Study on Stress-strain Constitutive Relationship of Super-long-age Plastic Concrete under Triaxial Compression

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Abstract. The triaxial stress-strain relationship of super long-aged (540d) plastic concrete mixed with clay and bentonite was studied. According to the test, the stress-strain curves of plastic concrete under different confining pressures are obtained. It is found that the stress-strain curves under the lateral confining pressure are basically the same (0.4 MPa, 0.6 MPa), (0.4 MPa, 0.8 MPa), and the slope of the rising section is basically the same. Compared with the peak pressure ratio (0.2 MPa, 0.4 MPa), the lateral confining pressure is larger. In addition, the calculation formula of the peak secant elastic modulus and the triaxial compressive stress-strain relationship in accordance with the lateral pressure of plastic concrete are also proposed. The fourth-order polynomial mathematical model. The establishment of the mathematical model provides a basis for the development of plastic concrete specifications and triaxial compression performance numerical analysis, and further promotes the engineering application of plastic concrete.

1. Introduction
Plastic concrete is made by adding materials such as clay, bentonite, fly ash and other admixtures to concrete to enhance plasticity and reduce strength. Because of its good impermeability and deformation performance, in recent years, water conservancy and damming and Engineering and high-rise building foundation anti-seepage projects have been widely used [1–2]. In engineering applications, plastic concrete is usually in a three-dimensional compression state, and the principal stress values in three directions are different ($\sigma_1 \neq \sigma_2 \neq \sigma_3$). Due to the complex nature of the plastic concrete constituent materials, it is more realistic and reasonable to directly determine the multiaxial mechanical properties of the plastic concrete by the test method [3]. Therefore, it is necessary to carry out a triaxial test that reflects the true mechanical properties of concrete.

At present, most domestic scholars have carried out ordinary concrete triaxial tests [4–8], and research on plastic concrete is relatively rare. For plastic concrete, foreign expert Ahmad Mahboubi et al. conducted a triaxial compression test on plastic concrete. The study shows that the shear strength and cohesion of plastic concrete have a positive correlation with age, and the internal friction angle is opposite [9]; Nicholas et al. carried out an experimental study on the carbon fiber reinforced plastic concrete panels and established I-type fatigue model [10]; Hinchberger et al. Carried out experimental research and theoretical analysis on the basic mechanical properties of plastic concrete, the causes of crack formation, the calculation of crack depth, and the prevention of crack propagation [11-15]; And the International Dam Conference suggested that the elastic modulus of the plastic concrete should be
1–5 times of the elastic modulus of the surrounding soil [16]. The domestic Tsinghua University scholar Deng Mingji combined with the Dongping Lake anti-seepage project to study the uniaxial compression mechanical properties of clay plastic concrete and bentonite plastic concrete [17]; Domestic scholars Shi Yan and others studied the mechanical and triaxial stress-strain relationship of plastic concrete with low liquid limit clay, and found that the strength of plastic concrete decreases with the increase of clay content [18]; Song Shuaiqi et al. studied The mechanical and impermeability properties of ash-plastic concrete in cement kiln [19]; Liu Lulu et al. studied the stress-strain characteristics of plastic concrete under cyclic loading and unloading [20].

In general, there are few true triaxial compression tests for plastic concrete. In this paper, the constitutive relationship of super-long-age plastic concrete mixed with clay and bentonite under triaxial compression is studied, and the triaxial compression constitutive relation is established. The fourth-order polynomial mathematical model has important theoretical significance and practical application value for the study of plastic concrete.

2. Experimental Procedure

2.1. Test raw Materials

The experiment uses 42.5 # ordinary portland cement, calcium bentonite produced in Pingqiao Henan Xinyang, silty clay in the lake area of Longzi lake in Zhendong new district, Zhengzhou city. The silty clay is retrieved and dried and ground to powder at a standard of 350 particle size. All the above indicators are in accordance with Ordinary Portland Cement (GB175-2007) [21]. The coarse aggregate used in the test is crushed stone with particle size of 5 ~ 20mm. The fine aggregate is natural river sand, and the fineness modulus is 2.7. It belongs to medium sand and the grading curve is in the second zone. The aggregates meet the requirements of Sand for Construction (GB/T14684 - 2011)[22] and Specifications for Hydraulic Concrete Construction (DL/T5144 - 2001)[23] standards.

2.2. Design Scheme of Mix Proportions of Plastic Concrete

In this experiment, clay and bentonite were used to replace part of the cement, and clay-bentonite-cement plastic concrete was made. The mass ratio method was used to design the mixture ratio. The sand ratio, water-to-binder ratio and unit water consumption were the three clear parameters in this test. The sand ratio is the ratio of the mass between sand and the total sand; the water-to-binder ratio is an indicator reflecting the ratio of the quality of water to cementitious materials (cement, bentonite and clay); the unit water consumption is the quality of the cement slurry and aggregate. After the trial distribution, the specific mix design was determined, as shown in Table 1.

| Number | Water binder ratio | Sand ratio | Water /kg | Cement content / kg | Clay / kg | Bentonite /kg | Sand /kg | Cobblestone /kg |
|--------|-------------------|------------|-----------|---------------------|-----------|---------------|----------|----------------|
| WB075  | 0.75              | 0.50       | 307.50    | 150                 | 180       | 80            | 666.25   | 666.25         |
| WB087  | 0.87              | 0.50       | 356.70    | 150                 | 180       | 80            | 614.65   | 641.65         |
| WB100  | 1.00              | 0.50       | 400       | 150                 | 180       | 70            | 625      | 625            |
| S04    | 1.00              | 0.40       | 370       | 120                 | 180       | 70            | 524      | 786            |
| S05    | 1.00              | 0.50       | 370       | 120                 | 180       | 70            | 655      | 655            |
| S06    | 1.00              | 0.60       | 370       | 120                 | 180       | 70            | 786      | 524            |
| CL180  | 0.87              | 0.50       | 322       | 120                 | 180       | 70            | 655      | 655            |
| CL220  | 0.87              | 0.50       | 357       | 120                 | 220       | 70            | 641.5    | 641.5          |
| CL260  | 0.87              | 0.50       | 391       | 120                 | 260       | 70            | 604.5    | 604.5          |
| B40    | 0.87              | 0.50       | 296       | 120                 | 180       | 40            | 707      | 707            |
| B70    | 0.87              | 0.50       | 322       | 120                 | 180       | 70            | 655      | 655            |
| B100   | 0.87              | 0.50       | 348       | 120                 | 180       | 100           | 651      | 651            |

*WB: water binder ratio, S: sand ratio, CL: clay, B: Bentonite.
2.3. Experimental Method
According to the mixing ratio set in Table 1, 12 sets of 150mm × 150mm × 150mm plastic concrete cube specimens were prepared, and the specimens were cured 540d under standard curing conditions after pouring. This test uses a LY-C tensile true triaxial test for triaxial compression test. The test scheme is a triaxial compression test with a side pressure fixed. The triaxial compressive failure test is to apply a three-way pressure. The direction of $a_1$ and $a_2$ is increased to a predetermined confining pressure value, and the pressure is continuously applied to the concrete test in the direction of $a_3$. Piece of damage, triaxial compressive strength is the peak of the stress-strain curve in the $a_3$ direction. In this test, combined with engineering experience, three confining pressure conditions were established, namely (1) $a_1 = 0.2$MPa, $a_2 = 0.4$MPa; (2) $a_1 = 0.4$MPa, $a_2 = 0.6$MPa; (3) $a_1 = 0.4$MPa, $a_2 = 0.8$MPa. Let q = 1.0 MPa, simplified to (0.2q, 0.4q), (0.4q, 0.6q), (0.4q, 0.8q).

3. Test Results and Analysis

3.1. Triaxial Compressive Strength and Stress-strain Curve Characteristics of Plastic Concrete
The results of the triaxial compression test of the test piece and the typical stress-strain curves under different confining pressures are shown in Table 2 and Figure 1.

| Number | triaxial compressive strength / MPa  | (0.2q, 0.4q) | (0.4q, 0.6q) | (0.4q, 0.8q) |
|--------|------------------------------------|--------------|--------------|--------------|
| WB075  | 12.18                              | 13.73        | 13.50        |
| WB087  | 10.41                              | 10.46        | 10.85        |
| WB100  | 8.97                               | 8.79         | 8.72         |
| S04    | 7.38                               | 7.49         | 7.54         |
| S05    | 6.07                               | 7.32         | 7.71         |
| S06    | 7.90                               | 8.43         | 8.54         |
| CL180  | 8.63                               | 10.16        | 10.57        |
| CL220  | 9.04                               | 9.82         | 10.40        |
| CL260  | 6.50                               | 7.59         | 6.93         |
| B40    | 9.93                               | 11.37        | 8.87         |
| B70    | 8.63                               | 10.16        | 10.57        |
| B100   | 8.25                               | 9.54         | 10.07        |

Figure 1. Stress-strain curves of plastic concrete under triaxial compression with constant lateral pressure
It can be seen from Table 2 that the plastic concrete triaxial compressive strengths with different mixing ratios are relatively different, and different confining pressure conditions have little effect on the concrete strength. It can be seen from Figure 1 that the three-axis compressive stress-strain curve under the condition of constant lateral pressure can be divided into two sections, namely the curve rising section and the horizontal section. When the three-way pressure is applied, the deformation of the specimen increases rapidly. However, as the pressure continues to increase, the principal stress gradually increases, and the lateral compressive stress constrains the lateral deformation of the plastic concrete. The generation and extension of cracks are delayed by the lateral constraint, the growth of the main strain becomes relatively slow, and the slope of the stress-strain curve increases. When the principal stress gradually increases to a large stress value, the plasticity of the plastic concrete specimen at this time the deformation begins to develop gradually, the strain increases rapidly, and the stress growth is slow. The corresponding stress-strain curve rises gently and the slope gradually decreases. When the stress-strain curve reaches the peak point, the test piece reaches the triaxial compressive strength and stress. It remains basically stable, the strain continues to increase, the stress-strain curve enters the horizontal section, and the principal stress $\sigma_2$ maintains the triaxial compressive strength.

3.2. Mathematical Model of Constitutive Relation of Plastic Concrete under Triaxial Compression
Plastic concrete is a nonlinear system composed of a variety of materials. If the linear elastic constitutive relation is used to describe the stress-strain relationship of plastic concrete, there is a large error. The mathematical model of the non-elastic constitutive equation is complex and involves many parameters. The process of derivation is quite cumbersome and is not conducive to popularization. Therefore, this paper proposes a mathematical model of the fourth-order polynomial form that conforms to the characteristics of the triaxial compression stress-strain curve of plastic concrete compression stress-strain curve is proposed.

3.2.1 The Relationship between the Peak Secant Modulus and the Peak Stress. It has been found that the elastic modulus of concrete increases with the increase of strength, showing a positive correlation, but the dispersion is large. In this paper, by analyzing the results of plastic concrete triaxial compression test, the formula for calculating the peak secant modulus and peak stress of plastic concrete is obtained, see equation (1).

$$E_f = a_1 \sigma_{3f} - a_2$$  \hspace{1cm} (1)

where $a_1 = 32.075 \sigma_1 - 37.746 \frac{\sigma_1}{\sigma_0} + 55.522$, $a_2 = -333.45 \sigma_1 + 387.36 \frac{\sigma_1}{\sigma_0} - 253.19$, $E_f$ is the peak elastic modulus of plastic concrete under fixed lateral pressure, $\sigma_{3f}$ is the triaxial compressive strength of plastic concrete, $\sigma_1$, $\sigma_2$ are lateral compressive stress of plastic concrete.

According to the empirical equation of uniaxial compression modulus proposed by the American Standard (ACI318-08) and the former Soviet Union specification and the Chinese standard (GB50010-2002), the plastic concrete test data were fitted based on the stress-strain curve equation of concrete under uniaxial compression. The empirical equation for the elastic modulus of the peak secant of the plastic concrete under fixed lateral pressure is obtained as:

$$E_f = b_1 \sqrt{\sigma_{3f}} + b_2$$  \hspace{1cm} (2)

Where $b_1 = 302.7 \sigma_1 - 199.8 \frac{\sigma_1}{\sigma_0} + 283.69$, $b_2 = -947.25 \sigma_1 + 641.04 \frac{\sigma_1}{\sigma_0} - 600.03$.

$$E_f = \frac{10^8}{\varepsilon_{3f}}$$  \hspace{1cm} (3)
Where $\epsilon_1 = 17426.5a_1 - 10747.2\frac{\sigma_1}{\sigma_0} + 4817.2$, $c_2 = -1913.15a_1 + 910.32\frac{\varepsilon_1}{\sigma_0} + 40.52$

By comparing and analyzing the difference between the calculated value and the experimental value of formula (1), formula (2) and formula (3), obtain the total average, mean square error and coefficient of variation of the ratio of the calculated values of the formulas to the experimental values, see Table 3. It can be found from Table 3 that the total mean value of the calculated value and the experimental value obtained by the formula (3) is 0.9992, the error is 0.08%, and the errors of the formula (1) and the formula (2) are 6.04% and 6.24%, respectively. In addition, the mean square error and coefficient of variation of the formula (3) are also the smallest, which can perfectly reflect the relationship between the peak secant elastic modulus and the peak stress. It is recommended to use the formula (3) as the calculation formula of the peak secant modulus.

### Table 3. The comparison between calculated values of elastic modulus and experimental values

| Equation number | Ratio of calculated values of elastic modulus to experimental ones | Average value | Mean square deviation | Coefficient of variation |
|-----------------|-----------------------------------------------------------------|--------------|----------------------|-------------------------|
| (1)             |                                                                 | 1.0604       | 0.2787               | 0.2628                  |
| (2)             |                                                                 | 1.0624       | 0.2841               | 0.2675                  |
| (3)             |                                                                 | 0.9992       | 0.2437               | 0.2439                  |

3.2.2 Mathematical model of constitutive relation of plastic concrete under triaxial compression. The stress-strain curve of the plastic concrete is fitted by the theoretical mathematical model. It is found that the quartic polynomial model agrees well with the measured curve. Therefore, the quartic polynomial mathematical model of stress-strain relation of plastic concrete under triaxial compression is put forward in this paper. That is:

$$y = Ax^4 + Bx^3 + Cx^2 + Dx \ (x \leq 1)$$

(4)

$$y = 1 (x > 1)$$

(5)

Where vertical and horizontal coordinates are $y = \frac{\sigma_3}{\sigma_{3f}}$, $x = \frac{\varepsilon_3}{\varepsilon_{3f}}$, $\varepsilon_{3f}$ is the peak compressive strain corresponding to $\sigma_{3f}$.

When $\varepsilon_3 = \varepsilon_{3f}$, $\sigma_3 = \sigma_{3f}$, that is, $x = 1$, $y = 1$. Substituting $x = 1$ and $y = 1$ into Eq. (4) yields:

$$A + B + C + D = 1$$

(6)

Both sides of the Eq. (4) make a derivative of $\varepsilon_3$ then:

$$\frac{dy}{d\varepsilon_3} = \left(4Ax^3 + 3Bx^2 + 2Cx + D\right)\frac{dx}{d\varepsilon_3}$$

$$\frac{1}{\sigma_{3f} d\varepsilon_3} \left(4Ax^3 + 3Bx^2 + 2Cx + D\right) \frac{1}{\varepsilon_{3f}}$$

Substituting $\varepsilon_3 = 0$, that is, $x=0$ into the equation:

$$\frac{d\sigma_3}{d\varepsilon_3} \bigg|_{\varepsilon_3=0} = \frac{\sigma_{3f} D}{\varepsilon_{3f}}$$

(7)

Where $E_0'$ is initial tangential elastic modulus of plastic concrete under triaxial compression and $E_0' = \frac{d\sigma_3}{d\varepsilon_3} \bigg|_{\varepsilon_3=0}$. 


It can be seen from Eq. (7) that the model coefficients D are related to $E_{o}^{'}/E_{f}$. Similarly, there is a close relationship between the model coefficients A, B, C and $E_{o}^{'}/E_{f}$. The initial tangent elastic modulus $E_{o}'$ is difficult to measure in the statistical analysis, so the measured uniaxial elastic modulus $E_{o}$ is used instead of $E_{o}'$. $E_{o}$ is calculated using Eq. (3). The curve equation of each specimen can be obtained by fitting based on the measured stress-strain curves, resulting in 33 values of A, B, C, D. And then these values are fitted to get equations of A, B, C, D as follows:

$$A = -0.1756\left(\frac{E_{o}}{E_{f}}\right)^{2} + 1.6224\frac{E_{o}}{E_{f}} - 0.1773$$

$$B = 0.1583\left(\frac{E_{o}}{E_{f}}\right)^{2} - 1.0175\frac{E_{o}}{E_{f}} - 7.9733$$

$$C = 0.1237\left(\frac{E_{o}}{E_{f}}\right)^{2} - 1.8483\frac{E_{o}}{E_{f}} + 11.294$$

$$D = -0.1064\left(\frac{E_{o}}{E_{f}}\right)^{2} + 1.2452\frac{E_{o}}{E_{f}} - 3.0482$$

$E_{o}$ is the uniaxial compressive elastic modulus of plastic concrete with 90d age.

3.3. Verify the Fourth-order Polynomial Mathematical Model

In order to verify the reliability of the mathematical model established, the experimental measured curves of the test pieces of each group are compared with the theoretical model curves obtained by using equations (4) and (5), as shown in Fig. 2 to Fig. 12. From the comparison of the theoretical model curve and the experimental measured curve in Figure 2 to Figure 12, it can be seen that the theoretical model curve under the lateral confining pressure of (0.4q, 0.6q) almost coincides with the measured curve, (0.2q, 0.4q), (0.4q, 0.8q) There is a slight deviation between the theoretical curve and the measured curve under the lateral confining pressure, but the overall consistency is consistent, which fully proves that the established polynomial mathematical model has good reliability.

Figure 2. The comparison between theoretical curves and measured curves of S04 models

Figure 3. The comparison between theoretical curves and measured curves of S05 models
Figure 4. The comparison between theoretical curves and measured curves of S06 models

Figure 5. The comparison between theoretical curves and measured curves of B40 models

Figure 6. The comparison between theoretical curves and measured curves of B100 models

Figure 7. The comparison between theoretical curves and measured curves of CL220 models
Figure 8. The comparison between theoretical curves and measured curves of CL180, B70 models

Figure 9. The comparison between theoretical curves and measured curves of CL260 models

Figure 10. The comparison between theoretical curves and measured curves of WB075 models

Figure 11. The comparison between theoretical curves and measured curves of WB087 models
Figure 12. The comparison between theoretical curves and measured curves of WB100 models

4. Conclusions
1. The slope of the 540 d plastic concrete in the rising section of the stress-strain curve under the medium-high lateral confining pressure is larger than the low confining pressure, but the compressive strength is not much different.

2. Refer to different specifications to obtain the calculation formula of the compressive peak secant modulus of plastic concrete. After comparison, it is recommended that the triaxial compression peak secant modulus be calculated by formula (3).

3. A quadratic polynomial mathematical model conforming to the triaxial compression constitutive relation of plastic concrete is fitted, and the model has good reliability.

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