Demonstration of Optimal Fixed-Point Quantum Search Algorithm in IBM Quantum Computer

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Quantum search algorithm can be described as the rotation of state vectors in a Hilbert space. The state vectors uniformly rotate by iterative sequences until they hit the target position. To optimize the algorithm, it is necessary to have the precise knowledge about some parameters like the number of target positions. Here we demonstrate the implementation of optimal fixed-point quantum search (OFPQS) algorithm in a five-qubit quantum computer developed by IBM Corporation. We perform the OFPQS algorithm for one and two-iterations and confirm the accuracy of our results by state tomography process.

Keywords: Search Algorithm, OFPQS Algorithm, IBM Quantum Experience

I. INTRODUCTION

Quantum versions of classical algorithms have always been a boon for the typical problems which are encountered in various branches of computation and information processing [1, 2] i.e. data security [3–4], cryptography [5–7], database handling [8], to name a few. Many of these exploit the simple but crucial fact that in quantum theory, a superposition of states can also exist at a time unlike classical domain where only a single state (string of bits) can be represented at an instance.

Grover’s search algorithm [9] for unsorted database is another brilliant example of that. This algorithm comprising of the iterative applications of the Grover’s operator has a beautiful geometrical understanding when the initial state being a superposition of all the states present in the database is expressed in a 2-dimensional Hilbert space with the orthogonal states are (a) superposition of unmarked or non-target states (b) superposition of marked or target states. Thus, Grover’s operator is a unitary operator which serves to rotate the given initial state towards the target state axis in this Hilbert space effectively increasing the amplitude of the target state in the initial superposition.

This algorithm when applied within an unsorted database of N items having M marked items, can perform the task in $\sqrt{\frac{N}{M}}$ applications of the Grover’s operator, decomposition of which depends upon the requirement of search problem. However, its usefulness is limited as a general problem doesn’t come up with the known number of marked items without which the number of iteration is unknown to us. This leads to us the souffle problem i.e. only a few iterations leave the state mostly comprising of the unmarked states while too many iterations cause the state vector to surpass the target state in the assumed 2-D Hilbert space.

Souffle problem is dealt with using the fixed-point quantum search (FPQS) algorithms which always increases the amplitude of the target state with each iteration. One such FPQS algorithm is Grover’s $\pi/3$ algorithm. But these algorithms lose the quadratic speed-up which is the astonishing and useful feature of quantum algorithms.

Furthermore, Yoder et al. [10] developed another algorithm which avoids the souffle problem with the quadratic speed up. This requires setting the success probability with a bound over it in the form of a tunable parameter $\delta$. In this case, we can achieve the quadratic speed-up as well in our process. This algorithm, abbreviated as OFPQS (Optimal FPQS) has been discussed in the Sec. II.

IBM Quantum Experience is widely used to perform different tasks in the field of quantum computation and quantum information [11–30]. Experimental test of Hardy’s paradox [31], topological quantum walks [32], quantum permutation algorithm [33] have been illustrated. Error correction with 15 qubit repetition code [34] and estimation of molecular ground state energy [35] have also been implemented using 16 qubit IBM quantum computer, ibmqx5. Alvarez-Rodrique et al. [36] have shown artificial life in quantum technologies. Current trends like quantum machine learning [37] has been performed on the quantum computer. An essential ingredient of quantum communication, quantum repeater has been designed by Behera et al. [38] using IBM quantum computer. One of the important quantum mechanical problems, quantum tunneling [39] has been simulated on the universal quantum simulator, ibmqx4.

In this letter, we demonstrate the OFPQS algorithm in IBM Q Composer by taking a uniform superposition of four 2 qubit states which has one target state in it. We follow the circuit proposed by Yoder et al. [10, 10] with the oracle designed for the desired state. Due to the limitations of space provided by IBM Q Composer, we have been able to perform the task only for one and two iterations of the Grover’s operator.

This paper is organized as follows. In Sec. II we briefly introduce the mathematical formalism used in the design of the circuit. In Sec. III we discuss the oracle used in this OFPQS algorithm for the mentioned initial state and the target state. Sec. IV comprises of the plots of the results obtained from the IBM Q simulator and real quan-
We present the results for one and two iterations of the Grover’s operator with a success probability initially set to some value close to unity. In Sec. IV we compare the results for both the cases and present the plausible reasons for the noticeable differences.

II. OFPQS ALGORITHM

Following are the steps to implement the aforementioned algorithm, for which the quantum circuit for one iteration is depicted in Fig. 1. We choose the lower bound on the success probability \( P_l \geq 1 - \delta^2 = 0.8 \); \( L \) is \( 2l + 1 \) and \( l \) is the number of Grover iterations.

We make the corresponding circuits for \( l = 1 \) and \( 2 \) i.e. one and two iterations. The value of \( \alpha_i \) and \( \beta_i \) are calculated using the following relations:

\[
\alpha_j = -\beta_{j-1} = 2 \cot^{-1}(\tan(2\pi j/L)\sqrt{1 - \gamma^2}) \tag{1}
\]

where \( j = 1, 2, 3... \) and

\[
\gamma = T_{1/L}(1/\delta) \tag{2}
\]

where \( T_{1/L}(x) = \cos(L\cos^{-1}x) \) is the \( L \)th Chebyshev polynomial of the first kind.

**Single Iteration:** We put \( l = 1 \) and \( \delta = 1/\sqrt{5} \) in Eq. (1). Thus, we obtain the values:

\[ \alpha_1 = -\beta_1 = 4.4597 \]

**Double Iteration:** We put \( l = 2 \) and \( \delta = 1/\sqrt{5} \) in Eq. (1). Following are the values of corresponding parameters:

\[ \alpha_1 = -\beta_2 = 1.7156 \]
\[ \alpha_2 = -\beta_1 = 3.5443 \]

III. THE ORACLE

Our demonstration is for the equal superposition of \(|00\rangle, |01\rangle, |10\rangle \) and \(|11\rangle \) and we design our oracle to search the target state \(|00\rangle \). Our oracle \( U \) is such that \( U|00\rangle|b\rangle = |00\rangle|b \oplus 1\rangle \) where \(|b\rangle \) is the ancilla qubit. It is obvious that our oracle is nothing but a variant of the Toffoli gate which works when both the control qubits are \(|0\rangle \). A Toffoli gate can be decomposed in terms of elementary single and two qubit gates [30].

It should be noted that the circuit which we have prepared in the IBM Q Composer is the reversed version of what has been discussed in the letter. It has been done so because the ibmqx4 processor allows C-NOT gate in only one direction where the control qubit remains downwards and target qubit is directed upwards. Hence we consider \( q[2] \) as our first qubit and \( q[1] \) as the second qubit. Thus, \( q[0] \) serves the purpose of the ancilla qubit.

IV. EXPERIMENTAL PROCEDURES AND RESULTS

We first executed our circuits on the quantum simulator of the IBM Q Composer and then run it on the actual quantum processor. Results in both the cases are obtained by taking 8192 shots. As predicted, in both the cases, simulator showed the results with the state \(|00\rangle \) having the highest probability (measurement basis-ZZ) which was more than 0.8 as we had set initially. However, actual runs on quantum processors showed a noticeable difference from the simulator results although they indeed had the highest probability for the \(|00\rangle \) state when measurement was performed in the ZZ basis. One more aspect was that the deviation from the simulator results was more for two Grover’s iterations than single iteration. The reason is that in case of two iterations, more number of quantum gates are used and also the depth of the quantum circuit is considerably higher than the single iteration case. Hence, it is natural to have more errors in the later case as each gate introduces some error in the system.

Quantum state tomography is performed to check the accuracy of the experimentally prepared quantum states. Figs. 2 and 3 represent the performance of the algorithm for one and two iterations respectively.

![Fig. 2](image-url)  
**Case-I:** One iteration (a) & (b) are the simulational real and imaginary parts of the density matrix elements of the \(|00\rangle \) state; (c) & (d) are the experimentally reconstructed density matrix elements for the \(|00\rangle \) state.
FIG. 1. The quantum circuit depicting one iteration of the OFPQS algorithm. The boxes coloured red represent the Oracle and the green ones are used to prepare the initial state $|++0\rangle$.

V. CONCLUSION

To conclude, we have experimentally demonstrated here the optimal fixed-point quantum search algorithm using the 5 qubit IBM quantum computer. We have designed the equivalent quantum circuit for one and two iterations of the algorithm in the real quantum processor ibmqx4. We have explicated the working of quantum algorithm by fixing a target quantum state, which has been prepared in our experiment. We have discussed some comments about the results obtained in both the cases. The two iteration case involves a large number of gates which introduce more noise and decoherence in the system. Hence, it is experimentally observed that the target state in case of one iteration is prepared with a more fidelity as compared to the two iteration case.

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Acknowledgements BKB acknowledges the support of DST Inspire Fellowship. We are extremely grateful to IBM team and IBM Quantum Experience project.