ABSTRACT
Reliable aerodynamic and aeroelastic design of wind turbines, aircraft wings and turbomachinery blades increasingly relies on the use of high-fidelity Navier-Stokes Computational Fluid Dynamics codes to predict the strongly nonlinear periodic flows associated with structural vibrations and periodically varying farfield boundary conditions. On a single computer core, the harmonic balance solution of the Navier-Stokes equations has been shown to significantly reduce the analysis runtime with respect to the conventional time-domain approach. The problem size of realistic simulations, however, requires high-performance computing. The Computational Fluid Dynamics COSA code features a novel harmonic balance Navier-Stokes solver which has been previously parallelised using both a pure MPI implementation and a hybrid MPI/OpenMP implementation. This paper presents the recently completed optimisation of both parallelisations. The achieved performance improvements of both parallelisations highlight the effectiveness of the adopted parallel optimisation strategies. Moreover, a comparative analysis of the optimal performance of these two architectures in terms of runtime and power consumption using some of the current common HPC architectures highlights the reduction of both aspects achievable by using the hybrid parallelisation with emerging many-core architectures.

Keywords
CFD, Harmonic Balance, Navier Stokes, Parallel, Hybrid, MPI, OpenMP, Power Efficiency

1. INTRODUCTION
Reliable aerodynamic and aeroelastic design of wind turbine and rotorcraft blades, aircraft wings and turbomachinery blades increasingly relies on the use of high-fidelity Navier-Stokes (NS) Computational Fluid Dynamics (CFD) codes to predict the strongly nonlinear periodic flows associated with structural vibrations and periodically varying farfield boundary conditions. Many examples of using NS codes for such simulations have been published [16, 15]. These simulations have high computational costs even when parallel or high performance computing is adopted. This is particularly true for unsteady time-domain (TD) three-dimensional (3D) NS equations, which can require many months of computational time on a single computer. The use of a frequency-domain (FD) technique can dramatically reduce the required runtime for simulation of unsteady periodic flows with respect to that of TD solvers. On a single computer core, the FD harmonic balance (HB) solution of the NS equations has been shown to reduce at least by an order of magnitude the analysis runtime with respect to the conventional time-domain approach [5, 14]. The problem size of realistic periodic flow problems, however, is such that, despite the above discussed reduction, the HB NS analysis still requires high-performance computing.

Computational hardware has been rapidly evolving over recent years, particularly with the advent of multi-core processors and many-core hardware. Clusters of shared-memory servers have been common for high-end computational resources but the advent of multi- and many-core processors is bringing significant shared-memory computational resources at the node level for these clusters. Distributed memory parallelisations (based on the message passing library MPI [7]) have been the primary method for parallelising scientific codes, such as the one considered in this paper, for the past fifteen years; primarily due to the availability of large distributed memory systems and the prohibitive cost of large shared memory systems. Shared memory parallelisations, generally undertaken using the OpenMP [6] shared memory library, have been restricted to a number of specialised high performance computer (HPC) systems or to very small numbers of processors.

The CFD structured multi-block COSA code features a novel HB NS solver which has been previously parallelised using both a pure MPI implementation and a hybrid MPI/OpenMP implementation [8, 9]. This paper presents the recently completed optimisation of both parallelisations aiming to optimise performance and thereby reduce time to solution, or improve computational efficiency, for a given simulation.

An increasing number of research programmes aimed at developing efficient hybrid parallelisation technologies are un-
The work presented in this paper builds on previous parallelisation work of the COSA HB NS solver [9, 8], and has been undertaken to 1) improve the efficiency of the MPI parallelisation through rationalisation of the messages sent between processes and optimisations of the MPI-I/O utilisations, and 2) improve the hybrid parallelisation performance by extending the amount of functionality covered by the OpenMP implementation, and re-engineering the OpenMP code to reduce overheads and improve performance.

The harmonic formulation of the HB NS equations and the main structural features of the COSA code are summarised in the next two Sections. We then describe the optimisations undertaken to the MPI and hybrid parallelisations in the following two sections, and finally discuss the benchmarking undertaken on a range of the world’s largest HPC resources, and draw some conclusions, in the final sections of the paper.

The OpenMP parallelisation can work for all three solvers, with different parallelisations available over the blocks in the multi-block grids, over the harmonics for the HB solver, and over the grid points for those problems that use low numbers of blocks or harmonics (for instance a single block, TD, simulation). However, the OpenMP code is limited to the size of shared-memory machine available.

The MPI parallelisation distributes the blocks of the multi-block grid over the available MPI processes to distribute the work of the simulation. Communication is required between the blocks where data on the edge of blocks (called cuts in COSA) needs to be communicated to neighbouring blocks (halo communications). The maximum number of processes that the MPI parallelisation can use is limited by the number of geometric partitions (grid blocks) in the simulation.

The hybrid parallelisation combines the MPI code with either the harmonic OpenMP parallelisation or the grid point OpenMP parallelisation, depending on the simulation being performed.

2. OPTIMISATIONS

Of the three different parallelisations of COSA the MPI version is the one currently used for production simulations. In general the efficiency of the original MPI implementation is acceptable, for example with test case 1 (described in Section 3) experiencing around 50% efficiency when run on 512 cores (compared to running on a single core on a Cray XE6), or around 70% efficiency if considering the performance without writing any data to file at the end of the simulation.

The hybrid parallelisation was implemented to enable further reduction in time to solution for a given problem beyond what could be achieved by using the pure MPI code by enabling the usage of more computational resources (by scaling beyond the number of blocks in the simulation). The MPI code is limited in the number of processing elements, or cores, it can use by the number of blocks in the simulation. For instance, for test case 1 and 2 that we used to benchmark COSA for this paper the MPI code is limited to 512 and 2048 cores respectively. The hybrid code can enable further resource to be used, provided the HPC machine that the code is being run on is a shared memory cluster (as is generally true of modern HPC machines), by underpopulating each node with MPI processes and enabling each MPI process to create a number of OpenMP threads to utilise the cores left free of MPI processes. If a machine is composed of 32-core nodes, such as the Cray XE6 described in Section 3, then we can, for example, place 4 MPI processes on to each node, let them each create 8 OpenMP threads, and we can utilise 128 nodes for a simulation with 512 blocks as compared to 16 nodes for the MPI code.

However, the existing hybrid code does not provide optimal performance, and therefore resource usage. For test case 1 we can reduce the runtime of the simulation by four times when using eight times the amount of resources as the MPI code can utilise. Therefore, both the MPI and hybrid parallelisations were optimised to improve performance and thereby reduce the time to solution required to solve a given simulation on a given amount of computational resources, as outlined in the following subsections.

2.1 MPI Optimisations

The initial focus of our optimisation work was on the MPI communications in COSA. The current code utilises non-

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1This is when using a small number of simulation iterations, 100. Normal simulations would run with thousands of iterations.
blocking MPI communications, but for a large simulation there can be as many as 5,000 messages sent between pairs of communicating processes at each Runge-Kutta step to communicate \textit{halo} or cut data to neighbouring processes. This is because the original implementation sends small parts of the boundary data to neighbouring processes at a time, with an example of this shown in the following pseudo code:

```plaintext
do i = 0, boundary length
    if(myblock1 .and. myblock2) then
        do n = 0, 2*nharms
            copy 1st part of q2 to q1
            copy 2nd part of q2 to q1
        end do
    else if(myblock1) then
        receive 1st part of q1 from remote process
        receive 2nd part of q1 from remote process
    else if(myblock2) then
        send 1st part of q2 to remote process
        send 2nd part of q2 to remote process
    end if
end do
```

Note that in the above pseudo code we can see that the MPI communications have already been partially optimised, as they don’t send a message for each element of the \textit{n} and \textit{ipde} loops, they aggregate the data to be sent or received into an array and then send that array, as shown in the following code (which implements one of the send steps in the pseudo code above):

```plaintext
data size = npde*(((2*nharms)+1)
temp index = 1
    do n = 0, 2*nharms
        do ipde = 1, npde
            copy 1st part of q2 to q1
            copy 2nd part of q2 to q1
        end do
    else if(myblock1) then
        receive 1st part of q1 from remote process
        receive 2nd part of q1 from remote process
    else if(myblock2) then
        send 1st part of q2 to remote process
        send 2nd part of q2 to remote process
    end if
end do
```

However, as the send and receive functionality is within a loop, and for that loop the send and receive processes do not change (the same sender and receiver are involved in all the communications for a given invocation of the loop) it is possible to reduce all these send and receives down to one send and one receive by further aggregating the data into a single send array and using a single receive array.

We performed a similar optimisation for the collective communications used in the code, where the original collective functionality was of the following form:

```plaintext
do i = 1, nbody
    temparray(1) = cl(n,i)
end do
```

As well as a collective operation being undertaken for each iterations of the \textit{nbody} loop this functionality is also called from within a loop over harmonics (which sets the variable \textit{n} in the above code), enabling us to rationalise the number of collective operations undertaken from \textit{nbody} * ((2 * nharms) + 1) to 1 for each Runge-Kutta step in the simulation by collecting all the data to be sent into a single buffer, performing one collective all reduce, and unpacking the results at the end of the communication, as shown next:

```plaintext
j = 1
do k = 0, 2*nharms
    do i = 1, nbody
        temparray(j) = cl(k,i)
        j = j + 1
    end do
    j = 1
end do
call realsumallreduce(temparray,j-1)
j = 1
do k = 0, 2*nharms
    do i = 1, nbody
        cl(k,i) = temparray(j)
        j = j + 1
    end do
    j = 1
end do
```

As with the previous communication aggregation we have performed this at the expense of extra memory requirements for the routine, however these are not significantly large so do not adversely impact the overall memory footprint of the code, even for high \textit{nbody} and harmonic sizes.

Finally, it was also evident from profiling that the MPI I/O functionality in COSA was consuming large amounts of runtime, especially as larger simulations were undertaken. COSA produces a number of different output files, but for optimisation there are two types of file that are important, as they are the largest and require the most time to write; the flowtec files and the restart file. COSA produces a single restart file at the end of the simulation (or more frequently
if requested by the user) which can be used to restart the simulation from the point the restart file was written. It also produces one flowtec file per real harmonic at the end of the simulation. The flowtec files contain the solution in a format suitable for use with the commercial CFD postprocessor and flow visualisation software TECPLP.

When large simulations are executed the output can be extremely large, with the restart file being many gigabytes (GB) in size and each flowtec file being close to a GB in size. The current code does use parallel I/O functionality, calling MPI I/O routines to perform the output from all processes at once. However, the I/O is performed, as shown in the example below, through individual writes of data elements to the file one at a time:

```fortran
    tempindex = tempindex + 1
    tempdata(tempindex) = x(i,j,n)
    tempindex = tempindex + 1
    tempdata(tempindex) = rho
    tempindex = tempindex + 1
    tempdata(tempindex) = uy
    tempindex = tempindex + 1
    call setupfile(fid(n),disp,MPI_DOUBLE_PRECISION)
    call mpi_file_write(fid(n),tempdata(1),tempindex-1, i, MPI_DOUBLE_PRECISION, MPI_STATUS_IGNORE, ierr)
    call setupfile(fid(n),disp,MPI_INTEGER)
    call mpi_file_write(fid(n), 4*(tempindex-1),1, & MPI_INTEGER,MPI_STATUS_IGNORE,ierr)
```

2.2 Hybrid Optimisations

Using profiling information from COSA we worked to optimise the OpenMP functionality by focussing on the core routines that are heavily used for these types of simulations. This necessitated removing OpenMP functionality from areas of code that were not heavily used, and therefore removing the OpenMP overheads for those sections of the code, and re-implementing some of the existing OpenMP functionality, including replacing some heavily used small shared arrays with private variables and reduction operations.

Furthermore, the loops in the code that compute over the harmonics of the simulation are contained within subroutines which are called for each block in the simulation, and in each subroutine there can be a number of separate loops over harmonics. Simply parallelising each harmonic loop with an OpenMP parallel do directive meant that there were a lot of places that OpenMP parallel regions are started and finished in the code. There is an overhead associated with starting and finishing a parallel region in OpenMP, therefore we re-engineered the OpenMP code we had added to reduce these overheads by hoisted OpenMP parallel regions to higher levels in the program (where appropriate).

We also implemented *first touch* initialisation functionality to ensure that data is initialised on the cores that will be processing it. The original code zeros all the data arrays when they are allocated, and the allocation is done by the MPI process. We have altered the zeroing of the arrays so it is done in parallel, following the parallelisation pattern that is used in the rest of the code.

Finally, there were also a number of parts of the code that had not been parallelised with OpenMP, primarily the I/O and MPI message passing code. The MPI communications are performed over a loop of the cut, or halo, data. Each cut is independent so they can be performed by separate threads. However, as they involve MPI communications then we need to ensure that we are using the threaded version of the MPI library using the function MPI_INIT_THREAD rather than the usual MPI_INIT function. Furthermore, we need to ensure that the MPI library being used can support individual OpenMP threads performing MPI communications (MPI_THREAD_MULTIPLE).

The MPI communication code within COSA also implicitly depends on the order that the messages are send and received to match data with its correct location on the re-
ceasing. Each cut is processed in order and the individual sections of the cut are sent sequentially. Therefore, simply parallelising this process using OpenMP will not guarantee that data is placed in the correct arrays once it is received. To address this issue we added extra functionality to calculate where the data for each part of the cut should be placed in the send and receive arrays for each processes communications, and combined this with a unique identifier in the MPI tag for each message, to enable threads to send and receive the MPI messages as required and ensure that they place the received data in the correct places in the data arrays.

The I/O undertaken through the MPI code used MPI I/O functionality. In general the I/O operations are independent for each block and each harmonic within the block. However, there are a number of collective operations (operations that all processes must be involved in) in the I/O functionality, particularly opening and closing files. To enable the OpenMP threads to be able to write to the restart and flowtec files independently we needed to ensure that all the threads are involved in the opening of the files so they each have a separate file handle to write. Therefore, we implemented a hybrid file opening and closing strategy as follows:

```fortran
!$OMP DO ORDERED
  do i=1,omp_get_num_threads()
!$OMP ORDERED
    call openfile(fid,'restart',iomode)
!$OMP END ORDERED
  end do
!$OMP END DO
```

Where `openfile` calls MPI_FILE_OPEN. The only other modification that need to be made to enable file writing from the OpenMP threads was to ensure that they could correctly calculate where the data for each harmonic need to be written to (rather than each block as was the case previously).

3. EXPERIMENTAL SETUP

We evaluated the performance of the MPI and hybrid parallelisations of COSA, and the optimisations we have undertaken, using a range of common large scale HPC platforms and a range of representative test cases.

3.1 Test cases

Two different simulations were used to evaluate the performance of COSA, outlined in the following sections.

3.1.1 Test case 1

This first test case is a HB analysis of a heaving and pitching wing designed to extract energy from an oncoming air stream. The 512-block grid has 262,144 cells, and 31 real harmonics are used. This HB analysis has the same memory requirements of a steady flow analysis with more than 8 million cells. Further details on the aerodynamics of this device and the analysis of its efficiency based on COSA time-domain simulations are reported in the articles [1,3].

3.1.2 Test case 2

The other test case is a HB flow analysis of the blade section at 90% span of a multi-megawatt horizontal axis wind turbine operating in yawed wind. The analysis has been performed using a fine grid with 2048 blocks, and 4,194,304 cells. In the simulations we have used 17 real harmonics. Further details on the time-domain and HB COSA analyses of this problem are reported in[2].

3.2 Computing Resources

We used three different large scale HPC machines to benchmark performance. The first was a Cray XE 6, HECToR, which is the UK National Supercomputing Service consists of 2816 nodes, each containing two 16-core 2.3 GHz Intel Xeon Opteron processors per node, with a total of 32 cores per node, with 1 GB of memory per core. This configuration provides a machine with 90,112 cores in total, 90TB of main memory, and a peak performance of over 800 TFlop/s. We used the PGI FORTRAN compiler on HECToR, compiled with the -fastsse optimisation flag.

The seconds was a Bullx B510, called HELIOS, that is based on Intel Xeon processors. A node contains 2 Intel Xeon E5-2680 2.7 GHz processors giving 16-cores and 64 GB memory. HELIOS is composed of 4410 nodes, providing a total of 70,560 cores and a peak performance of over 1.2 PFlop/s. The network is built using Infiniband QDR non-blocking technology and is arranged using a fat-tree topology. We used the Intel FORTRAN compiler on HELIOS, compiling with the -O2 optimisation flag.

The final resource was a BlueGene/Q, JUQUEEN at Forschungszentrum Juelich, which is an IBM BlueGene/Q system based on the IBM POWER architecture. There are 28 racks composed of 28,672 nodes giving a total of 458,752 compute cores and a peak performance of 5.9 PFlop/s. Each node has an IBM PowerPC A2 processor running at 1.6 GHz and containing 16 SMT cores, each capable of running 4 threads, and 16 GB of SDRAM-DDR3 memory. IBM’s FORTRAN compiler, xlf90, was used on JUQUEEN, compiling using the -qhot -03 -arch=qp -gtune=qp optimisation flags.

Further technical details of these systems are provided in Table 3.2. The power listed in the table is the nominal power per node calculated using the reported power consumed during the LINPACK benchmark for the Top500 list entries divided by the number of nodes in the system.

4. PERFORMANCE RESULTS

We evaluated the effect of our performance optimisation on COSA on the different systems using a range of MPI process counts per shared memory node on each system, and also comparing the best performing MPI benchmarks with the hybrid code using four OpenMP threads for every MPI process used. We also used different iteration counts for the

\[http://www.top500.org\]

\[The HECToR Top500 entry does not include power data so the Cray XE6 power figure is calculated using the data from the Gaea C2 entry in the list which is a comparable Cray XE6\]

\[The network latency and bandwidth figures have been calculated used the mpbench benchmark which is part of the llbench benchmark suite\]
test cases to enable evaluation of the performance across a full range of node counts (for the small number of iterations) and for a more realistic usage scenario on a reduced number of nodes (using a large number of iterations). Ideally the code would have benchmarked using a large number of iterations for all node counts, however the restrictions on the job queues on the systems we were using meant that the longest job we could run was 12 hours which limited the number of iterations that could be completed within this limit on a single node for the slowest system used.

There are two performance metrics we are evaluating for this code, overall time to solution and cost. The user is generally looking to obtain results as quickly as possible, but may also be interested in getting results as efficiently as possible so they can utilise the HPC resource they have in the more efficient way. Because each of the systems we are using for the benchmarking uses different processor, memory, network, and disk technology it is difficult to directly compare the performance based purely on runtime between the systems. Therefore, we are evaluating relative performance (and therefore cost) using the estimated electrical power required for an iteration of the simulations on each system.

4.1 MPI Results

Figure 1 presents the runtime of the optimised MPI code on all three systems, using 100 iteration of test case 1, for fully populated and under populated nodes. The Cray XE6 has 32 cores per node, meaning a full populated node has 32 MPI processes per node. The Bullx B510 has 16 MPI processes per node when fully populated. The BG/Q is slightly different as it has 16 hardware cores per node, but each core can run 4 separate threads very efficiently (which means each core can run 4 MPI processes), therefore a fully populated BG/Q node can have either 64 or 16 MPI processes depending on how a user wishes to utilise the hardware. The dotted lines on the graph represent the ideal runtimes for their corresponding case based on the single node runtime figure.

We can compare the performance of the different machines using the graph in Figure 1. We can see that, in terms of time to solution, the Bullx B510 gives the best performance. It is significantly faster than the Cray XE6, even when using the same number of nodes (for instance 16 nodes on each system) which involves using double the number of cores on the Cray compared to the Bull machine. We can also see that underpopulating nodes on the Cray and Bull does not improve performance, with underpopulated nodes requiring twice the number of nodes to achieve the same performance as the fully populated case.

The scaling of the code is better on the Bull (with the exception of the transition from 1 to 2 nodes) than on the Cray, and the scaling is also good on a node basis on the BG/Q system. Underpopulating nodes on the BG/Q does significantly improve performance, reducing the time to solution for the simulation at the cost of extra resources. However, we can see that when the BG/Q nodes are fully populated (i.e. using 64 MPI processes per node) we get better resource usage compared to the underpopulated case. If we compare the 64 MPI processes per node case with the 16 MPI processes per node results we can see that the same time to solution is achieved using 8 nodes (when fully populating) compared to 16 nodes (when underpopulating by a factor of 4). The downside of fully populating BG/Q nodes is that it is not always possible to run a given simulation on a set of fully populated nodes. For instance, it was only possible to run test case 1 on 4 or 8 nodes, few nodes and there was not enough memory to execute the simulations. The memory effect experienced is likely due the memory consumption associated with the MPI library rather than the COSA application itself.

We also ran the same simulation with a large number of iterations to evaluate the efficiency of the main computational part of the code (increasing the iterations reduces the impact of the initial and final I/O functionality on the overall runtime). Figure 2 presents the time to solution for the same testcase with 1000 iterations. Now we can see the overall runtime scaling has improved, with all three architectures exhibiting better than ideal scaling (on a node level scaling). For 1000 iterations it was only possible to run on the BG/Q with 64 processes per node on the maximum number of nodes for that test cases (8 nodes) as any fewer nodes required longer runtime than the queueing system would allow on that particular machine. However, we can see that the 8 node runtime for fully populated BG/Q has approximately the same runtime as 16 nodes of under-populated BG/Q.
We also present the efficiency of the node scaling of the code on the different machines in Table 2. The efficiency of the MPI code is calculated using the following equation:

$$EM_n = \frac{T_M}{T_M^s}$$  \hspace{1cm} (1)

Where $EM_n$ is the efficiency of the MPI code at $n$ nodes, $T_M$ is the runtime on $s$ (the smallest number of nodes used for the benchmark), and $T_M^s$ is the runtime on $n$ nodes. Note that on the BG/Q we could only run the 1000 iteration version of test case 1 on 8 nodes so we cannot calculate the scaling of this test case. We are using the most efficient configurations of these systems for this table (64 MPI processes per node on BG/Q; 32 and 16 processes per node for the Cray and Bull machines respectively). It is evident from this graph that the hybrid code can provide improved time to solution over the MPI code for all the architectures. On the BG/Q the hybrid code exhibits similar performance to the MPI code for the comparable node counts, and reduces the runtime by around 3 times when 4 times the number of nodes is used. For the Cray and Bull machines the hybrid code has significantly lower performance than the MPI code for comparable node counts, but still reduces time to solution compared to the MPI code when more resources are used (3.26 times faster for the Cray and 2.11 times faster for the Bull when using 4 times the resources).

As with the MPI benchmarking we have evaluated the performance using 1000 iterations instead of 100 iterations, as shown in Figure 2b. As with the MPI evaluation, increasing
the iterations has improved the overall scaling of the hybrid code, bring the results closer to those of the pure MPI code (shown as the dotted lines the graph).

We have also evaluated the efficiency of the Hybrid code is calculated using the following equation:

\[ E_{Hn} = \frac{T_{Mn}}{T_{Hn}} \]  

(2)

Where \( E_{Hn} \) is the efficiency of the hybrid code at \( m \) nodes, \( T_{Mn} \) is the runtime of the best MPI code on \( n \) (the maximum nodes it can use for test case 1), and \( T_{Hn} \) is the runtime of the hybrid code on \( m \) nodes (the maximum number of nodes the hybrid code can exploit using 4 OpenMP threads per MPI process using test case 1).

As shown in Table 2, we can see the hybrid code does not show as good performance as the MPI code, but is still giving acceptable performance for the Cray and BG/Q architectures considering that we are using four times the resources as the MPI code, and therefore we are reducing the amount of work for each thread to undertake by a factor of 4.

4.3 Resource Efficiency

As previously discussed, to enable us to evaluate the performance of COSA across a range of resources we are looking at the notional power consumption of the code when running a simulation. To perform this analysis we are using the power figures presented in Table 2, which as previously explained is the amount of power reported consumed by each node during LINPACK benchmarking. We have used this figure, along with the runtime of the simulation, to calculate the power consumed per iteration of the simulation, as shown in equation 3:

\[ P_i = \frac{T_i}{3600} \times P_n \]  

(3)

Where \( P_i \) is the power consumed per iteration in Watt hours, \( T_i \) is the total runtime of the simulation in seconds, \( P_n \) is the notional power per node figure, and \( n \) is the number of iterations undertaken.

Figure 3a displays the relative power usage of the simulation on the three systems. We can see that the BG/Q has the best power performance for the simulations we have undertaken, requiring less power to complete the simulation for all the configurations of fully populated or underpopulated nodes when compared to the other two systems. It is also evident that fully populating the nodes makes much more efficient use of the resources, albeit for an increased time to solutions.

We should note that these are estimated power figures, not recorded power usage, so the actual power consumed may vary when under populating nodes, which may mean the figures for the BG/Q 16 and 32 MPI processes per node are not accurate.

The Bull machine shows generally fixed power efficiency regardless of the number of nodes used (disregarding the 2 node results), which highlights the MPI code is scaling well across nodes on this machine, whereas the power consumed by the Cray increases as we increase the number of nodes, highlighting sub-optimal scaling of the code in this instance.

When considering the power efficiency of the hybrid code, shown in Figure 3b (where the dotted lines are the power per iteration of the most efficient MPI runs for comparison), we can see that for the Cray and Bull machines the hybrid code generally has considerably lower efficiency than the pure MPI code, with the hybrid code on the Cray now showing similar or better efficiency when compared to the hybrid code on the Bull, although the hybrid code on the Cray has similar efficiency compared to the pure MPI code when using 8 and 16 nodes, whereas on the Bull machine the efficiency is always much worse than the pure MPI code.

On the BG/Q the picture is different. Here the efficiency of the hybrid code still does not beat that of the pure MPI code, however it is much closer, and if compared to the underpopulated BG/Q MPI result (where 16 MPI processes are used per node) it is considerably better. Given that the hybrid code on BG/Q enables a user to exploit as many nodes as the underpopulated approach takes (4 times the number of nodes that the fully populated MPI code can use for this code) at an efficiency that is close to that of the fully populated case we can see that the hybrid code is extremely beneficial on the BG/Q.

If we consider the efficiency of the code using 1000 iterations for the same test case we can see a further improved picture, as we so with the runtime scaling of the code in the previous section. We can now see that the efficiency of the Cray has much improved, no longer increasing with as the nodes increase. A similar effect is also exhibited by the Bull and IBM machines, and it is also evident that the increased iterations have reduced the gap between the power consumption of the hybrid implementation on the Bull and Cray with that of the MPI implementation. We can also see that the performance of the hybrid code on the BG/Q is also much improved. The hybrid implementation uses approximately the same power as the fully populated MPI code despite scaling out to 4 times the node and reducing the runtime by around 3.5 times.

4.4 Large Benchmark

We have also evaluated the optimisations using test case 2 on the different computing resources we have access to. Test case 2 can utilise 2048 MPI processes using the pure MPI code, but only has half the real harmonics that test case 1 had so we are only utilising 2 OpenMP threads for the hybrid case, meaning we can use up to 4096 cores for the hybrid case. When we evaluate the optimised codes
using this test case we get a scaling efficiency of around 80% efficiency on the Cray and more than 90% efficiency on the Bull (we could only run one test on the BG/Q as lower node counts would not run for the fully populated configuration). The BG/Q required between 4 and 5 times less energy per iteration than the other two machines.

The hybrid version of this code exhibited around 80% efficiency when using 2 OpenMP threads per process for the machines used in this evaluation.

5. CONCLUSIONS

Through the optimisation we have undertaken on the parallelisations of COSA we are able to ensure that the code scales with excellent efficiency as the number of nodes are increased for the MPI parallelisation, with around or exceeding 100% efficiency across a range of systems providing a realistic number iterations are used in the simulation. It should be noted that this performance is including the full functionality of the code, including input and output of data.

We have demonstrated that on hardware which is designed for explicit multi-threading, such as the BG/Q, the hybrid code gives excellent performance when scaling beyond the number of nodes that the MPI code can use, enabling users to efficiently reduce the time to solution at almost ideal efficiency. When comparing to more traditional hardware, such as used in the AMD based Cray or the Intel based Bull machines the hybrid code does not provide as good efficiency when scaling the number of nodes. However, it should be noted that the hybrid code is, but its nature, going to be most efficient when using more resources than the pure MPI code. This is because the hybrid code undertakes parallelisation of the harmonics of each block in the simulation. If an MPI process has more than one block then it will encounter OpenMP overheads for each block it processes, whereas if the MPI code is maximally parallelised (i.e. 1 block per MPI process) then these overheads are minimised. If we examine the performance of the hybrid code in that scenario we can see that we are achieving between 70% and 90% efficiency, enabling time to solution to be significantly reduce for a minimal reduction in efficiency.

We have also observed the the BG/Q architecture gives significantly better power to iteration performance than the other two systems, albeit for a longer overall time to solu-
ation. However, the low memory per thread on the BG/Q (256 MB per thread when using 64 threads per node) is a significant barrier to memory applications utilising such a system, but we have shown that a hybrid approach can alleviate this problem by enabling all the resources on the node to be used by a code without having to have 64 MPI processes per node (with all the associated fixed memory requirements those MPI processes have).

We have further work to undertake to understand the performance difference between the Cray and Bull systems, particularly why the Bull system exhibits poorer hybrid performance than the rest. However, a preliminary hypothesis is that the higher performance of the individual nodes means that the test cases we are using are not sufficiently computationally demanding on the Bull system to warrant the hybrid parallelisations. It is also possible that the ratio of MPI processes to OpenMP threads was not optimal for this system. Furthermore, we have not investigated the thread assignment behaviour of any of these systems, simply using what is provided by default through the batch systems (assigning the requisite MPI processes per node and OpenMP threads per process) so it is possible that non-optimal thread and process binding in affecting performance.

6. ACKNOWLEDGMENTS
This work was supported by Dr M. Sergio Campobasso at Lancaster University.

Part of this work was funded under the HECToR Distributed Computational Science and Engineering (CSE) Service operated by NAG Ltd. HECToR - A Research Councils UK High End Computing Service – is the UK’s national supercomputing service, managed by EPSRC on behalf of the participating Research Councils. Its mission is to support capability science and engineering in UK academia. The HECToR supercomputers are managed by UoE HPCx Ltd and the CSE Support Service is provided by NAG Ltd. http://www.hector.ac.uk

Part of this work was supported by NAIS(the Centre for Numerical Algorithms and Intelligent Software) which is funded by EPSRC grant EP/G036136/1 and the Scottish Funding Council.

7. REFERENCES
[1] http://developer.download.nvidia.com/GTC/PDF/GTC2012/PresentationPDF/RobertStrzodka_Accelerated_ANSYS_Fluent_SC12.pdf
[2] A. Baker, M. Schulz, and U. Yang. On the performance of an algebraic multigrid solver on multicore clusters. In J. M. M. Palma, M. Dayde, O. Marques, and J. C. Lopes, editors, High Performance Computing for Computational Science - VECPAR 2010, volume 6449 of Lecture Notes in Computer Science, pages 102–115. Springer Berlin Heidelberg, 2011.
[3] I. Bethune. Improving the performance of cp2k on the cray xt. In Cray User Group 2010 (CUG2010), 2010.
[4] S. Boehmer, T. Cramer, M. Hafner, E. Lange, C. Bischof, and K. Hameyer. Numerical simulation of electrical machines by means of a hybrid parallelisation using mpi and openmp for finite-element method. Science, Measurement Technology, IET, 6(5):339–343, 2012.
[5] M. S. Campobasso and M. H. Baba-Ahmadi. Analysis of Unsteady Flows Past Horizontal Axis Wind Turbine Airfoils Based on Harmonic Balance Compressible Navier-Stokes Equations With Low-Speed Preconditioning, pages 729–745. ASME International, 2011.
[6] M. Forum. Openmp architecture review board. openmp fortran application program interface, version 1.1. available from: http://www.openmp.org.
[7] M. Forum. MPI: A message-passing interface standard. available at: http://www.mpi-forum.org.
[8] A. Jackson and M. S. Campobasso. Shared-memory, distributed-memory, and mixed-mode parallelisation of a cfd simulation code. Computer Science - R&D, 26(3-4):187–195, 2011.
[9] A. Jackson, M. S. Campobasso, and M. H. Baba-Ahmadi. On the Parallelization of a Harmonic Balance Compressible Navier-Stokes Solver for Wind Turbine Aerodynamics, pages 747–761. ASME International, 2011.
[10] Y. Liu. Hybrid parallel computation of openfoam solver on multi-core cluster systems.
[11] llcbench. http://icl.cs.utk.edu/projects/lclbench/.
[12] K. Madduri, K. Z. Ibrahim, S. Williams, E.-J. Im, S. Ethier, J. Shalf, and L. Oliker. Gyrokinetic toroidal simulations on leading multi- and manycore hpc systems. In Proceedings of 2011 International Conference for High Performance Computing, Networking, Storage and Analysis, SC ’11, pages 23:1–23:12, New York, NY, USA, 2011. ACM.
[13] G. R. Mudalige, M. B. Giles, I. Reguly, C. Bertolli, and P. H. J. Kelly. Op2: An active library framework for solving unstructured mesh-based applications on multi-core and many-core architectures ?.
[14] A. D. Ronch, M. Ghoreyshi, K. Badcock, S. Goertz, M. Widhalm, R. Dwight, and M. Campobasso. Linear frequency domain and harmonic balance predictions of dynamic deritvates. In 28th AIAA Applied Aerodynamics Conference, 2010.
[15] R. Steijl and G. Barakos. Sliding mesh algorithm for cfd analysis of helicopter rotor-fuselage aerodynamics. International Journal for Numerical Methods in Fluids, 58(5):527–549, 2008.
[16] M. Vahdati, A. I. Sayma, M. Imregun, and G. Simpson. Multibladerow forced response modeling in axial-flow core compressors. Journal of Turbomachinery, 129(2):412, 2007.
[17] M. Weiland, L. Mitchell, G. Gorman, S. Kramer, M. Parsons, and J. Southern. Mixed-mode implementation of petsc for scalable linear algebra on multi-core processors. CoRR, pages –1–1, 2012.
[18] M. A. A. S. Xiaooh Guo, G. Gorman. Developing hybrid openmp/mpi parallelism for fluidity-icom - next generation geophysical fluid modelling technology. In Cray User Group 2012: Greengineering the Future (CUG2012), 2012.