Determining the Sign of the $b \to s\gamma$ Amplitude

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The latest Belle and BaBar measurements of the inclusive $\bar{B} \to X_s l^+ l^-$ branching ratio have smaller errors and lower central values than the previous ones. We point out that these results indicate that the sign of the $b \to s\gamma$ amplitude is the same as in the SM. This underscores the importance of $\bar{B} \to X_s l^+ l^-$ in searches for new physics, and may be relevant for neutralino dark matter analyses within the MSSM.

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The branching ratio of the inclusive radiative $B$-decay is one of the most important constraints for a number of new physics models, because it is accurately measured and its theoretical determination is rather clean. The present world average $B(\bar{B} \to X_s \gamma) = (3.52\pm0.30) \times 10^{-4}$ agrees very well with the Standard Model (SM) prediction $B(\bar{B} \to X_s \gamma)_{\text{SM}} = (3.70 \pm 0.30) \times 10^{-4}$ [2]. A well-known way to avoid this constraint without excluding large new physics effects consists in having new physics contributions that approximately reverse the sign of the amplitude $A(\bar{b} \to s\gamma)$ with respect to the SM and leave $B(\bar{B} \to X_s \gamma)$ unaltered within experimental and theoretical uncertainties. Several authors pointed out that even a rather rough measurement of the inclusive $\bar{B} \to X_s l^+ l^-$ branching ratio could provide information on the sign of $A(\bar{b} \to s\gamma)$ [3].

Other observables that are sensitive to the sign of $A(\bar{b} \to s\gamma)$ are the forward-backward and energy asymmetries in inclusive and exclusive $b \to sl^+ l^-$ decays [3,4]. Very recently, the first measurement of the forward-backward asymmetry in $B \to K^{(*)} l^+ l^-$ was announced by the Belle Collaboration [5]. Within the limited statistical accuracy, however, the results were found to be consistent with both the SM and the “wrong-sign” $A(\bar{b} \to s\gamma)$ case.

The purpose of this Letter is to point out that the present measurements of $B(\bar{B} \to X_s l^+ l^-)$ already indicate that the sign of $A(\bar{b} \to s\gamma)$ is unlikely to be different from that in the SM. The experimental results that we consider are summarized in Tab. I.

The results in Tab. I are averaged over muons and electrons. The first range of the dilepton mass squared $q^2$ corresponds to the whole available phase-space for $l = \mu$, but includes a cut for $l = e$. Moreover, the intermediate $\psi$ and $\psi'$ are treated as background, and a Monte Carlo simulation based on perturbative calculations is applied for the unmeasured part of the $q^2$-spectrum that hides under the huge $\psi$ and $\psi'$ peaks (see Refs. [3,7] for more details). In the second range of $q^2$ in Tab. I theoretical uncertainties are smaller than in the first case (see below), but the experimental errors are larger due to lower statistics. As we shall see, the analyses in both regions lead to similar conclusions concerning the sign of $A(\bar{b} \to s\gamma)$.

| Range | Belle [6] | BaBar [7] | w.a. |
|-------|-----------|------------|-----|
| (a)   | 4.11 $\pm$ 0.83 $^{+0.71}_{-0.57}$ | 5.6 $\pm$ 1.5 $\pm$ 0.6 $\pm$ 1.1 | 4.5 $\pm$ 1.0 |
| (b)   | 1.493 $\pm$ 0.504 $^{+0.382}_{-0.283}$ | 1.8 $\pm$ 0.7 $\pm$ 0.5 | 1.60 $\pm$ 0.51 |

The Standard Model perturbative calculations are available at the Next-to-Next-to-Leading Order (NNLO) in QCD for both the considered ranges of $q^2$ — see Refs. [3,7] for the most recent phenomenological analyses and a list of relevant references. The dominant electroweak corrections are also known [8]. In the low-$q^2$ domain, non-perturbative effects are taken into account in the framework of the Heavy Quark Expansion as $\Lambda^2/m_b^2$ and $\Lambda^2/m_c^2$ corrections [10]. Analytical expressions for such corrections are also available for the full $q^2$ range, but they blow up in the vicinity of the intermediate $\psi$ peak. Consequently, a cut needs to be applied, and it is no longer clear what theoretical procedure corresponds to the interpolation that is performed on the experimental side. Thus, the relative theoretical uncertainty for the full $q^2$ range is larger than for the low-$q^2$ window.

The results of the SM calculations are given in the central column of Tab. I. For the low-$q^2$ domain, they correspond to the ones of Ref. [8] with updated input values $m_{\psi,\text{pole}} = 178.0 \pm 4.3$ GeV [11] and $B(\bar{B} \to X_s \bar{d}) = 10.61 \pm 0.16 \pm 0.06$ [12]. The dominant sources of uncertainty are the values of the top and bottom quark masses, as well as the residual renormalization scale de-
TABLE II: Predictions for $\mathcal{B}(\bar{B} \rightarrow X_s l^+ l^-)$ [$10^{-6}$] in the Standard Model and with reversed sign of $\tilde{C}_{7}^{\text{eff}}$ for the same ranges of $q^2$ as in Tab. I

| Range | SM | $\tilde{C}_{7}^{\text{eff}} \rightarrow \tilde{C}_{7}^{\text{eff}}$ |
|-------|----|------------------|
| (a)   | 4.4 ± 0.7 | 8.8 ± 1.0 |
| (b)   | 1.57 ± 0.16 | 3.30 ± 0.25 |

dependence. For the full $q^2$ range, we make use of the statement in Ref. 8 that the NNLO matrix elements for $\hat{s} = q^2/m_b^2 > 0.25$ are accurately reproduced by setting the renormalization scale $\mu_b = m_b/2$ at the Next-to-Leading Order (NLO) level.

To a very good approximation, the amplitude $A(b \rightarrow s\gamma)$ is proportional to the effective Wilson coefficient $\tilde{C}_{7}^{\text{eff}}(q^2 = 0)$ that determines the strength of the $s_L\gamma^\alpha b_L\gamma^\beta$ interaction term in the low-energy Hamiltonian. The sign of $A(b \rightarrow s\gamma)$ is therefore given by the sign of $\tilde{C}_{7}^{\text{eff}}(q^2 = 0)$. Both the value and the sign of this coefficient matter for the rare semileptonic decay. The results in the right column of Tab. I differ from those in the central column only by reversing the sign of $\tilde{C}_{7}^{\text{eff}}$ in the expression for the differential $\bar{B} \rightarrow X_s l^+ l^-$ decay rate

$$
\frac{d\Gamma[\bar{B} \rightarrow X_s l^+ l^-]}{d\hat{s}} = \frac{G_F^2 m_b^3}{48\pi^3} \left| V^\ast_{tb} V_{ts} \right|^2 \left( \frac{\alpha_{\text{em}}}{4\pi} \right)^2 \times \left\{ 1 - \hat{s}^2 \left[ \left| \tilde{C}_{9}^{\text{eff}} \right|^2 + \left| \tilde{C}_{10}^{\text{eff}} \right|^2 \right] \right\}.
$$

where $\tilde{C}_{9}^{\text{eff}}$ and $\tilde{C}_{10}^{\text{eff}}$ correspond to the low-energy interaction terms $(s_L\gamma^\alpha b_L)(\bar{l}_L\gamma^\beta)$ and $(s_L\gamma^\alpha b_L)(\bar{l}_L\gamma^\beta)$, respectively. The definitions of all the relevant effective coefficients can be found in Sec. 5 of Ref. 9. We stress that $\tilde{C}_{7}^{\text{eff}}$ depend on $q^2$ and do not depend on the renormalization scale, up to residual higher-order effects. For simplicity, some of the NNLO QCD, electroweak and non-perturbative corrections are omitted in Eq. 1. However, all those corrections are taken into account in our numerical results and plots.

The sensitivity of $\mathcal{B}(\bar{B} \rightarrow X_s l^+ l^-)$ to the sign of $\tilde{C}_{7}^{\text{eff}}$ is quite pronounced because the last term in Eq. 1 is sizeable and it interferes destructively (in the SM) with the remaining ones. One can see that the experimental values of the $\bar{B} \rightarrow X_s l^+ l^-$ branching ratio in Tab. I differ from the values in the right column of Tab. I by 3σ in both the low-$q^2$ window and the full $q^2$ range. This fact disfavors the sizeable corrections of the SM in which the sign of $\tilde{C}_{7}^{\text{eff}}$ gets reversed while $\tilde{C}_{9}^{\text{eff}}$ and $\tilde{C}_{10}^{\text{eff}}$ receive small non-standard corrections only.

In Fig. 1 we present constraints on additive new physics contributions to $\tilde{C}_{9,10}^{\text{eff}}$ placed by the low-$q^2$ measurements of $\bar{B} \rightarrow X_s l^+ l^-$ (Tab. I), once the $\bar{B} \rightarrow X_s\gamma$ bounds on $|\tilde{C}_{7}^{\text{eff}}|$ are taken into account. Similar plots have been previously presented in Refs. 14, 15. The two plots correspond to the two possible signs of the coefficient $\tilde{C}_{7}^{\text{eff}}$. The regions outside the rings are excluded. Surroundings of the origin are magnified in Fig. 2 for the non-standard case. The three lines correspond to three different values of $\mathcal{B}(\bar{B} \rightarrow X_s\gamma)$ (see the text).

FIG. 1: Model-independent constraints on additive new physics contributions to $\tilde{C}_{9,10}^{\text{eff}}$ at 90% C.L. for the SM-like (upper plot) and the opposite (lower plot) sign of $\tilde{C}_{7}^{\text{eff}}$. The three lines correspond to three different values of $\mathcal{B}(\bar{B} \rightarrow X_s\gamma)$ (see the text). The dot at the origin indicates the SM case for $\tilde{C}_{9,10}^{\text{eff}}$. 
FIG. 2: Same as in the lower plot in Fig. 1. Surroundings of the origin. The maximal MFV MSSM ranges for \( \tilde{C}_{9,\text{NP}}^{\text{eff}} \) and \( \tilde{C}_{10,\text{NP}}^{\text{eff}} \) are indicated by the dashed cross (according to Eq. (52) of Ref. [14]).

To conclude: We have pointed out that the recent measurements of \( B(B \to X_s l^+ l^-) \) by Belle and BaBar already indicate that the sign of the \( b \to s \gamma \) amplitude is unlikely to be different from that in the SM. This underscores the importance of \( B \to X_s l^+ l^- \) in searches for new physics, and may be relevant for neutralino dark matter analyses within the MSSM.

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