Roughness exponent in the fracture of fibrous materials

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March 22, 2022

In this paper, a computational model in (2+1)-dimensions which simulates the rupture process of a fibrous material submitted to a constant force $F$, is analyzed. The roughness exponent $\zeta$ at the boundary that separates two failure regimes, catastrophic and slowly shredding, is evaluated. In the catastrophic (dynamic) regime the initial strain creates a crack which percolates rapidly through the material. In the slowly shredding (quasi-static) regime several cracks of small size appear in all parts of the material, the rupture process is slow and any single crack percolates the sample. At the boundary between these two regimes, we obtained a value $\zeta \approx 0.42 \pm 0.02$ for the roughness exponent, in agreement with results provided by other simulations in three dimension. Also, at this boundary we observed a power law behavior on the number of cracks versus its size.

PACS # 62.20.Mk, 64.60.Fe, 05.40.+j

I. INTRODUCTION

The fracture process in disordered materials is a subject of intensive research and has attracted much scientific and industrial interest [1,3]. The fracture process is extremely sensitive to disorder, which may act as obstacles difficulting the propagation of cracks through the material. Thus, disorder has a strong influence on the roughness of the fracture surface. Several computational models have been constructed to study the phenomenon of fracture of these materials, such as, the extensively studied fuse [4] and the well-known fiber bundle models [3,4], created from the pioneer work of Daniels [11].

Experiments have shown that the fracture surface in disordered materials can often be described by self-affine scaling [1,3]. In this case, the roughness of the fracture surface can be characterized by a roughness exponent $\zeta$. Some experimental works have claimed that the roughness exponent $\zeta$ has a universal value of 0.8 [14,16]. However this universality was questioned by Milman et al. [17], which experimentally found a roughness exponent closer to 0.5. From the theoretical point of view, numerical models have been searched in order to evaluate the roughness exponent $\zeta$. Simulations have shown that $\zeta \approx 0.7$ in two dimensions [18,19] and that $\zeta$ ranges from 0.4 to 0.5 in three dimensions [20,21]. Nowadays there is a conjecture relating the smaller and the higher value of the roughness exponent $\zeta$ to the speed of crack propagation through the sample [21]. The greater value has been associated with a high speed of crack propagation and interpreted as a dynamic regime. In contrast, the smaller value of the exponent was related with a quasi-static regime, where the dynamic effects of the propagation are negligible. Experimentally, the smaller and the higher value of $\zeta$ are connected to the length scale at which the crack is examined. The smaller value is associated to small length scales and the higher value is connected to large length scales [14,22]. Several models used to study the characterization of the fracture surface by the roughness exponent do not consider the influence of the temperature on the fracture process. In this paper we present a fibrous model in (2+1) dimensions for which it is possible to obtain the fracture profile of a fibrous material submitted to a constant force $F$, for example, by a hanging weight on it. In this work the influence of the temperature $t$ on the fracture process was considered. We show that, at the boundary between the catastrophic and the slowly shredding regime, the roughness exponent $\zeta$ does not depend on the temperature $t$.

II. MODEL

Our model consists of a bundle of $N_0 = L \times L$ parallel fibers, all with the same elastic constant, $k$, distributed on a triangular lattice. In order to simulate the height of the sample, the fibers are divided in $\eta$ segments with the same length. The fiber bundle is fixed at both extremes by two parallel plates, one of these is fixed and in the other a constant force $F$ is applied. This force is equally and completely distributed in the fiber bundle, submitting all fibers to the same linear deformation $z = F/Nk$, where $N$ is the number of unbroken fibers. This type of distribution is called equal load sharing (ELS) [21,22]. Another type is the local load sharing (LLS), where the load of a broke fiber is transfer onto its two nearest unbroken fibers [23,24]. At our model, when the deformation $z$ reaches a critical value $z_c$, the failure probability of an isolated fiber is equal to one. The failure probability of a fiber $i$ is given by

$$P_i(\delta, t) = \frac{\delta}{n_i + 1} \exp \left[ \frac{(\delta^2 - 1)}{t} \right],$$

where $n_i$ is the number of unbroken neighboring fibers, $\delta = z/z_c = F/Nkz_c$ is the strain of the material, $t = K_B T/E_c$ is the normalized temperature, $K_B$ is the Boltzmann constant, $T$ is the absolute temperature and $E_c$ is the critical elastic energy. In this model, besides finding the failure probability of a fiber, we have to indicate in which segment it breaks. Since, each fiber of the
bundle can break at different parts a fracture surface is produce. Similar procedure was used in Ref [26] to obtain the fracture profile of a model for fracture on fibrous materials in (1+1) dimensions. The segment is randomly selected and the probability of the fiber to break in it is given by [26].

\[ \phi_j(m_j) = \frac{(m_j + 1)}{g}, \]  

(2)

where \( m_j \) is a vector which indicates how many times a segment \( j \) broke and \( g = \sum_j (m_j + 1) \). Eq. (2) simulates a concentration of tension near to the region where the fiber bundle is weaker.

At the beginning of the simulation, the bundle is submitted to an initial strain given by

\[ \delta_0 = \frac{z_o}{z_c} = \frac{F}{N_0 k z_c}. \]  

(3)

At each time step we randomly choose a fiber of a set of \( N_q = q N_u \) unbroken fibers. The number \( q \) represents a percentage of fibers and allow us to work with any system size. Then, using Eq. (1), we evaluate the fiber failure probability \( P_i \) and compare it with a random number \( r \) in the interval \([0,1]\). If \( r < P_i \) the fiber breaks. We then choose a segment \( j \) in the fiber and evaluate its probability \( \phi \) to break, using Eq. (2). If the probability \( \phi \) is higher than a random number \( f \) the fiber breaks in the chosen segment. If not, we analyze the neighboring segments \((j+1)\) and \((j-1)\) and again evaluate the probability \( \phi \). If the condition \( f < \phi \) does not hold to neither of the neighboring segments, we return to the initial segment and test the condition \( f < \phi \) for a new value of \( f \). This process continues until the condition \( f < \phi \) is true. Once defined the segment where the fiber breaks, we begin to test all neighboring unbroken fibers. The first segment tested in the neighboring unbroken fibers is the one in which the previous fiber broke. The failure probability \( P_i \) of these neighboring fibers increases due to the decreasing of \( n_i \) and a cascade of breaking fibers may begin. This procedure describes the propagation of a crack through the fiber bundle in the perpendicular direction to the applied force. The process of propagation stops when the test of the probability does not allow rupture of any other fiber on the border of the crack or when the crack meets another already formed crack. The same cascade propagation is attempted by choosing another fiber of the set \( N_q \). After all the \( N_q \) fibers have been tested, the strain \( \delta \) is increased if some fibers have been broken. Since the force is fixed, the greater the number of broken fibers, the larger is the strain on the intact fibers and the higher is their failure probability. Then, another set of \( N_q \) unbroken fibers is chosen and all the rupture process is restarted. The simulation terminates when all the fibers of the bundle are broken, \( i.e. \), when the bundle is divided into two parts.

III. RESULTS

We performed simulations considering \( L = 2000 \) \((N_0 = 4 \times 10^6 \) fibers\), the elastic constant \( k = 1 \), the critical deformation \( z_c = 1 \) and the number of segments \( \eta = 1000 \). The failure probability (Eq. (4)) can be written as

\[ P_i(z) = \frac{\Gamma(t, \delta)}{(n_i + 1)}, \]  

(4)

where the parameter \( \Gamma(t, \delta) \) is defined as

\[ \Gamma(t, \delta) = \delta \exp \left( \frac{\delta^2 - 1}{t} \right). \]  

(5)

For a triangular lattice (with coordination number 6) and \( \Gamma(t, \delta) \geq 6 \), the rupture of any fiber induces the rupture of the whole bundle, \( i.e. \), the bundle breaks with just one crack. Obviously, this crack forms a cluster which percolates through the entire system.

We can define the density of the percolation crack as

\[ \rho = \frac{N_{pc}}{N_0}, \]  

(6)

where \( N_{pc} \) is the number of broken fibers belonging to the percolating crack. Thus, when \( \Gamma(t, \delta) \geq 6 \), we have \( \rho = 0 \).

Figure 3 shows the density of the percolation crack \( \rho \) versus the initial strain \( \delta_0 \), for two different temperatures. Notice that, for high values of \( \delta_0 \), \( \rho = 1 \) and for low values of \( \delta_0 \) the density of the percolating cluster \( \rho \) jumps to zero. Thus, we may assume that there is a critical value \( \delta_{0c} \) which depends on the temperature \( t \). For the temperatures \( t = 0.5 \) and \( t = 2.0 \) used in our simulation the values obtained for \( \delta_{0c} \) are 1.11 and 1.27 respectively. Above \( \delta_{0c} \), there is a percolation crack and below it any single crack percolates the fiber bundle. The critical value, \( \delta_{0c} \), represents the transition between two failure regimes [22]: catastrophic and slowly shredding. In the catastrophic regime there are cracks that percolate the fiber bundle and in the slowly shredding any crack percolates the bundle. In Ref. [4] we have shown that these two regimes are separated by a second order phase transition and determined from (Eq. 3) the critical line separating these two failure regimes in the plane \( t \times \delta_0 \).

Figure 3 shows the fracture surface obtained for \( t = 2.0 \) and three different initial strains \( \delta_0 \). In Fig. 3(a) the fracture surface is very rough and this profile is characteristic of a shredding fracture, in which the speed of crack propagation is low due to a slow process of successive rupture of fibers in the material. In Fig. 3(c) the fracture surface presents little roughness and is characteristic of catastrophic fracture. Here the crack propagates with high speed and the breakage of a single fiber induces the rupture of the whole the bundle. For \( \delta_0 = 1.27 \)
[Fig. 2 (b)] the fracture occurs at the boundary between the two failure regimes. It can be seen from Fig. 3 that the rupture of the sample begins at different segments, since we did not used a deterministic starting notch in our simulations.

In order to evaluate the roughness exponent ζ different one-dimensional cuts in the fracture surface were considered. The roughness W of each cut was found by the method of the best linear least-square fitting described in [27]. In this method, the roughness W(ε) in the scale ε is given by

$$W(\varepsilon) = \frac{1}{M} \sum_{i=1}^{M} w_i(\varepsilon)$$

and the local roughness w(ε) is defined as

$$w^2 = \frac{1}{(2\varepsilon + 1)} \sum_{j=i-\varepsilon}^{i+\varepsilon} [h_j - (a_i x_j + b_i(\varepsilon))]^2.$$  

(7) 

(8)

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The roughness exponent ζ at the critical point δ0c for two temperatures t was calculated and a value of ζ ≃ 0.42 ± 0.02 was obtained. Our results indicate that at the critical point the value of ζ does not depend on the temperature t. Figure 3 shows the fits for roughness W at t = 0.5 and t = 2.0. We also verified that, as the initial strain δ0 decreases below δ0c, the rupture process and, consequently, the speed of crack propagation become more slow. In this situation the roughness exponent ζ tend to zero. In Fig. 4 we show the plot of the time to failure (in Monte Carlo step) T_f as function of the initial strain δ0 for t = 2.0. Notice that the time decreases with increase of the initial strain δ0. For δ0 > δ0c the rupture is catastrophic and the material breaks in the first time step. In this regime was not possible to find the roughness exponent ζ. We believe that in this regime the fracture surface ceases to be self-affine.

The roughness exponent ζ was calculated along the x and y direction. At both directions we verify that the roughness exponent has the same value. This behavior is expected for a large variety of materials, where the two directions have similar scaling properties [28, 29]. At the transition we have observed a power law behavior on the number of cracks versus its size.

The log-log diagram of the frequency of the cracks H_c versus their sizes S_c observed for t = 2.0 and δ0c = 1.27 is shown in Fig. 5. Note that at the beginning H_c seems to decay linearly so that we can assume a power law

$$H_c \sim S^{-\alpha},$$ 

(9)

where α = 2.032 ± 0.007. We observed that at the transition this exponent does not depend on the temperature.

IV. CONCLUSION

In conclusion, we have studied a model for fracture on fibrous materials in (2+1) dimensions which provides the fracture surface of the material, in contrast with previous models. We calculated the roughness exponent ζ and showed that it does not depend on the temperature t. Our results indicate a value of ζ ≃ 0.42 ± 0.02 at the boundary between the catastrophic (similar to the dynamic regime) and the slowly shredding regime (similar to the quasi-static regime). This value is the same obtained in other type of simulations in three dimensions [20, 21] that related the exponent ζ with the quasi-static regime. We also have shown that at the boundary there is a power law connecting the frequency H_c and the size S_c of the size. This power law behavior is characteristic of systems in the criticality and is a good indication that the fracture surface is a fractal.

It is a pleasure to acknowledge Professors A. T. Bernardes and J. G. Moreira for precious guidance and many interesting discussions. We also thank M. L. Martins and M. S. Couto for helpful criticism of the manuscript and the kind hospitality of the Departamento de Física, UFMG. The author also acknowledges the FAPEMIG (Brazilian agency) for financial support.

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FIG. 1. Density of the percolating cluster $\rho$ vs the initial strain $\delta_0$ for two different temperatures: $t = 0.5$ (circles) and $t = 2.0$ (squares). The data were averaged over 1000 statistically independent samples.

FIG. 2. Fracture surface for three different initial strains. In (a) we have: $\delta_0 = 0.4$, in (b) $\delta_0 = 1.27$ and in (c) $\delta_0 = 1.4$. In this particularly simulation we have used a total of $N_0 = 4 \times 10^4$ fibers.
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FIG. 3. Log-Log plot of the roughness $W$ as function of the scale $\varepsilon$ for two different temperatures: $t = 0.5$ (circles) and $t = 2.0$ (squares). The two straight lines have slope 0.42.

FIG. 4. Plot of the time to failure (in Monte Carlo step) $T_f$ vs the initial strain $\delta_0$ for $t = 2.0$. The data were averaged over 1000 statistically independent samples.

FIG. 5. Log-Log plot of the frequency of the cracks $H_s$ vs the size of the cracks $S_c$ for $t = 2.0$ and $\delta_c = 1.27$. The straight line has a slope 2.032. The data were averaged over 1000 statistically independent samples.