The oscillation effects on thermalization of the neutrinos in the universe with low reheating temperature

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(Dated: March 20, 2022)

Abstract

We study how the oscillations of the neutrinos affect their thermalization process during the reheating period with temperature $O(1)$ MeV in the early universe. We follow the evolution of the neutrino density matrices and investigate how the predictions of big bang nucleosynthesis vary with the reheating temperature. For the reheating temperature of several MeV, we find that including the oscillations makes different predictions, especially for $^4$He abundance. Also, the effects on the lower bound of the reheating temperature from cosmological observations are discussed.
I. INTRODUCTION

The standard big bang model assumes that the universe was once dominated by thermal radiation composed of photons, electrons, neutrinos, and their anti-particles. It is one of the main issues in theories beyond the standard cosmology where these particles came from, or equivalently, what reheated the universe. The reheating temperature, at which the universe becomes radiation-dominated, is therefore a very important parameter that discriminates among many scenarios on the thermal history of the universe. In the following we define the reheating temperature as that of the latest reheating process, if the universe experienced several reheating stages.

Recent observations of the cosmic microwave background radiation (CMB) has strongly suggested that the universe underwent inflation at an early stage. After inflation ended, the universe was dominated by the oscillation energy of the inflaton until it decayed and reheated the universe. The upper limit on the reheating temperature was obtained by constraining the relic abundance of the gravitinos, the superpartner of the graviton, which are inevitably present in the supersymmetric (SUSY) framework. Here we are interested in the relatively low reheating temperature, especially in the MeV range, and would like to put a lower limit on the reheating temperature.

The MeV-scale reheating is actually ubiquitous in theories beyond the standard cosmology. In the framework of the SUSY and superstring theories, there are many particles with very long lifetime, e.g., the moduli and the gravitinos mentioned above, since their interaction is so weak, typically suppressed by the Planck scale. These long-lived massive particles might have dominated over the radiation from the inflaton decay. If the masses of these particles are heavy enough, they decay and reheat the universe again just before the big bang nucleosynthesis (BBN) starts. Otherwise they often cause cosmological disaster known as “cosmological moduli problem” and “gravitino problem”. The simplest solution of these problems is to dilute the unwanted relics by producing large entropy at a later time. In either case, the reheating temperature is very low and typically around MeV.

Another example that prefers the low reheating temperature is the curvaton scenario in which the curvaton field dominates the universe and its isocurvature fluctuation is transformed into an adiabatic one. Furthermore, in the Affleck-Dine mechanism responsible
for the origin of the baryon asymmetry, it is known that non-topological solitons such as $Q$-balls are generally created. Since the decay process of the $Q$-balls is geometrically suppressed, they might dominate the universe, and such possibility has been extensively studied in many different scenarios.

What if the reheating temperature is several MeV? In contrast to electrons that are always (at least until the temperature drops below a few eV) in thermal contact with photons via electromagnetic forces, neutrinos interact with electrons and themselves only through the weak interaction. The decoupling temperature of the neutrinos should be around 3 MeV for the electron neutrinos and 5 MeV for the muon and tau neutrinos, respectively. The difference comes from the fact that the electron neutrinos have additional charged current interaction with electrons. Therefore the neutrinos might not be fully thermalized if the reheating temperature is in the MeV range. If this is the case, the expansion rate of the universe becomes smaller, which affects the light element abundances and the CMB angular power spectrum. In particular it has been widely believed or taken for granted that the predicted abundance of $^4$He decreases as the reheating temperature drops below a few MeV. This is because the smaller expansion rate delays the decoupling of the neutron-proton transformation, decreasing the neutron-to-proton ratio at the beginning of the BBN. Since almost all the neutrons are absorbed in the $^4$He nuclei, such a naive reasoning can explain the dependence of the $^4$He abundance on the reheating temperature. In this paper, however, we will see that this widespread picture is drastically changed if we take account of the neutrino oscillations.

Recent neutrino oscillation experiments have determined the mass differences and mixing angles with high precision and established that mixing angles are large. The crucial point is that a flavor eigenstate transforms itself into another one. Therefore we must take special care to calculate neutrino distribution functions and the resultant effective number of neutrinos. As pointed out in Refs., it is useful to follow the evolution of the neutrino density matrices when flavor mixings are present. Here we will solve momentum dependent Boltzmann equations for the neutrino density matrices. We will see that the predicted abundance of $^4$He is drastically changed, while the effective number of neutrinos does not change much. To put it simply, the reason for this is that the number density of the electron neutrinos is decreased due to flavor mixings, which makes the freeze-out temperature of the neutrons higher; this effect cancels and even overcomes that of the decrease in the
expansion rate. Thus MeV-scale reheating scenario is one of the examples in which the neutrino oscillations play a essential role.

The outline of this paper is the following. In the next section we formulate neutrino thermalization including flavor mixings, and derive a evolution equation of the neutrino density matrix. In the section II we will show how the predicted abundances of the light elements are modified when the reheating temperature is in the MeV range, and discuss their implications. Finally we present our conclusion in the section IV.

II. NEUTRINO THERMALIZATION

In this section, we illustrate the formulation needed to follow the neutrino thermalization process. The case without the neutrino oscillations is studied by Refs. [21, 22, 23]. Although subjects of study are different from this paper, issues of the neutrino spectrum evolution using momentum dependent Boltzmann equations in the early universe are treated in Refs. [18, 19, 20, 31, 32, 33, 34]. Our formulation almost goes in parallel with the no-mixing case and we use some of the useful techniques discussed in those papers. However, there is a very important exception that neutrino distribution functions have to be extended to neutrino density matrices [30] in order to include oscillations.

First of all, let us explain our assumptions on the reheating process and the neutrino oscillations. We refer to the massive particles which reheat the universe, or inflaton, as φ. We assume φ only decays into photons (they in turn produce electrons and positrons and they all thermalize very quickly by the electromagnetic interaction). In other words, the branching ratios to neutrinos or hadrons are assumed to be negligible and neutrinos are produced exclusively via the electron-positron annihilation. Then φ is characterized simply by its decay rate Γ. We parametrize it by the reheating temperature $T_R$ which is defined as

$$\Gamma = 3H(T_R),$$

(1)

where $H$ is the expansion rate of the universe. Here, we use the Friedmann equation $H^2 = \rho_{\text{tot}}/3M_{\text{Pl}}$ where the reduced Planck mass $M_{\text{Pl}} = 2.435 \times 10^{18}$ GeV. The total energy density $\rho_{\text{tot}}$ which consists of the radiation including photons, electrons, and three

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1 Hereafter we call φ as inflaton even if it is not responsible for inflation.
species of neutrinos, is expressed as \( \rho_{\text{tot}} = \left( g^* \pi^2 / 30 \right) T_R^4 \) where the relativistic degree of freedom \( g^* = 43/4 \). This leads to

\[
\Gamma = 3.26 \frac{T_R^2}{M_{\text{Pl}}} = 2.03 \left( \frac{T_R}{\text{MeV}} \right)^2 \text{sec}^{-1}.
\]  

(2)

It should be noted that, even if the neutrinos are not fully thermalized, we stick to Eq. (2) as the definition of \( T_R \) to avoid unnecessary confusion.

We consider three active flavors of neutrinos: \( \nu_e, \nu_\mu \) and \( \nu_\tau \). When the oscillations are neglected as in Refs. [21, 22, 23], there are only two sets of variables required to describe the neutrino evolution. They are the distribution functions for \( \nu_e \) and \( \nu_\mu \) which have to be distinguished since they interact differently with electrons; \( \nu_e \) interacts via both neutral and charged currents while \( \nu_\mu \) and \( \nu_\tau \) has only the former interaction. Since \( \nu_\mu \) and \( \nu_\tau \) interact with electrons identically, we do not need to solve for the distribution function of \( \nu_\tau \). It is same as \( \nu_\mu \)’s. On the contrary, when we include the oscillations among them, \( \nu_\mu \) and \( \nu_\tau \) also have to be distinguished because their oscillations between \( \nu_e \) are known to be different. Namely, we need to consider general three-flavor oscillations which require 9 real variables to fully describe our issue. However, if \( \theta_{13} \) is zero, a simplification to two-flavor oscillations is possible by using non-mixing mass eigenstates \( \nu'_\mu \) and \( \nu'_\tau \) instead of \( \nu_\mu \) and \( \nu_\tau \). Then \( \nu'_\mu \) and \( \nu_e \) are described by two-flavor oscillations and \( \nu'_\tau \) decouples from the oscillations. \( \nu'_\tau \) just interacts with \( e^\pm \) via neutral current and should behave as \( \nu_\tau \) (or \( \nu_\mu \)) in the no-mixing case.

Under those assumptions, the variables necessary for simulating thermalization of oscillating neutrinos are: the inflaton energy density \( \rho_\phi \), the photon temperature \( T \), the \( \nu_e-\nu'_\mu \) two-flavor neutrino density matrix \( \rho_p \) and the \( \nu'_\tau \) distribution function \( f_{\nu'_\tau}(p) \). \( \rho_p \) and \( f_{\nu'_\tau} \) are functions of neutrino momentum \( p \). \( \rho_p \) is defined by expectation value of the product of the creation and annihilation operators [30]:

\[
\left\langle a_{ij}^\dagger(p)a_{ij}(q) \right\rangle \equiv (2\pi)^3 \delta^{(3)}(p-q) \left[ \rho_p \right]_{ij}, \quad \{i,j\} = \{e,\mu\},
\]  

(3)

where \( a_{ij}(p) \) is the annihilation operator for negative-helicity neutrino of flavor \( i \) with momentum \( p \). Readers should bear in mind that the density matrix \( \rho_p \) is just an extension

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\(^2\) Similar simplification is shown to be useful to analyze the evolution of neutrino-antineutrino asymmetries by Refs. [35, 36, 37].
of the occupation number to the mixed neutrinos, and should not confuse with the energy density, to which we refer as $\rho_\nu$, $\rho_\phi$, etc. Each diagonal component of $\rho_p$ is the neutrino distribution of the corresponding flavor, while the off-diagonal ones represent more subtle information on the correlation. For anti-neutrinos we can similarly define the density matrix $\bar{\rho}_p$:

$$\langle b_i^\dagger(p)b_j(q) \rangle \equiv (2\pi)^3 \delta^{(3)}(p-q) [\bar{\rho}_p]_{ij}, \quad \{i,j\} = \{e,\mu\},$$

(4)

where $b_i(p)$ is the annihilation operator for positive-helicity neutrino of flavor $i$ with momentum $p$. However, unless the lepton asymmetry is very large, we do not have to distinguish neutrinos from anti-neutrinos. In this case they are related to each other as $\bar{\rho}_p = \rho_p^T$. We next derive the differential equations which govern their evolutions.

We use scale factor $a$ as a time variable and later we use $y \equiv pa$ instead of a momentum [19]. Then the time evolution equation for the neutrino density matrix $\rho_p$ is

$$Ha \frac{d\rho_p}{da} = -i[\Omega(p), \rho_p] + I_{\text{coll}}(p).$$

(5)

The matrix $\Omega(p)$ represents both the vacuum oscillations and the refractive term. Neglecting lepton asymmetry, which is usually as small as the baryon asymmetry, it is written as

$$\Omega(p) \equiv \Omega_V(p) - \frac{8\sqrt{2} G_F p}{3m_W^2} E,$$

(6)

where the Fermi coupling constant $G_F = 1.16637 \times 10^{-11}$ MeV$^{-2}$, $W$ boson mass $m_W = 80$ GeV. In the ultrarelativistic limit, $\Omega_V(p)$ is given by

$$\Omega_V(p) = \frac{1}{2p} U M^2 U^T,$$

(7)

where $M^2$, the neutrino mass matrix, and $U$, the matrix which relates mass eigenstates and flavor eigenstates, are

$$M^2 \equiv \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix}, \quad U \equiv \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} \\ -\sin \theta_{12} & \cos \theta_{12} \end{pmatrix}.$$

(8)

We use the solar neutrino oscillation experiment values for neutrino parameters: $m_2^2 - m_1^2 = 7.3 \times 10^{-5}$ eV$^2$ and $\sin^2 \theta_{12} = 0.315$ [38]. The second term in Eq. [10] comes from the

3 The most recent result [26] gives a slightly higher value of the mass squared difference: $m_2^2 - m_1^2 = 7.9^{+0.6}_{-0.5} \times 10^{-5}$ eV$^2$. However, we have confirmed that our results do not change for $m_2^2 - m_1^2$ in the error range.
non-local effect of the \( W \)-exchange interactions, and \( E \) is the energy density matrix of the charged leptons:

\[
E = \begin{pmatrix}
\rho_e + \rho_{\bar{e}} & 0 \\
0 & 0
\end{pmatrix} = \begin{pmatrix}
(7/60)\pi^2T^4 & 0 \\
0 & 0
\end{pmatrix}, \tag{9}
\]

where \( \rho_{e(\bar{e})} \) is the energy density of electrons (positrons) and we have assumed that neither muons nor taus exist in the plasma.

For the collision term \( I_{\text{coll}} \), we consider the processes \( \nu + e^\pm \leftrightarrow \nu + e^\pm \) and \( \nu + \bar{\nu} \leftrightarrow e^- + e^+ \). In calculating the collision term, we take electrons to be massless and neglect processes of scattering among neutrinos as Refs. \cite{21, 22, 23}. The contributions from each process are

\[
I_{\nu e e}(p_1) = \frac{1}{2E_1} \int \frac{d^3p_2}{2E_2} \frac{d^3p_3}{2E_3} \frac{d^3p_4}{2E_4} (2\pi)^4 \delta(4)(p_1 + p_2 - p_3 - p_4)
\times 2^5 G_F^2 \left[ 4(p_1 \cdot p_2)(p_3 \cdot p_4) F_{LL}(\nu^{(1)}, e^{(2)}, \nu^{(3)}, e^{(4)}) + 4(p_1 \cdot p_4)(p_2 \cdot p_3) F_{RR}(\nu^{(1)}, e^{(2)}, \nu^{(3)}, e^{(4)}) \right], \tag{10}
\]

\[
I_{\nu \bar{e} e}(p_1) = \frac{1}{2E_1} \int \frac{d^3p_2}{2E_2} \frac{d^3p_3}{2E_3} \frac{d^3p_4}{2E_4} (2\pi)^4 \delta(4)(p_1 + p_2 - p_3 - p_4)
\times 2^5 G_F^2 \left[ 4(p_1 \cdot p_4)(p_2 \cdot p_3) F_{LL}(\nu^{(1)}, \bar{e}^{(2)}, \nu^{(3)}, \bar{e}^{(4)}) + 4(p_1 \cdot p_3)(p_2 \cdot p_4) F_{RR}(\nu^{(1)}, \bar{e}^{(2)}, \nu^{(3)}, \bar{e}^{(4)}) \right], \tag{11}
\]

\[
I_{\bar{\nu} \nu e}(p_1) = \frac{1}{2E_1} \int \frac{d^3p_2}{2E_2} \frac{d^3p_3}{2E_3} \frac{d^3p_4}{2E_4} (2\pi)^4 \delta(4)(p_1 + p_2 - p_3 - p_4)
\times 2^5 G_F^2 \left[ 4(p_1 \cdot p_4)(p_2 \cdot p_3) F_{LL}(\nu^{(1)}, \bar{e}^{(2)}, \nu^{(3)}, \bar{e}^{(4)}) + 4(p_1 \cdot p_3)(p_2 \cdot p_4) F_{RR}(\nu^{(1)}, \bar{e}^{(2)}, \nu^{(3)}, \bar{e}^{(4)}) \right], \tag{12}
\]

where we define \( d^3p = d^3p/(2\pi)^3, E_i \equiv p_i^0, \) and

\[
F_{ab}(\nu^{(1)}, e^{(2)}, \nu^{(3)}, e^{(4)}) \equiv \frac{1}{2} [(1 - \rho_{\bar{p}_1})G_a \rho_{\bar{p}_3} G_b (1 - f_e(p_2)) f_e(p_4) + \text{h.c.} - \rho_{\bar{p}_1} G_a (1 - \rho_{\bar{p}_3}) G_b f_e(p_2)(1 - f_e(p_4)) + \text{h.c.}], \tag{13}
\]

\[
F_{ab}(\nu^{(1)}, \bar{e}^{(2)}, \nu^{(3)}, \bar{e}^{(4)}) \equiv \frac{1}{2} [(1 - \rho_{\bar{p}_1})G_a \rho_{\bar{p}_3} G_b (1 - f_{\bar{e}}(p_2)) f_{\bar{e}}(p_4) + \text{h.c.} - \rho_{\bar{p}_1} G_a (1 - \rho_{\bar{p}_3}) G_b f_{\bar{e}}(p_2)(1 - f_{\bar{e}}(p_4)) + \text{h.c.}], \tag{14}
\]

\[
F_{ab}(\nu^{(1)}, \bar{e}^{(2)}, e^{(3)}, \bar{e}^{(4)}) \equiv \frac{1}{2} [(1 - \rho_{\bar{p}_1})G_a (1 - \rho_{\bar{p}_3}) G_b f_{\bar{e}}(p_2)(1 - f_{\bar{e}}(p_4)) + \text{h.c.} - \rho_{\bar{p}_1} G_a \rho_{\bar{p}_2} G_b (1 - f_{\bar{e}}(p_3))(1 - f_{\bar{e}}(p_4)) + \text{h.c.}], \tag{15}
\]

with \( G_L = \text{diag}(g_L, \bar{g}_L) \) and \( G_R = \text{diag}(g_R, g_R) \). Here, \( \bar{g}_L = g_L - 1 = \sin^2 \theta_W - \frac{1}{2} \) and \( g_R = \sin^2 \theta_W \) where \( \sin^2 \theta_W = 0.23120 \) is the weak-mixing angle. \( f_e(f_{\bar{e}}) \) is the distribution
function of electrons (positrons). Hereafter we take $\bar{\rho}_p = \rho_p^T$ and $f_{\bar{e}} = f_e$. Note that these collision terms coincide with those found in Ref. [19] if oscillations are absent (i.e., if off-diagonal components in the density matrices are zero).

We further approximate that electrons obey the Boltzmann distribution and their Pauli blocking factors are neglected. Namely, in $F$’s, we replace $f_{\bar{e}}(p) \to \exp(-p/T)$ and $1 - f_{\bar{e}}(p) \to 1$. Then the collision terms above are reduced to one-dimensional momentum integration by the technique in Ref. [22] and the reduced expressions become equal to the ones in the reference 4 in the limit of the zero mixing angle.

In practice, since $\rho_p$ is $2 \times 2$ Hermitian matrix, it is convenient to expand it using Pauli matrices. Namely,

$$\rho_p = \sum_{i=0}^{3} P_i(p) \sigma_i / 2,$$

where

$$\sigma_0 \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 \equiv \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 \equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 \equiv \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

On the right hand side of Eq. (5), $i[\Omega, \rho_p]$ and $I_{\text{coll}}$ are expanded similarly. We solve for the evolution of $P_0 \sim P_3$ and the distributions of $\nu_e$ and $\nu'_\mu$ are in turn derived by $f_{\nu_e} = (P_0 + P_3)/2$ and $f_{\nu'_\mu} = (P_0 - P_3)/2$. The evolution equations are formally written as

$$Ha \frac{dP_i(y)}{da} = -i\Omega_i(y) + I_i(y),$$

where $i$ runs from 0 to 3, $\Omega_i \equiv \text{Tr}([\Omega, \rho_p] \sigma_i)$ and $I_i \equiv \text{Tr}(I_{\text{coll}} \sigma_i)$, and we have changed the variable $p$ to $y$.

We need to solve for the evolution of $\nu'_\tau$, too. To this end, it is most simple to obtain the time evolution of $f_{\nu'_\tau}$ from the $\nu'_\mu$-component of Eq. (5) with no mixing (which is given by omitting the first term on the right hand side) because the interactions of $\nu'_\tau$ with $e^\pm$ are same as those of $\nu'_\mu$.

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4 The right hand side of Eq. (A16) in Ref. [22] has to be multiplied by 2 to be a correct equation. Due to this error, the right hand side of Eq. (3) in Ref. [21] has to be multiplied by 2. The right hand side of Eq. (12) in Ref. [22] has to be multiplied by 8 since it had already contained a typo of factor 4. In this occasion, we correct a typo in the right hand side of Eq. (8) in Ref. [22]: it has to multiplied by 2 (so that it is same as Eq. (2) in Ref. [21].
For the evolution of $\rho_{\phi}$ and $T$, the equations are almost same as those found in Ref. [22]. We just need modifications due to our use of scale factor $a$ as a time variable and discrimination of $\nu_\mu$ from $\nu_\tau$. For $\rho_{\phi}$, it is given by

$$\frac{d\rho_{\phi}}{da} = -\frac{\Gamma}{aH} \rho_{\phi} - \frac{3}{a} \rho_{\phi}. \quad (19)$$

The equation of the total energy-momentum conservation is

$$\frac{d\rho_{\text{tot}}}{da} = -\frac{3}{a}(\rho_{\text{tot}} + P_{\text{tot}}), \quad (20)$$

where the total energy density and the total pressure are given by

$$\rho_{\text{tot}} \equiv \rho_{\phi} + \rho_\gamma + \rho_{e^\pm} + \rho_{\nu_e} + \rho_{\nu_\mu} + \rho_{\nu_\tau}$$

$$= \rho_{\phi} + \frac{\pi^2 T^4}{15} + \frac{2}{\pi^2} \int_0^\infty dp \frac{p^2}{E_e \exp(E_e/T) + 1}$$

$$+ \frac{1}{\pi^2 a^4} \int_0^\infty dy y^3 \left( f_{\nu_e} + f_{\nu_\mu} + f_{\nu_\tau} \right), \quad (21)$$

$$P_{\text{tot}} \equiv P_\gamma + P_{e^\pm} + P_{\nu_e} + P_{\nu_\mu} + P_{\nu_\tau},$$

$$= \frac{\pi^2 T^4}{45} + \frac{2}{3\pi^2} \int_0^\infty dp \frac{p^4}{E_e \exp(E_e/T) + 1}$$

$$+ \frac{1}{3\pi^2 a^4} \int_0^\infty dy y^3 \left( f_{\nu_e} + f_{\nu_\mu} + f_{\nu_\tau} \right), \quad (22)$$

with the electron energy $E_e \equiv \sqrt{m_e^2 + p^2}$. The evolution equation for $T$ is derived from Eq. (20):

$$\frac{dT}{da} = -\left( \frac{\partial \rho_\gamma}{\partial T} + \frac{\partial \rho_{e^\pm}}{\partial T} \right)^{-1} \left\{ \frac{4}{a} \rho_\gamma + \frac{3}{a} (\rho_{e^\pm} + P_{e^\pm}) - \frac{\Gamma}{aH} \rho_{\phi} \right. \right.$$

$$+ \frac{1}{\pi^2 a^4} \int_0^\infty dy y^3 \left( \frac{df_{\nu_e}}{da} + \frac{df_{\nu_\mu}}{da} + \frac{df_{\nu_\tau}}{da} \right) \left\} \right.$$  

$$\frac{dT}{da} = -\left( \frac{\partial \rho_\gamma}{\partial T} + \frac{\partial \rho_{e^\pm}}{\partial T} \right)^{-1} \left\{ \frac{4}{a} \rho_\gamma + \frac{3}{a} (\rho_{e^\pm} + P_{e^\pm}) - \frac{\Gamma}{aH} \rho_{\phi} \right.$$  

$$+ \frac{1}{\pi^2 a^4} \int_0^\infty dy y^3 \left( \frac{df_{\nu_e}}{da} + \frac{df_{\nu_\mu}}{da} + \frac{df_{\nu_\tau}}{da} \right) \left\} \right.$$  

Final, the expansion rate is

$$H \equiv \frac{1}{a} \frac{da}{dt} = \frac{\sqrt{\rho_{\text{tot}}}}{\sqrt{3}M_\text{Pl}}. \quad (24)$$

To integrate the differential equations, since the equations for $f_\nu(y)$ are stiff, we used semi-implicit extrapolation method [40]. Using the Ref. [40]'s implementation which incorporates adaptive stepsize control routine, we were able to evolve the neutrino density matrices very efficiently. We followed the evolution well after the electron-positron annihilation ends and $f_\nu(y)$’s become constant.
As for the initial condition, we have to make the inflaton energy density dominate the universe at first. As long as $\rho_\phi$ is much larger than radiation energy density ($\sim T^4$), evolution afterward does not depend on their precise values. In this paper, we adopt rather realistic relation between $\rho_\phi$ and $\rho_{\text{rad}}$,

$$\rho_{\text{rad}} = \frac{2\sqrt{3}}{5} \Gamma M_{\text{Pl}} \rho_\phi^{1/2},$$

(25)

which derived from the analytic solutions during the epoch of coherent oscillations [39].

III. RESULTS AND COSMOLOGICAL IMPLICATIONS

In this section, we present the results of our numerical calculation for neutrino thermalization and consider its implications for cosmology. We evolve the neutrino density matrices with various values of the reheating temperature $T_R$ and investigate how the neutrino distribution functions, neutrino energy densities and big bang nucleosynthesis depend on $T_R$. Along with the neutrino thermalization with oscillations, we show the results without oscillations which have been studied in Refs [21, 22, 23] and elucidate the neutrino oscillation effects on a low reheating temperature scenario. Our results when the oscillations are omitted turn out to be consistent with those of previous papers. We find that the inclusion of the oscillations most characteristically alters the $^4\text{He}$ synthesis and its abundance varies with respect to $T_R$ quite differently from the no oscillation case.

A. Neutrino distribution functions

We show the final neutrino distribution functions in Figs. (a)-(d) for the cases of $T_R = 15$ MeV and 2.5 MeV respectively with and without the oscillations. We see from Figs. (a) and (b) that when the reheating temperature is sufficiently high, all the neutrino species are thermalized regardless of the oscillations.

However, for the case of lower reheating temperature, the oscillations significantly matter as seen from comparing Figs. (c) and (d) : $f_{\nu_e}$ and $f_{\nu_{\mu}}$ are almost equalized by the solar mixing. When the oscillations are neglected, $f_{\nu_e}$ becomes much larger than $f_{\nu_{\mu}}$ as shown in Fig. (c) because $\nu_e$ is produced by the charged current interaction in addition to the neutral current interaction but $\nu_\mu$ and $\nu_\tau$ are produced only by the latter [21, 22]. When
FIG. 1: The final distribution functions of neutrinos. (a) and (c) are cases for no oscillations ($\nu_e$ is displayed by solid lines and $\nu_\mu$ by dashed lines) and (b) and (d) incorporate the oscillations ($\nu_e$ is displayed by solid lines, $\nu'_\mu$ by dashed lines and $\nu'_\tau$ by dot-dashed lines). The equilibrium distributions are drawn by dotted lines in order to show how much they are thermalized. For $T_R = 15$ MeV, in (a) and (b), whether the oscillations are present or not, all the lines overlap and this means every neutrino species is fully thermalized for high reheating temperature. For $T_R = 2.5$ MeV, in (c) and (d), distributions are away from equilibrium form. When the oscillations are taken into account, distributions of $\nu_e$ and $\nu'_\mu$ get close as seen in (d).

There are the flavor mixings, $\nu_e$ and $\nu'_\mu$ can convert into each other. $\nu'_\mu$ is now produced also by the oscillations from $\nu_e$ which exists more than $\nu'_\mu$ so $f_{\nu'_\mu}$ increases compared to no oscillation case. On the contrary, $f_{\nu_e}$ becomes smaller when the oscillations are included naturally because $\nu_e$ oscillates into $\nu'_\mu$. However, this deficit is to some extent filled by the $\nu_e$ production from the thermal plasma so the neutrinos are produced more in total under the existence of the oscillations. This is seen in Fig. 2 which shows clearly that $f_{\nu_e} + f_{\nu'_\mu}$
FIG. 2: We draw the sums of the distribution functions, $f_{\nu_e} + f_{\nu_\mu}$ (no oscillation) and $f_{\nu_e} + f_{\nu'_\mu}$ (including oscillation) with the dashed line and the solid line respectively. The latter is larger showing that the oscillations make the thermalization more efficient in total.

$$(\text{including oscillation}) > f_{\nu_e} + f_{\nu_\mu} \ (\text{no oscillation}).$$

B. Effective number of neutrinos

Let us discuss our results of the neutrino thermalization in terms of neutrino energy density. This is often expressed using the effective number of neutrinos $N_\nu$. This number is observationally relevant to the CMB power spectrum and large scale structure. It is given by

$$N_\nu \equiv \frac{\sum \rho_\nu}{\rho_{\nu,\text{std}}},$$

(26)

where the summation is taken for $\nu = \nu_e$, $\nu_\mu$, and $\nu_\tau$ when the oscillations are not included and $\nu = \nu_e$, $\nu'_\mu$, and $\nu'_\tau$ when we consider the oscillations. We define $\rho_{\nu,\text{std}}$ using the photon temperature $T$ as

$$\rho_{\nu,\text{std}} = \frac{7\pi^2}{120} \left\{ \left( \frac{4}{11} \right)^{1/3} T \right\}^4,$$

(27)
FIG. 3: The effective neutrino number $N_\nu$ as a function of the reheating temperature $T_R$ (shown on the bottom abscissa) or the decay width $\Gamma$ (shown on the top abscissa). The cases with and without the oscillations are drawn respectively by the solid and dashed lines. The horizontal line denotes $N_\nu = 3.04$ with which $N_\nu$ for high $T_R$ should coincide (see the text).

which corresponds to the neutrino energy density assuming that neutrinos are completely decoupled from the rest of the thermal plasma before the electron-positron annihilation takes place. If this assumption is exact, $N_\nu$ would be 3. It is actually a very good assumption but detailed calculations on the entropy transfer from electrons to neutrinos have shown that $\rho_\nu$’s are slightly larger than $\rho_{\nu,\text{std}}$ and $N_\nu = 3.04$ \[18, 19, 20, 32\].

We calculate $\rho_\nu$ by integrating the final neutrino distribution functions such as presented in Fig. 1 and derive $N_\nu$ as a function of the reheating temperature $T_R$. The result is shown in Fig. 3. For $T_R \gtrsim 10$ MeV, $N_\nu$ asymptotes the value 3.04 which indicates thermalized neutrino distributions. This is regardless of the neutrino oscillations and consistent with Fig. 1 (a) and (b) discussed in section III.A. For the smaller values of $T_R$, the inclusion of the oscillations make $N_\nu$ larger as expected from Fig. 2. This effect is most conspicuous for
\( T_R = 2 \sim 5 \text{ MeV} \) and changes \( N_\nu \) up to \( \sim 0.2 \).

Fig. 3 enables us to constrain \( T_R \) by using the limits on the effective number of neutrino species from cosmological observations such as CMB and galaxy surveys. Recent papers, Refs. \[41, 42, 43, 44\], derive the lower limit to be \( 0.9 \sim 1.9 \) (these are the limits obtained without resorting to BBN. Some of them have also reported more stringent limits obtained using data combined with observed \( Y_p \). However, since they assume Fermi-Dirac distribution for neutrinos and only modify the Friedmann equation when they calculate \( Y_p \), we cannot use those limits. This point is discussed in section III C in detail). If \( N_\nu > 0.9 \) is adopted, the bound on the reheating temperature is \( T_R > 1.69 \text{ MeV} \) with the oscillations and \( T_R > 1.74 \text{ MeV} \) for no oscillation case.

C. Light element abundances

We now investigate how the big bang nucleosynthesis is affected by the non-thermal neutrino distributions and/or the neutrino oscillations. We calculate the light element (D, \(^4\text{He}\) and \(^7\text{Li}\)) abundances as functions of \( T_R \), again with and without the neutrino oscillations. The cosmological effects of incomplete neutrino thermalization is most strikingly seen in \(^4\text{He}\) abundance since electron-type neutrinos play a special role in determining the rate of neutron-proton conversion during BBN. This has been already known from the previous papers Refs. \[21, 22\] in which the oscillations are neglected, but we find that the neutrino oscillations prominently matter in regard to the \( T_R \)-dependence of \(^4\text{He}\) abundance.

We show how \( Y_p \) varies with respect to \( T_R \) in Fig. 4. This is calculated by plugging the solutions of the evolution equations derived in section II into the Kawano BBN code \[45\] (with updated reaction rates compiled by Angulo et al. \[46\]). Required modifications are the temperature dependence of the neutron-proton conversion rates, \( \Gamma_{n \rightarrow p} \) and \( \Gamma_{p \rightarrow n} \), and the evolution equation for the photon temperature. The calculation of \( \Gamma_{n \leftrightarrow p} \) (see e.g. Ref. \[47\]) involves the integration of the electron neutrino distribution function \( f_{\nu_e} \) which does not necessarily take the Fermi distribution form in our case. For the photon temperature evolution, the contributions from \( \phi \) and neutrinos are supplemented in the same way as Eq. (23).

There are two effects caused by incomplete thermalization of neutrinos competing to make up the dependence of \( Y_p \) on \( T_R \) as shown in Fig. 4 slowing down of the expansion rate
FIG. 4: The $^4$He abundance (mass fraction) $Y_p$ as a function of the reheating temperature $T_R$ (shown on the bottom abscissa) or the decay width $\Gamma$ (shown on the top abscissa). The cases with and without the oscillations are drawn respectively by the solid and dashed curves. Thinner curves are calculated with fermi distributed neutrinos with $N_\nu$ of Fig. 3 (namely, only the change in the expansion rate due to the incomplete thermalization is taken into account). The horizontal line represents “standard” $Y_p$ calculated by BBN with neutrinos obeying the fermi distribution and $N_\nu = 3.04$. The baryon-to-photon ratio is fixed at $\eta = 5 \times 10^{-10}$.

and decreasing in $\Gamma_{n+p}$. The former is just a result of the decrease in the neutrino energy density (of all species). The latter is due to the deficit in $f_{\nu_e}$. They compete in a sense that they work in opposite way to determine the epoch of neutron-to-proton ratio freeze-out: the former makes it later and the latter makes it earlier. Then, the competition fixes the n-p ratio at the beginning of nucleosynthesis and eventually determine $Y_p$. Roughly speaking, for larger $T_R$, the former dominates to decreases $Y_p$ but, for smaller $T_R$, the latter dominates and increase $Y_p$. This is clearly seen in the case without the oscillations but not for the case including the oscillations because the incompleteness in the $\nu_e$ thermalization is made
FIG. 5: The weak interaction rate $\Gamma_{n\rightarrow p}$ and the expansion rate $H$ as functions of temperature, where $\Gamma_{n\rightarrow p}$ and $H$ first become equal. We plot for $T_R = 2.5$ MeV with and without the oscillations. For $T_R = 15$ MeV, the oscillations do not make any difference.

severer by the mixing (see the panels (c) and (d) in Fig. 1) and this effect dominates already at high $T_R$.

Before going forward, it may be worthwhile to look slightly more into the explanation of the $T_R$-dependence of $Y_p$. First, let us forget about modifying $\Gamma_{n\leftrightarrow p}$ or temperature evolution and just calculate $^4$He abundance using thermally distributed neutrinos with $N_{\nu}$'s indicated in Fig. 3 for each value of $T_R$. This corresponds to including the effect of slowing down the expansion rate due to the incomplete thermalization but neglecting the electron neutrino deficiency. Accordingly, lowering $T_R$ only acts to delay the n-p ratio freeze-out and decrease $Y_p$ (shown by the thinner curves in Fig. 4). In actual low-reheating temperature scenario, a lack of $\nu_e$ reduces $\Gamma_{n\leftrightarrow p}$. This counterbalances the effect of slowing-down expansion and boosts $Y_p$ in total at lower $T_R$. To see this is really the case, we plot $\Gamma_{n\rightarrow p}$ for some values of $T_R$ in Fig. 5. We see that $\Gamma_{n\rightarrow p}$ is smaller for lower $T_R$ which is attributed to less thermalized $\nu_e$. It is also instructive to calculate the neutron-to-proton ratio freeze-out temperature $T_{np}$, which we define by $\Gamma_{n\rightarrow p}(T_{np}) = H(T_{np})$, to confirm where the competition settles. This
is shown in Fig. 6 (a) and we see that at low $T_R$, the decrease in $\Gamma_{n\rightarrow p}$ wins to make $T_{np}$ higher (in the case with the oscillations, this seems to win for every $T_R$ and $T_{np}$ rises monotonically as $T_R$ decreases). We note that the figure well reproduces the profile found in Fig. 4. This resemblance becomes more meaningful by plotting instead the quantity $2/[1 + (n_p/n_n)_f] = 2/[1 + \exp(\Delta m/T_{np})]$, the usual estimation of $^4$He abundance from the neutron-to-proton ratio at the freeze-out value, which is shown in Fig. 6 (b). Although the figure is not exactly same as Fig. 4 because free decays of neutrons are not considered, we see that the $Y_p$’s dependence on $T_R$ is sufficiently understood from this estimation. When the neutron free decay is properly taken into account, the estimation for $Y_p$ decreases from the values indicated in Fig. 6 (b). For lower $T_R$, since the time between $T_{np}$ and the start of the nucleosynthesis ($T \approx 0.07$ MeV) is longer (this in turn is explained by the smaller expansion rate due to less neutrino energy densities), this decrease should be larger. Therefore, on including the neutron free decay, Fig. 6 (b) would be tilted toward left (smaller $T_R$) side and should look more like Fig. 4. In particular, the minimum found for the case without the oscillations should be located at lower $T_R$ when the free decay is included.

We have so far discussed the $^4$He synthesis features common to the low-$T_R$ universe with and without the neutrino oscillations, but we would rather like to emphasize that there is a striking difference between them. This is most clearly visible in Fig. 4 when we include...
the oscillations, $Y_p$ does not decrease if we lower $T_R$. This is somewhat surprising because, at the same time, $N_\nu$ becomes smaller (see Fig. 3). This means that, in the case with the oscillations, the effect of slowing down cosmic expansion (as represented by decreasing $N_\nu$) is completely overcome by the decrease in $\Gamma_{n\leftrightarrow p}$ for all $T_R$. The reason why this happens is that since the oscillations convert electron neutrinos into muon neutrinos, the deficiency in electron neutrinos is made severer (see Fig. 2). Moreover, why this matters for $^4$He synthesis is that it is exclusively sensitive to the $\nu_e$ distribution function which determines $\Gamma_{n\leftrightarrow p}$. On the other hand, the structure formation is affected only by the energy density so it does not distinguish neutrino flavors. Since only their sum matters, the oscillations scarcely make difference (see Fig. 3). Therefore, BBN, especially when the neutrino oscillations are taken into account, turns out to be unique probe of low reheating temperature scenario. Next, we proceed to compare the predictions of the scenario with the observed abundances.

On comparing the predictions of low reheating temperature scenario with the observed abundances, we need to vary the baryon-to-photon ratio, $\eta$, which is the input parameter for the standard BBN calculation, in addition to $T_R$. In Fig. 7 we show contour plots for abundances of light elements, D, $^4$He and $^7$Li, against $\eta$ and $T_R$. Since contours tend to be parallel to each other, we see that how abundances vary with respect to $T_R$ has little dependence on $\eta$. In particular, for $^4$He, features found in Fig. 4 seem to appear at every $\eta$. We notice, in Fig. 7 (c) and (d), that the oscillations almost do not make difference for D and $^7$Li abundances. In the figure, we also indicate observed values taken from Ref. [48] for $^4$He, from Ref. [49] for D, and from Ref. [50] for $^7$Li:

\begin{align*}
Y_p & = 0.238 \pm 0.002 \pm 0.005, \quad (28) \\
D/H & = 2.78^{+0.44}_{-0.38} \times 10^{-5}, \quad (29) \\
^7\text{Li}/H & = 1.23^{+0.68}_{-0.32} \times 10^{-10} \quad (95\%) \quad (30)
\end{align*}

In Eq. (28), the first uncertainty is statistical and the second one is systematic. Their root-mean-square, $\sqrt{\text{(stat.)}^2 + \text{(syst.})^2}$, is adopted as overall $1\sigma$ error. In this paper, we do not consider the $^7$Li data since its systematic error is under debate at present, but show it just for reference.\footnote{It is known that the baryon density derived from Eq. (28) is somewhat lower than one from Eq. (29). It was widely believed that $N_\nu < 3$ decreases $Y_p$ and ameliorates this tension (see e.g., Ref. [51]). However...
We immediately realize from Fig. 7 (a), (b), (c) that inclusion of the oscillations leaves smaller room for the low reheating temperature scenario. In other words, the parameter region allowed from D and $^4$He measurements is smaller for the case with the neutrino oscillation. We can see it more clearly by $\chi^2$-analysis whose results are shown in Fig. 8. The lower bound on $T_R$ at 95% confidence level in the $\eta$-$T_R$ plane is 1 MeV for the case of no oscillations but tightened to be 2 MeV for the case incorporating the oscillations.\footnote{Recently, analysis of the $^4$He abundance by Ref. \cite{52} suggests $Y_p = 0.249 \pm 0.009$. This is higher than the value of Eq. \cite{28} mainly due to the different treatments of stellar absorption. Although, at present, such large uncertainty does not allow us to derive any meaningful lower bound on $T_R$. However, higher $Y_p$ is interesting for MeV-scale reheating scenario. Should future research yield $Y_p > 0.25$, $T_R \sim O$(MeV) would be favored.}

IV. CONCLUSION

In this paper we have investigated the MeV-scale reheating scenario wherein the thermalization of neutrinos could be insufficient. We have paid particular attention to the oscillation effects on the thermalization processes of neutrinos, and solved numerically the momentum dependent Boltzmann equations for neutrino density matrix, fully taking account of neutrino oscillations. In contrast to the widespread picture, we have found that $^4$He abundance does increase while the effective neutrino number $N_\nu$ decreases. The reason is simple; the neutrino oscillations reduce the number density of $\nu_e$, due to which the neutron-proton transformation decouples earlier. This effect cancels and even overcomes that of the decrease in the expansion rate; only the latter effect has been usually taken into account when discussing the effect of $N_\nu$ on the light-element abundances. Therefore we would like to stress that it is indispensable to take into consideration the oscillation effects, to set a lower bound on the reheating temperature by using the BBN. As a reference value, we quote our results; $T_{RH} \gtrsim 2$ MeV or equivalently $N_\nu \gtrsim 1.2$ obtained by using the observational data on the $^4$He and D abundances.

What are then the distinct predictions of the MeV-scale reheating? Clearly, they are: both larger $Y_p$ and smaller $N_\nu$ compared to their standard values; if both the observed $Y_p$ and $N_\nu$ suggest the same $T_R$ by the relations shown in Figs. 3 and 4, they would serve as
FIG. 7: Contour plots for the light element abundances. $^4$He mass fraction is plotted in (a) with the oscillations and in (b) without. D and $^7$Li are plotted respectively in (c) and (d) where dotted lines express the case with the oscillations and dashed lines express the case without. Shaded areas represent uncertainties in the observed abundances expressed in eqs. (28) – (30) (for D and $^7$Li, they are drawn against the contours considering the oscillations). Darker areas are for $1\,\sigma$ and lighter for $2\,\sigma$.

decisive evidence for the MeV-scale reheating $^7$.

At last, let us comment on the validity and possible extension of the present work. As explained in section II, we have neglected the self-interactions of neutrinos. Such simplification is considered to be valid due to the following reason. Since self-interactions cannot change

$^7$ According to Ref. 54, we can determine both $Y_p$ and $N_\nu$ with future CMB observations such as Planck.
FIG. 8: $\chi^2$ contour plot using data of D and $^4$He. For no oscillation case, the allowed regions at 68% and 95% confidence levels are drawn with solid and dashed lines. For the case with oscillations, the 68% allowed region does not appear and only the 95% region is indicated by the shaded area.

the total energy stored in the neutrino sector, they affect only the momentum distribution of neutrinos. On the other hand, it should be noted that we have taken into consideration the neutrino-electron ($\nu e$) scattering, which also shifts the neutrino momentum distribution toward kinetic equilibrium at the rate of the same order of magnitude as the $\nu\nu$ scattering. However we have checked that our results do not change at all even if we increase the $\nu e$ scattering rate a few times larger than the standard one. Considering that the $\nu\nu$ scattering rate is further suppressed due to the deficit in the neutrino number, we are sure that the self-interactions have only a minor effect in the neutrino momentum distribution. Still, the self-interactions have a potential effect on the number density of $\nu_e$ through e.g., $\nu_e\bar{\nu}_e \leftrightarrow \nu_{\mu(\tau)}\bar{\nu}_{\mu(\tau)}$. Furthermore, nonzero $\theta_{13}$ can have a similar effect; in this case it is necessary to perform three generation analysis. Nevertheless we believe that our main conclusion is robust, since these extensions, too, are expected to decrease the number density of $\nu_e$, further increasing the $^4$He abundance. Of course the quantitative improvement should be necessary and the full analysis on these points will be presented elsewhere [55].
Acknowledgments.—F.T. would like to thank the Japan Society for Promotion of Science for financial support.

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