Numerical simulations of compact object binaries

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Abstract

Coalescing compact object binaries consisting of black holes and/or neutron stars are a prime target for ground-based gravitational wave detectors. This paper reviews the status of numerical simulations of these systems with an emphasis on recent progress.

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1. Introduction

Inspiralizing and coalescing compact object binaries consisting of black holes (BHs) and/or neutron stars (NSs) are of high importance to understand gravity and matter under extreme conditions. The strong gravitational fields require full nonlinear general relativity to describe them, and so compact object binaries allow exploration of curved spacetime in the nonlinear and highly dynamic regime. The matter density inside NSs is above the nuclear density. NSs therefore provide an avenue to study matter at super-nuclear densities, and compact object mergers allow such studies even in dynamic situations when the two objects collide.

Compact object binaries are therefore at the centre of attention for several scientific disciplines. The study of BH binaries (BH–BH) sheds light on properties of general relativity in the genuinely nonlinear, dynamic regime (in a clean vacuum environment), ranging from BH kicks (e.g. [1]) to the topological structure of event horizons [2–4]. Simulations of binaries involving one or two NSs (BH–NS, NS–NS) [5, 6] elucidate the connection between these systems and short gamma-ray bursts. Finally, gravitational waves emitted by compact object binaries are the most promising source for gravitational wave detectors like advanced LIGO [7–9], advanced Virgo [10, 11] and KAGRA [12]. Gravitational wave detectors require accurate waveform models and information about electromagnetic counterparts to optimally search for gravitational waves via matched filtering (event detection), and to extract the maximum amount of information about the source of gravitational waves, once such waves have been detected (parameter estimation).

Direct numerical simulations of the full Einstein equations are an important cornerstone in the study of coalescing compact object binaries. These simulations have continued to progress.
swiftly, resulting in a large number of notable recent results. This paper gives an overview of the current status of numerical simulations of compact object binaries. Our particular focus lies on recent advances, which have not yet been incorporated into longer or more specialized review articles (like [6, 13, 14]) or books (like [15]). Numerical simulations of higher dimensional gravity (e.g. [16, 17]) and alternative theories of gravity (e.g. [18, 19]) are advancing rapidly, but because of length limitations we shall restrict our attention to the ‘classical’ compact object binaries (BH–BH, BH–NS, NS–NS) in four spacetime dimensions in standard general relativity, the scenario of most direct relevance to gravitational wave detectors.

This paper is organized as follows. Section 2 reviews BH–BH simulations, starting with numerical methods, recent advances, applications to gravitational wave science and ending with BH–BH simulations embedded in gaseous or electromagnetic environments. Section 3 deals with binaries with NSs (BH–NS, NS–NS). We conclude in section 4 with some thoughts about the future of the field.

2. Black hole—black hole binaries

2.1. Numerical methods

Since the numerical relativity breakthroughs in 2005 [20–22], several frameworks have emerged to simulate inspiraling and merging BH–BH binaries. The major differentiating factors are the choices for the formulation of Einstein’s equations (either the BSSNOK system [23–25] or the generalized harmonic (GH) system [20, 26–28]) and the choice of the numerical evolution algorithm, finite-differences (FD) or pseudo-spectral methods. Pretorius’ first simulations employed the combination GH+FD [20, 27], and this code has since been applied to a variety of physical scenarios including high-energy collisions of BH–BH [29], eccentric BH–BH [30] and studies of gravity in different spacetime dimensions [17, 31].

However, in regard to the main focus of this section, BH–BH simulations of direct relevance to gravitational wave detectors, the combinations BSSNOK+FD and GH+Spectral have emerged as the leading frameworks, and so we shall describe these in some detail. The BSSNOK+FD framework is built around the BSSNOK [23–25] evolution equations coupled with FD approximation schemes. The GH+Spectral framework utilizes the generalized harmonic (GH) equations [20, 26–28] and multi-domain spectral methods. These frameworks differ in a large number of individual elements that are required to compute a gravitational waveform with numerical relativity. Table 1 lists the main choices that are made within the two frameworks for setting initial data, performing the evolution and analyzing the resulting data. Besides formulation and numerical algorithm used for the evolution equations, these choices include formulation and numerical methods for the initial data, diagnostics tools and factors that determine the physics of the scenario under consideration (e.g. low orbital eccentricity).

While table 1 conveys the broad picture, it must necessarily simplify and omit details. For instance, Pretorius’ breakthrough work [20] does not fit into these categories1.

The choices shown in table 1 are interrelated and only certain combinations are feasible. In particular, one cannot simply exchange specific elements between BSSNOK+FD and GH+Spectral. Let us illustrate some of these dependencies: moving puncture evolutions performed in the BSSNOK+FD framework require very specific gauge conditions (Gamma-driver and 1+log slicing [45–47]) to keep the punctures stable during the evolution. Because the BHs are not excised, moving puncture evolutions require initial data covering the entire

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1 Pretorius used the GH evolution system with an adaptive-mesh-refinement FD code, starting from initial data that contained nonsingular balls of the scalar field that collapsed to BHs early in the evolution.
Table 1. Ingredients into a BH–BH simulation, and the most common choices for the two major frameworks in use to solve BH–BH spacetimes.

|  | BSSNOK+FD | GH+Spectral |
|---|---|---|
| **Initial data** |  |  |
| **Formulation** | Conformal method using Bowen–York solutions [32–34] | Conformal thin sandwich [33, 35] |
| **Treatment of BH singularities** | Puncture data [36] | Quasi-equilibrium BH excision [37–39] |
| **Num. algorithm** | Pseudo-spectral [40] | Pseudo-spectral [41] |
| **Low-eccentricity orbital param’s** | Post-Newtonian inspiral [42] | Iterative eccentricity removal [43, 44] |
| **Evolution** |  |  |
| **Formulation** | BSSNOK [23–25] | Generalized harmonic with constraint damping [20, 26–28] |
| **Gauge conditions** | Evolved lapse and shift [45–47] | Harmonic ($\mathcal{H}^\nu = 0$) and/or evolved $H^\nu$ [48] |
| **Treatment of BH singularities** | Moving punctures [21, 22] | BH excision [49] |
| **Treatment of outer boundary** | Sommerfeld BC | Minimally-reflective, constraint-preserving [28, 50] |
| **Discretization** | High-order finite-differences [51, 52] | Pseudo-spectral methods |
| **Mesh-refinement** | Adaptive mesh refinement | Domain decomposition [41, 48] |
| **Diagnostics** | BH Apparent horizon finder, quasi-local spin measures |  |
| **GW extraction** | Newman Penrose scalars, Regge-Wheeler-Zerilli |  |
| **Infrastructure** | BAM, Cactus, Einstein Toolkit, Hahndol | SpEC |
| **Codes** | BAM, Hahndol, LazEv, Lean, Llama, MayaKranc, UIUC | SpEC |

initial-data hypersurface. Therefore, a moving puncture evolution cannot be started from the excision initial-data used for the spectral evolutions$^2$.

Puncture data cover the entire initial-data hypersurface, so it seems that GH+Spectral evolutions could use puncture data. However, this is also non-trivial: singularity excision requires careful control over the location and shape of the excision boundary [49, 56]. This is more difficult when the BH horizons change on short timescales, in particular during merger$^3$ and early in an evolution when the coordinate location of the BHs and the gauge settles down. Conformal thin sandwich initial data (as used in the GH+Spectral approach) reduce significantly such transients and are therefore more suitable for GH+Spectral than puncture initial data.

Trumpet initial data reduce initial transient apparent horizon motion [57] relative to standard puncture initial data; however, so far trumpet initial data have not been used regularly for production BH–BH simulations.

To summarize, some key properties of the two BH–BH frameworks are as follows.

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$^2$ Proposals to fill-in the BH interiors of excision initial data [53–55] have not been systematically pursued.

$^3$ Control of the excision boundary was one of the major obstacles in obtaining BH–BH mergers with GH+Spectral, cf [49, 56].
• BSSNOK+FD simulations are generally considered more robust and ‘easier’ because of the lack of BH excision and the larger expertise of how to stabilize FD methods. Moreover, code-infrastructure [58–60], including adaptive-mesh-refinement (Carpet [61, 62]) and apparent horizon finders [63], are publicly available. Several independent research groups have used these tools and have obtained a tremendous wealth of results (e.g. BH kicks). The major codes are listed in table 1 and more details are available in the related proceeding contributions describing the NINJA collaboration [64].

• Spectral methods, in contrast, require BH excision and are very sensitive to the presence of ill-defined elements in the problem (e.g. outer boundary conditions, handling of inter-domain boundaries where grids of different resolution touch, or filtering). Significant care and expertise is needed to analyze and control all relevant aspects of the evolution, and only one spectral BH–BH code exists: the Spectral Einstein Code SpEC [65] developed by the SXS collaboration between Cornell, Caltech, CITA and Washington State University.

• The key advantage of spectral methods is their fast (exponential) convergence. Therefore, high accuracy can be obtained with comparatively low computational cost. The high accuracy/lower cost permeates the entire ‘simulation pipeline’. Lower numerical noise (due to higher accuracy) makes it easier to perform certain fits that are required in iterative eccentricity removal (see section 2.2.2); the low computational cost makes it possible to perform longer simulations; the clean waveforms extracted at finite radius are more amenable to extrapolation to obtain the asymptotic waveform at infinitely large distance; high accuracy also makes it easier to compute and analyze quantities that depend on higher derivatives of the evolved fields, for instance BH vortices and tendencies [66, 67].

• To date, many more simulations have been performed with BSSNOK+FD than with GH+Spectral, but the GH+Spectral simulations are longer and more accurate. Both techniques have found their place, depending on the precise needs of the science to be studied. However, this conventional wisdom slowly changes, as the BSSN+FD simulations become more accurate and achieve longer simulations, whereas the GH+Spectral simulations become more robust and the number of GH+Spectral simulations increases [64].

While simulations work successfully, we caution the reader that both frameworks utilize heuristically determined ingredients. Gauge conditions and constraint damping terms contain parameters chosen by trial and error, and mesh structures are tuned based on user experience. Therefore, parameter choices working in one region of parameter space may require adjustments for other regions (e.g. [68–70]). Furthermore, constraint damping parameters impact the accuracy and quality of the simulation beyond their main purpose of preserving the constraints [71]. The presence of user-tuned parameters has further implications. On the negative side, it is not guaranteed that the current techniques will work in unexplored regions of parameter space. On the positive side, it is conceivable—even likely—that further tuning of code parameters will enhance accuracy and efficiency of future simulations compared to today’s runs.

2.2. Recent advances

Given the rather mature state of BH–BH simulations, recent advances are incremental, pushing and refining capabilities, and confirming assumptions that were made in previous years to achieve fast progress.
2.2.1. New records. Let us start with some ‘new records’. With increasing mass ratio \( q = M_1/M_2 \geq 1 \), fully numerical simulations become more challenging because the computational cost increases (the small BH must be resolved), and because experience of code tuning from comparable mass binaries is no longer applicable. Two groups pushed the mass ratio of BH–BH simulations up to \( q = 100 \). Sperhake et al [72] examine head-on collisions of a small BH with a large BH. Lousto and Zlochower [73] perform a numerical simulation of the last two orbits of a BH–BH binary of mass ratio \( q = 100 \). Both these calculations are very impressive, requiring not only a large amount of CPU resources, but also improvements and tuning of the computational infrastructure (e.g. location of mesh-refinement regions). The RIT group [74–76] extracts BH trajectories from numerical simulations of mass ratio up to \( q = 100 \). These trajectories are then used in perturbative calculations to compute the waveforms of high mass-ratio binaries.

Turning to spin, the long-standing bound

\[
\frac{S}{M^2} = \chi \lesssim 0.93
\]

was recently broken. Certain BH properties vary significantly between \( \chi = 0.93 \) and \( \chi = 1 \) and therefore angular momentum may not be the most useful measure of extremality. For instance, the rotational energy of a BH with \( \chi = 0.93 \) is only 59% of the rotational energy of a maximally rotating BH. When taking rotational energy as the metric, the bound (1) is far from extremality.

Equation (1) represents the maximal spin that is achievable with puncture initial data [77], and therefore, all BH–BH simulations within the BSSN-OK framework are limited by this bound. BH spins higher than the bound (1) are achieved with the more general initial-data formalism utilized within the GH+Spectral framework. Lovelace et al [78] construct initial data and perform short evolutions of BH–BH binaries with spins of 0.97. More recently, Lovelace et al [56, 79] compute complete inspiral/merger/ringdown simulations for equal mass BH–BH binaries with aligned spins of 0.97 and anti-aligned spins of 0.95. These simulations require delicate control of the excision boundaries and demonstrate that the SpEC code has significantly matured.

The \( \chi = 0.97 \) aligned spin simulation [56] also carries another new record. It is presently the longest published complete BH–BH simulation, lasting about 25 orbits before merger and ringdown, for an evolution time of 7000 M. The longest incomplete simulation (i.e. inspiral only) so far is presented by Le Tiec et al [80], lasting 34 orbits and 11 000 M. All these extremely long simulations were obtained with SpEC. The simulations presented in [80] range up to mass ratio 8, all in excess of 20 orbits, and are used to compare periastron advance in BH–BH binaries with several analytical approximation schemes.

The last year has also witnessed another increase in the largest known BH kick from inspiralling BH–BH4. To remind the readers, in 2007 the ‘BH super-kick configuration’ was discovered [82–84], with BH kicks close to 4000 km s\(^{-1}\). In 2011, Lousto and Zlochower [1] demonstrated that tilting the BH spins from the super-kick configuration toward partial alignment with the orbital angular momentum can increase the BH kicks to almost 5000 km s\(^{-1}\). Partial alignment of the BH spins with the orbital angular momentum allows the BHs to spiral closer to each other, where the higher velocities enhance the BH kick.

2.2.2. Improved techniques. Eccentricity removal has been extended to precessing binaries. Isolated BH–BH binaries originating from binary stars are expected to have vanishing orbital eccentricity when they enter LIGO’s frequency band [85, 86]. To achieve low eccentricity in

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4 Interactions of high-velocity BHs result in even larger kicks [81].
a numerical simulation requires careful choices for orbital frequency $\Omega_0$ and radial velocity $v_r$ of the individual BHs (or, equivalently, tangential and radial linear momentum of the BHs). These parameters can be chosen based on post-Newtonian information, either in closed form or by integrating post-Newtonian ordinary differential equations for two point-masses starting at large separation. The post-Newtonian coordinates and velocities are then used in the construction of BH–BH initial data. This approach [42, 87] is computationally inexpensive and achieves eccentricities of a few $0.001$ for low-spin and comparable mass BH–BH, and somewhat higher eccentricity for high spins and unequal mass BH–BH.

One can also adjust orbital parameters iteratively [43, 88]. One begins with a reasonable first guess for $\Omega_0$ and $v_r$ (perhaps based on post-Newtonian information), evolves for about two orbits, analyzes the orbital trajectories and then adjusts $\Omega_0$ and $v_r$. With the adjusted initial orbital parameters, one constructs a new BH–BH initial data set and performs a new evolution. This technique is computationally more expensive but achieves eccentricities as small as $\sim 10^{-5}$. Recently, Buonanno et al [44] extended this technique to precessing BH–BH systems. Buonanno et al [44] perform a post-Newtonian analysis demonstrating that spin impacts iterative eccentricity removal only for eccentricities $10^{-4}, \ldots, 10^{-3}$ (the exact bound depends on the BH spins and the initial BH separation). Furthermore, Buonanno et al [44] demonstrate that $e \sim 10^{-4}$ can indeed be reached for several BH–BH configurations with dimensionless spins of 0.5. This work uses the GH+Spectral approach with the SpEC code. Tichy et al [89] proposed an alternative iterative technique within the BSSNOK+FD framework and reached eccentricities of a few $\times 10^{-3}$. Very recently, Purrer et al [204] studied iterative eccentricity removal for moving puncture simulations based on the gravitational waveforms.

**Cauchy characteristic extraction** (CCE) [90–93] extracts certain data from a standard 3+1 evolution code (as described in section 2.1) and evolves these data with a separate characteristic code to future null infinity. At future null infinity, gravitational radiation is unambiguously defined and so this technique promises gauge invariant waveforms, improving on the more widely used technique to extrapolate finite-radius waveforms to infinite extraction radius [94]. In recent papers, Babiuc et al [92, 93] improve the PITT CCE code, and perform careful convergence tests. This code is publicly available as part of the Einstein Toolkit [60, 95]. With the recent improvements and public availability of the CCE code, I expect rapid increase in the use of this post-processing tool. While CCE is gauge-invariant, it requires initialization at the beginning of the numerical simulation. Bishop et al [96] investigated the impact of different initialization strategies and found some impact onto the CCE waveform. Furthermore, CCE cannot remove errors that were introduced in the underlying 3+1 numerical evolution. For instance, if the outer boundary conditions of the 3+1 evolution admit unphysical incoming radiation (or reflected outgoing radiation), then such radiation will appear in the CCE waveform.

### 2.2.3. Code validation and consistency checks.

Several papers confirmed that BH–BH simulations indeed work as expected thus further validating the numerical techniques.

Owen [97, 98] (extending work by Campanelli et al [99]) analyzes carefully a SpEC simulation of an equal-mass, non-spinning BH–BH. He confirmed with a multipolar analysis that the remnant BH settles down to Kerr with the correct quasi-normal mode fall-offs. The combination of gauge-invariant quantities with the high accuracy of the SpEC code allows Owen to confirm agreement to many significant digits, usually to better than 1 part in $10^3$ and sometimes to better than 1 part in $10^9$. References [97, 99] also confirm that the spacetime of a BH merger approaches Petrov-type D at late times after the BH–BH merger.
Hinder et al [100] perform a careful analysis of the asymptotic fall-off of the Newman–Penrose scalars, revisiting [101]. The fall-off rates agree with expectations,

$$\psi_n \sim \frac{1}{r^{5-n}}$$

where $n = 0, \ldots, 4$ labels the individual Newman–Penrose scalars, $\psi_0, \ldots, \psi_4$.

The NINJA-2 project [102] collected about 40 numerical waveforms, with some configuration contributing multiple times by different codes. These duplicate waveforms allow consistency checks between waveforms computed independently by different codes. Overlap calculations between (2, 2) modes of hybridized waveforms show disagreements broadly in line with the expected errors of the different numerical codes and hybridization procedures. Comparisons of higher order modes have not yet been performed and are planned for future work. For details, see the separate NINJA-2 contribution to the Amaldi Proceedings [64].

2.2.4. Future improvements. Several recent directions of research may have an impact on accuracy and efficiency of BH–BH simulations in the future. The conformal Z4-system [103] combines features from BSSNOK with the constraint damping of the GH equations, resulting in improved suppression of constraint violations (compared to BSSNOK). Witek et al [104] developed a generalized BSSNOK system, which encompasses BSSNOK as a special case. They found that their extension improves numerical behavior of BSSN. Existing BSSNOK codes can easily be adopted to either the conformal Z4-system or the generalized BSSNOK system. Bona et al [105] presented a Lagrangian for the Z4 system, which facilitates the use of symplectic integrators for Einstein’s equations.

Improved efficiency or reduced wall-clock run-time is in dire need, given that a high-quality BH–BH simulation requires months to complete. At least two approaches are under development. Several groups [95, 106, 107] work on porting numerical relativity codes to graphical-processing units (GPUs), usually within the CUDA framework, opening up the possibility that in the future GPUs may accelerate BH–BH simulations by a significant factor. Lau et al [108, 109] explore novel time-stepping algorithms which circumvent the Courant timestep limit, and promise the ability to take significantly larger timesteps than current codes (see also [110]).

2.3. BH–BH waveforms for gravitational wave astronomy

The primary motivation for the intense activity in BH–BH research lies in gravitational wave astronomy. Gravitational wave detectors require reasonably accurate waveform templates to detect GWs, and more accurate waveforms to extract detailed knowledge about the source properties (parameter estimation). As discussed in detail in the review [111], one first performs BH–BH simulations at discrete points in parameter space. One then constructs analytical waveform models (continuous in parameters) from these BH–BH simulations, which are used for GW data-analysis purposes. This sequence of steps has been carried out several times [112–119], based on different numerical simulations, different types of analytical waveform models and for different regions of parameter space (again, we refer to [111] for details).

The NINJA projects [64, 102, 120] give a good sense of the rate of progress of BH–BH simulations for GW modeling. The initial NINJA project [120] in 2008 consisted of about 20 numerical relativity simulations lasting on average about 12 GW cycles before merger, without any length and accuracy requirements. The current NINJA-2 project [64, 102] collected about 40 waveforms with an average of about 20 GW cycles. NINJA-2 also expands the coverage of parameter space by including more spinning waveforms, the simulations are more accurate and one carefully attaches post-Newtonian inspirals to the numerical waveforms. The second
major ongoing effort, the NR-AR collaboration [121] is assembling a yet larger number of waveforms of average length of about 30 GW cycles, and with yet higher accuracy than NINJA-2. Each one of these efforts takes about two years to complete, demonstrating the high complexity of computing, validating and collecting high-quality BH–BH waveforms.

One obstacle toward computation of high-quality waveforms is a shortage of experienced researchers capable of running BH–BH codes and improving the efficiency, robustness and automation of the codes. Furthermore, there were unexpected difficulties in scaling BSSN-OK simulations to greater length and higher accuracy and for the SpEC code to obtain robust and automatic mergers. Finally, computational resources are also constrained. As a rule of thumb, for ‘easy’ parameter choices (moderate spins $\sim 0.5$, moderate mass ratios $\sim 2$, moderate length, $\sim 10$ orbits), a single state-of-the-art BH–BH simulation requires on the order of 100 000 CPU-hours$^5$. This CPU-cost is of course dependent on the length $T/M$ of the simulation (measured in units of the total mass $M$). For given symmetric mass ratio $\nu$, initial orbital frequency $\Omega_i$ or number $N$ of orbits to merger, lowest order post-Newtonian expressions [122] yield

\[
\frac{T}{M} \approx \frac{5}{256} \nu^{-1} (M\Omega_i)^{-8/3}, \tag{3}
\]

\[
\frac{T}{M} \approx 5\nu^{3/5} (2\pi N)^{8/5}. \tag{4}
\]

Halving the initial orbital frequency $\Omega_i$ or doubling the number of orbits $N$ increases $T$ by a factor $\sim 6$ or $\sim 3$, respectively. The increase in CPU cost is even higher, to preserve phase accuracy over the longer inspiral. Higher mass ratio and higher spins increase the CPU cost further.

Equations (3) and (4) basically force a trade-off between the length of each simulation and the number of simulations that can be performed with limited CPU resources. Reasonable accuracy for event-detection can be achieved with $\sim 10$ orbits [123, 124]; however, optimal parameter extraction requires numerical simulations starting at much lower initial frequency [125–127]. The low starting frequency is necessary because the accuracy of the hybrid waveform is primarily limited by the errors of the 3.5th-order post-Newtonian waveforms that are attached before the start of the numerical simulation.

2.3.1. Exploration of parameter space. The entire parameter space for BH–BH binaries is nine dimensional: mass ratio, two spin vectors and two parameters related to eccentricity (eccentricity and phase at periastron). Essentially all efforts to explore this parameter space so far have focused on non-eccentric binaries, and even for non-eccentric binaries, only the following low-dimensional subspaces have been covered in detail (see the NINJA-2 contribution [64] for details and references).

- Non-spinning binaries with the mass ratio $1 \leq q \leq 10$.
- Equal mass binaries with equal spins parallel to the orbital angular momentum.
- Circular, non-precessing binaries form a three-dimensional parameter space (mass ratio and spin magnitudes of the two spins parallel with the orbital angular momentum). This space has not been covered extensively yet, with most efforts having been focused on the two one-dimensional subspaces just mentioned.

The vast seven-dimensional space of precessing binaries on circular orbits has received so far surprisingly little attention: Campanelli et al [128] compute one waveform with the mass $5$.

$^5$ The SpEC code is more efficient than the BSSN-OK codes. But SpEC-simulations focus on longer and more accurate simulations, resulting in a CPU cost of the same order of magnitude.

8
ratio $q = 1.25$, spins of 0.6 and 0.4, lasting about nine orbits. A few configurations are used to demonstrate BH–BH mergers in Szilagyi et al [48], but without discussion of gravitational waveforms. Buonanno et al [44] perform several inspiral simulations of precessing binaries to develop and test eccentricity removal. Sturani et al [118, 129] perform few-cycle long simulations, exploring a one-dimensional line in the seven-dimensional parameter space. While these simulations have been used to construct a phenomenological waveform model of precessing BH–BH [118, 129], the short length of numerical simulations will severely limit the accuracy of this model. Lousto and Zlochower [1] perform 42 simulations to explore BH kicks with partial spin/orbit alignment. Again, the short length of the simulations (about five orbits) makes them unsuitable for GW data-analysis purposes. Recently, several papers discuss techniques to represent waveforms of precessing binaries by expanding the waveforms in spherical harmonics with respect to a time-dependent frame which is aligned with the instantaneous radiation [130–132] (of which [130] presents a precessing $q = 3$ BH–BH simulation).

As this short survey illustrates, several groups have put their toe into the ocean of precessing binaries, but nobody has seriously braved it yet. The most significant current effort to explore precessing systems is the NR-AR collaboration [121], which aims to construct waveform models for precessing BH–BH systems. While an intermediate goal is to revisit the aligned spin case, this collaboration will compute several high-quality precessing waveforms.

2.4. BH–BH in non-vacuum

Fully relativistic hydro-dynamics simulations of BH–BH in gaseous environment have continued. Bode et al [133] and Bogdanovic et al [134] consider radiatively inefficient accretion flow on merging BH–BH binaries, modeled as a BH–BH embedded in a cloud of gas with Gaussian density profile centered on the center of mass. Farris et al [135] investigate the BH–BH analogue of Bondi accretion, embedding the BH–BH in the ambient gas of constant density.

Accretion disks are also under continued investigation. The authors of [137–139] study the behavior of a circumbinary disk after the BH–BH merger, when the disk responds to the remnant BH with reduced mass (due to energy loss through gravitational waves) and with non-zero velocity relative to the center of mass of the accretion disc (due to BH kicks). More recently, Bode et al [140] and Farris et al [141] investigate BH–BH binaries with a circumbinary disc.

Two groups are investigating the effects of electro-magnetic fields surrounding BH–BH binaries. This work started with solving the vacuum Maxwell equations coupled to GR by Palenzuela et al [142, 143] and Moesta et al [144]. More recently, a force-free treatment of the electro-magnetic fields was presented in [145–149]. The force-free approximation assumes the presence of a tenuous plasma which shortens out any electric field parallel to the magnetic field lines. Such a plasma is produced by pair-production and forms the basis for the Blandford–Znajek process [150] to extract energy from rotating BHs. Palenzuela et al [145], in particular, demonstrate how a merger of a spinning BH–BH can result in a total of three jets: During the inspiral, one jet associated with each BH, and after the merger, a third jet associated with the remnant BH. Moesta et al [149] consider the strength of this beamed emission relative to the uniform emission that also accompanies mergers of BH–BH in these cases.

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6 Zanotti et al [136] studied the standard Bondi accretion onto a single BH with general relativistic radiation hydrodynamics.
3. BH–NS and NS–NS systems

We now turn to simulations with at least one NS. These simulations have a more varied set of goals than the BH–BH simulations, among them:

(i) **Compute gravitational waveforms to aid GW detectors.** This requires simulations covering many inspiral orbits at high phase accuracy, a significant challenge for hydro-codes which are typically less accurate than vacuum BH–BH codes.

(ii) **Investigate properties of the merger.** In BH–NS binaries, does the NS get disrupted? Does an accretion disk form, and how large is it? Is the binary a viable progenitor for a short gamma-ray burst?

(iii) **Are these mergers a viable source for r-process elements** [151]? This requires the formation of unbound ejecta which return material into the interstellar medium.

A variety of physical effects may affect evolution of these binaries, and all of these effect must be studied numerically: impact of mass ratio, effect of the equation of state (EOS), magnetic fields, nuclear physics and neutrino transport. For BH–NS systems, one must also investigate the effect of the BH spin (both in magnitude and direction). NSs are generally assumed to be very slowly spinning, albeit some recent work approaches simulations of spinning NS [152, 153]. Several different groups investigate BH–NS and NS–NS systems, with each group generally focusing on a subset of these physical effects, as detailed in sections 3.1 and 3.2.

The numerical techniques for BH–NS and NS–NS combine a solver for the Einstein equations with appropriate solvers for the matter fields. Most commonly used is the BSSNOK evolution system for gravity, combined with high-resolution shock-capturing techniques for relativistic hydrodynamics, discretized with adaptive-mesh refinement FD. The SpEC collaboration combines a spectral treatment of the gravity sector (similar to their BH–BH simulations) with a treatment of matter on a separate Cartesian FD grid that covers only the region of space in which matter is present and that moves along with the NSs. Some recent papers describing numerical techniques and individual codes are given in [154–162]. Initial data for compact object binaries with NSs are described in [163–168].

Detailed investigations into the accuracy of BH–NS and NS–NS simulations are given in [169–171]. Broadly speaking, preserving low-phase errors during inspiral is difficult and simulations with NSs are less accurate than their BH–BH counterparts. No current code is clearly superior; in particular, the advantage of SpEC for vacuum simulations does not carry over and SpEC–hydro-simulations are of comparable accuracy than other codes. Resolving disk dynamics is another numerically challenging aspect, especially when the disks are long-lived. Finally, MHD simulations require a lower cutoff on the matter density. Because gamma-ray bursts involve outflows in regions of low Baryon density, the MHD caveats might become important (the IMEX treatment of Palenzuela et al [172] is a recent alternative).

3.1. BH–NS

One key objective of the study of BH–NS binaries lies in finding the region of parameter space where the NS is tidally disrupted. In this case, an accretion disk forms and the system is considered a possible candidate for a gamma-ray burst (although disruption may not be necessary for a gamma-ray burst, see [173, 174]). Moreover, the gravitational wave signature changes dramatically at disruption. Once the NS is disrupted and spread into an accretion disk, GW emission is drastically reduced.

Tidal disruption depends primarily on three parameters. (1) The compaction of the NS: more compact stars are harder to disrupt. (2) The mass of the BH: more massive BHs with
weaker tidal forces, reducing the tendency of disruption. (3) The distance between the NS and the BH: for spinning BHs, the innermost stable orbit of co-rotating geodesics moves inward; therefore, for BH–NS systems with the BH spin aligned with the orbital angular momentum, the orbit of the NS will reach a smaller distance, thus increasing the tendency of disruption.

A significant amount of attention has been devoted on determining the parameter space in which accretion disks form and on the properties of the formed accretion disks. Let us summarize some recent work. Shibata et al. [175] investigate polytropic NSs orbiting non-spinning BHs. For mass ratios \( q = M_{\text{BH}}/M_{\text{NS}} \) = 1.5 and 3, disruption occurs, but not so for \( q = 5 \). Kyutoku et al. [176] consider BHs with spin aligned or anti-aligned with the orbital angular momentum with spin magnitudes up to \( \chi_{\text{BH}} = S_{\text{BH}}/M_{\text{BH}}^2 = 0.5 \). For low mass ratio \( q \leq 3 \) the NS disrupts. For \( q = 5 \), disruption occurs only for prograde BH spin (this work uses a piecewise polytropic EOS, which is fitted to physical EOSs). The remnant disks have the mass of \( \sim 0.1 M_\odot \), and this work (as the other references mentioned) confirms that indeed NS disruption leads to a cutoff in the gravitational wave spectrum. A related study by Chawla et al. [177] agrees with the results of [176]. Focusing on the mass ratio \( q = 5 \), aligned co-rotating BH with the spin of \( \chi_{\text{BH}} = S_{\text{BH}}/M_{\text{BH}}^2 = 0.5 \) and a polytropic EOS, Chawla et al. find that an accretion disk forms, with essentially all material gravitationally bound. Chawla et al. [177] also incorporate magnetic fields, and for field strengths up to \( 10^{12} G \), the effect of the magnetic field was found to be marginal.

BH–NS binaries with relatively low mass ratio \( q \lesssim 5 \) are numerically easier to handle, and are interesting because these systems form accretion disks easily. However, population synthesis suggests that BHs are generally more massive [178]. Foucart et al. [170] perform simulations with more massive BHs, \( M_{\text{BH}} = 10 M_\odot \), and \( q = 7 \). For a non-spinning BH, or moderately co-rotating BH, no accretion disk forms at \( q = 7 \). Only at high spins \( \chi_{\text{BH}} \gtrsim 0.7 \) does an accretion disk develop. These simulations investigate BHs with spins as large as \( \chi_{\text{BH}} = 0.9 \), the largest to date.

To close our summary on the effect of the BH spin and mass ratio, we note that all simulations mentioned so far have the BH spin parallel to the orbital angular momentum. This assumption is removed by Foucart et al. [179] who vary the spin direction of the BH. Specifically, the angle \( \theta \) between BH spin and orbital angular momentum is varied between 0° and 80° (for \( q = 3 \), BH spin \( \chi_{\text{BH}} = 0.5 \) and a polytropic EOS). Angles \( \theta > 40° \) result in a reduction in the disk mass by about a factor of 2. For the same mass ratio, Foucart et al. [179] also investigate aligned BH spins with magnitude \( \chi_{\text{BH}} = 0.9 \). Consistent with expectations, such a high co-rotating BH spin coupled with the low mass ratio \( q = 3 \) results in a very large disk with mass approaching \( \sim 0.4 M_{\text{NS}} \).

A second focal point for recent work was the impact of the EOS of the NS matter. Duez et al. [180] investigate the polytropic equations of state with two different polytropic indices (\( \Gamma = 2 \) and \( \Gamma = 2.75 \)), and the Shen EOS with two treatments of the electron fraction. The binary has the mass ratio \( q = 3 \) and an aligned BH spin \( \chi_{\text{BH}} = 0.5 \). Duez et al. find that more compact NSs emit stronger gravitational waves and result in smaller disk masses. In particular, tidal tails depend on the EOS. Kyutoku et al. [181] investigate piecewise polytropic equations of state [182, 183]. They find that less compact NSs tend to disrupt at a larger separation, and that disk masses (and the characteristic GW cutoff frequency) correlate with the compaction of the NS.

Recently, the first MHD simulations of BH–NS binaries were performed. Chawla et al. [177] consider the field strength of \( 10^{12} G \) and find no appreciable difference to non-magnetic binaries, whereas Etienne et al. [184] report that a magnetic field of \( 10^{12} G \) results in an increased disk mass (no evidence for collimated outflows were observed, although [184] points out that
higher resolution simulations would be needed, that follow the post-merger dynamics for a longer period of time).

Stephens et al [185] and East et al [186] consider hyperbolic encounters of a NS with a BH (with relative velocity $1000 \text{ km s}^{-1}$ at a large distance). Stephens et al and East et al [185, 186] vary the EOS, impact parameter and BH spin. By far the biggest effect on the results has the impact parameter: depending on the periastron distance, the NS can be disrupted in the first approach, periodic mass transfer can occur, or the NS can pass the BH unharmed.

### 3.2. NS–NS

Binary NSs have seen an equal amount of activity lately as BH–NS binaries. The themes are quite similar to BH–NS, with intense efforts of the groups to extend the range of included physical effects.

EOS effects are exhaustively explored by Hotokezaka et al [187]. They simulate six different EOSs (parametrized by piecewise polytropic [182, 183]) for three different NS masses each and classify the results into prompt collapse to a BH, short-lived hypermassive NS, and long-lived hypermassive NSs. Hotokezaka et al report torus masses of up to $0.1M_\odot$ around the newly formed BH. Sekiguchi et al [188] explore an EOS with a phase transition to hyperons. This causes substantial differences in dynamics, observable in gravitational waves.

When a NS–NS merger results in a hypermassive NS (as opposed to prompt collapse), shocks during the merger will heat the remnant NS to high temperatures, and neutrino cooling will become important. Therefore, neutrino cooling must be modeled in order to determine reliably the timescale on which the hypermassive NS cools to a BH when the thermal pressure becomes insufficient to support the star. Sekiguchi et al [189] perform the first simulation of this process, incorporating neutrino cooling with a leakage scheme, and using a finite-temperature Shen EOS. They report the neutrino luminosities for the simulated NS–NS mergers and find indeed that the lifetime of low-mass hypermassive NS depends on the neutrino cooling.

MHD simulations have also been improved. Giacomazzo et al [190] and Rezzolla et al [191] focus on very long simulations of the post-merger accretion disk. While Giacomazzo et al [190] focus on disk dynamics, Rezzolla et al [191] report evidence of electromagnetic collimation along the rotation axis of the accretion disk. This is the first claim of the ‘missing link’ that connects the post-merger accretion disk with the eventual launching of the jets that power gamma-ray bursts. Two other research groups have published simulations of merging magnetized NS–NS systems [192, 193], and both these groups since then have published improved techniques to handle magnetic fields [160, 161, 172, 194]. It will be very interesting to see whether the impressive and important results of [191] can be confirmed by these groups.

Eccentric NS–NS systems were also studied for the first time. Gold et al [195] find that eccentric mergers result in larger disks, and that periastron passage leads to the excitation of f-modes in the NSs.

Finally, recently work has begun to compare NS–NS inspiral simulations with post-Newtonian approximations, and to fit analytical waveform models to the numerical NS–NS inspiral simulations. Such work is important for gravitational wave data analysis, because complete, phase-accurate waveforms (ranging from early post-Newtonian inspiral into merger) promise the most accurate data analysis for gravitational waves from NS–NS. Several NS–NS simulations were performed for these purposes all about 10 orbits long. Bernuzzi et al [171] perform a comparison with TaylorT4 and find significant phase differences (about 1 GW cycle), which cannot be explained by known tidal corrections. Baiotti et al [196, 197] investigate TaylorT4 and EOB models. Without free parameters, EOB and TaylorT4 both deviate by about one GW cycle from the numerical NS–NS simulation; in contrast, if the EOB
model is amended with one free fitting parameter (parameterizing the strength of higher order tidal effects), the NS–NS simulation can be fitted with an error of only 0.24 rad.

Read et al [198] and Lackey et al [199] analyze large sets of NS–NS simulations to determine what information about the EOS can be extracted from future gravitational wave observations. The best constrained parameter will be the compactness of the NS.

4. Conclusion

The recent progress in simulations of BH–BH, BH–NS and NS–NS systems is spectacular. What are the challenges going forward?

For BH–BH, the numerical methods are in good shape. Very challenging simulations were successfully performed as described in section 2.2, pushing large mass ratio, large spins and the number of GW cycles. The open question is whether the numerical techniques can handle a combination of these properties, e.g. high spin \( \gtrsim 0.9 \) and high mass ratio \( \gtrsim 4 \), and how well accuracy holds up when the number of GW cycles in the simulation is increased. Besides these unexplored regions of parameter space, the main challenge forward is the systematic exploration of the vast parameter space of possible binaries, at sufficient accuracy and length to allow gravitational wave detectors to reach optimal sensitivity and optimal accuracy in parameter estimation. The computational cost of BH–BH simulations, the possibility that the numerical waveforms may have to be much longer than current simulations for optimal parameter estimation, and the sheer size of the parameter space will necessitate compromises, in length of the simulations or in parameter space coverage (or both). The severity of these compromises will only be known after an initial exploration of the precessing parameter space and after the first attempts to fit analytical waveform models. The ease of fitting (currently unknown) will determine how many simulations are needed.

Efficiency improvements in the numerical codes will furthermore determine how quickly the parameter space can be sampled and how quickly on-demand simulations can be performed (e.g. in response to gravitational wave detectors observing a tentative event). Such efficiency improvements may come from novel computer algorithms like implicit time stepping [108, 109] or from utilizing novel computing paradigms like graphical processing units.

For BH–NS and NS–NS systems, the current frontier is exploration of all relevant physical effects. Given the complexity of the simulations and the varied micro-physics, it is imperative that different groups perform similar simulations to cross-check. Once qualitative features are explored, the field will turn toward quantitative sampling of the parameter space, with systematic and careful calculation of gravitational waveforms.

Numerical and implementation issues of compact object binaries seem to be a problem of the past, as attested by the stunning progress in numerical relativity. The deeper insight into formulation of the equations and the experience of what works and what does not make it now possible to consider entirely different numerical algorithms, like discontinuous Galerkin methods [200, 201], moving Voronoi meshes [202, 203], or novel time-stepping techniques [108].

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