Internal stress model for pre-primary stage of low-stress creep

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Abstract. Initial transient stage in low-stress creep experiments was observed in all such experiments. Recently, evidences were presented that this stage cannot be considered as a normal creep primary stage, though the shape of the creep curve is similar. The strain reached during this so called pre-primary stage is fully recoverable upon unloading; the internal stresses must play important role in the effect. Model of standard linear anelastic solid was modified by introduction of creeping body instead of viscous dashpot. Both power law and hyperbolic sine creep law were used to fit observed creep curves of model and structural materials. Mainly the model using hyperbolic sine creep law provides good fit to individual creep curves and sets of creep curves at different stresses.

1. Introduction
Transient stage with decreasing strain rate was observed in almost all low-stress creep experiments. Since the stage resembles the normal primary stage in conventional creep experiments, the interpretation of the transient deformation mechanisms was the same. Nevertheless, this approach met some serious difficulties [1]. Recently, evidences were presented [2] that the observed transient deformation cannot be considered a normal primary stage. One of the main differences is that the strain reached during this stage is almost fully reversible [3, 4], so it should be connected to the internal stresses field building.

The viscoelastic model of the standard anelastic material leads to the McVetty type of the creep equation [5], which is not capable to describe transient creep accurately. In this work, the model was adopted using creeping element instead of viscous dashpot; both power law and hyperbolic sine creep law were used to fit observed creep curves of model and structural materials. Mainly the model using hyperbolic sine creep law provides good fit to individual creep curves and sets of creep curves at different stresses.

2. Experimental procedure
Some previous experimental results were arbitrarily chosen to test the model capabilities. These were the Ni-15%Cr solid solution [6], the alumina fine grain advanced ceramics [7] and the P-91 type of the creep resistant steel [8, 4].

High strain sensitivity experimental techniques were used, mainly the helicoid spring specimen technique. The experimental procedures were described elsewhere [9] in more details. The important issue is the starting point of the experimental creep curves. Since most of the experiments use the “self-loaded” specimens, no initial strain upon loading is included in the creep curve. The creep curves recording starts in the time instant when the nominal temperature
of the test is reached. Since the heating up process is relatively fast, it can be considered as purely elastic initial state.

3. Model

The model consists of two elastic elements A and B, one creeping element C and one viscous element D, as depicted in figure 1. The viscous element D was added to introduce the steady state creep term into the final equations; in fact, no reasonable fit could be achieved without this term.

![Diagram of the model](image)

Particular elements are described by equations

\[
\dot{\varepsilon}_D = d \cdot \sigma \quad (1)
\]

\[
\varepsilon_A = a \cdot \sigma_A \quad (2)
\]

\[
\varepsilon_B = b \cdot \sigma_B \quad (3)
\]

\[
\dot{\varepsilon}_C = c \cdot (\sigma_B)^n \quad (4a)
\]

or

\[
\dot{\varepsilon}_C = c \cdot \sinh(g \cdot \sigma_B) \quad (4b)
\]

where \(a, b, c, d, g\) and \(n \neq 1\) are parameters.

![Figure 1. Mechanical model of transient creep strain.](image)

Model behavior is then described by differential equations

\[
\dot{\varepsilon} = \frac{ac}{a+b} \left( \sigma - \frac{\varepsilon - d \sigma t}{a} \right)^n + d \sigma \quad (5a)
\]

or

\[
\dot{\varepsilon} = \frac{ac}{a+b} \sinh \left( g \left( \sigma - \frac{\varepsilon - d \sigma t}{a} \right) \right) + d \sigma \quad (5b)
\]

for power law or hyperbolic sine law, respectively.

Initial conditions are given by the elastic state, that is at \(t = 0\) the elastic strain is \(\varepsilon(0) = a_b \sigma/(a + b)\) and \(\sigma_A = b \sigma/(a + b), \sigma_B = a_b \sigma/(a + b)\). Since this initial elastic strain is not recorded in the experimental creep curves, it must be subtracted from the final equations:

\[
\varepsilon = \frac{a^2 \sigma}{a + b} - \frac{a}{\left( \frac{(n-1)c}{a+b} \right) t + \left( \frac{a+b}{a \sigma} \right)^{n-1}} + d \sigma \quad (6a)
\]

or

\[
\varepsilon = \frac{a^2 \sigma}{a + b} \left( \frac{\exp \left( \frac{g \sigma}{a+b} t \right) - \exp \left( g \sigma \right) - \exp \left( \frac{b \sigma}{a+b} \right) - \exp \left( \frac{b \sigma}{a+b} t \right) + \exp \left( g \sigma \right) - \exp \left( g \sigma \right) - \exp \left( \frac{b \sigma}{a+b} \right)}{\exp \left( \frac{g \sigma}{a+b} t \right) + \exp \left( g \sigma \right) + \exp \left( \frac{b \sigma}{a+b} \right) + \exp \left( g \sigma \right)} \right) + d \sigma \quad (6b)
\]
4. Results

For practical reasons, the stress exponent \( n \) is limited to values between 3 (Alloy class metals) and 12 (experimental value for the P-91 steel [10]). Since all fitting procedures tend to adhere to one or other of these limits, the model equations were used with values \( n \) fixed to both limits. The fits are marked as “PL3” and “PL12, while the model using hyperbolic sine is marked “SH” in what follows.

4.1. Individual creep curves

An example of model creep curves fitted on experimental data for Ni-15%Cr solid solution is depicted in figure 2. It is clear that it is practically impossible to distinguish between results provided by particular model equations, so the fitting residuals are also plotted in figure 3. The statistical errors of the fits are summarized in the table 1.

![Figure 2. Creep curve of Ni-15%Cr solid solution at 700°C and 4.9 MPa, fitted by the three model equations](image)

![Figure 3. Residuals of the fit from figure 2](image)

| Material  | PL3  | PL12 | SH     |
|-----------|------|------|--------|
| NiCr      | \(9.7 \times 10^{-7}\) | \(1.1 \times 10^{-6}\) | \(1.2 \times 10^{-6}\) |
| Alumina   | \(8.9 \times 10^{-7}\) | \(7.7 \times 10^{-7}\) | \(7.3 \times 10^{-7}\) |
| P-91      | \(6.9 \times 10^{-6}\) | \(2.7 \times 10^{-6}\) | \(1.2 \times 10^{-6}\) |

It is clear, that while with NiCr and alumina all equations provides comparable results, with P-91 having more pronounced transient stage, the fit provided by PL3 is worse than that provided by PL12 and mainly SH.

4.2. Sets of creep curves

For the physical meaning of the model to be maintained, the parameters \( a, b, c, d \) and \( g \) should be stress independent. To prove it, the equations were fitted to the sets of creep curves with different stress provided by the helicoid spring specimen technique. Results are shown in figures 4 and 5.
It is clear that the PL12 equation fail to follow the shape of the creep curves. So the good results of the equation on individual creep curves is paid by strong stress dependence of the parameters. This fact contradicts the physical meaning of the model. Thus, the PL12 model does not work. On the other hand, the PL3 and SH models behave correctly. In most cases, the SH model provides better results than the PL3 one.

5. Conclusion
Simple elastoplastic model using elastic, viscous and creeping elements is able to describe the transient stage of the low stress creep of various material. The best fit is obtained, if the creeping element is governed by the hyperbolic sine creep law (equations 5b, 6b). If the power-law creep rule is used, for the high values of the stress exponent the strong stress dependence of parameters occur, which is not acceptable. On the other hand, model with power-law equation and stress exponent \(n = 3\) behaves correctly, but provides generally worse results than the model with hyperbolic sine equation.

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