The Electroweak Interactions in the Standard Model and beyond

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Abstract

We present a concise review of the status of the Standard Model and of the models of new physics.

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I. PRECISION TESTS OF THE STANDARD MODEL

The results of the electroweak precision tests as well as of the searches for the Higgs boson and for new particles performed at LEP and SLC are now available in nearly final form. Taken together with the measurements of \(m_t\), \(m_W\) and the searches for new physics at the Tevatron, and with some other data from low energy experiments, they form a very stringent set of precise constraints [1] to compare with the Standard Model (SM) or with any of its conceivable extensions. When confronted with these results, on the whole the SM performs rather well, so that it is fair to say that no clear indication for new physics emerges from the data [2].

All electroweak Z pole measurements, combining the results of the 5 experiments, are summarised in Table I. Information on the Z partial widths are contained in the quantities:

\[
\sigma_0^h = \frac{12\pi}{m_Z^2} \text{\Gamma}_{ee} \frac{\text{\Gamma}_{had}}{\Gamma_Z^2}, \quad R_\ell^0 = \frac{\sigma_0^h}{\sigma_0^\ell} = \frac{\text{\Gamma}_{had}}{\Gamma_\ell}, \quad R_q^0 = \frac{\text{\Gamma}_{q\bar{q}}}{\Gamma_{had}}. \tag{1}
\]

Here \(\Gamma_\ell\) is the partial decay width for a pair of massless charged leptons. The partial decay width for a given fermion species are related to the effective vector and axial-vector coupling constants of the neutral weak current:

\[
\Gamma_{f\bar{f}} = N_C^f \frac{G_F m_{Z}^3}{6\sqrt{2} \pi} \left( g_{Af}^2 C_{Af} + g_{Vf}^2 C_{Vf} \right) + \Delta_{ew/QCD}, \tag{2}
\]

where \(N_C^f\) is the QCD colour factor, \(C_{(A,V)f}\) are final-state QCD/QED correction factors also absorbing imaginary contributions to the effective coupling constants, \(g_{Af}\) and \(g_{Vf}\) are the real parts of the effective couplings, and \(\Delta\) contains non-factorisable mixed corrections.

Besides total cross sections, various types of asymmetries have been measured. The results of all asymmetry measurements are quoted in terms of the asymmetry parameter \(\mathcal{A}_f\), defined in terms of the real parts of the effective coupling constants, \(g_{Vf}\) and \(g_{Af}\), as:

\[
\mathcal{A}_f = 2 \frac{g_{Vf} g_{Af}}{g_{Vf}^2 + g_{Af}^2} = 2 \frac{g_{Vf}/g_{Af}}{1 + (g_{Vf}/g_{Af})^2}, \quad A_{FB}^0 = \frac{3}{4} \mathcal{A}_e \mathcal{A}_f. \tag{3}
\]

The measurements are: the forward-backward asymmetry \((A_{FB}^0 = (3/4) \mathcal{A}_e \mathcal{A}_f)\), the tau polarisation \((\mathcal{A}_\tau)\) and its forward backward asymmetry \((\mathcal{A}_e)\) measured at LEP, as well as the left-right and left-right forward-backward asymmetry measured at SLC \((\mathcal{A}_e\) and \(\mathcal{A}_f\), respectively). Hence
Table I: Summary of electroweak precision measurements at high $Q^2$. The first block shows the Z-pole measurements. The second block shows additional results from other experiments: the mass and the width of the W boson measured at the Tevatron and at LEP-2, the mass of the top quark measured at the Tevatron, and the contribution to $\alpha(m_Z^2)$ of the hadronic vacuum polarisation. The set of partial width and asymmetry results allows the extraction of the effective coupling.
constants. In particular, from the measurements at the Z, lepton universality of the neutral weak current was established at the per-mille level.

Using the effective electroweak mixing angle, $\sin^2 \theta_{\text{eff}}^f$, and the $\rho$ parameter, the effective coupling constants are given by:

$$g_{A_f} = \sqrt{\rho} T_3^f, \quad \frac{g_{V_f}}{g_{A_f}} = 1 - 4|q_f| \sin^2 \theta_{\text{eff}}^f,$$

where $T_3^f$ is the third component of the weak iso-spin and $q_f$ the electric charge of the fermion. The effective electroweak mixing angle is thus given independently of the $\rho$ parameter by the ratio $g_{Vf}/g_{Af}$ and hence in a one-to-one relation by each asymmetry result.

The various asymmetries determine the effective electroweak mixing angle for leptons with highest sensitivity. The results on $\sin^2 \theta_{\text{lept}}^\text{eff}$ are compared in Figure 1. The weighted average of these six results, including small correlations, is:

$$\sin^2 \theta_{\text{eff}}^\text{lept} = 0.23150 \pm 0.00016.$$  

Note, however, that this average has a $\chi^2$ of 10.5 for 5 degrees of freedom, corresponding to a probability of 6.2%. The $\chi^2$ is pushed up by the two most precise measurements of $\sin^2 \theta_{\text{eff}}^\text{lept}$, namely those derived from the measurements of $A_\ell$ by SLD, dominated by the left-right asymmetry $A^0_{LR}$, and of the forward-backward asymmetry measured in $b\bar{b}$ production at LEP, $A^{0,b}_{FB}$, which differ by about 2.9 standard deviations. No experimental effect in either measurement has been identified to explain this, thus the difference is presumably either the effect of statistics or an unidentified systematics or a hint for new physics, as further discussed below.

Also shown in table 1 are the results on $m_W$ obtained at LEP-2 and at the Tevatron, and the new world average of the top mass.

For the analysis of electroweak data in the SM one starts from the input parameters: as in any renormalisable theory masses and couplings have to be specified from outside. One can trade one parameter for another and this freedom is used to select the best measured ones as input parameters. As a result, some of them, $\alpha$, $G_F$ and $m_Z$, are very precisely known $^6$, some other ones, $m_{\text{light}}$, $m_t$ and $\alpha_s(m_Z)$ are far less well determined while $m_H$ is largely unknown.
Figure 1: Effective electroweak mixing angle $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ derived from measurement results depending on lepton couplings only (top) and also quark couplings (bottom) [1]. Also shown is the prediction of $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ in the SM as a function of $m_H$, including its parametric uncertainty dominated by the uncertainties in $\Delta \alpha_{\text{had}}^{(5)}(m_Z^2)$ and $m_t$, shown as the bands.

Note that the new combined CDF and DØ value for $m_t$ [3], as listed in Table [1] is higher than the previous average by nearly one standard deviation.

Among the light fermions, the quark masses are badly known, but fortunately, for the
calculation of radiative corrections, they can be replaced by $\alpha(m_Z)$, the value of the QED running coupling at the Z mass scale. The value of the hadronic contribution to the running, $\Delta\alpha_{\text{had}}^{(5)}(m_Z^2)$, reported in Table [I] is obtained through dispersion relations from the data on $e^+e^- \rightarrow \text{hadrons}$ at low centre-of-mass energies [4]. From the input parameters one computes the radiative corrections to a sufficient precision to match the experimental accuracy. Then one compares the theoretical predictions and the data for the numerous observables which have been measured, checks the consistency of the theory and derives constraints on $m_t$, $\alpha_s(m_Z^2)$ and $m_H$.

The computed radiative corrections include the complete set of one-loop diagrams, plus some selected large subsets of two-loop diagrams and some sequences of resummed large terms of all orders (large logarithms and Dyson resummations). In particular large logarithms, e.g., terms of the form $(\alpha/\pi \ln (m_Z/m_f))^{n}$ where $f_\ell$ is a light fermion, are resummed by well-known and consolidated techniques based on the renormalisation group. For example, large logarithms dominate the running of $\alpha$ from $m_e$, the electron mass, up to $m_Z$, which is a 6% effect, much larger than the few per-mille contributions of purely weak loops. Also, large logs from initial state radiation dramatically distort the line shape of the Z resonance observed at LEP-1 and SLC and must be accurately taken into account in the measurement of the Z mass and total width.

Among the one loop EW radiative corrections a remarkable class of contributions are those terms that increase quadratically with the top mass. The large sensitivity of radiative corrections to $m_t$ arises from the existence of these terms. The quadratic dependence on $m_t$ (and possibly on other widely broken isospin multiplets from new physics) arises because, in spontaneously broken gauge theories, heavy loops do not decouple. On the contrary, in QED or QCD, the running of $\alpha$ and $\alpha_s$ at a scale $Q$ is not affected by heavy quarks with mass $M \gg Q$. According to an intuitive decoupling theorem [7], diagrams with heavy virtual particles of mass $M$ can be ignored for $Q \ll M$ provided that the couplings do not grow with $M$ and that the theory with no heavy particles is still renormalizable. In the spontaneously broken EW gauge theories both requirements are violated. First, one important difference with respect to unbroken gauge theories is in the longitudinal modes of weak gauge bosons. These modes are generated by the Higgs mechanism, and their couplings grow with masses (as is also the case for the physical Higgs couplings). Second, the theory without the top quark is no more renormalisable because the gauge symmetry is broken if the b quark is left with no partner (while
its measured couplings show that the weak isospin is $1/2$). Because of non-decoupling precision tests of the electroweak theory may be sensitive to new physics even if the new particles are too heavy for their direct production.

While radiative corrections are quite sensitive to the top mass, they are unfortunately much less dependent on the Higgs mass. If they were sufficiently sensitive, by now we would precisely know the mass of the Higgs. However, the dependence of one loop diagrams on $m_H$ is only logarithmic: \( \sim G_F m_W^2 \log(m_H^2/m_W^2) \). Quadratic terms \( \sim G_F^2 m_H^2 \) only appear at two loops and are too small to be important. The difference with the top case is that $m_t^2 - m_b^2$ is a direct breaking of the gauge symmetry that already affects the relevant one loop diagrams, while the Higgs couplings to gauge bosons are "custodial-SU(2)" symmetric in lowest order.

We now discuss fitting the data in the SM. One can think of different types of fit, depending on which experimental results are included or which answers one wants to obtain. For example, in Table II we present in column 1 a fit of all Z pole data plus $m_W$ and $\Gamma_W$ (this is interesting as it shows the value of $m_t$ obtained indirectly from radiative corrections, to be compared with the value of $m_t$ measured in production experiments), in column 2 a fit of all Z pole data plus $m_t$ (here it is $m_W$ which is indirectly determined), and, finally, in column 3 a fit of all the data listed in Table I (which is the most relevant fit for constraining $m_H$). From the fit in column 1 of Table II we see that the extracted value of $m_t$ is in perfect agreement with the direct measurement (see Table I). Similarly we see that the experimental measurement of $m_W$ in Table II is larger by about one standard deviation with respect to the value from the fit in column 2. We have seen that quantum corrections depend only logarithmically on $m_H$.

In spite of this small sensitivity, the measurements are precise enough that one still obtains a quantitative indication of the mass range. From the fit in column 3 we obtain: \( \log_{10} m_H(\text{GeV}) = 2.05 \pm 0.20 \) (or $m_H = 113^{+62}_{-42}$ GeV). This result on the Higgs mass is particularly remarkable. The value of $\log_{10} m_H(\text{GeV})$ is right on top of the small window between $\sim 2$ and $\sim 3$ which is allowed, on the one side, by the direct search limit ($m_H \gtrsim 114$ GeV from LEP-2 [8]), and, on the other side, by the theoretical upper limit on the Higgs mass in the minimal SM, $m_H \lesssim 600 - 800$ GeV [9].

A different way of looking at the data is to consider the epsilon parameters. As well known these parameters vanish in the limit of tree level SM plus pure QED or pure QCD corrections. So they are a measure of the weak quantum corrections. Their experimental values are given
Table II: Standard Model fits of electroweak data. All fits use the Z pole results and $\Delta \alpha_{\text{had}}^{(5)}(m_Z^2)$ as listed in Table II also including constants such as the Fermi constant $G_F$. In addition, the measurements listed in each column are included as well. For fit 2, the expected W mass is also shown. For details on the fit procedure see [5].

| Fit | 1 | 2 | 3 |
|-----|---|---|---|
| Measurements | $m_W, \Gamma_W$ | $m_t$ | $m_t, m_W, \Gamma_W$ |
| $m_t$ (GeV) | $178.5^{+11.0}_{-8.5}$ | $177.2 \pm 4.1$ | $178.1 \pm 3.9$ |
| $m_H$ (GeV) | $117^{+162}_{-62}$ | $129^{+76}_{-50}$ | $113^{+62}_{-42}$ |
| $\log [m_H(\text{GeV})]$ | $2.07^{+0.38}_{-0.33}$ | $2.11 \pm 0.21$ | $2.05 \pm 0.20$ |
| $\alpha_s(m_Z)$ | $0.1187 \pm 0.0027$ | $0.1190 \pm 0.0027$ | $0.1186 \pm 0.0027$ |
| $\chi^2/dof$ | $16.3/12$ | $15.0/11$ | $16.3/13$ |
| $m_W$ (MeV) | | $80386 \pm 23$ |

The experimental values are compared to the SM predictions as function of $m_t$ and $m_H$ in Figure 2. We see that $\epsilon_3$ points to a light Higgs, that $\epsilon_b$ is a bit too large because of $A_{FB}^b$ and $\epsilon_2$ a bit too small because of $m_W$. Thus the whole picture of a perturbative theory with a fundamental Higgs is well supported by the data on radiative corrections. It is important that there is a clear indication for a particularly light Higgs: at 95% c.l. $m_H \lesssim 237$ GeV. This is quite encouraging for the ongoing search for the Higgs particle. More general, if the Higgs couplings are removed from the Lagrangian the resulting theory is non renormalisable. A cutoff $\Lambda$ must be introduced. In the quantum
corrections \( \log m_H \) is then replaced by \( \log \Lambda \) plus a constant. The precise determination of the associated finite terms would be lost (that is, the value of the mass in the denominator in the argument of the logarithm). A heavy Higgs would need some unfortunate conspiracy: the finite terms, different in the new theory from those of the SM, should accidentally compensate for the heavy Higgs in a few key parameters of the radiative corrections (mainly \( \epsilon_1 \) and \( \epsilon_3 \), see, for example, [13]). Alternatively, additional new physics, for example in the form of effective contact terms added to the minimal SM lagrangian, should accidentally do the compensation, which again needs some sort of conspiracy.

In Table III we collect the results on low energy precision tests of the SM obtained from neutrino and antineutrino deep inelastic scattering (NuTeV [10]), parity violation in Cs atoms (APV [11]) and the recent measurement of the parity-violating asymmetry in Moller scattering [12]. The experimental results are compared with the predictions from the fit in column 3 of Table III. We see the agreement is good except for the NuTeV result that shows a deviation by three standard deviations. The NuTeV measurement is quoted as a measurement of \( \sin^2 \theta_W = \)
Table III: Summary of other electroweak precision measurements, namely the measurements of the on-shell electroweak mixing angle in neutrino-nucleon scattering, the weak charge of cesium measured in an atomic parity violation experiment, and the effective weak mixing angle measured in Møller scattering, all performed in processes at low $Q^2$. The SM predictions are derived from fit 3 of Table II. Good agreement of the prediction with the measurement is found except for $\nu N$.

$1 - m^2_W/m^2_Z$ from the ratio of neutral to charged current deep inelastic cross-sections from $\nu_\mu$ and $\bar{\nu}_\mu$ using the Fermilab beams. There is growing evidence that the NuTeV anomaly could simply arise from an underestimation of the theoretical uncertainty in the QCD analysis needed to extract $\sin^2 \theta_W$. In fact, the lowest order QCD parton formalism on which the analysis has been based is too crude to match the experimental accuracy. In particular a small asymmetry in the momentum carried by the strange and antistrange quarks, $s - \bar{s}$, could have a large effect. A tiny violation of isospin symmetry in parton distributions, too small to be seen elsewhere, can similarly be of some importance. In conclusion we believe the discrepancy has more to teach about the QCD parton densities than about the electroweak theory.

When confronted with these results, on the whole the SM performs rather well, so that it is fair to say that no clear indication for new physics emerges from the data. However, as already mentioned, one problem is that the two most precise measurements of $\sin^2 \theta^\text{lept}_{\text{eff}}$ from $A_{\text{LR}}$ and $A_{\text{FB}}^{0,b}$ differ nearly three standard deviations. In general, there appears to be a discrepancy between $\sin^2 \theta^\text{lept}_{\text{eff}}$ measured from leptonic asymmetries ($(\sin^2 \theta^\text{eff}_l)_l$) and from hadronic asymmetries ($(\sin^2 \theta^\text{eff}_h)_h$), see also Figure 1. In fact, the result from $A_{\text{LR}}$ is in good agreement with the leptonic asymmetries measured at LEP, while all hadronic asymmetries, though their errors are large, are better compatible with the result of $A_{\text{FB}}^{0,b}$.

The situation is shown in Figure 3. The values of $(\sin^2 \theta^\text{eff}_l)$, $(\sin^2 \theta^\text{eff}_h)$ and their formal combination are shown each at the $m_H$ value that would correspond to it given the
Figure 3: The data for $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ are plotted vs $m_H$. For presentation purposes the measured points are shown each at the $m_H$ value that would ideally correspond to it given the central value of $m_t$ (updated from [15]).

central value of $m_t$. Of course, the value for $m_H$ indicated by each $\sin^2 \theta_{\text{eff}}^{\text{lept}}$ has an horizontal ambiguity determined by the measurement error and the width of the $\pm 1\sigma$ band for $m_t$. Even taking this spread into account it is clear that the implications on $m_H$ are sizably different. One might imagine that some new physics effect could be hidden in the Zb$b$ vertex. Like for the top quark mass there could be other non decoupling effects from new heavy states or a mixing of the b quark with some other heavy quark. However, it is well known that this discrepancy is not easily explained in terms of some new physics effect in the Zb$b$ vertex. In fact, $A_{\text{FB}}^{0,b}$ is the product of lepton- and b-asymmetry factors: $A_{\text{FB}}^{0,b} = (3/4)A_eA_b$. The sensitivity of $A_{\text{FB}}^{0,b}$ to $A_b$ is limited, because the $A_e$ factor is small, so that a rather large change of the b-quark couplings with respect to the SM is needed in order to reproduce the measured discrepancy.
(precisely a $\sim 30\%$ change in the right-handed coupling, an effect too large to be a loop effect but which could be produced at the tree level, e.g., by mixing of the b quark with a new heavy vectorlike quark \cite{16}). But then this effect should normally also appear in the direct measurement of $A_b$ performed at SLD using the left-right polarized b asymmetry, even within the moderate precision of this result, and it should also be manifest in the accurate measurement of $R_b \propto g_{Rb}^2 + g_{Lb}^2$. The measurements of neither $A_b$ nor $R_b$ confirm the need of a new effect. Even introducing an ad hoc mixing the overall fit is not terribly good, but we cannot exclude this possibility completely. Alternatively, the observed discrepancy could be due to a large statistical fluctuation or an unknown experimental problem. The ambiguity in the measured value of $\sin^2 \theta_{\text{lept}}^{\text{eff}}$ could thus be larger than the nominal error, reported in Equation 5, obtained from averaging all the existing determinations.

We have already observed that the experimental value of $m_W$ (with good agreement between LEP and the Tevatron) is a bit high compared to the SM prediction (see Figure 4). The value of $m_H$ indicated by $m_W$ is on the low side, just in the same interval as for $\sin^2 \theta_{\text{lept}}^{\text{eff}}$ measured from leptonic asymmetries. It is interesting that the new value of $m_t$ considerably relaxes the previous tension between the experimental values of $m_W$ and $\sin^2 \theta_{\text{lept}}^{\text{eff}}$ measured from leptonic asymmetries on one side and the lower limit on $m_H$ from direct searches on the other side \cite{17, 18}. This is also apparent from Figure 4.

The main lesson of precision tests of the standard electroweak theory can be summarised as follows. The couplings of quark and leptons to the weak gauge bosons $W^\pm$ and $Z$ are indeed precisely those prescribed by the gauge symmetry. The accuracy of a few per-mille for these tests implies that, not only the tree level, but also the structure of quantum corrections has been verified. To a lesser accuracy the triple gauge vertices $\gamma W^+ W^-$ and $ZW^+ W^-$ have also been found in agreement with the specific prediction of the $SU(2) \otimes U(1)$ gauge theory. This means that it has been verified that the gauge symmetry is unbroken in the vertices of the theory: the currents are indeed conserved. Yet there is obvious evidence that the symmetry is otherwise badly broken in the masses. Thus the currents are conserved but the spectrum of particle states is not at all symmetric. This is a clear signal of spontaneous symmetry breaking. The practical implementation of spontaneous symmetry breaking in a gauge theory is via the Higgs mechanism. The Higgs sector of the SM is still very much untested. What has been tested is the relation $m_W^2 = m_Z^2 \cos^2 \theta_W$, modified by computable radiative corrections. This relation means that the effective Higgs (be it fundamental or composite) is indeed a weak isospin
The Higgs particle has not been found but in the SM its mass can well be larger than the present direct lower limit $m_H \gtrsim 114$ GeV obtained from direct searches at LEP-2. The radiative corrections computed in the SM when compared to the data on precision electroweak tests lead to a clear indication for a light Higgs, not too far from the present lower bound. No signal of new physics has been found. However, to make a light Higgs natural in presence of quantum fluctuations new physics should not be too far. This is encouraging for the LHC that should experimentally clarify the problem of the electroweak symmetry breaking sector and search for physics beyond the SM.
II. OUTLOOK ON AVENUES BEYOND THE STANDARD MODEL

Given the success of the SM why are we not satisfied with that theory? Why not just find the Higgs particle, for completeness, and declare that particle physics is closed? The reason is that there are both conceptual problems and phenomenological indications for physics beyond the SM. On the conceptual side the most obvious problems are that quantum gravity is not included in the SM and the related hierarchy problem. Among the main phenomenological hints for new physics we can list coupling unification, dark matter, neutrino masses, baryogenesis and the cosmological vacuum energy.

The computed evolution with energy of the effective SM gauge couplings clearly points towards the unification of the electro-weak and strong forces (Grand Unified Theories: GUT’s) at scales of energy $M_{\text{GUT}} \sim 10^{15} - 10^{16}$ GeV which are close to the scale of quantum gravity, $M_{\text{Pl}} \sim 10^{19}$ GeV. One is led to imagine a unified theory of all interactions also including gravity (at present superstrings provide the best attempt at such a theory). Thus GUT’s and the realm of quantum gravity set a very distant energy horizon that modern particle theory cannot ignore. Can the SM without new physics be valid up to such large energies? This appears unlikely because the structure of the SM could not naturally explain the relative smallness of the weak scale of mass, set by the Higgs mechanism at $\mu \sim 1/\sqrt{G_F} \sim 250$ GeV with $G_F$ being the Fermi coupling constant. This so-called hierarchy problem is related to the presence of fundamental scalar fields in the theory with quadratic mass divergences and no protective extra symmetry at $\mu = 0$. For fermion masses, first, the divergences are logarithmic and, second, they are forbidden by the $SU(2) \times U(1)$ gauge symmetry plus the fact that at $m = 0$ an additional symmetry, i.e. chiral symmetry, is restored. Here, when talking of divergences, we are not worried of actual infinities. The theory is renormalisable and finite once the dependence on the cut off is absorbed in a redefinition of masses and couplings. Rather the hierarchy problem is one of naturalness. We should see the cut off as a parameterization of our ignorance on the new physics that will modify the theory at large energy scales. Then it is relevant to look at the dependence of physical quantities on the cut off and to demand that no unexplained enormously accurate cancellations arise.

The hierarchy problem can be put in very practical terms: loop corrections to the higgs mass squared are quadratic in $\Lambda$. The most pressing problem is from the top loop. With
\[ m_h^2 = m_{\text{bare}}^2 + \delta m_h^2 \] the top loop gives

\[ \delta m_{h,\text{top}}^2 \sim \frac{3G_F}{\sqrt{2\pi^2}} m_t^2 \Lambda^2 \sim (0.3\Lambda)^2 \] (10)

If we demand that the correction does not exceed the light Higgs mass indicated by the precision tests, \( \Lambda \) must be close, \( \Lambda \sim o(1 \, \text{TeV}) \). Similar constraints arise from the quadratic \( \Lambda \) dependence of loops with gauge bosons and scalars, which, however, lead to less pressing bounds. So the hierarchy problem demands new physics to be very close (in particular the mechanism that quenches the top loop). Actually, this new physics must be rather special, because it must be very close, yet its effects are not clearly visible (the "LEP Paradox" [19]). Examples of proposed classes of solutions for the hierarchy problem are:

**Supersymmetry.** In the limit of exact boson-fermion symmetry the quadratic divergences of bosons cancel so that only log divergences remain. However, exact SUSY is clearly unrealistic. For approximate SUSY (with soft breaking terms), which is the basis for all practical models, \( \Lambda \) is replaced by the splitting of SUSY multiplets, \( \Lambda \sim m_{\text{SUSY}} - m_{\text{ord}} \). In particular, the top loop is quenched by partial cancellation with s-top exchange.

**Technicolor.** The Higgs system is a condensate of new fermions. There are no fundamental scalar Higgs sector, hence no quadratic divergences associated to the \( \mu^2 \) mass in the scalar potential. This mechanism needs a very strong binding force, \( \Lambda_{TC} \sim 10^3 \, \Lambda_{\text{QCD}} \). It is difficult to arrange that such nearby strong force is not showing up in precision tests. Hence this class of models has been disfavoured by LEP, although some special class of models have been devised aposteriori, like walking TC, top-color assisted TC etc (for recent reviews, see, for example, [20]).

**Large compactified extra dimensions.** The idea is that \( M_{\text{Pl}} \) appears very large, that is gravity seems very weak because we are fooled by hidden extra dimensions so that the real gravity scale is reduced down to \( o(1 \, \text{TeV}) \). This possibility is very exciting in itself and it is really remarkable that it is compatible with experiment.

"Little Higgs" models. In these models extra symmetries allow \( m_h \neq 0 \) only at two-loop level, so that \( \Lambda \) can be as large as \( o(10 \, \text{TeV}) \) with the Higgs within present bounds (the top loop is quenched by exchange of heavy vectorlike new charge-2/3 quarks).
We now briefly comment in turn on these possibilities.

SUSY models are the most developed and most widely accepted. Many theorists consider SUSY as established at the Planck scale $M_{Pl}$. So why not to use it also at low energy to fix the hierarchy problem, if at all possible? It is interesting that viable models exist. The necessary SUSY breaking can be introduced through soft terms that do not spoil the good convergence properties of the theory. Precisely those terms arise from supergravity when it is spontaneously broken in a hidden sector. This is the case of the MSSM [21]. Of course, minimality is only a simplicity assumption that could possibly be relaxed. The MSSM is a completely specified, consistent and computable theory which is compatible with all precision electroweak tests. In this most traditional approach SUSY is broken in a hidden sector and the scale of SUSY breaking is very large of order $\Lambda \sim \sqrt{G_F^{-1/2} M_{Pl}}$. But since the hidden sector only communicates with the visible sector through gravitational interactions the splitting of the SUSY multiplets is much smaller, in the TeV energy domain, and the Goldstino is practically decoupled. But alternative mechanisms of SUSY breaking are also being considered. In one alternative scenario [22] the (not so much) hidden sector is connected to the visible one by ordinary gauge interactions. As these are much stronger than the gravitational interactions, $\Lambda$ can be much smaller, as low as 10-100 TeV. It follows that the Goldstino is very light in these models (with mass of order or below 1 eV typically) and is the lightest, stable SUSY particle, but its couplings are observably large. The radiative decay of the lightest neutralino into the Goldstino leads to detectable photons. The signature of photons comes out naturally in this SUSY breaking pattern: with respect to the MSSM, in the gauge mediated model there are typically more photons and less missing energy. The main appeal of gauge mediated models is a better protection against flavour changing neutral currents but naturality problems tend to increase. As another possibility it has been pointed out that there are pure gravity contributions to soft masses that arise from gravity theory anomalies [23]. In the assumption that these terms are dominant the associated spectrum and phenomenology have been studied. In this case gaugino masses are proportional to gauge coupling beta functions, so that the gluino is much heavier than the electroweak gauginos, and the wino is most often the lightest SUSY particle.

What is really unique to SUSY with respect to all other extensions of the SM listed above is that the MSSM or similar models are well defined and computable up to $M_{Pl}$ and, moreover, are not only compatible but actually quantitatively supported by coupling unification and GUT’s. At present the most direct phenomenological evidence in favour of supersymmetry is
obtained from the unification of couplings in GUTs. Precise LEP data on $\alpha_s(m_Z)$ and $\sin^2 \theta_W$ show that standard one-scale GUTs fail in predicting $\sin^2 \theta_W$ given $\alpha_s(m_Z)$ (and $\alpha(m_Z)$) while SUSY GUTs are in agreement with the present, very precise, experimental results. If one starts from the known values of $\sin^2 \theta_W$ and $\alpha(m_Z)$, one finds for $\alpha_s(m_Z)$ the results: $\alpha_s(m_Z) = 0.073 \pm 0.002$ for Standard GUTs and $\alpha_s(m_Z) = 0.129 \pm 0.010$ for SUSY GUTs to be compared with the world average experimental value $\alpha_s(m_Z) = 0.119 \pm 0.003$. Another great asset of SUSY GUT’s is that proton decay is much slowed down with respect to the non SUSY case. First, the unification mass $M_{GUT} \sim$ few $10^{16}$ GeV, in typical SUSY GUT’s, is about 20-30 times larger than for ordinary GUT’s. This makes p decay via gauge boson exchange negligible and the main decay amplitude arises from dim-5 operators with higgsino exchange, leading to a rate close but still compatible with existing bounds (see, for example, [24]). It is also important that SUSY provides an excellent dark matter candidate, the neutralino. We finally recall that the range of neutrino masses as indicated by oscillation experiments, when interpreted in the see-saw mechanism, point to $M_{GUT}$ and give additional support to GUTs [25].

In spite of all these virtues it is true that the lack of SUSY signals at LEP and the lower limit on $m_H$ pose problems for the MSSM. The lightest Higgs particle is predicted in the MSSM to be below $m_h \lesssim 135$ GeV (the recent increase of $m_t$ helps in this respect). The limit on the SM Higgs $m_H \gtrsim 114$ GeV considerably restricts the available parameter space of the MSSM requiring relatively large $\tan \beta$ ($\tan \beta \gtrsim 2 - 3$: at tree level $m_h^2 = m_Z^2 \cos^2 2\beta$) and rather heavy s-top (the loop corrections increase with log $\tilde{m}_t^2$). Stringent naturality constraints also follow from imposing that the electroweak symmetry breaking occurs at the right place: in SUSY models the breaking is induced by the running of the $H_u$ mass starting from a common scalar mass $m_0$ at $M_{GUT}$. The squared Z mass $m_Z^2$ can be expressed as a linear combination of the SUSY parameters $m_0^2$, $m_{1/2}^2$, $A_t^2$, $\mu^2$,... with known coefficients. Barring cancellations that need fine tuning, the SUSY parameters, hence the SUSY s-partners cannot be too heavy. The LEP limits, in particular the chargino lower bound $m_{\chi^+} \gtrsim 100$ GeV, are sufficient to eliminate an important region of the parameter space, depending on the amount of allowed fine tuning. For example, models based on gaugino universality at the GUT scale are discarded unless a fine tuning by at least a factor of 20 is not allowed. Without gaugino universality the strongest limit remains on the gluino mass: $m_{\tilde{g}}^2 \sim 0.7 m_{\tilde{g}_{\text{gluino}}} + \ldots$ which is still compatible with the present limit $m_{\tilde{g}_{\text{gluino}}} \sim 200$ GeV.
The non discovery of SUSY at LEP has given further impulse to the quest for new ideas on physics beyond the SM. Large extra dimensions \cite{28} and "little Higgs" \cite{29} models are the most interesting new directions in model building. Large extra dimension models propose to solve the hierarchy problem by bringing gravity down from $M_{Pl}$ to $m \sim o(1 \text{ TeV})$ where $m$ is the string scale. Inspired by string theory one assumes that some compactified extra dimensions are sufficiently large and that the SM fields are confined to a 4-dimensional brane immersed in a d-dimensional bulk while gravity, which feels the whole geometry, propagates in the bulk. We know that the Planck mass is large because gravity is weak: in fact $G_N \sim 1/M^2_{Pl}$, where $G_N$ is Newton constant. The idea is that gravity appears so weak because a lot of lines of force escape in extra dimensions. Assume you have $n = d - 4$ extra dimensions with compactification radius $R$. For large distances, $r >> R$, the ordinary Newton law applies for gravity: in natural units $F \sim G_N/r^2 \sim 1/(M^2_{Pl}r^2)$. At short distances, $r \lesssim R$, the flow of lines of force in extra dimensions modifies Gauss law and $F^{-1} \sim m^2(mr)^{d-4}/r^2$. By matching the two formulas at $r = R$ one obtains $(M_{Pl}/m)^2 = (Rm)^{d-4}$. For $m \sim 1 \text{ TeV}$ and $n = d - 4$ one finds that $n = 1$ is excluded ($R \sim 10^{15} \text{ cm}$), for $n = 2$ $R$ is at the edge of present bounds $R \sim 1 \text{ mm}$, while for $n = 4, 6, R \sim 10^{-9}, 10^{-12} \text{ cm}$. In all these models a generic feature is the occurrence of Kaluza-Klein (KK) modes. Compactified dimensions with periodic boundary conditions, as for quantization in a box, imply a discrete spectrum with momentum $p = n/R$ and mass squared $m^2 = n^2/R^2$. There are many versions of these models. The SM brane can itself have a thickness $r$ with $r < \sim 10^{-17} \text{ cm}$ or $1/r \gg 1 \text{ TeV}$, because we know that quarks and leptons are pointlike down to these distances, while for gravity there is no experimental counter-evidence down to $R < \sim 0.1 \text{ mm}$ or $1/R \gg 10^{-3} \text{ eV}$. In case of a thickness for the SM brane there would be KK recurrences for SM fields, like $W_n, Z_n$ and so on in the $TeV$ region and above. There are models with factorized metric $(ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + h_{ij}(y)dy^idy^j$, where $y (i,j)$ denotes the extra dimension coordinates (and indices), or models with warped metric $(ds^2 = e^{-2kR|\phi|\eta_{\mu\nu}dx^\mu dx^\nu - R^2\phi^2}$ \cite{30}. In any case there are the towers of KK recurrences of the graviton. They are gravitationally coupled but there are a lot of them that sizably couple, so that the net result is a modification of cross-sections and the presence of missing energy.

Large extra dimensions provide a very exciting scenario \cite{31}. Already it is remarkable that this possibility is compatible with experiment. However, there are a number of criticisms that can be brought up. First, the hierarchy problem is more translated in new terms rather than solved. In fact the basic relation $Rm = (M_{Pl}/m)^{2/n}$ shows that $Rm$, which one would apriori
expect to be $0(1)$, is instead ad hoc related to the large ratio $M_{Pl}/m$. In this respect the
Randall-Sundrum variety is more appealing because the hierarchy suppression $m_{W}/M_{Pl}$ could
arise from the warping factor $e^{-2kR|\phi|}$, with not too large values of $kR$. Also it is not clear
how extra dimensions can by themselves solve the LEP paradox (the large top loop corrections
should be controlled by the opening of the new dimensions and the onset of gravity): since $m_{H}$
is light $\Lambda \sim 1/R$ must be relatively close. But precision tests put very strong limits on $\Lambda$ In
fact in typical models of this class there is no mechanism to sufficiently quench the corrections.
No simple, realistic model has yet emerged as a benchmark. But it is attractive to imagine
that large extra dimensions could be a part of the truth, perhaps coupled with some additional
symmetry or even SUSY.

In the extra dimension general context an interesting direction of development is the study
of symmetry breaking by orbifolding and/or boundary conditions. These are models where a
larger gauge symmetry (with or without SUSY) holds in the bulk. The symmetry is reduced in
the 4 dimensional brane, where the physics that we observe is located, as an effect of symmetry
breaking induced geometrically by suitable boundary conditions. There are models where
SUSY, valid in $n > 4$ dimensions is broken by boundary conditions [32], in particular the model
of ref. [33], where the mass of the Higgs is computable and can be estimated with good accuracy.
Then there are ”Higgsless models” where it is the SM electroweak gauge symmetry which is
broken at the boundaries [34]. Or models where the Higgs is the 5th component of a gauge
boson of an extended symmetry valid in $n > 4$ [35]. In general all these alternative models for
the Higgs mechanism face severe problems and constraints from electroweak precision tests [36].
At the GUT scale, symmetry breaking by orbifolding can be applied to obtain a reformulation
of SUSY GUT’s where many problematic features of ordinary GUT’s (e.g. a baroque Higgs
sector, the doublet-triplet splitting problem, fast proton decay etc) are improved [37], [31].

In ”little Higgs” models the symmetry of the SM is extended to a suitable global group $G$ that
also contains some gauge enlargement of $SU(2) \otimes U(1)$, for example $G \supset [SU(2) \otimes U(1)]^2 \supset SU(2) \otimes U(1)$. The Higgs particle is a pseudo-Goldstone boson of $G$ that only takes mass at
2-loop level, because two distinct symmetries must be simultaneously broken for it to take mass,
which requires the action of two different couplings in the same diagram. Then in the relation
between $\delta m_{h}^2$ and $\Lambda^2$ there is an additional coupling and an additional loop factor that allow
for a bigger separation between the Higgs mass and the cut-off. Typically, in these models
one has one or more Higgs doublets at $m_{h} \sim 0.2 \text{ TeV}$, and a cut-off at $\Lambda \sim 10 \text{ TeV}$. The
top loop quadratic cut-off dependence is partially canceled, in a natural way guaranteed by
the symmetries of the model, by a new coloured, charge-2/3, vectorial quark $\chi$ of mass around
1 $TeV$ (a fermion not a scalar like the s-top of SUSY models). Certainly these models involve
a remarkable level of group theoretic virtuosity. However, in the simplest versions one is faced
with problems with precision tests of the SM [38]. Even with vectorlike new fermions large
corrections to the epsilon parameters arise from exchanges of the new gauge bosons $W'$ and
$Z'$ (due to lack of custodial $SU(2)$ symmetry). In order to comply with these constraints the
cut-off must be pushed towards large energy and the amount of fine tuning needed to keep
the Higgs light is still quite large. Probably these bad features can be fixed by some suitable
complication of the model (see for example, [39]). But, in my opinion, the real limit of this
approach is that it only offers a postponement of the main problem by a few TeV, paid by a
complete loss of predictivity at higher energies. In particular all connections to GUT’s are lost.

Finally, we stress the importance of the cosmological constant or vacuum energy problem [40]. The exciting recent results on cosmological parameters, culminating with the precise
WMAP measurements [41], have shown that vacuum energy accounts for about 2/3 of the
critical density: $\Omega_\Lambda \sim 0.65$, Translated into familiar units this means for the energy density
$\rho_\Lambda \sim (2 \times 10^{-3}$ eV$)^4$ or $(0.1$ mm$)^{-4}$. It is really interesting (and not at all understood) that
$\rho_\Lambda^{1/4} \sim \Lambda^2_{EW}/M_{Pl}$ (close to the range of neutrino masses). It is well known that in field theory
we expect $\rho_\Lambda \sim \Lambda^4_{cutoff}$. If the cut off is set at $M_{Pl}$ or even at $0(1$ TeV$)$ there would an
enormous mismatch. In exact SUSY $\rho_\Lambda = 0$, but SUSY is broken and in presence of breaking
$\rho_\Lambda^{1/4}$ is in general not smaller than the typical SUSY multiplet splitting. Another closely related
problem is ”why now?”: the time evolution of the matter or radiation density is quite rapid,
while the density for a cosmological constant term would be flat. If so, them how comes that
precisely now the two density sources are comparable? This suggests that the vacuum energy
is not a cosmological constant term, buth rather the vacuum expectation value of some field
(quintessence) and that the ”why now?” problem is solved by some dynamical mechanism.

Clearly the cosmological constant problem poses a big question mark on the relevance of
naturalness as a relevant criterion also for the hierarchy problem: how we can trust that we
need new physics close to the weak scale out of naturalness if we have no idea on the solution of
the cosmological constant huge naturalness problem? The common answer is that the hierarchy
problem is formulated within a well defined field theory context while the cosmological constant
problem makes only sense within a theory of quantum gravity, that there could be modification
of gravity at the sub-eV scale, that the vacuum energy could flow in extra dimensions or in different Universes and so on. At the other extreme is the possibility that naturalness is misleading. Weinberg \[42\] has pointed out that the observed order of magnitude of $\Lambda$ can be successfully reproduced as the one necessary to allow galaxy formation in the Universe. In a scenario where new Universes are continuously produced we might be living in a very special one (largely fine-tuned) but the only one to allow the development of an observer. One might then argue that the same could in principle be true also for the Higgs sector. Recently it was suggested \[43\] to abandon the no-fine-tuning assumption for the electro-weak theory, but require correct coupling unification, presence of dark matter with weak couplings and a single scale of evolution from the EW to the GUT scale. A ”split SUSY” model arises as a solution with a fine-tuned light Higgs and all SUSY particles heavy except for gauginos, higgsinos and neutralinos, protected by chiral symmetry. Or we can have a two-scale non-SUSY GUT with axions as dark matter. In conclusion, it is clear that naturalness can be a good heuristic principle but you cannot prove its necessity.

III. SUMMARY AND CONCLUSION

Supersymmetry remains the standard way beyond the SM. What is unique to SUSY, beyond leading to a set of consistent and completely formulated models, as, for example, the MSSM, is that this theory can potentially work up to the GUT energy scale. In this respect it is the most ambitious model because it describes a computable framework that could be valid all the way up to the vicinity of the Planck mass. The SUSY models are perfectly compatible with GUT’s and are actually quantitatively supported by coupling unification and also by what we have recently learned on neutrino masses. All other main ideas for going beyond the SM do not share this synthesis with GUT’s. The SUSY way is testable, for example at the LHC, and the issue of its validity will be decided by experiment. It is true that we could have expected the first signals of SUSY already at LEP, based on naturality arguments applied to the most minimal models (for example, those with gaugino universality at asymptotic scales). The absence of signals has stimulated the development of new ideas like those of large extra dimensions and ”little Higgs” models. These ideas are very interesting and provide an important reference for the preparation of LHC experiments. Models along these new ideas are not so completely formulated and studied as for SUSY and no well defined and realistic baseline has
so far emerged. But it is well possible that they might represent at least a part of the truth and it is very important to continue the exploration of new ways beyond the SM.

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