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Published in:
Applied Physics Letters

Link to article, DOI:
10.1063/5.0015166

Publication date:
2020

Document Version
Publisher's PDF, also known as Version of record

Link back to DTU Orbit

Citation (APA):
Piller, M., Sadeghi, P., West, R. G., Luhmann, N., Martini, P., Hansen, O., & Schmid, S. (2020). Thermal radiation dominated heat transfer in nanomechanical silicon nitride drum resonators. Applied Physics Letters, 117(3), [034101 ]. https://doi.org/10.1063/5.0015166
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Cite as: Appl. Phys. Lett. 117, 034101 (2020); https://doi.org/10.1063/5.0015166
Submitted: 25 May 2020 . Accepted: 10 July 2020 . Published Online: 22 July 2020

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ABSTRACT

Nanomechanical silicon nitride (SiN) drum resonators are currently employed in various fields of applications that arise from their unprecedented frequency response to physical quantities. In the present study, we investigate the thermal transport in nanomechanical SiN drum resonators by analytical modeling, computational simulations, and experiments for a better understanding of the underlying heat transfer mechanism causing the thermal frequency response. Our analysis shows that radiative heat loss is a non-negligible heat transfer mechanism in nanomechanical SiN resonators, limiting their thermal responsivity and response time. This finding is important for optimal resonator designs for thermal sensing applications as well as cavity optomechanics.

Since their emergence, nanomechanical resonators have shown distinct advantages in various fields of application due to their high amplitude and frequency response to external physical quantities. In order to achieve optimal performance, material properties are essential, and silicon nitride (SiN) has proven to be well suited for nanomechanical resonators. The large intrinsic stress results in unprecedented high quality factors based on so-called “damping dilution,” which has been observed in silicon nitride strings and drums. The combination of high quality factors and excellent optical properties has made nanomechanical SiN drums interesting devices for cavity optomechanics. Among other things, they have been used for fundamental research and as transducers between optical and radio waves or microwave signals. Recent developments of optimized trampoline and phononic crystal designs are pushing the quality factors of SiN resonators into the realm of room temperature quantum optomechanical experiments.

From cavity optomechanics experiments, it is well known that the local heating of the laser causes a frequency detuning of SiN drums. Such photothermal detuning has been extensively used for sensing applications, such as infrared absorption spectroscopy, nanoparticle analysis, single-molecule detection, and, recently, electromagnetic radiation detection. Generally, it has been assumed that heat transfer is dominated by conduction, as it was concluded for nanomechanical torsional paddle resonators. Recently, evidence for significant radiative heat transfer in large SiN drums has been presented. Despite the proliferation of nanomechanical SiN resonators, the underlying heat transfer mechanisms, which cause the thermal frequency response and ultimately determine the performance limit, have not been studied in detail.

In this work, we investigate the heat transfer in nanomechanical SiN drum resonators by means of computational simulations and experiments to gain a better understanding of the dominating mechanism. We assume the situation of an experiment under vacuum in which heat convection is negligible, and heat transfer occurs solely by radiation and conduction. Our study is conducted by local heating of SiN drums with a laser and analyzing the resulting frequency and time response. We show that the frequency response and the response time are dominated by radiative heat transfer, which is a function of the lateral size of the drums. This is an important finding to be considered for the optimal design of thermal sensors as well as cavity optomechanics experiments.

The relative frequency shift $\delta f$ for an even temperature change $\Delta T = T - T_0$ from an initial temperature $T_0$ of a resonator under tensile stress $\sigma$ with a resonance frequency $f(T)$, such as a drum or a string, is given by

$$
\delta f = \frac{\Delta f}{f(0)} = \frac{\Delta T}{T_0} \left[ \frac{f''(T)}{f'(T)} \right]_{T=T_0} = \frac{\Delta T}{T_0} \left[ \frac{d^2 f}{dT^2} \right]_{T=T_0}
$$
\[ \delta f = \frac{f(T)}{f(T_0)} = \sqrt{1 - \frac{zE(T - T_0)}{\sigma}}, \]  

(1)

with \( z \) being the thermal expansion coefficient and \( E \) Young’s modulus. This results in the relative temperature responsivity (relative frequency shift per change in temperature) of

\[ \left. \frac{1}{f(T_0)} \frac{\partial f(T)}{\partial T} \right|_{T=T_0} = -\frac{zE}{2\sigma}. \]  

(2)

From (2), it is obvious that the observed temperature-induced frequency detuning is enhanced for resonators with a low tensile stress \( \sigma \). Therefore, we performed our study with nanomechanical SiN drum resonators made of low-stress silicon nitride. The drums with a thickness of \( h = 50 \) nm are supported by a silicon frame with a thickness of \( 380 \) \( \mu \)m. We present the results from square drums of different sizes \( L \times L \) with \( L = 0.5 \) mm, 1.0 mm, 2.5 mm, 4.0 mm. We used a silicon wafer with silicon-rich SiN grown by low-pressure chemical vapor deposition. The square drum shapes were defined on the backside of the wafer by a standard photolithography process and etched by reactive ion etching. To finally release the drum structures, the silicon wafer was etched through from the backside with potassium hydroxide. To finally release the drum structures, the silicon wafer with silicon-rich SiN grown by low-pressure chemical vapor deposition.

The tensile stress of the drums was calculated from the fundamental frequency shift per change in temperature of some SiN drums by means of the physical vapor deposition process.

The experimental setup, schematically depicted in Fig. 1(a), shows the silicon nitride drum resonator on a piezo actuator and a laser-Doppler vibrometer (LDV) (MSA-500 from Polytec GmbH) to read out the vibrational motion. The signal from the vibrometer is fed into a lock-in amplifier (HF2LI from Zurich Instruments) with an integrated phase-locked loop to control the piezo actuator and to drive the drum at its resonance frequency. A power controllable laser diode (LPS-635-FC from Thorlabs GmbH), with a center wavelength of \( \lambda = 638 \) nm, was attached to the LDV unit to photothermally heat the drums. The exact laser power values were recorded using a silicon photodiode (S120C from Thorlabs GmbH). Simulations shown in this work are performed using COMSOL Multiphysics Version 5.5. All measurements were conducted in a high vacuum at a pressure below \( 1 \times 10^{-5} \) mbar.

Figure 1(b) schematically depicts the heat flux in a drum resonator when locally heated in its center. The incident laser with power \( P_{\text{abs}} \) is absorbed by the drum, producing a heating power \( P_{\text{abs}} = A_i P_{\text{abs}} \) for a wavelength specific absorbance \( A_i \). According to Fourier’s law, the resulting temperature gradient across the drum causes a conductive heat flux \( q_{\text{cond}} = -k dT/dx \) from the drum center toward the frame, for a specific thermal conductivity \( k \). The radiative heat transfer from the drum surface is given by the Stefan–Boltzmann law \( q_{\text{rad}} = \sigma_{\text{SB}} (T^4 - T_0^4) \), for the special case of having a large surrounding at temperature \( T_0 \) and the assumption of a gray surface with an emissivity \( e \) with the Stefan–Boltzmann constant \( \sigma_{\text{SB}} \) and the surface temperature \( T \) at a specific location on the drum. Taking into account thermal radiation, whereby part of the thermal power is emitted, leads to a reduced effective temperature of the drum. It is obvious from Eq. (1) that a lower average temperature results in a smaller frequency detuning \( \Delta f_i \), compared to the case of negligible thermal radiation \( \Delta f_{\text{rad}} \), as schematically depicted in Fig. 1(c).

The experimental results, measured for the relevant dimensions have been used.

Figure 2(a) shows a temperature profile simulation across the center of a 1 mm square drum for different emissivities. The temperature along the drum decreases with higher emissivities for the same incident power. In Fig. 2(b), the peak temperature \( T_{\text{peak}} \) at \( x = 0 \) \( \mu \)m is shown for different heating powers and emissivities, causing the resonance frequency to shift. Figure 2(c) shows the simulated frequency responses with different emissivities for an increasing absorbed power \( P_{\text{abs}} \). As predicted by the simplified model (1), the resonator frequency decreases with increasing absorbed laser power \( P_{\text{abs}} \), which corresponds to a rise of the drum’s effective temperature. It also shows that the slope of the frequency detuning, and hence the responsivity, becomes smaller when much heat is radiated due to a higher emissivity of the drum.

For small changes of temperature \( \Delta T \), that is for small \( P_{\text{abs}} \), the frequency response (1) can be linearized to a good approximation. From Fig. 2(b), it can be seen that this linear approximation is valid for \( P_{\text{abs}} < 10 \) \( \mu \)W. Considering the applied laser powers and the assumed \( A_i \approx 0.4\% \) for SiN, which is close to the reported absorbance value of 0.5\%\(^{25}\), our maximal absorbed power is \( P_{\text{abs}} \approx 1 \) \( \mu \)W. Figure 2(c) shows that the measured frequency detuning is indeed linear with the absorbed power. Based on this presented method, we measured the relative power responsivity \( \delta R = \delta f/P_{\text{abs}} \) for Al-coated and bare SiN drum resonators, shown in Figs. 3(a) and 3(b), respectively.

For the case of negligible thermal radiation, an analytical power responsivity model for the fundamental mode has been derived for the case of local heating in the center of a circular drum.\(^{22}\)

\[ \delta R = \frac{\delta f}{P_{\text{abs}}} \approx -\frac{zE}{8\pi n x \sigma} \left( \frac{2 - \nu}{1 - \nu} - 0.642 \right), \]  

(3)

with \( \nu \) being Poisson’s ratio. A comparison to finite element method (FEM) simulations shows that the analytical model is a good approximation for small power changes.
approximation for square drums.\(^2\) According to (3), the power responsivity is independent of the lateral drum size. This is the case for the Al-coated SiN drums, as can be seen in Fig. 3(a). Compared to the thermal conductivity of SiN of \(\kappa = 3 \text{ W m}^{-1} \text{ K}^{-1}\),\(^2\) the effective conductivity of the Al-coated drums is \(~32\) times higher; thus, the heat transfer in the Al-coated drums seems to be dominated by conduction. The fit of (3) for the Al-coated drums with different effective tensile stress is of good quality. In contrast, as seen in Fig. 3(b), the measured values with good agreement.

(Fig. 4(a). By increasing the modulation frequency of the heating laser, the recorded resonance frequency detuning from the phase-locked loop will start to decay. The response time \(\tau\) is evaluated for the frequency amplitude at \(f_{\text{3 dB}} = f_{\text{max}}/\sqrt{2}\). The extracted response times of Al-covered SiN drums are plotted in Fig. 4(b). Compared to the measurements of bare SiN drums, shown in Fig. 4(c), the Al-coated drums show a significantly faster response time, due to their higher thermal conductivity and, hence, faster thermalization. Besides the magnitude, the scaling of \(\tau\) with the lateral size is notably different for the two drum types.

Our theoretical model (see the supplementary material for the derivation) for the response time of square drums made of multiple layers, considering both thermal conduction and thermal radiation, yields

\[
\tau = \frac{2\pi^2}{\rho L^2} \left( \sum_{i} h_i \rho_i C_{p,i} + \sum_{i} 8 \sigma \varepsilon_i \rho_i T_0^3 \right)^{-1},
\]

with \(\rho\) being the mass density and \(C_p\) the specific heat capacity for each layer \(i\). The model (4) is in excellent agreement with the...
measured response times for both drum types, as seen in Figs. 4(b) and 4(c). In order to dissect the dominating heat transfer mechanism at play, we additionally plotted the model taking into account only heat conduction. In that case, (4) predicts a quadratic scaling with drum size \( L \) and is a good approximation in the case for the conduction-dominated Al-coated drums, as seen in Fig. 4(b). In comparison, only the full model (4), taking into account both conduction and radiation, predicts the measured response times accurately, as seen in Fig. 4(b). This is another clear sign that heat transfer in Al-coated SiN drums is dominated by conduction in contrast to the bare SiN drums that are radiation-limited. The higher radiative heat loss for larger drums leads to a reduced response time that levels off, compared to what would be expected from heat conduction alone.

In conclusion, it has been demonstrated that the thermal frequency response of nanomechanical SiN drum resonators is significantly affected by radiative heat transfer. This results in a size-dependent responsivity of bare SiN drums in contrast to Al-coated resonators where thermal conduction dominates and no size dependency on responsivity was observed. The dominating heat transfer mechanism of the resonator is also reflected in response time measurements. The Al-covered drums are dominated by thermal conduction and show a significantly faster response time due to the high thermal conductivity of the metal layer. An analytical model for this case agrees well with the measurements and shows that the response time scales quadratically with the drum size. For bare SiN drums, where thermal radiation plays a non-negligible role, measurements also showed good agreement with the response time model, predicting faster response times due to the additional radiative heat loss. Larger drums are more affected by radiation, exhibiting an even higher deviation for response times compared to the case of a thermal conduction-dominated heat transfer. The obtained results show the significance of thermal radiation in finding the optimal performance of SiN drum resonators for specific sensor applications and fundamental research.

See the supplementary material for calculations of emissivity and response time.

The authors wish to thank Johannes Hiesberger, Sophia Ewert, Patrick Meyer, and Michael Buchholz for their support with the sample fabrication as well as Hendrik Köhler and Miao-Hsuan Chien for many fruitful discussions. We would also like to thank Dr. Pavel Grincho of HMTI, Belarus, for his support. This work was supported by the European Research Council under the European Union’s Horizon 2020 research and innovation program (Grant Agreement Nos. 716087-PLASMЕCS and 875518-NIRD). We further acknowledge funding from Invisible-Light Labs GmbH.

**DATA AVAILABILITY**

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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