Mechanism of heavy ion fusion to superheavy nuclei

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Abstract. This article reviews different models for the description of fusion of heavy ions to superheavy nuclei by using adiabatic and diabatic potentials. The dynamics of fusion is basically different in the two types of models for fusion: In the adiabatic models the nuclei melt together, whereas in the diabatic models the nuclei transfer nucleons between each other up to the instant when the compound nucleus is formed. As final result we state that diabatic potentials seem more appropriate for the description of fusion of heavy nuclei than adiabatic potentials.

1. Introduction
Here we present a short discussion about the mechanism of heavy ion fusion. In the actual literature one can roughly divide the models in two different groups. The one type of models describes the fusion process as a melting of two nuclei and usually uses adiabatic internuclear potentials. The other type of models assumes that the nuclei stick together in a touching configuration and then exchange nucleons up to the time when the compound nucleus is formed. In this case, the nuclei first form a nuclear molecule, named dinuclear system by Volkov [1].

The potential between the nuclei is taken as a sudden one. Therefore, it sensitively depends on the choice of the internuclear potential which trajectory the nuclear system proceeds during the fusion process.

Fusion of two heavy nuclei to superheavy nuclei are reactions of the following kind:

cold fusion: $^{70}\text{Zn} + ^{208}\text{Pb} \rightarrow ^{277}\text{Cn} + n$,

hot fusion: $^{48}\text{Ca} + ^{238}\text{U} \rightarrow ^{283}\text{Cn} + 3n$.

The dynamics of fusion is described by two main degrees of freedom, namely, by (a) the relative motion of the fusing nuclei, (b) the mass and charge transfer between the nuclei. Additional degrees of freedom are the deformations and orientations, the neck degree of freedom, internal excitations and other ones. For the mass and charge transfer we use the following coordinates

$$\eta = (A_1 - A_2)/(A_1 + A_2), \quad \eta_Z = (Z_1 - Z_2)/(Z_1 + Z_2),$$

where $A_1$, $A_2$ and $Z_1$, $Z_2$ are the mass and charge numbers of the two clusters, respectively. In the case of $\eta = 0$ the nuclei with $A_1$ and $A_2 = A_1$ nucleons form a symmetric fragmentation; if $\eta = \pm 1$, the mass numbers $A_2$ or $A_1$, respectively, are zero which is interpreted as a fused system. So, the coordinate $\eta$ varies in the interval $(1, -1)$ and includes also the fused system.
The parameters of the two-center shell model. Here, $2R_0\lambda$ measures the length of the system. The deformation parameters are given by $\beta_i = a_i/b_i$ with $i = 1, 2$. The neck parameter is $\epsilon = E_0/E'$. $R_0$ denotes the radius of the spherical compound nucleus.

The potential energy determines the dynamics of the various models of the fusion process. It can be calculated with the Strutinsky method where the potentials also contain shell effects:

$$U = U_{\text{liquid drop}} + \delta U_{\text{shell}}. \quad (2)$$

The shell effects in nucleus-nucleus collisions are usually determined by the two-center shell model of Maruhn and Greiner [2] (for the coordinates see figure 1).

The details of this short publication can be taken from the article “Clustering Effects within the Dimuclear Model”, written by the authors for the second volume of the series “Clusters in Nuclei”, edited by Christian Beck.

2. Adiabatic and diabatic potentials

For the description of the fusion process one uses adiabatic and diabatic potentials. Adiabatic potentials represent the minimum of energy for a set of given collective coordinates. In figure 2 we show the adiabatic potential for the $^{110}\text{Pd} + ^{110}\text{Pd}$ collision as a function of the length $\ell = 2R_0\lambda$ of the nucleus system, where $R_0$ is the radius of the compound nucleus. The adiabatic potentials have an outside barrier, but in general no further inside barrier versus smaller $\lambda$ ($\sim R$). They allow the formation of the compound system as a melting of two nuclei. Adiabatic potentials as a function of $\eta$ are usually smallest for symmetric fragmentations ($\eta = 0$) and, therefore, give the largest fusion cross sections around $\eta = 0$ in contrast to the experimental data.

Diabatic potentials include the effects of relative motion. At every avoided crossing of the single-particle energies in the adiabatic two-center shell model the nucleons follow the so-called diabatic path, a process depending on the relative velocity of the nuclei and denoted as Landau-Zener effect for which the Pauli principle between the nuclei is responsible [3]. Diabatic two-center shell models can be treated with the maximum overlap method or the simpler method of maximum symmetry. The time-independent diabatic potential is then obtained on the basis of the adiabatic potential [4]

$$U_{\text{diab}}(R) = U_{\text{adiab}} + \sum_{\alpha} \varepsilon_{\alpha}^{\text{adiab}}(R)n_{\alpha}^{\text{diab}}(R) - \varepsilon_{\alpha}^{\text{adiab}}(R)n_{\alpha}^{\text{adiab}}(R), \quad (3)$$
Figure 2. The diabatic (solid curve), the diabatic time-dependent (dashed curve) and the adiabatic (dotted curve) potentials for $^{110}\text{Pd} + ^{110}\text{Pd}$ as a function of $\lambda$.

where $\varepsilon_{\alpha}^{\text{diab}}$, $\varepsilon_{\alpha}^{\text{adiab}}$ are the single-particle energies of the diabatic and adiabatic two-center shell models and $n_{\alpha}^{\text{diab}}$, $n_{\alpha}^{\text{adiab}}$ their occupation numbers, respectively. $R$ is the internuclear coordinate.

Diabatic potentials are shown in figures 2 and 3. They are strongly repulsive and forbid fusion via the internuclear coordinate. As demonstrated in figure 3, they are similar to potentials calculated with the double folding method by using frozen densities (sudden potentials).

It is also possible to investigate the time-dependence of the transition from a diabatic potential to an adiabatic one. The evolution in time can be related to the characteristic relaxation time for the shape degrees of freedom of the system. The potential is calculated as [5]

$$U(\lambda, t) = U_{\text{adiab}}(\lambda) + \Delta U_{\text{diab}}(\lambda, t),$$

$$\Delta U_{\text{diab}}(\lambda, t) = \sum_{\alpha} \varepsilon_{\alpha}^{\text{diab}}(\lambda) n_{\alpha}^{\text{diab}}(\lambda, t) - \varepsilon_{\alpha}^{\text{adiab}}(\lambda) n_{\alpha}^{\text{adiab}}(\lambda).$$

(4)

Here, the occupation number $n_{\alpha}^{\text{adiab}}(\lambda)$ varies with $\lambda$ according to a Fermi distribution with temperature $T(\lambda)$. The relaxation equations are given as

$$\frac{dn_{\alpha}^{\text{adiab}}(\lambda, t)}{dt} = -\frac{1}{\tau(\lambda, t)} \left( n_{\alpha}^{\text{adiab}}(\lambda, t) - n_{\alpha}^{\text{adiab}}(\lambda) \right),$$

(5)

where the relaxation time is chosen $\tau(\lambda, t) \simeq 5 \times 10^{-21}\text{s}$. The dashed curve in figure 2 is the time-dependent diabatic potential for $^{110}\text{Pd} + ^{110}\text{Pd}$ at a time $t_0 = 8 \times 10^{-21}\text{s}$ which corresponds to the reaction time. One recognizes that at this time a barrier of about 55 MeV towards the inside remains which strongly suppresses the fusion along the relative coordinate.
3. Comparison of models for fusion with adiabatic and diabatic potentials

3.1. Models using adiabatic potentials

These models minimize the potential energy and essentially apply an adiabatic dynamics in the internuclear distance. In this case the potential energy as a function of $R$ and $\eta$ allows a motion in $R$ up to the fused compound nucleus. The adiabatic potential energy surface yields large probabilities for the fusion of equal projectile and target nuclei ($\eta = 0$) which contradicts the experimental data.

In figure 4 we show adiabatic potentials for collisions of nuclei forming the compound nucleus $^{246}$Fm [6]. (The small crosses mean the touching points of the nuclei.) The corresponding fusion probabilities, given by the dotted curve in figure 6, result in the order of 0.1 (quite large). In contrast to the adiabatic potentials, the time-dependent diabatic potentials, drawn in figure 5 after the reaction, reveal barriers of about 50 MeV towards the inside [6]. With these potentials very small fusion probabilities are obtained. They are shown by the dashed curve in figure 6 which strongly deviates from the experimental values schematically presented by the solid curve in the same figure [6]. Therefore, we conclude that the fusion process can be described as a melting of nuclei along the relative coordinate with neither adiabatic nor diabatic potentials. These results force us to consider another fusion reaction mechanism by choosing a motion in the $\eta$ coordinate.

3.2. Dinuclear system (DNS) concept

Models using diabatic potentials describe the fusion dynamics in the following steps: First, the nuclei are captured in the minimum of the adiabatic potential where they touch each other and form an excited nuclear molecule. This molecule consists of two individual nuclei. Then, a transfer of nucleons starts to change the ratio between the masses (also charges) of the nuclei up to the moment when the smaller nucleus lost all its nucleons to the larger one which becomes

Figure 4. Adiabatic potential in different reactions leading to $^{246}$Fm as a function of $\lambda$ for a fixed $\varepsilon = 0.75$. The crosses denote the touching configurations.

Figure 5. Dynamical diabatic potential. The notations are the same as in figure 4.
Figure 6. Fusion probability $P_{CN}$ in the reactions leading to $^{246}$Fm with excitation energy 30 MeV as a function of the mass asymmetry in the entrance channel. The result of the adiabatic treatment of the fusion in $\lambda$ is presented by the dotted line. The upper limit of the fusion probability in $\lambda$ in the dynamical diabatic treatment is presented by the dashed line. The fusion probability in the $\eta$ channel with a closed fusion channel in $\lambda$ is presented by the solid line.

the compound nucleus. The dynamics is a collective motion in the $\eta$ (also $\eta_Z$) degree of freedom at a nearly fixed relative distance $R \approx 12$ fm which is roughly the sum of the nuclear radii. This concept is the dinuclear system model [1, 7] which treats the fusion as a transfer of nucleons at a fixed internuclear coordinate in contrast to the adiabatic models which explain the fusion as a melting of two nuclei along the relative coordinate at a nearly fixed $\eta$-value.

The potential at the touching point, in which the fusion process evolves by nucleon transfer, depends on the mass asymmetry coordinate $\eta$ as shown in figure 7 and is called driving potential [8]. As one can recognize in figure 7, the driving potential is strongly influenced by the deformation of the fragments. It has a barrier, called inner barrier, for asymmetric fragmentations and is essentially diminished for symmetric systems which favor the fission of the dinuclear system in two (nearly) equal fragments, denoted as quasifission since no compound nucleus was existing before. The inner barrier determines the fusion probability in competition with the probability of quasifission. In figure 8 we present the probabilities for fusion in the cold reactions $^A X + ^{208}$Pb and in the hot ones $^{48}$Ca + $^A Y$ as a function of the $Z_{CN}$-value of the formed superheavy compound nucleus [9]. They can be calculated by master equations or by a statistical quantum mechanical method.

4. The motion of the neck
Let us consider the dynamics of the neck degree of freedom. The neck parameter $\epsilon$ is defined by the ratio of the actual barrier height $E_0$ to the barrier height $E'$ of the corresponding two-center oscillator (see figure 1):

$$\epsilon = \frac{E_0}{E'}$$

(6)
The neck grows with decreasing $\epsilon$. First, we made classic dynamical calculations with an adiabatic potential obtained with the Strutinsky method with shell effects calculated with the two-center shell model [10]. Figure 9 shows the potential energy surface as a function of the neck and length parameters $\epsilon$ and $\lambda$, respectively. The mass parameters were determined with the Werner-Wheeler approximation by assuming an incompressible and irrotational flow.

The fission-type valley is reached after a very short time of $3 - 4 \times 10^{-22}$ s at $\lambda \sim 1.68$ (see figure 9). Then oscillations in this valley occur if the kinetic energy is small. There must exist a hindrance for the fast $\epsilon$-increase and $\lambda$-motion towards the compound system. The essential hindrance which we found is a large, microscopically with the cranking model calculated mass parameter of the $\epsilon$-motion.

The main contributions to the mass tensor $B_{ij}^{cr}$ result from the expression [10]

$$B_{ij}^{cr} \approx h^2 \sum_\alpha \frac{f_\alpha}{\Gamma_\alpha^2} \frac{\partial \varepsilon_\alpha}{\partial q_i} \frac{\partial \varepsilon_\alpha}{\partial q_j}$$

with $f_\alpha = -\frac{dn_\alpha}{d\varepsilon_\alpha}$.

Here, $\varepsilon_\alpha$ and $n_\alpha$ are the two-center shell model single-particle eigenvalues and occupation numbers, respectively, and $\Gamma_\alpha$ are the widths of the decaying excited single-particle states. A much larger neck mass parameter than the one of the Werner-Wheeler approach results ($cr$=cranking, $WW$= Werner-Wheeler) [10]:

$$B_{\lambda\lambda}^{cr} = B_{\lambda\lambda}^{WW}, \quad B_{\epsilon\epsilon}^{cr} \approx 30 \times B_{\epsilon\epsilon}^{WW}, \quad B_{\lambda\epsilon}^{cr} \approx 0.35 \times B_{\lambda\epsilon}^{WW}.$$
As one can see in figure 10, the system stays a sufficiently long time near the entrance configuration. This justifies the dinuclear system model to a large extent. Then, mainly thermal fluctuations in the mass asymmetry degree of freedom are responsible for the process of fusion.

5. Summary
Fusion reactions for the production of superheavy nuclei are explained with adiabatic and diabatic potentials. The dynamics of fusion evolves differently in these potentials.

In adiabatic potentials the nuclei melt together along the internuclear distance. This process yields larger fusion cross sections for symmetric target and projectile combinations in contradiction to known experimental data.

Since diabatic potentials are strongly repulsive, the nuclei build up a dinuclear system of two touching nuclei and exchange nucleons up to the point when the compound nucleus is formed. This yields smaller fusion cross sections for symmetric fragmentations in agreement with the experimental data.

The formation of a larger neck is hindered by a large, microscopically calculated mass parameter for the neck degree of freedom. Therefore, the system hangs for some time in the initial touching configuration and can be interpreted as nuclear molecule or a dinuclear system. It can be analytically proved that a quantization of the kinetic energy tensor yields strongly repulsive potentials between the two approaching nuclei due to the interaction with excited states [11].

Several signatures for repulsive internuclear potentials between fusioning nuclei are found...
**Figure 9.** Potential energy surface (units MeV), calculated in the \((\lambda, \epsilon)\)-plane for the reaction \(^{110}\text{Pd} + ^{110}\text{Pd}\) with shell corrections and \(\beta_i = 1\) (lowest part), without shell corrections and \(\beta_i = 1\) (middle part), and with shell corrections and \(\beta_i = 1.2\) (upper part). The dynamical trajectories in the lowest part starting from the touching configurations and with initial kinetic energies 0, 40 and 60 MeV are presented by solid, dashed and dotted lines, respectively.

\(\text{(e.g. Pauli principle)}\). The calculated fusion and quasifission cross sections strongly point to the correctness of the diabatic description of the fusion process in comparison with the experimental data. This means that the ideas of the dinuclear system concept seem to give the correct description of the fusion process.

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Figure 10. Upper part: Time-dependence of the neck parameter $\epsilon$ in the system $^{96}\text{Zr} + ^{96}\text{Zr}$ calculated with microscopical (solid curve) and Werner-Wheeler (dashed curve) mass parameters. Lower part: Trajectories in the $(\lambda, \epsilon)$-plane calculated for the system $^{136}\text{Xe} + ^{136}\text{Xe}$ with microscopical (solid curve) and Werner-Wheeler (dashed curve) mass parameters. The end points of the solid and dashed curves are at time $t = 2 \times 10^{-21}$ s and $t = 2 \times 10^{-22}$ s, respectively.

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