Model-based Estimation of Elastic Moduli by Local Displacement Observation of an Elastic Body

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Abstract  As the pathological characteristics of living tissue generally correlate with tissue stiffness, techniques such as ultrasound elastography play an important role in the medical field. One issue with elastography is the limited area of measurement. To determine the spatial distribution of elastic moduli, model-based estimation methods have been developed. However, these methods can only be applied to the observable area, and expanding the estimation area remains problematic. This paper introduces a method to estimate the spatial distribution of elastic moduli over the entire elastic body from locally observed deformation patterns. The reconstruction of elasticity from multiple deformation patterns is formulated as a minimization problem based on the sparseness of the gradient of tissue elasticity. Based on simulation experiments investigating the performance of the proposed method, we confirm that spatial variations in elasticity are effectively estimated and the area of elasticity reconstruction is extended.

Keywords: elastic modulus estimation, finite element modeling, sparse signal reconstruction.

1. Introduction

Tissue elasticity is an important characteristic in the medical field, because pathological changes in tissue generally correlate with changes in tissue stiffness. Most tumors or lesions become harder than normal tissue as the disease progresses. By measuring tissue elasticity, it is possible to provide a quantitative diagnosis based on the mechanical properties of tissue. For this purpose, elastography [1] was developed in the 1990s, and this approach is replacing palpations traditionally performed by clinicians. Ultrasound and magnetic resonance imaging (MRI) have been widely used for elastography [2, 3], particularly for diagnosis in the medical field.

To date, two major approaches based on different measured physical quantities have been reported. One approach is strain imaging [4] that determines stiffness by analyzing the strain in tissue under stress and is basically qualitative. This method has the advantages of simplicity and real-time measurements with high spatial resolution. However, accurate measurements of tissue stiffness are difficult because the magnitude of the applied external force is not proportional to the biological tissue strain. Moreover, the use of focused acoustic radiation involves the application of external excitation to human bodies by strong convergent ultrasonic waves. The second approach is shear-wave imaging [3, 5] that provides a stiffness image by quantifying the propagation speed of a shear wave. In this method, it is possible to evaluate the elastic modulus quantitatively using the shear wave velocity. Both strain and shear-wave speed images provide information related to underlying tissue stiffness. However, it is difficult to measure stiffness deep inside the body using elastography techniques. Therefore, a method that can measure deep-layer tissue stiffness in a noninvasive manner is required.

To determine the spatial distribution of elastic moduli, model-based estimation methods have also gained attention. In [6], the elastic modulus of tissue was estimated based on the Navier–Stokes equation by solving an optimization problem using a finite element (FE) model. Mesh adaptation techniques have also been investigated to improve the accuracy of tissue-elasticity reconstruction [7]. However, these methods can only be applied to an observable area, and expanding the estimation area remains an issue. The models used in medical virtual reality organ simulators [8, 9] require information about the elasticity of biological tissues. If such data could be obtained during surgery without additional hardware or time/cost burdens, various areas of biomed-
tical research and intraoperative support systems [10, 11] would benefit.

This paper proposes a method to estimate the spatial distribution of elastic moduli over the entire elastic body from locally observed deformation patterns. As the proposed method is a model-based estimation, the elastic modulus is estimated under the condition that the shape of the target is known. The reconstruction of elasticity from multiple deformation patterns is formulated as a minimization problem based on the sparseness of the gradient of tissue elasticity. This implementation makes it possible to extend the area of elasticity reconstruction. Additionally, a new application of vision-based elastography during surgery will be feasible, because the internal elastic properties may be reconstructed from partial observations of deformed surfaces.

We conducted simulation experiments to investigate the performance of the proposed method. The results show that our approach can estimate spatial variations in Young’s modulus over the entire elastic body, including unobservable areas. The basic concept has been reported previously [12]. This paper presents new experimental results to evaluate the estimation performance considering the influence of the model parameters. Details of the methods and experiments are described in Sections 2 and 3, respectively.

2. Method

As outlined in Fig. 1, the model assumes that the whole shape of the target elastic body is given, and that a suitable tetrahedral mesh can be configured. As computed tomography (CT) imaging or MRI is available, this configuration is clinically acceptable. We also assume that the elastic body has both observable and unobservable areas, and the positions of contact points are given. This situation represents an external force being applied to a specific contact point on the elastic body, but the magnitude of the force is unknown because measurement of external forces is difficult in clinical situations such as during diagnosis or surgery. We first obtain displacement information in the observable area, and then estimate Young’s moduli by solving a minimization problem based on the displacements of observed vertices. In the next two subsections, the problem is described and defined within the context of linear FE theory, and details of the proposed method are subsequently introduced.

2.1 Problem Definition

The FE model configured for the experiments introduces an array of \( m \) observed vertices out of \( n \) total vertices, and an \( l \)-dimensional vector \( E = [E_0, E_1, \ldots, E_{l-1}] \in \mathbb{R}^l \) of Young’s moduli. All tetrahedral elements of the FE model are divided into \( l \) local regions, and a uniform Young’s modulus \( E_i (k = 0, \ldots, l - 1) \) is assigned to each region. Figure 2 shows a simple FE model with 216 tetrahedral elements, which are divided into \( 6 \times 6 \) rectangular regions. Since each region has a uniform stiffness, this model represents 36-dimensional spatially distributed Young’s moduli. We also define \( f \in \mathbb{R}^{3m} \) as the external force vectors applied to the vertices, \( u \in \mathbb{R}^{3m} \) as the displacement of the vertices, and \( K \in \mathbb{R}^{3m \times 3m} \) as the stiffness matrix. Here, the Young’s moduli \( E \) are part of the set of parameters constituting \( K \). Linear FE theory uses the following relation:

\[
\mathbf{f} = \mathbf{Ku} \iff \mathbf{u} = \mathbf{K}^{-1}\mathbf{f} = \mathbf{Lu}.
\]

We assume that the displacements of the observed vertices, the observable part of \( \mathbf{u} \), and the positions of the contact points of the external force are given. The estimation target is the Young’s moduli vector \( E \) and an external force vector \( f \). Although Poisson’s ratio and the fixed conditions of the elastic body may be estimated in the proposed framework, 0.4 is homogeneously assigned to Poisson’s ratio of all tetrahedral elements to simplify the optimization problem. Also, some vertices are preconditionally fixed in this paper. As there are various methods of measuring the displacement, the vertex dis-
placements in an observable area are available as the given parameters. For example, surface deformations of elastic bodies based on camera images [13] and internal deformations based on ultrasonography [14] have been reported. In addition, assuming that the elastic body is an organ, the force exerted by forceps or the transducer of a probe can be considered as an external force. Therefore, the area to which the external force is applied, such as the contact points, is clearly defined.

2.2 Estimation Method

By labeling all vertices of the FE model as observed vertices or unobservable ones, equation (1) can be rewritten as

\[
\begin{pmatrix}
u_o \\
u_i
\end{pmatrix}
= \begin{pmatrix}
L_{oo} & L_{oi} \\
L_{io} & L_{ii}
\end{pmatrix}
\begin{pmatrix}
f_o \\
f_i
\end{pmatrix},
\]

(2)

where \(u_o \in \mathbb{R}^{3m}\) is a given vector quantity and \(u_i\) is an unknown vector. Extracting only the terms related to \(u_o\) yields

\[
u_o = (L_{oo} - L_{oi}) \begin{pmatrix}
f_o \\
f_i
\end{pmatrix} = L_o(E)f.
\]

(3)

The displacement of the observable vertices can be derived from equation (3). The estimation method now involves updating \(E\) and \(f\) from their initial values to those for which \((E,f)\) is closest to the observed displacement \(u_o\). Overall, the optimum elastic modulus distribution and the direction and magnitude of the external force are determined by solving the optimization problem so that the deformed state of the elastic body calculated from equation (3) becomes closest to the previously observed deformed state.

\[
E^* = \arg\min_{E,f} J(E,f) = \arg\min_{E,f} \|u_o - u'_o\|_2.
\]

(4)

2.2.1 Reconstruction from multiple deformation patterns

To extend the proposed method, we focus on the number of deformation states observed in advance. In regard to the number of observed patterns, the more patterns we observe that are associated with the deformed shape of the elastic body, the more detail about the physical condition can be developed. Therefore, the estimation accuracy is expected to improve if we can solve the minimization problem for multiple observed displacements \(u_o\).

In observing multiple patterns of deformed shapes of the FE model, the pre-observed displacement matrix \(U_o \in \mathbb{R}^{3m \times \omega}\) is defined as

\[
U_o = [u_{1o} u_{2o} \cdots u_{\omega o}],
\]

where \(\omega\) represents the number of patterns of the deformed shape of the FE model. By considering \(U_o\), equation (4) can be rewritten as

\[
E^* = \arg\min_{E,f} J(E,f) = \arg\min_{E,f} \|U_o - U'_o\|_F.
\]

(6)

where \(F\) denotes the Frobenius norm

\[
\|A\|_F = \sqrt{\sum_{i=1}^m \sum_{j=1}^n |a_{ij}|^2} = \sqrt{tr(A^tA)}.
\]

(7)

2.2.2 Sparseness of the gradient elasticity

To improve the estimation accuracy, we also focus on the sparseness of the gradient of tissue elasticity. Sparseness refers to the property of a matrix in which the elements are mostly zero. As tumors or lesions are harder than normal tissue and are generally confined to a small area, only a small region of the elastic body is hard; that is, the gradient of the tissue elasticity is sparse. By introducing the notion of sparsity, equation (6) can be rewritten as

\[
E^* = \arg\min_{E,f} J(E,f) = \arg\min_{E,f} \|U_o - U'_o\|_F + \lambda \|\Delta E\|_1,
\]

(8)

where \(\lambda\) is a parameter to control the sparseness of the gradient of elasticity. By considering sparseness, it is possible to add a constraint condition whereby the elasticity in most areas of the elastic body is uniform, so that improvements in estimation accuracy can be expected. We use the Covariance Matrix Adaptation Evolution Strategy (CMA-ES) [15] to solve the minimization problem.

3. Experiments and Results

Experiments were conducted to verify how accurately the proposed method estimates the spatial variation of elastic moduli and to determine whether the area of elasticity reconstruction can be extended. We implemented the proposed method using Visual C/C++ and conducted simulation experiments on a computer running Windows 10 Professional with an Intel Core i7-6700K and 16 GB of memory.

3.1 Experimental settings

We used a simple plate model in the simulation experiments to investigate the performance of the proposed method. The FE model has 98 vertices and 216 tetrahedral elements. As shown in Fig. 3, 14 fixed points (red) and 3 contact points (green) were configured. We assumed that an external force was exerted on the surface of an organ by forceps or by the transducer of a probe. Therefore, we regarded the magnitude and direction of the external force to be uniform over all neighboring vertices. At the contact points, a force of 9.8 N was directed along (a) the +y axis, (b) the −y axis, and (c) the +x axis.

The FE model was divided into 6 × 6 equal regions (Fig. 2). Each region had an identical rectangular shape composed of six tetrahedral elements. We supposed each region to be homogeneous; in other words, a minimal unit to represent Young’s modulus. Therefore, the pur-
pose of this experiment was to solve an optimization problem of 36-dimensional Young’s moduli $E_k$ ($k = 0, \ldots, 35$) using the proposed estimation framework. In this experimental design, we planned to perform elastic modulus estimation experiments using real objects. Therefore, we conducted the simulation experiments under the same conditions as in the models created by 3D printers. As shown in Fig. 3, Young’s moduli for the red and white areas were set at 35.8 kPa and 117.6 kPa, respectively. It was assumed that hard areas were tumors or lesions, and other areas were normal tissue. In the comparative experiments on sparseness, we used FE models with different ratios of hard portion in the elastic body.

In this method, the elastic modulus was estimated based on local observations. Therefore, it was necessary to choose which vertices of the FE model to observe. In the experiments, the observed vertices were set as shown in Fig. 4. The number of observed vertices was increased by five sequentially along the $-x$ axis on the upper surface, and the numbers of observed vertices were (a) 5, (b) 10, (c) 15, (d) 20, and (e) 25. For the initial values, all elements of $E$ were set at 1.0 and all elements of $f$ at 0.0.

### 3.2 Observation of multiple deformation patterns

We now describe three points. First, we applied force-$i$ ($i = 1, 2, 3$) to the FE model and estimated the spatial variation of Young’s moduli based only on $u_{io}$, where $u_{io}$ is the displacement corresponding to force-$i$. Next, to improve the estimation accuracy, we focused on the number of deformation patterns that were pre-observed and the sparseness of the gradient of elasticity.

The estimation results were evaluated using the root mean square error (RMSE) of the elastic moduli. When the original Young’s modulus vector is denoted as $E$ and the estimated Young’s modulus vector as $E'$, the RMSE is given by

$$\text{RMSE} = \sqrt{\frac{1}{7} \sum_{i=0}^{i=4} (E'_i - E_i)^2} \quad (9)$$

Each simulation experiment was repeated 30 times, and the $E'$ that gave the smallest value of $J(E', f)$ in equation (8) was taken as the representative vector.

In this section, we present results of the relationship between the number of observed vertices and the estimation accuracy, as well as the influence of the number of preliminary deformation observations on the estimation accuracy. First, force-1 was applied to the FE model as shown in Fig. 3, and the elastic modulus distribution was estimated. At this time, the sparsity coefficient was set at 0. The estimation results under this experimental condition are shown in Fig. 5 (1-Pattern Observation). When force-1 was applied to the FE model, the RMSE of Young’s modulus was 431.4 kPa when five vertices were
observed. In this case, the poor accuracy of elastic modulus estimation was caused by the model becoming stuck around a local minimum solution. This was a result of the small number of observation vertices, and the estimation of external force was not successful. As the number of observed vertices increased to 20, the RMSE decreased to 6.0 kPa. The RMSE tended to decrease as the number of observed vertices increased. In the case of force-1, the elastic modulus distribution was estimated with sufficient accuracy when 20 vertices were observed.

Next, we pre-observed various displacements: (a) $u_{1o}$, (b) $u_{2o}$, and (c) $u_{1o}$, $u_{2o}$, and $u_{3o}$ as shown in Fig. 3, and set the coefficient $\lambda$ in (6) at 0.0 (non-sparsity). The results of observing (a) one, (b) two, and (c) three patterns are shown in Fig. 5. When five vertices were observed, the RMSEs were 431.4, 464.1 and 305.9 kPa, respectively. As the number of observed vertices increased to 10, the RMSEs decreased to 169.3, 90.0, and 23.2 kPa, respectively. Therefore, as the number of observed deformation states increased, the estimation accuracy tended to improve. When $\lambda = 0$, highly accurate estimates (RMSE = 0.6 kPa) required 20 vertices to be observed.

3.3 Sparseness of elasticity gradient

We then verified the relationship between the sparsity coefficient $\lambda$ and the estimation accuracy. Under the assumption that the target to be estimated is an organ, we considered the situation in which a tumor constitutes a small part of the organ and is locally hard. In this case, the gradient of elasticity can be assumed to be sparse. In this section, we consider two validation items. The first is whether the estimation accuracy improves when the elasticity change is assumed to be sparse. The second is the tendency of change in the effective range of the sparsity coefficient with change in the ratio of the hard part in the elastic body.

As an example of the results, the output with a sparsity coefficient of 0.35 is shown in Fig. 6. By comparing these results with the non-sparse case shown in Fig. 5 ($\lambda = 0$), the former generally gave smaller RMSEs and better estimation accuracy. When the sparsity coefficient was 0.35, the RMSEs were 15.5, 9.6, and 0.8 kPa, respectively, when 10 vertices were observed. In this research, we propose that the elasticity is estimated by applying sparse modeling to the FE method. From this result, the estimation accuracy improves when the sparsity coefficient is non-zero. Therefore, we consider that the proposed method effectively improves the estimation accuracy. However, it is basically impossible to derive the optimum value of the sparsity coefficient from prior information. In practical situations, there is a need to change the sparsity coefficient step by step, and explore which value gives the best output.

Next, the sparsity coefficient was varied from 0–0.5 in intervals of 0.05 (11 patterns), and the ratio of the hard portion in the elastic body was set at five different values (see Fig. 7). Figure 8 shows the results for 10, 15, and 20 observed vertices. The experimental conditions were as shown in Figs. 3 and 4. In this experiment, we observed a tendency of decreased improvement of accuracy under the sparseness assumption when the ratio of the hard portion in the elastic body exceeded 25%.

3.4 Elastic modulus estimation for three-dimensional plate model

In this section, we examine the elastic modulus estimation for a three-dimensional plate model with 196 vertices and 648 tetrahedral elements in which the plate model of Fig. 3 is stacked in three stages (Fig. 9). This FE model was divided into $6 \times 6 \times 3$ rectangular regions that represented distribution of Young’s moduli. Whereas the previous experiments were conducted for models with two elastic modulus values, the three-dimensional model gave an elasticity estimation composed of five values. The elastic modulus values were set as shown in Fig. 9. We observed 25 points on the upper surface of the elastic body. An external force of 9.8 N was applied to three vertices of the lower surface of the model in the (a) –y axis direction, (b) +y axis direction, and (c) +x axis direction, in the same manner as in Fig. 3.

The results shown in Fig. 10 have an RMSE of 0.45 kPa. The index of the Young’s modulus parameter (shown on the horizontal axis in Fig. 10) can be determined in ascending order of the x, y, and z axes. These results suggest that the proposed method can be applied to an elastic body model with plural stiffness. Under this experimental condition, elastic modulus estimation can be achieved with high accuracy based on local displace-
ment observation.

4. Discussion

We have proposed a method to estimate the elastic modulus distribution of an elastic body based only on local displacement observations, and demonstrated that our method can effectively extend the estimated area. As shown in Fig. 10, by focusing on the sparsity of the elasticity gradient and the number of preliminary observations, it was possible to reconstruct the elastic modulus distribution with high accuracy. Figure 5 shows the influence of the number of preliminary observations on the estimation accuracy. These results verify that the RMSE tends to decrease as the number of observed vertices increases and as the number of observed deformation states increases. Therefore, in our method, it is desirable to increase the number of deformed states of elastic bodies.

Fig. 7 Ratios of the hard portion in the elastic body: (a) 2.8%, (b) 11.1%, (c) 16.7%, (d) 25.0%, and (e) 44.4%.

Fig. 8 RMSE color map diagram showing the relationship between the sparse coefficient and the ratio of the hard portion. The numbers of observed vertices are (i) 10, (ii) 15, and (iii) 20. The ratios of hard portion are (a) 2.8%, (b) 11.1%, (c) 16.7%, (d) 25.0%, and (e) 44.4%.

Fig. 9 Three-dimensional plate model. Blue area: 17.9 kPa, light blue area: 53.6 kPa, light red area: 71.5 kPa, red area: 89.4 kPa, white area: 35.8 kPa. Red dots: fixed points, blue dots: free points.

Fig. 10 Estimation result for 25 vertices observed when the number of pre-observations is 3 and the sparsity coefficient $\lambda = 0.2$. The FE model is equally divided into 108 areas, and has an elastic modulus distribution composed of 108 parameters.
that are observed beforehand. This indicates that the estimation accuracy may be affected by the external force conditions. Therefore, it is necessary to investigate the influence of the external force on the estimation. However, in this experiment, estimations were based solely on three deformation patterns, and we have not yet verified what kind of deformation is most effective and how many deformed state observations are necessary. Therefore, it is important to perform experiments to verify these relationships.

The estimation results under the assumption of sparsity (sparsity coefficient value of 0.35) are shown in Fig. 6, and those under an assumption of non-sparsity (sparsity coefficient = 0.0) are shown in Fig. 5. Figure 8 shows the RMSE as a color map diagram of the relationship between the sparsity coefficient and the ratio of the hard portion in the elastic body. Comparing the results under sparse and non-sparse assumptions, the estimation accuracy tends to improve when the gradient of elasticity is considered to be sparse. Therefore, introducing sparsity into the proposed method is effective. However, it is difficult to derive the optimum sparsity coefficient based on prior information. There is a need to change the sparsity coefficient step by step, and explore the value that gives the best output. Therefore, as shown in Fig. 8 after observing 20 vertices, when the effective range of the sparsity coefficient is wide, the method proposed in this research can be of practical use.

The numerical experiments performed in this study were limited to a simple elastic object. To further develop the proposed framework for clinical application, more experiments simulating real organs are needed. However, we have not yet conducted experiments on actual organ models with complex shapes, and verification has not been extended to more realistic conditions derived from human bodies. We consider that positional relationships between observed points, contact points, and fixed points are one of the major factors that influence the accuracy of estimation. For example, in cases that the hard region is far from the observed points and the contact point is close to the observed points, the estimation accuracy may be worse because the observed point is little affected by the external force. However, even under such conditions, we consider that increase of observation patterns will reduce estimation errors. To investigate the relationship between 3D shape, boundary conditions and estimation accuracy is interesting and will be our future work.

In the current algorithm design, elasticity estimation depends on the boundaries of the pre-conditioned local regions. In Fig. 2, the elastic model is divided into 36 rectangular regions with equal shapes. In Fig. 9, the elastic model is composed of 108 (= 6 × 6 × 3) regions, and each region is assumed to be homogeneous. This means that the spatial resolution of Young’s moduli estimation is restricted to the dimensions and the boundaries of the pre-conditioned regions. This low-dimensional setup was conducted because the CMA-ES algorithm works stably under around 100 dimensions of parameters in our optimization problem. In actual situations, a greater spatial resolution is needed to represent stiffness distribution of organs. However, to increase dimensions of parameters takes more computation time, and makes the optimization process unstable. Therefore, we are developing a method to estimate the elastic modulus without using information of the element boundaries of the estimation parameters. In future work, we will undertake the challenge of conducting experiments on models with complicated shapes, such as organs, and examine the proposed method from a more practical point of view.

Furthermore, we plan to conduct an experiment using the measured displacements of real elastic bodies. 3D soft printer models and real organ models from animals are available [16]. It is inevitable that noise will appear in the measurement data when performing experiments using real objects. In this study, we conducted simulation experiments to purely investigate the performance on the concept of sparse elasticity reconstruction. To examine the robustness of the proposed framework, we plan simulation experiments in which the displacement vector contains noise, as future work. Based on the results, we will consider reduction of the influence of noise on elastic modulus estimation. Moreover, although a linear FE method was used in this study, the deformation of organs is nonlinear. To simulate the physical properties of real organs or large deformations, a nonlinear elastic body model should be considered. We believe it is possible to apply nonlinear models such as corotational FEM [17] to the proposed method in the deformation calculations. In future work, we will develop an estimation algorithm for objects that exhibit nonlinear deformation.

5. Conclusion
We have developed a calculation method to estimate the elastic modulus of an extended area of tissue. In addition, we focused on the number of pre-observed deformation patterns and the sparseness of the gradient of tissue elasticity to improve the estimation accuracy. As a result, the proposed method of estimation offers improved accuracy and is very effective. In conclusion, we confirmed that the proposed method can estimate spatial variations in the elastic moduli and extend the area of elasticity reconstruction. In future work, we will use more complicated shape models to conduct simulation experiments and perform experiments using real-world
objects.

Acknowledgment

This research was supported by a JSPS Grant-in-Aid for Scientific Research (B) (Grant No. 15H03032). We thank Stuart Jenkinson, PhD, from Edanz Group (www.edanzediting.com/ac) for editing a draft of this manuscript.

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