Upper Limit for Tidal Turbines

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Introduction
Low-head hydropower such as tidal power offers a promising contributing to the world’s future renewable energy mix. For power extraction, hydrokinetic turbines are or will be installed at several promising sites on earth. For investment decisions and optimal installation and operation of the turbines, sophisticated models that capture the relevant physical effects are necessary.

A common method to model hydrokinetic turbines is the disc actuator theory as used by Garrett & Cummins [1] (in the following denoted by approach I), Whelan et al. [2], Houlsby et al. [3] (both approach II) and Pelz et al. [4] (approach III) for a single turbine in quasi-stationary open channel flow. The models differ by the considered physical effects: Approach I only considers the confinement due to finite water height, whereas approach II and III consider the water head drop above the turbine due to energy extraction as well. Approaches I and II imply an undisturbed turbine streamtube, whereas approach III considers the contraction of the turbine streamtube due to the water head drop. Pelz et al. [4] herein treat a generic vertical turbine from the channel bottom through the surface, which enables a strictly axiomatic treatment of the streamtube deformation.

They show, that neglecting the deformation of the turbine streamtube, i.e. approach I and II, the yields to an overestimated power extraction and for high blockages and energy extraction even physically impossible results which conflict with conservation laws.

It is the aim of this paper to discuss the results of Pelz et al. [4] in the context of submerged turbines. Like in the approach of Whelan et al. [2], a disc actuator spanning the whole width of the channel is considered. This allows the maximum range of submersion depths $\zeta \in [0,1]$ ($0 :=$ emerged, $1 :=$ on the channel bottom) for each blockage ratio $\sigma := A_T / A$, i.e. turbine area $A_T$ to channel cross section $A := bh_t$ (cf. figure 1).
Method

In order to compare the different approaches, a general system of equation (c.f. table 1) is established by formulating the continuity, energy and mass balance for five control volumes, i.e. the

(i) upstream turbine streamtube stretching from section [1] to section [+],
(ii) turbine wake streamtube stretching from section [-] to section [*],
(iii) turbine itself, with inlet section [+] and outlet section [-],
(iv) bypass streamtube stretching from section [1] to section [*], and
(v) mixing (or wake) streamtube stretching from section [*] to section [2]. (c.f. figure 2)

![Diagram of a submerged turbine including streamtube deformation](image)

Figure 2: model of a submerged turbine including streamtube deformation

The different models solely differ by the approach to model the pressure $p_i$ and the positions $z_+, z_-$ and areas $A_+, A_-$ of the streamtube at the sections [+ and -] in front and after the turbine (c.f. table 1). Approach I neglects the free surface, i.e. considers different $p_i$ at equal water height $h_i = h$. (This only reflects the confinement, as for free surface flows pressure and depths are not independent.) Recognizing this, approaches II and III consider different water heights $h_i$ and use hydrostatic pressure profiles $p_i = 0.5 q h_i$ (for the momentum equations) and total (hydrotastic and geodetic) pressure profiles $p_i = q H_i$ (for the energy equations). Further, approaches I and II neglect any streamtube deformation due to the water head drop, i.e. $A_{+/-} = A_0, z_{+/-} = z_0$, whereas approach III incorporates the effects of deformation, i.e. $A_{+/-} = \sigma h_{+/-}, z_{+/-} = h_{+/-}/2$ and the associated the flux terms in momentum and energy equation.

| Mass equations |
|----------------|
| [1] $\rightarrow$ [+]: $A_3 u_3 - A_4 u_4 = 0$ |
| [+]: $A_4 u_4 - A_- u_- = 0$ |
| [-]: $A_- u_- - A_4 u_4 = 0$ |
| [1]: $A_+ u_+ - A_- u_- = 0$ |
| [+] $\rightarrow$ [+] $A_+ u_+ - A_- u_- = 0$ |
| [-] $\rightarrow$ [*]: $A_- u_- - A_4 u_4 = 0$ |
| [1] $\rightarrow$ [*]: $(b h_1 - A_1) u_1 - (b h_1 - A_1) u_0 = 0$ |
| [*] $\rightarrow$ [2]: $(b h_2 - A_1) u_0 + A_1 u_0 - (b h_2) u_2 = 0$ |

| Energy equations |
|-------------------|
| [1] $\rightarrow$ [+]: $(p_1 + 0.5 g u_1^2) - (p_2 + 0.5 g u_2^2) = 0$ |
| [+]: $(p_1 + 0.5 g u_1^2) - (p_2 + 0.5 g u_2^2) = q g H_T$ |
| [-] $\rightarrow$ [*]: $(p_2 + 0.5 g u_2^2) - (p_1 + 0.5 g u_1^2) = 0$ |
| [1] $\rightarrow$ [*]: $(p_2 + 0.5 g u_2^2) - (p_1 + 0.5 g u_1^2) = 0$ |

| Momentum equations |
|---------------------|
| [1] $\rightarrow$ [*]: $(p_1 + g u_1^2) b h_1 - (p_2 + g u_2^2) (b h_1 - A_) - (p_1 + g u_1^2) A_ = F$ |
| [+]: $\rightarrow$ [-]: $(p_2 + g u_2^2) A_ - (p_2 + g u_2^2) A_ = F$ |
| [*] $\rightarrow$ [2]: $(p_2 + g u_2^2) b h_2 - (p_2 + g u_2^2) (b h_1 - A_) - (p_2 + g u_2^2) A_ = 0$ |

Table 1: general system of equation
As approach III is initially designed for vertical turbines penetrating the surface, a quick modification (approach IV) is used. By introducing an empirical influence factor $f \in [0,1]$, the grade of influence of the surface wave on the streamtube is described. The factor can be interpreted as followed: For $f = 0$ the upper edge of the turbine is below the area influenced by the surface wave, i.e. half the surface waves wavelength below the surface. Thus, the turbine streamtube is not disturbed as considered by approach II. For $f = 1$ the upper edge is fully exposed to the surface wave as in approach III, i.e. contraction $\Delta A = \Delta h$ and deflection $\Delta z = \Delta h/2$. For $0 < f < 1$ the turbine is within the influenced area but submerged, which leads to a reduced contraction $f \Delta A$ and deflection $f \Delta z$ of the streamtube (cf. figure 2).

**Experimental Validation**

Analogue to the procedure in Pelz et al. [4] semi-analytical simulations are performed to study the validity of the different models: Volume flow rate, water heights, drag force and blockage ratio measured in the test rig (c.f. figure 3) where reduced by either the drag force $D$ or the water height $h$, (equilibrium flow) and set as boundary conditions for the systems of equations of each approach.

![Channel Test Rig](image)

Figure 3: Channel Test Rig

By solving the equations in MATLAB the unknowns, including the previously omitted measured values, are calculated. To validate the prediction quality of the different models, the calculated drag force $D$ and water height $h_*$ are compared with the measurement. (c.f. figure 4). The experiments where conducted with perforated plates of different height, perforation and position at three different downstream Froude-numbers $Fr_2 = 0.2, 0.3$ and $0.4$ and different submersion depths.

It is evident that, similar to the results in Pelz et al. [4], approach I overpredicts the drag force for significant change in water head as due to its neglection, the resulting change in potential energy is interpreted as a change in kinetic energy and thus higher drag. For approaches II and III, which consider the water head, the predicted forces are in good agreement with the measurement.
Evaluating the measured and predicted water depth $h_\text{s}$ downstream of the turbine, the predictions of Garrett & Cummins [1] are higher than the measured depths. This is obvious due to the neglected water head drop, i.e. $h_\text{s} = h_1$. As for the measurements with vertical plates Pelz et al. [4] approach II underpredicts the water height for high blockage $\sigma$ and downstream Froude-number. This is equal to an overprediction of the beneficial energy extraction over the mixing losses and occurs due to the illicit neglection of any streamtube deformation. The results of approach Pelz et al. [4] by contrast show only a slight overprediction in these cases. This overprediction can be explained by the smaller influence of the surface on submerged turbines than on turbines which penetrate the surface as measured in [4].

The better accordance of approach III compared to approach II implies, that the influence is, although smaller, still significant! It is worth noticing, that even for cases with low blockage, high submersion and low Froude number - where no streamtube deformation is to expect - approach III provides predictions in good accordance with the measurement. This is easily explained, as in these cases surface effects are negligible anyway and all approaches yield similar results.

By introducing the empirical influence factor (approach IV) the predictions can be brought to perfect accordance with the measurement (cf. figure 4). The differences between approach III and IV are however quite small. This is due to the fact, that the resulting surface wave is longer if the turbine is at higher submersion depth as quick optical measurements (c.f. figure 5) show.
Consequently, the turbine is within the reach of the surface wave \((f \gg 0)\) as long the wave is significant. Vice versa, in cases where the turbine is without the reach of the surface wave, surface effects are negligible anyway.

**Simulation Results**

This can as well be seen by studying the simulation results analogous to Pelz et al. [4]. Figure 6 shows exemplarily the coefficient of performance \(C_p := \frac{P_T}{P_{avail}}\) (with the available power defined after Pelz [5]) for four blockage ratios \(\sigma = [0.2, 0.4, 0.6, 0.8]\) and two downstream Froude numbers \(Fr_2 = [0.1, 0.2, 0.4]\) over the dimensionless turbine head \(\bar{H}_T := \frac{H_T}{H_1}\) (c.f. [4]).

![Figure 6: prediction of the coefficient of performance of all four approaches](image)

It is evident, that approaches IV and III show almost identical results, independent of the chosen submersion depth \(\zeta\) over the whole range of conditions. Approach I and II lead to similar results only for low turbine head \(\bar{H}_T\), low \(\sigma\) or vanishing flow speeds \(Fr_2\). For higher \(\sigma, \bar{H}_T, Fr_2\) model II and especially model I overestimate the energy extraction. This is due to the fact that these models assume, that the turbine streamtube can expand freely without interfering with the water surface. Approach I even exceeds the upper limit \(C_{p,max} = 1/2\) [5], which conflicts with the first law of thermodynamics.

![Figure 7: prediction of the coefficient of performance of all four approaches](image)

Figure 7 shows the dimensionless volumetric efficiency \(\bar{\eta}_V := \frac{\eta_V}{\sigma}\), i.e. the fraction of the flow through the turbine stream tube under operation \((H_T > 0)\) in relation to the initial flow through the turbine aera \((H_T = 0)\). For blockages \(\sigma < 1\), the volumetric efficiency decreases with increasing \(H_T\), as the turbine opposes a higher resistance and more flow is bypassing the turbine. At some point \(H_T \gg 0\), the whole volume flow bypasses the turbine, leading to \(\eta_V \to 0\). For low \(H_T\) and/or low \(\sigma\), i.e. \(f \to 0\), approaches III and IV differ visibly, as approach IV follows approach II. (For the coefficient of performance and the mixing losses, this is as well the case. However, approaches II and III lead to almost identical results for these conditions anyway.) For larger blockage, especially with higher Froude number and turbine head, model I and model II overestimate again. For approach II at \(\sigma = 0.8\) the volumetric efficiency is even increasing with increasing turbine head until values of \(\eta_V > 1\) are reached, which is obviously wrong.
Conclusion

To summarize: the predictions with (III and IV) and without (I, II) streamtube deformation differ only for cases with high water head drop, i.e. high turbine heads at higher blockages and Froude-numbers. In these cases, however, the surface wave affects almost the whole water depth, i.e. \( f \rightarrow 1 \). Thus, the results for model IV tend to those of model III. Considering the flux terms due to streamtube deformation, the results keep physical over the whole range of turbine heads \( \bar{H}_T \), blockage ratios \( \sigma \) and Froude numbers \( Fr_2 \). Approaches I and II however overestimate both coefficient of performance and volumetric efficiency, even predicting non-physical results with \( C_p > 0.5 \) (c.f. [5]) for approach I and \( \bar{\eta}_V > 1 \) for approach II. Only for small turbine heads, especially for high submersion depths, the results of the semi-analytical model IV tend to those of model II, i.e. \( f \rightarrow 0 \). For these operation conditions however, all approaches lead to similar results anyway.

Thus, the model proposed by Pelz et al. in [4] is a suitable, fully axiomatic model for calculating the maximum power output of tidal turbines, not only for vertical turbines, but also for submerged turbines. It has however to be addressed, that real turbines often operate at conditions with low surface deformation, i.e. \( f \ll 1 \). Thus, a more detailed analysis of streamtube deformation and its effects for these cases using more and enhanced experimental data is subject of current research.
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