Dicke-type phase transition in a spin-orbit-coupled Bose–Einstein condensate

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Spin-orbit-coupled Bose–Einstein condensates (BECs) provide a powerful tool to investigate interesting gauge field-related phenomena. Here we study the ground state properties of such a system and show that it can be mapped to the well-known Dicke model in quantum optics, which describes the interactions between an ensemble of atoms and an optical field. A central prediction of the Dicke model is a quantum phase transition between a superradiant phase and a normal phase. We detect this transition in a spin-orbit-coupled BEC by measuring various physical quantities across the phase transition. These quantities include the spin polarization, the relative occupation of the nearly degenerate single-particle states, the quantity analogous to the photon field occupation and the period of a collective oscillation (quadrupole mode). The applicability of the Dicke model to spin-orbit-coupled BECs may lead to interesting applications in quantum optics and quantum information science.
Ultracold atomic gases afford unique opportunities to simulate quantum optical and condensed matter phenomena, many of which are difficult to observe in their original contexts\textsuperscript{1,2}. Over the past decade, much theoretical and experimental progress in implementing quantum simulations with atomic gases has been achieved, exploiting the flexibility and tunability of these systems. The recent generation of spin-orbit (SO) coupling in Bose–Einstein condensates (BECs)\textsuperscript{2-6} and Fermi gases\textsuperscript{7-9} has brought the simulation of a large class of gauge field-related physics into reach, such as the spin Hall effect\textsuperscript{10-12}. With such achievements, SO-coupled ultracold atomic gases have emerged as excellent platforms to simulate topological insulators, topological superconductors/superfluids and so on, which have important applications for the design of next-generation spin-based atomtronic devices and for topological quantum computation\textsuperscript{13,14}.

Recently, the ground state properties of a BEC with one-dimensional (1D) or two-dimensional (2D) SO coupling have been analysed theoretically. These investigations have predicted a plane wave or stripe phase for different parameter regimes\textsuperscript{16-23}, agreeing with the experimental observations\textsuperscript{3}. In the plane wave phase of such a SO-coupled BEC, the atomic spins collectively interact with the motional degrees of freedom in the external trapping field, providing a possible analogy to the well-known quantum Dicke model. The Dicke model\textsuperscript{24}, proposed nearly 60 years ago, describes the interaction between an ensemble of two-level atoms and an optical field\textsuperscript{25}. For atom–photon interaction strengths greater than a threshold value, the ensemble of atoms favours to interact with the optical field collectively as a large spin and the system shows an interesting superradiant phase with a macroscopic occupation of photons and non-vanishing spin polarization\textsuperscript{26-28}. Even though this model has been solved and is well understood theoretically, the experimental observation was achieved only recently by coupling a BEC to an optical cavity\textsuperscript{29}.

In this work, we experimentally investigate the ground state properties of the plane wave phase of a SO-coupled BEC and show that an insightful analogy to the quantum optical Dicke model can be constructed. The SO coupling in a BEC is realized with a Raman dressing scheme. The system exhibits coupling between momentum states and the collective atomic spin, which is analogous to the coupling between the photon field and the atomic spin in the Dicke model. This analogy is depicted in Fig. 1. By changing the Raman coupling strength, the system can be driven across a quantum phase transition from a spin-polarized phase, marked by a non-zero quasi-momentum, to a spin-balanced phase with zero quasi-momentum, akin to the transition from superradiant to normal phases in Dicke model. Measurements of various physical quantities in these two phases are presented.

**Results**

**Theoretical description of the SO-coupled BEC.** The Raman dressing scheme is based on coupling two atomic hyperfine states in such a way that a momentum transfer of $2\hbar k_x$ in the $x$ direction is accompanied with the change of the hyperfine states, where $\hbar k_x$ is the photon recoil momentum. The dynamics in the $y$ and $z$ directions are decoupled, which allows us to consider a 1D system in our following discussions (see Methods). The two coupled hyperfine states are regarded as the two orientations of a pseudo-spin 1/2 system. The Raman dressed BEC is governed by the 1D Gross–Pitaevskii (G-P) equation with the Hamiltonian $H_{SO} = H_s + H_t$. Here $H_s$ is the single-particle Hamiltonian and in the basis of the uncoupled states can be written as

$$H_s = \left( \frac{\hbar^2}{2m} (k_x + k_R)^2 + \frac{\Omega^2}{2} \frac{\hbar^2}{2m} (k_x - k_R)^2 - \frac{\delta^2}{2} \right) + V_t. \quad (1)$$

$\Omega$ is the Raman coupling strength and $\delta$ is the detuning of the Raman drive from the level splitting. The recoil energy is defined as $E_R = \hbar^2 k_x^2 / 2m$. $k_x$ is the quasi-momentum and $V_t = m \omega^2 \sigma^2 / 2$ is the external harmonic trap. The many-body interactions between atoms are described by

$$H_t = \text{diag} \left( \sum_{\sigma=\uparrow,\downarrow} \langle \sigma | \psi_\sigma |^2 \sum_{\sigma=\uparrow,\downarrow} \langle \sigma | \psi_\sigma |^2 \right). \quad (2)$$

where $g_{\sigma\bar{\sigma}}$ are the effective 1D interaction parameters (see Methods).\textsuperscript{30,31} The presence of the interatomic interactions is crucial to observe the Dicke phase transition. For $^{87}$Rb atoms, the differences between the spin-dependent nonlinear coefficients are very small and contribute only small modifications to the collective behaviour (see Methods).

The band structure of the non-interacting system with $\Omega < 4E_R$ and $\delta = 0$ has two degenerate local minima at quasi-momenta $\pm \bar{q}$, where $\bar{q} = k_R (1 - (\Omega / 4E_R)^2)^{1/2}$. The spin polarization of these two states is finite and opposite to each other. An ensemble of non-interacting atoms occupies both states equally and thus has zero average spin polarization and quasi-momentum. When the nonlinear interactions are taken into account, a superposition state with components located at both degenerate minima generally has an increased energy and is thus not the many-body ground state\textsuperscript{32}. The ground state of the BEC is obtained when the atoms occupy one of the degenerate single-particle ground states (L or R). This is depicted in the inset of Fig. 2a.

The mean field energy associated with a spin-flip in an interacting, harmonically trapped BEC is determined by the coupling between the atomic spin and the many-body ground state harmonic mode. This situation is similar to that of many two-level atoms interacting with a single photon field in an

**Figure 1** | Analogy between standard Dicke model and SO-coupled BEC. (a) Standard Dicke model describing the interaction of an ensemble of two-level atoms in an optical cavity. The optical mode in the cavity couples two atomic spin states. (b) SO-coupled BEC in an external trap. Two spins states are coupled by two counter-propagating Raman lasers.

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expressed in terms of the harmonic trap mode (see Methods). Setting $p_s = \hbar k_s = i(m \omega_o \hbar / 2)^{1/2}(a^\dagger - a)$, the N-particle Hamiltonian can be written as

$$H_{\text{Dicke}} = N \hbar \omega_o a^\dagger a + i \hbar k_R \sqrt{2 \hbar \omega_o / m} (a^\dagger - a) J_z + \Omega / \hbar I_k$$

where the uniform approximation has been adopted to treat the nonlinear interaction term, $G_j = n(g_{1j} - g_{2j})/N$, $n$ is the local density, $J_{xz}$ are the collective spin operators defined as $J_x = \hbar / 2 \sum \sigma_x^j$, $J_z = \hbar / 2 \sum \sigma_z^j$, $a^\dagger a$ is the occupation number of the harmonic trap mode. The differences between the interaction energies contribute an effective detuning term and a nonlinear term in the large spin operator, $J_z^2$. However, these terms are small for the experimental states chosen and thus are ignored in the following analysis. For $\delta = 0$, the Hamiltonian of the first line in equation (3) is equivalent to the Dicke model\cite{45}. A quantum phase transition between the normal phase and a superradiant phase can be driven by changing the Raman coupling strength $\Omega$.

The critical point for the phase transition can be derived using the standard mean field approximation\cite{24,25,26,27,28} yielding $\Omega_c = 4E_R$ (note that the $J_z^2$ term yields a small correction $-4G_j/N$ to $\Omega_c$, which is neglected here). When $\Omega < \Omega_c$, the Dicke model predicts that the dependence of the order parameter on the Raman coupling strength is $(\sigma_z) = \langle J_z \rangle / \hbar \Omega = \sqrt{1 - \Omega^2 / 16E_R^2}$. For $\Omega > \Omega_c$, one obtains $\langle J_z \rangle / \hbar \Omega = 0$ and $\langle \sigma_x \rangle = \langle J_x \rangle / \hbar \Omega = 1$ ($j = N/2$). This scaling is confirmed in our numerical simulation of the G-P equation. The ground state of the BEC is obtained through an imaginary time evolution. The spin polarization can be calculated as $\langle |\sigma_z| \rangle = \int dx \psi_\dagger \cdot \left( \left| \psi \right|^2 - \left| \psi_\dagger \right|^2 \right)$. The absolute value of $\langle |\sigma_z| \rangle$ is taken since $\pm \sigma_z$ are spontaneously chosen. The scaling of $\langle |\sigma_z| \rangle / (\hbar \Omega)$ is shown in Fig. 2 and clearly is consistent with the experimental data described in the following.

Experimental procedure. To experimentally probe this system, we adiabatically Raman dress a BEC of $^{87}$Rb atoms in the $|1\rangle$ $\equiv |F = 1, m_F = -1 \rangle$ and $|0\rangle$ $\equiv |F = 1, m_F = 0 \rangle$ hyperfine states (see Methods). A magnetic bias field is applied that generates a sufficiently large quadratic Zeeman splitting such that the $|F = 1, m_F = 1 \rangle$ state can be neglected. The system can thus be treated as an effective two-state system. To analyse the bare state composition, all lasers are switched off and the atoms are imaged after time-of-flight in a Stern–Gerlach field. This separates the bare states along the vertical axis of the images. The absolute value of the spin polarization, given by $|\langle N_z + N - N \rangle | / (N_z + N - N)$, and the quasi-momentum are directly measured for condensed atoms at $\pm q$. The experimentally measured absolute value of the spin polarization for various Raman coupling strengths $\Omega$ spanning the quantum phase transition is shown in Fig. 2a.

An experimental investigation of the collective choice of the atoms to occupy a single minimum in the dispersion relation is presented in Fig. 3. For this data, the BEC is adiabatically prepared in a Raman dressed state with $\Omega = 3E_R$ and a detuning $\delta$ of either $\pm 5.4E_R$. The detuning is then linearly swept to a final value in 50 ms. The left/right relative occupation of the BEC, defined by $(N_{+q} - N_{-q}) / (N_{+q} + N_{-q})$, provides a measure for the relative occupation of the dressed states at the quasi-momentum of the two dispersion minima. The data presented in Fig. 3 exhibits a hysteric effect dependent on the sign of the initial detuning $\pm \delta$. The width of the hysteresis depends on the chosen sweep rate of the detuning (see Methods). While these sweeps are slow enough for the BEC to follow a chosen minimum of the
The ground state energy excitation, while the photon field vanishes in the normal phase.

Collective excitations. It is well known that various physical quantities may change dramatically across a quantum critical point. As a particular example, we investigate the quadrupole collective excitation of a SO-coupled BEC. In BECs without SO coupling, the quadrupole excitation frequency only depends on the trapping geometry and on the ratio of the kinetic energy to the trapping energy. Measurements and G-P simulations of the quadrupole-mode frequencies for SO-coupled BECs are shown in Fig. 5 as a function of the Raman coupling strength. For this data, the quadrupole oscillation period is scaled by the period measured for an off-resonant case ($\delta \gg \Omega$). This removes a dependence on the trapping geometry that changes with the Raman coupling strength in the experiment. A peak in the oscillation period around the phase transition is observed. In the G-P simulations, a quadrupole oscillation amplitude of $0.2\hbar k_R$ was used, whereas the oscillation amplitude in the experimental results varied from $0.15\hbar k_R$ to $0.55\hbar k_R$. In SO-coupled systems, the hydrodynamic-mode frequencies depend strongly on the oscillation amplitude. This may contribute to the variation of the experimental data. The numerical results reveal that the oscillations of the BEC are undamped in both spin-balanced and -polarized phases, but show strong damping in the transition region. The behaviour of the quadrupole mode provides an additional signature of the quantum phase transition. Similar experimental behaviour has been observed for the collective dipole motion.

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Finite detuning. While the analysis above has focused on the case of $\delta = 0$, a finite detuning leads to the realization of the generalized Dicke Hamiltonian. For finite detuning, the sharp quantum phase transition vanishes and the BEC is always in a superradiant (spin-polarized) phase, as shown in Fig. 6. The inclusion of this parameter demonstrates the completeness in the mapping of the SO-coupled BEC to a Dicke-type system.

**Discussion**

We have demonstrated that varying the Raman coupling strength can drive a harmonically trapped SO-coupled BEC across a quantum phase transition from a spin-balanced to a spin-polarized ground state. This is a realization of the long-sought phase transition from a normal to a superradiant phase in the quantum Dicke model. The corresponding spin polarization and photon field occupation are calculated for the Dicke model and agree with the experimental measurements and the numerical simulations. The ground state energy is found to be consistent with our numerical simulations as well. Using SO-coupled BECs to study the Dicke model physics may have important implications for quantum many-body systems.
applications in quantum information and quantum optics including spin squeezing, quantum entanglement and so on.

Methods

Experimental set-up and parameters. For the experiments presented in this article, the BECs are held in a crossed optical dipole trap with frequencies \(\omega_x, \omega_y, \omega_z\) in the range of \(2\pi \times 12-34,134,178\) Hz, respectively. Depending on the Raman coupling strength and crossed dipole trap potential, BECs of \(2 \times 10^7\) to \(10 \times 10^8\) atoms are loaded into the dressed state. Small atom numbers are favourable in this trapping geometry to reduce collisions between atoms differing by \(2\hbar k_B\) during time-of-flight. The Raman beams, intersecting under an angle of \(\theta/2\), are individually oriented at an angle of \(\pi/4\) from the long axis of the BEC. They are operated between the D1 and D2 lines of \(^{87}\)Rb in the range of 782.5 – 790 nm. A 10-G magnetic bias field is applied along the long axis of the BEC and produces a quadratic Zeeman shift of \(-7.4 e\hbar\). The BEC is imaged after performing Stern-Gerlach separation during a 11.5-ms time-of-flight expansion. The images directly reveal the spin and momentum distributions.

For each value of \(\Omega\), \(\delta = 0\) is experimentally identified by finding the Raman laser detuning for which the spin polarization is minimized (normal phase) or for which the BEC switches between the two degenerate states (superradiant phase). This method compensates for the presence of the third atomic hyperfine state which the BEC switches between the two degenerate states (superradiant phase). Assuming the harmonic ground states along the transverse directions, \(\Omega\) is chosen near \(\delta = 0\). As seen in Fig. 3, this method can be applied to a single point in the dispersion relation, the quasi-momentum (shown in Fig. 8b), strongly deviates from the single-particle expectation and the BEC acquires excitations. While this loading method does not result in the occupation of the ground state at \(\delta = 0\), the BEC does choose a single minimum for detunings much smaller than the chemical potential \(\mu\), which is on the order of 0.53\(\mu\) for the data shown. Similarly results are obtained by adding sufficient time following the linear sweeps of \(\delta\) performed for Fig. 3.

The quadrupole frequency measurements begin with a BEC in the \(|F = 1\rangle\) state in the crossed dipole trap. The two Raman beam intensities are linearly increased, one after the other, over 30–150 ms. This loading method, performed with the Raman beams at 782.5 nm, generates large quadrupole and small dipole excitations. To simplify the analysis, only the first few oscillations of the quadrupole mode are fit to an undamped oscillation. In the numerical simulations, the quadrupole mode is excited by a sudden jump of the trap frequency along the SO coupling direction. Evolution time is added to observe the temporal oscillation of the condensate width that reveals the quadrupole-mode frequency.

Dimension reduction from 3D to 1D. The dynamics of SO Coupled BECs and their mapping to the Dicke model are studied on the basis of the 1D G-P equation, although the experimental system is three dimensional (3D) with strong confinement along two directions. The SO coupling is along the elongated direction of the trap, therefore the transverse degrees of freedom do not couple with the internal spin states. Assuming the harmonic ground states along the transverse directions, the effective 1D nonlinear interaction coefficients, \(g_{1D}\), can be approximately obtained from 3D through \(g_{1D} = g_{2D}/\hbar L_{\parallel}\), where \(g_{2D}\) are the harmonic characteristic lengths along the transverse directions and \(g_{2D} = 4\hbar \Gamma_{a} /N\hbar^2\), is the 3D nonlinear interaction coefficient. For the study of the quadrupole mode, we simulate the 2D G-P equation in the \(xy\) plane to compare with the experimental observations, where the dimension reduction gives \(g_{1D} = g_{2D}/L_{\parallel}\).

Mapping to the Dicke-type Hamiltonian. In the mean field approximation, the single-particle Hamiltonian is given by

\[
H_i = \frac{p_i^2}{2m} + \frac{\hbar \omega_i}{2} \sigma_i^x + \frac{\hbar \omega_{c}}{2} \sigma_i^+ \sigma_i^- + \frac{\hbar \delta}{2} \sigma_i^z + E_k.
\]

As discussed in the main text, the large nonlinear interactions enable the atoms to collectivity occupy the same many-body ground state, forming a single mode. For an ensemble of \(N\) interacting bosonic atoms, we define the collective spin operators \(J_{\parallel} = \hbar /2 \sum_i \sigma_i^+ \sigma_i^-\) and \(I_z = \hbar /2 \sum_i \sigma_i^z\). Substituting these collective operators into the \(N\)-particle Hamiltonian and using the harmonic mode operator, \(p_{\parallel} = i\hbar /\sqrt{m}\), we obtain

\[
H = \hbar \omega_{c} \left( a^\dagger a + \frac{1}{2} \right) + \hbar \delta \frac{2\hbar \omega_{c}}{m} \left( a^\dagger a - a \right) I_z + \hbar \hbar \frac{\delta}{N} L_{\parallel} + \hbar \Omega L_{\parallel} + N \hbar E_k,
\]

which corresponds to the generalized Dicke model.

The interaction term in the mean field approximation is

\[
H_{int} = \begin{pmatrix} 0 & |\psi_1|^2 & |\psi_1|^2 & 0 \\ |\psi_1|^2 & 0 & |\psi_1|^2 & 0 \\ |\psi_1|^2 & 0 & 0 & |\psi_1|^2 \\ 0 & 0 & 0 & 0 \end{pmatrix},
\]

which can also be formally expressed using collective operators. We define the variables \(G_i = m |g_i| / \delta; \quad G_{G} = m |g_i| / \delta \approx m |g_i| / \delta\) and

\(G_{G} = m |g_i| / \delta \approx m |g_i| / \delta\) (ref. 22), where \(N\) is the average density of the BEC. For the parameters used in our experiment, we have \(g_i \approx g_i \approx \delta / 2\). Because of the normalization condition of the BEC wave function, it is easy to see that \(|\psi_1|^2 + |\psi_2|^2 = N/2\) and \(|\psi_1|^2 + |\psi_2|^2 = 2m_{\parallel}N\hbar^2 /\hbar L_{\parallel}\) (for all atoms are in the spin-up state). With these notations, the Hamiltonian can be mapped to

\[
H_{\text{J}} = \hbar \omega_{c} \left( a^\dagger a + \frac{1}{2} \right) + \hbar \delta \frac{2\hbar \omega_{c}}{m} \left( a^\dagger a - a \right) I_z + \hbar \hbar \frac{\delta}{N} L_{\parallel} + 2G_i + \hbar \hbar \Omega L_{\parallel} + N \hbar E_k.
\]

Note that \(g_{2D} = 4\hbar \omega_{c} a_{\sigma} a_{\sigma} /\hbar L_{\parallel}\) is proportional to \(N\), thus all the terms in the above Hamiltonian scale as \(N\).
Dicke phase transition. The critical point for the phase transition of the Dicke model can be derived using the mean field coherent state, where the mean field ansatz of the ground state wave function is given by

$$|\psi\rangle = |\theta\rangle \otimes |x\rangle.$$  

Here the spatial coherent state $|x\rangle$ is defined by $a|x\rangle = x|a\rangle$, and the spin coherent state $|\theta\rangle$ is defined as

$$|\theta\rangle = e^{\theta i\hbar |i_1, i_2\rangle}$$

with $j = N/2$ for spin 1/2 atoms and $\theta \in [0, 2\pi]$. In the absence of the detuning term ($4G_i/N\hbar + \delta /\hbar |f_i\rangle$, the ground state energy of the Dicke Hamiltonian is

$$E(\theta, x) = \langle \psi | H_{\text{Dicke}} | \psi \rangle = -Nh\omega_x (u^2 + v^2) - Nh\omega_z \sqrt{\frac{2h\omega_z}{m}} \cos \theta + 2G_i$$

$$+ \frac{\Omega}{\hbar} \left( \frac{N}{2} \theta \sin \theta \right) + G_i \cos^2 \theta + \frac{Nh\omega_z}{2} + NE_R$$

where $u$ and $v$ are the real and imaginary parts of $x$, that is, $x = u + iv$. Minimizing $E(\theta, x)$ with respect to $u$ and $v$ leads to

$$u = 0$$

$$v = k_x \frac{\hbar}{2m\omega_x} \cos \theta.$$  

Thus the ground state energy becomes

$$E(\theta, x) = -NE_R \left[ \left( 1 - \frac{G_i}{NE_R} \right) \cos^2 \theta - \frac{\Omega}{2E_R} \sin \theta \right]$$

$$+ 2G_i + \frac{Nh\omega_z}{2} + NE_R.$$  

Further minimization of $E(\theta, x)$ with respect to $\theta$ leads to

$$-2\cos \theta \left( 1 - \frac{G_i}{NE_R} \right) \sin \theta + \frac{\Omega}{4E_R} = 0.$$  

Defining $\Omega = 4E_R(1 - G_i/NE_R)$, we obtain two different regions:

1. $\Omega > \Omega_c$: there is only one solution that minimizes the mean field energy: $\cos \theta = 0$ and $\sin \theta = -1$. In this case, the spin polarization is zero and the corresponding phase is the spin-balanced normal phase.

2. $\Omega < \Omega_c$: the energy is minimized for $\sin \theta = -\Omega/(4E_R(1 - G_i/NE_R)) \approx -\Omega/4E_R$ and there are two possible values for $\cos \theta = \pm \sqrt{1 - (\Omega/4E_R)^2}$, corresponding to a BEC occupying the left or right band minimum.

Ground state properties in the Dicke model. In the following, we neglect the constant mean field energy terms and the small $G_i$ terms, therefore $\Omega \approx 4E_R$. In the region $\Omega > \Omega_c$, the system is in the normal phase and $\nu = 0$. The mean photon number is

$$n_{\text{photon}} = |x|^2 = u^2 + v^2 = 0.$$  

The spin polarizations become

$$\langle j_x \rangle_{\frac{\hbar}{N}} = -\cos \theta = 0$$

$$\langle j_z \rangle_{\frac{\hbar}{N}} = \sin \theta = -1.$$  

The ground state energy per particle is

$$E_x = \frac{E}{N} = -\frac{1}{2} \Omega + \frac{2G_i}{N} + \frac{1}{2} Nh\omega_z.$$  

We denote the first two terms as the single-particle energy $E_x$. In the region $\Omega < \Omega_c$, the system is in the superradiant phase with

$$\cos \theta \approx \pm \sqrt{1 - \left( \frac{\Omega}{4E_R} \right)^2}$$

and the mean photon number

$$n_{\text{photon}} = v^2 \approx \frac{\hbar k_x^2}{2m\omega_x} \left( 1 - \frac{\Omega^2}{16E_R^2} \right) = \frac{\hbar}{2m\omega_x} \eta^2.$$  

where $q = \pm k_x \sqrt{1 - (\Omega/4E_R)^2}$ is the quasi-momentum of the BEC. Note that the average photon number depends on the trapping frequency and in the main text we rescale it to $N_p = 2m\omega_x n_{\text{photon}} / \hbar \approx q^2$. The spin polarization in the superradiant phase is

$$\langle j_x \rangle_{\frac{\hbar}{N}} = -\cos \theta \approx \pm \sqrt{1 - \left( \frac{\Omega}{4E_R} \right)^2}$$

$$\langle j_z \rangle_{\frac{\hbar}{N}} = \sin \theta \approx -\frac{\Omega}{4E_R}$$

Figure 8 | Effect of a small detuning on the superradiant phase using an alternative loading method. (a) Experimental determination of the L/R relative occupation of the BEC in the superradiant phase for $\Omega = 3E_R$. The Raman coupling is ramped on over 60 ms at a fixed laser detuning (which differs from the condition of fixed $\delta$) followed by a 150-ms wait time. For all but small detunings the system is loaded to a single point in the dispersion relation. (b) The accompanying quasi-momentum deviates from the expected single-particle value (solid lines) at small detunings where the BEC acquires excitations. This region is highlighted by the dashed red boxes. (c) Experimental images. The inset scale bar corresponds to 50 µm.
Finally, the ground state energy per particle is

$$E_g = \frac{E}{N} \approx -\frac{\Omega^2}{16\hbar \omega} + \frac{2G_1}{N} - \frac{1}{2}\hbar \omega_a.$$

We denote the first term as the single-particle energy $E_s$. The energy expressions for $\Omega < 4E_s$ and $\Omega > 4E_s$ are consistent at $\Omega = 4E_s$.

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**Author contributions**

C.H., Y.Z. and C.P. conceived the experiment and theoretical modelling; C.H., J.C. and P.E. performed the experiments; C.Q., Y.Z., M.G. performed the theoretical calculations; C.Z. and P.E. supervised the project.

**Additional information**

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