PHASE TRANSITION IN ANYON SUPERCONDUCTIVITY AT FINITE TEMPERATURE

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The magnetic response of the charged anyon fluid at temperatures larger than the fermion energy gap \( T \gg \omega_c \) is investigated in the self-consistent field approximation. In this temperature region a new phase, characterized by an inhomogeneous magnetic penetration, is found. The inhomogeneity is linked to the existence of an imaginary magnetic mass which increases with the temperature at \( T \gg \omega_c \). The system stability in the \( T \gg \omega_c \)-phase is proved by investigating the electromagnetic field rest-energy spectrum.

I. INTRODUCTION

In recent years there has been much interest in investigating (2+1)-dimensional gauge theories with Chern-Simons (CS) interactions. This interest is due, in part, to a variety of significant physical applications of these theories in QFT, as well as in condensed matter physics.

A well known example in QFT is the Deser, Jackiw and Templeton Parity Anomaly [1]. This result shows that the fluctuations of a massive Fermi field induces a CS term in the effective action of the gauge fields. In this context the photon acquires a topological mass [1], [2]. Parity violation [3] and vortex solutions in lower dimensions [4] are among the consequences of the presence of CS terms. On the other hand, it has been shown that when Dirac fermions are coupled to a Maxwell-Chern-Simons (MCS) gauge field, the Lorentz symmetry can be spontaneously broken by the dynamical generation of a magnetic field [5]. The effects of CS terms in supersymmetric models have been also investigated [6].

In condensed matter, CS models have been considered in the study of different physical applications. CS theories involving several vector potentials are known to be particularly appropriate for describing the quantum Hall effect [7], [8], [9]. A recent model of the fractional quantum Hall effect considers that the electrons are transformed into composite fermions by attaching two artificial statistical flux quanta to each electron [10]. The gauge field theory describing these composite fermions introduces the statistical gauge flux via CS fields [11].

When matter field is coupled to the CS gauge fields in (2+1)-dimensions, a suitable description for anyons is obtained [12], [13]. Anyons [14], [15] are particles with fractional statistics in (2+1)-dimensions. The anyon description within the CS gauge theory is equivalent to attaching flux tubes to the charged fermions. The Aharonov-Bohm phases resulting from the adiabatic transport of two anyons is the source of the fractional exchange statistics [15].

It has been argued that strongly correlated electron systems in two dimensions can be described by an effective field theory of anyons [16], [17]. Anyons can be also obtained as solitons which fractional spin in electron systems. Excitations with fractional spin in two dimensional systems necessarily obey fractional statistics [18]. An important feature of the anyon theory is that it violates parity and time-reversal invariances. Although there are claims that anyons could play a basic role in high-\( T_C \) superconductivity [19]-[21], at present no experimental evidences of P and T violation in high-\( T_C \) superconductivity have been confirmed. It should be pointed out, nevertheless, that it is possible to construct more sophisticated P and T invariant anyonic models [21].

Whether linked or not to high-\( T_C \) superconductivity, anyon superconductivity is an interesting effect in its own right, and deserves a deeper study. As it is known, anyon superconductivity has an origin different from the Nambu-Goldstone-Higgs like mechanism. The genesis of the anyon superconductivity is given by the spontaneously violation of commutativity of translations in the free anyon system [22]. This new mechanism may have wider applications than the original physical problem that motivated its study.

The superconducting behavior of anyon systems at \( T = 0 \) has been investigated by many authors [23], [24]-[25]. At \( T = 0 \), anyon superconductivity appears due to the exact cancellation between the bare and induced CS terms in the effective action of the theory [25]. However, at \( T \neq 0 \) this cancelation does not take place [25]. Hence, several
authors [27]-[31] have advocated that the superconducting phase evaporates at any finite temperature. In ref. [28], it has been independently claimed that the destruction of the anyon superconducting phase at finite temperature is connected to the existence of a long-range mode, found inside the infinite bulk at \( T \neq 0 \). This long range mode is the consequence of the existence of a pole \( \sim \left( \frac{1}{k^2} \right) \) in the polarization operator component \( \Pi_{00} \) at finite temperature. On the other hand, in Ref. [28] it has been argued that an anyon fluid should undergo a Kosterlitz-Thouless type transition rather than an immediate destruction of its superconducting state for all \( T > 0 \).

In previous works [23], we found that, contrary to some authors’ belief, the superconducting behavior, manifested through the Meissner effect in the charged anyon fluid at \( T = 0 \), does not disappear as soon as the system is heated. In papers [23] we showed that the presence of boundaries affects the dynamics of the two-dimensional system in such a way that the long-range mode, that accounts for a homogeneous field penetration [28], cannot propagate in the bulk. According to these results, the anyon system with boundaries exhibits a total Meissner effect at temperatures smaller than the fermion energy gap \( T \ll \omega_c \). In this case the magnetic field cannot penetrate the bulk farther than a very short distance \( \Lambda \sim 10^{-5} \text{cm} \) for \( T \sim 200 \text{ K} \) and electron densities characteristic of the high-\( T_c \) superconductors). Our main conclusion was that the magnetic behavior of the anyon fluid is not just determined by its bulk properties, but it is essentially affected by the sample boundary conditions. The importance of the boundary conditions in \((2+1)\)-dimensional models has been previously stressed in ref. [34].

It is natural to expect that at temperatures larger than the energy gap this superconducting behavior should not exist. At those temperatures the electron thermal fluctuations should make accessible the free states existing beyond the energy gap. As a consequence, the charged anyon fluid should not be a perfect conductor at \( T \gg \omega_c \). A signal of such a transition may be found studying the system magnetic response at those temperatures. The main goal in this paper is to investigate the characteristics of the magnetic response in this high temperature phase.

In what follows we show that at \( T \gg \omega_c \), an externally applied constant and homogeneous magnetic field can penetrate the sample, giving rise to a periodic inhomogeneous magnetic field within the bulk. The inhomogeneity of the magnetic response increases with the temperature. We also find that, contrary to the \( T \ll \omega_c \) case, a long-range penetration, associated to a massless mode of the electromagnetic field within the anyon fluid, can penetrate the bounded sample in the high temperature phase. Nevertheless, in the range of temperatures considered the effect of this homogeneous field penetration is negligible as compared to the inhomogeneous component.

These results corroborate the existence of a phase transition in the charged anyon fluid from a superconducting phase, at \( T \ll \omega_c \) (see Ref. [33]), to a non-superconducting phase, at \( T \gg \omega_c \). The inhomogeneous character of the magnetic response in the high temperature phase is linked to the inhomogeneity of the spatial distribution of the induced many-particle charge and current densities at \( T \gg \omega_c \).

We also prove that the inhomogeneous magnetic penetration is associated to an imaginary magnetic mass associated to one of the electromagnetic field modes within the charged anyon fluid. The appearance of an imaginary magnetic mass at \( T \gg \omega_c \) by no means indicates that the linear approximation used in these calculations is broken by the presence of tachyons. Tachyons, as it is known, correspond to imaginary rest-energy solutions. The rest energy and the magnetic mass spectrum of the MCS theory at finite density are not the same at \( T \gg \omega_c \), as proved in Sec. 4. For the MCS theory at finite density, we show that the rest energies of the electromagnetic field modes at \( T \gg \omega_c \) are real.

The plan for the paper is as follows. In Sec. 2, for completeness, as well as for the convenience of the reader, we review the many-particle \((2+1)\)-dimensional MCS model used to describe the charged anyon fluid in the self-consistent field approximation. Principal attention is given to the derivation of the high-temperature MCS effective action in the linear approximation. In Sec. 3 we study the magnetic response in the self-consistent field approximation of a charged anyon fluid confined to a half plane in the \((T \gg \omega_c)\)-phase. We find the analytical solution of the MCS field equations that satisfies the corresponding boundary conditions and minimizes the system free energy. The propagation modes of the magnetic field within the anyon fluid have three contributions: one that decays exponentially in space, other that accounts for an homogeneous field penetration, and a third that changes periodically in space within the sample.

When all the coefficients appearing in the magnetic field solution are evaluated for the range of temperatures and other characteristic parameter values, we find that the leading term in the magnetic response is an inhomogeneous magnetic field with a characteristic wavelength that decreases with the temperature. In Sec. 4 we investigate the dispersion equation for the Maxwell field in the high-temperature approximation. We solve this equation in arbitrary covariant gauges for the Maxwell and CS fields to obtain the magnetic masses of the charged anyon fluid in the high-temperature phase. As expected, the magnetic masses turn out to be equal to the inverse length scales which characterize the magnetic response of the anyon fluid in this phase. We then prove that the existence of an imaginary magnetic mass cannot be associated to a tachyonic mode in this many-particle system with CS interactions. In doing that, we find the rest energies of the electromagnetic field modes at \( T \gg \omega_c \). Sec. 6 contains the summary and discussion.
II. ANYON FLUID AT HIGH TEMPERATURE AND DENSITY

A. Many-particle model and energy gap

The non-relativistic charged anyon system in interaction with an electromagnetic field in 2+1 dimensions can be modeled by the MCS Lagrangian density

\[ L = -\frac{1}{4} F_{\mu\nu}^2 - \frac{N}{4\pi} \varepsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho + e n_e A_0 + i\psi^\dagger D_0 \psi - \frac{1}{2m} |D_\mu \psi|^2. \]  

(2.1)

In (2.1) \( \mu, \nu, \rho = 0, 1, 2 \) and \( k = 1, 2 \). \( A_\mu \) and \( a_\mu \) represent the electromagnetic field and the CS field respectively. The CS fields are simply changing the quantum statistics of the matter field, thus, they do not have independent dynamics. \( \psi \) represents non-relativistic spinless fermions. \( N \) is a positive integer which determines the magnitude of the CS coupling constant. \( n_e \) is a background neutralizing ‘classical’ charge density, and \( m \) is the fermion mass. The covariant derivative \( D_\nu \) (we use the metric \( g_{\mu\nu} = (1, -1, 1) \)) is given by

\[ D_\nu = \partial_\nu + i (a_\nu + e A_\nu), \quad \nu = 0, 1, 2 \]  

(2.2)

The many-particle system is implemented through the Grand Partition function

\[ Z = Tr \exp \left[ -\beta H(\mu) \right] \]  

(2.3)

where \( \beta \) is the inverse of the absolute temperature and the many-particle Hamiltonian density is given by

\[ H(\mu) = H - \mu N \]  

(2.4)

with \( H \) the canonical Hamiltonian density corresponding to the Lagrangian density (2.1) and \( \mu \) the chemical potential associated to the conserved particle number \( N = \psi^\dagger \psi \).

Doing the functional integrals in the momenta in (2.3) we obtain the effective-Lagrangian density of the many-particle anyon system

\[ L_{\text{eff}} = L + \psi^\dagger \mu \psi \]  

(2.5)

The mean-field Euler-Lagrange equations derived from (2.5) are

\[ -\frac{N}{4\pi} \varepsilon^{\mu\nu\rho} f_{\nu\rho} = \langle j^\mu \rangle \]  

(2.6)

\[ \partial_\nu F^{\mu\nu} = e \langle j^\mu \rangle - e n_e \delta^{\mu 0} \]  

(2.7)

To guarantee the electric neutrality of the system formed by the electron fluid and the background charge \( n_e \) we impose the condition

\[ \langle j^0 \rangle - n_e \delta^{00} = 0, \]  

(2.8)

where \( \langle j^0 \rangle \) is the many-particle system fermion density

\[ \langle j^0 \rangle = \frac{\partial \Omega}{\partial \mu}, \]  

(2.9)

with \( \Omega \) is the one-loop fermion thermodynamic potential

\[ \Omega = -\beta^{-1} \ln \det G_0^{-1}(p, \mu), \]  

(2.10)

given in terms of the free fermion thermal Green’s function

\[ G_0(p, \mu) = \frac{1}{ip_4 + \mu - \frac{p^2}{2m}} \]  

(2.11)

where \( p_4 = (2n + 1)\pi/\beta \), are the discrete frequencies \( (n = 0, 1, 2, ...) \) corresponding to fermion fields.
The existence of a different from zero fermion density, as it is required by the neutrality condition (2.8), generates through eqs.(2.6)-(2.7) a nontrivial background of CS magnetic field

\[ \mathcal{B} = \mathcal{F}_{21} = \frac{2\pi n_e}{N}. \]  

(2.12)

Then, the unperturbed one-particle Hamiltonian of the matter field represents, in this many-particle system, a particle in the background of that CS magnetic field \( \mathcal{B} \),

\[ H_0 = -\frac{1}{2m} \left[ (\partial_1 + i\mathbf{a}_1)^2 + \partial_2^2 \right] \]

(2.13)

In (2.13) we considered the background CS potential, \( \mathbf{a}_k \), \( k = 1, 2 \), in the Landau gauge

\[ \mathbf{a}_k = \mathcal{B}_x \delta_{k1} \]  

(2.14)

The eigenvalue problem defined by the Hamiltonian (2.13) with periodic boundary conditions in the \( x_1 \)-direction:

\[ H_0 \Psi_{nk} = \epsilon_n \Psi_{nk}, \quad n = 0, 1, 2, \ldots \text{ and } k \in \mathbb{Z} \]

(2.15)

has eigenvalues and eigenfunctions given respectively by

\[ \epsilon_n = \left( n + \frac{1}{2} \right) \omega_c \]  

(2.16)

\[ \Psi_{nk} = \frac{\mathcal{B}^{1/4}}{\sqrt{L}} \exp \left( -2\pi ikx_1/L \right) \varphi_n \left( x_2 \sqrt{\mathcal{B}} - \frac{2\pi k}{L \sqrt{\mathcal{B}}} \right) \]  

(2.17)

where \( \omega_c = \mathcal{B}/m \) is the cyclotron frequency and \( \varphi_n (\xi) \) are the orthonormalized harmonic oscillator wave functions.

Note that the energy levels \( \epsilon_n \) are degenerate (they do not depend on \( k \)). Therefore, for each Landau level \( n \), there exists a band of degenerate states. The cyclotron frequency \( \omega_c \) plays here the role of the energy gap between occupied Landau levels. It is easy to prove [9], [20], [22], that at \( T = 0 \) the system will be confined to a filled band, which is separated by an energy gap \( (\omega_c) \) from the free states; therefore, it is natural to expect that at \( T = 0 \) the system should superconduct. This result is already a well established fact on the basis of Hartree-Fock analysis [19] and Random Phase Approximation [20], [22]. At \( T \neq 0 \) it was proved in Refs. [33] that the existence of a natural scale (the cyclotron frequency \( \omega_c \)) in this theory, makes possible the realization of a superconducting phase at \( T \ll \omega_c \), for systems confined to a bounded region.

It is logical to expect that this energy scale, \( \omega_c \), separates two different physical phases of the system. As we will prove below, the superconducting state, found at \( T \ll \omega_c \), disappears when the system reaches temperatures large enough (i.e. at \( T \gg \omega_c \)) to move the electrons beyond the energy gap to the free-energy band.

B. Effective action in the linear approximation

The linear response of the medium can be found under the assumption that the quantum fluctuations of the gauge fields about the ground-state are small. In this case the one-loop fermion contribution to the effective action, obtained after integrating out the fermion fields, can be evaluated up to second order in the gauge fields. The effective action in terms of the quantum fluctuation of the gauge fields within the linear approximation [28], [29] takes the form

\[ \Gamma_{\text{eff}} (A_\mu, a_\mu) = \int dx \left( -\frac{1}{4} F_{\mu\nu}^2 - \frac{N}{4\pi} \varepsilon^{\mu\nu\rho\sigma} a_\mu \partial_\nu a_\sigma \right) + \Gamma^{(2)} \]

(2.18)

\( \Gamma^{(2)} \) is the one-loop fermion contribution to the effective action in the linear approximation.
\[
\Gamma^{(2)} = \int dx dy \left[ a_\mu (x) + e A_\mu (x) \right] \Pi^{\mu\nu} (x, y) \left[ a_\nu (y) + e A_\nu (y) \right].
\]  
(2.19)

In (2.19) \(\Pi_{\mu\nu}\) represents the fermion one-loop polarization operator in the presence of the CS background magnetic field \(b\). To calculate the different components of \(\Pi_{\mu\nu}\), it is convenient to consider its structure,

\[
\Pi^{\mu\nu} = A_1 \left( k^2 g^{\mu\nu} - k^\mu k^\nu \right) + i A_2 \varepsilon^{\mu\nu\rho} k_\rho \\
+ A_3 \left( \delta_{\rho\sigma} k^2 - k_\rho k_\sigma \right) \delta^{\mu\nu}
\]  
(2.20)

The \(\Pi_{\mu\nu}\) operator (2.20) is a second rank tensor, formed by the momentum space basic tensors \(k_\nu, g_{\mu\nu}\) and \(\varepsilon^{\mu\nu\rho}\). The structure of \(\Pi_{\mu\nu}\) appearing in (2.20) is chosen in such a way that it satisfies the symmetry properties of the theory. That is, the transversality condition \((k_\mu \Pi_{\mu\nu} = 0)\), which is a consequence of the gauge invariance of the theory; the rotational invariance in the two-dimensional space, and the invariance under simultaneous permutation of indices and arguments \((\Pi_{\nu\mu} (k) = \Pi_{\mu\nu} (-k))\).

When the system has a rotational symmetry in the 2+1 space (as it is the case of a relativistic invariant system [35]), the polarization operator can be expressed in terms of only one independent invariant coefficient \((A_1)\) in eq.(2.20)). In the non-relativistic case, the symmetry between time and space-components is broken, giving rise to an additional invariant coefficient \((A_3)\) in eq.(2.20)). In the anyon fluid, in particular, as the P and T-invariances are broken, a third independent coefficient arises \((A_2)\) in eq.(2.20)). The presence of a medium (which is the case when we have a statistical system), however, does not introduce here any additional independent coefficient [36], as it is the case in the relativistic formulation [35]. The reason is that a medium cannot produce any new symmetry breaking in the non-relativistic case, where the Minkowskian symmetry is already broken.

For the sake of convenience we write the independent coefficients: \(A_1, A_2\) and \(A_3\), in term of the following new set \((\Pi_0, \Pi_0', \Pi_1\) and \(\Pi_2)\) of coefficients

\[
A_1 = \frac{\Pi_0}{k^2} + \Pi_0', \quad A_2 = \Pi_1, \quad A_3 + A_1 = \Pi_2
\]  
(2.21)

From (2.20) and (2.21) we have that the new independent coefficients can be found from the polarization operator components

\[
\Pi^{00} = \left( \frac{\Pi_0}{k^2} + \Pi_0' \right) \left( k^2 - k_0^2 \right)
\]  
(2.22)

\[
\Pi^{02} = - \left( \frac{\Pi_0}{k^2} + \Pi_0' \right) k^0 k^2 - i \Pi_1 k^3
\]  
(2.23)

\[
\Pi^{22} = \Pi_2 k_1^0 - \left( \frac{\Pi_0}{k^2} + \Pi_0' \right) k_0^2
\]  
(2.24)

Taking into account that we will investigate the magnetic response of the charged anyon fluid to a uniform and constant applied magnetic field, we need the \(\Pi_{\mu\nu}\) leading behaviors for static \((k_0 = 0)\) and slowly \((k \sim 0)\) varying configurations. Then, from (2.22)-(2.24) it is clear that to find the independent coefficients \(\Pi_0, \Pi_0', \Pi_1\) and \(\Pi_2)\) in this limit we just need to calculate the \(\Pi_{\mu\nu}\) components \(\Pi_{00}, \Pi_{02}\) and \(\Pi_{22}\) in the \((k_0 = 0, k \sim 0)\)-limit.

The polarization operator components of the many-particle system are calculated using the fermion thermal Green’s function in the presence of the background field \(b\) [28].

\[
G (p_4, p) = \int_0^\infty \frac{d\rho}{\rho} \int_{-\infty}^\infty dx_2 \sqrt{\vec{b}} \exp \left( -ip_2 x_2 \right) \exp \left( ip_4 + \mu - \frac{\vec{b}}{2m} \right) \rho \\
\sum_{n=0}^{\infty} \varphi_n (\xi) \varphi_n (\xi') t^n
\]  
(2.25)

where

\[
t = \exp \left( \frac{\vec{b}}{m} \rho \right), \quad \xi = \frac{p_1}{\sqrt{\vec{b}}} + \frac{x}{2 \sqrt{\vec{b}}}, \quad \xi' = \frac{p_1}{\sqrt{b}} - \frac{x}{2 \sqrt{b}}
\]  
(2.26)
In the Landau gauge, the $\Pi_{\mu\nu}$ Euclidean components: $\Pi_{00}$, $\Pi_{02}$ and $\Pi_{22}$ are given by [29],

$$\Pi_{00} (k, \mu, \vec{b}) = -\frac{1}{\beta} \sum_{p_0} \frac{d\mathbf{p}}{(2\pi)^2} G(p) G(p - k),$$  \hspace{1cm} (2.27)

$$\Pi_{0j} (k, \mu, \vec{b}) = \frac{i}{2m\beta} \sum_{p_0} \frac{d\mathbf{p}}{(2\pi)^2} \{ G(p) \cdot D_{-j} G(p - k) + D_{j}^+ G(p) \cdot G(p - k) \},$$  \hspace{1cm} (2.28)

$$\Pi_{jk} (k, \mu, \vec{b}) = \frac{1}{4m^2 \beta} \sum_{p_0} \frac{d\mathbf{p}}{(2\pi)^2} \left\{ D_k^- G(p) \cdot D_j^- G(p - k) + D_j^+ G(p) \cdot D_k^+ G(p - k) + D_j^+ D_k^- G(p) \cdot G(p - k) + G(p) \cdot D_j^- D_k^+ G(p - k) \right\} - \frac{1}{2m} \Pi_4,$$  \hspace{1cm} (2.29)

where the notation

$$D_{\pm j} G(p) = \left[ ip_j \mp \frac{\vec{b}}{2} \varepsilon^{jk} \partial_p k \right] G(p),$$

$$D_{\pm j} G(p - k) = \left[ i (p_j - k_j) \mp \frac{\vec{b}}{2} \varepsilon^{jk} \partial_p k \right] G(p - k),$$  \hspace{1cm} (2.30)

has been used. The independent coefficients: $\Pi_0$, $\Pi_0'$, $\Pi_1$ and $\Pi_2$, found from (2.22)-(2.24), are functions of $k^2$, $\beta$ and $\vec{b}$.

C. One-loop polarization operator coefficients in the high-temperature approximation

The polarization operator coefficients $\Pi_0$, $\Pi_0'$, $\Pi_1$ and $\Pi_2$ corresponding to the static limit ($k_0 = 0$, $k \to 0$) in the frame $\mathbf{k} = (k, 0)$ can be found from (2.25), (2.27)-(22.9) through the following relations,

$$\Pi_0 + \Pi_0' k^2 = -\Pi^{00} (k_0 = 0, k \to 0)$$  \hspace{1cm} (2.31)

$$\Pi_1 k = i \Pi^{02} (k_0 = 0, k \to 0)$$  \hspace{1cm} (2.32)

$$\Pi_2 k^2 = \Pi_{22} (k_0 = 0, k \to 0)$$  \hspace{1cm} (2.33)

After summing in $p_0$ in eqs. (2.27)-(2.29), and using the relations (2.31)-(2.33), we find that the polarization operator coefficients, in the $k/\sqrt{\vec{b}} \ll 1$ limit, are given by

$$\Pi_0 + \Pi_0' k^2 = \frac{\beta \vec{b}}{8\pi} \sum_n \Theta_n + k^2 \left[ \frac{2m}{\pi \beta} \sum_n \Delta_n - \frac{\beta}{8\pi} \sum_n (2n + 1) \Theta_n \right]$$  \hspace{1cm} (2.34)

$$\Pi_1 = \left[ \frac{1}{\pi} \sum_n \Delta_n - \frac{\beta \vec{b}}{16\pi m} \sum_n (2n + 1) \Theta_n \right]$$  \hspace{1cm} (2.35)

$$\Pi_2 = \left[ \frac{1}{\pi m} \sum_n (2n + 1) \Delta_n - \frac{\beta \vec{b}}{32\pi m^2} \sum_n (2n + 1)^2 \Theta_n \right]$$  \hspace{1cm} (2.36)

where
\[ \Theta_n = \text{sech} \left( \frac{\mu_n - \mu}{2} \right), \quad \Delta_n = \left( e^{\frac{\mu_n - \mu}{2}} + 1 \right)^{-1} \]  

(2.37)

To find the high-temperature leading contributions of the one-loop polarization operator coefficients, the summations in (2.34)-(2.36) are carried out using a high-temperature asymptotic expansion in the Euler-MacLauring sum formula. Their high-temperature leading contributions are \[28\], \[31\].

In these expressions \( m = 2m_e \) (\( m_e \) is the electron mass).

III. MAGNETIC RESPONSE IN THE HIGH-TEMPERATURE APPROXIMATION

In the high temperature region, for temperatures above the energy gap \( T \gg \omega_c \), the electrons will be energized enough to reach the empty Landau level bands, as we mentioned above. Consequently, the electron confinement into a completely filled band is lost, being possible for the electrons to change their initial states. Hence, it is natural to expect, from a heuristic point of view, that in this high temperature phase the system cannot behave as a superconductor.

If the system is not a superconductor at \( T \gg \omega_c \), then it has to allow the penetration of an externally applied constant magnetic field. In other words, no Meissner effect can take place. In this section we will show that this is indeed the case.

A. Many-Particle System Linear Equations

To obtain the extremum equations corresponding to the anyon many-particle system we have to consider the variational problem derived from the effective action (2.18). This formulation is known in the literature as the self-consistent field approximation \[29\].

In component form, the field equations in the presence of the induced currents and the CS fields are

\[ \nabla \cdot \mathbf{E} = eJ_0 \]  

(3.1)

\[ - \partial_0 E_k + \varepsilon^{kl} \partial_l B = eJ^k \]  

(3.2)

\[ \frac{eN}{2\pi} b = \nabla \cdot \mathbf{E} \]  

(3.3)

\[ \frac{eN}{2\pi} f_{0k} = \varepsilon^{kl} \partial_l E_0 + \partial_0 B \]  

(3.4)

where \( f_{\mu\nu} \) is the CS gauge field strength tensor, defined as \( f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \), and \( J_{\text{ind}}^\mu \) is the current density induced by the many-particle system.

In solving eqs. (3.1)-(3.4) we confine our analysis to gauge field configurations which are static and uniform in the y-direction. Within this restriction we take a gauge in which \( A_1 = a_1 = 0 \). Then, the different current density components are

\[ J_{\text{ind}}^0 (x) = \Pi_0 \left[ a_0 (x) + eA_0 (x) \right] + \Pi_1' \partial_x (E + eE) + \Pi_1 \left( b + eB \right) \]  

(3.5)

\[ J_{\text{ind}}^1 (x) = 0, \quad J_{\text{ind}}^2 (x) = \Pi_1 (E + eE) + \Pi_2 \partial_x (b + eB) \]  

(3.6)

In the above expressions we used the following notation: \( E = f_{01}, \quad E = F_{01}, \quad b = f_{12} \) and \( B = F_{12} \). Eqs. (3.5)-(3.6) play the role in the anyon fluid of the London equations in BCS superconductivity. When the induced currents (3.5)-(3.6) are substituted in eqs. (3.1)-(3.2) we find, after some manipulation, a set of independent differential equations depending on the fields \( B \) and \( E \), along with the zero components of the gauge potentials, \( A_0 \) and \( a_0 \).
\[ \omega \partial_x^2 B + \alpha B = \gamma [\partial_x E - \sigma A_0] + \tau a_0, \]  
\[ \partial_x B = \kappa \partial_x^2 E + \eta E, \]  
\( (3.7) \)  
\( (3.8) \)

In obtaining these equations we have used the eqs. (3.3)-(3.4) to eliminate the CS magnetic and electric fields in terms of the Maxwell fields

\[ b = -\chi \partial_x E \]  
\[ \mathcal{E} = -\chi \partial_x B \]  
\( (3.9) \)  
\( (3.10) \)

The coefficients appearing in these differential equations depend on the components of the polarization operators through the relations

\[ \omega = \frac{2\pi}{N} \Pi_0', \quad \alpha = -e^2 \Pi_1, \quad \tau = e \Pi_0, \quad \chi = \frac{2\pi}{eN}, \quad \sigma = -\frac{e^2}{\gamma} \Pi_0, \quad \eta = -\frac{e^2}{\delta} \Pi_1, \]

\[ \gamma = 1 + e^2 \Pi_0 - \frac{2\pi}{N} \Pi_1, \quad \delta = 1 + e^2 \Pi_2 - \frac{2\pi}{N} \Pi_1, \quad \kappa = \frac{2\pi}{N \delta} \Pi_2. \]  
\( (3.11) \)

### B. Field Solutions at High Temperature

Deriving eq. (3.7) with respect to \( x \) and using eqs. (3.8) and (3.10), we obtain a higher order differential equation that involves only the electric field,

\[ a \partial_x^4 E + d \partial_x^2 E + c E = 0, \]  
\( (3.12) \)

In this equation, \( a = \omega \kappa, \quad d = \omega \eta + \alpha \kappa - \gamma - \tau \kappa \chi, \quad \) and \( c = \alpha \eta - \sigma \gamma - \tau \eta \chi. \)

Solving (3.12) we find

\[ E(x) = C_1 e^{-x \xi_1} + C_2 e^{x \xi_1} + C_3 e^{-x \xi_2} + C_4 e^{x \xi_2}, \]  
\( (3.13) \)

where

\[ \xi_{1,2} = \left[ -d + \sqrt{d^2 - 4ac} \right] / 2a \]  
\( (3.14) \)

The solutions for \( B, a_0 \) and \( A_0 \), can be obtained using eqs. (3.8), (3.9), (3.13) and the definition of \( E \) in terms of \( A_0 \).

\[ B(x) = -\gamma_1 \left( C_1 e^{-x \xi_1} - C_2 e^{x \xi_1} \right) - \gamma_2 \left( C_3 e^{-x \xi_2} - C_4 e^{x \xi_2} \right) + C_5 \]  
\( (3.15) \)

\[ a_0(x) = \chi \gamma_1 \left( C_2 e^{x \xi_1} - C_1 e^{-x \xi_1} \right) + \chi \gamma_2 \left( C_4 e^{x \xi_2} - C_3 e^{-x \xi_2} \right) + C_6 \]  
\( (3.16) \)

\[ A_0(x) = \frac{1}{\xi_1} \left( C_1 e^{-x \xi_1} - C_2 e^{x \xi_1} \right) + \frac{1}{\xi_2} \left( C_3 e^{-x \xi_2} - C_4 e^{x \xi_2} \right) + C_7 \]  
\( (3.17) \)

In the above formulas we introduced the notation \( \gamma_1 = (\xi_1^2 \kappa + \eta) / \xi_1, \quad \gamma_2 = (\xi_2^2 \kappa + \eta) / \xi_2. \)

As can be seen from the magnetic field solution (3.15), the real character of the inverse length scales (3.14) is crucial for the realization of the Meissner effect. At temperatures much lower than the energy gap \( (T \ll \omega_c) \) this is indeed the case, as it was shown in our previous works (see ref. [33]).

In the high temperature region \( (T \gg \omega_c) \) the polarization operator coefficients are given by eqs. (2.38). Using this approximation, and assuming that \( N = 2, \) together with the assumption \( n_c \ll m^2 \) (this approximation is in agreement with the typical values found in high-\( T_C \) superconductivity), we can calculate the coefficients \( a, c \) and \( d \) that define the behavior of the inverse length scales.
an imaginary $\xi$ have a damping behavior, but an oscillating one. On the other hand, the presence of the constant coefficient $C$ can be determined once the asymptotic condition for the magnetic field (i.e. the value of the coefficient $C_1$) and the coefficient $A$ are found.

On the other hand, the coefficients $C_0$ and $C_7$, associated with the asymptotic configurations of the potentials $A_0$ and $A_0'$ respectively, are related to $C_5$. It is a consequence of the fact that, in obtaining eq. (3.12), we took the derivative of eq. (3.7). Therefore, the solution of eq. (3.12) belongs to a wider class than the one corresponding to eqs. (3.7)-(3.10). To exclude redundant solutions we must require that they satisfy eq. (3.7) as a supplementary condition. Therefore, substituting the solutions (3.13), (3.15)-(3.17) into eq. (3.7), we obtain the relation
\[ e\pi_0 C_5 = -\pi_0 (C_6 + eC_7) \]
Eq. (3.27) establishes a connection between the linear combination of the coefficients of the long-range modes of the zero components of the gauge potentials, \((C_6 + eC_7)\), and the coefficient of the long-range mode of the magnetic field, \(C_5\). Note that if the induced CS coefficient \(\Pi_5\), or the Debye-screening coefficient \(\Pi_0\) were zero, there would be no link between \(C_5\) and \((C_6 + eC_7)\). This relation between the long-range modes of \(B\), \(A_0\) and \(a_0\) can be interpreted as a sort of Aharonov-Bohm effect, which occurs in this system at finite temperature [33]. At \(T = 0\), we have \(\Pi_0 = 0\), and this effect disappears.

After using the boundary conditions (3.23), it follows that they are not sufficient to determine the coefficients \(C_5\) and \(A\). We need another physical condition from where \(C_5\) and \(A\) can be found. Since, obviously, any meaningful solution have to be stable, the natural additional condition to be considered is the stability equation derived from the system free energy. With this goal we start from the free energy of the half-plane sample

\[
\mathcal{F} = \frac{1}{2} \int_{-L/2}^{L/2} dy \int_0^L dx \left\{ (E^2 + B^2) + \frac{N}{\pi} a_0 b - \Pi_0 (eA_0 + a_0)^2 \right\}
\]

\[-\Pi_0' (eE + \mathcal{E})^2 - 2\Pi_1 (eA_0 + a_0) (eB + b) + \Pi_2 (eB + b)^2 \]

(3.28)

where \(L\) and \(L'\) determine the two sample’s lengths.

In (3.28) we have to substitute the field solutions (3.16), (3.17), (3.24) and (3.25) together with the solutions for the CS fields (that can be found substituting (3.24) and (3.25) in eqs. (3.9) and (3.10) respectively)

\[
b(x) = \chi \xi_1 C_1 e^{-x \xi_1} - \chi \xi_2 (A \cos \xi_2 x + C_1 \sin \xi_2 x)
\]

(3.29)

\[
\mathcal{E}(x) = -\chi \xi_1 \gamma_1 C_1 e^{-x \xi_1} + \chi \xi_2 \gamma_2 (A \sin \xi_2 x - C_1 \cos \xi_2 x).
\]

(3.30)

Then, after using the boundary conditions (3.23) and the constraint equation (3.27); it is found that the leading contribution to the free-energy density \(f = \frac{\mathcal{F}}{A}, \quad (A = LL'\) being the sample area) in the sample’s length limit \((L \to \infty, L' \to \infty)\) is given as a function of \(A\) and \(C_1\) by

\[
f = \frac{1}{2} \left[ X_1 A^2 + X_2 C_1^2 + X_3 AC_1 + X_4 A + X_5 C_1 + X_6 \right]
\]

(3.31)

The coefficients \(X_i\) are expressed in terms of the polarization operator coefficients as

\[
X_1 = g \xi_2^2 + \mathcal{G}, \quad X_2 = g \gamma_1^2 + \mathcal{G}, \quad X_3 = -2g \gamma_1 \xi_2, \quad X_4 = -2g \xi_2 \gamma_2,
\]

\[
X_5 = 2g B \gamma_1, \quad X_6 = g B^2
\]

(3.32)

\[
g = 1 + \frac{e^2 \Pi_1^2}{\Pi_0} + e^2 \Pi_2
\]

(3.33)

\[
\mathcal{G} = \frac{1}{2} \left[ 1 + \frac{N}{\pi} \xi_2^2 \right] - \left( \frac{\Pi_0}{2 \xi_2} + \frac{\Pi_0'}{2} \right) \left( e + \xi_2 \gamma_2 \right)^2
\]

\[-\frac{\Pi_1}{\xi_2} (\xi_2 - \gamma_2) \left( e + \xi_2 \gamma_2 \right) + \frac{\Pi_2}{2} (\xi_2 - \gamma_2)^2 \]

(3.34)

The values of \(A\) and \(C_1\) are found by minimizing the corresponding free-energy density

\[
\frac{\delta f}{\delta A} = \frac{1}{2} (2X_1 A + X_3 C_1 + X_4) = 0
\]

(3.35)

\[
\frac{\delta f}{\delta C_1} = \frac{1}{2} (2X_2 C_1 + X_3 A + X_5) = 0,
\]

(3.36)
to be

\[ A = \frac{\gamma_2}{\gamma_1 + \gamma_2} \overline{B} \]  

(3.37)

\[ C_1 = -\frac{g\gamma_1^3}{(g\gamma_1^2 + \mathcal{G})(\gamma_1^2 + \gamma_2)} \overline{B} \]  

(3.38)

Taking into account the boundary conditions (3.23) we have that the long-range mode of the magnetic field \( C_5 \) is given by

\[ C_5 = \gamma_1 C_1 - \gamma_2 A + B = \frac{\gamma_1^2 \mathcal{G}}{(g\gamma_1^2 + \mathcal{G})(\gamma_1^2 + \gamma_2)} \overline{B} \]  

(3.39)

From (3.39) we see that for \( T \gg \omega_c \) the electromagnetic field long-range mode propagates into the sample, producing an anhomogeneous magnetic penetration. This result is different from the one obtained in the low-temperature limit \( (T \ll \omega_c) \) [33]. In that limit it was found that \( C_5 = 0 \), which implies that the long-range mode cannot propagate within the sample when a uniform and constant magnetic field is perpendicularly applied at the sample’s boundaries.

D. Inhomogeneous Magnetic Response

As it has been previously established, in the high-temperature limit the coefficients \( A, C_1 \) and \( C_5 \) are all different from zero, i.e., in the \( (T \gg \omega_c) \)-phase the magnetic response of the charged anyon fluid has an exponential decaying component, as well as, both homogeneous and inhomogeneous penetrations. To complete our study of the magnetic response at high temperature we still need to estimate the corresponding values of each component for the range of parameters and temperatures here considered.

At the densities under consideration, \( n_e \ll m^2 \), the estimated values of the coefficients \( A, C_1 \) and \( C_5 \) in the high-temperature approximation \( (T \gg \omega_c) \) are

\[ A \approx 10^3 \overline{B}, \quad C_1 \approx -10^{-11} \overline{B}, \quad C_5 \approx 10^{-4} \overline{B} \]  

(3.40)

In this approximation the leading contributions to the electric and the magnetic fields, (3.24) and (3.25), are then given respectively by

\[ E(x) = E_0(T) \sin \left( \frac{2\pi}{\lambda} x \right) \]  

(3.41)

\[ B(x) = \overline{B} \cos \left( \frac{2\pi}{\lambda} x \right) \]  

(3.42)

where

\[ E_0(T) = \frac{12\sqrt{2}m}{\xi_2} \left( \tanh \frac{\beta \mu}{2} + 1 \right)^{-1} \overline{B} \]  

(3.43)

\[ \lambda = \frac{2\pi}{\xi_2} \]  

(3.44)

From eq. (3.42), one sees that the magnetic response exponentially decaying component of the (the one associated with the coefficient \( \gamma_1 C_1 \) in the general solution (3.25)), as well as the uniform one (3.39), are negligible if compared with the inhomogeneous component associated with the coefficient \( A \).

Hence, at \( T \gg \omega_c \) the applied magnetic field penetrates the charged anyon fluid with a magnitude that changes sinusoidally with \( x \) and has an amplitude \( \overline{B} \). Moreover, the inhomogeneous magnetic field penetration (3.42) is characterized by a wavelength \( \lambda \), which is proportional to the inverse of the length scale magnitude \( \xi_2 \) (eq. (3.44)).
At $T \gtrsim \omega_c$, using that $\mu \simeq \frac{\pi n_c}{m}$ [28], one can estimate from (3.44) and (3.22) that $\lambda \simeq 0.4 \ A^\circ$. On the other hand, taking into account that $\xi_2$ increases with the temperature (see eq. (3.22)), we have, that the wavelength decreases with $T$. The high-temperature leading behavior for $\lambda$ is given by

$$\lambda \approx \frac{\pi}{24 \sqrt{2mT}}$$

(3.45)

Note that when an external constant and uniform magnetic field is applied to the charged anyon fluid in the ($T \gg \omega_c$)-phase, an inhomogeneous electric field (3.24) is induced within the medium. The amplitude of this induced electric field depends on the magnitude of the applied magnetic field, $B$, and the temperature. The $E$'s inhomogeneity also increases with the temperature through $\lambda$.

From the obtained results we conclude that for temperatures larger than the energy gap, the charged anyon fluid is in a new phase on which the superconductivity is lost (non Meissner effect is found in this phase).

The induction of inhomogeneous electric and magnetic fields within the charged anyon fluid at high temperature, indicates that some redistribution of the induced charge and currents occurs at $T \gg \omega_c$.

To verify this, let us calculate the induced electric charge density of the charged medium in the high-temperature limit, we find that the induced electric charge density presents an inhomogeneous spatial distribution with high-temperature leading contribution given by

$$\epsilon J_0 (x) = 24 \sqrt{2mB} \left[ \tanh \left( \frac{\beta \mu}{2} \right) + 1 \right]^{-1} \cos \left( \frac{2\pi}{\lambda} x \right)$$

(3.46)

As discussed above, in the high-temperature regime $\lambda \sim \sqrt{1/T}$, so the spatial inhomogeneity of the charge density (3.46) increases with the temperature.

In the same way, if we calculate the current density (3.6) in the high-temperature limit we find

$$\epsilon J_2 (x) = -96 \pi \sqrt{2} \sqrt{m/\beta B} \cos \left( \frac{\beta \mu}{2} \right) \left[ \tanh \left( \frac{\beta \mu}{2} \right) + 1 \right]^{-\frac{1}{2}} \sin \left( \frac{2\pi}{\lambda} x \right)$$

(3.47)

Obviously, the current density (3.47) is not a supercurrent confined to the sample's boundary.

IV. MAGNETIC MASS AND REST ENERGY IN THE CHARGED ANYON FLUID AT $T \gg \omega_C$

We have seen that the inverse length scales, $\xi_1$ and $\xi_2$, are basic elements in the determination of the magnetic response of the charged anyon fluid. In this Sec. we shall go one step forward in clarifying the physical interpretation of these parameters. We will show that the inverse length scales (3.21), (3.22) can be identified with the magnetic masses of the electromagnetic field within the fluid at $T \gg \omega_c$. A particularly important point in this Sec. is our proof that the existence at high temperature of an imaginary magnetic mass (corresponding to the inverse length (3.22)) is not linked to the presence of tachyons in the theory, or to the breaking of the linear approximation, as was suggested in ref. [28]. As shown below, the magnetic mass and the rest energy are not the same in the MCS theory (contrary to what happens in a Klein-Gordon-like theory).

To investigate the magnetic masses and the rest energies associated with the electromagnetic modes, we need to study the electromagnetic field dispersion equation. With this aim we start from the effective action (2.18) taken in the covariant gauges for the Maxwell and CS fields

$$\frac{1}{\alpha_1} \partial_\mu A^\mu = 0, \quad \frac{1}{\alpha_2} \partial_\mu a^\mu = 0$$

(4.1)

$\alpha_1$ and $\alpha_2$ being two independent gauge parameters.

The corresponding effective Lagrangian density for the Maxwell and CS field configurations can be represented as

$$L_{eff} = -\frac{1}{2} A^\mu (-k) \Delta^{-1}_{\mu\nu} (k) A^\nu (k) - \frac{1}{2} a^\mu (-k) D^{-1}_{\mu\nu} (k) a^\nu (k) - e A^\mu (-k) \Pi_{\mu\nu} (k) a^\nu (k)$$

(4.2)

where the matrices $\Delta^{-1}_{\mu\nu}$ and $D^{-1}_{\mu\nu}$ are given by

$$\Delta^{-1}_{\mu\nu} (k) = k^2 g_{\mu\nu} - \left( 1 - \frac{1}{\alpha_1} \right) k_\mu k_\nu + e^2 \Pi_{\mu\nu} (k)$$

(4.3)
\[ \mathcal{D}^{-1}_{\mu\nu}(k) = \frac{iN}{2\pi} \epsilon_{\mu\nu\rho}k^\rho + \frac{1}{\alpha^2}k_\mu k_\nu + \Pi_{\mu\nu}(k) \]  

(4.4)

\( \Pi_{\mu\nu} \) is the one-loop fermion polarization operator given from (2.20)-(2.21) by

\[ \Pi^{\mu\nu} = \begin{pmatrix} -A_1 k^2 & A_1 k \omega & -i \Pi_1 k \\ A_1 k \omega & -A_1 \omega^2 & i \Pi_1 \omega \\ i \Pi_1 k & -i \Pi_1 \omega & -A_1 \omega^2 + \Pi_2 k^2 \end{pmatrix} \]  

(4.5)

In (4.5) we are considering the frame \( \mathbf{k} = (k, 0) \), and the notation \( \omega = k_0 \) has been used.

The effective theory for the electromagnetic field in the charged anyon fluid, is found integrating the CS fields in the partition function (2.3) with Lagrangian density (4.2). The new effective Lagrangian density so obtained is

\[ \mathcal{L}'_{\text{eff}} = - \frac{1}{2} A^\mu \left[ \Delta^{-1}_{\mu\nu}(k_\rho) - N_{\mu\nu}(k_\rho) \right] A^{\nu}(k_\rho) \]  

(4.6)

where the matrix \( N_{\mu\nu}(k_\rho) \) is defined by

\[ N^{\mu\nu}(k_\rho) = e^2 \Pi^{\mu\lambda}(k_\rho) \mathcal{D}_{\lambda\rho}(k_\omega) \Pi^{\rho\nu}(k_\rho) \]  

(4.7)

The dispersion equation for the Maxwell field is then given by

\[ \det \left[ \Delta^{-1}_{\mu\nu}(\omega, k) - N_{\mu\nu}(\omega, k) \right] = 0 \]  

(4.8)

It is noteworthy to discuss now some of the characteristics of the dispersion equation in QFT. In general, the dispersion equation can be reduced to an equation of the form

\[ F_1(k, \omega) + F_2(k, \omega) - \mathcal{M}^2 = 0 \]  

(4.9)

where \( F_1(k, \omega) \) is a homogeneous function in \( \omega \) and \( F_2(k, \omega) \) is a homogeneous function in \( k \). The squared magnetic mass is then defined as the solution for \(-k^2\) found from the dispersion equation (4.9) evaluated at \( \omega = 0 \),

\[ F_2(k, \omega = 0) - \mathcal{M}^2 = 0 \]  

(4.10)

while the system rest energy (from where the existence of tachyons can be determined) has to be found as the \( \omega \) solution of (4.9) when \( k = 0 \),

\[ F_1(\omega, k = 0) - \mathcal{M}^2 = 0 \]  

(4.11)

In a Klein-Gordon-like theory, \( F_1(k, \omega) = \omega^2 \) and \( F_2(k, \omega) = -k^2 \), then the magnetic mass is just equal to the rest energy and given by \( \mathcal{M} \), but in general this does not have to necessarily be the case. For example, in relativistic systems of charged fermions at finite density (i.e. in the presence of a chemical potential \( \mu \)) the dispersion equation has a linear term in \( \omega \),

\[ \omega^2 + \mu \omega - (k^2 + \mathcal{M}^2) = 0 \]  

(4.12)

In that case the solution for \(-k^2\) with \( \omega = 0 \) is equal to \( \mathcal{M} \), while, as it is known, the rest energy for particles and antiparticles is given by

\[ \omega = -\frac{\mu}{2} \pm \sqrt{\left(\frac{\mu}{2}\right)^2 + \mathcal{M}^2} \]  

(4.13)

Furthermore, in the presence of a background magnetic field, the distinction between mass and rest energy becomes essential. For instance, in the context of string theory in a background magnetic field, it has been shown that the mass (defined in agreement with Wigner’s definition) of higher spin \((s \geq 1)\) charged boson particles does not coincide with their rest energy. This is due to the modification of the algebra of the global symmetries by the background field.

From the above discussion, it is also clear that in calculating the magnetic mass (eq. (4.10)) and the rest energy (eq. (4.11)) at finite temperature, we have to take the polarization operators coefficients in the static limit \((\omega = 0, k \sim 0)\) and in the plasmon limit \((k = 0, \omega \sim 0)\) respectively. Now, because of the lack of analyticity of the Green’s function about \( k_\mu = 0 \) at \( T \neq 0 \), it is known that in QFT these limits do not commute. In anyon theory at \( T \neq 0 \) one faces a similar situation, as it was shown in ref. [31]. Then, in each case we have to consider the polarization operator coefficients evaluated in the corresponding limit.

In anyon theory the CS interaction gives rise to a dispersion equation with a structure more complicated than that corresponding to a Klein-Gordon-like theory. As shown below, in this case the magnetic masses and the rest energies of the electromagnetic modes are different.
A. Electromagnetic field magnetic masses at high temperatures

To find the electromagnetic field magnetic masses we must solve the dispersion equation (4.8) at \( \omega = 0 \). Since we are interested in the magnetic masses at temperatures higher than the energy gap (\( T \gg \omega_c \)), we should consider in solving (4.8) the polarization operator coefficients in the high-temperature approximation (2.38).

Let us determine first the expression of the matrix \( N^{\mu\nu}(k) \) (eq. (4.7)) at \( \omega = 0 \). In this case the matrix \( D_{\lambda\rho}(k) \) appearing in eq. (4.7) takes the form

\[
D_{\lambda\rho}(k) = \frac{k^2}{\alpha D} \begin{bmatrix}
\Pi_2 k^2 & 0 & iHk \\
0 & (\Pi_2 A_1 k^2 + H^2) \alpha_2 & 0 \\
0 & 0 & -A_1 k^2 \\
\end{bmatrix}
\] (4.14)

In writing (4.14) we have used the following notation,

\[
D^{-1} = \frac{1}{\alpha D} \begin{bmatrix}
\Pi_2 A_1 k^2 + 2H\Pi_1 - \Pi_2 k^2 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -\Pi_2 Bk^2 \\
\end{bmatrix}
\] (4.15)

\[
H = -\frac{1}{\pi} + \Pi_1 (4.16)
\]

Thus, after taking the matrix products indicated in (4.7) with the polarization operator (4.5) evaluated at \( \omega = 0 \), we find,

\[
N^{\mu\nu}(k) = \frac{e^2}{\alpha D} \begin{bmatrix}
A_1 Bk^2 & 0 & ikC \\
0 & 0 & 0 \\
0 & 0 & -\Pi_2 Bk^2 \\
\end{bmatrix}
\] (4.17)

with

\[
B = (\Pi_2 A_1 k^2 + 2H\Pi_1 - \Pi_2 k^2)^4, \quad C = [A_1 k^2 (2\Pi_1 \Pi_2 - HH_2) + HH_1 k^2] k^4
\] (4.18)

Using (4.3) and (4.17) in the dispersion equation (4.8) we obtain

\[
\det [\Delta_{\mu\nu}^{-1}(\omega = 0, k) - N_{\mu\nu}(\omega = 0, k)] = -\frac{k^4}{\alpha_1} \left[ (k^2 + e^2 A_1 G_1 k^2) \left( 1 + e^2 G_1 \Pi_2 \right) + e^4 G_2^2 \right] = 0
\] (4.19)

with

\[
G_1 = \frac{B}{\alpha_2 D} + 1, \quad G_2 = \frac{C}{\alpha_2 D} + \Pi_1
\] (4.20)

For \( N = 2 \), taking into account that in natural units \( e^2 \sim 10^5 cm^{-1} \), and considering the characteristic values \( n_e = 2 \times 10^{14} cm^{-2} \) and \( m_e = 2.6 \times 10^{10} cm^{-1} \), we can estimate the relative orders between the polarization operator coefficients (2.38) at temperatures larger than \( \omega_c \) as

\[
\Pi_0 \approx -10^6 e^2 \Pi_1 \approx -10^7 e^4 \Pi_0 \approx 10^{20} e^4 \Pi_2
\] (4.22)

Taking into account the relations (4.22), the high-temperature leading contribution to the dispersion equation (4.19) is

\[
\det [\Delta(k) - N(k)] \approx -\frac{(\Pi_2 \Pi_0)^2}{\alpha_1 (\alpha_2 D)^2} k^{12} (k^6 + \sigma_1 k^4 + \sigma_2 k^2 + \sigma_3) = 0
\] (4.23)

where
\[ \sigma_1 = \frac{2}{\pi^2} (\Pi_2 \Pi_0 ')^{-1}, \quad \sigma_2 = \frac{\sigma_1^2}{4}, \quad \sigma_3 = \frac{\sigma_1^2}{4} e^2 \Pi_0 \]  

(4.24)

To find the magnetic masses of the electromagnetic field in the charged anyon fluid we need to find the zeros in \( k \) of the dispersion equation (4.23). In doing that, we have to solve a cubic equation in \( k^2 \) (see the polynomial into the parenthesis in eq. (4.23))

\[ y^3 + \sigma_1 y^2 + \sigma_2 y + \sigma_3 = 0 \]  

(4.25)

In (4.25) we made the variable change \( y = k^2 \). The roots of (4.25) are given by

\[ y_1 = A + B - \frac{\sigma_1}{3}, \quad y_{2,3} = -\frac{A + B}{2} \pm \frac{A - B}{2} \sqrt{3} \frac{\sigma_1}{3} \]  

with

\[ A = \frac{\sigma_1^2}{6} (1 - \vartheta_A)^{1/3}, \quad B = \frac{\sigma_1^2}{6} (1 - \vartheta_B)^{1/3}, \quad \vartheta_{A,B} = 3^2 \vartheta \mp 3i \sqrt{6} \vartheta, \]  

\[ \vartheta = \frac{e^2 \Pi_0}{\sigma_1} \]  

(4.27)

In the high-temperature limit \( \vartheta_{A,B} \ll 1 \), so we can take the expansions,

\[ (1 - \vartheta_{A,B})^{1/3} \approx 1 - \frac{1}{3} \vartheta_{A,B} - \frac{1}{9} \vartheta_{A,B}^2 \]  

(4.28)

Substituting with (4.27) and (4.28) in (4.26) we obtain the leading high-temperature approximation for the roots of eq. (4.25),

\[ y_1 = -e^2 \Pi_0, \quad y_{2,3} = -\frac{1}{\pi^2} (\Pi_2 \Pi_0 ')^{-1} \]  

(4.29)

The solutions of (4.23) obtained in the high-temperature approximation represents the electromagnetic field magnetic mass spectrum

\[ k^2 + M_j^2 = 0, \quad j = 1, 2, 3 \]  

(4.30)

The squared magnetic masses in the high-temperature approximation are found from (4.29) to be

\[ M_1^2 = 0 \]  

(4.31)

\[ M_2^2 = e^2 \Pi_0 \]  

(4.32)

\[ M_3^2 = \frac{1}{\pi^2} (\Pi_2 \Pi_0 ')^{-1} \]  

(4.33)

We should note that the magnetic masses (4.32)-(4.33) are gauge independent. That is, they do not depend on the gauge parameters \( \alpha_1 \) and \( \alpha_2 \). We can see that the equation (4.23), from where the magnetic masses are found, is independent of the gauge parameter \( \alpha_2 \), since \( \alpha_2 D \) does not depend on \( \alpha_2 \); and \( \alpha_1 \) appears only as a multiplicative factor.

From (4.31)-(4.33) we have that one of the infrared modes of the electromagnetic field in the anyon fluid is massless, \( M_1 = 0 \), while \( M_2^2 > 0 \) and \( M_3^2 < 0 \). The signs obtained for the square of the masses (4.32), (4.33), are a consequence of the fact that in the high-temperature limit, \( \Pi_0 > 0, \Pi_2 > 0 \) and \( \Pi_0 ' < 0 \) (eq. (2.38)).

Comparing eqs. (4.32), (4.33) with eqs. (3.21), (3.22) respectively, one can see that

\[ \xi_1 = M_2, \quad \xi_2 = M_3 \]  

(4.34)

Hence, the magnetic masses coincide with the inverse length scales, \( \xi_1 \) and \( \xi_2 \), which determine the magnetic response of the medium. This is precisely the physical meaning of these infrared masses (\( M_i \)). They cannot be interpreted, otherwise, as the rest energies of the electromagnetic field modes, as we will see below. The zero mode (4.31) is linked to the long-range component \( C_5 \) appearing in the magnetic response (3.25).
B. Electromagnetic field rest energies at high temperatures

The rest energies of the electromagnetic modes are found by solving the dispersion equation (4.8) for \( \omega \) at \( k = 0 \). We recall that the polarization operator coefficients have to be taken now in the plasmon limit (\( k = 0, \omega \sim 0 \)). Using this limit, we find that \( \Pi_0 (k = 0, \omega \sim 0) = 0 \), while the rest of the coefficients maintain the same functional behavior (2.38) obtained in the static limit [2]. Then the polarization operator is given by (4.5) with \( \Pi_0 = 0 \) and \( k = 0 \).

In this case the matrix \( \mathcal{D}_\mu^\nu (\omega) \) appearing in eq. (4.7) takes the form

\[
\mathcal{D}_\mu^\nu (\omega) = \frac{1}{D} \begin{bmatrix}
\frac{\alpha_2}{\alpha_1} \omega & 0 & 0 \\
0 & -\Pi_0 \frac{\omega^2}{\alpha_2} - iH \omega \frac{\omega}{\alpha_2} & 0 \\
0 & iH \omega \frac{\omega_1}{\alpha_2} & -\Pi_0 \frac{\omega^2}{\alpha_2}
\end{bmatrix}
\]

(4.35)

In writing (4.35) the following notation was used

\[
D = \det \mathcal{D}_\mu^\nu (k) = \frac{\omega^4}{\alpha_2 D} \left[ (\Pi_0 \gamma^\nu)^2 \omega^2 - H^2 \right]
\]

(4.36)

Thus, after taking the matrix products indicated in (4.7) we find,

\[
N_{\mu\nu} (\omega) = \frac{\alpha_2}{\omega D} \begin{bmatrix}
0 & 0 & 0 \\
0 & M & N \\
0 & -N & M
\end{bmatrix}
\]

(4.37)

with

\[
M = -\omega^6 \Pi_0 \gamma^\nu \left[ (\Pi_0 \gamma^\nu)^2 \omega^2 + (\Pi_1 - 2H) \Pi_1 \right],
\]

(4.38)

\[
N = i\omega^5 \left[ (2\Pi_1 - H) (\Pi_0 \gamma^\nu)^2 \omega^2 - H (\Pi_1)^2 \right]
\]

(4.39)

Using (4.3) and (4.37) in the dispersion equation (4.8) we obtain the general expression for the rest-energy equation

\[
\det \left[ \Delta^{-1}_{\mu\nu} (\omega) - N_{\mu\nu} (\omega) \right] = \frac{\omega^2}{\alpha_1} \cdot \left\{ \left[ 1 + \frac{\alpha_2}{\alpha_1} \omega^2 \right]^2 + \left[ i\frac{\omega_1}{\alpha_1} \omega - \frac{\alpha_2}{\alpha_1} N \right]^2 \right\} = 0
\]

(4.40)

Taking into account the relations (4.22), the high-temperature leading contribution to the rest-energy equation (4.40) is

\[
\det \left[ \Delta^{-1} (k) - N (k) \right] \simeq \frac{(\Pi_0 \gamma^\nu)^4 \omega^10}{\alpha_1 \alpha_2 D} \left( \omega^6 + \theta_1 \omega^4 + \theta_2 \omega^2 + \theta_3 \right) = 0
\]

(4.41)

where

\[
\theta_1 = -\frac{2H^2}{(\Pi_0 \gamma^\nu)^2}, \quad \theta_2 = \frac{1}{4} \theta_1^2, \quad \theta_3 = -\frac{1}{4} \theta_1^2 (\epsilon^2 \Pi_1)^2
\]

(4.42)

The \( \omega \) solutions of eq. (4.41) are

\[
\omega_1^2 = 0, \quad \omega_2^2 = (\epsilon^2 \Pi_1)^2, \quad \omega_3^2 = \left( \frac{N}{2\pi \Pi_0 \gamma^\nu} \right)^2
\]

(4.43)(4.44)(4.45)

From (4.43)-(4.45) we have that \( \omega_2^2, \omega_3^2 > 0 \); therefore, at \( T \gg \omega_c \) there is no negative squared rest energy, which means that the high temperature phase is stable. Hence, the existence of a negative squared magnetic mass, \( \mathcal{M}_3^2 \), simply indicates that there is an inhomogeneous magnetic penetration in the charged anyon fluid at \( T \gg \omega_c \).

Finally, we should point out that the rest energies \( \omega_{2,3} \) are also gauge independent (they do not depend on \( \alpha_1 \) and \( \alpha_2 \), and they are determined by the explicit (proportional to \( N \)) and induced (proportional to \( \Pi_1 \)) CS contributions.
V. CONCLUDING REMARKS

The particle energy spectrum of the anyon theory exhibits a band structure given by different Landau levels separated by an energy gap $\omega_c$. The energy gap is proportional to the background CS magnetic field $\tilde{B}$, which is induced in the charged anyon fluid to guarantee the electrical neutrality of the system. As it was shown in our previous works, at temperatures lower than the energy gap ($T \ll \omega_c$) a constant and uniform applied magnetic field cannot penetrate the anyon fluid (i.e. the Meissner effect takes place in that superconducting phase).

In this paper we have proved that at $T \gg \omega_c$ the charged anyon fluid does not exhibit a Meissner effect. Hence, we can conclude that the energy gap $\omega_c$ defines a scale that separates two phases in the charged anyon fluid: a superconducting phase at $T \ll \omega_c$, and a non-superconducting one at $T \gg \omega_c$. We expect that the critical temperature for this phase transition should be of order $\omega_c$. Nevertheless, the temperature approximation (2.38) is not suitable to perform the calculation needed to find the phase transition temperature.

We must emphasize that the scenario we have found here and in previous works for the anyon superconductivity at finite temperature is in agreement with the heuristic understanding of anyon superconductivity given by Wilczek in ref. [3]. There, Wilczek pointed out that the London arguments, which start from the role of the energy gap as an essential fact in the theory of superconductivity, seem to provide the base for anyon superconductivity. In the charged anyon fluid, there is no the charge-violating local order parameter which is familiar in the theories with spontaneously broken symmetry. In this system, instead, it is the background CS magnetic field $\tilde{B}$ what determines the energy gap ($\omega_c = \tilde{B}/m$) and plays the role of the order parameter in the anyon gas [24]. Then, it is natural to expect that the critical temperature of the superconducting phase is associated with the anyon fluid order parameter, i.e. with the energy gap.

As we have found in this paper, at $T \gg \omega_c$ the magnetic response of the charged anyon fluid exhibits an inhomogeneous penetration characterized by a wavelength which decreases with the temperature as $1/\sqrt{T}$ in its high-temperature leading order. That is, the spatial inhomogeneity of the many-particle magnetic response will increase with the temperature in this new phase.

The absence of the Meissner effect at $T \gg \omega_c$ is related to the appearing of an imaginary magnetic mass $\mathcal{M}_3$ in this phase (eq. (4.33)). The existence of an imaginary magnetic mass cannot be associated with a tachyonic mode in this many-particle system with CS interactions. The reason is that magnetic masses and the rest energies of the electromagnetic field modes are not the same within the charged anyon fluid.

It is important to note that in obtaining the inhomogeneous magnetic response at high temperatures, it was crucial that the polarization operator coefficient $\Pi_0'$ changes its sign from a positive value at $T \ll \omega_c$ [28], [31], [33], to a negative value at $T \gg \omega_c$ (eq. (2.38)). That is, because $\Pi_0'$ changes its sign, while $\Pi_2$ continues positive, we have that $\mathcal{M}_3$ is imaginary in eq. (4.33).

Finally, we should state that the results obtained in Ref. [3] and in this paper for the superconducting properties of the charged anyon fluid have been derived on the base of a linear approximation. If nonlinear effects, as for instance vortices, are considered, it is possible that a richer scenario for the superconducting phases of the anyon system will appear.

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