Study on failure mechanism of tunnel-type anchorage using discontinuous deformation analysis method

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Abstract: Tunnel-type anchorage are increasingly used in the construction of long-span suspension bridges. Numerical methods are a major means to study their bearing characteristics and failure modes. The current mainstream research method is still based on continuum mechanics, which cannot simulate the whole failure evolution process of tunnel-type anchorage-surrounding rock system. Therefore, discontinuous deformation analysis method based on discontinuous medium mechanics was used to carry out related research, comprehensively considering the influence of different rock mass quality and faults. The results show that: 1) In the case of soft rock and hard rock, the bearing capacity of the tunnel-type anchorage increases with the increase of the clamping angle, and the change in hard rock is more obvious. 2) Under the condition of soft rock, the envelope area of resistance body increases linearly with the increase of clamping angle. Because of gravity, the envelope area of the resistance body on the lower side of the tunnel-type anchorage is larger than that on the upper side. Under the four clamping angles of 2°, 4°, 6° and 8°, the envelope curves on the upper and lower sides of the tunnel-type anchorage are parallel to each other on their own sides. 3) Compared with the soft rock, the envelope areas of hard rock under the above four clamping angles are significantly increased, and the growth multiple is between 0.44 and 0.87. But in this case, the envelope area has no obvious change with the increase of clamping angle. 4) In soft rock, the envelope shape of the resistance body is a curve that is convex toward the outside of the anchorage; when it is hard rock, it tends to be a curve that is concave toward the inside of the anchorage. 5) Under the condition of hard rock, the integral deformation of surrounding rock on both sides of the tunnel-type anchorage is more obvious, which means that the load transfer range is wider and the clamping effect is more obvious. 6) The ultimate bearing capacity increases with the increase of the clamping angle, and shows a linear change trend. 7) Faults significantly weaken the bearing capacity of tunnel-type anchorage, but the impact of faults at different angles is slightly different. Finally, the whole process of deformation and failure of surrounding rock is intuitively displayed, and the load transfer mechanism can be peeped from it.

1. Introductions
The transportation construction in the western region is the key to the smooth implementation of the national strategy of the western development. There are many high mountains and canyons in the western region, which is a big test for the layout of local traffic routes. Suspension bridge is famous for its outstanding crossing ability, which is an excellent choice
for crossing high mountains and valleys. The suspension bridge transfers the huge bridge load
to the anchorage through the main cable, and the anchorage and rock jointly bear the load of
the main cable. Therefore, anchorage is the core structure of suspension bridge. Anchorage is
generally divided into two types, namely gravity-type anchorage and tunnel-type anchorage.
The structure of tunnel-type anchorage is more complex, which is a wedge structure with small
front end and large back end. Compared with gravity-type anchorage, it has the advantages of
environmental protection, small earthwork excavation and saving investment, so it is more and
more favored by engineering construction and design units. But the bearing mechanism of
tunnel-type anchorage is not clear, and a set of scientific design method and theory has not been
formed.

The bearing capacity and failure mode of tunnel-type anchorage are closely related to its
occurrence environment, which involves a lot of rock mechanics and engineering geological
problems. Therefore, the research on tunnel-type anchorage is also carried out from the aspects
of theoretical analysis, field test, indoor test and numerical simulation.

In terms of theoretical analysis, 'Code for design of highway suspension bridge' points out
that stress analysis of spatial structure should be carried out for tunnel-type anchorage, and the
compressive strength of concrete and tunnel wall and the anti-pulling force of anchorage should
be checked [1]. However, the formula for calculating the anti-pulling force is not given clearly.
The calculation model of gravity-type anchorage against sliding is often applied to the tunnel-
type anchorage. However, it ignores the clamping effect of the tunnel-type anchorage and
suddenly transforms into a th-pulling capacity, which makes the design too conservative.
The so-called clamping effect means that the special wedge shape of the tunnel-type anchorage
will force a larger range of surrounding rock to participate in the struggle against the pulling-out
load compared with the constant section tunnel-type anchorage [2]. Under the assumed
failure mode, scholars have derived the calculation formula of the ultimate bearing capacity of
tunnel-type anchorage based on the limit equilibrium theory [3-5].

In the aspect of scale model test, 1:50 scale tunnel-type anchorage model test was carried
out for Humen Bridge in Guangdong Province. The surrounding rock falls into the plastic state
when the design load is 4.8 times, so the design scheme of tunnel-type anchorage is not adopted
at last [6]. The 1:12.5 scale model test of Egongyan bridge was carried out, and the steel strand
broke when the load reached 4.6P, but it can still be confirmed that its anti-pulling capacity met
the design requirements [7]. The 1:30 model test of Baling River Suspension Bridge was carried
out, and the deformation of surrounding rock showed an inclined inverted plug shape, and the
displacement of the front-end face of the anchor body decreased approximately linearly [8]. A 1:25 model test was carried out on the Pulit bridge. Under the pulling-out force of 1P ~ 8p, the anchorage-rock system is in the stage of elastic deformation.
The rheological test was carried out under 6P, and no rheological deformation was found at
each measured point. When the maximum load is increased to 50P, the anchorage-rock system
is still not damaged [2]. In the tunnel-type anchorage project of Pulit bridge, the comparative
experimental study of cylindrical anchorage and circular truncated cone anchorage was carried
out [9]. The results show that the deformation range and ultimate load of the latter are obviously
larger than the former, proving the existence of clamping effect. In the above model tests, except
for the test in reference [9], the whole process test of elastic-plastic-ultimate failure of tunnel-
type anchorage model has not been realized, and there is still room for improvement in test
technology. By means of model test, it is still unable to clearly understand the stress distribution
and deformation characteristics, especially the bearing capacity and failure mode of tunnel-type
anchorage under the condition of different lithology and structural characteristics.

In the numerical simulation of tunnel-type anchorage, finite difference or finite element
commercial software is often used to study the bearing characteristics and stress deformation
response [10-12]. Based on the Cvisc model of FLAC3D, the rheological behavior of tunnel-
type anchorage is simulated and its long-term stability is evaluated [13-14]. A comprehensive
study on rock mechanics of tunnel-type anchorage of Siduhe suspension bridge was carried out.
Based on the observational data of scale model test, the macro mechanical parameters are
inversed and applied to the numerical simulation. Finally, the simulation results and monitoring
results are compared and analyzed [15]. Many scholars combine numerical simulation with
field scale model test to study the bearing characteristics and failure law of tunnel-type anchorage under external load. The two are complementary and verifiable to each other, and a lot of meaningful conclusions have been drawn [16-22]. The above studies are all based on the numerical analysis method of continuum mechanics, which is obviously difficult to simulate the whole process of fracture initiation, development, penetration and ultimate failure of tunnel-type anchorage.

Discontinuous deformation analysis (DDA) is a numerical analysis method based on discontinuous medium mechanics proposed by Shi [23]. Since it was proposed, great achievements have been made, such as explicit DDA [24-25], hydraulic coupling [26], lasting simulation [27], seismic dynamic response analysis [28] and crack propagation [29-30].

DDA adopts a rigorous open-close iteration algorithm for the contact treatment, which can accurately simulate the contact state between anchorage and rock mass. It allows large displacement and rotation of the block, and can simulate the whole process of deformation and failure of the anchorage-rock system driven by the main cable load. At the same time, it can accurately capture the main control effect of rock mass structure on the stability of anchor system. Therefore, based on DDA method, the numerical test research on the bearing characteristics and deformation and failure law of tunnel-type anchorage will be carried out, and the influence of different rock mass quality and fault factors will be comprehensively considered.

2. The basic principle of DDA

DDA defines the displacement function on the block, and the displacement of any point \((x, y)\) [31] can be expressed as

\[
\begin{bmatrix}
    u \\
    v
\end{bmatrix} = [T_i][D_i] = \begin{bmatrix}
    t_{i1} & t_{i2} & t_{i3} & \cdots & t_{in} \\
    t_{i2} & t_{i3} & t_{i4} & \cdots & t_{in} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    t_{in} & \cdots & t_{in} & \cdots & t_{in}
\end{bmatrix} \begin{bmatrix}
    d_1 \\
    d_2 \\
    \vdots \\
    d_n
\end{bmatrix}
\]

(1)

Where, \([T_i]\) is the displacement basis function of the \(i\)th block, and \([D_i]\) is the degree of freedom.

Assuming that there are \(n\) blocks in the block system, the following equilibrium equation can be established.

\[
\begin{bmatrix}
    K_{11} & K_{12} & K_{13} & \cdots & K_{1n} \\
    K_{21} & K_{22} & K_{23} & \cdots & K_{2n} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    K_{n1} & K_{n2} & K_{n3} & \cdots & K_{nn}
\end{bmatrix} \begin{bmatrix}
    D_1 \\
    D_2 \\
    \vdots \\
    D_n
\end{bmatrix} = \begin{bmatrix}
    F_1 \\
    F_2 \\
    \vdots \\
    F_n
\end{bmatrix}
\]

(2)

Where, \([K_i]\) is the stiffness matrix, \([D_i]\) and \([F_i]\) are the degree of freedom vector and load vector of block \(i\). \([K_i]\) depends on the material properties and local displacement function of block \(i\), \([K_{ij}](i \neq j)\) depends on the contact between blocks \(i\) and \(j\).

3. Failure mode and bearing characteristics of tunnel-type anchorage

The 1:10 scaled tunnel-type anchorage model as shown in Figure 1 is used to carry out the research based on DDA. The length of the tunnel-type anchorage is 6m, and its clamping angle is \(\alpha\) which takes the values as 2°, 4°, 6° and 8°. The same as the in-situ scale model test, the load is applied by the backward pushing method, that is, the axial thrust is applied on the back-
end face of the tunnel-type anchorage. As shown in figure 1, a fault with an angle of 20° to the longitudinal axis of the tunnel-type anchorage is set to study the influence of different angles on the failure mode and bearing characteristics when the fault exists. As shown in figure 2, the clamping angle is fixed to 6° and three faults with angles 8°, 21° and 34° are set up to study the influence of different fault angles on the bearing characteristics and failure mode of tunnel-type anchorage. The DDA model with clamping angle of 6° is given, as shown in figure 3, which is divided into 2294 blocks.

![Figure 1. Geological model of tunnel-type anchorage](image1.png)

![Figure 2. Faults with different angles](image2.png)

![Figure 3. DDA model of tunnel-type anchorage-surrounding rock system](image3.png)

### Table 1. Mechanical parameters used in calculation

|                | Elastic modulus $E$ (GPa) | Poisson's ratio $\mu$ | Friction angle $\varphi$ (°) | Cohesion $c$ (MPa) | Tensile strength $R_t$ (MPa) |
|----------------|---------------------------|-----------------------|-----------------------------|-------------------|-----------------------------|
| Anchorage      | 32.50                     | 0.28                  | —                           | —                 | —                           |
| Soft rock      | 0.80                      | 0.33                  | 33.00                       | 0.30              | 0.20                        |
| Hard rock      | 24.0                      | 0.22                  | 55.00                       | 1.80              | 1.50                        |
| Interface      | —                         | —                     | 50.20                       | 0.58              | 0.20                        |

### 3.1. Failure mode and bearing characteristics of tunnel-type anchorage under different rock mass quality

Next, the bearing characteristics of tunnel-type anchorage will be studied according to the two different lithology of soft rock and hard rock. It mainly includes the envelope range of resistance body, ultimate load and failure evolution process.

#### 3.1.1. The change of envelope range of resistance body of surrounding rock under soft rock condition

![Figure 4. Envelope range of resistance body](image4.png)

(a) 2°  
(b) 4°
The change of the envelope range of the resistance body with four different clamping angles of 2°, 4°, 6° and 8° is shown in figure 4. The envelope range here refers to that represented by the resistance body of surrounding rock whose x-direction displacement is greater than 0.015 m when the x-direction displacement of the tunnel-type anchorage reaches 0.16 m. All the envelope ranges involved in the paper are based on this standard. Obviously, the envelope range of the resistance body increases with the increase of the clamping angle, and their overall shapes are similar. The envelope area of resistance body increases linearly with the increase of clamping angle. Because of gravity, the envelope area on the lower side of the tunnel-type anchorage is larger than that on the upper side as shown in figure 5. According to the envelope range, the envelope curves of the upper and lower sides of the tunnel-type anchorage are given, as shown in figure 6. Under the four clamping angles, the envelope curves on the upper and lower sides of the tunnel-type anchorage are parallel to each other on their own sides. The overall shape of the envelope curve is convex towards the outside of the anchor body, which is just opposite to that of the tunnel-type anchorage.

As shown in figure 7, it is the ultimate load when the clamping angle is 0° to 8° and the ultimate load increases with the increase of the clamping angle, basically showing a linear change.

**Figure 4.** Variation of envelope range of rock resistance body with clamping angle

**Figure 5.** The change of envelope area with clamping angle

**Figure 6.** Variation of envelope curves with clamping angle
3.1.2. The whole evolution process of the envelope range of resistance body in soft rock

As shown in Figure 8, it is the whole evolution process of envelope range at 6 ° clamping angle. At time step 300, the surrounding rock at the back end of the lower side of the tunnel-type anchorage deforms first. When the loading continues to time step 350, the surrounding rock at the back end of the upper side of the anchorage also deforms. At time step 450, the rock mass deformation continues, and the rock mass deformation penetrates to the free boundary at time step 550. At time step 850, a stable deformation form of surrounding rock is formed. At time step 1400, due to the self-weight of rock mass, some surrounding rocks under the tunnel-type anchorage still participate in the resistance to the clamping effect. However, the obvious failure boundary of surrounding rock has been formed at this time, and this transfer effect will not continue, and it will be stable after small residual deformation.

3.1.3. The whole evolution process of the envelope range of resistance body in hard rock

As shown in Figure 9, it is the change of envelope range of resistance body under four 4 clamping angles for class II surrounding rock. Compared with the soft rock, the envelope areas of hard rock under the above four clamping angles are significantly increased, and the growth multiple is between 0.44 and 0.87. But in this case, the envelope area has no obvious change with the increase of clamping angle, as shown in Figure 10. Under the condition of hard rock, the integral deformation of surrounding rock on both sides of the tunnel-type anchorage is more
obvious, which means that the load transfer range is wider and the clamping effect is more obvious. On the contrary to the case of soft rock, the envelope boundary tends to be a trumpet shape concave to the side of tunnel-type anchorage.

Figure 9. Envelope range of different clamping angles in hard rock

Figure 10. Comparison of envelope area under different rock mass quality conditions

Figure 11. Load-displacement curve in both hard rock and soft rock

As shown in figure 11, it is the comparison of load-displacement curves corresponding to different clamping angles in both hard rock and soft rock. Under the same anchorage displacement, the corresponding load suffered by tunnel-type anchorage in hard rock is generally greater than that in soft rock, which indicates that the clamping effect of hard rock is better than that of soft rock. For the same lithology, the larger the clamping angle is, the stronger
the clamping effect is. Therefore, under the combination of hard rock and large clamping angle, the bearing capacity of tunnel-type anchorage is the best.

3.2. Variation of envelope range of resistance body of surrounding rock with fault

![Figure 12](image1.png)

Figure 12. Envelope range of surrounding rock with faults under different clamping angles

As shown in figure 12, it shows the change of the envelope range of the resistance body under the four kinds of clamping angles when there are faults. Generally, it shows an increasing trend with the increase of the clamping angle. When the clamping angle is 2°, the envelope range of the resistance body mainly surrounds the two sides of the anchor body, but does not extend to the fault boundary. Under the latter three clamping angles, obvious displacement discontinuity boundary is formed at the fault. The clamping effect is more obvious at the angle of 8°, because there is a deep deformation trend near the fault. Fault provides natural failure boundary for surrounding rock deformation, which will weaken the clamping effect of intact rock mass and reduce the safety of tunnel-type anchorage. Therefore, this kind of harmful fault should be avoided as far as possible in the process of determining tunnel-type anchorage location.

![Figure 13](image2.png)

Figure 13. Load-displacement curve under the same fault
The bearing characteristics of different clamping angles under the fixed fault is shown in figure 13. When the fault is contained, the bearing capacity with large clamping angle is also better, which is consistent with the previous conclusion. Similarly, the case without fault is better than that with fault.

3.3. Influence of different fault angles on envelope range

![Figure 14. Evolution process of the envelope range when the angle of fault is 8°](image)

Taking the clamping angle of 6° as an example, the influence of different fault angles on the clamping effect is studied. As shown in figure 14, it is the evolution process of the envelope range of the resistance body when the angle between the fault and the central axis of the tunnel-type anchorage is 8°. At time step 310, the deformation started from the lower side of the back end of the tunnel-type anchorage. At time step 500, the deformation of the tunnel-type anchorage rapidly advanced along the fault to the free face, and the surrounding rock on the upper side of the tunnel-type anchorage and near the fault also began to deform. At time step 570, the deformation of the upper side also advanced to the free face. As the load continues to increase, the deformation continues to expand to the depth of the rock mass. Figure 14 (d) shows the deformation at time step 777. Because of the small angle between the fault and tunnel-type anchorage, the range of resistance between them is not enough to resist the displacement of the tunnel-type anchorage. Therefore, more surrounding rock masses are forced to participate in resisting the deformation caused by the tunnel-type anchorage.
Figure 15. Evolution process of the envelope range when the angle of fault is 21°.

Figure 16. Evolution process of the envelope range when the angle of fault is 34°.
The evolution process of envelope range is shown in Figure 15 when the fault angle is 21°. At time step 520, the surrounding rock mass on the lower side forms a clear failure boundary along the fault, and the weakening effect of fault on the integrity of surrounding rock mass is obvious. When the clamping effect along the weak discontinuity is weakened, the constraint of the upper rock mass must be overcome for further deformation. With the completion of the deformation of the upper surrounding rock, the deformation will further extend towards the surrounding rock near the fault, as shown in Figure 15 (d).

As shown in Figure 16, the fault and the axis of the anchorage present a large angle of 34°. The initial deformation starts along the fault, but does not continue all the time, because the surrounding rock around the anchorage can still provide greater resistance, as shown in Figure 16 (b). When the anchorage moves further, the remaining rock within the fault boundary participates in the process of resisting its deformation, as shown in Figure 16 (c). The strength of the surrounding rock inside the fault is enough to resist the displacement of the anchorage, so there is no further deep diffusion of deformation near the fault, as shown in Figure 16 (d).

As shown in Figure 17, when the clamping angle is fixed at 6°, the influence of faults with different angles on the bearing characteristics of the anchorage is studied. The results show that the bearing capacity is the worst at the middle angle of 21° and that at the angles of 8° and 34° are slightly better. This is mainly affected by the size of intersection of the effective range of the clamping effect and the surrounding rock area near the inner side of the tunnel-type anchorage. It is obvious that there is too much surrounding rock in the range of clamping effect at 21°. Similarly, the bearing capacity without fault is better than that with fault.

4. Conclusions
The bearing characteristics, deformation evolution law and failure mode of tunnel-type anchorage are studied by DDA. DDA simulation results show that the clamping angle of tunnel-type anchorage plays an important role in the clamping effect. The rock mass in a certain range is enveloped to play the role of resistance body, and the surrounding rock mass needs to have enough strength to support the clamping effect. DDA can simulate the whole failure process of the tunnel-type anchorage-surrounding rock system, and can form a real failure boundary. Therefore, the loading process simulation and mechanical response based on DDA can better reflect the real situation. DDA can also better reflect the influence of discontinuities, which confirms the weakening effect of discontinuities on the clamping effect. But for the property of tunnel-type anchorage, which is a three-dimensional problem, three-dimensional DDA or three-dimensional numerical manifold method will be a better choice to study the failure mode of tunnel-type anchorage.

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