High-Multiplicity Processes
(Theoretical status)

J.Manjavidze, A.Sissakian
JINR, Dubna, Russia

Abstract

We wish to demonstrate that investigation of asymptotically high multiplicity (AHM) hadron
reactions may solve, or at least clear up, a number of problems unsolvable by other ways. We would
lean upon the idea: (i) the entropy is proportional to multiplicity and, by this reason, at the AHM
domain one may expect the equilibrium final state and (ii) the high-multiplicity processes becomes
hard. Last one means that the nonperturbative corrections are frozen during this processes and the
QCD predictions are available for them.

1 Introduction

The interest to multiparticle processes noticeably falls down during last decades and this trend is un-
derstandable: (a) the multiparticle processes are too complicated because of large number of involved
degrees of freedom and (b) there is not quantitative hadrons theory to describe the nonperturbative effect
of color charge confinement.

It may be surprising at first glance that the asymptotically high multiplicity (AHM) processes are
simple at least from theoretical point of view. Nevertheless this is so and our aim is to demonstrate
this suggestion. We will consider also the simplest idea of the trigger for AHM final state.

We wish to note firstly that AHM states are equilibrium since the corresponding entropy reach a
maximum in the AHM region. Then, a comparatively small number of simplest parameters is needed
for their description because of the Bogolyubov’s correlations relaxation principle. Note also that in
this case, following the ergodic hypothesis, one may restricted by event-by-event measurements, i.e. it is
sufficient to observe small number of events in the AHM domain. The cross sections in the AHM domain
are assumed extremely small, see e.g. and the last note seems important from experimental point of
view.

Besides, the AHM processes becomes hard and we may neglect (with exponential accuracy) the
background low-$p_t$ nonperturbative channel of hadrons production. Intuitively this is evident noting that
in all soft Regge-like theories, with interaction radii of hadrons $\sim \sqrt{\ln s}$, one can not ‘stir’ in the disk of
area $\sim \ln s$ arbitrary number of partons, since the high $p_t$ are cutted off. Then the Regge-like theories
are unable to describe the multiplicities $n > \ln^2 s$, if each parton in the disk is a source of $\sim \ln s$ hadrons.
At high energies the AHM domain is sufficiently wide $(\ln^2 s << n_{\text{max}} \sim \sqrt{s})$ and, therefore, should be
‘occupied’ by hard processes. Indeed, the mean transfer momentum of created particle grows with $n_c$.

One can say by this way that the confinement forces are frozen in the AHM processes and one can
expect that the pQCD predictions becomes highly precise, forgetting on a moment the infrared (low-$x$)
problem of QCD. We will return to this problem later.

So, the interest to AHM processes seems obvious, since (a) being hard the experimental investigation
of them may help to clear up the structure of fundamental Lagrangian and (b) the theoretical predictions
in this field are more or less evident in the asymptotically free theories. From our point of view the AHM
processes may be considered as a supplement to such popular physical programs as the search of Higgs
bosons, $s$-particles, $CP$ violation, neutrino oscillations, the quark-gluon plasma (QGP) physics, the early
Universe dynamics.

We want to add also that in the AHM domain:
(i) the space density of colored constituents becomes large;

1Permanent address: Inst. of Physics, Tbilisi, Georgia
(ii) the created particles system becomes ‘cold’ and the collective effects becomes transparent;
(iii) the non-perturbative effects becomes essential only at the last stage of the process.

It seems constructive to start investigation of hadron dynamics from AHM domain and, experienced in this field, go to the moderate multiplicities. The to-day experimental status of AHM is following. There is experimental data at TeVatron energies up to \( n_c \approx 5\bar{n}_c(s) \). The cross section \( \sigma_n \approx 10^{-5}\sigma_{\text{tot}} \) at this values of charged particles multiplicity \( n_c \). The events with higher multiplicity are unknown, if the odd cosmic data is not taken into account. The main experimental problem to observe AHM is smallness of corresponding cross sections and impossibility to formulate experimentally the ‘naive’ trigger tagging the AHM events by charged particles multiplicity. We think that this question is very important since otherwise the AHM problem becomes pure academic.

We suggest to reject tagging the event by the multiplicity noting that it is enough to have the ‘cold’ final state in the AHM domain. Then, to extract experimentally the AHM events the calorimetric measurements are enough. Moreover, remembering that the AHM states are equilibrium (or are near the equilibrium), the ‘rough’ measurements of small statistics should be enough. This practically solves the problem of experimental investigation of AHM events. For instance, such modern CERN experiments as ATLAS, CMS have from this point of view good calorimeters. We will return to this question later and bearing in mind such type ‘rough’ experiments we will follow corresponding ideology: formulating the theory we would try to generalize ordinary inclusive approach.

In Sec.2 we will give the physical interpretation of possible asymptotics over \( n \). In Sec.3 the hard Pomeron contribution in AHM domain is discussed. In Sec.4 we will describe the QGP formation signal at AHM. In Sec.5 the possible trigger for AHM is discussed.

## 2 Classification of asymptotics over \( n \)

It is better to start from pure theoretical problem assuming that the incident total CM energy \( E = \sqrt{s} \) is arbitrarily high and to consider asymptotics over the total multiplicity \( n \), assuming nevertheless that \( n \ll n_{\text{max}} = \sqrt{s}/m \), where \( m \approx 0.2 \text{ Gev} \) is the characteristic hadron mass, to exclude the influence of phase space boundaries. Last one means that in the sum

\[
\Xi_{\text{max}}(z, s) = \sum_{n=1}^{n_{\text{max}}} z^n \sigma_n(s)
\]

we should choose the real positive \( z \) so small that upper boundary is not important and we can summing up to infinity:

\[
\Xi(z, s) = \sum_{n=1}^{\infty} z^n \sigma_n(s).
\]

This trick allows introduce classification of \( \sigma_n \) asymptotics counting the singularities of \( \Xi(z, s) \) over \( z \).

So, if \( z_c \) is the solution of equation:

\[
n = z \frac{\partial}{\partial z} \ln \Xi(z, s)
\]

then in the AHM region

\[
\sigma_n \sim e^{-n \ln z_c(n, s)}.
\]

To do further step we will use the connection with statistical physics. By definition

\[
\sigma_n(s) = \int d\omega_n(q) \delta(p_a + p_b - \sum_{i=1}^{n} q_i)|A_n|^2,
\]

where \( A_n \) is the \( a + b \rightarrow (n \ \text{hadrons}) \) transition amplitude and \( d\omega_n(q) \) is the \( n \) particles phase space element. There is well known in the particle physics trick as this \((3n)\)-dimensional integrals may
be calculated. For this purpose one should use the Fourier transformation of the energy-momentum conservation \( \delta \)-function. Then, in the CM frame, if \( n \) is large,

\[
\sigma_n(s) = \int \frac{d\beta}{2\pi} e^{\beta \epsilon} \rho_n(\beta),
\]

(2.5)

where

\[
\rho_n(\beta) = \int d\omega_n(q) \prod_{i=1}^{n} e^{-\beta \varepsilon_i} |A_n|^2
\]

(2.6)

and \( \varepsilon_i \) is the \( i \)-th particles energy.

Obviously this trick is used to avoid the constraints from the energy-momentum conservation \( \delta \)-function. But it have more deep consequence. So, if we consider interacting particles \( a \) and \( b \) in the black-body environment, then we should use the occupation number \( \bar{n}_{ext} \) instead of ‘Boltzmann factor’ \( e^{-\beta \varepsilon_i} \). For bosons

\[
\bar{n}_{ext}(\beta \varepsilon) = \{e^{\beta \varepsilon} - 1\}^{-1}.
\]

(2.7)

In result, replacing \( e^{-\beta \varepsilon} \) on \( \bar{n}_{ext} \),

\[
\Xi(\beta, z) = \sum_n z^n \rho_n(\beta)
\]

(2.8)

would coincide \textit{identically} with big partition function of relativistic statistical physics, where \( \beta \) is the inverse temperature \( 1/T \) and \( z \) is the activity: the chemical potential \( \mu = T \ln z \).

We may use this \( S \)-matrix interpretation of the equilibrium statistical physics and consider \( \Xi(z, s) \) as the energy representation of the partition function. Then, following to Lee and Yang \[9\], \( \Xi(z, s) \) should be regular function of \( z \), for practically arbitrary interaction potentials, in the interior of unite circle. In the frame of natural physical assumption \[3, 10\], using estimation (2.3), one can find that

\[
\sigma_n(s) < O(1/n),
\]

(2.9)

i.e. \( \sigma_n(s) \) should decrease faster then any power of \( 1/n \).

First singularity may locate at \( z = 1 \). It is evident from definition (2.1) that \( \Xi(z, s) \) would be singular at \( z = 1 \) iff

\[
\sigma_n(s) > O(e^{-n}),
\]

(2.10)

i.e. in this case \( \sigma_n(s) \) should decrease slower then any power of \( e^{-n} \). It follows from estimation (2.3), in this case \( z_c(n, s) \) should be the decreasing function of \( n \). Note that such solution, at first glance, impossible. Indeed, by definition (2.1), \( \Xi(z, s) \) should be increasing function of \( z \) since all \( \sigma_n \) are positive. Then the solution of eq.(2.2) should be increasing function of \( n \). But, nevertheless, such possibility exist.

Remember for this purpose connection of \( z \) with chemical potential, \( z_c(n, s) \) defines the work needed for creation of additional particle. So, \( z_c(n, s) \) may be the decreasing function of \( n \) iff the vacuum is unstable and the transition from false, free from colorless particles (or being chiral invariant, etc.), vacuum to the true one means colorless particles (or of the chiral-invariance broken states, etc) creation. Just this case corresponds to the first order phase transition \[3, 10\].

This phenomena describes expansion of the domain of new phase with accelerating expansion of the new phase domains boundary, if the radii of domain is larger then some critical value \[1\]. So, \( z_c(n, s) \) should decrease with \( n \) since \( z \) is conjugate to physical particles number in the domain of new phase.

Following singularity in the equilibrium statistics may locate at \( z = \infty \) only. This is the general conclusion and means, as follows from (2.3), that

\[
\sigma_n(s) < O(e^{-n}).
\]

(2.11)

In this case \( \sigma_n(s) \) should fall down faster then any power of \( e^{-n} \). Note, investigation of the Regge-like theories gives just this prediction \[13\].

But considering the process of particles creation it is too hard restrict ourselves by equilibrium statistics. The final state may be equilibrium, as is expected in the AHM domain, but generally we consider
the process of incident (kinetic) energy dissipation into particles mass. At very high energies having the AHM final state we investigate the process of highly nonequilibrium states relaxation into equilibrium one.

Such processes have interesting property readily seen in the following well known model. So, at the very beginning of this century couple P. and T. Ehrenfest had offered a model to visualize Boltzmann’s interpretation of irreversibility phenomena in statistics \[13\]. The model is extremely simple and fruitful. It considers two boxes with \(2N\) numerated balls. Choosing number \(l = 1, 2, \ldots, 2N\) randomly one must take the ball with label \(l\) from one box and put it to another one. Starting from the highly ‘nonequilibrium’ state with all balls in one box it is seen tendency to equalization of balls number in the boxes. So, there is seen irreversible flow toward preferable (equilibrium) state. One can hope that this model reflects a physical reality of nonequilibrium processes with initial state very far from equilibrium. A theory of such processes with (nonequilibrium) flow toward a state with maximal entropy should be sufficiently simple to give definite theoretical predictions since there is not statistical fluctuation of this flaw.

Following this model one can expect total dissipation of incident energy into particle masses. In this case the mean multiplicity \(\bar{n}\) should be \(\simeq n_{\text{max}}\) \[3\]. But it is well known that experimentally \(\bar{n} << n_{\text{max}}\). Explanation of this phenomena is hidden in the constraints connected with the conservation laws of Yang-Mills field theory. It is noticeable also that this constraints are not so rigid as in integrable systems, where there is not thermalization \[14\]. In the AHM domain we expect the free from above constraints dynamics. So, the AHM state may be the result of dissipation process governed by ‘free’ (from QCD constraints) irreversible flow.

The best candidates for such processes are stationary Markovian ones. They well described by so-called logistic equation \[13\] and lead to inverse binomial distribution with generating function

\[
\Xi(z, s) = \sigma_{\text{tot}}(s) \left\{ \frac{z_s(s) - 1}{z_s(s) - z} \right\}^\gamma, \quad \gamma > 0.
\] (2.12)

The normalization condition \(\partial \ln \Xi(z, s)/\partial z|_{z=1} = n_j(s)\) determines the singularity position:

\[
z_s(s) = 1 + \gamma/n_j(s).
\] (2.13)

Note, \(z_s(s) \to 1\) at \(s \to \infty\) since \(n_j(s)\) should be increasing function of \(s\).

Mostly probable values of \(z\) tends to \(z_s\) from below with rising \(n\):

\[
z_{\text{c}}(n, s) \simeq z_s - \gamma/n = 1 + \gamma(\frac{1}{n_j(s)} - \frac{1}{n})
\] (2.14)

This means that the vacuum of corresponding field theory should be stable. It is evident that \(\sigma_n\) decrease in this case as the \(O(e^{-n})\):

\[
\sigma_n \sim e^{-\gamma n/n_j}.
\] (2.15)

The singular solutions of \[2.12\] type arise in the field theory, when the \(s\)-chanel cascades (jets) are described \[10\]. By definition \(\Xi(z, s)\) coincide with total cross section at \(z = 1\). Therefore, nearness of \(z_{\text{c}}\) to one defines the significance of corresponding processes. It follows from \[2.14\] that both \(s\) and \(n\) should be high enough to expect the jets creation. But the necessary condition is closeness to one of \(z_s\), i.e. the high energies are necessary, and high \(n\) simplifies only theirs creation. Note the importance of jet creation processes in early Universe, when the energy density is extremely high.

Expressing the ‘logistic grows law’ the singular structure \[2.12\] leads to following interesting consequence \[13\]. The energy conservation law shifts the singularity to the right. For instance, the singularity associated with two-jets creation is located at \(z_s^{(2)}(s) = z_{\text{c}}(s/4) > z_{\text{c}}(s)\). Therefore, the multi-jet events will be suppressed with exponential accuracy in the AHM domain if the energy is high enough, since at ‘low’ energies even the exponential accuracy may be insufficient for such conclusion \[1\]. One may assume that there should be the critical value of incident energy at which this phenomena may realized. So the AHM are able to ‘reveal’ the jet structure iff the energies are high enough. In this sense the AHM domain is equivalent of asymptotic energies.
Summarizing above estimations we may conclude that

\[ O(e^{-n}) \leq \sigma_n < O(1/n), \]

i.e. the soft Regge-like channel of hadron creation is suppressed in the AHM region in the high energy events with exponential accuracy.

### 3 Hard Pomeron

During last 50 years the contribution (Pomeron) which governs the \( s \)-asymptotics of the total cross section \( \sigma_{\text{tot}}(s) \) is stay unsolved. The efforts in the pQCD frame shows that the \( t \)-channel ladder diagrams from dressed gluons may be considered as the dynamical model of the Pomeron [17].

The BFKL Pomeron is arise in result of summation, at least in the LLA, of ladder gluon diagrams in which the virtuality of space-like gluons rise to the middle of the ladder. To use the LLA this virtualities should be high enough. Noting that the ‘cross-beams’ (time-like gluons) of the ladder are the sources of jets it is natural that in the AHM domain the jet masses \( q_i^2 \) are large enough and one can apply the LLA.

Consideration of Pomeron as the localized object allows conclude that the multi-Pomeron contribution is \( \sim 1/k! \), if \( k \) is the number of Pomerons. This becomes evident noting that in the \( t \) channel the distribution over \( k \) localized uncorrelated ‘particles’ should be Poissonian. This factorial damping should be taken into account in the AHM domain.

Following to our above derived conclusion, number of ‘cross-beams’ (jets) should decrease with increasing \( n \), and, therefore, in the AHM domain the BFKL Pomeron, with exponential accuracy, should degenerate into ladder with two ‘cross-beams’ only. The virtuality of time-like gluons becomes in this case \( \sim \sqrt{s} \). So, the bare gluons are involved at high energies at the AHM. This solution is in agreement with our general proposition that in the AHM domain the particles creation process should be stationary Markovian.

We conclude that the AHM processes gave unique possibility to understand as the BFKL Pomeron is builded up. But there is the problem, connected with masslessness of gluons. So, the vertices of time-like gluons emission are singular at the \( q_i^2 = 0 \). It is the well known low-\( x \) problem. In the BFKL Pomeron this singularity is canceled by attendant diagrams of the ‘real’ soft gluons emission. However, this mechanism should destroyed when number of created particles (i.e. of gluons) is fixed. Note, the solution of this problem is unknown.

We hope to avoid this problem noting that the heavy jets creation is dominate in the AHM domain. This idea reminds the way as the infrared problem is solved in the QED (The emission of photons with wave length much better then the dimension of measuring devise is summed up to zero.) The quantitative realization of this possibility for QCD is in progress now.

### 4 QGP

There is a question: can we modelling in the terrestrial conditions the early Universe. The hot, dense, pure from colorless particles quark-gluon plasma (QGP) is the best candidate for investigation of this fundamental problem [18].

In our opinion [6] the plasma is a state of unbounded charges. The ‘state’ assumes presence of some parameters characterizes the collective of charges and the ‘unbounded’ assumes that the state is not locally, in some scale, neutral (we discuss the globally neutral plasma).

The ordinary QED plasma assumes that the mean energies of charged particles is sufficiently high (higher then the energy of particles acceptance), i.e. the QED plasma is ‘hot’. The QCD plasma in opposite does not be ‘hot’ (in corresponding energy scale) since the thermal motion moves apart the color charges. This leads to sufficient polarization and further ‘boiling’ of vacuum, with creation of \( q\bar{q} \) pairs. So, the QCD plasma should be dense and at the same time ‘cold’ enough.

There is two principal possibilities to create such state. Mostly popular is QGP plasma formation in the heavy ion-ion collisions at high energies. It is believed that at the central (head-on) collisions
one can observe the QGP in the CM central region of rapidities. But there is not in to-day situation
the unambiguous (experimental) signals of QGP formation (number of the theoretical possibilities are
discussed in literature).

Other possibility opens the AHM region: since in this region the hard channel of particles creation is
favorable dynamically (at high energies) one can try to consider the collective of color charges on precon-
finement stage as the plasma state. Because of energy-momentum conservation this state would be ‘cold’.
We should underline that possible cold QGP (CQGP) formation is just the dynamical, nonkinematical,
effect: the estimation (2.16) means that the sufficient polarization of vacuum and its ‘boiling’ effects are
insufficient, are frozen, at the AHM. So, considered CQGP remains relativistic. This solves the problem
of unbounded charges formation

But the question – may we consider the collective of colored charges created in the AHM events as
the ‘state’ – remain opened. To-day situation in theory is unable to give answer on this question (even in
the QCD frame, this will be discussed). But we can show the experimentally controlled condition when
same parameters may be used to characterize this collective.

For instance, we may examine in what conditions the mean energy of colored particles may be consid-
ered as such parameter. Let us return to the definition (2.3) for this purpose. To calculate the integral
over \( \beta \) we will use the stationary phase method. Mostly probable values of \( \beta \) are defined by equation of
state:

\[
E = \partial \ln \rho(\beta, z)/\partial \beta. \tag{4.17}
\]

It is well known that this equation have positive real solution \( \beta_c \). Then, as was noted above, \( \beta_c \) coincide
with inverse temperature \( 1/T \) and this definition of \( T \) is obvious in the microcanonical formalism of
statistical physics.

The parameter \( \beta_c \) is ‘good’, i.e. has a physical meaning, iff the fluctuations near it are Gaussian
(It should be underlined that the value of fluctuations may be arbitrary, but the distribution should be
Gaussian). This is so if, for instance,

\[
\frac{\rho^{(3)}}{\rho} - 3 \frac{\rho^{(2)}}{\rho} \rho^{(1)} + 2 \left( \rho^{(1)} \right)^3 \approx 0, \tag{4.18}
\]

where, for identical particles,

\[
\rho^{(k)}(\beta_c, z) \equiv \frac{\partial^k \rho(\beta_c, z)}{\partial \beta_c^k} = \left( -z \frac{\partial}{\partial z} \right)^k \int d\omega_n(q) \prod_{i=1}^{k} \epsilon(q_i) f_k(q_1, ..., q_k; \beta_c, z) \tag{4.19}
\]

and \( f_k(q_1, ..., q_k; \beta_c, z = 1) \) is the \( k \)-particle inclusive cross section. Therefore, the (4.18) condition requires
smallness of energy correlation functions. It is the obvious in statistics energy correlations relaxation
condition near the equilibrium. One can find easily the same condition for higher correlation functions.

This conditions establish the equilibrium, when knowledge of one parameter (\( \beta_c \) in considered case)
is enough for whole systems description. The analogous conditions would arise if other parameters are
considered. For instance, the (baryon, lepton, etc.) charge correlations relaxation condition means the
‘chemical’ equilibrium. The quantitative expression of this phenomena is smallness of corresponding
correlation functions.

This conditions are controllable experimentally. But it is hard to expect that at finite values of \( n \),
where the cross sections are not too small, above derived conditions are hold, even in the AHM
region. Later we will find more useable from experimental point of view conditions making more accurate
analyses.

\(^2\) To amplify this effect it seems reasonable to create AHM in the ion-ion collisions
5 AHM events triggering

Following to our main idea we would consider following solution of the triggering problem \[2\]. Let \(\varepsilon_i\) be the energy of \(i\)-th particle (we did not distinguish particles), \(i = 1, 2, ..., n\), and let us introduce \(\varepsilon_{\text{max}} = \max \{\varepsilon_i\}\). Then, we have

\[ n \geq n_{\text{min}} = \frac{E}{\varepsilon_{\text{max}}} \tag{5.20} \]

if the equality

\[ \sum \varepsilon_i = E, \tag{5.21} \]

where \(E\) is the total incident energy, may be established experimentally. So, choosing \(\varepsilon_{\text{max}} \ll E\) we examine the high-multiplicity events. The approach assumes that it is unimportant to know \(n\) exactly in the high-multiplicity domain. We will try to adjust the theory to such formulation of experiment.

Note, the trigger shrinks the phase space volume:

\[ d^n \omega_{\varepsilon_{\text{max}}} = \prod_{i=1}^{n} \frac{d^3 q_i}{(2\pi)^3 2\varepsilon_i} \Theta(\varepsilon_i - \varepsilon_{\text{max}}), \tag{5.22} \]

So, the trigger depress the leading-particles creation first of all. The effectiveness of this trigger was shown using the PYTHIA Monte Carlo simulation.

Let \(\varepsilon_\mu\) be now the energy measured in \(\mu\)-th calorimeter cell (bean), \(\mu = 1, 2, ..., M\). Then, instead of (5.21), we should assume that

\[ \sum \varepsilon_\mu = E \tag{5.23} \]

and, following to our idea, \(\varepsilon_{\text{max}} = \max \{\varepsilon_\mu\}\). Let us assume also that \(\varepsilon_\mu = \sum_{i=1}^{n_\mu} \varepsilon_i\), where \(n_\mu\) is the number of particles in \(\mu\)-th cell,

\[ \sum n_\mu = n. \tag{5.24} \]

Then

\[ n \geq M \geq \frac{E}{\varepsilon_{\text{max}}} \equiv n_{\text{min}} \tag{5.25} \]

Assuming particles identity,

\[ d^n \omega_{\varepsilon_{\text{max}}} \sim \prod_{\mu} \left\{ \frac{n_\mu}{(2\pi)^3 2\varepsilon_i} \prod_{i=1}^{n_\mu} \Theta(\varepsilon_i - \varepsilon_{\text{max}}) \right\}. \tag{5.26} \]

Note, \(M \geq E/\varepsilon_{\text{max}}\).

The calorimeters can not overlap the whole range of rapidities. If we assume that \(E'\) is the measured by calorimeter energy \((E' \leq E)\) than in the inequality (5.23) we should change \(E \to E'\). In this case we examine AHM in the fixed domain of the rapidity, assuming that \(E'\) is fixed.

Offered trigger constraints (i) the ‘peripheral’ interactions and (ii) the ‘short-range’ fluctuation (in calorimeter cells dimension) of particles densities. It was assumed that the number of calorimeter cells \(M >> n\). This is important in the AHM domain. Besides the dimension of cells should be smaller then the cross section of the QCD jets. Otherwise the condition \(\varepsilon_\mu \leq \varepsilon_{\text{max}}\) may suppress jets creation in the AHM domain, where, as was shown above, the jets mass may be comparable with incident energy.

By this reason, having in mind the real calorimeters, the first-level trigger should be weakened. Noting that the events have a tendency become hard in the AHM domain one may ask to trigger the high (comparable with \(E\)) \(E_t\) events. The PYTHIA simulation shows that the total number (including particles nonregistered in calorimeter) grows with total energy \(E_t\) registered in calorimeter.

Number of events (normalized on incident particles flow) with given \(\varepsilon_{\text{max}}\) is

\[ N(E, \varepsilon_{\text{max}}) = \sum_{n=n_{\text{min}}}^{n_{\text{max}}} \sigma_n(E, \varepsilon_{\text{max}}) \tag{5.27} \]
where $\sigma_n(E, \varepsilon_{\text{max}})$ is the cross section with constraints on the particles energies. This constraint is fixed by $\Theta$-functions in (5.22). The PYTHIA simulation shows that the distribution $\sigma_n(E, \varepsilon_{\text{max}})$ have a maximum at $n = \bar{n}(E, \varepsilon_{\text{max}})$ rising with $\varepsilon_{\text{max}} \to 0$ and $\bar{n}(E, \varepsilon_{\text{max}}) \gg n_{\text{min}}$ at LHC energies.

The topological cross section as was shown above is the important quantity being sensible to structure of the QCD vacuum. On other hand, its experimental measurement is the problematic task in the AHM domain, even if $n_c$ only may be measured with definite accuracy. In considered example,

$$\sigma_n(s) = \int d\varepsilon_{\text{max}} \sigma_n(E, \varepsilon_{\text{max}}).$$

(5.28)

Using this definition one can try to find $\sigma_n(s)$ from integral quantity $N(E, \varepsilon_{\text{max}})$. For instance, differentiating over $\varepsilon_{\text{max}}$ we find:

$$\frac{\partial N(E, \varepsilon_{\text{max}})}{\partial \varepsilon_{\text{max}}} = n_{\text{min}} \sigma_{n_{\text{min}}}(E, \varepsilon_{\text{max}}) + \sum_{n=n_{\text{min}}}^{n_{\text{max}}} \frac{\partial \sigma_n(E, \varepsilon_{\text{max}})}{\partial \varepsilon_{\text{max}}}$$

(5.29)

Let us assume now that just $n_{\text{min}} = n$ is the independent quantity. Then $\varepsilon_{\text{max}} = \bar{\varepsilon} \equiv E/n$. With this definitions,

$$\frac{\partial N(E, \bar{\varepsilon})}{\partial \bar{\varepsilon}} = n \sigma_n(E, \bar{\varepsilon}) + \sum_{\nu=n}^{n_{\text{max}}} \frac{\partial \sigma_\nu(E, \bar{\varepsilon})}{\partial \bar{\varepsilon}}$$

(5.30)

remembering that

$$\bar{\varepsilon} \equiv \frac{E}{n} = \varepsilon_{\text{max}}.$$ (5.31)

Just last equality presents problem: the maximal energy $\varepsilon_{\text{max}}$ may exceed the arithmetic average $E/n$ in the deep AHM domain only. We are unable to examine this possibility quantitatively since the PYTHIA generator gives too big uncertainty in this region of multiplicities. But note that above described possibility is not interesting since it is too hard to achieve the region, where $\varepsilon_{\text{max}} = E/n$, experimentally.

## 6 Inclusive description

The calorimetric measurement introduces averaging over particles number, momentum, charges, etc. It is natural then to adjust the theory to such type of experiment.

Having in mind the ion-ion collisions also let us consider the $n$-into-$m$ particles transition. We find:

$$\rho_{nm}(\beta) = \frac{(-1)^{n+m}}{n!m!} N_m(\beta_i, \hat{\phi}) N_n(\beta_f, \phi^\dagger) \rho_0(\phi),$$

(6.32)

where $N_m(\beta, \hat{\phi})$ is the initial ‘temperature’ 1/$\beta_i$ $n$-particles number operator:

$$N_m(\beta, \hat{\phi}) = \int d\omega_m(q) \prod_{k=1}^{m} dx_k dy_k \times e^{-\beta \varepsilon(q)} e^{-iq_k(x_k-y_k)} \frac{\delta}{\delta \phi-(y_k)} \frac{\delta}{\delta \phi^\dagger(x_k)}$$

(6.33)

and

$$\rho_0(\phi) = \mathcal{Z}(\phi^\dagger) \mathcal{Z}^\dagger(-\phi_-)$$

(6.34)

is the vacuum-into-vacuum transition probability in the environment of external field $\phi$. 
Introducing new coordinates:

\[
\begin{align*}
    x_k &= R_k + r_k/2, \quad y_k = R_k - r_k/2, \\
    x'_k &= R'_k - r'_k/2, \quad y'_k = R'_k + r'_k/2
\end{align*}
\]  

(6.35)

we come naturally from (6.32) to definition of the Wigner functions [19]:

\[
\rho_{nm}(\beta) = \frac{1}{n!m!} \int d\omega_m(q') d\omega_n(q) \times \prod_{k=1}^{N} dR_k \prod_{k=1}^{N} dR'_{k} e^{-\varepsilon(q)(\beta_i + \beta_f)} \times W_{mn}(q, R; q', R'),
\]

(6.36)

where

\[
W_{nm}(q, R; q', R') = (-1)^{n+m} \prod_{k=1}^{n} N_+(q_k, R_k; \hat{\phi}) \times \prod_{k=1}^{m} N_-(q'_k, R'_k; \hat{\phi}) \rho_0(\phi).
\]

(6.37)

The Wigner function has formal meaning in quantum case (it is not positively definite), but in the classical limit it has the meaning of ordinary in statistics phase-space distribution function. It obey the Liouville equation conserving the phase space volume.

It is natural to introduce the generating functional weighing the operator \(N_{\pm}(R, q; \hat{\phi})\) by the arbitrary 'good' function \(z(R, q)\):

\[
N_{\pm}(\alpha, z; \hat{\phi}) \equiv \int dR d\omega_1(q) e^{-z^\beta z(R, q)} N_{\pm}(R, q; \hat{\phi}).
\]

(6.38)

In result, summation over all \(n, m\) gives the generating functional of Wigner functions in the temperature representation:

\[
\rho(\alpha, z) = e^{-N_+(\beta_i, z; \hat{\phi}) - N_-(\beta_f, z; \hat{\phi})} \rho_0(\phi) \equiv e^{-N(\beta, z; \hat{\phi})} \rho_0(\phi).
\]

(6.39)

Introducing the black-body environment and summing over \(n\) and \(m\) the structure of generating functional (6.39) is the same as in the real-time finite-temperature field theories. This is the simplest (minimal) choice of environment. One can consider another organization of the environment, e.g. may consist from the correlated particles as it happens in the heavy ion collisions (the nucleons in ion may be considered as the quasi-free particles at high energies).

So, we construct the two-temperature theory (for initial and final states separately). In such theory with two temperatures the Kubo-Martin-Schwinger (KMS) periodic boundary conditions applicability is not evident. Note, KMS boundary condition play the crucial role in Gibbs thermodynamics since the temperature in it is introduced just by this condition. In our approach the KMS boundary condition arise as the consequence of specially chosen environment (namely, of the black-body environment).

The temperature was introduced as the parameter conjugate to created particles energies. By this reason the uncertainty principle restrictions should be taken into account. Considering the Fourier-transformed probability \(\rho(\beta, z)\) as the observable quantity the phase-space boundaries are not fixed exactly, i.e. the 4-vector \(P\) can be defined with some accuracy only if \(\beta_i\) are fixed, and vice versa. It is the ordinary quantum uncertainty condition. In the particles physics namely the 4-vector \(P\) is defined by experiment. Let us find the condition when both \(P\) and \(\beta\) may be the well defined quantities, i.e.
may be used for description of $\rho(\alpha, z)$. This is necessary if we want to use the temperature formalism in particles physics also.

Note, in statistical physics such formulation of problem has no meaning since the interaction with thermostat is assumed. In result of this interaction the energy of system is not conserved, i.e. the systems word line belong to the energy surface (it can be thin if the interaction with thermostat is weak).

Considering the general problem of particles creation it is hard to expect that the constant $\beta_{(f)}(E)$ is a ‘good’ parameter, i.e. that the factorization conditions (4.18) are hold. Nevertheless there is a possibility to have the above factorization property in the restricted space-time domains of size $L$. It is the so called ‘kinetic’ phase of the process when the memory of initial state was disappeared, the ‘fast’ fluctuations was averaged over and we can consider the long-range ‘slow’ fluctuations only.

In this ‘kinetic’ phase one can use the ‘local equilibrium’ hypothesis in frame of which $\beta_{(f)}(E) \rightarrow \beta_{(f)}(R, E)$, where $R \in L_c$ and $L_c$ is the dimension of the measurement (calorimeter) cell. Note, we always can divide the external particles measuring device on cells since the free states are measured. Such description of nonstationary media seems favorable in comparison with traditional one. It is natural to take

$$L_c \ll L,$$

(6.40)

where $L$ is the characteristic thermal fluctuations dimension. It is assumed that $\beta_{(f)}(R, E) = \text{const.}$ if $R \in L_c$. In the equilibrium $L \rightarrow \infty$.

By definition, $1/\beta_{(f)}(R, E) = \bar{\varepsilon}(R)$ is the mean energy of particles in the cell with dimension $L_c$. The fluctuations in $\bar{\varepsilon}(R)$ vicinity should be Gaussian.

The quantum uncertainty principle dictates also the condition:

$$L_c >> L_q,$$

(6.41)

where $L_q$ is the characteristic scale of quantum fluctuations ($L_q \sim 1/m$ for massive theories).

The ‘infrared unstable’ situation means that

$$L_q >> L.$$

(6.42)

One should underline that $L$ defines the scale of thermodynamics fluctuations and, by this reason, the inequality (6.42) points to (unphysical) instability in the infrared domain.

So, if conditions (6.40, 6.41) are hold we can use the Wigner functions to describe the phase-space distributions, i.e. the formalism has right classical limit in this case.

To introduce the scales $L$, $L_c$ into formalism we can divide the $R$ 4-space on the cells of $L_c$ dimension:

$$\int dR = \sum_r \int_{L_c} dR,$$

(6.43)

where $r$ can be considered as the cells 4-coordinate. Assuming that the inequality (6.41) is hold we can assume that $\beta = \beta(r)$, $z = z(q, r)$, are the constants at least on the $L_c$ scale. With this definitions

$$\hat{N}_\pm(\beta, z; \phi) \equiv \sum_r \int d\omega_1(q)e^{-\beta(r)(\varepsilon(q)+\mu(r,q))} \times \int_{L_c} dR \hat{N}_\pm(R, q; \phi),$$

(6.44)

where

$$\mu(r, q) \equiv \frac{1}{\beta(r)} \ln z(r, q)$$

is the local chemical potential.

We can use described above generalization of inclusive description to investigate the tendency to equilibrium (total thermalization) in kinetic phase considering the energy correlations between various calorimeter cells.
On the more early pre-kinetic stages no thermodynamical shortened description can be applied and the pure quantum description (in terms of momenta only) should be used. For this purpose one should expand $\rho(\alpha, z)$ over operators $N^\pm$ and the integrations over $\alpha_i, \alpha_f$ gives ordinary energy-momentum conservation $\delta$-functions, i.e. defines the system on the infinitely thin energy sheet.

7 Conclusion

To trigger the AHM events the restricted from above created particles energies prescription was considered. Theoretically this means introduction into the phase space differential measure products of $\Theta$-functions, see (5.22). it is equivalent of introduction of local activities

$$z(q, r) = \Theta(\varepsilon_{max}(r) - \varepsilon(q))$$

(7.45)

So, the choice of $z(q, r)$ introduces the model of given calorimeter into the theory, i.e. allows to adjust the theory to given trigger.

It was consider the unsuccessful attempt to find topological cross sections $\sigma_n$ varying $\varepsilon_{max}$, i.e. $z$. This idea is natural noting definition:

$$\sigma_n(s) = \frac{1}{n!} \frac{d^n}{dz^n} \Xi(z, s)|_{z=0},$$

(7.46)

see (2.1). So the attempts to find $\sigma_n$ varying activity have a meaning. The realization of this idea crucially depends from concrete choice of calorimeters.

But to investigate the phase transition phenomena we may use other possibility. Noting that the system have a tendency to become equilibrium in the AHM domain and noting that the Gibbs free energy is $\sim \ln \Xi$, we can compare the heat capacity in the hadrons and photons (or leptons) systems in the AHM domain. It is the direct measurement of phase transition.

Acknowledgement

The discussions of the inside of BFKL Pomeron with L.Lipatov and E.Kuraev, and of the mini-jets creation in the semi-hard processes with E.Levin was fruitful. The modern experimental possibilities explanation given by G.Chelkov, Z.Krumshtein, V.Nikitin was important for us. The PYTHIA simulation results gave us kindly M.Gostkin. We are sincerely grateful to all of them. We would like to thank V.G.Kadyshevski to the kind interest and helpful discussions.

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