Gamma-ray bursts and gravitational radiation from black hole-torus systems

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Cosmological gamma-ray bursts (GRBs) are probably powered by systems harboring a rotating black hole. This may result from hypernovae or black hole-neutron star coalescence. We identify short/long bursts with hyper- and suspended-accretion states around slowly/rapidly rotating black holes [van Putten & Ostriker, ApJL, 552, L31, 2001]. Black holes may be activated into producing outflows by a surrounding torus magnetosphere, in the form of baryon poor jets as input to the observed GRB/afterglow emissions. Here, we attribute these outflows to a differentially rotating gap in an open flux-tube along the axis of rotation of the black hole [van Putten, Phys. Rep., 345, 1 (2001)]. A high incidence of the black hole luminosity into the surrounding matter is expected by equivalence in poloidal topology to pulsar magnetospheres. For long bursts, this predicts a large fraction of the black hole spin-energy emitted in gravitational waves by a quadrupole moment in the surrounding torus [van Putten & Levinson, ApJL, 555, L41, 2001]. This suggests that long GRBs may be the most powerful LIGO/VIRGO burst sources of gravitational waves in the Universe with an expected duration of 10-15s on a horizontal branch of 1-2kHz in the $f(\dot{f})$-diagram [van Putten, Phys. Rev. Lett., 87, 091101, 2001]. Gravitational wave-emissions from GRBs, therefore, promise calorimetric evidence for Kerr black holes.

1 Introduction

Cosmological gamma-ray bursts (GRBs) are the most enigmatic events in the Universe (Fig. 1). Their emissions are characteristically nonthermal in the few hundred keV range with a bi-modal distribution in durations, of short bursts around 0.3s and long bursts around 30s. Redshift determinations from long bursts indicate a cosmological origin, probably associated with the formation of young massive stars. GRBs, therefore, are probably most frequent within a redshift $z = 1 - 2$. The recent proposal that GRB relics could be found in some of the galactic soft X-ray transients (SXTs), notably so GRO J1655-40 and V4641 Sgr, further suggests an association with black holes of about $3 - 14 M_\odot$.

The inner engine producing the GRBs should be energetic and compact. Angular momentum forms a canonical energy reservoir, and GRB inner engines are probably no exception. This is consistent with breaking of spherical symmetry. Hypernovae, then, follow up on this idea by postulating massive young stars in binaries as their progenitors. High angular momentum is also present in coalescing black hole-neutron star systems.
Figure 1: Shown are the locations of 2704 GRBs from the GRBs from the BATSE Catalogue over a nine-year period. The projection is in galactic coordinates. (Courtesy of NASA Marshall Space Flight Center, Space Sciences Laboratory.) The isotropic distribution and a $< V/V_{max} >$ distinctly less than the Euclidean value of 1/2 (Schmidt 1999) are indicative for their cosmological origin. For long bursts, this is further evidenced by redshifts of order unity from GRB afterglow emissions and a probably association with star-forming regions (Paczynski 1998; Bloom et al. 2000). Hence, long bursts are expected to track closely the star-formation rate, which peaks at about $z = 1 - 2$. Remnants of GRBs are potentially found in some of the Soft X-ray transients, with notable candidates GRO J1655-40 (Israelian et al, 1999) and V4641Sgr (Orosz et al., 2001)
Here, I discuss some physical aspects of black hole-torus systems and a few observational predictions. Black holes may be active in accord with the Rayleigh criterion, in contact with a torus magnetosphere supported by baryonic matter. This is expected to be manifest in non-thermal emissions, in outflows along an open flux-tube on the axis of rotation of the black hole and various emissions from a surrounding torus:

- At supercritical magnetic field-strengths, outflows may spontaneously be created by frame dragging-induced potential differences, as follows from a perturbative calculation in Hawking’s approach. In competition with equilibration by charge separation, pair-creation is expected to be confined to a gap along the open flux-tube. Asymptotic boundary conditions on the horizon and infinity, linked by current continuity, define the net dissipation in the gap, and hence a powerful outflow whenever the black hole spins more rapidly than twice the angular velocity of the torus.

- The torus is expected to be luminous in several channels, notably so in gravitational radiation, neutrino emissions and Poynting flux winds. Powered by the spin-energy of the black hole, the fluence in gravitational waves may reach about 1% of the black hole mass. The coupling to the rotational energy of the black hole operates by equivalence in poloidal topology to pulsar magnetospheres. Detection by LIGO/VIRGO of these emissions may result in calorimetric evidence of Kerr black holes, whenever the fluence is determined to be in excess of the rotational energy of a rapidly rotating neutron star.

These prospects suggest several observational predictions.

Increasing evidence towards clustering in the true energy in GRB emissions indicates a standard opening angle of about $\theta_H \approx 35^\circ$ of the open magnetic flux-tube on the horizon of the black hole. This predicts a cut-off $\theta_j \leq 35^\circ$ in the observed opening angles $\theta_j$ on the celestial sphere in any large statistical sample. Furthermore, with outflows created in long and short bursts alike, HETE-II may detect afterglows also to short GRBs (see also ). The latter is expected to differ mostly in net fluence, as their inner engine operates for a significantly shorter time than in long GRBs.

Gravitational radiation appears to be a major channel in the emissions from the torus in long bursts, along with emissions in neutrinos and winds coming off the torus. Calculations in the suspended-accretion state indicate a net luminosity in gravitational waves of about one-third the net luminosity of the black hole. This amounts to a fluence $E_{gw}$ about 1% of the mass-energy of the central black hole. The frequency $f_{gw}$ in these emissions at twice the Keplerian
frequency of the torus, as it develops a quadrupole moment in its mass distribution, is expected to be about 1-2kHz for a black hole mass of about $10M_\odot$. This range overlaps with the design bandwidth of 0.1-1.5kHz of the upcoming Laser Interferometric Gravitational Wave Observatories LIGO/VIRGO.

This raises an unanticipated prospect: calorimetric evidence for Kerr black holes from the emission in gravitational waves from the torus. Indeed, consider the product $\alpha = 2\pi E_{gw} f_{gw}$, which expresses a measure for the ratio of rotational energy to the linear size of the inner engine. It appears that $\alpha$ from black hole-torus systems may reach values in excess of those attainable by rapidly rotating neutron stars. A LIGO/VIRGO detection of a large $\alpha$, therefore, individually or as an average over a sample of detections, would be evidence for the Kerr relationship $E_{\text{rot}} \sim M/3$ between the rotational energy $E_{\text{rot}}$ and the mass $M$ of a rapidly rotating black hole.

The proposed association of gamma-ray bursts to black hole-torus systems will be reviewed in §2, and prospects for GRBs as potential LIGO/VIRGO sources is outlined in §3. We close with a comment on the potential for calorimetric evidence of Kerr black holes in GRBs.

2 A theory of gamma-ray bursts from black hole-torus systems

A black hole-torus system is of compact dimension, consistent with the short time-variability in the GRB light-curves and the proposed GRB-SXT association. The mass in the surrounding torus or disk will be limited, in both the hypernovae and binary black hole-neutron star coalescence scenario. This introduces relatively short time-scales of accretion, leaving a central Kerr black hole as the major energy reservoir. This poses two questions: what accounts for the duration in long GRBs and how can the rotational energy of the black hole create baryon poor jets?

2.1 Formation of black hole-disk or torus systems

A black hole-torus system may form from binary black hole-neutron star coalescence. Here, the neutron star gradually approaches the black hole by angular momentum loss in gravitational radiation. The neutron star will then be subject to tidal interactions, which may lead to break-up outside the inner most stable circular orbit (ISCO) when the central black hole is sufficiently small in mass. For this to happen, canonical estimates provide a bound of $3.7M_\odot$ on non-rotating black holes and a bound of $28M_\odot$ on rapidly rotating black holes. This indicates a substantially wider window of mass for the rotating case. It follows that a torus is more likely to form around a Kerr black hole.
Figure 2: Shown is the distribution of black hole masses in X-ray novae. The top four are XTE J118+408 (McClintock et al., 2001), V4641 Sgr (Orosz et al., 2001), 4U 1543-47 (Orosz et al., 1998) and Nova Vel 1993 (Filippenko et al., 1999); the lower seven are from Bailyn et al. (1998). This mass distribution manifests a certain diversity in black hole masses of about $3 - 14M_\odot$. [Reprinted from van Putten, Physics Reports, 345 ©2001 Elsevier B.V.]

The collapsar, failed supernova or hypernova scenario envisions the collapse of the center of a young massive star with high angular momentum. The origin of the angular momentum is most likely orbital angular momentum from the progenitor binary system. While the details of orbital angular momentum transfer into the collapsing star are somewhat uncertain, collapse of a rapidly rotating object is expected to result in a compact core surrounded by matter stalled against an angular momentum barrier. If the core forms a black hole in prompt collapse then, furthermore, the black hole will have a minimum mass, sufficient to account for the angular momentum $J_H$ in view of the Kerr constraint $J_H^2 \leq M^2$ (in geometrical units, with $M$ denoting the Schwarzschild radius $Gm/c^2$, where $G$ is Newton’s constant, $m$ the mass of the black hole and $c$ the velocity of light). For example, a Lane-Emden relationship with polytropic index $n = 3$ for the progenitor star gives $M \geq 10M_\odot$, consistent with the observed range of $3 - 14M_\odot$ in SXTs shown in Fig. 2.
In both scenarios, a magnetized neutron star or young massive star - represented by a magnetic moment density – will result in a disk or torus endowed with a net poloidal flux. It may be appreciated that the mass in the disk or torus a finite in a much more stricter sense than in analogous configurations believed to exist in active galactic nuclei. With magnetic regulated accretion, accretion of $0.1M_\odot$ becomes fairly rapid on a time-scale of a second or less onto onto a $10M_\odot$ mass black hole. Depleting the surroundings of baryonic matter inevitably prevents any further activity of the black hole. This suggests that additional physical processes should account for the relatively long duration in long bursts. In a recent proposal, the magnetic moment density of the surrounding torus is believed to permit a suspended-accretion state for the duration of spin-down of the central black hole. A bi-modal distribution of durations then occurs when the ratio of black hole-to-disk or torus mass is large.

2.2 The lowest energy state of the black hole

The black hole will be surrounded by a torus magnetosphere, supported by the accretion disk. The black hole will adjust to a lowest energy state by developing an equilibrium magnetic moment

$$\mu_H \simeq aBJ_H,$$  

where $a = J_H/M$ denotes the specific angular momentum of a black hole with mass $M$ and angular momentum $J_H$ and $B$ denotes the (average) poloidal magnetic field. This results from a minimum in the energy $\mathcal{E}(q) = (1/2)Cq^2 - \mu_H B$, where $C \simeq 1/r_H$ denotes the capacitance of the black hole, $q$ the charge on the horizon, and Carter’s identity $\mu_H = qJ_H/M$ (“no fourth hair”). The minimum of $\mathcal{E}$ at $q \simeq BJ_H$ recovers Wald’s result. Similar results are found in a largely force-free magnetosphere. This equilibrium magnetic moment preserves an essentially maximal and uniform horizon flux. This serves to preserve a strong coupling to the magnetosphere and, hence, to the inner face of the surrounding torus. It also permits the black hole to support an open flux-tube to infinity along its axis of rotation, particularly so in a suspended accretion state. Frame-dragging will act on this flux-tube, to to produce a differentially rotating gap for the creation of baryon poor outflows.

2.3 Hyper- and suspended-accretion states for a ring

GRBs show a bi-modal distribution in durations, as show in Fig. 3. We attribute this to different modes of angular momentum losses in black hole plus disk or torus systems.
As magnetic fields can be very efficient in mediating angular momentum transport, magnetic regulated accretion times tend to be short. Indeed, the accretion of a magnetized ring is illustrative, showing evolution towards a finite-time singularity

$$\varpi = (1 - t/t_f)^p$$

of its radius $\varpi = R(t)/R(0)$, where $R(0)$ denotes the initial radius and $t_f$ the final time of collapse. Here, $p = 1/2$ and 2 for a split monopole geometry (SMG) and toroidal field geometry (TFG), respectively. The final time in both field geometries is governed by the ratio of kinetic-to-magnetic energy $\delta E_k/\delta E_B$ - a free parameter, at present not well constrained from first principles or current numerical simulations. For a fiducial value of $10^2$, SMG applied to the initial evolution ring and TFG applied to the final evolution of the ring indicates accretion times $t_f$ of a few seconds or less – consistent with the timescale of short bursts. This analysis does not take into account any action by the black hole back onto the ring, i.e.: the results apply to the accretion onto slowly rotating or Schwarzschild black holes.

What then, may account for the duration of long bursts? A ring surrounding a rapidly rotating black hole will support, by a magnetic moment density, poloidal magnetic field-lines connected the ring to the horizon. These field-lines permit energy and angular momentum transport from the spin of the black hole into the ring. This process operates by equivalence in poloidal topology to pulsar magnetospheres, wherein the ring and the horizon of the black hole are, respectively, equivalent to a pulsar and infinity. The angular velocity of the equivalent pulsar is the relative angular velocity between the ring and the black hole. When the black hole spins more rapidly than the ring, the ring receives angular momentum like a pulsar being spun-up when infinity wraps around it.

The ring – an element of the disk or torus – will assume a suspended-accretion state arises when radiative losses in energy and angular momentum to infinity are replenished by gain from the central black hole. For a balance on magnetic torques alone, we find a critical angular velocity $\Omega^*_H$ of the black hole:

$$\Omega^*_H = \Omega_T[1 + (f_w/f_H)^2 \ln(\theta_0/2)]^{-1}$$

for fractions $f_w$ and $f_H$ of magnetic flux which connect to infinity (in an outgoing Poynting flux-wind) and the black hole (in an ingoing Poynting flux-wind), respectively, where $\theta_0$ denotes the minimum poloidal angle in TFG.
Figure 3: Shown is the bi-modal distribution of GRB durations of the 4B Catague, set by a T90 duration parameter, on lightcurves integrated over all 4 channels ($E > 20\text{keV}$). (Courtesy of NASA Marshall Space Flight Center, Space Sciences Laboratory.) The population of long bursts is probably associated with young massive stars and, hence, with a redshift of $z = 1 - 2$. This indicates a redshift corrected mean value of the intrinsic duration of about 10-15s. In van Putten & Ostriker (2001), short bursts are identified with magnetic regulated hyperaccretion onto slowly rotating black holes, and long bursts with rapidly rotating black holes in a state of suspended accretion. Long bursts are potential LIGO/VIRGO sources of gravitational radiation by gravitational radiation from the torus, derived from the spin-energy of the black hole. This indicates a mean duration of 10-15s of gravitational wave-emissions commensurate with the redshift corrected GRB-event, for a cosmologically nearby sample within the detection sensitivity of LIGO/VIRGO.
This state lives as long as the black hole spins rapidly

\[ t_{\text{long}} = 88s \left( \frac{M}{10M_\odot} \right) \left( \frac{M/M_d}{100} \right) \left( \frac{E_k/E_B}{100} \right) g^2(\theta_0), \]

where \( g(\theta_0) \) denotes a geometrical factor of order unity. We conclude that a bimodal distribution in duration occurs due to hyper- and suspended-accretion states whenever the ratio \( M/M_d \) is large.

2.4 Outflows from a differentially rotating gap

Frame-dragging introduces differential rotation along the axis of rotation of a Kerr black hole. Flux-tubes on a differentially rotating space-time background thus tend to develop potential differences by Faraday-induction. These potentials may drive large-scale currents. See \(^{29}\) for a perturbative discussion about a Wald field in Hawking’s approach. At the same time, the magnetosphere tends to equilibrate locally by charge-separation. These two processes are generally in competition with one another, subject to current continuity linking asymptotic boundary conditions on the horizon and at infinity. In one proposal for the boundary conditions, discussed below, these considerations indicate that outflows may be created by a macroscopic gap in an open flux-tube along the axis of rotation of the black hole.

An open flux-tube supported by the black hole is endowed with ingoing boundary conditions on the horizon of the black hole and outgoing boundary conditions at infinity. In field theory, these asymptotic boundary conditions assume conjugate radiative-radiative boundary conditions. In the continuum limit of a plasma which is asymptotically in charge-separated equilibrium, these become slip-slip boundary conditions: the angular velocities of the flux-surfaces on the horizon may differ from that of the black hole, and the angular velocity at infinity may be non-zero. In contrast, a flux-surfaces supported by baryonic matter are fixed to its angular velocity, namely that of the disk or torus (a no-slip boundary condition). This will hold to within a fair approximation over an appreciable scale relative to the system size. Recall that this well-known corotation law is based on the singular limit of perfect conductivity; deviations of order unity will arise over distance scales of order \( 1/\epsilon \), upon deviations from the corotation charge-density to order \( \epsilon \).

Equilibration towards a force-free state introduces an asymptotic null-condition on the current carried by the flow going into the black hole: \( j^2 \to 0 \) upon approaching the horizon, where \( j^b \) denotes the four-current. This expresses the condition that the current becomes asymptotically convective: \( j^r = \pm \alpha j^t \), where \( \alpha \) denotes the redshift factor on-axis of the Kerr black
Figure 4: Images of closed (top left) and open (bottom left) topology of flux-tubes in the solar atmosphere from the Solar Heliospheric Observatory (Courtesy of the SOHO/EIT consortium, ESA-NASA). The footpoints of the tubes are rooted in the surface of the Sun with no-slip boundary conditions. The open tube represents a violent coronal mass ejection. The present proposal builds on potentially similar structures in a torus magnetosphere around a black hole, wherein the first corresponds to field-lines connecting the torus (with no-slip boundary conditions) to the black hole (with slip boundary conditions) as schematically indicated in ($\alpha$); the second corresponds to field-lines extending from the black hole to infinity (with slip boundary conditions on the horizon) and from the torus to infinity (with no-slip boundary conditions on its surface) as schematically indicated in ($\beta$). Since open flux-tubes ($\beta$) form from closed loops ($\alpha$), these fluxes are the same in magnitude and opposite in sign for the black hole and the torus. This co-axial flux-structure permits current closure at infinity.
hole. In approaching the horizon, drift-currents are suppressed by a divergent Lorentz factor. Here, we shall consider the proposal that the boundary condition at infinity is similar in an ultrarelativistic outflow. It would be of interest to study this proposal self-consistently with the micro-physics in the gap.

Frame-dragging appears explicitly in the expression for the electric charge-density \( \rho \) in the equilibrium charge-separated limit. Indeed, the equilibrium charge-density is associated with a time-like coordinate which is orthogonal to the azimuthal Killing vector. Hence, this charge-density corresponds to the density-at-infinity as seen by zero angular momentum observers (ZAMOs); the “true” angular velocity of a flux-surface is that relative to a local ZAMO with angular velocity \(-\beta\). Consequently, we have the expression \( \rho = -(\Omega + \beta)B/2\pi \) as a modified Goldreich-Julian density. The asymptotical condition \( j^2 = 0 \) on the horizon and infinity now expresses electric current mediated by convection of this modified Goldreich-Julian density. Integrating over an effective area corresponding to a given flux surface \( A_\phi = \text{const.} \), we find current sources \( I_- = \Omega_-A_\phi \) at infinity and \( I_+ = (\Omega_H - \Omega_+)A_\phi \) on the horizon. Here, \( \Omega_- \) and \( \Omega_+ \) denote the Boyer-Lindquist angular velocities of the two asymptotically equilibrated sections attached to infinity and the horizon, respectively. Current continuity enforces the condition \( \Omega_- = \Omega_H - \Omega_+ \).

Global current closure may obtain over the surrounding torus. Here, we appeal to a potential similarity to solar flares, as observed by the Transient Region Corona Experiment (TRACE) and the Solar Heliospheric Observatory (SOHO). While magnetic field-lines form closed loops when supported by compactly supported current sources, these loops are occasionally unstable, and flare as open prominences, as shown in the left column of Fig. 4. This suggests that open flux-tubes might form from loops in the torus magnetosphere connecting the black hole and the torus, sketched in the right column of Fig. 4. If so, this gives rise to an inner open flux-tube supported by the magnetic moment of the black hole and outer open flux-tube supported by the magnetic moment of the inner face of the torus. Creating open flux-tubes in the fashion is accompanied by an algebraic constraint: the inner and outer flux-tubes carry a magnetic flux which is equal in magnitude and opposite in sign. Applying the same asymptotic boundary condition \( j^2 = 0 \) to the outflow from the torus – and this is expected to be an approximation to within order unity – we obtain global current closure in the form of \( I_- = I_+ = I_T = \Omega_T A_\phi \), where \( \Omega_T \) denotes the angular velocity of the torus. The result is a differentially rotating gap between forementioned two equilibrium sections, one attached to infinity and the other attached to the horizon, with a Faraday-induced potential drop \( \Delta V = (\Omega_+ - \Omega_-)A_\phi \). The power dissipated in this gap becomes

\[
P = \Omega_T(\Omega_H - 2\Omega_T)A_\phi^2.
\]
Thus, a gap forms with macroscopic dissipation whenever the black hole spins more rapidly than \textit{twice} the angular velocity of the torus\footnote{Of some interest is the formation of the gap ($\Omega_H > 2\Omega_T$) while being in a state of hyper-accretion ($\Omega_H < \Omega_T^*$). Attributing the power released in the gap to the input to the observed GRB and afterglow emissions, suggests the possibility for afterglow emissions to short bursts. Unless the environment to short bursts is dramatically different from long bursts, HETE-II should see afterglows to short bursts as well.\footnote{2.5 Clustering and spread in GRB emissions}}.

Of some interest is the formation of the gap ($\Omega_H > 2\Omega_T$) while being in a state of hyper-accretion ($\Omega_H < \Omega_T^*$). Attributing the power released in the gap to the input to the observed GRB and afterglow emissions, suggests the possibility for afterglow emissions to short bursts. Unless the environment to short bursts is dramatically different from long bursts, HETE-II should see afterglows to short bursts as well.

### 2.5 Clustering and spread in GRB emissions

Recent analysis of achromatic breaks in a sub-sample of GRB lightcurves indicates that these emissions are beamed, and that their true fluence $E_{\text{grb}}$ is standard (with a dynamic range of about one decade) at few times $10^{50}$ ergs. At the same time, the opening angle displays a rather wide dynamic range, between a few degrees and a few tens of degrees.

In\footnote{In this event, the opening angle of the open flux-tube on the horizon commensurate with the true emissions in GRBs is about $35^\circ$. Collimation of this flux-tube down to an opening angle $\theta_j$ on the celestial sphere may derive from winds coming off the torus, possibly so along forementioned outer flux-tube. This will account for a true output in outflow of about $E_{\text{grb}}(\theta_H/\theta_j)\epsilon^{-1}$, where $\epsilon \sim 0.15$ denotes the efficiency of kinetic energy to gamma-rays. Though substantial, this output remains well below the energy deposit into the torus by the black hole. This predicts a bound $\theta_j \leq 35^\circ$ (6) in any large sample of observations. We attribute variations in the opening angle $\theta_j$ to a diversity in torus parameters.\footnote{3 GRBs: the tip of the iceberg?}}\footnote{For long bursts, the equivalence in poloidal topology to pulsar magnetospheres indicates a high incidence of the black hole luminosity into the torus. Only a small fraction of less than one percent of the black hole output is associated with the true output in GRB-afterglow emissions.}, we consider a geometrically standard inner region in the vicinity of the black hole, when the torus is thick relative to the size of the black hole. In this event, the opening angle of the open flux-tube on the horizon commensurate with the true emissions in GRBs is about $35^\circ$. Collimation of this flux-tube down to an opening angle $\theta_j$ on the celestial sphere may derive from winds coming off the torus, possibly so along forementioned outer flux-tube. This will account for a true output in outflow of about $E_{\text{grb}}(\theta_H/\theta_j)\epsilon^{-1}$, where $\epsilon \sim 0.15$ denotes the efficiency of kinetic energy to gamma-rays. Though substantial, this output remains well below the energy deposit into the torus by the black hole. This predicts a bound $\theta_j \leq 35^\circ$ (6) in any large sample of observations. We attribute variations in the opening angle $\theta_j$ to a diversity in torus parameters.

### 3 GRBs: the tip of the iceberg?

For long bursts, the equivalence in poloidal topology to pulsar magnetospheres indicates a high incidence of the black hole luminosity into the torus. Only a small fraction of less than one percent of the black hole output is associated with the true output in GRB-afterglow emissions.
3.1 Gravitational radiation from a torus around a black hole

The torus processes the input from the black hole by emission in various channels. This can be estimated in a suspended accretion state, including gravitational radiation, neutrino emissions and Poynting flux-winds. Gravitational radiation will be emitted as the torus develops non-axisymmetries, which features several aspects which suggest considering long GRBs as potential sources for LIGO/VIRGO. Namely, the torus is strongly coupled to the spin-energy of the black hole; lumpiness in the torus will produce gravitational radiation at twice the Keplerian angular frequency, i.e., in the range of 1-2kHz; the emission in gravitational radiation should dominate over emissions in radio waves; the true rate of GRBs should be frequent as inferred from their beaming factor of a few hundred. These gravitational wave emissions from the torus are powered by the spin energy of the black hole. This sets it apart from such emissions in neutron star-neutron star mergers or by fragmentation in collapse towards supernovae.

Non-axisymmetries in the torus are expected from dynamical and, potentially, radiative instabilities. Notably so, a geometrically thick torus, consistent with the recent indication that long GRBs may be standard, is generally subject to the Papaloizou-Pringle instability. If the torus reaches a mass on the order of that of the central black hole, it will be unstable to self-gravity. Of interest is further the possibility of a Chandrasekhar-Friedman-Schutz instability, or radiative instabilities since lumps of matter radiate preferentially on inner orbits. It would be of interest to study these radiative instabilities in further detail. The resulting gravitational wave-emissions may be quasi-periodic (QPO). This may be reminiscent of the observed QPOs in accretion disks in X-ray binaries, some of which have been attributed to general relativistic effects in orbital motions.

A detailed calculation in the suspended-accretion state gives the estimate $L_{gw} \simeq L_H/3$ for the luminosity in gravitational waves as a fraction of the black hole luminosity $L_H$. This gives a fluence in gravitational waves

$$E_{gw} \simeq 1\% M$$

in terms of the mass $M$ of a rapidly spinning Kerr black hole. This suggests to consider black hole-torus systems as potential LIGO/VIRGO sources of gravitational waves. The abundance of GRBs on a cosmological scale suggests to consider an interesting contribution to the stochastic background in gravitational radiation.
3.2 Calorimetric evidence for Kerr black hole

The existence of black hole remains circumstantial, in particular for the population of stellar mass black holes. Obtaining evidence based on first principles is a challenging task which occasionally drives new observational strategies. Perhaps, then, LIGO/VIRGO may contribute by calorimetry on gravitational wave emissions. Successful detection of a burst in gravitational waves of a duration commensurate with the redshift corrected duration of 10-15s of long bursts would provide evidence for a compact and high-angular momentum inner engine. Additionally, a fluence in gravitational waves in excess of the rotational energy of a rapidly spinning neutron star would support the presence of a central Kerr black hole. Kerr black holes have the unique property of storing up to about a third of their mass in rotational energy. This has no baryonic counterpart.

This may be pursued by the observable combination

$$\alpha = 2\pi E_{gw} f_{gw}. \quad (8)$$

Here, $\alpha$ is dimensionless upon use of geometrical units, with energy expressed in terms of the corresponding Schwarzschild radius and frequency in $1/cm$. It may be noted that $\alpha$ is a compactness parameter reminiscent of the dimensionless Kerr parameter $a/M$. Indeed, for large specific angular momenta $a$, the ratio $a/M$ becomes effectively a measure for the stored rotational energy to the linear size of the black hole. The estimates above show that a black hole-torus system may produce $\alpha = 0.01 - 0.015$, while a neutron star satisfies $\alpha < 0.007$. This indicates an opportunity to detect $\alpha$ in excess of that permitted by a neutron star, to serve as calorimetric evidence of a Kerr black hole. Notice that $E_{gw}$ requires a distance estimate to the source. In practice, this may require statistical analysis on a sample of detections.

Acknowledgment. The author thanks the Korean Institute for Advanced Study for their hospitality and for hosting a very stimulating meeting. He also thanks G.E. Brown, C.W. Lee and A. Levinson for continuing conversations. This work is partially supported by NASA Grant No. 5-7012 and an MIT C.E. Reed Award.

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