Optimization of Energy Harvesting MISO Communication System with Feedback

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Abstract—Optimization of a point-to-point (p2p) multiple-input single-output (MISO) communication system is considered when both the transmitter (TX) and the receiver (RX) have energy harvesting (EH) capabilities. The RX is interested in feeding back the channel state information (CSI) to the TX to help improve the transmission rate. The objective is to maximize throughput by a deadline, subject to the EH constraints at the TX and the RX. The throughput metric considered is an upper bound on the ergodic rate of the MISO channel with beamforming and limited feedback. Feedback bit allocation and transmission policies that maximize the upper bound on the ergodic rate are obtained. Tools from majorization theory are used to simplify the formulated optimization problems. Optimal policies obtained for the modified problem outperform the naive scheme in which no intelligent management of energy is performed.

Index Terms—Energy harvesting, Limited feedback, MISO, Offline optimization.

I. INTRODUCTION

Powering up terminals in communication networks by renewable ambient energy reduces the carbon footprint of the information and communication technologies, which can no longer be neglected with the exponential growth in the number of communication devices. Another advantage of EH technology is that, it increases the autonomy of battery-run communication devices. In traditional wireless networks nodes get their energy from the power grid by always or periodically connecting to it. While it is easy to connect the terminals to the grid in some networks, in others, such as sensor networks, it cannot be done once after the deployment. Therefore, in such networks a node’s lifetime, and hence, the network lifetime, is constrained by the limited initial energy in the battery. Providing EH capabilities to the communication nodes is an attractive solution to the network lifetime problem [1]. An EH node can scavenge energy from the environment (typical sources are solar, wind, vibration, thermal, etc.) [2]. With EH nodes in the network, in principle, one can guarantee perpetual lifetime without the need of replacing batteries.

However, EH poses a new design challenge as the energy sources are typically sporadic and random. The main challenge lies in ensuring the Quality of Service (QoS) constraints of the network given the random and time varying energy sources. This calls for the intelligent management of various parameters involved in a communication system.

Recently, a significant number of papers have appeared studying the optimal transmission schemes for EH communication systems under different assumptions regarding the node’s knowledge about the underlying EH process. Offline optimization framework deals with systems in which non-causal knowledge of the EH process is available. Within this frame work, optimal transmission schemes are studied for the p2p fading channel [3], broadcast channel [4], [5], [6] and relay channel [7], [8]. See [9] for an extensive overview.

To the best of our knowledge, a common aspect of all prior works on EH communication networks is that the TX is assumed to have access to perfect CSI. Knowledge of the CSI at the TX is beneficial in designing the optimal channel adaptation techniques and the TX filters in multi-antenna systems. However, recent studies have demonstrated that, although feedback enhances the system performance, feedback resources, namely power and bandwidth, are limited, and must be spent wisely [10]. As a result, an important question arises: How do the EH constraints affect the design of feedback enabled wireless networks?

In this paper, we study the optimization of a feedback enabled EH MISO channel, where feedback is used to improve the rate through array gain. The system model and the main assumptions in this paper are given in Section III. In Section IV, we consider the optimization of the feedback policy under EH constraints at the RX, while the TX is assumed to have a constant power supply. The motivation is to address the following: In the case of EH, the available energy at the RX varies over time. Should the RX feedback same quality of CSI at all times? If so, can the CSI feedback quality be improved by using more bandwidth in the low energy scenario? In the second part of this paper (Section V), we assume that both the TX and the RX harvest energy. In this case, the transmission power policy and the feedback policy are coupled, and need to be jointly optimized. Results from multivariate majorization theory are used to devise simple algorithms. We start by giving a brief preliminary description of majorization theory in Section II. Numerical results are presented in Section VI to validate the analysis. Finally, Section VII concludes the paper.

Notation: Boldface letters are used to denote matrices and vectors. The transpose and conjugate transpose of matrix $A$ is denoted by $A^T$ and $A^H$, respectively. We use $d_{i,j}$ to denote the element at the $i$-th row and $j$-th column of matrix $D$, and $|S|$ to denote the cardinality of the set $S$. The set of integers from $m$ to $n$, $m < n$, is represented by $\left[m : n\right]$. The algorithm with name “Algo” is represented as $\text{[output arguments]}= \text{Algo (input arguments)}$. A circularly-symmetric complex Gaussian
determined random variable $\eta$ with zero mean and variance $\sigma^2$ is denoted by $\eta \sim \mathcal{CN}(0, \sigma^2)$.

II. PRELIMINARIES

In this section, the basic notion of majorization is introduced and some important inequalities on convex functions that are used in this work are stated. The readers are referred to \cite{[11], [12]} for a complete reference. We start by stating the Edmundson–Madansky’s inequality.

Theorem 1: \cite{[11]} If $f$ is a convex function and $x$ is a random variable with values in an interval $[a, b]$, then
\[
E[f(x)] \leq \frac{b - \mu}{b - a} f(a) + \frac{\mu - a}{b - a} f(b),
\]
where $\mu$ is the mean of $x$.

Majorization theory formalizes the notion that the components of a vector $x$ are “less spread out” than the components of a vector $y$.

Definition 1: Let $x = [x_1, \ldots, x_n], y = [y_1, \ldots, y_n], x, y \in \mathbb{R}^n$ and let $x_{(i)}$ denote the $i$-th largest component of $x$. Then $x$ is said to be majorized by $y$, denoted by $x \preceq y$, if
\[
\sum_{i=1}^{l} x_{(i)} \leq \sum_{i=1}^{l} y_{(i)}, \quad \forall l \in [1:n-1]
\]
\[
\sum_{i=1}^{n} x_{(i)} = \sum_{i=1}^{n} y_{(i)}.
\]

Definition 2: \cite{[12]} 2.A.1. An $n \times n$ matrix $D$ with elements $d_{i,j}$ is doubly stochastic if
\[
d_{i,j} \geq 0, \quad \forall i, j \in [1:n],
\]
\[
\sum_{i=1}^{n} d_{i,j} = 1, \quad \forall j \in [1:n] \text{ and } \sum_{j=1}^{n} d_{i,j} = 1, \quad \forall i \in [1:n].
\]

Theorem 2: \cite{[12]} 4.A.1, 4.B.1] For $x, y \in \mathbb{R}^n$, the following conditions are equivalent:

- $x \preceq y$.
- $x = yD$ for some doubly stochastic matrix $D$.
- For all continuous concave functions $g : \mathbb{R} \to \mathbb{R}$, $\sum_{i=1}^{n} g(x_i) \geq \sum_{i=1}^{n} g(y_i)$.

Definition 3: \cite{[12]} 15.A.2. Let $X$ and $Y$ be $m \times n$ real matrices. Then $X$ is said to be majorized by $Y$, written $X \preceq Y$, if $X = YD$, where the $n \times n$ matrix $D$ is doubly stochastic.

Theorem 3: \cite{[12]} 15.A.4] Let $X$ and $Y$ be $m \times n$ real matrices. Then, $X \preceq Y$ if and only if
\[
\sum_{i=1}^{n} g(x_{(i)}) \geq \sum_{i=1}^{n} g(y_{(i)}),
\]
for all continuous concave functions $g : \mathbb{R}^m \to \mathbb{R}$; here $x_{(i)}$ and $y_{(i)}$ denote the $i$-th column vector of $X$ and $Y$, respectively.

III. SYSTEM MODEL

We consider a p2p MISO fading channel as shown in Fig. \[1\] where both the TX and the RX harvest energy from the environment. Each node is equipped with an individual energy buffer, i.e., a rechargeable battery, that can store the locally harvested energy.

Figure 1. MISO channel with feedback, where both the TX and the RX harvest and store ambient energy.

A. Energy Harvesting Model

The total observation time is divided into $K$ equal length EH intervals. At the beginning of the $k$-th EH interval, $k \in [1 : K]$, energy packets of size $e_k^x, e_k^r$ units arrive at the TX and the RX, respectively. At each node, this energy is first stored in an infinite size energy buffer, and used only for communication purposes, i.e., TX sending data, and the RX feeding back the CSI. We assume that all $e_k^x, e_k^r$'s are known in advance by both terminals. This model is suitable for an EH system in which the time-varying harvested energy can be accurately predicted \cite{[9]}.

B. Communication System Model

Each EH interval consists of $L$ data frames, each of length $T$ channel uses. We assume a block fading channel model. The channel is constant during $T$ channel uses of each frame, but changes in an independent and identically distributed (i.i.d.) fashion from one frame to another. The time frame structure is shown in Fig. \[2\] The TX has $M > 1$ antennas, while the RX has a single antenna. The received signal in a given channel use is given by
\[
y = h^H w s + \eta,
\]
where $h \in \mathbb{C}^{M \times 1}$ represents the vector of channel coefficients from TX to the RX with i.i.d. $\mathcal{CN}(0, 1)$ elements, $w \in \mathbb{C}^{M \times 1}$ denotes the beamforming vector, the input symbol maximizing the achievable ergodic rate in the $k$-th EH interval is $s \sim \mathcal{CN}(0, p_k)$, and $\eta \sim \mathcal{CN}(0, 1)$ represents the noise at the RX.

C. Feedback Model

We assume that the RX perfectly estimates the channel state at the beginning of each data frame, and feeds back the quantized CSI to the TX within the same frame. In the $k$-th EH interval, the frame structure is as follows: The RX in $\tau_k$ channel uses sends the CSI through a feedback channel (uplink) which is modeled as an additive white Gaussian noise.
(AWGN) channel. In the remaining $T - \tau_k$ channel uses, TX sends data to the RX (downlink) exploiting the obtained CSI. The feedback model represents the Time-Division Duplex (TDD) system in which uplink and downlink use the same band in a time-sharing fashion, but the communication devices are not self-calibrated, and hence, induce non-reciprocal effects [13]. In the above model, although the feedback overhead incurs a cost in the downlink bandwidth, a similar trade-off in the resource allocation between the CSI feedback quality and uplink data rate also arise in a Frequency-Division Duplex (FDD) system [14]. Hence, the analytical results obtained in this paper are applicable in general settings, and for instance, can be used to address the trade-off between CSI quality and effective data rate in an FDD system.

In the $k$-th EH interval, quantization of the channel state is performed using a codebook $C_k$ known at both the TX and RX. The receiver uses Random Vector Quantization (RVQ). The codebook consists of $M$-dimensional unit vectors $C_k \triangleq \{ f_1, \ldots, f_{2^b_k} \}$, where $b_k$ is the number of bits used for quantization. The RX chooses the beamforming vector according to $w_k = \arg \max_{f \in C_k} ||h^H f||^2$, where $h \triangleq \frac{h}{||h||}$. We assume that the length of the EH interval is very large compared to the channel coherence time (i.e., $L$ is very large). As a result, the achievable ergodic rate in the $k$-th EH interval is given by

$$R_k = \left(1 - \frac{\tau_k}{T}\right) E[||h||^2, \nu_k] \left[ \log_2 \left( 1 + \frac{p_k}{1 - \frac{\tau_k}{T} ||h||^2 \nu_k} \right) \right],$$

(2)

where $\nu_k = ||h^H w_k||^2$. Note that $\nu_k$ and $||h||^2$ are independent [15]. By using the AWGN feedback channel model, the number of feedback bits $b_k$ can be related to the energy used by the RX, $q_k$, and the number of channel uses $\tau_k$ as follows:

$$b_k = \tau_k \log_2 \left( 1 + \frac{q_k}{\tau_k \sigma^2} \right),$$

(3)

where $\sigma^2$ is the noise variance in the uplink. For analytical tractability, we neglect the practical constraint that $b_k$ should be an integer. Using the ergodic rate expression given in [15], Equation (27)] and [3], the ergodic rate $R_k \triangleq \hat{R}(p_k, q_k, \tau_k)$ is found to be

$$R_k = \left(1 - \frac{\tau_k}{T}\right) \log_2 e \left( e^{\rho_k} \sum_{l=0}^{M-1} E_{l+1} (\rho_k) - \int_{\nu_k=0}^{1} \left(1 - (1 - \nu_k)^{M-1}\right) N_k \frac{\nu_k}{\nu_k} \frac{d}{d\nu_k} \left( e^{\rho_k} \nu_k \right) d\nu_k \right),$$

(4)

where $\rho_k = \left(1 - \frac{\tau_k}{p_k}\right)$, $N_k = \left(1 + \frac{q_k}{\tau_k \sigma^2}\right)$, and $E_n(x) \triangleq \int_0^\infty e^{-nx} x^{-x} dx$ is the $n$-th order exponential integral.

### D. Optimization Problem

The problem of maximizing the sum throughput by the end of the $K$-th EH interval can be formulated as

$$\max_{p_k, q_k, \tau_k} \sum_{k=1}^{K} R_k$$

(5a)

s.t. $L \sum_{l=1}^{i} q_l \leq L \sum_{l=1}^{i} e^\tau_l, \quad \forall l \in [1 : K],$

(5b)

$L T \sum_{l=1}^{i} p_l \leq L T \sum_{l=1}^{i} e^\tau_l, \quad \forall l \in [1 : K],$

(5c)

$\tau_k \in [0, T), \quad p_k \geq 0, \quad q_k \geq 0, \quad \forall k \in [1 : K].$

(5d)

The constraints (5b) and (5c) guarantee the energy neutrality of the system, i.e., at each node, energy consumed can not be more than the energy harvested till that time. Also note that $\tau_k$ impacts the achievable rate $R_k$ in each EH interval.

Coming up with simple algorithms to solve the optimization problem is desirable in EH networks as the nodes may not have the computational and energy resources for running complex optimization algorithms. However, the ergodic rate expression used in the above optimization problem is not in closed form and offers little insight into the convexity of the problem which is required to reduce the complexity of optimization. This motivates the use of convex bounds on (4) as the objective function in the following optimization problems. Solving these modified problems provides an upper bound on the throughput. Since the constraints in the original and the modified optimization problems are the same, the solution for the modified problem is also feasible in the original problem, and if used in evaluating the exact rate expression in (4), we obtain a lower bound on the throughput. In some settings, we show that the bounds used are very close to the ergodic rate.

Before tackling the above problem, first, we consider a special case in which only the RX harvests energy. Later, the general case with both the TX and the RX harvesting energy is studied.

### IV. EH Receiver

In this setting, the RX harvests energy from the environment, whereas the TX is connected to the power grid so that it has a fixed power supply at all times. Therefore, there are no EH constraints at the TX, and constraints (5c) can be ignored. However, there is now a constraint on the average transmission power at each data frame of the $k$-th EH interval i.e., $p_k \leq p, \forall k$. The expected value $\nu_k$ is given by [15, 16]

$$E[\nu_k] = 1 - 2^{\beta_k} \beta \left( e^{\beta_k} \frac{M}{M - 1} \right),$$

(6)

where $\beta(x, y)$ denotes the beta function. Using the quantization error bound in [16 Lemma 6], (6) can be bounded as

$$E[\nu_k] \leq \nu_k^u \triangleq 1 - \left( \frac{M - 1}{M} \right)^{1 - \frac{1}{M}}.$$
Applying Jensen’s inequality on (2), substituting (7) and (3), and using the fact that \( E[\|h\|^2] = M \), an upper bound on the ergodic rate \( R_k^u \triangleq R^u(p_k, q_k, \tau_k) \) is obtained as

\[
R_k^u = t_k \log_2 \left( 1 + \frac{p_k M}{t_k} \left( 1 - \frac{M - 1}{M} \left( 1 + \frac{q_k}{\tau_k \sigma^2} \right) \right) \right),
\]

where \( t_k \triangleq (1 - \frac{\tau_k}{\tau}) \).

We now illustrate the tightness of the upper bound. Applying the Jensen’s inequality on (2), \( R_k^u - R_k \) can be lower bounded as

\[
R_k^u - R_k \geq t_k \log_2 \left( 1 + \frac{p_k M}{t_k} \nu_k \right) - t_k E[\|h\|^2] \log_2 \left( 1 + \frac{p_k}{t_k} \|h\|^2 E[\nu_k] \right).
\]

Since (2) is a concave function of \( \nu_k \) and \( \nu_k \in [0, 1] \), applying Theorem 1 on (2), we have

\[
R_k \geq t_k E[\|h\|^2] \log_2 \left( 1 + \frac{p_k}{t_k} \|h\|^2 \right) E[\nu_k]
\]

Now using (10), \( R_k^u - R_k \) can be upper bounded as

\[
R_k^u - R_k \leq t_k \log_2 \left( 1 + \frac{p_k M}{t_k} \nu_k \right) - t_k E[\|h\|^2] \log_2 \left( 1 + \frac{p_k}{t_k} \|h\|^2 \right) E[\nu_k].
\]

Since both \( \lim_{b_k \to \infty} \nu_k = 1 \) and \( \lim_{b_k \to \infty} E[\nu_k] = 1 \), and using (9) and (11), we have,

\[
\Delta R_k \triangleq \lim_{b_k \to \infty} R_k^u - R_k = t_k \log_2 \left( \frac{t_k + p_k M}{t_k + p_k \|h\|^2} \right).
\]

Further, for all feasible \( \tau_k \), in the low power regime,

\[
\lim_{p_k \to 0} \Delta R_k = 0,
\]

and in the high power regime,

\[
\lim_{p_k \to \infty} \Delta R_k = t_k \log_2 \left( \frac{M - E[\|h\|^2] \log_2 \|h\|^2} {t_k + p_k \|h\|^2} \right) \leq \log_2 M - E[\|h\|^2] \log_2 \|h\|^2.
\]

From the above analysis, it can be seen that when the RX has enough harvested energy to send large number of feedback bits, in the low power regime the bound is tight, and in the high power regime the difference is bounded by a constant. For example, it is 0.1958 for \( M = 4 \), and also note that \( \lim_{M \to \infty} \log_2 M - E[\|h\|^2] \log_2 \|h\|^2 = 0 \).

Using (8) as the objective function, the modified optimization problem can be written as follows,

\[
\max_{p_k, q_k, \tau_k} \U_k = \sum_{k=1}^{K} R_k^u
\]

s.t.

\[
L \sum_{i=1}^{l} q_i \leq \sum_{i=1}^{l} e_i^r, \forall l \in [1 : K],
\]

\[
p_k \leq p, \quad \text{and} \quad p_k \geq 0, \quad \forall k \in [1 : K],
\]

\[
\tau_k \in [0, T], \quad \text{and} \quad q_k \geq 0, \quad \forall k \in [1 : K],
\]

where \( p \) is the power constraint at the transmitter.

As the objective function is monotonic in \( q_k \) and \( p_k \), the constraint in (15b) must be satisfied with equality for \( l = K \), and the first constraint in (15c) must be satisfied with equality, i.e., \( p_k = p, \forall k \); otherwise, we can always increase \( q_k, p_k \), and hence, the objective function, without violating any constraints. Now it remains to optimize over the variables \( q_k \) and \( \tau_k \).

The feasible set is represented as

\[
\tilde{\mathcal{F}} = \{ q, \tau | q_k, \tau_k \text{ satisfy (15b), (15d) } \forall k \}
\]

where \( q = [q_1, \ldots, q_K] \) and \( \tau = [\tau_1, \ldots, \tau_K] \). To show that the above problem is a convex optimization problem, we make use of the following lemma.

**Lemma 1**: If the function \( f(x, y) : \mathbb{R}_+^2 \to \mathbb{R}_+ \) is concave, and \( g(y, z) : \mathbb{R}_+^2 \to \mathbb{R}_+ \) is concave and monotonically increasing in each argument, then the function \( h(x, y, t) = (1 - \frac{t}{T}) g \left( \frac{y}{1 - \frac{t}{T}}, \frac{f(x, y)}{1 - \frac{t}{T}} \right) \) is concave \( \forall (x, y, t) \in \mathbb{R}_+^2, t \in [0, T) \).

**Proof**: The proof is similar to that of showing the perspective of a concave function is concave. See Appendix.

**Proposition 1**: The objective function of the optimization problem (15) is concave.

**Proof**: See Appendix.

Since the objective function in (15) is concave and the constraints are linear, it has a unique maximizer [17]. Using the concavity of the objective function, we show that the optimal energy allocation vector is the most majorized feasible energy vector.

**Proposition 2**: The global optimum of (15) is obtained at \((q^*, \tau^*)\), where \( q^* \leq q, \forall (q, \tau) \in \tilde{\mathcal{F}} \), and \( \tau^*_k \) is the solution of the following equation

\[
\frac{\partial R_k^u}{\partial \tau_k}(q^*_k, \tau^*_k) = 0, \quad \forall k \in [1 : K].
\]

**Proof**: Consider the following equivalent form of (15), where the optimization is performed in two steps.

\[
\max_{q} \mathcal{U}(q) \text{ s.t. } (q, \tau) \in \tilde{\mathcal{F}},
\]

where \( \mathcal{U}(q) \) is obtained by

\[
\hat{\mathcal{U}}(q) = \max_{\tau} \mathcal{U}(q, \tau) \text{ s.t. } (q, \tau) \in \tilde{\mathcal{F}},
\]

Since \( \hat{\mathcal{U}}(q) \) is a concave function over the convex set \( \tilde{\mathcal{F}} \), the function \( \hat{\mathcal{U}}(q) \) is concave, where the domain of \( \hat{\mathcal{U}} \) is the set \( \tilde{\mathcal{F}} \) [17 3.2.5]. \( \hat{\mathcal{U}} = \sum_{k=1}^{K} R_k^u \) is continuous, differentiable and concave in \( \tau_k \in [0, T) \). Furthermore, for given \( q_k \), \( R_k^u \) approaches \( \log_2 (1 + p) \) and 0, as \( \tau_k \) approaches 0 and \( T \), respectively. Therefore, the unique maximizer of (19) lies in \([0, T]) \), and it is obtained at

\[
\frac{\partial \hat{\mathcal{U}}}{\partial \tau_k}(q^*_k, \tau^*_k) = 0, \quad \forall k \in [1 : K].
\]

From above, as \( \tau_k^* \) is only a function of \( q_k \),

\[
\hat{\mathcal{U}}(q) = \sum_{k=1}^{K} \hat{R}_k^u
\]
where \( \hat{R}_k^u \triangleq \hat{R}_k^u (q_k) = R_k^u (q_k, \tau^*_k (q_k)) \). Using (21) and Theorem 2 \( \mathbb{U} (q^*) \subseteq \mathbb{U} (q), \forall q \in \mathbb{R} \). Finding the optimal energy allocation vector \( q^* \) under the EH constraints turns out be a well known problem, and the algorithm to construct \( q^* \) is given in various works [13–20]. The proof that the algorithm constructs the most majorized feasible energy vector is given in [20]. Since the optimal energy allocation vector is \( q^* \), the optimal \( \tau^* \) is obtained by (17).

A brief description of the algorithm tailored to this work is given next, while the details can be found in [18]–[20]. There is no closed form expression for the solution of (17), hence we resort to numerical methods to obtain \( \tau^* \). Fig. 3 shows the behavior of \( \tau^*_k \) as a function of the allocated energy \( q^*_k \).

### A. Optimal Energy Allocation

From Definition 1 we can see that the components of the most majorized energy vector are "less spread out" than any other feasible energy vector. Therefore, the algorithm essentially tries to make the energy vector as equalized as possible. This is done by spreading the energy to future intervals. However, note that the energy arriving in later possible. This is done by spreading the energy to future intervals. However, note that the energy arriving in later

Figure 3. Optimal number of channel uses for sending feedback.



\[
\frac{\text{Input}}{\text{Optimal Energy Allocation (OEA) algorithm}}
\]

\[
\text{Output}: \text{Energy allocation } q^*, \text{Energy band indices} \; S = \{B_0, B_1, \ldots, B_{|S|}\}
\]

// initialization

\[
B_0 := 0;
\]

for \( i = 1 \rightarrow K \) do

- for \( k = K : -1 : (B_{i-1} + 1) \) do
  - \( o^*_i = \frac{\sum_{j = 0}^{k} e^*_j}{k-B_{i-1}} \), \( l \in \{B_{i-1} + 1, \ldots, k\} \)
  - if \( \sum_{l=1}^{k} o^*_i \leq \sum_{l=1}^{i} e_i, l = 1, \ldots, K \) then
    - \( B_k = k \);
    - Save \( \{o^*_1, \ldots, o^*_k\} \)
  - break;

do
end

end

\\

V. EH TRANSMITTER AND RECEIVER

In this section, we consider the general case where both the TX and the RX harvest energy. Note that if the TX is silent in the \( k \)-th interval, i.e., \( p_k = 0 \), there is no incentive for the RX to send feedback in this interval. Therefore, without loss of optimality we only consider EH profiles where \( e^*_k > 0 \). Otherwise, if there is an EH profile such that \( e^*_k = 0, k \in [1 : m-1] \), then \( p_k = 0, k \in [1 : m-1] \) due to the constraints in (5). In these intervals the RX simply accumulates the harvested energy, and without loss of optimality we can have a new EH profile with \( e^*_k = e^*_{k+1} + e^*_{k+2}, \forall i \in [1 : K - m + 1] \), and \( e^*_k = \sum_{k=1}^{m} e^*_k \) and \( e^*_k = e^*_{k+m-1}, \forall i \in [1 : K - m + 1] \) for further analysis. The ergodic rate upper bound in (8) is not concave, but concave in each variable given the other variables are fixed. To obtain a simple algorithm and an upper bound on the throughput, we follow a similar approach as in the previous section, and use a concave upper bound on (8) as the objective function for throughput optimization.

This bound is obtained by using a hypothetical system in which the transmission power is 1 watt higher than the actual transmission power of the system, which is \( p_k/t_k \). Plugging this into the upper bound in (8), a new upper bound \( R^u_k \triangleq R^{ab} (p_k, q_k, \tau_k) \) on the ergodic rate is obtained:

\[
R^u_k = t_k \log_2 \left( 1 + \left( 1 + \frac{p_k}{t_k} \right) \frac{f_k}{t_k} \right),
\]  

(22)

where \( t_k \triangleq 1 - \frac{1}{\gamma_k} \) and \( f_k \triangleq M - (M - 1) \left( 1 + \frac{p_k}{t_k} \right)^{-\frac{1}{\gamma_k}} \). We now illustrate the tightness of the upper bound in (22) in the low and high power regimes. For all feasible \( \tau_k, p_k \) and \( q_k \), we can see that \( 0 < t_k \leq 1 \) and \( 1 \leq f_k \leq M \). Consider

\[
R^{ab}_k = R^u_k = t_k \log_2 \left( \frac{t_k^2 + t_k f_k + p_k f_k}{t_k + p_k f_k} \right) - t_k \log_2 (t_k)
\]

(23)

Note that (23) is decreasing in \( p_k \) for fixed \( \tau_k \) and \( q_k \). Since \( \tau_k, f_k \) are bounded, for fixed \( \tau_k \) and \( q_k \), in the low power
lies in respectivley. Therefore, the unique maximizer of (28), where $p_k$ is obtained as
\[
\lim_{p_k \to 0} R_k^{ub} - R_k^b = t_k \log_2 \left( 1 + \frac{f_k}{t_k} \right) \leq \log_2 (1 + M),
\] (24)
and in the high power regime,
\[
\lim_{p_k \to \infty} R_k^{ub} - R_k^b = -t_k \log_2(t_k) \leq 0.5.
\] (25)
From the above analysis, it can be seen that, (23) decreases as the power is increased, and it is bounded by a constant in the high power regime. By using (22), the modified throughput maximization problem is formulated as
\[
\max_{p_k, q_k, \tau_k} U_1 = \sum_{k=1}^{K} R_k^{ub}
\] (26a)
s.t. \[ L \sum_{i=1}^{l} q_i \leq \sum_{i=1}^{l} e_i^r, \forall l \in [1 : K], \] (26b)\[ LT \sum_{i=1}^{l} p_i \leq \sum_{i=1}^{l} e_i^r, \forall l \in [1 : K], \] (26c)\[ \tau_k \in [0, T), p_k \geq 0, q_k \geq 0, \text{ and } \forall k \in [1 : K]. \] (26d)
Since the objective function is monotonic in $q_k$ and $p_k$, the constraints in (26b) and (26c) must be satisfied with equality for $l = K$, otherwise, we can always increase $q_k, p_k$, and hence the objective function, without violating any constraints. The feasible set is represented as $\mathcal{J} = \{(p, q, \tau) | p_k, q_k, \tau_k \text{ satisfy (26b), (26c) } \forall k \}$, where $p = (p_1, \ldots, p_K), q = (q_1, \ldots, q_K)$ and $\tau = (\tau_1, \ldots, \tau_K)$.

**Proposition 3:** The objective function in the optimization problem (26) is concave.

**Proof:** See Appendix.

Since the objective function in (26) is concave and the constraints are linear, it has a unique maximizer \[17\]. Consider the following equivalent form of (26), where the optimization is performed in two steps.
\[
\max_{p, q} \tilde{U}_1 (p, q) \text{ s.t. } \forall (p, q, \tau) \in \mathcal{J},
\] (27)
where $\tilde{U}_1 (p, q)$ is obtained by
\[
\tilde{U}_1 (p, q) = \max_{\tau} U_1 (p, q, \tau) \text{ s.t. } \forall (p, q, \tau) \in \mathcal{J}.
\] (28)
Since $\tilde{U}_1 (p, q)$ is a concave function over the convex set $\mathcal{J}$, the function $\tilde{U}_1$ is concave with domain $\mathcal{J} = \{(p, q) | (p, q, \tau) \in \mathcal{J} \}$ \[17\]. \[3.2.5\], $U_1 = \sum_{k=1}^{K} R_k^{ub}$ is continuous, differentiable and concave in $\tau_k \in [0, T)$. Furthermore, for given $p_k$ and $q_k$, $R_k^{ub}$ approaches $\log_2 (2 + p_k)$ and 0, as $\tau_k$ approaches 0 and $T$, respectively. Therefore, the unique maximizer of $\tilde{U}_1$, $\tau_k^*$, $\forall k$ lies in $[0, T)$, and it is obtained as
\[
\frac{\partial \tilde{U}_1}{\partial \tau_k} |_{\tau_k^*} = \frac{\partial R_k^{ub}}{\partial \tau_k} |_{\tau_k^*} = 0, \forall k \in [1 : K].
\] (29)
As $\tau_k^*$ is only a function of $q_k$ and $p_k$, (27) can be written as
\[
\max_{p_k, q_k} \tilde{U}_1 = \sum_{k=1}^{K} R_k^{ub} \text{ s.t. } \forall k, (p_k, q_k) \in \mathcal{J},
\] (30)
where $R_k^{ub} = R_k^{ub} (p_k, q_k) = R_k^{ub} (p_k, q_k, \tau_k^* (p_k, q_k))$.

In order to get an insight on how the optimal solution of (27) may look like, consider a simple scenario in which there is only a sum power constraint at the TX and the RX, i.e., the constraints in (26b), (26c) has to be satisfied for only $l = K$. In this case, by Jensen’s inequality, the uniform power allocation at the TX and the RX is optimal. However, due to the EH constraints, this may not be feasible. Using this intuition, we can see that the optimal policy tries to equalize the powers as much as possible, while satisfying the EH constraints. Next, we consider the case in which the EH profiles at the TX and the RX are similar, and show that the optimization problem is considerably simplified.

**A. Similar EH Profiles**

The EH profiles are similar in the sense that the most majorized feasible vectors obtained from the EH profiles of the TX and RX, $p^*$ and $q^*$, have the same structure, i.e., if $p_i^* = c_1, \forall i \in [m : n]$, then $q_i^* = c_2, \forall i \in [m : n]$ for some constants $c_1, c_2 \geq 0$. We now give a formal definition.

**Definition 4:** By using the OEA algorithm, let $[q^*, S_r] = \text{OEA}(K, \{e_i^r / L\})$ and $[p^*, S_t] = \text{OEA}(K, \{e_i^t / LT\})$. EH profiles at the TX and the RX are said to be similar if $S_r = S_t$.

From Section III, we can see that the definition of majorization for the vector case does not directly extend to the matrix case. If OEA algorithm is used at the TX and RX separately, we get the most individually majorized power vectors, which in general may not be the optimal solution of (27). However, we now show that if the EH profiles are similar, the above mentioned approach is indeed optimal.

**Proposition 4:** If the EH profiles at the TX and the RX are similar then $(q^*, p^*, \tau^*)$ is the global optimum of (26), where $q^* \preceq q, p^* \preceq p, \forall (q, p, \tau) \in \mathcal{J}$, and $\tau_k^*$ is the solution of
\[
\frac{\partial R_k^{ub}}{\partial \tau_k} |_{(p_k^*, q_k^*, \tau_k^*)} = 0, \forall k \in [1 : K].
\] (31)
**Proof:** See Appendix.

**B. Different EH Profiles**

Unfortunately, we could not find a simple algorithm to solve (26) in a general setting where the EH profiles are not similar. In (30), if one variable is fixed, optimizing over the other variable has a directional or staircase water-filling interpretation \cite{13, 18}, however, the difficulty lies in the fact that there is no closed form expression for $R_k^{ub}$. Nonetheless, based on the convexity of the objective function, some properties of the optimal solution are given below.

**Lemma 2:** Under the optimal policy, the transmission power $p_k$, and the energy used to send the feedback $q_k$ are non-decreasing in $k, \forall k \in [1 : K]$.

**Lemma 3:** Under the optimal policy, at the time instants at which $R_k^{ub}$ changes, the energy buffer of either the TX or the RX is emptied.

The proofs of the above lemmas are given in Appendix.

\footnote{In this section, with slight abuse of terminology we use the terms RX power and RX energy interchangeably.}
VI. NUMERICAL RESULTS

We start by considering the case in which the RX harvests energy, while the TX has a constant power supply. We assume that the RX is equipped with a solar EH device. Following \[21\], solar irradiance data is taken from the database reported in \[22\]. Each EH interval is of duration $\Delta = 1$ hour, $T = 200$ ms, resulting in $L = 18000$ frames. The harvested power from the irradiance data can be calculated as, $p_{\text{harv}} = I[\text{Watt}/m^2] \times \text{Area}[m^2] \times \rho$, where $\rho$ is the efficiency of the harvester. A hypothetical solar panel of variable area is assumed. The area of the panel is adjusted such that we have the EH profile shown in Fig. 4 at the RX. In Fig. 4 the harvested power to noise ratio (HPN) in each EH interval $\frac{e_q}{\Delta}$ is shown.

Using this EH profile, throughput of different feedback policies is shown in Fig. 5. In Fig. 5 OEA represents the proposed policy in which the energy vector is obtained by using the OEA algorithm, and then the optimal time span of feedback $\tau_k^*$ is obtained by solving (20). In the greedy scheme, the consumed energy is equal to the harvested energy in that interval, i.e., $q_k = e_q^*/L$, and then optimization is performed only over $\tau_k$, given $q_k$. The performance of the above policies when the feedback bits are rounded to the largest previous integer is also shown. We can see that the proposed approach outperforms the greedy policy by a positive number (area), while keeping the same shape and multiplexing gain (pre-log factor) is reduced.

Finally, we consider a case with non-similar EH profiles, where the EH profiles are generated independently at the TX and the RX, and they are i.i.d. with exponential distribution. EH profiles are verified not to be similar according to Definition 4. Similarly to Fig. 7, in Fig. 8 the mean HPN at the TX is varied by multiplying the EH profile by a constant, while keeping the same shape. Since we could not find a simple algorithm in this case, CVX solver is used to solve the optimization problem \[17\], and is denoted as CVX in Fig. 8. As we can see, the heuristic of using the OEA approach performs quite well even in the non-similar EH profile scenario. The energy allocation at the TX and the
remark that the result obtained in Proposition 4 is general, and, for example, it can be used in a network setting in which a concave utility is to be maximized in the presence of EH nodes with similar harvesting profiles and infinite size energy buffers. Numerical results show that the proposed policies not only outperform the greedy policy, but also achieve performances very close to the theoretical upper bound. Our work sheds light on the design of feedback-enabled multi-antenna systems when the nodes depend on EH devices for their energy.

**APPENDIX**

A. Proof of Lemma 7

Let \( X_1 = [x_1 \ y_1 \ t_1]^T, \ X_2 = [x_2 \ y_2 \ t_2]^T, \) we have

\[
\begin{align*}
& h(\lambda X_1 + (1 - \lambda) X_2) \\
& = \Theta g \left( \frac{\lambda y_1 + (1 - \lambda) y_2 \ f(x, t)}{\Theta} \right) \\
& \geq (a) \Theta g \left( \frac{\lambda y_1 + (1 - \lambda) y_2 \ f_1}{\Theta} \right) \\
& = \Theta g \left( \frac{\Theta_1 y_1}{\Theta_1 \alpha_1} + \Theta_2 y_2 + \Theta_1 f_1}{\Theta_1 \alpha_1} + \Theta_2 f_2}{\Theta_2 \alpha_2} \right) (32) \\
& \geq (b) \Theta_1 g \left( \frac{y_1}{\alpha_1} \right) + \Theta_2 g \left( \frac{y_2}{\alpha_2} \right) \\
& = \lambda h(X_1) + (1 - \lambda) h(X_2),
\end{align*}
\]

where \( \frac{\lambda x_1 + (1 - \lambda) x_2}{\Theta} \), \( \frac{(1 - \lambda) t_2}{\Theta} \) and \( \frac{(1 - \lambda) \left(1 - \frac{1}{\Theta} \right)}{\Theta} \) are concave, and \( g(y, z) \) is monotonically increasing in each argument,

(a) follows from the fact that \( f(x, t) \) is concave, and \( g(y, z) \) is monotonically increasing in each argument,

(b) follows from the fact that \( \frac{\Theta_1 \alpha_1}{\Theta_2 \alpha_2} = 1, \) and \( g(y, z) \) is concave.

B. Proof of Proposition 4

Reproducing the ergodic rate bound in (8) with \( p_k = P, \forall k, \) we have

\[
R^u(q_k, \tau_k) = t_k \log_2 \left(1 + \frac{P f_k}{t_k} \right),
\]
where $t_k \triangleq 1 - \frac{\tau_k}{\tau}$, $f_k \triangleq (M - (M - 1))(1 + \frac{\tau_k}{\tau_{k+1}})\frac{\tau_{k+1}}{\tau_k}$. Since $b_k$ in (33) is concave in $q_k$ and $\tau_k$, it can be easily seen that $2^{-\frac{\tau_k}{\tau}} = (1 + \frac{\tau_k}{\tau_{k+1}})\frac{\tau_{k+1}}{\tau_k}$ is convex, and hence, $f_k$ is concave. Using Lemma 1 with $g(y, z) = \log_2 (1 + z)$ and $f_k$, we can see that $R_{k}^{ib}$ is concave. Since the objective function in (13) is the summation of $R_{k}^{ib}$’s, it is also concave.

C. Proof of Proposition 3

First, we show that $g(y, z) = \log_2 (1 + (1 + y)z), (y, z) \in \mathbb{R}_+^2$ is concave for $y \geq 0, z \geq 1$. The Hessian of $g$ is given by

$$J = \frac{1}{\beta} \left( \begin{array}{cc} -z^2 & 1 \\ 1 & -(1 + y)^2 \end{array} \right),$$

where $\beta = \log_2 (1 + (1 + y)z)^2 > 0$. Consider $u^T Ju = -\frac{1}{\beta} \left( a^2 z^2 + b^2 (1 + y)^2 - 2ab \right)$, where $u = [a b]^T \in \mathbb{R}^2$. It can be easily seen that $u^T Ju \leq 0$ for $ab \leq 0$. For $ab > 0$, since $z(1 + y) \leq 1$, $u^T Ju = -\frac{1}{\beta} \left( (az - b(1 + y))^2 + 2ab(z(1 + y) - 1) \right) \leq 0$. As $\beta$ is negative semidefinite, $g(y, z)$ is concave. Reproducing the ergodic rate bound in (22), we have

$$R_{k}^{ib} = t_k \log_2 \left( 1 + \left( 1 + \frac{p_k}{t_k} \frac{f_k}{t_k} \right) \right),$$

where $t_k$ and $f_k$ are as defined before.

By following the similar steps in Proposition 1, $f_k$ can be shown to be concave. Using Lemma 1 with $g(y, z)$ and $f_k$, we can see that $R_{k}^{ab}$ is concave. Since the objective function in (20) is the summation of $R_{k}^{ab}$’s, it is also concave.

D. Proof of Proposition 4

First, $(p^*, q^*)$ is shown to be the solution of (30) and $\tau^*$ is obtained by (31). Before solving (30), we prove that

$$(p^*, q^*) = \arg \max_{g(p_k, q_k)} \sum_{k=1}^{K} g(p_k, q_k)$$

s.t. $\forall k, (p_k, q_k) \in \tilde{\mathcal{J}}, g \in \mathcal{C}$,

where $\mathcal{C}$ is the set of all continuous concave functions. As (30) is a special case of (36), $(p^*, q^*)$ is also the solution of (30).

Before starting, we note that the notations and properties of the OEA algorithm discussed in Section IV-A are used throughout the proof. By contradiction, let us assume that there exists a $[p^T \ q^T]^T \neq [p^* \ q^*]^T$ and $(\hat{p}, \hat{q})$ be the solution of (30). Then, by Theorem 3 we have,

$$[p^T \ q^T]^T \leq [\hat{p}^T \ \hat{q}^T]^T, \forall (p, q) \in \tilde{\mathcal{J}}.$$ (37)

Since $(p^*, q^*) \in \tilde{\mathcal{J}}$, by (37) and Definition 3

$$[p^T \ q^T]^T = [p^* \ q^*]^T \mathbf{D}.$$ (38)

By the feasibility constraint in (26b),

$$\sum_{j=B_{i-1}+1}^{B_{i}} \hat{q}_j \leq V_i = \sum_{j=B_{i-1}+1}^{B_{i}} e_{j}^*/L, \quad (39)$$

where $B_{i}$’s are the energy band indices as explained in Section IV-A.

Applying (39) for $i = 1$, and remembering that $B_0 = 0$, we get

$$\sum_{j=1}^{B_1} \hat{q}_j = \sum_{j=1}^{B_1} \sum_{k=1}^{N} q_{k,j} \leq V_1.$$ (40)

By (P1) and (P2) in Section IV-A, $q_{i}^* = q_{i}^*(1) + L_i$, where

$$L_i = 0 \quad \forall i \in [1 : B_1],$$

$$L_i > 0 \quad \forall i \in [B_1 + 1 : K].$$ (41)

From (40) and (41), we get

$$\sum_{j=1}^{B_1} \sum_{i=1}^{K} q_{i,j} \leq V_1.$$ (42)

Using the fact that $\mathbf{D}$ is doubly stochastic and by (P1), $B_1 q_{(1)} = V_1$, and we have

$$\sum_{j=1}^{B_1} \sum_{i=1}^{K} L_i d_{i,j} \leq 0.$$ (43)

From (41) and (42), we get

$$d_{i,j} = 0, \forall i \in [B_1 + 1 : K], \forall j \in [1 : B_1].$$ (44)

As $\mathbf{D}$ is doubly stochastic, using (P1) and (44),

$$\hat{q}_j = \sum_{i=1}^{B_1} q_{i,j} = q_{(1)}, \forall j \in [1 : B_1].$$ (45)

Since $\mathbf{D}$ is doubly stochastic, using (44), we get

$$\sum_{i=1}^{B_1} \sum_{j=1}^{K} d_{i,j} = B_1, \sum_{i=1}^{B_1} d_{i,j} = 1, \forall j \in [1 : B_1].$$ (46)

We can rewrite (46) as

$$\sum_{i=1}^{B_1} \sum_{j=1}^{K} d_{i,j} = B_1, \sum_{i=1}^{B_1} \sum_{j=1}^{K} d_{i,j} = 1, \forall j \in [1 : B_1].$$ (47)

from which it follows that

$$\sum_{i=1}^{B_1} \sum_{j=1}^{K} d_{i,j} = 0,$$ (48)

and hence,

$$d_{i,j} = 0, \forall i \in [1 : B_1], \forall j \in [B_1 + 1 : K].$$ (49)

Then applying (39) for $i = 2,$

$$\sum_{j=B_{i-1}+1}^{B_{i}+1} \hat{q}_j = \sum_{j=B_{i-1}+1}^{B_{i}+1} \sum_{k=1}^{N} q_{k,j} \leq V_2.$$ (50)

By (P1) and (P2), we have $q_{i}^* = q_{i}^*(2) + L_i$, where

$$L_i < 0 \quad \forall i \in [1 : B_1],$$

$$L_i = 0 \quad \forall i \in [B_1 + 1 : B_2],$$

$$L_i > 0 \quad \forall i \in [B_2 + 1 : K].$$ (51)
From (50) and (51),
\[ \sum_{j=B_1+1}^{B_2} \sum_{i=1}^{K} L_i d_{i,j} + \sum_{j=B_1+1}^{B_2} \sum_{i=1}^{K} q_{(2)}^{j} d_{i,j} \leq V_2. \] (52)

Since \( D \) is doubly stochastic, by (P1), we obtain \( (B_2 - B_1) q_{(2)}^{j} = V_2 \), and using (49) and (51) in (52), we get
\[ \sum_{j=B_1+1}^{B_2} \sum_{i=1}^{K} L_i d_{i,j} \leq 0, L_i > 0. \] (53)

From (51) and (53) it can be concluded that
\[ d_{i,j} = 0, \forall i \in [B_2 + 1 : K], \forall j \in [B_1 + 1 : B_2]. \] (54)

As \( D \) is doubly stochastic, using (P1) together with (49) and (54), we have
\[ \hat{q}_j = q_{(2)}^{j} \sum_{i=B_1+1}^{B_2} d_{i,j} = q_{(2)}^{j} q_j^{s}, \forall j \in [B_1 + 1 : B_2]. \] (55)

Again, since \( D \) is doubly stochastic, using (49) and (54),
\[ \sum_{i=B_1+1}^{B_2} \sum_{j=B_1+1}^{K} d_{i,j} = B_2 - B_1, \]
\[ \sum_{i=B_1+1}^{B_2} \sum_{j=B_1+1}^{K} d_{i,j} = 1, \forall j \in [B_1 + 1 : B_2]. \] (56)

We can rewrite (56) as
\[ \sum_{i=B_1+1}^{B_2} \sum_{j=B_1+1}^{K} d_{i,j} = \sum_{i=B_1+1}^{B_2} \sum_{j=B_1+1}^{K} d_{i,j} + \sum_{i=B_1+1}^{B_2} \sum_{j=B_1+1}^{K} d_{i,j} \] (57)

From (57) we can see that
\[ \sum_{i=B_1+1}^{B_2} \sum_{j=B_1+1}^{K} d_{i,j} = 0, \] (58)
and hence,
\[ d_{i,j} = 0, \forall i \in [B_1 + 1 : B_2] \text{ and } \forall j \in [B_2 + 1 : K]. \] (59)

Continuing this approach for \( i = 3, \ldots, (|S_r| - 1) \), we get \( \hat{q} = q^{s} \).

Since the EH profiles are similar, i.e., \( S_r = S_t \), replacing \( \hat{q} \) by \( \tilde{p} \) and \( e_j^{s} \) by \( e_j^{f}/T \) in the above proof, we reach the similar conclusion for \( \tilde{p} \), i.e., \( \hat{q} = p^{s} \).

Therefore, \[ \lfloor \hat{p}^{T}, \hat{q}^{T} \rfloor = \lfloor p^{T}, q^{T} \rfloor. \]
\[ \hat{p}_k \] is doubly stochastic, using (P1) together with (49) and (51) in (52), we get
\[ \hat{p}_k = p_k, \tilde{p}_{k+1} = p_{k+1}, \]
\[ \hat{q}_k = q_k, \tilde{q}_{k+1} = q_{k+1}. \] (60)

Since the objective function is concave, by Jensen’s inequality, the new policy strictly increases the objective. Finally considering the case where \( p_k < p_{k+1}, q_k > q_{k+1} \), we can construct another feasible policy,
\[ \tilde{p}_k = p_k, \tilde{p}_{k+1} = p_{k+1}, \]
\[ \tilde{q}_k = q_k, \tilde{q}_{k+1} = q_{k+1}. \] (61)

The function \( R^{ub} \) with variables \( p, q, \tau \) can be written as,
\[ R^{ub}(p, q, \tau) = t \log_2 \left( 1 + \left( \frac{1}{\tau} + \frac{p}{q} \right) f \right), \] (62)
where \( f \triangleq M - (M - 1) \left( 1 + \frac{q}{\tau p} \right)^{1/\tau} \), \( \tau \triangleq 1 - \frac{r}{p} \) and \( 0 \leq \tau < T \). The second order partial derivative of \( R^{ub}(p, q, \tau) \) is given by,
\[ \frac{\partial^2 R^{ub}}{\partial p \partial q} = \frac{1}{2} \left( \frac{q}{pt} + \frac{p}{qt} \right)^2. \] (63)

Since \( f \) is monotonic in \( q \), (63) is positive. As \( \frac{\partial^2 R^{ub}}{\partial p \partial q} > 0 \), by the definition of derivative,
\[ R^{ub}(p, q, \tau) + R^{ub}(p + \delta, q + \alpha, \tau) > R^{ub}(p + \delta, q, \tau) + R^{ub}(p, q + \alpha, \tau), \delta, \alpha > 0. \] (64)

Since (64) holds for all \( 0 \leq \tau < T \), we have
\[ \tilde{R}^{ub}(p, q) > \tilde{R}^{ub}(p + \delta, q + \alpha) > R^{ub}(p, q + \alpha), \] (65)
where \( \tilde{R}^{ub} \) is obtained by,
\[ \tilde{R}^{ub}(p, q) = \max_{\tau} \tilde{R}^{ub}(p, q, \tau). \] (66)

Finally, using (61) and (65) we can see that the newly constructed policy strictly increases the objective.

\[ F. \text{ Proof of Lemma 2} \]

Let us assume that the transmission rates in the \( k \)-th and the \( k + 1 \)-th intervals are different, i.e., \( R^{ub}(p_k, q_k) \neq R^{ub}(p_{k+1}, q_{k+1}) \). Before the \( k+1 \)-th interval, the energy in the buffers of TX and the RX are \( \Delta_r = \sum_{i=1}^{k} e_i^{f} - L \sum_{i=1}^{k} p_i \) and \( \Delta_t = \sum_{i=1}^{k} e_i^{f} - L \sum_{i=1}^{k} p_i \), respectively. W.l.o.g., we assume that \( \Delta_r \leq \Delta_t \). We can construct another feasible policy
\[ \tilde{p}_k = p_k + \delta, \tilde{p}_{k+1} = p_{k+1} - \delta, \]
\[ \tilde{q}_k = q_k + \delta, \tilde{q}_{k+1} = q_{k+1} - \delta, \] (67)
where \( \delta \) is chosen such that \( \delta < \Delta_r \text{ and } \tilde{q}_k < \tilde{q}_{k+1} \). Now, (67) can be written as
\[ \tilde{p}_k = \alpha p_k + (1 - \alpha) p_{k+1}, \tilde{p}_{k+1} = (1 - \alpha) p_k + \alpha p_{k+1}, \]
\[ \tilde{q}_k = \alpha q_k + (1 - \alpha) q_{k+1}, \tilde{q}_{k+1} = (1 - \alpha) q_k + \alpha q_{k+1}, \] (68)
where \( \alpha = 1 - \delta/\left( q_{k+1} - q_k \right) \). Using Jensen’s inequality
\[ \sum_{j=k}^{k+1} \tilde{R}^{ub}(\tilde{p}_j, \tilde{q}_j) > \sum_{j=k}^{k+1} \tilde{R}^{ub}(p_j, q_j), \] (69)
which concludes the proof.

\[ E. \text{ Proof of Lemma 2} \]

Assume that at least one of the \( p_k, q_k \) is not monotonically increasing in \( k \). Without loss of generality (w.l.o.g) we consider the cases in which \( p_k > p_{k+1}, q_k \geq q_{k+1} \) and \( p_k < p_{k+1}, q_k > q_{k+1} \). In the case of \( p_k > p_{k+1}, q_k \geq q_{k+1} \), we can construct a new feasible policy,
\[ \tilde{p}_k = \frac{p_k + p_{k+1}}{2}, \]
\[ \tilde{q}_k = \frac{q_k + q_{k+1}}{2}. \] (60)
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