Modified strip projection method

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Abstract. The diffraction image of a quasicrystal admits a finite group $G$ as a symmetry group, and the quasicrystal can be regarded as a quasiperiodic packing of copies of a $G$-cluster $C$, joined by glue atoms. The physical space $E$ containing $C$ can be embedded into a higher-dimensional space $\mathbb{R}^k$ such that, up to an inflation factor, $C$ is the orthogonal projection of the set $\{(\pm 1, 0, ..., 0), (0, \pm 1, 0, ..., 0), ... (0, ..., 0, \pm 1)\}$. The projections of the points of $\mathbb{Z}^k$ lying in the strip $S = E + [-1/2, 1/2]^k + t$ obtained by shifting a hypercube $[-1/2, 1/2]^k + t$ along $E$ is a quasiperiodic packing of partially occupied copies of $C$, but unfortunately, the occupation of clusters is very low. In our modified strip projection method we firstly determine for each point $x \in \mathbb{Z}^k \cap S$ the number $n(x)$ of all the arithmetic neighbours of $x$ lying in the strip $S$, and project the points of $\mathbb{Z}^k \cap S$ on $E$ in the decreasing order of the occupation number $n(x)$. In the case when $n(x)$ represents more than $p\%$ of all the points of the cluster $C$ we project all the arithmetic neighbours of $x$ (lying inside or outside $S$). We choose $p$ such that to avoid the superposition of the fully occupied clusters. The projection of a point $x$ with $n(x)$ less than $p\%$ of all the points of the cluster $C$ is added to the pattern only if it is not too close to the already obtained points.
1. Introduction

Quasicrystals are materials with perfect long-range order, but with no three-dimensional translational periodicity. The discovery of these solids in the early 1980’s and the challenge to describe their structure led to a great interest in quasiperiodic sets of points \[11, 12\]. The diffraction image of a quasicrystal contains a set of sharp Bragg peaks invariant under a finite non-crystallographic group of symmetries \( G \), called the symmetry group of quasicrystal (in reciprocal space). In the case of quasicrystals with no translational periodicity this group is the icosahedral group \( Y \) and in the case of quasicrystals periodic along one direction (two-dimensional quasicrystals) \( G \) is one of the cyclic groups \( C_8 \) (octagonal quasicrystals), \( C_{10} \) (decagonal quasicrystals) and \( C_{12} \) (dodecagonal quasicrystals).

Real structure information obtained by high resolution transmission electron microscopy suggests us that a quasicrystal with symmetry group \( G \) can be regarded as a quasiperiodic packing of copies of a well-defined \( G \)-invariant cluster \( C \), joined by glue atoms \[7\]. From a mathematical point of view, a \( G \)-cluster is a finite union of orbits of \( G \), in a fixed linear representation of \( G \). A mathematical algorithm for generating quasiperiodic packings of interpenetrating copies of \( G \)-clusters was obtained by author in collaboration with Verger-Gaugry several years ago \[1\]. This algorithm based on strip projection method \[6, 8, 9, 10\] works for any finite group \( G \) and any \( G \)-cluster, but in the case of a multi-shell cluster the dimension of the involved superspace is rather high, and the occupation of the clusters occurring in the generated pattern is too low. Some mathematical results recently obtained by author \[3, 5\] simplify the computer program and allow to use strip projection method in the superspaces required by this approach. Now, our aim is to present a way to increase the occupation of clusters occurring in the generated quasiperiodic set.

2. Two-dimensional packings of clusters

Let \( G \) be one of the cyclic groups \( C_8, C_{10}, C_{12} \). Each group \( C_n \) can be defined as

\[
C_n = \{ a \mid a^n = e \} = \{ e, a, a^2, ..., a^{n-1} \}
\]

and the formula

\[
a(\alpha, \beta) = \left( \alpha \cos \frac{2\pi}{n} - \beta \sin \frac{2\pi}{n}, \alpha \sin \frac{2\pi}{n} + \beta \cos \frac{2\pi}{n} \right)
\]

define an \( \mathbb{R} \)-irreducible representation in \( \mathbb{R}^2 \). The orbit generated by \((\alpha, \beta) \neq (0,0)\)

\[
C_n(\alpha, \beta) = \{(\alpha, \beta), a(\alpha, \beta), a^2(\alpha, \beta), ..., a^{n-1}(\alpha, \beta)\}
\]

contains \( n \) points (vertices of a regular polygon with \( n \) sides). Let

\[
C_2 = \{v_1, v_2, ..., v_k, -v_1, -v_2, ..., -v_k\}
\]
from $R^k$ are orthogonal and have the same norm
\[ \langle w_1, w_2 \rangle = v_1 v_{21} + v_2 v_{22} + \ldots + v_{1k} v_{2k} = 0 \]
\[ ||w_1|| = \sqrt{v_{11}^2 + v_{12}^2 + \ldots + v_{1k}^2} = ||w_2||. \]
We identify the physical space with the two-dimensional subspace
\[ E_2 = \{ \alpha w_1 + \beta w_2 \mid \alpha, \beta \in \mathbb{R} \} \]
of the superspace $R^k$ and denote by $E_2^\perp$ the orthogonal complement
\[ E_2^\perp = \{ x \in R^k \mid \langle x, y \rangle = 0 \text{ for all } y \in E_2 \}. \]
The orthogonal projection on $E_2$ of a vector $x \in R^k$ is the vector
\[ \pi_2 x = \left( x, \frac{w_1}{\kappa} \right) \frac{w_1}{\kappa} + \left( x, \frac{w_2}{\kappa} \right) \frac{w_2}{\kappa}, \]
where $\kappa = ||w_1|| = ||w_2||$, and the orthogonal projector corresponding to $E_2^\perp$ is
\[ \pi_2^\perp : R^k \longrightarrow E_2^\perp \quad \pi_2^\perp x = x - \pi_2 x. \]
We describe $E_2$ by using the orthogonal basis \{$(\kappa^{-2}w_1, \kappa^{-2}w_2)$\}. Therefore, in view of (3) the expression in coordinates of $\pi_2$ is
\[ \pi_2 : R^k \longrightarrow R^2 \quad \pi_2 x = (\langle x, w_1 \rangle, \langle x, w_2 \rangle). \]
The projection $W_{2,k} = \pi_2^\perp(\Lambda_k)$ of the unit hypercube
\[ \Lambda_k = \left\{ (x_1, x_2, \ldots, x_k) \mid -\frac{1}{2} \leq x_i \leq \frac{1}{2} \text{ for all } i \in \{1, 2, \ldots, k\} \right\} \]
is a polyhedron (called the window of selection) in the $(k-2)$-dimensional subspace $E_2^\perp$, and each $(k-3)$-dimensional face of $W_{2,k}$ is the projection of a $(k-3)$-dimensional face of $\Lambda_k$. The vectors $e_1 = (1, 0, 0, \ldots, 0)$, $e_2 = (0, 1, 0, \ldots, 0)$, ..., $e_k = (0, 0, \ldots, 0, 1)$ from $R^k$ form the canonical basis of $R^k$, and each $(k-3)$-face of $\Lambda_k$ is parallel to $(k-3)$ of these vectors and orthogonal to three of them. There exist eight $(k-3)$-faces of $\Lambda_k$ orthogonal to the distinct vectors $e_{i_1}, e_{i_2}, e_{i_3}$, and the set
\[ \left\{ x = (x_1, x_2, \ldots, x_k) \mid x_i \in \{-1/2, 1/2\} \text{ if } i \in \{i_1, i_2, i_3\} \text{ and } x_i = 0 \text{ if } i \notin \{i_1, i_2, i_3\} \right\} \]
contains one and only one point from each of them. There are
\[ \binom{k}{3} = \frac{k(k-1)(k-2)}{6} \]
sets of $2^k$ parallel $(k-3)$-faces of $\Lambda_k$, and we label them by using the elements of the set
\[ I_{2,k} = \{(i_1, i_2, i_3) \in \mathbb{Z}^3 \mid 1 \leq i_1 \leq k-2, \ i_1 + 1 \leq i_2 \leq k-1, \ i_2 + 1 \leq i_3 \leq k \}. \]
In \( \mathbb{R}^3 \) the cross-product of two vectors \( \mathbf{v} = (v_x, v_y, v_z) \) and \( \mathbf{w} = (w_x, w_y, w_z) \) is a vector orthogonal to \( \mathbf{v} \) and \( \mathbf{w} \), and can be obtained by expanding the formal determinant

\[
\mathbf{v} \times \mathbf{w} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix}
\]

where \( \{\mathbf{i}, \mathbf{j}, \mathbf{k}\} \) is the canonical basis of \( \mathbb{R}^3 \). For any vector \( \mathbf{u} = (u_x, u_y, u_z) \), the scalar product of \( \mathbf{u} \) and \( \mathbf{v} \times \mathbf{w} \) is

\[
\mathbf{u} \langle \mathbf{v} \times \mathbf{w} \rangle = \begin{vmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix}
\]

(16)

In a very similar way, a vector \( \mathbf{y} \) orthogonal to \( l = k - 1 \) vectors

\[
\mathbf{u}_i = (u_{i1}, u_{i2}, u_{i3}, ..., u_{ik}) \quad i \in \{1, 2, 3, ..., l\}
\]

from \( \mathbb{R}^k \) can be obtained by expanding the formal determinant

\[
y = \begin{vmatrix} e_1 & e_2 & e_3 & ... & e_k \\ u_{11} & u_{12} & u_{13} & ... & u_{1k} \\ u_{21} & u_{22} & u_{23} & ... & u_{2k} \\ ... & ... & ... & ... & ... \\ u_{l1} & u_{l2} & u_{l3} & ... & u_{lk} \end{vmatrix}
\]

(19)

containing the vectors of the canonical basis in the first row. For any \( x = (x_1, x_2, ..., x_k) \in \mathbb{R}^k \), the scalar product of \( x \) and \( \mathbf{y} \) is

\[
\langle x, y \rangle = \begin{vmatrix} x_1 & x_2 & x_3 & ... & x_k \\ u_{11} & u_{12} & u_{13} & ... & u_{1k} \\ u_{21} & u_{22} & u_{23} & ... & u_{2k} \\ ... & ... & ... & ... & ... \\ u_{l1} & u_{l2} & u_{l3} & ... & u_{lk} \end{vmatrix}
\]

(20)

For example,

\[
y = \begin{vmatrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & ... & e_k \\ 0 & 0 & 0 & 1 & 0 & 0 & ... & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & 0 & 0 & 0 & ... & 1 \\ v_{11} & v_{12} & v_{13} & v_{14} & v_{15} & v_{16} & ... & v_{1k} \\ v_{21} & v_{22} & v_{23} & v_{24} & v_{25} & v_{26} & ... & v_{2k} \end{vmatrix} = (-1)^{k-1} \begin{vmatrix} e_1 & e_2 & e_3 \\ v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \end{vmatrix}
\]

(21)

is a vector orthogonal to the vectors \( e_4, e_5, ..., e_k, w_1, w_2, \) and

\[
\langle x, y \rangle = (-1)^{k-1} \begin{vmatrix} x_1 & x_2 & x_3 \\ v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \end{vmatrix}
\]

(22)
for any $x \in \mathbb{R}^k$. The vector $y$ belongs to $E_2^\perp$, and since $e_i - \pi_2^\perp e_i$ is a linear combination of $w_1$ and $w_2$, it is also orthogonal to $\pi_2^\perp e_4$, $\pi_2^\perp e_5$, ..., $\pi_2^\perp e_k$. Therefore, $y$ is orthogonal to the $(k-3)$-faces of $W_{2,k}$ labelled by $(1,2,3)$. Similar results can be obtained for any $(i_1,i_2,i_3) \in I_{2,k}$.

Consider the strip corresponding to $W_{2,k}$ (see figure 1)

$$S_{2,k} = \{ x \in \mathbb{R}^k \mid \pi_2^\perp x \in W_{2,k} \}$$

and define for each $(i_1,i_2,i_3) \in I_{2,k}$ the number

$$d_{i_1i_2i_3} = \max_{\alpha \in \{-1/2,1/2\}} \left| \begin{array}{ccc} \alpha_1 & \alpha_2 & \alpha_3 \\ v_{1i_1} & v_{1i_2} & v_{1i_3} \\ v_{2i_1} & v_{2i_2} & v_{2i_3} \end{array} \right|.$$  

A point $x \in \mathbb{R}^k$ belongs to the strip $S_{2,k}$ if and only if

$$-d_{i_1i_2i_3} \leq \left| \begin{array}{ccc} x_{i_1} & x_{i_2} & x_{i_3} \\ v_{1i_1} & v_{1i_2} & v_{1i_3} \\ v_{2i_1} & v_{2i_2} & v_{2i_3} \end{array} \right| \leq d_{i_1i_2i_3} \text{ for any } (i_1,i_2,i_3) \in I_{2,k}. \quad (25)$$

The set defined in terms of the strip projection method \[6 \ 8 \ 9 \ 10\]

$$\Omega = \pi_2(S_{2,k} \cap \mathbb{Z}^k) = \{ \pi_2 x \mid x \in S_{2,k} \cap \mathbb{Z}^k \}$$

can be regarded as a packing of translated partially occupied copies of $C_2$. Since

$$\pi_2 e_i = (\langle e_i, w_1 \rangle, \langle e_i, w_2 \rangle) = (v_{1i}, v_{2i}) = v_i$$

we get

$$\pi_2(\{ x \pm e_1, x \pm e_2, ..., x \pm e_k \} \cap S_{2,k})$$

$$\subseteq \{ \pi_2 x \pm v_1, \pi_2 x \pm v_2, ..., \pi_2 x \pm v_k \} = \pi_2 x + C_2$$

that is, the neighbours of any point $\pi_2 x \in \Omega$ belong to the translated copy $\pi_2 x + C_2$ of $C_2$.

A larger class of aperiodic patterns can be obtained by translating the strip $S_{2,k}$. For each $t \in \mathbb{R}^k$ the set

$$\Omega = \pi_2((t + S_{2,k}) \cap \mathbb{Z}^k) = \{ \pi_2 x \mid x-t \in S_{2,k} \text{ and } x \in \mathbb{Z}^k \}$$

(29)
avoid the superposition of the fully occupied clusters. The projection of a point microscopy show the presence of a significant percentage of fully occupied clusters. Concerning the quasicrystal structure obtained by high resolution transmission electron microscopy the occupation of the corresponding cluster is very low. On the other hand, the images of the sets obtained by high resolution transmission electron microscopy show the presence of a significant percentage of fully occupied clusters. Some particular examples can be seen in figures 1-3.

3. Modified strip projection method

The algorithm based on the strip projection method presented in the previous section is very efficient. Hundreds of points of $\Omega$ can be obtained in only a few minutes for rather complicated $G$-clusters $C_2$, but, as one can remark in figures 1-3, for most of the points of $\Omega$ the occupation of the corresponding cluster is very low. On the other hand, the images concerning the quasicrystal structure obtained by high resolution transmission electron microscopy show the presence of a significant percentage of fully occupied clusters.

The number $n(x)$ of the neighbours of a point $\pi_2 x \in \Omega$ occurring in $\Omega$ corresponds to the number of the points of the set $\{x \pm e_1, x \pm e_2, \ldots, x \pm e_k\}$ belonging to the strip $t + S_{2,k}$. In our modified strip projection method we firstly determine $n(x)$ for all the points of $\mathbb{Z}^k$ lying in the fragment of the strip we intend to project, and then we project the points in the decreasing order of the occupation number $n(x)$. In the case when $n(x)$ represents more than $p\%$ of all the points of the cluster $C_2$ we project all the arithmetic neighbours of $x$ (lying inside or outside the strip $t + S_{2,k}$). We choose $p$ such that to avoid the superposition of the fully occupied clusters. The projection of a point $x$ with $n(x)$ less than $p\%$ of all the points of the cluster $C_2$ is added to the pattern only if it is not too close to the already obtained points. We get in this way a discrete set $\tilde{\Omega}$ containing fully occupied copies of the cluster $C_2$.

In the structure analysis of quasicrystals, the experimental diffraction image is compared with the diffraction image of the mathematical model, regarded as a set of scatterers. In order to compute the diffraction image of a discrete set $\Omega$ one has to use the Fourier transform and to identify $\Omega$ with the Dirac comb $\sum_{\omega \in \Omega} \delta_{\omega}$, which will also be denoted by $\Omega$. The set $\Omega$ defined in terms of the strip projection method is an infinite
set, but the set $\tilde{\Omega}_0$ we can effectively generate is evidently a finite set, obtained by starting from a finite fragment $\Omega_0$ of $\Omega$. This is not very bad since any quasicrystal has a finite number of atoms. Nevertheless, a fragment of $\Omega$ or $\tilde{\Omega}$ can not be an acceptable model for a quasicrystal unless it is large enough.

The diffraction pattern corresponding to $\Omega_0$ is related to the function

$$\Omega^*_0 : \mathbb{R}^2 \rightarrow [0, \infty), \quad \Omega^*_0(\xi) = |\mathcal{F}[\Omega_0](\xi)|^2 = \left| \sum_{\omega \in \Omega_0} e^{i\omega \cdot \xi} \right|^2$$

(30)

where $\mathcal{F}[\Omega_0]$ means the Fourier transform of the distribution $\Omega_0 = \sum_{\omega \in \Omega_0} \delta_\omega$. In figure 3 we present a fragment $\Omega_0$ of the set $\Omega$ corresponding to a $C_{12}$-cluster and the set $\{ \xi \in \mathbb{R}^2 \mid \Omega^*_0(\xi) > \frac{1}{1000} \Omega^*_0(0) \}$ in order to illustrate the shape and symmetry properties of the diffraction image of $\Omega_0$. The case of the set $\tilde{\Omega}_0$ corresponding to $\Omega_0$, obtained by using the modified strip projection method in $\mathbb{R}^6$, is presented in figure 4.

It is an open problem if the diffraction properties of the sets obtained by using the modified strip projection method are similar to those of sets obtained by the non-

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**Figure 3.** Left: A set $\Omega_0$ containing 923 points defined by starting from a $C_{12}$-cluster $C_2$. Centre: The cluster $C_2$. Right: For the diffraction pattern $\Omega^*_0$ of $\Omega_0$ please see [5].

**Figure 4.** Left: The set $\tilde{\Omega}_0$ containing 1019 points corresponding to the set $\Omega_0$ from the previous figure, defined by using the modified strip projection method. Centre: The cluster $C_2$. Right: For the diffraction pattern $\tilde{\Omega}^*_0$ of $\tilde{\Omega}_0$ please see [5].

For diffraction pattern please see [5].
modified strip projection method

A mathematical answer seems to be difficult, but some suggestions in this direction can be obtained by analysing larger fragments. By using our computer program the fragment containing 923 points presented in figure 3 and its diffraction pattern are obtained in one minute. In two hours one can obtain about 16000 points. The fragment generated by the modified method presented in figure 4 can be obtained in two minutes.

4. Quasiperiodic packings of icosahedral clusters

The icosahedral group $Y = 235$ can be defined in terms of generators and relations as

$$Y = \langle a, b \mid a^5 = b^2 = (ab)^3 = e \rangle$$

and the rotations $a, b : \mathbb{R}^3 \rightarrow \mathbb{R}^3$

$$a(\alpha, \beta, \gamma) = (\frac{1}{2} - \alpha - \frac{1}{2} \beta + \frac{1}{2} \gamma, \frac{1}{2} \alpha + \frac{1}{2} \beta + \frac{1}{2} \gamma, -\frac{1}{2} \alpha + \frac{1}{2} \beta + \frac{1}{2} \gamma)$$

$$b(\alpha, \beta, \gamma) = (-\alpha, -\beta, \gamma)$$

where $\tau = (1 + \sqrt{5})/2$, generate an irreducible representation of $Y$ in $\mathbb{R}^3$. In the case of this representation there are the trivial orbit $Y(0, 0, 0) = \{(0, 0, 0)\}$ of length 1, the orbits

$$Y(\alpha, \alpha \tau, 0) = \{g(\alpha, \alpha \tau, 0) \mid g \in Y\} \quad \text{where} \quad \alpha \in (0, \infty)$$

of length 12 (vertices of a regular icosahedron), the orbits

$$Y(\alpha, \alpha, \alpha) = \{g(\alpha, \alpha, \alpha) \mid g \in Y\} \quad \text{where} \quad \alpha \in (0, \infty)$$

of length 20 (vertices of a regular dodecahedron), the orbits

$$Y(\alpha, 0, 0) = \{g(\alpha, 0, 0) \mid g \in Y\} \quad \text{where} \quad \alpha \in (0, \infty)$$

of length 30 (vertices of an icosidodecahedron), and all the other orbits are of length 60.

If the set symmetric with respect to the origin

$$C_3 = \{v_1, v_2, \ldots, v_k, -v_1, -v_2, \ldots, -v_k\}$$

where $v_1 = (v_{11}, v_{21}, v_{31}), \ldots, v_k = (v_{1k}, v_{2k}, v_{3k})$, is a finite union of orbits of $Y$ then the vectors

$$w_1 = (v_{11}, v_{12}, \ldots, v_{1k})$$

$$w_2 = (v_{21}, v_{22}, \ldots, v_{2k})$$

$$w_3 = (v_{31}, v_{32}, \ldots, v_{3k})$$

from $\mathbb{R}^k$ are orthogonal

$$\langle w_1, w_2 \rangle = \langle w_2, w_3 \rangle = \langle w_3, w_1 \rangle = 0$$

and have the same norm $\kappa = ||w_1|| = ||w_2|| = ||w_3||$. They allow us to identify the physical space with the three-dimensional subspace

$$E_3 = \{ \alpha w_1 + \beta w_2 + \gamma w_3 \mid \alpha, \beta, \gamma \in \mathbb{R} \}$$
of the superspace $\mathbb{R}^k$. The orthogonal projection on $E_3$ of a vector $x \in \mathbb{R}^k$ is the vector
\[
\pi_3 x = \langle x, \frac{w_1}{\kappa} \rangle \frac{w_1}{\kappa} + \langle x, \frac{w_2}{\kappa} \rangle \frac{w_2}{\kappa} + \langle x, \frac{w_3}{\kappa} \rangle \frac{w_3}{\kappa}
\] (40)
The orthogonal projector corresponding to the orthogonal complement
\[
E_3^\perp = \{ x \in \mathbb{R}^k \mid \langle x, y \rangle = 0 \text{ for all } y \in E_3 \}
\] (41)
is $\pi_3^\perp : \mathbb{R}^k \to E_3^\perp$, $\pi_3^\perp x = x - \pi_3 x$. If we describe $E_3$ by using the orthogonal basis $\{\kappa^{-2}w_1, \kappa^{-2}w_2, \kappa^{-2}w_3\}$ then the expression in coordinates of $\pi_3$ is
\[
\pi_3 : \mathbb{R}^k \to \mathbb{R}^3 \quad \pi_3 x = (\langle x, w_1 \rangle, \langle x, w_2 \rangle, \langle x, w_3 \rangle).
\] (42)

The projection $W_{3,k} = \pi_3^\perp(\Lambda_k)$ of the unit hypercube $\Lambda_k$ is a polyhedron in the $(k - 3)$-dimensional subspace $E_3^\perp$, and each $(k - 4)$-dimensional face of $W_{3,k}$ is the projection of a $(k - 4)$-dimensional face of $\Lambda_k$. Each $(k - 4)$-face of $\Lambda_k$ is parallel to $(k - 4)$ of the vectors $e_1, e_2, \ldots, e_k$ and orthogonal to four of them. There exist sixteen $(k - 4)$-faces of $\Lambda_k$ orthogonal to the distinct vectors $e_{i_1}, e_{i_2}, e_{i_3}, e_{i_4}$, and the set
\[
\left\{ x = (x_1, x_2, \ldots, x_k) \left| \begin{array}{c}
 x_i \in \{-1/2, 1/2\} \text{ if } i \in \{i_1, i_2, i_3, i_4\} \\
 x_i = 0 \text{ if } i \not\in \{i_1, i_2, i_3, i_4\}
\end{array} \right. \right\}
\] (43)
contains one and only one point from each of them. There are
\[
\binom{k}{4} = \frac{k(k-1)(k-2)(k-3)}{24}
\] (44)
sets of $2^4$ parallel $(k - 4)$-faces of $\Lambda_k$, and we label them by using the elements of the set
\[
\mathcal{I}_{3,k} = \left\{ (i_1, i_2, i_3, i_4) \in \mathbb{Z}^4 \left| \begin{array}{cc}
 1 \leq i_1 \leq k - 3, & i_1 + 1 \leq i_2 \leq k - 2, \\
 i_2 + 1 \leq i_3 \leq k - 1, & i_3 + 1 \leq i_4 \leq k
\end{array} \right. \right\}.
\] (45)

A point $x = (x_1, x_2, \ldots, x_k) \in \mathbb{R}^k$ belongs to the strip $S_{3,k}$ if and only if
\[
-x_{i_1} \leq v_{i_1} x_{i_2} v_{i_3} x_{i_4} \leq d_{i_1 i_2 i_3 i_4}
\] (46)
for each $(i_1, i_2, i_3, i_4) \in \mathcal{I}_{3,k}$, where
\[
d_{i_1 i_2 i_3 i_4} = \max_{\alpha_j \in \{-1/2, 1/2\}} \left| \begin{array}{cccc}
 \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\
 v_{1i_1} & v_{1i_2} & v_{1i_3} & v_{1i_4} \\
 v_{2i_1} & v_{2i_2} & v_{2i_3} & v_{2i_4} \\
 v_{3i_1} & v_{3i_2} & v_{3i_3} & v_{3i_4}
\end{array} \right|
\] (47)

For each $t \in \mathbb{R}^k$, the pattern defined in terms of the strip projection method
\[
\Omega = \pi_3((t + S_{3,k} \cap \mathbb{Z}^k) = \{ \pi_3 x \mid x - t \in S_{3,k} \text{ and } x \in \mathbb{Z}^k \}
\] (48)
can be regarded as a quasiperiodic packing of copies of the starting cluster $\mathcal{C}_3$. 

\[\text{Modified strip projection method}\]
The algorithm based on the strip projection method presented above is very efficient. In the case of a three-shell $Y$-cluster formed by the vertices of a regular icosahedron, a regular dodecahedron and an icosidodecahedron we use a 31-dimensional superspace, $W_{3,k}$ is a polyhedron lying in the 28-dimensional subspace $E_{3}^{\perp}$ bounded by 31465 pairs of parallel 27-dimensional faces, but we obtain 400-500 points in less than 10 minutes [2]. With the modification indicated in the previous section we can obtain quasiperiodic packings of multi-shell icosahedral clusters containing a significant percentage of fully occupied clusters.

5. Concluding remarks

Some of the most remarkable tilings and discrete quasiperiodic sets used in quasicrystal physics are obtained by using strip projection method in a superspace of dimension four, five or six, and the projection of a unit hypercube as a window of selection [4, 6, 9]. The mathematical results presented above allow one to use this very elegant method in superspaces of dimension much higher, and to generate discrete quasiperiodic sets with a more complicated structure by starting from the symmetry group $G$ and the local structure described by a covering cluster $\mathcal{C}$. In our approach the window (which, generally, is a polyhedron with hundreds or thousands faces) is described in a simple way and we have to compute only determinants of order three or four, independently of the dimension of the superspace we use. These mathematical results have allowed us to obtain some very efficient computer programs for our algorithm [5]. Hundreds of points of our mathematical models can be obtained in only a few minutes.

The quasiperiodic set generated by starting from a $G$-cluster $\mathcal{C}$ is a packing of partially occupied copies of $\mathcal{C}$, but for most of these copies the occupation is very low. The main purpose of the paper is to present a modified version of the strip projection method. We project certain points lying outside the strip and do not project certain points lying inside the strip in order to favour the appearance of fully occupied clusters. More exactly, we start from the pattern generated by the standard strip projection method and help the clusters with occupation above a certain threshold ($p\%$) to complete their configuration up to a fully occupied clusters. We project a minimum number of points lying outside the strip, and avoid to project certain points lying inside the strip.

The symmetry group $G$ corresponding to a real quasicrystal can be deduced from the diffraction images, and the covering cluster $\mathcal{C}$ can be chosen by analyzing the real structure information obtained by high-resolution transmission electron microscopy. Our modified strip projection method allows one to generate a mathematical model, and to compute the corresponding diffraction image by using the Fourier transform. If the agreement with the experimental data is not acceptable one has to look for a more suitable covering cluster $\mathcal{C}$. 

\textit{Modified strip projection method}
The computer program in FORTRAN 90 used in the case of figure 3

! PLEASE INDICATE HOW MANY POINTS DO YOU WANT TO ANALYSE
INTEGER, PARAMETER :: N = 6000

! PLEASE INDICATE THE DIMENSION M OF THE SUPERSPACE
INTEGER, PARAMETER :: M = 6

! PLEASE INDICATE THE RADIUS OF THE PATTERN
REAL, PARAMETER :: R = 9.0

INTEGER I, J, K, L, I1, I2, I3, JJ, J1, J2
REAL D1, D2, D3, PR
 REAL, DIMENSION(M) :: V, W, TRANSLATION, WJ, EPSILON
 INTEGER, DIMENSION(N) :: CLUSTER
 REAL, DIMENSION(2,M) :: BASIS
 REAL, DIMENSION(2,2) :: C12
 REAL, DIMENSION(1:M-2,2:M-1,3:M) :: STRIP
 REAL, DIMENSION(N,M) :: POINTS, STRIPOINTS
 REAL, DIMENSION(N + M) :: XPOINT, YPOINT
 REAL, DIMENSION(200,200) :: FOURIER
 REAL, DIMENSION(2,40000) :: PATTERN
 COMPLEX II
 II=(0,1)

EPSILON = 0.0001

! PLEASE INDICATE THE COORDINATES OF A POINT OF THE CLUSTER
 BASIS(1,1) = 1.0
 BASIS(2,1) = 0.0

! PLEASE INDICATE THE TRANSLATION OF THE STRIP YOU WANT TO USE
 TRANSLATION = 0.1

C12(1,1) = SQRT(3.0) / 2.0
C12(1,2) = -1.0 / 2.0
C12(2,1) = 1.0 / 2.0
C12(2,2) = SQRT(3.0) / 2.0
DO J = 2, 6
   DO I = 1, 2
      BASIS(I,J) = C12(I,1) * BASIS(1,I-1) &
                   + C12(I,2) * BASIS(2,I-1)
   END DO
END DO
END DO
END DO
STRIP=0
DO I1 =1, M-2
DO I2 =I1+1, M-1
DO I3 =I2+1, M
DO D1 =-0.5, 0.5
DO D2 =-0.5, 0.5
DO D3 =-0.5, 0.5
PR = D1 * BASIS(1,I2) * BASIS(2,I3) + &
    D3 * BASIS(1,I1) * BASIS(2,I2) + &
    D2 * BASIS(1,I3) * BASIS(2,I1) - &
    D3 * BASIS(1,I2) * BASIS(2,I1) - &
    D1 * BASIS(1,I3) * BASIS(2,I2) - &
    D2 * BASIS(1,I1) * BASIS(2,I3)
IF ( PR > STRIP(I1,I2,I3) ) STRIP(I1,I2,I3) = PR
END DO
END DO
END DO
IF( STRIP(I1,I2,I3) .EQ. 0 ) STRIP(I1,I2,I3)=N * SUM( BASIS(1,:) ** 2)
END DO
END DO
END DO
PRINT*, 'COORDINATES OF THE POINTS OF THE ONE-SHELL C12-CLUSTER:'
DO J = 1, M
PRINT*, J, BASIS(1,J), BASIS(2,J)
END DO
PRINT*, '* STRIP TRANSLATED BY THE VECTOR WITH COORDINATES:'
PRINT*, TRANSLATION
PRINT*, '* PLEASE WAIT A FEW MINUTES OR MORE,&
DEPENDING ON THE NUMBER OF ANALYSED POINTS'
POINTS = 0
STRIPOINTS = 0
POINTS(1,:) = ANINT( TRANSLATION)
STRIPOINTS(1,:) = ANINT( TRANSLATION)
K = 1
L = 0
DO I = 1, N
    V = POINTS(I, :) - TRANSLATION
    JJ = 1
    DO I1 =1, M-2
    DO I2 =I1+1, M-1
DO I3 = I2 + 1, M
    PR = V(I1) * BASIS(1, I2) * BASIS(2, I3) + &
         V(I3) * BASIS(1, I1) * BASIS(2, I2) + &
         V(I2) * BASIS(1, I3) * BASIS(2, I1) - &
         V(I3) * BASIS(1, I2) * BASIS(2, I1) - &
         V(I1) * BASIS(1, I3) * BASIS(2, I2) - &
         V(I2) * BASIS(1, I1) * BASIS(2, I3)
    IF (ABS(PR) > STRIP(I1, I2, I3)) JJ = 0
END DO
END DO
END DO

IF (JJ .EQ. 1) THEN
    I3 = 1
    DO J = 1, L
        WJ = ABS(POINTS(I,:) - STRIPOINTS(J,:))
        IF (ALL(WJ < EPSILON)) I3 = 0
    END DO
    IF (I3 == 1 .AND. SUM(V * V) < R**2) THEN
        L = L + 1
        STRIPOINTS(L,:) = POINTS(I,:)
    ELSE
        END IF
    END IF
    DO I1 = 1, M
        DO I2 = -1, 1, 2
            W = POINTS(I, :)
            W(I1) = W(I1) + I2
            I3 = 0
            DO J = 1, K
                WJ = ABS(W - POINTS(J,:))
                IF (ALL(WJ < EPSILON)) I3 = 1
            END DO
            IF (I3 == 0 .AND. K < N) THEN
                K = K + 1
                POINTS(K, :) = W
            ELSE
                END IF
        END DO
    END DO
END IF
ELSE
    END IF
END IF
ELSE
    END IF
END DO
CLUSTER = 0
DO I = 1, L
DO J1 = 1, M
    DO J2 = -1, 1, 2
    W = STRIPOINTS(I,:) - TRANSLATION
    W(J1) = W(J1) + J2
    JJ = 1
    DO I1 =1, M-2
        DO I2 =I1+1, M-1
            DO I3 =I2+1, M
                PR = W(I1) * BASIS(1,I2) * BASIS(2,I3) + &
                    W(I3) * BASIS(1,I1) * BASIS(2,I2) + &
                    W(I2) * BASIS(1,I3) * BASIS(2,I1) - &
                    W(I3) * BASIS(1,I2) * BASIS(2,I1) - &
                    W(I1) * BASIS(1,I3) * BASIS(2,I2) - &
                    W(I2) * BASIS(1,I1) * BASIS(2,I3)
                IF( ABS(PR) > STRIP(I1,I2,I3) ) JJ = 0
            END DO
        END DO
    END DO
    IF( JJ .EQ. 1 ) CLUSTER(I) = CLUSTER(I) + 1
END DO
END DO
END DO
IF( JJ .EQ. 1 ) CLUSTER(I) = CLUSTER(I) + 1
END DO
END DO
DO J = 1, L
XPOINT(J) = SUM( STRIPOINTS(J,:) * BASIS(1,:) )
    YPOINT(J) = SUM( STRIPOINTS(J,:) * BASIS(2,:) )
END DO
PRINT*, 'NUMBER OF ANALYSED POINTS :', K
    PRINT*, 'NUMBER OF OBTAINED POINTS :', L
DO I = 1, 100
DO J = 1, 100
    D1=0.0
    DO I1 = 1, L
        D1=D1+EXP( II * (-1.5+I*0.03)*XPOINT(I1)+ &
                    II * (-1.5+J*0.03)*YPOINT(I1) )
    END DO
    FOURIER(I,J)=(ABS(D1))**2
END DO
END DO
I2=0
DO I = 1, 100
DO J = 1, 100
IF(FOURIER(I,J)>0.001*L**2) THEN
I2=I2+1
PATTERN(1,I2)=-1.5+I*0.03
PATTERN(2,I2)=-1.5+J*0.03
ELSE
END IF
END DO
END DO
PRINT*, 'INDICATE THE NAME OF A FILE WITH EXTENSION tex FOR RESULTS'
WRITE(4,60)
60   FORMAT('\\documentclass{article} &
\begin{document} &
\begin{figure} &
\setlength{\unitlength}{1.5mm} &
\begin{picture}(50,20)(0,0) ' &
'\put(32.0,20.0){{\circle*{0.2}} }')
DO J = 1, L
IF( CLUSTER(J) < M+1 ) THEN
WRITE(4,65) 10+XPOINT(J), 20+YPOINT(J)
65   FORMAT( '\put( F10.5,F10.5,){\circle*{0.2} }')
ELSE
END IF
END DO
DO J = 1, L
IF( CLUSTER(J) > M ) THEN
WRITE(4,70) 10+XPOINT(J), 20+YPOINT(J)
70   FORMAT( '"put( F10.5,F10.5,')\{\circle{0.4} }')
ELSE
END IF
END DO
DO J = 1, 6
WRITE(4,72) 32+BASIS(1,J), 20+BASIS(2,J)
72   FORMAT( '"put( F10.5,F10.5,')\{\circle*{0.2} }')
WRITE(4,73) 32-BASIS(1,J), 20-BASIS(2,J)
73   FORMAT( '"put( F10.5,F10.5,')\{\circle*{0.2} }')
END DO
WRITE(4,75)
75   FORMAT(' \setlength{\unitlength}{1.8mm}''
DO J = 1, I2
WRITE(4,80) 45+10*PATTERN(1,J), 17+10*PATTERN(2,J)
80   FORMAT( '"put( F10.5,F10.5,')\{\circle*{0.1} }')
END DO
WRITE(4,90)
90 FORMAT( 'end{picture} &
        \caption{Quasiperiodic set obtained by using &
        the strip projection method } &
        \end{figure} &
\end{document}'
PRINT*, '* COMPILE THE OBTAINED FILE AND SEE THE "dvi" FILE'
END
The computer program in FORTRAN 90 used in the case of figure 4

! PLEASE INDICATE HOW MANY POINTS DO YOU WANT TO ANALYSE
INTEGER, PARAMETER :: N = 6000

! PLEASE INDICATE THE DIMENSION M OF THE SUPERSPACE
INTEGER, PARAMETER :: M = 6

! PLEASE INDICATE THE RADIUS OF THE PATTERN
REAL, PARAMETER :: R = 9.0

INTEGER I, J, K, L, I1, I2, I3, JJ, J1, J2, L1
REAL D1, D2, D3, PR, XP, YP
REAL, DIMENSION(M) :: V, W, TRANSLATION, WJ, EPSILON
INTEGER, DIMENSION(N) :: CLUSTER
REAL, DIMENSION(2,M) :: BASIS
REAL, DIMENSION(2,2) :: C12
REAL, DIMENSION(1:M-2,2:M-1,3:M) :: STRIP
REAL, DIMENSION(N,M) :: POINTS, STRIPOINTS
REAL, DIMENSION(N + M) :: XPOINT, YPOINT
REAL, DIMENSION(200,200) :: FOURIER
REAL, DIMENSION(2,40000) :: PATTERN
COMPLEX II
II=(0,1)

EPSILON = 0.0001

! PLEASE INDICATE THE COORDINATES OF A POINT OF THE CLUSTER
BASIS(1,1) = 1.0
BASIS(2,1) = 0.0

! PLEASE INDICATE THE TRANSLATION OF THE STRIP YOU WANT TO USE
TRANSLATION = 0.1

C12(1,1) = SQRT(3.0) / 2.0
C12(1,2) = -1.0 / 2.0
C12(2,1) = 1.0 / 2.0
C12(2,2) = SQRT(3.0) / 2.0
DO J = 2, 6
  DO I = 1, 2
    BASIS(I,J) = C12(I,1) * BASIS(1,J-1) &
    + C12(I,2) * BASIS(2,J-1)
! BASIS(I,6+J) = C12(I,1) * BASIS(1,5+J) &
! + C12(I,2) * BASIS(2,5+J)

END DO
END DO
STRIP=0
DO I1 =1, M-2
DO I2 =I1+1, M-1
DO I3 =I2+1, M
DO D1 =-0.5, 0.5
DO D2 =-0.5, 0.5
DO D3 =-0.5, 0.5
PR = D1 * BASIS(1,I2) * BASIS(2,I3) + &
    D3 * BASIS(1,I1) * BASIS(2,I2) + &
    D2 * BASIS(1,I3) * BASIS(2,I1) - &
    D3 * BASIS(1,I2) * BASIS(2,I1) - &
    D2 * BASIS(1,I1) * BASIS(2,I3)
IF ( PR > STRIP(I1,I2,I3) ) STRIP(I1,I2,I3) = PR
END DO
END DO
END DO
IF( STRIP(I1,I2,I3) .EQ. 0 ) STRIP(I1,I2,I3)=N * SUM( BASIS(1,:) ** 2)
END DO
END DO
END DO
PRINT*, 'COORDINATES OF THE POINTS OF THE ONE-SHELL C12-CLUSTER:'
DO J = 1, M
PRINT*, J, BASIS(1,J), BASIS(2,J)
END DO
PRINT*, '* STRIP TRANSLATED BY THE VECTOR WITH COORDINATES:'
PRINT*, TRANSLATION
PRINT*, '* PLEASE WAIT A FEW MINUTES OR MORE,&
    DEPENDING ON THE NUMBER OF ANALYSED POINTS'
    POINTS = 0
STRIPPOINTS = 0
POINTS(1,:) = ANINT( TRANSLATION)
STRIPPOINTS(1,:) = ANINT( TRANSLATION)
K = 1
L = 0
DO I = 1, N
    V = POINTS(I, :) - TRANSLATION
    JJ = 1
DO I1 = 1, M-2
DO I2 = I1+1, M-1
DO I3 = I2+1, M
PR = V(I1) * BASIS(1,I2) * BASIS(2,I3) + &
V(I3) * BASIS(1,I1) * BASIS(2,I2) + &
V(I2) * BASIS(1,I3) * BASIS(2,I1) - &
V(I3) * BASIS(1,I2) * BASIS(2,I1) - &
V(I1) * BASIS(1,I3) * BASIS(2,I2) - &
V(I2) * BASIS(1,I1) * BASIS(2,I3)
IF( ABS(PR) > STRIP(I1,I2,I3) ) JJ = 0
END DO
END DO
END DO
IF( JJ .EQ. 1 ) THEN
I3 = 1
DO J = 1, L
WJ = ABS(POINTS(I,:) - STRIPOINTS(J,:))
IF( ALL(WJ < EPSILON) ) I3 = 0
END DO
IF( I3 == 1 .AND. SUM( V * V) < R **2 ) THEN
L = L + 1
STRIPOINTS(L,:) = POINTS(I,:)
ELSE
END IF
END IF
DO I1 = 1, M
DO I2 = -1, 1, 2
W = POINTS(I, :)
W(I1) = W(I1) + I2
I3 = 0
DO J = 1, K
WJ = ABS(W - POINTS(J,:))
IF( ALL( WJ < EPSILON) ) I3 = 1
END DO
IF ( I3 == 0 .AND. K < N ) THEN
K = K + 1
POINTS(K, :) = W
ELSE
END IF
END DO
END DO
ELSE
END IF
Modified strip projection method

END DO
CLUSTER = 0
DO I = 1, L
DO J1 = 1, M
DO J2 = -1, 1, 2
W = STRIPOINTS(I,:) - TRANSLATION
W(J1) = W(J1) + J2
JJ = 1
DO I1 = 1, M-2
DO I2 = I1+1, M-1
DO I3 = I2+1, M
PR = W(I1) * BASIS(1,I2) * BASIS(2,I3) + &
W(I3) * BASIS(1,I1) * BASIS(2,I2) + &
W(I2) * BASIS(1,I3) * BASIS(2,I1) - &
W(I3) * BASIS(1,I2) * BASIS(2,I1) - &
W(I1) * BASIS(1,I3) * BASIS(2,I2) - &
W(I2) * BASIS(1,I1) * BASIS(2,I3)
IF ( ABS(PR) > STRIP(I1,I2,I3) ) JJ = 0
END DO
END DO
END DO
IF( JJ .EQ. 1 ) CLUSTER(I) = CLUSTER(I) + 1
END DO
END DO
I1 = L
DO I = 1, I1
IF ( CLUSTER(I) > M) THEN
DO J1 = 1, M
DO J2 = -1, 1, 2
W = STRIPOINTS(I,:)
W(J1) = W(J1) + J2
I3 = 0
DO J = 1, L
WJ = ABS(W - STRIPOINTS(J,:))
IF( ALL( WJ < EPSILON)) I3 = 1
END DO
IF ( I3 == 0 ) THEN
L = L + 1
STRIPOINTS(L, :) = W
ELSE
END IF
END DO
ELSE
END IF
END DO
L1=0
DO I = 1, I1
IF ( CLUSTER(I) > M) THEN
L1=L1+1
XPOINT(L1) = SUM( STRIPOINTS(I,:) * BASIS(1,:))
YPOINT(L1) = SUM( STRIPOINTS(I,:) * BASIS(2,:))
ELSE
END IF
END DO
DO J = I1+1, L
L1=L1+1
XPOINT(L1) = SUM( STRIPOINTS(J,:) * BASIS(1,:))
YPOINT(L1) = SUM( STRIPOINTS(J,:) * BASIS(2,:))
END DO
D1=4.0
DO I =2, M
IF(((BASIS(1,1)+BASIS(1,I))**2 + (BASIS(2,1)+BASIS(2,I))**2 ) < D1 ) &
D1=(BASIS(1,1)+BASIS(1,I))**2 + (BASIS(2,1)+BASIS(2,I))**2
END DO
DO I =2, M
IF(((BASIS(1,1)-BASIS(1,I))**2 + (BASIS(2,1)-BASIS(2,I))**2 ) < D1 ) &
D1=(BASIS(1,1)-BASIS(1,I))**2 + (BASIS(2,1)-BASIS(2,I))**2
END DO
DO I = 1, I1
IF ( CLUSTER(I) <= M) THEN
D2=4.0
XP = SUM( STRIPOINTS(I,:) * BASIS(1,:))
YP = SUM( STRIPOINTS(I,:) * BASIS(2,:))
DO J = 1, L1
IF( ((XP - XPOINT(J))**2 + (YP - YPOINT(J))**2 ) < D2 ) &
D2=(XP - XPOINT(J))**2 + (YP - YPOINT(J))**2
END DO
IF( D2 > 0.9*D1 ) THEN
L1=L1+1
XPOINT(L1)=XP
YPOINT(L1)=YP
ELSE
Modified strip projection method

END IF
ELSE
END IF
END DO

DO I = 1, 100
DO J = 1, 100
   D1=0.0
   DO I1 = 1, L1
      D1=D1+EXP( II * (-1.5+I*0.03)*XPOINT(I1)+ &
                II * (-1.5+J*0.03)*YPOINT(I1) )
   END DO
END DO
FOURIER(I,J)=(ABS(D1))**2
END DO
END
I2=0
DO I = 1, 100
   DO J = 1, 100
      IF(FOURIER(I,J)>0.0015*L**2) THEN
         I2=I2+1
         PATTERN(1,I2)=-1.5+I*0.03
         PATTERN(2,I2)=-1.5+J*0.03
      ELSE
      END IF
   END DO
END DO
PRINT*, 'NUMBER OF ANALYSED POINTS :', K
PRINT*, 'NUMBER OF OBTAINED POINTS :', L1

PRINT*, 'INDICATE THE NAME OF A FILE WITH EXTENSION tex FOR RESULTS'
WRITE(4,60)
60 FORMAT('documentclass{article} &
 \begin{document} &
 \begin{figure} &
 \setlength{\unitlength}{1.5mm} &
 \begin{picture}(50,20)(0,0) ' &
\put(32.0,20.0){\circle*{0.2}} ') &
DO J = 1, L1
   IF( CLUSTER(J) < M+1 ) THEN
      WRITE(4,65) 10+XPOINT(J), 20+YPOINT(J)
65 FORMAT( 'put( 'F10.5','F10.5,')\circle*{0.2} )'} &
ELSE
END IF
END DO

DO J = 1, L1
IF( CLUSTER(J) > M ) THEN
WRITE(4,70) 10+XPOINT(J), 20+YPOINT(J)
70 FORMAT( 'put( 'F10.5','F10.5,)\circle*{0.2} ' )
ELSE
END IF
END DO

DO J = 1, 6
WRITE(4,72) 32+BASIS(1,J), 20+BASIS(2,J)
72 FORMAT( 'put( 'F10.5','F10.5,)\circle*{0.2} ' )
WRITE(4,73) 32-BASIS(1,J), 20-BASIS(2,J)
73 FORMAT( 'put( 'F10.5','F10.5,)\circle*{0.2} ' )
END DO

WRITE(4,75)
75 FORMAT( 'setlength\unitlength{1.8mm}')

DO J = 1, I2
WRITE(4,80) 45+10*PATTERN(1,J), 17+10*PATTERN(2,J)
80 FORMAT( 'put( 'F10.5','F10.5,)\circle*{0.1} ' )
END DO

WRITE(4,90)
90 FORMAT( '\end{picture} &
\caption{Quasiperiodic set obtained by using &
the modified strip projection method} &
\end{figure} &
\end{document}')

PRINT*, '* COMPILE THE OBTAINED FILE AND SEE THE '.dvi' FILE'
END
Acknowledgment

This research was supported by the grants CERES 4-129 and CEx05-D11-03.

References

[1] Cotfas N and Verger-Gaugry J-L 1997 A mathematical construction of $n$-dimensional quasicrystals starting from $G$-clusters J. Phys. A: Math. Gen. 30 4283-91
[2] Cotfas N 2005 Preprint [math-ph/0504044]
[3] Cotfas N 2006 Discrete quasiperiodic sets with predefined covering cluster Phil. Mag. 86 895-900
[4] Cotfas N 2006 Discrete quasiperiodic sets with predefined local structure J. Geom. Phys. 56 2415-2428
[5] Cotfas N 2006 http://fpcm5.fizica.unibuc.ro/~ncotfas
[6] Elser V 1986 The diffraction pattern of projected structures Acta Cryst. A 42 36-43
[7] Janot C and de Boissieu M 1994 Quasicrystals as a hierarchy of clusters Phys. Rev. Lett. 72 1674-7
[8] Kalugin P A, Kitayev A Y and Levitov L S 1985 6-dimensional properties of AlMn alloy J. Physique Lett. 46 L601-7
[9] Katz A and Duneau M 1986 Quasiperiodic patterns and icosahedral symmetry J. Phys. (France) 47 181-96
[10] Kramer P and Neri R 1984 On periodic and non-periodic space fillings of $\mathbb{E}^m$ obtained by projection Acta Crystallogr. A 40 580-7
[11] Kramer P and Papadopolos Z (eds.) 2003 Coverings of Discrete Quasiperiodic Sets. Theory and Applications to Quasicrystals (Berlin: Springer)
[12] Moody R V 1997 Meyer sets and their duals The Mathematics of Long-Range Aperiodic Order ed. R V Moody (Dordrecht: Kluwer) pp 403-41