Evidence for bouncing evolution before inflation after BICEP2

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The BICEP2 collaboration reports a detection of primordial cosmic microwave background (CMB) B-mode with a tensor-scalar ratio $r = 0.20^{+0.07}_{-0.05}$ (68% C.L.). However, this result is in tension with the recent Planck limit, $r < 0.11$ (95% C.L.), on constraining inflation models. In this Letter we consider an inflationary cosmology with a preceding nonsingular bounce which gives rise to observable signatures on primordial perturbations. One interesting phenomenon is that both the primordial scalar and tensor modes can have a step feature on their power spectra, which nicely cancels the tensor excess power on the CMB temperature power spectrum. By performing a global analysis, we obtain the 68% C.L. constraints on the parameters of the model from the Planck+WP and BICEP2 data together: the jump scale $\log_{10}(k_B/\text{Mpc}^{-1}) = -2.4 \pm 0.2$ and the spectrum amplitude ratio of bounce-to-inflation $r_B \equiv P_B/A_0 = 0.71 \pm 0.09$. Our result reveals that the bounce inflation scenario can simultaneously explain the Planck and BICEP2 observations better than the standard $\Lambda$CDM model, and can be verified by the future CMB polarization measurements.

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Introduction.—Recently, the BICEP2 collaboration announced the detection of primordial B-mode polarization on the CMB. This significant measurement implies that, if all the B-mode polarization signals are contributed by primordial gravitational waves, the corresponding tensor-to-scalar ratio is constrained as $r < 0.11$ (95% C.L.). However, this result is in tension with the recent Planck limit, $r < 0.11$ (95% C.L.) on standard inflation models since the excess power in the CMB temperature power spectrum was not observed by the Planck experiment.

In order to lessen the pressure on inflation models and the tension with Planck data, we consider an important extension of inflationary cosmology, which may introduce a nonsingular bounce to connect a contracting phase of the universe with the inflationary stage. It is well known that the big bang singularity issue can be avoided in the framework of bouncing cosmologies. The scenario of bounce inflation has been applied to suppress CMB anisotropies on large angular scales. By virtue of the effective field description, a bounce inflation solution can be achieved by matter fields with the null energy condition violation, such as the quintom bounce. This scenario was also realized in the frame of loop quantum cosmology (namely see ref. and references therein).

In this Letter, we aim at searching for key observational signals for the bounce inflation scenario which are expected to be sensitive to cosmological CMB measurements. Specifically, we perform an estimate on the power spectrum of primordial gravitational waves generated in the matter-bounce inflation scenario and find that its amplitude presents a nontrivial jump feature at a critical length scale. A similar property was also found in the power spectrum of primordial curvature perturbation as pointed out in ref. Using the Planck and BICEP2 data, we perform a global analysis on this bounce inflation scenario and find that it can better interpret the recent CMB observations when compared with the $\Lambda$CDM.

Formalism.—We begin with a brief discussion of primordial perturbations in the frame of a flat FRW Universe. The relic gravitational waves generated in very early universe is a basic prediction in the modern cosmology. A standard process of generating primordial power spectrum suggests that, metric fluctuations initially emerge inside a Hubble radius, and then leave it in a primordial epoch, and finally reenter at late times. The dynamics of primordial gravitational waves is convenient to be investigated by tracking a Fourier mode $v_k$ along the cosmic evolution. In the context of General Relativity, the corresponding equation of motion in the Fourier space is given by

$$v_k'' + (k^2 - \frac{a''}{a})v_k = 0 ,$$

where $a$ is the scale factor of the universe and the prime denotes the derivative with respect to the comoving time $\eta \equiv \int dt/a$. Specifically, the scale factor often scales as $a(t) = a_B(t/\tau_B)^{1/\epsilon}$, where the subscript “$B$” denotes any reference time which will be referred as the bouncing time later. For a constant $\epsilon$, there is $a''/a = (\nu^2 - 1/4)/\eta^2$, with $\nu = \pm (\epsilon - 3)/(2\epsilon - 2)$.

We assume cosmological perturbations originate from vacuum fluctuations, which suggests, $v_k(t) \simeq$
phenomenologically parameterize the power spectrum for a model of matter-bounce inflation. When $k < k_b$, the perturbation can be parameterized as $v_k(\eta) \approx \frac{1}{\sqrt{2k}}(k\eta)^{\nu}$, at super-Hubble scales with $|k\eta| \ll 1$. Now we match these two asymptotic solutions at the moment of Hubble crossing $|k\eta| \sim 1$, and then obtain the tensor mode on super-Hubble scales as

$$v_k(\eta) \approx \frac{1}{\sqrt{2k}}(k\eta)^{\frac{1}{2} - \nu}.$$

From the definition of the power spectrum $P_T \equiv \frac{\Lambda}{\pi} k^3 |\tilde{u}_k|^2$, one easily learns that the scale invariance requires $|\nu| = 3/2$ which has to be achieved in a period of matter contraction [17, 18] or by inflation. However, the comoving Hubble rate evolves as $|H| \approx 2/|\eta|$ during matter contraction while takes another form $|H| \approx |1/\eta|$ during inflation. As a result, if there is a matter contraction before inflation, the amplitude of the power spectrum for primordial gravitational waves would undergo a jump around the scale $k_b$ comparable to the bounce scale. A detailed calculation reveals that $P_T \approx H^2/4\pi^2$ when $k < k_b$ while $P_T \approx 2H^2/\pi^2$ when $k \geq k_B$ for the model of matter-bounce inflation.

In analogy with the method developed in [12], we phenomenologically parameterize the power spectrum for primordial tensor perturbations as follows,

$$P_T = P_m^m + \frac{P_T^m - P_m^m}{2} \left\{ 1 + \tanh \left[ T \log_{10} \left( \frac{k}{k_b} \right) \right] \right\},$$

with $P_T^m \equiv H^2/4\pi^2$ and $P_T^I \equiv 2H^2/\pi^2$ being introduced. In addition, the power spectrum of primordial curvature perturbation can be parameterized as

$$P_\zeta = P_m + \frac{P_{\zeta,i} - P_m^m}{2} \left\{ 1 + \tanh \left[ T \log_{10} \left( \frac{k}{k_b} \right) \right] \right\}.$$

Particularly, $P_{\zeta,i} = H^2/8\pi^2$, the power spectrum during inflation and $P_m$ is the spectrum before the bounce which is required to be less than $P_{\zeta,i}$.

Since primordial density fluctuations rely on the model parameters during the bounce, the amplitude of its power spectrum before the bounce can be any arbitrary value lower than that during inflation [19, 20]. Therefore, the observational constraint on primordial density perturbations is pretty loose [13]. However, for primordial tensor fluctuations, their dynamics only depend on the evolution of the scale factor and hence, once we have determined the background evolution, the power spectrum of primordial gravitational waves can be fixed.

As in usual, the power spectrum during inflation can be parameterized as the power low format via $P_{\zeta,i} = A_s k^{n_s - 1}$, in which $A_s$ and $n_s$ are the amplitude and the spectral index correspondingly. Similar to the analysis of primordial gravitational waves, one can introduce a bounce-to-inflation ratio of power spectrum, $r_B \equiv P_m/A_s$ to characterize the spectrum obtained before the bounce. Moreover, the parameter $k_b$ denotes the occurrence scale of the jump feature in the power spectrum (in unit of Mpc$^{-1}$), and $T$ depicts the slope of this jump and thus is associated with the bounce duration. Apparently, these three parameters are highly correlated. We try to constrain them simultaneously, but the results are not good enough. Therefore, in our numerical calculations we fix $T = 5$ which is the best fit value we obtain and constrain the other two parameters.

**Results.**—We perform a global fitting using the CosmoMC package [21], a Markov Chain Monte Carlo code, which has been modified to calculate the theoretical CMB power spectra in the bounce inflation scenario. We assume adiabatic initial conditions and a flat universe. We vary the following cosmological parameters $(\Omega_b h^2$, $\Omega_c h^2$, $\tau$, $\Theta_s$, $n_s$, $A_s$, $r)$, where $\Omega_b h^2$ and $\Omega_c h^2$ are the baryon and cold dark matter densities, $\tau$ is the optical depth to reionization, $\Theta_s$ is the ratio (multiplied by 100) of the sound horizon at decoupling to the angular diameter distance to the last scattering surface, $n_s$ is the spectral index, $r$ is the tensor to scalar ratio of the power spectrum and $A_s$ is the primordial amplitude at the pivot scale $k_0 = 0.05$ Mpc$^{-1}$. Furthermore, we have two more parameters $k_b$ and $r_B$ which are related to the bounce model.

Particularly, we use the low-$\ell$ and high-$\ell$ CMB temperature power spectrum data from the Planck with the low-$\ell$ WMAP9 polarization data (Planck+WP). We marginalize over the nuisance parameters that model the unresolved foregrounds with wide priors. For the BICEP2 data, we use their BB power spectrum into our analyses.

In table I we list the minimal $\chi^2$ values for different best fit models from different data combinations. $\chi^2(P,\text{low-}\ell)$ is for the Planck low-$\ell$ TT power spectrum only, $\chi^2(P)$ is for the Planck+WP data, and $\chi^2(B)$ is for the BICEP2 BB spectrum.

| Model          | $\chi^2(P,\text{low-}\ell)$ | $\chi^2(P)$ | $\chi^2(B)$ |
|----------------|------------------------------|------------|------------|
| $r = 0$        | $-6.7$                       | $9805.7$   | $56.0$     |
| $r = 0.162$    | $0.7$                        | $9814.3$   | $9.0$      |
| $r = 0$        | $-10.9$                      | $9804.3$   | $53.7$     |
| $r = 0.183$    | $-9.2$                       | $9805.6$   | $7.0$      |

TABLE I: The $\chi^2$ values for different best fit models from different data combinations.
When including tensor fluctuations in the calculation, the $\chi^2$ value of the best fit model from the BICEP2 data significantly decreases to $\chi^2(B) = 9.0$. The BICEP2 data strongly favor a non-zero amplitude of the primordial tensor power spectrum, namely the 68% C.L. limit is $r = 0.162 \pm 0.034$. This result is consistent with that from the BICEP2 collaboration [1]. However, the non-zero $r$ model will bring the extra power on CMB low-$\ell$ temperature power spectrum, which leads to the worse fit to the Planck data, especially to the low-$\ell$ data, as shown in the left panel of figure. Therefore, the standard $\Lambda$CDM model can not simultaneously fit to the Planck and BICEP2 data very well, due to the excess power on CMB TT spectrum at large scales.

Next, we consider the bounce inflation model. We use the Planck+WP data alone to constrain the parameters $k_b$ and $r_B$. We find the best fit values of $\log_{10}(k_b/\text{Mpc}^{-1}) = -2.6$ and $r_B = 0.8$ with the minimal $\chi^2(P) = 9804.3$, which means the bounce model can only slightly improve the fit to the Planck data with $\Delta \chi^2 \sim 1.4$. This result is slightly worse than some other works [22, 23], due to our moderate suppression in the bounce model, which is shown in the left panel of figure. In figure we show the one-dimensional distributions on bounce parameters $k_b$ and $r_B$, and obtain the 95% limits $\log_{10}(k_b/\text{Mpc}^{-1}) < -2.1$ and $0 < r_B < 1$. The bounce model with no suppression is still consistent with the Planck+WP data. Again, similar to the $\Lambda$CDM, this bounce inflation model with $r = 0$ can not fit the BICEP2 data as well, $\chi^2(B) = 53.7$. Afterwards, we include the BICEP2 data and the tensor fluctuations into the analyses. Although in the bounce inflation, the theoretical CMB primordial BB power spectrum is suppressed at large scales, as shown in the right panel of figure, the BICEP2 experiment can only measure the BB power spectrum at scales $\ell > 30$, where the suppression effect is very small. Therefore, the median value of the tensor to scalar ratio $r$ in the bounce inflation model is similar with that obtained in the standard $\Lambda$CDM model, $r = 0.183 \pm 0.072$ at 68% confidence level, as shown in figure. Meanwhile, we find that the suppression effect is obvious at very large scales $\ell < 20$. We expect that the Planck team will soon release the CMB polarization data which may cover the BB power spectrum at these scales. Therefore, it is very promising to examine the bounce inflation scenario in near future.

More importantly, adding the BICEP2 data sig-
significantly improves the constraints on parameters of the bounce inflation. The 68% C.L. constraints are: \( \log_{10}(k_{b}/\text{Mpc}^{-1}) = -2.4 \pm 0.2 \) and \( r_{B} = 0.71 \pm 0.09 \), while the 95% limits are: \( -2.8 < \log_{10}(k_{b}/\text{Mpc}^{-1}) < -2.1 \) and \( 0.54 < r_{B} < 0.88 \). In figure 2 we show the two-dimensional contours between \( k_{b} \), \( r_{B} \) and \( r \). Since we have two free parameters to describe the suppression effect of the bounce model, when \( k_{b} \) is increasing, the other parameter \( r_{B} \) also becomes larger in order to compensate this effect. Therefore, the correlation of \( r_{B} \) with \( k_{b} \) is positive. On the other hand, the model with a non-zero \( r \) brings the extra CMB TT power spectrum, which allows a large suppression, corresponding to an increasing \( k_{b} \). So we find that there is a tiny positive correlation between \( k_{b} \) and \( r \).

Additionally, the \( \chi^{2} \) values for the best fit model from the Planck+WP and BICEP2 data are \( \chi^{2}(P) = 9805.6 \) and \( \chi^{2}(B) = 7.0 \), respectively. The bounce inflation with \( r = 0.183 \) can fit the BICEP2 data well, while it can also explain the Planck+WP data with the similar \( \chi^{2} \) value, especially for the Planck low-\( \ell \) TT data (see table 1). The reason is that the extra CMB TT power spectrum at large scales, due to the non-zero tensor fluctuations, can be canceled by the suppression effect brought by the bounce, which is significantly different from the standard \( \Lambda \)CDM case. Based on these results, we conclude that when using Planck data alone, the bounce model can only slightly improve the fit to the data, comparing with the \( \Lambda \)CDM model. However, after including the BICEP2 data, the bounce inflation can fit to the Planck+WP and BICEP2 data simultaneously, and then is more favored by the data than the \( \Lambda \)CDM model with \( \Delta \chi^{2}_{\text{min}} \approx -12 \), corresponding to \( \sim 3.5\sigma \) confidence level.

Conclusions.—Since a nonsingular bounce is expected to occur at an extremely high energy scale in very early universe, it is hardly to detect directly by experiments. To search for a bounce, the associated observational consequences are significant in cosmological surveys. In the present Letter, we study the evolution of primordial gravitational waves in a combined scenario of matter bounce and inflation. We interestingly discover a novel jump feature on the power spectrum of these tensor modes at large scales, which could be verified by the Planck polarization data in the near future. The same feature was found to exist in the spectrum of primordial density perturbations. Importantly, this jump feature on the primordial scalar and tensor spectrum could alleviate problems of the excess power in the CMB temperature power spectrum.

Recently, the BICEP2 collaboration reports a 3\( \sigma \) detection of the non-zero tensor-to-scalar ratio, which brings too much extra power in CMB TT power spectrum to fit in the standard \( \Lambda \)CDM framework. When we consider the bounce inflation model, the suppression effect could partially cancel those excess power at large scales. We perform a global analysis to constrain the jump features of both the scalar and tensor fluctuations from the Planck+WP and BICEP2 data. Our results reveal that the CMB data favor the bounce inflation model at about 3.5\( \sigma \) confidence level, namely \( \log_{10}(k_{b}/\text{Mpc}^{-1}) = -2.4 \pm 0.2 \) (68% C.L.) and \( r_{B} = 0.71 \pm 0.09 \) (68% C.L.), when using Planck+WP and BICEP2 together. The bounce inflation model can simultaneously explain the Planck and BICEP2 data very well.

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