Spin Quantum Number or Rest Angular Momentum

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Abstract

The aim of this letter (considering the fundamental origins of Klein-Gordon and Dirac equations, Thomas precession, and the photon spin) is to point out that since the origin of spin angular momentum refers to relativity than quantum theory, it is better to call it as the “rest angular momentum”.
Spin quantity which was introduced short after formulation of quantum mechanics, in addition to its theoretical significance, has a wide variety of applications in different technological fields (e.g. NMR and new hot subject of research spintronics). Although the mathematical formulation (Lie algebra) of the subject is well-known, the fundamental discussions related to the spin can be hardly found in modern (text)books and papers. In this letter, with an overview of the fundamental origins of Klein-Gordon and Dirac equations, Thomas precession, and the photon spin, it is pointed out that the origin of spin angular momentum refers to relativity than quantum theory.

As is clear in the history of the discovery of the electron spin [1], spin degree of freedom cannot be considered as the electron rotation. If the electron is to rotate around itself, its surface velocity should be more than the velocity of light in vacuum?! Although some experts have tried to find a realistic and causal model for spin angular momentum [2], they haven't succeeded to satisfy the physics society yet; the reason is that the so-called classical models are "toy" models than be considered realistic.

In 1926, Klein [3] and Gordon [4] found a relativistic generalization of Schrödinger equation which could remain invariant under Lorentz transformations. Possibility of finding negative probability density in Klein-Gordon equation made Dirac search for another relativistic version of Schrödinger equation. He found a first order equation which is invariant under Lorentz transformations. Dirac equation could remove negative probability density and simultaneously contains spin with no postulates. Since Dirac aim was not to find spin and just wanted to reach a relativistic equation and since in non-relativistic quantum mechanics, which is based on Schrödinger equation, one can only include spin quantity by hand (based on Pauli approach), it seems the relativistic consideration/generalization is the main origin of leading to spin degree of freedom.

In the same year (1926), Uhlenbeck and Goudsmit, who had introduced the idea of electron spin for the first time, showed that if the electron had a $g$ factor of 2, the anomalous Zeeman Effect could be explained correctly. In 1927, Thomas showed that the reason for a $g$ factor of 2 ($g = 2$) originated from a relativistic kinematical effect [5]. (Note: Now, the $g$ factor value and its origin can be explained and computed based on modern relativistic quantum field theory). This fact that the Thomas work was based on relativistic corrections confirms this idea that spin has relativistic origin.

In classical electrodynamics the total angular momentum of a distribution of electromagnetic fields in vacuum can be written in the form [6]:

$$
\vec{J} = \frac{1}{4\pi \varepsilon} \int d^3 x \vec{X} \times (\vec{E} \times \vec{B}) = \frac{1}{4\pi \varepsilon} \int d^3 x \left[ \vec{E} \times \vec{A} + \sum_{j=1}^{3} E_j (\vec{X} \times \nabla) A_j \right] = \vec{S} + \vec{L} \quad (1)
$$

The first term is an intrinsic position-independent expression; but, the second term depends on the position. The Hermitian quantized (operatorial) form of the first term is:
\[ \bar{S} = \frac{1}{8\pi c Vol} \int d^3x (\vec{E} \times \vec{A}) - [\vec{A} \times \vec{E}] \]  \hspace{1cm} (2)

Fourier expansion of vector potential operator leads to [7]:

\[ \bar{S} = \sum_k \sum_{m=1,1} (\hbar m) a^+_k a_{km} \left( \frac{\vec{k}}{k} \right) \]  \hspace{1cm} (3)

\[ \Rightarrow \bar{S}(a^+_{km=\pm 1}|0\rangle) = (\pm \hbar) \left( \frac{\vec{k}}{k} \right) (a^+_{km=\pm 1}|0\rangle) \]

Fourier coefficients are creation and annihilation operators and have no \( m = 0 \) component. The photon spin vector has projections along the direction of the \( \vec{k} \) vector of +\( \hbar \) (right circularly polarized photons) or of −\( \hbar \) (left circularly polarized photons). Although the spin number of photon is \( s = 1 \), there aren't \( 2s + 1 = 3 \) states available; this is due to the fact that photon has no rest mass and no rest frame.

Once again and based on the above argument on the electromagnetic field which is a purely relativistic field whose quantum particle is photon, it is deduced that the spin which is an intrinsic (position-independent) kind of angular momentum has relativistic origin.

We should also mention that even by considering modern quantum theory of fields for studying the subject through spin-statistics theorem, relativistic considerations are of great importance; this is because the spin-statistics theorem is proved by means of micro-causality (micro-locality) relations whose origins are in relativity than quantum theory.

In conclusion, spin isn't a pure quantum mechanical quantity but it has relativistic origin. It seems it is better to call spin as the”rest angular momentum”.

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References

[1] Interested readers, for more information, among many interesting historical notes and papers, can refer to the book “Sin-itiro Tomonaga, The Story of Spin, (University of Chicago Press 1997)” in which the main discoveries during a period of 40 years have been reviewed.

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