Oscillatory inflation in non-minimal derivative coupling model

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Abstract
Inflation during rapid oscillation of a scalar field in non-minimal derivative coupling model is discussed. Cosmological perturbations originated in this stage are studied and the consistency of the results with observational constraints coming from Planck 2013 data are investigated.

1 Introduction

In the past three decades various models have been proposed for inflation [1], where in many of them inflation is driven by a canonical scalar field, \( \phi \) (dubbed inflaton), rolling slowly in an almost flat potential. Higgs boson may be a natural candidate for inflaton [2]. Inspired by this idea, the authors of [3], by introducing a non-minimal coupling between kinetic term of the scalar field and the Einstein tensor, tried to consider the inflaton as the Higgs boson, without violating the unitarity bound. This model is specified by the action

\[
S = \int \left( \frac{M^2}{2} R - \frac{1}{2} \Delta^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) \sqrt{-g} d^4 x, \tag{1}
\]

where \( \Delta^{\mu \nu} = g^{\mu \nu} - \frac{1}{M^2} G^{\mu \nu} \), and \( G^{\mu \nu} = R^{\mu \nu} - \frac{1}{2} R g^{\mu \nu} \) is the Einstein tensor. The minus sign before the Einstein tensor prevents ghost presence in the theory. \( M \) is a coupling constant with the dimension of mass, and \( M_P = 2.435 \times 10^{18} \text{GeV} \) is the reduced Planck mass. Inflation [3], rapid oscillation [4], reheating [5], and late time acceleration [6], have been recently studied in the context of this non minimal derivative coupling model. To find more features of this model see [7].

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In the literature, it is often assumed that inflation nearly ceases after the slow-roll and the inflaton enters a rapid oscillation phase during which the radiation is generated. But in [8] the possibility of continuation of inflation during rapid oscillation phase for a potential satisfying a non-convexity inequality was proposed. Inflation continues as long as the scalar field is trapped in the convex core. This effect was reported and numerically confirmed in [9], where it was shown that only a few number of e-folds is realized in this era. Due to the few number of e-folds in the rapid oscillation phase, it is expected that cosmological perturbations, as the seed of structure formation, were originated in the slow-roll regime. However perturbations originated in the rapid oscillation era may have imprints on cosmological scales provided that one considers an adequate period of inflation during rapid oscillation. This may happen in more complicated models such as hybrid inflation, as was asserted in [8].

In this work we study inflation during rapid oscillation in non-minimal derivative coupling model proposed in [3]. Conditions required for this oscillation and also inflation in this stage are discussed and the possibility that the inflation ceases is studied. Cosmological perturbations created in this era are computed and the consistency of the results with observational constraints coming from Planck 2013 [10] are investigated.

2 Oscillatory inflation

We consider gravitational enhanced friction model (1) in the spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) space-time. The scalar field equation of motion is

\[
(1 + 3 \frac{H^2}{M^2}) \ddot{\phi} + 3H(1 + 3 \frac{H^2}{M^2} + 2 \frac{\dot{H}}{M^2}) \dot{\phi} + V'(\phi) = 0,
\]

(2)

where \( H = \frac{\dot{a}}{a} \) is the Hubble parameter and a dot is the differentiation with respect to the cosmic time \( t \). The energy density and the pressure for this homogeneous and isotropic scalar field can be expressed as

\[
\rho_\phi = (1 + \frac{9H^2}{M^2}) \frac{\dot{\phi}^2}{2} + V(\phi),
\]

(3)

and

\[
P_\phi = (1 - \frac{3H^2}{M^2} - \frac{2\dot{H}}{M^2}) \frac{\dot{\phi}^2}{2} - V(\phi) - 2H \dot{\phi} \ddot{\phi},
\]

(4)

respectively. The Friedmann equation reads

\[
H^2 = \frac{1}{3M_P^2} \rho_\phi.
\]

(5)

The slow roll solution and the associated inflation were studied in [3]. Here we consider rapid oscillatory solution for the scalar field, with time...
dependent amplitude $\Phi(t)$ (the highest point of oscillation at which $\dot{\phi} = 0$) and also time dependent frequency $\omega(t) = \frac{1}{T(t)}$. $T(t)$ is the period of the oscillation

$$T = 2 \int_{-\Phi}^{\Phi} \frac{d\phi}{\dot{\phi}}.$$  \hspace{1cm} (6)

The rapid oscillation phase is characterized by

$$H(t) \ll \frac{1}{T(t)}$$  \hspace{1cm} (7)

and

$$\left| \frac{\dot{H}}{H} \right| \ll \frac{1}{T},$$  \hspace{1cm} (8)

implying that the Hubble parameter is much smaller than the time dependent frequency and changes insignificantly during one oscillation: $H(t') \approx H(t)$ for $t \leq t' \leq t + T(t)$. From (5) and (8), it is clear that like $H$, $\rho_\phi$ remains approximately a constant during one period. We take this nearly constant as the value of the energy density at the amplitude, $\Phi$, where $\dot{\phi}\big|_{\phi=\Phi} = 0$ (see fig. (1)). Therefore the energy density during one oscillation can be expressed in terms of the value of the potential at the corresponding amplitude

$$\rho_\phi \approx V(\Phi).$$  \hspace{1cm} (9)

Therefore for a power law potential we expect that $\left| \frac{\dot{\Phi}}{\Phi} \right| \ll \frac{1}{T}$. To elucidate more this subject, in fig. (1), the rapid oscillating scalar field is depicted numerically by using eqs. (3), (2), and (5) for a quadratic potential, showing that the amplitude of oscillation changes very slowly during one oscillation. A more detailed discussion about this solution may be found in [11] and [4]. Also, in fig. (2) the oscillation of the scalar field for the potential $V(\phi) = \lambda |\phi|^{0.0392}$ is numerically shown (the reason for this choice will be revealed when we will determine our parameters from astrophysical data in the third section).

The adiabatic index of the scalar field, defined by $\gamma = w + 1$ where $w$ is the equation of state parameter (EoS): $w = \frac{p_\phi}{\rho_\phi}$, in the rapid oscillation
Figure 1: $\varphi := \frac{\phi}{M_P}$ in terms of dimensionless time $\tau = mt$, for $\frac{m^2}{M_P^2} = 10^8$ with $\{\varphi(1) = 0.01, \dot{\varphi}(1) = 0\}$, for the quadratic potential $\frac{1}{2} m^2 \dot{\phi}^2$.

Figure 2: $\varphi := \frac{\phi}{M_P}$ in terms of dimensionless time $\tau = M_P t$, for $M = 10^{-9} M_P$ and $\lambda = 1.76 \times 10^{-8} M_P^{-1 - 0.0392}$ with initial conditions $\{\varphi(1) = 10^{-6}, \dot{\varphi}(1) = 0\}$.

The phase is effectively given by

$$\gamma = \frac{\langle P_\phi + \rho_\phi \rangle}{\langle \rho_\phi \rangle}$$

$$= \frac{\langle (1 + \frac{3H^2}{M_P^2}) \dot{\phi}^2 - \frac{d}{dt} (\frac{\phi^2}{2}) \rangle}{\langle \rho_\phi \rangle}$$

$$= \left(1 + \frac{3H^2}{M_P^2}\right) \frac{\langle \dot{\phi}^2 \rangle}{\langle \rho_\phi \rangle}$$

$$= \frac{2 \left(1 + \frac{3H^2}{M_P^2}\right) \langle \rho_\phi - V(\phi) \rangle}{\left(1 + \frac{3H^2}{M_P^2}\right)}$$

$$= \frac{2 \left(1 + \frac{3H^2}{M_P^2}\right)}{\left(1 + \frac{3H^2}{M_P^2}\right) V(\Phi)} \int_{\Phi}^{\Phi} \sqrt{V(\Phi) - V(\phi)} d\phi$$

$$= \int_{\Phi}^{\Phi} \frac{d\phi}{\sqrt{V(\Phi) - V(\phi)}} \left(1 + \frac{3H^2}{M_P^2}\right) V(\Phi)$$

(10)
\[ \langle .. \rangle = \frac{\int_{t-T}^{t+T} \text{d}t}{T} \text{ is the average over an oscillation with period } T. \] To obtain (10), we have used (3), (4) and (9), and have taken into the account the fact that \( \dot{\phi} \) vanishes at \( |\phi| = \Phi \). (10) is valid only for time scale much larger than the period \( T: t \gg T \), over which the average was taken. For even power law potentials

\[ V(\phi) = \lambda \phi^q, \quad (11) \]

where \( \lambda \in \mathbb{R} \), the adiabatic index becomes

\[ \gamma = \frac{2q}{q+2} \frac{1 + \frac{3H^2}{M^2}}{1 + \frac{9H^2}{M^2}}. \quad (12) \]

In the minimal coupling, \( M \to \infty \), we recover the result derived in [11] (see also [8], [9], [12] and references therein), \( \gamma = \frac{2q}{q+2} \). We have also

\[ \langle \rho_\phi \rangle = \frac{\rho_\phi(t+T) - \rho_\phi(t)}{T} \simeq \rho_\phi, \quad (13) \]

where \( t \gg T \) has been used. By taking the average of the continuity equation, namely \( \langle \dot{\rho}_\phi + 3H(P_\phi + \rho_\phi) \rangle = 0 \) we obtain

\[ \dot{\rho}_\phi + 3H\gamma \rho_\phi = 0, \quad (14) \]

where \( \gamma \) is given by (10). All the quantities in the above equation must be regarded as their averaged value in the sense explained above and the equation is valid for large time with respect to the period \( T \). For a constant \( \gamma \), the system composed of the equations (14) and (5) may be solved analytically. For the power law potential, this occurs for minimal model and also when we consider the high friction regime [3],

\[ \frac{H^2}{M^2} \gg 1, \quad (15) \]

leading to \( \gamma = \frac{2q}{3q+6} \). In this situation analytical solutions for the energy density, the scale factor, and the Hubble Parameter are

\[ \rho_\phi \propto a^{-3\gamma}, \quad (16) \]

\[ a \propto t^{\frac{2}{3\gamma}}, \quad (17) \]

\[ H = \frac{2}{3\gamma}t, \quad (18) \]

respectively. Here, in contrast to the slow roll, \( |\dot{H}| \) and \( H^2 \) may be of the same order of magnitude. From the continuity equation one can find that the amplitude, \( \Phi \), satisfies

\[ \dot{\Phi} + \frac{3\gamma H}{q} \Phi = 0 \quad (19) \]
whose solution is
\[ \Phi(t) \propto a^{-\frac{1}{3}} \propto t^{-\frac{2}{3}}. \] (20)

Hereafter we restrict ourselves to the high friction regime [15], where as we have seen, analytical solution for the problem can be found. Note that in derivation of these solutions we have employed the conditions (5), (7) and \( t \gg T \). So the domain of validity of our result is where the solutions satisfy these conditions. Using (18), we find that if \( HT \ll 1 \) is satisfied, then (5) and \( t \gg T \) are also fulfilled. In the case of the power law potential, the period is determined as

\[ T = 2 \int_{-\Phi}^{\Phi} \frac{d\phi}{\sqrt{\rho_\phi - V(\phi)}} = \sqrt{\frac{18H^2}{M^2}} \int_{-\Phi}^{\Phi} \frac{d\phi}{\sqrt{\lambda\Phi^q - M^2}} = 2\sqrt{\frac{18\pi H^2}{\lambda M^2}} q \Gamma\left(\frac{q+2}{2q}\right) \Phi^{\frac{q-2}{2}}. \] (21)

where \( H^2 = \frac{\lambda}{3M_P^2} \Phi^q \), derived from (5), (9), and (11) has been used. Hence, \( HT \ll 1 \) can be rewritten in terms of \( \Phi \) as

\[ \Phi^{\frac{q+2}{2}} \ll \left( \frac{q \Gamma\left(\frac{q+2}{2q}\right)}{\sqrt{8\pi \Gamma\left(\frac{1}{q}\right)}} \right) \frac{M_P^2 M}{\sqrt{\lambda}}. \] (22)

The presence of the scale \( M \) reduces the scale of the scalar field with respect to the minimal case in which the same procedure gives [13]

\[ \Phi \ll \left( \frac{q \Gamma\left(\frac{q+2}{2q}\right)}{\sqrt{8\pi \Gamma\left(\frac{1}{q}\right)}} \right) \sqrt{3M_P}. \] (23)

This can also be rewritten in terms of the Hubble parameter as

\[ H^{\frac{q+2}{2}} \ll \left( \frac{q \Gamma\left(\frac{q+2}{2q}\right)}{\sqrt{8\pi \Gamma\left(\frac{1}{q}\right)}} \right) \frac{1}{\lambda^\frac{q}{2}} \frac{M_P^2}{M^2} \frac{\Phi^q}{M^{q-2}}. \] (24)
Therefore the domain of validity of our solutions is given by (22) or (24) which specifies a bound for the Hubble parameter (and consequently for the energy density) during rapid oscillation.

Note that in the slow roll we had \( \ddot{\phi} \ll \dot{3}H \dot{\phi} \), and also \( \rho_{\phi} \approx V(\phi) \) which together with (3) imply

\[
\left(1 + 9 \frac{H^2}{M^2}\right) \frac{\dot{\phi}^2}{2} \ll V(\phi). \tag{25}
\]

In the high friction regime, these conditions are satisfied (for details see [4]) when \( \phi^{q + 2} \gg \frac{2M^2 M_{Pl}^4}{\lambda} \), which is opposite to (22). In the high friction regime (25) leads to

\[
\frac{9H^2}{2M^2} \frac{\dot{\phi}^2}{2} \ll V(\phi) \sim 3M_{Pl}^2 H^2 \text{ resulting } \dot{\phi}^2 \ll \frac{2}{3} M^2 M_{Pl}^2. \]

In contrast to this result, in quasi periodic stage, (10) and (5) result in

\[
\langle \dot{\phi}^2 \rangle \approx \frac{P}{M^2} \gamma M_{Pl}^2. \tag{26}
\]

Inflation occurs when \( \ddot{a} > 0 \) or in terms of the adiabatic index: \( \gamma < \frac{2}{3} \), which leads to \( q \in (-2, \infty) \). Note that in the minimal case, where \( \gamma = \frac{2q}{q+2} \), inflation takes place only for the short range \( q \in (-2, 1) \). Inflation continues as long as \( \gamma < \frac{2}{3} \), which from the fourth equality in (10) leads to

\[
\frac{1 + \frac{3H^2}{M^2}}{1 + \frac{9H^2}{M^2}} \langle \rho_{\phi} - V \rangle < \frac{1}{3} < \rho_{\phi}. \tag{27}
\]

In our high friction regime, this reduces to the simple inequality

\[
\langle V(\phi) \rangle > 0, \tag{28}
\]

while in the minimal case, \( \frac{H^2}{M^2} \rightarrow 0 \), a more complicated inequality, \( \langle V(\phi) \rangle > \frac{2}{3} \langle \rho_{\phi} \rangle \), arises, which using \( \langle \phi^2 \rangle = \langle \phi V'(\phi) \rangle \), leads to [8]: \( \langle V(\phi) - \phi V'(\phi) \rangle > 0. \)

For simple power law potentials and in the rapid oscillation phase \( \gamma \) is a constant and consequently inflation does not cease without taking into the account another formalism such as particle production. Indeed if one considers interaction between the scalar field and other components such as radiation, the energy of the scalar field is released and, depending on the coupling, rapid oscillatory inflation may be promptly terminated in this situation. This possibility is discussed in [14], where inflation and reheating are studied in the framework of an effective action consisting of a Galileon scalar field.

Inflation ends also for more complicated potential such as the potential suggested by Damour-Mukhanov [8]

\[
V(\phi) = v \left( \left( \frac{\dot{\phi}^2}{\phi_{c}^2} + 1 \right)^{\frac{2}{q}} - d \right), \tag{29}
\]
where \( d \) is a positive real number, \( v > 0 \) and \( \phi_c \) are real parameters with dimension [mass]\(^4\) and [mass] respectively (note that for large \( \phi, \phi \gg \phi_c \), (29) reduces to a simple power law potential). To see this, we follow the same steps as \[15\]. By using

\[
\langle V(\phi) \rangle = \frac{\int_{-\phi}^{\phi} V(\phi) \, d\phi}{\int_{-\phi}^{\phi} d\phi},
\]

we find that the inflation continues as long as \( \int_{-\phi}^{\phi} V(\phi) \, d\phi > 0 \), which for the potential (29) gives

\[
\int_{-1}^{1} \left( (b^2 x^2 + 1) \frac{q}{2} - d \right) \, dx > 0.
\]

(31)

We have defined \( b = \frac{\phi}{\phi_c} \) and \( x = \frac{\phi}{\Phi} \). (31) results in that the inflation continues whenever

\[
d < g(b, q),
\]

where

\[
g(b, q) = \frac{G(b, q)}{2b \Gamma \left( \frac{2q+3}{2} \right) (1 + q) \Gamma \left( -\frac{q}{2} \right)}
\]

(33)

in which \( G(b, q) = -\pi^\frac{3}{2} (q+1) \sec \left( \frac{\pi q}{2} \right) + 2 \left( b^2 \right)^{\frac{1+q}{2}} 2F1(-\frac{q}{2}, -\frac{1+q}{2}, -\frac{1-q}{2}, -b^{-2}) \times \Gamma \left( -\frac{q}{2} \right) \Gamma \left( \frac{2+3q}{2} \right) \). \( 2F1 \) is the Gauss hypergeometric function. The inflation ceases at \( t_{end} \), i.e. when this inequality is violated such that \( d = g(b_{end}, q) \) and \( d > g(b(t > t_{end}), q) \). In fig. (3), \( g(b, q) \) in (33) is numerically depicted for \( q = 0.0392 \) (our reason for this choice will be revealed in the next section) in terms of \( b \), which shows that for \( d > 1 \), the inflation ends for some real value of \( b \). For \( d \simeq 1 \), the inflation ends for \( \Phi \sim \phi_c \).

We use the same definition for the number of e-folds, from a specific time \( t_* \) until the end of inflation, as \[9\]

\[
N = \ln \left( \frac{a_{end} H_{end}}{a_s H_s} \right).
\]

(34)

In this definition \( N \) is a measure of \( \ln(aH) \) increase during inflation. We have \( N = \ln \left( \frac{a_{end}}{a_s} \right) + \ln \left( \frac{H_{end}}{H_s} \right) \), and as \( H < 0 \), \( N \) is less than the more usual definition of e-folds number i.e. \( N = \ln \left( \frac{a_{end}}{a_s} \right) = (1 + \frac{q}{2})N \). If \( H \) changes insignificantly during inflation, like in the slow roll and in the de Sitter inflation, \( N \simeq \hat{N} \). By substituting (17) and (18) in (34) we arrive at

\[
N = \frac{q}{2} \left( \frac{2}{3\gamma} - 1 \right) \ln \left( \frac{\Phi_s}{\Phi_{end}} \right)
\]

(35)
Figure 3: $g(b, 0.0392)$ in the equation (33) in terms of $b$.

which in the high friction regime gives (see (12))

$$N = \ln \frac{\Phi_*}{\Phi_{end}},$$

while for the minimal case

$$N_{\text{min}} = \frac{1 - q}{3} \ln \frac{\Phi_*}{\Phi_{end}},$$

in agreement with [9]. By comparing these results, we deduce that with a same $\frac{\Phi_*}{\Phi_{end}}$ our model can provide more e-folds than the minimal case. Note that in an intermediate regime, where high friction condition does not hold, obtaining an analytical solution for $a$ and $H$ is not feasible, and we are unable to obtain a simple form for $N$.

Now let us specify a lower bound for efold number during inflation. Take $t_k$ as the time where a length scale $\lambda_k = \frac{a}{k}$, attributed to the wavenumber $k$, exited the Hubble radius during inflation:

$$k = \frac{1}{\lambda_k} = a(t_k)H(t_k),$$

where we have taken $a(t_0) = 1$ and $t_0$ denotes the present time. Large scale structure observations are limited to scales of about 1 Mpc (which we denote $\lambda_{\text{minimum}}$) to the present Hubble radius (denoted by $\lambda_{\text{max}}$). These observable scales crossed the Hubble radius during the following visible e-foiding (see (34))

$$N_{\text{vis}} = \ln \left( \frac{\lambda_{\text{max}}}{\lambda_{\text{minimum}}} \right) = \ln \left( \frac{H_0^{-1}}{1\text{Mpc}} \right).$$
By inserting $H_0 = 67.3 \text{ km/s/Mpc}^{-1}$ in (39), we obtain $N_{\text{vis}} = 8.4$. Hence all relevant scales exited the Hubble radius during 8.4 e-folding after $\frac{1}{H_0}$'s exit. Hence $N > 8.4$.

In the minimal case, from (37) and (23) we find

$$N_{\text{min}} < \frac{1 - q}{3} \ln \left( \sqrt{\frac{3}{8\pi}} \frac{q \Gamma \left( \frac{q+2}{2q} \right)}{\Gamma \left( \frac{1}{q} \right)} \frac{M_P}{\Phi_{\text{end}}} \right)$$

(40)

$\Phi_{\text{end}}$ depends on the chosen potential, e.g. for (29) with $d = 1$, $\Phi_{\text{end}} \sim \phi_c$ [8]. If we take $\Phi_{\text{end}}$ of the same order as the electroweak scale, $\Phi_{\text{end}} \sim 10^{-17} m_P$ (where $m_P$ is the Planck mass), for $q > 0$, we obtain $N_{\text{min}} < 11.3$. By increasing the scale of $\Phi_{\text{end}}$ this value decreases, for example for $\Phi_{\text{end}} \sim 10^{-6} m_P$, and $q > 0$, we obtain $N_{\text{min}} < 3.01$. Therefore in the minimal model $N_{\text{vis}} = 8.4$ may be consistent with rapid oscillating scalar phase and with the potential (29) provided that we assume the extreme case (i.e. $\Phi_{\text{end}}$ is reduced to the electroweak scale and $\Phi$ is augmented to the right hand side of (23)). In the nonminimal case, as the model is capable to provide more e-folds than the minimal situation, the theory may become more viable at least in the context of perturbations generation.

Following (38), a wavenumber had the possibility to exit the Hubble radius during rapid oscillation phase, provided that the condition $k \ll \frac{1}{T(t_k)}$ holds. To study this condition, we proceed as follows: The largest scale of our observable universe is of the same order of magnitude as $\lambda_{\text{max}} = \frac{1}{H_0}$. Using (21) and (23) one can find an upper bound for $T$ during rapid oscillation: $T(t) < T_u$. Hence $H_0 T_u \ll 1$ guarantees the compatibility of our assumptions with the horizon exit of $\lambda_{\text{max}}$ during rapid oscillation phase. This can be expressed as

$$H_0 \ll \frac{1}{\sqrt{3}} \left( \frac{\sqrt{8\pi} \Gamma \left( \frac{1}{q} \right)}{q \Gamma \left( \frac{2q+2}{2q} \right)} \right)^{-\frac{q+2}{2q}} \left( M_P^2 M_\lambda \right)^{\frac{q+2}{2q}}$$

(41)

If (41) holds and the model provided enough efolds ($N > N_{\text{vis}}$) after this exit, then we can claim that other large cosmological observable scales had also the possibility to exit the Hubble radius during this stage of inflation. In the next part we will study perturbations generation and, based on astrophysical data, find an estimation for parameters of our model as well as for $N$.

3 Cosmological perturbations

To study the scalar and the tensor fluctuations we decouple the spacetime into two components, the background and the perturbation. The back-
ground is described by the homogeneous and isotropic FLRW metric corresponding to the oscillatory inflation in the context of non minimal derivative coupling model studied in the previous section. To study quantum perturbations in rapid oscillation stage we have to use the Mukhanov-Sasaki equation. Mukhanov-Sasaki equation for scalar and tensor perturbations in non-minimal derivative coupling model was obtained in [16]

\[
\frac{d^2 v_{(s,t)k}}{dt^2} + \left( c_{s,t,k}^2 k^2 - \frac{1}{z_{s,t}} \frac{d^2 z_{s,t}}{dt^2} \right) v_{(s,t)k} = 0. \tag{42}
\]

c_s and c_t are the sound speed for the scalar and the tensor mode respectively and k is wave number for mode function v_k. The conformal time η is defined by

\[
η(t) = \int^t dt' \frac{a(t')}{a(t')}, \tag{43}
\]

and, z_s and z_t are given by

\[
z_s = a(t) \frac{M_p \Gamma}{H} \sqrt{2 \Sigma} \quad z_t = a(t) M_p \sqrt{\frac{\epsilon^\lambda_{ij} \epsilon'^\lambda_{ij}}{2}} \sqrt{1 - \alpha}. \tag{44}
\]

The polarization tensor is normalized to \( \epsilon^\lambda_{ij} \epsilon'^\lambda_{ij} = 2 \delta_{\lambda \lambda'} \). \( \Gamma \) and \( \Sigma \) are defined as

\[
\Gamma = \frac{1 - \alpha}{1 - 3\alpha} \quad \Sigma = M^2 \alpha \left[ 1 + \frac{3H^2}{M^2} \left( \frac{1 + 3\alpha}{1 - \alpha} \right) \right]. \tag{45}
\]

In the above, \( \alpha = \frac{\dot{\phi}^2}{2M^2 M_p^2} \) and \( c_{s,t} \) is given by relation

\[
c_s^2 = \frac{H^2}{1 - \alpha} \quad c_t^2 = \frac{1 + \alpha}{1 - \alpha}, \tag{46}
\]

where \( \epsilon_s \) is

\[
\epsilon_s = \frac{1}{a(t)} \frac{\dot{a}(t) \Gamma}{H} \left( \frac{1 - \alpha}{1 - 3\alpha} \right) - (1 + \alpha). \tag{47}
\]

The equation (42) was studied in the slow roll approximation (\( \alpha \approx 0 \)) for quasi de-Sitter background in [16]. From (26) we find that \( \alpha \) is nearly constant

\[
\alpha \approx \frac{\gamma}{2} = \frac{q}{3q + 6}. \tag{48}
\]

By using (47) and the Raychaudhuri equation,

\[
- \frac{\dot{H}}{H^2} (1 - \alpha) = \frac{M^2}{H^2} \alpha + 3\alpha - \frac{\dot{\alpha}}{\dot{H}}, \tag{49}
\]

we find that \( \epsilon_s \) in high friction limit becomes

\[
\epsilon_s = -6\alpha \left( \frac{1 - \alpha}{1 - 3\alpha} \right) \left( 1 - \frac{\dot{H}}{H^2} \right) + \alpha \left( -\frac{15\alpha^2 - 2\alpha + 9}{(1 - 3\alpha)^2} \right). \tag{50}
\]
In the rapid oscillation stage $a(t)$ is a power law function of time (see (17)), therefore $\epsilon = \frac{\dot{H}}{H^2} \approx \frac{q}{q+2}$, which shows that $\varepsilon_s$ is approximately a constant.

Using (45) and (46), one can show that $c_s$ becomes

$$c_s^2 \approx \frac{(1-3\alpha)^2}{3\alpha(1-\alpha)(1+3\alpha)} \varepsilon_s.$$  \hspace{1cm} (51)

Therefore, $c_s$, is approximately a constant too. We can calculate $c_s$ and $c_t$ as functions of $q$

$$c_s^2 \approx \frac{q^3 + 8q^2 + 19q + 18}{3(q+1)(q+2)(q+3)} \quad \quad c_t^2 = \frac{2q + 3}{q + 3}.$$ \hspace{1cm} (52)

In fig. (4) $c_s$ is plotted with respect to $q$. This figure shows that sound speed for power law potentials is restricted to the range $0 < c_s < 1$. $z$ in the rapid oscillation era is

$$z_s = a(t)M_p\left(\frac{1-\alpha}{1-3\alpha}\right)\sqrt{6\alpha\left(1+3\alpha\right)}.$$ \hspace{1cm} (53)

We have $a(\eta) \propto \eta^{-(\frac{q+2}{2})}$, thus we can write $z$ in the form

$$z_{s,t} = \beta_{s,t}a(\eta),$$ \hspace{1cm} (54)

where

$$\beta_s \approx M_p\left(\frac{q + 3}{3}\right)\sqrt{\frac{2q(q+1)}{q+2}} \quad \beta_t = M_p\sqrt{\frac{\epsilon_i^j\epsilon_j^i}{2}}\sqrt{\frac{2q + 6}{3q + 6}}.$$ \hspace{1cm} (55)

So the conformal time derivative of $z$ is given by

$$\frac{1}{z_{s,t}} \frac{d^2z_{s,t}}{d\eta^2} = \left(\frac{q}{2} + 1\right)(\frac{q}{2} + 2)\eta^{-2}.$$ \hspace{1cm} (56)
Hence the mode function satisfies
\[
\frac{d^2 v(s,t)k}{d\eta^2} + \left(c_{s,t}k^2 - \left(\frac{q}{2} + 1\right)\left(\frac{q}{2} + 2\right)\eta^{-2}\right)v(s,t)k = 0,
\]
whose solution is
\[
v(s,t)k(\eta) = |\eta|^{\frac{1}{2}}[C^{(1)}(k)H^{(1)}(c_{s,t}k|\eta|) + C^{(2)}(k)H^{(2)}(c_{s,t}k|\eta|)].
\]

\(C^{(1)}(k)\) and \(C^{(2)}(k)\) are the constants of integration and \(H^{(1)}\) and \(H^{(2)}\) are Hankel functions of the first and second kind of order \(\nu = \frac{3}{2} + q\) respectively.

We adopt the Bunch-Davies vacuum by imposing the condition that the mode function approaches the vacuum of the Minkowski spacetime in the short wavelength limit \(a_k \ll 1\), where the mode is well within the horizon. In the rapid oscillation epoch we have \(aH \propto \frac{1}{|\eta|}\) resulting \(k|\eta| \gg 1\). In this limit the Bunch-Davies mode function is given by \(v_k(\eta) \approx \frac{1}{\sqrt{2c_{s,t}k}} e^{-ic_{s,t}k}\eta\).

This must be the asymptotic form of (58), therefore
\[
v(s,t)k(\eta) = \frac{\sqrt{\pi}}{2} e^{i(\nu + \frac{1}{2})\pi / 2} (-\eta)^{\frac{\nu}{2}} H^{(1)}(-c_{s,t}k\eta).
\]

In the limit \(\frac{k}{aH} \rightarrow 0\) the asymptotic form of mode function (59) is given by
\[
v(s,t)k(\eta) \rightarrow e^{i(\nu + \frac{1}{2})\pi / 2} \Gamma(\nu) \frac{1}{\Gamma(\nu)} (-c_{s,t}k\eta)^{-\nu + \frac{1}{2}}.
\]

To obtain the power spectrum for scalar (tensor) perturbation we follow the steps of \([17]\) and substitute (60) in
\[
P_{s,t}(k) \frac{1}{2} = \sqrt{\frac{k^3}{2\pi^2}} \left|\frac{v(s,t)k}{z_{s,t}}\right|,
\]
which yields
\[
P_{s,t}(k) \frac{1}{2} = \sqrt{\frac{k^3}{2\pi^2}} \frac{2^{(\nu - \frac{3}{2})}}{\beta_{s,t}a} \sqrt{2c_{s,t}k} (-c_{s,t}k\eta)^{-(\nu + \frac{1}{2})}.
\]

We rewrite the conformal time as
\[
\eta = \int \frac{dt}{a(t)} = \int \frac{da}{a^2 H} = -\frac{1}{aH} + \int \epsilon da / a^2 H,
\]
but in the rapid oscillation epoch \(\epsilon\) is a constant, therefore
\[
\eta = -\frac{1}{aH} \frac{1}{1 - \epsilon}.
\]
By substituting (64) into the equation (62), we arrive at

\[ P_{s,t}(k) = \frac{2^{(\nu - \frac{5}{2})}}{\pi} \frac{\Gamma(\nu)}{\Gamma(\frac{3}{2})} \frac{k}{c_{s,t}} \beta_{s,t} \alpha^{\nu - \frac{5}{2}} \frac{e^{\nu} k}{aH} \left( 1 - \epsilon \right)^{\nu - \frac{5}{2}} \].

(65)

At the horizon crossing \( c_{s} k = aH \),

\[ P_{s,t}(k) = \frac{2^{(\nu - \frac{5}{2})}}{\pi} \frac{\Gamma(\nu)}{\Gamma(\frac{3}{2})} \frac{H}{c_{s,t} \beta_{s,t}} (1 - \epsilon)^{\nu - \frac{5}{2}}. \]

(66)

The above relation may be written as

\[ P_{s,t}(k) = A_{s,t}(q) \frac{H}{M_{P}} |_{c_{s,t} k = aH}, \]

(67)

where

\[ A_{s}(q) = \frac{3^{\frac{7}{2}} 2^{q - \frac{7}{2}} \Gamma(\frac{3}{2} + \frac{q}{2}) (q + 2)^{(-q + 1)} / (q + 1)}{\pi \Gamma(\frac{3}{2}) \sqrt{q(q + 1)(q + 3)}} \] \( \frac{q^{3} + 8q^{2} + 19q + 18}{(q + 1)(q + 2)(q + 3)} \) \].

(68)

and

\[ A_{t}(q) = \frac{3^{\frac{7}{2}} 2^{q - \frac{7}{2}} \Gamma(\frac{3}{2} + \frac{q}{2}) (q + 2)^{(-q + 1)} / (q + 1)}{\pi \Gamma(\frac{3}{2}) (q + 3)^{-\frac{1}{2}} (2q + 3)^{\frac{1}{2}}}. \]

(69)

The ratio of the tensor to scalar spectrum is given by

\[ r = \frac{P_{t}}{P_{s}} = (\frac{A_{t}}{A_{s}})^{2} = \frac{\sqrt{3}q(q + 1)(q + 3)^{\frac{3}{2}} \left( \frac{q^{3} + 8q^{2} + 19q + 18}{(q + 1)(q + 2)(q + 3)} \right)^{\frac{3}{2}}}{27(2q + 3)^{3}}. \]

(70)

Now we can calculate the spectral index by differentiating the power spectrum with respect to \( k \) at horizon crossing

\[ n_{s} - 1 = \frac{d \ln P_{s}}{d \ln k} |_{c_{s,t} k = aH}. \]

(71)

At the horizon crossing, we have \( \frac{d \ln k}{dt} = H(1 - \epsilon) \), so

\[ n_{s} - 1 = \frac{d \ln H^{2}}{d \ln k} = -2 \epsilon \frac{1}{1 - \epsilon} = -q. \]

(72)

Now, equipped with these results, we are capable to use astrophysical data to fix the parameter \( q \). For the pivot mode \( k_{s} = 0.05 Mpc^{-1} \), the power spectrum and the spectral index are determined from Planck 2013 data as (for \( 68\% CL \), or 1\( \sigma \) error)[10]

\[ P_{s} = (2.200 \pm 0.056) \times 10^{-9} \]

\[ n_{s} = 0.9608 \pm 0.0054 \]

(73)
Therefore (72) leads to
\[ q = 0.0392 \pm 0.0054. \]  
(74)

For \( q = 0.0392 \), the tensor scalar ratio is
\[ r \approx 0.0387, \]  
(75)

which is in agreement with Planck data which put an upper bound on \( r \), \( r < 0.11(95\% CL) \) [10].

Now we are able to determine the range of the parameters required for validity of the high friction and rapid oscillation assumptions. \( A_s(q = 0.0392) = 0.7993 \) and (67) give the energy density of the scalar field at horizon crossing as:
\[ \rho_* \approx 1.032 \times 10^{-8} M_p^4 \simeq 36.28 \times (10^{16} \text{GeV})^4, \]  
(76)

which is compatible with the fact that our model does not enter in the quantum gravity regime. Using (74), the rapid oscillation condition (22) reduces to
\[ \Phi_* \ll 0.0393 \frac{M_p^2 M}{\sqrt{\lambda}}. \]  
(77)

Where \( \Phi_* \) is the scalar field amplitude at the horizon crossing. In high friction regime one has \( \frac{H^2}{M^2} \gg 1 \), and in the rapid oscillation stage \( \lambda \Phi_*^{0.0392} = \rho_* = 3M_p^2 H_*^2 \) holds. By collecting all these results together we find
\[ \Phi_* \ll 386.8 M \]  
(78)

\[ M^2 \ll 0.344 \times 10^{-8} M_p^2, \]

and
\[ \tilde{\lambda} \frac{1}{0.0392} \tilde{M} \gg \left( \frac{1.032 \times 10^{-8}}{386.8} \right) \frac{1}{0.0392} \]  
(79)

where the dimensionless parameters \( \tilde{M} \) and \( \tilde{\lambda} \) are defined through \( \lambda = \tilde{\lambda} M_p^{1/4} \), and \( M = \tilde{M} M_p \). By inserting (74) and \( H_0 = 67.3 \text{km/s Mpc}^{-1} \) [10] in (11), we derive
\[ \tilde{\lambda} \frac{1}{0.0392} \tilde{M} \frac{386.8}{0.0392} \gg 1.102 \times 10^{-60}. \]  
(80)

Note that there is an interval of 5.4 e-folds between the exits of \( H_0 \) and \( k_s = 0.05 \text{Mpc}^{-1} \), from the Hubble radius: \( \ln \left( \frac{k_s}{H_0} \right) \simeq 5.4 \). In our computations we have assumed that the high friction condition is still valid until the end of rapid oscillatory inflation, hence \( M^2 \ll H^2_{\text{end}} \), which puts a stronger constraint on \( M \)
\[ M^2 \ll 0.344 \times 10^{-8} e^{-0.0392 N} M_p^2. \]  
(81)
To derive (81),
\[ \mathcal{N} = \ln \left( \frac{\Phi_*}{\Phi_{end}} \right) = \frac{1}{q} \ln \left( \frac{\rho_*}{\rho_{end}} \right) \]  
was used. Note that \( N \simeq 1.02N \), where \( N = \ln \left( \frac{\Phi_{end}}{\Phi_*} \right) \). The smallness of \( q \) gives us the option to choose \( M \) (in (81)) such that \( \Lambda = (M^2M_P)^{1/3} \) and consequently \( \Lambda = (H^2M_P)^{1/3} \) (the cut-off scale during inflation [3],[18]) become much larger than the TeV scale.

The number of e-folds from the horizon crossing (of the pivot mode \( k_* \)), until the end of inflation in the rapid oscillation stage can be determined from (36). Due to the smallness of \( q \), we may have \( \Phi_{end} \gg 1 \), while \( H_* \) and \( H_{end} \) have the same order of magnitude. For example for \( N = 60 \), we have \( \frac{\Phi_{end}}{\Phi_*} = e^{60} \), while \( H_* = 3.2H_{end} \). This looks like the slow roll situation where \( H \) decreases very slowly during inflation. The ratio \( \frac{\Phi_{end}}{\Phi_*} \) cannot be fixed via our derived relations. A formal upper bound for \( N \) may be extracted from (78), \( N < \ln \left( \frac{386.8M}{\Phi_{end}} \right) \). The rapid oscillation condition puts an upper bound on the scalar field amplitude (see (22)). Therefore this condition cannot be violated during the expansion (in contrast to the slow roll conditions), hence the end of inflation and \( \Phi_{end} \) may not be determined in terms of the actual parameters of our scalar field model with a power law potential as may be usually done in the slow roll models. Besides, to minimize the uncertainties in the evaluation of \( N \), one needs to study the evolution of the universe after the inflation specially the reheating era. If we consider a prompt reheating, then the energy scale at the end of inflation may be approximated as the reheating temperature which must be less the GUT scale. In this situation by taking \( \rho_{end} = (10^{16} GeV)^4 \), we obtain a lower bound for e-folds number as \( N > 91 \). \( N \) reduces by adopting larger values for \( \rho_{end} \). For example by setting \( \{ \lambda = 1.76 \times 10^{-8}M_P^40.0035, M = 10^{-6}M_P, \Phi_{end} = 10^{-32}M_P \} \), which lie on the allowed domain for the rapid oscillation in high friction regime, one obtains \( \rho_{end} = 3.45 \times (10^{16} GeV)^4 = 0.0081 \times 10^{-8}M_P^4 \) and \( N = 60 \). In fig. (5), \( N \) is depicted in terms of \( x := \frac{\rho_{end}}{10^{-8}M_P^4} \). At the end let us note that if like [9], one takes \( \Phi_{end} \sim 5 \times 10^{-17}M_P \), and consider an extreme value for \( \Phi_* \), i.e. \( \Phi_* = 0.023M_P \) (derived from (78)), the number of e-folds from \( t_* \) (time of horizon crossing) to \( t_{end} \) becomes \( \mathcal{N} = \ln \left( \frac{0.023M_P}{5 \times 10^{-17}M_P} \right) = 35.4 \), which is approximately three times the number obtained in [9] and [8], as is expected.

4 conclusion

Inflation driven by an oscillating scalar field with a power law potential, \( V(\phi) \propto \phi^q \), in the context of non minimal derivative coupling model was studied. In high friction regime, conditions required for such evolution were
discussed. It was shown that, in contrast to the minimal case, \( q \) is not restricted to a tighten limited range. The number of e-folds, from a specific time (horizon crossing of a pivot scale) in inflationary era until the end of inflation, was discussed. Our results indicate that more e-folds can be produced with respect to the minimal case, giving the opportunity to observable cosmological scales to exit the Hubble radius during inflation. Also, the conditions required for the end of inflation were discussed.

We considered cosmological perturbations originated in this era and computed the power spectrum, the scalar spectral index, and the tensor to scalar ratio. By confronting our results with the Planck 2013 data, we specified the range of the model parameters and investigated the consistency of our results.

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Figure 5: \( N \) in terms of \( x := \frac{\rho_{\text{end}}}{10^{-8}M_P^2} \).
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