ASTROPHYSICAL EXPLOSIONS DRIVEN BY A ROTATING, MAGNETIZED, GRAVITATING SPHERE

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ABSTRACT

We present the results of a numerical magnetohydrodynamic simulation that demonstrates a mechanism by which magnetic fields tap the rotational energy of a stellar core and expel the envelope. Our numerical setup, designed to focus on the basic physics of the outflow mechanism, consists of a solid, gravitating sphere, which may represent the compact core of a star, surrounded by an initially hydrostatic envelope of ionized gas. The core is threaded by a dipolar magnetic field that also permeates the envelope. At the start of the simulation, the core begins to rotate at 10% of the escape speed. The magnetic field is sufficiently strong to drive a magnetorotational explosion, whereby the entire envelope is expelled, confirming the expectation of analytical models. Furthermore, the dipolar nature of the field results in an explosion that is enhanced simultaneously along the rotation axis (a jet) and along the magnetic equator. While the initial condition is simplified, the simulation approximates circumstances that may arise in astrophysical objects such as Type II supernovae, gamma-ray bursts, and proto-planetary nebulae.

Subject headings: MHD — planetary nebulae: general — stars: evolution — stars: mass loss — stars: rotation — supernovae: general

1. INTRODUCTION

Over the last two decades, magnetic fields have been identified as the principle, universal agent for creating collimated astrophysical outflows. When a magnetic field is anchored in a rapidly rotating object near the bottom of a gravitational potential, the field can act as a drive belt, tapping rotational kinetic energy and launching plasma back up the potential well. This magnetorotational (MR) scenario for jet launching has been explored by numerous authors, both analytically (Blandford & Payne 1982; Pelletier & Pudritz 1992) and numerically (Ouyed et al. 1997; Krasnopolsky et al. 1999). These authors showed that this mechanism can produce steady flows of matter, in the sense that the outflow engine still operates while the flow is observable—as in jets from young stellar objects (Reipurth & Bally 2001) and active galactic nuclei (Begelman et al. 1984).

On the other hand, this mechanism can also operate in a transient event, linked to a rapid evolution of the source and driving an explosion. In this case, the engine loses a significant fraction of its power by the time the outflow (or its interaction with the environment) is detected. The proposed mechanism for gamma-ray bursts (GRBs; Piran 2005) and supernovae (SNe) lies in the explosive regime. There is also growing evidence (Bujarrabal et al. 2001) that collimated outflows from planetary nebulae (PNs) are explosive.

Transient MR explosions have not been explored in as much detail as the steady state models. In a collapsing star, differential rotation near the core may amplify the field linearly (when turbulence is unimportant; Kluzniak & Ruderman 1998; Wheeler et al. 2002) or exponentially (when turbulence is important; Akiyama et al. 2003; Blackman et al. 2006). When differential rotation twists a poloidal magnetic field, generating the toroidal field $B_t$, enough rotational energy may be tapped to power a supernova explosion. The role of toroidal field pressure in then driving an outflow has been highlighted by Lynden-Bell (1996), who explored magnetostatic “magnetic towers” in order to understand jet properties (see also Li et al. 2001), and Uzdensky & McFadyen (2006) has extended this work to exploding stars. Numerical simulations by LeBlanc & Wilson (1970) and more recently by Ardeljan et al. (2005) support the idea that MR explosions can be important for driving SNe and GRBs.

However, the inherent time dependence and general complexity of the system present a significant challenge to our understanding of the basic mechanism and identification of key parameters. It is clear that an MR explosion is ultimately driven by the rotational kinetic energy extracted from the material that is left behind (the stellar remnant). But, notably, there is still some uncertainty as to whether an accretion disk must form inside the star (as in Uzdensky & McFadyen 2006) or whether the rotation of the stellar core (e.g., a protopulsar; Wheeler et al. 2002) alone will drive an MR explosion.

In this Letter, we present a magnetohydrodynamic (MHD) simulation of an MR explosion. While our setup is simple, similar to the analytical “protopulsar jet” model described by Wheeler et al. (2002), our simulation captures the nonlinear dynamics, as the magnetic field is twisted at the shear layer between the core and the envelope. This heuristic approach enables us to better understand the basic MR physics and complements the more phenomenological approach of previous, more complex, numerical studies.

2. PHYSICAL EXPERIMENT AND NUMERICAL METHOD

2.1. Physics

The model consists of a gravitating, magnetized sphere (the “core,” representing a stellar core) surrounded by an overlying envelope. The core is solid in the sense that no mass flows in or out of the surface, and it rotates as a solid body. The core is also a conductor with a rotation axis–aligned dipole magnetic field anchored to the surface. This magnetic field couples to the envelope, whose behavior is characterized by the ideal MHD equations. This core-envelope model approximates the conditions in the interior of a star where there is a very steep
density gradient, for example, in the proximity of a nearly degenerate core in an evolved star. There is an implicit assumption that the envelope self-gravity is negligible.

We assume an initial state in which the envelope is not rotating and rests in hydrostatic equilibrium, and whose density falls off as $r^{-2}$. The initial dipole magnetic field is force-free and permeates the envelope. At a time defined as $t = 0$ in the simulations, the core begins rotating at a constant rate of 10% of the escape speed. This initiates a shear at the boundary between the surface of the core and the base of the envelope.

The strong initial shear and the purely poloidal magnetic field approximate the conditions in a real star immediately following an abrupt change in structure, such as a core collapse (or envelope expansion). If the collapse is rapid enough, $B_0$ subsequently amplified at the core-envelope interface will be much stronger than $B_0$ generated elsewhere before the collapse, and the envelope rotation will be negligible. The envelope is unlikely to be at rest following such a change, and any bulk motion could quantitatively (but not qualitatively) change our results. For example, the ram pressure of an infalling envelope would change the magnetic energy required for an explosion by approximately a factor of 2. Finally, the field geometry is reasonable, since pulsars and magnetic white dwarfs are thought to have dipolar fields (although tilted with respect to their rotation axes).

The poloidal magnetic field connecting the core to the envelope is twisted by the shear, generating an azimuthal component to the magnetic field, $B_\phi$, near the interface, which increases approximately linearly in time. The magnetic pressure force associated with $-\nabla B_\phi^2$ will be directed generally outward from the core, so when $B_\phi$ becomes dynamically important, the field will expand, driving the ionized envelope material in front of it. If the expansion is powerful enough, it can be explosive and drive off the entire envelope. We intentionally inhibit any sort of wind from the surface of the core, so the outflow in this explosion is composed of material that originally overlies the core (i.e., envelope material). The flow thus depends on the initial conditions existing above the surface of the core and is inherently a transient phenomenon. This contrasts with any steady state wind theory whose solution depends only on the boundary conditions at the wind driving source.

2.2. Parameters

The key parameters in the system are the initial magnetic, rotational kinetic, gravitational potential, and thermal energy densities near the core-envelope interface. It is instructive to compare the characteristic speeds associated with these energies, namely, the Alfvén speed $v_A = B/(4\pi\rho)^{1/2}$, where $B$ is the dipole field strength at the equator of the core and $\rho$ is the mass density at the base of the envelope, the rotation speed at the equator ($v_{\text{rot}}$), the escape speed at the core surface ($v_{\text{esc}}$), and the sound speed at the base of the envelope ($c_\rho$), respectively.

The assumption of initial hydrostatic equilibrium couples the thermal energy to the potential energy, effectively reducing the number of free parameters (by requiring the initial $c_\rho \approx 0.53v_{\text{esc}}$). Also, since the simulations can be scaled to any system with similar ratios of energies, there are only two fundamental parameters, which can be cast as the dimensionless velocity ratios $v_A/v_{\text{esc}}$ and $v_{\text{rot}}/v_{\text{esc}}$. Here, we present results from a simulation with $v_A/v_{\text{esc}} = 1.0$ and $v_{\text{rot}}/v_{\text{esc}} = 0.1$.

The key parameter that determines whether or not the envelope will be driven off is $\chi \equiv (v_Av_{\text{rot}})^{1/2}/v_{\text{esc}}$. We have run a limited parameter study thus far (not presented in this Letter), which suggests that the threshold for explosion occurs near $\chi \approx 0.2$. The model presented here has $\chi \approx 0.3$.

2.3. Numerics

We employ the 2.5-dimensional MHD code of Matt (2002), and the reader can find details of the code there and also in Matt et al. (2002) and Matt & Balick (2004). The code solves the ideal (nonresistive) MHD equations using a two-step Lax-Wendroff, finite difference scheme (Richtmyer & Morton 1967) in cylindrical $(\sigma, \phi, z)$ geometry. The formulation of the equations allows for a polytropic equation of state (we adopt $\gamma = 5/3$), includes a source term in the momentum and energy equations for point-source gravity, and assumes axisymmetry ($\partial/\partial\phi = 0$ for all quantities).

The computational domain consists of five nested grids (or “boxes”) in the cylindrical $\sigma$-$z$ plane. Each box contains $401 \times 400$ grid points (in $\sigma$ and z, respectively) with constant grid spacing. The boxes are nested concentrically, so that the inner (first) box represents the smallest domain at the highest resolution. The next outer box represents twice the domain size with half the spatial resolution (and so on for other boxes). The simulation ended before the explosion propagated beyond the fourth box. A circular boundary with a radius of 30.5 grid points and centered on the origin represents the surface of the core. Thus, the innermost box has a spatial resolution of 0.033 core radii, $R_*$, and a domain size of 13.1$R_*$, and the fourth box has a resolution and domain size of 0.26$R_*$ and 105$R_*$, respectively.

We use standard outflow conditions on the outer box boundaries, appropriate for wind studies. We also use standard boundary conditions on the rotation axis ($\sigma = 0$) and reflection symmetry on the equator ($z = 0$). The simulation domain, then, consists of a $\sigma$-$z$ slice through a single quadrant. The surface of the core is represented by a circular inner boundary, centered at the origin ($\sigma = z = 0$). Here, we enforce boundary conditions on a three-grid cell layer, and we have performed several tests to ensure that this inner boundary behaves as appropriate for the surface of a solid, rotating, conducting, and magnetized sphere.

For the current study, we initialize the entire computational domain with a spherically symmetric hydrostatic envelope, whose density falls off as $r^{-2}$ (where $r$ is the spherical radius). The envelope is initially stagnant (no rotation or motion) and is threaded everywhere by a dipole magnetic field that is anchored to the core, as shown in the upper left panel of Figure 1.

3. RESULTS: NATURE OF THE EXPLOSION

At the start of the simulations, the dipolar magnetic field begins to wrap up. For the chosen parameters, $B_0$ becomes dynamically important after a few tenths of a rotation of the core. The panels of Figure 1 show the evolution during the first 4.5 rotations of the core. The expansion of newly generated $B_0$ is evident, as it drives the envelope material outward. This also expands the poloidal magnetic field (lines), although some field lines near the equator remain closed, where material is forced and held in corotation with the core. The region near the core is evacuated by the expansion of the magnetic field, indicative of a transient event rather than the beginning of a steady wind.

Figure 2 shows the density in the system after 4.5 rotations of the core, on a scale that is 10 times larger than in Figure 1, and the data have been reflected about the rotation axis and equator to better illustrate the flow. In the figure, one sees a high-
density swept-up shell of envelope material that is moving outward from the core and bounded by a shock.

The shell has a quadrupolar shape. This symmetry is due to the coupling of the rotation to the dipole field, which produces a $B$, that has a maximum strength at midlatitudes near the core, so magnetic pressure forces direct outward from the core, and also toward the pole and equator (see Matt & Balick 2004). This shape can be qualitatively understood as an explosion driven by detonating two doughnut-shaped charges placed at midlatitudes around the north and south hemispheres, which results in a convergent blast wave on the equator and pole.

By 4.5 rotations of the core, the shell material near the rotation axis and equator is traveling faster than its own internal fast magnetosonic wave speed as well as that of the ambient envelope ahead of the shell. We expect the expansion speed of the shell to roughly correlate with $(v_{\text{ms}}^2 + v_v^2)^{1/2}$. At 4.5 rotations, the speed of the shell near the rotation axis roughly equals this. However, the flow speed of the shell material accelerates throughout the simulation and does not seem to reach an asymptotic velocity by the end. The simulations were stopped after 5.6 rotations of the core, since the steepening shock front began to produce numerical instabilities at that time.

The total energy (gravitational potential plus magnetic plus kinetic plus thermal) integrated over the entire simulation grid (excluding the core) increases monotonically in time and becomes positive at around 2.5 rotations. This means that there is enough energy for the entire envelope to escape from the gravitational potential well of the core. Indeed, by the end of the simulations, the shell material near the axis and equator is traveling faster than the local escape speed. On the other hand, material in the shell at midlatitudes is traveling substantially below its local escape speed. However, since this midlatitude material is still accelerating outward at the end of the simulation, and since the total energy in the system is well above zero, it appears likely that the entire envelope will escape from the core (if the simulation were able to run long enough). This confirms the general picture of Wheeler et al. (2002). In addition, the structure of the outflow suggests that, for lower $\chi$, a partial ejection of the envelope may occur (a “failed” explosion) in which their is not only a jet (as in Wheeler et al. 2002) but also an equatorial flow, leaving behind the material at midlatitudes.

Finally, the magnetic field configuration after 4.5 rotations is apparent in Figure 3, where we have exploited the axisymmetry of the system to create a three-dimensional rendering of data from the fourth simulation grid. For scale, the outer iso-density surface corresponds to the outer edge of the shell in Figure 2. A comparison between Figures 2 and 3 reveals that the “hollow” region interior to the dense shell is actually filled with magnetic energy, mostly due to the $B_\phi$ component. The shell is like a magnetically inflated “balloon,” which expands as the spinning core adds magnetic energy inside. Thus, the spin energy of the core is ultimately being transferred to the kinetic energy of the envelope (and the core should spin down as a result). The interior region is magnetically (Poynting flux) dominated, as all material there flows more slowly than the fast magnetosonic speed. The flow along the axis is a magnetic tower jet, similar to what is seen in other simulations (e.g., Kato et al. 2004), while the flow near the equatorial plane is more of a “magnetic pancake,” a feature of the rotating dipole field and an interesting new result of this study.

4. APPLICATION TO REAL STARS

Our numerical setup is simple, which allows us to understand the basic physical principles at work. The requirements for the mechanism to operate are (1) a shear layer in which the inner region is rotating at a fraction of breakup speed and (2) a magnetic field that results in an Alfvén speed comparable to the escape speed. Because we focus on the MR mechanism and ignore other physics (e.g., neutrino heating), we can scale our results to various astrophysical systems. This is instructive, as it determines what conditions would be necessary for an
Fig. 3.—Three-dimensional rendering revealing the explosion mechanism, viewed from ∼30° above the magnetic equator. The two blue surfaces are contours of constant density, each at the same density value. The dense shell of swept-up envelope material (see Fig. 2) exists between the two surfaces. Gold wires trace the magnetic field lines and illustrate that the field is most highly twisted in the low-density region interior to the shell.

MR explosion alone to drive off the envelope. Here, we briefly apply our results to the progenitor stars of SNe and PNs.

A simple model of a core-collapse SN suggests that core material conserves magnetic flux and angular momentum during collapse and naturally forms a rapidly rotating, highly magnetized proto–neutron star. We adopt a neutron star (core) mass of 1.4 $M_\odot$ and a core radius of $R_c = 10^6$ cm. If the overlying envelope contains 1 between and 1 $R_c$, our simulation corresponds to a dipole field strength at the core surface equal to $B = 3 \times 10^{13}$ G. Note that the envelope mass is comparable to the core mass, but the self-gravity of the envelope near the core should not be important, and so the simulation still applies. In our simulation, we find that the core loses rotational energy at a constant rate of $B^2 R_c^3/3 = 3 \times 10^{48}$ ergs per rotation, which would extract most of the rotational energy of the core (∼$10^{51}$ ergs) in 1 s. These numbers are consistent with previous work and should be sufficient to eject the envelope (e.g., Wheeler et al. 2002; Ardeljan et al. 2005; Blackman et al. 2006).

The evolutionary phase preceding PN formation is marked by high mass-loss rates, leading to an expansion of the stellar envelope, which overlies a proto–white dwarf. A shear layer at the core-envelope boundary is expected (Blackman et al. 2001), and as the envelope density decreases, the Alfvén speed should rise. There may be a threshold density below which the remaining envelope may be expelled via an MR explosion. We adopt a white dwarf (core) mass of 0.5 $M_\odot$ and $R_c = 10^6$ cm. At this time, the envelope is essentially an extension of the massive outflow, and if it contains 0.05 between $r = R_c$ and $10^3$ AU, our simulation corresponds to $B = 9 \times 10^6$ G. Most of the rotational energy of the core (∼$10^{48}$ ergs) should be transferred to the envelope in a few hundred years. The linear momentum of the swept-up shell will be $\sim 10^{39}$ g cm s$^{-1}$, which is consistent with the observations of proto-PNs by Bujarrabal et al. (2001).

Finally, we note that the large-scale field in our simulations is strong enough to suppress turbulence due to shear instabilities near the core, and the field therefore is neither subject to decay nor the exponential growth that is important in some other models. We have instead focused on the physics of how the field actually drives the outflow.

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