Quark-Exchange Mechanism of $\gamma d \rightarrow np$ Reaction At 2-6 GeV

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Abstract
Within the constituent quark model, we examine the extent to which the deuteron photo-disintegration at 2-6 GeV can be described by the quark-exchange mechanism. With the parameters constrained by the $np$ scattering, the calculated differential cross sections disagree with the data in both magnitude and energy-dependence. The results can be improved if we use a smaller size parameter for quark wavefunctions. We also find that the on-shell approximation used in a previous investigation is not accurate.

1. Introduction
It has been well established that nuclear dynamics at low energies can be described in terms of hadronic degrees of freedom. Within the context of Quantum Chromodynamics (QCD), this hadronic picture is expected to break down at sufficiently high energies where the basic degrees of freedom must be quarks and gluons. It is therefore interesting and important to know in which energy region the transition from hadronic picture to quark-gluon picture takes place. This has been the motivation of the experiments on deuteron photo-disintegration($\gamma d \rightarrow np$) conducted at SLAC\textsuperscript{1,2} around 1990 and at Jefferson Laboratory\textsuperscript{3,4} in the past few years. More measurements are expected in the near future.

The limitation of the hadronic picture of deuteron photo-disintegration at energies above about 1.5 GeV was first suggested in Ref.5. The data at 1-6 GeV accumulated at SLAC and JLab\textsuperscript{4} to follow the quark counting rule\textsuperscript{6} in the kinematic region where the momentum transfer of the reaction becomes larger than a critical value. On the other hand, one expects that the non-perturbative dynamics, such as those due to meson-exchange, still play an important role in the transition region. It
is obviously a highly non-trivial problem to interpret these data. Attempts in this direction have been made in Refs.7-9.

![Fig. 1. The quark-exchange mechanism of $\gamma d \rightarrow np$ reaction.](image)

In this contribution we would like to report on our recent investigation of the deuteron photo-disintegration at 2-6 GeV. We are motivated by the quark-exchange model of Frankfurt et al.\textsuperscript{9} Based on the quark-exchange mechanism illustrated in Fig.1, the authors of Ref.9 succeeded in describing the data at 90 degrees both in magnitude and energy-dependence. On the other hand, their predictions at other angles failed to describe the data unless a phenomenological factor is included.

Because of the complexities of the quark-exchange amplitude, several simplifications were made in the calculation of Ref.9. First the on-shell approximation was used to simplify the loop-integration over the quark propagator. Then the $\gamma d \rightarrow np$ cross section is cast, with some arguments, into a product of a factor depending on a convolution over the deuteron wavefunction and a factor depending on an appropriately evaluated elastic $np$ cross sections. No explicit calculation in terms of quark-gluon degrees of freedom is needed. While the simplicity of their final expression of the differential cross section is very appealing, it is possible that the difficulties they encountered as well as the successes they achieved could be due to the use of these simplifications. The purpose of this work is to address this question within a model in which calculations of the quark-exchange amplitude can be performed exactly.

2. Quark-Exchange Amplitude
We first observe that apart from the photon-quark interaction vertex,
the exchange mechanism illustrated in Fig.1 is the same as that studied in the constituent quark model of nucleon-nucleon interaction. We therefore will perform the calculation within the same theoretical framework. Thus our approach is not fully relativistic, like all of the previous studies of multi-quark systems. But this is the only framework within which an exact calculation of the quark-exchange amplitude can be performed. It is obvious that our intention here is not a realistic confrontation with the data. Rather it is aimed at addressing the theoretical questions mentioned above.

In the center of mass frame, the $\gamma d \rightarrow np$ amplitude can be written as (suppressing the spin-isospin indices)

$$t(\vec{p}, \vec{k}) = \langle \chi^{(-)}_{\vec{p}} | X(\vec{k}) | \Phi_d \rangle,$$

(1)

where $\vec{k}$ is the incident photon momentum, $\vec{p}$ the final proton momentum, $\Phi_d$ the deuteron wavefunction, and $\chi^{(-)}_{\vec{p}}$ the final neutron-proton scattering wavefunction. The operator $X(\vec{k})$ represents the quark-exchange mechanism. The matrix element Eq.(1) can be written as

$$t(\vec{p}, \vec{k}) = \int d\vec{P}_1 X^{(-)*}(\vec{P}_F) T(\vec{P}_F, \vec{k}),$$

(2)

with

$$T(\vec{P}_F, \vec{k}) = \int d\vec{P}_1 \int d\vec{P}_2 \delta(\vec{P}_1 - \vec{P}_F) \int d\vec{P}_1' \int d\vec{P}_2' \delta(\vec{P}_1' + \vec{P}_2' + \vec{k})$$

$$X(\vec{P}_1, \vec{P}_2, \vec{P}_1', \vec{P}_2', \vec{k}) \Phi_d(\vec{P}_1 - \vec{P}_2').$$

(3)

where $\vec{P}_1$ and $\vec{P}_2$ ($\vec{P}_1'$ and $\vec{P}_2'$) denote the momenta of the initial (final) two three-quark clusters. By assuming that the wavefunctions of all three-quark clusters in Fig.1 are of Gaussian form ($\phi(r) \sim \exp(-r^2/2b^2)$) in coordinate-space), we can integrate out most of the quark momenta analytically and obtain

$$X(\vec{P}_1, \vec{P}_2, \vec{P}_1', \vec{P}_2', \vec{k}) = \delta(\vec{P}_1 + \vec{P}_2 - \vec{P}_1' - \vec{P}_2' - \vec{k}) \frac{1}{2^{3/2} \pi b^3} b^{1/2} g^3$$

$$\int dq_{12}^2 dq_{56}^2 e^{-b^2 q_{12}^2} e^{-b^2 q_{56}^2}$$

$$\int dq_2 dq_5 V(q) G(q_p) h(q_p, k)$$

$$\exp[-b^2 \{a_{kk} k^2 + a_{qq} q^2 + a_{kq} k \cdot q + a_{kp} k \cdot P_F + a_{qq} q \cdot P_F + a_{kq} q \cdot P_I + a_{kp} q \cdot P_I + a_{qP_F} q \cdot P_F + a_{qP_I} q \cdot P_I + a_{pF} P_F + a_{pI} P_I \}]$$

(4)
where \( a_{kk} = 19/24, a_{kq} = -1/2, a_{qq} = 3/2, a_{qk} = -5/2, a_{q_pq} = a_{q_pq} = 3, a_{k_pP} = 3/2, a_{q_pP} = -1, a_{q_pP} = -3, a_{P_pP} = 7/6, a_{k_pP} = -1, a_{q_pP} = 2, a_{q_pP} = 3, a_{P_pP} = -2, \) and \( a_{P_pP} = 7/6 \). The parameter \( b \) in Eq. (4) defines the range of Gaussian function:

In Eq. (4), \( V(q) \) is a gluon-exchange interaction, \( G(q_p) \) is a quark propagator, and \( h(q_p, k) = e_q \bar{u}_q k_\mu \gamma^\mu u_{\bar{q} - \bar{k}} \) describes the photon-quark coupling. For simplicity, we follow a quark-model study of \( \pi N \) scattering to set

\[
V(q) = \sum_{i>j} \left[ -\pi \alpha_s (\lambda_i \cdot \lambda_j) (\sigma_i \cdot \sigma_j) \frac{\Lambda_g^2}{\Lambda_g^2 + q^2} \right],
\]

where \( \lambda_i \) is the color SU(3) generator, \( \alpha_s = 1 \) and \( \Lambda_g = 1.087 \) GeV were determined from fitting the \( \pi N \) scattering data. The quark propagator is taken to be

\[
G(q_p) = \frac{1}{W - \sqrt{M_5 + q_p^2 - \sqrt{m_q^2 + q_p^2 + i\varepsilon}}} = \frac{P}{W - \sqrt{M_5 + q_p^2 - \sqrt{m_q^2 + q_p^2}}}
- i\pi\delta(W - \sqrt{M_5 + q_p^2 - \sqrt{m_q^2 + q_p^2}})
\]

where \( P \) denotes taking the principal-value integration, \( m_q = 330 \) MeV, and \( M_5 = 5m_q \).

To carry out the multi-dimensional integration in Eq. (4), we use the method of Ref. 15 to expand the gluon-exchange interaction and deuteron wavefunction in terms of Gaussians: \( V(q) = \sum_i^{N_V} \sum_j^{N_G} g_i e^{-\alpha_i q^2} \) and \( \Phi_d(q) = \sum_j^{N_G} g_j e^{-\beta_j q^2} \). We find that very precise expansions can be obtained with \( N_V = 35 \) and \( N_G = 50 \). Substituting these expansions into Eqs. (3), and (4), we then obtain

\[
T(\vec{P}_F, \vec{k}) = \sum_{i}^{N_V} \sum_{j}^{N_G} \int d\vec{q}_p G(q_p) h(q_p, k) v_i g_j \exp(B) \int d\vec{q} d\vec{P}_I \exp \left( -\frac{\vec{x} \cdot \delta \vec{x}}{2} + \vec{s} \cdot \vec{x} \right)
\]

where \( \vec{x} = (\vec{q}, \vec{P}_I) \) is a two-component vector and

\[
B = -b^2 \left\{ a_{kk} k^2 + a_{kq} k \cdot q_p + a_{k_pP} k \cdot P_F + a_{q_Pq} q_p^2 + a_{q_Pq} q_p^2 + a_{q_Pq} q_p \cdot P_F + a_{P_pP} P_F^2 \right\},
\]

\[
\hat{A} = \begin{pmatrix} 2b^2 (a_{kq} + \frac{q_P^2}{m_q}) \\ a_{k_pP} \\ a_{q_Pq} \end{pmatrix},
\]

\[
\vec{s} = b^2 (a_{kq} \vec{k} + a_{q_Pq} \vec{q}_p + a_{q_Pq} \vec{P}_F, a_{k_pP} \vec{k} + a_{q_Pq} \vec{q}_p + a_{P_pP} \vec{P}_F).
\]
All integrations in Eq.(7) except that over the propagating quark momentum $\vec{q}_p$ can be carried out analytically by using the following formula for a multidimensional gaussian

$$
\int d\vec{x} e^{-\frac{\vec{A} \cdot \vec{x}}{2}} = \left( \frac{2\pi)^N}{\text{det}(A)} \right)^{d/2} e^{\frac{1}{2} \vec{x}^T \vec{A}^{-1} \vec{x}}
$$

(8)

where $d$ is the dimension of the space and $N$ is the number of variables, $\vec{x} = \{x_1, x_2, ..., x_N\}$ is a $N$-component vector. Schematically, Eq.(7) is then reduced into

$$
T(\vec{P}_F, \vec{k}) = \sum_{i} \sum_{j} v_i g_j \int d\vec{q}_p G(\vec{q}_p) h(q_p, k) I_{i,j}(\vec{P}_F, \vec{k}, \vec{q}_p)
$$

(9)

where $I_{ij}(\vec{P}_F, \vec{k}, \vec{q}_p)$ are from integrating analytically various Gaussian functions. We then carry out the remaining integration over the propagating quark momentum $\vec{q}_p$ in Eq.(9) numerically by using the standard subtraction method to handle the singularity of $G(q_p)$ of Eq.(6).

Fig. 2. Some results from our fits to np elastic scattering data. The data are from Refs.12-13.

3. Results

The first step to evaluate the $\gamma d \to np$ amplitude Eq.(2) is to construct a np model for generating the scattering wavefunction $\chi_{\vec{p}}^{(-)}$. It can be written in terms of np scattering t-matrix

$$
\chi_{\vec{p}}^{(-)}(p') = \delta(\vec{p} - \vec{p}') + t(\vec{p}, \vec{p}', E) \frac{1}{E - 2E_N(p') + i\epsilon}
$$

(10)
where $E = 2E_N(p)$ is the total energy. We have found that the $np$ elastic scattering data in the relevant 4-12 GeV region can be fitted by parameterizing the amplitude $t(p', p, E)$ as a sum of the exchanges of $\pi, \sigma, \rho, \omega,$ and Pomeron. Our fits at four energies are displayed in Fig. 2. The details will be given in Ref. 16.

Fig. 3. The $\gamma d \rightarrow np$ differential cross sections. No $np$ final state interaction is included. The solid curves are from an exact calculation of Eq. (7), while the dashed curves are from taking the on-shell approximation used in Ref. 9. The data are from Refs. 1-4.

To explore the dynamical content of the quark-exchange amplitude $T(\vec P_F, \vec k)$, we first perform calculations with the final $np$ scattering neglected; i.e. setting $\chi_{\vec k}^{(\sim)}(\vec p') = \delta(\vec p - \vec p')$ in evaluating Eq. (2). The deuteron wavefunction is generated from the Paris potential. With the specifications given in section 2, the only free parameter of the calculations is then the range parameter $b$ of the quark wavefunctions. We find that the predicted cross section is very sensitive to this parameter. If we set $b = 0.52$ fm used in the quark model of nucleon-nucleon interaction, the predicted cross sections are an order of magnitude smaller than the data. This is the first major difficulty of the quark-exchange model considered here. To get reasonable magnitudes at 90$^\circ$ degrees, we need to use a much smaller value $b = 0.25$ fm. The calculated differential cross
sections are the solid curves shown in Fig.3.

We now turn to investigating the validity of the on-shell approximation used in Ref.9. This amounts to performing our calculations by neglecting the principal-value part of the quark propagator Eq.(6). The calculated results are the dashed curves in Fig.3. The difference between the solid and dashed curves indicate very sizable contributions from the principal-value parts of the integrations. Our results strongly suggest that the on-shell approximation used in Ref.9 is not accurate.

![Figure 4](image)

**Fig. 4.** The $\gamma d \rightarrow np$ differential cross sections. The solid curves are from our full calculations, while the dashed curves are obtained when the final $np$ scattering is neglected. The data are from Refs.1-4.

Our complete calculations are the solid curves in Fig.4. The dashed curves are obtained when the $np$ final state interactions are omitted. We see that the $np$ final state interaction, constrained by the data shown in Fig.2, do not change our predictions significantly except at 70 degrees. Clearly our complete calculations are not able to describe the data.

To end, we emphasize that this is a very first step toward a realistic calculation of $\gamma d \rightarrow np$ reactions in terms of quark-gluon degrees of freedom. To firmly assess the quark-exchange model, several improvements must be made in our calculations. First, we find that our predictions are sensitive to the choice of the quark mass. This suggests that the use
of the quark propagator Eq.(6) with a constituent quark mass 330 MeV may not be correct. We may need to include the momentum-dependence of quark mass, as suggested by some QCD model calculations\textsuperscript{17}. Second, the gluon-exchange interaction Eq.(5), taken from a $\pi N$ study, needs to be improved to include more realistic momentum-dependence and other components of the gluon-exchange interaction. Third, we find that the main contribution to the cross sections is from the high momentum component of the deuteron wavefunction in the region where the employed nonrelativistic Paris potential may not be valid. It is necessary to generate the deuteron wavefunction from a model which can also describe $np$ scattering data at high energies, such as those shown in Fig.2. It may be possible to obtain such a model by extending the $\pi NN$ model of Ref.\textsuperscript{18} to higher energies. Finally, the size parameter $b$ of the three-quark clusters needs to be chosen from a more fundamental consideration.

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