Recent results on exclusive hadronic cross sections measurements at $B_{ABAR}$

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Abstract. The $B_{ABAR}$ Collaboration has an intensive program studying hadronic cross sections in low-energy $e^+e^-$ annihilations, accessible via initial-state radiation. Our measurements allow significant improvements in the precision of the predicted value of the muon anomalous magnetic moment. These improvements are necessary for shedding light on the current 3 sigma difference between the predicted and the experimental values. We have published results on a number of processes with two to six hadrons in the final state, and other final state are currently under investigation. We report here on the most recent results obtained by analysing the entire $B_{ABAR}$ dataset, including the $\pi^+\pi^-\pi^0\pi^0$, $K^0_SK^0_L\pi^0$, $K^0_SK^0_L\eta$, and other final states.

1. PEP-II and $B_{ABAR}$: more than just a B factory
The $B_{ABAR}$ experiment collected 0.5 $ab^{-1}$ of data over $\sim 10$ years, at centre of mass energies on or just below the mass of the $\Upsilon(4S)$ meson. Although the primary objective was the study of $B$ meson properties – which it accomplished very successfully – the accelerator also produced significant numbers of charm quarks and $\tau$ leptons, resulting in many $B_{ABAR} \tau$ and charm publications.

However the detector can also be used for the study of light quark pair production at low energies (from threshold up to $\sqrt{s} \sim 3$GeV) through the process of Initial State Radiation (ISR), in which a Bremsstrahlung photon is emitted from the electron or positron, leading to an $e^+e^-$ collision at reduced energy. This is shown in Figure 1, as a Feynman diagram and with a typical event. The clearly-separated photon balances the hadronic system.

The effective centre of mass energy is given by

$$s' = s(1 - 2E_\gamma/\sqrt{s})$$

and is measured on an event by event basis. It is not as precisely known as the full CMS energy $s$, so some unfolding is needed, but this effect is not large. Event rates are low, but the enormous size of the $B_{ABAR}$ data sample means that numbers of events are large enough.

Photons can also be produced by electromagnetic radiation from final state particles (FSR), and the two amplitudes can in principle interfere. However the amplitude falls with energy so much that this effect is very small.

The apparatus and its performance have been described previously [1]. In this analysis the Electromagnetic calorimeter, with 6,580 CsI(Tl) crystals, is of paramount importance as it enables the ISR photon to be clearly distinguished and its energy accurately measured. It
Figure 1. The ISR process: Feynman diagram and an observed event

Figure 2. Radiative corrections to $g - 2$

also provides excellent reconstruction for $\pi^0$ and $\eta$ mesons. The DIRC ring imaging Cherenkov detector is also significant as it provides superb $K/\pi$ separation.

2. The $g - 2$ puzzle

One motivation for exploring this region of low energy $e^+e^-$ scattering is the reported anomaly in the measurement of the magnetic moment of the muon. The value $a_\mu = g - 2$ is measured[2] as $(11 659.1 \pm 5.4 \pm 3.3) \times 10^{-10}$ but the Standard Model calculation[3] gives $(11 659.4 \pm 5.1) \times 10^{-10}$. This is a 3.4 standard deviation ‘tension’ between experiment and the Standard Model. Although such effects have a history of disappearance, it deserves serious investigation and the BNL muon storage ring has been moved to FNAL where measurements will reduce the experimental statistical and systematic errors.

This must be matched by improvement in the theoretical errors. The difference from the Dirac value of 2 is due to higher order corrections to the $\mu - \mu - \gamma$ vertex, including the loop correction shown in Figure 2.

The corrections from QED and weak interactions are calculable, but if the particle in the loop is a quark then the amplitude cannot be calculated perturbatively as the strong coupling constant is large at this low energy. While lattice calculations offer interesting possibilities, in practice this correction needs to be obtained from experimental measurements. This can be done using the optical theorem which links the $\gamma - X - \gamma$ loop correction to the cross section for the reaction $e^+e^- \rightarrow \gamma^* \rightarrow X$.

One thus needs to measure the value of the ratio $R = \sigma(\text{hadrons})/\sigma(\mu^+\mu^-)$ as a function of
energy, with the range below 2 GeV being of greatest importance. Although this is an inclusive quantity, it has to be measured through the different exclusive channels, which are then summed, as these have very different experimental efficiencies.

3. The reaction \( e^+e^- \rightarrow \pi^+\pi^- \)

The most important contribution to \( a_\mu \) is the \( \pi^+\pi^- \) channel, and results are shown in Figure 3. The spectrum is dominated by the broad \( \rho(770) \) resonance, and other resonances are needed to fit the data at high energies.

Use of data driven techniques drives the systematic uncertainties below the 1% level. For example, the effective luminosity is obtained from measuring \( \mu\mu\gamma \) events rather than relying on Monte Carlo simulations (though these have been performed, and are consistent). Efficiencies for particle identification - which uses information from the DIRC and other detectors, and also the results of a kinematic fit - are determined by Monte Carlo data corrected by using clearly identifiable control channels.

This analysis also includes the next order correction \( \pi^+\pi^-\gamma\gamma \) events, where the second \( \gamma \) may be visible or it may have been emitted along the beam pipe.

From this we obtain the pion contribution to vacuum polarisation (from threshold up to 1.8 GeV) of \( (514.1 \pm 2.2 \pm 3.1) \times 10^{-10} \). The relative statistical error is large above the 1.8 GeV limit, but the absolute error is small, and for these high energies a QCD calculation can be used.

4. The reaction \( e^+e^- \rightarrow K^+K^- \)

A similar analysis has been performed where the two tracks are identified as kaons [5]. The cross section is shown in Figure 4. It is dominated by the narrow \( \phi \) resonance. We have fitted the mass and the width of the peak, and the results agree with standard values [6]. Results are in excellent agreement with previous low energy experiments at CMD-2 [7] and SND [8], but our statistics are significantly higher than theirs. This analysis has also been used to extract the kaon form factor.

Integration gives the \( K^+K^- \) contribution to \( a_\mu \) as \( (22.93 \pm 0.18 \pm 0.22 \pm 0.03) \times 10^{-10} \) where the 3rd error is a systematic arising from the parametrisation of the \( \phi \) meson.

Figure 3. Cross section for the reaction \( e^+e^- \rightarrow \pi^+\pi^- \) as a function of energy. Taken from [4]
Figure 4. Cross section for the reaction $e^+e^→ K^+K^−$ as a function of energy. Taken from [5]

Figure 5. (a) Left: the invariant mass distribution of $K_S^0$ candidates. (b) Right: The cross section for the reaction $e^+e^→ K_S^0K_L^0$ as a function of energy. Taken from [9]

5. The reactions $e^+e^→ K_S^0K_L^0, K_S^0K_L^0\pi^+\pi^−, K_S^0K_S^0\pi^+\pi^−$ and $K_S^0K_S^0K^+K^−$

The analysis has been extended to neutral kaons [9]. $K_S^0$ particles are easy to identify, from pairs of $\pi^+\pi^−$ tracks forming a distinct vertex. The mass distribution is shown in Figure 5a. Note the scale of the $x$ axis and the logarithmic $y$ axis. $K_S^0$ particles are harder. They do not decay in the detector but interact to give a cluster in the EM calorimeter, which appears similar to a photon cluster. However the efficiency can be comprehensively studied from $\phi → K_S^0K_L^0$ decays.

The $K_S^0K_L^0$ reaction is dominated by $\phi$, like $K^+K^−$, and again fitted results agree with standard values. The $\phi'(1680)$ appears as a positive peak, shown in close-up in Figure 5b, whereas for $K^+K^−$ it produces an edge, pointing to differences in the phases of the $\phi$ and $\phi'$ in the two channels.

The analysis also gives first measurements of the channels $e^+e^→ K_S^0K_L^0\pi^+\pi^−$, which is dominated by $K^*$ production with some $\phi\pi\pi$, and of $K_S^0K_S^0\pi^+\pi^−$ and $K_S^0K_S^0K^+K^−$. Full details are in the reference [9].
6. The reactions \( e^+e^- \rightarrow K_S^0K_L^0\pi^0 \), \( K_S^0K_L^0\eta \) and \( K_S^0K_L^0\pi^0\pi^0 \)

Recently the analysis has been extended to states with neutral pions and kaons[10]. Some results are shown in Figure 6.

The \( K_S^0K_L^0\pi^0 \) system is dominated by \( KK^*(892) \) (red circles) with a small \( \phi\pi^0 \) contribution, and a \( J/\psi \) signal. The \( K_S^0K_L^0\eta \) system is dominated by \( \phi \) (red circles). Again, a small \( J/\psi \) peak appears.

The measurement of the \( K_S^0K_L^0\pi^0\pi^0 \) state gives the total contribution of \( KK\pi\pi \) to \( a_\mu \) as \((0.85 \pm 0.05) \times 10^{-10}\) - previously it was \((1.35 \pm 0.39) \times 10^{-10}\). This is an order of magnitude improvement in the error, as the channels are now all measured with no reliance on isospin relations.

7. The reaction \( e^+e^- \rightarrow \pi^+\pi^-\eta \)

The \( \pi^+\pi^-\eta \) channel has also been studied [11]. The \( \eta \) is detected through the decay mode \( \eta \rightarrow \gamma\gamma \) so the event comprises 3 photons and two tracks, and the \( \eta \) peak is very clean, with low backgrounds. The regions \( E_{\gamma\gamma} \) above and below are treated separately.

The cross section is shown as a function of energy in Figure 7. Data from previous experiments, and an earlier \( B_{s\bar{s}}B_{s\bar{s}} \) result, are shown for comparison. The results agree, but the precision of the new data is clearly superior.
Figure 8. The cross section for $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ as a function of energy, together with results from previous experiments. Taken from [12]

The cross section is well described by models including contributions from the $\rho(770)$, $\rho(1450)$ and $\rho(1700)$. There is a peak at 3.1 GeV (removed from the plot) which can be used to measure the branching ration $BR(J/\psi \rightarrow \pi^+\pi^-\eta) = (0.042 \pm 0.008)\%$.

8. The reaction $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$

Finally, we have performed an analysis of the four pion state by selecting the full 431 fb$^{-1}$ sample of $e^+e^-$ collisions, and 150,000 signal events were selected. Results are shown in Figure 8. The superiority of the $BaBar$ data (black) over previous experiments is clear.

Our precision gives a significant improvement to the evaluation of the $a_\mu$ theory uncertainty for this channel, which was one of the larger contributors to the the previous total theory error. We now have a contribution to $a_\mu$ $(18.1 \pm 0.1 \pm 0.6) \times 10^{-10}$ in the range 0.85 to 1.8 GeV. The uncertainties correspond to a total relative precision of 3.2%. Hence, the relative precision of the final $BaBar$ measurement alone is a factor 2.5 higher than the precision of the world data set without $BaBar$.

9. Conclusions

Use of the ISR technique with an identified photon has enabled $BaBar$ to do precision studies in low energy $e^+e^-$ annihilation, over a large number of channels. The high luminosity and the particle identification and energy/momentum measurement more than compensate for any drawbacks, and the data quality and quantity surpasses the results of the low energy storage rings.

These measurements have been used to improve the uncertainty on the theoretical calculation of the muon $g - 2$ value. This shows that, if the currently observed anomaly persists when new results are obtained from the forthcoming run at FNAL, this will not be explainable by uncertainties in the hadronic vacuum polarisation.
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