Measurement of the ground-state hyperfine splitting of antihydrogen

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Abstract. The ASACUSA collaboration at the Antiproton Decelerator of CERN is planning to measure the ground-state hyperfine splitting of antihydrogen using an atomic beam line, consisting of a cusp trap as a source of partially polarized antihydrogen atoms, a radiofrequency spin-flip cavity, a superconducting sextupole magnet as spin analyser, and an antihydrogen detector. This will be a measurement of the antiproton magnetic moment, and also a test of the CPT invariance. Monte Carlo simulations predict that the antihydrogen ground-state hyperfine splitting can be determined with a relative precision of $\sim 10^{-7}$. The first preliminary measurements of the hyperfine transitions will start in 2011.

1. Motivation
The validity of the Charge-Parity-Time reversal (CPT) invariance is assumed – either implicitly or explicitly – in all our mainstream quantum field theories and in the Standard Model of particle physics. This assumption is rooted in a mathematical theorem, which states that any quantum field theory which respects Lorentz invariance, locality and unitarity (i.e. conservation of probability) must be symmetric under the CPT transformations. However, some of the three conditions of this CPT theorem might be violated in certain scenarios, for example around microscopic black holes at the Planck scale, or in string theory. Therefore it is important to test experimentally the validity of CPT invariance.

CPT symmetry implies that the properties of a particle and its antiparticle, or more generally, a system and its anti-system, should be the same or exactly opposite. These properties, among others, are the mass, charge, spin, magnetic moment, or various energy levels. Thus one way to test the CPT invariance is to study antimatter systems and compare them to their corresponding matter counterpart. However, even without such comparisons, the apparent abundance of matter in the Universe is a hint that the matter-antimatter symmetry might not be perfect. One possible explanation to the lack of antimatter in the Universe is either CP or CPT violation.

2. Theoretical background
The 1s ground state of hydrogen is split due to the interaction between the electron spin and the proton spin, and can have quantum numbers $(F, M) = (0, 0), (1, -1), (1, 0)$ and $(1, 1)$, where $F$ is the total spin, and $M$ is the projection of $F$. The states with quantum numbers $F = 0$ (singlet state) and $F = 1$ (triplet state) have different energies even at zero external magnetic
field. This ground-state hyperfine splitting (GS-HFS) frequency has been measured to a very high precision of $10^{-12}$ [1]:

$$\nu_{\text{HFS}} = 1420.405\ 751\ 766\ 7(9)\ \text{MHz}. \tag{1}$$

Thus measuring the $\nu_{\text{HFS}}$ of the antimatter counterpart of hydrogen, the antihydrogen atom ($\bar{\text{H}}$), which consists of an antiproton ($\bar{p}$) and a positron ($e^+$), would allow us to test the CPT invariance to a similarly high precision.

The ground-state hyperfine splitting frequency in hydrogen (antihydrogen) is proportional in the leading order to the proton (antiproton) spin magnetic moment $\mu_p$:

$$\nu_{\text{HFS}} = \frac{16}{3} \left( \frac{m_p}{m_p + m_e} \right)^3 \frac{m_e}{m_p} \frac{\mu_p}{\mu_N} c R_\infty (1 + \delta), \tag{2}$$

where $m_p$ and $m_e$ are the proton (antiproton) and the electron (positron) masses, respectively, $\mu_N$ is the nuclear magneton, and $R_\infty$ is the Rydberg constant. The correction term $\delta$ is $\sim 1100$ ppm, and it is mainly due to higher-order QED corrections. It is theoretically calculated with a precision of slightly under 1 ppm; this uncertainty comes from the proton structure-dependent corrections [2]. The antiproton spin magnetic moment $\mu_{\bar{p}}$, on the other hand, is known to a precision of only 0.3% from experiments with antiprotonic lead [3] and later from experiments with antiprotonic helium [4]. Therefore a measurement of $\nu_{\text{HFS}}$ of antihydrogen (which has never been done before) with a precision of only $10^{-6}$ can already lead to an improvement of the value of $\mu_{\bar{p}}$ by three orders of magnitude.

The group of V.A. Kostelecký has developed an extension of the standard model of elementary particles that includes both CPT violating and Lorentz invariance violating terms in the Lagrangian of the quantum field theory [5]. These terms result in a correction to the energies of the 1s and 2s hyperfine sublevels:

$$\Delta E^H(m_J, m_I) = a_e^0 + a_p^p - c_{00}^p m_e - c_{00}^p m_p + (-b_3^0 + d_{50}^0 m_e + H_{12}^0)m_J/|m_J| + (-b_3^p + d_{50}^p m_p + H_{12}^p)m_I/|m_I|, \tag{3}$$

where $m_J$ and $m_I$ are the characteristic spin quantum numbers of the electron (positron) and the proton (antiproton), respectively, at large external magnetic fields. The parameters $a, d$ and $H$ change sign for antihydrogen, therefore the GS-HFS transition frequencies in hydrogen and antihydrogen can be different.

An important feature of this theory is that the parameters $a$ and $b$ are not dimensionless; instead, they have a dimension of energy (or frequency). From this it follows that it is not the relative but the absolute precision of an experiment that matters when one is comparing matter and antimatter systems for CPT tests. Therefore the $\bar{\text{H}}$ GS-HFS measurement can be competitive with the oft-quoted ‘most sensitive CPT limit’ of $|m_{K^0} - m_{\bar{K}^0}|/m_{K^0} < 10^{-18}$ [6]. This is because the $R^{0,\bar{K}^0}$ measurement corresponds to only $\sim 100$ kHz on the absolute scale, while the $\bar{\text{H}}$ GS-HFS measurement with a relative precision of only $10^{-6}$ would have an absolute precision of $1.42$ GHz $\times 10^{-6} = 1.42$ kHz.

### 3. Proposed experimental method

The experimental resolution of the ground-state hyperfine transitions is inversely proportional to the interaction time of the (anti)hydrogen atoms with the radiofrequency (RF) field. Thus the most precise measurements of the transition frequencies of hydrogen have been done with hydrogen masers, where the atoms can be kept in the RF field for several minutes. Unfortunately,
antimatter atoms cannot be kept in a maser cavity, since they would annihilate when colliding with the cavity walls.

The neutral antihydrogen atoms could be kept in the RF field by a confining magnetic field gradient. However, the required fields would be very strong and would shift and broaden the energy levels too much, which would make high-precision spectroscopy measurements very difficult.

Thus the ASACUSA collaboration at CERN’s Antiproton Accelerator (AD) plans to use an atomic beam line [7, 8] to measure the \( ^3\)He GS-HFS. This method is based on the same Stern-Gerlach principle as the classic experiments which first measured the hyperfine splitting of hydrogen. It has the advantage that it can work with relatively high-temperature (50-100 K) atoms, while trapping would require ultra-cold (<1 K) atoms. A schematic view of the proposed atomic spectrometer line is shown in Fig. 1.

The antihydrogen atoms will be produced in a cusp trap i.e. a pair of anti-Helmholtz coils [9]. The currents running in the opposite direction in the two coils produce zero field in the centre of the trap, but the magnetic field strength increases linearly with the distance from the trap centre. Such an inhomogeneous magnetic field can confine not only charged but also neutral particles. Thus the produced antihydrogen atoms, which are most likely created in higher-lying Rydberg states, can be trapped long enough to de-excite to the ground state before they eventually leave the trap and reach the radiofrequency resonator [10].

Figure 2 shows the energy levels of the four hyperfine states of antihydrogen as a function of the external magnetic field. Two hyperfine states are so-called low-field seekers, while the other two are high-field seekers. When moving in the inhomogeneous magnetic field of the cusp trap, these two types will have different trajectories: the low-field seekers will be focussed, while the high-field seekers will be defocussed as they leave the trap (see Fig. 1). Thus the emerging \( ^3\)He beam will be at least partially polarized: more low-field seeker atoms will reach the radiofrequency cavity as high-field seekers.

Figure 2 also shows the three allowed hyperfine transitions which can convert one of the two low-field seeker states into one of the two high-field seeker states (or vice versa). Two of these transitions, \( \pi_1 \) and \( \sigma_1 \), lie close to each other in weak magnetic fields, therefore a single radiofrequency resonator operating around 1.42 GHz can be used to flip the spin of the \( ^3\)He atoms as they pass through it. A superconducting sextupole magnet will be placed after this cavity, which will focus the low-field seeker atoms onto an antihydrogen detector, while it will defocus the high-field seekers. This forms the basis of the spectroscopy method: when the radiofrequency field is on resonance with one of the hyperfine transitions, fewer atoms will reach the antihydrogen detector. The advantage of the sextupole magnet is that its magnetic field gradient increases linearly with the distance from the magnet axis, therefore all low-field seeker
Figure 2. Frequencies (i.e. energies) of the four hyperfine states of antihydrogen as a function of the external static magnetic field $B$. The allowed transitions which convert low-field seekers into high-field seekers ($\pi_1$, $\sigma_1$, $\pi_2$) are also drawn.

Figure 3. Simulated trajectories of low-field seeker antihydrogen atoms emerging from the cusp trap, then passing through the radiofrequency cavity and the sextupole magnet, and finally hitting an antihydrogen detector (not shown). The internal diameter of the sextupole magnet is 10 cm.

Atoms with the same velocity will be focussed into a single focal point, independently of where they enter the magnet.

4. Monte Carlo simulations
Simulations using the GEANT4 toolkit [11] have been carried out to determine the necessary size and field strength of the sextupole magnets, and to estimate the expected count rates and experimental resolution. It was assumed that the cusp trap can provide an antihydrogen beam with a polarization ratio of 4:1 1 m downstream of the cusp trap, i.e. where the RF resonator will be connected. Such a simulation with the cusp trap is shown in Fig. 3. The sextupole magnet needs to have a large internal diameter of 10 cm in order to increase the transmission efficiency for the $\overline{H}$ atoms. The simulation results showed that if the antihydrogen atoms have a temperature of 5 K, then the sextupole magnet has to have a pole tip field of 0.35 T and an effective length (field FWHM) of 22 cm. The necessary pole tip field strength increases linearly with the temperature of the atoms. Since the superconducting magnet can reach a maximum pole tip field of 3 T, atoms of up to 50 K can be used in the experiment.
5. Radiofrequency resonator

As explained above, the purpose of the radiofrequency resonator is to induce a spin flip of the antihydrogen atoms. Ideally, the spin flip should be a ‘π-pulse’ i.e. when the spin of of the atoms makes exactly one half of a Rabi oscillation. This way the width $\Delta \nu$ of the GS-HFS line in the RF spectrum can reach the theoretical minimum, which can be calculated for monoenergetic (monovelocitic) atoms as [12]:

$$\Delta \nu = \frac{0.799}{T},$$

(4)

where $T$ is the observation time of the transition i.e. the time it takes for one atom to pass through the RF resonator and interact with the oscillating magnetic field. It can be readily seen that the longer the observation time, the narrower the transition line.

The resonator has to be tunable within the range of 1420–1425 MHz. This will be achieved by using a low-$Q$ cavity in which the frequency of the oscillating field can be changed simply by changing the frequency of the external frequency source.

5.1. Oscillating magnetic field

The spin flip of the antihydrogen atom has to be induced by a magnetic field oscillating at 1.42 GHz. With such an oscillating field, the $\pi_1$ and the $\sigma_1$ transitions in Fig. 2 can be observed. Ideally, the oscillating magnetic field should be perfectly homogenous in all directions. However, Maxwell’s equations forbid to generate a perfectly homogenous oscillating magnetic (or electric) field in a volume whose dimensions are comparable or larger than the half-wavelength of the field. This is exactly the case in this setup, since the size of the volume in which the oscillating field has to be present is at least $\sim 10 \times 10 \times 10 \text{ cm}^3$, while the half-wavelength of the radiofrequency field is $\lambda_{\text{HFS}}/2 \simeq 10.5 \text{ cm}$. Therefore at least in one direction the magnetic field cannot be homogenous.

To generate the required oscillating magnetic field, a cylindrical cavity with a double stripline, i.e two parallel conducting plates, has been chosen [13, 14] (see Fig. 4). The length of the plates (measured along the beam) has to be an integer multiple of the desired half-wavelength, while the width and the distance between the plates can be arbitrary. The plates are placed inside a cylindrical vacuum chamber, which has two openings on the front and back plates. These openings are covered with fine metallic meshes, which allow the $\overline{\text{H}}$ atoms to enter and leave the cavity, but keep the RF field from leaking out.
Figure 4 shows that the oscillating magnetic field of the double stripline is very homogenous perpendicularly to the beam (X-Y plane): the field inhomogeneity is ±2%. Along the beam (Z direction), however, the field strength follows a sin(Z) distribution: the field is zero in the center plane, and has maxima at the front and back walls of the resonator. Thus the magnetic field points at opposite directions in the two half-volumes. When the frequency of the oscillating field is on resonance with one of the GS-HFS transitions, the effects of the two half-volumes cancel each other. Consequently, one gets zero signal on resonance (independently of the velocity of the atoms), and gets a non-zero signal away from the resonance: the peak of each transition in the GS-HFS spectrum is split into two, as it can be seen in Figs. 5 and 6. This can be thought of as a Doppler splitting, since the standing wave in the cavity is a superposition of two travelling waves, one moving in the direction of the H beam, and the other moving the opposite way. The splitting is thus proportional to the velocity (i.e. energy) of the antihydrogen atoms.

5.2. Static magnetic field

In order to prevent the spontaneous Majorana spin flips of the antihydrogen atoms inside the cavity, a homogeneous static magnetic field has to be created there using one or more pairs of Helmholtz coils. Thus the GS-HFS frequency at zero external magnetic field needs to be obtained from the $\pi_1$ and $\sigma_1$ transition frequencies (see Fig. 2) measured at non-zero magnetic field(s). To excite the $\sigma_1$ transition, the static magnetic field and the oscillating magnetic field have to be parallel. To excite the $\pi_1$ transition, however, they have to be perpendicular to each other. Therefore in order to excite both transitions while keeping the static field unchanged, the static and the oscillating fields have to be at an angle of 45° with respect to each other.

6. Determination of the GS-HFS frequency

Of the two above-mentioned GS-HFS transitions, $\sigma_1$ has only a weak, approximately second-order dependence on the external magnetic field strength:

$$\Delta \nu_{\sigma_1} \simeq 2.76 \text{ kHz/G}^2$$

for small external fields. Therefore this transition is not significantly broadened by the inhomogeneities of the external magnetic field. Figure 5 illustrates this with simulated resonance profiles at 1 G external magnetic field with various field inhomogeneities. It can be seen that even a 10% inhomogeneity hardly distorts the line shape.

The $\pi_1$ transition line, on the other hand, has a very strong, approximately linear dependence on the external magnetic field:

$$\Delta \nu_{\pi_1} \simeq 1400 \text{ kHz/G} + 1.38 \text{ kHz/G}^2$$

for small external fields. Therefore this transition is severely broadened by the magnetic field inhomogeneities. Figure 6 shows that at an inhomogeneity of 0.5%, the double peak structure is already washed out, and an inhomogeneity of 2% already makes the transition line too broad to reliably detect it.

Knowing the above, the zero-field GS-HFS frequency $\nu_{\text{HFS}}$ can be determined in two ways. If one can create a static magnetic field in the RF resonator with an inhomogeneity of not more than 1%, then both the $\sigma_1$ and the $\pi_1$ transition frequencies can be measured at the same external magnetic field of e.g. 1 G, and from these two frequencies the zero-field GS-HFS frequency can be calculated. The Monte Carlo simulations showed that with an external magnetic field of 1 G ± 5 mG (i.e. magnetic field inhomogeneity of 1%), $\nu_{\text{HFS}}$ can be obtained with an accuracy of ~100 Hz, which corresponds to a relative precision of $\sim 10^{-7}$.

A static magnetic field with such a good homogeneity, however, is not easily achievable, since the cusp trap and the sextupole magnet produce a strong (~100 G) inhomogeneous stray field at
the position of the RF resonator. Therefore in the first stage of the experiment, only the ‘easy’ \( \sigma_1 \) transition will be measured at different external magnetic field strengths, and \( \nu_{\text{HFS}} \) will be obtained by extrapolating the measured frequencies to zero. This method has the disadvantage that the achievable precision is lower: according to the Monte Carlo simulations, by measuring \( \nu_{\sigma_1} \) at three different magnetic fields between 1–10 G with a constant absolute inhomogeneity of 0.1 G, \( \nu_{\text{HFS}} \) can be determined with a a relative precision of \( \sim 5 \times 10^{-7} \).

7. Project status
In 2010, antihydrogen atoms could be produced for the first time in the cusp trap. The average production rate was \( \sim 30 \ \text{H/s} \) in Rydberg states [15]. This was an important step towards the measurement of the ground-state hyperfine splitting of antihydrogen. In 2011, the \( \text{H synthesis} \) process will be further optimized to increase the number of atoms produced. The RF resonator, the sextupole magnet and the antihydrogen detector will be installed, but the magnetic shielding and the Helmholtz coils around the resonator will not yet be in their final form. Nevertheless, this setup will be enough to attempt the first preliminary antihydrogen spin flip experiment. If successful, it will be the first ever spectroscopy measurement of an atomic system made entirely of antimatter. Afterwards the final, high-resolution scans of the transition lines will commence from 2012.
Figure 6. Simulated resonance profiles of the π₁ transition at an external magnetic field \( B \) of 1 G but at different magnetic field inhomogeneities \( \Delta B/B \).

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