Stationary solutions of $N = 2$ supergravity

Klaus Behrndt, Dieter Lüst and Wafic A. Sabra

Humboldt-Universität, Institut für Physik
Invalidenstraße 110, 10115 Berlin, Germany

Abstract

We discuss general bosonic stationary configurations of $N = 2$, $D = 4$ supergravity coupled to vector multiplets. The requirement of unbroken supersymmetries imposes constraints on the holomorphic symplectic section of the underlying special Kähler manifold. The corresponding solutions of the field equations are completely determined by a set of harmonic functions. As examples we discuss rotating black holes, Taub-NUT and Eguchi-Hanson like instantons for the $STU$ model. In addition, we discuss, in the static limit, worldsheet instanton corrections to the $STU$ black hole solution, in the neighbourhood of a vanishing 4-cycle of the Calabi-Yau manifold. Our procedure is quite general and includes all known black hole solutions that can be embedded into $N = 2$ supergravity.

behndt@qft2.physik.hu-berlin.de
luest@qft1.physik.hu-berlin.de
sabra@qft2.physik.hu-berlin.de
1 Introduction

During the recent years there has been an enormous progress in the understanding of non-perturbative phenomena in $N = 2$ supersymmetric field theories \(^1\) as well as in $N = 2$ string vacua in four-dimensions. In particular, strong-weak coupling and string/string duality symmetries \(^2\) among type II compactifications on Calabi-Yau threefolds and heterotic vacua on $K3 \times T^2$ with $N = 2$ supersymmetry in four dimensions were successfully tested for a class of models \(^3\). Moreover, various types of (non-)perturbative transitions among different $N = 2$ string vacua were explored. Because of all this progress, the unification of possibly all superstring theories can very likely be realized by common underlying framework, often called $M$-theory.

Transitions among $N = 2$ string vacua generically can take place at those points in the moduli spaces where non-perturbative BPS states become massless. For example, considering $N = 2$ type II string vacua on Calabi-Yau three-folds, electrically or magnetically charged black holes become massless at the conifold points in the Calabi-Yau moduli spaces, where certain homology cycles of the Calabi-Yau spaces shrink to zero size. Therefore it is very important to find the most general solutions of the effective $N = 2$ supergravity action coupled to $N = 2$ matter multiplets. By constructing non-trivial space-time dependent solutions of $N = 2$ supergravity one can hope to obtain more information about the dynamical nature of the non-perturbative transitions. For instance, for generic non-trivial black hole solutions, not only the four-dimensional metric but also the four-dimensional gauge fields as well as the moduli fields are expected to be space-time dependent functions. Then one deals with internal Calabi-Yau spaces whose shapes vary over every point in the four-dimensional space-time; in this way one gets a very interesting link between the structures of the internal space and of four-dimensional space-time. In particular, one can try to determine those points in space-time where the internal Calabi-Yau periods shrink to zero size and where the transitions due to massless black holes take place. Therefore, the black hole solutions may provide an interesting link between external and internal singularities.

In this paper we will study stationary (i.e. time independent) solutions of $N = 2$ supergravity coupled to $N = 2$ vector multiplets. In recent times there has been a considerable progress in the understanding of extreme solutions of $N = 2$ supergravity in four dimensions.\(^4\) In contrast to $N = 4, 8$ theories, where the dynamics is highly restricted by supersymmetry, for $N = 2$ solutions one may expect a much richer structure. E.g. black holes of $N = 4, 8$ supersymmetric theories are expected not to receive quantum corrections whereas solutions of $N = 2$ supergravity will alter significantly when quantum corrections are taken into account. This opens the possibility to address, besides the already mentioned problems, many interesting questions: Are black holes stable? What happens with the singularity? Are quantum correction encoded in the Hawking temperature?

\(^{4}\)A recent discussion of non-extreme Calabi-Yau black holes appeared in \[^{5}\].
To find general solutions that include quantum corrections is in general a difficult problem - to solve the equations of motion of quantum gravity seems to be hopeless. One interesting first starting point is to look at the entropy of non-singular static black holes. Here, a major breakthrough came with the observation, that the entropy can be obtained by extremizing the central charge of the underlying $N = 2$ super-algebra with respect to the moduli. It was also shown that the scalar fields follow attractor equations with fixed points on the horizon. These fixed points are independent of the moduli (scalar fields) at infinity and are determined by the conserved charges and topological data only. If the scalar fields take this value throughout the entire space-time, i.e. they are constant, one gets the so-called double extreme black holes.

All these results are covariantly formulated in terms of the underlying special geometry, i.e., all quantities can be expressed in terms of the symplectic sections. The central statement is that the symplectic section has to fulfil constraints, the so-called stabilisation equations. Specifically, these equations are satisfied if the symplectic vector $\Pi = (Y^I, M_I(Y))$, where $Y^I = \bar{Z} L^I$, with $Z$ being the $N = 2$ central charge and $(L^I, M_I)$ being the symplectic period vector of $N = 2$ supergravity (covariantly holomorphic section), fulfils the following set of equations

$$i(Y^I - \bar{Y}^I) = p^I, \quad i(F_I(Y) - \bar{F}_I(\bar{Y})) = q_I; \quad (1)$$

here the $p^I$ ($q_I$) are the integer valued magnetic (electric) charge vectors. Solutions of these equations determine the double extreme black holes completely or the entropy for any static black hole with a non-singular horizon. For the double extreme black holes, the equations are also called “stabilisation equations”. The motivation was the following. The scalar fields at infinity take constant values (moduli), which are not protected by any kind of gauge symmetry. Therefore, one can argue that dynamically a given configuration will “choose” those values for which the mass (=central charge) becomes minimal. As long as the solution is non-singular one can show that for these values the scalar fields become constant everywhere and these constants ensure that the mass is minimal. The solution (black hole) has been stabilised. By means of the equations it is straightforward to compute the classical entropy of static $N = 2$ black holes as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections as well as to incorporate quantum corrections. So the full entropy depends in the heterotic case on the space-time instanton numbers, whereas in the type II compactifications the entropy depends on the topological data of the Calabi-Yau spaces, like the intersection numbers, the Euler number and the rational worldsheets. In the meanwhile, these results have been extended in several directions. For example, extreme solutions with non-constant scalar fields were constructed for the axion-free $STU$ model with cubic prepotential, and for supergravity models based on quadratic prepotentials. In these models it was found that the static solutions are expressed in terms of constrained harmonic functions. Later it was in fact shown that for general static extreme $N = 2$ black holes, the solutions are completely specified by the Kähler potential of the underlying special geometry where the imaginary part of the holomorphic sections are given in terms of a set of constrained harmonic functions. In addition, for
certain cases a microscopic understanding of the $N = 2$ entropy formulae was achieved by wrapping various branes around the internal Calabi-Yau cycles [15].

The aim of the present paper is to discuss a generalisation of this approach, which can be used for any static or stationary $N = 2$ solution. It will be shown, that a supersymmetric configuration is given, if the symplectic holomorphic section $\Omega = (X^I, F_I)$ satisfies the equations

$$i(X^I - \bar{X}^I) = \tilde{H}^I(x^\mu), \quad i(F_I - \bar{F}_I) = H_I(x^\mu)$$

where the symplectic vector $\left(\tilde{H}^I(x^\mu), H_I(x^\mu)\right)$ defines the gauge fields. In addition, imposing the equations of motion and Bianchi identities these functions have to be harmonic. These equations can be seen as a consistent bosonic truncation of $N = 2$ supergravity, off-shell if the $H$'s are arbitrary and on-shell for harmonic $H$'s.

The above equations (2) are very interesting, since they combine the structure of the internal space, e.g. a Calabi-Yau threefold, on the left hand side with space-time properties on the right hand side. Consequently, as discussed at the beginning, singularities in space-time are related to special points in the internal space, like walls of the Kähler cone or conifold singularities.

Moreover, the equations (2) provide black hole solutions for cases where the horizon is singular. But for these singular solutions, the scalar fields cannot be stabilised. Again, one would expect that the moduli will dynamically arrange each other in such a way that the mass becomes minimal. But this minimum is not described by the equations (2). Instead, these equations describe the complete supersymmetric configuration, in general not with an extremized central charge. Therefore, they open especially the possibility to investigate singularities, e.g. for topology change of the internal space (all kinds of phase transitions). In addition one can discuss solutions, that have not enough charges for a non-singular horizon, or solutions which do not have any horizon like Taub-NUT spaces, or naked singularities like for supersymmetric rotating black holes. Note, that the equations (2) allow to include all perturbative and non-perturbative corrections - only higher derivative corrections are not (yet) taken into account.

As already said, the $N = 2$ black hole solutions constructed so far, describe static configurations. In this case the metric depends on the $N = 2$ Kähler potential. We will show that stationary, but non-static solutions are obtained if the metric also depends on the $U(1)$ Kähler connection, which is also a symplectic invariant quantity in the context of $N = 2$ special geometry. Depending on the choices for the harmonic functions and on the considered prepotentials, one gets for example non-static rotating $N = 2$ black holes, $N = 2$ Taub-NUT spaces or $N = 2$ Eguchi-Hanson like instantons. Note that similar solutions were also discussed previously in the context of $N = 2$ supersymmetric non-linear $\sigma$-models [13].

The paper is organised as follows. In the next chapter we will collect some formulae and expressions of $N = 2$ supergravity which will be important for the following discussion. In section three we will determine the stationary solutions by the requirement that half of
the $N = 2$ supersymmetries are unbroken in the considered background. In addition, the field equations constrain the functions, which appear in the solutions of the gravitino and gaugino variations, to be harmonic. Then, in the fourth chapter we will apply our formalism to construct examples of stationary $N = 2$ solutions. Specifically, after considering possible choices for the harmonic functions, we will explicitly discuss the case of pure supergravity, and rotating black holes, Taub-NUT spaces and Eguchi-Hanson instantons in the so-called STU model. We also discuss worldsheet instanton corrections to the static STU black hole solution, in the neighbourhood of a vanishing 4-cycle of the Calabi-Yau manifold. Some conclusions will close the paper. Our conventions are collected in the appendix.

2 Special geometry and $N = 2$ supergravity

The structure of $N = 2$ supergravity theories coupled to vector and hypermultiplets is governed by special geometry. In this section, we briefly review some of the essential formalism of special geometry which will be relevant for our discussion. The complex scalars $z^A$ of the $N = 2$ vector multiplets coupled to supergravity are coordinates which parametrise a special Kähler manifold. This is a Kähler-Hodge manifold, with an additional constraint on the curvature \[ R_{A B C D} = g_{A B} g_{C D} + g_{A D} g_{C B} - C_{A C E} C_{B D L} g^{E L}, \] (3)

where $g_{A \bar{B}} = \partial_A \partial_{\bar{B}} K$, is the Kähler metric with $K$ the Kähler potential and $C_{A B C}$ is a completely symmetric covariantly holomorphic tensor. Kähler-Hodge manifolds are characterised by a $U(1)$ bundle whose first Chern class is equal to the Kähler class. This implies that, locally, the $U(1)$ connection can be represented by

$$ Q = -\frac{i}{2} (\partial_A K dz^A - \partial_{\bar{A}} K d\bar{z}^A). $$ (4)

An intrinsic definition of special Kähler manifold can be given in terms of a flat $2n + 2$ dimensional symplectic bundle over the Kähler-Hodge manifold, with the covariantly holomorphic sections

$$ V = \begin{pmatrix} L_I \\ M_I \end{pmatrix}, \quad I = 0, \ldots, n $$

$$ D_A V = (\partial_A - \frac{1}{2} \partial_A K) V = 0, $$ (5)

obeying the symplectic constraint

$$ i \langle V | \bar{V} \rangle = i (\bar{L}_I M_I - L_I \bar{M}_I) = 1 $$ (6)
where the symplectic inner product is understood to be taken with respect to the metric 
\[
\begin{pmatrix}
0 & -1 \\
1 & 0
\end{pmatrix}.
\]
One also defines
\[U_A = D_AV = (\partial_A + \frac{1}{2} \partial_A K)V = \left(\frac{f_A^I}{h_{AI}}\right).\] (7)
In general, \(D_A\) is the covariant derivative with respect to the Levi-Civita connection and the connection \(\partial_A K\). Thus, for a generic field \(\phi^i\) which transforms under the Kähler transformation, \(K \rightarrow K + f + \bar{f}\), by the \(U(1)\) transformation \(\phi^A \rightarrow e^{-(\frac{f}{2} + \frac{\bar{f}}{2})}\phi^A\), we have
\[D_A\phi^B = \partial_A\phi^B + \Gamma^B_{AC}\phi^C + \frac{p}{2} \partial_A K\phi^B.\] (8)
One also defines the covariant derivative \(D_A\) in the same way but with \(p\) replaced with \(\bar{p}\). The sections \((L^I, M_I)\) has the weights \(p = -\bar{p} = 1\) and \(C_{ABC}\) has the weights \(p = -\bar{p} = 2\).

In general, one can write
\[M_I = N_{I,J}L^J,\]
\[h_{AI} = \bar{N}_{I,J}f^I_A.\] (9)
The complex symmetric \((n + 1) \times (n + 1)\) matrix \(N\) encodes the couplings of the vector fields in the corresponding \(N = 2\) supergravity theory.

It can be shown [17]-[23] that the condition (3) can be obtained from the integrability conditions on the following differential constraints
\[D_AV = U_A,\]
\[D_AU_B = iC_{ABC}g^{CL}\bar{U}_L,\]
\[D_A\bar{U}_B = g_{A\bar{B}}\bar{V},\]
\[D_A\bar{V} = 0,\] (10)
and [24]
\[\langle V, U_A \rangle = 0.\] (11)
It is well known that the above constraints can in general be solved in terms of a holomorphic function of degree two [17]. However, there exists symplectic sections for which such a holomorphic function does not exist. This, for example, appears in the study of the effective theory of the \(N = 2\) heterotic strings [22]. Thus it is more natural to use these differential constraints as the fundamental equations of special geometry.

The Kähler potential can be constructed in a symplectic invariant manner as follows. Define the sections \(\Omega\) by
\[V = \left(\frac{L^I}{M_I}\right) = e^{\frac{K}{2}}\Omega = e^{\frac{K}{2}}\left(\frac{X^I}{F_I}\right).\] (12)
It immediately follows from (13) that $\Omega$ is holomorphic;
$$\partial_A X^I = \partial_A F_I = 0 .$$  
(13)

Using (13), one obtains
$$K = - \log \left( i \langle \Omega | \bar{\Omega} \rangle \right)$$
$$= - \log \left[ i ( \bar{X}^I F_I - X^I \bar{F}_I ) \right] .$$  
(14)

Exploiting the relations (6), (10) and (9), the following symplectic expression can be obtained for the Kähler metric
$$g_{AB} = - i \langle U_A | \bar{U}_B \rangle = - 2 f^I_A \text{Im} N_{IJ} \bar{f}^J_B .$$  
(15)

For our purposes, it is also useful to display the following relations
$$g^{AB} f^I_A \bar{f}^J_B = - \frac{1}{2} ( \text{Im} N )^{IJ} - \bar{L}^I L^J .$$  
(16)

and
$$F_I \partial_\mu X^I - X^I \partial_\mu F_I = 0 .$$  
(17)

which is a consequence of (11).

It should be mentioned that the dependence of the gauge couplings on the scalars characterising homogenous special Kähler manifolds of $N = 2$ supergravity theory can also be determined from the knowledge of the corresponding embedding of the isometry group of the scalar manifold into the symplectic group à la Gaillard and Zumino [25, 26].

The $N = 2$ supergravity action includes one gravitational, $n$ vector and hypermultiplets. However, for our purposes, the hypermultiplets are assumed to be constants. In this case, the bosonic $N = 2$ action is given by
$$S_{N=2} = \int \sqrt{-g} \, d^4 x \left( - \frac{1}{2} R + g_{AB} \partial^\mu z^A \partial_\mu \bar{z}^B + i \left( \bar{N}_{IJ} F^{-I}_{\mu\nu} F^{-J \mu\nu} - \bar{N}_{IJ} F^{+I}_{\mu\nu} F^{+J \mu\nu} \right) \right) .$$  
(18)

where
$$F^{\pm I \mu\nu} = \frac{1}{2} \left( F^{I \mu\nu} \pm i \varepsilon^{\mu\nu\rho\sigma} F^{\rho\sigma}_I \right) .$$  
(19)

The important field strength combinations which enter the chiral gravitino and gauginos supersymmetry transformation rules are given by
$$T^-_{\mu\nu} = M I F^I_{\mu\nu} - L^I G_{\mu\nu} = 2 i (\text{Im} N_{IJ}) L^I F^{-j}_{\mu\nu}$$
$$G^{-A}_{\mu\nu} = - g^{AB} \bar{f}^J_B (\text{Im} N)_{IJ} F^{J}_{\mu\nu} .$$  
(20)

with
$$G_{I \mu\nu} = \text{Re} N_{IJ} F^{j}_{\mu\nu} - \text{Im} N_{IJ} F^{j}_{\mu\nu} .$$  
(21)
The supersymmetry transformation for the chiral gravitino $\psi_{\alpha\mu}$ and gauginos $\lambda^{A\alpha}$ in a bosonic background of $N=2$ supergravity are given by

\[
\delta \psi_{\alpha\mu} = \nabla_{\mu} \epsilon_{\alpha} - \frac{1}{4} T_{\rho\sigma}^{\alpha} \gamma^\rho \gamma^\sigma \gamma_{\mu} \epsilon_{\alpha} \epsilon_{\beta},
\]

\[
\delta \lambda^{A\alpha} = i \gamma^\mu \partial_{\mu} z^A \epsilon_{\alpha} + G_{\rho\sigma}^{A} \gamma^\rho \gamma^\sigma \epsilon_{\alpha} \epsilon_{\beta}
\]

where $\epsilon_\beta$ is the chiral supersymmetry parameter, $\epsilon^{ab}$ is the $SO(2)$ Ricci tensor and the space-time covariant derivative $\nabla_{\mu}$ also contains the Kähler connection

\[
Q_{\mu} = -i \left( \partial_{\mu} K \partial_{\mu} z^A - \partial_{\mu} K \partial_{\mu} \bar{z}^A \right),
\]

Therefore we have

\[
\nabla_{\mu} \epsilon_{\alpha} = \left( \partial_{\mu} - \frac{1}{4} w^{ab}_{\mu} \gamma_a \gamma_b + \frac{i}{2} Q_{\mu} \right) \epsilon_{\alpha}
\]

where $w^{ab}_{\mu}$ is the spin connection.

### 3 Stationary solutions

In this section we will first describe the stationary solution. In a second and third subsection we show that this solution is supersymmetric, i.e. the gravitino and gaugino variations vanish. Since the hyperscalars are trivial in our model, the hyperino variation is identically fulfilled. We will see that supersymmetry does not restrict the functions $(\tilde{H}^I, H^I)$ to be harmonic. This property is a consequence of the equations of motions and the Bianchi identities of the gauge fields.

#### 3.1 The solution

First, we investigate the gauge field part, which can be written as

\[
\int ( \text{Im} \mathcal{N}_{IJ} F^I \cdot F^J + \text{Re} \mathcal{N}_{IJ} F^I \cdot \ast F^J ) = \int F^I \ast G_I
\]

where we have ignored space-time indices, i.e., $F^I \cdot F^J \equiv F^I_{\mu\nu} F^J_{\mu\nu}$ and $G^I_{\mu\nu}$ is defined in (21). The equations of motion and the Bianchi identities for the gauge fields are given by

\[
\partial_{\mu} (\epsilon^{\mu\nu\rho\lambda} G_{I\rho\lambda}) = 0, \quad \partial_{\mu} (\epsilon^{\mu\nu\rho\lambda} F^I_{\rho\lambda}) = 0.
\]

Assuming that our solution is time independent, the spatial components are given by

\[
F^I_{mn} = \frac{1}{2} \epsilon_{mn} \partial_{p} \tilde{H}^I, \quad G_{Imn} = \frac{1}{2} \epsilon_{mn} \partial_{p} H_I
\]
where \( m, n, p = 1, 2, 3 \) and \((\bar{H}^I, H_I)\) are harmonic functions. Note, that the timelike components \( F^I_{0m} \) and \( G_{10m} \) are fixed in terms of the spatial ones using (21). As next step we consider the metric. We are interested in time independent supersymmetric solutions. It has been shown by Tod \[28\] that this requires an IWP metric \[27\] which can be expressed in the following form
\[
d s^2 = -e^{2U}(dt + \omega_m dx^m)^2 + e^{-2U}dx^m dx^m .
\]
(28)

If in addition \( \omega_m = 0 \) the metric becomes static. A natural guess for the functions \( e^{-2U} \) and \( \omega_m \) follows from the requirement of duality invariance. Note, there is no requirement for symplectic invariance, only the duality subgroup represents an isometry. In special geometry there are only two duality invariant expressions: the Kähler potential \( K \) and the \( U(1) \) Kähler connection \( Q_\mu \). Below we show, that
\[
e^{-2U} = e^{-K} \equiv i(\bar{X}^IF_I - X^F\bar{I}),
\]
\[
\frac{1}{2} e^{2U} \epsilon_{mpn} \partial_n \omega_p = Q_m \equiv \frac{1}{2} e^K (\bar{F}_I \partial_m X^I - \bar{X}^I \partial_m F_I + \text{c.c.})
\]
\[
= \frac{1}{2} e^{2U} (H_I \partial_m \bar{H}^I - \bar{H}^I \partial_m H_I)
\]
(29)
gives a supersymmetric configuration. Similar expressions for \( e^{-2U} \) and \( \omega_m \) have been suggested in \[31\]. Having in mind that the scalar fields are given by \( z^A = X^A/X^0 \), we have expressed the solution completely in terms of the section \((X^I, F_I)\), which is fixed by (2).

### 3.2 Gravitino variation

Before we start to show that (29) provides a supersymmetric configuration, we first collect some general formulae (our notations are given in the appendix). The Vielbeins for the IWP metric are
\[
e_0^0 = e^U , \quad e_0^m = e^U \omega_m , \quad e_m^n = e^{-U} \delta_m^n ,
\]
\[
e_0^- = e^{-U} , \quad e_m^a = -e^U \omega_m , \quad e_m^- = e^U \delta_m^a .
\]
(30)

For the anti-selfdual spin connections one obtains
\[
\omega_0^-_{0i} = -\frac{i}{2} e^{2U} V_i , \quad \omega_0^-_{ij} = \frac{1}{2} e^{2U} e^{ijk} \bar{V}_k,
\]
\[
\omega_m^-_{0i} = -\frac{i}{2} (e^{2U} \omega_m V_i - i \epsilon^{imk} \bar{V}_k),
\]
\[
\omega_m^-_{ij} = \frac{1}{2} (e^{2U} \omega_m e^{ijk} \bar{V}_k - 2i \delta_m[i \bar{V}_j]),
\]
(31)
with
\[ \vec{V}_m = Q_m - i \partial_m U = Q_m - \frac{i}{2} \partial_m K. \] (32)

Now we show that the configuration defined by (27) – (29) with the holomorphic sections given by (4) is supersymmetric. We first transform the curved indices of the gauge fields as well as the the \( \gamma \)-matrices into flat ones. The gauge fields are defined by vanishing variations of (25) and after taken into account that \((F^I \ast G_I)_{\text{curv.}} = \det |e^I_{\mu}| (F^I \ast G_I)_{\text{fl.}}\) we get
\[ \partial_\mu (\epsilon^{\mu \rho \lambda} e^{-2U} G_{I \rho \lambda}) = 0, \quad \partial_\mu (\epsilon^{\mu \rho \lambda} e^{-2U} F^I_{\rho \lambda}) = 0. \] (33)

As solution for the spatial components we obtain
\[ F^I_{ij} = \frac{1}{2} e^{2U} \epsilon_{ijp} \partial_p H^I, \quad G^I_{ij} = \frac{1}{2} e^{2U} \epsilon_{ijp} \partial_p H^I. \] (34)

Again the time components can be obtained by using the relation (21). Inserting these fields into (20) and using the anti-self-duality condition (104) we get for the graviphoton field strength
\[ T^{-ij} = \frac{1}{2} e^{3U} \epsilon_{ijm} \left( F^I \partial_m \tilde{H}^I - X^I \partial_m H^I \right), \quad T^{-0m} = \frac{i}{2} e^{3U} \left( F^I \partial_m \tilde{H}^I - X^I \partial_m H^I \right). \] (35)

Also, using the constraint (4) and the relation (17) one finds
\[ \partial_m e^{-2U} = (X^I + \tilde{X}^I) \partial_m H^I - (F^I + \tilde{F}^I) \partial_m \tilde{H}^I \] (36)

and in terms of (23) and (32) one gets
\[ \vec{V}_m = i e^{2U} (F^I \partial_m \tilde{H}^I - X^I \partial_m H^I) = 2 e^{-U} T^{-0m}. \] (37)

Thus, we can write for the spin connections (31)
\[ \omega^{-0i}_0 = i e^U T^{-0i}, \quad \omega^{-ij}_0 = i e^U T^{-ij}, \]
\[ \omega^{-0i}_m = i e^U \omega_m T^{-0i} + e^{-U} e^{imk} T^{-0k}, \]
\[ \omega^{-ij}_m = i (e^U \omega_m T^{-ij} + 2 e^{-U} \delta^i_{m[j} T^{-j0]). \] (38)

Transforming the curved index of \( \gamma_\mu \) into a flat one \((\gamma_0 \rightarrow e^U_0 \gamma_0 = e^U \gamma_0 \) and \( \gamma_m \rightarrow e^U_0 \gamma_0 + e^U_m \gamma_m = e^U \gamma_0 + e^{-U} \gamma_m\), the time component of the gravitino supersymmetry variation (22) gives
\[ \delta \psi_{0\alpha} = -\frac{1}{4} e^U T^{-ab} \gamma_a \gamma_b \left( i \epsilon_\alpha + \gamma_0 \epsilon_{\alpha \beta} \epsilon^\beta \right). \] (39)

This variation vanishes if
\[ \epsilon_\alpha = i \gamma_0 \epsilon_{\alpha \beta} \epsilon^\beta. \] (40)
As a consequence generically one half of supersymmetry is broken. For static black holes with a non-singular horizon we know, that on the horizon the complete $N = 2$ supersymmetry is restored \[29\]. In order to investigate, whether also in the present case supersymmetry restoration occurs at special points in space time (note, there is no horizon in general), one has to look on the variation of the gravitino field strength, i.e. at the points of enhanced supersymmetry $[\nabla_\mu, \nabla_\nu] \epsilon_\alpha$ should vanish identically (if $\delta \psi_\mu \alpha = \nabla_\mu \epsilon_\alpha$). For a recent discussion see e.g. \[30\].

Next, using the relation (40) and the chirality properties of the spinor (105), we obtain from the spatial components of (22)

\[
\delta \psi_{m \alpha} = \left( \partial_m + \frac{1}{2} \partial_m U + i Q_m \right) \epsilon_\alpha .
\] (41)

Hence, in order to have non-trivial solutions the following integrability constraint has to be fulfilled

\[
\partial_{[m} Q_{n]} = 0 \quad (42)
\]
where $Q_m$ is the Kahler connection.

### 3.3 Gaugino variation

This variation is given by

\[
\delta \lambda^{\alpha A} = i \gamma^\mu \partial_\mu z^A \epsilon^\alpha + G^{-A}_{\rho \nu} \gamma^\rho \nu \epsilon^{\alpha \beta} \epsilon_\beta
\] (43)

with $G^{-A}_{\rho \nu} = -g^{AB} \bar{f}_B \text{Im} N_{IJ} F^{-J}_{\rho \nu}$ where $g^{AB}$ is the inverse Kähler metric and $\bar{f}_B = (\partial_B + \frac{1}{2} \partial_B K) \bar{L}^I$ (see (6)).

Multiplying (13) with $f_A^I$ and using the relation (16) yields

\[
f_A^I \delta \lambda^{\alpha A} = i \gamma^\mu \partial_\mu z^A \left[ (\partial_A + \frac{1}{2} \partial_A K) L^I \right] \epsilon^\alpha + \frac{1}{2} (F^{-I}_{\mu \nu} - i \bar{L}^I T^{-I}_{\mu \nu}) \gamma^\mu \gamma^\nu \epsilon^{\alpha \beta} \epsilon_\beta .
\] (44)

Using the definition of the Kähler potential and $L^I = e^{K/2} X^I$ one obtains

\[
\partial_\mu z^A \partial_A e^{-K} = \bar{X}^J \partial_\mu H_J - \bar{F}_J \partial_\mu \bar{H}^J ,
\] (45)
\[
\partial_\mu z^A \partial_A L^I = - \frac{1}{2} e^{3/2 K} X^I \partial_\mu z^A \partial_A e^{-K} + e^{K/2} \partial_\mu X^I .
\]

Furthermore, in terms of (103), (40) and transforming the curved $\gamma$-indices into flat ones ($\gamma^m \partial_m z^A \rightarrow e^U \gamma^m \partial_m z^A$) one gets

\[
f_A^I \delta \lambda^{\alpha A} = \left( -2 e^U (\bar{T}_0^- X^I - T_0^- \bar{X}^I) + i e^{2U} \partial_m X^I + 2 i F_{0m}^{-I} \right) \gamma^m \epsilon^\alpha .
\] (46)
Using the relations (9) and (7) we find
\[ \bar{N}_{IJ} f^I_A \delta \lambda^\alpha_A = \left( -2e^U (\bar{T}_{0m} F_J - T_{0m} \bar{F}_J) + i e^{2U} \partial_m F_J + 2iG_{-0m}^m \right) \gamma^m \epsilon^\alpha. \] (47)

From (34) it follows that
\[ F^I_{0m} \equiv \frac{1}{2} (F^I_{0m} - i (*F^I)_{0m}) = \frac{1}{2} (F^I_{0m} + \frac{i}{2} e^{2U} \partial_m \tilde{H}^I), \] (48)

\[ G^I_{10m} \equiv \frac{1}{2} (G^I_{10m} - i (*G^I)_{0m}) = \frac{1}{2} (G^I_{10m} + \frac{i}{2} e^{2U} \partial_m H^I), \]

and thus it can be easily seen that that the real parts of the expressions multiplying the \( \gamma^m \epsilon^\alpha \) term in (46) and (47) vanish. Consequently, \( \delta \lambda^\alpha_A = 0 \), because the matrix \( \bar{N}_{IJ} \) has always a non-trivial imaginary part (otherwise the gauge field part would be pure topological) and the imaginary part is invertible (\( \text{Im} \bar{N} \cdot V = 0 \leftrightarrow V = 0 \)).

Thus, we have shown that the configuration (27) – (29) with the holomorphic section constrained by (2) defines a supersymmetric bosonic configuration, which breaks generically one half of \( N = 2 \) supersymmetry.

4 Examples

In this section we describe some explicit solutions. First we start with a classification of harmonic functions, that yield different solutions, like rotating black holes, Taub-NUT or Eguchi-Hanson instantons and static solutions like extreme Reissner Nordstrom black holes.

As simplest example we start with pure supergravity. Then, in terms of the STU model we will discuss various solutions for choices of the harmonic functions.

4.1 Classification of solutions

A general real harmonic function is given by the real or imaginary part of a general complex harmonic function
\[ H = \sum_n \left( C_n + \frac{P_n}{\sqrt{(x - (x^0_n + i \gamma_n))^2 + (y - (y^0_n + i \beta_n))^2 + (z - (z^0_n + i \alpha_n))^2}} \right). \] (49)

We have dropped here the symplectic index; \( C_n \) and \( P_n \) are complex constants. Such a choice describes a multicenter solution, with the constituents located at \( (x^0_n, y^0_n, z^0_n) \) and \( n \) counts all centers. Note, that the integrability constraints (42) will give us restrictions for the parameters entering the harmonic functions. In the following, we will ignore the multicenter case. The singularity structure is determined by the parameters \( (\gamma, \beta, \alpha) \), e.g.,
if all are nontrivial, one obtains a non-singular solution. If, however, we set $\gamma = 0$, we obtain a solution with two singular points located at $y = z = 0$ and $x = \pm \sqrt{\beta^2 + \alpha^2}$. More interesting for us are the cases of the harmonic functions with $\gamma = \beta = 0$ or $\alpha = \beta = \gamma = 0$. The first case has a ring singularity at $z = 0$, $x^2 + y^2 = \alpha^2$ and corresponds to rotating black holes. The second case gives a spherically symmetric configuration and defines Taub-NUT spaces, which are asymptotically not flat. In the multicenter case when all the constants $C_n$ vanish one obtains a multi-instanton solution, in the simplest case the Eguchi-Hanson instantons. Finally, a vanishing Taub-NUT charge yields static solutions. Of course, generically all the above mentioned solutions can appear in a superposition. For an analog classification of axion-dilaton black holes see [31].

4.2 Pure supergravity

The case of pure supergravity provides a toy model which illustrates our methods. The holomorphic prepotential of this theory is given by

$$ F = -\frac{i}{4}(X^0)^2 $$

where $X^0$ is the graviphoton complex scalar field. Eq (2), gives in this case the following two equations

$$ i(X^0 - \bar{X}^0) = \tilde{H}^0, \quad \frac{1}{2}(X^0 + \bar{X}^0) = H_0 $$

which implies that $X^0$ is set by a complex harmonic function

$$ X^0 = H_0 - \frac{i}{2}\tilde{H}^0 $$

and the solution is then given by

$$ ds^2 = -\frac{1}{X^0\bar{X}^0}(dt + \omega_m dx^m)^2 + X^0\bar{X}^0 dx^2. $$

If we choose for $X^0$ the complex harmonic function

$$ X^0 = 1 + \frac{M + iN}{r} $$

where $M$ and $N$ are real constants and $r$ is the radial distance in spherical coordinates. Here, $\omega$ has only one component $\omega_\phi$ which is given by the following equation

$$ \frac{1}{r^2 \sin \theta} \partial_\theta \omega_\phi = H_0 \partial_r \tilde{H}^0 - \tilde{H}^0 \partial_r H_0 $$

where

$$ H_0 = 1 + \frac{M}{r}; \quad \tilde{H}^0 = -\frac{2N}{r}. $$
Therefore one obtains the following metric
\[ ds^2 = -\frac{1}{\left(1 + \frac{M}{r}\right)^2 + \frac{N^2}{r^2}} \left( dt + 2N \cos \theta d\phi \right)^2 + \left(1 + \frac{M}{r}\right)^2 + \frac{N^2}{r^2} \right) d\Omega^2 \] (57)
where \( d\Omega^2 = (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) \). This is the extreme electromagnetic generalization of the Taub-Nut metric considered by Hawking and Hartle [32].

Next, we choose a different harmonic function for \( X^0 \),
\[ X^0 = 1 + \frac{m}{\sqrt{x^2 + y^2 + (z - i\alpha)^2}} \] (58)
In this case, it is convenient to use the so-called oblate spheroidal coordinates, which are defined by
\[ x + iy = \sqrt{r^2 + \alpha^2 \sin^2 \theta} e^{i\phi}; \quad z = r \cos \theta, \] (59)
where the flat spatial metric becomes
\[ dx^m dx^m = \frac{r^2 + \alpha^2 \cos^2 \theta}{r^2 + \alpha^2} dr^2 + (r^2 + \alpha^2 \cos^2 \theta) d\theta^2 + (r^2 + \alpha^2) \sin^2 \theta d\phi^2 \] (60)
and
\[ X^0 = 1 + \frac{m(r + i\alpha \cos \theta)}{r^2 + \alpha^2 \cos^2 \theta} \] (61)
In this case we obtain for \( \omega_\phi \) and \( e^{-K} \) the following
\[ e^{-K} = \frac{(r + m)^2 + \alpha^2 \cos^2 \theta}{r^2 + \alpha^2 \cos^2 \theta}, \quad \omega_\phi = \frac{(2mr + m^2)\alpha \sin^2 \theta}{r^2 + \alpha^2 \cos^2 \theta} \] (62)
Thus one obtains the BPS-saturated Kerr-Newman metric [27].

4.3 Solution for the symplectic section of the STU model

The classical STU model is given by the prepotential
\[ F_{cl}(X) = -\frac{X^1 X^2 X^3}{X^0}. \] (63)
For a supersymmetric configuration, \( X^I \) are given as solutions of (2). Since these equations are similar to the stabilisation equations after replacing \((\bar{Z}_L^I, \bar{Z}_M_I)\) with \((X^I, F_I)\) and the charges by harmonic functions, we can take known results from double extreme black hole solutions and replace the charges by harmonic functions. For the model at hand the double extreme solution has been discussed in [9]. If one first defines the “symmetrized \( \epsilon \)-tensor” with non-vanishing components given by
\[ 1 = d_{123} = d^{123} = d_{213} = (\text{all permutations}) \] (64)
\(^{e}\text{Note, that in these references the solution is given in a different symplectic basis, where } X^0 = 1.\)
the solution for the \textit{STU} model can be written as
\[ e^{-4U} = e^{-2K} = -(\tilde{H}^I H_I)^2 + (d_{ABC} \tilde{H}^B \tilde{H}^C d^{ADE} H_D H_E) + 4 \tilde{H}^0 H_1 H_2 H_3 - 4 \tilde{H}^1 \tilde{H}^2 \tilde{H}^3 \]
and the scalar fields are
\[ z^A = \frac{X^A}{X^0} = \frac{2 \tilde{H}^A H_A - (\tilde{H}^I H_I)}{(d_{ABC} \tilde{H}^B \tilde{H}^C + 2 \tilde{H}^0 H_A)} \]
(65)
with no summation over the index \( A = 1, 2, 3 \) but over \( I = 0, 1, 2, 3 \). These equations define the complete symplectic coordinates \( X^I; X^0 \) can be obtained by inserting \( z^A \) into the Kähler potential (29).

### 4.4 Rotating black holes

By choosing different harmonic functions \((\tilde{H}^I, H_I)\) in (49) one can obtain different types of solutions. For example, as mentioned earlier, rotating black holes could be obtained if we choose for our solution harmonic functions with \( \gamma = \beta = 0 \). This provides an axial-symmetric configuration with the \( z \)-direction as rotational axis. Here it is more convenient to go to the appropriate coordinate system. As for the pure gravity case we use the spheroidal coordinates, in terms of which, the general (one-center) complex harmonic function is given by
\[ H = C + \frac{Q}{\sqrt{x^2 + y^2 + (z - i\alpha)^2}} = C + \frac{Q(r + i\alpha \cos \theta)}{r^2 + \alpha^2 \cos^2 \theta} \]
(67)
and therefore the ring singularity is at \( r = \cos \theta = 0 \).

Next, rotating black holes are asymptotically flat, which implies
\[ e^{2U} \to 1 \quad \text{and} \quad \omega_m \to 0 \quad \text{for} \quad r \to \infty \, . \]
(68)

The first condition in the above equation puts constraints on the additive constants in the harmonic functions. More interesting is the second part. The axial symmetry requires that nothing depends on the coordinate \( \phi \). Transforming (29), which defines \( \omega_m \), into spheroidal coordinates, we obtain \footnote{As consequence of the axial symmetry \( \omega_m \) has only a \( \phi \) component and the determinant of the 3-d metric is \( \sqrt{g} = (r^2 + \alpha^2 \cos^2 \theta) \sin \theta \).}
\[ \frac{1}{(r^2 + \alpha^2) \sin \theta} \partial_{\theta} \omega_\phi = H_I \partial_r \tilde{H}^I - \tilde{H}^I \partial_r H_I \, , \]
\[ -\frac{1}{\sin \theta} \partial_r \omega_\phi = H_I \partial_\theta \tilde{H}^I - \tilde{H}^I \partial_\theta H_I \, . \]
(69)

Therefore, in order to have asymptotic flat solutions we have to choose the harmonic functions in such a way that the rhs for the first equation in (39) vanishes faster than...
$1/r^2$ and at least as $1/r^2$ for the second equation. This is possible if one takes the following Ansatz for \( (\tilde{H}^I, H_I) \)

\[
\tilde{H}^0 = \text{Im} \left( \frac{m_0}{\sqrt{x^2+y^2+(z-i\alpha)^2}} \right) = \frac{m_0 \alpha \cos \theta}{r^2 + \alpha^2 \cos^2 \theta},
\]

\[
\tilde{H}^A = \text{Re} \left( h^A + \frac{p^A}{\sqrt{x^2+y^2+(z-i\alpha)^2}} \right) = h^A + \frac{p^A}{R},
\]

\[
H_0 = \text{Re} \left( h_0 + \frac{q_0}{\sqrt{x^2+y^2+(z-i\alpha)^2}} \right) = h_0 + \frac{q_0}{R},
\]

\[
H_A = \text{Im} \left( \frac{m_A}{\sqrt{x^2+y^2+(z-i\alpha)^2}} \right) = \frac{m_A \alpha \cos \theta}{r^2 + \alpha^2 \cos^2 \theta},
\]

where we introduced $R = (r^2 + \alpha^2 \cos^2 \theta)/r$. Furthermore, in order to get an asymptotic Minkowski space \((U \to 0)\) we have the constraint

\[
-4h_0h^1h^2h^3 = 1,
\]

i.e. we have to choose one of the $h$'s to be negative, e.g. $h_0 < 0$. It can also be seen as a fixing of $h_0$ and the additional three parameter $h^A$ parameterize the scalar fields at infinity

\[
z^A_\infty = -i \frac{2h_0h^A}{\sqrt{-4h_0h^1h^2h^3}}.
\]

Thus asymptotically, all axions vanish. It is also useful to determine our section at infinity.

From (2) one sees that $X^0_\infty$ has to be real and thus $X^A_\infty$ are pure imaginary ($z^A_\infty$ is pure imaginary) and one obtains

\[
X^0_\infty = \sqrt{-\frac{h^1h^2h^3}{4h_0}}, \quad X^A_\infty = -i \frac{h^A}{2}.
\]

Inserting this expression into the prepotential we find

\[
F^0_\infty = -i \frac{h_0}{2}, \quad F^A_\infty = \frac{1}{2}h_0d_{ABC}h^B_\infty h^C_\infty.
\]

Next, we need to ensure that the integrability constraint (12) is satisfied. This constraint in terms of the harmonic functions takes the form

\[
2\partial_{[n}Q_{m]} = \partial_{[n} \left( e^{2U} (H_I \partial_{m]} \tilde{H}^I - \tilde{H}^I \partial_{m]} H_I) \right) = 0
\]

which is solved if

\[
\partial_{[n} \tilde{H}^I \partial_{m]} H_J = 0.
\]

For our harmonic functions we have

\[
\partial_n \tilde{H}^0 \partial_m H_0 \sim m_0 q_0 \text{Im}(\partial_n \frac{1}{r_1} \partial_m \frac{1}{r_1}) ,
\]

\[
\partial_n \tilde{H}^A \partial_m H_B \sim p^A m_B \text{Im}(\partial_n \frac{1}{r_1} \partial_m \frac{1}{r_1}) ,
\]

\[
\partial_n \tilde{H}^0 \partial_m H_A \sim m_0 m_B \partial_n \text{Im}(\frac{1}{r_1}) \partial_m \text{Im}(\frac{1}{r_1}) ,
\]

\[
\partial_n \tilde{H}^A \partial_m H_0 \sim p^A q_0 \partial_n \text{Re}(\frac{1}{r_1}) \partial_m \text{Re}(\frac{1}{r_1})
\]

\[16\]
where $r_1 = \sqrt{x^2 + y^2 + (z - i \alpha)^2}$. So, after antisymmetrizing the derivatives, the condition (74) is identically fulfilled.

In order to give the parameters that enter the harmonic functions a physical meaning we go to the asymptotic flat region.

First from the asymptotic behavior of the metric we get the mass and the angular momentum

$$-g_{00} = e^{2U} = 1 - \frac{2M}{r} \pm .. = 1 + \frac{2(q_0 h^1 h^2 h^3 + 1/2 h_0 p^A d_A B C h^B h^C)}{r} \pm .. ,$$

$$-g_{0\phi} = e^{2U} \omega_{\phi} = \frac{2J \sin^2 \theta}{r} \pm .. = \frac{2(h^A m_A - h_0 m^0) \alpha \sin^2 \theta}{r} \pm .. .$$

(78)

Thus, the mass is positive only, if some of the parameters are negative and we will assume that $H_0 < 0 \forall r$, i.e. $h_0, q_0 < 0$. By using of (71) the mass and angular momentum of our solution is given by

$$M = |Z| = X^0_\infty q_0 - F^\infty_A p^A , \quad J = (h_0 m^0 - h^A m_A) \alpha$$

(79)

where $Z$ is the central charge and $q_0, h_0 < 0$. Finally, for the gauge fields we get asymptotically

$$(^* F^0)_0 r = 2 \frac{m^0 \alpha \cos \theta}{r^3} \pm .. , \quad (^* F^A)_0 r = \frac{p^A}{r^3} \pm .. ,$$

$$(^* G^0)_0 r = \frac{q_0}{r^2} \pm .. , \quad (^* G_A)_0 r = 2 \frac{m^A \alpha \cos \theta}{r^3} \pm .. .$$

(80)

Therefore, the black hole carries 3 magnetic ($p^A$), one electric ($q_0$) charges; as well as 3 electric ($1/2 m^0 \alpha$) and one magnetic ($1/2 m^0 \alpha$) dipole momenta.

These expressions suggest now the following interpretation. The rotating black hole is a bound state that consist of 4 constituents each determined by two harmonic functions ($\tilde{H}_I, H_I$), i.e. 3 magnetically charged and one electrically charged. For every consituent the gyromagnetic ratio is 1. E.g. extracting the $(\tilde{H}_1, H_1)$ part and making all others trivial, one finds for the dipol moment

$$\mu_1 = \frac{1}{2} g J \frac{p^1}{M}$$

(81)

with $g = 1$. This is an expected result, since every constituent is a so-called $a = \sqrt{3}$ black hole, which should be equivalent to Kaluza-Klein states; for a recent review see [33].

Already the BPS bound for the mass (79) suggest this bound state interpretation. For our solution here, the central charge on the rhs. is real, i.e. $M = |Z|$ and the total mass is a direct sum of the masses of their consituents. Also we note that the angular momentum does not enter into the BPS bound. This has a serious consequence: the black hole exhibits a naked singularity; there is no horizon hiding the ring singularity. Different

$^9$So, strongly speaking these objects are not really black, but for convenience we will use the name black hole for any asymptotically flat 0-brane (no spatial translation isometries).
speculations have been made in the literature to overcome this shortcoming. First, in forming macroscopic black holes one could argue that the starting point is a non-extreme black hole, which will then evaporate until it becomes extremal. In this process it can also lose angular momentum and reaches the minimal mass as a static supersymmetric black hole, see e.g. [34]. Other arguments suggests that it could be the wrong way in giving a black hole (or general any kind of 0-branes) angular momentum. Alternatively, the angular momentum could come from fermionic hairs [35]. Or, one could construct a rotating black hole solution, which asymptotically does not differ, but has at the core a much milder singularity and in addition has a Regge-bounded angular momentum [36].

One can also speculate about a different point of view. As we will discuss later there is a limit in which the ring singularity could be hidden, namely in the massless case. In this limit an additional singularity appears at a finite radius - where in the internal space an expected topology change takes place. By doing this one avoids the naked singularity, but obtains a massless black hole singularity which is not well understood either. On the other hand this opens the interpretation that rotating black holes can consistently exist only as massless gauge bosons. So far this point of view is rather speculative and we will leave it for further investigations.

4.5 Taub-NUT and Eguchi-Hanson instantons

As a next example we will discuss the case $\alpha = \beta = \gamma = 0$, but $Q_m \neq 0$, which includes Taub-NUT spaces and Eguchi-Hanson instantons. We take for the harmonic functions

\[
\tilde{H}^0 = \tilde{h}^0 + \frac{m^0}{r}, \quad \tilde{H}^A = \tilde{h}^A + \frac{m^A}{r}, \quad (82)
\]

\[
H_0 = h_0 + \frac{n_0}{r}, \quad H_A = h_A + \frac{n_A}{r},
\]

where $r$ is now the standard 3-d radius. In this case $\omega_m$ is defined by

\[
\frac{1}{r^2 \sin \theta} \partial_\theta \omega_\phi = - \frac{(h_I m^I - \tilde{h}^I n_I)}{r^2}, \quad (83)
\]

As solution one finds

\[
\omega_\phi = 2l \cos \theta, \quad l = \frac{1}{2} (h_I m^I - \tilde{h}^I n_I), \quad (84)
\]

where $l$ as Taub-NUT charge. Also, in this case the integrability constraint (12) is identically fulfilled.

This solution is consistently defined only after removing the Dirac-singularity, which appears at $\theta = \pi$ or 0. As usual, one has to introduce different coordinate patches for the north and south hemisphere without a singularity. To do this consistently one has to assume that the time is periodic ($t \simeq t + 8\pi n$). Also, since $\omega_m$ does not vanish at infinity,
this solution is not asymptotically flat. As Euclidean solutions the Taub-NUT space is one example for a gravitational instanton.

To find further solutions, we generalize the harmonic functions to the multi-center case

\[ \tilde{H}^0 = \tilde{h}^0 + m^0 \sum_i^k \frac{1}{r_i}, \quad \tilde{H}^A = \tilde{h}^A + m^A \sum_i^k \frac{1}{r_i}, \]

\[ H_0 = h_0 + n_0 \sum_i^k \frac{1}{r_i}, \quad H_A = h_A + n_A \sum_i^k \frac{1}{r_i} \]

(85)

with \( r_i^2 = |\vec{x} - \vec{x}_i|^2 \). We have taken the same charge for every center in order to satisfy the integrability constraint (82). This is a generalization of the \((k - 1)\)-multi-instanton configuration; for non-vanishing \( h' \)'s related to multicenter Taub-NUT and for vanishing \( h' \)'s to Eguchi-Hanson instantons. As for the rotating case, this solution can be seen as a bound state where every constituent is represented by one harmonic function. One has however to keep in mind that these solutions here are generalisations in the sense, that they couple to further scalars as well as gauge fields. In the standard form, these instantons have a self-dual curvature tensor [37] and for vanishing constant part \((h' \)'s) it is flat space time for \( k = 1 \), for \( k = 2 \) it is an Eguchi-Hanson instanton and in general it is a \((k - 1)\)-multi instanton configuration (see also [38]).

4.6 Static limit and quantum corrections

Finally we want to discuss the static limit, which is defined by \( Q_m = \omega_m = 0 \). Going back to the rotating black hole solution this limit can be obtained by setting \( \alpha = 0 \), i.e. vanishing angular momentum. The four harmonic functions are

\[ H_0 = h_0 + \frac{q_0}{r}, \quad \tilde{H}^A = h^A + \frac{p^A}{r} \]

(86)

with \( A = 1, 2, 3 \) and the metric is given by

\[ ds^2 = -e^{2U} dt^2 + e^{-2U} dx^m dx^m, \quad e^{-2U} = \sqrt{-4H_0\tilde{H}^1\tilde{H}^2\tilde{H}^3} \]

(87)

the scalar fields are

\[ z^A = -i \frac{2H_0\tilde{H}^A}{\sqrt{-4H_0H^1H^2H^3}}, \quad S = -i z^1, \quad T = -i z^2, \quad U = -i z^3 \]

(88)

and the gauge fields are defined in [27]. Again one harmonic function has to be negative definite, e.g. \( H_0 < 0 \ \forall r \). We see, that all axionic parts in the scalar fields vanished in the static limit, which is a consequence of the simple choice for the harmonic functions and has nothing to do with static limit. The axion-free solutions are especially interesting since they allow a clear brane interpretation. The role of the axions in the brane picture is not yet completely understood. Perhaps they are important for an understanding of
tilted branes \[39\]. On the type IIA side the model at hand has the interpretation of an intersection of three 4-branes and a 0-brane, or in the \(M\)-brane picture, it is an intersection of three 5-branes with a boost along the common string. The 11-d solution is given by

\[
\begin{align*}
\text{ds}_{11d}^2 &= \frac{1}{(H^1H^2H^3)^{1/3}} \left[ du dv + H_0 du^2 + H^A d\chi^A + H^1 H^2 H^3 d\vec{x} d\vec{x} \right], \\
F &= * \left( e^{ABC} d(1/H^A) \wedge d\chi^B \wedge d\chi^C \wedge du \wedge dv \right)
\end{align*}
\]

where \(\chi^A\) are three 2-d line elements and \(u, v = z \pm t\) where \(z\) is the common string direction. This configuration is compactified first to \(D = 5\) by wrapping the 5-branes around 4-cycles of the torus. As consequence we get the magnetic string, which, when wrapped around the 5th direction, yields the black hole solution \(87\). Equivalently, after compactification one can see that the 4-d black hole as consisting of 4 “elementary black holes”, the so-called \(a = \sqrt{3}\) black holes \[41\].

The mass of this solution is (using the relation \(71\))

\[
M = -q_0 h^1 h^2 h^3 + \frac{p^A d_{ABC} h^B h^C}{8 h^1 h^2 h^3}
\]

which coincides with the mass of the rotating black hole \(78\), note \(q_0 < 0\). The \(h^i\)s define the vev’s of the scalar fields and for certain values the black hole mass becomes extremal (minimal). This defines the double extreme limit \[7\]. Calculating \(\partial/\partial h^A M = 0\) and using the ansatz

\[
h_0 = \frac{q_0}{c}, \quad h^A = \frac{p^A}{c}
\]

one finds

\[
c^4 = -4 q_0 p^1 p^2 p^3.
\]

This gives the minimal mass as long as \(q_0 < 0, p^A > 0\). Plugging these values into the scalar fields \(88\) the space-time dependence drops out completely, they are constant and given only by the conserved charges. In this case the mass coincides with the Bekenstein-Hawking entropy \[42\] that measures the area of the horizon at \(r = 0\)

\[
S = \frac{A}{4} = \pi \left( r^2 e^{-2U} \right)_{r=0}.
\]

On the other hand, allowing different signs between the \(h^i\)s and the charges one finds massless black holes \[13\]. A simple example is given by \[14\]

\[
e^{-2U} = \sqrt{(1 + \frac{q}{r})(1 - \frac{q}{r})(1 + \frac{p}{r})(1 - \frac{p}{r})}
\]

i.e. we took \(p^1 = -q_0 = q, p^3 = -p^2 = p\) and all \(h^i\)s are trivial. Since there is no \(1/r\) term, the mass vanish identical. However, as consequence, additional singularities appear at \(r = p, q\). From the brane picture these negative charges are defining anti-branes, or
after compactification, the bound states contains constituents with negative mass. As, e.g., recently discussed in [43], we have to expect that these singular points signal a topology change (see also below).

These singularities create a gravitational potential that is repulsive to all matter (“anti-gravity”) [46]. This is a welcomed feature because, as consequence, this massless black hole at rest is not stable. Instead any further matter will accelerate it until the speed of light is reached [17]. Also, as speculated earlier, the potential barrier could hide the naked singularity of rotating black holes, with the consequence that rotating supersymmetric black holes (or 0-branes) have to be massless and cannot be at rest. We have, however, to admit that these singular points are not yet well understood in the black hole picture.

At these points a Kähler class modulus (88) vanishes indicating a vanishing 4-cycle. In our example $z^1$ and $z^3$ vanish at $r = q$ or $r = p$ resp. and near these points our classical solution breaks down, it cannot be trusted anymore. It is well-defined as long as all scalar fields $z^A$ are large enough, which is equivalent to the requirement $|H_0| \gg H^A \gg 1, \forall r$. The first inequality ensures that all 4-cycles have everywhere large radii whereas the last one ensures, that $\alpha'$ corrections (e.g. $R^2$ terms) can be neglected. Since near these singular points the solution is describable by a conformal exact model [48], [43] it is unlikely that these singularities can be cured by $\alpha'$ corrections (higher derivative terms). We have however to expect that near a vanishing 4-cycle quantum corrections on the heterotic side or worldsheet instanton corrections on the type II side becomes important. Let us discuss the structure of these corrections on the type II side.

On the type II side (heterotic) quantum corrections are encoded in the topological structure of the Calabi-Yau (CY) threefold upon which we have to compactify. The complete prepotential is given by [18]

$$F(X^A) = (X^0)^2 \left[ -\frac{1}{6} C_{ABC} z^A z^B z^C \right] - \frac{i\chi(3)}{2(2\pi)^3} + \frac{i}{(2\pi)^3} \sum_{d_1, \ldots, d_n} n_{d_1, \ldots, d_n} \text{Li}_3(e^{2\pi i d_4 z^4}) \right]. \quad (95)$$

Here $C_{ABC}$ are the classical intersection numbers, $\chi$ is the Euler number of the CY manifold, $n_{d_1, \ldots, d_n}$ are the numbers of genus zero rational curves (instanton numbers) and $d_4$ are the degrees of these curves. These numbers are determined by the CY-threefold of the compactified effective Type II string theory. For this prepotential we have now to solve the equations [2]. In general this is hopeless. But we are mainly interested in corrections around a vanishing 4-cycle, say $z^3$.

Again, in solving the algebraic constraint (2) we can adopt results for the double extreme case and find as solution for the symplectic section [11]

$$X^0 = \frac{\lambda}{2}, \quad X^A = -i \frac{H^A}{2} \quad \text{and} \quad z^A = -i \frac{H^A}{\lambda}. \quad (96)$$

The parameter $\lambda$ is fixed by the equation $F_0 - \bar{F}_0 = -i H_0$ and expanding the solution
around \( z^3 \approx H^3 \approx 0 \) one finds

\[
\lambda = \pm \sqrt{-\frac{1}{6}C_{ABC}H^AH^BH^C}{H_0} + \sum_{d_3} n_{d_3}^r \frac{(d_3H^3)^2}{8\pi H_0} \pm \ldots \quad (97)
\]

Inserting this into \( e^{-2U} \) yields

\[
e^{-2U} = \sqrt{-H_0 \frac{1}{6}C_{ABC}H^AH^BH^C - \frac{1}{8} \sum_{d_3} n_{d_3}^r (d_3H^3)^2 \log \left( \frac{(d_3H^3)^2H_0}{-\frac{1}{6}C_{ABC}H^AH^BH^C} \right) \pm \ldots \quad (98)
\]

The black hole (87) includes now all perturbative and non-perturbative corrections in the neighbourhood of a vanishing 4-cycle. Following the procedure described in [11] one could easily calculate further corrections. But the general structure is the same, the black hole receives polynomial and logarithmic corrections. Also in the entropy, the instanton corrections yield additional logarithmic corrections. For a microscopic discussion see also the second ref. of [11].

If the Kähler potential behaves smooth at this point \( (z^3 \approx 0) \), the corrections regularize the metric. But the system can undergo a phase transition, if derivatives of the Kähler potential, like the Kähler metric or gauge couplings, behave non-smoothly. We have to expect different types of phase transitions, mild forms like the flop transition as recently discussed in [49], but also drastic ones like conifold transitions. Physically, at these points an internal cycle of the CY vanishes and “beyond” this point a new, topologically different cycle emerges.

We will leave further discussions of these interesting phenomena for future investigations. Especially, it would be very interesting to understand in our framework the connection to the massless black holes described in the context of the resolution of conifold singularities [50].

5 Conclusions

The stabilization equations for the double extreme black holes has been a very fruitful framework in addressing quantum corrections for supersymmetric black holes. They allow to calculate the entropy for all black holes with non-singular horizons and describe black hole solutions that have been stabilized at their minimal mass.

In this paper we generalized this framework to incorporate general solutions of \( N = 2 \) supergravity, which include not only black holes (non-singular and singular ones) but also Taub-NUT and Eguchi-Hanson instantons.

Our starting point was a general Ansatz for a stationary metric, which should be completely determined by duality invariant quantities, namely by the Kähler potential and the \( U(1) \) Kähler connection (see section 3.1). Next, we showed that solutions of our constraint
equations (2) define supersymmetric configurations (section 3.2 and 3.3). Note, although these configurations are supersymmetric (vanishing gravitino and gaugino variation), they do not solve the equations of motion. They are off-shell supersymmetric configurations defined by arbitrary functions \((\tilde{H}^I, H_I)\). However, the gauge field equations and Bianchi identities are fulfilled only, if the section \((\tilde{H}^I, H_I)\) consists of harmonic functions. As a result, we get an on-shell supersymmetric solution. Since the supersymmetry variations are only first order differential equations, the harmonic property of the functions can never be important. Therefore, equations (2) can be seen as defining a consistent supersymmetric truncation to a bosonic theory, i.e. vanishing gravitino and gaugino variations. As usual, this truncation breaks one half of supersymmetry.

These equations have also a nice physical meaning, they make the interplay between the internal and external space obvious. Whenever, one approaches a singular/special point in space time one moves in the internal space to singular/special points too and vice versa. One cannot separate the internal and external space. It is the gravity, that unifies both parts.

In a second part, we discussed explicit solutions, starting with the pure supergravity case. Different types of solutions are related to different choices of harmonic functions. We discussed some examples for the \(STU\) model, like rotating black holes, Taub-NUT and Eguchi-Hanson instantons and generalization for the Reissner-Nordstrøm black hole.

Since our approach is not restricted to special types of solutions, it can be used to investigate singularities. For a static black hole we have discussed an example. The classical solution becomes singular at points where a 4-cycle vanishes, around which a 5-branes is wrapped. However, if one takes into account all perturbative and non-perturbative corrections and expand the solution around this point, one finds that the black hole solution behaves non-singular as long as the Kähler potential remains non-singular. In addition to polynomial corrections, the black hole receives logarithmic corrections. Although, for a wide range of models the Kähler potential stays non-singular, their derivatives, like Kähler metric or gauge couplings, will not be smooth indicating a phase transition.

Of course, all examples can only be seen as a starting point for further investigations. The framework described in the paper could provide a solid basis for addressing questions related to singular points of the Calabi-Yau, e.g. different types of phase transitions. For the rotating black holes one could ask, whether quantum corrections could remove/hide the naked singularity in the supersymmetric limit.
Appendix: conventions and notation

For the metric we are using the signature \((-++,++)\) and for the indices we take: \(\mu, \nu, = 0, 1, 2, 3\), whereas \(m, n, \cdots = 1, 2, 3\). Where confusions can arise, we will denote the underlined indices \(\underline{\mu}, \underline{\nu}\) as curved ones and \(a, b, \cdots = 0, 1, 2\) as flat ones. Antisymmetrized indices are defined by: \([ab] = \frac{1}{2}(ab - ba)\).

The spin connections is

\[
\omega_{\mu}^{ab} = 2e^{v[a}\partial_{[\mu}e^{b]}_{v]} - e^{\rho}_{a}e^{\sigma b}e_{\rho p}\partial_{[\sigma}e^{p]}_{[\rho]} \cdot \tag{99}
\]

We define (anti) selfdual components like

\[
F_{ab}^{\pm} = \frac{1}{2}(F_{ab} \pm i*F_{ab}) \tag{100}
\]

with

\[
*F_{ab} = \frac{1}{2}\varepsilon^{abcd}F_{cd} \tag{101}
\]

and \(\varepsilon^{0123} = 1 = -\varepsilon_{0123}\) (\(**F = -F\)).

For \(\gamma\) matrices we are using the relation

\[
\gamma^a \gamma^b = -\eta^{ab} + \frac{i}{2}\gamma_5 \varepsilon^{abcd} \gamma^c \gamma^d \tag{102}
\]

with \(\gamma_5 = -i\gamma_0 \gamma_1 \gamma_2 \gamma_3\) (\(\gamma_5^2 = 1\)). Using these definitions we find for any antiselfdual tensor the identity

\[
T^{-ab}\gamma_a \gamma_b = 2(1 - \gamma_5)T^{-0m}\gamma_0 \gamma_m \tag{103}
\]

and

\[
T^{-mn} = -i\varepsilon_{mnp}T^{-0p} \cdot \tag{104}
\]

For the chiral spinors (gaugino and gravitino) we have

\[
\gamma_5 \left( \begin{array}{c} \lambda^\alpha \\ \psi_\alpha \end{array} \right) = - \left( \begin{array}{c} \lambda^\alpha \\ \psi_\alpha \end{array} \right) , \quad \gamma_5 \left( \begin{array}{c} \lambda_\alpha \\ \psi^\alpha \end{array} \right) = \left( \begin{array}{c} \lambda_\alpha \\ \psi^\alpha \end{array} \right) \cdot \tag{105}
\]

Note, that our conventions slightly differ from the ones used in \[51\]. Our signature and \(\varepsilon\)-tensor definition has the consequence that also the spinor chiralities are opposite.

Acknowledgements

We would like to thank I. Gaida, S. Mahapatra, A. Van Proeyen, and especially T. Mohaupt for interesting discussions. The work is supported by the Deutsche Forschungsgemeinschaft (DFG) and by the European Commision TMR programme ERBFMRX-CT96-0045. W.A.S is partially supported by DESY-Zeuthen.
References

[1] N. Seiberg and E. Witten, *Nucl. Phys.* B426 (1994) 19, [hep-th/9407087](https://arxiv.org/abs/19); *Nucl. Phys.* B431 (1994) 484, [hep-th/9408099](https://arxiv.org/abs/9408099).

[2] S. Kachru and C. Vafa, *Nucl. Phys.* B450 (1995) 69, [hep-th/9505103](https://arxiv.org/abs/9505103); S. Ferrara, J.A. Harvey, A. Strominger, C. Vafa, *Phys. Lett.* B361 (1995) 59, [hep-th/9505162](https://arxiv.org/abs/9505162).

[3] V. Kaplunovsky, J. Louis and S. Theisen, *Phys. Lett.* B357 (1995) 71, [hep-th/9506110](https://arxiv.org/abs/9506110); A. Klemm, W. Lerche and P. Mayr, *Phys. Lett.* B357 (1997) 313; [hep-th/9506112](https://arxiv.org/abs/9506112); G. Curio, *Phys. Lett.* B366 (1996) 131, [hep-th/9509042](https://arxiv.org/abs/9509042); G. L. Cardoso, G. Curio, D. Lüst and T. Mohaupt, *Phys. Lett.* B382 (1996) 241, [hep-th/9603108](https://arxiv.org/abs/9603108); G. L. Cardoso, G. Curio and D. Lüst, *Perturbative couplings and Modular forms in N = 2 string models with a Wilson line*, to appear in *Nucl. Phys. B*; I. Antoniadis, E. Gava, K.S. Narain, T.R. Taylor, *Nucl. Phys.* B455 (1995) 109, [hep-th/9507113](https://arxiv.org/abs/9507113).

[4] D. Kastor and K. Z. Win, *Non-extreme Calabi-Yau Black holes*, [hep-th/9705090](https://arxiv.org/abs/9705090).

[5] S. Ferrara, R. Kallosh and A. Strominger, *Phys. Rev.* D52 (1995) 5412, [hep-th/9508072](https://arxiv.org/abs/9508072).

[6] S. Ferrara and R. Kallosh, *Phys. Rev.* D54 (1996) 1514, [hep-th/9602136](https://arxiv.org/abs/9602136); S. Ferrara, R. Kallosh *Phys. Rev.* D54 (1996) 1525, [hep-th/9603090](https://arxiv.org/abs/9603090).

[7] R. Kallosh, M. Shmakova and W.K. Wong, *Phys. Rev.* D54 (1996) 6284; [hep-th/9607077](https://arxiv.org/abs/9607077).

[8] S.-J. Rey, *Classical and quantum aspects of BPS black holes in N = 2, D = 4 heterotic string compactifications*, [hep-th/9610157](https://arxiv.org/abs/9610157); S. Ferrara, G.W. Gibbons, R. Kallosh, *Black holes and critical points in moduli space*, [hep-th/9702103](https://arxiv.org/abs/9702103).

[9] K. Behrndt, R. Kallosh, J. Rahmfeld, M. Shmakova and W.K. Wong, *Phys. Rev.* D54 (1996) 6293, [hep-th/9608059](https://arxiv.org/abs/9608059); G. Lopes Cardoso, D. Lüst and T. Mohaupt, *Phys. Lett.* B388 (1996) 266, [hep-th/9608099](https://arxiv.org/abs/9608099).

[10] K. Behrndt, G.L. Cardoso, B. de Wit, R. Kallosh, D. Lüst and T. Mohaupt, *Nucl. Phys.* B488 (1997) 236, [hep-th/9610105](https://arxiv.org/abs/9610105).

[11] K. Behrndt and I. Gaida, *Subleading contributions from instanton corrections in N=2 supersymmetric black hole entropy*, [hep-th/9702168](https://arxiv.org/abs/9702168); K. Behrndt, G. Lopes Cardoso and I. Gaida, *Quantum N = 2 supersymmetric black holes in the S–T model*, [hep-th/9704093](https://arxiv.org/abs/9704093); I. Gaida, *Gauge symmetry enhancement and N = 2 supersymmetric quantum black holes in heterotic string vacua*, [hep-th/9705150](https://arxiv.org/abs/9705150).

[12] K. Behrndt, *Phys. Lett.* B396 (1997) 77, [hep-th/9610232](https://arxiv.org/abs/9610232).
[13] W.A. Sabra, *Classical entropy of N = 2 black holes: the minimal coupling case*, to appear in *Mod. Phys. Lett. A* (1997), [hep-th/9611210]. K. Behrndt and W. A. Sabra, *Static N = 2 black holes for quadratic prepotentials*, [hep-th/9702010], to appear in *Phys. Lett. B* (1997).

[14] W. A. Sabra, *General static N = 2 black holes*, [hep-th/9703101]. W. A. Sabra, *Black holes in N = 2 supergravity and harmonic functions*, [hep-th/9704147].

[15] K. Behrndt and T. Mohaupt, *Entropy of N = 2 black holes and their M-brane description*, [hep-th/9611140].

[16] E. Kiritsis, C. Kounnas and D. Lüst, *Int. J. Mod. Phys. A* 9 (1994) 1361; G. L. Cardoso and D. Lüst, *Phys. Lett.* B345 (1995) 220.

[17] B. de Wit and A. Van Proeyen, *Nucl. Phys. B245* (1984) 89; E. Cremmer, C. Kounnas, A. Van Proeyen, J. P. Derendinger, S. Ferrara, B. de Wit and L. Girardello, *Nucl. Phys. B250* (1985) 385; B. de Wit, P. G. Lauwers and A. Van Proeyen, *Nucl. Phys. B255* (1985) 569; S. Cecotti, S. Ferrara, and L. Girardello, *Int. J. Mod. Phys. A* 4 (1989) 2475.

[18] P. Candelas and X. de la Ossa, *Nucl. Phys. B355* (1991) 455; P. Candelas, X. de la Ossa, P. Green and L. Parkers, *Nucl. Phys. B359* (1991) 21.

[19] A. Strominger, *Commun. Math. Phys.* 133 (1990) 163.

[20] R. D’Auria, S. Ferrara and P. Frè, *Nucl. Phys. B359* (1991) 705.

[21] L. Castellani, R. D’Auria, and S. Ferrara, *Class. Quantum Grav.* 1 (1990) 317.

[22] A. Ceresole, R. D’Auria, S. Ferrara and A. van Proyen, *Nucl. Phys. B444* (1995) 92.

[23] S. Ferrara and A. Strominger, in Strings ’89, ed. R. Arnowitt, R. Bryan, M. J. Duff, D. Nanopulos and C. N. Pope, World Scientific, Singapore, (1990) 245.

[24] B. Craps, F. Roose, W. Troost and A. Van Proeyen, *What is Special Kähler Geometry?*, [hep-th/9703082].

[25] M. K. Gaillard and B. Zumino, *Nucl. Phys. B193* (1981) 221.

[26] P. Frè and P. Soriani, *Nucl. Phys. B371* (1992) 659; W. A. Sabra, *Nucl. Phys. B486* (1997) 629.

[27] G. Neugebauer, Habilitationschrift, Jena (1969); Z. Perjés, *Phys. Rev. Lett.* 27 (1971) 1668; W. Israel and G.A. Wilson, *J. Math. Phys.* 13 (1972) 865.

[28] K. P. Tod, *Phys. Lett. B121* (1983) 241; *Class. Quantum Grav.* 12 (1995) 1801.
[29] G.W. Gibbons, in: Supersymmetry, Supergravity and Related Topics, World Scientific, Singapore 1985);

[30] R. Kallosh and J. Kumar, Supersymmetry enhancement of D-p-branes and M-branes, hep-th/9704189.

[31] E. Bergshoeff, R. Kallosh and T. Ortín, Nucl. Phys. B478 (1996) 156, hep-th/9605059.

[32] J. B. Hartle and S. W. Hawking, Commun. Math. Phys. 26 (1972) 87.

[33] M.J. Duff, James T. Liu and J. Rahmfeld, Dipole moments of black holes and string states, hep-th/9612013.

[34] M. Cvetic and I. Gaida, Duality invariant nonextreme black holes in toroidally compactified string theory, hep-th/9703134.

[35] M. J. Duff and J. Rahmfeld, Nucl. Phys. B481 (1996) 332, hep-th/9605085.

[36] A. Dabholkar, J. P. Gauntlett, J.A. Harvey, D. Waldram, Nucl. Phys. B474 (1996) 85, hep-th/9511053.

[37] G.W. Gibbons and S. Hawking, Phys. Lett. B078 (1978) 430.

[38] T. Eguchi, P.B. Gilkey and A.J. Hanson, Phys. Rep. 66 (1980) 213.

[39] K. Behrndt and M. Cvetic, BPS saturated bound states of tilted p branes in type II string theory, hep-th/9702205.

[40] A. A. Tseytlin, Nucl. Phys. B475 (1996) 149, hep-th/9604035, G. Papadopoulos and P. K. Townsend, Phys. Lett. B30 (1996) 273, hep-th/9603087.

[41] J. Rahmfeld, Phys. Lett. B372 (1996) 198, hep-th/9512089.

[42] J. Bekenstein, Lett. Nuov. Cimento 4 (1972) 737, Phys. Rev. D7 (1973) 2333; Phys. Rev. D9 (1974) 3292; S. Hawking, Nature 248 (1974) 30, Comm. Math. Phys. 43 1975.

[43] K. Behrndt, Nucl. Phys. B455 (1995) 188., hep-th/9506106.

[44] M. Cvetic and D. Youm, Phys. Lett. B359 (1995) 87, hep-th/9507160; K.-L. Chan and M. Cvetic, Phys. Lett. B375 (1996) 98, hep-th/9512188.

[45] R. Mann, Black holes of negative mass, gr-qc/9705007.

[46] R. Kallosh and A. Linde, Phys. Rev. D52 (1995) 7137, hep-th/9505997.

[47] T., Ortín, Phys. Rev. Lett. 76 (1996) 3890 , hep-th/9602067.
[48] G.T. Horowitz and A.A. Tseytlin, *Phys. Rev.* **D51** (1995) 2896, hep-th/9409021.

[49] A. Chou, R. Kallosh, J. Rahmfeld, S.-J. Rey, M. Shmakova and W. K. Wong, *Critical points and phase transitions in 5-d compactification of M-theory*, hep-th/9704143.

[50] A. Strominger, *Nucl. Phys.* **B451** (1995) 96, hep-th/9504090. B. R. Greene, D. R. Morrison and A. Strominger, *Nucl. Phys.* **B451** (1995) 109, hep-th/9504145.

[51] L. Andrianopoli, M. Bertolini, A. Ceresole, R. D’ Auri, S. Ferrara, P. Fré and T. Magri, *N = 2 Supergravity and N = 2 super Yang-Mills theory on general scalar manifold*, hep-th/9605032.