Studies in Structure formation in theories with a repulsive long range gravitational force

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Abstract

This article reports on emergence of structures in a class of alternative theories of gravity. These theories do not have any horizon, flatness, initial cosmological singularity and (possibly) quantization problems. The model is characterised by a dynamically induced gravitational constant with a “wrong” sign corresponding to repulsive gravitation on the large scale. A non-minimal coupling of a scalar field in the model can give rise to non-topological solitons in the theory. This results in domains (gravity-balls) inside which an effective, canonical, attractive gravitational constant is induced.

We consider simulations of the formation and evolution of such solutions. Starting with a single gravity-ball, we consider its fragmentation into smaller (lower mass) balls - evolving by mutual repulsion. After several runs, we have been able to identify two parameters: the strength of the long range gravitational constant and the size of the gravity balls, which can be used to generate appropriate two point correlations of the distribution of these balls.
Standard Big - Bang [SBB] theory provides a framework within which one could address a variety of problems in Cosmology. Particularly regarded as “well accounted” are the primordial light element synthesis and the cosmic microwave background radiation. However, when applied to large scale structure formation, the status of SBB is not quite as respectable. More than 90% of the gravitating content of the universe is required to be non-luminous and in an as yet unknown form. Attempts to account for large scale structure in the Universe by a suitable combination of hot and cold dark matter has met with debatable success [see eg. Padmanabhan 1993, for a review]. Fixing the total amount of dark matter by reference to the closure density of the universe and fixing the hot component by requiring potential wells to form at large scales, one is not left with sufficient residual [cold] dark matter to account for enough power to accommodate smaller [galactic and cluster] scale structures. Further, the proximity of the density of the universe to the closure density poses the flatness problem. Related to this problem is the difficulty in obtaining the homogeneous and isotropic Robertson Walker solution dynamically. The FRW metric is a very special metric with measure zero in the space of anisotropic solutions.

On the quantum theoretical side, we have further problems. There is no viable quantum field theory of gravity. If we treat the Einstein - Hilbert action as a fundamental quantum action, we get a non-renormalizable theory. A promising program, originally proposed by Sakharov and much exalted in the early 80’s, was developed by Adler and Zee [see eg. Adler 1982]. The idea was to generate gravitation from an effective action induced by quantized (renormalizable) matter fields on a curved spacetime background. Explicit calculations, made for a large class of renormalizable models [Zee 1982], realize the prescription. While the sign of the gravitational constant turns out to be sensitive to the infrared details of the theory, a large class of models give an induced gravitational constant with a “wrong” sign. In a previous article [Lohiya 1995] we considered a non-minimally coupled scalar field in addition to an induced negative gravitational constant. The model is described by the action:

\[
S = \int d^4x \sqrt{-g}[ -\epsilon \phi^2 R + \frac{1}{2} g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) + \beta_{ind} R + \Lambda_{ind} + L_m ]
\] (1.1)

\(\beta_{ind}\) and \(\Lambda_{ind}\) being the induced gravitational and cosmological constants, \(L_m\) the matter action for the rest of the matter fields and \(V(\phi)\) the effective potential for the scalar field. Derrick’s theorem is not an impediment to the existence of non topological soliton solutions in such a model. For such solutions, \(\phi\) is constant inside a sphere and rapidly goes to zero near its surface. Such solutions would have an effective canonical attractive gravitation in their interior and have repulsive gravitation outside. We have been exploring the possibility that such non topological domains - gravity balls (g-balls) - of the size of a typical halo of a galaxy [or larger], play an essential role in cosmology.

Large scale dynamics of an FRW universe with a gravitational constant of the “wrong sign” is not saddled with any horizon, flatness or initial singularity problems. We have
further shown [Batra 1995, Lohiya et al 1995] that the resulting dynamics of a hot FRW universe can consistently account for the right amount of Helium synthesis. Studies on primordial metallicity are still in progress.

The standard theory of density perturbations gets drastically modified in such a model. In standard cosmology one picks up a seed perturbation and hopes to construct schemes that could follow its growth to the non linear regime: namely scales over which a typical galaxy could form. With a repulsive gravitational constant,

density perturbations do not grow at all in a standard FRW model [Lohiya and Savita 1995]. Instead, one expects the universe to homogenize - density perturbations freezing to a constant in co moving coordinates. One could follow the evolution of a large dense cloud expanding in an FRW spacetime while fragmenting into smaller sections as the overall density perturbation approaches a constant. At some time during its evolution g - balls could form in different pockets of such a cloud. These would then be the sites where canonical non - linear perturbations would grow into a typical galaxy. Alternatively one can entertain the possibility that at some epoch in the past, a large g-ball is formed. The stability of such a ball at any epoch would sensitively depend upon the effective potential of the scalar field and the bulk parameters of the universe, namely, the temperature and the trace of the stress energy tensor of matter fields. Such an unstable g - ball could split into smaller, stable g - balls in a random manner. For the purpose of the present paper, we have considered successive fragmentations of a cloud in a random manner. In the alternative picture, our considerations would equally well describe a large g - ball, when destabilized at any epoch, splitting into two balls in a randomly chosen direction - separated by a distance which would be of the order of the size of a typical ball. We have written a simple, user-friendly simulation code in Pascal which we have run on PC 486 DX-2 and Pentium machines. The next section describes the program in its essential detail and in the last section we describe the result of some 100 runs.

Section II:

A gravity ball is a non topological soliton solution in a theory described by eqn[1.1]. The effective gravitational constant is negative outside the ball and has the canonical positive value inside. We assume that the parameters of the effective potential would support the existence of g - balls as large as a typical galactic halo. The simulation described assumes that at some epoch a large gravity ball forms. The centre of the ball is chosen as the initial point of our simulation. The energy $m_b$ of the ball bears a relation to the size $r_o$ of the ball. The relation is an option available in the code. The default relation used most in our simulations is $m_b \approx r_o^2$. We assign an integer number $m$ to signify the mass of the initial ball. Balls at any epoch are labelled by another integer label, $i$, which runs from 1 to $n$: the number of balls at any epoch. The program prescribes a procedure “splitmass” in which we randomly split the mass $m(i)$ of the $i$ th ball, distributing it over two balls: the $i$ th and the $n + 1$ th ball. For most simulations we randomise the mass of the final balls after a split, such that one of them has a mass equal to 50% to 75% of the
initial ball and the other ball has the balance 50% to 25%. The speed of both the final balls just after the split is the same as that of the parent \(i\)th ball before the split. Energy and momentum conservation are thus automatically taken care of in this procedure. The split points \([\text{ball centres}]\) are placed in a randomly oriented direction at a distance proportional to \(\sqrt{m[i]_{\text{final}}} + \sqrt{m[n+1]}\). The proportionality constant is a parameter “splitrad” that can be varied in our simulations. From the moment the balls split, their dynamics are governed by the following set of equations in co-moving coordinates of the FRW metric: [see eg. Borner [1988] chapter 12.):

\[
\vec{v}_i = \ddot{\vec{x}}_i [i = 1, \ldots, N]
\]

\[
\ddot{\vec{v}}_i + 2H(t)\vec{v}_i = \vec{F}_i
\]

\[
\vec{F}_i = \sum_{j \neq i} Gm_j \frac{\vec{x}_i - \vec{x}_j}{|\vec{x}_i - \vec{x}_j|^3}
\]

We integrate these equations numerically as a set of difference equations. As described in the previous section, the simulation equally well describes a highly correlated cloud which evolves in an FRW spacetime - fragmenting into smaller sections that evolve by mutual repulsion.

The instance of splitting is randomised by choosing an arbitrary prescription that ensures that the probability of splitting of higher mass balls at any given epoch is greater than that for lower mass balls. This is prescribed in a separate procedure which again can be altered if desired. We ran several simulations where the instance of splitting of a ball of given mass was not random. Instead, we chose a monotonic functional dependence of the splitting time with mass that ensured the higher mass balls split faster than the lower mass balls. Splitting stops when all balls have mass \(m_i = 1\). Thus the final number of balls equals the mass assigned to the initial ball. We thus assume that a ball with mass \(m_i = 1\) is stable. The evolution may be continued even after all splitting is over.

The simulation is displayed by standard graphics displays. We have built in user-friendly menu options for pausing, changing the orientation of the axes, changing the scale, recording the configurations of all particles for thin real time slices for replays, loading previous simulations and / or going back to the current simulation. The simulation continuously displays the current status of a particular run, viz. the current number of balls, the number of time steps, the value of the simulation time and the maximum distance over which the points in a run have spread. The three dimensional depth is displayed by assigning colour to all the points: red dots representing the farthest - the colour going to blue for the nearest (to the starting plane).

The purpose of the entire exercise is to explore the possibility that the g - balls would be regions where matter would condense to give galactic structures. The minimum distance over which the balls are split is taken to be the size of a typical galactic halo. This fixes the
distance scale for the simulation. To set the time scale, we note that the scale factor scales as the cosmic time $t$ for most part in the matter dominated era. The Hubble parameter $H_o$ scales as $t^{-1}$. Taking $H_o^{-1} \approx 10^{10}$ years we may consider the original g - ball to form well after matter and radiation decouple, (say at $H^{-1} \approx 10^8$ years) and match the final configuration of the simulation with current data on galaxy-galaxy correlation functions. The inverse of the Hubble parameter at the start sets the time scale for our simulation. For example, a time run of 100 units for the above mentioned range of $H_o$ gives a scale of $10^8$ years per unit.

We performed some preliminary runs for a chosen value of the gravitational constant. It was found that, after an initial rapidly changing profile, the distribution starts simply scaling conformally. This is as expected from the linear theory of perturbations [Lohiya, Savita 1995]. We ran several trials initially to ascertain a cutoff distance beyond which we could ignore gravitational terms [the contribution to the rhs in eqn(2.1)]. This was done by running a particular simulation and storing the system configuration for different times. Intermediate configurations were then re-loaded with different distance cutoffs. The distance cutoff for which the final configuration does not change much can thus be determined by trials. We can thus introduce a cutoff distance in our gravitational term - ignoring contributions from balls beyond that distance. This considerably improves the speed of the program.

On terminating the simulation, one can opt for inbuilt procedures that can output positions and velocities in formats suitable to be read off by appropriate statistical packages. We built in a specific procedure that evaluates two point auto - correlation function. We enclose the entire simulation within a sphere centered around the initial starting point, the radius of the sphere being of the same order of magnitude as the average distance over which our simulations spread. We ensure that any runaway particle gets reflected back from the surface of the sphere. This amounts to the assumption that any particles lost from the sphere are compensated by incoming particles escaping from neighbouring, similarly evolving, domains in a homogeneous, isotropic universe. The autocorrelation function is evaluated by contrasting the final distribution with a random distribution of $m$ points within the sphere.

The input parameters are thus the value of gravitational constant and the size parameter of the gravity balls. A complete listing of the program can be made available on request. We have also designed some test files which test the randomisation and approximation procedures for known closed orbits for canonical attractive gravity. For any given value of input parameters chosen in the main program, one can run corresponding test files to determine appropriate increments for time to be used in the difference equations to get the desired accuracy.

III Results:

The purpose of this exercise was to explore the possibility that a typical g - ball can be the region inside which non - linear perturbations grow. Density perturbations ($\delta \rho / \rho$)
with repulsive gravity approach a constant in time. One could fix this constant by matching the perturbations with the COBE result at the recombination epoch. The large scale distribution of galaxies could be generated by the bifurcation and evolution of the gravity balls.

Figures I describes a typical simulation snapshot. In figures II to V we plot the two point correlation functions for the specified ranges of distances for different values of the gravitational constant. In every simulation we can identify distance ranges over which the correlation function falls off as $r^{-1.7}$ to $r^{-2}$. One can of course identify distance ranges over which the fall off is around the observed $r^{-1.8}$.

The distance at which the autocorrelation function is unity is sensitive to the chosen gravitational constant. Assuming the size of a typical galactic halo to be some 25 kpc, and recalling this to be the distance between the g - balls at the time of splitting, the distance scale in our simulations is fixed. Assuming that the simulation terminates at epoch $t_o$, the number of time steps for a typical simulation being 100, the initial time and the time unit are then fixed at $10^{-2}t_o$. This would be the case if the gravity balls form and evolve at such an initial time. The units of time and distance then fix the gravitational constant. At present we do not have any rigorous theory of formation of gravity balls or their bifurcation. If these form at any epoch $t_i$, our simulations ends at $100t_i$. A careful study of parameters that could fix the epoch of formation of non - topological solitons is thus necessary and is under progress.

We find our results quite encouraging and are in the process of designing programs for much larger distributions that could be used in larger and faster machines.

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Figure Captions:

Figure I: A typical snapshot of a simulation after the development freezes in comoving coordinates.

Figures II - V: Two point autocorrelation function for different simulations.

Note: The figures will be posted on request, either as PostScript files or by FAX.