SUPERRIGIDITY FROM CHEVALLEY GROUPS INTO ACYLINDRICALLY HYPERBOLIC GROUPS VIA QUASI-COCYCLES.

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Abstract. We prove that every homomorphism from the elementary Chevalley group over a finitely generated unital commutative ring associated with reduced irreducible classical root system of rank at least 2, and ME analogues of such groups, into acylindrically hyperbolic groups has an absolutely elliptic image. This result provides a non-arithmetic generalization of homomorphism superrigidity of Farb–Kaimanovich–Masur and Bridson–Wade.

1. Main result

The celebrated Farb–Kaimanovich–Masur superrigidity theorem [KM96, FM98] states that every homomorphism from an (irreducible) higher rank lattice into $\text{MCG}(\Sigma_g)$, the mapping class group of a closed oriented surface $\Sigma_g$ of genus $g$, has finite image. Later, Bridson and Wade [BW11] have showed that the same superrigidity remains true if the target group is replaced with $\text{Out}(F_N)$, the outer automorphism group of a (non-abelian) free group $F_N$ of finite rank. In an unpublished manuscript of [Mimb], the present author obtained a similar homomorphism superrigidity from (commutative) universal lattices and symplectic universal lattices, that means, groups of the form $\text{SL}(n,\mathbb{Z}[x_1,\ldots,x_k])$ with $n \geq 3$; and $\text{Sp}(2n,\mathbb{Z}[x_1,\ldots,x_k])$ with $n \geq 2$, where $k$ finite. In this paper, we present a full generalization of this homomorphism superrigidity, as follows.

Theorem 1.1. Let $\Phi$ be a reduced irreducible classical root system of rank at least 2. Let $R$ be a finitely generated, unital, commutative, and associative ring. Let $\Gamma$ and $\Lambda$ be as follows:

- The $\Gamma$ is a quotient of a finite index subgroup of the (simply connected) elementary Chevalley group $E(\Phi, R)$;
- The $\Lambda$ is measure equivalent to $\Gamma$ with the $L_2$-integrability condition on a corresponding ME cocycle from $\Gamma$ to $\Lambda$.

Then the following hold true.

(i) For any acylindrically hyperbolic group $G$, every group homomorphism from $\Lambda$ into $G$ has an absolutely elliptic image $H$. That means, any acylindrical...
Remark 1.2. That for any finite index subgroup has a normal subgroup that surjects onto \( R, N > 0 \) such that for every two points \( x, y \in S \) with \( d(x, y) \geq R \), there are at most \( N \) elements \( g \in G \) which satisfy \( d(x, gx) \leq \epsilon \) and \( d(y, gy) \leq \epsilon \). And an acylindrical \( G \)-action by isometries on a hyperbolic geodesic space \( S \) is said to be non-elementary if the limit set of \( G \) on the Gromov boundary \( \partial S \) contains more than 2 points. The class of acylindrical hyperbolic groups contains \( \text{MCG}(\Sigma_g) \) and \( \text{Out}(F_N) \) for \( g \geq 1 \) and \( N \geq 2 \) (for more examples, see [Osi] Appendix A).

Remark 1.3. The acylindrical hyperbolicity can be also characterized in terms of hyperbolically embedded subgroups, see [Osi] Theorem 1.2. The reader familiar with relative hyperbolic groups might think that the conclusion in item (i) of Theorem 1.1 may be stated in terms of such subgroups (not in terms of the absolute ellipticity). However, Theorem 7.7 in [MO] implies that it is impossible in general.
2. STRATEGY OF THE PROOF OF THEOREM 1.1

The proof of Theorem 1.1 consists of the following two parts.

**Theorem 2.1.** Let $\Lambda$ be as in Theorem 1.1. Then $\Lambda$ has property (TT)$_{wm}$.

**Proposition 2.2.** Let $\Lambda$ be a group. If $\Lambda$ has property (TT)$_{wm}$, then every homomorphism from $\Lambda$ into an acylindrically hyperbolic group has an absolutely elliptic image.

In this paper, we introduce the notion of Property (TT)$_{wm}$ above, which is defined in terms of quasi-(1-)cocycles into weakly mixing unitary representations. It is strictly stronger than Kazhdan’s property (T). We discuss it in details in Section 3.

Another generalization of use of quasi-cocycles to homomorphism superrigidity, observed in [Mimb], is obtained by Burger and Iozzi [BI]. They introduce the $\ell_2$-stability, and prove certain type of homomorphism superrigidity of $\ell_2$-stable groups into $\text{MCG}(\Sigma_g)$. Some infinite groups coming from products of trees have the $\ell_2$-stability, and they never have property (T) (thus, nor (TT)$_{wm}$). Hence, their result applies to more groups. The $\ell_2$-stability, on the other hand, is not stable under the measure equivalence.

3. PROPERTY (TT)/T AND PROPERTY (TT)$_{wm}$

As we mentioned in Section 2, we employ property (T)-like properties for the proof of Theorem 1.1. We refer the reader to [BdlHV08] for a comprehensive treatise on Kazhdan’s property (T).

Let $\Gamma$ be a countable discrete group, and $(\pi, \mathcal{H})$ be a unitary $\Gamma$-representation. A map $c: \Gamma \to \mathcal{H}$ is called a quasi-(1-)cocycle into $\pi$ if

$$\sup_{g, h \in \Gamma} \| c(gh) - c(g) - \pi(g)c(h) \| < \infty.$$ 

This quantity is called the defect of $c$. Quasi-(1-)cocycles are related to (second) bounded cohomology, see [Mon01] for details. Recall that $\Gamma$ has Kazhdan’s property (T) if and only if any genuine cocycle (namely, any quasi-cocycle with defect 0) into any unitary representation $\pi$ is bounded (this is the Delorme–Guichardet theorem. see [BdlHV08] Theorem 2.12.4 and Proposition 2.2.9); and that $\Gamma$ is said to have Monod’s property (TT) if any quasi-cocycle into any unitary representation $\pi$ is bounded. Recall in addition that a unitary representation $\pi$ is said to be weakly mixing if it does not contain any finite dimensional subrepresentation.

**Definition 3.1.** (1) A group $\Gamma$ is said to have property (TT)/T (“property TT modulo T”) if any quasi-cocycle into any unitary $\Gamma$-representation which does not contain trivial representation is bounded.

(2) A group $\Gamma$ is said to have property (TT)$_{wm}$ (“property TT for weakly mixing representations”) if any quasi-cocycle into any unitary $\Gamma$-representation which is weakly mixing is bounded.

Clearly, the following implications hold:

$$(\text{TT}) \Rightarrow (\text{TT})/\text{T} \Rightarrow (\text{TT})_{\text{wm}} \Rightarrow (\text{T}).$$
Indeed, the right implication follows from the fact that any non-Kazhdan group admits an unbounded genuine cocycle into some weakly mixing representation (this follows from [BdlHV08, Theorem 2.12.9] and a Guichardet-type argument [BdlHV08, Proposition 2.12.2(ii)]). The right implication cannot be reversed, because any infinite hyperbolic group fails to have property (TT)$_{wm}$ (also see Section 4). Burger and Iozzi [BI] construct a counterexample to the converse implication to the left one. The $\ell_2$-stability in the sense in [BI] is implied by property (TT)$_{wm}$ by the next proposition. It might be open whether the middle implication can be reversed.

**Proposition 3.2.** (1) Property (TT)/T and property (TT)$_{wm}$, respectively, pass to group quotients and to finite index subgroups.

(2) If $\Gamma$ has property (TT)/T, and if $\Lambda$ is measure equivalent to $\Gamma$ with the $L_2$-integrability condition on an ME cocycle from $\Gamma$ to $\Lambda$, then $\Lambda$ has property (TT)$_{wm}$.

**Proof.** Item (1) is straightforward from the pull-back and induction of quasi-cocycles. Item (2) follows from the $L_2$-induction of quasi-cocycle for the ME coupling $(\Lambda, \Gamma)$ (this is possible by the $L_2$-integrability of an ME-cocycle), and from a deep fact in [MS06, Theorem 4.4]. Here we observe that if the $L_2$-induction for the ME coupling $(\Lambda, \Gamma)$ of a weakly mixing unitary $\Lambda$-representation is a unitary $\Gamma$-representation which does not contain trivial representation (see [Fur11, Subsection 4.1.1]).

The next lemma is the key observation to proving property (TT)/T for elementary Chevalley groups, and is a development of [Mim11, Proposition 6.6]. Here we say that a pair $\Gamma \supseteq Z$ of a group and a subset is said to have relative property (TT) if any quasi-cocycle into any unitary $\Gamma$-representation is bounded on $Z$. For two subsets $e \in X \subseteq \Gamma$, $Y \subseteq \Gamma$ of a group, we say that $Y$ is **boundedly generated** by $X$ if there exists an integer $N$ such that $Y \subseteq X^N$ holds true (the right-hand side denotes the set of all the products of possibly overlapping $N$ elements in $X$).

**Lemma 3.3.** Let $\Gamma$ be a countable discrete group, $\Gamma_0$ be a subgroup of $\Gamma$, and $Z$ be a subset of $\Gamma$. Assume that the triple $(\Gamma, \Gamma_0, Z)$ satisfies the following three conditions.

(i) The $Z$ generates $\Gamma$;

(ii) For any $h \in \Gamma_0$, $h^{-1}Zh \subseteq Z$;

(iii) The $\Gamma$ is boundedly generated by $\Gamma_0 \cup Z$.

Assume, furthermore, that the pair $\Gamma \supseteq Z$ has relative property (TT) and that $\Gamma$ has property (T). Then, $\Gamma$ has property (TT)/T.

**Proof.** Let $(\pi, \mathcal{H})$ be a unitary $\Gamma$-representation that does not contain trivial representation, and $c: \Gamma \to \mathcal{H}$ is a quasi-cocycle. Let $C < \infty$ be the maximum of the defect of $c$ and $\sup_{z \in Z} \|c(z)\|$.

We claim that $c(\Gamma_0)$ is bounded. Indeed, for any $h \in \Gamma_0$ and $z \in Z$, we have that by (ii),

\[
\|\pi(z)c(h) - c(h)\| \leq \|c(\pi(z)h) - c(h) - c(z)\| + C
\]

\[
\leq \|c(h(h^{-1}z)h) - c(h) - c(z)\| + C \leq \|c(h) - c(h)\| + 4C = 4C.
\]
Suppose, in the contrary, that \( c(\Gamma_0) \) is unbounded, and take \( \{h_n\}_n \subseteq \Gamma_0 \) such that \( \|c(h_n)\| \to \infty \). Then, \( \{c(h_n)/\|c(h_n)\|\}_n \) forms a sequence of almost invariant vectors by (i), but this is absurd because \( \Gamma \) has property (T) (recall the original definition of property (T) in terms of spectral gaps, see [BdlHV08, Definition 1.1.3]). Hence, \( c(\Gamma_0) \) is bounded.

Finally, we deduce the boundedness of \( c(\Gamma) \) with the aid of bounded generation (item (iii)).

\[ \square \]

The next theorem is the essential part in the present paper, and may be regarded as a strengthening of [EJZK, Theorem 1.1] for elementary Chevalley groups. Note that, however, because we employ property (T) for such groups in the proof below, our theorem is deeply based on their result.

**Theorem 3.4.** Let \( \Phi \) and \( R \) be as in Theorem [1.1]. Then \( E(\Phi, R) \) has property (TT)/T.

One of the main ingredients of the proof of this theorem is to utilize a very weak form of bounded generation (other than condition (iii) in Lemma 3.3), as we will discuss in the proof.

Proof. Set \( \Gamma = E(\Phi, R) \), \( \Gamma_0 = \Gamma \), and \( X = X(\Phi, R) \), which denotes the set of all elementary root unipotents (for instance, if \( \Phi = A_{n-1} \), then \( X \) is the set of all elementary matrices in \( E(n, R) \)). Let

\[ Z := \bigcup_{h \in \Gamma_0} h^{-1}Xh \subseteq \Gamma. \]

We claim that this triple \((\Gamma, \Gamma_0, Z)\) fulfills all of the assumptions in Lemma 3.3. Indeed, items (i)–(iii) are by definition (for (iii), observe that \( \Gamma_0 = \Gamma \)). Property (T) for \( \Gamma \) is a celebrated result by Ershov, Jaikin-Zapirain, and Kassabov [EJZK, Theorem 1.1].

It remains to verify that \( \Gamma \supseteq Z \) has relative property (TT) to prove the claim above. To see the assertion above, firstly, we show relative property (TT) for \( \Gamma \supseteq X \). This follows from the combination of [EJZK, Theorem 7.10 and Corollary 7.11] (see also arguments above Theorem 7.12 there and in the proof of it) with [Oza11, Proposition 3: (1) \( \Rightarrow \) (2)]. Here we do not go into the definition of (relative) property (TTT) of Ozawa, but we only remark that this implies (relative) property (TT). Secondly, we wish to deduce relative property (TT) for \( \Gamma \supseteq Z \) from that for \( \Gamma \supseteq X \). This deduction works if the following condition is fulfilled:

(condition): \( Z \) is boundedly generated by \( X \).

For \( \Phi = A_{n-1} \), [PW95, Lemma 2.3, Lemma 2.6, and Corollary 2.7] confirms this condition. For \( \Phi = C_n \), [Kop78, Section 1] does. In general case, Stepanov [Ste, Corollary 9.2] ensures the condition above. In fact, Stepanov pointed out to the author that to prove the (condition) above, we do not need to make full power of his result. It suffices to combine the normality of \( E(\Phi, R) \) in the Chevalley group established by Taddei with the argument in the proof of [Ste, Theorem 9.1].

Thus, we have showed that the triple \((\Gamma, \Gamma_0, Z)\) above fits the bill. \[ \square \]
Remark 3.5. It might be open whether there exist unbounded \textit{quasi-homomorphisms} (namely, quasi-cocycles into $(1_{\Gamma}, \mathbb{R})$) on $E(\Phi, R)$, even for $\Phi = A_{n-1}$ and $R = \mathbb{Z}[[x]]$. A partial result is given by the author [Mim10].

Another remark is on \textit{noncommutative universal lattices}. When $\Phi = A_{n-1}$, it is possible to drop the commutativity assumption on $R$. The group $E(n, \mathbb{Z}(x_1, \ldots, x_k))$ with $n \geq 3$ and $k$ finite is called the noncommutative universal lattice. Osin pointed out that it might be open whether noncommutative universal lattices are acylindrically hyperbolic. The gap to apply Theorem 3.4 is that the lack of the weak bounded generation, namely, there is \textit{no} reason to believe that the (condition) holds in this case. The current status of the question of whether noncommutative universal lattices have property (TT)$T$ might be open. In a forthcoming work [Mima], on the other hand, we show that noncommutative universal lattices with $n \geq 4$ have the fixed point property relative to $L_p$-spaces for any $p \in [1, \infty)$.

\textbf{Proof of Theorem 2.1.} Combine Theorem 3.4 with Proposition 3.2. □

4. \textsc{Comparison with acylindrical hyperbolicity}

In contrary to groups with property (TT)$_{wm}$, Harmenstäd [Ham08] showed that any non-elementary subgroup $H$ of $\text{MCG}(\Sigma_g)$ has plenty of unbounded quasi-cocycles into $(\lambda_H, \ell_2(H))$ (the left-regular representation). It turns out that her theorem applies to any acylindrically hyperbolic groups, see [Osi, Theorem 8.3].

Before proceeding into the proofs, we recall that property (TT)$_{wm}$ implies property (T), and that group homomorphism from a discrete group with property (T) into a discrete amenable group must have finite image.

\textbf{Proof of Proposition 2.2.} By [Osi] Theorem 1.1], the image $H$ is either elliptic, virtually $\mathbb{Z}$, or acylindrically hyperbolic. Because $\Lambda$ and hence $H$ have, in particular, property (T), the second option is impossible. Suppose, in contrary, that $H$ is acylindrically hyperbolic. Then by the argument above (see also [HO13, Corollary 1.5] and the main theorem of [BBF]), there must exist unbounded quasi-cocycles into $(\lambda_H, \ell_2(H))$. Note that because such an $H$ is infinite, $\lambda_H$ is weakly mixing (in fact, strongly mixing). This contradicts property (TT)$_{wm}$ for $H$ (see also Proposition 3.2).

Therefore, $H \leq G$ must be absolutely elliptic. □

\textbf{Proof of Theorem 2.1.} Item (i) immediately follows from Theorem 2.1 and Proposition 2.2. To show item (ii), we employ subgroup classifications of $\text{MCG}(\Sigma_g)$ and of $\text{Out}(F_N)$. Theorem 8.10 in [DGO] shows that if $H \leq \text{MCG}(\Sigma_g)$ is not virtually abelian, then there exists a finite index subgroup $H_0 \leq H$ which maps onto an acylindrically hyperbolic group. Hence, by item (i), which we have verified, and Proposition 3.2, the image of $\Lambda$ must be virtually abelian. Again, property (T) implies that the image must be finite. In the $\text{Out}(F_N)$-target case, we need more care. However, the argument in the proof of [BW11, Proposition 2.1] remains to work without any essential change. Indeed, the absolute ellipticity excludes all fully irreducible automorphism classes, and then property (T) may take place of the $\mathbb{Z}$-aversion. Finally, [BW11, Corollary 2.9] implies that $H_0$ (and hence $H$) are finite, which completes our proof. □
Remark 4.1. Osin pointed out to the author that, in order to show only that $E(\Phi, R)$ in Theorem 1.1 itself is not acylindrically hyperbolic, we do not need to appeal to property (T) for that group. More precisely, he provided the author with the following lemma.

Lemma 4.2 (Osin). Let $\Gamma$ be a group, $\Gamma_0$ a subgroup of $\Gamma$, $Z$ a subset of $\Gamma$. Assume that $(\Gamma, \Gamma_0, Z)$ satisfies (i'): the subgroup $\langle Z \rangle \leq \Gamma$ generated by $Z$ is not virtually cyclic; and conditions (ii) and (iii) in Lemma 3.3. If, furthermore, $\Gamma \supseteq Z$ has relative property (TT), then $\Gamma$ is not acylindrically hyperbolic.

We exhibit a sketch of Osin’s proof. For the terminologies below, see [Osi].

Proof. By the way of contradiction, assume that $\Gamma$ acts acylindrically and non-elementarily on a hyperbolic geodesic space $S$. By [Osi, Theorem 1.1], there are 3 cases: $\Gamma_0$ is non-elementary with respect to the action on $S$; $\Gamma_0$ is elliptic; or, $\Gamma_0$ is virtually cyclic and contains a loxodromic element $h$.

We deal with the first case. By [DGO, Lemma 6.5 and Theorem 6.14], we can find a loxodromic element $h \in \Gamma_0$ such that $E(h) = \langle h \rangle \times K(\Gamma)$. Here $E(h)$ is the maximal elementary (= virtually cyclic) subgroup of $\Gamma$ containing $h$, and $K(\Gamma)$ is the maximal finite normal subgroup of $\Gamma$. By (i'), there exists $z \in Z$ such that $z \notin E(h)$. Let $q$ be the quasi-cocycle from $\Gamma$ into $(\lambda_{\Gamma/E(h)}, \ell_2(\Gamma/E(h)))$, provided by [HO13], which extends the natural homomorphism $E(h) \to E(h)/K(\Gamma) \cong \mathbb{Z}$. Then, from the construction of $q$ in [HO13] (see the formula above Lemma 4.7 therein), $\|q(h^{-n}zh^n)\| \to \infty$ as $n \to \infty$. This contradicts (ii) and relative property (TT) in the assumption.

Then, we discuss the second and third cases. In the second case, by the construction of quasi-cocycles in [HO13], we can easily obtain an unbounded quasi-cocycle $q$ from $\Gamma$ into $(\lambda_{\Gamma}, \ell_2(\Gamma))$ that is bounded on $\Gamma_0$. This contradicts (iii) and relative property (TT). The third case may be reduced to the second one. Indeed, in that case, $E(h) \hookrightarrow \Gamma$ by [DGO, Theorem 6.8]. Now by [Osi] Theorem 5.4], we can construct a non-elementary acylindrical action of $\Gamma$ on another hyperbolic space such that $E(h)$ (and hence $\Gamma_0$) is elliptic with respect to this action.

From Lemma 4.2, if we know that all of the finite index subgroups of $\Gamma$ have finite abelianization, then we have the same conclusion as items (i) and (ii) of Theorem 1.1 for $\Gamma = E(\Phi, R)$, without appealing to property (T) (this condition on abelianization is needed to exclude the case that the image is virtually $\mathbb{Z}$). To have the full result in Theorem 1.1 for $\Lambda$, we may need to employ property (TT)/T and property (TT)$_{wm}$, as we have seen in the present paper. This is similar to the original situation in the Farb–Kaimanovich–Masur and the Bridson–Wade superrigidity that the case of non-uniform lattices is easier and that the case of uniform lattices is harder.

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