Black hole masses of weak emission line quasars based on continuum disc-fitting method

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ABSTRACT

We study observed optical-ultraviolet spectral energy distribution of 10 weak emission-line quasars (WLQs) which lie at redshifts $z = 0.19$ and $1.43 < z < 3.48$. The theoretical models of their accretion disc continua are created based on the Novikov-Thorne equations. It allows us to estimate masses of their supermassive black holes ($M_{\text{BH}}$) and accretion rates. We determine the virial factor for WLQs and note its anti-correlation with the full width at half maximum ($\text{FWHM}$) of H$\beta$ emission-line ($f \propto \text{FWHM}^\alpha$, $\alpha = -1.99 \pm 0.67$). By comparison with the estimated previously BH masses, the underestimation of $M_{\text{BH}}$ is noticed with a mean factor 7.4 dependent on the measured full width. We propose the new formula to estimate $M_{\text{BH}}$ in WLQs based on their observed FWHM(H$\beta$) and luminosities at 5100Å. In our opinion, WLQs are also normal quasars visible in a reactivation stage.

Keywords: Active galactic nuclei - Black hole - Quasars

1. INTRODUCTION

Weak emission-line quasars (WLQs) are unsolved puzzle in the model of active galactic nuclei (AGN). Typical equivalent width ($\text{EW}$) of C IV emission line is extreme weak ($\lesssim 10$Å) compared to normal quasars and very weak or absent Ly$\alpha$ emission (Fan et al. 1999; Diamond-Stanic et al. 2009). Diamond-Stanic et al. (2009) concluded that WLQs have optical continuum properties similar to normal quasars, although Ly$\alpha$+N V line luminosities are significantly weaker, by a factor of 4. An explanation for the weak or absence of emission lines has not been found so far. Potential explanations include a radiatively inefficient accretion flow (Yuan & Narayan 2004) and an cold accretion disc (Laor & Davis 2011) with a small accretion rate. An extremely high accretion rate where we have inefficient photoionized flux is proposed by Leighly et al. (2007a), Leighly et al. (2007b). Parallel to that, Wu et al. (2011) postulate presence of shielding gas between the accretion disc and a Broad Line Region (BLR) which could absorb the high-energy ionizing photons from the accretion disc. The last but not least model suggests an unusual BLR i.e., anemic in construction and gas abundance (Shemmer et al. 2010; Nikolajuk & Walter 2012). This explanation supports the idea that WLQs can also be in the early stage of AGN evolution (Hryniewicz et al. 2010; Liu & Zhang 2011; Bañados et al. 2014; Meusinger & Balafkan 2014). Several works about X-ray properties of WLQ have recently appeared (e.g. Ni et al. 2018; Marlar et al. 2018). The main conclusion arising from these works is that WLQs are X-ray weak. Ni et al. (2018) suggest that it may be caused from the shielding gas which prevents the observer to see central X-ray emitting region. The mentioned above research suggest that WLQs are intrinsically weak cause by several explanations.

Knowledge about the values of BH masses and the accretion rates is crucial in understanding of an accretion flows phenomena. The most robust technique is the reverberation mapping method (RM, Blandford & McKee 1982; Peterson 1993, 2014; Fausnaugh et al. 2017; Bentz & Manne-Nicholas 2018; Shen et al. 2019). The method is based on the study of the dynamics surrounding a black hole gas. In this way, we are able to determine the supermassive black hole (SMBH) mass:

$$M_{\text{BH}} = \frac{v_{\text{BLR}}^2 R_{\text{BLR}}}{G} = f \frac{\text{FWHM}^2 R_{\text{BLR}}}{G} \quad (1)$$

where $M_{\text{BH}}$ is the black hole mass, $G$ - the gravitational constant, $R_{\text{BLR}}$ is a distance between the SMBH and a cloud in the Broad Line Region. $v_{\text{BLR}}$ is a velocity of the cloud inside the BLR. This speed is unknown and we express our lack of knowledge in the form of the Full Width Half Maximum (FWHM) of an emission-line and $f$ - the virial factor, which describes a distribution of the BLR clouds. In the RM the $R_{\text{BLR}}$ is determined as the
time delay between the continuum change and the BLR response. This technique requires a significant number of observations. The modification of this method is the scaling method. The relation between $R_{\text{BLR}}$ and continuum luminosity ($\nu L_\nu$) was observed (Kaspi et al. 2005; Bentz et al. 2009). Thus, the method is powerful and eagerly used because of its simplicity (e.g. Kaspi et al. 2000; Peterson et al. 2004; Vestergaard & Peterson 2006; Plotkin et al. 2015).

The non-dynamic method, which is the spectra disc-fitting method, is based on well grounded model of emission from an accretion disc (AD) surrounded black hole (e.g. Shakura & Sunyaev 1973; Novikov & Thorne 1973). The most important parameter in such models is the mass of black hole and the accretion rate. The spin of the black hole and the viewing angle are also taken into account. In this technique, a Spectral Energy Distribution (SED) of an AGN is fitted to the model and one is able to constrains these four parameters. More advanced disc spectra models, which take into account an irradiation effect, limb-darkening/brightening effects, the departure from a blackbody due to radiative transfer in the disc atmosphere, the ray-tracing method to incorporate general relativity effects in light propagation, can be fitted (e.g. Hubeny et al. 2000; Loska et al. 2004; Sadowski et al. 2009; Czerny et al. 2011; Laor & Davis 2011; Czerny et al. 2019). Neither the RM nor the scaling method are adequate in WLQs due to weakness of emission-lines in these objects. This paper is organised as follows. Section 2 explains the methods and procedures used to fitting continuum of accretion disc to photometric points of WLQs. Section 3 presents our results.

## 2. Sample Selection and Method

### 2.1. Sample selection and dereddening

The sample contains 10 WLQs, which positions cover a wide range of redshift from 0.19 to 3.48 (see Tab. 1). Four objects, namely SDSS J083650.86+142539.0 (hereafter J0836), SDSS J141141.96+140233.9 (J1411), SDSS J141730.92+073320.7 (J1417), and SDSS J144741.76-020339.1 (J1447) were analysed by Shen et al. (2011); Plotkin et al. (2015). Three next sources – SDSS J111453.34+021924.3 (J1141) and SDSS J123743.08+630144.9 (J1237) were studied by Diamond-Stanic et al. (2009), and SDSS J094533.98+100950.1 (J0945) by Hryniewicz et al. (2010). The quasar SDSS J152156.48+520238.5 (J1521) was inspected by Just et al. (2007); Wu et al. (2011). The number of objects in the sample from the SDSS campaign (Abazajian et al. 2009) has been increased by two next WLQs: PG 1407+265 and PHL 1811. The first one object is the first observed WLQ in history and intensively examined by McDowell et al. (1995). PHL 1811 is the low redshift source classified also as NLS1 galaxy (Leighly et al. 2007a,b). Photometric points of WLQs at visible wavelengths are collected based on the Sloan Digital Sky Survey (SDSS) optical catalogue Data Release 7, which contains u, g, r, i, and z photometry (Abazajian et al. 2009). In the case of PHL 1811, we based on measurements of fluxes in B and R colours, and performed by the Dupont 100" telescope at Las Campanas Observatory (LCO) (Prochaska et al. 2011). The flux at U band was observed by the UVOT telescope onboard Swift satellite (Page et al. 2014). Near-infrared photometry in the W1-W4 bands are taken from the Wide-field Infrared Survey Explorer (WISE) Preliminary Data Release (Wright et al. 2010; Wu et al. 2012). Those data were supplied by photometry in the J, H, Ks colours obtained from the Extended Source Catalog of the Two Micron All Sky Survey (2MASS) (Skrutskie et al. 2006). Crucial points for the project are those detected in near- and far-ultraviolet (NUV, FUV, respectively) wavelengths. They are provided by the Galex Catalogue Data Release 6 (Bianchi et al. 2017; Seibert et al. 2012). A basic observational properties of the WLQ sample and sources of their photometry points are listed in Tab. 1.

To check if our disc-fitting method works with respect to WLQs correctly, we are running a method on a sample of normal type 1 quasars. For this purpose, we select the sample of objects taken from the Large Bright Quasar Survey (LBQS) Hewett et al. (1995, 2001). It is one of the largest published spectroscopic survey of optically selected quasar at bright apparent magnitudes. It contains data, including positions and spectra of 1067 quasars. Additionally, Vestergaard & Osmer (2009) give black hole masses and Eddington accretion rates estimates of 978 LBQS (see their Table 2). The disc fitting method gives results that we can trust as long as the bend point in SED and the spectrum in the ultraviolet are visible. Therefore, we have chosen 27 quasars with the presence of a well visible big blue bump. The sample of the normal quasars are observed at redshift between 0.254 and 3.36, logarithm of masses of super-massive black holes, log $M_{BH}$ ($M_\odot$), from 8.09 to 10.18, and luminosities, log $L_{bol}$ (erg s$^{-1}$), in the range 45.25 to 47.89. Photometric points of selected quasars come from the same catalogues mentioned earlier.
The obtained Spectral Energy Distribution of all objects are corrected for Galactic reddening with an extinction law. This extinction curve is usually parameterized in terms of the V-band extinction $A_V$ and a measure of the relative extinction between B and V-band: $R_V = A_V/E(B-V)$. The value of $R_V$ varies from 2.6 to 5.5 in the measurements of the diffuse interstellar medium with a mean value of 3.1 (Cardelli et al. 1989; Fitzpatrick 1999). $A_V$ values are taken from NED\(^1\) based on the dust map created by Schlegel et al. (1998). Cardelli et al. (1989) extinction curve has cutoff at 1250 Å and some photometric points we use go back to shorter wavelengths. Nevertheless, the extinction law examined in the range of 900-1200 Å seems to follow the Cardelli et al. law (Hutchings & Giasson 2001). In this way, we extrapolate the curve down to 900Å for our FUV photometric points by using the same formula. Dereddered SEDs of our WLQs are shown in Fig. 2.

2.2. Method

2.2.1. Model of Accretion Disc

The primary goal of this work is to fit SED of quasars by the simple geometrically thin and optically thick accretion disc (AD) model described by Novikov & Thorne (NT) equations. In the simplest approach, the AD continuum can be illustrated by the Shakura & Sunyaev model, nevertheless this attitude does not include a non-zero spin. The solution to this problem has resulted in the NT equations that we use in our numerical code. As the spin of the black hole increases, the innermost stable circular orbit (ISCO) decreases and the disc produces more high-energy radiation. The output continuum of the NT model is fully specified by four parameters, which we determine: These 4 parameters are: the black hole mass – $M_{\text{BH}}$, the mass accretion rate – $\dot{M}$, the dimensionless spin$^2$ – $a_*$, and the line-of-sight inclination angle – $i$ at which an observer looks at the AD. The mass of the black hole is expressed in units of mass of the Sun ($M_\odot$), and the accretion rate in the form of the Eddington rate, i.e. $\dot{m} = \dot{M}/\dot{M}_{\text{Edd}} \propto \dot{M}/M_{\text{BH}}$.

We construct a grid of 360000 models of AD, for evenly spaced values of $M_{\text{BH}}$, $\dot{m}$, $a_*$, and $i$. The log $M_{\text{BH}}$ range is from 6.0 to 12.0, the Eddington accretion rate covers the band 0–1, and the dimensionless spin $0 \leq a_* \leq 0.9$ with the step 0.1. The inclination is fixed for 6 values that cover a range from 0° to 75° with the step of 15° (see Tab. 2).

To find the best-fit model and to evaluate the quality of the fit, we used a simple $\chi^2$ procedure, which is based on directly matching the photometric points to the AD model. In our approach, we calculate $\chi^2 = \sum_{i=1}^{n} (O_i - E_i)^2/\sigma_i$ for each quasar, where $O_i$ and $E_i$ are observed and modelled monochromatic luminosity $L_\lambda$, respectively, read out at wavelength corresponding to the $i$th photometric point. $\sigma_i$ is the observed error, and $n$ – total number of observed data for the quasar. Satisfactory fits are defined as those showing reduced $\chi^2 < 4.5$.

In order to use information on yet determined black hole masses, accretion rates, and their errors in our WLQ sample we carry out a more sophisticated statistical analysis using the Bayesian method, which is the conditional distribution of the uncertain quantity given the data. The value of BH masses and accretion rates ($M_{\text{BH}}^{\text{lit}}$ and $\dot{m}_{\text{lit}}$, respectively) of 9 WLQ were collected by different authors (see Tab. 3, Col. 9) and those values for PG 1407 was determined by us (see Subsec. 2.2.2). Note, that both $M_{\text{BH}}^{\text{lit}}$ and $\dot{m}_{\text{lit}}$ of WLQs are based on the FWHM(line) determination. Authors use equations with factors suitable for normal quasars which show strong lines and broad FWHM. However, this is not true for many WLQs. For this reason, both values $M_{\text{BH}}^{\text{lit}}$ and $\dot{m}_{\text{lit}}$ could be calculated wrongly.

The Bayesian inference method requires the knowledge of the prior probability distribution $P(H|I)$, which represents our beliefs about a hypothesis $H$ before some evidence, $I$, is taken into account. In our calculations $H$ is $j$th model, $\text{mod}_j = \text{mod}(M_{\text{BH}}^{\text{lit}}, \dot{m}_{\text{lit}}, a_*, i_j)$, from among 360000 models we put in (note that both of the analyses we have conducted i.e. $\chi^2$ and the Bayesian, are based on the same set of constructed grid of AD models). $I$ means any prior information about this $j$th model. In our case, the information should be $M_{\text{BH}}^{\text{obs}}$, $\dot{m}_{\text{obs}}$, etc., which are observed. Nevertheless, we do not have those real parameters and therefore $I$ is based on earlier calculated $M_{\text{BH}}^{\text{lit}}$, $\dot{m}_{\text{lit}}$ and their errors. Assuming a Gaussian probability distribution for $M_{\text{BH}}^{\text{lit}}$ with standard deviations equal to $\sigma_M$, the prior can be written as $P(H|M_{\text{BH}}^{\text{lit}}) \propto \exp\left(\frac{-(M_{\text{BH}} - M_{\text{BH}}^{\text{lit}})^2}{2\sigma_M^2}\right)$. The prior probability related to $\dot{m}$ takes a similar Gaussian form. We do not have a prior knowledge on either BH spin or inclination. We assume delta function probability distribution for both parameters.

For each $j$th model, we also derive its likelihood function $L(\text{mod}_j) \propto \exp\left(-\chi^2/2\right) = P(D|\text{mod}_j, I)$, where $D$ is the set of photometric points measured for each WLQ quasar. Note, that there is no free parameters.

\(^1\) The NASA/IPAC Extragalactic Database (NED) is operated by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration (ned.ipac.caltech.edu).

\(^2\) $a_* = \frac{\sigma_M}{\sigma_{\text{lit}}}$
Table 1. Sample of Weak Emission-Line Quasars and the sources of their photometry points

| Name           | RA        | Dec       | z_{spec} | Av  | Photometry data                                      |
|----------------|-----------|-----------|----------|-----|------------------------------------------------------|
| SDSS J083650.86+142539.0 | 129.211935 | +14.427527 | 1.749   | 0.129 | WISE (W1,W2,W3), SDSS (u,g,r,i,z), Galex (NUV,FUV)   |
| SDSS J094533.98+100950.1 | 146.391610 | +10.163912 | 1.683   | 0.062 | 2MASS (J,H,K), SDSS (u,g,r,i,z), Galex (NUV,FUV)    |
| SDSS J114153.34+021924.3 | 175.472251 | +02.323508 | 3.55    | 0.065 | WISE (W1,W2), SDSS (g,r,i,z)                        |
| SDSS J13743.08+630144.9  | 189.429435 | +63.029141 | 3.49    | 0.032 | WISE (W1,W2,W3), SDSS (g,r,i,z)                     |
| SDSS J141141.96+140233.9 | 212.929408 | +14.042742 | 1.754   | 0.064 | WISE (W1,W2,W3), SDSS (u,g,r,i,z)                   |
| SDSS J141730.92+073320.7 | 214.378855 | +07.555744 | 1.716   | 0.084 | WISE (W1,W2,W3,W4), SDSS (u,g,r,i,z)                |
| SDSS J144741.76-020339.1 | 221.920448 | -02.060986 | 1.430   | 0.163 | WISE (W1,W2,W3,W4), SDSS (u,g,r,i,z), Galex (FUV)   |
| SDSS J152516.48+520238.5 | 230.485324 | +52.044062 | 2.238   | 0.052 | WISE (W1,W2,W3), 2MASS (J,H,K), SDSS (u,g,r,i,z)    |
| PHL 1811           | 328.756274 | -09.373407 | 0.192   | 0.133 | 2MASS (J,H,K), LCO (B,R), Swift (U), Galex (NUV,FUV) |
| PG1407+265         | 212.349634 | +26.305865 | 0.940   | 0.043 | 2MASS (J,H,K), SDSS (u,g,r,i,z)                     |

Note—The coordinates (Col. 2 and 3), spectral redshift (Col. 4) and foreground Galactic extinction measured at V colour (5) are taken from NED. The column (6) contains references to the names of the relevant catalogues and photometric points.

Table 2. Parameter values for the grid of the AD models

| Parameter      | Δ | min-max values |
|----------------|---|----------------|
| log $M_{BH}$   | 0.1 | 6–12           |
| $\dot{m}$     | 0.01 | 0–1            |
| $a_*$          | 0.1 | 0–0.9          |
| $i$            | 15° | 0°–75°         |

Finally, the posterior probability is determined for each model, as the product of the likelihood and the priors on $M_{BH}$ and $\dot{m}$ (for details see Capellupo et al. 2015, Appendix A). It is given by:

$$P(H|D,I) = N \times \exp \left( -\frac{\chi^2}{2} \right) \times$$

$$\times \exp \left( -\frac{(M_{BH} - M_{BH}^{\text{fit}})^2}{2\sigma_{M_{BH}}^2} \right) \times$$

$$\times \exp \left( -\frac{(\dot{m} - \dot{m}_{\text{fit}})^2}{2\sigma_{\dot{m}}^2} \right)$$

(2)

where $N$ is the normalization constant.

Bayesian analysis identifies which model has the highest probability of explaining the observed SED assuming knowledge of BH mass and accretion rate. The results for model with the highest posterior probability is shown in Tab. 4. We find that the number of sources with satisfactory fit are the same (i.e. have the same high probability) when we use the $\chi^2$ or Bayes’ theorem. Thus and finally, for further analysis we take the masses $M_{BH}$ calculated from $\chi^2$ evaluation as the masses that are not loaded with uncertainty in the determination of $FWHM$.

2.2.2. BH mass and accretion rate determination in PG 1407

We base on the equation (7.28) from Netzer (2013) to estimate the BH mass: $M_{BH} \propto (\nu L_{\nu}(3000 \text{Å}))^{0.58}$ $FWHM$ ($\text{Mg II}$)$^2$. The level of continuum at 3000Å is $\nu L_{\nu} = 4.02 \times 10^{46}$ erg s$^{-1}$ and we determine it using spectrum taken from McDowell et al. (1995). Based on the provided spectrum we also estimate $FWHM$ of Mg II emission-line. Using Fe II template taken from Vestergaard & Wilkes (2001) we subtract contribution of this pseudo-continuum from magnesium and fit one Gaussian function to it. The $FWHM$(Mg II) calculated in this way is equal to $4300\pm 1400$ km s$^{-1}$. Finally, the calculated black hole mass is $M_{BH}^{\text{Mg(II)}} = (3.69^{+2.79}_{-0.85}) \times 10^9$ M$_{\odot}$ and we take this value as $M_{BH}^{\text{fit}}$ (Tab. 3).

In our project we need $FWHM$ of H$\beta$ of PG1407+265, as well. However, this line is almost undetectable weak (McDowell et al. 1995). In order to determine this value, we convert width of magnitude to appropriate one based on equation (6) by Wang et al. (2009) that states log($FWHM$(Mg II)) $\propto 0.81 \times \log(FWHM$(H$\beta$)). Thus, $FWHM$(H$\beta$) = $5400\pm 2240$ km s$^{-1}$. Additionally, we calculate $M_{BH}$ using the mentioned width and estimated by us $\nu L_{\nu}(5100 \text{Å}) = 3.16 \times 10^{46}$ erg s$^{-1}$. The mass of black hole is $M_{BH}^{H\beta} = (2.62^{+2.61}_{-0.73}) \times 10^9$ M$_{\odot}$.

The accretion rate in PG 1407 is estimated using relationship $L_{Bal}/L_{Edd} = J(L) \times (\nu L_{\nu}(5100 \text{Å}))^{0.5} \times$
same grid of 366000 models (see Sec. 2) to obtain \( \nu_L \) formula (1), which is proportional to line is a 1:1 identity line. Vestergaard & Osmer (2009) masses are given in mass units of the Sun. Violet solid line is a 1:1 identity line. Vestergaard & Osmer (2009) of errors means that the continuum fitting method applies to quasars.

There are many approaches of estimation the accretion efficiency, \( \eta \). In order to bring the masses into conformity, it is necessary to adopt \( \eta = 0.15 - 0.20 \). There are many approaches of estimation the accretion efficiency. For a non-rotating BH \( \eta \) computed in Schwarzschild metric \( \simeq 0.057 \). Shankar et al. (2009) suggest \( \eta = 0.05 - 0.1 \) in relation to AGNs. Observational constrains on growth of BHs made by Yu & Tremaine (2002) give us reasonable argument that \( \eta \) should be \( \gtrsim 0.1 \). Even more, Cao & Li (2008) proposed \( \eta = 0.18 \) for AGNs with BH masses above \( 10^9 \, M_\odot \). We take the same value of \( \eta = 0.18 \) in relation to WLQ and LBQS.

Our sample of weak emission-line quasars contains 10 objects. In Fig. 2 we present the best fits of disc continua that match the quasar SED. On the x axis is the logarithmic value of frequency in Hertz, while on the y axis is the logarithmic value of \( \nu L_\nu \) in erg s\(^{-1}\). The accretion disc continuum is marked with a solid purple line, the photometric points that are taken for fitting are shown by green crosses with errors, while the photometric points that have not been taken for fitting are shown in blue points. The rejection of photometric points is caused by a bigger influence of a molecular/dust torus to IR luminosity. In object J0386 and J1147 point in UV range could suggest an absorption seen in some quasars (e.g. KVRQ 1500-0031 Heintz et al. (2018), SDSS J080248.18+551328.9 Ji et al. (2015); Liu et al. (2015)).

Our results are collected in Tab. 3. The values contain the black hole masses, accretion rates, spins, inclinations of each WLQ, and \( \chi^2 \) values. They are in Columns (1)-(6). Columns (7) and (8) contain the literature values of the black hole masses and the accretion rates, respectively. Column (9) contains references to the above values, as appropriate. Fig. 3 shows the mass distribution of black holes. Identity 1:1 line is marked as solid purple.

The presented mass comparison suggests that literature determinations of black hole masses, \( M_{BH} \), based on \( FWHM(H_\beta) \) are generally underestimated. We also determine the difference between \( M_{lbh} \) and \( M_{bh} \) values. The \( \gamma \)-factor is calculated (\( \gamma = M_{BH}/M_{BH}^{L} \)). In Fig. 4 we present the relationship between the logarithmic value of \( FWHM(H_\beta) \) in km s\(^{-1}\) (see Tab. 5) and the logarithmic value of the \( \gamma \)-factor. The green solid line shows the best fit between those variables. The fit is made using the nonlinear least-squares (NLLS) Marquardt-Levenberg algorithm which takes into account errors in both \( x \) and \( y \) directions. The relationship is:

\[
\log \gamma = (-1.993 \pm 0.667) \times \log \left( \frac{FWHM(H_\beta)}{10^3 \text{ km s}^{-1}} \right) + (1.976 \pm 0.395)
\]

For a better assessment of our calculations, we have determined the Spearman coefficient, which is \( r_s = -0.681 \) and the linear correlation coefficient,
Figure 2. The best fit of SED to photometric points of WLQs. Green crosses with errors – fitted points, blue stars – rejected. Violet solid line represents the theoretical curve of continuum model.
Table 3. The best fit parameters

| Name   | $M_{\text{BH}}$          | $\dot{m}$       | $a_*$ | $i$ | $\chi^2$/d.o.f | $M_{\text{BH}}^{\text{lit}}$ | $\dot{m}_{\text{lit}}$ | ref. |
|--------|--------------------------|-----------------|-------|-----|----------------|-----------------------------|-------------------------|------|
|        | ($1.00 \pm 1.64$) $\times 10^{10}$ | 0.035 $\pm 0.034$ | 0.00 $\pm 0.10$ | 0.260 $\pm 0.520$ | 4.42 | ($3.89 \pm 13.11$) $\times 10^8$ | 0.87 $\pm 1.36$ | 1/1     |
| J0836  | ($6.30 \pm 1.97$) $\times 10^9$  | 0.146 $\pm 0.020$ | 0.70 $\pm 0.25$ | 1.040 $\pm 0.200$ | 1.19 | ($3.08 \pm 0.27$) $\times 10^9$ | 0.51 $\pm 0.15$ | 2/1     |
| J0945  | ($7.85 \pm 2.20$) $\times 10^9$  | 0.058 $\pm 0.021$ | 0.20 $\pm 0.13$ | 0.520 $\pm 0.086$ | 1.13 | ($5.25 \pm 2.25$) $\times 10^9$ | 0.34 $\pm 0.42$ | 3/3     |
| J141   | ($7.95 \pm 2.58$) $\times 10^9$  | 0.066 $\pm 0.025$ | 0.00 $\pm 0.00$ | 0.520 $\pm 0.130$ | 1.23 | ($3.55 \pm 1.53$) $\times 10^8$ | 0.92 $\pm 0.50$ | 1/1     |
| J1447  | ($2.50 \pm 0.36$) $\times 10^9$  | 0.169 $\pm 0.026$ | 0.96 $\pm 0.02$ | 0.260 $\pm 0.260$ | 3.35 | ($1.12 \pm 0.51$) $\times 10^8$ | 1.30 $\pm 0.78$ | 1/1     |
| J1521  | ($7.90 \pm 0.85$) $\times 10^{10}$ | 0.114 $\pm 0.013$ | 0.20 $\pm 0.20$ | 0.000 $\pm 0.520$ | 1.61 | ($6.26 \pm 0.73$) $\times 10^9$ | 0.81 $\pm 0.27$ | 4/4     |
| PHL 1811 | ($1.30 \pm 0.49$) $\times 10^9$  | 0.223 $\pm 0.027$ | 0.00 $\pm 0.00$ | 0.26 $\pm 0.780$ | 1.59 | ($1.14 \pm 2.57$) $\times 10^8$ | 1.30 $\pm 0.02$ | 5/5     |
| PG 1407 | ($5.00 \pm 1.92$) $\times 10^9$  | 0.100 $\pm 0.100$ | 0.20 $\pm 0.10$ | 1.300 $\pm 0.260$ | 0.26 | ($3.69 \pm 2.75$) $\times 10^9$ | 0.45 $\pm 0.23$ | a/a     |

Note—Estimation of $M_{\text{BH}}$ (Col. 2) and $M_{\text{BH}}^{\text{lit}}$ (Col. 7) are in units of $M_{\odot}$. † based on Mg II. * errors of $\dot{m}$ are estimated by us. Numbers refer to data sources 1) Plotkin et al. (2015), 2) Hryniewicz et al. (2010), 3) Shemmer et al. (2010), 4) Wu et al. (2011), 5) Leighly et al. (2007a), 6) McDowell et al. (1995), a) this work.
r = -0.710. The dashed blue line in Fig. 4 represents the best fit obtained by Mejía-Restrepo et al. (2018). They use a sample of 37 Type I AGNs, which lie in the range of redshifts ~ 1.5. It can be seen in Fig. 4 that two objects J1141 and J1237 are close to 1:1 line, this means that their mass values should be corrected by a small γ-factor, while the remaining WLQs have too low masses based on \( FWHM(\text{Hβ}) \) values. Objects J1141, J1237, PG1407 should be changed by factor γ < 4, and PHL1811, J0836, J1411, J1417, J1521 by factor γ > 10. The mean γ-factor is 7.40, median is 12.02. Eq. (3) allows us to correct the black hole masses, \( M_{\text{BH}}^{\text{fit}} \), determined so far and based on \( FWHM \) values of Hβ line.

Black hole masses determined from Bayesian analysis are presented in Tab. 4. No significant differences/similar results of black hole masses from both \( \chi^2 \) methods and Bayesian analysis suggest that the determined global parameters are correct and describe the overall SED shape of these objects.

### 4. DISCUSSION

The virial factor (f in Eq. 1) is often assumed to be constant with values of 0.6-1.8 (e.g. Peterson 2004; Onken et al. 2004; Nikolajuk et al. 2006), where 0.75 corresponds to a spherical geometry of the BLR. Generally, f depends on non-virial velocity components (e.g. winds), the relative thickness (\( H/R_{\text{BLR}} \)) of the Keplerian BLR orbital plane, the line-of-sight inclination angle (i) of this plane, and the radiation pressure (Wills & Browne 1986; Gaskell 2009; Denney et al. 2009, 2010; Shen & Ho 2014; Runnoe et al. 2014) and it should be a function of those phenomenons. The analysis carried out by Mejía-Restrepo et al. (2018) indicates a low influence of radiation pressure on the f-factor, however this mechanism cannot be excluded. Whether or not we skip the radiation pressure influence, the line-of-sight incli-

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**Table 4. Masses from Bayesian analysis**

| Name   | \( M_{\text{BH}}^{\text{Bay}} \) [M\(_\odot\)] |
|--------|----------------------------------|
| J0836  | \((2.00 \pm 1.09) \times 10^{9}\) |
| J0945  | \((6.30 \pm 3.96) \times 10^{9}\) |
| J1141  | \((4.00 \pm 8.10) \times 10^{9}\) |
| J1237  | \((6.30 \pm 3.96) \times 10^{9}\) |
| J1411  | \((7.90 \pm 2.02) \times 10^{9}\) |
| J1417  | \((7.90 \pm 1.55) \times 10^{9}\) |
| J1447  | \((1.30 \pm 0.61) \times 10^{9}\) |
| J1521  | \((1.30 \pm 0.64) \times 10^{11}\) |
| PHL 1811 | \((4.00 \pm 4.90) \times 10^{10}\) |
| PG 1407 | \((2.00 \pm 1.80) \times 10^{9}\) |

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**Table 5. The Full Width at Half Maximum of Hβ emission line and the γ-factor**

| Name   | \( FWHM(\text{Hβ}) \) [km s\(^{-1}\)] | \( \log \gamma \) | ref. |
|--------|-------------------------------|-----------------|-----|
| J0836  | \(2880 \pm 1877\) | 1.41 \(\pm 0.18\) | 1   |
| J0945  | \(4278 \pm 598\) | 0.75 \(\pm 0.13\) | 1   |
| J1141  | \(5000 \pm 1500\) | 0.06 \(\pm 0.25\) | 2   |
| J1237  | \(5200 \pm 1500\) | 0.30 \(\pm 0.13\) | 2   |
| J1411  | \(3966 \pm 1256\) | 1.17 \(\pm 0.31\) | 1   |
| J1417  | \(2784 \pm 759\) | 1.35 \(\pm 0.27\) | 1   |
| J1447  | \(1924 \pm 933\) | 1.35 \(\pm 0.15\) | 1   |
| J1521  | \(5750 \pm 750\) * | 1.11 \(\pm 0.22\) | 3   |
| PHL 1811 | \(1943 \pm 19\) | 1.05 \(\pm 0.19\) | 4   |
| PG 1407 | \(5400 \pm 810\) * | 0.14 \(\pm 0.43\) | a   |

**Note:** \( \gamma = M_{\text{BH}}/M_{\text{BH}}^{\text{fit}} \). \* errors are estimated by us. \* Hβ is weak and almost not visible, its \( FWHM \) is estimated based on \( FWHM(\text{Mg II}) \). Numbers refer to \( FWHM \) sources: 1) Plotkin et al. (2015), 2) Shemmer et al. (2010), 3) Wu et al. (2011), 4) Leighly et al. (2007a), a) this work.

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**Figure 3.** Comparison of SMBH masses (\( M_{\text{BH}}^{\text{fit}} \)) of WLQs based on \( FWHM \)estimations with \( M_{\text{BH}} \), that come from spectral fitting method. Violet solid line is identity 1:1 line. Green crosses represent \( M_{\text{BH}} \) calculated using \( FWHM \) of Hβ and blue stars base on Mg II (it concerns only PG1407 – \( M_{\text{BH}}^{\text{fit}} \) from our estimation and J0945 – \( M_{\text{BH}}^{\text{fit}} \) the mass estimated by Hryniewicz et al. 2010).

**Figure 4.** Comparison of SMBH masses (\( M_{\text{BH}}^{\text{fit}} \)) of WLQs based on \( FWHM \)estimations with \( M_{\text{BH}} \), that come from spectral fitting method. Violet solid line is identity 1:1 line. Green crosses represent \( M_{\text{BH}} \) calculated using \( FWHM \) of Hβ and blue stars base on Mg II (it concerns only PG1407 – \( M_{\text{BH}}^{\text{fit}} \) from our estimation and J0945 – \( M_{\text{BH}}^{\text{fit}} \) the mass estimated by Hryniewicz et al. 2010).

The radiation pressure on the \( f \)-factor, however this mechanism cannot be excluded. Whether or not we skip the radiation pressure influence, the line-of-sight incli-
In this way, the corrected formula for the SMBH masses in WLQ is:

$$\frac{M_{BH}}{10^8 M_\odot} = (4.78^{+7.39}_{-2.55}) \times \left(\frac{\nu L_\nu(5100)}{10^{42} \text{erg s}^{-1}}\right)^{0.5} \times \left(\frac{FWHM(H_\beta)}{10^3 \text{km s}^{-1}}\right)^{-0.01 \pm 0.67}$$

We note a low dependence on $FWHM(H_\beta)$. Such behaviour can be realized in two cases. The BLR is dominated by outflows e.g. jets, winds which are perpendicular to the line-of-sight. In the second case, the BLR produced $H_\beta$ is elongated and parallel to the accretion disc and an observer is above it.

Mejia-Restrepo et al. (2018) find that the dependence of $M_{BH}$ on the observed $FWHM$ of the Balmer lines for AGNs is close to linear rather ($M_{BH} \propto FWHM(H_\beta)^{-0.83 \pm 0.11}$, where $f$ is a function of the Full Width) than quadratic ($M_{BH} \propto FWHM^2$ with $f = const$). In our case (see Fig. 4), this relationship for WLQs is even stronger ($M_{BH} \propto FWHM(H_\beta)^{-0.01 \pm 0.67}$), but still compatible with the Mejia-Restrepo et al. result within $2\sigma$ error. A similar or even the same behaviour of normal AGNs and WLQs suggests that both kind of sources have the same dim nature of the velocity component and the same geometry of the BLR. We conclude that WLQs are normal quasars in an reactivation stage.

Systematic underestimation of $FWHM$ (and $M_{BH}$) may be caused by a strong influence of the Fe II pseudo-continuum in optics. Such phenomena is noticed by Plotkin et al. (2015) for their sample of WLQ, which have larger $R_{opt,FeII}$ and narrower $H_\beta$ than most reverberation mapped quasars. Despite the mentioned contamination, the trend described by Eq. (3) should be preserved with slope smaller than zero.

The underestimation of $M_{BH}$ could also appears in those cases where the SMBH masses are calculated using Mg II instead $H_\beta$ line (blue stars in Fig. 3). However, our sample (2 objects – PG1407, J0945) is too small to bear this remark out. Following Plotkin et al. (2015), we do not attempt estimations of $M_{BH}$ or $\dot{m}$ using C IV or Mg II, because there are indications for non-virialized motion of gas (Baskin & Laor 2005; Trakhtenbrot & Netzer 2012; Kratzer & Richards 2015). Thus, Plotkin et al. (2015) suggest that using either C IV or MgII could bias the mass measurements in WLQs.

5. CONCLUSIONS

In this work we have studied the accretion disc continua of 10 WLQs. The SMBH masses of those objects are estimated previously based on the scaling method ($M_{BH}$). We create grid of 366000 models using the Novikov-Thorne formulas. We adopt four parameters
($M_{\text{BH}}$, $\dot{m}$, spin of BH and the line-to-sight inclination) to describe the observed SED and compare obtained BH masses with those got from the literature.

Our main findings are:

1. Using the Novikov-Thorne model, we can describe very well the SED of WLQs.

2. The SMBH masses of WLQs, which are estimated based on $FWHM(\text{H}\beta)$, are underestimated. On average, the masses are undervalued 7.4 times. The median of this $\gamma$ correction factor is 12.

3. We propose the formula to estimate $M_{\text{BH}}$ in WLQs based on their observed $FWHM(\text{H}\beta)$ and luminosities at 5100Å (Eq. 5). Our results suggest that selected WLQs have the accretion rates in the range $\sim 0.1$-0.4.

4. We support Mejía-Restrepo et al. result and confirm that the virial factor, $f$, depends on $FWHM$ ($\propto FWHM(\text{H}\beta)^{-1.99\pm0.67}$). The BLR is non-spherical region.

5. We suggest that WLQs are normal quasars in an reactivation stage.

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