Operator product expansion for B-meson distribution amplitude and dimension-5 HQET operators

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Abstract

When the bilocal heavy-quark effective theory (HQET) operator for the \( B \)-meson distribution amplitude has a light-like distance \( t \) between the quark and antiquark fields, the scale \( \sim 1/t \) separates the UV and IR regions, which induce the cusp singularity in radiative corrections and the mixing of multiparton states in nonperturbative corrections, respectively. We treat these notorious UV and IR behaviors simultaneously using the operator product expansion, with the local operators of dimension \( d \leq 5 \) and radiative corrections at order \( \alpha_s \) for the corresponding Wilson coefficients. The result is derived in the coordinate space, which manifests the Wilson coefficients with Sudakov-type double logarithms and the higher-dimensional operators with additional gluons. This result yields the \( B \)-meson distribution amplitude for \( t \) less than \( \sim 1 \text{ GeV}^{-1} \), in terms of \( \Lambda = m_B - m_b \) and the two additional HQET parameters as matrix elements of dimension-5 operators. The impact of these novel HQET parameters on the integral relevant to exclusive \( B \) decays, \( \lambda_B \), is also discussed.
For the exclusive $B$-meson decays, such as $B \to \pi \pi, \rho \gamma, \ldots$, systematic methods have been developed using QCD factorization based on the heavy-quark limit [1–3]. Among the building blocks in the corresponding factorization formula of the decay amplitude, essential roles are played by the light-cone distribution amplitudes (LCDAs) for the participating mesons, which include nonperturbative long-distance contributions. In particular, in addition to the LCDAs for the light mesons $\pi, \rho$, etc., produced in the final state, those for the $B$ meson [4] also participate in processes where large momentum is transferred to the soft spectator quark via hard gluon exchange [1–3, 5–8]. The leading quark-antiquark component of the $B$-meson LCDA is defined as the vacuum-to-meson matrix element [9]*:

$$\tilde{\phi}_+(t, \mu) = \frac{1}{iF(\mu)} \langle 0 | \bar{q}(t) n | e^{i g \int_0^{\Lambda} d\lambda n \cdot A(\lambda n) \gamma_5} h_v(0) | \bar{B}(v) \rangle = \int d\omega e^{-i\omega t} \phi_+(\omega, \mu). \quad (1)$$

Here, the bilocal operator is built of the $b$-quark and light-antiquark fields, $h_v(0)$ and $\bar{q}(t) n$, linked by the Wilson line at a light-like separation $tn$, with $n_{\mu}$ as the light-like vector ($n^2 = 0, n \cdot v = 1$), and $v_\mu$ representing the 4-velocity of the $B$ meson; a difference between (1) and the familiar pion-LCDA [11] is that $h_v(0)$ is an effective field in the heavy-quark effective theory (HQET) [12]. $\mu$ denotes the scale where the operator is renormalized, and $F(\mu)$ is the decay constant in HQET, $F(\mu) = -i \langle 0 | \bar{q}(0) \gamma_5 h_v(0) | \bar{B}(v) \rangle$. The RHS in (1) defines the momentum representation, with $\omega v^+$ denoting the LC component of the momentum of the light antiquark.

The “IR structure” of the $B$-meson LCDA was studied [13] using constraints from the equations of motion (EOM), $\bar{q} \slashed{D} v = v \cdot \slashed{D} h_v = 0$, and heavy-quark symmetry, $\bar{\psi} h_v = h_v$. These constraints are solved to give (1) as $\phi_+(\omega) = \phi_+^{(WW)}(\omega) + \phi_+^{(g)}(\omega)$, where the first term is expressed by the analytic formula $\phi_+^{(WW)}(\omega) = \omega \theta(2\Lambda - \omega)/(2\Lambda^2)$ in terms of a HQET parameter [12], $\Lambda = m_B - m_b$, which represents the mass difference between the $B$-meson and $b$-quark. The second term, $\phi_+^{(g)}$, is obtained as a certain integral of the matrix element of the three-body LC operator, $\langle \bar{q}(t) n | G_{a\beta}(u) \sigma_{\lambda\gamma} h_v(0) \rangle$, where the nonperturbative gluons participate as the field strength tensor $G_{a\beta}$; see (9) below for the integro-differential equation, which is derived from the relevant EOM constraints and gives $\phi_+(\omega) = \phi_+^{(WW)}(\omega) + \phi_+^{(g)}(\omega)$ as its solution (see also [14]). In addition, the “UV structure” of the $B$-meson DA was studied [15] by calculating the 1-loop renormalization of the bilocal operator in (1). The “vertex-type” correction around a “cusp” between the two Wilson lines, the light-like Wilson line of (1) and the time-like Wilson line from $h_v(0) = P \exp[ig \int_{-\infty}^0 d\lambda v \cdot A(\lambda v)] h_v(-\infty v)$, produces the “radiation tail”, given by $\phi_+^{(g)}(\omega, \mu) \sim -C_F \alpha_s \ln(\omega/\mu)/(\pi \omega)$ for $\omega \gg \mu$, where $C_F = (N_c^2 - 1)/(2N_c)$. This implies that the moments $\int_0^{\infty} d\omega \omega^j \phi_+^{(g)}(\omega, \mu)$, which would correspond to matrix elements of the local operators $\bar{q}(0)(n \cdot D)^j \gamma_5 h_v(0)$, are divergent, reflecting “cusp singularity” [15, 16].

However, these results do not represent the whole story. As we shall demonstrate explicitly, a full description of (1) actually involves a complicated mixture of the IR and UV structures; e.g., the above-mentioned functional forms $\phi_+^{(WW)}(\omega)$, $\phi_+^{(g)}(\omega)$ from the EOM

*For three-quark LCDAs for the $\Lambda_b$ baryon, see [10].

†For simplicity the Wilson lines connecting the constituent fields are suppressed, and $\langle \cdots \rangle \equiv \langle 0 | \cdots | \bar{B}(v) \rangle$. 

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constraints are subject to additional effects from radiative corrections when combined with the UV structure and are profoundly modified, in particular, for large values of $\omega$ (see the discussion below (9)). To investigate the behavior of the $B$-meson DA incorporating both IR and UV structures, we first calculate the radiative corrections, taking into account hard and soft/collinear loops. The one-particle-irreducible 1-loop diagrams (1LDs) for the 2-point function $\langle \bar{q}(tn)\gamma_5 h_v(0) \rangle$ of (1) yield

$$1\text{LDs} = \frac{\alpha_s C_F}{2\pi} \int_0^1 d\xi \left\{ - \left( \frac{1}{2\varepsilon_{UV}^2} + \frac{L}{\varepsilon_{UV}} + L^2 + \frac{5\pi^2}{24} \right) \delta(1-\xi) + \left( \frac{1}{\varepsilon_{UV}} - \frac{1}{\varepsilon_{IR}} \right) \left( \frac{\xi}{1-\xi} \right)_+ \right. $$

$$ - \left( \frac{1}{\varepsilon_{IR}^2} + L \right) \langle \bar{q}(\xi tn)\gamma_5 h_v(0) \rangle - t \left( \frac{1}{\varepsilon_{IR}} + 2L - 1 -\xi \right) \langle \bar{q}(\xi tn) v \cdot \overrightarrow{D} \gamma_5 h_v(0) \rangle \right\} + \cdots,$$

(2)

in $D = 4 - 2\varepsilon$ dimensions and Feynman gauge, where $L \equiv \ln [i(t - i0)\mu e^{i\pi}]$ with the $\overline{MS}$ scale $\mu$ and the Euler constant $\gamma_E$, and the “$-i0$” prescription comes from the position of the pole in the relevant propagators in the coordinate-space (see (3) below). The “vertex-type” correction that connects the light-like Wilson line and the $\bar{q}(tn)$ field in (1) is associated with only the massless degrees of freedom; thus, the correction yields the term with the “canceling” UV and IR poles, $1/\varepsilon_{UV} - 1/\varepsilon_{IR}$, from the scaleless loop-integral, and with the “plus”-distribution $(\xi/(1-\xi))_+$ as the splitting function. This term is identical to the corresponding correction for the case of the pion LCDA, where the contributions from all 1-loop diagrams have the same $1/\varepsilon_{UV} - 1/\varepsilon_{IR}$ structure with the Brodsky-Lepage kernel [11] as the total splitting function. However, the other terms in (2) have “non-canceling” UV and IR poles: another vertex-type correction, connecting the light-like Wilson line and the $h_v(0)$ field in (1), gives [15] the terms proportional to $\delta(1-\xi)$, which contain the double as well as single UV pole, corresponding to the cusp singularity mentioned above. The “ladder-type” correction, connecting the two quark fields in (1), gives all the remaining terms in (2), which contain the IR poles and are associated with not only the bilocal operator in (1), but also the higher dimensional operators (see the discussion below (3)); note that the ellipses in (2) are expressed by the operators involving two or more additional covariant derivatives.

The renormalized LCDA is obtained by subtracting the UV poles from (2) with the trivial quark self-energy corrections complemented. Here, the term with the plus-distribution $(\xi/(1-\xi))_+$ is analytic (Taylor expandable) at $t = 0$, similar to the pion LCDA, but the other terms are not analytic due to the presence of logarithms $L, L^2$ [15, 17]. In particular, the nontrivial dependence of the latter terms on $t\mu$ through $L$ implies that the scale $\sim 1/t$ separates the UV and IR regions. Thus, we have to use the operator product expansion (OPE) to treat the different UV and IR behaviors simultaneously: the coefficient functions absorb all the singular logarithms, while, for the local operators to absorb the IR poles, we have to take into account many higher dimensional operators. Such OPE with local operators is useful when the separation $t$ is less than the typical distance scale of quantum fluctuation, i.e., when $t \lesssim 1/\mu$. The OPE result can be evolved from a low $\mu$ to a higher scale using the Brodsky-Lepage-type kernel and the Sudakov-type operator with the cusp anomalous dimension [15, 17] associated with the single-UV-pole terms in (2).
An OPE for the $B$-meson LCDA (1) was discussed in [18], taking into account the local operators of dimension $d \leq 4$ and the NLO ($O(\alpha_s)$) corrections to the corresponding Wilson coefficients in a “cutoff scheme”, where an additional momentum cutoff $\Lambda_{UV} \gg \Lambda_{QCD}$ was introduced, and the OPE, in powers of $1/\Lambda_{UV}$, was derived for the regularized moments, \[\int_0^{\Lambda_{UV}} d\omega \omega^j \phi_+ (\omega, \mu),\] in particular, for the first two moments with $j = 0, 1$. In this Letter, we derive the OPE for (1), taking into account the local operators of dimension $d \leq 5$ and calculating the corresponding Wilson coefficients at NLO accuracy. Following the discussion above, we carry out the calculation for $t \lesssim 1/\mu$ in the coordinate space and in the MS scheme, so that there is no need to introduce any additional cutoff.

The most complicated part of this calculation is the reorganization of contributions from (many) Feynman diagrams in terms of the matrix element of gauge-invariant operators, including higher dimensional operators. In particular, to derive the Wilson coefficients associated with the dimension-5 operators such as $\bar{q}G_{\alpha\beta}^{\mu} \gamma_5 h_\nu$, we have to compute the 1-loop diagrams for the 3-point function, as well as those for the 2-point function as in (2), where the former diagrams are obtained by attaching the external gluon line to the latter diagrams in all possible ways. To perform the calculation in a systematic and economic way, we employ the background field method [19]. We decompose the quark and gluon fields into the “quantum” and “classical” parts, where the latter part represents the nonperturbative long-distance degrees of freedom inside the $B$ meson as a background field and satisfies the classical EOM. The quark and gluon propagators for the quantum part contain their coupling with an arbitrary number of background fields, and each building block of the Feynman diagrams obeys the exact transformation property under the gauge transformation for the background fields. We use the Fock-Schwinger gauge, $x^\mu A^{(c)}_\mu (x) = 0$, for the background gluon field $A^{(c)}_\mu$. This gauge condition is solved to give $A^{(c)}_\mu (x) = \int_0^1 dw x^\beta G^{(c)}_{\beta \mu} (wx)$ [19], which allows us to reexpress each Feynman diagram in terms of the matrix element of the operators associated with the field strength tensor. Using this framework, the tree-level matching to derive our OPE can be performed replacing each constituent field in (1) with the corresponding background field. The classical bilocal operator can be Taylor expanded, and we obtain the OPE at the tree level with the local operators of dimension-3, -4, and -5 (see the $O(\alpha_s^0)$ terms in (4) below).

For the 1-loop matching, we calculate the 1-loop corrections to the 2- and 3-point functions with the insertion of the bilocal operator in (1), taking into account the mixing of operators of dimension $d \leq 5$. Apparently, the mixing through the UV region of the loop momenta arises only in the 2-point function, and the result can be immediately obtained from (2); however, additional mixing can arise in both 2- and 3-point functions accompanying the IR poles. We perform the loop integration in the coordinate space using the Schwinger (“heat kernel”) representation of the Feynman amplitudes under the background fields [19]. For the calculation of the 3-point function, the Fock-Schwinger gauge ensures that the Wilson line in (1), as well as the heavy-quark field, does not couple directly to the background gluons, while a massless quark or gluon line couples to them. For example, the “ladder-type” correction diagrams for the 2- and 3-point functions are obtained by connecting the two quark fields in (1) using the gluon propagator in the background gluon.
fields [19], and, after some manipulations using the heat-kernel expansion, we encounter the (coordinate-space) loop integrations of the type

\[
\int_0^\infty dr \left[ \frac{e^{\xi r^2}}{4} \right] \frac{1}{\Gamma \left( \frac{D}{2} - m \right)} \frac{r^{1-m+n}}{(-r^2 - 2tr + i0)^{\frac{D}{2}-m}} = \frac{(-1)^{n+1} \Gamma(n)(2t)^m+n-2}{\Gamma(m+n-1)} \left[ \frac{1}{2 \varepsilon_{IR}} + L + \frac{1}{2} S_{n-1} - S_{m+n-2} \right],
\]

in \( D (= 4 - 2\varepsilon) \) dimensions, with \( n, m = 1, 2, \ldots \), and \( S_n = \sum_{k=1}^n (1/k) \). Here, the integral over the proper time \( r \), associated with the propagation of the heavy quark in the corresponding diagrams, is UV finite but IR divergent as \( D \to 4 \), yielding the RHS. The 2-point function receives the contribution of the dimension-3 operator, \( \bar{q} \gamma_5 h_v \), accompanying (3) with \( n = m = 1 \) as its coefficient. The higher dimensional operators with additional covariant derivatives and/or the gluon field strength tensor contribute to 2- as well as 3-point functions, accompanying (3) with \( n + m \geq 2 \). Those latter contributions manifest the mixing of operators of dimension-4 and higher, accompanying the IR poles. Note that this type of mixing mechanism for “higher twist” operators through loop effects is forbidden if the corresponding loop subdiagram is composed of massless degrees of freedom only.

After working out the loop integral of all 1-loop diagrams for the 2- and 3-point functions, we reorganize the result in terms of a complete set of gauge-invariant operators of dimension \( d \leq 5 \), using some extension of the technique of [9, 13] based on the EOM and heavy-quark symmetry. We subtract the UV poles to renormalize the bilocal operator of (1) and also reorganize the local operators to absorb the IR poles. (The details of the above-mentioned procedures to treat the 2- and 3-point functions at 1-loop will be presented elsewhere [20].) Combining the result with the tree-level result obtained above, we obtain the OPE for the bilocal operator in (1), \( \bar{q}(t n) P e^{ig f_a^d \alpha_i (\lambda n)} \bar{q} \gamma_5 h_v(0) = \sum_i C_i(t, \mu) \mathcal{O}_i(\mu) \), to the desired accuracy as

\[
\bar{q}(t n) P e^{ig f_a^d \alpha_i (\lambda n)} \bar{q} \gamma_5 h_v(0) = \left[ 1 - \frac{\alpha_s C_F}{4\pi} \left( 2L^2 + 2L + \frac{5\pi^2}{12} \right) \right] \mathcal{O}_1^{(3)}
\]

\[
- i t \left\{ \left[ 1 - \frac{\alpha_s C_F}{4\pi} \left( 2L^2 + 2L + \frac{5\pi^2}{12} \right) \right] \mathcal{O}_1^{(4)} - \frac{\alpha_s C_F}{4\pi} \left( 4L - 3 \right) \mathcal{O}_2^{(4)} \right\}
\]

\[
- \frac{t^2}{2} \left\{ \left[ 1 - \frac{\alpha_s C_F}{4\pi} \left( 2L^2 + \frac{2}{3}L + \frac{5\pi^2}{12} \right) \right] \mathcal{O}_1^{(5)} - \frac{\alpha_s C_F}{4\pi} \left( 4L - \frac{10}{3} \right) \mathcal{O}_2^{(5)} + \mathcal{O}_3^{(5)} \right\}
\]

\[
+ \frac{\alpha_s}{4\pi} \left[ C_F \left( -4L + \frac{10}{3} \right) + C_G \left( 7L - \frac{13}{2} \right) \right] \mathcal{O}_4^{(5)} + \frac{\alpha_s}{4\pi} \left( -\frac{4}{3}C_F + C_G \right) (L - 1) \mathcal{O}_5^{(5)}
\]

\[
+ \frac{\alpha_s}{4\pi} \left( -\frac{2}{3}C_F + C_G \right) (L - 1) \mathcal{O}_6^{(5)} + \frac{\alpha_s}{4\pi} \left( \frac{1}{3}C_F + \frac{1}{4}C_G \right) (L - 1) \mathcal{O}_7^{(5)} \right\}.
\]

Here and below, \( C_G = N_c, \mu \) is the \( \overline{MS} \) scale, and \( \alpha_s \equiv \alpha_s(\mu) \). A basis of local operators of dimension-\( d \), \( \mathcal{O}_k^{(d)} \) (\( k = 1, 2, \ldots \)), is defined as

\[
\mathcal{O}_1^{(3)} = \bar{q} \gamma_5 h_v, \quad \left\{ \mathcal{O}_k^{(4)} \right\} = \left\{ \bar{q}(i n \cdot \overline{D}) \gamma_5 h_v, \quad \bar{q}(i v \cdot \overline{D}) \gamma_5 h_v \right\}, \quad (5)
\]
\{ \mathcal{O}_k^{(5)} \} = \{ \bar{q}(in \cdot \overrightarrow{D})^2 \gamma_5 h_\nu, \ \bar{q}(iv \cdot \overrightarrow{D})(in \cdot \overrightarrow{D}) \gamma_5 h_\nu, \ \bar{q}(iv \cdot \overrightarrow{D})^2 \gamma_5 h_\nu, \}

\bar{q}igG_{\alpha\beta}v^\alpha n^\beta \gamma_5 h_\nu, \ \bar{q}igG_{\alpha\beta}\gamma^\alpha v^\beta \gamma_5 h_\nu, \ \bar{q}igG_{\alpha\beta}\gamma^\alpha v^\beta \gamma_5 h_\nu, \ \bar{q}gG_{\alpha\beta}\sigma^\alpha \gamma_5 h_\nu \}

(6)

with another light-like vector, \( \vec{n}^2 = 0 \), as \( \nu_\mu = (n_\mu + \vec{n}_\mu)/2 \). The double logarithm \( L^2 \) in the coefficient functions originates from the cusp singularity (see (2)). The 1-loop corrections for the 3-point function induce only \( \mathcal{O}^{(5)}_{4,5,6,7} \) associated with the field-strength tensor, while those for the 2-point function induce all ten operators of (5), (6) through the use of the EOM.

Taking the matrix element \( \langle \cdots \rangle \equiv \langle 0 | \cdots | \mathcal{B}(v) \rangle \) of (4), we can derive the OPE form of the \( B \)-meson LCDA (1). The matrix elements of the local operators in (5), (6) are known to be related to a few nonperturbative parameters in the HQET, using the EOM and heavy-quark symmetry as demonstrated in [9, 13]: \( \langle \mathcal{O}^{(4)} \rangle = 4iF(\mu)\Lambda/3 \), \( \langle \mathcal{O}^{(5)} \rangle = iF(\mu)\Lambda \), where \( F \) and \( \Lambda \) were introduced below (1), and all seven matrix elements \( \langle \mathcal{O}_k^{(5)} \rangle \) for the dimension-5 operators (6) can be expressed by \( F, \Lambda \) and two additional HQET parameters \( \lambda_E \) and \( \lambda_H \), which are associated with the chromoelectric and chromomagnetic fields inside the \( B \) meson as \( \langle \bar{q}gE \cdot \boldsymbol{\sigma} \gamma_5 h_\nu \rangle = F(\mu)\lambda_E^2(\mu) \) and \( \langle \bar{q}gH \cdot \boldsymbol{\sigma} \gamma_5 h_\nu \rangle = iF(\mu)\lambda_H^2(\mu) \), respectively, in the rest frame where \( v = (1, 0) \). As a result, we obtain the OPE form for the LCDA (1),

\[
\tilde{\phi}_\pm(t, \mu) = 1 - \frac{\alpha_s C_F}{4\pi} \left( 2L^2 + 2L + \frac{5\pi^2}{12} \right) - \frac{4\Lambda}{3} \left[ 1 - \frac{\alpha_s C_F}{4\pi} \left( 2L^2 + 4L - \frac{9}{4} + \frac{5\pi^2}{12} \right) \right] \\
- t^2 \Lambda^2 \left[ \frac{1 - \alpha_s C_F}{4\pi} \left( 2L^2 + \frac{16}{3} - \frac{35}{9} + \frac{5\pi^2}{12} \right) - \frac{2\Lambda^2}{3} \left[ 1 - \frac{\alpha_s C_F}{4\pi} \left( 2L^2 + 2L - \frac{2}{3} + \frac{5\pi^2}{12} \right) \right] \\
+ \frac{\alpha_s C_G}{4\pi} \left( \frac{3}{4} - \frac{1}{2} \right) \right] - \frac{t^2 \Lambda_H^2(\mu)}{6} \left[ 1 - \frac{\alpha_s C_F}{4\pi} \left( 2L^2 + \frac{2}{3} + \frac{5\pi^2}{12} \right) - \frac{\alpha_s C_G}{8\pi} (L - 1) \right],
\]

(7)

which takes into account the Wilson coefficients to \( O(\alpha_s) \) and a complete set of the local operators of dimension \( d \leq 5 \); the cusp singularity in the former leads to the double logarithms \( L^2 \), while the constraints on matrix elements of the latter from the EOM and heavy-quark symmetry allow us to represent the result completely in terms of only three HQET parameters, \( \Lambda, \lambda_E \) and \( \lambda_H \). Thus, (7) “merges” the UV [15] and IR structures [13] peculiar to the \( B \)-meson LCDA, so that it embodies novel behaviors that are completely different from those of the pion LCDA: \( \mu \) and \( t \) are strongly correlated due to the logarithmic contributions, \( L = \ln [i(t - i0)\mu e^{\gamma_E}] \), from radiative corrections, so that the DA is not Taylor expandable about \( t = 0 \). The DA receives the contributions from (many) higher dimensional operators, in particular, from those associated with the long-distance gluon fields inside the \( B \)-meson.

Fourier transforming (7), we obtain the momentum representation, \( \phi_\pm(\omega, \mu) \) in (1), and can also evaluate the regularized-moments, \( M_j = \int_0^{\Lambda_{UV}} d\omega \omega^3 \phi_\pm(\omega, \mu) \), with the cutoff \( \Lambda_{UV} \gg \Lambda_{QCD} \); \( M_j \) corresponds to the “regularized coefficients” of the \( t^j \) term in the Taylor expansion of the DA (1) and thus represents the \( t^j M_j = O(M_j/\Lambda_{UV}^j) \) effects on \( \tilde{\phi}_\pm(t, \mu) \) for \( t \lesssim 1/\Lambda_{UV} \). Here, without going into the detail [20], we note that, for the first two moments \( M_0 \) and \( M_1 \), the contributions from the first line in (7), associated with the matrix elements
of the dimension-3 and -4 operators (5), coincide with the result obtained in [18], while the second and third lines\textsuperscript{1} in (7), which are generated from the dimension-5 operators (6), yield the new power-correction terms down by \( \Lambda_{QCD}/\Lambda_{UV} \), as controlled by the corresponding Wilson coefficients. A similar pattern is observed for the asymptotic behavior of \( \phi_+(\omega, \mu) \) for \( \omega \gg \Lambda_{QCD} \). In view of our coordinate-space OPE approach, the UV divergence in the moments [9,15,18], \( M_j \to \infty \) as \( \Lambda_{UV} \to \infty \), reflects the non-analyticity of (4) and (7) at \( t = 0 \) due to the presence of logarithms \( L \) and \( L^2 \) in the Wilson coefficients (see also the discussion in [17]).

Our OPE results (4) and (7) give a model-independent description of the \( B \)-meson LCDA when \( t \lesssim 1/\mu \) (\( \lesssim 1/\Lambda_{QCD} \)), taking into account the UV and IR structures simultaneously. It is instructive to draw a comparison with the previous results mentioned below (1), concerning UV [15] or IR structure [13]. In [15], the renormalization group (RG) evolution of (1) was derived. The corresponding evolution kernel is determined by the (single) UV poles in (2) as the sum of the cusp anomalous dimension and the plus-distribution terms. One can prove that our LCDA (7) satisfies the RG equation of [15], taking into account the sum of the cusp anomalous dimension and the plus-distribution terms. One can prove that our LCDA (7) satisfies the RG equation of [15], taking into account \( d\bar{\Lambda}/d\mu = 0 \) and [21]

\[
\mu \frac{d}{d\mu} \left( \frac{\lambda_E^2(\mu)}{\lambda_H^2(\mu)} \right) = - \frac{\alpha_s}{4\pi} \left( \frac{8}{3} C_F + \frac{3}{2} C_G \right) \left( \frac{4}{3} C_F - \frac{3}{2} C_G \right) \left( \frac{8}{3} C_F + \frac{3}{2} C_G \right) \left( \frac{\lambda_E^2(\mu)}{\lambda_H^2(\mu)} \right). \tag{8}
\]

Thus, our results, (4) and (7) via the 1-loop matching, are completely consistent with the UV structure obtained at the 1-loop level in [15]. As a result, the scale dependence of the \( B \)-meson LCDA (1) with \( t \lesssim 1/\Lambda_{QCD} \) is represented by the two-step evolution: it is governed by the solution of the evolution equation in terms of the Sudakov-type and Brodsky-Lepage-type scale dependence from the high scale \( \mu_i \simeq \sqrt{m_b \Lambda_{QCD}} \), associated with the QCD factorization formula, to the scale \( \mu \lesssim 1/t \), while that for the lower scale is governed by the anomalous dimensions of local operators like (8). Here, in principle, we can take advantage of the RG improvement as usual. To achieve the control at the next-to-leading logarithmic accuracy, we take into account the 2-loop as well as 1-loop cusp anomalous dimension [16] for the Sudakov-type evolution from \( \mu_i \) to \( \mu \) (see, e.g., [6]); similarly, the evolution for the lower scale requires the 2-loop anomalous dimensions of \( \lambda_E^2 \) and \( \lambda_H^2 \), but they are unknown at present.

On the other hand, the connection of the present result (7) with the IR structure of [13] is not straightforward: from the EOM and heavy-quark symmetry constraints, a set of relations between the two- and three-particle LCDAs was obtained in [13]. Using these relations, we can derive the following integro-differential equation for the LCDA (1),

\[
\int_0^t \frac{d\tilde{\phi}_+(t)}{dt} + (2i\Lambda + 1) \tilde{\phi}_+(t) - \frac{1}{t} \int_0^t dt' \tilde{\phi}_+(t') + 2t^2 \int_0^1 du \left\{ (u + 1) \tilde{\Psi}_A(t, u) + u \tilde{\Psi}_V(t, u) + \tilde{X}_A(t, u) - \frac{u}{t^2} \int_0^t dt'' t'' \left[ \tilde{\Psi}_A(t'', u) - \tilde{\Psi}_V(t'', u) \right] \right\} = 0 , \tag{9}
\]

\textsuperscript{1}They also allow us to derive \( M_2 \), up to the corrections that are suppressed for \( \Lambda_{UV} \gg \Lambda_{QCD} \) [20]; from dimension counting [18], the local operators of dimension \( d \geq 3 \) contribute to \( M_j \) as \( \sim \Lambda_{UV}^d (\Lambda_{QCD}/\Lambda_{UV})^{d-3} \).
where $\tilde{\Psi}_A(t, u)$, $\tilde{\Psi}_V(t, u)$, and $\tilde{X}_A(t, u)$ are the three-particle LCDAs introduced in [13] (see also [14]): when $t \lesssim 1/\Lambda_{QCD}$, we substitute (7) for $\tilde{\phi}_+$, while the three-particle LCDAs are given as $\tilde{\Psi}_A = \frac{1}{2} \lambda_E^2$, $\tilde{\Psi}_V = \frac{1}{3} \lambda_H^2$, and $\tilde{X}_A = 0$ in terms of matrix elements of local operators of dimension $d \leq 5$ [13], omitting the contributions from higher dimensional operators as well as from radiative corrections. Then the $O(\alpha_s^3)$ contribution to the LHS of (9) vanishes up to the corrections of $O((t\Lambda_{QCD})^3)$ which are beyond the present accuracy, but the $O(\alpha_s)$ contribution proves to yield a nonzero result as $(\alpha_s C_F/4\pi) [-8L + it\Lambda (32L/3 + 3/2) + \cdots]$. Thus, (9) receives corrections at order $\alpha_s$; namely, the relations of [13] are satisfied by the nonperturbative matrix elements of local operators in the OPE (4), but the $O(\alpha_s)$ loop effects in the corresponding Wilson coefficients prevent those relations from being exact for the DA (7) (see also the discussion in [17, 22]). Such a violation of relations of the type (9) at order $\alpha_s$ in perturbation theory is peculiar to the heavy-meson LCDAs in the HQET and does not arise for the case of the (higher twist) LCDAs for the light mesons, $\pi, \rho$, etc. [23].

Our OPE form (7) allows us to parameterize all nonperturbative contributions in the $B$-meson LCDAs (1) for $t \lesssim 1/\Lambda_{QCD}$ by a few HQET parameters. Here, we evaluate (7) at the scale $\mu = 1$ GeV, using the information available for those HQET parameters. First of all, we note that $\Lambda = m_B - m_b$ in (7) is defined by the $b$-quark pole mass $m_b$. Following [18], we eliminate $\tilde{\Lambda}$ in favor of a short-distance parameter, $\Lambda_{DA}$, free from IR renormalon ambiguities [24] and written as $\Lambda = \Lambda_{DA}(\mu) [1 + (7/16\pi)C_F\alpha_s] - (9/8\pi)\mu C_F\alpha_s$, to one-loop accuracy; $\Lambda_{DA}(\mu)$ can be related to another short-distance mass parameter whose value is extracted from analysis of the spectra in inclusive decays $B \to X_{\gamma} \gamma$ and $B \to X_u l \nu$, leading to $\Lambda_{DA}(\mu = 1$ GeV) $\approx 0.52$ GeV [18]. For the novel parameters associated with the dimension-5 operators, we use the central values of

$$\lambda_E^2(\mu) = 0.11 \pm 0.06 \text{ GeV}^2, \quad \lambda_H^2(\mu) = 0.18 \pm 0.07 \text{ GeV}^2,$$

at $\mu = 1$ GeV, which were obtained by QCD sum rules [9]; no other estimate exists for $\lambda_E$ or $\lambda_H$. We calculate (7) for imaginary LC separation, performing the Wick rotation $t \to -i\tau$ [9, 17].

The results for $\tilde{\phi}_+(-i\tau, \mu = 1$ GeV) using (7) are shown as a function of $\tau$ in Fig. 1: the RHS shows the behaviors for small and moderate $\tau$ regions, and the LHS displays also the region with larger $\tau$. The wide-solid curve shows the whole contributions of (7), while the narrow-solid curve shows the result for $\alpha_s \to 0$; the NLO perturbative corrections are at the 10-30% level for moderate $\tau$, while they are very large for $\tau \to 0$ because of singular logarithms $L^2$ and $L$. The dashed and dot-dashed curves show the contributions of the first two terms and the first line in (7), respectively, associated with the operators of dimension $d = 3$ and $d \leq 4$, while the dotted curve gives the results of (7) when $\lambda_E = \lambda_H = 0$. For moderate $\tau$, the contributions from the dimension-4 operators suppress the DA by 30-40%, but the dimension-5 operators, in contrast, lead to enhancement by 10-20% with significant effects from $\lambda_E$ and $\lambda_H$.

In the large $\tau$ region, the dimension-4 and -5 operators change their roles such that they enhance and suppress the DA, respectively, because of the growth of the double logarithm $L^2$ in the NLO Wilson coefficients; this fact would eventually call for treating the higher order
perturbative corrections beyond NLO, in particular, resummation of the large logarithms in the Wilson coefficients. We also note that the contributions from the dimension-$d$ operators grow as $\sim \tau^{d-3}$ with increasing $\tau$, and, beyond a certain value of $\tau$, the contributions from the dimension-4 or -5 operators become larger than those from the lower-dimensional operators; this indicates that the hierarchy of contributions is lost, and the OPE breaks down. These considerations with quantitative results in Fig. 1 show that our $B$-meson LCDA (7) indeed works up to moderate LC distances $\tau$ of order $1 \text{ GeV}^{-1} \sim 1/\mu$; in this region, the hierarchy among the dashed, dot-dashed, and wide-solid curves demonstrates convergence of the OPE (4) and the corresponding accuracy of our LCDA (7).

We find that the behavior of the wide-solid curve for small and moderate $\tau$ in Fig. 1 is similar to that of the previous results [17, 18] for the $B$-meson LCDA, obtained by taking into account perturbative as well as nonperturbative QCD corrections in a systematic framework. For example, the two-dot-dashed curve in Fig. 1 shows the behavior of the two-component ansatz by Lee and Neubert [18], which is given in momentum space as

$$\phi_{1}^{\text{LN}}(\omega, \mu) = N \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0} + \theta(\omega - \omega_t) \frac{C_F \alpha_s}{\pi \omega} \left[ \left( \frac{1}{2} - \ln \frac{\omega}{\mu} \right) + \frac{4 \bar{\Lambda}_D A_3}{3 \omega} \left( 2 - \ln \frac{\omega}{\mu} \right) \right], \quad (11)$$

where the second term reproduces the correct asymptotic behavior of the DA (1) for $\omega \gg \Lambda_{\text{QCD}}$, and the first term represents the nonperturbative component modeled by an exponential form [9]; $\omega_t$ is chosen such that (11) is continuous. The other parameters, $N$ and $\omega_0$, are fixed by matching the first two $(j = 0, 1)$ cut-moments $\int_0^{\Lambda_{UV}} d\omega \omega^2 \phi_{1}^{\text{LN}}(\omega, \mu)$ with the OPE for the corresponding cut-moments $M_{0,1}$ derived in [18], where the operators of dimension $d \leq 4$ and the corresponding Wilson coefficients at NLO are taken into account.

Figure 1: The $B$-meson LCDA $\tilde{\phi}_+(−i\tau, \mu)$ at $\mu = 1 \text{ GeV}$ using the OPE-based form (7): the wide-solid curve represents the whole contributions, and the dashed and dot-dashed curves show the contributions associated with the operators of dimension $d = 3$ and $d \leq 4$, respectively. The narrow-solid and dotted curves show the cases when $\alpha_s \to 0$ and $\lambda_{E,H} \to 0$, respectively. The two-dot-dashed curve plots the result using the Lee-Neubert ansatz (11).
(see the discussion below (7)). The (central) values of these parameters are \(\omega_1 = 2.33\) GeV, \(N = 0.963\), and \(\omega_0 = 0.438\) GeV at \(\mu = 1\) GeV [18]. For \(\tau \lesssim 1\) GeV\(^{-1}\) in Fig. 1, the Lee-Neubert ansatz (11) shows behavior similar to (7) with \(\lambda_E = \lambda_H = 0\) substituted. Indeed, when Fourier transformed to the coordinate space, (11) produces, with good accuracy, the terms in the first line of (7), which are associated with the operators of dimension \(d \leq 4\); moreover, the first term of (11) also produces particular contributions associated with the operators of dimension-5 and higher (see (12) below), and the sizes of those contributions are actually rather close to those of the terms proportional to \(\tilde{\Lambda}^2\) in (7) for \(\tau \lesssim 1\) GeV\(^{-1}\).

The wide-solid curve in Fig. 1 represents the model-independent behavior of the \(B\)-meson LCDA (1) based on the most accurate OPE (4), (7) at present. However, in the long-distance region, \(\tau \gg 1\) GeV\(^{-1}\), the contributions associated with the operators of any higher dimension become important, and the OPE diverges; thus, one has to rely on a certain model for the large \(\tau\) behavior and connect the model-independent descriptions at small and moderate \(\tau\) to that model in a reasonable manner. The results in Fig. 1 suggest the possibility of connecting the behavior for \(\tau \leq \tau_c\) (\(\tau_c \sim 1\) GeV\(^{-1}\)) given by our OPE form (7) to that for \(\tau \geq \tau_c\), given by the coordinate-space representation of the first term of (11),

\[
\int_0^\infty d\omega e^{-\omega \tau} N \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0} = \frac{N}{(\tau \omega_0 + 1)^2} .
\]

Here, \(N\) and \(\omega_0\) can be determined such that both the resulting total DA \(\tilde{\phi}_+(-i\tau, \mu)\) and its derivative \(\partial \tilde{\phi}_+(-i\tau, \mu) / \partial \tau\) are continuous at \(\tau = \tau_c\). Namely, we perform the matching of \(\tilde{\phi}_+(-i\tau_c, \mu) = \int_0^\infty d\omega e^{-\omega \tau_c} \phi_+(\omega, \mu)\), as well as of \(\partial \tilde{\phi}_+(-i\tau_c, \mu) / \partial \tau_c = -\int_0^\infty d\omega e^{-\omega \tau_c} \omega \phi_+(\omega, \mu)\), between (7) and (12), and this is formally analogous to the matching used for (11). In the LHS of Table 1, associated with the central values of (10), we show the values of \(N\) and \(\omega_0\) obtained by solving our matching relations for \(\mu = 1\) GeV. (The RHS of Table 1 shows the results that would be obtained by solving the similar matching relations with \(\lambda_E = \lambda_H = 0\), and we find that, indeed, \(\tau_c \simeq 0.7\) GeV\(^{-1}\) gives the behavior close to (11).) In Fig. 2, the wide-solid and two-dot-dashed curves are same as those in Fig. 1, and the dotted, solid-gray, and dashed curves show the behavior of (12) for \(\tau \geq \tau_c\) with \(\tau_c = 0.6, 1.0,\) and \(1.4\) GeV\(^{-1}\), respectively, using the corresponding values of \(N\) and \(\omega_0\) in the LHS of Table 1; these three curves behave as \(\sim N/(\omega_0^2 \tau^2)\) at large \(\tau\), with larger \(N/\omega_0^2\) than those of (11) and the RHS of Table 1. Indeed, we can show that \(N/\omega_0^2 = (9/4\tilde{\Lambda}_{DA}^2) \{ 1 + \tau_c \tilde{\Lambda}_{DA} [\lambda_E^2/\tilde{\Lambda}_{DA}^2 + \lambda_H^2/(2\tilde{\Lambda}_{DA}^2) - 1] \} + \cdots\), using our matching relations, and thus the contributions of \(\lambda_E\) and \(\lambda_H\) enhance \(N/\omega_0^2\).

Using these results, we calculate the first inverse moment of the LCDA,

\[
\lambda_B^{-1}(\mu) = \int_0^\infty d\omega \frac{\phi_+(\omega, \mu)}{\omega} = \int_0^{\tau_c} d\tau \tilde{\phi}_+(-i\tau, \mu) + \int_{\tau_c}^\infty d\tau \tilde{\phi}_+(-i\tau, \mu) ,
\]

which is of particular interest for the QCD description of exclusive \(B\)-meson decays. We substitute (7) and (12) into the first and the second terms in the RHS, respectively, and the results are shown in Table 1 for each value of \(\tau_c\). The “stable” behavior observed for \(0.6\) GeV\(^{-1} \lesssim \tau_c \lesssim 1\) GeV\(^{-1}\) in the LHS of Table 1 and in Fig. 2 suggests that \(\lambda_B^{-1}(\mu = 1\) GeV\) \(\simeq 2.7\) GeV\(^{-1}\), i.e., \(\lambda_B(\mu = 1\) GeV\) \(\simeq 0.37\) GeV. This value of \(\lambda_B\) is somewhat
\[ \lambda_E^2 = 0.11 \text{ GeV}^2, \quad \lambda_H^2 = 0.18 \text{ GeV}^2 \]

Table 1: Parameters of the model function (12) for different values of \( \tau_c \), determined by continuity at \( \tau = \tau_c \) with the OPE-based LCDA (7) for \( \mu = 1 \) GeV, and the results of the inverse moment \( \lambda_B^{-1}(\mu) \) at \( \mu = 1 \) GeV, with the first and second numbers in the parentheses denoting the contributions from the first and the second terms in the RHS of (13).

| \( \tau_c [\text{GeV}^{-1}] \) | \( N \) | \( \omega_0 [\text{GeV}] \) | \( \lambda_B^{-1} [\text{GeV}^{-1}] \) |
|----------------|-----|----------------|----------------|
| 0.4 | 0.816 | 0.257 | 3.11 (0.23 + 2.88) |
| 0.6 | 0.850 | 0.306 | 2.70 (0.35 + 2.35) |
| 0.8 | 0.852 | 0.308 | 2.69 (0.47 + 2.22) |
| 1.0 | 0.858 | 0.313 | 2.66 (0.58 + 2.08) |
| 1.2 | 0.910 | 0.349 | 2.51 (0.67 + 1.84) |
| 1.4 | 1.09 | 0.456 | 2.22 (0.76 + 1.46) |
| 1.6 | 1.81 | 0.777 | 1.87 (0.83 + 1.04) |
| 0.4 | 0.832 | 0.301 | 2.69 (0.23 + 2.46) |
| 0.6 | 0.899 | 0.394 | 2.19 (0.35 + 1.84) |
| 0.8 | 0.966 | 0.461 | 1.99 (0.46 + 1.53) |
| 1.0 | 1.11 | 0.572 | 1.79 (0.56 + 1.23) |
| 1.2 | 1.55 | 0.839 | 1.56 (0.64 + 0.92) |
| 1.4 | 4.43 | 1.95 | 1.32 (0.71 + 0.61) |
| 1.6 | 9.82 | −4.55 | 1.11 (0.77 + 0.34) |

Figure 2: The \( B \)-meson LCDA \( \tilde{\phi}_+(−i\tau, \mu) \) at \( \mu = 1 \) GeV. The solid-black and two-dot-dashed curves show the results using the OPE-based form (7) and the Lee-Neubert ansatz (11), respectively. The dotted, solid-gray and dashed curves plot the model function (12) for \( \tau \geq \tau_c \) with \( \tau_c = 0.6, 1.0 \) and \( 1.4 \) GeV\(^{-1} \), respectively, determined by continuity with (7).

smaller than the previous estimates that include nonperturbative and/or perturbative QCD corrections [17,18,25] (e.g., (11) gives \( \lambda_B(\mu = 1 \) GeV) \( \simeq 0.48 \) GeV), although consistent with them within their theoretical errors. A value of \( \lambda_B \) that is as small as our value was adopted in [1,7]. Note that in the RHS of Table 1 with \( \lambda_{E,H} = 0 \), the stable behavior is not seen as clearly as in the LHS, and \( \lambda_B \) assumes larger values than in the latter.

The above results demonstrate that the novel HQET parameters, \( \lambda_E \) and \( \lambda_H \), associated with the dimension-5 quark-antiquark-gluon operators, could lead to smaller value of \( \lambda_B \). In particular, this effect can be more significant: using the values of \( \lambda_E \) and \( \lambda_H \) corresponding to the upper bound of (10), we find that the solid curve in Fig. 2 becomes further enhanced.
in the moderate $\tau$ region, so that (13) gives $\lambda_B(\mu = 1 \text{ GeV}) \sim 0.2 \text{ GeV}$ or smaller. This finding calls for more precise nonperturbative estimates of $\lambda_E$ and $\lambda_H$. We also note that in the RHS of (13) evaluated in Table 1, the second term is much larger than the first term. This suggests that $\lambda_B$ is rather sensitive to the functional form that models the LCDA in the long-distance region; for example, a functional form motivated by $\phi_{_{\text{WW}}}$, which is mentioned below (1), provides an interesting possible alternative to (12). Moreover, the Sudakov-type resummation for the Wilson coefficients may give rise to additional important contribution in the large $\tau$ region. Systematic investigations of these points, as well as the effect of the RG evolution of the LCDA, are beyond the scope of this Letter and will be presented elsewhere.

To summarize, we have derived the OPE that embodies both the notorious UV and IR behaviors of the $B$-meson LCDA, including all contributions from the local operators of dimension $d \leq 5$ and the corresponding Wilson coefficients at NLO accuracy. This OPE allows us to parameterize all nonperturbative contributions in terms of three HQET parameters and provides us with the most accurate description of the $B$-meson LCDA for distances less than $\sim 1/\Lambda_{\text{QCD}}$. We have also used the model-independent behaviors from our OPE to constrain the long-distance behavior of the LCDA and estimate the first inverse moment $\lambda_B^{-1}$ as the integral of the LCDA over entire distances. The results demonstrated the impact of the novel HQET parameters, associated with the matrix elements of the dimension-5 quark-antiquark-gluon operators.

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