ANALYTICAL DETERMINATION OF RESIDUAL STRESSES FROM SURFACE TOPOGRAPHY CREATED BY LASER CUTTING TECHNOLOGY

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Abstract
This paper describes a numerical modeling method that utilizes stress equations derived from surface topography created by laser cutting technology. The explanatory ability of the geometric phenomena of the created surface topography is an essential analytical tool for the physical-mathematical formulation of the principle of laser cutting processes. As experimental material was used structural steel of type S355J0 and CP-Ti Grade 2. A cutting machine from Prima Industry - ZAPHIRO / CV5000 was used. The new derived relations for the determination of the residual stress were determined and verified by experiment. The objective of this paper is to contribute to the understanding of process-related stresses and deformations generated by laser cutting technology, particularly by ultrasound testing of stress inside the material and significantly influenced by surface topography.

Keywords
residual stress, laser cutting, surface topography, stress - strain, ultrasound, deformation parameters

1 INTRODUCTION

Rapidly expanding industry with so many increasing requirements for production quality presupposes and requires a continuous and everlasting innovation and introduction of new technologies that meet the demanding criteria given by a flexible free market, as well as the demands of the environment. The production technologists are now confronted with a need to adapt the existing technologies to the specific properties of new materials, or elsewhere to develop and invent the new ones. One of the few tools that can satisfy the requirements set by the technologist and thus to adapt to the new trends in engineering materials with special properties is the laser beam or rather the laser beam cutting (LBC) technology. The authors dealt with a comparison of the most frequently used thermal cutting technologies (oxygen cutting, plasma cutting, laser cutting) with a focus on experimental measurement and evaluation of characteristics of the heat affected zone. The smallest heat affected zone (that has been evaluated on the base of microhardness measurements and metallographic analysis) was achieved after the laser cutting, whereas there were no large differences when comparing different thicknesses of a material (low carbon steel) and the values of microhardness were comparable to the plasma cutting. The laser cutting process has several advantages over the conventional cutting methods. Lasers are used for cutting since the 1970s. The principle of this kind of material cutting consists in focusing the laser beam on a workpiece, the material heats up so much that it melts or evaporates. Just heat is one of the dominant factors causing the distortion and residual stresses during laser cutting process. Residual stresses remain after all applied boundary conditions have been removed. The basic cause of residual stress is non-uniform strain due to some previously performed operations. This non-uniform strain will arise from plastic deformation, phase transformations, or lattice mismatch. Residual stress may be decreased or eliminated by annealing or by certain mechanical treatments [1 - 6]. In laser beam fusion cutting, solidification of the material and its respond to the ambient material give rise to permanent deformations and residual stresses in surface layers of the cutting surface. The size and course of deformation and residual stresses depends on several factors. One of the most important factors are stress-strain and heat-technical characteristics such as the elastic limit Re, proportional limit Rr, Young’s modulus E, yield strength R, Rp.2, breaking strength Rm, elongation at failure A, specific heat C or density of the material . The prediction and numerical derivation of residual stresses is not an easy task due to the complexity involved in the laser cutting process. The principal difference of the presented approach to the problem of interpretation - in comparison to the current approaches of identification of residual stress - is the consideration of the final surface state to achieve the complete understanding of the deformation processes. Thus, in our opinion much greater emphasis should be put on the deformation and stress-strain properties of the cut material. Analytical solutions can be obtained only for simple conditions. Models with analytical solutions are very helpful in understanding of the physical processes involved, but it is difficult to derive them. Therefore, in real life, numerical solutions are sought when analytical solutions are not available. Finite difference and finite element methods are the most commonly used numerical
approaches for modeling of residual stresses. Besides the experimental analyses of residual stress distributions, the use of "prediction" methods based on mathematical modeling of technological processes generating stress fields has recently started. This paper deals with the development of the new method for estimation of residual and flow stresses in steel plates being cut by laser [7 - 14].

2 EXPERIMENTAL PROCEDURE

The experiment was carried out within the framework of an academic cooperation between the Institute of Geonics AS CR, v. v. i. in cooperation with the Slovak firm Metakov, Ltd., Spisska Nova Ves (Mr. Slivka), supported by the IT4Innovations Centre of Excellence project, reg. no. CZ.1.05/1.1.00/02.0070 supported by the Operational Programme ‘Research and Development for Innovations’ funded by Structural Funds of the European Union and state budget of the Czech Republic, where our main goal was to develop the numerical method of residual stress determination for laser cutting technology.

2.1 Material

Flat specimens were made from structural steel of type S355J0 in accordance with DIN EN 10027-1 (DIN EN 10027-1 : Designation systems for steels - Part 1: Steel names), with a pearlite-ferrite matrix. It is a non-alloyed fine grain structural steel with a guaranteed cold weldability. It is suitable for welded structures with higher strength, for machinery parts, and transport equipments, for the production of low stress rotating parts. The mechanical parameters are given in Table 1. In order to compare the residual stress after the laser cutting process, it was also used CP-Ti Grade 2 (Table 2) according to ASTM (ASTM B265-99 Standard Specification for Titanium and Titanium Alloy Strip, Sheet, and Plate).

Table 1

| Parameter       | S355J0 | AlMg3 | CP-Ti Grade 2 |
|-----------------|--------|-------|---------------|
| Thickness h     | 20     | 8     | 10            |
| Lens (inch)     | "9"   | "7.5" | "7.5"         |
| Traverse speed  | 830    | 1200  | 550           |
| Power P (W)     | 4600   | 5000  | 3500          |
| Assist gas      | O₂     | N₂    | N₂            |
| Pressure p (bar)| 0.6    | 15    | 17            |
| Stand-off distance z (mm) | 1.2 | 0.8 | 1.5 |
| Focus (mm)      | 6      | -8    | -8            |
| Nozzle diameter d (mm) | 2.5 | 2   | 2             |

Table 2

Note: Mean value of yield limit $R_{p0.2}$ = 241 MPa was determined in a 29 percent probability interval; mean value of relative elongation $A$ = 31 % was determined in a 19 percent probability interval.

2.2 Cutting experiment

In order to identify residual stresses, the samples of 150 × 150 mm size were cut in different thicknesses. The cutting process was carried out at optimal machine settings, i.e. the settings proposed by a control system after entering the parameters of the material being cut. For the realization of experiments, a cutting machine from Prima Industry - ZAPHIRO / CV5000 was used. It is high-speed 2D laser cut machine. The selected laser source was a CO₂ laser with a wavelength of 10.6 μm. A total of 30 experiments were carried out using the mentioned laser machine. The parameters for cutting of used materials are given in Table 3.

The cutting quality was evaluated according to the international standard UNI EN ISO 9013. The surface topography was analyzed by means of a non-contact profilometer Talysurf CLI 2000 and a contact profilometer SURFTEST SJ401 (Fig. 1).

Figure 1. Principle of the surface roughness measurement

2.3 Ultrasonic measurement of residual stresses

The pulse–echo-overlap method was used for the measurement of ultrasonic waves. The echo signals were captured using the normal beam transducer of longitudinal wave mode (a dual element transducer with a diameter and a center frequency of 6.35 mm and 10 MHz). The transducer broadcasts the ultrasonic pulse at the surface of the specimen. The ultrasonic pulse travels through the specimen and reflects off the opposite face. The transducer receives the reflected echoes. The ultrasonic transducer operates as both transmitter and
receiver in one unit. Ultrasonic velocity measurements have been carried out across the cut thickness at different depths \( h_{\text{cut}} \).

### 3 MODELLING OF DEFORMATIONS AND STRESSES ACCORDING TO THE SURFACE TOPOGRAPHY

The best way how to proceed to a comprehensive examination of the deformation of solids is to study the surface geometry after laser cutting. The main parameters of the wall cut geometry are suggested as follows: surface roughness profile parameter \( R_a \), deviation of trace \( Y_{\text{ret}} \), \( \delta \) – deviation angle of trace and depth of cut \( h \) (Fig. 2).

**Figure 2. Determination of deformation parameters \( R_x \), \( Y_{\text{ret}} \), \( \delta \), \( h \) at selected point \( X \) on cut wall after CO2 laser cutting.**

An analytical solution for the profile topographical function is very complex; this is the reason why an impact of an initial zone has not been taken into account, and why many authors replaced a functionality of the actually proved roughness \( R_{\text{ar}} = f(h) \) by easier definable functions. This process has thus led to an idealization and distortion. The laboratory results are in many cases different; the researchers use different thicknesses, whereas they do not investigate the entire surface – they neglect an entry area (initial zone). We derived a more realistic approximation for the calculation of the actual surface roughness \( R_{\text{ar}} \) with the use of an auxiliary function, which is dependent on the flow stress in the radial direction \( R_{\text{rad}} = f (G_{\text{det}}) \) given by Eq. (1):

\[
R_{\text{rad}} = R_{\text{ar}} \cdot 10^{0.05 \cdot \sigma / E_{\text{mat}}}
\]  

(1)

Physical equations are in general applicable for all materials. The equations are designed to include the direct effect of technological parameters \( v_p \), \( W_{\text{las}} \), \( d \), \( p \) in order to find the optimized process parameters, as well as for their on-line control. A typical course of the topographical function is shown in Fig. 3 with subsequent verification according to the values, measured by optical and mechanical profilometer.

**Figure 3. Course of main topographical function for the material EN S355J0 by Eq. (11) for the process parameters \( v_p = 1000 \text{ mm} \cdot \text{min}^{-1}, W_{\text{las}} = 3000 \text{ W}, p = 0.6 \text{ MPa, } d = 2 \text{ mm; verified for various experimental measurements, with correlation coefficient } R^2 = (0.95 - 0.95).**

The course of \( \delta = f (h, t) \) is shown in Fig. 4, with the following symbols, \( h_{\text{in}} \) – depth of initial zone, \( h_{\text{lim}} \) – depth at the point of elastic limit, \( h_{\text{max}} \) – depth at the yield point, \( h_{\text{max}} \) – depth at the point of \( \delta_{\text{max}} = 90^\circ \). The residual stresses can be expressed by the following relationships Eq. (2) at the general level \( h \), Eq. (3) at the neutral plane \( h_{\text{ret}} \) and Eq. (4) in direct functional relationship to the angle of deviation \( \delta \):

\[
\sigma_{r_{\text{est}}} = \sqrt{E_{\text{mat}}} \cdot \sin \left( \frac{\arccos \delta}{180} \right)^2
\]  

(2)

\[
\sigma_{r_{\text{est}0}} = \sqrt{E_{\text{mat}}} \cdot \sin \left( \frac{\arccos \delta_0}{180} \right)^2
\]  

(3)

\[
\sigma_{r_{\text{est}}0} = R_{p,0.2} \cdot \sin \left( \frac{\arccos \delta_0}{180} \right)^2
\]  

(4)

The reason for that can be explained by Eq. (2), which defines the stress-strain state in the cut. Theoretically, these equations can be supplemented by the equations relative to other elements of the surface topography given in Fig. 3. Compared to existing methods of solving, the authors have focused on the parameter \( \delta \) and other functionally related parameters to the surface topography, i.e. roughness \( R_a \) and retardation of the trace \( Y_{\text{ret}} \) according to the depth of cut \( h \). Therefore, it is possible to use the new derived relations for the determination of the residual stress from the beginning to the limit depth of cut and in the immediate vicinity of the cutting kerf. According to Eq. (2), there is also a functional relation between the cut depth \( h \) and material type. Apart from other functional relations to the technology parameters, the rate of external deformation stress affects the main parameters of the surface geometry. To a large extent, it is affected by the residual stress intensity induced in the process of material disintegration. This can be calculated according to the following implicit Eq. (5):

\[
(\delta, R_a, Y_{\text{ret}}) = f (h, E_{\text{mat}}, R_{p,0.2}, \sigma_{\text{det}}, \sigma_{\text{res}})
\]  

(5)
The model of the cutting kerf in Fig. 5 illustrates both, i.e. the intensity of residual stress distribution as calculated according to Eq. (2) for material of type EN S355J0 and changes in the longitudinal ultrasonic velocity $\Delta V_{\text{total}}$.

Changes in the total longitudinal ultrasonic velocity $V_{\text{total}}$ were measured in horizontal lines in the direction of depth cut of the sample by the pulsed ultrasound method. By subtracting the the pre-measured static values $V_{\text{lm}}$, there were determined differences in values for the given material, the functional relationships $\Delta V_{\text{c}} = \Delta V_{\text{total}}$ were evaluated and consequently compared with calibrated values. The cutting kerf position corresponds to the maximum achievable cut depth at the level $h_{\text{max}} = 18.5$ mm where $\delta = \delta_{\text{max}} = 90^\circ$. The value of residual stress $\sigma_{\text{res}}$ reaches here a maximum value of 450 MPa. According to Eq. (2), the residual stress decreases exponentially and at a certain speed on both sides of the cutting kerf towards the material of both parts of the sample. The measured values of the longitudinal ultrasonic velocity at equal distances of 12 mm go into negative values $\Delta V_{\text{c}}$. In the case of the left side of the sample, this is concerned with achieving the yield point at $h_{\text{y}} = 6.7$ mm. This would demonstrate the assumed existence of the neutral stress-strain plane, where the pressure and the tensile stress are balanced in the cut and it holds that $d_0 = d_0$. When comparing the change of ultrasound velocity at two sides of the cutting kerf, it is evident that both profiles are almost identical. This indicates that the residual stress distributions developed at both sides of the cutting kerf are similar. The thermal contribution is defined by the plastic compressive strain, thus, if the cutting temperature exceeds the phase transition temperature (approx. 730 °C for steel) the resulting stresses tend to be compressive. It can be assumed that the compressive residual stresses are more advantageous in terms of the structure of the material under investigation than the tension residual stresses, as they do not encourage cracking. However, if the compressive residual stresses are very high, deformations occur and, as a result, the mechanical properties of the material deteriorate. In the case of the neutral depth of the cut, we expect that the compressive and tensile component will be in equilibrium, and in the case of the particular cut depth, we expect theoretically to predominate the compressive component. In the experimental section below, the prediction is verified.

4 EXPERIMENTAL VERIFICATION

The experimental work was focused to check the validity of Eq. (2) on samples of titanium ($h_{\text{max}} = 60$ mm) and steel EN S355J0 ($h_{\text{max}} = 20$ mm). The sample thickness was chosen according to the calculations in order to get a minimum value of 75 for the deviation angle of trace $\delta$ achieved by the cutting process of all selected materials. The total velocity change for longitudinal wave $V_{\text{total}}$ was measured in equidistant lines at the cutting depth of both parts of the sample by the pulsed ultrasound method, always after cooling the sample for five minutes. According to the static value $V_{\text{lm}}$ being measured in advance, there were determined differential values $\Delta V_{\text{c}}$ for the given material, afterwards there were evaluated functional relationships $\Delta V_{\text{c}} = f (\sigma_{\text{res}}, h)$ and consequently compared with calibrated values. The deviation angle was determined indirectly from the dependence of $\delta = \arctg (Y_{\text{ret}} / h)$. The deviation of trace $Y_{\text{ret}}$ from the radial plane and the depth line was measured at regular intervals on both parts of the sample. For the control and comparison of the residual stress calculation $\sigma_{\text{res}}$ according to Eq. (2), there were taken into account the modified relations according to other authors [15, 16, 17] based on the von Mises hypothesis. Very interesting in relation to the residual stress is also the solution Eq. (8) according to [18]. Equation (6) obtained from [16] has been modified and used in the form Eq. (7) to express the residual stress in the vicinity of the cutting kerf at the maximum deep level $h_{\text{max}}$ for reaching the maximum $\delta_{\text{max}}$.

$$
\sigma_{\text{res}} = \frac{E}{(1 + \nu) \sin^2 \psi} \left( \frac{d_0 - d_0}{d_0} \right)$$

(6)

where $E$ is the Young’s modulus of the substrate material, $\nu$ is Poisson’s ratio, $\psi$ is the tilt angle and $d_0$ represent the spacing measured at each tilt angle, i.e. the spacing between outer $d_0$ and inner diameter $d_0$ of the hole.

$$
\sigma_{\text{res}} = \left( \frac{10^{-6} \cdot E_{\text{res}}}{(1 + \mu) \cdot \sin \left( \arcsin \left( \frac{\pi}{180} \right) \right)^2 \cdot h_{\text{ret}}} \right)^{-1}
$$

(7)

According to Kreutzer:

$$
\sigma_{\text{res}} = \frac{4 \pi (1 - \mu) d_{\text{Em}}}{G \cdot b}
$$

(8)

$$
d_{\text{Em}} = \frac{G}{4 \pi (1 - \mu^2) \sqrt{E_{\text{res}}}}
$$

(9)

where $d_{\text{Em}}$ is the dislocation dipole, $G$ is the shear modulus, $b_{\text{cub}}$ is the Burgers vector for a cubic lattice, and $\gamma_{\text{Em}} = \sigma_{\text{res}}$ is the critical shear stress.

Based on quantitative stress analysis using the ultrasonic technique, it is possible to use the formulas for calculating the function $v_i$ in the form of Eq. (10).

$$
v_{\text{Ltotal}} = v_{\text{Lm}} + \beta \cdot \sigma_{\text{res}} \Rightarrow v_{\text{Ltotal}} = v_{\text{Lm}} + \Delta v_{\text{L}}
$$

(10)
where $v_{Lm}$ is the ultrasound wave velocity at steady state ($\sigma_{res} = 0$)

$$\Delta v_L = v_{L_{total}} - v_{Lm}$$ (11)

and

$$\Delta v_L = \beta \cdot \sigma_{res}$$ (12)

and the acoustoelastic constant

$$\beta = \frac{\Delta v_L}{\sigma_{res}}$$

The values of $\beta$ for each material are as follows: for titanium $\beta_T = 4 \text{ m}^2\text{·s}^{-1}\cdot\text{MPa}^{-1}$ and for used steel $\beta_T = 1.184 \text{ m}^2\text{·s}^{-1}\cdot\text{MPa}^{-1}$. Residual stresses at each point were calculated by using the relation Eq. (11) for the ultrasonic wave velocity change. Moreover, it should be also noted that there was an interesting agreement with the results obtained by Yilbas [15, 16, 17]. Comparison of the values determined by the above given relations and the measured values are given in graphic form on the following charts, Fig. 6, 7, 8 and 9.

The figures show the values of calculated residual stress $\sigma_{res}$ and the values being measured by ultrasound $\sigma_{resM}$. First of all, this concerns the changes of wave velocity ($+/−v_L$). There is achieved a very good correlation between the measured and calculated residual stress according to Eqs. (3), (7), and (8). The correlation coefficient range is $R^2 = 0.85$ - 0.99.

### RESULTS AND DISCUSSION

The proposed new method allows to calculate the values of the residual stress distribution in the vicinity of the cutting kerf after the cutting process is finished through the geometrical characteristics of surface topography. The results show that there is a possibility to focus on the expected geometrical and physical-mechanical distribution similarity of the angle of internal friction $\delta$ at the shear plane depending on the depth of cut. This angle takes effect on the newly created surface as the well-measurable deviation angle of the trace caused by the retardation of the cut trace $Y_{ret}$ in relation to the radial plane below the tool. Together with the mechanical parameter of cuttability of materials $K_{cut}$ and immediate

![Figure 8. Comparison of residual stress for steel.](image)

![Figure 9. a) Course of values of $v_{L_{total}}, \Delta v_L$ in relation to arc $\delta$; b) Course of values of $v_{L_{total}}, \Delta v_L$ in relation to ores for steel.](image)
depth of cut $h$ (t), these elements of the surface topography express the instantaneous and equilibrium stress-strain state of the cutting kerf. The surface deformation features are determined by the surface deformation capacity according to the material type and the used cutting technology. Using the analytical expression for the balance, e.g. by Eq. (2), it is possible to transform the equation for calculation of $\sigma_{\text{res}} = f(\delta)$ by Eq. (2) to the use of other variables represented in the equation of balance. As it is for example, transformation into the function $\sigma_{\text{res}} = f(Ra)$ according the Eqs. (7) and (8). Therefore, it is possible to generally write that $\sigma_{\text{res}} = f(\delta, V_{\text{cut}}, Ra, h, K_{\text{cut}})$. The existence of residual stress in the material can be demonstrated by measuring the velocity change of ultrasonic longitudinal waves ($+/-$) $\Delta v_L$ compared to the static value of the given material.

6 CONCLUSION

The objective of this study is to propose the new method of numerical derivation of flow and residual stresses caused by laser cutting technology. In the context of this work, the ultrasonic testing method was used to examine and study the stresses locked inside materials as a result of manufacturing process. In order to perform ultrasound measurement, the location for the measurement was first defined. In this case, the measurement of ultrasound velocity was made at regular intervals over the sample. The velocity change was observed all over the sample. The wave velocity is different at each measuring point of the sample. Residual stress in the sample affect the wave velocity in the material. The results using the ultrasound measurement on both sides of the cutting kerf showed a possible pattern of compressive and tensile stresses. There has to be the balance defined by the neutral plane, when the yield point is being reached. The surface topography has a significant effect on the ultrasound measurement. Thanks to this model, it is possible to predict limit depths, surface roughness and main deformation functions in many materials as it can be seen in Fig. 4, 5 and 6.

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