Research on error mechanism analysis and compensation method of Fiber-Optical Current Transformer

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Abstract. The structure of Fiber-Optical Current Transformer (FOCT) is completely different from that of traditional electromagnetic transformer, and the error analysis method is quite different from that of traditional transformer. However, there is no systematic calculation method for the error of FOCT at present. Starting from the working principle and structure of fiber-optical current transformer, this paper theoretically analyses and deduces all kinds of error sources of FOCT, puts forward the error calculation method of FOCT, calculates the overall error based on the actual parameters of a 200 kV Fiber-Optical Current Transformer, and proposes a method to compensate the measurement error of FOCT by changing the initial phase angle of λ/4 wave plate. The initial phase angle of the λ/4 wave plate with the best error compensation effect is 86 degrees.

1. Introduction
Fiber-Optical Current Transformer (FOCT) is an electronic current transformer based on Faraday magneto-optic effect. It has the advantages of high measurement accuracy, large dynamic range, good electromagnetic compatibility and flexible installation. It can measure AC and DC currents at the same time. It can meet the needs of continuous development in the fields of power metering, protection and control, fault recording and dynamic observation of power grid. It is used more and more widely in the current smart grid[1-6].

Compared with the traditional electromagnetic current transformer, FOCT has more environmentally sensitive optical and circuit structures, and the error generation mechanism is completely different. At the same time, it is found that the adaptability of FOCT to the environment is not good in practical engineering application. Under the conditions of temperature change and low current measurement, the error of FOCT will increase sharply. Therefore, it is necessary to study the error characteristics of FOCT in different environments[7-10].

In reference [11], the frequency characteristics of the transformer are tested, and the error mechanism of the transformer under high frequency is analyzed. Literature [12] analyzed the main error sources of FOCT and conducted an accuracy experiment of the transformer prototype. However, the above literature does not systematically analyze and quantify the overall error of FOCT based on the structure of the transformer and the external environment of the transformer.

Based on the working principle and structure of FOCT, this paper theoretically analyses and deduces all kinds of error sources of FOCT, proposes the error calculation method of FOCT, calculates the overall error combining with the actual parameters of a 200kV FOCT. The method of compensating...
the measurement error of the FOCT by changing the initial phase angle of the $\lambda/4$ wave plate is proposed, and the initial phase angle of the $\lambda/4$ wave plate with the best error compensation effect is obtained.

2. The principle of FOCT

Fiber-Optical Current Transformer (FOCT) is a new type of electronic current transformer based on Faraday magneto-optical effect and Ampere loop theorem. The system structure consists of three parts: optical fiber sensing ring, polarization-maintaining optical fiber delay loop and signal processing unit, as shown in Figure 1. The working principle of FOCT is that the light emitted by the light source is changed into linearly polarized light by the coupler and polarizer. The linearly polarized light is injected into the PMF delay line at 45 degrees and propagates along the X and Y axes of the PMF respectively. After passing through the $\lambda/4$ wave plate, the linearly polarized light turns into left-handed and right-handed circularly polarized light respectively and enters the optical fiber sensing ring. Faraday magneto-optic effect is produced by the measured current in the current-carrying conductor, which offsets the two circularly polarized beams and generates Faraday phase shift. After the two circularly polarized lights are reflected by the mirror, the polarization modes are interchanged and pass through the sensing fiber again, and the phase difference is doubled by the Faraday effect, and the phase shift generated at this time is $\phi_F = 4VNI$. After passing through the $\lambda/4$ wave plate, the circularly polarized light returns to linearly polarized light and interferes at the polarizer. The light carrying the phase difference signal enters the photodetector, and finally passes through photoelectric conversion, filter amplification, A/D conversion, demodulation, and output[13-15].

![Figure 1. Principle structure diagram of FOCT.](image)

According to Faraday’s magneto-optical effect, the magnitude of the current transmitted in the wire is proportional to the phase difference. Therefore, the measured current can be calculated by detecting the optical phase difference signal. Due to the strong anti-interference ability of the optical system of FOCT, the signal entering the photoelectric detector only carries the phase difference. The expression of the signal after photoelectric conversion by photoelectric detector is:

\[
I_{out} = 0.5K_p \cdot P_o \cdot (1+\cos(\phi_F \mp \phi_{mod}))
\]

In formula (1), $K_p$ is the photoelectric conversion coefficient of the detector, $P_o$ is the output light intensity of the light source, and $\phi_F = 4VNI$ is the phase difference of the magneto-optical Faraday effect (where $N$ is the number of turns of the sensing fiber and $V$ is the Verdet constant of the sensing fiber). $I$ is the value of the transmission current in the wire. $\phi_{mod}$ is the modulation phase shift produced by the phase modulator.

The measured current is induced into the optical signal by Faraday magneto-optic effect, and the optical signal containing the information of measured current value is transmitted to the signal processing unit in the form of output light intensity for corresponding demodulation processing. After the dual optical path detection method, the output signal of FOCT is as follows:

\[
\mu_o = \sin(\phi_F) \approx \phi_F = 4VNI
\]

3. Error analysis of FOCT
The error of FOCT refers to the difference between the measured value and the measured value of FOCT caused by the internal structure defect of transformer or the change of external factors such as components and environment. According to the working principle and structure of FOCT, the error of normal FOCT mainly comes from the following devices: 1. Polarizer; 2. Phase modulator; 3. \( \lambda/4 \) wave plate; 4. Sensing optical fiber; 5. Signal processing unit.

3.1. Polarizer extinction ratio error

The extinction ratio of polarizer has a great influence on the output of FOCT, and the bigger extinction ratio will cause larger measurement error. If the extinction ratio coefficient of the polarizer is \( \varepsilon \), the output signal of FOCT considering the extinction ratio of the polarizer is as follows:

\[
I_{out} = 0.5K_p \cdot P_o \left[ \left(1 + \varepsilon^2\right) + \left(1 - \varepsilon^2\right) \cos(\phi_p + \phi_{mod}) \right]
\]

(3)

The output signal of FOCT is:

\[
u_o = \left(1 - \varepsilon^2\right)^2 / \left(1 + \varepsilon^2\right)^2 \cdot 4VNI
\]

(4)

The error caused by extinction ratio of polarizer is as follows:

\[E = \frac{1}{4NV_0} \times u_o - I = \left[\left(1 - \varepsilon^2\right)^2 / \left(1 + \varepsilon^2\right)^2 - 1\right] I
\]

(5)

3.2. Phase modulator error

After phase modulation, the interference output of optical system becomes formula (1) \( \Phi_{mod} \) is the modulation phase shift produced by the phase modulator, usually set to \( \pi/2 \) to improve system detection sensitivity. At this time, the interference output of the optical path system is:

\[I_{out} = 0.5K_p \cdot P_o \cdot \left[1 + \sin(\phi_p)\right]
\]

(6)

When the fast and slow axis modulation phase shift errors of the phase modulator are inconsistent, an additional modulation phase shift error will be added to the interference output of the optical system. At this time, the interference output of the optical system is as follows:

\[I_{out} = 0.5K_p \cdot P_o \cdot \left[1 + \cos(\phi_p + \phi_{mod} + \Delta\phi)\right]
\]

(7)

Among them, \( \Delta\phi = \phi_1 - \phi_2 \), \( \phi_1 \) is fast axis modulation phase shift error and \( \phi_2 \) is slow axis modulation phase shift error.

The output signal of FOCT is:

\[u_o = 4VNI + \Delta\phi
\]

(8)

The error caused by the phase modulator is:

\[E = \frac{1}{4NV_0} \times u_o - I = \frac{\Delta\phi}{I}
\]

(9)

3.3. \( \lambda/4 \) wave plate error

The error of \( \lambda/4 \) wave plate is related to ambient temperature. The phase delay angle error of \( \lambda/4 \) wave plate affected by ambient temperature is as follows:

\[\Delta\theta_{wave \ plate} = 0.09(25 - T)
\]

(10)

The intensity expression of FOCT output is as follows:
The output signal of FOCT is:

\[ I_{\text{out}} = 0.5K_p \cdot P_o \cdot 4VNI \cdot \sin(\theta_o + 0.09(25-T)) \]  \hspace{1cm} (11)

The output signal of FOCT is:

\[ u_o = 4VNI \cdot \sin(\theta_o + 0.09(25-T)) \]  \hspace{1cm} (12)

The error caused by the temperature on the \( \lambda/4 \) wave plate is:

\[ E_{1/4} = \frac{1}{4NV_o} \times u_o - I \]

\[ = \left[ \sin\left(\frac{\pi}{2} + 0.09(25-T)\right) - 1 \right] I \] \hspace{1cm} (13)

### 3.4. The sensing loop error

The sensing loop error is the most important component of FOCT error. The error caused by the sensing loop is mainly caused by the error caused by the influence of temperature on the Verdet constant and the error caused by the linear birefringence effect of the sensing loop.

#### 3.4.1. Errors caused by the effect of temperature on the Verdet constant.

The size of the Verdet constant will change with ambient temperature. Since SiO\(_2\) is the main component of the optical fibers that make up the FOCT optical path, the relationship between the change of Verdet constant and temperature is as follows:

\[ \frac{1}{V_0} \frac{\partial V}{\partial T} = 7.0 \times 10^{-5} / \degree C \] \hspace{1cm} (14)

In the formula, \( V_0 \) is the Verdet constant at 25°C at room temperature; \( T \) is the ambient temperature. For the sensing fiber with wavelength of 1310 nm, the Verdet constant at 25°C is \( V_0 = 0.999 \times 10^{-6} \) rad/A. The relationship between the Verdet constant and the ambient temperature is obtained as follows:

\[ V = V_0 \times \left[ 1 + 7.0 \times 10^{-5} (T - 25) \right] \] \hspace{1cm} (15)

The output signal of FOCT is:

\[ u_o = 4V_0 \times \left[ 1 + 7.0 \times 10^{-5} (T - 25) \right] NI \] \hspace{1cm} (16)

The error caused by the influence of temperature on the Verdet constant is as follows:

\[ E_v = \frac{1}{4NV_0} \times u_o - I = \left[ 1 + 7.0 \times 10^{-5} (T - 25) - 1 \right] I \] \hspace{1cm} (17)

#### 3.4.2. Errors caused by linear birefringence of sensing fibers.

The linear birefringence error of sensing fibers is divided into three parts: intrinsic birefringence, bending birefringence and temperature birefringence.

The intrinsic linear birefringence is a kind of linear birefringence caused by residual stress in non-circular core. In this paper, a low linear birefringence fiber is used. Its intrinsic linear birefringence is low. Generally, it is chosen as \( \delta_0 = 0.006 \text{rad/m} \).

Bending-induced linear birefringence is a linear birefringence caused by the bending stress applied to the sensing fiber when the sensing ring is bent into a ring. Its magnitude can be expressed as follows:

\[ \delta_e = \frac{\pi n^3}{2\lambda} (1 + \nu) (p_{12} - p_{11}) \left( \frac{r}{R} \right)^2 \] \hspace{1cm} (18)

Among them, \( p_{12}, p_{11} \) is the photoelastic tensor of the core, \( \nu \) is Poisson's ratio, \( \lambda \) is the wavelength of incident light, \( n \) is the refractive index of the core, \( R \) is the outer radius of the core.
Temperature-induced birefringence is a linear birefringence change caused by the stress produced by the different thermal expansion coefficients of the core and cladding of the sensing fiber when the ambient temperature changes. Its magnitude can be expressed as follows:

$$
\delta_T = \frac{\pi n^2 (1+\nu) (p_{12} - p_{11}) \Delta \alpha}{6 \lambda (1-2\nu)} (T - T_0)
$$  \hspace{0.5cm} (19)

Among them, $\Delta \alpha$ is the difference of thermal expansion coefficient between core and cladding.

By accumulating the Jones matrix of the optical path model of FOCT, the output of the photoelectric converter can be obtained as follows:

$$
I_{\text{out}} = 0.5K_p \cdot P_r \left[ \sin \left( \frac{\varphi_f}{2} \sin \frac{2\Delta}{\Lambda} \right) \right]
$$  \hspace{0.5cm} (20)

The output signal of FOCT is:

$$
u_o = 2VNI \sin \frac{2\Delta}{\Lambda}
$$  \hspace{0.5cm} (21)

Since the linear birefringence is much larger than the circular birefringence caused by the measured current, $\Delta \approx \delta$, the FOCT output signal considering the linear birefringence is as follows:

$$
u_o = 2VNI \sin \left[ \frac{4\pi R N (\delta_0 + \delta_n + \delta_T)}{2\pi R N (\delta_0 + \delta_n + \delta_T)} \right]
$$  \hspace{0.5cm} (22)

The error caused by linear birefringence is as follows:

$$
E_{\text{linear birefringence}} = \left[ 1 \sin \left( \frac{4\pi R N \delta}{2\pi R N \delta} \right) - 1 \right] I
$$  \hspace{0.5cm} (23)

3.5. Error of signal processing unit

The error of signal processing unit is mainly caused by the error of photoelectric converter and closed-loop control system, but compared with the error of optical path, the error of electronic signal conversion is much smaller than that of optical propagation process. The error caused by signal processing unit is neglected in this paper.

3.6. Error calculating method of FOCT

The polarizer extinction ratio error, phase modulator modulation phase shift error, $\lambda/4$ wave plate temperature error, Verdet constant temperature error, and linear birefringence error are coupled to obtain the output signal of the photoelectric converter as follows:

$$
I_{\text{out}} = 0.5K_p \cdot P_r \left[ \left( 1 + e^1 \right)^2 + \left( 1 - e^1 \right)^2 \sin \theta \left( V_0 \sin \left( \frac{4\pi R N \delta}{2\pi R N \delta} \right) I + \Delta \phi \right) \right]
$$  \hspace{0.5cm} (24)

In the formula,

$$
\theta = \theta_0 + 0.09 (25 - T)
$$

$$
V = V_0 \left[ 1 + 7 \times 10^{-3} \left( T - 25 \right) \right]
$$

Then the output signal of FOCT is:

$$
u_o = \left( \frac{1 - e^1}{1 + e^1} \right)^2 \sin \theta \cdot \left[ 2VNI \sin \left( \frac{4\pi R N \delta}{2\pi R N \delta} I + \Delta \phi \right) \right]
$$  \hspace{0.5cm} (25)

Then the error of FOCT is:
\[ E = \left[ \frac{1}{1 + \varepsilon^2} \right]^{\frac{1}{2}} \sin \theta \cdot 2N \sin \left( \frac{4\pi NR\delta}{2\pi NR\delta} \right) \frac{1}{4N_0} - 1 \] \[ I + \left[ \frac{1}{1 + \varepsilon^2} \right]^{\frac{1}{2}} \sin \theta \cdot \Delta \phi \] \quad (26)

4. Measurement error calculation and analysis of FOCT

Based on the actual parameters of 200KV FOCT prototype, the measurement errors of FOCT are theoretically analyzed and calculated. The parameters and constants of the transformer used in this paper are shown in Table 1.

**Table 1. Calculating required coefficients and constants.**

| Parametric                  | numerical | Parametric                  | numerical |
|-----------------------------|-----------|-----------------------------|-----------|
| Turn Number of Transformer  | N = 4     | Wavelength of incident light | \( \lambda = 1310 \text{nm} \) |
| Core refractive index       | n = 1.456 | Poisson ratio               | \( \nu = 0.17 \) |
| Core diameter               | r = 4.5 \( \mu \text{m} \) | Reference Verdet Constant   | \( V_v = 9.999 \times 10^{-7} \text{rad/A} \) |
| Bending radius of sensing ring | R = 0.1585m | Thermal expansion coefficient difference | \( \Delta \alpha = 10^{10} \) |
| Fiber Core Photoelastic Tensor | \( p_{11} = 0.121 \) \( p_{12} = 0.27 \) | Extinction ratio coefficient | \( \varepsilon = 0.0076 \) |

From equation (5), the error caused by extinction ratio of polarizer is as follows:

\[ E = -2.31 \times 10^{-4} I \]

The experimental results show that the modulation phase shift error caused by the phase modulator is as follows:

\[ E = -0.0401 \]

From equation (13), the error caused by \( \lambda/4 \) wave plate is as follows:

\[ E_{\lambda/4} = -1.23 \times 10^{-4} I \]

From equation (15), we can see that the Verdet constant of the test temperature at 15\(^\circ\)C is as follows:

\[ V = V_v \left[ 1 + 0.7 \times 10^{-4} \Delta T \right] = 1.0006 \times 10^{-6} \text{rad/A} \]

From equation (17), the error caused by the temperature effect of the Verdet constant is as follows:

\[ E_v \left[ 1 + 7.0 \times 10^{-5} (T - 25) - 1 \right] = -7.0 \times 10^{-4} I \]

Formula (18) shows that the bending-induced linear birefringence of a unit length optical fiber is as follows:

\[ \delta_v = \frac{\pi n^3}{2\lambda} (1 + \nu) \left( p_{12} - p_{11} \right) \left( \frac{E}{R} \right)^2 = 5.201 \times 10^{-4} \text{rad/m} \]

From equation (19), the temperature-induced linear birefringence of a unit length optical fiber is as follows:

\[ \delta_t = \frac{\pi n^3}{6\lambda} \left( 1 + \nu \right) \left( p_{12} - p_{11} \right) \Delta \alpha \cdot \Delta T = 3.259 \times 10^{-4} \text{rad/m} \]

Therefore, the total linear birefringence of an fiber-optical current transformer per unit length is as follows:

\[ \delta = \delta_v + \delta_t = 6.846 \times 10^{-3} \text{rad/m} \]

From formula (23), the error caused by the linear birefringence effect of optical fibers is as follows:

\[ E_{\text{linear birefringence}} = -4.96 \times 10^{-4} I \]

From formula (26), the total error of fiber-optical current transformer prototype is as follows:
It can be seen that the main sources of FOCT errors are the errors caused by extinction ratio of polarizer and temperature effect of Verdet constant. At the same time, the errors caused by $\lambda/4$ wave plate and linear birefringence effect of sensing fiber can not be ignored.

5. Research on measurement error compensation method of FOCT

Formula (26) shows that the measurement error of FOCT is related to extinction ratio of polarizer, modulation error of phase modulator, initial phase angle of $\lambda/4$ wave plate, Verdet constant and linear birefringence of fiber itself. The Verdet constant and linear birefringence of optical fibers are fixed values. The extinction ratio of polarizer and phase modulator have reached a high precision, which makes it difficult to improve. The initial phase angle of the $\lambda/4$ wave plate can be controlled by changing the length of the optical fiber selected at the time of fabrication. In addition, the errors caused by $\lambda/4$ wave plate and the errors caused by the Verdet constant are all affected by temperature, and the error caused by the Verdet constant is larger by temperature. The measurement error of transformer can be compensated by controlling the initial phase angle of $\lambda/4$ wave plate.

In order to achieve the best compensation effect, the initial phase angle of the $\lambda/4$ wave plate should be within the temperature range of -40 ~70°C to minimize the measurement error of FOCT, and the accuracy is satisfied. Because the ideal initial phase angle of the $\lambda/4$ wave plate is 90 degrees, the ratio curve of FOCT can be drawn by formula (26) when the initial phase angle of the $\lambda/4$ wave plate is selected from 80 degrees to 100 degrees in the temperature range of -40 ~70°C.

Figure 2. Relation Curve of FOCT Ratio Error and Initial Phase Angle of $\lambda/4$ Wave plate at Different Temperatures.

Fig. 2 shows that when the initial phase angle of the $\lambda/4$ wave plate is 86.39 degrees, the FOCT can meet the accuracy requirement of 0.2% in most temperature ranges from -40°C to 70°C, and the ratio error is relatively small. The maximum value of the ratio error is -0.342% at -40°C. In order to facilitate processing and manufacturing, the initial phase angle of $\lambda/4$ wave plate is generally taken as an integer, and the initial phase angle of $\lambda/4$ wave plate should be taken near 86.39 degrees. Therefore, the ratio error and ambient temperature curve of FOCT can be plotted by formula (26) when the initial phase angle of $\lambda/4$ wave plate is selected from 84 to 90 degrees.
Figure 3. Relation Curve of FOCT Ratio Error with Environmental Temperature for Different Initial Phase Angle of λ/4 Waveplate.

Fig. 3 shows that when the initial phase angle of the λ/4 wave plate is 86 degrees, the accuracy of FOCT is 0.2 grade in most temperature ranges from -40 to 70°C, and the maximum specific error is -0.37% at 70 degrees.

The measurement error of FOCT can be compensated by changing the initial phase angle of λ/4 wave plate, and the compensation effect is the best when the initial phase angle of λ/4 wave plate is 86 degrees. At this time, FOCT meets the accuracy requirements in most temperature ranges and can accurately measure the current value.

6. Conclusion

Based on the structure and working principle of FOCT, the error factors affecting FOCT are analysed in this paper. The error of FOCT is mainly affected by polarizer extinction ratio, modulation angle error of phase modulator, temperature effect on λ/4 wave plate, temperature effect on Verdet constant and linear birefringence of sensing loop.

In this paper, the mechanism of FOCT error formation is analyzed, and the calculation method of each error influence factor is obtained quantitatively. After coupling, the calculation method of FOCT error is obtained.

Taking a 200kV FOCT as an example, this paper calculates the error using the error calculation method. At the same time, a method to compensate the measurement error of FOCT by changing the initial phase angle of λ/4 wave plate is proposed, and the initial phase angle of λ/4 wave plate with the best error compensation effect is 86 degrees.

7. References

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