Multirate Mass Transfer Approach for Double-Porosity Poroelasticity in Fractured Media

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Abstract Natural and anthropogenic fractured aquifers and reservoirs are dual porosity matrix-fracture systems, where the fracture network provides highly-conductive flow pathways and the low-permeability matrix stores most of the fluid. The coupling between flow and mechanical deformation in fractured media is often modeled using the classical theory of dual-porosity poroelasticity (DPP), based on Barenblatt’s hypothesis of pressure equilibrium inside the rock matrix blocks. Equilibrium can be expected if the matrix blocks are small and the matrix diffusion time is comparable to the flow time scales along the fractures. In practice, matrix blocks may be large enough so that diffusion time scales are long, and the equilibrium hypothesis breaks down. Here, we study nonequilibrium effects in coupled flow and deformation in fractured media. We compare analytical predictions and modeling results of coupled flow and deformation in heterogeneous fractured porous media. The theoretical analysis is a nonequilibrium, dual-porosity model. We use this theory to (a) Reveal the limitations of classical DPP formulations. (b) Obtain the scalings for drainage and displacement to be expected for coupled flow and deformation in highly heterogeneous, fractured media. (c) Identify what behavior to expect in fractured aquifers and reservoirs regarding flow and deformation. We observe strong tailing in fluid fluxes and land subsidence that cannot be captured by the classical DPP approach or a single porosity effective medium approach. We show that theoretical predictions from the multirate DPP model and high-fidelity models agree, even for highly heterogeneous matrix-fracture systems, and reproduce the observed nonequilibrium effects.

1. Introduction

In naturally fractured aquifers and reservoirs, complex fracture networks created by geological processes dominate the flow of water and hydrocarbons, and control the transport of heat and solutes through them. Fractured reservoirs can also be anthropogenic: in enhanced geothermal systems, the permeability of deep hot rock is created and enhanced through hydrofracturing and hydroshearing-a process where fluid is injected at high pressures, inducing shear failure and slip on preexisting faults and fractures. In unconventional oil and gas reservoirs, hydraulic fracturing is used to create a stimulated reservoir volume, with a fracture network whose geometry and connectivity depends on the rock brittleness and stimulation protocols. The multiscale heterogeneity that is inherent to fracture-matrix systems for any field-scale problem hampers our ability to model and predict flow and transport processes in such settings.

Hydromechanical coupling is important in fractured and stress-sensitive reservoirs (Rutqvist & Stephansson, 2003). Stress states affect solute transport and multiphase flow in fractured media, both at the fracture (da Silva et al., 2019) and field scales (Kang et al., 2019). Geomechanical constraints may be the limiting factor in the development of large-scale CO2 geological storage projects (Szulczewski et al., 2012), where it is essential to ensure caprock integrity (Birkholzer et al., 2009; 2015; Castelletto et al., 2013; Vilarrasa et al., 2010) and to reduce the risk of induced seismicity (Juanes et al., 2012; Vilarrasa & Carrera, 2015; White & Foxall, 2016; Zoback & Gorelick, 2012). The coupling between flow and rock deformation has also been proposed as an effective monitoring tool to understand and predict surface deformations and subsidence in large-scale CO2 geological storage projects (Morris et al., 2011; Verdon et al., 2013).

Land subsidence and its connection with induced seismicity has been a concern in oil and gas production for decades (Ferronato et al., 2008; Segall, 1989). Understanding the various mechanisms that control long-term reservoir compaction and subsidence rates is particularly important in environmentally sensitive regions, such as the gas fields in the Netherlands (Fokker et al., 2018; de Waal & Schouten, 2020; van Eck...
et al., 2006; van Thienen-Visser & Fokker, 2017; Wees et al., 2014). Lastly, understanding the evolution of spatial patterns of seismicity induced by fluid injection remains a challenge for hydro-mechanical modeling of geometrically complex reservoirs and fault zones (Ellsworth, 2013; Raleigh et al., 1976). This is particularly important for the development of enhanced geothermal systems that rely on stimulating permeability through hydroshearing and some degree of hydrofracturing (Brodsky & Lajoie, 2013; Deichmann & Giardini, 2009; Grigoli et al., 2018; Kim et al., 2018; Majer et al., 2007).

Fully coupled hydromechanical modeling of porous and fractured media is typically based on the classical linear theory of poroelasticity (Verruijt, 2016), and incorporated into modeling workflows for single- and multiphase flow (Chin et al., 2000; Lewis et al., 1991; Lewis & Sukirman, 1993; Settari & Mourits, 1998; Settari & Walters, 2001). In fractured media, discrete fractures have been incorporated as lower-dimensional objects with rigorous coupling between matrix and fracture flow and deformation that allow the continuity of fluid flow and displacements (Jha & Juanes, 2014; Segura & Carol, 2008a, 2008b).

For naturally fractured reservoirs where the density of fractures is large enough so that discrete fractures cannot be characterized or modeled explicitly, the current upscaling paradigm for flow-transport-deformation relies on the double-porosity approach (Barenblatt et al., 1960; Wilson & Aifantis, 1982): a small-scale structure of highly permeable fractures and poorly permeable matrix with large storage capacity. Continuum equivalent theories date back to Barenblatt’s dual-porosity concept (Barenblatt et al., 1960), in which the porous matrix blocks and the fractures are envisioned as two separate continua in terms of flow, but linked through the fluid exchange between the matrix blocks and secondary fractures. In the context of poroelasticity, this is the so-called dual-porosity poroelasticity (DPP) paradigm (Bai et al., 1993; Elsworth & Bai, 1992; Wilson & Aifantis, 1982), thoroughly analyzed by (Berryman & Wang, 1995), (Berryman & Pride, 2002), and (Bai, 1999), and recently revisited by (Mehrabian & Abousleiman, 2014), (Zheng et al., 2016), and (Presho et al., 2011). Analogous to the double-porosity concept for solute and heat transfer, DPP recognizes nonequilibrium in deforming fractured media: the possibility that, over short timescales, pressures may be different in the rock matrix and fracture system. Mass transfer between mobile and immobile medium portions contribute to nonlocal behavior and history-dependence in two-phase flow (Geiger et al., 2013; Tecklenburg et al., 2013) and non-Fickian mass and heat transport (Geiger & Emmanuel, 2010) in porous and fractured media, which can be quantified by a sound characterization of the nonequilibrium mass transfer in the immobile medium regions. We propose to explore this modeling strategy in the context of poroelastic continua with nonequilibrium pore pressure drainage.

Effective parameters of a multiporosity mixture are to be determined from the intrinsic properties of its individual porous components, under the assumption that the individual components are Gassmann materials (each poroelastic component to deformation follows) a simple linear relationship between volumetric strain and water content change with total stress and pore pressure (Bai, 1999; Berryman & Pride, 2002; Mehrabian & Abousleiman, 2014).

In this paper, we study nonequilibrium effects in coupled flow and deformation in fractured media. Nonequilibrium effects are not captured by classical dual-porosity formulations, which assume local equilibrium inside the matrix blocks. Local equilibrium (constant pressure inside each matrix block), which is assumed in the classical dual–porosity models, is not satisfied in general in heterogeneous fractured systems. If the block sizes are large enough, or the matrix permeabilities are small enough, the time scales for fluid drainage to the highly conductive fracture network, which control poroelastic effects, prevent the matrix from reaching internal pressure equilibrium. In general, there is a wide range of time scales for flow, due to permeability heterogeneity, which leads to a wide range of time scales for deformation. While this nonequilibrium effect has been proposed as a mechanism to explain anomalous time–dependent subsidence in mature gas fields (de Waal & Schouten, 2020; Fokker et al., 2018; van Eck et al., 2006; van Thienen-Visser & Fokker, 2017), it has been seldom studied in detail.

We compare analytical predictions and modeling results of coupled flow and deformation in heterogeneous fractured porous media. The theoretical analysis is a nonequilibrium, multirate model for dual-porosity poroelasticity. We use this theory to (a) Reveal the limitations of classical DPP formulations. (b) Obtain the scalings to be expected for coupled flow and deformation in highly heterogeneous, fractured media. c) Identify what behavior to expect in fractured aquifers and reservoirs regarding flow and deformation. The
evolution of fluid fluxes and displacements may help interpret the flux/production data in connection with long-term land subsidence.

Numerical simulations address the problem from two perspectives: (a) High-fidelity models, where we describe the coupled flow and deformation for both the rock matrix and a network of discrete fractures that is explicitly represented by lower–dimensional (1D) objects; (b) Effective models based on the classical theories of dual–porosity poroelasticity (DPP) (Wilson & Aifantis, 1982). We show that high-fidelity models and theoretical predictions of the non-equilibrium model agree, even for highly heterogeneous matrix-fracture systems. We also show that the classical DPP approach is unable to capture the local non–equilibrium effects, which prevent these effective models from capturing the observed tailing in fluid fluxes and land subsidence.

The paper is organized as follows: Section 2 presents the mathematical model of single- and dual-porosity poroelasticity, for single-phase flow. This section also describes the theoretical model of nonequilibrium multiporosity poroelasticity, with single and multirate mass transfer. We also discuss the predicted scalings for the simplified case of uncoupled consolidation. Section 3 presents the numerical results, which are organized along two model problems: (a) Single fracture and matrix interaction, which serves as a validation of the theory and models, and illustrates the limitations of the classical DPP models. (b) Consolidation of a synthetic heterogeneous matrix-and-fracture network model.

2. Background and Methodology

2.1. Mathematical Model of Single-Phase Flow Through Deformable Porous Media

The Biot equations for flow and quasi-static deformation in a single-porosity heterogeneous medium can be written as (Biot, 1941; Rice & Cleary, 1976):

\[ \nabla \cdot \mathbf{\sigma} = 0, \]

\[ \beta \frac{\partial \mathbf{\varepsilon}}{\partial t} + b \frac{\partial \mathbf{\varepsilon}}{\partial t} - \nabla \cdot \left( \frac{k}{\mu_f} \nabla p \right) = 0, \]

where \( b \) is the Biot coefficient, \( \mathbf{\varepsilon} \) is the volumetric strain, \( \mathbf{\varepsilon} = \text{tr} (\mathbf{\varepsilon}) \) (\( \varepsilon \) is the infinitesimal strain tensor), \( k \) is the intrinsic permeability of the porous medium, \( \mu_f \) is the fluid dynamic viscosity, and \( p \) is the fluid pressure. Note that fluid density \( \rho_f \) is assumed here to be constant. The storage coefficient \( \beta \) is defined by:

\[ \beta = \phi c_f + (b - \phi)c_s, \]

where \( \phi \) is porosity, \( c_f \) is fluid compressibility and \( c_s \) is the compressibility of the solid. The latter is given by:

\[ c_s = (1 - b)/K, \]

where \( K \) is the bulk modulus of the porous matrix. It can be expressed in terms of the Young modulus \( E \) and Poisson ratio \( \nu \) as:

\[ K = \frac{E}{3(1 - 2\nu)}. \]

The Biot coefficient is defined following Skempton (1960) and Nur and Byerlee (1971):

\[ b = 1 - \frac{K}{K_s}, \]

where \( K_s \) is the bulk modulus of the solid grains. The constitutive definitions of linear elasticity and infinitesimal strains are based on the effective Cauchy stress tensor \( \mathbf{\sigma}' \):

\[ \mathbf{\sigma} = \mathbf{\sigma}' - bpI, \]
\[ \mathbf{\sigma}' = \mathbf{D} \cdot \mathbf{\varepsilon}, \]

(6b)

where \( \mathbf{I} \) is the identity matrix, \( \mathbf{\sigma} \) [Pa] is the total stress tensor (tensile stresses are positive), \( \mathbf{D} \) is the tensor of elastic constants, and the infinitesimal strain tensor is \( \mathbf{\varepsilon} = \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \), with \( \mathbf{u} \) the displacement field. The effective stress tensor is a linear function of infinitesimal strains, \( \mathbf{\sigma}' = 2G\mathbf{\varepsilon} + Lr(\mathbf{\varepsilon})\mathbf{I} \), where \( L = \frac{vE}{(1 + \nu)(1 - 2\nu)} \) [Pa] and \( G = \frac{E}{2(1 + \nu)} \) [Pa] are the Lamé constants. We assume plane strain elasticity, so that Hooke's law reduces to:

\[
\begin{pmatrix}
\sigma'_{xx} \\
\sigma'_{yy} \\
\sigma'_{xy}
\end{pmatrix} = \frac{E}{(1 + \nu)(1 - 2\nu)}
\begin{pmatrix}
1 - \nu & \nu & 0 \\
\nu & 1 - \nu & 0 \\
0 & 0 & 1 - 2\nu
\end{pmatrix}
\begin{pmatrix}
\mathbf{\varepsilon}_{xx} \\
\mathbf{\varepsilon}_{yy} \\
\mathbf{\varepsilon}_{xy}
\end{pmatrix}
\]

(7)

### 2.2. Discrete Fracture Modeling

Discrete fractures are introduced in the model as quasi one-dimensional objects that have no inherent stiffness. This means, we assume that the fracture is a thin poroelastic medium that deforms according to the volumetric strain at the fracture-matrix interface. This means, we solve the mass conservation equation for fluid along a fracture of effective width \( d_f \) [m]:

\[
d_f \frac{\partial}{\partial t} (\beta p_f + b \mathbf{v}_f) + \frac{d}{dt} \left( \rho_f d_f \frac{\mathbf{k}_f}{\mu_f} \frac{dp_f}{dt} \right) = 0,
\]

(8)

where \( t \) [m] is the tangential coordinate along the fracture, \( \phi_f \) [-] is the fracture porosity, \( k_f \) [m²] is the fracture permeability and \( p_f \) [Pa] is the fracture pore pressure. We impose \( p = p_f \) along the internal boundaries of each matrix sub-domain, so we allow fluid flow between the fractures and the surrounding media. One-way mechanical coupling between the matrix and fracture is introduced such that at the interface between the fracture and matrix domains, \( \mathcal{\Omega}_{fm} \), we assume continuity in pressure:

\[
p_f = p_m, \mathbf{x} \in \mathcal{\Omega}_{fm}
\]

(9a)

and in flux:

\[
\int_{\mathcal{\Omega}_{fm}} d\mathbf{a} \frac{\mathbf{k}_f}{\mu_f} \mathbf{n}_f \cdot \nabla p_f = \int_{\mathcal{\Omega}_{fm}} d\mathbf{a} \frac{\mathbf{k}_m}{\mu_f} \mathbf{n}_m \nabla p_m,
\]

(9b)

where \( \mathbf{n}_f \) is the unit normal vector pointing from the fracture to the matrix, and \( \mathbf{n}_m \) the unit normal vector pointing from the matrix to the fracture.

### 2.3. Classical Model of Dual-Porosity Poroelasticity

We adopt the framework of poroelasticity for dual-porosity media proposed by Wilson and Aifantis (1982) and Elsworth and Bai (1992). These models are based on Barenblatt’s concept of a small-scale structure of highly permeable fractures and poorly permeable matrix with large storage capacity, envisioned as two separate continua in terms of flow (Barenblatt et al., 1960).

The effective stress law for dual-porosity poroelastic media is defined as (Berryman & Wang, 1995; Wilson & Aifantis, 1982):

\[
\mathbf{\sigma}' = \mathbf{\sigma} + (\chi_m b_m p_m + \chi_f b_f p_f) \mathbf{I},
\]

(10)

where \( \mathbf{\sigma} \) and \( \mathbf{\sigma}' \) are the total and effective Cauchy stress tensors (tractions are positive), \( p \) is the pore fluid pressure, and the subscripts \( m \) and \( f \) refer to the matrix and fractures, respectively. The pressure ratio
factors, $b_m$ and $b_f$, are effective matrix and fracture Biot coefficients, and $X_{m,f}$ are the matrix and fracture volume fractions. (Wilson & Aifantis, 1982) proposed effective parameters based on the Skempton coefficient:

$$b_m = \frac{K^*}{K} \left(1 - \frac{K}{K_f}\right), \quad b_f = 1 - \frac{K^*}{K},$$

(11)

where $K^*$, $K$, and $K_f$ are the bulk moduli of fractured rock, intact porous matrix and solid grains, respectively (Bai, 1999). Alternative models to compute effective parameters for dual-porosity poroelastic media have been proposed by (Berryman & Pride, 2002) and by (Zheng et al., 2016).

Based on the effective stress law (Equation 10), the mass conservation and equilibrium equations of DPP, assuming an incompressible fluid, become:

$$\mathbf{\nabla} \cdot \mathbf{\sigma} = 0,$$

(12a)

$$\chi_m \beta_m \frac{\partial p_m}{\partial t} + \chi_m b_m \frac{\partial \varepsilon_m}{\partial t} - \mathbf{\nabla} \left( \frac{\chi_m k_m}{\mu_f} \mathbf{\nabla} p_m \right) = -\gamma (p_m - p_f),$$

(12b)

$$\chi_f \beta_f \frac{\partial p_f}{\partial t} + \chi_f b_f \frac{\partial \varepsilon_f}{\partial t} - \mathbf{\nabla} \left( \frac{\chi_f k_f}{\mu_f} \mathbf{\nabla} p_f \right) = \gamma (p_m - p_f),$$

(12c)

where $k_{m,f}$ and $\beta_{m,f}$ are the effective matrix and fracture permeabilities and total compressibilities, respectively.

The constitutive equations reduce to the effective stress law and linear elasticity:

$$\mathbf{\sigma} = \mathbf{\sigma}' = (\chi_m b_m p_m + \chi_f b_f p_f) \mathbf{I},$$

(13a)

$$\mathbf{\sigma}' = 2G\varepsilon + L\text{tr}(\varepsilon) \mathbf{I},$$

(13b)

together with the assumption of vanishing fracture compliance, so that the fracture and matrix strains are equal, $\varepsilon = \varepsilon_m = \varepsilon_f$. The matrix-fracture mass transfer coefficient, $\gamma$, is given by (Presho et al., 2011):

$$\gamma = s k_m \left( \frac{1}{\ell^2_x} + \frac{1}{\ell^2_y} \right),$$

(14)

where $s$ is a fitting parameter, $k_m$ is matrix permeability, $\mu_f$ is fluid viscosity, and $(\ell_x, \ell_y)$ are characteristic horizontal and vertical matrix block sizes.

The mechanical compatibility between the matrix and fracture domain deformations has led to alternative formulations of DPP. In the above stress law, adopted here, the fracture deformation is essentially ignored, and the fracture domain deforms with the matrix volumetric strains (Bai, 1999; Mehrabian & Abousleiman, 2014). A perhaps more rigorous alternative formulation was proposed by (Elsworth & Bai, 1992), with separate effective stress laws for the two domains. In this alternative approach, the constraint coupling fracture and matrix is one of local stress equilibrium (equal total stresses), with strains that are the sum of the individual domain strains. (Bai, 1999) proposed parameter conversions between both formulations, under the assumption of vanishing fracture compliance.

The DPP model accounts for a single mass transfer rate $\gamma$ between the fracture and matrix domain. In natural fractured media, however, one encounters a distribution of mass transfer rates due to the spatial heterogeneity of the hydraulic properties in the matrix domain. As we show in Section 4, the classical DPP model is not able to capture observed tailing behaviors in outflow and displacement. Thus, in Section 3, we present a multirate DPP model that explicitly accounts for these features.

### 2.4. Numerical Methodology

We solve Equation 1 monolithically coupled, together with the fracture fluid flow using Equations 8–9c, employing the finite element software COMSOL Multiphysics (COMSOL, 2016). We use triangular elements
with quadratic Lagrange interpolation for displacements and linear approximation for pore pressures. For fracture 1D linear elements, we also employ linear discretization for pressure and impose continuity between the fracture pressure \( p_f \) and the surrounding poroelastic media. The time discretization is implicit, based on the second-order backward differentiation formula (BDF2).

### 2.4.1. Fracture-Matrix Implementation

The above mathematical model is discretized using the standard fracture flow implementation in COMSOL Multiphysics (COMSOL, 2016), which is valid for fractures that are embedded in a less-permeable matrix. The formulation does not introduce new pressure degrees of freedom, but rather adds the weak form of Equation 8 to the overall variational formulation of the Darcy flow problem. In that sense, pressures at the fracture at interpolated from the matrix pressures (there is pressure continuity across the fracture), and the fracture becomes a fast-flow, low-dimensional region embedded in the low-permeability matrix. While valid for fractures that are at least as permeable as the background matrix, this formulation is not appropriate for sealing faults or fractures that are less permeable than the matrix, which require the introduction of additional degrees of freedom to allow pressure and displacement discontinuity across the fracture while enforcing flux continuity (Cueto-Felgueroso et al., 2017, 2018; Jha & Juanes, 2014; Segura & Carol, 2008a, 2008b). We do not allow slip of the fractures, so that displacements are also continuous across them.

### 2.4.2. Classical Dual Porosity Implementation

The classical DPP equations are implemented as an effective equilibrium equation for the matrix and fracture deformations, and two separate pressure equations, Equations 12–13. There are two separate pressures as unknowns, that is, the matrix- and fracture-domain pressures, \( p_m \) and \( p_f \), which are coupled through a mass-exchange source term. The presence of one common equilibrium equation implies that rock deformations in the dual-porosity model are equivalent to single-porosity ones with the modified effective stress law (Equation 13).

### 3. Multi-Porosity NonEquilibrium Model of Poroelasticity

In this section, we present the multi-porosity nonequilibrium model of poroelasticity that relaxes the assumption that the processes of deformation and pressure propagation in the matrix are in local equilibrium. Figure 1 shows a schematic illustration of the dual porosity nonequilibrium phenomenon. The fractures are highly permeable, embedded into a low permeability matrix. The zoom-in view shows the pressure distribution in matrix blocks for a consolidation scenario due to the application of a load at the right medium boundary. Fracture-matrix interaction under such conditions cannot be described by a local equilibrium approach. In the following, we derive the upscaled model using volume averaging, and provide model solutions and scalings of outflow for a heterogeneous consolidation scenario.

#### 3.1. Model Development

We consider a fracture–matrix system with distinct hydraulic and material properties in fracture and matrix, and consider the flow problem in each of the domains separately. We write the governing Equation 1 for each domain as:

\[
\beta_f \frac{\partial p_f}{\partial t} + b_f \frac{\partial \epsilon_{df}}{\partial t} - \nabla \cdot \left( \frac{k_f}{\mu_f} \nabla p_f \right) = 0, \tag{15}
\]

\[
\beta_m \frac{\partial p_m}{\partial t} + b_m \frac{\partial \epsilon_{dm}}{\partial t} - \nabla \cdot \left( \frac{k_m}{\mu_f} \nabla p_m \right) = 0. \tag{16}
\]

At the interface \( \partial \Omega_{fm} \) between the fracture and matrix domains, we specify pressure continuity and flux continuity, as expressed by Equation 9.

We consider a representative elementary volume \( \Omega_r \) of volume \( V_r \), in which a representative number of fracture and matrix blocks are contained. This means that the length scale \( \ell_r \) of \( \Omega_r \) needs to be much larger than...
a characteristic fracture length scale. The representative elementary volume is the union of the fracture
domain \( \Omega_f \) and unconnected matrix domains \( \Omega_m \), see schematic in Figure 1. We assume that the matrix
domains are convex.

We define the average pressures in the fracture and matrix domains by the following moving averages (Davit
et al., 2013; Whitaker, 1998):

\[
\bar{p}_f = \frac{1}{V_f} \int_{\Omega_f} d\rho_f(x + r), \quad \bar{p}_m = \frac{1}{V_m} \int_{\Omega_m} d\rho_m(x + r).
\]

(17)

Averaging Equation 15 over the fracture domain gives:

\[
\beta_f \frac{\partial \bar{p}_f}{\partial t} + b_f \frac{\partial \bar{e}_f}{\partial t} = \nabla \cdot \left( \frac{k_f}{\mu_f} \nabla \bar{p}_f \right) = \frac{1}{V_f} \int_{\partial \Omega_{fm}} d\alpha \frac{k_f}{\mu_f} n_f \cdot \nabla \bar{p}_f,
\]

(18a)

where we use a volume averaging theorem that decomposes the average of the derivative into the sum of two
terms, which can be seen as the decomposition of the surface terms in the divergence theorem. Details can
be found in Whitaker (1998) and Davit et al., (2013) and references therein. Note that the surface integral on
the right side integrates over all fracture-matrix interfaces within the representative elementary volume. We
assume that the matrix does not contribute to the overall pressure propagation, this means, pressure in the
matrix is propagated predominantly perpendicular to and across the fracture-matrix interface. The validity
of this assumption is confirmed by detailed numerical simulations as reported below. Furthermore, matrix

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Figure 1. Schematic of dual-porosity poroelastic fractured media.
blocks interface only with the fractures, not with other matrix blocks, see also Barenblatt et al. (1960). Thus, averaging over the immobile regions gives:

$$\beta_m \frac{\partial \bar{p}_m}{\partial t} + b_m \frac{\partial \bar{E}_m}{\partial t} = \frac{1}{V_m} \sum_{\alpha_j \in \Omega_{jm}} \int \frac{d \alpha}{\mu_f} \frac{k_m}{\mu_f} \mathbf{n}_m \cdot \nabla p_m. \quad (18b)$$

The right side is a direct application of the divergence theorem. Flux conservation between fracture and matrix implies that:

$$\frac{1}{V_r} \sum_{\alpha_j \in \Omega_{jm}} \int \frac{d \alpha}{\mu_f} \frac{k_f}{\mu_f} \mathbf{n}_r \cdot \nabla p_f = \frac{1}{V_m} \sum_{\alpha_j \in \Omega_{jm}} \int \frac{d \alpha}{\mu_f} \frac{k_m}{\mu_f} \mathbf{n}_m \cdot \nabla p_m. \quad (18c)$$

The DPP model discussed in the previous section assumes that equilibrium is established immediately within each domain in response to flux across the interface, see Appendix A. This is a valid approximation for times that are larger than the characteristic matrix time scale. In the following, we do not make this assumption, but consider a closure that is based on the assumption that pressure in the mobile REV portion equilibrates instantaneously, while the matrix domain is in nonequilibrium. This means, we consider situations, for which the pressure within the matrix cannot be assumed to equilibrate locally on the time scales of interest. Combining Equations 18a and 18b using flux continuity Equation 18c, we can write:

$$\beta_f \frac{\partial \bar{p}_f}{\partial t} + b_f \frac{\partial \bar{E}_f}{\partial t} - \nabla \left( \frac{k_f}{\mu_f} \bar{p}_f \right) = -\beta_m \frac{Z_m}{Z_f} \frac{\partial \bar{p}_m}{\partial t} + b_m \frac{Z_m}{Z_f} \frac{\partial \bar{E}_m}{\partial t}, \quad (19)$$

where the volume fractions $Z_{m,f} = V_{m,f} / V$. In order to close this equation, we need to express the right side in terms of the average fracture pressure $\bar{p}_f$. The matrix pressure $p_m$ is obtained by solving Equation 16 with the boundary condition Equation 9a. We assume that the fracture pressure equilibrates fast across the fracture cross section and thus set the boundary condition:

$$p_m = \bar{p}_f, \quad \mathbf{x} \in \partial \Omega_{jm} \quad (20)$$

$$\bar{E}_{v,f} = \bar{E}_{v,m}, \quad \mathbf{x} \in \partial \Omega_{jm} \quad (21)$$

The average volumetric strain in the fracture is not computed explicitly, but set equal to the volumetric strain of the matrix at the fracture-matrix intersection. This means, the volumetric strain in the fracture is determined by the deformation properties of the matrix. Thus, in the following we drop the sub-indices for the Young modulus $E$ and Poisson ratio $\nu$ as it is understood that they refer to the matrix.

In order to analytically close the set of equations, we assume that the strain terms in fracture and matrix are linearly proportional to pressure. This means:

$$b_i \frac{\partial \bar{E}_i}{\partial t} = \xi_i \frac{\partial \bar{p}_i}{\partial t}, \quad (22)$$

where $i = m,f$. The $\xi_i$ are compressibility coefficients defined below. The approximation Equation 22 corresponds to the case of uncoupled consolidation. For three dimensional uncoupled consolidation, the compressibility coefficients $\xi_i (i = m, f)$ are given by (Verruijt, 2016):

$$\xi_i = b_i^3 \frac{3(1 - 2\nu)}{E}. \quad (23)$$

For uniaxial strain load, the $\xi_i$ can be expressed as (Merxhani, 2016):

$$\xi_i = b_i^3 m_i, \quad (24)$$

$$m_i = \frac{(1 - 2\nu)(1 + \nu)}{(1 - \nu)E}. \quad (25)$$

where $m_i$ represents the drained uniaxial compressibility of a laterally confined porous medium. We now define the effective storage coefficients:
\[ S_i = \beta_i + \xi_i \text{, } \text{for } f, m. \]  

(26)

Thus, Equations 16 and 19 simplify to:

\[ S_m \frac{\partial p_m}{\partial t} - \nabla \cdot \left( \frac{k_m}{\mu_f} \nabla p_m \right) = 0, \]  

(27)

\[ S_f \frac{\partial p_f}{\partial t} - \nabla \cdot \left( \frac{k_f}{\mu_f} \nabla p_f \right) = -\frac{\chi_m}{\chi_f} S_m \frac{\partial \bar{p}_m}{\partial t}. \]  

(28)

As outlined above the boundary condition at the fracture–matrix interface is given by Equation 20. Under these conditions, we can write the solution of Equation 27 by using the Duhamel theorem as:

\[ p_m(y, t) = p_{m,0} + \int_0^t dt' g(y, t - t') \left[ \bar{p}_f(x, t') - p_{m,0} \right]. \]  

(29)

The Green function \( g(y, t) \) solves the diffusion Equation 27 for the boundary condition \( \delta(y) \) with \( y \in \partial \Omega_{jm} \). The uniform initial pressure \( p_{m,0} \) in the matrix is given in terms of the average vertical hydrostatic stress \( \sigma_v \) as (Rice & Cleary, 1976):

\[ p_{m,0} = -B \sigma_v. \]  

(30)

where \( B \) is the Skempton coefficient. For uniaxial strain load and if the porous medium is restricted in the perpendicular directions, the Skempton coefficient is (Merxhani, 2016):

\[ B = \frac{m_v}{bm_v + \beta / b}. \]  

(31)

The average pressure in the matrix can be written as:

\[ \bar{p}_m(x, t) = p_{m,0} \left[ 1 - \frac{1}{V_m} \int \Omega_m d y \right] + \frac{1}{V_m} \int \partial \Omega_m \frac{1}{V_m} \int d y' g(y', t), \]  

(32)

where we defined the memory function:

\[ \varphi(t) = \frac{1}{V_m} \int \partial \Omega_m \frac{1}{V_m} \int d y' g(y', t). \]  

(33)

Combining Equations 28 and 32, we obtain the following closed form equation for the average pressure in the fracture:

\[ S_f \frac{\partial \bar{p}_f}{\partial t} - \nabla \cdot \left( \frac{k_f}{\mu_f} \nabla \bar{p}_f \right) = \frac{\chi_m}{\chi_f} S_m \varphi(t) p_{m,0} - \frac{\chi_m}{\chi_f} S_m \frac{\partial }{\partial t} \int_0^t dt' \varphi(t - t') \bar{p}_f(x, t'). \]  

(34)

Note that relation Equation 29 can be used to reconstruct the pressure distribution in the matrix based on the knowledge of the average pressure \( \bar{p}_f \) in the fracture.

To illustrate the physical meaning of the memory function, we consider the situation that pressure in the fracture is equal to 0. In this case, we obtain from Equation 32 that \( \varphi(t) \) is proportional to the flux \( j_m(t) = -\frac{\partial p_m(t)}{\partial t} \) from the matrix to the fracture:

\[ j_m(t) = p_{m,0} \varphi(t). \]  

(35)

### 3.1.1. Single Rate Mass Transfer

We first assume that the medium consists of equally sized matrix blocks with identical mechanical and hydraulic properties. This means, there is a single characteristic mass transfer time scale \( \tau_m \). Note that this notion of single rate mass transfer is different from the one used for example by (Haggerty & Gorelick, 1995), for whom the term single rate refers to a single first-order mass transfer rate.
For general (convex) geometries of the matrix blocks, no closed form expression for the memory function is available. However, we can make some general statements on its form and asymptotic behavior. To this end, we first note that the Green function $g(x,t)$ of the diffusion Equation 16 has the scaling form:

$$g(x,t) = f(x / \ell_m, t / \tau_m).$$

(36)

where $\ell_m$ is a characteristic matrix length scale, and

$$\tau_m = \frac{\ell_m^2 \mu S_m}{k_m},$$

(37)

the characteristic time for the response of the matrix to a perturbation on the interface. This response is in general not instantaneous, as assumed by the dual porosity approach. As we will see in the following, nonequilibrium effect can be observed in the outflow for observation times $t < \tau_m$. From Equation 36, we obtain for the memory function $\phi(t)$ defined by Equation 33 the scaling form:

$$\phi(t) = \frac{1}{\tau_m} \phi(t / \tau_m).$$

(38)

To understand the behavior of the memory function, we recall that it is proportional to the flux from the matrix to the fracture if the fracture pressure is 0, see Equation 35. Thus, for times $t \ll \tau_m$ the behavior of $\phi(t)$ is given the diffusive flux across the boundary of a semi-infinite domain. As the width of the diffusive interface at the boundary scales as $\sqrt{t}$, the pressure gradient and thus the flux across the interface scale as $1/\sqrt{t}$. This implies that the memory function scales as $\phi(t) \propto t^{-1/2}$. For times $t \gg \tau_m$, the flux and thus the memory function drop to 0 exponentially fast.

For the fracture-matrix configurations under consideration here, the diffusion problem Equation 16 can be approximated by one-dimensional diffusion into slabs. In this case, the memory function is given in Laplace space by (Dentz et al., 2011; Haggerty & Gorelick, 1995):

$$\phi^*(\lambda) = \frac{1}{\sqrt{\lambda \tau_m}} \tanh \left( \sqrt{\lambda \tau_m} \right).$$

(39)

The Laplace transform is defined in (Abramowitz & Stegun, 1972). Henceforth, Laplace transformed quantities are marked by an asterisk and the Laplace variable is denoted by $\lambda$.

### 3.1.2. Multirate Mass Transfer

Consider now matrix block permeabilities that vary within the representative elementary volume, while the characteristic matrix sizes are approximately constant. This means that the characteristic matrix time scale $\tau_m$ given by Equation 37 is distributed and determined by the matrix permeability. Its statistical properties at the REV scale are representative of the whole medium. It is characterized by the unimodal distribution $\psi(t)$ of $\tau_m$. Thus, the aggregate memory function $\phi(t)$ is given by the weighted sum of the memory functions for the individual matrix blocks and reads as:

$$\phi(t) = \int_0^\infty d\tau \psi(\tau) \phi(t / \tau) / \tau.$$ 

(40)

This memory function accounts for mass exchange over multiple times scales $\tau_m$, or equivalently rates. The information about heterogeneity on the scale of the representative elementary volume is fully contained in the distribution $\psi(t)$ of characteristic time scales. Note that our notion of multirate mass transfer refers to multiple diffusion rates or times $\tau_m$, while other authors refer to multirate mass transfer in the context of multiple first-order mass transfer rates (Haggerty & Gorelick, 1995).

### 3.2. Heterogeneous Consolidation—Scales and Scalings

We derive scalings for a heterogeneous consolidation scenario, characterized by a no-flow boundary at $x = 0$, fixed fracture pressure $\bar{p}_f = 0$ at $x = \ell_f$ and zero initial fracture pressure $\bar{p}_f = 0$ at $t = 0$ (Figure 2).
Loading of the medium gives rise to an initial pressure \( p_{m,0} \) at \( t = 0 \). We focus on the outflow at \( x = \ell_f \) and the displacement within the matrix.

To analyze this scenario, we consider Equation 34 averaged over the dimensions perpendicular to the mean flux in \( x \)-direction, which gives:

\[
\frac{\partial \bar{p}_f}{\partial t} - D_f \frac{\partial^2 \bar{p}_f}{\partial x^2} = \kappa \bar{p}(t) - \kappa \frac{\partial}{\partial t} \int_0^t \bar{p}(t-t') \bar{p}_f(x,t') \, dt'.
\]  

(41)

We define the diffusivity \( D_f \) and storage ratio \( \kappa \) as:

\[
D_f = \frac{k_f S_f}{\mu_f}, \quad \kappa = \frac{S_m S_f}{\chi_f S_f}.
\]  

(42)

Note that \( \kappa \) quantifies the ratio of the storage capacities of fracture and matrix, which is much larger than 1. The outflow \( j \) is defined by:

\[
\frac{d}{d_x} \left| \frac{k_f}{\mu_f} \frac{\partial \bar{p}_f}{\partial x} \right|_{x=\ell_f},
\]  

(43)

where \( d_f \) is the fracture aperture. The characteristic time scale for draining the fracture domain is:

\[
\tau_f = \frac{\ell_f S_f}{k_f}.
\]  

(44)

The total displacement \( v \) in the matrix is obtained from volume conservation as:

\[
v = v_0 - \frac{1}{w_0} \int_0^t \int_0^w j(t') \, dt'.
\]  

(45)

where \( v_0 \) is the initial displacement corresponding to the initial pressure \( p_{m,0} \), and \( w \) is the width of the matrix. Analytical Laplace space solutions for this scenario are given in Appendix B. Appendix C gives the analytical solutions for the corresponding classical dual porosity model. In the following, we discuss the expected scalings for single rate and multirate nonequilibrium scenarios.
3.2.1. Single Rate Mass Transfer

We analyze the predicted scaling behaviors for the outflow in the heterogeneous consolidation problem. To this end, we consider different time regimes defined by $\tau_f$, the characteristic fracture time scale and $\tau_m$, the characteristic matrix time scale, and the volume ratio $\kappa$. The scale $\tau_m$ denotes the time needed to drain the matrix into a fracture at 0 pressure, the scale $\tau_f$ is the time required to drain an isolated fracture. In order to clarify the impact of $\kappa$ on the time evolution, we first consider the situation that $t \gg \tau_m$. At these times, the matrix has discharged into the fracture and fluid may reenter in and drain from the matrix repeatedly before completely draining the fracture. This causes a retardation effect on the drainage of the fracture, which can be seen as follows. For $t \gg \tau_m$, the memory function can be approximated by an exponential decay at large times. In the first scenario, an early time regime exists in which the outflow is limited by both drainage through the fracture and fluid may reenter in and drain from the matrix repeatedly before completely draining the fracture. Thus, we expect that the flux decreases slower than $t^{-1/2}$. In fact, in Appendix B, the scale $\tau_m$ is the time that the outflow starts decreasing from its initial value.

We now consider the situation that $\tau_f \gg \tau_m$. For times $\tau_m \ll \tau_f$, the outflow from the fracture domain is due to fast mass transfer between fracture and matrix and flow along the fracture. It scales as $j \sim t^{1/2}$. For times $\tau_f \gg \tau_m$, the outflow decreases exponentially fast, see also Appendix D. At early times $\tau_m / \kappa^2 \ll t \ll \tau_m$, the outflow is limited both by the discharge from the matrix into the fracture and the fracture into the outlet. Thus, we expect that the flux decreases slower than $t^{-1/2}$. In fact, in Appendix D, we find that $j \sim t^{-1/4}$.

We consider the situation that the matrix drains slower than the fracture, this means $\tau_f \ll \tau_m$. For times $\tau_f \ll \tau_m$, the response of the fracture to a pressure change can be considered instantaneous and the pressure in the fracture can be set equal to 0, the pressure at the outflow boundary. Drainage from the matrix into the fracture is the process that limits the outflow. Thus, the flux through the outlet is given by $j = \hat{P}_{mf} / \kappa^2$. Recall that the memory function is equal to the drainage flux from the matrix at 0 fracture pressure. This means, the outflow scales as $j \sim t^{-1/2}$ for $\tau_f \ll t \ll \tau_m$, and decreases exponentially fast for times $t > \tau_m$. If $\tau_m / \kappa^2 \ll \tau_f$, an early time regime exists in which the outflow is limited by both drainage from matrix and fracture. As for the previous scenario, this leads to the scaling $j \sim t^{-1/4}$.

In both scenarios, it is possible to observe a short time regime with the scaling $j \sim t^{-1/2}$. One generally observes an intermediate regime with the scaling $j \sim t^{1/2}$ and an exponential decay at large times. In the first scenario, for which $\tau_m \ll \tau_f$, the intermediate and longtime regimes are determined by fracture drainage, in the second scenario by drainage of the matrix. The short time regimes are conditioned by both drainage through the fracture and the matrix. The full derivations for these scalings are given in Appendix D1.

3.2.2. Multi-Rate Mass Transfer

The memory function for a distribution of permeabilities between matrix blocks is given by Equation 40. We can rewrite this expression by performing the variable transform $s = t / \tau$. This gives:

$$\phi(t) = \int_0^\infty ds \phi(t / s) \phi(s) / s.$$  (47)

We consider the case of a broad distribution of characteristic time scales, which is quantified here by a power-law distribution of $\psi(t) \sim (t / \tau_0)^{-1-a}$ for $t \gg \tau_0$ and $0 < \alpha < 1$. Thus, we obtain for the memory function for $t \gg \tau_0$:

$$\phi(t) \sim (t / \tau_0)^{-1-a} \int_0^\infty ds s^a \phi(s).$$  (48)
where we changed variables according to \( r \to s = r^{-1} \). The integral on the right side is finite, which means that \( \varphi \sim (t / r_0)^{-1/a} \).

At early times, the outflow is dominated by the fast draining matrix blocks and drainage through the fracture. The matrix blocks that drain faster than the characteristic time \( \tau_0 \) lead to a retardation of the outflow through fracture, which behaves as \( j \sim t^{-1/2} \) for times \( \tau_0 \ll t \ll \tilde{\tau}_f \). For times \( t \gg \tilde{\tau}_f \), drainage through the fracture can be considered instantaneous and the outflow is limited by the slowly draining matrix blocks. Thus, it is given by \( j \sim \varphi(t) \), which scales as \( j \sim t^{-1/a} \). The derivations of these scalings and the scalings for different distributions of matrix time scales are given in Appendix D2.

4. Heterogeneous Consolidation of Fractured Media

In this section, we numerically study consolidation in fractured media due to fracture-matrix interaction in systems of different complexity, and compare the behaviors of outflow and displacement with the predictions of the multi-porosity model presented in the previous section. We first consider a single fracture matrix system in order to illustrate the fundamental mechanisms discussed in the previous section. Then, we consider consolidation in a discrete fracture network embedded in a matrix with heterogeneous mechanical and hydraulic properties.

4.1. Single Fracture-Matrix System

As a first example to illustrate the multirate nature of fracture-matrix interactions, we study the coupled flow and deformation of a rock mass composed of a single fracture separating two homogeneous matrix blocks (Figure 2). We consider a domain containing a solid poroelastic medium and a vertical fracture that splits the domain into two halves. The fracture length is \( \ell_f = 10^3 \) m, its effective aperture is \( d_f = 10^{-1} \) m. We consider two geometries given by the matrix widths of (G1) \( \ell_m = 100 \) m and (G2) 5 m. We impede horizontal displacements at the right and left boundaries, and vertical displacements at the lower boundary. We impose no-flow conditions at all boundaries except for the fracture outlet at the top of the domain, which is allowed to drain by specifying a reference pressure \( p_f = 0 \). At time \( t = 0 \) we apply a vertical uniform load of \( \sigma_v = 10 \) MPa at the top boundary. The direct model is solved numerically using COMSOL. The load is maintained throughout the simulation until the overpressure is completely drained and consolidation reaches steady-state conditions. This setup constitutes a uniaxial strain scenario because the porous matrix is restricted in the lateral directions.

The porosity, density, Young modulus, and Poisson ratio of the rock matrix are the same for fracture and matrix and given by \( \phi = 0.1 \), \( \rho = 2700 \) kg/m³, \( E = 10 \) GPa, and \( \nu = 0.25 \), respectively. The fluid density, dynamic viscosity, and compressibility are \( \rho_f = 1000 \) kg/m³, \( \mu_f = 10^{-3} \) Pas, \( c_f = 4 \cdot 10^{-10} \) Pa⁻¹, respectively. The matrix permeability is \( k_m = 10^{-16} \) m². The solid phase is incompressible, this means \( c_s = 0 \), and the Biot coefficient for poroelastic coupling is \( b = 1 \), which leads to a storativity \( \beta = \phi \cdot c_f = 4 \cdot 10^{-11} \) Pa⁻¹. For each geometry (wide, G1, and narrow, G2, matrix blocks), we consider two scenarios of fracture permeability defined by the values \( (S1) k_f = 10^{-8} \) m² and \( (S2) 10^{-12} \) m². The model parameters are summarized in Table 1. We focus on the evolution of integrated drainage flux (unit thickness) and average matrix displacement at the top boundary. We compare the direct numerical simulation of the discrete fracture-matrix system to the predictions of the DPP model summarized in Section 2.3, and the multi-porosity model derived in Section 3. The direct numerical simulations take about 3 min using two Intel i7 processors with 3.6 Gz clock speed.

The prediction of the DPP model is obtained by numerically solving Equations 10–13. The transfer coefficient \( \gamma \) of the DPP model, given by Equation 14, is a fitting parameter which here is set to \( s = 2 \).

The predictions of the nonequilibrium model are based on numerical Laplace inversion of the explicitly analytical solution given in Appendix B, which can be obtained in less than a second. As the considered setup is a uniaxial strain scenario, the Skempton coefficient \( B_s \) which relates initial pore pressure in the matrix and the initial load is given by Equation 31. For this setup, the effective storage parameters \( S = S_m = S_f \).

It results from the linear relation (Equation 22) between pressure and deformation, and is obtained from expression (Equation 26). The initial matrix pressure \( p_{m,0} \) is given by Equation 30.
Figures 3a and 3b show the drainage flux and vertical displacement during consolidation for the geometry G1 and the permeability scenarios S1 and S2. The characteristic time scales are $\tau_m = 142$ d, and (S1) $\tau_f = 2.8 \cdot 10^{-1}$ d and (S2) $\tau_f = 2.8 \cdot 10^{3}$ d. For S1, $\tau_m \gg \tau_f$, this means that the fracture discharges much faster than the matrix, that is, drainage is limited by matrix pressure diffusion. The drainage flux is essentially equal to the outflow from the matrix, which is determined by the memory function $\phi(t)$, as predicted in Section 3.2.1. Hence, the outflow decays as $t^{-1/2}$ until time $\tau_m$. For $t \gg \tau_m$ it decays exponentially fast. For times smaller than $\tau_m / \kappa^2$ (not shown), the outflow is constant because the matrix pressure remains at its initial value. For scenario S2, the role of time scales is reversed, $\tau_m \ll \tau_f$, which implies that fracture-matrix mass transfer is faster than the discharge along the fracture. At times $\tau_m / \kappa^2 \ll t \ll \tau_f$, the outflow is limited both by the matrix and fracture flow processes, and it scales as $t^{-1/4}$. For times $t > \tau_f$ we observe a crossover behavior that scales approximately as $t^{-1/2}$, which is characteristic of the drainage of the fracture. For times $t > \tau_f$ the flux is cut off exponentially fast because both fracture and matrix are drained. The behavior of the vertical deformation is analogous in both scenarios. The nonequilibrium model predicts the full evolution of drainage flux and vertical displacement and fully explains the observed scalings, as discussed also in Section 3.2.1. The DPP model correctly predicts the cutoff behavior but is unable to describe the flux behavior at times $t < \tau_m$, which is dominated by pressure redistribution within the matrix. In fact, as

| Parameter | Value | Units | Description |
|-----------|-------|-------|-------------|
| $E$       | 10    | GPa   | Young modulus |
| $\nu$     | 0.25  | –     | Poisson ratio |
| $b_f = b_m$ | 1    | –     | Biot coefficient |
| $\rho'$   | 2700  | kg/m$^3$ | rock density |
| $\phi_f' = \phi_m$ | 0.1  | –     | porosity |
| $c_s$     | 0     | Pa$^{-1}$ | solid compressibility |
| $k_m$     | $10^{-16}$ | m$^2$ | matrix permeability |
| $k_f$     | $(S1)10^{-8}$ and $(S2)10^{-12}$ | m$^2$ | fracture permeability |
| $\rho_f$  | 1000  | kg/m$^3$ | fluid density |
| $\mu_f$   | 0.001 | Pas   | fluid viscosity |
| $c_f$     | $4 \cdot 10^{-10}$ | Pa$^{-1}$ | fluid compressibility |
| $\sigma_v$| 10    | MPa   | vertical stress |
| $d_f$     | 0.1   | m     | fracture aperture |
| $\ell_m$  | (G1) 100 and (G2) 5 | m | horizontal block size |
| $\ell_f$  | 1000  | m     | vertical block size |
| $s$       | 2     | –     | transfer coefficient fitting parameter [Eq. 14] |
| $S$       | $1.23 \cdot 10^{-10}$ | Pa$^{-1}$ | effective storage coefficient [Eq. 26] |
| $\kappa$  | (G1) $2 \cdot 10^3$ and (G2) $10^2$ | – | volume ratio [Eq. 42] |
| $P_{m,0}$ | $6.8 \cdot 10^6$ | Pa | initial matrix pressure [Eq. 30] |
| $\tau_m$  | (G1) 142 and (G2) 0.4 | d | matrix time scale [Eq. 37] |
| $\tau_f$  | $(S1)1.42 \cdot 10^{-4}$ and (S2) 1.42 | d | fracture time scale [Eq. 44] |

Note. The first section details the mechanical parameters, the second the medium characteristics, the third the fluid characteristics, the fourth the vertical stress at the boundary. The fifth details the geometry of the fracture matrix system and the sixth section the relevant parameters of the dual- and multi-porosity models.
outlined above, the DPP model assumes that the pressure in the matrix is equilibrated instantaneously, and therefore uniform inside the matrix blocks. The pressure and thus the outflow predicted by the DPP model are approximately constant up to the characteristic matrix time scale $\tau_m$, and then the matrix pressure drops exponentially (see also Appendix A).

Figures 3c and 3d show the drainage flux and vertical deformation during consolidation for the geometry G2 and the permeability scenarios S1 and S2. The characteristic time scales are $\tau_m = 0.4 \text{ d}$, and (S1) $\tilde{\tau}_f = 1.42 \cdot 10^{-3} \text{ d}$ and (S2) $\tilde{\tau}_f = 1.42 \cdot 10^2 \text{ d}$. For S1, we have the separation of time scale $\tau_m \gg \tilde{\tau}_f$. The fracture discharges faster than the matrix and the outflow is determined by discharge from the matrix into the fracture. We observe the characteristic $t^{-1/2}$-scaling and cutoff at time $\tau_m$. Again similar to G1, in scenario S2, the order of time scales is reversed, $\tau_m \ll \tilde{\tau}_f$. For times $t \ll \tau_m$, the outflow is limited by both matrix and fracture flows and we observe the slow $t^{-1/4}$-scaling. In the time regime $\tau_m \ll t \ll \tilde{\tau}_f$, diffusion along the
fracture limits the outflow, and we identify the characteristic $t^{-1/2}$-scaling, which becomes an exponential cutoff for $t > \hat{\tau}$. The vertical displacement behaves analogously.

The behavior of drainage fluxes and vertical displacements and their scalings are fully predicted by the nonequilibrium model discussed in Section 3.2.1. Their analytical solutions are given in Appendix B. As for geometry G1, the DPP model predicts correctly the cutoff behavior for both the scenarios. In scenario S1, the matrix dominates the outflow. The DPP model, which assumes instantaneous pressure equilibration in the matrix is not able to quantify the outflow behavior for times $t < \tau_m$. Also, for scenario S2 and times $t < \tau_m$, the outflow behavior in the DPP model is approximately constant and not able to model the observed $t^{-1/4}$-decay. For $\tau_m \ll t \ll \hat{\tau}$, the matrix has fully discharged and the outflow behavior is dominated by diffusion along the fracture, which is correctly predicted by the DPP model. Thus, the DPP model, if parameterized by the correct transfer coefficient $s$, is able to predict the outflow behavior only in situations and at times for which the matrix processes can be considered at equilibrium, this means for times larger than the characteristic matrix time scale $\tau_m$.

4.2. Consolidation of a Synthetic Discrete Fracture Network Embedded in a Heterogeneous Matrix

We consider a $200 \times 200$ m$^2$ square domain of a solid poroelastic medium, containing a fracture network of orthogonal, equally-spaced fractures that split the medium into $400$ matrix block of $10 \times 10$ m$^2$ (Figure 4). As boundary conditions, we impede $x$-displacements along the left and right boundaries, and $y$-displacements along the bottom boundary. We consider a consolidation problem, where the matrix is fully saturated with fluid, and at $t = 0$ a vertical load $\sigma_v = 10$ MPa is applied along the top boundary. The fluid is only allowed to drain through the fracture outlets on the top boundary. We therefore impose a constant reference pressure, $p = 0$, along the upper seepage boundary, and no-flow conditions on the bottom and lateral boundaries.

The mechanical and fluid properties are shown in Table 2. The medium characteristics are also those of Table 2, except for the permeability values $k_m$ in the matrix, which are randomly distributed, and the permeability of the fracture, which here is set to $k_f = 10^{-8}$ m$^2$. The effective fracture aperture is $d_f = 10^{-3}$ m. All

Figure 4. Consolidation of a heterogeneous matrix–fracture network model. Model geometry and problem setup. The domain is a 200 m side square with orthogonal fractures that divide the medium in 400 matrix block of $10 \times 10$ m$^2$. As the fractures are much more permeable than the rock matrix, when the load is applied the matrix sub-domains drain to the fractures. Flow through the fracture network drains toward the top boundary of the model.
The permeability $k_m$ of each square matrix block is drawn randomly from the Gamma distribution:

$$p_k(k) = \frac{1}{\Gamma(\alpha) k_0} \left( \frac{k}{k_0} \right)^{\alpha - 1} \exp\left(-\frac{k}{k_0}\right).$$

(49)

The characteristic permeability is set to $k_0 = 10^{-14}$ m$^2$. Figure 5 shows a representative fracture network for $\alpha = 0.1$, together with the empirical and theoretical permeability distributions for the exponents $\alpha = 0.1, 0.3$ and 0.6. The probability of low permeability values increases as the exponent $\alpha$ decreases. As a consequence, for the same medium size we have better statistics of small permeabilities as the exponent decreases.

The direct problem is solved numerically using COMSOL. We simulate $10^4$ days of consolidation for 5 different realizations of the matrix block permeabilities, characterized by $\alpha = 0.1, 0.3$ and 0.6 in the Gamma distribution (Equation 49). We consider the outflow in single realizations, as well as the average flux over five realizations. We verified that the average does not change significantly with an increasing number of realizations, which is discussed in more detail at the end of this section. For comparison, we consider also a scenario with a single matrix permeability of $k_m = k_0 = 10^{-14}$ m$^2$, which is assigned to all matrix blocks. Note that $k_0$ is the upper cut-off value of the permeability distribution (Equation 49). The computation time for each scenario is about 30 min using two Intel i7 processors with 3.6 Ghz clock speed.

The expected scaling behaviors are obtained following the analysis presented in Section 3.2.2, which requires determining the scaling behavior of the memory function $\varphi(t)$. To this end, we recall that $\varphi(t)$ is given by Equation 40 in terms of the distribution $\psi(t)$ of characteristic matrix time scales. The Gamma distribution (Equation 49) of matrix permeabilities implies that the time scales $\tau_m = \mu_f S_m d_m^2 / k$ are distributed according to the inverse Gamma distribution

$$\psi(t) = \frac{1}{\Gamma(\alpha) \tau_0} \left( \frac{t}{\tau_0} \right)^{-1-\alpha} \exp(-t_0 / t),$$

(50)

of the parameters remain constant for all the domain, except for the matrix permeability, which is randomly assigned to each matrix block, as follows.

| Parameter | Value | Units | Description |
|-----------|-------|-------|-------------|
| $E$       | 10    | GPa   | Young modulus |
| $\nu$     | 0.25  | –     | Poisson ratio |
| $b_f = b_m$ | 1     | –     | Biot coefficient |
| $\rho$    | 2700  | kg/m$^3$ | rock density |
| $\phi_f = \phi_m$ | 0.1 | – | porosity |
| $c_s$     | 0     | Pa$^{-1}$ | solid compressibility |
| $k_m$ distribution | $10^{-8}$ | m$^2$ | matrix permeability |
| $k_f$    | $10^4$ | m$^2$ | fracture permeability |
| $\rho_f$  | $10^4$ | kg/m$^3$ | fluid density |
| $\mu_f$   | $0.001$ | Pa$s$ | fluid viscosity |
| $c_f$     | $4.10^{-10}$ | Pa$^{-1}$ | fluid compressibility |
| $\sigma_v$ | 10 | MPa | vertical stress |
| $d_f$     | 0.001 | m | fracture aperture |

Note. The first section details the mechanical parameters, the second the medium characteristics, the third the fluid characteristics, the fourth the vertical stress at the boundary. The fifth section details the geometry of the fracture matrix system.
where $\tau_0 = \mu_J S_m d^2 / k_0$. This means it behaves as $\psi(t) \sim t^{-1-\alpha}$ for $t \gg \tau_0$. According to Equation 48 the memory function scales accordingly as $\phi(t) \sim (t / \tau_0)^{-1-\alpha}$. The expected evolution of the drainage flux is discussed in Section 3.2.2. Note that we do not solve the full multirate model, but compare the numerical data to the expected analytical scaling behaviors.

Figure 6a shows the evolution of the average drainage fluxes at the top boundary for each network. We find the scalings predicted in Section 3.2.2. At short times, the drainage flux is dominated by the matrix blocks with relatively large permeability values, which equilibrate relatively fast, and fluid transport through the fractures. This yields the characteristic $t^{-1/2}$-scaling. At longer times, pressure diffusion from matrix blocks with lower permeability start to control the outflow time scales, which leads to the predicted power-law scaling as $t^{-1-\alpha}$. The scenario characterized by a single matrix permeability shows the behaviors discussed in Section 3.2.1. At short times, the drainage flux decays as $t^{-1/2}$. For times larger than the characteristic matrix time scale $\tau_m = 1.42 \times 10^{-2}$ d, it decays exponentially fast. This comparison shows that the characteristic $t^{-1-\alpha}$-scaling observed in the heterogeneous scenarios is due to matrix blocks whose permeabilities are smaller than $k_0$, and emphasizes the impact of matrix blocks with low permeability on the tailing behavior. Note that here we consider a regular fracture network aligned with the direction of the applied load. Ashworth and Doster (2020) showed that the poroelastic response in fractures and matrix differ depending on the fracture orientation. While the orientation is expected to have an impact also for the disorder scenario under consideration here, we conjecture that the expected late time scaling will persist because it is caused by the broad distribution of matrix permeabilities.

We now consider the fluctuations around the ensemble mean behavior shown in Figure 6a. Figures 6b–6d show the drainage fluxes in single realizations for different exponents $\alpha$ in the distribution (Equation 49) of permeabilities. Note that the probability of encountering a value smaller than $k_0$ increases with decreasing exponent $\alpha$. The outflows for single realizations are compared with their respective averages. The observed fluctuations about the average are due to the finite size of the flow domain. Due to the finite domain size, only a finite sample of permeability values can be assigned to the matrix blocks. Thus, a single realization is not necessarily representative of the full spectrum of permeability values, and differences between realizations give rise to fluctuations in the outflow behavior. Furthermore, as we found above, the long-time power-law behavior reflects the low-end behavior of the permeability distribution. For small $\alpha$, the probability to sample a low permeability value is higher than for a large value of $\alpha$. Thus, for the same domain size, the low end of the permeability distribution is better represented for a small than for a large value of $\alpha$. For this reason, the fluctuations that we observe in Figures 6b–6d increase with increasing $\alpha$. In general, we...
observe relatively small fluctuations about the mean for the chosen domain size, and an average over only five realizations provides a stable estimate of the predicted power-law tail.

5. Conclusions

We presented analytical and numerical results of nonequilibrium effects in coupled single-phase flow and deformation in fractured media. Nonequilibrium effects, due to the relaxation time needed for pore pressure to equilibrate diffusively inside matrix blocks, are not captured by the classical DPP models formulations, which assume local equilibrium inside the matrix blocks. Using hydro-mechanical numerical simulations of flow and rock deformation in fractured media, where matrix blocks are discretized as poroelastic continua and fractures are represented explicitly as lower-dimensional fast flow conduits, we show that local equilibrium is not satisfied in general in heterogeneous fractured systems. If the block sizes are large enough, or the contrast between fracture and matrix conductivities is sufficiently large, the time scales for fluid drainage to the highly conductive fracture network prevent the matrix blocks from reaching internal pressure equilibrium. Since pore pressure diffusion controls poroelastic deformations, heterogeneity in hydraulic properties...
leads to tailing in mechanical displacements and subsidence due to pressure depletion, with scalings that are not predicted by the classical equilibrium upscaling theories. There is a wide range of time scales for flow, due to permeability heterogeneity, which leads to a wide range of time scales for deformation. While this nonequilibrium effect has been proposed as a mechanism to explain anomalous time-dependent subsidence in mature gas fields, it has been seldom studied in numerical or analytical detail.

The proposed nonequilibrium multiporosity theory was compared with numerical simulations in scenarios of consolidation in simple matrix-fracture synthetic configurations. The agreement between analytical predictions and numerical simulations is remarkable. The fluid fluxes and displacements predicted by the classical dual-porosity theory, however, deviate significantly in cases where the hypothesis of local pressure equilibrium breaks down. This lack of pressure equilibrium has been demonstrated in the simple case of a single fracture dividing a rock matrix into two blocks. For a narrow system, the aggregate poroelastic behavior can be reasonably approximated by the classical DPP theory, while nonequilibrium effects dominate the flow and deformation behavior as the width of the matrix blocks increases.

The proposed dual-porosity approach assumes that the fracture domain is a well-connected network, so that flow can occur entirely through the fracture network. The existence of a connected fracture network with a distribution of characteristic block sizes is essential to define meaningful time scales for the matrix-fracture interactions. The degree of connectivity is usually very low in some applications, such as enhanced geothermal systems (EGS) and other natural reservoirs. The proposed model is not directly applicable to those scenarios with poor connectivity of the fractures. The assumption of null-stiffness fractures may also be a limitation of the proposed effective approach. Considering fracture stiffness and its specific mechanical response may be important to introduce a pressure-dependent fracture permeability, which based of laboratory tests would vary at most one order of magnitude due to effective stress changes. The proposed methodology could in principle be extended to dual-porosity formulations that incorporate microscale mechanical constraints and compatibility conditions between the fracture and matrix domains.

Our results reveal the limitations of classical DPP formulations, which provides good estimates for the outflow at times that are larger than the characteristic matrix time scales, but is not able to capture nonequilibrium effects at shorter times. We obtain the scalings to be expected for coupled flow and deformation in highly-heterogeneous, fractured media, and characterize the evolution of fluid fluxes and displacements may help interpret the flux/production data in connection with tailing and anomalous patterns of long-term land subsidence in mature reservoirs and aquifers with significant fluid drawdowns. The proposed model opens a line of research toward effective models of poroelasticity that capture the interplay between flow and mechanical deformation in highly heterogeneous and fractured porous media.

Appendix A: Quasi-Equilibrium of Pressure in the Immobile Domain

Following Barenblatt et al. (1960), the interfacial flux (Equation 18c) is approximated by the Darcy-type flux

$$Q_{i} = \gamma (\overline{p}_f - \overline{p}_m),$$

where $\gamma$ is the matrix-fracture mass transfer coefficient discussed in Section 2.3. Thus, we can write the system of Equation 18 in terms of the bulk pressures as

$$\chi_f \overline{\beta}_f \frac{\overline{\varepsilon}_f}{\overline{c}_t} + \chi_f \overline{\beta}_f \frac{\overline{\varepsilon}_f}{\overline{c}_t} - \nabla \cdot \left( \frac{\chi_f k_f}{\mu_f} \nabla \overline{p}_f \right) = -\gamma (\overline{p}_f - \overline{p}_m),$$

$$\chi_m \overline{\beta}_m \overline{p}_m + \chi_m b_m \frac{\overline{\varepsilon}_m}{\overline{c}_t} = \gamma (\overline{p}_f - \overline{p}_m),$$

where $\gamma = \kappa / \mu V$, and $\chi_i = V_i / V$, is the volume fraction of phase $i = f, m$. This formulation is equivalent to the classical dual porosity approach discussed above, in the absence of fluid flow in the matrix. This means, the matrix acts as a deformable storage region. It emphasizes that the hydraulic and mechanical parameters in the classical dual porosity model (Equation 12) are effective quantities that are weighted by the volume fractions of the respective phase, while the pressures and deformations are phase averages.
Appendix B: Analytical Solutions for the Non-Equilibrium Dual Porosity Model

In order to solve Equation 41, we consider its Laplace transform, which reads as

$$\lambda \hat{p}_f^* - D_f \frac{\partial^2 \hat{p}_f^*}{\partial x^2} = \kappa \varphi^* p_{m,0} - \kappa \varphi^* \hat{p}_f^*.$$  \hspace{1cm} (B1)

We write this equation as

$$\Lambda \hat{p}_f^* - D_f \frac{\partial^2 \hat{p}_f^*}{\partial x^2} = \kappa \varphi^* p_{m,0}.$$  \hspace{1cm} (B2)

with the auxiliary function

$$\Lambda = \hat{\lambda} \left( 1 + \kappa \varphi^* \right).$$  \hspace{1cm} (B3)

Thus, we obtain the explicit Laplace space solution

$$p_f^* = \frac{\kappa p_m \varphi^* \Lambda}{\Lambda^2 - \frac{\cosh(\Lambda \tau_f x/L)}{\cosh(\Lambda \tau_f)}},$$  \hspace{1cm} (B4)

Inserting (Equation B4) into the Laplace transform of expression (Equation 43) gives for the outflow

$$j^* = j_0 \tau^*_f \varphi^* \frac{\tanh(\Lambda \tau_f)}{\sqrt{\Lambda \tau_f}},$$  \hspace{1cm} (B5)

where we defined the characteristic flux

$$j_0 = \frac{d_j k_f \kappa}{\mu_f \ell_f}.$$  \hspace{1cm} (B6)

The initial flux is obtained from the initial value theorem of the Laplace transform as

$$j(t = 0) = \lim_{\lambda \to 0} \hat{j}_f^* (\hat{\lambda}).$$  \hspace{1cm} (B7)

We find

$$j(t = 0) = j_0 \sqrt{\tau_f / \tau_m} = \frac{2 \sqrt{k_m k_f p_{m,0}}}{\mu_f}.$$  \hspace{1cm} (B8)

Appendix C: Analytical Solution for the Classical Dual Porosity Model

Under the conditions of uniaxial strain and negligible mass transfer in the direction of the principal pressure gradient, the governing Equation 12 for the classical dual porosity model can be written as

$$\chi_m \frac{\partial p_m}{\partial t} = -\gamma (p_m - p_f).$$  \hspace{1cm} (C1a)

$$\chi_f \frac{\partial p_f}{\partial t} - \nabla \cdot \left( \frac{\chi_f k_f}{\mu_f} \nabla p_f \right) = \gamma (p_m - p_f).$$  \hspace{1cm} (C1b)

The pressure $p_m$ in the matrix is obtained by using separation of variables in order to solve the ordinary differential Equation C1a,

$$p_m(x, t) = p_{m,0} \exp(k_m t) + \int_0^t k_m \exp(-k_m u) p_f(x, u) du.$$  \hspace{1cm} (C2)

where the exchange rate is given by $k_m = \gamma / \chi_m S_m$. Thus, we can combine the system (Equation C1) in the single equation
\[
\frac{\partial p_f}{\partial t} - \nabla \cdot \left( D_f \cdot \nabla p_f \right) = \kappa p_m \phi_k \exp(-k_m t) - \kappa \int_0^t dt' k_m \exp[-k_m(t-t')]p_f(x,t'),
\]  
(C3)

Note that this equation can be considered a special case of the multirate nonequilibrium model (Equation 34) for the choice \( \phi(t) = k_m \exp(-k_m t) \). Averaging over the dimensions perpendicular to the main flux gives

\[
\frac{\partial p_f}{\partial t} - D_f \frac{\partial^2 p_f}{\partial x^2} = \kappa p_m \phi_k \exp(-k_m t) - \kappa \int_0^t dt' k_m \exp[-k_m(t-t')]p_f(x,t').
\]  
(C4a)

For the boundary conditions of the consolidation scenario of Section 3.2, we obtain the analytical Laplace space solutions

\[
p_f^* = \kappa p_m \phi \frac{\Lambda}{\Lambda - 1} \left[ 1 - \frac{\cosh(\sqrt{\Lambda \tau_f} x / L)}{\cosh(\sqrt{\Lambda \tau_f})} \right].
\]  
(C5)

\[
\tau = j_0 \tau_f \phi \frac{\tanh(\sqrt{\Lambda \tau_f} x / L)}{\sqrt{\Lambda \tau_f}}.
\]  
(C6)

for the pressure distribution in the fracture and the outflow, respectively, where \( \phi^*(\lambda) \) and \( \Lambda \) is given explicitly by

\[
\phi^*(\lambda) = \frac{1}{1 + \lambda k_m}, \quad \Lambda = \lambda \left( 1 + \frac{\kappa}{1 + k_m \lambda} \right).
\]  
(C7)

Note that \( j_0 \) is defined by Equation B6 and \( \tau_f = L^2 / D_f \).

For \( \tau_f \ll k_m^{-1} \), this means if the fracture domain drains much faster than the matrix domain, we obtain for \( t \gg \tau_f \) that

\[
j(t) = j_0 k_m \tau_f \exp(-k_m t).
\]  
(C8)

For \( \tau_f \gg \tau_m \), this means if the matrix domain drains much faster than the fracture domain, we obtain for \( \tau_m \ll t \ll \tau_f (1 + \kappa) = \tilde{\tau}_f \) the behavior

\[
j(t) \sim t^{-1/2}
\]  
(C9)

and for \( t > \tilde{\tau}_f \) an exponential decrease to 0.

**Appendix D: Scalings**

In the case \( \Lambda \tau_f \gg 1 \), which corresponds to times that are small compared to the characteristic time \( \tau_f \) to drain the fracture, we can approximate the outflow by

\[
j = j_0 \tau_f \phi^*(\Lambda \tau_f)^{-1/2}.
\]  
(D1)

For \( \Lambda \tau_f \ll 1 \), which corresponds to times that are large compared to the fracture drainage time \( \tau_f \), outflow can be approximated by

\[
j = j_0 \tau_f \phi^*.
\]  
(D2)

In the following, we consider these cases and the corresponding time behaviors for the single and multirate mass transfer scenarios. All scenarios, we consider in the following are characterized by storage ratios \( \kappa \gg 1 \) much larger than 1.
D1. Single Rate Mass Transfer

We first consider $t \ll \tau_m$, this means times that are smaller than the characteristic time to drain the matrix. This time regime corresponds in Laplace space to $\tau_m \gg 1$. In this case, the memory function behaves as

$$\phi^* = 1 / \sqrt{\lambda \tau_m} \ll 1.$$  \hspace{1cm} \text{(D3)}

If additionally $\lambda / \sqrt{\lambda \tau_m} \gg 1$, which implies that $t \gg \tau_m / \kappa^2$, we can set

$$\Lambda \tau_f = \kappa (\tau_f / \tau_m) \sqrt{\lambda \tau_m}.$$  \hspace{1cm} \text{(D4)}

For $\Lambda \tau_f \gg 1$, which here implies $t \ll \tau_f$, we obtain for the outflow from Equation D1

$$j^* = j_0 (\tau_f / \kappa)^{1/2} (\lambda \tau_m)^{-3/4}.$$  \hspace{1cm} \text{(D5)}

Inverse Laplace transform gives the behavior

$$j = j_0 \sqrt{\tau_f / \tau_m} (t / \tau_m)^{-1/4} / \Gamma(3 / 4).$$  \hspace{1cm} \text{(D6)}

This outflow behavior can be observed in the time regime

$$\tau_m \ll t \ll \min\{\tau_m, \tau_f\}.$$  \hspace{1cm} \text{(D7)}

For $\Lambda \tau_f \ll 1$, which implies $t \gg \tau_f (1 + \kappa)$, we obtain for the outflow from Equation D2

$$j^* = j_0 \tau_f / \sqrt{\lambda \tau_m}.$$  \hspace{1cm} \text{(D8)}

Inverse Laplace transform gives

$$j = j_0 \tau_f / \tau_m \Gamma(1 / 2) \left( t / \tau_m \right)^{-1/2} / \Gamma(3 / 4).$$  \hspace{1cm} \text{(D9)}

This behavior can be observed in the time regime

$$\tau_m \max\{1 / \kappa^2, \kappa^2 (\tau_f / \tau_m)^2\} \ll t \ll \tau_m.$$  \hspace{1cm} \text{(D10)}

This regime exists if $\kappa^2 (\tau_f / \tau_m)^2 \ll 1$, this means $\tau_f \ll \tau_m / \kappa$, which implies that the fracture drains much faster than the matrix.

We now consider the case $t \gg \tau_m$, this means times, for which the matrix has been drained. In this case, the memory function can be approximated by

$$\phi^* = 1.$$  \hspace{1cm} \text{(D11)}

This implies that $\Lambda \tau_f = \lambda \tau_f (1 + \kappa)$. Thus, for $\lambda \tau_f (1 + \kappa) \gg 1$, the outflow is according to Equation D1 given by

$$j^* = j_0 \lambda (1 + \kappa)^{-1/2} (\lambda \tau_f)^{-1/2}. $$ \hspace{1cm} \text{(D12)}

Inverse Laplace transform gives

$$j = j_0 \lambda (1 + \kappa)^{-1/2} \left( t / \tau_f \right)^{-1/2} / \Gamma(3 / 4).$$  \hspace{1cm} \text{(D13)}

This behavior can be observed in the time regime

$$\tau_m \ll t \ll \tau_f (1 + \kappa).$$  \hspace{1cm} \text{(D14)}

This time regime exists if $\tau_f (1 + \kappa) \gg \tau_m$, this means if the matrix drains faster than the fracture domain. For $t \gg \max\{\tau_m, (1 + \kappa) \tau_f\}$, the outflow decays to zero exponentially fast.
D2. Multirate Mass Transfer

We consider two scenarios for the distribution $\psi(t)$ of matrix time scales. The first scenario assumes that $\psi(t) \sim t^{-1-\alpha}$ with $0 < \alpha < 1$ for $t \gg \tau_0$ with $\tau_0$ a characteristic time scale. This means the drainage scales have a fractal distribution and is referred to as fractal distribution of time scales. The second scenario assumes that $\psi(t) \sim t^{\alpha-1}$ with $\alpha > 0$ for $t \ll \tau_0$, and that it decays exponentially fast for $t \gg \tau_0$. This scenario is referred to as truncated power-law distribution of time scales.

D2.1. Fractal Distribution of Time Scales

We consider the case the $\psi(t)$ behaves as

$$\psi(t) \sim t^{-1-\alpha}$$

with $0 < \alpha < 1$ and for $t \gg \tau_0$. This implies that the memory function has the Laplace transform (Dentz & Berkowitz, 2003)

$$\phi^*(\lambda) = 1 - C(\lambda \tau_0)^\alpha$$

for $\lambda \tau_0 \ll 1$. This means that we can approximate $\Lambda$ defined by Equation B3 as

$$\Lambda = \lambda (1 + \kappa) + \ldots,$$

where the dots denote subleading contribution.

For $\Lambda \tau_f = \lambda \tau_f (1 + \kappa) \gg 1$, we then obtain for the Laplace transform of the outflow according to Equation D1

$$j^* = \frac{j_0 \tau_f}{(1 + \kappa)^{1/2}} (\lambda \tau_f)^{-1/2}.$$  \hspace{1cm} (D18)

Inverse Laplace transform gives

$$j = \frac{j_0 \tau_f}{\Gamma(1/2)(1 + \kappa)^{1/2}} \left( \frac{t}{\tau_f} \right)^{-1/2}.$$  \hspace{1cm} (D19)

This behavior can be observed in the time regime $\tau_0 \ll t \ll \tau_f (1 + \kappa)$.

For $\Lambda \tau_f = \lambda \tau_f (1 + \kappa) \ll 1$, we obtain for the Laplace transform of the outflow according to Equation D1

$$j^* = j_0 \tau_f [1 - C(\lambda \tau_f)^\alpha].$$  \hspace{1cm} (D20)

Inverse Laplace transform gives

$$j = \frac{j_0 \tau_f}{\Gamma(\alpha) \tau_0} \left( \frac{t}{\tau_0} \right)^{1-\alpha}.$$  \hspace{1cm} (D21)

D2.2. Truncated Power-Law Distribution of Matrix Time Scales

We now consider the case that $\psi(t)$ behaves as the power-law

$$\psi(t) \sim (t / \tau_0)^{\alpha-1}$$  \hspace{1cm} (D22)

with $\alpha > 0$ for times smaller than a certain time scale $\tau_0$ and is cut off exponentially fast for $t \gg \tau_0$. For $t \ll \tau_0$ we thus obtain for the memory function the approximation

$$\phi(t) \sim (t / \tau_0)^{\alpha-1} \int_{t/\tau_0}^\infty ds s^{-\alpha} \phi(s).$$  \hspace{1cm} (D23)

where the lower integral limit reflects cutoff of $\psi(t)$ for $t \gg \tau_0$. The integral on the right side exists in the limit $t \to 0$ only for $\alpha < 1 / 2$ because $\phi(s) \sim s^{-1/2}$ for $s \to 0$. This means, $\phi(t) \sim (t / \tau_0)^{\alpha-1}$ for $t \ll \tau_0$ and $0 < \alpha < 1 / 2$. For $\alpha > 1 / 2$, the memory function behaves as the memory function for single rate mass transfer according to $\psi(t) \sim (t / \tau_0)^{-1/2}$. This implies that the Laplace transform of $\phi(t)$ is

$$\phi^*(\lambda) = (\lambda \tau_0)^{-\alpha}.$$  \hspace{1cm} (D24)
for \( \lambda \tau_0 \gg 1 \) and \( 0 < \alpha < 1 / 2 \). Under these conditions, we can approximate \( \Lambda = \lambda^*(1 + \kappa(\lambda \tau_0)^{-\alpha}) \). If \( \kappa(\lambda \tau_0)^{-\alpha} \gg 1 \), which implies \( \lambda \tau_0 \ll \kappa^{1/\alpha} \), we can set

\[
\Lambda \tau_f = \kappa(\tau_f / \tau_0)(\lambda \tau_0)^{1-\alpha}.
\]

(D25)

For \( \Lambda \tau_f \gg 1 \), we obtain from Equation D1 for the Laplace transform of the outflow

\[
j^* = j_0(\tau_0 \tau_f / \kappa)^{1/2}(\lambda \tau_0)^{(\alpha+1)/2}
\]

(D26)

Inverse Laplace transform gives

\[
j = j_0 \sqrt{\frac{\tau_f}{\tau_0 \kappa}} \left( \frac{t}{\tau_0} \right)^{(\alpha-1)/2} \frac{\Gamma(\alpha+1/2)}{\Gamma(\alpha)}
\]

(D27)

This behavior can be observed in the time regime

\[
\tau_0 / \kappa^{3/2} \ll t \ll \tau_0 \min[1, (\kappa \tau_f / \tau_0)^{(1/(1-\alpha)}].
\]

(D28)

For \( \Lambda \tau_f \ll 1 \), we obtain for the Laplace transform of the outflow from Equation D2

\[
j = j_0 \tau_f (\lambda \tau_0)^{-\alpha}
\]

(D29)

Inverse Laplace transform gives the behavior

\[
j = j_0 \frac{\tau_f}{\tau_0 \Gamma(\alpha)} \left( \frac{t}{\tau_0} \right)^{(\alpha-1)/2}
\]

(D30)

This behavior can be observed in the time regime

\[
\tau_0 \max[1 / \kappa^{3/2}, (\kappa \tau_f / \tau_0)^{(1/(1-\alpha)}] \ll t \ll \tau_0.
\]

(D31)

For times \( t \gg \tau_0 \), the matrix is drained and the outflow decays to zero exponentially fast.

### Data Availability Statement

No data were used in producing this manuscript.

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