Effect of moment of inertia to H type vertical axis wind turbine aerodynamic performance

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Abstract. The main aerodynamic performances (out power, out power coefficient, torque, torque coefficient and so on) of H type Vertical Axis wind Turbine (H-VAWT) which is rotating machinery will be impacted by moment of inertia. This article will use NACA0018 airfoil profile to analyze that moment of inertia through impact performance of H type VAWT by utilizing program of Matlab and theory of Double-Multiple Streamtube. The results showed that the max out power coefficient was barely impacted when moment of inertia is changed in a small area, but the lesser moment of inertia’s VAWT needs a stronger wind velocity to obtain the max out power. The lesser moment of inertia’s VAWT has a big out power coefficient, torque coefficient and out power before it gets to the point of max out power coefficient. Out power coefficient, torque and torque coefficient will obviously change with wind velocity increased for VAWT of the lesser moment of inertia.

1. Introduction
It is important that wind resources is developed and utilized, specially, now the non-renewable has less and less with economic development. The H type vertical axis wind turbine (H-VAWT) have a simple construction, easily installed, insensitive to yaw and can handle a wind field that rapidly changes directions, making it ideally suited for application in the built environment.

In recently years, many of scholars has do a lot of studies about VAWT. Doctor Wang L B calculated the performance of VATT by used the unsteady vortex panel method models in the 2006 year; the fluent and moving grid technology is used by professor Yang C X to detailed and roundly study performances of VAWT in the 2009 year; professor Zhang L is used to couple method to be numerical simulation for VATT in the 2011 year; Yutaka Hara come from Tottori university studies the dependence of Vertical Axis Wind Turbines in Pulsating Winds in the 2012 year. But the article which is especial to study influence of moment of inertia to performance of VAWT is very less.

The biggish moment of inertia is disadvantage of VAWT’s start, effecting reasonable design (rate of high and radius) and result of brake. Therefore, it is necessary that study the influence of moment of inertia of VAWT. This paper will analyze performance of different moment of inertia of VAWT by utilizing theory of Double-Multiple Streamtube and program of Matlab, and wish as reference of VAWT’s design in the future.

2. Theory of Double-Multiple Streamtube
In the 1981 year, Paraschivoiu proposed a model to analyze and predict aerodynamics of VAWT, in this model\(^6\), the wind turbine is divided into two parts (up streamtube and down streamtube), and every streamtube is applied momentum equation and blade element theory, so we can obtain induce drag formation as follow:

\[
f_{up}u = \pi \eta (1 - u)
\]

\[
f_{up} = \frac{N_c}{8\pi R} \int_{-\pi}^{\pi} \left( C_n \cos \theta - C_i \frac{\sin \theta}{\cos \delta} \right) \left( \frac{W}{V} \right)^2 d\theta
\]

According to analyze motion, momentum theory and relation of torque and rotate speed, we can obtain torque coefficient \(C_Q\) and power coefficient \(C_p\) of up steamtube section:

\[
\overline{C_Q} = \frac{N_c H}{2\pi S} \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} C_l \left( \frac{W}{W_\infty} \right)^2 \left( \frac{\eta}{\cos \delta} \right) d\zeta d\theta
\]

\[
C_{p1} = (R\omega/V_\infty) \overline{C_Q}
\]

As the above, we can calculate the induce velocity of up streamtube and apply the blade element theory and momentum theory for every down streamtube to get induce drag coefficient, since we can obtain torque coefficient \(C_{Q2}\) and power coefficient \(C_{p2}\) of down streamtube section:

\[
\overline{C_{Q2}} = \frac{N_c H}{2\pi S} \int_{-\pi/2}^{\pi/2} \int_{-\pi/2}^{\pi/2} C_l \left( \frac{W}{W_\infty} \right)^2 \left( \frac{\eta}{\cos \delta} \right) d\zeta d\theta
\]

\[
C_{p2} = (R\omega/V_\infty) \overline{C_{Q2}}
\]

Therefore, the power coefficient of whole wind turbine as follow:

\[
C_p = C_{p1} + C_{p2}
\]

3. Moment of inertia of wind turbine

This article is only considered the effect of blade’s moment of inertia for VAWT. The energy equation is expressed by moment of inertia \(J\) as follow:

\[
E = \frac{1}{2} J \omega^2
\]

According to theorem of kinetic energy of moment of inertia, we can obtain the equation:

\[
Q(\theta_2 - \theta_1) = \Delta E
\]

The following equation will express the relation of torque and power:

\[
Q\omega = P
\]

And:
- \(P\)—rated output power;
- \(Q\)—rated moment of force;
- \(E\)—kinetic energy;
- \(\theta\)—angle of rotation.
4. Program MATLAB codes
This article will build relationship between theory of Double-Multiple Streamtube and theorem of kinetic energy of moment of inertia, then compiling MATLAB codes, the detailed process as figure 1:

![Flow chart for MATLAB program.](image)

Figure 1. flow chart for MATLAB program.

5. Example
All of calculation of this article based on parameters list , as following table 1:

| Name       | parameter      | unit |
|------------|----------------|-----|
| Airfoil    | NACA0018       |     |
| chord length | 0.34          | m   |
| high       | 4              | m   |
| diameter   | 5              | m   |
| Number of blade | 3           |     |
| Rotation speed | 150         | rpm |

6. Result and analysis
According to the above parameter list ,this article assumes that the moment of inertia of VAWT is J, then build three teams difference moment of inertia ,0.9J,0.85J,0.75J,respectively ,its aim to observe the performance of VAWT when change the moment of inertia in the small area.

In figure 2, the torque coefficient’s variation is showed with tip-speed rate \( \lambda \) changes at the different moment of inertia. The torque coefficient is impacted barely When the moment of inertia changed in a small area, in other words, the wind turbine’s stress do not related with moment of inertia; after the max out put power coefficient, because dynamic stalled and Circumference of a circle effect with wind velocity increased, the performance of VAWT is unsteady for the lesser moment of inertia’s.
Figure 2. Relation of tip-speed rate and torque coefficient.

In figure 3, it's showed the max output power coefficient is not impacted when moment of inertia changed in the small area, of course this condition don't consider the dynamic stalled and wind shear.

Table 2. The Max $C_p$ for different moment of inertia's wind turbine

| Moment of inertia | 56kg.m$^2$ | 50.4kg.m$^2$ | 47.6kg.m$^2$ | 42kg.m$^2$ |
|-------------------|-----------|-------------|-------------|-----------|
| Max $C_p$         | 0.38      | 0.39        | 0.39        | 0.39      |

Before get the max power coefficient, the power coefficient relative steady and the VAWT of the lesser moment of inertia has good start performance, but the steady of performance is bad with wind velocity increased; its because that the process is unsteady when wind turbine is rotated, that is to say, the angular velocity is different at the different azimuth angle, the angular velocity has obvious wave with azimuth angle’s difference for the lesser moment of inertia’s wind turbine$^{[3]}$.

Figure 3. Relation of tip-speed rate and power coefficient.

In figure 4 showed that the lesser moment of inertia’s VAWT need more stronger wind velocity to get the max output power, this option agree with figure 4 showed.
7. Conclusion

1) The max output power coefficient is not impacted barely when the moment of inertia changed in a small area for VAWT;
2) The lesser moment of inertia’s VAWT own biggis h torque coefficient, power coefficient before get the max power coefficient, so it has trend of easy start;
3) The performance is not steady for the lesser moment of inertia’s VAWT with wind velocity increased.

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