QUASAR-CLUSTER ASSOCIATIONS AND GRAVITATIONAL LENSING BY LARGE-SCALE MATTER CLUMPS

XIANG-PING WU

Department of Physics, University of Arizona, Tucson, AZ 85721; and Beijing Astronomical Observatory, Chinese Academy of Sciences, Beijing 100080, People’s Republic of China

AND

LI-ZHI FANG

Department of Physics, University of Arizona, Tucson, AZ 85721

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ABSTRACT

Motivated by the significant overdensity of background bright quasars recently detected behind the foreground clusters of galaxies on a scale of $10^9$, we have investigated the possibility of attributing the quasar-cluster associations to gravitational lensing by large-scale matter inhomogeneities. Based on the conventional lensing models, we have shown that the reported quasar overdensity is unlikely to be generated by cluster matter alone. The situation does not change even if all of the clusters of galaxies that follow their spatial two-point correlation function are taken into account, while matter clumps on scales of greater than 20 Mpc are also found to be unable to provide the required mass surface density, since their density contrast is strictly limited by the anisotropy measurements of the cosmic background radiation. Moreover, we have pointed out that the influence of a nonzero cosmological constant on the quasar-cluster associations is very minor. We conclude that either the observed quasar number counts have been seriously contaminated by the magnification bias of matter inhomogeneities of the universe or there should exist some intercluster matter on a scale of less than $\sim 20$ Mpc, e.g., from cluster-galaxy correlation, whose mean cosmic density is about an order of magnitude higher than that of clusters of galaxies.

Subject headings: galaxies: clusters: general — gravitational lensing — large-scale structure of universe

1. INTRODUCTION

A significant overdensity of background bright optical/radio quasars has recently been detected on a scale of 10’ around foreground Zwicky/Abell clusters (Rodrigues-Williams & Hogan 1994; Wu & Han 1995; Rodrigues-Williams & Hawkins 1995; Seitz & Schneider 1995). Although these unusual quasar-cluster associations are generally believed to be the result of statistical lensing of quasars by a foreground gravitational potential, cluster matter alone cannot explain the observed amplitudes of the quasar overdensity behind the clusters. It then seems that the matter inhomogeneities on even larger scales (>10 Mpc) traced by galaxy clusters should be taken into account in the explanation of the reported quasar-cluster associations.

While the overdensity of background quasars on a scale similar to that found around foreground galaxies was detected a few year ago (Fugmann 1988, 1990; Bartelmann & Schneider 1993b, 1994), the large-scale matter clumps that galaxies are associated with were also advocated in order to produce the quasar-galaxy associations. Using $N$-body simulations of galaxy formation, Bartelmann & Schneider (1993a) did find a correlation of high-redshift quasars with low-redshift galaxies in the scenario of magnification bias by the matter of galaxies and their surrounding large-scale structures. Their results indicate that galaxies, and probably clusters of galaxies, contribute a minor effect to the quasar overdensity on a scale of arcminutes. Interestingly, the recent work by Wu, Zhu, & Fang (1995) shows that even on a small scale of arcseconds, the quasar-galaxy associations are actually generated mainly by the cluster matter rather than by the galaxies.

Compared to the quasar-galaxy associations, the quasar-cluster associations deal with the matter distributions on scales ranging from $\sim 1$ to $\sim 10$ Mpc, on which galaxy clusters are strongly correlated, revealed by their two-point correlation function $\xi(r)$. It is timely and necessary to address the following question: Is galaxy cluster clustering described by $\xi(r)$ able to provide enough gravitational matter to act as a lens for the reported quasar-cluster associations? A definite answer to such a question today relies on the numerical study of various models of formation of large-scale structure of the universe. However, an analytic investigation, as we will undertake in this Letter, of the quasar-cluster associations in the scenario of gravitational lensing by various matter clumps may supply us with a very useful clue to the matter distribution on large-scale structure of the universe.

2. QUASAR ENHANCEMENT FACTOR

The overdensity of background quasars at an angular distance $\theta$ around a foreground galaxy cluster is described by the enhancement factor $q(\theta)$ (Narayan 1989):

$$q(\theta) = \frac{N(<m + 2.5 \log S/\mu(\theta))}{N(<m)} = \frac{N(>S/\mu(\theta))}{N(>S)} \mu(\theta)^{-1},$$

(1)

where $N$ is the intrinsic quasar number count above a limiting magnitude $m$ or a flux threshold $S$, and $\mu(\theta)$ is the lensing magnification introduced by the foreground gravitational potential. This equation accounts for both the magnification effect (2.5 log $\mu$ or $S/\mu$) and the area distortion (1/$\mu$) because of light deflection by foreground matter. To compare with the measurements of quasar-cluster associations that search for quasar number excess over a range with radius $\theta$ around the
cluster center, the average enhancement factor \( \langle q(\theta) \rangle \) is employed: \[ q(\theta) = 2 \int_0^\theta q(\theta) d\theta/\theta^2. \]

If we assume that the observed quasar number counts are not significantly contaminated by gravitational lensing due to the matter clumps in the universe, then the number-magnitude relation \( N(<B) \) from Boyle, Shanks, & Peterson (1988) and the source counts \( N(>S) \) at 5 GHz from Langston et al. (1990) can be adopted for the optically selected and the radio-selected quasars, respectively. However, it should be noted that the radio counts \( N(>S) \) contain both quasars and galaxies, and the fraction of quasars in \( N(>S) \) varies with the flux threshold. Therefore, the employment of \( N(>S) \) in the study of quasar-cluster associations can only be regarded as an approximate estimate of \( \langle q \rangle \). Furthermore, quasars are treated as pointlike sources, which would be suitable for clusters and large-scale matter clumps as lenses.

The observational data of the four searches for quasar-cluster associations actually provide the variations of \( q(\theta) \) with the search distance and/or the limiting magnitude. We don’t intend to fit the curves from the theoretical modeling of lensing systems as did Wu & Han (1995) and Rodrigues-Williams & Hawkins (1995). Instead, we adopt the one significant result of \( q(\theta) \) measured at a fixed \( \theta \) and a limiting magnitude (or flux). Rodrigues-Williams & Hogan (1994) have explicitly given the enhancement factor within 6 Zwicky radii and \( B < 18.5 \). The enhancement of \( q(\theta) \) versus \( \theta \) begins nearly unity at \( \theta > 5^2 \) in Wu & Han (1995), and therefore, \( \theta = 4^2 \) seems to be a reasonable “edge” for their sample. Rodrigues-Williams & Hawkins (1995) show the variations of \( q(\theta) \) with the quasar limiting magnitude for \( \theta \approx 0':12 \). We take the most significant value at \( B = 18 \). Seitz & Schneider (1995) choose the quasars from the 1 Jy radio source sample at 5 GHz but actually use the optically identified sources, which are mostly quasars or BL Lac objects at high redshifts. Their most significant signal appears at \( B = 19 \) and \( z_s \approx 1 \). We adopt the value of \( q(\theta) \approx 1.3 \) at 9 Zwicky radii. In particular, we keep both the limiting magnitude and the radio flux threshold, which helps to determine the reliability of using radio source counts \( N(>S) \) in the evaluation of \( q \). Unfortunately, we cannot read out the uncertainties from their data but learn that the result has a very high significance of up to 98\%. Table 1 summarizes these four measurements, in which the error bars in \( q(\theta) \) are the \((1\sigma)N^{1/2}\) errors arising from the estimates of both the quasar density over the association area and the mean quasar density.

3. MODELING OF QUASAR OVERDENSITY BEHIND CLUSTERS

Now we work with the lensing models of the matter inhomogeneities associated with the foreground clusters and test what would be required to explain the reported quasar-cluster associations. We adopt a flat cosmological model with a cosmological constant \( \Lambda \), i.e., \( \Omega_0 + \Lambda_0 = 1 \), and \( H_0 = 50 h_{50} \) km s\(^{-1}\) Mpc\(^{-1}\).

Clusters of galaxies were naturally thought to be the deflectors for the observed quasar-cluster associations. We can evaluate the contribution to \( q(\theta) \) from clusters by modeling the cluster matter as a singular isothermal sphere, which is characterized uniquely by its one-dimensional velocity dispersion \( \sigma_v \). Though this profile is the simplest, it is a more or less reasonable model for dark matter distribution in clusters of galaxies. We count both the primary quasar images and secondary ones, if any, that are gravitationally magnified by a factor of \( \mu(\theta) = [1 - \theta_\theta/\theta^2]^{-1} \), where the Einstein radius is \( \theta_e = 4\pi(\sigma_v/c)^2D_{ls}/D_s \), where \( D_{ls}, D_s \), and \( D_m \) denote the angular diameter distances to foreground lenses, to background sources, and from lenses to sources, respectively. In Table 1 we list the cluster velocity dispersion that is required to produce the observed enhancement for each of the four measurements. Apparently, the resulting \( \sigma_v \) is substantially larger than any realistic values for clusters of galaxies. Taking the mean cluster velocity dispersion to be 1000 km s\(^{-1}\), we can estimate that the gravitational mass \( (M \sim \sigma_v^2) \) responsible for the quasar-cluster associations is an order of magnitude higher than the presently known total cluster mass. Meanwhile, it is seen that the real matter clumps that generate the quasar-cluster associations must deviate from the \( r^{-2} \) distribution, since one cannot use a single velocity dispersion parameter to reproduce all the observed \( q \).

If the large-scale matter inhomogeneities traced by clusters of galaxies contribute an additional mean surface mass density \( \Sigma \) to cluster matter, the Einstein radii \( \theta_\theta \) of the background quasars will be increased by a factor of \((1 - \Sigma/\Sigma_{crit})^{-1}\), where \( \Sigma_{crit} = (c^2/4\pi G)(D_s/D_m)^2 \) is the critical surface mass density (Turner, Ostriker, & Gott 1984). The image magnification now reads \( \mu(\theta) = [1 - \theta_e/\theta^2]^{-1} (1 - \Sigma/\Sigma_{crit})^{-1} \). Therefore, if the large-scale matter inhomogeneities have a mass density comparable to the critical one, the magnification factor can be greatly enhanced. Alternatively, \( \Sigma_{crit} \) would be smaller in a cosmological constant-dominated universe than in the matter-dominated universe, i.e., the same uniform matter sheet would act as a more efficient lens in a \( \Lambda \)-dominated universe (Wu et al. 1995). Table 1 gives the mean surface mass density \( \Sigma \) of the large-scale inhomogeneities that are needed to explain the quasar overdensity around clusters. Recall that the surface mass density at the cluster center with core radius \( r_c \) is \( 0.087(\sigma_v/10^2 \text{ km s}^{-1})^2 (r_c/0.25 \text{ Mpc})^{-1} h_{50} \) g cm\(^{-2}\), and the minimum critical density for a source at \( z_s = 2 \) is \( \Sigma_{crit} = (0.41, 0.28) h_{50} \) g cm\(^{-2}\) for \( \Omega_0 = (1.0, 0.2) \). Thus, the surface mass densities of the large-scale matter clumps should be comparable to the one at the cluster center in order for the clumps to act as the lenses for the observed quasar enhancement around clusters. It is noticed that \( \Sigma \) deduced from the radio bright quasar associations with clusters is a factor of \( \sim 2 \) larger than the one from the optical quasar samples. As we have mentioned before, this is because of the contamination of radio galaxies in the radio source counts \( N(>S) \) we adopted. The data of Seitz & Schneider (1995) illustrate this effect very well: the optical quasar number-magnitude relation \( N(<B) \) results in a \( \Sigma \) that is a factor of about \( 2 \) smaller than the value given by the radio source counts \( N(>S) \) for the same set of quasar data.

In summary, the \( r^{-2} \) mass distributions for clusters of galaxies fail in reproduction of the observed quasar-cluster associations, no matter how massive they would be. A uniform surface matter sheet is found to be an accepted model as long as its surface density reaches a value comparable to the one at the cluster center. The introduction of the cosmological constant does not significantly reduce the demand for such a high-density inhomogeneity on a large scale.

4. CONTRIBUTIONS OF LARGE-SCALE MATTER CLUMPS

We now discuss the mass contributions from the cluster-cluster correlation. The matter clustering that clusters of galaxies trace on a large scale can be quantitatively described
by the cluster spatial two-point correlation function \( \xi(r) = (r/r_c)\left(1 + (1 + z_J)^{1/3}\right) \), where the correlation length is \( r_c \approx 40 \text{ Mpc} \ h_{50}^{-1} \) (Postman, Huchra, & Geller 1992). Since \( \xi(r) \) diverges at \( r = 0 \), we truncate \( \xi(r) \) when \( r < r_c \). The probability of finding a cluster in the surface element \( 2\pi\xi d\xi \) at a distance \( \xi \) from a cluster on the plane perpendicular to the line of sight is

\[
dP(\xi) = 4\pi n \xi d\xi \int_{\xi}^{\infty} \left[1 + \xi(r)(1 + z_J)^{1/3}\right] \frac{r \, dr}{\sqrt{r^2 - \xi^2}},
\]

where \( n \) is the mean cluster number density and \( e \) accounts for the evolution of \( \xi(r) \). The expected mass contribution from all the clusters following \( \xi(r) \) can be computed by the integration of \( m(\xi) \) \( dP(\xi) \) over \( \xi \) from 0 to \( \infty \). Here \( m(\xi) \) is the cluster mass within \( d\xi \) of \( \xi \).

We again adopt a singular isothermal sphere model for an individual cluster with velocity dispersion \( \sigma \). Moreover, the cluster matter distribution is truncated at the cluster gravitational radius \( R_c \), so that the cluster mass is \( M_c = 2\pi R_c \sigma^2 / G \). We consider only those excess populations relative to the "background" clusters of mean density \( n \), i.e., we take out the factor "1" in \( 1 + \xi(r)(1 + z_J)^{1/3}\). The expected mean mass density over the area \( \pi \xi^2 \) provided by clusters with mass \( M_c \) is \( \int m(\xi) \, dP(\xi) / (\pi \xi^2) \), which reads

\[
\sum \langle \xi_0 \rangle = 4nM_c r_c (1 + z_J)^2 F(\xi_0, r_c, R, r_0),
\]

where

\[
F = \left(\frac{r_c}{\xi_0}\right)^{0.8} \int_0^{r_c + \xi_0} \frac{m(\xi)}{M_c} k(r_0, \xi) \left(\frac{\xi}{\xi_0}\right)^{0.2} d\left(\frac{\xi}{\xi_0}\right),
\]

and

\[
k(r_0, \xi) = \begin{cases} \int_{r_0/\xi}^{\infty} \frac{dx}{x^{0.8} \sqrt{x^2 - 1}}, & \xi < r_0; \\ 1.84, & \xi \geq r_0. \end{cases}
\]

Summing the contributions from all kinds of clusters with different mass and number density gives rise to the expected mass surface density: \( \Sigma(\xi_0) = 4\Omega \rho_0 (1 + z_J) r_c F \), in which \( \Omega \) represents the fraction of the total cluster matter in the matter (with the critical mass density \( \rho_c \)) of the universe. For the typical cluster radii of \( R_c = 3\,\text{--}\,5 \ h_{50}^{-1} \) Mpc and the smallest cluster separation of \( r_c = 5\,\text{--}\,10 \ h_{50}^{-1} \) Mpc, numerical computation shows that \( F \approx 2\,\text{--}\,3 \) over the range of \( \xi = 1\,\text{--}\,20 \ h_{50}^{-1} \) Mpc, which is comparable to the search distances from cluster centers in the measurements of quasar-cluster associations. Therefore,

\[
\sum = 0.01\Omega \left(\frac{1 + z_J}{1.15}\right)^{0.8} \frac{F}{3} h_{50} \text{ g cm}^{-2},
\]

where we have assumed a stable clustering model \( e = -1.2 \) and converted the comoving surface mass density into a physical one by multiplying by a factor of \( (1 + z_J)^3 \). These two factors do not significantly alter our subsequent result, since the clusters involved are at relatively low redshift. It appears that even if we take \( \Omega = \Omega_0 = 1 \), the cluster matter provided by the cluster-cluster correlation is of an order of magnitude lower than the surface mass density required to produce the observed overdensity of background quasars around foreground clusters.

The mass surface density from matter inhomogeneities on scales larger than the coherence length (~50 Mpc) of the cluster-cluster correlation can be estimated by

\[
\sum = \int_0^\infty [\rho(r) - \rho_0] dr \sim \rho_0 \delta R = 1.45 \times 10^{-3} \delta \left(\frac{R}{100 \text{ Mpc}}\right) h_{50} \text{ g cm}^{-2},
\]

where \( \delta \) is the mean present density contrast over a scale of \( R \). However, the evaluation of \( \delta \) on a scale larger than ~10 \ h_{50}^{-1} \ Mpc is sharply constrained by the measurements of temperature fluctuation \( \Delta T/T \) of the cosmic background radiation on various angular scales. Using the simple model for a spherical density perturbation (Fang & Wu 1993), we can set an upper limit on \( \Sigma \) in terms of the recent results of \( \Delta T/T \). It turns out that the resulting \( \Sigma \) from any mass clumps on scales greater than ~20 \ h_{50}^{-1} \ Mpc is at least an order of magnitude smaller than the mass surface density required to explain the quasar-cluster associations. Therefore, it is unlikely that the observed quasar-cluster associations can be attributed to the lensing effect by large-scale (>20 \ h_{50}^{-1} \ Mpc) structures of the universe.
5. DISCUSSION AND CONCLUSIONS

We have shown that the strong associations of background quasars with foreground clusters on a scale of $\sim 10^9$ cannot be interpreted as the statistical lensing by clusters of galaxies. The situation does not improve even when all the cluster matter that follows the two-point cluster-cluster correlation function is involved, while the matter contribution from large-scale structures ($>20 \ h_{50}^{-1} \text{ Mpc}$) is strongly constrained by the measurements of the temperature anisotropies of the cosmic background radiation. We are limited to very few possibilities in solving the puzzle of quasar-cluster associations.

An intuitive speculation is that the reported quasar-cluster associations are statistical variations arising from the quasar/cluster selections. Rodrigues-Williams & Hogan (1994) and Seitz & Schneider (1995) have already pointed out that the patchy dust obscuration cannot explain their observations. Alternatively, the background quasar clustering is detected only at $r < 60 \ h_{50}^{-1} \text{ Mpc}$ (Mo & Fang 1993). This clustering scale is comparable to the angular separation in the quasar-cluster associations but is much smaller than the spatial separation of the selected quasars. Cluster-cluster autocorrelation seems to be another possibility. However, if the background quasars are detected randomly in the sky, there would be no angular correlation between quasars and clusters, even if the clusters are autocorrelated, which has been shown by Rodrigues-Williams & Hogan (1994) using their data and also by our simulations.

Another way out of the difficulty is that there exists a large amount of unseen matter between clusters of galaxies on a scale of less than $\sim 20 \text{ Mpc}$. Recall that we did not include the contribution of the intercluster matter in the above discussion. It is hard to figure out the distribution of this dark matter, but it should be massive enough to provide a surface density of as high as that required by the quasar-cluster associations. We will employ the $N$-body simulation of formation of clusters and large-scale structures to study the issue further (Wu, Fang, & Jing 1996a).

It may be possible that the observed background quasar counts deviate significantly from their intrinsic ones. The fact that quasars are strongly associated with the foreground galaxies and clusters indicates that the observations may preferentially select those quasars whose angular positions appear to be close to the foreground matter clumps. Unfortunately, previous studies (Schneider 1987, 1992; Pei 1995 and references therein) about the magnification bias on the observed quasar number counts reached a controversial result, depending mainly on our current knowledge of the distribution of lensing objects in the universe. It deserves to be investigated whether or not the quasar counts have been seriously contaminated by the lensing effect due to large-scale matter inhomogeneities. Meanwhile, our theoretical prediction of $q$ depends sensitively on the adopted quasar counts (Boyle et al. 1988), which may have large uncertainties. Recall that a different quasar number-magnitude relation is derived by Hawkins & Veron (1993).

Finally, we have tested the possibility of attributing the quasar-cluster associations to the cluster environmental effect from the gravitational matter of cluster-galaxy correlation and will present the result elsewhere (Wu et al. 1996b).

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