Mean-field Phase Diagram of Two-dimensional Electrons with Disorder in a Weak Magnetic Field

Igor S. Burmistrov
Landau Institute for Theoretical Physics, Kosygina str. 2, 117940 Moscow, Russia

Mikhail A. Baranov
Institute for Theoretical Physics, University of Hannover, D-30167, Germany

We study two-dimensional interacting electrons in a weak perpendicular magnetic field with the filling factor $\nu \gg 1$ and in the presence of a quenched disorder. In the framework of the Hartree-Fock approximation, we obtain the mean-field phase diagram for the partially filled highest Landau level. We find that the CDW state can exist if the Landau level broadening $1/2\tau$ does not exceed the critical value $1/2\tau_c = 0.038\omega_H$. Our analysis of weak crystallization corrections to the mean-field results shows that these corrections are of the order of $(1/\nu)^{2/3} \ll 1$ and therefore can be neglected.

PACS numbers: 72.10.-d, 73.20.Dx, 73.40.Hm

I. INTRODUCTION

A two-dimensional electron gas (2DEG) in a perpendicular magnetic field was a subject of intensive studies, both theoretical and experimental, for several decades. The behaviour of the system in a strong magnetic field where only the lowest Landau level is occupied, has been investigated in detail. Several attempts were made in order to incorporate the case with larger filling factors $\nu > 1$ into the theory. Usually in these approaches, the ratio of the characteristic Coulomb energy (at distances of the order of the magnetic length) to the cyclotron energy, has been assumed to be small. However, in a weak magnetic field this is not the case, and the characteristic Coulomb energy exceeds the cyclotron energy. An attempt to investigate the situation with large Coulomb energy was made in Ref. 1.

The progress in understanding of the clean 2DEG in a weak magnetic field was achieved by Aleiner and Glazman. They have derived the low-energy effective theory on the partially filled highest Landau level by using the small parameter $1/\nu \ll 1$. By treating the effective interaction within the Hartree-Fock approximation, Koulakov, Fogler, and Shklovskii predicted a unidirectional charge-density-wave (CDW) state (stripe phase) for the half-filled highest Landau level at zero temperature and in the absence of disorder. Moessner and Chalker extended the ideas of Fukuyama, Platzmann and Anderson to the case of a partially filled highest Landau level and showed the existence of the mean-field CDW state on the half-filled Landau level below some temperature $T_0$.

Recently, the existence of compressible states near half-filling with anisotropic transport properties was demonstrated experimentally for high Landau levels. This stimulates an extensive study of the clean 2DEG in a weak magnetic field and pinning of stripes by disorder. In the clean case, the properties of the CDW states can be described on the basis of the low energy effective theory for smooth “elastic” deformations. Recently, attempts were made to derive such a theory microscopically starting from the mean-field solution. The effects of a quenched disorder on the unidirectional CDW state (stripe phase) were investigated in the framework of the phenomenological elasticity theory, and a rich variety of different regimes, which depend on the strength of disorder, were found. However, to identify the phenomenological parameters of the theory, a successive microscopic theory should be developed.

At present, a thorough microscopic analysis of the effects of disorder on the mean-field transition from the uniform state to the CDW one, as well as on the phase diagram of the mean-field CDW states, is absent. The main objective of the present paper is to investigate these effects on the existence of the mean-field CDW states in 2DEG in a weak perpendicular magnetic field $H$ (filling factor $\nu \gg 1$). For the considered case of a large number of the occupied Landau levels, the mean-field analysis is legitimate because the fluctuations of the order parameter are strongly suppressed. On the other hand, the mean-field approach cannot be applied to the critical region in the direct vicinity of the phase transition. This region is however small and does not lead to any significant uncertainty in our results for critical temperatures of the transitions.

We assume the presence in the system of a weak quenched disorder, i.e. the elastic collisions time satisfies the condition $\tau_0 \gg \omega_H^{-1}$, where $\omega_H = eH/m$ is the cyclotron frequency, $e$ the electron charge, and $m$ the effective electron mass. Under this condition, the Landau level broadening $1/2\tau$, which is of the order of $1/2\tau_c$, is much less than the spacing $\omega_H$ between them. This case can be realized in high mobility samples which were used for experimental studies of the anisotropic magnetotransport. Keeping in mind that the relation $T_0 \sim 1/\tau$ is usually hold, one expect a much pronounced influence of the quenched disorder on the properties of electrons on the partially filled highest Landau level even for a small level broadening $1/2\tau \ll \omega_H$.

One of the main results of our paper is that at zero...
temperature the mean-field CDW state is destroyed when the Landau level broadening exceeds the critical value \(1/2\tau_c = 4T_0/\pi\). At nonzero temperatures the quenched disorder leads to the decrease of the temperature of the CDW instability as compared to the clean case. The physical reason is that the scattering on impurities breaks the CDW correlations, and therefore results in the destruction of the coherent CDW state. This is somewhat similar to the suppression of the critical temperature in conventional superconductors by magnetic impurities or in anisotropic superconductors by nonmagnetic impurities.

The paper is organized as follows. In Sec. II we introduce the formalism that allows us to evaluate the free energy of the CDW state in the presence of disorder. In Sec. II we investigate the instability of the CDW state towards the formation of the CDW state, and present the mean-field phase diagrams at the half-filling and arbitrary filling. The weak crystallization corrections to the mean-field solution are presented in Sec. IV. Sec. V contains the comparison of the theory with the recent experimental and numerical results. We end with conclusions in Sec. VI.

II. FREE ENERGY OF THE CDW STATES

We consider two-dimensional interacting electrons in the presence of a weak quenched disorder and a weak perpendicular magnetic field. The parameter that characterizes the strength of the Coulomb interaction is \(r_s = \sqrt{2e^2/\varepsilon v_F}\) with \(v_F\) being the Fermi velocity and \(\varepsilon\) the dielectric constant of a media. We assume that the Coulomb interaction between the electrons is weak, \(r_s \ll 1\), and the magnetic field obeys the condition \(N\tau_s \gg 1\), where \(N = \lfloor \nu/2 \rfloor\) is the integer part of \(\nu/2\). In this case it is possible to construct an effective field theory for the electrons on the highest partially filled Landau level by integrating out all other degrees of freedom. We also assume that the electrons on the partially filled highest Landau level are spin-polarized. This assumption is based on the calculations that shows the existence of fractional states, composite fermions, and skyrmions only on the lowest and the first excited Landau levels, as well as on the experimental observations.

In order to study the transition from the uniform state to the CDW one we employ the Landau expansion of the free energy in powers of the CDW order parameter \(\Delta(q_j)\), where the vectors \(q_j\) that characterize the CDW state, have the same length \(q_j = q\). We perform the expansion up to the forth order in the CDW order parameter under the assumption \(N\tau_s^2 \gg 1\). In this case the Hartree-Fock approximation is well justified because the corrections are small in the parameter \(a_B/l_H = 1/N\tau_s^2 \ll 1\), where \(a_B = \varepsilon/me^2\) is the Bohr radius and \(l_H = 1/\sqrt{m\omega_H}\) the magnetic length.

A. Formalism

The thermodynamical potential of the spin-polarized 2DEG projected on the \(N\)th Landau level in the presence of the random potential \(V_{\text{dis}}(\mathbf{r})\) and the magnetic field is given by

\[
\Omega = -\frac{T}{N_F} \int \mathcal{D}[\psi, \bar{\psi}] \int \mathcal{D}[V_{\text{dis}}] \mathcal{P}[V_{\text{dis}}] \exp(S[\psi, \bar{\psi}, V_{\text{dis}}]),
\]

where the action \(S[\psi, \bar{\psi}, V_{\text{dis}}]\) in the Matsubara representation has the form

\[
S = \int \sum_{\omega_n} \left\{ \bar{\psi}_{\omega_n}^{\alpha}(\mathbf{r}) \left[ i \omega_n + \mu - \mathcal{H}_0 - V_{\text{dis}}(\mathbf{r}) \right] \psi_{\omega_n}^{\alpha}(\mathbf{r}) - \frac{T}{2} \sum_{\omega_m, \omega_n, \nu_i} \int \bar{\psi}_{\omega_n}^{\alpha}(\mathbf{r}) \psi_{\omega_m}^{\alpha}(\mathbf{r}) U_0(\mathbf{r}, \mathbf{r}') \bar{\psi}_{\omega_m}^{\alpha}(\mathbf{r}') \psi_{\omega_n}^{\alpha}(\mathbf{r}') + \nu_i(\mathbf{r}') \right\}.
\]

Here \(\psi_{\omega_n}^{\alpha}(\mathbf{r})\) and \(\bar{\psi}_{\omega_n}^{\alpha}(\mathbf{r})\) are the annihilation and creation operators of an electron on the \(N\)th Landau level, \(T\) the temperature, \(\mu\) the chemical potential, \(\omega_n = \pi T(2n + 1)\) the Matsubara fermionic frequency, and \(\nu_n = 2\pi Tn\) the bosonic one. The free Hamiltonian \(\mathcal{H}_0\) for 2D electrons with mass \(m\) in the perpendicular magnetic field \(H = eB/2m\) is \(\mathcal{H}_0 = (-i\nabla - eA)^2/(2m)\). The screened electron-electron interaction \(U_0(\mathbf{r}, \mathbf{r}')\) on the \(N\)th Landau level takes into account the effects of interactions with electrons on the other levels, and has the form (see Ref.\(\alpha\) for the clean case and Ref.\(\beta\) for the weakly disordered case)

\[
U_0(q) = \frac{2\pi e^2}{\varepsilon q} \left( 1 + \frac{2}{qa_B} \left( 1 - \frac{\pi}{6\omega_H} \right) (1 - J_0^2(qR_c)) \right),
\]

where \(R_c = l_H\sqrt{\tau}\) is the cyclotron radius on the \(N\)th Landau level and \(J_0(x)\) the Bessel function of the first kind. The range of the screened electron-electron interaction is determined by the Bohr radius \(a_B\). We also assume the Gaussian distribution for the random poten-
tial $V_{\text{dis}}(\mathbf{r})$

$$\mathcal{P}[V_{\text{dis}}(\mathbf{r})] = \frac{1}{\sqrt{2\pi g}} \exp \left( -\frac{1}{g} \int \mathcal{V}_{\text{dis}}^2(\mathbf{r}) \right),$$  

(4)

where $g = 1/\pi \rho \tau_0$, $\rho$ is the thermodynamical density of states, and introduce $N_r$ replicated copies of the system labeled by the replica indices $\alpha = 1, \ldots, N_r$ in order to average over the disorder.

B. The Hartree-Fock decoupling and the average over disorder

The CDW ground state is characterized by the order parameter $\Delta(\mathbf{q})$ that is related to the electron density

$$\langle \rho(\mathbf{q}) \rangle = L_x L_y n_L F_N(q) \Delta(q).$$  

(5)

Here $L_x L_y$ is the area of the 2DEG, $n_L = 1/2\pi \ell_H^2$ the number of states on one Landau level, and the form-factor $F_N(q)$ is

$$F_N(q) = L_N \left( \frac{2^2 \ell_H^2}{2} \right) \exp \left( -\frac{q^2 \ell_H^2}{4} \right),$$  

(6)

where $L_N(x)$ is the Laguerre polynomial. For the case $N \gg 1$, one can use the following asymptotic expression for the form-factor

$$F_N(q) = \mathcal{J}_0(q R_c), \quad q R_c \ll \frac{R_c^2}{\ell_H} = \nu$$  

(7)

After the Hartree-Fock decoupling of the interaction term in the action we obtain

$$S = -\frac{N_r \Omega \Delta}{T} +$$

$$+ \int \sum_{\omega_n} \psi_{\alpha n}^\dagger(\mathbf{r}) \left[ i \omega_n + \mu - \mathcal{H}_0 - V_{\text{dis}}(\mathbf{r}) + \lambda(\mathbf{r}) \right] \psi_{\alpha n}(\mathbf{r}),$$

$$\Omega = \frac{n_L (L_x L_y)}{2} \int_q U(q) \Delta(q) \Delta(-q),$$  

(9)

where the potential $\lambda(\mathbf{r})$ results from the perturbation of the uniform electron density by the charge density wave, and is connected with the CDW order parameter as follows

$$\lambda(\mathbf{q}) = L_x L_y U(q) F_N^{-1}(q) \Delta(q),$$  

(10)

and $U(q) = -n_L U_{HF}(q)$ with the Hartree-Fock potential $U_{HF}(q)$ given by

$$U_{HF}(q) = U_0(q) F_N^2(q) - \int_p e^{-i \mathbf{q} \cdot \mathbf{p}} \frac{\mathcal{V}_{\text{dis}}^2}{n_L} U_0(\mathbf{p}) F_N^2(\mathbf{p}).$$  

(11)

The averaging over the random potential $V_{\text{dis}}(\mathbf{r})$ in Eq. (11) is straightforward and results in the following quartic term

$$\frac{q}{2} \int \sum_{\omega_n \omega_m} \psi_{\omega_n}^\dagger(\mathbf{r}) \psi_{\omega_n}(\mathbf{r}) \psi_{\omega_m}^\dagger(\mathbf{r}) \psi_{\omega_m}(\mathbf{r})$$  

(12)

in the action. This term can be decoupled by means of the Hubbard-Stratonovich transformation with the Hermitian matrix field variables $Q_{nm}(\mathbf{r})$

$$\int \mathcal{D}[Q] \exp \left[ -\frac{1}{2g} \text{tr} Q^2(\mathbf{r}) + i \psi_{\alpha n}^\dagger(\mathbf{r}) Q(\mathbf{r}) \psi_{\alpha n}(\mathbf{r}) \right],$$  

(13)

where the symbol $\text{tr}$ denotes the matrix trace over the Matsubara and replica indices. The measure for the functional integral over the matrix field $Q$ is defined as: the integral equals unity when the fermionic fields $\psi^\dagger$ and $\psi$ vanish. Note also that in Eq. (13) we introduce the matrix notations according to

$$\psi_{\alpha n}^\dagger(\mathbf{r}) = \sum_{\omega_n \omega_m} \psi_{\alpha n}^\dagger(\mathbf{r}) \psi_{\alpha n}(\mathbf{r}) \psi_{\alpha n}^\dagger(\mathbf{r}).$$  

(14)

After making all these steps, the action becomes

$$S = -\frac{N_r \Omega \Delta}{T} - \frac{1}{2g} \int \text{tr} Q^2 +$$

$$+ \int \psi_{\alpha n}^\dagger(\mathbf{r}) (i \omega_n + \mu - \mathcal{H}_0 + \lambda) \psi_{\alpha n}(\mathbf{r}),$$  

(15)

where $\omega$ is the frequency matrix $(\omega)_{nm} = \omega_n \delta_{nm} \delta_{\alpha \beta}$.

C. The saddle-point in the $Q$ field

The $Q$ matrix field can be naturally split into the transverse $V$ and the longitudinal $P$ components as follows $Q = V^{-1} PV$. The longitudinal component $P$ has the block-diagonal structure in the Matsubara space, $P_{\nu \nu}^\alpha = \Theta(nm)$, where $\Theta(x)$ is the Heaviside step function, and corresponds to massive modes. The transverse component $V$ is a unitary rotation and describes massless (diffusive) modes (see Refs. 24, 25 for details).

This decomposition of the variable $Q$ into $P$ and $V$ is motivated by the saddle-point structure of the action (15) at zero temperature ($\omega_n \to 0$) and in the absence of the potential $\lambda(\mathbf{r})$. The corresponding saddle-point solution has the form $Q_{sp} = V^{-1} P_{sp} V$, where the matrix $P_{sp}$ is

$$(P_{sp})_{mn}^\alpha = P_{sp}^{\alpha mn} \delta_{nn} \delta_{\alpha \beta}$$  

(16)

with $P_{sp}^{\alpha mn} \rho \tau_0$ obeying the equation

$$\pi \rho \tau_0 P_{sp}^{\alpha mn} = i G_{0\alpha}^m(\mathbf{r}, \mathbf{r})$$  

(17)

This equation is equivalent to the self-consistent Born approximation equation 26. The Green function $G_{0\alpha}^m(\mathbf{r}, \mathbf{r}')$ is determined as

$$G_{0\alpha}^m(\mathbf{r}, \mathbf{r}') = \sum_k \phi_{Nk}^\alpha(\mathbf{r}) G_0(\omega_n) \phi_{Nk}^\alpha(\mathbf{r}')$$  

(18)
where $\epsilon_N = \omega_H (N + 1/2)$ and $\phi_{nk}(r)$ are the eigenvalues and eigenfunctions of the Hamiltonian $H_0$, and $k$ denotes pseudomomentum.

In the case of a small disorder $\omega_H \tau_0 \gg 1$ the solution of equation (17) has the form\[ P_{sp}^n = \frac{\text{sign} \omega_n}{2\tau}, \quad \tau = \pi \sqrt{\frac{\rho}{m} \frac{\tau_0}{\sqrt{\omega_H \tau_0}}}. \] (20)

The fluctuations of the $V$ field are responsible for the localization corrections to the conductivity (in the weak localization regime they correspond to the maximally crossed diagrams). However, in the considered case, these corrections are of the order of $1/N \ll 1$ and, therefore, can be neglected. For this reason we simply put $V = 1$.

The presence of the potential $\lambda$ results in a shift of the saddle-point value (20) due to the coupling to the fluctuations $\delta P = P - P_{sp}$ of the $P$ field. The corresponding effective action for the $\delta P$ field follows from Eq. (9) after integrating out fermions:

$$S[\delta P, \lambda] = \int_r \text{tr} \ln G^{-1}_0 - \frac{N_r \Omega_0}{T} - \frac{1}{2g} \int_r \langle (P_{sp} + \delta P)^2 \rangle + \int_r \text{tr} \ln \left[ 1 + (i\delta P + \lambda)G^{-1}_0 \right].$$ (21)

By using the identity

$$\langle (\delta P_{m_1 m_2}^\beta(q) \delta P_{m_3 m_4}^\gamma(-q) \rangle = \frac{g \delta_{m_1 m_3} \delta_{m_2 m_4} \delta^{\alpha \beta} \delta^{\gamma \delta}}{1 + g \pi_0 \delta_m(m_3 - m_1; q)}$$

where the bare polarization operator $\pi_0^m(n; q)$ is

$$\pi_0^m(n; q) = -n_L G_0(\omega_m + \nu_n)G_0(\omega_m)F_N^2(q).$$ (25)

D. Thermodynamical potential

To find the expansion of the thermodynamical potential $\Omega$ in powers of the CDW order parameter $\Delta(q)$, it is convenient to introduce a new variable $\tilde{\delta P} = \delta P + i\lambda$ and expand $\text{tr} \ln$ in the action (21) in powers of this new field $\tilde{\delta P}$. Then the thermodynamical potential can be written in the form

$$\Omega = \Omega_0 + \Omega_\Delta + \delta \Omega,$$ (26)

where

$$\Omega_0(\mu) = \int_r \text{tr} \ln G^{-1}_0 - \frac{1}{2g} \int_r P_{sp}^2$$ (27)

As a result, the thermodynamical potential can be written as

$$\Omega = -\frac{T}{N_r} \ln \int \mathcal{D}[\delta P] I[\delta P] \exp S[\delta P, \lambda],$$ (22)

where, following Ref. 22, the integration measure $I[\delta P]$ is

$$\ln I[\delta P] = -\frac{1}{(\pi\rho)^2} \int \sum_{n,m} [1 - \Theta(n;m)] \delta P^{\alpha \beta}_{nm} \delta P^{\beta \gamma}_{nm}.$$ (23)

The quadratic in $\delta P$ part of the action (21) together with the contribution (24) from the integration measure determine the propagator of the $\delta P$ fields (see Ref. 22 for details)

The mean-field thermodynamical potential of the homogeneous state, and

$$\delta \Omega = -\frac{T}{N_r} \ln \int \mathcal{D}[\tilde{\delta P}] \exp \tilde{S}[\tilde{\delta P}, \lambda]$$ (28)

takes into account the fluctuations of the massive longitudinal field $\delta P$ and their interaction with the CDW order parameter (potential $\lambda$). The action $\tilde{S}[\tilde{\delta P}, \lambda]$ has the form

$$\tilde{S} = S^{(2)}[\lambda] + S_{\text{int}}[\tilde{\delta P}, \lambda] + S^{(2)}[\tilde{\delta P}] + \sum_{n=3}^{\infty} S^{(n)}[\tilde{\delta P}],$$ (29)

with

$$S^{(2)}[\lambda] = \frac{N_r}{2g} \sum_{\omega_n} \int_r \lambda(r) \lambda(r),$$ (30)

$$S_{\text{int}}[\tilde{\delta P}, \lambda] = -\frac{i}{g} \int_r \lambda(r) \text{tr} \tilde{\delta P}(r),$$ (31)
Another important observation is that the propagator of \( \delta P \) is shown in Fig. 1. The corresponding diagram in the usual “cross technique” is shown in Fig. 1. A square denotes a propagator, a circle is the vacuum and dashed lines are for impurities.\(^\text{24}\)

FIG. 1: Second-order contribution to the thermodynamic potential. Solid line denotes electron Green function, dashed lines are impurity lines and vertexes are \( \lambda (r) \)

By using Eqs. (28)-(32) we can write

\[
\delta \Omega = -\frac{T}{2g} \sum_{\omega_n} \int_r \lambda(r)\lambda(r) - \frac{T}{N_r} \ln \langle \exp \tilde{S}_{\text{int}} \rangle ,
\]

where \( \langle \cdots \rangle \) denotes the average over \( \tilde{P} \) with respect to the action \( \tilde{S}[\delta \tilde{P}, 0] \). This equation allows us to find the contributions to the thermodynamical potential \( \Omega \) up to any order of the CDW order parameter \( \Delta(q) = F_N(q)U(q)^{-1}\lambda(q)/L_xL_y \).

In this paper we will work only with the expansion up to the fourth order term (the Landau expansion). This implies that our consideration is valid only close to the transition point where the value of the order parameter is small and one can truncate the series (28) after several first terms. It should be mentioned, however, that we should avoid a direct vicinity of the phase transition (the critical region, for more details see Sec. III) where the fluctuations of the order parameter break the mean-field approach.

1. Second-order contribution

The second order contribution to the thermodynamical potential \( \delta \Omega \) is

\[
\delta \Omega^{(2)} = -\frac{T}{2g} \sum_{\omega_n} \int_r \lambda(r)\lambda(r) - \frac{T}{N_r} \langle S^2_{\text{int}} \rangle_0 ,
\]

where \( \langle \cdots \rangle_0 \) stands for the average over \( \tilde{P} \) with respect to the action \( \tilde{S}^{(2)}[\delta \tilde{P}] \). We replace the average over the full action \( \tilde{S}[\delta \tilde{P}, 0] \) by the average over the quadratic part \( \tilde{S}^{(2)}[\delta \tilde{P}] \) only because the higher order in \( \delta \tilde{P} \) terms lead to the contributions that are proportional to \( N_r^2 \), and therefore vanish in the replica limit \( N_r \to 0 \).

With the help of Eqs. (10), (24), and (31), we obtain

\[
\frac{\delta \Omega^{(2)}}{L_x^2L_y^2} = n_L\frac{T}{2} \sum_{\omega_n} \int_q \frac{U^2(q)\Delta(q)\Delta(-q)}{1 + g\pi_0^2(0, q)} ,
\]

The corresponding diagram in the usual “cross technique” is shown in Fig. 2.

2. Third-order contribution

The contribution of the third power of the CDW order parameter to the thermodynamical potential \( \delta \Omega^{(3)} \) can be written as

\[
\delta \Omega^{(3)} = -\frac{T}{3!N_r} \langle S^3_{\text{int}} \rangle^{(c)}_0 - \frac{T}{3!N_r} \langle S^3_{\text{int}} \rangle^{(c)}_0 ,
\]

where the superscript \( (c) \) indicates that only connected diagrams are taken into account. Here we omit again the terms that vanish in the replica limit \( N_r \to 0 \). After performing the averaging over \( \tilde{P} \) with the help of Eqs. (10), (24), (31), and (32), we obtain

\[
\frac{\delta \Omega^{(3)}}{L_x^2L_y^2} = (2\pi)^2 n_L \frac{T}{3} \sum_{\omega_n} \prod_{j=1}^3 \int_q \frac{U(q)\Delta(q)\Delta(0, q)}{1 + g\pi_0^2(0, q)} \exp\left(\frac{i}{2}(q_1^yq_2^y - q_1^xq_2^x)\right) ,
\]

The contribution \( \delta \Omega^{(3)} \) corresponds to the diagram in Fig. 2.

3. Fourth-order contribution

The forth order contribution \( \delta \Omega^{(4)} \) is

\[
\delta \Omega^{(4)} = -\frac{T}{4!N_r} \langle S^4_{\text{int}} \rangle^{(c)}_0
\]

\[
= -\frac{T}{4!N_r} \langle S^4_{\text{int}} \rangle^{(c)}_0 ,
\]

where \( \langle \cdots \rangle_0 \) denotes electron Green function, dashed lines are impurity lines and vertexes are \( \lambda (r) \).
where again only terms which is proportional to $N_e$ are kept. By using Eqs. (10), (24), (31), and (32), we find

\[
\frac{\delta \Omega^{(4)}}{L_x L_y^3} = (2\pi)^2 n_L \frac{T}{4} \sum_{\omega_n} \prod_{j=1}^{4} \left[ \int_{q_j} \frac{U(q_j) \Delta(q_j) G_0(\omega_n)}{1 + g \pi_0^\omega (0, q_j)} \right] \times \delta(q_1 + q_2 + q_3 + q_4) \left[ 1 - g \pi_0^\omega (0, q_1 + q_2) \right] \times \exp \frac{i}{2} \left( q_1^2 q_2^2 - q_3^2 q_4^2 \right) \exp \frac{i}{2} \left( q_1 q_2 - q_3 q_4 \right).
\]

In the usual “cross technique” the contribution $\delta \Omega^{(4)}$ corresponds to the diagram shown in Fig. 3.

E. Free energy

The free energy of the CDW state can be written in the form

\[
\mathcal{F} = \mathcal{F}_0 + \Omega(\mu) - \Omega(\mu_0) + (\mu - \mu_0)N_e,
\]

where $\mathcal{F}_0$ is the free energy of the normal (homogeneous) state, $N_e$ the total number of electrons, $\mu$ and $\mu_0$ the chemical potentials of the CDW state and the normal state respectively.

In order to find the free energy of the CDW state to the forth order in the CDW order parameter we expand $\Omega(\mu)$ around the point $\mu$ to the second order in $\mu - \mu_0$. This results in

\[
\mathcal{F} = \mathcal{F}_0 + \Omega(\mu) - \Omega(\mu_0) - \frac{1}{2} (\mu - \mu_0)^2 \partial^2 \Omega_0 / \partial \mu^2.
\]

The difference $\mu - \mu_0$ of the chemical potentials in the CDW and the normal states is

\[
\mu - \mu_0 = \frac{\partial \delta \Omega}{\partial N_e} \left( \frac{\partial N_e}{\partial \mu} \right)^{-1},
\]

and from Eq. (26) we obtain

\[
\mathcal{F} = \mathcal{F}_0 + \delta \mathcal{F} + \frac{1}{2} \left( \frac{\partial \delta \Omega}{\partial \mu} \right)^2 \left( \frac{\partial N_e}{\partial \mu} \right)^{-1}.
\]

With the expression (35) for $\delta \Omega$, this gives

\[
\mathcal{F} = \mathcal{F}_0 + \Omega_{\Delta} + \delta \Omega + \frac{n_L(L_y)}{2} \sum_{\omega_n} \int_q \frac{U^2(q) G_0^\omega(\omega_n)}{1 + g \pi_0^\omega (0, q)} \Delta(q) \Delta(-q) \left[ T \sum_{\omega_n} G_0^\omega(\omega_n) \right]^{-1}.
\]

F. Free energy of the triangular CDW state

The CDW order parameter for the triangular lattice symmetry (bubble phase) can be written in the form

\[
\Delta(q) = \frac{(2\pi)^2}{L_x L_y} \Delta(q) \sum_{j=1}^{3} \left[ \delta(q - Q_j) + \delta(q + Q_j) \right],
\]

where the vectors $Q_j$ have the angle $\pi/3$ between each other and obey the condition $Q_1 + Q_2 + Q_3 = 0$.

By using Eqs. (43), (44), (48), and (12), we obtain the following expression for the free energy of the triangular CDW state

\[
\mathcal{F}^t = \mathcal{F}_0 + 4 \frac{L_x L_y}{2 \pi T} T_0(Q) \left[ a_2 \Delta^2 + a_3 \Delta^3 + a_4 \Delta^4 \right].
\]

Here the three coefficients $a_1, a_2$, and $a_3$ of the Landau expansion are as follows

\[
a_2 = 3 \left[ 1 - \frac{T_0(Q)}{\pi^2 T} \sum_n \frac{1}{\xi_n^2 + \gamma^2(Q)} \right],
\]

where

\[
\xi_n = n + \frac{1}{2} + \frac{1}{4 \pi T} - i \frac{\mu}{2 \pi T}, \quad \gamma(Q) = \frac{F_N(Q)}{4 \pi T}
\]

and $T_0(Q) = U(Q)/4$.

\[
a_3 = i 8 \frac{T_0^2(Q)}{\pi^3 T^2} \cos \left( \frac{\sqrt{3} Q^2}{4} \right) \sum_n \frac{\xi_n^3}{\xi_n^2 + \gamma^2(Q)}.
\]

and
The high order terms in the Landau expansion.

parameters of the forming CDW state are determined by a homogeneous state to the CDW state. As usual, the specific frequency sums in expressions for free energy of the triangular and unidirectional CDW states (46)-(55) leads to the results obtained in Refs. 7-9.

III. MEAN-FIELD PHASE DIAGRAM

A. Instability line

The vanishing of the coefficient in front of the quadratic term in the Landau expansion of the free energy signals about the instability of the normal state towards the formation of the CDW. This instability corresponds to the second order phase transition from the homogeneous state to the CDW state. As usual, the specific parameters of the forming CDW state are determined by the high order terms in the Landau expansion.

From Eqs. (47) and (54) we obtain the following equation

\[ T \] for the instability line. The solution \( T(Q) \) of this equation depends on the modulus \( Q \) of the vector that characterizes the CDW state. The temperature \( T_2 \) of the second order phase transition corresponds to the maximal value of \( T(Q) \):

\[ T_2 = \max_Q T(Q) \] and the corresponding value \( Q_0 \), \( T_2 = T(Q_0) \), determines the period of the CDW state. The Hartree-Fock potential (11) has minima at those vectors \( Q_k \) for which the form-factor \( F_N(Q_k) \) vanishes. In the clean case this corresponds to \( Q_0 = \min Q_k = r_0 / R_c \), where \( r_0 \approx 2.4 \) is the first zero of the Bessel function of the first kind. It can be seen from Eq. (56) that a weak disorder does not shift the vector \( Q_0 \) (see Appendix). Thus the equation for the

\[ a_4 = \frac{24T^3_0(Q)}{\pi^4 T^3} \left\{ \frac{1}{2} \sum_n \frac{\xi_n^4}{\xi_n^2 + \gamma^2(Q)} \right\} \left\{ 3D_n(0) + \frac{1}{2} D_n(2Q) \right\} \left( 1 + \cos \frac{\sqrt{3}Q^2}{2} \right) \left( D_n(Q) + D_n(\sqrt{3}Q) \right) + \frac{1}{2} D_n(2Q) \] with

\[ D_n(Q) = \frac{\xi_n^2 - \gamma^2(Q)}{\xi_n^2 + \gamma^2(Q)} \] the free energy of the unidirectional CDW state reads

\[ F^u = F_0 + \frac{L_z L_y}{2\pi^2 T_0} \left\{ b_2 \Delta^2 + b_4 \Delta^4 \right\} \] Here the coefficients \( b_2 \) and \( b_4 \) of the Landau expansion are

\[ b_2 = \frac{a_2}{3} = \left\{ 1 - \frac{T_0(Q)}{\pi^2 T} \sum_n \frac{1}{\xi_n^2 + \gamma^2(Q)} \right\} \] and

\[ b_4 = \frac{4T^3_0(Q)}{\pi^4 T^3} \left\{ \sum_n \frac{\xi_n^4}{\xi_n^2 + \gamma^2(Q)} \right\} \left\{ D_n(0) + \frac{1}{2} D_n(2Q) \right\} + 2 \left\{ \sum_n \frac{\xi_n^4}{\xi_n^2 + \gamma^2(Q)} \right\} \left\{ \sum_n \xi_n^{-2} \right\}^{-1} \] (50)
temperature of the second order phase transition into the CDW state reads

\[ \frac{T}{T_0} = \frac{2}{\pi^2} \Re \psi \left( \frac{1}{2} + \frac{1}{4\pi T \tau} + i \frac{\mu}{2\pi T} \right) \] (58)

where \( \psi'(z) \) is the derivative of digamma function, \( \Re \) the real part, and \( T_0 \equiv T_0(Q_0) \) the temperature of the transition in the clean case.

Eq. (58) contains the chemical potential \( \mu \) that, together with the temperature \( T \) and the broadening of Landau levels 1/2\( T \tau \), determines the filling factor \( \nu_N = \nu - 2N \) of the partially filled highest Landau level. However, in order to find this relation one needs to know the density of states in the system. This question about the density of states is a very subtle \(^7\) and beyond the scope of the present paper. For this we use the chemical potential \( \mu \) rather than filling factor \( \nu_N \).

Eq. (58) can be solved analytically in the two extreme cases: when temperature \( T \) is closed to the temperature \( T_0 \) of instability in the absence of disorder, and when the temperature \( T \) is close to zero.

In the first case, the broadening of the Landau level 1/2\( T \tau \) and the chemical potential \( \mu \) are small compared to the temperature \( T_0 \) of the instability in the clean case, and, therefore, the leading order expansion in powers of 1/2\( T \tau \) and \( \mu/T_0 \) is legitimate. It appears that the presence of disorder decreases the temperature of instability linearly:

\[ \frac{T}{T_0} = 1 - \frac{7\zeta(3)}{\pi^3 T_0 \tau} \frac{\mu^2}{4T_0^2} , \quad \frac{1}{2\tau}, \mu \ll 2\pi T. \] (59)

In the opposite case \( T \to 0 \), one has 1/2\( T \tau, \mu \gg 2\pi T \), and Eq. (58) reduces to

\[ \frac{\pi}{8T_0 \tau} = \frac{1}{1 + 4\pi^2 \tau^2 \mu^2} \left[ 1 - \left( 1 - 12\pi^2 \tau^2 \right)^{\frac{\pi^2 T^2}{4T_0^2}} \right]. \] (60)

We see from Eq. (60) that at zero temperature the second order phase transition can occur only when the broadening of the Landau level is smaller than some critical value, 1/2\( T \tau \leq 1/2T_c = 4T_0/\pi \).

For other cases Eq. (58) can be solved numerically, and the corresponding instability (spinodal) line is shown in Fig. 4.

B. Half-filled Landau level (\( \nu_N = 1/2 \))

We now consider the case of the half-filled \( N \)th Landau level (\( \nu_N = 1/2 \)), that is related to the recent experiments. In this case the chemical potential is zero, \( \mu = 0 \), provided the density of states is symmetric around the center of the \( N \)th Landau level. As follows from Eq. (58), the temperature of the second order phase transition for this case can be found from the equation

\[ \frac{T}{T_0} = \frac{2}{\pi^2} \zeta \left( \frac{2}{2} + \frac{1}{4\pi T \tau} \right), \] (61)

where \( \zeta(2, z) = \sum_{m=0}^{\infty} (m + z)^{-2} \) is the generalized Riemann zeta function. The analytical solutions of this equation in the cases of high and low temperature can be obtained from Eqs. (58) and (60) by putting \( \mu \) to zero. The entire behavior of the spinodal line, obtained numerically from Eq. (61), is shown in Fig. 4.

We mention that at \( \nu_N = 1/2 \) the coefficient \( a_3 \) vanishes due to the particle-hole symmetry. It means that the transition from the normal state into the CDW state...
FIG. 6: Phase diagram at zero temperature near $\nu = 1/2$. The solid line is obtained from Eq. (63), the dashes are the spinodal line and the dots are obtained from Eq. (64).

is of the second order for both cases of unidirectional and triangular CDW symmetry. Therefore, to find the structure of the CDW state, one has to take into account the fourth order terms in the the Landau expansion. In the vicinity of the spinodal line, it follows from Eqs. (50) and (51) with $\mu = 0$ that

$$a_4 = \frac{12 T_0^3}{\pi^4 T^3} \left[ -7 \zeta(4, u) + 12 \Phi_0(u) + 6 \Phi_2(u) + 8 \Phi_4(u) \right]$$

and

$$b_4 = \frac{2 T_0^3}{\pi^4 T^3} \left[ -3 \zeta(4, u) + 4 \Phi_0(u) + 2 \Phi_2(u) \right],$$

where we introduce the new variable $u = 1/2 + 1/4\pi T\tau$ and the new function

$$\Phi_a \left( \frac{1}{2} + z \right) = \frac{1}{z \mathcal{J}_0^2(ar_0)} \left[ -3 \zeta(4, u) + 4 \Phi_0(u) + 2 \Phi_2(u) \right]$$

with $\Im$ being the imaginary part. With these expressions we minimized $F_{\tau}^{\text{un}}$ with respect to the order parameter $\Delta$ and found that the unidirectional CDW state has lower free energy.

C. Phase diagram at zero temperature

In this section we analyze the zero temperature phase diagram in the case where the Landau level broadening is close to its critical value $1/2\tau_c = 4 T_0 / \pi$. Under these conditions, the CDW order parameter $\Delta$ is small, and one can use the Landau expansions at $\nu = 1/2$ and $\nu = 3/2$ at zero temperature. The coefficients of these expansions are

$$a_2 = 3 \left( 1 - \frac{8 T_0 \tau}{\pi} H_1(\mu \tau) \right), \quad a_3 = 2 \pi \left( \frac{8 T_0 \tau}{\pi} \right)^2 H_2(\mu \tau),$$

and

$$a_4 = 3 \pi^2 \left( \frac{8 T_0 \tau}{\pi} \right)^3 H_3(\mu \tau, QR_c)$$

for the triangular CDW state, and

$$b_2 = \frac{a_2}{3}, \quad b_4 = \frac{\pi^2}{2} \left( \frac{8 T_0 \tau}{\pi} \right)^3 H_4(\mu \tau, QR_c)$$

for the unidirectional CDW state. Here we introduce four functions $H_i(z)$ as

$$H_1(z) = \frac{1}{1 + 4 z^2}, \quad H_2(z) = 4 z H_1(z),$$

$$H_3(z, r) = \frac{1 + 108 z^2 - 5 \frac{6}{(1 + 4 z^2)^3} + 3 R_0(z, r) + 2 R_1(z, r)}{2},$$

and

$$H_4(z, r) = \frac{28 z^2 - 1}{(1 + 4 z^2)^3} + 2 R_0(z, r) + R_2(z, r),$$

where

$$R_a(z, r) = \frac{2 H_1(z)}{\mathcal{J}_0^2(ar)} - \frac{1}{\mathcal{J}_0^2(ar)} \arctan \frac{2 \mathcal{J}_0(ar)}{1 + 4 z^2 - \mathcal{J}_0^2(ar)}.$$

These expressions result in the following equation on the line of the first order transition from the uniform to the triangular CDW state

$$\frac{\pi}{8 T_0 \tau} = H_3(\mu \tau) + \frac{H_4^2(\mu \tau)}{9 H_3(\mu \tau, QR_c)}.$$

As before, the maximum of the solutions $1/\tau(Q)$ of Eq. (72) with respect to $Q$ should be found. It appears that the magnitude is not exactly at $Q = Q_0$ as in the clean case, but at some shifted value $Q_0 + \delta Q$ with the shift $\delta Q = -0.02 (\mu \tau)^2 R_c^{-1}$ for small $\mu \tau \ll 1$. The existence of the shift is a feature of the disordered case. Below in the limit $\mu \tau \ll 1$, we will neglect this shift. In this case Eq. (72) can be written as

$$\frac{\pi}{8 T_0 \tau} = 1 - 2.94 (\mu \tau)^2.$$

By comparing the free energies of the triangular and the unidirectional CDW states, we can find the line of the first order transition between them

$$\frac{\pi}{8 T_0 \tau} = H_1 - \frac{H_2^2(2 H_1 + H_4) [3 H_4 + \sqrt{H_1^2 + 2 H_1 H_2}]}{2 H_3 (2 H_3 - 3 H_4)^2}.$$
For the case $\mu \tau \ll 1$ Eq. (74) can be simplified as
\[
\frac{\pi}{8T_0 \tau} = 1 - 18.44(\mu \tau)^2. \tag{75}
\]
For other values of $\mu \tau$ Eqs. (72) and Eq. (74) were solved numerically, and the results are shown in Fig. 6. When the parameter $8/\pi T_0 \tau$ decreases at a fixed value of the chemical potential, the CDW order parameter grows, and hence, we go beyond the applicability of to the Landau expansion.

IV. WEAK CRYSTALLIZATION CORRECTIONS

The CDW order parameter $\Delta(r)$ introduced in Eq. (5) can be thought of as a saddle-point solution for the plasmon field that appears in the Hubbard-Stratonovich transformation of the electron-electron interaction in the action (22). The Landau expansions (46) and (53) for the free energy and average over the fluctuations $\delta T$ in the Landau expansion of the free energy of the CDW states were derived under the assumption that one can neglect the fluctuations of the CDW order parameter. This is legitimate for $N \gg 1$ and not very close to the transition (outside the critical region). However, when one approaches the instability line, the fluctuations of the CDW order parameter increase. To analyse the effects of the order parameter fluctuation, we introduce, following the original ideas of Brazovskii, the fluctuations of the CDW order parameter $\Delta(r) \to \Delta(r) + \delta(r)$ in the Landau expansion of the free energy and average over the fluctuations $\delta(r)$. We present below the results of the corresponding analysis only for the most interesting case of the half-filled Landau level.

We find that the transition from the uniform to the unidirectional CDW state becomes of the first order, and takes place at the lower temperature that can be found from the following equation (see Eq. (61) for comparison)
\[
\frac{T}{T_0} = \frac{2}{\pi^2 \zeta} \left( 2 \frac{1}{2} + \frac{1}{4 \pi T \tau} \right) - g \left( \frac{1}{4 \pi T \tau} \right) N^{-2/3}, \tag{76}
\]
Here function $g(z)$ is defined as
\[
g(z) = 3 \left[ \frac{3 \pi r_0}{16} \right]^{2/3} \left[ \frac{\lambda_0(z)}{f(z)} \right]^{2/3} \left[ \frac{2 \lambda_0(z) + \lambda_2(z)}{4 \lambda_0(z) - \lambda_2(z)} \right]^{1/3}, \tag{77}
\]
where we introduce the following three functions
\[
f(z) = \frac{2}{\pi^2} \left[ \beta_1 z \left( 2 \frac{1}{2} + z \right) + \beta_2 z^2 \zeta \left( 4 \frac{1}{2} + z \right) \right], \tag{78}
\]
\[
\lambda_a(z) = \frac{2}{\pi^4} \left[ -\zeta \left( 4 \frac{1}{2} + z \right) + 2 \Phi_a \left( 4 \frac{1}{2} + z \right) \right], \tag{79}
\]
The constants $\beta_i$ are given by
\[
\beta_1 = \frac{T_0'(Q_0)}{T_0(Q_0)} = 2.58, \quad \beta_2 = (J'_0(Q_0))^2 = 0.27, \tag{80}
\]
and function $\Phi_a$ is defined by Eq. (65).

We mention that the function $\pi^2 g(z)/2 z (2.1/2 + z)$ decreases monotonically from the value 0.35 at $z = 0$ to zero at $z \to \infty$. Therefore, we obtain the following inequality for the shift $\delta T$ of the mean-field transition temperature $T$
\[
\frac{\delta T}{T} \leq 3 \left( \frac{\pi r_0}{16 \sqrt{\beta_1}} \right)^{2/3} N^{-2/3}, \quad N \gg 1 \tag{81}
\]
(the equality corresponds to the clean case).

The appearance of a noninteger powers in Eq. (40) results from the fact that the momentum dependence of the correlation function for the order parameter fluctuations contains $(Q - Q_0)^2$ rather than $Q^2$ (see Ref. 26).

Eq. (29) was derived under the assumption that the main contribution in the momentum space comes from the region $Q \approx Q_0$. This assumption is justified under the following condition (27)
\[
\frac{g \left( \frac{1}{4 \pi T \tau} \right)}{r_0^2 f \left( \frac{1}{4 \pi T \tau} \right)} \ll N^{2/3}, \tag{82}
\]
The combination of functions in the left hand side of inequality (82) decreases monotonically from 0.023 to 0 while $z$ increases from zero to infinity and, hence, the condition (82) is hold.

According to Eq. (81), the fluctuations reduce the transition temperature by the amount of the order of $N^{-2/3} \ll 1$ and, therefore, in the considered case of the weak magnetic field ($N \gg 1$) their effects can be neglected. These results indicate that the critical region for the considered transition is indeed small, and the mean-field approach gives a good approximation for $N \gg 1$.

V. DISCUSSIONS

A. Comparison with experimental results

Now we discuss the possible applications of our theory to the recent experiments. Although our mean-field theory was derived for the case of a large number of the occupied Landau level $N \gg 1$, and neglects corrections of the order of $1/N$, while experimentally one has $N = 2, 3, 4$, we however expect that Eq. (58) gives a good estimation for the temperature of the transition from the uniform to the CDW state, even for $N = 2, 3, 4$. We have complementary assurance that it can really be the case because Eq. (58) can be obtained without introducing the CDW order parameter and considering the mean-field...
theory but as the equation that determines the temperature $T(Q)$ at which the two-particle vertex function at wave vector $Q$ diverges.\textsuperscript{29}

We restrict ourselves by discussion of the experiments without an in-plane magnetic field.\textsuperscript{9,10} The theory for the half-filled highest Landau level contains two physical parameters: the temperature $T_0$ and the broadening of the Landau level $1/2\tau$. The estimate of the temperature of instability $T_0$ in the absence of disorder is more subtle. The theory can provide an estimate for $U(Q_0)$ and, correspondingly, for the value of $T_0$ only for a weak magnetic field. In this case the value of $U(Q_0)$ can be found analytically\textsuperscript{5} with the following result for $T_0$

$$T_0 = \frac{\alpha}{4} \omega_H , \quad N_{rs} \gg 1, \ r_s \leq 1 , \quad (83)$$

where the numerical parameter $\alpha$ only slightly depends on $r_s$ in the range $0.1 < r_s < 1$, and approximately $\alpha \approx 0.03$. The broadening of the Landau level $1/2\tau$ can be estimated from the mobility at zero magnetic field. However, the results obtained in Sec. \textbf{III.B} impose the restriction on the value of the sample mobility at zero magnetic field. In order to observe the CDW states at the partially filled Landau level with index $N$ the mobility at zero magnetic field should satisfy the condition $\mu > eN/2\alpha^2 n_e$ where $n_e$ denotes the electron density of 2DEG. For typical values of the electron density of 2DEG we can obtain the following estimate $\mu > N \cdot 10^6 \text{cm}^2/\text{Vs}$.

In the experiments of Lilly et al., the samples were relatively clean (mobilities exceed $9 \cdot 10^6 \text{cm}^2/\text{Vs}$), and the anisotropy was observed for $N = 2$ below $T = 150 \text{mK}$. For the temperature $T = 25 \text{mK}$, the anisotropy in the resistance was found to disappear with the decreasing the magnetic field for $N = 4$. We plot experimental points in Fig. 5 under the assumption that Eq. (83) remains valid even for $N = 2, 3, 4,$ and 5. As it can be seen from this Figure, for a constant magnetic field the highest temperature at which anisotropy appears, corresponds to $N = 2$. As the magnetic field decreases at some constant temperature, the anisotropy disappears at some value of $N$ due to the disorder induced transition from the CDW state to the uniform. It should be mentioned that without the effects of disorder the anisotropy in the resistance should remains up to $N = 12$ at $T = 25 \text{mK}$. Therefore, the disorder plays an important role even in the high mobility samples.

\textbf{B. Comparison with numerical results}

The problem of the formation of the CDW state on the second Landau level with $\nu \nu = 1/2$ at zero temperature in the presence of a quenched disorder was studied numerically in Ref.\textsuperscript{24}. The system of 12 electrons interacting via the Coulomb interaction $U(q) = 2\pi e^2/q$ in the presence of the quenched disorder was projected on the second Landau level ($N = 2$). The effects of interactions with electrons on the other Landau level was not taken into account. The system was diagonalized numerically. It was found that the CDW state transforms into a uniform liquid state as the dimensionless disorder strength $\omega_H \sqrt{n_L/2\pi \rho_0}$ exceeds 0.12.

In order to be able to compare the results of the presented above mean-field theory with the numerical results, we perform the evaluation of the temperature $T_0$ in the case for which the numerical results were obtained (instead screened interaction\textsuperscript{25} we use $U(q) = 2\pi e^2/q$). Under this circumstances our theory gives the value 0.14.

The small discrepancy may be attributed to two factors: on the one hand, the finite number of electrons in numerical calculations and, on the other hand, insufficient of the Hartree-Fock approximation for the problem with the Coulomb interaction $U(q) = 2\pi e^2/q$. In the later case one should take into account the diagrams beyond the Hartree-Fock theory. Nevertheless, the comparison demonstrates that such corrections are small.

We emphasize that our theory which takes into account the screening of electron-electron interaction by electrons on the other Landau levels gives much smaller value 0.038 of the dimensionless disorder strength for the transition from the uniform liquid state to the CDW state.

\textbf{VI. CONCLUSIONS}

For the system of a two-dimensional interacting electrons in the presence of a weak disorder and a weak magnetic field, we investigated the effect of disorder on the existence of the mean-field CDW states in the framework of the Hartree-Fock approximation. In the considered case of large filling factors $\nu \gg 1$, we obtained that the mean-field CDW instability exists if the disorder is rather weak, $1/\tau \leq 8T_0/\pi$. We found that at half-filling the unidirectional CDW state appears, and the presence of disorder does not change the vector of the CDW. Near half-filling, the unidirectional CDW state is energetically more favorable than the triangular one. We obtained that the weak crystallization corrections to the mean-field result are of the order of $(1/\nu)^{2/3} \ll 1$ and thus can be neglected. We discussed the applications of our theory to the recent experimental and numerical results.

\textbf{Acknowledgments}

I.B. is grateful to M.V. Feigel’man and M.A. Skvortsov for stimulating discussions. This research was supported by Forschungszentrum Jülich (Landau Scholarship), Russian Foundation for Basic Research (RFBR), and Deutsche Forschungsgemeinschaft (DFG).

\textbf{APPENDIX A: INSTABILITY VECTOR}

In this appendix we prove that the weak disorder does not change the vector at which the instability towards the
formation of the CDW state grows. Let us consider the solution $T + \delta T$ of Eq. (53) for the vector $Q = Q_0 + \delta Q$, where $\delta Q \ll Q_0$. We will now show that the shift $\delta T$ is always negative, and hence, the maximal instability solution is always negative, and hence, the maximal instability temperature corresponds to the vector $Q = Q_0$, as it is in the clean case.

For a small deviation $\delta Q$ we can write

$$T_0(Q) = T_0(1 - \beta_1(\delta Q R_c)^2), \quad \mathcal{J}_0^2(Q R_c) = \beta_2(\delta Q R_c)^2. \quad (A1)$$

The shift $\delta T$ results in the substitution

$$\xi_n \rightarrow \xi_n - \frac{\delta T}{4\pi T^2} - i \frac{\delta T \mu}{2\pi T^2}, \quad (A2)$$

in Eq. (53), and we obtain

$$\frac{\delta T}{T} = -(\delta Q R_c)^2 \frac{\beta_1 g_2 + \beta_2 z^2 y_4}{g_2 - 2(z g_1 + y g_3)}, \quad (A3)$$

where $z = 1/4\pi T \tau$ and $y = \mu/2\pi T$. Here we introduce the four functions $g_a(z, y)$ and $g_a(z, y)

It can be easily seen that the rhs of Eq. (53) is negative for all possible values of $z$ and $y$.

---

* Also at Kurchatov Institute, Kurchatov Square 1, 123182 Moscow, Russia
1 For a review, see The Quantum Hall effect, ed. by R.E. Prange and S.M. Girvin (Springer-Verlag, Berlin, 1987)
2 A.H. MacDonald and S.M. Girvin, Phys. Rev. B 33, 4009 (1986)
3 L. Belkhir and J.K. Jain, Solid State Commun. 94, 107 (1995); R. Morf and N. d’Ambrumenil, Phys. Rev. Lett. 74, 5116 (1995)
4 A.P. Smith, A.H. MacDonald and G. Gumbs, Phys. Rev. B 45, 8829 (1992)
5 I.L. Aleiner and L.I. Glazman, Phys. Rev. B 52, 11296 (1995)
6 A.A. Koukakov, M.M. Fogler, and B.I. Shklovskii, Phys. Rev. Lett. 76, 499 (1996), Phys. Rev. B 54, 1853 (1996)
7 H. Fukuyama, P.M. Platzmann, and P.W. Anderson, Phys. Rev. B 19, 5211 (1979)
8 R. Moessner and J.T. Chalker, Phys. Rev. B 54, 5006 (1996)
9 M.P. Lilly, K.B. Cooper, J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. 82, 394 (1999)
10 R.R. Du, D.C. Tsui, H.L. Stormer, L.N. Pfeiffer, and K.W. West, Solid State Commun. 109, 389 (1999)
11 For a review, see M.M. Fogler, arXiv: cond-mat/ 0111001 and references therein.
12 A. Lopatnikova, S.H. Simon, B.I. Halperin, and X.-G. Wen, Phys. Rev. B 64, 155301 (2001)
13 D.G. Barci, E. Fradkin, S.A. Kivelson, and V. Oganesyan, Phys. Rev. B 65, 245319 (2002); D.G. Barci and E. Fradkin, Phys. Rev. B 65, 245320 (2002)
14 S. Scheidt and F. von Oppen, Europhys. Lett. 55, 260 (2001)
15 W. Pan, R.R. Du, H.L. Stormer, D.C. Tsui, L.N. Pfeiffer, K.W. Baldwin, and K.W. West, Phys. Rev. Lett. 83, 820 (1999); M.P. Lilly, K.B. Cooper, J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, Phys. Rev. Lett. 83, 824 (1999)
16 A.A. Abrikosov and L.P. Gor’kov, Zh. Eksp. Teor. Fiz. 39, 1781 (1960)
17 A.I. Larkin, Pis’ma Zh. Eksp. Teor. Fiz. 2, 205 (1965)
18 I.S. Burmistrov, Zh. Eksp. Teor. Fiz. 122, 150 (2002) [JETP 95, 132 (2002)], arXiv: cond-mat/0203022
19 X.-G. Wu and S.L. Sonohdi, Phys. Rev. B 51, 14725 (1995)
20 For details of the HF decoupling see e.g. Ref. 2 and Ref. 16.
21 R.L. Stratonovich, Dokl. Akad. Nauk SSSR 2, 1097 (1957) [Sov. Phys. Doklady 2, 416 (1957)]; J. Hubbard, Phys. Rev. Lett. 3, 77 (1959)
22 K.B. Efetov, A.I. Larkin, D.E. Khmel’nitzkii, Zh. Eksp. Teor. Fiz. 79, 1120 (1980) [Sov. Phys. JETP 45, 1980]
23 A.M. Finkel’stein, Pis’ma Zh. Eksp. Teor. Fiz. 37, 436 (1983) [JETP Lett. 37, 517 (1983)]; Zh. Eksp. Teor. Fiz. 84, 168 (1983) [Sov. Phys. JETP 57, 97 (1983)]; Zh. Eksp. Teor. Fiz. 86, 367 (1984) [Sov. Phys. JETP 59, 212 (1984)]
24 A.M.M. Pruisken, Nucl. Phys. B 235, 277 (1984)
25 A.M.M. Pruisken, M.A. Baranov, and B. Škorić, Phys. Rev. B 60, 16807 (1999)
26 T. Ando and Y. Uemura, J. Phys. Soc. Japan 36, 959 (1974); ibid 36, 1521 (1974); T. Ando, ibid 37, 1233 (1974)
27 For a review, see I.V. Kukushkin, J.P. Eisenstein, L.N. Pfeiffer, and K.W. West, Phys. Rev. B 54, 11296 (2003)
28 A.M.M. Pruisken, M.A. Baranov, and B. Škorić, Phys. Rev. B 60, 16807 (1999)
29 For the clean case see Ref. 16 and for the weakly disordered case I.S. Burmistrov and M.A. Baranov unpublished
30 D.N. Sheng, Z. Wang, and B. Friedman, Phys. Rev. B 64, 195316 (2002)