Theoretical study of the spin polarization measurement using non-contact Andreev reflection

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Abstract. We investigated the non-contact Andreev reflection of the normal-conductor/superconductor junction as a tool for spin polarization measurement of ferromagnetic metals. The conductance due to the Andreev reflection is obtained by numerically solving the corresponding one-dimensional Bogoliubov-de Gennes equation with use of the recursion-transfer-matrix method. We showed that the conductance suppression due to the exchange splitting of conduction band in ferromagnets. We also derived the analytical expression of the zero-bias conductance due to the Andreev reflection through the square barrier which agrees well with the exact numerical values.

1. Introduction

Andreev reflection (AR) at the interface between the ferromagnetic metal (FM) and superconductor (SC) has been attracted much attention because it provides a powerful tool for measuring the spin polarization of conduction electrons of the ferromagnetic metal. In 1995 de Jong et al. predicted that the AR of a FM/SC point contact is suppressed due to the exchange splitting of conduction band in the FM[1]. In the contact region the number of spin-up transmitting channels ($N_\uparrow$) is larger than that of spin-down transmitting channels ($N_\downarrow$), i.e., $N_\uparrow \geq N_\downarrow$. They suppose that there is no partially-transmitting channels and neglect mixing of channels for simplicity. When the SC is in the normal conducting state, all scattering channels (transverse modes in the point contact at the Fermi level) are fully transmitted, yielding the conductance

$$G_{FN} = \frac{e^2}{h} (N_\uparrow + N_\downarrow),$$

where subscript $FN$ stands for the FM/normal-conductor(NC) junction. When the SC is in the superconducting state, all the spin-down incident electrons in $N_\downarrow$ channels are Andreev reflected and give a double contribution to the conductance, $2(e^2/h)N_\downarrow$. However, spin-up incident electrons in some channels cannot be Andreev reflected since the density of states for spin-down electrons is smaller than that for spin-up electrons. Then only a fraction ($N_\downarrow/N_\uparrow$) of the $N_\uparrow$ channels can be Andreev reflected and the resulting conductance is $2(e^2/h)(N_\downarrow/N_\uparrow)N_\uparrow$. The total conductance at zero bias voltage ($V = 0$) is given by the sum of these two contributions:

$$G_{FS} = \frac{e^2}{h} 2 \left( N_\downarrow + \frac{N_\downarrow}{N_\uparrow} N_\uparrow \right) = 4N_\downarrow.$$
Figure 1. (a) The vacuum barrier we considered. The solid line shows the triangular barrier and the dashed line shows the square barrier. (b) Energy diagram of the FM(NM) / SC(NC) junction is schematically shown. The distance between the ferromagnetic metal and the superconductor is taken to be $L = 0.5, 1, 1.5, \text{ and } 2 \text{ Å.}$

Taking the ratio of Eq.(2) to Eq.(1), we obtain the normalized conductance

$$
\frac{G_{FS}}{G_{FN}} = 2 \left( 1 - P \right),
$$

where $P$ is the spin-polarization of transmitting channels defined as $P = (N_{\uparrow} - N_{\downarrow}) / (N_{\uparrow} + N_{\downarrow})$. For non-magnetic metal (NM) the spin-polarization is zero, i.e., $P = 0$. Eq.(3) shows that the normalized conductance is a monotonic decreasing function of $P$ and it vanishes if the FM is half metallic ($P = 1$).

In 1998, Soulen et al [2, 3] and independently by Upadyay et al [4] proposed the measurement technique called Point contact Andreev reflection (PCAR), which is based on the theory of Ref.[1]. The PCAR technique is much easier to put into practice than the tunneling technique. With no restrictions on the sample geometry, one can avoid the complex fabrication steps. They put a superconducting tip on a ferromagnet and measure the bias voltage dependence of the normalized conductance. By analyzing the observed normalized conductance curves they can estimate the spin polarization of FM [5, 6].

However, in the PCAR measurement, the surface of the FM is damaged by the superconducting tip. The surface damage will cause the interfacial scattering as well as the suppression of the spin polarization at surface. The above difficulty can be overcome by considering the non-contact AR through the vacuum, like the STM. Here we propose the non-contact Andreev reflection of the FM/SC junction as an effective tool for spin polarization measurement of ferromagnetic metals.

2. Model and Method
The system we considered is a one-dimensional tunnel junction with vacuum barrier shown in Fig. 1 (a). The work functions for FM(NM) and SC(NC) electrodes are denoted by $W_F$ and $W_S$, respectively. Two electrodes are separated by vacuum tunnel barrier of thickness $L$.

We employ the Stoner model for the conduction electrons in FM electrode, where the spin asymmetry is given by the exchange field $h$. We neglect the proximity effect for simplicity. Then the system is described by the following Bogoliubov-de Gennes (BdG) equation [7]:

$$
\begin{pmatrix}
H_0(x) - h(x)\sigma & \Delta(x) \\
\Delta^*(x) & -H_0(x) - h(x)\sigma
\end{pmatrix}
\begin{pmatrix}
f_{\sigma}(\vec{r}) \\
g_{\sigma}(\vec{r})
\end{pmatrix}
= E
\begin{pmatrix}
f_{\sigma}(\vec{r}) \\
g_{\sigma}(\vec{r})
\end{pmatrix},
$$

(4)
Figure 2. (a) Normalized conductances of the FM/SC junction through the triangular barrier for \( h = 0, 0.3, 0.6, 0.9, \) and \( 0.99 \) µF. The distance between the FM and the SC electrodes is set to be 1.5 Å. (b) The zero-bias conductance of the FM/SC junction with \( L = 1.5 \) Å is plotted against the exchange field \( h \). The conductance is normalized by that for \( h = 0 \). The filled circles indicate the results obtained by numerically solving the BdG equation of Eq.(4) with use of the recursion-transfer-matrix method. The line shows the system with the square barrier and the plot is the one with the triangular barrier.

where \( H_0(x) \equiv -(\hbar^2/2m)\nabla^2 - \mu_F + V - Fx \) is the single particle Hamiltonian, \( E \) is the quasiparticle (QP) energy measured from the Fermi energy \( \mu_F \), and \( \sigma = +(-) \) represents the up-(down-)spin band. The vacuum potential barrier \( V \) is given by \( V = W_F + \mu_F \) where \( W_F \) is the work function of the FM(NM) surface. The electric field \( F \) is given by \( F = (W_F - W_S)/L \) where \( W_S \) is the work function of the SC(NC) tip. The exchange field function \( h(x) \) is given by \( h(x) = h_0[1 - \Theta(x)] \) where \( h_0 \) represents the exchange field in the ferromagnetic metal and \( \Theta(x) \) is the Heaviside step function. The superconducting gap function is expressed as \( \Delta(x) = \Delta_0 \Theta(x - d) \), where \( \Delta_0 \) represents the superconducting gap in the superconductor.

The probabilities of the AR, \( A_{\sigma}(E) \), and normal reflection, \( B_{\sigma}(E) \) is obtained by solving the scattering problem with appropriate boundary conditions. Using the probabilities \( A_{\sigma}(E) \) and \( B_{\sigma}(E) \) the conductance is expressed as

\[
G = \frac{e^2}{h} \sum_{\sigma} [1 + A_{\sigma}(eV) - B_{\sigma}(eV)].
\]

The material parameters we used are the following. The superconducting gap is \( \Delta_0 = 1.5 \) meV, Fermi energy \( \mu_F = 3.8 \) eV, the work function of FM(NM) electrode is \( W_F = 5.2 \) eV and that of SC(NC) electrode is \( W_S = 4.0 \) eV.

3. Results

In Fig. 1 (b), we plot the normalized conductance of NM/SC junctions with \( L = 0.5, 1, 1.5, \) and 2Å. The conductance due to the AR below the superconducting gap \( \Delta_0 = 1.5 \) meV is suppressed by the vacuum tunnel barrier and decreases with increasing \( L \). In Fig. 2 (a) we plot the normalized conductance due to the AR for various values of exchange field \( h \). The distance between two electrodes is set to be 1.5 Å. For \( h = 0 \), the normalized conductance takes the value of 0.08 and it decreases with increasing the exchange field \( h \). The obtained results are quite similar to those obtained in the PCAR system and imply that the non-contact AR can be used as a tool for spin polarization measurement of ferromagnetic metals.
In Fig. 2 (b) we plot the zero-bias conductance shown in Fig. 2 (a) against the exchange field $h$ by filled circles. The zero-bias conductance is normalized by that for $h = 0$. We also calculate the zero-bias conductance for the junction with rectangular potential indicated by dotted line in Fig. 1 (a). The height of the potential is assumed to be $(W_F + W_S)/2$. Under the Andreev approximation we obtain the analytic form of the normalized zero-bias conductance,

$$G_{FS}/G_{NS} = \frac{4\sqrt{1 - \eta^2}(V_0 - 1)^2}{\xi},$$

(6)

where

$$\xi = \left\{ \left( \sqrt{1 + \eta} - \sqrt{1 - \eta} \right) V_0 \sqrt{V_0 - 1} \cosh(L\rho) \sinh(L\rho) \right\}^2 + \left\{ \left( \sqrt{1 - \eta^2} + 1 \right) (V_0 - 1) + \left( \sqrt{1 - \eta^2} + V_0 - 1 \right) V_0 \sinh^2(L\rho) \right\}^2.$$  

(7)

Here we introduce the parameters $\eta = h_0/\mu_F$ and $V_0 = V/\mu_F$. In Fig. 2 (b) we plot Eq. (6) by the solid line. One can see that the approximated results of Eq.(6) agree with the exact ones indicated by filled circles very well. Equation (6) is the basis of the non-contact measurement of spin polarization we propose.

4. Conclusion

We theoretically studied the non-contact AR of the FM / SC junction as a tool for spin polarization measurement of FMs. We show that the conductance due to the AR decreases with increasing the exchange field. We derived the analytical expression of the zero-bias conductance due to the AR through the square barrier and showed that the approximated results agree with the exact ones obtained by using the recursion-transfer-matrix method. We found that the conductance with the triangular barrier is similar to the one with square barrier in the range of $L$ we considered. Our results support that the non-contact AR will be a powerful tool for measuring the spin polarization of FMs.

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