SwiftAgg: Communication-Efficient and Dropout-Resistant Secure Aggregation for Federated Learning with Worst-Case Security Guarantees

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Abstract

We propose SwiftAgg, a novel secure aggregation protocol for federated learning systems, where a central server aggregates local models of $N$ distributed users, each of size $L$, trained on their local data, in a privacy-preserving manner. Compared with state-of-the-art secure aggregation protocols, SwiftAgg significantly reduces the communication overheads without any compromise on security. Specifically, in presence of at most $D$ dropout users, SwiftAgg achieves a users-to-server communication load of $(T+1)L$ and a users-to-users communication load of up to $(N-1)(T+D+1)L$, with a worst-case information-theoretic security guarantee, against any subset of up to $T$ semi-honest users who may also collude with the curious server. The key idea of SwiftAgg is to partition the users into groups of size $D+T+1$, then in the first phase, secret sharing and aggregation of the individual models are performed within each group, and then in the second phase, model aggregation is performed on $D+T+1$ sequences of users across the groups. If a user in a sequence drops out in the second phase, the rest of the sequence remain silent. This design allows only a subset of users to communicate with each other, and only the users in a single group to directly communicate with the server, eliminating the requirements of 1) all-to-all communication network across users; and 2) all users communicating with the server, for other secure aggregation protocols. This helps to substantially slash the communication costs of the system.

Index Terms

Federated learning, Communication-efficient secure aggregation, Secret sharing, Dropout resiliency.

I. INTRODUCTION

Federated learning (FL) is an emerging distributed learning framework that allows a group of distributed users (e.g., mobile devices) to collaboratively train a global model with their local private data, without sharing the data \cite{1, 2, 3}. Specifically, in a FL system with a central server and $N$ users, during each training iteration, the server sends the current state of the global model to the users. Receiving the global model, each user then trains a local model with its local data, and sends the local model to the server. By aggregating the local models, the server can update the global model for the next iteration. While the local datasets are not directly shared with the server, several studies have shown that a curious server can launch model inversion attacks to reveal information about the training data of individual users from their local models (see, e.g., \cite{4, 5}). Therefore, the key challenge to protect users’ data privacy is to design secure aggregation protocols, which allow the aggregation of local models to be computed without revealing each individual model. Moreover, as some users may randomly drop out of the aggregation process (due to low batteries or unstable connections), the server should be able to robustly recover the aggregated local models of the surviving users, in a privacy-preserving manner.

As such motivated, a secure aggregation protocol SecAgg was proposed in \cite{6}, where each user’s local model is hidden under masks computed from pair-wise random seeds. These masks have an additive structure and can be canceled out when aggregated at the server, hence the exact model aggregation can be recovered without compromising each user’s data privacy. To deal with user dropouts, each user secret shares its private seed with other users, such that the pair-wise masks between dropped and surviving users can be reconstructed at the server, and removed from the final aggregation result. One of the major challenges for SecAgg to scale out is the communication cost. First, the secret sharing among users requires all-to-all communication, which incurs quadratic cost in the number of users $N$ and is often not even feasible in practical scenarios; second, every user has to communicate its masked model to the server, yielding significant communication latency at the server as $N$ increases (especially for deep models with hundreds of millions of parameters).

There has been a series of works that aim to improve the communication efficiency of SecAgg (see, e.g., \cite{7, 8, 9}). In \cite{7}, TurboAgg was proposed to perform secure aggregation following a circular topology, achieving a communication cost of $O(LN \log N)$ at the server and $O(L \log N)$ at each user, for a model size $L$. SecAgg$^+$ was proposed in \cite{6} to consider a $k$-regular communication graph among users instead of the complete graph, where $k = O(\log N)$.

It is shown that SecAgg$^+$
TABLE I: Communication loads of secure aggregation frameworks in federated learning.

| Approach         | Server comm. | Per user comm. |
|------------------|--------------|----------------|
| SecAgg[12]       | $O(NL + N^2)$ | $O(L + N)$     |
| SecAgg+ [8]      | $O(NL + N \log N)$ | $O(L + \log N)$ |
| TurboAgg         | $O(NL \log N)$ | $O(L \log N)$ |
| Choi et al. [9]  | $O(N \log N + L)$ | $O(N \log N + L)$ |
| LightSecAgg[11]  | $O(NL)$      | $O(L)$         |
| Proposed SwiftAgg | $(T + 1)L$    | $(T + D + 1)L$ |

requires a communication of $O(LN + N \log N)$ at the server and $O(L + \log N)$ at each user. In [9], another similar idea was proposed in which a sparse random graph is used as communication network instead of the complete graph. While these approaches improve the communication efficiency of SecAgg, they only provide probabilistic privacy guarantees as opposed to the worst-case guarantee of SecAgg. Besides communication, other secure aggregation protocols have been proposed to reduce the computation complexity of SecAgg [10], [11].

In this paper, we propose a new scheme for secure aggregation in federated learning called SwiftAgg, which reduces the communication loads and is robust against user dropouts. In SwiftAgg, we first partition the users into groups of size $T + D + 1$ (See Fig. 2). Then in the first phase, users within each group secret share their local models and aggregate the shares locally. In the second phase, $T + D + 1$ sequences of users are arranged, such that in each sequence, there is one user from each group. Then, in-group aggregated shares are sequentially aggregated in each sequence, which is finally sent to the server. In the second phase, if one user in a sequence drops out, the rest of the sequence remain silent.

SwiftAgg simultaneously achieves the following advantages compared to the existing works:

1) It does not require all-to-all communication among users, which is often not feasible in many scenarios.
2) It requires very low communication cost per user and at the server (See Table II for comparison).
3) It is resilient to user dropouts.
4) It achieves worst-case information-theoretic security against a curious server and any subset of $T < N - D$ colluding users.

In addition, Table II shows the comparison between different frameworks in secure aggregation problem in terms of the communication loads. For a fair comparison, we consider two metrics: server communication and per user communication. Server communication indicates the total size of all messages which are sent or received by the server, and per user communication denotes the total size of all messages which are sent by each user.

The rest of the paper is organized as follows. In Section II, we formally formulate the problem in the proposed scheme. In Section III, we state the main result. In Section IV, we present the proposed scheme using a motivating example and the general form. Finally, we present the detailed proofs for the correctness of the proposed scheme and the privacy.

Notation: Matrices are denoted by upper boldface letters. For $n \in \mathbb{N}$ the notation $[n]$ represents set $\{1, \ldots, n\}$. Furthermore, the cardinality of set $S$ is denoted by $|S|$. In addition, we denote the difference of two sets $A, B$ as $A \setminus B$, that means the set of elements which belong to $A$ but not $B$. $H(X)$ denotes the entropy of random variable $X$ and $I(X; Y)$ is the mutual information of two random variables $X$ and $Y$.

II. PROBLEM FORMULATION

We consider the secure aggregation problem, for a federated learning system consisting of a server and $N$ users $U_1, \ldots, U_N$. For each $n \in [N]$, user $n$ has a private local model of length $L$, denoted by $W_n \in \mathbb{F}^L$, for some finite field $\mathbb{F}$. Each user $n$ also has a collection of random variables $Z_n$, whose element is selected uniformly at random from $\mathbb{F}^L$, and independently from each other and from the local models. Users can send messages to each other and also to the server, using error-free private communication links. $M_{n \rightarrow m}^{(L)} \in \mathbb{F}^* \cup \{\bot\}$ denotes the message that node $n \rightarrow m$. In addition, $X_{n}^{(L)} \in \mathbb{F}^* \cup \{\bot\}$ denotes the message sent by node $n$ to the server. The symbol $\bot$ represents the case no message is sent.

The message $M_{n \rightarrow m}^{(L)}$ is a function of $W_n$, $Z_n$ and the messages that node $n$ has received from other nodes so far. We denote the corresponding encoding function by $f_{n \rightarrow m}^{(L)}$. Similarly, $X_{n}^{(L)}$ is a function of $W_n$, $Z_n$, and the messages that node $n$ has received from other nodes so far. We denote the corresponding encoding function by $g_{n}^{(L)}$. For a subset $S \subseteq [N]$, we let $X_S = \{X_{n}^{(L)}\}_{n \in S}$ represent the set of messages the server receives from users in $S$. We assume that a subset $D \subset [N]$ of users drop out, i.e., stay silent (or send $\bot$ to other nodes and the server) during the protocol execution. We denote the number of dropped out users as $D = |D|$.

We also assume that a subset $T \subset [N]$ of the users, whose identities are not known before the execution of the aggregation protocol, are semi-honest. It means that users in $T$ follow the protocol faithfully; however, they are curious and may collude with each other to gain information about the local models of the honest users. We assume $|T| \leq T$, for some security parameter $2 \leq T < N - D$.

A secure aggregation scheme consists of the encoding functions $f_{n \rightarrow n'}^{(L)}$ and $g_{n}^{(L)}$, $n, n' \in [N]$, such that the following conditions are satisfied:
1. Correctness: The server is able to recover $W = \sum_{n \in [N] \setminus D} W_n$, using $X_{[N] \setminus D} = \{X_n^{(L)}\}_{n \in [N] \setminus D}$. More precisely, 
\[
H\left(\sum_{n \in [N] \setminus D} W_n \mid X_{[N] \setminus D}\right) = 0. \tag{1}
\]

2. Privacy Constraint: Receiving $X_{[N] \setminus D}$, the server should not gain any information about local models of the honest users, beyond the aggregation of the local models, even if it colludes with semi-honest users in $T$. Formally, 
\[
I\left(W_n, n \in [N] \setminus T : X_{[N] \setminus D}, \bigcup_{k \in T} \{M_{k \rightarrow k', k' \in [N]}^{(L)}\}, \{W_k, Z_k, k \in T\} \mid \sum_{n \in [N] \setminus \{D \cup T\}} W_n\right) = 0.
\]

It is also possible for SwiftAgg to guarantee privacy when there are $T$ semi-honest users who collude with each other, and the server is curious, but does not collude with the semi-honest users. The privacy constraint here is that the server should not gain any information beyond the aggregation, and the semi-honest users should also not gain any information about the local models.

For a secure aggregation scheme satisfying the above two conditions, we define user-to-user communication load and uplink communication load as follows:

**Definition 1** (Normalized user-to-user communication load). denoted by $R_{\text{user}}^{(L)}$, is defined as the the aggregated size of all messages communicated between users, normalized by $L$, i.e., 
\[
R_{\text{user}}^{(L)} = \frac{1}{L} \sum_{n,n' \in [N]} H(M_{n \rightarrow n'}^{(L)}).
\]

**Definition 2** (Normalized uplink communication load). denoted by $R_{\text{uplink}}^{(L)}$, is defined as the the aggregated size of all messages sent from users to the server, normalized by $L$, i.e., 
\[
R_{\text{uplink}}^{(L)} = \frac{1}{L} \sum_{n \in [N]} H(X_n^{(L)}).
\]

We say that the pair of $(R_{\text{uplink}}, R_{\text{user}})$ is achievable, if there exist a sequence of secure aggregation schemes with rate tuples $(R_{\text{uplink}}^{(L)}, R_{\text{user}}^{(L)})$, $L = 1, 2, \ldots$, such that 
\[
R_{\text{uplink}} = \lim_{L \rightarrow \infty} \sup \frac{R_{\text{uplink}}^{(L)}}{L}, \quad R_{\text{user}} = \lim_{L \rightarrow \infty} \sup \frac{R_{\text{user}}^{(L)}}{L}. \tag{2}
\]

The capacity region of a secure aggregation problem, denoted by $\mathcal{C}_{N,D,T}$, is defined as the convex closure of all achievable rate tuples $(R_{\text{uplink}}, R_{\text{user}})$.

**III. MAIN RESULT**

We first present the main result of the proposed secure model aggregation scheme in the following theorem.

**Theorem 1.** Consider a secure aggregation problem, with $N$ users and one server, where up to $T$ users are semi-honest and up to $D$ users may drop out. Let 
\[
\mathcal{R} = \{(R_{\text{uplink}}, R_{\text{user}}) \mid R_{\text{uplink}} \geq (T + 1), R_{\text{user}} \geq (N - 1)(T + D + 1)\}
\]

Then $\mathcal{R} \subset \mathcal{C}_{N,D,T}$.

**Proof.** The proof can be found in Section [IV-D] \[\square\]

To achieve the communication loads in (4), we propose SwiftAgg, a novel secure aggregation scheme, which partitions the users into disjoint groups and operates in two main phases: (i) Intra-group secret sharing and aggregation; and (ii) Inter-group communication and aggregation. Finally, communication with the server is required so that the server can obtain the aggregation of local models.

Compared to the existing schemes in secure aggregation, SwiftAgg simultaneously reduces both the uplink and the user-to-user communication cost (Table I), while the correctness and the privacy constraint are satisfied via information-theoretic approaches.

**IV. THE PROPOSED SCHEME**

In this section, we propose SwiftAgg which reduces the communication load of the secure aggregation problem in federated learning. We first introduce the main idea of this method using a simple motivation example.
A. Motivation Example

Consider a secure aggregated problem consisting of one server and \( N = 12 \) users, \( U_1, U_2, \ldots, U_{12} \). There is \( D = 1 \) user dropout and up to \( T = 2 \) semi-honest users that may collude with each other to gain some information about the local models of other users. User \( n \) contains its local model \( W_n \) which is a vector with a length of \( L \), \( n \in [12] \). Also, each user has two random vectors, \( Z_n = \{Z_{n,1}, Z_{n,2}\} \) which are chosen uniformly at random from \( \mathbb{F}^L \). Each user takes the following steps:

1) **Grouping**: The set of users are arbitrarily partitioned into \( \Gamma = 3 \) groups with a size of \( D + T + 1 = 4 \), denoted by \( G_1, G_2, G_3 \). Figure \[1\] represents one example of this partitioning, where \( G_1 = \{U_1, U_2, U_3, U_4\} \), \( G_2 = \{U_5, U_6, U_7, U_8\} \), and \( G_3 = \{U_9, U_{10}, U_{11}, U_{12}\} \). We also order the users in each group arbitrarily. For simplicity of exposition, we may refer to user \( n \) based on its location in a group of users. If user \( n \) is the \( t \)th user in group \( \gamma \), we call it as user \( (\gamma, t) \).

For example in Fig \[1\], user 8 is the same as user \( (2, 4) \). We use indices \( n \) or \( (\gamma, t) \) interchangeably.

2) **Intra-Group Secret Sharing and Aggregation**: User \( n \) forms the following polynomial.

\[
F_n(x) = W_n + Z_{n,1}x + Z_{n,2}x^2, \tag{4}
\]

where \( F_n(0) = W_n \) is the local model of user \( n, n \in [12] \).

Let \( \alpha_t \in \mathbb{F}, t \in [4] \), are four distinct constants. We assign \( \alpha_t \) to user \( t \) of all groups, i.e., users \( (\gamma, t), \gamma = 1, \ldots, \Gamma \).

In this step, each user \( (\gamma, t) \) sends the evaluation of its polynomial function at \( \alpha_t \), i.e., \( F(\gamma, t)(\alpha_t) \), to user \( (\gamma, t') \), for \( t' \in [4] \). For example, in Fig \[1\], user \( (2, 1) \), which is indeed \( U_5 \), sends \( F(2, 1)(\alpha_1) = F_5(\alpha_1) \), \( F(2, 1)(\alpha_2) = F_5(\alpha_2) \), \( F(2, 1)(\alpha_3) = F_5(\alpha_3) \), \( F(2, 1)(\alpha_4) = F_5(\alpha_4) \), to user \( (2, 1) \) (or user \( U_5 \) which is basically itself), user \( (2, 2) \) (or user \( U_6 \)), user \( (2, 3) \) (user \( U_7 \)), and user \( (2, 4) \) (user \( U_8 \)) respectively. If a user \( (\gamma, t) \) drops out and stays silent, \( F(\gamma, t)(\alpha_t') \) is just presumed to be zero.

Each user \( (\gamma, t) \) calculates

\[
Q_{(\gamma, t)} = F_{(\gamma, 1)}(\alpha_t) + F_{(\gamma, 2)}(\alpha_t) + F_{(\gamma, 3)}(\alpha_t) + F_{(\gamma, 4)}(\alpha_t).
\]

In this example, assume that \( U_7 \) or user \( (2, 3) \) drops out and does not send its share to other users in the second group. Other users within the group treat its share as zero. In this phase, within each group, at most 6 communication take place.

3) **Inter-Group Communication and Aggregation**: In this phase, user \( (1, t) \), \( t \in [4] \), calculates the following message.

\[
S_{(1,t)} = Q_{(1,t)} \tag{5}
\]

and sends \( S_{(1,t)} \) to user \( (2, t) \).

User \( (2, t) \), \( t \in [4] \), calculates \( S_{(2,t)} \) as

\[
S_{(2,t)} = S_{(1,t)} + Q_{(2,t)}, \tag{6}
\]

upon receiving \( S_{(1,t)} \) and sends it to user \( (3, t) \). If user \( (2, t) \) does not receive \( S_{(1,t)} \), it also remains silent for the rest of the protocol. In this particular example that user 7 drops out, it sends no message to user 11, and thus user 11 also remains silent.

4) **Communication with the Server**: User \( t \) of the last group, i.e., user \( (3, t) \) calculates

\[
S_{(3,t)} = S_{(2,t)} + Q_{(3,t)}, \tag{7}
\]

and sends \( S_{(3,t)} \) to the server, \( t \in [4] \). Clearly in this example, user 11 remains silent and sends nothing (or null message \( \perp \)) to the server.

5) **Recovering the result**: Let us define

\[
F(x) \triangleq \sum_{n=1}^{12} F_n(x) = \sum_{n=1}^{12} W_n + x \sum_{n=1}^{12} Z_{n,1} + x^2 \sum_{n=1}^{12} Z_{n,2}.
\]

One can verify that \( S_{(3,t)} \), for \( t = 1, 2, 4 \) that are received by the server are indeed equal to \( F(\alpha_1) \), \( F(\alpha_2) \), \( F(\alpha_4) \).

Since \( F(x) \) is a polynomial function of degree 2, based on Lagrange interpolation rule the server can recover all the coefficients of this polynomial. In particular, the server can recover \( F(0) = \sum_{n=1}^{12} W_n \). Thus, the server is able to recover the aggregation of local models of surviving users and the correctness constraint is satisfied.

The privacy constraint will be proven formally later in Subsection \[IV-E\] for the general case.

B. General case

In this subsection, we formally describe SwiftAgg. Consider a network consisting of one server and \( N \) users, \( U_1, U_2, \ldots, U_N \), where up to \( T \) of them are semi-honest which may collude with each other to gain some information about other users. Furthermore, \( D \) users may drop out, and their indices are denoted by \( D \). The \( n \)th user contains its local model \( W_n \in \mathbb{F}^L \) and
a set of random variables \( Z_n = \{Z_{n,j}, j \in [T]\} \) which are chosen independently and uniformly at random from \( \mathbb{F}^L \). In this setting, the server here wants to recover the aggregated local models of the surviving users, i.e., \( W = \sum_{n \in [N] \setminus T} W_n \), while the individual models remain private. To reach this goal, \textit{SwiftAgg} takes the following steps.

1) **Grouping:** The set of \( N \) users are arbitrarily partitioned into \( \Gamma \) groups with a size of \( \nu \triangleq D + T + 1 \), denoted by \( \mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_\Gamma \), and it is divisible by \( \nu \). Figure 2 represents an overview of the proposed grouping method. We also order the users in each group arbitrarily. For simplicity, we refer to user \( n \) based on its location in a group of users. If user \( n \) is the \( t \)th user in group \( \gamma \), we call it as user \((\gamma, t)\).

2) **Intra-Group Secret Sharing and Aggregation:** User \( n \in [N] \) forms the following polynomial.

\[
F_n(x) = W_n + \sum_{j=1}^{T} Z_{n,j} x^j.
\]  

(8)

This polynomial function is designed such that \( F_n(0) = W_n \). Each user uses its polynomial function \( F_n(.) \) to share its local model with other users. Let \( \alpha_t \in \mathbb{F}, t \in [\nu] \), are \( \nu \) distinct constants. We assign \( \alpha_t \) to user \( t \) of all groups, i.e., users \((\gamma, t)\), \( \gamma = 1, \ldots, \Gamma \).

In this step, each user \((\gamma, t)\) sends the evaluation of its polynomial function at \( \alpha_{t'} \), i.e., \( F_{(\gamma, t)}(\alpha_{t'}) \), to user \((\gamma, t')\), for \( t' \in [\nu] \). If a user \((\gamma, t)\) drops out and stays silent, \( F_{(\gamma, t)}(\alpha_{t'}) \) is just presumed to be zero.

Each user \((\gamma, t)\) calculates

\[
Q_{(\gamma, t)} = \sum_{t' \in [\nu]} F_{(\gamma, t')}(\alpha_t).
\]  

(9)

Note that in this phase, within each group, at most \( \binom{\nu}{2} \) communication take place.

3) **Inter-group Communication and Aggregation:** In this phase, user \( t \) of group \( \gamma \) calculates a message denoted by \( S_{(\gamma, t)} \) and sends it to user \( t \) of group \( \gamma + 1 \), for \( \gamma = 1, \ldots, \Gamma - 1 \).

User \((1, t), t \in [\nu]\), in group one sets

\[
S_{(1, t)} = Q_{(1, t)}
\]  

(10)

and sends \( S_{(1, t)} \) to user \((2, t)\).

User \((\gamma, t)\) in group \(\gamma, 2 \leq \gamma \leq \Gamma - 1\), calculates \( S_{(\gamma, t)} \) as

\[
S_{(\gamma, t)} = S_{(\gamma-1, t)} + Q_{(\gamma, t)},
\]  

(11)

upon receiving \( S_{(\gamma-1, t)} \). If user \((\gamma, t)\) does not receive \( S_{(\gamma-1, t)} \), it also remains silent for the rest of the protocol.

4) **Communication with the Server:** User \( t \) of the last group, i.e., user \((\Gamma, t)\) computes

\[
S_{(\Gamma, t)} = S_{(\Gamma-1, t)} + Q_{(\Gamma, t)},
\]  

(12)

and sends it to the server, for \( t \in [\nu] \).

5) **Recovering the result:** Having received the outcomes of a subset of users in \( \mathcal{G}_T \) with a size of at least \( T + 1 \), the server can recover the aggregated local models.
C. Proof of correctness

To prove the correctness we must show that the server can recover \( \sum_{n \in [N] \setminus \mathcal{D}} W_n \) from the messages received from group \( \Gamma \). Using the recursive equations (10) to (12), \( S_{(\gamma, t)}(\alpha_t) \) is either a null message, or it is equal to

\[
S_{(\gamma, t)}(\alpha_t) = \sum_{\gamma' = 1}^{\gamma} \sum_{n \in \mathcal{D}} W_n + \sum_{j=1}^{T} \alpha_t^j \sum_{\gamma' = 1}^{\gamma} \sum_{n \in \mathcal{D}} Z_{n,j}.
\]

(13)

Thus if user \( t \) in group \( \Gamma \) sends a message to the server, it is equal to \( S_{(\gamma, t)}(\alpha_t) \). From (13), it is easy to see that \( S_{(\gamma, t)}(\alpha_t) = F(\alpha_t) \), where

\[
F(x) = \sum_{n \in [N] \setminus \mathcal{D}} W_n + \sum_{j=1}^{T} x^j \sum_{n \in [N] \setminus \mathcal{D}} Z_{n,j}.
\]

(14)

\( F(x) \) is a polynomial of degree \( T \), with \( F(0) = \sum_{n \in [N] \setminus \mathcal{D}} W_n \). Thus if the server receives at least \( T + 1 \) messages from the last group, it can use Lagrange interpolation to recover \( F(x) \) and \( \sum_{n \in [N] \setminus \mathcal{D}} W_n \).

Recall that, in \( \text{SwiftAgg} \), for any user \( (\gamma, t) \) in \( \mathcal{D} \), all the messages \( S_{(\gamma', t)}(\alpha_t) \), \( \gamma \leq \gamma' \leq \Gamma \) is null. In particular, for any user \( (\gamma, t) \) in \( \mathcal{D} \), \( S_{(\Gamma, t)}(\alpha_t) \) is null. Thus at most \( D \) users in the last group send null messages to the server. Since the size of each group is \( D + T + 1 \), the server receives at least \( T + 1 \) values \( S_{(\Gamma, t)}(\alpha_t) \) for distinct \( \alpha_t \), and thus can recover \( F(x) \).

D. The communication loads

According to (14), the total number of messages that are needed to be received by the server is \( (T+1) \). Thus, the normalized uplink communication load in \( \text{SwiftAgg} \) is \( R_{\text{uplink}}^{(L)} = (T+1) \). In each group, at most \( \nu (\nu - 1) \) messages are sent by the members, and there are \( \frac{N}{\nu} \) groups. In addition, at most \( \nu \) messages are sent between two consecutive groups. Thus, the normalized user-to-user communication load of \( \text{SwiftAgg} \) is upper-bounded by \( R_{\text{user}}^{(L)} \leq (N - 1)\nu \), where \( \nu = T + D + 1 \).

E. Proof of privacy

In this section, we prove the privacy constraint in (2). As mentioned before, the privacy must be guaranteed even if the server colludes with any set \( T \subset [N] \) of at most \( T \) semi-honest users. Let us define \( W_{N \setminus T} \triangleq \{ W_n, n \in [N] \setminus T \} \), and \( Z^{(t)}_n \triangleq \sum_{j=1}^{T} Z_{n,j} \alpha_t^j \) for \( n \in [N] \) and \( t \in [\nu] \).

**Lemma 2.** Consider \( \mathbf{R}_1, \ldots, \mathbf{R}_T \) are chosen independently and uniformly at random from \( \mathbb{F}^L \). For an arbitrary vector \( \mathbf{Y} \in \mathbb{F}^L \), consider \( \mathbf{H}(x) = \mathbf{Y} + \sum_{j=1}^{T} \mathbf{R}_j x^j \). Let \( \mathbf{H} \triangleq [\mathbf{H}(\beta_1), \mathbf{H}(\beta_2), \ldots, \mathbf{H}(\beta_T)] \), where \( \beta_i \) for \( i \in [T] \) are some arbitrary and distinct points from the field. Then, \( I(\mathbf{Y}; \mathbf{H}) = 0 \) [13].

**Corollary 3.** Assume that user \( U_n \) is denoted by \( (\gamma, t) \). In \( \text{SwiftAgg} \), the local model of \( U_n \) is shared using polynomial function \( \mathbf{F}_n(x) \) in (5). In other words, \( \mathbf{F}_{(\gamma, t)}(\alpha_t) \) for \( t' \in [\nu] \setminus \{t\} \) are delivered to user \( (\gamma, t') \). According to (8) and directly from Lemma 2 we have \( I(\mathbf{W}_n; \{\mathbf{W}_n + Z^{(t')}_n, t' \in \{t\}\}) = 0 \).
Lemma 4. Assume that the random noise in inter-group message $S_{(\gamma,t)}$, (10), is denoted by $\tilde{Z}_{(\gamma,t)}$, i.e.,

$$\tilde{Z}_{(\gamma,t)} \triangleq \sum_{j=1}^{T} \alpha_{\ell,j} \sum_{\gamma' = 1}^{\gamma} \sum_{n \in \mathcal{G}_{\gamma',\ell}} Z_{n,j}.$$ 

Then, we have $I(\tilde{Z}_{(\gamma,t)}; \tilde{Z}_{(\gamma+1,t)}) = 0$, for $t \in \nu$ and $\gamma \in \Gamma - 1$.

Proof. In each group, there are $\nu$ users each of which uses $T$ random vectors chosen uniformly and independently from $\mathbb{F}_L^{\nu}$ in its shares. Thus we have, $\tilde{Z}_{(\gamma+1,t)} = Z_{(\gamma,t)} + \sum_{i \in \mathcal{G}_{\gamma+1 \downarrow \ell}} Z_{i}^{(\theta)}$. Since the random vectors are i.i.d., we can conclude that $I(\tilde{Z}_{(\gamma,t)}; \tilde{Z}_{(\gamma+1,t)}) = 0$. \hfill \qed

According to Lemma 4, we can conclude that $I(\tilde{Z}_{(\gamma-\ell,t)}; \tilde{Z}_{(\gamma,t)}) = 0$ for $\ell \in [i-1]$, $\gamma \in \Gamma$.

To prove the privacy constraint we have to show (2) for any set $T$ of semi-honest users. Let us define

$$M_T \triangleq \bigcup_{n \in T} \{M_n^{(L)}, n' \in [N]\}.$$ 

Also, we define $R_n$ as a set of random noises which are used in the messages sent to $U_n$, where $n \in [N]$. In other words, assume $U_n$ is called user $(\gamma,t)$, we define

$$R_n \triangleq \{Z_{n}^{(t'}), n \in \mathcal{G}_{\gamma}, \tilde{Z}_{(\gamma-1,t)}\}. $$

Thus, $R_T \triangleq \bigcup_{n \in T} R_n$, denotes the set of random noises which are used in the messages received by semi-honest users. In addition, $S_T \triangleq \{S_{(I,T)}; t \in [\nu] \downarrow \nu\}$ represents the set of messages that the server receives from users in Group $\Gamma$. From the definition of privacy constraint, we have

$$I(W_N \setminus T; M_T, S_T; \{W_k, Z_k, k \in T\}) = \sum_{n \in [N] \setminus \{\nu \downarrow \nu\}} W_n$$

$$= I(W_N \setminus T; \{W_k, Z_k, k \in T\}) + I(W_N \setminus T; M_T, S_T; \sum_{n \in [N] \setminus \{\nu \downarrow \nu\}} W_n, \{W_k, Z_k, k \in T\})$$

$$= (a) I(W_N \setminus T; M_T, S_T; \sum_{n \in [N] \setminus \{\nu \downarrow \nu\}} W_n, \{W_k, Z_k, k \in T\}), \quad (15)$$

where in (a) we use the independence of the local models and independence of random vectors from the local models.

Without loss of generality assume that the semi-honest users are denoted by $\tilde{U}_1, \tilde{U}_2, \ldots, \tilde{U}_T$. We call these semi-honest users as users $(\gamma_1, t_1), (\gamma_2, t_2), \ldots, (\gamma_T, t_T)$ respectively, where $1 \leq \gamma_1 \leq \gamma_2 \leq \cdots \leq \gamma_T \leq \Gamma$. In addition, we define $K_{N,T} \triangleq \{\{W_k, Z_k, k \in T\}; \sum_{n \in [N] \setminus \{\nu \downarrow \nu\}} W_n\}$ and $I_{\Gamma} \triangleq \{n : n \in \mathcal{G}_{\gamma, T} \}$ for $\gamma \in \Gamma$.

The set of messages which are received by $\tilde{U}_i$, i.e., $M_{\tilde{U}_i}$, consists of two kinds of messages. In other words, $M_{\tilde{U}_i} = \{F_{(n,t_i)}, n \in I_{\gamma_i}\}, S_{(\gamma_i-1,t_i)}\}$, where

$$F_{(n,t_i)}, n \in I_{\gamma_i} \} = \{W_n + Z_{n}^{(t_i)}, n \in \gamma_i\}, \quad (16)$$

is a set of intra-group messages and

$$S_{(\gamma_i-1,t_i)} = \sum_{m \in \bigcup_{j=1}^{i-1} X_j} W_m + \tilde{Z}_{(\gamma_i-1,t_i)}, \quad (17)$$

is the message received from the previous group. According to the definitions and the fact that the random vectors are chosen uniformly and independently from $\mathbb{F}_L$, we can verify the following facts.

**Fact 1.** $S_{(\gamma_i-1,t_i)}$ is independent from $\{F_{(n,t_i)}, n \in I_{\gamma_i}\}$. Furthermore, $\tilde{Z}_{(\gamma_i-1,t_i)}$ is independent from $\{Z_{n}^{(t_i)}, n \in I_{\gamma_i}\}$, for $t_i \in \nu$ and $\gamma_i \in [2 : \Gamma]$.

**Fact 2.** From (17), we know that $S_{(\gamma_i,t_i)} = S_{(\gamma_i-1,t_i)} + \sum_{n \in I_{\gamma_i}} F_{(n,t_i)}$. Thus, according to Lemma 4 and using the fact that there is at least one honest and non-dropped user in each group, we can conclude that $S_{(\gamma_i,t_i)}$ is independent form $\{S_{(\gamma_i-1,t_i)}, t \in \nu, \gamma_i \in [2 : \Gamma]\}$.

**Fact 3.** Since the local models and random vectors of different users are independent, the independence of $\{F_{(n,t_i)}, n \in I_{\gamma_i}\}$ from $\{F_{(n,t_i)}, n' \in I_{\gamma_i}\}$ for $\gamma_i \neq \gamma_i'$, $t_i, t_i'$ can be concluded. A similar argument also leads to the independence of $\{Z_{n}^{(t_i)}, n \in I_{\gamma_i}\}$ from $\{Z_{n'}^{(t_i)}, n' \in I_{\gamma_i}\}$ for $\gamma_i \neq \gamma_i'$, $t_i, t_i'$ can be concluded.

**Fact 4.** Based on Shamir’s secret sharing and Corollary 3 for any $n \in I_{\gamma_i}, \{F_{(n,t_i)}, t_i \in T, T \subset \nu, |T| \leq T\}$ are independent. In addition, for any $\gamma_i \in [\Gamma], \{S_{(\gamma_i,t_i)}, t_i \in T, T \subset \nu, |T| \leq T\}$ are independent. Similarly, we can conclude that for any $n \in I_{\gamma_i}, \{Z_{n}^{(t_i)}, t_i \in T, T \subset \nu, |T| \leq T-1\}$ are independent, and for any $\gamma_i \in [\Gamma], \{\tilde{Z}_{(\gamma_i,t_i)}, t_i \in T, T \subset \nu, |T| \leq T-1\}$ are independent.
**Fact 5.** For $\gamma_i \in [\Gamma]$, consider $\mathcal{Z}_{1,\Gamma} = \{\mathcal{Z}_{n,\Gamma}^{i}, n \in \mathcal{I}_i, t \in T'\}$, where $\mathcal{I}_i$ is a subset of $\{\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_\gamma\}$ with a size of up to $T' - 1$, and $|T'| \leq T - 1$. We can consider different cases: (I) If there is at least one group like $\gamma' \in [\Gamma]$ that $\mathcal{I}_{\gamma'} \notin \mathcal{I}_i$ then we can conclude that $\mathcal{Z}_{(\gamma_i, t)}$ is independent from $\mathcal{Z}_{1,\Gamma}$. The reason is that there is a non-empty set of honest and non-dropped users that $\mathcal{Z}_{(\gamma_i, t)}$ includes a summation of their local models and random vectors, (II) $\mathcal{I}_i = \bigcup_{j=1}^{i} \mathcal{I}_j$ and users in $T'$ have the same order $t$, then $\mathcal{Z}_{(\gamma_i, t)} = \sum_{n \in \mathcal{I}_i} \mathcal{Z}_{n,\Gamma}^{i}$ and users in $T'$ have different orders, then based on Shamir's secret sharing $\mathcal{Z}_{(\gamma_i, t)}$ is independent from $\mathcal{Z}_{1,\Gamma}$.

**Lemma 5.** Let $(\gamma_1, t_1), (\gamma_2, t_2), \ldots, (\gamma_T, t_T)$ be $T$ the semi-honest users in the setting. Then, for $i \in [T]$ we have

$$I(W_{N,T}; M_{\mathcal{G}_i} | K_{N,T}, \{M_{\mathcal{G}_j}, j \in [i-1]\}) = 0.$$  

**Proof.** For $i = 1$, using the independence of the local models and random variables, we have

$$I(W_{N,T}; M_{\mathcal{G}_1} | K_{N,T}, \{M_{\mathcal{G}_j}, j \in [i-1]\}) = I(W_{N,T}; \{F_{(n,t_i)} \mid n \in \mathcal{I}_1\}, S_{(\gamma_1-1, t_i)} | K_{N,T})$$

\[= H(F_{(n,t_i)} \mid n \in \mathcal{I}_1), S_{(\gamma_1-1, t_i)} | K_{N,T}) - H(F_{(n,t_i)} \mid n \in \mathcal{I}_1), S_{(\gamma_1-1, t_i)} | W_{N,T}) \]

\[ \leq H(F_{(n,t_i)} \mid n \in \mathcal{I}_1), S_{(\gamma_1-1, t_i)} | K_{N,T}) - H(F_{(n,t_i)} \mid n \in \mathcal{I}_1), S_{(\gamma_1-1, t_i)} | W_{N,T}) \]

The last term follows from the fact that $H(F_{(n,t_i)} \mid n \in \mathcal{I}_1), S_{(\gamma_1-1, t_i)} | W_{N,T})$ and $H(F_{(n,t_i)} \mid n \in \mathcal{I}_1), S_{(\gamma_1-1, t_i)} | W_{N,T})$ have the same size and the uniform variables maximize the entropy. Therefore, $I(W_{N,T}; M_{\mathcal{G}_1} | K_{N,T}) = 0$.

For $i \in [2 : T]$ we have

$$I(W_{N,T}; M_{\mathcal{G}_i} | K_{N,T}, \{M_{\mathcal{G}_j}, j \in [i-1]\})$$

\[= I(W_{N,T}; \{F_{(n,t_i)} \mid n \in \mathcal{I}_1\}, S_{(\gamma_1-1, t_i)} | K_{N,T}) - H(F_{(n,t_i)} \mid n \in \mathcal{I}_1), S_{(\gamma_1-1, t_i)} | W_{N,T}) \]

\[\leq H(F_{(n,t_i)} \mid n \in \mathcal{I}_1), S_{(\gamma_1-1, t_i)} | K_{N,T}) - H(F_{(n,t_i)} \mid n \in \mathcal{I}_1), S_{(\gamma_1-1, t_i)} | W_{N,T}) \]

\[\leq H(F_{(n,t_i)} \mid n \in \mathcal{I}_1), S_{(\gamma_1-1, t_i)} | K_{N,T}) - H(F_{(n,t_i)} \mid n \in \mathcal{I}_1), S_{(\gamma_1-1, t_i)} | W_{N,T}) \]

where in (a) the first and the third terms follow from the fact that $H(\mathcal{X} | \mathcal{Y}) = H(\mathcal{X})$ and (b) follows from the above mentioned facts. In addition, the last term follows from the fact that uniform variables maximize entropy.

Thus, $I(W_{N,T}; M_{\mathcal{G}_i} | K_{N,T}, \{M_{\mathcal{G}_j}, j \in [i-1]\}) = 0.$

According to Lemma 5, (15) can be written as follows.

$$I(W_{N,T}; M_{\mathcal{T}}, S_{\Gamma} | K_{N,T})$$

\[= \sum_{i=1}^{T} I(W_{N,T}; M_{\mathcal{G}_i} | K_{N,T}, \{M_{\mathcal{G}_j}, j \in [i-1]\}) + I(W_{N,T}; S_{\Gamma} | K_{N,T}, M_{\mathcal{T}}) \]

\[= I(W_{N,T}; \{Z_{(\Gamma, t')} \mid t' \in [\nu] \cup (\mathcal{T} \cup \mathcal{D})\} | K_{N,T}, M_{\mathcal{T}}) \]

\[= H(Z_{(\Gamma, t')} | K_{N,T}, M_{\mathcal{T}}) - H(Z_{(t', \mathcal{T})} | K_{N,T}, M_{\mathcal{T}}) = 0. \]

If $T$ semi-honest users are located in group $\Gamma$, each term in (18) is equal to 0, as for any $t' \in [\nu] \cup (\mathcal{T} \cup \mathcal{D})$, $\{Z_{(t', \mathcal{T})} | t' \in [\nu] \cup (\mathcal{T} \cup \mathcal{D})\}$ can be uniquely reconstructed from $(K_{N,T}, M_{\mathcal{T}})$. Otherwise using argument similar to that in the proof of Lemma 5 and the above mentioned facts, both terms in (18) are equal to $H(Z_{(t', \mathcal{T})} | t' \in [\nu] \cup (\mathcal{T} \cup \mathcal{D}))$ and the result is 0. Therefore,

$$I(W_{N,T}; M_{\mathcal{T}}, S_{\Gamma}; W_k, Z_k, k \in \mathcal{T}) \sum_{n \in [N] \cup (\mathcal{T} \cup \mathcal{D})} W_n = 0,$$

which proves the privacy constraint.
In this paper we propose SwiftAgg, which is a secure aggregation protocol for model aggregation in federated learning. Via partitioning users into groups and careful designs of intra and inter group secret sharing and aggregation schemes, SwiftAgg is able to achieve correct aggregation in presence of $D$ dropout users, with the worst-case security guarantee against $T$ users colluding with a curious server. Compared with previous secure aggregation protocols, SwiftAgg does not require an all-to-all communication network among users, and significantly slashes the communication load of the server.

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