Solid particle acceleration in a high Reynolds number channel flow: DNS and LES with stochastic modelling of subgrid acceleration

Rémi Zamansky, Ivana Vinkovic and Mikhail Gorokhovski
Laboratoire de Mécanique des Fluides et d’Acoustique CNRS UMR 5509
Ecole Centrale de Lyon, 36, av. Guy de Collongue, 69134 Ecully Cedex, France
E-mail: ivana.vinkovic@univ-lyon1.fr

Abstract. Inertial particle acceleration statistics are analyzed using DNS in the case of a turbulent channel flow. Along with effects recognized in homogeneous isotropic turbulence, an additional effect is observed due to high and low speed vortical structures aligned with the channel wall. In response to those structures, the inertial particles experience strong streamwise acceleration variations. DNS is also used in order to assess LES-SSAM (Subgrid Stochastic Acceleration Model), in which an approximation to the instantaneous non-filtered velocity field is given by simulation of both, filtered and residual, accelerations. Advantages of this approach in predicting particle dynamics in the channel flow at a high Reynolds number are shown.

1. Introduction
Understanding the Lagrangian behavior of inertial particles in turbulent channel flows has important implications for many environmental systems, from sediment transport to atmospheric dispersion of pollutants or solid deposition in marine flows. Previous experimental [1] and numerical [2] studies on particle-laden channel flows have examined particle deposition, trapping, segregation or the modification of particle velocity statistics due to the presence of coherent structures. It is recognized that inertial effects cause particle segregation and clustering [3]. In a high Reynolds number “free” turbulence [4, 5], these phenomena were linked to particle acceleration statistics. In our paper, such statistics are of interest in the case of a high Reynolds number channel flow. This may help in understanding complex interactions between particle dynamics and wall flow structures.

Lagrangian measurements [5, 6, 7] or computations [2, 8] provided insight into inertial particle accelerations and their statistics, including the acceleration probability density functions (PDF), in both homogeneous and inhomogeneous flows. These studies showed that, in the case of homogeneous isotropic turbulence, solid particles with low inertia exhibit highly non-Gaussian acceleration PDF with a high probability of intense acceleration events. These acceleration PDF tend to narrow and to Gaussianize as particle inertia increases [4]. In [4], the authors suggested that the trend of acceleration PDF to Gaussianity, as well as a monotonic decrease of acceleration variance, are both a consequence of preferential concentration (dominant at small Stokes numbers) and filtering (for particles with larger Stokes numbers).

One could expect the same tendencies in the presence of the wall: preferential concentration and filtering may decrease the variance of particle acceleration. However, close to the wall,
the flow is very different from homogeneous isotropic turbulence. There are typical streaks, aligned with the wall, characterized by alternating high and low longitudinal speed regions. Low inertia particles do not see this alternation of regions; once such particles get trapped in the structure, they travel with it. Highly inertial particles do not respond to this intermittency neither, due to the filtering effect. Only particles with intermediate inertia may respond to the spanwise alternation of high and low speed regions; this may cause an additional agitation of those particles. In this paper, particle acceleration statistics in the channel flow are explored, in comparison with the effects described by [4] for the case of homogeneous isotropic turbulence.

For single phase flows, large-eddy simulation (LES) has become a suitable tool that produces acceptable results at much lower computational costs than direct numerical simulation (DNS). However, application of LES in turbulent channel flow becomes too expensive when the Reynolds number is high and the resolution of strong subgrid gradients is needed, specifically in the near-wall region, in the case of wall-bounded flows. An alternative to standard LES, referred to as LES-SSAM approach was proposed in [12, 13]. In this approach, the exact Navier-Stokes equation is replaced by a model equation in which the instantaneous total acceleration is viewed as a sum of two model accelerations. One represents the filtered acceleration closed by Smagorinsky model, the other one emulates the residual (subgrid) acceleration. The latter is done by two independent stochastic processes, for the norm of this acceleration and for its orientation. The new stochastic model for the subgrid acceleration was proposed in [14], and was assessed in the case of turbulent channel flow in the framework of LES-SSAM approach. The stochastic model for the norm of the fluid acceleration is based on the fragmentation process [15] in order to represent the long-range interactions across the channel. The fluid acceleration orientation is also simulated stochastically as a Brownian motion on a unit sphere, starting from random orientation on the wall plane, and then going toward stochastic isotropy away from the wall. Such a development of LES-SSAM approach [14] is applied to particle-laden channel flows in this paper. The accuracy of the method will be tested in regard of particle velocity and acceleration statistics. Results will be compared with DNS and classical LES. The tests will be run for different types of particles given by Stokes numbers from 1 to 125. The article is organized as follows. In the next section, the numerical method is described. Then, we present some results obtained by DNS. In the last section, the LES-SSAM approach is compared to DNS and classical LES. Finally, conclusions are stated.

2. Numerical method

2.1. Flow

The flow considered is an incompressible turbulent channel flow. The computational domain consisting of two infinite parallel walls is illustrated in Figure 1. Periodic boundary conditions are imposed on the fluid velocity field in x (streamwise) and z (spanwise) directions and no-slip boundary conditions are imposed at the walls.

Three different sets of numerical simulations for the fluid are treated here: direct numerical simulation of channel flow (DNS), standard large-eddy simulation (LES) and large-eddy simulation coupled with stochastic forcing of subgrid acceleration (LES-SSAM). For the case of LES the classical Smagorinsky model is used [11].

Details on the simulation characteristics are given in Table 1, where \( N_i \) and \( L_i \) are the number of grid points and the domain length in direction \( i \). The size of the grid is also given. The superscript “+” denotes quantities expressed in wall units, normalized by the friction velocity \( u_\tau \) and the viscosity \( \nu \). For all simulations, the Reynolds number based on the \( u_\tau \), the channel half height \( h \) and \( \nu \), is \( Re_\tau \sim 587 \). In the case of DNS, this corresponds to a Reynolds number based on the mean velocity \( U \) at the center of the channel of \( Re \sim 12500 \). The dimensions of the computational domain have been changed in LES with respect to DNS in order to match previous LES of turbulent channel flow. It has been checked that the difference in size does not
Figure 1. Channel flow. $x$, $y$, $z$ represent the streamwise, the wall-normal and the spanwise directions respectively.

affect the particle statistics presented here.

| Type          | $Re_r$ | $Re$   | $N_x \times N_y \times N_z$ | $L_x \times L_y \times L_z$ | $\Delta x^+ \times \Delta y^+ \times \Delta z^+$ |
|---------------|--------|--------|-----------------------------|-----------------------------|---------------------------------|
| DNS           | 587    | 12490  | $384 \times 257 \times 384$ | $3/2\pi h \times 2h \times 3/4\pi h$ | $7.2 \times (0.04 \sim 7.2) \times 3.6$ |
| LES           | 587    | 14160  | $64 \times 65 \times 64$    | $3\pi h \times 2h \times \pi h$  | $87 \times (0.71 \sim 29) \times 29$ |
| LES-SSAM      | 587    | 12760  | $64 \times 65 \times 64$    | $3\pi h \times 2h \times \pi h$  | $87 \times (0.71 \sim 29) \times 29$ |

The incompressible Navier-Stokes equations in a turbulent channel flow are solved using a Galerkin spectral approximation (Fourier Chebyshev) and a variationnal projection method on a divergence free space [16]. Steady state fluid statistics have been compared with other DNS in [14].

2.2. Particles
Particles are injected into the flow at low concentrations in order to consider dilute systems. Particle-particle interactions are neglected as well as gravity or the influence of particles on the carrier fluid. Particle-wall collisions are assumed to be elastic. Furthermore, particles are considered to be point-wise, spherical, rigid and to obey the following Lagrangian dimensionless equation of motion:

$$St \frac{d\bar{v}_p}{dt} = (\bar{u} - \bar{v}_p) f(Re_p)$$  (1)

$$\frac{d\bar{x}_p}{dt} = \bar{v}_p$$
Here, $\vec{v}_p$ and $\vec{x}_p$ are the dimensionless particle velocity and position. The solid particle - fluid interaction is modelled by a drag force with the correction term $f(Re_p) = 1 + 0.15Re_p^{0.687}$ suggested by [17]. $Re_p$ is the local and instantaneous particle Reynolds number based on the local relative velocity, the particle diameter $d_p$ and the fluid viscosity.

$St$ is the Stokes number given by:

$$St = \frac{\tau_p}{\tau_f}$$  \hspace{1cm} (2)

with $\tau_p = \frac{\rho_p d_p^2}{18 \rho \nu}$ and $\tau_f = \frac{\nu}{u_x^2}$, $\rho_p$ and $\rho$ being respectively the particle and fluid density. The Stokes number characterizes the response time of a particle to fluid solicitation.

Once the steady state for the fluid is obtained, 200000 particles are released at randomly chosen locations within the channel, then tracked at each time step. Five sets of particles are considered with $St = 1$, $St = 5$, $St = 15$, $St = 25$ and $St = 125$.

The initial velocities of the particles are set equal to the interpolated fluid velocities at each particle location. A high order three dimensional Hermite interpolation is used for computing the fluid velocity $\vec{u}(\vec{x}_p, t)$ at the particle position. The time-integration of the particle motion Equation (1) is performed using a second-order Adams-Bashforth method with the same time step as the DNS. Once the particles released, the simulations are run over several particle timescales $\tau_p$. Particle statistics are sampled starting from $t^+ = 1000$, counted from particle release. For all simulations, velocity statistics for the solid phase are at stationary state.

In this study, the acceleration is evaluated using the velocity time derivative along particle trajectories. Even though we use a three dimensional Hermite interpolation for computing the fluid velocity at particle position, as suggested in [18] numerical errors are generated when a particle crosses a grid point. Therefore, as in [19], the acceleration is estimated by a convolution of the Lagrangian velocity with the derivative of a Gaussian kernel. This ensures both time derivation and filtering. The filter width, of the order of $\tau_f$, is such that there is agreement between fluid Lagrangian and Eulerian acceleration statistics.

In the case of DNS, for a lower Reynolds number, particle velocity statistics have been compared with the benchmark [20]. For higher Reynolds numbers, velocity statistics have also been compared to experiments [21]. Details about these comparisons and other results based on particle velocity statistics may be found in [22].

3. DNS acceleration statistics

In this section we present some results obtained by DNS.

3.1. Acceleration variance

Figure 2 shows the streamwise acceleration variance of solid particles as a function of $y^+ = y u_\tau / \nu$. For comparison, other DNS [23, 8] and experiments are also illustrated [7]. In [23], the fluid acceleration in a turbulent channel flow at $Re_\tau = 180 \sim 600$ without gravity is studied. From Figure 2, it is seen that the peak of acceleration variance obtained in our DNS is lower than the one computed by [23]. The measurements reported in [7] for inertial particles in the turbulent boundary layer at $Re_{\lambda_0} = 100$ and $St_0 = 0.035$ are plotted, where $Re_{\lambda_0}$ is the Reynolds number based on the Taylor-scale and the streamwise velocity root-mean-square (RMS), and where the subscript zero refers to the free-stream conditions. This corresponds to $Re_\tau \sim 470$ and $St \sim 0.7$ and is closest to our $St = 1$ and $Re_\tau \sim 600$ simulation. Although gravity was not taken into account here, the DNS results are close to the measurements [7]. The DNS results of [8] are also shown in Figure 2 for the case without gravity, $St = 0.87$ and $Re_\tau = 300$. It can be seen that the present DNS is close to the DNS reported in [8] except close to the wall, where we slightly underestimate the peak of inertial particle acceleration variance.
From Figure 2 it is seen that as the distance to the wall increases, the acceleration variance decreases. In addition to this, as particle inertia increases the acceleration variance departs from the fluid. This is in accordance with previous studies in homogeneous isotropic flows [4, 5, 24] and results from the simultaneous effect of preferential concentration and filtering. For $St = 5$, the streamwise acceleration variance presents the highest peak close to the wall. This peak is even higher than the fluid streamwise acceleration variance. To gain insight on this behavior, we analyze the acceleration statistics of the fluid seen by the solid particles.

![Figure 2](image-url)

**Figure 2.** Streamwise acceleration variance profile for solid particles with different Stokes numbers and for the fluid (dashed line). [7] for $St \sim 0.7$ and $Re_{\tau} \sim 470$ (full diamonds), [8] for $St = 0.87$ and $Re_{\tau} = 300$ (full triangles), [23] for fluid particles at $Re_{\tau} = 600$ (full squares).

Figure 3 illustrates the streamwise acceleration variance of the fluid seen by the solid particles. The variance for the fluid seen by the solid particles is higher than the variance of the non-conditional fluid. This is especially the case close to the wall, at the position of the peak of streamwise acceleration variance. The peak of fluid streamwise acceleration at the solid particle position increases as the Stokes number increases. Whereas, the peak of solid particle acceleration first increases from $St = 1$ to $St = 5$ and then decreases as the Stokes number increases.

These observations suggest that in the case of wall bounded flow, solid particles are entrained preferentially by regions with relatively high streamwise acceleration variance. The contrary is observed in homogeneous isotropic turbulence [4, 5, 24], where previous studies found that inertial particles tend to cluster in regions of the fluid experiencing relatively low fluid accelerations. This behavior of the solid particle streamwise acceleration variance suggests that because of inertia particles respond less and less to the increasingly varying fluid solicitations. This results in a net decrease of solid particle streamwise acceleration variance.
3.2. Acceleration PDF
In Figures 4 and 5 we compare the normalized PDFs of streamwise and wall-normal acceleration, respectively, at $y^+ \sim 100$ for different Stokes numbers with the normalized PDFs obtained by using the fluid acceleration on the particle position. For reference, the normalized PDF for the non-conditional fluid acceleration is plotted as well. As expected, when the Stokes number increases, the tails of the normalized solid particle PDFs become narrower. For all Stokes numbers, the normalized PDFs of the fluid seen by the solid particles overlap almost perfectly with the normalized PDFs of the non-conditional fluid. The same conclusions can be drawn for other distances to the wall and for the spanwise component of the acceleration. For simplicity reasons, these plots are not shown here. The overlap suggests similarity of structure in terms of statistical distribution of fluid acceleration seen by the solid particles. The scaling factor is given by the acceleration RMS, which is different for each Stokes number.

These results show that in wall bounded flows, effects predicted in [4] are present with a new aspect which is additional agitation of particles by alternation of high and low speed streaks. Wall bounded flows present spatial alternation of high and low speed vortical structures aligned with the channel wall. As in homogeneous isotropic turbulence, inertial particles are ejected from high vorticity regions toward high strain (high dissipation rate) regions. Inertial particles are swept by these regions and due to the random alternation of high and low speed streaks, experience strong streamwise velocity fluctuations. Thereby, in these regions, inertial particles see fluid with high streamwise acceleration variance. Slightly inertial particles (with a response time $\tau_p$ similar to the characteristic fluid timescale) may well respond to the fluid solicitations and therefore experience an increase of the streamwise acceleration variance relatively to the
Figure 4. Streamwise acceleration PDF at $y^+ = 100$, for (from top to bottom) $St = 1, 5, 15, 25, 125$. Solid particles (line), fluid particles (triangle), fluid seen by solid particles (square).

Fluid. Due to the filtering effect of inertia, very inertial particles ignore the wall turbulent structures and therefore present a more homogeneous concentration.

Here, the significance of wall structures on particle acceleration statistics has been shown. In the frame of LES, wall structures are under-resolved. Therefore, in the next section, classical LES is coupled with a stochastic model for subgrid acceleration in order to introduce flow intermittency at subgrid scales.

4. LES-SSAM applied to particles
Here we briefly describe the stochastic subgrid model for the acceleration used in the LES-SSAM approach. The LES-SSAM is then compared to classical LES and DNS in the case of particle laden turbulent channel flows.

4.1. LES-SSAM approach
In order to take into account the non-resolved acceleration in standard LES, the approach proposed in [12] and [13] is used here. In this approach, the total instantaneous acceleration is given by two contributions: $a_i = \bar{a}_i + a'_i$. The first component is the filtered total acceleration where the overbar denotes the filtering operation. The second component represents the total acceleration in the residual field. When both parts are modeled, their sum gives an approximation to the non-filtered velocity field.

In the case of channel flow, a stochastic model for the subgrid acceleration was proposed in [14]. This model is based on the norm/orientation decomposition of the fluid acceleration. The
stochastic model for the norm of the fluid acceleration is based on the fragmentation process in order to represent the long-range interactions across the channel. The fluid acceleration orientation is simulated by a random walk on a sphere in order to reproduce the relaxation towards isotropy with increasing wall distance. In comparison with standard LES, it was showed that the LES-SSAM approach gives better prediction of velocity spectra on small spatial scales and better statistics of fluid acceleration [14]. In the following section, we apply this model to the case of particle laden channel flow. We assume that the modelled subgrid acceleration can be attributed to the acceleration of the fluid seen by the solid particle.

4.2. Results and comparisons

Figures 6 and 7 show the solid particle mean and RMS velocity profiles obtained from the LES-SSAM model and compared to the DNS and LES results. The LES-SSAM model gives a better estimation of the mean solid particle velocity than classical LES (Figure 6). This is illustrated by the average relative distance of LES and LES-SSAM to the DNS, given in the inset of Figure 6. In addition to this, there is also an improvement of the prediction of the wall-normal (Figure 7) velocity RMS profile.

A quantitative indicator of the improvements made by LES-SSAM is the average streamwise particle flux:

\[ <u_p> = \frac{1}{2h} \int_0^{2h} u_p(y)dy \]
Figure 6. Mean streamwise velocity profile of solid particles for (from top to bottom) $St = 1, 5, 15, 25, 125$. DNS (line), LES (squares), LES-SSAM (triangles). The average relative distance of LES (squares) and LES-SSAM (triangles) to DNS is given in the inset.

This flux is given in Figure 8 as a function of the Stokes number. The inset shows the relative distance to DNS. A better global estimation of the flux is achieved by LES-SSAM.

Figures 9 and 10 illustrate the RMS and PDF of the solid particle acceleration. The profile of the RMS is shown for the spanwise component, whereas the PDFs are given for the wall-normal component at $y^+ \sim 100$. As described in the previous section, the PDFs obtained by the DNS display stretched tails. As the Stokes number increases the acceleration PDFs tend to Gaussianize and the RMS of solid particle acceleration decreases. This evolution is well reproduced by LES-SSAM, while standard LES predicts much higher tails for the normalized PDF and much lower values of the acceleration RMS. The overall improvement achieved by the model is illustrated by the average relative distance of LES and LES-SSAM to DNS, given in the inset of Figure 10. For other distances to the wall or other components of the acceleration we observe either the same improvement or no difference with classical LES. These results point out the importance of predicting subgrid scale acceleration for estimating solid particle velocity and acceleration statistics.

5. Conclusion
In this study, numerical simulations of particle laden turbulent channel flow are performed for five different Stokes numbers and a high Reynolds number. Acceleration statistics obtained by DNS show that in wall bounded flows, effects predicted in homogeneous isotropic turbulence [4] are completed by a new aspect which is interaction of particles with high and low speed
streaks. As in homogeneous isotropic flows, inertial particles are ejected from high vorticity regions toward high strain (high dissipation rate) regions. Due to the proximity of high and low speed streaks, particles experience strong streamwise acceleration variations. Depending on their inertia particles may well respond to those fluid solicitations (experiencing an increase of the streamwise acceleration variance) or ignore the wall turbulent structures (presenting in that case a more homogeneous concentration).

In order to take the intermittency of subgrid flow into account, a model for the subgrid acceleration is introduced according to [14]. The capability of LES-SSAM for particle-laden turbulent channel flow is assessed by comparing the results with DNS and classical LES. The LES-SSAM model gives a better estimation of the solid particle velocity and acceleration statistics. These improvements are due to two factors. First, particle velocity statistics are mainly influenced by the large scale flow which, as shown in [14], are better simulated. Second, with LES-SSAM the influence of small scales is taken into account, which is important for particle acceleration statistics.

Along with fundamental interest, the LES-SSAM approach has a practical relevance, when significant physics take place on subgrid scales. Examples include atomization, droplet evaporation and combustion.

6. Acknowledgments
M. Buffat is acknowledged for the development of the computational code. The authors express their gratitude to F. Laadhari who kindly provided his initial fields for the DNS. This work
was granted access to the HPC resources of CINES under the allocation 2009-c200902560 made by GENCI (Grand Equipement National de Calcul Intensif). Numerical simulations were also performed on the P2CHPD parallel cluster. The authors are therefore grateful to C. Pera for the administration of the computational tool.

References
[1] Kaftori D, Hestroni G and Banerjee S 1995 Phys. Fluids 7 1095
[2] Marchioli C and Soldati A 2002 J. Fluid Mech. 468 283
[3] Kiger K T and Pan C 2002 J. of Turbulence 3 1
[4] Bee J, Biferale L, Boffetta G, Celani A, Cencini M, Lanotte A, Musacchio S and Toschi F 2006 J. Fluid Mech. 550 349
[5] Ayyalasomayajula S, Warhaft Z and Collins L R 2008 Phys. Fluids 20 095104
[6] Qureshi N M, Arrieta U, Baudet C, Cartellier A, Gagne Y and Bourgoin M 2008 Eur. Phys. J. B 66 531
[7] Gerashchenko S, Sharp N S, Neuscamman S and Warhaft Z 2008 J. Fluid Mech. 617 255
[8] Lavezzo V, Soldati A, Gerashchenko S, Warhaft A and Collins L R 2010 J. Fluid Mech. 658 229
[9] Meneveau C and Katz J 2000 Annu. Rev. Fluid Mech. 32 1
[10] Piomelli U and Balaras E 2002 Annu. Rev. Fluid Mech. 34 349
[11] Sagaut P 2002 Large Eddy Simulation for Incompressible Flows: An introduction 2nd edition (Springer Verlag)
[12] Sabel’nikov V, Chtab A and Gorokhovski M 2007 Advances in Turbulence XI 117 209
[13] Sabel’nikov V, Chtab-Desportes A and Gorokhovski M 2011 Eur. Phys. J. B 80 177
[14] Zamansky R, Vinkovic I and Gorokhovski M 2010 J. Turbulence 11 1
[15] Gorokhovski M and Saveliev V 2008 J. Phys. D: Appl. Phys. 41 085405
[16] Buffat M, Le Penven L and Cadou A 2011 Comput. Fluids 42 62

Figure 8. Streamwise particle flux $< u_p >$ obtained by DNS (line), LES (squares), LES-SSAM (triangles). The average relative distance of LES (squares) and LES-SSAM (triangles) to DNS is given in the inset.
Figure 9. Spanwise acceleration RMS profile of solid particles for (from top to bottom) $St = 1, 5, 15, 25, 125$. DNS (line), LES (squares), LES-SSAM (triangles).

[17] Schiller L, Naumann A 1933 Vereines Deutscher Ingenieure 77 318
[18] Choi J-I, Yeo K and Lee C 2004 Phys. Fluids 16 779
[19] Mordant N, Crawford A M and Bodenschatz E 2004 Physica D 193 245
[20] Marchioli C, Soldati A, Kuerten J G M, Arcen B, Tanière A, Goldensoph G, Squires K D, Cargnelutti M F and Portela L M 2008 Int. J. Multiphase Flow 34 879
[21] Lelouvetel J, Bigillon F, Doppler D, Vinkovic I and Champagne J-Y 2009 Water Resour. Res 45 doi:10.1029/2007WR006520
[22] Vinkovic I, Doppler D, Le Louvetel J and Buffat M 2010 Int. J. Multiphase Flow 37 187
[23] Yeo K, Kim B-G and Lee C 2010 J. Fluid Mech. 659 405
[24] Calzavarini E, Volk R, Bourgoin M, Lévêque E, Pinton J-F and Toschi F 2009 J. Fluid Mech. 630 179
Figure 10. Wall-normal acceleration PDF of solid particles, at $y^+ = 100$, for (from top to bottom) $St = 1, 5, 15, 25, 125$. DNS (line), LES (squares), LES-SSAM (triangles). The average relative distance of LES (squares) and LES-SSAM (triangles) to DNS is given in the inset.