Extracting the three- and four-graviton vertices from binary pulsars and coalescing binaries

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Using a formulation of the post-Newtonian expansion in terms of Feynman graphs, we discuss how various tests of General Relativity (GR) can be translated into measurement of the three- and four-graviton vertices. In problems involving only the conservative dynamics of a system, a deviation of the three-graviton vertex from the GR prediction is equivalent, to lowest order, to the introduction of the parameter $\beta_{\text{PPN}}$ in the parametrized post-Newtonian formalism, and its strongest bound comes from lunar laser ranging, which measures it at the 0.02% level. Deviation of the three-graviton vertex from the GR prediction, however, also affects the radiative sector of the theory. We show that the timing of the Hulse-Taylor binary pulsar provides a bound on the deviation of the three-graviton vertex from the GR prediction at the 0.1% level. For coalescing binaries at interferometers we find that, because of degeneracies with other parameters in the template such as mass and spin, the effects of modified three- and four-graviton vertices is just to induce an error in the determination of these parameters and, at least in the restricted PN approximation, it is not possible to use coalescing binaries for constraining deviations of the vertices from the GR prediction.

PACS numbers: 04.30.-w, 04.80.Cc, 04.80.Nn

I. INTRODUCTION

Binary pulsars, such as the Hulse-Taylor [1] and the double pulsar [2], are wonderful laboratories for testing General Relativity (GR). They have given the first experimental confirmation of the existence of gravitational radiation [3, 4], provide stringent tests of GR and allow for comparison with alternative theories of gravity, such as scalar-tensor theories [5–14] (see [15–17] for reviews). Another very sensitive probe of the non-linearities of GR is given by the gravitational wave (GW) emission during the last stages of the coalescence of compact binary systems made of black holes and/or neutron stars, which is one of the most promising signals for GW interferometers such as LIGO and Virgo, especially in their advanced stage, and for the space interferometer LISA. Various investigations have been devoted to the possibility of using the observation of coalescing binaries at GW interferometers to probe non-linear aspects of GR [9, 10, 18–24].

Compact binary systems probe both the radiative sector of the theory, through the emission of gravitational radiation, and the non-linearities intrinsic to GR which are already present in the conservative part of the Lagrangian. In a field-theoretical language, these non-linearities can be traced to the non-abelian vertices of the theory, such as the three- and four-graviton vertices.

It is therefore natural to ask whether from binary pulsars or from future observations of coalescing binaries at interferometers one can extract a measurement of these vertices, much in the same spirit in which the triple and quartic gauge boson couplings have been measured at LEP2 and at the Tevatron [25–28].

In this paper we tackle this question. The organization of the paper is as follows. In Section II we discuss how to “tag” the contribution of the three- and four-graviton vertices to various observables in a consistent and gauge-invariant manner, and we compare it with other approaches, such as the parametrized post-Newtonian (PPN) formalism [16]. In particular, we find that the introduction of a modified three-graviton vertex corresponds – in the conservative sector of the theory and at first Post-Newtonian order (1PN) – to the introduction of a value for the PPN parameter $\beta_{\text{PPN}}$ different from the value $\beta_{\text{PPN}} = 1$ of GR. However, a modified three-graviton vertex also affects the radiative sector of the theory, which is not the case for the PPN parameter $\beta_{\text{PPN}}$. We also discuss subtle issues related to the possible breaking of gauge invariance which takes place when one modifies the vertices of the theory. In Section III we present our computations with modified vertices, and in Section IV we compare these computations with experimental results obtained from the timing of binary pulsars and with what can be expected from the detection of gravitational waves (GWs) at ground-based interferometers or with the space interferometer LISA. Section V contains our conclusions.

II. TAGGING THE THREE- AND FOUR-GRAVITON VERTEXES

Our aim is to quantify how well the non-linearities of GR can be tested by various existing or planned experiments/observations. Historically, there have been several approaches to this problem and, basically, one can identify two complementary strategies. The first is to develop a purely phenomenological approach in which deviations from GR are expressed in terms of a number of parameters, without inquiring at first whether such a deformation of GR can emerge from a fundamental theory. An
example of such an approach is the parametrized post-
Newtonian (PPN) formalism. In its simpler version, it
consists of writing the 1PN metric generated by a source,
treated as a perfect fluid with density $\rho(x)$ and velocity
field $v(x)$, in the form

$$g_{00} = -1 + 2U - 2\beta_{PPN}U^2, \quad (1)$$

$$g_{0i} = -\frac{1}{2}(4\gamma_{PPN} + 3)V_i, \quad (2)$$

$$g_{ij} = (1 + 2\gamma_{PPN}U)\delta_{ij}, \quad (3)$$

where

$$U(x) = \int d^3x' \frac{\rho(x')}{|x - x'|}, \quad (4)$$

$$V_i(x) = \int d^3x' \frac{\rho(x')v_i(x')}{|x - x'|}, \quad (5)$$

and the standard PPN gauge has been used [16, 29]
(we use units $c = 1$). General Relativity corresponds
to $\beta_{PPN} = 1$ and $\gamma_{PPN} = 1$. More phenomenological
parameters can be introduced by working at higher PN
orders, see [16]. One then investigates how deviations of
$\beta_{PPN}$ and $\gamma_{PPN}$ from their GR values affect various
experiments. Writing $\beta_{PPN} = 1 + \beta$ and $\gamma_{PPN} = 1 + \gamma$, the
best current limits (at 68% c.l.) are

$$\gamma = (2.1 \pm 2.3) \times 10^{-5} \quad (6)$$

from the Doppler tracking of the Cassini spacecraft, and

$$4\beta - \gamma = (4.4 \pm 4.5) \times 10^{-4} \quad (7)$$

from lunar laser ranging. This bound comes from the
Nordtvedt effect, i.e. from the fact that, in a theory with
$\beta$ and $\gamma$ generic, the weak equivalence principle is viol-
ated and the Earth and the Moon can fall toward the
Sun with different accelerations, which depend on their
gravitational self-energy. The effect is studied by moni-
toring the Earth-Moon distance with lunar laser ranging.
The perihelion shift of Mercury gives instead the bound
$|\beta| < 3 \times 10^{-3}$ [16, 30].

To get these bounds, we do not need to know the funda-
mental theory that gives rise to values of $\beta_{PPN}$ and $\gamma_{PPN}$
that differ from their GR values. However, it is of course
interesting to see that consistent field theories exist that
give rise to values of $\beta_{PPN}$ and $\gamma_{PPN}$ different from one.
For instance, a Brans-Dicke theory with parameter $\omega_{BD}$
gives $\beta_{PPN} = 1$ and $\gamma_{PPN} = (1 + \omega_{BD})/(2 + \omega_{BD})$, with the
GR value $\gamma_{PPN} = 1$ recovered for $\omega_{BD} \to \infty$, while more
general tensor-scalar theories can produce both $\gamma_{PPN} \neq 1$
and $\beta_{PPN} \neq 1$. However, in the PPN approach, one can
also explore other possibilities, such as PPN parameters
that correspond to preferred-frame effects or to viola-
tion of the conservation of total momentum, which are
not necessarily well-motivated in terms of current field-
theoretical ideas on possible extensions or UV complet-
ions of GR. It is also important to observe that the pa-
rameters $\beta_{PPN}$ and $\gamma_{PPN}$ are gauge-invariant, and there-
fore observable, because they have been defined with
respect to a specific gauge, namely the standard PPN
 gauge in which the metric takes the form (1)–(3).

A second, complementary, approach to the problem
is to study a specific class of field-theoretical extensions
of GR. A typical well-motivated example is provided by
multiscalar-tensor theories. These have been studied in
detail and compared with experimental tests of relativis-
tic gravity in refs. [6, 9–14]. This approach has the ad-
van telephone that one is testing a specific and well-defined
fundamental theory. On the other hand, an experimen-
tal bound on the parameters of a given scalar-tensor the-
ory, such as for instance the bound $\omega_{BD} > 40000$ on the parameter
$\omega_{BD}$ of Brans-Dicke theory obtained from the
tracking of the Cassini spacecraft [31] is, strictly speak-
ing, only a statement about that particular extension of
GR and not about GR itself.

In this paper we quantify how well GR performs with
respect to experiments of relativistic gravity by studying
how much these experiments constrain the values of the
non-abelian vertices of the theory, in particular the three-
graviton vertex and the four-graviton vertex. We proceed
as follows. After choosing a gauge (the De Donder gauge,
corresponding to harmonic coordinates) we multiply the
three-graviton vertex by a factor $(1 + \beta_3)$ and the four-
graviton vertex by a factor $(1 + \beta_4)$, with constants $\beta_3$
and $\beta_4$. For $\beta_3 = \beta_4 = 0$ we recover GR. Observe that,
since $\beta_3$ and $\beta_4$ are defined with respect to a given gauge
choice, they are gauge-invariant by definition. This is in
fact the same logic used to define in a gauge-invariant
manner the PPN parameters $\beta_{PPN}$ and $\gamma_{PPN}$.

We then use a Feynman diagram approach to com-
pute the modifications induced by $\beta_3$ and $\beta_4$ on various
observables in classical GR. Diagrammatic approaches
and field-theoretical methods have been in use in clas-
sical GR for a long time, see e.g. [13, 32, 33]. We make
use of the effective field theory formulation proposed in
[34], which provides a clean and systematic separation
of the effects that depend on (model-dependent) short-
distance physics from long-wavelength gravitational dyna-
metics, and in the non-relativistic limit (after perform-
ing a multipole expansion) has manifest power counting
in the typical velocity $v$ of the source.

In the next section we see how these deformed vertices
give additional terms in the PN effective Lagrangian.
In particular we find that, in the conservative sector of
the theory, the introduction of $\beta_3$ is phenomenologically
equivalent, at 1PN level, to the introduction of a non-
trivial value of $\beta_{PPN}$ given by $\beta_{PPN} = 1 + \beta_3$. However,
$\beta_3$ also affects the radiative sector of the theory, i.e. the
Lagrangian describing the interaction between the mat-
ter fields and the gravitons radiated at infinity.

Before entering into the technical aspects, however, let
us clarify the meaning of the introduction of $\beta_3$ and $\beta_4$.
In ordinary GR, with $\beta_{3,4} = 0$, coordinate transforma-
tion invariance ensures that the negative norm states de-
couple. After gauge fixing (in the De Donder gauge for
instance), the kinetic terms for all of the ten components
of the metric are invertible, but four of them have the
wrong sign, i.e. they give rise to negative norm states. In the De Donder gauge the six positive-norm states that diagonalize the kinetic term are

\[ \hat{h}_{ij} \equiv h_{ij} + \frac{1}{2} \delta_{ij}(h_{00} - \delta^{lm}h_{lm}), \quad (8) \]

while the four “wrong-sign” components are given by the spatial vector \( h_{0i} \), and by the scalar

\[ \hat{h}_N \equiv h_{00} - \delta^{lm}h_{lm}. \quad (9) \]

In standard GR the existence of these negative-norm states do not create difficulties because they are coupled to four integrals of motion (energy, and the three components of angular momentum), so they cannot be produced. In contrast, the remaining six “healthy” components couple to the source multipole moments. After complete gauge fixing one finds that among the six positive-norm states, four obey Poisson-like equations, so they do not radiate (even though they are non-radiative physical degrees of freedom), while the remaining two are the radiative degrees of freedom representing GW’s [35].

Allowing \( \beta_3 \neq 0 \) has the effect that the negative norm state \( \hat{h}_N \) now couples, already at lowest order, to a non conserved quantity, namely to a combination of the Newtonian kinetic and potential energy of the binary system. This means that, in general, a modification of GR in which we just change the strength of the three-graviton vertices cannot be taken as a fundamental field theory, neither at the quantum level, nor even at the classical level, since the negative-norm state contributes to the classical radiated power (a related concern is that, for \( \beta_3 \neq 0 \), the energy-momentum tensor is in general not conserved).

A consistent classical and quantum field theory could in principle emerge from a simultaneous modification of all the vertices of the theory, such as the three-, four- and higher-order graviton vertices, together with a related modification of the graviton-matter couplings. As a trivial example, an overall rescaling of the gauge coupling in a Yang-Mills theory, or of Newton’s constant in GR, results in a combined modification of all the vertices, but obviously introduces no pathology. Anyway, our approach to the problem is purely phenomenological. We introduce \( \beta_3 \) and \( \beta_4 \) simply as “tags” that allow us to track the contribution of the three- and four-graviton vertices throughout the computations. As long as \( |\beta_3| \ll 1 \) and \( |\beta_4| \ll 1 \), the corrections that they induce to the radiated power are small compared to the standard GR result, so the total radiated power is given by the GR result plus a small correction, and in particular the total radiation emitted is positive. At this phenomenological level the introduction of modified vertices is therefore acceptable, and provides a simple and, most importantly, gauge invariant manner of quantifying how well different observations constrain the non-linear sector of GR, in a way which is intrinsic to GR itself, without reference to any other specific field theory.

In this sense, our approach is close in spirit to the phenomenological PPN approach, and can be seen as an extension of it where the radiative sector of the theory is also modified. Another approach which is related to ours is the one proposed in ref. [23, 24]. They consider the phase of the GW emitted during the coalescence of compact binaries, which up to 3.5PN has the form

\[ \Psi(f) = 2\pi f t_c - \Phi_c + \sum_{k=0}^{7} [\psi_k + \psi_{kl} \ln f] f^{(k-5)/3}, \quad (10) \]

where \( f \) is the GW frequency, and \( t_c \) and \( \Phi_c \) are the time and the phase at merger. The seven non-zero coefficients \( \psi_k \) with \( k = 0, 2, 3 \ldots 7 \) and the two non-zero coefficients \( \psi_{kl} \) with \( k = 5 \) and \( 6 \) are known from the PN expansion, in terms of the two masses \( m_1 \) and \( m_2 \). In ref. [23, 24] they study how the template is affected if these coefficients are allowed to vary, so that two of them, the 0PN coefficient \( \psi_0 \) and the 1PN coefficient \( \psi_2 \), are used to fix the masses \( m_1 \) and \( m_2 \) of the two stars, while varying any of the remaining coefficients with \( k \geq 3 \) provides a test of GR. In the case of coalescing binaries at interferometers, our introduction of \( \beta_3 \) and \( \beta_4 \) is a particular case of a more general analysis in which one treats the quantities \( \psi_k \) and \( \psi_{kl} \) as free parameters, but it has a sharper field-theoretical meaning since \( \beta_3 \) and \( \beta_4 \) measure the deviation from the three- and four-graviton vertices from the GR prediction. For the same reason, we are also able to compare the effect of \( \beta_3 \) on the waveform of coalescing binaries with its effect on binary pulsar timing and on solar system experiments, while in the phenomenological approach in which the parameters \( \psi_k \) and \( \psi_{kl} \) of the GW phase are taken as free parameters, a modification of the waveform of coalescing binaries cannot be related to a modification of the binary pulsar timing formula.

Another issue is whether a modification of the vertices of this form (typically with \( \beta_3 \), \( \beta_4 \), etc. not independent, but related to each other by some consistency conditions) could emerge from a plausible and consistent extension of GR. Actually, a typical UV completion of GR at an energy scale \( \Lambda \) will rather generate corrections to the vertices that are suppressed by inverse powers of \( \Lambda \), so it would give rise to an energy dependent \( \beta_3 \), e.g. \( \beta_3 = E^2/\Lambda^2 \), which furthermore, at the energy scales that we are considering and for any sensible choice of \( \Lambda \), would be utterly negligible. Still, let us remark that this kind of behavior is not a theorem. It assumes the UV-IR decoupling typical of effective field theories, and one can exhibit counterexamples. For instance, in non-commutative Yang-Mills theories there is a UV-IR mixing, such that low-energy processes receive contributions from loops where very massive particles are running, and these contributions are independent of the mass of these particles [36]. Anyway, again our aim here is not to test any given consistent extension of GR, but rather provide a simple and phenomenologically consistent way of quantifying how well various experiments can test the non-linearities of GR, and quantify how the results of different experiments compare among themselves.

It is also interesting to observe that, even when \( \beta_3 \) and \( \beta_4 \) are non-zero, the graviton remains massless at the
classical level, since $\beta_3$ and $\beta_4$ affect interaction terms, but not the kinetic term. The breaking of diffeomorphism invariance induced by $\beta_3$ and $\beta_4$ could in principle generate a graviton mass at the one-loop level. However, even if we are using the language of quantum field theory, in the end we are only interested in the classical theory, since quantum loops are suppressed by powers of $\hbar/L$, where $L$ is the angular momentum of the system, so they are completely negligible for a macroscopic system.[55]

III. EFFECTIVE LAGRANGIAN FROM A MODIFIED THREE-GRAVITON VERTEX

To perform our computations we use the effective field theory formulation proposed in ref. [34]. Computations of the conservative dynamics at 2PN level have been performed using this effective field theory technique [37, 38] and the results are in agreement with the classic 2PN results of refs. [39, 40], while the full 3PN result for non-spinning particles to date has only been obtained with the standard PN formalism using dimensional regularization [32, 33] (see ref. [46], or Chapter 5 of ref. [17], for a pedagogical introduction to the PN expansion and for a more complete list of references). Spin-spin contributions at 3PN order in the conservative two-body dynamics have recently been computed both with the effective field theory techniques [41, 42] and with the ADM Hamiltonian formalism [43] (see also [44, 45] for other applications of the EFT technique related to gravitational radiation).

In the formalism of ref. [34], after integrating out length-scales shorter than the size of the compact objects, the action becomes

$$ S = S_{EH} + S_{pp}, $$

where $S_{EH}$ is the Einstein-Hilbert action and

$$ S_{pp} = -\sum_a m_a \int d\tau_a $$

is the point particle action. Here $a = 1, 2$ labels the two bodies in the binary system and $d\tau_a = \sqrt{g_{\mu\nu}(x_a) dx_a^\mu dx_a^\nu}$. One then observes that, in the binary problem, the gravitons appearing in a Feynman diagram can be divided into two classes: the forces between the two bodies with relative distance $r$ and relative speed $v$ are mediated by gravitons whose momentum $k^\mu$ scales as $(k^0 \sim v/r, |k| \sim 1/r)$. These are called “potential gravitons” and are off-shell, so they can only appear in internal lines. The gravitons radiated to infinity rather have $(k^0 \sim v, |k| \sim v/r)$. One then writes $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and separates $h_{\mu\nu}$ into two parts, $h_{\mu\nu}(x) = \bar{h}_{\mu\nu}(x) + H_{\mu\nu}(x)$ with $\bar{h}_{\mu\nu}(x)$ describing the radiation gravitons and $H_{\mu\nu}(x)$ the potential gravitons. One fixes the de Donder gauge and, expanding the action in powers of $\bar{h}_{\mu\nu}$ and $H_{\mu\nu}(x_0)$, one can read off the propagators and the vertices, and write down the Feynman rules of the theory. Then, using standard methods from quantum field theory, one can construct an effective Lagrangian that, used at tree level, reproduces the amplitudes computed with the Feynman graphs. For a classical system, whose angular momentum $L \gg \hbar$, only tree graphs contribute, and reproduce the classical Lagrangian that is usually derived from GR using the PN expansion. For instance, the conservative dynamics of the two-body problem, at 1PN level, is given by the Einstein–Infeld–Hoffmann Lagrangian,

$$ L_{EIH} = \frac{1}{8} m_1 v_1^4 + \frac{1}{8} m_2 v_2^4 + \frac{G_N m_1 m_2}{2r} \left[ 3(v_1^2 + v_2^2) - 7v_1 \cdot v_2 - (\hat{r} \cdot \mathbf{v}_1)(\hat{r} \cdot \mathbf{v}_2) - \frac{G_N (m_1 + m_2)}{r} \right]. \quad (13) $$

In the language of the effective field theory of Ref. [34], this result is obtained from Feynman diagrams involving the exchange of potential gravitons. In particular, the terms linear in $G_N$ in eq. (13) are obtained from a single exchange of potential gravitons between two matter lines, see Fig. 4 of Ref. [34], while the term proportional to $G_N^2$ is obtained from the sum of the two graphs shown in Fig. 1. In the derivation of the conservative 1PN Lagrangian, the three-graviton vertex only enters through the graph in Fig. 1a. Multiplying this vertex by a factor $(1 + \beta_3)$ and repeating the same computation as in ref. [34] we get the additional contribution to the conservative part of the Lagrangian

$$ \Delta L_{cons} = -\beta_3 \frac{G_N^2 m_1 m_2 (m_1 + m_2)}{r^2}. \quad (14) $$

Comparing this result with the Lagrangian whose equations of motion are the same as the equations of motion of a test particle in the PPN metric (1)–(3) (see eq. (6.80) of ref. [29]) we find that, to 1PN order and as far as the conservative dynamics is concerned, the introduction of $\beta_3$ gives rise to a PPN theory with $\beta = 1 + \beta_3$, i.e. $\beta = 3\beta$, while $\gamma = 1$ as in GR. Therefore the bound on $\beta$ from the perihelion of Mercury translates into $|\beta_3| < 3 \cdot 10^{-3}$, while eq. (7) translates into the bound (at 68% c.l.)

$$ |\beta_3| < 2 \cdot 10^{-4}. \quad (15) $$

This bound reflects the fact that $\beta_3$, just as the PPN parameter $\beta$, violates the weak equivalence principle. The

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**FIG. 1:** The diagrams that give the terms proportional to $G_N^2$ in the 1PN Lagrangian. Dashed lines denote potential gravitons, solid lines the point-like sources.
introduction of $\beta_3$, however, also affects the radiative sector of the theory, something that is not modeled in the phenomenological PPN framework since the latter by definition is only concerned with the motion of test masses in a deformed metric, and therefore only modifies the conservative part of the dynamics. Note however that, in the framework of multiscalar-tensor theories, the extension of the PPN formalism introduced in Ref. [11] allows for a consistent treatment of both the conservative dynamics (including the case of strongly self-gravitating bodies) and of radiative effects.

It is clearly interesting to see what bounds on $\beta_3$ can be obtained from experiments that probe the radiative sector of GR, such as the timing of binary pulsars or the observation of the coalescence of compact binaries at interferometers. The effective Lagrangian describing the interaction of the binary system with radiation gravitons is obtained by computing the three graphs in Fig. 2 (corresponding to Fig. 6 of ref. [34]), and the introduction of $\beta_3$ affects the $HHh$ vertex in Fig. 2c.

Computing these graphs as in ref. [34], but with our modified three-graviton vertex, we find

$$L_{\text{rad}} = \frac{1}{2M_{\text{Pl}}} [Q_{ij} R_{00ij} + q R_{00i} + \beta_3 (3V h_{00} + Z^{ij} h_{ij})] ,$$

(16)

where $Q_{ij}$ is the quadrupole moment of the source and we define

$$q = \frac{1}{3} \sum_a m_a x_a^3 ,$$

(17)

$$V(r) = \frac{G_N m_1 m_2}{r} ,$$

(18)

$$Z^{ij}(r) = \frac{G_N m_1 m_2 r^i r^j}{r^3} ,$$

(19)

where $r = x_1 - x_2$. The term $Q_{ij} R_{00ij}$ in eq. (16) is the usual quadrupole interaction. The second term, $q R_{00i}$, is non-radiating when $\beta_3 = 0$, but we will see that for $\beta_3 \neq 0$ it contributes to the radiated power when the orbit is non-circular. The last two terms in eq. (16) are the explicit $\beta_3$-dependent terms induced by the modification of the three-graviton vertex in Fig. 2c.

Finally, we have omitted a term where $h_{00}$ is coupled to the conserved energy (at this order) and $h_{0i}$ is coupled to the conserved angular momentum, since these terms do not generate gravitational radiation.

The back-reaction of GW emission on the source can be computed as usual from the energy balance equation

$$P_{\text{GW}} = -\dot{E} ,$$

(20)

where $P_{\text{GW}}$ is the power radiated in GWs and $E$ the orbital energy of the system. To obtain the expression for the radiated power for $\beta_3 \neq 0$ we cannot simply use the quadrupole formula of GR, since the introduction of $\beta_3$ generates new contributions. To take them into account we proceed as in ref. [34], and we compute the imaginary part of the graph shown in Fig. 3. The vertices of the graph can be read from eq. (16). When $\beta_3 = 0$ the only relevant vertex comes from $Q_{ij} R_{00ij}$, and the computation of the imaginary part gives back the Einstein quadrupole formula [34]. In our case we have various possible vertices, and we must compute the imaginary part of

$$-\frac{i}{8M_{\text{Pl}}^2} \sum_{a,b=1}^4 \int dt_1 dt_2 I_{ij}^{a}(t_1) I_{kl}^{b}(t_2) \langle S_{ij}^{a}(t_1) S_{kl}^{b}(t_2) \rangle ,$$

(21)

where $I_{ij}^{a}$ depends on the matter variables and $S_{ij}^{a}$ on the gravitational field. When both vertices of the diagram in Fig. 3 are proportional to the quadrupole, one obtains the usual GR result

$$P_{QQ} = \frac{G_N}{3} \langle \ddot{Q}_{ij} \ddot{Q}_{ij} \rangle ,$$

(22)

as already found in [34]. Computing the other contributions we find that the terms $P_{Qq}$ and $P_{qq}$ vanish identically. In fact, the $QQ$ and $qQ$ graphs vanish because $Q_{ij}$ is traceless, while the $qq$ graph vanishes because $\delta_{ij} \delta_{kl}$ gives zero when contracted $\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} - \frac{1}{2} \delta_{ij} \delta_{kl}$, which is the tensor that comes out from the two-point function $\langle R_{00ij} R_{00ij} \rangle$. The $QV$, $qZ$ and $VZ$ graphs vanish for similar reasons, so the only relevant contributions come from the $QZ$ and $qV$ graphs, and we find

$$P_{QZ} = -2\beta_3 G_N \langle \ddot{Q}_{ij} Z_{ij} \rangle ,$$

(23)

and

$$P_{qV} = -6\beta_3 G_N \langle \ddot{q} V \rangle .$$

(24)

As for the $VV$ and $ZZ$ graphs, they give a contribution that, from the point of view of the multipole expansion, is of the same order as the quadrupole radiation but proportional to $\beta_3^2$, and can be neglected.

We can now use these results to perform the comparison with binary pulsars and with interferometers.

IV. COMPARISON WITH EXPERIMENT

As we already saw in eq. (15), solar system experiments, and in particular lunar laser ranging, give the

![FIG. 2: The diagrams that contribute to the matter-radiation Lagrangian. Dashed lines denote potential gravitons, wiggly lines radiation gravitons, and solid lines the point-like sources.](image-url)
bound $|\beta_3| < 2 \cdot 10^{-4}$. In this section we study the bounds on $\beta_3$ that can be obtained from binary pulsars and from the detection of coalescing binaries at interferometers.

**A. Binary pulsars**

Since $\beta_3$ modifies the emitted power already at Newtonian order, the energy of the orbit in eq. (20) can now be directly computed using the Keplerian equations of motion. We see that this test of GR is conceptually different from the tests based on solar system experiments. The latter only probe the conservative part of the Lagrangian, i.e. the $\beta_3$-dependent term given in eq. (14), while binary pulsars are sensitive to the $\beta_3$ dependence given in the radiation Lagrangian (16) (even if the effect of $\beta_3$ on the conservative dynamics will also enter, through the determination of the masses of the stars from the periastron shift, see below).

Using the Keplerian equations of motion for an elliptic orbit of eccentricity $e$ we get

$$P_{QQ} = \frac{32G_N \mu^2 M^3}{5a^2(1-e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right),$$

(25)

$$P_{QZ} = \frac{32G_N \mu^2 M^3}{5a^2(1-e^2)^{7/2}} \left(\frac{5}{2} - \frac{175}{24} e^2 + \frac{85}{96} e^4\right),$$

(26)

$$P_{QV} = -\frac{32G_N \mu^2 M^3}{5a^2(1-e^2)^{7/2}} \left(\frac{5}{16} e^2 + \frac{5}{64} e^4\right),$$

(27)

where $M = m_1 + m_2$ is the total mass, $\mu = m_1 m_2/M$ is the reduced mass, and we will also use the notation $\nu = \nu/\nu_{\text{GR}}$ for the symmetric mass ratio. From the energy balance equation we then get the evolution of the orbital period $P_b$,

$$\frac{\dot{P}_b}{P_b} = -\frac{96}{5} G_N^{2/3} \nu^{5/3} \left(\frac{P_b}{2\pi}\right)^{-8/3} \left[f(e) + \beta_3 g(e)\right],$$

(28)

where

$$f(e) = \frac{1}{(1-e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right),$$

(29)

$$g(e) = \frac{1}{(1-e^2)^{7/2}} \left(\frac{5}{2} + \frac{335}{48} e^2 + \frac{155}{192} e^4\right).$$

(30)

The term proportional to $f(e)$ is the standard GR result [47], while the term proportional to $g(e)$ is the extra contribution due to $\beta_3$.

In order to measure $\beta_3$ from the dynamics of binary pulsars, we must also determine the dependence on $\beta_3$ of the periastron shift $\omega$ and of the Einstein time delay $\gamma_E$, since these two observables are used to determine the masses of the two compact stars. In particular the periastron shift fixes the total mass $M$ of the system, while the Einstein time delay $\gamma_E$ measures a different combination of masses, see e.g. eqs. (6.56) and (6.93) of ref. [17]. Using the conservative Lagrangian with the modification (14) and repeating the standard textbook computation of the periastron shift $\omega$, we find that the value $\omega_{\beta_3}$ computed in a theory with $\beta_3 \neq 0$ is related to the GR value $\omega_{\text{GR}}$ by

$$\omega_{\beta_3} = \left(1 - \frac{\beta_3}{3}\right)\omega_{\text{GR}},$$

(31)

while the Einstein time delay is unchanged because it is not affected by the post-Keplerian parameters. So, if $\beta_3 \neq 0$, the true value of the total mass $M$ of the binary system, that enters in eq. (28), is not the one that would be inferred from the periastron shift using the predictions of GR, but rather we get

$$M_{\beta_3} = \left(1 + \frac{\beta_3}{2}\right)M_{\text{GR}}.$$

(32)

Similarly, using eq. (6.93) of ref. [17], for the symmetric mass ratio $\nu$ we get

$$\nu_{\beta_3} = (1 + w_{\beta_3})\nu_{\text{GR}},$$

(33)

where

$$w = \frac{\kappa}{3} \sqrt{1 + 4\kappa - 2 \frac{1}{(1 + 4\kappa)^{1/2}}(1 + \kappa)}$$

(34)

and

$$\kappa = \frac{2\pi}{\beta_3} \left(1 - \nu_{\text{GR}}\right)^{1/3}.$$  

(35)

Putting everything together and keeping only the linear order in $\beta_3$ we finally find that the ratio between the value of $\dot{P}_b$ computed at $\beta_3 \neq 0$ and the value of $\dot{P}_b$ computed in GR is

$$\frac{\dot{P}_b(\beta_3)}{\dot{P}_b_{\text{GR}}} = 1 + \beta_3 \tilde{g}(e),$$

(36)

where

$$\tilde{g}(e) = \frac{g(e)}{f(e)} + \frac{5}{6} + w.$$  

(37)

Observe that the term $g(e)/f(e)$ comes from the effect of $\beta_3$ on the radiative sector of the theory, while the term
(5/6) + w comes from the effect of $\beta_3$ on the conservative sector, i.e. on the mass determination. Inserting the numerical values for the Hulse-Taylor pulsar, we get $\bar{g}(e) \simeq 3.21$. Note that $g(e)/f(e) \simeq 2.38$, so $\bar{g}(e)$ is dominated by the effect of $\beta_3$ on the radiative sector of the theory. For this binary pulsar, after correcting for the Doppler shift due to the relative velocity between us and the pulsar induced by the differential rotation of the Galaxy, the ratio between the observed value $\bar{P}_b^{\text{obs}}$ and the GR prediction $\bar{P}_b^{\text{GR}}$ is $\bar{P}_b^{\text{obs}}/\bar{P}_b^{\text{GR}} = 1.0013(21)$. Interpreting this as a measurement of $\beta_3$ we finally get $3.21/3 = 0.0013(21)$, i.e.,

$$\beta_3 = (4.0 \pm 6.4) \cdot 10^{-4},$$

so the three-graviton vertex is consistent with the GR prediction at the 0.1% level. This bound is slightly worse, but comparable, to the one from lunar laser ranging, eq. (15). It should be stressed, however, that eq. (38) is really a test involving the radiative sector of GR, while eq. (15) only tests the conservative sector. For comparison, observe that in the Standard Model the triple gauge boson couplings are measured to an accuracy of about 3% [28].

For the double pulsar we find $\bar{g}(e) \simeq 3.3$. Since $\bar{P}_b$ for the double pulsar is presently measured at the 1.4% level [8], we get a larger bound compared to eq. (38). However, further monitoring of this system is expected to bring the error on $\bar{P}_b$ down to the 0.1% level.

### B. Binary coalescences at interferometers

We now compare these results with what can be expected from the detection of a binary coalescence at GW interferometers. In this case one can determine the physical parameters of the inspiraling bodies, by performing matched filtering of theoretical waveform templates. In the matched filtering method any difference in the time behavior between the actual signal and the theoretical template model will eventually cause the two to go out of phase, with a consequent drop in the signal-to-noise ratio (SNR). The introduction of $\beta_3$ and $\beta_4$ affects the template, and in particular the accumulated phase

$$\phi = 2\pi \int_{t_{\text{min}}}^{t_{\text{max}}} f(t) dt,$$

where $f(t)$ is the time-varying frequency of the source, and the subscript min (max) denotes the values when the signal enters (leaves) the detector band-width. Thus in principle a detection of a GW signal from coalescing binaries could be translated into a measurement of the three- and four-graviton vertices. In this section we investigate the accuracy of such a determination.

With respect to the timing of binary pulsars, there are at least three important qualitative differences that affect the accuracy at which these systems can test the non-linearities of GR. First, coalescing compact binaries in the last stage of the coalescence reach values of $v/c \sim 1/3$, and are therefore much more relativistic than binary pulsars, which rather have $v/c \sim 10^{-3}$. Second, the leading Newtonian result for $\phi$ is of order $(v/c)^{-5}$ so it is much larger than one, and to get the phase with a precision $\Delta\phi \ll 1$, as needed by interferometers, all the corrections at least up to $O(v^6/c^6)$ to the Newtonian result must be included, so higher-order corrections are important even if they are numerically small relative to the leading term. In other words, even if PN corrections are suppressed by powers of $v/c$ with respect to the leading term, they can be probed up to high order because what matters for GW interferometers is the overall value of the PN corrections to the phase, and not their value relative to the large Newtonian term. These two considerations should suggest that interferometers are much more sensitive than pulsar timing to the non-linearities of GR.

On the other hand, for binary pulsars we can measure not only the decay of the orbital period due to GW emission, but also several other Keplerian observables, that provide a determination of the geometry of the orbit, as well as post-Keplerian observables, such as the periastron shift and the Einstein time delay, which fix the masses of the stars in the binary system. This is not the case for the detection of coalescences at interferometers. With interferometers the parameters that determine the waveform, such as the masses and spins of the stars, must be determined from the phase of the GW itself, and one must then carefully investigate the degeneracies between the determination of $\beta_3$ (or of $\beta_4$) and the determination of the masses and spins of the stars. This effect clearly goes in the direction of degrading the accuracy of parameter reconstruction at GW interferometers, with respect to binary pulsar timing, so in the end it is not obvious a priori which of the two, GW interferometers or binary pulsar timing, is more sensitive to the non-linearities of GR. This question is answered in what follows.

Repeating with $\beta_3 \neq 0$ the standard computation of the orbital phase for a circularized orbit, we find that $\beta_3$ modifies the orbital phase $\phi(t)$ already at 0PN (i.e. Newtonian) level, where to linear order in $\beta_3$ we get

$$\phi^{0\text{PN}} = -\frac{\Theta^{5/8}}{\nu} (1 + b_0 \beta_3),$$

with $b_0 = -5/2$, and $\Theta$ is defined as

$$\Theta = \frac{\nu(t_c - t)}{5GM} (1 - b_0 \beta_3),$$

where $t_c$ is the time of coalescence. Combining the factors $\nu$ and $M$ which enter in the definition of $\Theta$ with the explicit factor $1/\nu$ in eq. (40) we recover the well-know result that the Newtonian phase depends on the masses of the stars only through the chirp mass $M_\ast = \nu^{3/5}M$. From eq. (40) we immediately understand the crucial role that degeneracies have for interferometers. In fact, since
\( M_c \) is determined from eq. (40) itself, using only the 0PN phase (40) it is impossible to detect the deviation from the prediction of GR induced by \( \beta_3 \). A non-zero value of \( \beta_3 \) would simply induce an error in the determination of \( M_c \).

The same happens at 1PN level. In fact, at 1PN order and with \( \beta_3 \neq 0 \) the phase has the general form

\[
\Phi^{1\text{PN}} = \frac{\Theta^{5/8}}{\nu} \left[ (1 + b_0 \beta_3) + a_1(\nu)(1 + b_1 \beta_3) \Theta^{-1/4} \right],
\]

where, as before, \( b_0 = -5/2 \) is the 0PN correction proportional to \( \beta_3 \), while

\[
a_1(\nu) = \frac{3715}{8064} + \frac{55}{96} \nu
\]

is the 1PN GR prediction [17, 46], and \( b_1 \) (which is possibly \( \nu \)-dependent) parametrizes the 1PN correction due to \( \beta_3 \). (For simplicity, we only wrote explicitly the term linear in \( \beta_3 \) since \( \beta_3 \) is much smaller than one, but all our considerations below can be trivially generalized to terms quadratic in \( \beta_3 \), just by allowing the function \( b_1(\nu) \) to depend also on \( \beta_3 \).) In general we expect \( b_1 \) to be \( O(1) \), and we will see below that for our purposes this estimate is sufficient.

Using \( M_c \) and \( \nu \) as independent mass variables, in place of \( m_1 \) and \( m_2 \), we see that while the effect of \( \beta_3 \) on the 0PN phase can be rescaled into \( M_c \), its effect on the 1PN phase can be rescaled into a rescaling of \( \nu \). Observe that, in the detection of a single coalescence event, GW interferometers do not measure the functional dependence of \( \nu \) of the 1PN phase, but only its numerical value for the actual value of \( \nu \) of that binary system, so we cannot infer the presence of a term proportional to \( \beta_3 \) from the fact that it changes the functional form of the \( \nu \)-dependence from the one obtained by eq. (43). Thus, even at 1PN order, it is impossible to detect the deviations from GR induced by \( \beta_3 \). A non-zero \( \beta_3 \) would simply induce an error on the determination of \( M_c \) and \( \nu \), i.e. on the masses of the two stars.

We then examine the situation at 1.5PN order. Let us at first neglect the spin of the two stars. Then the 1.5PN phase with \( \beta_3 \neq 0 \) has the generic form

\[
\Phi^{1.5\text{PN}} = \frac{\Theta^{5/8}}{\nu} \left[ (1 + b_0 \beta_3) + a_1(\nu)(1 + b_1 \beta_3) \Theta^{-1/4} + a_2(1 + b_2 \beta_3) \Theta^{-3/8} \right],
\]

where \( a_2 = -3\pi/4 \) is the 1.5PN GR prediction and \( b_2 \) is the (possibly \( \nu \)-dependent) 1.5PN correction due to \( \beta_3 \). Again, we will not need its exact value, and we will simply make the natural assumption that it is \( O(1) \).

However, for the purpose of determining \( \beta_3 \), neglecting spin is not correct. Indeed, at 1.5PN order the spin of the bodies enters through the spin-orbit coupling, and the evolution of the GW frequency \( f \) with time is given by [49]

\[
\frac{df}{dt} = \frac{12}{5} \pi^{8/3} M_5^{5/3} f^{11/3} \left[ 1 - \frac{24}{5} a_1(\nu) x + (4\pi - \beta_{1S}) x^{3/2} \right],
\]

where \( x \equiv (\pi f)^{2/3} \), while \( \beta_{1S} \) describes the spin-orbit coupling and is given by

\[
\beta_{1S} = \frac{1}{12} \sum_{a=1}^{2} \left[ 115 \frac{m_a^2}{M^2} + 75 \nu \right] \hat{L} \cdot \hat{x}_a,
\]

where \( \hat{L} \) is the orbital angular momentum, \( \hat{x}_a = \mathbf{S}_a / m_a \), and \( \mathbf{S}_a \) is the spin of the \( a \)-th body. In principle \( \beta_{1S} \) evolves with time because of the precession of \( \mathbf{L} \), \( \mathbf{S}_1 \) and \( \mathbf{S}_2 \). However, it turns out that in practice it is almost conserved, and can be treated as a constant [48]. Integration of eq. (45) then shows that, in the 1.5PN phase, \( \beta_{1S} \) is exactly degenerate with \( \beta_3 \) in eq. (44). Furthermore, observe that \( \beta_{1S} \), depending on the spin configuration, can reach a maximum value of about 8.5 [48] (and its maximum value remains large even in the limit \( \nu \to 0 \), while \( \beta_3 \) is already bound by laser ranging at the level of \( 2 \times 10^{-4} \) and by pulsar timing at the level of \( 10^{-3} \) (which tests the radiative sector, as do GW interferometers). Thus, the effect of \( \beta_3 \) at 1.5PN is simply reabsorbed into a (very small) shift of \( \beta_{1S} \).

At 2PN order \( \beta_3 \) is degenerate with the parameter \( \sigma \) that describes the spin-spin interaction

\[
\sigma = \frac{\nu}{48} \left[ 721 (\hat{L} \cdot \hat{x}_1) - 247 \hat{x}_1 \cdot \hat{x}_2 (\hat{L} \cdot \hat{x}_2) \right] + \frac{1}{96} \sum_{a=1}^{2} \left[ 719 (\hat{L} \cdot \hat{x}_a)^2 - 233 \hat{x}_a^2 \right].
\]

The first term is the one which is usually quoted in the literature, first computed in [49] (see also [50, 51]). The term in the second line, computed recently in [52], is however of the same order, and must be included.

The first term is proportional to \( \nu \), and reaches a maximum value \( \sigma_{\text{max}}(\nu) \sim 10\nu \). In a coalescence with very small value of \( \nu \), this term is therefore suppressed; e.g. in an extreme mass-ratio inspiral (EMRI) event at LISA where a BH of mass \( m_1 = 10 M_\odot \) falls into a supermassive BH with \( m_2 = 10^6 M_\odot \), one has \( \nu = 10^{-5} \) and the term in the first line has a maximum value \( \sim 10^{-4} \). If this standard term gave the full answer, a value of \( \beta_3 \) in excess of this value could therefore give an effect that cannot be ascribed to \( \sigma \). However, the presence of the new term recently computed in [52] spoils this reasoning, since it is not proportional to \( \nu \). The conclusion is that, just as with \( \beta_{1L} \) at 1.5PN order, the effect of \( \beta_3 \) at 2PN order is just reabsorbed into a small redefinition of \( \sigma \), and therefore simply induces an error in the reconstruction of the spin configuration (observe also that fixing \( \beta_{1S} \) does not allow us to fix the spin combinations that appear in \( \sigma \)).

One could in principle investigate the effect of \( \beta_3 \) on higher-order coefficients, such as the 2.5PN term \( \psi_5 \) and
the 3PN term $\psi_6$ in eq. (10), which at LISA can be measured with a precision of order $10^{-2}$ [24]. Unfortunately, it is difficult to translate them into bounds on $\beta_3$ because at 2.5PN order one finds that $\beta_3$ is degenerate with a different combination of spin and orbital variables, which is not fixed by the 1.5PN spin-orbit term (see Table II of ref. [33]), while at 3PN order the spin contribution is not yet known. The conclusion is therefore that interferometers cannot measure the three- and higher-order graviton vertex, since the effect of a modified vertex is simply reabsorbed into the determination of the masses and spin of the binary system. The conclusion that interferometers are not competitive with pulsar timing for measuring deviations from GR was also reached in Ref. [10], although in a different context. In fact, Ref. [10] was concerned with multiscalar-tensor theories, whose leading-order effect is the introduction of a term corresponding to dipole radiation (a “minus one”-PN term).

We now examine what can be said about the four-graviton vertex, parametrized by $\beta_4$. In the conservative part of the Lagrangian $\beta_4$ contributes through the diagrams of Fig. 4. However, these contributions only affect the equations of motion at 2PN order, so there is no hope to see them in solar system experiments, where the velocities at play are very small. For the same reason, no significant bound can be obtained from binary pulsars; from a simple order of magnitude estimate we find that the Hulse-Taylor pulsar can only give a limit $\beta_4 < O(10)$, which is not significant.

At GW interferometers, $\beta_4$ enters into the phase for the first time at 1PN order, through the diagram in Fig. 5. However, it suffers exactly of the same degeneracy issues as $\beta_3$, so it cannot be measured to any interesting accuracy at present or future interferometers, at least with the technique discussed here. Note however that in this paper we have worked in the restricted PN approximation, in which only the harmonic at twice the source frequency is retained. Higher-order harmonics however break degeneracies between various parameters in the template [54], and it would be interesting to investigate whether their inclusion in the analysis allows one to put a bound on $\beta_3$ and $\beta_4$ from binary coalescences.

V. CONCLUSION

We have proposed to quantify the accuracy by which various experiments probe the non-linearities of GR, by translating their results into measurements of the non-abelian vertices of the theory, such as the three-graviton vertex and the four-graviton vertex. This is similar in spirit to tests of the Standard Model of particle physics, where the non-abelian vertices involving three and four gauge bosons have been measured at LEP and at the Tevatron.

We have shown that, at a phenomenological level, this can be done in a consistent and gauge-invariant manner, by introducing parameters $\beta_3$ and $\beta_4$ that quantify the deviations from the GR prediction of the three- and four-graviton vertices, respectively. We have found that, in the conservative sector of the theory, i.e. as long as one neglects the emission of gravitational radiation at infinity, the introduction of $\beta_3$ at 1PN order is phenomenologically equivalent to the introduction of a parameter $\beta_{PPN} = 1 + \beta_3$ in the parametrized PN formalism. Strong bounds on $\beta_3$ therefore come from solar system experiments, and most notably from lunar laser ranging, that provides a measurement at the 0.02% level.

The modification of the three-graviton vertex however also affects the radiative sector of the theory, and we have found that the timing of the Hulse-Taylor pulsar gives a bound on $\beta_3$ at the 0.1% level, not far from the one obtained from lunar laser ranging. Conceptually, however, the two bounds have different meanings, since lunar laser ranging only probes the conservative sector of the theory, while pulsar timing is also sensitive to the radiative sector.

We have then studied the results that could be obtained from the detection of coalescences at interferometers, and we have found that, even if $\beta_3$ already modifies the GW phase at the Newtonian level and $\beta_4$ at 1PN order, their effect can always be reabsorbed into other parameters in the template, such as the mass and spin of the two bodies so, rather than detecting a deviation from the GR prediction, one would simply make a small error in the estimation of these parameters.

Acknowledgments. The work of UC, SF, MM and HS is supported by the Fonds National Suisse. We thank Alvaro de Rujula for a stimulating discussion and Thibault Damour and the referees for useful comments. HS would like to thank Steven Carlip and Bei-Lok Hu, and UC would like to thank Yi-Zen Chu, for helpful discussions.
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[55] At the quantum level, if a mass is generated, it will be power divergent with the cutoff, and will have to be fine tuned order by order in perturbation theory. However, the fact that quantum divergences have to be subtracted order by order is a generic problem of the standard quantum extension of GR, independently of $\beta_3$. In any case, again, our approach is purely phenomenological, and the introduction of $\beta_3$ is simply a tool for tracking a specific contribution to the computation.