Precision Analysis of Hartmann-Shack Wave-front Sensor with Modal Reconstruction

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Abstract. Shack-Hartmann Wave-front Sensor is a popular instrument used to measure wave-front. And micro-lens array is the important component of it. In this paper we compared the reconstruction precision of square configuration sub-aperture with that of hexagon configuration through simulation. The result shows that the main factors influence the reconstruction precision is the layout of edge sub-apertures in both configurations. When the two configurations have the similar value area ratio of edge sub-apertures, hexagon configuration has higher reconstruction precision.

1. Foreword
Hartmann-Shack wave-front sensor (H-S) is a popular instrument used in adaptive optics, and its operational principle is showed in figure 1. The wave-front passes through and is separated by the micro-lens array then focuses on the CCD (Charge Coupled Device), which is located on focal plane of micro lens. Making a comparison between the reference positions we could get spot centroids and get the offset and wave-front partial derivative in each sub-aperture. Then the wave-front would be acquired with modal reconstruction\cite{1}.

![Figure 1. Shack-Hartmann Wave-front Sensor.](image-url)
Micro-lens array is used to separate the wave-front into many sub-apertures. Many articles have analysed the influence of different micro-lens array configurations on reconstruction precision [2]. Jin Tao [3] shows us the difference between square and loop configurations in the arrangements, complete detection ability and the accept ability on light undulation. But in that paper only a special number of sub-apertures is referred and the result is limited in the special case. So in this paper, we consider a different number of sub-apertures and compare the reconstruction precision with square and hexagon configurations, in the end of this paper, the reasons for the reconstruction precision difference are given.

![Figure 2. Two configurations of sub-apertures in micro-lens array.](image)

2. Two Kinds of Micro-lens Array Configurations

The sub-apertures configurations used in this paper are shown in figure 2. The first picture is the square configuration which is composed of many uniform squares arranged in order. The hexagon configuration is shown in the second picture. For each sub-aperture having equal area, the number of sub-aperture in both configurations is ensured. The number in the two pictures is 208 and 199 separately. Light axis is located in the centre of sub-aperture.

3. Reconstruct Wave-front

Each order of Zernike has evidence discrepancy means, so it is usually used to represent the wave-front [4]. The wave-front could be expressed by:

$$\Phi(x, y) = \sum_{k=1}^{\infty} a_k Z_k(x, y)$$  \hspace{1cm} (1)

Where $a_k$ represent the Zernike coefficient and $Z_k(x, y)$ is the Zernike polynomial. So we can obtain the partial derivative:

$$\Phi'_x(x, y) = \sum_{k=1}^{\infty} a_k Z'_k(x, y)$$  \hspace{1cm} (2)
\[ \Phi^j(x, y) = \sum_{k=1}^{\infty} a_k Z_{kj}(x, y) \]  

Eq. (2), (3) could be written as:

\[
\begin{bmatrix}
G_x(1) \\
G_y(1) \\
\vdots \\
G_x(m) \\
G_y(m)
\end{bmatrix}
= \begin{bmatrix}
D_{x1}(1) & \cdots & D_{xn}(1) \\
D_{y1}(1) & \cdots & D_{yn}(1) \\
\vdots & \ddots & \vdots \\
D_{x1}(m) & \cdots & D_{xn}(m) \\
D_{y1}(m) & \cdots & D_{yn}(m)
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
\vdots \\
a_n
\end{bmatrix}, \text{That is } G = DA \tag{4}
\]

So

\[ A = D^{-1}G \tag{5} \]

That is to say if we acquire \( A \) through Eq. (5), then we can reconstruct the wave-front described by Eq. (1).

In practice, we can calculate the spot centroid in each sub-aperture and compare it with reference point, and then we will get the offset in each sub-aperture. The offset divided by focus length is wave-front derivate.

We evaluate the reconstruction precision through \( t \)-the wave-front error coefficient. And \( t \) is defined as:

\[ t = \frac{C_{\text{rms}}}{T_{\text{rms}}} \tag{6} \]

Where, \( C_{\text{rms}} \) and \( T_{\text{rms}} \) are the RMS of the residual wave-front and insert wave-front separately. The coefficient is less, the precision is higher.

4. Simulation Processing

We use the first twenty Zernike polynomials except the first one to describe and reconstruct the wave-front. Comparing the simulation results of both configurations, we would acquire the major factors influencing the reconstruction precision. Simulation process is:

1. Selecting the Nth Zernike as the insert wave-front.
2. The measured derivate is the average slope in each sub-aperture [5]. that is:

\[ P_f = \frac{1}{S_d} \int_D \nabla \Phi dxdy \tag{7} \]

\( S_d \) is the sub-aperture area and \( \Phi \) is the insert wave-front. Through Eq. (7) we would get the \( G \) shown in Eq. (4) without the CCD read noise error and the CCD discrete sample error [6]. The data in the simulation processing is precise, not less than e-10, and then the error caused by the restore precision is also ignored. So the differences of reconstruction error are only caused by different micro-lens array configurations.

3. Selecting the first N Zernike polynomials to form the reconstruction matrix, and then eliminating the mode confusion error and mode truncation error [7].

4. Acquiring the coefficient through Eq.5 and reconstruct the wave-front expressed in Eq.1.

5. The reconstruction wave-front subtracted from the insert one is the residual discrepancy. Selecting 2,500 sample points on residual wave-front randomly and getting the \( C_{\text{rms}} \). In the same way, we get the \( T_{\text{rms}} \). So \( C_{\text{rms}} \) to \( T_{\text{rms}} \) ratio is the wave-front error coefficient.

5. Simulation Result

Through the process described in the forth section, we acquired that \( t \) was more than 10% when the order was more than 20. So we only discuss the first twenty Zernike polyamines. The result is shown in figure 3.
The differences of reconstruction precision in both configurations have relationship with the number and the valid area of edge sub-apertures. The valid area is the area of edge sub-apertures in the incident pupil, and the ratio of the valid area to the total edge sub-apertures area represents the measured precision of spot centroids. From figure 3 and table 1 we know the configuration which has the higher ratio has higher reconstruction precision. When both configurations have the same ratio, such as figure 3(c), the hexagon configuration still has the higher precision.

Figure 3. Wave-front error coefficient with both configurations.

a square: 112, hexagon: 109

b square: 208, hexagon: 199

c square: 256, hexagon: 253
Table 1. The effective number and area of edge sub-apertures.

| Configuration | Effective Edge Sub-apertures | Effective Edge Sub-apertures Area |
|---------------|-----------------------------|----------------------------------|
|               | Number          | Sum  | Sum  | Sum  | Sum  |
| Square (112)  | 24              | 21.4%| 86.8%|      |      |
| Hexagon (109) | 18              | 16.5%| 77.9%|      |      |
| Square (208)  | 44              | 21.2%| 80.1%|      |      |
| Hexagon (119) | 24              | 12.1%| 91%  |      |      |
| Square (256)  | 56              | 21.9%| 78.8%|      |      |
| Hexagon (253) | 54              | 21.3%| 78.1%|      |      |

The amount of edge sub-apertures is also a factor influences the reconstruction precision. The ratio of the number to the total indicates the edge sub-apertures affect on the reconstruction precision. In the figure 3 (a), the square configuration with higher valid area ratio has higher reconstruction precision. On the contrary, the precision of the square configuration in figure 3 (b) is lower. But the difference between the two configurations in figure 3 (a) is less than that in figure 3 (b). The reason for this is that the ratio difference between the two configurations in figure 3 (a) is lower than that in figure 3 (b).

The results acquired base on the assumption that the sub-apertures area in both configurations is equal. Without considering the condition, the function of $t$ and Zernike order is shown in figure 4. The difference of $t$ in both configurations is less as the number of sub-apertures is increasing, but the hexagon always has higher configuration precision.

In sum, the main reasons for the reconstruction precision difference between both configurations are the number and the value area ratio of edge sub-apertures. The reconstruction precision is higher with the number and the ratio is less. The hexagon configuration still has higher reconstruction precision when the two factors in the both configurations are similar.

References

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