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Measurement of elastic properties of materials employing 3-D DIC in a Cornu’s experiment

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Abstract

Measurement of elastic properties, especially the Poisson’s ratio, utilizing non-contact techniques in a tensile experiment is very challenging. This is primarily due to the poor spatial resolution and the large strain noise inherent to these techniques. The difficulty level increases many folds when Poisson’s ratio of less elongating, stiffer, and/or brittle materials, like ceramics and ablatives, is measured. This paper reports a newer approach that employs 3-D digital image correlation (3-D DIC) in a Cornu’s experiment to enable accurate measurement of elastic properties in a single test. The deflection field obtained from 3-D DIC in the form of anticlastic surfaces during Cornu’s experiment is utilized for determining Poisson’s ratio. In the same experiment, the elastic modulus is estimated using the center point deflection method. The proposed methods are validated with standard materials and extended to newly developed materials. Cornu’s method with 3-D DIC can provide the elastic properties with ease and has many advantages over other conventional techniques.

1. Introduction

Poisson’s ratio and elastic modulus are the two important inputs for stress analysis of mechanical, civil and aerospace structures. Reliable stress analysis depends upon the accurate determination of these parameters. Poisson’s ratio, in general, is determined experimentally by employing various direct and indirect methods [1]. In the direct methods, it is calculated directly from the experimental results. The indirect methods however utilize other elastic constants such as the elastic modulus (E), shear modulus (G), and bulk modulus (K) to evaluate Poisson’s ratio wherein multiple tests are needed. In this paper, an attempt is made to measure Poisson’s ratio and elastic modulus directly in a single and simple experiment. It is also well-known that elastic properties can be determined using static or dynamic experimental methods [1, 2]. However, the scope of this paper is limited to measuring the elastic properties through a static method assuming the materials to be isotropic and homogeneous. In the following, a brief review of the static experimental methods of measuring elastic material properties is presented.

The static methods of measurement involve tensile testing and flexural testing of the specimens. In tensile tests, the longitudinal extension and lateral contraction measured from strain gages (SG) are generally used for direct estimation of the Poisson’s ratio and elastic modulus. The primary drawback of this method is that it is only useful for materials that allow the mounting of the gages. Moreover, mounting of the SG must not influence the local strain fields of the materials under testing. Therefore, it is not suitable for lower modulus and brittle materials such as silica tiles, ceramic ablatives, and elastomeric insulation foams. To alleviate such issues, non-contact methods can be used, for example, marker or grid methods [1, 4]. These techniques track inscribed markers on the surface of the specimen to measure the extensions in the longitudinal and lateral directions in tensile tests. The marker and grid methods are particularly useful for highly elongating materials like elastomers. However, it is not suitable for hard and stiff materials owing to its poor spatial resolution.
The spatial resolution related issues in the marker or grid methods can be resolved by using the state-of-the-art Digital Image Correlation (DIC) technique [5]. Until now, DIC has been exploited only in tensile tests for measuring the elastic properties [4, 6–10]. In the tension tests, strain is evaluated mostly in the longitudinal and lateral directions to estimate Poisson’s ratio and elastic modulus. For lesser elongating materials (with smaller Poisson’s ratio) which fail below 0.1 to 0.2% elongation, e.g. ceramics, ablative, ceramic-composites, carbon-carbon (C-C), insulation tiles and glass, the total lateral contraction is in the order of 0.01% to 0.02% or less. Hence use of DIC in such tests suffers from large strain errors. Often 2-dimensional DIC (2-D DIC) has been used for such planar specimen tests. If 2-D DIC is used for the above materials, then even a small out-of-plane displacement would corrupt the strain fields tremendously [11]. Ultimately it would lead to the wrong calculation of the elastic properties. Hence, estimation of the elastic properties of low modulus and/or brittle materials utilizing a conventional tensile testing machine and the techniques like DIC remains a challenge. Furthermore, performing tensile tests with brittle and low modulus materials seems non-feasible, because gripping the specimens in a tensile machine is difficult. The gripping devices may crush or damage the specimens during testing. Therefore, a technique in which testing of specimens can be achieved without gripping is desirable.

Towards this end, testing a specimen under flexure/bending would be an attractive option for assessing the elastic properties of materials. The famous anticlastic behavior of beams and plates under pure bending can be exploited to measure the Poisson’s ratio [12] (see also reference [13]). The anticlastic bending produces hyperbolic deflection profiles in the specimen [14–17]. Cornu, a French scientist, was the first to use such information to calculate the Poisson’s ratio [18]. The conventional Cornu’s method was limited to transparent materials alone. Other optical techniques were thus devised to measure Poisson’s ratio of non-transparent materials, e.g. holography [19–21] and electronic speckle pattern interferometry (ESPI) [22]. Even if these techniques can measure micron level deflections, a complex experimental setup with laser safety is needed. Moreover, the fringe maps produced in the above techniques do not provide any quantitative information, hence data interpretation becomes involved. One of the main problems with interferometry techniques is the elimination of rigid body displacements which would almost corrupt the anticlastic fringe patterns. Moreover, the bending tests exhibit deflections of the order of millimeters. Speckle decorrelation would be a major issue in the laser interferometry techniques under such large deflections. Therefore, interferometry techniques are harder to implement in conjunction with Cornu’s method. It must be noted that elastic modulus also can be easily determined from the flexural deflections in a Cornu’s experiment [18]. However, such attempts have been rarely made with laser interferometry techniques, particularly in a Cornu’s setup.

This paper revisits the well-known Cornu’s experiment utilizing 3-D DIC so that the challenges associated with the contact and non-contact techniques are addressed. Such an improvement in Cornu’s experiment enables accurate measurement of the Poisson’s ratio and the elastic modulus simultaneously. Stereo or 3-D DIC has been established as a reliable method of measuring 3-D displacements with micron level of accuracy [5]. This special feature of 3-D DIC has been exploited in this paper for implementing Cornu’s method. The anticlastic bending behavior mapped with 3-D DIC facilitates the measurement of Poisson’s ratio directly. Moreover, the same Cornu’s experiment can provide the elastic modulus [18], if a proper test arrangement is used. The displacement/deflection based strategies developed in this paper are entirely different from the non-contact strain/extension based measurements. Moreover, Cornu’s experiment with 3-D DIC provides quantitative data about the anticlastic behavior which is difficult to achieve with interferometric techniques. This paper demonstrates the complete elastic characterization of materials using 3-D DIC in a Cornu’s setup. Recently, a patent has been granted based on the experimental setup and methodology presented in this paper [23].
The paper is organized as follows. The initial part of this paper exhaustively deals with the determination of Poisson’s ratio followed by a small section on simultaneous measurement of both Poisson’s ratio and elastic modulus. This arrangement is planned to accommodate two different options of determining the Poisson’s ratio. The measurement principles for Poisson’s ratio are described in the next section followed by the materials and methods section. In the materials and methods section, the measurement procedures for Poisson’s ratio are also elaborated. Subsequently, the results validating these procedures are discussed. Afterwards, the capability of 3-D DIC in determining both elastic modulus and Poisson’s ratio in Cornu’s experiment has been demonstrated. In the penultimate section, the measurement accuracy in obtaining the elastic properties is discussed. The paper is summarized in the conclusion.

2. Measurement principles of poisson’s ratio

The anticlastic behavior of rectangular beams is explained in this section considering the coordinate systems shown in figure 1. The deflection of the beam (w) is along the z-axis whereas the plane of bending is the xz plane and the neutral axis passes along the y-axis. The beam before and after deformation are shown in dotted and solid lines respectively in figure 1. The centroidal planes of the beam are represented as the xy, xz, and yz planes in figure 1.

A counter-clockwise (CCW) moment is applied on the beam (figure 1(a)) such that the top fibers are under compression and the bottom fibers are in tension. Due to the Poisson effect, the top fibers would try to expand in the yz plane and the reverse would happen to the bottom fibers. Thereby there would be bending in the yz plane too which is shown in figure 1(b). In the illustration in figure 1, the first principal curvature is concave downwards in xz plane with a radius of curvature \( R_1 \). The opposite would happen in the yz plane with a convex upwards bending with a radius of curvature \( R_2 \). This kind of bending in a beam is known as anticlastic bending [14–17], because the radii of curvatures are having opposing signs. Timoshenko and Goodier [12] have derived the equations for the pure flexural deformation of a beam as follows

\[
\begin{align*}
    u &= \frac{xz}{R}, \\
    v &= -\frac{\nu z y}{R}, \\
    w &= -\frac{1}{2R}[x^2 + \nu(z^2 - y^2)],
\end{align*}
\]

where \( u, v \) and \( w \) are the displacements along the three x, y, and z axes respectively and \( \nu \) is the Poisson’s ratio.

To infer the pattern of deflection in the xy plane, \( z \) and \( w \) can be held constant. Considering these conditions, equation (3) can be modified to obtain [12]

\[
x^2 - \nu z^2 = k,
\]

where \( k \) is an arbitrary constant. The loci of the curves in equation (4) with different values of \( k \) would provide a family of hyperbolas. Such hyperbolas are shown in figure 2(a), where a beam is under 4-point bending. A three-dimensional view of the anticlastic surfaces resulting from such a bending in a plate is shown in figure 2(b), where the two principal curvatures \( R_1 \) and \( R_2 \) are also shown. Similar derivations can be made for plate configurations also [12, 14–17]. The anticlastic information from a pure bending of a beam or a plate can be used to measure the Poisson’s ratio in two ways as described below. Both these methods have been proposed by Cornu.

Figure 2. (a) The pictorial depiction of hyperbolic deflection profiles at the center of a beam under 4-point bending as a result of anticlastic bending and (b) 3-D view of anticlastic bending of a plate with CCW moment.
2.1. Poisson’s ratio using the angle between the asymptotes of the hyperbolas

The deflection \( w \) (out-of-plane displacement) profile in a 4-point bending setup results in a family of hyperbolas as per equation (4). The acute angle between the asymptotes \( 2\alpha \) as shown in figure 2(a) can be measured from the deformation patterns which directly relates to Poisson’s ratio as [12, 19]

\[
\nu = \frac{1}{\tan^2(90 - \alpha)}. \quad (5)
\]

Poisson’s ratio is very sensitive to the angle between the asymptotes, hence accurate determination of this angle holds the key for successful measurement. Unfortunately, if the hyperbolic pattern shifts or gets disturbed due to rigid body displacements, then it is hard to obtain the angle between the asymptotes, such a case will be highlighted in an upcoming section. Moreover, it is important to locate the iso-deflection contours for the measurement of asymptotic angles which can be directly obtainable from 3-D DIC. However, holography and ESPI directly provide fringe patterns without any spatial and numerical information. If the fringe spacing is denser it is hard to locate fringes with similar deformation in these techniques. Contrary to this if the fringe density is less, then the larger width of the fringes would make it difficult to identify the iso-deformation asymptotes correctly. Therefore, the fringe pattern needs to be interpreted in numerical terms Such a requirement in the interferometric techniques can be accomplished with phase map generation and unwrapping which are very complex and error prone as compared to the direct interpretations in DIC.

2.2. Poisson’s ratio directly using the two principal curvatures

Cornu has devised an equipment with a Newton’s ring apparatus to image the variable air thickness caused due to bending of a beam when a plano-convex lens is placed over it. The interference patterns create elliptic profiles from which the two principal radii of curvatures \( R_1 \) and \( R_2 \) can be measured. These two principal radii of curvatures provide Poisson’s ratio as [18, 24]

\[
\nu = -\frac{R_1}{R_2}, \quad (6)
\]

where \( R_1 \) and \( R_2 \) are the smaller and larger radius of curvatures in the experiment. Here, the sign of the radius of curvature must be taken into account.

Cornu’s method implementing this principle is available as commercial equipment which is used in undergraduate physics laboratories for determining Poisson’s ratio of glass, perspex and other transparent materials.
materials. However, since these are limited to transparent materials, they cannot be used for characterizing opaque materials.

3. Material and methods

In this section, the material and experimental details are described followed by the measurement procedures which implement the above measurement principles in a 3-D DIC setup to obtain Poisson’s ratio alone. In a subsequent section, the simultaneous measurement of elastic modulus and Poisson’s ratio would be dealt with.

3.1. Materials

Three types of materials e.g. (i) metallic alloys, (ii) elastomers, and (iii) ceramics, with known properties, were tested to validate the procedure for Poisson’s ratio. The metal alloys have Poisson’s ratio ranging from 0.28 to 0.35, the elastomers have Poisson’s ratio larger than 0.35 and the ceramics have Poisson’s ratio below 0.25. Hence, Poisson’s ratio in a wide range (soft/ductile to brittle/hard) was planned for validation purpose. Two newly built materials namely medium density ablatives (MDA) and polyurethane foam (PUF) also have been tested whose properties were not known. However, both Poisson’s ratio and elastic modulus of these materials have been determined in the same experiment simultaneously. Hence, the procedures and results for these materials would be presented in a forthcoming section.

The specimen sizes must cater to the experiential setup and must have dimensions in line with the beam or plate assumptions. Several materials with various dimensions, which have been experimented, are listed in Table 1. Herein, the breadth to thickness ratios are also provided which gives an idea about the nature of the specimens (a beam or a plate; \( b/t > 10 \) represents a plate).

3.2. Experiment details

A 3-D DIC system (supplied by M/s Correlated Solutions [26]) was used in the 4-point bending experiments for determining the elastic properties (figure 3). The schematic of the loading arrangement is shown in the inset in figure 3. The specimens were supported at the knife edges with almost equal overhangs to achieve a simply supported boundary condition. The loading span was half of the span of the specimens (length between the knife-edge supports) and the distance between the knife-edge to the loading points was a quarter of the span as per ASTM standard [27]. The markings were made on the specimens with a scriber or marker pen to maintain proper alignment of the specimens on the loading fixture.

In DIC, random patterns, being the primary signal carriers, were created on one of the surfaces of the specimens using a spray paint gun. Initially, a white background was created on which black paint was sprayed to create a random distribution of black dots. The random pattern was cured before testing. For specimens made with LDA tiles, MDA tiles and PUF the natural background was sufficient, hence only black paint was sprayed over these specimens. A pair of cameras (PointGrey make with 5 megapixel resolution) were mounted on a tripod vertically above the specimen as shown in figure 3 to view the area of interest (AOI). Fixed focal length

| Sl. No. | Materials tested          | Length of sp (l) mm | Breadth (b) mm | Thickness (t) mm | b/t ratio | Loading span (L) mm |
|--------|---------------------------|---------------------|----------------|-----------------|-----------|--------------------|
| 1      | Mild Steel                | 200                 | 35             | 1.56            | 22.5      | 80                 |
| 2      | Aluminium Alloy (AA2014-T6) | 200                  | 30             | 1               | 30        | 80                 |
| 3      | Aluminium Alloy (AA2219)  | 245                 | 35             | 1.51            | 22.5      | 100                |
| 4      | Polycarbonate (PS1 [25])  | 200                 | 35             | 3.03            | 11.67     | 80                 |
| 5      | Glass                     | 200                 | 36             | 1.9             | 18.94     | 80                 |
| 6      | Araldite Epoxy             | 200                 | 35             | 3.2             | 10.94     | 80                 |
| 7      | Light density ablative (LDA)| 150                 | 24             | 5.6             | 4.3       | 60                 |
| 8      | Medium Density ablative (MDA)| 250                | 35             | 5               | 7         | 100                |
| 9      | Polyurethane Foam (PUF)    | 250                 | 35             | 15              | 2.33      | 100                |
imaging lenses (Schneider make with 35 mm focal length) were used for the experiments which had negligible distortions. The stereo cameras were connected to a laptop for image acquisition. Prior to the testing of the specimens, the stereo system was calibrated following standard procedures of VIC3D and VICSNAP [26]. The details about calibration and other theoretical descriptions of DIC can be found in reference [5]. After the calibration was completed, the specimen on the loading fixture were loaded in static mode by adding dead weights. During the experiments, images were stored in a computer for offline analysis. Images at three different loads were captured for each specimen to estimate the Poisson’s ratio. The displacements were computed from the stored images using VIC3D with a subset size of 101 pixels and a step size of 11 pixels. Strain calculation was not performed to reduce the processing time. The post-processing features of VIC3D were used to obtain the desired information for calculating the elastic properties. The procedures adopted for the measurement of Poisson’s ratio are presented next.

3.3. Measurement procedure
In section 2, the basic principles of measuring Poisson’s ratio were introduced. Now, the procedures followed for estimating this parameter using the outputs from 3-D DIC would be described.

During the loading of any specimen, rigid body displacements are inevitable. Such rigid body displacements disturb the anticlastic information and therefore it may introduce uncertainties in the measurement of the Poisson’s ratio. The VIC3D software facilitates the removal of rigid body displacements. Hence this feature of VIC3D was used to remove the rigid body displacements while estimating Poisson’s ratio. Thereby, the deflection data with rigid body displacements can be converted into a purely local deformation field. Such a process greatly simplified the interpretation of data. More importantly, the Poisson’s ratio measured from such local deflection fields was the most accurate. In the following, the procedures for the measurement of (i) the angle between the asymptotes and (ii) the radii of principal curvatures, using the post-processing module of VIC3D are described. The data presented in the following sections were taken from a typical AA2219 specimen to illustrate the measurement procedures.

3.3.1. Measurement of angle between the asymptotes
To effect the current procedure, initially, the deflection or the w-displacement profile in the post-processing module of VIC3D is displayed to visualize the iso-deflection contours. The number of contours is such adjusted that the iso-deflection contours are the part of two asymptotes of a hyperbola as shown in figures 4(a) and (d). Then two straight lines are drawn over the asymptotes. The coordinates of the endpoints of the two straight lines must be known to find out the angle between them. It is important that the two straight lines should intersect at

![Image](https://example.com/image.png)
the center of the specimen in the AOI. Keeping this requirement in view, the center points were marked on each specimen before the experiments which assisted in drawing the asymptotes accurately.

In figures 4(a) and (d), the endpoints of the two lines show the deflection values which are of almost similar magnitude. Such numerical information is difficult to obtain with interferometric techniques from the raw fringe outputs. In figures 4(a) and (d) the deflection contours show two different cases. In the first case in figure 4(a) the absolute deflections are shown which includes the rigid body displacements of the specimen. The rigid body displacements were the natural outcomes from the loading. In the second case (figure 4(d)), these rigid body displacements were removed to obtain the local deformation contours.

Now, the coordinates of the endpoints of the two straight lines can be found out from the X (figures 4(b) and (e)) and Y (figures 4(c) and (f)) contour maps in the AOI. Such spatial information can easily be extracted from VIC3D by displaying the corresponding contours. The X and Y metric contours are the undeformed coordinates of the specimen. Once the coordinates of the endpoints are known it is easy to determine the value of \( \phi \) given in equation (5). In figures 4(b), (c), (e) and (f) the X and Y contours are shown for two different cases representing the absolute and local relative deflection profiles. Two lines \( L_0 \) and \( L_1 \) are shown in figures 4(b), (c), (e) and (f) to extract the deflection and the spatial coordinates. Considering the co-ordinates of the endpoints of each line, the slope of each line can be obtained as

\[
\theta_0 = \tan^{-1}\left( \frac{Y_{20} - Y_{10}}{X_{20} - X_{10}} \right) \quad \text{and} \quad \theta_1 = \tan^{-1}\left( \frac{Y_{21} - Y_{11}}{X_{21} - X_{11}} \right),
\]

where \((X_{10}, Y_{10})\) and \((X_{20}, Y_{20})\) are the endpoint coordinates and \(i\) is 0 or 1 depending upon the chosen line. Now, the semi-angle between the asymptotes \( \alpha \) can be found as

\[
\alpha = \frac{\Delta \theta}{2} = \frac{\theta_1 - \theta_0}{2} = \frac{1}{2} \left[ \tan^{-1}\left( \frac{Y_{21} - Y_{11}}{X_{21} - X_{11}} \right) - \tan^{-1}\left( \frac{Y_{20} - Y_{10}}{X_{20} - X_{10}} \right) \right].
\]

In equation (8), if the angle \( \alpha \) is less than 45°, then the formula in equation (5) can be directly used. Otherwise, it has to be subtracted from 90° and to be used in equation (5) to obtain the Poisson’s ratio.

To find the effect of rigid body motion on the estimated Poisson’s ratio with this procedure, the Poisson’s ratios obtained by considering the absolute deflection profiles (with rigid body displacements) and the local deflections profiles (without rigid body displacements) can be compared. The deflection contours in figures 4(a) and (d) are for a typical AA2219 specimen. The Poisson’s ratio obtained by considering the lines in figures 4(a) and (d) are 0.421006 and 0.33285, respectively. It is now clear that the rigid body displacements make the deflection profile asymmetric and if these deflection data are used for estimating Poisson’s ratio it provides an incorrect value (expected value is 0.33). Such issues are difficult to handle with other optical interferometry techniques. Fortunately, in figure 4(a), some form of information about an asymmetric hyperbola is available. If the rigid body displacements are dominating in nature, it is impossible to obtain even such deflection profiles. This would occur for cases where the loading setup is not sturdy and the loads are larger.

### 3.3.2. Measurement of the radii of principal curvatures

To implement the second method given by equation (6), the endpoints of two lines on the specimens, one parallel to the length of the specimen (X-axis) and the other perpendicular to it (Y-axis) passing through the center were marked prior to the experiments. In this procedure, the radii of curvatures are strictly calculated along the above-said lines to find out the anticlastic behavior and thereby the Poisson’s ratio. Since the \( w \)-displacement (deflection) profile is laid on the specimen images in VIC3D software, it is easy to draw lines on the deflection contours by aligning them to the already available marks on the specimen. This ensures the intersection of the two chosen lines at the center in an orthogonal manner. Now, the deflections along these lines can be stored to find the radius of curvature. The deflection profiles of the specimen surface along the X and Y axis denoted as \( \delta_x \) and \( \delta_y \) can be fitted to a quadratic equation as

\[
\delta_x = a_1x^2 + a_2x + a_3 \quad \text{and} \quad \delta_y = b_1y^2 + b_2y + b_3
\]

where \( a_i \) and \( b_i \) are arbitrary constants obtained from the curve fit.

Using calculus, the principal radii of curvatures can be calculated as

\[
R_x = \frac{1 + \delta_x'^{2}}{\delta_x''} \approx \frac{1}{\delta_x''}
\]

\[
R_y = \frac{1 + \delta_y'^{2}}{\delta_y''} \approx \frac{1}{\delta_y''}
\]

where \( \delta_x' \) and \( \delta_y'' \) denote the first and second derivative of the fitted deflection curves w.r.t. \( x \) or \( y \) depending upon \( \delta_x \) or \( \delta_y \). The approximated values of the radius of curvature hold when the curvatures are large enough.
which is the case with all the experiments in this paper. Now, equation (6) can be used to find the Poisson’s ratio as

\[ \nu = -\frac{R_1}{R_2} = -\frac{\delta_y}{\delta_x} = -\frac{b_1}{a_1}. \]  

Here, \( b_1 < a_1 \) as per the co-ordinate axes chosen, since the small radius of curvature \( (R_1) \) is along the \( X \)-axis.

To illustrate the current procedure, the deflection contours of the same specimen as in the previous section were considered as shown in figures 5(a) and (c). The deflection contours in figures 5(a) and (c) are the contours with and without rigid body displacements respectively. Both the cases are shown to find out the influence of rigid body displacements on the curvatures, thereby the Poisson’s ratio. The fitted curves to the deflections profiles along the \( X \)-axis and \( Y \)-axis for the case with rigid body displacements are plotted in figure 5(b) whereas figure 5(d) shows these fit curves after removal of rigid body displacements. Figures 5(b) and (d) show that there is almost no difference in the radii of curvatures (coefficients \( a_1 \) and \( b_1 \) as per equation (9)) when deflection profiles with and without rigid body displacements are used. For the case with rigid body displacement, the Poisson’s ratio is 0.3206 and without rigid body displacement, it is 0.3214. The Poisson’s ratios obtained in both the cases are closer to the expected value of 0.33. Hence, the removal of rigid body displacements does not play any role in the curvature method of determining Poisson’s ratio utilizing equation (6). However, in the previous (angle between asymptotes) method the accuracy gets influenced by the rigid body displacements.

4. Results for Poisson’s ratio

Since a single experimental setup has the ability to provide both (i) angle between the asymptotes and (ii) the two principal curvatures, Poisson’s ratio is measured in both the ways for comparison. Such a comparison would provide clues about an accurate and easiest method for finding Poisson’s ratio. In section 3.2, it has been mentioned that three images were captured for each specimen at three different loads for estimating the

Figure 5. The deflection contours in the AOI at the center of a typical specimen obtained as (a) the actual deflection (with rigid body displacements) and (c) the local deflection (without rigid body displacements). The two principal curvatures were measured on the specimen surface along the length of the specimen \( (X \text{-} \text{axis}) \) and along the transverse direction \( (Y \text{-} \text{axis}) \) for the cases (b) with rigid body displacements and (d) without rigid body displacements, respectively. In (b) and (d) the quadratic fits to the principal curvatures are also presented.
Poisson’s ratio. Herein, the averaged value of the Poisson’s ratio calculated after analyzing the said three images are reported for the materials listed in table 1. The Poisson’s ratio measured with both the methods is presented in table 2.

The Poisson’s ratio obtained for the metal alloys in serial numbers 1 to 3 are in good agreement with the values reported in [28] (page-33, table 2.1). The Poisson’s ratio for aluminium alloys and MS are given as 0.33 and 0.29 which are almost matching with those obtained with the proposed method in table 2. The Poisson’s ratio of the polycarbonate sheet (PS1 sheets supplied by M/s Micro Measurements, USA) is close to the reported value of 0.38 in [25]. The Poisson’s ratio for glass falls in the range of 0.2 to 0.25 and the value of 0.235 obtained here is closer to the value of 0.24 as reported in [28] (page-33, table 2.1). These results show that Poisson’s ratios for the well-known materials are in good agreement with the book values. This provided the necessary confidence to measure Poisson’s ratios for two other materials Araldite epoxy and LDA for which Poisson’s ratio was unknown. Araldite belongs to the elastomer category and LDA is a ceramic ablative. The Poisson’s ratio for Araldite is around 0.4 and for LDA it is less than 0.25, as expected from the behavior of such materials. Indeed, Araldite and LDA tiles are not standard materials. These materials are more of process dependent, therefore the Poissons ratios for these materials are not compared with any literature. The values for these materials are reported for the sake of demonstration only. The ongoing discussions suggest that the proposed procedures have the potential to determine Poisson’s ratio of any material that can be tested in a 4-point bending setup or simply a bending experiment.

The comparison of the two procedures shows that the values obtained with the angle between the asymptotes slightly overestimate the Poisson’s ratios than the curvature method (see table 2). The difference is slightly pronounced in the case of Araldite. It was seen in the case of the elastomers that the spacing between the isodeflection contours was small, hence the drawing of correct asymptotes was somewhat ambiguous. Moreover, a small amount of twisting owing to specimen flatness also can change the two principal curvatures. The contribution of both might be the reason for the difference seen between the two methods in Araldite. However, in the case of LDA a completely opposite behavior was seen. Light density ablatives (LDA) being brittle and non-homogeneous, the deflection profiles were not as smoother as compared to the metals and elastomers. Thereby, obtaining the correct angle between the asymptotes of the hyperbolic deflection profiles was difficult. Hence, the choice of the asymptotes would be approximate in such cases which might lead to the observed behavior. Furthermore, maintaining the flatness of specimens is also difficult in such materials, which might also contribute to the uncertainties in Poisson’s ratio estimation.

Among the two procedures, the curvature method is easier to implement and free of any ambiguities too. Moreover, the curvature method is less sensitive to the errors introduced by a user while drawing lines on the specimen. Therefore, it produces more repeatable and accurate data in lesser time as compared to the angle
method. However, the specimen flatness and the fixture alignment should be perfectly maintained to obtain accurate Poisson’s ratios. The Poisson’s ratios reported above in Table 2 are after removal of rigid body displacements, hence from the local deflection profiles alone. It should be noted that for comparison of the accurate results from both the above methods, it is necessary to remove the rigid body displacements. Otherwise, the results in the case of the angle method would always be erroneous. For the angle method, the choice of the iso-deflection hyperbolic contours also plays a main role in the accuracy of measurement. Therefore, more experience or skill is needed for a user who wishes to implement this method. Nevertheless, the angle method can be utilized as a complementary tool to verify the results obtained from the curvature method.

**Table 3.** Elastic modulus measured for mild steel (MS) and aluminium AA2219.

| Material name | Expt No | Elastic modulus (GPa) | Average value (GPa) |
|---------------|---------|----------------------|---------------------|
| Mild Steel    | 1       | 198.62               |                     |
|               | 2       | 198.54               | 199.1               |
|               | 3       | 200.12               |                     |
| AA2219        | 1       | 73.25                |                     |
|               | 2       | 73.35                | 73.1                |
|               | 3       | 72.71                |                     |

**Table 4.** The results of Poisson’s ratio and elastic modulus measured for MDA tile and PUF specimens.

| Material name | Specimen No | Expt No | Poisson’s Ratio Value | Mean | Elastic modulus Value (MPa) | Mean (MPa) |
|---------------|-------------|---------|-----------------------|------|----------------------------|------------|
| MDA           | 1           | 1       | 0.232                 |      | 1665                       |            |
|               | 2           | 0.222   | 0.223                 |      | 1904                       | 1893.3     |
|               | 3           | 0.215   |                       |      | 1911                       |            |
|               | 2           | 0.226   |                       |      | 2058                       |            |
|               | 2           | 0.235   | 0.2283                |      | 2066                       | 2057.3     |
|               | 3           | 0.224   |                       |      | 2038                       |            |
| PUF           | 1           | 1       | 0.353                 |      | 6.33                       |            |
|               | 2           | 0.356   | 0.359                 |      | 6.28                       | 6.3        |
|               | 3           | 0.368   |                       |      | 6.3                        |            |
|               | 2           | 0.379   |                       |      | 6.39                       |            |
|               | 2           | 0.376   | 0.374                 |      | 6.59                       | 6.34       |
|               | 3           | 0.367   |                       |      | 6.26                       |            |

Figure 6. Load (P) versus deflection (δ) plots obtained from typical specimens of (a) mild steel (MS) and AA2219 and (b) MDA tile and PUF samples for estimating elastic modulus. Here, AA2219 is denoted in an abbreviated form for the subscript of P as PAA.
5. Simultaneous measurement of Poisson’s ratio and elastic modulus

The 4-point bending experiment described in the experimental section, also provides an opportunity to evaluate elastic modulus in the same experiment. More precisely, in the testing setup, the flexural modulus of the materials can be measured. Interestingly, there is no need of repeating the experiment, the deflection data from the same experiment can be used directly for measuring elastic modulus. However, it is important to note that the actual deflection of the specimen is measured accurately. The method of removing the rigid body displacements as described in the previous section provides local deflection profiles. Therefore these are not suitable for determining elastic modulus. Hence, a test fixture where large-scale rigid body displacements are not present would be necessary for the measurement of elastic modulus. Towards this end, a rugged test fixture (figure 3) was designed and realized to minimize the rigid body displacements during this test. In the previous section, the test fixture was designed only for measuring Poisson’s ratio, therefore the rigid body displacements were more prominent. The new testing device was fabricated to overcome these effects so that it would be suitable for measuring elastic modulus.

The center point deflection ($\delta$) obtained from the improved test setup (figure 3) is used for estimating the elastic modulus as per the following equation

$$\delta = \frac{Pa(3L^2 - 4a^2)}{48EI}.$$  \hspace{1cm} (13)

where $P$ is the total load applied, $L$ denotes the total span of the beam, $a$ is the distance between the simply supported point to the load application point (here $a = L/4$), $I = bt^3/12$ represents the moment of inertia of the cross-section of the beam and $E$ is the elastic modulus. For determining $E$, the load versus deflection data is directly used from the experiments, thereby equation (13) is reformulated in the following form

$$E = \epsilon \left( \frac{P}{\delta} \right).$$  \hspace{1cm} (14)

where $\epsilon$ is a constant given by ‘11L$^3$/64bt$^3$’ and $P/\delta$ is obtained from the slope of the load-deflection curve in N/mm after linear curve fitting of the experimental data. This procedure would be utilized for evaluating the elastic modulus.

The main objective of this section is to evaluate both elastic modulus and Poisson’s ratio from a single experiment. The demonstration of Poisson’s ratio would be carried out for the two new materials, MDA tiles and PUF slabs as listed in table 1 (rows 9 and 10). In the meantime, elastic modulus would be determined for four materials, viz mild steel (MS), AA2219, MDA and PUF samples. It is noted that Poisson’s ratio of MS and AA2219 are already reported in the previous section. From the experience gained during Poisson’s ratio measurement, it was noticed that the curvature method was easier and accurate than the angle method. Therefore, while reporting the Poisson’s ratio for the MDA and PUF samples, the curvature method alone was followed.

Herein, the results for elastic modulus are presented first followed by the Poisson’s ratio. For validating the proposed measurement scheme for elastic modulus, initially mild steel (MS) and an aluminium alloy (AA2219) have been tested because the properties of these materials are well known. The load versus deflection plots for these two materials are reported in figure 6 (a). The elastic modulus for these two materials after using the slope of the load-deflection plots is presented in table 3. The measured elastic moduli are in good agreement with the...
values reported in the literature [28](page-33, table 2.1), wherein the elastic modulus for aluminium alloys and steel are reported to be 72 GPa and 193 to 207 GPa, respectively. The same measurement procedure was further extended for the MDA tile and PUF specimens. The resulting load-deflection plots for these specimens are shown in figure 6(a). The elastic moduli estimated for these materials are provided in table 4. The values obtained are quite repeatable for the three experiments conducted on each specimen. There was a slight variation between the specimens. Indeed, these materials are porous in nature wherein material homogeneity cannot be ensured entirely over the blocks from which the specimens were cut. However, the surface finish was good enough for measuring the parameters. With these uncertainties of material homogeneity, the specimen to specimen variation of elastic modulus was around 8% for the MDA tiles whereas the scatter was much lesser (≈1%) for PUF samples.

As mentioned earlier, the Poisson’s ratio for the MDA tiles and PUF samples were estimated using the curvature method alone. The plots of the two principal curvatures obtained for the typical specimens of MDA tiles and PUF samples are shown in figures 7(a) and (b), respectively. The equations for the deflection profiles in the two principal directions are also displayed inside these plots. The Poisson’s ratios were estimated using these deflection equations. The resulting Poisson’s ratios are reported in table 4 which are again repeatable for the three experiments conducted on each specimen. Small variation was seen from specimen to specimen owing to the reasons already mentioned. The specimen to specimen variation was around 4% for the PUF samples whereas for MDA tiles the scatter was ≈ 2%.

The results of this section demonstrated the ability of a single Cornu’s experimental setup for evaluating both the elastic properties. Such a possibility was never attempted with Cornu’s setups utilizing laser interferometry. Moreover, the scope of measuring the elastic properties for a wide variety of materials starting from transparent to opaque or soft to hard have not been demonstrated earlier using Cornu’s experiment.

### 6. Estimation of measurement accuracy

Until now, more emphasis was put towards measuring the elastic properties for a variety of materials to demonstrate the capability of the developed methodology. In this section, the measurement accuracy has been estimated by observing the repeatability of the results. Towards this, the uncertainty of measurement in both Poisson’s ratio and elastic modulus is assessed by repeating the experiments six times for AA2219, mild steel and polycarbonate (details provided in table 1). The six experiments were conducted for only one specimen for each material. Such a design of experiment is planned to understand the contribution of the experimental errors alone which includes the uncertainties in the loading as well as the optical setups. Otherwise, if several specimens are tested, the specimen to specimen scatter of material property would mix with the experimental uncertainties. In such a circumstance, it would be difficult to segregate the contribution of experimental errors alone.

| Table 5. The Poisson’s ratio and elastic modulus measured for AA2219, mild steel and polycarbonate for assessing the uncertainty in measurements. |
|---|---|---|---|---|
| Material name | Expt no | Poisson’s ratio Value (SD) | Mean (SD) | Elastic modulus Value in MPa | Mean (SD) in MPa |
| AA2219 | 1 | 0.3226 (0.0059, ≈ 1.8%) | 0.3267 | 72 471 | 72925 |
| (Threshold | 2 | 0.3261 (0.0073, ≈ 2.2%) | (0.0032) | 72 984 | (234) |
| Load ≥ 6 N | 3 | 0.3261 (0.0093, ≈ 2.8%) | ≈ 1% | 72 900 | ≈ 0.3% |
| | 4 | 0.3246 (0.0046, ≈ 1.4%) | | 73 043 | |
| | 5 | 0.3304 (0.0081, ≈ 2.5%) | | 73 122 | |
| | 6 | 0.3307 (0.0069, ≈ 2.1%) | | 73 023 | |
| Mild Steel | 1 | 0.2882 (0.0079, ≈ 2.7%) | 0.2899 | 198 058 | 199462 |
| (Threshold | 2 | 0.2921 (0.0078, ≈ 2.7%) | (0.0017) | 199 330 | (725) |
| load ≥ 20 N | 3 | 0.2891 (0.0069, ≈ 2.4%) | ≈ 0.6% | 199 870 | ≈ 0.4% |
| | 4 | 0.2881 (0.0071, ≈ 2.5%) | | 199 669 | |
| | 5 | 0.2899 (0.0058, ≈ 2.0%) | | 199 892 | |
| | 6 | 0.2917 (0.0091, ≈ 3.1%) | | 199 955 | |
| Poly— carbonate | 1 | 0.3803 (0.0036, ≈ 0.9%) | 0.3833 | 2502.2 | 2505.7 |
| (Threshold | 2 | 0.3913 (0.0102, ≈ 2.6%) | (0.0048) | 2495.4 | (6.2) |
| load ≥ 3 N | 3 | 0.3849 (0.0061, ≈ 1.6%) | ≈ 1.25% | 2509.1 | ≈ 0.3% |
| | 4 | 0.3809 (0.0044, ≈ 1.1%) | | 2505.1 | |
| | 5 | 0.3848 (0.0073, ≈ 1.9%) | | 2509.6 | |
| | 6 | 0.3775 (0.0065, ≈ 1.7%) | | 2512.5 | |
For measuring the Poisson’s ratio, the curvature method was deployed as was the case in section 5. In each experiment, the specimen was loaded in 10 to 13 steps. It was observed that Poisson’s ratio values were consistent beyond a threshold load for the three materials. These threshold loads are reported in table 5 (column-1). Poisson’s ratio was measured for the load steps beyond these threshold loads. For calculating the mean and standard deviation (SD) of the Poisson’s ratio, data from at least 10 load steps have been used. The mean and SD in Poisson’s ratio thus obtained in each experiment are reported in table 5 (column-3). Subsequently, mean and SD of Poisson’s ratio were obtained considering all six experiments for each material (table 5, column-4). In the case of elastic modulus, only one value can be measured from one experiment which is reported in table 5 (column-5). Finally, mean and SD of elastic modulus were calculated based on the six experiments (table 5, column-6).

It can be seen that the SD of Poisson’s ratio in each experiment was approximately 3%. However, the SD considering all the six experiments was less than 1.5%. The SD for elastic modulus was below 0.5% which is much lesser as compared to the uncertainty in Poisson’s ratio. Due to the lower level of experimental uncertainties, the accuracy in the measurement of elastic properties can be expected to be better. However, if the data in table 4 is considered, the SD in both the elastic properties for each material is around 4%. It must be noted that in MDA tile and PUF experiments only 3 to 4 load steps were used. Moreover, these specimens did not have a good surface finish for DIC based measurements as compared to the metallic and polymeric specimens. Hence, the experimental uncertainty is expected to be more in such samples. Therefore, for reporting the experimental uncertainties for any material, it is suggested that one specimen is at least tested 5 times with more than 10 load steps without damaging the specimen as per the experimental design presented here. Such an exercise would help in estimating the experimental uncertainties with good confidence.

7. Conclusion

New procedures implementing 3-D DIC in a Cornu’s experiment have been successfully demonstrated to measure the elastic properties of several materials. The procedures were purely based on the flexural deflections of the specimens. The introduction of 3-D DIC in Cornu’s setup simplifies the complexities of such an experiment which are generally faced while utilizing an interferometric technique. Moreover, it eliminates the problems associated with the strain and extension based methods when special materials are characterized. The extension of 3-D DIC to Conru’s experiment has the following further advantages:

(i) Elastic properties can be measured for almost all kinds of materials in a single experiment which saves considerable time.

(ii) The test and measurement schemes are simple (no holding and gripping issues), therefore suitable for low modulus, brittle as well as stiffer materials.

(iii) Loads much lower than the yield stress are used to determine the material properties which fully satisfy the elastic as well as non-destructive criteria.

(iv) Rigid body displacements are easily removable, therefore it increases the accuracy of Poisson’s ratio measurement.

(v) A single experiment provides both angles between the asymptotes and principal radii of curvatures. The comparison of the Poisson’s ratios obtained with both methods would increase the experimental confidence.

In this paper, materials with positive Poisson’s ratios only are tested. However, the methodology can be extended to materials with negative Poisson’s ratios too. Moreover, the procedures can be implemented to obtain the secant modulus and Poisson’s ratio in the plastic range. The limitation with the present method is that elastic properties of ultimately flexible materials cannot be measured, since 4-point bending cannot be performed on such materials. Nevertheless, flexible materials can be characterized easily in tensile tests, hence these materials need not be tested in flexure.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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