RECONSTRUCTING SIMULATED CMB POLARIZATION POWER SPECTRA WITH THE ANALYTICAL BLIND SEPARATION METHOD

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Abstract

The Analytical Blind Separation (ABS) method is dedicated to extract the CMB signal from its contaminants without any prior information about foregrounds. In this study, we apply the ABS method to simulated polarization sky maps in order to test its ability in the recovery of the CMB E- and B-mode power spectra. We simulate multi-frequency Q and U microwave sky maps in the presence of two polarized foreground components (synchrotron and thermal dust), together with an uncorrelated Gaussian distributed instrumental noise, measured with a hypothetical PICO-like experiment operating in 10-frequency bands in the range of 30 to 321 GHz. Both full and partial sky surveys are taken into account in our analysis. For the full sky case with the tensor-to-scalar ratios of $r = 0$ and $r = 0.05$, the ABS can accurately recover the E-mode and B-mode power spectra with a relative error below 20% with respect to the input CMB power spectra for the full multipole range ($2 \leq \ell \leq 1050$), close to theoretical expectations. For the partial sky analysis with a E/B separation proposed by Smith & Zaldarriaga (2007) to avoid “E-to-B” leakage, the results are comparable to the full sky case. We find that the ABS is able to estimate both CMB E-mode and B-mode power spectra within 1σ at all scales for full sky, and for most of the scales ($50 \leq \ell \leq 1050$) for partial sky. For low-$\ell$ multipoles ($\ell \leq 50$), a noticeable divergence occurs in the partial sky analysis, which might be due to uncertainties originated from both the ABS and the E/B separation. A more detailed analysis for the partial sky case is thus needed for estimating primordial B-modes unbiasedly and accurately.

Subject headings: Cosmology: Cosmic Microwave Background, techniques: image processing, method: data analysis

1. INTRODUCTION

Cosmic Microwave Background (CMB) experiments detect a mixture of different sky microwave emissions in which the CMB signal is embedded in. Multi-frequency coverage is needed for reliably separating the CMB signal from foreground contamination. The presence of instrumental noise also hampers the separation of the CMB. The separation of these different astrophysical foregrounds and instrumental noise from the CMB signal is a fundamental task to be performed in order to accurately extract the CMB power spectra, enabling the estimation of the cosmological parameters with suitable precision. In the past decades, ground-based and balloon-borne CMB experiments, like, for instance, Boomerang (de Bernardis et al. 2000), MAXIMA (Halverson et al. 2002), DASI (Halverson et al. 2002), VSA (Watson et al. 2003), CBI (Mason et al. 2003), ACBAR (Kuo et al. 2004), BEAST (Meinhold et al. 2005), ODwyer et al. (2005), SPT (Sievers et al. 2013), ACT (Das et al. 2014), POLARBEAR (Polarbear Collaboration et al. 2014), SPT-pol (Keisler et al. 2015), as well as satellite-based experiments, like Relikt 1 (Strukov et. al 1992), COBE (Smoot et. al 1992), Mather et al. (1994), WMAP (Bennett et al. 2013), Hinshaw et al. (2013) and Planck satellites (Planck Collaboration et al. 2014), focused on measuring both the CMB spectrum and angular distribution. In particular, recent results from the Planck satellite also show a precise measurement of the CMB E-mode polarization field (Planck collaboration et al. 2016c), which, just like the CMB temperature field, is generated by the scalar primordial density perturbations.

As a consequence of these experiments, large amounts of high quality data are now available, allowing tight constraints on the determination of cosmological parameters. On the other hand, the CMB field is not only characterized by temperature fluctuations, but also by
polarized anisotropies, since a small fraction of the CMB photons is linearly polarized. They convey important information that allows the disentanglement of cosmological models. Therefore, besides the temperature power spectrum, measuring the CMB polarization signal is a requirement to fully understand the physics encoded in the CMB field. The $B$-mode polarization field, for instance, encodes different cosmological information, particularly on large scales, since it arises from tensor perturbations due to the primordial gravitational waves (Zaldarriaga & Seljak 1997; Kamionkowski et al. 1997a; Hu & Dodelson 2002), which can probe inflation (Grishchuk 1975, 1976, 1977; Starobinsky 1979, 1980; Lyth & Riotto 1999).

Thus, CMB polarization is a new frontier of CMB observations. Therefore, many experiments targeting to measure the CMB polarization field have been or are being developed, such as WMPol (Levy et al. 2008), QUBIC (Battistelli et al. 2011), BICEP3 (Kang J. H. et al. 2018), Ali-CPT (Li et al. 2017), CLASS (Essinger-Hoxmeier T. et al.), Simons Observatory (Ade et al. 2019), CMB-S4 (Abazajian et al. 2016), CORE (Delabrouille et al. 2017), LiteBIRD (Matsumura et al. 2014), EPIC/CMPol (Bock et al. 2009), PIXIE (Kogut et al. 2011), PRISM (André et al. 2014), PICO (Hanany et al. 2019).

Unfortunately, foregrounds pose strong limitations for experiments that aim to measure CMB polarization. For this reason, besides designing experiments to avoid these signals as much as possible, there is a need to build tools to separate the CMB signal from its contaminants in a reliable way. On small scales, CMB $B$-mode signal is dominated by the lensed $B$-mode polarization produced by the conversion of the $E$-mode polarization signal by the gravitational lensing of the CMB photons during their travel from the last scattering surface to observers (Zaldarriaga & Seljak 1998; Hu 2002; Lewis & Challinor 2006).

In particular, $B$-mode measurements, both cosmological or due to the CMB lensing, are limited by several polarized foregrounds (Planck Collaboration et al. 2016a; Planck collaboration et al. 2016b; Planck Collaboration et al. 2018), mainly synchrotron and dust emissions. Moreover, other contamination effects must be taken into account, including the so-called $E$-$B$ mixing, which arises from $E$-$B$ decomposition on an incomplete sky coverage (Tegmark, & de Oliveira-Costa 2001) and instrumental noise. The measurement and accuracy of the $B$-mode power spectrum are thus dependent on both our ability to decompose the CMB polarization signal with a partial sky coverage and the efficiency of the component separation methods.

Various methods have been proposed in the literature to extract the emission of foreground astrophysical components out of multi-frequency CMB observations. Some of these methods require a prior knowledge of the components frequency dependence, being thus referred to as “non-blind” methods of component separation, such as the Wiener Filtering (WF; Bunn et al. 1994; Tegmark & Efstathiou 1996; Bouchet et al. 1999), the Maximum Entropy Method (MEM; Hobson et al. 1998), and the Gibbs sampling approach that has been implemented by Jewell et al. (2004); Wandelt et al. (2004); Eriksen et al. (2004); Larson et al. (2007); Planck Collaboration et al. (2016a). The main problem with these “non-blind” methods is precisely due to our poor knowledge about the frequency dependence of foregrounds. Therefore, to avoid residual contamination in a cleaned CMB map, several “blind” approaches have been proposed, such as the methods using Independent Component Analysis (ICA; Baccigalupi et al. 2004) and Correlated Component Analysis (CCA; Bonaldi et al. 2004). The Internal Linear Combination (ILC) approach has been extensively addressed by both WMAP and Planck surveys to obtain a foreground-cleaned map (Tegmark & Efstathiou 1996; Tegmark et al. 2002; Bennett et al. 2003; Saha et al. 2006; Delabrouille et al. 2009; Remazeilles et al. 2011; Leach et al. 2008; Fernández-Cobos et al. 2012). Moreover, important foreground template matching approaches include the Spectral Estimation via Expectation Maximisation (SEVEM) (Martínez-González et al. 2003; Leach et al. 2008; Fernández-Cobos et al. 2012) and the Spectral Matching Independent Component Analysis (SMICA) (Delabrouille et al. 2003; Martínez-González et al. 2003; Aumont & Macías-Pérez 2007; Cardoso et al. 2008), both of which are effectively incorporated in Planck data analysis pipeline.

Recently, an Analytical method of Blind Separation (ABS) of the CMB signal from foregrounds is proposed by Zhang et al. (2016). The ABS method, contrary to most other approaches, does not involve heavy computations. It is a pure “blind” approach in the sense that it does not rely on any assumption about the characteristics of the foreground components. It simply relies on the measured cross-band power between different frequency bands from which the CMB power spectra can be solved analytically, avoiding multiple parameter fitting procedure. On previous works (Zhang et al. 2016; Yao et al. 2018), a complete description of the mathematical formalism and numerical techniques applied in our analysis are provided and implemented. For instance, the ABS method has already been successfully tested against simulated temperature Planck maps (Yao et al. 2018). As an extension of this work, here, we test the ABS method on simulated CMB polarization maps, targeting the reconstruction of the $E$- and $B$-mode power spectra. In particular, we are interested in CMB $B$-mode power spectrum obtained from measurements from partial sky surveys, as it will be the case of most future experiments.

We use thermodynamic units throughout this paper, corresponding to a constant CMB power spectrum across frequencies.

2. CMB $E/B$ Decomposition

It is well known that the CMB linear polarization field, described by the Stokes parameters $Q(\hat{n})$ and $U(\hat{n})$, can be decomposed into the rotationally invariant $E$-mode and $B$-mode components (Seljak & Zaldarriaga 1997; Zaldarriaga & Seljak 1997; Kamionkowski et al. 1997a,b). It is also known that these two parameters can completely describe the linearly polarized CMB field, $P_\pm(\hat{n}) = Q(\hat{n}) \pm iU(\hat{n})$, which are spin-2 quantities. In the ideal case of full-sky coverage, these fields are easily expanded over spin-weighted harmonic functions basis (Seljak & Zaldarriaga 1997):

$$P_\pm(\hat{n}) = \sum_{\ell m} a_{\pm 2, \ell m} \pm 2 Y_{\ell m}(\hat{n}).$$

(1)
However, since the Stokes parameters description of the polarization field is frame-dependent, one can decompose the linear polarization into rotation-invariant $E$ and $B$ components for convenience, which are defined in the harmonic space by the coefficients $a_{\pm2,\ell m}$ in terms of

\[
E_{\ell m} = -\frac{1}{2} [a_{2,\ell m} + a_{-2,\ell m}],
\]

\[
B_{\ell m} = -\frac{1}{2i} [a_{2,\ell m} - a_{-2,\ell m}].
\]

Just as the temperature map can be expanded in terms of spherical harmonics, one can expand $E$- and $B$-fields through

\[
E(\hat{n}) = \sum_{\ell m} E_{\ell m} Y_{\ell m}(\hat{n}),
\]

\[
B(\hat{n}) = \sum_{\ell m} B_{\ell m} Y_{\ell m}(\hat{n}).
\]

Finally, the power spectra are straightforward calculated as

\[
C^{EE}_\ell = \frac{1}{2\ell + 1} \sum_m \langle E_{\ell m} E^*_{\ell m} \rangle,
\]

\[
C^{BB}_\ell = \frac{1}{2\ell + 1} \sum_m \langle B_{\ell m} B^*_{\ell m} \rangle,
\]

where the brackets denote the ensemble average over all realizations.

If the polarization field is not measured over the full sky, then the decomposition into $E$ and $B$ modes is not unique, since there are modes that satisfy the properties of both the $E$ and $B$ modes simultaneously (Tegmark, & de Oliveira-Costa 2001). One has to remove the resulting $E$-$B$ leakage to correctly estimate the $E$- and $B$-mode power spectra (Bunn et al. 2003; Bunn 2011; Lewis 2000; Caò & Pang 2011; Louis et al. 2013; Grant et al. 2009; Smith 2006; Smith & Zaldarriaga 2007; Zhao & Baskaran 2010; Kim & Næsset 2010; Liu et al. 2018; Liu et al. 2019; Liu 2019).

Even though various methods for eliminating $E$-$B$ mixing have been proposed, residuals are still present in the maps and should be carefully taken into account (Santos et al. 2017).

For the purpose of dealing with the leakage in a partial sky analysis, we define a new set of fields $\mathcal{E}$ and $B$ (Zaldarriaga & Seljak 1997):

\[
\mathcal{E}(\hat{n}) = -\frac{1}{2} \left[ \bar{\partial} \partial \mathcal{P}_+(\hat{n}) + \partial \bar{\partial} \mathcal{P}_-(\hat{n}) \right],
\]

\[
B(\hat{n}) = -\frac{1}{2i} \left[ \bar{\partial} \partial \mathcal{P}_+(\hat{n}) - \partial \bar{\partial} \mathcal{P}_-(\hat{n}) \right],
\]

where $\bar{\partial}$($\partial$) corresponds to the spin-raising (lowering) operator for an arbitrary function $f(\hat{n})$ with spin $s$,

\[
\bar{\partial}_s f(\hat{n}) \equiv -\sin^s \theta \left( \frac{\partial}{\partial \theta} + \frac{i}{\sin \theta} \frac{\partial}{\partial \phi} \right) \sin^{-s} \theta f(\hat{n}),
\]

\[
\partial_s f(\hat{n}) \equiv -\sin^{-s} \theta \left( \frac{\partial}{\partial \theta} - \frac{i}{\sin \theta} \frac{\partial}{\partial \phi} \right) \sin^s \theta f(\hat{n}).
\]

Expanding the $\mathcal{E}(\hat{n})$ and $B(\hat{n})$ components in spherical harmonics, we obtain:

\[
\mathcal{E}_{\ell m} \equiv \sum_{\ell m} \mathcal{E}_{\ell m} Y_{\ell m}(\hat{n}), \quad B_{\ell m} \equiv \sum_{\ell m} B_{\ell m} Y_{\ell m}(\hat{n}),
\]

where the $\mathcal{E}_{\ell m}$ and $B_{\ell m}$ coefficients can then be written as:

\[
\mathcal{E}_{\ell m} = \int \mathcal{E}(\hat{n}) Y^*_{\ell m}(\hat{n}) d\hat{n},
\]

\[
B_{\ell m} = \int B(\hat{n}) Y^*_{\ell m}(\hat{n}) d\hat{n}.
\]

These new coefficients are related to the original $E_{\ell m}$ and $B_{\ell m}$ by the simple relations (Zaldarriaga & Seljak 1997):

\[
\mathcal{E}_{\ell m} = N_{\ell,2} E_{\ell m}, \quad B_{\ell m} = N_{\ell,2} B_{\ell m},
\]

where $N_{\ell,2} = \sqrt{(\ell + s)/(\ell - s)}$.

The corresponding power spectra therefore become:

\[
C^{\mathcal{E}\mathcal{E}}_\ell = \frac{1}{2\ell + 1} \sum_m \langle \mathcal{E}_{\ell m} \mathcal{E}^*_{\ell m} \rangle = N_{\ell,2}^2 C^{EE}_\ell,
\]

\[
C^{BB}_\ell = \frac{1}{2\ell + 1} \sum_m \langle B_{\ell m} B^*_{\ell m} \rangle = N_{\ell,2}^2 C^{BB}_\ell.
\]

Now, we introduce a window function, $W(\hat{n})$, to define the masked $E$- and $B$-mode maps, $\mathcal{E}W(\hat{n}), BW(\hat{n})$, with pseudo multipoles coefficients defined by Elstathion (2004):

\[
\hat{\mathcal{E}}_{\ell m} = \int d\hat{n} W(\hat{n}) \mathcal{E}(\hat{n}) Y^*_{\ell m}(\hat{n}),
\]

\[
\hat{B}_{\ell m} = \int d\hat{n} W(\hat{n}) B(\hat{n}) Y^*_{\ell m}(\hat{n}).
\]

For this paper, we will use the method (Smith 2006; Smith & Zaldarriaga 2007) to avoid the “$E$-to-$B$” leakage (hereafter SZ method). Following Smith (2006); Smith & Zaldarriaga (2007), for the $B$-modes, the pure-pseudo multipoles coefficients can be rewritten as:

\[
\hat{B}_{\ell m} = -\frac{1}{2i} \int d\hat{n} P_+(\hat{n}) \left[ \bar{\partial} \partial (W(\hat{n}) Y_{\ell m}(\hat{n})) \right]^* - P_-(\hat{n}) \left[ \partial \bar{\partial} (W(\hat{n}) Y_{\ell m}(\hat{n})) \right]^*.
\]
The $E$-mode case is calculated in an analogous way. In order to avoid numerical divergences in calculating the derivatives of the window function, we must use a proper sky apodization. The Gaussian smoothing method was shown to induce the smallest leakage in the final $B$-map, being our apodization choice throughout this paper (see Wang et al. 2016 & Kim 2011 for detailed information).

We then use the UPB77 Planck mask with the Gaussian apodization as shown in Fig. 1. Finally, in order to estimate the pseudo power spectrum, we recall Eqs. 38, 39 and 42 of Zhao & Baskaran 2010.

![Gaussian smoothed UPB77 mask](image_url)

**Fig. 1.** The Gaussian smoothed UPB77 mask [Wang et al. 2016] [Kim 2011].

### 3. THE ABS Method

The ABS method provides an analytical and blind way to recover the CMB power spectra from contaminated multi-frequency maps, relying on the measured cross-band power between different frequency bands. Data contained in the multipole bin $\ell$ can be written as:

$$D_{ij}^{\text{obs}}(\ell) = f_i f_j D_{ij}^{\text{cmb}}(\ell) + D_{ij}^{\text{fore}}(\ell) + \delta D_{ij}^{\text{noise}}(\ell).$$

For the cases treated in this paper, $D_{ij}^{\text{obs}}(\ell)$ represents the cross-band power spectra of the decomposed CMB polarized components at the $i$- and $j$-th frequency channels, where $i, j = 1, 2 \cdots N_f$ and $N_f$ is the total number of frequency channels. It is clear from (21) that we have three main contributions to the signal: $D_{ij}^{\text{fore}}$ is the cross band power matrix of the foregrounds, $D_{ij}^{\text{cmb}}$ represents the CMB signal (the same in every frequency band), and $D_{ij}^{\text{noise}}(\ell)$ is the noise contribution to the total “E”- and “B”-mode power spectra. Here, $f_i = 1$ for all channels, in units of thermodynamic temperature.

In order to elucidate the ABS method, let us begin with the simple case of no instrumental noise, in which the CMB power spectrum can be analytically derived as (Zhang et al. 2016):

$$D_{ij}^{\text{cmb}} = \left( \sum_{\mu=1}^{M+1} G_{\mu}^2 \lambda_{\mu}^{-1} \right)^{-1},$$

for $M < N_f$, where $M \equiv \text{rank}(D_{ij}^{\text{fore}})$, which depends on the number of independent foreground components.

The order of $D_{ij}^{\text{obs}}(\ell)$ is $N_f$. In the above equation, $G_{\mu} = \mathbf{f} \cdot \mathbf{E}^\mu$, with the vector $\mathbf{f} = (f_1, \ldots, f_N)^T$. The $\mu$-th eigenvector and the associated eigenvalue of $D_{ij}^{\text{obs}}(\ell)$ are represented by $\mathbf{E}^\mu$ and $\lambda_{\mu}$, respectively. The eigenvectors are normalized as $\mathbf{E}^\mu \cdot \mathbf{E}^\nu = \delta_{\mu\nu}$.

However, in a more realistic scenario in the presence of instrumental noise, by modifying Eq. (22) to account for noise, one can derive

$$\hat{D}_{ij}^{\text{cmb}} = \left( \sum_{\mu=1}^{\lambda_{\text{cut}}} G_{\mu}^2 \lambda_{\mu}^{-1} \right)^{-1} - S .$$

The new variables are related to the previous ones as:

$$\hat{D}_{ij}^{\text{obs}} = \frac{D_{ij}^{\text{obs}}}{\sqrt{\sigma_{\text{noise}}^2 + \sigma_{\text{noise}}^2}},$$

$$\hat{f}_i = \frac{f_i}{\sqrt{\sigma_{\text{noise}}^2}}, \quad \hat{\mathbf{E}}^\mu = \mathbf{f} \cdot \mathbf{E}^\mu,$$

where the matrix $\hat{D}_{ij}^{\text{obs}}$ has $\hat{\mathbf{E}}^\mu$ and $\hat{\lambda}_{\mu}$ as the $\mu$-th eigenvector and corresponding eigenvalue, respectively.

The instrumental noise can indeed lead to noise dominated eigenmodes with eigenvalues of $|\hat{\lambda}_{\mu}| \lesssim 1/2$ in $\hat{D}_{ij}^{\text{obs}}$ (see the details in Zhang et al. 2016 & Yao et al. 2018). Therefore, to avoid nonphysical eigenmodes, a threshold corresponding to $\lambda_{\text{cut}}$ in Eq. (23) is taken into account. Moreover, the free shift parameter $S$ is responsible for making the numerical calculations stable, especially in the recovery of the CMB $B$-mode power spectrum as the underlying $B$-mode signal might be smaller than the instrumental noise level in measuring the polarization signal.

### 4. SIMULATED POLARIZATION SKY MAPS

First of all, let us consider a hypothetical experiment capable of measuring the CMB polarization signal in ten frequency bands, as described in Tab. 4. The experiment frequency range is chosen here, similarly to Planck experiment (with three additional frequency channels), bearing in mind the spectral dependence of the foregrounds we considered (synchrotron and thermal dust), which present distinct behaviors in this frequency range. While synchrotron emission dominates the low-frequency part of the spectrum, thermal dust dominates the higher part (see Fig. 2).

Polarized $Q$ and $U$ maps are then generated for these ten frequency bands using the LensPix software [Lewis 2005, Lewis et al. 2011] with a resolution of $N_{\text{side}} = 1024$. Most of the calculations were done using HEALPix [Górski et al. 2005] software. The predicted CMB power spectra ($E$- and $B$-modes) were obtained by running the public available CAMB [Lewis et al. 2000] code, considering the best fit parameters from the standard cosmological model (Planck Collaboration et al. 2016b) for both $r = 0$ and $r = 0.05$. We then add to two polarized foreground components generated from the nominal PySM model [Thorne et al. 2017] as the fiducial foreground model for thermal dust and synchrotron. Moreover, Gaussian instrumental noises, un-
correlated from pixel to pixel and from channel to channel, are also included in the simulations. Finally, all \( Q \) and \( U \) maps are degraded to the lowest resolution of 28.3’ by a beam smoothing process.

In this study, 50 independent noise maps are generated for each frequency band, in order to properly test the ABS method for the polarized CMB signal in the presence of noise. In summary, together with the pure CMB \( Q \) and \( U \) signals, the two different foreground contamination components and the instrumental noises, with level for each frequency band specified in Table 1, are mixed in the maps. The frequency channels and noise levels are then chosen according to a future CMB designed experiment. We deliberately selected 10 frequency channels (according to the spectral dependence of our two foregrounds) from the experimental setup developed for the Probe of Inflation and Cosmic Origins (PICO) experiment.

For three of the frequency bands shown in Table 1, the \( Q \) and \( U \) total maps, as well as each \( E \)- and \( B \)-mode power spectra without any beam convolution, can be seen in Figs. 3 and 4, respectively.

Finally, we apply the ABS method to our simulated maps into two different cases: full sky and partial sky observations. In the former case, the approach is straightforward and no complications arise. In the latter one, we follow the steps below, in order to test the accuracy of the ABS approach in reconstructing the \( E \)- and \( B \)-mode power spectra in a specific partial-sky analysis with the SZ method. We:

(A) apply the Planck 2015 component separation common polarization mask (UPB77 with \( f_{\text{sky}} = 74.68\% \)) to the final \( Q \) and \( U \) simulations: CMB + foregrounds + noise for each frequency band;

(B) reconstruct the \( B \)-mode power spectra for every frequency maps by means of the SZ method;

(C) extract the final CMB \( B \)-mode spectrum using the ABS method;

(D) repeat the above procedure for other 50 independent realizations of the instrumental noise, but keeping the CMB signal and the foregrounds fixed;

(E) and, finally, calculate the mean and the standard deviation of the estimated \( B \)-mode power spectrum based on the results from steps (C) and (D).

5. RESULTS

Here, we present the results obtained with the ABS method in the recovery of the CMB \( E \)- and \( B \)-mode power spectra, based on the simulated polarization maps made by the fiducial survey specification. We consider the means and the associated statistical errors of ABS-derived CMB \( E \)-mode and \( B \)-mode power spectra from 50 independent noise realization simulations. Moreover, we adopt a binned result of \( \Delta \ell = 50 \).

5.1. The full sky case

In this Section, we show the results obtained by applying the ABS approach to the case of full-sky observations. The estimated CMB power spectra were obtained by averaging over the results from sky maps with 50 independent realizations of instrumental noise. The associated statistical errors are obtained from their dispersion. The CMB signal and foreground components are fixed among the realizations.

First of all, as an example, we show the distribution of the eigenvalues for \( r = 0 \) (see Eq. 23) according to the multipole \( \ell \) in Fig. 5. For a given \( \ell \), there are 10 eigenvalues, as \( D_{ij}^{\text{obs}}(\ell) \) corresponds to 10 frequency channels. Thus, the simulated microwave sky is expected to be completely decomposed into these eigenmodes.

As the first step, we recover the \( E \)- and \( B \)-mode power spectra considering no tensor-to-scalar ratio, \( r = 0 \). We compare the ABS result with the power spectra obtained from the pure CMB \( Q \) and \( U \) maps without foregrounds and noise, but smoothed to 28.3’ (dubbed as the “true” power spectrum) for \( \ell_{\text{max}} = 1050 \), as shown in Figs. 6 and 7. In order to calculate the differences between the recovered and the true spectra, we show in the lower panel of Figs. 6 and 7 the relative error, in percentage, calculated as: \( \frac{D_{ij}^{\text{cmb}} - D_{ij}^{\text{real}}}{D_{ij}^{\text{real}}} - 1 \) where \( D_{ij}^{\text{cmb}} \) is the estimator described in Eq. 23.

For full sky, the recovery of both polarized power spectra using the ABS method is in agreement with the “true” one for all multipole range within 20%, even though the 1-\( \sigma \) confidence level for the reconstructed \( B \)-mode power spectrum is quite large at low-\( \ell \) region. However, for the recovered \( E \)-mode spectrum in the range of \( 50 \leq \ell \leq 1050 \), the agreement is within 8%.

\( ^{1} \) https://zzz.physics.umn.edu/ipsig/start

\begin{table}[h]
\centering
\caption{The considered experimental setup.}
\begin{tabular}{ccc}
\hline
Band center (GHz) & Beam FWHM (arcmin) & noise level (\( \mu \text{K}_{\text{CMB}} \)-arcmin) \\
\hline
030 & 28.3 & 12.4 \\
043 & 22.2 & 7.9 \\
075 & 10.7 & 4.2 \\
090 & 9.5 & 2.8 \\
108 & 7.9 & 2.3 \\
129 & 7.4 & 2.1 \\
155 & 6.2 & 1.8 \\
223 & 3.6 & 4.5 \\
268 & 3.2 & 3.1 \\
321 & 2.6 & 4.2 \\
\hline
\end{tabular}
\end{table}
The total $Q$ (upper panel) and $U$ (lower panel) simulated full sky maps (without any beam convolution): CMB + synchrotron + thermal dust + noise. From left to right: 30, 129 and 321 GHz. The scale is in $\mu$K.

The same calculations are repeated for $r = 0.05$ to demonstrate, in the case of the $B$-modes, our present ability to distinguish between both cases of $r = 0$ and $r = 0.05$ and for the $E$-modes, as sake of completeness (see Figs. 8 & 9). We see that the difference between the “true” spectrum and the recovered one is also below 20%, in the case of the $E$-mode reconstruction for the full multipole range.

Once again, this relative error decreases to 8% for $50 \leq \ell \leq 1050$. In the case of the $B$-mode, we found an improvement in the reconstructed power spectrum in comparison to the “true” one, being the relative error between the two below 10%. Compared to the result for $r = 0$, the better performance of the ABS method in the $B$-mode case is expected due to the higher CMB signal for low multipoles when $r = 0.05$.

Moreover, in Fig. 10 we show the null test which is important for $B$-mode detection, in which we consider no CMB signal on the $Q$ and $U$ maps, but only foregrounds and a Gaussian noise. The null test is expected to be an important check for verifying the validity of our estimator, especially for detecting the extremely faint primor-
Fig. 5.— Eigenvalues (called egv above) of $D_{\text{obs}}^{ij}(\ell)$ for each of the 10 frequency bands (represented in different symbol colors), considering CMB, foregrounds (synchrotron and thermal dust) and one noise realization. The threshold $\tilde{\lambda}_{\text{cut}} = 1/2$ is shown by the black-solid line. The eigenvalues, $\tilde{\lambda}_{\mu}$, are shown in absolute value. Due to the instrumental noise, $D_{\text{obs}}^{ij}(\ell)$ is not strictly positive, leading to some small negative eigenvalues (red dots). Eigenvalues from the $E$-mode (upper) and $B$-mode (lower) analysis are shown, respectively.

dial $B$-mode polarization signal. Even though a small underestimate in the null spectra is found below $\ell = 150$, the result is still robust enough, with the deviations for $\ell \geq 150$ below $10^{-4} \mu K^2$.

In order to avoid the Galactic foreground, responsible for part of the $B$-mode contribution to the power spectrum, especially for low $\ell$'s, we mask this sky region as a next step. Moreover, as it is well-known, future ground-based experiments are supposed to measure the polarization field only in parts of the sky, typically less than 5% of the celestial sphere. In the next section, we will therefore perform a partial sky analysis.

5.2. The partial sky case

In this section, we use the same CMB simulation, foregrounds and noise realizations as in the previous section, whereas we now consider the $E/B$ decomposition in partial sky for each analyzed frequency band, using both the ABS foreground cleaning approach and the pseudo power spectrum reconstruction through the SZ method.

The result for $r = 0$ can be seen in Figs. 11 & 12. We notice that the performance of the ABS method in recovering the $E$-mode and $B$-mode power spectra for $50 \leq \ell \leq 1050$ is comparable with the full sky case. For $E$-modes, the difference between the recovered power spectrum and the “true” one is below 17% for the mentioned multipole range, being the recovered power spectrum mainly underestimated. For the $B$-modes, once again, we find the relative error to be less than 21% for $50 \leq \ell \leq 1050$ and less than 15% for $100 \leq \ell \leq 1050$.

For smaller multipoles, especially accounting for the first bin, a noticeable divergence occurs. The uncertainties, in the partial sky case, originate from both the ABS method and the SZ pseudo power spectrum recovery. It is important to note that the performance of SZ reconstruction method deteriorates rapidly below $\ell = 50$, for analytically apodized windows, especially for masks containing holes (as for our case) (Ferté et al. 2013). The uncertainties from the SZ method at $\ell \lesssim 50$ are known to
increase rapidly with increasing the angular scale. This explains the underestimate/overestimate in the recovered\(E\) and \(B\)-mode power spectra in the multipole range of \(\ell \leq 50\), being however the recovered \(EE\) and \(BB\) spectrum still in agreement with the “true” ones within 1-\(\sigma\) confidence level. In the same way, the results for the SZ methodology considering high \(\ell\)'s deteriorate rapidly (for a more detailed explanation see the appendix). As for comparison, we also plot the pseudo power spectra in Figs. 11 & 12 for our cleaned CMB simulation using the SZ methodology.

Since the objective of this paper is to test the ABS method for foreground removal, we will not pursue a more sophisticated approach for polarization reconstruction on partial sky here, such as variance-optimized SZ windows in pixel space \citep{Ferte2013,Smith2006,Smith2007}. We will leave the optimization of \(E/B\) reconstruction in low-\(\ell\) region to a future work. At present, we are unable to comment on precisely the performance of the ABS method at low/high multipoles on a partial sky, due to the large uncertainties coming from the SZ reconstruction. In this work, the optimal multipole range to be considered for the partial sky analysis is \(100 \leq \ell \leq 1000\), in which the performance of the ABS method agrees with the results for the full sky case.

Finally, we repeat the calculations for \(r = 0.05\), which are shown in Figs. 13 & 14. Once more, we find a divergence between the first bin with respect to the “true” spectrum. In this case, for the \(E\)-mode, the pseudo power spectrum reconstruction for the clean CMB simulation (no foreground) is also well underestimated, which agrees with our result for the final ABS reconstruction (see Fig. 13 blue and green curves, respectively). However, the “true” spectrum lies outside the 1-\(\sigma\) confidence level. Nevertheless, in this case, excluding the low multipoles (for \(\ell \leq 50\)), we again find an agreement with the input one within 17% for the \(E\)-modes and within 21% for the \(B\)-modes, in the range of \(50 \leq \ell \leq 1050\).

Finally, in Fig. 15, we see the results for the null test in the partial sky analysis.

6. CONCLUSIONS

In this study, we have tested the ability of the ABS method to recover CMB \(E\)-mode and \(B\)-mode power spectra from both foreground and instrumental noise contaminated multi-frequency maps. The ABS estimator has been applied to the simulated maps, considering a hypothetical future experiment with 10-frequency bands in the range of 30 to 321 GHz. Taking into account 50
Fig. 11.— Upper panel: CMB binned $E$-mode power spectrum estimated from the ABS approach from a hypothetical experiment specified in Tab. 4 considering $r = 0$ (green curve). The red curve corresponds to the CMB $E$-mode input power spectrum ("true" spectrum). The blue curve corresponds to the pseudo power spectrum recovery from the one CMB sky realization without noise/foregrounds using the SZ $E/B$ separation methodology. The associated 1-$\sigma$ statistical errors are also shown as the shadow region, based on 50 independent realizations of the instrumental noise. Lower panel: in comparison with the "true" one, the relative error, $D_{\text{rec}}^{\ell}/D_{\text{real}}^{\ell} - 1$, is shown in percentage level. The symbol colors and sizes illustrate deviations from 0% in respect to the "true" spectrum. Notice that the relative error was not plotted for the first bin in the lower panel, since it is well underestimated.

Fig. 12.— Same as in Fig. 11 for the $B$-mode power spectrum ($r = 0$), however, in this case, the relative error of the first bin are not plotted in the lower panel, since it is well overestimated.

Fig. 13.— For completeness, the same as in Fig. 11 but for $r = 0.05$.

Fig. 14.— Same as in Fig. 12 ($r = 0.05$).

The recovery of the pseudo power spectra using the SZ methodology faces difficulties, among others, due to the mask shape, which would induce too big uncertainties especially for $\ell \lesssim 50$. The performance of the ABS method in the case of partial sky for $\ell < 50$ must be studied in the light of a more optimized pseudo power spectrum reconstruction. We leave this analysis for a future work. As in the near future there will be high-resolution, ground-based instruments observing small patches of the sky ($f_{\text{sky}} \lesssim 5\%$), it is very important to develop tools capable of dealing with these data sets. The results obtained with the ABS method are promising, which is expected to be further improved by considering more optimal methods of pseudo power spectrum reconstruction in partial sky, in order to achieve desired accuracy level.
of detecting primordial $B$-mode signal.

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APPENDIX

A NOTE ON $E/B$ SEPARATION

In this appendix we look in detail at the performance of our $E/B$ separation method with the pseudo-$C_\ell$ estimator. The $E/B$ separation pipeline contributes to the error of the reconstructed power spectra. It is important to understand the efficiency and behavior of the pseudo-$C_\ell$ estimator used in this work. This would help us to understand the contribution of the $E/B$ separation method to the total error. For this purpose, we performed 300 simulations of pure CMB signal (without noise or foreground). The input power spectra used in the simulations and the Gaussian apodized mask are the same as the ones used throughout this work. We consider, as for illustration, the case of $r = 0$.

The resulting spectra are shown in Fig. 16. One can see that the mean reconstructed spectra (shown in solid green) above $\ell = 50$ exactly agree with the input power spectra (shown in red). It is also clear that the SZ-derived $E$- and $B$-mode pseudo power spectra are overestimated/underestimated, respectively, in the case of $r = 0$.

We find that for the range $100 \leq \ell \leq 1000$ the errors for both the $E$- and $B$-mode power spectra are consistent with the cosmic variance limit. This means that the pipeline is optimal in the range $100 \leq \ell \leq 1000$. But the error sharply increases below $\ell = 100$ and above $\ell = 1000$. It is evident from the calculations presented here that the SZ method does not provide a reliable estimation for the power spectra at the largest angular scales, i.e., $\ell \leq 50$, due to the large uncertainty and possible bias of this methodology in the low-$\ell$ regime.

This behavior would also explain the partial sky results previously shown. The large uncertainty and deviation of the reconstructed power spectra in the first bin are mainly due to the effects from the $E/B$ separation method. In other words, the discrepancy observed at low $\ell$s for the ABS reconstruction in partial sky would be attributed to the $E/B$ separation method. These results
also suffer from similar increase in the error at high-ℓ, which will have a large contribution from the E/B separation method itself.

One possible way to improve the low-ℓ estimation is to use a variance-optimized window function with the SZ method proposed by [Ferte et al. (2013)]. The construction of a variance-optimized window function is beyond the scope of this work and we hope to address this problem in the future.

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