Thermality of the Hawking flux

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ABSTRACT: Is the Hawking flux “thermal”? Unfortunately, the answer to this seemingly innocent question depends on a number of often unstated, but quite crucial, technical assumptions built into modern (mis-)interpretations of the word “thermal”. The original 1850’s notions of thermality — based on classical thermodynamic reasoning applied to idealized “black bodies” or “lamp black surfaces” — when supplemented by specific basic quantum ideas from the early 1900’s, immediately led to the notion of the black-body spectrum, (the Planck-shaped spectrum), but without any specific assumptions or conclusions regarding correlations between the quanta. Many (not all) modern authors (often implicitly and unintentionally) add an extra, and quite unnecessary, assumption that there are no correlations in the black-body radiation; but such usage is profoundly ahistorical and dangerously misleading. Specifically, the Hawking flux from an evaporating black hole, (just like the radiation flux from a leaky furnace or a burning lump of coal), is only approximately Planck-shaped over a bounded frequency range. Standard physics (phase space and adiabaticity effects) explicitly bound the frequency range over which the Hawking flux is approximately Planck-shaped from both above and below — the Hawking flux is certainly not exactly Planckian, and there is no compelling physics reason to assume the Hawking photons are uncorrelated.

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1 Introduction

Stephen Hawking predicted, (now some 40 years ago), that semi-classical black holes will emit quantum radiation with a temperature proportional to their surface gravity, and will slowly evaporate due to subtle quantum effects [1, 2]. Direct experimental tests of this phenomenon have so far been impractical, and the best laboratory data comes from analogue systems such as surface waves in a water tank [3, 4], and more recently, phonons in a BEC [5]. Faced with this extreme paucity of both experimental and observational data, the community has focussed almost entirely on gedanken-experiments, with well over 5000 theory papers generated to date.

While there is almost universal agreement that the predicted Hawking flux will actually occur for a general relativistic black hole, and almost universal agreement that the associated back-reaction will slowly reduce the black hole’s mass, there is relatively little agreement (in fact, considerable disagreement) regarding the endpoint of the Hawking evaporation process. In particular, the question of the thermality of the Hawking flux, (and the precise sense in which it is thermal), is a crucial and important ingredient in the so-called “information puzzle”, and its recent “firewall” variant [6–17]; this has important implications regarding the quantum unitarity of the Hawking evaporation process [18–22]. Let us first consider the spectral shape, and then the correlation structure, of the Hawking flux.
2 Spectrum of the Hawking flux

Hawking’s 1974 derivation [1, 2], the many and quite varied subsequent rederivations thereof, and the modern adiabatic variants of Hawking’s original calculation [23, 24], all agree that the shape of the spectrum is approximately a Planckian black-body spectrum, but with certain key modifications and limitations. Specifically, the Planckian shape of the Hawking spectrum will, at an absolute minimum, be modified by at least three distinct physical effects:

1. greybody factors;
2. adiabaticity constraints;
3. available phase space.

Let us consider these effects in turn.

— Greybody factors: These arise when the Hawking flux is backscattered by the non-trivial gravitational field between the quasi-local horizon (the apparent/trapping horizon) and spatial infinity; the existence of these greybody effects is entirely standard and well-known, though quantitative estimates are sometimes tricky. Early work dates back to the mid-to-late 1970’s [25–27], and interest in these quantities is active and ongoing, see for instance [28–30]. A key feature is to note that the greybody factors will always suppress the Hawking flux.

— Adiabaticity constraints: These arise from including back-reaction and depend on the fact that the spacetime geometry must be slowly evolving on the time-scale set by the frequency of the Hawking photon in order for Hawking’s calculation, or any of its variants, to apply. Implications of the adiabaticity condition were carefully analyzed and discussed in both references [23] and [24]. Specifically, to obtain an approximately Planckian spectrum the surface gravity must be “slowly evolving” in the sense that:

\[ |\dot{\kappa}| \ll \kappa^2. \quad \text{(2.1)} \]

This is the constraint that a photon at the peak of the Hawking spectrum should not see any significant change in the surface gravity during one oscillation period. More generally, let us now consider the conditions for the validity of Hawking’s “exponential approximation” for the relative e-folding of the affine null parameters between past and future null infinity (scri\(^{-}\) and scri\(^{+}\)) [1, 2].
Whenever the surface gravity is time-varying, $\dot{\kappa} \neq 0$, then the exponential approximation is at best valid over a bounded time interval of width [23, 24]:

$$\Delta t \ll \frac{1}{\sqrt{|\dot{\kappa}|}}. \quad (2.2)$$

See the discussion surrounding equation (10) of reference [23] or equation (3.22) of reference [24] for extensive technical details. But then Hawking’s argument, (or its variants), can be applied only to wave packets which can be localized within this time interval. This implies the wave-packet must be built out of modes of frequency at least $\omega_{\text{min}} = \sqrt{|\dot{\kappa}|}$. In particular, a suitable extension of Hawking’s argument leading to a Planckian spectrum only works for the limited range of frequencies

$$\omega \gtrsim \omega_{\text{min}} = \sqrt{|\dot{\kappa}|}. \quad (2.3)$$

Thus this adiabaticity argument provides an infrared frequency cutoff on the Hawking flux. If Hawking’s argument (suitably extended) is to apply not just in the exponential Boltzmann tail, but also to include the peak of the Planck spectrum, then one recovers the condition $|\dot{\kappa}| \ll \kappa^2$.

— **Phase space effects:** There is also a phase space ultraviolet frequency cutoff in the Hawking flux. At its crudest, the emitted photon energy can never exceed the available mass energy: $\omega < m$. A slightly safer statement for charged or rotating black holes is $\omega < m - m_{\text{extremal}}$.

More carefully, applying (standard flat-space) Lorentz kinematics in the asymptotic region, it is easy to see that for a black hole of mass $m_i$ emitting, (in its rest frame), a photon of energy $\omega$, and thereby reducing its mass to $m_f$, one has:

$$\omega = \frac{m_i^2 - m_f^2}{2m_i}. \quad (2.4)$$

Applied to the photons in the Hawking flux this yields the purely kinematic bound:

$$\omega \leq \frac{m^2 - m_{\text{extremal}}^2}{2m}. \quad (2.5)$$

Thus this argument provides an ultraviolet frequency cutoff on the Hawking flux.

— **Combined effects:** Combining the adiabaticity and phase space constraints we see that the Hawking flux can (at best) be approximately Planckian (up to greybody factors) only over the limited frequency range:

$$\omega \in \left(\sqrt{|\dot{\kappa}|}, \frac{m^2 - m_{\text{extremal}}^2}{2m}\right). \quad (2.6)$$
Even within this range, where the (suitably extended modern variants of the) Hawking calculation can be trusted, greybody factors (barrier transmission probabilities) will to some extent suppress the Hawking flux below that of an ideal Planck spectrum.

3 Schwarzschild black holes

For the specific case of the Schwarzschild black hole $m_{\text{extremal}} = 0$, so the phase space cutoff is simply $\omega < m/2$. Indeed, for Schwarzschild black holes the phase space cutoff never intersects the peak ($\omega \sim \kappa$) of the approximately Planck-shaped spectrum while one remains within the semi-classical regime.

Because the phase space cutoff is so high, (compared to the location of the Hawking peak at $\omega \sim \kappa$), it is perfectly acceptable, (at least as a zeroth-order approximation), to approximate the Hawking flux by a complete Planck spectrum, integrate over all of phase space, and so get the Stefan–Boltzmann law, ($\dot{m} = -\sigma T^4 A_{\text{horizon}}$). Then, introducing Planck quantities for simplicity, we have $\kappa \sim m^2 P / m$. Thereby we deduce:

$$\dot{\kappa} = -\frac{\dot{m}m^2 P}{m^2} = - \left\{ \left( \frac{m^2 P}{m^2} \right)^4 \times 4\pi(2m)^2 \right\} \frac{m^2 P}{m^2} \sim -\frac{m^6 P}{m^4}. \quad (3.1)$$

Consequently the Hawking flux from a Schwarzschild black hole is approximately Planckian (up to greybody factors) over the rather broad frequency interval

$$\omega \in \left( \frac{m^3 P^4}{m^2}, \frac{m}{2} \right). \quad (3.2)$$

This interval is non-empty for macroscopic black hole masses, and even for mesoscopic black hole masses all the way down to the Planck scale. Furthermore

$$\left| \frac{\dot{\kappa}}{\kappa^2} \right| \sim \frac{m^6 P^4}{m^2 P^4/m^2} = \frac{m^2 P}{m^2}. \quad (3.3)$$

So, as claimed, the peak in the Planck blackbody spectrum is indeed contained in the approximately Planckian interval all the way down to the Planck scale, (where one should stop believing semiclassical physics anyway).

Thus we see that for a Schwarzschild black hole these three bounds on the Planckian nature of the Hawking flux are (numerically) not particularly stringent. But they do however provide important qualitative information — at the very least they serve as a suitable antidote to the often made, (and often repeated, but utterly incorrect), assertion that the Hawking flux is exactly Planckian.

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1 The applicability of the Stefan–Boltzmann law is intrinsically an approximate result; in view of the physical arguments above it cannot be exact.
4 Effective temperature

The net effect of these greybody factors, adiabaticity constraints, and phase space constraints is to modify the spectrum of the Hawking flux:

\[ n(k) = \frac{f(\omega)}{\exp(h\omega/k_B T_H) - 1}; \quad f(\omega) \in (0, 1). \]  

(4.1)

Here \( f(\omega) \) is some dimensionless suppression factor encoding all three effects. This allows one (in the quite usual manner) to define an effective temperature in terms of the total energy flux:

\[ \sigma T_{\text{effective}}^4 = \int n(k) \frac{\hbar \omega}{(2\pi)^3} \frac{d^3 k}{d^3 k}. \]  

(4.2)

Setting \( z = \hbar \omega / k_B T_H \), and \( f(\omega) \rightarrow f(z) \), one sees

\[ T_{\text{effective}}^4 = \frac{\int f(z) z^3 e^z - 1 dz}{\int z^3 e^z - 1 dz} T_H^4 \leq T_H^4. \]  

(4.3)

That is, \( T_{\text{effective}} \leq T_H \), so the effective temperature of the outgoing Hawking flux has been suppressed below the naive Hawking temperature.

This then changes (both qualitatively and quantitatively) the entropy budget in the Hawking evaporation process. As the black hole evaporates, its Bekenstein entropy \([31]\) decreases as

\[ dS_B = -\frac{|dM|}{T_H}, \]  

(4.4)

whereas the entropy change of the outgoing radiation can best be estimated as

\[ dS_H = +\frac{|dM|}{T_{\text{effective}}}. \]  

(4.5)

(The outgoing radiation, since it is not exactly Planckian, should really be analyzed using non-equilibrium thermodynamics; but use of the effective temperature is a well-known stand-in for such effects.) Overall one has

\[ dS_{\text{total}} = |dM| \left( \frac{1}{T_{\text{effective}}} - \frac{1}{T_H} \right) \geq 0. \]  

(4.6)

So the Hawking evaporation process actually increases the total entropy of the universe. This is perhaps somewhat unexpected from the standard point of view, but is utterly unavoidable as soon as one takes proper cognisance of greybody, adiabaticity, and phase space effects.
5 Wick rotation and the Hawking flux

Ultimately the origin of the often-made but mistaken assertion that the Hawking flux is \textit{exactly} Planckian seems to trace back to an over-enthusiastic and uncritical adoption of Wick rotation (Euclidean quantum gravity) techniques \cite{32}. Certainly the Wick rotation of a static black hole \((t \to it, \text{ in the manifestly static coordinate system where } g_{ti} = 0)\), combined with the condition that there be no conical singularity at the Euclideanized version of the Lorentzian-signature Killing horizon, picks out the surface gravity as being physically important, being related (via periodicity in imaginary time) to a notion of temperature — but this is by construction an \textit{intrinsically equilibrium} argument for a black hole in \textit{exact thermal equilibrium with a heat bath} at the Hawking temperature \cite{32}.

- By construction the heat bath has an exactly Planckian spectrum, simply because it is \textit{assumed} to be in exact thermal equilibrium; the greybody factors quietly drop out.

- By construction the situation is static; there are no adiabaticity conditions since \(\dot{\kappa} \equiv 0\).

- By construction there are no phase-space constraints; since (typically) one is completely ignoring back-reaction.

But this Wick-rotated Euclideanized system tells you relatively little regarding the non-equilibrium emission of the Hawking flux into vacuum; the Unruh quantum vacuum state is radically different from the Hartle–Hawking quantum vacuum state. (Wick rotation automatically puts one into the Hartle–Hawking quantum vacuum state, not the physically relevant Unruh vacuum state.) While the Wick rotation trick provides a “quick and dirty” way of relating surface gravity to Hawking temperature \cite{33}, it misses much of the essential physics of the evaporation process. Once one considers a real black hole evaporating into vacuum, the Hawking flux is no longer exactly Planckian — the shape of the spectrum must at the very least be modified by the three effects considered above.

6 Correlations in the Hawking flux

Are the Hawking quanta in any way correlated with each other? This quite deceptively innocent question can easily initiate a firestorm of quite inconclusive debate.
The original 1850’s notions of thermality, based on classical thermodynamic reasoning applied to black bodies, (such as, for instance, the traditional “leaky cavity” or “lamp-black” surfaces), made no intrinsic assumptions regarding correlations in the outgoing radiation. But modern abuse of the word “thermal” often implicitly makes assumptions about a lack of correlations. It is essential to realise that the physical distinction between “Planckian” (Planck-shaped spectrum) and “thermal” is both important and subtle. No-one seriously doubts that burning a lump of coal in a leaky furnace results in an approximately Planckian spectrum, (an approximately “black body” spectrum), nor that this process implies correlations in the outgoing radiation — which then cannot be exactly “thermal” in the technical sense that this word has come to be used (or rather abused) in the modern literature.

Now Hawking’s original 1974 calculation [1, 2], (and its modern variants, see for instance references [23] and [24]), certainly demonstrate that a collapsing ball of matter will excite the quantum vacuum state, and that the outgoing radiation is approximately blackbody, that is, has an approximately Planck-shaped spectrum, (at least up to greybody, adiabaticity, and phase space effects, as discussed above). But the considerably stronger statement that the outgoing Hawking quanta are completely uncorrelated, (the modern misuse of the word “thermal”), depends on a separate and very much stronger implicit assumption: That in a semi-classical astrophysical black hole an event horizon forms to permanently hide any possible correlations, in such a way that they never again become visible to the external universe — but the possibility of doing this depends on delicate issues of global geometry — including what will happen in the infinite future [34]. In contrast apparent horizons or trapping horizons, while they may temporarily hide correlations, are unlikely to do so permanently. Without an event horizon, whose very existence is delicately predicated on assumptions being made about the infinite future, a black hole defined in terms of apparent or trapping horizons will behave much more like a furnace; a leaky furnace with a small hole in it, the original 1850’s classical thermodynamic definition of a “black body”.

7 Analogue Hawking flux

To really drive home the point that the existence of possible correlations in the Hawking flux is logically independent of the existence of the Hawking flux simply consider an acoustic black hole (dumb hole) [35]. (For various theoretical developments see [36–42] and [43–45]. For a laboratory implementation using surface waves see [3, 4]. For a more recent laboratory implementation using BECs see [5].)
There is widespread agreement that an acoustic horizon (defined by the normal component of fluid velocity exceeding the local speed of sound) will emit an approximately Planckian spectrum of Hawking phonons; but there is absolutely no requirement that the acoustic horizon be an *event* horizon — in fact by accelerating or decelerating the fluid flow it is easy to make acoustic horizons appear and disappear at will. Any horizon that can completely disappear (without any trace of its prior existence, and without any way of permanently hiding correlations) will qualitatively behave like a leaky furnace with a small hole in it, the original 1850’s classical thermodynamic definition of a “black body”. So in these analogue systems, not only is there no reason to believe that there is any “information puzzle”, but in contrast there is every reason to believe that ordinary unitary evolution and standard physics applies.

Consequently, even if one could somehow prove that the Hawking photons coming from a specifically general relativistic black hole were uncorrelated, this would merely be a side-effect of the specific details of general relativistic black holes, (as opposed to the generic features analogue black holes); it would have *nothing* to do with the fundamental physics underlying Hawking radiation itself.

8 Discussion

In short, the so-called “information puzzle”, (often somewhat excessively referred to as the “information paradox”), is intimately reliant on the assumed existence of an *event* horizon, and much of the force of the information puzzle simply goes away once one uses *apparent* horizons or *trapping* horizons to define what we mean by a black hole [34, 46, 47]. This observation is closely related to Hawking’s recent arguments regarding the necessity of making careful physical distinctions between the mathematical concepts of *event* horizon and *apparent* horizon [46]: “The absence of event horizons means that there are no black holes — in the sense of regimes from which light can’t escape to infinity. There are, however, apparent horizons which persist for a period of time.” Similarly, a decade ago Hawking asserted [47]: “The way the information gets out seems to be that a true event horizon never forms, just an apparent horizon.”

The physical picture that then emerges matches quite nicely with certain proposals for the Hawking radiation process, both somewhat older and more recent, that make no intrinsic reference to *event* horizons per se [48–56]. Exact thermality of the Hawking flux, and a total absence lack of correlations in the Hawking flux, is often asserted in the scientific literature — but neither assertion holds up to any level of scrutiny. The spectrum of the Hawking flux is certainly not exactly Planckian, and the effective temperature (suitably defined) of the Hawking flux is not equal to, but is instead bounded above, by the Hawking temperature. Whether or not correlations exist in the
Hawking flux is contingent upon the assumed existence of event horizons (as opposed to apparent/trapping horizons). Certainly event horizons are not necessary for the development of a Hawking flux, and the often assumed survival of classical event horizons in semi-classical physics is an assumption that is increasingly in doubt [34, 46, 47].

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References

[1] S. W. Hawking, “Black hole explosions”, Nature 248 (1974) 30.
[2] S. W. Hawking, “Particle creation by black holes”, Commun. Math. Phys. 43 (1975) 199 [Erratum-ibid. 46 (1976) 206].
[3] S. Weinfurtner, E. W. Tedford, M. C. J. Penrice, W. G. Unruh and G. A. Lawrence, “Measurement of stimulated Hawking emission in an analogue system”, Phys. Rev. Lett. 106 (2011) 021302 [arXiv:1008.1911 [gr-qc]].
[4] S. Weinfurtner, E. W. Tedford, M. C. J. Penrice, W. G. Unruh and G. A. Lawrence, “Classical aspects of Hawking radiation verified in analogue gravity experiment”, Lect. Notes Phys. 870 (2013) 167.
[5] Jeff Steinhauer, “Observation of self-amplifying Hawking radiation in an analog black hole laser”, arXiv:1409.6550 [cond-mat].
[6] A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, “Black holes: Complementarity or firewalls?”, JHEP 1302 (2013) 062 [arXiv:1207.3123 [hep-th]].
[7] A. Almheiri, D. Marolf, J. Polchinski, D. Stanford, and J. Sully, “An apologia for firewalls”, JHEP 1309 (2013) 018 [arXiv:1304.6483 [hep-th]].
[8] S. L. Braunstein, “Black hole entropy as entropy of entanglement, or it’s curtains for the equivalence principle” [arXiv:0907.1190v1 [quant-ph]]; published as: S. L. Braunstein, S. Pirandola and K. Zyczkowski, “Better Late than Never: Information Retrieval from Black Holes?, Physical Review Letters 110 (2013) 101301.
[9] R. Bousso, “Complementarity is not enough”, Phys. Rev. D 87 (2013) 124023 [arXiv:1207.5192 [hep-th]].
[10] Y. Nomura, J. Varela, and S. J. Weinberg, “Complementarity endures: No firewall for an infalling observer”, JHEP 1303, 059 (2013) [arXiv:1207.6626 [hep-th]].

[11] S. D. Mathur and D. Turton, “Comments on black holes I: The possibility of complementarity”, JHEP 1401 (2014) 034 [arXiv:1208.2005 [hep-th]].

[12] T. Banks and W. Fischler, “Holographic space-time does not predict firewalls”, arXiv:1208.4757 [hep-th].

[13] R. Brustein, “Origin of the blackhole information paradox”, arXiv:1209.2686 [hep-th].

[14] S. Hossenfelder, “Comment on the black hole firewall”, arXiv:1210.5317 [gr-qc].

[15] K. Larjo, D. A. Lowe, and L. Thorlacius, “Black holes without firewalls”, Phys. Rev. D 87 (2013) 10, 104018 [arXiv:1211.4620 [hep-th]].

[16] S. B. Giddings, “Nonviolent nonlocality”, Phys. Rev. D 88 (2013) 064023 [arXiv:1211.7070 [hep-th]].

[17] E. Verlinde and H. Verlinde, “Passing through the firewall”, arXiv:1306.0515 [hep-th].

[18] D. N. Page, “Is black hole evaporation predictable?”, Phys. Rev. Lett. 44 (1980) 301.

[19] S. W. Hawking, “The Unpredictability of Quantum Gravity”, Commun. Math. Phys. 87 (1982) 395.

[20] T. Banks, L. Susskind, and M. E. Peskin, “Difficulties for the evolution of pure states into mixed states”, Nucl. Phys. B 244 (1984) 125.

[21] J. Preskill, “Do black holes destroy information?”, In Black holes, membranes, wormholes and superstrings, pp 22-39. (Houston 1992 Proceedings) [hep-th/9209058].

[22] S. W. Hawking, “Virtual black holes”, Phys. Rev. D 53, 3099 (1996) [hep-th/9510029].

[23] C. Barceló, S. Liberati, S. Sonego, and Matt Visser, “Minimal conditions for the existence of a Hawking-like flux”, Phys. Rev. D 83 (2011) 041501 [arXiv:1011.5593 [gr-qc]].

[24] C. Barceló, S. Liberati, S. Sonego, and Matt Visser, “Hawking-like radiation from evolving black holes and compact horizonless objects”, JHEP 1102 (2011) 003 [arXiv:1011.5911 [gr-qc]].

[25] D. N. Page, “Particle Emission Rates from a Black Hole: Massless Particles from an Uncharged, Nonrotating Hole”, Phys. Rev. D 13 (1976) 198.

[26] D. N. Page, “Particle Emission Rates from a Black Hole. 2. Massless Particles from a Rotating Hole”, Phys. Rev. D 14 (1976) 3260.
[27] J. D. Bekenstein and A. Meisels, “Einstein A and B coefficients for a black hole”, Phys. Rev. D 15 (1977) 2775.

[28] Jorge Escobedo, “Greybody Factors: Hawking Radiation in Disguise”, M.Sc. Thesis, U Amsterdam, 2008.

[29] Petarpa Boonserm and Matt Visser, “Bounding the greybody factors for Schwarzschild black holes”, Phys. Rev. D 78 (2008) 101502 [arXiv:0806.2209 [gr-qc]].

[30] P. Boonserm, T. Ngampitipan and M. Visser, “Regge-Wheeler equation, linear stability, and greybody factors for dirty black holes”, Phys. Rev. D 88 (2013) 041502 [arXiv:1305.1416 [gr-qc]].

[31] J. D. Bekenstein, “Black holes and entropy”, Phys. Rev. D 7, 2333 (1973).

[32] G. W. Gibbons and S. W. Hawking, “Action Integrals and Partition Functions in Quantum Gravity”, Phys. Rev. D 15 (1977) 2752.

[33] M. Visser, “Dirty black holes: Thermodynamics and horizon structure”, Phys. Rev. D 46 (1992) 2445 [hep-th/9203057].

[34] M. Visser, “Physical observability of horizons”, arXiv:1407.7295 [gr-qc].

[35] W. G. Unruh, “Experimental black hole evaporation”, Phys. Rev. Lett. 46 (1981) 1351.

[36] M. Visser, “Acoustic propagation in fluids: An unexpected example of Lorentzian geometry”, gr-qc/9311028.

[37] M. Visser, “Acoustic black holes: Horizons, ergospheres, and Hawking radiation”, Class. Quant. Grav. 15 (1998) 1767 [gr-qc/9712010].

[38] C. Barceló, S. Liberati and M. Visser, “Analogue gravity”, Living Rev. Rel. 8 (2005) 12 [Living Rev. Rel. 14 (2011) 3] [gr-qc/0505065].

[39] M. Visser, “Survey of analogue spacetimes”, Lect. Notes Phys. 870 (2013) 31 [arXiv:1206.2397 [gr-qc]].

[40] C. Barceló, S. Liberati, S. Sonego and M. Visser, “Causal structure of acoustic spacetimes”, New J. Phys. 6 (2004) 186 [gr-qc/0408022].

[41] C. Barceló, S. Liberati, S. Sonego, and Matt Visser, “Hawking-like radiation does not require a trapped region”, Phys. Rev. Lett. 97 (2006) 171301.

[42] Matt Visser, “Hawking radiation without black hole entropy”, Phys. Rev. Lett. 80 (1998) 3436 [gr-qc/9712016].
[43] C. Barceló, S. Liberati and M. Visser,
“Probing semiclassical analog gravity in Bose-Einstein condensates with widely tunable interactions”, Phys. Rev. A 68 (2003) 053613 [cond-mat/0307491].

[44] C. Barceló, S. Liberati and M. Visser,
“Towards the observation of Hawking radiation in Bose-Einstein condensates”,
Int. J. Mod. Phys. A 18 (2003) 3735 [gr-qc/0110036].

[45] C. Barceló, S. Liberati and M. Visser,
“Analog gravity from Bose-Einstein condensates”,
Class. Quant. Grav. 18 (2001) 1137 [gr-qc/0011026].

[46] S. W. Hawking, “Information preservation and weather forecasting for black holes”,
arXiv:1401.5761 [hep-th].

[47] S. W. Hawking, abstract of a talk given at the GR17 conference in Dublin, Ireland, 2004.

[48] T. A. Roman and P. G. Bergmann, “Stellar collapse without singularities?”,
Phys. Rev. D 28 (1983) 1265.

[49] A. Ashtekar and M. Bojowald, “Black hole evaporation: A paradigm”,
Class. Quant. Grav. 22 (2005) 3349 [gr-qc/0504029].

[50] S. A. Hayward, “Formation and evaporation of regular black holes”,
Phys. Rev. Lett. 96 (2006) 031103 [gr-qc/0506126].

[51] S. A. Hayward, “The Disinformation problem for black holes”, gr-qc/0504037.

[52] Matt Visser, “Black holes in general relativity”, arXiv:0901.4365 [gr-qc].

[53] S. Hossenfelder and L. Smolin,
“Conservative solutions to the black hole information problem”,
Phys. Rev. D 81 (2010) 064009 [arXiv:0901.3156 [gr-qc]].

[54] V. P. Frolov,
“Information loss problem and a ‘black hole’ model with a closed apparent horizon”,
JHEP 1405 (2014) 049 [arXiv:1402.5446 [hep-th]].

[55] W. Israel, “A massless firewall”, arXiv:1403.7470 [gr-qc].

[56] J. M. Bardeen, “Black hole evaporation without an event horizon”,
arXiv:1406.4098 [gr-qc].