Mechanical and mathematical model used in the study of automobile oscillations

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Abstract. In this paper is presented an equivalent mechanical model of the vehicle, useful to find the movement of the vehicle and also is presented the matrix form of the mathematical model found for this mechanical model. This mathematical model is solved via the computer and graphical results are presented in case of linear motion with 30km/h velocity while the vehicle passes with both wheels from the left side over an obstacle.

1. Introduction
Nowadays, due to advanced techniques of computing, enabling complex mathematical models resolving and cinematic and dynamic simulations, it is really feasible to fully define the dynamic behaviour of a mechanical system even in the initial stage of a project. In this sense, mechanical and mathematical models are useful for analyzing the behavior of the car in different displacement situations. Mechanical models in the technical literature in the field are based on different levels of simplification of the real model ([1], [2], [3], [4], [5]) and thus allow for a more or less significant appreciation of the behavior of the real system. These theoretical models are confirmed by values of parameters that are significantly closer to the values obtained by road testing [6].

Complex mathematical models can be solved by computer and allow simulations of different modes of movement of the car. Thus, using such mechanical models and analyzing the solutions of these mathematical models, constructive measures can be taken, from the early stages of design, to improving the dynamic behavior of the automobile.

The means through which these simulations can be made are:
- the computer-aided simulation of mechanical systems (in our case – the vehicle), using the equivalent dynamic models concordantly with some real situations;
- the simulation on computer of mechanical systems using some special utility software, like ADAMS etc.

This paper presents the study of vehicles bouncing oscillations by using the first of the above methods. Thus, the next steps are to be taken into account:
- the determination of equivalent mathematical model, also the determination of moving equations suitable of some character of moving;
- the moving equations system resolving (to find the solutions of this equations system) suitable of some character of moving;
- the parameters analyzing: the vertical oscillations or displacement of unsuspended mass \( u_i(t) \), the displacement of suspended mass \( u(t) \), the roll angle \( \phi(t) \) and the pitch angle \( \psi(t) \).
2. The mechanical and mathematical model used to study the vehicles motion

2.1. The mechanical model
For studying oscillating vibrations, the model shown in Figure 1 will be used.

![Figure 1](image_url)

**Figure 1.** The simplified model of vehicle.

2.2. The mathematical model
For the equivalent mechanical model presented above, we obtained the motion equations given by the next matrix form:

\[ [M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F\} \]

with the notations:
- \( \{X\} = [u_1, u_2, u_3, u_4, \psi] \) - the displacements matrix;
- \( [M] \) - the matrix of the mass and the inertia moments;
- \( [K] \) - the matrix of spring constants of elements binding the five masses;
- \( [C] \) - the matrix of damping constants of elements binding the five masses;
- \( \{F\} \) - the matrix of the forces and the exterior moments;

\[
\begin{align*}
X_s &= k \cdot u \cdot \sin \alpha_0 - \left( R_k + k \cdot m \cdot \frac{\pi}{4} \right) \\
Y_s &= 0 \\
Z_s &= -u \cdot \cos \alpha_0
\end{align*}
\]

2.3. The simulation on the computer of equivalent mathematical model of vehicle
The numerical data used in numerical simulations are concordantly with the physical parameters of NUBIRA-WAGON, made by DAEWOO AUTOMOBILE ROMÂNIA S.A CRĂIOVA.

The analysis was made for the case of linear movement of the vehicle and was considered that it passes with both wheels from the left side over an obstacle which has the form and dimensions as represented in Figure 2.
The operating condition shall be analyzed with the rate of travel of 30 km/h, without longitudinal and transversal dip of the rolling path; the following considerations will be made:
- the vehicle comes in contact with the obstacle with the front wheel at the moment \( t_0 = 0 \);
- the vehicle passes over the obstacle with a width of \( L = 0.3 \)m with the front wheel, at the moment \( t_1 \) given by the relation:
\[
    t_1 = \frac{L}{v} [s] \tag{3}
\]
- the vehicle touches the obstacle with the back wheel at the moment \( t_2 \) given by the relation:
\[
    t_2 = \frac{L + A}{v} [s] \tag{4}
\]
- the vehicle passes over the obstacle of a width of \( L = 0.3 \)m with the rear wheel, at the moment \( t_3 \) given by the relation:
\[
    t_3 = \frac{L + A}{v} + \frac{L}{v} = \frac{2L + A}{v} [s] \tag{5}
\]

The analysis of the seven parameters: the vertical movements of the four wheels \( u_1(t) \), \( u_2(t) \), \( u_3(t) \), \( u_4(t) \), the vertical movement of the car body \( u_5(t) \), of the pitching angle \( \phi(t) \) and of the rolling angle \( \psi(t) \) will be made during the time periods: \( t_0...t_1; t_1...t_2; t_2...t_3; t_3=t3 \). The Maple V program was used for the solutions determination.

In case of movement with 30 km/h speed:
- \( t_0=0; t_1=0.036[s]; t_2=0.308[s]; t_3=0.344[s] \);

During the period \( (t_0...t_1) = (0...0.036) \) [s] the solutions for the system are determined for:
- the initial conditions: \( u_1(t)=0; u_2(t)=0; u_3(t)=0; u_4(t)=0; u_5(t)=0; f(t)=0, y(t)=0; \)
- the dimension of obstacles: \( e_1=0; e_2=60mm; e_3=0; e_4=0; \)

It is obtained the solutions of the system, based on which their variation curves were marked as follows:

In the figure 3 is presented the vertical displacement variation of the right front wheel (Fig 3a) and left front wheel (Fig 3b) during the time period \( (0...0.036) \) s.

If the left wheels are getting over obstacle during the time period of \( (0...0.036) \) s, the uplift of this wheel at approximately 30 mm creates an imbalance to the entire vehicle, i.e.:
- the right rear and left front wheels have a vertical displacement equal to 0.5 mm, respectively 1.18 mm, whereas the left rear wheel descends with -2.52mm;
- the car body, i.e. its mass point, initially has a downward movement down to 0.56 mm, then at the end of this period it has a positive movement up to 0.6 mm, and the rolling and pitching angles reach relatively small negative values (/0.48° respectively -0.2°).
**Figure 3.** Vertical displacement of the front axle wheels.

In the figure 4 is presented the variation of vertical displacement of right rear wheel $u_3(t)$ and vertical displacement of left rear wheel $u_4(t)$ during the time period $(0...0.036)$ s.

In the figure 5 are presented the variation of the vertical displacement of rigid body $u_5(t)$ during the time period $(0...0.036)$ s.

**Figure 4.** Vertical displacement of the rear axle wheels.

**Figure 5.** Vertical displacement of the cars body.

In the figure 6 is presented the variation of pitching angle during the time period $(0...0.036)$ s and in the figure 9 is presented the variation of rolling angle during the time period $(0...0.036)$ s:
3. Conclusion
This mathematical model is a simple but useful model for vehicle stability and maneuverability analysis. The values obtained are accurate values, compared to the measurements made on a real vehicle [6], having the same parameters as those used in the numerical simulation, so this can be considered a validation for this model. Once this model is confirmed, other types of oscillations can be analyzed.

Car behavior analysis can be improved by taking into account tangential forces in the contact pad, more than the wheel-to-road link and body-to-body forces, which can lead to models with a higher degree of complexity but with a accuracy of higher results.

References
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