An inverse problem of thickness design for bilayer textile materials under low temperature

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Abstract. The human heat-moisture-comfort level is mainly determined by heat and moisture transfer characteristics in clothing. With respect to the model of steady-state heat and moisture transfer through parallel pore textiles, we propose an inverse problem of thickness design for bilayer textile material under low temperature in this paper. Adopting the idea of regularization method, we formulate the inverse problem solving into a function minimization problem. Combining the finite difference method for ordinary differential equations with direct search method of one-dimensional minimization problems, we derive three kinds of iteration algorithms of regularized solution for the inverse problem of thickness design. Numerical simulation is achieved to verify the efficiency of proposed methods.

1. Introduction

With the rapid developments in sciences and technologies, people have increasing requirements on functional clothing, such as strong, durable clothing in the past, meanwhile both fashionable and comfortable clothing in the recent years. As for the human-body-comfort textiles, it is hoped that the textiles are of fast decalescence, fast heat radiation, soft or stand-up apparel. Of course, clothing should be light and keeps body warm under low temperature, meanwhile sweat vaporizes fast and body feels cool under high temperature. Hence people need take full account of the functional material design based mainly on heat and moisture transfer characteristics.

We should consider the physical characteristics (e.g. sorption, condensation), structure characteristics (e.g. porous media, parallel pore structure, pellets accumulation pore structure; single-layered or multi-layered clothing) in textile materials and moreover various heat and mass transfer, momentum transfer (e.g. conduction, convection, radiation and molecular diffusion, etc.) [1-2]. From 1930s, researchers discussed a few models of heat and moisture transfer and corresponding numerical simulations of thermal and water vapor concentration in porous media [3-14]. Henry first proposed the establishment of coupled heat and moisture transfer model base on differential element in 1939 [3] and further analyzed the model in 1948 [4], where two parabolic partial differential equations were used to describe the process of heat and moisture transfer and coupling term was used to describe the fiber absorption and desorption of moisture and the latent heat.
Different textile fibers have different heat capacity and thermal conductivity, consequently, heat and moisture transfer in fabric will be different. In addition, the process of moisture and humidity changes have latent heat release and absorption, which also affect heat and moisture transfer in the fabric. Although considerable previous work has been carried out on the diverse aspects of simultaneous heat and moisture transfer in the literature both mathematically (in mathematical modeling way) and experimentally, little has been done on the coupling of heat and moisture transfer with phase change and condensation before 2000. From 2000, some researchers (e.g. Fan and Li [5-13], Huang [14]) have already put forward a few mathematical models of coupled heat and moisture transfer through porous clothing assemblies and fibrous insulation. Based on these models, they have designed different numerical methods to solve these problems [11-14], such as finite difference method, finite volume method, finite element method and controllability volume-time-domain recursive method, and the numerical results are well matched with experimental results. On the basis of present research results and the analysis of the basic features of various models, we can see that coupled models of heat and moisture transfer are the most promising theoretical model, so it is necessary that we should study well-posed conditions and numerical algorithms for such problems. 

Recently, a kind of coupled model of heat and moisture transfer was established on the basis of ordinary differential equations [15]. Textile is certainly of a complex multi-pore structure, which can be described as the parallel pore structure and pellets accumulation pore structure. The heat and moisture transfer through textile is affected by the textile structure. In [15], Based on the parallel pore structure textile and a system of human-textile-environment, a model of heat and moisture transfer through the parallel pore textiles is established. It is a system of coupled ordinary differential equations on temperature, water vapor pressure and water vapor mass flux through textile and condensation on the surface of textile. According to this model, we propose an inverse problem of thickness design for single layer textile material [16], and propose numerical algorithms of the inverse problem, numerical simulation is achieved in order to verify the validity of proposed methods.

In this paper, we propose an inverse problem of thickness design for bilayer textile material with parallel pore structure. In this humanbody-clothing-environment system, thickness of the outer fabric is known but thickness of the inner fabric is unknown and need determining. We design numerical algorithm for the inverse problem of thickness design. The numerical results indicate the correctness of mathematical models and validity of numerical algorithms.

2. Mathematical formulation

In this section, we consider an inverse problem of thickness design for bilayer textile material based on the steady-state model presented in [15]. A clothing assembly consisting of a layer of inner fabric, a layer of outer fabric was considered in a cold climate in this paper. Such clothing assembly, as well as human body and environment, constitutes a system of Humanbody-Clothing-Environment.

2.1. Assumptions and restrictions

The following assumptions are made:

(A1) Each fabric material is isotropic.

(A2) Volume changes of the fibers due to changing moisture and water content are neglected.

(A3) Local thermal equilibrium exits among all phases.

(A4) Diffusion within the fiber is considered to be so rapid that moisture content at the fiber surface is in sorptive equilibrium with that of the surrounding air. We only consider the condensation of water on the fibre surface.
Figure 1. Schematic diagram of the body-clothing-environment system.

(A5). Free convection in the inner fabric is negligible.

(A6). Temperature, water vapor pressure, mass flux of water vapor and heat fluxes on the boundary of inner fabric and outer fabric are all continuous.

2.2. Mathematical formulation of the inverse problem of thickness design

Under the above assumptions, we can establish the following mathematical equations for the coupled heat and mass transfer in the parallel pore fabric according to the conservation of heat energy and mass balance.

The model of the inner fabric is given as follows

$$\begin{align*}
\frac{k_1 \varepsilon_1 r_1}{r_3} \cdot \frac{p_v}{T^{3/2}} \cdot \frac{dp_v}{dx} + m_v(x) &= 0 \\
\frac{dm_v}{dx} + \Gamma(x) &= 0 \\
\kappa_1 \frac{\partial^2 T}{\partial x^2} + \lambda \Gamma(x) &= 0 \\
\Gamma(x) &= -\frac{k_2 \varepsilon_2 r_2}{r_3} \cdot (p_{sat} - p_v) \cdot \frac{1}{\sqrt{T}},
\end{align*}$$

where $0 \leq x \leq L$, and the boundary conditions are prescribed as follows

$$\begin{align*}
T(0) &= T_L \\
m_v(0) &= m_{v,0}.
\end{align*}$$

Meanwhile the model of the outer fabric is given as follows

$$\begin{align*}
\frac{k_1 \varepsilon_2 r_3}{r_2} \cdot \frac{p_v}{T^{3/2}} \cdot \frac{dp_v}{dx} + m_v(x) &= 0 \\
\frac{dm_v}{dx} + \Gamma(x) &= 0 \\
\kappa_2 \frac{\partial^2 T}{\partial x^2} + \lambda \Gamma(x) &= 0 \\
\Gamma(x) &= -\frac{k_2 \varepsilon_2 r_2}{r_3} \cdot (p_{sat} - p_v) \cdot \frac{1}{\sqrt{T}},
\end{align*}$$

where $L \leq x \leq L + L_0$, and boundary value conditions are prescribed as follows

$$\begin{align*}
T(L + L_0) &= T_R \\
\kappa_2 T'(L + L_0) &= \frac{T_s - T(L + L_0)}{R_0 + (1/h_2)} \\
m_v(L + L_0) &= m_{v,R} \\
p_v(L + L_0) &= p_{v,R}.
\end{align*}$$

Nomenclatures and Symbols:

$T(x)$ is temperature(K);

$p_v(x)$ is water vapor pressure(Pa);
\[ m_v(x) \] is mass flux of water vapor \((kg/(m^2 \cdot s))\);
\[ \Gamma(x) \] is the rate of condensation \((kg/(m^3 \cdot s))\).
\[ \varepsilon_i, r_i \] and \(\tau_i\) \((i=1,2)\) denote the porosity of textile surface(\%), the radius of cylindrical pore(m) and the effective tortuosity of the textile, respectively.
\[ \kappa_1 \] and \(\kappa_2\) are thermal conductivities of inner and outer fabric respectively.\((W/(m \cdot K))\);
\[ R_{t0} \] is thermal resistance of outer textile material\((K \cdot m^2/W))\);
\[ h_t \] is the convective heat transfer coefficient between the outer surface of the textile material and the environment\((W/(K \cdot m^2))\);
\[ \lambda \] is latent heat of sorption and condensation of water vapor\((J/kg))\);
\[ T_{eR} \] is temperature of the out boundary of outer textile material;
\[ T_e \] is the environmental temperature;
\[ m_{v,R} \] is mass flux of water vapor of the out boundary of outer textile material;
\[ p_{v,R} \] is water vapor pressure of the out boundary of outer textile material;
\[ k_1 \] and \(k_2\) are constants which are related with molecular weight and gas constant.

The saturation vapor pressure within the parallel pore is given as follows [8]:
\[ p_{sat}(T) = 100 \cdot e^{18.956-\frac{4030}{(T-273.16)+235}} \]

Let \(k_{3,i} = \frac{\kappa_i}{\lambda}, A_i = \frac{\varepsilon_i r_i}{\tau_i}\) \((i = 1, 2)\).

The model(3) combined with boundary conditions(4) constitute the terminal value problem of coupled ordinary differential equations, it is usually called a direct problem \((DP\) in abbreviation).

In above heat and moisture transfer process, heat conductivity, condensation and diffusion are involved in the double layer parallel cylindrical pore fabric.

Next, we consider an inverse problem of thickness design for double-layer textile material with parallel cylindrical pore structure.

Suppose that the environmental temperature and relative humidity are given as follows:
\[(T^e, RH^e) \in [T_{min}, T_{max}] \times [H_{min}, H_{max}],\]
where \(T_{min}\) and \(T_{max}\) are minimum average temperature and maximum average temperature at a specific place during a specific time period respectively; Similarly \(H_{min}\) and \(H_{max}\) are minimum average relative humidity and maximum average relative humidity respectively.

Suppose that the structure and type of both layer fabric are known. The structure of textile includes the radius of pore, porosity of textile surface and effective tortuosity of the fabric.

The literatures on clothing thermal comfort have indicated that the comfort indexes in the clothing microclimate, which is located between the skin surface and the inner surface of fabric, are given as follows (refer to [2]): temperature \((32 \pm 1)\)\(^\circ\)C, relative humidity \((50\% \pm 10\%)\), wind speed \((0.25 \pm 0.15)m/s\).

According to the requirements of clothing thermal comfort, we intend to determine the thickness \(L\) of inner fabric. Thus, the inverse problem of thickness design can be formulated as follows:

**Inverse Problem of Thickness Design (IPTD in abbreviation):** Given the environmental temperature and relative humidity and the above comfort indexes, according to the boundary conditions \((2)\) and right boundary conditions \((4)\), we intend to determine the thickness \(L\) of inner fabric, where \(T_R\) and \(p_{v,R}\) are both related with environmental temperature and humidity.

3. Numerical Algorithms of the IPTD

With regard to the inverse problem of thickness design for bilayer fabric, we construct numerical algorithm by adopting idea of regularization method. The similar algorithm was constructed to solve the inverse problem of thickness design for single-layer fabric in [16].
3.1. Regularized Solution of the IPTD

In order to obtain the regularized solution, we discretize the combination of environmental temperature and humidity as $(T_e^i, RH_e^j)$ ($i = 1, 2, \ldots, k; j = 1, 2, \ldots, m$). Under this combination, we can obtain the numerical solution of the temperature at inner side of outer fabric, $T_{ib}^{i,j}$, and the water vapor pressure at inner side of outer fabric, $p_{iv,b}^{i,j}$. As temperature and water vapor pressure are both continuous on the boundary, we can use $T_{ib}^{i,j}$ and $p_{iv,b}^{i,j}$ as the right boundary value of inner fabric. Let $RH_{i,j,0}(x)$ be relative humidity of inner fabric, thus we can easily obtain its numerical solution through the model (1) and following conditions:

$$\begin{align*}
T(0) &= T_L \\
T(L) &= T_b^{i,j} \\
M_v(0) &= m_{v,0} \\
p_v(L) &= p_v^{i,j}
\end{align*}$$ (5)

Suppose that $RH_0^*$ is experience value of relative humidity in comfortable state.

We can attribute the inverse problem to the following least squared problem:

$$\min \sum_{i=1}^{k} \sum_{j=1}^{m} (RH_{i,j,0}(x) - RH_0^*)^2$$

Since above least squared problem doesn’t exist unique solution or the solutions are unstable, we should adopt regularization idea to improve the least squared method.

In this respect, we define the following function:

$$J(x) = \alpha \cdot x^2 + \sum_{i=1}^{k} \sum_{j=1}^{m} (RH_{i,j,0}(x) - RH_0^*)^2$$

This function is different from the least squared function, as it is added a penalty term on the least squared function, where $\alpha > 0$ is a regularization parameter. Set $M = [0, L]$, which is called the permissible solution set. If $x_{reg}$ satisfies

$$J(x_{reg}) = \min_{x \in M} J(x)$$

then it is called the regularized solution of the IPTD, or the generalized solution.

3.2. Iteration Algorithms of the Regularized Solution

According to the idea that direct problem solving is combined with search iteration of one-dimension minimization problem, we construct the iteration algorithm:

$$x_{n+1} = x_n + \Delta_n \cdot d_n, n = 1, 2, \ldots$$

to solve the minimization problem such that $J(x_{n+1}) < J(x_n)$, where $x_1$ is arbitrarily given, and $x_n$ satisfies:

$$\begin{align*}
\frac{k_1 T_1}{\tau_1} \cdot \frac{p_v}{p_{sat}} \cdot \frac{dp_v}{dx} + m_v(x) &= 0, 0 < x < x_n \\
\frac{dM_v}{dx} + \Gamma(x) &= 0, 0 < x < x_n \\
k_1 \frac{dT}{dx} + \Lambda \Gamma(x) &= 0, 0 < x < x_n \\
\Gamma(x) &= \frac{k_2 T_1}{\tau_1} \cdot (p_{sat} - p_v) \cdot \frac{1}{\sqrt{x}}, 0 < x < x_n
\end{align*}$$ (6)
3.3. Search Method of The One-Dimensional Minimization Problem

The optimization problem involved in this paper is a single variable problem. As we know, \( RH_{i,j,0}(x) \) is relative humidity of inner fabric which is a numerical solution calculated by coupled ordinary differential equations, and it is difficult to obtain the derivative of \( RH_{i,j,0}(x) \), hence we have to use the direct search method. Taking this actual situation into account, we use Hooke-Jeeves pattern search algorithm[17-18], direct search algorithm by Cai[19] and Golden Section Method [20] respectively to solve the above optimization problem.

(1) Hooke-Jeeves’ Pattern Search Algorithm

Step 1. \( x_1 \) is given. Set initial step \( \Delta_1 > 0 \), acceleration factor \( \gamma \geq 1 \), reduced rate \( \beta \in (0,1) \), permissible error \( \varepsilon > 0 \), search direction \( e_1 = 1 \), \( e_2 = -1 \). set \( y_1 = x_1, i = 1 \).

Step 2. If \( J(y_1 + \Delta_1 \cdot e_1) < J(y_1) \), then

\[
y_2 = y_1 + \Delta_1 \cdot e_1
\]

carry out the step 4; otherwise, carry out the step 3.

Step 3. If \( J(y_1 + \Delta_1 \cdot e_2) < J(y_1) \), then

\[
y_2 = y_1 + \Delta_1 \cdot e_2
\]

carry out the step 4; otherwise, if \( J(y_1 + \Delta_1 \cdot e_2) \geq J(y_1) \), then

\[
y_2 = y_1,
\]

carry out the step 4.

Step 4. If \( J(y_2) < J(x_1) \), carry out the step 5; otherwise, if \( J(y_2) \geq J(x_1) \), carry out the step 6.

Step 5. \( x_{i+1} = x_i; y_1 = x_{i+1} + \gamma (x_{i+1} - x_i); i = i + 1 \); go to the step 2.

Step 6. If \( \Delta_1 \leq \varepsilon \), then stop, \( x^* = x_i \); otherwise, \( \Delta_1 = \beta \cdot \Delta_1; y_1 = x_i; x_{i+1} = x_i; i = i + 1 \); go to the step 2.

(2) Direct search algorithm proposed by Cai

Step 1. \( \eta \in \left[ \frac{1}{2}, 1 \right) \) is given. Set initial values \( x^0_i, x^0_j \in [0, L], x^0_i \neq x^0_j \). Suppose \( J(x^0_1) < J(x^0_2) \) (otherwise, exchange \( x^0_1 \) and \( x^0_2 \), \( k = 0 \).

Step 2. If \( \frac{|x^1_i - x^1_j|}{1-\eta} \geq L \), continue; otherwise, choose the initial values again.

Step 3. If \( |x^0_i - x^0_j| \leq \varepsilon \), go to the step 9.

Step 4. \( x^k_i = x^k_i - \eta(x^k_j - x^k_i) \)

Step 5. If \( J(x^k_i) \leq J(x^k_1) \) and \( x^k_i \in [0, L] \), then \( x^{k+1}_i = x^k_i; x^{k+1}_j = x^k_j; k = k + 1 \); go to the step 3.

Step 6. If \( J(x^k_i) > J(x^k_j) \) or \( x^k_i \notin [0, L] \), then \( x^k_{i+\eta} = x^k_i + \eta(x^k_j - x^k_i) \).

Step 7. If \( J(x^k_1) \leq J(x^k_2) \), then \( x^{k+1}_0 = x^k_0; x^{k+1}_i = x^k_i; k = k + 1 \); go to the step 3.

Step 8. If \( J(x^k_1) \leq J(x^k_1) < J(x^k_2) \), then \( x^{k+1}_0 = x^k_0; x^{k+1}_j = x^k_j; k = k + 1 \); go to the step 3.

\[
\begin{align*}
T(0) &= T_L \\
T(x_n) &= T_{sij} \\
m_v(0) &= m_{v,0} \\
p_{v}(x_n) &= p_{v,b},
\end{align*}
\]
Step 9. $x^* = x_k^1$, stop.

(3) Golden Section Method

Step 1. An initial interval $[a_1, b_1]$ is given. Set permissible error $\varepsilon > 0$. Choose the explosive point $\lambda_1$ and $\mu_1$, calculate the function values $J(\lambda_1)$ and $J(\mu_1)$:

$$\lambda_1 = a_1 + (1 - \beta)(b_1 - a_1), \mu_1 = a_1 + \beta(b_1 - a_1).$$

where $\beta = (\sqrt{5} - 1)/2$. $k = 1$
Step 2. If $b_k - a_k < \varepsilon$, then stop, $x^* = \frac{1}{2}(a_k + b_k)$; otherwise, if $J(\lambda_k) > J(\mu_k)$, go to the step 3; if $J(\lambda_k) \leq J(\mu_k)$, go to the step 4.
Step 3. $a_{k+1} = \lambda_k, b_{k+1} = b_k, \lambda_{k+1} = \mu_k, \mu_{k+1} = a_{k+1} + \beta(b_{k+1} - a_{k+1})$,
calculate $J(\mu_{k+1})$ go to the step 5.
Step 4. $a_{k+1} = a_k, b_{k+1} = \mu_k, \mu_{k+1} = \lambda_k, \lambda_{k+1} = a_{k+1} + \beta(b_{k+1} - a_{k+1})$,
calculate $J(\lambda_{k+1})$, go to the step 5.
Step 5. $k = k + 1$, go to the step 2.

4. Numerical simulations

In this section, numerical simulation is carried out to verify the validity of above numerical method and the efficiency of the numerical algorithms.

4.1. Micro-structure and Type of Textile Materials

We choose down material as inner fabric, and nylon as outside fabric. Structure and type parameters of these two textile materials are listed in Table 1.

Table 1. Geometric and physical parameters of textile materials

| Material          | Down (inner fabric) | Nylon (outer fabric) |
|-------------------|----------------------|----------------------|
| Thickness (m)     | Unknown and need determining | $2.73 \times 10^{-4}$ (fixed in advance) |
| Radius (m)        | $1.0 \times 10^{-5}$  | $3.71 \times 10^{-7}$  |
| Porosity          | 0.915                | 0.85                 |
| Heat Conductivity | 0.024                | 0.25                 |
| Effective tortuosity | 1.2              | 1.1                |

4.2. The parameters in simulation process

We consider numerical solution for the heat and mass transfer model consisting of down material and nylon batting under the given boundary conditions of temperature and water vapor concentration in order to verify the correctness of heat and moisture model and efficiency of the proposed algorithm.

Suppose that the initial mass flux of water vapor is $m_w(0) = 3.3084 \times 10^{-5} kg/(m^2 \cdot s)$; The temperature of the inner side of fabric is assumed to be $32^\circ C$ to guarantee that temperature
in microclimate is in the comfort index interval, that is \( T(0) = 305.16 \, K \); The convective heat transfer coefficient between the outer surface of the textile material and the environment is \( h_t = 8 \, \text{W/}(m^2 \cdot K) \); Thermal resistance of outer textile material is \( R_{t0} = 0.0315 \, \text{K} \cdot m^2/\text{w} \);

The mass flux of water vapor between the outside fabric and environment is assumed to be \( \dot{m}_v(L + L_0) = 1.5 \times 10^{-5} \, \text{kg}/(m^2 \cdot s) \); In the model, \( k_1 = 0.00006, k_2 = 0.00007 \) are constants which are related with molecular weight and gas constant; \( T_e \) is the environmental temperature; \( T_R \) and \( p_{vR} \) are related to the temperature and moisture of environment.

The latent heat of sorption or condensation of water vapor is determined by moisture level in the fabrics, and it has nothing to do with the type of material, here we choose \( \lambda = 2260 \times 10^3 \, \text{J/kg} \).

We assume that heat conductivity of textiles is a constant due to small changes in the environment:

\[ T_e \in [-10^oC, 10^oC], RH_e \in [30\%, 90\%] \]

4.3. Measurements on Environmental Temperature and relative Humidity

Two different environmental conditions under low temperature are chosen for simulation.

Case 1. Environmental conditions: \( T_e \in [-10^oC, 0^oC], RH_e \in [40\%, 90\%] \)

Case 2. Environmental conditions: \( T_e \in [0^oC, 10^oC], RH_e \in [30\%, 85\%] \)

We make isometry subdivision on the interval \([T_{min}, T_{max}]\) and \([H_{min}, H_{max}]\) respectively, where \([T_{min}, T_{max}]\) is decomposed into \( k \) equal portions, and \([H_{min}, H_{max}]\) is decomposed into \( m \) equal portions. In simulation, we choose \( k = 10, m = 10 \).

4.4. The uniqueness of regularized solution from the Graphs of \( J(x) \)

Let’s study the property of function \( J(x) \) for different materials and environmental conditions through the following function graphs.

The graphs of \( J(x) \) for down material in case 1 and case 2 are shown in Figure 2 and Figure 3 respectively.

**Figure 2.** The Graph of Function \( J(x) \) in case 1.  
**Figure 3.** The Graph of Function \( J(x) \) in case 2.

From above two graphs, it’s easy to see that the minimum point of function \( J(x) \) is unique in the interval \([0, 0.01]\). This property illustrates that the regularized solution of the inverse problem of thickness design for double-layer textile materials is uniquely determined.
4.5. Realization of thickness determination of the inner textile material

Using above iteration algorithm for forward and inversion problems in section 3 and 4, we can obtain approximate values of thickness of the inner textile material respectively in Case 1 and Case 2.

Some parameters in above three kind of search methods of the one-dimensional minimization problems are given as follows:

(1) Hooke-Jeeves’ Pattern Search Algorithm
Acceleration factor $\gamma = 3$, reduced rate $\beta = 0.5$, permissible error $\varepsilon = 10^{-4}$.

(2) Cai’s Method
$x_0^1 = 0.001, x_0^2 = 0.008, \eta = 0.5, \varepsilon = 10^{-5}$.

(3) Golden Section Method
Initial interval $[a_1, b_1] = [0.001, 0.008], \varepsilon = 10^{-5}$.

In Case 1, the numerical solutions of the thickness of down material are shown in table 2.

Table 2. Approximate values of down material’s thickness determined by three algorithms in Case 1

| Algorithms          | Hooke-Jeeves’ Method | Cai’s Method | Golden Section Method |
|---------------------|----------------------|--------------|-----------------------|
| Initial Values(m)   | Thickness(mm)        | Thickness(mm)| Thickness(mm)         |
| 0.0005              | 4.8625               |              |                       |
| 0.001               | 5.025                | 4.869        | 4.867                 |
| 0.005               | 4.875                |              |                       |
| 0.008               | 4.875                |              |                       |

In Case 2, the numerical solutions of the thickness of down material are shown in the table 3.

Table 3. Approximate values of down material’s thickness determined by three algorithms in Case 2

| Algorithms          | Hooke-Jeeves’ Method | Cai’s Method | Golden Section Method |
|---------------------|----------------------|--------------|-----------------------|
| Initial Values(m)   | Thickness(mm)        | Thickness(mm)| Thickness(mm)         |
| 0.0005              | 4.45                 |              |                       |
| 0.001               | 4.2125               | 4.5          | 4.209                 |
| 0.005               | 4.2125               |              |                       |
| 0.008               | 4.075                |              |                       |

4.6. The Validation of the Numerical Results

In order to verify correctness and rationality of the above numerical results, we take the inversion results of thickness of down material as the thickness of inner textile material, and obtain the numerical results of relative humidity $RH_{i,j,0}(x)$ ($i = 1, 2, \cdots, k; j = 1, 2, \cdots, m$) in the microclimate area under the combination of environmental temperature and humidity. Then we will try to verify whether $RH_{i,j,0}(x)$ belong to the comfort index interval. If $RH_{i,j,0}(x)$ belong to the comfort index interval generally, then it indicates that the numerical results are reasonable.
Under the environmental condition 1 (Case 1), we show in figure 4 that the relative humidity $RH_{i,j,0}(x)$ ($i = 1, 2, \ldots, k; j = 1, 2, \ldots, m$) in the microclimate area vary as the environmental temperature and humidity $T_e^i \in [-10^\circ C, 0^\circ C], RH_e^j \in [40\%, 90\%]$ change. The relative humidity in the microclimate area $RH_{i,j,0}(x) \in [49\%, 52\%]$, which belong to the comfort index interval.

Under the environmental condition 2 (Case 2), we show in figure 5 that the relative humidity $RH_{i,j,0}(x)$ ($i = 1, 2, \ldots, k; j = 1, 2, \ldots, m$) in the microclimate area vary as the environmental temperature and humidity $T_e^i \in [0^\circ C, 10^\circ C], RH_e^j \in [30\%, 85\%]$ change. The relative humidity in the microclimate area $RH_{i,j,0}(x) \in [48\%, 56\%]$, which belong to the comfort index interval.

According to the figure 4 and figure 5, we prove that the determined thickness results of down material are correct and reasonable.

5. Conclusions and Prospects

On the basis of the stationary model of heat and moisture transfer through textiles, we present numerical algorithms for the inverse problems of thickness design for bilayer textile materials under low temperature. The associated direct problems are solved by FDM. Meanwhile, by means of three kinds of direct search methods of minimization of a regularized function, the IPTD are effectively solved.

The numerical results prove that the formulation of the IPTD is correct and the numerical algorithms are efficient in solving the IPTD. Therefore we are sure that the research results are necessary fundamentals to theoretical support and scientific explanations for laminated textile materials design experiments.

Hooke-Jeeves’ method, Cai’s method and Golden Section Method can applied to solve the IPTD for bilayer textile materials. The Golden Section Method is much simpler than the other two methods. As for Hooke-Jeeves’s pattern search algorithm, different initial values lead to different iterations and computational time, so its convergence is much sensitive to the initial value.

In this paper, we study the stationary model of heat and moisture transfer through textiles by formulation of the IPTD and corresponding numerical algorithms. It is necessary that we further study well-posedness of the IPTD and mathematical analysis of proposed numerical methods, which will be discussed in forthcoming papers.
6. Acknowledgments

The research is partially supported by National Natural Science Foundation of China (Contract Grant No. 10561001 and 11071221). The first-named author would like to extend his sincere thanks to the organizing committee of International Conference on Inverse Problems, held in Hong Kong City University on December 13-17 2010, for the financial support and the Outstanding Presentation IOP Award which I was luckily granted. Meanwhile he appreciates the inspiring discussion with Professor Y. Li in Donghua University and Hong Kong Polytechnic University, Professor J.T. Fan in Zhejiang Sci-Tech University and Hong Kong Polytechnic University and Prof W Sun in Hong Kong City University.

References

[1] Yao M 1996 Science of Textile Materials(2nd ed) (Beijing: China Textile Press).
[2] Huang J H 2008 Clothing Comfort (Beijing: Scientific Press).
[3] Henry P S H 1939 Diffusion in absorbing media Pro.R.Soc. vol 17, 215-41.
[4] Henry P S H 1948 The diffusion of moisture and heat through textiles Discuss. Faraday. Soc. 3 243-57.
[5] Li Y and Zhu Q Y 2003 A model of coupled liquid moisture and heat transfer in porous textiles with consideration of gravity Numer Heat Transfer, Part A. 43 501-23.
[6] Li Y and Luo Z X 2000 Physical mechanisms of moisture transfer in hygroscopic fabrics under humidity transients J. Textile Res. 91 302-16.
[7] Fan J T, Luo Z X and Li Y 2000 Heat and moisture transfer with sorption and condensation in porous clothing assemblies and numerical simulation Int. J. Heat Mass Transfer. 43 2989-3000.
[8] Wu H J and Fan J T 2008 Study of Heat and Moisture Transfer Within Multi-layer Clothing Assemblies Consisting of Different Types of Battings Int. J. Thermal Sciences 47 641-647.
[9] Fan J T, Cheng X and Sun W 2004 An improved model of heat and moisture transfer with phase change and mobile condensates in fibrous insulation and comparison with experimental results Int. J. Heat Mass Transfer 47 2343-52.
[10] Fan J T and Wen X 2002 Modeling heat and moisture transfer Int. J. Heat Mass Transfer 45 4045-55.
[11] Li Y and Luo Z X 1999 An improved mathematical simulation of the coupled diffusion of moisture and heat in wool fabrics J. Textile Res. 69 760-68.
[12] Li F Z, Li Y and Liu Y X 2004 Numerical simulation of coupled heat and mass transfer in hygroscopic fabrics considering the influence of atmospheric pressure Numer Heat Transfer, Part B 45 249-62.
[13] Li F Z, Li Y and Cao Y L 2009 The control volume time recursive expansion algorithm for solving coupled heat and moisture transfer through fabrics J. Nanjing University of Aeronautics and Astronautics 41 319-23.
[14] Huang H X, Ye C H and Sun W W 2008 Moisture transport in fibrous clothing assemblies J. Engineering Math. 61 35-54.
[15] Xu D H, Cheng J X and Zhou X H 2010 A model of heat and moisture transfer through the parallel pore textiles Proc. Textile Bioengineering and Informatics Symposium (TBIS 2010, Shanghai) 1151-56.
[16] Xu D H, Cheng J X and Zhou X H 2010 An inverse problem of thickness design for single layer textile material under low temperature J.Math-for-Industry 2 139-146.
[17] Hooke R and Jeeves T A 1961 Direct search solution of numerical and statistical problems J. Assoc. Comput. Mach. 8 212-29.
[18] Torczon V 1997 On the convergence of pattern search algorithms SIAM. J. Optim. 7 1-25.
[19] Cai Z J and Chen D Q 2006 The direct search algorithm of nonlinear optimization and its convergence proof Journal of Pudan University(Natural Science Edition) 45 396-403.
[20] Chen B L 2005 Optimization Problems and Algorithms (Beijing: Tsinghua University Press)