Limits on the low-energy antinucleon annihilations from the Heisenberg principle

A. Bianconi.\textsuperscript{a}

\textsuperscript{a}Dip. di Chimica e Fisica per l’Ingegneria e per i Materiali, Università di Brescia and INFN, Sez. di Pavia, Italy

Abstract

Here a short synthesis is presented of the work, developed in the last two years by the Brescia Collaboration, on the phenomenology of antinucleon-nucleon and antinucleon-nucleus annihilation at small momenta (below 300 MeV/c in the laboratory), with special stress on the role of general principles.

Our work\textsuperscript{[1]} in the last two years was mainly devoted to the study of the strong shadowing characterizing $\bar{p}$ annihilations on light nuclei. The nuclear shadowing is evident in the fact that the $\bar{p}p$ total annihilation cross sections are larger than the corresponding $\bar{p}D$ and $\bar{p}^4$He ones for $\bar{p}$ momenta in the laboratory $k < 70 \text{ MeV/c}\textsuperscript{[2]}$. In previous works we used the word “inversion”. A related phenomenon was studied in antiprotonic atoms\textsuperscript{[3–6]} and also pionic atoms\textsuperscript{[7]}.

Since “shadowing” can be defined as the amount of departure from the Impulse Approximation, to quantify its real presence we need first to produce an impulse approximation estimation of the measured nuclear data. Not to overrate the shadowing effects, two more points must be taken into account: (1) a correct estimation of Coulomb enhancement effects, and (2) center of mass effects. When data are represented with respect to the center of mass momentum\textsuperscript{[1]}, the shadowing effect is smaller, although apparently all the available nuclear annihilation cross sections become quite similar at $k_{cm} << 100 \text{ MeV/c}$. To fully understand the relevance of this similarity, we remind that at low energies Coulomb effects are expected to be quite strong and enhance $\bar{p}^4$He annihilation rates about twice with respect to $\bar{p}p$ ones. This traditional estimate of Coulomb effects\textsuperscript{[8]} is based on the approximation of pointlike particles. Within an optical potential framework we re-calculated these Coulomb corrections, taking into account the finite size of the $\bar{p}$ and nuclear charge distributions. One of the results\textsuperscript{[1]} was that the advantage of the Helium charge with respect to the Hydrogen case was not that large, in the laboratory frame.
This advantage is however stronger with respect to center of mass momenta. The role of the reference frame can be understood taking into account that low energy exoenergetic reaction cross sections are supposed to be roughly $\sim 1/k_{cm}$ for neutral projectiles, and $\sim 1/k_{cm}^2$ for charged projectiles. E.g., $k_{lab} = 100$ MeV/c means $k_{cm} \approx 100$ MeV/c for $\bar{p}^4$He, and $k_{cm} = 50$ MeV/c for $\bar{p}p$.

![Fig. 1. Antineutron (empty circles) and antiproton (full circles) total annihilation cross sections (mb) measured by the Obelix experiment[9,2]. The empty crosses reproduce the two low-energy $\bar{n}p$ total annihilation points measured in [10]. Error bars are not reported here. The two dotted lines correspond to optical potential fits (see text for details). The solid line represents the $\bar{p}p$ annihilation cross section after Coulomb effects have been subtracted. The lower energy part of this curve has been calculated by extrapolating the optical potential fit of the $\bar{p}p$ data and by removing the electrostatic part of the potential. For $k > 30$ MeV/c the Coulomb effects have been subtracted from the actual $\bar{p}p$ points, not from the potential fit (for this reason the solid curve is larger than the dotted curve for $k > 130$ MeV/c).

For calculating a PWIA $\bar{p}$-nucleus annihilation rate, we need $\bar{p}p$ and $\bar{p}n$ annihilation rates in a suitable momentum range. The latter have been supposed to be equal to the $\bar{n}p$ measured annihilation rates[9] (see fig.3). We have subtracted Coulomb effects from the measured $\bar{p}p$ annihilation rates, leading to
the “uncharged $\bar{p}p$ annihilation cross section” shown by the solid line in fig.1. After calculating the total nuclear annihilation cross section, the result has been rescaled by the nuclear enhancement Coulomb factor. This procedure is needed for the following reasons. When a $\bar{p}$ annihilates on a nucleus, the Coulomb forces have two effects: (1) a focusing of the $\bar{p}$ wave in the reaction region, and (2) an increase of the $\bar{p}$ kinetic energy. Both effects take place on an atomic scale $\sim r_B >> R_{\text{nucleus}}$. So $\bar{p}p$ and $\bar{p}n$ processes are equally “Coulomb affected” when the proton and neutron are bound to the same nucleus.

Last, the PWIA calculated annihilation cross section has been renormalized to the measured value at $k \approx 350$ MeV/c. This permits to remove the eclipse effect from the calculations. This effect is well known, and reduces the annihilation rates by a slowly energy dependent factor at all energies, leading to a reaction rate proportional to $A^{2/3}$. This effect can be considered a component of the shadowing, but clearly it is not what we are interested in. In fig.2 we show the results of the IA fit on deuteron. We produce three curves. One takes into account the real deuteron composition, the other two assume a deuteron composed by two neutrons or two protons (in both cases with total deuteron electric charge 1). The comparison between the three curves, and between the solid one and the data suggests that (1) not to take into account the actual proton/neutron composition of the nucleus introduces large errors for $k < 200$ MeV/c, and (2) remarkable shadowing is anyway present for $k < 100$ MeV/c.

Rather independently of the mechanism underlying the annihilation process, it had been previously demonstrated that in the framework of the multiple scattering theory[5], of variational methods[5] and of optical potential treatments[1,13,14,4] one can predict such shadowing effects. The effect seems to be present also in pionic atoms[7].

The fact that different methods lead to similar results suggested us to investigate the problem from a more general and qualitative, although less precise, point of view. In our work we have shown that due to the quantum uncertainty principle the $\bar{n}$-nucleus cross sections should be almost $A$-independent, apart for fluctuations due to nuclear surface effects. Consequently the $\bar{p}$-nucleus cross sections should depend on the target because of its electric charge only. The underlying argument is that most of the existing models[15] or phenomenological analyses[16–18] establish that the annihilation process takes place when the centers of mass of the antinucleon and of the target nucleus are at a relative distance $d$ such that $R_{\text{nucleus}} < d < R_{\text{nucleus}} + \Delta$, where $\Delta \sim 1$ fm (or smaller, depending on the model) and $\Delta$ does not depend too much on the target. So the annihilation is equivalent to a measurement of the projectile-target relative distance with uncertainty $\Delta < 1$ fm, and this measurement is incompatible with a relative momentum $<< 200$ MeV/c.

We distinguish between two classes of nuclear reactions. On one side, inelastic
Fig. 2. PWIA calculation of the $\bar{p}$-Deuteron total annihilation cross sections, together with data points taken from references [2] (full squares), [11] (empty stars), [12] (full circles). Continuous line: full PWIA calculation with Coulomb correction and renormalization of the curve to the point at 340 MeV/c. Dashed line: the Deuteron is supposed to be composed by two neutrons (with overall nuclear charge $Z=1$). Dotted line: the Deuteron is supposed to be composed by two protons (with overall nuclear charge $Z=1$).

reactions where the entire nucleus is involved, as in compound nucleus reactions, but the underlying projectile-nucleon processes are elastic (e.g. neutron induced nuclear reactions). In this case the characteristic reaction region coincides approximately with the target nucleus. Then the uncertainty $\Delta$ coincides approximately with the nuclear radius. On the other side, we find reactions where a strong inelasticity is present at the projectile-nucleon level. In this case reactions deep inside the nuclear volume are rare, the reaction region is a shell at the surface of the target nucleus, with thickness $\Delta$, and $\Delta$ is approximately the same for all the possible targets.

The consequence of the limitations imposed by the uncertainty principle is that for antinucleon momenta $k << 1/\Delta$ the total reaction cross section be-
Fig. 3. S-wave projectile wavefunction $\chi(r) = r \cdot \Psi(r)$ in the reaction region and around it. In the free motion region $\chi$ (continuous line) has the form $\sin[k(r - \alpha)]$. Here $\alpha$ is assumed real for graphical reasons. Reactions are possible for $r < m$, a matching radius ($R_{\text{nucleus}} + \Delta$ in the text), however conspicuous damping of the projectile wavefunction is reached in the “effective reaction region”. The (approximate) lower limit $r = s$ of this region is defined by the condition $\chi(s) \ll \chi(m)$. What happens for $r < s$ is not relevant. The parameter $\Delta$ used in the text coincides with $|s - m|$ in this example. In the matching point $r = m$ the free motion wave must join regularly with the internal wave (dashed line). The scattering length $\alpha$ represents the “virtual source” of the free motion wave, i.e. the position where the extrapolation of the free motion wave reaches the $r$-axis. The logarithmic derivative $\chi'/\chi$ in the matching point is clearly inversely proportional to the size $\Delta$ of the effective reaction region. For a fixed value of $\chi$ at the matching point $m$, the larger the derivative $\chi'$ in $m$ the larger the peak value of the wavefunction in the free motion region. In this example the wavelength is 10 fm, and is much larger than $\Delta$, with the consequence that the peak value of the free motion function is much larger than its value at the matching point. With a wavelength of e.g. 0.5 fm (i.e. $\ll \Delta$) this would not be true.
comes much smaller than its possible unitarity limit. In the limit of very small momenta this is also established by the well known\[^8\] low energy limit for the phase shifts: \( \delta_l \propto k^{2l+1} \) for \( k \to 0 \). The unitarity limit is reached when a partial wave is completely absorbed in the reaction process, which means \( \exp(i\delta_l) = 0 \), i.e. \( \Im(\delta_l) = \infty \), so the unitarity limit cannot be attained at small enough \( k \). Uncertainty considerations suggest that for \( k >> 1/\Delta \) it is possible, for strong enough reactions, to saturate the unitarity limit, while for \( k << 1/\Delta \) we are in the situation where \( \delta_l = O(k^{2l+1}) \), whatever the strength of the reaction.

On the ground that the projectile wavefunction \( \chi \equiv r \Psi(r) \) is completely damped within a range \( \Delta \) (i.e. \(|\chi|\) is large for \( r > R_{\text{nucleus}} + \Delta \) and very small for \( r < R_{\text{nucleus}} \)) it is straightforward to demonstrate that for the scattering length \( \alpha \) we have (approximately):

\[
\Im(\alpha) \approx -\Delta,
\]

\[
\Re(\alpha) \approx +R_{\text{nucleus}}.
\]

These results derive from the observation that the \( \chi \) damping requirement implies, for the logarithmic derivative of the projectile wavefunction, \(|\chi'/\chi| \approx 1/\Delta\). This is an obvious geometrical consequence of the damping requirement \( \chi(R - \Delta) << \chi(R) \), but in more physical terms it is a consequence of the uncertainty principle. When the absolute value of the logarithmic derivatives of the free motion wavefunction \(|\chi'/\chi|_{r=R_{\text{nucleus}}+\Delta} = |k \cdot \cotg\{k(R_{\text{nucleus}} + \Delta - \alpha)\}|\) is matched with the corresponding quantity for the wavefunction in the annihilation region \(|\chi'/\chi|_{r=R_{\text{nucleus}}+\Delta} = 1/\Delta\), the previous values for the scattering length are obtained in the limit \( k << 1/\Delta \). An illustration of this is given in fig.3.

A paradoxical consequence is that a smaller \( \Delta \) corresponds to what would be a stronger reaction at large energies, so that at low energies “stronger” interactions lead to a smaller reaction rate.

The above values of course are deduced from approximate equations, so they represent just estimates, however they suggest that the antineutron annihilation cross sections should not show a systematic increase with the target mass number \( A \). Such an increase could be present for antiproton annihilations, but because of Coulomb effects only. When going to any specific target nucleus, non-systematic effects will be present, related with the structure of the nuclear surface.

The exposed mechanism has an interesting consequence in the case of optical potential analyses: an increase of the strength of the imaginary part of the optical potential can lead to a decrease of the consequent reaction rate at small momenta. In the above language, an increase in the potential strength leads
to a decrease in the size parameter $\Delta$, since the absorption of the projectile wavefunction takes place in a shorter range. Also modification of other parameters (radius, diffuseness, etc) leads to consequences that are not necessarily the most obvious ones. A relevant example is given next.

In fig.1 we have shown, together with data, two optical potential fits. The $\bar{p}p$ fit uses a slight modification of an optical potential used by another group[17] to fit elastic data at 200-600 MeV/c. It is a Woods-Saxon shape with $V_I = -8000$ MeV, $V_R = -46$ MeV, $R_I = 0.52$ fm, $R_R = 1.89$ fm, $a_I = a_R = 0.15$ fm. The Coulomb potential is the potential of a spherical charge distribution with radius $R_c = \sqrt{R_p^2 + R_\bar{p}^2} = 1.25$ fm. The interesting point is that the $\bar{n}p$ fit is obtained either by increasing $V_I$ to 14000 MeV, or by increasing $R_I$ to 0.75 fm (in addition to removing Coulomb effects). In both cases a “more effective” annihilating potential leads to a smaller cross section.

It is interesting to observe that the fit by an energy independent optical potential can reproduce very well the $\bar{p}p$ annihilation cross section, and follows the “average” trend of the $\bar{n}p$ one, but is unable to reproduce the broad peak that is present at 150-350 MeV/c over this “background”. At very low momenta the $\bar{n}p$ cross section rises again, reaching the “uncharged $\bar{p}p$” one, represented by the solid line in fig.1. So we speak of a “regular background”, dominant in the $\bar{p}p$ case, also relevant in the $\bar{n}p$ case, that can be reproduced by an energy independent optical potential, and of a “gap/peak” structure that corresponds to a more complicate physics than pure absorption. This structure is evidently characteristic of the isospin-1 channel.

To conclude, we may say that the nuclear annihilation rates at low energies can be partly explained by the difference between $\bar{p}p$ and $\bar{n}p$ interactions. On the other side, a strong shadowing is present. We think this shadowing to be due to the “inversion” behavior of the strongly absorbing processes at low energies, inversion behavior that can be justified as a manifestation of the Heisenberg uncertainty principle. This also has the consequence that probably, despite appearances, $\bar{n}p$ interactions are stronger than $\bar{p}p$ ones at low energies. We also stress that the ratio between $\bar{n}p$ and $\bar{p}p$ annihilation rates presents an oscillating behavior that cannot be justified by regular absorption only.

References

[1] A.Bianconi, G.Bonomi, M.P. Bussa, E.Lodi Rizzini, L.Venturelli and A.Zenoni: nucl-th/9910031, nucl-th/9912025, Phys. Rev. C 62 (2000) 014611; nucl-th/0002013, Phys. Lett. B 483 (2000) 353; nucl-th/0003006, in print on Eur. Phys. Jour. A; nucl-th/0007058.
[2] A. Zenoni et al, Phys. Lett. B 461 (1999) 405; A. Zenoni et al, Phys. Lett. B 461 (1999) 413; A. Bianconi et al, Phys. Lett. B 481 194.

[3] M. Augsburger et al, Phys. Lett. B 461 (1999) 417.

[4] C.J. Batty, E. Friedman and A. Gal, Phys. Rep. 287 (1997) 385; Same authors: nucl-th/0010006.

[5] S. Wycech, A.M. Green and J.A. Niskanen, Phys. Lett. B 152 (1985), 308. G.Q. Liu, J.M. Richard and S. Wycech, Phys. Lett B260 (1991) 15.

[6] J. Carbonell and K.V. Protasov, Hyp. Int. 76 (1993) 327.

[7] E. Friedman and G. Soff, J. Phys. G: Nucl. phys. 11 (1985) L37.

[8] E.g.: L.D. Landau and E.M. Lifshits: A course in theoretical physics, vol. III “quantum mechanics”.

[9] F. Iazzi et al, Phys. Lett. B 475 (2000) 378.

[10] G.S. Mutchler et al, Phys. Rev. D 38 (1988) 742.

[11] T. Kalogeropoulos and G.S. Tzanakos, Phys. Rev. D 22 (1980) 2585.

[12] R. Bizzarri et al, Il Nuovo Cimento A (1980) 225.

[13] K.V. Protasov, “Workshop on hadron spectroscopy 99”, March 8-12, 1999, LNF, Frascati (Italy), 463; K.V. Protasov, G. Bonomi, E. Lodi Rizzini and A. Zenoni, in print on Eur. Phys. Jour. A (1999).

[14] Ye. S. Golubeva and L.A. Kondratiuk, Nucl. Phys. B (proc. suppl.) 56A (1997) 103.

[15] Reviews on several aspects of antinucleon-nucleon processes can be found, e.g., in: C.B. Dover, T. Gutsche, M. Maruyama and A. Faessler, Progr. Part. Nucl. Phys. 29 (1992) 87. C. Amsler and F. Myhrer, Ann. Rev. Nucl. Part. Sci. 41 (1991) 219. B.O. Kerbikov, L.A. Kondratyuk and M.G. Saposnikov, Sov. Phys. Usp. 32 (1989) 739. W. Weise, Nucl. Phys. A558 (1993) 219c. A.M. Green and J.A. Niskanen, International Review of Nuclear Physics, Vol. 1, pg. 570, World Scientific, 1984. J.J. De Swart and R. Timmermans, “The Antibaryon - Baryon interactions”, Proceedings of the LEAP’94 Conference, World Scientific 1994, p. 20. Data up to 1996 are reviewed in: G. Bendiscioli and D. Kharzeev, Rivista del Nuovo Cimento 17 (1994) 1.

[16] J. Haidembauer, T. Hippchen, K. Holinde and J. Speth, Z. Phys. A 334 (1989), 467.

[17] W. Brückner et al, Z. Phys. A 339 (1991), 379; W. Brückner et al, Z. Phys. A335 (1990) 217.

[18] A.S. Jensen, in “Antiproton-nucleon and antiproton-nucleus interactions”, eds. F. Bradamante, J.-M. Richard and R. Klapish, Ettore Majorana international science series, Plenum Press 1990, p. 205.