Research on vibration response prediction method of high speed space vehicle structure based on LSSVM

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Abstract. Considering effects of boundary conditions on the vibration of space vehicle structure caused by the environment noise, this paper dedicates to study the vibration prediction method of the high-speed space vehicle structure. Based on the mapping relationship of the structural vibration response under the same excitation but different boundary conditions in the frequency domain, a novel LSSVM (Least Squares Support Vector Machine) based prediction method is proposed to predict the vibration response of the high speed space vehicle structure under different constraints. Simulation studies are conducted to prove the effectiveness of the proposed vibration prediction method. The results have considerable significance for improving the effectiveness of the ground tests of high speed space vehicle and prediction accuracy of vibration environment condition in flight process, and provide important references for the dynamic environment design of high speed vehicle.

1. Introduction

During the actual flight process, high speed space vehicle need to withstand the pulsation thrust from propellant engine, atmospheric noise and the surface fluctuating pressure in transonic flight, which will cause structural vibration response and affect the normal working performance of instruments and equipment [1]. Therefore, in the design process of high speed space vehicle, vibration and noise environmental conditions have to be proposed accurately according to its structure and flight trajectory characteristics so that the adaptability and reliability of instruments and equipment can be verified by ground vibration and noise tests.

In the design process of traditional spacecraft (represented by carrier rocket and missile weapons), the vibration environment conditions are usually formulated by methods such as analogy, induction, envelopment and extrapolation, basing on the data of telemetry and ground tests; and the corresponding standard design specifications are created [2]. However, in the design process of the new high speed space vehicle represented by reenter vehicle and gliding missile, there is no enough flight telemetry data and design experience because of the unique structural form and ballistic characteristics, so the structural vibration response cannot be evaluated accurately. Moreover, although results of ground vibration and noise test on high speed space vehicle can provide certain references for the vibration and noise environment conditions, ground tests are restricted by actual conditions and difficult to fully simulate the flight state. Particularly, the constraint states of boundary conditions have a significant impact on the structure vibration response, and are prone to cause overtesting or undertesting problems. This has become a key technique in the overall design of a new type of high speed space vehicle [3].
With the rapid improvement of computer level and the development of modern statistical learning theory, sample learning is an important way to establish system models and conduct data deduction so as to analyze and predict complex systems. For example, BP neural network algorithm has simple structure and good performance for the prediction of nonlinear model, but there are problems of slow convergence speed and local minimum value [4]. Support Vector Machine (SVM) can guarantee the global optimal solution, but is difficult to implement for large-scale training samples [5]. Random forests algorithm can deal with high-dimensional data without feature selection, but the obtained model always is over fitted if there is large noise [6]. Least Squares Support Vector Machine (LSSVM) is a kind of excellent machine learning methods which can mine the intrinsic interdependence relationship of the observation data and use these laws to predict the unknown data [7]. LSSVM has been widely used in model identification [8], fault diagnosis [9], data prediction [10-11] and so on. Therefore, a new LSSVM based method can be proposed to investigate the similarity and difference of the same structure under different boundary conditions.

In this paper, a novel LSSVM based vibration prediction method of the high-speed space vehicle structure is investigated. Firstly, structural dynamics model of high speed space vehicle is analyzed in the frequency domain, and the mapping relationship of the structural vibration response under the same excitation but different boundary conditions is obtained in Section 2. Then, a novel prediction method based on LSSVM is proposed to analyze the mapping relationship of structural vibration response under different boundary constraints and the procedure to predict the vibration response of the high speed space vehicle structure under different constraints is developed in Section 3. Furthermore, simulation studies are conducted to prove the proposed vibration prediction method in Section 4. Finally, some conclusions are provided in Section 5. The proposed method can predict the vibration response of high speed space vehicle structure in flight state, and the analysis results can improve significantly the effectiveness of the ground tests of high speed space vehicle and prediction accuracy of vibration environment condition in flight process, so as to provide important references for the dynamic environment design of high speed vehicle.

2. Mapping relationship of structural vibration response in the frequency domain

The expression of structural dynamics model of high speed space vehicle in the time domain can be expressed as [12]:

\[
M \ddot{x} + C \dot{x} + K x = F
\]

where M, C and K are mass matrix, damping matrix and stiffness matrix of the structural system with the order of nxn, respectively. F is the external excitation vector of the structure system with the order of nx1. \( \ddot{x} \), \( \dot{x} \) and \( x \) are acceleration vector, velocity vector and displacement vector with the order of nx1, separately.

Equation (1) can be transformed to the frequency domain as follows:

\[
X(\omega) = H(\omega)F(\omega)
\]

Where \( \omega \) is the frequency, \( X(\omega) \) is structure response in frequency domain, \( F(\omega) \) is external excitation in frequency domain, and \( H(\omega) = (K + i\omega C - \omega^2 M)^{-1} \) is frequency domain response function between \( X(\omega) \) and \( F(\omega) \). When the structure of the vehicle is in two different boundary constraints (i.e., A state and B state), the structural vibration responses can be expressed as follows:

\[
\begin{align*}
X_A(\omega) &= H_A(\omega)F_A(\omega) \\
X_B(\omega) &= H_B(\omega)F_B(\omega)
\end{align*}
\]

When the structure of a high speed space vehicle is subjected to the same external excitation, namely \( F_A(\omega) = F_B(\omega) \), Equation (3) can be transformed as:

\[
X_B(\omega) = H_{AB}(\omega)X_A(\omega)
\]
In Equation (4), \( H_{ab}(\omega) = H_a(\omega)\left[H_A(\omega)H_A(\omega)^{\dagger}\right]^{-1}H_A(\omega) \) is the mapping function of the vehicle structure under the same excitation but different constraints in the frequency domain. It can be indicated that \( H_{ab}(\omega) \) is only related to the inherent characteristics of the boundary constraint state \((A \text{ and } B)\) and the structural dynamic characteristics.

From Equation (4), it is shown that when the structure of high speed space vehicle is subjected to the same external excitation, there is a definite mapping relationship in the frequency domain between the structural vibration response under the ground constraint condition and the free flight boundary condition. And the mapping relationship is only related to the structure and constraint state, but is independent of the external load excitation. Based on the mapping relationship model in the frequency domain, the vibration response of the flight state can be predicted through the ground vibration and noise test results of high speed space vehicle structure, and more accurate vibration environment conditions can be provided for the design of instruments and equipment.

However, high speed space vehicle is a complicated system consisting of thermal protection structure, support structure, electrical system, propulsion system and drive mechanism. Especially, it also has large wings and control-flaps. Thus, in the process of the dynamic response analysis, it often involves multiple elements (i.e., shell, solid, constrain element) to construct the finite element model, which lead to the mass, damping and stiffness matrices in Equation (1) become more complicated and mapping relationship in Equation (4) can hardly be obtained directly by numerical simulation and theoretical deduction.

### 3. Mapping relationship and vibration prediction method based on LSSVM

LSSVM is an improved small sample learning method based on statistical learning theory, which adopts the principle of structural risk minimization and transforms the solution of quadratic programming problem into solving linear least squares problem in high dimensional space so as to guarantee the strong generalization and efficiency of the network algorithm. LSSVM has obvious advantages in solving small sample, high dimension and nonlinear problems. Moreover, LSSVM can solve the problem of complex model and long training time in the regression analysis of complex systems. Therefore, LSSVM provides a crucial theoretical basis for the study on the prediction of vibration response of the high speed space vehicle [13, 14].

For the given sample data set \( \{(x_1, y_1), \ldots, (x_k, y_k)\} \subset \mathbb{R}^d \times \mathbb{R} \), LSSVM can be expressed as the following optimization problem:

\[
\begin{align*}
\text{Min } & \Phi(w, e) = \frac{1}{2} w^T w + \frac{1}{2} \gamma \sum_{i=1}^{k} e_i^2 \\
\text{s. t. } & y_i = w^T \Phi(x_i) + b + e_i, \quad i = 1, 2, \ldots, k
\end{align*}
\]

(5)

In Equation (5), \( e_i \subset \mathbb{R} \) is error term.

In order to solve the optimization problem in Equation (5), the following linear equations can be obtained by using the Lagrange method:

\[
\begin{bmatrix}
K + \frac{1}{\gamma} I \\
L^T
\end{bmatrix}
\begin{bmatrix}
a \\
b
\end{bmatrix}
= \begin{bmatrix}
y \\
0
\end{bmatrix}
\]

(6)

Where \( y = (y_1, y_2, \ldots, y_k)^T \), \( L \) is a unit column vector with the length of \( k \). \( K = [K_{ij} = K(x_i, y_j)]_{i,j=1}^{k} \) is the kernel matrix, and \( K(x_i, y_j) = \langle \Phi(x_i), \Phi(y_j) \rangle \), \( I \) is a unit matrix with the order of \( k \).

In order to obtain the model parameters \((a, b)\) in Equation (6), the decision function can be expressed as:

\[
f(x) = \sum_{i=1}^{k} a_i K(x_i, x) + b
\]

(7)
When modeling $H_{AB}(\omega)$ in Equation (4), the optimization algorithm can be used to select the model parameters, and the corresponding error criteria can be carried out to optimize the generalization ability of the mapping relationship modeling.

As shown in Equation (4), considering the same load excitation, there is a determined frequency domain mapping relationship of structural vibration response between the ground constraint and the free flight state. Therefore, the following analysis process based on the LSSVM method can be used to predict the vibration response of the high speed space vehicle structure under different constraints. The corresponding block diagram is shown in figure 1.

![Block diagram of LSSVM based vibration prediction method.](image)

- Establish simulation model of high speed vehicle under ground constraint and free flight state
- Select the vibrational feature points on the vehicle structure as a focal point for establishing the mapping relationship
- Based on the load equivalent criterion, apply multiple sets of white noise power spectral excitations with different energy to the simulation model separately, and consider the vibration response of structural vibration feature points as the learning samples of the LSSVM method.
- Obtain the mapping relation model of the structural vibration feature points between ground constraint and the free flight state by using the LSSVM method which can be expressed as:
  \[
  \hat{Y}_n(X_n) = \text{diag} \left[ a_{kn}^T \left[ K(X_n) \right]_{kn} \right] \ast I_n + b_n
  \]
- Predict the vibration response of the flight state (or ground constraint state) vibration feature points by treating response data of the vibration feature points of the vehicle structure in the ground constraint state (or flight state) as the working samples and input vectors of the mapping relationship.

4. Simulation study on a high speed space vehicle
In this simulation study, an US HTV-2 high speed space vehicle as shown in figure 2 is taken as an example to verify the effectiveness of the proposed method. The full length of the vehicle is 4m, and
the width of the vehicle is 2 m and the structure material is aluminum alloy with the modulus of elasticity 71GPa and the density 2800kg/m³. The ground boundary constraints are defined when the lower surface of flying craft is fixed on the ground at the front 1/3 and the rear 1/4 position. Simulation model under the ground boundary constraint and free flight condition is established as shown in figure 3. The vibration velocity on the head vertex is selected as the vibration feature point for analysis and prediction.

Figure 2. US HTV-2 high speed space vehicle. Figure 3. Simulation model of high speed space vehicle [15].

Figure 4. Noise pressure loads applied to the vehicle surface.

The flight state is simulated by the free boundary and the ground vibration test state is simulated by the fixed constrain boundary. 20 sets of noise pressure loads, similar to 4 excitations shown in figure 4, are applied to the surface of the high speed space vehicle. Each excitation holds different noise pressure amplitude in different frequency range which can represent noise pressure loads applied to the vehicle in different flight state. Then, 20 groups of vibration velocity responses on the head point of the vehicle can be obtained by simulation study. 16 groups of them are used to establish the mapping relationship model between the fixed constrained state and the free state of a high speed vehicle by machine learning method, and the other 4 groups are used to verify the mapping prediction relationship. The vibration prediction and simulation results under 4 sets of input loads (shown in...
figure 4) are displayed in figure 5, where the lines are for the simulation results and the dash lines for the residual curve between the predictive results and the simulation results.

![Graphs](image1.png)

(a) Vibration contrast under excitation 1  (b) Vibration contrast under excitation 2  

![Graphs](image2.png)

(c) Vibration contrast under excitation 3  (d) Vibration contrast under excitation 4

Figure 5. Four groups of vibration prediction results under 4 different excitation conditions.

It can be demonstrated by figure 5 (a)-(d) that vibration responses are different under excitations but the residual between the predictive results and the simulation results are fluctuating in a small range in all cases. The root mean square error (RMSE) between the predicted values and the simulation values is shown in table 1 which shows that the maximum RMSE is 1.23 mm/s which indicate that the residual between the predictive results and the simulation results are very small. The analysis results show that the predicted value of vibration velocity response of the focus point at the top of the vehicle is basically the same with the response value of the flight state simulation analysis. Therefore, the effectiveness of the mapping method of structural vibration response is verified.

Table 1. RMSE between the predicted and the simulation values.

| Excitation | 1    | 2    | 3    | 4    |
|------------|------|------|------|------|
| RMSE (mm/s)| 1.17 | 1.23 | 0.98 | 0.68 |
5. Conclusions
The vibration response prediction of high speed space vehicle is an essential technology that affects the structure design and the electronic equipment selection. The mapping model in the frequency domain is established when high speed space vehicle structure is under the same excitation but different boundary conditions. The LSSVM based method of calculating and analyzing the response relationship of structures under different constrains boundaries can realize the prediction of the vibration environment of high speed space vehicle in free flight state, and is verified by simulation studied. The proposed method can improve the effectiveness of high speed space vehicle ground vibration test and vibration predict accuracy under flight condition, and therefore, can provide further reference for the dynamic environment design of high speed space vehicle.

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References
[1] Xu M and Zhang N C 2012 Based on aerodynamic/aeroacoustic/structure coupling simulation study Struct. Env. Eng. 39(1) 12-17
[2] Zhu G S 2012 Planning and control of large scale ground test Qual. Reliab. 6 24-28
[3] Qiu J B, Zhang Z P, Li H B, Zhang Z, Han L and Ren F 2015 The consistency of dynamic responses of a full scale spacecraft between ground tests and its space missions Struct. Env. Eng. 42(1) 1-11
[4] Zhou Z H and Cao C G 2004 Neural network and its application (Beijing :Tsinghua University Press)
[5] Vapnik V N 2000 The Nature of Statistical Learning Theory, 2nd ed (New York: Springer-Verlag)
[6] Breiman L 2001 Random forests Mach. Learn. 45(1) 5-32
[7] Suykens J A K and Vandewalle J 1999 Least squares support vector machine classifiers Neural Process. Lett. 9 293–300
[8] Zhang W, Hu C H, Jiao L C and Shang R H 2007 Forgetting-factor least square support vector machine and application on drift forecasting of gyro J. Astronaut. 28(2) 448-451
[9] Yu X K, Li Y H, Wang J Y, Zhang P and Lu J M 2004 A novel aero-engine identification model based on support vector machines J. Aerosp. Power 19(5) 684-688
[10] Zhong Y R, Chen C X, Ren X Y and Wang X J 2016 Fault diagnosis based on combination of BAPSO and LSSVM Comput. Eng. Des. 37(11) 3075-3079
[11] Chi E N and Li C X 2016 Forecast of fluctuating wind velocity using LSSVM with optimized combination kernel and Morlet wavelet kernel J.Vib. Shock 35(18) 52-57
[12] Long Y H, Fang X H, Liu S Z and Li Z K 1991 Overall Design-Liquid Ballistic Missile and Launch Vehicle Series (Beijing: Journal of Astronautics)
[13] Suykens J A K and Vandewalle J 2000 Sparse least squares support vector machine classifiers Eur. Symp. Artif. Neur. Networks (Bruges Belgium)
[14] Yan G R, Dong L L and Yu L 2013 Experiment and simulations of stiffened panel buckling with composite damage and interfacial delamination Chin. J. Appl. Mech. 30(1) 13-18
[15] Li Q Y and Shi J H 2012 Recent developments of hypersonic technology Struct. Env. Eng. 39(5) 55-64