Low-energy hadronic cross sections measurements at BABAR, and implication for the $g - 2$ of the muon

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Abstract

The BABAR Collaboration has an intensive program studying the cross sections of hadron production in low-energy $e^+e^-$ annihilation, accessible via initial-state radiation. Our measurements allow a significant improvement in the precision of the predicted value of the muon anomalous magnetic moment. These improvements are necessary for shedding light on the current $> 3$ sigma difference between the predicted and the experimental values. We have published results on a number of processes with two to six hadrons in the final state, and other final states are currently under investigation. We report here on the most recent results obtained by analysing the entire BABAR dataset.

1 The muon gyromagnetic factor and “anomalous” moment

As a result of more than three decades of intense efforts to validate every corner of the standard model (SM) of elementary particles and their interactions, and to submit it to a redundant metrology with an always increasing precision, the SM has only become more and more “standard”, with some very few exceptions that include the “tension” between the theoretical prediction and the unique precise experimental measurement of the “anomalous” magnetic moment of the muon, $a_\mu$, which is the relative deviation of the gyromagnetic factor, $g_\mu$, from the value of $g = 2$ for a pointlike Dirac particle, i.e. $a_\mu \equiv (g_\mu - 2)/2$. 

1
Hadronic Vacuum Polarisation (VP)  
Hadronic light-by-light Scattering  
Weak Interactions

Figure 1: Examples of diagrams contributing to the calculation of $a_\mu$. Up: QED diagrams of various orders in $\alpha$. Bottom: VP, LbL and weak-interaction contributions [9].

| $\alpha$ from | $a_\mu^{QED}$ $(10^{-10})$ |
|--------------|-----------------|
| $a_e$        | $11 658 471.885 \pm 0.004$ |
| Rubidium Rydberg constant | $11 658 471.895 \pm 0.008$ |

Table 1: Values of $a_\mu^{QED}$ computed using values of $\alpha$ extracted from the measured value of $a_e$ and from atomic physics measurements [4].

2 $a_\mu$: predictions and measurement

Since the first measurement (for the electron) [1] and its interpretation within the QED framework [2], both the prediction and the measurement of $a_\mu$ have undergone a tremendous improvement in precision, to the point that hadronic vacuum polarization (VP) i.e. modifications of the photon propagator, hadronic light-by-light scattering (LbL) and weak interactions must be taken into account (Fig. 1). Understanding the value of $a_\mu$ necessitates a precise knowledge of the value of the fine structure constant $\alpha$. From the development [4] of $a_e$ and of $a_\mu$ [3],

$$a_e = \frac{\alpha}{2\pi} - 0.3 \left( \frac{\alpha}{\pi} \right)^2 + 1.2 \left( \frac{\alpha}{\pi} \right)^3 - 1.9 \left( \frac{\alpha}{\pi} \right)^4 + 9.2 \left( \frac{\alpha}{\pi} \right)^5 + 1.7 \times 10^{-12} (\text{QCD} + \text{weak}),$$

$$a_\mu = \frac{\alpha}{2\pi} + 0.8 \left( \frac{\alpha}{\pi} \right)^2 + 24. \left( \frac{\alpha}{\pi} \right)^3 + 131. \left( \frac{\alpha}{\pi} \right)^4 + 753. \left( \frac{\alpha}{\pi} \right)^5 + 7.1 \times 10^{-8} (\text{QCD} + \text{weak}),$$

we see that due to the $\mu$-to-$e$ mass difference, the development for $a_e$ converges extremely rapidly and that the non-QED contributions are very small: a precise value of $\alpha$ can be extracted from $a_e$ and then injected in the calculation of $a_\mu$. The value of $a_\mu$ so obtained has a very small uncertainty and is compatible with that obtained using a value of $\alpha$ from atomic physics (Table 1): the QED contribution, which has been computed up to the 5th order in $\alpha$ [4], is under excellent control. Table 2 presents the sizable contributions to the prediction and the comparison with experiment as of 2014 [5].

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1 I have truncated the numerical factors.
Leading hadronic vacuum polarization (VP)
Sub-leading hadronic vacuum polarization
Hadronic light-by-light (LbL)
Weak (incl. 2-loops)

| Theory | 11 659 180.3 | ± 4.2 | ± 2.6 |
|--------|--------------|--------|--------|
| Experiment (E821 @ BNL) | 11 659 209.1 | ± 5.4 | ± 3.3 |

Exp. − theory +28.8 ± 8.0

Table 2: Contributions to the prediction for $a_\mu$ $(10^{-10})$ and comparison with experiment as of 2014 [5].

- The QED contribution is the main contributor to the value of $a_\mu$, while the uncertainty is dominated by the hadronic contributions (VP and LbL);
- The uncertainties of the prediction and of the measurement are of similar magnitude;
- The measured value exceeds the prediction with, assuming Gaussian statistics, a significance of $\approx 3.6$ standard deviations.

As QCD is not suited to precise low energy calculations, the VP contribution to $a_\mu$ is computed from the “dispersion integral” (9 and references therein):

$$a_{\mu,VP} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int \frac{R(s) \times \hat{K}(s)}{s^2} ds,$$

where $R(s)$ is the the cross section of $e^+e^-$ to hadrons at center-of-mass (CMS) energy squared $s$, normalized to the pointlike muon pair cross section $\sigma_0$: $R(s) = \sigma_{e^+e^-\to hadrons}/\sigma_0$, and $\hat{K}(s)$ is a known function that is of order unity on the $s$ range $[(2m_\pi c^2)^2, \infty]$. Technically, the low energy part of the integral is obtained from experimental data (up to a value often chosen to be $E_{cut} = 1.8$ GeV), while the high-energy part is computed from perturbative QCD (pQCD). Due to the presence of the $s^2$ factor at the denominator of the integrand, the precision of the prediction of $a_\mu$ relies on precise measurements at the lowest energies, and the channels with the lightest final state particle rest masses, $\pi^+\pi^-, \pi^+\pi^0\pi^-, \pi^+\pi^-2\pi^0, \pi^+\pi^-\pi^+\pi^-$, $KK$ are of particular importance.

3 **BABAR** measurements: the ISR method

The **BABAR** experiment [21,22] at the SLAC National Accelerator Laboratory has committed itself over the last decade to the systematic measurement of the production of all hadronic final states using the initial-state radiation (ISR) process. The cross section of the $e^+e^-$ production of a final state $f$ at a CMS energy squared $s'$ can be obtained from the differential cross section of the ISR production $e^+e^-\to f \gamma$ through the expression:

$$\frac{d\sigma_{e^+e^-\to f \gamma}}{ds'}(s') = \frac{2m}{s} W(s, x) \sigma_{e^+e^-\to f}(s'),$$

where $W(s, x)$, the probability density to radiate a photon with energy $E_\gamma = x\sqrt{s}$, is a known “radia-
tor” function [6], and $\sqrt{s}$ is here the CMS energy of the initial $e^+e^-$ pair, which is close to 10.6 GeV for **BABAR**. In contrast with the energy scans that provided the earlier experimental information on the variations of $R$ (see Figs. 50.5 and 50.6 in Ref. [5] and references in their captions), this ISR method
Figure 2: Left: $\mu^+\mu^-$ cross section as a function of the $\mu^+\mu^-$ invariant mass compared to the QED prediction, as a sanity check for the BaBar NLO analyses [28–30]. Right: The BaBar acceptance for the $K^+K^-$ analysis as a function of the $K^+K^-$ invariant mass [30].

makes an optimal use of the available luminosity and allows a consistent measurement over the full energy range with the same accelerator and detector conditions. In addition, in the case of BaBar the $e^+e^-$ initial state is strongly boosted longitudinally so the detector acceptance stays sizable down to threshold (Fig. 2 right).

The observation of the hadronic final state alone, if kinematically compatible with a system recoiling against a single massless particle, would allow the reconstruction of the event and the measurement of $s'$, but when in addition the ISR photon is observed ($\gamma$-tagging), a powerful background rejection and a good signal purity can be achieved. We have performed most of these measurements using a leading-order (LO) method, in which the final state $f$ and the ISR photon are reconstructed regardless of the eventual presence of additional photons. For these analyses the differential luminosity is obtained from the luminosity of the collider, known with a typical precision of 1%, and involves a computation of the detection efficiency that relies on Monte Carlo (MC) simulations [23–27], [32–38]. This experimental campaign has lead BaBar to improve the precision of the contribution to $a_\mu$ of most of the relevant channels by a large factor, typically close to a factor of three.

A list of the contributions $a^f_\mu$ to $a^{VP}_\mu$ for a number of individual hadronic final states $f$, available at the time, can be found in Table 2 of Ref. [12].

4 BaBar NLO ($e^+e^- \to f \gamma (\gamma)$) results

BaBar has also developed a new method that was applied to the dominant channel $\pi^+\pi^-$ [28,29] and more recently to the $K^+K^-$ channel [30]. The control of the systematics below the % level made it necessary to perform the analysis at the NLO level, that is, to take into account the possible radiation of an additional photon, be it from the initial (ISR) or from the final (FSR) state. The impossibility to control the global differential luminosity with the desired precision, in particular the MC-based efficiency, lead us to derive the value of $R$ from the ratio of the ISR production of the final state $f$ to the ISR production of a pair of muons, $\mu^+\mu^-$. Most of the systematics, including those related to the absolute luminosity, of the ISR photon reconstruction and of additional ISR radiation, cancel in the ratio. Figure 3 shows the obtained form-factor (here squared) distributions extracted from the cross-section distributions, together with fits using the GS parametrization of the VDM model. The values of $a^{\pi^+\pi^-}_\mu$ and of $a^{K^+K^-}_\mu$ integrated over the most critical range, that is, from threshold to

\footnote{A review on the PHOKHARA and AfkQed event generators used in our GEANT4-based simulations can be found in section 21 of Ref. [7].}
used in these precise ππ measurements at 1.8 GeV are more precise than the average of the previous measurements (Table 3).

Even though neither the time-integrated luminosity nor the absolute acceptance/efficiency were used in these precise ππ and KK cross-section measurements, we checked that we understand them by comparing the μμ cross section distribution we observe to the QED prediction: a good agreement is found (Fig. 2 left) within 0.4 ± 1.1%, which is dominated by the large uncertainty on the time-integrated luminosity (±0.9%).

These NLO analyses were performed assuming that the FSR corrections for the hadronic channel are negligible, as theoretical estimates are well below the systematic uncertainties in the cross section [28,30]. We have validated this assumption by an experimental study of the ISR-FSR interference in BABAR. This effect is due to the interference between ISR and FSR, which enables the separate measurement of the magnitudes of the ISR and of the FSR amplitudes [31]. For the pion channel, results match a model where final state radiation originates predominantly from the quarks that subsequently hadronize into a pion pair, while for the muon control channel, good consistency is found with QED.

5 Recent BaBar LO (e+e− → fγ) results

Recently BABAR obtained results on channels with two neutral kaons K0S K0L, K0S K0L π±π−, K0S K0S π±π− and K0S K0S K+K− [36] (Fig. 4 up), on K0S K+π−π0 and K0S K+π−η (preliminary) (Fig. 4 bottom), and updated the pp analysis to the full statistics [37] (Fig. 4 center left). The pp measurement has also been extended up to 6.5 GeV [34] (Fig. 4 center center) and the K+K− measurement to 8 GeV [38] (Fig. 4 center right) by untagged analyses.

pQCD is found to fail to describe the K+K− form factors extracted from our cross section mea-
Figure 4: Recent LO results. Magenta: First measurements. Up: channels with two neutral kaons \([36]\). Center: \(p\bar{p}\) with \([37]\) and without \([34]\) \(\gamma\) tagging, and \(K^+K^-\) without \([38]\) \(\gamma\) tagging. Bottom: \(K_S^0K^+\pi^-\eta\), the neutral meson \(h^0\) being either a \(\pi^0\) or an \(\eta\) (preliminary).

measurements (Fig. 5), but there is some hint that the discrepancy is getting better at higher mass, which kind-of supports the use of pQCD for the calculation of the dispersion integral above \(E_{\text{cut}}\). Note that given the improvement in precision of the hadronic cross sections, the most recent prediction \([15]\) restricts the \(s\) range over which pQCD is used to \([4.5 – 9.3]\) GeV and \([13\,\text{GeV} – \infty]\).

A summary of the \(\text{BaBar}\) measurements is provided in Fig. 6 and Table 4. The analyses of the \(\pi^+\pi^-\pi^0\pi^0\) \([35]\), of the \(\pi^+\pi^-\pi^0\) \([23]\) and of the \(\pi^+\pi^-\eta\) \([26]\) channels are presently being updated with the full available statistics: stay tuned.
Figure 5: Comparison of the $BaBar K^+K^-$ results with Chernyak-Zhitnitsky $^{[16]}$ pQCD predictions. With (left, $^{[30]}$) and without (right, $^{[38]}$) $\gamma$ tagging.

Figure 6: Summary of the $BaBar$ measurements (Courtesy of Fedor V. Ignatov, April 2016). Beware that some channels have the charmonia contribution removed while some others have not. The $\pi^+\pi^-\pi^0\pi^0$ $^{[35]}$ and $K^0_S K^+\pi^-\pi^0$ entries are preliminary. The NLO measurements are denoted by an additional $^\gamma$. 
Table 4: Summary of the \textit{B} \textit{A} \textit{B} \textit{A} \textit{R} results on ISR production of exclusive hadronic final states (The superseded results have been removed). Channels above the horizontal line have been mentioned in this paper.

| Channels | \( \int \mathcal{L} dt \ (\text{fb}^{-1}) \) | Method | Reference |
|----------|--------------------------------|--------|-----------|
| \( K_0^0 K^+ \pi^- \pi^0, K_0^0 K^+ \pi^- \eta \) | 454 | LO | preliminary |
| \( K^+ K^- \) | 469 | LO, no tag | 38 |
| \( K_0^0 K_0^0, K_0^0 K_0^0 \pi^+ \pi^-, K_0^0 K_0^0 \pi^+ \pi^- \eta, K_0^0 K_0^0 K^+ K^- \) | 469 | LO | 30 |
| \( \bar{p} p \) | 454 | LO | 37 |
| \( \bar{p} p \) | 469 | LO, no tag | 34 |
| \( K^+ K^- \) | 232 | NLO | 39 |
| \( \pi^+ \pi^- \) | 232 | NLO | 28 [20] |
| 2((\pi^+ \pi^-)\pi^0) | 454 | LO | 32 |
| \( K^+ K^- \pi^+ \pi^-, K^+ K^- \pi^0 \pi^0, K^+ K^- K^+ K^- \) | 454 | LO | 33 |
| \( K^+ K^- \eta, K^+ K^- \pi^0, K^+ K^- \pi^\pm \) | 232 | LO | 27 |
| \( \pi^+ \pi^- \pi^0 \pi^0 \) | 232 | LO | 35 preliminary |
| 2((\pi^+ \pi^-)\pi^0) (including \( \pi^+ \pi^- \eta \)), 2((\pi^+ \pi^-)\eta) | 232 | LO | 26 |
| \( K^+ K^- \pi^+ \pi^- \pi^0, K^+ K^- \pi^+ \pi^- \eta \) | | | |
| \( \Lambda T\), \( \Lambda \Sigma^{0}, \Sigma^{0} \Sigma^{0} \) | 232 | LO | 25 |
| 3((\pi^+ \pi^-)), 2((\pi^+ \pi^-)\pi^0), K^+ K^- 2((\pi^+ \pi^-)) | 232 | LO | 24 |
| \( \pi^+ \pi^- \pi^0 \) | 89 | LO | 23 |

Figure 7: Recent predictions of the value of \( a_\mu \) in chronological order [8–15], after the experimental value [3] is subtracted. Blue : \( e^+e^- \)-based; Green : \( \tau \) spectral function-based; Black: \( e^+e^- \) and \( \tau \) combinations.
6 What about $a_\mu$ then?

The time evolution of the prediction of $a_\mu$ with the availability of experimental results of increasing precision and with the development of combination techniques is shown in Fig. 7.

- After $\rho - \gamma$ mixing is taken into account, the discrepancy between the combinations based on $e^+e^-$ results and those based on the $\tau$ decay spectral functions [10] is resolved [14].

- The discrepancy between the prediction and the measurement still sits close to 3 – 4 standard deviations.

- Given that the precision of most of BaBar measurements is now dominated by the contribution of the systematics, it will most likely be difficult to achieve major improvements at a future super-$B$ factory.

- Thanks to the high-precision results obtained up to the end of 2014, the uncertainty on $a_{\mu}^{VP}$ is now smaller than $4 \times 10^{-10}$ [15]. That work includes a NNLO correction for $a_{\mu}^{VP}$ [17] and a NLO contribution to $a_{\mu}^{LbL}$ [18]. Given the spread of the values predicted by the available models of light-by-light scattering, the global uncertainty on $a_{\mu}^{LbL}$ is of the same order of magnitude [9,15].

- Indeed, new measurements of $a_\mu$ at Fermilab [19] and at J-PARC [20] are eagerly awaited.

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