A Fast Randomized Algorithm for Finding the Maximal Common Subsequences

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ABSTRACT
Finding the common subsequences of $L$ multiple strings has many applications in the area of bioinformatics, computational linguistics, and information retrieval. A well known result states that finding a Longest Common Subsequence (LCS) for $L$ strings is NP-hard, e.g., the computational complexity is exponential in $L$. In this paper, we develop a randomized algorithm, referred to as Random-MCS, for finding a random instance of Maximal Common Subsequence (MCS) of multiple strings. A common subsequence is maximal if inserting any character into the subsequence no longer yields a common subsequence. A special case of MCS is LCS where the length is the longest. We show the complexity of our algorithm is linear in $L$, and therefore is suitable for large $L$. Furthermore, we study the occurrence probability for a single instance of MCS, and demonstrate via both theoretical and experimental studies that the longest subsequence from multiple runs of Random-MCS often yields a solution to LCS.

KEYWORDS
Longest Common Subsequence, Maximal common subsequence, randomized algorithm, string pattern discovery

1 INTRODUCTION
Data discovery and pre-processing in many data science projects often require laborious effort and creativity from the data scientist. Developing methods that can automatically generate insights from raw data is an important topic in automated machine learning [6] in order to eliminate human bottleneck and make machine learning available to non-experts. As string or text is a common form of data representation, comparing strings so that information regarding to what is common and what is unique among the strings can be extracted and summarized is an important pre-processing task.

A subsequence of a string $S$ is a character sequence that can be derived from $S$ by deleting some characters without changing the order of the remaining characters. Consider the case of $L$ strings where $L$ is large. A common subsequence of $L$ strings can be thought of as a common pattern shared by all strings. Unlike substrings, subsequences are not required to occupy consecutive positions within the original strings.

For string comparison, we consider two types of common subsequences of the $L$ strings. The Longest Common Subsequence (LCS) is a subsequence common to all the $L$ strings that has a maximal length. The Maximal Common Subsequence (MCS) is defined as maximal if and only if inserting any character into the subsequence can no longer yield a common subsequence. By definition, a LCS is a MCS with the maximal length. Furthermore, there may exist many MCSs of different lengths, and many LCSs of the same maximal length. For example, for the given two strings 'fabedc' and 'acdef', the set of MCSs are \{ f, acd, ae \} where acd is the LCS.

Finding LCS for multiple strings has important applications in many areas, including bioinformatics, computational linguistics, and information retrieval [1, 3, 18]. The problem is, however, NP-hard [15] as the number of strings $L$ becomes large. Much of the literature addresses the simple case of two or three strings [8, 10, 16]. Several methods have been proposed to improve the computation efficiency for the general case of $L$ strings, either by using parallelization [4, 14, 19] or assuming a special string structure [9]. Reviews of various methods can be found in [2, 12].

In this paper, we attack the problem of string comparison from the angle of MCS instead of LCS. The problem of finding MCS is much less studied compared to LCS. All methods from the existing literature only consider the case of two strings. For example, methods are presented by [10] to find MCS and constrained MCS. A dynamic programming approach is presented in [7] to find the shortest MCS. More recently, [17] proposes an computationally efficient way to find a MCS but his method can only find one MCS.

We develop a fast randomized algorithm to find MCS solutions of $L$ strings and show the computational complexity is linear in $L$, thus much more amenable for the analysis of a large number of strings than algorithms developed for LCS. Furthermore, as each run of our algorithm returns a random MCS and LCS is the longest MCS, we can run our algorithms multiple times and then take the longest MCS from the returned solutions to approximate LCS. We study this both theoretically and empirically. Our main contributions are summarized as follows:

- We develop a randomized algorithm, referred to as RandomMCS, for finding a random MCS solution of multiple strings.
- We extend an existing algorithm for finding MCS of two strings [17] to the case of $L$ strings.
- For a set of $L$ strings with common length $n$, we show the computational complexity of our RandomMCS algorithm is $O(n^3L)$ and our extension to the algorithm in [17], is $O(nL \log n)$, both are linear in the number of strings $L$.
- We carry out simulation studies to understand the performance of our proposed approach.
- We analyze the occurrence probability of a MCS solution returned from RandomMCS.
- We demonstrate via both theoretical analysis and experimental studies that the longest subsequence from multiple runs of our algorithm often yields a LCS.

The rest of the paper is organized as follows. In Section 2, we present the relevant background for our work. In Section 3, we propose our method and illustrate it using a toy example. In Section 4, we analyze the occurrence probability for a specific MCS and
show the computational complexity of our algorithm is linear in the number of strings \( L \). We carry out simulations to understand the performance of our algorithm empirically and present an application of our work to Automated Machine Learning (AutoML) in Section 6. We conclude and discuss future work in Section 7.

2 BACKGROUND

In this section, we shall first formally define Longest Common Subsequence (LCS) and Maximal Common Subsequence (MCS) for \( L \) strings. Then we discuss previous work on finding LCS and MCS. We shall introduce the following notations used throughout the paper. We denote the empty string by "" and denote the empty set by \( \emptyset \). To make presentation clear, we put quote "" around single characters to differentiate them from variables but sometimes omit the "" for strings with multiple characters. We use calligraphic letters to indicate sets, i.e., \( \mathcal{A}, \mathcal{M} \), etc. Throughout the paper, strings are represented using upper case letters. We use \( @ \) to represent string join, and reserve the letter \( L \) to indicate the number of strings in consideration.

2.1 Definitions

In the following, we are given a set \( \mathcal{A} \) of \( L \) strings: \( \{A_1, A_2, \ldots, A_L\} \), where each \( A_l \) is a string with \( n_l \) characters represented by \( A_l = a_{1l}a_{2l} \ldots a_{nl} \).

Definition 2.1. A sequence of characters \( C \) is a common subsequence for (strings in) \( \mathcal{A} \), if \( C \) is contained in each \( A_l, l = 1, 2, \ldots, L \) in the same character order.

To avoid confusion, we differentiate a subsequence from a substring where a substring a consecutive block of characters from a string. For a subsequence, we often concatenate its characters and use a string to represent it.

Definition 2.2. Define LCS(\( \mathcal{A} \)) as the longest common subsequence contained in each string \( A_l \) in \( \mathcal{A}, l = 1, \ldots, L \).

Definition 2.3. Define MCS(\( \mathcal{A} \)) as a subsequence contained in each string \( A_l \) in \( \mathcal{A} \) with the property such that an addition of any character to MCS(\( \mathcal{A} \)) no longer yields a common subsequence for \( \mathcal{A} \).

Example. The solution set of MCS for \( \mathcal{A} = \{\text{TEGAP}, \text{GAEP}, \text{GAEPR}\} \) is \{\( \text{GAP}, \text{EP} \}\}. Out of these two solutions, "GAP" is the LCS.

2.2 Algorithms for Finding LCS and MCS

Dynamic programming is a common technique used for finding LCS. For example, consider the LCS of two strings of length \( n \), \( X = x_1x_2 \ldots x_n \) and \( Y = y_1y_2 \ldots y_n \). If \( x_n = y_n \), then LCS(\( X, Y \)) = LCS(\( X_{n-1}, Y_{n-1} \)) @\( x_n \). If \( x_n \neq y_n \), then LCS(\( X, Y \)) = max(LCS(\( X_{n-1}, Y \)), LCS(\( X, Y_{n-1} \))) where \( X_{n-1} \) and \( Y_{n-1} \) represent the previous \( n-1 \) elements of \( X \) and \( Y \) respectively. It can be shown the complexity of using dynamic programming for finding LCS is \( O(n^2) \). For the general case of \( L \) strings, the extension of the dynamic programming algorithm will have a time complexity of \( O(n^L) \), which implies the problem is NP-hard [15]. An algorithm of a running time of \( O(r + n) \log n \) is proposed by [11] where \( r \) is the total number of ordered pairs of positions at which the two sequences match. In the worst case \( r \) can be \( O(n^2) \).

There are several proposed methods for finding MCS. It has been shown by [7] the problem of finding all shortest MCSs for \( L \) strings is NP-hard for large \( L \). All proposed algorithms focus only on two strings and no computationally effective methods have been proposed in the general case of \( L \) strings. Our algorithm targets the general case.

3 ALGORITHMS TO FIND MULTIPLE MCSS OF \( L \) STRINGS

3.1 Intuition

Our algorithm is inspired by Lemma 2 from [17] which states a necessary and sufficient condition for a subsequence \( W \) being maximal for two strings. We shall extend the lemma to the case of \( L \) strings. In the following, we denote the set of \( L \) strings of interest by \( \mathcal{A} = \{A_1, \ldots, A_L\} \).

Definition 3.1. For a string \( A \), define \( |A| \) as the number of characters in \( A \). For each \( k = 1, \ldots, |A| \), define \( A(0,k] \) as the prefix of \( A \) starting from position 1 to \( k \). Define \( A(k,|A|] \) as the suffix of \( A \) starting from position \( (k + 1) \) to \( |A| \). Define \( A(0,k] \) = "" for \( k = 0 \) and \( A(k,|A|] \) ="" for \( k = |A| \) where "" is the empty string.

Definition 3.2. Let \( W \) be a subsequence contained in string \( A \), then for any \( k = 0, \ldots, |W| \), define Middle\( (A, W, k) \) as the remaining substring obtained from \( A \) by deleting both the shortest prefix containing \( W(0,k] \) and the shortest suffix containing \( W(k,|W|] \).

Example. The following gives a simple example of this function. Middle\( (\text{TEGAP}, \text{E}', k = 0) \) is \"T" since when \( W = \text{E}' \) and \( k = 0 \), the shortest prefix in \( \text{TEGAP} \) containing \( W(0,k] \) ="", and the shortest suffix containing \( W(k,|W|] \) = \"E'\" is \"EGAP\" (this example is also shown in the first line in Cell 3 of Figure 2).

Theorem 3.3. For any common subsequence \( W \) of \( \mathcal{A} \), \( W \) is maximal if and only if for any \( 0 \leq k \leq |W| \), the set of \( L \) substrings Middle\( (A_l, W, k) \), derived from \( A_l, l = 1, \ldots, L \), are disjoint (i.e. do not share any common characters).

Proof. If \( W \) is maximal, then for each \( k = 0, 1, \ldots, |W| \), the \( L \) substrings Middle\( (A_l, W, k) \), derived from \( A_l, l = 1, \ldots, L \), have to be disjoint. This is because if this is not true, then there exists a common character \( c \) shared by the \( L \) substrings Middle\( (A_l, W, k) \), \( l = 1, \ldots, L \). Therefore, by (string) joining \( W(0,k], c \), and \( W(k,|W|] \), we can construct a longer common subsequence that contains \( W \) which contradicts the condition that \( W \) is maximal. The converse is true since it validates the condition of \( W \) being maximal.

The contra-positive of the above lemma can be stated as follows.

Theorem 3.4. For any common subsequence \( W \) of \( \mathcal{A} \), \( W \) is not maximal if and only if there exist \( k, 0 \leq k \leq |W| \) such that the set of \( L \) substrings, Middle\( (A_l, W, k) \), derived from \( A_l, l = 1, \ldots, L \) share at least one common character.

Theorems 3.3 and 3.4 are in fact the basis of our algorithm since it can be used to constructively obtain a MCS. Suppose we start \( W \) as the empty set, according to Theorem 3.4, if \( W \) is not maximal, then we can find a character that is common to the set of \( L \) strings \( \mathcal{A} \) to add to \( W \). This step can be performed iteratively until \( W \) become maximal, i.e., the set of \( L \) substrings, Middle\( (A_l, W, k) \), each from
Algorithm 1

Therefore, the updated common subsequence is the string join of the given strings. The following gives examples of this function. For the given \( \mathcal{A} = \{ \text{TEGAP, GAEP} \} \) and a subsequence \( W = 'A' \), \( \text{BreakPoints}(\mathcal{A}, W) \) will return the set \( \{0, 1\} \). This is because when \( k = 0 \), according Definition 3.2, \( \text{Middle}(\text{TEGAP}', 'A', 0) = 'E' \) and \( \text{Middle}(\text{GAEP}', 'A', 0) = 'E' \). Hence, since there is a common character 'E' shared by 'TEG' and 'GAEP', the evaluation of existence of common characters in line 4 of Algorithm 1 will succeed. Likewise, when \( k = 1 \), \( \text{Middle}(\text{TEGAP}', 'A', 1) = 'P' \) and \( \text{Middle}(\text{GAEP}', 'A', 1) = 'P' \), sharing a common character 'P'. Therefore \( \text{BreakPoints}(\mathcal{A}, W) \) will return the set \( \{0, 1\} \).

On the contrary, for \( \mathcal{A} = \{ \text{TEGAP, GAEP} \} \) and \( W = 'G' \), \( \text{BreakPoints}(\mathcal{A}, W) \) will return an empty set. This is because for each \( k = 0, 1, 2 \), \( \text{Middle}(\text{TEGAP}', 'G', k) \) and \( \text{Middle}(\text{GAEP}', 'G', k) \) do not share any common characters.

Algorithm 2 presents the pseudo-code of our algorithm for finding a random solution of MCS. The function is written in a recursive fashion and has an optional starting value of \( W \) which we shall explain further in Section 3.4. The termination condition of the algorithm is expressed in line 2 which validates \( W \) as a MCS by Theorem 3.3. Line 3-7 applies Theorem 3.4 (which states the contrapositive of Theorem 3.3) to constructively search for the possible common characters to update a previous common subsequence \( W \).

In line 5, when we randomly select a character from the common set, we can utilize the minimum frequency discussed in the example following Definition 3.5 as the optional weights. We have found via simulation studies in Section 5 that this performs better for finding the long MCSs.

Algorithm 2 A randomized algorithm, RandomMCS, to find a single MCS for \( L \) strings \( \mathcal{A} = \{ A_1, \ldots, A_L \} \)

Input: A set of strings \( \mathcal{A} \)

Optional Input: An initial starting value of \( W \) with default \( W = \emptyset \)

Output: A random MCS \( M \) of \( \mathcal{A} \)

1: \( \text{position} \leftarrow \text{BreakPoints}(\mathcal{A}, W) \)
2: if \( \text{position} = \emptyset \) return \( W \)
3: else \( k \leftarrow \text{a random element (index value) from the set position} \)
4: \( \mathcal{A}' = \{ \text{Middle}(A_i, W, k), i = 1, \ldots, L \} \)
5: \( c \leftarrow \text{a random character from the set commonChar}(\mathcal{A}') \)
6: \( W \leftarrow W(0, k] \oplus c \oplus W(k, |W|) \)
7: return \( \text{RandomLCS}(\mathcal{A}, W) \)

3.3 A Toy Example

We shall illustrate our \texttt{RandomLCS} algorithm for finding a random MCS solution using a toy example consisting of two simple strings: \{TEGAP, GAEP\}. We show two runs of the algorithm with different MCS solution output in Figure 1 and 2 respectively. The solutions are different due to the inherent randomness in the algorithm design.

Each figure consists of cells that show a certain state of the algorithm through iterations, linked by arrows illustrating the state progression. To make the presentation clear, we label each cell with an index value shown in the upper right corner of the cell. Characters in red within each cell represent the current value of the common subsequence \( W \) which will be updated through the progression to produce a final MCS solution. The small red frames around the characters indicate the prefix and suffix to be eliminated when computing \( \text{Middle}(A_i, W, k) \) for a certain \( k \) value (see Definition 3.2), i.e. \( \text{Middle}(A, W, k) \) is the remaining characters excluding the characters in the red frames. The outgoing branches from a cell represent the candidate indices \( k = 0, \ldots, |W| \) of current common subsequence \( W \), in an attempt to update \( W \) by inserting new characters (line 2 of Algorithm 1). A branch will expire if condition in line 4 of Algorithm 1 is not satisfied, that is, no common characters are found to perform the update.

In Figure 1, we want to find the MCS for the list \{TEGAP, GAEP\} shown in Cell 1. Notice that the two strings share 4 common characters 'E', 'G', 'A', 'P'. Initialize \( W = '' \). Next in Step 1, we choose one of the four characters 'P' as the first character to be
inserted in $W$, and update $W = 'P'$. We now move to Cell 2 where $|P|$ is marked red. Since the length of $|W| = |'P'| = 1$, we have two places to insert characters in $W$, $k = 0, 1$, corresponding to the two branches from Cell 2, resulting Cell 3 and Cell 4, respectively.

We will discuss Cell 4 first, which corresponds to the case of $W = 'P'$. We refer to Figure 1 for the result of the analysis of the base algorithm. The starting point. Branches from Cell 3 will finally lead to Cell 7 (corresponding to the two places to insert characters in $W$, $k = 0, 1$), sharing $W_k = 'P'$. We now move to Cell 2 where $|P|$ is marked red. Since the length of $|W| = |'P'| = 1$, we have two places to insert characters in $W$, $k = 0, 1$, corresponding to the two branches from Cell 2, resulting Cell 3 and Cell 4, respectively.

3.4 Constrained MCS

A constrained MCS is a MCS that must include a predefined subsequence $W_0$. It is in fact straightforward to modify our algorithm to obtain constrained MCS, simply by using $W_0$ as the starting value (the optional input in the pseudo-code shown in Algorithm 2). This is due to the nature of our algorithm design as it incrementally inserts a new character to update an existing common subsequence until it becomes maximal. For instance, consider the constrained MCS problem for the input string set $A = \{TEGAP, GAEP\}$ that has to contain $GP$. Using $GP$ as the optional input in Algorithm 2, the derivation process is identical to Figure 1 when Cell 3 is used as the starting point. Branches from Cell 3 will finally lead to $GP$ as the MCS output.

We comment here that [17] presented an algorithm for the constrained MCS problem in the case of two strings. However, the modification from the base algorithm used to derive a single MCS solution is significant.

4 ANALYSIS OF RandomMCS ALGORITHM

In this section, we analyze the performance of RandomMCS algorithm. First, for each MCS solution, we study the probability of the solution being returned from one run of the algorithm. We analyze LCS as a special instance of MCS and discuss the probability of a LCS being returned from the algorithm. Next, we analyze the computational complexity of our algorithm and compare it to previous approaches. As previous approaches for finding MCS only applies to two strings, we also propose an extension of a previous solution to the case of multiple strings.
4.1 Probability Analysis

As a set of strings may have many MCSs, we denote the set of MCSs as \( M \). Note that one run of our RandomMCS algorithm will yields exactly one random MCS \( M \) from the set \( M \), a natural question to ask is what is the probability value of \( M \) being returned from a single run.

**Theorem 4.1.** For a given MCS \( M \) in the solution set \( M \), the probability of \( M \) being returned as the solution from RandomMCS depends only on \( M \) and the solution set \( M \). For a given subsequence \( W_0 \), let \( M(W_0) \) be the set of MCSs that contains \( W_0 \) as a subsequence. Then for any \( M \in \mathcal{M}(W_0) \), the probability that \( M \) being returned as the solution from constrained RandomMCS depends only on \( M \) and \( M(W_0) \). This implies that the probability is conditionally independent of the set of \( L \) strings, \( \mathcal{A} \).

**Proof.** Notice that each character insertion to an existing common subsequence \( W \) (line 3-6 of Algorithm 2) is carried out by two random selections. The first is the choice of a breakpoint position \( k \) (line 3) and the second is the choice of a common character \( c \) (line 5). Both random selections depend only on the current \( W \) and the set of MCSs. Therefore, the random selection is conditionally independent of the original set of strings given \( M \). Hence the result.

**Example.** We evaluate the occurrence probability of each MCS being returned from one run of RandomMCS using examples in Figure 1 and 2 where the set of strings under consideration are \{TEGAP, GAEP\}. The solution set of MCS is \{GAP, EP\}. Starting with an empty string \( W \), notice that we have 4 common characters \{E, G, A, P\} in the beginning and all of them share the same probability 1/4 to be selected. If the first selected character is \('G' \) or \('A' \), the final MCS produced must be \('GAP' \). Likewise, the MCS is \('EP' \) when the first character selected is \('E' \). But when the first character is \('P' \), the returned solution depends on the second selected character. In this case, the choice of first two characters are \{GP, AP, EP\} and all of them have the same occurrence probability of 1/3. In total, the probability of \('GAP' \) is \( 1/4 + 1/4 + 1/4 \cdot 2/3 = 2/3 \) and that of \('EP' \) is \( 1/4 + 1/4 \cdot 1/3 = 1/3 \). In this case, we can see our algorithm favors the longer MCS (the LCS) since it has a higher probability.

**Theorem 4.2.** Let \( C \) be an upper bound of the number of unique common characters for string set \( \mathcal{A} \), i.e., \(|\text{CommonChars}(\mathcal{A})| \leq C \). If \( M \in \mathcal{M} \) is a MCS that has a distinguishing subsequence with length bounded by \( D \) and the character is selected uniformly random in line 5 of Algorithm 2, then it is easy to show that

\[
P(M) \geq C^{-D}.
\]

This implies the occurrence probability of \( M \) is bounded below.

**Proof.** Let \( S \) be a distinguishing subsequence for a MCS \( M \) with length bounded by \( D \), which implies that \( M \) is the only MCS containing \( S \). Therefore, if \( S \) is selected as the common subsequence after at most \( |D| \) character insertions to the initial empty string, then \( M \) would be returned as the output MCS from the RandomMCS algorithm. It is now clear that the probability of returning \( M \) is bounded by the probability of selecting \( S \) as the common subsequence after at most \( |D| \) character insertions into the initial empty string. If the characters are chosen uniformly, then this probability is bounded by \( C^{-D} \).

For our toy example where the string set is \{TEGAP, GAEP\} and the solution set of MCS is \{GAP, EP\}. Notice that the number of unique common characters is \( C = 4 \). In addition, either \('G' \) or \('A' \) is a distinguishing subsequence for MCS \('GAP' \), therefore, the probability of \( GAP \) is bounded by \( 2C^{-D} = 2 \cdot 4^{-3} = 1/2 \). Obviously this is a loose lower bound since we have shown before that the actual probability is 2/3.

For a specific MCS \( M \), if the occurrence probability of \( M \) is bounded below by a value \( p \), then with enough independent runs of RandomMCS algorithm we can recover \( M \) with a high probability. In fact, for an arbitrarily small \( \epsilon \), if we set

\[
T = \left\lceil \frac{\log \epsilon}{\log(1 - p)} \right\rceil,
\]

then

\[
P(M \text{ does not appear in } T \text{ runs}) \leq \epsilon.
\]

As LCS is a special case of MCS, this implies that if the condition of Theorem 4.2 holds for a LCS, then we can recover the LCS with high probability with enough runs of the algorithm. Hand-waving arguments suggest that our algorithm favors longer MCS as it will likely to contain more characters and more positions (from Algorithm 1) to be selected to \( W \). In fact in the extreme case where a MCS contains is formed by multiple occurrences of a single distinct character, it will not be returned unless the character is selected at the first time. In Section 5, we shall study empirically the occurrence probability of a MCS and correlate that with its length.

4.2 Complexity Analysis

**Theorem 4.3.** For a set of \( L \) strings \( \mathcal{A} = \{A_1, A_2, \ldots, A_L\} \), let \( n_1 \) be the string length of \( A_1, l = 1, \ldots, L \). Define \( n_0 = \min(n_1, n_2, \ldots, n_L) \) as the minimum string length, then the time complexity for one run of Algorithm RandomMCS (Algorithm 2) to find a MCS solution for \( \mathcal{A} \) is \( O(n_0^2 \sum_{i=1}^{L} n_i) \). Therefore, when all strings are of equal length \( n \), the time complexity is \( O(n^2 L) \).

**Proof.** It is easy to show that the computational complexities of BreakPoint (Algorithm 1) and commonChar (Definition 3.5) are \( O(n_0^2 \sum_{i=1}^{L} n_i) \) and \( O(n_0 L) \), respectively. The algorithm RandomMCS may replicate BreakPoint evaluations at most \( n_0 \) times. Hence the result.

The above theorem states that the time complexity of our algorithm is linear in the number of strings \( L \), as opposed to exponential in \( L \) for algorithms to find LCS. It is therefore much more amenable for the case of large number of strings.

4.3 Comparison with Previous Approaches

We compare our approach to previous approaches for finding MCSs.

4.3.1 Extension of MCS Calculation. All previous approaches for finding MCSs are developed for the case of two strings [7, 10, 17]. The recent algorithm in [17] can be extended to the case of multiple strings in the following manner. The original algorithm maintains a sequence of index pairs that tracks the matches between two
strings. We extend their technique and maintain a sequence of L-tuple indices that tracks the matches between the L strings. These L-tuple indices break the original strings into blocks, where additions to the sequence of L-tuples are searched within the matched blocks. We present the pseudo code in the appendix.

4.3.2 Computational Complexity Comparisons. For two strings with equal length n, [17] has the highest efficiency among all proposed algorithms for finding a MCS for two strings. The complexity is $O(n \log n)$. Our extension to the case of L strings (see appendix) also enjoys the highest efficiency with a complexity $O(Ln \log n)$. However, since the algorithm maintains a certain order when traversing the strings, it can only find one MCSs (or two MCSs if we reverse the order of strings), which may not be desirable when there are multiple MCSs. [7] focuses on finding MCS first, and obtain all MCSs and the LCS for two strings with length $m$ and $n$ with a complexity $O(mn(m + n))$. [10] developed an algorithm for the constrained LCS for two strings with lengths $m$ and $n$ and a complexity $O(mn)$. [11] provides an algorithm to compute the LCS for 2 strings in the complexity $O(n \log(n))$, but it is only for the special best case scenario with a short LCS. The following table summarizes the computational complexities of these different methods.

| Algorithm           | Target                | Complexity     |
|---------------------|-----------------------|----------------|
| RandomMCS           | MCS, L strings        | $L^n$          |
| Our extension to Sakai (2019) | MCS, L strings    | $O(Ln \log n)$ |
| Sakai (2019) [17]   | MCS, 2 strings        | $O(n \log n)$  |
| Fraser & Irving (1995) [7] | MCSs, 2 strings     | $mn(m + n)$    |
| Hirschberg (1975) [10] | CLCS, 2 strings   | $mn$           |
| Hunt & Szymanski (1977) [11] | LCS, 2 strings | $\geq O(n \log(n))$ |

| Table 1: Comparison of Computational Complexity (CLCS stands for constrained LCS) |

5 SIMULATION STUDY

In this section, we perform simulation studies to understand the performance of our RandomMCS algorithm. First, we would like to understand empirically if the longest MCS from multiple runs of RandomMCS would yield a solution to LCS. Second, we study empirically the computational complexity of our algorithm.

5.1 Less than 5 Strings

In this setting, our simulations are run with the number strings varies from 2 to 4, with string lengths ranging from 20 to 50. We also vary the alphabet size from 5 to 100. For this experiment, we use the basic dynamic programming method to compute LCS, and run our RandomMCS algorithm 1000 times to select the longest one and compare the result with the real LCS. The reason that we stop at 4 strings is due to the explosion of the computational time used for finding LCS using dynamic programming when the number of strings exceeds 5.

To simplify the evaluation, strings are generated using random characters from the alphabet. We also consider two kinds of randomization when implementing RandomMCS. For the first kind, when we randomly insert a character into a common sequence (line 5 of Algorithm 2), we uniformly choose the character from the common set. For the second kind, we use frequency weighting to select the character with a weight that is proportional to the (least) number of times the character appears in each string.

Some sample results are described as follows. For $L = 4$ random strings each with length $n = 50$ from an alphabet of size $B = 6$, both LCS algorithm and the longest MCS solution from 1000 iterations of RandomMCS yield the same string with length 15. The longest MCS solution took 3sec, and the LCS solution takes 8sec. For $L = 4, n = 50$ and alphabet size $B = 50$, longest MCS from our algorithm also yields the same result as the real LCS. In fact, we have not encountered a case where they disagree. Furthermore, the 1000 repetitions are unnecessary for finding LCS using our algorithm as the real LCS tends to have a high occurrence probability being returned (close to 40%) in many instances. Finally, we do not find significant differences in the performance between the two types of random selection.

5.2 A Large Number of Strings

When the number of strings $L$ gets large, existing algorithms for finding LCS fails to work well due to the high computational time. We use the following approach to evaluate our algorithm in this instance. Our simulation is designed in such a way that finding the longest common subsequence is challenging.

Our simulation generates $L = 1000$ strings of length 60 in the following manner. First, we generate 4 common subsequences that are contained in each of the 1000 strings: $S_1, S_2, S_3, S_4$ with increasing lengths 3, 6, 9, and 12 respectively, from an alphabet size of 15. Next, we insert these subsequences into a string of 60 characters in the following way. First we randomly pick 3 indices to situate $S_1$, then we randomly pick 6 indices to situate $S_2$ from the remaining 57 indices, then we randomly pick 9 indices to situate $S_3$ from the remaining 51 indices, and finally we randomly pick 12 indices to situate $S_4$ from the remaining 42 indices. This way all the subsequences $S_1, S_2, S_3, S_4$ will be intermingled in each string which makes the problem of finding LCS challenging. Notice that the total number of characters in $S_1, S_2, S_3, S_4$ is 30. In the last step of the string generation, we insert 30 random characters into the remaining 30 slots, with an expanded alphabet size of 30 (which includes the original alphabet set of size 15 for $S_1, S_2, S_3, S_4$).

For two random strings with a common length $n$ where characters are randomly generated from an alphabet, let the expected length of their LCS be $e$. It has been shown that $\lim_{n \to \infty} e/n < 1$ [5, 13]. Therefore it is easy to conclude that the expected length of LCS of $L$ such random strings will decrease to 0 exponentially fast with $L$. Since in the last step where we generated 30 completely random characters, with a large $L$, we expect the common subsequence from these 30 random characters will be negligible (or empty). Therefore, by design, we expect the long subsequences in $S_1, S_2, S_3, S_4$ will remain as MCS and $S_4$ will be LCS since it is the longest.

The result of our simulation is as follows. With 200 runs of RandomMCS, the empirical estimate of the occurrence probabilities for each $S_i, i = 1, \ldots, 4$, is: 0.27 for $S_4$, 0.23 for $S_3$, 0.11 for $S_2$ and a zero probability value for $S_1$. The reason that $S_1$ is no longer a MCS...
is due to the intermingling of \( S_1, S_2, S_3, S_4 \) among themselves during the process of situting \( S_1, S_2, S_3, S_4 \), as the mixing creates spurious common subsequences and \( S_1 \) is short enough to be absorbed by other MCS solutions. In fact, it is absorbed in one of returned MCS solutions with length 4 (so an extra character was included) and a probability value of 2%. The intermingling also creates other MCS solutions which accounts for the remaining 38% of the returned MCS solutions with lengths ranging from 4 to 11. We also varied the alphabet size in the experiment, and found that the intermingling will decrease with larger alphabet size and therefore it would be easier to locate LCS.

To understand the impact of frequency weighting in the random character selection (line 5 of Algorithm 2) and the number of characters in the long common sequence on the performance of RandomMCS algorithm, we perform the following 2 by 2 experiments. We have two settings for the weights: uniform or frequency based; and two configuration for \( S_4 \) (the longest common subsequence with length 12): a single alphabet and the original 8 distinct alphabets generated by random. The following table shows the occurrence probabilities of \( S_4 \) in the returned 200 MCS solutions. It is clear when \( S_4 \) is made of all identical characters (i.e., alphabet of size 1), there is a significant drop in the probability of locating the LCS. Nonetheless, random character selection using frequency-weighting performs a lot better. The uniform weights fails to discover \( S_4 \), and the longest returned MCS has a length 9. This is because in the case of uniform weights, the unique alphabets in the long LCS is one of the many to be selected at random with no frequency weighting and this character is shared by many other MCSs.

We also observe that time to run RandomMCS 200 times is about 150sec for \( L = 1000 \) in our experiment, which is about 50 times for \( L = 4 \) and 1000 runs (recall the latter instance took about 3sec). This is in agreement with our theoretical analysis of RandomMCS which shows a time complexity linear in \( L \).

| single alphabet \( S_4 \) | uniform weights | frequency-based weights |
|--------------------------|----------------|------------------------|
| 0%                       | 5%             |
| 28%                      | 27%            |

Table 2: Occurrence probabilities for \( S_4 \), the longest common subsequence by design

Our empirical results indicate that LCS typically has a non-negligible occurrence probability among all solutions of MCS and thus will very likely be found by running RandomLCS repeatedly. However, the performance depends on the nature of LCS and how random search is carried out in the algorithm.

### 6 Applications to Auto Machine Learning

In this section, we illustrate how methods we developed for finding MCSs can be applied to string pre-processing. Developing automated methods for data pre-processing is an important topic in automated machine learning, or AutoML, where the objective is to automate the end-to-end process of applying machine learning to real-world problems [6]. We demonstrate how our method can be used to develop a good understanding of string columns in tabular data, and extract important features for downstream machine learning tasks.

#### 6.1 Data Understanding

Tabular data is a common form of data representation. It is organized by rows and columns where rows represent individual records and columns are the associated attributes. For large data tables with many rows and columns, it is difficult to obtain a good understanding of the data content without laborious manual examination. For columns with string values, we can apply our methods to understand the patterns that are common across all column values and extract important information or features for downstream machine learning.

The dataset we use for demonstration contains broadband home router data records of customers from a network carrier during a 30-day period. It consists of 27 columns and 238330 rows, where columns are device ID and type, associated network node and type, the customer information, and time series of several KPIs. Among the 27 columns, there are 8 columns are either strings or DateTime. For each of these columns, we apply our algorithm to uncover the longest common subsequence from 100 runs of RandomMCS algorithm. The resulting patterns are shown in Table 3, where we post-processed these common subsequences and represented them in the form of regular expressions where \( * \) (the asteroid sign) indicates any number of characters. As a result, the contents in the string columns become much more apparent with this information.

| Colname       | Pattern                                      |
|---------------|----------------------------------------------|
| network.type  | 2"CN*                                        |
| software.version | *                                              |
| day           | 2015-12-*                                    |
| customer.attr1 | *                                              |
| pop.location  | POP-*                                        |
| linecard.id   | 2"CN*"-"-"                                  |
| sid           | BB*                                          |
| device.id     | "0"-Home Hub '0 Type "-*"                    |

Table 3: String pattern discovered using our algorithm

#### 6.2 Feature Extraction

We can often use the extracted column string patterns in tabular data to engineer new features.

It is clear that from Table 3 that some columns have a clear pattern while others do not. For example, both software.version and customer.attr1 do not have a common pattern. On the other hand, the column of device.id shows a clear pattern where it can be represented by the string join of 6 sub-fields, each is a combination of some common characteris shared across the values and a varying substring indicated by asteriod (*). These subfields can be extracted to represent possibly more informative features for characterizing the device.id. This feature extraction step can be automated once patterns are found and the extracted features can be used for downstream machine learning. In fact, our methods can also be applied...
to auto-detect field separators from an ASCII file and then extract the columns.

7 CONCLUSION AND FUTURE WORK

In this paper, we develop a randomized algorithm, referred to as Random-MCS for finding the maximal common subsequence (MCS) of multiple strings. We show the complexity of our algorithm is linear in the number of strings L. Furthermore, we demonstrate via both theoretical and experimental studies that the longest subsequence from multiple runs of Random-MCS often yields a solution to LCS. As for future work, we want to improve the probability bound for a single MCS solution and extend our algorithm to the case when the set of strings is polluted with dirty data.

A EXTENSION OF ALGORITHM 1 IN SAKAI (2019) [17] TO THE CASE OF L STRINGS

The $I^*(A, c, i)$ denotes the least index such that c does not appear in $A$, $I^*(A, c, i)$ denotes the greatest index such that c does not appear in $A$. The idea is to cut the strings into segments backward and determine the MCS forward. The index sets $idxP$ and $idxR$ mean the previous indices and rear indices. For example, the indices $idxP[j]$ and $idxR[j]$ determine a segment of the j string. So $idxP$ and $idxR$ cut a segment from every string. The Algorithm 4 Common will return −1 if no common character exists in all the L segments and return c and j if the common char c appears in the j string first.

The Algorithm 3 OneMCS finds a specific MCS for L strings in the complexity $O(n\log(n))$. Inspired by [17], we extend the algorithm from 2 strings to L strings. If it is hard to understand the algorithm OneMCS, please read [17] first.

REFERENCES

[1] TK Attwood and JBC Findlay. 1994. Fingerprinting G-protein-coupled receptors. Protein Engineering, Design and Selection 7, 2 (1994), 195–203.
[2] Lasse Bergroth, Harri Hakonen, and Timo Raita. 2000. A survey of longest common subsequences algorithms. In Proceedings Seventh International Symposium on String Processing and Information Retrieval. SPIRE 2000. IEEE, 39–48.
[3] Guillaume Bourque and Pavel A Pevzner. 2002. Genome-scale evolution: reconstructing gene orders in the ancestral species. Genome research 12, 1 (2002), 26–36.
[4] Yixin Chen, Andrew Wan, and Wei Liu. 2006. A fast parallel algorithm for finding the longest common sequence of multiple biosequences. BMC bioinformatics 7, 4 (2006), 54.
[5] Václav Chvatal and David Sankoff. 1975. Longest common subsequences of two random sequences. Journal of Applied Probability 12, 2 (1975), 360–315.
[6] Matthias Feurer, Aaron Klein, Katharina Eggensperger, Jost Springenberg, Manuel Hutter, and Frank Hutter. 2015. Efficient and robust automated machine learning. In Advances in neural information processing systems. 2962–2970.
[7] Campbell B. Fraser and Robert W. Irving. 1995. Approximation Algorithms for the Shortest Common Supersequence. Nordic J. of Computing 2, 3 (Sept. 1995), 303–325. http://dl.acm.org/citation.cfm?id=642129.642130
[8] Koji Hakata and Hiroshi Imai. 1992. Algorithms for the longest common subsequence problem. Genome Informatics 3 (1992), 53–56.
[9] Koji Hakata and Hiroshi Imai. 1998. Algorithms for the longest common subsequence problem for multiple strings based on geometric maxima. Optimization Methods and Software 10, 2 (1998), 233–260.
[10] Daniel S Hirschberg. 1975. A linear space algorithm for computing maximal common subsequences. Commun. ACM 18, 6 (1975), 341–343.
[11] James W Hunt and Thomas G Szymanski. 1977. A fast algorithm for computing longest common subsequences. Commun. ACM 20, 5 (1977), 350–355.
[12] G. Kawade, S. Sahu, S. Upadhye, N. Korde, and M. Motghare. 2017. An analysis on computation of longest common subsequence algorithm. In 2017 International Conference on Intelligent Sustainable Systems (ICISS). 982–987. https://doi.org/10.1109/ISSI.2017.8389525
[13] Marcos Kiwi, Martin Loebl, and Jiří Matoušek. 2005. Expected length of the longest common subsequence for large alphabets. Advances in Mathematics 197, 2 (2005), 480–498.
[14] Dmitry Korkin, Qingguo Wang, and Yi Shang. 2008. An efficient parallel algorithm for the multiple longest common subsequence (mLcs) problem. In 2008 57th International Conference on Parallel Processing. IEEE, 354–363.

Algorithm 3 OneMCS returns a single solution for MCS

Input: A List of String List
Output: Single MCS W
1: Initialize $W = \{A, S\}, W_p$ and $W_r$ are vectors with length $L$, $k = 0$.
2: for $i$ in 1 : n
3:     $W[i] = \{0, |A_i| - 2\}$
4:     while $k < |W|-1$
5:         for $i$ in 1 : n
6:             $W_p[i] = W[i][k], W_r[i] = W[i][k + 1]$
7:         while common($A, W_p, W_r$) == -1
8:             for $j$ in 1 : $|W_r|$
9:                 $W[j][k + 1] = W[j] - 1, W_r[j] = W[j][k + 1]$
10:        first = 0
11:       first = 1 if $\exists j$ such that $W_r[j] == W_p[j]$
12:       if first == 1
13:           for $j$ in 1 : n
14:               $W[j][k + 1] = I^-W[j], W[k + 1], W_p[j] + 1$
15:       $k = k + 1$
16: else
17:     idx, c = common($A, W_p, W_r$)
18:     $W=W[1, k] \oplus c @ W[k + 1, |W|]$
19: for $j$ in 1 : n
20:     if $j == idx$
21:         $W[j] = W[j][k] \oplus W_r[j] - 1 \oplus W[j][k + 1, :]$
22:     else
23:         $W[j] = W[j][k] \oplus I^-(W[j], c, W_r[j]) - 1 \oplus W[j][k + 1, :]$
24: return $W$

Algorithm 4 Common returns the common character and its index

Input: A List of String $A = \{A_1, ..., A_L\}$, Previous Index idxP, Rear Index idxR
Output: The list of indices
1: for $j$ in 1 : L:
2:     if idP[j] $\geq$ idR[j]
3:         return idP[j], idxR[j]
4:     for $j$ in 1 : L:
5:         c = $A_i[idxR[j]]$
6:     for $i$ in 1 : L:
7:         if $i == j$
8:             continue
9:     for $j$ in 1 : L:
10:        if $i == j$
11:            break
12:       if $i == L - 1$
13:           return $(j, c)$
14:    if $i == L - 2$ and $i == L - 2$
15:    return $(j, c)$
[15] David Maier. 1978. The complexity of some problems on subsequences and supersequences. *Journal of the ACM (JACM)* 25, 2 (1978), 322–336.

[16] William J Masek and Michael S Paterson. 1980. A faster algorithm computing string edit distances. *Journal of Computer and System sciences* 20, 1 (1980), 18–31.

[17] Yoshifumi Sakai. 2019. Maximal common subsequence algorithms. *Theoretical Computer Science* (2019).

[18] Alexey Sorokin. 2016. Using longest common subsequence and character models to predict word forms. In *Proceedings of the 14th SIGMORPHON Workshop on Computational Research in Phonetics, Phonology, and Morphology*. 54–61.

[19] Qingguo Wang, Dmitry Korkin, and Yi Shang. 2010. A fast multiple longest common subsequence (MLCS) algorithm. *IEEE Transactions on Knowledge and Data Engineering* 23, 3 (2010), 321–334.