Compositeness and the asymmetries of leptons at the $Z^0$ peak

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Abstract

We study the effects on the leptonic asymmetries $A_{FB}$ and $A_{LR}$ coming from a model of compositeness. We consider the effects coming from the self-energies and the vertex correction to $Zl^+l^-$. Thus we use the Altarelli parametrization of the oblique corrections. We get the asymptotic limits of these corrections in terms of the parameters $(m^*, \Lambda, f, f')$ and we get bounds for the quotient $m^*/\Lambda$ with different values of $(f, f')$. We conclude that both asymmetries produce bounds for such quotient when $f'$ overweight to $f$, and this fact is related with the breakdown of the custodial symmetry.

1 Introduction

Compositeness is regarded as a solution to many problems which are unsolved in the Standard Model, like the fermion mass spectrum, the generation of masses and the large number of parameters. The idea is that some underlying structure can explain the quark and lepton family pattern. At this stage, we shall search for new excited fermions ($\psi^*_F$), and for new interactions originated from the constituents.

A signal for a composite structure of fermions could be the production of excited leptons or quarks, either pairwise, due to their normal gauge couplings, or singly, due to radiative transitions between normal and excited fermions. For instance, excited fermions could be produced in $Z$ decays; in fact, there are strong limits coming from the $Z^0$ partial widths.
Mass regions up to 45 GeV for $u^*$ and $d^*$ as well as 35 – 40 GeV for $l^*$ and $\nu^*$ have been excluded \[4\]. However, the $l^*$ limits can be pushed to 46 GeV by searching the decay mode $Z \rightarrow l^*\bar{l}^* \rightarrow l^+l^-\gamma\gamma$ \[4\].

Moreover, recently the possibility has been studied that the significant excess from QCD predictions found by the collider detector at Fermilab (CDF) in the inclusive jet cross section for transverse energies $E_T \geq 200$ GeV could be explained by the production of excited bosons or excited quarks in the mass region of 1600 GeV and 500 GeV \[5\] \[6\], respectively. Other studies have already been done in order to get the low energy scale $\Lambda$ for compositeness by precise comparison between currently available experimental data and calculations in the composite model of quarks \[7\].

On the other hand, radiative corrections are the appropriate place to indirectly look for new physics at low energies \[7\] \[9\], because when heavy fermions are coupled to longitudinal components of the gauge bosons there is no decoupling of the high energy physics from the low energy physics. In ref. \[9\], radiative corrections at the $Z$ scale have been considered in order to bound substructure. Our purpose in this paper is to find bounds for the masses of possible excited leptons by using the radiative corrections involved in the forward-backward and left-right asymmetries for the lepton average.

2 The model

In general, the normal fermions ($\psi_f$) and excited fermions ($\psi_F^*$) at low energies can be described by an effective Lagrangian like \[3\]

$$L_{\text{eff}} = \sum_{i=\gamma,Z,W} \bar{v}_i \psi_F^* \gamma^\mu V_{\mu}^i \psi_F + \frac{e}{\Lambda} \bar{\psi}_F \sigma^{\mu\nu} (c_{iFf} - d_{iFf} \gamma_5) \psi_f V_{\mu\nu}^i + \text{h.c.} ,$$

(1)

where $V_{\mu\nu}^i$ are the field strengths for the gauge bosons and $\Lambda$ denotes the compositeness scale. In particular, we choose a specific model which assumes that excited leptons form a weak isodoublet,

$$\bar{L} = (\bar{\nu}^*, \bar{l}^*)$$

(2)

which couples to the ordinary left handed lepton doublet by the $SU(2)_L \otimes U(1)_Y$ invariant interaction Lagrangian

$$L = \frac{g_f}{\Lambda} \bar{L} \sigma^{\mu\nu} \tau^i l_L W_{\mu\nu} + \frac{g'_f}{\Lambda} \bar{L} \sigma^{\mu\nu} Y_{\tau^i} B_{\mu\nu} + \text{h.c.} .$$

(3)
Where \( g \) and \( g' \) are the \( SU(2) \) and \( U(1) \) coupling constants respectively, \( \tau \) denotes the Pauli matrices and \( Y \) is the hypercharge.

As for the coefficients \( c_{iFF} \) and \( d_{iFF} \), in general, \( g - 2 \) measurements imply \(|c| = |d|\), and also, the absence of dipole moments for the electron and the muon requires \( c \) and \( d \) to be relatively real \cite{10}. Thus, we have the following relations:

\[
\begin{align*}
  c_{\gamma l^* l} & = -\frac{1}{4} (f + f') , \\
  c_{\gamma\nu^* \nu} & = \frac{1}{4} (f - f') , \\
  c_{Zl^* l} & = -\frac{1}{4} (f \cot \theta_W + f' \tan \theta_W) , \\
  c_{Z\nu^* \nu} & = \frac{1}{4} (f \cot \theta_W + f' \tan \theta_W) , \\
  c_{W\pm \nu^* l} & = \frac{f}{2\sqrt{2} \sin \theta_W} .
\end{align*}
\]  

(4)

Now, since the excited fermions are doublets, their couplings to photon, \( Z \) and \( W \) bosons are defined by the following renormalizable Lagrangian

\[
\mathcal{L} = \bar{L} \gamma_{\mu} (g_2 \tau \cdot W^\mu + g'_2 Y B^\mu) L ,
\]  

(5)

where the coefficients \( v_i \) from eq.(1) satisfy

\[
\begin{align*}
  v_{\gamma l^* l} & = -e , \\
  v_{\gamma\nu^* \nu} & = 0 , \\
  v_{Zl^* l} & = -\frac{e}{2s_W c_W} (1 - 2s_W^2) , \\
  v_{Z\nu^* \nu} & = \frac{e}{2s_W c_W} , \\
  v_{W\pm \nu^* l} & = \frac{e}{\sqrt{2} s_W} .
\end{align*}
\]  

(6)

3 **Self-energy contributions**

Once we have the framework setup (eqs. (1)-(6)), we can calculate the contribution to the self-energies of the gauge bosons by using dimensional regularization, where the pole \( d = 4 \) is identified with \( \ln(\Lambda^2/m_Z^2) \) \cite{8}. In order
to simplify the analysis we assume the same mass for the excited leptons, $m_{\nu}^* = m_l^* = m^*$; this is justified by the fact that we are considering the excited fermion in a linear vectorial representation of the $SU(2) \times U(1)$ gauge group. The states with equal mass for $m_{\nu}^*$ and $m_l^*$ do not break the symmetry.

In general the self-energies due to the lagrangians of dimension 5 and 4 are:

$$\Sigma_{VV}(q^2) = -\frac{\alpha}{9\pi \Lambda^2} F^{(5)}(m^*, q^2)$$

$$\Sigma_{VV}(q^2) = -\frac{\alpha}{12\pi^2} F^{(4)}(m^*, q^2)$$

where the functions $F(m^*, q^2)$ can be written as [4]

$$F^{(5)}(m^*, q^2) = 6m^* [1 - \left(\frac{m^* - q^2}{q^2}\right) \ln\left(\frac{m^* - q^2}{q^2}\right)]$$

$$+ 3m^* q^2 [1 + \left(\frac{m^* - q^2}{q^2}\right) \ln\left(\frac{m^* - q^2}{q^2}\right)]$$

$$- q^4 [8 + 3 \ln\left(\frac{\Lambda^2}{m^* q^2}\right) - 3 \left(\frac{m^* - q^2}{q^2}\right) \ln\left(\frac{m^* - q^2}{q^2}\right)],$$

$$F^{(4)}(m^*, q^2) = q^2 \ln\left(\frac{\Lambda^2}{m^* q^2}\right) + 4m^* + \frac{5}{3} q^2$$

$$- 2(2m^* + q^2) \sqrt{\frac{4m^* + q^2}{q^2}} - 1 \arctan\left(\frac{1}{\sqrt{\frac{4m^* + q^2}{q^2}} - 1}\right).$$

We have considered the ordinary lepton masses negligible. Therefore, we find the vacuum polarization tensors for the gauge bosons:

$$-\Pi_Z(m_Z^2) = \frac{\alpha(1 - 2s^2_W + 2s^4_W)}{6\pi m_Z^2 s_W c_W^2} F^{(4)}(m^*, m_Z^2)$$

$$+ \frac{\alpha}{72\pi m_Z^2 \Lambda^2} (f \cot \theta_W + f' \tan \theta_W) F^{(5)}(m^*, m_Z^2),$$

$$-\Pi_{\gamma Z}(m_Z^2) = \frac{\alpha(1 - 2s^2_W)}{6\pi s_W c_W m_Z^2} F^{(4)}(m^*, m_Z^2)$$

$$+ \frac{\alpha f}{72\pi \Lambda^2 m_Z^2} (f \cot \theta_W + f' \tan \theta_W) F^{(5)}(m^*, m_Z^2),$$

$$-\Pi_\gamma(m_Z^2) = \frac{\alpha}{3\pi m_Z^2} F^{(4)}(m^*, m_Z^2)$$

4
\[
\frac{\alpha}{72\pi \Lambda^2 m_Z^2} (f^2 + f'^2) F^{(5)}(m^*, m_Z^2) ,
\]
where \( \Pi_W(0) = 0 \) and \( \Sigma'_Z(m_Z^2) \) is obtained directly from \( \Pi_Z(m_Z^2) \). The asymptotic limits \( (m^*, \Lambda) \to \infty \) in the functions \( F(m^*, q^2) \) are

\[
\frac{F^{(5)}(m^*, q^2)}{\Lambda^2} = 9 \left( \frac{m^*}{\Lambda} \right)^2 q^2 ,
\]
\[
F^{(4)}(m^*, q^2) = q^2 \ln \left( \frac{\Lambda}{m^*} \right)^2 .
\]

In such limit, \( \Sigma^{(5)}_{V_i V_j}(q^2) \) approaches a constant value and the physics described by (3) is non-decoupled.

4 The asymmetries and the new physics

On the other hand, the forward-backward asymmetry \( A_{FB} \) and the left-right asymmetry \( A_{LR} \) for leptons in the \( Z \) decays are given by

\[
A_{FB}^l = \frac{g^l_{V} g_A^l}{g^l_{V} + g_A^l} g_{V} g_A^l ,
\]
\[
A_{LR}^l = \frac{2g^l_{V} g_A^l}{g^l_{V} + g_A^l} .
\]

where a \( Zll \) amplitude is defined as \(-ig\gamma_{\mu} (g^l_{V} - g_A^l \gamma_5) / 4c_W \) and the superscripts \( e, l \) denote electron, lepton respectively. In our case, new physics is included in the constants \( g^l_{V,A} \) as

\[
g^l_{V,A} = g_{V,A}^{SM} + \delta g^l_{V,A} \]

where the superindex SM denotes the SM coupling with the contribution at one loop level of the top quark and Higgs scalar boson. In this way, \( \delta g^l_{V,A} \) only contains the new physics contribution. With this prescription we find that \( A_{FB}^l \) and \( A_{LR}^l \) can be written as

\[
A_{FB}^l = A_{FB}^{l,SM}(1 + 2\delta^{NP}) ,
\]
\[
A_{LR}^l = A_{LR}^{l,SM}(1 + \delta^{NP}) ,
\]
where $\delta_{NP}$ has the contribution of new physics and

$$
\delta_{NP} = \frac{\delta g^l_A}{g^SM_A} + \frac{\delta g^l_V}{g^SM_V} - 2\frac{g^SM_V \delta g^l_V + g^SM_A \delta g^l_A}{(g^SM_V)^2 + (g^SM_A)^2}.
$$

Additionally, these couplings $\delta g^l_{V,A}$ can be expressed in terms of the $\epsilon_i$ parameters [11],

$$
\delta g_V = \frac{2s^2_W}{c^2_W-s^2_W}\epsilon_3 - \left(\frac{1}{4} + \frac{s^2_W}{c^2_W-s^2_W}\right)\epsilon_1 - \epsilon_l/2,
$$

$$
\delta g_A = -\frac{1}{4}\epsilon_1 - \epsilon_l/2.
$$

Here, $\epsilon_l$ is the $Zl^+l^-$ vertex correction given in [9]. The other two parameters $\epsilon_1$ and $\epsilon_3$ can be written as a function of self-energies as given by eq. (9) [11].

We consider the asymmetries for electron and leptonic average. The SM predicted values are [13]

$$
A_{FB}^{SM} = 0.0168,
$$

$$
A_{LR}^{SM} = 0.1485
$$

for $\alpha = 1/128.75$, $m_W = 80.35$, $M_H = 100$ GeV and $m_t = 175$ GeV. And the experimental values are [12] :

$$
A_{FB}^l = 0.0174 \pm 0.0010,
$$

$$
A_{LR} = 0.1542 \pm 0.0037
$$

Notice that the standard model value $A_{LR}$ goes 1.54$\sigma$ out from experimental data. Figure 1 shows the $A_{FB}$ as a function of the scale $\Lambda$ for values of $m^* = 100$ GeV (a) and $m^* = 500$ GeV (b). Different values of the parameters $f$ and $f'$ have been considered. Figure 2 shows $A_{LR}$ as a function of the scale $\Lambda$ for the excited lepton mass of $m^* = 500$ GeV (a) and $m^* = 1000$ GeV (b). In figure 3 we have plotted $A_{FB}$ of the electron versus $m^*/\Lambda$, as $\Lambda$ is the energy scale for new physics, $m^*$ must be less than $\Lambda$, and we have $m^*/\Lambda$ between 0 and 1. In figure 4 we show the left-right asymmetry versus the quotient $m^*/\Lambda$ for different values of $f$ and $f'$ couple constants.

We can see that both forward-backward and left-right asymmetries produce bounds for such quotient when $f'$ overweight considerably to $f$, it means that custodial symmetry is strongly broken and therefore radiative contributions become higher.
5 Conclusion

In conclusion, we have evaluated the contribution of excited lepton states, up to one-loop level, to the oblique parameters. We have included this contribution into the leptonic asymmetries $A_{FB}$ and $A_{LR}$, and we have compared our results with the precise data on the electroweak observables obtained by LEP and LSD Collaborations [12] [13]. Therefore, we can extract bounds on the compositeness scale and the excited lepton mass in the context of the phenomenological model under consideration. Further, in equation (3) the term proportional to $B_{\mu\nu}$ breakdowns the custodial symmetry and it is proportional to the constant $f'$. Otherwise the new physics is allowed in the experimental region (1σ), which is satisfied when custodial symmetry is strongly broken. In order to get this requirement we need that $f'$ becomes bigger than $f$. With this prescription we obtain bounds on the rate $m^*/\Lambda$.

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Figure Captions

Figure 1. $A_{FB}$ as a function of the scale $\Lambda$ for values of the excited lepton mass of $m^* = 100\text{GeV}$ (a) and $m^* = 500\text{GeV}$ (b). From the upper solid line we use the values $(f' = f = 1)$, $(f' = -f = 1)$, $(f = 0, f' = 1)$ up to $(f = 1, f' = 0)$ the lower dot-dot-solid line. The horizontal solid lines are the experimental limits.

Figure 2. As figure 1 for $A_{LR}$.

Figure 3. $A_{FB}$ as a function of the ratio $m^*/\Lambda$. From the upper solid line we use the values $(f = 1, f' = 7)$, $(f = 0, f' = 6)$, $(f = -1, f' = 3)$ up to $(f = 1, f' = -1)$ the lower dashed line. The horizontal solid lines are the experimental limits.

Figure 4. $A_{LR}$ as a function of the ratio $m^*/\Lambda$. From the upper solid line we use $(f = 1, f' = 7)$, $(f = 0, f' = 6)$, $(f = -1, f' = 3)$, $(f = -2, f' = 2)$, $(f = 1, f' = 2)$, up to $f = 1, f' = -1$ (lower dashed line). The horizontal solid lines are the experimental limits.
Figure 1
Figure 2
Figure 3
Figure 4