Outlook for Charged Higgs Physics

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Almost all extensions of the Standard Model predict the existence of charged Higgs bosons. This talk focuses on the minimal supersymmetric extension of the Standard Model (MSSM), which is relatively predictive. The outlook for detecting supersymmetric particles and Higgs bosons at the LHC are discussed, as are the prospects for finding indirect effects of supersymmetric Higgs bosons at low energies, e.g., in $K$ decays. The outlook for discovering observable effects of CP-violating supersymmetric phases at high energies or in $B$ decays is also mentioned.

Prospects for Charged Higgs Discovery at Colliders
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1. Beyond the Minimal Higgs Model

Rare is the theorist who advocates the minimal Higgs model for electroweak symmetry breaking (EWSB), based on a single elementary doublet of Higgs fields. It is well-known that quantum corrections render the required small mass scale \( m_W \) or \( m_{GUT} \) very unnatural, and this motivates many non-minimal Higgs models, such as composite Higgs scenarios and supersymmetric extensions of the Standard Model. Accordingly, (almost) all models of EWSB are non-minimal, featuring either one or more additional Higgs doublets, and possibly other Higgs fields, either singlets or triplets of weak SU(2). The only scenarios without charged Higgs bosons are those where the supplementary Higgs fields are all singlets, but the preferred scenarios are those with extra Higgs doublets, and hence charged Higgs bosons.

The minimal two-Higgs-doublet model (THDM) has five physical Higgs bosons: two charged (\( H \) ) and three neutral, two of which are scalars (the lighter \( h \) and the heavier \( H \)) and one pseudoscalar (\( A \)). The most studied type of THDM is the minimal supersymmetric extension of the Standard Model (MSSM) \([1]\), in which the effective Higgs potential has specific restrictions, resulting from the specification in supersymmetry of the quartic Higgs couplings in terms of the gauge couplings, and the pattern of supersymmetric radiative corrections. There was considerable discussion of more general THDMs during this workshop, but in this talk I focus on variations of the MSSM, since these are the most predictive.

2. Supersymmetric Higgs Models

This concentration on supersymmetry merits a few more words of motivation. There are many reasons to like supersymmetry: its intrinsic beauty, its use in rendering the hierarchy \( m_W \) or \( m_{GUT} \) more natural \([2]\), its help in promoting the unification of the gauge couplings \([3]\), the fact that it predicts a relatively light Higgs boson weighing \(< 150 \) GeV \([4]\), as suggested by precision electroweak data \([5]\), the fact that it provides a plausible candidate for the cold dark matter \([6]\), and the fact that it is an (almost) essential ingredient in string theory. Fig. 1 displays the latest indications concerning the Higgs mass \([7]\), showing the likelihood function obtained by combining the negative results of Higgs searches at LEP and (more recently) the Tevatron with the indications on the Higgs mass provided by precision electroweak data. Taken together, they suggest strongly that \( m_h \geq (114;140) \) GeV, in good agreement with the prediction of supersymmetry for the mass of the lighter scalar Higgs boson. We will see later what indications there may be on the possible masses of the heavier supersymmetric Higgs bosons.

Enthused now by supersymmetry, we now focus on specific supersymmetric models. Within the MSSM framework, the properties of the Higgs sector are characterized at the classical level by the supersymmetric coupling \( \mu \) between the Higgs doublets and the related bilinear soft supersymmetry-breaking parameter \( B \), by the ratio of Higgs vacuum expectation values (VEVs), and by the supersymmetry-breaking contributions \( m_0 \) to the masses of the two Higgs doublets. At the loop level, there is also sensitivity to the supersymmetry-breaking contributions \( m_0;m_{1,2} \) to the masses of the other MSSM particles, and to the soft trilinear supersymmetry-breaking parameters \( A \). It is often assumed that these parameters are universal at the GUT scale, a framework referred to as the constrained MSSM (CMSSM). This is not the same as minimal supergravity (mSUGRA), in
Figure 1: The $\chi^2$ function for the mass of the Higgs boson in the Standard Model [7], combining the negative results of direct searches at LEP [8] and the Tevatron collider [9] with the indications from precision electroweak measurements [5].

which there are additional relations for the gravitino mass: $m_{3\tilde{g}} = m_0$, and between the bilinear and trilinear supersymmetry-breaking parameters: $B = A m_0$.

One may also consider scenarios with non-universal supersymmetry-breaking scalar masses. Universality for the different sfermions with the same quantum numbers e.g., $d$ squarks, is favoured by upper limits on flavour-changing neutral interactions beyond the Standard Model [10, 11]. These arguments do not extend to squarks with different quantum numbers, or to squarks and sleptons, but these are also expected to be universal in various GUT models, e.g., $m_{\tilde{d}_R} = m_{\tilde{e}_L}$ in SU(5), all squark and slepton masses equal in SO(10). However, there are no arguments to forbid non-universal supersymmetry-breaking contributions to the masses of the Higgs masses. If one allows these to vary independently of the common scalar masses $m_0$ of the squarks and sleptons, there are one or more extra parameters compared to the CMSSM, depending whether the Higgs masses are assumed to be equal (NUHM1) or allowed to differ (NUHM2) [12].

In what follows, we will use the CMSSM, NUHM1 and NUHM2 as guides to the outlook for charged Higgs physics.

### 3. Constraints on Supersymmetry

There are important constraints on supersymmetric models from the absence of sparticles at LEP, which tell us that the selectron and chargino weigh more than about 100 GeV [13], and the Tevatron [14], which tells us that squarks and gluinos weigh more than about 300 GeV. There are also important indirect constraints from the LEP lower limit on the Higgs mass of 114 GeV [8], and from the measured rate for $b \to s \gamma$ [15] (other $B$-decay constraints such the upper limit on
Outlook

$B_s \to \mu^+ \mu^-$ decay are also important at larger $\tan \beta$). The density of astrophysical cold dark matter: $0.094 < \Omega_{CDM} h^2 < 0.124$ [16] is also an important constraint. The most studied supersymmetric candidate for the cold dark matter particle is the lightest neutralino $\chi$. Assuming this to be the case, the relic density range tightly constrains one combination of the supersymmetric model parameters, restricting them to a narrow strip in the sample CMSSM $(m_{1/2}, m_0)$ plane shown in Fig. 2 [17]. We see that relatively heavy sparticle masses would be compatible with the above constraints.

![Figure 2: The $(m_{1/2}, m_0)$ planes for (left) $\tan \beta = 10$ and (right) $\tan \beta = 50$, assuming $\mu > 0$, $A_0 = 0$, $m_t = 175$ GeV and $m_\tau (m_\tau)_{\overline{MS}} = 4.25$ GeV [17]. The near-vertical (red) dot-dashed lines are the contours for $m_h = 114$ GeV, and the near-vertical (black) dashed line is the contour $m_{\chi} = 104$ GeV. Also shown by the dot-dashed curve in the lower left is the region excluded by the LEP bound $m_\tau > 99$ GeV. The medium (dark green) shaded region is excluded by $b \to s \gamma$, and the light (turquoise) shaded area is the cosmologically preferred region. In the dark (brick red) shaded region, the LSP is the charged $\tilde{t}_1$. The region allowed by the E821 measurement of $a_\mu$ at the 2-σ level, is shaded (pink) and bounded by solid black lines, with dashed lines indicating the 1-σ ranges.](image)

4. The LHC

In the immediate future, the best outlook for charged Higgs physics lies with the LHC, which
Outlook

started operation on Sept. 10th, shortly before this meeting. The CERN accelerator team was quickly able to circulate beams in both directions, to capture and bunch one beam with the RF system, and to store it for tens of minutes \[21\]. These successes augur well for the successful future operation of the LHC. All the experiments recorded events, notably beam ‘splash’ events in which the beam struck an upstream beam stop and generated large numbers of particles that passed through the detectors. These enabled timing, alignment and calibration issues to be resolved, and demonstrated that the detectors were operational and ready to take collisions.

There was therefore particularly strong disappointment on Sept. 19th when, during the presentation of this talk, an electrical problem in the interconnect between two LHC magnets caused a massive Helium leak and collateral damage that put a premature end to LHC operations for the rest of the year. At the time of writing, the source of the problem has been understood, and it does not seem to be a major design problem \[22\]. The damage is being made good, and precautions are being taken to avoid such an incident in the future and to mitigate the consequences of any similar incident, should one ever recur. The current plan is to complete the repairs during the normal CERN winter accelerator shutdown period, then cool down the repaired LHC sectors. Barring any unforeseen problems, it should be possible to restart operations in the Summer of 2009, aiming at collisions at 10 TeV in the centre of mass.

5. How soon might Supersymmetry be Detected?

A likelihood analysis of the CMSSM and the NUHM1, implementing all the constraints mentioned previously, including the controversial $g_\mu - 2$ constraint, was recently performed in \[23\]. Fig. 3(a) displays in the CMSSM ($m_{1/2}$ $m_0$) plane the best-fit point (black dot), and the regions of supersymmetric parameter space favoured at the 68 and 95% confidence levels (blue and red hatched regions, respectively). Also shown are the regions of the ($m_{1/2}$ $m_0$) plane excluded by previous LEP and Tevatron searches (black hatched regions), and the regions where the LHC could discover supersymmetry at the 5-σ level with the indicated amounts of luminosity: 50/pb at 10 TeV in the centre of mass, and 100/pb or 1/fb at the design energy of 14 TeV in the centre of mass. We see that the best-fit point could be discovered with just 50/pb at 10 TeV, that 100/pb at 14 TeV would suffice to discover supersymmetry in the 68% confidence-level (C.L.) region, and that 1/fb at 14 TeV would cover almost all the 95% C.L. region in the CMSSM. Fig. 3(b) displays the corresponding analysis for the NUHM1, which reaches rather similar conclusions, though the favoured regions extend to somewhat larger values of $m_{1/2}$. It should be emphasized, though, that these conclusions depend crucially on the implementation of the questionable $g_\mu - 2$ constraint.

As shown in Fig. 4, there are several channels in which supersymmetry could be detected at the LHC, notably including various jet and lepton channels accompanied by the classic missing-energy signal of the escaping dark matter neutralinos. We also see that searches for the lightest supersymmetric Higgs boson in supersymmetric decays could also play an interesting role, though perhaps not in the most favoured regions of parameter space.

The best-fit spectra in the CMSSM and NUHM1 are shown in Fig. 5 \[23\]. We see that the favoured slepton, squark, gluino and lighter neutralino and chargino masses are quite similar in the two scenarios, though somewhat lighter in the NUHM1 because the best-fit value of $m_{1/2}$ is slightly lower. On the other hand, the heavier neutralinos and chargino are significantly heavier in
Outlook

Figure 3: The \((m_0, m_{\tilde{\chi}_0^0})\) planes for (left) the CMSSM and (right) the NUHM1 \cite{Haber:2010bw}, displaying the best-fit points (black dots), the 68\% C.L. regions (blue hatching), the 95\% C.L. regions (red hatching) and the region excluded by LEP \cite{LEP} and Tevatron \cite{Tevatron} searches (black hatching). Also shown are the 5-\sigma supersymmetry discovery reaches at the LHC assuming 50/pb of data at 10 TeV, 100/pb at 14 TeV, and 1/fb at 14 TeV, as estimated for the indicated values of the supersymmetric model parameters.

Figure 4: The same \((m_0, m_{\tilde{\chi}_0^0})\) planes for (left) the CMSSM and (right) the NUHM1 as in Fig. 3 \cite{Haber:2010bw}, this time displaying the LHC discovery reaches in several different channels with 1/fb at 14 TeV, and the light Higgs discovery potential in supersymmetric cascades with 2/fb at 14 TeV, as estimated for the indicated values of the supersymmetric model parameters.

As can be seen in Fig. 4, one of the most promising channels for discovering a supersymmetric signal is in the same-sign dilepton signal (SS). An early measurement of the edge of the dilepton mass spectrum might constrain significantly the supersymmetric parameter space \cite{Haber:2010bw}, and thereby refine the prospects for discovering the heavier Higgs bosons at the LHC.

the NUHM1, and the heavier Higgs bosons are significantly lighter. We will discuss later how this might affect the prospects for discovering the heavier MSSM Higgs bosons at the LHC.
6. Will the heavier Supersymmetric Higgs Bosons be discovered at the LHC?

There have been many studies of the prospects for discovering the heavier Higgs bosons at the LHC, usually presented as surveys of some \( (m_A, \tan \beta) \) plane for fixed values of the other MSSM parameters \cite{24}. Early studies indicated that the heavier Higgs bosons might be detectable with 300/fb of data, no matter what the value of \( \tan \beta \) if \( m_A < 200 \) GeV, and for \( \tan \beta > 10 \) if \( m_A < 600 \) GeV. The most promising search channels seem to be \( H \! \to \! t \bar{b} \) or \( H \! \to \! \mu \mu \) and \( \tau \tau \). However, these analyses needed to be validated with the latest and best available simulations of ATLAS \cite{25} and CMS \cite{26}. Some updated sensitivities to charged Higgs bosons estimated assuming 30/fb of data are shown in Fig. 6 \cite{27}. We see that varying \( \mu \) can have a significant effect on the regions of the \( (m_A, \tan \beta) \) plane that are accessible, and this is also true of the other MSSM parameters displayed in the legends.

These and some previous analyses were made with specific choices of the low-energy MSSM parameters that are not necessarily possible within the CMSSM or NUHM1(2) frameworks. Nor do these analyses necessarily take into account all the phenomenological constraints used in the previous analysis, specifically the cold dark matter constraint \cite{28}. This point is demonstrated in Fig. 7 where we see explicitly that most of a sample \( (m_A, \tan \beta) \) plane for certain fixed values of \( \mu, m_{1\nu} \), and \( m_0 \) is compatible with \( m_h \) and \( b + \gamma \), but not necessarily with \( g_{\mu \mu} \). More importantly, we see that only two near-vertical, very narrow strips of the sample \( (m_A, \tan \beta) \) plane on either side of the (blue) line where \( m_X = m_{A=2} \) are compatible with the cold dark matter constraint. The band between the two strips is under-dense, which might be permissible if there is some additional source of cold dark matter. On the other hand, the exterior regions are over-dense and hence forbidden. However, one can construct an acceptable \( (m_A, \tan \beta) \) plane in the NUHM2 by varying \( m_{1\nu} \) with \( m_A \), so as to maintain the appropriate relic density by keeping \( m_X \) slightly \( \leq m_{A=2} \) \cite{29}. 

Figure 5: \textit{The supersymmetric spectra at the best-fit points for (left) the CMSSM and (right) the NUHM1 [23]. Note that the heavier Higgs bosons, squarks sleptons, gluinos and lowest-lying inos are somewhat lighter in the NUHM1, whereas the highest-lying inos are somewhat lighter in the CMSSM.}
Figure 6: Discovery reach for (left) a light and (right) a heavy charged Higgs boson in the \((\mu, \tan \beta)\) plane for a maximal Higgs mixing scenario, in CMS with 30 fb⁻¹ [27].

\begin{align*}
m_{1/2} & = \frac{9}{8} m_A \quad \text{for} \quad m_0 = 800 \text{ GeV; } \mu = 1000 \text{ GeV} \quad (6.1) \\
\mu & = 250 \text{ to } 400 \text{ GeV for } m_{1/2} = 500 \text{ GeV; } m_0 = 1000 \text{ GeV} \quad (6.2)
\end{align*}

These have been used to make global fits to the electroweak and \(B\)-decay observables with the supersymmetric model parameters varying across these \((m_A, \tan \beta)\) planes, treating the relic neutralino density as a fixed constraint. These serve as a more suitable NUHM2 framework for assessing the outlook for heavy Higgs boson searches at the LHC and elsewhere.

The left panels of Fig. 8 display the plane (6.1), and the right panels show the plane (6.2) [28]. The black regions are excluded by the LEP Higgs searches, the regions shaded blue (yellow) are favoured by the global fit at the 68% (95%) confidence levels, and the best-fit points in these planes are indicated by red crosses. Also shown in the top panels are the regions accessible to LHC searches for the heavy MSSM Higgs bosons in the channels \(H \rightarrow t\bar{t}\) and \(H \rightarrow t\bar{t}n\). We see that the \(H \rightarrow t\bar{t}\) search is likely to be effective at large \(\tan \beta\) and small \(m_A\), whereas the \(H \rightarrow t\bar{t}n\) searches should be able to extend to lower \(\tan \beta\) and larger \(m_A\), covering (most of) the 68% confidence-level regions.

The lower panels of Fig. 8 display the sensitivities of various \(B\)-physics observables in the planes (6.1, 6.2) [28], including \(B_u \rightarrow \tau\nu\) (which is sensitive directly to the charged Higgs boson), \(B_s \rightarrow \mu\mu\) (which is sensitive to the closely-related heavy neutral Higgs bosons \(H \rightarrow A\)), and \(b \rightarrow s\gamma\) (which is sensitive to cancellations between heavy Higgs contributions and sparticle exchanges). We see that these have good prospects for exploring large areas of the \((m_A, \tan \beta)\) planes, including the regions favoured at the 68% confidence level. Thus these indirect searches play important roles in the outlook for charged Higgs physics.
Figure 7: The \((m_A, \tan \beta)\) plane in the NUHM2 with the indicated choices of \(\mu, m_{1/2}, m_0\) and \(A_0\) \cite{28}. The region excluded by \(b \rightarrow s \gamma\) is shaded dark (green), and that favoured by \(g_\mu - 2\) is shaded light (pink). The near-vertical strips with the neutralino relic density in the range favoured by astrophysics are shaded light (turquoise): they lie on either side of the (blue) line where \(m_\chi = m_A = 2\). The LEP Higgs constraint is shown as a (red) dot-dashed line.

7. Charged Higgs Effects in \(K\) Physics?

It has recently been pointed out \cite{29} that there is an interesting opportunity to study charged Higgs physics in \(K\) decays by probing lepton universality in \(K \to e\nu\) decays, via the ratio

\[
R^{LFV}_K = \frac{\Sigma_i \Gamma(K \to e\nu_i)}{\Sigma_i \Gamma(K \to \mu\nu_i)}; \tag{7.1}
\]

The present experimental range for the new physics contribution to \(R^{LFV}_K\) is \((0.063, 0.017)\), and the NA62 experiment at CERN is set to reduce this range by an order of magnitude. There are two potentially-important charged-Higgs contributions to \(R^{LFV}_K\): from lepton-flavour-violating contributions to \(\mathcal{H} \nu_\tau\) vertices:

\[
\mathcal{H} \nu_\tau \rightarrow \frac{g_2^2}{8} \frac{m_\tau}{M_W} \Delta_R^2 \tan^2 \beta \beta'; \quad \beta' = e ; \mu ; \tag{7.2}
\]

and from non-universality in the lepton-flavour-conserving \(H\) vertices:

\[
\mathcal{H} \nu \rightarrow \frac{g_2^2}{8} \frac{m_\tau}{M_W} \tan^2 \beta \quad 1 + \frac{m_\tau}{m_\mu} \Delta_R^2 \tan \beta \beta'; \quad \beta' = e ; \mu ; \tag{7.3}
\]
Figure 8: The \((M_A, \tan \beta)\) planes for the NUHM2 benchmark surfaces \((6.1, 6.2)\) displaying (top) the 5-\(\sigma\) discovery contours for \(H = A^+ \tau^- \tau^+\) and \(H \tau^+ \nu\) detection in the CMS detector, and (bottom) the sensitivities provided by \(B\) decays. \(B_s \rightarrow \mu^+ \mu^-\), \(b \rightarrow s \gamma\), and \(B_u \rightarrow \tau \nu\) [28].

In combination, these yield

\[
R_K^{LFV} = j \left( \frac{m_K^2}{M_H^4} \frac{m_{\tau}}{m_e} \frac{\Delta_{11}^{11}}{\Delta_{11}^{11}} \tan^3 \beta \right)^2 + \left( \frac{m_K^2}{M_H^4} \frac{m_{\tau}}{m_e} \frac{\Delta_{11}^{11}}{\Delta_{11}^{11}} \tan^6 \beta \right) \tag{7.4}
\]

The first term in (7.4) gets contributions from mixing in both the left- and right-handed slepton
Left-handed mixing is constrained strongly by the upper limit on $\tau \!\to\! e\gamma$ decay, so the dominant combination may come from mixing in the right-handed slepton sector [30]. This does not arise in a minimal SU(5) GUT, even including the seesaw mechanism for neutrino masses. However, it may arise in non-minimal GUTs, and make a contribution to (7.1) that is observable in the NA62 experiment, despite the upper limit on $\tau \!\to\! e\gamma$ decay. Maybe NA62 will find the first evidence for a charged Higgs boson?

8. CP Violation in Higgs Physics?

As discussed previously, upper limits on flavour violation beyond the Standard Model suggest that the supersymmetry-breaking scalar masses and trilinear scalar couplings are universal at high scale for all sparticles with the same quantum numbers, in which case the following are independent parameters:

$$m_{\tilde{u}_{R},\tilde{d}_{R}}^2 \propto \mathbf{1}_3; \quad A_{u,d,e} \propto \mathbf{1}_3; \quad M_{1,2,3}$$  \hspace{1cm} (8.1)

In this case, all squark mixing is due to the Cabibbo-Kobayashi-Maskawa (CKM) matrix, a scenario known as minimal flavour violation (MFV). This framework has a total of 19 parameters, of which 6 violate CP [31], namely:

$$\text{Im} A_{u,d,e}; \quad \text{Im} M_{1,2,3}$$  \hspace{1cm} (8.2)

It is often assumed that the parameters $\text{Im} M_{1,2,3}$, $\text{Im} A_{u,d,e}$ are universal, but non-universality is completely compatible with MFV. The scenario with all 6 CP-violating phases left free is the maximally CP-violating, minimally flavour-violating (MCPMFV) variant of the MSSM [31].

In the presence of CP violation, there is mixing between the scalar Higgs bosons $h, H$ and the pseudoscalar $A$, and one may label the mass eigenstates $H_{1,2,3}$ in order of increasing mass [32]. In this case, since the pseudoscalar state is no longer a mass eigenstate, it is better to use the charged Higgs mass as a parameter to characterize the MSSM Higgs sector, e.g., by displaying ($m_H \!\times\! \tan\beta$) planes instead of ($m_A \!\times\! \tan\beta$) planes. Fig. 9 shows some effects of CP-violating phases on the masses and couplings of the MSSM Higgs bosons [33], under the simplifying assumption that the phases $\text{Im} A_{u,d,e}$ and $\text{Im} M_{1,2,3}$ are each universal. We see interesting level-crossing phenomena in the masses, and possible suppressions of the $H_{1,2,3}$ couplings, which could have important consequences for phenomenology. For example, the LEP exclusion of a neutral Higgs below 114 GeV would no longer be valid in a CP-violating scenario, and there would be new challenges for Higgs searches at the LHC, because of the different pattern of Higgs couplings [34].

The CP-violating phases in the MCPMFV are strongly constrained by the experimental upper limits on electric dipole moments (EDMs), notably those of Thallium, the neutron and Mercury [35]. However, there is the possibility of non-trivial cancellations between the contributions of different phases [36]. The three EDM constraints cannot force all the 6 CP-violating phases of the MCPMFV model to be small simultaneously.

The MCPMFV phases may have important implications for the heavy Higgs contributions to $B$-physics observables, such as $B_s$ mixing, $B_s \!\to\! \mu\mu$, $B_u \!\to\! \tau\nu$ and $b \!\to\! s\tau$ [31]. Even after taking into account the constraints these observables impose, it is possible that there might be an important
Outlook

(a) 

\[ \arg (A_t) = \arg (A_b) \ [\text{deg}] \]

(b) 

\[ \arg (A_t) = \arg (A_j) \ [\text{deg}] \]

\[ \tan \beta = 5 \]

\[ M_{\tilde{\mu}^+} = 150 \text{ GeV} \]

\[ M_{\tilde{\text{SUSY}}} = 0.5 \text{ TeV} \]

\[ g^2_{H1ZZ}, g^2_{H2ZZ}, g^2_{H3ZZ} \]

Figure 9: The dependences on one CP-violating phase of (top) the two lightest neutral Higgs mass eigenstates \( H_1, H_2 \) and (bottom) the couplings of the three neutral Higgs bosons to pairs of Z bosons, for the indicated values of other supersymmetric parameters [33].

contribution from MCPMFFV phases to CP-violating observables such as the rate and CP-violating asymmetry in \( b \to s \gamma \) decay, as seen in Fig. 10.

One should be on the lookout not just for CP-conserving effects of charged Higgs bosons, but also for the possibility that they may manifest CP violation beyond the CKM model.

9. Summary

The existence of charged Higgs bosons is generic in physics beyond the Standard Model. In this talk I have focused on the MSSM's one particularly predictive example, but similar considerations apply to many other proposed extensions of the Standard Model.
Figure 10: The branching ratio $B(B \to X_s \gamma)$ (left) and the CP asymmetry $A_{CP}^{dir}(B \to X_s \gamma)$ (right) as functions of the common gaugino mass phase $\Phi_M$ for four values of $\tan \beta$, taking the trilinear coupling phase $\Phi_A^{GUT} = 0^\circ$. The region allowed experimentally at the 2-$\sigma$ level is bounded by two horizontal lines in the left frame. In the right frame, points satisfying this constraint are denoted by open squares: see [31] for details.

The LHC offers real prospects for direct detection of charged Higgs bosons, and is coming on line. After its brilliant startup and the subsequent disappointment, we are all crossing our fingers for high-energy collisions in 2009.

However, the LHC is not the only game in town: there are also opportunities to probe charged Higgs bosons indirectly in low-energy physics. Examples include tests of lepton universality in $K^+ \to \nu$ decays, a detectable contribution to $B_u \to \tau \nu$ decay, CP-violating effects, etc.

Floreat charged Higgs physics!

References

[1] H. P. Nilles, Phys. Rept. 110 (1984) 1; H. E. Haber and G. L. Kane, Phys. Rept. 117 (1985) 75.

[2] L. Maiani, All You Need To Know About The Higgs Boson, Proceedings of the Gif-sur-Yvette Summer School On Particle Physics, 1979, pp.1-52; G. ’t Hooft, in Recent Developments in Gauge Theories, Proceedings of the NATO Advanced Study Institute, Cargèse, 1979, eds. G. ’t Hooft et al. (Plenum Press, NY, 1980); E. Witten, Phys. Lett. B 105 (1981) 267.

[3] J. R. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B 249 (1990) 441 and Phys. Lett. B 260 (1991) 131; U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. B 260 (1991) 447; C. Giunti, C. W. Kim and U. W. Lee, Mod. Phys. Lett. A 6 (1991) 1745; P. Langacker and M. x. Luo, Phys. Rev. D 44 (1991) 817.

[4] J. R. Ellis, D. V. Nanopoulos, K. A. Olive and Y. Santoso, Phys. Lett. B 633 (2006) 583 [arXiv:hep-ph/0509331].
[5] LEP Electroweak Working Group, http://lepewwg.web.cern.ch/LEPEWWG/; Tevatron Electroweak Working group, http://tevewwg.fnal.gov/.

[6] J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos, K. A. Olive and M. Srednicki, Nucl. Phys. B 238 (1984) 453.

[7] H. Flaecher, M. Goebel, J. Haller, A. Hoecker, K. Moenig and J. Stelzer, arXiv:0811.0009 [hep-ph].

[8] R. Barate et al. [ALEPH, DELPHI, L3, OPAL Collaborations and LEP Working Group for Higgs boson searches], Phys. Lett. B 565 (2003) 61 [arXiv:hep-ex/0306033]; S. Schael et al. [ALEPH, DELPHI, L3, OPAL Collaborations and LEP Working Group for Higgs boson searches], Eur. Phys. J. C 47 (2006) 547 [arXiv:hep-ex/0602042].

[9] Tevatron New Phenomena and Higgs Working Group, combining CDF and D0 upper limits, http://tevnphwg.fnal.gov/results/SM_Higgs_Summer_08/.

[10] J. R. Ellis and D. V. Nanopoulos, Phys. Lett. B 110 (1982) 44.

[11] R. Barbieri and R. Gatto, Phys. Lett. B 110 (1982) 211.

[12] J. R. Ellis, K. A. Olive and P. Sandick, Phys. Rev. D 78 (2008) 075012 [arXiv:0805.2343 [hep-ph]], and references therein.

[13] LEP SUSY Working Group, ALEPH, DELPHI, L3 and OPAL Collaborations, http://lepsusy.web.cern.ch/lepsusy/Welcome.html.

[14] V. M. Abazov et al. [D0 Collaboration], arXiv:0901.0646 [hep-ex], and references therein.

[15] E. Barberio et al. [Heavy Flavour Averaging Group (HFAG)], hep-ex/0603003, http://slac.stanford.edu/xorg/hfag/.

[16] J. Dunkley et al. [WMAP Collaboration], arXiv:0803.0586 [astro-ph].

[17] These figures are updated versions of those in J. R. Ellis, K. A. Olive, Y. Santoso and V. C. Spanos, Phys. Lett. B 565 (2003) 176 [arXiv:hep-ph/0303043].

[18] G. W. Bennett et al. [Muon G-2 Collaboration], Phys. Rev. D 73 (2006) 072003 [arXiv:hep-ex/0602035].

[19] M. Passera, W. J. Marciano and A. Sirlin, Phys. Rev. D 78, 013009 (2008) [arXiv:0804.1142 [hep-ph]].

[20] M. Davier, http://tau08.inp.nsk.su/prog.php.

[21] For a summary of the LHC start-up, see:
http://press.web.cern.ch/press/PressReleases/Releases2008/PR08.08E.html

[22] For an analysis of the LHC incident, see:
http://press.web.cern.ch/press/PressReleases/Releases2008/attachments/CERN_081205b_LHCrestart.pdf.

[23] O. Buchmueller et al., JHEP 0809 (2008) 117 [arXiv:0808.4128 [hep-ph]].

[24] Z. Kunszt and F. Zwirner, Nucl. Phys. B 385 (1992) 3 [arXiv:hep-ph/9203223].

[25] G. Aad et al. [ATLAS Collaboration], arXiv:0901.0512.

[26] G. L. Bayatian et al. [CMS Collaboration], CMS Technical Design Report, Volume II: Physics Performance, CERN-LHCC-2006-021, CMS-TDR-008-2 J. Phys. G34, 995 (2007); see:
http://cdmsdoc.cern.ch/cms/cpt/tdr/.
[27] M. Hashemi, S. Heinemeyer, R. Kinnunen, A. Nikitenko and G. Weiglein, arXiv:0804.1228 [hep-ph].

[28] J. Ellis, T. Hahn, S. Heinemeyer, K. A. Olive and G. Weiglein, JHEP 0710 (2007) 092
[arXiv:0709.0098 [hep-ph]].

[29] A. Masiero, P. Paradisi and R. Petronzio, Phys. Rev. D 74 (2006) 011701 [arXiv:hep-ph/0511289];
JHEP 0811 (2008) 042 [arXiv:0807.4721 [hep-ph]].

[30] J. Ellis, S. Lola and M. Raidal, arXiv:0809.5211 [hep-ph].

[31] J. R. Ellis, J. S. Lee and A. Pilaftsis, Phys. Rev. D 76 (2007) 115011 [arXiv:0708.2079 [hep-ph]].

[32] For a code to calculate many CP-violating effects in the MSSM, see: J. S. Lee, A. Pilaftsis,
M. S. Carena, S. Y. Choi, M. Drees, J. R. Ellis and C. E. M. Wagner, Comput. Phys. Commun. 156
(2004) 283 [arXiv:hep-ph/0307377]; J. S. Lee, M. Carena, J. Ellis, A. Pilaftsis and C. E. M. Wagner,
arXiv:0712.2360 [hep-ph]. See also T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein,
in Proceedings of the 15th International Conference on Supersymmetry and the Unification of
Fundamental Interactions (SUSY07), Karlsruhe, Germany, 26 Jul - 1 Aug 2007 arXiv:0710.4891
[hep-ph].

[33] M. S. Carena, J. R. Ellis, A. Pilaftsis and C. E. M. Wagner, Phys. Lett. B 495 (2000) 155
[arXiv:hep-ph/0009212]; Nucl. Phys. B 625 (2002) 345 [arXiv:hep-ph/0111245].

[34] M. S. Carena, J. R. Ellis, S. Mrenna, A. Pilaftsis and C. E. M. Wagner, Nucl. Phys. B 659 (2003) 145
[arXiv:hep-ph/0211467].

[35] K. A. Olive, M. Pospelov, A. Ritz and Y. Santoso, Phys. Rev. D 72 (2005) 075001
[arXiv:hep-ph/0506106].

[36] J. R. Ellis, J. S. Lee and A. Pilaftsis, JHEP 0810 (2008) 049 [arXiv:0808.1819 [hep-ph]].