Modulating quantum evolution of moving-qubit by using classical driving

Qilin Wang,1 Jianhe Yang,2 Rongfang Liu,3 Hong-Mei Zou,1† Ali Mortezapour,4 Dan Long,1 Jia Wang,1 and Qianqian Ma1
1Synergetic Innovation Center for Quantum Effects and Application, Key Laboratory of Low-dimensional Quantum Structures and Quantum Control of Ministry of Education, School of Physics and Electronics, Hunan Normal University, Changsha, 410081, People’s Republic of China.
2School of Physics and Electronics, Central South University, Changsha 410083, People’s Republic of China.
3School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China.
4Department of Physics, University of Guilan, P.O. Box 41335-1914, Rasht, Iran.
(Dated: February 7, 2023)

In this work, we study quantum evolution of an open moving-qubit modulated by a classical driving field. We obtain the density operator of qubit at zero temperature and analyze its quantum evolution dynamics by using quantum speed limit time (QSLT) and a non-Markovianity measure introduced recently. The results show that both the non-Markovian environment and the classical driving can speed up the evolution process, this quantum speedup process is induced by the non-Markovianity and the critical points only depend on the qubit velocity. Moreover, the qubit motion will delay the evolution process, but this negative effect of the qubit velocity on the quantum speedup can be suppressed by the classical driving. Finally, we give the corresponding physical explanation by using the decoherence rates.

PACS numbers: 03.65.Yz, 03.67.Lx, 42.50.-p, 42.50.Pq.

I. INTRODUCTION

Quantum mechanics restriction on the evolution speed of quantum systems is called quantum speed limit, which is the fundamental law of nature[1–6]. In the past decades, it has been attracting considerable attention and played remarkable roles in various areas of quantum physics, including quantum communication, quantum optimal control, quantum computation and non-equilibrium thermodynamics[7–19]. Quantum speed limit time (QSLT) is defined as the minimal evolution time of a quantum system. For a unitary evolution, there are two common bounds of the QSLT. One is expressed as $\tau_\text{qsl} = \pi \hbar/(2\Delta E)$, where $\Delta E$ represents the energy fluctuation of the initial state, which is proposed by Mandels and Tamm (i.e. the MT bound). The other is $\tau_\text{qsl} = \pi \hbar/(2E)$, where $\tau_\text{qsl}$ depends on the average energy $E$, which is derived by Margolus and Levitin (i.e. the ML bound). Combination of the two bounds yields, the QSLT of the two orthogonal pure states in the closed system as $\tau_\text{qsl} = \max\{\pi \hbar/(2\Delta E), \pi \hbar/(2E)\}$[8, 15, 20–22]. In addition, the MT-QSL (i.e. the MT-QSL bound based on the relative purity), the NI-QSL (i.e. the quantum speed limit without using the von Neumann trace inequality) and the quantum speed limit in a non-equilibrium environment as well as the QSL bound in terms of the quantum fisher information has also been investigated in succession[23–32]. In particular, a new bound of the quantum speed limit different from the MT and the ML bounds is also proposed by employing the gauge invariant and geometric properties of quantum mechanics[33].

On the other hand, the measure of the non-Markovianity in dynamic processes for an open two-level system has been presented in Refs.[34–38]. In recent years, the quantum speed limit and the non-Markovian dynamic process of an open quantum system has been widely concerned. For example, the authors in Ref.[39] acquired the unified bound of an open system by using the Bures angle based on the ML and MT bounds and found that the non-Markovian effects could speed up the quantum evolution. The quantum speedup in open quantum systems and the relationship between the quantum speedup with the formation of a system-environment bound state is also studied in[40–44]. In addition, quantum speedup dynamics process can be also obtained by using the coherent driving, the correlated channel and the Markovian and non-Markovian noise channels[45–47]. Recently, we have studied the QSLT and the non-Markovianity of the atom in Jaynes-Cummings model coupling with the Lorentzian reservoir and the Ohmic reservoir, in which we characterized the non-Markovianity by using the positive derivative of the trace distance, the probability of the atomic excited state and the negative decoherence rate, respectively[48, 49].

These researches mentioned above are based on a stationary qubit model, but one is not making the atom completely stationary in the present experiments such as cavity QED and cooling technology[50–52]. Therefore, it seems logical to consider the motion of the qubits. Recently, A. Mortezapour and D. Park et al studied the entanglement and coherence of an open moving-qubit by assuming the length of the cavity is close to infinity, and their results showed that the entanglement and coherence can be protected from decay by suitable adjusting

†These authors contribute equally to this article.

Electronic address: zhmzc1997@hunnu.edu.cn
Electronic address: mortezapour@guilan.ac.ir
the velocity of the qubit \cite{53} \cite{56}. The authors in \cite{57} investigated recently the entanglement dynamics of an open moving-biparticle system driven by classical-field and the results showed that the classical driving can not only protect the entanglement, but also effectively eliminate the influence of the qubit velocity and the detuning on the quantum entanglement. At the same time, Y. J. Zhang and W. Han studied the evolution process of an open stationary system driven by external classical field and found that the classical field can effectively speed up the evolution of the stationary qubit \cite{58}. Inspired by these works, we construct a model of an open moving-qubit driven by external classical field in order to understand the influence of the classical field and the velocity of moving-qubit in an infinite cavity at zero-temperature on quantum evolution process.

In fact, we try to regulate the quantum evolution process of the moving-qubit by the external classical field. Considering both the external classical field and the qubit movement will make this model more complicated, but we first obtain an analytical solution of such a qubit in the dressed-state basis when the cavity has Lorentzian spectral density by using the dressed-state. Afterward, we investigate in detail the effect of the classical field and the velocity of qubit on the QSLT. We find that both the strong qubit-cavity coupling and the classical field can accelerate the evolution process, but the velocity of moving-qubit can delay the evolution process. Namely, we provide a method to suppress the influence of the velocity of moving-qubit on the quantum evolution process by external classical field when the qubit is not completely stationary, which is the second goal of this work. In addition, we investigate a non-Markovianity measure introduced recently in the dynamic process and the relationship between the non-Markovianity and the QSLT, which is the third goal of this work.

This paper is organized as follows: In Section 2, we present the physical model and analytical solution of an open moving-qubit driven by the external classical field. In section 3, we have provided a useful preliminary to QSLT and non-Markovianity and then calculated them for the proposed system. In section 4, we give the results and discussions. Finally, a simple conclusion of this paper is given in section 5.

II. PHYSICAL MODEL AND ANALYTICAL SOLUTION

Authors in Ref. \cite{59} proposed a method treating an open moving-qubit model, in which the reservoir is modeled by a leaky cavity and the leakage of radiation from the cavity is replaced by the coupling to the outside world. The structure of the environment consists of two completely reflective mirrors at \(z = l\) and \(-L\), and a semitransparent mirror at \(z = 0\). It can be said that the environment consists of two consecutive cavities in intervals \((L, 0)\) and \((0, l)\). The model is shown in Fig. 1.

![FIG. 1: The reservoir is modelled by a leaky cavity with the Lorentzian spectral density. We also assume that the reservoir is big enough, that is the cavity with partially reflecting mirror is imbedded in large ideal cavity.](image)

Based on \cite{59}, we structure a moving-qubit interacting with the multimode reservoir, where the qubit is driven by the classical field. In this model, the classical field is polarized along the x-axis and propagates along the y-axis, and the qubit interacts with the second cavity located in the range \((0, l)\) and \(l \rightarrow \infty\), the qubit also moves along the z-axis at a constant velocity \(v\) as shown in Fig. 2. The Hamiltonian is given by (\(\hbar = 1\))

\[
\hat{H} = \frac{1}{2} \omega_0 \sigma_z + \sum_k \omega_k a_k^\dagger a_k + \Omega e^{-i\omega_f t} \sigma_+ + \Omega e^{i\omega_f t} \sigma_- + \sum_k g_k (f_k(z) a_k \sigma_+ + f_k^\dagger(z) a_k^\dagger \sigma_-),
\]

where \(\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|, \sigma_+ = |e\rangle\langle g|, \) and \(\sigma_- = \sigma_+^\dagger\) associated with the upper level \(|e\rangle\) and lower level \(|g\rangle\). \(\omega_0\) and \(\omega_f\) are the transition frequency of the qubit. \(a_k^\dagger(a_k)\) and \(\omega_k\) are the creation (annihilation) operator and the frequency of the \(k\)-th mode of the cavity, respectively. And \(\omega_f\) represents the frequency of the classical field. In addition, \(g_k\) denotes the coupling constant between the qubit and the \(k\)-th mode of the cavity, and \(\Omega\) designates the coupling strength between the qubit and the classical field. The parameter \(f_k(z)\) describes the shape function induced by the velocity of the qubit along the z-axis, which it is given by

\[
f_k(z) = f_k(vt) = \sin[k(z - l)] = \sin[\omega_k(\beta t - \tau_0)],
\]

where \(\beta = v/c, \tau_0 = l/c, v\) and \(c\) are respectively the velocities of the moving-qubit and the light, \(l\) is the length of the right side cavity. Note that the shape function is not zero when \(z = 0\), while it is zero when \(z = l\) (perfect boundary) \cite{55}.

In the dressed-state basis \(\{|E\rangle = \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle), |G\rangle = \frac{1}{\sqrt{2}}(|g\rangle - |e\rangle)\}\) and exploiting the unitary operator \(U_1 = \exp[-i\omega_f \sigma_z l/2]\), the Hamiltonian in Eq. (1) is equiva-
[The text is too long to summarize effectively, but it appears to discuss a quantum mechanical system, specifically a qubit moving in a cavity, with various interactions and terms involving operators and constants. The equations and diagrams suggest a deep-level understanding of quantum mechanics, likely related to quantum computing or quantum information theory.]
In this section, we briefly review the QSLT and the non-Markovianity for an open quantum system. The bound of the minimal evolution time from an initial state \( \rho_0 \) to a final state \( \rho(\tau) \) is defined as the quantum speed limit time (QSLT) of a system, where \( \tau \) is an standard evolution time. As a measure of statistical distance between quantum states, the QSLT is obtained as follows [39], and it describes the bounds on all possible evolution times of an open quantum system from its initial state to its final state.

\[
\tau_{qsl} = \max \left\{ \frac{1}{\mathcal{V}_\tau^{op}}, \frac{1}{\mathcal{V}_\tau^{tr}}, \frac{1}{\mathcal{V}_\tau^{hs}} \right\} \sin^2 \left[ B(\rho_0, \rho(\tau)) \right],
\]

(18)

where \( \mathcal{V}_\tau^{op} = \frac{1}{\tau} \int_0^\tau dt \mathcal{L}(\rho(t)) ||_{op} \), \( \mathcal{L}(\rho(t)) \) is the time-dependent non-unitary dynamical operator, and \( || \cdot ||_{op} \) represents operator norm. \( \mathcal{V}_\tau^{tr} = \frac{1}{\tau} \int_0^\tau dt \mathcal{L}(\rho(t)) ||_{tr} \) indicates trace norm. \( \mathcal{V}_\tau^{hs} = \frac{1}{\tau} \int_0^\tau dt \mathcal{L}(\rho(t)) ||_{hs} \) shows Hilbert-Schmidt norm. Owning to the relationship \( \frac{1}{\mathcal{V}_\tau^{op}} > \frac{1}{\mathcal{V}_\tau^{tr}} > \frac{1}{\mathcal{V}_\tau^{hs}} \), it is easy to prove that the ML bound based on the operator norm provides the sharpest bound of quantum speed limit time of open quantum systems [39].

Regarding the Eq. (14) and Eq. (15), and supposing \( \rho_0 = |E><E| \), the QSLT is obtained as [63]

\[
\frac{\tau_{qsl}}{\tau} = \frac{1 - |C_1(\tau)|^2}{\int_0^\tau |\partial_\tau C_1(t)|^2 dt}.
\]

(19)

In Eq. (19), \( \tau_{qsl} \) is the minimal evolution time, \( \tau \) is the standard evolution time. \( \frac{\tau_{qsl}}{\tau} \approx 1 \) represents that the minimal evolution time is equal to the standard evolution time, which is called as the no-speedup evolution process. \( \frac{\tau_{qsl}}{\tau} < 1 \) indicates that the minimal evolution time is less than the standard evolution time, which is called as the speedup evolution process. And the smaller value of \( \frac{\tau_{qsl}}{\tau} \) will be corresponding to the greater capacity for potential promotion of the evolution speed.

The non-Markovianity measure \( \mathcal{N} \) is defined as [10]

\[
\mathcal{N} = \max_{\rho_1(0),\rho_2(0)} \int_{t>0} \sigma[t, \rho_1(0), \rho_2(0)] dt,
\]

(20)

where \( \sigma[t, \rho_1(0), \rho_2(0)] = \dot{D}[\rho_1(t), \rho_2(t)] \) is the time change rate of the trace distance. \( D[\rho_1(t), \rho_2(t)] = \frac{1}{\tau} \tr[||\rho_1(t) - \rho_2(t)||] \) and indicates the distinguish ability between the two states \( \rho_{1,2}(t) \) evolving from their respective initial forms \( \rho_{1,2}(0) \). \( \sigma[t, \rho_1(0), \rho_2(0)] \) < 0, corresponds to all dynamical semigroups and all time-dependent Markovian processes, a process is non-Markovian if there exists a pair of initial states and at
certain time $t$ such that $\sigma[t, \rho_1(0), \rho_2(0)] > 0$. We should take the maximum over all initial states $\rho_{1,2}(0)$ to calculate the degree of non-Markovianity. Similar to Refs. [58], by drawing a sufficiently large sample of random pairs of initial states, the optimal state pair is attained for the initial states $\rho_1(0) = |E\rangle\langle E|$ and $\rho_2(0) = |G\rangle\langle G|$ in the dressed-state basis. For the Eq. (15), it can be proven that the optimal pair of initial states to maximize $N$ are $\rho_1(0) = |E\rangle\langle E|$ and $\rho_2(0) = |G\rangle\langle G|$. The trace distance between of the evolved states can be written as $D[\rho_1(t), \rho_2(t)] = |C_1(t)|^2$. Thus the $N$ in Eq. (20) can be rewritten as

$$N = \frac{1}{2} \left[ \int_0^\tau |\partial_t C_1(t)|^2 dt + |C_1(\tau)|^2 - 1 \right], \quad (21)$$

Form Eq. (19) and Eq. (21), the relationship between the QSLT and the non-Markovianity can be obtained as

$$\frac{\tau_{\text{qsl}}}{\tau} = \frac{1 - |C_1(\tau)|^2}{1 - |C_1(\tau)|^2 + 2N}. \quad (22)$$

Eq. (22) shows that the QSLT is equal to the standard evolution time when $N = 0$, but the QSLT is smaller than the standard evolution time when $N > 0$. That is, the non-Markovianity in the dynamics process can lead to the faster quantum evolution and the smaller QSLT.

IV. RESULTS AND DISCUSSION

In this section, we will use the QSLT and the non-Markovianity to research the quantum evolution dynamics of open moving-qubit modulated by a classical driving field in detail.

In order to obtain the quantum-accelerated evolution process manipulated by classical field under weak and strong coupling regimes, we draw Fig. 3. Fig. 3 exhibits the curves of the $\tau_{\text{qsl}}$ versus the driving strength $\Omega$ when $\beta = 0$ in the weak and strong coupling regimes, respectively. From Fig. 3(a), we find that the $\tau_{\text{qsl}}$ will be equal to the standard evolution time $\tau$ when the classical driving strength $\Omega$ is small. The $\tau_{\text{qsl}}$ is less than the standard evolution time only when the classical driving strength is greater than a certain critical value in which a sudden transition from standard evolution process to speedup process will occur. For different spectral width $\lambda$, the classical driving has different critical values $\Omega_c$ and the speedups processes have also obvious differences. The smaller $\lambda$ is, the smaller the critical driving strength $\Omega_c$ is, the smaller the $\tau_{\text{qsl}}$ is, the faster the qubit evolves. This fact indicates that, in the weak coupling regime, the larger driving strength can speed up the evolution process and the quantum evolution process relies mainly on the classical driving. Fig. 3(b) shows that, in the strong coupling regime, the smaller driving strength can also speed up the evolution process and the $\tau_{\text{qsl}}$ will reduces rapidly and then oscillate due to the flowback information of the non-Markovian environment when the driving strength increases. For different spectral width $\lambda$, there is the same critical driving strength $\Omega_c$ and the oscillation periods of the curves are the same, but the evolution curves of speedup process are not very different. This indicates that, in the strong coupling regime, both of the classical driving and the non-Markovian environment can speed up the quantum evolution. Namely, the quantum evolution process is determined by both the classical driving and the non-Markovian environment. One important point to be noted is that, if there is not the classical driving, the quantum evolution of an open system would never be accelerated in the weak coupling regime, because the information flows irreversibly from the system to the reservoir under the dissipation of Markovian reservoir. In contrast, the quantum evolution can be accelerated in the strong coupling regime, because the information can feed back into the system from the reservoir under the memory and feedback of non-Markovian reservoir. However, if there exists the classical driving, the quantum evolution of an open system will be also accelerated in the weak coupling regime, this shows that the classical driving can be equivalent to a non-Markovian environ-
qubit moves with different velocities, the qubit can also evolve at the same speed by adjusting the classical driving strength, shown as the points at which the gray dotted line intersects these curves of $\tau_{\text{qsl}}/\tau$ in Fig. 4(b). The results show that the classical field strength can be served as a controlling tool to regulate the effect of the velocity of the moving qubit on the quantum evolution process when the atom cannot be completely stationary in the experiments of cavity QED and cooling technology.

Fig. 4 shows the effect of velocity of moving-qubit and the classical driving on the $\tau_{\text{qsl}}$ in the weak and strong coupling regimes. From Fig. 4(a), we know that the evolution process of moving-qubit will be speed up in the weak-coupling regime($\lambda = 3\gamma$) when $\Omega > \Omega_c$. The critical value $\Omega_c$ is dependent on the velocity ratio $\beta$ and the larger velocity ratio $\beta$ corresponds to the larger critical value $\Omega_c$ and the slower evolution process. That is to say, the quantum evolution process depends on both the classical driving and the velocity of moving-qubit. Comparing with Fig. 4(a), the difference of Fig. 4(b) is that the effect of velocity on the quantum evolution process is more obvious and tangible in the strong-coupling regime($\lambda = 0.01\gamma$) than that in the weak-coupling regime($\lambda = 3\gamma$). And, the $\tau_{\text{qsl}}$ will reduce rapidly and then oscillate due to the flowback information of the non-Markovian environment when the driving strength increases. Therefore, the velocity of moving-qubit will delay the evolution process while the classical driving can speed up the evolution process under both weak and strong coupling regimes. Namely, when the decay rate of the open system.

FIG. 4: $\tau_{\text{qsl}}$ for an open system driven by an external classical field as a function of the parameter variable $\Omega/\gamma$. (a) in the weak-coupling regime($\lambda = 3\gamma$). (b) in the strong-coupling regime($\lambda = 0.01\gamma$). The transition frequency $\omega_0 = 1.53 \times 10^9$. The coupling strength $\gamma = 1$. The actual evolution time $\tau = 1$. The detuning $\Delta = 0$.

In order to explore the physical mechanism of the influence of the velocity of moving-qubit on the $\tau_{\text{qsl}}$, we draw the curve of the decoherence rate $\Gamma(t)$ in Fig. 5. As it is abundantly clear, the information and energy are exchanged between the qubit and the cavity, $\Gamma(t) > 0$ indicates that the information flows irreversibly from the qubit to the environment, but $\Gamma(t) < 0$ shows that the information flows back from the environment to the qubit. It is seen that in the weak-coupling regime($\lambda = 3\gamma$) in Fig. 5(a), the decoherence rate $\Gamma(t)$ is always greater than zero, which implies that the information flows to environment without flowback. The greater the velocity of moving-qubit, the smaller the decoherence rate and,
consequently, the slower the qubit evolution. Note that, the impact of the velocity on the decoherence rate is minor and then there is no flowback information that affects the moving-qubit, which is corresponding to those monotonous decay curves of the $\tau_{qsl}$ in Fig. 4(a). However, the decoherence rate $\Gamma(t)$ in Fig. 5(b) is negative in some moments, which signifies that the information flows back to the qubit from the environment in the strong-coupling regime ($\lambda = 0.01\gamma$). The faster the qubit, the smaller the decoherence rate, the slower the qubit evolves. In particular, the decoherence rate is heavily dependent on the velocity in the strong-coupling regime owing to the flowback information from the environment which affects the moving-qubit, such a behavior reveals the physical mechanism behind the curves in Fig. 4(b).

In order to shed light on the dependency relationship of the $\tau_{qsl}$ and the non-Markovianity $N$, we draw Fig. 7. Fig. 7 simultaneously depicts the $\tau_{qsl}$ and the non-Markovianity $N$ for an open system driven by an external classical field as a function of the parameter variable $\Omega/\gamma$. (a) in the weak-coupling regime ($\lambda = 3\gamma$) and (b) in the strong-coupling regime ($\lambda = 0.01\gamma$). The coupling strength $\gamma = 1$. The velocity ratio $\beta = 0$. The transition frequency $\omega_0 = 1.53 \times 10^9$. The actual evolution time $\tau = 1$. The detuning $\Delta = 0$.

Fig. 6 investigates the effects of the driving classical field on the non-Markovianity $N$ when qubits have different velocities in the weak and strong coupling regimes. That a suitable classical driving strength can transform the original Markovian dynamics into non-Markovian dynamics. Moreover, as a common result in both weak and strong regimes, increasing the velocity of qubit leads to the critical driving strength increase. Furthermore, it is observed that the larger the value of qubit velocity, the smaller the non-Markovianity. This implies that moving qubit can significantly interfere with the backflow of information from the environment to qubits. And, the non-Markovianity increases non-monotonically when $\Omega > \Omega_c$.

FIG. 6: The non-Markovianity $N$ for an open system driven by an external classical field as a function of the parameter variable $\Omega/\gamma$. (a) in the weak-coupling regime ($\lambda = 3\gamma$) and (b) in the strong-coupling regime ($\lambda = 0.01\gamma$). The transition frequency $\omega_0 = 1.53 \times 10^9$. The coupling strength $\gamma = 1$. The actual evolution time $\tau = 1$. The detuning $\Delta = 0$.

FIG. 7: $\tau_{qsl}$ and the non-Markovianity $N$ for an open system driven by an external classical field as a function of the parameter variable $\Omega/\gamma$. (a) in the weak-coupling regime ($\lambda = 3\gamma$) and (b) in the strong-coupling regime ($\lambda = 0.01\gamma$). The coupling strength $\gamma = 1$. The velocity ratio $\beta = 0$. The transition frequency $\omega_0 = 1.53 \times 10^9$. The actual evolution time $\tau = 1$. The detuning $\Delta = 0$. 

In order to shed light on the dependency relationship of the $\tau_{qsl}$ and the non-Markovianity $N$, we draw Fig. 7. Fig. 7 simultaneously depicts the $\tau_{qsl}$ and the non-Markovianity $N$ with respect to the driving strength $\Omega$ when $\beta = 0$ in the both weak and strong coupling regimes. From Fig. 7(a), it is observed that when $\tau_{qsl}$ is equal to 1, the $N$ is equal to 0. As the $N$ increases, the $\tau_{qsl}$ decreases, which means that the evolution of the qubit is speeding up. Moreover Fig. 7(b) demonstrates that, in the strong-coupling regime, the curves of $\tau_{qsl}$ and $N$ show oscillating behaviors due to the feedback and memory of environment. Obtaining such results are rooted in Eq. (22). Furthermore Fig. 7 reveals that switching from the Markovian regime to non-Markovian regime gives rise to the quantum speedup process. It is also seen that both driving field and strong coupling can enhance the non-Markovianity and speed up the evolution of qubit.
V. CONCLUSIONS

In summary, we investigated quantum evolution of an open moving-qubit modulated by a classical driving field. Firstly, we constructed a model of an open moving-qubit driven by the external classical field, where the environment at zero temperature has the Lorentzian spectral density. Secondly, we obtained an analytical solution of the density operator of this qubit in the dressed-state basis. Thirdly, we analyzed the quantum evolution dynamics by using the QSLT and the non-Markovianity. The results showed that both the non-Markovian environment and the classical driving can speed up the quantum evolution and increase the non-Markovianity in the evolution process, while the qubit motion will delay the quantum evolution and decrease the non-Markovianity. Moreover, the quantum speedup process is induced by the non-Markovianity and the critical points only depend on the qubit velocity. In a way, we can utilize the classical driving to suppress the negative effect of the qubit velocity on the speedup evolution if the qubit is not completely stationary in the experiment. That is to say, the controllable operation of quantum evolution can be realized by adjusting the classical driving strength, the qubit-cavity coupling and the qubit velocity. Also, we give the corresponding physical explanation by using the decoherence rates. The potential candidates that can effectively utilize this approach include cavity QED [64, 65], trapped ions [66], superconducting qubits [67, 68].

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Grant No.11374096).

Data Availability Statement

The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

[1] Seth Lloyd. Ultimate physical limits to computation. Nature, 406(6799):1047–1054, 2000.
[2] J. Anandan and Y. Aharonov. Geometry of quantum evolution. Physical Review Letters, 65(14):1697–1700, 1990.
[3] Lev Vaidman. Minimum time for the evolution to an orthogonal quantum state. American Journal of Physics, 60(2):182–183, 1992.
[4] Shulong Luo. How fast can a quantum state evolve into a target state? Physica D: Nonlinear Phenomena, 189(1):1–7, 2004.
[5] H. P. Breuer, and F. Petruccione, editor. The Theory of Open Quantum Systems. Oxford University Press, New York, 2002.
[6] Michael A. Nielsen and Isaac L. Chuang, editor. Quantum Computation and Quantum Information. Cambridge University Press, Cambridge, 2010.
[7] Man-Hong Yung. Quantum speed limit for perfect state transfer in one dimension. Physical Review A, 74(3):030303, 2006.
[8] Jacob D. Bekenstein. Energy cost of information transfer. Physical Review Letters, 46(10):623–626, 1981.
[9] T. Caneva, M. Murphy, T. Calarco, R. Fazio, S. Montangero, V. Giovannetti, and G. E. Santoro. Optimal control at the quantum speed limit. Physical Review Letters, 103(24):240501, 2009.
[10] Victor Mukherjee, Alberto Carlini, Andrea Mari, Tommaso Caneva, Simone Montangero, Tommaso Calarco, Rosario Fazio, and Vittorio Giovannetti. Speeding up and slowing down the relaxation of a qubit by optimal control. Physical Review A, 88(6):062326, 2013.
[11] Gerhard C. Hegerfeldt. Driving at the quantum speed limit: Optimal control of a two-level system. Physical Review Letters, 111(26):260501, 2013.
[12] Gerhard C. Hegerfeldt. High-speed driving of a two-level system. Physical Review A, 90(3):032110, 2014.
[13] C. Avinadav, R. Fischer, P. London, and D. Gershoni. Time-optimal universal control of two-level systems under strong driving. Physical Review B, 89(24):245311, 2014.
[14] Seth Lloyd. Computational capacity of the universe. Physical Review Letters, 88(23):237901, 2002.
[15] L. B. Levitin. Physical limitations of rate, depth, and minimum energy in information processing. International Journal of Theoretical Physics, 21:299–309, 1982.
[16] Sebastian Deffner and Eric Lutz. Generalized clausius inequality for nonequilibrium quantum processes. Physical Review Letters, 105(17):170402, 2010.
[17] Shen-Shuang Nie, Feng-Hua Ren, Run-Hong He, Jing Wu and Zhao-Ming Wang. Control cost and quantum speed limit time in controlled almost-exact state transmission in open systems. Physical Review A, 104(5):052424, 2021.
[18] Nikolai Il’in and Oleg Lychkovskiy. Quantum speed limit for thermal states. Physical Review A, 103(6):062204, 2021.
[19] Eoin O’Connor, Giacomo Guarnieri and Steve Campbell. Action quantum speed limits. Physical Review A, 103(2):022210, 2021.
[20] G. N. Fleming. A unitarity bound on the evolution of nonstationary states. Il Nuovo Cimento A (1965-1970), 16(2):232–240, 1973.
[21] Kamal Bhattacharyya. Quantum decay and the mandelstam-tamm-energy inequality. Journal of Physics A: Mathematical and General, 16:2993, 1999.
[22] Norman Margolus and Lev B. Levitin. The maximum speed of dynamical evolution. Physica D: Nonlinear Phenomena, 120(1):188–195, 1998.
[23] A. del Campo, I. L. Egusquiza, M. B. Plenio, and S. F. Huelga. Quantum speed limits in open system dynamics. Physical Review Letters, 110(5):050403, 2013.
[24] S.-X. Wu, Y. Zhang, C.-S. Yu and H.-S. Song. The initial-
state dependence of the quantum speed limit. *Journal of Physics A: Mathematical and Theoretical*, 48:045301, 2014.

[25] Xiangji Cai and Yujun Zheng. Quantum dynamical speedup in a nonequilibrium environment. *Physical Review A*, 95(5):052104, 2017.

[26] Marco Schirò and Aditi Mitra. Transient orthogonality catastrophe in a time-dependent nonequilibrium environment. *Physical Review Letters*, 112(24):246401, 2014.

[27] Francesco Peronaci, Marco Schirò, and Massimo Capone. Transient dynamics of d-wave superconductors after a sudden excitation. *Physical Review Letters*, 115(25):257001, 2015.

[28] P. Bhupathi, Peter Groszkowski, M. P. DeFeo, Matthew Ware, Frank K. Wilhelm, and B. L. T. Plourde. Transient dynamics of a superconducting nonlinear oscillator. *Physical Review Applied*, 5(2):024002, 2016.

[29] S. Oviedo-Casado, J. Prior, A. W. Chin, R. Rosenbach, S. F. Huelga, and M. B. Plenio. Phase-dependent exciton transport and energy harvesting from thermal environments. *Physical Review A*, 93(2):020102, 2016.

[30] Fernando C. Lombardo and Paula I. Villar. Nonunitary geometric phases: A qubit coupled to an environment with random noise. *Physical Review A*, 87(3):032338, 2013.

[31] Nicolás Mirkin, Fabricio Toscano, and Diego A. Wisniacki. Comment on “modified quantum-speed-limit bounds for open quantum dynamics in quantum channels”. *Physical Review A*, 97(4):046101, 2018.

[32] Du Kang-Ying, Ma Ya-Jie, Wu Shao-Xiong, Yu Chang-Shui. Quantum speed limit for the maximum coherent state under the squeezed environment. *Chinese Physics B*, 30(9):090308, 2021.

[33] Shuning Sun and Yujun Zheng. Distinct Bound of the Quantum Speed Limit via the Gauge Invariant Distance. *Physical Review Letters*, 123(18):180403, 2019.

[34] Elsi-Mari Laine, Jyrki Piilo, and Heinz-Peter Breuer. Measure for the non-markovianity of quantum processes. *Physical Review A*, 81(6):062115, 2010.

[35] Hao-Sheng Zeng, Ning Tang, Yan-Ping Zheng, and Guo-You Wang. Equivalence of the measures of non-markovianity for open two-level systems. *Physical Review A*, 84(3):032118, 2011.

[36] Zhi He, Hao-Sheng Zeng, Yan Li, Qiong Wang, and Chunmei Yao. Non-markovianity measure based on the relative entropy of coherence in an extended space. *Physical Review A*, 96(2):022106, 2017.

[37] Felipe F. Fanchini, Göktuğ Karpat, Leonardo K. Castelano, and Daniel Z. Rossatto. Probing the degree of non-markovianity for independent and common environments. *Physical Review A*, 88(1):012105, 2013.

[38] Claudia Benedetti, Matteo G. A. Paris, and Sabrina Maniscalco. Non-markovianity of colored noisy channels. *Physical Review A*, 89(1):012114, 2014.

[39] Sebastian Deffner and Eric Lutz. Quantum speed limit for non-markovian dynamics. *Physical Review Letters*, 111(1):010402, 2013.

[40] Liu H-B, Yang W L, An J-H and Xu Z-Y. Mechanism for quantum speedup in open quantum systems. *Physical Review A*, 93(2):020105, 2016.

[41] Mirkin N, Toscano F and Wisniacki D A. Quantum speed limit bounds in an open quantum evolution. *Physical Review A*, 94(5):052125, 2016.

[42] Xu K, Zhang Y-J, Xia Y-J, Wang Z D and Fan H. Hierarchical-environment-assisted non-Markovian speedup dynamics control. *Physical Review A*, 98(2):022114, 2018.

[43] Ahansaz B and Ektessabi A. Quantum speedup, non-Markovianity and formation of bound state. *Scientific Reports*, 9:14946, 2019.

[44] Wang J, Wu Y N and Xie Z Y. Role of flow of information in the speedup of quantum evolution. *Scientific Reports*, 8:16870, 2018.

[45] Ying-Jie Zhang, Xiang Lu, Hai-Feng Lang, Zhong-Xiao Man, Yun-Jie Xia, Heng Fan. Quantum speedup dynamics process without non-Markovianity. *Quantum Information Processing*, 20(3):1-21, 2021.

[46] N. Awasthi, S. Haseli, U. C. Johri, S. Salimi, H. Dolkatkhah, A. S. Khorashad. Quantum speed limit time for correlated quantum channel. *Quantum Information Processing*, 19(1):1-17, 2020.

[47] Natasha Awasthi, Joshi Devraj Kumar and Surbhi Sachdev. Variation of quantum speed limit under Markovian and non-Markovian noisy environment. *Laser Physics Letters*, 19(3):035201, 2022.

[48] Hong-Mei Hou, Jianhe Yang, Danping Lin and Fang M-F. Quantum speedup process of atom in dissipative cavity. *Journal of Physics B: Atomic, Molecular and Optical Physics*, 53(13):135502, 2020.

[49] Hong-Mei Hou, Rongfang Liu, Dan Long, Jianhe Yang and Danping Lin. Ohmic reservoir-based non-Markovianity and Quantum Speed Limit Time. *Physica Scripta*, 95(8):085105, 2020.

[50] Shuo Zhang, Chun-Wang Wu, and Ping-Xing Chen. Dark-state laser cooling of a trapped ion using standing waves. *Physical Review A*, 85(5):053420, 2012.

[51] Shuo Zhang, Jian-Qi Zhang, Qian-Heng Duan, Chu Guo, Chun-Wang Wu, Wei Wu, and Ping-Xing Chen. Ground-state cooling of a trapped ion by quantum interference pathways. *Physical Review A*, 90(4):043409, 2014.

[52] Shuo Zhang, Jian-Qi Zhang, Jie Zhang, Chun-Wang Wu, Wei Wu, and Ping-Xing Chen. Ground state cooling of an optomechanical resonator assisted by a λ-type atom. *Optics Express*, 22(23):28118–28131, 2014.

[53] Ali Mortezaipur, Mahdi Ahmadi Borji, and Rosario Lo Franco. Protecting entanglement by adjusting the velocities of moving qubits inside non-markovian environments. *Laser Physics Letters*, 14(5):055201, 2017.

[54] DaeKil Park. Protection of entanglement in the presence of markovian or non-markovian environment via particle velocity: Exact results. *arXiv:1703.09341*, 2017.

[55] Ali Mortezaipur, Mahdi Ahmadi Borji, DaeKil Park and Rosario Lo Franco. Non-markovianity and coherence of a moving qubit inside a leaky cavity. *Open Systems & Information Dynamics*, 24(03):1740006, 2017.

[56] Golkar, Mohammad K. Tavassoly and Alireza Nourmandipour. Entanglement dynamics of an arbitrary number of moving qubits in a common environment. *Journal of the Optical Society of America B*, 37(2):400-411, 2020.

[57] Qilin Wang, Rongfang Liu, Hong-Mei Zou, Dan Long, Jianhe Yang and Danping Lin. Ohmic reservoir-based non-Markovianity and Quantum Speed Limit Time. *Physica Scripta*, 95(8):085105, 2020.

[58] Ying-Jie Zhang, Xiang Lu, Hai-Feng Lang, Zhong-Xiao Man, Yun-Jie Xia, Heng Fan. Quantum speedup dynamics process without non-Markovianity. *Quantum Information Processing*, 20(3):1-21, 2021.

[59] Roy Lang, Marlan O. Scully, and Willis E. Lamb. Why is the laser line so narrow? A theory of single-quismode
laser operation. *Physical Review A*, 7(5):1788–1797, 1973.

[60] Heinz-Peter Breuer, Elsi-Mari Laine, and Jyrki Piilo. Measure for the degree of non-markovian behavior of quantum processes in open systems. *Physical Review Letters*, 103(21):210401, 2009.

[61] Hong-Mei Zou, Mao-Fa Fang, Bai-Yuan Yang, You-Neng Guo, Wei He, and Shi-Yang Zhang. The quantum entropic uncertainty relation and entanglement witness in the two-atom system coupling with the non-markovian environments. *Physica Scripta*, 89(11):115101, 2014.

[62] Mao-Fa Fang Hong-Mei Zou. Discord and entanglement in non-markovian environments at finite temperatures. *Chinese Physics B*, 25(9):090302, 2016.

[63] Xu Z Y, Luo S, Yang W L, Liu C and Zhu S Q. Quantum speedup in a memory environment. *Physical Review A*, 89: 012307, 2014.

[64] Benjamin T. H. Varcoe, Simon Brattke, Matthias Weidinger, and Herbert Walther. Preparing pure photon number states of the radiation speed up. *Nature*, 403(6771):743–746, 2000.

[65] McKay D C, Naik R, Reinhold P, Bishop L S and Schuster D I. High-Contrast Qubit Interactions Using Multimode Cavity QED. *Physical Review Letters*, 114(8):080501, 2015.

[66] Daniel Jonathan and Martin B. Plenio. Light-shift-induced quantum gates for ions in thermal motion. *Physical Review Letters*, 87(12):127901, 2001.

[67] J. Q. You and Franco Nori. Atomic physics and quantum optics using superconducting circuits. *Nature*, 474(7353):589–597, 2011.

[68] Xiu Gu, Anton Frisk Kockum, Adam Miranowicz, Yuxi Liu, Franco Nori. Microwave photonics with superconducting quantum circuits. *Physics Reports*, 30(718-719):1-102, 2017.