Stability analysis and stabilisation of an amplifier with non-linearity compensation

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\textbf{ABSTRACT}

The rapid development of wireless communication system has led to pressing need for highly linear amplifier design and implementation as amplifier is one of the most critical devices in transmit and receive chain. In recent years, a number of new and emerging amplifier linearisation techniques using special distortion correction have been receiving more and more attention due to their significant advantages. One of the techniques is the negative impedance compensation. The design results showed that high gain, good linearity and wider bandwidth can be achieved by using the novel compensation method. However, as some kind of feedback can be introduced when applying the negative impedance compensation, the stability of the whole system should be investigated carefully. In this paper, stability analysis and stabilisation have been performed for the amplifier with negative impedance compensation. The simulation results showed that stable circuit behaviour can be achieved by using the capacitive compensation in the negative impedance circuit.

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\section{1. Introduction}

The applications of a number of advanced modulation techniques have increased the capacity and speed of modern wireless and mobile communication systems significantly. The wide dynamic signals used in these modulation techniques need highly linear transmit and receive system architecture. As amplifier is key component in both transmit and receive systems, its linearity becomes more and more important. The traditional amplifier linearisation techniques (Kenington, 2000), such as predistortion, feedforward and Cartesian feedback, are not applicable to all systems, for example, handset (Kim, Ko, & Lee, 2000). Some new techniques based on special distortion cancellation and compensation such as digital feedback (Woo et al., 2007), special transistor combination and arrangement (Kim et al., 2000; Yum, Leung, Hou, & Xue, 2006; Kang et al., 2006; Nakatsugawa, 2001), signal injection (Cheng & Leung, 2003; Mizusawa & Kusunoki, 2005), compensation using auxiliary amplifier (Liang, Ho, Hsieh, & Chan, 2005; Zhang, 2011; Wu, Lidgey, & Hayatleh, 2007; Wu, Lidgey, Hayatleh, & Hart, 2010; Ali, Wu, Callaghan, & Rapajic, 2014) have been reported.
In implementation of radio frequency (RF) and microwave amplifier linearisation stability can be a great concern because the degree of linearity improvement may be limited by the accuracy and stability of the circuitry with linearisation (Breed, 2010). In above-mentioned new linearisation methods, because auxiliary transistors or amplifiers are used to compensate the non-linear distortion generated by the main amplifier, some kind of feedback could be introduced. Therefore, the stability of the amplifiers with compensation should be considered carefully. However, stability analysis and stabilisation of the amplifiers with linearisation is quite challenging due to high complexity of the circuitry. Several researchers have reported their work on the stability analysis of RF amplifier with traditional envelope feedback and Cartesian feedback (Kheirkhahi, Naghshtabrizi, & Larson, 2011; Dowson & Lee, 2004). In this paper, the stability of the amplifier with the negative impedance compensation reported in Wu et al. (2007, 2010), Ali et al. (2014) is investigated by using effective stability analysis methods such as the Routh–Hurwitz criterion (Surhone, Timpledon, & Marseken, 2010; Jagan, 2008). The certain region to guarantee the stability of the amplifier can be determined by using additional capacitance compensation, which provides improved flexibility for the amplifier design.

2. Negative impedance compensation

2.1. Principle

In order to compensate the non-ideal characteristics of amplifiers, a novel amplifier design method based on the negative impedance compensation has been reported in Wu et al. (2007, 2010), Ali et al., 2014. The principle of the method is described below.

Usually, distortion generated by a non-linear amplifier can be represented either at the input or at the output, which leads to two distortion representation methods: the input-referred method and output-referred method. Consequently, amplifier linearisation techniques can be categorised into the input-referred and output-referred technique. The negative impedance compensation is a type of input-referred method.

Consider the negative feedback amplifier shown in Figure 1. If the amplifier is assumed ideal in all respects, the input impedance $Z_i = \infty$, the output impedance $Z_o = 0$, the open-loop gain $A = \infty$ and the input current $i_i = 0$. Then, the closed-loop gain, $G$, can be written as:

$$ G = \frac{V_o}{V_{in}} = -\frac{R_F}{R_G} $$

or

Figure 1. A classic feedback amplifier configuration.
For a practical amplifier where the non-ideal open-loop gain, $A$, is finite or non-linear and $Z_i \neq \infty$, $Z_o = 0$, $A \neq \infty$, $i \neq 0$, a straightforward circuit analysis for Figure 1 shows that

$$\frac{-V_o}{R_F} = \frac{V_{in}}{R_G} \quad (2)$$

Comparing Equation (3) with (2), it can be seen that there is an additional $V_o$ term in Equation (3). Actually, this term can be regarded as a distortion current, $i_d$, caused by the non-ideal behaviour of the amplifier, given by

$$i_d = -\frac{V_o}{A} \left( \frac{1}{Z_i} + \frac{1}{R_G} + \frac{1}{R_F} \right) \quad (4)$$

As shown in Figure 2, if an anti-distortion current $i_{ad}$ is injected into the inverting terminal, then the input current to the inverting terminal, $i_i$, is given by,

$$i_i = i_G + i_F + i_{ad} \quad (5)$$

Use of Equation (3) gives

$$\frac{-V_o}{R_F} - \frac{V_o}{A} \left( \frac{1}{Z_i} + \frac{1}{R_G} + \frac{1}{R_F} \right) = \frac{V_{in}}{R_G} + i_{ad} \quad (6)$$

Clearly, if $i_{ad}$ is chosen so that

$$i_{ad} = -\frac{V_o}{A} \left( \frac{1}{Z_i} + \frac{1}{R_G} + \frac{1}{R_F} \right) = V_- \left( \frac{1}{Z_i} + \frac{1}{R_G} + \frac{1}{R_F} \right) \quad (7)$$

where if $V_-$ is the voltage at the inverting input terminal, then the distortion current, $i_d$, will be cancelled.

For most amplifiers, $R_F$, $R_G$ are such that $R_F$, $R_G \ll |Z_i|$, in which case Equation (7) simplifies to

$$i_{ad} \approx V_- \left( \frac{1}{R_G} + \frac{1}{R_F} \right) = \frac{V_-}{R_G / / R_F} \quad (8)$$

![Figure 2](image_url) Negative feedback amplifier of Figure 1 with distortion correction at the input.
Equation (8) reveals that injecting a current \( i_{ad} = V_\text{in}/(R_G/R_F) \) into the inverting terminal is equivalent to connecting a negative resistance to the terminal as shown in Figure 3. The negative resistance is given by

\[
R_n = -\left(\frac{R_G}{R_F}\right).
\]  

(9)

2.2. Circuit implementation

Figure 4 shows a general circuit for negative resistance compensation, \( A_1 \) being the main amplifier and \( A_2 \) the complementary amplifier. In this circuit, assuming the input resistance for both amplifiers are high enough to be neglected, the current equation at the inverting input terminal is given by

\[
i_F = i_G + i_N.
\]  

(10)

If the gain of \( A_2 \) is set to 2 and \( R_N = R_F//R_G \), then \( i_N \) equals \( i_{ad} \) of Equation (8) and the distortion is theoretically almost cancelled out.

Negative resistance can be implemented, in practice, using the special amplifier topology shown in Figure 5, where the amplifier \( A_2 \) in Figure 4 is realised by a non-inverting configuration with \( R_1 = R_2 \) to have a gain of 2. Assuming the amplifier has a high open-loop input resistance and high open-loop gain the voltage between the
inverting and non-inverting terminals is negligibly small, the input impedance of the circuit can be obtained as

\[ R_{in} = R_n = \frac{-R_F}{R_G}. \]  

(11)

In practice, the negative resistance is slightly reduced because the input resistance of the amplifier is finite. The closed-loop gain of the negative resistance circuit is only +6 dB, and when used to linearise the amplifier of Figure 1, it is connected to the ‘virtual-ground’ inverting input of the amplifier. The negative resistance circuit only sees very small input signals, and the output signal is also very small; hence, it is possible to assume that any distortion effects that the second amplifier may introduce are minimal. Also, it should be noted that this negative resistance realisation using a combination of positive and negative feedback is stable when connected to the low impedance node of the inverting terminal of the main amplifier. This fact is further validation of the appropriateness of the choice of topology to realise \( R_n \).

3. Stability analysis and stabilisation of amplifier with the negative impedance compensation

3.1. Analysis of stability for amplifier with compensation

In practical applications of the negative impedance compensation method, stability could be an important issue due to the auxiliary amplifier. Therefore, it is crucial to carefully analyse the impact of the additional negative element on the stability of the amplifier.

As mentioned in Section 2, the negative impedance compensation technique can be implemented by connecting the negative resistance circuit shown in Figure 5 to the main amplifier shown in Figure 1. The amplifier \( A_1 \) in the main amplifier and the amplifier \( A_3 \) in the negative resistance circuit may be implemented not only by an operational amplifier but also by a differential amplifier, etc. Therefore, in this paper, the complementary metal-oxide-semiconductor (CMOS) differential amplifier shown in Figure 6 as described in Allen & Holberg (2011) has been used to simulate the amplifiers, where the open-loop gain has been characterised as 40 dB (i.e. \( A_o = 100 \)) and the unity-gain bandwidth as 300 MHz.

![Figure 5. An implementation of negative resistance.](image)
As an example to test the stability of an amplifier with the negative impedance method, the main amplifier in Figure 1 has been designed to have a closed-loop gain of $-4$. The values of the components to realise the gain are chosen as $R_F = 2 \, \text{k}\Omega$, $R_G = 500 \, \Omega$. For the negative resistance circuit shown in Figure 5, the two resistors, $R_1$ and $R_2$, have been considered as $R_1 = R_2 = 10 \, \text{k}\Omega$. Also, according to the negative impedance compensation theory and using Equation (9) in Wu et al. (2007), the value of the compensation component, $R_N$, can be calculated as $R_N = 390 \, \Omega$. The stability can be tested by injecting step signal at the input and observing the output. In this case, a $-0.1$-V step signal with $0.1 \, \mu\text{s}$ delay was injected at the input. As shown in Figure 7, the example amplifier delivers an output with excessive overshoot (up to 0.614 V) and ringing (till 0.75 $\mu\text{s}$).

The simulation result in Figure 7 shows that the amplifier with the proposed negative impedance compensation may have stability problem for some values of its parameters.

![Figure 6. CMOS differential amplifier.](image1)

![Figure 7. Simulation result of the step response for a linearised amplifier.](image2)
In this paper, Routh–Hurwitz stability criterion is used to carry out stability analysis and find the stable region for the parameters of the amplifier with the compensation.

### 3.2. Stability analysis and stabilisation

In order to examine the stability of the amplifier with the negative impedance compensation, the Routh–Hurwitz criterion can be used.

As a very popular and useful stability analysis method, the Routh–Hurwitz criterion allows us to compute the number of roots of the characteristic equation in the right half-plane without actually computing the values of the roots. Thus, stability can be determined without the added computational burden of determining characteristic root locations. This gives us a design method for determining values of certain system parameters that will lead to closed-loop stability (Dorf & Bishop, 2014). At present, many stability analysis software tools based on the Routh–Hurwitz method are being used (Vo, Brandstetter, Dong, Thieu, & Vo, 2016).

For a transfer function with following general form

\[ T(s) = \frac{N(s)}{D(s)}. \] (12)

If the denominator of the transfer function is a fourth-order polynomial,

\[ D(s) = a_0 + a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4. \] (13)

According to the Routh–Hurwitz criterion, all the coefficients must satisfy following conditions if the system is stable

\[ a_3 > 0 \] (14a)

\[ a_3 a_2 > a_4 a_1 \] (14b)

\[ a_3 a_2 a_1 > a_4 a_1^2 + a_2^2 a_0. \] (14c)

Stability analysis using above approach has been performed for the amplifier in Figures 4 and 5. In order to stabilise the amplifier, \( R_F/R_G (R_N) \), \( R_1 \) and \( R_2 \) in Figure 5 are replaced by the conductance–capacitance parallel structures:

\[ Y_A = \frac{1}{R_N} + j\omega C_A = G_A + j\omega C_A, \]

\[ Y_B = \frac{1}{R_1} + j\omega C_B = G_B + j\omega C_B \text{ and } Y_C = \frac{1}{R_2} + j\omega C_C \]

\[ = G_C + j\omega C_C, \]

where \( C_A \), \( C_B \) and \( C_C \) are chosen as adjustable parameters to determine the stability region.

Using the one-pole model of op-amp, the transfer function of the amplifier in Figure 4 can be derived as

\[ \frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{A_m \omega T_1 \omega T_2 G_B + A_m \omega T_1 (\omega T_2 C_B + G_B + G_C) s + A_m \omega T_1 (C_B + C_C) s^2}{a s^4 + b s^3 + c s^2 + d s + e} \]

\[ = -\frac{N'(s)}{D'(s)}, \] (15)
\[ a = R_F C_A C_C + R_F C_A C_B \]
\[ b = A_{m1} C_B + A_{m1} C_C + R_F G_A C_C + R_F G_A C_B - R_F C_A C_C \omega T_2 + R_F C_A G_C + R_F C_A G_B \]
\[ c = \omega T_1 C_B + \omega T_1 C_C + A_{m1} \omega T_2 C_B + A_{m1} G_B + A_{m1} C_B - R_F G_A C_C \omega T_2 + R_F G_A G_C + R_F G_A G_B - R_F C_A G_C \omega T_2 \]
\[ d = \omega T_1 \omega T_2 C_B + \omega T_1 G_B + \omega T_1 C_C + A_{m1} \omega T_2 G_B - R_F G_A G_C \omega T_2 \]
\[ e = \omega T_1 \omega T_2 G_B \]
\[ A_m = R_F / R_G \]
\[ A_{m1} = A_m + 1. \]

As can be seen, \( D'(s) \), the denominator of Equation (15), can be used as a fourth-order characteristic polynomial for stability analysis based on the Routh–Hurwitz criteria. Then, by choosing \( C_A, C_B \) and \( C_C \) as adjustable parameters in the coefficients of \( D'(s) \) and using Equation (14a)–(14c) the stable region of the amplifier with the negative impedance compensation can be determined by a group of compensating capacitors.

To demonstrate the above method, stability analysis and stabilisation have been performed for the example in Section 3.1. For the closed-loop gain of \(-4 A_m \) and \( A_{m1} \) in Equation (15) are equal to 4 and 5, respectively. The unity-gain bandwidth \( \omega T_1 = \omega T_2 = 2\pi f_T = 1.885 \times 10^9 \) rad/s. As described above, the amplifier can be stabilised using parallel capacitance \( C_A, C_B \) and \( C_C \). Using MATLAB, separate stability regions have been generated using the stability criterion in Equation (14a)–(14c) as shown in Figures 8–10. To illustrate the global relationship of \( C_A, C_B \) and \( C_C \) in the stability analysis, a 3-dimensional plot was drawn as shown in Figure 11. The shape of the plot provides clear information about the stable region. From the figures, it can be identified that when \( C_B = C_C = 0 \), the amplifier is unstable, which has proved the result in the example in Section 3.1.

Figure 11 reveals that the stability of the amplifier can be guaranteed by choosing a group of \( C_A, C_B \) and \( C_C \) in the stable region. In the example with the CMOS differential amplifier in Section 3.1, in order to stabilise the amplifier with compensation, the circuit has been modified with the conductance–capacitance parallel structures including the capacitors \( C_A, C_B \) and \( C_C \), where the main and auxiliary amplifiers still remain the same as Figure 4. Figure 12 shows the simulation result with \( C_A = 3.3 \) pF, \( C_B = 3.3 \) pF and \( C_C = 2.8 \) pF. As can be seen from the figure, the stability has been significantly improved compared with the result shown in Figure 7.

The stability analysis using above method has also been performed for the highly linear amplifier designed by the author in Wu et al. (2007). The amplifier was implemented with BFR520 9-GHz wideband transistor (http://www.nxp.com). For example, the SPICE simulation result showed that the boundary of \( C_B \) for a stable region is 9 pF when \( C_A = 8 \) pF and \( C_C = 0 \) pF. Normally, the stable region for a practical circuit could be smaller than that in Figure 11 due to the use of high-level transistor models.

In summary, the stability region of an amplifier with the special non-linearity compensation could be obtained by the following steps:
For a designed amplifier with non-linearity compensation, identify the values of the relevant parameters such as the gain and bandwidth for both the main and the auxiliary amplifier as well as the resistors.

Figure 8. Stable region for the linearised amplifier for $a_3 > 0$.

Figure 9. Stable region for the linearised amplifier for $a_3 a_2 > a_4 a_1$.

(1) For a designed amplifier with non-linearity compensation, identify the values of the relevant parameters such as the gain and bandwidth for both the main and the auxiliary amplifier as well as the resistors.
(2) Based on above parameters, determine a predictive stable region specified by $C_A$, $C_B$ and $C_C$ by using the simulation tool presented in the paper.

(3) Test and verify the predictive region by using a specific circuit simulation software such as SPICE with high-level device models.

4. Conclusion

In this paper, stability analysis and stabilisation for the amplifier with the negative impedance compensation have been performed. As can be seen, the stable region of the amplifier with the compensation can be determined by a group of compensating capacitors. The simulation results show that the stability of the amplifier can be improved significantly. The research is ongoing. In order to improve the accuracy, reliability and practicability of the proposed method, the future work will include:

(1) Add the algorithm for analysing both gain margin and phase margin to the design tool.

(2) Develop more effective function for determining suitable values and ranges of the compensating capacitors to meet required stability criteria. Such a function could be realised by considering different ranges, values and analytical expressions of $R_{n}$, $R_1$ and $R_2$ as well as practical amplifier parameters based on more complicated device models.
(3) Build an interface between the stability analysis tool and the existing circuit simulation software such as SPICE, etc. so that the result of the stabilisation can be evaluated using standard technology.
The design, implementation and testing of the amplifier with the negative impedance compensation is in progress. It is expected that the stability analysis and stabilisation will be carried out in the process to examine the validity of the proposed method practically.

Disclosure statement
No potential conflict of interest was reported by the authors.

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