EVANESCENT WAVES ARE NOT SUPERLUMINAL

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ABSTRACT

It is demonstrated that an electromagnetic pulse, which is made to tunnel through a barrier, would not be photo-detected before an identical pulse, which travels the same distance in vacuum.

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In the last few years several experiments were conducted on tunneling of Electromagnetic (EM) signals through EM barriers\textsuperscript{1,4}. In Refs. 1 and 2 microwave pulses were forced to overcome barriers as evanescent waves. Steinberg et al\textsuperscript{3} investigated the tunneling of a single-photon across Multilayer Dielectric Mirrors (MDM). Speilman et al\textsuperscript{4} made 12 fs classical optical pulses to propagate through MDM. Both the classical and the single-photon experiments appear to indicate superluminal Electromagnetic tunneling.

In these experiments a single EM signal (a classical pulse or a single-photon) was made to split into two equivalent signals. A barrier was inserted along the path of one signal, and its "time of flight" was compared with the other reference signal. The detection was essentially a measurement of the product of the two signals: In the classical case a nonlinear detector was employed, while in the quantum twin photons case the overlap of the wave packets was being detected.

In the present communication we readdress the issue of whether the EM signal, which tunnel through the barrier, that is converted into an evanescent wave, may be considered as moving faster than light. We concentrate here on the classical case, where electromagnetic pulses can clearly be represented by EM wave packets. We will only relate to the more dramatic single-photon case by pointing out to the possible analogy between a photon and a classical pulse. The important questions of different velocities like group velocity, signal velocity etc. are being circumvented, by theoretically focusing on the direct detection of photo-emission.

We start by considering the following experimental setup, which admittedly may be only a Gedanken experiment. A classical electromagnetic pulse, which propagates along the \( x \) axis with its electric field linearly polarized in the \( y \) direction is split at the point \( x = x_0 \) into two identical pulses. The two pulses are now propagating in parallel also along the \( x \) axis, with the above polarizations. Each of the pulses then travels the same spatial distance \( l \), where it reaches a photodetector at the point \( x_1 \). One of the pulses, which we label as the reference or "free" pulse, travels all the distance \( l \) in vacuum. While along the other path, also in vacuum, of the second pulse, which we label
as the barrier pulse, a barrier of thickness $d$ is introduced, say between $x_1$ and $x_2$ with $x_2 - x_1 = d$ (See Fig. 1(a)). A barrier is understood here as a region in space, where in the range of frequencies of the pulse, EM waves can travel only as evanescent waves, and the transmitted wave is attenuated without any absorption.

The EM signals will be represented by the electric field

$$E(r; t) = \gamma E(x; t),$$

where $r$ is the radius vector, and $t$ is the time. The reference field propagating along the $x$ axis is given by

$$E_r(x; t) = Z \int_{t_0 - \tau_0}^{t_0 + \tau_0} d! f(!, !_0) \cos ! t \ 0(x \ x_0) = \gamma 0,$$

(1)

Here $f(!, !_0)$ is the spectral function of the pulse, $t_0$ is the time when the field is maximum at the entrance position $x_0$, and $c$ is the speed of light in vacuum. The spectral function of the angular frequency $!$ is taken to be symmetric and centered at $!_0$. Its spectral width, $!$, is extremely narrow, i.e., $! \ll !_0$, however the time duration of the pulse, $\tau = 1$, is assumed to be much shorter than $T = \tau c$, the EM time retardation between $x_0$ and the detector position at $x_1$. It is convenient to introduce the analytic signal,

$$\bar{E}_r(x; t) = \int_{t_0}^{t_0 + \tau_0} d! f(!, !_0) \exp i! t \ 0(x \ x_0) = \gamma 0,$$

in terms of which $E_r(x; t) = \Re \bar{E}_r(x; t)$, where $\Re$ stands for the real part of.

The detector at $x_1$ is considered to be a broadband or fast photoionization detector. The probability that the first electron will be ejected by the incoming signal at the time $t$ is given by

$$P_{\text{exc}}(t) = \int_{t_0}^{t_0 + \tau_0} dt \bar{E}_r(x; t) \bar{E}_r(x; t) :$$

Here $\tau$ is a parameter which characterizes the detector, and $\Re$ denotes the complex conjugate. The lower limit of the time integration is taken arbitrarily to be $t_0$, which is
far in the past with respect to the arrival time of the pulse to the detector. If we set, for convenience, $t_0 = 0$, then for positive $t$ we get for the detector of the reference signal

$$P_r(t) = \int_0^t e^{i(t^0 - T)} dt^0$$

We return now to the detection of the barrier pulse. The analytic signal along the barrier path, for $x > x_2$, i.e. to the right of the barrier is

$$\overline{E_b}(x; t) = \int_0^{R_1} d! f(! !) \exp i! [t \ t_0 \ (x \ x_0)=c]g,$$

where $(!)$ is transmission coefficient through the barrier. While to the left of the barrier, for $x < x_1,$

$$\overline{E_b}(x; t) = \int_0^{R_1} d! f(! !)$$

$$\exp i! [t \ t_0 \ (x \ x_0)=c]g + (!) \exp i! [t \ t_0 + (x \ x_0)=c]g),$$

where $(!)$ is the reflection function. For a given physical barrier the functions and are determined electromagnetically for a propagating plane wave of frequency $\omega$.

Thus for an identical broadband detector at $x_1$, on the barrier path, the probability that the first electron will be ejected at time $t$ is evidently

$$P_{exc}(t) = \int_0^{R_1} dt^0 \overline{E_b}(x_1; t^0)\overline{E_b}(x_1; t^0).$$

In terms of the complex transmission function

$$(!) = B (!) \exp [i (!)],$$

where $B = j (!) j$ and are real functions of the frequency, we get for the detector of the barrier signal

$$P_b(t) = \int_0^{Z_1} d! f(! !) B (!) \exp [i (t^0 - T) + i (!)]^2 :$$

In the present set-up the relevant question is which detector will receive, or rather - since the detectors must be treated quantum mechanically - which ionization probability
is larger, \(P_r(t)\) of the reference path, or \(P_b(t)\) of the barrier channel. This way we avoid the questions related to whether the velocity of the pulse is "faster than light". We shall demonstrate that, at least for the classical experiments, when the barrier prohibits propagating EM waves, i.e. allows only evanescent waves through it, the answer is \(P_r(t) > P_b(t)\). That is, the free channel detector is more probable to receive, and thus, in this sense the evanescent wave is not superluminal.

We start with the analysis of an experimental set-up analog to that of Ref. 4. It is clear that similar analysis would be relevant to the other classical cases of Refs. 1,2. We consider a barrier made of a quarter wave stake (aka MDM) of the form \(^6\) (vacuum) \(\times\) (vacuum) \(\times\) (vacuum) \(\times\) (high ref. index material) \(\times\) (high ref. index material), and low index material (titanium dioxide, \(n=2.4\)), and low index material (fused silica, \(n=1.46\)). The pulse is assumed to have a Gaussian spectral function \(f\), centered at the frequency \(0\) \(375\) THz (wave-length \(0\) \(0.8\) m in vacuum), and with a bandwidth \(28\) THz (FWHM). The thickness of the alternate layers of the MDM is taken to be optically equivalent to one quarter of the central wave length. The angle of incidence of the barrier pulse on the MDM is taken to be zero. The complex transmission coefficient, \((1)\), of this MDM is depicted in Fig. 1(b). Here, the magnitude, \(B(1)\) (solid line), and the phase, \((1)\), (dashed line) are plotted as functions of the frequency; and also shown, for comparison, is the spectral function of the pulse (dash-dot line). It is evident that \(B\) is symmetric with respect to the central frequency, \(0\), while \(\phi\) is antisymmetric. Furthermore, the phase is practically linear over the range where \(f\) is appreciable, and to a good extent its slope represents the apparent time delay seen in the experiments, e.g. \(5\) fs of Ref. 4.

We now calculate numerically the relative probabilities for ejection of an electron of the two identical detectors (same) using \(P_r(t) = \text{of Eq. (2)}\), and \(P_b(t) = \text{of Eq. (3)}\). The results are plotted in Fig. 2, with the time measured relative to the vacuum retardation time \(T = (x_1 - x_0)c\). It is clearly demonstrated that, at all times, the probability of detecting an electron injected by the vacuum or reference pulse, is much greater than that of the barrier pulse. The plateau reached on the vacuum channel is about \(10^4\) times larger than that of the barrier channel, and is not seen in the figure. We therefore conclude
that direct detection of the split pulses does not unveil any superluminal behavior of the evanescent wave.

It would be interesting to verify this conclusion for a general barrier, which compels propagating waves to tunnel as evanescent waves through the barrier. It should be demonstrated that for any tunnel barrier, which is represented by a causal (!), with attenuated transmitted pulse, the difference in probability, \( P(t) = P_c(t) - P_b(t) \), is positive. From Eqs. (2,3) we have

\[
P(t) = \frac{Z_k}{Z_1} \int_0^T dt' \int_0^1 d!_1 \int_0^1 d!_2 \exp[i!_1(t^0 - T)] \exp[i!_2(t^0 - T)] P(t^0 - t') P(t^0 - t') \quad (4)
\]

We were not able to provide a general proof that \( P(t) > 0 \), however this seems to be plausible when in Eq. (4) \( j(!) = B(!) \) is much smaller than one. As a matter of fact, this is indeed the case, i.e. \( B << 1 \), when the coincidence experiments\(^{12,4} \) apparently indicate "faster than light" results.

At this point we wish to indicate the difference between the previous experiments, and the one which is analyzed here. Consider e.g. Spielmann et al.\(^4 \) experiment for comparison: The two channels for their split pulse are the same as the reference pulse and the so-called barrier pulse in our case. While we put two photodetectors at the end of the two channels (of equal lengths), and compare the direct times of light, Spielmann et al.\(^4 \) use one nonlinear crystal as a detector. They measure the intensity of the Second Harmonic Generation (SHG) signal, due to time coincidence of the two pulses, which are directed to the detector. Their control parameter is the arm length of the reference pulse, which varies to produce coincidence. They detect essentially the reference and barrier signals\(^7 \), i.e.,

\[
P_{\text{SHG}}(t) / \lim_{T \to \infty} \int_0^T dt' \int_0^1 dx E_r(x,t^0 + t') P_r(x,t^0 + t') P_b(x,t^0 + t') \quad (5)
\]

where \( t \) is the relative delay time of the pulses. This measurement of the product of the pulses is clearly sensitive to the distortion, which the pulse suffers passing through the
While in our proposed scheme the response of the detectors is evidently connected to the transfer times of the two pulses, in the coincidence experiments the measurement corresponds to indirect evidence on the times of flight.

To conclude we wish to remark that the comparison with the case of Steinberg et al is harder to make, since they deal with single photons, which should be analyzed by quantum mechanics. The only relevant statement we may offer is that, if in some sense, single photons could have been precisely described in terms of wave packets, analogous to the classical ones outlined here, the same arguments would be applicable for a similar Gedanken experiment with two photodetectors for the twin photons.

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Figure captions

Fig. 1

(a) A schematic description of the suggested setup. An electromagnetic pulse is split at \( x_0 \) into two identical pulses. One pulse is propagating in vacuum to \( x_l \), where a photodetector (PD) is located. The second pulse is made to tunnel a barrier along its path, and reaches a second detector, at the same distance \( l \). The photoionization probability of the detectors is calculated as a function of time.

(b) The magnitude (solid line) and phase in units of \( 2 \pi \) (dashed line) of the transmission function through the Multilayer Dielectric Mirror are plotted as a function of the frequency. Also shown for reference, a schematic plot of the spectral function of the incident pulse (in the dash-dot line).

Fig. 2

The calculated relative probabilities of photoionization of the two detectors are plotted vs. time (measured with respect to the vacuum retardation time). It is seen that at all times the probability for photoionization of the reference detector is much larger than that of the barrier detector.
Magnitude and Phase of the Transmission Function
Probability for Photoionization

"Reference" Pulse

"Barrier" Pulse