P-brane Black Holes and Post-Newtonian Approximation

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Abstract

We analyze p-brane black hole solutions with 'block-orthogonal' intersection rules. The post-Newtonian parameters $\beta$ and $\gamma$ corresponding to 4-dimensional section of the metric are calculated. A family of solutions with $\gamma = 1$ is singled out. Some examples of solutions (e.g. in $D = 11$ supergravity) are considered.

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1 Introduction

Exact spherically symmetric solutions describing generalized analogues of black holes in an arbitrary number of dimensions have an interesting history and have recently received renewed attention mainly in efforts to obtain a framework for a unified theory, for example in the context of strings or $p$-branes.

The first multidimensional generalization of such type was considered by D. Kramer [1] and rediscovered by A.I. Legkii [2], D.J. Gross and M.J. Perry [3] (and also by A. Davidson and D. Owen [4], see also [5]). In [6] the Schwarzschild solution was generalized to the case of $n$ internal Ricci-flat spaces and it was shown that a black hole configuration takes place when the scale factors of the internal spaces are constants. In [7], an analogous generalization of the Tangherlini solution [8] was obtained. These solutions were also extended to the electrovacuum [4, 10, 12] and dilatonic [13, 12] cases. (We remind that the multidimensional $O(d+1)$-symmetric analogue of the well-known Reissner-Nordström charged black hole solution was obtained earlier by R. C. Myers and M. J. Perry [11].) A theorem was proved in [12] that cuts all non-black-hole configurations as non-stable under even monopole perturbations.

In [15] the extremely-charged dilatonic black hole solution was generalized to the multicenter (Majumdar-Papapetrou) case when the cosmological constant is non-zero. (The $D = 4$ Majumdar-Papapetrou solutions with conformal scalar and electromagnetic fields were considered already in [16].) In [17, 18], the Majumdar-Papapetrou type solutions with composite intersecting $p$-branes (in theories with fields of forms) [1] corresponding to Ricci-flat internal spaces were obtained and generalized to the case of Einstein internal spaces. Earlier some special classes of these solutions were considered in [29–34]. The obtained solutions take place when certain orthogonality relations (on coupling parameters, dimensions of branes, total dimension) are imposed. In such a situation, one may have a new class of cosmological and spherically symmetric solutions [35]. Special cases were also considered in [36–39]. Solutions with a horizon were studied in detail in [40–43], [44]. In [43, 44] some propositions related to i) the interconnection between Hawking temperature and singularity behaviour and ii) multitemporal configurations were proved. It should be noted that the multidimensional and multitemporal generalizations of the Schwarzschild and Tangherlini solutions were considered in [44, 47] wherein generalized Newton’s formulas for the multitemporal case were obtained.

The plan of this letter is as follows. In Section 2, we consider $p$-brane black hole (BH) solutions with ‘block-orthogonal’ intersection rules [18] (see also [11]) and provide an example of a black hole solution in $D = 11$ supergravity [13]. The metric of this solution contains the Reissner-Nordström metric as a 4-dimensional section. In Section 3, the post-Newtonian parameters $\beta$ and $\gamma$ corresponding to 4-dimensional section of the metric are calculated and a family of solutions with $\gamma = 1$, corresponding to electro-magnetic pairs of $p$-branes is singled out. Some comments of a more general nature are given in the last Section.

\[\text{5 For non-composite electric case see [20, 21], for composite electric case see [22], for solutions with intersections governed by Lie algebras see [14]. For other solutions with $p$-branes see also [23–28] and references therein.}\]
2 \( p \)-brane black holes

Our starting point is the action

\[
S = \int_M d^Dz \sqrt{|g|} \{ R[g] - h_{\alpha \beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta - \sum_{a \in \Delta} \frac{1}{n_a} \exp[2\lambda_a(\varphi)] (F^a)^2 \},
\]

(2.1)

where \( g = g_{M N} dz^M \otimes dz^N \) is the metric of signature \((-+,+\ldots,+)\) \((M,N = 1,\ldots,D)\), \( \varphi = (\varphi^a) \in \mathbb{R}^l \) is a vector from dilatonic scalar fields, \( (h_{\alpha \beta}) \) is a non-degenerate symmetric \( l \times l \) matrix \((l \in \mathbb{N})\),

\[
F^a = dA^a = \frac{1}{n_a} F^a_{M_1 \ldots M_{na}} dz^{M_1} \wedge \ldots \wedge dz^{M_{na}}
\]

(2.2)

is a \( n_a \)-form \((n_a \geq 2)\) on a \( D \)-dimensional manifold \( M \), and \( \lambda_a \) is a 1-form on \( \mathbb{R}^l \) : \( \lambda_a(\varphi) = \lambda_{a \alpha} \varphi^\alpha, a \in \Delta, \alpha = 1,\ldots,l \). In (2.1) we denote \( |g| = |\det(g_{MN})| \),

\[
(F^a)^2_g = F^a_{M_1 \ldots M_{na}} F^{a}_{N_1 \ldots N_{na}} g^{M_1 N_1} \ldots g^{M_{na} N_{na}},
\]

(2.3)

\( a \in \Delta \), where \( \Delta \) is some finite set. Varying this action with respect to \( g, \varphi \) and \( A^a \) we obtain the equations of motion in the form,

\[
R_{MN} - \frac{1}{2} g_{MN} R = T_{MN},
\]

(2.4)

\[
\triangle[g] \varphi^\alpha - \sum_{a \in \Delta} \frac{\lambda^a}{n_a!} e^{2\lambda_a(\varphi)} (F^a)^2_g = 0,
\]

(2.5)

\[
\nabla_{M_1}[g](e^{2\lambda_a(\varphi)} F^a_{M_1 \ldots M_{na}}) = 0,
\]

(2.6)

\( a \in \Delta; \alpha = 1,\ldots,l \). Here, \( \lambda^a = h^{\alpha \beta} \lambda_{a \beta} \) where \( (h^{\alpha \beta}) \) is a matrix inverse to \( (h_{\alpha \beta}) \),

\[
T_{MN} = T_{MN}[\varphi, g] + \sum_{a \in \Delta} e^{2\lambda_a(\varphi)} T_{MN}[F^a, g],
\]

(2.7)

where,

\[
T_{MN}[\varphi, g] = h_{\alpha \beta} \left( \partial_M \varphi^\alpha \partial_N \varphi^\beta - \frac{1}{2} g_{MN} \partial_P \varphi^\alpha \partial^P \varphi^\beta \right),
\]

(2.8)

\[
T_{MN}[F^a, g] = \frac{1}{n_a!} \left[ -\frac{1}{2} g_{MN} (F^a)^2_g + n_a F^a_{MM_{2\ldots M_{na}}} F^{a}_{NN_{2\ldots N_{na}}} \right],
\]

(2.9)

and \( \triangle[g], \nabla[g] \) are the Laplace-Beltrami and covariant derivative operators respectively corresponding to \( g \). In the following we shall be interested in \( p \)-brane black hole solutions to the equations (2.4)-(2.9) [18], defined on the manifold

\[
M = (R_0, +\infty) \times S^{d_0} \times \mathbb{R} \times M_2 \times \ldots \times M_n,
\]

(2.10)

where \( R_0 > 0 \) and \( S^{d_0} \) is the \( d_0 \)-dimensional unit sphere. Black hole (BH) solution may be obtained also from general spherically symmetric solutions [17]. In explicit form BH
solution reads
\[ g = U \left\{ \frac{dR \otimes dR}{1 - 2\mu/R} + R^2 d\tilde{\Omega}_{d_0}^2 - U_1 \left( 1 - \frac{2\mu}{R^d} \right) dt \otimes dt + \sum_{i=2}^n U_i \tilde{g}^i \right\}, \] (2.11)
\[ U = \prod_{s \in S} H_s^{2d(I_s)\nu_s^2/(D-2)}, \] (2.12)
\[ U_1 = \prod_{s \in S} H_s^{-2\nu_s^2}, \] (2.13)
\[ U_i = \prod_{s \in S} H_s^{-2\nu_s^2} \delta_{I_s}, \quad i > 1, \] (2.14)
\[ \varphi^\alpha_s = \sum_{s \in S} \nu_s^2 \chi_s \lambda^\alpha_{a_s} \ln H_s, \] (2.15)
\[ F^a_s = \sum_{s \in S_e} \delta^a_{a_s} d\Phi^s \wedge \tau(I_s) + \sum_{s \in S_m} \delta^a_{a_s} e^{-2\lambda_{a_s} \varphi} \ast [d\Phi^s \wedge \tau(I_s)]. \] (2.16)

Here, we have set \( \bar{d} = d_0 - 1, \) \( R_0^d = 2\mu, a \in \Delta, \alpha = 1, \ldots, l, \) and
\[ \Phi^s = \frac{\nu_s}{H_s^s}, \] (2.17)
\[ H_s = 1 + \frac{P_s}{R^d}, \] (2.18)
\[ H'_s = \left( 1 - \frac{P'_s}{H_s R^d} \right)^{-1} = 1 + \frac{P'_s}{R^d + P_s - P'_s}, \] (2.19)
\[ (P'_s)^2 = P_s(P_s + 2\mu), \] (2.20)

where \( P_s > 0 \) are constants, \( s \in S. \) In (2.11), \( g^i = g^{i_{m,n_i}}(y_i)dy^{m_i}_i \otimes dy^{n_i}_i \) is the Euclidean Ricci-flat metric on \( M_i, i = 2, \ldots, n \) and \( \tilde{g}^i = p_i^*g^i \) is the pullback of \( g^i \) to the \( M \) by the canonical projection: \( p_i : M \to M_i, i = 0, 2, \ldots, n; \) \( g^0 = d\Omega_{d_0}^2 \) is canonical metric on \( M_0 = S_{d_0} \) and in (2.16) \( \ast \ast = \ast [g] \) denotes the Hodge operator on \( (M,g). \)

The set \( S \) (generalized \( p \)-brane set) is by definition
\[ S = S_e \sqcup S_m, \quad S_v = \sqcup_{a \in \Delta} \{a\} \times \{v\} \times \Omega_{a,v}, \] (2.21)
\( v = e, m \) where \( \sqcup \) means the union of non-intersecting sets and \( \Omega_{a,e}, \Omega_{a,m} \subset \Omega, \) where \( \Omega = \Omega(n) \) is the set of all non-empty subsets of \( \{1, \ldots, n\}. \) Thus any \( s \in S \) has the form
\[ s = (a_s, v_s, I_s), \] (2.22)
where \( a_s \in \Delta, v_s = e, m \) and \( I_s \in \Omega_{a_s,v_s}. \) The sets \( S_e \) and \( S_m \) define electric and magnetic \( p \)-branes. In (2.13)
\[ \chi_s = +1, -1 \] (2.23)
for \( s \in S_e, S_m \) respectively. In (2.14)
\[ \delta_{it} = \sum_j \delta_{ij} \] (2.24)
is the indicator of \(i\) belonging to \(I\): \(\delta_{iI} = 1\) for \(i \in I\) and \(\delta_{iI} = 0\) otherwise.

All the manifolds \(M_i\) are assumed to be oriented and connected and the volume \(d_i\)-forms

\[
\tau_i \equiv \sqrt{|g^i(y_i)|} \, dy_i^1 \wedge \ldots \wedge dy_i^{d_i},
\]

are well-defined for all \(i = 1, \ldots, n\). Here \(d_i = \dim M_i\), \(i = 0, \ldots, n\) \((M_0 = S^{d_0})\), \(D = 1 + \sum_{i=0}^{n} d_i\), and for any \(I = \{i_1, \ldots, i_k\} \in \Omega\), \(i_1 < \ldots < i_k\), we denote

\[
\tau(I) \equiv \tau_{i_1} \wedge \ldots \wedge \tau_{i_k};
\]

\[
M_I \equiv M_{i_1} \times \ldots \times M_{i_k},
\]

\[
d(I) \equiv \sum_{i \in I} d_i.
\]

In our solution, since

\[
1 \in I_s
\]

for all \(s \in S\) all brane manifolds \(M_I\) contain the time submanifold \(M_1 = \mathbb{R}\). Due to (2.16), the dimension of \(p\)-brane worldsheet \(d(I_s)\) is defined by

\[
d(I_s) = n_s - 1, \quad d(I_s) = D - n_s - 1,
\]

for \(s \in S_e, S_p\) respectively. (For a \(p\)-brane: \(p = p_s = d(I_s) - 1\)). The parameters \(\nu_s\) appearing in the solution satisfy the relations

\[
\sum_{s \in S} B_{ss'} \nu_{s'}^2 = 1,
\]

with

\[
B_{ss'} \equiv d(I_s \cap I_{s'}) + \frac{d(I_s)d(I_{s'})}{2 - D} + \chi_s \chi_{s'} \lambda_{\alpha a_s} \lambda_{\beta a_{s'}} h^{\alpha \beta},
\]

and we assume that

\[
S = S_1 \sqcup \ldots \sqcup S_k,
\]

\(S_i \neq \emptyset, i = 1, \ldots, k\), and

\[
B_{ss'} = 0,
\]

for all \(s \in S_i, s' \in S_j, i \neq j; i, j = 1, \ldots, k\). Eq. (2.33) means that the set \(S\) is a union of \(k\) non-intersecting (non-empty) subsets \(S_1, \ldots, S_k\) and according to (2.34) the matrix \(B = (B_{ss'})\) (2.32) has a block-diagonal structure and is the direct sum of \(k\) blocks \(B^{(i)} = (B_{ss'}, s, s' \in S_i), i = 1, \ldots, k\), i.e.

\[
B = \text{diag}(B^{(1)}, \ldots, B^{(k)}).
\]

(It is tacitly assumed that \(S\) is ordered, \(S_1 < \ldots < S_k\), and the order in \(S_i\) is inherited by the order in \(S\).) The parameters \(P_s\) coincide inside blocks,

\[
P_s = P_{s'},
\]
for all $s, s' \in S_i, i = 1, \ldots, k$.

The solution given above describes non-extremal charged intersecting generalized $p$-branes with block-diagonal matrix $B$ and agrees in some particular cases with those given in Refs. [40, 41, 42], \((d_1 = \ldots = d_n = 1), \) [43] for the non-composite case and [34] for the case of diagonal $B$ (i.e. when $|S_1| = \ldots = |S_k| = 1$). Note also that the special case of our solution with the parameters $\nu_s^2$ coinciding inside blocks (i.e. $\nu_s^2 = \nu_s'^2$ for all $s, s' \in S_i, i = 1, \ldots, k$) was analyzed also in [44].

We now note that the metric \((2.11)\) has a horizon at $R = 2\mu$. The Hawking temperature corresponding to the solution is (see also [42, 43] for orthogonal case) found to be

\[
T_H = \frac{d}{4\pi(2\mu)^{1/d}} \prod_{s \in S} \left( \frac{2\mu}{2\mu + P_s} \right)^{\nu_s^2}. \tag{2.37}
\]

Therefore, for fixed $P_s > 0$ and $\mu \to +0$, we deduce $T_H(\mu) \to 0$ for the extremal black hole configurations \([18]\) satisfying

\[
\xi = \sum_{s \in S} \nu_s^2 - d^{-1} > 0. \tag{2.38}
\]

**Remark.** Relation \((2.37)\) may be obtained using e.g. formulae from [48]. One finds for static metrics written in the form

\[
g = -\exp[2\gamma(u)] dt \otimes dt + \exp[2\alpha(u)] du \otimes du + \ldots \tag{2.39}
\]

the following expression for the Hawking temperature of a surface $u = u^*$ where $\exp(\gamma) = 0$, assumed to be a horizon:

\[
T_H = \frac{1}{2\pi} \lim_{u \to u^*} \exp(\gamma - \alpha) \left| \frac{d\gamma}{du} \right|. \tag{2.40}
\]

It is instructive to give some examples that show the behaviour of our solution in some simple cases.

**Example 1: Reissner-Nordström solution.** Our solutions contain the Reissner-Nordström solution (for a charged black hole) as a special case, namely,

\[
g = H^2 \left\{ \frac{dR \otimes dR}{1 - 2\mu/R} + R^2 d\hat{\Omega}_2^2 \right\} - H^{-2} \left( 1 - \frac{2\mu}{R} \right) dt \otimes dt
\]

\[
F = \nu d(H')^{-1} \wedge dt \tag{2.41}
\]

where $\nu^2 = 2$ and

\[
H = 1 + \frac{P}{R}, \quad H' = 1 + \frac{P'}{R + P - P'}, \tag{2.42}
\]

\((P')^2 = P(P + 2\mu)\) and $P > 0$ is constant. Introducing the new radial variable $r = R + P$, we may rewrite this solution in a more familiar form, namely,

\[
g = -f dt \otimes dt + r^2 d\hat{\Omega}_2^2 + f^{-1} dr \otimes dr, \tag{2.43}
\]

\[
F = \nu \frac{P'}{r^2} dt \wedge dr, \tag{2.44}
\]
where \( \nu^2 = 2 \),

\[
f = 1 - \frac{2GM}{r} + \frac{(P')^2}{r^2},
\]

with \( GM = \mu + P \). (Here \( G \) is gravitational constant, \( M \) is mass and \( P' \) is charge.)

**Example 2: D=11 supergravity.** Consider the truncated bosonic sector of \( D = 11 \) supergravity (truncated means without Chern-Simons terms). The action (2.1) in this case reads

\[
S_{tr} = \int_M d^{11}z \sqrt{|g|} \{ R[g] - \frac{1}{4!} F^2 \}. 
\]

where \( \text{rank} F = 4 \). In this particular case, consider the dyonic black-hole solutions with electric 2-brane and magnetic 5-brane defined on the manifold

\[
M = (2\mu, +\infty) \times S^2 \times \mathbb{R} \times M_2 \times M_3, 
\]

where \( \text{dim} M_2 = 2 \) and \( \text{dim} M_3 = 5 \). The metric and 4-form field then read as follows,

\[
g = H^2 \left\{ \frac{dR \otimes dR}{1 - 2\mu/R} + R^2 d\hat{\Omega}_2^2 \right\} - H^{-2} \left( 1 - \frac{2\mu}{R} \right) dt \otimes dt + \hat{g}^2 + \hat{g}^3, 
\]

\[
F = \nu_1 d(H')^{-1} \wedge dt \wedge \tau_2 + \nu_2 * (d(H')^{-1} \wedge dt \wedge \tau_3), 
\]

where \( \nu_1^2 = \nu_2^2 = 1 \) the metrics \( g^2 \) and \( g^3 \) are Ricci-flat and the functions \( H \) and \( H' \) are defined by (2.43).

The solution (2.49), (2.50) satisfies not only equations of motion for the truncated model, but also the equations of motion for \( D = 11 \) supergravity with the bosonic sector action

\[
S = S_{tr} + c \int_M A \wedge F \wedge F 
\]

\((c = \text{const}, F = dA)\), since the only modification related to ”Maxwells” equations

\[
d \ast F = \text{const} \ F \wedge F, 
\]

is trivial due to \( F \wedge F = 0 \) (since \( \tau_1 \wedge \tau_3 = 0 \)).

This solution describes two \( p \)-branes (electric 2-brane and magnetic 5-brane) with equal charges intersecting on the time manifold. In the extremal case, \( \mu \rightarrow 0 \), this solution corresponds to the so-called \( A_2 \)-dyon solution from [18]. We see that the 4-dimensional section of the metric (2.49) coincides with the Reissner-Nordström metric (2.41).

### 3 Post-Newtonian approximation

Let \( d_0 = 2 \) and consider the 4-dimensional section of the metric (2.1), namely,

\[
g^{(4)} = U \left\{ \frac{dR \otimes dR}{1 - 2\mu/R} + R^2 d\hat{\Omega}_2^2 - U_1 \left( 1 - \frac{2\mu}{R} \right) dt \otimes dt \right\}, 
\]

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in the range $R > 2\mu$. We imagine that some real astrophysical objects (e.g. stars) may be described by the 4-dimensional physical metric (3.53), i.e. they are traces of extended multidimensional objects (charged $p$-branes). Introducing a new radial variable $\rho$ by the relation

$$R = \rho \left(1 + \frac{\mu}{2\rho}\right)^2,$$

($\rho > \mu/2$), we may rewrite the metric (3.53) in the 3-dimensional conformally-flat form,

$$g^{(4)} = U \left\{-U_1 \left(1 - \frac{\mu}{2\rho}\right)^2 dt \otimes dt + \left(1 + \frac{\mu}{2\rho}\right)^4 \delta_{ij} dx^i \otimes dx^j\right\},$$

(3.55)

where $\rho^2 = |x|^2 = \delta_{ij} x^i x^j$ ($i, j = 1, 2, 3$).

For possible physical applications, one should calculate the post-Newtonian parameters $\beta$ and $\gamma$ (Eddington parameters) using the following standard relations

$$g^{(4)}_{00} = -(1 - 2V + 2\beta V^2) + O(V^3),$$

(3.56)

$$g^{(4)}_{ij} = \delta_{ij}(1 + 2\gamma V) + O(V^2),$$

(3.57)

$i, j = 1, 2, 3$, where,

$$V = \frac{GM}{\rho}$$

(3.58)

is Newton’s potential, $G$ is the gravitational constant and $M$ is the gravitational mass. From (3.53)-(3.58) we deduce the formulas

$$GM = \mu + \sum_{s \in S} \nu_s^2 P_s \left(1 - \frac{d(I_s)}{D - 2}\right)$$

(3.59)

and

$$\beta - 1 = \frac{1}{2(GM)^2} \sum_{s \in S} \nu_s^2 (P'_s)^2 \left(1 - \frac{d(I_s)}{D - 2}\right),$$

(3.60)

$$\gamma - 1 = -\frac{1}{GM} \sum_{s \in S} \nu_s^2 P_s \left(1 - 2 \frac{d(I_s)}{D - 2}\right).$$

(3.61)

The parameter $\beta$ is defined by the charges $P'_s$ of $p$-branes (or more correctly by the charge densities). It follows from (3.60) and the inequalities $d(I_s) < D - 2$ (for all $s \in S$) that

$$\beta \geq 1,$$

(3.62)

and $\beta = 1$ (as for the Schwarzschild solution) only if all $P'_s = 0$, i.e. in the pure vacuum case. As an example, for the Reissner-Nordström solution (2.44) we deduce,

$$\beta = 1 + \frac{(P')^2}{2(GM)^2},$$

(3.63)

$$\gamma = 1.$$
Examples 1 and 2 imply that the same parameters appear in the 4-dimensional section of the $D = 11$ supergravity solution.

The results obtained above suggest that there exist non-trivial p-brane configurations with $\gamma = 1$. That this is indeed the case is shown by the following Proposition.

**Proposition.** Let the set of p-branes consist of several pairs of electric and magnetic branes and let any such pair $(s, \bar{s} \in S)$ correspond to the same colour index, i.e. $a_s = a_{\bar{s}}$ and the charge parameters are equal, i.e. $P_s = P_{\bar{s}}$. Then,

$$\gamma = 1.$$  \hspace{1cm} (3.65)

The Proposition can be readily proved using the relation $d(I_s) + d(I_{\bar{s}}) = D - 2$ following from (2.30).

Further, for small $P_s$ ($P_s \ll \mu$) we obtain

$$P_s \sim (P_s')^2/(2\mu), \quad GM \sim \mu$$  \hspace{1cm} (3.66)

and hence

$$\gamma - 1 \sim - \frac{1}{2(GM)^2} \sum_{s \in S} \nu_s^2 (P_s')^2 \left( 1 - 2 \frac{d(I_s)}{D - 2} \right).$$  \hspace{1cm} (3.67)

As a last example of our procedure, let us consider the special case of one p-brane. Here we have,

$$\nu_s^{-2} = d(I_s) \left( 1 - \frac{d(I_s)}{D - 2} \right) + \lambda_{a_s}^2,$$  \hspace{1cm} (3.68)

and with the help of Eqs. (3.60), (3.61) and (3.68) we see that for large enough values of the dilatonic coupling constant it is possible to perform a ‘fine tuning’ of the parameters $(\beta, \gamma)$ near the point $(1,1)$ even if the ‘charges’ $P_s'$ are big.

4 Conclusions

In this Letter, we analyzed the basic features of p-brane black hole solutions and described some of the better known black hole spacetimes as limiting cases of p-brane black holes. A restriction to the 4–dimensional sector of these metrics allowed the determination of the post–Newtonian parameters $\beta, \gamma$ and in particular, it was shown how a new family of solutions having $\gamma = 1$ could be singled out.

Our results allow a number of more general comments to be made. Firstly, there is the question of the stability of our p–brane black holes under small perturbations away from our static configurations. It is known that in the multidimensional case, the class of metrics which are stable against monopole perturbations is of measure zero in the space of all possible spherically symmetric solutions [12]. Therefore a more general problem of stability of multidimensional black holes involving other types of perturbations has to be formulated and studied.
Secondly, it would be of interest to develop more general aspects of $p$–brane black holes addressing issues such as the area theorem and the laws of $p$–brane black hole thermodynamics in this generalized context. Does the nondecreasing of area in the four dimensional sector imply an area theorem for the full multidimensional case? We leave such matters for the future.

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References

[1] D. Kramer, Acta Physica Polonica 2, F. 6, 807 (1969).
[2] A.I. Legkii, in Probl. of Grav. Theory and Elem. Particles (Atomizdat, Moscow) 10, 149 (1979) (in Russian).
[3] D.J. Gross and M.J. Perry, Nucl. Phys. B 226, 29 (1983).
[4] A. Davidson and D. Owen, Phys. Lett. B 155, 247 (1985).
[5] G. W. Gibbons and D. L. Wiltshire, Annals of Physics, Bf 167 (1986) 201.
[6] K.A. Bronnikov, V.D. Ivashchuk in Abstr. VIII Soviet Grav. Conf (Erevan, EGU, 1988) p. 156.
[7] S.B. Fadeev, V.D. Ivashchuk and V.N. Melnikov, Phys. Lett. A 161, 98 (1991).
[8] F.R. Tangherlini, Nuovo Cimento 27, 636 (1963).
[9] S.B. Fadeev, V.D. Ivashchuk and V.N. Melnikov, Chinese Phys. Lett. 8, 439 (1991).
[10] V.D. Ivashchuk and V.N. Melnikov, Class. Quantum Grav., 11, 1793 (1994).
[11] R. C. Myers and M. J. Perry, Annals of Physics (N.Y.) 172, 304 (1986).
[12] K.A. Bronnikov and V.N. Melnikov, Annals of Physics (N.Y.) 239, 40 (1995).
[13] U. Bleyer and V.D. Ivashchuk, Phys. Lett. B 332, 292 (1994).
[14] V.D. Ivashchuk and V.N. Melnikov, Grav. and Cosmol. 1, No 3, 204 (1995).
[15] V.D. Ivashchuk and V.N. Melnikov, Extremal Dilatonic Black Holes in String-like Model with Cosmological Term, Phys. Lett. B 384, 58 (1996).
[16] The first MP-type solution with a conformal scalar field was considered in, N.M. Bocharova, K.A. Bronnikov and V.N. Melnikov, Vestnik MGU (Moscow Univ.), 6, 706 (1970)(in Russian); also the first MP-type solution with conformal scalar and electromagnetic fields was found in, K.A. Bronnikov and V.N. Melnikov, in Problems of Theory of Gravitation and Elementary Particles , 5, 80 (1974) (in Russian); K.A. Bronnikov, Acta Phys. Polonica , B4, 251 (1973).
[17] V.D. Ivashchuk and V.N. Melnikov, Sigma-Model for Generalized Composite p-branes, hep-th/9705036; Class. and Quant. Grav. 14, 11, 3001 (1997).

[18] V.D. Ivashchuk and V.N. Melnikov, Mujumdar-Papapetrou Type Solutions in Sigma-model and Intersecting p-branes, hep-th/9702121, to appear in Class. and Quantum Gravity.

[19] M.A. Grebeniuk and V.D. Ivashchuk, Solutions in Sigma Model and Intersecting p-branes Related to the Lie Algebras, hep-th/9805113; Phys. Lett. B 442 (1998) 125.

[20] V.D. Ivashchuk and V.N. Melnikov, Intersecting p-Brane Solutions in Multidimensional Gravity and M-Theory, hep-th/9612084; Grav. and Cosmol. 2, No 4, 297 (1996).

[21] V.D. Ivashchuk and V.N. Melnikov, Phys. Lett. B 403, 23 (1997).

[22] V.D. Ivashchuk, M. Rainer and V.N. Melnikov, Multidimensional Sigma-Models with Composite Electric p-branes, gr-qc/9705005; Gravit. and Cosmol. 4, No1 (13) (1998).

[23] A. Dabholkar, G. Gibbons, J.A. Harvey, and F. Ruiz Ruiz, Nucl. Phys. B 340, 33 (1990).

[24] G.T. Horowitz and A. Strominger, Nucl. Phys. B 360, 197 (1990).

[25] M.J. Duff and K.S. Stelle, Phys. Lett. B 253, 113 (1991).

[26] R. Güven, Phys. Lett. B 276, 49 (1992); Phys. Lett. B 212, 277 (1988).

[27] M.J. Duff, R.R. Khuri and J.X. Lu, Phys. Rep. 259, 213 (1995).

[28] K.S. Stelle, Lectures on Supergravity p-Branes, hep-th/9701088, hep-th/9608117.

[29] G. Papadopoulos and P.K. Townsend, Phys. Lett. B 380, 273 (1996).

[30] A.A. Tseytlin, Harmonic Superpositions of M-branes, hep-th/9604035; Nucl. Phys. B 475, 149 (1996).

[31] J.P. Gauntlett, D.A. Kastor, and J. Traschen, Overlapping Branes in M-Theory, hep-th/9604179; Nucl. Phys. B 478, 544 (1996).

[32] I.Ya. Aref’eva and O.A. Rytchkov, Incidence Matrix Description of Intersecting p-brane Solutions, hep-th/9612233.

[33] R. Argurio, F. Englert and L. Houri, Intersection Rules for p-branes, hep-th/9701042.

[34] I.Ya. Aref’eva M.G. Ivanov and O.A. Rytchkov, Properties of Intersecting p-branes in Various Dimensions, hep-th/9702077.

[35] V.D. Ivashchuk and V.N. Melnikov, Multidimensional Classical and Quantum Cosmology with Intersecting p-branes, hep-th/9708157; J. Math. Phys., 39, 2866 (1998).

[36] H. Lü, C.N. Pope, and K.W. Xu, Liouville and Toda Solitons in M-Theory, hep-th/9604058.

[37] K.A. Bronnikov, M.A. Grebeniuk, V.D. Ivashchuk and V.N. Melnikov, Integrable Multidimensional Cosmology for Intersecting p-branes, Grav. and Cosmol. 3, No 2(10), 105 (1997).
[38] M.A. Grebeniuk, V.D. Ivashchuk and V.N. Melnikov, Integrable Multidimensional Quantum Cosmology for Intersecting p-Branes, *Grav. and Cosmol.* 3, No 3 (11), 243 (1997), gr-qc/9708033.

[39] K.A. Bronnikov, U. Kasper and M. Rainer, Intersecting Electric and Magnetic p-Branes: Spherically Symmetric Solutions, gr-qc/9708058.

[40] M. Cvetic and A. Tseytlin, *Nucl. Phys. B* 478, 181 (1996).

[41] I.Ya. Aref’eva, M.G. Ivanov and I.V. Volovich, Non-Extremal Intersecting p-Branes in Various Dimensions, hep-th/9702079; *Phys. Lett. B* 406, 44 (1997).

[42] N. Ohta, Intersection Rules for Non-extreme p-branes, hep-th/9702164.

[43] K.A. Bronnikov, V.D. Ivashchuk and V.N. Melnikov, The Reissner-Nordström Problem for Intersecting Electric and Magnetic p-Branes, gr-qc/9710054; *Grav. and Cosmol.* 3, No 3 (11), 203 (1997).

[44] K.A. Bronnikov, Block-orthogonal Brane systems, Black Holes and Wormholes, hep-th/9710207; *Grav. and Cosmol.* 4, No 2 (14), (1998).

[45] V.D. Ivashchuk and V.N. Melnikov, *Int. J. Mod. Phys. D* 4, 167 (1995).

[46] E. Cremmer, B. Julia, and J. Scherk, *Phys. Lett. B* 76, 409 (1978).

[47] V.D. Ivashchuk and V.N. Melnikov, Cosmological and Spherically Symmetric Solutions with Intersecting p-branes, submitted to *J. Math. Phys.*.

[48] R. Wald, “General Relativity”, Univ. of Chicago Press, Chicago, 1984.