Calculation of initial dynamically unstabilized sections in the elements of liquid rocket engines turbopump units

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Abstract. The article considers an analytical approach to the relative characteristic parameters determining in the initial hydrodynamically unstabilized sections. Using the momentum equation, analytical equations are obtained for the dynamic boundary layer thickness determining. Laminar and turbulent flow regimes were considered. It is noted that the initial sections are characteristic for the flow around such surfaces as: blades of turbines and centrifugal pumps, nozzle elements, cavities between the casing and the rotor of the turbopump unit. In the flowing parts of liquid propellant rocket engines turbopump units, relatively short spatial channels, characterized by the presence of non-merged dynamic and temperature boundary layers prevail. The initial section significantly affects the flow regime, length loss and heat exchange processes. The analytical dependencies obtained cover a wide range of flow regimes and are in good agreement with the experimental data of other authors. An approach is given to determine the hydrodynamic parameters for the initial sections that affect the overall flow pattern in complex spatial channels with different degrees of curvature in the presence of a pressure gradient.

1. Introduction

When designing and analysing structures of feed units for liquid-propellant rocket engines, it is necessary to consider spatial flows in the elements of the hydraulic path. The hydraulic path is a channels complex of various shapes. In a turbopump unit (TPU), these are the cavities of the turbines and pumps, or rather, the rotor and stator elements. In this study, an analysis of the geometric and operating parameters of gas turbines and TPU pumps hydraulic path is carried out. This analysis revealed that the path contains mainly areas of dynamically unstabilized flow (areas with a developing dynamic boundary layer). These areas are characterized by the presence of a flow core and a developing boundary layer $\delta$ which changes with the $x$ channel coordinate.

The elements of the flow paths are relatively short channels with curved flow surfaces. To a large extent, the flow regime is influenced by the initial section, and, as a consequence, affects the hydrodynamic losses. In the initial section, dynamic and temperature spatial boundary layers develop and dynamically unstabilized flow is observed.

In the papers of the authors [1-3], an analytical approach is proposed, and it is noted that the parameters' distribution of the dynamic boundary layer is significantly influenced by heat exchange...
processes (since the viscosity parameter largely depends on temperature), but the initial section influence of the flow is not taken into account.

The problem of identifying a dynamic initial section was solved by many authors [4-10], however, it has not been unambiguously solved until now [5]. Methods for determining the length coefficient of a hydrodynamic section, such as experimental, analytical and numerical integration of the equations of motion, lead to results that differ from each other by almost 4 times [10].

2. Research task
To determine the numerical values of the characteristic quantities of the dynamic and temperature boundary layers (such as the displacement thickness, the thickness of the momentum loss and the thickness of the energy loss), as well as to calculate the dynamic and thermal parameters of technical systems, it is necessary to obtain equations for determining the thickness of the dynamic boundary layer, for the laminar and developed turbulent flows depending on the distance from the input edge of the channel [11, 12].

Consider the momentum equation of the boundary layer obtained by T. Karman:

\[ \rho \frac{d}{dx} \int_0^I (U - u) dy - \rho \frac{dU}{dx} \int_0^I u dy = \tau_{\text{wall}} + \int \frac{dp}{dx}, \]

(1)

Using the profiles of the velocity distribution in the boundary layer and the equation for the momentum integral (1), we obtain:

\[ I = \rho \int_0^I (U - u) dy = \rho U^2 \int_0^I \left( \frac{1 - u}{U} \right) dy. \]

(2)

The upper limit of integration is replaced by the dynamic boundary layer thickness \( \delta \), since for the integration condition \( y \geq \delta \) the velocity \( U = u \) and the integrand vanish. Taking into account the obtained expressions for the momentum loss thickness for laminar and turbulent flows

\[ \int_0^\delta \frac{u}{U} \left( 1 - \frac{u}{U} \right) dy = \delta^{m\text{a}}, \]

we write the equation (2) in the following form:

\[ I = \rho U^2 \delta^{m\text{a}}. \]

(3)

3. Laminar boundary layer
Let us consider the features of the laminar flow. We approximate the distribution of the laminar dynamic boundary layer by the function:

\[ \frac{u}{U} = 1 - \left( \frac{1 - \frac{y}{\delta}}{m} \right)^m, \]

taking into account the obtained expression for the momentum loss thickness for the longitudinal flow of the laminar flow case in the boundary layer [1, 3], we transform the equation (3):

\[ I = \rho U^2 \delta^{m\text{a}} = \rho U^2 \frac{\delta m}{(m+1)(2m+1)}. \]

(4)

According to [12], the tangential friction stress is defined as:
\[ \tau_{\omega} = 0.332 \rho U^2 \left( \frac{v}{Ux} \right)^{1/2}. \]  

(5)

Then, taking into account the momentum equation (4) and friction shear stress (5), the equation of momentum leads to the differential equation:

\[ \rho U^2 \frac{m}{(m+1)(2m+1)} \frac{d\delta}{dx} = 0.332 \rho U^2 \left( \frac{v}{Ux} \right)^{1/2}. \]  

(6)

After reducing and separating the variables, we get:

\[ d\delta = \frac{0.332}{m} \left( \frac{v}{Ux} \right)^{1/2} dx. \]  

(7)

After integrating equation (7), we obtain:

\[ \delta = \frac{0.332 \cdot 2}{m} \left( \frac{v}{U} \right)^{1/2} x^2 + C. \]  

(8)

Based on the boundary conditions at \( x = 0 \), respectively \( C = 0 \), then the laminar boundary layer thickness depending on the distance from the input edge:

\[ \delta = \frac{0.664}{m} \left( \frac{v}{U} \right)^{1/2} x^2 \]  

\[ + \frac{0.664}{m} \frac{1}{(m+1)(2m+1)} \left( \frac{m}{(m+1)(2m+1)} \right) \frac{1}{\text{Re}^2} x. \]  

(9)

The equation (9) determines the thickness dependence of the dynamic boundary layer depending on the \( x \) coordinate (the length of the flow section around the surface or element) and on the external flow parameter (Re criterion). Figure 1 shows the graphical dependence determined by equation (9) for various values of the dynamic laminar boundary layer profile distribution degrees.

![Figure 1. Dependence of the laminar dynamic layer thickness on the profile degree and surface coordinates.](image-url)
Figure 1 shows that in the initial dynamically unstabilized section, the regime and flow parameters of the external flow significantly affect the parameter $\delta/x$. This affects the losses in the boundary layer and shows the need to take into account the initial section. Initial dynamically unstabilized sections are predominant due to the geometrical and operating parameters of the liquid-propellant engine feed units implemented in the flow paths. This is due to relatively short sections and high flow rates: blades, working disks, inlet and outlet units of turbines and pumps, etc. Note that for $m = 2$ the values obtained from dependence (9) coincide with the expression obtained by G. Schiechting.

The transition of a laminar boundary layer into a turbulent one is characterized by the form parameter $H = \frac{\delta^*}{\delta} \approx 2.6$.

Moreover, when passing from laminar to turbulent flow, this form parameter decreases from 2.6 in the laminar region to 1.4 in the turbulent region [30].

4. Turbulent boundary layer
Consider a turbulent boundary layer, approximate the distribution of a turbulent dynamic boundary layer by the function

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{m},$$

taking into account the obtained equation for the momentum loss thickness for the longitudinal flow in the case of turbulent flow in the boundary layer, we obtain [8, 9]:

$$I = \rho U^2 \delta^{**} = \rho U^2 \frac{\delta m}{(m+1)(m+2)}.$$  (12)

Let us use the law of friction on a plate for a turbulent boundary layer according to [12]:

$$\tau_\omega = 0.0225 \rho U^2 \left(\frac{v}{U \delta}\right)^{\frac{1}{4}}.$$  (13)

Taking into account the momentum equation (12) and the law of friction (5), the equation of momentum leads to the differential equation

$$\rho U^2 \frac{m}{(m+1)(m+2)} \frac{d\delta}{dx} = 0.0225 \rho U^2 \left(\frac{v}{U \delta}\right)^\frac{1}{4}.$$  (14)

After reducing and separating the variables, we get:

$$\delta^\frac{1}{4} d\delta = \frac{0.0225}{m} \frac{1}{(m+1)(m+2)} \left(\frac{v}{U}\right)^{\frac{1}{4}} dx.$$  (15)

After integrating the equation (15) and further transformation, we obtain an equation for determining the thickness of the turbulent boundary layer depending on the distance from the initial edge:
\[
\delta = \frac{0.0572}{\left(\frac{m}{(m+1)(m+2)}\right)^{4/5} \left(\frac{m}{(m+1)(m+2)}\right)^{4/5} x^5 + C.}
\] (16)

If a turbulent boundary layer is realized immediately from the initial edge, then proceeding from the boundary conditions at \( x=0 \), respectively \( C=0 \), then

\[
\delta = \frac{0.0572}{\left(\frac{m}{(m+1)(m+2)}\right)^{4/5} \left(\frac{m}{(m+1)(m+2)}\right)^{4/5} x^5 = \frac{0.0572}{\left(\frac{1}{Re_x}\right)^{4/5} x.}
\] (17)

Note that a turbulent boundary layer is formed only at some critical distance \( x_c \) from the initial edge, i.e. at \( x \neq 0 \). At this critical point the boundary layer already has a certain thickness since it is realized in the transition from the laminar boundary layer. Then from (16)

\[
\delta = \frac{0.0572}{\left(\frac{m}{(m+1)(m+2)}\right)^{4/5} \left(\frac{m}{(m+1)(m+2)}\right)^{4/5} x + k\delta_x,}
\] (18)

where \( k \) is the coefficient which decreases the boundary layer in the transition from laminar to turbulent from the condition (10).

Figure 2 shows a graphical dependence determined by equation (17) for various values of distribution degree of the dynamic turbulent boundary layer profile.

![Figure 2](image)

Figure 2. Dependence of the turbulent dynamic layer thickness on the degree of the profile and the surface coordinate.

It should be noted that the dependence obtained by equation (17) coincides with the equation obtained by G. Schlichting. However, in the flow parts of the liquid-propellant engine feed units, flow parameters can differ significantly and the profile of the dynamic boundary layer diagram changes, which entails a change in the boundary layer parameters.
5. Conclusion
The expressions obtained for determining the thicknesses of the laminar (9) and dynamic (17, 18) boundary layers should be used to determine the relative characteristic thicknesses of the dynamic spatial boundary layer. The obtained analytical dependencies are in good agreement with the results of experimental studies of other authors. The initial section of the flow affects the nature of the flow, the distribution of dynamic parameters and, as a consequence, the losses along the length of the channel. The distributions of dynamic parameters in the boundary layer and in the main non-viscous core affect the heat transfer and energy parameters of the entire unit.

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