Possible Classification of the Chiral Scalar $\sigma$-Nonet

Muneyuki ISHIDA

Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan

(Received July 22, 1998)

We recently observed the iso-singlet scalar $\sigma(600)$-particle and the iso-doublet scalar $\kappa(900)$-particle, through the re-analyses of $\pi\pi$- and $K\pi$-scattering phase shifts, respectively. First, assuming the two iso-singlet states, the $\sigma(600)$ and the established $f_0(980)$, being the ideal mixing states $n\bar{n}$ and $s\bar{s}$, respectively, of a single scalar nonet, it is pointed out that the mass value of the iso-singlet flavor-octet state, obtained from the orthogonal transformation, satisfies, together with the $\kappa(900)$ and the iso-triplet scalar $a_0(980)$-particle, the Gell-Mann Okubo mass-formula. Furthermore, it is argued that, by investigating their properties of masses and widths, this scalar $\sigma$-nonet, together with the pseudo-scalar $\pi$-nonet, realizes the linear representation of $SU(3)$ chiral symmetry.

§1. Introduction
—Observation of $\sigma(600)$ and $\kappa(900)$—

In previous works by our group, we have observed the scalar $q\bar{q}$-mesons with $I=0$ and $I=1/2$, $\sigma(600)$ and $\kappa(900)$, respectively, by the re-analyses of the $\pi\pi$- and $K\pi$-scattering phase shifts. The mass and width of the $\sigma(\kappa)$-meson are determined with the values of $m_\sigma = 540 - 675$ MeV and $m_\kappa = 905\pm 65$ MeV ($m_\kappa = 905\pm 30$ MeV and $\Gamma_{\kappa\rightarrow\pi\pi} = 545\pm 235$ MeV). Existence of these scalar mesons has been conventionally neglected for many years. The reason which led us to different results using the same data are two-fold: Technically, we have applied a new $S$-matrix parametrization method, the interfering Breit-Wigner amplitude (IA-method) for the analyses, where the amplitude is represented directly by the physically meaningful parameters, masses and widths of resonances, while physically, we have introduced a negative background phase $\delta_{BG}$ of a hard core type. This $\delta_{BG}$ represents a very strong repulsive force between pions (or pion and kaon), which strongly cancels the attractive force due to intermediate $\sigma$- (or $\kappa$-) production. The obtained $\chi^2$ value is greatly improved compared with that in the conventional analysis without the $\sigma$ (and $\kappa$) meson. The physical reason for missing the existence of these new scalar particles in the conventional phase shift analyses is due to overlooking this cancellation mechanism, which is guaranteed by current algebra and PCAC.

In addition to these new particles, there are the other established scalar mesons below 1 GeV: the iso-singlet $f_0(980)$ and the iso-triplet $a_0(980)$. The purpose of this paper is to investigate the possibility for classification of these scalar particles into a

---

* Other recent phase shift analyses also suggest the existence of light $\sigma$ and $\kappa$ particles.

** In a previous work we estimated $m_\sigma$ with the value of $585\pm 20$ MeV for the “standard” phase shift of the CERN-Münich b-analysis.
single scalar nonet, and furthermore, to study whether it is possible to ascribe this scalar \( \sigma \)-nonet to the chiral partner of the pseudo-scalar \( \pi \)-nonet in \( SU(3) \) chiral symmetry.

In §2, the Gell-Mann Okubo (GMO) mass formula is shown to be approximately satisfied for the octet members of this scalar \( \sigma \)-nonet, where, in order to identify the octet members, the OZI-rule concerning the decay property of \( f_0(980) \) is assumed. In §3.1, by investigating the relations among their masses and widths, they are suggested to have the properties of the scalar nonet predicted by the \( SU(3) \) linear \( \sigma \) model (LoM \(^{21-25}\)). Being based on these results, in §3.2 the situations of the validity of OZI rule, which was assumed in §2, is examined somewhat quantitatively. Section 4 is devoted to concluding remarks.

### §2. Scalar \( \sigma \)-nonet mass formula

Experimentally, \( f_0(980) \) has a considerably small \( \pi \pi \)-width regardless of its large phase volume, while having a rather large \( K \bar{K} \)-width in spite of the fact that its mass is quite close to the \( K \bar{K} \)-threshold. Assuming the approximate validity of the OZI rule, this fact seems to suggest that \( f_0(980) \) consists of almost pure \( s \bar{s} \)-component.

Here we simply assume that \( \sigma(600) \) and \( f_0(980) \) are the ideal mixing states of a single scalar nonet and that the squared-mass matrix takes a diagonal form in these ideal bases. The ideal-mixing states are related to the octet state \( \sigma_8 \) and the singlet state \( \sigma_1 \) through the orthogonal transformation:

\[
\begin{pmatrix}
\sigma(600) \\
f_0(980)
\end{pmatrix}
= \begin{pmatrix}
\sigma_n \\
\sigma_s
\end{pmatrix}
= O \begin{pmatrix}
\sigma_8 \\
\sigma_1
\end{pmatrix},
\quad \sigma_n \equiv \frac{u \bar{u} + d \bar{d}}{\sqrt{2}}, \quad \sigma_s \equiv s \bar{s},
\tag{2.1}
\]

where \( O \) is the matrix of the orthogonal transformation, given by

\[
O \equiv \begin{pmatrix}
\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} \\
-\sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}}
\end{pmatrix}.
\tag{2.2}
\]

Through the transformation \( O \), the elements of the squared-mass matrix in the octet-singlet bases are numerically given by

\[
\begin{pmatrix}
m_{\sigma_8}^2 \\
m_{\sigma_1}^2
\end{pmatrix}
= O \begin{pmatrix}
m_{\sigma(600)}^2 & 0 \\
0 & m_{f_0(980)}^2
\end{pmatrix} O
= \begin{pmatrix}
(0.87 \text{ GeV})^2 & -(0.54 \text{ GeV})^2 \\
-(0.54 \text{ GeV})^2 & (0.74 \text{ GeV})^2
\end{pmatrix},
\tag{2.3}
\]

where we have used the experimental values \( m_{\sigma} = 0.59 \text{ GeV} \) and \( m_{f_0(980)} = 0.98 \text{ GeV} \).\(^*\)

\(^*\) The fact that \( m_{\sigma_1} \) is smaller than \( m_{\sigma_8} \) is in contrast with the case of pseudoscalar \( \eta-\eta' \) mass splitting. This fact reflects the property of the \( U_A(1) \)-breaking interaction (see, §3). In this connection, note that the famous nonet mass formula \(^{26}\) of the vector mesons, \( m_{\rho}^2 = m_{\omega}^2 \) and \( m_{\phi}^2 - m_{K^*}^2 = m_{K^*}^2 - m_{\rho}^2 \), is valid in the case \( m_{\eta_1}^2 = m_{\eta_8}^2 \).
The mass $m_{\sigma^8}$ can also be determined theoretically by using the Gell-Mann Okubo (GMO) relation,

$$m_\kappa^2 = (3m_{\sigma^8}^{\text{theor}} + m_{a_0}^2)/4,$$

as

$$m_{\sigma^8}^{\text{theor}} = 0.88 \text{ GeV},$$

where we have used the experimental values $m_\kappa = 0.91 \text{ GeV}$ and $m_{a_0(980)} = 0.98 \text{ GeV}$. This value of $m_{\sigma^8}^{\text{theor}}$ is quite close to $m_{\sigma^8} = 0.87 \text{ GeV}$, obtained phenomenologically in Eq. (2.3). This fact supports our classification that $\sigma(600)$, $f_0(980)$, $\kappa(900)$ and $a_0(980)$ form a single scalar nonet.

§3. Chiral symmetry and properties of the $\sigma$-nonet in relation with the $\pi$-nonet

3.1. Chiral symmetry and the mass and width of the scalar nonet

We assume that our scalar $\sigma$-nonet is a composite $q\bar{q}$-system as a chiral partner of the pseudoscalar $\pi$-nonet and that in the low energy region, where the structure of composite mesons can be neglected, they can effectively be described by the linear $\sigma$ model ($L\sigma M$). Let $B \equiv s + i\phi$ ($s \equiv \lambda_i s_i/\sqrt{2}$ and $\phi \equiv \lambda_i^\dagger \phi_i/\sqrt{2}$ denoting the scalar and pseudoscalar meson nonet, respectively), the Lagrangian of $SU(3)L\sigma M^{25, 24}$ is

$$L_{\sigma M} = \frac{1}{2}\langle \partial_\mu B \partial^\mu B^\dagger \rangle - \frac{\mu^2}{2} \langle BB^\dagger \rangle - \frac{\lambda_1}{4} \langle (BB^\dagger)^2 \rangle - \frac{\lambda_2}{2} \langle (BB^\dagger)^2 \rangle + \kappa_d (\det B + \det B^\dagger) + \langle f_s \rangle,$$  

(3.1)

where $\langle \rangle$ represents the trace. Here $f$ is proportional to the current-quark mass matrix in QCD and the form $f = \text{diag} \{f_n, f_n, f_s\}$ guarantees the PCAC. In the process of spontaneous chiral symmetry breaking, $s$ acquires the vacuum expectation value $s_0 \equiv \Sigma = \text{diag} \{a, a, b\}$, and $s\phi\phi$-couplings appear. The pseudoscalar decay constants $f_\pi$ and $f_K$, and their ratio are represented by

$$f_\pi = \sqrt{2}a, \ f_K = \frac{a + b}{\sqrt{2}}; \ \frac{f_K}{f_\pi} = \frac{a + b}{2a}.$$  

(3.2)

The six model parameters contained in Eq. (3.1) are determined by the masses of $\pi, \eta, \eta'$, $\sigma$ and $\kappa$, and the decay constant $f_\pi$, and thus we can predict the masses and widths of the scalar mesons. These are given in Table I. The predicted properties are very sensitive to the value of $f_K/f_\pi$; as shown in Fig. 1. The deviation of the value of $f_K/f_\pi$ from 1 represents the degree of $SU(3)$ breaking by $s_0$, as can be seen from Eq. (3.2). We prefer the region in which this ratio satisfies $1.329 < f_K/f_\pi < 1.432$ (which is somewhat larger than the experimental value $1.22$) indicated by the two vertical lines in the figure, where the value $m_{\kappa}^{\text{theor}}$ reproduces the experimental value within its uncertainty. In this region, $\Gamma_\sigma$ and $\Gamma_\kappa$ are obtained with much larger
values than those of $\Gamma_{\sigma'}$ and $\Gamma_{\delta}$. The reason that $\Gamma_{\kappa \rightarrow K\pi}$, in spite of its comparatively smaller phase space, becomes as large as $\Gamma_{\sigma \rightarrow \pi\pi}$ is due to the contribution to the coupling constant $g_{\kappa K\pi}$ from the determinant-type interaction in Eq. (3.1).

The predicted widths of $\sigma$ and $\kappa$ are consistent with the experimental values. The predicted masses and widths of the other members, $\delta(I = 1)$ and $\sigma'(ss)$, are close to those of $a_0(980)$ and $f_0(980)$, respectively. The $\sigma'$ and $\delta$ states have large $K\bar{K}$-coupling constants.\(^{2)}\) (Especially the $\sigma'$ strongly couples to the $K\bar{K}$ channel.) This suggests that these states may also be interpreted as $K\bar{K}$-molecule states.\(^{20)}\)

In $L\sigma M$, $\sigma$ and $\delta$ have almost the same quark content. Despite this fact, $m_{\delta}$

---

\(^{2)}\) In the case $f_{\pi} = 93$ MeV, $f_{K}/f_{\pi} = 1.394$ and $m_{\sigma} = 585$ MeV, for $\sigma'$, $\mathcal{L}_{\text{int}} = g_{\sigma'\pi\pi}\sigma'\pi^2 + g_{\sigma'KK}\sigma'(K^+K^- + K^0\bar{K}^0)$, $g_{\sigma'\pi\pi} = -0.02$ GeV, and $g_{\sigma'KK} = -4.97$ GeV.

For $\delta$, $\mathcal{L}_{\text{int}} = g_{\delta\pi\eta}(\pi^-\delta^+ + \pi^+\delta^- + \pi^0\delta^0)\eta + g_{\delta\bar{K}K}(\delta^0K^+ - \delta^-K^- + \delta^+\bar{K}^0K^0)\sqrt{2} + g_{\delta\pi\pi}\pi^2 + g_{\delta\bar{K}K}K^0\bar{K}^0 + g_{\delta\eta}\eta^2$, $g_{\delta\pi\eta} = -3.12$ GeV, and $g_{\delta\bar{K}K} = -3.19$ GeV.
Table I. The properties of the scalar meson nonet predicted by SU(3)LσM, compared with experiments. The underlined values of $m_{\sigma}$ and $m_{\kappa}$, along with $f_{\pi}$, $m_{\pi}$, $m_{\eta}$ and $m_{\eta'}$ are used as inputs. The region of the value of $m_{\kappa}^{\exp}$ corresponds to the region in which the ratio of the decay constants satisfies $1.329 < f_{K}^{L\sigma M}/f_{\pi}^{L\sigma M} < 1.432$. The properties of $\delta$ and $\sigma'$ become close to those of the observed resonances $a_{0}(980)$ and $f_{0}(980)$, respectively, taken as the experimental candidates. The quantity $I_{\sigma'-\to K\bar{K}}^{\mathrm{theor}}$ is the partial width $I_{\sigma'-\to K\bar{K}}^{\mathrm{theor}}$. The value of $I_{\sigma'-\to K\bar{K}}^{\mathrm{theor}}$ is highly dependent on $m_{\sigma'}^{\mathrm{theor}}$, since $m_{\sigma'}^{\mathrm{theor}}$ is close to $K\bar{K}$-threshold.

|   | $m_{\sigma}^{\mathrm{theor}}/\text{MeV}$ | $m_{\sigma}^{\exp}/\text{MeV}$ | $I_{\sigma}^{\mathrm{theor}}/\text{MeV}$ | $I_{\sigma}^{\exp}/\text{MeV}$ |
|---|---|---|---|---|
| $\sigma$ | $535 \sim 650$ | $535 \sim 650$ | $400 \sim 800$ | $385 \pm 70$ |
| $\kappa$ | $905^{+65}_{-30}$ | $905^{+65}_{-30}$ | $300 \sim 600$ | $545^{+235}_{-110}$ |
| $\delta = a_{0}(980)$ | $900 \sim 930$ | $982.7 \pm 2.0$ | $110 \sim 170$ | $57 \pm 11$ |
| $\sigma' = f_{0}(980)$ | $1030 \sim 1200$ | $993.2 \pm 9.5$ | $0 \sim 300$ | $67.9 \pm 9.4$ |

becomes much larger than $m_{\sigma}$. This phenomenon is explained by the properties of the instanton-induced $U_{A}(1)$-breaking determinant-type interaction.\(^{(*)}\)

Thus, it may be plausible to regard $\sigma(600)$, $\kappa(900)$, $a_{0}(980)$, and $f_{0}(980)$ as members of the scalar nonet, forming with the members of $\pi$-nonet a linear representation of the SU(3) chiral symmetry.\(^{(**)}\)

All the above results are obtained in the tree level bases of the LσM. Following the renormalization procedure of Chan and Haymaker,\(^{24}\) we have made a preliminary estimate of the one-loop effects with the present renewed experimental data. The width at the one-loop level is defined from the imaginary part of the inverse propagator of the relevant particle, and it is almost equal to the value given in Table I. The masses of the scalar nonet (especially the $m_{\sigma'}$) is affected by the comparatively large one-loop effect, and $m_{\sigma'}^{1\text{-loop}}$ becomes much larger than $m_{f_{0}(980)}$. We might expect that introduction of the form factor, reflecting the composite structure of the relevant meson system, decreases the effects and leads to the improvement.

Another problem of our scalar assignment is, as mentioned above, our theoretical value of $f_{K}/f_{\pi}$ is\(^{(***)}\) somewhat larger than the experimental one.

3.2. The GMO mass formula, OZI rule and effective linear $\sigma$ model

In §2 it is shown that the GMO mass formula is approximately satisfied for the octet members of our $\sigma$-nonet of scalar mesons, where $\sigma(600)$ and $f_{0}(980)$ are assumed to be ideal mixing states of isoscalars $f_{8}$ and $f_{1}$. This assumption was motivated by reasoning concerning the decay properties of $f_{0}(980)$, based on the approximate validity of the OZI rule.

In §3.1, by using the LσM at the tree level, the properties of the $\sigma$-nonet are analyzed quantitatively. In this sub-section, based on the results of §3.1, the validity of the GMO mass formula and the OZI rule in LσM is examined.

In the LσM Eq. (3.1), since the explicit symmetry breaking term $\langle f_{8} \rangle$ is introduced in the $T_{3}^{2}$-breaking pattern, the GMO mass formula is expected to be

\(^{(*)}\) However, see another viewpoint on this problem given by Jaffe.\(^{30}\)

\(^{(**)}\) This interesting assignment was suggested and insisted upon repeatedly by M. D. Scadron.\(^{28}\)

\(^{(***)}\) A possible solution may be given by taking into account the pseudoscalar-axial vector mixing effect, which induces the field renormalization of pseudoscalars (see Ref. 16).
satisfied. On the other hand, the validity of the OZI rule for sφφ couplings is generally not guaranteed in the LσM. In Eq. (3.1), the two terms with the coefficients λ₁ and κ₃ give generally OZI-forbidden sφφ-couplings. In the following, among their component couplings, we consider only the coupling for the octet pseudoscalar meson (that is, ⟨φ⟩ = 0), since only the ππ and KK decay channels are relevant. These terms include the following:

\[-2\kappa₃(s'φ²) + \kappa₃(s')⟨φ²⟩ - \lambda₁(Σs')⟨φ²⟩.\]  (3.3)

In Eq. (3.3), the second and third terms are OZI-forbidden. They are explicitly represented in the ideal bases (σₙ and σₚ), and Eq. (3.3) is rewritten into the form

\[-2\kappa₃(s'φ²) - \{\sqrt{2}(λ₁a - κ₃d)σₙ + (λ₁b - κ₃d)σₚ}\}⟨φ²⟩.\]  (3.4)

On the other hand, in the ideal bases, the elements of the squared mass matrix of the iso-singlet scalar mesons are given by

\[
\begin{align*}
m_{σₙσₙ}^2 &= μ² + λ₁(2a² + b²) + 4λ₁a² + 6λ₂a² - 2κ₃b \\
m_{σₚσₚ}^2 &= μ² + λ₁(2a² + b²) + 2λ₁b² + 6λ₂b² \\
m_{σₙσₚ}^2 &= 2\sqrt{2}a(λ₁b - κ₃d).
\end{align*}
\]  (3.5)

In the case of λ₁b = κ₃d, the value of m²σₙσₚ vanishes and the physical σ(σ') becomes identical to the ideal state σₙ(σₚ). Moreover, in this case for σₚ, as shown in Eq. (3.4), the OZI-forbidden second and third terms in Eq. (3.3) cancel each other, and the OZI-allowed first term predicts g₃σₚπ = 0, giving the vanishing decay width Γ₃σₚπ = 0. In Fig. 1 this case corresponds to the zero-points on the f₉/f₇-axis of the Γ₃σₚπ-curves. Furthermore, in this case for σₙ, the second and third terms approximately cancel each other, and their sum becomes small comparatively to the first term, and the OZI-rule is almost satisfied.

The region of the values of f₉/f₇ chosen in §3.1 is close to this case (λ₁b = κ₃d), and thus the OZI-forbidden term is expected to be small. For example, in the case that f₇ = 0.093 GeV, f₉/f₇ = 1.394 and mₗ = 0.585 GeV, the parameters a, b, λ₁ and κ₃d are determined, respectively, as 0.0658 GeV, 0.1176 GeV, 13.03 and 1.518 GeV. The coefficients of the OZI-forbidden terms for σₚ and σₙ in Eq. (3.4) are given, respectively, by λ₁b - κ₃d = 0.0135 GeV and λ₁a - κ₃d = -0.662 GeV. These values are much smaller than the coefficients 2κ₃d = 3.04 GeV of the OZI-allowed first term in Eq. (3.4), which may be considered as a typical OZI-allowed coupling term in LσM.

Thus we may conclude that the situation of the approximate validity of the OZI rule, which was the basis of our assignment of σ and f₇(980) in §2, is actually realized in SU(3) LσM with our classified members.

*) Due to the T₃²-breaking pattern of f, s acquires the vacuum expectation value s₀ = Σ = \text{diag}(a, a, b) of the T₃²-breaking pattern. Correspondingly, actual scalar and pseudoscalar mass spectra include, through the λ₁, λ₂ and κ₃d-terms, higher order effects of the T₃²-breaking of the f matrix, and strictly GMO mass formula is expected to be valid only approximately in the LσM.
In this paper we have investigated the possibility of classification of the new scalar nonet, $\sigma(600)$, $\kappa(900)$, $a_0(980)$, and $f_0(980)$. First by assuming the approximate validity of the OZI rule for the decay property of $f_0(980)$, the $\sigma(600)$ and $f_0(980)$ were supposed to be ideal mixing states, $n\bar{n}$ and $s\bar{s}$, respectively, of the scalar $\sigma$-nonet. It was then pointed out that the mass value of the iso-singlet flavor-octet state, obtained from the orthogonal transformation, satisfies the Gell-Mann Okubo mass formula. Furthermore, it was shown that the experimental masses and widths of members of this scalar $\sigma$-nonet are consistent with those of the scalar nonet predicted by (the tree level calculation of) the $\Lambda\sigma\text{M}$, as shown in Table I.

This result implies that the chiral symmetry plays a stronger role than ever thought in understanding the strong interaction, not only for deriving the low energy theorems through the non-linear realization, but also for explaining the spectroscopy and reactions related to all the mesons with masses below and around $\sim 1$ GeV through the linear realization.

We now give supplementary discussions related to the present problem. It is often argued that the validity of the $\Lambda\sigma\text{M}$ and the existence of the $\sigma$-meson as a chiral partner of the $\pi$-meson are not acceptable, since the phenomenological pattern of the ten low energy coefficients of the $O(p^4)$-level of chiral perturbation theory (ChPT)\(^\text{31-35}\) is not reproduced by the $\Lambda\sigma\text{M}$. Surely, the framework of ChPT is useful for relating phenomenologically the various low energy phenomena concerning the Nambu-Goldstone $\pi$-meson octet mutually, with the ten parameters. However, the above argument seems too excessive and not appropriate, since it is based only on the results of analyses of indirect experiments with much lower energy than $m_\sigma$. Whether or not a resonance exists should be investigated directly by experiments with sufficiently high energy to produce the relevant resonance. As a matter of fact, $\sigma(600)$ can now be directly observed both in $\pi\pi$ scattering and in production processes.\(^\text{36-40} \text{3, 41-43}\) Furthermore, the parameters in the $\Lambda\sigma\text{M}$ describe the physics in the resonance energy region as well as the low energy region, and the contribution of the $\sigma$-meson to low energy quantities can be predicted with no new free parameters.\(^\text{*}\)

On the other hand it has been discussed\(^\text{45}\) that in the framework of ChPT the effect of the $\sigma$-meson can be taken into account through the $O(p^4)$ and $O(p^6)$ Lagrangian. However, such an approach seems clearly to have no predictive power regarding the properties of the scalar $\sigma$-meson nonet itself as a chiral partner of the $\pi$-nonet.

Finally we would like to mention that, as we have pointed out previously,\(^\text{1, 27}\) the $\sigma$-nonet treated in this paper should be discriminated from the $^3P_0$-nonet. The $\sigma$-nonet is assigned the quantum numbers $(L, S) = (0, 0)$ in the “relativistic $LS$-
coupling scheme.”

Acknowledgements

I am grateful to Professor M. Oka for his continual encouragement. The contents
of the present work are closely connected with a series of works done in cooperation
with the theoretical physics group of Nihon University. I would like to thank Pro-
fessor S. Ishida for many useful comments, and I would like to express my sincere
appreciation to all members of the NU-group. I would also like to thank Professor
K. Takamatsu, Professor T. Tsuru and Doctor T. Ishida for suggesting many ideas.
I am grateful to Professor K. Yazaki, Professor S. Wada, Professor J. Arafune, Pro-
fessor K. Ohta and Professor T. Kobayashi for their valuable comments, especially
on the mass formula given in §2.

References

1) S. Ishida, M. Y. Ishida, H. Takahashi, T. Ishida, K. Takamatsu and T. Tsuru,
Prog. Theor. Phys. 95 (1996), 745.
2) S. Ishida, T. Ishida, M. Y. Ishida, K. Takamatsu and T. Tsuru, Prog. Theor. Phys. 98
(1997), 1005.
3) Plenary talk of S. Ishida on “On Existence of \(\sigma(600)\)-Particle — Its Physical Implications
and Related Problems ——”, Int. Conf. on Hadron 97, Brookhaven National Laboratory,
ed. by S. U. Chung and H. J. Willutzki, AIP conference proceedings 432, Upton NY, 1997.
4) S. Ishida, M. Y. Ishida, T. Ishida, K. Takamatsu and T. Tsuru, “Re-analysis of \(\pi\pi/K\pi-
phase shift and Existence of \(\sigma(600)/\kappa(900)\)-Particle,” in Proc. of Int. Conf. on Hadron 97,
ed. by S. U. Chung and H. J. Willutzki, AIP conference proceedings 432, Upton NY, 1997.
5) S. Ishida, M. Y. Ishida, T. Ishida, K. Takamatsu and T. Tsuru, Prog. Theor. Phys. 98
(1997), 621.
6) N. N. Achasov and G. N. Shestakov, Phys. Rev. D49 (1994), 5779.
7) R. Kamiński, L. Lesniki and J.-P. Maillet, Phys. Rev. D50 (1994), 3145.
8) N.A. Törnqvist, Z. Phys. C68 (1995), 647.
N.A. Törnqvist and M. Roos, Phys. Rev. Lett. 76 (1996), 1575.
9) M. Harada and F. Sannino and J. Schechter, Phys. Rev. D54 (1996), 1991.
10) E. Beveren, T. A. Rijken, K. Metzger, C. Dullemen, G. Rupp and J. E. Ribeiro, Z. Phys.
C30 (1986), 615.
11) D. Morgan, “Status of the \(0^+\) Nonet”, Proc. of Argonne conf. 1975, p. 45.
12) K. L. Au, D. Morgan and M. R. Pennington, Phys. Rev. D35 (1987), 1633.
D. Morgan and M. R. Pennington, Phys. Rev. D48 (1993), 1185.
13) M. R. Pennington, in Proceedings of Int. Conf. Hadron ’95, Manchester UK, July 1995
(Word Scientific, Singapore), p. 3.
14) G. Grayer et al., Nucl. Phys. B75 (1974), 189.
B. Hyams et al., Nucl. Phys. B64 (1973), 134.
15) D. Aston et al., Nucl. Phys. B296 (1988), 493.
P. Estabrooks et al., Nucl. Phys. B133 (1978), 490.
N. Awaji, Doctor Thesis of Nagoya University (1986).
16) M. Y. Ishida and S. Ishida, “Existence of \(\sigma(600)/\kappa(900)\)-Particle and New Chiral Scalar
Nonet, “Chiralons”,” in Proc. of Int. Conf. on Hadron 97, ed. by S. U. Chung and H. J.
Willutzki, AIP conference proceedings 432, Upton NY, 1997.
17) O. Endo, I. Shimodaya and J. Hiura, Prog. Theor. Phys. 31 (1964), 1157.
P. Darriulat, G. Igo, H. G. Pugh and H. D. Holmgren, Phys. Rev. 137 (1965), B315.
18) M. Taketani et al., Prog. Theor. Phys. Suppl., No. 39 (1990); No. 42 (1968). In particular
see Chapter 7 (S. Otsuki, No. 42, p. 39) and also Chapter 6 (N. Hoshizaki, No. 42, p. 1).
19) V. G. Neudatchin, Yu. F. Smirnov and R. Tamagaki, Prog. Theor. Phys. 58 (1977), 1072.
M. Oka and K. Yazaki, Prog. Theor. Phys. 66 (1981), 556, 572.
Possible Classification of the Chiral Scalar $\sigma$-Nonet

20) M. Y. Ishida, Prog. Theor. Phys. \textbf{96} (1996), 853.
21) J. Schwinger, Ann. of Phys. \textbf{2} (1957), 407. M. Gell-Mann and M. Lévi, Nuovo Cim. \textbf{16} (1960), 705.
22) G. Kramer, Phys. Rev. \textbf{177} (1968), 2515.
23) J. L. Basdevant and B. W. Lee, Phys. Rev. \textbf{D2} (1970), 1680.
24) L. H. Chan and R. W. Haymaker, Phys. Rev. \textbf{D7} (1973), 402; Phys. Rev. \textbf{D10} (1974), 4143, 4170.
25) G. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. \textbf{3} (1969), 531.
26) S. Okubo, Phys. Lett. \textbf{5} (1963), 165.
27) M. Y. Ishida, Prog. Theor. Phys. \textbf{96} (1996), 853.
28) M. Y. Ishida, Nucl. Phys. \textbf{A629} (1998), 148c.
29) J. Weinstein and N. Isgur, Phys. Rev. \textbf{D27} (1983), 588.
30) R. L. Jaffe, Phys. Rev. \textbf{D15} (1977), 267, 281.
31) S. Weinberg, Physica \textbf{96A} (1979), 327.
32) J. Gasser and H. Leutwyler, Ann. Phys. \textbf{158} (1984), 142; Nucl. Phys. \textbf{B250} (1985), 465, 517.
33) T. H. R. Skirme, Proc. Roy. Soc. London Ser. \textbf{A260} (1961), 127; Nucl. Phys. \textbf{31} (1962), 556.
34) A. Pich, CERN-TH.6978/93; hep-ph/9308351; Proc. of Particle and fields, Guanajuato 1992, p. 95.
35) J. F. Donoghue, E. Golowich and B. R. Holstein, \textit{Dynamics of the Standard Model}, (Cambridge university press, 1992).
36) H. Shimizu, \textit{contributed paper on Particle and Nuclei XIII International Conference - PANIC XIII, Perugia, Italy, June 1993}.
37) D. Alde et al., Phys. Lett. \textbf{B397} (1997), 350.
38) T. Ishida, in \textit{Proceedings of Int. Conf. Hadron '95, Manchester UK, July 1995} (World Scientific, Singapore), p. 451.
39) T. Ishida, Doctor thesis, “On Existence of $\sigma$(555) Particle—Study in $pp$-central collision reaction and Re-analysis of $\pi\pi$-scattering phase shift—,” Univ. of Tokyo (1996); KEK Report 97-8(1997).
40) J. E. Augstin et al., Nucl. Phys. \textbf{B320} (1989), 1.
41) K. Takamatsu, M. Y. Ishida, S. Ishida, T. Ishida, and T. Tsuru, “$\sigma$-Particle in Production Processes,” in \textit{Proc. of Int. Conf. on Hadron 97}, ed. by S. U. Chung and H. J. Willutzki, \textit{AIP conference proceedings 432, Upton NY, 1997}.
42) M. Y. Ishida, S. Ishida and T. Ishida, “Relation Between Scattering and Production Amplitudes — Case of Intermediate $\sigma$-Particle in $\pi\pi$-System —,” in \textit{Proc. of Int. Conf. on Hadron 97}, ed. by S. U. Chung and H. J. Willutzki, \textit{AIP conference proceedings 432, Upton NY, 1997}.
43) C. Amsler et al. (Crystall Barrel Collaboration), Phys. Lett. \textbf{B355} (1995), 425.
44) E. P. Shabalin, Yadern. Fiz. \textbf{41} (1985), 260; \textbf{48}(1988), 272; \textbf{49}(1989), 588 (Sov. J. Nucl. Phys. \textbf{42} (1985), 164; \textbf{48}(1988), 172; \textbf{49}(1989), 365).
45) U. G. Meissner, Comm. Nucl. Part. Phys. \textbf{20} (1991), 119.