( m_\tilde{g} = 300 \) GeV.

Figure 4: SUSY–QCD corrections \( \delta \sigma^g \) and \( \delta \sigma^g \) as a function of \( \cos \theta_\tilde{t} \) for \( e^+ e^- \rightarrow \tilde{t}_1 \tilde{t}_2 \), \( \sqrt{s} = 2 \) TeV, \( m_{\tilde{t}_1} = 400 \) GeV, \( m_{\tilde{t}_2} = 800 \) GeV, and \( m_\tilde{g} = 600 \) GeV.

Figure 5: SUSY–QCD corrections \( \delta \sigma^g / \sigma^{\text{tree}} \) and \( \delta \sigma^g / \sigma^{\text{tree}} \) for \( e^+ e^- \rightarrow \tilde{t}_2 \bar{\tilde{t}}_2 \) as a function of \( \cos \theta_\tilde{t} \) for \( \sqrt{s} = 2 \) TeV, \( m_{\tilde{t}_1} = 400 \) GeV, \( m_{\tilde{t}_2} = 800 \) GeV, and \( m_\tilde{g} = 600 \) GeV.
3 Squark Decays into Charginos and Neutralinos

In the following we discuss the SUSY–QCD corrections for the decays:

\[
\tilde{t}_i \rightarrow b \tilde{\chi}^+_j, \quad \tilde{b}_i \rightarrow t \tilde{\chi}^-_j, \quad \tilde{t}_i \rightarrow t \tilde{\chi}^0_k, \quad \tilde{b}_i \rightarrow b \tilde{\chi}^0_k,
\]

with \( i, j = 1, 2 \) and \( k = 1 \ldots 4 \). The supersymmetric QCD corrections were calculated for \( m_q = 0 \) and \( \tilde{\chi}^0_1 \) being a photino in ref.\(^{10}\) and taking into account squark mixing, quark masses (i.e. Yukawa couplings), and general gaugino–higgsino mixing of charginos and neutralinos in refs.\(^{4}\) and\(^{5}\). The decay width at tree–level for \( \tilde{t}_i \rightarrow b \tilde{\chi}^+_j \) is given by:

\[
\Gamma^0(\tilde{t}_i \rightarrow b \tilde{\chi}^+_j) = \frac{g^2 \kappa(m_{\tilde{t}_i}^2, m_b^2, m_{\tilde{\chi}^+_j}^2)}{16\pi m_{\tilde{t}_i}^2} \left( \left[ (\ell_{ij}^e)^2 + (k_{ij}^e)^2 \right] X - 4 \ell_{ij}^e k_{ij}^e m_b m_{\tilde{\chi}^+_j} \right)
\]

with \( X = m_{\tilde{t}_i}^2 - m_b^2 - m_{\tilde{\chi}^+_j}^2 \) and \( \kappa(x, y, z) = [(x - y - z)^2 - 4yz]^{1/2} \). The \( \tilde{t}_i^* - b - \tilde{\chi}^+_j \) couplings \( \ell_{ij}^e \) and \( k_{ij}^e \) read, for instance, for \( \tilde{t}_1 \rightarrow b \tilde{\chi}^+_1 \):

\[
\ell_{ij}^e = -V_{j1} \cos \theta_t + \frac{m_t}{\sqrt{2} m_W \sin \beta} V_{j2} \sin \theta_t, \quad \quad k_{ij}^e = \frac{m_b}{\sqrt{2} m_W \cos \beta} U_{j2} \cos \theta_t,
\]

where \( U \) and \( V \) are the matrices diagonalizing the charged gaugino–higgsino mass matrix\(^{11}\).
The $O(\alpha_s)$ SUSY–QCD corrected decay width can be written as:

$$\Gamma = \Gamma^0 + \delta \Gamma^{(v)} + \delta \Gamma^{(w)} + \delta \Gamma^{(c)} + \delta \Gamma^{(\text{real gluon})},$$

where the superscript $v$ again denotes the vertex correction (Figs. 7 a,b) and $w$ the wave–function correction (Figs. 7 c-g). $\delta \Gamma^{(c)}$ corresponds to the shift from the bare to the on–shell couplings, taking into account the renormalization of the quark mass and the squark mixing angle. $\delta \Gamma^{(\text{real gluon})}$ is the correction due to real gluon bremsstrahlung and cancels the infrared divergencies.

The procedure of the calculation is completely analogous to that discussed just before in section 2. The complete formulae for the different correction parts in Eq. (32) are given in ref. We want to note that, contrary to the production process $e^+e^- \rightarrow \tilde{q}_i\tilde{q}_j$, the corrections to the decay widths of $\tilde{q}_i \rightarrow q'\tilde{\chi}_j^\pm$ and $\tilde{q}_i \rightarrow q\tilde{\chi}_k^0$ are different in the dimensional regularization and in the dimensional reduction scheme. (At first order the difference is finite.) This is because of the quark wave–function correction due to gluon exchange, Fig. 7c. The quark self–energy corresponding to Fig. 7c is given by:

$$\Pi^{(q)}(k^2) = \frac{\alpha_s}{3\pi} \left[ 2\frac{k^2}{k} B^1 + 2(\frac{k}{k} - 2m_q) B^0 - r(\frac{k}{k} - 2m_q) \right]$$ \hspace{1cm} (33)
with \( B^n = B^n(k^2, \lambda^2, m_{q_i}^2) \) and the gluon mass \( \lambda \to 0 \). This leads to the quark wave–function renormalization constants due to gluon exchange

\[
\delta Z^{L(g)} = \delta Z^{R(g)} = \frac{2}{3} \frac{q_0}{m} \left[ B^0 + B^1 - 2m_q^2(B^0 - B^1) - \frac{r}{2} \right] \quad (34)
\]

with \( B^n = B^n(m_{q_i}^2, \lambda^2, m_{q_i}^2), \dot{B}^n = \dot{B}^n(m_{q_i}^2, \lambda^2, m_{q_i}^2) \). \( \delta Z^L \) and \( \delta Z^R \) are defined by the usual relation between the unrenormalized quark field \( q^0 \) and the renormalized one, \( q^0 = (1 + \frac{1}{2} \delta Z^L P_L + \frac{1}{2} \delta Z^R P_R) q \). Note the dependence on \( r \) in Eqs. (33) and (34), where \( r = 0 \) in the dimensional reduction and \( r = 1 \) in the dimensional regularization scheme. Note, however, that there is no such difference for the squark self–energy graph due to gluon exchange.

3.1 Numerical Results

Let us first discuss the decay \( \tilde{t}_1 \to b\tilde{\chi}_1^\pm \), where we take \( m_{\tilde{t}_1} = 100 \) GeV, \( \tan \beta = 2, m_{\tilde{t}_2} = 600 \) GeV, \( m_{\tilde{b}_1} = 450 \) GeV, \( m_{\tilde{b}_2} = 470 \) GeV, and \( \cos \theta_{\tilde{b}} = -0.9 \). We study three cases: \( M \ll |\mu| (M = 95 \) GeV, \( \mu = -800 \) GeV), \( M \sim |\mu| (M = 100 \) GeV, \( \mu = -100 \) GeV), and \( M \gg |\mu| (M = 300 \) GeV, \( \mu = -89 \) GeV). We use the GUT relations: \( M' \approx 0.5 \) M, \( m_{\tilde{g}} \approx 3.5 \) M.

In Fig. 8 the dependence of the SUSY–QCD corrections on the stop mass is exhibited for \( \cos \theta_{\tilde{t}} = 0.6 \). Notice the pronounced dependence on the nature of the chargino. The corrections are largest (\( \sim -25\% \)), if the chargino is higgsino–like (\( |\mu| \ll M \)) due to the large top Yukawa coupling. If \( \tilde{\chi}_1^\pm \) is gaugino–like \( (M \ll |\mu|) \) the corrections are between +20\% and −10\%.

In Fig. 9 we show the SUSY–QCD corrected widths together with the tree–level widths as a function of \( \cos \theta_{\tilde{t}} \) for \( m_{\tilde{t}_1} = 200 \) GeV and the other parameters as in Fig. 8. Again, the corrections are biggest in the case of a higgsino–like chargino. The behaviour of the \( \cos \theta_{\tilde{t}} \) dependence reflects the fact that if \( \tilde{t}_1 \sim \tilde{t}_R \) (\( \cos \theta_{\tilde{t}} \sim 0 \)) it strongly couples to the higgsino component of \( \tilde{\chi}_1^\pm \), and if \( \tilde{t}_1 \sim \tilde{t}_L \) (\( \cos \theta_{\tilde{t}} \sim \pm 1 \)) it strongly couples to the gaugino component.

In Fig. 10 we show \( \delta \Gamma/\Gamma^0 \) as a function of \( m_{\tilde{t}_1} \) for \( \tilde{t}_1 \to \tilde{t}\tilde{\chi}_1^0 \), taking \( m_{\tilde{\chi}_1^0} = 80 \) GeV, \( \beta = 2, m_{\tilde{t}_2} = 600 \) GeV, and \( \cos \theta_{\tilde{t}} = 0.6 \). Again we observe that if \( \tilde{\chi}_1^0 \) is higgsino–like (\( |\mu| \ll M \)) the corrections are about −20\%.

We have also studied the dependence on the gluino mass. In Fig. 11 we show a plot where \( \delta \Gamma/\Gamma^0 \) is exhibited for \( \tilde{t}_1 \to b\tilde{\chi}_1^\pm \) and \( \tilde{t}_1 \to t\tilde{\chi}_1^0 \) as a function of \( m_{\tilde{g}} \) for \( m_{\tilde{t}_1} = 300 \) GeV, \( \cos \theta_{\tilde{t}} = 0.6, \tan \beta = 2, \mu = -100 \) and −800 GeV. \( M \) is fixed by \( M \approx 0.3 m_{\tilde{g}} \). Notice that the SUSY–QCD corrections are still important for \( m_{\tilde{g}} \sim 1 \) TeV and no decoupling of the gluino mass can be seen. This is also the case if we relax the condition \( M \approx 0.3 m_{\tilde{g}} \) and keep the chargino (neutralino) mass fixed.
Figure 8: SUSY–QCD corrections to the width of $\tilde{t}_1 \rightarrow b\tilde{\chi}_1^+$ as a function of $m_{\tilde{t}_1}$, for $m_{\tilde{\chi}_1^+} = 100$ GeV, $\cos\theta_1 = 0.6$, $\tan\beta = 2$, and various $(M, \mu)$ [GeV] values.

Figure 9: Tree–level (dashed lines) and SUSY–QCD corrected (solid lines) decay widths of $\tilde{t}_1 \rightarrow b\tilde{\chi}_1^+$ as a function of $\cos\theta_1$, for $m_{\tilde{t}_1} = 200$ GeV, $m_{\tilde{\chi}_1^+} = 100$ GeV, $\tan\beta = 2$, and various $(M, \mu)$ [GeV] values.

Figure 10: SUSY–QCD corrections to the width of $\tilde{t}_1 \rightarrow t\tilde{\chi}_2^0$ as a function of $m_{\tilde{t}_1}$, for $m_{\tilde{\chi}_2^0} = 80$ GeV, $\cos\theta_1 = 0.6$, $\tan\beta = 2$, and various $(M, \mu)$ [GeV] values.

Figure 11: SUSY–QCD corrections to the widths of $\tilde{t}_1 \rightarrow b\tilde{\chi}_1^+$ (solid lines) and $\tilde{t}_1 \rightarrow t\tilde{\chi}_2^0$ (dash-dotted lines) as a function of $m_{\tilde{g}}$, for $m_{\tilde{t}_1} = 300$ GeV, $\cos\theta_1 = 0.6$, $\tan\beta = 2$, $M \sim 0.3 m_{\tilde{g}}$. 

$\delta\Gamma/\Gamma^0$ [%] $\Gamma$ [GeV]

$\delta\Gamma/\Gamma^0$ [%] $\delta\Gamma/\Gamma^0$ [%] $m_{\tilde{t}_1}$ [GeV] $m_{\tilde{g}}$ [GeV] $m_{\tilde{g}}$ [GeV]
4 Dimensional Reduction Technique

The regularization by dimensional reduction was proposed by\(^\text{[1]}\). It means that only the space–time dimensions (the coordinates \(x^\mu\) and momenta \(p^\mu\)) are continued to \(D = 4 - \epsilon\) dimensions, whereas the vector fields and spinors remain four–dimensional. Following\(^\text{[2]}\) it is convenient to write the four–dimensional vector field \(V^\mu\) as \(V^\mu = (V_i, V^\sigma)\), where \(V_i\) is a \(D\)–dimensional vector, and \(V^\sigma\) is \(\epsilon\)–dimensional behaving as \(\epsilon\) scalars. Moreover, one has \(\gamma^\mu = (\gamma^i, \gamma^\sigma)\). Note that \(x^\mu = (x^i, 0)\), \(\partial^\mu = (\partial^i, 0)\), and \(p^\mu = (p^i, 0)\). As a consequence the lagrangian \(\mathcal{L}\) can be decomposed as \(\mathcal{L} = \mathcal{L}^{(D)} + \mathcal{L}^{(\epsilon)}\), where \(\mathcal{L}^{(D)}\) is the lagrangian of the conventional dimensional regularization. Therefore, to each interaction term of a vector field there is a corresponding “\(\epsilon\) scalar” interaction term, except for the vector–scalar–scalar interaction because \(V^\mu \phi^* \bar{\partial}_\mu \phi = V^i \phi^* \partial_i \phi\) (and no \(\epsilon\) term), with \(\phi\) being a scalar field. Therefore, in this case there is no difference between dimensional regularization and dimensional reduction. There is, however, a difference in the case of the interaction of a fermion with a vector field. For instance, the fermion self–energy, Fig. 7c, receives a contribution due to \(\epsilon\) scalars in the loop of \(\frac{g^2}{3\pi}(k - 2m_q)\). This is just the expression which cancels the \(r\)–dependent term in Eq. (33) for \(r = 1\) in order to get the result of dimensional reduction \((r = 0)\). Thus at the one–loop level the “\(\epsilon\)–scalar” technique is equivalent to performing the algebra in the numerator of the integrand in four dimensions and making the integration in \(D\) dimensions, or equivalently taking \(D = 4 - r\epsilon\) with \(r \to 0\), as we did in our calculations.

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