Polyakov loop potential at finite density

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The Polyakov loop potential serves to distinguish between the confined hadronic and the deconfined quark-gluon plasma phases of QCD. For \( N_f = 2 + 1 \) quark flavors with physical masses we determine the Polyakov loop potential at finite temperature and density and extract the location of the deconfinement transition. We find a cross-over at small values of the chemical potential running into a critical end-point at \( \mu/T > 1 \).

Introduction

In recent years much progress has been made in our understanding of the phase structure of QCD at finite temperature and density. This understanding has been achieved with a variety of methods ranging from first principle lattice and continuum computations to elaborate model studies. At vanishing density all these methods by now converge quantitatively leaving only a few open fundamental questions, e.g. the order of the phase transitions in different regions of the Columbia plot. In turn, at finite density, progress has been hampered by several intricate problems. On the lattice one has to face the sign problem density, progress has been hampered by several intricate different regions of the Columbia plot. In turn, at finite tal questions, e.g. the order of the phase transitions in verge quantitatively leaving only a few open fundamental model studies.

The phase diagram with functional methods

In the past decade continuum quark and gluon correlations functions have been computed with the help of functional equations for the effective action of QCD. These works have been mostly performed in (background) Landau gauge,

\[
\bar{D}_\mu A_\mu = 0 , \quad \text{with} \quad \bar{D}_\mu = \partial_\mu - ig\bar{A}_\mu , \quad (1)
\]

where \( \bar{A} \) is chosen to be the expectation value of the gauge field, \( \bar{A} = \langle A \rangle \). The present work also utilizes the gauge \([1]\). Correlation functions in ordinary Landau gauge are directly related to those in background Landau gauge by simply substituting plain momentum \( p^2 \) with background covariant momentum, \( p^2 \rightarrow -D^2 \) \([3,4]\). In this approach the Polyakov loop variable

\[
L = \frac{1}{N_c} \text{tr}_{\text{fund}} P(\bar{x}) , \quad \text{with} \quad P(\bar{x}) = \mathcal{P} \, e^{ig \int_0^\beta dx_\tau A_\mu(x_\tau, \bar{x})} ,
\]

in the fundamental representation, evaluated at the minimum of the Polyakov loop effective potential \( V[A_0] \), is an order parameter for confinement \([3,5]\). The effective potential is defined from the effective action \( \Gamma \), evaluated at constant background fields \( A_0^{\text{const}} \) and vanishing gauge fields,

\[
V[A_0^{\text{const}}] := \frac{1}{\beta V} \Gamma[A_0^{\text{const}}, 0] .
\]

The minimum of \( V[A_0] \) singles out the expectation value of the gauge field in the background Landau gauge, \( \langle A_0 \rangle \). The related order parameter satisfies

\[
L(\langle A_0 \rangle) \geq \langle L[A_0] \rangle \quad (4)
\]

within an appropriate (re)normalization of \( \langle L[A_0] \rangle \), see \([3,6]\). This inequality holds true for both, Yang-Mills theory and fully dynamical QCD. In the presence of a phase transition both sides vanish at \( T_c \) and the inequality \([1]\) is saturated below \( T_c \). In turn, in the presence of a cross-over we expect the cross-over temperature computed from \( L(\langle A_0 \rangle) \) to be lower than the one computed from \( \langle L[A_0] \rangle \).

The effective potential, or its \( A_0 \)-derivative, can be computed from the functional DSE and FRG equations, see Fig. \([1]\) and Fig. \([2]\) respectively. For the FRG this has been put forward in Yang-Mills theory, \([3,10]\), and in QCD at finite temperature and imaginary chemical potential in \([7]\). There, the effective potential \( V[A_0] \) is computed solely from the scale-dependent propagators. More recently, a similar computation of the Polyakov loop potential has also been performed in Coulomb gauge, \([8,9]\). Related lattice computations can be found in \([10,13]\).
In the physical case of $SU(3)$ we restrict ourselves to $2\pi T \varphi / g \tau^3$ in the Cartan subalgebra generated by $\tau^3, \tau^8$. The corresponding eigenvalues are given by

$$
\varphi_{\text{ad}} \in \{ \pm \varphi, \pm \frac{\varphi}{2}, 0, 0 \}, \quad \varphi_{\text{fund}} \in \{ \pm \frac{\varphi}{2}, 0 \},
$$

(7)

for more details see [4, 9]. Then, the shifted Matsubara frequencies $p_0 + g A_0$ read after diagonalization,

$$
2\pi T(n + \varphi_{\text{ad}}), \quad \text{and} \quad 2\pi T \left( n + \frac{1}{2} + \varphi_{\text{fund}} \right),
$$

(8)

for ghost, gluon in the adjoint representation and the quark in the fundamental representation respectively.

The additive nature of the loop representation in Fig. 1 and Fig. 2 leads to the simple form

$$
V(\varphi) = V_{\text{glue}}(\varphi) + V_{\text{quark}}(\varphi).
$$

Here, $V_{\text{glue}}$, contains all contributions from the gluon and ghost diagrams in the DSE and FRG, see Figs. [1].

In the present approximation, i.e. without the backreaction of $V[A_0]$ to the chromo-electric gluon, all diagrams contributing to $V_{\text{glue}}$ involve only traces and contractions in the adjoint representation, and hence the eigenvalues $\varphi_{\text{ad}}$ in (9, 7). In turn, the matter contribution, $V_{\text{quark}}$, involves only traces and contractions in the fundamental representation, and hence the eigenvalues $\varphi_{\text{fund}}$ in (6, 7). With (8) this leads to the periodicities

$$
V_{\text{glue}}(\varphi + 2) = V_{\text{glue}}(\varphi), \quad V_{\text{quark}}(\varphi + 2) = V_{\text{quark}}(\varphi),
$$

(10)

for the physical case of $SU(3)$. For comparison we also quote the $SU(2)$-case where we have

$$
V_{\text{glue}}(\varphi + 1) = V_{\text{glue}}(\varphi), \quad V_{\text{quark}}(\varphi + 2) = V_{\text{quark}}(\varphi).
$$

(11)

We observe that the periodicity of $V_{\text{quark}}$ is independent of $N_c$ in contrast to that of the glue part. The latter dependence reflects the fact that $V_{\text{glue}}$ is center-symmetric and hence invariant under $Z_{N_c}$-transformations. For the simple case of $N_c = 2$ the Cartan is one-dimensional and a center transformation entails $\varphi \rightarrow 1 - \varphi$ with center-symmetric point $\varphi = 1/2$. Evidently this is not the symmetry of the quark potential $V_{\text{quark}}$ due to its periodicity, see (11). The Polyakov loop in the fundamental representation in $SU(2)$ reads

$$
L(\varphi) = \cos(\pi \varphi),
$$

(12)

and vanishes at the center-symmetric point $\varphi = 1/2$.

For $N_c = 3$ and higher $N_c$ a center transformation is a rotation in the Cartan. Accordingly, the explicit center-breaking in the quark potential is only visible for general

1 At finite chemical potential, this involves a center average.
gauge fields in the Cartan sub-algebra, i.e. $A_0 = A_3^0 + A_8^0 \tau^8$, which are not considered here. Interestingly, for $SU(3)$ the quark potential has the same periodicity w.r.t. $\varphi$ as the glue potential in contradistinction to $SU(2)$. This may be a helpful property for model applications at finite density, \cite{15-22}, and shall be studied elsewhere. The Polyakov loop in three-color QCD reads

$$L(\varphi) = \frac{1}{3} (1 + 2 \cos(\pi \varphi)),$$

which vanishes at the confining values $\varphi = 2/3, 4/3$ in the fundamental period $\varphi \in \{0, 2\}$. This gives us direct access to an order parameter potential for the confinement-deconfinement phase transition in a DSE-approach to the phase structure of QCD as put forward in \cite{23,24}. In the following we will exploit this approach at finite temperature and density thus providing first insights into the Polyakov loop potential at finite density.

**DSE for the quark and gluon propagators**

In order to determine the $N_f = 2 + 1$ quark and gluon propagators at finite temperature and chemical potential we have solved their corresponding DSEs given diagrammatically in Figs. 3 and 4. In the gluon DSE we work with an approximation neglecting unquenching effects in the Yang-Mills part of the equation. Consequently this part can be replaced by the inverse quenched propagator denoted by the diagram with the box labelled 'YM' in Fig. 4. This approximation is valid on the few percent level \cite{24}. For the quenched gluon propagator one may use corresponding lattice results \cite{14,26-28} or input from a FRG calculation within Yang-Mills theory \cite{14,24}. We have checked that our results for the potential and the respective critical temperatures are hardly affected by this choice. This is a direct consequence of the inheritance of the above-mentioned renormalisation scheme in the quenched case \cite{4}, allowed by the absence of two-loop diagrams in the matter sector of the DSE. To make contact with the results of Ref. \cite{24} in the following we use the direct access to an order parameter potential for the confinement-deconfinement phase transition in a DSE-framework \cite{24}, are compared with very recent unquenched lattice data from \cite{25}. The unquenched results (solid lines), predicted in the DSE framework \cite{24}, are compared with very recent unquenched lattice data from \cite{25}. We observe large

![FIG. 3. The DSE for the quark propagator.](image)

![FIG. 4. The truncated gluon DSE for $N_f = 2 + 1$ QCD. The first term is the inverse quenched propagator.](image)

![FIG. 5. Quenched and unquenched gluon dressing functions $Z^L$ (upper plot) and $Z^T$ (lower plot), see (14), compared to gauge-fixed unquenched lattice data from \cite{25}.](image)
unquenching effects in the longitudinal part of the propagator and somewhat smaller effects in the magnetic part. For both dressing functions the prediction from the functional framework is nicely matched by the lattice data. We believe these results provide solid justification for the vertex construction and the truncation of the gluon DSE used in our work.

Results

The DSE for the potential depicted in Fig. 1 is used to compute \( \partial_\varphi V(\varphi) \). Upon \( \varphi \)-integration this yields the Polyakov loop potential \( V(\varphi) \) as a function of temperature and chemical potential. In Fig. 6 and Fig. 7 we show the dimensionless potential \( V(\varphi)/p_{SB} \) with \( V(0) = 0 \) and \( p_{SB} = \frac{3\pi^2}{36} T^4 + \frac{3}{2} T^2 \mu^2 + \frac{3}{16\pi^2} \mu^4 \). The pressure is hidden in the integration constant [4] and will be discussed elsewhere.

We have computed the Polyakov loop potential \( V(\varphi) \) in 2+1 flavor QCD at the physical pion mass. The confining minimum with vanishing Polyakov loop, \( L(\varphi) = 0 \), is at \( \varphi = 2/3 \), see [13]. In turn, for \( \varphi = 0 \) we have \( L(\varphi = 0) = 1 \). One clearly sees the transition from the confining regime at low temperature/small chemical potential to the deconfined phase at high temperature/large chemical potential. The sharper cross-over transition as a function of chemical potential with fixed \( T = 115 \) MeV reflects the proximity of the critical endpoint.

Fig. 8 shows the Polyakov loop (2) evaluated at the minimum \( \langle A_0 \rangle \) of the effective potential \( V[A_0] \). For small chemical potential or densities the deconfinement transition is a smooth cross-over. There is no unique definition of the cross-over temperature \( T_{conf} \). In the present work we use the inflection point of the Polyakov loop,

\[
\partial_T L[\langle A_0 \rangle]_{T_{conf}} \geq \partial_T L[\langle A_0 \rangle].
\]

i.e., the maximum of the thermal derivative. Other definitions include the inflection point of the expectation value \( \langle A_0 \rangle \), and that of the dual chiral condensate as computed in [24] for 2+1 flavors. In [24] the cross-over temperature is computed from the susceptibility and differs slightly from the dual \( T_{conf} \) computed here. Also the quark masses have been slightly larger than the physical ones; this has been corrected in the present work. The cross-over sharpens with increasing chemical potential and finally turns into a first order transition at \( (T_*, \mu_*) = (101 \text{ MeV}, 174 \text{ MeV}) \). Note that the critical point \( (T_*, \mu_*) \) as well as the first order line does not depend on the definition of the cross-over temperature. In Fig. 9 we show \( T_{conf} \) together with the chiral transition temperature \( T_\chi \) which is obtained from the inflection point of the light-quark condensate. The shaded area shows the width of the deconfinement cross-over defined by 80% of the inflection peak. Interestingly, all transition temperatures, \( T_{conf} \) and \( T_\chi \) agree within this width for the whole phase diagram. Since definitions of \( T_{conf} \) with either Polyakov loop potential or dressed Polyakov loop are based on different properties of the quark and gluon
correlation functions, this provides a highly non-trivial check of the self-consistency of the present approximation. Nevertheless, at very large chemical potential the present scheme may not be sufficient, see Ref. [24] for a more detailed discussion.

In this work we presented the first results for the Polyakov loop potential at finite chemical potential in QCD with $N_f = 2 + 1$, evaluated from a combination of functional and lattice methods. Besides providing input for model applications, our results serve as a benchmark prediction for future evaluations of the potential with different methods.

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