Open quantum dynamics induced by light scalar fields

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Abstract

We consider the impact of a weakly coupled environment comprising a light scalar field on the open dynamics of a quantum test mass. The light scalar is assumed to couple to matter either through a non-minimal coupling to gravity or, equivalently, through a Higgs portal. We introduce a novel approach for deriving the quantum master equation describing the evolution of the single-particle matrix element of the density operator from first principles. Our approach draws on the techniques of non-equilibrium quantum field theory, including the Feynman-Vernon influence functional and thermo field dynamics, as well as a method of LSZ-like reduction. In addition, we show that non-Markovian effects, namely the violation of time-translational invariance due to finite-time effects, require us to introduce time-local counterterms in order to renormalize the resulting loop corrections consistently. This complementary and robust approach provides cutoff-independent quantitative predictions and has the potential to shed new light on, for example, the divergences encountered in the context of gravitationally induced decoherence in general relativity. The resulting master equation features corrections to the coherent dynamics, as well as decoherence and momentum diffusion. We comment on the possibilities for experimental detection and the related challenges, and highlight possible pathways for further improvements.

Keywords: light scalar fields, open quantum dynamics, non-equilibrium quantum field theory

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I. INTRODUCTION

The fundamental explanation for the evolution of the Universe on its largest scales remains a mystery, and there are many suggestions that light fields, beyond those present in the Standard Model (SM) of particle physics, could play an important role, not least in connection to the cosmological constant problem, the nature of ‘dark energy’ [1, 2] and the missing ‘dark matter’ mass in the Universe [3, 4]. If such cosmologically light fields exist, they can provide an environment to which ordinary matter is coupled as an open (quantum) system.

In this work, we restrict our attention to the introduction of light scalar fields. Light scalar fields often arise in modifications to general relativity [1, 2, 5] and in attempts to understand the dark energy problem, where they may be introduced directly to explain the observed accelerated expansion [6], as well as the dark matter problem, where such fields can form condensates around...
galaxies and play the role of the ‘missing mass’ in the Universe \[3, 4, 7–15\].

Additional scalar fields will, in general, develop interactions with the SM fields, unless there is a symmetry to prevent such couplings (or we invoke fine-tuning), and they can therefore mediate long-range fifth forces. The latter can have implications on all scales, from the cosmological to the sub-atomic [21]. However, any such forces are tightly constrained within the Solar System [22]. In order to have avoided experimental and observational detection to date, the couplings between the light scalars and SM matter must be weak, at least locally. If the scalar field theory is non-linear then it is not necessary to fine-tune these couplings, which can instead be suppressed dynamically in the local environment through so-called screening mechanisms [5, 23].

The aim of this work is to develop an approach for describing the open quantum dynamics of matter fields, such as cold atoms, induced by an environment comprising a light scalar field. Our main result is a renormalized master equation, describing the open dynamics of a toy, scalar matter field in the single-particle momentum subspace. It features a coherent shift (i.e. a correction to the unitary dynamics of the ‘atoms’), decoherence and momentum diffusion. The reason for considering atoms as concrete motivation, or quantum objects more generally, such as molecules and optomechanical systems, is that they are at the forefront of precision metrology, using relatively small and cheap table-top experiments. They are exploited to explore a wide range of phenomena. These include probing the time evolution of fundamental constants and searching for new forces [24–28], testing gravitationally induced decoherence [29–31], realizing macroscopic quantum superpositions [32] to probe the so-called collapse models [33–37], and constraining models of dark matter and dark energy, using dielectric nanospheres levitated in laser beams [38, 39], as well as atom-interferometry searches for fifth forces mediated by light scalar fields [40–46].

In the context of a toy model, we derive the relevant quantum master equation describing the open dynamics by means of the Feynman-Vernon influence functional approach [47–50]. This and similar field-theoretic approaches have been used previously, e.g., in studies of quantum Brownian motion [51, 52], interacting quantum field theories (both in vacuum [53] and at finite temperature [54]) and decoherence during inflation in the early Universe [55–61] (see also Refs. [62, 63] for the Hamiltonian approach), as well as quarkonium suppression in heavy-ion collisions [64, 65]. In order to make connection with the relevant matrix elements of the density operator, we make use of the operator-based formulation of non-equilibrium field theory known as thermo field dynamics [66–68] (see also Ref. [69]). By this means, we are able to project out the single-particle matrix

\[1\] Attempts have also been made to understand galactic dynamics through modifications of gravity mediated by a light scalar field [10, 20].
elements of interest in the low-energy limit in a manner analogous to the LSZ reduction \cite{70} that is well-known in scattering-matrix theory. This approach has the advantage that it allows us to establish a direct connection between powerful QFT techniques, including those that can account for finite-temperature effects, and master equations of a form customary in atomic and condensed matter physics. In addition, we are able to renormalize the loop corrections appearing in the master equation consistently in the non-Markovian regime, where finite-time effects need to be taken into account. We show that these finite-time effects, and the associated violation of time-translational invariance, mean that we must introduce time-local counterterms (for earlier discussions of renormalization in the case of open quantum systems, see, e.g., Refs. \cite{71,72}). In this way, we are not required to introduce time-dependent ultra-violet cut-offs that can obscure the origin of divergences in the master equations.

For a light and weakly coupled environment, the effects that we describe are, as one would anticipate, small and far beyond the reach of current atom-interferometry searches. However, the resulting master equation serves as an important guide for future experimental design, illustrating, for instance, the need to maximize the momentum difference in the coherences between the states. Perhaps most importantly, the approach that we develop in this work is sufficiently general and robust that it will lead to new studies of quantum decoherence due to other long-range forces. In the context of gravitational decoherence, it provides a new method of renormalization, involving time-local counterterms, that should be compared with those that have been applied previously in the literature \cite{74,75}. We leave this interesting application for future work.

We begin in Sec. II by introducing the light scalar-field models that we have in mind, discussing, in particular, the ways in which they can be coupled to SM matter. In addition, we describe their potential screening mechanisms, focusing on the so-called chameleon mechanism, which we take as an archetype of this class of theories. We then proceed, in Sec. III, to describe the derivation of the quantum master equation of the single-particle matrix element of the reduced density operator for a toy system, through which the effects of the scalar environment can be analyzed. This section includes discussions of the Feynman-Vernon influence functional and the LSZ-like reduction technique that we employ, as well as our approach to the renormalization of the loop corrections in the non-Markovian regime. We discuss the possible implications of our results for experiments in Sec. IV. Our conclusions are presented in Sec. V.
II. LIGHT SCALAR FIELDS

As discussed in the introduction, two types of couplings are commonly introduced between light, gauge-singlet scalar fields and SM matter: (i) Higgs-portal couplings and (ii) conformal (non-minimal) couplings to the Ricci scalar.

Higgs-portal couplings are often considered in the context of dark matter theories, and in extensions of the SM more generally, where they allow hidden sectors to communicate with the SM via a scalar mediator, which may itself play a role as a dark matter component. The new scalar couples directly to the Higgs field through terms of the form

\[ L \supset -\frac{\alpha_n}{n} m^{2-n} X^n H^\dagger H, \quad n \in \{1, 2\}, \]

where the \( \alpha_n \) are dimensionless constants, \( m \) is a mass scale, \( H \) is the SM Higgs doublet and \( X \) is the additional scalar.

On the other hand, conformal couplings are commonly considered in the context of dark energy and modified theories of gravity, although they have recently been discussed also for light scalar dark matter [76–80], where such non-minimal couplings can impact upon the production mechanism of the relic abundance. In the case of modified gravity, matter fields move on geodesics of a spacetime metric \( \tilde{g}_{\mu\nu} = A^2(X) g_{\mu\nu} \), where \( \tilde{g}_{\mu\nu} \) is the so-called Jordan-frame metric (see, e.g., Ref. [81]) and \( A^2(X) \) is a coupling function. While the Higgs-portal and non-minimal gravitational couplings would appear very different at a first glance, they are, in fact, equivalent up to second order in the scalar field \( X \). The two descriptions are related, up to field redefinitions, by a change of conformal frame [82].

An additional light scalar field, which couples to matter, will mediate a fifth force, and the latter will arise at tree-level so long as the coupling function \( A^2(X) \) or, equivalently, the Higgs-portal interaction does not respect a \( Z_2 \) symmetry \( (X \rightarrow -X) \). This can be seen by considering the geodesic equation for a matter particle moving with respect to the metric \( \tilde{g}_{\mu\nu} = A^2(X) g_{\mu\nu} \).

Alternatively, it can be seen by computing the contribution of scalar exchange to two-to-two fermion scattering in the Higgs-portal picture [82], wherein the coupling to fermions results from the mass-mixing with the would-be SM Higgs field. The existence of long-range fifth forces is constrained both by experiments and observations, and, for a simple Yukawa force law, any such force must couple to matter five orders of magnitude more weakly than gravity within the Solar System [83].

However, the scalar field can have a much more varied phenomenology in general, which can allow it to evade local tests of gravity. Specifically, whenever the scalar field theory contains
non-linearities, either in the potential, in the coupling to matter or in the kinetic structure, its properties can change depending on the environment in such a way that the fifth force it mediates can be suppressed dynamically [2]. This gives rise to the phenomenon of screening. There is a zoo of theories which behave in this way, and this includes the Galileons [84], chameleons [85, 86] and symmetrons [87, 88] (see Refs. [89–93] for similar/related models), all of which fall within the Horndeski class of scalar-tensor theories [94, 95].

For the sake of concreteness, we consider a specific chameleon model in this work, but the results that follow can be adapted readily to other models.

### A. Einstein-frame action

For our purposes, it is convenient to work in the Einstein frame, wherein the gravitational sector is of canonical Einstein-Hilbert form and the action involving the conformally coupled scalar can be written in the general form [85, 86]

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_{Pl}^2 R - \frac{1}{2} g_{\mu\nu} \partial_\mu X \partial_\nu X - V(X) \right] + \int d^4x \sqrt{-g} A^4(X) \tilde{L}_m(\{\tilde{\phi}_i\}, A^2(X) g_{\mu\nu}) .
\] (2)

Herein, \(g_{\mu\nu}\) is the Einstein-frame metric, \(R\) is the associated Ricci scalar, \(M_{Pl}\) is the reduced Planck mass, \(V(X)\) is the potential for the scalar \(X\) and \(\tilde{L}_m\) is the Lagrangian density of the SM/matter fields, which we denote by the set \(\{\tilde{\phi}_i\}\) and which feel the rescaled metric \(\tilde{g}_{\mu\nu} = A^2(X) g_{\mu\nu}\).

Throughout this article, we use the \((-,-,+,+)+\) signature convention.

We will take the Jordan-frame matter action to be that of a single scalar field of mass \(\tilde{m}^2\):

\[
\tilde{L}_m = -\frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} - \frac{1}{2} \tilde{m}^2 \tilde{\phi}^2 .
\] (3)

The Einstein-frame Lagrangian then takes the form

\[
\mathcal{L}_m \equiv A^4(X) \tilde{L}_m = -\frac{1}{2} A^2(X) g^{\mu\nu} \partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} - \frac{1}{2} A^4(X) \tilde{m}^2 \tilde{\phi}^2 .
\] (4)

After redefining the scalar matter field via

\[
\phi \equiv A(X) \tilde{\phi} ,
\] (5)

we obtain

\[
\mathcal{L}_m = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \phi^2 g^{\mu\nu} \partial_\mu \ln A(X) \partial_\nu \ln A(X) + \phi g^{\mu\nu} \partial_\mu \phi \partial_\nu \ln A(X) - \frac{1}{2} A^2(X) \tilde{m}^2 \phi^2 .
\] (6)
Assuming that the coupling to matter is controlled by some mass scale $\mathcal{M}$ and that $X/\mathcal{M} \ll 1$, the coupling function can be expanded as

$$A^2(X) = a + b \frac{X}{\mathcal{M}} + c \frac{X^2}{\mathcal{M}^2} + \mathcal{O}\left(\frac{X^3}{\mathcal{M}^3}\right),$$

(7)

where $a$, $b$ and $c$ are model-specific coefficients. Keeping only operators up to dimension-four (since the higher-dimension operators are suppressed by higher powers of the scale $\mathcal{M}$), the matter action then contains

$$\mathcal{L}_m = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \left(a + b \frac{X}{\mathcal{M}} + c \frac{X^2}{\mathcal{M}^2}\right) \tilde{m}^2 \phi^2.$$  

(8)

We take the following effective potential for the $X$ field:

$$V^{\text{eff}}(X) = \pm \frac{1}{2} \mu^2 X^2 + \frac{\lambda}{4!} X^4 + A(X) \rho^{\text{ext}},$$

(9)

where $\rho^{\text{ext}}$ is the covariantly-conserved energy-density of some external matter source (e.g. the vacuum chamber in an atom-interferometry experiment), approximated here as a pressureless perfect fluid. The interplay between the bare mass $\mu$ (which may be tachyonic and drive spontaneous symmetry breaking), the self-interactions and the coupling to the background density will, in general, lead to a non-trivial background field configuration $\langle X \rangle \neq 0$, satisfying the classical equation of motion

$$\Box \langle X \rangle \mp \mu^2 \langle X \rangle - \frac{\lambda}{3!} \langle X \rangle^3 = \left| \frac{dA(X)}{dX} \right|_{X = \langle X \rangle} \rho^{\text{ext}}.$$  

(10)

Expanding $X = \langle X \rangle + \chi$, and if we assume that the background is constant, i.e. $\rho^{\text{ext}}$ and $\langle X \rangle$ are constant, we can redefine the mass of the matter scalar via

$$m^2 \equiv \left(a + b \frac{\langle X \rangle}{\mathcal{M}} + c \frac{\langle X \rangle^2}{\mathcal{M}^2}\right) \tilde{m}^2,$$

(11)

and the full Lagrangian of the two-scalar system becomes

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 - \frac{1}{2} \alpha_1 m \chi \phi^2 - \frac{1}{4} \alpha_2 \chi^2 \phi^2 - \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} \mathcal{M}^2 \chi^2 - \frac{\lambda}{4!} (\chi^4 + 4 \langle X \rangle \chi^3) - V(\langle X \rangle),$$

(12)

where (up to terms of order $\langle X \rangle^2 / \mathcal{M}^2$)

$$\alpha_1 \equiv \frac{m}{\mathcal{M}} \left[ \frac{b}{a} \left(1 - \frac{b}{a} \frac{\langle X \rangle}{\mathcal{M}}\right) + 2 \frac{c}{a} \frac{\langle X \rangle}{\mathcal{M}} \right],$$

(13a)

$$\alpha_2 \equiv 2 \frac{c}{a} \frac{m^2}{\mathcal{M}^2},$$

(13b)
and we have defined the squared mass

\[ M^2 \equiv \frac{\lambda}{2} \langle X \rangle^2 \pm \mu^2 + c \frac{\phi_{\text{ext}}}{M^2}. \]  

(14)

In advance of our later analysis, we can now isolate the free and (self-)interacting parts of the action for the fluctuations \( \phi \) of the atom and \( \chi \) of the chameleon, defining

\[ S_\phi[\phi] = \int_x \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right], \]  

(15a)

\[ S_\chi[\chi] = \int_x \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - \frac{1}{2} M^2 \chi^2 \right], \]  

(15b)

\[ S_{\chi, \text{int}}[\chi] = \int_{x \in \Omega_t} \left[ -\frac{\lambda}{4!} (\chi^4 + 4 \langle X \rangle \chi^3) \right], \]  

(15c)

\[ S_{\text{int}}[\phi, \chi] = \int_{x \in \Omega_t} \left[ -\frac{1}{2} \alpha_1 m \chi \phi^2 - \frac{1}{4} \alpha_2 \chi^2 \phi^2 \right], \]  

(15d)

where we omit the term arising from \( V_{\text{eff}}(\langle X \rangle) \) [cf. Eq. (45)] and use the shorthand notation

\[ \int_x \equiv \int d^4 x. \]  

(16)

The terms in \( S_{\text{int}}[\phi, \chi] \) should be compared with Eq. (1), illustrating the connection with portal couplings discussed earlier. Here, we have accounted for the fact that we will later restrict the interactions to take place over a finite period of time between the preparation of the initial state of the system and its subsequent measurement. The spacetime integrals in the interaction parts of the action are therefore restricted to the hypervolume \( \Omega_t = [0, t] \times \mathbb{R}^3 \). We emphasize that the free parts nevertheless have support for all times (see Refs. [96, 97]). Alternatively, one could extend the limits of the time integrals to the full real line and replace the coupling constants \( \alpha_1, \alpha_2 \) and \( \lambda \) by time-dependent couplings, which then parametrize the effective switching on and off of the interactions by the experimental apparatus. More generally, we might introduce a switching function that reflects the realistic preparation of the system.

B. Chameleons

The scalar field theory described by Eq. (2) is a chameleon model if, in the presence of a non-relativistic matter distribution, the scalar field is stabilized at the minimum of its effective potential and the mass of small fluctuations about this minimum depends on the local energy density. It is this variation in the mass which allows the scalar fifth force to be suppressed, or screened, in local tests of gravity. Specifically, near to dense sources of matter, the minimum of the effective potential \( V_{\text{eff}}(X) \) [see Eq. (9)] lies at a finite value \( \langle X \rangle = X_{\text{min}} \) and the mass becomes large, such that the fifth
force is Yukawa-suppressed. Instead, in the cosmological vacuum, the fluctuations are essentially massless, allowing them to propagate a long-range force of a strength comparable to gravity. It is common to consider polynomial potentials of the form \( V_{\text{eff}}(X) = \Lambda^4(\Lambda/X)^n + A(X)\rho^{\text{ext}} \), with integer \( n \), giving a chameleon model if \( n > 0 \) or if \( n \) is an negative even integer strictly less than \(-2\).

In this work, we consider the case \( n = -4 \), giving the effective potential

\[
V_{\text{eff}}(X) = \frac{\lambda}{4!}X^4 + A(X)\rho^{\text{ext}},
\]

with

\[
A(X) = e^{X/\mathcal{M}}.
\]

This corresponds to taking \( a = 1, \ b = c = 2 \) and \( \mu = 0 \) in the results of the preceding subsection, giving, for instance, the coupling constants

\[
\alpha_1 = \frac{2}{\mathcal{M}}, \quad \alpha_2 = \alpha_1^2.
\]

The quartic chameleon model, with the potential given by Eq. (17), is currently under pressure from atom-interferometry and torsion-balance experiments, with only a small window of parameter space remaining for \( \lambda \) close to one and \( \mathcal{M} \) close to the Planck mass [21]. However, we shall consider this archetypal model with screening, since it is a particularly useful prototype for evaluating the effect of the chameleon fluctuations on the dynamics of the matter fields.

In a constant-density environment, the expectation value of the chameleon field is given (to leading order in \( \rho^{\text{ext}}/\mathcal{M} \)) by

\[
X_{\text{min}}(\rho^{\text{ext}}) = -\left(\frac{6\rho^{\text{ext}}}{\lambda\mathcal{M}}\right)^{1/3},
\]

and the corresponding mass of small fluctuations around the minimum is

\[
m_{\text{min}}^2(\rho^{\text{ext}}) = \left(\frac{\lambda}{2}\right)^{1/3} \left(\frac{3\rho^{\text{ext}}}{\mathcal{M}}\right)^{2/3}.
\]

The situation is summarized in Fig. 1. The solid blue, solid red and black dashed lines correspond to the effective potential \( V_{\text{eff}}(X) \) at zero external density \( \rho^{\text{ext}} = 0 \), the contribution from the linear coupling to matter at non-zero density and the effective potential at non-zero density, respectively. The panes (a) and (b) correspond to the potential in regions with smaller and higher matter densities, respectively. In higher-density environments, the chameleon is more massive; in lower density environments, it is less massive.
FIG. 1: The effective potential $V_{\text{eff}}$ at zero density (solid blue), the contribution from the linear coupling to matter at non-zero density (solid red) and the effective potential at non-zero density (black dashed). The panes (a) and (b) correspond to the potential in regions with smaller and higher matter densities $\rho^{\text{ext}}$, respectively.

To see how this variation in the mass of the scalar leads to a suppression of the scalar fifth force, we now consider situations where the background matter density is not uniform. Specifically, we consider a uniform sphere of constant density $\rho^{\text{ext}}$ and radius $R$ embedded in a background of lower density $\rho^{\text{bg}}$. For sufficiently large spheres, the scalar field can reach the value $X_{\min}$ which minimizes its effective potential at the centre of the sphere. It therefore has a large mass in the interior of the sphere and does not roll from $X_{\min}$ apart from in a thin shell near the surface. It is only matter in this thin shell (whose thickness is proportional to $1/m_{\min}$) that sources the chameleon fifth force in the exterior. As only a small fraction of the mass of the sphere gives rise to the fifth force, it is much weaker than we would naively expect. This is known as the thin-shell effect.

To see this mathematically, a sphere of radius $R$ and density $\rho^{\text{ext}}$, embedded in a background of density $\rho^{\text{bg}}$ has a thin-shell at position $R_{TS}$, if there exists a real solution to

$$1 - \frac{R_{TS}^2}{R^2} = \left(\frac{M}{M_{\text{Pl}}}\right)^2 \frac{6M_{\text{Pl}}^2}{R^2\rho^{\text{ext}}} \left(\frac{X_{\min}(\rho^{\text{bg}}) - X_{\min}(\rho^{\text{ext}})}{M}\right).$$

If there is no real solution to this equation, the field cannot minimize its potential inside the sphere, and the chameleon fifth force is unscreened. The chameleon field profile around the sphere is

$$\langle X \rangle = \begin{cases} X_{\min}(\rho^{\text{ext}}), & 0 < r < R_{TS}, \\ X_{\min}(\rho^{\text{ext}}) + \frac{R_{TS}^2}{R^2} \rho^{\text{ext}} \frac{r^3 - 3R_{TS}^2 r + 2R_{TS}^3}{R^3}, & R_{TS} < r < R, \\ X_{\min}(\rho^{\text{bg}}) - \frac{R_{TS}^2}{R^2} \rho^{\text{ext}} \left(1 - \frac{R_{TS}^3}{R^3}\right) \frac{R}{r} e^{-m_{\min}(\rho^{\text{bg}}) r}, & R < r. \end{cases}$$

The fifth force felt by a test particle outside the sphere is $\vec{F} = -\vec{\nabla} \langle X \rangle / M$. It is now clear that as $R_{TS}$ approaches $R$, the fifth force is suppressed. In the limit where $R_{TS} \to 0$, which corresponds to
small (or very diffuse) spheres, the force is unsuppressed. More details on how chameleons screen
can be found in Ref. [21].

III. OPEN DYNAMICS

We now turn our attention to deriving the quantum master equation describing the open dy-
namics of the matter field $\phi$ in contact with an environment composed of the chameleon and its
fluctuations $\chi$.

A. The Feynman-Vernon influence functional

Our aim is to trace out the states of the conformally coupled scalar field $\chi$, so as to leave us with
an open scalar system, whose reduced density matrix evolves subject to a quantum master equation.
In order to derive this master equation, we could proceed directly at the operator level, by taking
a partial trace of the quantum Liouville equation of the coupled scalar system. Alternatively, but
equivalently, we can make use of the path-integral description provided by the Feynman-Vernon
influence functional [17], having the advantage that we can exploit the power of the diagrammatic
expansions to which functional approaches lend themselves.

Our starting point is the reduced density operator of the scalar system

$$\hat{\rho}_\phi(t) = \text{tr}_\chi \hat{\rho}(t).$$

We can evaluate the partial trace on the right-hand side by inserting complete sets of eigenstates
of the Heisenberg-picture field operators $\hat{\chi}$ at time $t$. In this way, we obtain

$$\hat{\rho}_\phi(t) = \int d\chi^\pm t \, \delta(\chi^+ t - \chi^- t) \hat{\rho}[\chi^\pm t],$$

where

$$\hat{\rho}[\chi^\pm t] \equiv \langle \chi^\pm t | \hat{\rho}(t) | \chi^- t \rangle,$$

which includes a functional delta function that sets $\chi^+ = \chi^-$ at the time $t$. We have introduced
the measure

$$\int d\chi^\pm t \equiv \int d\chi^+ t \, d\chi^- t$$

and suppress the spatial dependence of the field eigenstates $|\chi^\pm t\rangle$ for notational convenience. Notice
that we have introduced two copies of the eigenstates at the time $t$, distinguished by the superscript
FIG. 2: Schematic representation of the time evolution of the system density matrix. Here, + and − label the two branches of the closed-time-path contour.

±; this amounts to the usual doubling of degrees of freedom needed to write a path-integral representation of the trace of an operator, giving rise to the Schwinger-Keldysh closed-time-path formalism [98, 99] (see also Subsec. III B). We can now take matrix elements of the reduced density operator in the basis of \( \phi \) field eigenstates, defining the reduced density functional

\[
\rho_\phi[\phi^\pm_t; \tau] \equiv \langle \phi^+_t | \hat{\rho}_\phi(\tau) | \phi^-_t \rangle = \int d\chi^\pm_t \delta(\chi^+_t - \chi^-_t) \rho[\phi^+_t, \chi^+_t; \tau],
\]

where

\[
\rho[\phi^+_t, \chi^+_t; \tau] \equiv \langle \phi^+_t, \chi^+_t | \hat{\rho}(\tau) | \phi^-_t, \chi^-_t \rangle.
\]

In order to proceed further, we assume that the state of the full system at the initial time \( t_i \) is a product state of the form

\[
\hat{\rho}(t_i) = \hat{\rho}_\phi(t_i) \otimes \hat{\rho}_\chi(t_i).
\]

The state of the system at time \( t \) is then given by

\[
\rho_\phi[\phi^\pm_t; \tau] = \int d\phi^\pm_t \mathcal{I}[\phi^\pm_t, \phi^\mp_t; t, t_i] \rho_\phi[\phi^\pm_t; t_i],
\]

where

\[
\mathcal{I}[\phi^\mp_t, \phi^\pm_t; t, t_i] = \int_{\phi^\pm_t} D\phi^\pm e^{i \hat{S}_{\text{eff}}[\phi^\pm_t]}
\]

is the influence functional (IF) propagator. The latter arises from inserting complete sets of field and conjugate-momentum eigenstates into Eq. (28) at all intermediate times from the final time \( t \), to the initial time \( t_i \) and back again, giving rise to the closed-time path [98, 99], pictured in Fig. 2. The master equation follows straightforwardly from taking the time-derivative of Eq. (31); specifically,

\[
\partial_t \rho_\phi[\phi^\pm_t; \tau] = i \frac{1}{\hbar} \partial_t \hat{S}_{\text{eff}}[\phi; \tau] \rho_\phi[\phi^\pm_t; \tau].
\]
The effective action $\tilde{S}_{\text{eff}}$ arises from tracing out the $\chi$ field. It takes the general form

$$\tilde{S}_{\text{eff}}[\phi; t] = \tilde{S}_\phi[\phi; t] + \tilde{S}_{\text{IF}}[\phi; t],$$

(34)

where we use a  to indicate functionals (and later operators, cf. Subsec. IIIB) that depend on both of the doubled field variables $\phi^+$ and $\phi^-$. Contributions to the effective action associated with unitary evolution can be written in terms of the usual action, e.g.,

$$\tilde{S}_\phi[\phi; t] = \sum_{a=\pm} a S_\phi[\phi^a; t] = S_\phi[\phi^+; t] - S_\phi[\phi^-; t].$$

(35)

On the other hand, the influence action $\tilde{S}_{\text{IF}}[\phi; t]$, which, in general, describes non-unitary dynamics, cannot be constructed in this way. Instead, it will contain terms that mix + and − field variables. Its explicit form is defined by means of the Feynman-Vernon influence functional [49]

$$\tilde{F}[\phi; t] = \exp \left\{ \frac{i}{\hbar} \tilde{S}_{\text{IF}}[\phi; t] \right\}$$

$$= \int d\chi^+ d\chi^- \delta(\chi^+ - \chi^-) \rho_{\chi}[\chi^\pm; t_i] \int_{\chi^+_i}^{\chi^-_f} D\chi^\pm \exp \left\{ \frac{i}{\hbar} \left( \tilde{S}_\chi[\chi; t] + \tilde{S}_{\chi,\text{int}}[\chi; t] + \tilde{S}_{\text{int}}[\phi, \chi; t] \right) \right\}.$$  

(36)

If the system is weakly coupled to the environment, $\tilde{S}_{\text{IF}}$ can be obtained perturbatively by expanding the right-hand side of Eq. [36], in our case with respect to the small parameters $\{\lambda, m/\mathcal{M}, X/\mathcal{M}\} \ll 1$. Expanding to quadratic order in the action, we obtain

$$\tilde{S}_{\text{IF}}[\phi] = \sum_{a=\pm} a \left[ \langle S_{\text{int}}[\phi^a, \chi^a] \rangle + \langle S_{\chi,\text{int}}[\chi^a] \rangle \right] + \frac{i}{2\hbar} \sum_{a,b=\pm} ab \left[ \langle S_{\text{int}}[\phi^a, \chi^b] S_{\text{int}}[\phi^b, \chi^b] \rangle' + \langle S_{\chi,\text{int}}[\chi^a] S_{\chi,\text{int}}[\chi^b] \rangle' + 2 \langle S_{\chi,\text{int}}[\chi^a] S_{\text{int}}[\phi^b, \chi^b] \rangle' \right],$$

(37)

where

$$\langle A[\chi^a] \rangle \equiv \int d\chi^+ d\chi^- \delta(\chi^+ - \chi^-) \rho_{\chi}[\chi^\pm; t_i] \int_{\chi^+_i}^{\chi^-_f} D\chi^\pm A[\chi^a] \exp \left\{ \frac{i}{\hbar} \tilde{S}_\chi[\chi; t] \right\}$$

(38)

and

$$\langle A[\chi^a] B[\chi^b] \rangle' \equiv \langle A[\chi^a] B[\chi^b] \rangle - \langle A[\chi^a] \rangle \langle B[\chi^b] \rangle.$$

(39)

We suppress the time arguments of the contributions to the action when convenient to do so.

We take the initial state of the environment to be a thermal state (with respect to the fluctuations $\chi$ around the background field $\langle X \rangle$, taken here to be in equilibrium with the vacuum chamber), in which case

$$\rho_{\chi}[\chi^\pm; t_i] = \frac{1}{\text{Tr} e^{-\beta H_\chi}} \langle \chi^+_i | e^{-\beta H_\chi} | \chi^-_i \rangle,$$

(40)
where $\beta = 1/T$ is the inverse thermodynamic temperature and $\hat{H}_\chi$ is the free Hamiltonian of the $\chi$ fluctuations. We work in units where the Boltzmann constant is unity. The various correlation functions can be evaluated by means of Wick’s theorem, and we have (see, e.g., Ref. [100])

\[
\begin{align*}
\chi_x \chi_y & = \langle T [\chi_x \chi_y] \rangle = \Delta^{++}_{xy} = \Delta^F_{xy} \\
\chi_x \chi_y & = \langle \hat{T} \chi_x \chi_y \rangle = \Delta^{--}_{xy} = \Delta^D_{xy} \\
\chi_x \chi_y & = \langle \chi_x \chi_y \rangle = \Delta^{+-}_{xy} = \Delta^<_x = \langle \Delta^< \rangle_{xy} \\
\chi_x \chi_y & = \langle \chi_x \chi_y \rangle = \Delta^{+-}_{xy} = \Delta^<_y = \Delta^{<y} = \langle \Delta^< \rangle_{xy} ,
\end{align*}
\]

where $\Delta^F_{xy}$ is the Feynman (Dyson) propagator, $\Delta^\pm_{xy}$ are the Wightman propagators, and $T$ and $\hat{T}$ are the time- and anti-time-ordering operators, respectively. We have used the shorthand notation

\[
\int_k \equiv \int \frac{d^4k}{(2\pi)^4} .
\]

The thermal contributions are encoded in the on-shell terms proportional to the Bose-Einstein distribution function

\[
f(k^0) = \frac{1}{e^{\beta k_0} - 1} .
\]

Here, $\text{sgn}(k^0) = \theta(k^0) - \theta(-k^0)$ is the signum function, and the form of the positive-frequency Wightman propagator $\Delta^>_y$ follows from the identity

\[
f(-k^0) = -[1 + f(k^0)] .
\]

The various terms in $\hat{S}_{\text{IF}}$, Eq. (37), can then be expressed as follows:

\[
\begin{align*}
\langle S_{\text{int}}[\phi^a, \chi^a] \rangle & = -\frac{m^2}{M^2} \int_x (\phi^a_x)^2 \Delta^F_{xx} , \\
\langle S_{\text{int}}[\phi^a, \chi^a] S_{\text{int}}[\phi^b, \chi^b] \rangle & = \langle S_{\text{int}}[\phi^a, \chi^a] S_{\text{int}}[\phi^b, \chi^b] \rangle = \frac{m^4}{M^2} \int_{xy} (\phi^a_x)^2 (\phi^b_y)^2 \Delta^F_{xy} , \\
\langle S_{\chi, \text{int}}[\chi^a] \rangle & = -\frac{\lambda}{4!} \int_x \left[ 3(\Delta^F_{xx})^2 + \langle X \rangle^4 \right] , \\
\langle S_{\chi, \text{int}}[\chi^a] S_{\chi, \text{int}}[\chi^b] \rangle & = \frac{\lambda^2}{(4!)^2} \int_{xy} \left[ 24(\Delta^F_{xy})^4 + 72(\Delta^F_{xx})^2 (\Delta^F_{xy})^2 + 96 \langle X \rangle^2 (\Delta^F_{xy})^3 \\
&+ 144 \langle X \rangle^2 (\Delta^F_{xx})^2 \Delta^F_{xy} + 9(\Delta^F_{xx})^4 + 6 \langle X \rangle^4 (\Delta^F_{xx})^2 + \langle X \rangle^8 \right] ,
\end{align*}
\]
\begin{align}
\langle S_{\chi,\text{int}}[\chi^a] S_{\chi,\text{int}}[\chi^b] \rangle' &= \frac{\lambda^2}{(4!)^2} \int_{xy} \left[ 24 (\Delta_{xy}^{ab})^4 + 72 (\Delta_{xy}^F)^2 (\Delta_{xy}^{ab})^2 + 96 \langle X \rangle^2 (\Delta_{xy}^{ab})^3 \right. \\
&\quad + 144 \langle X \rangle^2 (\Delta_{xy}^F)^2 \Delta_{xy}^{ab} \right], \\
\langle S_{\chi,\text{int}}[\chi^a] S_{\text{int}}[\phi^b, \chi^b] \rangle' &= \langle S_{\chi,\text{int}}[\chi^a] S_{\text{int}}[\phi^b, \chi^b] \rangle = \frac{\lambda \langle X \rangle m^2}{2 \mathcal{M}} \int_{xy} \Delta_{xx}^F \Delta_{xy}^{ab} (\phi_x^b)^2, \quad \text{(45f)}
\end{align}

where we have restored the factors arising from \( V_{\text{eff}} ((X)) \), omitted in Eq. \((15c)\), in order to illustrate that these do not contribute to \( \hat{S}_{\text{IF}} \). Hereafter, we leave it implicit that all time integrals run over the domain \([0, t]\). We note that we work in a regime where \( \alpha_2^2 \ll \alpha_2^1 \), i.e. where \( \langle X \rangle / \mathcal{M}, m / \mathcal{M} \ll 1 \).

Putting everything together, we have

\[ \hat{S}_{\text{IF}}[\phi; t] = \frac{-m^2}{2 \mathcal{M}^2} \sum_{a, b = \pm} \int_x a(\phi_x^a)^2 \Delta_{xx}^F + \frac{i}{\hbar} \int_{xy} a_{ab} = \pm \ \left\{ \frac{m^4}{\mathcal{M}^2} (\phi_x^a)^2 (\phi_y^b)^2 \Delta_{xy}^{ab} \right. \]
\[ + \frac{\lambda^2}{24} \left[ (\Delta_{xy}^{ab})^4 + 3(\Delta_{xy}^F)^2 (\Delta_{xy}^{ab})^2 + 4 \langle X \rangle^2 (\Delta_{xy}^{ab})^3 + 6 \langle X \rangle^2 (\Delta_{xy}^F)^2 \Delta_{xy}^{ab} \right] \]
\[ + \left. \frac{\lambda \langle X \rangle m^2}{\mathcal{M}} \Delta_{xx}^F \Delta_{xy}^{ab} (\phi_y^b)^2 \right\}. \quad \text{(46)} \]

We hereafter work in natural units, setting \( \hbar = 1 \).

The terms in square brackets in the second line of Eq. \((16)\) do not involve any fields \( \phi \). Since it involves only \( \chi \) propagators, the sum over all \( a \) and \( b \) is equivalent to a sum over underlinings in the sense of the largest time equation \([101, 102]\), yielding a vanishing contribution, that is to say

\[ \forall \ n \in \mathbb{N}_0 : \sum_{a, b = \pm} a_b (\Delta_{xy}^{ab})^n = 0. \quad \text{(47)} \]

Thus, we have that

\[ \hat{S}_{\text{IF}}[\phi; t] = \frac{-m^2}{\mathcal{M}^2} \sum_{a, b = \pm} \int_x a(\phi_x^a)^2 \Delta_{xx}^F + \frac{i}{\hbar} \int_{xy} a_{ab} = \pm \ \left\{ \frac{m^4}{\mathcal{M}^2} (\phi_x^a)^2 (\phi_y^b)^2 \Delta_{xy}^{ab} \right. \]
\[ + \left. \frac{\lambda \langle X \rangle m^2}{\mathcal{M}} \Delta_{xx}^F \Delta_{xy}^{ab} (\phi_y^b)^2 \right\}. \quad \text{(48)} \]

B. Operator-based approach

In order to guide the path-integral approach of Subsec. \([11\text{A}]\) it is helpful to consider the corresponding canonical, operator-based formulation. This is known as thermo field dynamics (TFD) \([66, 68]\) (see also Ref. \([69]\)), and the doubling of field degrees of freedom needed to describe relativistic quantum statistical systems requires us to construct this canonical formalism over a doubled Hilbert space \( \mathcal{H} \equiv \mathcal{H}^+ \otimes \mathcal{H}^- \). The usual scalar field operator \( \hat{\phi} \) is then embedded by
defining the plus- and minus-type field operators

$$\hat{\phi}^+(x) \equiv \hat{\phi}(x) \otimes I, \quad \hat{\phi}^-(x) \equiv I \otimes \hat{\phi}^T(x), \quad (49)$$

with analogous expressions for the embeddings of the usual scalar creation and annihilation operators. Here, the $T$ indicates time reversal. The interaction-picture field operators can then be written in the usual plane-wave decompositions

$$\hat{\phi}(x) = \int d\Pi_k \left[ \hat{a}^\pm_k e^{\mp iE_k t \pm i k \cdot x} + \hat{a}^\pm_k e^{\pm iE_k t \mp i k \cdot x} \right], \quad (50)$$

where we use the notation

$$\int d\Pi_k = \int \frac{1}{2E_k}, \quad \int_k = \int \frac{d^3k}{(2\pi)^3}, \quad (51)$$

for the Lorentz-invariant phase space integrals. When no time arguments are provided, it is assumed that the operators and states are evaluated at $t = 0$. For notational convenience, we also suppress the superscript $\phi$ on the energies in this subsection. Notice that the minus-type field is built from the time-reversed field operator, reflecting the anti-time-ordering of the negative branch of the closed-time path in the path-integral formulation. We also note that, by virtue of the Kronecker product structure, the plus- and minus-type operators commute with one another.

The plus- and minus-type creation and annihilation operators act in the corresponding Hilbert spaces, that is, by acting on the doubled vacuum state $|0\rangle \rangle \equiv |0\rangle \otimes |0\rangle$, we have

$$\hat{a}^{\pm\dagger}_k |0\rangle \rangle = |k\rangle \otimes |0\rangle \equiv |k_+\rangle, \quad \hat{a}^{\pm}_k |0\rangle \rangle = |0\rangle \otimes |k\rangle \equiv |k_-\rangle, \quad (52)$$

and

$$\hat{a}^{\pm}_k |p_+, p_-\rangle \rangle = (2\pi)^3 2E_k \delta^{(3)}(p - k) |p_-\rangle \rangle, \quad \hat{a}^{\pm}_k |p_+, p_+\rangle \rangle = (2\pi)^3 2E_k \delta^{(3)}(p - k) |p_+\rangle \rangle, \quad (53)$$

where $|p_+, p_-\rangle \rangle \equiv |p\rangle \otimes |p\rangle$ and so on.

The density operator of an isolated system can be embedded as

$$\hat{\rho}^+(t) \equiv \hat{\rho}(t) \otimes \hat{1}, \quad (54)$$

where $\hat{1}$ is the unit operator. Its trace can then be expressed in the following form:

$$\text{tr} \hat{\rho}(t) = \langle\langle 1 | \hat{\rho}^+(t) | 1 \rangle\rangle, \quad (55)$$

where the state (see Ref. [67])

$$|1\rangle \equiv |0\rangle + \int d\Pi_{p_1} |p_{1+}, p_{1-}\rangle \rangle + \frac{1}{2!} \int d\Pi_{p_1} d\Pi_{p_2} |p_{1+}, p_{2+}, p_{1-}, p_{2-}\rangle \rangle + \ldots \ldots \quad (56)$$

...
The trace of an operator $\hat{O}(t)$ can be written

$$\text{tr}\,\hat{O}(t)\dot{\hat{\rho}}(t) = \langle\langle 1|\hat{O}^+(t)\hat{\rho}^+(t)|1\rangle\rangle .$$

(57)

In this way, one is able to recast the quantum Liouville equation (in the Schrödinger picture)

$$\partial_t\hat{\rho}(t) = -i[\hat{H},\hat{\rho}(t)]$$

(58)

in the Schrödinger-like form

$$\partial_t\hat{\rho}(t)|1\rangle\rangle = -i\hat{H}\hat{\rho}(t)|1\rangle\rangle ,$$

(59)

where

$$\hat{H} = \hat{H} \otimes \hat{1} - \hat{1} \otimes \hat{H}$$

(60)

is the Liouvillian operator.

Moving to the interaction picture, we take a density operator of the form

$$\hat{\rho}(t) = \int d\Pi_k d\Pi_{k'} \rho(k,k';t)|k;t\rangle\langle k';t| .$$

(61)

We recall that state and basis vectors evolve respectively with the interaction and free parts of the Hamiltonian in the interaction picture. We assume that the single-particle term dominates in the low-energy, non-relativistic limit for the matter scalar and set the occupancy of all multi-particle states to zero. We are therefore interested in the matrix element

$$\langle p';t|\hat{\rho}(t)|p;t\rangle = \rho(p,p';t) .$$

(62)

Notice that all of the basis states and operators are evaluated at equal times, and the matrix element $\rho(p,p';t)$ is therefore picture-independent (see Refs. [96, 97]). In the TFD language, this can be written as

$$\text{tr}\,|p';t\rangle\langle p;t|\hat{\rho}(t) = \langle\langle 1|(|p';t\rangle\langle p;t| \otimes \hat{1})(\hat{\rho}(t) \otimes \hat{1})|1(t)\rangle\rangle ,$$

(63)

and we draw attention to the time-dependence of the state $|1(t)\rangle\rangle$. Using the fact that

$$\langle\langle 1|(|p';t\rangle\langle p;t| \otimes \hat{1}) = \langle\langle p_+;p_-';t|$$

(64)

and

$$\langle\langle 1|(|p';t\rangle\langle p;t| \otimes \hat{1})|1(t)\rangle\rangle = \int d\Pi_k d\Pi_{k'} \rho(k,k';t)|k_+,k_-';t\rangle\rangle ,$$

(65)
it follows that
\[ \partial_t \rho(p, p'; t) = -i \int d\Pi_k d\Pi_{k'} \rho(k, k'; t) \langle \| \Phi(p, p' \rangle | H(t) | \Phi(k, k') \rangle \}. \] (66)

This can be rewritten as
\[ \partial_t \rho(p, p'; t) = -i \int d\Pi_k d\Pi_{k'} e^{i(E_k - E_{k'})t} \rho(k, k'; t) \langle \| \hat{a}_p^+(t) \hat{a}_{p'}^-(t) \hat{H}(t) \hat{a}_{k}^{\dagger}(t) \hat{a}_{k'}^{\dagger}(t) \| 0 \rangle \}. \] (67)

In our case, \( \hat{H} \rightarrow \hat{H}_{\text{eff}} = -\partial_t \hat{S}_{\text{eff}} \), the effective Liouvillian that comes from tracing out the chameleon degrees of freedom. (We emphasise that \( \hat{H}_{\text{eff}} \) is non-Hermitian.) Allowing for the fact that \( \hat{H}_{\text{eff}} \) is a non-local, but time-ordered operator, and after accounting for the free-phase evolution of the right-most creation operators, the expectation value on the right-hand side of Eq. (67) can be time-ordered as
\[ \partial_t \rho(p, p'; t) = -i \int d\Pi_k d\Pi_{k'} e^{i(E_k - E_{k'})t} \rho(k, k'; t) \langle \| \hat{a}_p^+(t) \hat{a}_{p'}^-(t) \hat{H}_{\text{eff}}(t) \hat{a}_{k}^{\dagger}(t) \hat{a}_{k'}^{\dagger}(t) \| 0 \rangle \}. \] (68)

Continuing, we can recast the expression in terms of field operators by using
\[ \hat{a}_p^+(t) = +i \int_x e^{-i p \cdot x} \partial_{t, E_p} \hat{\phi}^+(t, x), \quad \hat{a}_{p'}^+(t) = -i \int_x e^{-i p' \cdot x} \partial_{t, E_{p'}} \hat{\phi}^+(t, x), \] (69)
where
\[ \partial_{t, E_p} \equiv \frac{\partial}{\partial t} - i E_p, \] (70)
along with
\[ \hat{a}_p^-(t) = -i \int_x e^{i p \cdot x} \partial_{t, E_p} \hat{\phi}^-(t, x), \quad \hat{a}_{p'}^-(t) = +i \int_x e^{i p' \cdot x} \partial_{t, E_{p'}} \hat{\phi}^-(t, x). \] (71)

Specifically, we have
\[ \partial_t \rho(p, p'; t) = -i \lim_{x^{0(t)} \rightarrow t^+} \int_{y^{0(t)} \rightarrow 0^-} d\Pi_k d\Pi_{k'} e^{i(E_k - E_{k'})t} \rho(k, k'; t) \int_{\| y^0 \rangle \langle y' \|} e^{-i(p \cdot x - p' \cdot x') + i(k \cdot y - k' \cdot y')} \times \partial_{x^{0(t)}, E_p} \hat{\phi}^+(x) \hat{\phi}^-(y) \hat{H}_{\text{eff}}(t) \hat{\phi}^+(y') \hat{\phi}^-(y') \| 0 \rangle \rangle, \] (72)

where \( x^{0(t)} \) approaches \( t \) from above and \( y^{0(t)} \) approaches 0 from below to ensure that the time-ordering of the operators is equivalent to the original ordering in Eq. (68). Notice that the role of the differential operators is to chop off external propagators and replace them with plane-wave factors. The procedure outlined here for projecting into the single-particle subspace therefore amounts to an LSZ-like reduction [70] of the four-point function
\[ \langle \| \Phi(p, p') \rangle \hat{H}_{\text{eff}}(t) \hat{\phi}^+(y) \hat{\phi}^-(y') \| 0 \rangle \). \] (73)
In the path-integral language, equation 72 can be recast as
\[
\partial_t \rho(p, p'; t) = -i \lim_{\substack{x(0) \to t^+ \\ y(0) \to 0^-}} \int d\Pi_k d\Pi_{k'} e^{i(E_k - E_{k'})t} \rho(k, k'; t) \int_{xx'yy'} e^{-i(p x - p' x') + i(k y - k' y')} \
\times \partial_x \partial_{x'} \partial_{p \pm} \partial_{p' \pm} \partial_{p \pm} \partial_{p' \pm} \int D\phi^\pm e^{iS_{\phi}^\pm(x)} \phi^+(x) \phi^-(x') \hat{H}_{\phi}^\pm(y) \phi^+(y) \phi^-(y') ,
\]
which can be expanded in terms of the 2 \times 2 matrix propagator. The latter is of the diagonal form
\[
D_{ab}^{\phi} = \begin{bmatrix} D_x & 0 \\ 0 & D_y \end{bmatrix} ,
\]
since, by virtue of the fact that Eq. 72 is a vacuum expectation value, there can be no +- contractions, and the off-diagonal elements of this 2 \times 2 matrix propagator are zero. We emphasise that, while \(\langle 1 | \hat{\phi}_x^+ \hat{\phi}_y^- | 0 \rangle > \) (as arises at zero temperature when we do not restrict to the single-particle subspace), \(\langle 0 | \hat{\phi}_x^+ \hat{\phi}_y^- | 0 \rangle = 0\).

The Feynman and Dyson propagators of the \(\phi\) field are
\[
\langle 0 | T[\hat{\phi}_x^+ \hat{\phi}_y^-] | 0 \rangle > = D_{xy}^{++} = D_{xy}^f = -i \hbar \int \frac{e^{ik \cdot (x - y)}}{k^2 + m^2 - i\epsilon} , \quad (76a)
\]
\[
\langle 0 | T[\hat{\phi}_x^- \hat{\phi}_y^-] | 0 \rangle > = D_{xy}^{--} = D_{xy}^D = +i \hbar \int \frac{e^{ik \cdot (x - y)}}{k^2 + m^2 + i\epsilon} , \quad (76b)
\]
Since we have restricted to the single-particle subspace, we necessarily assume that the \(\phi\) field remains at zero temperature and therefore out-of-equilibrium with the environment formed by the conformally coupled scalar \(\chi\) [cf. Eq. 41].

C. Quantum master equation

After projecting into the single-particle subspace, as described in Subsec. III B, we arrive at the following quantum master equation:
\[
\partial_t \rho(p, p'; t) = i \lim_{\substack{x(0) \to t^+ \\ y(0) \to 0^-}} \int d\Pi_k d\Pi_{k'} e^{i(E_k - E_{k'})t} \rho(k, k'; t) \int_{xx'yy'} e^{-i(p x - p' x') + i(k y - k' y')} \
\times \partial_x \partial_{x'} \partial_{p \pm} \partial_{p' \pm} \partial_{p \pm} \partial_{p' \pm} \int D\phi^\pm e^{iS_{\phi}^\pm(x)} \phi^+(x) \phi^-(x') \left(\partial_t \hat{S}_{\phi}^\pm[\phi; t] \right) \phi^+(y) \phi^-(y') ,
\]
which we have expressed in terms of the partial time derivative of \(\hat{S}_{\phi}\). Substituting for the explicit form of the effective action from Eqs. 34, 35 and 48 in Subsec. III A and performing the
remaining Wick contractions, we arrive at

\[
\partial_t \rho(p, p'; t) = -i\left(E^\phi_p - E^\phi_{p'}\right)\rho(p, p'; t) - \frac{im^2}{\mathcal{M}} \left(\frac{1}{E^\phi_p} - \frac{1}{E^\phi_{p'}}\right)\rho(p, p'; t)
\]

\[
\times \left\{\frac{\Delta F}{\mathcal{M}^2} + \left[\frac{m^2}{\mathcal{M}} D^{\mathcal{F}}_{\mathcal{F}} + \frac{\lambda}{2} \left\langle X \right\rangle \frac{\Delta F}{\mathcal{M}^2} \int_{x^0} \sin[\mathcal{M}(x^0 - t)]\right\}\right. \\
- \frac{4m^4}{\mathcal{M}^2} \int_{x^0} \int_k \left\{ \rho(p, p'; t) \cos\left[E^\phi_p(t - x^0)\right] \exp\left[-iE^\phi_{p-k}(t - x^0)\right] \right. \\
- \rho(p - k, p' - k; t) \frac{\exp[i(E^\phi_{p-k} - E^\phi_p)(t - x^0)]}{2E^\phi_k 2E^\phi_{p-k} 2E^\phi_{p-k}} \right\} \\
\left. \times \left[ \exp\left[-iE^\chi_k(t - x^0)\right] + 2 \cos\left[E^\chi_k(t - x^0)f\left(E^\chi_k\right)\right] + (p \leftrightarrow p')^* \right]\right. \\
- \frac{4m^4}{\mathcal{M}^2} \rho(p, p'; t) \sum_{s = \pm} \int_{x^0} \int_{k_1k_2} \cos\left[(E^\phi_{k_1} + E^\phi_{k_1-k_2} + sE^\chi_{k_2})(x^0 - t)\right] \\
\times s[1 + f(sE^\chi_{k_2})] .
\]

The right-hand side of this expression is represented diagrammatically in Fig. 3. Figures 3(a) and (b) correspond to the \(\chi\) tadpole insertion arising from the first term in the second line of Eq. (78), and (c) and (d) to the \(\chi\) (lollipop) tadpole from the third term in the second line. Figures 3(e) and (f) correspond to the \(\phi\) tadpoles, arising from the second term in the second line, and the remaining Figs. 3(g)–(i) are the \(\chi - \phi\) bubble diagrams, appearing in the third to fifth lines of Eq. (78). The final two lines of Eq. (78) contains a contribution from the absorptive part of the disconnected vacuum diagram shown in Fig. 3(j). The disconnected diagrams could be absorbed order-by-order in a redefinition of the matrix element \(\rho(p, p'; t) \rightarrow \rho(p, p'; t)(1 + \text{disconnected diagrams})\), taking into account that the time-derivative on the left-hand side counts at a finite order in the coupling constants. For our present discussions, however, we leave the vacuum diagram explicit throughout for completeness.

The terms in the third to seventh lines of Eq. (78) include decay (i.e. \(\phi \rightarrow \chi\chi\)) and production processes (i.e. \(\chi\chi \rightarrow \phi\)), which we would not expect to be present for realistic atoms, which are complex and stable configurations of many elementary particle fields rather than a simple scalar one. The decay and production processes arise here, because such processes are permitted in the simple scalar field theory that we have used as a toy proxy for the atom.
\[ \partial_t \rho(p, p'; t) = -i(E^\phi_p - E^\phi_{p'})\rho(p, p'; t) - \frac{im^2}{\mathcal{M}} \left( \frac{1}{E^\phi_p} - \frac{1}{E^\phi_{p'}} \right) \rho(p, p'; t) \]
\[
\times \left\{ \frac{\Delta F_{xx}}{\mathcal{M}} + \left[ \frac{m^2}{\mathcal{M}} D_{xx} + \frac{\lambda}{2} \langle X \rangle \Delta F_{xx} \right] \frac{\cos(Mt) - 1}{M^2} \right\}
\]
\[
+ \frac{4m^4}{M^2} \sum_{s = \pm} \int_k \left\{ \frac{1}{E^\phi_{2k} E^\phi_{-k} (sE^\chi_k + E^\phi_{-k})^2 - (E^\phi_p)^2} \right\}
\]
\[
\times \left[ \exp \left[ -i(sE^\chi_k + E^\phi_{-k})t \cos (E^\phi_p t) \right] \right.
\]
\[
- \left. iE^\phi_p \exp \left[ -i(sE^\chi_k + E^\phi_{-k})t \sin (E^\phi_p t) \right] [1 + f(sE^\chi_k)] \right]
\]
\[
+ \rho(p - k, p' - k; t) \frac{1}{2E^\chi_k E^\phi_{-k} E^\phi_{p' - k} sE^\chi_{p' - k} + E^\phi_{p' - k} - E^\phi_p} \]
\[
\times \left[ 1 - \exp \left[ i(sE^\chi_k + E^\phi_{-k} - E^\phi_p)t \right] f(sE^\chi_k) \right] - \left( p \leftrightarrow p' \right),
\]
\[
\frac{4m^4}{M^2} \rho(p, p'; t) \sum_{s = \pm} \int \int_{k_1 k_2} \sin \left[ \frac{(E^\phi_{k_1} + E^\phi_{k_1 - k_2} + sE^\chi_{k_2})t}{E^\chi_{k_1} + E^\phi_{k_1 - k_2} + sE^\chi_{k_2}} \right] \frac{1}{2E^\phi_{k_1} 2E^\phi_{k_1 - k_2} 2E^\chi_{k_2}} s[1 + f(sE^\chi_{k_2})],
\]
\[ (79) \]

Here, we have introduced the dummy parameter \( s = \pm \) in order to simplify the sum over the two energy flows in the thermal contributions (see, e.g., Ref. [100]), making use of the identity in Eq. [44]. Specifically, we have written

\[ \exp[-iE^\chi_k(t - x^0)] + 2 \cos [E^\chi_k(t - x^0)] f(E^\chi_k) = \sum_{s = \pm} s \exp[-isE^\chi_k(t - x^0)] [1 + f(sE^\chi_k)]. \]
\[ (80) \]

Notice that all of the contributions on the right-hand side of Eq. [79] that arise from non-local insertions, i.e. all but the first term in the second line, which do not depend on the momentum flow through the diagram, vanish identically in the limit \( t = 0 \). This is as one would expect, since the non-local insertions contain a residual time integral whose support vanishes in the limit \( t \to 0 \). In addition, the right-hand side is real in the limit \( p' \to p \) and consistent with \( \rho(p, p', t) = \rho^\ast(p', p, t) \), again as it should be.

The disconnected vacuum diagram of the final line of Eq. [79] vanishes at \( t = 0 \). The \( T = 0 \) part also vanishes in the limit \( t \to \infty \), since

\[ \frac{1}{\pi} \sin \left[ \frac{(E^\phi_{k_1} + E^\phi_{k_1 - k_2} + E^\chi_{k_2})t}{E^\phi_{k_1} + E^\phi_{k_1 - k_2} + sE^\chi_{k_2}} \right] \to \delta(E^\phi_{k_1} + E^\phi_{k_1 - k_2} + E^\chi_{k_2}), \]
\[ (81) \]

the argument of which is strictly positive. At any finite time \( t \), the \( T = 0 \) contribution accounts for particle creation out of the vacuum, as permitted by the uncertainty principle. The \( T \neq 0 \)
part accounts for particle-number changing interactions with the $\chi$ thermal bath. Note that this is also vanishing in the limit $t \to \infty$ for the present setup, since $M < 4m^2$. As noted above, such number-changing processes would not be present for real atoms.

We recall that we have worked in terms of states with a rescaled mass defined by Eq. (11), which depends on the background value of the chameleon field. If we are sensitive to the absolute value of the mass of the matter field, i.e. we can predict the phase evolution based on the mass measured in a vanishing ambient value of the chameleon field, then we can capture the leading effect on the dynamics by expanding

$$E_\phi p - E_\phi p' = \tilde{E}_\phi p - \tilde{E}_\phi p' + \frac{\tilde{m}^2 \langle X \rangle}{\mathcal{M}} \left( 1 + \frac{\langle X \rangle}{\mathcal{M}} \right) \left( \frac{1}{\tilde{E}_\phi p} - \frac{1}{\tilde{E}_\phi p'} \right) - \frac{\tilde{m}^4 \langle X \rangle^2}{2\mathcal{M}^2} \left( \frac{1}{(E_\phi p')^3} - \frac{1}{(E_\phi p)^3} \right) + O\left( \frac{\langle X \rangle^3}{\mathcal{M}^3} \right),$$

where $\tilde{E}_\phi = \sqrt{p^2 + \tilde{m}^2}$. A quantitative estimate of the leading effect will be given later in Sub-sec. IV A. In addition, we can expand the term

$$- \frac{im^2 \lambda \langle X \rangle}{2\mathcal{M}} \left( \frac{1}{E_\phi p} - \frac{1}{E_\phi p'} \right) \rho(p, p'; t) \Delta_{xx}^F \frac{\cos(Mt) - 1}{M^2} = - \frac{i\tilde{m}^2 \lambda \langle X \rangle}{2\mathcal{M}} \left[ \left( 1 + \frac{\langle X \rangle}{\mathcal{M}} \right) \left( \frac{1}{E_\phi p} - \frac{1}{E_\phi p'} \right) - \frac{\tilde{m}^2 \langle X \rangle}{\mathcal{M}} \left( 1 + \frac{\langle X \rangle}{\mathcal{M}} \right) \left( \frac{1}{(E_\phi p')^3} - \frac{1}{(E_\phi p)^3} \right) \right] \rho(p, p'; t) \Delta_{xx}^F \frac{\cos(Mt) - 1}{M^2} + O\left( \frac{\langle X \rangle^3}{\mathcal{M}^3} \right).$$

We remark, however, that a naive expansion about $m^2 \sim \tilde{m}^2$ cannot be made in the time-dependent exponentials in the third to seventh lines of Eq. (79).

### D. Renormalization

The terms in Eq. (79) yield quadratic and logarithmic ultra-violet divergences. At this order, there are three relevant counterterms: the mass counterterms for the $\phi$ and $\chi$ fields and the tadpole counterterm for the $\chi$ field.

The tadpole divergences in the second line of Eq. (79) can be renormalized by the standard counterterms, calculated in vacuum. On the other hand, the logarithmic divergence arising in the third to fifth lines has acquired a non-trivial modulation by virtue of the fact that the interactions have a finite domain of support in time, as have the divergences arising from the vacuum diagram in the final line. In particular, we see that the contributions vanish identically in the limit $t \to 0$. As such, and were we to subtract the usual $t$-independent vacuum divergence, the divergence would
FIG. 3: Diagrammatic representation of the various terms contributing to the right-hand side of the quantum master equation (78): (a)–(d) $\chi$ tadpoles [line 2]; (e) and (f) $\phi$ tadpoles [line 2]; (g)–(i) $\chi$-$\phi$ bubbles [lines 3–5]; and (j) the disconnected vacuum diagram [lines 6 and 7]. Solid lines represent $\phi$ propagators, dashed lines represent $\chi$ propagators, crossed boxes indicate insertions of the matrix element of the density operator $\rho$, and crosses indicate insertions of the background field $\langle X \rangle$. 
persist for all times, except in the limit $t \to \infty$. It follows therefore that the contributions from the relevant counterterms must also vanish in the limit $t \to 0$ and carry the same $t$-dependence.

In order to see this more explicitly, it is instructive to rewrite the divergent terms in the third to fifth lines of Eq. (79) in terms of the more familiar expression for the self-energy. Proceeding in this way and ignoring the thermal corrections (i.e. the terms which vanish at zero temperature), since these are ultra-violet finite and therefore not relevant to the renormalization, we find that

$$\partial_t \rho(p, p'; t) \supset \rho(p, p'; t) \left\{ \frac{1}{E_p} \int_{p^0} \sin \left[ \frac{(p^0 - E_p) t}{p^0 - E_p} \right] i \Pi^{(T=0)}(\text{non-loc})(-p^2) - (p \leftrightarrow p')^* \right\}, \quad (84)$$

where

$$i \Pi^{(T=0)}(\text{non-loc})(-p^2) = \left( - \frac{2im^2}{M} \right)^2 \int_k \frac{-i}{k^2 + M^2 - i\epsilon} \frac{-i}{(k-p)^2 + m^2 - i\epsilon} \quad (85)$$

is the usual non-local, bubble self-energy. (Here, we refer to self-energies that depend on the external momentum flow as non-local, and those that are independent of the external momentum flow, i.e. tadpoles, as local.) We see that the convolution integral over $p^0$ accounts for the finite-time effects, and it must be present also for the counterterm if the divergence is to be removed for all times $t$. It follows then that the relevant counterterm $\delta m^2$ must be local in time, that is it must depend explicitly on a time coordinate, i.e.

$$\delta \hat{S}_{IF} \supset - \frac{1}{2} \sum_{a=\pm} a \int_{xy \in \Omega_t} \delta m^2_{xy} \phi_a^\dagger \phi_a^a, \quad (86)$$

with $\delta m^2_{xy}$ having the double Fourier transform

$$\delta m^2(p, p') = \int_x \int_y e^{-ip\cdot x} e^{ip'\cdot y} \delta m^2_{xy} = (2\pi)^4 \delta^4(p - p') \text{Re} \Pi^{(T=0)}(-p^2) \bigg|_{p = \bar{p}}, \quad (87)$$

which maintains a non-trivial dependence on $p^0$ and therefore $x^0 - y^0$. (More generally, the counterterms may be non-local in time, i.e. depend on multiple time coordinates, as in the case of the coupling-constant counterterms.) Here,

$$i \Pi^{(T=0)}(\text{non-loc})(-p^2) = i \Pi^{(T=0)}_{\text{loc}} + i \Pi^{(T=0)}_{\text{non-loc}}(-p^2) \quad (88)$$

also contains the local, tadpole self-energy given by

$$i \Pi^{(T=0)}_{\text{loc}} = - \frac{2im^2}{M^2} \Delta F^{(T=0)} = - \frac{2im^2}{M^2} \int_k \frac{-i}{k^2 + M^2 - i\epsilon} \quad (89)$$

However, since $i \Pi_{\text{loc}}$ is independent of the four-momentum $p$, it yields a time-independent contribution to the mass counterterm $\delta m^2_{xy}$. We note that only the dispersive (real) part of the one-loop self-energy is subtracted, since this is where the divergence resides.
We should have anticipated that the counterterm must, in general, be non-local in time. This follows from the fact that the external preparation and measurement of the system take place over a finite time, breaking both time-translational invariance and Lorentz invariance. It is for this reason that we have taken the subtraction point for the renormalization to be at a fixed three-momentum $\vec{p}$. One cannot, for instance, straightforwardly apply on-shell renormalization, since the loop corrections do not depend only on the Lorentz scalar $p^2$. The counterterm is independent of the three-momentum and, therefore, spatially local. We note that, in the limit $t \to \infty$, we have

$$\lim_{t \to \infty} \frac{1}{\pi} \frac{\sin \left[ \left( p^0 - E^\phi \right) t \right]}{p^0 - E^\phi} \to \delta(p^0 - E^\phi),$$

(90)

setting $-p^2 = m^2$ on-shell, such that we recover on-shell renormalization. The $t$-dependent factors should be compared with those that arise in the modified Feynman rules of the interaction-picture formulation of non-equilibrium field theory [96, 97].

The addition of bi-local terms (that is terms that depend on two spacetime coordinates) to the action of an open system is not so unusual. Specifically, we can regard the time-local counterterm above as a correction to the usual bi-local source (see, e.g., Refs. [96, 103, 104]) that can be used to encode the impact of the environment on the quadratic fluctuations of the open system on approaches based on the (two-particle irreducible) quantum effective action [105], as embedded in the Schwinger-Keldysh closed-time-path formalism [98, 99].

The full form of the counterterm action (including only the terms relevant at the order we are working) is then

$$\delta \hat{S}_{\text{IF}} = - \sum_{a = \pm} \alpha \left[ \int_x \delta \alpha \chi^a_x + \frac{1}{2} \int_{xy} \delta m^2_{xy} \phi^a_x \phi^a_y + \frac{1}{2} \int_{xy} \delta M^2_{xy} \chi^a_x \chi^a_y \right],$$

(91)

with

$$\delta m^2_{xy} = - \frac{2m^2}{\mathcal{M}^2} \Delta^{(T=0)}_{xx} \delta^{(4)}_{xy} + \int_{pp'} e^{ip_x - ip'_x} (2\pi)^4 \delta^4(p - p') \text{Re} \Gamma^{(T=0)}_{\text{non-loc}}(-p^2) \bigg|_{p = \bar{p}},$$

(92a)

$$\delta M^2_{xy} = - \left[ \frac{2m^2}{\mathcal{M}^2} \Delta^{(T=0)}_{xx} + \frac{\lambda}{2} \Delta^{(T=0)}_{xx} \right] \delta^{(4)}_{xy} + \int_{pp'} e^{ip_x - ip'_x} (2\pi)^4 \delta^4(p - p') \text{Re} \Sigma^{(T=0)}_{\text{non-loc}}(-p^2) \bigg|_{p = \bar{p}},$$

(92b)

$$\delta \alpha = - \frac{m^2}{\mathcal{M}} D^{(T=0)}_{xx} - \frac{\lambda \langle X \rangle}{2} \Delta^{(T=0)}_{xx},$$

(92c)

for any given regularization procedure (e.g. dimensional regularization). Here, we have introduced the non-local chameleon self-energy

$$i \Sigma_{\text{non-loc}}(-p^2) = \left( - \frac{2im^2}{\mathcal{M}} \right)^2 \int_j \frac{-i}{k^2 + m^2 - i\epsilon} \frac{-i}{(k - p)^2 + m^2 - i\epsilon}.$$
After making the subtraction, all that remains of the tadpole diagrams in the second line of Eq. [79] are the thermal parts for the \( \chi \) field, given by the integral

\[
\Delta_{xx}^{F(T\neq 0)} \equiv 2 \int d\Pi_k f(E_k^x) = \frac{T^2}{2\pi^2} \int_{M/T}^\infty d\xi \frac{\sqrt{\xi^2 - (M/T)^2}}{e^\xi - 1},
\]

which reduces to \( T^2/12 \) in the limit \( M = 0 \) and \( \sim T^2/29 \) for \( M/T \sim 1 \). The vacuum diagram is renormalized by the two mass counterterms at the level of its two sub-diagrams: the non-local one-loop \( \phi \) and \( \chi \) self-energies. Since the loop integrals arising from the non-local diagrams cannot be performed in closed form, we do not present these explicitly.

IV. DISCUSSION

The expression \([79]\) after the renormalization [see Eq. (91)] constitutes the main result of this article, i.e. the one-loop master equation describing the open quantum dynamics of a scalar matter field \( \phi \) induced by the light scalar \( \chi \). To discuss the content and implications of this dynamics, we first rewrite the master equation as

\[
\partial_t \rho(p, p'; t) = -[i(u(p, p'; t) + \Gamma(p, p'; t)] \rho(p, p'; t) + \int_k \gamma(p, p', k; t)\rho(p - k, p' - k; t),
\]

where we have defined

\[
u(p, p'; t) \equiv E_p^x - E_{p'}^x + \frac{m^2}{\mathcal{M}} \left( \frac{1}{E_p^x} - \frac{1}{E_{p'}^x} \right) \Delta_{xx}^{F(T\neq 0)} \left\{ \frac{1}{\mathcal{M}} + \frac{\lambda}{2} \langle X \rangle \cos(Mt) - 1 \right\},
\]

\[- \left\{ \frac{1}{E_p^x} \int_p \sin\left(\frac{p^0 - E_p^0}{p^0 - E_p^x}\right) \left[ \text{Re} \Pi_{\text{non-loc}}(-p^2) \right. \right. \]

\[- \left. \left. - \text{Re} \Pi_{\text{non-loc}}(T=0)(-p^2) \right|_{p = p} \right] - (p \leftrightarrow p'), \]

\[
\Gamma(p, p'; t) \equiv \frac{1}{E_p^x} \int_p \sin\left(\frac{p^0 - E_p^0}{p^0 - E_p^x}\right) \text{Im} \Pi_{\text{non-loc}}(-p^2) + (p \leftrightarrow p'),
\]

\[
\gamma(p, p', k; t) \equiv \frac{4m^4}{\mathcal{M}^2} \sum_{s = \pm} \left\{ \frac{1}{2E_k^x2E_p^{\pm}\pm - E_{p'}^x} \frac{s}{sE_k^x + E_p^{\pm} - E_{p'}^x} \right. \]

\[
\times \left(1 - \exp\left[i(sE_k^x + E_p^{\pm} - E_{p'}^x)\right] \right)f(sE_k^x) - (p \leftrightarrow p'), \]

We have omitted the contribution from the disconnected vacuum diagram. We note that the non-local self-energy \( \Pi_{\text{non-loc}}(-p^2) \) appearing here contains also the thermal corrections, i.e.

\[
i\Pi_{\text{non-loc}}(-p^2) = \left( -2im^2 \right) \int_k \left[ \frac{-i}{k^2 + \mathcal{M}^2 - i\epsilon} + 2\pi f(|k^0|)\delta(k^2 + \mathcal{M}^2) \right] \frac{-i}{(k-p)^2 + m^2 - i\epsilon}.
\]
The coefficients $u$ and $\Gamma$ are real, whereas $\gamma$ is complex. This decomposition provides a clear interpretation of the resulting master equation: $u$ corresponds to coherent evolution, resulting from the mass shifts, $\Gamma$ corresponds to decays and, together with the real part of $\gamma$, is responsible for decoherence; $\gamma$ also accounts for momentum diffusion, due to the coupling between the different momentum states. We note that $u(p, p; t) = 0$, as it should, since diagonal elements of the density matrix must be real.

The master equation (95) is time-local, but with time-dependent coefficients. In general, such master equations do not necessarily preserve the trace and positivity of the density matrix, unless they are in the Lindblad form \cite{106}. In order to assess whether Eq. (95) can be recast in Lindblad form and if it therefore corresponds to a completely positive and trace preserving map, we would need to evaluate the phase-space integrals that appear in the coefficients (96) numerically. We leave such dedicated numerical studies for future work.

A. Quantitative estimates

Before concluding, we now provide some quantitative estimates of the effects induced by the light scalar. We consider the effects on a system of atoms in a chameleon scalar environment in an idealized experimental set up. Tests of the coherence and decoherence of atomic systems typically take place in high-quality vacuum chambers. If we assume the vacuum chamber is spherical then we can predict the form of the background chameleon field profile inside the chamber. The walls of the chamber are dense, and so the chameleon is massive, and we can take the chameleon field to be constant in the walls of the chamber. Inside the vacuum chamber, the chameleon can be much lighter, and the field evolves towards the value that minimizes the effective potential, Eq. (17), when $\rho_{\text{ext}}$ is the density of the residual gas in the chamber. However, over a large part of the chameleon parameter space, and for typical vacuum chamber sizes, there is not enough space for the chameleon to reach the minimum of the effective potential. In this case, the field adjusts its value so that its Compton wavelength becomes of order the size of the vacuum chamber, and, at the centre of the spherical chamber, the expectation value of the chameleon field and the mass of its fluctuations at the centre of the sphere are given by \cite{40}

$$\langle X \rangle = \frac{q}{\sqrt{\lambda L}}, \quad M = \frac{q}{\sqrt{2}L},$$  \hspace{1cm} (98)$$

\footnote{The possible mapping of time-local master equations to master equations in the Lindblad form has been considered in Ref. \cite{107} by coupling the system described by the non-Lindblad master equation to an ancilla.}
FIG. 4: Schematic of the experimental setup. The green line represents the profile of the chameleon across a cross-section of the spherical vacuum chamber (in grey).

where \( q = 1.287 \) \( ^{40} \) \( ^{85} \) and \( L \) is the radius of a spherical vacuum chamber. The form of the chameleon profile inside the chamber is illustrated schematically in Fig. 4.

In experiments with microscopic test masses, one will typically be in the regime \( \tilde{m}/M \ll 1 \). Therefore, in order to make a conservative estimate, we assume that we are sensitive to the dominant coherent shift arising from the change in the effective mass of the matter field due to the background value of the chameleon and thermal corrections due to the chameleon fluctuations. The leading contributions to this shift scale like the first power of \( \tilde{m}/M \) in contrast to \( \Gamma \) and \( \gamma \), which scale as \( (\tilde{m}/M)^2 \), cf. Eqs. (96b) and (96c).

Motivated primarily by using the atom-interferometry precision measurements, we rewrite the free-phase part of Eq. (96a) via Eq. (82), performing the non-relativistic expansion of the \( 1/\tilde{E} \) terms to find

\[
\Delta u \approx \frac{\langle X \rangle}{2M} \tilde{m} \left[ 1 - \frac{\lambda T^2}{29M^2} \right] \frac{v^2}{c}.
\]  

Here, we have defined the characteristic velocity scale by

\[
v^2 = \frac{\|p\|^2 - |p'|^2}{\tilde{m}^2},
\]

taken the maximum value of \( 1 - \cos(Mt) = 2 \), used the relation (94) for \( \Delta_{xx}^{F(T \neq 0)} \) and restored the dimensions, so that \( [u] = \text{Hz} \) for \( T \) expressed in mass units and \( \langle X \rangle \) expressed in units of inverse length, as it should for a rate of change. The thermal correction here corresponds to the thermal shift in the background chameleon field to smaller values, arising from the third term in the second line of Eq. (79).

For illustration, we consider the quantum test mass to be the \(^{87}\text{Rb} \) isotope with \( \tilde{m} = 87 m_u \) (where \( m_u \) is an atomic mass unit) and choose \( \lambda = 1/10 \) and \( M = M_{Pl} \), \( L = 1 \) m for the radius of the vacuum chamber and \( v = 10 \) m s\(^{-1} \) (velocities of up to 6 m s\(^{-1} \) were reported in atomic transport experiments \(^{108} \)) for the characteristic velocity scale. We further consider the vacuum
chamber to be in a thermal equilibrium at temperature $T = 1 \text{ mK}$. The chosen values imply $M \approx 10^{-16} m_u$ and $M/T \sim 1$, in which case the thermal shift to the chameleon background field value is subleading. We then obtain $\Delta u \approx 10^{-23}$ Hz, which is far out of reach of the current atom-interferometry sensitivity of order $10^{-8}$ Hz\footnote{This value is inferred from Ref. \cite{109}, which reported a phase measurement with statistical uncertainty of $10^{-8}$ rad obtained after $\sim 1$ day of integration time and for a duration of the order of $1 \text{ s}$ for each experimental run.}

The conservative value of $\mathcal{M} = M_{\text{Pl}}$ is motivated by constraints on chameleon theories \cite{110}, and the value of $\lambda < 1$ was chosen to remain well within the regime of perturbative validity. We reiterate, however, that the chosen parameters are in tension with the current experimental bounds for the chameleon potential considered here, as discussed earlier in Subsec. II A. Even so, our aim is to provide a conservative estimate of the order of magnitude of the effects. We see that the effects induced by the quantum fluctuations of the quartic chameleon considered here are negligibly small and potentially out of reach of any near-future experiment. This is consistent with the fact that already the classical effects, i.e. effects which depend on the classical background field value $\langle X \rangle$ are constrained to be very small.

Finally, we would like to comment on the difference of the current analysis with that of Ref. \cite{40}, where a proposal of testing the modification of gravitational acceleration $g \rightarrow g + \delta g$ induced by the chameleon scalar fields has been put forward. Specifically, the change $\delta g$ predicted in Ref. \cite{40} scales as $\delta g \propto \partial X V(X)$, i.e. with the gradient of the chameleon potential. This is a classical effect which arises because the length of the paths that atoms explore in the interferometer depends on the fifth force due to the gradients of the background chameleon field configuration. This is not the same experimental setup as considered in this work, as in order to detect the classical fifth force due to the chameleon, a macroscopic source mass must also be placed inside the vacuum chamber. For the choices of the chameleon potential $\lambda \approx 1/10$ and $\mathcal{M} \sim M_{\text{Pl}}$, and a vacuum chamber similar to the simplified chamber we consider here, a phase shift of order $10^{-3}$ would be induced if the interferometry experiment were performed with Rubidium atoms held within 1 cm of a massive sphere of radius 1 cm and density $1 \text{ g cm}^{-3}$. This should be contrasted with the prediction of Eq. (96a) for the coherence shift, which scales with the background field value instead, i.e. $u \propto \langle X \rangle$, see also Fig. 1.

It therefore seems that, while atom interferometry provides a powerful tool in the search for the fifth forces due to light scalar fields, a direct detection of the effects induced by the quantum fluctuations of the light fields remains extremely challenging. Here, it would be interesting to apply the present theory to the other light scalar field models discussed in Sec. II in order to
obtain quantitative estimates. While it is likely that the induced effects will remain elusive also in these cases, we note that possible pathways for improvements include using higher test masses to increase the ratio \( \tilde{m}/M \), provided they do not affect the screening mechanism. In this context, we note that a Bell test was recently performed with levitated nanoparticles containing \( 10^{10} \) silica atoms \[111\], amounting to an increase of \( \tilde{m} \) by 10 orders of magnitude as compared to single-atom interferometry. Other possibilities include, for example, considering the effects induced in other detection platforms, in particular in optical atomic clocks, which provide the most accurate measurement tool currently being developed, with reported relative precision reaching \( 10^{-19} \) \[112\]. Alternatively, one could consider modifying the experimental setup so that each branch of the interferometer experiences a different background value of the light field together with its gradient. In such a case, the description provided here [Eq. (95)] has to be extended so as to account for the spatially inhomogeneous background provided by the screening field. We leave this and related extensions for future work.

V. CONCLUSIONS AND OUTLOOK

Light scalar fields, which couple to matter either conformally or through a Higgs portal, are well motivated as extensions of the Standard Model and/or general relativity. Models such as the chameleon, discussed here, possess a screening mechanism that allows them to avoid local searches for fifth forces while remaining light on cosmological scales and coupling to matter with at least gravitational strength. In this work, we have derived a quantum master equation from first principles, which allows us to describe how an environment comprising such a scalar field affects the dynamics of a quantum test particle. This is, to the best of our knowledge, the first work in which the quantum dynamics of both an atomic test particle and the (conformally coupled) light scalar are studied; excepting Ref. \[113\], previous work looking at the effects of screened scalar fields in, for example, atom-interferometry experiments treated the scalar only classically.

Herein, we have used a second scalar field as a proxy for the quantum probe, imagining, for instance, an atom in an atom-interferometry experiment. While this simple scalar field theory provides a convenient playground in which to develop the necessary techniques, it nevertheless has some shortcomings as a toy model of an atom, such as allowing for decay and production processes of such a complex object, and the extension of this work to more realistic models of probe systems may be presented elsewhere.

Beginning from the path-integral approach of the Feynman-Vernon influence functional, we have
employed an LSZ-like reduction technique, constructed in the operator-based framework of thermo field dynamics, which allows us to make use of diagrammatic techniques from (non-equilibrium) quantum field theory, while also making concrete connection with the single-particle matrix elements of the density operator of interest for quantum probes. In addition, we have shown that the consistent renormalization of the resulting master equation in the non-Markovian regime requires the introduction of time-local counterterms, allowing us to make quantitative predictions that are independent of any ultra-violet cut-off.

The resulting master equation describes the coherent dynamics of the quantum test particle, as well as decoherence and momentum diffusion. Having access to quantitative estimates of these effects, we have confirmed that their experimental observation remains a challenge. Even so, the present formalism allows us to identify possible pathways for improvements in future searches, including the use of heavier test masses and the studies of the effects in optical atomic clocks or non-homogeneous scalar-field backgrounds. Importantly, the present work provides a robust and complementary approach for studying open quantum dynamics, which may shed new light on, e.g., the contentious area of gravitational decoherence. We leave such studies for future work.

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[1] T. Clifton, P. G. Ferreira, A. Padilla and C. Skordis, Modified Gravity and Cosmology, Phys. Rept. 513 (2012) 1 [1106.2476].
[2] A. Joyce, B. Jain, J. Khoury and M. Trodden, Beyond the Cosmological Standard Model. Phys. Rept. 568 (2015) 1 [1407.0059].

[3] W. Hu, R. Barkana and A. Gruzinov, Cold and fuzzy dark matter. Phys. Rev. Lett. 85 (2000) 1158 astro-ph/0003365.

[4] L. Hui, J. P. Ostriker, S. Tremaine and E. Witten, Ultralight scalars as cosmological dark matter, Phys. Rev. D95 (2017) 043541 1610.08297.

[5] P. Bull et al., Beyond ΛCDM: Problems, solutions, and the road ahead, Phys. Dark Univ. 12 (2016) 56 1512.05356.

[6] E. J. Copeland, M. Sami and S. Tsujikawa, Dynamics of dark energy, Int. J. Mod. Phys. D15 (2006) 1753 hep-th/0603057.

[7] M. S. Turner, Coherent Scalar Field Oscillations in an Expanding Universe. Phys. Rev. D28 (1983) 1243.

[8] W. H. Press, B. S. Ryden and D. N. Spergel, Single Mechanism for Generating Large Scale Structure and Providing Dark Missing Matter. Phys. Rev. Lett. 64 (1990) 1084.

[9] S.-J. Sin, Late time cosmological phase transition and galactic halo as Bose liquid. Phys. Rev. D50 (1994) 3650 hep-ph/9205208.

[10] J. Goodman, Repulsive dark matter. New Astron. 5 (2000) 103 astro-ph/0003018.

[11] P. J. E. Peebles, Fluid dark matter, Astrophys. J. 534 (2000) L127 astro-ph/0002495.

[12] A. Arbey, J. Lesgourgues and P. Salati, Quintessential haloes around galaxies, Phys. Rev. D64 (2001) 123528 astro-ph/0105564.

[13] J. Lesgourgues, A. Arbey and P. Salati, A light scalar field at the origin of galaxy rotation curves, New Astron. Rev. 46 (2002) 791.

[14] L. Amendola and R. Barbieri, Dark matter from an ultra-light pseudo-Goldsone-boson, Phys. Lett. B642 (2006) 192 hep-ph/0509257.

[15] H.-Y. Schive, T. Chiuheh and T. Broadhurst, Cosmic Structure as the Quantum Interference of a Coherent Dark Wave, Nature Phys. 10 (2014) 496 1406.6586.

[16] E. Gessner, A new scalar tensor theory for gravity and the flat rotation curves of spiral galaxies, Astrophys. Space Sci. 196 (1992) 29.

[17] J. Khoury, Alternative to particle dark matter. Phys. Rev. D91 (2015) 024022 [1409.0012].

[18] C. Burrage, E. J. Copeland and P. Millington, Radial acceleration relation from symmetron fifth forces, Phys. Rev. D95 (2017) 064050 [1610.07529].

[19] C. A. O’Hare and C. Burrage, Stellar kinematics from the symmetron fifth force in the Milky Way disk. Phys. Rev. D98 (2018) 064019 [1805.05226].

[20] C. Burrage, E. J. Copeland, C. Käding and P. Millington, Symmetron Scalar Fields: Modified Gravity, Dark Matter or Both?. 1811.12301.

[21] C. Burrage and J. Sakstein, Tests of Chameleon Gravity. Living Rev. Rel. 21 (2018) 1 [1709.09071].

[22] E. Adelberger, J. Gundlach, B. Heckel, S. Hoedl and S. Schlamminger, Torsion balance experiments: 32
A low-energy frontier of particle physics, *Progress in Particle and Nuclear Physics* 62 (2009) 102

[23] K. Koyama, *Cosmological Tests of Modified Gravity*, *Rept. Prog. Phys.* 79 (2016) 046902

[24] J. Schmiedmayer and H. Abele, *Probing the dark side*, *Science* 349 (2015) 786

[25] Y. V. Stadnik and V. V. Flambaum, *Manifestations of dark matter and variations of fundamental constants in atoms and astrophysical phenomena*, *arXiv:1509.00966* (2015).

[26] J. C. Berengut, D. Budker, C. Delaunay, V. V. Flambaum, C. Frugiele, E. Fuchs et al., *Probing new light force-mediators by isotope shift spectroscopy*, *arXiv:1704.05068* (2017).

[27] Y. V. Stadnik, *Manifestations of Dark Matter and Variations of the Fundamental Constants of Nature in Atoms and Astrophysical Phenomena*, Ph.D. thesis, University of New South Wales, 2017.

[28] M. Safronova, D. Budker, D. DeMille, D. F. J. Kimball, A. Derevianko and C. Clark, *Search for new physics with atoms and molecules*, *arXiv:1710.01833* (2017).

[29] A. Bassi, A. Großardt and H. Ulbricht, *Gravitational decoherence*, *Classical and Quantum Gravity* 34 (2017) 193002.

[30] J. Minár, P. Sekatski and N. Sangouard, *Bounding quantum-gravity-inspired decoherence using atom interferometry*, *Phys. Rev. A* 94 (2016) 062111.

[31] J. Minár, P. Sekatski, R. Stevenson and N. Sangouard, *Testing unconventional decoherence models with atoms in optical lattices*, *arXiv:1601.05381* (2016).

[32] F. Fröwis, P. Sekatski, W. Dür, N. Gisin and N. Sangouard, *Macroscopic quantum states: measures, fragility and implementations*, *arXiv:1706.06173* (2017).

[33] A. Bassi, K. Lochan, S. Satin, T. P. Singh and H. Ulbricht, *Models of wave-function collapse, underlying theories, and experimental tests*, *Rev. Mod. Phys.* 85 (2013) 471.

[34] M. Bahrami, M. Paternostro, A. Bassi and H. Ulbricht, *Proposal for a noninterferometric test of collapse models in optomechanical systems*, *Phys. Rev. Lett.* 112 (2014) 210404.

[35] J. Li, S. Zippilli, J. Zhang and D. Vitali, *Discriminating the effects of collapse models from environmental diffusion with levitated nanospheres*, *Phys. Rev. A* 93 (2016) 050102.

[36] A. Vinante, M. Bahrami, A. Bassi, O. Usenko, G. Wijts and T. H. Oosterkamp, *Upper bounds on spontaneous wave-function collapse models using millikelvin-cooled nanocantilevers*, *Phys. Rev. Lett.* 116 (2016) 090402.

[37] M. Tóroš and A. Bassi, *Bounds on collapse models from matter-wave interferometry*, *arXiv:1601.03672* (2016).

[38] J. Bateman, I. McHardy, A. Merle, T. R. Morris and H. Ulbricht, *On the existence of low-mass dark matter and its direct detection*, *Scientific reports* 5 (2015) 8058.

[39] G. Winstone, M. Radenmacher, R. Bennett, S. Buhmann and H. Ulbricht, *Direct measurement of short-range forces with a levitated nanoparticle*, *arXiv:1712.01426* (2017).

[40] C. Burrage, E. J. Copeland and E. A. Hinds, *Probing Dark Energy with Atom Interferometry*, *JCAP*.
[41] P. Hamilton, M. Jaffe, P. Haslinger, Q. Simmons, H. Müller and J. Khoury, *Atom-interferometry constraints on dark energy*, Science 349 (2015) 849 [1502.0388].

[42] S. Schlögel, S. Clees and A. Füzfa, *Probing Modified Gravity with Atom-Interferometry: a Numerical Approach*, Phys. Rev. D93 (2016) 104036 [1507.03081].

[43] M. Jaffe, P. Haslinger, V. Xu, P. Hamilton, A. Upadhye, B. Elder et al., *Testing sub-gravitational forces on atoms from a miniature, in-vacuum source mass*, Nature Physics 13 (2017) 938 [1612.05171].

[44] B. Elder, J. Khoury, P. Haslinger, M. Jaffe, H. Müller and P. Hamilton, *Chameleon dark energy and atom interferometry*, Phys. Rev. D94 (2016) 044051 [1603.06587].

[45] P. Brax and A.-C. Davis, *Atomic interferometry test of dark energy*, Phys. Rev. D 94 (2016) 104069.

[46] P. Brax, S. Fichet and G. Pignol, *Bounding quantum dark forces*, arXiv:1710.00850 (2017).

[47] R. P. Feynman and F. L. Vernon, *The theory of a general quantum system interacting with a linear dissipative system*, Annals of physics 24 (1963) 118.

[48] A. O. Caldeira and A. J. Leggett, *Path integral approach to quantum brownian motion*, Physica A 121A (1983) 587.

[49] E. A. Calzetta and B.-L. Hu, *Nonequilibrium Quantum Field Theory*. Cambridge University Press, Cambridge UK, 2008.

[50] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems*. Oxford University Press, Oxford, 2002.

[51] B. L. Hu, J. P. Paz and Y. Zhang, *Quantum brownian motion in a general environment: Exact master equation with nonlocal dissipation and colored noise*, Phys. Rev. D 45 (1992) 2843.

[52] B. L. Hu, J. P. Paz and Y. Zhang, *Quantum brownian motion in a general environment. ii. nonlinear coupling and perturbative approach*, Phys. Rev. D 47 (1993) 1576.

[53] J. F. Koksma, T. Prokopec and M. G. Schmidt, *Decoherence in an Interacting Quantum Field Theory: The Vacuum Case*, Phys. Rev. D81 (2010) 065030 [0910.5733].

[54] J. F. Koksma, T. Prokopec and M. G. Schmidt, *Decoherence in an Interacting Quantum Field Theory: Thermal Case*, Phys. Rev. D83 (2011) 085011 [1102.4713].

[55] F. Lombardo and F. D. Mazzitelli, *Coarse graining and decoherence in quantum field theory*, Phys. Rev. D 53 (1996) 2001.

[56] F. C. Lombardo and D. L. Nacir, *Decoherence during inflation: The generation of classical inhomogeneities*, Phys. Rev. D 72 (2005) 063506.

[57] F. C. Lombardo, *Influence functional approach to decoherence during inflation*, Brazilian Journal of Physics 35 (2005) 391.

[58] D. Boyanovsky, *Effective field theory during inflation: Reduced density matrix and its quantum master equation*, Phys. Rev. D92 (2015) 023527 [1506.07395].

[59] D. Boyanovsky, *Effective field theory during inflation. II. Stochastic dynamics and power spectrum*
suppression. *Phys. Rev.* **D93** (2016) 043501 [1511.06649].

[60] D. Boyanovsky, *Fermionic influence on inflationary fluctuations*, *Phys. Rev.* **D93** (2016) 083507 [1602.05609].

[61] D. Boyanovsky, *Imprint of entanglement entropy in the power spectrum of inflationary fluctuations*, *Phys. Rev.* **D98** (2018) 023515 [1804.07967].

[62] C. Burgess, R. Holman, G. Tasinato and M. Williams, *Eft beyond the horizon: Stochastic inflation and how primordial quantum fluctuations go classical*, *JHEP* **2015** (2015) 90.

[63] T. J. Hollowood and J. I. McDonald, *Decoherence, discord, and the quantum master equation for cosmological perturbations*, *Phys. Rev. D* **95** (2017) 103521.

[64] N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, *Quarkonium suppression in heavy-ion collisions: an open quantum system approach*, *Phys. Rev.* **D96** (2017) 034021 [1612.07248].

[65] N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, *Heavy quarkonium suppression in a fireball*, *Phys. Rev.* **D97** (2018) 074009 [1711.04515].

[66] T. Takahasi and H. Umezawa, *Thermo field dynamics*, *Collect. Phenom.* **2** (1975) 55.

[67] T. Arimitsu and H. Umezawa, *A General Formulation of Nonequilibrium Thermo Field Dynamics*, *Prog. Theor. Phys.* **74** (1985) 429.

[68] T. Arimitsu and H. Umezawa, *Non-Equilibrium Thermo Field Dynamics*, *Prog. Theor. Phys.* **77** (1987) 32.

[69] F. C. Khanna, A. P. C. Malbouisson, J. M. C. Malbouisson and A. E. Santana, *Thermal Quantum Field Theory: Algebraic Aspects and Applications*. World Scientific, Singapore, 2009.

[70] H. Lehmann, K. Symanzik and W. Zimmermann, *On the formulation of quantized field theories*, *Nuovo Cim.* **1** (1955) 205.

[71] C. Agon, V. Balasubramanian, S. Kasko and A. Lawrence, *Coarse Grained Quantum Dynamics*, *Phys. Rev.* **D98** (2018) 025019 [1412.3148].

[72] D. Boyanovsky, *Effective Field Theory out of Equilibrium: Brownian quantum fields*, *New J. Phys.* **17** (2015) 063017 [1503.00156].

[73] C. Agn and A. Lawrence, *Divergences in open quantum systems*, *JHEP* **04** (2018) 008 [1709.10095].

[74] C. Anastopoulos and B. L. Hu, *A master equation for gravitational decoherence: probing the textures of spacetime*, *Classical and Quantum Gravity* **30** (2013) 165007.

[75] M. P. Blencowe, *Effective field theory approach to gravitationally induced decoherence*, *Phys. Rev. Lett.* **111** (2013) 021302.

[76] T. Markkanen and S. Nurmi, *Dark matter from gravitational particle production at reheating*, *JCAP* **1702** (2017) 008 [1512.07288].

[77] Y. Ema, R. Jinno, K. Mukaida and K. Nakayama, *Gravitational particle production in oscillating backgrounds and its cosmological implications*, *Phys. Rev.* **D94** (2016) 063517 [1604.08899].

[78] Y. Ema, K. Nakayama and Y. Tang, *Production of Purely Gravitational Dark Matter*, *JHEP* **09** (2018) 135 [1804.07471].
M. Fairbairn, K. Kainulainen, T. Markkanen and S. Nurmi, *Despicable Dark Relics: generated by gravity with unconstrained masses*, 1808.08236.

G. Alonso-Lvarez and J. Jaeckel, *Lightish but clumpy: scalar dark matter from inflationary fluctuations*, JCAP 1810 (2018) 022 1807.09755.

Y. Fujii and K.-I. Maeda, *The Scalar-Tensor Theory of Gravitation*, ch. 3, pp. 61–76. Cambridge University Press, Cambridge, 2003.

C. Burrage, E. J. Copeland, P. Millington and M. Spannowsky, *Fifth forces, Higgs portals and broken scale invariance*, JCAP 1811 (2018) 036 1804.07180.

E. G. Adelberger, B. R. Heckel and A. E. Nelson, *Tests of the gravitational inverse square law*, Ann. Rev. Nucl. Part. Sci. 53 (2003) 77 hep-ph/0307284.

A. Nicolis, R. Rattazzi and E. Trincherini, *The Galileon as a local modification of gravity*, Phys. Rev. D79 (2009) 064036 0811.2197.

J. Khoury and A. Weltman, *Chameleon cosmology*, Phys. Rev. D69 (2004) 044026 astro-ph/0309411.

J. Khoury and A. Weltman, *Chameleon Fields: Awaiting Surprises for Tests of Gravity in Space*, Phys. Rev. Lett. 93 (2004) 171104 astro-ph/0309300.

K. Hinterbichler and J. Khoury, *Symmetron Fields: Screening Long-Range Forces Through Local Symmetry Restoration*, Phys. Rev. Lett. 104 (2010) 231301 1001.4525.

K. Hinterbichler, J. Khoury, A. Levy and A. Matas, *Symmetron Cosmology*, Phys. Rev. D84 (2011) 103521 1107.2112.

H. Dehnen, H. Frommert and F. Ghaboussi, *Higgs field and a new scalar - tensor theory of gravity*, Int. J. Theor. Phys. 31 (1992) 109.

T. Damour and A. M. Polyakov, *The String dilaton and a least coupling principle*, Nucl. Phys. B423 (1994) 532 hep-th/9401069.

M. Pietroni, *Dark energy condensation*, Phys. Rev. D72 (2005) 043535 astro-ph/0505615.

K. A. Olive and M. Pospelov, *Environmental dependence of masses and coupling constants*, Phys. Rev. D77 (2008) 043524 0709.3825.

C. Burrage, E. J. Copeland and P. Millington, *Radiative Screening of Fifth Forces*, Phys. Rev. Lett. 117 (2016) 211102 1604.06051.

C. Deffayet, S. Deser and G. Esposito-Farese, *Generalized Galileons: All scalar models whose curved background extensions maintain second-order field equations and stress-tensors*, Phys. Rev. D80 (2009) 064015 0906.1967.

G. W. Horndeski, *Second-order scalar-tensor field equations in a four-dimensional space*, Int. J. Theor. Phys. 10 (1974) 363.

P. Millington and A. Pilaftsis, *Perturbative nonequilibrium thermal field theory*, Phys. Rev. D88 (2013) 085009 1211.3152.
Gradient Expansion, \textit{Phys. Lett.} \textbf{B724} (2013) 56 [1304.7249].

[98] J. S. Schwinger, \textit{Brownian Motion of a Quantum Oscillator}, \textit{J. Math. Phys.} \textbf{2} (1961) 407.

[99] L. V. Keldysh, \textit{Diagram technique for nonequilibrium processes}, \textit{Zh. Eksp. Teor. Fiz.} \textbf{47} (1964) 1515.

[100] M. Le Bellac, \textit{Thermal field theory}. Cambridge University Press, 1996.

[101] G. ’t Hooft and M. J. G. Veltman, \textit{Diagrammar}, \textit{NATO Sci. Ser. B} \textbf{4} (1974) 177.

[102] R. L. Kobes and G. W. Semenoff, \textit{Discontinuities of Green Functions in Field Theory at Finite Temperature and Density}, \textit{Nucl. Phys.} \textbf{B260} (1985) 714.

[103] E. Calzetta and B. L. Hu, \textit{Nonequilibrium Quantum Fields: Closed Time Path Effective Action, Wigner Function and Boltzmann Equation}, \textit{Phys. Rev.} \textbf{D37} (1988) 2878.

[104] J. Berges, \textit{Introduction to nonequilibrium quantum field theory}, \textit{AIP Conf. Proc.} \textbf{739} (2005) 3 [hep-ph/0409233].

[105] J. M. Cornwall, R. Jackiw and E. Tomboulis, \textit{Effective Action for Composite Operators}, \textit{Phys. Rev.} \textbf{D10} (1974) 2428.

[106] M. J. W. Hall, J. D. Cresser, L. Li and E. Andersson, \textit{Canonical form of master equations and characterization of non-markovianity}, \textit{Phys. Rev. A} \textbf{89} (2014) 042120.

[107] M. R. Hush, I. Lesanovsky and J. P. Garrahan, \textit{Generic map from non-lindblad to lindblad master equations}, \textit{Phys. Rev. A} \textbf{91} (2015) 032113.

[108] S. Schmid, G. Thalhammer, K. Winkler, F. Lang and J. Denschlag, \textit{Long distance transport of ultracold atoms using a 1d optical lattice}, \textit{New J. Phys.} \textbf{8} (2006) 159.

[109] B. Estey, C. Yu, H. Müller, P.-C. Kuan and S.-Y. Lan, \textit{High-resolution atom interferometers with suppressed diffraction phases}, \textit{Phys. Rev. Lett.} \textbf{115} (2015) 083002.

[110] C. Burrage and J. Sakstein, \textit{A compendium of chameleon constraints}, \textit{JCAP} \textbf{1611} (2016) 045 [1609.01192].

[111] I. Marinković, A. Wallucks, R. Riedinger, S. Hong, M. Aspelmeyer and S. Gröblacher, \textit{Optomechanical bell test}, \textit{Phys. Rev. Lett.} \textbf{121} (2018) 220404.

[112] G. E. Marti, R. B. Hutson, A. Goban, S. L. Campbell, N. Poli and J. Ye, \textit{Imaging optical frequencies with 100 µHz precision and 1.1 µm resolution}, \textit{Phys. Rev. Lett.} \textbf{120} (2018) 103201.

[113] P. Brax and S. Fichet, \textit{Quantum Chameleons}, [1809.10166].

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