Simulation of vortex lattice melting in a dirty square superconducting plate

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Abstract. Vortex lattice melting in a dirty mesoscopic square superconducting plate is studied using the molecular dynamics (MD) method. We include an impurity potential in the MD method and investigate vortex number and impurity number dependences of the melting transition temperature. We find strong vortex number dependence and weak pinning potential dependence of the melting temperature.

1. Introduction
A single vortex has a quantized flux (Φ0 = ch/2e) and there is screening current around it. It appears between the lower critical field (Hc1) and the upper critical field (Hc2) in a type II superconductor. Above Hc2, the superconductivity becomes unstable and vortices disappear. In the intermediate field regime Hc1 < H < Hc2, vortices form a triangular lattice which is known as an Abrikosov lattice. Increasing the temperature, the vortex lattice becomes unstable. Then vortex lattice melts into a vortex liquid state. In H – T phase diagram, there is a melting line of the vortex lattice. Because of large thermal and quantum fluctuation, this melting transition clearly appears in the cuprate high temperature superconductors [1,2].

Recently, Ooi et al [3] found that melting transition temperature of vortex lattices in a square superconducting plate (Bi2212) oscillates with increasing magnetic field, and it becomes maximum when the vortex number is a square number. Then, using the molecular dynamics (MD) method [4-6], Kato and Kitago [7] investigated the vortex lattice melting transition in a square superconducting plater without impurities. They found the vortex motion becomes large as temperature increases and showed the melting transition temperature can be determined from the rapid increase of the standard deviation of the position of the vortices.

In this study, we investigate vortex number dependence and impurity dependence of vortex lattice melting in a dirty square superconducting plate, using the MD method.

2. Method
In the MD method, we consider following equations of motion of vortices [4-6],
\[
\eta \frac{dr_i}{dt} = f_i^{\text{imp}} + f_{vi} + f_0
\]

where \( r_i \) is a position of an \( i \)-th vortex, and \( \eta \) is the viscosity. \( f_i^{\text{imp}} \) is a sum of pinning forces from impurities, which is given as,

\[
f_i^{\text{imp}} = \sum_j \frac{f_p}{r_p} |r_i - r_j|^3 \Theta \left( \frac{r_p - |r_i - r_j|}{\lambda} \right) r_j
\]

Here \( f_p \) and \( r_p \) are a strength and a range of the pinning force, and \( r_j^0 \) is the position of \( j \)-th impurity. \( \Theta \) is the step function. \( \lambda \) is the penetration depth. \( f_{vi} \) is a sum of vortex-vortex interaction forces from other vortices.

\[
f_{vi} = \sum_j f_j^{\text{vv}}
\]

\[
f_j^{\text{vv}} = f_s \sum_j K_1 \left( \frac{r_j}{\lambda} \right) \hat{r}_j
\]

\[
f_0 = -\frac{\Phi_0^2}{8\pi^2 \lambda^3}
\]

where \( f_0 \) is the vortex-vortex interaction force, \( \Phi_0 \) is the quantized flux, and \( K_1(k) \) is the MacDonald function. This force depends on temperature \( T \) through temperature dependence of \( \lambda = \lambda_0/\sqrt{1 - T/T_c} \). \( \lambda_0 \) is the penetration depth at \( T = 0 \) and \( T_c \) is the critical temperature, at which the penetration depth diverges. \( f_0 \) is the thermal fluctuation force, which satisfies following equation,

\[
\left\langle f_{vi}(t_1) \cdot f_{vi}(t_2) \right\rangle = 2\eta k_B T \delta(t_1 - t_2)
\]

where \( k_B \) is the Boltzmann constant. Temperature dependence of vortex motion comes from temperature dependences of the thermal fluctuation \( f_0 \) and the vortex-vortex interaction.

3. Numerical results

We consider a mesoscopic \((3\lambda_0 \times 3\lambda_0)\) square superconducting plate with impurities under the periodic boundary condition. For investigation of the experiments in Ref.3, we should impose the fixed boundary condition. However, with the periodic boundary condition in the square shape, vortices are also confined in the square shape and they are forced to form lattices that match with the square shape, as shown below.

First, we focus on cases where vortex number is from 4 to 18 and number of impurities is 100. We set radius and strength of the impurities as \( r_p = 0.03\lambda_0 \) and \( f_p = -0.1f_0 \). The number of vortices may correspond to the external magnetic field. As a measure of vortex motion, we use an averaged variance \( \sigma^2 \) of a vortex position \( r_i \)

\[
\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} \left( r_i - \langle r_i \rangle \right)^2
\]

where \( \langle \ldots \rangle \) is a time average. In figure 1, we show \( \sigma \) as a function of temperature when vortex number \( N \) is from 4 to 18. Here \( \sigma \) is normalized by an average of vortex distance \( l \), where \( l = \sqrt{(3\lambda_0 \times 3\lambda_0)/N} \) for \( N \) vortices. Around \( T/T_c \approx 0.99 \), \( \sigma/l \) is larger for larger \( N \). This behavior is explained as follows. For larger \( N \), \( l \) is small but vortex number dependence of the vortex motion in liquid phase is rather small, and therefore \( \sigma/l \) becomes relatively larger. But for \( 0.1 < T/T_c < 0.8 \), \( \sigma \) is smaller for larger
$N$, except $N = 4$ and 16. This is because for smaller $N$, vortices moves rather freely in the solid phase and therefore $\sigma/l$ becomes larger. However, for $N = 4$ and 16, $\sigma$ becomes small. Especially $\sigma$ is smallest for $N = 16$. This result shows vortex lattice is relatively stable for 4 and 16 vortices. In order to clarify this point, we plot $\sigma/l$ as a function of vortex number $N$ in Figure 2.

![Figure 1](image1.png)

**Figure 1.** Standard deviation of the vortex position as function of temperature in the $3\lambda_0 \times 3\lambda_0$ square superconducting plate with 100 impurities. $N$ is from 4 to 18.

![Figure 2](image2.png)

**Figure 2.** The standard deviation of vortex position as a function of the number of vortices.

In figure 2, when the number of vortices is 4, 8, 12, and 16, $\sigma/l$ becomes small for lower temperature where the vortex lattice is formed and therefore vortex lattice for these vortex numbers becomes relatively stable. In order to investigate the stable lattice, we show trajectories of vortices when $N = 16$ at $T/T_c = 0.1$, 0.5, 0.9, and 0.98 in figure 3. For lower temperature vortices form a $4 \times 4$...
square lattice. As we can see from figure 1, the melting transition occurs between $T/T_c = 0.9$ and $T/T_c = 0.98$.

![Figure 3. Trajectories of vortices at $T/T_c = 0.1, 0.5, 0.9$ and $0.98$.](image)

For larger number of vortices, we show $\sigma/l$ as functions of vortex number for various temperatures in figure 4. At low temperature, there are large dips of $\sigma/l$ when the number of the vortices is 4, 8, 16, 24, 36, 48, 64, 80 or 100. Also, there are small dips at $N=12, 20, 28, 32, 40, 44, 52, 56, 60, 68, 72, 76, 84, 88, 92$, and 96. All of these numbers are multiple of 4. And for large dip numbers are close to square numbers. We can say when the vortex number is a multiple of 4, the vortex lattice becomes rather stable. This result shows that vortex lattices with $N = 4m$ ($m$ is an integer) relatively stable. This may come from our periodic boundary condition, which is suitable for square and rectangular lattices. For more realistic calculation, we must impose a fixed boundary condition.
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Figure 5. Standard deviation of vortex position as a function of temperature of 4 and 17 vortices in the $3\lambda_0 \times 3\lambda_0$ square superconducting plate with 100, 500, 1000 impurities.

Next, in order to investigate the effect of impurities on the vortex motion, we compare vortex motions for $N = 4$ and 17 vortices in the superconducting plate with 100, 500 and 1000 impurities. Here, $r_p = 0.03\lambda_0$ and $f_p = -0.1f_0$. In figure 5, we show $\sigma$ as a function of temperature for $N = 4$ and 17 and $N_{\text{imp}} = 100, 500$ and 1000. In this figure, we cannot see any difference between $N_{\text{imp}} = 100, 500$ and 1000 cases. However, in the higher temperature region, there are small differences, which are shown in figure 6. From figure 6, we can see vortex motion becomes a little bit smaller for larger impurity number.

Figure 6. $\sigma/\lambda$ as a function of temperature for 17 vortices (a) and 4 vortices (b) in the $3\lambda_0 \times 3\lambda_0$ square superconducting plate with 100, 500, and 1000 impurities.

4. Summary
We have investigated vortex lattice melting in dirty superconductors, using the molecular dynamics method. We have found that when vortex number $N$ is a multiple of 4 and close to a square number, vortex state become rather stable and melting temperature becomes higher.

In future, we will investigate vortex motion in a dirty square superconducting plate under the fixed boundary condition that is more realistic compare to present periodic boundary condition. In the fixed boundary condition, we might use a mirror image method because of the condition for the supercurrent. In this study, effects of impurities on vortex motion is small. This may come from that our parameters for impurities rather are small. Impurity potential dependence on the vortex motion is also a future problem.
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