TOWARDS A COMPLETE TWISTORIZATION
OF THE HETEROISTIC STRING

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Abstract. In $D = 3, 4, 6$ and 10 space–time dimensions considered is a string model invariant under transformations of $N = 1$ space–time supersymmetry and $n = D - 2$ local worldsheet supersymmetry with the both Virasoro constraints solved in the twistor form. The twistor solution survives in a modified form even in the presence of the heterotic fermions.

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1 Introduction

Recently progress has been made in understanding the geometrical origin of the fermionic \( \kappa \)-symmetry \([1]\) of \( D = 3, 4, 6 \) and 10 Brink–Schwarz superparticles \([2]\), and Green–Schwarz superstrings \([3]\). This has been achieved through the development of a formulation of these theories possessing space–time supersymmetry and local \( n = D - 2 \) worldsheet supersymmetry the latter turning to the \( \kappa \)-symmetry on the mass shell \([4]\). Such a doubly supersymmetric formulation provides a natural ground for the twistor transform \([5]\) in describing superparticle and superstring dynamics \([6]\) since twistor components (being commuting spinors) arise as superpartners of the Grassmann coordinates of target superspace. The motion of the superparticle and superstring determines the embedding of the worldsheet supersurface into the target superspace (and vice versa) in such a way that for dynamical characteristics of the objects the Cartan–Penrose representation of the light–like vector as a bilinear combination of spinors arises. A hope has been that this approach may contain the advantages of both the Green–Schwarz and the Neveu–Schwarz–Ramond formulation of the superstrings and at the same time will be free of their problems. (On closely related Lorentz–harmonic approaches see \([7]\), \([8]\)).

Up to now the twistor–like formulation has been developed for describing \( D = 3, 4, 6 \) and 10 \( N = 1 \) superparticles \([4]\), \([9]\), \([11]\), \( N = 1 \) null superstrings \([12]\) and \( N = 1 \) heterotic strings \([13]\), \([14]\) though the latter encountered the problem of incorporating chiral fermions \([17]\). At the same time a problem arose with generalizing these results to construct \( n = D - 2 \) worldsheet superfield versions of the \( N = 2 \) superstrings \([15]\), \([21]\). The difficulty is to not allow the auxiliary fields in the superfield action of the string, such as Lagrange multipliers, to propagate. For \( D = 3 \), type IIA superstring it was solved in \([21]\), while for getting an appropriate D=10, type IIA superstring action the authors of ref. \([20]\) proposed to perform a dimensional reduction of a \( D = 11 \), \( N = 1 \) twistor–like supermembrane theory. This problem of the manifest doubly supersymmetric description of \( N = 2 \) dynamics is apparently connected with the problem of solving for the both Virasoro constraints in the twistor form. Component actions of this kind for describing \( N = 1 \) as well as \( N = 2 \) superstrings have been studied in \([10]\), and relevant superfield equations of motion of \( N = 2, D = 4 \) superstring were discussed in \([15]\). At the same time all known superfield versions of \( D = 3, 4, 6 \) and 10 \( N = 1 \) superstrings deal with the twistorization of only one Virasoro constraint corresponding to the supersymmetric (for example, the right–moving) sector of the heterotic string, while the Virasoro constraint corresponding to the left–moving sector is taken into account in the “old–fashion” manner.

In the present paper we propose a doubly twistorized supersymmetric version of an
N = 1 superstring in D = 3, 4, 6 and 10 space–time. We will also show that in spite of the fact that the left–moving (non–supersymmetric) sector of the heterotic string is a relic of a bosonic string in D = 26, where the conventional twistor relation does not take place, it is nevertheless possible to keep the twistor solution to the corresponding Virasoro constraint in the presence of the chiral fermions.

The aim of the paper is twofold. From the one hand side it seems of interest to put the both Virasoro constraints of the heterotic string onto an equal “twistor” footing, and from the other hand the construction of this version testifies, though indirectly, to the existence of the n = D − 2 worldsheet superfield formulation of type IIA and IIB superstrings, since the former may be thought of as a truncated theory of the latter.

2 Geometry of (0, D − 2) worldsheet superspace

To construct a superfield action one has to specify the geometrical properties of the superspace where the superfields of the theory are determined.

In the case under consideration we deal with a worldsheet superspace parametrized by two bosonic and n = D − 2 (D = 3, 4, 6, 10) fermionic coordinates z \(M = (\xi^\mu, \eta^A)\), where \(\mu = 0, 1;\) and \(A = 1, 2, ..., n\) are to be identified with indices of an n-dimensional representation of an internal \(SO(n)\) group. For physical reasons the objects such as supervielbeins \(E^A = dz^M E^A_M, SO(1,1) \times SO(n)\) superconnection \(\Omega = dz^M \Omega_M\), torsion \(T^A = E^B \wedge E^C T^A_{BC} = \mathcal{D}E^A\) and curvature \(R^A_B = E^D \wedge E^C R^A_{DCB} = d\Omega^A_B + \Omega^A_C \wedge \Omega^C_B\) determining supergeometry should be constrained. This is achieved by imposing the following conditions \[22\], \[14\]:

\[
T^{++} = 0, \quad (1)
\]
\[
T^{--} = iE^{-A} \wedge E^{-A}, \quad (2)
\]
\[
T^{-A} = E^{--} \wedge E^{++} T^{-A}_{--++}, \quad (3)
\]

or in terms of the covariant derivatives:

\[
[D_{++}, D_{-A}] = R_{+++A}, \quad (4)
\]
\[
[D_{--}, D_{-A}] = 0, \quad (5)
\]
\[
\{D_{-A}, D_{-B}\} = -2i\delta_{AB}D_{--}, \quad (6)
\]

\(1\) \(D = d + \Omega = E^A(\nabla_A + \Omega_A) = E^A\mathcal{D}_A\) denotes a covariant exterior derivative, and \(A, B, C\) stand for \((++, --, -A)\), where the indices of the \(SO(n)\) structure subgroup are distinguished from the \(\eta^A\)–coordinate indices by the sign \((-)\) indicating the conformal weight \(\frac{1}{2}\) of corresponding right–moving spinors.
\[ [D_{--}, D_{++}] = T^{A}_{-A}D_{-A} + R_{-++}. \] (7)

In (4)–(7) we skipped the \( SO(n) \) curvature components since when constructing the twistor superstring action we shall deal with the superfields being singlets of \( SO(n) \).

Eqs. (4), (5) mean that in a superconformal gauge the right–moving sector of the heterotic geometry is inert under supersymmetry transformations, and eqs. (6), (7) imply that \( (0,n) \) superspace is to be superconformally flat [22], [14].

Under infinitesimal superdiffeomorphisms \( \delta z^M = \Xi^M(z) \), local \( SO(1,1) \times SO(n) \) tangent space rotations \( L_{AB} \) and super–Weyl transformations \( W(z) \) the inverse supervielbeins \( E_{A}^{M} \) are transformed as follows:

\[
\begin{align*}
\delta E_{A}^{M} &= - L_{B}^{A} E_{B}^{M} + \nabla_{A} \Xi^{M}, \\
\delta_{W} E_{++}^{M} &= - 2 W E_{++}^{M}, \\
\delta_{W} E_{--}^{M} &= - 2 W E_{--}^{M} - i E_{--}^{M} \nabla_{-} W, \\
\delta_{W} E_{-A}^{M} &= - W E_{-A}^{M}.
\end{align*}
\] (8)

Reducing the transformations (8), (9) to that of supergravity transformations and partially solving the constraints one may, at least locally, represent the covariant derivatives \( \nabla_{A} = E_{A}^{M} \partial_{M} \) (which act on the scalar superfields) in the following form [12]:

\[
\begin{align*}
\nabla_{-A} &= E D_{-A} \equiv E(\partial_{-A} - i V_{-A}^{\mu} \partial_{\mu}), \\
\nabla_{--} &= E^{2} D_{--} \equiv E^{2} V_{--}^{\mu} \partial_{\mu} = E^{2} \frac{1}{n} D_{-A} V_{-A}^{\mu} \partial_{\mu}, \\
\nabla_{++} &= E^{-2} (D_{++} + 2i H^{(-4)} D_{--} - D_{-A} H^{(-4)} D_{-A}),
\end{align*}
\] (10)

where \( E \) and \( H^{(-4)} \) are independent superfields, \( V_{-A}^{\mu} \) are subject to constraints [12]

\[
D_{-A} V_{-B}^{\mu} + D_{-B} V_{-A}^{\mu} = 2 \frac{1}{n} \delta_{AB} D_{-C} V_{-C}^{\mu},
\] (11)

and derivatives \( D_{-A}, D_{--}, D_{++} \) obey the global supersymmetry algebra

\[
\{ D_{-A}, D_{-B} \} = - 2i D_{-}, \quad [ D_{-A}, D_{++} ] = [D_{--}, D_{++}] = 0.
\] (12)

For further constructing and studying the twistor string model it is essential that in view of (8)–(13) one may choose a Wess–Zumino gauge in which

\[
\begin{align*}
E &= 1, \quad V_{-A}^{\mu} = \eta_{A} e_{-}^{\mu}(\xi), \\
D_{++} + 2i H^{(-4)} D_{--} &= e_{++}^{\mu}(\xi) \partial_{\mu} + \eta^{--} \Psi_{-}^{-} \psi_{-}^{--}(\xi, \eta) \eta^{--}(\xi) \partial_{\mu},
\end{align*}
\] (13)

where \( e_{++}^{\mu}(\xi) \) are \( d = 2 \) graviton vielbeins, and the superfunction \( \Psi_{-}^{-} = \psi_{-}^{--}(\xi, \eta) \) (being a relic of \( H^{(-4)} \)) contains as components a gravitino field \( \psi_{-}^{--}(\xi) \), an \( SO(n) \) gauge field \( A_{-}^{--}(\xi) \) and fields \( B_{[ABC...]}(\xi) \) which may be identified as gauge fields of additional symmetries being part of \( n > 2 \) local supersymmetry in \( d = 2 \) [23] (brackets imply antisymmetrization).
3 \( N = 1 \) twistor superstring action in (0,n) worldsheet superspace

We use the derivatives (10), (12) to construct the following doubly twistorized \( n = D - 2 \) superfield action of the \( N = 1 \) superstring propagating (for simplicity) in a flat target superspace parametrized by bosonic vector coordinates \( X^m \) and fermionic coordinates \( \Theta^\alpha \) the latter being Majorana or Majorana–Weyl spinors depending on the dimension \( D \) of space–time:

\[
S = \int d^2 \xi d^n \eta [ \mathcal{P}_{Am}(D_{-A}X^m - iD_{-A}\bar{\Theta}\gamma^m \Theta) \\
+ \mathcal{P}^{MN}(i\partial_M X^m \partial_N \bar{\Theta}\gamma^m \Theta - \frac{1}{n} E^{++}_M E^{-}_N D_{-A}\bar{\Theta}\gamma^m D_{-A}\Theta\bar{\Lambda}_+\gamma^m \Lambda_+ + \partial_M Q_N) \\
+ \mathcal{P}_m(E^2 \nabla_{++} X^m - iE^2 \nabla_{++} \bar{\Theta}\gamma^m \Theta - \bar{\Lambda}_+\gamma^m \Lambda_+)],
\]

where \( \mathcal{P}_{mA}, \mathcal{P}^{MN} = (-1)^{MN+1}\mathcal{P}^{N,M} \), and \( \mathcal{P}_m \) are Lagrange multipliers, \( Q_N \) is an abelian gauge worldsheet superfield forcing the pullback into the worldsheet superspace of the Wess–Zumino form \( B = idX^m \wedge d\bar{\Theta}\gamma^m \Theta - \frac{1}{n} E^{++} \wedge E^{--} D_{-A}\bar{\Theta}\gamma^m D_{-A}\Theta\bar{\Lambda}_+\gamma^m \Lambda_+ \) to be a closed form on the mass shell [14], [16]; and \( \Lambda_+ \) is a bosonic spinor superfield whose space–time chirality may be the same or opposite to that of \( \Theta \). The action (16) is invariant under the following local transformations of \( \Lambda_+ \) (see, for example, [13]):

\[
\delta \Lambda_+^\alpha = \frac{1}{2}(\bar{\Lambda}_+\gamma^m \Lambda_+)(\gamma^m U_-)^\alpha - (\bar{\Lambda}_+ U_-)\Lambda_+^\alpha,
\]

where \( U_-^\alpha \) is a spinor superfield parameter. Transformations (17) are two–level reducible, so among \( 2D - 4 \) spinor components of \( \Lambda_+ \) only \( D - 1 \) are independent, which is equal to the number of independent components of the light–like vector. Instead of \( \Lambda_+ \) we could alternatively use “multitwistors” \( \Lambda^{+\hat{A}} \), where \( \hat{A} \) are indices of an \( n \)–dimensional representation of another local \( SO(n) \) group. Then to reduce the number of their components to that of the light–like vector one should include into the action (16) the constraints [11], [14]

\[
\bar{\Lambda}^{+\hat{A}}\gamma^m \Lambda^{+\hat{B}} + \bar{\Lambda}^{+\hat{B}}\gamma^m \Lambda^{+\hat{A}} = \frac{2}{n}\delta_{A\hat{B}}\bar{\Lambda}^{+\hat{C}}\gamma^m \Lambda^{+\hat{C}}.
\]

To analyze eq. (16) it is convenient to rewrite it in a form being closer to a twistor \( N = 1 \) superstring action studied previously in [14], [16], [12]

\[
S = \int d^2 \xi d^n \eta [(\mathcal{P}_{Am}(D_{-A}X^m - iD_{-A}\bar{\Theta}\gamma^m \Theta) + \mathcal{P}^{MN}(\partial_M Q_N \\
+ i\partial_M X^m \partial_N \bar{\Theta}\gamma^m \Theta - \frac{1}{n} E^{++}_M E^{-}_N D_{-A}\bar{\Theta}\gamma^m D_{-A}\Theta(\nabla_{++} X^m - i\nabla_{++} \bar{\Theta}\gamma^m \Theta)) \\
+ \mathcal{P}_m(E^2 \nabla_{++} X^m - iE^2 \nabla_{++} \bar{\Theta}\gamma^m \Theta - \bar{\Lambda}_+\gamma^m \Lambda_+)],
\]
where \( \hat{P}_m \) is a redefined Lagrange multiplier of (16).

The first two lines in (19) constitute the action discussed in detail in [16]. From the analysis performed therein we know that the twistor solution to the Virasoro constraint 
\[
(\nabla_- X^m - i \nabla_- \bar{\Theta} \gamma^m \Theta)^2|_{\eta=0} = 0
\]
corresponding to the right–moving sector of the \( N = 1 \) superstring arises as a consequence of the equation of motion of \( P_{mA} \) (which turns out to be a completely auxiliary variable) and the constraints (3):
\[
\begin{align*}
D_- A X^m - i D_- A \bar{\Theta} \gamma^m \Theta &= 0, \\
\nabla_- X^m - i \nabla_- \Theta \gamma^m \Theta &= D_- C \bar{\Theta} \gamma^m D_- C \Theta, \\
D_- A \bar{\Theta} \gamma^m D_- B \Theta + D_- B \Theta \gamma^m D_- A \Theta &= \frac{2}{n} \delta_{AB} D_- C \bar{\Theta} \gamma^m D_- C \Theta.
\end{align*}
\] (20)

As for the Lagrange multiplier \( P^M N \), upon fixing a gauge of corresponding local symmetries and solving for relevant equations of motion it contains only one non–zero component
\[
p^{\mu\nu} \eta^n = T \varepsilon^{\mu\nu} \eta^n,
\] (21)
where \( T \) is identified with string tension [24], [16].

The last term in (16), (19) is introduced for getting the twistor solution to the second Virasoro constraint 
\[
(\nabla_+ X^m - i \nabla_+ \bar{\Theta} \gamma^m \Theta)^2|_{\eta=0} = 0:
\]
\[
(\nabla_+ X^m - i \nabla_+ \bar{\Theta} \gamma^m \Theta) = E^{-2} \Lambda_+ - \gamma^m \Lambda_+.
\] (22)

For the induced metric \( G_{\mu\nu} = (\partial_\mu X^m - i \partial_\mu \bar{\Theta} \gamma^m \Theta)(\partial_\nu X^m - i \partial_\nu \bar{\Theta} \gamma^m \Theta) \) on the string worldsheet to be non–degenerate require that
\[
(\Lambda_+ + \gamma^m \Lambda_+)(D_- A \bar{\Theta} \gamma^m D_- A \Theta) \neq 0.
\] (23)

Otherwise we would get a twistor–like formulation of a null superstring [12].

With this in mind we will show that the all auxiliary variables containing in the last term of (19), including the gravitino and \( SO(n) \) field, are either zero or expressed in terms of the superstring coordinates \( x^m(\xi) = X^m|_{\eta=0} \) and \( \theta^\alpha(\xi) = \Theta^\alpha|_{\eta=0} \) or their derivatives, and thus do not lead to appearance of superfluous dynamical degrees of freedom in the theory.

To get equations of motion from action (19) we may substitute for \( \nabla_+ \) its form in the Wess–Zumino gauge (15). Then, upon redefining \( P_{Am} \) (in view of (20)) one may reduce the last term in (19) to the following form
\[
S_\Lambda = \int D^2 \xi d^n \eta \hat{P}_m \left( e^\mu_+ (\partial_\mu X^m - i \partial_\mu \bar{\Theta} \gamma^m \Theta) + \frac{i}{n} \eta^{-A} \Psi^- A - D_- B \bar{\Theta} \gamma^m D_- B \Theta - \Lambda_+ \gamma^m \Lambda_+ \right).
\] (24)
Note that $\Psi_{A}^{--}$ and $\Lambda_{+}$ do not contribute to the other parts of action (19). Thus, varying (24) with respect to $\Lambda_{+}$ and $\Psi_{A}^{---}$ we get

$$\left(\hat{P}_{m}\gamma^{m}\Lambda_{+}\right)^{\alpha} = 0 \implies \hat{P}_{m} = \Phi(\xi, \eta)\bar{\Lambda}_{+}^{+}\gamma_{m}\Lambda_{+},$$  \hspace{1cm} (25)$$

$$\left(\eta^{-A}\hat{P}_{m}\right)D_{-B}\bar{\Theta}\gamma^{m}D_{-B}\Theta = 0,$$  \hspace{1cm} (26)$$

where $\Phi(\xi, \eta)$ is a superfield factor.

Taking into account the non–degeneracy condition (23), from (25) and (26) we see that the only non–zero component in $\Phi(\xi, \eta)$ is $\eta^{n}a_{(-4)}(\xi)$, and hence $\hat{P}_{m}$ is reduced to

$$\hat{P}_{m} = \eta^{n}\hat{p}_{m}^{--} = \eta^{n}a_{(-4)}(\xi)\bar{\lambda}_{+}^{+}\gamma_{m}\lambda_{+},$$  \hspace{1cm} (27)$$

where $\lambda_{+} = \Lambda_{+}|_{\eta=0}$. This means that the term (24) does not contribute to the equations of motion of $X^{m}$ and $\Theta$ obtained from (19) starting from the next to the leading components in $\eta$–expansion of $X^{m}$ and $\Theta$. This allows one to apply the reasoning of refs. [14], [16] and show that the only nontrivial components of $P_{A}^{m}$ are

$$P_{A}^{m} = \eta^{-A_{2}}...\eta^{-A_{n}}\varepsilon_{A_{2}...A_{n}B}(\delta_{AB}\hat{p}_{++}^{m} + p_{AB++}^{m})$$  \hspace{1cm} (28)$$

(where $p_{AB++}^{m}$ is symmetric and traceless with respect to $A, B$), and that all higher components of $X^{m}$ and $\Theta$ are expressed in terms of their leading components.

Now, varying (19) or (24) over $\hat{P}_{m}$ we get

$$\partial_{++}X^{m} - i\partial_{++}\bar{\Theta}\gamma^{m}\Theta + \frac{i}{n}\eta^{-A}\Psi_{A}^{--}D_{-B}\bar{\Theta}\gamma^{m}D_{-B}\Theta = \bar{\lambda}_{+}^{+}\gamma_{m}\lambda_{+} \quad (\partial_{\pm\pm} \equiv e_{\pm\pm}^{\mu} \partial_{\mu}).$$  \hspace{1cm} (29)$$

This superfield equation can be solved by an iteration procedure. At $\eta^{-A} = 0$ we obtain the twistor solution to the second Virasoro constraint:

$$\partial_{++}x^{m} - i\partial_{++}\bar{\theta}\gamma^{m}\theta = \bar{\lambda}_{+}^{+}\gamma_{m}\lambda_{+}.$$  \hspace{1cm} (30)$$

At the first order in $\eta$’s powers we have

$$\partial_{++}\zeta^{m}_{-A} - i\partial_{++}\bar{\theta}\gamma^{m}\lambda_{-A} + i\bar{\theta}\gamma^{m}\partial_{++}\lambda_{-A} + \frac{1}{n}\psi_{A}^{--}\bar{\lambda}_{-B}^{+}\gamma_{m}\lambda_{-B} = 2\lambda_{+}^{+}\gamma_{m}s_{A},$$  \hspace{1cm} (31)$$

where $\lambda_{-A} = D_{-A}\Theta|_{\eta=0}$, $s_{A} = D_{-A}\Lambda_{+}|_{\eta=0}$, and $\zeta^{m}_{A} = D_{-A}X^{m}|_{\eta=0} = i\bar{\theta}\gamma^{m}\lambda_{-A}$, which follows from equations of motion (20). Multiplying (31) by $\bar{\lambda}_{+}^{+}\gamma_{m}\lambda_{+}$ and taking into account (23) we find that the gravitino field is expressed through target superspace coordinates $\theta$ and twistor–like commuting spinors $\lambda_{+}$, $\lambda_{-A}$:

$$\psi_{A}^{--} = -2i\frac{(\partial_{++}\bar{\theta}\gamma^{m}\lambda_{-A})(\bar{\lambda}_{+}^{+}\gamma_{m}\lambda_{+})}{(\lambda_{+}^{+}\gamma_{n}\lambda_{+})(\lambda_{-B}^{+}\gamma_{n}\lambda_{-B})}.$$  \hspace{1cm} (32)$$
Substituting (34) back into eq. (31) we have the sufficient number of equations to express \( s_A \) in terms of \( x^m, \theta, \lambda_+ \) and \( \lambda_- \). This is possible due to the local symmetry (17) allowing one to eliminate all extra degrees of freedom of \( \Lambda_+ \).

Proceeding further to the third \( \eta \)'s order we can find the expression for the \( SO(n) \) gauge field \( A_{-\bar{B}} \) and \( D_- A D_+ \Lambda_+ |_{\eta=0} \) component of \( \Lambda_+ \) in terms of the leading components of \( X^M, \Theta \) and \( \Lambda_+ \). Following this way one may convince oneself that \( \Psi^{--} \) and \( \Lambda_+ \) do not contain independent components. It means that adding the new term to the superstring action (14) or (19) does not lead to extra, undesirable, degrees of freedom and the model still remains equivalent to a classical \( N = 1 \) Green–Schwarz superstring.

Indeed, because all components of the superfield Lagrange multipliers \( P^{MN} \), \( \hat{P}_m \) and \( P_{mA} \) except (21), (27) and (28) vanish, we may integrate (19), or (16) over \( \eta \) and get a component action

\[
S = \int d^2 \xi [p_{m-}(\partial_+ x^m - i \partial_+ \bar{\theta} \gamma^m \theta - \bar{\lambda}_+ \gamma^m \lambda_+) + p_{m+}(\partial_- x^m - i \partial_- \bar{\theta} \gamma^m \theta - \frac{1}{n} \bar{\lambda}_- \gamma^m \lambda_-) + \frac{1}{e} \epsilon^{\mu} \gamma^m \partial_\mu \bar{\theta} \gamma^m \theta + \frac{1}{n} (\bar{\lambda}_- \gamma^m \lambda_-) (\bar{\lambda}_+ \gamma^m \lambda_+) + p_{mAB++}^m \bar{\lambda}_- \gamma^m \lambda_-] \tag{33}
\]

where \( e = \text{det}(e^{\pm \pm}) \), and \( p_{m\pm} \), \( p_{AB++}^m \) are appropriately redefined Lagrange multipliers.

Varying (33) over \( p_{m\pm} \), \( e^{\mu \pm \pm} \) and \( \lambda_\pm \) one finds that

\[
p_{m+}^m = \bar{\lambda}_+ \gamma^m \lambda_+, \quad p_{m-}^m = \frac{1}{n} \bar{\lambda}_- \gamma^m \lambda_- \tag{34}
\]

(from which it follows that \( a_{(-4)} \) in eq. (27) is zero). Substituting (34) back into eq. (33) we may get rid of \( p_{\pm}^m \) and obtain a form of the twistor–like action for the \( N = 1 \) superstring analogous to that considered in [14, 8]:

\[
S = \int d^2 \xi \epsilon^{\mu}[\frac{1}{n} (\bar{\lambda}_- \gamma^m \lambda_-) (\partial_+ x^m - i \partial_+ \bar{\theta} \gamma^m \theta) + \bar{\lambda}_+ \gamma^m \lambda_+ (\partial_- x^m - i \partial_- \bar{\theta} \gamma^m \theta) + \frac{1}{e} \epsilon^{\mu} \gamma^m \partial_\mu \bar{\theta} \gamma^m \theta + \frac{1}{n} (\bar{\lambda}_- \gamma^m \lambda_-) (\bar{\lambda}_+ \gamma^m \lambda_+) + p_{AB++}^m \bar{\lambda}_- \gamma^m \lambda_-] \tag{35}
\]

This form of the string action turns out to be convenient for the straightforward incorporation of heterotic fermions into the dynamics of the model at the component level. Just add to (33) a term:

\[
S_{\psi'} = \int d^2 \xi \psi^I_+ e^\mu_- \partial_\mu \psi^I_+, \tag{36}
\]

where index \( I \) corresponds to an internal symmetry group, which is \( SO(32) \) or \( E_8 \times E_8 \) for the conventional \( D = 10 \) heterotic string because of quantum consistency reasons [3].

The contribution of the heterotic fermions (36) modifies the corresponding left–moving Virasoro constraint

\[
(\partial_+ x^m - i \partial_+ \bar{\theta} \gamma^m \theta)^2 + \psi^I_+ \partial_+ \psi^I_+ = 0. \tag{37}
\]
A twistor–like solution to eq. (37) which directly follows from action (35) plus (36) is
\[
(\partial_{++} x^m - i \partial_{++} \bar{\theta} \gamma^m \theta) + \frac{1}{2} \frac{\bar{\lambda}_{-A} \gamma^m \lambda_{-A}}{(\lambda_{-B} \gamma^n \lambda_{-B})(\lambda_{+\gamma_n} \lambda_{+})} \psi_{+}^{I} \partial_{++} \psi_{+}^{I} = \bar{\lambda}_{+} \gamma^m \lambda_{+}.
\] (38)

4 Conclusion

We have constructed the doubly twistorized version of the classical $N = 1$ superstring in $d = 2$, $n = D - 2$ worldsheet superspace which at least at the component level can be generalized to include the chiral fermions. To write down the action we introduced in eq. (16) the terms containing the bosonic spinor superfield $\Lambda_{+}$. By somewhat “mysterious” reasons a bulk of variables involved in the $n = D - 2$ superfield description either vanish or are expressed through the relevant dynamical characteristics of the string on the mass shell. This happens, in particular, to the gravitino and other gauge fields of $d = 2$, $n = D - 2$ supergravity entering the action. So it is not required to use extended supersymmetry transformations to gauge these fields away, and the former may be used for matching the number of bosonic and fermionic coordinates of the superstring itself, the role played by the fermionic $\kappa$–symmetry in the conventional superstring approach.

As we have already mentioned, in (16) we could equally well use a set of spinor superfields $\Lambda_{+\hat{A}}(\xi, \eta)$ ($\hat{A} = 1, \ldots, D - 2$) satisfying eq. (18). In the latter case the model considered may be regarded as a reduced version of a twistor–like $N = 2$ closed superstring (of type A or B), where $\Lambda_{+\hat{A}}$ is identified with $D_{+\hat{A}} \Theta^2$, and $D_{+\hat{A}}$ is an odd covariant derivative acting in a right–moving sector of an enlarged $(D - 2, D - 2)$ worldsheet superspace of the $N = 2$ superstring (with $\Theta^2$ corresponding to the second space–time supersymmetry). For $D = 3$, type IIA twistor superstring of [21] this is indeed the case. It seems of interest to establish the analogous relationship between the present model and that of ref. [21].

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