Model-Based and Data-Driven Control of Event- and Self-Triggered Discrete-Time Linear Systems

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Abstract—The present paper considers the model-based and data-driven control of unknown discrete-time linear systems under event-triggering and self-triggering transmission schemes. To this end, we begin by presenting a dynamic event-triggering scheme (ETS) based on periodic sampling, and a discrete-time looped-functional approach, through which a model-based stability condition is derived. Combining the model-based condition with a recent data-based system representation, a data-driven stability criterion in the form of linear matrix inequalities (LMIs) is established, which also offers a way of co-designing the ETS matrix and the controller. To further alleviate the sampling burden of ETS due to its continuous/periodic detection, a self-triggering scheme (STS) is developed. Leveraging precollected input-state data, an algorithm for predicting the next transmission instant is given, while achieving system stability. Finally, numerical simulations showcase the efficacy of ETS and STS in reducing data transmissions as well as practicality of the proposed co-design methods.

Index Terms—Data-driven control, discrete-time systems, event-triggering scheme (ETS), linear matrix inequalities (LMIs), self-triggering scheme (STS).

I. INTRODUCTION

SAMPLED-DATA control has received considerable attention in the robotics and smart manufacturing thanks to its convenience in system design and analysis [1]. Traditional sampled-data schemes, which are mostly time-triggered, feature low computational overhead and easy deployment, but they may incur a sizable number of “redundant” transmissions occupying network resources [2]. Therefore, periodic sampling/transmission schemes are not appealing in networked control systems (NCSs) when communication resources (e.g., energy of wireless transmitting nodes or network bandwidth) are limited. Recently, research efforts have focused on developing transmission schemes that can use minimal communication resources while maintaining acceptable control performance (e.g., [3] for a survey).

A paradigm called the event-triggering scheme (ETS) [4] has been proven efficient. In ETS, the system state is continuously [4] or periodically [5] monitored as in the time-triggered control, but measurements are transmitted only when deemed “important.” This leads to a considerable decrease in resource occupancy while maintaining the system performance. In recent years, many variants of ETS have been proposed to further reduce transmissions (e.g., dynamic ETSs [6], [7]). However, it is difficult for these ETSs to realize continuous or periodic supervision of the system state. To overcome this difficulty, the concept of self-triggering scheme (STS) was proposed. Its core idea is to predict the next triggered instant based on a function constructed using the current sampled information as well as the system knowledge. Thus, in STS, the dedicated hardware in ETS is replaced with online [8], or offline [9] computations. In NCSs under STS, sensors are allowed to be completely shut off between adjacent sampling instants, leading to additional energy savings compared to the ETS. Due to this advantage, the STS has been implemented in various fields [10]. It should be mentioned that most existing ETS and STS are designed for continuous-time systems. However, in the context of NCSs, continuous-time systems are often controlled via digital computers, in which case one typically first discretizes a continuous-time system and works with the resulting discrete-time counterpart. Current literature has few results on discrete-time ETS/STS (e.g., [11] and [12]).

All above-mentioned triggering schemes are model based, namely, they require explicit knowledge of system models. Nonetheless, obtaining accurate system models can be
computationally demanding and oftentimes impossible in real-world applications. Naturally, an interesting question is how to co-design a controller and a triggering scheme without any knowledge of the system model. Measured data sequences, in practice, can be easily obtained. One solution to the above question is to first estimate a model based on measured data, also known as system identification [13], and subsequently, perform model-based system analysis and controller design (e.g., [14] for a survey). Yet, such a two-stage approach comes with an unavoidable drawback; that is, it is hard to provide an accurate model with guaranteed uncertainty bounds from limited and noisy data [15], [16]. An alternative approach, the so-called data-driven control, recently received increasing attention. Data-driven control is aimed at learning control laws directly from data without resorting to any prior system identification steps. Under this umbrella, various results have been presented, including state-feedback and optimal control [17], [18], robust control [19], [20], control of time-delay systems [21], predictive control [22], [23], and more can be found in the survey [24]. By wedging the data-driven system representation in [20] with the model-based approach in [25], a data-based stability condition for continuous-time sampled-data control systems was derived in [26], along with a controller design proposal. This data-driven framework has been extended to discrete-time sampled-data systems [27]. Data-driven ETS for continuous-time sampled-data systems with delays has been recently investigated in [28]. It remains an untapped field to co-design a data-driven controller and ETS/STS for unknown discrete-time sampled-data systems.

These developments have motivated our work in this article, which is focused on data-driven control of discrete-time sampled-data systems under ETS and STS. As demonstrated in [6], under the dynamic ETS that contains an additional dynamic variable, the triggering events can be reduced significantly compared to static ETS [4] for continuous-time systems. In this article, we develop a discrete-time dynamic ETS based on periodic sampling, which is reminiscent of the ETS [5] for continuous-time systems. For stability analysis of continuous-time sample-data systems, the looped-functional approach was proposed in [29] and subsequently explored by [28]. Looped-functionals often yield markedly improved stability conditions relative to common Lyapunov functionals, since the looped-functional is only required to be monotonically decreasing at sampling points but not necessarily between these points.

We generalize the looped-functional approach to discrete-time systems, by developing a discrete-time looped-functional (DLF) alternative, using which we derive model-based stability conditions for ETS. Combining this condition and the data-based representation of discrete-time systems in [20], a data-based stability condition is established, which provides a data-driven co-design method of the controller and ETS parameters.

On the other hand, a model-based discrete-time STS is designed, which can predict the next transmission instant without requiring online observation of state measurements between transmission times. For unknown discrete-time systems, it remains a key challenge to precompute the next execution time of sensor and controller using data, while ensuring stability under STS. To address this issue, we rewrite the discrete-time sampled-data system as a switched system. Using the data-driven parametrization of switched systems in [27], a data-driven algorithm for precomputing the next transmission instant is derived, which does not require any explicit model knowledge. Specially, the proposed STS law can be deduced to a special case of the dynamic ETS. Subsequently, the co-designed controller and triggering parameters under the ETS are employed to guarantee the stability of the system under the corresponding STS. In a nutshell, the main contributions of the present paper are summarized as follows.

\begin{enumerate}
  \item A dynamic ETS based on periodic sampling for discrete-time systems, where the triggering condition depends on previously released data and current sampled data.
  \item Model- and data-based stability conditions for discrete-time systems under the dynamic ETS using a novel DLF approach, as well as model/data-driven methods for co-designing the controller and triggering matrices.
  \item A model-based STS to predict the next transmission instant, and, building on this approach, a data-driven STS using only some precollected data from the system.
\end{enumerate}

While we only focus on closed-loop stability in this article, it is straightforward to extend our results to obtain performance guarantees, e.g., on the closed-loop $L_2$-gain, using similar arguments as in [5].

The remainder of this article is structured as follows. In Section II, we recall the problem setting as well as a discrete-time system representation which relies on noisy data. In Section III, an ETS control method for sampled-data systems is put forth, along with a DLF approach. Then, an STS is developed in Section IV. In both Sections III and IV, we present results for the model-based as well as the data-driven case. Section V validates the merits and practicality of our methods and conditions using a numerical example. Section VI draws concluding remarks.

\textbf{Notation:} Throughout this article, $\mathbb{N}$, $\mathbb{R}^+$, $\mathbb{R}^n$, and $\mathbb{R}^{n \times m}$ denote the sets of all non-negative integers, non-negative real numbers, $n$-dimensional real vectors, and $n \times m$ real matrices, respectively. Then, we define $\mathbb{N}_{[a,b]} := \mathbb{N} \cap [a, b)$, $a, b \in \mathbb{N}$. We write $P > 0$ ($P \succeq 0$) if $P$ is a symmetric positive (semi)definite matrix; $\text{diag}\{\cdots\}$ denotes a block-diagonal matrix; $\text{Sym}(P)$ represents the sum of $P^T$ and $P$. We write $[-]$ if elements in the matrix can be inferred from symmetry. Let $0$ ($I$) denote zero (identity) matrices of appropriate dimensions. Notation “$*$” represents the symmetric term in (block) symmetric matrices. We use $\| \cdot \|$ to stand for the Euclidean norm of a vector.

\section{Preliminaries}

Consider the following discrete-time linear system:

\begin{equation}
    x(t+1) = Ax(t) + Bu(t), \quad x(0) = x_0 \in \mathbb{R}^n
\end{equation}

for $t \in \mathbb{N}$, where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^m$ is the control input, and $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are the system matrices. Such discrete-time systems can model NCSs equipped with digital devices, where the sensor, the controller, and the actuator act at discrete instants. We consider in this
article that the system matrices $A$ and $B$ are unknown, but some precollected state-input measurements $(x(T))_{T=0}^{q}$ and $(u(T))_{T=0}^{q-1}$ $(T \in \mathbb{N}, q \in \mathbb{N}_{[1,\infty)}$) satisfy the following dynamic:

$$x(T + 1) = Ax(T) + Bu(T) + B_w w(T)$$

are available at discrete time instants $T \in \{0, 1, \ldots, q\}$. Here, $B_w \in \mathbb{R}^{n_w \times n_u}$ is a known matrix, which has full column rank and models the influence of the disturbance on the collected data. The measured data are corrupted by an unknown noise (perturbation) sequence $(w(T))_{T=0}^{q-1}$, where $w(T) \in \mathbb{R}^{n_w}$ captures, e.g., process noise or unmodeled system dynamics. This noise only affects the data generated for the controller design and will be neglected in the closed-loop operation of our ETS and STS scheme, but we note that an extension in this direction is straightforward. The available measurements can be stacked to form the following data matrices:

$$X_{+} := [x(1) \ x(2) \ \cdots \ x(q)]$$

$$X := [x(0) \ x(1) \ \cdots \ x(q-1)]$$

$$U := [u(0) \ u(1) \ \cdots \ u(q-1)]$$

$$W := [w(0) \ w(1) \ \cdots \ w(q-1)]$$

where $X_{+}$, $X$, and $U$ are known, but $W$ is unknown. Then, it is evident that

$$X_{+} = AX + BU + B_w W.$$  

(3)

For the proposed approach, the required data are collected offline and can be transmitted once by the sensor for the controller design step. Therefore, we can neglect network effects on these data and we can assume that they are collected with sampling period $1$. In practice, the noise is typically bounded. We make the following standing assumption on the noise.

**Assumption 1 (Noise Bound):** The noise sequence $(w(T))_{T=0}^{q-1}$ collected in the matrix $W$ belongs to

$$W \in \mathbb{R}^{n_w \times q} \quad \left\{ \begin{array}{l}
W^T \begin{bmatrix} I & Q_d & S_d \end{bmatrix} W^T \geq 0
\end{array} \right.$$  

(4)

for some known matrices $Q_d < 0 \in \mathbb{R}^{q \times q}$, $S_d \in \mathbb{R}^{q \times n_w}$, and $R_d = R_d^T \in \mathbb{R}^{n_w \times n_w}$.

Assumption 1 provides a general framework to model bounded additive noise, which has been used in similar forms in [17], [20], [26], and [27]. Based on (3) and Assumption 1, we define $\Sigma_{AB}$ to be the set of all pairs $[A \ B]$ adhering to the measured data and the noise bound, namely

$$\Sigma_{AB} := \{[A \ B] | X_{+} = AX + BU + B_w W, \ W \in \mathcal{W} \}.$$  

Then, an equivalent expression of $\Sigma_{AB}$ is provided in the form of a quadratic matrix inequality (QMI).

**Lemma 1 (Data-Based Representation) [20, Lemma 4]:** The set $\Sigma_{AB}$ is equal to

$$\Sigma_{AB} = \left\{ [A \ B] \in \mathbb{R}^{n \times (n + n_w)} \left| \begin{bmatrix} [A \ B]^T & \cdot \end{bmatrix}^T \Theta_{AB}[\cdot]^T \geq 0 \right. \right\}$$

where $\Theta_{AB} := \begin{bmatrix} -X & 0 \\ -U & 0 \\ X_{+} & B_w \end{bmatrix} \begin{bmatrix} Q_d & S_d \\ * & R_d \end{bmatrix} [\cdot]^T$.

**III. EVENT-TRIGGERED CONTROL**

In this section, we propose an ETS for the discrete-time system in Section III-A. As well as a novel looped-functional

![Fig. 1. Structure of data-driven sampled-data systems under ETS.](image-url)
for discrete-time sampled-data systems in Section III-B.

The stability of the event-triggered system is analyzed in Section III-C. Based on this, Section III-D derives a data-driven analysis and co-design method of the controller and the ETS without any explicit knowledge of the system matrices.

A. Discrete-Time Dynamic ETS

We use a discrete-time dynamic event-triggering module to dictate the transmission instants \( t_k \) \( k \in \mathbb{N} \), as depicted in Fig. 1. In our dynamic ETS, the system states first are periodically sampled at discrete instants \( t_k + h j \) \( j \in \mathbb{N} \), where the sampling interval \( h \) is a constant satisfying \( 1 \leq h \leq h \leq h \) for given lower and upper bounds \( h, h \in \mathbb{N} \). In the next step of the ETS, the sampled signal \( x(t_k + h j) \) is checked for the following triggering law:

\[
\eta(t_k^j) + \theta \rho(t_k^j) < 0
\]

(6)

where \( \theta > 0 \) is to be designed; \( t_k^j := t_k + h j \) for all \( j \in \mathbb{N} \); with \( m_k = (t_{k+1} - t_k)/h - 1; \rho(t_k^j) \) is a discrete-time function defined by

\[
\rho(t_k^j) := \sigma_1 x^\top(t_k^j) \Omega x(t_k^j) + \sigma_2 x^\top(t_k) \Omega x(t_k) - e^\top(t_k^j) \Omega e(t_k^j)
\]

(7)

where \( \Omega > 0 \) is some weight matrix; \( \sigma_1 \geq 0 \) and \( \sigma_2 \geq 0 \) are triggering parameters to be designed; \( e(t_k^j) := x(t_k^j) - x(t_k) \) denotes the error between sampled signals \( x(t_k^j) \) at the current sampling instant and \( x(t_k) \) at the latest transmission instant; and, \( \eta(t) \) is a dynamic variable, satisfying \( \eta(t) = \eta(t_{k}^j) \) for \( t \in (t_{k}^j, t_{k+1}^j - 1) \) and the following difference equation:

\[
\eta(t_{k+1}^j) - \eta(t_k^j) = -\lambda \eta(t_k^j) + \rho(t_k^j)
\]

(8)

where \( \eta(0) \geq 0 \) and \( \lambda > 0 \) are given parameters.

Once condition (6) is violated, the triggering module sends the sampled state \( x(t_{k+1}^j) \) to the controller, and a new control input is computed which is held via a zero-order hold (ZOH) element in the interval \( [t_{k+1}, t_{k+2} - 1] \). Subsequently, the triggering module is updated with the latest transmitted data, and monitors the next sample system state. In summary, our discrete-time dynamic ETS can be formulated as follows:

\[
t_{k+1} = t_k + h \cdot \min \left\{ j \geq 0 \mid \eta(t_k^j) + \theta \rho(t_k^j) < 0 \right\}.
\]

(9)

In Table 1, all hyperparameters in the dynamic ETS are listed. In Fig. 2, an example is presented to illustrate the proposed triggering transmission scheme. On the side of the sensor, discrete-time data are sampled periodically at a constant sampling period \( h \). Then, the sampled data are sent to the controller at the instants \( 0h, 3h, \) and \( 5h \), when the condition (6) is satisfied, but none of the others.

The following lemma states that the extra dynamic variable satisfies \( \eta(t_{k}^j) \geq 0 \) \( \forall j \in \mathbb{N} \). Under the condition (9) with the given parameters \( \eta(0) \geq 0, \theta > 0, \) and \( \lambda > 0 \). The proof of Lemma 2 is similar to [28, Lemma 2], which is omitted here.

**Lemma 2 (Non-Negativity):** Let \( \eta(0) \geq 0, \ Omega > 0, \) and \( \lambda > 0, \theta > 0 \) be constants satisfying \( 1 - \lambda - (1/\theta) \geq 0 \). Then, it holds that \( \eta(t_{k}^j) \geq 0 \) \( \forall j \in \mathbb{N} \), \( k \in \mathbb{N} \), under the condition (9).

**Remark 1 (Connection to Existing Works):** The transmission scheme (9) is a discrete-time analog of the continuous-time dynamic ETSs proposed by [6] and [28]; that is, our ETS is equivalent to the continuous-time periodic dynamic ETS [28] when choosing the lower bound of the sampling period sufficiently small, and further turns to the continuous-time dynamic ETS [6] by additionally setting \( \sigma_2 = 0 \). It can be seen from (9), our ETS becomes the discrete-time dynamic ETS proposed in [12] by setting \( h = 1 \) and \( \sigma_2 = 0 \); when, in addition, the parameter \( \theta \) approaches infinity, the condition (9) further boils down to the static ETSs [32, 33]; moreover, it degenerates to the time-triggering scheme [29] when \( \theta \to \infty \) and \( \sigma_1 = \sigma_2 = 0 \). Thus, our triggering scheme in (9) unifies and generalizes several existing discrete-time ETSs, and is expected to further reduce transmissions and save transmission resources, which is shown in Table IV of Section V. Finally, beyond these concrete differences, the present framework has the following additional features in contrast to most of the above works: First, we develop data-driven triggering schemes that are applicable to unknown systems using only measured data. In contrast, most existing approaches assume exact model knowledge and cannot handle uncertainty which may arise, e.g., from an estimation step (compare the discussion in Section I). Second, we not only propose an ETS but also an STS in order to control sampled-data systems with significant reduction of transmissions (Section IV).

B. Discrete-Time Looped-Functional Approach

This section develops a DLF approach for stability analysis. To begin with, the definition of DLF is given below.

**Definition 1 (DLF):** Consider any series of time instants \( \{t_j^k\} \). A functional \( V_j(x, t) : \mathbb{R}^n \times [t_j^k, t_{j+1}^k - 1] \to \mathbb{R} \) is called a DLF, if it satisfies

\[
V_j(x(t_{j+1}^k), t_{j+1}^k) = V_j(x(t_j^k), t_j^k) \quad \forall j \in \mathbb{N}.
\]

(10)

Since the property that \( V_j(x(t_{j+1}^k), t_{j+1}^k) = V_j(x(t_j^k), t_{j+1}^k) \) shares the characteristics of the looped-functional in [29] but is defined at discrete time instants, we call \( V_j(x(t), t) \) a DLF. Similar to the continuous looped-functional approach [29, Th. 1], the following stability theorem can be
obtained by using the (discontinuous and not necessarily positive definite) DLF.

Lemma 3: Choose a function $V_a : \mathbb{R}^n \to \mathbb{R}^+$ with scalars $c_2 > c_1 > 0$, and $p > 0$ satisfying $\forall x \in \mathbb{R}^n$, $c_1||x||^p \leq V_a(x) \leq c_2||x||^p$. Then, the following statements are equivalent.

1) The increment of the function $V_a$ is strictly negative at all $\{t_j^k\}$, that is, $\Delta V_a := V_a(x(t_{j+1}^k)) - V_a(x(t_j^k)) < 0 \ \forall x(t_j^k) \neq 0$.

2) There exists a DLF $V_l$, such that for all $t \in [t_j^k, t_{j+1}^k - 1]$, $\Delta V(t) := V_l(x(t + 1)) - V_l(x(t)) < 0 \ \forall x(t_j^k) \neq 0$, and $\{t_j^k\}$, where $V_l := V_l(x(t)) + V_l(x(t), t)$.

Moreover, if either one of the two statements is true, the origin of the system (5) is asymptotically stable.

Proof: Let $j \in \mathbb{N}_{[0,m]}$, $k \in \mathbb{N}$, and $t \in \mathbb{N}_{[t_j^k, t_{j+1}^k - 1]}$. 2) $\Rightarrow$ 1). Assume that 2) is satisfied. Summing up $\Delta V(t)$ over $[t_j^k, t_{j+1}^k - 1]$ yields

$$\sum_{t = t_j^k}^{t_{j+1}^k - 1} \Delta V(t) = \sum_{t = t_j^k}^{t_{j+1}^k - 1} \left[ V_a(x(t + 1)) - V_a(x(t)) \right]$$

$$+ \sum_{t = t_j^k}^{t_{j+1}^k - 1} \left[ V_l(x(t + 1), t + 1) - V_l(x(t), t) \right] < 0.$$

According to (10), it follows that $\Delta V_a < 0$.

1) $\Rightarrow$ 2). Assume 1) is satisfied. Similar to [29, Th. 1], the following functional is introduced for $t \in \mathbb{N}_{[t_j^k, t_{j+1}^k - 1]}$:

$$V_l(x(t), t) = -V_a(x(t)) + \frac{t}{t_{j+1}^k - t_j^k} \Delta V_a$$

which satisfies (10). Then, we have that

$$\Delta V(t) = V_a(x(t + 1)) - V_a(x(t)) - V_a(x(t + 1)) + V_a(x(t)) + \frac{t + 1}{t_{j+1}^k - t_j^k} \Delta V_a - \frac{t}{t_{j+1}^k - t_j^k} \Delta V_a$$

$$= \frac{1}{t_{j+1}^k - t_j^k} \Delta V_a < 0.$$

This proves the equivalence between 1) and 2).

Asymptotic stability: From condition 1), we have that $||x(t_j^k)|| \to 0$ as $k \to \infty$. Finally, similar to [34] and [35], there exists $\delta < \infty$ yielding $||x(t)|| \leq \delta||x(t_j^k)||$ for all $t \in \mathbb{N}_{[t_j^k, t_{j+1}^k - 1]}$, which implies that the system (5) is asymptotically stable under 1) or 2).

Remark 2: Lemma 3 was inspired by the continuous-time Lyapunov stability theorem, Lemma 3 provides less conservative stability conditions by constructing a proper DLF for discrete-time sampled-data systems, since the added DLF is not necessarily a positive definite functional. This can be clearly seen from Fig. 3. Note that the discrete Lyapunov functional $V_a(t)$ is not required to decrease at each time $t$, but only at ordered and nonadjacent discrete points $t_k$. To the best of our knowledge, a DLF as in Lemma 3 has not yet been introduced in discrete-time sampled-data control systems.

C. Model-Based Stability Analysis

In this section, we develop a model-based stability condition for the sampled-data system (5) under the transmission scheme (9), where matrices $A$ and $B$ are assumed known. This provides the theoretical basis for our data-driven stability analysis and co-design in Section III-D, which guarantees the stability of system (5) with predesigned controller $u(t) = Kx(t_k)$. Our triggering strategy (9) comprises a time-trigger with sampling interval $h$. Increasing values of $h$ lead to fewer data transmissions under our triggering strategy, where $h$ is directly related to the stability properties. Based on this observation, we derive a stability criterion for system (5) under (9) using the DLF approach. Before moving on, a useful result is given.

Lemma 4 (Summation Inequality): For any vector $\theta \in \mathbb{R}^m$, matrices $R = R^T \in \mathbb{R}^m \times n > 0$, $N \in \mathbb{R}^m \times 2n$, scalars $\alpha \leq \beta \in \mathbb{N}$, and a sequence $\{x(s)\}_{s = \alpha}^{\beta}$, the following summation inequality holds true:

$$\sum_{i = \alpha}^{\beta} y_i^T(i)Ry(i) \leq (\beta - \alpha)\theta^TNR^{-1}N^T\theta + \text{Sym}\left\{\theta^TN\Pi\right\}$$

where $y(i) := x(i+1) - x(i)$, $R := \text{diag}\{R, 3R\}$, $\Pi := [x^T(\beta) - x^T(\alpha), x^T(\beta) + x^T(\alpha) - \sum_{i = \alpha}^{\beta} (x^T(i)/[\beta - \alpha + 1])^T$.

Lemma 4 can be cast as a special case of [36, Lemma 2], whose proof is omitted here. Based on Lemmas 3 and 4, we have the following model-based stability condition.

Theorem 1 (Model-Based Condition): For given scalars $\tilde{h} > h > 1$, $\sigma_1 \geq 0$, $\sigma_2 \geq 0$, $\lambda > 0$, and $\theta > 0$ satisfying $1 - \lambda - (1/\theta) \geq 0$, asymptotic stability of system (5) is achieved under the triggering condition (9), and $\eta(t_j^k)$ converges to the origin for $\eta(0) \geq 0$, if there exist matrices $P > 0$, $R_1 > 0$, $R_2 > 0$, $\Omega > 0$, $\Sigma$, $N_1$, $N_2$, $F$, such that the following linear matrix inequalities (LMIs) hold for $h \in [h, \tilde{h}]$:

$$\begin{bmatrix} \Xi_0 + h\Xi_\xi + \Psi + \mathcal{O} & hN_\xi \\ -hR_\xi^{*} \end{bmatrix} < 0, \xi = 1, 2$$

where $\Psi := \text{Sym}\{F(AL_1 + BKL_7 - L_2)\}$, and

$$\Xi_0 := \text{Sym}\{\Pi_1^TSP_2 - \Pi_3^TSP_4 + N_1\Pi_9 + N_2\Pi_{10}\} + L_2^TPL_2$$

$$- L_1^TPL_1 + (L_2 - L_1)\tau(R_2 - R_1)(L_2 - L_1)$$

$$\Xi_1 := \text{Sym}\{\Pi_3^TSP_6\} + (L_2 - L_1)\tau R_2(L_2 - L_1)$$

$$\Xi_2 := \text{Sym}\{\Pi_3^TSP_8\} + (L_2 - L_1)\tau R_1(L_2 - L_1)$$

$$\mathcal{O} := \sigma_1L_1^T\Omega_3 + \sigma_2L_2^T\Omega_7 - (L_3 - L_7)^T\Omega_3(L_3 - L_7)$$

$$\mathcal{R}_1 := \text{diag}\{R_1, 3R_1\}, \mathcal{R}_2 := \text{diag}\{R_2, 3R_2\}$$
According to Lemma 3, we calculate the forward difference $\Delta x$:

$$\Delta x(t) = x(t+1) - x(t),$$

where the notation $\xi(t)$ is given as $\xi(t) := [x^T(t), x^T(t+1), x^T(t_j), x^T(t_{j+1})]$. By Lemma 4, the summation terms (14) satisfy

$$\sum_{i=t}^{t+1} y^T(i)R_1 y(i) \leq \xi^T(t) R_1 \xi(t),$$

(15) where $F$ is a fixed matrix of dimensions $7n \times n$.

In light of (9), when currently sampled data are not transmitted, Lemma 2 asserts that $\eta(t_j) \geq 0$ for $\lambda > 0$, $\eta(0) \geq 0$, and $\theta > 0$ satisfying $1 - \lambda - (1/\theta) \geq 0$. Hence, from (7) and (8), it holds that

$$\eta(t_{j+1}) - \eta(t_j) \leq -\lambda \eta(t_j) + \rho(t_j) \leq \xi^T(t) \eta(t),$$

(17) Consequently, we have that by summing up (14)–(17)

$$\Delta V_1(t) = \xi^T(t) \left[ \left( t - t_j \right) - \frac{t_{j+1} - t}{h} \right] \sum_{i=t_j}^{t} y^T(i) R_2 y(i) \leq \xi^T(t) \eta(t),$$

(18) where $\xi(t_j) = \xi_0 + \Psi + \Omega h \xi + h N \Sigma^{-1} N^T$ for $\xi = 1, 2$.

Using the Schur Complement Lemma, it can be deduced that $\xi_1(h) < 0$ and $\xi_2(h) < 0$ are equivalent to the LMIs in (11), which are affine, and consequently convex, with respect to $h$. Thus, LMI (11) at the vertices of $h \in [\hat{h}, \tilde{h}]$ ensure $\Delta V(x, t) < 0$ for all $h \in [\hat{h}, \tilde{h}]$. It follows from Lemma 3 that:

$$\sum_{i=t_j}^{t_{j+1}-1} \Delta V(x, t) = V_1(t_{j+1}) - V_1(t_j) + \eta(t_{j+1}) - \eta(t_j) \not= 0,$$

(19) which ensures $V_1(t_{j+1}) + \eta(t_{j+1}) < V_1(t_j) + \eta(t_j)$ for all $j \in \mathbb{N}_{[0, m]}$ and $k \in \mathbb{N}$. We conclude that System (5) and the dynamic variable $\eta(t)$ converge to the origin asymptotically under our transmission scheme when $k \to \infty$, since $V_1(t) > 0$, $\eta(t) > 0$, and $x(t)$ is bounded during $t \in \mathbb{N}_{[t_j, t_{j+1}-1]}$, thereby completing the proof.
Remark 3 (Discussion): Theorem 1 provides a stability criterion for the discrete-time system (5) under the dynamic triggering scheme (9) using the DLF approach in Lemma 3. The latter is a discrete-time version of the looped-functional approach [29] that has been applied to deduce less conservative stability conditions. In [12] and [33], related stability criteria have been presented for the event-triggered control of discrete-time systems. However, under these results, the lower bound of the interexecution interval is \( t_{k+1} - t_k = 1 \). Recently, a switching dynamic event-triggered control method for discrete-time systems was proposed by [11], where a guaranteed lower bound larger than 1, that is, \( t_{k+1} - t_k > 1 \), is beneficial for reducing the amount of transmissions. Our periodic-sampling-based dynamic ETS (9) explained in Section III-A only requires that \( t_{k+1} - t_k \geq h \), where \( h \) is the periodic sampling interval. Based on Theorem 1, we can determine a possibly large value of \( h \) leading to stability, thus, saving communication resources.

Remark 4 (DLF): A special case of DLF in the form of quadratic matrix functions is employed in (13). It can be proven that \( V_i (x(t_i^j), t_i^j) = V_i (x(t_{i+1}^j), t_{i+1}^j) = 0 \), which satisfies condition (10) in Definition 1. In (13), only the system state and its singlesummation terms are considered. Less conservative stability criteria can be derived if higher-order summation terms, e.g., multiple summation of the system state, are included. This is left for future research.

D. Data-Based Stability Analysis and Controller Design

We now derive a data-based stability certificate for the event-triggered control system (5) with unknown system matrices \( A \) and \( B \), as well as a data-driven method for co-designing the controller gain \( K \) and the triggering matrix \( \Omega \). Motivated by [26], the main idea is to employ a system expression using the data \( (x(t), \tilde{u}(t))_{t=0}^{T_i} \) to replace the model-based representation in (1). Following this line, the database system representation in Lemma 1, combined with the model-based stability condition in Theorem 1, is employed to obtain a data-based stability condition. We begin with an algebraically equivalent system to (5).

Assume that \( G \in \mathbb{R}^{n \times n} \) is nonsingular, and let \( x(t) = Gz(t) \). The system (5) is restructured as follows:

\[
  z(t+1) = G^{-1}AGz(t) + G^{-1}BK_c z(t_k)
\]

for \( t \in \mathbb{N}_{\llbracket h, h_{k+1} \rrbracket} \), where \( K_c := KG \). The system (20) exhibits the same stability behavior as (5), and the triggering condition (9) remains effective. Based on Theorem 1, we have the following theoretical result.

Theorem 2 (Data-Driven Condition and Controller Design): For given scalars \( h > h_1 > 1 \), \( \sigma_1 \geq 0 \), \( \sigma_2 \geq 0 \), \( \epsilon, \lambda > 0 \), and \( \theta > 0 \) satisfying \( 1 - \lambda - (1/\theta) \geq 0 \), there exists a controller gain \( K \) such that asymptotic stability of system (5) is achieved under the triggering condition (9) for any \( [A \ B] \in \Sigma_{AB} \), and \( \eta(t_i^j) \) converges to the origin, if there exist a scalar \( \epsilon > 0 \), and matrices \( P > 0 \), \( R_1 > 0 \), \( R_2 > 0 \), \( \Omega_c > 0 \), \( S, N_1, N_2, G, K_c \), such that the following LMIs hold for all \( h \in \{h, \tilde{h}\} \):

\[
  \begin{bmatrix}
    T_1 & \mathcal{T}_2 + \mathcal{F} \\
    * & T_3 + \Xi_0 + h\Xi_1 + \tilde{\Psi} + \tilde{\Omega} + h\mathcal{N}_c - h\mathcal{R}_c
  \end{bmatrix} < 0
\]

where \( \zeta = 1, 2 \), \( \tilde{\Psi} := \text{Sym}[-DG_L] \), and \( \tilde{\Omega} := \sigma_1 L_3^T \Omega L_3 + \sigma_2 L_7^T \Omega L_7 - (L_3 - (L_7^T \Omega L_3 - L_7)) \).

\[
  \mathcal{D} := (L_4 + \epsilon L_2)^T, \quad \mathcal{F} := [L_4^T \mathcal{G}^T, (\mathcal{L}_7^T \mathcal{K}_c^T)^T]
\]

\[
  T_1 := \epsilon V_1 \Theta_{AB} V_1^T, \quad T_2 := \epsilon V_1 \Theta_{AB} V_2^T, \quad T_3 := \epsilon V_2 \Theta_{AB} V_2^T
\]

\[
  V_1 := [I \ 0], \quad V_2 := [0 \ \mathcal{D}]
\]

Moreover, \( K = K_c G^{-1} \) and \( \Omega = G^{-1} \Omega G^{-1} \) are desired the controller gain \( K \) and triggering matrix.

Proof: Replacing \( x \) of the functional in (12) with the state \( z \), the following functional is built for the system (20):

\[
  V(z, t) = V_a(z(t)) + V_i(z(t), t) + I[\tilde{\eta}(t_{i+1}^j) - \eta(t_i^j)].
\]

Using the descriptor method again, the system (20) can be represented as follows:

\[
  0 = 2\tilde{\xi}_z^j(t) D(AG_L + BK_c L_7 - GL_2) \tilde{\xi}(t)
\]

where \( \epsilon \) is a given constant and \( \tilde{\xi}(t) \ := \{z^T(t), \ z^T(t+1), \ z^T(t^j), \ z^T(t^j) / \{t - t^j + 1\}, \ \sum_{i=t^j}^{t_{i+1}} \{z^T(i) / \{t - t^j + 1\}\}, \ \sum_{i=t^j}^{t_{i+1}} \{z^T(i) / \{t - t^j + 1\}\}\}^T \).

Triggering condition (9) and Lemma 2 prove that \( \eta(t_{i+1}^j) - \eta(t_i^j) \leq \tilde{\xi}_z^j(t) \Omega I \tilde{\xi}(t) \) as in (17), which directly ensures the following inequality with \( x(t) = Gz(t) \):

\[
  \eta(t_{i+1}^j) - \eta(t_i^j) \leq \tilde{\xi}_z^j(t) \Omega I \tilde{\xi}(t).
\]

Imitating (18), it can be obtained that

\[
  \Delta V(z, t) \leq \tilde{\xi}_z^j(t) \begin{bmatrix}
    0 & \mathcal{T}_2 + \mathcal{F}
  \end{bmatrix}
\]

where \( \tilde{\xi}_z(h) := \text{Sym}(D(AG_L + BK_c L_7)) + \Xi_0 + h\Xi_1 + \tilde{\Psi} + \tilde{\Omega} + h\mathcal{N}_c R_c^{-1}N_c^T \).

According to the data-based representation in Lemma 1, it holds for any \( [A \ B] \in \Sigma_{AB} \) that

\[
  \begin{bmatrix}
    [A \ B]^T \\
    I
  \end{bmatrix} \Theta_{AB} \begin{bmatrix}
    [A \ B]^T \\
    I
  \end{bmatrix} \geq 0.
\]

By the full-block S-procedure [37], we have \( \tilde{\xi}_z(h) < 0 \) and \( \tilde{\xi}_z(t_{i+1}^j) < 0 \) for any \( [A \ B] \in \Sigma_{AB} \) if there exists a scalar \( \epsilon > 0 \) such that for \( \zeta = 1, 2 \)

\[
  \begin{bmatrix}
    0 & \mathcal{F}
  \end{bmatrix}
\]

where \( \mathcal{F} := \mathcal{V}_1 (\Theta_{AB} V_1^T) \mathcal{V}_1 \mathcal{V}_2 \Theta_{AB} V_2^T \), and \( \mathcal{V}_1 \mathcal{V}_2 \Theta_{AB} V_2^T < 0 \).
Finally, the Schur Complement Lemma ensures that the inequalities in (26) are equivalent to LMIs in (21). Similar to Theorem 1, we have a conclusion that LMIs (21) at the vertices of \( h \in [h, h] \) are sufficient stability conditions for system (20) under the triggering condition (9) for any \([A \ B] \in \Sigma_{AB} \), and that \( \eta(t_j^\circ) \) converges to the origin. Since \( G \) is nonsingular, system (20) exhibits the same stability behavior as (5).

**Remark 5 (Model-Based Design Under ETS):** A model-based co-design method under the ETS (9) can be derived by replacing (21) with the condition \( \tilde{\mathcal{Y}}(h) < 0 \) for \( \varsigma = 1, 2 \).

**Remark 6 (Conservatism):** The matrix \( D \) is expressed as \([I_n, e_1 n, 0_{n \times 3n}]\). The free selected scalar \( \epsilon \) contained in the matrix \( D \) is introduced for enhancing the feasibility of Theorem 2. By appropriate selection of \( \epsilon \) (e.g., via line-search in an interval), a sampling period \( h \) as large as possible can be obtained by solving the LMIs in (21). A future task is to introduce more free variables into Theorem 2 to further reduce the conservatism.

**Remark 7 (Novelty):** Up to date, there are few research efforts devoted to data-driven control of discrete-time systems under aperiodic sampling, in particular data-driven control under ETSs. Theorem 2, which is based on Theorem 1, provides a stability condition and a co-design method of the controller and the ETS for unknown sampled-data control systems. A possibly large sampling interval \( h \) and a triggering matrix \( \Omega \) for the triggering condition (9) can be searched for by using Theorem 2. The application of Theorem 2 is simple, requiring only a solution of LMIs which can be constructed based on noisy data. Besides, when \( \theta \) approaches infinity, \( \sigma_1 = 0 \), and \( \sigma_2 = 0 \), the triggering condition (9) degenerates to the periodic sampling scheme studied in [27] and [31]. In comparison to the results in [27] and [31], Theorem 2 provides larger values of \( h \), which shows the role of the DLF in reducing conservatism of stability conditions. The comparison results are listed in Table II for a numerical example.

### E. Discussion on Parameters

In this section, we discuss how to find parameters \( \sigma_1, \sigma_2, \lambda, \) and \( \theta \) as required for the application of Theorem 2 and the implementation of the proposed data-driven ETS.

As mentioned by [3], larger values of \( \sigma_1 \) and \( \sigma_2 \) can reduce the transmission frequency. Therefore, it is desirable to choose \( \sigma_1 \) and \( \sigma_2 \) as large as possible, as long as the LMIs in Theorem are feasible. In practice, one can start with small values of \( \sigma_1 \) and \( \sigma_2 \) for which the LMIs are feasible and then gradually increase them as much as desired.

In what follows, we provide a proposition to demonstrate the influence of scalars \( \lambda \) and \( \theta \) on the triggering threshold. By using the triggering condition in (6), the dynamics in (8), and Lemma (2), the following proposition is easily derived and, therefore, its proof is omitted.

**Proposition 1 (Influence of Parameters on Triggering Threshold):** For given matrix \( \Omega > 0 \) and scalars \( \lambda > 0, \theta > 0 \) satisfying \( 1 - \lambda - (1/\theta) \geq 0 \), the inequality in (6) is equivalent to

\[
e^\top(t_j^\circ)\Omega e(t_j^\circ) > \sigma_1 x^\top(t_j^\circ)\Omega x(t_j^\circ) + \sigma_2 x^\top(t_k)\Omega x(t_k) + \eta(t_j^\circ)/\theta)
\]

where the variable \( \eta(t_j^\circ) \geq 0 \) satisfies (8).

The previous proposition shows that the triggering threshold function is increasing for decreasing values of \( \theta \). This suggests to choose \( \theta \) as small as possible for the purpose of maximizing the transmission interval \( t_{k+1} - t_k \). According to the limitations \( \lambda > 0, \theta > 0, \) and \( 1 - \lambda - (1/\theta) \geq 0 \) (and the fact that the LMIs in Theorem 2 are independent of \( \lambda \) and \( \theta \)), a reasonable minimum value is that \( \theta \) is close to 1 while \( \lambda \) approaches 0, based on Proposition 1. In Section V, we explore the impact of the precise choice of \( \theta \) and \( \lambda \) on the number of transmissions (NTs) and on the performance.

### IV. Self-Triggered Control

As highlighted in Section I, STS does not rely on extra hardware to continuously monitor the system states [38], but rather predicts the next sampling instant based on a local function and previous data. In this section, we study data-driven STS with unknown matrices \( A \) and \( B \), as depicted in Fig. 4. The challenge, here, is to predict the next transmission instant using the already transmitted system states and historical noisy measurements (i.e., \( \{x(T)\}_{T=t_0}^{t_0}, \{u(T)\}_{T=t_0}^{t_0} \)) as in Section II) without explicit knowledge of the system matrices \( A \) and \( B \). To this end, we begin by designing a model-based STS.

#### A. Model-Based STS

In order to apply data-driven control arguments similar to those in Section III, we first need to define a lifted version of the original system (5) as suggested in [27]. To this end, let us define for \( s > 0, s \in \mathbb{N} \)

\[
B^s := \begin{bmatrix} A^{s-1}B & A^{s-2}B & \cdots & B \end{bmatrix}
\]

\[
K^s := \begin{bmatrix} K & K^\top & \cdots & K^\top \end{bmatrix}^\top.
\]

We exploit that discrete-time sampled-data systems can be viewed as switched systems, which is a well-known fact in [39]

\[
x(t_k + s_k) = (A^{s_k} + B^{s_k}K^{s_k})x(t_k), \quad s_k \in \mathbb{N}[1, \bar{s}]
\]

where \( s_k = t_{k+1} - t_k \) and \( \bar{s} > 1 \in \mathbb{N} \).
Fig. 5. Evolution of transmission series.

The idea of the proposed STS is to build a function \( \Gamma(x(t_k)) \) for computing the next transmission instant \( t_{k+1} \) based on the current state \( x(t_k) \) of system (5)

\[
t_{k+1} = t_k + \Gamma(x(t_k), s_k).
\]  

(29)

We employ the following condition to find \( \Gamma(x(t_k), s_k) \):

\[
\sigma_1 x^T(t_k + s_k) \Sigma x(t_k + s_k) + \sigma_2 x^T(t_k) \Sigma x(t_k) - e^T(s_k) \Sigma e(s_k) \geq 0
\]  

(30)

where \( \Omega > 0 \) is some weight matrix; \( \sigma_1 \) and \( \sigma_2 \) are parameters to be designed; \( e(s_k) = x(t_k + s_k) - x(t_k) \) denotes the error between the sampled signals \( x(t_k) \) at the latest transmission instant and \( x(t_k + s_k) \) at time \( t_k + s_k \). According to (28), the condition (30) can be reformulated in the form of a QMI

\[
Q(x(t_k), s_k) = \left( (A^{s_k} + B^s K^{s_k}) x(t_k) \right)^T \times \left( \sigma_1 I - \sigma_2 \Omega \right) \geq 0.
\]  

(31)

If (31) is satisfied, the time \( t_k + s_k \) is declared to be the next transmission instant, that is, \( t_{k+1} = t_k + s_k \). When the sampled state \( x(t_k + s_k) \) is transmitted to the controller, a ZOH is used to maintain it within the interval \([t_k + 1, t_k + s_k - 1]\). Simultaneously, the self-triggering module is updated and employed to predict the next transmission instant \( t_{k+2} \). Consequently, the function \( \Gamma(x(t_k)) \) is designed to be

\[
\Gamma(x(t_k)) = \max_{s_k \in \mathbb{N}} \left\{ s_k > 0 \mid Q(x(t_k), s_k) \geq 0 \right\}.
\]  

(32)

In Fig. 5, an example illustrating the STS is given. The next transmission instant is predicted using the current transmitted measurement; that is, \( t_{k+1} \) is determined by the system state at time \( t_k \).

Remark 8 (Relationship Between ETS and STS): The STS condition in (30) is a static triggering scheme that is similar to [32] and [33], discussed in Remark 1. When the parameter \( \theta \) approaches infinity and \( j h = s_k \), the ETS condition in (9) boils down to (30). Both STS and ETS are based on the current transmitted signal \( x(t_k) \). However, the next transmission instant in STS is determined by predicted states, while the ETS uses currently sampled data. Extending the above STS by considering a dynamic triggering scheme is an interesting issue for future research.

B. Data-Driven STS

Although the system matrices \( A \) and \( B \) are unknown, the measurements \( \{x(T)\}_{T=0}^{T+1} \) and \( \{u(T)\}_{T=0}^{T+1} \) of the perturbed system (2) are available. Based on the model-based function in (32), a data-based discrete-time STS is proposed. We introduce two scalars \( \kappa > 0 \) and \( s > 0 \) satisfying \( \kappa + s = \varrho \) for the follow-up descriptions. Our idea is to rebuild a self-triggering function using the data \( \{x(T)\}_{T=0}^{T+1} \) and \( \{u(T)\}_{T=0}^{T+1} \) to replace the \([A^t B^t]\)-based representation (32). To that end, we recall the data-driven parametrization of the lifted matrix \([A^t B^t]\) in [27]. Similar to the system expression in (28), we first rewrite the perturbed system (2) as follows:

\[
x(T + s) = A^t x(T) + B^t \begin{bmatrix} u(T) \\ \vdots \\ u(T + s - 1) \end{bmatrix} + \begin{bmatrix} A^{s-1} B_w \\ \vdots \\ B_w \end{bmatrix} \begin{bmatrix} w(T) \\ \vdots \\ w(T + s - 1) \end{bmatrix}.
\]  

(33)

Recall that the measured data \( \{x(T)\}_{T=0}^{T+1} \) and \( \{u(T)\}_{T=0}^{T+1} \) are corrupted by the unknown noise \( \{w(T)\}_{T=0}^{T+1} \). Let us define the following matrices containing the measurements:

\[
X^s_+ = \begin{bmatrix} x(s) & x(1 + s) & \cdots & x(\kappa + s - 1) \end{bmatrix},
\]

\[
U^s = \begin{bmatrix} u(0) & u(1) & \cdots & u(\kappa - 1) \\
\vdots & \vdots & \ddots & \vdots \\
u(s - 1) & u(s) & \cdots & u(\kappa + s - 2) \end{bmatrix}.
\]

We further define the following lifted disturbance:

\[
W^1 = \begin{bmatrix} w(0) & w(1) & \cdots & w(\kappa - 1) \end{bmatrix},
\]

\[
W^s = \begin{bmatrix} \cdots \\
\vdots \\
w(s - 1) & w(s) & \cdots & w(\kappa + s - 2) \end{bmatrix}.
\]

\[
W^s = \begin{bmatrix} A^{s-1} B_w & \cdots & B_w \end{bmatrix} W^s, \quad \text{for } s > 1.
\]

Then, it is clear that

\[
X^s_+ = A^t X + B^t U^s + B^t w W^s
\]  

(34)

where \( B^t w := B w, B^t w := I \) for \( s > 1 \). Similar to Assumption 1, we make the following assumption on the noise.

Assumption 2 (Lifted Noise Bound): The noise sequence \( \{w(T)\}_{T=0}^{T+1} \) collected in the matrix \( W^s \) satisfies \( W^s \in \mathcal{W}^s \) with

\[
\mathcal{W}^s = \left\{ W^s \in \mathbb{R}^{n_w \times k} \mid \begin{bmatrix} W^s & I \\ \begin{bmatrix} Q^d_w & S^d_w \\ R^d_w \end{bmatrix} \end{bmatrix} \begin{bmatrix} W^s & I \\ \begin{bmatrix} Q^d_w & S^d_w \\ R^d_w \end{bmatrix} \end{bmatrix} \geq 0 \right\}
\]

for some known matrices \( Q^d_w < 0 \in \mathbb{R}^{k \times k}, S^d_w \in \mathbb{R}^{k \times n_w}, \) and \( R^d_w = R^d_w \in \mathbb{R}^{n_w \times n_w} \), where \( n_w := n_w, n_s := n_w \) for \( s > 1 \). Define the set of all pairs \([A^t B^t]\) consistent with the model (34) and Assumption 2 as the same as [27]

\[
\Sigma_A^B \in \mathbb{R}^{n \times (n + sm)} \mid 
X^s_+ = A^t X + B^t U^s + B^t w W^s, \quad W^s \in \mathcal{W}^s.
\]  

(35)

Analogously to Lemma 1, we obtain the following equivalent expression of \( \Sigma_A^B \) in the form of a QMI:

\[
\Sigma_A^B \in \mathbb{R}^{n \times (n + sm)} \mid \begin{bmatrix} A^t B^t \\ I \end{bmatrix} \begin{bmatrix} A^t B^t \end{bmatrix} \geq 0
\]  

(36)
where

$$\Theta^f_{AB} = \begin{bmatrix} Q^f_e & S^f_e \\ R^f_e \end{bmatrix} := \begin{bmatrix} -X & 0 \\ -U^\mu & 0 \end{bmatrix} \begin{bmatrix} Q^d & S^d \\ R^d \end{bmatrix} \top.$$

Having obtained a data-based representation of system (28), we can now translate the model-based self-triggering function (32) that depends on $[A^t \ B^t]$ to a data-based one. The following technical assumption on the matrix $\Theta^f_{AB}$ is required for the subsequent derivation.

**Assumption 3:** The matrix $\Theta^f_{AB}$ is invertible and has $n_w$ positive eigenvalues.

In practice, Assumptions 3 is satisfied when the data are sufficiently rich and $B_w$ is invertible [26]. Based on Assumption 3, we have the following theorem.

**Theorem 3 (Data-Driven Self-Triggering Condition):** For given scalars $\sigma_1 \geq 0$, $\sigma_2 \geq 0$, matrix $\Omega > 0$, controller gain $K$, and $x(t_k)$ from system (28), $\bar{Q}(x(t_k), s)$ in (31) satisfies

$$\bar{Q}(x(t_k), s) \geq 0 \tag{37}$$

for any $[A^t \ B^t] \in \Sigma^t_{AB}$, if there exists a scalar $\gamma > 0$, such that the following LMI holds for some $s \in \mathbb{N}, s \geq 1$:

$$\bar{Q}(x(t_k)) - \gamma \tilde{G}^a(x(t_k)) \geq 0 \tag{38}$$

where

$$\bar{Q}(x(t_k)) := \begin{bmatrix} I & 0 \\ 0 & x^\top(t_k) \end{bmatrix} \begin{bmatrix} (\sigma_1 - 1)\Omega & \Omega \\ * & (\sigma_2 - 1)\Omega \end{bmatrix} \begin{bmatrix} I \\ 0 \end{bmatrix} \top$$

$$\tilde{G}^a(x(t_k)) := \begin{bmatrix} I & 0 \\ 0 & x^\top(t_k) \end{bmatrix} x^\top(t_k) \begin{bmatrix} \Theta^f_{AB} \top \end{bmatrix} \top$$

$$\Theta^f_{AB} := \begin{bmatrix} -\bar{R}_e^\top \bar{S}_e^\top \\ \bar{S}_e \end{bmatrix} \begin{bmatrix} \tilde{Q}^f_e \\ \bar{S}_e \end{bmatrix} \top \begin{bmatrix} \bar{R}_e \end{bmatrix} \top = \begin{bmatrix} \bar{Q}_e & \bar{S}_e \\ \bar{S}_e^\top & \bar{R}_e \end{bmatrix}^{-1}.$$

**Proof:** The matrix $\bar{Q}(x(t_k), s)$ in (31) is rewritten as follows:

$$\bar{Q}(x(t_k), s) = \begin{bmatrix} (A^t + B^t K) x(t_k) \end{bmatrix} \top \begin{bmatrix} \bar{Q}_e & \bar{S}_e \\ \bar{S}_e^\top & \bar{R}_e \end{bmatrix}^{-1}. \tag{39}$$

Applying the dualization lemma [40, Lemma 4.9] to the system representation in (36) under Assumption 3, it can be proven that $[A^t \ B^t] \in \Sigma^t_{AB}$ if and only if

$$[A^t \ B^t] \top \Theta^f_{AB} [A^t \ B^t] \geq 0. \tag{40}$$

Immediately, through the full-block S-procedure [37], we have $\bar{Q}(x(t_k)) \geq 0$ for any $[A^t \ B^t] \in \Sigma^t_{AB}$ if there exists a scalar $\gamma > 0$ such that the LMI (38) holds. End the proof.

**Remark 9 (Explanation of the Data-Driven STS Condition):** Theorem 3 offers a data-driven triggering condition based on the model-based one in (32). The key idea is that we leverage the data-based representation in (36) to robustly verify the STS condition for all $[A^t \ B^t]$ consistent with the data. As a result, the model-based triggering STS function in (32) is translated into a data-driven one as follows:

$$\bar{\Gamma}(x(t_k), s_k) := \max_{s_k \in \mathbb{N}} \left\{ s_k \geq 1 \left| \bar{Q}(x(t_k)) - \gamma \tilde{G}^a(x(t_k)) \geq 0 \right. \right\}. \tag{41}$$

Overall, our data-driven STS is given by

$$t_{k+1} = t_k + \bar{\Gamma}(x(t_k), s_k) \tag{42}$$

under Assumptions 2 and 3 for system (28).

Note that, in Theorem 3, the data-driven condition $\bar{Q}(x(t_k)) - \gamma \tilde{G}^a(x(t_k)) \geq 0$ sufficiently guarantees the model-based one $\bar{Q}(x(t_k), s) \geq 0$ in (32) that is consistent with system stability characteristics. A smaller triggering interval may be produced by (41), since (41) only provides a sufficient condition for (32).

**Remark 10 (Co-Design Under STS):** As mention in Remark 8, the proposed STS (30) is a special case of ETS (9). Hence, the computed controller gain $K$ and the triggering matrix $\Omega$ by Theorem 2 still guarantee stability of system (28) under the data-driven STS condition in (38) in Theorem 3 with $s = \bar{h}$.

**Remark 11 (Summary of Data-Driven STS Algorithm):** According to (42), the next transmission instant $t_{k+1}$ of system (5) can be computed using only collected data $\{x(T)\}_{T=0}^{T+1}$ and $\{u(T)\}_{T=0}^{T+1}$. Note that the matrix $\Theta^f_{AB}$ in (42) needs to be determined in advance from the given noise bound. To that end, we recall [27, Algorithm 1], which can be used to construct a lifted noise bound as in Assumption 2 based on pointwise bound $\|w(T)\| \leq \bar{w}$ for all $T = 0, \ldots, s + 2$ with some $\bar{w} > 0$. This leads to a lifted system parametrization as in (36). Then, we continuously check the data-driven self-triggering condition using the matrices $\Theta^f_{AB}$ from [27, Algorithm 1]. The next triggering instant can be determined by checking the LMI (38) as soon as the current transmission instant and state become available.

**Remark 12 (Motivation):** This article combines the data-driven control and ETS/STS for discrete-time sampled-data systems, where the co-design problem of the controller and the triggering matrix without any knowledge of the system model is solved. To the author’s knowledge, the only alternative to this approach in the current literature would be system identification, e.g., least-squares estimation of the system matrices [16], followed by discrete-time ETS/STS in [11] and [12]. However, while the proposed approach guarantees that the closed-loop system under the designed ETS/STS is stable, such an identification-based ETS/STS does in general not provide such guarantees, in particular when the data are affected by noise. This is due to the fact that: 1) providing tight estimation bounds in system identification is in challenging, compare, e.g., [15] and 2) [11], [12] only provide nominal results, that is, error bounds arising from identification based on noisy data are not handled systematically.

**Remark 13 (Comparison):** Based on the adaptive control method, some existing approaches have investigated the data-driven ETSs without using system models, e.g., for nonlinear systems over fading channels in [41] and load frequency control of multiarea interconnected power systems in [42]. Those methods have been numerically proved to utilizes communication resources more efficiently compared to periodic transmission strategy. In the comparison with [41] and [42], the main differences to our works are given as follows. First, the triggering parameters in the data-driven ETS (9) are predetermined according to Section III-E, to avoid repeated experiments for optimizing the parameters in [41] and [42]. Second, the real-time iterative computations of the control input and the system parameter estimations by using real-time data in [41] and [42] are not required in our data-driven
event-triggered control method. Instead, a static co-design method for computing the fixed feedback controller gain and the triggering parameter using only historical data is proposed in this article, while guaranteeing system stability. In this sense, the proposed method has less computational burden. Third, the novel data-driven STS predicts the next triggering/exection instant of the controller, which saves the energy cost for continuous or periodic supervision of the system in [41] and [42]. Recently, the data-driven design of state-feedback controllers under a static ETS [32], [33] and a dynamic ETS has been investigated by [43], [44], and [45]. A key benefit of the proposed data-driven ETS (9) compared to [43], [44], and [45] is that a nontrivial interevent time (i.e., \( t_{k+1} - t_k > 1 \)) can be predetermined by choosing the sampling interval \( h > 1 \). Besides, a data-driven ETS based on model predictive control was developed by [46] in the absence of noise. In contrast to the methods in [43], [44], and [46], the proposed ETS and STS are both designed in a more realistic scenario with noise-corrupted data.

V. EXAMPLE AND SIMULATION

In this section, a numerical example from [47] is employed to certificate the effectiveness and merits of our proposed methods. All numerical computations were performed using MATLAB, together with the SeDuMi toolbox [48].

Example 1: Consider the linear system used in [47]

\[
\dot{x}(v) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} x(v) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(v), \quad v \geq 0.
\]

Discretizing the system leads to

\[
x(t + 1) = A(T_k)x(t) + B(T_k)u(t), \quad t \in \mathbb{N}
\]

where \( T_k > 0 \) is a discretization interval, and

\[
A(T_k) := e^{T_k} = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, \quad B(T_k) := \int_0^{T_k} e^{A(\tau)} \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} d\tau.
\]

We assume that the linear sampled-data state-feedback controller \( u(t) = Kx(t_k) \) is used to control the system. For \( t \in \mathbb{N}[t_k, t_{k+1}-1] \), the system (43) can be written as follows:

\[
x(t + 1) = A(T_k)x(t) + B(T_k)Kx(t_k).
\]

Upper Bounds of \( h \) for Known \( A \) and \( B \): Similar to [27] and [31], the controller gain \( K = \begin{bmatrix} -3.75 & 11.5 \end{bmatrix} \) is employed. By setting \( \eta_1 = 0, \eta_2 = 0 \), with \( \theta \rightarrow \infty \) (that is, \( \eta(T) = 0 \)), the proposed triggering scheme (9) reduces to a periodic transmission scheme. We first consider the model-based stability analysis. Leveraging Theorem 1 with discretization interval \( T_k = 0.01 \), the maximum sampling interval \( h \) leading to closed-loop stability is \( h = 173 \) with \( h = 1 \). Compared with the model-based results of \( h = 111 \) obtained by [49], \( h = 122 \) by [27], \( h = 133 \) by [50], \( h = 136 \) by [31], and \( h = 169 \) by [25], Theorem 1 in our paper provides improvements of 55.8%, 41.8%, 30.0%, 27.2%, and 2.3%, respectively. This shows the merits of the proposed DLF approach in reducing the conservatism of stability conditions.

Upper Bounds of \( h \) for Unknown \( A \) and \( B \): Next, assume that the matrices \( A \) and \( B \) are unknown. We set the discretization interval as \( T_k = 0.1 \) and generated \( \varrho = 800 \) measurements \( \{x(T)\}_{T=0}^{T=\varrho} \). The data-generating input was sampled uniformly from \( u(T) \in [-1, 1] \). The measured data were perturbed by a disturbance distributed uniformly over \( w(T) \in [-\bar{w}, \bar{w}]^2 \) for \( \bar{w} > 0 \). Such disturbance \( w(T) \) fulfills Assumption 1 with \( Q_d = I, S_d = 0, \) and \( R_d = \bar{w}^2 \varphi I \) (\( \bar{w} = 800 \)). The matrix \( B_w \) was taken as \( B_w = 0.01I \), which has full column rank. Using Theorem 2 and \( K = \begin{bmatrix} -3.75 & 11.5 \end{bmatrix} \) and setting parameters \( \bar{h}_1 = 0, \bar{h}_2 = 0, \) and \( \varrho = 2 \), values of \( \bar{h} \) with \( h = 1 \) for different realizations of \( \bar{w} \) were computed and presented in Table II. The results come from different approaches in [27] and [31] under the same levels of disturbance are also given. According to the comparison in Table II, Theorem 2 provides larger values of \( \bar{h} \), that is, our method reduces the conservatism if compared to [27, Th. 11], [27, Th. 20], and [31, Th. 3]. Furthermore, we design a controller leading to a possibly large sampling bound \( \bar{h} \) with \( h = 1 \). By virtue of [27, Corollary 13], [27, Corollary 23], and Theorem 2, corresponding values of \( \bar{h} \) were computed and listed in Table III. Again, Theorem 2 provides the largest \( \bar{h} \), which leads to the same conclusion as by Table II. It should also be mentioned that the use of different lengths of the data trajectories in constructing the data-based representation \( \Sigma_{AB} \) has an impact on the conservatism of the stability conditions. As shown in [30], a relatively long trajectory that contains rich system information effectively achieves a desired performance in the simulations. In the following, the proposed data-driven ETS (9) and STS (42) are applied for system (44), respectively, with the same \( \varrho = 800 \) measurements.

A. Data-Driven ETS and STS Control

Data-Driven ETS Control: For data-driven control of system (5) with unknown matrices \( A \) and \( B \) under our transmission scheme (9), we now employ Theorem 2 to co-design the controller gain \( K \) and the triggering matrix \( \Omega \) using the same measurements (for \( \bar{w} = 0.001 \)) as above. Set the sampling interval \( h = 2 \), discretization period \( T_k = 0.1 \), and \( \bar{h} = 2 \). According to Section III-E, the maximum values of triggering parameters \( \bar{h}_1 \) and \( \bar{h}_2 \) are computed as \( \bar{h}_1 = 0.9 \) and \( \bar{h}_2 = 0.9 \). Meanwhile, the controller and triggering matrices

\[
\begin{array}{cccccccc}
\bar{w} & 0.001 & 0.002 & 0.005 & 0.01 & 0.02 & 0.05 \\
\hline
[27, Theorem 11] & 12 & 12 & 11 & 11 & - & - \\
[27, Theorem 20] & 16 & 15 & 1 & - & - & - \\
[31, Theorem 3] & 13 & 13 & 12 & 12 & - & - \\
Theorem 2 & 17 & 16 & 15 & 13 & 12 & 8 \\
\end{array}
\]

\[
\begin{array}{cccccccc}
\varrho & 0.001 & 0.002 & 0.005 & 0.01 & 0.02 & 0.05 \\
\hline
[27, Corollary 13] & 15 & 11 & 7 & 5 & 3 & - \\
[27, Corollary 23] & 19 & 17 & 12 & 8 & 5 & 1 \\
Theorem 2 & 54 & 45 & 34 & 27 & 16 & 9 \\
\end{array}
\]
are co-designed as follows:

\[ K = \begin{bmatrix} -0.1537 & 1.6465 \end{bmatrix}, \quad \Omega = \begin{bmatrix} 0.0001 & 0.0008 \\ 0.0008 & 0.0091 \end{bmatrix}. \]

We then simulate system (5) under the triggering scheme in (9) for \( t \in [0, 500] \), as well as the dynamic variable \( \eta(\tau_j^k) \), with initial condition \( x(0) = [3 \ 2]^T \) and choosing \( \theta = 5, \lambda = 0.2 \) (for a compromise between the system performance and the transmission frequency). Their trajectories are depicted in Fig. 6. Evidently, both the system states and the dynamic variable converge to the origin, which demonstrates the feasibility of our designed controller gain \( K \) and the triggering matrix \( \Omega \). It is also worth pointing out that only six measurements were transmitted to the controller under our proposed triggering scheme in (9), while 250 measurements were sampled. This validates the effectiveness of the proposed scheme in saving communication resources, while maintaining stability.

**Testing System Performance and Transmission Frequency:**
In what follows, we examine the influence of parameters \( \theta \) and \( \lambda \) on the system performance and the transmission ratio. A cost function of system performance \( \mathcal{P}(x) \) and a ratio of transmitted data \( \mathcal{J} \) are defined as follows:

\[ \mathcal{P}(x) := \sum_{k=0}^{\infty} \sum_{j=0}^{m_k} x^T(\tau_j^k)x(\tau_j^k), \]

\[ \mathcal{J} := \frac{\text{Number of transmitted data}}{\text{Number of sampled data}}. \]

We set \( \epsilon = 2, \sigma_1 = \sigma_2 = 0.5, \) and \( h = 2 \). Employing the same measured data in data-driven ETS, the controller, and triggering matrices are computed by Theorem 2 as follows:

\[ K = \begin{bmatrix} -0.2908 & -4.0340 \end{bmatrix}, \quad \Omega = \begin{bmatrix} 0.0001 & 0.0007 \\ 0.0007 & 0.0104 \end{bmatrix}. \]

Fig. 7 depicts values of \( \mathcal{P}(x) \times 0.1 \) and \( \mathcal{J} \times 100 \) for different triggering parameters \( \theta \) and \( \lambda \). There are two phenomena to be drawn. The one is that when \( \theta \) and \( \lambda \) increase the system performance index decreases and the transmission ratio increases. The main reason is that the proposed ETS in (9) becomes a static scheme, when \( \theta \) and \( \lambda \) are large enough to have no effect on the trigger threshold. The other remarkable phenomenon from the two curves in Fig. 7 is that the transmission savings are always at the expense of the system performance. A compromise between these two indexes can be found by appropriately choosing \( \theta \) and \( \lambda \) in the ETS.

**Data-Driven STS Control:** The data-based STS (42) for unknown systems is tested in the following. First, according to [27, Algorithm 1], the bound on the lifted disturbance in Assumption 2 can be computed by using above measurements with \( \kappa = 750 \) and \( \bar{s} = 50 \) (by dividing \( \phi = \kappa + \bar{s} \)). Then, it is straightforward to obtain the data-based matrices \( \hat{G}^s(x(t_k)) \) for \( s = jh \) for \( s \in \mathbb{N}_{[1, 3]} \) in (38) based on the matrix \( \Theta_{AB}^s \) from the bound in Assumption 2. Applying the same controller gain and the triggering matrix as in the ETS control (which also guarantees the stability under the STS when using the same parameters in the ETS scheme, since as discussed in Remark 8 the STS is a special case of the ETS), the system state trajectory under the data-based STS (42) with \( s = jh \) is depicted in Fig. 8 for \( t \in [0, 500] \). All states converge to the origin, thereby validating the practicality of our proposed co-design method and STS. Note that, in Fig. 8, only 17 out of 250 samples were transmitted to the controller. This illustrates the usefulness of the STS in reducing transmissions while ensuring stability. Moreover, note that in Figs. 6 and 8, more data
were generated by the STS compared with the ETS. The main reason is that the STS law (42) is more conservative than that of ETS (9).

**Compared With Existing ETSs:** As mentioned in Remark 1, the proposed ETS (9) becomes the existing ETSs [6], [12], [32], [33], by certain parameter settings given in Table IV. We simulate the systems under ETS (9) with different groups of parameters \((h, \theta, \lambda, \sigma_1, \sigma_2)\) in Table IV as well as the controller and ETS matrices for the data-driven ETS. The NTs for each ETS is listed in Table IV. It supports the statement in Remark 1 that our ETS has an advantage of reducing transmissions over listed ETSs, while maintaining system stability.

**B. Comparison With Identification-Based ETS and STS Control**

In this part, we compare the data-driven approaches described above to an alternative approach, consisting of least-squares identification of the system matrices and subsequent model-based ETS and STS as presented in Sections III and IV. In the system identification step, the following least-squares problem is considered:

\[
\arg\min_{\hat{A}, \hat{B}} \sum_{T=0}^{T} \|x(T + 1) - Ax(T) - Bu(T)\|_2^2.
\]

The least-squares solution \([\hat{A}, \hat{B}]\) is given by

\[
[\hat{A}, \hat{B}]^\top = \left(\left[X^T \ U^T\right] - \left[X^T \ U^T\right]^{-1} \left[X^T \ U^T\right]^\top \left[X^T \ U^T\right]\right)^{-1} \left[X^T \ U^T\right]^\top \ X^T
\]

which is the estimation of the real system matrices \(A \) and \(B\). Next, we will show the the results of identification-based ETS and STS control.

**Identification-Based ETS and STS Control:** Using the least-squares approach and the same \(\varphi = 800\) measurements as in the data-driven control, the system matrices are estimated as

\[
\hat{A} = \begin{bmatrix} 1.0000 & 0.0995 \\ 0.0000 & 0.9906 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} 0.0005 \\ 0.0100 \end{bmatrix}.
\]

Subsequently, by solving the model-based approach in Remark 5 with the same parameters as in data-driven control, the controller and the triggering matrices were computed as follows:

\[
K = \begin{bmatrix} -0.0944 & -1.3662 \end{bmatrix}, \quad \Omega = \begin{bmatrix} 0.0001 & 0.0006 \\ 0.0006 & 0.0092 \end{bmatrix}.
\]

The trajectories of system (9) under the identification-based ETS (9) and STS (32) were depicted in Figs. 9 and 10, respectively, over \(t \in [0, 500]\). The simulation results show that \(x(t)\) approaches to zero as \(t \to \infty\) under the identification-based ETS or STS with the above designed \(K \) and \(\Omega\). In light of the comparisons with the simulation results of the data-driven ETS and STS, almost the same amount of data (4 out of 250 samples) were transmitted to the controller under the identification-based ETS as in Fig. 9 compared to the data-driven ones in Fig. 6, and less data (5 out of 250 samples) were generated under the STS in Fig. 10 than the one in Fig. 8, while stability characteristics at the approximate level are guaranteed for all approaches. However, in contrast to the direct data-driven design, the identification-based ETS and STS approaches do not provide stability guarantees.

**VI. Conclusion**

In this article, we proposed data-based ETS and STS for discrete-time systems leveraging a novel looped-functional approach and a data-driven system representation. We also developed methods for co-designing the controller gain and the triggering matrix for the discrete-time ETS and STS systems. Finally, a numerical example was presented to corroborate the role of our triggering schemes in saving communication resources, as well as the merits and effectiveness of our co-designing methods. A foreseeable direction is to extend the data-driven ETS and STS to more general system classes, e.g., multiagent systems [51], [52] and switching systems with unknown system matrices.
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