Ill-posedness of waterline integral of time domain free surface Green function for surface piercing body advancing at dynamic speed

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Abstract
The three-dimensional time domain free surface Green function is the sum of the potentials of a Rankine source and its image source and a wave integral. The wave integral is harmonic in the fluid domain upper bounded by the mean free surface plane \( z = 0 \) and is highly oscillatory when field and source points are close to the mean free surface plane. It is obtained that the limit form of the wave integral in the dimensionless formulation is the elementary function

\[
\int_0^\infty \sqrt{\lambda} J_0(\lambda) \sin(s \sqrt{\lambda}) d\lambda = \frac{s}{\sqrt{2}} \sin\left(\frac{s^2}{4}\right)
\]
on the mean free surface plane. The function \( J_0 \) is the zero order Bessel function of the first kind.

The velocity potential of the fluid motion problem of a surface piercing body advancing in waves is defined by a normal velocity boundary integral equation, which contains waterline contour integral of the normal derivative of the time domain free surface Green function. This elementary function form implies the ill-posedness of the boundary integral equation due to the unboundedness of the waterline integral.

Keywords: Time domain free surface Green function, potential flow, waterline integral, surface piercing body, translational wave body motion

1. Introduction

As a fundamental formulation in linear hydrodynamics, the velocity potential of a wave-body motion problem is expressed as an integral of time domain free surface sources distributed on wetted body surface and waterline contour. The time domain free surface Green function represents the potential of a free surface source with unit strength.

From [9, 16], the time domain free surface Green function with respect to a field point \( q = (x, y, z) \) and a source point \( p = (\xi, \eta, \zeta) \) is expressed as

\[
\hat{G} = \frac{\delta(t - \tau)}{|(x, y, z) - (\xi, \eta, \zeta)|} - \frac{\delta(t - \tau)}{|(x, y, z) - (\xi, \eta, -\zeta)|} + G
\]
for the Dirac delta function $\delta$ and the wave integral
\[
G = 2 \int_0^\infty \sqrt{gk} \sin \left( (t - \tau) \sqrt{kg} \right) e^{k(z + \zeta)} J_0(k|x, y) - (\xi, \eta)|)dk. \tag{2}
\]
Here $g$ is the gravity and $J_0$ the zero order Bessel function of the first kind.

Therefore to solve the linear hydrodynamics problem [1, 2, 11, 13], it is necessary to provide the Green function approximations [6, 7, 8, 12, 13, 15] and then transform the potential sources integral formulation into a normal velocity boundary integral equation to derive the source strength. To do so, it is necessary to evaluate the waterline integral involved in the boundary integral equation. The difficulty on the numeral evaluation of the waterline integral is well known but the rigorously analytic discussion on the integral is missing. In contrast to earlier investigations based on well-posedness assumption of the boundary integral equation, we use rigorous analysis to show the ill-posedness of the equation. That is, the waterline integral of the normal velocity boundary integral equation is unbounded when the field point approaches to the waterline.

The wave-body motion problem can also be modelled mathematically by using frequency domain free surface Green functions if the body is advancing at a uniform speed. Therefore velocity potential of the fluid motion problem can be derived numerically based on the numerical evaluations of the frequency domain free surface Green function (see [4, 5, 14]).

2. Limit time domain free surface Green function

When both field and source points are in the mean free surface or $z = \zeta = 0$, the wave integral reduces to
\[
G = 2 \int_0^\infty \sqrt{gk} \sin \left( (t - \tau) \sqrt{kg} \right) J_0(kR)dk \tag{3}
\]
\[
= 2 \sqrt{\frac{g}{R^3}} \int_0^\infty \sqrt{\lambda} \sin \left( s\sqrt{\lambda} \right) J_0(\lambda) d\lambda \tag{4}
\]
for $R = |(x, y) - (\xi, \eta)|$, $\lambda = kR$ and $s = (t - \tau) \sqrt{\frac{g}{R}}$.

It has been displayed in [16, Equations. (22.18), (22.19) and (22.21)] that the dimensionless wave integral can be expressed by using Bessel functions of the first kind:
\[
\int_0^\infty \sqrt{\lambda} \sin \left( s\sqrt{\lambda} \right) J_0(\lambda) d\lambda = \frac{\pi s^3}{16 \sqrt{2}} \left( J_\frac{3}{4} \left(\frac{s^2}{8}\right) - J_{-\frac{3}{4}} \left(\frac{s^2}{8}\right) \right) + J_{\frac{1}{4}} \left(\frac{s^2}{8}\right) J_{-\frac{1}{4}} \left(\frac{s^2}{8}\right). \tag{5}
\]
From this formula, we obtain that the integral can be expressed as the following elementary function
\[
\int_0^\infty \sqrt{\lambda} \sin \left( s\sqrt{\lambda} \right) J_0(\lambda) d\lambda = \frac{s}{\sqrt{2}} \sin \left(\frac{s^2}{4}\right) \tag{6}
\]
or
\[
G = \sqrt{2} \frac{g(t - \tau)}{R^2} \sin \left(\frac{(t - \tau)^2 g}{4 R^4}\right). \tag{7}
\]

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Figure 1: Comparison (c) between numerical data (a) of the dimensionless wave integral $F(s) = \int_0^\infty \sqrt{\lambda} \sin \left( s \sqrt{\lambda} \right) J_0(\lambda) d\lambda$ and its analytic expression (b) $\frac{s}{\sqrt{2}} \sin \left( \frac{s^2}{4} \right)$ when both field point and source point are on the mean free surface plane $z = 0$. 
The comparison between the left-hand and right-hand sides of (6) is presented in Figure 1.

The asymptotic behaviour of the wave integral

$$- \frac{s}{\sqrt{2}} \leq \int_0^\infty \sqrt{\lambda} \sin \left(s \sqrt{\lambda}\right) J_0(\lambda) d\lambda \leq \frac{s}{\sqrt{2}} \quad \text{and} \quad s \to \infty$$

was predicted in [7].

3. Mathematical modelling

The mathematical model for the wave-body motion by using the Green function is traditional. Let a surface piercing body advance in waves. The velocity potential $\phi$ of the fluid motion problem satisfies the Laplace equation

$$\Delta \phi = 0 \quad \text{in the fluid domain},$$

the free surface boundary condition

$$\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial z} = 0$$

and the initial value condition

$$\phi|_{t=0} = 0, \quad \frac{\partial \phi}{\partial t}|_{t=0} = 0.$$

As in [11, 13] in the extension of the Bird [3] waterline integral, the application of Green’s theorem and the conditions (9)-(11) implies that the velocity potential $\phi$ of the fluid motion satisfies the following boundary integral equation

$$4\pi \phi + \int_0^t d\tau \int_{S_B(\tau)} \left( \frac{\partial \hat{G}}{\partial n} - \hat{G} \frac{\partial \phi}{\partial n} \right) dS = -\frac{1}{g} \int_0^t d\tau \oint_{\Gamma(\tau)} \left( \phi \frac{\partial \hat{G}}{\partial \tau} - \hat{G} \frac{\partial \phi}{\partial \tau} \right) U_N dl,$$

where $n = (n_1, n_2, n_3)$ is the unit normal vector field of the mean wetted body surface $S_B(\tau)$ pointing into fluid domain and $U_N$ is the two-dimensional normal velocity in the mean free surface plane of $\Gamma(\tau)$, the waterline defined by the intersection of $S_B(\tau)$ and the mean water surface plane $z = 0$. The contour integral is in the anticlockwise direction.

With the use of the boundary integral of the inner flow inside the body surface and the existence of the Dirichlet problem as in Lamb [10], the boundary integral equation (12) is formulated as (see [13] for details)

$$4\pi \phi(q, t) + \int_0^t d\tau \int_{S_B(\tau)} \sigma(\mathbf{p}, \tau) \hat{G}(q, \mathbf{p}, t - \tau) dS$$

$$= \frac{1}{g} \int_0^t d\tau \oint_{\Gamma(\tau)} \sigma(\mathbf{p}, \tau) G(q, \mathbf{p}, t - \tau) U_N(q, \mathbf{p}, \tau) dl.$$
for field point \( q = (x, y, z) \) in the fluid domain and source point \( p = (\xi, \eta, \zeta) \in S_B(\tau) \) or \( p = (\xi, \eta, 0) \) when \( p \in \Gamma(\tau) \). The unknown \( \sigma \), the strength of the source point \( p \in \Gamma(\tau) \), is assumed to be obtained from the normal velocity boundary integral equation

\[
4\pi \frac{\partial \phi}{\partial n} + \int_0^t d\tau \int_{S_B(\tau)} \sigma(p, \tau) \frac{\partial G(q,p,t-\tau)}{\partial n} dS = \frac{1}{g} \int_0^t d\tau \int_{\Gamma(\tau)} \sigma(p, \tau) \frac{\partial G(q,p,t-\tau)}{\partial n} U_n(p, \tau) U_N(p, \tau) dl,
\]

by moving the field point \( q \) to \( S_B(t) \) and to \( \Gamma(t) \), since the normal velocity \( \frac{\partial \phi}{\partial n} \) on the fluid boundary \( S_B(t) \) is known. Here the normal derivative operator is applied to the field point \( q \) and \( n = (n_1, n_2, n_3) \) is the unit normal vector of \( S_B(t) \) at \( q \).

In the next section, Equation (14) is shown to be ill-posed in the sense that the waterline integral on the right-hand side of (14) is infinite at some field point \( q \in \Gamma(t) \) if \( n_3 \neq 0 \) and the surface piercing body advances at a dynamic speed.

### 4. Ill-posedness

Consider the surface piercing body advancing at a continuous dynamic speed \( U = U(t) > 0 \) for \( t > 0 \) in the \( Ox \) direction. Hence the waterline at \( \tau \) is expressed as

\[
\Gamma(\tau) = \Gamma(0) + \left( \int_0^\tau U(s) ds, 0, 0 \right).
\]

For the field point \( q(t) \in \Gamma(t) \), the wave integral \( G(q, p, t-\tau) \) is continuous with respect to \( (p, \tau) \) except at the singular point \( (p, \tau) = (q(t), t) \). In order to show the ill-posedness of the waterline integral of (14), it is sufficient to show the unboundedness of the integral on a small panel \( [p_1(\tau), p_0(\tau)] \times [t-\epsilon, t] \) covering the singular point \( (q(t), t) \). The dynamics behaviour of the waterline (15) gives the expression of the panel elements

\[
q(t) = q(0) + \left( \int_0^t U(s) ds, 0, 0 \right) \in \Gamma(t),
\]

\[
p_1(\tau) = q(0) + t\epsilon + \left( \int_0^\tau U(s) ds, 0, 0 \right) \in \Gamma(\tau),
\]

\[
p_0(\tau) = q(0) - t\epsilon + \left( \int_0^\tau U(s) ds, 0, 0 \right) \in \Gamma(\tau),
\]

\[
t = (t_1, t_2, 0) - \text{ the unit tangential vector of } \Gamma(0) \text{ at } q(0)
\]

for \( t > \tau > t - \epsilon > 0 \). Assume \( t_2 \neq 0 \) and \( \Gamma(0) \) being smooth at \( q(0) \) in order to use the approximation \( q(0) = \frac{1}{2}(p_1(0) + p_0(0)) \) since \( \epsilon > 0 \) is sufficiently small.

The single panel part of the waterline integral on the right hand side of (14) is

\[
\frac{1}{g} \int_{t-\epsilon}^t \int_{p_0(\tau)}^{p_1(\tau)} \sigma(p, \tau) \frac{\partial G(q, p, t-\tau)}{\partial n} U_n V_N dld\tau
\]

\[
= \frac{\sigma(q(t), t) (n_1 U(t))^2}{g} \int_{t-\epsilon}^t \int_{p_0(\tau)}^{p_1(\tau)} \frac{\partial G(q, p, t-\tau)}{\partial n} dld\tau
\]

(16)
by the smallness assumption of $\epsilon > 0$ and the continuity of $\sigma$ and $U$. Thus we only need to consider the panel integral of $\frac{\partial G}{\partial n}$

$$
\int_{t-\epsilon}^{t} \int_{\mathbf{p}(\tau)}^{\mathbf{p}(\tau)} \frac{\partial G(\mathbf{q}, \mathbf{p}, t - \tau)}{\partial n} \, d\tau \\
= \int_{t-\epsilon}^{t} \int_{\mathbf{p}(\tau)}^{\mathbf{p}(\tau)} \left( (n_1, n_2, 0) \cdot \frac{\mathbf{q}(t) - \mathbf{p}}{R} \frac{\partial G(\mathbf{q}, \mathbf{p}, t - \tau)}{\partial \mathbf{R}} + n_3 \frac{\partial G(\mathbf{q}, \mathbf{p}, t - \tau)}{\partial z} \right) \, d\tau \\
= \int_{t-\epsilon}^{t} \int_{\mathbf{p}(\tau)}^{\mathbf{p}(\tau)} \left( (n_1, n_2, 0) \cdot \frac{\mathbf{q}(t) - \mathbf{p}}{R} \frac{\partial G(\mathbf{q}, \mathbf{p}, t - \tau)}{\partial \mathbf{R}} - \frac{n_3}{g} \frac{\partial^2 G(\mathbf{q}, \mathbf{p}, t - \tau)}{\partial t^2} \right) \, d\tau
$$

(17)

after the use of the free surface boundary condition of the Green function. Since $\epsilon$ is small, the previous integral equals

$$
n_{1} U(t) \int_{t-\epsilon}^{t} \int_{-\epsilon}^{\epsilon} \frac{(t - \tau)}{R} \frac{\partial G(\mathbf{q}, \mathbf{p}, t - \tau)}{\partial \mathbf{R}} \, d\tau - \frac{n_3}{g} \int_{t-\epsilon}^{t} \int_{-\epsilon}^{\epsilon} \frac{\partial^2 G(\mathbf{q}, \mathbf{p}, t - \tau)}{\partial t^2} \, d\tau
$$

(18)

for the source point

$$
\mathbf{p} = \mathbf{q}(0) + t\mathbf{l} + \int_{0}^{T} U(s) \, ds.
$$

(19)

The smallness assumption of $\epsilon$ gives the validity of the approximation

$$
R = |\mathbf{q} - \mathbf{p}| = |t\mathbf{l} + \int_{0}^{T} U(s) \, ds| = |t\mathbf{l} + (t - \tau)U(t)|, \quad t - \epsilon < \tau < t.
$$

(20)

This yields the estimates

$$
\frac{1}{R} \leq \frac{2}{(t - \tau)U(t)} \quad \text{when} \quad |t_1\mathbf{l}| < \frac{1}{2}(t - \tau)U(t),
$$

and

$$
\frac{1}{R} \leq \frac{2|t_1\mathbf{l}|}{(t - \tau)|t_2|U(t)} \quad \text{when} \quad |t_1\mathbf{l}| \geq \frac{1}{2}(t - \tau)U(t),
$$

or

$$
\frac{1}{R} \leq \frac{2}{U(t)} \left( 1 + \frac{|t_1\mathbf{l}|}{|t_2|} \right) \frac{1}{t - \tau}.
$$

(21)

This estimate and the smallness assumption imply the validity of the approximation of the trigonometric functions

$$
\sin \frac{g(t - \tau)^2}{4R} = \frac{g(t - \tau)^2}{4R} \quad \text{and} \quad \cos \frac{g(t - \tau)^2}{4R} = 1.
$$

(22)

Let us begin with panel integral of the following partial derivative of the wave integral

$$
\frac{\partial^2 G}{\partial t^2} = 3\sqrt{2}g^2(t - \tau) \cos \left( \frac{(t - \tau)^2g}{4R} \right) - \sqrt{2}g^3(t - \tau)^3 - \sin \left( \frac{(t - \tau)^2g}{4R} \right)
$$

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Upon the observation
\[ dR = \frac{(q - p) \cdot (-t) dl}{R} = \frac{ldl}{R} \leq \frac{dl}{|t_2|} \text{ for } l > 0. \]

and the use of the approximations (20) and (22) and the estimate (21), the half panel integral is calculated as

\[
\frac{1}{\sqrt{2}} \int_{t_1}^{t} \int_{t_1}^{t} \frac{\partial^2 G(q, p, t - \tau)}{\partial t^2} dld\tau \\
= \int_{t_1}^{t} d\tau \int_{t_1}^{t} \left( \frac{3g^2(t - \tau)^2 \cos(t - \tau)g}{2R^3} - \frac{g^3(t - \tau)^3 \sin(t - \tau)g}{4R^4} \right) dl \\
\geq |t_2| \int_{t_1}^{t} d\tau \int_{t_1}^{t} \left( \frac{3g^2(t - \tau)^2 \cos(t - \tau)g}{2R^3} - \frac{g^3(t - \tau)^3 \sin(t - \tau)g}{4R^4} \right) dR \\
= |t_2| \int_{t_1}^{t} \left[ -3 \frac{g^2(t - \tau)}{4R^2} + \frac{g^4(t - \tau)^5}{64R^4} \right]_{t_0}^{t_1} d\tau \\
= |t_2| \int_{t_1}^{t} \left[ -3 \frac{g^2(t - \tau)}{4(t - \tau)U(t)^2} + \frac{g^4(t - \tau)^5}{64(t - \tau)U(t)^4} \right] d\tau \\
\geq |t_2| \int_{t_1}^{t} \left[ -3 \frac{g^2(t - \tau)}{4|t_2|^2} \right] d\tau - |t_2| \int_{t_1}^{t} \left[ -3 \frac{g^2}{4(t - \tau)U^2} + \frac{g^4(t - \tau)^2}{64U^4} \right] d\tau \\
= -|t_2| \left[ 3 \frac{g^2}{4U^2} \ln(t - \tau) \right]_{\tau=t_1}^{\tau=t_2} - \frac{|t_2| |g^4\epsilon^2|}{128U^4} - \frac{3g^2}{8|t_2|}. \\
\]

Similarly, we have the result for another half panel integral

\[
\frac{1}{\sqrt{2}} \int_{t_1}^{t} \int_{t_1}^{t} \frac{\partial^2 G(q, p, t - \tau)}{\partial t^2} dld\tau \geq -|t_2| \left[ 3 \frac{g^2}{4U^2} \ln(t - \tau) \right]_{\tau=t_1}^{\tau=t_2} - \frac{|t_2| |g^4\epsilon^2|}{128U^4} - \frac{3g^2}{8|t_2|}. \\
\]

Hence, we have the whole panel integral result the second time derivative of \(G\)

\[
\frac{1}{\sqrt{2}} \int_{t_1}^{t} \int_{t_1}^{t} \frac{\partial^2 G(q, p, t - \tau)}{\partial t^2} dld\tau \geq -|t_2| \left[ 3 \frac{g^2}{2U^2} \ln(t - \tau) \right]_{\tau=t_1}^{\tau=t_2} - \frac{|t_2| |g^4\epsilon^2|}{64U^4} - \frac{3g^2}{4|t_2|}. \\
\]

On the other hand, the panel integral involving the derivative

\[
\frac{\partial G}{\partial R} = -2\sqrt{2} \frac{g(t - \tau)g}{R^3} \sin \left( \frac{(t - \tau)g}{4R} \right) - \sqrt{2} \frac{g^2(t - \tau)^3 g}{4R^4} \cos \left( \frac{(t - \tau)^2 g}{4R} \right) \\
\]
is calculated as

\[
\frac{1}{\sqrt{2}} \left| \int_{t-\epsilon}^{t} \int_{-\epsilon}^{\epsilon} \frac{t-\tau}{R} \frac{\partial G(\mathbf{q}, \mathbf{p}, t-\tau)}{\partial n} dl d\tau \right|
\]

\[
= \int_{t-\epsilon}^{t} d\tau \int_{-\epsilon}^{\epsilon} \frac{t-\tau}{R} \left( \frac{2g(t-\tau)}{R^3} \sin \frac{(t-\tau)^2g}{4R} + \frac{g^2(t-\tau)^3}{4R^4} \cos \frac{(t-\tau)^2g}{4R} \right) dl
\]

\[
= \int_{t-\epsilon}^{t} d\tau \int_{-\epsilon}^{\epsilon} \frac{3g^2(t-\tau)^4}{4R^5}
\]

\[
\leq \int_{t-\epsilon}^{t} \int_{-\epsilon}^{\epsilon} \frac{3g^2}{4\sqrt{|t_2|}} \left( \frac{2}{U(t)} \left( 1 + \frac{|t_1|}{|t_2|} \right) \right)^{4+\frac{\epsilon}{2}} \frac{dl d\tau}{\sqrt{(t-\tau)|t|}}
\]

\[
= \epsilon \frac{6g^2}{\sqrt{|t_2|}} \left( \frac{2}{U(t)} \left( 1 + \frac{|t_1|}{|t_2|} \right) \right)^{4+\frac{\epsilon}{2}}
\]

(24)

after the use of (20)-(22).

Thus the combination of (17), (18), (23) and (24) gives the estimate

\[
\left| \int_{t-\epsilon}^{t} \int_{\Gamma(t-\tau)} \frac{\partial G(\mathbf{q}, \mathbf{p}, t-\tau)}{\partial n} dl d\tau \right|
\]

\[
\geq \left| \int_{t-\epsilon}^{t} \int_{-\epsilon}^{\epsilon} \frac{3g^2}{2U^2} \ln(t-\tau) \left. \frac{\partial^2 G(\mathbf{q}, \mathbf{p}, t-\tau)}{\partial t^2} \right|_{\tau=\tau} \right| - \left| n_1 U(t) \int_{t-\epsilon}^{t} \int_{-\epsilon}^{\epsilon} \frac{(t-\tau) \partial G(\mathbf{q}, \mathbf{p}, t-\tau)}{\partial R} dl d\tau \right|
\]

\[
\geq \int_{t-\epsilon}^{t} \int_{-\epsilon}^{\epsilon} \frac{3g^2}{2U^2} \ln(t-\tau) \left. \frac{\partial^2 G(\mathbf{q}, \mathbf{p}, t-\tau)}{\partial t^2} \right|_{\tau=\tau} \left| n_1 \right| U(t) \left( \epsilon \frac{6g^2}{\sqrt{|t_2|}} \left( \frac{2}{U(t)} \left( 1 + \frac{|t_1|}{|t_2|} \right) \right)^{4+\frac{\epsilon}{2}} \right).
\]

For \( \epsilon > 0 \) sufficiently small. This gives the unboundedness of the integral (25) since

\[
- \ln(t-\tau) \bigg|_{\tau=\tau} = \lim_{\tau \to -0} \left( -\ln(t-\tau) + \ln \epsilon \right) = +\infty
\]

and hence shows the ill-posedness of the waterline integral in the boundary integral equation (14).

5. Conclusion

The linearised problem of a surface piercing body advancing at a dynamic speed in the \( Ox \) direction is known to be modelled by a boundary integral equation given by time domain free surface sources integrated on the wetted body surface and the waterline contour (see, for example, [13]). Therefore, to find the velocity potential of the wave body motion problem becomes to derive the strength distribution of the time domain free surface source potential
or the time domain free surface Green function. The boundary integral equation is described (13). The strength $\sigma$ for the sources on the waterline $\Gamma(\tau)$ has to be solved by the boundary integral equation (14) with field point $q$ in the waterline contour $\Gamma(t)$ in order to use the normal velocity boundary condition applied to the wetted body surface $S_B(\tau)$.

The present study shows that the strength $\sigma$ of the source on the waterline contour $\Gamma(\tau)$ is not obtainable from the normal velocity boundary integral equation (14) because the waterline integral for $q \in \Gamma(t)$ is infinite under the condition that the normal vector $n = (n_1, n_2, n_3)$ of the of the body surface $S_B$ at $q \in \Gamma(t)$ is not parallel to the plane $z = 0$ or $n_3 \neq 0$.

It is popular to move the field point $q$ down from the waterline to obtain approximate value of the waterline integral in earlier studies. However, the ill-posedness of the waterline integral shows that the approximation is valid only when body surface is perpendicular to the free surface plane $z = 0$ at the intersection contour $\Gamma$.

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