Theory of macroscopic quantum tunnelling and dissipation in high-$T_c$ Josephson junctions

Shiro Kawabata$^{1,2}$, Satoshi Kashiwaya$^3$, Yasuhiro Asano$^4$, Yukio Tanaka$^5$, Takeo Kato$^6$ and Alexander A Golubov$^1$

$^1$ Faculty of Science and Technology, University of Twente, PO Box 217, 7500 AE Enschede, The Netherlands
$^2$ Nanotechnology Research Institute (NRI), National Institute of Advanced Industrial Science and Technology (AIST), Tsukuba, Ibaraki 305-8568, Japan
$^3$ Nanoelectronics Research Institute (NeRI), AIST, Tsukuba, Ibaraki 305-8568, Japan
$^4$ Department of Applied Physics, Hokkaido University, Sapporo 060-8628, Japan
$^5$ Department of Applied Physics, Nagoya University, Nagoya 464-8603, Japan
$^6$ The Institute for Solid State Physics (ISSP), University of Tokyo, Kashiwa 277-8581, Japan

E-mail: s-kawabata@aist.go.jp

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Abstract

We have investigated macroscopic quantum tunnelling (MQT) in in-plane high-$T_c$ superconductor Josephson junctions and the influence of the nodal-quasiparticle and zero energy bound states (ZES) on MQT. We have shown that the presence of ZES at the interface between the insulator and the superconductor leads to strong Ohmic quasiparticle dissipation. Therefore, the MQT rate is noticeably suppressed in comparison with $c$-axis junctions in which ZES are completely absent.

1. Introduction

A mesoscopic single Josephson junction is an interesting physical object for testing quantum mechanics at a macroscopic level. In current-biased Josephson junctions, measurements of macroscopic quantum tunnelling (MQT) are performed by switching the junction from its metastable zero-voltage state to a non-zero voltage state (see figure 1(d)). Until now, experimental investigations of MQT have been focused on $s$-wave (low-$T_c$) junctions only. This is due to a naive preconception that the existence of low energy quasiparticles in the $d$-wave order parameter of a high-$T_c$ cuprate superconductor [1] may preclude the possibility of observing MQT.

Recently we have theoretically investigated the effect of the nodal-quasiparticle on MQT in $d$-wave $c$-axis junctions (e.g. Bi2212 intrinsic Josephson junctions [12, 13] and cross whisker junctions [14]) [2, 3]. We have shown that the effect of the nodal-quasiparticle gives rise to super-Ohmic dissipation [4, 5] and the suppression of MQT due to the nodal-quasiparticle is very weak.

The first experimental observation of MQT in high-$T_c$ Josephson junction was made by Bauch et al, using a YBCO grain boundary bi-epitaxial junction [6, 7]. Recently, Inomata et al [8], Jin et al [9] and Kashiwaya et al [10, 11] have

![Figure 1. Schematic of an in-plane d-wave Josephson junction: (a) $d_0/d_0$, (b) $d_0/d_{\pi/4}$ and (c) $d_{\pi/4}/d_{\pi/4}$. In the case of $d_0/d_{\pi/4}$ and $d_{\pi/4}/d_{\pi/4}$ junctions, zero energy bound states (ZES) are formed near the boundary between superconductor $d_{\pi/4}$ and insulating barrier I. (d) Potential $U(\phi)$ versus the phase difference $\phi$ between two superconductors. $U_0$ is the barrier height and $\omega_p$ is the Josephson plasma frequency. (This figure is in colour only in the electronic version)
experimentally observed MQT in c-axis (Bi2212 intrinsic) junctions. They reported that the effect of the nodal-
quasiparticle on MQT is negligibly small and the thermal-to-
quantum crossover temperature is relatively high (0.5–1 K)
compared with the case of low-Tc and YBCO bi-epitaxial
junctions. In Jin et al’s experiment, $O(N^2)$ ($N$ being
the number of the stacked junctions) enhancement of the
MQT rate was reported. This enhancement is attributed to
collective motion of the phase differences in the intrinsic
junctions [15–17].

In this paper we will theoretically investigate MQT
in d-wave in-plane junctions parallel to the $ab$-plane (see
figure 1) [18]. In such junctions, ZES [19] are formed near
the interface between the superconductor and the insulating
barrier. ZES are generated by the combined effect of multiple
Andreev reflections and the sign change of the d-wave order
parameter symmetry, and are bound states for the quasiparticle
at the Fermi energy. Below, we will show that ZES give
rise to Ohmic-type strong dissipation so MQT is considerably
suppressed compared with the c-axis and the $d_0/d_0$ junction
cases.

2. Effective action

By using the method developed by Eckern et al [20] the
partition function of the system can be described by a
functional integral over the macroscopic variable (the phase
difference $\phi$):

$$Z = \int D\phi(\tau) \exp\left(-\frac{S_{\mathrm{eff}}[\phi]}{\hbar}\right).$$

(1)

In the high barrier limit, i.e. $z_0 \equiv mw_0/h^2k_F \gg 1$ ($m$ is the
mass of the electron, $w_0$ is the height of the delta function
potential I and $k_F$ is the Fermi wavelength), the effective action
$S_{\mathrm{eff}}$ is given by

$$S_{\mathrm{eff}}[\phi] = \int_0^{\beta/2} d\tau \int_0^{\beta/2} d\tau' \exp \left(\frac{\phi(\tau) - \phi(\tau')}{\beta} + U(\phi)\right) + S_o[\Phi],$$

$$S_o[\Phi] = -\int_0^{\beta/2} d\tau \int_0^{\beta/2} d\tau' \beta \phi(\tau) - \phi(\tau'),$$

(2)

In this equation $\beta = 1/k_BT$, $M = C(h/2e)^2$ is the mass ($C$
the capacitance of the junction) and the potential $U(\phi)$ is
described by

$$U(\phi) = \frac{\hbar}{2e} \int_0^\beta d\tau I_1(\lambda,\phi) - \phi I_{\mathrm{ext}}.$$

(3)

where $I_1$ is the Josephson current and $I_{\mathrm{ext}}$ is the external
bias current. The dissipation kernel $\alpha(\tau)$ is related to the
quasiparticle current $I_{qp}$ under constant bias voltage $V$ by

$$\alpha(\tau) = \frac{\hbar}{e} \int_0^\infty \frac{d\omega}{2\pi} e^{-\omega t} I_{qp}(V = \hbar\omega/e),$$

(4)
at zero temperature.

Below, we will derive the effective action for the three
types of d-wave junction ($d_0/d_0$, $d_0/d_{\gamma/4}$ and $d_{\gamma/4}/d_{\gamma/4}$) in
order to investigate the effect of the nodal-quasiparticles and
ZES on MQT. In the case of the $d_0/d_0$ junction, node-to-
node quasiparticle tunnelling can contribute to the dissipative
quasiparticle current. However, ZES are completely absent.
These behaviours are qualitatively identical with that for c-axis
Josephson junctions [2,3]. On the other hand, in the case of
$\bar{d}_0/d_{\gamma/4}$ and $d_{\gamma/4}/d_{\gamma/4}$ junctions, ZES are formed around the
surface of the superconductor $d_{\gamma/4}$. Therefore node-to-ZES
($d_0/d_{\gamma/4}$ and ZES-to-ZES ($d_{\gamma/4}/d_{\gamma/4}$) quasiparticle tunnelling
becomes possible.

First, we will calculate the potential energy $U$ in the
effective action (2). As mentioned above, $U$ can be described by
the Josephson current through the junction in the high
barrier limit. In order to obtain the Josephson current we start
from the Bogoliubov–de Gennes (BdG) equation [19]:

$$\int \exp[A(\tau') - A(\tau)] - \frac{\partial A(\tau')}{\partial A(\tau)} = E(\tau'),$$

(5)

where $\phi$ is the phase of the order parameter, $u(\tau)$ is the
amplitude of the wavefunction for the electron (hole)-like
quasiparticle, $h(\tau) = -\hbar^2 \nabla^2 / 2m - \mu + w_0(\tau)$, and $\Delta(\tau - \tau') = \Omega^{-1} \sum \Delta_k \exp[i \cdot (\tau - \tau')]$ is the order parameter ($\Omega$
the volume of the superconductor). In the superconductor
regions ($d_0$ and $d_{\gamma/4}$), the BdG equation (5) can be transformed into
the eigenequation

$$\begin{pmatrix} \xi_k & \Delta_k \exp[i \phi] \\ \Delta_k \exp[-i \phi] & -\xi_k \end{pmatrix} \begin{pmatrix} u_k \\ v_k \end{pmatrix} = E \begin{pmatrix} u_k \\ v_k \end{pmatrix},$$

(6)

where $\xi_k = h^2 k^2 / 2m + h^2 \pi^2 / 2m - \mu^2 / \Delta_k^2$
and $\Delta_k = \Delta_0 \cos \theta_k = \Delta_{d_0} \theta_k$ for $d_0$ and $\Delta_0 \sin \theta_k = \Delta_{d_{\gamma/4}} \theta_k$ for $d_{\gamma/4}$, where $\cos \theta_k = k / k_F$. The Andreev reflection coefficient for the electron
(hole)-like quasiparticle $r_{\mathrm{eh}}$ ($r_{\mathrm{he}}$) is calculated by solving
the eigenequation (6) together with the appropriate boundary
conditions. By substituting $r_{\mathrm{eh}}$ ($r_{\mathrm{he}}$) into the formula of the
Josephson current for unconventional superconductors (the
Tanaka–Kashiwaya formula) [19]

$$I_j = \frac{e}{\hbar} \sum \int d\phi \left( \frac{\Delta_+}{\Delta_\pm} r_{\mathrm{he}} - \frac{\Delta_-}{\Delta_\pm} r_{\mathrm{eh}} \right),$$

(7)

we can obtain $\varphi$ dependence of the Josephson current. Here
$\Delta_\pm = \Delta_{d_0} / \Delta_0$, $\Omega_\pm = \sqrt{(R_0 \Delta_0)^2 - \Delta_0^2}$, $\Omega_0 = (2n + 1) \pi / \beta h$
is the fermionic Matsubara frequency. In the case of low
temperatures ($\beta^{-1} \ll \Delta_0$) and the high barrier limit ($z_0 \gg 1$), we get

$$I_j(\varphi) \approx \begin{cases} I_1 \sin \varphi & \text{for } d_0/d_0 \\
I_2 \sin 2\varphi & \text{for } d_0/d_{\gamma/4} \\
I_3 \sin \varphi / 2 & \text{for } d_{\gamma/4}/d_{\gamma/4} \end{cases}$$

(8)

where $I_1 \equiv 3\pi \Delta_0 / 10 e R_N$, $I_2 \equiv \pi^2 h \beta \Delta_0^2 / 35 e^3 N_c$, $R_N^2$ and
$I_3 \equiv 3 \pi z_0 \Delta_0 / e R_N$ ($R_N = 3 \pi \Delta_0^2 / 2 e^2 N_c$ is the normal state
resistance of the junction and $N_c$ is the number of channels at the
Fermi energy).
By substituting the Josephson current into equation (3), we can obtain the analytical expression of the potential $U$, i.e.,

$$
U(\phi) \approx \begin{cases} 
-\frac{\hbar I_1}{2e} \left( \cos \phi + \frac{I_{\text{int}}}{I_2} \phi \right) & \text{for } d_0/d_0 \\
-\frac{\hbar I_1}{4e} \left( -\cos 2\phi + \frac{2I_{\text{int}}}{I_2} \phi \right) & \text{for } d_0/d_{\pi/4} \\
-\frac{\hbar I_1}{e} \left( \cos \phi + \frac{1}{2} \frac{I_{\text{int}}}{I_2} \phi \right) & \text{for } d_{\pi/4}/d_{\pi/4}.
\end{cases}
$$

(9)

As in the case of the s-wave and the c-axis junctions [2], $U$ can be expressed as a tilted washboard potential (see figure 1(d)).

3. Dissipation due to nodal-quasiparticles and ZES

Next we will calculate the dissipation kernel $\alpha(\tau)$ in the effective action (2). In the high barrier limit, the quasiparticle current is given by [19]

$$
I_{qp}(V) = \frac{2e}{h} \sum_{\sigma} [n_{\sigma}(E, \theta) + n_{\sigma}(E + eV, \theta)]
\times \int_{-\infty}^{\infty} \left[ f(E) - f(E + eV) \right] dE,
$$

(10)

where $\tau_s \approx \cos \theta/\pi$ is the transmission coefficient of the barrier $I$, $N_{\sigma(E, \theta)}$ is the quasiparticle surface density of states ($L = d_0$ and $R = d_0$ or $d_{\pi/4}$) and $f(E)$ is the Fermi–Dirac distribution function. The explicit expression of the surface density of states was obtained by Matsumoto and Shiba [21]. In the case of $d_0$, there are no ZES. Therefore the angle $\theta$ dependence of $N_{\sigma}(E, \theta)$ is the same as the bulk and is given by

$$
N_{\sigma}(E, \theta) = \frac{1}{\sqrt{E^2 - \Delta_{\sigma}(\theta)^2}}.
$$

(11)

On the other hand, $N_{\sigma_{\pi/4}}(E, \theta)$ is given by

$$
N_{\sigma_{\pi/4}}(E, \theta) = \frac{1}{\sqrt{E^2 - \Delta_{\sigma_{\pi/4}}(\theta)^2}} + \pi |\Delta_{\sigma_{\pi/4}}(\theta)| \delta(E).
$$

(12)

The delta function peak at $E = 0$ corresponds to ZES. Because of the bound state at $E = 0$, the quasiparticle current for the $d_0/d_{\pi/4}$ and $d_{\pi/4}/d_{\pi/4}$ junctions is drastically different from that for the $d_0/d_0$ junctions in which no ZES are formed. By substituting equations (11) and (12) into equation (10), we can obtain the analytical expression of the quasiparticle current $I_{qp}(V)$. In the limit of low bias voltages ($eV \ll \Delta_0$) and low temperatures ($B^{-1} \ll \Delta_0$), this can be approximated as

$$
I_{qp}(V) \approx \begin{cases} 
\frac{3\pi^2}{2} \frac{eV^2}{\sqrt{2} \Delta_0 R_N} & \text{for } d_0/d_0 \\
3\frac{\pi^2}{2} \frac{V}{\sqrt{2} \Delta_0 R_N} & \text{for } d_0/d_{\pi/4} \\
\frac{2\pi^2}{35} \frac{(\Delta_0/\epsilon)^2}{R_N} & \text{for } d_{\pi/4}/d_{\pi/4}.
\end{cases}
$$

(13)

In the calculation of $I_{qp}$ for the $d_{\pi/4}/d_{\pi/4}$ junctions, we have replaced the ZES delta function $\delta(E)$ in equation (12) with the Lorentz type function, i.e.,

$$
\delta(E) \to \frac{1}{\pi} \frac{\epsilon}{\epsilon^2 + E^2}.
$$

(14)

in order to avoid a mathematical difficulty and model the real systems (which include, for example, disorder and many-body effects). It is apparent from equation (13) that, in the case of $d_0/d_0$ junctions, the dissipation is of the super-Ohmic type as in the case of c-axis junctions [2]. This can be attributed to the effect of the node-to-node quasiparticle tunnelling. Thus the quasiparticle dissipation is very weak. On the other hand, in the case of $d_0/d_{\pi/4}$ junctions, node-to-ZES quasiparticle tunnelling gives the Ohmic dissipation which is similar to that in normal junctions [20]. Therefore the dissipation for $d_0/d_{\pi/4}$ junctions is stronger than that for $d_0/d_0$ junctions. Moreover, in the case of $d_{\pi/4}/d_{\pi/4}$ junctions, ZES-to-ZES quasiparticle tunnelling dominates the quasiparticle dissipation. The broadening of the ZES peak $\epsilon$ is typically one order of magnitude smaller than $\Delta_0$. Therefore, due to the prefactor $(\Delta_0/\epsilon)^2$ in equation (12), the quasiparticle dissipation in the $d_{\pi/4}/d_{\pi/4}$ junctions becomes enormously stronger than that for the $d_0/d_0$ and $d_0/d_{\pi/4}$ cases.

From equation (4), the asymptotic form of the dissipation kernel is given by

$$
\alpha(\tau) \approx \begin{cases} 
\frac{3\pi^2}{2} \frac{eV^2}{\sqrt{2} \Delta_0 R_N} & \text{for } d_0/d_0 \\
3\frac{\pi^2}{2} \frac{V}{\sqrt{2} \Delta_0 R_N} & \text{for } d_0/d_{\pi/4} \\
\frac{2\pi^2}{35} \frac{(\Delta_0/\epsilon)^2}{R_N} & \text{for } d_{\pi/4}/d_{\pi/4}.
\end{cases}
$$

(15)

The result for the $d_0/d_0$ junction is in agreement with previous works [4, 5, 22, 23].

4. MQT in in-plane d-wave junctions

Let us move to calculation of the MQT rate $\Gamma$ for d-wave Josephson junctions based on the standard Caldeira and Leggett theory [24]. At zero temperature $\Gamma$ is given by

$$
\Gamma \approx A \exp \left( -\frac{S_B}{\hbar} \right),
$$

(16)

where $S_B \equiv S_{\text{eff}}[\phi_B]$ and $\phi_B$ is the bounce solution. Following the procedures above, we obtain the analytical formulae of the MQT rate for in-plane d-wave junctions as

$$
\Gamma \approx \frac{A}{\Gamma_0} \begin{cases} 
\exp \left[ -\left( \frac{c_0}{2} \frac{3\pi}{2} \frac{\hbar \eta}{\Delta_0 R_N} + \frac{18 \delta M}{5 \hbar} \right) \frac{U_0}{M\omega_p} \right] & \text{for } d_0/d_0 \\
\exp \left[ -\left( \frac{3\pi}{2} \frac{\hbar \eta}{\Delta_0 R_N} \right) \frac{\eta(1 - x^2)}{2\sqrt{2\pi}^2} \right] & \text{for } d_0/d_{\pi/4} \\
\exp \left[ -\frac{2\pi^2}{35} \frac{(\Delta_0/\epsilon)^2}{R_N} \eta(1 - x^2) \right] & \text{for } d_{\pi/4}/d_{\pi/4},
\end{cases}
$$

(17)

where $c_0 = \int_0^{\infty} dy y^4 \ln(1 + 1/y^2)/\sinh^2(\pi y) \approx 0.0135$, $\zeta(3)$ is the Riemann zeta function, $\eta \equiv R_Q/R_N$ is the dissipation parameter, $U_0$ is the barrier height of the potential $U$, $\omega_p$ is the
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Josephson plasma frequency, $x = I_{ph}/I_i$ ($i = 1, 2, 3$), and
\[
\Gamma_0 = 12\omega_p \frac{3U_b}{2\pi\hbar\omega_p} \exp\left(-\frac{36U_b}{5\hbar\omega_p}\right) \tag{18}
\]
is the MQT rate without the dissipation. In equation (17)
\[
\delta M = \frac{3}{24\sqrt{2}} \frac{\hbar^2}{\Delta_0} \int_{-1}^{1} dy \frac{1 + y}{\sqrt{1 - y^2}} \int_{0}^{\pi/2} dz \ z^2 K_1(z|y|)^2
\]
is the renormalized mass due to the high frequency components ($\omega \gg \omega_p$) of the quasiparticle dissipation.

In order to compare the influence of ZES and the nodal-quasiparticle on MQT more clearly, we will estimate the MQT rate (17) numerically. For a mesoscopic bi-crystal YBCO Josephson junction [25] ($\Delta_0 = 17.8$ meV, $C = 20 \times 10^{-15}$ F, $R_N = 100 \Omega$, $x = 0.95$), the MQT rate is estimated as
\[
\frac{\Gamma}{\Gamma_0} \approx \begin{cases} 
83\% & \text{for } d_0/d_0 \\
25\% & \text{for } d_0/d_{3/4} \\
0\% & \text{for } d_{3/4}/d_{3/4}
\end{cases} \tag{20}
\]
As expected, the node-to-ZES and ZES-to-ZES quasiparticle tunnelling in $d_0/d_0$ and $d_{3/4}/d_{3/4}$ junctions gives strong suppression of the MQT rate compared with the $d_0/d_0$ junction cases. Moreover in the $d_{3/4}/d_{3/4}$ cases, MQT is almost completely depressed.

5. Summary

In conclusion, MQT in in-plane high-$T_c$ superconductors has been theoretically investigated and the formulae for the MQT rate, which can be used to analyse experiments, have been analytically obtained. Node-to-node quasiparticle tunnelling in $d_0/d_0$ junctions gives rise to weak super-Ohmic dissipation as in the case of $c$-axis junctions [2]. For $d_0/d_{3/4}$ junctions, on the other hand, we find that node-to-ZES quasiparticle tunnelling leads to Ohmic dissipation. Moreover, in the case of $d_{3/4}/d_{3/4}$ junctions, ZES-ZES quasiparticle tunnelling gives very strong Ohmic dissipation so MQT is drastically suppressed.

In this paper we have only considered the case of a high barrier limit ($\varepsilon_0 \gg 1$). In low barrier cases, the ZES become split into two finite energy Andreev levels due to ZES resonance [19]. Moreover, the energy of the split Andreev levels depends on the phase difference $\phi$ and the influence of the proximity effect becomes more important. To take into account such effects, the present approach should be considerably modified. This issue will be investigated in future articles.

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