NEW CONSTRUCTIONS OF YANG–BAXTER SYSTEMS

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Abstract. The quantum Yang–Baxter equation admits generalisations to systems of Yang–Baxter type equations called Yang–Baxter systems. Starting from algebra structures, we propose new constructions of some constant as well as the spectral-parameter dependent Yang–Baxter systems. Besides, we also present explicitly the commutation algebra structure associated to the constant type in dimension two.

AMS Contemporary Math. 442 (2007) 193–200.

1. Preliminaries

Yang–Baxter systems emerged from the study of quantum integrable systems, as generalisations of the quantum Yang–Baxter equation (QYBE) related to nonultralocal models [6, 5]. In deriving the relations for the quantum monodromy matrices, it is common to assume that the quantum integrable models under investigation are ultralocal, i.e. the quantised Lax operators corresponding to different sites of the lattice commute. This is no longer the case for nonultralocal models where the relations for the monodromy matrices satisfy certain conditions involving a collection of several Yang–Baxter type equations which form the so-called Yang–Baxter systems.

It is convenient to describe Yang–Baxter systems in terms of Yang–Baxter commutators. Let \( V, V', V'' \) be finite dimensional vector spaces over a field \( k \). Consider three linear maps \( R : V \otimes V' \to V \otimes V' \), \( S : V \otimes V'' \to V \otimes V'' \) and \( T : V' \otimes V'' \to V' \otimes V'' \). Then, a constant Yang–Baxter commutator is a map \([R, S, T] : V \otimes V' \otimes V'' \to V \otimes V' \otimes V''\) defined by

\[
[R, S, T] := R_{12}S_{13}T_{23} - T_{23}S_{13}R_{12}
\]

where \( R, S, T \) are understood as constant \( n^2 \times n^2 \) matrices (\( n \) being the dimension of \( V \)) and \([R, S, T] \) a \( n^3 \times n^3 \) matrix. Note that \([R, R, R] = 0\) is just a short-hand notation for writing the constant QYBE

\[
R_{12}R_{13}R_{23} = R_{23}R_{13}R_{12}
\]

Similarly, a coloured or spectral-parameter dependent Yang–Baxter commutator is the \( n^3 \times n^3 \) matrix

\[
[[R, S, T]] = [[R, S, T]](u, v, w) := R_{12}(u, v)S_{13}(u, w)T_{23}(v, w) - T_{23}(v, w)S_{13}(u, w)R_{12}(u, v)
\]

where the \( n^2 \times n^2 \) matrices \( R, S, T \) now depend upon the spectral parameters. \([[R, S, T]] = 0\) then denotes the coloured or spectral-parameter dependent QYBE

\[
R_{12}(u, v) R_{13}(u, w) R_{23}(v, w) = R_{23}(v, w) R_{13}(u, w) R_{12}(u, v)
\]

2000 Mathematics Subject Classification. 16W30, 81R50.
Equation (1.4) is also known as the two-parameter form of the QYBE since the matrix $R$ depends upon two spectral parameters, and reduces to the one-parameter form
\begin{equation}
R_{12}(u) R_{13}(u + v) R_{23}(v) = R_{23}(v) R_{13}(u + v) R_{12}(u)
\end{equation}
for $R(u, v) = R(u - v)$ and to the constant QYBE (1.2) for $R(u, v) = R$.

Let us now have a look at a Yang–Baxter system. A system of linear maps
\begin{align*}
W & : V \otimes V \rightarrow V \otimes V,
Z & : V' \otimes V' \rightarrow V' \otimes V',
X & : V \otimes V' \rightarrow V \otimes V',
\end{align*}
is called a $WXYZ$–system or a Yang–Baxter system \cite{7} if the following conditions hold:
\begin{align}
[W, W, W] &= 0, \\
[Z, Z, Z] &= 0, \\
[W, X, X] &= 0, \\
[X, X, Z] &= 0.
\end{align}
In \cite{11} it was observed that $WXYZ$–systems with invertible $W, X$ and $Z$ can be used to construct dually paired bialgebras of the FRT type leading to quantum doubles. The above is one type of a constant Yang–Baxter system that has recently been studied in \cite{9} and also shown to be closely related to entwining structures \cite{11}. Other types of constant Yang–Baxter systems are those that are related to quantised braid groups \cite{6} and the generalised reflection algebras \cite{2}. A number of solutions are known for the first two types but none for the third type. Yet another type of Yang–Baxter system is the 'coloured' Yang–Baxter system \cite{8} for which hardly any solutions are known.

In this paper, we provide new construction of solutions (with Yang–Baxter operators coming from algebra structures) for Yang–Baxter systems related to the generalised reflection algebras as well as the coloured ones.

### 2. Constant Yang–Baxter systems for generalised reflection algebras

In the quantisation of nonultralocal models \cite{4}, a quantum version of the relations for the monodromy matrices $T$ was given in the form
\begin{align*}
A_{12} T_1 B_{12} T_2 &= T_2 C_{12} T_1 D_{12}
\end{align*}
where $A, B, C, D$ are numerical $n^2 \times n^2$ matrices and $T_1 = T \otimes 1$, $T_2 = 1 \otimes T$. This is a reflection-type algebra \cite{2} that has been used for the description of open spin chains \cite{10}. In this spirit, we consider the algebra (introduced in \cite{4}) generated by elements $L^k_j$, $j, k \in \{1, 2, ..., n\}$ satisfying the relations
\begin{align}
A_{12} L_1 B_{12} L_2 &= L_2 C_{12} L_1 D_{12}
\end{align}
where $L = \{L^k_j\}_{j,k=1}^n$. These types of algebras include well-known algebras such as the reflection algebras, quantised function algebras and braid groups among others. The algebra (2.7) has to satisfy certain consistency conditions given in terms of the following Yang–Baxter system:
\begin{align}
[A, A, A] &= 0, & [D, D, D] &= 0, \\
[A, C, C] &= 0, & [D, B, B] &= 0, \\
[A, B^+, B^+] &= 0, & [D, C^+, C^+] &= 0, \\
[A, C, B^+] &= 0, & [D, B, C^+] &= 0.
\end{align}
where $X^+ = PXP$, and $P$ is the permutation matrix.

The next theorem presents new solutions for this Yang–Baxter system.

**Theorem 2.1.** Let $X$ be a commutative $k$-algebra and $\lambda, \lambda' \in k$. The following is a Yang–Baxter system:

$A, B, C, D : X \otimes X \rightarrow X \otimes X$,

$A(a \otimes b) = \lambda 1 \otimes ab + ab \otimes 1 - b \otimes a,$

$B(a \otimes b) = C(a \otimes b) = 1 \otimes ab + ab \otimes 1 - b \otimes a,$

$D(a \otimes b) = \lambda' 1 \otimes ab + ab \otimes 1 - b \otimes a.$

**Proof.** $[A, A, A] = 0$ and $[D, D, D] = 0$ follow from Theorem 1.1 of [3].

$[A, C, C] = 0$ and $[D, B, B] = 0$ follow from Theorem 5.2 of [9].

Notice that $B^+(a \otimes b) = P B(b \otimes a) = P(1 \otimes ba + ba \otimes 1 - a \otimes b)ba \otimes 1 + 1 \otimes ba - b \otimes a = ab \otimes 1 + 1 \otimes ab - b \otimes a = B(a \otimes b)$.

Other relations follow immediately. \(\square\)

Next, we look at the explicit form of the above solution in dimension two. We consider the algebra $A = \frac{k[X]}{(X^2 - \sigma)}$, where $\sigma \in \{0, 1\}$ is a scalar. Then $A$ has the basis $\{1, x\}$, where $x$ is the image of $X$ in the factor ring. We consider the basis $\{1 \otimes 1, 1 \otimes x, x \otimes 1, x \otimes x\}$ of $A \otimes A$ and represent the operator $A$ of Theorem 2.1 in this basis:

$A(1 \otimes 1) = \lambda 1 \otimes 1$

$A(1 \otimes x) = \lambda 1 \otimes x$

$A(x \otimes 1) = (\lambda - 1)1 \otimes x + x \otimes 1$

$A(x \otimes x) = \sigma(\lambda + 1)1 \otimes 1 - x \otimes x$

which in matrix form reads

\[
A_{12} = \begin{pmatrix}
\lambda & 0 & 0 & \sigma(\lambda + 1) \\
0 & \lambda & (\lambda - 1) & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\]

Similarly,

\[
B_{12} = C_{12} = \begin{pmatrix}
1 & 0 & 0 & 2\sigma \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\]

\[
D_{12} = \begin{pmatrix}
\lambda' & 0 & 0 & \sigma(\lambda' + 1) \\
0 & \lambda' & (\lambda' - 1) & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\]
Note that $A$ and $D$ satisfy the constant QYBE (1.2). The generators $L^k_j$ of the algebra (2.7) can be arranged in the $2 \times 2$ matrix
\[ L = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \]
and $L_1 = L \otimes 1$, $L_2 = 1 \otimes L$ are now $4 \times 4$ matrices. Then, the commutation relations among the generators $\{a, b, c, d\}$ are obtained by evaluating both sides of the matrix equation (2.7). This yields two cases (with $\lambda \neq -1, \lambda' \neq -1$): either $\lambda = \lambda'$ or $ac = 0$. In the case $\lambda = \lambda'$, we obtain
\[
\begin{align*}
    a^2 &= 0, \quad c^2 = 0 \\
    ca &= \lambda ac, \quad cb = -\lambda bc, \quad cd = \lambda dc \\
    [a, b] &= 2\sigma ac - (\lambda + 1)\sigma dc, \quad [a, d] = (\lambda - 1)bc \\
    [d, b] &= (\lambda + 1)\sigma ac - 2\sigma cd, \quad 2\sigma ad + b^2 - \sigma d^2 = 0
\end{align*}
\]
For $ac = 0$ case, the commutation relations are
\[
\begin{align*}
    ac = 0 &= ca, \quad a^2 = 0 = c^2 \\
    cb &= -\lambda' bc, \quad cd = \lambda' dc \\
    ab &= -\lambda' ba, \quad ad = \lambda' da \\
    [d, b] &= -2\sigma cd \\
    da &= bc, \quad ba = \sigma dc \\
    2\sigma ad + b^2 - \sigma d^2 &= 0
\end{align*}
\]
It is curious to note that if $ac \neq 0$ and $\lambda + 1 \neq 0, \lambda' + 1 \neq 0$, then the equality (2.7) requires that $\lambda$ be equal to $\lambda'$ thus leading to the independent relations (2.12). This is consistent with Theorem 2.1 since if $\lambda = \lambda'$ then the operator $A = D$ and we still obtain a solution to the Yang–Baxter system (2.8) as a special case. On the other hand, if $\lambda \neq \lambda'$ and $\lambda + 1 \neq 0, \lambda' + 1 \neq 0$, then (2.7) requires that $ac = ca = 0$ leading to relations (2.13). It is intriguing that, in this case, $\lambda$ disappears from the computations and the algebra depends solely on $\lambda'$ and $\sigma$.

3. Coloured Yang–Baxter systems

The commutation relations for the elements of the quantised Lax operators depending on spectral parameters are given by
\[
A_{12}(u, v) L_{1}(u) B_{12}(u, v) L_{2}(v) = L_{2}(v) C_{12}(u, v) L_{1}(u) D_{12}(u, v)
\]
These algebras were given in [4] and were proved useful in finding a commuting subalgebra that facilitates the construction of quantum Hamiltonian of a model. Here $A, B, C, D$ are spectral-parameter dependent $n^2 \times n^2$ matrices satisfying the coloured Yang–Baxter system [8]
\[
\begin{align*}
    [A, A, A] &= 0, & [D, D, D] &= 0 \\
    [A, C, C] &= 0, & [D, B, B] &= 0 \\
    [A, B^{++}, B^{++}] &= 0, & [D, C^{++}, C^{++}] &= 0 \\
    [A, C, B^{++}] &= 0, & [D, B, C^{++}] &= 0
\end{align*}
\]
where $X^{++}(u, v) := PX(v, u)P$. We present here a new method to obtain families of solutions for the above system, starting from algebra structures.
Let $X$ be a commutative $k$-algebra. We consider the maps $A, C : X \otimes X \to X \otimes X$ defined as follows:

\[
A(u, v)(a \otimes b) = \alpha(u, v)1 \otimes ab + \beta(u, v)ab \otimes 1 - \gamma(u, v)b \otimes a
\]

\[
C(u, v)(a \otimes b) = \eta(u, v)1 \otimes ab + \zeta(u, v)ab \otimes 1 - \delta(u, v)b \otimes a
\]

where $\alpha, \beta, \gamma, \eta, \zeta, \delta$ are $k$-valued functions on $k \times k$.

We impose the conditions $[[A, A, A]] = 0$ and $[[A, C, C]] = 0$, and find functions which satisfy a system of functional equations. According to [9], the condition $[[A, A, A]] = 0$ implies that the functions $\alpha, \beta, \gamma$ satisfy the following system of equations:

\[
(\beta(v, w) - \gamma(v, w))(\alpha(u, v)\beta(u, w) - \alpha(u, w)\beta(u, v))
+ (\alpha(u, v) - \gamma(u, v))(\alpha(v, w)\beta(u, w) - \alpha(u, w)\beta(v, w)) = 0
\]  
(3.16)

\[
\beta(v, w)(\beta(u, v) - \gamma(u, v))(\alpha(u, w) - \gamma(u, w))
+ (\alpha(u, v) - \gamma(u, v))(\alpha(v, w)\gamma(u, v) - \beta(u, v)\gamma(u, w)) = 0
\]  
(3.17)

\[
\alpha(u, v)\beta(v, w)(\alpha(u, w) - \gamma(u, w)) + \alpha(v, w)\gamma(u, w)(\gamma(u, v) - \alpha(u, v))
+ \gamma(v, w)(\alpha(u, v)\gamma(u, w) - \alpha(u, w)\gamma(u, v)) = 0
\]  
(3.18)

\[
\alpha(u, v)\beta(v, w)(\beta(u, w) - \gamma(u, w)) + \beta(v, w)\gamma(u, w)(\gamma(u, v) - \beta(u, v))
+ \gamma(v, w)(\beta(u, v)\gamma(u, w) - \beta(u, w)\gamma(u, v)) = 0
\]  
(3.19)

\[
\alpha(u, v)(\alpha(v, w) - \gamma(v, w))(\beta(u, w) - \gamma(u, w))
+ (\beta(u, v) - \gamma(u, v))(\alpha(u, w)\gamma(w, v) - \alpha(v, w)\gamma(u, w)) = 0
\]  
(3.20)

The condition $[[A, C, C]] = 0$ implies that the functions $\alpha, \beta, \gamma, \eta, \zeta, \delta$ satisfy the following system of equations:

\[
(\alpha(u, v) - \gamma(u, v))(\zeta(u, w)\eta(v, w) - \eta(u, w)\zeta(v, w))
+ (\alpha(u, v)\zeta(u, w) - \beta(u, v)\eta(u, w))(\zeta(v, w) - \delta(v, w)) = 0
\]  
(3.21)

\[
(\gamma(u, v) - \beta(u, v))(\eta(u, w)\delta(v, w) - \delta(u, w)\eta(v, w))
+ \alpha(u, v)(\delta(u, w) - \zeta(u, w))(\eta(v, w) - \delta(v, w)) = 0
\]  
(3.22)

\[
\gamma(u, v)(\zeta(u, w)\delta(v, w) - \delta(u, w)\zeta(v, w)) + \beta(u, v)\delta(u, w)(\zeta(v, w) - \delta(v, w))
+ \alpha(u, v)(\delta(u, w) - \zeta(u, w))\zeta(v, w) = 0
\]  
(3.23)

\[
(\beta(u, v) - \gamma(u, v))(\eta(u, w) - \delta(u, w))\zeta(v, w)
+ (\beta(u, v)\delta(u, w) - \gamma(u, v)\zeta(u, w))(\delta(v, w) - \eta(v, w)) = 0
\]  
(3.24)
\[\gamma(u, v)(\delta(u, w)\eta(v, w) - \eta(u, w)\delta(v, w)) + \alpha(u, v)\delta(u, w)(\delta(v, w) - \eta(v, w)) + \alpha(u, v)(\eta(u, w) - \delta(u, w))\zeta(v, w) = 0\] (3.25)

Using Theorem 2.1 from [9] and by simplifying the computations, we obtain the following solutions for the system of equations (3.16) – (3.25):

1) \(\alpha(u, v) = p(u - v), \beta(u, v) = q(u - v), \gamma(u, v) = pu - qv, \eta(u, v) = pu - q'v, \zeta(u, v) = qu - p'v, \delta(u, v) = pu - p'v\), where \(p, p', q, q' \in k\);

2) \(\alpha(u, v) = p(u - v), \beta(u, v) = q(u - v), \gamma(u, v) = pu - qv, \eta(u, v) = p(\lambda u - \mu v), \zeta(u, v) = q(\lambda u - \mu v), \delta(u, v) = p\lambda u - q\mu v\), where \(p, q, \lambda, \mu \in k\).

These solutions lead to the following theorem:

**Theorem 3.1.** Let \(X\) be a commutative \(k\)-algebra and \(p, p', q, q', \lambda, \mu \in k\).

The coloured operators \(A, B, C, D : k\otimes k \rightarrow \text{End}_k(X\otimes X)\) are solutions for the coloured Yang–Baxter system in the following two cases:

1) \(A(u, v)(a\otimes b) = p(u - v)1\otimes ab + q(u - v)ab\otimes 1 - (pu - qv)b\otimes a,\)
\(B(u, v)(a\otimes b) = (p'u - qv)1\otimes ab + (q'u - pv)ab\otimes 1 - (p'u - pv)b\otimes a,\)
\(C(u, v)(a\otimes b) = (pu - q'v)1\otimes ab + (qu - p'v)ab\otimes 1 - (pu - p'v)b\otimes a,\)
\(D(u, v)(a\otimes b) = p'(u - v)1\otimes ab + q'(u - v)ab\otimes 1 - (p'u - q'v)b\otimes a;\)

2) \(A(u, v)(a\otimes b) = p(u - v)1\otimes ab + q(u - v)ab\otimes 1 - (pu - qv)b\otimes a,\)
\(B(u, v)(a\otimes b) = q(\mu u - \lambda v)1\otimes ab + p(\mu u - \lambda v)ab\otimes 1 - (q\mu u - p\lambda v)b\otimes a,\)
\(C(u, v)(a\otimes b) = p(\lambda u - \mu v)1\otimes ab + q(\lambda u - \mu v)ab\otimes 1 - (p\lambda u - q\mu v)b\otimes a,\)
\(D(u, v)(a\otimes b) = p(u - v)1\otimes ab + q(u - v)ab\otimes 1 - (pu - pv)b\otimes a.\)

**Proof.** Let us observe that \(C^{++}(u, v) = -B(u, v)\) in both cases of the theorem. Everything now follows from the above analysis. \(\square\)

Finding other solutions for the system of equations (3.16) – (3.25) is an open problem. We now present the above solutions in dimension two. Consider the algebra \(A\) of the previous section and working in the same basis for \(A\otimes A\), we obtain the following matrix solutions for case 1 of Theorem 3.1:

\[
A(u, v) = \begin{pmatrix}
qu - pv & 0 & 0 & \sigma(q + p)(u - v) \\
0 & p(u - v) & (q - p)v & 0 \\
0 & (q - p)u & q(u - v) & 0 \\
0 & 0 & 0 & qu - pv
\end{pmatrix}
\] (3.26)

\[
B(u, v) = \begin{pmatrix}
qu' - pv & 0 & 0 & \sigma(p' + q')(u - (p + q)v) \\
0 & p'(u - v) & (p - q)v & 0 \\
0 & (q' - p'u) & q'u - pv & 0 \\
0 & 0 & 0 & pv - p'u
\end{pmatrix}
\] (3.27)

\[
C(u, v) = \begin{pmatrix}
qu - q'v & 0 & 0 & \sigma(p + q)(u - (p' + q')v) \\
0 & pu - q'v & (p' - q')v & 0 \\
0 & (q - p)u & qu - p'v & 0 \\
0 & 0 & 0 & p'v - pu
\end{pmatrix}
\] (3.28)
\( D(u, v) = \begin{pmatrix}
q'u - p'v & 0 & 0 & \sigma(q' + p')(u - v) \\
0 & p'(u - v) & (q' - p')v & 0 \\
0 & (q' - p')u & q'(u - v) & 0 \\
0 & 0 & 0 & q'v - p'u
\end{pmatrix} \)

Similarly, for case 2 we have

\( A(u, v) = \begin{pmatrix}
qu - pv & 0 & 0 & \sigma(q + p)(u - v) \\
0 & p(u - v) & (q - p)v & 0 \\
0 & (q - p)u & q(u - v) & 0 \\
0 & 0 & 0 & qv - pu
\end{pmatrix} \)

\( B(u, v) = \begin{pmatrix}
p\mu u - q\lambda v & 0 & 0 & \sigma(p + q)(\mu u - \lambda v) \\
0 & q(\mu u - \lambda v) & (p - q)\lambda v & 0 \\
0 & (p - q)\mu u & p(\mu u - \lambda v) & 0 \\
0 & 0 & 0 & p\lambda v - q\mu u
\end{pmatrix} \)

\( C(u, v) = \begin{pmatrix}
q\lambda u - p\mu v & 0 & 0 & \sigma(p + q)(\lambda u - \mu v) \\
0 & p(\lambda u - \mu v) & (q - p)\mu v & 0 \\
0 & (q - p)\lambda u & q(\lambda u - \mu v) & 0 \\
0 & 0 & 0 & q\mu v - p\lambda u
\end{pmatrix} \)

\( D(u, v) = \begin{pmatrix}
pu - qv & 0 & 0 & \sigma(q + p)(u - v) \\
0 & q(u - v) & (p - q)v & 0 \\
0 & (p - q)u & p(u - v) & 0 \\
0 & 0 & 0 & pv - qu
\end{pmatrix} \)

4. Conclusions

In this work, we have investigated new constructions of the constant and coloured Yang–Baxter systems from the viewpoint of Yang–Baxter operators from algebra structures. A solution for the constant Yang–Baxter system related to the generalised reflection algebra is presented and the commutation algebra structure in dimension two is exhibited in detail. Furthermore, for the coloured Yang–Baxter systems, we have given a construction that involves solving a nontrivial system of functional equations. We obtain two families of such solutions and it remains an open problem to find and classify solutions associated to a certain system of functional equations.

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