Universality of Charge and Spin Response in Doped Antiferromagnets

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Recent results for the finite-temperature static and dynamical properties of the planar $t - J$ model, obtained by a novel numerical method for small correlated systems, are reviewed. Of particular interest are $T > 0$ charge and spin response functions: optical conductivity $\sigma(\omega)$ and spin susceptibility $\chi(\mathbf{q}, \omega)$, which show very universal features at intermediate doping. In spin dynamics the universality is best seen in the local spin correlation function, which appears to be $T$-independent and only weakly doping dependent. Results apply directly to the inelastic neutron scattering and to the NMR relaxation experiments in cuprates. Analysing $\sigma(\omega)$ we find that the current correlation function $C(\omega)$ is nearly $T$-and moreover $\omega$-independent in the regime $T < J$, leading to the $\sigma(\omega)$ behavior, very close to the marginal Fermi liquid concept and experiments. These properties are interpreted in connection with the large entropy, related to the large degeneracy of the low-lying states and quantum frustration in the doped antiferromagnets. On the other hand, results for the uniform susceptibility $\chi_0(T)$, as well as for the $\sigma(\omega)$, reveal qualitative changes on entering the underdoped regime.

I. INTRODUCTION

Strongly correlated electrons have been in recent years intensively investigated theoretically, mainly in connection with the open question of the mechanism of superconductivity as well as the unusual metal properties in cuprates. Here we will discuss only the anomalous normal-state properties of cuprates at $T > T_c$. These are generally attributed to correlation effects, nevertheless a unifying theory still remains a challenge.

In this article we will address several questions related to the theoretical understanding of the normal-state properties of cuprates:

a) Can strong correlations alone account for the unusual normal-state properties, as manifested by the charge response: optical conductivity $\sigma(\omega)$ and d.c. resistivity $\rho_0$, and the spin response: dynamical spin susceptibility $\chi(\mathbf{q}, \omega)$, related to the NMR relaxation $T_1$, etc.? 

b) What is the origin of the unconventional universality within the ‘normal’ metallic state of cuprates, in particular in the ‘optimum-doping’ regime, obtained by doping the reference antiferromagnetic (AFM) insulator with the mobile hole-type charge carriers?

c) How many relevant energy scales exist in the doped AFM?

d) Are the simplest prototype models, such as the $t - J$ model (and analogous Hubbard model), enough to reproduce qualitatively and quantitatively experimental results in cuprates?

f) Can we learn something by studying small correlated systems at $T > 0$?

From the recent experiments testing the normal-state properties such as the d.c. resistivity $\rho_0$, optical conductivity $\sigma(\omega)$, neutron scattering $\sigma(\mathbf{q}, \omega)$, and the NMR and NQR relaxation $T_1$, a unifying picture seems to emerge. These properties mainly depend on the extent of doping, and the regimes have been classified into the underdoped, the ‘optimal’ (intermediate-doping), and the overdoped, respectively. Whereas the ‘optimal’ doping refers in a narrow sense usually to samples with the highest $T_c$, the same broader region of doping, with effective hole concentration approx. $c_h = 0.15 - 0.25$, also exhibits most universal properties. Down to the lowest $T \lesssim T_c$ resistivity follows the remarkable $\rho_0 \propto T$ law, the uniform susceptibility $\chi_0$ is varying monotonously with $T$, etc. On the other hand, the underdoped materials (effective $c_h \lesssim 0.15$) are characterized by the onset of the ‘pseudogap’ at $T < \tilde{T}$, which shows up in the kinks in $\rho(T)$, $\chi_0(T)$ etc.

We investigate in this review mainly the intermediate-doping (optimal) regime. We discuss finite-$T$ (normal-state) properties of the doped AFM within the $t - J$ model $\hat{H} = -t \sum_{\langle ij \rangle s} (c^\dagger_{is} c_{js} + H.c.) + J \sum_{\langle ij \rangle} (\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j)$

\begin{equation}
H = -t \sum_{\langle ij \rangle s} (c^\dagger_{is} c_{js} + H.c.) + J \sum_{\langle ij \rangle} (\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j),
\end{equation}

where $c^\dagger_{is}(c_{is})$ are projected fermionic operators, prohibiting double occupancy of sites. The ground state of the $t - J$ model and related properties have been intensively studied both by analytical and numerical methods $\hat{A}$, nevertheless some basic questions, e.g. concerning possible pairing in the ground state and the origin of anomalous metallic properties at finite $T > 0$, remain unsolved.

II. FERMI LIQUIDS

Normal metals are expected to follow the concept of the Landau Fermi liquid. The excitations in this case should be quasiparticles with well defined $\mathbf{q}$ and energy $\epsilon_\mathbf{q}$ near the Fermi energy, with the damping $\tau(\mathbf{q})^{-1} \propto (\epsilon_\mathbf{q} - E_F)^2$. Consequences are well known, e.g. for the dynamical susceptibility

\begin{equation}
\omega \to 0 : \quad \chi''(\mathbf{q}, \omega)/\omega \neq f(T),
\end{equation}

...
leading to the Korringa law NMR $T_{1}^{-1} \propto T$. Also, the Drude form is expected for $\sigma(\omega)$ with $\tau^{-1} \propto T^{2}$ etc.

The normal-state properties of cuprates, however, do not follow the Landau scenario. Experiments have been mainly described within two alternative concepts. The nearly antiferromagnetic Fermi liquid has been introduced by Moriya and Millis et al. [11] to describe the spin fluctuations in the doped AFM, mainly in connection with the anomalous NMR relaxation phenomena. The low-$\omega$ spin response is modeled with the overdamped spin fluctuations peaked at the AFM wavevector $\vec{Q} = (\pi, \pi)$,

$$\chi''(\vec{q}, \omega) = \frac{\omega \chi_0 \xi^{2}}{\omega^{2} + \Gamma^{2}[1 + |\vec{q} - \vec{Q}|^{2}]^{2}}$$

AFM correlation length $\xi(T)$ is assumed to be critical in the doped regime, in particular with $\xi^{2}(T) \propto 1/T$ in the $z = 2$ dynamical RPA-type theory [4], while the postulated mapping to the quantum critical regime of the nonlinear sigma model would imply $\xi(T) \propto 1/T^{1/2}$.

Alternative scenario has been summarized within the marginal Fermi liquid (MFL) hypothesis [11]. Here, the spin and charge susceptibility have been postulated in the anomalous form

$$\chi''(\vec{q}, \omega) \sim C \frac{\omega}{T}, \quad |\omega| < T,$$

$$\chi''(\vec{q}, \omega) \sim C \text{sgn}{\omega}, \quad |\omega| > T,$$

leading to the marginality of the quasiparticle self energy

$$\Sigma(\omega) \sim \lambda [2\omega \ln \frac{\pi T - i\omega}{\omega} - i\pi^{2} T].$$

Consistent with Eq.(2), the low-frequency $\sigma(\omega)$ has been written in the generalized Drude form introducing an anomalous effective relaxation rate $\tau^{-1} \sim 2\pi \lambda (\omega + \pi T)$.

### III.FINITE TEMPERATURE METHOD

$T > 0$ properties of the $t-J$ model have been so far calculated only by the high-temperature series expansion [12] and via full or partial diagonalization of quite small systems. Recently the present authors introduced a novel numerical method, based on the Lanczos diagonalization method combined with random sampling [13]. The latter method has the advantage that it allows the study of dynamical and static response functions within the most challenging regime $T, \omega < J$, where one could hope for a universal behavior, as observed in the experiments on cuprates. It should be noted that analogous Hubbard model seems more feasible for $T > 0$ studies, due to the possible application of the Quantum Monte Carlo methods [14]. Nevertheless, in the doped case the well known minus sign problem so far mainly prevented calculations within the regime $T, \omega < J$ [14].

Let us present briefly our method for $T > 0$ calculations, showing in more detail the evaluation of the statical expectation value of an operator $A$,

$$\langle A \rangle = Z^{-1} \sum_{n}^{N} \langle n | e^{-\beta H} | n \rangle, \quad Z = \sum_{n}^{N} \langle n | e^{-\beta H} | n \rangle,$$

where $\beta = 1/T$ (we use $k_{B} = \hbar = 1$ furtheron) and the sum runs over the chosen complete basis set of orthonormal wavefunctions $|n\rangle$, $n = 1, N$, spanning the Hamiltonian $H$. If we could perform the full diagonalization of the problem and find all eigenstates $|\Psi_{l}\rangle$ and corresponding energies $E_{l}$, we would express the result in a usual way,

$$\langle A \rangle = \sum_{l}^{N} e^{-\beta E_{l}} \langle l|A|l\rangle / \sum_{l}^{N} e^{-\beta E_{l}}.$$

We choose an alternative approach. With each basis function $|n\rangle$ we start a Lanczos procedure $|\phi_{0}^{n}\rangle = |n\rangle$, generating an orthonormal set of functions $|\phi_{m}^{n}\rangle, m = 0, M$,

$$H|\phi_{0}^{n}\rangle = a_{n0}|\phi_{0}^{n}\rangle + b_{n1}|\phi_{1}^{n}\rangle,$$

$$H|\phi_{m}^{n}\rangle = b_{nm}|\phi_{m-1}^{n}\rangle + a_{nm}|\phi_{m}^{n}\rangle + b_{nm+1}|\phi_{m+1}^{n}\rangle,$$

For the chosen number of Lanczos steps $M \ll N$ we then diagonalize the tridiagonal matrix of coefficients $a_{nm}, b_{nm}$ to find the energies $\epsilon_{nm}$ and the corresponding eigenfunctions $|\psi_{m}^{n}\rangle$. Within the restricted basis of these functions one can write the approximation for Eq.(9) as

$$\langle A \rangle = Z^{-1} \sum_{n}^{N_{0}} \sum_{m}^{M} \langle n | \psi_{m}^{n} \rangle e^{-\beta \epsilon_{nm}} \langle \psi_{m}^{n} | A | n \rangle,$$

$$Z = \sum_{n}^{N_{0}} \sum_{m}^{M} |\langle n | \psi_{m}^{n} \rangle|^{2} e^{-\beta \epsilon_{nm}}.$$

It is evident that the average evaluated via Eq.(10) is equivalent to Eq.(9) for the sampling over the full basis set $N_{0} = N$ and $M = M - 1$.

Our claim is that very accurate results can be obtained via Eq.(9) even for a severely reduced number of Lanczos steps $M \ll N$ and for a partial random sampling of the basis states $N_{0} \ll N$, instead of the full sampling. Let us first investigate the method for the full sampling $N_{0} = N$. We note that the Lanczos procedure Eq.(10) represents an iterative action of the operator $H$ on the initial function $|n\rangle$. Performing the (high temperature) expansion in $\beta$ of the numerator and of the denominator of Eq.(10), it is easy to prove that both power series are correct up to the order $M$. On the other hand, $\langle A \rangle$ evaluated via Eq.(9) is accurate also for $T = 0$ (provided $M > M_{0}$), since it is a standard experience that the Lanczos procedure converges quite rapidly, typically in $M = M_{0} \sim 50 \ll N$. 

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steps, from an arbitrary initial function $|n\rangle$ (not orthogonal to the ground state) to the ground state energy $E_0$ and the corresponding wavefunction $|\Psi_0\rangle$. Hence, at full sampling our Lanczos-based method represents an optimum interpolation scheme between the high temperature $T \to \infty$ and the ground state $T = 0$ result.

The second important ingredient is the reduction of the sampling to the partial random one with $N_0 \ll N$. This step cannot be justified rigorously, but seems plausible in analogy with the statistical Monte Carlo methods. The most severe test is expected to be the result at $T = 0$, which can be anyhow tested via an alternative direct evaluation with the known $|\Psi_0\rangle$. The general experience is that good convergence is achieved for $N_0 \ll N$.

Even more appealing is the application of the method to the calculation of dynamical quantities. We choose as an example the optical conductivity, expressed within the linear response theory as

$$
\sigma(\omega) = \frac{1}{\omega} \left( 1 - e^{-\beta \omega} \right) \text{Re} \int_0^\infty dt \, e^{i\omega t} C(t),
$$

$$
C(t) = \langle j(t)j \rangle = Z^{-1} \sum_n \langle n|e^{(-\beta + it)H} je^{-iHt}|n\rangle. \quad (10)
$$

In order to get the result at arbitrary $\beta$ a double sum over all eigenstates $|\Psi_i\rangle$, $|\Psi_f\rangle$ is required. Instead, we use the approach described above for $\langle A \rangle$ generalized to dynamical $C(t)$,

$$
C(t) = Z^{-1} \sum_{n=1}^{N_0} \sum_{m,k} \langle n|\tilde{\psi}_m^n e^{-\beta \varepsilon_{nm} t} e^{iHt} \psi_k^n |m, k\rangle \times \langle \psi_k^n |j| n\rangle \frac{\tilde{\tilde{\psi}}_k^n}{\sqrt{\langle n|j^2| n\rangle}}. \quad (11)
$$

Here, $|\tilde{\psi}_k^n\rangle$ and $\tilde{\varepsilon}_{nk}$ are generated by the Lanczos procedure analogous to Eq.$(\hat{1})$ via the orthonormal basis set $|\tilde{\psi}_k^n\rangle$, but with the initial condition

$$
|\tilde{\psi}_0^n\rangle = j|n\rangle / \sqrt{\langle n|j^2| n\rangle}. \quad (12)
$$

While for $N_0 = M = N$ the expression $(\hat{1})$ is equivalent to the exact one, the arguments for using $N_0 \ll N$ and $M \ll N$ are analogous to those described above for the static case. E.g., for the full sampling $N_0 = N$ Eq.$(\hat{1})$ yields correct series in $-\beta + it$ and $it$, respectively, up to the $M^{th}$ order. Accordingly, $\sigma(\omega)$ has correct frequency moments $\langle \omega^p \rangle$, $p = 0, M$, for $\beta \to 0$ etc. Again for $M > M_0$ the method yields meaningful result also for $T = 0$, which can be tested via the usual Lanczos method for the $T = 0$ dynamical quantities $(\hat{1})$.

It is important to stress that computational requirements of the method for a system of the given size are comparable to those for the ground state evaluations via the Lanczos method, whereby the CPU time clearly increases with the number of random samples $N_0$. The CPU time and memory requirements are also enhanced due to the necessary reorthogonalization of Lanczos functions and due to the evaluation of matrix elements in Eq.$(\hat{1})$.

Let us finally comment on the finite size effects, which due to the restricted system sizes still remain the main obstacle to achieve reliable results for low $T$. It is a plausible observation that the method yields macroscopic-like results for $T > T^*$, where $T^*$ is roughly determined by the average level separation in the low-energy many-body spectrum, being dependent on the system size. For $T < T^*$ the finite-size effects become pronounced, e.g. dynamical quantities in general reveal (spurious) peak structures etc. It is however very important to note that within the $t - J$ model, the $T^*$ is smallest within the intermediate (optimum) doping regime $2/16 \leq c_h \leq 4/16$, where we typically reach $T^* \sim 0.1 t$ for systems $N = 16, 18$. The $T^*$ is larger both within the underdoped and the overdoped region (for the fixed system size).

IV. SPIN SUSCEPTIBILITY

Let us consider the dynamical spin response, as given by the susceptibility $\chi(\tilde{q}, \omega)$ and the corresponding dynamical spin correlation function $S(\tilde{q}, \omega)$

$$
\chi''(\tilde{q}, \omega) = (1 - e^{-\beta \omega}) S(q, \omega),
$$

$$
S(q, \omega) = \text{Re} \int_0^\infty dt \, e^{i\omega t} (S(q,t)^* S_q(t)). \quad (13)
$$

It has been shown by the present authors $(\hat{1})$ that $\chi''(q, \omega)$ shows the coexistence of the high-frequency ($\omega \propto t$) free-fermion-like contribution and the low-$\omega$ spin-fluctuation contribution. At the same time, $\chi''(q, \omega < T)$ reveals a pronounced $T$ dependence, consistent with the MFL form, and (even quantitatively) with the NMR relaxation in cuprates.

It appears quite helpful to concentrate on the local spin correlation function $S_L(\omega)$ $(\hat{1})$ and its symmetric part $\tilde{S}(\omega)$

$$
S_L(\omega) = \frac{1}{N} \sum_q S(q, \omega),
$$

$$
\tilde{S}(\omega) = S_L(\omega) + S_L(-\omega) = (1 + e^{-\beta \omega}) S_L(\omega). \quad (14)
$$

It should be pointed out that $S_L(\omega)$ and the related susceptibility $\chi_L(\omega)$ are directly measured (in cuprates) by neutron scattering $(\hat{1})$. The NMR relaxation as well yields the information on $S_L(\omega \to 0)$. Provided that the AFM spin fluctuations $q \sim Q = (\pi, \pi)$ are dominant.

An important restriction for $\tilde{S}(\omega)$ is the sum rule

$$
\int_0^\infty \tilde{S}(\omega) d\omega = \pi \langle (\lambda^z)^2 \rangle = \frac{\pi}{4} (1 - c_h), \quad (15)
$$

where $c_h = N_h/N$ is the hole concentration.
We perform the evaluation of $\bar{S}(\omega)$ via Eq. (14) by calculating $S(q, \omega)$, using the finite-$T$ diagonalization method for small systems, in this case for the $t - J$ model on the square lattice with $N = 16, 20$ sites. We fix $J/t = 0.3$ to remain in the regime of cuprates. We stress again that the method yields macroscopic-like results for $T > T^* \gtrsim 0.1$.

In Fig. 1 we display the $\bar{S}(\omega)$ for $c_h = 1/20, 3/16$ and several $T$ in the range $0.1 < T/t < 0.7$. It is immediately evident that $\bar{S}(\omega)$ at ‘optimal’ doping $c_h = 3/16$ is essentially $T$-independent in a wide $T$-range, although one crosses the exchange-energy scale $T \sim J$. For the underdoped case $c_h = 1/20$ the behavior is analogous for higher $T > T_0 \sim 0.7 J$, consistent with the quantum critical regime within the AFM. Deviations at lower $T < T_0$ (where the renormalized classical regime is expected in the AFM) could be an indication for the onset of a ‘pseudogap’.

FIG. 1. Local spin correlation function $\bar{S}(\omega)$ for $c_h = 1/20, 3/16$ and various $T$.

To follow the doping dependence we present in Fig. 2 the variation with $c_h$ at fixed $T = 0.2 t < J$, plotting the integrated intensity

$$I_S(\omega) = \int_0^\omega \bar{S}(\omega')d\omega'.$$  

Again, we notice that for the chosen $T$ the results are most reliable at the intermediate doping. The most striking message is that the initial slope of $I_S(\omega)$ and consequently $S_L(\omega \to 0)$ are nearly doping independent for $0 \leq c_h \leq 0.25$. This is well consistent with the NMR (NQR) relaxation rates $T_1^{-1}$ as measured in La$_{2-x}$Sr$_x$CuO$_4$ in the range $x = 0 - 0.15$.

FIG. 2. Integrated spectra $I_S(\omega)$ at fixed $T = 0.2$ and various $c_h$.

Only for the overdoped systems with $c_h > 0.25$ the low-frequency behavior changes qualitatively, where the latter part is strongly suppressed as expected in (more) normal Fermi liquids. $I_S(\omega > J)$ is doping dependent even for $c_h < 0.25$, consistent with the $c_h$-dependence of the sum rule, Eq. (15). In addition, at the intermediate doping $\bar{S}(\omega)$ decreases smoothly (see Fig. 1) up to $\omega \sim 4t$, this being the consequence of a free-fermion-like component. On the other hand, in the underdoped regime the dynamics is restricted to $\omega < 3J < t$.

How can one explain the universality of $\bar{S}(\omega)$ at intermediate $c_h$? First, we note that up to $c_h \sim 0.3$ the dominant scale of spin fluctuations remains related to $J$. From the explicit expression in terms of the eigenstates of the system

$$\bar{S}(\omega) = (1 + e^{-\beta\omega})^4 \sum_{n,m} e^{-\beta E_n^z} |⟨n|S_{z}^x|m⟩|^2 \times$$

$$\delta(\omega - E_m + E_n),$$  

one would conjecture (quite generally) a $T$-independence of response for $\omega \gg T$. Although such plausibility arguments have been used previously, their validity clearly depends on the character and on the density of low-lying many-body states. To explain also the $T$-independence of $\bar{S}(\omega < T)$, we only need to recall the sum rule Eq. (15) and to assume that there is no characteristic scale $\omega_c < T$, which could introduce an additional low-$\omega$ structure in $\bar{S}(\omega)$.

A natural scale for the AFM is the (gap) frequency $\omega_c \sim c/\xi$, where $c$ is the spin wave velocity and $\xi$ the AFM correlation length. Here originates the essential difference between the undoped and the ‘optimally’ doped AFM. While for an AFM in the renormalized classical regime $\xi$ is exponentially large for $T \ll J$, and consequently $\omega_c < T$, in the doped case $\xi < 1/\sqrt{c_h}$ is determined predominantly by $c_h$, so $\xi$ is rather $T$-independent for $T < J$, excluding $\omega_c < T < J$. In other
words, the observed anomalous low-ω behavior within the t – J model does not originate in a strong T-dependence of $\xi$, as conjectured by the nearly AFM Fermi liquid, Eq. (13).

We conclude the discussion of spin dynamics by the consequences of the universality of $\bar{S}(\omega)$. The local susceptibility is given by

$$\chi'_L(\omega) = \tanh \left( \frac{\omega}{2T} \right) \bar{S}(\omega).$$

(18)

Since neutron scattering probes only $\omega < J$, one can simplify Eq. (18) further by $\bar{S}(\omega) \sim \bar{S}_0$. Such form has been recently used to describe experiments [1]. It is also qualitatively consistent (but as well more restricted in the form) with the MFL Ansatz, Eq. (11).

V. OPTICAL CONDUCTIVITY

Let us investigate in an analogous way the dynamical conductivity $\sigma(\omega)$, Eq. (14). Results for this quantity by the present authors [13], obtained by the finite-T method, can be summarized as follows: a) in the intermediate-doping regime $\sigma(\omega)$ shows a non-Drude fall-off, consistent with the $\omega$-dependent relaxation rate within the MFL concept Eq. (5) [11], deduced from experiments in cuprates [13], and b) qualitative as well as quantitative results for $\sigma(\omega)$ and $\rho(T)$ agree well with the experimental ones.

Unlike $\bar{S}(\omega)$, $C(\omega)$ does not obey a $T$-independent sum rule. Nevertheless, motivated by the universal spin dynamics we reexamine in an analogous manner our results on $\sigma(\omega)$ [13]. In Fig. 3 we present the corresponding integrated spectra $I_C(\omega) = \int_0^\infty C(\omega')d\omega'$ at fixed doping in the ‘optimal’ regime for various $T \leq t$ [11]. We establish that for $T \leq J$ spectra $I_C(\omega)$ are essentially independent of $T$, at least for the available $T > T^*$. At the same time the slope of $I_C(\omega < 2t)$ is nearly constant, i.e. $C(\omega) \sim C_0$ in a wide $\omega$-range, $C_0$ being weakly $J$-dependent (tested for $J/t = 0.2, 0.6$).

We note that such $C(\omega) \sim C_0$ implies a nonanalytic behavior of $\sigma(\omega \to 0)$, starting with a finite slope at $\omega = 0$. Moreover, with $C(\omega) = C_0$ we can claim a simple universal form for $\omega < 2t$

$$\sigma(\omega) = C_0 \frac{1 - e^{-\beta\omega}}{\omega}.$$  

(19)

At first sight, it appears rather surprising that this form can be well fitted (for $\omega, T \ll t$) with a Drude-type form with an MFL-like effective relaxation rate $\tau^{-1} = 2\pi\lambda(\omega + \eta T)$ with specific $\lambda \sim 0.09$ and $\eta \sim 2.7$ [13]. The form Eq. (19) for $\sigma(\omega)$ trivially reproduces the remarkable linear law $\rho \propto T$ in cuprates [1], as well as the non-Drude falloff at $\omega > T$.

FIG. 3. Integrated current-correlation spectra $I_C(\omega)$ for $c_h = 3/16$ and various $T$.

A quantitative comparison (assuming $J/t = 0.3$ and $t = 0.4$ eV) of $\sigma(\omega)$ with the experimental results [13] is quite remarkable [18], realizing the simplicity of the model. It is however evident that the expression (19) is more restricted than the MFL Ansatz, hence it should be retested in more detail with experiments.

It should be mentioned that $C(\omega) \sim C_0$ has been derived for a single hole conductivity within the retraceable path approximation [12], with a restricted validity for $T > t$ (or possibly $T > J$). We find this behavior only for the intermediate doping, hence new arguments are needed. We can follow the analysis analogous to $\bar{S}(\omega)$, Eq. (17), expressing $C(\omega)$ in terms of the eigenstates. As before, the $T$-dependence of $C(\omega > T)$ seems plausible. The fact that $C(\omega < 2t) \sim C_0$ requires, however, that the current relaxation is very fast, i.e. determined only by the incoherent hopping and by the interhole collisions. The spin fluctuation scale $J$ does not enter directly, i.e. even for $T \ll J$ the spin system only serves as a random bath for the charge degrees of freedom (holons). This conclusion remains valid as far as there is no characteristic frequency $\omega_c < T$ (e.g. the ‘pseudogap’) in the system.

On the other hand, our results also indicate the possible qualitative changes when entering the underdoped regime. E.g. the $c_h = 2/18$ doping seems to represent a crossover to the latter behavior, where in contrast (e.g. for $c_h = 1/20$) we find a pronounced $T$- and $\omega$-dependence of $C(\omega)$ at $T \leq J$. This qualitative change can be again attributed to a ‘pseudogap’ appearing at larger $T$ in the underdoped systems [1].

VI. ENTROPY

Above arguments, both for $S_L(\omega)$ and $C(\omega)$, require a large density (degeneracy) of the low lying many-body
states, apparently being a crucial feature for the most challenging intermediate-doping regime. To quantify this statement we calculate the entropy density $s = S/N$, using again the finite-$T$ diagonalization method, being less space and time consuming for static quantities [13]. In Fig. 4 we present the results for $s(c_h)$, obtained for $N = 16$ and $N = 20$ at different $T$. The main lesson from Fig. 4 is that the ‘optimal’ cases with $c_h \sim 0.15 - 0.3$ are characterized by the largest entropy $s$ at low $T < J$, e.g. $s > 0.2$/site for $T \sim 0.2 J$, being almost one half of $s(T = \infty)$ for an AFM. This implies a very large degeneracy of the low-lying states, which could be attributed to the spin subsystem, frustrated by the hole motion. Calculated entropy, large even at moderate $T$ and strongly dependent on doping, seems to be (also quantitatively) consistent with the specific heat measurements on cuprates [20].

\begin{center}
\vspace{2cm}
FIG. 4. Entropy density $s$ vs. $c_h$ for three $T$ and two system sizes $N$.
\end{center}

\vspace{2cm}

VII. CONCLUSIONS

Our results for the $t-J$ model seem to indicate that the anomalous, but universal, dynamics in the intermediate-doping regime of correlated systems is a consequence of an extreme degeneracy and a collapse of the low-lying quantum states (introduced by doping the magnetic insulator). This leads to a diffusive-like charge and spin response, where $T$ represents the only relevant energy (frequency) scale. On the other hand, our results do not exclude the possible onset of coherence (related to ‘pseudogaps’ and superconductivity) at lower $T < T^* \sim 0.1 t$.

It is evident that the ‘optimal’ doping regime is characterized by the competing influence of two quantum processes: AFM fluctuations and hole hopping. Outside this regime, either one of the two processes should dominate.

\begin{center}
\vspace{2cm}
FIG. 5. Uniform susceptibility $\chi_0$ (in units of $1/t$) vs. $T$ for various $c_h$ (increasing in steps of 0.05), as obtained by calculations on the system with $N = 20$ sites for $c_h > 0$, and $N = 26$ sites for $c_h = 0$, respectively.
\end{center}

To show that such physics is indeed incorporated within the $t-J$ model, we present our results for the uniform susceptibility $\chi_0(T)$ at various hole dopings $c_h$. From Fig.5 we establish that $\chi_0(T)$ is most monotonous within the intermediate-doping regime. On the other hand, for $c_h < 0.15$ the susceptibility starts to reveal a shoulder at $T$, which could be interpreted as an increase of the pseudogap scale, but as well as a sign of the dominating role of the AFM fluctuations.

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