New Deformation of $\mathcal{N} = 4$ Super-Yang-Mills Theory

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Abstract: We propose a new algebraic deformation of $\mathcal{N} = 4$ SYM via decomposition of spinor and scalar fields in vector supermultiplet. This decomposition generates degrees of freedom of usual quarks and leptons and the deformation model is a low energy effective model. We show that supersymmetry is broken in certain limit and the deformation model reduces to a SM-like model, or a QCD-like model. Meanwhile, gauge symmetry can be spontaneously broken by nontrivial supersymmetry vacuum.

Keywords: Super Yang-Mills theory, Supersymmetry, Supersymmetry breaking.

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1. Introduction

Today, more and more physicists have believed that the supersymmetry (SUSY), which is treated as maximum symmetry allowed by S-matrix\textsuperscript{1}, is fundamental symmetry of action of quantum field theory\textsuperscript{2}. During the past two decades, many SUSY models, such as supergravity, super Yang-Mills theory (SYM) (and consequently, the supersymmetric version of standard model (SSM)) have been widely studied. All of these SUSY models include large number of field degrees of freedom, and most of them are out of our current observable world. For instance, in SSM, besides of well-known standard model particles: gluon, quark, lepton, ..., there are also their superpartners: gluino, squark, slepton, .... However, all of these superpartners were not found by current experiment. Traditionally, it is believed that those missed degrees of freedom are very heavy when SUSY is spontaneously broken. So that they did not be observed in current experiment spectrum. Unfortunately, so far we still can not find a simple and wide-accepted mechanism to spontaneously break SUSY even though large number of remarkable results have been achieved\textsuperscript{3, 4, 5}. Consequently, we still can not satisfactorily interpret how these particles are missed. This may be because the mechanism of SUSY spontaneously breaking is very complicated, but are there other possibilities? This is just purpose of this paper.

Our work was motivated by Maldacena’s conjecture on Ads/CFT corresponds\textsuperscript{6}, which states that $\mathcal{N} = 4$ SYM in four dimension is dual to IIB string theory on $AdS_5 \times S_5$. This conjecture provides a principle method to deal with strong coupling case of quantum field theory. In particular, it seem to emerge a possibility, that we can directly calculate nonperturbative effect of QCD from QCD. This purpose has been partly achieved in some literature\textsuperscript{7}. For example, the spectrum of glueball has been calculated\textsuperscript{7} and agree with result of lattice QCD. However, it is still very difficult to deal with interaction of state involving quarks. The essential reason is that, in $\mathcal{N} = 4$ SYM, fermion fields are adjoint representation of gauge group rather than fundamental representation of gauge group. The usual method to construct SSM from SYM is to add extra chiral supermultiplets, which are fundamental representation of gauge group and represent quark, lepton and
their superpartners. Unfortunately, we can not obtain this type of chiral superfields from string theory and its dimensional reduction. It seem to imply that the quark and lepton \(^1\) can not be defined (perturbatively at least) in string theory. However, whether can quark or lepton be generated by vector multiplet itslef via deformation of SYM or spontaneously breaking of SYM? We will suggest a new deformation of \(\mathcal{N} = 4\) SYM in this paper and show that this can indeed be achieved.

Our idea is originated by knowledge of group theory: The adjoint representation of \(SU(N)\) group can be generated by directly multiplication of \(N\) and \(\bar{N}\) representation of \(SU(N)\) group. Then vector supermultiplet of \(\mathcal{N} = 4\) SYM can be decomposed into a scalar field, four spin \(1/2\) fermions and a vector field. Here all of these scalar field and fermions are fundamental representation of gauge group. This idea proposes a simple algebra deformation on \(\mathcal{N} = 4\) SYM and the deformation indeed yields similar field degrees of freedom of SM-like model or of QCD-like model. R-symmetry, furthermore, will correspond to generation symmetry of SM-like model, or flavor symmetry of QCD-like model. The simple dimensional analysis requires a scale parameter \(M\) appearing in deformation model. It indicates that the deformation model will be low energy effective model, and its Lagrangian will be expanded in power of \(1/M^2\). When energy is much lower than scale \(M\), only part of Lagrangian is survived. Thus this method provides a natural mechanism to “spontaneously break” SUSY. Meanwhile, the new scalar fields in deformed model are no longer Higgs bosons, rather, they are Goldstone bosons corresponding to SUSY spontaneously breaking, or even they do not appear as dynamical degrees of freedom in certain condition.

In the next section, we propose a simple algebra deformation on \(\mathcal{N} = 4\) SYM. It yields a low energy effective theory. In section 3, We discuss some properties of this deformation model. Section 4 is devoted a brief summary and discussion.

### 2. New Deformation of \(\mathcal{N} = 4\) SYM

The supermultiplets of \(\mathcal{N} = 4\) super Yang-Mills theory contains a spin-1 gauge boson \(A^a_\mu\), four spin \(1/2\) Majorana spinors \(\Psi^a_A\) and six scalar fields \(\Phi^{AB}_a\) (they belong to an antisymmetric tensor representation of \(SU(4)\) group so that \(A, B\) are antisymmetric). Here \(A, B = 1, 2, 3, 4\) denote indexes of \(SU(4)\) R-symmetry, \(a, b = 1, ..., N^2 - 1\) denote indexes of adjoint representation of gauge group. The Lagrangian density of \(\mathcal{N} = 4\) SYM can compactly be written in the manifestly \(SU(4)\)-invaraint form

\[
\mathcal{L} = \frac{1}{2} \sum_{AB} T r \{ (D_\mu \Phi_{AB})(D^\mu \Phi_{AB}) \} + i \sum_A T r (\bar{\Psi}_A D^A \Psi_A) \\
- \sqrt{2} \text{Re} \sum_{AB} T r (\bar{\Psi}_A \gamma_5 [\Phi_{AB}, \Psi_B]) - \frac{1}{8} \sum_{ABCD} T r ([\Phi_{AB}, \Phi_{CD}]^2) \\
- \frac{1}{4} F^{a}_{\mu \nu} F^{a}_{\mu \nu} + \frac{g^2 \theta}{64 \pi^2} \epsilon^{\mu \nu \rho \sigma} F^{a}_{\mu \nu} F^{a}_{\rho \sigma},
\]

\(^1\)In this paper, quark and lepton denote spin \(1/2\) fermion which belong to fundamental representation of gauge group.
where

\[
D_\mu \Psi_A = \partial_\mu \Psi_A + i[A_\mu, \Psi_A],
\]
\[
D_\mu \Phi_{AB} = \partial_\mu \Phi_{AB} + i[A_\mu, \Phi_{AB}].
\] (2.2)

The Majorana spinor fields and scalar fields in the vector supermultiplets can be decomposed to a set of scalar fields \( \varphi_i \) and four sets of Majorana spinors \( \lambda_{Ai} \), \( \bar{\lambda}_{Ai} \), \( \Phi_{AB} \), and \( \bar{\Phi}_{AB} \):

\[
\Psi_{Aij} = \frac{1}{\sqrt{2}} (\phi_i \lambda_{Aj} + \lambda_{Ai} \phi_j - \frac{2}{N} \phi_i \lambda_{Ai} \delta_{ij}),
\]
\[
\bar{\Psi}_{Aij} = \frac{1}{\sqrt{2}} (\phi_i \bar{\lambda}_{Aj} + \bar{\lambda}_{Ai} \phi_j - \frac{2}{N} \phi_i \bar{\lambda}_{Ai} \delta_{ij}),
\]
\[
(\Phi_{AB})_{ij} = M^{-2}(\bar{\lambda}_{Ai} \gamma_5 \lambda_{Bj} - \bar{\lambda}_{Bj} \gamma_5 \lambda_{Ai}) - \frac{1}{2} Z_{AB} U_{ij}(\varphi) - \frac{1}{2}(1 - \frac{2}{N}) Z_{AB} \delta_{ij},
\] (2.3)

where \( i, j = 1, ..., N \) are indexes of gauge group,

\[
\phi_i = \frac{\varphi_i}{\sqrt{\sum_i \varphi_i^2}} \implies \sum_i \phi_i^2 = 1,
\]
\[
U_{ij} = \delta_{ij} - 2 \phi_i \phi_j \implies U^{-1} = U,
\] (2.5)

and \( Z_{AB} \) is central charge associating with extended supersymmetric algebra

\[
\{Q_{Aa}, Q_{Bb}\} = e_{ab} Z_{AB} \quad e = i \sigma_2 = \begin{pmatrix} 0, & 1 \\ -1, & 0 \end{pmatrix}.
\] (2.6)

Since \( \lambda_A \) are fundamental representation of \( SU(N) \) gauge group, its gauge transformation \( \lambda_A \rightarrow \lambda'_A = \Omega(x) \lambda_A (\Omega^\dagger \Omega = 1) \) makes \( \lambda_{Ai} \) no longer be Majorana spinor in general gauge. In this paper, it is convenient to adopt a specific version of Wess-Zumino gauge (hereafter we call it as SWZ gauge) in which \( A_\mu \) is antisymmetric, i.e., its nonzero elements form an adjoint representation of \( SO(N) \) subgroup of \( SU(N) \) group. In SWZ gauge, \( \lambda_A \) will be Majorana spinor, and the covariant derivative

\[
D_\mu \lambda_A = \partial_\mu \lambda_A + i A_\mu \lambda_A,
\]
\[
D_\mu \bar{\lambda}_A = \partial_\mu \bar{\lambda}_A - i \bar{\lambda}_A A_\mu
\] (2.7)

are well-defined. In addition, since \( \Psi_A \) is Majorana spinor, eq. (2.3) requires that \( \phi \) is real field in SWZ gauge.

Now inserting eq. (2.3) into Lagrangian of \( N = 4 \) SYM, we can derive effective Lagrangian of deformation model.

1. Using the properties of Majorana spinors, \( \lambda_{Ai} \gamma_\mu \lambda_{Aj} = -\bar{\lambda}_{Aj} \gamma_\mu \lambda_{Ai} \), and defining \( \mathcal{M}_{ij} = \phi_i \phi_j \), we have \( \bar{\lambda}_A \mathcal{M} \lambda_A = \bar{\lambda}_A (\partial \mathcal{M}) \lambda_A = \bar{\lambda}_A (\mathcal{M} \partial \mathcal{M}) \lambda_A = 0 \). So that

\[
Tr \left( \bar{\Psi}_A \mathcal{D} \Psi_A \right) = \bar{\lambda}_A \mathcal{D} \lambda_A + \left( \frac{1}{2} + \frac{1}{N} \right) \bar{\lambda}_A \gamma_\mu (\partial^\mu \mathcal{M} - \mathcal{M} \partial^\mu \mathcal{M}) \lambda_A + (1 - \frac{2}{N}) \bar{\lambda}_A (\mathcal{M} \partial \mathcal{M} + \partial \mathcal{M}) \lambda_A.
\] (2.8)
The kinetic term of spinor \( \lambda_A \) can be diagonalized via field redefinition
\[
\lambda_A \rightarrow (1 + aM)\lambda_A,
\]
(2.9)
with \( a = -1 + \sqrt{\frac{N}{2(N-1)}} \). Then eq. (2.8) becomes
\[
Tr(\bar{\Psi}_A \mathcal{D} \Psi_A) = \bar{\lambda}_A \mathcal{D} \lambda_A + (1 + 2a)\bar{\lambda}_A \gamma_\mu (D^\mu U U^\dagger U - U^\dagger D^\mu U) \lambda_A
\]
(2.10)
where
\[
D_\mu U = \partial_\mu U + iA_\mu U - iU A_\mu.
\]
(2.11)
Furthermore, in order to make the coupling between spinor and gauge boson be standard form, we can let that
\[
A_\mu = A'_\mu + ibU^\dagger D'_\nu U, \quad b = \frac{1}{2} \left( -1 + \sqrt{\frac{N-1}{2N}} \right),
\]
(2.12)
where
\[
D'_\nu U = \partial_\nu U + iA'_\mu U - iU A'_\nu.
\]
(2.13)
Then finally we have
\[
Tr(\bar{\Psi}_A \mathcal{D} \Psi_A) = \bar{\lambda}_A \mathcal{D}' \lambda_A
\]
(2.14)
2. Inserting eq. (2.3) into \( F^a_{\mu \nu} F^{a \mu \nu} \) and \( \varepsilon^{\mu \rho \sigma} F^a_{\mu \nu} F^{a \rho \sigma} \), and using eq. (2.12), we have
\[
Tr(F_{\mu \nu} F^\mu \nu) = (1 + 2b + 2b^2)Tr(F'_{\mu \nu} F'^\mu \nu) - 2b(1 + b)Tr(F'_{\mu \nu} U^\dagger F'^\mu \nu U)
\]
\[
-2b^2Tr \left( D'_\mu U D'_\nu U D'_{\mu} U^\dagger D'^\nu U - D'_{\mu} U^\dagger D'_{\nu} U D'_{\mu} U^\dagger D'^\nu U \right)
\]
\[
+ 4ibTr \left( F'_{\mu \nu} D'_\mu U^\dagger D'_{\nu} U \right)
\]
(2.15)
\[
\varepsilon^{\mu \nu \rho \sigma} Tr(F_{\mu \nu} F_{\rho \sigma}) = (1 - 2b + 10b^2)\varepsilon^{\mu \nu \rho \sigma} Tr(F'_{\mu \nu} F'_{\rho \sigma})
\]
\[
+ 2b(1 - 5b)\varepsilon^{\mu \nu \rho \sigma} Tr(F'_{\mu \nu} U^\dagger F'_{\rho \sigma} U)
\]
3. \[
Tr\{ (D_\mu \Phi_{AB})(D^\mu \Phi_{AB})^\dagger \} = \frac{f^2}{4} Tr \left( D'_\mu U^\dagger D'^\mu U \right) + \text{four fermion terms (suppressed by } M^{-4})
\]
(2.16)
with \( f^2 = \sum_{AB} (1 + 2b)^2 |Z_{AB}|^2 \).

4. The deformation of \( \sqrt{2} \{ (\Phi_{AB})_{ij} [\bar{\Psi}_{Aij} \gamma_5 \Psi_{Bki} - \bar{\Psi}_{Aki} \gamma_5 \Psi_{Bjk}] \} \) only yields some four fermion terms which is suppressed by \( M^{-2} \).
5. The deformation of the term $\sum_{ABCD} Tr (\Phi_{AB}^\dagger \Phi_{CD}^2)$ is nothing other than generating four fermion terms (suppressed by $Z^2/M^4$), six fermion terms (suppressed by $Z/M^6$) and eight fermion terms (suppressed by $M^{-8}$).

Finally, the Lagrangian of deformation model is written

$$\mathcal{L} = i \sum_A \bar{\lambda}_A \mathcal{D} \lambda_A - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{g^2 \theta}{64\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F^{a\rho\sigma} + \frac{f^2}{4} Tr \left( D_\mu U^\dagger D^\mu U \right)$$

$$- b(1 + b) Tr \left( F_{\mu\nu} F^{\mu\nu} M - F_{\mu\nu} M F^{\mu\nu} \right) - \frac{i}{2} b Tr \left( F_{\mu\nu} U^\dagger D^\mu U \right)$$

$$+ \frac{b^2}{4} Tr \left( D_\mu U^\dagger D_\nu U D^\mu U^\dagger D^\nu U - D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U \right)$$

$$- \frac{g^2 \theta}{8\pi^2} b(1 - 5b) Tr \left( F_{\mu\nu} F_{\rho\sigma} \right) - \frac{i}{g} \Omega \partial_\mu \Omega^\dagger.$$  (2.17)

3. Characterization of The Deformation Model

In this section we discuss some important properties of the deformation model parametered by Lagrangian (2.17).

Gauge invariance

The Lagrangian (2.17) is manifest gauge invariant under the following gauge transformation

$$\lambda_A \rightarrow \Omega(x) \lambda_A, \quad \Omega(x) \in SU(N),$$

$$U \rightarrow \Omega U \Omega^\dagger, \quad \mathcal{M} \rightarrow \Omega \mathcal{M} \Omega^\dagger,$$

$$A_\mu \rightarrow \Omega A_\mu \Omega^\dagger + \frac{i}{g} \Omega \partial_\mu \Omega^\dagger.$$  (3.1)

In the previous section, the Lagrangian (2.17) is written in SWZ gauge. It can be written in general gauge via the above gauge transformation. In general gauge, however, $\lambda_A$ is neither Majorana spinor nor Dirac spinor.

Symmetry breaking, Low energy limit

Two dimensional parameters, $M$ and $Z_{AB}$, present in Lagrangian (2.17). So that this deformation model no longer is a renormalizable theory, rather, it is a low energy effective model. In other words, it includes the coupling of multi fermions and is similar to extended Nambu-Jola-Lasinio model. In Lagrangian (2.17), although the supersymmetry has become highly nonlinear, it is still kept. The conformal invariance, however, has been broken when dimensional parameters are introduced in deformation (2.3).

Another interesting issue is low energy limit of this deformation model, i.e., for energy scale $\mu \ll M$ or $M \rightarrow \infty$. In this limit, the terms on multi fermion coupling can be
ignored. Then the supersymmetry is manifest broken in low energy limit and we obtain a SM-like model except for the terms associating coupling of scalar fields. Up to the gauge transformation, there are $N$ scalar field in this deformation model. They are dynamical degrees of freedom of deformation model. However, the coupling among them fields are nonlinear and their decay constant $f$ is dimensional. Therefore, these scalar fields are not Higgs bosons in this SM-like model (for $M \to \infty$), rather, they become Goldstone bosons corresponding to SUSY breaking.

Since there is another dimensional parameter $Z = \sqrt{\sum_{AB} |Z_{AB}|^2}$ in deformation model, the limit of $M \to \infty$ is not unique. In general, there are three possible limits:

i $Z \ll \mu \ll M$. From kinetic term of scalar fields we can see that the physical scalar fields are obtained via field rescaling $\phi \to \phi/f \sim \phi/Z$. Then dynamics is dominant by Goldstone bosons in this case.

ii $Z \sim \mu \ll M$. In this case, both of the interaction of Goldstone bosons and one of other fields are important.

iii $Z \sim M \gg \mu$. This case is very interesting and important. All interaction associating Goldstone bosons are frozen out in this case. Then remain dynamics is parametered by the following Lagrangian

$$L = i \sum_A \bar{\lambda}_A D\lambda_A - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{g^2 \theta}{64 \pi^2} \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a - f^2 \text{Tr} ([A^\mu, <\mathcal{M}>][A^\nu, <\mathcal{M}>]).$$

(3.2)

Here $<\mathcal{M}>$ denotes expectation value of $\mathcal{M}$. It can acquire a non-vanish value, and then, gauge symmetry is spontaneously broken (detail discussion see next subsection). In this limit, therefore, $\mathcal{N} = 4$ SYM reduces to a SM-like model.

**Gauge symmetry spontaneously breaking**

In original $\mathcal{N} = 4$ SYM, the condition of SUSY unbroken,

$$[\Phi_{AB}, \Phi_{CD}] = 0,$$

(3.3)

allows many degenerate vacuums. In particular, it allows that the vacuum expectation value of scalar fields takes some nonzero values. For example, we can set $<\mathcal{M}> = c_i T^i$ and require at least one of real coefficients $c_i$ nonzero, where $T^i$ are symmetric generators of $SU(N)$ group. From decomposition (2.3) we can see that this is indeed a supersymmetric vacuum. It is well-known as a special direction is taken in isospin space, and gauge symmetry is spontaneously broken partly. Consequently, part of gauge bosons acquires a mass and others (one of parallel to this direction) still remain massless. The former corresponds to a Higgs vacuum, and the later corresponds to a confine vacuum. This mechanism, as showing in many literature[8], reveals a method to spontaneously break gauge symmetry but without extra Higgs bosons. In addition, since we focus our attention on the case with $Z \to \infty$ (or $f \to \infty$), we must expect that the expectation value $<\mathcal{M}>$ (or $<\Phi_{AB}>$) is small fluctuation only and is restricted as $Z < \mathcal{M} >\sim m_W$. 


Generation and masses of fermions

It is different from supersymmetry and conformal invariance, that R-symmetry of $\mathcal{N} = 4$ SYM is remained in deformation model even when the limit $M \to \infty$ is taken. In low energy limit, R-symmetry become generation symmetry of SM-like model, or we can call it as general “flavour” symmetry. It indicates that there should be four generation in the SM-like model. All of these fermion are massless. However, it is because our decomposition (2.3) is too simple. If we replace decomposition of scalar fields $\Phi_{AB}$ by

$$ (\Phi_{AB})_{ij} = M^{-2}e^{ia(\phi_i - \phi_j)}(\bar{\lambda}_A j_5 \lambda_{Bi} - \bar{\lambda}_{Bj} j_5 \lambda_{Ai}) - \frac{1}{2} Z_{AB} \tilde{U}_{ij}(\varphi) - \frac{1}{2}(1 - \frac{2}{N}) Z_{AB} \delta_{ij}, $$

(3.4)

with $\tilde{U}_{ij} = e^{ia(\phi_i - \phi_j)}U_{ij}$ and real constant $a$. Then Yukawa coupling terms of $\mathcal{N} = 4$ SYM will yield mass term of fermions,

$$ (\Phi_{AB})_{ij}[\bar{\Psi}_A j_5 \Psi_{Bki} - \bar{\Psi}_{Aki} j_5 \Psi_{Bjk}] = \left( \frac{1}{2(N-1)} + \frac{1}{N} - 2 \right) Z_{AB} \bar{\lambda}_A < e^{ia(\phi_i - \phi_j)}\mathcal{M} > \gamma_5 \lambda_B + ...,$n

(3.5)

and expectation value $< e^{ia(\phi_i - \phi_j)}\mathcal{M} >$ does not vanish. We can see that the mass gap among different generation is created by central charge $Z_{AB}$, and the mass gap among different fermions in same generation is created by expectation value $< e^{ia(\phi_i - \phi_j)}\mathcal{M} >$. Since $< Z\mathcal{M} > \sim m_W$, the masses of heavier fermions are also order to $m_W$.

Toward QCD

If $< \mathcal{M} > = 0$, we achieve a theory with asymptotically freedom. In other words, Lagrangian (3.2) exactly becomes Lagrangian of QCD with four flavors massless fermions. Even though gauge symmetry is spontaneously broken by a nontrivial SUSY vacuum, the theory possesses a confine vacuum (and a Higgs vacuum). It is difficult to separate the confine theory from whole theory and beyond the scope of this present paper. Alternately, there is another simple method to obtain a QCD-like model via deformation of $\mathcal{N} = 4$ SYM.

To replace eq. (2.3) by the following decomposition

$$ \Psi_{Ai} = \frac{1}{\sqrt{2}}(\phi_1 \lambda_{A} + \lambda_{A} \phi_1), $$

$$ \bar{\Psi}_{Ai} = \frac{1}{\sqrt{2}}(\phi_1 \bar{\lambda}_{A} + \bar{\lambda}_{A} \phi_1), $$

(3.6)

$$ (\Phi_{AB})_{ij} = M^{-2}(\bar{\lambda}_A j_5 \lambda_{Bi} - \bar{\lambda}_{Bj} j_5 \lambda_{Ai}) - \frac{1}{2} Z_{AB} U_{ij}(\varphi) - \frac{1}{2}(1 - \frac{2}{N}) Z_{AB} \delta_{ij}, $$

and impose condition $\sum_i \phi_i \lambda_{Ai} = 0$ as traceless condition of spinors $\Psi_{A}$, we obtain a rather simple deformation model

$$ \mathcal{L} = i \sum_A \bar{\lambda}_A \gamma_\mu F_{\mu} - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{g^2}{64\pi^2} e e^{\mu \nu \rho \sigma} F_{\mu \nu} F_{\rho \sigma} $$
\[ + \frac{1}{4} \sum_{AB} |Z_{AB}|^2 Tr \left( D_\mu U^\dagger D^\mu U \right) \]

+ four fermion terms (suppressed by \( M^{-2} \) and \( M^{-4} \))

+ six fermion terms (suppressed by \( M^{-6} \))

+ eight fermion terms (suppressed by \( M^{-8} \)) \quad (3.7)

It must be stressed that the condition \( \sum \phi_i \lambda_{Al} = 0 \) can be imposed only for \( N > 2 \). The reason is follows: In order to keep spinor \( \lambda_{Al} \) as independent dynamical degrees of freedom, not all of scalar fields \( \phi_i \) are independent, i.e., they must be solution of equation \( \sum \phi_i \lambda_{Al} = 0 \). Notice that Majorana \( \lambda_{Al} = (eQ^*_A, Q_A)^T \) (where \( Q_A \) are Weyl spinor) have two independent components. If we want to obtain nonzero solution of \( \phi_i \) for \( N = 2 \), equation \( \sum \phi_i \lambda_{Al} = 0 \) requires \( \det |Q_{A1}, Q_{A2}| = 0 \). Then not all of \( \lambda_{Al} \) will be independent.

For \( N \geq 3 \), it is always possible to find some \( \phi_i \) which satisfy equation \( \sum \phi_i \lambda_{Al} = 0 \) and keep all of Majorana spinor \( \lambda_{Al} \) independent. If we now take the limit \( Z \sim M \to \infty \) and \( \langle M \rangle = 0 \), we still achieve QCD with four massless flavors. However, we can take another special limit: \( Z = 0, M \to \infty \). This limit also yields an theory with asymptotically freedom and with four flavor massless fermions no matter whether expectation value \( \langle M \rangle \) vanishes or not. This theory, in terms of Ads/CFT corresponds, can help us to calculate low energy behaviour of QCD from QCD directly.

4. Conclusion and Discussion

To conclude, we propose a new algebraic deformation of \( \mathcal{N} = 4 \) SYM via decomposing fermion fields and scalar fields in vector supermultiplets. This decomposition generates degrees of freedom of usual quarks and leptons and a scale is introduced in the decomposition. The deformation model is a low energy effective (Nambu-Jona-Lasinio-like) model. In appropriate limit, supersymmetry is broken and the deformation model reduces to a SM-like model, or a QCD-like model. The scale, meanwhile, corresponds to scale of supersymmetry breaking. In the SM-like model, four generation fermions are required and the mass gaps among these generation are created by central charge \( Z_{AB} \) of supersymmetry algebra. There are no Higgs bosons. The gauge symmetry is spontaneously broken by nontrivial supersymmetric vacuum. This mechanism indeed yields masses of part of gauge bosons and mass gap among fermions of same generation. We also show how to obtain a QCD-like model via deformation of \( \mathcal{N} = 4 \) SYM.

The essential idea of supersymmetry is symmetry between fermion and boson. However, in traditional SUSY model we have to introduce extra bosons or fermions as superpartner of known fermions or bosons for achieving supersymmetry. On the contrary, in this paper, we reveal a mechanism that SUSY can be a symmetry between known fermions and bosons. Then it is not necessary to introduce extra chiral supermultiplet when we construct SSM from SYM. It also indicates that we do not need so many dynamical degrees of freedom in any SUSY models.

There is problem on fermion mass matrix \( W_{AB,ij} = Z_{AB} < e^{ia(\phi_i, -\phi_j)} M_{ij} > \). It is \( (4 \times 4) \otimes (N \times N) \) dimension matrix and is antisymmetric for R-symmetry indexes \( A, B \) as
well as gauge group indexes $i$, $j$. This matrix has $2N$ real, positive and different eigenvalues. So that every two generation fermions are degenerated. This result is not supported by spectrum of standard model. However, it should be pointed out that the decomposition eqs. (2.3), (3.4) and (3.6) are not unique. There are many other possibilities. For example, we can take decomposition of Majorana spinor $\Phi_A$ as

$$\Psi_{Aij} = \frac{1}{\sqrt{2M'}} Z_{AB} \phi_i \lambda_{Bj} + ...$$

(4.1)

This decomposition adds an extra term $\sum C Z_{AC} Z_{CB} \delta_{ij} / M'$ to mass matrix $W_{A,B,ij}$. Then diagonalization on this term releases the degeneration among those fermions and may push mass of the fourth generation fermion to very heavy if $M' < M$. It also means that a complete study on this type of deformation of $\mathcal{N} = 4$ is still needed.

In this paper, our study on deformation of $\mathcal{N} = 4$ SYM can be easily extended to other $\mathcal{N}$ values. Of course, it is unambiguous that $\mathcal{N} = 4$ SYM is very special. It plays a role connecting superstring theory in ten dimensions and standard model in four dimensions. In addition, our study can be extended to two different directions:

1. The idea of Ads/CFT corresponds can be used in this deformation model. For example, an important issue is to find the corresponding deformation of supergravity in 5-d Ads space. Then we can calculate strong coupling limit of standard model by Ads/CFT corresponds. In particular, it provides a feasible method to calculate meson dynamics in low energy from QCD directly. Another interesting issue is to study holographic renormalization group[10] of the deformation model.

2. If we take gauge group of $\mathcal{N} = 4$ SYM including $SU(3)_c \times SU(2)_L \times U(1)_Y$ group, such as $SU(5)$, we can study whether we can obtain a complete standard model via deformation of SYM with extended supersymmetry. This deformation may be more complicated than one suggested in this paper, but essential idea is still that the matter fields should be generated by decomposition of superpartner of gauge bosons.

The studies on these aspects will be seen in future papers.

The deformation of this paper is entirely algebraic. It is not doubted that this deformation on $\mathcal{N} = 4$ SYM acquires considerable success. However, we believe that this deformation should be inspired by certain underlying dynamical mechanism. Unfortunately, it has to still be an unsolved problem so far.

References

[1] S. Coleman and J. Mandula, Phys. Rev. 159 (1967) 1251.

[2] S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150;N. Sakai, Z. Phys. C11 (1981) 153;E. Witten, Nucl. Phys. B188 (1981) 513;S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D24 (1981) 1681.

[3] Supergravity provides a tree level SUSY breaking effects: A. Chamseddine, R. Arnowitt and P. Nath, Phys. Rev. Lett. 49 (1982) 970;R. Barbieri, S. Ferrara and C. Savoy, Phys. Lett. B199 (1982) 334;L. Hall, J. Lykken and S. Weinberg, Phys. Rev. D27 (1982) 2359.
[4] Gauge mediation models use messenger fields to communicate the SUSY breaking at loop level: M. Dine and A. Nelson, Phys. Rev. D48 (1993) 1277; M. Dine, A. Nelson and Y. Shirman, Phys. Rev. D51 (1995) 1362; M. Dine, A. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D53 (1995) 2658.

[5] SUSY breaking associating to extra dimensions exhibits several possibilities: I. Antoniadis, Phys. Lett. B246 (1990) 377; P. Horava and E. Witten, Nucl. Phys. B460 (1996) 506; ibid, B475 (1996) 94; I. Antoniadis and M. Quiros, Nucl. Phys. B505 (1997) 109; H. P. Nilles, M. Olechowski and M. Yamaguchi, Phys. Lett. B415 (1997) 24; ibid, Nucl. Phys. B530 (1998) 43; E. A. Mirabelli and M. E. Peskin, Phys.Rev. D58 (1998) 065002; L. Randall and R. Sundrum, Nucl.Phys. B557 (1999) 79; I. Antoniadis, E. Dudas and A. Sagnotti, Phys.Lett. B464 (1999) 38; T. Gherghetta and A. Pomarol, Nucl.Phys. B602 (2001) 3.

[6] J. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231.

[7] J. Polchinski and M. J. Strassler, Phys. Rev. Lett. 88 (2002) 031601; J. Polchinski and L. Susskind, String Theory and The Size of Hadrons, hep-th/0112204.

[8] R. C. Brower, S. D. Mathur and C.-I. Tan, Nucl. Phys. B587 (2000) 249.

[9] C. Vafa and E. Witten, Nucl. Phys. B431 (1994) 4; J. Polchinski and M. J. Strassler, The String Dual of a Confining Four-Dimensional Gauge Theory, hep-th/0003139.

[10] E. Verlinde and H. Verlinde, JHEP 0005 (2000) 034; J. Kalkkinen, D. Martelli and W. Muck, JHEP 0104 (2001) 036; N. Hambli, Phys. Rev. D64 (2001) 024001.