Standard Reference for Zero Temperature from Quantum Supersymmetry is Possible?

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Abstract. Supersymmetry at a susy harmonic oscillator, \( H(\omega_1, \omega_1) \), can be broken or restored in certain conditions and parameters, that are linked with thermal interaction and with a polynomial interactions of creation and annihilation operators. All possibles supersymmetric harmonic oscillators represented by a point \( (\omega_1, \omega_1) \) in the frequency space of the system, are in a two dimensional surface parametrized by the \( (\omega_2, \alpha_2) \), which we call s-surface, where \( \alpha_2 \) is the interaction parameter. The temperature in the s-surface are intended to be zero. Interaction with the thermal bath represented by the tilde Hilbert space from the doubling Hilbert space, establishes thermal oscillations that push the system from the s-surface. In such a way we can define the set of all supersymmetric harmonic oscillator or in a equivalent way the s-surface as a global standard reference for zero temperature.

1. Introduction

Supersymmetry can be broken due thermal effects or due some interactions, as we demonstrated in the work: supersymmetry breaking at finite temperature in a SUSY harmonic oscillator with Interaction ref[1]. The statistical average of the Hamiltonian at finite positive temperature is not zero. So, considering this referred SUSY model, it is possible to define a s-surface, parameterized with two parameters, that maps all supersymmetric harmonic oscillators.

Although there are some controversies in the literature, refs [2], [3], [4], [5], [6] and [7], of whether supersymmetry is broken at finite temperature, in this paper we follow the viewpoint that supersymmetry (SUSY) is broken at positive temperature even when unbroken at \( T = 0 \), ref[2]; an example is the supersymmetric harmonic oscillator, that exists only in \( T = 0 \), as we can see by the ref [1]. We will consider this model here to define the s-surface.

2. zero temperature and the s-surface

Despite of the controversy question about Lorentz transformations of thermodynamic quantities [9, 10, 11], there is agreement that zero temperature is a Lorentz invariant. Also
is invariant the condition to break the supersymmetry in the SUSY harmonic oscillator, or in an equivalent way, the condition of belonging in the s-surface: Which means that the bosonic frequency $\omega_b$ and fermionic frequencies $\omega_f$ can change by Lorentz transformation, but the equality $\omega_b = \omega_f$ remains, it is invariant. In such a way the set of all supersymmetric harmonic oscillators, or in an equivalent way, the s-surface, we define as a standard reference for zero temperature for all Lorentz referential. Each point P over the s-surface represents a supersymmetric harmonic oscillator $(\omega_1, \omega_1)$, whose temperature is zero. The SUSY harmonic oscillator can be pushed out of s-surface, $(\omega_1, \omega_1) \rightarrow (\omega_1, \omega_2)$ due to thermal fluctuations or by some polynomial interactions of creation and annihilation operators. Through similar interaction it is possible to move back the system to the s-surface $(\omega_3, \omega_3)$ by a convenient choice of parameters; revealing an interesting magneto caloric effect at the supersymmetric harmonic oscillator.

3. The s-surface solution

The s-surface follow directly from the condition over $H = H_0 + H_{int}$ where

$$H_0 = \omega_1 a^\dagger a + \omega_2 b^\dagger b,$$

and

$$H_{int} = \alpha a^\dagger b^\dagger b - \alpha a b^\dagger b.$$ (1)

where $a^\dagger$ and $b^\dagger$ are respectively the bosonic and fermionic creation operators and the dual $a$ and $b$, are respectively the annihilation operators. Though $H$ is a supersymmetric oscillator for some parameter $\alpha$, defined by $(\alpha)^2 = \omega_2 \omega_1 - (\omega_1)^2$. $H_0$ is not a supersymmetric harmonic oscillator due to the fact that $\omega_1 \neq \omega_2$.

From $H$ with no constrain in the parameter $\alpha$, after we perform the Bogoliubov transformation ref[1],

$$a_2 = a + \frac{\alpha}{\omega_1} b^\dagger b; \quad a_2^\dagger = a^\dagger + \frac{\alpha}{\omega_1} b^\dagger b,$$

$$b_2 = (\exp[\frac{\alpha}{\omega_1} (a^\dagger - a)]) b; \quad b_2^\dagger = b^\dagger (\exp[\frac{\alpha}{\omega_1} (a - a^\dagger)]).$$ (2)

that preserves the algebra,

$a^\dagger \wedge a = -1/2; \quad a^\dagger \bullet a = n_b + 1/2$

$b^\dagger \bullet b = 1/2; \quad b^\dagger \wedge b = n_f - 1/2$

$n_b \in N$ and $n_f \in \{0, 1\}$.

we obtain the harmonic oscillator with the bosonic and fermionic frequencies $\omega_1$ and $\omega_3$, respectively.

$$H = \omega_1 a^\dagger a + \omega_3 b^\dagger b$$ (3)

The condition to be supersymmetric harmonic oscillator is $\omega_3 = \omega_1$, leading the solution bellow that define the s-surface,

$$\omega_{1\pm} = \frac{1}{2} (\omega_2 \pm (\omega_2^2 - 4\alpha^2)^{1/2});$$

This summarize that supersymmetry can be broken or restored through some definition of parameters, from a polynomial interaction that could be a external magnetic field.
Figure 1. S- Surface showing the parameters which is consistent to unbroken SUSY($T = 0$K).

4. Susy Harmonic Oscillator at Finite Temperature by Thermo field Dynamics - TFD

To introduce the temperature following the algorithm of TFD, we double the Hilbert space ref[1], writing the generator of time translation by: \[ \hat{H} = \omega_1(a\dagger a + b\dagger b) - \omega_1(\tilde{a}\dagger \tilde{a} + \tilde{b}\dagger \tilde{b}) \]
That allows us to calculate the thermal vacuum and then the statistical average of any operator in the thermal vacuum, in particular the Hamiltonian operator of the supersymmetric oscillator with interaction. The unitary transformation that leads the vacuum to the thermal vacuum and any operator to a thermal operator preserving the algebra in eq. in (3), is also called Bogoliubov transformation, \[ A(\beta) = e^{-iG}A(0)e^{iG}, \quad |0(\beta)\rangle = e^{-iG}|0\rangle, \]
where \[ G = -i\theta(\beta)(\tilde{b}\dagger \tilde{b} - b\dagger b) - i\theta(\beta)(\tilde{a}\dagger \tilde{a} - a\dagger a), \]
is the generator of the Bogoliubov transformation, and \[ \beta = \frac{1}{\kappa T}, \]
\[ \kappa \]
is the Boltzmann constant. with tan \[ \theta(\beta) = e^{-\beta\omega_1/2}. \]

The thermal energy of the thermal vacuum is given by \[ E_0(\beta) = \langle 0(\beta)|\omega_1 a\dagger a + \omega_1 b\dagger b|0(\beta)\rangle = \]
\[ \omega_1 \left( \frac{e^{-\beta\omega_1}}{1 - e^{-\beta\omega_1}} + \frac{e^{-\beta\omega_1}}{1 + e^{-\beta\omega_1}} \right). \]
This shows that the supersymmetric is broken at \[ T > 0, \]
Fig.2.

5. Conclusion

Supersymmetry is broken when temperature is introduced in the model, we try to use an interaction to restore the supersymmetry but it is possible only in some parts of the ensemble not in the whole ensemble. Although the fermionic and bosonic quantum corrections in a supersymmetric theory tend to cancel, the thermal effects are additive and the thermal energies are positive. Leading to non-zero statistical average of the Hamiltonian at finite temperature. All possibile supersymmetric harmonic oscillators represented by a point \( (\omega_1, \omega_2) \) in the frequency space of the system, are in a two dimensional surface, which we call s-surface. The temperature in the s-surface are intended to be zero. Interaction with the thermal bath represented by
Figure 2. Plot of the vacuum energy $E_0/\omega_1$, as function of $T/\omega_1$. Taken from [1].

the tilde Hilbert space from the doubling Hilbert space, establishes thermal oscillations that push the system from the s-surface. Zero temperature is Lorentz invariant, which implies that supersymmetric Harmonic Oscillator and the s-surface is also Lorentz invariant. In such a way we can define the supersymmetric harmonic oscillator or in an equivalent way, the s-surface as a standard reference for zero temperature for all Lorentz referential. At $T \neq 0$ the supersymmetry is broken, and now we have a bosonic and fermionic oscillator model with the respective frequency ($\omega_1, \omega_2$). The susy harmonic oscillator was pushed out of the s-surface due to thermal fluctuations.

Another possible and interesting application of this model are the study of the break of symmetries perceived by the wave function of the valence neutron field, specially in some conditions like those appointed at the work Searching Neutrino-Nucleus interaction in Mössbauer Spectroscopy [12].

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