On the Symmetries of BF Models and Their Relation with Gravity

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Abstract The perturbative finiteness of various topological models (e.g. BF models) has its origin in an extra symmetry of the gauge-fixed action, the so-called vector supersymmetry. Since an invariance of this type also exists for gravity and since gravity is closely related to certain BF models, vector supersymmetry should also be useful for tackling various aspects of quantum gravity. With this motivation and goal in mind, we first extend vector supersymmetry of BF models to generic manifolds by incorporating it into the BRST symmetry within the Batalin-Vilkovisky framework. Thereafter, we address the relationship between gravity and BF models, in particular for three-dimensional space-time.

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1 Introduction

1.1 Motivation

As was realized in recent years, there exists a close relationship between BF models and gravity in three dimensions [1] as well as in higher dimensions [2]. (For a review of BF models [1, 3] and other topological field theories [4, 5], see reference [6].) The perturbative finiteness of BF models can be traced back to the invariance of the gauge-fixed action under vector supersymmetry (VSUSY) [7]-[10]. Since a symmetry of this type also exists for gravity [11], it should impose some constraints on the quantum theory of gravitation and thereby prove to be useful for its formulation.

Before presenting an outline of the present paper, we first review the results which are known for the three-dimensional case.

1.1.1 Local VSUSY of the 3d BF model

Let $G$ be a matrix Lie group and let $\mathcal{G}$ denote the associated Lie algebra. The basic variables of the 3d BF model with symmetry group $G$ are a connection 1-form $A$ and a 1-form potential $B$, both taking their values in $\mathcal{G}$. More explicitly, $A = A_\mu dx^\mu = A^a_T dx^\mu$ (similarly for $B$) where the matrices $T_a$ belong to $\mathcal{G}$ and satisfy the relations

$$[T_a, T_b] = f_{ab}^c T_c, \quad \text{Tr} (T_a T_b) = \delta_{ab}. \tag{1.1}$$

The arena of the model is a smooth manifold $\mathcal{M}_3$ of dimension 3. The action functional is given by

$$S_{BF} (A, B) = - \int_{\mathcal{M}_3} \text{Tr} (BF) , \tag{1.2}$$

where $F = dA + \frac{1}{2}[A, A] = dA + A^2$ denotes the curvature (field strength) of $A$. This functional is invariant under local gauge transformations of $A$ and $B$ parametrized at the infinitesimal level by $\mathcal{G}$-valued fields $c$ and $\phi$, respectively\footnote{Strictly speaking, these transformations only leave the action invariant if the manifold is compact without boundary or if the symmetry parameters vanish on the boundary \footnote{See appendix for further details.}.}. In the standard BRST framework, these parameters are turned into ghost fields and the symmetry transformations are described by the BRST operator $s$. All fields are then characterized by a total degree which is the sum of their form degree and their ghost-number and all commutators are assumed to be graded with respect to this degree\footnote{See appendix for further details.}. The $s$-variations of the basic fields and ghosts read as

$$sA = - D_A c, \quad sc = - c^2$$

$$sB = - D_A \phi - [c, B], \quad s\phi = - [c, \phi], \tag{1.3}$$

$$sT_a = f_{ac} b^c, \quad s\phi = - i \gamma^a \sigma_a \phi - \omega^a \phi - 2 \omega^a c^a.$$
where \( D_A c \equiv dc + [A, c] \) denotes the Yang-Mills covariant derivative.

After performing the gauge-fixing in a Landau-type gauge \([8, 9, 10, 12]\) (or in non-covariant versions of the latter \([13]\)), the gauge-fixed action is invariant under VSUSY-transformations. At the infinitesimal level, these transformations are described by an operator \( \delta_\tau \) where \( \tau \equiv \tau^\mu \partial_\mu \) is a \( s \)-invariant vector field of ghost-number zero. The variation \( \delta_\tau \) acts as an antiderivation (odd operator) which lowers the ghost-number by one unit and which anticommutes with \( d \). Its action on the ghost fields \( c \) and \( \phi \) is given by \([8]\)

\[
\delta_\tau c = i_\tau A \quad , \quad \delta_\tau \phi = i_\tau B ,
\]

(1.4)

where \( i_\tau \) denotes the interior product with respect to the vector field \( \tau \) (see appendix for technical details concerning operators, vector fields and differential forms). The operators \( s \) and \( \delta_\tau \) satisfy a graded algebra of Wess-Zumino type,

\[
[s, \delta_\tau] = \mathcal{L}_\tau + \text{equations of motion} ,
\]

(1.5)

where \( \mathcal{L}_\tau \) denotes the Lie derivative (see (A.9)) with respect to the vector field \( \tau \).

To be more precise, we should note two points. First, invariance under VSUSY was shown to exist solely on manifolds admitting vector fields that are covariantly constant with respect to some background metric, the vector field \( \tau \) being any one of these \([14]\). VSUSY therefore represents a rigid symmetry. This restriction has its origin in a particular way of implementing the gauge-fixing and introducing VSUSY-transformations. As we shall see in the present paper, VSUSY can hold as a truly local symmetry on a generic manifold if it is implemented in a different way.

Second, VSUSY only holds as an exact invariance as long as one does not incorporate external sources (coupling to the non-linear \( s \)-variations of the fields). Once the latter are included into the action, VSUSY is expressed by a broken Ward identity, the breaking term being linear in the quantum fields and thus unproblematic in quantum theory.

### 1.1.2 3d gravity

The basic variables of 3d gravity are the dreibein 1-forms \((e^i)_{i=1,2,3}\) and the Lorentz connection 1-form \( \omega = (\omega_{ij})_{i,j=1,2,3} \) which takes its values in the Lie algebra \( so(3) \) or \( so(1,2) \), i.e. \( \omega_{ij} = -\omega_{ji} \). The action functional \([15]\)

\[
S_{\text{grav}}(e, \omega) = -\int_{\mathcal{M}_3} \varepsilon_{ijk} e^j R^{ik} ,
\]

(1.6)

in which \( R = d\omega + \omega^2 \) denotes the curvature 2-form, is invariant under diffeomorphisms (general coordinate transformations) and under local Lorentz transformations. At the infinitesimal level, these symmetries are parametrized, respectively, by
a vector field \( \xi = \xi^\mu \partial_\mu \) and an antisymmetric matrix \( \Omega \). In the BRST formalism, the latter parameters represent ghost fields and the s-variations have the form

\[
s\omega = -D_\omega \Omega + \mathcal{L}_\xi \omega \quad , \quad s\Omega = -\Omega^2 + \mathcal{L}_\xi \Omega
\]

(1.7)

Here, \( D_\omega \Omega = d\Omega + [\omega, \Omega] \) denotes the Lorentz covariant derivative, \( \mathcal{L}_\xi = i_\xi d - di_\xi \) represents the Lie derivative with respect to the ghost vector field \( \xi \) and the vector field \([\xi, \xi]\) is the graded Lie bracket of \( \xi \) with itself: \([\xi, \xi]^\mu = 2\xi^\nu \partial_\nu \xi^\mu\).

The action (1.6) can be gauge-fixed by considering a background metric and by choosing the Landau gauge. The gauge-fixed action is then invariant under VSUSY-transformations parametrized by a Killing vector field \( \tau = \tau^\mu \partial_\mu \) \([1]\): the latter only act non-trivially on the ghost field \( \xi \) according to

\[
\delta_\tau \xi = \tau
\]

(1.8)

and they satisfy the algebra (1.5).

### 1.1.3 Relation between 3d gravity and BF models

We now describe the correspondence between 3d gravity and BF models \([\ ]\) (see also \([\ ]\)). As symmetry group of the BF model, one chooses \( G = SO(3) \) or \( G = SO(1,2) \) as in the last subsection. Then, the connection 1-form \( A = (A_{ij})_{i,j=1,2,3} \) and the 1-form potential \( B = (B_{ij})_{i,j=1,2,3} \) both represent antisymmetric matrices of 1-forms. The correspondence between the degrees of freedom involved in both theories can be made more precise by writing

\[
B_{jk} = \varepsilon_{ijk} e^i \quad , \quad A_{jk} = \omega_{jk} \quad (\text{hence } F_{jk} = R_{jk})
\]

\[
\phi_{jk} = i_\xi B_{jk} \quad , \quad c = \Omega + i_\xi \omega
\]

(1.9)

where \( \varepsilon_{ijk} \) denotes the components of the totally antisymmetric tensor. The action (1.6) then goes over into the action (1.2):

\[
S_{BF} (A, B) = -\frac{1}{2} \int_{M_3} B_{jk} F^{jk}.
\]

(1.10)

Furthermore, the VSUSY-variations (1.8) imply the variations (1.4). The transformation laws of \( \omega \) and \( e^i \) as given by (1.7) become

\[
sA = -D_A c + i_\xi F
\]

\[
sB = -D_A \phi - [c, B] + i_\xi D_A B
\]

If the equations of motion \( F = 0 = D_A B \) of the BF model are taken into account, the latter transformation laws coincide with those given in eqs. (1.3). In summary, the
actions of both models coincide exactly and the symmetry transformations coincide on-shell.

Some comments concerning these results are in order. First, we note that the reparametrization \( (c, \phi) \to (\Omega, \xi) \) considered in eqs.(1.3) represents a field-dependent change of the generators of the BRST differential algebra: this explains the appearance of the equations of motion upon passage from one model to the other one. Second, we remark that there is a problem of invertibility with the change of generators \( \xi^\mu \to \phi = i_Ω B \equiv \xi^\mu B_\mu \) since one cannot express \( \xi^\mu \) in terms of \( \phi \), unless the 3-bein \( e^i_\mu = \frac{1}{2} \varepsilon_{ijk} B^j_\mu \) represents a nonsingular matrix in whole space-time. As a matter of fact, this invertibility problem reappears in the perturbative approach to quantum theory, since the BF model is perturbed around the configuration \( B = 0 \) whereas the corresponding configuration in gravity, i.e. \( e^i = 0 \), represents a singular metric. (For a general discussion, see references [18] and the remarks made in [5].)

1.2 Program

The results we just summarized suggest the following line of investigation:

1. to generalize the results concerning VSUSY of BF models (in three and, more generally, in higher dimensions) to generic manifolds, not just those admitting covariantly constant vector fields,

2. to promote the three-dimensional on-shell results to off-shell results,

3. to extend this correspondence to the higher-dimensional case and to exploit its consequences.

In the present paper, we will discuss VSUSY on generic 3- and 4-manifolds for BF models which, in addition, involve a cosmological term in their action. (The higher-dimensional case is analogous to the 4-dimensional one in that it involves the phenomenon of “ghosts for ghosts” which does not occur in the 3-dimensional case.) Moreover, some results concerning the relationship between gravity and BF models will be presented.

At different stages of the discussion, the equations of motion of the models appear in the transformation laws of fields and therefore we will follow the Batalin-Vilkovisky (BV) approach [19] to the description of symmetries. Henceforth, antifields are to be introduced into the formalism from the beginning on. They play the role of external sources coupled to the BRST-variations of the basic fields. In a final step, the antifields are redefined according to the Batalin-Vilkovisky prescription in order to implement the gauge-fixing. This procedure of gauge-fixing does not to interfere with vector supersymmetry thanks to the fact that the latter is incorporated directly into the BRST operator.
2 3d BF model with cosmological term

2.1 The model and its symmetries

To start with, we consider an arbitrary gauge group which will be specialized to $SO(3)$ or $SO(1,2)$ when discussing gravity. The notation is the one introduced in subsection 1.1.1.

The action for a BF model with “cosmological constant” $\alpha$ on a 3-manifold $M_3$ reads

$$S_{\text{inv}}(A, B) = -\int_{M_3} \text{Tr} \left( BF + \frac{\alpha}{3} B^3 \right),$$

(2.1)

where $\alpha$ represents a real dimensionless parameter. The equations of motion of this model are given by

$$F + \alpha B^2 = 0, \quad DB = 0,$$

(2.2)

where $D$ denotes the covariant derivative: $D \cdot = d \cdot + [A, \cdot]$.

The symmetries of the action (2.1) can be expressed in terms of horizontality conditions [20, 21] which have the form

$$\tilde{F} = F - \alpha [B, \phi] - \alpha \phi^2$$

$$\tilde{D} \tilde{B} = DB,$$

(2.3)

where the “tilded” quantities are defined by

$$\tilde{A} \equiv A + c, \quad \tilde{F} \equiv \tilde{d} \tilde{A} + \tilde{A}^2, \quad \tilde{d} \equiv d + s$$

$$\tilde{B} \equiv B + \phi, \quad \tilde{D} \cdot \equiv \tilde{d} \cdot + [\tilde{A}, \cdot].$$

(2.4)

Relations (2.3) yield the BRST transformations

$$s A = -Dc - \alpha [\phi, B]$$

$$s B = -[c, B] - D\phi$$

$$s c = -c^2 - \alpha \phi^2$$

$$s \phi = -[c, \phi],$$

(2.5)

where the ghosts $c$ and $\phi$ parametrize local gauge transformations of the potentials $A$ and $B$, respectively.

Following the lines of the BV-formalism, we now include antifields $A^*, B^*$ and $c^*, \phi^*$ associated to the basic fields $A, B$ and to the ghosts $c, \phi$. It is convenient to
introduce the complete ladders (generalized fields or extended forms \[22\])

\[
\mathcal{A} = c + A + B^* + \phi^* \\
\mathcal{B} = \phi + B + A^* + c^*,
\]

(2.6)
as well as the extended differential

\[
\delta = d + s.
\]

(2.7)

As usual, the exterior derivative \(d\) is assumed to anticommute with the BRST operator\[s\]. The generalized field strengths associated to \(\mathcal{A}\) and \(\mathcal{B}\) are defined by

\[
\mathcal{F} \equiv \delta \mathcal{A} + \mathcal{A}^2, \quad \mathcal{D} \mathcal{B} \equiv \delta \mathcal{B} + [\mathcal{A}, \mathcal{B}].
\]

(2.8)
The action of the BRST operator \(s\) is again defined in terms of horizontality conditions, namely the “zero curvature” conditions \[22, 23, 12\]

\[
\mathcal{F} + \alpha \mathcal{B}^2 = 0, \quad \mathcal{D} \mathcal{B} = 0,
\]

(2.9)

which imply the nilpotency of the extended operator \(\delta\) and thus of \(s\). These horizontality conditions have the same form as the equations of motion (2.2), with \(A\) and \(B\) substituted by \(\mathcal{A}\) and \(\mathcal{B}\), and they are equivalent to

\[
sA = -(dA + A^2 + \alpha \mathcal{B}^2), \quad sB = -(dB + [A, B]).
\]

(2.10)
The action of the nilpotent BRST operator \(s\) on the fields and antifields is found by expanding relations (2.10) with respect to the ghost-number. In doing so, we obtain

\[
s \chi = -c^2 - \alpha \phi^2 \\
sA = -Dc - \alpha [\phi, B] \\
sB^* = -(F + \alpha \mathcal{B}^2) - [c, B^*] - \alpha [\phi, A^*] \\
s\phi^* = -DB^* - [c, \phi^*] - \alpha ([\phi, c^*] + [B, A^*])
\]

and

\[
s \phi = -[c, \phi] \\
sB = -D \phi - [c, B] \\
sA^* = -DB - [c, A^*] - [B^*, \phi] \\
sc^* = -DA^* - [c, c^*] - [B^*, B] - [\phi^*, \phi].
\]

(2.11)

\(3\)Unlike the BRST differential considered up to now, the operator \(s\) introduced in (2.7) will act on the antifields as well. It will turn out to be the “linearized Slavnov-Taylor operator” which will be defined later on. For simplicity, we shall keep the notation \(s\) for this operator and refer to the corresponding variations of fields and antifields as “BRST-transformations”.

\(4\)We note that, up to field redefinitions, these equations are the most general ones which are compatible with conservation of the total degree, provided one imposes the discrete symmetry \(A \rightarrow A, B \rightarrow -B.\)
If all antifields are set to zero, we recover the BRST-transformations (2.3) which have been deduced from the horizontality conditions (2.3). However, in addition, we also get the field equations (2.2).

The BRST operator \( s \) defined by (2.9) or by (2.11) and (2.12) can be interpreted as the “linearized Slavnov-Taylor operator” \( S_S \) associated to a certain action \( S(A, B, c, \phi, A^*, B^*, c^*, \phi^*) \): the latter operator has the form

\[
S_S X \equiv (S, X) = \sum_\varphi \int \left( \frac{\delta S}{\delta \varphi^*} \frac{\delta X}{\delta \varphi} + \frac{\delta S}{\delta \varphi} \frac{\delta X}{\delta \varphi^*} \right) \quad \text{with} \quad \varphi \in \{A, B, c, \phi\},
\]

(2.13)

where \((\cdot, \cdot)\) denotes the Batalin-Vilkovisky (BV) bracket whose properties are depicted in the appendix, see eq.(A.18). Indeed, given the \( s \)-variations (2.11) and (2.12), we can find an action \( S \) solving the functional differential equations (see (A.19))

\[
S_S \varphi \equiv \frac{\delta S}{\delta \varphi^*} = s \varphi \quad , \quad S_S \varphi^* \equiv \frac{\delta S}{\delta \varphi} = s \varphi^*.
\]

(2.14)

The solution is given by the \( BF \)-like action

\[
S = - \int_{\mathcal{M}_3} \operatorname{Tr} \left( B(dA + A^2) + \frac{\alpha}{3} B^3 \right)|_3,
\]

(2.15)

where the integral is performed over all contributions of form degree 3 [22]. When expanded into components, this action takes the familiar form

\[
S = - \int_{\mathcal{M}_3} \operatorname{Tr} \left( BF + \frac{\alpha}{3} B^3 \right) + \sum_\varphi \int_{\mathcal{M}_3} \operatorname{Tr} (\varphi^* s \varphi) \equiv S_{\text{inv}}(A, B) + S_{\text{antifields}}(\varphi, \varphi^*).
\]

(2.16)

Moreover, the action \( S \) which solves the differential equations (2.14) obeys the (non-linear) \textit{Slavnov-Taylor identity} or \textit{BV master equation}

\[
\mathcal{S}(S) \equiv \frac{1}{2} (S, S) = \sum_\varphi \int \frac{\delta S}{\delta \varphi^*} \frac{\delta S}{\delta \varphi} = 0.
\]

(2.17)

This follows from the nilpotency of \( s \), which implies the nilpotency of the operator \( S_S \) defined by (2.13) and (2.14), and from the following identity that results from (A.25):

\[
S_X S_S + (S_S)^2 X = 0 \quad \text{for any functional} \quad X(\varphi, \varphi^*).
\]

(2.18)

Indeed, by applying (2.18) to \( X = \varphi \) or \( X = \varphi^* \) and by using \( S_S^2 = 0 \), we find

\[
\frac{\delta S(S)}{\delta \varphi^*} = 0 = \frac{\delta S(S)}{\delta \varphi},
\]

(2.19)

i.e. \( S(S) \) does not depend on the fields and antifields, hence it vanishes.
2.2 Including diffeomorphisms and VSUSY

2.2.1 Diffeomorphisms as dynamical symmetry or as external symmetry

Diffeomorphism (or general coordinate) invariance is already present, as a dynamical symmetry of the model, in the gauge invariances as defined by the BRST-transformations \((2.5)\) or \((2.11)-(2.12)\). This can be seen \([1, 17]\) by rewriting the ghosts \(c\) and \(\phi\) as (c.f. \((1.9)\))

\[
c = i_\xi A , \quad \phi = i_\xi B ,
\]

where \(\xi\) denotes a vector field of ghost-number one. In fact, by substituting expressions \((2.20)\) in the gauge transformations of \(A\) and \(B\) as given by \((2.5)\), we obtain the infinitesimal transformations

\[
\delta_\xi A = L_\xi A - i_\xi (F + \alpha B^2) \\
\delta_\xi B = L_\xi B - i_\xi DB .
\]

Up to terms involving equations of motion, these variations represent the action of diffeomorphisms generated by the vector field \(\xi\). This shows that diffeomorphisms constitute a subgroup of the group of gauge transformations of the theory.

Nevertheless, it turns out to be useful to introduce diffeomorphisms in an independent and explicit way, i.e. as “external” (non-dynamical) symmetries generated by the vector field \(\xi\). They will act by virtue of the Lie derivative \(L_\xi\) on all fields including ghosts and antifields. The ghost vector field \(\xi\) itself transforms non-trivially under BRST-variations, but to start with, its transformation law will \emph{not} be specified.

2.2.2 Vector supersymmetry

Let us recall some facts concerning vector supersymmetry \([9, 10, 12]\) viewed as an extra symmetry besides BRST invariance. The VSUSY-transformations parametrized by a vector field \(\tau\) are given by \([12]\)

\[
\delta_\tau A = i_\tau A , \quad \delta_\tau B = i_\tau B ,
\]

\(^5\)We note that our procedure is reminiscent of four-dimensional topological Yang-Mills theories \([24]\) where the generic shifts of the gauge field \(A\) given by \(\delta A = \psi\) involve, as a special case, infinitesimal gauge transformations corresponding to \(\psi = -dc\). Nevertheless, both the shift and gauge symmetries are included into the BRST operator.
which relations are equivalent to
\[
\begin{align*}
\delta_\tau c &= i_\tau A, \quad \delta_\tau \phi = i_\tau B, \\
\delta_\tau A &= i_\tau B^*, \quad \delta_\tau B = i_\tau A^*, \\
\delta_\tau B^* &= i_\tau \phi^*, \quad \delta_\tau A^* = i_\tau c^*, \\
\delta_\tau \phi^* &= 0, \quad \delta_\tau c^* = 0.
\end{align*}
\] (2.23)

These variations represent a generalization (involving antifields) of the transformation laws (1.4). From eqs. (2.10) (2.22) and the commutation relations \([d, \delta_\tau] = 0 = [s, i_\tau]\), it follows that the algebra \([\delta_\tau, s] = L_\tau\) is satisfied on the extended forms \(\mathcal{A}\) and \(\mathcal{B}\) and thereby on all fields \(\phi\) and antifields \(\phi^*\).

The functional (2.13) or (2.16) is invariant under the VSUSY-transformations (2.22) or (2.23) up to terms which are linear in the quantum fields and thus controllable in the quantized theory. This result also holds after performing the gauge-fixing using the BRST operation (2.10), provided specific gauge conditions are considered and provided the manifold \(\mathcal{M}_3\) admits covariantly constant vector fields \(\tau\), as emphasized in subsection 1.1.1.

### 2.2.3 “Diffeomorphisms imply VSUSY”

Let us now show how local VSUSY appears in a natural way, within the BRST symmetry, as soon as (external) diffeomorphism invariance is included in the latter. A convenient way to incorporate external diffeomorphisms is to consider the extended forms
\[
\hat{\mathcal{A}} = \dot{c} + \hat{A} + \hat{B}^* + \dot{\phi}^* , \quad \hat{\mathcal{B}} = \hat{\phi} + \hat{B} + \hat{A}^* + \dot{c}^*
\]
defined by \([10, 23]\)
\[
\hat{\mathcal{A}} = e^{-i\xi} \mathcal{A} , \quad \hat{\mathcal{B}} = e^{-i\xi} \mathcal{B},
\] (2.24)
with the extended forms \(\mathcal{A}\) and \(\mathcal{B}\) given by (2.6). More explicitly:
\[
\begin{align*}
\dot{\phi}^* &= \phi^*, & \dot{c}^* &= c^*, \\
\hat{B}^* &= B^* - i\xi \phi^*, & \hat{A}^* &= A^* - i\xi c^*, \\
\hat{A} &= A - i\xi B^* + \frac{1}{2} i\xi^2 \phi^*, & \hat{B} &= B - i\xi A^* + \frac{1}{2} i\xi^2 c^*, \\
\dot{c} &= c - i\xi A + \frac{1}{2} i\xi^2 B^* - \frac{1}{6} i\xi^3 \phi^* , & \dot{\phi} &= \phi - i\xi B + \frac{1}{2} i\xi^2 A^* - \frac{1}{6} i\xi^3 c^* .
\end{align*}
\] (2.25)

By applying the operator \(e^{-i\xi}\) to the horizontality conditions (2.9) and using the last of relations (A.11), we obtain
\[
\begin{align*}
0 &= e^{-i\xi} (\delta \mathcal{A} + \mathcal{A}^2 + \alpha \mathcal{B}^2) = (s + d - L_\xi + i_\nu) \hat{\mathcal{A}} + \hat{\mathcal{A}}^2 + \alpha \hat{\mathcal{B}}^2 \\
0 &= e^{-i\xi} (\delta \mathcal{B} + [\mathcal{A}, \mathcal{B}]) = (s + d - L_\xi + i_\nu) \hat{\mathcal{B}} + [\hat{\mathcal{A}}, \hat{\mathcal{B}}] ,
\end{align*}
\] (2.26)
where we have introduced the even (ghost-number 2) vector field
\[ v \equiv s\xi - \xi^2 \quad \text{with} \quad \xi^2 \equiv \frac{1}{2}[\xi,\xi]. \quad (2.27) \]

Equations (2.26) can be rewritten as
\[ \hat{\mathcal{F}} + \alpha\hat{\mathcal{B}}^2 = 0 \quad , \quad \hat{\mathcal{D}}\hat{\mathcal{B}} = 0, \quad (2.28) \]
where
\[ \hat{\mathcal{F}} \equiv \hat{\delta}\hat{A} + \hat{A}^2 \quad , \quad \hat{\mathcal{D}}\hat{\mathcal{B}} \equiv \hat{\delta}\hat{B} + [\hat{A},\hat{B}], \quad (2.29) \]
with \[ \hat{\delta} \equiv e^{-i\xi} \delta e^{i\xi} = d + s - \mathcal{L}_\xi + i_v. \quad (2.30) \]

Hence, they again have the same form as the equations of motion or as the horizontality conditions (2.9). They determine the action of the BRST operator on the extended forms \( \hat{A} \) and \( \hat{B} \):
\[ s\hat{A} = - (d\hat{A} + \hat{A}^2 + \alpha\hat{B}^2 - \mathcal{L}_\xi\hat{A} + i_v\hat{A}) \]
\[ s\hat{B} = - (d\hat{B} + [\hat{A},\hat{B}] - \mathcal{L}_\xi\hat{B} + i_v\hat{B}) . \quad (2.31) \]

Thus, they also provide the variations of all component fields,
\[ s\hat{c} = c^2 - \alpha\hat{c}^2 + \mathcal{L}_\xi\hat{c} - i_v\hat{A} \]
\[ s\hat{\phi} = \hat{\phi}^2 - \alpha\hat{\phi}^2 + \mathcal{L}_\xi\hat{\phi} - i_v\hat{B} \]
\[ s\hat{\phi}^* = - (\hat{\phi} + \alpha\hat{B}^2) - \hat{\phi}^*_\xi - \alpha\hat{\phi}^* + \mathcal{L}_\xi\hat{\phi}^* \]
\[ s\hat{\phi} = - (\hat{\phi} + \alpha\hat{B}^2) - \hat{\phi}^*_\xi - \alpha\hat{\phi}^* + \mathcal{L}_\xi\hat{\phi}^* \]
\[ s\hat{\phi} = \hat{\phi} - \hat{\phi}^* - \hat{\phi}^* - \hat{\phi}^* \]
\[ s\hat{\phi} = \hat{\phi} - \hat{\phi}^* - \hat{\phi}^* - \hat{\phi}^* \]
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\[ s\hat{\phi} = \hat{\phi} - \hat{\phi}^* - \hat{\phi}^* - \hat{\phi}^* \]
\[ s\hat{\phi} = \hat{\phi} - \hat{\phi}^* - \hat{\phi}^* - \hat{\phi}^* \]
where \( \hat{\mathcal{F}} \equiv d\hat{A} + \hat{A}^2 \) and \( \hat{\mathcal{D}} \equiv d + [\hat{A}, \cdot] \). Note that the \( s \)-operator can be decomposed according to
\[ s = s_g + s_\xi + s_v , \quad (2.33) \]
where the action of \( s_g \) and \( s_v \) have the form (2.11)(2.12) and (2.23), respectively, while the action of \( s_\xi \) is given by the Lie derivative \( \mathcal{L}_\xi \).

So far, the vector field \( v \equiv s\xi - \xi^2 \) determining the \( s \)-variation of \( \xi \) was not yet specified. By requiring the nilpotency of \( s \) on \( \hat{A}, \hat{B} \) as well as on \( \xi, \) we do not obtain any constraint on \( v \), except that it must transform according to
\[ sv = [\xi, v] . \quad (2.34) \]
Henceforth, we conclude that \((2.27)\) (with \(v\) subject to the transformation law \((2.34)\)) can be interpreted as the most general BRST-transformation of the diffeomorphism ghost \(\xi\) which is compatible with nilpotency:

\[
s\xi = \xi^2 + v . \tag{2.35}
\]

We thus see how the local VSUSY-transformations \((2.23)\) appear in the BRST operator once the diffeomorphisms have been incorporated: they are nothing but the symmetry transformations of the fields \(\hat{\phi}\) and \(\hat{\phi}^\ast\) whose corresponding ghost is the even vector field \(v\) of ghost-number 2.

To simplify the notation, we will drop the symbol \(\hat{\ }\) on fields and extended forms in the remainder of this subsection. An action \(S\) which is invariant under the BRST symmetry defined by the transformations \((2.32)-(2.35)\) can be constructed along the lines followed at the end of subsection 2.1. The Slavnov-Taylor identity to be satisfied by the action \(S\) now takes the extended form

\[
S(S) \equiv \frac{1}{2} (S, S) + \Delta S = 0 , \tag{2.36}
\]

where the BV antibracket \((\cdot, \cdot)\) is given by \((A.18)\) and where the operator \(\Delta\) is defined by

\[
\Delta \equiv \int d^3 x \left( s\xi^\mu \frac{\delta}{\delta \xi^\mu} + sv^\mu \frac{\delta}{\delta v^\mu} \right) \quad \text{with} \quad \Delta^2 = 0 . \tag{2.37}
\]

The corresponding linearized Slavnov-Taylor operator reads

\[
S_S \cdot \equiv (S, \cdot) + \Delta \cdot , \tag{2.38}
\]

and we have the following identities resulting from \((A.20)\) and \((A.25)\):

\[
\Delta S(S) = -S_S \Delta S , \quad S_S S(S) = 0 \\
(X, S(S)) + (S_S)^2 X = 0 \quad \text{for any functional} \quad X(\varphi, \varphi^\ast, \xi, v) . \tag{2.39}
\]

Following the arguments of subsection 2.1, an action obeying the Slavnov-Taylor identity \((2.36)\) is found by solving the functional differential equations

\[
S_S \varphi \equiv \frac{\delta S}{\delta \varphi} = s\varphi , \quad S_S \varphi^\ast \equiv \frac{\delta S}{\delta \varphi^\ast} = s\varphi^\ast , \tag{2.40}
\]

where the BRST operator \(s\) is now given by \((2.32)-(2.35)\). The solution represents an extension of expression \((2.15)\),

\[
S = - \int_{M_3} \text{Tr} \left( B \left( d\varphi + \varphi^2 \right) + \frac{\alpha}{3} B^3 - B(\mathcal{L}_\xi \varphi - i_v \varphi) \right) \bigg|_3 , \tag{2.41}
\]

\(^6\)As a matter of fact, a similar Slavnov-Taylor identity incorporating all of the symmetries had initially been considered for 3d Chern-Simons theory in flat space, see the second of references \[\].
or, in components,
\[
S = - \int \text{Tr} \left( BF + \frac{\alpha}{3} B^3 \right) + \sum_{\varphi} \int \text{Tr} \left( \varphi^* \bar{s}_\varphi \right) - \int \text{Tr} \left( \phi^* i_v B + c^* i_v A + A^* i_v B^* \right) \\
\equiv S_{\text{inv}}(A, B) + S_{\text{antifields}}(\varphi, \varphi^*).
\]
(2.42)

Here, \( \bar{s}_\varphi \) denotes the BRST-variations \( (2.32) \) taken at \( v = 0 \), i.e. without vector supersymmetry, the effect of the latter appearing explicitly in the third integral.

We note that the action involves a contribution that is quadratic in the antifields. This term reflects the fact that the algebra of gauge symmetries, diffeomorphisms and vector supersymmetry only closes on-shell, i.e. by virtue of the equations of motion \([7, 14]\).

To verify that the Slavnov-Taylor identity \( (2.36) \) is satisfied, we again proceed as in subsection \( 2.1 \), by applying the last of the identities \( (2.39) \) and using \( S_S^2 = 0 \). As before, this implies that \( S(S) \) is independent of the fields and antifields \( \varphi, \varphi^* \). Hence \( S(S) \) can only depend on the variables \( \xi \) and \( v \),
\[
S(S) = F(\xi, v) = \int d^3 x a_\mu \xi^\mu ,
\]
(2.43)
where we used the fact that the functional \( F \) has to be linear in \( \xi \), as well as independent of \( v \) due to ghost-number conservation (the coefficient \( a_\mu \) is field independent). The second of the identities \( (2.39) \) then yields the consistency condition \( \Delta F = 0 \) whose solution is \( a_\mu = 0 \).

The main conclusions of the present section are the following. First, by incorporating diffeomorphisms into the BRST operator, we have shown that the presence of VSUSY as a local symmetry is natural in the sense that it belongs to the most general BRST algebra (compatible with nilpotency) for the present set of fields. Second, the inclusion of VSUSY-transformations into the \( s \)-variations allows for their discussion on generic manifolds.

### 2.3 Correspondence between 3d gravity and BF models

#### 2.3.1 Action and BRST symmetry

We now choose \( \text{so}(3) \) or \( \text{so}(1, 2) \) as symmetry algebra in order to establish the relationship between the associated BF models and 3d gravity, extending off-shell the on-shell result presented in subsection \( 1.1.3 \). The correspondence appeared there via the substitution \( (1.13) \), in particular via the reparametrization \( \phi = i_\xi B \) of the ghost \( \phi \) in terms of the diffeomorphism ghost \( \xi \). In view of relations \( (2.25) \), we infer that the off-shell extension of the former equation reads
\[
\hat{\phi} = 0 , \quad \text{i.e.} \quad \phi = i_\xi B - \frac{1}{2} i_\xi^2 A^* + \frac{1}{6} i_\xi^3 c^* .
\]
(2.44)
From the transformation law of $\hat{\phi}$ as given in (2.32), we see that the necessary and sufficient condition for setting $\hat{\phi}$ to zero consistently is to set $v$ to zero – which condition for its part is compatible with the transformation law of $v$, see eq.(2.34). Thus, it is necessary to keep VSUSY out of the BRST operator.

For $\hat{\phi} = 0 = v$, the $s$-variations (2.32) and (2.35) reduce to

$$
\begin{align*}
\hbar sA &= -\hat{\partial} c + \mathcal{L}_\xi \hat{A} \\
\hbar sA^* &= -\hat{\partial} \hat{B} - [\hat{c}, \hat{A}^*] + \mathcal{L}_\xi \hat{A}^* \\
\hbar s\hat{B} &= -[\hat{c}, \hat{B}] + \mathcal{L}_\xi \hat{B} \\
\hbar s\hat{B}^* &= -(\hat{F} + \alpha \hat{B}^2) - [\hat{c}, \hat{B}^*] + \mathcal{L}_\xi \hat{B}^* \\
\hbar sc &= -\hat{c}^2 + \mathcal{L}_\xi \hat{c} \\
\hbar sc^* &= -\hat{D}\hat{A}^* - [\hat{c}, \hat{c}^*] - [\hat{B}^*, \hat{B}] + \mathcal{L}_\xi \hat{c}^*
\end{align*}
$$

(2.45)

Furthermore, the action (2.42) reduces to

$$
S = -\int \text{Tr} \left( \hat{B}\hat{F} + \frac{\alpha}{3} \hat{B}^3 \right) + \sum_{\hat{\phi} = \hat{A}, \hat{B}, \hat{c}} \int \text{Tr} (\hat{\phi}^* s\hat{\phi}) ,
$$

(2.46)

with $s\hat{\phi}$ given by the previous set of equations.

With notation (1.3) for the Lorentz connection and the 3-bein,

$$
\begin{align*}
\hat{A}_{jk} &= \omega_{jk} \\
\hat{c}_{jk} &= \Omega_{jk} \\
\hat{B}_{jk} &= \varepsilon_{ijk}e^i ,
\end{align*}
$$

(2.47)

the $s$-variations (2.45) become the BRST-transformations of gravity, i.e. eqs.(1.7) and the action (2.46) becomes the action for gravity involving a cosmological term, i.e. the action based on the invariant functional

$$
S_{\text{inv}}(e, \omega) = -\int \varepsilon_{ijk} \left( \varepsilon^i R^{jk} + \frac{\alpha}{3} \varepsilon^i e^j e^k \right) .
$$

(2.48)

Since diffeomorphisms now represent a dynamical symmetry – the vector field $\xi$ being a dynamical Faddeev-Popov field – one has to introduce an antifield $\xi^*$ coupled to the BRST-variation of $\xi$, i.e. add the term $\int \xi^* s\xi$ to the action (2.46). BRST invariance is then expressed by the Slavnov-Taylor identity (2.17) in which $\hat{\phi}$ now takes the values $\hat{A}, \hat{e}, \hat{\Omega}$ and $\xi$.

These results generalize the relations found in references [1, 17] where the antifields have not been taken into account.

### 2.3.2 Vector supersymmetry

Since we could not include VSUSY-variations into the BRST-transformations when considering the gravity theory variables, we now have to deal with this symmetry separately, i.e. as an extra symmetry expressed by a separate Ward identity. Thus, we

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7 The $s$-variation of $\xi_\mu$ is determined by $s\xi_\mu = \delta S / \delta \xi^\mu$. 

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consider the \( s \)-variations \((2.32)-(2.33)\) with \( v = 0 \) and the VSUSY-transformations \((2.22)\) with an even vector field \( \tau \) as parameter. Moreover, we assume that \( \dot{\phi} = 0 \) as in the last section, since this truncation can be performed in a consistent way for \( v = 0 \).

Before evaluating the action of the operator \([\delta_\tau, s]\) on \( \hat{A} \) and \( \hat{B} \), we have to specify the VSUSY-variation of \( \xi \): in view of the \( v \)-dependent contribution to \( s\xi \) (see eq.\((2.33)\)), we postulate

\[
\delta_\tau \xi = k\tau ,
\]

where \( k \) denotes a constant. It follows that \([\delta_\tau, i\xi] = ki_\tau \) and thereby

\[
[\delta_\tau, e^{-i\xi}] = -ki_\tau e^{-i\xi} .
\]

By virtue of the definition \((2.24)\), we have

\[
\delta_\tau \hat{A} = \delta_\tau (e^{-i\xi} A) \\
= [\delta_\tau, e^{-i\xi}] A + e^{-i\xi} i_\tau A \\
= [\delta_\tau, e^{-i\xi}] A + [e^{-i\xi}, i_\tau] A + i_\tau \hat{A} .
\]

Since the second commutator in the last line vanishes, we obtain

\[
\delta_\tau \hat{A} = (1 - k)i_\tau \hat{A} ,
\]

an analogous result holding for \( \hat{B} \). From this relation and eqs.\((2.31), (A.10)\), we readily find

\[
[\delta_\tau, s] \hat{A} = \mathcal{L}_\tau \hat{A} - (1 - k) i_{[\xi, \tau]} \hat{A}
\]

and analogously for \( \hat{B} \). Choosing \( k = 1 \), we get the \( \delta_\tau \)-variations

\[
\delta_\tau \xi = \tau \quad , \quad \delta_\tau \hat{A} = 0 \quad , \quad \delta_\tau \hat{B} = 0 ,
\]

which satisfy the algebra

\[
[\delta_\tau, s] = \mathcal{L}_\tau \quad , \quad s^2 = 0 \quad , \quad [\delta_{\tau_1}, \delta_{\tau_2}] = 0 .
\]

If we pass over from the BF model to gravity by virtue of the identifications \((2.47)\), we recover the VSUSY-variations \((1.8)\) of gravity, i.e. the result of reference \[11\], now generalized by the presence of the antifields.

As we already noted at the end of subsection \[1.1.3\], the relation between both versions of 3d gravity, namely the topological (i.e. BF) version and the conventional one, as expressed by eq.\((2.44)\), is not one-to-one unless the 3-bein coefficients (or the metric) represent a nonsingular matrix.
2.4 Gauge-fixing

As usual [10], the theory is gauge-fixed by introducing pairs of antighosts and Lagrange multipliers associated to each of the gauge invariances. We will consider the BF version of the theory. Since the gauge transformations of $A$ and $B$ represent irreducible symmetries in three dimensions, it suffices to consider one pair of antighosts and multipliers for each of them. These pairs of 0-forms are denoted by $\bar{c}, \pi$ and $\bar{\phi}, \lambda$, respectively and we have the BRST-transformation laws

$$
\begin{align*}
 s\bar{c} &= \pi, \quad s\pi = 0 \\
 s\bar{\phi} &= \lambda, \quad s\lambda = 0.
\end{align*}
$$

(2.56)

In the remainder of this subsection, we will again omit the hats on field s and antifields in order to simplify the notation. Following the Batalin-Vilkovisky procedure [19], we first complete the set of antifields by introducing antifields associated to the new fields, namely the 3-forms $\bar{c}^*, \pi^*, \bar{\phi}^*$ and $\lambda^*$. The latter admit the transformation laws [12]

$$
\begin{align*}
 s\bar{c}^* &= 0, \quad s\pi^* = \bar{c}^* \\
 s\bar{\phi}^* &= 0, \quad s\lambda^* = \bar{\phi}^*
\end{align*}
$$

(2.57)

and enter the so-called non-minimal action

$$
S_{\text{nm}}(\varphi, \pi, \lambda, \varphi^*, \bar{c}^*, \bar{\phi}^*) = S(\varphi, \varphi^*) + \int \text{Tr} \left( \bar{c}^* \pi + \bar{\phi}^* \lambda \right),
$$

(2.58)

where $S$ is the action (2.42) or (2.41). This non-minimal action solves the same Slavnov-Taylor identity (2.36) as $S$, the summation in (2.13) now including the new fields.

Next, we consider the “gauge fermion” functional for a Landau gauge,

$$
\Psi(A, B, \bar{c}, \bar{\phi}) = \int \text{Tr} \left( \bar{c} d \star A + \bar{\phi} d \star B \right),
$$

(2.59)

where the Hodge duality operator $\star$ is given by eq.(A.5). The fields $\varphi, \bar{c}, \bar{\phi}, \pi, \lambda$ are to be denoted collectively by $\Phi$ and the corresponding antifields by $\Phi^*$. We redefine the antifields according to

$$
\tilde{\Phi}^* = \Phi^* + \frac{\delta \Psi}{\delta \Phi},
$$

(2.60)

where $\tilde{\Phi}^*$ is to be viewed as the external source associated to the BRST variation of the field $\Phi$. The non-trivial reparametrizations read as

$$
\begin{align*}
\tilde{A}^* &= A^* + d\bar{c} \\
\tilde{B}^* &= B^* + d\bar{\phi}
\end{align*}
$$

(2.61)
According to the BV prescription, the *gauge-fixed action* is given by the non-minimal action (2.58) in which the antifields are reparametrized in terms of the sources $\Phi^*$ by virtue of eq.(2.60):

$$
S_{\text{gauge-fixed}}(\Phi, \Phi^*) = S_{\text{nm}}(\Phi, \Phi^* = \Phi^* - \frac{\delta \Psi}{\delta \Phi}).
$$

(2.62)

Thus, we obtain

$$
S_{\text{gauge-fixed}} = S_{\text{inv}} + S_{\text{gf}} + S_{\text{ext}} + S_{\text{quadr}},
$$

(2.63)

with

$$
S_{\text{gf}} = - \int \text{Tr} \left( \pi d \ast A + \lambda d \ast B - \bar{\phi} d \ast \bar{s}A - \bar{\phi} d \ast \bar{s}B - (\ast d \bar{c}) i_v(\ast d \bar{\phi}) \right)
$$

$$
S_{\text{ext}} = \sum \Phi \int \text{Tr} \left( \bar{\Phi}^* \bar{s} \Phi \right) - \int \text{Tr} \left( \bar{\phi}^* i_v B + \bar{c}^* i_v A + \bar{B}^* i_v (\ast d \bar{c}) + \bar{A}^* i_v (\ast d \bar{\phi}) \right)
$$

$$
S_{\text{quadr}} = - \int \text{Tr} \left( \bar{A}^* i_v \bar{B}^* \right),
$$

where $\bar{s}$ is the BRST operator (2.32) taken at $v = 0$. This action obviously fulfills the same Slavnov-Taylor identity as the non-minimal action.

We note that VSUSY continues to hold as a *local* invariance of the gauge-fixed theory. This is in contrast to the results obtained by alternative implementations of the gauge-fixing procedure [14] for which VSUSY only holds as a rigid invariance, the possible vector fields $v$ being covariantly constant (assuming that such vector fields exist on the manifold under consideration). The more general validity of VSUSY invariance in our approach is due to its inclusion in the Slavnov-Taylor identity whereas it is left outside in the other schemes. As for the usual gravitational formulation discussed in section 2.3, one is precisely in the latter situation since we were forced to keep VSUSY outside of the BRST operator: VSUSY then represents a rigid symmetry, $v$ being a Killing vector field with respect to a background metric [11] (assuming again that the manifold under consideration admits such vector fields at all).

### 2.5 Summary

Let us briefly summarize our approach and results.

- **BF model without diffeomorphisms**: As is well known, the gauge transformations of the BF model can be obtained from the horizontality conditions $\mathcal{F} + \alpha \mathcal{B}^2 = 0 = \mathcal{D} \mathcal{B}$ which are equivalent to the $s$-variations of $\mathcal{A}$ and $\mathcal{B}$ given by eqs.(2.10).
• **Local VSUSY of the BF model on a generic manifold (or inclusion of diffeomorphisms and VSUSY):** By applying the operator $e^{-i\xi}$ to the previous horizontality conditions and defining $v \equiv s\xi - \xi^2$ (i.e. $s\xi = \xi^2 + v$), one can derive the $s$-variations of the reparametrized fields $\hat{A} \equiv e^{-i\xi}A, \hat{B} \equiv e^{-i\xi}B$, see eqs. (2.31), which include diffeomorphisms and local VSUSY.

• **From the BF model to 3d gravity:** We set $v = 0$ (in order to eliminate VSUSY from the $s$-variations) and $\hat{\phi} = 0$ (in order to express the gauge transformation of the 1-form potential $B$ in terms of diffeomorphisms). By virtue of the identification (2.47), the invariant action of the BF model and its local symmetries then yield those of gravity. The VSUSY-transformations of gravity are recovered directly from the VSUSY-variations of the BF model as given by eqs. (2.22), by virtue of the reparametrization $\hat{A} \equiv e^{-i\xi}A, \hat{B} \equiv e^{-i\xi}B$, the identification (2.47) and the postulate $\delta_s\xi = \tau$.

• **Gauge-fixing:** Applying the Batalin-Vilkovisky gauge-fixing procedure to the BF theory, with VSUSY included into the BRST-transformations, we have seen that this symmetry holds as a local invariance of the action. Yet, this result is no longer valid if VSUSY is kept out of the BRST operator, as expected for the conventional formulation of 3d gravity.

3 4d BF model with cosmological term

3.1 The model and its symmetries

The model is now defined on a 4-manifold $\mathcal{M}_4$ and the potential $B$ represents a 2-form. The action reads

$$S_{\text{inv}}(A, B) = -\int_{\mathcal{M}_4} \text{Tr} \left(BF + \frac{\lambda}{2}B^2\right),$$

where the real dimensionless parameter $\lambda$ is again referred to as cosmological constant. The equations of motion of this model have the form

$$F + \lambda B = 0 , \quad DB = 0 .$$

In this case, we consider the generalized fields

$$A = c + A + B^* + B_1^* + B_0^* \quad (\text{with } B_i^* \equiv (B_i)^*)$$

$$B = B_0 + B_1 + B + A^* + c^* ,$$

where $B_1$ denotes the ghost parametrizing the local gauge symmetry of $B$ while $B_0$ represents the “ghost for the ghost” $B_1$. 

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The horizontality conditions have the same form as the equations of motion,
\[ \mathcal{F} + \lambda \mathcal{B} = 0, \quad D\mathcal{B} = 0 \] (3.4)
and define a nilpotent BRST operator. Expansion with respect to the ghost-number yields the BRST-transformations of fields and antifields:
\[
\begin{align*}
s c &= -c^2 - \lambda B_0 \\
s A &= -Dc - \lambda B_1 \\
s B^* &= -(F + \lambda B) - [c, B^*] \\
s B_1^* &= -DB^* - [c, B_1^*] - \lambda A^* \\
s B_0^* &= -DB_1^* - (B^*)^2 - [c, B_0^*] - \lambda c^* \\
s B_0 &= -[c, B_0] \\
s B_1 &= -DB_0 - [c, B_1] \\
s B &= -DB_1 - [c, B] + [B^*, B_0] \\
s A^* &= -DB - [c, A^*] - [B^*, B_1] + [B_1^*, B_0] \\
s c^* &= -DA^* - [c, c^*] - [B^*, B] - [B_1^*, B_1] - [B_0^*, B_0].
\end{align*}
\] (3.5)

If the antifields are set to zero, we recover the BRST-transformations of \( A, B \) and of their ghosts, as well as their equations of motion.

### 3.2 Diffeomorphisms and vector supersymmetry

Proceeding along the lines of the three dimensional theory, one introduces new generalized fields \( \hat{A} = e^{-i\xi} A \) and \( \hat{\mathcal{B}} = e^{-i\xi} \mathcal{B} \) as in eqs.\( (2.24) \), which define new fields \( \hat{\phi} \) and antifields \( \hat{\phi}^* \).

Applying the operator \( e^{-i\xi} \) to the horizontality conditions (3.4), we obtain the \( s \)-variations of the new fields:
\[
\begin{align*}
s \hat{c} &= -c^2 - \lambda \hat{B}_0 + \mathcal{L}_\xi \hat{c} - i_v \hat{\dot{A}} \\
s \hat{A} &= -\hat{D}\hat{c} - \lambda \hat{B}_1 + \mathcal{L}_\xi \hat{A} - i_v \hat{\dot{B}}^* \\
s \hat{B}^* &= -(\hat{F} + \lambda \hat{B}) - [\hat{c}, \hat{B}^*] + \mathcal{L}_\xi \hat{B}^* - i_v \hat{\dot{B}}_1^*.
\end{align*}
\]
\[
s\hat{B}^* = -\hat{D}\hat{B}^* - [\hat{c}, \hat{B}^*_1] - \lambda\hat{A}^* + \mathcal{L}_\xi\hat{B}_1^* - i_v\hat{B}_0^*
\]
\[
s\hat{B}_0^* = -\hat{D}\hat{B}_0^* - \frac{1}{2}(\hat{B}^*)^2 - \lambda\hat{c}^* + \mathcal{L}_\xi\hat{B}_0^*
\]

and
\[
s\hat{B}_0 = -[\hat{c}, \hat{B}_0] + \mathcal{L}_\xi\hat{B}_0 - i_v B_1
\]
\[
s\hat{B}_1 = -\hat{D}\hat{B}_0 - [\hat{c}, \hat{B}_1] + \mathcal{L}_\xi\hat{B}_1 - i_v B
\]
\[
s\hat{B} = -\hat{D}\hat{B}_1 - [\hat{c}, \hat{B}] - [\hat{B}^*, \hat{B}_0] + \mathcal{L}_\xi\hat{B} - i_v \hat{A}^*
\]
\[
s\hat{A}^* = -\hat{D}\hat{A} - [\hat{c}, \hat{A}^*] - [\hat{B}^*, \hat{B}_1] - [\hat{B}_1^*, \hat{B}_0] + \mathcal{L}_\xi\hat{A} - i_v \hat{c}^*
\]
\[
s\hat{c}^* = -\hat{D}\hat{c} - [\hat{c}, \hat{c}^*] - [\hat{B}^*, \hat{B}] - [\hat{B}_1^*, \hat{B}_1] - [\hat{B}_0^*, \hat{B}_0] + \mathcal{L}_\xi\hat{c}^*,
\]

where \(\hat{F} = d\hat{A} + \hat{A}^2\) and \(\hat{D} = d \cdot + [\hat{A}, \cdot]\). The BRST-transformations of the diffeomorphism and supersymmetry ghosts again have the form
\[
s\xi = \xi^2 + v, \quad sv = [\xi, v],
\]
where the latter relation follows by requiring nilpotency of \(s\) on \(\xi\).

### Slavnov-Taylor identity:

In the following, we will again omit the hats on fields and antifields. An action which is invariant under the latter BRST-transformations that involve VSUSY can be constructed along the lines of section 2. This action satisfies the Slavnov-Taylor identity (2.36) in which the integration is now performed over the 4-manifold \(M_4\) and it is explicitly given by
\[
S = -\int_{M_4} \text{Tr} \left( BF + \frac{1}{2} B^2 \right) + \sum_{\varphi} \int_{M_4} \text{Tr} \left( \varphi^* s\varphi \right)
- \int_{M_4} \text{Tr} \left( A^* i_v B^* + (B^*)^2 B_0 \right),
\]
where \(s\varphi\) is the part of \(s\varphi\) that does not depend on antifields. The antifield-dependent part of \(s\varphi\) is taken into account by the third integral containing terms that are quadratic in the antifields. As we can see, a quadratic term of the form \((B^*)^2 B_0\) is present even if VSUSY is excluded from the BRST operator, as usual for BF models in dimensions greater than three [12].

### 3.3 Gauge-fixed action

We shall introduce the gauge-fixing term in the action following the Batalin-Vilkovisky (BV) procedure as in section 2. By constructing the BV pyramid [19, 14], it
can easily be seen that it is necessary to introduce a set of Lagrange multipliers \((\pi^0, \pi_{0}^{-1}, \pi_{1}^{-1})\) and antighosts \((\bar{c}_0^{-1}, \bar{c}_0^{-2}, \bar{c}_0^0)\) for the reducible gauge symmetry of \(B\). We also have to introduce a Lagrange multiplier \(\pi\) and an antighost \(\bar{c}\) for the gauge symmetry of \(A\). These fields transform as

\[
s\bar{c} = \pi , \quad s\bar{c}_1 = \pi_1^0 , \quad s\bar{g} = \pi_0^{-1} , \quad s\bar{\phi} = \pi_0^1 ,
\]

all Lagrange multipliers being s-inert. As before, an antifield is introduced for each Lagrange multiplier and each antighost. Moreover, all fields are collectively denoted by \(\Phi\) and the corresponding antifields by \(\Phi^*\).

The non-minimal action reads

\[
S_{nm}(\Phi, \Phi^*) = S(\varphi, \varphi^*) + \int_{M_4} \text{Tr} \left( \bar{c}^* \pi + \bar{c}_1^* \pi_1^0 + \bar{g}^* \pi_0^{-1} + \bar{\phi}^* \pi_0^1 \right)
\]

and as “gauge fermion” we choose the functional

\[
\Psi(\Phi) = \int_{M_4} \text{Tr} \left( (d\bar{c}) \star A + (d\bar{c}_1) \star B + (d\bar{g}) \star B_1 + (d\bar{\phi}) \star \bar{c}_1 \right).
\]

An external source is associated to each field \(\Phi\) by virtue of the reparametrization (2.60) and the gauge-fixed action is obtained from the non-minimal action by virtue of the prescription (2.62). The result again has the form (2.63) and explicit expressions for all contributions can readily be obtained from (3.9) and (3.10).

The generalization to any dimension \(d \geq 5\) can be achieved in a straightforward way by introducing the appropriate ghosts for ghosts, antighosts and Lagrange multipliers.

## 4 Conclusions

Our first two conclusions concern VSUSY of BF models. First, we are naturally led to the existence of a local vector supersymmetry for BF models within the BV framework if we use the formalism of extended differential forms and if we include diffeomorphism symmetry into the BRST-transformations. Second, VSUSY introduced in this way is valid on generic manifolds and still holds exactly as a local invariance after carrying out the gauge-fixing procedure. These results which we discussed in 3 and 4 space-time dimensions obviously extend to BF models in higher dimensions. They generalize in a substantial way the previous results [7, 14] using an approach for which VSUSY only holds, after gauge-fixing, as a rigid symmetry generated by covariantly constant vector fields. In fact, the latter approach is restricted to manifolds admitting such covariantly constant vectors.

Our further conclusions concerning 3d field theories are as follows. For three dimensions and specific gauge groups, the BF model can be related directly to
gravity. Thus, we could show that VSUSY still holds, as expected [11], within the gravity framework. However, VSUSY then turns out to be excluded from the BRST symmetry corresponding to the invariances with respect to diffeomorphisms and local Lorentz transformations. Together with the BRST operator $s$, it still obeys the usual [7, 14, 11] algebra (2.55). As a consequence, after carrying out the gauge-fixing procedure, VSUSY can only hold as a rigid symmetry generated by a Killing vector field, as shown in reference [11]. Thus, for 3d gravity, VSUSY can only be included into the BRST operator if the theory is described in the topological framework, i.e. to a BF model. Of course, this conclusion only applies in the 3d case, since higher dimensional gravity is not topological. Rather it is related to a constrained BF model whose investigation deserves a separate study [26].

Appendix: Notation and Useful Formulas

All fields that we consider are vector fields or Lie algebra-valued $p$-forms on a $d$-dimensional manifold $\mathcal{M}_d$ (see the beginning of subsection 1.1.1). In the sequel, we will summarize our notation concerning all of these fields and functionals thereof.

Differential forms and grading

The total degree of a Lie algebra-valued $p$-form $\omega^g_p$ of ghost-number $g$ is defined by

$$[\omega^g_p] = p + g . \quad (A.1)$$

If the total degree is even (odd), the form is said to be even (odd) and its grading function

$$\text{Grading } (X) = (-1)^{|X|} \quad (A.2)$$

then takes the values +1 and −1, respectively. For instance, the gauge connection $A$ is odd, since it is a 1-form with ghost-number 0 and the Faddeev-Popov ghost $c$ is odd too, since it is a 0-form with ghost-number 1.

The commutator of Lie algebra-valued forms is graded by the total degree, i.e.

$$[X, Y] = XY - (-1)^{|X||Y|} YX . \quad (A.3)$$

Thus, the graded commutator of $X$ and $Y$ amounts to an anticommutator if both quantities are odd and to a commutator otherwise.

The exterior derivative $d$, which acts as an antiderivation increasing the form degree by one unit, is defined in local coordinates by

$$d = dx^\mu \partial_\mu . \quad (A.4)$$
E.g. for the Faddeev-Popov ghost \(c\) (which is of ghost-number 1), we have \(dc = dx^\mu \partial_\mu c = -\partial_\mu c dx^\mu\).

The BRST differential \(s\) also acts as an antiderivation which increases the ghost-number (and thus the total degree) by one unit. A linear operator acting on products like a derivation (antiderivation) is called an even (odd) operator. The commutator of two such operators is always assumed to be graded according to (A.3), e.g. \([s, d]\) = \(sd + ds\).

The Hodge dual of a \(p\)-form \(\omega\) is the \((d - p)\)-form \(*\omega\) defined by

\[
*\omega = \frac{1}{(d - p)!} \bar{\omega}_{\mu_1...\mu_{d-p}} dx^{\mu_1}...dx^{\mu_{d-p}}
\]

where \(\bar{\omega}_{\mu_1...\mu_{d-p}} = \frac{1}{p!} \varepsilon_{\mu_1...\mu_p \nu_{d-p+1}...\nu_d} \omega^{\nu_{d-p+1}...\nu_d} \).

Here and elsewhere in the text, the wedge product symbol has been omitted. Moreover, a background metric \((g_{\mu\nu})\) has been used on \(\mathcal{M}_d\), as well as the totally antisymmetric tensor of Levi-Civita:

\[
\varepsilon_{\mu_1...\mu_d} = g_{\mu_1 \nu_1}...g_{\mu_d \nu_d} \varepsilon^{\nu_1...\nu_d} , \quad \varepsilon^{1...d} = 1 , \quad \varepsilon_{1...d} = \det (g_{\mu\nu}) .
\]

The following formulas are quite useful [27]:

\[
* * \omega^g_p = (-1)^{p(d-p)} \det (g_{\mu\nu}) \omega^g_p , \quad \text{Tr} \left( \omega^g_p * \phi^h_p \right) = (-1)^{(p+d)(g+h)+gh} \text{Tr} \left( \phi^h_p * \omega^g_p \right) .
\]

Since the Hodge star operator maps a form of total degree \(p + g\) to a form of total degree \((d - p) + g\), it represents an even operator if the space-time dimension is even and an odd operator otherwise.

**Vector fields, inner product and Lie derivative**

For a vector field \(w = w^\mu \partial_\mu\) on \(\mathcal{M}_d\), the total degree is given by its ghost-number \(g\). It is said to be even (odd) if \(g\) is even (odd).

The Lie bracket \([u, v]\) of two vector fields \(u\) and \(v\) is again a vector field: this bracket is assumed to be graded according to (A.3) so that its components are given by

\[
[u, v]^\mu = u^\nu \partial_\nu v^\mu - (-1)^{[u][v]} u^\nu \partial_\nu v^\mu .
\]

The interior product \(i_w\) with respect to the vector field \(w = w^\mu \partial_\mu\) is defined in local coordinates by

\[
i_w \varphi = 0 \quad \text{for 0-forms } \varphi , \quad i_w (dx^\mu) = w^\mu .
\]
If \( w \) is even, the operator \( i_w \) acts as an antiderivation (odd operator), otherwise it acts as a derivation (even operator).

The Lie derivative \( \mathcal{L}_w \) with respect to \( w \) acts on differential forms according to

\[
\mathcal{L}_w \equiv [i_w, d] = i_w d + (-1)^{|w|} di_w
\]  

(A.9)

and we have the graded commutation relations

\[
[\mathcal{L}_u, i_v] = i_{[u,v]} \ , \ [\mathcal{L}_u, \mathcal{L}_v] = \mathcal{L}_{[u,v]} .
\]  

(A.10)

In the main body of the text, the quantity \( \xi = \xi^\mu \partial_\mu \) always denotes a vector field of ghost-number 1 (representing the ghost for diffeomorphisms). We then have the following identities involving the vector fields \( \xi \) and \( \xi^2 \equiv \frac{1}{2}[\xi, \xi] \) as well as the previously introduced operators:

\[
e^{i\xi}(XY) = (e^{i\xi}X)(e^{i\xi}Y)
\]

\[
e^{-i\xi}d e^{i\xi} = d - \mathcal{L}_\xi - i\xi^2
\]  

(A.11)

\[
[s, e^{i\xi}] = i s \xi e^{i\xi} \ , \ [s, e^{-i\xi}] = -i s \xi e^{-i\xi} .
\]

Functional calculus with differential forms

The functional derivative \( \delta F/\delta \omega \) of a functional \( F \) depending on differential forms \( \omega,... \) is defined as a left derivative by

\[
\delta F = \int \delta \omega \ \frac{\delta F}{\delta \omega} ,
\]  

(A.12)

where \( \delta F \) is the variation of \( F \) induced by the variation \( \delta \omega \). In particular, for a \( p \)-form \( \omega_p \), we have

\[
\frac{\delta \omega_p(x)}{\delta \omega_p(y)} = \delta_{d-p,p}(y, x) ,
\]  

(A.13)

where the right-hand side is a Dirac-type distribution defined by

\[
\int_{y \in \mathcal{M}_d} \omega_p(y) \delta_{d-p,p}(y, x) = \omega_p(x) .
\]  

(A.14)

In order to discuss the grading of \( \delta F/\delta \omega \), we first recall that the integral of a \( d \)-form over \( \mathcal{M}_d \) is defined by

\[
\int_{\mathcal{M}_d} \omega^g_d = \frac{1}{d!} \int_{\mathcal{M}_d} d^d x \ \varepsilon^{\mu_1...\mu_d} \omega^g_{\mu_1...\mu_d} ,
\]  

(A.15)

where the right-hand side represents the integral written in local coordinates. Thus, for an odd dimension \( d \), this prescription changes the grading of the integrand. For
instance, if the integral is an action functional, then \( g = 0 \) and the integrand is of odd degree, whereas the integral is of even degree. This fact implies that the integration operator itself has a grading and therefore the grading of a functional derivative also depends on the space-time dimension:

\[
\begin{align*}
\text{Grading of integral symbol} & : \int_{\mathcal{M}_d} (-1)^d \\
\text{Grading of functional derivative} & : \left(\frac{\delta F}{\delta \omega}\right) : (-1)^{d+|F|+|\omega|}.
\end{align*}
\]  

(Batalin-Vilkovisky Algebra)

In the following, we adapt the formalism of Batalin and Vilkovisky (BV) \[19\] to the case where the fields \( \varphi \) are differential forms on a \( d \)-dimensional manifold. The antifield associated to the field \( \varphi \) is denoted by \( \varphi^* \), the total degrees of the fields and antifields being related by

\[
|\varphi| + |\varphi^*| = d - 1.
\]  

(A.17)

With derivatives operating from the left, the BV antibracket of two functionals \( X \) and \( Y \) depending on \( \varphi \) and \( \varphi^* \) is defined by

\[
(X, Y) = \sum_{\varphi} \int_{\mathcal{M}_d} \left( (-1)^{|X||\varphi^*|} \frac{\delta X}{\delta \varphi^*} \frac{\delta Y}{\delta \varphi} + (-1)^{|X|+|\varphi|+d(|\varphi|+1)} \frac{\delta X}{\delta \varphi} \frac{\delta Y}{\delta \varphi^*} \right).
\]  

(A.18)

Here, the summation is performed on all fields (forms) \( \varphi \) and \( [Q] \) denotes the total degree of the quantity \( Q \). For \( Y = \varphi \) and \( Y = \varphi^* \), we obtain respectively

\[
(X, \varphi) = (-1)^{|X||\varphi^*|} \frac{\delta X}{\delta \varphi^*}, \quad (X, \varphi^*) = (-1)^{|X|+|\varphi|+d(|\varphi|+1)} \frac{\delta X}{\delta \varphi}.
\]  

(A.19)

Our definition of the bracket \((X, Y)\) corresponds to the normalization requirements

\[
\frac{1}{2}(S, S) = \sum_{\varphi} \int \frac{\delta X}{\delta \varphi^*} \frac{\delta Y}{\delta \varphi},
\]

\[
\frac{1}{2} \partial_\lambda (S, S) = (\partial_\lambda S, S),
\]

(A.20)

where \( S \) denotes a functional of even total degree and \( \lambda \) an even or odd parameter.

Keeping in mind our definitions (A.12) and (A.2) of the functional derivative and grading, as well as the properties (A.16), we can derive the graded anti-symmetry property

\[
(X, Y) + (-1)^{|X|+1}(-1)^{|Y|+1}(Y, X) = 0
\]  

(A.21)
and the graded Jacobi identity

$$(X, (Y, Z)) + (-1)^{([X]+1)([Y]+[Z])}(Y, (Z, X)) + (-1)^{([Z]+1)([X]+[Y])}(Z, (X, Y)) = 0 .$$

(A.22)

The latter results from the following simple identity which is valid for any functional $S$ of even total degree:

$$(S, (S, S)) = 0 .$$

(A.23)

Indeed, it suffices to consider $S = xX + yY + zZ$ where the gradings of the coefficients $x$, $y$ and $z$ are equal to those of $X$, $Y$ and $Z$, respectively, then to differentiate with $\partial^3/\partial x \partial y \partial z$ at $x = y = z = 0$ while using the differentiation formula

$$\partial_\lambda (X, Y) = (\partial_\lambda X, Y) + (-1)^{[\lambda([X]+1)}(X, \partial_\lambda Y) .$$

(A.24)

A useful special case of the Jacobi identity is

$$(X, (S, S)) + 2 (S, (S, X)) = 0 ,$$

(A.25)

where the total degree of $X$ is arbitrary.

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