Constraint on $K\bar{K}$ compositeness of the $a_0(980)$ and $f_0(980)$ resonances from their mixing intensity

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1. Introduction
2. $a_0(980)$-$f_0(980)$ mixing
3. Compositeness
4. Constraint on their structure
5. Summary

[1] T. S. and S. Kumano, arXiv:1409.2213 [hep-ph] (revision coming soon).
1. Introduction
1. Introduction

++ Exotic hadrons and their structure ++

- Quark models tell us that ordinary hadrons consist of $qqq$ and $q\bar{q}$.

- However, exotic hadrons --- not same quark component as ordinary hadrons = not $qqq$ nor $q\bar{q}$ --- might exist at somewhere in the hadron spectrum.

  - They should be "color" singlet as well.

- Does QCD allow their existence? And why?

- Actually there are several candidates for exotic hadrons.

- Baryons $(p, n, \Lambda, ...)$
- Mesons $(\pi, K, \varphi, ...)$
- Penta-quarks
- Tetra-quarks
- Hybrids
- Glueballs
- Hadronic molecules
1. Introduction

++ The lightest scalar meson nonet ++

- One of the important candidates for exotic hadrons is the member of the lightest scalar meson nonet: $\sigma$, $\kappa$, $f_0(980)$ and $a_0(980)$.
- **Inverted spectrum** from the $q\bar{q}$ configuration.
- **In a bag model**, the interaction between quarks inside a compact $qq\bar{q}\bar{q}$ system is attractive especially in the scalar channel. Jaffe (1977).
- **In a quark model**, $K\bar{K}$ molecules can appear as weakly bound $s$-wave states. Weinstein and Isgur (1982).

- Their structure is still controversial.
1. Introduction

++ Uniqueness of hadronic molecules ++

- **Hadronic molecules** should be **unique**, because they would have **large spatial size** compared to other (compact) hadrons.

![Hadronic molecules diagram]

- The uniqueness comes from the fact that **hadronic molecules** are composed of color-singlet hadrons themselves.

- Actually **the deuteron** was proved to be a **proton-neutron bound state** by considering **general wave equations** (not QCD!).

--- Field renormalization const. $Z$ in the weak binding: Weinberg (1965).

$$a = \frac{2(1-Z)}{2-Z} R + \mathcal{O}(m^{-1}_\pi), \quad r_e = -\frac{Z}{1-Z} R + \mathcal{O}(m^{-1}_\pi), \quad R \equiv \frac{1}{\sqrt{2\mu B}} = 4.318 \text{ fm}$$

$$a = 5.419 \pm 0.007 \text{ fm}, \quad r_e = 1.7513 \pm 0.008 \text{ fm}$$

--> **Consistent with** $Z \approx 0$!
1. Introduction

++ Identifying hadronic molecules ++

- The Weinberg’s study indicates that:
  - Hadronic molecules may be able to be identified without relying directly upon QCD, since constituents are color singlet.
  - In the weak binding, $Z$ can be determined model independently.

- An extension to unstable systems and an application to $a_0(980)$ and $f_0(980)$ were done to study whether they are $K\bar{K}$ molecules.

--- They formulated in terms of the spectral density:

$$W_{a_0(f_0)} = \int_{-50 \text{ MeV}}^{50 \text{ MeV}} w_{a_0(f_0)}(E) \, dE.$$ 

the “probability” for finding the bare state (missing channels such as $q\bar{q}$ and $qq\bar{q}\bar{q}$).

Baru et al. (2004).
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| Ref. | $M_R$ | $\Gamma_{\pi\eta}$ | $\bar{g}_{K\bar{K}}$ | $E_f$ | $r_e$ | $a$ | $k_1$ | $k_2$ | $W_{a_0}$ | $W_{f_0}$ |
|------|-------|---------------------|---------------------|-------|-------|-----|-------|-------|----------|----------|
| [18] | 1001  | 70                  | 0.224               | 9.6   | -7.1  | -0.16 - i0.59 | -104 + i55 | 104 - i111 | 0.49      | 0.29      |
| [19] | 999   | 146                 | 0.516               | 7.6   | -3.1  | -0.07 - i0.69 | -134 + i71 | 134 - i199 | 0.24      | 0.24      |
| [20] | 1003  | 153                 | 0.834               | 11.6  | -1.9  | -0.16 - i1.05 | -129 + i44 | 129 - i250 | 0.29      | 0.29      |
| [20] | 992   | 145.3               | 0.56                | 0.6   | -2.8  | -0.01 - i0.76 | -126 + i73 | 126 - i212 | 0.36      | 0.36      |
| [21] | 984.8 | 121.5               | 0.41                | -18.0 | -3.9  | 0.18 - i0.61 | -102 + i97 | 102 - i199 | 0.23      | 0.23      |
| [21] | 973   | 253                 | 2.84                | -154  | 0.56  | 1.09 - i0.89 | -69 + i604 | 69 - i804 | 0.17      | 0.17      |
| [24] | 996   | 128.8               | 1.31                | +4.6  | -1.22 | -0.14 - i1.99 | -84 + i17  | 84 - i351 | 0.14      | 0.14      |

Channels such as $q\bar{q}$ and $q\bar{q}q\bar{q}$. 

Hadrons and Hadron Interactions in QCD @ Yukawa Inst. (Feb. 15th - Mar. 21st, 2015)
1. Introduction

++ Identifying hadronic molecules ++

- The Weinberg’s study indicates that:
  - Hadronic molecules may be able to be identified without relying directly upon QCD, since constituents are color singlet.
  - In the weak binding, $Z$ can be determined model independently.

- An extension to unstable systems and an application to $a_0(980)$ and $f_0(980)$ were done to study whether they are $K\bar{K}$ molecules.

- The “probability” for finding the bare state, $W_{a,f}$, is small compared unity [or $(2/\pi) \times \arctan(2) \approx 0.70$].
  - The evidence that $a_0(980)$ and $f_0(980)$ have large $K\bar{K}$ components inside them.

- Remark: They defined $W$ as a real value, although both $a_0(980)$ and $f_0(980)$ are resonances!
  - Need check whether this treatment is justified.
1. Introduction

++ The $a_0(980)$-$f_0(980)$ mixing ++

- The $a_0(980)$-$f_0(980)$ mixing was predicted as a phenomenon caused by the threshold difference between charged and neutral $K\bar{K}$ loops. Achasov, Devyanin and Shestakov (1979).

--- Namely, in the energy between the $K^+K^-$ and $K^0\bar{K}^0$ thresholds (987 ~ 995 MeV) the mixing effect is unusually enhanced:

$$\Lambda_{K^+K^-} + \Lambda_{K^0\bar{K}^0} = \mathcal{O} \left( \sqrt{\frac{m_{K^0}^2 - m_{K^+}^2}{m_{K^0}^2 + m_{K^+}^2}} \right)$$

<--> Natural size: $\mathcal{O}[m_{K^0}^2 - m_{K^+}^2]/(m_{K^0}^2 + m_{K^+}^2)$ [cf. $\rho(770)$-$\omega(782)$ mixing]

- The $a_0(980)$- and $f_0(980)$-$K\bar{K}$ coupling constants are the model parameters of the mixing amplitude.
1. Introduction

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- Recently the mixing was measured in Exp. by using the $J/\psi$ decay, and its intensity $\xi_{fa}$ is Ablikim et al. [BES III] (2011).

\[
\xi_{fa} \equiv \frac{\text{Br}(J/\psi \rightarrow \phi f_0(980) \rightarrow \phi a_0^0(980) \rightarrow \phi \pi^0 \eta)}{\text{Br}(J/\psi \rightarrow \phi f_0(980) \rightarrow \phi \pi \pi)} = 0.60 \pm 0.20(\text{stat}) \pm 0.12(\text{sys}) \pm 0.26(\text{para})\%,
\]

$\xi_{fa}|_{\text{upper limit}} = 1.1\% \ (90\% \ C.L.)$

---> Investigate their structure!
1. Introduction

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- Recently the mixing was measured in Exp. by using the $J/\psi$ decay, and its intensity $\xi_{fa}$ is

$$\xi_{fa} \equiv \frac{\text{Br}(J/\psi \to K^+ f_0(980) K^-)}{\text{Br}(J/\psi \to K^+ a_0(980) K^-)}$$

$$= 0.60 \pm 0.20 \text{(stat)} \pm 0.12 \text{(sys)} \pm 0.26 \text{(para)} \%,$$

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- The $a_0(980)$-$f_0(980)$ mixing was predicted as a phenomenon caused by the threshold difference between charged and neutral $K\bar{K}$ loops. 

- Recently the mixing was measured experimentally by using the $J/\psi$ decay, and its intensity is $\xi_{fa}$.

\[ \xi_{fa} \equiv \frac{Br(J/\psi \to K^+K^-)}{B} \]

\[ = 0.60 \pm 0.20{\rm (stat)} \pm 0.12{\rm (sys)} \pm 0.26{\rm (para)}\%, \]

\[ \xi_{fa}|_{\text{upper limit}} = 1.1\%\ (90\% \text{ C.L.)} \]

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1. Introduction

++ The $a_0(980)$-$f_0(980)$ mixing ++

- The $a_0(980)$-$f_0(980)$ mixing was predicted as a phenomenon caused by the threshold difference between charged and neutral $\bar{K}K$ loops. (Achasov, Devyanin and Shestakov (1979).

- Recently the mixing was measured in Exp. by using the $J/\psi$ decay, and its intensity is

\[ \xi_{fa} \equiv \frac{\text{Br}(J/\psi \rightarrow \tau^+\tau^-)}{\text{B}} \]

\[ = 0.60 \pm 0.20\text{(stat)} \pm 0.12\text{(sys)} \pm 0.26\text{(para)}\% , \]

\[ \xi_{fa}|_{\text{upper limit}} = 1.1\% \quad (90\% \text{ C.L.}) \]

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1. Introduction

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- Recently the mixing was measured in Exp. by using the $J/\psi$ decay, and its intensity is

$$\xi_{fa} \equiv \frac{\text{Br}(J/\psi \rightarrow K^+ f_0) - \text{Br}(J/\psi \rightarrow K^- a_0)}{\text{Br}(J/\psi \rightarrow K^+ a_0) + \text{Br}(J/\psi \rightarrow K^- f_0)}$$

$$= 0.60 \pm 0.20\,(\text{stat}) \pm 0.12\,(\text{sys}) \pm 0.26\,(\text{para})\%,$$

$$\xi_{fa}|_{\text{upper limit}} = 1.1\% \quad (90\% \ C.L.)$$

- The scenario that both $a_0(980)$ and $f_0(980)$ are $K\bar{K}$ molecules seems to be excluded.

--> Investigate their structure!
1. Introduction

++ The $a_0(980)$-$f_0(980)$ mixing ++

- The $a_0(980)$-$f_0(980)$ mixing was predicted as a phenomenon caused by the threshold difference between charged and neutral $KK$ loops. Achasov, Devyanin and Shestakov (1979).

- Recently the mixing was measured in Exp. by using the $J/\psi$ decay, and its intensity $\xi_{fa}$ is Ablikim et al. [BES III] (2011).

- Contradicts the evidence that $a_0(980)$ and $f_0(980)$ have large $KK$ components?

- However, these theoretical values of $\xi_{fa}$ are calculated using effective models of QCD such as quark models.

- The scenario that both $a_0(980)$ and $f_0(980)$ are $KK$ molecules seems to be excluded.

Constrain their structure?

The mixing intensity in the $\eta(1405)$ decay is larger. Ablikim et al. (2012). This decay, however, seems to be affected by $KK^*$ loop. Aceti et al. (2012).
1. Introduction

++ Motivation ++

- We want to know the structure of \(a_0(980)\) and \(f_0(980)\).
  - An application of the Weinberg’s study indicates that the “probability” of finding the bare state \((q\bar{q}, q\bar{q}q\bar{q})\) is small.

--> They should have large \(K\bar{K}\) component.

--- However, the “probability” \(W\) was defined as a real value even for the resonances \(a_0(980)\) and \(f_0(980)\).

- The Exp. of the \(a_0(980)-f_0(980)\) mixing implies that both \(a_0(980)\) and \(f_0(980)\) are simultaneously \(K\bar{K}\) molecules seems to be excluded.

--- However, the conclusion relies on effective models of QCD.

--> Investigate their \(K\bar{K}\) structure without relying directly on QCD nor effective models.

- For this purpose, we formulate:
  - The \(a_0(980)-f_0(980)\) mixing intensity.
  - The \(K\bar{K}\) compositeness for the \(a_0(980)\) and \(f_0(980)\) resonances.

and constrain their structure in terms of the \(K\bar{K}\) component.
2. $a_0(980)$-$f_0(980)$ mixing
2. \( a_0(980) - f_0(980) \) mixing

++ Amplitude of \( a_0(980) - f_0(980) \) mixing ++

- Calculate the \( a_0(980) - f_0(980) \) mixing amplitude \( \Lambda \) from diagrams:

--- Parameters: only the \( a_0(980) - K\bar{K} \) and \( f_0(980) - K\bar{K} \) coupling constants.

- The Flatte parameterization is used for the propagators: Flatte (1976).

\[
\frac{1}{D_a(s)} \equiv \frac{1}{s - M_a^2 + i\sqrt{s}[\Gamma_{\pi\eta}(s) + \Gamma_{K\bar{K}}^a(s)]'}, \quad \frac{1}{D_f(s)} \equiv \frac{1}{s - M_f^2 + i\sqrt{s}[\Gamma_{\pi\pi}(s) + \Gamma_{K\bar{K}}^f(s)]}
\]

--- Parameters: \( M_a, M_f \) and \( a_0-K\bar{K}, \pi\eta \) and \( f_0-K\bar{K}, \pi\pi \) coupling consts.

--> The propagators with the mixing is expressed as:
2. $a_0(980)$-$f_0(980)$ mixing

++ Formulation of their mixing ++

- The $a_0(980)$-$f_0(980)$ mixing intensity $\xi_{fa}$ was experimentally defined:

$$\xi_{fa} \equiv \frac{\text{Br}(J/\psi \to \phi f_0(980) \to \phi a_0(980) \to \phi \pi^0\eta)}{\text{Br}(J/\psi \to \phi f_0(980) \to \phi \pi\pi)}$$

- Therefore, we define the mixing intensity as the ratio of two branching fractions of a parent particle $X$:

$$\xi_{fa} \equiv \frac{\Gamma(X \to Y f_0(980) \to Y a_0(980) \to Y \pi^0\eta)}{\Gamma(X \to Y f_0(980) \to Y \pi\pi)}$$

- Assuming that the phenomena on $a_0(980)$ and $f_0(980)$ takes place particularly at the $KK$ thresholds, we obtain

$$\xi_{fa} = \frac{\int dM_{\pi\eta} M^2_{\pi\eta} \Gamma^a_{\pi\eta}(M^2_{\pi\eta})|P_{f \to a}(M^2_{\pi\eta})|^2}{\int dM_{\pi\pi} M^2_{\pi\pi} \Gamma^f_{\pi\pi}(M^2_{\pi\pi})|P_{f}(M^2_{\pi\pi})|^2}.$$
2. $a_0(980)$-$f_0(980)$ mixing

**++ Exercise ++**

- The $a_0(980)$-$f_0(980)$ mixing intensity $\xi_{fa}$ can be evaluated by using Flatte parameters from Exp. fittings. --- Errors only for $\bar{K}K$ coup.

| Collaboration  | $M_a$ [MeV] | $\bar{g}_{a\bar{K}K}$ [GeV] | $\bar{g}_{a\pi\eta}$ [GeV] |
|---------------|-------------|-----------------------------|-----------------------------|
| CLEO (2011)   | 998         | 3.97 $\pm$ 0.77             | 4.25                        |
| KLOE (2009)   | 982.5       | 2.84 $\pm$ 0.41             | 2.46                        |
| CB (2008)     | 987.4       | 2.94 $\pm$ 0.12             | 2.87                        |
| SND (2000)    | 995         | 5.93 $^{+10.54}_{-2.39}$    | 3.11                        |
| E852 (1999)   | 1001        | 2.36 $\pm$ 0.13             | 2.47                        |

| Collaboration  | $M_f$ [MeV] | $\bar{g}_{f\bar{K}K}$ [GeV] | $\bar{g}_{f\pi\pi}$ [GeV] |
|---------------|-------------|-----------------------------|-----------------------------|
| CDF (2011)    | 989.6       | 4.02 $^{+1.01}_{-1.37}$     | 2.65                        |
| KLOE (2006)   | 977.3       | 2.45 $\pm$ 0.17             | 1.21                        |
| Belle (2006)  | 950         | 4.07 $^{+0.76}_{-0.95}$     | 2.28                        |
| BES (2005)    | 965         | 5.80 $^{+0.22}_{-0.23}$     | 2.83                        |
| FOCUS (2005)  | 957         | 3.39 $^{+0.62}_{-0.76}$     | 2.15                        |
| SND (2000)    | 969.8       | 7.88 $^{+1.09}_{-0.86}$     | 3.19                        |

$\xi_{fa} = 0.60 \pm 0.20_{(stat)} \pm 0.12_{(sys)} \pm 0.26_{(para)}\%$, $\xi_{fa}|_{\text{upper limit}} = 1.1\%$  

*Ablikim et al. (2011).*

--- Many combinations of Flatte params. reproduce the Exp. value!

**Red:** consistent with Exp.

**Blue:** above the upper limit.

---
3. Compositeness
3. Compositeness

++ Compositeness for two-body systems ++

- The Weinberg’s study on deuteron indicates that **hadronic molecules may be able to be identified without relying directly upon QCD**, since **constituents are color singlet**.

- In this context, **the compositeness** was recently introduced so as to observe **the two-body components** inside a resonance as well as a bound state.

Hyodo, Jido, Hosaka (2012), Aceti-Oset (2012), Hyodo (2013), Nagahiro-Hosaka (2014), ... .
See also T.S., Hyodo and Jido arXiv:1411.2308.
3. Compositeness

++ Physical meaning of compositeness ++

- Compositeness \( (X) \) = amount of the two-body components in a resonance as well as a bound state.

Ex.) \( \Lambda(1405) \):

- Compositeness can be defined as the contribution of the two-body component to the normalization of the total wave function.

\[
\langle \Lambda(1405) | \Lambda(1405) \rangle = X_{KN} + X_{\pi\Sigma} + \cdots + Z = 1
\]

--- For a bound state with zero width --\> Interpreted as a probability:

Molecule \( \leftrightarrow X \approx 1, \ Z \approx 0 \). Elementary \( \leftrightarrow Z \approx 1, \ X \approx 0 \).
3. Compositeness

++ Formulation ++

- The two-body wave function for a general separable interaction:

  \[
  \tilde{\Psi}(\vec{q}) = \frac{g}{s_{\text{pole}}} - \frac{\omega(\vec{q}) + \omega'(\vec{q})}{s_{\text{pole}}} \]

  \(\omega(\vec{q}) \equiv \sqrt{m^2 + \vec{q}^2}, \quad \omega'(\vec{q}) \equiv \sqrt{m'^2 + \vec{q}^2}\)

--- \(g\): the coupling constant of the resonance to the two-body state. \(s_{\text{pole}}\): the pole position of the resonance in the complex \(s\) plane.

- The compositeness is defined as the “norm” for the two-body w.f.:

  \[
  X \equiv \int Dq \left[ \tilde{\Psi}(\vec{q}) \right]^2 = -g^2 \left[ \frac{dG}{ds} \right]_{s=s_{\text{pole}}}, \quad Dq = \frac{d^3q}{(2\pi)^3} \frac{\omega(\vec{q}) + \omega'(\vec{q})}{2\omega(\vec{q})\omega'(\vec{q})}
  \]

--- \(G(s)\) is the two-body loop function = Green function.
3. Compositeness

++ Formulation ++

- The two-body wave function for a general separable interaction:

\[
\tilde{\Psi}(\vec{q}) = \frac{g}{s_{\text{pole}} - \left[\omega(\vec{q}) + \omega'(\vec{q})\right]^2}, \quad \omega(\vec{q}) \equiv \sqrt{m^2 + \vec{q}^2}, \quad \omega'(\vec{q}) \equiv \sqrt{m'^2 + \vec{q}^2}
\]

--- \( g \): the coupling constant of the resonance to the two-body state.

\( s_{\text{pole}} \): the pole position of the resonance in the complex \( s \) plane.

- The elementariness is defined as the bare state \( \Psi_0 \) contribution:

\[
Z \equiv \langle \Psi^* | \Psi_0 \rangle \langle \Psi_0 | \Psi \rangle = -g^2 \left[ G^2 \frac{dV}{ds} \right]_{s=s_{\text{pole}}}
\]

--- Measures genuine compact systems and missing channels.

- The sum of the compositeness \( X \) and the elementariness \( Z \) coincides with the normalization of the total wave function:

\[
\langle \Psi^* | \Psi \rangle = X + Z = 1
\]
3. Compositeness

++ Formulation ++

- The compositeness / elementariness has following properties:
  T. S., T. Hyodo and D. Jido, arXiv:1411.2308.

- **Model dependent.** --> we employ the following expressions:

  \[ X \equiv \int Dq \left[ \vec{\Psi}(q) \right]^2 = -g^2 \left[ \frac{dG}{ds} \right]_{s=s_{pole}} \]

  \[ Z \equiv \langle \Psi^* | \Psi_0 \rangle \langle \Psi_0 | \Psi \rangle = -g^2 \left[ G^2 \frac{dV}{ds} \right]_{s=s_{pole}} \]

--- which correctly reproduces the Weinberg’s relation for the scattering length and effective range in the weak binding limit (with non-rel. Green function).

- **Correct normalization even for resonances:** \( \langle \Psi^* | \Psi \rangle = X + Z = 1 \)

- **Cut-off for the Green function is **not necessary.**

- From the pole position \( s_{pole} \) and the residue \( g \) as the coupling constant, one can calculate the compositeness without knowing the details of the interaction.

\[ T_{ij}(s) \approx \frac{g_i g_j}{s - s_{pole}} \]
3. Compositeness

++ Model calculation ++

- Compositeness $X$ and elementariness $Z$ for scalar mesons in the chiral unitary approach. $\rightarrow$ Complex values for resonances!

\[ X_i = -g_i^2 \left[ \frac{dG_i}{ds} \right]_{s=s_{\text{pole}}} \]

\[ Z = -\sum_{i,j} g_i g_j \left[ G_i G_j \frac{dV_{ij}}{ds} \right]_{s=s_{\text{pole}}} \]

\[ \langle \Psi^* | \Psi \rangle = \sum_i X_i + Z = 1 \]

| $f_0(500)$ or $\sigma$ | $f_0(980)$ | $\alpha_0(980)$ | $K^*_0(800)$ or $\kappa$ |
|------------------------|------------|----------------|-------------------------|
| $\sqrt{s_{\text{pole}}}$ | 471 – 181$i$ MeV | 987 – 18$i$ MeV | 979 – 53$i$ MeV | 750 – 227$i$ MeV |
| $X_{\pi\pi}$ | –0.16 + 0.35$i$ | 0.01 + 0.01$i$ | – | – |
| $X_{K\bar{K}}$ | –0.01 – 0.01$i$ | 0.74 – 0.11$i$ | 0.38 – 0.29$i$ | – |
| $X_{\pi\eta}$ | – | – | –0.06 + 0.10$i$ | – |
| $X_{\pi K}$ | – | – | – | 0.32 + 0.36$i$ |
| $X_{\eta K}$ | – | – | – | –0.01 + 0.00$i$ |
| $Z$ | 1.17 – 0.34$i$ | 0.25 + 0.10$i$ | 0.68 + 0.18$i$ | 0.70 – 0.36$i$ |

- We interpret complex compositeness / elementariness on the basis of the similarity to the wave function of the bound state:

1. $\text{Re}(X) \sim 1$, $\text{Im}(X) \sim |Z| \ll 1 \quad \iff \quad$ Dominated by a molecular state.

2. $|X_i| \ll 1 \quad \iff \quad$ $i$-th channel component is very small.
3. Compositeness

++ Model calculation ++

- Compositeness \( X \) and elementariness \( Z \) for scalar mesons in the chiral unitary approach.  --> Complex values for resonances!

\[
X_i = -g_i^2 \left[ \frac{dG_i}{ds} \right]_{s=s_{pole}} \\
Z = - \sum_{i,j} g_i g_j \left[ G_i G_j \frac{dV_{ij}}{ds} \right]_{s=s_{pole}} \\
\langle \Psi^* | \Psi \rangle = \sum_i X_i + Z = 1
\]

| \( \sqrt{s_{pole}} \) | \( f_0(500) \) or \( \sigma \) | \( f_0(980) \) | \( a_0(980) \) | \( K_0^*(800) \) or \( \kappa \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( \sqrt{s_{pole}} \) | 471 – 181i MeV | 987 – 18i MeV | 979 – 53i MeV | 750 – 227i MeV |
| \( X_{\pi \pi} \) | –0.16 + 0.35i | 0.01 + 0.01i | | |
| \( X_{K\bar{K}} \) | –0.01 – 0.01i | 0.74 – 0.11i | 0.38 – 0.29i | |
| \( X_{\pi \eta} \) | | | –0.06 + 0.10i | |
| \( X_{\pi K} \) | | | | 0.32 + 0.36i |
| \( X_{\eta K} \) | | | | –0.01 + 0.00i |
| \( Z \) | 1.17 – 0.34i | 0.25 + 0.10i | 0.68 + 0.18i | 0.70 – 0.36i |

- We interpret complex compositeness / elementariness on the basis of the similarity of the wave function of the bound state:

--> \( f_0(980) \) in this model is dominated by the \( K\bar{K} \) component.
3. Compositeness

++ Exercise ++

- The $K\bar{K}$ ompositeness of $a_0(980)$ and $f_0(980)$ from the Flatte params.

--- We employ the Flatte parametrization to calculate the pole position and their residue by the analytical continuation.

\[
\frac{1}{D_a(s)} \equiv \frac{1}{s - M_a^2 + i\sqrt{s}[\Gamma_{\pi\eta}^{a}(s) + \Gamma_{K\bar{K}}^{a}(s)]},
\]

\[
\frac{1}{D_f(s)} \equiv \frac{1}{s - M_f^2 + i\sqrt{s}[\Gamma_{\pi\pi}^{f}(s) + \Gamma_{K\bar{K}}^{f}(s)]}
\]

| Collaboration   | $M_a$ [MeV] | $\bar{g}_{aK\bar{K}}$ [GeV] | $\bar{g}_{a\pi\eta}$ [GeV] |
|-----------------|-------------|-----------------------------|-----------------------------|
| CLEO (2011)     | 998         | 3.97 ± 0.77                 | 4.25                        |
| KLOE (2009)     | 982.5       | 2.84 ± 0.41                 | 2.46                        |
| CB (2008)       | 987.4       | 2.94 ± 0.12                 | 2.87                        |
| SND (2000)      | 995         | 5.93 +10.54 ± 2.39          | 3.11                        |
| E852 (1999)     | 1001        | 2.36 ± 0.13                 | 2.47                        |

| Collaboration   | $M_f$ [MeV] | $\bar{g}_{fK\bar{K}}$ [GeV] | $\bar{g}_{f\pi\pi}$ [GeV] |
|-----------------|-------------|-----------------------------|-----------------------------|
| CDF (2011)      | 989.6       | 4.02 +1.01 ± 1.37           | 2.65                        |
| KLOE (2006)     | 977.3       | 2.45 ± 0.17                 | 1.21                        |
| Belle (2006)    | 950         | 4.07 +0.76 ± 0.95           | 2.28                        |
| BES (2005)      | 965         | 5.80 +0.22 ± 0.31           | 2.83                        |
| FOCUS (2005)    | 957         | 3.39 +0.62 ± 0.76           | 2.15                        |
| SND (2000)      | 969.8       | 7.88 ± 1.09 ± 0.86          | 3.19                        |
3. Compositeness

++ Exercise ++

- The $K\bar{K}$ compositeness of $a_0(980)$ and $f_0(980)$ from the Flatte params.

--- We employ the Flatte parametrization to calculate the pole position and their residue by the analytical continuation.

\[
\frac{1}{D_a(s)} \equiv \frac{1}{s - M_a^2 + i\sqrt{s}[\Gamma^a_{\pi\eta}(s) + \Gamma^a_{K\bar{K}}(s)]}, \quad \frac{1}{D_f(s)} \equiv \frac{1}{s - M_f^2 + i\sqrt{s}[\Gamma^f_{\pi\pi}(s) + \Gamma^f_{K\bar{K}}(s)]}
\]

- Compared with the previous work by Baru et al., we obtain complex $KK$ compositeness, which is however necessary to get correct normalization (with appropriate interaction and elementariness).

--- cf. Baru et al. used the following:

\[
X = \int_{0}^{\infty} w(E) dE, \quad w(E) \equiv 4\pi\sqrt{2\mu^3} \frac{\sqrt{E}|g(E)|^2}{|E + B|^2}
\]

Hyodo (2013).
3. Compositeness

++ Exercise ++

- The $K\bar{K}$ compositeness of $a_0(980)$ and $f_0(980)$ from the Flatte params.

--- We employ the Flatte parametrization to calculate the pole position and their residue by the analytical continuation.

\[
\frac{1}{D_a(s)} \equiv \frac{1}{s - M_a^2 + i\sqrt{s}[\Gamma_a^\pi\eta(s) + \Gamma_a^{K\bar{K}}(s)]}, \quad \frac{1}{D_f(s)} \equiv \frac{1}{s - M_f^2 + i\sqrt{s}[\Gamma_f^\pi\pi(s) + \Gamma_f^{K\bar{K}}(s)]}
\]

- The imaginary part of the $K\bar{K}$ compositeness is not small, so we cannot clearly conclude the structure of $a_0(980)$ and $f_0(980)$.

- The real part of the $K\bar{K}$ compositeness for $f_0(980)$ is non-negligible compared to unity, which might imply a larger $K\bar{K}$ component inside $f_0(980)$. 

![Graph showing the real and imaginary parts of $a_0(980)$ and $f_0(980)$]
4. Constraint on the structure of $a_0(980)$ and $f_0(980)$
4. Constraint on their structure

++ Constructing a relation ++

- Again we consider the structure of the $a_0(980)$-$f_0(980)$ mixing:

--- The $a_0(980)$ w.f. ($\to a_0(980)$-$K\bar{K}$ coupling const.) \times the $K\bar{K}$ loops
\times the $f_0(980)$ w.f. ($\to f_0(980)$-$K\bar{K}$ coupling const.).

- Therefore, the mixing intensity is sensitive to the $K\bar{K}$ component both in $a_0(980)$ and in $f_0(980)$.

- Especially, for a small mixing amplitude $\Lambda$, we expect:

\[ \xi_{fa} \sim |\Lambda|^2 \sim |g_ag_f|^2 \propto |X_aX_f| \]

--- Small $\xi_{fa}$ may imply small $|X_aX_f|$.

--- The $a_0(980)$ w.f. ($\to a_0(980)$-$K\bar{K}$ coupling const.) \times the $K\bar{K}$ loops
\times the $f_0(980)$ w.f. ($\to f_0(980)$-$K\bar{K}$ coupling const.).
4. Constraint on their structure

++ Constructing a relation ++

- Again we consider the structure of the $a_0(980)$-$f_0(980)$ mixing:

--- The $a_0(980)$ w.f. (--> $a_0(980)$-$K\bar{K}$)

x the $f_0(980)$ w.f. (--> $f_0(980)$-$K\bar{K}$)

- Therefore, the mixing intensity is sensitive to the $KK$ component both in $a_0(980)$ and in $f_0(980)$.

- Especially, for a small mixing

\[ \xi_{fa} \sim |\Lambda|^2 \sim |g_{af}|^2 \propto |X_aX_f| \]

--> Small $\xi_{fa}$ may imply small $|X|$.

- Notice: in general $X_{a,f}$ should be complex, and $|X|$ cannot be interpreted as a probability.

- However, $|X|$ will have a piece of information on the structure. Especially, $|X| \ll 1$ implies that molecular component is very small.
4. Constraint on their structure

++ Test with Flatte parameters ++

- Now we examine the relation between the $a_0(980)$-$f_0(980)$ mixing intensity $\xi_{fa}$ and the product of the two $KK$ compositeness $|X_a X_f|$, with the Flatte parameters from Exp. fittings.

| Collaboration | $M_a$ [MeV] | $\bar{g}_{aKK}$ [GeV] | $\bar{g}_{a\pi\pi}$ [GeV] |
|---------------|-------------|----------------|----------------|
| CLEO (2011)   | 998         | 3.97 ± 0.77    | 4.25           |
| KLOE (2009)   | 982.5       | 2.84 ± 0.41    | 2.46           |
| CB (2008)     | 987.4       | 2.94 ± 0.12    | 2.87           |
| SND (2000)    | 995         | 5.93 ± 0.54    | 3.11           |
| E852 (1999)   | 1001        | 2.36 ± 0.13    | 2.47           |

| Collaboration | $M_f$ [MeV] | $\bar{g}_{fKK}$ [GeV] | $\bar{g}_{f\pi\pi}$ [GeV] |
|---------------|-------------|----------------|----------------|
| CDF (2011)    | 989.6       | 4.02 ± 1.01    | 2.65           |
| KLOE (2006)   | 977.3       | 2.45 ± 0.17    | 1.21           |
| Belle (2006)  | 950         | 4.07 +0.76     | 2.28           |
| BES (2005)    | 965         | 5.80 +0.19     | 2.83           |
| FOCUS (2005)  | 957         | 3.39 +0.76     | 2.15           |
| SND (2000)    | 969.8       | 7.88 +0.86     | 3.19           |

- There is not a clear proportional connection, but there is actually a tendency.

$\xi_{fa} = 0.60 \pm 0.20_{(stat)} \pm 0.12_{(sys)} \pm 0.26_{(para)} \%$  
$\xi_{fa}|_{upper\ limit} = 1.1\%$  
Ablikim et al. (2011).
4. Constraint on their structure

++ Test with Flatte parameters ++

- Now we examine the relation between the $a_0(980)$-$f_0(980)$ mixing intensity $\xi_{fa}$ and the product of the two $K\bar{K}$ compositeness $|X_a X_f|$, with the Flatte parameters from Exp. fittings.

| Collaboration | $M_a$ [MeV] | $\bar{g}_{aKK}$ [GeV] | $\bar{g}_{a\pi\pi}$ [GeV] |
|---------------|-------------|----------------------|----------------------|
| CLEO (2011)   | 998         | 3.97 ± 0.77          | 4.25                 |
| KLOE (2009)   | 982.5       | 2.84 ± 0.41          | 2.46                 |
| CB (2008)     | 987.4       | 2.94 ± 0.12          | 2.87                 |
| SND (2000)    | 995         | 5.93 ± 0.54          | 3.11                 |
| E852 (1999)   | 1001        | 2.36 ± 0.13          | 2.47                 |

| Collaboration | $M_f$ [MeV] | $\bar{g}_{fKK}$ [GeV] | $\bar{g}_{f\pi\pi}$ [GeV] |
|---------------|-------------|----------------------|----------------------|
| CDF (2011)    | 989.6       | 4.02 ± 1.03          | 2.65                 |
| KLOE (2006)   | 977.3       | 2.45 ± 0.17          | 1.21                 |
| Belle (2006)  | 950         | 4.07 ± 0.76          | 2.28                 |
| BES (2005)    | 965         | 5.80 ± 0.22          | 2.83                 |
| FOCUS (2005)  | 957         | 3.39 ± 0.76          | 2.15                 |
| SND (2000)    | 969.8       | 7.88 ± 0.86          | 3.19                 |

- There is not a clear proportional connection, but there is actually a tendency.
4. Constraint on their structure

++ In a more general way ++

- We further see the relation between $\xi_{fa}$ and $|X_a X_f|$ in a more general way. --> 4 of Flatte parameters are fixed as

\[
M_a = 990 \text{ MeV}, \quad g_{a\pi\eta} = 3.0 \text{ GeV}, \quad M_f = 970 \text{ MeV}, \quad g_{f\pi\pi} = 2.4 \text{ GeV}
\]

---Rough average of Exp. params.

while the $a_0(980)$-$K\overline{K}$ and $f_0(980)$-$K\overline{K}$ coupling consts. are allowed to be arbitrary. (generated by random num.)

- There is an upper limit of $|X_a X_f|$ for each $\xi_{fa}$.

--- Especially, from

\[
\xi_{fa}|_{\text{upper limit}} = 1.1\%
\]

we have $|X_a X_f| < 0.47$.

--- Especially, from

\[
\xi_{fa} = 0.60 \pm 0.20_{(\text{stat})} \pm 0.12_{(\text{sys})} \pm 0.26_{(\text{para})} \%
\]

\[
\xi_{fa}|_{\text{upper limit}} = 1.1\%
\]

Ablikim et al. (2011).
4. Constraint on their structure

++ Favored | $X_a$ |-| $X_f$ | area ++

Border line for Exp.:

$\xi_{fa}|_{\text{upper limit}} = 1.1\%$

Favored | $X_a$ |-| $X_f$ | area from Exp.:

$\xi_{fa} = 0.60 \pm 0.20_{\text{(stat)}} \pm 0.12_{\text{(sys)}} \pm 0.26_{\text{(para)}}\%$

Unfavored area: Inevitably $\xi_{fa} > 1.1\%$
4. Constraint on their structure

++ Favored \( |X_a| \sim |X_f| \sim 1 \) is not favored.

--> We find that

“both \( a_0(980) \) and \( f_0(980) \) are \( K\bar{K} \) molecules” is questionable.

“\textbf{One of them} has large \( K\bar{K} \) component” is not ruled out.

--- Especially \( |X_f| \geq 0.3 \) for every \( |X_a| \). --> Not small \( K\bar{K} \) in \( f_0 \)?

\[
|X_f| = 0.60 \pm 0.20_{\text{stat}} \pm 0.12_{\text{sys}} \pm 0.26_{\text{para}}\%
\]

Border line for Exp.: \( \xi_{fa} |\text{upper limit} = 1.1\% \)

“Both \( a_0(980) \) and \( f_0(980) \) are \( K\bar{K} \) molecules” is not favored.
5. Summary
5. Summary

++ Summary ++

- Hadronic molecules are unique because they are composed of color-singlet hadrons themselves. --> They may be able to be identified w/o relying directly on QCD.

- We have formulated the $a_0(980)$-$f_0(980)$ mixing intensity. --> Many combinations of Flatte params. reproduce the Exp. value!

- We have formulated the compositeness as the “norm” for the two-body w.f., and obtained correct normalization even for resonances. --> The $K\bar{K}$ compositeness for $a_0(980)$ and $f_0(980)$ becomes complex, and their imaginary parts are not negligible, which did not appear in the previous study by Baru et al..

- The $a_0(980)$-$f_0(980)$ mixing intensity can constrain their $K\bar{K}$ compositeness via the $a_0(980)$- and $f_0(980)$-$K\bar{K}$ coupling constants. --> From Exp. value of the mixing intensity, “both are simultaneously $K\bar{K}$ molecules” is questionable.
Thank you very much for your kind attention!