Solving high order thinking problem with a different way in trigonometry

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Abstract. This research is a descriptive-analytical study that aims to describe how student solve high order thinking (HOT) problem. The research subject was a student who was chosen based on HOT problem-solving in a non-routine way and took a mathematics course at Science Education of Universitas Sarjanawiyata Tamansiswa Yogyakarta DS Setiana¹. Data were collected using observation, documentation and interviews methods, then will be analyzed descriptively. Based on the analysis that has been done, it was found that: (1) the strategy used by the student to solve the HOT problem was obtained from the internet; (2) there are errors when student applied the strategy that has been obtained in solving the HOT problem; (3) the student have difficulties when solving HOT problem in the same way. The results of this analysis are expected to provide a little description for educational activists related to the solving of HOT problem by students.

1. Introduction

Improve the ‘thinking curriculum’ with a way that can foster skills for all students is a major educational challenge [1]. In addition, high-level thinking is very important for students to have because it is one of the aspects of job assessment. Job readiness skills are grouped into three skill groups: basic academic skills, high order thinking skills, personal qualities [2]. This makes the government must think seriously about how to improve students' high-level thinking skills as one way to increase human resources. In general terms, HOT refers to cognitive activities that are beyond the stage of recall and comprehension/understanding [3–5]. HOT skills will be achieved optimally if supported by right learning, while the right learning to improve HOT skills is problem-based learning [6–10]. Problem-based learning is a learning approach that begins with focusing students with mathematical problems [11].

Explicitly, HOTS is implemented in aspects of the curriculum because curriculum standards emphasize the ability of students to apply, analyze, assess, and create knowledge through the teaching and learning process in schools [12]. On the other hand, in solving a mathematical problem can use a variety of strategies and allow several correct answers [11]. The HOT problem can be said as a problem that can be solved through the process of analyzing, evaluating or creating [13]. The problem is that not all students can solve HOT problems. For example, given a problem that can be seen in Figure 1. The problem in Figure 1 is a problem that must be solved through several steps. In addition, the problem is solved by connecting trigonometric concepts and plane concepts, especially the triangle.

There are many strategies to solving the problem in Figure 1, both of these strategies are: (1) solving with sinus theorem; (2) solving with cosine theorem. These strategies are the fastest strategies for solving the problem given. But sometimes the students use different ways to solve it, using the base and height
of the triangle. The problem is that it is not easy to find the height of the triangle, because it requires the application of the Pythagorean theorem. In fact, the Pythagorean theorem can be applied if at least two sides are known. Finally, sinus or cosine theorem is needed to solve the problem even though it starts in a different way. This wasting time certainly. Moreover, it can make students make mistakes in their calculations.

Determine the area of the triangle below!

![Diagram of a triangle with sides AB, AC, and BC, and an angle of 75° at vertex A and a side of 10 cm]

**Figure 1.** HOT problem in trigonometry

Errors in the selection of strategies to solve the problem in Figure 1 show that students cannot yet filter correct and efficient strategies. In other words, students are accustomed to using routine and familiar strategies without thinking of other strategies that are more effective and efficient. Students with the ability at this stage are at the level of applying those classified into Middle Order Thinking (MOT) [13, 14]. Based on the problems that have been explained, it can be said that the selection of strategies when resolving HOT problems is one indicator in determining the level of student cognitive processes. This study will describe how the student solves HOT problem with non-routine strategies as well as problems that occur when resolving the HOT problem. Furthermore, the results of this study can be used as a reference for further research related to solving HOT problems by students.

2. Method

This research is a descriptive-analytical study that aims to describe how student solve high order thinking (HOT) problem of trigonometry. The research subject was a student who was chosen based on HOT problem-solving in a different way than another student. The subject of the research was a student who took a mathematics course at Science Education of Universitas Sarjanawiyata Tamansiswa Yogyakarta. The data in qualitative research takes the form of words or pictures rather than numbers. Often the descriptive data contains quotations said by informants to illustrate and substantiate the presenting findings. Data can include; transcripts, field notes, photographs, video recordings, audio recordings, personal documents and memos [15]. In this study, data were collected using observation, documentation and interviews methods, then will be analyzed descriptively. In addition, several studies were used as references to support the results of the analysis.

3. Result and Discussion

The data obtained in this study are: (1) the behavior of the research subject (S) in trigonometric learning; (2) HOT problem test results from S at the level of evaluating and (3) S answer scripts when interviewed about how and why S used non-routine methods to solve the HOT problem. Furthermore, S tries to solve the HOT problem in the same way at different times. But, it will be explained first about the design of the HOT problem that is done by S. HOT problem can be seen in Figure 2.
Based on the picture, is it true
\[ \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta? \]

**Figure 2. HOT problem in evaluation level**

Figure 2 shows the HOT problem given to S. This problem includes 'evaluating' which is a checking category at the cognitive level. 'Checking' is testing the consistency or deficiency of a product based on internal criteria (the criteria that correspond to the product). 'Checking' leads to testing activities that are inconsistent or failures of an operation or product [13]. The problem in Figure 2 provides an opportunity for students to check the truth of a theorem in geometry.

3.1. A strategy that student used to solve the HOT problem

There are many strategies that can be used to solve HOT problems in Figure 2 (HP). However, not all students can choose an efficient strategy for solving HP. Actually, it doesn't matter when students choose a different strategy because the HOT problem does have many problem-solving strategies and some correct answers. But, if this strategy complicates the student or takes longer than other strategies, the strategy becomes inefficient.

In solving HP, S uses the concept of polar coordinates and applies them when doing algebraic proofs. The steps carried out by S are: (1) changing P, Q, O into polar coordinates; (2) use the polar coordinates in the distance theorem; (3) apply some algebraic operations to get the desired form. Algebraic proof by the student based on these steps can be seen in Figure 3.

**Figure 3. S work description when solving HP**
Figure 3 shows the S work when solving HP. E1 and E2 are the main mistakes that S makes when solving HP. E1 indicates that S made a mistake when understanding HP. E2 indicates that S made an error when applying the distributive theorem so that this error makes the proof is incorrect. However, the proof in Figure 3 can be done if P and Q are passed by the same circle and there were no errors in the computation which could result in the proofing being incorrect.

3.2. Why the student used that strategies?
The strategy used by S gives a new question, such as: why does S use the strategy? How did S get the strategy? how can S make mistakes when implementing the strategy? Then, is S able to use the same strategy to work on HP at different times? Therefore, further discussion is needed related to this matter. The discussion obtained was based on a comparison of data from interviews, observation, and documentation.

Based on the interviews that have been done, it was obtained that S used the strategy because it was assumed easier than other strategies such as geometric proof. Moreover, S explained that the strategy was studied independently through the internet. It can be understood that S has self-regulated learning because taken the initiative to learn things that have not been instructed. One of the characteristics of individuals having self-regulated learning in them is that the individual is able to design his own learning according to the needs or goals of the individual concerned [16–18]. However, the self-regulated learning that utilizes the internet as a learning resource should be mentored by the teacher so that there is no mistake in receiving knowledge. Unfortunately, S did not ask or clarify the results have been discovered on the internet to the teacher. It can be a difficult source of S when given problems with the same cognitive level but in a different form.

The idea that the strategy is easier than other strategies cannot be said to be right or wrong, because it is easy or not to solve a problem depending on who resolves it. But, if the strategy is not efficient, then the possibility of errors will also be higher. Factually, S in trigonometric learning tends to choose a strategy that takes a long time to solve the HOT problem. It can be seen in learning that happens which S uses a strategy that takes longer when solving a simple problem. For example, students are asked to investigate whether \( \cos(-30^\circ) = \cos 30^\circ \). Then some students presented it in front of the class. S as one of the participants, uses the steps: (1) change the degree \((-30^\circ \) to \((360 - 30)^\circ = 330^\circ\); (2) find the value of \(\cos 30^\circ\) and \(\cos 330^\circ\); (3) compare the results in the second step to investigate the truth of \(\cos(-30^\circ) = \cos 30^\circ\). This strategy is certainly not false and even tends to show a strengthening of the concept. Indeed, a strategy like this can be used as a daily assessment. But on examination, of course, the negative impact is more than the positive impact for students because it takes a long time to solve it. The solve of a problem that is getting longer than the probability of making mistakes are also getting bigger. This results in a failure to solve the problem. The main reason underlying student failure in problem solving is that they cannot monitor themselves (in this case it is not appropriate to choose a strategy) during problem solving [19].

Based on the explanation above, it can be said that S obtained the strategy used to solve HP through the internet. But the character S who is accustomed to choosing an inefficient strategy provides more opportunities to make mistakes so that errors such as E1 and E2 arise. But besides the negative impact, the selection of a strategy like this also has a positive side, namely the existence of learning independence, training creative thinking skills of students and strengthening basic concepts in solving a problem. Some behaviours that show creative thinking are able to produce some ideas, answers or questions from different perspectives [20]. Another hand, when children discover different strategies, they feel comfortable in reasoning processes [21]. Therefore, choosing a strategy like this is very suitable to be used as a daily assessment of students.

In the previous discussion, E1 indicates a conceptual error when S solves the HP. In Figure 2, S assumes that \( \overline{OP} = \overline{OQ} = r \) so that the \( \triangle OPQ \) triangle is an isosceles triangle. This is certainly impossible because Figure 2 shows that \( \overline{PQ} \) is perpendicular to \( \overline{OQ} \). Moreover, there is no information showing the lines have the same size. Therefore, the only rational reason is that S thinks that P and Q are the coordinates on a circle with center \((0,0)\) and radius \(r\). In addition, S also assumes that \( \overline{PQ} \) is not perpendicular to \( \overline{OQ} \). The geometry interpretation of this situation can be seen in Figure 4.
Figure 4 shows the initial thinking S when solving the HP which is adjusted to Figure 3. It is clear that S defines some new coordinates, O and S, which are then converted to polar coordinates. Moreover, there are differences in the location of the coordinates with the coordinates in Figure 2. Therefore, at this step S makes a deviation when understanding HP. These deviations have a mismatch between the solutions made by S and the context of the HP. That is, S has not been able to associate the basic concepts that already exist in HP with the concept of solving HP through the chosen strategy. At this step, S has not categorized into conceptual knowledge [13, 14].

![Geometry interpretation of the initial understanding of HP by S](image_url)

Figure 4. Geometry interpretation of the initial understanding of HP by S

E2 indicates procedural errors when S solves HP. This error occurs when S applies the distributive theorem. At this step, S has not been able to apply the theorems that exist to resolving HP so that S has not been categorized into procedural knowledge [14, 22]. In Figure 3, E2 consists of two errors, E2a and E2b. E2a occurs because S is not careful in solving the HP. While E2b shows a deviation, another form appears in this section. Moreover, there is no computation that shows how \(-2r^2\) is found in the HP solving algorithm. This shows an intentional effort by S so that the HP is resolved. Why? Let’s discuss this. In fact, based on the interviews that have been done, S intentionally added \(-2r^2\) so that the HP can be resolved in accordance with S’s expectations. In general, this thought is not so clear. But if we studied more, it can be said that S does a knowledge deviation exactly as before. The thing that should be done when someone does a computation error is to check what has been done.

In addition to E1 and E2, another surprising thing is that S can’t finish using the same strategies at different times. And S is only finished in step in Figure 4, and that’s not right. This situation shows that S make independent learning by memorizing so that S is unable to solve the HP at different times. In other word, something that has been learned doesn’t last long in the cognitive structure of S. The independent learning become meaningless and sometimes to be obstacles for S in different learning contexts. Meaningful learning is a cognitive process in the association, the creation of new knowledge with relevant concepts that already exist in a person’s cognitive structure, which it’s needed in successful problem solving [23].

Then what should the teacher do when these problems occur? Generally, it can be said that the mistakes that occur when S solve the HP because of S learning behaviour that has not been well directed. It is the task of a teacher to direct the student in the correct direction. Scaffolding can be used by the teacher to solve some problems in learning. In addition, scaffolding can be an effective technique in improving learning outcomes, learning activities, in solving a mathematical problem [24, 25]. The concept of scaffolding is used to define and explain the role of adults or peers who support children's learning and development. Although not giving the right steps about how the teaching process will occur, scaffolding provides an understanding of the interactions between adults and children [26]. Scaffolding
has 3 levels, namely: (1) environmental provisions; explaining, reviewing, and restructuring; (3) developing conceptual thinking. Environmental provisions are preparing a learning environment. Explaining is giving the concept to be learned. Reviewing is refocusing students' attention. And restructuring is rebuilding understanding capabilities. Developing conceptual thinking is teacher interaction directed to the development of conceptual thinking [27]. Teachers can implement learning based on these levels to minimize student errors in solving a mathematical problem, especially those that correspond to problems previously described.

4. Conclusion
The strategy used by the student to solve the HOT problem was obtained from the internet. There are errors when student applied the strategy that has been obtained in solving the HOT problem. The student have difficulties when solving HOT problem in the same way. Therefore, there needs to be further action regarding these results. For example, do more in-depth research on the effect of the internet on HOT skills in trigonometry or other discussions.

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