Constituent Quark Picture out of QCD in two–dimensions — on the Light–Cone

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Abstract

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Abstract

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Key words: Fock–space, non–perturbative, light–cone, hadron spectrum.

Introduction. The idea of formulating mechanics and field theory on null–plane surfaces was first stimulated by Dirac in 1949 [1] with his introduction of the three independent schemes of Hamiltonian dynamics: the (conventional) instant form, the front form and the point form. The second of these schemes was exploited in 1985 by Pauli and Brodsky [2] in their formulation of Discretised Light–Cone Quantisation (DLCQ). DLCQ is a non–perturbative Hamiltonian field theoretic method. The hope is that within this precise treatment of Quantum Chromodynamics (QCD) something like a constituent quark or parton picture of hadrons can emerge. The ingredients for this are first that a unique vacuum state can be found and second that the low energy hadron spectrum can be described in terms of a low number of quark (and gluon) excitations above this state. The first condition is rigorously satisfied in the front form if zero modes are ignored [3]. The second follows from the related positive definiteness of the light–cone momentum operator. It is the concrete realisation of this in a spectrum which we examine here in QCD in one space and one time dimension (QCD$_{1+1}$). This theory has the advantage of being superrenormalisable.

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In DLCQ space is a ‘box’; in 1+1 dimensions $x^- = \frac{1}{\sqrt{2}} (x^0 - x^1)$ is restricted to a finite interval of length $2L$. Choosing appropriate boundary conditions discretises the Fourier momenta. Thus the field theory problem is reduced to one of finite dimensional matrices which can be directly diagonalised [3]. The first application to QCD\textsubscript{1+1} was by Hornbostel in [4]. Mass spectra and wavefunctions of baryons and mesons were numerically obtained for finite harmonic resolution $K = \frac{L}{\pi} P^+$ where $P^+$ is the total light–cone momentum. $K$ is dimensionless and itself regulates the size of the Fock–space. The continuum limit, $L \to \infty$, must be achieved maintaining finite momentum $P^+$ thus $K$ must become ‘infinite’, namely large enough to enable reasonable extrapolation. This can make overwhelming demands on CPU time. So a second advantage of a Fock–space truncation lies in the computer time and memory saved for numerical calculations.

Fock–space truncation, also called ‘Tamm–Dancoff truncation’, has already seen some justification. Ground state hadrons in QCD\textsubscript{1+1} have been shown in [4] to consist only of a very low number of particles: the ground state of an SU($N$) meson can be described by a single quark–antiquark ($q \bar{q}$) pair and the ground state of an SU($N$) baryon simply by $N$ quarks. This minimal Fock–space truncation is already known to breakdown even for the first excited state. For example, the first excited state of an SU($N$) meson was recently calculated analytically in [5] using the Light–Front Tamm–Dancoff (LFTD) method of [6], a related approach exploiting the advantages of the front form with a Fock–space truncation. They worked in ‘next to leading’ order in Fock–space truncation.

One aim of the present work is to show that, within DLCQ, the same level of truncation works not only for the first but for a number of the lowest lying excited states. In SU(2) and SU(3) colour group gauge theory we will find that a truncation to a low number of particles reproduces a large number of the lowest lying states to a good approximation with calculations taking about one hour CPU time. There is a so–called ‘zero mode problem’ in the front form. Here we ignore these modes to study the physics of the trivial vacuum, reasonable here since for finite $N$, SU($N$) gauge theory has no symmetry–breaking in (1+1) dimensions.

DLCQ in Brief. We shall be cursory in our presentation of the basic elements of DLCQ as the literature is now quite extensive, for example [3,4,7]. The light–cone coordinate convention is for any Lorentz vector $V^\mu$ define $V^\pm = V^\mp \sqrt{2} (V^0 \pm V^1)$. We start with the Lagrangian of QCD with one quark flavour

$$\mathcal{L} = -\frac{1}{4} F^{a\mu\nu} F_{a\mu\nu} + \frac{1}{2} (\bar{\Psi} i\gamma^\mu D_\mu \Psi + \text{h.c.}) - m \bar{\Psi} \Psi,$$  \hspace{1cm} (1)
with the field-strength tensor $F_{\mu\nu}^a = \partial_\mu A_{\nu}^a - \partial_\nu A_{\mu}^a + gf^{abc}A_{\mu}^bA_{\nu}^c$ and the covariant derivative $D_\mu = \partial_\mu - igA_\mu^aT^a$ in the fundamental representation. Thus the colour matrices $T^a$ are related to the Pauli matrices for SU(2) and to the Gell–Mann matrices for SU(3). For the $\gamma$–matrices we choose the chiral representation [8]. The quark field $\Psi$ is just a two component spinor in two dimensions $\Psi = (\Psi_{L,c_i}, \Psi_{R,c_i})^t$, where $L$ and $R$ represent chirality and $c_i$ colour. In one space and one time dimension there is no spin.

We solve this theory by addressing the eigenvalue equation [2]

$$2P^+P^-|\Psi\rangle = M^2|\Psi\rangle. \tag{2}$$

Here $P^+$, $P^-$ are the Poincaré generators of respectively space and time translations: the light–cone momentum and the light–cone energy. The eigenvalue $M^2$ is the Lorentz invariant mass–squared of the eigenstate $|\Psi\rangle$. The link between (1) and (2) is the energy–momentum tensor. Since the generators are constants of the motion under evolution in light–cone time $x^+$ they can be written in terms of the independent fields specified at a given time, say $x^+ = 0$. Ignoring zero modes\footnote{The zero mode $A_0^{+a} \equiv \frac{1}{2}\int_{-\Lambda}^{+\Lambda} dx^- A^{+a}(x^-)$ leads to the gauge invariant Wilson loop around light–cone space $x^-$ [9]. So it cannot actually be gauged away. Its role has been explored elsewhere e.g. [10,11].}, we choose the light–cone gauge $A^{+a} = 0$. Thus there is only one independent field: the fermion component $\Psi_{R,c_i}$. Both the gluon field $A^{-a}$ and the other fermion component are constrained by, respectively, their Maxwell and Dirac equations which are trivially implemented. Quantisation is achieved by imposing anticommutation relations on this independent field $\{\Psi_{R,c_i}(0,x^-),\Psi_R^{\dagger c_j}(0,y^-)\} = \frac{1}{\sqrt{2}}\delta_{c_j}^c\delta(x^- - y^-)$ for a fixed light–cone time $x^+ = y^+ = 0$. Introducing the plane wave expansion as ‘initial data’

$$\Psi_{R,c_i}(0,x^-) = \frac{1}{\sqrt{2}\sqrt{2L}} \sum_n \left( b_{n,c_i} e^{-ik_n^+ x^-} + d_{n,c_i} e^{+ik_n^+ x^-} \right), \tag{3}$$

we obtain as the non–vanishing anticommutators $\{b_{n,c_i}^{\dagger}, b_{m,c_j}\} = \{d_{n,c_i}^{\dagger}, d_{m,c_j}\} = \delta_{c_i}^c\delta_{n,m}$. As in [4], we impose antiperiodic boundary conditions for the fermion field: $\Psi_{R,c_i}(x^- + 2L) = -\Psi_{R,c_i}(x^-)$. This gives $n \in \{\frac{1}{2}, \frac{3}{2}, ..., \frac{\Lambda}{2}\}$. The cut–off $\Lambda$ drops out after normal ordering, reflecting superrenormalisability. The generators $P^\pm$ can thus be expressed in terms of the Fock–modes $b_{n,c_i}$ and $d_{n,c_i}$. Then : $P^+ :$ is just proportional to the number operator in quarks and antiquarks even in the interacting theory and : $P^- :$ has a kinetic term also proportional to the number operator and an interaction term bilinear in the quark current $j^{+a}(x^-) = \frac{2}{g}\Psi_R^{\dagger c_i}(T^a)^{c_i}_{c_j}\Psi_{R,c_j}$. The detailed expression for the interaction term can be found in [4,7]. Its main feature is the linear Coulomb potential between the quark currents obtained from elimination of $A^{-a}$.\footnote{The zero mode $A_0^{+a} \equiv \frac{1}{2}\int_{-\Lambda}^{+\Lambda} dx^- A^{+a}(x^-)$ leads to the gauge invariant Wilson loop around light–cone space $x^-$ [9]. So it cannot actually be gauged away. Its role has been explored elsewhere e.g. [10,11].}
We now discuss how the wavefunctions are represented in the Fock–basis. For a given colour group SU($N$), the colour–singlet state $|\Psi\rangle$ depends on the baryon number $B$ and the harmonic resolution $K$. Group theoretic aspects of the following procedure can be found in [12]. To construct an SU($N$) meson state ($B = 0$) at a fixed harmonic resolution $K$, we begin with a two particle colour–singlet state: $|\text{meson}\rangle = \delta_{c_1} \delta_{c_2} b_{n_1}^c n_1 d_{n_2,c_2}^l |0\rangle$. To this we can append colour–singlet $q\bar{q}$ creation operators with equal or increasing momentum until the total momentum $K$ is saturated by the sum of the parton momenta $\sum_i n_i$. Thus the resulting state must be multiplied by a Kronecker $\delta_{K, \sum_i n_i}$. For an SU($N$) one–baryon state ($B = 1$) we proceed analogously. As a basic state we contract $N$ quarks with the antisymmetric epsilon–tensor of rank $N$: $|\text{baryon}\rangle = \varepsilon_{c_1,c_2,\cdots,c_N} b_{n_1}^{c_1} b_{n_2}^{c_2} \cdots b_{n_N}^{c_N} |0\rangle$. Now, as above, we can append as many $B = 0$ operators, $q\bar{q}$ pairs, as the total momentum $K$ allows. The Hilbert space so constructed is overcomplete and not orthonormal. This is dealt with in the code as in [4] by weeding out states with zero inner product $\langle i | j \rangle$.

These are the tools for directly solving Eq. (2). For this purpose a computer code was set up to construct the Fock–space and calculate $\tilde{M}^2$ directly for given $K$, $N$ and $B$. The dimensionless parameter $\lambda = (1 + \pi m^2 / g^2)^{-1/2} \in [0, 1]$ allows features of the spectrum over an entire range of couplings or masses to be seen in one finite domain plot. Eventually, Eq. (2) can be rewritten purely in terms of dimensionless quantities as

$$\tilde{M}^2 |\Psi\rangle \equiv \frac{\pi M^2 / g^2}{1 + \pi m^2 / g^2} |\Psi\rangle = (1 - \lambda^2) K \tilde{T} |\Psi\rangle + \lambda^2 K \tilde{V} |\Psi\rangle,$$

where $\tilde{M}^2$ is a dimensionless invariant mass–squared, and $\tilde{T}$ and $\tilde{V}$ are sums over the Fock–modes with all dimensionful quantities, such as $m, g$ and $L$, stripped off. Their detailed form can be found in [4,7]. For historical reasons, the spectrum is normalised in units of $g/\sqrt{\pi}$ which is the mass of the lowest boson in the Schwinger model though no such state exists in the massless fermion limit of this theory.

Fock–space Truncation in a Typical DLCQ Spectrum. To be concrete, consider the mass spectrum with full Fock–space allowed at fixed momentum $K = 33/2$. This value was chosen because the lowest lying states in the spectrum became stable for various couplings in this region of $K$, but remained computable in a reasonable amount of CPU time (the reader is referred to [7] for details). For an SU(3) baryon $K = 33/2$ means storing 812 states and thus calculating $812^2$ matrix elements. This procedure, on a DEC OSF/1 V1.3, takes about 70 minutes the least part being devoted to diagonalisation. This time can be further reduced by a factor of three by Fock–space truncation. The Fock–space truncations we use in this letter are defined in Table 1.

A Fock–space truncation which includes only sector 1 is a one–sector trun-
cation. Taking sectors 1 and 2 into account is called a two-sector truncation. When we say a hadron state is well-described by a particular sector truncation we mean two things: (1) that the invariant mass-squared in the truncated approximation, denoted $\tilde{M}_{\text{tr}}^2$, agrees with that calculated with the full available Fock-space for finite $K$, and (2) that the result agrees with other methods, such as semi-analytic [13] or lattice [14], where such data is available and reliable. As mentioned [4], the ground state of each SU($N$) hadron is well-described by a one-sector truncation. This has been cross-checked in [7] where it is also clear that excited states are poorly described in this approximation. This is a consequence of the absence of interactions between the different Fock-space sectors. Some of these interactions are implemented when the second Fock-space sector is taken into account.

Let us turn then to a typical excitation spectrum computed with DLCQ in the two-sector truncation. There is no need to show both mesons and baryons for SU(2) and SU(3) as they are all qualitatively similar. We just give as an example in Fig. 1 only the SU(3) baryon spectrum.

In such plots there are places where finite $K$ artifacts are dominant: in particular the mass gaps between the bunches of states at low $\lambda$ and the mass gap at $\lambda = 1$. This was seen by studying how the gaps change for varying $K$. The detailed reasons for these artifacts we come to later. But not all the gaps are artifacts: those between the lowest states for intermediate $\lambda$ values are stable against variation in $K$. Another reason to regard these gaps as physical is that they emerge from the continuum spectrum which these lowest lying states already roughly form in the free theory limit $\lambda = 0$. Thus this low energy regime is one where the real physics, well understood already in [4], is perceptible even at finite $K$. The physics is dominated by the linear Coulomb potential giving increasing repulsion (with increasing $\lambda$ or $g$) between partons in the baryon. On the other hand, with a two-sector truncation pair-production terms are allowed in the interaction [4,7]. Thus the invariant mass of the hadron should eventually decrease as $\lambda \rightarrow 1$ since the Coulomb energy is lost into the production of more partons. This we observe except for states with $\tilde{M}_{\text{tr}}^2 > 80$ and coupling $\lambda > 0.6$. This is, in this case, a consequence of the Fock-space truncation: no further partons can be created to absorb the increasing Coulomb energy. Thus we encounter the first place where the Fock-space truncation has harmed the physics. Even without this kind of Fock-space truncation, this would still occur, maybe at higher states, for any finite $K$, as $K$ itself is a truncation in the number of allowed partons. At $\lambda = 1$ we see a group of the lowest states going to zero. This is the massless fermion limit being manifested: in 1+1 dimensions these particles are in colinear motion, so one can always find a frame in which the total mass of the system is zero. As mentioned, the finite $K$ artifacts are mainly at the extreme regimes in $\lambda$ -- in the gaps at $\lambda = 0$ and $\lambda = 1$. With higher $K$ these turn into a continuum indicating the continuous momentum that can be assigned to free or infinitely
coupled constituent quarks.

As mentioned, for $0.3 < \lambda < 1.0$ we find a mass gap between the ground state and the first excited state in the SU(3) baryon. This seems stable as $K \to \infty$ [4,7]. In this region we propose that real physics is best approximated for finite $K$. However, how do these results in this region compare with those from a calculation at ‘full’ (at given $K$) Fock–space?

**Success of Truncation at Finite $K$.** We now make a quantitative comparison between the two–sector truncation and the full Fock–space for fixed momentum $K$. The actual spectra are not presented here but can be found in [7]. We define the quantity $\Delta_i = \tilde{M}^2_{\text{trunc},i} - \tilde{M}^2_i$, which is the difference between the mass–squared of the $i$th state of the two–sector calculation and the corresponding quantity from the full calculation. This difference is shown in Fig. 2 for the first 20 states over the whole range of $\lambda$ for the SU(2) and SU(3) hadrons.

The most obvious feature in these plots is the ‘valley’ in Fig. 2c corresponding to the SU(3) meson. This occurs in the non–interacting theory limit. The effect is due to the removal of a lowest energy $qqqq$ state as a result of the two–sector approximation. The energy of this missing state in the free theory is slightly above some of the $qqqq$ states. Because $K$ is finite there is no density of states around it that would better match the energy of these neighbouring $qqqq$ states once this state is removed in the two-sector truncation. In the truncated space spectrum this missing state is replaced as the seventh state by a $qqqq$ state with slightly lower energy so that the difference $\Delta_7$ is negative. Hence the negative valley. Some higher states are of course degenerate in the full–calculation and remain in degeneracy in the two-sector truncation. Thus they remain comparable in energy and the valley disappears beyond the seventh state until a similar thing occurs again at the fifteenth state. With higher $K$ all the states in the spectrum would be closer together and the removal of a higher Fock–space state would not cause such mismatches.

In general then, we can say at worst the lowest six, and at best the lowest ten, states agree for the two choices of Fock–space size in the whole regime of $\lambda$ by a two–sector truncation. Even for moderate values of $\lambda$ ($\lambda < 0.6$) the lowest 20 states fit perfectly. Evidently in the strong coupling regime the numerics for the higher states in a two–sector truncation are poor. The reason for this behaviour has been discussed above: suppression of pair–production by the Fock–space truncation. For the SU(2) hadrons the maximum difference for the highest considered state ($\Delta = 14$) means a relative difference compared to the full calculation of less than 8% . For the SU(3) hadrons the maximum difference ($\Delta = 22$) means less than 10% relative difference.

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3 We have done the calculation with a truncation at six particles and observed the valley to vanish with maximum absolute deviation $\Delta_{20} = 0.5$. 
The conclusion of this analysis is that at least for the first five excited states in a given baryon number spectrum there is a broad range of couplings for which the truncated Fock–space computation is consistent at least with the full calculation. We next give some actual numbers for invariant masses.

**Numerical Results for First 5 Mass Eigenstates.** The numerical results of our two–sector truncation are presented in Table 2 for the lowest five excited states and the ground state for moderately large values of dimensionless coupling which we indicate in terms of $\lambda$ for two cases: the SU(2) meson and the SU(3) baryon. The intuitive picture this approximation would seek to justify is: for the meson of a pair of $q\bar{q}$ states in relative motion, for the baryon of ground state $qqq$ state combining with a $q\bar{q}$ pair. To what extent is this a valid physical picture for these states? To judge we compare our results for the ground state with those of [14] for coupling $\lambda = 0.82$, 0.58 and 0.33. Hamer [14] gets respectively $\tilde{M}_0^2 = 2.46$, 3.90 and 4.21. We get $\tilde{M}_{true,0}^2 = 2.33$, 3.86 and 4.19 as shown in Table 2. The comparison is reasonable. It must be pointed out that Hamer [14] worked in the continuum limit ($K \to \infty$ in our case) by extrapolating his values obtained for a finite grid size via a Padé method. It may seem inappropriate to compare Hamer’s continuum result [14] to our results at finite $K$. We have checked that for very small fermion mass $m$, indeed our ground state results are poor compared to the lattice result\(^4\). However, as stated above, for intermediate values of the coupling, $0.33 < \lambda < 0.82$ (corresponding to $0.40 < m/g < 1.6$), the comparison is excellent. In other words finite volume artifacts are small in this range of coupling for the lowest lying states. Alternately one could correctly regard finite $K$ as a truncation of the Fock–space a priori, and thus the question becomes: how good is that as an approximation to the continuum limit? Again, for the given coupling range it is very good.

For large $\lambda$ or small fermion mass $m$ the finite volume effects grow, as is also known from the Schwinger model [15]. Extrapolation using results over a range of $K$ can be applied to improve the DLCQ results, as will be shown in a following paper [16]. But here quantitative comparison with the results of Light–Front Tamm–Dancoff [5] for ground states and first excited states can be made. Our extrapolation results and comparison to these authors will be presented elsewhere [16].

The remaining values in Table 2 for the five excited states are presented in order for future comparison with other methods such as lattice gauge theory or LFTD. The inherent advantage of DLCQ is that a huge portion of the excited state spectrum is obtained with the same (and relatively small) computational

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\(^4\) Hamer [14] worked in units of $m/g$ and $M/g$. We have simply converted his results into our units.

\(^5\) Our results are about 50% smaller than the continuum limit results of [14].
effort that goes into the ground state. This we believe is DLCQ’s advantage over other methods. A flaw may be that the plane wave basis implicit in DLCQ may be too restricted a class of basis functions to give the higher states with any reasonable accuracy. Nonetheless, the plots of Fig. 2 give some cause for confidence in the results of Table 2: there is internal consistency in the results of DLCQ. Beyond the fifth state there is some reason to put less trust in the accuracy of a two-sector truncation at such a low \( K = 33/2 \), as evidenced by the anomalous valley discussed earlier. However, the Fock–space truncation now permits calculations at higher \( K \) followed by an extrapolation to the continuum within reasonable CPU time. This is treated in a separate work [16].

**Conclusions.** We have used DLCQ to compute the spectrum of invariant masses of mesons and baryons in one–flavour SU(2) and SU(3) gauge theory in (1+1) dimensions. With a two–sector truncation on the Fock–space we find excellent agreement for the masses as compared to a full Fock–space computation for the first 20 states. For a region of coupling ranging from moderately weak to strong we find an absence of finite \( K \) artifacts enabling us to make numerical ‘predictions’ for up to the first six states in the SU(2) meson and SU(3) baryon spectra. Hopefully comparable computations from lattice or LFTD will be done in the near future enabling a check of these numbers. For QCD in 3+1 dimensions one expects chiral symmetry–breaking to impact on the spectrum of the lighter mesons. The neglected zero modes may play a pivotal role in this. More seriously, renormalisation is a significant problem to be overcome in higher dimensions. Finally, confinement–generation in 3+1 dimensions remains a big unknown in a DLCQ treatment. Modulo these features, it is a reasonable question to ask how much of the above insight should work, say, for the intermediate physical mesons where the constituent quark model is known to work well. The key mechanism in our spectra was played by the linearly confining potential. If such a potential were accessible in 3+1 dimensions via DLCQ treatment of QCD, then there is hope that Fock–space truncation should work. In other words, a similarly intuitive picture of the intermediate mass hadrons consistent with the parton model would emerge but expressed in terms of fundamentally *QCD degrees of freedom*. The method deserves further examination in more complicated theories.

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Table 1
Definition of the two distinct Fock–space sectors under consideration for SU(2) and SU(3) mesons ($B = 0$) and baryons ($B = 1$).

| sector | $B = 0$ | $B = 1$ | $B = 0$ | $B = 1$ |
|--------|---------|---------|---------|---------|
| 1      | $q\bar{q}$ | $qq$    | $q\bar{q}$ | $qq$    |
| 2      | $qqq\bar{q}$ | $qqq\bar{q}$ | $qqq\bar{q}$ | $qqq\bar{q}$ |

Fig. 1. **SU(3) baryon mass spectrum in the two–sector truncation.** The mass spectrum of the SU(3) baryon with a two–sector truncation defined in Table 1. The units are the dimensionless mass $\tilde{M}_{\text{tru}}^2$ versus the dimensionless coupling $\lambda$. The harmonic resolution used was $K = 33/2$. 
Table 2
Numerical results of our two-sector truncation calculation for the ground state and the lowest five excited states. The results are shown for the SU(2) meson and the SU(3) baryon. All entries are in units of $\tilde{M}_{\text{tru},i}^2$ as defined by Eq. (4). The values of coupling are presented in dimensionless units of $\lambda$.

| SU(2) meson | SU(3) baryon |
|-------------|--------------|
| $\lambda$  | 0.82 0.58 0.33 0.28 0.18 0.14 | 0.82 0.58 0.33 0.28 0.18 0.14 |
| $\tilde{M}_{\text{tru},0}^2$ | 2.33 3.86 4.19 4.26 4.21 4.17 | 5.41 9.83 10.24 10.25 9.86 9.65 |
| $\tilde{M}_{\text{tru},1}^2$ | 5.14 6.40 4.46 5.21 4.69 4.48 | 12.78 17.25 13.62 12.84 10.95 10.31 |
| $\tilde{M}_{\text{tru},2}^2$ | 7.47 8.40 6.18 5.79 5.00 4.75 | 15.01 21.50 15.08 13.89 11.76 11.17 |
| $\tilde{M}_{\text{tru},3}^2$ | 8.53 10.09 6.77 6.18 5.07 4.79 | 15.17 22.46 15.40 14.41 12.79 12.36 |
| $\tilde{M}_{\text{tru},4}^2$ | 8.79 11.44 7.30 6.73 5.86 5.65 | 15.63 24.76 16.47 15.49 13.39 12.77 |
| $\tilde{M}_{\text{tru},5}^2$ | 9.05 12.43 7.37 6.76 5.88 5.66 | 15.97 26.00 17.29 15.84 13.89 13.45 |
Fig. 2. Difference of the mass spectra in a two-sector truncation and the full calculation. The dimensionless difference $\Delta_i$ for the first 20 states is plotted over all $\lambda$. The harmonic resolution for both the two-sector truncation and the full Fock-space is in the cases a)–c) $K = 16$, for the SU(3) baryon in d) $K = 33/2$. 