DISK-FED GIANT PLANET FORMATION

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ABSTRACT

Massive giant planets, such as the ones being discovered by direct imaging surveys, likely experience the majority of their growth through a circumplanetary disk. We argue that the entropy of accreted material is determined by boundary layer processes, unlike the “cold-” or “hot-start” hypotheses usually invoked in the core-accretion and direct-collapse scenarios. A simple planetary evolution model illustrates how a wide range of radius and luminosity tracks become possible, depending on details of the accretion process. Specifically, the protoplanet evolves toward “hot-start” tracks if the scale height of the boundary layer is \( \lesssim 0.24 \), a value not much larger than the scale height of the circumplanetary disk. Understanding the luminosity and radii of young giant planets will thus require detailed models of circumplanetary accretion.

Key words: accretion, accretion disks – planets and satellites: formation – planets and satellites: gaseous planets – planets and satellites: physical evolution

1. INTRODUCTION

We are now in an era where giant planets orbiting other stars can be directly imaged. The current sample is small, but in the coming years the next generation of direct detection instrumentation (e.g., SPHERE, Vigan et al. 2010; GPI, Macintosh et al. 2008; HiCIAO, Yamamoto et al. 2013) will grow this sample over the coming years, and these campaigns recently yielded an actively accreting protoplanet (Sallum et al. 2015). Since sub-stellar mass objects do not generate any internal luminosity from nuclear burning, they passively cool over time, such that there is a large degeneracy between luminosity, age, and mass. This issue is particularly acute at young ages when the planets may not have cooled significantly from their formation conditions so that even for a fixed age and mass a planet could admit a large range of luminosities, depending on its initial thermal content (e.g., Spiegel & Burrows 2012).

One can turn this issue on its head and with dynamical constraints on a planet’s mass learn about its initial thermal content and perhaps its formation. The original approach assumed that giant planets started cooling from an arbitrary high entropy state (Stevenson 1982; Burrows et al. 1997), with such “hot-start” models now typically associated with the outcome of fragmentation in the protoplanetary disk (Boss 1997). By contrast, Marley et al. (2007) used the standard core-accretion picture (Pollack et al. 1996; Bodenheimer et al. 2000) and argued that newly formed planets would be significantly cooler than the “hot-start” models, with initial cooling times typically \( \gtrsim 10^8 \) years in these “cold-start” models. Observationally, the luminosity of a handful of directly imaged exoplanets with mass constraints are inconsistent with the “cold-start” scenario (Marleau & Cumming 2014), possibly in opposition to the core-accretion model. On the other hand, it appears difficult to form giant planets \( (\lesssim 10 \ M_J) \) through gravitational instability (Rafikov 2005; Kratter et al. 2010).

While there have been attempts to blur these formation channels into “warm-start” scenarios (e.g., Mordasini et al. 2012; Spiegel & Burrows 2012; Mordasini 2013), all such models implicitly assume that accretion takes place in a spherically symmetric manner. Accreting material would then be processed by a shock at the planet’s surface, which plays a key role in determining the initial entropy of the planet (e.g., Marley et al. 2007). However, this is unlikely to be how giant planets accrete their mass in a realistic scenario. Excess angular momentum of the accreting material is likely to form a circumplanetary disk through which material can accrete and be thermally processed. In this work, we argue that disk accretion can drive the entropies of forming giant planets up to traditional “hot-start” values, even in the core-accretion framework, and that this indeed likely to happen.

2. OVERVIEW OF THE ACCRETION PARADIGM

Most current models of planet formation within the core-accretion scenario assume that the planet remains embedded in the disk until the disk disperses at a later time. However, it is well known that once a planet grows to become massive enough to perturb the disk then it can open a gap (e.g., Lin & Papaloizou 1993). A rough estimate of the mass at which the planet can open a gap is given by the “thermal mass,” when the planet’s Hill sphere—\( R_H = a (M_p/3M_J)^{1/3} \)—exceeds the local scale height of the disk (\( H \)). For a typical passively heated protoplanetary disk with temperature \( T \propto R^{-1/2} \) (Kenyon & Hartmann 1987), this qualitatively requires that

\[
M_p \gtrsim 0.07 \ M_J \left( \frac{a}{1 \ au} \right)^{3/4} \left( \frac{T_{\text{in}}}{200 \ K} \right)^{3/2} \left( \frac{M_\star}{1 \ M_\odot} \right)^{-1/2}.
\]

Therefore, it is likely that a giant planet in the region 1–30 au will accrete the majority of its mass after gap opening. Once the gap has opened, simulations suggest that the incoming accretion streams possess enough angular momentum to form a circumplanetary disk (e.g., D’Angelo et al. 2002; Ayliffe & Bate 2009; Martin & Lubow 2011; Szulágyi et al. 2014). Thus, the entropy of the accreting planetary material is no longer associated with the parent protoplanetary disk, but rather

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is processed by the circumplanetary disk and is associated with the accretion process that transfers material from the disk to the planet. Two such mechanisms exist: magnetospheric and boundary layer accretion. Gas giant planets are hypothesized to have magnetic fields, and indeed Jupiter has a field strength of ~5 G. In order to determine whether accretion is controlled by magnetic fields (in the magnetospherical accretion model) or whether the disk extends all the way to the planet’s surface, we must determine the magnetospheric truncation radius.

For a formation time $t_{\text{form}} = M_p/\dot{M}$, which we expect to be $\lesssim 10$ Myr, the truncation radius is (Königl 1991; Livio & Pringle 1992)

$$\left( \frac{R_t}{R_p} \right) \approx 0.23 \left( \frac{B_p}{5 \text{ G}} \right)^{4/7} \left( \frac{R_p}{10^{10} \text{ cm}} \right)^{5/7} \times \left( \frac{t_{\text{form}}}{1 \text{ Myr}} \right)^{2/7} \left( \frac{M_p}{1 M_J} \right)^{-3/7}.$$  

(2)

For field strengths similar to gas giants in our solar system, this indicates that the magnetic field cannot truncate the disk and accretion will proceed all the way to the star. Alternatively, one would need a planetary field strength in excess of $B_p \gtrsim 65$ G for the truncation radius to exceed the planetary radius (see also Quillen & Trilling 1998; Fendt 2003; Lovelace et al. 2011; Zhu 2015). The expected magnetic field strengths of young massive giant planets are thought to be ~5–12 times Jupiter’s field strength (Christensen et al. 2009). Thus, it is highly likely that the circumplanetary disk will accrete onto the planet through a boundary layer (e.g., Lynden-Bell & Pringle 1974).

Since the gas disk lifetime limits the accretion timescale for gas giants to $\lesssim 10$ Myr, we know the accretion rates onto the protoplanets must be high with values of the order of $\sim 10^{-9} - 10^{-8} M_\odot$ yr$^{-1}$, as expected. These accretion rates can lead to extremely large accretion luminosities (e.g., Rafikov 2008b; Owen 2014; Zhu 2015) of the order of $\sim 10^{-3} L_\odot$. Approximately half of this is released in the disk and the remaining fraction in the boundary layer. Such accretion luminosities are many orders of magnitude larger than the internal luminosities of planets in “cold-start” scenarios ($\lesssim 10^{-5} L_\odot$) and higher than the majority of “hot-start” models ($\lesssim 10^{-3} L_\odot$; e.g., Marley et al. 2007). The ratio of accretion luminosity to internal cooling luminosity is

$$\frac{GM_p \dot{M}}{F_0 R_p^3} = 270 \left( \frac{M_p}{1 M_J} \right) \left( \frac{\dot{M}}{10^{-8} M_\odot \text{ yr}^{-1}} \right) \times \left( \frac{L_0}{10^{-4} L_\odot} \right)^{-1} \left( \frac{R_p}{10^{10} \text{ cm}} \right)^{-1},$$  

(3)

where $F_0$ and $L_0$ are the internal flux and luminosity of a passively cooling planet with mass $M_p$ and radius $R_p$. Therefore, for a planet forming through disk accretion, the disk and boundary layer will strongly irradiate the surface of the planet, not unlike the earliest stages of star formation (e.g., Adams & Shu 1986; Rafikov 2008a). External irradiation of a gas giant with an internal luminosity many orders of magnitude smaller is also similar to the hot-Jupiter problem. The irradiation pushes the radiative–convective boundary deeper into the planetary interior, preventing the radiation from escaping as easily, thus suppressing cooling and contraction (Guillot et al. 1996; Burrows et al. 2000; Arras & Bildsten 2006). Therefore, by accreting through a disk, the planet cooling will be suppressed and it will retain a higher entropy than if it were cooling passively.

3. SIMPLE DISK-FED PLANETARY FORMATION MODEL

We assume that the bulk of the planet’s mass is contained in a convective envelope surrounding the core. We can evaluate the binding energy of this envelope by assuming that the envelope mass exceeds the core mass and that its structure is described by a polytrope with $n = 3/2$, so that the total binding energy ($E_p$) is

$$E_p = \frac{3}{7} \frac{GM_p^2}{R_p}.$$  

(4)

where $M_p$ and $R_p$ are the planet’s mass and radius, respectively. As the planet accretes, its total binding energy evolves. Following Hartmann et al. (1997), who studied the evolution of accreting low-mass stars, we assume the bulk of the planet remains convective and describes the evolution as

$$\frac{dE_p}{dt} = \left( \epsilon - 1 \right) \frac{GM_p \dot{M}}{R_p} - L_{\text{rad}}.$$  

(5)

The accretion efficiency parameter, $\epsilon$, represents the internal energy of the accreted matter; $\dot{M}$ is the accretion rate; and $L_{\text{rad}}$ describes the radiative losses from the planetary surface. The boundary layer can be described using a “slim-disk” model (e.g., Abramowicz et al. 1988; Popham & Narayan 1991). Global integration of the slim-disk energy equation (Popham 1997) shows

$$L_d + C_p \dot{M}_c^2 = \frac{GM_p \dot{M}}{R_p} \left( 1 - j \frac{\Omega_p}{\Omega_K} + \frac{1}{2} \frac{\Omega_p^2}{\Omega_K^2} \right),$$  

(6)

where $L_d$ is the luminosity radiated away by the disk surface layers, $C_p$ is the heat capacity, $\Omega_p$ is the angular velocity of the planet, $\Omega_K$ is the Keplerian angular velocity at the radius of the planet, and $j$ is the angular momentum flux normalized to $M \Omega_K R_p^2$. Here, we do not attempt to model the evolution of the planetary rotation rate, assuming it is not close to break-up, but note that given an explicit boundary layer model the evolution of the planet’s angular velocity could be computed as well. The second term of the left-hand side represents the energy advected into the planet and the right-hand side represents the energy dissipated by the disk and boundary layer, where we neglect the role of the $\Omega_p/\Omega_K$ term. Therefore, the fraction $\epsilon$ of the accretion luminosity advected into the star is (Popham 1997)

$$\epsilon = C_p \dot{M}_c^2 \left( \frac{GM_p \dot{M}}{R_p} \right)^{-1} = C_p \left( \frac{H_p}{R_p} \right)^2,$$  

(7)

where $H_p$ is the scale height of the boundary layer at the planetary radius.

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5 Assuming the disk’s luminosity is large compared to $(H/R) L_p$. 

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Finally, assuming that the boundary layer obscures an area $4\pi R_p H_p$ of the planetary surface, $L_{\text{rad}}$ is given by

$$L_{\text{rad}} = 4\pi R_p^2 \left(1 - \frac{H_p}{R_p}\right) \int_0^{\pi/2} F_p(\theta)d\cos\theta \tag{8}$$

where $F_p(\theta)$ is the flux emerging from the planetary surface at an angle $\theta$ to the pole. As discussed above, the accretion luminosity often exceeds the internal luminosity by several orders of magnitude. Rafikov (2008a) showed that the integral in Equation (8) can be written as

$$\int_0^{\pi/2} F_p(\theta)d\cos\theta = F_0 f\left(\frac{GM_p M}{F_0 R_p^3}\right). \tag{9}$$

Calculation of $F_p(\theta)$ can be performed assuming emission from a standard $\alpha$-disk (e.g., Adams & Shu 1986; Popham 1997; Rafikov 2008a). The function $f$ is not analytic; however, given an opacity law of the form $\kappa \propto \rho^{\alpha}\theta^B$, $f$ can be calculated following Rafikov (2008a). A power-law fit to the opacity law in the planetary regime is given by Rogers & Seager (2010) with $\alpha = 0.68$ and $\beta = 0.45$. Adopting this fit, the function $f$ is shown in Figure 1, indicating that irradiation from the disk can lead to cooling luminosities several tens of percent lower than for a passively cooling luminator. Therefore, we write $L_{\text{rad}}$ as

$$L_{\text{rad}} = f L_0 \left(1 - \frac{H_p}{R_p}\right), \tag{10}$$

where $L_0$ is the luminosity of an isolated planet.

In order to fully evolve our accreting planet–disk system, we need to know the scale height of the boundary layer at the planetary surface. Boundary layers are poorly understood; since $dS/2dR > 0$, they are stable to the MRI and an $\alpha$-viscosity prescription may lead to unphysical solutions (Pringle 1977; Popham & Narayan 1992). Recently, Belyaev et al. (2013a, 2013b) showed that angular momentum transport can occur through waves arising from the sonic instability in boundary layers; however, a simple boundary layer model including this mechanism does not exist. We can still make some reasonable estimate of the scale height by using the disk scale height just outside the boundary layer. Since one would expect enhanced dissipation in the boundary layer due to the sharper angular momentum gradients present, we might expect the scale height to be larger in the boundary layer than in the disk. For an actively heated disk, where one equates radiative cooling with local viscous dissipation in a Keplerian disk, the scale height in the disk, $H_d^d$, is given by

$$\frac{H_d^d}{R_p} \approx 0.15 \mu^{-1/2} \left(\frac{M_p}{M_\odot}\right)^{-3/8} \left(\frac{M_\odot}{10^{-8} M_\odot\text{ yr}^{-1}}\right)^{1/8} \times \left(\frac{R_p}{10^{10}\text{ cm}}\right)^{1/8}, \tag{11}$$

where $\mu$ is the mean particle weight in amu. In obtaining Equation (11), we have neglected the “standard” $(1 - \sqrt{R_p/R})$ factor. This factor arises if one applies a zero-torque boundary condition at the boundary-layer/disk interface, resulting in a decrease in the surface density toward the planet’s surface. This specific boundary condition is unlikely to be appropriate for the boundary-layer/disk interface that is more likely to have an angular velocity and thus associated torque of a full Keplerian disk (e.g., Popham & Narayan 1991). Since Equation (11) is weakly sensitive to the input parameters, we assume that the scale height of the boundary layer$^6$ remains constant for the entire evolution and choose values $\gtrsim 0.15$. Combining Equations (5) and (10), we obtain the evolution equation for the radius of an accreting convective planet:

$$\frac{\dot{R}_p}{R_p} = \frac{7}{3} \frac{M}{M_p} \left[\frac{C_p}{\left(\frac{H_d^*}{R_p}\right)^2} - \frac{1}{7}\right] - 7 \frac{L_{\text{rad}} R_p}{3 GM_p^2}. \tag{12}$$

To integrate Equation (12), one needs to know the passive luminosity of the planet as a function of planet mass and radius. To calculate this, we use the MESA code (Paxton et al. 2011) to produce a series of hydrostatic models as a function of planet mass and radius, assuming a 10 $M_\oplus$ rocky core and the Freedman et al. (2008) opacities. The resulting luminosities (left panel) and Kelvin–Helmholtz timescales ($t_{KH}$; right panel) are shown in Figure 2.

We can now understand how the protoplanet will evolve from Equation (12). The first term on the rhs is $\sim 1/t_M$, where $t_M$ is the mass evolution timescale, while the second term is approximately $1/t_{KH}$. Therefore, if $t_{KH} < t_M$, the protoplanet will cool and shrink. If $t_M < t_{KH}$, then the evolution of the planet depends on the internal energy of the incoming material, and as such there is a critical $H_d^*/R_p$ above which the planet increases in radius as it accretes; otherwise, it shrinks. This critical boundary layer height is $H_d^*/R_p = \sqrt{(\gamma - 1)/7\gamma}$, which is $\approx 0.24$ for $\gamma = 5/3$ and $\approx 0.2$ for $\gamma = 7/4$. Given typical temperatures in the boundary layer, we expect the gas to be monotonic, so that for $H_d^*/R_p < 0.24$ the protoplanet’s radius will increase if $t_M < t_{KH}$. This critical value of the boundary layer scale height is not much larger than the estimated disk scale height (Equation (11)); thus, we may expect the planet’s radius to increase rather than decrease during circumplanetary disk accretion. The left panel of Figure 2 shows that in the giant planet regime $M_p \gtrsim 10^{30}$ g, luminosity increases with increasing radius as the mass

$^6$ Since we only evaluated the scale height in the disk not the boundary layer, we cannot verify this assumption.
increases. Therefore, disk accretion through a boundary layer can potentially drive the planet to high luminosities, comparable or even larger than typical values in the “hot-start” scenarios.

4. RESULTS

We consider the evolution of a protoplanet accreting through a boundary layer. The planet is taken to be an initially $50 \ M_\oplus$.
The radius evolution of a protoplanet with a final mass of \(1 M_J\) (top) and \(10 M_J\) (bottom), accreting at a constant rate with a formation time of 1 Myr (left) and 10 Myr (right), are shown in Figure 3. Three initial cooling times of \(10^6\) years, \(10^7\) years, and \(10^8\) years are each shown with \(H_p/R_p\) values of 0.15, 0.25, 0.35, and 0.45.

The accretion of material by a protoplanet via a circumplanetary disk and boundary layer can result in a large range of initial planet properties, from “hot-start” to “cold-start” conditions. For reference, “hot-start” and the “cold-start” radii from Spiegel & Burrows (2012) are shown as the points. In all cases, \(t_{KH} > t_f\) initially, and the protoplanet follows an evolutionary path determined by \(H_p/R_p\). Therefore, above the critical value of \(H_p/R_p\) the protoplanet grows in radius and below it shrinks in radius, as expected. For the highest \(H_p/R_p\) values considered, the protoplanet is driven to high enough luminosities that radiative cooling dominates and contraction ensues. This decrease in radius causes the planets to follow a convergent evolutionary path, and the final properties of planets with \(H_p/R_p \gtrsim 0.3\) are insensitive to their initial properties. However, protoplanets forming with \(H_p/R_p \lesssim 0.2\) do not cool appreciably, and final properties post-accretion bear the signature of the initial Kelvin–Helmholtz timescales. For massive planets (10 \(M_J\) final masses), the radius starts to increase again once they reach about 2 \(M_J\). This can be understood from Figure 2, which shows that at high masses, the cooling time increases as one increases mass at a fixed radius. So a planet whose evolution was dominated by cooling at lower masses can evolve into a region where cooling becomes subdominant and the thermal content of the accreted material drives the evolution instead. Unsurprisingly, faster accreting protoplanets experience stronger evolution, as mass accretion dominates over cooling more prominently.

Finally, we can compute the luminosity evolution of our planets, including at late times after accretion stopped. We do this by continuing to evolve the planet according to Equation (12); however, we set \(M = 0\) and allow the planet to cool over its entire surface. The comparison of model tracks with the handful of directly imaged giant planets for which their luminosity and age have been measured is shown in Figure 4. Planetary formation through boundary layer accretion can drive planets onto “hot-start”-like evolutionary paths, which can successfully match the luminosity of young directly imaged exoplanets.

5. DISCUSSION AND SUMMARY

A protoplanet embedded in a protoplanetary disk is likely to open a gap at modest masses; thus, a giant planet is likely to accrete the majority of its mass through a circumplanetary disk. This scenario is unlike the direct-collapse (gravitational fragmentation) framework where the high entropy disk material is turned into a planetary mass object on a short-timescale, in an essentially adiabatic fashion. It is also unlike the final stages of the standard core-accretion framework, which assumes that the protoplanet accretes spherically, shocks, and then radiates away the disk material’s entropy, accreting material with the same temperature as the protoplanet’s atmosphere, leading to comparatively small, low-luminosity planets (e.g., Marley et al. 2007). In the disk-accretion scenario considered here, we argue that the entropy of the accreting material depends on how the material is transported from the disk to the planet. Unless the protoplanet has a very strong magnetic field (\(>65\) G), this will occur through a boundary layer, and the fraction of heat added to the protoplanet directly depends on the thermal structure of the boundary layer, with puffier boundary layers advecting more heat into the planet. There is a critical value of the height of the boundary layer above which the protoplanet becomes inflated by accretion and
driven to high luminosities, \( H_p/R_p \gtrsim 0.24 \), which is only slightly larger than the scale height of the circumplanetary disk feeding the boundary layer. To the extent that boundary layers are hotter than their circumplanetary disk, one may expect in the majority of cases that circumplanetary disk accretion will inflate the protoplanet, driving it to high luminosities in line with those expected from “hot-start” or direct-collapse scenarios.

Before concluding, we note that our models of disk-fed planet formation are highly idealized. We have assumed an \( n = 3/2 \) polytrope. For the largest boundary layers we are adding extremely high entropy gas on top of lower entropy gas so that the envelope’s outer layers may become stably stratified. Accretion over a long period of time could result in a complicated layered structure of convective and radiative zones, meaning that an \( n = 3/2 \) polytrope may no longer be a good description of the protoplanetary structure. If such a stably stratified region were very thin, it may cool quickly, resulting in low-luminosity planets fairly soon after accretion ceases. Simple models—a constant opacity, hydrostatic radiative envelope (cf. Stevenson 1982)—suggest a thick radiative layer with large thermal inertia, so that the inflated radii should remain that way for a long time after accretion stops. More detailed models are needed to address this reliably. One obvious avenue for progress is thus to couple boundary layer accretion models with detailed structure calculations for the planetary interior.

In conclusion, disk-fed accretion may be the dominant mechanism shaping the properties of young giant planets, where the thermal structure of the boundary layer controls the amount of heat advected into the planet during late-stage growth. Such a process will naturally arise in the core-accretion scenario once the planet becomes massive enough to open a gap in the protoplanetary disk, and angular momentum conservation leads to the formation of a disk around the protoplanet. It therefore seems that further development of boundary layer accretion theory is warranted to understand giant planet formation and to interpret the wealth of direct imaging data expected to become available over the next few years.

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