X(1576) as Diquark-Antidiquark Bound State

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Abstract

We propose that the broad 1−− resonance structure recently discovered by BES in $J/ψ \rightarrow K^+K^-π^0$ is the P-wave excitation of a diquark-antidiquark bound state. This interpretation implies that there exists a negative parity, vector nonet. A rough estimate of the mass spectrum of the nonet is presented, and the prediction for the mass of $X(1576)$ is consistent with the experimental data. The OZI allowed strong decays are studied, it can decay into two pseudoscalars or one pseudoscalar plus one vector meson. A crucial prediction is that $X(1576)$ should dominantly decay into $K^+K^-$, $K_LK_S$, $φπ^0$. The observation of $I_3 = 1$ or $I_3 = −1$ states which predominantly decays into strange mesons could provide another important test to our proposal. To search the charged $I_3 = 1$ isospin partner of $X(1576)$, careful search in $J/ψ \rightarrow K^+K_Lπ^−$, $J/ψ \rightarrow K^+K_Sπ^−$ and $J/ψ \rightarrow φπ^+π^−$ is suggested.

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I. INTRODUCTION

A broad 1−− resonant structure X(1576) in \( J/\psi \to K^+K^−\pi^0 \) has been reported by the BES collaboration recently\(^1\). Its pole position is determined to be \((1576^{+49}_{−55}^{+98}_{−91})\text{MeV}\)-\(i(409^{+11}_{−12}^{+32}_{−12}^{−67})\text{MeV})\), and the product branching ratio \( Br(J/\psi \to X(1576)\pi^0)Br(X(1576) \to K^+K^-) = (8.5±0.6^{+2.7}_{−3.6})\times10^{-4} \). Therefore the branching fraction of \( J/\psi \to X(1576)\pi^0 \) must be much larger than \( O(10^{-4}) \). Considering the branching ratio of the \( J/\psi \) electromagnetic decay is usually of the order \( O(10^{-4}) \), so we determined that the decay \( J/\psi \to X(1576)\pi^0 \) is mainly via the hadronic decay, where both isospin and \( G \)-parity are conserved. Then \( X(1576) \) is of even \( G \)-parity and its isospin \( I = 1 \), and the quantum numbers of this structure are \( I^G(J^P) = 1^{+}(1^{−−})[2, 3] \). There is no obvious standard \( q\bar{q} \) candidate for this state.

Since the decay products \( K^+K^- \) contain a pair of strange quark, it may contain a pair of hidden strange quark, and the isospin triplet nature of this resonance requires that it at least contain additionally a pair of nonstrange quark, so it is reasonable to expect that \( X(1576) \) is a diquark-antidiquark bound state. The combined effects of the negative parity and the total angular momentum \( J = 1 \) require a unit of orbital angular momentum excitation. Thus we are led to the following assumption about the structure of \( X(1576) \):

\[
X(1576) = \frac{1}{\sqrt{2}}\left( |(ds)[\bar{d}s]|_{P-wave} - |(su)[\bar{s}u]|_{P-wave} \right) \tag{1}
\]

In the ref.\(^4\) Maiani et al. pointed out that the exotic states \( X(3872) \) and \( X(3940) \) can be well explained if they are S-wave diquark-antidiquark bound state \( |cq)[\bar{c}\bar{q}]|_{S-wave} \). Furthermore, they proposed that the new state \( Y(4260) \) may be the first orbital excitation of a diquark-antidiquark bound state, \( Y(4260) = |cs][\bar{c}\bar{s}]|_{P-wave}. \) If these are really what happens in nature, it is reasonable to expect the P-wave excitation of the four quark state \( |(q_1q_2)[\bar{q}_3\bar{q}_4]|_{P-wave}(q_i \text{ is light quark with } i=1-4) \) should be seen experimentally, \( i.e. \), the P-wave excitation of the nonet of light scalar \( (J^{PC} = 0^{++}) \) mesons \( \sigma_0(600), f_0(980), a(980), \kappa(800) \). In our scheme, \( X(1576) \) is exactly the P-wave excitation of \( a_0(980) \), and there exists analogously an nonet of vector mesons with \( J^P = 1^− \). Henceforth, this nonet is denoted by \( X \). Since the width of \( a_0(980) \) is very large, the width of \( X(1576) \) should also be large. Thus we qualitatively understood the reason why the observed width of \( X(1576) \) in the diquark-antiquark picture is so huge. In this letter, we would like to give a rough mass estimate of these states, and the prediction about the mass of \( X(1576) \) is consistent with its experimental value. The decay properties of these states are discussed,
which can decay into two pseudoscalars or one pseudoscalar plus one vector meson, and some distinctive predictions are given.

II. MASS SPECTRUM OF THE VECTOR NONET WITH $J^{PC}=1^{--}$

The weight diagram for the nonet is shown in fig.1, and we define $[q_1 q_2] \equiv \frac{1}{2}(q_1 q_2 - q_2 q_1)$, then the composition of the states of the nonet is as followings:

\[
\begin{align*}
X^+_a &= ([su][d\bar{s}])_{P\text{-wave}}, & X^-_a &= ([ds][\bar{s}u])_{P\text{-wave}}, & X^0_a &= \frac{1}{\sqrt{2}}(([ds][d\bar{s}])_{P\text{-wave}} - ([su][\bar{s}u])_{P\text{-wave}}), \\
X^+_\kappa &= ([ud][d\bar{s}])_{P\text{-wave}}, & X^-_\kappa &= ([ds][\bar{u}d])_{P\text{-wave}}, & X^0_f &= \frac{1}{\sqrt{2}}(([ds][d\bar{s}])_{P\text{-wave}} + ([su][\bar{s}u])_{P\text{-wave}}), \\
X^0_\kappa &= ([ud][\bar{s}u])_{P\text{-wave}}, & \bar{X}^0_\kappa &= ([su][\bar{u}d])_{P\text{-wave}}, & X^0_\sigma &= ([ud][\bar{u}d])_{P\text{-wave}}
\end{align*}
\]

where for the two isosinglets, the states with definite strange quark pair are introduced by assuming ideal mixing. The physical states $X_f$ and $X_\sigma$ are mixing of $X^0_f$ and $X^0_\sigma$ with mixing angle $\theta$,

\[
X_f = \cos \theta X^0_f + \sin \theta X^0_\sigma, \quad X_\sigma = -\sin \theta X^0_f + \cos \theta X^0_\sigma
\]

We will assume that the quarks prefer to form the "good" diquark when possible. States
dominated by that configuration should be systematically lighter, more stable, and therefore more prominent than the states formed from other types of diquarks. The residual QCD interaction and the spin-orbit interaction will mix the \( S = 0 \) "good" diquark with \( S = 1 \) "bad" diquark ("good" and "bad" diquarks in Jaffe’s terminology[7]), and a more sophisticated treatment would have to consider these effects quantitatively. However the effects only give a second order correction to the mass and other properties, so we restrict to the "good" diquark in this first analysis.

Most quark model treatments of multiquark spectroscopy use the colormagnetic short range hyperfine interaction as the dominate mechanism for possible binding[8, 9, 10]. Here we follow the same procedure, and the colormagnetic hyperfine interaction is:

\[
H' = -\sum_{i>j} C_{ij} \vec{\lambda}_i \cdot \vec{\lambda}_j \vec{\sigma}_i \cdot \vec{\sigma}_j
\]  

Here \( \vec{\sigma} \) and \( \vec{\lambda} \) are the Pauli and Gell-Mann matrices, \( i \) and \( j \) run over the constituent quarks and antiquarks. The coefficient \( C_{ij} \) are dependent on the quark masses and properties of the spatial wave functions of the quarks and antiquarks in the system. In the SU(3) flavor symmetry limit, \( C_{ij} \equiv C \), and the standard treatment using the colorspin \( SU(6)_{cs} \) algebra gives the hyperfine energy contribution[12, 13]:

\[
E' = \frac{C}{2} [D(\text{tot}) - 2D(Q) - 2D(\overline{Q}) + 16N]
\]  

where \( D = C_6 - C_3 - \frac{8}{3} S(S + 1) \), and \( D(\text{tot}) \), \( D(Q) \), \( D(\overline{Q}) \) denote the \( D \) of the total system, the subsystem of the quarks and the antiquarks respectively. \( C_6 \) and \( C_3 \) are the quadratic Casimir operators of \( SU(6)_{cs} \) and \( SU(3)_c \) respectively, \( S \) is the spin and \( N \) is the total number of the quarks and antiquarks. Rich phenomenology based on the colormagnetic hyperfine interaction have been developed[10, 11, 12, 13], and a fit of charmed baryons gives the consistent quark mass:

\[
m_u \approx m_d \approx 360\text{MeV}, \quad m_s \approx 540\text{MeV}, \quad m_c \approx 1710\text{MeV}
\]  

and the strength factors

\[
C_{qq} = 20\text{MeV}, \quad C_{qs} = 12.5\text{MeV}, \quad C_{ss} = 10\text{MeV}
\]  

Because the diquark and antidiquark are in P-wave and are separated by a distance larger than the range of the colormagnetic force, the color hyperfine interaction operates only within the diquark(antidiquark), but is not felt between the clusters.
There are three contributions to the mass of the states, i.e., the masses of the constituent quarks, the colormagnetic hyperfine interaction energy, and the energy due to the P-wave excitation. We estimate the contribution of the constituent quark mass from the decay products $(K^+K^-)$, since their quark content is the same as that of the parent state $(X(1576))$. Following the Ref. [5], the mass contribution due to the orbital angular momentum can be estimated from the mass spectrum of the $q\bar{q}$ mesons with $L = 0$ and $L = 1$.

The mass of the $S = 1$ states $K^*(892)$, $K_1(1270)$ and $K_2^*(1430)$ can be described by the following equation:

$$M(S, L, J) = K + 2AS \cdot \vec{L} + B\frac{L(L + 1)}{2}$$

where the second term is the spin-orbit interaction and the third term is the mass contribution of the orbital angular momentum. Then we find

$$B = \frac{m_{K_1} + m_{K_2} - 2m_{K^*}}{2} \approx 458\text{MeV}$$

Then the mass contribution of the P-wave excitation for this set of mesons equals $B$ which is approximately $458\text{MeV}$.

The mass of the charm mesons $D^*(2007)^0$, $D_1(2420)^0$, $D_2^*(2460)^0$ can also be described by the formula Eq.(8) with different parameters $K, A, B$. In this case, the parameter $B$ is about $433\text{MeV}$, then the P-wave excitation energy for this set of charm mesons approximately is $433\text{MeV}$. From above, we can see that the P-wave excitation energy changes slowly with the meson mass variations within the range about $1 \to 2.5\text{GeV}$.

For simplicity, we approximately take the P-wave excitation energy of the nonet $X$ to be the average of the P-wave excitation energy of $K^*(892)$, $K_1(1270)$, $K_2^*(1430)$ and that of $D^*(2007)^0$, $D_1(2420)^0$, $D_2^*(2460)^0$, i.e., $E_P \approx \frac{458+433}{2} \approx 445.5\text{MeV}$. By using this $E_P$, we determine the masses of $X$ as follows:

1. The mass of $X_\alpha(I = 1)$ and $X_\alpha^0$

   $$m_{X_\alpha} = m_{X_\alpha^0} = 2(m_K + 16C_{qs}) + E_P - 8C_{qs} - 8C_{qs} = 2m_K + E_P + 16C_{qs} = 1632.854\text{MeV}$$

   (10)

   This prediction is consistent with the experiment data, the pole position of $X(1576)$ is: $(1576^{+49+98}_{-55-91})\text{MeV} - i(409^{+11+32}_{-12-67})\text{MeV}$. Because of the large decay width, it is very difficult to precisely determine the mass of this resonance by experiments.

2. The mass of $X_\kappa(X_\kappa^+, X_\kappa^0$ and $\overline{X}_\kappa^0)$

   $$m_{X_\kappa} = m_{X_\kappa^0} - 8C_{qq} + 8C_{qs} + m_q - m_s = m_{X_\alpha} - 240 = 1392.854\text{MeV}$$

   (11)
If we use the experiment central value for the mass of \( X_a \), the peak mass of \( X_\kappa \) is 1336 MeV.

3. The mass of \( X_\sigma^0 \)

\[
m_{X_\sigma^0} = m_{X_a} - 16C_{qq} + 16C_{qs} + 2m_q - 2m_s = m_{X_a} - 480 = 1152.354 \text{ MeV} \tag{12}
\]

If the experimental value for the mass of \( X_a \) is input, the peak mass of \( X_\sigma^0 \) is 1096 MeV. The spectrum is similar to that of the light scalar nonet, which is inverted with respect to the q\( \bar{q} \) nonet.

III. THE DECAY OF THE VECTOR NONET \( X \)

The dominant decay mode of the four quark states is that they dissociate into two colorless q\( \bar{q} \) mesons \([5, 13]\), which means that a quark-antiquark pair is switched between the diquark and antidiquark, then form a pair of colorless q\( \bar{q} \) states. This mechanism has successfully described the decay of the scalar nonet \([6]\), also has been used to discuss the decay of other four quark states, and the predictions for the decay width are close to the experiment \([4, 5, 6]\).

The nonet \( X \) can decay into two pseudoscalars or one pseudoscalar and one vector meson. In the exact SU(3) flavor limit, the decay amplitude can be described with a single parameter \( g \), which describes the tunneling from the bound diquark-antidiquark pair configuration to the meson-meson pair. The parameters for the two pseudoscalars channel and one pseudoscalar and one vector meson channel should be different, are denoted as \( g_1 \) and \( g_2 \) respectively.

A. \( X \rightarrow \text{pseudoscalar} + \text{pseudoscalar} \)

We can describe the decay process by a single switch amplitude, \( e.g. \), the decay of \( X_a^+ \)

\[
[su]_3 \cdot [\bar{d}s]_3 \rightarrow (s\bar{s})_1_c (u\bar{d})_1_c - (sd)_1_c (u\bar{s})_1_c \tag{13}
\]

where the subscripts indicate color configuration. Taking into account the conservation of \( C \)–parity and \( G \)–parity, we can further write the invariant three mesons effective coupling:

\[
ig_1 X_a^{+\mu} [K^- \partial_\mu K^0 - K^0 \partial_\mu K^-] \tag{14}
\]

Here the coupling constant \( g_1 \) is dimensionless, and we will introduce \( \eta_q \) and \( \eta_s \) in the following, which are defined by \( \eta_q = \frac{2}{3} \eta_1 + \frac{1}{3} \eta_8 \), \( \eta_s = \frac{1}{\sqrt{3}} \eta_1 - \frac{1}{\sqrt{3}} \eta_8 \). The physical states
\( \eta, \eta' \) are related to \( \eta_8 \) and \( \eta_1 \) via the usual mixing formula
\( \eta_8 = \eta \cos \theta_p - \eta' \sin \theta_p, \eta_1 = \eta \sin \theta_p + \eta' \cos \theta_p \) with the mixing angle \( \theta_p = 16.9^\circ \pm 1.7^\circ \) [14]. From the effective lagrangian [14], we find the decay amplitude:
\[
\mathcal{M}(X_a^+ \to K^+ K^0) = g_1 \varepsilon^\mu(X_a^+)(\varepsilon^\mu(K^+) - \varepsilon^\mu(K^0))
\]
Here \( \varepsilon^\mu(X_a^+) \) is the polarization vector of \( X_a^+ \), \( \varepsilon^\mu(K^+) \) is the four momentum vector of \( K^+ \).

The decay of the other member of the nonet can be investigated in the same way, and the effective lagrangian for the relevant decays is as followings,

\[
\mathcal{L}_{eff} = ig_1 \left[ X_a^{+\mu} [K^- \partial_\mu K^0 - K^0 \partial_\mu K^-] + X_a^{-\mu} [-K^+ \partial_\mu K^0 + K^0 \partial_\mu K^+] \right.
\]
\[
+ X_a^{+\mu} [K^- \partial_\mu \pi^- - \pi^- \partial_\mu K^0 + \frac{1}{\sqrt{2}} K^- \eta_8 \partial_\mu (\pi^0 + \eta_8) - \frac{1}{\sqrt{2}} K^- \partial_\mu (\pi^0 + \eta_8)]
\]
\[
+ X_a^{+\mu} [-K^+ \partial_\mu \pi^+ + \pi^+ \partial_\mu K^- - \frac{1}{\sqrt{2}} K^+ \eta_8 \partial_\mu (\pi^0 + \eta_8)]
\]
\[
+ X_a^{+\mu} [K^0 \partial_\mu \pi^- - \pi^- \partial_\mu K^0 - \frac{1}{\sqrt{2}} K^0 \partial_\mu (\pi^0 + \eta_8)]
\]
\[
+ X_a^{+\mu} \frac{1}{\sqrt{2}} [K^0 \partial_\mu K^- - K^- \partial_\mu K^0 + K_L \partial_\mu K_S - K_S \partial_\mu K_L]
\]
\[
+ X_a^{+\mu} \frac{1}{\sqrt{2}} [K^0 \partial_\mu K^- + K^- \partial_\mu K^0 + K_L \partial_\mu K_S - K_S \partial_\mu K_L]\] (16)

From the above lagrangian, we can calculate the width of various decay channels following standard procedure. The decay width is expressed as
\[
\Gamma(X \to P_1 + P_2) = \frac{g_1^2}{6\pi} C_{X \to P_1 P_2} \frac{2}{(1 - \beta) \sqrt{2\pi}} \int_{m_X - \delta}^{m_X + \delta} \frac{d|m|}{m^2} \exp\left[-\frac{(m - m_X)^2}{2(\Gamma_X/2)^2}\right]
\]
where because of the large decay width, the mass distribution has been considered by using an exponential function [15]. \( |p| \) is the decay momentum \( |p| = \sqrt{m^2 - (m_1 + m_2)^2}/(m^2 - (m_1 - m_2)^2) \), \( \delta = 1.645 \frac{m_X}{2} \), \( \beta = 10\% \) [13]. \( m_1 \) and \( m_2 \) are respectively the mass of two pseudoscalars \( P_1 \) and \( P_2 \). \( C_{X \to P_1 P_2} \) is a numerical coefficient which can be found from the effective lagrangian [16], and coefficient \( C_{X \to P_1 P_2} \) for various decays are listed in Table I. In this table, we have not shown the decay channels which can be obtained from the channels appearing in the table by making charge conjugation, e.g., for \( X_a^- \to K^- K^0 \), the corresponding numerical coefficient is 1. From Table I, we can see that the dominant decay modes of \( X_a^0(X(1576)) \) are \( K^+ K^- \) and \( K_L K_S \), and
\[
\frac{\Gamma(X_a^0(X(1576)) \to K^+ K^-)}{\Gamma(X_a^0(X(1576)) \to K_L K_S)} \approx 1
\]
(18)
Some interesting relations can be found, such as:

\[ \frac{1}{2}(\sqrt{\frac{3}{2}} \cos \theta_p - \sqrt{\frac{3}{2}} \sin \theta_p)^2 \]  

However, \( X_a^0(X(1576)) \) can not decay into \( \pi^+\pi^- \),

\[ \Gamma(X_a^0(X(1576)) \to \pi^+\pi^-) \approx 0 \quad (19) \]

Dominant \( K^+K^- \) and \( K_LK_S \) decays is a distinctive signature of the validity of the present model. Some interesting relations can be found, such as:

\[
\begin{align*}
\Gamma(X_a^+ \to K^+K_L) &\approx \Gamma(X_a^+ \to K^+K_S) \approx \Gamma(X_a^0 \to K^+K^-) \approx \Gamma(X_a^0 \to K_LK_S) \\
\Gamma(X_f^0 \to K^+K^-) &\approx \Gamma(X_f^0 \to K_LK_S) \approx \Gamma(X_a^0 \to K^+K^-) \\
\Gamma(X_k^+ \to \pi^+K^0) &\approx 2\Gamma(X_k^+ \to K^+\pi^0) \approx \Gamma(X_k^0 \to K^+\pi^-) \approx 2\Gamma(X_k^0 \to K^0\pi^0) \\
\Gamma(X_k^+ \to K^+\eta) &\approx \Gamma(X_k^0 \to K^0\eta), \quad \Gamma(X_k^+ \to K^+\eta') \approx \Gamma(X_k^0 \to K^0\eta') \\
\tilde{\Gamma}(X_k^+ \to \pi^+K^0) &\approx \tilde{\Gamma}(X_k^+ \to K^+\pi^0) + \tilde{\Gamma}(X_k^+ \to K^+\eta) + \tilde{\Gamma}(X_k^+ \to K^+\eta') \\
\tilde{\Gamma}(X_k^- \to \pi^-K^-) &\approx \tilde{\Gamma}(X_k^0 \to \pi^-K^-) + \tilde{\Gamma}(X_k^0 \to K^0\eta) + \tilde{\Gamma}(X_k^0 \to K^0\eta')
\end{align*}
\]

where \( \tilde{\Gamma} \) denotes the decay width neglecting phase space correction (i.e., ignoring the effect of the factor \(|\tilde{p}|^3 \) in Eq. (17)). It can be easily checked that the first four equations are consistent with the isospin symmetry, and the last two equations in Eq. (20) express the flavor cross symmetry [3]. The effective lagrangian [16] describes the decays allowed by the OZI rule, and the contributions of the other couplings which violate the OZI rule are neglectable in the first order. Since \( X_a^\pm \) and \( X_a^0 \) form a isospin triplet, the pole position of these states should be approximately equal. Under this approximation and using Eq. (17), we can further
obtain the following ratio:

$$\Gamma(X_a^+ \to K^+K_L) : \Gamma(X_a^+ \to K^+K_S) \approx 1 : 1$$

(21)

We can search the other members of the nonet $X$ in $J/\psi$ decay, e.g., we can search $X_a^+$ which is the $I_3 = 1$ isospin partner of $X_0^0(X(1576))$ in $J/\psi \to X_a^+\pi^- \to K^+K_L\pi^-$ or $J/\psi \to X_a^+\pi^- \to K^+K_S\pi^-$. However, since $X_0^0(X(1576)) \to K^+K^-$ has been observed, this prediction is naturally the outcome of isospin conservation, and any rational proposal about the nature of $X(1576)$ should produce this result. So this prediction can not distinguish the different models about $X(1576)$, and we should search some particular signals which are almost unique in our model. With this idea in mind, we will investigate another strong decay mode $X \to \text{pseudoscalar} + \text{vector}$. Generally the width of these resonances is very large, so it is likely that some members of the vector nonet disappear into the continuum and can not be observed.

### B. $X \to \text{pseudoscalar} + \text{vector}$

The OZI allowed decays can be described by the effective lagrangian:

$$L_{\text{eff}} = g_2 \varepsilon^{\mu\nu\alpha\beta} \{ (X_a^+)_\mu \rho^-_{\alpha\beta} \eta_\nu + \phi_{\alpha\beta} \pi^- - K_{\alpha\beta}^* K^0 - K_{\alpha\beta}^{*0} K^- \}
\begin{align*}
+ (X_a^-)_\mu \rho^+_{\alpha\beta} \pi^+ + K^*_{\alpha\beta} K^0 - K_{\alpha\beta}^{*0} K^+ \\
+ (X_\pi^-)_\mu \rho^-_{\alpha\beta} \pi^- + \frac{1}{2} \sqrt{2} K^-_{\alpha\beta} (\mp \pi^0 + \eta_\nu) + \frac{1}{2} (\rho^0_{\alpha\beta} + \omega_{\alpha\beta}) K^- \\
+ (X^-)_\mu \rho^+_{\alpha\beta} \pi^+ + \frac{1}{2} \sqrt{2} K_\alpha^+ (\mp \pi^0 + \eta_\nu) + \frac{1}{2} (\rho^0_{\alpha\beta} + \omega_{\alpha\beta}) K^+ \\
+ (X_\rho^-)_\mu [-K^*_{\alpha\beta} \pi^+ - \rho^+_{\alpha\beta} K^- + \frac{1}{2} (\rho^0_{\alpha\beta} + \omega_{\alpha\beta}) K^0 + \frac{1}{2} K_{\alpha\beta}^0 (\pi^0 + \eta_\nu)] \\
+ (X_\eta^-)_\mu [-K^*_{\alpha\beta} \pi^+ - \rho^+_{\alpha\beta} K^- + \frac{1}{2} (\rho^0_{\alpha\beta} + \omega_{\alpha\beta}) K^0 + \frac{1}{2} K_{\alpha\beta}^0 (\pi^0 + \eta_\nu)] \\
+ (X_\rho^-)_\mu \frac{1}{2} \sqrt{2} [K^*_{\alpha\beta} K^+ - K^{*0}_{\alpha\beta} K^- + K_{\alpha\beta}^0 K^0 + K_{\alpha\beta}^{*0} K^0 + \sqrt{2} \rho_{\alpha\beta} \eta_\nu + \sqrt{2} \phi_{\alpha\beta} \pi^0] \\
+ (X_\eta^-)_\mu \frac{1}{2} \sqrt{2} [K^*_{\alpha\beta} K^+ + K^{*0}_{\alpha\beta} K^- + K_{\alpha\beta}^0 K^0 + K_{\alpha\beta}^{*0} K^0 - \sqrt{2} \omega_{\alpha\beta} \eta_\nu - \sqrt{2} \phi_{\alpha\beta} \eta_\nu] \\
+ (X_\sigma^-)_\mu \rho^0_{\alpha\beta} \pi^+ + \rho^+_{\alpha\beta} \pi^+ + \rho^0_{\alpha\beta} \pi^0 - \omega_{\alpha\beta} \eta_\nu \}
\end{align*}
$$

where $(X_a^+)_\mu$ is the field strength, and it is defined by $(X_a^+)_\mu = \partial_\mu (X_a^+)_\nu - \partial_\nu (X_a^+)_\mu$, the meanings of $(X_a^0)_\mu$, $\rho^0_{\alpha\beta}$ etc are similar. The dimension of the constant $g_2$ is $(\text{mass})^{-1}$. 


M$$^{-1}$$). Generally the decay width is

$$\Gamma(X \rightarrow P + V) = \frac{4g^2}{3\pi} D_{X \rightarrow PV} \frac{2}{(1 - \beta)\sqrt{2\pi}} \int_{m_X - \delta}^{m_X + \delta} d\tilde{m} \ |\tilde{p}|^3 \exp\left[-\frac{(m - m_X)^2}{2(\Gamma_X/2)^2}\right]$$

(23)

Here $$|\tilde{p}|$$ is the momentum of the vector meson $$V$$ or that of the pseudoscalar $$P$$, $$|\tilde{p}| = \sqrt{(m^2 - (m_P + m_V)^2)/(m^2 - (m_P - m_V)^2)}$$, $$\delta = 1.64\Gamma_X/2$$, $$\beta = 10\%$$ [15]. $$m_P, m_V$$ are respectively the mass of the pseudoscalar $$P$$ and the vector meson $$V$$. Being similar to $$C_{X \rightarrow P_1 P_2}, D_{X \rightarrow PV}$$ is also a numerical coefficient, which can be read from the lagrangian [22], and $$D_{X \rightarrow PV}$$ for various decay channels are listed in Table II.

The decay channels which can be obtained from the channels appearing in Table II by making charge conjugation, are not shown. From this table, we can learn that $$X_0^a(X(1576))$$ can decay into $$K^{*+}K^-, K^{*-}K^+, K^*0K_L, K^*0K_S, \overline{K}^0K_L, \overline{K}^0K_S, \rho^0\eta, \rho^0\eta', \phi\pi^0$$. Since the pole position of $$X_0^a(X(1576))$$ is below the threshold of $$\rho^0\eta$$, the process $$X_0^a(X(1576)) \rightarrow \rho^0\eta'$$ only occurs from the tail of its mass distribution. Some interesting relations can be obtained,

$$\Gamma(X_a^+ \rightarrow K^{*+}\overline{K}^0) \approx \Gamma(X_a^+ \rightarrow \overline{K}^{*0}K^+) \approx 2\Gamma(X_a^0 \rightarrow K^{*+}K^-) \approx 2\Gamma(X_f^0 \rightarrow K^{*+}K^-)$$
$$\Gamma(X_a^0 \rightarrow K^{*+}K^-) \approx \Gamma(X_a^0 \rightarrow K^{-}K^+) \approx \Gamma(X_a^0 \rightarrow K^{*0}\overline{K}^0) \approx \Gamma(X_a^0 \rightarrow \overline{K}^{*0}K^0)$$
$$\Gamma(X_f^0 \rightarrow K^{*+}K^-) \approx \Gamma(X_f^0 \rightarrow K^{-}K^+) \approx \Gamma(X_f^0 \rightarrow K^{*0}\overline{K}^0) \approx \Gamma(X_f^0 \rightarrow \overline{K}^{*0}K^0)$$
$$\Gamma(X_a^+ \rightarrow \rho^+\eta) \approx \Gamma(X_a^0 \rightarrow \rho^0\eta) \approx \Gamma(X_f^0 \rightarrow \omega\eta)$$
$$\Gamma(X_a^+ \rightarrow \rho^+\eta') \approx \Gamma(X_a^0 \rightarrow \rho^0\eta') \approx \Gamma(X_f^0 \rightarrow \omega\eta')$$
$$\Gamma(X_a^+ \rightarrow \phi\pi^+) \approx \Gamma(X_a^0 \rightarrow \phi\pi^0)$$

(24)

$$\Gamma(X_a^+ \rightarrow \rho^+K^0) \approx \Gamma(X_a^0 \rightarrow \rho^-K^+)$$
$$\Gamma(X_a^+ \rightarrow \rho^0K^0) \approx \Gamma(X_a^+ \rightarrow \omega\pi^+) \approx \frac{1}{2}\Gamma(X_a^0 \rightarrow \rho^+K^0)$$
$$\Gamma(X_a^0 \rightarrow \rho^+K^0) \approx \Gamma(X_a^0 \rightarrow \omega\pi^+) \approx \frac{1}{2}\Gamma(X_a^0 \rightarrow \rho^-K^+)$$
$$\Gamma(X_a^+ \rightarrow K^{*0}\pi^+) \approx 2\Gamma(X_a^+ \rightarrow K^{*+}\pi^0) \approx \Gamma(X_a^0 \rightarrow K^{*+}\pi^-) \approx 2\Gamma(X_a^0 \rightarrow K^{*0}\pi^0)$$
$$\Gamma(X_a^+ \rightarrow K^{*+}\eta) \approx \Gamma(X_a^0 \rightarrow K^{*0}\eta), \ \Gamma(X_a^0 \rightarrow K^{*+}\eta') \approx \Gamma(X_a^0 \rightarrow K^{*0}\eta')$$
$$\Gamma(X_a^0 \rightarrow \rho^-\pi^+) \approx \Gamma(X_a^0 \rightarrow \rho^+\pi^-) \approx \Gamma(X_a^0 \rightarrow \rho^0\pi^0)$$

(25)

$$\tilde{\Gamma}(X_a^+ \rightarrow K^{*+}\overline{K}^0) + \tilde{\Gamma}(X_a^+ \rightarrow \overline{K}^{*0}K^+) \approx \tilde{\Gamma}(X_a^+ \rightarrow \rho^+\eta) + \tilde{\Gamma}(X_a^+ \rightarrow \rho^+\eta') + \tilde{\Gamma}(X_a^+ \rightarrow \phi\pi^+)$$
$$\tilde{\Gamma}(X_a^0 \rightarrow K^{*+}K^-) + \tilde{\Gamma}(X_a^0 \rightarrow K^{-}K^+) + \tilde{\Gamma}(X_a^0 \rightarrow K^{*0}\overline{K}^0) + \tilde{\Gamma}(X_a^0 \rightarrow \overline{K}^{*0}K^0)$$
$$\approx \tilde{\Gamma}(X_a^0 \rightarrow \rho^0\eta) + \tilde{\Gamma}(X_a^0 \rightarrow \rho^0\eta') + \tilde{\Gamma}(X_a^0 \rightarrow \phi\pi^0)$$
TABLE II: the numerical coefficient $D_{X\rightarrow PV}$ entering Eq. (23) for the decay of the nonet $X$

| Decay | $D_{X\rightarrow PV}$ | Decay | $D_{X\rightarrow PV}$ |
|-------|----------------------|-------|----------------------|
| $X_+ \rightarrow K^{*+}K^0$ | 1 | $X^{+}_a \rightarrow \overline{K}^{*0}K^{+}$ | 1 |
| $X^{+}_a \rightarrow \rho^+\eta$ | $(\sqrt{\frac{4}{3}} \sin \theta_p - \sqrt{\frac{2}{3}} \cos \theta_p)^2$ | $X^{+}_a \rightarrow \rho^+\eta'$ | $(\sqrt{\frac{4}{3}} \cos \theta_p + \sqrt{\frac{2}{3}} \sin \theta_p)^2$ |
| $X^{+}_a \rightarrow \phi\pi^+$ | 1 | $X^{+}_a \rightarrow \rho^+K^0$ | 1 |
| $X^{+}_a \rightarrow K^{*0}\pi^+$ | 1 | $X^{+}_a \rightarrow K^{*+}\pi^0$ | $\frac{1}{2}$ |
| $X^{+}_a \rightarrow K^{*+}\eta$ | $\frac{1}{2}(\sqrt{\frac{4}{3}} \sin \theta_p + \sqrt{\frac{2}{3}} \cos \theta_p)^2$ | $X^{+}_a \rightarrow K^{*+}\eta'$ | $\frac{1}{2}(\sqrt{\frac{4}{3}} \cos \theta_p - \sqrt{\frac{2}{3}} \sin \theta_p)^2$ |
| $X^{+}_a \rightarrow \rho^0K^0$ | $\frac{1}{2}$ | $X^{+}_a \rightarrow \omega K^+$ | $\frac{1}{2}$ |
| $X^{+}_a \rightarrow K^{*+}\pi^0$ | $\frac{1}{2}$ | $X^{+}_a \rightarrow \rho^0\pi^+$ | $\frac{1}{2}$ |
| $X^{+}_a \rightarrow K^{*0}\eta^0$ | $\frac{1}{2}(\sqrt{\frac{4}{3}} \cos \theta_p - \sqrt{\frac{2}{3}} \sin \theta_p)^2$ | $X^{+}_a \rightarrow K^{*+}\pi^0$ | $\frac{1}{2}$ |
| $X^{+}_a \rightarrow \rho^0\eta'$ | $\frac{1}{2}$ | $X^{+}_a \rightarrow \rho^0\pi^0$ | $\frac{1}{2}$ |
| $X^{+}_a \rightarrow K^{*+}\eta_1$ | $\frac{1}{2}(\sqrt{\frac{4}{3}} \cos \theta_p + \sqrt{\frac{2}{3}} \sin \theta_p)^2$ | $X^{+}_a \rightarrow \phi\pi^0$ | 1 |
| $X^{+}_a \rightarrow K^{*0}\pi^0$ | $\frac{1}{2}$ | $X^{+}_a \rightarrow \phi\eta^0$ | $\frac{1}{2}$ |
| $X^{+}_a \rightarrow \rho^+\pi^+$ | 1 | $X^{+}_a \rightarrow \rho^0\pi^+$ | 1 |
| $X^{+}_a \rightarrow \omega\eta$ | $\frac{1}{2}(\sqrt{\frac{4}{3}} \cos \theta_p - \sqrt{\frac{2}{3}} \sin \theta_p)^2$ | $X^{+}_a \rightarrow \omega\eta'$ | $\frac{1}{2}(\sqrt{\frac{4}{3}} \cos \theta_p + \sqrt{\frac{2}{3}} \sin \theta_p)^2$ |

\[
\tilde{\Gamma}(X^{+}_a \rightarrow \rho^+K^0) + \tilde{\Gamma}(X^{+}_a \rightarrow K^{*0}\pi^+ \sim \tilde{\Gamma}(X^{+}_a \rightarrow K^{*+}\pi^0) + \tilde{\Gamma}(X^{+}_a \rightarrow K^{*+}\eta) + \tilde{\Gamma}(X^{+}_a \rightarrow K^{*+}\eta') + \tilde{\Gamma}(X^{+}_a \rightarrow \rho^0K^0) + \tilde{\Gamma}(X^{+}_a \rightarrow \omega K^+) + \tilde{\Gamma}(X^{+}_a \rightarrow \rho^0\pi^+\pi^0) + \tilde{\Gamma}(X^{+}_a \rightarrow \omega\eta) + \tilde{\Gamma}(X^{+}_a \rightarrow \omega\eta') + \tilde{\Gamma}(X^{+}_a \rightarrow \phi\eta) + \tilde{\Gamma}(X^{+}_a \rightarrow \phi\eta'), \tag{26}
\]

where $\tilde{\Gamma}$ denotes the partial decay width neglecting phase space. We can see that Eq. (24) and Eq. (25) are consistent with the isospin symmetry. The first equation in Eq (26) is exactly...
the Eq.(4) of the Ref.[3], and the equations in Eq.(26) reflect the flavor cross symmetry in the decay of the four quark states. Using Eq.(23) and the pole position of $X(1576)$: $(1576^{+49+98}_{-55-91} \pm 409^{+11+32}_{-12-67})\text{MeV}$, we can further obtain,

$$\Gamma(X^+_a(X(1576)) \to K^{*+K^0}) : \Gamma(X^+_a(X(1576)) \to K^{*+0K^0}) : \Gamma(X^+_a(X(1576)) \to \rho^+\eta)$$

$$: \Gamma(X^+_a(X(1576)) \to \rho^+\eta') : \Gamma(X^+_a(X(1576)) \to \phi\pi^+) : \Gamma(X^+_a(X(1576)) \to K^*\pi^-) \approx 1 : 1 : 0.47 : 0.175 : 1.24$$

$$\Gamma(X^0_a(X(1576)) \to K^{*+K^-}) : \Gamma(X^0_a(X(1576)) \to K^{*0K^0}) : \Gamma(X^0_a(X(1576)) \to \rho^0\eta)$$

$$: \Gamma(X^0_a(X(1576)) \to \rho^0\eta') : \Gamma(X^0_a(X(1576)) \to \phi\pi^0) \approx 1 : 1 : 0.94 : 0.35 : 2.48$$

(27)

The above ratios shows that in our four quark state scenario, the decay $X^0_a(X(1576)) \to \phi\pi^0$ is favorable, which is the distinctive feature of our four quark state interpretation. We expect the $I_3 = 1$ state $X^+_a$ should also appear in $J/\psi \to X^+_a \pi^- \to \phi\pi^+\pi^-$, $J/\psi \to X^+_a \pi^- \to K^{*0K^0}K^+\pi^-$ and $J/\psi \to X^+_a \pi^- \to K^{*+K^-}\pi^-$. Experimental search of the channel $X^0_a(X(1576)) \to \text{pseudoscalar} + \text{vector}$ is necessary so that the existence of $X(1576)$ can be reexamined.

IV. CONCLUSION AND DISCUSSION

We propose that $X(1576)$ recently reported by BES collaboration can be interpreted as the diquark-antidiquark bound state in P-wave excitation. This implies that there exists a vector nonet $X$, and $X(1576)$ is a member of the nonet. We estimate the mass spectrum of the nonet by considering both the colormagnetic hyperfine interaction energy and the P-wave excitation energy. The theoretical prediction for the mass of $X(1576)$ is $1632.854\text{MeV}$, which is consistent with the experimental data: $(1576^{+49+98}_{-55-91})\text{MeV}$-$i(409^{+11+32}_{-12-67})\text{MeV}$. The strong coupling of $X(1576)$ to its decay channel $K^+K^-$ may affect both the imaginary part of the pole position and its real part[16], this effect is ignored in the work, which is need to be studied further. Because the experimental error on the pole position is large, we expect the prediction will also be consistent with the experimental data if this effect is taken into account. The diquark here is taken as ”good” diquark, generally the ”bad” diquark is involved. However, the lowest lying and more stable states are dominated by the ”good” diquark configuration[7, 13]. Dealing with the mixing effects exactly from quark model is in progress.

OZI allowed strong decay of the nonet are investigated in detail. Both two pseudoscalars
decay channel and one pseudoscalar plus one vector meson channel are discussed. We find out that in our four quark state scheme, the dominant decay modes of $X_0^a(X(1576))$ are $K^+K^-$, $K_LK_S$, $\phi\pi^0$, but not $\pi^+\pi^-$, and this is an important test for our proposal. We predict that the positive and negative charged isospin partner of $X_0^a(X(1576))$ dominantly decay into strange mesons. Since these two states are connected by charge conjugation, we concentrate on the positive charged $I_3 = 1$ states $X^+_a$. In order to search these states, we suggest to analyze the $J/\psi$ decay data in $J/\psi \to K^+K_L\pi^-$, $J/\psi \to K^+K_S\pi^-$ and $J/\psi \to \phi\pi^+\pi^-$. The observation of $X^+_a$ is another crucial test of our scheme. The decays of the other members of the nonet are also discussed, which can provide important clues to the experimental search of these states. Similar to $X_0^a$ (i.e., $X(1576)$), the width of these states should be broad too, and hence it is also difficult to observe them experimentally.

Diquark is in $3_c$ configuration, so diquark and antidiquark cannot be observed individually. As the distance between diquark and antidiquark gets large, a $q\bar{q}$ will be created from the vacuum, then the state decays into baryon-antibaryon. But the central values of the mass distribution of the nonet are generally below the threshold of the baryon-antibaryon pair, the decay width should be small.

These states can mix with the ordinary $q\bar{q}$ states, if they have the same quantum numbers (e.g., $X_0^a(X(1576))$ can mix with $\rho(1450)$, $\rho(1700)$ and so on). The mixing effects interfere in the spectrum and the decay properties, and a full consideration including mixing effects is notoriously a difficult problem in exotic hadron spectrum. More sophisticated treatment of $X(1576)$ which takes these effects into account is expected. However, as a first step to understand $X(1576)$, we expect that such mixing effects in most cases are small from previous work on multiquark states\cite{[7, 13]}, and the results obtained in this letter are at least correct qualitatively.

Recently BES performs a partial wave analysis of $J/\psi \to \phi\pi^+\pi^-$ and $J/\psi \to \phi K^+K^-$ from a sample of $58M$ $J/\psi$ events in the BESII detector. There is a strong peak in the $\phi\pi$ mass distribution which centers at $1500\text{MeV}/c^2$ with a full-width of $200\text{MeV}/c^2$\cite{[17]}, and this peak was also reported about twenty years ago with quantum number $J^P = 1^-$\cite{[18]}. It is very likely there is some component of $X^+_a$ in this $\phi\pi$ peak. Finally, we would like to mention that if our predictions are not consistent with future experimental results, $X(1576)$ should have a different structure. More experimental facts about $X(1576)$ are needed in order to clarify these issues.
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