Abstract

We analyse the impact of quantum gravity on the possible solutions to the strong CP problem which utilize the spontaneously broken discrete symmetries, such as parity and time reversal invariance. We find that the stability of the solution under Planck scale effects provides an upper limit on the scale $\Lambda$ of relevant symmetry breaking. This result is model dependent and the bound is most restrictive for the seesaw type models of fermion masses, with $\Lambda < 10^6 \text{ GeV}$. 

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1. Introduction. It is well known that the instanton effects bring about a periodic structure of QCD vacuum. This leads to $CP$-violation by strong interaction, characterized by the $\bar{\Theta}$ parameter defined as

$$\bar{\Theta} = \Theta_{QCD} + \Theta_{QFD}$$

Here $\Theta_{QCD}$ is the coefficient of the $P$- and $CP$-violating gluonic anomaly term $G\tilde{G}$ and $\Theta_{QFD} = \text{argDet}\hat{m}_u\hat{m}_d$ is a fermionic contribution, where $\hat{m}_u$ and $\hat{m}_d$ are the up and down quark mass matrices. This $CP$-violation manifests itself in an appearance of the neutron dipole electric moment, which is known experimentally to be less than $10^{-25}$ e cm leading to a phenomenological upper bound on $\bar{\Theta}$ of about $10^{-9}$ [3]. However, in the standard model $\bar{\Theta}$ receives an infinite renormalization even if it is put to zero at tree level by hand [4]. Understanding the smallness of the $\bar{\Theta}$ without fine tuning of parameters is known as the strong $CP$-problem. There are two widely discussed approaches to solving this problem:

i) The Peccei-Quinn mechanism [5] where the whole Lagrangian of QCD plus QFD is required to obey a global $U(1)_{PQ}$ invariance with nonvanishing color anomaly, which dynamically fixes $\bar{\Theta} = 0$. The spontaneous breaking of this symmetry leads to the existence of a pseudo-Goldstone boson - axion [6, 7].

ii) The discrete symmetry approach where a combination of discrete symmetries such as $P$ or $CP$ is used to set $\bar{\Theta} = 0$ naturally [8, 9, 10, 11, 12, 13, 14] at the tree level. In such a theory a finite $\bar{\Theta}$ arises at the higher loop level and one has to show that $\bar{\Theta} < 10^{-9}$.

An essential common ingredient of both these approaches is the presence of global symmetries, either $U(1)_{PQ}$ or $P/CP$, that guarantee the smallness of $\bar{\Theta}$. However, these are not dynamical symmetries. There is no physical ground to exclude that they are violated by higher dimensional effective operators, that could originate from new interactions existing at some high scale $M$. These operators must be of non-renormalizable type so that at $M \to \infty$ their effects disappear. The ultimate scale for such higher order operators can be regarded as a Planck scale $M_{Pl}$, where the gravity becomes as strong as other interactions. This is a product of one’s experience with the quantum gravitational effects related to virtual black holes [15] or wormholes [16] which are likely not to respect global symmetries. It is
therefore important to include all higher dimensional operators, consistent with the local invariance, in the effective low energy theory before discussing whether the model solves the strong CP-problem. Since in most models the extra global symmetries imposed on the Lagrangian are not automatic symmetries, one could argue that perhaps the renormalizable terms in the low energy theory should also be allowed to break these symmetries. In the absence of detailed calculations of the non-perturbative quantum gravity effects it is hard to argue for or against this. In this paper (as also in refs. [22, 23]) we will assume that only the Planck scale induced non-renormalizable terms are relevant. Certainly, they also have the nice property of vanishing in the limit of zero gravity. Of course, no such apology is needed if the theory is automatically invariant under these global symmetries.

Such a study for the PQ models was performed in recent papers [22]. It has been shown that barring an unnaturally high degree of suppression of the strength of the higher dimensional operators, the scale of $U(1)_{PQ}$ symmetry, $V_{PQ}$, must be less than 100 GeV in the simplest theories of the invisible axion [1], whereas the lower bound on $V_{PQ}$ coming from various physical and astrophysical data is larger by many orders of magnitude [1]. This result considerably diminishes our belief in Peccei-Quinn symmetry as a solution to the strong CP-problem.

Some authors [17] put forward the idea that wormholes themselves may set $\Theta$ to 0 or $\pi$ dynamically, thereby avoiding the strong CP-problem. However, as of now there is no universal agreement on the validity of this point.

In a self-consistent picture the global symmetry should originate as an accidental symmetry of the theory at lower energies, being automatically respected by the renormalizable piece of the Lagrangian due to the certain field content. The well-known examples are the lepton and baryon number conservation in the standard model. The quantum gravity effects can violate them only through the $d = 5$ and $d = 6$ operators with $M \sim M_{Pl}$ [18] - all renormalizable terms are automatically invariant under these symmetries. (In the context of grand unification these operators can appear at the lower scale, with $M \sim M_{GUT}$ [19].) Neither $U(1)_{PQ}$ nor $P$ and $CP$ are automatic in general, though there are a few attempts to introduce $U(1)_{PQ}$ as an accidental global symmetry, at the price of enlarging the local symmetry of the theory [20]. It has been argued recently [21], that $P$ or $CP$ may also appear automatically as discrete gauge symmetries in theories with dimensional compactification.

It is amusing to notice that the original Peccei-Quinn model [4] with low scale axion [4] is rather stable against Planck scale corrections. It is however ruled out by experimental data.
In this paper we consider the effects of gravity on the second class of solutions. The original idea [8] was to use discrete symmetries such as $P$ or $T$ in order to have $\Theta$-term vanishing at tree level and to keep it finite and calculable in perturbation theory. The challenge in this approach is to come out with a simple enough model which gives $\bar{\Theta}$ sufficiently small. The original models [8, 9] suggested to illustrate the philosophy behind them ended up using some ad hoc further symmetries needed for the consistency of the program. One would not have expected these symmetries to exist for any other reason. Yet another difficulty of these models is that the Higgs sector involved in electroweak symmetry breaking is nonminimal, in which case the natural suppression of flavour-changing neutral currents (FCNC) [24] does not occur. A more realistic scheme was suggested by Nelson [10] and generalized by Barr [11], which utilize $CP$ invariance to put $\Theta_{QCD} = 0$, and special field content to achieve also $\Theta_{QFD} = 0$ at the tree level after spontaneous breaking of $CP$. The key ingredients of these models, which also avoid a problem of FCNC, are:

i) the presence of extra heavy fermions which are mixed with ordinary quarks,

ii) the hypothesis that spontaneous $CP$-violation takes place only in these mixing terms.

The $\bar{\Theta}$-term effectively arises only at the 1-loop level. It is less than $10^{-9}$ if the Yukawa coupling constants are sufficiently small, less than $10^{-3}$.

However, the simplest possibility which utilizes the heavy fermions came in paper [12] and subsequently in papers [13, 14]. The idea of refs. [12, 13] is based on the universal seesaw mechanism [24, 25]: the quark and lepton masses appear due to their mixing with heavy fermions, in direct analogy with the well-known seesaw picture for neutrinos [27]. In this picture the solution to the strong $CP$-problem can be implemented through the spontaneous violation of $P$-parity only [12], as soon as one deals with left-right symmetric model $SU(2)_L \otimes SU(2)_R \otimes U(1)$, No other additional symmetry is required. Alternatively, one can use a concept of $CP$-invariance (without $P$-parity) even in the context of the $SU(2) \otimes U(1)$ model [13]. This possibility, however, requires also some extra symmetries (e.g. horizontal family symmetry, as also in the model of Nelson [10]). One can show that in these models, with reasonable assumptions about the new scales and parameters, the
effective $\Theta$ arising in loop effects is small enough ($< 10^{-9}$).

A different way to use the concept of parity was suggested in ref. [14]. The electroweak gauge symmetry of standard model was doubled by introducing the mirror world with new weak interactions being right-handed, which repeats the whole pattern of fermion masses of our left-handed world at some higher scale. This is a result of spontaneous violation of $P$-parity between ordinary and mirror worlds. The strong interactions are the same in both sectors, so the contributions of mirror fermions cancel the infinite renormalization of $\Theta$ within the standard model and $\Theta$ is guaranteed to be negligibly small, less than $10^{-19}$.

In this paper we discuss the impact of the Planck scale effects on the models of refs. [12], [13] and [14], which in the following are referred to as BM, B and BCS models, respectively. We show that these effects provide an upper bound on the scale of relevant symmetry breaking ($P$ or $CP$), with interesting phenomenological consequences.

2. The BM model. This model is based on the gauge $SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)$ symmetry with the quark fields in following representations:

\begin{align}
q_{Li} &\ (1/2, 0, 1/3), \quad U_{Ri} \ (0, 0, 4/3), \quad D_{Ri} \ (0, 0, -2/3) \\
q_{Ri} &\ (0, 1/2, 1/3), \quad U_{Li} \ (0, 0, 4/3), \quad D_{Li} \ (0, 0, -2/3)
\end{align}

(2)

where the $SU(2)_{L,R}$ isospins $I_{L,R}$ and $U(1)$ hypercharge $Y$ are shown explicitly (the indices of the colour $SU(3)_c$ are omitted), and $i = 1, 2, 3$ is the family index. The Higgs sector consists of only two doublets

\begin{align}
H_L \ (1/2, 0, 1) \\
H_R \ (0, 1/2, 1)
\end{align}

(3)

Obviously, the fields in first rows of eqs. (2)-(3) have the usual standard model content with respect to $SU(2)_L \otimes U(1)$ whereas the fields in second rows form the analogous set of $SU(2)_R \otimes U(1)$. The most general Yukawa couplings, consistent with gauge invariance, are

\footnote{By adding the obvious lepton fields to the quarks of eq. (2) the theory is free of gauge anomalies. As far as strong CP-problem is concerned, we do not consider them here.}
essentially the standard model ones:

\[
\mathcal{L}_L = \Gamma_{Lq}^i \bar{q}_{Li} U_{Ri} \tilde{H}_L + \Gamma_{Ld}^i \bar{q}_{Li} D_{Ri} H_L + h.c.
\]

\[
\mathcal{L}_R = \Gamma_{Rq}^i \bar{q}_{Ri} U_{Li} \tilde{H}_R + \Gamma_{Rd}^i \bar{q}_{Ri} D_{Li} H_R + h.c.
\]

(4)

For the singlet quarks \( Q = U, D \) the mass terms \( \hat{M}_q^{ij} \bar{Q}_{Li} Q_{Rj} \) are also allowed, unless they are suppressed by some additional symmetry. Imposing the discrete left-right symmetry \( P_{LR} \), which is essentially parity \[28\]:

\[
q_L \leftrightarrow q_R, \quad Q_L \leftrightarrow Q_R, \quad H_L \leftrightarrow H_R, \quad W_\mu^L \leftrightarrow W_\mu^R
\]

(5)

we have \( \Gamma_{Lq} = \Gamma_{Rq} = \Gamma_q \ (q = u, d) \), and the mass matrices \( \hat{M}_q \) are forced to be hermitean.

The VEVs \( < H_L^0 > = v_L \) and \( < H_R^0 > = v_R \), with \( v_R \gg v_L = 174 \text{ GeV} \), violate the \( P_{LR} \) invariance and break the gauge symmetry down to \( U(1)_{em} \). As a result, the whole \( 6 \times 6 \) mass matrices of quarks take the form

\[
\mathcal{M} = \begin{pmatrix}
0 & \Gamma v_L \\
\Gamma v_L & \hat{M}
\end{pmatrix}
\]

(6)

where \( \Gamma = \Gamma_{u,d} \) and \( \hat{M} = \hat{M}_{U,D} \) for the up- and down-type quarks, respectively. Notice, that the \( \text{Det} \mathcal{M} \sim \text{Det} \Gamma \) is real and therefore \( \Theta_{QFD} = 0 \). Since the \( \Theta_{QCD} \) is absent from the beginning due to parity invariance, we have \( \tilde{\Theta} = 0 \) naturally at tree level. Then all that remains to do is to identify properly the fermion mass eigenstates and show that the effective \( \tilde{\Theta} \) arising with radiative corrections is sufficiently small. In fact, the structure described above reflects the spirit of both BM and BCS models, which are in fact two limiting cases, corresponding to \( \hat{M} \gg v_R \) and \( \hat{M} \to 0 \), respectively.

However, the quantum gravitational effects can induce the higher dimensional operators violating explicitly the global \( P_{LR} \) invariance and thereby effectively contributing to \( \tilde{\Theta} \). These operators should be cutoff by Planck scale \( M_{Pl} \), so that their effects disappear at \( M_{Pl} \to \infty \). The leading order terms allowed by gauge symmetry are the following:

\[
\mathcal{L}_5 = \frac{1}{M_{Pl}} \bar{q}_{Li} \left( \alpha_{u}^{ij} \tilde{H}_L \tilde{H}_R^\dagger + \alpha_{d}^{ij} H_L H_R^\dagger \right) q_{Rj} + h.c.
\]

(7)
\[ L'_5 = \frac{1}{M_{Pl}} \bar{Q}_L \hat{Q}_R \beta_{RQ}^{ij} H_R^i H_R^j + \beta_{LQ}^{ij} H_L^i H_L^j + h.c. \]  

(8)

\[ L_6 = \frac{1}{M_{Pl}^2} \bar{q}_L (\gamma_{Ld}^{ij} \hat{u} R_j \bar{H}_L + \gamma_{Lu}^{ij} \hat{D}_R j H_L) H_R^i H_R^j + (L \leftrightarrow R) + h.c. \]  

(9)

where the \( \alpha, \beta \) and \( \gamma \)'s are in general the complex constants of the order of one. Notice, that for these operators to be \( P \)-invariant, the matrices \( \alpha_q \) and \( \beta_Q \) must be hermitean, and \( \gamma_{Lq} = \gamma_{Rq}^\dagger \). Since we expect that the Planck scale effects are not to respect the \( P \)-invariance, we assume the above matrices to be arbitrary.

Let us study now the impact of these operators on the \( \bar{\Theta} \) parameter. It is convenient to assume that \( \hat{M} \gg v_R \), in which case the ordinary light quarks are essentially \( q \)'s, whereas \( Q \)'s form a heavy states mixed with the latter through the non-diagonal terms in eq. (6). The mass matrices of the \( q \)'s, induced due to this, so called universal seesaw mixing \cite{25, 26}, are the following:

\[ \hat{m} = v_L v_R \Gamma \hat{M}^{-1} \Gamma^\dagger \]  

(10)

The inter-family hierarchy (hierarchy between eigenvalues of \( \hat{m} \)) can be related either with corresponding hierarchy in \( \Gamma \)'s or with the inverted hierarchy \cite{29} of the eigenvalues of \( \hat{M} \)'s. As we have seen above, \( \bar{\Theta} \) is vanishing at tree level. It was shown in ref. \cite{12} that a finite and small \( \bar{\Theta} \) arises at the two loop level, whose magnitude can be less than \( 10^{-9} \) for the reasonable choice of parameters in the theory. However, the Planck scale operator (7) will change the mass matrix (6) to the form:

\[ \mathcal{M} + \Delta \mathcal{M} = \begin{pmatrix} \alpha v_L v_R / M_{Pl} & \Gamma v_L \\ \Gamma^\dagger v_R & \hat{M} \end{pmatrix} \]  

(11)

(Other contributions are neglected). Since the coefficients \( \alpha \) are in general complex, the effective \( \bar{\Theta} \) is induced:

\[ \bar{\Theta} \simeq \frac{1}{M_{Pl}} \text{Tr}(\alpha \Gamma^{-1} \dot{M} \Gamma^{-1}) = \frac{v_L v_R}{M_{Pl}} \text{Tr}(\alpha \hat{m}^{-1}) \]  

(12)

Obviously, the dominant contribution in eq. (12) comes from the light quarks \( u \) and \( d \) with masses \( \sim \)few MeV. Then the condition \( \bar{\Theta} < 10^{-9} \) constrains the scale of right-handed current \( v_R \). Demanding that both the moduli and phases of the \( \alpha \)'s are \( O(1) \) (certainly,
one should not exclude the possibility of an order of magnitude suppression), and barring unforeseen conspiracies, we therefore conservatively estimate an upper limit on $v_R$ of about $10^6$ GeV. As long as the seesaw formula (10) is assumed to be valid, i.e. $\hat{M} \gg \Gamma v_R$, this limit is rather independent of the details of the model. It equally applies to the original version of BM model \cite{12}, where the heavy $Q$ fermion masses $M$ are assumed to be of the same order and the inter-family hierarchy is related to the hierarchy of Yukawa couplings in eq.(4), as well as to the inverse hierarchy model \cite{29}, where all $\Gamma$’s are assumed to be $O(1)$ and the inter-family hierarchy is originated from the hierarchy in $\hat{M}$’s.

3. The BCS model. This model also utilizes the discrete $P_{LR}$ symmetry acting on the set of fermions as in eq. (2) and scalars as in eq. (3). However, the mass terms $\hat{M}_Q$ of $Q$’s are put to zero due to additional axial symmetry $U(1)_A$. The $U(1)_A$ hypercharges are defined as following: $Y_A = Y$ for the fields of the first rows of eqs. (2) and (3), and $Y_A = -Y$ for the second rows. It is obvious that incorporating this symmetry, the theory remains free of gauge anomalies. It can be local or global.\footnote{The original version \cite{14} of the BCS model is based on the local symmetry $SU(3)_c \otimes [SU(2)_L \otimes U(1)_L] \otimes [SU(2)_R \otimes U(1)_R]$, where $SU(2)_L \otimes U(1)_L$ acting on the fields $q_L, Q_R$ and $H_L$ corresponds to the standard model of electroweak interactions and $SU(2)_R \otimes U(1)_R$ with the fields $q_R, Q_L$ and $H_R$ corresponds to the parallel mirror world, a complete replica of ours, but with new weak interactions being right-handed. These two worlds communicate only via the same colour $SU(3)_c$. No doubt that apart from nice ”mirror” philosophy behind it, such a presentation is completely equivalent to that we consider above: $U(1)_L \otimes U(1)_R = U(1) \otimes U(1)_A$ with $Y = Y_L + Y_R$ and $Y_A = Y_L - Y_R$. Moreover, in our case $U(1)_A$ can be global as well. It cannot serve us as a Peccei-Quinn symmetry, being free of gauge anomalies. As we show below, the impact of the Planck scale physics for the case of global $U(1)_A$ is different from the case of the local one.}

Forbidding the explicit mass terms, the $U(1)_A$ symmetry has nothing against the Yukawa couplings in eq. (4). As far as the fermion mass spectrum and mixing is concerned, this model completely operates with the parameters of the standard model. Two fermion sectors are completely decoupled in the mass matrix (6): $\hat{m} = \Gamma v_L$ is a mass matrix of the ordinary quarks $q_L$ and $Q_R$, whereas the mass matrix of the mirror ones $q_R$ and $Q_L$ is just rescaled by the factor $v_R/v_L$. At the tree level their contributions in $\bar{\Theta}$ cancel each other: $\bar{\Theta} = \text{argDet}\Gamma^{\dagger} = 0$, and the non-vanishing contribution to $\bar{\Theta}$ arising only at higher loops
is extremely small. Indeed, it was shown by Ellis and Gaillard [4] that in standard model $\bar{\Theta}$, once put to zero at tree level, arises only at the 3-loop level and is about $10^{-19}$. The divergent contributions appear only at the 6-loop level. However, in BCS scenario these are cancelled by contributions of mirror quarks: $v_R$, as the scale of the mirror (or parity) symmetry breaking provides the natural cutoff. This scale can be arbitrarily large and so leaves us with a little hope of detecting the mirror fermions.

The Planck scale operators, however, provide an upper bound on $v_R$. Let us consider first the case of $U(1)_A$ symmetry being local, as it was suggested in the original version of the BCS model. In this case the effective $d = 5$ operators of eq. (8) are forbidden by local symmetry and the dominant contributions to $\bar{\Theta}$ come from the $d = 6$ operators of the eq. (9). One has:

$$\bar{\Theta} \simeq \frac{v_R^2}{M_{Pl}^2} \text{Tr}(\gamma \Gamma^{-1})$$  

(13)

Then, by considering the contributions of the light quarks being dominant, the condition $\bar{\Theta} < 10^{-9}$ constrains $v_R$ to be less than about $10^{12} - 10^{13}$ GeV.

In the case of $U(1)_A$ symmetry being global both $d = 5$ operators (7) and (8) are active. Then the fermion mass matrices take the form:

$$\mathcal{M} + \Delta \mathcal{M} = \begin{pmatrix} \alpha v_L v_R / M_{Pl} & \Gamma v_L \\ \Gamma^\dagger v_R & \beta v_R^2 / M_{Pl} \end{pmatrix}$$  

(14)

so that the condition

$$\bar{\Theta} \simeq \frac{v_R^2}{M_{Pl}^2} \text{Tr}(\alpha \Gamma^\dagger \beta \Gamma^{-1}) < 10^{-9}$$  

(15)

implies $v_R < 10^9 - 10^{10}$ GeV. This limit makes mirror world accessible at SSC/LHC, since in this case the mirror partner of electron cannot be heavier than about 10 TeV.

4. The B model. This model is also based on the field content of eqs. (2)-(3), but in addition it utilizes also concept of local horizontal symmetry $SU(3)_H$ [30]: the fermions of the first row in eq. (2) transform as triplets of $SU(3)_H$ and of the second row as anti-triplets, while the scalars in eq. (3) are $SU(3)_H$ singlets. In fact, this is exactly the field content

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6Obviously, the same limit applies to the general case of the model without $U(1)_A$ symmetry when the mass terms of $Q$’s are allowed but are assumed to be less than $v_R.$
of ref. \[25\] where the universal seesaw mechanism was suggested for the generation of the quark and charged lepton masses. Clearly, \(SU(3)_H\) is free of gauge anomalies. The matrices \(\Gamma\) of the Yukawa coupling constants are forced now to be \(SU(3)_H\) singlets (i.e. proportional to the unit \(3 \times 3\) matrix). The explicit mass terms \(\hat{M}_Q\) are forbidden by horizontal symmetry, but they appear due to Yukawa couplings \(G_{nQ}\bar{Q}LQ\xi_n\), where \(\xi_n\) \((n = 1, 2, ..)\), are some scalar fields in representations 3 and \(\bar{6}\) of \(SU(3)_H\), introduced for the breaking of horizontal symmetry. Therefore, the mass matrices of the heavy fermions \(Q = U, D\) have the form:

\[
\hat{M}_Q = \sum G_{nQ} < \xi_n >
\]

Provided that \(\hat{M}_Q > v_R\), mass matrix of the ordinary quarks \(q\) appears due to their seesaw mixing with \(Q\)’s. Since the Yukawa couplings \(\Gamma\) are the same for each family, the inter-family hierarchy between \(q\)’s is necessarily related to the inverse hierarchy of the masses of \(Q\)’s, which, on the other hand, reflects the hierarchy of the horizontal symmetry breaking.

The presence of chiral horizontal symmetry \(SU(3)_H\) makes it unnatural to impose the left-right parity. However, CP-invariance can be imposed, which implies that all the Yukawa couplings can be taken to be real. The spontaneous \(CP\)-violation occurs in a sector of heavy fermions due to relative phases of the VEVs \(< \xi_n >\) \([13]\), and is transferred to the mass matrix \(\hat{m}\) of \(q\)’s due to seesaw mechanism. However, \(\hat{\Theta}\) remains vanishing at tree level. It appears in radiative corrections and can be rendered to be less than \(10^{-9}\) under certain assumptions on the parameters of the theory.

Let us include now the Planck scale effects. Instead of \(d = 5\) operators \((7)\), which are forbidden by horizontal symmetry, one has to consider the \(d = 6\) operators

\[
\frac{1}{M_{Pl}^2} \bar{q}_L \left( \alpha_a^u \tilde{H}_L \tilde{H}_R^\dagger + \alpha_a^d \tilde{H}_L \tilde{H}_R^\dagger \right) q_R \xi_n^\dagger + h.c.
\]

Accounting for these operators, after the similar considerations as in BM model one can deduce the limit \(v_R < 10^{10}\) GeV. This constraint holds also true if instead of \(SU(3)_H\) one considers the left-right horizontal symmetry \(SU(3)_{HL} \otimes SU(3)_{HR}\), with the scalars \(\xi_n\) being in representations \((3, 3)\). The \(P\) parity is natural in this case, under which \(\xi_n \leftrightarrow \xi_n^\dagger\). Then the strong \(CP\)-problem can be solved due to \(P\)-parity only, without imposing \(CP\)-invariance, since the tree-level features of the model are essentially the same as in BM model.
On the other hand, if one deals with $CP$-invariance and $SU(3)H$ symmetry, there is no need for $SU(2)_L \otimes SU(2)_R \otimes U(1)$ symmetry and the strong $CP$-problem can be solved at the level of $SU(2)_L \otimes U(1)$ \cite{13}. The fermion content of the theory with respect to $SU(2)_L \otimes U(1) \otimes SU(3)_H$ remains the same apart from that now $q_R = u_R, d_R$ are electroweak singlets, as well as $Q = U, D$. The scalar $H_R$ is also absent: the Yukawa couplings in the second equation (4) are changed to the $SU(3)_H$-invariant mass terms. Then the leading Planck scale operators consistent with the $SU(2)_L \otimes U(1) \otimes SU(3)_H$ symmetry are the $d = 5$ ones

$$\frac{1}{M_{Pl}} \bar{q}_L (\alpha_u^n u_R \bar{H}_L + \alpha_d^n d_R H_L) \xi_n^\dagger + h.c.$$ (18)

Repeating the above considerations concerning their contribution to $\bar{\Theta}$, one can readily obtain the upper limit on the horizontal symmetry breaking scale: $< \xi_n > < 10^6$ GeV, which is essentially also the scale of $CP$-violation. This limit makes the flavour-changing effects, related to the horizontal gauge bosons, available for the experimental search in $CP$-violation phenomena or rare decays \cite{31}. On the other hand, recalling that in this model the hierarchy between ordinary quark and lepton families should be the inverse with respect to the hierarchy in heavy fermions, this bound simply means that the lightest among the latter are expected to be in 100 GeV to 1 TeV range and so within the reach of new accelerators.

5. Conclusion. We have shown that if the non-perturbative Planck scale effects are assumed to break all global symmetries of nature, viable mechanisms to solve the strong $CP$-problem can be constructed implementing the natural physical symmetries such as parity or time reversal invariance. The upper limit on the $\bar{\Theta}$ parameter imposes upper limits on the scale at which these symmetries break. In two examples of such models \cite{12, 13}, this scale is less than $10^6$ GeV whereas in a third class of models \cite{14}, the upper limit is $10^{12}$ to $10^{10}$ GeV. The same consideration can be applied to the general class of models \cite{10, 11}. In particular, in the original model of Nelson \cite{10} one can deduce the limit $< \xi > < 10^9$ GeV on the scale of horizontal symmetry breaking.

Many physical consequences can follow from these considerations. New particles and phenomena can be within the reach of new experiments, in particular, at SSC/LHC. In models \cite{12, 13} the obtained upper limits can be rephrased (in model dependent way) in
lower bounds on neutrino masses. On the other hands, the same Planck scale effects could help us to adivde certain difficulties of the models under consideration. Let us comment on two of them:

i) As is well known, the spontaneous breaking of discrete symmetries such as $P$- and $CP$-parities leads to the formation of domain walls in the early universe. It has recently been argued [32], that the same quantum gravity effects can also induce the explicit $P$ and $CP$ violating Planck scale non-renormalizable terms in the Higgs potential, that can cause the decay of the domain walls and thereby adivde the associated cosmological disaster. Of course, in the BCS type models the upper limit on the parity breaking scale is high enough so that one could inflate away the domain walls. The point is that the reheating temperature after inflation must be less than the symmetry breaking scale so that the walls do not reappear. As for the BM and B type models, one would require a low scale inflation for the same purpose, which might not be a best case for the inflationary ideology.

ii) In the BCS type models there is another potential danger coming from the fact that in the limit of exact $U(1)_A$ symmetry the lightest of heavy mirror quarks (and leptons) is stable. Since one would expect that a baryon asymmetry is associated with the mirror world as well, the cosmological abundance of the latter should be much above the allowed experimental limit. To inflate them away, one should remember that still low scale inflation is needed: for example, in the case where the $U(1)_A$ symmetry is global, their masses cannot be more than about $10^4 - 10^5$ GeV due to upper limit $v_R < 10^{10}$ GeV. However, the Planck induced symmetry breaking operators mix the ordinary and mirror fermions (see eq. (14)), making the latter unstable and thereby rendering them harmless. For the case of local $U(1)_A$ symmetry, however, one again has to rely on inflation to achieve the same goal.

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