Two dimensional deformation in microstretch thermoelastic half space with microtemperatures and internal heat source

Praveen Ailawalia*, Sunil Kumar Sachdeva²,³ and Devinder Pathania⁴

Abstract: The purpose of this paper is to study the two dimensional deformation due to internal heat source in a microstretch thermoelastic solid with microtemperatures (MTSM). A mechanical force is applied along the interface of fluid half space and microstretch thermoelastic half space. The normal mode analysis has been applied to obtain the exact expressions for component of normal displacement, microtemperature, normal force stress, microstress tensor, heat flux moment tensor, and couple stress for MTSM. The effect of internal heat source, micropolarity, and microstretch on the above components has been depicted graphically.

1. Introduction

The dynamical interaction between the thermal and mechanical has great practical applications in modern aeronautics, astronautics, nuclear reactors, and high-energy particle accelerators. Classical elasticity is not adequate to model the behavior of materials possessing internal structure. Furthermore, the micropolar elastic model is more realistic than the purely elastic theory for studying the response of materials to external stimuli. Eringen and Suhubi (1964a, 1964b) developed a...
nonlinear theory of microelastic solids. Later Eringen (1965, 1966a, 1996b) developed a theory for the special class of microelastic materials and called it the “linear theory of micropolar elasticity”. Under this theory, solids can undergo macro-deformations and microrotations. Eringen (1990) developed a theory of thermo-microstretch elastic solids in which he included microstructural expansions and contractions. The material points of microstretch solids can stretch and contract independently of their translations and rotations. Microstretch continuum is a model for Bravais lattice with a basis on the atomic level and a two-phase dipolar solid with a core on the macroscopic level. For example, composite materials reinforced with chopped elastic fibers, porous media whose pores are filled with gas or inviscid liquid, other elastic inclusions and “solid-liquid” crystals, etc., should be characterizable by microstretch solids. Eringen (1968) developed a theory of microstretch elastic solid in which he included microstructural expansions and contractions, Singh and Kumar (1998) studied wave propagation in a generalized thermo-microstretch elastic solid, Kumar and Rupender (2008) studied the reflection at free surface of magneto-thermo-microstretch elastic solid, Tomar and Khurana (2009) discussed reflection and transmission of elastic waves from a plane interface between two thermo-microstretch solid half-spaces. Marin (2010) discussed Lagrange identity method for microstretch thermoelastic materials, Othman and Lotfy (2010) studied the plane waves of generalized thermo-microstretch elastic half space under three theories, Kumar and Partap (2009) presented the analysis of free vibrations for Rayleigh–Lamb waves in a microstretch thermoelastic plate with two relaxation times, Othman, Lotfy, and Farouk (2010) studied generalized thermo-microstretch elastic medium with temperature-dependent properties for different theories, Kumar and Kansal (2011) studied fundamental solution in the theory of thermo-microstretch elastic diffusive solids, Othman and Lotfy (2011) studied the effect of rotation on plane waves in generalized thermo-microstretch elastic solid with one relaxation time, Kumar, Sharma, and Sharma (2011) discussed the generalized thermoelastic waves in microstretch plates loaded with fluid of varying temperature. Abbas and Othman (2012) studied the plane waves in generalized thermo-microstretch elastic solid with thermal relaxation using finite element method, Kumar and Rupender (2009) discussed the propagation of plane waves at imperfect boundary of elastic and electromicrostretch generalized thermoelastic solids.

Grot (1969) discussed a theory of thermodynamics of elastic bodies with microstructure whose microelements possess microtemperatures. Říha (1976) studied heat conduction in materials with microtemperatures. Iesan and Quintanilla (2000) studied a theory of thermoelasticity with microtemperatures. Iesan (2001) proposed the theory of micromorphic elastic solids with microtemperatures. Exponential stability in thermoelasticity with microtemperatures was studied by Casas and Quintanilla (2005). Scalia and Svanadze (2006) gave the solutions of the theory of thermoelasticity with microtemperatures. Magaña and Quintanilla (2006) discussed the time decay of solutions in one-dimensional theories of porous materials. Aouadi (2008) discussed some theorems in the isotropic theory of microstretch thermoelasticity with microtemperatures. Ieșan and Quintanilla (2009) discussed thermoelastic bodies with inner structure and microtemperatures. Scalia, Svanadze, and Tracínó (2010) studied basic theorems in the equilibrium theory of thermoelasticity with microtemperatures. Quintanilla (2011) discussed the growth and continuous dependence in thermoelasticity with microtemperatures. Steeb, Singh, and Tomar (2013) studied time harmonic waves in thermoelastic material with microtemperatures. Chiriță, Ciarletta, and D’Apice (2013) studied the theory of thermoelasticity with microtemperatures. Singh, Kumar, and Kumar (2014) discussed a problem in microstretch thermoelastic diffusive medium. Kumar and Kaur (2014) studied the reflection and refraction of plane waves at the interface of an elastic solid and microstretch thermoelastic solid with microtemperatures (MTSM).

In the present problem, the authors have discussed deformation due to internal heat source and a mechanical force which is applied along the interface of fluid half space and microstretch thermoelastic half space with microtemperatures. The normal mode analysis has been applied to obtain the exact expressions for component of normal displacement, microtemperature, normal force stress, microstress tensor, heat flux moment tensor, and couple stress for MTSM. The effect of internal heat source, micropolarity, and microstretch on the above components has been depicted graphically.
The behavior of a thermo-microstretch isotropic material with microtemperatures without body forces, body couples, stretch force, heat sources, and first heat source moment is governed by the following equations given by Eringen (1990) and Ieşan (2007) as,

\[ t_{ij} = \rho \ddot{u}_j \]  
\[ m_{ij} + \varepsilon_{jk} \dot{t}_{jk} - \mu_1 \varepsilon_{ij} w_{rj} = \rho J \dot{\varphi}_j \]  
\[ h_{ij} - s = \frac{\rho \dot{\varphi}}{2} \]  

The constitutive relations are,

\[ t_y = \lambda U_{y, \delta^y} + \mu (u_{ij} + u_{ji}) + K (u_{ij} - \varepsilon_{ij} \varphi_r) - v T \delta_{ij} + \lambda_0 \dot{\varphi}_i \delta_{ij} \]  
\[ m_j = \alpha \varphi_{rj} \delta_{ij} + \beta \varphi_{ij} + \gamma \varphi_{rj} + b_0 \varepsilon_{ijm} \dot{\varphi}_m \]  
\[ \lambda^*_i = a_0 \dot{\varphi}_i^* + b_0 \varepsilon_{ijm} \varphi_{j,m} \]  
\[ q_i = -k_i w_{ij} \delta_{ij} - k_i w_{ij} - k_i w_{ij} \]  
\[ h_i = a_0 \dot{\varphi}_i^* - \mu_2 w_i \]  
\[ s = \lambda_0 e_{rr} - v_1 T + \lambda_1 \varphi^*; \quad i,j,m = 1,2,3 \]  

using Equations 4–9 in Equations 1–3, we get the equations,

\[ (\mu + K) u_{ij} + (\lambda + \mu) u_{ij} - K \varepsilon_{ij} \varphi_r + \lambda_0 \dot{\varphi}_i - v T_j = \rho \ddot{u}_j \]  
\[ \gamma \varphi_{ij} + K \varepsilon_{ij} u_r - 2K \varphi_r - \mu_1 \varepsilon_{ij} w_r = \rho J \dot{\varphi}_i \]  
\[ a_0 \dot{\varphi}_i^* + v_1 T - \lambda_1 \varphi^* - \lambda_0 u_{ij} - \mu_2 w_{ij} = \frac{\dot{\varphi}}{2} \]  
\[ K^* T_{ij} - \rho c^* T - v_1 T_0 \phi^* - v T_0 u_{ij} + k_i w_{ij} = Q_1 \]  
\[ k_i w_{ij} + (k_a + k_{ij}) w_{ij} + \mu_1 \varepsilon_{ij} \dot{\varphi}_r - \mu_2 \dot{\varphi}_i^* - b w_i - k_i w_i - k_{ij} T_{ij} = 0 \]  

where

\[ \nu = (3 \lambda + 2 \mu + K) \alpha_1 + v_1 = (3 \lambda + 2 \mu + K) \alpha_4, \alpha_1, \alpha_4 \]  
\[ \gamma, \alpha, \beta, \lambda, \nu, K, \gamma, \mu \]  
\[ \lambda^* \]  
\[ q_i \]  
\[ k_i \]  
\[ c^* \]  
\[ Q_1 \]  
\[ T_{ij} \]  
\[ T_0 \]  
\[ \varphi^* \]  

The equations of motion and stress components in fluid (Ewing, Jardetzky, & Press, 1957) are:

\[ \lambda^* U_{ij} = \rho^* \ddot{U}_j \]
\[ t'_y = \lambda f_{ij,\delta_j} \]  
(16)

where \( \bar{\mathbf{u}}' = (u'_f) \) is the displacement vector, \( \lambda \) is the fluid constant, and \( \rho' \) is the density of fluid.

We consider a normal force of magnitude \( F_1 \) acting along the interface of microstretch thermoelastic medium with microtemperatures (medium I) occupying the region \( 0 \leq z \leq \infty \) and a non-viscous fluid (medium II) in the region \( -\infty \leq z \leq 0 \) is shown in Figure 1.

A homogeneous isotropic, microstretch thermoelastic solid half space with microtemperatures is considered. We have restricted our analysis to the plane strain parallel to \( xz \) plane with displacement vector \( u_i = (u_x, 0, u_z) \), microtemperature vector \( w_i = (w_x, 0, w_z) \), and microrotation vector \( \varphi_i = (0, \varphi_z, 0) \).

For convenience, the following non-dimensional variables are used:

\[ x' = \frac{x}{L}, \quad z' = \frac{z}{L}, \quad u'_i = \frac{u_i}{L}, \quad w'_i = \frac{Lw_i}{c_L}, \quad t' = \frac{t}{t_0}, \quad t'_y = \frac{t'_f}{v_t}, \quad \varphi'_i = \varphi_i, \quad \varphi''_i = \varphi''_i, \]
\[ m'_y = \frac{m}{L^2 t_0}, \quad q'_0 = \frac{q_0}{L^2}, \quad \lambda''_y = \frac{h'}{L^2 t_0}, \quad T' = \frac{T}{T_0}, \]
\[ F'_1 = \frac{F_1}{L^2 v_t}, \quad Q'_1 = \frac{Q_1}{q_0}, \]

where \( L = \left( \frac{b}{\rho c_T} \right)^{\frac{1}{2}}, \quad c_1 = \frac{c_L^2 + 2K c_T}{\rho}. \)

Assuming the scalar potential functions \( \psi_i(x, z, t), \psi_j(x, z, t), \psi_3(x, z, t), \) and \( \psi_4(x, z, t) \) defined by the relation in non-dimensional form as,

\[ u_1 = \frac{\partial \psi_1}{\partial x} - \frac{\partial \psi_2}{\partial z}, \quad u_3 = \frac{\partial \psi_3}{\partial z} + \frac{\partial \psi_4}{\partial x}; \quad w_1 = \frac{\partial \psi_3}{\partial x} - \frac{\partial \psi_4}{\partial z}; \quad w_3 = \frac{\partial \psi_3}{\partial z} + \frac{\partial \psi_4}{\partial x}. \]  
(17)

using above non-dimensional variables and relation given by Equation 17, Equations 10–14 reduce to (after dropping superscripts),

\[ (A_1 + 1) \nabla^2 - A_2 \frac{\partial^2}{\partial t^2} \psi_1 + A_3 \varphi''_i - A_4 T = 0 \]  
(18)

\[ \nabla^2 - A_2 \frac{\partial^2}{\partial t^2} \psi_2 + A_5 \varphi_2 = 0 \]  
(19)

\[ \nabla^2 - 2A_6 - A_7 \frac{\partial^2}{\partial t^2} \varphi_2 - A_8 \nabla^2 \psi_2 + A_9 \nabla^2 \varphi_4 = 0 \]  
(20)

Figure 1. Geometry of the problem.
\[ (V^2 - A_9 - A_{10} \frac{\partial^2}{\partial t^2}) \phi^* - A_{11} V^2 \psi_4 - A_{12} V^2 \psi_3 + A_{13} T = 0 \]  
\[ (V^2 - A_{14} \frac{\partial}{\partial t}) T - A_{15} \frac{\partial \phi^*}{\partial t} - A_{16} V^2 \psi_1 + A_{17} V^2 \psi_3 = Y \dot{Q}_1 \]  
\[ (V^2 (1 + A_{18}) - A_{19} - A_{20} \frac{\partial}{\partial t}) \psi_3 - A_{21} \frac{\partial \phi^*}{\partial t} - A_{22} T = 0 \]  
\[ (V^2 - A_{19} - A_{20} \frac{\partial}{\partial t}) \psi_4 + A_{23} \frac{\partial \psi_2}{\partial t} = 0 \]

where
\[ A_1 = \frac{\mu + \mu K}{\mu K}, \quad A_2 = \frac{\mu K}{\mu K}, \quad A_3 = \frac{\mu K}{\mu K}, \quad A_4 = \frac{\nu \mu K}{\mu K}, \quad A_5 = \frac{\mu K}{\mu K}, \quad A_6 = \frac{\nu \mu K}{\mu K}, \quad A_7 = \frac{\nu \mu K}{\mu K}, \quad A_8 = \frac{\nu \mu K}{\mu K}, \quad A_9 = \frac{\nu \mu K}{\mu K}, \]
\[ A_{10} = \frac{\nu \mu K}{\mu K}, \quad A_{11} = \frac{\nu \mu K}{\mu K}, \quad A_{12} = \frac{\nu \mu K}{\mu K}, \quad A_{13} = \frac{\nu \mu K}{\mu K}, \quad A_{14} = \frac{\nu \mu K}{\mu K}, \quad A_{15} = \frac{\nu \mu K}{\mu K}, \quad A_{16} = \frac{\nu \mu K}{\mu K}, \quad A_{17} = \frac{\nu \mu K}{\mu K}, \quad A_{18} = \frac{\nu \mu K}{\mu K}, \quad A_{19} = \frac{\nu \mu K}{\mu K}, \quad A_{20} = \frac{\nu \mu K}{\mu K}, \quad A_{21} = \frac{\nu \mu K}{\mu K}, \quad A_{22} = \frac{\nu \mu K}{\mu K}, \quad A_{23} = \frac{\nu \mu K}{\mu K}, \quad A_{24} = \frac{\nu \mu K}{\mu K}, \quad A_{25} = \frac{\nu \mu K}{\mu K} \]

2. Analytic solution
The solution of the considered physical variable can be decomposed in terms of normal mode and can be considered in the following form,

\[ (\psi^*, \phi^*, T, \varphi_2, t_y, q_y, u_y, \psi_4, \psi_3, \psi_2, \psi_1, t_{\bar{Y}}(z, x, t)) = (\bar{\psi}(z), \bar{\phi}(z), \bar{T}(z), \bar{\varphi_2}(z), t_y, q_y, u_y, \bar{\psi}_4, \bar{\psi}_3, \bar{\psi}_2, \bar{\psi}_1, t_{\bar{Y}}(z)) e^{i \omega T + i a x} \]

where \( \omega \) is the complex frequency, \( a \) is the wave number in x-direction, and \( \bar{\psi}(z), \bar{\phi}(z), \bar{T}(z), \bar{\varphi_2}(z), t_y, q_y, u_y, \bar{\psi}_4, \bar{\psi}_3, \bar{\psi}_2, \bar{\psi}_1, t_{\bar{Y}}(z) \) are the amplitudes of field quantities.

Using normal mode in Equations 18–24, we get,

\[ (D^2 - B_9) \bar{\psi}_1 + B_3 \bar{\phi}^* - B_3 \bar{T} = 0 \]  
\[ (D^2 - B_9) \bar{\psi}_2 + A_5 \bar{\phi}_2 = 0 \]  
\[ (D^2 - B_{10}) \bar{\psi}_2 - A_6 (D^2 - \alpha^2) \bar{\psi}_2 + A_6 (D^2 - \alpha^2) \bar{\psi}_4 = 0 \]  
\[ (D^2 - B_{11}) \bar{\phi}^* - A_{11} (D^2 - \alpha^2) \bar{\psi}_1 - A_{12} (D^2 - \alpha^2) \bar{\psi}_3 + A_{13} \bar{T} = 0 \]  
\[ (D^2 - B_{12}) \bar{T} - A_{15} \alpha \bar{\phi}^* - A_{16} (D^2 - \alpha^2) \bar{\psi}_1 + A_{17} (D^2 - \alpha^2) \bar{\psi}_3 = Y \bar{Q}_1 \]  
\[ (D^2 - B_{13}) \bar{\psi}_3 - B_6 \bar{\phi}^* - B_7 \bar{T} = 0 \]  
\[ (D^2 - B_{14}) \bar{\psi}_4 + A_{23} \alpha \bar{\psi}_2 = 0 \]

where
\[ B_1 = \frac{B_1}{A_{12}}, \quad B_2 = \frac{B_2}{A_{12}}, \quad B_3 = \frac{B_3}{A_{12}}, \quad B_4 = \frac{B_4}{A_{12}}, \quad B_5 = \frac{B_5}{A_{12}}, \quad B_6 = \frac{A_{15} \alpha}{A_{12}}, \quad B_7 = \frac{A_{17} \alpha}{A_{12}}, \quad B_8 = \frac{A_{13} \alpha}{A_{12}}, \quad B_9 = \frac{A_{16} \alpha}{A_{12}}, \quad B_{10} = \alpha^2 + A_{10} \alpha^2, \quad B_{11} = \alpha^2 + 2 A_{11} \alpha^2, \quad B_{12} = \alpha^2 + A_{12} \alpha^2, \quad B_{13} = \alpha^2 + A_{13} \alpha^2, \quad B_{14} = \alpha^2 + A_{14} \alpha^2, \quad B_{15} = \alpha^2 + A_{15} \alpha^2, \quad B_{16} = \alpha^2 + A_{16} \alpha^2 \]

and constitutive relations (4–7) become,

\[ \ddot{t}_{xx} = (A_{25} D^2 - \alpha^2 A_{24}) \bar{\psi}_1 + i \alpha (A_{25} - A_{24}) D \bar{\psi}_2 - \bar{T} + A_{26} \bar{\phi}^* \]
\[
\begin{align*}
\ddot{t}_{zz} &= ia(A_{27} + A_{28})D\dot{\psi}_1 - \left( A_{27}D^2 + \alpha^2 A_{28} \right)\dot{\psi}_2 - K\dot{\phi}_2 \\
\ddot{q}_{xx} &= \left( A_{29}a^2 - A_{30}D^2 \right)\dot{\psi}_3 + ia(A_{29} - A_{30})D\dot{\psi}_4 \\
\ddot{q}_{zz} &= \left( A_{30}a^2 - A_{29}D^2 \right)\dot{\psi}_3 + ia(A_{30} - A_{29})D\dot{\psi}_4 \\
\ddot{m}_{yz} &= A_{33}D\dot{\psi}_2 - iaA_{34}\dot{\phi}^* \\
\ddot{x}_2^* &= A_{35}D\dot{\phi}^* - iaA_{34}\dot{\phi}_2
\end{align*}
\]
\[
S = \left[ B_{12}B_{13}B_{14} - B_2A_1, \sigma^2 B_4B_{11} + B_4B_{13}A_{13}A_{15} \omega - B_6B_4A_{13}A_7 \sigma^2 - B_7B_4A_{12}A_{15} \sigma^2 \omega \\
+ B_6B_2B_{12}A_1 \sigma^2 + \sigma^2 B_2B_2(A_{11}A_{17} + A_{12}A_{16}) - \sigma^2 B_2A_{11}B_{12}B_{13} \\
- \sigma^2 B_2A_{13}A_{15}B_{13} + \sigma^2 B_2B_6(A_{11}A_{17} + A_{12}A_{16}) - B_3B_{13}A_{11}A_{15} \sigma^2 \omega + B_3B_{13}B_{11}A_{16} \sigma^2 \right]
\]

\[
E = -(B_{10} + B_{14}) - A_6A_{23} \omega - B_9 + A_2A_6
\]

\[
F = \left[ B_{10}B_{14} + A_6A_{23} \sigma^2 + B_9(B_{10} + B_{14}) + B_7A_6A_{23} \sigma^2 - A_2A_6(B_{14} + \sigma^2) \right]
\]

\[
G = \left[-B_9B_{10}B_{14} - B_9A_6A_{23} \sigma^2 + \sigma^2 A_2A_6B_{14} \right]
\]

\[
B_{15} = YA_{12}B_3 \left[ -A_3B_{13}B_2 + B_7A_{12}B_2 \sigma^2 - \sigma^2 A_2B_7B_3 - B_4B_{13}B_{11} \right]
\]

In a similar manner, we can show that \( \phi^*(z) \), \( \psi_3(z) \), and \( T(z) \) satisfy the equation,

\[
(D^6 + PD^6 + QD^4 + RD^2 + S)(\phi^*(z), \psi_3(z), T(z)) = B_{15}Q_1 \tag{42}
\]

which can be factorized as follows,

\[
(D^2 - r_1^n)(D^2 - r_2^n)(D^2 - r_3^n)(D^2 - r_4^n)\psi_1(z) = B_{15}Q_1 \tag{43}
\]

where \( r_i^n \) (\( n = 1, 2, 3, 4 \)) are the roots of Equation 42.

and \( \psi_1(z) \) and \( \psi_2(z) \) satisfy the equation,

\[
(D^6 + ED^4 + FD^2 + G)(\psi_1(z), \bar{\psi}_2(z)) = 0 \tag{44}
\]

which can be factorized as follows,

\[
(D^2 - h_1^n)(D^2 - h_2^n)(D^2 - h_3^n)\psi_2(z) = 0 \tag{45}
\]

where \( h_i^n \) (\( n = 1, 2, 3 \)) are the roots of Equation 44.

The series solution of Equation 42 has the form,

\[
\tilde{\psi}_1(z) = \sum_{n=1}^{4} [M_n(a, \omega)e^{-r_i^nz}] + N \tag{46}
\]

\[
\phi^*(z) = \sum_{n=1}^{4} [M_n'(a, \omega)e^{-r_i^nz}] + N_1 \tag{47}
\]

\[
\tilde{T}(z) = \sum_{n=1}^{4} [M''_n(a, \omega)e^{-r_i^nz}] + N_2 \tag{48}
\]

\[
\bar{\psi}_3(z) = \sum_{n=1}^{4} [M'''_n(a, \omega)e^{-r_i^nz}] + N_3 \tag{49}
\]
The series solution of Equation 44 has the form,

\[ \tilde{\psi}_2(z) = \sum_{n=1}^{3} [L_n(a, \omega)e^{-h_n z}] \]  

(50)

\[ \tilde{\phi}_2(z) = \sum_{n=1}^{3} [L'_n(a, \omega)e^{-h_n z}] \]  

(51)

\[ \tilde{\psi}_4(z) = \sum_{n=1}^{3} [L''_n(a, \omega)e^{-h_n z}] \]  

(52)

where \( M_n(a, \omega), M'_n(a, \omega), M''_n(a, \omega), M'''_n(a, \omega) \), and \( L_n(a, \omega), L'_n(a, \omega), L''_n(a, \omega) \) are the specific function depending upon \( a \) and \( \omega \).

Using Equations 46–49 in Equations 25, 28–30, we get the following relations,

\[ M'_n(a, \omega) = H_{1n} M_n(a, \omega) \]  

(53)

\[ M''_n(a, \omega) = H_{2n} M_n(a, \omega) \]  

(54)

\[ M'''_n(a, \omega) = H_{3n} M_n(a, \omega) \]  

(55)

similarly, using Equations 50–52 in Equations 26–27 and 31, we get the following relations,

\[ L'_n(a, \omega) = R_{1n} L_n(a, \omega) \]  

(56)

\[ L''_n(a, \omega) = R_{2n} L_n(a, \omega) \]  

(57)

Thus we have,

\[ \tilde{\psi}_\ast(z) = \sum_{n=1}^{4} [H_{1n} M_n(a, \omega)e^{-r_n z}] + N_1 \]  

(58)

\[ \tilde{T}(z) = \sum_{n=1}^{4} [H_{2n} M_n(a, \omega)e^{-r_n z}] + N_2 \]  

(59)

\[ \tilde{\psi}_3(z) = \sum_{n=1}^{4} [H_{3n} M_n(a, \omega)e^{-r_n z}] + N_3 \]  

(60)

\[ \tilde{\phi}_2(z) = \sum_{n=1}^{3} [R_{1n} L_n(a, \omega)e^{-h_n z}] \]  

(61)

\[ \tilde{\psi}_4(z) = \sum_{n=1}^{3} [R_{2n} L_n(a, \omega)e^{-h_n z}] \]  

(62)

\[ \tilde{t}_{xx}(z) = \sum_{n=1}^{4} [H_{4n} M_n(a, \omega)e^{-r_n z}] + \sum_{n=1}^{3} [R_{3n} L_n(a, \omega)e^{-h_n z}] - N_4 \]  

(63)
\[ \tilde{t}_{x_2}(z) = \sum_{n=1}^{4} [H_{5n}M_n(a, \omega)e^{-r,z}] + \sum_{n=1}^{3} [R_{4n}L_n(a, \omega)e^{-h,z}] \]  
(64)

\[ \tilde{t}_{z_2}(z) = \sum_{n=1}^{4} [H_{6n}M_n(a, \omega)e^{-r,z}] - \sum_{n=1}^{3} [R_{3n}L_n(a, \omega)e^{-h,z}] - N_5 \]  
(65)

\[ \tilde{q}_{x_2}(z) = \sum_{n=1}^{4} [H_{7n}M_n(a, \omega)e^{-r,z}] + \sum_{n=1}^{3} [R_{5n}L_n(a, \omega)e^{-h,z}] + N_6 \]  
(66)

\[ \tilde{q}_{x_2}(z) = \sum_{n=1}^{4} [H_{8n}M_n(a, \omega)e^{-r,z}] + \sum_{n=1}^{3} [R_{6n}L_n(a, \omega)e^{-h,z}] \]  
(67)

\[ \bar{m}_{y_2}(z) = \sum_{n=1}^{4} [H_{10n}M_n(a, \omega)e^{-r,z}] + \sum_{n=1}^{3} [R_{8n}L_n(a, \omega)e^{-h,z}] + N_8 \]  
(69)

\[ \tilde{\lambda}_3(z) = \sum_{n=1}^{4} [H_{11n}M_n(a, \omega)e^{-r,z}] + \sum_{n=1}^{3} [R_{8n}L_n(a, \omega)e^{-h,z}] \]  
(70)

where

\[ N = \frac{B_1N}{S} \tilde{Q}, N_1 = \frac{A_1B_1}{B_3} \tilde{Q} - \frac{\omega B_1}{B_3}, N_2 = \frac{i(iN_1 + N_2)iN_4}{B_3}, N_6 = (A_{25}a^2N + N_2 - A_{26}N_1), \]

\[ N_4 = (A_{25}a^2N + N_2 - A_{26}N_1), N_6 = (A_{25}a^2N_3), N_7 = (A_{30}a^2N_3), N_8 = (-iaA_{34}N_1), \]

\[ H_{1n} = -\frac{(A_{17}a^2 - B_1r^2)v}{B_3}, H_{3n} = \frac{(B_{H_2} + B_{H_3})}{B_3}, H_{4n} = (A_{25}r_n^2 - a^2A_{25}) - H_{2n} + A_{26}H_{4n}, \]

\[ H_{5n} = -i\omega r_n(A_{27} + A_{28}), H_{6n} = (A_{24}r_n^2 - a^2A_{25}) - H_{2n} + A_{26}H_{1n}, H_{7n} = (a^2A_{29} - A_{30}r_n^2)H_{3n}, \]

\[ H_{8n} = i\omega r_n(A_{31} + A_{32})r_nH_{3n}, H_{9n} = (a^2A_{30} - A_{29}r_n^2)H_{3n}, \]

\[ R_{1n} = \frac{B_1(B_{H_1}r_n^2)}{B_3}, R_{2n} = \frac{A_{17}a^2w}{(B_{H_1} - B_{H_2})}, R_{3n} = i\omega r_n(A_{24} - A_{25})h_n, R_{4n} = -(A_{25}r_n^2 + a^2A_{28} + KR_{1n}), \]

\[ R_{5n} = -i\omega r_n(A_{29} - A_{30})h_nR_{2n}, R_{6n} = (h_nA_{31} + A_{32}a^2)R_{2n}, R_{7n} = -A_{33}h_nR_{1n}R_{8n} = -iA_{34}R_{1n}, \]

\[ B_{16} = (A_{12}(B_{12} + B_8) + A_{13}A_{17} - (A_{11}A_{17} + A_{12}A_{16})B_3), \]

\[ B_{17} = (A_{11}B_{12} - A_{13}A_{17})B_8 + a^2B_3(A_{11}A_{17} + A_{12}A_{16}). \]

Similarly for medium II (i.e., fluid half space), the solutions are of the form,
where $M_5(a, \omega)$ and $M_5'(a, \omega)$ are the specific functions depending upon $a$ and $\omega$ and $r_5$ is the root of characteristic equation,

$$(D^2 - a^2 + \omega^2)\ddot{u}_1(z) = 0$$

where $l = \frac{\omega^2}{\dot{c}}$ and $r_5 = \sqrt{a^2 - \omega^2}$

Thus we have,

$$\ddot{u}_1(z) = HM_5(b, \omega) e^{-r_5 z}$$

$$\ddot{t}_{zz}(z) = IM_5(b, \omega) e^{-r_5 z}$$

$$\ddot{t}_{xz}(z) = 0$$

where $H = \frac{r_5^2 - \omega^2}{\dot{c}^2}$ and $I = \frac{(\omega/\dot{c}) e^{-r_5 z}}{\omega^2} = \frac{1}{\dot{c}^2}$. 

3. Applications

In this section, we determine the parameters $M_n$; ($n = 1, 2, 3, 4, 5$) and $L_n$; ($n = 1, 2, 3$). In the physical problem, we should suppress the positive exponentials that are unbounded at infinity. Constants $M_1, M_2, M_3, M_4, M_5$ and $L_1, L_2, L_3$ have to be selected such that boundary conditions at the surface $z = 0$ take the form,

$$t_{zz} = t_{zz}' - F_1 e^{\alpha t + i \alpha z}, \quad t_{xz} = t_{xz}', \quad m_{yz} = 0, \quad \lambda_3 = 0, \quad q_{zz} = 0, \quad q_{xz} = 0, \quad \frac{\partial T}{\partial z} = 0, \quad \frac{\partial u_3}{\partial t} = \frac{\partial u_z}{\partial t}$$

where $F_1$ is the magnitude of mechanical force.

Using the expressions of $t_{zz}, t_{xz}, t_{zz}', t_{xz}', m_{yz}, \lambda_3, q_{zz}, q_{xz}, T, u_3, u_z$ into above boundary conditions (77),

give the following equations satisfied by the parameters,

$$\sum_{n=1}^{6} [H_{6n}M_{n}] - \sum_{n=1}^{3} [R_{3n}L_{n}] - IM_5 = N_5 - F_1$$

$$\sum_{n=1}^{4} [H_{5n}M_{n}] + \sum_{n=1}^{3} [R_{4n}L_{n}] = 0$$

$$\sum_{n=1}^{4} [H_{10n}M_{n}] + \sum_{n=1}^{3} [R_{7n}L_{n}] = -N_8$$

$$\sum_{n=1}^{4} [H_{11n}M_{n}] + \sum_{n=1}^{3} [R_{8n}L_{n}] = 0$$

$$\sum_{n=1}^{4} [H_{9n}M_{n}] - \sum_{n=1}^{3} [R_{5n}L_{n}] = -N_7$$
After solving these non-homogeneous system of equations, we get the values of constants $M_1, M_2, M_3, M_4, M_5, L_1, L_2, L_3$ and hence obtain the component of normal displacement, microtemperature, normal force stress, microstress tensor, heat flux moment tensor, and couple stress at the interface of fluid half space and MTSM.

4. Special case

(1) If we neglect micropolarity effect i.e. $\alpha = \beta = \gamma = b_0 = \mu = K = J = 0$, we obtain the results for microstretch thermoelastic solid with microtemperatures without microrotational effect (TSMWM).

(2) If we neglect microstretch effect i.e. $\alpha_0 = \lambda_0 = \lambda_1 = \nu_1 = b_0 = \mu_2 = J_0 = 0$, we obtain the results for thermoelastic solid with microtemperatures without microstretch effect (TSMWS).

(3) If we neglect both micropolarity effect and microstretch effect i.e. $\alpha = \beta = \gamma = 0, \mu = K = J = \alpha_0 = \lambda_0 = \lambda_1 = \nu_1 = b_0 = \mu_2 = J_0 = 0$, we obtain the results for thermoelastic solid with microtemperatures (TSM).

5. Numerical results and discussions

In order to illustrate the theoretical results obtained in the preceding section, we present some numerical results for the physical constants,

The values of micropolar constants are (Eringen, 1984):

\[
\lambda = 9.4 \times 10^{10} \text{ N/m}^2, \quad \mu = 4.0 \times 10^{10} \text{ N/m}^2, \quad \rho = 1.74 \times 10^3 \text{ kg/m}^3, \quad K = 10^{10} \text{ Nm}^{-2}, \quad \gamma = 7.79 \times 10^{-10} \text{ N}, \quad J = 0.0000002 \times 10^{-14} \text{ m}^2, \quad \beta = 0.32 \times 10^{10} \text{ N/m}^2 \text{ K}, \quad b_0 = 0.0098 \times 10^{10} \text{ N}.
\]

The values of thermal parameters are (Dhaliwal & Singh, 1980):

\[
c^* = 0.104 \times 10^4 \text{ Nm/kg/K}, \quad T_o = 298 \text{ K}, \quad K^* = 1.7 \times 10^2 \text{ Ns}^{-1} \text{ K}^{-1}, \quad \alpha_1 = 0.05 \text{ K}^{-1}, \quad \alpha_2 = 0.05 \text{ K}^{-1}, \quad \tau_1 = 0.613 \times 10^3 \text{ s}.
\]

The values of microstretch parameters are (Kumar & Kaur, 2014):

\[
j_0 = 0.000019 \times 10^{-13} \text{ m}^2, \quad \lambda_0 = 0.21 \times 10^{13} \text{ N/m}^2, \quad \lambda_1 = 0.007 \times 10^{12} \text{ N/m}^2, \quad \alpha_0 = 0.008 \times 10^{-7} \text{ N}, \quad b = 0.15 \times 10^{-10} \text{ N}.
\]

The values of microtemperature parameters are (Kumar & Kaur, 2014):

\[
k_1 = 0.0035 \text{ Ns}^{-1}, \quad k_2 = 0.045 \text{ Ns}^{-1}, \quad k_3 = 0.055 \text{ NK}^{-1} \text{s}^{-1}, \quad k_4 = 0.065 \text{ Ns}^{-1} \text{ m}^3, \quad k_5 = 0.076 \text{ Ns}^{-1} \text{ m}^3, \quad k_6 = 0.096 \text{ Ns}^{-1} \text{ m}^3, \quad \mu_1 = 0.0085 \text{ N}, \quad \mu_2 = 0.0095 \text{ N}.
\]

The physical constants for water are given by Ewing et al. (1957):

\[
\sum_{n=1}^{4} [H_{8n} M_n] + \sum_{n=1}^{3} [R_{bn} L_n] = 0
\]

\[
\sum_{n=1}^{4} [H_{2n} r_n M_n] = 0
\]

\[
\sum_{n=1}^{4} [-r_n M_n] + i\alpha \sum_{n=1}^{3} L_n - HM_5 = 0
\]
\[ \lambda' = 2.14 \times 10^9 \text{N/m}^2, \rho' = 10^3 \text{kg/m}^3. \]

The computations are carried out for the value of non-dimensional time \( t = 0.2 \) in the range \( 0 \leq x \leq 10 \) and on the surface \( z = 1.3 \). The numerical values for normal displacement, microtemperature, normal force stress, microstress tensor, heat flux moment tensor, and couple stress are shown in Figures 2–7 for mechanical force with magnitude.

\[ F_1 = 1.0, \quad Q_0 = 1, \quad \omega = \omega_0 + i\xi, \quad \omega_0 = -0.3, \quad \xi = 0.1, \quad Q_1 = 10 \quad \text{and} \quad a = 0.9 \]

(a) MTSM by solid line with centered symbol ◆.

(b) TSMWM by solid line with centered symbol ■.

(c) TSMWS by dashed line with centered symbol ▲.

(d) TSM by dashed line with centered symbol ×.

6. Discussion

The variation of normal displacement for MTSM, TSMWM, and TSMWS is similar in nature. These values decrease sharply in the entire range. The values of normal displacement for TSM increase in the range \( 0 \leq x \leq 2.3 \) and then the values approach zero with a straight curve. The variations of microtemperature for MTSM and TSMWS are opposite in nature which shows that microstructure has significant effect on microtemperature. The values of microtemperature for TSM are very less and lie in a very short range. These variations of normal displacement and microtemperature are shown in Figures 2 and 3, respectively.

Figure 4 depicts that the variations of normal force stress are opposite in nature for both MTSM and TSMWM. This concludes that micropolarity effect is more prominent in the study of normal force stress. The variations of normal force stress for TSMWS and TSM are similar in nature. The values are
also quite close to each other. The values for these medium (TSMWS and TSM) decrease sharply and then follow a straight curve to converge. With difference in magnitude, the variation of microstress tensor for MTSM and TSMWM is similar in nature. These values decrease uniformly and then approach to zero with increase in horizontal distance. The variation of microstress tensor is shown in Figure 5.

Figure 6 shows that the variations of heat flux moment tensor are similar in nature for all mediums. There is difference in magnitude among all the solids which proves the effect of micropolarity and microstress in the medium. It is again observed that the values of heat flux moment tensor for TSM are very less and hence as compared to other medium, the variation lies in a very short range.

In the absence of stretch effect, the variation of couple stress is effected to a great extent as visible in Figure 7. The values increase in the range $0 \leq x \leq 4.0$ and then show a constant behavior. The variations are sharper for TSMWS in comparison to MTSM.
7. Conclusion

Both micropolarity and stretch effect have a significant effect on the normal displacement, micro-temperature, normal force stress, microstress tensor, heat flux moment tensor, and tangential couple stress. The values of all the quantities for a generalized TSM are less in magnitude as compared to the medium with micropolarity and stretch effect. Micropolarity does not show appreciable effect on microstress tensor but microstretch has a significant effect on couple stress. Such type of problems is very useful in the study of earthquake engineering, seismology, and volcanic eruptions. It helps us to study the effect of a heat source in the medium and the deformation caused in the medium due to the heat source.

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Author details
Praveen Ailawalia
E-mail: praveen_2117@rediffmail.com
Sunil Kumar Sachdeva
E-mail: sunilsachdeva.daviet@gmail.com
Devinder Pathania
E-mail: despathania@yahoo.com
ORCID ID: http://orcid.org/0000-0002-3324-9633

1 Department of Applied Sciences and Humanities, M.M. University, Sadopur, Ambala City, Haryana, India.
2 Department of Applied Sciences, D.A.V. Institute of Engineering and Technology, Kabir Nagar, Jalandhar, Punjab, India.
3 Punjab Technical University, Jalandhar, Punjab, India.
4 Department of Applied Sciences, Guru Nanak Engineering College, Ludhiana, Punjab, India.

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References
Abbas, I. A., & Othman, M. I. A. (2012). Plane waves in generalized thermo-microstretch elastic solid with thermal relaxation using finite element method. International Journal of Thermophysics, 33, 2407–2423. http://dx.doi.org/10.1007/s10765-012-1340-8
Anoudi, M. (2008). Some theorems in the isotropic theory of microstretch thermoelasticity with microtemperatures. Journal of Thermal Stresses, 31, 469–682. http://dx.doi.org/10.1080/01495730801981772
Chirita, S., Ciarletta, M., & D’Apice, C. (2013). On the theory of thermoelasticity with microtemperatures. Journal of Mathematical Analysis and Applications, 397, 349–361. http://dx.doi.org/10.1016/j.jmaa.2012.07.061
Dholewal, R. S., & Singh, A. (1980). Dynamic coupled thermoelasticity. New Delhi: Hindustan Publication Corporation.
Eringen, A. C. (1965). Linear theory of micropolar elasticity (ONR Technical Report No. 29). School of Aeronautics, Aeronautics and Engineering Science, Purdue University, West Lafayette, IN.
Eringen, A. C. (1966). A unified theory of thermomechanical materials. International Journal of Engineering Science, 4, 179–202. http://dx.doi.org/10.1016/0020-7225(66)90022-X
Eringen, A. C. (1968). Micropolar elastic solids with stretch. In Mustafa Inan Anisina, Ari. Kitapevi Matbaassi, Istanbul (pp. 1–18).
Eringen, A. C. (1984). Plane waves in nonlocal micropolar elasticity. International Journal of Engineering Science, 22, 1113–1121. http://dx.doi.org/10.1016/0020-7225(84)90112-S
Eringen, A. C. (1990). Theory of thermo-microstretch elastic solids. International Journal of Engineering Science, 28, 1291–1301. http://dx.doi.org/10.1016/0020-7225(90)90076-U
Eringen, A. C. (1996b). Linear theory of micropolar elasticity. Journal of Mathematics and Mechanics, 15, 909–923.
Eringen, A. C., & Suhubi, E. S. (1964). Nonlinear theory of simple micro-elastic solids I. International Journal of Engineering Science, 2, 189–203. http://dx.doi.org/10.1016/0020-7225(64)90004-7
Eringen, A. C., & Suhubi, E. S. (1964). Nonlinear theory of simple micro-elastic solids II. International Journal of Engineering Science, 2, 389–404.
Ewing, W. M., Jardetzky, W. S., & Press, F. (1957). Elastic waves in layered media. New York, NY: McGraw Hill.
Grot, R. A. (1969). Thermodynamics of a continuum with microstructure. International Journal of Engineering Science, 7, 801–816. http://dx.doi.org/10.1016/0020-7225(69)90062-7
Jesan, D. (2001). On a theory of micromorphic elastic solids with microtemperatures. Journal of Thermal Stresses, 24, 737–752.
Ieşan, D., & Quintanilla, R. (2000). On a theory of thermoelasticity with microtemperatures. *Journal of Thermal Stresses, 23*, 199–215.

Ieşan, D. (2007). Thermoelasticity of bodies with microstructure and microtemperatures. *International Journal of Solids and Structures, 44*, 8648–8662. [http://dx.doi.org/10.1016/j.ijsolstr.2007.06.027](http://dx.doi.org/10.1016/j.ijsolstr.2007.06.027)

Ieşan, D., & Quintanilla, R. (2009). On thermoelastic bodies with inner structure and microtemperatures. *Journal of Mathematical Analysis and Applications, 354*, 12–23. [http://dx.doi.org/10.1016/j.jmaa.2008.12.017](http://dx.doi.org/10.1016/j.jmaa.2008.12.017)

Kumar, R., & Kaur, M. (2014). Reflection and refraction of plane waves at the interface of an elastic solid and microstretch thermoelastic solid with microtemperatures. *Archive of Applied Mechanics, 84*, 571–590. [http://dx.doi.org/10.1007/s00041-014-0818-1](http://dx.doi.org/10.1007/s00041-014-0818-1)

Kumar, R., & Partap, G. (2009). Analysis of free vibrations for Rayleigh–Lamb waves in a microstretch thermoelastic plate with two relaxation times. *Journal of Engineering Physics and Thermophysics, 82*, 35–46.

Kumar, R., & Rupender, R. (2008). Reflection at free surface of magneto-thermo-microstretch elastic solid. *Bulletin of the Polish Academy of Sciences, 56*, 263–271.

Kumar, R., & Rupender, R. (2009). Propagation of plane waves at the imperfect boundary of elastic and electro-microstretch generalized thermoelastic solids. *Applied Mathematics and Mechanics, 30*, 1445–1454. [http://dx.doi.org/10.1007/s10483-009-1110-6](http://dx.doi.org/10.1007/s10483-009-1110-6)

Kumar, S., Sharma, J. N., & Sharma, Y. D. (2011). Generalized thermoelastic waves in microstretch plates loaded with fluid of varying temperature. *International Journal of Applied Mechanics, 3*, 563–586. [http://dx.doi.org/10.1142/S1758825111001335](http://dx.doi.org/10.1142/S1758825111001335)

Maghre, A., & Quintanilla, R. (2006). On the time decay of solutions in one-dimensional theories of porous materials. *International Journal of Solids and Structures, 43*, 3414–3427. [http://dx.doi.org/10.1016/j.ijsolstr.2005.06.077](http://dx.doi.org/10.1016/j.ijsolstr.2005.06.077)

Marin, M. (2010). Lagrange identity method for microstretch thermoelastic materials. *Journal of Mathematical Analysis and Applications, 363*, 275–286. [http://dx.doi.org/10.1016/j.jmaa.2009.08.045](http://dx.doi.org/10.1016/j.jmaa.2009.08.045)

Othman, M. I. A., & Lotfy, Kh. (2013). Effect of rotation on plane waves in generalized thermo- microstretch elastic solid with one relaxation time. *Materials and Structures, 7*, 43–62. [http://dx.doi.org/10.1080/15736101111141430](http://dx.doi.org/10.1080/15736101111141430)

Othman, M. I. A., Lotfy, Kh., & Farouk, R. M. (2010). Generalized thermo-microstretch elastic medium with temperature dependent properties for different theories. *Engineering Analysis with Boundary Elements, 34*, 229–237. [http://dx.doi.org/10.1016/j.enganabound.2009.10.003](http://dx.doi.org/10.1016/j.enganabound.2009.10.003)

Quintanilla, R. (2011). On growth and continuous dependence in thermoelasticity with microtemperatures. *Journal of Thermal Stresses, 34*, 911–922. [http://dx.doi.org/10.1080/014957339.2011.586278](http://dx.doi.org/10.1080/014957339.2011.586278)

Singh, D., Kumar, A., & Kumar, R. (2014). A problem in microstretch thermoelastic diffusive medium. *Journal of Engineering Science, 36*, 891–912. [http://dx.doi.org/10.1016/S0020-7225(97)00099-2](http://dx.doi.org/10.1016/S0020-7225(97)00099-2)

Singh, B., & Kumar, R. (1998). Wave propagation in a generalized thermo-microstretch elastic solid. *International Journal of Engineering Science, 36*, 721–753. [http://dx.doi.org/10.1016/0020-7225(97)00099-2](http://dx.doi.org/10.1016/0020-7225(97)00099-2)

Othman, M. I. A., & Lotfy, Kh. (2013). On the plane waves of generalized thermo-microstretch elastic half space under three theories. *International Communications in Heat and Mass Transfer, 37*, 192–200. [http://dx.doi.org/10.1016/j.icheatmasstransfer.2009.09.017](http://dx.doi.org/10.1016/j.icheatmasstransfer.2009.09.017)

Othman, M. I. A., Lotfy, Kh., & Farouk, R. M. (2010). Generalized thermo-microstretch elastic medium with temperature dependent properties for different theories. *Engineering Analysis with Boundary Elements, 34*, 229–237. [http://dx.doi.org/10.1016/j.enganabound.2009.10.003](http://dx.doi.org/10.1016/j.enganabound.2009.10.003)