Interacting scenarios with dynamical dark energy: observational constraints and alleviation of the $H_0$ tension

Supriya Pan,$^1$ Weiqiang Yang,$^2$ Eleonora Di Valentino,$^3$†
Emmanuel N. Saridakis,$^{1,5,6,*}$ and Subenoy Chakraborty$^7$

$^1$Department of Mathematics, Presidency University, 86/1 College Street, Kolkata 700073, India.
$^2$Department of Physics, Liaoning Normal University, Dalian, 116029, P. R. China.
$^3$Jodrell Bank Center for Astrophysics, School of Physics and Astronomy, University of Manchester, Oxford Road, Manchester, M13 9PL, UK.
$^4$Department of Physics, National Technical University of Athens, Zografou Campus GR 157 73, Athens, Greece
$^5$Department of Astronomy, School of Physical Sciences, University of Science and Technology of China, Hefei 230026, P. R. China
$^6$Chongqing University of Posts & Telecommunications, Chongqing, 400065, P. R. China
$^7$Department of Mathematics, Jadavpur University, Kolkata 700032, West Bengal, India

We investigate interacting scenarios which belong to a wider class, since they include a dynamical dark energy component whose equation of state follows various one-parameter parametrizations. We confront them with the latest observational data from Cosmic Microwave Background (CMB), Joint light-curve (JLA) sample from Supernovae Type Ia, Baryon Acoustic Oscillations (BAO), Hubble parameter measurements from Cosmic Chronometers (CC) and a gaussian prior on the Hubble parameter $H_0$. In all examined scenarios we find a non-zero interaction, nevertheless the non-interacting case is allowed within 2σ. Concerning the current value of the dark energy equation of state for all combination of datasets it always lies in the phantom regime at more than two/three standard deviations. Finally, for all interacting models, independently of the combination of datasets considered, the estimated values of the present Hubble parameter $H_0$ are greater compared to the $\Lambda$CDM-based Planck’s estimation and close to the local measurements, thus alleviating the $H_0$ tension.

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I. INTRODUCTION

After almost 20 years from the detection of late-time universe acceleration, and due to the appearance of a huge amount of data, people are still looking for the actual underlying theory that could explain it. In general, there are two widely accepted approaches that could accommodate it. The first one is the introduction of some hypothetical dark energy fluid [1] in the context of Einstein’s gravitational theory. The second one is to consider modified or alternative gravitational theories where the extra geometrical terms may reproduce the effects of dark energy [2 3 4].

On the other hand, a mutual interaction between the dark matter and dark energy sectors was initially introduced in order to investigate the cosmological constant problem [5]. However, later on it was found that it could also alleviate the cosmic coincidence problem in a natural way [6 7 8 9 10 11], and this led to many investigations of interacting cosmology [12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50], including however all well-studied parametrizations of the dark-energy equation-of-state parameter. Hence, in the present work we are interested in performing a systematic confrontation of interacting dark energy scenarios, considering however all well-studied parametrizations of the dark-energy equation-of-state parameter. Only such a complete and consistent analysis can extract safe results about the observational validity of the examined scenarios.

We consider interacting scenarios in which the dark energy equation of state is parametrized with forms that include one free parameter. Such one-parameter mod-
els are more economical comparing to the two-parameter ones, and moreover recently it was found that these one-parameter dynamical dark-energy parametrizations are very efficient in alleviating the $H_0$ tension in the simple non-interacting framework [33]. This motivates us to consider a wider picture in which the interaction should be allowed too, and to check whether the $H_0$ tension is still released, since it has been argued that an allowance of a non-gravitational interaction between dark matter and dark energy naturally increases the error bars on $H_0$ (due to the existing correlation between $H_0$ and the coupling parameter of the interaction models) and consequently alleviates the corresponding tension [69, 70, 79, 81]. Thus, essentially the present work aims to investigate whether the release of $H_0$ tension discussed in [33] is influenced by the presence of an interaction between dark matter and dark energy.

The work has been organized in the following manner. In section II we describe the basic equations for any interacting dark energy model at the background and perturbative levels. Additionally, we present various one-parameter $w_x$ parametrizations. Section III deals with the observational data that we consider in this work. In section IV we describe the main observational results extracted for all the examined scenarios. Moreover, in section V we compute the Bayesian evidences of the models with respect to the reference ΛCDM paradigm. Finally, in section VI we conclude the present work with a brief summary of all findings.

II. COSMOLOGICAL EQUATIONS IN INTERACTING SCENARIOS

The universe is well described by the homogeneous and isotropic Friedman-Lemaitre-Robertson-Walker (FLRW) line element given by

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 + \sin^2 r d\phi^2 \right],$$  \hspace{1cm} (1)

where $a(t)$ is the expansion scale factor and $K = 0, -1, +1$ corresponds respectively to flat, open and closed spatial geometry. Since observations imply almost spatial flatness, we shall restrict ourselves to $K = 0$ throughout the work.

The total matter content of the universe constitutes to radiation, baryons, pressure-less dark matter and a dark energy fluid (that may be real fluid or an effective one arising from modified gravity). Moreover, we allow the dark matter and dark energy to have a mutual (non-gravitational) interaction, while the remaining two fluids follow the usual conservation laws. Hence, the Friedmann equation is given by

$$H^2 = \frac{8\pi G}{3} (\rho_r + \rho_b + \rho_c + \rho_x),$$  \hspace{1cm} (2)

in which $H \equiv \dot{a}/a$ is the Hubble rate, and $\rho_i$ is the energy density of the $i$-th fluid sector (with $i = r$ for radiation, $i = b$ for baryons, $i = c$ for cold or pressure-less dark matter and $i = x$ for dark energy). The conservation equation of the total fluid $\dot{\rho}_{\text{tot}} = \rho_r + \rho_b + \rho_c + \rho_x$, is given by

$$\dot{\rho}_{\text{tot}} + 3H (\rho_{\text{tot}} + p_{\text{tot}}) = 0,$$  \hspace{1cm} (3)

where $p_{\text{tot}}$ is the total pressure of the fluids defined as $p_{\text{tot}} = p_r + p_b + p_c + p_x$. Since radiation and baryons satisfy their own conservation equations, namely, $\dot{\rho}_b + 3H \rho_b = 0$ and $\dot{\rho}_r + 4H \rho_r = 0$, then the conservation equation for the total fluid (3) gives rise to

$$\dot{\rho}_{\text{Dark}} + 3H (p_{\text{Dark}} + \rho_{\text{Dark}}) = 0,$$  \hspace{1cm} (4)

where $\rho_{\text{Dark}} = \rho_c + \rho_x$ and $p_{\text{Dark}} = p_c + p_x$.

In interacting cosmology one splits the conservation equation for the dark sector (4) into

$$\dot{\rho}_c + 3H \rho_c = -Q(t),$$  \hspace{1cm} (5)

and

$$\dot{\rho}_x + 3H (1 + w_x) \rho_x = Q(t),$$  \hspace{1cm} (6)

by introducing a new function $Q(t)$, that actually characterizes the rate of energy transfer between these dark fluids. Thus, whenever the interaction $Q$ is prescribed, using the conservation equations (5), (6) as well as the Friedmann equation (2), one can determine the dynamics of this interacting scenario.

Since the nature of both dark fluids is unknown, there is an ambiguity in the choice of the interaction function. Thus, in general one considers phenomenological choices for $Q$, and through observational confrontation results to the best interaction model. In the present work we will focus on a well motivated interaction that induces stable perturbations [99]:

$$Q = 3\xi H (1 + w_x) \rho_x,$$  \hspace{1cm} (7)

with $\xi$ the coupling parameter characterizing the interaction strength.

Let us briefly describe the perturbation equations for an interacting dark energy model following [100, 101, 102]. The scalar perturbations of the FLRW metric read as

$$ds^2 = a^2(\tau) \left[ -d\tau^2 + 2\delta_i B d\tau dx^i + \left( (1 - 2\psi) \delta_{ij} + 2 \partial_i \partial_j E \right) dx^i dx^j \right],$$  \hspace{1cm} (8)

where $\tau$ is the conformal time and the $\phi$, $B$, $\psi$ and $E$ are the gauge-dependent scalar perturbation quantities. Additionally, for an interacting universe the conservation equations become [103, 104, 105]

$$\nabla_{\mu} T^\mu_A = Q^\mu_A, \quad \sum_A Q^A_A = 0,$$  \hspace{1cm} (9)
where $A$ is used to represent either pressure-less dark matter (then $A = c$) or dark energy (then $A = x$). Here, the quantity $Q_A^\mu$ takes the following expression

$$Q_A^\mu = (Q_A + \delta Q_A)u^\mu + a^{-1}(0, \partial f_A),$$

(10)

relative to the four-velocity $u^\mu$, in which $Q_A$ presents the background energy transfer (i.e. $Q_A = Q$) and $f_A$ is the momentum transfer potential. We restrict ourselves to the simplest possibility following the earlier works [103, 104, 105], i.e. we assume that the momentum transfer potential is zero in the rest frame of the dark matter, from which one can derive that $k^2 f_A = Q_A(\theta - \theta_c)$ (here $k$ is the wave number; $\theta = \theta_0^\mu$ is the volume expansion scalar of the total fluid, and $\theta_c$ is the the volume expansion scalar for the CDM fluid).

We proceed by applying the synchronous gauge to derive the perturbation equations for the interacting scenarios. Thus, in the synchronous gauge we have $\phi = B = 0$, $\psi = \eta$, and $k^2 E = -h/2 - 3\eta$ (h and $\eta$ are the metric perturbations, see [101] for details). Additionally, we assume the absence of an anisotropic stress, and we define the density perturbations for the fluid $A$ by $\delta_A = \delta \rho_A / \rho_A$. The resulting perturbation equations become

$$\delta_A' = -(1 + w_x) \left( \theta_x + \frac{h'}{2} \right) - 3Hw_x \theta_x \frac{\theta_x}{k^2}$$

$$+ 3H(c_{sx}^2 - w_x) \left[ \delta_x + 3H(1 + w_x) \frac{\theta_x}{k^2} \right]$$

$$+ \frac{aQ}{\rho_x} \left[ -\delta_x + \frac{\theta_x}{Q} + 3H(c_{sx}^2 - w_x) \frac{\theta_x}{k^2} \right],$$

(11)

$$\theta_x' = -H(1 - 3c_{sx}^2)\theta_x + \frac{c_{sx}^2}{1 + w_x} k^2 \delta_x$$

$$+ \frac{aQ}{\rho_x} \left[ \theta_x - \frac{(1 + c_{sx}^2)\theta_x}{1 + w_x} \right],$$

(12)

$$\delta_c' = - \left( \theta_c + \frac{h'}{2} \right) + \frac{aQ}{\rho_c} (\delta_c - \frac{\delta Q}{Q}),$$

(13)

$$\theta_c' = -H\theta_c,$$

(14)

where $H = aH$ is the conformal Hubble rate and in [11], [12], [13] the factor $\delta Q/Q$ incorporates the perturbations for the Hubble rate $\delta H$. We mention that using $\delta H$ one can easily find the gauge invariant perturbation equations [17].

We close this section by introducing the $w_x$ parametrizations having only one free parameter $w_0$, namely the present value of the dark energy equation of state [59]:

Model I: $w_x(a) = w_0 a[1 - \log(a)]$,\hspace{1cm}(15)

Model II: $w_x(a) = w_0 a \exp(1 - a)$,\hspace{1cm}(16)

Model III: $w_x(a) = w_0 a[1 + \sin(1 - a)]$,\hspace{1cm}(17)

Model IV: $w_x(a) = w_0 a[1 + \arcsin(1 - a)]$.\hspace{1cm}(18)

Thus, in summary we consider the interaction model [7] with four different dark energy equations of state given in [15]-[18]. From now on we identify the interaction model (7) with $w_x$ of [15] as IDE1, interaction model (7) with [16] as IDE2, interaction model (7) with [17] as IDE3, and finally the interaction model (7) with [18] as IDE4.

III. OBSERVATIONAL DATA

In this section we describe the observational data that we use to investigate the interacting dark energy models, and we provide a brief description on the methodology.

- The data from cosmic microwave background (CMB) observations are very powerful to analyze the cosmological models. Here we use the high-ℓ temperature and polarization as well as the low-ℓ temperature and polarization 2015 CMB angular power spectra from the Planck experiment (Planck TT, TE, EE + lowTEB) [106, 107].

- We include the Joint light-curve analysis (JLA) sample from Supernovae Type Ia data [108].

- We use the Baryon acoustic oscillations (BAO) distance measurements from the following references [109, 110, 111].

- We consider the measurements of the Hubble parameter at various redshifts from the Cosmic Chronometers (CC) [112].

- We adopt a gaussian prior on the Hubble constant (R19) $H_0 = 74.02 \pm 1.42$ as obtained from SH0ES [113].

| Parameter  | Prior         |
|------------|---------------|
| $\Omega_\Lambda h^2$ | [0.005, 0.1] |
| $\Omega_c h^2$ | [0.01, 0.99] |
| $\tau$ | [0.01, 0.8] |
| $n_s$ | [0.5, 1.5] |
| log[10$^{10}$ $A_s$] | [2.4, 4] |
| 1000$\mu$C | [0.5, 10] |
| $w_0$ | $[-2, 0]$ |
| $\xi$ | [0, 2] |

TABLE I: The table shows the priors imposed on various free parameters of the interacting scenarios during the statistical analysis.

In order to extract the observational constraints on the model parameters of the interaction scenarios, we use the efficient cosmological code cosmomic [114, 115], a markov chain monte carlo package which (i) has a convergence diagnostic and (ii) supports the Planck 2015
TABLE II: 68% and 95% confidence-level constraints on the interacting scenario IDE1 with the dark energy equation of state $w_s(a) = w_0[a(1 - \log(a))]$ (Model I) for various observational datasets. Here $\Omega_{m0}$ is the present value of the total matter density parameter $\Omega_m = \Omega_h + \Omega_s$, and $H_0$ is in units of km/sec/Mpc.

![Image of the table and diagram]
the scalar spectral index, $A_S$ is the amplitude of the primordial scalar power spectrum, $w_0$ is the current value of the dark energy parameter, and $\xi$ is the coupling parameter of the interaction. In Table I we summarize the flat priors on the model parameters during the statistical analysis.

IV. RESULTS AND IMPLICATIONS

In this section we extract the observational constraints on the present four interacting dark energy scenarios where dark energy has a time-dependent equation-of-state parameter. For all interacting scenarios we have performed several analyses using the observational data described in section III.

A. IDE1: Interacting dark energy with $w_x = w_0 a [1 - \log(a)]$

The summary of the observational constraints for this interaction scenario using different observational datasets is presented in Table II while the 2-D contour plots and the 1-D marginalized posterior distribution are shown in Figs. 1 and 2. We mention that in the figures we do not include the sole CMB case, since its parameter space is larger than the other datasets, however we note that the qualitative nature of the correlations between the parameters for CMB alone and other cases are similar. Moreover, we notice that the addition of CC to the CMB+BAO+JLA combination does not add extra constraining power, and hence the constraints from CMB+BAO+JLA and CMB+BAO+JLA+CC are actually the same in the fourth and fifth columns of Table II.

From the results we observe that for both CMB and CMB+BAO $\xi = 0$ is consistent within 68% CL. After the inclusion of JLA and JLA+CC to the combined dataset CMB+BAO, we find that an interaction of about $\xi = 0.003 \pm 0.002$ is suggested at 68% CL. In addition, if we combine CMB with R19 (we can safely do it since the tension on $H_0$ is less than 2σ as we can see in Fig. 3) we find an improved constraint (statistically very mild) but always in agreement with $\xi = 0$, while combining CMB+BAO+R19 gives slightly relaxed constraints Therefore, we conclude that for
CMB+BAO+JLA, CMB+BAO+JLA+CC, CMB+R19 and CMB+BAO+R19, \( \xi \neq 0 \) at 1σ, however within 95% CL, \( \xi \) is consistent with zero.

Concerning the current value of the dark energy equation of state \( w_0 \), for all combination of datasets it always lies in the phantom regime at more than two/three standard deviations. If we compare these results with those without interaction obtained in [83], we see that they are perfectly in agreement and very robust, even in those cases where an interaction different from zero is favoured.

Regarding the estimated values of \( \Omega_{m0} \), one can clearly see that for CMB alone case, \( \Omega_{m0} \) is really small compared to what Planck estimates [117]. This is due to the geometrical degeneracy existing between \( w_0, \Omega_{m0} \) and \( H_0 \). Since a phantom dark energy equation of state is preferred by the Planck data, the position of the peaks in the high multipoles damping tail will be shifted, and we need a lower value for the matter density and an higher Hubble constant to put them back in the measured position. When BAO data are added to CMB, the mean value of \( \Omega_{m0} \) increases with reduced error bars, because this dataset fixes very well the amount of matter density in the universe. For this reason, all the other dataset combinations we considered have \( \Omega_{m0} \) greater than the CMB alone case, even if always lower than the Planck one [117].

Finally, concerning the estimation of \( H_0 \), we see that for CMB data alone it takes a very high mean value compared to the \( \Lambda \)CDM-based Planck’s estimation [116] and the error bars are quite large (as one can see \( H_0 = 81^{+13}_{-14} \) at 68% CL for CMB alone). This is an implication of the strong anti-correlation between \( w_0 \) and \( H_0 \). How-

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**FIG. 3:** Whisker plot with the 68% CL constraints on the Hubble constant for all interacting models and all combination of datasets considered in this work. The grey vertical band corresponds to the R19 value for the Hubble constant, \( H_0 \), as measured by SH0ES in [113], and the red vertical band is the one estimate by the Planck 2018 release [116].
ever, when the BAO data are added to CMB, the error bars on $H_0$ are significantly decreased and its estimated mean value shifts towards a lower value ($H_0 = 71.0 \pm 1.5$ at 68% CL for CMB+BAO), i.e. perfectly in agreement with the direct measurements \cite{143, 148, 149} within 2σ. The inclusion of JLA (or JLA+CC) to CMB+BAO further decreases the error bars on $H_0$ and further shifts its lower value, increasing the $H_0$ tension, but still less than 3σ.

**B. IDE2: Interacting dark energy with**

$$w_x(a) = w_0a \exp(1 - a)$$

The observational summary for this interaction scenario is displayed in Table \ref{tab:4}, while the 2-D contour plots and 1-D parameter distributions are shown in Figs. \ref{fig:6} and \ref{fig:7}. From the analyses we deduce that for CMB data alone the non-interacting case $\xi = 0$ is consistent within 68% CL, however when BAO data are added to CMB then an indication of interaction is found at more than 68% CL. Surprisingly when JLA data are added to the previous dataset CMB+BAO, then we again find that $\xi = 0$ consistent within 68% CL. Moreover, for the remaining datasets the indication of an interaction is still present at more than 1σ. We note that due to the addition of R19 with CMB and CMB+BAO, slight improvements in the coupling parameter $\xi$ are observed, although such improvements are statistically very mild. In fact, R19 sets a lower bound on $\xi$ for both CMB+R19 and CMB+BAO+R19. The reason is that the inclusion of R19 with CMB and CMB+BAO breaks the degeneracies between $H_0$ and other cosmological parameters.

Concerning the dark energy equation-of-state parameter at present, for all the datasets a phantom value $w_0 < -1$ is always supported for more than 95% CL. Hence, in summary, as we can see from the results, most of the datasets indicate a non-zero interaction together with the existence of a phantom dark energy. A similar observation on the estimated values of $\Omega_m$, specially for its lower value for CMB alone analysis, is found as in IDE1.

Regarding the estimations of the Hubble parameter $H_0$, we see that for CMB alone $H_0$ acquires a very high value with very large error bars compared to the Planck one within minimal ΛCDM model \cite{116} (in particular $H_0 = 84^{+14}_{-14}$ at 68% CL with CMB alone). This is due to the strong correlation between $w_0$ and $H_0$. When external datasets are added, as for instance BAO, JLA, CC, R19, and their combinations, the estimations of $H_0$ decrease with significant reduction in the error bars, but they can still relieve the tension with $H_0$ within 3 standard deviations.

**C. IDE3: Interacting dark energy with**

$$w_x(a) = w_0a[1 + \sin(1 - a)]$$

The summary of the observational constraints for this interaction scenario using different observational datasets is presented in Table \ref{tab:5}, and in Figs. \ref{fig:8} and \ref{fig:9} we show the 2-D contour plots and 1-D posterior distributions for some of the free parameters and dataset combinations.

Concerning the coupling parameter our analysis reveals some interesting features. In particular, as we can see, for CMB data alone we have $\xi = 0$ within 68% CL and hence it is consistent with a non-interacting cosmology. Nevertheless, as soon as external datasets, namely BAO, JLA, CC or R19 are added in different combinations (such as CMB+BAO, CMB+BAO+JLA, CMB+BAO+JLA+CC and CMB+R19) we see that a non-zero interaction is favoured at more than 1σ. However, we mention that within 95% CL these combinations of observational datasets allow for a non-interacting cosmology. We note that the R19 has similar effects when combined with CMB and CMB+BAO, as already reported previously for IDE2.

Concerning the current value of the dark energy equation of state $w_0$, we find a similar character to what we already found in IDE1 and IDE2. In particular, the results show that irrespectively of the observational datasets that we have used in this work, $w_0$ remains less than $-1$ at more than 95% CL, i.e. in the phantom region, for the CMB only case, and several standard deviations (more than five) for the combinations with the other cosmological probes. If we compare this Table with the results shown in \cite{83} for the same model without interaction, we can see that the constraints are very robust, and only the upper limit of $w_0$ for the CMB alone case is slightly removed to less phantom values. Furthermore we mention that in this scenario the CC dataset does not improve the constraints at all. Let us note that similar to IDE1 and IDE2, for CMB alone analysis, $\Omega_m$ acquires a lower value compared to Planck \cite{117}.

Finally, regarding the $H_0$ parameter we again find a similar behaviour to what we observed for IDE1 and IDE2. Since to a phantom dark energy equation of state corresponds a higher value of the Hubble parameter, due to their negative correlation, the highly negative $w_0$ values that we obtain are accompanied with a high value of $H_0$ with large asymmetric error bars. Specifically, we find $H_0 = 84^{+12}_{-5}$ at 68% CL, which is much higher than the recent ΛCDM-based estimation by Planck \cite{116}, but in agreement with the direct measurements $H_0 = 73.24 \pm 1.74$ of \cite{118}, $H_0 = 73.48 \pm 1.66$ of \cite{149} or $H_0 = 74.03 \pm 1.42$ of \cite{113}. However, after the inclusion of the external datasets such as BAO, JLA, CC and R19 we find that $H_0$ decreases with respect to its estimation from CMB alone, and additionally its error bars are significantly reduced.
TABLE III: 68% and 95% confidence-level constraints on the interacting scenario IDE2 with the dark energy equation of state $w_a = w_0 a + \alpha$ (Model II) for various observational datasets. Here $\Omega_{m0}$ is the present value of the total matter density parameter $\Omega_m = \Omega_r + \Omega_c$, and $H_0$ is in units of km/sec/Mpc.

![Graphical representation of Table III](image)

FIG. 4: The 68% and 95% CL contour plots between various combinations of the model parameters of scenario IDE2, using different observational astronomical datasets. Additionally we display the one-dimensional marginalized posterior distributions of some free parameters.

In summary, we find that the alleviation of the $H_0$ tension is more robust in this scenario compared to IDE1 and IDE2 (see also Fig [3]). Indeed, one can notice the estimated values of $H_0$ from different combination of observational datasets as follows:

- **CMB+BAO**: $H_0 = 73.5^{+1.6}_{-1.7}$ at 68% CL ($H_0 = 73.5 \pm 3.2$ at 95% CL);  

- **CMB+BAO+JLA**: $H_0 = 70.06^{+0.95}_{-0.98}$ at 68% CL ($H_0 = 70.1^{+2.0}_{-1.8}$, at 95% CL);
FIG. 5: The 68% and 95% CL contour plots between various combinations of the model parameters of scenario IDE2 using only the CMB+BAO and CMB+BAO+R19 datasets, and the corresponding one-dimensional marginalized posterior distributions.

TABLE IV: 68% and 95% confidence-level constraints on the interacting scenario IDE3 with the dark energy equation of state $w(a) = w_0 a[1 + \sin(1 - a)]$ (Model III) for various observational datasets. Here $\Omega_{m0}$ is the present value of the total matter density parameter $\Omega_m = \Omega_0 + \Omega_c$, and $H_0$ is in units of $\text{km/sec/Mpc}$.

- CMB+BAO+JLA+CC: $H_0 = 70.1 \pm 1.0$ at 68% CL ($H_0 = 70.1^{+1.9}_{-1.8}$ at 95% CL), where the first one is perfectly in agreement with [13], and the last two alleviate the tension at about 2$\sigma$.

- IDE4: Interacting dark energy with $w_x(a) = w_0 [1 + \arcsin(1 - a)]$}

The summary of the observational constraints for this interacting scenario using different observational datasets is displayed in Table V and in Figs. 8 and 9 we present.
FIG. 6: The 68% and 95% CL contour plots between various combinations of the model parameters of scenario IDE3, using different observational astronomical datasets. Additionally we display the one-dimensional marginalized posterior distributions of some free parameters.

| Parameters | CMB | CMB+BAO | CMB+BAO+JLA | CMB+BAO+JLA+CC | CMB+R19 | CMB+BAO+R19 |
|------------|-----|---------|-------------|----------------|--------|-------------|
| $\Omega_b h^2$ | 0.121 ± 0.004 | 0.120 ± 0.004 | 0.119 ± 0.004 | 0.119 ± 0.004 | 0.120 ± 0.004 | 0.120 ± 0.004 |
| $\Omega_c h^2$ | 0.0214 ± 0.0016 | 0.0214 ± 0.0016 | 0.0214 ± 0.0016 | 0.0214 ± 0.0016 | 0.0214 ± 0.0016 | 0.0214 ± 0.0016 |
| $\Omega_{m0}$ | 0.225 ± 0.010 | 0.225 ± 0.010 | 0.225 ± 0.010 | 0.225 ± 0.010 | 0.225 ± 0.010 | 0.225 ± 0.010 |
| $\tau$ | 0.275 ± 0.015 | 0.275 ± 0.015 | 0.275 ± 0.015 | 0.275 ± 0.015 | 0.275 ± 0.015 | 0.275 ± 0.015 |
| $n_s$ | 0.969 ± 0.004 | 0.969 ± 0.004 | 0.969 ± 0.004 | 0.969 ± 0.004 | 0.969 ± 0.004 | 0.969 ± 0.004 |
| $\ln(10^{10} A_s)$ | 3.033 ± 0.023 | 3.033 ± 0.023 | 3.033 ± 0.023 | 3.033 ± 0.023 | 3.033 ± 0.023 | 3.033 ± 0.023 |
| $w_0$ | -0.50 ± 0.04 | -0.50 ± 0.04 | -0.50 ± 0.04 | -0.50 ± 0.04 | -0.50 ± 0.04 | -0.50 ± 0.04 |
| $\xi$ | 0.009 ± 0.002 | 0.009 ± 0.002 | 0.009 ± 0.002 | 0.009 ± 0.002 | 0.009 ± 0.002 | 0.009 ± 0.002 |

TABLE V: 68% and 95% confidence-level constraints on the interacting scenario IDE4 with the dark energy equation of state $w_\phi(a) = w_0 + a [1 + \arcsin(1 - a)]$ (Model IV) for various observational datasets. Here $\Omega_{m0}$ is the present value of the total matter density parameter $\Omega_m = \Omega_b + \Omega_c$, and $H_0$ is in units of km/sec/Mpc.

the 2-D contour plots and the 1-D posterior distributions. The behaviour of this interaction scenario has some similarities to that of IDE1. Looking at the results we find also in this case that for the analysis with CMB only and CMB+BAO datasets, the coupling parameter is consistent with $\xi = 0$ within the 68% CL. The addition of JLA or CC, namely the combinations of datasets CMB+BAO+JLA and CMB+BAO+JLA+CC, gives instead an indication for $\xi \neq 0$ at more than 68% CL, but always in agreement with zero within 2$\sigma$. We
mention that similarly to the previous IDE1 scenarios, here we also see that the inclusion of R19 with CMB and CMB+BAO improves the constraints on $\xi$, however it gives just an upper bound on it.

Concerning the dark energy equation-of-state parameter at present we extract similar conclusion to the previous interacting scenarios IDE3, namely here too we find that $w_0 < -1$ at more than 95% CL for the CMB only case, and several standard deviations for its combination with the external datasets. Furthermore, for this scenario the CMB only case has a slightly less phantom $w_0$ than the same case without interaction, as can be seen in [83]. As already reported in earlier IDE models, $\Omega_{m0}$ for CMB alone case obtains a lower value, contrary to Planck observations [117].

Now we focus on the trend on the Hubble parameter $H_0$. For the datasets we use, in this case it is again anti-correlated with $w_0$, as we can see in Fig. 8. We note that similar to the previous interaction scenarios, the CMB only fit returns very high value of $H_0$ with large error bars, that are reduced after the inclusion of the external datasets such as BAO, JLA and CC. For this scenario we also conclude that the tension with the direct measurements [113, 118, 119] is solved for CMB and CMB+BAO cases, while with the addition of JLA and JLA+CC it is at about 2$\sigma$. For this reason we can safely add the R19 measurement to the CMB and CMB+BAO, and we show the results in the last two columns of Table V.

V. BAYESIAN EVIDENCE

In this section we compute the Bayesian evidences of all the examined interacting models in order to compare their observational soundness with respect to some reference model, and in particular with $\Lambda$CDM cosmology. We use the publicly available code MCEvidence [120, 121] for computing the evidences, since the code directly accepts the MCMC chains of the analysis. We refer to Ref. [83] for the discussions on Bayesian evidence analysis, and in Table VI we provide the revised Jeffreys scale by Kass and Raftery [122].

For all the examined scenarios we compute the values of $\ln B_{ij}$, which are summarized in Table VII. From this Table one can see that $\Lambda$CDM paradigm is most of the time preferred over the present IDE models, with
FIG. 8: The 68% and 95% CL contour plots between various combinations of the model parameters of scenario IDE4, using different observational astronomical datasets. Additionally we display the one-dimensional marginalized posterior distributions of some free parameters.

| \( \ln B_{ij} \) | Strength of evidence for model \( M_i \) |
|------------------|------------------------------------------|
| 0 \( \leq \ln B_{ij} < 1 \) | Weak                                      |
| 1 \( \leq \ln B_{ij} < 3 \) | Definite/Positive                        |
| 3 \( \leq \ln B_{ij} < 5 \) | Strong                                    |
| \( \ln B_{ij} \geq 5 \) | Very strong                               |

TABLE VI: Revised Jeffreys scale [122] that quantifies the comparison of the models.

the exception of the CMB+R19 combination, where we see a weak/positive evidence for all IDE models against \( \Lambda \)CDM. This is expected since the number of free parameters of all IDE models is eight, namely two more compared to the six parameters \( \Lambda \)CDM.

VI. CONCLUDING REMARKS

Interacting scenarios have attracted the interest of the literature, since they are efficient in alleviating the coincidence problem, and additionally they seem to alleviate the \( H_0 \) tension and \( \sigma_8 \) tensions. In the present work we investigated interacting scenarios which belong to a wider class, since they include a dynamical dark energy component whose equation of state follows various one-parameter parametrizations. In particular, our focus was to see if a non-zero interaction is favoured, and if the \( H_0 \) tension is still alleviated.

We considered a well known interaction in the literature of the form \( Q = 3H\xi(1 + w_x)\rho_x \), and we took the dark energy equation-of-state parameter \( w_x \) to have the expressions: \( w_x(a) = w_{0a}[1 - \log(a)] \) (Model IDE1), \( w_x = w_{0a}\exp(1 - a) \) (Model IDE2), \( w_x(a) = w_{0a}[1 + \sin(1 - a)] \) (Model IDE3), and \( w_x(a) = w_{0a}[1 + \arcsin(1 - a)] \) (Model IDE4). Additionally, we used the latest observational data from CMB, JLA, BAO, Hubble parameter measurements from CC, and a gaussian prior on \( H_0 \) labeled as R19 from SH0ES [113].

Our analysis shows that the coupling strength for all interacting scenarios is quite small, and thus the models are consistent with the non-interacting \( w_x \)-cosmology. In particular, all scenarios are in agreement with \( \xi = 0 \).
FIG. 9: The 68% and 95% CL contour plots between various combinations of the model parameters of scenario IDE4 using only the CMB+BAO and CMB+BAO+R19 datasets, and the corresponding one-dimensional marginalized posterior distributions.

within 2σ, but an indication for ξ greater than zero appears at 1σ when JLA and JLA+CC are added to CMB+BAO, or when R19 is added to both CMB and CMB+BAO in the IDE1 and IDE2 scenarios.

Concerning the current value of the dark energy equation of state \( w_0 \), for all interacting scenarios and for all combination of datasets it always lies in the phantom regime at more than two/three standard deviations. Moreover, we find a robust anti-correlation between \( w_0 \) and \( H_0 \).

However, the most striking feature, and one of the main results of the present work, is that for all interacting models, independently of the combination of datasets considered, the estimated values of the Hubble parameter \( H_0 \) are greater compared to the ΛCDM-based Planck’s estimation [117] and close to the local measurements of \( H_0 \) from Riess et al. 2016 [118], Riess et al. 2018 [119] and Riess et al. 2019 [113]. This is triggered by the aforementioned anti-correlation between \( w_0 \) and \( H_0 \) and the strongly phantom values we obtain for \( w_0 \). The alleviation of \( H_0 \) tension is independent of the interaction model due to the absence of correlation between \( \xi \) and \( H_0 \), as shown in the two dimensional joint contours obtained for all observational datasets.

In summary, the extended interacting scenarios that include dark energy sectors with a dynamical equation of state with only one free parameter, are very efficient in alleviating the \( H_0 \) tension.

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TABLE VII: The values of $\ln B_{ij}$, where $j$ stands for the reference model $\Lambda$CDM and $i$ for the IDE models. The negative sign indicates that the reference model is favored over the IDE models.

| Dataset | Model  | Strength of evidence for reference model $\Lambda$CDM |
|---------|--------|------------------------------------------------------|
| CMB     | IDE1   | $-2.8$ Definite/Positive                             |
| CMB+BAO | IDE1   | $-4.6$ Strong                                        |
| CMB+BAO+JLA | IDE1 | $-7.3$ Very Strong                                   |
| CMB+BAO+JLA+CC | IDE1 | $-6.7$ Very Strong                                    |
| CMB+R19 | IDE1   | $+1.0$ Weak for IDE1                                  |
| CMB+BAO+R19 | IDE1 | $-0.7$ Weak                                          |
| CMB     | IDE2   | $-4.3$ Strong                                        |
| CMB+BAO | IDE2   | $-4.9$ Strong                                        |
| CMB+BAO+JLA | IDE2 | $-8.3$ Very Strong                                   |
| CMB+BAO+JLA+CC | IDE2 | $-8.9$ Very Strong                                    |
| CMB+R19 | IDE2   | $+1.6$ Definite/Positive                              |
| CMB+BAO+R19 | IDE2 | $-2.2$ Definite/Positive                              |
| CMB     | IDE3   | $-2.1$ Definite/Positive                              |
| CMB+BAO | IDE3   | $-7.6$ Strong                                        |
| CMB+BAO+JLA | IDE3 | $-8.8$ Strong                                        |
| CMB+BAO+JLA+CC | IDE3 | $-9.5$ Strong                                        |
| CMB+R19 | IDE3   | $+2.0$ Definite/Positive                              |
| CMB+BAO+R19 | IDE3 | $-1.1$ Definite/Positive                              |
| CMB     | IDE4   | $-2.0$ Definite/Positive                              |
| CMB+BAO | IDE4   | $-5.2$ Definite/Positive                              |
| CMB+BAO+JLA | IDE4 | $-9.6$ Strong                                        |
| CMB+BAO+JLA+CC | IDE4 | $-9.7$ Strong                                        |
| CMB+R19 | IDE4   | $+0.9$ Weak for IDE4                                  |
| CMB+BAO+R19 | IDE4 | $-2.3$ Definite/Positive                              |

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Cosmological constraints on interacting dark energy are an important area of research. Various authors have contributed to this field, including W. Yang and L. Xu, who, in their work "Is the Interacting Dark Matter Scenario an Alternative to Dark Energy?" published in Astron. Astrophys. 507, 47 (2009) [arXiv:0807.4590 [astro-ph]], have explored the implications of interactions between dark energy and dark matter.

M. B. Gavela, D. Hernandez, L. Lopez Honorez, S. Basilakos and M. Plionis, in their study "Dark coupling," published in JCAP 0907, 034 (2009) [arXiv:0901.1611 [astro-ph.CO]], have also contributed to the understanding of these interactions.

H. M. Sadjadi, in his work "w(d) = -1 in interacting quintessence model," published in Mon. Not. Roy. Astron. Soc. 472, 4736 (2017) [arXiv:1609.02287 [gr-qc]], has further explored the dynamics of interacting dark energy.

R. C. Nunes and E. M. Barboza, in their paper "Testing modified gravity in the dark sector interaction with LISA," published in JCAP 1705, no. 05, 040 (2017) [arXiv:1702.04189 [astro-ph.CO]], have investigated the implications of modified gravity in the context of dark energy.

G. S. Sharov, S. Bhattacharya, S. Pan, R. C. Nunes and S. Chakraborty, in their study "A new interacting two fluid model and its consequences," published in Mon. Not. Roy. Astron. Soc. 466, no. 3, 3497 (2017) [arXiv:1701.00780 [gr-qc]], have explored new models of interacting dark energy.

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The field of interacting dark energy continues to evolve, with ongoing research aimed at understanding the nature of dark energy and its interaction with other components of the universe.
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