Improved grey correlation theory to quantitatively characterize the effect of pore parameters on compressive strength of cement Pisha sandstone

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Abstract: In order to objectively characterize the correlation degree of the influence of different pore parameters of cement solidified soft rock on compressive strength. The initial data of pore parameters and compressive strength are normalized by five different dimensionless methods, and the correlation degree of each parameter is calculated. Based on the mode theory and spearman theory, an improved grey correlation calculation model is established to evaluate the correlation degree of pore parameters. The results show that the correlation degree between pore parameters and compressive strength of cement Pisha sandstone is in the following order: >0~0.1μm aperture ratio, bound fluid index, >1μm aperture ratio, free fluid index, T2 spectrum area, porosity and >0.1~1μm aperture ratio. The spearman rank correlation coefficients of sample C4 were 0.964, 0.536, 0.893 and 0.964 respectively; The spearman rank correlation coefficients of sample C7 were 0.964, 0.607, 0.964 and 0.893 respectively. It can be seen that "Range Analysis" is the best normalization method among the five dimensionless methods, and its evaluation results are most in line with the actual situation. This study provides a new mathematical method for quantitatively characterizing the grey correlation effect of pore parameters of cement-based materials on compressive strength.

1. Introduction
The Pisha sandstone is mainly developed in the Paleozoic Permian, Mesozoic Triassic, Jurassic and Cretaceous. It is mainly distributed in the Ordos Plateau bordering Shanxi, Shaanxi and Inner Mongolia. The area is about 16700 km², and the erosion modulus is as high as 20 ~ 40 000 t / (km² ·a). The soil and water loss is very serious, which has an extremely negative impact on the local ecological environment\cite{1}. Pisha sandstone has weak cementation, high porosity, low structural strength and poor diagenesis. Some scholars have modified the Pisha sandstone and explored the law of strength development and the evolution of pore structure\cite{2,3}. Domestic and foreign scholars have introduced mathematical methods into the division and evaluation of pore parameters and achieved great results.
Dyman et al. [4] quantitatively studied the pore structure characteristics of Cretaceous sandstone in Montana by factor analysis and cluster analysis; Dubois et al. [5] compared the application of artificial neural network, fuzzy logic and Bayesian discrimination in rock pore junction classification. Literature [6] studied the microscopic pore structure of sandstone based on fractal theory; Literature [7] proposed the automatic classification technology of pore structure based on information entropy-fuzzy spectral clustering algorithm. However, the pore structure of Pisha sandstone is complex and anisotropic, so it is difficult for a single mathematical evaluation method to objectively and truly characterize the influence of pore structure on strength. For this reason, based on the nuclear magnetic resonance test, this paper uses the modal theory and spearman theory [8] to optimize the dimensionless treatment method of pore parameters of cement Pisha sandstone, and applies grey system theory to establish a multi-index grey correlation model, so as to explore the influence of pore structure parameters of modified Pisha sandstone on compressive strength.

2. Materials and methods

2.1. Materials

The Pisha sandstone is taken from the north district of Kangbashi, Ordos City, Inner Mongolia. The remolded Pisha sandstone samples are naturally air-dried, rolled and screened through the square hole of 2.36mm. The physical index of the Pisha sandstone is shown in Table 1, and the gradation curve is shown in figure 1. According to XRD phase analysis (fig 2), it is found that the main mineral composition of Pisha sandstone is quartz, montmorillonite, mica, plagioclase, calcite, microcline and kaolinite. The cement is Mengxi P·O42.5 ordinary Portland cement, the water is ordinary tap water, and the cement solidified Pisha sandstone samples with cement content of 4% and 7% are prepared.

| Natural moisture content/% | Natural density/g cm$^{-3}$ | Air-dried moisture content/% | Optimal moisture content/% | Maximum dry density/g cm$^{-3}$ | Liquid limit /% | Plastic limit /% | Plastic index |
|---------------------------|-----------------------------|-----------------------------|---------------------------|-------------------------------|----------------|----------------|---------------|
| 4.83–6.92                | 1.63–1.69                   | 1.9–2.1                     | 14.5                      | 1.77                          | 27.6           | 18.3           | 9.3           |

Fig 1  Grading curve of Pisha sandstone  
Fig 2  XRD phase analysis of Pisha sandstone

2.2. Methods

2.2.1. NMR test

The MacroMR12-150H-I nuclear magnetic resonance (NMR) analysis system produced by Suzhou Newmai Science and Technology Co., Ltd. was used to test the characteristic parameters of NMR pore structure of the samples with curing age of 7, 14, 21 and 28 days. The relationship between transverse relaxation time and porosity of NMR is as follows [9]:
According to Equation (1), the transverse relaxation time $T_2$ depends on $S/V$, so $T_2$ spectrum can reflect the pore size and distribution interval of the test sample. The larger $T_2$ value of transverse relaxation time indicates the macropore structure, while the smaller $T_2$ value corresponds to the small pore component.

2.2.2. Improved grey correlation analysis

(1) Initial data dimensionless

Suppose $X$ is a set of grey correlation factors, $x_0 \in X$ is the reference column, $x_i = (x_i(1), x_i(2), \ldots, x_i(n))$; $x_i \in X(i = 1, \ldots, N)$ is the comparison column, $x_i = (x_i(1), x_i(2), \ldots, x_i(n))$, where $n$ is the number of elements in the set $x$. In order to make different types of data comparable, dimensionless processing is needed. The common dimensionless methods are as follows [10]:

(a) Mean Value Analysis:

$$X_i = \frac{x_i}{\bar{x}}$$  \hspace{1cm} (2)

(b) Standardization Analysis:

$$X_i = \frac{x_i - \bar{x}}{\sigma}$$  \hspace{1cm} (3)

(c) Minimization Analysis:

$$X_i = \frac{x_i}{x_{\text{max}}}$$  \hspace{1cm} (4)

(d) Maximation Analysis:

$$X_i = \frac{x_i}{x_{\text{min}}}$$  \hspace{1cm} (5)

(e) Range Analysis:

$$X_i = \frac{x_i - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}}$$  \hspace{1cm} (6)

In formula (2) ~ (6), $\bar{x}$, $\sigma$, $x_{\text{max}}$ and $x_{\text{min}}$ are the average, standard deviation, maximum and minimum of the set $x$, respectively.

(2) Grey correlation degree

The grey correlation coefficient between the comparison column and the reference column is:

$$\rho = \frac{\min_{i \in \mathbb{N}} \max_{k \in \mathbb{N}} \Delta_i(k) + \rho \max_{i \in \mathbb{N}} \max_{k \in \mathbb{N}} \Delta_i(k)}{\max_{i \in \mathbb{N}} \min_{k \in \mathbb{N}} \Delta_i(k) + \rho \max_{i \in \mathbb{N}} \max_{k \in \mathbb{N}} \Delta_i(k)}$$  \hspace{1cm} (7)

In the formula $k = 1, 2, \ldots, n$, $\min_{i \in \mathbb{N}} \Delta_i(k)$ is the minimum difference between the two poles, $\max_{i \in \mathbb{N}} \Delta_i(k)$ is the maximum difference between the two poles, $\rho$ is the resolution coefficient, the value is between 0 and 1, generally 0.5.

The correlation degree between the comparison column and the reference column is calculated as:

$$\gamma = \frac{1}{n} \sum_{k=1}^{n} \xi(k)$$  \hspace{1cm} (8)

The larger the correlation degree, the higher the correlation degree between the parameters of the comparison column and the reference column.

(3) Grey correlation optimization

In the grey correlation analysis, different dimensionless processing is used for the initial data, and the grey correlation ranking obtained is different [10]. In order to make the evaluation results more
objective and in line with the actual situation, this paper uses the modal theory to count the grey correlation order frequency and determine the reference order of the comparison column, then the spearman rank correlation coefficient $\gamma_s$ is determined based on spearman theory, its value is between -1 and 1, and the closer it is to 1, the higher the positive correlation and the better consistency of the two kinds of sorting, so that the dimensionless processing method which accords with the reality can be selected. The spearman grade correlation coefficient $\gamma_s$ is calculated according to the following formula:

$$
\gamma_s = 1 - \frac{6 \sum_i d_i^2}{n(n^2-1)}
$$

In the formula, $d_i$ is the difference between the associated sorting number and the reference sorting number of the $i$ evaluation method, $n$ is the total number of evaluation objects.

3. Results and Discussion

3.1. Pore structure parameters

Based on NMR, the microscopic pore structure of cement Pisha sandstone samples with curing age of 7 to 28 days was tested, as shown in Table 1. It can be seen from Table 1 that with the increase of age, porosity, $T_2$ spectrum area, free fluid index and pore diameter ratio of $> 0.1 \mu m$ all decrease, while irreducible fluid index and pore diameter ratio of $> 0 \sim 0.1 \mu m$ increase. Thus it can be seen that there is a negative correlation between porosity and irreducible fluid index to some extent. Taking the sample with 4% cement as an example, after 28 days of hydration, the bound fluid index increased from 64.6% to 66.0%, and the porosity decreased from 35.4% to 34.0%.

| Sample | Curing age/d | Porosity/% | $T_2$ Spectrum area | Bound fluid index/% | Free fluid index/% | $>0$~$0.1\mu m$ Aperture ratio/% | $>0.1$~$1\mu m$ Aperture ratio/% | $>1\mu m$ Aperture ratio/% |
|--------|--------------|-------------|---------------------|---------------------|---------------------|-------------------------------|-------------------------------|-----------------------------|
| C4     | 7            | 33.3        | 48584.11            | 64.6                | 35.4                | 45.02                         | 21.89                         | 33.09                       |
|        | 14           | 32.9        | 48672.30            | 65.1                | 34.9                | 47.68                         | 19.80                         | 32.52                       |
|        | 21           | 32.6        | 48316.09            | 65.3                | 34.7                | 48.34                         | 19.20                         | 32.46                       |
|        | 28           | 29.9        | 44687.13            | 66.0                | 34.0                | 49.48                         | 18.76                         | 31.76                       |
| C7     | 7            | 31.0        | 46086.19            | 63.2                | 36.8                | 48.24                         | 17.58                         | 34.18                       |
|        | 14           | 30.5        | 46086.70            | 63.7                | 36.3                | 50.94                         | 15.23                         | 33.82                       |
|        | 21           | 29.5        | 44995.46            | 64.5                | 35.5                | 52.71                         | 13.90                         | 33.39                       |
|        | 28           | 28.4        | 43514.80            | 65.3                | 34.7                | 53.44                         | 13.75                         | 32.81                       |

Note: C4 and C7 represent solidified Pisha sandstone with cement content of 4% and 7% respectively, the same below.

3.2. Grey correlation degree

As space is limited, take sample C4 as an example. After different dimensionless normalization of the initial data, the grey correlation between pore parameters and compressive strength is calculated by using the formula (7) ~ (8), as shown in Table 3. It can be seen from Table 3 that there are some differences in the grey correlation degree obtained by different dimensionless treatments.
Table 3  Grey correlation degrees under different dimensionless conditions

| Parameters           | Mean Value Analysis | Standardization Analysis | Minimization Analysis | Maximization Analysis | Range Analysis |
|----------------------|---------------------|--------------------------|-----------------------|------------------------|---------------|
| Porosity/%           | 0.6020              | 0.5739                   | 0.6168                | 0.5122                 | 0.5312        |
| T₂ Spectrum area     | 0.6076              | 0.5893                   | 0.6136                | 0.5192                 | 0.5379        |
| Bound fluid index/%  | 0.6463              | 0.8077                   | 0.6839                | 0.5426                 | 0.8148        |
| Free fluid index/%   | 0.6266              | 0.5674                   | 0.6528                | 0.5251                 | 0.5404        |
| >0~0.1μm Aperture ratio/% | 0.6656            | 0.9314                   | 0.7084                | 0.5571                 | 0.9395        |
| >0.1~1μm Aperture ratio/% | 0.6035           | 0.5862                   | 0.6158                | 0.4861                 | 0.4354        |
| >1μm Aperture ratio/% | 0.6289              | 0.5858                   | 0.6536                | 0.5250                 | 0.5586        |

3.3. optimal grey correlation

On the basis of obtaining the grey correlation degree between pore parameters and compressive strength, the modal theory is applied to obtain the distribution of pore parameter sequence frequency of sample C4, as shown in Table 4. It can be seen from Table 4 that the sequence frequency distribution of other pore parameters is relatively dispersed, except for the bound fluid index and the proportion of >0~0.1μm pore radius.

Table 4  Distribution of pore parameter sequence frequency based on modal theory

| Parameters           | First order | Second order | Third order | Fourth order | Fifth order | Sixth order | Seventh order |
|----------------------|-------------|--------------|-------------|--------------|------------|-------------|---------------|
| Porosity/%           | 0           | 0            | 0           | 0            | 1          | 3           | 1             |
| T₂ Spectrum area     | 0           | 0            | 1           | 0            | 3          | 0           | 1             |
| Bound fluid index/%  | 0           | 5            | 0           | 0            | 0          | 0           | 0             |
| Free fluid index/%   | 0           | 0            | 1           | 3            | 0          | 0           | 1             |
| >0~0.1μm Aperture ratio/% | 5           | 0            | 0           | 0            | 0          | 0           | 0             |
| >0.1~1μm Aperture ratio/% | 0           | 0            | 0           | 1            | 0          | 2           | 2             |
| >1μm Aperture ratio/% | 0           | 0            | 3           | 1            | 1          | 0           | 0             |

Combined with the sequence frequency distribution in Table 4, the ranking results of the correlation degree between pore parameters and compressive strength determined by the number theory are used as a reference (Table 5). As can be seen from Table 5, the grey relational orders obtained by different dimensionless treatments are different. At the same time, the grey relational ranking under each dimensionless treatment can be determined from Table 2 as shown in Table 5.

Table 5  Grey relational ranking and reference ranking under different dimensionless treatments

| Parameters           | Mean Value Analysis | Standardization Analysis | Minimization Analysis | Maximization Analysis | Range Analysis | Sequence of reference |
|----------------------|---------------------|--------------------------|-----------------------|------------------------|---------------|------------------------|
| Porosity/%           | 7                   | 6                        | 5                     | 6                      | 6             | 6                      |
| T₂ Spectrum area     | 5                   | 3                        | 2                     | 2                      | 5             | 5                      |
| Bound fluid index/%  | 2                   | 2                        | 2                     | 2                      | 2             | 2                      |
| Free fluid index/%   | 4                   | 7                        | 4                     | 3                      | 4             | 4                      |
| >0~0.1μm Aperture ratio/% | 1                   | 1                        | 1                     | 1                      | 1             | 1                      |
| >0.1~1μm Aperture ratio/% | 6                   | 4                        | 6                     | 7                      | 7             | 7                      |
| >1μm Aperture ratio/% | 3                   | 5                        | 3                     | 4                      | 3             | 3                      |

According to the formula (9), the spearman rank correlation coefficients of grey correlation ranking and reference ordering of sample C4 under five different dimensionless treatments were calculated, which were 0.964, 0.536, 0.893 and 0.964 respectively. It can be seen that the Spearman rank correlation coefficient obtained after the dimensionless treatment of "Range Analysis" was the highest, indicating that the evaluation results obtained by this method were most consistent with the actual situation.
addition, after calculation (process is brief), the spearman rank correlation coefficients of sample C7 under five different dimensionless methods are 0.964, 0.607, 0.964, 0.893, 1 respectively. The spearman rank correlation coefficient obtained by "Range Analysis" dimensionless method is the highest, which further shows that the "Range Analysis" dimensionless treatment is the optimal normalization method, which is consistent with the research results of reference[10]. Therefore, the improved correlation model obtained by using "Range Analysis" dimensionless normalization to the initial data of cement Pisha sandstone is the most consistent with the objective reality.

4. Conclusions
(1) Based on the modal theory, the grey correlation degree between pore parameters and compressive strength of cement Pisha sandstone is determined. The degree of grey correlation is in the following order: >0~0.1μm aperture ratio, bound fluid index, >1μm aperture ratio, free fluid index, $T_2$ spectrum area, porosity and >0.1~1μm aperture ratio.

(2) The spearman theory was used to calculate the spearman rank correlation coefficients of sample C4 under five dimensionless treatment methods, which were 0.964, 0.536, 0.893 and 0.964 respectively, and the spearman rank correlation coefficients of sample C7 were 0.964, 0.607, 0.964 and 0.893 respectively. The maximum spearman rank correlation coefficient of C4 and C7 is 1, and the dimensionless treatment method is "Range Analysis". Therefore, the evaluation result obtained by the infinite method of "Range Analysis" is the best normalization method, which accords with the actual situation.

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