Structural transformation of reinforced concrete structural system at sudden loss of stability of one of its elements

Nataliya Fedorova¹ and Sergey Savin²

¹Moscow State University of Civil Engineering, Yaroslavskoe shosse, 26, Moscow, 129337, Russia
²South West State University, street 50 let Oktyabrya, 94, Kursk, 305040, Russia

E-mail: ¹FedorovaNV@mgsu.ru

Abstract. Reconstruction or changing of the functional purpose of operated buildings and structures buildings are requires an assessment of its residual life, which should be made on the basis of the modern standards and building codes, the requirements of which were greatly expanded by the range of possible loads and impacts, including especial emergency impacts. Analysis of collapses and accidents of buildings and structures, that occurred in Russia and abroad in recent years, shows that the progressing in time accumulation of damage in structural elements (for example, in columns, in compressed belts of trusses, etc.) may lead to sudden loss of stability of one of the bearing structural elements of the system. The instant structural transformation of the system, which occurs at this moment, may produce total loss of stability of the whole structural system. At the same time this process in each element may be determined by the different physical phenomena: internal force acting or kinematic impacts from nearby elements. Academicians of RAACS A.V. Alexandrov and V.I. Travush proposed to use the work $A(M,Q)$ of nodal bending moments and shear forces as an energy criterion to assess the kind of the buckling of each element of the structural system. In this case bifurcation, which is determined by the internal force acting, should be considered as the active loss of stability, the work of which is negative (i.e. $A < 0$), buckling caused by the kinematic impacts from nearby elements is called passive loss of stability and has the positive work $A > 0$. Following these studies, in this paper the analytical relationships to assess the kind of buckling of individual nonlinear deformable concrete elements of the frame structural system with different variants of boundary conditions are obtained. The solution of these equations is given and it can be used to assess stability of reinforced concrete (RC) columns of the frame, damaged by corrosion. Herewith the constructive elements are considered as part of the whole structural system. Structural transformation of RC frame at sudden buckling of one of the bearing structural element subjected corrosion is analysed. The results of this study can be used to calculate frames of the reconstructed buildings and structures (in particular at the emergency impacts) for assessment of their sustainability, protection against the progressive collapse, as well as evaluation of their residual life of operated structures.

1. Introduction

During the last decades, scientific direction, connected with the development and improvement of methods of calculation of buildings and structures against the progressive collapse and development of measures and recommendations for protection against such phenomena, are actively developed in
Russia [1-7] and abroad [8-17]. In the general case, these studies deal with the assessment of survivability of structural systems of buildings and structures under force and environmental impacts [18]. Here, as it determined in GOST 27.002-89, we mean the survivability as a property of an object to resist the development of critical failures, caused by defects and damage in installed system maintenance and repairs, or the property of an object to maintain limited functionality at impacts, which are not included by operating conditions, or a property of an object to maintain limited functionality at defects or damage of a certain type, as well as failure of some components.

Analysis of collapses and accidents of buildings and structures, that occurred in Russia and abroad in recent years, shows that the progressing in time accumulation of damage in structural elements (for example, in columns, in compressed belts of trusses, etc.) may lead to sudden loss of stability of one of the bearing structural elements of the system. The instant structural transformation of the system, which occurs at this moment, may produce total loss of stability of the whole structural system. At the same time this process in each element may be determined by the different physical phenomena: internal force acting or kinematic impacts from nearby elements. Academicians of RAACS A.V. Alexandrov and V.I. Travush proposed to use the work \( A(M, Q) \) of nodal bending moments and shear forces as an energy criterion to assess the kind of the buckling of each element of the structural system. In this case bifurcation, which is determined by the internal force acting, should be considered as the active loss of stability, the work of which is negative (i.e. \( A < 0 \)), buckling caused by the kinematic impacts from nearby elements is called passive loss of stability and has the positive work \( A > 0 \).

In the present paper, we propose methodology and algorithm of structural system transformation analysis for RC structural system, when the structural element with the critical value of corrosion damage lose stability.

2. **Evolving in time damage of reinforced concrete element, exposed to corrosion**

According to studies by V. M. Bondarenko [18, 20], corrosion damage of reinforced concrete elements depends on the characteristics of physical or chemical effects, the composition of the concrete, as well as the level of the stress state \( \sigma / R_b \), and may evolve in time and through the depth of the lesion according to three scenarios:

1. entropy, fading in time, with the stabilization at the certain critical (limiting) depth of damage (depth of neutralization) with colmatation transformation of the concrete structure;
2. linear process with constant speed of the damage front advancing, without stabilization, for which the parameter \( \delta_{cr} \) changes the initial meaning and becomes some empirical parameter of the damage process;
3. avalanche process, intensifying in time with the inevitable failure of materials and structures.

Here \( \sigma \) is normal stresses, acting in the section; \( R_b \) is ultimate compressive strength of concrete; \( \delta_{cr} \) is empirical parameter of damage process. Parameter \( \delta_{cr} \) is linked with value of loading and takes the ultimate depth value of damage of compressed concrete for colmatation type of damage.

In accordance with V. M. Bondarenko hypothesis of phenomenological similarity of kinetics of not equilibrium processes of damage and creep strain development of concrete, these types of processes can be presented by unified mathematical equations (1), the graphical representation of which is shown in figure 1, (a):

\[
\frac{d[\Delta L(t, t_0)]}{dt} = \alpha[\Delta L(t, t_0)]^n, \tag{1}
\]

where \( \Delta L(t, t_0) \) is current value of damage depth, determined from formula:

\[
\Delta L(t, t_0) = \frac{L_{cr} - L(t, t_0)}{L_{cr}(t_0)}, \tag{2}
\]
Figure 1. Scheme of damage kinetic of compressed concrete (a); cross section of RC element exposed to corrosion (b); general form of the corrosion damage function $K(z)$ (c): 1 - field of avalanche process of damage ($m > 0$); 2 – field of fading damage process; 3 – boundary line ($m = 0$)

Let us introduce designation $\Delta L(t, t_0) = f(t)$.

Then equation (1) takes following form

$$\frac{df}{dt} = -\alpha f^{-m}.$$  \hspace{1cm} (3)

Dividing variables and then integrating expression (3), we obtain:

$$\frac{f^{1-m}}{1-m} = -\alpha t + C,$$  \hspace{1cm} (4)

where $C$ is integration constant.

In result, unknown function $f$ takes the form:

$$f = \left[\alpha(m-1) t + C\right]^{1/(1-m)},$$  \hspace{1cm} (5)

Using formula (2) we find

$$L(t, t_0) = L_{ult} \left\{1 - \left[\alpha(m-1) t + C\right]^{1/(1-m)}\right\}.$$  \hspace{1cm} (7)

Integration constant can be found using initial conditions
\[ \Delta L(t_0, t_0) = [\alpha(m - 1) \cdot t_0 + C]^{1/(m-1)} = \frac{L_{ult} - L(t_0 \cdot t_0)}{L_{ult}} = 1 - \frac{L(t_0 \cdot t_0)}{L_{ult}}. \]

Therefore

\[ C = \left[ 1 - \frac{L(t_0 \cdot t_0)}{L_{ult}} \right]^{1-m} - \alpha(m - 1) \cdot t_0. \] (8)

Introducing (8) in (7), we obtain

\[ L(t, t_0) = L_{ult} \left\{ 1 - \left[ \alpha(m - 1)(t - t_0) + \left(1 - \frac{L(t_0 \cdot t_0)}{L_{ult}} \right)^{1-m} \right]^{1/(1-m)} \right\}. \] (9)

Since the initial equation (1) based on the constancy of loading condition, thermodynamical, physical and chemical factors of external impacts, than \( L_{ult}, \alpha, t_0, L(t_0) \) are constant and \( m \) is parameter of time \( t \), which changes abruptly, i.e.

for \( t_0 \leq t_1 \leq t_{cr}, \quad m = m_0, \) (10)

for \( t_1 \leq t_{cr}^d, \quad m = m_1, \) (11)

then function \( L(t, t_0) \) takes the form

\[ L(t, t_0) = L_{ult} \left\{ 1 - \left[ \alpha(m_0 - 1)(t - t_0) + \left(1 - \frac{L(t_0 \cdot t_0)}{L_{ult}} \right)^{1-m_0} \right]^{1/(1-m_0)} \right\}, \quad t_0 \leq t < t_1; \] (12)

and

\[ L(t, t_0) = L_{ult} \left\{ 1 - \alpha(m_1 - 1)(t - t_0) - \left(1 - \frac{L(t_0 \cdot t_0)}{L_{ult}} \right)^{1-m_1} \right\}, \quad t \geq t_1. \] (13)

It’s should be noted, that \( t_0 \) and \( t_1 \) are time of beginning of observation for the structural element after first loading step and time after additional loading (for example after instant failure or loss of stability of some structural element [21]) respectively.

Relationships (21) and (13) are illustrated by graphics in figure 1, (a).

For the specific problem of survivability assessment of structure with the accumulated of corrosion damage the damaged depth of the section of reinforced concrete element \( \delta(t, t_0) \) or the initial measure of creep of concrete \( C(t, t_0) \) can be used instead the parameter \( L \). Moreover, for \( m > 0 \) relationship (9) corresponds to entropically fading kinetic of process \( L \); for \( m < 0 \) it corresponds to not equilibrium avalanche development of process; for \( m = 0 \) it corresponds to linear changes in time (filtration kinetic).

If the function of depth of corrosion damage in time is determined, reduced bending rigidity \( B_{red}(\delta(t)) \) (figure 1, (b)) can be written in the form:

\[ B_{red}(\delta(t)) = E_t \cdot I_{red}(\delta(t)). \] (14)

where \( E_t \) is variable strain modulus of concrete, which depends on deformation, caused by loading and creep [20].
\( I_{\text{red}}(\delta(t)) \) is reduced inertia moment of section:

\[
I_{\text{red}}(\delta(t)) = A_r r_c^2 + \sum_{i=1}^{l} A_{\text{icn},i} r_c^2 + I_n + \sum_{i=1}^{l} I_{\text{icn},i} + \frac{\omega_t}{\psi_s} \cdot \alpha \cdot A_{r} r_c^2 ,
\]

(15)

where \( A_r \) is area of part of section without corrosion damage; \( A_{\text{icn},i} \) is area of \( i \)-th part of section exposed to corrosion, which is determined in accordance to function of corrosion damage \( K(z) \) as it is shown in figure 1, (c); \( A_{r} \) is area of reinforcement cross section; \( r_c, r_{\text{icn},i} \) are distance from center of mass of not damaged section to center of mass of whole section and distance from center of mass of \( i \)-th part of section damaged by corrosion to center of mass of whole section, which is determined in accordance to function of corrosion damage \( K(z) \), respectively; \( r_s \) is distance from center of mass of reinforcement cross section to center of mass of whole section; \( I_n \) is inertia moment of not damaged section with respect to own center of mass; \( I_{\text{icn},i} \) is inertia moment of \( i \)-th part of damaged section with respect to own center of mass; \( \omega_t \) is coefficient of corrosion damage of reinforcement; \( \psi \) is coefficient of influence of tensiled concrete; \( \alpha = E_i / E_s \) is coefficient of reduction of reinforcement to concrete.

3. Stability of reinforced concrete rod element

Let us consider compressed reinforced concrete rod, which have an arbitrary boundary condition at one edge and rigidly clamped on the opposite. Following to [22], stability equation in not dimensional parameters takes the form:

\[
\frac{d^4 \bar{w}}{d\xi^4} + k^2 \frac{d^2 \bar{w}}{d\xi^2} = 0 ,
\]

(16)

where

\[
\xi = \frac{x}{l} , \quad \bar{w} = \frac{w}{l} , \quad k^2 = \frac{Nl^2}{B_{\text{red}}} ,
\]

(17)

In the formula (17), \( B_{\text{red}} \) is reduced bending rigidity of RC element, determined in accordance with (14); \( x \) is coordinate of undeformed axis, which takes count from rigidly clamped edge; \( w = w(x) \) is deflection of axis from undeformed state; \( l \) is length of rod element; \( N \) is longitudinal force.

Solution of the equation (16) in accordance to the initial parameters method takes the form:

\[
\bar{w} = \bar{w}_0 + \bar{w}' \xi + \bar{w}'' \frac{1 - \cos k \xi}{k^2} + \bar{w}''' \left( \frac{\xi}{k^2} - \frac{\sin k \xi}{k^3} \right) ,
\]

\[
\bar{w}' = \bar{w}'_0 + \bar{w}'' \frac{\sin k \xi}{k} + \bar{w}''' \frac{1 - \cos k \xi}{k^2} ,
\]

\[
\bar{w}'' = \bar{w}''_0 \cos k \xi + \bar{w}''' \frac{\sin k \xi}{k} ,
\]

\[
\bar{w}''' = -k \bar{w}''_0 \sin k \xi + \bar{w}''' \cos k \xi .
\]

(18)

In these equation, the not dimensional shear force in the initial section should be related with the deformed axis of a rod element.

Let us consider rod element, the initial section of which is rigidly clamped and opposite section supported against displacement and rotation by elastic supports \( C_1 \) and \( C_2 \) respectively. Supposing, that rotating angles are too small, the relationships of not dimensional shear force and bending moment take the form:
\[
\begin{align*}
\bar{Q} &= \bar{Q} + P \frac{d\bar{w}}{d\xi} = -C_1\bar{w} + k^2\bar{w}' = -\bar{w}'', \\
\bar{M} &= -C_2\bar{w}' = -\bar{w}'.
\end{align*}
\] (19)

Here, index ‘n’ corresponds to forces, related with undeformed axis of rod element.

Substituting (18) to (19), we obtain for section \( \xi = 1 \) two equation in the general case:

\[
\begin{align*}
\bar{w}_0(-C_1) + \bar{w}_0'(k^2 - C_1) + \bar{w}_0''(C_1 \cos k - 1 + 2k \sin k) + \bar{w}_0''(C_1 \left( \frac{\sin k}{k^3} - \frac{1}{k^2} \right) - 2 \cos k + 1) &= 0, \\
\bar{w}_0' C_2 + \bar{w}_0''(C_2 \sin k - \cos k) + \bar{w}_0''(C_2 \left( \frac{1 - \cos k}{k^2} \right) - \frac{\sin k}{k}) &= 0.
\end{align*}
\] (20)

It is necessary to impose additional boundary condition, corresponding specific problem. In particular, in the paper [22] it was obtained solution to compressed rod with fixed clamping of the initial section and arbitrary boundary condition on the opposite edge:

\[
A_1(k)C_1 C_2 + A_2(k)C_1 + A_3(k)C_2 + A_4(k) = 0, \tag{21}
\]

where

\[
A_1(k) = \frac{2(1 - \cos k) - k \sin k}{k^4}, \quad A_2(k) = \frac{k \cos k - \sin k}{k^2}; \quad A_3(k) = \frac{\sin k}{k}; \quad A_4(k) = -\cos k.
\]

Assigning single values to displacements, rotation angles or forces in edge section, we obtain functions of parameter \( k \), which can be applied to calculate rod structural system by the forces method or the displacements method. In particular, we obtain by the displacements method in the matrix form:

\[
r(k) \cdot Z = 0, \tag{22}
\]

where \( r \) is matrix of the reactive forces; \( Z \) is matrix of unknown displacements. Parameter \( k_c \) can be obtained if the determinant of the matrix \( r \) equal to zero.

It should be noted, that parameter \( k \) can be determined only approximately (for example, by the Newton method) if structural system has the degree of indetermination by the method of displacement equal or more than four.

4. Stability analysis of flat RC frame with accordance of influence of corrosion damage

Let us consider an example of stability analysis of the flat frame, which is shown in figure 2, (a). The source data: \( A_i = 28 \text{ mm}^2 \), concrete has strength class B25 [23], \( b = 50 \text{ mm} \), \( h = 100 \text{ mm} \), \( l = 1 \text{ m} \), \( h = 0.5 \text{ m} \), \( \alpha_1 = \alpha_3 = 0.5 \), \( \alpha_2 = 1 \). The left column is exposed to corrosion. There is no corrosion damage at the initial moment \( t_0 \). In the first approximation we suppose that variable strain modulus not changes and the one can be calculated in accordance with [23] as strain modulus for continuous loading and normal condition of wetness. Then \( E_i = E_{b1} = 0.86 \times 10^5 \text{ MPa} \) and reduced bending rigidities take value \( B_{red,1} = B_{red,2} = \ldots = B_{red,5} = 3.571 \times 10^4 \text{ N\cdotm}^2 \).

Applying the displacement method in relation with (18) and (19), we obtain approximately \( k_{cr,1} = 2.221 \) for moment \( t_0 \) and \( \delta(t) = z^* = 0 \). Therefore, critical force takes the value:

\[
P_{cr} = \frac{k_{cr,1}^2}{\alpha_2 l^2} B_{red,1} = 1401 \text{ kN}.
\]

This value differs by 1.3 % from result, obtained by the FEM in the SCAD Office at dividing of each rod to ten finite elements (\( P_{cr/FEM} = 1383.2 \text{ kN} \)).
Let us determine the bifurcation type for each column. Since the displacements method equations are system of homogenous linear algebraical equation, then solution can be obtained only approximately (for example, by the Newton method). However, if we need to obtain sign of nodal moments and shear forces work, that is enough to know values of ratios $Z_i / Z_j$. Here $i$ is index corresponding to arbitrary node displacement, $j$ is index corresponding to fixed node displacement.

Then we obtain for flat frame, which is shown in figure 2, (b):

$$
Z_i = \begin{pmatrix}
 r_1 & r_{12} & r_{13} \\
 r_{21} & r_2 & r_{23} \\
 r_{31} & r_{32} & r_3
\end{pmatrix}^{-1}
\begin{pmatrix}
 -r_{14} \\
 -r_{24} \\
 -r_{34}
\end{pmatrix} \cdot Z_j
= \begin{pmatrix}
 2.073 \\
 8.232 \cdot 10^{-4} \\
 2.073
\end{pmatrix} \cdot Z_j
$$

![Diagram](image)

**Figure 2.** Computational model (a); computational model of the displacement method (b); buckling form (c); computational model of the left column (d)

Taking in count values of the ratios $Z_i / Z_j$, we obtain

$$
A_1(M, Q)/Z_4^2 = \frac{4B_{red,1}}{l} \left( \mu_1(k_1) \frac{Z_1}{Z_4} \right)^2 - \frac{3\mu_1(k_1)}{l} \frac{Z_1}{Z_4} + \frac{3\mu_1(k_1)}{l^2} = -8.473 \cdot 10^5 < 0,
$$

$$
A_2(M, Q)/Z_4^2 = \frac{4B_{red,2}}{l} \left( \mu_1(k_2) \frac{Z_2}{Z_4} \right)^2 - \frac{3\mu_1(k_2)}{l} \frac{Z_2}{Z_4} + \frac{3\mu_4(k_2)}{l^2} = -1.864 \cdot 10^7 < 0,
$$

$$
A_3(M, Q)/Z_4^2 = \frac{4B_{red,3}}{l} \left( \mu_1(k_3) \frac{Z_3}{Z_4} \right)^2 - \frac{3\mu_1(k_3)}{l} \frac{Z_3}{Z_4} + \frac{3\mu_4(k_3)}{l^2} = -8.473 \cdot 10^5 < 0,
$$

where
\[ \mu_1(k_i) = \frac{k_i \cdot \tan(k_i) - k_{ij}}{8 \cdot \tan(k_i) \cdot \tan(k_i/2) - k_i/2}, \quad \mu_3(k_i) = \frac{k_i^2 \tan(k_i/2)}{12 \cdot \tan(k_i/2) - k_i/2}, \quad \mu_4(k_i) = \frac{k_i^3}{24 \cdot \tan(k_i/2) - k_i/2}. \]

Since the work of the nodal moments and shear forces of all columns are negative, then all column subjected active bifurcation, as it is shown in figure 2, (c). For comparison we calculate the single left column using computational model as it is shown in figure 2, (d) and appropriating to elastic supports the following values:

\[ C_1 = 12 \frac{B_{\text{red},1} \cdot \mu_4(k_i)}{l^3}, \quad C_2 = 4 \frac{B_{\text{red},1} \cdot \mu_3(k_i)}{l}. \]

Substituting these values into (21) and taking in count source data, we obtain \( k_{cr} = 3.14 = \pi \). The results of computation of \( P_{cr} \) for the left column at different values of \( \delta(t) \) for computational model by figure 2, (b) and computational model by figure 2, (d) are presented in table 1 and figure 3, where \( P_{cr,r} \) corresponds to single rod and \( P_{cr,fr} \) corresponds to rod of the flat frame.

**Table 1.** Stability analysis of left column of the frame

| \( \delta(t) \) (cm) | \( B_{\text{red}} \) \((10^4 \cdot \text{N} \cdot \text{m}^3)\) | \( P_{cr,r} \) (kN) | \( P_{cr,fr} \) (kN) |
|----------------------|------------------|-----------------|-----------------|
| 0.005                | 3.574            | 1410            | 710.2           |
| 0.01                 | 3.064            | 1208            | 685.8           |
| 0.015                | 2.696            | 1063            | 670.5           |
| 0.02                 | 2.43             | 957             | 658.6           |
| 0.025                | 2.235            | 880             | 649.4           |
| 0.025                | 2.082            | 819             | 641.9           |

**Figure 3.** Graphs of \( P_{cr}(t), \delta(t) \) for single rod and rod of the flat frame

Analysis of the graphs, presented in figure 3, shows that value \( k_{cr} \) for not damaged single rod \( (\delta(t) = 0) \) is higher than that one for rod of the frame. It is determined kinematic actions of nearby structural elements. However, when depth of corrosion damage increase (i.e. decreasing of bending rigidity), nearby elements of system hinder to the damaged element to lose stability.

**5. Conclusions**

The analytical relationships to assess the type of buckling of individual nonlinear deformable concrete elements of the frame structural system with different variants of boundary conditions are obtained. The solution of these equations is given and it can be used to assess stability of reinforced concrete
(RC) columns of the frame, damaged by corrosion. Herewith the constructive elements are considered as part of the whole structural system. Structural transformation of RC frame at sudden buckling of one of the bearing structural element subjected corrosion is analysed.

The results of this study can be used to calculate frames of the reconstructed buildings and structures (in particular at the accidental impacts [24]) for assessment of their sustainability, protection against the progressive collapse, as well as evaluation of their residual life of operated structures, subjected such actions.

References
[1] Travush V I, Fedorova N V 2017 Russian J. of Building Construction and Architecture. – 2017. – Vol. 1 No. 33, 6-14
[2] Klyuyeva N V, Koren’kov P A 2016 Promyshlennoye i grazhdanskoye stroitel'stvo, Vol. 2, 44-48
[3] Fedorova N V, Koren’kov P A 2016 Stroitel'stvo i rekonstruktksiya, Vol. 6, 90-100
[4] Fedorova N V, Khalina T A 2017 Promyshlennoye i grazhdanskoye stroitel'stvo, Vol. 5, 32-36
[5] Kolchunov V I, Savin S Y 2017 J. Appl. Eng. Sc., Vol. 15, No. 3, 325-331. doi:10.5937/iaaes15-14602
[6] Shapiro G I, Obukhova L V, Eysman Yu A, Sirotina YeV 2007 Promyshlennoye i grazhdanskoye stroitel'stvo, Vol. 9
[7] Almazov V O, Kkhoy Kao Zuy 2013 Dinamika progressiruyushcheho razrusheniya monolitnykh mnogoetazhnykh karkasov [Dynamics of the progressive destruction of monolithic multi-storey skeletons] (Moscow: Izdatel'stvo ASV)
[8] Richard M Bennett 1988 J. Structural Safety, Vol. 5, iss. 1, 67-77. doi:10.1016/0167-4730(88)90006-9
[9] Daigoro Isobe 2017 Progressive Collapse Analysis of Structures. Numerical Codes and Applications (Oxford: Butterworth–Heinemann).
[10] Setareh Amiri, Hamed Saffari, Javad Mashjadi 2018 Eng. Failure Analysis, Vol. 84, 300-310. doi:10.1016/j.engfailanal.2017.11.011
[11] YA Al-Salloum, H Abbas, TH Almusallam, T Ngo, P Mendis 2017 J. of King Saud Univ. – Eng. Sc., Vol. 29, Is. 4, 313-320. doi:10.1016/j.jksues.2017.06.005
[12] J Weng, KH Tan, CK Lee 2017 J. Eng. Structures, Vol. 151, 136-152. doi:10.1016/j.istructe.2017.08.024
[13] Waleed Mohamed Elsayed, Mohamed AN Abdul Moaty, Mohamed E Issa 2016 HBRC J., Vol. 12, Is. 3, 242-254. doi:10.1016/j.hbrcj.2015.02.005
[14] Hongyu Wang, Youpo Su, Qingshen Zeng 2011 Systems Eng. Proc., Vol. 1, 48-54. doi:10.1016/j.sypro.2011.08.009
[15] Meng-Hao Tsai, Tseui-Chiang Huang 2011 Proc. Eng., Vol. 14, 377-384. doi:10.1016/j.proeng.2011.07.047
[16] Jie Li, Hao Zhou 2017 Proc. Eng., Vol. 199, 1246-1251. doi: 10.1016/j.proeng.2017.09.262
[17] Emanuele Brunesi, Fulvio Parisi 2017 J. Eng. Structures, Vol. 152, 579-596. doi:10.1016/j.istructe.2017.09.043
[18] Bondarenko V M, Klyuyeva N V 2008 Izvestiya vuzov. Stroitel'stvo, Vol. 1, 4-12.
[19] Aleksandrov A V, Travush V I, Matveev A V 2002 Promyshlennoye i grazhdanskoye stroitel'stvo, Vol. 3, 16-19
[20] Bondarenko V M 2016 Silovoye deformatsirovaniye, korrozionnyye povrezhdeniya i energosoprotivleniya zhelezobetona (Kursk: Publ. of the South-West State Univ.)
[21] SP Protection of buildings and structures against progressive collapse. Design code. General positions
[22] Gordon V A, Kolchunov V I 2006 Stroitelnaya mehanika I raschet sooruzheniy, Vol. 4, 33-38
[23] SP 63.13330.2012 Concrete and won concrete construction. Design requirements
[24] SP 296.1325800.2017 Buildings and structures. Accidental actions