Hadronic Matter is Soft

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The stiffness of the hadronic equation of state has been extracted from the production rate of \( K^+ \) mesons in heavy ion collisions around 1 A GeV incident energy. The data are best described with a compressibility coefficient \( \kappa \) around 200 MeV, a value which is usually called “soft”. This is concluded from a detailed comparison of the results of transport theories with the experimental data using two different procedures: (i) the energy dependence of the ratio of \( K^+ \) from Au+Au and C+C collisions and (ii) the centrality dependence of the \( K^+ \) multiplicities. It is demonstrated that input quantities of these transport theories which are not precisely known, like the kaon-nucleon potential, the \( \Delta N \rightarrow NK^+\Lambda \) cross section or the life time of the \( \Delta \) in matter do not modify this conclusion.

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Since many years one of the most important challenges in nuclear physics is to determine \( E/A(\rho,T) \), the energy/nucleon in nuclear matter in thermal equilibrium as a function of the density \( \rho \) and the temperature \( T \). Only at equilibrium density, \( \rho_0 \), the energy per nucleon \( E/A(\rho = \rho_0, T = 0) = -16 \) MeV is known by extrapolation of the Weizsäcker mass formula to infinite matter. This quest has been dubbed “search for the nuclear equation of state (EoS)".

Modelling of neutron stars or supernovae have not yet constrained the nuclear equation of state \( E/A(\rho,T) \). Therefore, the most promising approach to extract \( E/A(\rho,T) \) are heavy ion reactions in which the density of the colliding nuclei changes significantly. Three principal experimental observables have been suggested in the course of this quest which carry - according to theoretical calculations - information on the nuclear EoS: (i) the strength distribution of giant isoscalar monopole resonances \( G^+ \), (ii) the in-plane sidewards flow of nucleons in semi-central heavy ion reaction at energies between 100 A MeV and 400 A MeV \( G^+ \), and (iii) the production of \( K^+ \) mesons in heavy ion reactions at energies around 1 A GeV \( G^+ \). Although theory has predicted these effects qualitatively, a quantitative approach is confronted with two challenges: a) The nucleus is finite and surface effects are not negligible, even for the largest nuclei and b) in heavy ion reactions the reacting system does not come into equilibrium. Therefore complicated non-equilibrium transport theories have to be employed and the conclusion on the nuclear equation of state can only be indirect.

(i) The study of monopole vibrations has been very successful, but the variation in density is minute. Therefore, giant monopole resonances are sensitive to the energy which is necessary to change the density of a cold nucleus close to the equilibrium point \( \rho_0 \). According to theory the vibration frequency depends directly on the force which counteracts to any deviation from the equilibrium and therefore to the potential energy. The careful analysis of the isoscalar monopole strength in non-relativistic \( G^+ \) and relativistic mean field models has recently converged \( G^+ \) due to a new parametrization of the relativistic potential. These calculations allow now for the determination of the compressibility \( \kappa = 9\rho^2 \frac{d}{d\rho} \frac{E/A(\rho,T)}{\rho_0} \) which measures the curvature of \( E/A(\rho,T) \) at the equilibrium point. The values found are around \( \kappa = 240 \) MeV and therefore close to what has been dubbed “soft equation of state”.

(ii) If the overlap zone of projectile and target becomes considerably compressed in semi-central heavy-ion collisions, an in-plane flow is created due to the transverse pressure on the baryons outside of the interaction region with this flow being proportional to the transverse pressure. In order to obtain a noticeable compression, the beam energy has to be large as compared to the Fermi energy of the nucleons inside the nuclei and hence a beam energy of at least 100 A MeV is necessary. Compression goes along with excitation and therefore the compressional energy of excited nuclear matter is encoded in the in-plane flow. It has recently been demonstrated \( G^+ \) that transport theories do not agree quantitatively yet and therefore former conclusions \( G^+ \) have to be considered as premature.

(iii) The third method is most promising for the study of nuclear matter at high densities and is subject of this Letter. \( K^+ \) mesons produced far below the \( NN \) threshold cannot be created in first-chance collisions between projectile and target nucleons. They do not provide sufficient energy even if one includes the Fermi motion. The effective energy for the production of a \( K^+ \) meson in the \( NN \) center of mass system is 671 MeV as in addition to the mass of the kaon a nucleon has to be converted into a \( \Lambda \) to conserve strangeness. Before nucleons can create a \( K^+ \) at these subthreshold energies, they have to accumulate energy. The most effective way to do this is the conversion of a nucleon into a \( \Delta \) and to produce in a subsequent collision a \( K^+ \) meson via \( \Delta N \rightarrow NK^+\Lambda \). Two effects link the yield of produced \( K^+ \) with the den-
sity reached in the collision and the stiffness of the EoS. If less energy is needed to compress matter (i) more energy is available for the $K^+$ production and (ii) the density which can be reached in these reactions will be higher. Higher density means a smaller mean free path and therefore the $\Delta$ will interact more often increasing the probability to produce a $K^+$ and hence, it has a lower chance to decay before it interacts. Consequently the $K^+$ yield depends on the compressional energy. At beam energies around $1$ A GeV matter becomes highly excited and mesons are formed. Therefore this process tests highly excited hadronic matter. At beam energies $> 2$ A GeV first-chance collisions dominate and this sensitivity is lost.

In this Letter we would like to report that for the third approach different transport theories have converged. Two independent experimental observables, the ratio of the excitation functions of the $K^+$ production for Au+Au and for C+C [12, 14], and a new observable, the dependence on the number of participants of the $K^+$ yield show that nucleons interact with a potential which corresponds to a compressibility of $\kappa \leq 200$ MeV in infinite matter in thermal equilibrium. This value extracted for hadronic matter at densities around 2.5 times the normal nuclear matter density is very similar to that extracted at normal nuclear matter density. A key point of this paper is to demonstrate that the different implementation of yet unsolved physical questions, like the $N\Delta \to K^+\Lambda N$ cross section, the $K\Lambda$ interaction as well as the life time of the nuclear resonances in the hadronic environment do not affect this conclusion.

In order to determine the energy which is necessary to compress infinite nuclear matter in thermal equilibrium by heavy ion reactions in which no equilibrium is obtained one chooses the following strategy: The transport theory calculates the time evolution of the quantal particles described by Gaussian wave functions. The time evolution is given by a variational principle and the equations one obtains for this choice of the wave function are identical to the classical Hamilton equations where the classical two-body potential is replaced by the expectation value of the real part of the $G$-Matrix which is supplemented by the Coulomb interaction between the charged particles. The former can be further subdivided in a part containing the contact Skyrme-type interaction only, a contribution due to a finite range Yukawa-potential, and a momentum-dependent part with

$$V^{ij} = V^{ij}_{\text{Skyrme}} + V^{ij}_{\text{Yuk}} + V^{ij}_{\text{md}} + V^{ij}_{\text{Coul}}$$

$$= t_1 \delta(\vec{x}_i - \vec{x}_j) + t_2 \delta(\vec{x}_i - \vec{x}_j)|p^z|^{-1}(\vec{x}_i) +$$

$$t_3 \exp\left\{-|\vec{x}_i - \vec{x}_j|/\mu\right\} \left\{Z_i Z_j e^2 \left|\vec{x}_i - \vec{x}_j\right| + Z_i Z_j e^2 \right\} +$$

$$t_4 \ln^2(1 + t_5 (\vec{p}_i - \vec{p}_j)^2) \delta(\vec{x}_i - \vec{x}_j)$$

with $Z_i, Z_j$ the charges of the baryons $i$ and $j$. For more details we refer to Ref. [9].

We include in this calculation all inelastic cross sections which are relevant for the $K^+$ production. For details of these cross sections we refer to [10]. Unless specified differently, the change of the $K^+$ mass due to the kaon-nucleon ($K\Lambda$) interaction according to $m_K^2(\rho) = m_K^2(1 - 0.075 \rho)$ is taken into account, in agreement with recent self-consistent calculations of the spectral function of the $K^+$ [11]. The $\Lambda$ potential is $2/3$ of the nucleon potential, assuming that the $s$ quark is inert.

In order to minimize the experimental systematical errors and the consequences of theoretical uncertainties it is better to compare ratios of cross sections rather than the absolute values [12]. We have made sure that the standard version of IQMD reproduces the excitation function for Au+Au as well as for C+C quite well [13]. These ratios are quite sensitive to the nuclear potentials because the compression obtained in the Au+Au collisions is considerable (up to $3\rho_0$) and depends on the nuclear equation of state whereas in C+C collisions the compression
is small and almost independent on the stiffness of the EoS.

Figure 1 shows the comparison of the measured ratio of the $K^+$ multiplicities obtained in Au+Au and C+C reactions [12] together with transport model calculations as a function of the beam energy.

We see clearly that the form of the yield ratio depends on the potential parameters (hard EoS: $\kappa = 380$ MeV, thin lines and solid symbols, soft EoS: $\kappa = 200$ MeV, thick lines and open symbols) in a quite sensible way and that the prediction in the standard version of the simulation (squares) for a soft and a hard EoS potential differ much more than the experimental uncertainties. The calculation of Fuchs et al. [14] given in the same graph, agrees well with our findings.

This observation is, however, not sufficient to determine the potential parameters uniquely because in these transport theories several not precisely known processes are encoded. Therefore, it is necessary to verify that these uncertainties do not render this conclusion premature. Figure 1, top, shows as well the influence of the unknown $N\Delta \rightarrow K^+\Lambda N$ cross section on this ratio. We confront the standard IQMD option (with cross sections for $\Delta N$ interactions from Tsushima et al. [10]) with another option, $\sigma(N\Delta) = 3/4\sigma(NN)$ [17], which is based on isospin arguments and has been frequently employed. Both cross sections differ by up to a factor of ten and change significantly the absolute yield of $K^+$ in heavy ion reactions but do not change the shape of the ratio.

The middle part demonstrates the influence of the kaon-nucleon potential which is not precisely known at the densities obtained in this reaction. The uncertainties due to the $\Delta$ life time are discussed in the bottom part. Both calculations represent the two extreme values for this lifetime [10] which is important because the disintegration of the $\Delta$ resonance competes with the $K^+$ production.

Thus we see that these uncertainties do not influence the conclusion that the excitation function of the ratio is quite different for a soft EoS potential as compared to a hard one and that the data of the KaoS collaboration are only compatible with the soft EoS potential. The only possibility to change this conclusions is the assumption that the cross sections are explicitly density dependent in a way that the increasing density is compensated by a decreasing cross section. It would have a strong influence on other observables which are presently well predicted by the IQMD calculations.

We would like to add that the smoothness of the excitation function also demonstrates that there are no density isomers in the density regions which are obtained in these reactions because the $K^+$ excitation function would be very sensitive to such an isomeric state [10].

The conclusion that nuclear matter is best described by a soft EoS, is supported by another variable, the dependence of the $K^+$ yield on the number of participating nucleons $A_{\text{part}}$. The prediction of the IQMD simulations in the standard version for this observable is shown in Fig. 2.

The top of the figure shows the kaon yield $M_{K^+}/A_{\text{part}}$ for Au+Au collisions at 1.5 A GeV as a function of the participant number $A_{\text{part}}$ for a soft EoS using different options: standard version (soft, $KN$), calculations without kaon-nucleon interaction (soft, no $KN$) and with the isospin based $N\Delta \rightarrow N\Lambda K^+$ cross section (soft, $KN$, $\sigma^*$). These calculations are confronted with a standard calculation using the hard EoS potential. The scaling of the kaon yield with the participant number can be parameterized by $M_{K^+} = A_{\text{part}}^\alpha$.

All calculations with a soft EoS show a rather similar value of $\alpha$ - although the yields are very different - while the calculation using a hard equation shows a much smaller value. Therefore we can conclude that also the slope value $\alpha$ is a rather robust observable.

The bottom of Fig. 2 shows that $\alpha$ depends smoothly on the compressibility $\kappa$ of the EoS. Whether we include the momentum dependence of the nucleon nucleon...
interaction (with mdi) or not (without mdi) does not change the value of $\alpha$ as long as the compressibility is not changed - in stark contrast to the in-plane flow. Again, the measured centrality dependence for Au+Au at 1.5 A GeV from the KaoS collaboration [18], $\alpha = 1.34 \pm 0.16$, is only compatible with a soft EoS potential.

This finding is also supported by a more recent analysis [19, 20] of the in-plane flow which supersedes the former conclusion that the EoS is hard [21] (made before the momentum-dependent interaction has been included in the calculations). Due to the strong dependence of the in-plane flow on the potential range parameter and its dependence on the particles observed these conclusions are much less firm presently. Comparisons of the out-of-plane squeeze of baryons also show a preference for a soft equation of state with momentum dependent interactions [22].

In conclusion, we have shown that the two experimental observables which are most sensitive to the potential parameters of the nucleon-nucleon interaction are only compatible with those parameters which lead in nuclear matter to a soft hadronic EoS. This conclusion is robust. Uncertainties of the input in these calculations, like the $KN$ potential at high densities, the lifetime of the $\Delta$ in matter and the $\Delta N \to NK^+A$ cross section do not influence this conclusion. The potential parameter $\kappa$ is even smaller than that extracted from the giant monopole vibrations. Thus the energy which is needed to compress hadronic matter of $\kappa \leq 200$ MeV is close to the lower bound of the interval which has been discussed in the past.

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FIG. 2: Dependence of the $K^+$ scaling on the nuclear equation of state. We present this dependence in form of $M_{K^+}/A_{part}$. On the top the dependence of $M_{K^+}/A_{part}$ of $\alpha$ is shown for different options: a “hard” EoS with $KN$ potential (solid line), the other three lines show a “soft” EoS, without $KN$ potential and $\sigma(N\Delta)$ from Tsushima [17] (dotted line), with $KN$ potential and the same parametrization of the cross section (dashed line) and with $KN$ potential and $\sigma(N\Delta) = 3/4\sigma(NN)\times 0.16$. On the bottom the fit exponent $\alpha$ is shown as a function of the compressibility for calculations with momentum-dependent interactions (mdi) and for static interactions ($t_4 = 0$, dashed line).

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