A novel digital phase tracking algorithm for a high resolution fibre Bragg grating based sensor system

BT Meggitt+, W Li, WJO Boyle & YM Gebremichael

+EM Technology, London, UK

School of Engineering and Mathematical Sciences, City University, London EC1V 0HB, UK

Abstract. The development of a high-resolution, optical fibre Bragg grating sensor system is described that incorporates a novel digital, condition-based, phase-tracking algorithm for unwrapping the $2\pi$ phase ambiguity and permits the continuous up/down phase-tracking of multiple fringe changes. An all-fibre, Michelson interferometric phase-measurement approach is utilised for the determination of strain and temperature induced Bragg wavelength shifts. The system incorporates orthoconjugate mirrors and a stabilised reference channel to compensate for polarisation and random thermal induced drifts and has the potential for resolving sub-µstrain and mK temperature changes with a resolutions of $<100n\varepsilon/\sqrt{Hz}$ (at 1Hz) and $<10mK$ respectively.

1. Background

Fibre optic Bragg grating sensors (FBGs) have become an established method for sensing environmental changes such as strain, temperature pressure, etc. and their fabrication and theory of operation are now well documented in the literature [1][2]. The FBG sensors respond to measurand changes by producing a discernable shift, $\delta\lambda_{\text{Bragg}}$, in the reflected central wavelength, $\lambda_{\text{Bragg}}$. Although it is possible to determine the wavelength shift directly using tunable Fabry-Perot filters or spectrometer methods [3-6], phase-domain processing can provide a significantly higher measurement sensitivity [7]. In the latter approach, the FBG sensor reflected wavelength, $\lambda_{\text{Bragg}}$, is used as the ‘source’ for an unbalanced, path-length modulated optical fibre interferometer. The change in the output-phase of the interferometer cosine transfer function is then used as a measure of the wavelength change, $\lambda_{\text{Bragg}}$. However, differential phase measurement schemes suffer a $2\pi$-phase ambiguity, which limits the sensor unambiguous measurement range to within one fringe. Previous reported work [8] used an all-fibre system to determine FBG sensor wavelength shifts, utilising a modulated, single mode fibre based Mach-Zehnder interferometer operating in the transmission mode to detect the differential output between the signal and reference FBG sensor, with the reference used to compensate for time-varying path length fluctuations in the processing interferometer. In this configuration, two cascaded two-beam interferometers with different optical path difference (OPD) were used to extend the $2\pi$ phase ambiguity associated with such differential phase measurements to a certain range. While good measurement resolutions were observed, the operation of the processing interferometer in this configuration lead to signal fading due to time-varying fibre birefringence changes between the two interferometer fibre arms. Digital phase tracking techniques have also been implemented in a phase modulated all fibre Michelson interferometer to overcome fringe order
ambiguity electronically for vibration sensing application [9]. In the work reported here, we describe a method for overcoming these performance limitations and present results showing its application and performance limitations in the measurement of temperature, longitudinal strain and vibration perturbations.

2. Theoretical considerations

In the approach used here, (Figure 1), an unbalanced all-fibre Michelson interferometer was utilised as the wavelength discriminator device, where one fibre arm of the interferometer was wound around a cylindrical PZT to phase modulate its OPD over one fringe in an active homodyne method. In order to avoid signal-fading effects [10-15], phase-conjugate Faraday fibre pigtailed mirrors were used as the reflecting elements, one in each interferometer arm. Since for a two beam interferometer the phase-change is related to the wavelength-shift by 

\[
\delta \phi = 2n \Delta L \delta \lambda_{\text{Bragg}} / \lambda_{\text{Bragg}}^2
\]

the phase-sensitivity of the interferometer increases proportionately with the arm imbalance \(\Delta L\), but with the fringe visibility \(v_{\text{vis}}\) decreasing in an exponential manner. An arm imbalance approximately equal to the coherence length, \(L_{\text{coh}}\), is used (i.e. \(L_{\text{coh}} = \lambda_{\text{Bragg}} / \Delta \lambda_{\text{Bragg}} \approx 10\)mm for \(\lambda_{\text{Bragg}} \approx 1550\)nm, \(\Delta \lambda_{\text{Bragg}} \approx 0.25\)nm) in order provide a high signal sensitivity with an acceptable output signal to noise ratio. In addition, the OPD of the interferometer imbalance should be greater than the coherence length of the broadband source (~50µm for SLD, FFS sources, \(\lambda_s = 1550\)nm, \(\Delta \lambda_s = 40\)nm) such that no interference effects are observable at its output of the Michelson interferometer, which is readily achieved. The output interference waveforms from the reference and sensor FBG gratings seen at the detector outputs have the form,

\[
I_s(\lambda_s) = A_s(1 + v \cos(\phi(t) + \phi_s))
\]

\[
I_R(\lambda_R) = A_R(1 + v \cos(\phi(t) + \phi_R))
\]

where \(A_s\) and \(A_R\) are proportional to the input optical power spectrum, \(v\) is the visibility of the interference signal, \(\lambda_s, R\) is the reflected wavelength from the sensing and reference gratings, \(\phi(t)\) is the random phase difference induced by variations in the interferometer OPD. The relative phase change, \(\delta \phi(\lambda_s)\), as a function of sensor grating wavelength changes, \(\delta \lambda_s\), between the signal and reference interference signals, induced by e.g. strain and temperature changes, is given by:

\[
\delta \phi(\lambda_s) = \frac{4 \pi n \Delta L}{\lambda_s^2} \delta \lambda_s
\]

where \(\Delta L\) the path difference, \(n\) is the fibre refractive index, \(\lambda_s\) and \(\lambda_{\text{Bragg}}\) are the central wavelength and the wavelength shift of the reflected light respectively. It is apparent from the above equation that the value of the phase shift may be amplified many times for higher sensitivity and lower operational range when a large path imbalance, \(\Delta L\), is used. This however reduces interferometric fringe visibility. In this application, with an optical imbalance of 3mm between the arms of the interferometer, (which corresponds to an OPD of ~10mm) and a Bragg sensor at 1550nm, a \(2\pi\) phase change (from equation 4) gives an equivalent Bragg wavelength shift of ~270pm. The strain and temperature phase responsivity can also be calculated from equation 4 such that

\[
\frac{\delta \phi(\lambda_s)}{\Delta \varepsilon} = \frac{4 \pi n \Delta L}{\lambda_s^2} (1 - \rho_s)
\]

\[
\frac{\delta \phi(\lambda_s)}{\Delta T} = \frac{4 \pi n \Delta L}{\lambda_s^2} (\varepsilon + \alpha)
\]

since \(\Delta \lambda_s = (1 - \rho_s) \lambda_s \Delta \varepsilon\), where \(\alpha (\approx 0.26)\) is the strain optic coefficient of the fibre and \(\Delta \varepsilon\) is the strain change, giving a strain-phase responsivity, \(\delta \phi / \delta \varepsilon \approx 0.026\) rad/µε (1.12pm/µε or ~240µε/fringe).
Similarly, the temperature induced wavelength shift is given by $\Delta \lambda_B = (\alpha + \xi) \lambda_B \Delta T$ where $\alpha (~0.54 \times 10^{-6}/{}^\circ C)$ is thermal expansion coefficient of the fibre core; $\xi (~8.3 \times 10^{-6}/{}^\circ C)$ is the fibre thermo-optic coefficient. Thus the temperature phase responsivity can be calculated by from equation 5 to give $\delta \phi / \delta T \approx 0.32 \text{rad/}{}^\circ K$ (equivalent to 13 pm/{}^\circ C or ~200 {}^\circ C/fringe). The ultimate phase detection sensitivity will depend on the stability of the fibre interferometer and the measurement technique, and it can be shown that for a measured phase sensitivity of ~2$\pi$/2x10$^3$ (equivalent to 0.2° phase angle), the wavelength shift resolution, $\delta \lambda_{\text{Bragg}} \approx 0.14$ pm, which gives a ~0.10 $\mu$e and ~10 mK temperature resolution.

3. Experimental arrangement

The differential FBG sensor system developed in this work is illustrated in Figure 1. The system uses a referenced interferometric detection configuration (a stable/shielded reference grating in conjunction with a sensor grating) and a pair of Faraday rotator mirrors spliced at each end of the interferometer arms to overcome random phase drifts due to the instability related to the unbalanced phase processing interferometer.

![Figure 1](Image)

**Figure 1.** Drift compensated fibre Bragg grating sensor system with sensing and reference grating elements and polarisation insensitive Michelson processing interferometer.

To minimise the effects of polarisation fading further, an erbium doped fibre fluorescent source was used because of its intrinsic un-polarized output radiation and substantially improved spectral stability [16]. The sensing and reference fibre Bragg grating elements were placed in the backward arm of an unbalanced, polarization-compensated fibre Michelson interferometer. Each of the two grating elements reflected a narrow-band portion of the interferometer output signal monitored by dc-coupled photodetectors. As shown in Figure 1, the phase difference between the interferometer output signals is monitored using the pseudo-heterodyne signal-processing scheme [17]. The phase modulation was achieved by applying a saw tooth ramp signal at a frequency of 30Hz to 300Hz with the voltage set to produce a 2$\pi$ peak-to-peak phase excursion.
4. Reference grating - Thermal compensation

In order to improve the stability of the reference sensor, and thus the phase measurement, a compensated aluminium-silicon structure was designed for the development of the athermal reference FBG element [18][19]. A differential expansion ‘jacket’ was formed over the reference FBG grating by adhering two short lengths of aluminium tubing either side of a pre-strained grating and then sliding over a longer piece of silica tubing and fixing it in position illustrated in Figure 2. The lengths of the silica and aluminium tubing are designed such that as the temperature increases, a compressive strain is exerted on the grating by the jacket arrangement which is equal and opposite to the thermal effects on the fibre sensor length and so cancelling the thermal expansion and thermo-optic effect in the fibre.

![Figure 2](image)

In order to achieve the intended compensation, the design criteria was such that:

\[
\frac{\Delta \lambda_B}{\lambda_B} \text{thermo} + \frac{\Delta \lambda_B}{\lambda_B} \text{strain} = 0
\]

(6)

It can be shown that the relationship between the differential expansion of the aluminium-silica structure and the thermo-optic properties of the fibre grating is given as;

\[
L_2 = L_1 \left( \alpha_{Al} - \alpha_{Sil} + \frac{2(\xi + \alpha_{Sil})}{\lambda_{Bragg}} \right)
\]

(7)

where \(L_1\) is the effective length of the aluminium tube, \(L_2\) is the length of the silica tube, \(\alpha_{Al}\) is the expansion coefficient of the aluminium tube, \(\alpha_{Sil}\) is the expansion coefficient of the silica tube and fibre, \(\xi\) is the thermo-optic coefficient of the fibre and \(\lambda_{Bragg}\) is the central wavelength of the Bragg grating. Athermal reference grating prototypes were fabricated and tested using this design approach which showed a reduction in thermal response of ~x5 over the normal grating. With further finesse in the design and improved fabrication techniques, it should be possible to reduce the thermal response by >x10.

5. Phase measurement and processing

The analogue interferometer signals were digitised by a 12-bit ADC data acquisition card and read into a microprocessor based processing system using software algorithms programmed in a LabView environment. Since the phase-carrier signals contained high frequency components due to the fast fly-back of the ramp signal used to drive the PZT, as shown in Figure 3(a), it was necessary to filter the signals using a low-pass filter. The resulting filtered signals are shown in Figure 3(b). Any DC component to the sinusoidal signals was then subtracted. Further more, it was important to maintain uniform amplitude for the sinusoidal waveforms and software based, automatic unity gain control was implemented so that high phase integrity was maintained.
6. A Digital fringe-tracking algorithm - Phase unwrapped measurement –

In order to overcome the $2\pi$ phase ambiguity problem of such interferometric fringe measurement systems, a software algorithm has been devised that uses the single, defined condition-state function $D(t)$ to measure the absolute, unwrapped phase-change, $\Delta\phi(t)+N(t)$, by determining both the wrapped, ambiguous phase, $\Delta\phi(t)$, and the fringe order, $N$. $D(t)$ is formed using the AND function to combine the signal and reference wave amplitudes to produce the binary-state waveform, i.e. $I_{\text{ref}}$ and $I_{\text{sig}} > 0$, $D(t) = 1$; $I_{\text{ref}}$ or $I_{\text{sig}} < 0$, $D(t) = 0$ (Figure 4(a)).

The measure of unwrapped phase-difference, $\Delta\psi(t)$, is taken as the width of the function $D(t)$, averaged over one period, where $\Delta\psi(t) \Rightarrow \pi$ to 0 as $\Delta\phi(t) \Rightarrow 0$ to $2\pi$. The condition-state waveform, phase-difference, $\Delta\psi(t)$, is related to signal-reference phase-difference by, $\Delta\phi(t) = 2\pi(1/2+\Delta\psi(t) S(t))$, where $S(t)=\pm 1$ depending on the value of the condition-statement, binary number generator algorithm, $T(t)$, defined as,
Here, D(t) is the present and D(t-1) the previous value evaluated at reference-wave phase values $\phi_R=0$ and $\phi_R=\pi/2$ (say). The unwrapped phase, $\Phi(t)$, is then given by,

$$\Delta \Phi(t) = 2 \pi \times (\Delta \psi(t) + (N + 1/2))$$

T(t) represents a four bit binary number (0 to 15 possible decimal values) and can be shown to have only four critical cases, which determine the processing action to be taken when the phase-difference $\Delta \psi(t)$ changes over the $\pi$ and zero fringe-phase boundaries, in an increasing or decreasing direction. A conditioning test is carried out to determine whether of the system is in one of the four possible critical states and depending on the result, an accumulator, N, is then incremented, decremented or left unchanged and S is assigned a value +1 or –1. It can be shown that the four critical values for T(t), corresponding to Cases (1) to (4) below, are:

Case (1): T(t) = 0001  Case (2): T(t) = 0010  
Case (3): T(t) = 1101  Case (4): T(t) = 1110

Case (1): the phase, $\Delta \phi(t)$, is increasing and the sensor signal just comes off from being completely out of phase ($\Delta \phi(t)=\pi$) with the reference. The accumulator count N(t) remains the same and the value of $\Delta \psi(t)$ is subtracted (S(t)=-1).

Case (2): $\Delta \phi(t)$ is decreasing and the sensor signal just comes into being out of phase ($\Delta \phi(t)=\pi$) with the reference. The accumulated count N(t) remains the same and the value of $\Delta \psi(t)$ is added (S(t)=+1).

Case (3): $\Delta \phi(t)$ is increasing and the sensor signal comes off from being in phase ($\Delta \phi(t)=0$) with the reference. The accumulated count N(t) increases by 1 and the value of $\Delta \psi(t)$ is subtracted (S(t)=-1), i.e. N=N+1.

Case (4): $\Delta \phi(t)$ is decreasing and the sensor signal comes onto being completely in phase ($\Delta \phi(t)=0$) with the reference. The accumulator count N has 1 is subtracted from the count N(t) and the value of $\Delta \psi(t)$ is added (S(t)=+1), i.e. N=N-1. The processing scheme was compiled and tested in a LabView algorithm with a 10$\pi$ (5 fringe) sinusoidal modulation applied to a signal waveform. It is seen from the results shown in Figure 5 that $\Delta \psi(t)$ (upper) is a folded (wrapped) function and that T(t) (central) provides the 4 digital states values necessary to give S(t), $\Delta \phi(t)$, N(t) and hence the (lower) absolute, unwrapped phase, $\Phi(t)$.

7. Experimental results and analysis

The performance of the FBG sensor system was characterised for both strain and temperature sensor perturbations using a micrometer adjustment (static strain), a steel cantilever beam (dynamic strain - vibration) and an emersion heated surface (temperature).

7.1 Strain measurements:

The new fringe counting/tracking capability of the software based digital phase tracker for extended (unambiguous, $\Delta \phi>2\pi$) phase measurement was demonstrated by applying an incremental strain of >7 interferometric fringes to a fibre Bragg grating sensor element. Figure 5a shows the phase change recoded by the digital phase tracker when the grating was repeatedly strained between two micrometer stages in near equal $\pm \pi$ radian increasing and decreasing increments over a 1740µm longitudinal strain range. Figure 5b shows the linearity of new digital phase-tracker and its ability to continuously accumulate the phase change over the multiple fringe intervals with the strain/phase change is ‘seamless’ with good linearity. The detection of a damped oscillation by the new system was also demonstrated by setting the cantilever beam into harmonic oscillation. The resulting output waveform at a harmonic
frequency of ~10Hz shows the performance of the system, both in drift-compensation and dynamic responses, as shown in Figure 6b.

**Figure 5.** Incremental strain response showing; (a) phase-tracking of fibre grating repeatedly loaded/unloaded over 1740µε (~7 fringes); (b) linearity of phase change against strain.

**Figure 6.** Output response of FBG: (a) a strain perturbation of 0.6µε, ~2.5 minutes period, and (b) vibration damping at harmonic frequency of ~10Hz.
7.2 Temperature Measurements:
The temperature response of the high resolution FBG sensor system was tested against a K-type thermocouple sensor system (voltage-temperature responsivity ~10mV/°C) where both sensors were attached to the outside surface of an emersion heated vessel and the outer surface temperature varied from ~10°C to ~90°C (ΔT>800°C). Figure 7a shows the results obtained with the FBG temperature sensor where it is seen that the phase excursion of the sensor exceeded 7π radians (i.e. ~3.5 interferometer fringes) with a temperature response of ~40°C/radian. As shown in Figure 7b, a similar response curve was demonstrated by the thermocouple but with a lower signal-to-noise ratio (SNR). Again this demonstrates the seamless nature of the fringe-to-fringe transition of the new phase-tracking algorithm.

To investigate the resolution and long-term drift of the FBG sensor system, the sensor grating was attached to a large metal plate and covered with an enclosure to isolate it from environmental effects. The temperature equivalent-drift over 5 minutes was ±20mK, while the short-term measurement resolution showed fluctuations of ±5mK, when using a 10 sec time constant. This is equivalent to ±0.1µε and without averaging, the long-term drift is ~0.01 fringes, i.e. 1.2µε, or in terms of temperature this corresponds to a resolution of ~140mK. This shows the greatly improved resolution when using a phase measurement when compared to other FGB processing methods, e.g. processing using a tuneable filter technique where resolutions of ~10µε or ~1°C are commonly obtained.

8. Summary and conclusions
A prototype high-resolution, differential interferometric sensor system for the detection of strain/temperature-induced Bragg grating wavelength shifts has been demonstrated. The unbalanced Michelson based fibre optic processing interferometer is designed to be polarization insensitive by making use of Faraday rotator mirror reflectors that eliminated signal fading effects. Two fibre Bragg gratings were used in the system, a sensor and reference gratings to provide a differential phase measurement scheme to greatly reduce interferometer induced phase noise. Good sinusoidal fringes were produced with signal-harmonic suppressed to −15dB. A novel software-based signal processing system was developed which gave a “seamless” transmission across the fringe boundaries. A strain resolution
close to $\pm 1.25\mu\varepsilon$ and a temperature resolution $\pm 15\text{mK}$ were obtained with a 0.3 second time constant over a 5 minute period.

Acknowledgements
This work was carried out under ESTEC Contract No 13958/99/NL/MM, whose support is gratefully acknowledged.

References
1. K.T.V. Grattan and B.T. Meggitt (Eds), “Optical Fiber Sensor Technology: Advanced Applications,” Kluwer Academic Publishers. Dordrecht, The Netherlands 2000, pp. 79 – 187.
2. W. W. Morey, J. R. Dunphy and G. Meltz, “Multiplexed fibre grating sensors”, Proc. Distributed and Multiplexed Fibre Optic Sensors Boston, 1991 Proc. SPIE 1586, pp216.
3. A. D. Kersey, T. A. Berkoff and W. W. Morey, “Multiplexed fibre Bragg grating strain-sensor system with fibre Fabry-Perot wavelength filter”, Opt. Lett., vol.18, pp1370, 1993.
4. S. M. Melle, K. Liu and R. M. Measures, “A passive wavelength demodulation system for guided-wave Bragg grating sensors”, IEEE Photon. Technol. Lett., vol.4, pp516, 1992.
5. H. Geiger, M. G. Xu, N. C. Eaton and J. P. Dakin, “Electronic tracking system for multiplexed fibre grating sensors”, Electron. Lett., vol.31, pp1006, 1995.
6. D. A. Jackson, A. B. Ribeiro, L Reekie and J. L. Archambault, “Simple multiplexing scheme for a fibre-optic grating sensor network”, Opt. Lett., vol.18, pp1192–4, 1993.
7. A. D. Kersey, T. A. Berkoff, and W. W. Morey, “High resolution fibre-grating based strain sensor with interferometric wavelength-shift detection”, Electron. Lett., vol.28, pp236, 1992.
8. Y.J. Rao, D.A. Jackson, L Zhang and I. Bennion, “Extended dynamic range detection system for in-fibre Bargg grating strain sensors based on two cascaded interferometric wavelength scanners”, Meas. Sci. Technol., vol. 8, pp.1043-1049, 1997.
9. B.T. Meggitt, A.C. Lewin and D.A. Jackson, “A fibre-optic non-contacting reference grade vibration sensor”, SPIE Vol. 1120, pp. 307-312, 1989.
10. H. C. Lefevre, “Single-mode fibre fractional wave devices and polarization controller”, Electron. Lett., vol.16, pp778, 1980.
11. A. D. Kersey, M. J. Marrone and A. Dandridge, “Dependence of visibility on input polarization in interferometric fibre optic sensors”, Opt. Lett., vol.13, pp288, 1988.
12. A. D. Kersey, “Optimisation and stabilization of visibility in interferometric fibre sensors using input polarization control”, J. Lightwave Technol., vol.6, pp1599, 1988
13. A. D. Kersey, M. J. Marrone and M. A. Davis, “Polarization-insensitive fibre optical Michelson interferometer”, Electron. Lett., vol.27, pp518, 1991.
14. M. Martinelli, “A universal compensation for polarization changes induced by birefringence on a retracing beam”, Optics Communication, vol.72, p341, 1989.
15. N. C. Pistoni and M. Martinelli, “Birefringence effects suppression in optical fibre sensor circuits”, Proc. 7th Int. Conf. On Optical Fibre Sensors (OFS ’90), Sydney, Australia, December1990, p125.
16. C. P. Wysocki, M. J. F. Digonnet and B. Y. Kim, “Wavelength stability of a high throughput broadband, Er-doped superfluorescent fibre source pumped near 980nm”, Opt. Lett., vol.16, pp961, 1991.
17. D. A. Jackson, A. D. Kersey and M. Corke, “Pseudo-heterodyne detection scheme for optical interferometers”, Electron. Lett., vol.18, pp1081, 1982.
18. G. W. Yoffe, P. A. Krug, F. Ouellette and D. A. Thorncraft, “Passive temperature-compensating package for optical fibre gratings”, Applied optics, Vol. 34, No 30, p. 6859-6861, 1995.
19. Yu-Lung Lo and Chih-Ping Kuo, “Packaging a fiber Bragg grating with metal coating for an athermal design”, Journal of Lightwave Technol. Vol 21, No 5, p. 1377- 1383, 2003.