Calculations of the atomic structure for the low-lying states of actinium

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We employ a technique that combines the configuration interaction method with the single-double coupled-cluster method to perform calculation of the energy levels, transition amplitudes, lifetimes, g-factors, and magnetic dipole and electric quadrupole hyperfine structure constants for many low-lying states of neutral actinium. We find very good agreement with existing experimental energy levels and make accurate predictions for missing levels. It has been noted that some of the levels have been previously misidentified; our analysis supports this claim. If spectroscopy is performed with actinium-225, our calculations will lead to values for nuclear structure constants. The accuracy of this can be constrained by comparing with actinium-227.

I. INTRODUCTION

Actinium is a radioactive element with atomic number $Z = 89$, and has three valence electrons above a radon-like core. The most stable isotope, $^{227}$Ac, has a half-life of 28 yr; the nearest even isotopes, $^{226,228}$Ac, have lifetimes on the order of hours. Its complicated electron structure and short half-life have made both theoretical and experimental investigations of its atomic properties quite difficult. The data for the electron spectrum of Ac presented in the NIST database [1] miss many levels and there are indications that some of the data are not accurate [2], see also Refs. [3–5].

Several isotopes of actinium have extensive use in medical applications. In particular, $^{225}$Ac, an alpha-emitter with a 10 day half-life, is a very promising. The emitted radiation is sufficiently energetic to destroy cancer cells, but has a range short enough to be essentially contained and not damage nearby cells. Work on producing and studying $^{225}$Ac is ongoing at the Los Alamos National Laboratory [6, 7], Brookhaven National Laboratory [8], Leuven [5] and other centres. The trapping and transport of these radioactive isotopes, however, is rather challenging. A detailed knowledge of the energy levels and transition amplitudes is extremely important for the development and testing of trapping and cooling schemes.

We perform accurate calculations for neutral actinium using a technique based on the combination of the configuration interaction method with the linearized single-double–coupled-cluster method (SD), as developed in Ref. [9]. This method is similar to that developed in Ref. [15], and good agreement between the two approaches has been demonstrated [9]. This technique has proven to be effective and very accurate for few-valence-electron systems [9, 15–19].

This method is similar to the combination of many-body perturbation theory (MBPT) with the CI method, as developed in Ref. [20]. Such a method has been used widely for a large number of applications and has been proven to be both efficient and accurate for few-valence atoms and ions. The SD technique allows one to accurately take into account the core–valence and core-core electron correlations, while the CI method accounts for the valence–valence correlations. The CI+SD method includes a more accurate treatment of the core–valence correlations, as well as the screening (by the core electrons) of the Coulomb interaction between valence electrons compared to the CI+MBPT technique [9].

In the SD method, the many-body wave function is expressed as an expansion that contains all single and double excitations from the Hartree-Fock reference wave function, see, e.g., Ref. [21]. The coefficients of the expansion are found by solving the set of SD equations. First, the SD equations are solved self consistently for the core electrons to determine the core excitation coefficients. Then, the SD equations are solved for valence states. This a complete procedure for the case of a single-valence electron. For systems with more than one valence electron, however, the interactions between the valence electrons must also be taken into account. This is done
The single-particle operator, \( \hat{\Sigma} \), that would correspond to the conventional CI method. Without these, the above equations of the Hartree-Fock equations. The radiative potential has the form

\[
V^{\text{rad}}(r) = V_U(r) + V_B(r) + V_e(r),
\]  

where \( V_U \) is the Uehling potential (lowest order vacuum polarization correction), and the self-energy corrections \( V_s \) and \( V_e \) are the potentials arising from the magnetic and electric electron form-factors, respectively. The \( V_U \) and \( V_e \) terms can be added directly to the Hartree-Fock potential, while \( V_s \) leads to corrections to the Dirac equation (see Ref. [23]). Relaxation is included for both the Breit and QED potentials by including them in the iterations of the Hartree-Fock equations.

**B. Matrix elements and core polarisation**

To calculate the matrix elements for the transition amplitudes we use the time-dependent Hartree-Fock (TDHF) method, as in Refs. [24–26]. In the presence of external fields, the Hartree-Fock core is polarized and its potential should be modified:

\[
\hat{V}^{N-3} \to \hat{V}^{N-3} + \delta V.
\]

To calculate \( \delta V \), the wave function is written in the TDHF framework as

\[
\psi = \psi_0 + \eta e^{-i\omega t} + \zeta e^{i\omega t},
\]

where \( \psi_0 \) is the unperturbed wave function, \( \eta \) and \( \zeta \) are corrections due to the external field, and \( \omega \) is the frequency of the external field (frequency of the transition). Then, the set of TDHF equations

\[
\begin{align*}
(\hat{h}^{\text{HF}} - \varepsilon_c - \omega)\eta_c &= -(\hat{h}_\text{ext} + \delta V - \delta \varepsilon_c)\psi_c \quad (8) \\
(\hat{h}^{\text{HF}} - \varepsilon_c + \omega)\zeta_c &= -(\hat{h}^{\dagger}_\text{ext} + \delta V^{\dagger} - \delta \varepsilon_c)\psi_c, \quad (9)
\end{align*}
\]

are solved self-consistently for the core orbitals. Here, the index \( c \) denotes a state in the core, \( \hat{h}_\text{ext} \) is the operator of the external field interaction, and \( \delta \varepsilon = \langle \psi_0 | \delta V | \psi_0 \rangle \) is the correction to the energy due to the external field. Core polarization is included into the calculation of the matrix elements via a redefinition of the external field operators, e.g.,

\[
d_{E1} \to \tilde{d}_{E1} = d_{E1} + \delta V_{E1},
\]

where \( d_{E1} = -e r \) is the operator of the E1 interaction, and \( \delta V_{E1} \) is the correction to the Hartree-Fock core potential due to the action of the external E1 photon field. This method is equivalent to the random phase approximation (RPA) method [27], and includes core-polarization effects to all-orders.

We also take into account two non-correlation corrections; those due to the Breit and radiative quantum electrodynamics (QED) effects. The effective Breit Hamiltonian, which includes magnetic and retardation corrections (in the zero energy transfer limit) can be expressed as

\[
\hat{h}^B(r_1, r_2) = \frac{-1}{2r_{12}} \left( \alpha_1 \cdot \alpha_2 + \frac{\alpha_1 \cdot r_{12}}{r_{12}^2} \cdot \frac{\alpha_2 \cdot r_{12}}{r_{12}^2} \right),
\]  

where \( r_{12} = r_1 - r_2 \).
| Level | Energy (cm$^{-1}$) |
|-------|------------------|
| N     | Conf. | Term  | $E_{\text{exp}}$ | $E_{\text{calc}}$ | $\Delta$ | $g$  |
| 1     | $7s^67p$ | $^2D_{3/2}$ | 0 | 0 | 0 | 0.8001 |
| 2     | $7s^67d$ | $^2D_{5/2}$ | 2231 | 2339 | $-108$ | 1.2002 |
| 3     | $7s^67p$ | $^2P_{1/2}$ | 7565 | 9899 | 228 | 0.4088 |
| 4     | $7s^6d^2$ | $^4F_{3/2}$ | 9864 | 9288 | 576 | 1.0298 |
| 5     | $7s^6d^2$ | $^4F_{5/2}$ | 10906 | 9974 | 932 | 1.2333 |
| 6     | $7s^6d^2$ | $^4F_{7/2}$ | 12078 | 11726 | 352 | 1.3143 |
| 7     | $7s^6d^2$ | $^2D_{3/2}$ | 12345 | 13301 | 14320 |
| 8     | $7s^6d^2$ | $^4P_{1/2}$ | 12583 | 12847 | 16841 |
| 9     | $7s^6d^2$ | $^4D_{3/2}$ | 13301 | 14810 | 9302 |
| 10    | $7s^6d^2$ | $^4D_{5/2}$ | 14985 | 13958 | $-245$ | 0.4778 |
| 11    | $7s^7p$ | $^4P_{5/2}$ | 14941 | 15746 | 7905 |
| 12    | $7s^7d^2$ | $^4G_{7/2}$ | 15746 | 16345 | 670 |
| 13    | $7s^7d^2$ | $^4D_{3/2}$ | 17200 | 17049 | 151 | 0.0440 |
| 14    | $7s^7d^2$ | $^4D_{5/2}$ | 17198 | 17329 | 1293 |
| 15    | $7s^7p$ | $^4D_{5/2}$ | 17329 | 17612 | 124 | 1.1984 |
| 16    | $7s^7d^2$ | $^4D_{7/2}$ | 17684 | 17715 | $-31$ | 1.2502 |
| 17    | $7s^7d^2$ | $^4S_{1/2}$ | 17951 | 18108 | $-157$ | 1.2684 |
| 18    | $7s^7d^2$ | $^4D_{9/2}$ | 18066 | 18747 | 6781 |
| 19    | $7s^7p$ | $^4D_{9/2}$ | 18747 | 19612 | 51 | 0.8590 |
| 20    | $7s^7d^2$ | $^4D_{11/2}$ | 18980 | 19012 | 32 |
| 21    | $7s^7d^2$ | $^{6}D_{3/2}$ | 19028 | 20288 | 13858 |
| 22    | $7s^7d^2$ | $^{6}D_{5/2}$ | 20640 | 20680 | 1333 |
| 23    | $7s^7d^2$ | $^{6}D_{7/2}$ | 22402 | 20669 | 1733 | 2.5826 |
| 24    | $7s^7d^2$ | $^{6}D_{9/2}$ | 22801 | 20877 | 1924 | 1.5966 |
| 25    | $7s^7d^2$ | $^{6}D_{11/2}$ | 21196 | 21170 | 26 | 1.0394 |
| 26    | $7s^7d^2$ | $^{6}D_{13/2}$ | 21918 | 21918 | 0.0 |
| 27    | $7s^7d^2$ | $^{6}D_{15/2}$ | 23899 | 22281 | 1618 | 1.2640 |
| 28    | $7s^7d^2$ | $^{6}D_{17/2}$ | 23917 | 23379 | 538 | 0.9769 |
| 29    | $7s^7d^2$ | $^{6}D_{19/2}$ | 24385 | 23657 | 728 |
| 30    | $7s^7d^2$ | $^{6}D_{21/2}$ | 23828 | 23875 | 4778 |
| 31    | $7s^7d^2$ | $^{6}P_{1/2}$ | 23476 | 24015 | $-539$ | 1.1600 |
| 32    | $7s^7d^2$ | $^{6}P_{3/2}$ | 24909 | 24532 | 437 | 1.1513 |
| 33    | $7s^7p$ | $^{6}P_{3/2}$ | 24692 | 25421 | 308 |
| 34    | $7s^7d^2$ | $^{6}P_{5/2}$ | 25729 | 25738 | 1.2375 |
| 35    | $7s^7d^2$ | $^{6}P_{7/2}$ | 26066 | 26235 | $-168$ | 1.0389 |
| 36    | $7s^7p$ | $^{6}P_{9/2}$ | 26533 | 26595 | $-62$ | 1.2333 |
| 37    | $7s^7d^2$ | $^{6}P_{11/2}$ | 26836 | 27029 | $-192$ | 0.7184 |
| 38    | $7s^7d^2$ | $^{6}P_{13/2}$ | 27097 | 27097 | 0.0 |
| 39    | $7s^7p$ | $^{8}S_{1/2}$ | 27353 | 27353 | 0.0 |
| 40    | $7s^7d^2$ | $^{8}S_{3/2}$ | 27634 | 27634 | 0.0 |
| 41    | $7s^7p$ | $^{8}P_{1/2}$ | 28000 | 28000 | 0.0 |
| 42    | $7s^7d^2$ | $^{8}P_{3/2}$ | 28568 | 27966 | 602 | 0.9936 |
| 43    | $7s^7p$ | $^{8}P_{5/2}$ | 28156 | 28156 | 0.6878 |
III. RESULTS AND DISCUSSION

In Table I, we present our calculations of the energy levels and g-factors for several of the low-lying states for Ac. Also presented are the accepted experimental values (taken from the NIST database [1]), and the difference between these and our calculations (in the column Δ). For most levels, the agreement is better than a few percent.

As noted, it is possible that a number of these levels have been previously misidentified in the literature. The three largest deviations from experiment are likely explained by misidentification of the levels [3, 10], see footnotes of Table I. In Ref. [3], the authors employed a multi-configuration Dirac-Fock calculation (including Breit and QED effects) to calculate the spectrum of the low-lying states of neutral actinium. Our calculations agree reasonably with those in Ref. [3], however, our work represents a significant improvement in accuracy. This becomes more noticeable at higher energies.

Table II presents our calculated hyperfine structure constants, and comparison with experimental values. The magnetic dipole constant A is relatively unstable in the calculations for the ground state, while the electric quadrupole constant B is comparatively stable. This is due primarily to the role of d-states. The direct contribution is small, because matrix elements for magnetic hyperfine structure are small for d-states; the value of A comes from many-body corrections where s and p states play role. In contrast, s and p_{1/2} states do not contribute to B (due to their too small total angular momentum j: ⟨1/2|Q|1/2⟩ = 0), but d-states contribute significantly.

### TABLE I: continued from previous page

| N  | Conf. | Term  | $E_{\text{exp}}$ | $E_{\text{calc}}$ | $\Delta$ | g  |
|----|-------|-------|-----------------|-----------------|----------|----|
| 53 | 7s7p6d | $^2P_{3/2}^0$ | 27010 | 28169 | −1159 | 1.3212 |
| 54 | 6d$^3$ | $^4P_{5/2}^0$ | 28201 | 1.5024 |
| 55 | 6d$^3$ | $^4P_{3/2}^0$ | 28402 | 1.0156 |
| 56 | 6d$^3$ | $^4D_{3/2}^0$ | 28793 | 1.0036 |
| 57 | 7s$^2$5f | $^2P_{3/2}^0$ | 29063 | 0.8621 |
| 58 | 7p6d$^2$ | $^4P_{9/2}^0$ | 29651 | 1.1646 |
| 59 | 7p7p$^2$ | $^4P_{1/2}^0$ | 30727 | 2.5566 |
| 60 | 6d$^3$ | $^2D_{5/2}^0$ | 30787 | 1.2923 |
| 61 | 7p6d$^2$ | $^4P_{9/2}^{10}$ | 30397 $^2P$ | 30803 | −407 | 0.4273 |
| 62 | 7p6d$^2$ | $^4P_{9/2}^0$ | 31333 | 161 | 1.0242 |
| 63 | 7s$^2$5f | $^2P_{7/2}^0$ | 31366 | 1.1238 |
| 64 | 7s8s6d | $^2D_{3/2}^0$ | 31558 | 0.9099 |
| 65 | 7s$^2$7d | $^2D_{5/2}^0$ | 31874 | 1.1915 |
| 66 | 7s8s6d | $^4D_{3/2}^0$ | 31937 | 1.1587 |
| 67 | 7p6d$^2$ | $^2G_{9/2}^0$ | 32166 | 1.1722 |
| 68 | 7s7p$^2$ | $^2P_{3/2}^0$ | 32495 | 1.6364 |
| 69 | 7s8s6d | $^4D_{5/2}^0$ | 32611 | 1.3124 |
| 70 | 7p6d$^2$ | $^2D_{5/2}^0$ | 32697 | 0.9900 |
| 71 | 7p6d$^2$ | $^2S_{1/2}^0$ | 32747 | 1.5230 |
| 72 | 6d$^3$ | $^4F_{5/2}^0$ | 32902 | 1.0395 |
| 73 | 7p6d$^2$ | $^4F_{5/2}^0$ | 32934 | 1.0626 |
| 74 | 6d$^3$ | $^2D_{3/2}^0$ | 33365 | 0.9511 |
| 75 | 7p6d$^2$ | $^2D_{9/2}^0$ | 33506 | 1.1784 |
| 76 | 7s$^2$9p | $^2P_{9/2}^0$ | 33551 | 1.331 |
| 77 | 7p6d$^2$ | $^2F_{5/2}^0$ | 33635 | 1.0619 |
| 78 | 7p6d$^2$ | $^4D_{3/2}^0$ | 34208 | 1.0352 |
| 79 | 7p6d$^2$ | $^4D_{5/2}^0$ | 34290 | 1.3224 |
| 80 | 6d$^3$ | $^4D_{3/2}^0$ | 34409 | 1.0191 |
| 81 | 7s7p$^2$ | $^4D_{5/2}^0$ | 34514 | 1.4652 |
| 82 | 7p6d$^2$ | $^4D_{9/2}^0$ | 35115 | 1.3015 |
| 83 | 7s8s6d | $^2D_{3/2}^0$ | 35290 | 1.2158 |
| 84 | 7p6d$^2$ | $^4D_{9/2}^0$ | 35461 | 1.1724 |
| 85 | 7s6d7d | $^4G_{5/2}^0$ | 36150 | 0.6868 |
| 86 | 7s6d7d | $^2D_{3/2}^0$ | 36218 | 0.8778 |

1 It is likely that this level does not originate from a transition to the ground state but from the $^2D_{5/2}$ state at 2231 cm$^{-1}$; the correct energy would then be 2463 cm$^{-1}$, with $J = 3/2$, 5/2, or 7/2 [10]. Most likely, this corresponds to one of the levels denoted here as $N = 38, 39, or 40$.

2 This level actually has $J = 5/2$ [2], and most likely corresponds to the level denoted here as $N = 34$. 
TABLE II. Magnetic dipole (A) and electric quadrupole (B) hyperfine structure constants for several states of Ac. The values for A are given in units of $\mu_0$ MHz, where $\mu$ is the nuclear magnetic moment (in nuclear magnetons), $I$ is the nuclear spin, and $Q$ is the magnetic quadrupole moment (in barns). These numbers are presented since they are independent of the nuclear parameters (besides the effects of finite nuclear size, which are well below the assumed accuracy). The corresponding experimental values are found using the known parameters for $^{227}$Ac, $\mu = 1.1$, $Q = 1.7$ b, and $I = 3/2$.

| N | Configuration | Energy | $A/\mu_0$ MHz | $B/Q$ MHz |
|---|---------------|--------|----------------|----------|
| 1 | $7s^6d$ $^2D_{3/2}$ | 0 | −51.7 | 372 |
| 2 | $7s^76d$ $^2D_{5/2}$ | 2339 | 211.2 | 464 |
| 34 | $7s7p6d$ $^2D_{5/2}$ | 22281 | 2161 | 128 |
| 42 | $7s7p6d$ $^2F_{1/2}$ | 25421 | 2176 | 0 |
| 44 | $7s7p6d$ $^2F_{3/2}$ | 26235 | −1391 | 54.2 |

Actually presented in the tables are $A/(\mu/I)$, and $B/Q$, where $\mu$ is the nuclear magnetic moment (in nuclear magnetons), $I$ is the nuclear spin, and $Q$ is the magnetic quadrupole moment (in barns). These numbers are presented since they are independent of the nuclear parameters (besides the effects of finite nuclear size, which are well below the assumed accuracy). The corresponding experimental values are found using the known parameters for $^{227}$Ac, $\mu = 1.1$, $Q = 1.7$, and $I = 3/2$. If measurements of the hyperfine structure are performed for $^{228}$Ac [4], these calculations can be used to extract the nuclear parameters $\mu$ and $Q$. The accuracy of these predictions can be gauged from the comparison with $^{227}$Ac.

In Table III, we present calculations of the reduced matrix elements for the electric dipole ($E1$) amplitudes between several of the lowest states of Ac. In order to control the accuracy of the calculations, we also performed all calculations using the CI+MBPT method, as developed in Ref. [20], which includes a set of dominating correlation diagrams to the second-order in perturbation theory. By comparing the results of the CI+MBPT calculations with the all-order CI+SD calculations, a reasonable estimate of the uncertainty due to missed higher-order correlations can be formed. For nearly all transitions, the difference between these calculations is less than 10%.

The total decay probability for state $i$ is given $\Gamma_i = \sum \gamma_{ij}$, where the summation is over all lower state $j$, and (considering only $E1$ transitions) the partial transition probability is (in atomic units)

$$\gamma_{ij} = \frac{4}{3} (\alpha \omega_{ij})^3 \frac{|\langle j|\hat{d}_{E1}|i\rangle|^2}{2J_i + 1}. \quad (10)$$

Here, $\langle j|\hat{d}_{E1}|i\rangle$ is the reduced matrix element for the $E1$ amplitude, $\omega_{ij} = E_i - E_j$, and $\alpha \approx 1/137$ is the fine structure constant. The lifetime for the state $i$ is then $1/\Gamma_i$.

To convert the rate from atomic units to ordinary units we should multiply the result in Eq. (10) by (2$\text{Ry}/\hbar$). Correspondingly, to find the lifetime we should multiply $1/\Gamma$ in atomic units by $(\hbar/2\text{Ry}) \approx 2.4189 \times 10^{-17}$ s.

In Table IV, we present our calculations of the lifetimes for several of the lowest-lying odd states. The presented errors take into account the uncertainties in the calculated frequencies (where experimental values were not available), the uncertainties in the $E1$ matrix elements. For the higher states, the uncertainties also reflect the fact that only the dominating transitions were included. To determine the uncertainties, we conservatively take the uncertainties in the frequencies and $E1$ amplitudes to be 10% and 20%, respectively. We calculate the lifetime of the state $7s^27p$ $^2P_{1/2}$ (denoted as state 3 in Table I) to be $4 \times 10^{-3}$ s, with an uncertainty of about 50% (which comes mostly from the calculation of the frequency).

For the even state $7s6d^2$ $^4F_{3/2}$ ($N = 4$ in Table I), we calculate the lifetime to be $4 \times 10^{-3}$ s. We note, however, that the smallness of the energy interval between this state and the lower $7s^27p$ $^2P_{1/2}$ state ($\gamma \approx 1000 \text{cm}^{-1}$) leads to instability in the calculation for the frequency. Therefore, without an experimental determination of the frequency, this should be considered an order-of-magnitude estimate. The even states $^2D_{5/2}$, $^4F_{5/2}$, $^4F_{7/2}$, $^4F_{9/2}$, and $^2G_{9/2}$, enumerated in Table I as 2, 5, 6, 7, and 19, respectively, are expected to be metastable.

IV. CONCLUSION

We have performed accurate calculations of the energy levels, $g$-factors, transition matrix elements, and lifetimes for several of the low-lying states of neutral actinium using an all-order method based on the combination of the configuration interaction technique with the singles–doubles–coupled-cluster method. Our calculations indicate good agreement with experiment for known levels, and we have provided predictions for many previously unidentified levels. The calculations will help shed light on some potential misidentified levels in the literature, and will aid in the indenification of new levels as experimental work continues.

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TABLE III. Calculated reduced $E1$ matrix elements for transitions to the three lowest even states of Ac, and the corresponding experimental and calculated frequencies. The column $N$ is given for ease of reference to Table I. All values are given in atomic units$^a$.

| Even state | $N$ | Odd state | $\omega_{\text{exp}}$ | $\omega_{\text{calc}}$ | $\langle \alpha \mid d E1 \mid \beta \rangle$ |
|------------|-----|-----------|-------------------------|-------------------------|---------------------------------|
| $7s^6d^2$  | 1→3 | $7s^5p$   | 0.0345                  | -1.7547                 |                                  |
|            | 18  | $7s^5d_5/2$ | 0.0784                  | 0.0777                  | 0.5696                           |
|            | 30  | $7s^5p_1/2$ | 0.1021$^b$              | 0.0942                  | 0.1229                           |
|            | 42  | $7s^5d_3/2$ | 0.1172                  | 0.1158                  | 3.9843                           |
|            | 51  | $7s^5d_1/2$ | 0.1283                  | -0.5594                 |                                  |
|            | 71  | $7p^2d_2$  | 0.1492                  | -0.0052                 |                                  |
|            | 8   | $7s^5p_3/2$ | 0.0562                  | -0.5593                 |                                  |
|            | 12  | $7s^5p_0/2$ | 0.0625                  | 0.0636                  | -1.2368                          |
|            | 21  | $7s^5d_2$  | 0.0808                  | 0.0802                  | 0.4973                           |
|            | 26  | $7s^5p_1/2$ | 0.0866                  | 0.0864                  | -2.6502                          |
|            | 31  | $7s^5p_3/2$ | 0.1039$^b$              | 0.0951                  | 0.3725                           |
|            | 38  | $7s^5d_0$  | 0.1086                  | -4.8447                 |                                  |
|            | 15  | $7s^5p_2$  | 0.0681                  | 0.0690                  | 1.6453                           |
|            | 23  | $7s^5d_3$  | 0.0818                  | 0.0825                  | -1.5012                          |
|            | 25  | $7s^5d_1$  | 0.0854                  | -0.4511                 |                                  |
|            | 32  | $7s^5d_0$  | 0.0966                  | 0.0965                  | 3.4260                           |
|            | 7s$^5d^2$ | 2→3       | 2$→3$                   | 2$→3$                   |                                  |
|            | 18  | $7s^5p_1/2$ | 0.0238                  | -2.8160                 |                                  |
|            | 30  | $7s^5p_0/2$ | 0.0677                  | 0.0670                  | -0.3091                          |
|            | 42  | $7s^5d_1/2$ | 0.0914$^b$              | 0.0835                  | -0.5364                          |
|            | 51  | $7s^5d_2$  | 0.1066                  | 0.1052                  | -0.2580                          |
|            | 71  | $7p^2d_2$  | 0.1176                  | 0.0854                  |                                  |
|            | 8   | $7s^5d_3$  | 0.1385                  | -1.4866                 |                                  |
|            | 12  | $7s^5p_2$  | 0.0456                  | 1.2645                  |                                  |
|            | 21  | $7s^5d_0$  | 0.0518                  | 0.0529                  | 2.2099                           |
|            | 26  | $7s^5d_1$  | 0.0702                  | 0.0696                  | 0.0814                           |
|            | 31  | $7s^5d_2$  | 0.0760                  | 0.0757                  | 1.7204                           |
|            | 38  | $7s^5d_0$  | 0.0932$^b$              | 0.0845                  | -1.2487                          |
|            | 15  | $7s^5d_0$  | 0.0979                  | 0.0979                  | 2.6283                           |
|            | 23  | $7s^5d_1$  | 0.0574                  | 0.0583                  | 1.0373                           |
|            | 25  | $7s^5d_2$  | 0.0711                  | 0.0718                  | -0.3365                          |
|            | 32  | $7s^5d_0$  | 0.0859                  | 0.0858                  | 2.7861                           |
|            | 7s$^5d^2$ | 4→3       | 4$→3$                   | 4$→3$                   |                                  |
|            | 18  | $7s^5d_1$  | 0.0364                  | 0.0367                  | 3.0645                           |
|            | 30  | $7s^5d_2$  | 0.0601$^b$              | 0.0532                  | -0.5028                          |
|            | 42  | $7s^5p_1/2$ | 0.0752                  | 0.0749                  | -0.3120                          |
|            | 51  | $7s^5p_2$  | 0.0873                  | -0.1871                 |                                  |
|            | 71  | $7p^2d_2$  | 0.1082                  | 0.4365                  |                                  |
|            | 8   | $7s^5d_3$  | 0.0153                  | 0.4046                  |                                  |
|            | 12  | $7s^5p_2$  | 0.0205                  | 0.0226                  | 2.5071                           |
|            | 21  | $7s^5d_0$  | 0.0388                  | 0.0393                  | 1.3461                           |
|            | 26  | $7s^5d_1$  | 0.0446                  | 0.0454                  | -0.8559                          |
|            | 31  | $7s^5d_2$  | 0.0619$^b$              | 0.0542                  | 0.6252                           |
|            | 38  | $7s^5d_0$  | 0.0676                  | -0.3042                 |                                  |
|            | 15  | $7s^5d_1$  | 0.0261                  | 0.0280                  | -1.1636                          |
|            | 23  | $7s^5d_2$  | 0.0398                  | 0.0415                  | -0.5022                          |
|            | 25  | $7s^5d_0$  | 0.0445                  | 0.0969                  |                                  |
|            | 32  | $7s^5d_0$  | 0.0546                  | 0.0555                  | -0.2601                          |

$^a$ 1 au = 219475 cm$^{-1}$ = 27.211 eV, and corresponds to a wavelength of 45.563 nm. The conversion factor for the reduced matrix elements is $e_a B$.

$^b$ Experimental identification uncertain; see Table 1.
TABLE IV. Calculated lifetimes of the low-lying odd states of Ac. Where available, experimental values of the transition frequency were used. The assigned errors include the uncertainty in the calculated frequencies, $E1$ amplitudes, and missing transitions for the higher states.

| $N$  | State     | Energy (cm$^{-1}$) | $\tau$ (10$^{-7}$ s) |
|------|-----------|------------------|---------------------|
| 3    | $7s^2\gamma p$ $2p^{1/2}_p$ | 7565 | 4(2) |
| 8    | $7s^2\gamma p$ $2p^{3/2}_p$ | 12345 | 9(4) |
| 12   | $7s7p6d$ $4f^{3/2}_v$ | 13713 | 13958 | 1.7(7) |
| 15   | $7s7p6d$ $4f^{5/2}_v$ | 14941 | 15141 | 3(1) |
| 18   | $7s7p6d$ $4d^{5/2}_v$ | 17200 | 17049 | 1.5(6) |
| 21   | $7s7p6d$ $4d^{7/2}_v$ | 17736 | 17612 | 8(5) |
| 25   | $7s7p6d$ $4d^{9/2}_v$ | 18747 | 5(3) |

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