FOCUS

Decision support model for the selection of asphalt wearing courses in highly trafficked roads

Daniel Jato-Espino · Irune Indacochea-Vega · László Gáspár · Daniel Castro-Fresno

Published online: 20 March 2018
© Springer-Verlag GmbH Germany, part of Springer Nature 2018

Abstract
The suitable choice of the materials forming the wearing course of highly trafficked roads is a delicate task because of their direct interaction with vehicles. Furthermore, modern roads must be planned according to sustainable development goals, which is complex because some of these might be in conflict. Under this premise, this paper develops a multi-criteria decision support model based on the analytic hierarchy process and the technique for order of preference by similarity to ideal solution to facilitate the selection of wearing courses in European countries. Variables were modelled using either fuzzy logic or Monte Carlo methods, depending on their nature. The views of a panel of experts on the problem were collected and processed using the generalized reduced gradient algorithm and a distance-based aggregation approach. The results showed a clear preponderance by stone mastic asphalt over the remaining alternatives in different scenarios evaluated through sensitivity analysis. The research leading to these results was framed in the European FP7 Project “DURABROADS” (No. 605404).

Keywords AHP · Fuzzy logic · Monte Carlo methods · Multi-criteria decision-making · Road management · TOPSIS

1 Introduction

Roads were one of the greatest contributors to the changing environment during the second half of the twentieth century in European countries. These infrastructures have become essential for daily life as they play a crucial role in transporting people and goods and providing access to services. In consequence, roads have an important influence on their surrounding economic activity, while generating social benefits, either direct or indirect, for the parties communicated (Collins and Africa 2017; Dukicin Vuckovic 2017; Joumard and Nicolas 2010). They also produce relevant environmental impacts due to the materials and processes involved in their construction and use. Furthermore, roads must be designed to withstand the vehicle loads of their installation site, especially if they are intended to support high traffic levels. According to the TEN-T road network information system (European Comission 2014), the number of equivalent single-axle loads (ESALs) for highly trafficked European roads would be above 25 million for a period of analysis of 24 years. Among the different layers forming road structures, the wearing course is the most sensitive one to these loads, because of its direct exposure to them.

Under these circumstances, which entail considering several conflicting factors, the need for a decision system for the selection of wearing courses from an integral point of view is fully justified. Multi-criteria decision analysis (MCDA) is a branch of operations research aimed at helping to make better decisions by applying analytical methods to solve complex problems characterized by having multiple criteria. In other words, MCDA supports the resolution of problems consisting of the evaluation of a group of alternatives \(A_i (i = 1, 2, \ldots, m)\) with respect to a set of criteria \(C_j (j = 1, 2, \ldots, n)\), in order to select the best solution among those contemplated.
Some authors have previously analysed several issues related to road management characterized by the presence of multiple conflicting criteria or attributes from different perspectives. Chou (1990) designed a decision-making tool to help engineers to design reliable pavements according to the values of several mechanical parameters. Davis and Campbell (1995) developed a decision support system based on the contribution of several criteria to an objective function for ranking different pavement materials. Cafiso et al. (2002) checked the applicability of the analytic hierarchy process (AHP) for pavement maintenance management. Chang et al. (2005) used the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) to compare different preventive treatments for pavement maintenance according to economic and technical criteria. Filippo et al. (2007) proposed a fuzzy AHP model to prioritize the restoration of paved highways from an environmental point of view. Based on an overview of existing multi-attribute decision support approaches, Zavadskas et al. (2007) selected the COPRAS method to assess different road design alternatives. Some of the same authors carried out a deeper review on the use of decision support tools in bridges and road quality management (Zavadskas et al. 2008). They concluded that multi-attribute analysis might be especially helpful in management and planning tasks, while cost–benefit analysis is mainly used for final project selection. Wu et al. (2008) combined multi-objective optimization and prioritization of criteria using the AHP method to create a decision support model for pavement preservation budgeting. Van Leest et al. (2009) compared various types of road pavements according to factors such as costs, risks, safety or emissions. Brauers et al. (2008) employed the multi-objective optimization on the basis of the ratio analysis (MOORA) to select the best alternative of highway design according to five objectives related to economy, environment and longevity. Sivilevicius led the development of two research papers (Sivilevicius et al. 2008; Sivilevicius 2011) aimed at assessing the quality of asphalt mixing plants (AMP) using multi-attribute models. Bian and Cai (2012) applied the AHP method to rank the performance of asphalt pavement crack repairing materials and select the most appropriate one according to the evaluation result. Lidicker et al. (2013) solved a multi-criteria optimization problem to minimize the life-cycle costs and greenhouse gas emissions of pavement resurfacing. Moretti et al. (2013) measured the global environmental impact of road works from cradle to grave through the weighted sum model (WSM). Kucukvar et al. (2014) studied four alternatives of pavement mixtures according to environmental and socio-economic indicators using an intuitionistic fuzzy decision-making approach based on the TOPSIS method. Jato-Espino et al. (2014) proposed a hybrid model based on the MIVES and AHP methods to assist the selection procedure of urban pervious pavements. Noori et al. (2014) presented a stochastic optimization approach based on multiple criteria for the selection of reflective cracking mitigation techniques.

The above-mentioned studies did not jointly addressed these infrastructures from the triple point of view of sustainability, which is crucial to ensure the selection of cost-effective road materials in harmony with environmental preservation and social welfare. For this reason, this paper aimed at developing a decision support model to facilitate the choice of wearing courses in highly trafficked European roads. To this end, a comprehensive approach based on the combination of the AHP and TOPSIS methods was conceived. Data to characterize the performance of various wearing courses were generated by combining the information obtained from both the literature sources and the opinions provided by a panel of recognized international experts in the topic under study. Other complements such as fuzzy logic, the generalized reduced gradient (GRG) algorithm, Monte Carlo methods and distance-based aggregation were also introduced to deal with some specifics of this decision-making problem. Finally, sensitivity analysis was performed to gain insight into how changing some of the inputs used to build the model affected the final ranking of alternatives.

2 Methodology

The proposed multi-criteria decision-making methodology was outlined as an algorithm consisting of five main steps, as depicted in Fig. 1: (1) definition of the decision-making problem, (2) processing of questionnaires, (3) weighting of criteria, (4) assessment of alternatives and (5) sensitivity

![Fig. 1 Algorithm of the multi-criteria decision-making methodology](image-url)
analysis. The next subsections describe in detail all the operations required to accomplish each of these five steps.

2.1 Definition of the decision-making problem

To ensure the choice of wearing courses meeting the principles of sustainability, they were assessed according to the concept of lifetime engineering. Lifetime engineering is based on using technical performance parameters, so that roads are capable of fulfilling economic, environmental and social requirements throughout their whole life cycle (Sarja 2010). These are conflicting aspects, since the satisfaction of some of them might result in the dissatisfaction of some others. This fact justified the need for a methodology based on multi-criteria decision-making theory to properly analyse all these aspects together.

The economic requirement was characterized through the cradle-to-grave costs involved by wearing courses. Since these variables are subject to continuous market fluctuations, they were defined through ranges of values expressing different degrees of likelihood of achieving a certain cost. The main environmental impacts associated with road pavements were summarized in the consumption of non-renewable resources (fuel and aggregates) and greenhouse gas emissions, whose main contributor is carbon dioxide (CO\(_2\)). As in the economic requirement, these factors were also evaluated throughout the life cycle of the materials involved and according to ranges of estimates. From the point of view of the users of the wearing courses, the social aspects to consider were grouped into two criteria: comfort and safety. The first group referred to indicators concerning driving quality, while safety represented the interaction of the pavement surface with both the wheels of vehicles and drivers’ visibility. Finally, key technical indicators were proposed based on methodologies for new and reconstructed pavements, as well as pavement performance monitoring methods (Litzka et al. 2008). These indicators were related to the mechanical behaviour of the wearing courses in terms of deformation and disintegration.

The breakdown of these four requirements into more specific levels (criteria and indicators) resulted in a hierarchical tree-shaped structure as shown in Table 1. This set of indicators was subjected to discussion among the members of the project in which this study was framed (DURABROADS, Ref. 605404), in order to gather their opinions about those originally proposed and suggest the addition or removal of some of them. There were only two variations in relation to the initial proposal. Firstly, the technical requirement was divided into two criteria, disintegration and deformation resistance, which were further broken down into two (fatigue and thermal cracking) and one (rutting resistance) indicators, respectively. Secondly, the environmental requirement included a fourth criterion, namely recyclability, which was represented through an indicator about the recyclability rate of the asphalt mixtures. In the end, the technical requirement was summarized as shown in Table 1, since the experts suggested that the characterization of specific functional variables might be difficult to approach, while recyclability was finally discarded because the alternatives were found to be very homogenous in these terms, such that the contribution of this indicator to the analysis would have been insignificant.

The alternatives to be assessed with respect to this decision-making tree were established from the specifications found in the European Standard BS EN 13108 “Bituminous mixtures. Material specifications” (BSI 2016) and a survey of members of the DURABROADS project about the most widely used asphalt wearing courses in the European regions to which they belong. As a result, the five different alternatives shown in Table 2 emerged.
Table 2 Set of alternatives for the selection of wearing courses

| Alternative                  |
|-----------------------------|
| A1                          | Asphalt concrete (AC)   |
| A2                          | Very thin asphalt concrete (BBTM) |
| A3                          | Hot rolled asphalt (HRA) |
| A4                          | Porous asphalt (PA)      |
| A5                          | Stone mastic asphalt (SMA) |

Table 3 Linguistic scales of opinion for weighting the criteria and assessing the alternatives

| Weighting of criteria       | Assessment of alternatives |
|-----------------------------|-----------------------------|
| Absolutely less important   | Extremely poor              |
| Much less important         | Very poor                  |
| Less important              | Poor                        |
| Slightly less important     | Medium poor                |
| Equally important           | Fair                        |
| Slightly more important     | Medium good                |
| More important              | Good                        |
| Much more important         | Very good                  |
| Absolutely more important   | Extremely good             |

2.2 Processing of questionnaires

Since part of the methodology relied on the opinions of a panel of experts in road management, well-prepared questionnaires were needed for both outlining the decision-making problem and capturing the expertise of the respondents. They were conceived to be concise, understandable and easy to fill in. Under these premises, two types of questionnaires were created to gather the information required to carry out the steps of weighting of criteria and assessment of alternatives.

They both were developed in MS Excel spreadsheets (Microsoft Corporation 2013), in order to use a familiar format for all the parties involved. A short introduction describing the aim of the questionnaires and the way they should be filled in was provided to put the addressees into context. The procedure was very simple, since the experts only had to answer questions such as: How important is criterion $j_1$ with respect to criterion $j_2$ and how is the behaviour of alternative $i$ with respect to criterion $j$?, according to the two scales of options listed in Table 3.

Several partners of the DURABROADS project and other representatives from both private and public sectors with extensive knowledge of the road industry formed the panel of experts who provided their opinions concerning the weights of criteria and the rating of alternatives, which resulted in 52 institutions represented by 81 different experts. After discarding those questionnaires sent back without being completely filled in, the valid outputs were reduced to 74 and 25 valid judgments for weighting the criteria and assessing the alternatives summarized in Tables 1 and 2, respectively.

2.3 Weighting of criteria

This phase sought to process the valid questionnaires according to the importance given to the elements shown in Table 1, in order to obtain their relative weights. To this end, the pairwise comparisons provided by the experts as given in Table 3 were related to the preference scale of the analytic hierarchy process (AHP).

2.3.1 Analytic hierarchy process (AHP)

The analytic hierarchy process, originally created by Saaty (1980), is one of the most widely used methods to establish the weights of a set of criteria defining a decision-making problem. Saaty (1980) proposed the numeric scale shown in Table 4 to quantify the linguistic terms used to establish the pairwise comparisons between two elements.

Table 4 Saaty’s comparison scale

| Linguistic term ($j_1$ with respect to $j_2$) | Numerical value |
|---------------------------------------------|-----------------|
| Absolutely less important                   | 1/9             |
| Much less important                         | 1/7             |
| Less important                              | 1/5             |
| Slightly less important                     | 1/3             |
| Equally important                           | 1               |
| Slightly more important                     | 3               |
| More important                              | 5               |
| Much more important                         | 7               |
| Absolutely more important                   | 9               |

The arrangement of the values used to compare a set of criteria yields an $n \times n$ reciprocal matrix $[M]$ consisting of elements that verify the expression $A_{j_1,j_2} \times A_{j_2,j_1} = 1$. The consistency of these comparisons is measured through the maximum eigenvalue of $[M]$ ($\lambda_{\text{max}}$). Hence, $[M]$ is completely consistent when $\lambda_{\text{max}} = n$, while it becomes increasingly inconsistent as the eigenvalue grows, according to Eq. (1):

$$C.R. = \frac{C.I.}{R.I.} < 0.1,$$

where $C.R.$ is the consistency ratio, $C.I.$ is the consistency index, and $R.I.$ is the random consistency index. A matrix is consistent when the ratio between $C.I.$ and $R.I.$ is less than 0.1, such that $C.I.$ is expressed as formulated in Eq. (2):
\[ C.I. = \frac{\lambda_{\text{max}} - n}{n - 1}. \]  

(2)

\( R.I. \) represents an average \( C.I. \) for a large number of randomly generated matrices of the same order. Table 5 shows the average value of \( R.I. \) for a sample size of 500 matrices.

The measurement of the consistency of pairwise comparison matrices is a widely discussed topic in the literature, which provides multiple evidence of the theoretical drawbacks associated with its original characterization based on Eqs. (1) and (2) and Table 5 (Bozóki and Rapcsák 2008; Dijkstra 2013; Grzybowski 2016; Peláez and Lamata 2003). Hence, forcing the comparison matrix to be consistent has been argued to be artificial and create certain dependencies that might lead to loose information and yield poor priori-
ties (Bana e Costa and Vansnick 2008; Grzybowski 2012; Koczkodaj 1993). However, using reciprocal matrices might result in less pairwise comparisons, improving the response rate for the questionnaire and increasing the accuracy of the responses provided by the experts addressed (Miller 1956). To deal with this duality, the consistency of valid questionnaires was checked by applying Eq. (1). Those questionnaires showing inconsistencies were not discarded, but were made consistent by adjusting them through nonlinear optimization.

2.3.2 Generalized reduced gradient (GRG) algorithm for nonlinear optimization

The GRG algorithm, proposed by Abadie and Carpentier (1969) as an extension of the reduced gradient method (Wolfe 1963), was developed to solve nonlinear programming problems of the form of Eq. (3):

\[
\begin{align*}
\text{Minimize } & \quad f(X), \quad X \in \mathbb{R}^n \\
\text{subject to: } & \quad g_i(X) = 0, \quad 1 \leq i \leq m \\
& \quad X_{\text{min}} \leq X \leq X_{\text{max}},
\end{align*}
\]

(3)

where \( X \) is a vector of \( n \) variables, \( f(X) \) is the objective function, and \( g_i(X) \) are nonlinear constraints. Kao (1998) highlighted the GRG algorithm as one of the best deterministic methods for the solution of nonlinear programming problems. Although the improvement of consistency in pairwise comparisons using optimization methods has been previously addressed in the literature (Koczkodaj and Szarek 2010), existing approaches are either linear or too complex in terms of computer modelling to be very widespread yet (Benítez et al. 2012; Bozóki et al. 2011). These factors are against the nonlinear nature of the problem under study and hinder the automation of the entire methodology, respectively.

The working principle of the GRG algorithm consists of transforming nonlinear problems into several linearized sub-problems by approximating its constraints and then solving each sub-problem with linear restrictions using the reduced gradient method (Yeniay 2005). This conversion is carried out by representing some of the variables contained in \( X \), called basics, through a subset of independent variables called non-basics (de Carvalho et al. 2008). Further details on the GRG method can be consulted in Lasdon et al. (1978).

The approach taken in this study was simpler, since only the objective function was nonlinear. Let \([M]\) be the inconsist-
ent comparison matrix provided by an expert with respect to a set of criteria \( C_j = \langle C_1, C_2, \ldots, C_n \rangle \) (see Eq. (4)).

\[
\begin{bmatrix}
C_1 & C_2 & \cdots & C_n \\
\end{bmatrix} =
\begin{bmatrix}
C_1 & 1 & x_{12} & \cdots & x_{1n} \\
1 & x_{21} & 1 & \cdots & x_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
C_n & 1/x_{n1} & 1/x_{n2} & \cdots & 1
\end{bmatrix}
\]

(4)

In addition, let \( [M]^{\prime} \) be the consistent matrix being sought. The aim was to minimize the differences between the elements forming the upper right triangles of both matrices, while fulfilling Eq. (1) and remaining within their lower and upper bounds (see Table 4). In other words, the goal was to estimate the real views that some experts were not able to provide due to the rigidity of the discrete comparison scale proposed by Saaty. To this end, the differences between both matrices were measured through the root-mean-square error (RMSE), which is a metric regularly employed to model errors in statistical analyses (Chai and Draxler 2014). Therefore, the problem was stated as expressed in Eq. (5):

\[
\text{Minimize } \sqrt{\frac{1}{n} \sum_{j=1}^{n} \left( \ln x_{j1} - \ln x_{j1}^{\prime} \right)^2}
\]

subject to: \[ C.R. \leq 0.1 \]

\[
\ln x_{j1}^{L.B.} < \ln x_{j1}^{\prime} < \ln x_{j1}^{U.B.}
\]

(5)

Since the scale shown in Table 4 is based on reciprocal val-
ues, the numerical judgments provided by the experts were transformed into a logarithmic scale before applying Eq. (5), in order to equalize the differences between lower and higher levels of importance. The resolution of this problem obliged the comparison matrix to be consistent (first constraint), while respecting the responses provided by the experts as much as possible (second constraint). The second restriction was a reflection of the difficulties often associated with the choice between terms such as “more important” or “slightly

Table 5 Random consistency index

| Matrix size (n) | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    |
|----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| R.I.       | 0.00  | 0.58  | 0.90  | 1.12  | 1.24  | 1.32  | 1.41  | 1.45  | 1.49  |
more important" when responding to this kind of questionnaires. Moreover, the combination of both restrictions acted as a quality measure, enabling the discarding of those questionnaires proving to be too inconsistent.

2.3.3 Distance-based aggregation

The next step consisted of aggregating all the questionnaires returned by the experts into a single one reflecting the consensual view of the entire panel. As a result of the previous step, some elements forming the comparison matrix were no longer discrete and became continuous, which means that there might be intermediate degrees of importance in addition to those shown in Table 4. For this reason, the Euclidean distance (see Eq. 6), which is the most common metric when measuring similarities between clusters (Xing et al. 2003), was proposed for assessing the affinity between the points of view of the experts:

\[ s_{ek} = \sqrt{\sum_{j=1}^{n} (x_{j1,j2,e_k} - x_{j1,j2,e_l})^2}, \]

where \( s_{ek} \) is the distance between the thoughts of experts \( e_k \) and \( e_l \), while \( x_{j1,j2,e_k} \) and \( x_{j1,j2,e_l} \) are the numerical expressions of their judgments regarding the relative importance of criterion \( j_1 \) with respect to \( j_2 \).

The calculation of the Euclidean distance for each expert with respect to the remaining experts resulted in a symmetric \( p \times p \) matrix \([P]\) (see Eq. 7), such that \( p \) is the number of experts. \([P]\) reflected the proximity between the points of view of each pair of experts.

\[
[P] = \begin{bmatrix}
e_1 & e_2 & \ldots & e_p \\
e_1 & 0 & s_{e_1e_2} & \ldots & s_{e_1e_p} \\
e_2 & s_{e_2e_1} & 0 & \ldots & s_{e_2e_p} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
e_p & s_{e_pe_1} & s_{e_pe_2} & \ldots & 0
\end{bmatrix}
\]

The next task was to give a weight to each expert according to the similarity of thought they showed with respect to the remaining experts. Thus, the opinions of those experts having shorter distances were more important when determining the final weights of criteria and vice versa. This was accomplished by calculating the weighted inverse of the sum of the distances from each expert to the remaining experts, as represented in Eq. (8).

\[
w_{ek} = \frac{1}{\sum_{k=1}^{p} s_{ek}} \left( \frac{1}{\sum_{k=1}^{p} s_{ek}} \right).
\]

In accordance with the studies carried out by Aczél and Saaty (1983) and Aczél and Alsina (1987), the weighted geometric mean (the weighted mean of \( g \) numbers expressed as the \( g \)th root of their product), not the often used weighted arithmetic mean, was used to aggregate the individual opinions of the experts into a single consensual judgment \((x_{j1,j2,c})\) through Eq. (9):

\[
x_{j1,j2,c} = \left( \prod_{k=1}^{p} x_{j1,j2,e_k} w_{ek} \right)^{1/\sum_{k=1}^{p} w_{ek}}.
\]

These consensual judgments were then arranged in a consensual comparison matrix \([M_c]\) as expressed in Eq. (10):

\[
[M_c] = \begin{bmatrix}
C_1 & C_2 & \ldots & C_n \\
C_1 & 1 & x_{12,c} & \ldots & x_{1n,c} \\
x_{21,c} & 1 & \ldots & \ldots & \ldots \\
x_{n1,c} & x_{n2,c} & \ldots & \ldots & 1
\end{bmatrix}
\]

The final calculation of the weights of criteria was preceded by the normalization of the elements of \([M_c]\) according to Eq. (11):

\[
x_{j1,j2,cn} = \frac{x_{j1,j2,c}}{\sqrt{\sum_{j=1}^{n} x_{j2,c}^2}}.
\]

Finally, the values contained in the normalized consensual comparison matrix enabled the determination of the weights of criteria \( C_j = \langle C_1, C_2, \ldots, C_n \rangle \) using Eq. (12):

\[
w_j = \frac{1}{\sum_{j=1}^{n} \sqrt{\sum_{j=1}^{n} x_{j1,j2,c}}}
\]

2.4 Assessment of alternatives

The aim of this phase was to rank the alternatives from the processing of their ratings with respect to the criteria. In this respect, Table 1 highlights by containing two different types of criteria: qualitative and quantitative. Qualitative variables were processed using fuzzy logic by combining the knowledge acquired from the literature and the opinions provided by the group of experts, both expressed in linguistic terminology. Instead, quantitative variables were modelled through Monte Carlo simulations according to ranges of likely numerical values according to specialized literature.

Once the ratings of the alternatives were expressed and processed in one of the two ways mentioned above, they were used as inputs to establish their ranking using the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS). TOPSIS is a compensatory aggregation method, which means that a decrease in a certain criterion might be
compensated by an increase in another. Although the compensation of some of the elements included in Table 1 might seem undesirable and there are operators to prevent these situations (Jato-Espin et al. 2016), the extra parameters and formulations required to implement them led to not considering additional approaches to deal with this matter.

2.4.1 Literature review

A scientific review was carried out to assess the performance of the wearing courses under consideration with respect to the indicators defined in Table 1. The studies conducted by Nicholls et al. (2012) and Nikolaides (2008, 2014) were taken as the main references to rate wearing courses from a functional point of view, since they enabled the comparative analysis of all the alternatives considered in Table 2 in terms of their noise, ride and water-related performance, as well as their disintegration, deformation and skid resistance. Unlike these indicators, which were directly rated from the values found in the bibliography and the opinions provided by the experts, the life cycle cost and the environmental indicators were calculated for a period of analysis of 24 years (EAPA 2007; Kim 2014; OECD 2005) using the concept of equivalent uniform annual cost (EUAC) and the values found in both the Inventory of Carbon and Energy (ICE)(Hammond and Jones 2008) and the research conducted by Chehovits and Galehouse (2010), respectively. The EUAC of each alternative, which stands for their average annual cost and takes into consideration the loss of value of money throughout time, was calculated for a discount rate of 4% according to Eq. (13):

$$\text{EUAC} = \frac{\text{PWC} \cdot \text{DR}}{(1 - \frac{1}{(1+\text{DR})^Y})}, \quad (13)$$

where PWC is the present worth of costs, DR the discount rate, and Y the years of analysis.

2.4.2 Characterization

Fuzzy logic to model linguistic ratings

Qualitative variables were those too complex or of such nature that their quantification was not possible. The ratings of this kind of variables were defined according to linguistic terms, which are very useful when characterizing vague situations. Zadeh (1965) developed the concept of fuzzy logic to account for the imprecision and ambiguity (i.e. the fuzziness) inherent to language statements.

One of the most significant and intuitive ways to handle fuzziness is the use of fuzzy numbers, whose definition includes the concept of membership degree. Zadeh (1965) proposed that the range of membership values of an element of a set may vary within the interval [0, 1], instead of having to be limited to one of the pair of values {0, 1}. Thereby, given a fuzzy set $F$, a fuzzy number can be characterized by a membership function $\mu_{T_1}(f)$ that represents the grade of membership of $f$ in $F$ (Lin 2010). For the sake of simplicity, triangular fuzzy numbers (TFNs) were chosen to model qualitative variables. The membership function of a triangular fuzzy number $T_1 = (\alpha, \beta, \gamma)$ can be represented as shown in Eq. (14):

$$\mu_{T_1}(f; \alpha, \beta, \gamma) = \begin{cases} \frac{f-\alpha}{\beta-\alpha}, & \alpha \leq z \leq \beta \\ \frac{\gamma-f}{\gamma-\beta}, & \beta \leq z \leq \gamma \\ 0, & \text{otherwise} \end{cases} \quad (14)$$

where $\alpha$, $\beta$ and $\gamma$ are the lower, middle and upper values of the triangular fuzzy number $T_1$. Table 6 shows the scale of the triangular fuzzy numbers used in this study to represent linguistic terms.

Again, the ratings provided by the panel of experts regarding the performance of these qualitative variables were synthesized into a single one, but taking into account that in this case there were ratings proceeding from the literature as well.

Let $r_{ij}$ be the rating of a certain alternative $A_i$ with respect to a criterion $C_j$. The distance between the points of view of two experts $r_{e_k}$ and $r_{e_l}$ who have expressed their linguistic ratings $r_{ij}$ through two triangular fuzzy numbers $T_1 = (\alpha_{T_1}, \beta_{T_1}, \gamma_{T_1})$ and $T_2 = (\alpha_{T_2}, \beta_{T_2}, \gamma_{T_2})$ was approximated using the vertex method (Jahanshahloo et al. 2006):

$$s_{e_ke_l} = \sqrt{\frac{1}{3} \left[ (\alpha_{T_1} - \alpha_{T_2})^2 + (\beta_{T_1} - \beta_{T_2})^2 + (\gamma_{T_1} - \gamma_{T_2})^2 \right]}, \quad (15)$$

where $s_{e_ke_l}$ is the distance between the thoughts of experts $e_k$ and $e_l$ with respect to a variable defined using the TFNs $T_1$ and $T_2$. 

| Table 6 | Linguistic terms for rating qualitative variables |
|-----------------|-----------------|
| Extremely poor | (1, 1, 2)        |
| Very poor       | (1, 2, 3)        |
| Poor            | (2, 3, 4)        |
| Medium poor     | (3, 4, 5)        |
| Fair            | (4, 5, 6)        |
| Medium good     | (5, 6, 7)        |
| Good            | (6, 7, 8)        |
| Very good       | (7, 8, 9)        |
| Extremely good  | (8, 9, 9)        |
The weight of each expert and the consensual rating for the whole panel of experts were calculated according to Eqs. (8) and (9), respectively. The rating acquired from the literature was incorporated into the process through the geometric mean as formulated in Eq. (16):

\[ r_{\tilde{ij}} = \sqrt{r_{ij}^E \times r_{ij}^L}, \]  

where \( r_{\tilde{ij}} \) is the final rating of alternative \( A_i \) with respect to criterion \( C_j \), \( r_{ij}^E \) is the consensual rating provided by the panel of experts, and \( r_{ij}^L \) is the rating taken from specialized literature.

In order to produce a simple and manageable value, those variables described through triangular fuzzy numbers were expressed by their canonical representation based on the graded mean integration method (Chou 2003). Given a triangular fuzzy number \( \tilde{T} = (\alpha, \beta, \gamma) \), its graded mean integration representation was defined as in Eq. (17):

\[ P(\tilde{T}) = \frac{1}{6} (\alpha + 4 \times \beta + \gamma). \]  

Thus, Eq. (17) enabled the conversion from the triangular fuzzy numbers obtained in Eq. (16) to crisp numbers, which is very useful in simplifying the TOPSIS method.

### 2.4.3 Technique for order of preference by similarity to ideal solution (TOPSIS)

The TOPSIS method, originally developed by Hwang and Yoon (1981), is based on the principle that the preferred alternative to a given multi-criteria problem is characterized by having not only the shortest distance to the positive ideal solution \( (A^+) \), but also the longest distance to the negative ideal solution \( (A^-) \). Handling the duality of these two concepts is not a trivial matter, since the nearest alternative to the positive ideal solution is not necessarily the same as the farthest from the negative ideal solution. The TOPSIS method, which arose to deal with this dilemma, is structured in a series of steps as follows:

1. **Define the decision-making matrix** The decision-making matrix shows the ratings \( r_{ij} \) of the set of alternatives \( A_i (i = 1, 2, \ldots, m) \), either qualitative or quantitative, with respect to the criteria \( C_j (j = 1, 2, \ldots, n) \).

\[
\begin{array}{c|cccc}
C_1 & C_2 & \ldots & C_n \\
\hline
A_1 & r_{11} & r_{12} & \ldots & r_{1n} \\
A_2 & r_{21} & r_{22} & \ldots & r_{2n} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
A_m & r_{m1} & r_{m2} & \ldots & r_{mn} \\
\end{array}
\]  

(18)

2. **Normalize the decision-making matrix** Normalized ratings \( u_{ij} \) are calculated as:

\[ u_{ij} = \frac{r_{ij}}{\sqrt{\sum_{i=1}^{m} r_{ij}^2}}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n \]  

(19)

3. **Construct the normalized weighted decision-making matrix** Normalized weighted ratings \( V_{ij} \) are determined as:

\[ v_{ij} = w_j \times u_{ij}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n, \]  

(20)

where \( w_j \) is the weight of the criterion, such that \( \sum_{j=1}^{n} w_j = 1 \).

4. **Determine the positive ideal solution \( (A^+) \) and negative ideal solution \( (A^-) \)**

This technique was combined with the remaining techniques and models included in the proposed methodology. Hence, the generation of triangularly distributed random numbers yielded a vector containing \( t \) different ratings \( r_{ij} \), such that \( r \) is the number of simulations carried out with triangularly distributed random numbers, instead of a single number.
A^+ = \left\{ \left( \max_{i} v_{ij} \forall j \in J \right) \left| \left( \min_{i} v_{ij} \forall j \in J' \right) \right. \right\} \tag{21}

A^- = \left\{ \left( \min_{i} v_{ij} \forall j \in J \right) \left| \left( \max_{i} v_{ij} \forall j \in J' \right) \right. \right\} \tag{22}

where \( J \) is associated with benefit criteria and \( J' \) is associated with cost criteria.

(5) Calculate the distance of each alternative from \( A^+ \) and \( A^- \) Separation measures are determined using the \( n \)-dimensional Euclidean distance:

\[ d^+_i = \sqrt{\sum_{j=1}^{n} (v_{ij} - v^+_j)^2}, \quad i = 1, 2, \ldots, m \tag{23} \]

\[ d^-_i = \sqrt{\sum_{j=1}^{n} (v_{ij} - v^-_j)^2}, \quad i = 1, 2, \ldots, m, \tag{24} \]

where \( v^+_j \) and \( v^-_j \) are the positive and negative ideal normalized weighted values for the criterion \( j \), respectively.

(6) Calculate the relative closeness from each alternative to the ideal solution The relative closeness of the alternative \( A_i \) with respect to the ideal solution is defined as:

\[ RC_i = \frac{d^-_i}{d^+_i + d^-_i}, \quad i = 1, 2, \ldots, m. \tag{25} \]

Since both \( d^+_i \) and \( d^-_i \) are zero or greater than zero, then \( 0 \leq RC_i \leq 1 \).

2.5 Sensitivity analysis

In the context of the decision-making problem addressed in this study, sensitivity analysis consisted of determining how and how much specific changes in the weights of criteria and ratings of alternatives modified the relative closeness and how much specific changes in the weights of criteria this study, sensitivity analysis consisted of determining how.

In the context of the decision-making problem addressed in this study, sensitivity analysis consisted of determining how and how much specific changes in the weights of criteria and ratings of alternatives modified the relative closeness and how much specific changes in the weights of criteria this study, sensitivity analysis consisted of determining how and how much specific changes in the weights of criteria and ratings of alternatives modified the relative closeness.

Network also studied the impact of climate change on roads (Tabaković et al. 2014). They reached similar conclusions to the Joint Research Centre (Nemry and Demirel 2012), which identified several impacts of different nature and severity depending on the region:

- Frequent freeze–thaw cycles in Northern countries.
- General warming in summer and more days with extreme maximum temperatures in Southern, Western and Central Europe.
- Increase in the intensity of daily rainfall and the probability of extreme precipitation throughout Europe, especially in some regions located in Northern Europe.

Table 7 summarizes the expected effects of climate change on asphalt wearing courses after reviewing these data sources. In addition to future climate change impacts, another scenario (1a) was added to reflect the lower durability of asphalt surfacing in Northern countries (OECD 2005).

3 Results and discussion

This section presents and discusses the results obtained in the three calculation phases of the methodology: weighting of criteria, assessment of alternatives and sensitivity analysis. The first was developed in MS Excel for convenience, since it was the format in which questionnaires were received, while the two others were computed in MATLAB R2014b (The MathWorks 2014), because of the need to loop through 3D matrices.

3.1 Weighting of criteria

The application of the proposed methodology for processing and minimizing the inconsistencies of the questionnaires returned by the experts (see Eqs. 5, 6, 7, 8 and 9) yielded the consensus numerical values shown in Table 8 for the pairwise comparisons among the elements shown in Table 1. The consensus comparison matrices were consistent in all cases (\( C.R. \leq 0.1 \)), which is logical considering that each individual comparison matrix was made consistent using the GRG algorithm, whenever appropriate.

To illustrate how the pairwise comparisons provided by the experts were transformed after applying the distance-based aggregation approach, Fig. 2 depicts the ranges of values found in the questionnaires for the most challenging level of comparisons (the four elements represented by the requirements), including the position of the consensus values achieved in Table 8. The average \( C.R. \) reached with respect to this level was 0.118, with 50.6% of the original comparisons being inconsistent by an average deviation of 0.099 from the threshold sought (\( C.R. = 0.1 \)). However,
Table 7  Sensitivity analysis scenarios and likely impact on asphalt wearing courses

| Region | Scenario | Description | Impacts on wearing courses |
|--------|----------|-------------|----------------------------|
| North  | 1a       | Lower durability of materials | ↓ Durability in LCC and LCA, ↑ Technique |
| North  | 1b       | Climate change effects | ↑ Disintegration resistance, ↑↑ Safety |
| South  | 2a       | Short-term climate change | ↑↑ Deformation resistance, ↑ Disintegration resistance, ↓ Safety |
| South  | 2b       | Long-term climate change | ↑↑ Deformation resistance, ↑ Disintegration resistance, ↓ Safety |
| West   | 3a       | Short-term climate change | ↑ Technique, ↑ Safety |
| West   | 3b       | Long-term climate change | ↑ Technique, ↑ Safety |
| Centre | 4a       | Short-term climate change | ↑ Deformation resistance, ↑↑ Disintegration resistance, ↑ Safety |
| Centre | 4b       | Long-term climate change | ↑ Deformation resistance, ↑↑ Disintegration resistance, ↑ Safety |

Table 8  Pairwise comparison values for the selection of wearing courses

| Level      | Comparison | Numerical value | C.R.  |
|------------|------------|----------------|-------|
| Requirements | $R_1$ versus $R_2$ | 0.709 | 0.002 |
| Requirements | $R_1$ versus $R_3$ | 0.876 | |
| Requirements | $R_1$ versus $R_4$ | 0.484 | |
| Requirements | $R_2$ versus $R_3$ | 1.249 | |
| Requirements | $R_2$ versus $R_4$ | 0.603 | |
| Requirements | $R_3$ versus $R_4$ | 0.619 | |
| Criteria   | $C_{2.1}$ versus $C_{2.2}$ | 1.643 | 0.000 |
| Criteria   | $C_{2.1}$ versus $C_{2.3}$ | 1.530 | |
| Criteria   | $C_{2.2}$ versus $C_{2.3}$ | 0.902 | |
| Criteria   | $C_{3.1}$ versus $C_{3.2}$ | 0.221 | 0.000 |
| Indicators | $I_{1.1.1}$ versus $I_{1.1.2}$ | 0.477 | 0.000 |
| Indicators | $I_{2.1.1}$ versus $I_{2.1.2}$ | 0.450 | 0.000 |
| Indicators | $I_{3.1.1}$ versus $I_{3.1.2}$ | 1.812 | 0.000 |
| Indicators | $I_{3.2.1}$ versus $I_{3.2.2}$ | 2.458 | 0.000 |
| Indicators | $I_{4.1.1}$ versus $I_{4.1.2}$ | 1.000 | 0.000 |

since none of these comparisons was inconsistent enough to prevent the GRG algorithm to find a solution, they all were taken into account in the calculation of the consensual values. Their position in Fig. 2 reaffirmed the convenience of adopting this course of action, proving not be affected by the existence of outliers, which were considered only marginally due to their distance to the majority of comparisons collected. This fact was especially noticeable in the comparison between $R_3$ and $R_4$, where the consensual value was remarkably separated from the median of the range of values provided by the experts.
As an example of using the GRG algorithm, Eq. (26) represents the inconsistent comparison matrix \((C \cdot R. = 0.275)\) returned by one expert regarding the importance of the four requirements:

\[
\begin{array}{ccccc}
R_1 & R_2 & R_3 & R_4 \\
1 & 7 & 5 & 1/3 \\
1/7 & 1 & 5 & 1/5 \\
1/5 & 1/5 & 1 & 1/5 \\
3 & 5 & 5 & 1 \\
\end{array}
\]  
\[(26)\]

After applying Eq. (5), the matrix was made consistent \((C \cdot R. = 0.1)\) while respecting as much as possible the original opinions provided by the expert (see Eq. (27)):

\[
\begin{array}{ccccc}
R_1 & R_2 & R_3 & R_4 \\
1 & 5.151 & 5.446 & 0.416 \\
0.194 & 1 & 3.611 & 0.205 \\
0.184 & 0.277 & 1 & 0.157 \\
2.404 & 4.878 & 6.369 & 1 \\
\end{array}
\]  
\[(27)\]

The use of Eqs. (11) and (12) from the values shown in Table 8 enabled the calculation of the weights of each element of the hierarchical decision-making tree, as shown in Table 9. The preponderance of the technical requirement over the others was noteworthy, which can be explained by considering that a road with an adequate mechanical behaviour is likely to present good economic and social performances, too. The importance of the second requirement clearly confirmed the increasing ecological awareness that exists in the field of road engineering. Moreover, users’ safety was the most relevant social factor when planning the construction of asphalt wearing courses, which is in line with the concerns of the Commission (2006) in terms of road management.

### 3.2 Assessment of alternatives

Table 10 shows the ratings of each of the alternatives assessed with respect to the set of indicators. Quantitative indicators were defined according to the range of values they might adopt (minimum, most likely and maximum), while qualitative indicators were expressed by their canonical representation, once Eq. (17) was applied.

According to Tervonen and Lahdelma (2007), a number of Monte Carlo simulations of 10,000 was set to generate the triangularly distributed vectors for the quantitative indicators, since this number of iterations was suggested to produce highly accurate results in many real-life applications. The set of ratings \(r_{ij}\) thus obtained was used to build the decision-making matrices required to feed the TOPSIS method. Figure 3a shows the relative closeness (RC) of each of the alternatives to the ideal solution after following the steps of the TOPSIS algorithm.

The overall performance of the alternatives was represented through their cumulative probability functions, in order to capture the variability that characterizes both the economic and environmental indicators. Hence, the final
decision depends on the attitude of the road designer towards uncertainty, because some alternatives might outperform others according to the market fluctuations and the environmental conditions of each case. However, it is clear that the most likely ranking is SMA > HRA > BBTM > AC > PA.

The combined interpretation of Table 9 and Fig. 3b explains the reasons why the aforementioned ranking was achieved. The excellent behaviour of SMA in terms of technique, which was the most important requirement according to Table 9, was the principal cause of the first position of this alternative. The results also showed the importance of having a balanced behaviour with respect to conflicting criteria. In this sense, HRA achieved a notable overall performance by virtue of its at least decent ratings across the four requirements considered. In contrast, PA was severely affected by its poor disintegration resistance and negative environmental impact, in spite of being the best option from the social point of view and having a great deformation resistance. Similarly, the overall performance of BBTM, which was the cheapest and greenest wearing course, was strongly influenced by its low disintegration and fair deformation resistances.

### 3.3 Sensitivity analysis

The results of the sensitivity analysis for the selection of wearing courses (see Fig. 4) reaffirmed the supremacy of SMA, which attained the highest $R_i$ for each of the scenarios proposed. Only the long-term consideration of climate change in South European countries decreased its superiority, since the increasing significance of CO$_2$ emissions enabled BBTM and HRA to slightly reduce the difference. The main variations caused by the sensitivity analysis were related to the PA wearing course outranking AC and/or BBTM in several scenarios (1b, 3a, 3b and 4a) in which safety became even more relevant. In fact, only its weak disintegration resistance prevented PA from outperforming HRA, too. In contrast, the poor behaviour of AC and BBTM in terms of skid resistance and disintegration resistance, respectively, made them less suitable in some scenarios for Western, Central and Northern European countries.

### 4 Conclusions

This study proposed and applied a new decision support model for the selection of asphalt wearing courses based on the combination of the AHP and TOPSIS methods, including several additional complements such as fuzzy logic, Monte Carlo methods, GRG algorithm and distance-based aggregation. The synergetic performance of these components enabled building a comprehensive and robust methodology capable of dealing with aspects such as vagueness, uncertainty, inconsistency and engagement of experts’ views, which are very common in complex decision-making environments.

The results showed the usefulness of the model and the clarity of vision it can provide when selecting the most suitable wearing course according to sustainable development criteria. Although the proper management of roads can have great positive impacts on economy, environment and society, there are few methodologies intended to assist this kind of selection processes, which further increases the importance and interest of the proposed model. Furthermore, the structuring of the decision-making problem in a hierarchical tree enables partial conclusions to be obtained about the performance of the alternatives with respect to a certain aspect or factor influencing them.

The automation capacity of the model was demonstrated through the sensitivity analysis performed to represent different European regions. The architecture and algorithms forming the methodology were programmed to avoid altering the system operation when varying the inputs, which is a crucial issue to enable the use of this model by non-experts in the underlying analytical theory and methods. In addition, its flexibility allows the introduction of the set of weights and ratings known or calculated by each user, depending on
Fig. 4  Overall performance of wearing courses for the sensitivity analysis scenarios a 1a, b 1b, c 2a, d 2b, e 3a, f 3b, g 4a, h 4b
the data sources available. Further research in this line should consider the design of a Web-based interface capable of linking all the operations required to solve the addressed problem in an interactive and visual way, enabling the choice of all or some of the methods and techniques included in the proposed model, in order to promote its use among practitioners and decision-makers.

Acknowledgements The research leading to these results has received funding from the European Union Seventh Framework Programme (FP7/2007–2013) under Grant Agreement No. 605404. This paper reflects only the authors’ views, and the Community is not liable for any use that may be made of the information contained therein. The authors wish to thank the participants of the DURABROADS Work Package 2 (ACCIONA Infraestructuras S.A., European Union Road Federation and Inzenerirbuve SIA) for their inestimable contribution to the research.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

References

Abadie J, Carpentier J (1969) Generalization of the Wolfe reduced gradient method to the case of nonlinear constraints. In: Fletcher R (ed) Optimization. Academic Press, London (U.K.), pp 37–47

Aczél J, Alsina C (1987) Synthesizing judgements: a functional equations approach. Math Model 9(3–5):311–320. https://doi.org/10.1016/0270-0255(87)90487-8

Aczél J, Saaty TL (1983) Procedures for synthesizing ratio judgments. J Math Psychol 27(1):93–102. https://doi.org/10.1016/0022-2496(83)90028-7

Bana e Costa CA, Vansnick J (2008) A critical analysis of the eigenvalue method used to derive priorities in AHP. Eur J Oper Res 187(3):1422–1428. https://doi.org/10.1016/j.ejor.2008.09.022

Benítez J, Delgado-Galván X, Izquierdo J, Pérez-García R (2012) Improving consistency in AHP decision-making processes. Appl Math Comput 219(5):2432–2441. https://doi.org/10.1016/j.amc.2012.08.079

Bian F, Cai H (2012) Choice of crack repairing material for asphalt pavement based on AHP. J Test Eval 40(7):1–4. https://doi.org/10.1520/JTE20120154

Bozóki S, Fülop J, Koczokdaj WW (2011) An LP-based inconsistency monitoring of pairwise comparison matrices. Math Comput Model 54(1–2):789–793. https://doi.org/10.1016/j.mcm.2011.03.026

Bozóki S, Rapcsák T (2008) On Saaty’s and Koczokdaj’s inconsistencies of pairwise comparison matrices. J Global Optim 42(2):157–175. https://doi.org/10.1007/s10898-007-9236-z

Brauers WKM, Zavadskas EK, Peldschus F, Turskis Z (2008) Multiobjective decision-making for road design. Transport 23(3):183–193. https://doi.org/10.3846/1648-4142.2008.23.183-193

BSI (2016) BS EN 13108 - Bituminous mixtures. Material specifications. https://doi.org/10.3403/BSEN13108

Cañizo S, Di Graziano A, Kerali HR, Odoki JB (2002) Multicriteria analysis method for pavement maintenance management. Transp Res Rec 1816(1816):73–84

Chai T, Draxler RR (2014) Root mean square error (RMSE) or mean absolute error (MAE)? arguments against avoiding RMSE in the literature. Geosci Model Dev 7(3):1247–1250. https://doi.org/10.5194/gmd-7-1247-2014

Chang J, Chen D, Hung C (2005) Selecting preventive maintenance treatments in texas : using the technique for order preference by similarity to the ideal solution for specific pavement study-3 sites. Transp Res Rec 1933(1933):62–71

Chehovits J, Galehouse L (2010) Energy usage and greenhouse gas emissions of pavement preservation processes for asphalt concrete pavements. National Centre for Pavement Preservation, Okemos

Chou C (2003) The canonical representation of multiplication operation on triangular fuzzy numbers. Comput Math Appl 45(10–11):1601–1610. https://doi.org/10.1016/S0898-1221(03)00139-1

Chou YT (1990) Reliability design procedures for flexible pavements. J Transp Eng 116(5):602–614

Collins F, Africa B (2017) Infrastructure development as a key driver of sustainable social and economic development. 24th Annual FIDIC-GAMA conference, pp 1–5

Davis CF, Campbell GM (1995) Selection of pavement markings using multicriteria decision making. Transp Res Rec 1509(1509):28–37

de Carvalho EP, dos Santos A, Ma TF (2008) Reduced gradient method combined with augmented lagrangian and barrier for the optimal power flow problem. Appl Math Comput 200(2):529–536. https://doi.org/10.1016/j.amc.2007.11.025

Dijkstra TK (2013) On the extraction of weights from pairwise comparison matrices. Central Eur J Oper Res 21(1):103–123. https://doi.org/10.1007/s10100-011-0212-9

Dukićin Vuckovic S, Dordević J, Milankovic Jovanov J, Ivanovic Bibic L, Protić B, Dordević T, Ivkov M (2017) The development of transport infrastructure and attitudes of the local population: a case study from the republic of serbia. Geografisk Tidsskrift-Danish J Geogr. https://doi.org/10.1080/00167223.2017.1419369

EAPA (2007) Long-life asphalt pavements-technical version. European Asphalt Pavement Association, Brussels

EEA (2014) Adaptation of transport to climate change in Europe (No. 8). European Environment Agency, Luxembourg. https://doi.org/10.2800/56672

European Comission (2014) Core network corridors (No. 13)

European Commission (2006) Examples of assessed road safety measures. ROSEBUD 1:7–66

Filippo S, Martins Ribeiro PC, Kahn Ribeiro S (2007) A fuzzy multi-criteria model applied to the management of the environmental restoration of paved highways. Transport Res Part D Transport Environ 12(6):423–436. https://doi.org/10.1016/j.trd.2007.05.004

Grzybowski AZ (2012) Note on a new optimization based approach for estimating priority weights and related consistency index. Expert Syst Appl 39(14):11699–11708. https://doi.org/10.1016/j.eswa.2012.04.051

Grzybowski AZ (2016) New results on inconsistency indices and their relationship with the quality of priority vector estimation. Expert Syst Appl 43:197–212. https://doi.org/10.1016/j.eswa.2015.08.049

Hammersley JM, Handscomb DC (1964) Monte carlo methods. Springer, London

Hammond GP, Jones CI (2008) Embodied energy and carbon in construction materials. Proc Inst Civil Eng Energy 161(2):87–98. https://doi.org/10.1068/ener.2008.161.2.87

Hwang CL, Yoon K (1981) Multiple attribute decision making: methods and applications. Springer, New York

Jahanshahloo GR, Lotfi FH, Izadikhah M (2006) Extension of the TOPSIS method for decision-making problems with fuzzy data. Appl Math Comput 181(2):1544–1551. https://doi.org/10.1016/j.amc.2006.02.057

Jato-espino D, Rodriguez-Hernandez J, Andrs-Valeri VC, Ballester-Muñoz F (2014) A fuzzy stochastic multi-criteria model for the selection of urban pervious pavements. Expert Syst Appl 41(15):6807–6817. https://doi.org/10.1016/j.eswa.2014.05.008

Jato-espino D, Blanco-Fernandez E, Carpio-Garcia J, Castro-Fresno D (2016) Decision aid system founded on nonlinear valuation,
