The Analysis of Students’ Mathematical Thinking based on Their Mathematics Self-Concept

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Abstract. This research aims to describe the students’ mathematical thinking based on their mathematics self-concept. In addition, this study also aims to determine whether there are differences in mathematical thinking between students who have positive mathematics self-concept (MSC) with students who have negative MSC and to know how much influence MSC to the students’ mathematical thinking. There are four indicators of students’ mathematical thinking which is used in this study. First, Specializing, i.e. trying the problem by looking at the example, paying attention to a simple case. Second, Generalising, i.e. looking for patterns and relationships. Third, Conjecturing, i.e. predicting relationships and results. Fourth, Convincing, i.e. finding and communicating the reason why something is right.

This research used a descriptive method. The population of this study is all senior high school students in Subang, West Java. Instruments in this research are the test of students’ mathematical thinking and MSC questionnaire. Based on the finding of this research, generally, students who have positive MSC, have good mathematical thinking abilities. Mann-Whitney U test results showed that significantly there are differences in mathematical thinking between students who have positive MSC with students who have negative MSC. Variables of MSC give positive and big influence on the students’ mathematical thinking.

1. Introduction

The objectives of mathematics education in school include formal objectives which emphasize organizing, reasoning, and shaping students' personality; and material objectives which emphasize problem-solving skill and applying mathematics ([1]). In achieving those material objectives, numerous researchers suggest that mathematics learning inside the classroom should imitate a mathematician ([2]). When a mathematician works on the mathematical problem, the activity will include exemplifying, specializing, completing, deleting, correcting, comparing, sorting, organizing, changing, varying, reversing, altering, generalizing, conjecturing, explaining, justifying, verifying, convincing, refuting. Those activities, according to [3], are several of the activities that eventually will build students' mathematical thinking skill. Therefore, the objectives of mathematics education in school will be achieved, if the learning process is able to construct a mathematical thinking skill.

There are four basic processes in the construction of mathematical thinking skill, according to [4]. Those four processes are (1) specializing, is the students’ skill in finishing numerous exercises by looking at the example; (2) generalizing, is the students skill in identifying patterns and connections; (3) conjecturing, can be defined as the students’ skill in predicting correlation and result; and (4) convincing or students’ skill in finding and communicating reasons why something is considered as true.
Mathematical thinking skill is closely related with mathematical literacy proposed by PISA. Stacey, in [5], states that the framework used by PISA to measure mathematical literacy includes several elements from mathematical thinking skill such as reasoning element, modeling and making connections among ideas. For that reason, it can be said that mathematical thinking skill is one skill that supports the mastery of other disciplines outside mathematics such as science, technology, economics, and even the advancement in the field of economy.

One of the processes that shape the mathematical thinking skill is convincing. This process demands students to have sufficient self-confidence in defending and convincing others about the truth from conjecture that they have decided ([4]). According [6] report there are several aspects that relate to mathematics self-belief, such as mathematics self-efficacy (MSE), which is individual’s self-belief in handling and solving mathematics problem effectively, mathematics self-concept (MSC) which is someone’s self-belief in accomplishing mathematics activities by him or her own self, mathematics anxiety is the feeling of being depressed and worried when dealing with mathematics, and engagement, the presence connection between mathematical and other activity both inside and outside classroom. Marsh (in [7]) defines mathematics self-concept (MSC) as an individual’s perception towards his or her own reasoning and mathematics skills.

According to the research conducted by Adegoke ([8]) written in a Journal of Studies in Education, it is found that students’ accomplishment in mathematics subject has a significant and positive correlation with their mathematics self-concept (MSC). The finding is parallel with what has been revealed by Marsh & Hau, 2013; Marsh, 2015; Liu, Wang & Perkins, 2018 (in [8]) which mention that the students' academic skills have a positive and significant correlation with their academic self-concept. In the meantime, Stacey (in [5]) declares that people will need to use their mathematical thinking skill when trying to solve particular problems using mathematics. Consequently, it can be said that the students’ achievement in mathematics subject is also can be identified base on their mathematical thinking skill. Therefore, it is assumed that there is a correlation between students’ mathematical thinking skill and their mathematics self-concept (MSC). This research aims to elaborate the analysis of mathematical thinking skill based on the senior high school students’ mathematics self-concept (MSC). In other words, the objective of this research is to uncover the students’ mathematical thinking skill based on the mathematics self-concept (MSC) they already have. Moreover, this research tries to investigate whether there is a difference in their mathematical thinking skill between the students who have positive mathematics self-concept (MSC) and they who have negative MSC or no as well as to uncover how huge the influence of the mathematics self-concept towards the mathematical thinking skill.

2. Methodology
This research is a descriptive one conducted in Subang. The population for this research is all senior high school students in Subang. The selection of senior high school students as the population is because they are going to make the decision on some study related to transformation and infinite series and line materials, with the composition one item for transformation and eight items
for infinite series and line. The test instrument is considered as valid through product moment r test with 5% coefficient level and has 0.805 Cronbach alpha coefficient correlation.

In the meantime, the mathematics self-concept (MSC) questionnaire is designed by modifying mathematics self-concept questionnaire used by PISA. The questionnaire used by PISA is a close-ended questionnaire with strongly agree, agree, disagree, and strongly disagree options consisting of five statements: (1) I am not good enough at mathematics, (2) I get a good score in mathematics, (3) I learn mathematics very fast, (4) I always believe that mathematics is one of the coolest subjects, and (5) I understand every mathematics material, even the most difficult one. The modification is performed through inserting numerous new statements and completing statement sentences by making them suitable for learning materials so that the number of statements in one questionnaire is thirty items; fifteen items for each transformation and infinite series and line. The mathematics self-concept (MS) questionnaire instrument is also considered as valid through Spearman r test with 5% significant level and 0.877 Cronbach alpha reliability coefficient.

3. Findings and Discussion

Data for this research consist of students’ mathematical thinking skill test score and their mathematics self-concept (MSC) questionnaire score. The students’ mathematical thinking skill scores are classified into three categories that low, average, and high. This classification is carried out by creating the frequency distribution table with the number of class is eleven. The following is the table of students’ mathematical thinking skill score frequency distribution for each group.

Table 1. Table of Students’ Mathematical Thinking (SMT) Score Frequency Distribution

| No | Group   | SMT Score | Number of Students | %  |
|----|---------|-----------|--------------------|----|
| 1  | Low     | 1 ≤ SMT Score ≤ 11 | 11                | 35 |
| 2  | Average | 12 ≤ SMT Score ≤ 22 | 8                 | 26 |
| 3  | High    | 23 ≤ SMT Score ≤ 32 | 12                | 39 |

By looking at the above table 1, it can be seen that the majority of the students have high mathematical thinking skill. According to the data, the mean of the overall mathematical thinking skill score is 17.42 with 10.17 standard deviation. Meanwhile, the percentage of the students who get mathematical thinking skill score above average is 51.61%. It demonstrates that more than half of the students have relatively high mathematical thinking skill.

As it is mentioned before, mathematical thinking skill has four indicators. The following table displays the students’ means score for each indicator.

Table 2. The Students’ Mathematical Thinking (SMT) Mean Score for each Indicator

| Students’ Mathematical Thinking (SMT) Indicators | Specializing | Generalizing | Conjecturing | Convincing |
|------------------------------------------------|--------------|--------------|--------------|------------|
| Mean                                           | 4.90         | 6.48         | 3.02         | 3.02       |
| Standard Deviation                            | 2.68         | 3.77         | 2.65         | 2.65       |

Based on Table 2, students are more capable when solving generalizing test items. These items request students to complete a line of numbers and to decide the pattern of those numbers. The generalizing aspect is an aspect of mathematical thinking skill that is naturally owned by every person. It is parallel with Gestalt's cognitive psychology theory who mentions that every person has a tendency to complete and to fill incomplete experiences to make them meaningful ([10]; [11]). The third year students of the senior high school are those who have already been familiar with the material of lines of numbers since they were in elementary school, and the more complicated version of this material then is relearned in senior high school. By doing so, when there is a recalling act of that material, the level of difficulty faced by the students will be lower compared with other types of test items.

Watson & Mason ([12]) mention that the test items containing generalizing indicator can help students to increase their ability in solving complicated problems. This statement is reinforced by [4] who demonstrate that before coming to the generalizing process, the students should step the process of
specializing, conjecturing, and symbolizing as it displayed in figure 1. This phenomenon tells us that generalizing skill is a series of process that constructs mathematical thinking skill. This makes the fact that the highest students' mean score lays in the test items that measure their generalizing skill is not surprising. This data echoes the aforementioned data that uncovers more than half of the students have higher mathematical thinking skill.

![Figure 1. The correlation between generalizing and specializing](image)

The mean score of each student's mathematics self-concept (MSC) is classified into two categories that are students with positive mathematics self-concept and students with negative mathematics self-concept. This classification is performed by creating two interval classes with the length of each class is 1.5. The following table displays the frequency distribution of students' mathematics self-concept (MSC) score in each group.

| No | Group          | MSC mean score | Numbers of student | SMT mean score | SMT Score standard deviation |
|----|----------------|----------------|--------------------|----------------|------------------------------|
| 1  | Positive       | $1.00 \leq \overline{MSC} < 2.50$ | 9                  | 21.89          | 8.28                         |
| 2  | Negative       | $2.50 \leq \overline{MSC} \leq 4.00$ | 22                 | 15.59          | 10.70                        |

In Table 3 above, the majority of the students have negative mathematics self-concept. The students who have negative mathematics self-concept get a lower mean score of mathematical thinking compared with those who have positive mathematics self-concept. This phenomenon is parallel with the theory proposed by Marsh, 1994; Managet & Eiklend, 1998 (in [13]) that there is a significant positive correlation between students' mathematics self-concept and their achievement in mathematics subject. This mathematics self-concept is closely related with the students’ self-confidence towards their mathematics mastery. There are students who still think that mathematics is difficult, and it, in turn, makes mathematics difficult for them. However, there are several students who have enough self-confidence when they are dealing with particular mathematics learning materials; in turn, this contributes to shaping their positive mathematics self-concept.

This research also uncovers the students’ mathematics self-concept towards each learning material. The following table displays data about the students’ mathematics self-concept for each learning material.

| Table 4. Students’ Mathematics Self-Concept (MSC) for each Learning Material |
|-------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                              | Transformation  | Infinite series |                  |                  |                  |                  |
| MSC                          | Numbers of student | SMT mean score | SMT Score standard deviation | Numbers of student | SMT mean score | SMT Score standard deviation |
|                              |                  |                  |                  |                  |                  |                  |
Based on the previous table 4, it can be observed that for transformation material, there is an anomaly where students with negative mathematics self-concept have higher MTS mean score compared with those who have positive mathematics self-concept. It is possibly because of the presence of a student who actually has good mathematical thinking skill, but less confident about her skill, in her daily life, she is introvert and rarely interacts with her peers. Moreover, there is also a student has good self-confidence but her mathematical thinking skill is low. In her daily life, this student is the one who is delighted to socialize with her peers and has a good artistic talent and is usually involved in the modern dance competition so that she has a high self-confidence. [14] and [15] state that the characteristic of students who have a tendency of mastering music and rhyme, tend to use their right side of their brain in thinking. [14] also mentions that those students usually have a good social life but find it is difficult to learn things that are more technical. Because of there are students whom their ways of thinking are dominated by their right side of their brain in this research, it causes although those students have a positive mathematics self-concept, their mathematical thinking skill is low. In this transformation material, students are only given one test item that measures only one indicator of mathematical thinking skill, which is specializing indicator, so that the score of this material does not represent the overall students' mathematical thinking skill. Therefore, this anomaly is not applicable to the students' mathematical thinking skill.

In the infinite series material, there is no such anomaly. It is because this material has the average level of difficulty. This material has already been known by the students since they were in their elementary school (although in the most simple form). The test items for this material measure all indicators of mathematical thinking skill. Consequently, this data reinforces the previous statement that claims, students with positive mathematics self-concept have higher mathematical thinking skill compared with those with negative mathematics self-concept.

A research indeed needs to draw the conclusion that can be generalized and be responsibly proven statistically. Therefore, in this research, it is also conducted proportion examination and examination of the difference of two means. The data used to perform both hypothesis examinations are the mathematical thinking skill score and the score of mathematics self-concept questionnaire for infinite series and line learning materials. Proportion examination was carried out to test whether the number of students with positive mathematics self-concept is significantly different from those with negative mathematics self-concept or not. The following is the data tabulation from each group.

| Table 5. Number of Students in each Group Data Tabulation |
|-----------------------------------------------------------|
| SMT Infinite Series | MSC Infinite Series | Negative | Positive | Total |
| SMT Infinite Series | Low | 8 | 1 | 9 |
| Average | 7 | 2 | 9 |
| High | 6 | 7 | 13 |
| Total | 21 | 10 | 31 |

The above table 5 demonstrates that the majority of students with negative mathematics self-concept have low mathematical thinking skill. The second biggest number of students is the group of students with positive mathematics self-concept and have high mathematical thinking skill. The difference of the attained score between students with positive mathematics self-concept and those with negative one is not too far. The Fisher's Exact Test proportion examination was carried out to investigate the significance of the difference between both groups of students. The upcoming table shows the result of the test.
Table 6. Fisher’s Exact Test

|                      | Value | df | Exact Sig. (2-sided) |
|----------------------|-------|----|----------------------|
| Fisher’s Exact Test  | 4.599 |    | .093                 |
| N of Valid Cases     | 31    |    |                      |

By giving a close attention to the previous table, it can be inferred that the proportion of students with positive mathematics self-concept and those with negative one is significant. In other words, it can be said that the number of the sample employed to test the difference of means is not too different significantly. The examination was also continued to test the data normality. The following table displays the result for the test.

Table 7. Normality Test

| MSC Infinite Series | Kolmogorov-Smirnova Statistic | Df | Sig. | Shapiro-Wilk Statistic | Df | Sig. |
|---------------------|-------------------------------|----|------|------------------------|----|------|
| SMT Infinite Series |                               |    |      |                        |    |      |
| Negative            | .167                          | 21 | .129 | .921                   | 21 | .092 |
| Positive            | .307                          | 10 | .008 | .789                   | 10 | .011 |

a. Lilliefors Significance Correction

According to the table 7 above, it can be observed that the group of students who have negative mathematics self-concept has abnormal data distribution. As a result, mean difference examination was performed using U Mann-Whitney test.

Table 8. U Mann-Whitney Test

| SMT Infinite Series | Mann-Whitney U | Z  | Asymp. Sig. (2-tailed) |
|---------------------|---------------|----|-----------------------|
|                     | 44.500        | -2.562 | .010                 |

The result of U Mann-Whitney uncovers that there is a significant difference in mathematical thinking skill between students with positive mathematics self-concept and those with negative mathematics self-concept. The table 8 is parallel with descriptive statistics in table 3 and table 7. The students with positive mathematics self-concept will have better mathematical thinking skill when it is being compared with those who have negative mathematics self-concept. This echoes what has been published in the PISA 2012 report that states, in every nation joint in OECD, the mathematics self-concept (MSC) is positively associated with their performance in mathematics. Furthermore, [8] also declares that there is a positive correlation between the students’ mathematics achievement and their mathematics self-concept (MSC). It corresponds to the previous research reports conducted by Marsh & Hau (2103); Marsh (2105); Liu, Wang & Perkins (2008) that reveal, the students’ academic achievement has a significant and positive correlation with their academic self-concept.

This research has unearthed that there is a significant difference of mathematical thinking skill between students who have positive mathematics self-concept and those who have the negative one. This difference indicates that the students’ mathematics self-concept will influence the students’ mathematical thinking skill. The level of influence given by the mathematics self-concept can be determined using Cohen effect size. Based on the result of the computation, it is revealed that the value of effect size from both variables is 1.039, from which can be concluded that mathematics self-concept...
variable positively and relatively hugely influences mathematical thinking skill. This finding is identical
with research conducted by [16] that analyzed TIMSS data and found that there is a positive correlation
between self-concept and mathematics subject achievement of the students in 16 different nations. Three
years earlier than that research, [17] found that there is a reciprocal effect between academic
achievement and academic self-concept, both variables are influencing each other, in which the change
in the students’ academic self-concept can cause the change in their academic achievement and vice
versa. Their research then is reinforced by [18] who declared that academic self-concept holds a crucial
role in increasing students' academic achievement.

The positive correlation between mathematics self-concept and mathematics achievement is also
revealed in the research carried out by [19] and [20]. The students' mathematics achievement can be
measured in various ways. The mathematical thinking test score that is measured in this research is one
of the students’ mathematics achievements so that it can be said that this research has confirmed the
correlation between the students’ mathematics self-concept and their mathematical thinking skill. This
relationship implies that if you want to improve students' mathematical thinking skills, the teacher can
improve students' mathematics self-concept first. Because it will be easier to touch the cognitive domain
of students if the students' affective is understood by the teacher. Therefore, at the beginning of the
lesson, the teacher should first measure the mathematics self-concept that is owned by the students, so
that the delivery of material can be given using an appropriate model.

4. Conclusion
Taking into account the result of data analysis elaborated on the previous section, it can be concluded
that the students with positive mathematics self-concept have higher mathematical thinking skill
contrasted with those who have negative mathematics self-concept. The result of U Mann-Whitney test
demonstrates that the mathematical thinking skill of the students who have positive mathematics self-
concept is significantly different from those who have negative mathematics self-concept. The
mathematics self-concept variable gives positive and significant influence on the mathematical thinking
skill. This research provides a recent reference about the interconnectedness of students’ mathematical
thinking skill and their mathematics self-concept. It is highly recommended for further researchers to
investigate the method to develop a learning model for the students with diverse mathematical thinking
skill and mathematics self-concept.

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