Cluster mass estimation through fair galaxies

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ABSTRACT

We analyse a catalogue of simulated clusters within the theoretical framework of the Spherical Collapse Model (SCM), and demonstrate that the relation between the infall velocity of member galaxies and the cluster matter overdensity can be used to estimate the mass profile of clusters, even though we do not know the full dynamics of all the member galaxies. In fact, we are able to identify a limited subset of member galaxies, the ‘fair galaxies’, which are suitable for this purpose. The fair galaxies are identified within a particular region of the galaxy distribution in the redshift (line-of-sight velocity versus sky-plane distance from the cluster centre). This ‘fair region’ is unambiguously defined through statistical and geometrical assumptions based on the SCM. These results are used to develop a new technique for estimating the mass profiles of observed clusters and subsequently their masses. We tested our technique on a sample of simulated clusters; the mass profiles estimates are proved to be efficient from 1 up to 7 virialization radii, within a typical uncertainty factor of 1.5, for more than 90 per cent of the clusters considered. Moreover, as an example, we used our technique to estimate the mass profiles and the masses of some observed clusters of the Cluster Infall Regions in the Sloan Digital Sky Survey catalogue. The technique is shown to be reliable also when it is applied to sparse populated clusters. These characteristics make our technique suitable to be used in clusters of large observational catalogues.

Key words: galaxies: clusters: general – galaxies: kinematics and dynamics – large-scale structure of Universe.

1 INTRODUCTION

The clusters of galaxies are the largest dynamically relaxed structure we observe in the Universe. The knowledge of their dynamics and mass distribution is strategic in modern cosmology, since it constrains different cosmological models (White & Frenk 1991; Bahcall & Cen 1993, Borgani et al. 1997). Several methods are currently used to estimate the cluster masses and mass profiles, based on the dynamical analysis of the member galaxies (Kaiser 1987; Regős & Geller 1989; Diaferio & Geller 1997; Girardi et al. 1998), on the observations of the X-ray emitting gas (Cowie, Henriksen & Mushotzky 1987; Borgani et al. 2004) and on the gravitational lensing effects (Grossman & Narayan 1989; Narayan & Bartelmann 1996). However, the estimation of cluster masses remains difficult, mainly because most of the matter in clusters is actually dark matter, which cannot be easily observed.

Within this framework, the dynamics of the cluster outskirts is particularly relevant. In the outskirts, the matter is not yet in equilibrium and is affected by an overall infall motion towards the cluster centre. As discussed by Cupani, Mezzetti & Mardirossian (2008, hereafter CMM08), the dynamics of this ‘non-equilibrium’ region turns out to be quite well described by the Spherical Collapse Model (SCM; Gunn & Gott 1972; Silk 1974; Peebles 1977; Gunn 1978; Peebles 1980). CMM08 focused on the turnaround radius $r_t$ (i.e. the radius where the mean velocity of the infalling matter balances the Hubble flow) and studied the dependence of the overdensity $\delta$ on the radial coordinate $r$ in the region surrounding $r_t$. They demonstrated that the large majority of clusters are compatible with a single mass profile when their mass $M$ is expressed in units of the turnaround mass $M_t$.

The results of CMM08 provide a description of the overall shape of the cluster mass profile in the non-equilibrium region. In principle, a way to reconstruct the individual features of these profiles is provided by the relation between the overdensity and the velocity of the infall motion in clusters (Regős & Geller 1989):

$$\omega_{inf} \equiv \frac{v_{inf}}{H_0 r} \approx \Omega_{M,0}^{0.6} F(\delta).$$

(1)

Here, $v_{inf}$ is the infall velocity (i.e. the bulk peculiar velocity of matter towards the cluster centre, defined as positive when directed inwards), $H_0$ is the present-day Hubble parameter, $\Omega_{M,0}$ is the present-day matter density parameter and $F$ is an analytical function of the overdensity $\delta$ (CMM08; Yahil 1985; Villumsen & Davis 1986). If
\( v_{\text{inf}} \) and \( F \) are known, one can compute the mass profile by inverting equation (1):

\[
M(r) = \frac{4}{3} \pi \rho_{\text{cr}} \Omega_{M,0} r^3 [1 + \delta(r)] = \frac{4}{3} \pi \rho_{\text{cr}} \Omega_{M,0} r^3 \left[ 1 + F^{-1} \left( \Omega_{M,0}^{-0.6} \omega_{\text{inf}}(r) \right) \right],
\]

(2)

where \( \rho_{\text{cr}} \) is the critical matter density and \( F^{-1} \) is the inverse function of \( F \).

To compute the mass profile via equation (2), one must reconstruct the overall infall pattern of clusters using the member galaxies as tracers. Such reconstruction may result to be difficult for two reasons:

(i) the radial velocities of member galaxies are affected by the presence of local substructures inside the clusters; therefore, the values of \( v_{\text{inf}} \) are distributed along \( r \) in a blurred band (see also CMM08 for details) and

(ii) the infall velocities of galaxies cannot be easily inferred from observations, owing to projection effects.

Nevertheless, in the present work we wish to show that the infall velocity approach can be used to estimate the overdensity and mass profiles of galaxy clusters. We focus on the dynamical quantities of galaxies which can be directly inferred from observation, namely the sky-plane distance from the cluster centre, \( r_{\text{sp}} \), and the line-of-sight velocity, \( u_{\text{los}} \) (all these quantities are the moduli of the corresponding vectors). The technique we put forward is based on the identification of a ‘fair region’ (FR) in the redshift space \( (r_{\text{sp}}, u_{\text{los}}) \). The galaxies which lie within this region will proved to be suitable to reconstruct the total matter distribution in clusters via equation (2).

We will discuss our technique using a catalogue of simulated clusters, to be able to know the overall kinematical properties of the galaxy distribution. The results of this analysis are consistent with the formulation of the SCM discussed by Cupani, Mezzetti and Mardroian, in preparation (hereafter CMM10) and hold within a range of values of the present-day matter density parameter \( (0.2 \leq \Omega_{M,0} \leq 0.4) \), for a spatially flat Universe). Our mass estimation technique relies only on the observable kinematic properties of cluster member galaxies, and it is therefore suitable to be applied to observed clusters. As an example, we will discuss the results of our technique when applied to a subset of clusters from the Cluster Infall Regions in the Sloan Digital Sky Survey (CIRS) catalogue (hereafter RD Rines & Diaferio 2006).

In Section 2, we describe the simulated data catalogue we use for our analysis. In Section 3, we discuss thoroughly the statistics of the member galaxy velocities. We first provide an empirical definition of the FR and then we justify this definition using the SCM. In Section 4, we detail our mass estimation technique and its reliability. Finally in Section 5, we summarize the recipe for estimating cluster masses and mass profiles and draw the conclusion of the present work.

## 2 The Cluster Catalogue

We performed our analysis on the same data catalogue used by CMM08. These data were produced via a large cosmological hydrodynamical simulation of a \( \Lambda \) cold dark matter (LCDM) Universe run by Borgani et al. (2004), while the catalogue of objects (clusters and galaxies) was extracted by Biviano et al. (2006). We refer to these papers for the details.

### Table 1.

|                        | Simulation | WMAP+BAO+SN |
|------------------------|------------|-------------|
| \( \sigma_8 \)        | 0.8        | 0.812 ± 0.026 |
| \( h \)                | 0.7        | 0.705 ± 0.013 |
| \( \Omega_{M,0} h^2 \) | 0.147      | 0.1358 ± 0.0037 |
| \( \Omega_{\text{bar},0} \) | 0.04      | 0.0456 ± 0.0015 |

The simulation was run assuming a flat \( \Lambda \)CDM cosmology with \( \Omega_M + \Omega_{\Lambda} = 1 \), where \( \Omega_M \) is the matter density parameter and \( \Omega_{\Lambda} \) is the cosmological constant density parameter. The adopted values of the main parameters (namely the normalization of the power spectrum \( \sigma_8 \), the Hubble constant \( h \), the present-day matter density parameter \( \Omega_{M,0} \) and the present-day density parameter of baryonic matter \( \Omega_{\text{bar},0} \)) are listed in Table 1. In the same table, we also listed the observed values of these parameters, obtained by combining the last measurements of the Wilkinson Microwave Anisotropy Probe (WMAP) with the distance measurements from the baryonic acoustic oscillations (BAO) and from the Type Ia supernovae (SN) (Komatsu et al. 2008). The two sets of values in Table 1 evidence a quite good agreement.

The catalogue we use (Biviano et al. 2006) contains 114 clusters and 9631 galaxies. The clusters differ in size, with virialization radii \( r_{v} \) in the range \( r_{v} = (0.9/2.2) \, h^{-1} \text{Mpc} \), virialization masses \( M_{v} \) in the range \( (8.0 \times 10^{14}/1.3 \times 10^{15}) \, h^{-1} \, M_{\odot} \), and number of member galaxies \( n_{\text{gal}} \) between 17 and 403. The detailed frequency distributions of \( r_{v} \) and \( M_{v} \) are shown in Fig. 1.

\[ \text{Figure 1. Statistics of cluster sizes in the simulated data catalogue. Upper panel: frequency distribution of the virialization radii} \ r_{v}; \text{lower panel: frequency distribution of the virialization masses} \ M_{v}. \text{\( \tilde{n}_{\text{clus}} \) is the number of clusters per frequency bin.} \]
The analysis of dynamics in clusters was performed using the cluster catalogue in two ways: superimposing all clusters into a synthetic object, and studying all clusters one by one. The former approach is needed to increase the statistical significance of the results, as pointed out by Vedel & Hartwick (1998), while the latter approach is used to test the accuracy of the results when they are applied to single-cluster analysis. Throughout the discussion, we identify the clusters with the subscript $i$ and the galaxies with the subscript $g$.

We took into account the particles [dark matter (DM), gas and stellar particles] of the simulation by Borgani et al. (2004) and the member galaxies subsequently identified by Biviano et al. (2006). The particles were grouped into concentric shells in each cluster $i$, using the value of $r_{sp;g}$ as normalization radius, while the member galaxies were attributed to shells according to their radial distance from the cluster centre. The shell subdivision is needed to make comparable objects of different scale. The shells were defined using a logarithmical spacing covering the whole radial extent from the virialization core to the far outskirts of clusters. We considered in our analysis the dynamically relevant quantities obtained from the simulation, namely the mass of the particles, $m$, and the six phase-space coordinates of both particles and galaxies $(x, y, z, v_x, v_y, v_z)$. The position coordinates $x, y$ and $z$ are referred to the cluster centre, while the velocity coordinates $v_x, v_y$ and $v_z$ are peculiar velocities corrected for the motion of the cluster centre. We also used the simulated data as ‘mock observations’, i.e. restricting to only two out of six phase-space coordinates: we adopted the $x$ direction as the line of sight and defined the sky-plane distance as $r_{sp} = \sqrt{x^2 + z^2}$ and the line-of-sight proper velocity as $v_{los;g} = v_z + H_0 r_{sp}$. The masses of the shells were computed by integrating the masses of all the enclosed particles. The velocities of the shells are the averages of the velocities of all the enclosed galaxies.

3 ANALYSIS OF THE REDSHIFT SPACE

Fig. 2 represents a cluster $C$ with an infalling galaxy $G$ as seen by a distant observer $O$; in the same figure, one can see the angles $\epsilon$ and $\alpha$ used in the following relations. The total peculiar velocity vector $v_g$ of the galaxy is split into two orthogonal components, the radial velocity vector $v_{r,g}$ and the tangential velocity vector $v_{t,g} = v_g - v_{r,g}$. Within the widely accepted assumptions of SCM, the overall infall motion of galaxies can be considered as isotropic:

\begin{equation}
\langle v_{t,g}(r) \rangle = v_{inf}(r), \quad \langle v_{r,g}(r) \rangle = 0.
\end{equation}

Angle brackets denote the average over all galaxies located at a given radial distance $r$ from the cluster centre. It is worth noting that $\langle v_{r,g} \rangle = 0$ does not generally imply $\langle v_{t,g} \rangle = 0$; on the contrary, a non-zero tangential velocity modulus is expected in most of cases. In fact, as can be seen from the simulations and from the SCM (CMM08; CMM10; see also Section 3.1),

\begin{equation}
v_{inf} \simeq \langle v_{t,g} \rangle.
\end{equation}

The existence of random tangential motions does not conflict with the presence of an overall infall motion of galaxies, since all non-radial contribution are averagely suppressed due to isotropy. Hereinafter, we will take into account only the moduli of the velocity components.

The relation between the phase-space coordinates $r$, $v_t$ and $v_r$, and the observational redshift-space coordinates $r_{sp}, v_{los}$ is obtained from Fig. 2 using trigonometry. Since $v_{los;g} = H_0 r_{sp}$ is a proper velocity, the radial Hubble flow term $H_0 r_{sp}$ must be taken into account. Therefore,

\begin{equation}
r_{sp;g} = r_g \cos \epsilon_g,
\end{equation}

\begin{equation}
v_{los} = (H_0 r_{sp} - v_{r;g}) \sin \epsilon_g + v_{t;g} \cos \alpha_g \cos \epsilon_g.
\end{equation}

Equations (5) and (6) form an underdetermined system and cannot be used to compute $r_g$ and $v_{r;g}$ when only $r_{sp;g}$ and $v_{los}$ are known. To overcome this issue, we focus on the subset of all galaxies having $r_{sp;g} \simeq r_g$ and satisfying the condition $|v_{los;g}| \simeq v_{inf}$ (see below). We will refer to these galaxies as ‘tracking galaxies’ (hereinafter TGs), since they are the best candidates to estimate the overdensity and the mass profile using only the observed redshift-space coordinates. By definition, the TGs have

\begin{equation}
\omega_{inf}(r) \simeq \langle |v_{los}(r_{sp})| \rangle.
\end{equation}

Substituting equations (5) and (7) into equation (2), one can compute the cluster mass profile using only the observable properties of the TGs:

\begin{equation}
M(r) \simeq M_{inf} \left[ r_{sp}, \langle |v_{los}(r_{sp})| \rangle \right] = \frac{4}{\pi} \rho_c r_{sp} \Omega_M, g_{sp}^3 \left[ 1 + F^{-1} \left[ \Omega_0 \left( \frac{|v_{los}(r_{sp})|}{H_0 r_{sp}} \right) \right] \right].
\end{equation}

The statistical properties of the TG subset are essential to determine how the infall velocity approach can be used to estimate the cluster mass profiles. TGs are not randomly distributed among other galaxies, but tend to be gathered in an identifiable region of the redshift space. By definition, the TGs have $\epsilon_g \simeq 0$, which substituted into equation (6) yields $v_{los;g} \simeq v_{t;g} \cos \alpha_g$. Combining this result with equation (4), one obtains that $(\epsilon_g) \simeq 0$. Therefore, at any given sky-plane distance, the TGs satisfy the condition

\begin{equation}
|v_{los}(r_{sp})| \simeq v_{inf} \simeq \langle v_{t,g}(r_{sp}) \rangle.
\end{equation}

Equation (9) suggests that TGs are unlikely to be found in the region of the redshift space where $u_{los} \simeq 0$, since both $v_r$ and $v_t$ are generally non-zero.

These considerations are supported by the analysis of the simulated data of our catalogue. We represented in Fig. 3, the dependence between $v_{r,g}$ and $v_{t,g}$ (left-hand column) and between $|u_{los;g}|$ and $v_{t,g}$ (right-hand column) as extracted from our data catalogue. The galaxies are represented with grey dots, while the TGs are represented as black dots. The TGs were operatively defined as satisfying the conditions

\begin{equation}
r_{sp;g} \simeq r_g \cos \epsilon_g \simeq 1 - \eta_{TG}.
\end{equation}

Figure 2. Galaxy kinematics in clusters: C is the cluster centre, G is an infalling galaxy and O is a distant observer.
3.1 The fair region of the redshift space

We can roughly define as ‘FR’ the region of the redshift space where the TGs are mostly concentrated. In fact, an effective definition of the FR is required to estimate the mass profile of clusters with the member galaxies via the infall velocity approach. We will obtain such definition combining a statistical analysis of the simulated data catalogue with the theoretical predictions provided by the SCM (CMM10).

According to the condition of equation (9), the FR is defined as the region where the mean peculiar radial velocity is almost equal to the mean tangential velocity. The \( v_r \) and \( v_t \) distribution of our galaxies along \( r \) is represented in Figs 5 and 6, respectively. To determine the grey-scale, we superimposed an orthogonal grid on each plane and counted the galaxies in each cell of the grid, in order to produce a two-dimensional histogram. The grey-scale represent the number density of galaxies for unit surface \( D \). Both distribution are interpolated with the SCM prediction for \( \Omega_M = 0.3 \) (thick solid line) and for \( \Omega_M = 0.2 \) and \( \Omega_M = 0.4 \) (lower and upper dashed lines, respectively). Despite the notable dispersion due to the variance among clusters (see CMM08), the SCM predicts quite well the main profile of both \( v_r \) and \( v_t \). The dependence on \( \Omega_M \) is overshadowed by the data dispersion and is generally negligible. To determine the location of the FR, we computed the joint probability for galaxies to be located in the same grid cell of Figs 5 and 6. Fig. 7 represents the distribution obtained by this procedure (upper panel), compared to the distribution of TGs from the simulated data catalogue. Here, the grey-scale represents the probability parameter \( P \), obtained by normalizing \( D \) to unity over the whole plane. The two panels show a remarkable agreement, indicating that the condition in equation (9) is suitable to identify the FR.

The empirical definition of the FR is theoretically confirmed by the SCM results (CMM10). In a spherically symmetric scenario,
the FR is theoretically defined as the region where the parameters \( \Delta r = 1 - r_{\text{g}}/r \) and \( \Delta v = 1 - u_{\text{los}}/v_r \) are minimized, in agreement with the definition of the TGs. Considering equations (5) and (6), we can write

\[
\Delta r = 1 - \cos \epsilon, \quad (12)
\]

\[
\Delta v = 1 - \left( \frac{H_0}{v_c} - 1 \right) \sin \epsilon + K_v \cos \alpha \cos \epsilon, \quad (13)
\]

where \( K_v \equiv v_c/v_r \). Equations (12) and (13) describe how the infall velocity profile predicted by the SCM is biased due to projection on to the sky plane. This bias depends both on the variance in galaxy dynamics (parametrized by \( K_v \) and \( \alpha \)) and on the galaxy displacement with respect to the sky plane (parametrized by \( \epsilon \)).

The extension of the FR can be therefore determined by taking the extremal values of \( \Delta r \) and \( \Delta v \) obtained by varying \( K_v \) and \( \epsilon \) and \( \alpha \) around their mean values. In fact, CMM10 showed that the mean value of \( K_v \) is quite close to unity in the non-equilibrium region for \( 0.2 \leq \Omega_{M,0} \leq 0.4 \), in agreement with equation (4). In particular, if \( \Omega_{M,0} = 0.3 \), \( (K_v(\bar{r}_v)) \approx 0.95 \) for \( \bar{r}_v \geq 3 \). So, in order to operatively define the FR, we used the following conditions:

\[
\frac{0.95}{K_v^*_{\text{FR}}} \leq K_v \leq 0.95K_v^*_{\text{FR}}, \quad K_v^*_{\text{FR}} = 1.5; \quad (14)
\]

\[-\eta_{\text{FR}}^* \leq \epsilon, \quad \alpha \leq \eta_{\text{FR}}^*, \quad \eta_{\text{FR}}^* = 0.2. \quad (15)\]

The parameter \( K_v^*_{\text{FR}} \) mimics the variance of \( K_v \) around its mean value 0.95, while the parameter \( \eta_{\text{FR}}^* \) mimics a small discrepancy of \( \epsilon \), and \( \alpha \) around their mean value 0. The boundaries of the FR obtained through equations (14) and (15) are shown as dashed lines in Fig. 8, superimposed to the grey-scale distribution of TGs (same as in the lower panel of Fig. 7). The agreement between the theoretical prediction and the data distribution of our catalogue is remarkable. This result shows that the FR, whose existence was originally inferred from the distribution of the TGs in the redshift space, is due to the projection of the infall velocity profile on to the sky plane. The SCM is shown to work for \( 0.2 \leq \Omega_{M,0} \leq 0.4 \); this allows to identify the FR in different cosmologies.

4 MASS ESTIMATION

The definition FR in the redshift space provides a way to select the galaxies which are suitable to reconstruct the total matter distribution of clusters. The galaxies lying within the FR will be referred to as ‘fair galaxies’ (hereinafter FGs or FG subset). In this section, we will prove that the infall velocity approach is suitable to estimate the mass profiles of clusters using the FG subset.

Our technique can be outlined as follows:

(i) identification of the FG subset via the identification of the FR provided in Section 3.1;

(ii) detection of the mass profiles, using the redshift-space coordinates of the FGs.

We used the simulated data catalogue to test the reliability of such approach, comparing the estimated mass profiles obtained with our technique with the actual mass profiles of the simulated clusters. The simulated data were transformed into ‘mock observations’ as described in Section 2. All galaxies lying within the dashed lines in Fig. 8 were regarded as FGs, and their observable redshift-space
was defined, according to CMM08, as the inverse of
\[ r = \frac{1}{\sigma(M)} \]
and \( r \) is used instead of the unknown quantities \( \sigma(M) \) and \( M \), when handling the FG subset. If the \( \delta M \) will be used here-instead of \( r \), we have no appreciable data beyond this distance, we approximated the average distribution in the upper panel of Fig. 9 with a linear fitting algorithm as \( R(r) = 5.5 \times 10^{-2} r + 0.6 \). This fit is in agreement with the data distribution and is used to correct the radial position of FGs in our simulated data catalogue.

The first raw estimate of the cluster mass profiles is obtained by substituting the values of \( r_{G,G} \) and \( u_{los,g} \) into equation (8):
\[ M_{raw,g} = M_{inf}(r_{G,G}, u_{los,g}, v) \]
(18)
The function \( F^{-1} \) was defined, according to CMM08, as the inverse of the Meiksin approximation (Villumsen & Davis 1986):
\[ F(\delta) = \frac{\delta}{\frac{3}{2}} \left( 1 + \frac{\delta}{\frac{3}{2}} \right)^{-1/2} \]
(19)
The upper panel of Fig. 9 shows the distribution obtained computing the ratio between \( M_{raw,g} \) and the real mass values \( M_{g} \) extracted from the simulation, for each galaxy in the FG subset (grey dots). The average and the 1σ uncertainty region of the distribution along the radial coordinate are shown in both panels with thick solid and dashed lines, respectively.

The final corrected estimate of the cluster mass profiles, \( M_{corr,g} \), was obtained by smoothing the distribution of \( M_{G,G} \) values for each cluster, taking its running median along the radial coordinate. We computed the running median within a window defined to contain...
approximately one-third of the galaxies in each cluster. The lower panel of Fig. 9 shows the distribution obtained computing the ratio between $M_{\text{corr}}$ and the real mass values $M_i$ extracted from the simulation, for each galaxy in the FG subset (light-blue dots). Also in this case, the average and the 1σ uncertainty regions of the distribution along the radial coordinate are also shown (thick solid line and dashed lines, respectively). The combination of correction and running median smoothing yields very good results. The overall distribution is very close to unity from 1 to 7 virialization radii. The variance of the distribution decreases when moving outwards, confirming once more the reliability of the infall velocity approach in the cluster non-equilibrium region.

The overall accuracy of our technique was evaluated by computing the statistical moments of the distribution of $M_{\text{corr}}/M_i$ values. We denote with $\mu_M$ and $\sigma_M$ the logarithmic mean and the logarithmic variance computed among the set of values $M_{\text{corr}}/M$:

\[
\mu_M = \exp \left[ \frac{1}{n_{\text{gal}}} \sum_{g=1}^{n_{\text{gal}}} \left( \ln M_{\text{corr},g} - \ln M_g \right) \right],
\]

\[
\sigma_M = \sqrt{\exp \left[ \frac{1}{n_{\text{gal}}} \sum_{g=1}^{n_{\text{gal}}} \left( \ln M_{\text{corr},g} - \ln M_g - \mu_M \right)^2 \right]}.
\]

These moments were computed under different conditions, by taking into account the effects of the observational error and of the radial aperture in cluster observation. In particular, (i) we mimicked the observational error associated to the line-of-sight velocity measurement by substituting $u_{\text{los},g}$ with a perturbed line-of-sight velocity $\tilde{u}_{\text{los},g}$:

\[
\tilde{u}_{\text{los},g} = u_{\text{los},g} + \Delta u_{\text{los},g}, \quad \Delta u_{\text{los},g} \equiv \text{rand}(0, \Delta u_{\text{los}}^*).
\]

Here, rand $(\mu, \sigma)$ is a function which generates random values having a Gaussian distribution with mean $\mu$ and variance $\sigma$. We set $\Delta u_{\text{los}}^*$ alternatively to 0, 100, 200 and 300 km s$^{-1}$; (ii) we defined different intervals of $r_{\text{los}}$, namely $[1, 7]$, $[1, 3.5]$ and $[1, 2.2]$, respectively, corresponding to the extent of uniform data coverage in our data catalogue, to the region encompassed by turnaround radius (CMM08; CMM10), and to the transition region between the cluster core and the non-equilibrium region (where the infall approach is expected to be less reliable; see CMM08). The cluster core was rejected in all cases to avoid the contamination from virialization-related phenomena, which undermine the reliability of infall approach.

The values of $\mu_M$ and $\sigma_M$ obtained with different choices of $\Delta u_{\text{los}}^*$ and of the aperture interval are listed in Table 2. The accuracy is very good in all cases, confirming the reliability of the infall velocity approach applied to the FG subset. The variance $\sigma_M$ is only slightly affected by the observational error on the line-of-sight velocity and by the radial coverage in the galaxy distribution, confirming the robustness of the estimation.

Our technique is also suitable to compute the virialization mass and turnaround mass of clusters. $M_{\text{corr, v}}$ and $M_{\text{corr, t}}$ were obtained by interpolating the distribution of $M_{\text{corr}}$ values at $r_{\text{los}} = 1$ and at the radius where the overdensity equals the turnaround value $\delta_t = 15$ (CMM08; CMM10). The interpolation was done with a linear fitting algorithm. In fact, this procedure is ineffective when the running median does not cover the radial extent from the virialization radius to the turnaround radius. We denote with $f_{\text{fail, v}}$ and $f_{\text{fail, t}}$, respectively, the fraction of clusters for which the estimation of $M_{\text{corr, v}}$ and $M_{\text{corr, t}}$ is impossible. Focusing on the remaining cases, we denote with $f_{\text{agree, v}}$ and $f_{\text{agree, t}}$, respectively, the fraction of clusters for which the estimated values are consistent with the actual values $M_i$ and $M_t$ within the uncertainty. The values of $f_{\text{fail, v}}, f_{\text{agree, v}}, f_{\text{fail, t}}$ and $f_{\text{agree, t}}$ obtained for different choices of $\Delta u_{\text{los}}^*$ are listed in Table 3. The percentage of failure is slightly increased when the observational error in the line-of-sight velocity is introduced, as an example.

### Table 2. Accuracy of mass estimation technique, evaluated by taking into account a mimicked observational error $\Delta u_{\text{los}}^*$ in the line-of-sight velocity and the radial interval of aperture in observation.

| $\Delta u_{\text{los}}^*$ (km h$^{-1}$) | Interval | $n_{\text{gal}}$ | $\mu_M$ | $\sigma_M$ | $\mu_M\sigma_M$ | $\mu_M/\sigma_M$ |
|--------------------------------------|----------|-----------------|---------|-----------|-----------------|-----------------|
| 0                                    | $[1.0, 7.0]$ | 3693            | 0.97    | 1.46      | 1.42            | 0.67            |
|                                      | $[1.0, 3.5]$ | 1938            | 0.96    | 1.56      | 1.49            | 0.62            |
|                                      | $[1.0, 2.2]$ | 743             | 0.93    | 1.59      | 1.48            | 0.58            |
|                                      | $[1.0, 7.0]$ | 3746            | 0.99    | 1.48      | 1.46            | 0.67            |
| 100                                  | $[1.0, 3.5]$ | 1942            | 0.96    | 1.58      | 1.52            | 0.61            |
|                                      | $[1.0, 2.2]$ | 752             | 0.92    | 1.62      | 1.49            | 0.57            |
|                                      | $[1.0, 7.0]$ | 3840            | 1.02    | 1.47      | 1.50            | 0.70            |
| 200                                  | $[1.0, 3.5]$ | 1504            | 1.01    | 1.58      | 1.59            | 0.64            |
|                                      | $[1.0, 2.2]$ | 798             | 0.97    | 1.60      | 1.57            | 0.60            |
|                                      | $[1.0, 7.0]$ | 3761            | 1.07    | 1.51      | 1.61            | 0.71            |
| 300                                  | $[1.0, 3.5]$ | 1488            | 1.05    | 1.62      | 1.69            | 0.65            |
|                                      | $[1.0, 2.2]$ | 802             | 1.00    | 1.67      | 1.67            | 0.60            |

Note: $\mu_M$ and $\sigma_M$ are the moments of the distributions in the lower panel of Fig. 9, computed for different intervals of radial aperture.

### Table 3. Accuracy of the virialization and turnaround mass estimate, evaluated by taking into account a mimicked observational error $\Delta u_{\text{los}}^*$ in the line-of-sight velocity and the radial interval of aperture in observation.

| $\Delta u_{\text{los}}^*$ (km h$^{-1}$) | $f_{\text{fail, v}}$ | $f_{\text{agree, v}}$ | $f_{\text{fail, t}}$ | $f_{\text{agree, t}}$ |
|--------------------------------------|---------------------|-----------------------|---------------------|----------------------|
| 0                                    | 0.10                | 0.90                  | 0.06                | 0.72                 |
| 100                                  | 0.15                | 0.93                  | 0.04                | 0.69                 |
| 200                                  | 0.19                | 0.91                  | 0.03                | 0.59                 |
| 300                                  | 0.19                | 0.91                  | 0.04                | 0.54                 |

Note: $f_{\text{fail, v}}$ and $f_{\text{fail, t}}$ are the fractions of clusters for which the estimation is impossible (computed among all clusters). $f_{\text{agree, v}}$ and $f_{\text{agree, t}}$ are the fractions of clusters for which the estimated value is in agreement with the actual value within the uncertainty (computed among the clusters for which the estimation is possible).
effect of the higher noise in the galaxy velocity distribution. Nevertheless, the estimation of both \( M_v \) and \( M_t \) is significantly accurate even for relatively high values of \( \Delta u_{\text{gal}}^* \).

### 4.1 Single-cluster estimations

Some characteristics of our mass estimation technique can be better appreciated by comparing the mass profiles estimated for single clusters separately. In this section, we will discuss in detail the results obtained for nine simulated clusters from our data catalogue and for nine observed clusters from the CIRS catalogue by RD.

Fig. 10 shows the mass profiles of the nine simulated clusters. The grey solid lines represent the actual profiles \( M_g \), while the black solid and dashed lines represent the estimated profile \( M_{\text{corr},g} \) together with its uncertainty band. We adopted the 2\( \sigma \) uncertainty on the running median as the uncertainty on this estimate, since it was observed to mimic well the dispersion in the actual distribution of \( M_{G,G} \) values. The distributions of values \( M_{\text{raw},g} \) are also shown with grey dots. This subset of nine clusters gives a good representation of the common results obtained when estimating the mass of single clusters from the whole catalogue. In most cases, our technique is able to produce a mass profile extended from the virialization core up to the extreme outskirts of clusters (clusters 1, 2, 4, 6, 7 and 9), even when only a few FGs are identified (clusters 1 and 7). If the FGs are not evenly distributed, the estimated profile turns out to be radially limited (clusters 3 and 8) but the estimate is generally correct within the covered interval. The definition of the FR which is the same for all clusters (as described in Section 3.1) does not mask the individual features of different clusters, which are correctly reconstructed through the features of the FG subset. In fact, our technique is able to discern the presence of remarkable changes of slope in the mass profiles (clusters 1, 2, 3, 6 and 9), which cannot be predicted using the results of CMM08. In some cases, the procedure of correction and smoothing produces an undesired plateau in the outermost regions (clusters 9). Only in few cases, the poor quality of data prevents our technique from properly reconstructing the mass profile (cluster 8).

The values of the virialization mass and the turnaround mass estimated for the nine simulated clusters in Fig. 10 are listed in Table 4, along with the corresponding values extracted from the simulations. In most cases, the agreement is fairly good, even when the overall profile is poorly reconstructed, thanks to the linear fitting algorithm used to compute \( M_{\text{corr},v} \) and \( M_{\text{corr},t} \).

Fig. 10 shows the mass profiles of the nine CIRS clusters. These clusters were chosen among the ones with the highest radius of complete radial coverage in the survey. As before, the black solid and dashed lines represent the estimated profile \( M_{\text{corr},g} \) together with its uncertainty band, while the distributions of values \( M_{\text{raw},g} \) are shown with grey dots. The virialization radii \( r_v \) of these clusters, needed to normalize the galaxy radial distribution, were computed iteratively as the radii where the overdensity estimated by our technique equals the virialization value \( \delta_v \approx 101/\Omega_0 - 1 \) (Bryan & Norman...
Table 4. Estimated virialization masses and turnaround masses for the nine simulated clusters in Fig. 10, compared with the corresponding values extracted from the simulation.

| Cluster | $M_\nu (10^{14} M_\odot)$ | $M_{corr, \nu} (10^{14} M_\odot)$ | $M_\tau (10^{14} M_\odot)$ | $M_{corr, \tau} (10^{14} M_\odot)$ |
|---------|-----------------|-----------------|-----------------|-----------------|
| 1       | 1.37            | 1.87$\pm$0.60  | 2.48            | 2.86$\pm$0.67  |
| 2       | 1.75            | 1.80$\pm$0.35  | 3.87            | 5.05$\pm$0.52  |
| 3       | 2.07            | 1.70$\pm$0.13  | 8.13            | 9.24$\pm$2.08  |
| 4       | 3.36            | 1.98$\pm$0.66  | 7.04            | 6.44$\pm$0.67  |
| 5       | 3.64            | 3.22$\pm$1.17  | 6.40            | 7.19$\pm$2.44  |
| 6       | 4.08            | 3.53$\pm$1.14  | 8.25            | 7.77$\pm$1.87  |
| 7       | 1.76            | 1.38$\pm$0.41  | 5.26            | 5.97$\pm$5.46  |
| 8       | 2.17            | 2.25$\pm$0.49  | 3.45            | 3.09$\pm$3.28  |
| 9       | 4.11            | 4.43$\pm$1.47  | 7.12            | 6.45$\pm$1.57  |

Figure 11. Mass profiles for nine observed clusters from the CIRS catalogue (RD). In each panel, we represented the distribution of $M_{raw; g}$ values (grey dots) and the estimated mass profile $M_{corr; g}$ (black solid lines) together with its uncertainty band (black dashed lines, see Section 4). The virialization radii $r_\nu$ and the turnaround radii $r_\tau$ are also shown (vertical dotted lines, leftmost and rightmost in each panel, respectively).

The CIRS clusters are generally more populated than those extracted from our simulation, allowing a lower uncertainty in the mass profile estimation. A fairly good agreement with the estimates of RD is generally observed at all radii. Compared to the results of RD, our estimate is generally less accurate in the cluster core and better accurate in the outskirts. In most cases, our technique is able to reconstruct the mass profiles well beyond the turnaround radius.

The values of virialization mass and turnaround mass estimated for the nine CIRS clusters in Fig. 11 are listed in Table 5, along with the corresponding values estimated by RD. Only in this case, we adopted as virialization mass the value $M_{corr, 200}$, which corresponds to the commonly used overdensity value $\delta_{200} = 200/\Omega_0 - 1$, to allow better comparison with the values $M_{200}$ estimated by RD. We point out that such estimation of $M_{corr, 200}$ is meant only to provide a reference value and must not be regarded as reliable, because $r_{200}$...
is smaller $r_c$ and falls outside the effectiveness interval of our technique. Nevertheless, the values of $M_{200}$ and $M_{\text{cor},200}$ are in quite good agreement within the uncertainties. A fairly good agreement is observed also between $M_1$ and $M_{\text{cor},1}$. In some cases (clusters A971, A1436, A1885), our estimates of the turnaround mass are significantly higher than those of RD. This is not unexpected, since the profile of these clusters as estimated by RD are truncated before actually reaching $r_c$, resulting in a possible systematic underestimation of $M_1$.

5 DISCUSSION AND CONCLUSION

We discussed a new approach to estimate mass profiles and masses of galaxy clusters, based on the relation existing between the cluster overdensity and the infall velocity of member galaxies (Regős & Geller 1989). Our technique is simple, it only needs to know sky-plane positions and the line-of-sight velocities of the galaxies, and it can be applied to observed clusters.

Our analysis was performed and checked on a simulated catalogue of clusters (Borgani et al. 2004; Biviano et al. 2006) in the theoretical framework of the SCM which is widely accepted in literature (CMM10). We found the existence of an FR in the redshift space and demonstrated that the galaxies belonging to this region, i.e. the FGs, are able to effectively identify the mass profiles of clusters and to measure the corresponding total masses.

Our technique consists in:

(i) identification of the FG subset via the identification of the FR in the redshift space;
(ii) detection of the mass profiles using the redshift-space coordinates of FGs.

We showed that it is possible to estimate the cluster mass profiles from 1 up to 7 virialization radii, within a typical uncertainty factor of 1.5, for more than 90 per cent of clusters. The technique is reliable even with few identified FGs.

Our technique was tested to be accurate for clusters in the mass range provided by our simulation, corresponding to virialization masses between $8.0 \times 10^{13}$ and $1.3 \times 10^{15} h^{-1} M_{\odot}$. In this paper, we chose for the cosmological matter density parameter the value $\Omega_{M,0} = 0.3$, which is also the value adopted by the simulation we used. Our results can be adapted to different values $\Omega_{M,0}$ ranging from 0.2 to 0.4.

We applied our technique in a subset of clusters taken from the CIRS catalogue (RD). The mass profiles and the masses we obtained are in fairly good agreement with previous literature values.

In future, we aim to apply the present technique to clusters of large observational catalogues.

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