Exactly solved models and beyond: a special issue in honour of R J Baxter’s 75th birthday

Abstract
This is an introduction to Exactly Solved Models and Beyond, a special issue collection of articles published in J. Phys. A: Math. Theor. in honour of R J Baxter’s 75th birthday.

(Some figures may appear in colour only in the online journal)

Rodney J Baxter’s pioneering contributions to the study of exactly solved models in statistical mechanics, dating back to the early 1970s, continue to have a profound impact in both mathematics and physics. His body of work includes both finding remarkable new solutions of key models and the invention of powerful techniques for calculating their physical properties. Baxter’s concepts of commuting transfer matrices, functional relations and corner transfer matrices have inspired developments across a broad spectrum of mathematical physics. The notion of Yang–Baxter integrability originating from lattice models led to profound advances in quantum field theory, in knot theory and in the development of quantum groups. Such integrable models have played a central role in the AdS/CFT correspondence in string theory and are also being realized in experiments in low-dimensional physics.

Accordingly Baxter has received a number of distinctions and awards throughout his career. These include:

• Pawsey Medal, Australian Academy of Science, 1975
• Elected Fellow of the Australian Academy of Science, 1977
• Boltzmann Medal, International Union of Pure and Applied Physics, 1980
• Elected Fellow of the Royal Society of London, 1982
• Lyle Medal, Australian Academy of Science, 1983
• Dannie Heineman Prize, American Physical Society, 1987
• Elected Royal Society Research Professor at Cambridge, 1992
• Harrie Massey Medal, British Institute of Physics, 1994
• Centenary Medal, Australian Government, 2003
• Lars Onsager Prize, American Physical Society, 2006
• Lars Onsager Lecture and Medal, Norwegian University of Science and Technology, 2006
• Royal Medal, Royal Society of London, 2013

Baxter’s work has involved finding brilliant solutions to highly non-trivial mathematical problems. Colleagues who have glanced over Rodney’s shoulder while he calculates will attest to the Baxter wizardry at deriving and manipulating formulae. When Baxter started his
research 50 years ago the classical culture of ‘working with formulas’ had been seemingly forgotten by many mathematicians in their pursuit of abstractions. Baxter is one of the few who stimulated the renaissance of this culture in modern mathematics. To perpetuate the culture that his work has also inspired at the Australian National University for well over half a century, the Rodney Baxter Endowment has been established to provide a Baxter Fellowship to support a prominent theoretician, a rising-star or established leader, to visit the ANU each year for a period of three months.

This special issue is a collection of articles in honour of Baxter’s 75th birthday. Previous publications celebrating Baxter milestones are the surveys marking his 50th \cite{1} and 60th \cite{2} birthdays. The collection of research papers \cite{3} was based around the conference Baxter2000 held in Canberra, Australia in August 2000. The title of the present collection of articles is shared with the conference Exactly Solved Models and Beyond held in Cairns, Australia in July 2015\footnote{See http://baxter2015.anu.edu.au}.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{baxter.jpg}
\caption{Rodney J Baxter taking a break from ‘doing sums’. Baxter’s name is associated with many mathematical terms, concepts and models in statistical mechanics and beyond. Photo credit: Elizabeth Baxter.}
\end{figure}
This special issue includes some of Baxter’s academic and personal reminiscences covering three career highlights [4]. There are also two review articles, one on the early history of the integrable chiral Potts model [5], the other on the impact of Yang–Baxter integrable models in experiments, from condensed matter to ultracold atoms [6].

Moreover, [7] contains a comprehensive review of all known solutions of the star-triangle relation. Indeed the spread and depth of the articles in this special issue are testament to the broad impact of Baxter’s work and to the vitality of work in the related areas of mathematical physics. There are a number of contributions on various aspects of the Yang–Baxter or star-triangle relation [7–11], the tetrahedron equation [12, 13] and fusion in the one-dimensional Hubbard [14] and RSOS [15] models. There are also contributions on CSOS [16], SOS [17], non-unitary RSOS [18], non-Abelian anyons [19], dimer [20], six-vertex [21, 22], eight-vertex [23], spin-boson [24], generalized Rabi [25] and dilute orientated loop [26] models.

New results are also presented for scalar products, form factors and correlation functions in integrable models [27–32]. Other topics included are Baxter’s Q-operators [33, 34], Q-colourings of the triangular lattice [35], periodic Temperley–Lieb algebras [36], the random-cluster model on isoradial graphs [37], discrete-time exclusion processes [38], susceptibility of the square lattice Ising model [39], diffusion processes [40], compressed self-avoiding walks, bridges and polygons [41], and discrete holomorphicity in the chiral Potts model [42].

An exact solution is given for three interacting friendly walks in the bulk [43] and topological defects are considered for the Ising model [44].

In other contributions, the Bethe Ansatz method is established for an XXZ type model associated to quantum toroidal $gl_1$ [45], the off-diagonal Bethe Ansatz scheme is discussed for the prototypical XXZ spin [46] and by the same method Bethe states are constructed for the integrable spin-$s$ chain with generic open boundaries [47]. The counting of Bethe roots is also discussed [48, 49].

Contributions on the more mathematical side include a general method to produce flat connections for the two-boundary quantum Knizhnik–Zamolodchikov equations [50], the study of generalizations of the Rogers–Ramanujan q-series [51], quantum Bäcklund transforms and topological quantum field theory [52], dynamics of a q-difference Painlevé equation [53], matrix product formula for Macdonald polynomials [54], invariants of the vacuum module associated with the Lie superalgebra $gl(1|1)$ [55], and the discussion of diagonals of rational functions occurring in lattice statistical mechanics and enumerative combinatorics [56]. Closer to the physics side, there are contributions on integrable pairing models [57, 58], bosons in a four-well ring [59] and multi-component Fermi gases [60].

This special collection of articles reflects the profound and ongoing influence of Baxter’s work, which we are sure will continue to inspire further developments in this important and widely influential area of mathematical physics and beyond for many decades to come.

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