Scaling up Memory-Efficient Formal Verification Tools for Tree Ensembles

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\textbf{Abstract.} To guarantee that machine learning models yield outputs that are not only accurate, but also robust, recent works propose formally verifying robustness properties of machine learning models. To be applicable to realistic safety-critical systems, the used verification algorithms need to manage the combinatorial explosion resulting from vast variations in the input domain, and be able to verify correctness properties derived from versatile and domain-specific requirements.

In this paper, we formalise the VoTE algorithm presented earlier as a tool description, and extend the tool set with mechanisms for systematic scalability studies. In particular, we show a) how the separation of property checking from the core verification engine enables verification of versatile requirements, b) the scalability of the tool, both in terms of time taken for verification and use of memory, and c) that the algorithm has attractive properties that lend themselves well for massive parallelisation.

We demonstrate the application of the tool in two case studies, namely digit recognition and aircraft collision avoidance, where the first case study serves to assess the resource utilisation of the tool, and the second to assess the ability to verify versatile correctness properties.

\textbf{Keywords:} Formal verification \cdot Tree ensembles \cdot Safety-critical.

\section{Introduction}

With the exposure of a wide variety of artificial intelligence (AI) and machine learning (ML) techniques, the volume of works that study explainability in AI and promote methods to analyse ML-based approaches with respect to trustworthiness of explanations are increasing exponentially \cite{1}. A growing number of surveys dealing with the notion of explainability \cite{2,15} show that the concept has many dimensions, e.g., interpretability, accountability, and transparency. When trustworthiness is discussed in this context, one considers that explainability increases trust in the decisions made by intelligent systems. However, there are concerns about the “risk of being forced to explain the explanations” \cite{1}, which calls for formalised approaches to quantitative assessments of explainability.
In our work, we begin with the classic premise that trustworthiness in safety-critical applications is of prime concern, and that methods that increase the confidence in the systems are worth exploring. Applying formal verification to show that a system exhibits its intended behaviour is promoted as a means of increasing its trustworthiness. When ML components are envisaged in a system, formal verification may additionally increase the degree of explainability of a system’s behaviour. However, to make a meaningful contribution in this direction, one must first apply the essential rigour that is the backbone of performing comparative performance studies. The use of ML methods makes some of these rigorous analyses harder to perform due to the multiple dependencies on the data used to train models, the internal workings of the increasingly complex and opaque ML models, the set of properties of interest that can be defined in slightly different ways, and large system state spaces demanding large amounts of resources during analysis.

This paper studies formal verification of tree ensembles, a class of ML models considered less opaque than other ML models, thus possessing more intrinsic explainability. The paper is centred around the tool VoTE, that uses abstract interpretation to reduce the search space when verifying a given property, and was presented informally earlier [19]. We aim to show that, in addition to building on well-known mathematical concepts (conservative approximations and abstraction-refinement) that make the verification algorithm sound and complete, the tool set can be extended towards useful practical properties that are a result of architectural and algorithmic design decisions. Most importantly, that systematic performance analysis is enabled in presence of multiple input languages for the trained ML models. Also, verification of different requirements are possible due to the decoupling of the property checking and the core abstraction-refinement engine. These basic tool characteristics are exploited for systematic empirical experiments showing its time and memory scalability, and ability to verify different types of requirements. The contributions of the paper are as follows:

3 Source code and data is available at https://github.com/john-tornblom/VoTE-scaling-experiments

- We describe extensions of VoTE that show how benchmarking of its performance can be performed transparently by providing a model-as-input translation validation mechanism, which facilitates systematic comparisons with other tools in a fair setting.
- We use time and memory as metrics of scalability, and quantify the resource efficiency of VoTE in comparison with a state of art baseline. Using insights from this comparison, we then show that parallelisation of VoTE computations is possible with little additional effort.
- We illustrate that VoTE is capable of verifying properties different from robustness (treated earlier [19]) through the modularity of the property checker. This is done by replicating the verification of ten properties of an aircraft collision avoidance system previously studied in the context of neural networks [11].
The paper is structured as follows. Section 2 introduces the fundamental concepts used in the paper, e.g., tree ensembles, robustness, and abstract transformers, and Section 3 relates our contributions to earlier work. In Section 4 we formalise the VoTE core algorithm, and propose a couple of extensions to VoTE and its analysis environment. In Section 5 we use two case studies to show the scalability and versatility of VoTE, and Section 6 concludes the paper.

2 Background

In this section, we introduce decision trees, and describe how they are used as main building blocks in a more advanced class of ML models called tree ensembles. We then define robustness against input perturbations, a property used in Section 5.1 for our systematic performance comparisons since the state of art so far is focused on this property. Finally, the section ends with a brief description of abstraction interpretation.

2.1 Decision Trees

In machine learning, decision trees are used as predictive models to capture statistical properties of a system. Let \( \{X_1, \ldots, X_k\} \) be a partition of the \( n \)-dimensional input domain \( Q^n \). A decision tree is then defined as a set of \( k \) pairs \( T = \{(X_1, \tilde{y}_1), \ldots, (X_k, \tilde{y}_k)\} \), where \( \tilde{y}_i \) is an \( m \)-dimensional value from the output domain \( Q^m \), and a prediction function \( t : Q^n \rightarrow Q^m \) that maps values from the input domain to values from the output domain, i.e.,

\[
t(\bar{x}) = \begin{cases} 
\tilde{y}_1 & \bar{x} \in X_1, \\
\vdots & \\
\tilde{y}_k & \bar{x} \in X_k.
\end{cases}
\] (1)

2.2 Tree Ensembles

Decision trees are known to suffer from overfitting, i.e., the model becomes too specialized towards training data, and the prediction function generalizes poorly when confronted by previously unseen inputs. To counteract this phenomenon, several types of tree ensembles have been proposed, e.g., random forests [4] and gradient boosting machines [8]. The different types of tree ensembles are normally distinguished by the used learning algorithms, while their prediction functions have similar structure. Let \( F = \{T_1, \ldots, T_B\} \) be a set of \( B \) decision trees. The prediction function of a tree ensemble is then defined as

\[
f(\bar{x}) = p \left( \sum_{i=1}^{B} t_i(\bar{x}) \right),
\] (2)

where \( p \) is a post-processing function. In random forests, the post-processing function divides the sum of trees with the number of trees, while gradient boosting machines trained on classification problems use the softmax function to post-process the sum of trees.
2.3 Classifiers

When tree ensembles are trained to predict probabilities, they can be combined with the argmax function to form a classifier. Let \( f(\bar{x}) = (y_1, \ldots, y_m) \) be a tree ensemble trained to predict the probability \( y_i \) that a given input maps to a class \( i \), where \( m \) is the number of classes. A classifier \( f_c \) is then defined as

\[
f_c(\bar{x}) = \arg\max_{i \in \{1, \ldots, m\}} y_i.
\] (3)

2.4 Robustness against Input Perturbations

In the context of machine learning, robustness describes the ability of a system to maintain the correct prediction despite noisy or adversarial input. Let \( f_c \) be a classifier subject to verification, \( X \subset Q^n \) a set of input samples with label \( l \in \{1, \ldots, m\} \), \( \epsilon \in Q \geq 0 \) a robustness margin, and \( \Delta = \{\delta \in Q : -\epsilon < \delta < \epsilon\} \) a set of possible input perturbations. We denote by \( \delta \) an \( n \)-tuple of elements drawn from \( \Delta \). The classifier \( f_c \) is robust with respect to \( X \) and \( \epsilon \) iff

\[
\forall \bar{x} \in X, \forall \delta \in \Delta^n, \quad f_c(\bar{x}) = f_c(\bar{x} + \delta) = l.
\] (4)

2.5 Abstract Interpretation

Abstract interpretation is a framework to facilitate sound and efficient reasoning about programs being analysed by a compiler [5]. The idea is to transform the source code of a program that computes values in a concrete domain into functions that operate in one or more abstract domains where some analyses of interest are faster than in the concrete domain, but potentially less precise.

For example, an abstraction function \( \alpha : \wp(Q) \to A \) is used to map a set of rational numbers \( X \subseteq Q \) to an abstract element \( \hat{x} \in A \), where \( \wp(Q) \) denotes the power set over \( Q \). Analogously, a concretisation function \( \gamma : A \to \wp(Q) \) is used to map an abstract value \( \hat{y} \in A \) to a set of rational numbers \( Y \subseteq Q \). Abstraction and concretisation mappings that have a certain property called Galois connection lead to sound reasoning with abstract interpretation [5], i.e.,

\[
\forall X \in \wp(Q), X \subseteq \gamma(\alpha(X)),
\]

\[
\forall \hat{x} \in A, \hat{x} = \alpha(\gamma(\hat{x})).
\] (5)

To perform the analysis, the program is interpreted in the abstract domain by evaluating sequences of abstract values, operators, and transformers. An abstract transformer \( \hat{f} : A \to A \) is conservative with respect to a concrete function \( f : Q \to Q \) iff

\[
\forall X \in \wp(Q), \forall x \in X, \quad f(x) \in \gamma(\hat{f}(\alpha(X)))).
\] (6)

An example operator on the abstract domain we use in Section [4] is the join operator \( \sqcup \), which is conservative with respect to the set intersection operator \( \cap \), i.e., \( \gamma(\hat{x}_1) \cap \gamma(\hat{x}_2) \subseteq \gamma(\hat{x}_1 \cap \hat{x}_2) \).
3 Related Works

The formal methods community has a long tradition of rigorous and systematic tool evaluations, often arranged in the form of competitions [3]. These arrangements help to ensure that tools are evaluated on a fair and transparent basis, and that experiments are reproducible. Unfortunately, there is no well-established competition for formal verification of machine learning models. However, Nicolaee et al. [14] present the Adversarial Robustness Toolbox, a library devoted to the evaluation of different learning and verification algorithms in the context of robustness. In this paper, we broaden their scope beyond the robustness property.

Liu et al. [13] present a comparison between many different verification algorithms designed specifically for neural networks, as well as evaluating their runtime performance when verifying robustness in a digit recognition case study [12], and domain-specific requirements of an aircraft collision avoidance system [11]. In this paper, we provide the means to perform transparent and systematic evaluations for tools designed specifically to verify tree ensembles. We reuse the above two case studies, but with some additional work. In particular, we verify robustness of the entire test set in the digit recognition case study, as opposed to a single image as was done in the named study. Since the data used to train the aircraft collision avoidance system is not publicly available (but only the resulting neural network), we use the published models as an oracle to sample data that we use to train tree ensembles. We then verify the properties of the created tree ensembles using the same domain-specific properties originally suggested by Katz et al. [11] to verify neural networks.

For tree ensembles, only a few formal verification tools have been published. Our own earlier work presents the tool VoTE with some heuristic studies, but no comparative scalability analyses [20]. Later work [19] presents the adaptation of the tool with abstraction-refinement, with an emphasis on robustness verification. In this paper, we formalise the algorithm used in the tool engine, and extend the tool suite with means for systematic studies and rigorous comparisons with related state-of-art. Analysis of memory complexity is exploited for parallel executions that show memory efficiency results and timing improvements. Moreover, we extend the set of properties verified to show the versatility of the tool’s property checking component.

Einziger et al. [7] present the tool VeriGB for verifying the robustness of gradient boosting machines. They encode the verification problem as an SMT formula, and use an SMT solver for verification. They design the SMT formula to facilitate parallel analyses for different counter-examples. Similarly, Davos et al. [6] also use an SMT solver to verify tree ensembles, but partition the input space into regions which are analysed in parallel. In this paper, we take a similar approach to parallelism as Davos et al.

Ranzato and Zanella [17] present a tool called Silva, which is based on a similar abstraction-refinement approach as proposed in [19]. Their paper is, to the best of our knowledge, the only work where run-time performance of several verifiers for tree ensembles is assessed. However, their work did not include
memory efficiency as a metric. Also, their analysis of timing did not compare verification on same artefacts (there were differences in models and timeouts), and used different metrics for comparative tools. This is rectified in our work with the extension to the VoTE tool suite so that systematic comparisons can be performed.

4 Formalising and Extending VoTE

In this section, we begin by providing an algorithmic overview of VoTE that formalises the earlier tool description [19] in the abstract interpretation framework. This helps us to understand the empirical results from scalability assessments conducted later in Section 5.1. We then go on to propose a couple of extensions to VoTE and its analysis environment, which facilitate systematic scalability analyses (with alternative tools plugged in), and the ability to parallelise the analyses across multiple CPU cores.

4.1 The VoTE Algorithm

Let \( \hat{f} : A^n \rightarrow A^m \) be a transformer that is conservative with respect to the tree ensemble prediction function \( f \). The VoTE algorithm can then be characterised as a Boolean function as defined by Algorithm 1, where \( F = \{T_1, \ldots, T_B\} \) is the set of trees in the ensemble, \( T_i = \{(X_1, \bar{y}_1), \ldots, (X_k, \bar{y}_k)\} \) the \( i \)-th tree in the ensemble, \( \hat{x} \) an abstracted input region, \( \mathcal{P}(Q^n) \xrightarrow{\alpha} A^n \) a Galois connection, and \( c \) a property checker. The property checker ensures that concrete input values captured by the abstract input region \( \hat{x} \) are in the relevant relation to concrete output values captured by an abstract output region \( \hat{y} \). It either returns Pass when the inputs and outputs are in the relevant relationship, Fail when the absence of the desired relationship can be proved, and Unsure when this cannot be proved [19].

Given the input region \( \hat{x} \) defined by the abstraction function \( \alpha \) applied to a set of values from the input domain of a model, the VoTE algorithm uses the transformer \( \hat{f} \) to compute the abstract output region \( \hat{y} \) that approximates the image of the model for the given input region (line 2). Next, the algorithm invokes the property checker, which checks if the property is satisfied for the given inputs and outputs. If the property checking is inconclusive, the algorithm selects an arbitrary tree \( T \) from the set \( F \), and uses the removed tree’s input partition \( \{X_1, \ldots, X_k\} \) (see Equation 1) to refine \( \hat{x} \) into smaller input regions (line 10). This procedure is then recursively applied to smaller and smaller input regions, ending with a precise abstraction of the output when the set \( F \) is empty. In other words, the right hand side of Equation 6 returns a singleton, being the lowest level of abstraction.

The algorithm does not require any particular order in the selection of trees (line 9). Indeed, different selection orders can influence the time taken for verifying the model. In the implementation of VoTE, trees are selected in the order that they appear in the serialised model given as input to the tool. This is a
Algorithm 1 The Boolean verification algorithm implemented in VoTE, which takes as input a set of trees ($F$), an abstract input region ($\hat{x}$), and a property checker ($c$).

1: function VoTE($F, \hat{x}, c$)
2: $\hat{y} \leftarrow \hat{f}(\hat{x})$ \hspace{1cm} \triangleright \text{Interpret tree ensemble}
3: $o \leftarrow c(\hat{x}, \hat{y})$ \hspace{1cm} \triangleright \text{Check the property}
4: if $o = \text{Pass}$ then
5: \hspace{1cm} return True \hspace{1cm} \triangleright \text{Property checker is unsure}
6: else if $o = \text{Fail}$ then
7: \hspace{1cm} return False \hspace{1cm} \triangleright \text{Select a tree}
8: end if
9: $T \in F$ \hspace{1cm} \triangleright \text{Refine the abstraction}
10: return $\bigwedge_{(X_i, \bar{y}_i) \in T} \text{VoTE}(F \setminus \{T\}, \hat{x} \cap \alpha(X_i), c)$ \hspace{1cm} \triangleright \text{Select a tree}
11: end function

recurring question for many tools with heuristics governed by internal choices. In this case, the order can be changed by simply rearranging the trees in the serialised model given as input to the tool. Similarly, the order of selection of regions to refine ($X_i$ in line 10) is arbitrary in the algorithm. In the actual implementation of VoTE, a heuristic is applied (and has been described earlier [20]).

Note that the parameterisation of the property checker makes it easy to plug-and-play properties without modifying the core engine, an important capability when applying formal verification to realistic case studies with diverse requirements. We also note that the algorithm is recursive with a maximum recursion depth equal to the number of trees in the ensemble, meaning that the space complexity grows linearly with respect to the number of trees.

4.2 Extensions to VoTE

For a rigorous performance evaluation of a verification tool, we need to benchmark its scalability and verification outcomes in comparison with relevant baselines using relevant metrics. This can be challenging when tools use different formats for their input. To address this, we develop a translation validation scheme that validates the correctness of format conversions. Let $F$ be a set of trees serialised in an unsupported format, $conv$ a function that translates $F$ into a supported format, and its inverse being $conv^{-1}$. We can then validate that $conv$ translates $F$ correctly by checking that $F = conv^{-1}(conv(F))$. Hence to enable systematic evaluations of scalability, a new module has been added to the tool set for such input validations in a new benchmarking context. In Section 5.1, we illustrate this with a state-of-art tool (Silva) that uses a different input format compared to VoTE.

Given the attractive space complexity, we note that the VoTE algorithm can benefit from massive parallelisation. In particular, we are able to partition the set of inputs relevant to a property as a set $P$ with disjoint abstractions $\hat{x}_i$. We
can then invoke VoTE in parallel for each element in $P$, leading to the same outcome as a sequence of invocations to VoTE, but potentially faster.

For example, when verifying robustness as defined by Equation 4, we create a set $P$ with abstractions of hyperrectangles centred around points $\bar{x}_i \in X$, and then invoke the VoTE algorithm in parallel for each abstracted hyperrectangle and the robustness property checker $c_{\text{robust}}$ as defined by Algorithm 2, where $\bigwedge$ denotes parallel invocations.

**Algorithm 2** The parallel robustness verification algorithm, leveraging VoTE as its core verification engine.

```
1: function $\text{IsRobust}(F,X,\epsilon)$
2:   $\Delta \leftarrow \{\delta \in \mathbb{Q} : -\epsilon < \delta < \epsilon\}$     \hspace{1em} $\triangleright$ Set of possible input perturbations
3:   $P \leftarrow \{\alpha(\{\bar{x}_i + \delta\}) : \delta \in \Delta^n, \bar{x}_i \in X\}$     \hspace{1em} $\triangleright$ Set of disjoint abstractions
4:   return $\bigwedge_{\bar{x}_i \in P} \text{VoTE}(F,\bar{x}_i,c_{\text{robust}})$     \hspace{1em} $\triangleright$ Parallel invocations to VoTE
5: end function
```

5 Scalability studies

In this section, we use two case studies to show the scalability and versatility of VoTE. Scalability is shown both with respect to time and memory used for verification. For systematic benchmarking, we exploit the extensions in Section 4 to measure verification time in presence of the presented input validation scheme. This serves to bring comparable algorithms and relevant case studies to run on an equal footing. The case studies are from earlier works, a digit recognition system (Section 5.1 and 5.2), and an aircraft collision avoidance system (Section 5.3). The baseline used to illustrate our input validation scheme is the recently published verifier Silva [17]. We use a compute cluster running CentOS 7.8.2003, where each node is equipped with an Intel Xeon Gold 6130 CPU with 32 cores, and up to 384 GiB RAM.

5.1 Digit Recognition Case Study

This case study serves to assess the scalability of VoTE in terms of computational resources when verifying the robustness of models trained on the MNIST dataset [12]. The dataset contains 70,000 grey-scale images of hand-written digits with a resolution of $28 \times 28$ pixels at 8 bpp, split into a 85% training set and a 15% test set. Normally, the intention is to use the training set together with different learning algorithms and parameters to synthesise models. To be comparable with related work, however, we reuse models trained by Ranzato and Zanella [17] in their evaluation of Silva. In particular, we reuse gradient boosting machines trained using CatBoost [16] with the MultiClass loss function, and
random forests trained using scikit-learn [15] with the Gini impurity splitting criterion. These models are fed to our input format conversion tool Silva2VoTE, and validated using the translation validation scheme proposed in Section 4.2 to ensure correctness of the translations, as illustrated by Figure 1.

![Diagram](image)

**Fig. 1.** Our translation validation scheme applied to check equivalence of VoTE and Silva input models.

While applying the scheme, we notice that leaf values in VoTE-models are associated with 64 bit floating-point tuples, while leaves in Silva-models are associated with 32 bit integer tuples, which are normalised into 64 bit floating-point tuples during analysis. Consequently, we add leaf normalisation to the VoTE input chain so both tools can interpret the exact same input in the same manner. With these careful configurations in the experimental setup, we execute the translation validation system for each model, and observe no discrepancies.

**Scalability metrics.** Silva and VoTE both report the total elapsed time taken during verification, but use different measurement techniques. To bring them on the same footing, we use the GNU `time` command to launch experiments and measure the elapsed wall time. Furthermore, we use GNU `time` to measure the maximum resident memory during each experiment, i.e., the maximum amount of memory allocated on RAM (which excludes swap memory).

**Tool parameters.** When evaluating robustness, both Silva and VoTE are parameterised with a robustness margin (denoted $\epsilon$ in Equation 4). In addition, Silva is parameterised with a timeout which limits the amount of CPU time the tool spends on verifying the robustness on a particular image. To evaluate both tools on a fair basis, we ensure that VoTE analyses are stopped after the same timeout (as used in the earlier evaluation of Silva). To sum up, we use the same parametric values as previous works [19,17], i.e., $\epsilon = 1$, and a timeout of 60 seconds per image. All experiments are executed using the version of Silva with the git hash 9db65e58, and VoTE tagged as version 0.2.1.

**Scalability outcomes on a single core.** Table 1 lists the elapsed time, peak memory consumption, and number of unsolved images (due to timeouts), when Silva and VoTE are used to verify the robustness of random forests (RF) and
gradient boosting machines (GBM) with different number of trees (B) and tree depths (d) on a single CPU core.

Table 1. Verification of tree ensembles using Silva and VoTE on a single core.

| Model type | Model size | Elapsed time (s) Silva | Memory (MB) Silva | VoTE Silva | Unsolved |
|------------|------------|------------------------|-------------------|------------|----------|
| RF         | 25         | 3                      | 254               | 65         | 0        |
| RF         | 25         | 5                      | 1,680             | 21         | 4,479    | 96       | 0        |
| RF         | 50         | 5                      | 10,664            | 21         | 4,479    | 96       | 0        |
| RF         | 75         | 5                      | 9,710             | 21         | 4,479    | 96       | 0        |
| GBM        | 50         | 10                     | 2                 | 2          | 81       | 141      | 0        |
| GBM        | 75         | 10                     | 2                 | 2          | 81       | 141      | 0        |
| GBM        | 100        | 10                     | 8                 | 2          | 81       | 141      | 0        |
| GBM        | 150        | 10                     | 7,745             | 2           | 81       | 141      | 0        |

Contrary to measures from earlier work [17] (which were not carried out in the systematic manner as we describe above), we observe that VoTE is typically faster in this case study, and with one exception, consumes less memory than Silva, especially when verifying the larger models. In the most time consuming experiment, VoTE is 1.2 times faster, and consumes 755 times less memory than Silva. Furthermore, Silva times out on a few more images than VoTE (as shown in the Unsolved column). In those cases where Silva and VoTE solve the same number of images, both tools report the same number of robust samples.

5.2 Parallelised Robustness Verification

We next run the same experiments with the parallel algorithm proposed in Section 4.2, now leveraging 32 cores. Due to the large memory consumption observed when verifying robustness with Silva in the previous section, in this section we restrict the experiments to VoTE only. Table 2 lists the elapsed time and peak memory consumption of VoTE when utilising a single CPU core vs. using 32 CPU cores, presented in the same format as before.

When VoTE runs analyses in parallel, we observe near-linear speedups with respect to CPU core count. However, we also observe time-outs on a few more images compared to the single-core experiments, which we believe are caused by inter-core interference. In the most time consuming experiment, VoTE with multi-core capabilities is 26.5 times faster, consumes 1.5 times more memory, and times out on 11 more images compared to the same single core experiment.

5.3 Aircraft Collision Avoidance Case Study

We now turn our attention to an early prototype of an aircraft collision avoidance system called ACAS Xu from earlier works [11]. This case study has more diverse
Table 2. Verification of tree ensembles using VoTE, executed on one and 32 cores.

| Model type | Model size | Elapsed time (s) | Memory (MB) | Unsolved |
|------------|------------|------------------|-------------|----------|
|            |            | 1 core | 32 cores | 1 core | 32 cores | 1 core | 32 cores |
| RF         | 25         | 5 3  | 1 65  | 82  | 0 0  |
| RF         | 25         | 10 21 | 4 96  | 117 | 0 0  |
| RF         | 50         | 5 1,234 | 84 67 | 95 6 13 |
| RF         | 50         | 10 4,101 | 249 127 | 166 20 36 |
| RF         | 75         | 5 7,860 | 296 68 | 104 84 95 |
| GBM        | 50         | 10 2  | 1 141 | 153 | 0 0  |
| GBM        | 75         | 5 2  | 1 68  | 76 0 0  |
| GBM        | 75         | 10 8  | 2 117 | 130 | 0 0  |
| GBM        | 100        | 10 83 | 13 155 | 178 | 0 0  |
| GBM        | 150        | 10 6,795 | 282 231 | 267 68 79 |

and application-specific properties than the robustness property studied in the digit recognition case study. It thus highlights the flexibility of VoTE when confronted with more diverse requirements.

System description. The ACAS Xu system is in part developed using a dynamic programming process that yields a large lookup table. Each entry in the table maps state variables such as speed and distance between vehicles to costs associated with different actions. The action with the lowest cost is then given to the vehicle operator as an advisory, and can be one of the following: clear of conflict, turn weak left, turn weak right, turn strong left, or turn strong right. Since the lookup table is very large, the mappings are compressed using a ML model. In previous works [10], Julian et al. investigate the use of neural networks for the purpose of table compression. In what follows, we use gradient boosting machines in the same context.

Katz et al. [11] define ten properties ($\phi_{1-10}$) that the ACAS Xu system shall satisfy. For example, property $\phi_8$ states that, for large vertical separation and a previous weak left advisory, the system shall either output clear of conflict, or continue advising weak left. These ten properties alone are not sufficient for a complete safety argument, but are mere sanity checks that ought to hold. In this paper, we use these properties solely to illustrate VoTE versatility.

Dataset. Unfortunately, the data used to train the ACAS Xu system by Julian et al. [10] and later used by Katz et al. [11] is not publicly available. Only the implementation in the form of 45 neural networks are shared, each responsible for providing action scores in disjoint input regions. Subsequent work by Julian and Kochenderfer [9] include published data for a simplified variant of the ACAS Xu system, but without a formal specification of the requirements. Rather than adapting $\phi_{1-10}$ to the simplified system, we use the published neural networks as an oracle to sample data. In particular, we sample $2 \cdot 10^6$ input tuples uniformly across each disjoint input region, and execute the corresponding neural network
for each sample to obtain an advisory. Since the neural networks are known to violate some of the intended requirements, we test each input/output pair against the available specification ($\phi_{1-10}$), and resample any input-output pairs that violate some requirement. Hence, we obtain training data that we know will satisfy the requirement. Our goal is to show that a formal verifier will discover violations of requirements on any data on which the model is not trained.

**Systematic model synthesis.** We train gradient boosting classifiers with 200 trees of depth 10 on 50% of the sampled data using CatBoost. We set the learning rate to 0.5, and leave the remaining training parameters at their default value. We then validate the model to observe the accuracy of the classifiers on the remaining 50% of the data. This shows accuracies above 0.95 on all trained models. Since the CatBoost classification algorithm uses the argmax function to select a label, and the specification associates the minimal score with the best action, we negate the leaf values associated with each tree after training. We then validate that the trained models behave similarly to the oracle by plotting advisories for different downranges (distance in the direction of the vehicle) and crossranges (distance perpendicular to the direction of the vehicle).

Figure 2 depicts the different advisories given by the oracle and one of the trained gradient boosting machines, i.e., clear of conflict (COC), turn weak left (WL), turn weak right (WR), turn strong left (SL), or turn strong right (SR).

![Advisories](image)

**Fig. 2.** Advisories provided by the original neural network (top) and the gradient boosting machine (bottom).

We observe that the space captured by our sample-based model captures more or less the same behaviour as the original neural network. Thus, we can expect the same requirements to be valid for this system.

**Requirement verification.** We execute the verification of all trained models using the same equipment as in Section 5.1 with single core. Table 3 lists the
number of models that passed/failed each property, and the elapsed time and peak memory consumption measured using the GNU `time` command.

Table 3. Verification outcome for up to 45 gradient boosting machines of ten properties in the ACAS Xu case study, executed on a single CPU core.

| Property | $\phi_1$ | $\phi_2$ | $\phi_3$ | $\phi_4$ | $\phi_5$ | $\phi_6$ | $\phi_7$ | $\phi_8$ | $\phi_9$ | $\phi_{10}$ |
|----------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Pass     | 45      | 36      | 42      | 42      | 1       | 0       | 0       | 0       | 1       | 0       |
| Fail     | 0       | 4       | 0       | 0       | 0       | 1       | 1       | 0       | 1       | 0       |
| Elapsed time (s) | 35  | 24      | 26      | 26      | 1       | 1       | 26      | 634     | 1       | 1       |
| Memory (MB) | 208  | 208     | 208     | 208     | 209     | 209     | 187     | 208     | 208     | 208     |

Most properties take on average about a second per applicable model to verify, with the exception of property $\phi_7$ and $\phi_8$, which takes about half a minute and 10 minutes to verify, respectively.

6 Conclusions

This paper has begun the trajectory towards systematic and rigorous performance evaluation of formal verifiers for tree ensembles, bringing both time and memory into focus. Our work indicates that a formal underpinning for verifiers adds to trustworthiness of the verification outcomes and ultimately the transparency of the deployed models.

Our study of two of the latest tools for tree ensembles on equivalent models and similar metrics and running conditions helps to shed light on the scalability of the tools, both in terms of time and memory. Our extension of VoTE with the translation validation scheme for an input model is a generic mechanism that can be applied to all models used across various tools when launching rigorous performance studies. This work also shows that the memory footprint is a highly relevant scalability metric for model verifiers, and a key to parallelisation as demonstrated with the near-linear speedup in VoTE’s context.

While the ACAS Xu case study demonstrates that architectural design decisions in VoTE affect its ability to verify diverse requirements, clearly more work is needed to see whether this holds in new domains with new requirements. On the algorithmic front, more extensive studies with variations of the heuristics used for selection of refinement orders would be useful.

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