SUSY and the mass difference of $B_d^0 - \overline{B}_d^0$

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Abstract

We present an overview of the loop corrections to the mass difference $\Delta m_{B_d}/m_{B_d}$ within the minimal supersymmetric standard model. We include the complete mixing matrices of the charginos and neutralinos as well as of the scalar partners of the left and right handed third generation quarks. We show that the SUSY contribution to the mass difference in the $B_d^0$ system is comparable to the Standard Model one and can be even larger for the charged Higgs contribution and for a certain supersymmetric parameter space.

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We present an overview of the loop corrections to the mass difference $\Delta m_{B^0_d}/m_{B^0_d}$ within the minimal supersymmetric standard model. We include the complete mixing matrices of the charginos and neutralinos as well as of the scalar partners of the left and right handed third generation quarks. We show that the SUSY contribution to the mass difference in the $B^0_d$ system is comparable to that of the Standard Model and can be even larger than that of the charged Higgs for parts of the supersymmetric parameter space.

1. Introduction

The mass difference $\Delta m_{B^0_d}/m_{B^0_d} \approx 5.9 \times 10^{-14}$ GeV \(^1\) is an experimentally well known value. In the standard model (SM), where the W bosons and up-type quarks run in the relevant box diagrams, it was found that in the $B^0_d$ system the top quark leads to the most important contribution.

Nowadays, from CDF and D∅ we know that the top quark mass is about 180 GeV \(^2\). Because of such a large value, we have to reconsider the influence of one of the most favoured models beyond the SM, its minimal supersymmetric extension (MSSM) \(^3\), to $\Delta m_{B^0_d}$. As is well known, the particle spectrum of the MSSM is enhanced by at least a factor of two and therefore many more particles contribute to the mass difference of the $B^0_d$ system.

In this talk we present an overview of the contribution of all particles within the MSSM to this electroweak parameter. We present the results only in a very general way with a short discussion and refer the interested reader to our articles \(^5\) for more detailed and complete calculations.

2. The SM and $\Delta m_{B^0_d}$

In the SM, the contribution of the W bosons and up-type quarks to the mass difference of the $B^0_d$ system is given by \(^6\)

$$\frac{\Delta m_{B^0_d}}{m_{B^0_d}} = \frac{G_F}{6\pi^2} f_B^2 B_B \eta_l m_W^2 (K_{31}^* K_{33})^2 S(x_t) \quad (1)$$

$f_B, B_B$ are the structure constant and the Bag factor obtained by QCD sum rules and $\eta_l$ a QCD correction factor.

3. The MSSM and $\Delta m_{B^0_d}$

The first obvious thing to do to obtain $\Delta m_{B^0_d}$ within the MSSM is to replace the W bosons by their fermionic partners and the up quarks by their scalar partners. Unfortunately due to mass mixing effects and the rich particle spectrum of the MSSM things are not quite that simple.

First of all due to new scalar fermion-fermion-gaugino couplings the fermionic partners of the W bosons mix with the fermionic partners of the charged Higgses when the neutral scalar Higgses obtain their vacuum expectation values (vev). The mass eigentstates are known as charginos.

Second, the scalar partners of the left and right handed up-type quarks will mix. This mixing is proportional to the up-type quark masses; since the top quark mass is very large it cannot be neglected for the third generation. Although the mixing of the scalar partners of the left and right handed bottom quarks is proportional to the bottom quark mass, we did not neglect it since for

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* Talk presented by H. König
large values of $\tan \beta = v_2/v_1$ (the ratio of the vevs) it can become important too.

As a result, after a lengthy but straightforward calculation we obtain the following result when charginos and scalar up quarks are running in the loop:

$$\frac{\Delta m_{B^0_2}}{m_{B^0_2}} = \frac{G_F^2}{4\pi^2} f^2_B B_B m_W^4 (K_{31}^* K_{33})^2 \quad (2)$$

$$[Z_{11}^W - 2Z_{31}^W + \tilde{Z}_{33}^W]$$

As we can see from the structure of eq.(2) if all the scalar quark masses are degenerate, the result is identically 0 (GIM mechanism). Since we neglected all quark masses beside the top and bottom quark masses, we made use of $Z_{11} = Z_{12} = Z_{21} = Z_{22}$ and $Z_{13} = Z_{31} = Z_{32} = Z_{33}$. In this case, only the mass difference between the first and third generations of the scalar quarks comes into play.

It was shown more than 10 years ago that loop diagrams induce flavour changing couplings of the gluinos (the fermionic partners of the gluons) to the down quarks and their scalar partners \[8,9\]. Since the gluinos couple strongly their contribution to $\Delta m_{B^0_2}$ was thought to be the most important one. The result is given by:

$$\frac{\Delta m_{B^0_2}}{m_{B^0_2}} = \frac{\alpha_s^2}{54} f^2_B B_B (K_{31}^* K_{33})^2 \quad (3)$$

$$[Z_{11}^g - 2Z_{31}^g + \tilde{Z}_{33}^g]$$

In the MSSM there are also the neutralinos (the mass eigenstates of the fermionic partners of the photon, the Z boson and the neutral Higgses) and the scalar partners of the down quarks within the relevant box diagrams. Since the neutralinos couple only weakly their contribution has been neglected in the literature so far. We show that this is illegitimate for a certain range of supersymmetric parameter space. The calculation is very lengthy and we obtain:

$$\frac{\Delta m_{B^0_2}}{m_{B^0_2}} = \frac{G_F^2}{(4\pi)^2} f^2_B B_B m_Z^4 (K_{31}^* K_{33})^2 \quad (4)$$

$$[Z_{11}^N - 2Z_{31}^N + \tilde{Z}_{33}^N]$$

Finally we also want to comment on the charged Higgs boson contribution to the mass difference in the $B^0_2$ system. In the case where we neglect the bottom quark mass, that is $m_b \tan \beta \ll m_t \cot \beta$ we obtain:

$$\frac{\Delta m_{B^0_2}}{m_{B^0_2}} = \frac{G_F^2}{16\pi^2} m_t^4 \cot \beta f^2_B B_B (K_{31}^* K_{33})^2 \quad (5)$$

$$\{ \tilde{F}_{H^+H^+} + 2\tan^2 \beta [\tilde{F}_{H^+W^+} \tilde{F}_{H^+W^+}] + 4(m_W/m_t)^2 \tilde{F}_{H^+W^+} \}$$

When one has $m_b \tan \beta \sim m_t \cot \beta$ one should not neglect the bottom quark mass when calculating the box diagram; this complicates greatly the calculations.

4. Discussions

We now present those contributions for different values of gaugino, gluino and scalar quark masses and charged Higgs masses as well as the bilinear Higgs mass term $\mu$. We also vary $\tan \beta$ and the symmetry-breaking scales $m_S$.

In Fig. 1, we show the chargino and gluino contributions. The global behaviour is clear: for small gluino mass and small values of $m_S$, the gluino contribution is important no matter what values the other parameters have. On the other hand, for large gluino mass and large values of $m_S$, the chargino contribution vastly dominates. The only exception to this rule is for very large values of $\tan \beta$ ($\sim 30$ or higher). This is due to the fact that such large values of $\tan \beta$ can push down the mass of one of the scalar b-quark eigenstates; well below the scalar top-quark eigenstates.

The effects of the mixing of the scalar partners with the top and bottom quarks are more important for large values of $m_S$: the contributions from the charginos don’t decrease as quickly with the mixing. For small values of $m_S$, there is also an enhancement.

In Fig. 4 we show the neutralino contribution and compare them with the chargino contribution. The global behaviour is clear: for small values of $\tan \beta$ ($\sim 20$ or less) the neutralino contribution is small compared to that of the chargino.
Figure 1. The ratios $\Delta m_{\text{Chargino}} / \Delta m_{\text{SM}}^{B_0^d}$ and $\Delta m_{\text{Gluino}} / \Delta m_{\text{SM}}^{B_0^d}$ as a function of the scalar mass $m_S$ for $\tan \beta = 1$ (solid); $\tan \beta = 2$ (dash); $\tan \beta = 5$ (dash-dot); $\tan \beta = 20$ (dot). The negative values for large $m_S$ are the chargino contributions; those of large amplitudes for small $m_S$ are the gluino contributions with $m_{\tilde{g}} = 100$ GeV; those of small amplitudes for small $m_S$ are the gluino contributions with $m_{\tilde{g}} = 200$ GeV.

On the other hand, when $\tan \beta \sim 50$, the neutralino contribution can be much larger than that of the chargino for the smallest possible values of $m_S$. Unfortunately, as we can see on the figure, this contribution falls very quickly as $m_S$ increases and becomes negligible as soon as $m_S$ is a few tens of GeV’s above its minimal value (for smaller values of $m_S$ the square of one of the mass eigenvalues of the scalar bottom quark becomes negative).

Figure 2. The ratios $\Delta m_{\text{Neutralino}} / \Delta m_{\text{SM}}^{B_0^d}$ for $\tan \beta = 2, 5$ (dotted line, the two lines are on top of each other) and $\tan \beta = 50$ (dash) and $\Delta m_{\text{Chargino}} / \Delta m_{\text{SM}}^{B_0^d}$ for $\tan \beta = 2$ (very long dash-dot), $\tan \beta = 5$ (long dash-dot), and $\tan \beta = 50$ (dash-dot) as a function of $m_S$ with $m_{g_2} = \mu = 200$ GeV.

Finally in Fig. 3 we show the charged Higgs contribution. We see that for small values of $\tan \beta$ and light Higgs, this contribution can exceed that of the SM. For $\tan \beta = 1$, even for very large Higgs masses, this contribution is still 20% of the SM contribution. However, this contribution goes down very quickly when $\tan \beta$ increases. Given our approximation ($m_b \tan \beta \ll m_t \cot \beta$) we cannot exceed $\tan \beta \sim 5$ and still trust our result.

Last but not least one must not forget that in eqs. $K_{31}K_{33}$ have not necessarily the same

Such high values for $\tan \beta$ are preferred in models, which require the Yukawa couplings $h_t$, $h_b$ and $h_\tau$ to meet at one point at the unification scale.
values as in the SM. The Kobayashi–Maskawa matrix in the couplings of the charginos to quarks and scalar quarks is multiplied by another matrix \( V_u \), which can be parametrized by \( \epsilon_u \), so that \( K \equiv V_u \cdot K_{SM} \). If \( \epsilon_u \ll 1 \) then \( K \sim K_{SM} \). However, with \( \epsilon_u = 0.3 \), \( K_{31}^* K_{33} \) is enhanced by a factor of 3 over the SM value.

We have a similar matrix in the gluino–down quark–scalar down quark couplings parametrized by \( \epsilon_d \). For \( \epsilon_d = 0.1 \), \( K_{31}^* K_{33} \) is identical to the SM values, whereas \( \epsilon_d = 0.3 \) enhances it by 9.

Considering that these values are to be squared in the mass difference of the \( B_0^0 \) system we can use that enhancement to put limits on \( \epsilon_u,d \). In the case at hand, \( \epsilon_u \) has to be smaller than 0.2 and \( \epsilon_d \) smaller than 0.1 to keep the results lower than the measured value of \( \Delta m_{B_0^0}/m_{B_0^0} \). This is not very constraining yet but it is already better than the limit one can get from current data on rare Kaon decays [10].

5. Conclusion

In this talk we presented the contributions of all particles within the MSSM to the mass difference in the \( B_0^0 \) system via box diagrams. In the calculations we included the mixing of the charginos and neutralinos and the mixing of the scalar top and bottom quarks.

We have shown that for reasonable values of the SUSY parameters the contribution of the box diagrams with charginos and scalar up quarks can be of the same order as those of the SM diagrams, but with opposite sign. The same goes for the contribution of the gluino and scalar down quarks box diagrams, which has the same sign as the SM contribution.

We have shown that in the case of charginos and scalar top quark, the mixing becomes important and leads to an enhancement. In the case of gluinos and scalar bottom quark the mixing is less important and even for higher values of \( \tan \beta \) the results are reduced only by a few per cent.

Since we have shown that despite the smallness of the weak coupling constant compared to the strong coupling constant charginos and scalar up quarks cannot be neglected, we included the contribution of the neutralinos and scalar down quarks and showed that it is in general small but can be important for large values of \( \tan \beta (\sim 50) \) and the smallest possible values of \( m_S \), given \( m_{g_2} \) and \( \mu \).

The contribution from the charged Higgs boson to the mass difference in the \( B_0^0 \) system can be very important for small values of \( \tan \beta \) and small Higgs masses. When the Higgs mass becomes large (\( \sim 500 \) GeV) and/or \( 2 \leq \tan \beta \), this contribution becomes small and even negligible compared to the chargino contribution.

We hope that our study will guide experiments at the upcoming \( B \)-factories.

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