Heat transfer in an infinite layer with fractal distribution of heating elements

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Abstract. The effect of inhomogeneous temperature distribution at boundary on convection in an infinite horizontal plane layer is investigated. Direct numerical simulation of the compressible non-isothermal flow in a cubic cell with rigid horizontal walls is performed under periodic vertical boundary conditions. Laminar and weakly non-linear flow regimes are determined. The heated area on the cell bottom obeys regular or fractal distributions. The intensities of heat flux through the layer are compared for different heterogeneous distributions of heating elements at a specific temperature gradient in the case when the area of heated surface remains constant. Fractal geometry shows the appearance of the multiscale structure of the flow and the enhancement of heat transfer.

1. Introduction

At present, energy saving and increasing energy efficiency in various sectors of the economy is a priority of science, engineering and technology. The increase in energy efficiency is tightly linked to the improvement of the characteristics of heat exchangers. The complexity of a heat conduction problem is dependent upon the properties of interacting media and contact surface conditions. In fluid and gaseous media, two mechanisms are responsible for heat transfer: diffusion and convection. The value of diffusion heat flux is defined by the thermal conductive properties of the medium. Convective mechanism is determined by the pattern of medium motion. This gives researchers an opportunity to manage heat transfer so that its efficiency increases. The intensification of the vortex structure is one of the approaches used to increase the efficiency of heat transfer. This can be achieved by changing a surface heater shape and a heat-transfer agent [1, 2]. For the flat horizontal heated surface, inhomogeneous heating can be used. Therefore, even in the case of stable stratification, a vortex flow will arise and an increase in convective heat transfer will take place [3].

Large-scale circulation makes the greatest contribution to advective heat transfer throughout the whole region. However, in a thin boundary layer, the effective transfer can be ensured only through high turbulence. Turbulence implies the presence of a multiscale flow that arises as a result of instability at large Reynolds numbers [4]. A significant influence on the occurrence of such a flow can be provided by the inhomogeneity of temperature on the heated surface [5]. It is possible to imagine a heat-conducting surface consisting of heat-conducting and heat-insulating areas. Depending on the configuration (geometry) of the distribution of these regions, different
convective flows over the surface and in the entire volume will form. Typically, a laminar flow over an inhomogeneously heated surface loses its stability and leads to the appearance of secondary structures [6]. These structures can play an important role and are therefore studied in detail experimentally and theoretically [7]. For the given temperature gradient and the area of the heat-conducting region, one can consider the problem of finding the optimal geometry for the formation of a developed turbulent flow, which in combination with large-scale circulation will provide the maximum heat flow. In this sense, fractal geometry may be of particular interest. There are also applied studies aimed at improving the efficiency of heat transfer, and the use of a fractal is one of the simplest and cheapest examples of their implementation [8].

The aim of this work is to numerically study the possibility of increasing the efficiency of heat transfer due to the inhomogeneity of the thermal conductive properties of the heater. We are focused on investigating the fractal distribution of heating elements. Depending on the dimension of a fractal, it is possible to obtain different energy distribution of pulsations across the spectrum, which in turn will enhance cascade processes and turbulent heat transfer. We consider several variants of the geometry of the heated region with a regular distribution of identical elements and a distribution that repeats the fractal - the "Sierpinsky carpet".

2. Mathematical model
Convection of the compressible non-isothermal viscous flow is described by the following equations:

\[
\begin{align*}
\frac{D \ln \rho}{Dt} &= -\nabla \cdot U \\
\frac{DU}{Dt} &= -c_v(\gamma - 1)(\nabla \ln \rho + \nabla T) + g + 2\nu(\mathbf{S} \cdot \nabla \ln \rho + \nabla \cdot \mathbf{S}) \\
\frac{DT}{Dt} &= -T(\gamma - 1)\nabla \cdot U + \frac{K}{\rho c_v} \Delta T + \frac{2\nu}{c_v} \mathbf{S}^2
\end{align*}
\]

where \( D/Dt = \partial/\partial t + \mathbf{U} \cdot \nabla \) is the total advective derivative, \( \nu \) is the kinematic viscosity, \( K \) is the heat conductivity, \( \rho \) is the density, \( U \) is the velocity, \( g \) is the gravitational acceleration, \( \gamma = c_p/c_v \) is the ratio of specific heats at constant pressure and volume, respectively. The traceless rate-of-strain tensor \( \mathbf{S} \) is given by

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{1}{3} \delta_{ij} \nabla \cdot \mathbf{U}. \tag{2}
\]

Application of Boussinesq equations is a common approach to describing turbulent flows. The system of equations (1) is more complex, but allows us to investigate convection both in liquids and gases with relatively large variations in density caused by a change of temperature.

To introduce non-dimensional variables, the following characteristic values are used: velocity \( U_0 \), layer height \( L \), temperature of cold surface \( T_0 \) (at the top), time \( L/U_0 \), density \( \rho_0 \), and sound speed \( a_0^2 = \gamma c_v T(\gamma - 1) \). Then the system (1) takes the form:

\[
\begin{align*}
\frac{D \ln \rho}{Dt} &= -\nabla \cdot U \\
\frac{DU}{Dt} &= - \left( M^2 \gamma \right)^{-1} (T \nabla \ln \rho + \nabla T) - Fr^{-2} e_z + 2Re^{-1}(\mathbf{S} \cdot \nabla \ln \rho + \nabla \cdot \mathbf{S}) \\
\frac{DT}{Dt} &= -T(\gamma - 1)\nabla \cdot U + \gamma (RePr)^{-1} \rho^{-1} \Delta T + 2M^2 \gamma (\gamma - 1)Re^{-1}\mathbf{S}^2,
\end{align*}
\]

where Mach number \( M = U_0/a_0 \), Froude number \( Fr = U_0/\sqrt{gL} \), Reynolds number \( Re = LU_0/\nu \), Prandtl number \( Pr = \nu/\chi \). The choice of these control parameters allows us to control a
transition from compressible (gas) to incompressible (fluid) medium by changing M, as well as a transition from laminar to turbulent flow by changing Re.

At the initial time, the medium is assumed to be stationary and homogeneous in temperature, so that $U_i = 0$, $T = 1$. The hydrostatic conditions allow us to formulate the initial density distribution $\ln(\rho(x, y, z)) = -Fr^{-2} \gamma M^2 z$. On horizontal rigid boundaries, the no-slip condition is satisfied, i.e. $U_i = 0$. At the upper boundary $T = 1$, and at the lower boundary $T = \theta$ in the heat-conducting region of the boundary and $\partial_z T = 0$ in the thermoisolated part, where $\theta$ is the relative temperature of a heater. The boundary condition for density $\partial_z \ln(\rho) = T^{-1} (Fr^{-2} \gamma M^2 + \partial_z T)$ comes from hydrostatic balance. Periodic boundary conditions are applied to the vertical boundaries of the studied domain.

The flow and heat transfer are calculated for 4 different heater configurations (see figure 1). The first heater (on the left) has a fractal configuration, which we are mostly interested in. For comparison, we consider three configurations with a regular distribution different in number and size of elements. The heaters have been selected in such a way that the total area of all the elements is equal for each configuration type. We introduce the parameter $D$ to denote: $D = f$ - fractal heater, $D = 1$ - 1 solid square heater, $D = 9$ - 9 squares heater, $D = 36$ - 36 squares heater.

Equations (3) are numerically integrated using the sixth-order central differences for spatial derivatives and advance in time using a fifth-order Runge-Kutta-Felberga explicit scheme with adaptive time step. We control the time step so that the relative error doesn’t exceed $10^{-4}$. The code is parallelized for MPI clusters. Simulations are performed using 144 nodes. We choose the resolution of $216^3$ meshpoints for more accurate discretization of the fractal distribution.

3. Results

In our simulations we used the following dimensionless parameters: $\theta = 3$, $M = 0.1$, $Fr = 0.088$, $Re = 180$, $\gamma = 5/3$, $Pr = 1$ and a variable parameter $D$. This approach is fully justified for implementing the basic idea – study of the characteristic structure of the flow and its effect on the heat transfer with different forms of heaters, which have an equal total surface area of heat exchange.

The main attention has focused on studying the four characteristics of the numerical solutions obtained in this work: the heat flux through the layer $Q$, the root-mean-square value of the velocity over the whole volume $u_{rms}$, the spectrum of kinetic energy in the horizontal plane, and the intensity of large-scale circulation.

Figure 2 shows $u_{rms}(t)$ and $Q(t)$ for different values of $D$. For $u_{rms}$, only $D = f$ exhibits quasi-periodic nature. Also, $D = f$ demonstrates an essential increase of $Q$, compared to $D = 1$, $D = 9$, $D = 36$.
Figure 2. Evolution of $u_{\text{rms}}$ (left) and $Q(t)$ (right) through upper (dashed line) and lower (solid line) boundaries for 4 values of $D$.

Figure 3. Vertical temperature distribution at $y = 0.5L$ for $D = f$, $D = 1$ and $D = 9$ (from left to right).

of about $50\%$. The same level of $Q$ is produced by $D = 9$. Increasing number of squares to $D = 36$ gives the unexpected value of $Q$, which becomes lower by $10\%$. $D = f$ with the quasi-periodic behavior of $u_{\text{rms}}$ and $Q$ demonstrates the weak nonlinear nature of convective flow.

Temperature distributions are given in figure 3, where one can see a more complicated structure of these distributions when $D = f$ in comparison with the linear gradient. The more the number of squares used in configuration, the weaker the ascending fluid flow above the heaters. The vertical and horizontal slices of the velocity field are displayed in figure 4 at $y = 0.5L$ and $z = 0.5L$. Velocity fields demonstrate a pattern of the multiscale vortexes for $D = f$. The vertical distribution of velocity for $D = f$ suggests that the entire amount of the fluid at the cell bottom is included in the ascending stream, and the horizontal distribution of velocity for the same $D$ represents at least five vortexes, of which the largest has a size of about $0.4L$. Velocity distributions for other values of $D$ represent similar large-scale circulations without any, even small, vortex. The spectra of total energy for each type of heater are shown in figure 5. The most developed spectrum is presented for $D = f$. For $D = 9$ and $D = 36$, we have obtained discrete spectra, where only a few scales are non zero.
Figure 4. Vertical slice of the flow structure at $y = 0.5L$ (upper row) and its horizontal slice at $z = 0.5L$ (lower row) for $D = f$, $D = 1$ and $D = 9$ (from left to right).

Figure 5. Kinetic energy spectrum: (left) for values $D = f$ and $D = 1$, in log-log scale, (right) $D = 9$ and $D = 36$, in log-linear scale. Energy of each scale for $D = 36$ is multiplied by a factor of 300 for comparison purposes.
4. Conclusions
We have investigated convection in an infinite plane layer over the inhomogeneous heating elements arranged in regular and fractal manner. Direct numerical simulations were performed for the compressible non-isothermal flow in the cubic cell with rigid horizontal walls under periodic vertical boundary conditions. Laminar and weak nonlinear flow regimes were considered at a given Reynolds number Re = 180. The comparative analysis of different heterogeneous heater distributions was carried out for the specific temperature gradient (heating at the bottom) in the case when the area of heated surface remains constant.

It is shown that this fractal heterogeneity, in comparison with all other cases of homogeneous heaters, provides a higher heat flux level throughout the entire layer, a more developed spatial (in horizontal plane) oscillation spectrum and the quasistochastic nature of the flow over the course of time. The results obtained at this stage are fundamental and do not pretend for practical application. We plan to investigate cascade process intensification and turbulent heat transfer when using a fractal heater thorough both numerical simulations and laboratory experimental studies.

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