Large Scale Structure and Cosmological Waveguides

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Abstract. A sort of gravitational waveguide effect in cosmology could explain some anomalous phenomena which cannot be understood by the current gravitational lensing models as the existence of "twins" objects with similar spectra and redshifts posed on the sky with large angular distance. Furthermore, the huge luminosities of quasars could be explained using filamentary or planar cosmological structures acting as waveguides. We describe the gravitational waveguide theory and then we discuss possible realizations in cosmology.

1. Introduction

Besides traditional fields of astronomy and astrophysics, gravitational lensing can be today considered a fundamental tool to investigate the large scale structure of the universe and to test cosmological models. In some sense, we can say that a sort of "gravitational" astronomy is coming up. One of the most interesting characteristics of gravitational lensing is that it acts on all scales. It provides a great amount of cosmological and astrophysical applications like the determination of the Hubble parameter $H_0$ via the measurement of time delay $\Delta t$ between the observed lightcurves of multiply imaged extragalactic sources; the possibility of weighing the mass and describing the potential of lensing galaxies and galaxy clusters from the observation of multiply imaged quasars, arcs and arclets. Furthermore, the gravitational lensing plays a leading role in searching for dark matter at

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large scales since the frequency of multiply imaged sources (e.g., quasars) depends on the cosmological density parameter $\Omega_L$ of compact objects. Particularly promising are the multiply macro-imaged quasars whose lensing galaxy should have a large optical depth for lensing effects (at least 20 objects of this kind have been identified till now). The above kinds of analysis are possible if we have a model explaining the way of forming images such as the above-mentioned arcs, rings or simply double images and predicting the effects of the deflector. The gravitational lensing may be explained through the action of a weak gravitational field on the light rays. The action of media with corresponding refraction index is completely determined by the Newtonian gravitational potential which deflects and focuses the light rays. In optics, however, there exist other types of devices, like optical fibers and waveguides which use the same deflection phenomena. The analogy with the action of a gravitational field onto light rays may be extended considering also these other structures. In other words, it is possible to suppose the existence of a sort of gravitational waveguides [1]. On the other hand, structures like cosmic strings, texture and domain walls, which are produced at phase transition in inflationary models, can evolve into today observed filaments, clusters and groups of galaxies and behave in a variety of ways with respect to gravitational lensing effects. For example, a filament of galaxies can be considered a sort of waveguide preserving total luminosity of a source, if we have locally an effective gravitational potential of the form $\Phi(r) \sim r^2$, while the planar structures generated by the motion of cosmic strings (the so-called "wakes") can yield cosmological structures where the total flux of light is preserved and the brightness of objects at high redshift, whose radiation passes through such structures, appears higher to a far observer. In this note, we construct a waveguide model using the paraxial approximation and consider the effect of gravitational systems which, combined in filaments or in planes, results as waveguides. Finally, we discuss the eventual cosmological realization of such structures and the connection with observations.

2. The propagation of light in a weak gravitational field

The discussion of gravitational waveguide properties can be done starting from the electromagnetic field theory in a gravitational field described by the metric tensor $g_{\mu\nu}$ [2]. In this context, the behaviour of the electromagnetic field, without sources, may be described by the Maxwell equations

$$\frac{\partial F_{\alpha\beta}}{\partial x^\gamma} + \frac{\partial F_{\beta\gamma}}{\partial x^\alpha} + \frac{\partial F_{\gamma\alpha}}{\partial x^\beta} = 0; \quad \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\beta} \left( \sqrt{-g} F^{\alpha\beta} \right) = 0,$$

where $F^{\alpha\beta}$ is the electromagnetic field tensor and $g$ is the determinant of the four-dimensional metric tensor. For a static gravitational field, these
equations can be reduced to the usual Maxwell equations describing the electromagnetic field in media where the dielectric and magnetic tensor permeabilities are connected with the metric tensor \( g_{\mu\nu} \) by the equation

\[
\varepsilon_{ik} = \mu_{ik} = -g_{00}^{-1/2}[\det g_{ik}]^{-1/2}g_{ik}; \quad i, k = 1, 2, 3.
\]

(2)

If one has an isotropic model, the metric tensor is diagonal and the refraction index of "media" may be introduced by mimicking the gravitational field \( n(\mathbf{r}) = (\varepsilon \mu)^{1/2} \), (it is worthwhile to note that such a situation can be easily reproduced in cosmology). For weak gravitational fields, the metric tensor components are expressed in terms of the Newton gravitational potential \( \Phi \) as

\[
g_{00} \simeq 1 + 2 \frac{\Phi(\mathbf{r})}{c^2}; \quad g_{ik} \simeq -\delta_{ik} \left( 1 - 2 \frac{\Phi(\mathbf{r})}{c^2} \right);
\]

(3)

where we are assuming the weak field limit, \( \Phi/c^2 \ll 1 \), and the slow motion approximation \( |v| \ll c \). Then, due to relations (2) and (3), the refraction index \( n(\mathbf{r}) \) can be expressed in terms of the gravitational potential \( \Phi(\mathbf{r}) \) produced by some matter distribution. Such a weak field situation is realized for cosmological structures which give rise to the gravitational lensing effects connected to several observable phenomena (multiple images, magnification, image distortion, arcs and arclets). The same scheme can be applied also to string and planar-like distributions of matter giving rise to gravitational waveguide effects.

3. The Helmholtz Equation

A waveguide solution can be obtained by reducing the Maxwell equations in media to the Helmholtz scalar equations for the fields \( \mathbf{E} \) and \( \mathbf{H} \). Let us take into account the media contribution by the relations

\[
\mathbf{D}_\omega(\mathbf{r}) = \varepsilon(\omega, \mathbf{r})\mathbf{E}_\omega(\mathbf{r}), \quad \mathbf{B}_\omega(\mathbf{r}) = \mu(\omega, \mathbf{r})\mathbf{H}_\omega(\mathbf{r}),
\]

(4)

considering the single field components from Eq.(1). The subscript \( \omega \) means the Fourier amplitudes of the fields, i.e.

\[
\mathbf{E}(\mathbf{r}, t) = \int \mathbf{E}_\omega(\mathbf{r})e^{-i\omega t}d\omega,
\]

(5)

and analogously for \( \mathbf{D}, \mathbf{B}, \mathbf{H} \). Then, taking the Fourier transforms with respect to the time variable, and with a little algebra on the Maxwell equations in vectorial form, we get

\[
\triangle \mathbf{E}_\omega(\mathbf{r}) + \frac{\omega^2}{c^2} n^2(\omega, \mathbf{r}) \mathbf{E}_\omega(\mathbf{r}) = -\nabla \left[ \frac{\mathbf{E}_\omega(\mathbf{r}) \nabla \varepsilon(\omega, \mathbf{r})}{\varepsilon(\omega, \mathbf{r})} \right].
\]

(6)
where, as above, \( n^2(\omega, r) = \mu(\omega, r)\varepsilon(\omega, r) \), is the refractive index, and \( \Delta \) is the ordinary Laplace operator which can be used in the weak field approximation which we are considering. One can neglect the term in the rhs of Eq.\( (6) \) if it is much less than both terms in the lhs of the same relation. In fact, since \( \Delta = \nabla \cdot \nabla \) for distances of an order of the light wavelength \( \lambda \), both terms in the lhs of Eq.\( (6) \) (independently of the light polarization) are of the order
\[
|\Delta E_\omega(r)| \sim \lambda^{-2}E_\omega(r), \quad \frac{\omega^2}{c^2}n^2(\omega, r)E_\omega(r) \sim \lambda^{-2}E_\omega(r).
\]
(7)
The term depending on the light polarization interaction for the same distances is of the order
\[
\nabla \left[ \frac{E_\omega(r)\nabla\varepsilon(\omega, r)}{\varepsilon(\omega, r)} \right] \sim \lambda^{-2}\frac{\delta\varepsilon}{\varepsilon}E_\omega(r),
\]
(8)
where \( \delta\varepsilon \) is the change of the dielectric permeability for distances of the order of wavelength \( \lambda \). Comparing Eqs.\( (7) \) and \( (8) \), we conclude that for \( \frac{\delta\varepsilon}{\varepsilon} \ll 1 \), we can neglect the term depending on the light polarization interaction with respect to the other two terms. In this approximation, we get the scalar Helmholtz equation for all the decoupled components of the electric vector field, i.e.
\[
\Delta E_\omega(r) + \frac{\omega^2}{c^2}n^2(\omega, r)E_\omega(r) = 0.
\]
(9)
The same holds for magnetic vector field. If one has a solution of the Helmholtz equation \( E_\omega^{(0)}(r) \), either exact or approximate one, the influence of the light polarization interaction may be taken into account using the Born method of iteration. In fact, the Green function given by Eq.\( (6) \), \( G(r, r', \omega) \), satisfies the equation
\[
\left[ \Delta + \frac{\omega^2}{c^2}n^2(\omega, r) \right] G(r, r', \omega) = \delta(r - r').
\]
(10)
Then the solution of the equation with the polarization term has the form
\[
E_\omega(r) = E_\omega^{(0)}(r) + \int G(r, r', \omega)\nabla \left[ \frac{E_\omega^{(0)}(r)\nabla\varepsilon(\omega, r)}{\varepsilon(\omega, r)} \right] dr.
\]
(11)
Eq.\( (10) \) is equivalent in form to the equation for the Green function of the Schrödinger equation, if the energy constant \( E \) is equal to zero. In fact, if we write down the Hamiltonian operator \( \hat{H} = -\frac{\hbar^2}{2m}\Delta + U(r) \), with \( \hbar = m = 1 \), and the equation for the Green function of the Schrödinger equation \( G_s(r, r', E) \) which is the matrix element of the operator \( (\hat{H} - E)^{-1} \)
in the coordinate representation \( G_s(r, r', E) = \langle r | (\hat{H} - E)^{-1} | r' \rangle \), which comes from the equation
\[
\left\{ -\frac{1}{2} \Delta + U(r) - E \right\} G_s(r, r', E) = \delta(r - r').
\]
(12)

The comparison of this equation with Eq.(10) shows that they are identical for \( E = 0 \) with the replacement
\[
U(r) = - \frac{\omega^2}{c^2} \frac{n^2}{n_0}(\omega, r), \quad -2G(r, r') = G_s(r, r', 0).
\]
(13)
Thus, we have shown that if one knows the Green function \( G_s(r, r', E) \) of the Schrödinger equation for the unit mass particle moving in a potential like that in Eq.(13), the Green function of the Helmholtz equation (10) is given by the equality \( G(r, r') = -\frac{1}{2} G_s(r, r', E = 0) \). Since the Green function for the Schrödinger equation are studied for many potentials, the results obtained in quantum mechanics can be applied for our purposes to study polarization and waveguiding effects since they are formally identical.

4. The gravitational waveguide model

Let us now consider the scalar Helmholtz equation for some arbitrary monochromatic component of the electric field
\[
\frac{\partial^2 E}{\partial z^2} + \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + k^2 n^2(r) E = 0 ,
\]
(14)
where \( k \) is the wave number. The coordinate \( z \), in Eq.(14), is the longitudinal one, and it can measure the space distance along the gravitational field structure produced by a mass distribution with an optical axis. Such a coordinate may also correspond to a distance along the light path inside a planar gravitational field structure produced by a planar matter–energy distribution in some regions of the universe. Let us consider a solution of the form
\[
E = n_0^{-1/2} \Psi \exp \left( ik \int_{-\infty}^{\infty} n_0(z') dz' \right) ; \quad n_0 \equiv n(0, 0, z),
\]
(15)
where \( \Psi(x, y, z) \) is a slowly varying spatial amplitude along the \( z \) axis, and \( \exp(ikn\zeta) \) is a rapidly oscillating phase factor. Its clear that the beam propagation is along the \( z \) axis. We rewrite Eq.(14), neglecting second order derivative in longitudinal coordinate \( z \), and obtain a Schrödinger–like equation for \( \Psi \):
\[
i\lambda \frac{\partial \Psi}{\partial \zeta} = -\frac{\lambda^2}{2} \left( \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} \right) + \frac{1}{2} \left[ n_0^2(z) - n^2(x, y, z) \right] \Psi ,
\]
(16)
where \( \lambda \) is the electromagnetic radiation wavelength and we adopt the new
variable \( \xi = \int z' \, dz' / n_0(z') \), normalized with respect to the refraction
index (for our application, \( n_0(z) \approx 1 \) so that \( \xi \) coincides essentially with \( z \)).

At this point, it is worthwhile to note that if one has the distribution of
the matter in the form of cylinder with a constant (dust) density \( \rho_0 \),
the gravitational potential inside has a parabolic profile providing waveguide
effect for electromagnetic radiation analogous to optical waveguides
realized in fiber optics. In this case, Schrödinger–like equation is that of
two–dimensional quantum harmonic oscillator for which the mode solutions
exist in the form of Gauss–Hermite polynomials. In the case of inhomogeneous
longitudinal dust distribution in the cylinder (that is \( \rho(z) \)), the
Schrödinger-like equation describes the model of two-dimensional parametric
oscillator for which the mode solutions, in the form of modified Gaussian
and Gauss–Hermite polynomials, exist with parameters determined by the
density dependence on longitudinal coordinate. As a side remark, it is
interesting to stress that, considering again Eq.(16), the term in square
brackets in the rhs plays the role of the potential in a usual Schrödinger
equation; the role of Planck constant is now assumed by \( \lambda \). Since the
refraction index can be expressed in terms of the Newtonian potential when
we consider the propagation of light in a gravitational field, we can write
the potential in (16) as

\[
U(r) = \frac{2}{c^2} [\Phi(x, y, z) - \Phi(0, 0, z)].
\]

(17)

The waveguide effect depends specifically on the shape of potential (17): for
example, the radiation from a remote source does not attenuate if \( U \sim r^2 \);
this situation is realized supposing a "filamentary" or a "planar" mass
distribution with constant density \( \rho \). Due to the Poisson equation, the
potential inside the filament is a quadratic function of the transverse
coordinates, that is of \( r = \sqrt{x^2 + y^2} \) in the case of the filament and of \( r = x \)
in the case of the planar structure (obviously the light propagates in the
"remaining" coordinates: \( z \) for the filament, \( z, y \) for the plane). In other
words, if the radiation, travelling from some source, undergoes a waveguide
effect, it does not attenuate like \( 1/R^2 \) as usual, but it is, in some
sense conserved; this fact means that the source brightness will turn out
to be much stronger than the brightness of analogous objects located at
the same distance (i.e. at the same redshift \( Z \)) and the apparent energy
released by the source will be anomalously large. To fix the ideas, let us
estimate how the electric field (15) propagates into an ideal filament whose
internal potential is

\[
U(r) = \frac{1}{2} \omega^2 r^2, \quad \omega^2 = \frac{4\pi G \rho}{c^2}
\]

(18)

where \( \rho \) is constant and \( G \) is the Newton constant. A spherical wave from
a source, \( E = (1/R) \exp(ikR) \), can be represented in the paraxial approxi-
mation as
\[ E(z,r) = \frac{1}{z} \exp \left( ikz + \frac{ikr^2}{2z} - \frac{r^2}{2z^2} \right), \]
where we are using the expansion
\[ R = (z^2 + r^2)^{1/2} \approx z \left( 1 + \frac{r^2}{2z^2} \right), \quad r \ll z. \]
It is realistic to assume \( n_0 \simeq 1 \) so that \( \xi = z \). Let us consider now that the starting point of the filament of length \( L \) is at a distance \( l \) from a source shifted by a distance \( a \) from the filament axis in the \( x \) direction. The amplitude \( \Psi \) of the field \( E \), entering the waveguide is
\[ \Psi_{in} = \frac{1}{l} \exp \left[ \frac{ikl - 1}{l^2} (x - a)^2 + y^2 \right], \]
and so we have \( R = (l^2 + y^2 + (x - a)^2)^{1/2} \). We can calculate the amplitude of the field at the exit of the filament by the equation
\[ \Psi_f(x,y,l+L) = \int dx_1 dy_1 G(x,y,l+L,x_1,y_1,l)\Psi_{in}(x_1,y_1,l), \]
where \( G \) is the Green function of Eq.(16). For the potential (18), \( G \) is the propagator of the harmonic oscillator. The integral (22) is Gaussian and can be exactly evaluated:
\[ \Psi_f = \frac{\omega l}{\omega^2 \cos \omega L + (l + i\lambda) \sin \omega l} \times \exp \left( -\frac{(x^2 + y^2)(\omega kl)^2 - \omega k(i + kl) \cot \omega l - a^2 \omega k(i + kl) \cot \omega L}{2(1 - ikl - ik \omega l^2 \cot \omega l)} \right) \]
\[ \times \exp \left( -\frac{2xawk(1 + kl)}{2 \sin \omega L(1 - ikl - ik \omega l^2 \cot \omega L)} \right), \]
The parameter \( l \) drops out of the denominator of the pre–exponential factor if the length \( L \) satisfies the condition \( \tan \omega L = -\omega l \). Eq.(23) is interesting in two limits. If \( \omega l \ll 1 \), we have
\[ \Psi_f = \frac{1}{i\lambda} \exp \left\{ -\frac{l + i\lambda}{2\lambda^2 l} [(x + a)^2 + y^2] \right\}, \]
which means that the radiation emerging from a point with coordinate \((a,0,0)\) is focused near a point with coordinates \((-a,0,l+L)\) (that is the radius has to be of the order of the wavelength). This means that, when the beam from an extended source is focused inside the waveguide in such a way that, at a distance \( L \), an inverted image of the source is formed, having the very same geometrical dimensions of the source. The waveguide
“draws” the source closer to the observer since, if the true distance of the observer from the source is \( R \), its image brightness will correspond to that of a similar source at the closer distance

\[
R_{\text{eff}} = R - l - L .
\]  

(25)

If we do not have \( \omega l \ll 1 \), we get (neglecting the term \( i\lambda/l \) compared with unity)

\[
\Psi_f = \sqrt{1 + (\omega l)^2} \frac{i\lambda}{2\lambda^2} \exp \left\{ - \frac{1 + (\omega l)^2}{2\lambda^2} \left[ y^2 + \left( x + \frac{a}{\sqrt{1 + (\omega l)^2}} \right)^2 \right] \right\},
\]

(26)

from which, in general, the size of the image is decreased by a factor \( \sqrt{1 + (\omega l)^2} \). The amplitude increases by the same factor, so that the brightness is \( (R/R_{\text{eff}}) \) times larger. In the opposite limit \( \omega l \gg 1 \), we have \( \tan \omega L \to \infty \), so that \( L \simeq \pi/\omega \), that is the shortest focal length of the waveguide is

\[
L_{\text{foc}} = \frac{\pi c^2}{4G\rho},
\]

(27)

which is the length of focusing of the initial beam of light trapped by the gravitational waveguide. All this arguments apply if the waveguide has (at least roughly) a cylindrical geometry. The theory of planar waveguide is similar but we have to consider only \( x \) as transverse dimension and not also \( y \). The cosmological feasibility of a waveguide depends on the geometrical dimensions of the structures, on the connected densities and on the limits of applicability of the above idealized scheme. In the next section, we shall discuss these features and the possible candidates which could give rise to observable effects.

5. Cosmic structures as waveguides

The gravitational waveguide effect has the same physical reason that has the gravitational lenses effect which is the deflection of light by a gravitational field acting as refractive media. However, there are essential differences producing specific predictions for observing the waveguide effect. The gravitational lenses are usually considered as compact objects with strong enough gravitational potential. The light rays deflected by gravitational lenses move outside the matter which forms the gravitational lens itself. The gravitational waveguide as well as optical waveguide is noncompact long structure which may contain small matter density and deflection of the light by each element of the structure is very small. Due to very large
scale sizes of the structure (we give an estimation below), the electromagnetic radiation deflection by the gravitational waveguide occurs and, in principle, it may be observed. We will mention, for example, a possibility of brilliancy magnification of the long distanced objects (like quasars) with large red shift as consequence of the waveguiding structure existence between the object and the observer. This effect exists also for a gravitational lens located between the object and the observer, but the long gravitational waveguide may give huge magnification, since the radiation propagates along the waveguide with functional dependence of the intensity on the distance which does not decrease as $\sim 1/R^2$, characteristic for free propagation. The gravitational lens, being a compact object, collects much less light by deflecting the rays to the observer than the gravitational waveguide structure transporting to the direction of observer all trapped energy (of course, one needs to take into account losses for scattering and absorption). From that point of view, it is possible that enormous amount of radiation emitted by quasars is only seemingly existing. The object may radiate a reasonable amount of energy but the existing waveguide structure transmits the energy in high portion to the observer. Similar ideas, related to gravitational lensing, were discussed in [3] but the singular lens or even few aligned strong lenses cannot produce effect of many orders of magnification of brilliancy. The waveguide effect may explain the anomalous high luminosity observed in quasars. In fact, quasars are objects at very high redshift which appear almost as point sources but have luminosity that are about one hundred times than that of a giant elliptical galaxy (quasars have luminosity which range between $10^{38} - 10^{41}$ W). For example, PKS 000-330 has one of the largest known redshifts ($Z = 3.78$) with a luminosity of $10^{40}$ W. Such a redshift corresponds to a distance of 3700 Mpc, if it is assumed that its origin is due to the expansion of the universe and the Hubble constant is assumed $H = 75 km/s/Mpc$. This means that light left the quasar when the size of the universe was one-fifth of its present age where no ordinary galaxies (including the super giant radio-galaxies) are observed. The quasars, often, have both emission and absorption lines in their spectra. The emission spectrum is thought to be produced in the quasar itself; the absorption spectrum, in gas clouds that have either been ejected from the quasar or just happen to lie along the same line of sight. The brightness of quasars may vary rapidly, within a few days or less. Thus, the emitting region can be no larger than a few light-days, i.e. about one hundred astronomical units. This fact excludes that quasars could be galaxies (also if most astronomers think that quasars are extremely active galactic nuclei). The main question is how to connect this small size with the so high redshift and luminosity. For example, H.C. Arp discovered small systems of quasars and galaxies where some of the components have widely discrepant redshifts [4]. For this reason, quasar high redshift could be produced by some unknown process and not being simply of cosmologi-
cal origin. This claim is very controversial. However there is a fairly widely accepted preliminary model which, in principle, could unify all the forms of activities in galaxies (Seyfert, radio, Markarian galaxies and BL Lac objects). According to this model, most galaxies contain a compact central nucleus with mass $10^7 - 10^9 \, M_\odot$ and diameter $< 1$ pc. For some reason, the nucleus may, some times, release an amount of energy exceeding the power of all the rest of the galaxy. If there is only little gas near the nucleus, this leads to a sort of double radio source. If the nucleus contains much gas, the energy is directly released as radiation and one obtains a Seyfert galaxy or, if the luminosity is even larger, a quasar. In fact, the brightest type 1 Seyfert galaxies and faintest quasars are not essentially different in luminosity ($\sim 10^{38} \, W$) also if the question of redshift has to be explained (in fact quasar are, apparently, much more distant). Finally, if there is no gas at all near such an active nucleus, one gets BL Lac objects. These objects are similar to quasar but show no emission lines. However the mechanism to release such a large amount of energy from active nuclei or quasars is still unknown. Some people suppose that it is connected to the releasing of gravitational energy due to the interactions of internal components of quasars. This mechanism is extremely more efficient than the releasing of energy during the ordinary nuclear reactions. The necessary gravitational energy could be produced, for example, as consequence of the falling of gas in a very deep potential well as that connected with a very massive black hole. Only with this assumption, it is possible to justify a huge luminosity, a cosmological redshift and a small size for the quasars. An alternative explanation could come from waveguiding effects. As we have discussed, if light travels within a filamentary or a planar structure, whose Newtonian gravitational potential is quadratic in the transverse coordinates, the radiation is not attenuated, moreover the source brightness is stronger than the brightness of analogous object located at the same distance (that is at the same redshift). In other words, if the light of a quasar undergoes a waveguiding effect, due to some structure along the path between it and us, the apparent energy released by the source will be anomalously large, as the object were at a distance given by $\frac{R}{R_{\text{eff}}}$. Furthermore, if the approximation $\omega l \ll 1$ does not hold, the dimensions of resulting image would be decreased by a factor $\sqrt{1 + (\omega l)^2}$ while the brightness would be $\left(\frac{R}{R_{\text{eff}}}\right)^2$, larger, then explaining how it is possible to obtain so large emission by such (apparently) small objects. In conclusion, the existence of a waveguiding effect may prevents to take into consideration exotic mechanism in order to produce huge amounts of energy (as the existence of a massive black hole inside a galactic core) and it may justify why it is possible to observe so distant objects of small geometrical size. Another effect concerning the quasars may be directly connected with multiple images in lensing. The waveguide effect does not disappear if the axis of "filament" or if the guide plane is bent smoothly in space. As in the case of gravitational lenses, we
can observe "twin" images if part of the radiation comes to the observer directly from the source, and another part is captured by the bent waveguide. The "virtual" image can then turn out to be brighter than the "real" one (in this case we may deal with "brothers" rather than "twins" since parameters like, spectra, emission periods and chemical compositions are similar but the brightnesses are different). Furthermore, such a bending in waveguide could explain large angular separations among the images of the same object which cannot be explained by the current lens models (pointlike lens, thin lens and so on). Now the issue is: are there cosmic structures which can furnish workable models for waveguides? Have they to be "permanent" structures or may the waveguide effect be accidental? For example an alignment of galaxies of similar density and structure, due to cosmic shear and inhomogeneity, may be available as waveguide just for a limited interval of time? In general both points of view may be reasonable and here we will outline both of them. Furthermore we have to consider the problem of the abundance of such structures: are they common and everywhere in the universe or are they peculiar and located in particular regions (and eras)?

We have to do a first remark on the densities of waveguide structures which allow observable effects. Considering Eq. (27) and introducing into it the critical density of the universe \( \rho_c \approx 10^{-29} \text{ g/cm}^3 \) (that is the value for which the density parameter is \( \Omega = 1 \)), we obtain \( L_{foc} \approx 5 \times 10^4 \text{ Mpc} \) which is an order of magnitude larger than the observable universe and it is completely unrealistic. On the contrary, considering a typical galactic density as \( \rho \approx 10^{-24} \text{ g/cm}^3 \), we obtain \( L_{foc} \approx 100 \text{ Mpc} \), which is a typical size of large scale structure.

However, an important issue has to be taken into account: the absorption and the scattering of light by the matter inside the filament or the planar structure increase with density and, at certain critical value the waveguide effect, can be invalidated. For the smaller frequency of broadcast range (due to strong dependence of the absorption cross section on the electromagnetic wavelength) \( \sigma \approx \sigma_T (\omega/\omega_0)^4 \), where Thomson cross-section \( \sigma_T = 6 \cdot 10^{-25} \text{ cm}^2 \) and the characteristic atomic frequency is \( \omega_0 \approx 10^{16} \text{ s}^{-1} \), the ratio \( \omega/\omega_0 \ll 1 \), and the absorption is small. It means that the absorption length \( L_a = m_p/\rho \sigma \), (where the mass of proton \( m_p \) is approximately equal to the hydrogen atom mass) is larger than the focusing length \( L_a < L_{foc} \) for the electromagnetic waves of broadcast range. Thus, the magnification of electromagnetic waves may be not masked by essential energy losses due to light absorption and scattering processes. However, no restrictions exist practically if the radio band and a thickness of the structure \( r > 10^{14} \text{ cm} \) are considered.

In such a case, the relative density change between the background and the structure density is valid till \( \delta \rho/\rho \leq 1 \). This means that we have to
stay in a linear perturbation density regime. By such hypotheses, practically all the observed large scale structures like filaments, walls, bubbles and clusters of galaxies can result as candidates for waveguiding effect if the restrictions on density, potential and waveband are respected (in optical band, such phenomena are possible but the density has to be chosen with some care). Concerning the second point of view (that is the existence of temporary waveguiding effects), it could be related to the observation of objects possessing anomalously large (compared with their neighbours) angular motion velocities (an analysis in this sense could come out in mapping galaxies with respect to their redshift and proper velocities). Such a phenomenon could mean that one observes not the object itself, but its image transmitted through the moving gravitational waveguide. The waveguide itself could change its form or it could be due to temporary alignments of lens galaxies. In this case, the image of the object could move with essentially different angular velocity than that of the observable neighbour objects whose light reaches the observer directly (not through the waveguide). The discovery of long distanced objects with anomalous velocity (and brightness) could support the hypothesis of gravitational waveguide effect, while the evolution of the waveguide, its destruction or change of the axis direction (from the orientation to the Earth) could produce the effect of the disappearance (or the appearance) of the observed object. For this analysis, it is crucial to consider long period astronomical observations and deep pencil beam surveys of galaxies and quasars. In a forthcoming papers, the authors are constructing a simulation where given random distributions of quasars and waveguides, it is possible to reproduce the observed luminosity function of quasar distribution explaining its anomalies. Finally we want to stress that our treatment does not concern only electromagnetic radiation: actually a waveguide effect could be observed also for streams of neutrinos, or other particles which gravitationally interact with large scale structures filaments or planes.

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