Event-by-event fluctuations of magnetic and electric fields in heavy ion collisions

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Abstract

We show that fluctuating proton positions in the colliding nuclei generate, on the event-by-event basis, very strong magnetic and electric fields in the direction both parallel and perpendicular to the reaction plane. The magnitude of $E$ and $B$ fields in each event is of the order of $m_n^2 \approx 10^{18}$ Gauss. Implications on the observation of electric dipole in heavy ion collisions is discussed, and the possibility of measuring the electric conductivity of the hot medium is pointed out.

Key words: heavy ion collisions, chiral magnetic effect, fluctuations

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1. Introduction

The importance of the strong magnetic field created in heavy ion collisions has been recently emphasized in Refs. [1, 2, 3], where the chiral magnetic effect has been proposed and studied. The chiral magnetic effect results in the electric current along the direction of the magnetic field $\vec{B}$. Consequently, the measurable charge separation may be observed in the direction perpendicular to the reaction plane ($y$ axis in Fig. 1) – the dominant direction of the magnetic field averaged over events [1, 4]. Recently, the STAR collaboration has published [5] first results on the charge-dependent two-particle...
correlations with respect to the reaction plane, which are qualitatively consistent with the chiral magnetic effect expectations: this conclusion, however, is under intensive debates (see, e.g., Refs. [6, 7, 8, 9, 10]).

As was mentioned earlier, according to early estimates (see Refs. [1, 4]) the magnetic field perpendicular to the reaction plane in heavy ion collisions is dominant and its magnitude may reach very high values up to dozens of $m_\pi^2 \approx 10^{18}$ Gauss. However, in all previous analysis the fluctuation of the magnetic field in each event was disregarded, and instead the averaged value over many events was used to calculate $\langle \vec{B} \rangle$. The importance of the event-by-event calculation becomes apparent after calculating the $x$ component of the magnetic field at $\vec{x} = 0$ (denoted by a black dot in Fig. 1). Indeed, averaging over many events we obtain $\langle B_x \rangle = 0$ (as we explain later), suggesting that the $x$ component of the magnetic field can be neglected. In this Letter, we argue that owing to fluctuating proton positions in both nuclei, $B_x$ becomes comparable to $B_y$ on the event-by-event basis. We also calculate the electric field $E$ and show that the $x$ and $y$ components of $B$ and $E$ fields are of the same order of magnitude, at least at the early stage of the collision, when the hot medium response may be neglected.

In the next Section we describe our approach in detail and present our results. Comments and summary are presented in Section 3 and Section 4, respectively.

![Figure 1: The transverse plane of a peripheral heavy ion collision with the impact parameter $b$. We calculate the magnetic $B$ and electric $E$ fields at $t = 0$ (time of the collision) and $\vec{x} = 0$ (denoted by a black dot) and we also average over a transverse area denoted by a grey circle.](image-url)
2. Calculation

The electric and magnetic fields at a position $\vec{x}$ and observation time $t$ can be calculated in each event according to the following equations \[1\]

\[
e E(t, \vec{x}) = \alpha_{EM} \sum_n \frac{1 - v_n^2}{R_n^3 \left(1 - [\vec{R}_n \times \vec{v}_n]^2 / R_n^2\right)^{3/2}} \vec{R}_n,
\]

\[
e B(t, \vec{x}) = \alpha_{EM} \sum_n \frac{1 - v_n^2}{R_n^3 \left(1 - [\vec{R}_n \times \vec{v}_n]^2 / R_n^2\right)^{3/2}} \vec{v}_n \times \vec{R}_n,
\]

where sums are performed over all protons in both nuclei. The fine-structure constant $\alpha_{EM} = e^2 / 4\pi \approx 1/137$ and $\vec{R}_n = \vec{x} - \vec{x}_n(t)$, where $\vec{x}_n$ is a position of proton moving with the velocity $\vec{v}_n$. If $\vec{v}_n$ is non-zero only in the $z$ direction, then $(\vec{R}_n \times \vec{v}_n)^2 = R_n^2 v_{n,z}^2$, where the transverse vector between a proton and the observation point $\vec{R}_{n,\perp} = \vec{x}_{\perp} - \vec{x}_{n,\perp}$ does not depend on time $t$.

In our calculation we sample proton positions $\vec{x}_n$ at $t = 0$ according to the Woods-Saxon distribution with the standard parameters \[12\]. We also assume that both nuclei are infinitely thin; and that all target and projectile protons move with the same velocity $\vec{v}_{\text{targ}} = [0, 0, v_z]$ and $\vec{v}_{\text{proj}} = [0, 0, -v_z]$, respectively. The value of $v_z$ is defined by the collision energy $\sqrt{s}$ and the proton mass $m_p$, $v_z^2 = 1 - (2m_p/\sqrt{s})^2$. Finally, when calculating $\vec{B}$ and $\vec{E}$ at a given point $\vec{x}$ we take into account only those protons which are not closer than $r_{\text{cut}} = 0.3$ fm to the observation point $\vec{x}$. This cut-off is to be implemented owing to the singularities of Eqs. \[1\] at $R_n \to 0$. The value $r_{\text{cut}} = 0.3$ fm was fixed as an effective distance between partons in a nucleon. By a variation of $r_{\text{cut}}$ in the range from 0.3 fm to 0.6 fm we affirm weak cut-off dependence of the fields.

We performed our calculations at $t = 0$ and at $\vec{x} = 0$ (denoted by a black dot in Fig. \[1\]). The results for magnetic field in $AuAu$ collisions at $\sqrt{s} = 200$ GeV are presented in Fig. \[2\].

As seen from Eq. \[1\] $\langle B_x \rangle = 0$ at $\vec{x} = 0$. However, in each event the $x$ component of the magnetic field can be huge and vanishes only after averag-

\[1\]It should be noted that the retardation effects are already taken into account and $\vec{R}_n$ depends explicitly on the observation time $t$.

\[2\]Indeed, one nucleus gives $\langle B_z \rangle \sim \langle y_n \rangle = 0$, in contrast to $\langle B_y \rangle \sim \langle x_n \rangle \neq 0$ if $b > 0$, see Fig. \[1\].
The mean absolute value of the magnetic field at \( t = 0 \) and \( \vec{x} = 0 \) as a function of impact parameter \( b \) for \( AuAu \) collision at \( \sqrt{s} = 200 \) GeV. Fluctuation of proton positions lead to non-zero values of \( \langle |B_x| \rangle \) that are comparable to \( \langle |B_y| \rangle \).

To study the magnitude of the magnetic field disregarding its direction from one event to another we consider the average (over events) absolute value of the magnetic and electric fields, \( \langle |B_{x,y}| \rangle \) and \( \langle |E_{x,y}| \rangle \). Due to fluctuations of the proton positions we obtain comparable numbers for \( \langle |B_x| \rangle \) and \( \langle |B_y| \rangle \) suggesting that on the event-by-event basis we should expect huge fields both in \( x \) and \( y \) directions. Since the chiral magnetic effect leads to the electric current along the magnetic field, our result indicate that, in principle, the chiral magnetic effect may take place not only in the \( y \) direction but also in the \( x \) direction.

The results for the electric field are shown in Fig. 3.

The symmetry of the system presented in Fig. 1 implies that at \( \vec{x} = 0 \) the average value of the electric field \( \langle E_x \rangle = \langle E_y \rangle = 0 \). However, as seen in Fig. 3 fluctuations lead to \( \langle |E_x| \rangle \approx \langle |E_y| \rangle \) with magnitude of the order of \( m^2_{\pi} \). It is interesting to notice that \( x \) and \( y \) components of the electric field

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3First we calculate, e.g., \( B_x \) in an event from all protons and after that we take the absolute value. Next we calculate average over events.
are almost identical to the $x$ component of the magnetic field
\[ \langle |B_x| \rangle \approx \langle |E_x| \rangle \approx \langle |E_y| \rangle. \quad (2) \]

This conclusion can be drawn easily from the analysis of Eqs. (1). The fields $B_x$ and $E_{x,y}$ are driven only by fluctuations of the proton positions and since $v_z \approx 1$ it is evident that Eq. (2) is valid. For peripheral collisions, the $y$ component of the magnetic field is influenced not only by the fluctuations, but also (mainly) by the geometry of the collision, as seen from Fig. 2, where $\langle |B_y| \rangle$ and $\langle B_y \rangle$ are compared. Thus we expect $\langle |B_y| \rangle$ to be larger than the fields in Eq. (2).

To provide more complete information on the fluctuating values of $B$ and $E$ fields, we show in Fig. 4 the event-by-event histograms for $B_{x,y}$ and $E_{x,y}$ at the impact parameter $b = 8$ fm. As expected $B_y$ distribution is shifted away from zero and $B_x$, $E_x$ and $E_y$ distributions are practically indistinguishable, consistent with Eq. (2). We checked that for $b = 0$ histograms look very similar, the only difference is $B_y$ that is centered around zero being indistinguishable from $B_x$, $E_x$ and $E_y$. 
Figure 4: Normalized event-by-event histograms of $x$ and $y$ components of magnetic $B$ and electric $E$ fields at $t = 0$ and $\vec{x} = 0$ at the impact parameter $b = 8$ fm. As expected for peripheral collisions, $B_y$ distribution is shifted away from zero. $B_x$, $E_x$ and $E_y$ histograms are practically indistinguishable.

3. Comments

Several comments are in order.

(i) We performed analogous calculations for CuCu collisions at the top RHIC energy and for PbPb collisions at the LHC energy. We found very similar qualitative behavior - in fact at $t = 0$ and $\vec{x} = 0$ the following approximate scaling holds for $\langle |B_{x,y}| \rangle$, $\langle |E_{x,y}| \rangle$

$$\frac{\text{Field}}{m^2_\pi} \propto \frac{\sqrt{s}}{m_p} f(b/R_A),$$

(3)

where $R_A$ is the radius of the appropriate nucleus, $\sqrt{s}$ is the center of mass energy and $m_p$ is a proton mass. In the first approximation, the function $f(x)$ is universal.

(ii) To check the uniformity of the magnetic and electric fields in each event, we integrated $E$ and $B$ over the transverse spherical domain $\Omega$ (denoted in Fig. 1 by a grey circle), i.e.,

$$\langle |B_{x,\Omega}| \rangle = \left\langle \left| \int_{\Omega} B_x(\vec{x}_\perp) d^2\vec{x}_\perp \right| \right\rangle / (\pi r_D^2),$$

(4)
and analogously for $B_y$, $E_x$ and $E_y$. Here $r_D$ is the radius of the domain which we varied from $r_D = 1$ fm to $r_D = 2$ fm. As seen from Fig. 5 for $b = 4$ fm the ratio $R = \langle |B_{y,\Omega}| \rangle / \langle |B_{x,\Omega}| \rangle$ is changing from 1.6 to 2.5 (with increasing $r_D$), as compared to 1.14 at $\vec{x} = 0$. For $b = 8$ fm $R$ is changing from 3.2 to 5.6, as compared to 1.6 at $\vec{x} = 0$. The ratio $\langle |E_{y,\Omega}| \rangle / \langle |E_{x,\Omega}| \rangle$ is always very close to 1, and again the $x$ and $y$ components of $E$ are almost identical to the $x$ component of $B$.

As seen in Fig. 5 for small $b$ the ratio $R$ weakly depends on the size of the domain in contrast to large $b$, where the $r_D$ dependence is significant. We performed our calculation up to $r_D = 2$ fm (size of the domain equals 4 fm), which in peripheral collisions covers almost the whole interaction region. In practical applications to the chiral magnetic effect, the fluctuations of the magnetic field are important only in the region with a characteristic size corresponding to the size of a sphaleron. At least at early stage of a collision, where the influence of the magnetic field is significant, this size is expected to be of the order of 1 fm (the characteristic size of a sphaleron is less than the magnetic screening length, which is $(\alpha_s T)^{-1}$ in the weak coupling limit). This suggests, as seen in Fig. 5, that, indeed, $\langle |B_{x}| \rangle$ and $\langle |B_{y}| \rangle$ are of the same order and should be treated equally in calculations of the chiral magnetic.
(iii) In an actual experiment, the reconstructed reaction plane (the so-called event plane) is, in general, tilted with respect to the one defined by the geometry of colliding nuclei. Thus a possible experimental measurement that is sensitive to the direction of the magnetic (electric) field in each event, will reveal the components $B'_x (E'_x)$ and $B'_y (E'_y)$, that are linear combinations of the original ones $B_x (E_x)$ and $B_y (E_y)$. This mixing provides an additional reinforcement to our conclusion that the different field components are comparable in magnitude.

(iv) Since the electric conductivity is significantly larger than the one associated with the magnetic field (at least an order of magnitude, see Refs. [13, 14]) it is quite clear that the current created by the electric field $j_E$ dominates over the one due to the chiral magnetic effect $j_B$. As seen from Fig. 2 and Fig. 3 we expect the ratio $\langle |\vec{j}_E| \rangle / \langle |\vec{j}_B| \rangle$ to be maximal for central collisions and minimal (but still larger than 1) for very peripheral collisions.

(v) Our results suggest that the strong electric current should not pose a significant problem for the correlation observable $\langle \cos(\phi_1 + \phi_2 - 2\Psi_{RP}) \rangle$ proposed in Ref. [15] to measure the chiral magnetic effect. Indeed, this observable is sensitive to the difference between correlations in-plain and out-of-plain and, in the first approximation, $j_E$ seems to be the same in $x$ and $y$ directions, in contrast to $j_B$ created in peripheral collisions (but not in central collision, where we expect $j_{B,x} \approx j_{B,y}$). This point, however, should be investigated in more elaborated studies, that take into account realistic temperature- and density- dependence of the electric conductivity.

(vi) In principle, by measuring the charge separation\footnote{For instance, in the way proposed in Ref. [16].} in central collisions one may perform rough estimate of the electric conductivity $\sigma$ of the matter created in heavy ion collisions. Assuming the dominance of the electric current in charge separation process in central collisions, and estimating effective lifetime $t_{eff}$ and spatial extensions of the hot spot, where the electric field is almost constant and homogeneous $E_{eff}$, we obtain $\sigma \sim \frac{Q}{t_{eff} S_{\perp} E_{eff}}$. Here $Q$ is the dipole charge and $S_{\perp}$ is the area perpendicular to the dipole axis. This problem is currently under our investigation.

(vii) Our estimates of the magnetic and electric fields are valid only at the early stage of the collision. At later stages, the magnetic response from the created medium becomes increasingly important \cite{17, 18}, and may lead
to substantial increase of the magnetic field over the electric one. For quantitative analysis of this effect, more elaborated studies are required [19].

4. Summary

In this Letter we argue that owing to the fluctuating proton positions in the relativistic heavy ion collisions, the strength of the magnetic and electric fields are comparable (both in the direction parallel and perpendicular to the reaction plane) in the event-by-event analysis. Thus, for the observables, that are sensitive to the electric and magnetic fields, it is essential to take into account event-by-event field fluctuations.

Our result in combination with the lattice QCD calculations of the electric and the chiral magnetic conductivities suggest, that the possible charge separation measured separately in-plane and out-of-plane will be dominated by the electric current, which in principle allows to measure the electric conductivity of the hot medium created in heavy ion collisions. However, since the electric field is not correlated with reaction plane, the difference between correlations in-plain and out-of-plain in peripheral collisions should be insensitive to the electric field.

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