6Li AND GAMMA RAYS: COMPLEMENTARY CONSTRAINTS ON COSMIC-RAY HISTORY

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Received 2004 July 29; accepted 2004 December 29

ABSTRACT

The rare isotope 6Li is made only by cosmic rays, predominantly in $\alpha\alpha \rightarrow 6$ Li fusion reactions with interstellar medium (ISM) helium. Consequently, this nuclide provides a unique diagnostic of the history of cosmic rays in our Galaxy. The same hadronic cosmic-ray interactions also produce high-energy $\gamma$-rays (mostly via $pp \rightarrow \pi^0 \rightarrow \gamma\gamma$). Thus, hadronic $\gamma$-rays and 6Li are intimately linked. Specifically, 6Li directly encodes the local cosmic-ray fluence over cosmic time, while extragalactic hadronic $\gamma$-rays encode an average cosmic-ray fluence over lines of sight out to the horizon. We examine this link and show how 6Li and $\gamma$-rays can be used together to place important model-independent limits on the cosmic-ray history of our Galaxy and the universe. We first constrain $\gamma$-ray production from ordinary Galactic cosmic rays, using the local 6Li abundance. We find that the solar 6Li abundance demands an accompanying extragalactic pionic $\gamma$-ray intensity that exceeds that of the entire observed extragalactic $\gamma$-ray background (EGRB) by a factor of 2–6. Possible explanations for this discrepancy are discussed. We then constrain Li production using recent determinations of the EGRB. We note that cosmic rays created during cosmic structure formation would lead to pre-Galactic Li production, which would act as a “contaminant” to the primordial 7Li content of metal-poor halo stars; the EGRB can place an upper limit on this contamination if we attribute the entire EGRB pionic contribution to structure-forming cosmic rays. Unfortunately, the uncertainties in the determination of the EGRB are so large that the present $\gamma$-ray data cannot guarantee that the pre-Galactic Li contribution is small compared to primordial 7Li; thus, an improved determination of the EGRB will shed important new light on this issue. Our limits and their more model-dependent extensions will improve significantly with additional observations of 6Li in halo stars and with improved measurements of the EGRB spectrum by GLAST.

Subject headings: cosmic rays — gamma rays: theory — nuclear reactions, nucleosynthesis, abundances

Online material: color figure

1. INTRODUCTION

The origin and history of cosmic rays has been a subject of intensifying interest. For more than a decade, a large body of work has focused on the light elements Li, Be, and B (LiBeB) as signatures of cosmic-ray interactions with the diffuse gas (for a recent review, see Cassé et al. 2001). LiBeB abundances in Galactic halo stars have been used to probe the history of cosmic rays in the (proto-)Galaxy. More recently, a great deal of attention has been focused on high-energy $\gamma$-rays also produced in interactions during cosmic-ray propagation. Here we draw attention to the tight connection between these observables, particularly between $\gamma$-rays and 6Li.

The abundances of LiBeB nuclei encode the history of cosmic-ray exposure in local matter. In the past 15 years or so, measurements of LiBeB in the Sun and in the Galactic disk have been joined by LiBeB observations in halo stars; these offer particularly valuable information about cosmic-ray origins and interactions in Galactic and proto-Galactic matter. In particular, different scenarios for cosmic-ray origin lead to different LiBeB trends, which have been modeled and compared with observations (see, e.g., Vangioni-Flam & Cassé 2001; Fields & Olive 1999b; Ramaty et al. 2000 and references therein). For the purposes of this paper, the details of these models are less important than the following basic distinction: all LiBeB species are produced as cosmic rays interact with interstellar gas and fragment (“spall”) heavy nuclei, e.g., $p + O \rightarrow 9$ Be. However, the fusion processes $\alpha + \alpha \rightarrow 6$ Li yield lithium isotopes exclusively and indeed dominate the cosmic-ray production of Li (Steigman & Walker 1992; Montmerle 1977c). This makes cosmic-ray lithium production particularly “clean,” since its evolution depends uniquely on its exposure to cosmic rays and unlike Be and B production does not depend on the ambient heavy-element abundances.

Cosmic-ray interactions provide the only known source for the nucleosynthesis of 7Li (Vangioni-Flam et al. 1999; Fields & Olive 1999b), as well as 9Be and 10B, making these species ideal observables of cosmic-ray activity. The story is more complex for 11B, which can also be produced in core-collapse supernovae by the “neutrino process” (e.g., Woosley et al. 1990; Yoshida et al. 2004). Finally, 7Li has the most diverse lineage. In the early Galaxy, and hence in halo stars, 7Li is dominated by the contribution from primordial nucleosynthesis (e.g., Cyburt et al. 2003b and references therein), with a small contribution from cosmic-ray fusion as well as the neutrino process (Ryan et al. 2000). At late times, and hence in disk stars including the Sun, 7Li has important and probably dominant contributions from longer-lived, low-mass stars (although the specific site remains controversial; Romano et al. 2001; Travaglio et al. 2001). In this paper we build on the work of Suzuki & Inoue (2002) to point out the possible importance of another, pre-Galactic source of cosmic-ray 7Li and 6Li, which could confound

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2 In fact, a pre-Galactic component of 6Li can be produced in some scenarios in which dark matter decays via hadronic (Dimopoulos et al. 1988) or electromagnetic (Jedamzik 2000; Kawasaki et al. 2001; Cyburt et al. 2003a) channels. Such scenarios are constrained via their effects on the other light elements, but some level of 6Li production is difficult to rule out completely.
Cosmic-ray interactions with interstellar gas not only produce LiBeB but also inevitably produce $\gamma$-rays. Cosmic rays in the Galactic disk today lead to pronounced emission seen in the Galactic plane (Hunter et al. 1997). Cosmic-ray populations in (and between!) external galaxies would contribute to a diffuse extragalactic $\gamma$-ray background (EGRB). The existence of an EGRB was already claimed by some of the first $\gamma$-ray observations (Fichtel et al. 1973). The most recent high-energy (i.e., roughly in the 30 MeV to 30 GeV range) $\gamma$-ray observations are those of the EGRET experiment on board the Compton Gamma-Ray Observatory, and the EGRET team also found evidence for an EGRB (Sreekumar et al. 1998). The intensity, energy spectrum, and even the existence of an EGRB are not trivial to measure, as this information only arises as the residual after subtracting the dominant Galactic foreground from the observed $\gamma$-ray sky. The procedure for foreground subtraction is thus crucial, and different procedures starting with the same EGRET data have arrived at an EGRB with a lower intensity and different spectrum (Strong et al. 2004) or have even failed to find evidence for an EGRB at all (Keshet et al. 2004). Despite these uncertainties, we find that the EGRB (or limits to it) and Li abundances are mutually very constraining.

Whether or not an EGRB has yet been detected, at some level it certainly should exist. EGRET detections of individual active galactic nuclei (blazars) as well as the Milky Way and the LMC together guarantee that unresolved blazars (e.g., Stecker & Salamon 1996; Mukherjee & Chiang 1999), and to a lesser extent normal galaxies (Pavlidou & Fields 2002), will generate a signal at or near the levels claimed for the EGRB. Many other EGRB sources have been proposed, but a promising one has been the subject of intense interest recently, namely, $\gamma$-rays originating from a cosmological component of cosmic rays. This as-yet putative cosmic-ray population would originate in shocks (Miniati et al. 2000; Keshet et al. 2003; Ryu et al. 2003) associated with baryonic infall and merger events during the growth of large-scale cosmic structures. Diffusive shock acceleration (e.g., Blasi et al. 2005; Kang et al. 2002; Jones & Ellison 1991; Furlanetto & Loeb 2004) would then generate a population of relativistic ions and electrons. Gamma-ray emission would then follow from inverse Compton scattering of electrons off of the ambient photon backgrounds and from $\pi^0$ production due to hadronic collisions (Loeb & Waxman 2000). The most recent semianalytical and numerical calculations (Gabici & Blasi 2003; Miniati 2002) suggest that this “structure-forming” component of the EGRB is likely to be below the blazar contribution, but the observational and theoretical uncertainties here remain large. Upcoming $\gamma$-ray observations by GLAST (Gehrels & Michelson 1999) will shed welcome new light on this problem.

The link between the nucleosynthesis and $\gamma$-ray signatures of cosmic-ray history has been pointed out by others in multiple contexts. We note in particular the prescient work of Montmerle (1977a, 1977b, 1977c), who in a series of papers considered the implications of a hypothetical population of “cosmological cosmic rays” in addition to the usual Galactic cosmic rays (GCRs). Montmerle’s analysis is impressive in its foresight and its breadth. Montmerle (1977a) develops the formalism for a homogeneous population of cosmological cosmic rays (assumed to be created instantaneously at some redshift) and describes their propagation in an expanding universe, as well as their light-element and $\gamma$-ray production. He identifies the tight connection between $^6$Li and extragalactic $\gamma$-rays and exploits this connection to use the available EGRB data to constrain Li production for a variety of different assumptions. A particularly pertinent case involves an EGRB near the levels discussed today (“normalization 2” in Montmerle’s parlance), coupled with a cosmic baryon density close to modern values (e.g., Spergel et al. 2003; Cyburt et al. 2003b). Under these conditions, Montmerle (1977b) finds that cosmological cosmic-ray activity at a level sufficient to explain the EGRB also leads to a present $^6$Li abundance that is about an order of magnitude smaller than the solar abundance. This result foreshadows an important conclusion that we find: if the solar $^6$Li abundance is produced by GCRs, then the associated pionic $\gamma$-ray production exceeds the entire EGRB by at least a factor of 2.

More recently, studies of structure formation cosmic rays (SFCRs) have focused primarily on their $\gamma$-ray signatures. However, recently Suzuki & Inoue (2002) also proposed using $^6$Li as a diagnostic of shock activity in the Local Group. These authors note that the resulting $^6$Li abundances in halo stars could be used to probe the shocks and resulting cosmic rays in proto-Galactic matter. We also draw on this idea, with an emphasis on the fact that pre-Galactic Li production would be (by itself) difficult to distinguish observationally from the primordial $^7$Li production from big bang nucleosynthesis (BBN).3 Thus, we attempt to use EGRB data to constrain this possibility but find that current data are unable to rule out a significant contribution to halo star Li abundances from SFCRs.

Our work thus follows these pioneering efforts, further emphasizing and formally exploring the intimate connection between cosmic-ray nucleosynthesis and high-energy $\gamma$-ray astrophysics. In § 2 we formally show and discuss the generality and tightness of the $^6$Li-$\gamma$ connection, and in § 3 the relevant observations are reviewed. In § 4 we use the theory of LiBeB production by GCRs to deduce the minimal contribution the EGRB. In § 5 we exploit the link to use the observed EGRB to limit the SFCR contribution to pre-Galactic lithium. Discussion and conclusions appear in § 6.

2. THE GAMMA-RAY–LITHIUM CONNECTION: FORMALISM

Before doing a detailed calculation, let us first establish a simple, back-of-the-envelope connection between $\gamma$-rays and lithium. We know that low-energy ($\sim$10–100 MeV $\nu e^{-1}$) hadronic cosmic rays produce lithium through $\alpha \alpha \rightarrow ^6$Li + . . . , but higher energy ($>$280 MeV $\nu e^{-1}$) cosmic rays also produce $\gamma$-rays via neutral pion decay: $pp \rightarrow \pi^0 \rightarrow \gamma \gamma$. Because they share a common origin in hadronic cosmic-ray interactions, we can directly relate cosmic-ray lithium production to “pionic” $\gamma$-rays. The cosmic-ray production rate of $^6$Li per unit volume is $q^{6}$Li = $\sigma_o \alpha \rightarrow ^6$Li $\Phi_e n_o$, where $\Phi_e$ is the net cosmic-ray He flux, $n_o$ is the interstellar He abundance, and $\sigma_o \alpha \rightarrow ^6$Li is the cross section for $^6$Li production, appropriately averaged over the cosmic-ray energy spectrum (detailed definitions and normalization conventions appear in Appendix A). Thus, the $^6$Li mole fraction $Y_o = n_o/n_{H}$ is just $Y^{6}$Li $\sim \int [dt/n_o] q^{6}$Li, where $\sigma_o \alpha \rightarrow ^6$Li $\Phi_e n_o$, $n_{H}$,$\Phi_e$ $\sim$ (He/H)ISM.

On the other hand, the cosmic-ray production rate of pionic $\gamma$-rays is just the pion production rate times a factor of 2, that is, $q_{\gamma} = 2\sigma_o \pi \rightarrow \gamma \Phi_\pi n_o$, $\sim$ (He/H)ISM. Integrated over a line of sight toward the cosmic particle horizon, this gives an EGRB intensity

\[ q_{\gamma} \]
\(I_\gamma \sim c \int dt \Phi/4\pi \sim 2\sigma_{pp} \gamma \phi_{pp} \). Thus, we see that both the \(^6\)Li abundance and the \(\gamma\)-ray intensity have a common factor of the (time-integrated) cosmic-ray flux, and so we can eliminate this factor and express each observable in terms of the other:

\[
Y^{(6)\text{Li}} = \frac{\sigma^{(6)\text{Li}}}{n_{\text{gas}}} \frac{2\pi}{\sigma_{pp}} I_\gamma.
\]

From equation (1) we see that the connection between cosmic-ray lithium production and pionic \(\gamma\)-ray flux is straightforward. This rough argument shows the intimacy of the connection between cosmic-ray lithium production and pion decay, \(\Phi_{\gamma} \approx \Phi_{\text{pp}} \gamma\Phi_{\phi_{\text{pp}}} \), which we can solve to get

\[
I_{\gamma}(t) = \int_0^t dt' q_{\gamma}^{\text{com}}(t').
\]

of the sources over the line of sight to the horizon. We are interested in particular in the case of hadronic sources, so that \(q_{\gamma}^{\text{com}} = a \Phi_{\gamma}\) is the total (energy-integrated) comoving rate of hadronic \(\gamma\)-ray production per unit volume; here \(a\) is the usual cosmic scale factor, which we normalize to a present value of \(a_0 = a(t_0) = 1\). A formal derivation of equation (8) appears in Appendix B, but one can arrive at this result from elementary considerations. Namely, note that the comoving number density of photons produced at any point is just \(n_{\gamma}^{\text{com}} = \int_0^\infty q_{\gamma}^{\text{com}} dt'\). We neglect photon absorption and scattering processes, and thus particle number conservation along with homogeneity and isotropy together demand that the comoving number density of ambient photons at any point is the same as the comoving number density of photons produced there. Furthermore, the total (energy-integrated) photon intensity is also isotropic and thus by definition is \(I_{\gamma} = n_{\gamma}^{\text{com}}c/4\pi\), which is precisely what we find in equation (8).

The comoving rate of pionic \(\gamma\)-ray production per unit volume at point \(s\) is

\[
q_{\gamma}^{\text{com}}(s, t) = \sigma_{\gamma} \Phi_p(s, t) n_{\text{gas}}^{\text{com}}(s, t) = \mu(s, t) \sigma_{\gamma} \Phi_p(s, t) n_{\text{gas}}^{\text{com}}(s, t),
\]

where \(n_{\text{gas}}\) is the (comoving) hydrogen density and \(\Phi_p = 4\pi \int \Phi_p(\epsilon) d\epsilon\) is the total (integrated over energy \(\epsilon\)) omnidirectional cosmic-ray proton flux. The flux-averaged pionic \(\gamma\)-ray production cross section is

\[
\sigma_{\gamma} = \frac{2\xi_\alpha \xi_\pi \sigma_{\gamma_0}}{2 \xi_\alpha} \int_0^\infty \frac{d\epsilon \Phi_p(\epsilon) \epsilon \sigma_{\gamma_0}^\text{p}(\epsilon)}{\int_0^\infty \Phi_p(\epsilon) d\epsilon},
\]

where the factor of 2 counts the number of photons per pion decay, \(\sigma_{\gamma_0}^\text{p}\) is the cross section for pion production, \(\xi_\alpha\) is the pion multiplicity, and the factor \(\xi_\alpha = 1.45\) accounts for \(p\alpha\) and \(\alpha\alpha\) reactions (Dermer 1986).

Then we have

\[
I_{\gamma}(t) = \frac{n_{\text{gas}} \sigma_{\gamma} Y_{\text{H}^0}}{4\pi} \int_0^t dt' \mu(s, t') \Phi_p(s, t'),
\]

where

\[
\Phi_p(s, t) = \int_0^\infty dt' \mu(s, t') \Phi_p^{\text{com}}(s, t', t')
\]

is the mean value of the cosmic-ray fluence along the line of sight, where the average is weighted by the gas fraction and the ratio \(n_{\text{gas}}^{\text{com}}(s)/n_{\text{gas}}(0)\) of the local baryon density along the photon path. Note that the \(\gamma\)-ray sources are sensitive to the overlap of the cosmic-ray flux with the diffuse hydrogen gas density and thus need not be homogeneous. Even so, we still assume the ERGB intensity to be isotropic, which corresponds to the assumption that the line-of-sight integral over the sources averages out any fluctuations.

One further technical note is that \(I_{\gamma} = I_{\gamma}(\epsilon > 0) = \int_0^\infty d\epsilon\), \(I_{\gamma}(\epsilon)\) represents the total pionic \(\gamma\)-ray flux, integrated
over photon energies. While this quantity is well defined theoretically, real observations have some energy cutoff and thus report $I_x(\epsilon_0) \equiv \int_{\epsilon_0}^{\infty} d\epsilon \frac{dI_x(\epsilon)}{d\epsilon}$, typically with $\epsilon_0 = 100$ MeV. However, the spectrum of pionic $\gamma$-rays will be shifted toward lower energies if they originate from a nonzero redshift. Thus, it is clear that the $\gamma$-ray intensity $I_x$, integrated above some energy $\epsilon_0 \neq 0$, will be redshift dependent. A way to eliminate this $z$-dependence is to include all pionic $\gamma$-rays, that is, to take $I_x(\epsilon) \equiv I_x(\epsilon_0)$, i.e., to take $\epsilon_0 = 0$. As discussed in more detail in Appendix A, the $^6\text{Li}$-$\gamma$ proportionality is only exact for $I_x(\epsilon_0)$, as this quantity removes photon redshifting effects, which spoil the proportionality for $\epsilon_0 \neq 0$. Thus, we have to use information on the pionic spectrum to translate between $I_x(\epsilon_0)$ and $I_x(\epsilon_0)$; these issues are discussed further in §3.1.

Thus, we see that the lithium abundance and the pionic $\gamma$-ray intensity (spectrum integrated from zero energy) arise from very similar integrals, which we can express via the ratio

$$I_x(t) \approx \frac{n_b c}{4\pi Y_{0,\text{CR}} Y_{0,\text{ISM}}} \frac{\sigma_{\alpha\alpha} F_{\gamma}(t)}{F_{p}(t)} Y_{i,0},$$

(13)

where $i$ denotes $^6\text{Li}$ or $^7\text{Li}$. Note that this “$\gamma$-to-lithium” ratio has its only significant space and time dependence via the ratio $F_{\gamma}(t)/F_{p}(t)$ of the line-of-sight baryon-averaged fluence to the local fluence.$^5$

The relationship expressed in equation (13) is the main result of this paper, and we bring this tool to bear on Li and $\gamma$-ray observations, using each to constrain the other. To do this, it is convenient to write equation (13) in the form

$$I_{x}(t) \equiv I_{0,\gamma} \frac{Y_{x}(t)}{Y_{i,0}} \frac{F_{\gamma}(t)}{F_{p}(x,t)},$$

(14)

where the scaling factor,

$$I_{0,\gamma} \equiv \frac{n_b c}{4\pi Y_{0,\text{CR}} Y_{0,\text{ISM}}} \frac{\sigma_{\alpha\alpha} Y_{i,0}},$$

(15)

is independent of time and space and only depends, via the ratio of cross sections, on the shape of the cosmic-ray population considered. Table 1 presents the values of $I_{0,\gamma}$ for the different spectra that are considered in the following sections. Values of the scaling factor were obtained by using photon multiplicity $\xi_{\gamma} = 2$, $\xi_{\gamma} = 1.45$, baryon number density $n_b = 2.52 \times 10^{-27}$ cm$^{-3}$, cosmic ray (CR) and interstellar medium (ISM) helium abundances $Y_{0,\text{CR}} = Y_{0,\text{ISM}} = 0.1$, and solar abundances as in §3.2. For the $^7\text{Li}$ and lithium production cross sections we used the fits taken from Dermer (1986) and Mercer et al. (2001) and from those obtained the ratios of flux-averaged cross sections for different spectra. These are also presented in Table 1.

Table 1 shows that the different cosmic-ray spectra lead to very different Li-to-$\gamma$ ratios. For example, the $^6\text{Li}$-to-$\gamma$ ratio $\sigma_{\alpha\alpha}^{\text{pp}} / \sigma_{\text{pp}}^{\text{p}}$ is almost a factor of 5 higher in the SFCR case than in the GCR case. The reason for this stems from the different threshold behaviors and energy dependences of the Li and $^7\text{Li}$ production cross sections. Li production via $\alpha\alpha$ fusion has a threshold around 10 MeV nucleon$^{-1}$, above which the cross section rapidly rises through some resonant peaks. Then beyond $\sim 15$ MeV nucleon$^{-1}$, the cross section for $^6\text{Li}$ drops exponentially as $\exp(-E/16 \text{MeV} \text{ nucleon}^{-1})$ (Mercer et al. 2001), rapidly suppressing the importance of any projectiles with $E > 16$ MeV nucleon$^{-1}$. Thus, as has been widely discussed, Li production is a low-energy phenomenon for which the important projectile energy range is roughly 10–70 MeV nucleon$^{-1}$.

On the other hand, $pp \to ^7\text{Li}$ production has a higher threshold of 280 MeV, and the effective cross section $\xi_{\gamma} \sigma_{\text{pp}}^{\text{p}}$ rises with energy up to and beyond 1 GeV. Neutral pion production is thus a significantly higher energy phenomenon.

These different cross section behaviors are sensitive to the differences in the two cosmic-ray spectra we adopt. On one hand, we adopt a GCR spectrum that is a power law in total energy, $\phi_p(E) \propto (m_p E)^{-2.75}$, a commonly used (e.g., Dermer 1986) approximation to the locally observed (i.e., propagated) spectrum. This spectrum is roughly constant for $E < m_p$. Thus, there is no reduction in cosmic-ray flux between the Li and $^7\text{Li}$ thresholds. Furthermore, the flux only begins to drop far above the $^7\text{Li}$ threshold at 280 MeV, so that there is significant pion production over a large range of energies, in contrast to the intrinsically narrow energy window for Li production. As a result of the effects, $\sigma_{\alpha\alpha}^{\text{pp}} / \sigma_{\text{pp}}^{\text{p}} \ll 1$ for the GCR case.

In contrast, the SFCR flux is taken to be the standard result for diffusive acceleration due to a strong shock, namely, a power law in momentum $\phi_p(E) \propto \rho(E)^{-2}$. This goes to $\propto E^{-1}$ at $E \lesssim m_p$ and $\propto E^{-2}$ at higher energies. This spectrum thus drops by a factor of 28 between the Li and $^7\text{Li}$ thresholds and continues to drop above the $^7\text{Li}$ threshold, offsetting the rise in the pion cross section. This behavior thus suppresses $^7\text{Li}$ production relative to the GCR case, and thus we have a significantly higher $\sigma_{\alpha\alpha}^{\text{pp}} / \sigma_{\text{pp}}^{\text{p}}$ ratio. As we see below, these ratios, and the differences between them, are critical in deriving quantitative constraints.

### 3. OBSERVATIONAL INPUTS

We have seen that the EGRB intensity and lithium abundances are closely linked. Here we collect information on both observables.

#### 3.1. The Observed Gamma-Ray Background and Limits to the Pionic Contribution

Ever since $\gamma$-rays were first observed toward the Galactic poles as well as in the plane (Fichtel et al. 1973), the existence of emission at high Galactic latitudes has been regarded as an indication of an EGRB. However, any information regarding the intensity, energy spectrum, and even the existence of the EGRB is only as reliable as the procedure for subtracting the Galactic foreground. Such procedures are unfortunately nontrivial.

| Cosmic-Ray Population | $I_{0,\gamma}$ ($\text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$) | $I_{0,\gamma}$ ($\text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$) | $\sigma_{\alpha\alpha}^{\text{pp}} / \sigma_{\text{pp}}^{\text{p}}$ | $\sigma_{\alpha\alpha}^{\text{pp}} / \sigma_{\text{pp}}^{\text{p}}$ | $\gamma_{^6\text{Li}} / ^7\text{Li}$ |
|-----------------------|----------------|----------------|----------------|----------------|----------------|
| GCR                   | 9.06 $\times 10^{-5}$ | 8.36 $\times 10^{-4}$ | 0.21 | 0.28 | 1.3 |
| SFCR                  | 1.86 $\times 10^{-5}$ | 1.15 $\times 10^{-4}$ | 1.02 | 2.03 | 2.0 |

$^5$ In fact, the ratio also depends on the shape of the cosmic-ray spectrum (assumed universal), which determines the ratio of cross sections. We take this into account below when we consider different cosmic-ray populations.
and model dependent. The EGRET team (Sreekumar et al. 1998) used an empirical model for tracers of Galactic hydrogen and starlight and found evidence for an EGRB that dominates polar emission. Other groups have recently presented new analyses of the EGRET data. In a semiempirical approach using a model of Galactic γ-ray sources, Strong et al. (2004) also find evidence for an EGRB, but with a different energy spectrum and a generally lower intensity than the Sreekumar et al. (1998) result. Finally, Keshet et al. (2004) find that the Galactic foreground is sufficiently uncertain that its contribution to the polar emission can be significant, possibly saturating the observations. Consequently, the Keshet et al. (2004) analysis is unable to confirm the existence of an EGRB in the EGRET data; instead, they can only place upper limits on the EGRB intensity.

It was recently shown by Prodanović & Fields (2004a) that a model-independent limit on the fraction of EGRB flux that is of pionic origin (γ-rays that originate from π^0 decay) can be placed. Their limit comes from noticing that the EGRB shows no strong evidence of the distinctive pionic γ-ray spectral peak at m_{π0}/2, the “pion bump.” Thus, by comparing the shapes of the observed EGRB and theoretical pionic γ-ray spectrum, they were able to maximize the pionic flux so that it stays below the observed one. This procedure allowed them to place constraints on the maximal fraction of EGRB that can be of pionic origin.

For the pionic γ-ray source function, Prodanović & Fields (2004a) used a semianalytical fit from the Pfrommer & Ensslin (2004) paper and the Dermer (1986) model for the production cross section. A key feature of the pionic γ-ray spectrum is that it approaches a power law at both high and low energies, going to e^{-α} for ε ≪ m_{π0}/2 and to e^{-αZ} for ε ≫ m_{π0}/2. In Dermer’s model, the γ-ray spectral index α is equal to the cosmic-ray spectral index. Prodanović & Fields (2004a) adopted the value α = 2.2 for pionic extragalactic γ-rays, which is consistent with blazars and SFCRs as their origin. In this simple analysis, Prodanović & Fields (2004a) used a single-redshift approximation; that is, they assumed that these γ-rays are all coming from one redshift, and thus their limit on the maximal pionic fraction is a function of z.

To obtain the EGRB spectrum from EGRET data, a careful subtraction of Galactic foreground is needed. Prodanović & Fields (2004a) considered two different EGRB spectra and obtained the following limit: for the Sreekumar et al. (1998) spectrum they found that the pionic fraction of the EGRB (integrated spectra above 100 MeV) can be as low as about 40% for cosmic rays that originated at present to about 90% for z = 10; for the more shallow spectrum of Strong et al. (2004) they found that the pionic fraction can go from about 30% for z = 0 up to about 70% for z = 10. However, the Keshet et al. (2004) analysis of the EGRET data implies that the Galactic foreground dominates the γ-ray sky, so that only an upper limit on the EGRB can be placed, namely, I_{γ}(>100 MeV) ≤ 0.5 × 10^{-5} cm^{-2} s^{-1} sr^{-1}. Thus, in this case we were not able to obtain the pionic fraction.

However, to be able to connect the pionic γ-ray intensity I_{γ}, with the lithium mole fraction Y_L, as shown in equation (13), I_{γ}, must include all of the pionic γ-rays; that is, the spectrum has to be integrated from energy ε_0 = 0. The upper limit to the pionic γ-ray intensity above energy ε_0 for a given redshift can be written as

\[
I_{γ}(> ε_0) = f_ε(> ε_0, z)I_{γ}^{\text{obs}}(> ε_0)
\]

\[
= N_{\text{max}} \int_{ε_0} \varphi(ε(1+z)) \, dε,
\]

where f_ε(> ε_0, z) is the upper limit to the fraction of pionic γ-rays (Prodanović & Fields 2004a), I_{γ}^{\text{obs}}(> ε_0) is the observed intensity above some energy, and \varphi(ε(1+z)) is the semianalytical fit for the pionic γ-ray spectrum (Pfrommer & Ensslin 2004), which is maximized with the N_{\text{max}} normalization constant. An upper limit to the pionic γ-ray intensity that covers all energies, I_{γ}(>0, z), follows immediately from the above equations:

\[
I_{γ}(>0, z) = f_ε(> ε_0, z)I_{γ}^{\text{obs}}(> ε_0) \int_{ε_0} \varphi(ε(1+z)) \, dε.
\]

Now this is something that is semiobservational and can be easily obtained from the γ-ray intensity observed above some energy and from the Prodanović & Fields (2004a) and Pfrommer & Ensslin (2004) results.

### 3.2. Lithium Abundances

Given the EGRB intensity, we infer the amount of associated lithium production. It is of interest to compare this to the solar abundance and also to the primordial abundance of 7Li. We take the solar Li isotope abundances from Anders & Grevesse (1989): (6Li/H)_{solar} = 1.53 × 10^{-10} and (7Li/H)_{solar} = 1.89 × 10^{-9}. These are derived from meteoritic data and thus reflect conditions in the presolar nebula and in particular are not plagued by the well-known deficit of Li in the solar photosphere. However, it is worth noting that the Galactic chemical evolution history of Li includes not only the sources we have mentioned, but also sinks. Main-sequence stars destroy both Li isotopes in all but their outermost layers, and for stars in the mass range 1–4 and 6–10 M_{⊙} there may be no additional Li production (Romano et al. 2001). These stars thus act as Li sinks and contribute to Galactic astration of Li, similar to but less severe than the astration of deuterium. Consequently, the solar Li isotopic abundances are, strictly speaking, a lower limit to the total Galactic production, with some additional production (up to a factor of ~2 higher, using deuterium as a guide; Cyburt et al. 2003b) being hidden by astration.

Metal-poor halo stars (extreme Population II) serve as a “fossil record” of pre-Galactic lithium. Ryan et al. (2000) find a pre-Galactic abundance of

\[
\frac{(\text{Li})}{H}_{\text{pre-Gal, obs}} = (1.23^{+0.34}_{-0.16}) \times 10^{-10},
\]

based on an analysis of very metal poor halo stars. On the other hand, one can use the WMAP (Spergel et al. 2003) baryon density and BBN to predict a “theoretical” (or cosmic microwave background–based [“CMB-based”]) primordial 7Li abundance (Cyburt et al. 2003b):

\[
\frac{(\text{Li})}{H}_{\text{BBN, th}} = (3.82^{+0.73}_{-0.60}) \times 10^{-10}.
\]

These abundances are clearly inconsistent. Possible explanations for this discrepancy include unknown or underestimated systematic errors in theory and/or observations or new physics; these are discussed thoroughly elsewhere (see, e.g., Cyburt et al. 2004 and references therein). For our purposes, we acknowledge this discrepancy by comparing pre-Galactic...
lithium production by cosmic rays with both the observed and CMB-based Li abundances.

4. 6Li AND GAMMA RAYS FROM GALACTIC COSMIC RAYS

We have shown that 6Li abundances and extragalactic γ-rays are linked because both sample cosmic-ray fluence. We now apply this formalism to γ-ray and 6Li data. In this section we turn to the hadronic products of GCRs, which are believed to be the dominant source of 6Li (Vangioni-Flam et al. 1999; Fields & Olive 1999b), but a subdominant contribution to the EGRB.

4.1. Solar 6Li and Gamma-Rays

We place upper limits on the lithium component of GCR origin by using the formalism established in earlier sections. To be able to find $I_\gamma / Y_{\text{Li}}$ from equation (13), we assume that the ratio of cosmic-ray fluence along the line of sight (weighted by the gas fraction) to the local cosmic-ray fluence is $F_p(t)/F_p(\infty, t) \approx 1$. That is, we assume that the Milky Way fluence is typical of star-forming galaxies, i.e., that the γ-luminosities are comparable: $L_{\text{MW}} \approx \langle L \rangle_{\text{gal}}$. Note that in the most simple case of a uniform approximation (cosmic-ray flux and gas fraction the same in all galaxies), the two fluences would indeed be exactly equal.

Taking the solar 6Li abundance and $\langle \sigma \rho_c \rangle / \langle \sigma \rho_p \rangle = 0.21$ for the ratio of GCR flux averaged cross sections, we now use equation (14) to find that $I_\gamma(\varepsilon > \varepsilon_0) = 9.06 \times 10^{-5} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$ is the hadronic γ-ray intensity that is required if all of the solar 6Li is made via Galactic cosmic rays.

We wish to compare this 6Li-based pionic γ-ray flux to the observed EGRB intensity, $I_{\text{obs}}(\varepsilon > \varepsilon_0)$. However, equation (14) gives the hadronic γ-ray intensity integrated over all energies, whereas the observed one is above some finite energy. Thus, we have to compute

$$I_\gamma(\varepsilon > \varepsilon_0) = I_\gamma(\varepsilon > 0) \int_{\varepsilon_0}^{\infty} d\varepsilon I_{\text{pc},\varepsilon} \left( \int_{\varepsilon_0}^{\infty} d\varepsilon I_{\gamma,\varepsilon} \right),$$

where $L_{\gamma}$ is the average galactic γ-ray luminosity (by photon number) and $n_{\text{gal}}(z)$ is the mean comoving number density of galaxies. The key assumptions for the luminosity $L_{\gamma}$ are (1) that supernova explosions provide the engines powering cosmic-ray acceleration, so that the cosmic-ray flux $\Phi \propto \psi$ scales with the supernova rate and thus the star formation rate $\psi$; (2) that the targets come from the gas mass, which evolves following the “closed box” prescription; and (3) that the Milky Way luminosity represents that of an average galaxy. With these assumptions we have that $L_{\gamma} \propto \mu \psi$ and thus that $q_{\gamma,\text{GR}}(z, \varepsilon) \propto \mu \varepsilon$, where $\mu$ is the cosmic star formation rate.

Following Pavlidou & Fields (2002), the specific form of $I_{\gamma,\varepsilon}$ is expressed in terms of the present-day Milky Way gas mass fraction $\rho_{0,\text{MW}}$, cosmic star formation rate $\dot{\psi}(z)$, Milky Way γ-ray (number) luminosity $L_{\gamma,\text{MW}}(z, E)$, and cosmology $\Omega_{\Lambda}$ and $\Omega_m$ and is integrated up to $z_*$, the assumed starting redshift for star formation.

For this calculation we adopt the following values: $\rho_{0,\text{MW}} = 0.14$, $\Omega_{\Lambda} = 0.7$, $\Omega_m = 0.3$, and $z_* = 5$. For the cosmic star formation rate we use the dust-corrected analytical fit from Cole et al. (2001). Finally, we need the (number) luminosity of pionic γ-rays, which we can write as

$$L_{\gamma,\text{MW}}(z, E) = \gamma_p N_p \propto \Phi M_{\text{gas}},$$

where $n_p$ is the proton number density in the Galaxy, $N_p$ is the total number of protons in the Galacticon ISM, and $q_{\gamma,\text{c}}$ is the source function of γ-rays that originate from pion decay adopted from PM (2004). That is, in equation (21) we have the ratio of two integrals for which the integrands are identical; thus, normalizations and constants cancel out. Therefore, instead of using the complete form of $L_{\gamma,\text{MW}}(z, E)$, we need only use the spectral shape of the pionic γ-ray source function (PM (2004), that is, only the part that is energy and redshift dependent.

Finally, then, we find

$$I_{\gamma}(\varepsilon > 0.1 \text{ GeV}) = 3.22 \times 10^{-5} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1},$$

which we can now compare to the observed EGRB values $I_{\text{obs}}(\varepsilon > 0.1 \text{ GeV})$ that are given in the first column of Table 2. As one can see, our pionic EGRB γ-ray intensity is between 2 and 6 times larger than the entire observed value! Moreover, the total observed high-latitude (b > 30°) emission (Kniffen et al. 1996) is $I_{\text{obs}}(b > 30°) = 1.5 \times 10^{-5} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}$, consistent with the Sreekumar et al. (1998) value. Thus, the GCR-based 6Li demand for pionic γ-rays exceeds the entire observed high-latitude signal by a factor of 2, independent of any prescription for Galactic foreground subtraction. The discrepancy is thus model-independent.

At this point, it is worth noting that we have assumed that 6Li is produced solely by α fusion processes. However, we note that spallation processes of the kind $p + \alpha + \text{CNO} \rightarrow ^6\text{Li}$. By ignoring these processes thus far, we have overestimated the α contribution to 6Li and in turn overestimated its inferred EGRB contribution. However, the spallation processes are negligible at low metallicities and are even subdominant for an ISM with solar metallicity; for our adopted GCR spectrum, the two are equal when $(O/H)_{\text{ISM}} = 0.44(O/H)_{\odot}$. As an extreme lower limit on 6Li, one can assume that the ISM was always at solar metallicity, in which case one finds that the 6Li contribution to 6Li is 6Li $> 0.3 \times 6\text{Li}_{\odot}$. However, over the Galactic history the ISM would have had a lower average metallicity; in the simplest picture, the appropriate mean metallicity to take is half of solar, in which case we find 6Li $> 0.5 \times 6\text{Li}_{\odot}$. Using this α contribution to 6Li, we find that the required spallon γ-ray background is just consistent with the full observed background as estimated by Sreekumar et al. (1998) but still exceeds more recent estimates of the EGRB by up to a factor of 3. Furthermore, we recall that the lack of a pionic spectral signature in the EGRB demands that there is a comparable additional nonpionic contribution as well.

We thus conclude that the solar 6Li abundance, if made by GCRs as usually assumed, seems to demand a large diffuse pionic γ-ray contribution, significantly above the entire EGRB level. How might this discrepancy be resolved? One explanation follows by dropping our assumption that $F_{\text{RMW}}(f_0) = F_{\text{TBM}}(f_0)$, i.e., that the baryon-weighted Milky Way GCR fluence is the same as the cosmic mean for star-forming galaxies. Note that we have $F = \int dt \mu \Phi \propto \int dt \langle \psi M_{\text{gas}} \rangle$, where $\psi$ is the global
galactic star formation rate (assuming $\Phi \propto \psi$) and $M_{\text{gas}}$ is the galactic gas content. If our Galaxy has an above-average star formation rate and/or gas mass, this will increase the local $^6\text{Li}$ production relative to the average over all galactic populations and thus lead to an overestimate of the EGRB.

In connection with this it is noteworthy to compare our $^6\text{Li}$-based estimate of the galactic EGRB contribution to the work of Pavlidou & Fields (2002). That calculation adopted the same model for the redshift history of cosmic-ray flux and interstellar gas and so only differed from the present calculation in the normalization to Galactic values. Pavlidou & Fields (2002) normalized to the present Galactic $\gamma$-ray luminosity. This amounts to a calibration not to the time-integrated cosmic-ray flux, but rather to the instantaneous cosmic-ray fluence, and which contains fewer uncertainties than the factors entering in the Pavlidou & Fields (2002) result. Yet, surprisingly, the $^6\text{Li}$-based fluence result gives a high pionic EGRB, while the more uncertain normalization gives an acceptable result.

Can we independently test whether our Galaxy has had an above-average cosmic-ray exposure? This presents a challenge, as we require an integral measure of cosmic-ray activity, which is readily available locally but difficult to obtain in external galaxies. The best candidates are the LiBeB isotopes; $^9\text{Be}$ is ideal for the reasons we have outlined but is not accessible in stars bright enough to be seen in even the nearest external galaxies. The best hope, then, would be for measurements of $^9\text{Be}$, in the Local Group or beyond. In the SMC, such measurements have placed interesting limits on boron abundances (Brooks et al. 2002), although the presence of the neutrino-process production of $^{11}\text{B}$ makes boron observations more difficult to interpret than beryllium.

Another explanation for the high intensity of equation (25) stems from noting that the required EGRB intensity is much larger for GCRs than that one would infer from SFCRs, because of the large difference in the $^\pi^\Lambda/\Lambda$ ratio for these two spectra. Were the Milky Way spectrum atypically skewed to high energies, we would overestimate the $^\pi^\Lambda/\Lambda$ ratio and thus the EGRB contribution. This possibility (which we regard as less likely than the previous one) could be tested by observations of cosmic-ray spectra in external galaxies, e.g., by $\gamma$-ray observations of Local Group galaxies such as GLAST should perform (Pavlidou & Fields 2001).

Finally, a related but more unconventional view would be that $^6\text{Li}$ is in fact primarily made by SFCRs themselves, rather than by GCRs. This suggestion is further discussed and constrained in § 5.

We close this subsection by noting that if the astrospace of $^6\text{Li}$ is taken into consideration (see § 3.2), one might use a $^6\text{Li}$ abundance larger than solar. In that case, one would find that the accompanying pionic EGRB $\gamma$-ray intensity is more than 2–6 times larger than the observed EGRB.

### 4.2. The Observed EGRB and Nonprimordial Lithium

We can exploit equation (14) in both directions. Here we use the observed EGRB spectrum to constrain the $^6\text{Li}$ abundance produced via GCRs. By comparing this Galactic $^6\text{Li}$ component to the observed solar abundance, we can then place an upper limit on the residual $^6\text{Li}$ that (presumably) was produced by SFCRs. As described in § 3.1, with the observed EGRB spectrum in hand we can place an upper limit on its fraction of pionic origin. In the case of SFCR-produced pionic $\gamma$-rays, we can place constraints directly only in the presence of a model for the SFCR redshift history. Since a full model is unavailable, in § 5 we adopt the “single-redshift approximation.” However, in the case of GCRs we have a better understanding of the redshift history of the sources. Therefore, we follow Pavlidou & Fields (2002) to calculate the pionic differential $\gamma$-ray intensity for some set of energies,

$$I_{\gamma,E} = \frac{c}{4\pi H_0 \psi_{\text{MW}}} \int_0^{z_{\text{f}}} dz \frac{\hat{\rho}_0(z) L_{\gamma}(1 + z) E}{\sqrt{\Omega_\Lambda + \Omega_M (1 + z)}} \times \left[ \frac{1}{\mu_{0,\text{MW}}} - \frac{1}{\mu_{\text{f},\text{MW}}} - 1 \right] \frac{f_{\gamma}}{f_{\gamma}} \frac{d\gamma}{dE} \frac{dE}{d\gamma} \hat{\rho}_0(z) \right],$$

where $L_{\gamma}$ is in units of $s^{-1} \text{cm}^{-2} \text{GeV}^{-1}$ and $\psi_{\text{MW}}$ is the present Milky Way star formation rate. For the pionic $\gamma$-ray luminosity $L_{\gamma}$, we, as before, use the pionic $\gamma$-ray source function adopted from Pfremmer & Ensslin (2004; $\alpha_{\gamma} = 2.75$ for the GCR spectrum); however, we let the normalization be determined by maximizing the pionic contribution to the EGRB. The adopted parameters, cosmology, and cosmic star formation rate we keep the same as in § 4.1.

Once we obtain the spectrum, we can then fit it with

$$\ln(I_{\gamma} E^2) = -14.171 - 0.546 \ln E - 0.131(\ln E)^2 + 0.032(\ln E)^3,$$
Thus, the pionic $\gamma$-ray flux above 0.1 GeV is $I_{\gamma,p}(>0.1 \text{ GeV}) = 0.83 \times 10^{-5} \text{ cm}^{-2} \text{s}^{-1} \text{ sr}^{-1}$. From equation (21), it now follows that the total flux is $I_{\gamma,p}(>0) = 2.31 \times 10^{-5} \text{ cm}^{-2} \text{s}^{-1} \text{ sr}^{-1}$. As before, we can now use equation (14) to find the GCR $^6$Li mole fraction,

$$
\left( \frac{Y_{\gamma,Li}}{Y_{\gamma,Li}} \right)_{GCR} = \frac{I_{\gamma,p}(>0)}{9.06 \times 10^{-5} \text{ photons cm}^{-2} \text{s}^{-1} \text{ sr}^{-1}} = 0.25,
$$

and thus, SFCR-produced $^6$Li can be at most (neglecting the $^6$Li astation) the residual $^6$Li,

$$
\left( \frac{Y_{\gamma,Li}}{Y_{\gamma,Li}} \right)_{SFCR} = 1 - \left( \frac{Y_{\gamma,Li}}{Y_{\gamma,Li}} \right)_{GCR} = 0.75.
$$

With the appropriate scaling between $^7$Li and $^6$Li, as given in Table 1, we can then determine the total elemental Li = $^7$Li + $^6$Li abundance and compare it to the primordial values from equations (19) and (20):

$$
\left( \frac{\text{Li}}{\text{H}} \right)_{SFCR} = 3.45 \times 10^{-10} = 0.90 \left( \frac{\text{Li}}{\text{H}} \right)_{p,thy}
$$

$$
= 2.81 \left( \frac{\text{Li}}{\text{H}} \right)_{p,obs}.
$$

So far we have been determining the maximized pionic fraction of the EGRB based only on the shape of the pionic spectrum. However, in the case of normal galaxies we have a better understanding of what that fraction should be. That is, we can normalize the pionic spectrum to that of the Milky Way and then integrate over the redshift history of sources. Following Pavlidou & Fields (2002 and references therein) we set up the normalization by requiring that $\int_0^{1 \text{ GeV}} dE L_{\gamma,p}(z = 0, E) = \int_0^{1 \text{ GeV}} dE L_{\gamma,p,\text{MW}}(E) = 2.85 \times 10^{42}\text{ s}^{-1}$. With the analytical fit of the shape of the pionic spectrum from Pfrommer & Ensslin (2004), this now gives

$$
L_{\gamma,p}(z = 0, E) = 9.52 \times 10^{44} \text{ s}^{-1} \text{ GeV}^{-1}
$$

$$
\times \left[ \frac{2 \epsilon}{m_{\pi^+}} \delta_\epsilon + \frac{2 \epsilon}{m_{\pi^+}} \delta_\epsilon \right]^{-\alpha_\gamma/\beta_\gamma},
$$

where $\alpha_\gamma = 2.75$ for the GCR spectrum and $\delta_\epsilon = 0.14\alpha_\gamma^{-1.6} + 0.44$. Now we can use equation (26) to obtain the pionic spectrum, which is plotted in Figure 1 (lower dashed line). We use star formation rate $\psi_{\text{MW}} = 3.2 M_\odot \text{ yr}^{-1}$ (McKee 1989). Finally, we find that in this case, when the pionic spectrum is normalized to the Milky Way, the GCR $^6$Li mole fraction that accompanies it is

$$
\left( \frac{Y_{\gamma,Li}}{Y_{\gamma,Li}} \right)_{GCR} = 0.14,
$$

which then gives $(\text{Li}/\text{H})_{SFCR} = 3.96 \times 10^{-10} = 1.03 (\text{Li}/\text{H})_{p,thy} = 3.22 (\text{Li}/\text{H})_{p,obs}$, which is of course a weaker limit than the maximal pionic case.

We thus see that in a completely model-independent analysis, current observations allow the possibility that SFCRs are quite a significant source of $^6$Li and of $\gamma$-rays. Indeed, we cannot exclude that SFCR-produced lithium can be a potentially large contaminant of the pre-Galactic Li component of halo stars,
which would exacerbate the already troublesome disagreement with CMB-based estimates of primordial \(^6\)Li. Consequently, we conclude that models for SFCR acceleration and propagation should include both \(\gamma\)-ray and \(^6\)Li production, and more constraints on SFCRs, both theoretically (e.g., space and time histories) and observationally (e.g., EGRB and possibly diffuse synchrotron measurement), will clarify the picture we have sketched.

Note that had we also considered the possibility of astration of \(^6\)Li (§ 3.2), we would have found a greater \(^6\)Li residual and thus had an even larger SFCR-produced component.

5. \(^6\)Li AND GAMMA RAYS FROM COSMOLOGICAL COSMIC RAYS

In this section we turn to the as-yet unobserved cosmological component of cosmic rays and to the synthesis of lithium by SFCRs. This lithium component would be the first made after BBN. Any Li that is produced this way prior to the most metal poor halo stars would amount to a pre-Galactic Li enrichment and thus would be a nonprimordial Li component, unaccompanied by beryllium and boron production. This structure formation Li would be an additional “contaminant” to the usual components in halo stars, the \(^7\)Li abundance due to primordial nucleosynthesis, and the \(^6\)Li and \(^7\)Li contribution due to GCRs (Ryan et al. 2000). Moreover, the pre-Galactic but nonprimordial component would by itself be indistinguishable from the true primordial component and thus would lead to an overestimate of the BBN \(^7\)Li production.

Our goal in this section is to exploit the \(\gamma\)-ray connection to constrain the structure formation Li contamination. Unfortunately, we currently lack a detailed understanding of the amount and time history of the SFCRs (and resulting \(\gamma\)-rays and Li). Thus, we make the conservative assumption that all SFCRs, and the resulting \(\gamma\)-rays and Li, are generated prior to any halo stars. Furthermore, we assume that the pionic contribution to the EGRB is entirely due to SFCRs. This allows us to relate observational limits on the pionic EGRB to pre-Galactic Li.

With this assumption and a SFCR composition \(\xi^{CR}_{p}/\xi^{CR}_{\gamma} \approx 0.1\), we can now use the appropriate scaling factor from Table 1 to rewrite equation (13) as

\[
I_{\gamma}(\epsilon > 0, z) = \frac{\xi^{CR}_{\gamma}}{4\pi} \frac{2}{\alpha^{\gamma} Y_{\alpha^{\gamma}} Y_{\gamma^{\gamma}} \sigma^{\gamma}_{L}} \left( \frac{6Li}{H} \right) n_{B} c \]

\[
= 1.86 \times 10^{-5} \text{ photons cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \left( \frac{6Li}{6Li_{\odot}} \right) .
\]

or

\[
\left( \frac{6Li}{6Li_{\odot}} \right) = 0.538 \frac{I_{\gamma}(0 > 0)}{10^{-5} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}} ,
\]

where we used the solar lithium mole fraction \(Y(6Li_{\odot}) = 1.09 \times 10^{-10}\).

To set up an extreme upper limit on pre-Galactic SFCR \(^6\)Li, we assume that the entire pionic extragalactic \(\gamma\)-ray background came from SFCR-made pions and was created prior to any halo star. As mentioned in § 4, the method used in subtraction of the Galactic foreground is crucial for obtaining the EGRB spectrum. What is more, the EGRB spectrum is an important input parameter in the Prodanović & Fields (2004a) analysis, whose estimates of the maximal pionic \(\gamma\)-ray flux we use here. Our results for the SFCR lithium upper limits are collected in Table 2. The results depend on the choice of the EGRB spectrum as well as the redshift of origin of cosmic rays according to the single-redshift approximation used by Prodanović & Fields (2004a) to obtain the maximal pionic EGRB fraction. Note that we considered only the two most extreme redshifts to illustrate the results. In the Table 2, \(z\) is the redshift, \(I_{\gamma}(0 > 0)\) is the upper limit for the pionic \(\gamma\)-ray intensity above zero energy determined from equation (18), as explained in § 4, \((LI/H)_{max}^{SFCR}\) is the upper limit to total \((6Li + 7Li)\) lithium abundance that can be of SFCR origin, and \(Li^{theo}_{obs}\) and \(Li^{obs}\) are the theoretical and observational primordial lithium abundances, respectively, as given in equations (20) and (19).

Note that for the case of the Keshe et al. (2004) EGRB, since a spectrum was unavailable, the procedure described in § 3.1 for maximizing the pionic fraction of the EGRB could not be used. Thus, to place an upper limit on SFCR lithium, we assumed that the entire EGRB can be attributed to decays of pions, that is, assume \(I_{\gamma} = I_{\gamma}^{obs}\). For the Sreekumar et al. (1998) and Strong et al. (2004) EGRB spectra, we use the upper limits to \(I_{\gamma}^{obs}\) obtained by Prodanović & Fields (2004a). Once the \(I_{\gamma}^{obs}\) is set, we can use equation (34) to find the SFCR \(^6\)Li upper limit.

To find the total halo star contribution, we must also include \(^7\)Li, which is in fact produced more than \(^6\)Li in \(\alpha\) fusion; as seen in Table 1, \((7Li/6Li)_{SFCR} = \langle \sigma^{\gamma}_{\alpha}/\sigma^{\gamma}_{\alpha} \rangle \approx 2\). The total SFCR elemental Li production appears in Table 2, both in terms of the absolute \(Li/H\) abundance and its ratio to the different measures of primordial Li (§ 3.2).

From Table 2 we see that the maximal possible SFCR contribution to halo star lithium could be quite substantial. If the pre-Galactic SFCR component is dominantly produced at high redshift (i.e., as in the \(z > 10\) results), then the maximum allowed Li production can exceed the primordial Li production (however it is estimated), in some cases by a factor up to 25! The situation is somewhat better if the pre-Galactic SFCR production is at low redshift, but here it is difficult to understand how this would predate the halo star component of our Galaxy. The high-redshift result is thus the more likely one, but also somewhat troubling in that the limit is not constraining. The indirect limits on SFCR Li in § 4 are somewhat stronger, but these also hold the door open for a significant level of pre-Galactic synthesis.

We caution that the lack of a strong constraint on SFCR Li production is not the same as positive evidence that the production was large. Recall that we have made several assumptions that purposefully maximize the SFCR contribution; to the extent that these assumptions fail, the contribution falls, perhaps drastically. A more detailed theoretical and observational understanding of the SFCR history, and of the EGRB, will help to clarify this situation. Moreover, given that the halo star Li is already found to be below the CMB-based \(^7\)Li BBN results, we are already strongly biased to believe that the pre-Galactic SFCR component is not very large. Thus, one might be tempted instead to go the other way and use Li abundances to constrain SFCR activity.

We thus now go the other way and use solar \(^6\)Li to constrain the SFCR \(\gamma\)-ray flux. Again, given our incomplete knowledge of SFCRs, we must adopt a simplifying assumption about the degree of \(^6\)Li production that is due to SFCR. To be conservative, we make the extreme assumption that all of the solar \(^6\)Li is produced by SFCRs, and thus find via equation (32) that the \(\gamma\)-ray flux is \(I_{\gamma}(0 > 0) > 1.86 \times 10^{-5} \text{ photons cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}\). From equation (18) we can determine \(I_{\gamma}(0 > 0.1 \text{ GeV}) < 0.23 \times 10^{-5} < I_{\gamma}(0 > 0.1 \text{ GeV}) < 1.43 \times 10^{-5} \text{ photons cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}\), depending on the redshift of pionic \(\gamma\)-rays, which is below
the observed level as determined by Sreekumar et al. (1998) and a factor of 2–14 lower than the prediction based on GCRs. Thus, for a given observed intensity $I_{\gamma}^{\text{obs}}(>\epsilon_0)$ we can now use equation (18) to constrain the hadronic fraction of the EGRB, that is, calculate $f_h(>\epsilon_0, z)$, which is also presented in Table 2.

However, since Li probably suffers some level of astration, the use of the solar $^6$Li abundance does not give us the uppermost limit to the required pionic $\gamma$-ray flux $I_{\gamma}^{\nu}(>0)$. Thus, if one were to compensate for the depletion, the pionic fraction $f_{\gamma}(>\epsilon_0, z)$ would become even larger.

Indeed, this may suggest a solution to the EGRB overproduction by GCRs, seen in §4. If $^6$Li is mostly made by SFCRs, then the associated $\gamma$-ray production is in line with the observed background. In this case, $^6$Li would still be of cosmic-ray origin, but not dominated by GCR production; this situation would be similar to that suggested by Suzuki & Inoue (2002), who found that GCR-created $^6$Li only becomes comparable to the SFCR component near solar metallicities. Such a scenario faces tests regarding $^6$Li and other LiBeB abundances and their Galactic evolution. A detailed discussion of this scenario will appear in a forthcoming study.

6. DISCUSSION

The main result of this paper is to identify and quantify the tight connection between $^6$Li and the EGRB as measures of cosmic-ray history. Specifically, these two observables provide measures of average, gas-weighted cosmic-ray fluence. Moreover, the observables are complementary, in that $^6$Li samples local fluence, while the EGRB encodes the cosmic mean fluence.

We present scaling laws that relate $^6$Li and the EGRB intensity for different cosmic-ray spectra appropriate for GCR and SFCR populations. Using these scalings, and assuming that our local $^6$Li measurements are typical, we can test the self-consistency of $^6$Li and EGRB observations in a relatively model-independent manner. We find that if SFCRs dominate the pionic EGRB, then the associated $^6$Li production can be a significant and perhaps dominant contribution to the solar abundance. On the other hand, we find that if $^6$Li production is dominated by GCRs, then the associated $\gamma$-ray production is enormous, at least a factor of 2 above the observed intensity.

Furthermore, using the EGRB we use two different lines of argument to place an upper limit on the SFCR contribution to pre-Galactic lithium in halo stars. Such a component of lithium would be confused with the true primordial abundance and thus would exacerbate the existing deficit in halo star Li relative to the CMB-based expectations of BBN theory. Unfortunately, current EGRB data are such that our model-independent upper limit (which must assume, among other things, that all SFCRs are created prior to any halo stars) is very weak. In particular, we cannot exclude the possibility that a significant portion of pre-Galactic lithium is due to SFCRs. We thus find that the nucleosynthesis aspects of SFCRs are important and deserve further, more detailed study.

A full understanding of the implications of the relationships among $^6$Li, diffuse $\gamma$-rays, and cosmic-ray populations thus awaits better observational constraints (both light elements and especially the EGRB) as well as a more detailed study of SFCRs. Having shown the importance of both the $^6$Li and $\gamma$-ray constraints, it is our hope that these observables will be calculated in models of Galactic and structure formation cosmic rays and that both $^6$Li and the EGRB will be used in concert to constrain cosmic-ray interactions with diffuse matter. The results of these models will go far to address some of the questions that this study has raised.

We are grateful for illuminating discussions with Vasiliki Pavlidou. We also thank Takeru Suzuki for helpful comments regarding $^6$Li spallation production. This material is based on work supported by the National Science Foundation under grant AST 00-92939.

APPENDIX A

NOTATION AND NORMALIZATION CONVENTIONS

The interactions of cosmic-ray species $i$ with target nucleus $j$ produces species $k$ at a rate per target particle of

$$\Gamma_k = \int_{E_{n,k}} dE \sigma_{ij-k}(E) \phi_i \equiv \sigma_{ij-k} \Phi_i.$$  \hfill (A1)

Here $E$ is the cosmic-ray energy per nucleon, $\sigma_{ij-k}$ is the energy-dependent production cross section with threshold $E_{\text{th},k}$, and $\phi_i$ is the cosmic-ray flux. The rate per unit volume for $i + j \rightarrow k$ is thus $q_k = \Gamma_k n_j$.

Note that the flux in equation (A1) is position and time dependent. To isolate this dependence, it is useful to define a total, energy-integrated flux

$$\Phi_i = \int_{E_{n,\text{min}}} dE \phi_i,$$  \hfill (A2)

where we choose the lower integration limit to always be the minimum threshold $E_{\text{th},\text{min}}$ for all reactions considered; in our case this is the $\alpha + \alpha \rightarrow ^7$Li threshold of 8.7 MeV nucleon$^{-1}$. From equations (A1) and (A2) it follows that

$$\sigma_{ij-k} = \frac{\Gamma_k}{\Phi_i}$$  \hfill (A3)

represents a flux-averaged cross section. Also note that if the spectral shape of $\phi_i$ is constant (as we always assume), then so is $\sigma_{ij-k}$, and the flux $\Phi_i$ contains all of the time and space variation of $\Gamma_k$.

Finally, two conventions are useful for quantifying abundances. Species $i$, with number density $n_i$, has a “mole fraction” (or baryon fraction) $Y_i = n_i/n_H = Y_i/Y_H$. It is also convenient to introduce the “hydrogen ratio” $Y_i = n_i/n_H = Y_i/Y_H$. 


COSMIC GAMMA RADIATION TRANSFER

APPENDIX B

The expression for $\gamma$-ray intensity in a Friedmann universe is well known (Stecker 1969), but usually expressed in redshift space. For our purposes, the result expressed in the time domain is critical, and indeed is more fundamental, so we give a derivation based on the Boltzmann equation. For this appendix we adopt units in which $c = 1$.

The differential photon (number) intensity $f$ is directly related via

$$I(p, x, t) = p^2 f(p, x, t)$$

(B1)

to the $\gamma$-ray distribution function $f(p, x, t) = d^3 N/d^3 p d^3 x$. Here $p$ and $x$, as well as the volume elements, are physical quantities (and thus subject to change with cosmic expansion). The distribution function is related to the photon sources via the relativistic Boltzmann equation

$$p^\mu \partial_\mu f - \Gamma^{\alpha\beta}_\mu p^\alpha p^\beta f = E \frac{dq}{d^3 p},$$

(B2)

where gravitational effects enter through the Affine connection $\Gamma$, where $E = p = |p|$ and where the source function (number of photons created per unit volume per unit time) is $q$.

For an isotropic FRW universe we have $f = f(E, t)$, and thus

$$\partial_t f - \frac{\dot{a}}{a} E \partial_E f = \frac{q(E)}{4\pi E^2},$$

(B3)

where $q(E) = dq/dE$ and where we neglect photon scattering and absorption.

We now note that a given photon’s energy $E$ drops as a result of redshifting as $a^{-1}$. It is thus useful to define a comoving energy $\epsilon = aE$; with $a(t_0) = 1$, we see that $\epsilon$ is also the present-day (observed) photon energy. Changing variables from $f(E, t)$ to $f(\epsilon, t)$, and similarly for $q$, the energy-dependent $\partial_t$ term drops out; this is physically reasonable, since we do not allow for scattering processes, and thus a photon’s energy can only change as a result of redshifting. We then have

$$\partial_t f = a^2 \frac{q(\epsilon/a)}{4\pi \epsilon^2},$$

(B4)

which, for any fixed comoving energy $\epsilon$, integrates to

$$f(\epsilon, t) = \frac{1}{4\pi \epsilon^2} \int_0^t dt' a^2 \left(\frac{\epsilon}{a}\right).$$

(B5)

Equation (B1) then gives the intensity

$$I(\epsilon, t) = \frac{1}{4\pi a(t)^2} \int_0^t dt' \frac{1}{a(t')} q_{\text{com}} \left[\epsilon a(t') \right],$$

(B6)

where $q_{\text{com}} = a^3 q$ is the comoving source rate. Equation (B6) is the usual expression (which is often then expressed in terms of an integral over redshift). Finally, if we integrate over the entire energy spectrum and evaluate at the present epoch $t_0$ (when $a_0 = 1$), we have

$$I(>0, t_0) = \int_0^\infty d\epsilon I(\epsilon, t_0) = \frac{1}{4\pi} \int_0^{t_0} dt' q_{\text{com}}(>0),$$

(B7)

where $q_{\text{com}}(>0) = \int_0^\infty du q(u)$ is the total source rate, integrated over energy.

We see from equation (B7) that the energy-integrated intensity is the same as one would find from uniform sources in a nonexpanding universe (which have been “switched on” for a duration $t$). This result is physically sensible, because the two effects of cosmic expansion are to introduce a particle horizon and redshifting. The energy integration removes the effect of redshifting, so that the only effect is that of the particle horizon, which acts to set the integration timescale.
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