Average and instantaneous velocities of energy of evanescent modes

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Many theoretical and experimental investigations have presented a conclusion that evanescent electromagnetic modes can superluminally propagate. However, in this paper, we show that the average energy velocity of evanescent modes inside a cut-off waveguide is always less than or equal to the velocity of light in vacuum, while the instantaneous energy velocity can be superluminal, which does not violate causality according to quantum field theory: the fact that a particle can propagate over a space-like interval does preserve causality provided that here a measurement performed at one point cannot affect another measurement at a point separated from the first with a space-like interval.

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I. INTRODUCTION

How long does it take a particle to tunnel through a potential barrier? This is the issue of tunneling times that has been controversial for decades. Over the last 20 years, the investigations on the issue of tunneling times have experienced a strong stimulus by the results of analogous experiments with evanescent electromagnetic waves (also called evanescent modes) [1-6]. As a result, nowadays, there have been many theoretical and experimental reports on the propagation of evanescent electromagnetic modes at speeds exceeding the speed of light in vacuum [7-23], and the experimental results have shown that the phase time do describe the barrier traversal time [7].

However, to understand the physical meanings of these superluminal behaviors, there has given rise to much controversy. Part of the controversy stems from the fact that these
superluminal velocities are the group velocities of evanescent modes. In view of which, in this paper we will study both an average and an instantaneous energy velocity of evanescent modes inside a cut-off waveguide, and show that the average velocity is always less than or equal to the velocity of light in vacuum, while the instantaneous one can be superluminal. Moreover, we show such results are in agreement with quantum field theory.

II. AVERAGE ENERGY VELOCITY OF EVANESCENT MODES INSIDE A CUT-OFF WAVEGUIDE

As we know, a single purely evanescent mode cannot transmit any time averaged power, which is due to the fact that the electric and magnetic fields are in quadrature (90° out of phase). This is the case in an infinitely long evanescent region. In a finite region, however, there will be some reflection of the forward attenuating evanescent mode at the exit [24]. Because the cutoff waveguide has a purely imaginary characteristic impedance, the reflection coefficient will have a phase shift associated with it. The reflected backward evanescent mode has a phase shift so that the total electric field is no longer in quadrature with the magnetic field. Thus the interference between the forward and backward evanescent modes gives rise to a nonzero time average power flow. This power flow is not due to a propagating wave but can be seen as the beating between two evanescent cavity modes. In addition to the purely reactive pulsations of energy, there is a time averaged contribution due to the fact that net energy escapes through the boundary during each cycle.

Assume that a hollow rectangular waveguide is made of perfect conductors, choosing a Cartesian coordinate system \((x, y, z)\) by taking the \(z\)-axis along the length of the waveguide, and the \(x\)- and \(y\)-axes parallel to the broad and narrow sides of the waveguide, respectively. The waveguide is a straight pipe with a part of reduced cross section from \(z=0\) to \(z=L\), its cross dimensions are \(d \times h\) for \(z<0\) and \(z>L\), while \(a \times b\) for \(0 \leq z \leq L\), where \(a>b, d>h\). Assume that one has \(0 \leq x \leq a\) and \(0 \leq y \leq b\) in our coordinate system. Let \(d \to +\infty, h \to +\infty\), and
then for $z<0$ and $z>L$, the cut-off frequency of the waveguide is taken to be zero, approximatively.

As we know, when a pulse of electromagnetic energy encounters a discontinuity in a waveguide, it is partially reflected and partially transmitted. In the process some of the energy may end up in higher-order evanescent modes which, being nonpropagating, are confined to the vicinity of the obstacle. Far away from the obstacle, its effects can be completely described by its reflection coefficient, its impedance, or its scattering matrix. A key simplification is that the scatterer does not alter the polarization state of the incident wave so that one may use scalar analysis. For simplicity, let us assume that the waveguide supports only the dominant $\text{TE}_{10}$ mode, and then the cut-off frequencies of the waveguide can be written as

$$\omega_c = \begin{cases} \frac{c\pi}{d} \approx 0, & \text{for } z < 0 \text{ or } z > L \\ \frac{c\pi}{a}, & \text{for } 0 < z < L \end{cases},$$

where $c$ is the velocity of light in vacuum. In our coordinate system, the electric and magnetic fields of the $\text{TE}_{10}$ mode can be written as $E = (0, E_y, 0)$ and $H = (H_x, 0, H_z)$, respectively. Let $\Psi = E_y, H_x, H_z$, these field quantities satisfy the wave equation

$$\left(\frac{d^2}{dz^2} + k^2 - k_c^2\right)\Psi = 0,$$  

where $k_c = \omega_c/c$, $k = |k| = \omega/c$, $k$ is the wave number vector of the $\text{TE}_{10}$ mode, and $\omega$ is the frequency. In terms of $\beta = -ik_z = \sqrt{k_c^2 - k^2}$, the wave number vector can be expressed as $k = (\pm k_c, 0, i\beta)$. Assume that the frequency $\omega$ satisfies $c\pi/a > \omega > c\pi/d \approx 0$, and then the waveguide within the region of $0 \leq z \leq L$ becomes a cut-off waveguide. From now on, let $\omega_c$ just denote the cut-off frequency of the cut-off waveguide (i.e., the waveguide within the region of $0 \leq z \leq L$), which implies that $\omega_c = c\pi/a$ and $\omega_c > \omega > 0$. At the entrance of the
cut-off waveguide (i.e., at \( z=0 \)), an incident electromagnetic pulse (as the TE\(_{10} \) mode) will be partially reflected and partially transmitted. Let \( R \) and \( T \) denote the reflection and transmission coefficients, respectively, they satisfy the normalization condition of \( |R|^2 + |T|^2 = 1 \). For \( z<0 \) and \( z>L \), the solutions of Eq. (2) can be rewritten as, formally

\[
\psi(x,z,t) = \begin{cases} 
\Phi(x,z,t) + R\Phi(x,z,t), & z < 0 \\
T\Phi(x,z,t), & z > L
\end{cases}
\]  

(3)

For the moment the cut-off waveguide plays the role of a photonic potential barrier. In view of the fact that, for the TE\(_{10} \) mode, the waveguide can be imagined as just consisting of two vertical plates with the separation \( d \) for \( z<0 \) and \( z>L \), and with the separation \( a \) for \( 0 \leq z \leq L \). Therefore, our discussions can be shown in Fig.1 (it depicts the waveguide’s longitudinal profile that is parallel to the broadside of the waveguide).

At the entrance and exit of the cut-off waveguide (i.e., at \( z=0 \) and \( z=L \)), both the electromagnetic fields and their first derivatives satisfy the condition of continuity, such that for \( 0 < z < L \) (i.e., inside the cut-off waveguide), one can prove that the electromagnetic fields, as the evanescent TE\(_{10} \) modes, can be written as, respectively
\[
E_x = (iH_0 \omega \mu_0 / k_x) \sin k_x x \exp(-i \omega t) [A \exp(\beta z) + B \exp(-\beta z)] \\
H_x = (H_0 \beta / k_x) \sin k_x x \exp(-i \omega t) [-A \exp(\beta z) + B \exp(-\beta z)] \\
H_z = H_0 \cos k_x x \exp(-i \omega t) [A \exp(\beta z) + B \exp(-\beta z)]
\]

where \( \varepsilon_0 \) and \( \mu_0 \) are respectively the vacuum permittivity and permeability satisfying

\[
\mu_0 \varepsilon_0 = \frac{1}{c^2}, \quad H_0 \quad \text{is a constant amplitude,} \quad \beta = \sqrt{k_x^2 - k_z^2}, \quad \text{and}
\]

\[
T = \frac{-2i(k/\beta) \exp(-ikL)}{[1-(k/\beta)^2] \sinh \beta L - 2i(k/\beta) \cosh \beta L} \\
A = \frac{T}{2} [1 + \frac{ik}{\beta}] \exp(ikL - \beta L) \\
B = \frac{T}{2} [1 - \frac{ik}{\beta}] \exp(ikL + \beta L)
\]

For the moment, inside the cut-off waveguide the Poynting vector \( S_z \) and energy density \( w \) are respectively,

\[
S_z = \frac{1}{2} \operatorname{Re}(E \times H^*)_z = \frac{1}{2} \operatorname{Re}(-E_x H_z^*), \quad w = \frac{1}{4} (\varepsilon_0 |E|^2 + \mu_0 |H|^2).
\]

The average velocity of energy inside the cut-off waveguide is

\[
\bar{v}_e(z) = \frac{\int_0^z \int_0^b S_z \, dx \, dy}{\int_0^z \int_0^b w \, dx \, dy}, \quad 0 < z < L,
\]

Substituting Eqs. (4)-(6) into Eq. (7), one has

\[
\bar{v}_e(z) = \frac{P_z}{W} = c \frac{2k_x^2}{k_z^2} \left[ \frac{k_x^2}{\beta^2} \cosh 2\beta(z - L) + \frac{k_x^2 (\beta^2 - k_z^2)}{k_z^2} \right]^{-1} \\
P_z = \int_0^a \int_0^b S_z \, dx \, dy = |TH_0|^2 \int_0^a \int_0^b \mu_0 \, c k_x^2 / 4k_z^2 \\
W = \int_0^a \int_0^b w \, dx \, dy = \frac{1}{8} |TH_0|^2 \int_0^a \int_0^b \mu_0 \left[ \frac{k_x^2}{\beta^2} \cosh 2\beta(z - L) + \frac{k_x^2 (\beta^2 - k_z^2)}{k_z^2} \right].
\]

One can show that \( |\bar{v}_e(z)| \leq c \), i.e., the average energy velocity of evanescent modes cannot be superluminal.

Nevertheless, some people might argue that, because the Poynting vector \( S_z \) stands for the flow of momentum along the waveguide, to obtain the average energy velocity, one should calculate the energy density \( w \) via the field quantities that make a contribution to the
Poynting vector $\mathbf{S}$, that is
\[
W = \left( \varepsilon_0 |\mathbf{E}_z|^2 + \mu_0 |\mathbf{H}_z|^2 \right)/4,
\]
which implies that the resulting average energy velocity is larger than the one given by Eq. (8), in particular, it might become superluminal. However, if Eq. (9) were right, using Eq. (9) one has
\[
\frac{P_z}{W} = \frac{c}{k_z^2} \left[ \frac{k_z^2}{\beta^2} \cosh 2\beta(z-L) + \frac{(\beta^2-k_z^2)^2}{k_z^2} \right]^{-1}
\]
\[
P_z = \int_0^z \int_0^b S_z \, dx \, dy = \left| TH_0 \right|^2 ab\mu_0 c k_z^2 / 4k_z^2
\]
\[
W = \int_0^z \int_0^b w \, dx \, dy = \frac{1}{16} \left| TH_0 \right|^2 ab\mu_0 \left[ \frac{k_z^2}{\beta^2} \cosh 2\beta(z-L) + \frac{(\beta^2-k_z^2)^2}{k_z^2} \right]
\]
using Eq. (10) one can show that the result of $|\mathbf{S}_z(z)|c$ is also valid.

Therefore, the average energy velocity of evanescent modes is always subluminal (or equal to $c$), which implies that for expectation values or ensemble average, Einstein causality is preserved.

It is worthy of note that the energy velocity of evanescent modes inside a cutoff waveguide has also been studied in Ref. [24]. However, some results in Ref. [24] might be questionable. For example, the equation (38) in Ref. [24] is not valid for the energy velocity of evanescent modes, which is due to the fact that the evanescent modes satisfy $\omega < \omega_1$ such that the energy velocity becomes an imaginary number.

III. INSTANTANEOUS ENERGY VELOCITY OF EVANESCENT MODES INSIDE A CUT-OFF WAVEGUIDE

To investigate the instantaneous energy velocity of evanescent modes inside the above-mentioned cutoff waveguide, let us study the probability for a photon to propagate a space-like interval along the cut-off waveguide. In Chapter III, the natural units of measurement $(\hbar = c = 1)$ is applied, repeated indices must be summed according to the
Einstein rule, and the space-time metric tensor is chosen as \( g^{\mu\nu} = \text{diag}(1,-1,-1,-1) \), \( \mu, \nu = 0,1,2,3 \), \( x^\mu = (t,x) = (t,x,y,z) \), and so on.

Let \( |0\rangle \) denote the vacuum state of a field operator \( \varphi(x^\mu) = \varphi(t,x,y,z) \), according to quantum field theory, the quantity \( G(x^\mu - x'^\mu) \equiv \langle 0 | \varphi(x^\mu) \varphi(x'^\mu) | 0 \rangle \) represents a transition probability amplitude from the quantum state of \( \varphi(x'^\mu) | 0 \rangle \) to the one of \( \varphi(x^\mu) | 0 \rangle \), such that \( |G(x^\mu - x'^\mu)|^2 \) is associated with the probability for a particle to propagate over a spacetime interval of \( (x^\mu - x'^\mu) \). From now on, all the field quantities inside the waveguide become operators in the field-quantization level. In terms of the electric field operator \( E_y \) inside the cut-off waveguide, let us study the quantity \( G(x^\mu - x'^\mu) \equiv \langle 0 | E_y(x^\mu) E_y(x'^\mu) | 0 \rangle \) without loss of generality. Likewise, the function \( G(x^\mu - x'^\mu) \) is related to the probability amplitude for photons to propagate from \( x'^\mu = (t',x',y',z') \) to \( x^\mu = (t,x,y,z) \) inside the waveguide, it can be regarded as a correlation function for photons inside the cut-off waveguide, too. If \( G(x^\mu - x'^\mu) \neq 0 \) for a space-like interval \( (x^\mu - x'^\mu) \), the probability for photons to propagate superluminally along the cut-off waveguide does not vanish, such that the instantaneous energy velocity of evanescent modes can be superluminal. In the present case, the 4-dimensional wave-number vector of photons in the free space inside the waveguide is \( k^\mu = (\omega,k_x,k_y,k_z) = (\omega,\pi/a,0,k_z) \), that is, \( k_x = \omega_z = \pi/a \) (note that the natural units of measurement \( (\hbar = c = 1) \) is applied). Applying QED and combining with our conditions, one can obtain

\[
G(x^\mu - x'^\mu) = \left( \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial t^2} \right) \mathcal{D}(x^\mu - x'^\mu),
\]

where
\[ D(x^\mu - x'^\mu) = C \int \frac{dk_z}{2\pi} \frac{1}{2\omega} \exp[-i\omega(t-t') + ik_z(z - z')] , \] (12)

where \( \omega = \sqrt{\omega_c^2 + k_z^2} \) is the frequency of photons and \( C = \exp[i(\pi/\alpha)(x-x')]/(2\pi)^2 \) is a phase factor. Using Eqs. (11) and (12) one has

\[ G(x^\mu - x'^\mu) = -\frac{\partial^2}{\partial t^2} D(x^\mu - x'^\mu) , \] (13)

For the cut-off waveguide, one has \( 0 < \omega = \sqrt{\omega_c^2 + k_z^2} < \omega_c \), which implies that \( k_z = i\beta \) (\( \beta \) is a real number satisfying \(-\omega_c < \beta < \omega_c \)). Seeing that the waveguide is placed along the \( z \) axis, let us take \( x'' = (0,0,0,0) \) and \( x'' = (t,0,0,z) \) with \( t, z \geq 0 \) for simplicity, and then one has \( C = 1/(2\pi)^2 \). By substituting \( x'' = (0,0,0,0) \), \( x'' = (t,0,0,z) \), \( k_z = i\beta \), \( 0 < \omega < \omega_c \) and Eq. (12) into Eq. (13), one can obtain

\[ G(x''') = (1/16\pi^2) \int_0^{\alpha_z} d\beta \sqrt{\omega_c^2 - \beta^2} \exp(-i\sqrt{\omega_c^2 - \beta^2}t - \beta z) . \] (14)

Applying the integral representation of the Hankel function of the second kind

\[ H^{(2)}_0(r) = \frac{2}{\pi} \int_0^{\pi/2} d\theta \exp(-ir\sin\theta) , \] (15)

one can obtain (notice that \( x'' x_\mu = c^2t^2 - z^2 = t^2 - z^2 \) in the natural units of \( \hbar = c = 1 \)):

\[ G(x''') = \frac{\alpha_z}{32\pi^2 \sqrt{x'' x_\mu}} \left[ H^{(2)}_2(\sqrt{x'' x_\mu}) - tH^{(2)}_2(\sqrt{x'' x_\mu}) \right] . \] (16)

Obviously, \( G(x'' - x''') = G(x''') \neq 0 \) for the space-like interval of \( x'' x_\mu < 0 \). As we know, for a timelike interval \( x'' x_\mu = t^2 - z^2 > 0 \), there is always an inertial frame in which \( z = 0 \); while, for a spacelike interval \( x'' x_\mu = t^2 - z^2 < 0 \), there is always an inertial frame in which \( t = 0 \). Then one can show the asymptotic behaviors of \( G(x''') \) as follows:

\[ G(x''') \sim \begin{cases} (t)^{-3/2} \exp(-i\omega t), & \text{as timelike interval } x'' x_\mu = t^2 \to +\infty, \\ (z)^{-3/2} \exp(-\omega \sqrt{z^2}), & \text{as spacelike interval } x'' x_\mu = -z^2 \to -\infty. \end{cases} \] (17)

Therefore, the probability for photons to propagate superluminally along the cut-off
waveguide does not vanish, which implies that the instantaneous energy velocity of the evanescent modes can be superluminal.

**IV. CONCLUSIONS AND DISCUSSIONS**

Up to now, we have come to the conclusion that the instantaneous energy velocity of evanescent modes can be superluminal, while the average energy velocity is always subluminal (or equal to c). However, such superluminal behavior does not violate causality. In fact, according to quantum field theory (see for example, Refs. [25] and [26]), a non-zero propagator or non-zero transition probability amplitude for a spacelike interval implies that a particle can propagate over the spacelike interval, but this spacelike propagation does not destroy causality provided that the commutator of two observables with a spacelike interval vanishes. In other words, causality is maintained as long as a measurement performed at one point cannot affect a measurement at another point whose separation from the first is spacelike.

On the other hand, just as S. Weinberg discussed in Ref. [27], although the relativity of temporal order raises no problems for classical physics, it plays a profound role in quantum theories. The uncertainty principle tells us that when we specify that a particle is at position $x_1$ at time $t_1$, we cannot also define its velocity precisely. In consequence there is a certain chance of a particle getting from $(t_1, x_1)$ to $(t_2, x_2)$ even if the spacetime interval is spacelike, that is, $|x_1 - x_2| > c |t_1 - t_2|$. To be more precise, the probability of a particle reaching $(t_2, x_2)$ if it starts at $(t_1, x_1)$ is nonnegligible as long as

$$0 < (x_1 - x_2)^2 - c^2(t_1 - t_2)^2 \leq (\hbar/mc)^2,$$

(18)

where $\hbar$ is Planck’s constant (divided by $2\pi$), and $m$ the particle’s mass (and then $\hbar/mc$ is the particle’s Compton wavelength). We are thus faced with our paradox: if one observer sees a particle emitted at $(t_1, x_1)$, and absorbed at $(t_2, x_2)$, and if $(x_1 - x_2)^2 - c^2(t_1 - t_2)^2$ is
positive (but less than or equal to \((\hbar/mc)^2\)), then a second observer may see the particle absorbed at \(x_2\) at a time \(t_2\) before the time \(t_1\) it is emitted at \(x_1\). There is only one known way out of this paradox. The second observer must see a particle emitted at \(x_2\) and absorbed at \(x_1\). But in general the particle seen by the second observer will then necessarily be different from that seen by the first observer (it is the antiparticle of the particle seen by the first observer). In other words, to avoid a possible causality paradox, one can resort to the particle-antiparticle symmetry. The process of a particle created at \((t_1, x_1)\) and annihilated at \((t_2, x_2)\) as observed in a frame of reference, is identical with that of an antiparticle created at \((t_2, x_2)\) and annihilated at \((t_1, x_1)\) as observed in another frame of reference.

In our case, the antiparticle of the photon is the photon itself. Therefore, the process that a photon propagates superluminally from A to B as observed in a frame of reference, is equivalent to the process that the photon propagates superluminally from B to A as observed in another frame of reference, where causality is preserved provided that the measuring results from the same observer are consistent and do not conflict with any physical law. More generally, general relativity tells us that causality is always preserved provided that there does not form a closed timelike curve. In fact, one can define an effective rest mass of guided photons inside a waveguide as \(m = h\omega_c/c^2\) (\(\omega_c\) is the cut-off frequency of the waveguide), and then the Compton wavelength of guided photons is \(\lambda_c = h/mc\) [28]. The walls of a cut-off waveguide localizes photons along the cross direction with a greater precision than the Compton wavelength of the photons, such that the quantum-mechanical effects cannot be ignored. As a result, the superluminal behavior of evanescent modes attributes to a purely quantum-mechanical effect, and it preserves quantum-mechanical causality or the so-called weak causality [29], that is, it preserves Einstein causality for
expectation values or ensemble average only, not for individual probabilistic process.

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