An improved ranking method for fuzzy numbers using integral value of inverse function

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Abstract. Ranking fuzzy numbers is very important decision making in process, procedure, analysis and application. In practice, many problems in real situation which need handling and evaluating for problems have fuzzy data, so that ranking fuzzy number can be used to make decision precisely. Vincent and Dat proposed an improved Liou and Wang’s approach for left, right, and total integral values of the fuzzy number. However, the improved by Vincent and Dat fails to rank trapezoidal fuzzy numbers having equal the compensation of areas. This paper proposes an improved ranking method to overcome shortcomings of Vincent and Dat’s ranking method. The improved ranking method for ranking trapezoid fuzzy numbers, presenting the left, right, and total integral values from inverse function of trapezoid fuzzy numbers. Finally, the comparative example is given here to illustrate the advantages of the improved ranking method for ranking trapezoid fuzzy numbers

1. Introduction
The defuzzification of fuzzy number plays an important role in the decision support system under fuzziness various applications in real life. In practice, various problems in real life that need handling and evaluating for problems have fuzzy data, such that ranking fuzzy number can be used to make decision precisely. So far, a lot of literature has discussed the ranking function. Such as, Liou and Wang [1] proposed integral total ranking based inverse of membership function. Kumar et al. [2] presented integral value based on the exponential fuzzy number of membership function. Kumar et al. [3] discussed Liou and Wang’s ranking method modification based on the non-normal fuzzy number of membership function. Vincent et al. [6] introduced centroid ranking method and epsilon deviation degree of membership function. Ebrahimnejad [4] improved the Kumar’s ranking method based on the trapezoidal fuzzy number and applied in fuzzy transportation problem. Vincent and Dat [5] developed Liou and Wang’s ranking method with left and right membership function of triangular and trapezoidal fuzzy number. Rezvani [7] proposed the concept of mellin’s transform based exponential fuzzy number of membership function. Vague value ranking based on protection ambiguity of fuzzy number [8]. centroid of centroid plus index of optimistic to rank fuzzy number was proposed by Peddi
[9]. The newly magnitude method with index of optimistic was proposed by Vincent et al. [10] to overcome Ezzati et al.’s ranking method. Wang [11] presented the preference relative ranking method based on triangular and trapezoidal fuzzy numbers. Dempster-Shafer theory (DST) ranking method with fuzzy target based trapezoidal fuzzy number [12]. The trapezoidal bipolar VIKOR ranking method that applied in fuzzy TOPSIS of MADM [13].

Based on the ranking methods, Liou and Wang’s ranking method is cited by many researchers [2, 5, 4]. Liou and Wang’s ranking method value still have shortcomings in ranking fuzzy numbers. These ranking cannot differentiate between normal and non-normal in either triangular and trapezoidal fuzzy numbers [15]. Liou and Wang’s ranking method also failed to fuzzy number with compensation of areas [14]. Based on the shortcomings of Liou and Wang’s ranking method that have been mentioned. Vincent and Dat [5] overcame with total integral value base left and right of membership function. However, Vincent and Dat’s ranking method still has shortcomings for ranking the generalized trapezoidal fuzzy numbers with compensation of areas that have different hight. Therefore, we proposes the development of Vincent and Dat’s ranking by using left and right inverse function of membership function with index of optimistic. Finally, several comparative examples are presented to show that the improve ranking is capable to revise the shortcomings of Liou and Wang’s and Vincent and Dat’s ranking methods.

2. The previous ranking methods

This section, the previous ranking methods are provided

Let $H: f(R) \rightarrow R, f,\,$ is fuzzy sets which are defined on the real numbers set. Liou and Wang [1] introduced a ranking method of $\mathcal{A} = (a_1, a_2, a_3, a_4; 1)$ as follows

$$f^i_\mathcal{A}(\mathcal{A}) = \frac{1}{2}[\alpha(a_3 + a_4) + (1 - \alpha)(a_1 + a_2)]$$

where $\alpha = [0,1]$ is optimistic index. $\alpha$ is used a decision makers to relate degree of optimism where $\alpha = 1$ is decision of optimistic, $\alpha = 0.5$ is decision of moderate and $\alpha = 0$ is a decision of pessimistic. Let $\mathcal{M} = (\mathcal{A}, \mathcal{B}) \subseteq f,\,$ where $\mathcal{A} = (a_1, a_2, a_3, a_4; 1)$ and $\mathcal{B} = (b_1, b_2, b_3, c_4; 1)$ are trapezoid fuzzy numbers. The ranking function proposed by [3], i.e.

a) if $\mathcal{A} > \mathcal{B}$ then $H(\mathcal{A}) > H(\mathcal{B})$

b) if $\mathcal{A} < \mathcal{B}$ then $H(\mathcal{A}) < H(\mathcal{B})$

c) if $\mathcal{A} = \mathcal{B}$ then $H(\mathcal{A}) = H(\mathcal{B})$

Vincent and Dat [5] developed Liou and Wang’s is named total integral value ranking method based on left and right membership function of $\mathcal{A} = (a_1, a_2, a_3, a_4; 1)$ as follows

$$f^i_\mathcal{A}(\mathcal{A}) = \frac{1}{2}[\alpha(a_3 + a_4) + (1 - \alpha)(a_2 + a_1) - 2x_{\min}]$$

where where $x_{\min} = \inf P, P = \bigcup_{i=1}^{4} P_i, P_i = \{\mu_i > 0\}, \alpha = [0,1]$ is optimistic index.

3. Improved ranking method

The improved Liou and Wang’s and Vincent and Dat’s ranking methods propose to rank trapezoid fuzzy numbers. This improved ranking method uses left and right inverse function of trapezoid membership function with index of optimistic function as follows:
**Definition 1** Let \( \tilde{A} = (a_1, a_2, a_3, a_4; 1) \) is a trapezoid fuzzy number, \( \mu_{\tilde{A}} \) can be presented by

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
\frac{(x-a_1)}{(a_2-a_1)}, & a_1 \leq x < a_2, \\
1, & a_2 \leq x \leq a_3, \\
\frac{(x-a_3)}{(a_4-a_3)}, & a_3 < x \leq a_4, \\
0, & \text{otherwise}.
\end{cases}
\]

(3)

It is clear that from Eq. (3) that if \( v_{\tilde{A}} = \frac{(x-a_1)}{(a_2-a_1)} \) with \( v_{\tilde{A}}: [0,1] \rightarrow [a_1, a_2] \) and \( v_{\tilde{B}} = \frac{(x-a_3)}{(a_4-a_3)} \) with \( v_{\tilde{B}}: [0,1] \rightarrow [a_3, a_4] \), then inverse function of \( v_{\tilde{A}} \) and \( v_{\tilde{B}} \) are \( v_{\tilde{A}}^{-1}: [a_1, a_2] \rightarrow [0,1] \) and \( v_{\tilde{B}}^{-1}: [a_3, a_4] \rightarrow [0,1] \), respectively. Hence, Based on Liou and Wang [1] \( v_{\tilde{A}}^{-1} \) and \( v_{\tilde{B}}^{-1} \) are integrable on the closed interval \([0,1]\). Obvious that, \( \mu_{\tilde{A}}^{-1} = \int_0^1 v_{\tilde{A}}^{-1}(y) \, dy \) and \( \mu_{\tilde{B}}^{-1} = \int_0^1 v_{\tilde{B}}^{-1}(y) \, dy \) exist. So, with \( y \in [0,1] \)

\[
\int_0^1 v_{\tilde{A}}^{-1}(y) \, dy = \int_0^1 [a_1 + (a_2 - a_1)y] \, dy = \frac{1}{2} (a_1 + a_2)
\]

(4)

and

\[
\int_0^1 v_{\tilde{B}}^{-1}(y) \, dy = \int_0^1 [a_3 + (a_4 - a_3)y] \, dy = \frac{1}{2} (a_3 + a_4)
\]

(5)

by Eq. (4) and (5), the improved ranking from Vincent and Dat’s ranking method can be defined as

\[
S'_{L}(\tilde{A}) = a_2 - x_{\min} - \int_0^1 v_{\tilde{A}}^{-1}(y) \, dy = -\frac{1}{2} (a_1 + 3a_2) - x_{\min}
\]

(6)

\[
S'_{R}(\tilde{A}) = a_3 - x_{\min} + \int_0^1 v_{\tilde{A}}^{-1}(y) \, dy = \frac{1}{2} (3a_3 + a_4) - x_{\min}
\]

(7)

where \( x_{\min} = \inf \mu P, P = \bigcup_{i=1}^{k} P_i, P_i = \{ \frac{\mu_{A_i}}{x} > 0 \} \). Thus, with \( \alpha \in [0,1] \), by (6) and (7) the improved ranking of trapezoid fuzzy number \( \tilde{U} \) can be obtained with

\[
S_{\alpha}(\tilde{A}) = aS'_{R} + (1 - \alpha)S'_{L} = \left( \frac{\alpha}{2} \right) [a(3a_3 + a_4) - (1 - \alpha)(a_1 + 3a_2)] - x_{\min}
\]

(8)

Let \( \tilde{A} = (a_1, a_2, a_3, a_4; 1) \) and \( \tilde{B} = (b_1, b_2, b_3, b_4; 1) \) be two trapezoid fuzzy numbers. The Algorithm 1 can be used to order \( \tilde{A} \) and \( \tilde{B} \).

**Algorithm 1**

**Step 1:** Determine \( S'_{L} \) via (6) and \( S'_{R} \) via (7)

**Step 2:** Determine \( S'_{T} \) via (8)

**Step 3:** Ordering of trapezoid fuzzy numbers \( \tilde{U}_1 \) and \( \tilde{U}_2 \) as follows:

(a) if \( S'_{L}(\tilde{A}) < S'_{L}(\tilde{B}) \), then \( \tilde{A} < \tilde{B} \);

(b) if \( S'_{T}(\tilde{A}) > S'_{T}(\tilde{B}) \), then \( \tilde{A} > \tilde{B} \);

(c) if \( S'_{T}(\tilde{A}) = S'_{T}(\tilde{B}) \), then \( \tilde{A} = \tilde{B} \)

4. Comparative of numerical example

In this section presents the shortcomings of existing ranking methods i.e. Vincent and Dat’s ranking [5] and Liou and Wang’s ranking [1] and also to overcome both of these existing ranking methods by using the proposed ranking method.

**Example 1** Given trapezoid fuzzy numbers i.e. \( \tilde{A} = (2,3,4,5; 1) \) and \( \tilde{B} = (4,5,6,7; 1) \). Clearly, \( \tilde{A} \neq \tilde{B} \) and \( h_{\tilde{A}} = h_{\tilde{B}} \). By using the existing ranking function of Vincent and Dat’s ranking [5] in Eq. (1), Liou
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and Wang’s ranking [1] in Eq. (2) and the proposed method in Eq. (7) can be obtained \( T_\alpha (A) = 3\alpha - 2 \), \( T_\alpha (B) = 3\alpha - 2 \), \( T_\alpha (A) = 2\alpha - 2.5 \), \( T_\alpha (B) = 4\alpha - 4.5 \) and \( T_\alpha (A) = 5.2 - 6\alpha \), \( T_\alpha (B) = 9.3 - 12\alpha \) respectively. For alpha = 0 the ranking value of two fuzzy numbers by using Vincent and Dat’s ranking are \( T_0 (A) = 5.2 \) and \( T_0 (B) = 9.3 \) i.e. \( A \neq B \), by using Liou and Wang’s ranking are \( T_0 (A) = 4.5 \) i.e. \( A > B \), then by using proposed ranking method are \( T_0 (A) = 5.2 \) and \( T_0 (B) = 9.3 \) i.e. \( A > B \). For alpha = 0.5 the ranking value of two fuzzy numbers by using Vincent and Dat’s ranking are \( T_{0.5} (A) = 5.2 \) and \( T_{0.5} (B) = 9.3 \) i.e. \( A > B \), then by using proposed ranking method are \( T_{0.5} (A) = 5.2 \) and \( T_{0.5} (B) = 9.3 \) i.e. \( A > B \). For alpha = 1 the ranking value of two fuzzy numbers by using Vincent and Dat’s ranking are \( T_1 (A) = 5.2 \) and \( T_1 (B) = 9.3 \) i.e. \( A > B \), then by using proposed ranking method are \( T_1 (A) = 5.2 \) and \( T_1 (B) = 9.3 \) i.e. \( A > B \). In the other word, for \( \alpha \in [0,1] \), The improved ranking is capable to overcome the shortcoming of existing ranking function with \( h_A \neq h_B \).

5. Conclusion
The total integral ranking is proposed to overcome the shortcomings of the existing ranking method i.e. Vincent and Dat’s ranking and Lio and Wang’s ranking. The improved ranking method develops of Vincent and Dat’s ranking by using left and right inverse function of membership function with index of optimistic. By using the comparative example is obtained that the total integral ranking is capable to give ranking that different with the existing ranking metho and also the improved ranking has consistently for ranking trapezoid fuzzy number.

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