Continuously tunable acoustic metasurface with rotatable anisotropic three-component resonators

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We propose a tunable acoustic metasurface consisting of identical units. Units are rotatable anisotropic three-component resonators, which can induce non-degenerate dipolar resonance, causing an evident phase change in low frequencies. Compared with the monopole resonance widely used in Helmholtz resonators, the polarization direction of the dipole resonance is a new degree of freedom for phase manipulation. The proposed metasurface is constructed by identical units that are made with real (not rigid) materials. The phase profile can continuously change by rotating the anisotropic resonators. We present a wide-angle and broad-band acoustic focusing by the metasurface under a water background.

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Acoustic metasurfaces (AMs) that can freely manipulate wavefronts have received extensive attention in recent years.1–4 Most of the classical acoustic metasurfaces (CAMs) have two ways to modulate the phase change inside the unit cells. One way is to change the effective spatial path of waves via space-coiling structures;5–7 the second way is to obtain phase delay from resonances.8,9 These CAMs have realized many interesting functions, including abnormal reflection20 or refraction,9 acoustic focusing,10,11 asymmetric transmission,10,11 acoustic accelerating beam,8,12 acoustic holography,13 acoustic illusion14 and cloaking.15 Although CAMs hold great potential for wavefront manipulation, they are limited to meet the requirements of broadband and alterable functionalities due to fixed microstructures. To overcome this limitation, researchers propose tunable acoustic metasurfaces (TAMs),16–27 whose acoustic response can be modulated by tunable units, like actively controlled transducer arrays, independently adjustable units or reconﬁgurable microstructures.

Currently, there are two main types of TAMs: active and passive. Most of the TAMs still rely on actively controlled units, like transducer arrays5,16,17 and active membrane metamaterials,19,20 which are complex and expensive. Recently, researchers have paid more attention to passive TAMs consisting of reconﬁgurable micro-structures.22–27

Based on the matched screw-and-nut physical mechanism, Zhao et al. designed a class of the tunable space-coiling metasurface with individual unit components of a helix screwed inside a plate for transmitted24 and reﬂected wavefront modulation.25 With the nested Helmholtz resonant structure, Zhai et al. designed a TAM for ﬁltering and imaging.22 Wang et al. proposed a TAM consisting of annular resonators to modulate the transmitted wavefront.27 Besides, a TAM composed of tunable Helmholtz resonators has been reported by Tian et al.23 So far, the research works about TAM are still limited.

In this letter, we design a TAM based on identical anisotropic resonant units,28–30 each of which is a modiﬁed three-component composite. The anisotropic resonant unit is proven to be a dipolar resonator with two non-degenerate eigenmodes, which could be controlled by the rotation angle of the elliptical rotor. The reﬂected phase shift of the unit will have a span of 2π just by changing its rotation angle. In a wide frequency range, this TAM can focus the reﬂected waves of different incident angles on a ﬁxed focal length. With the same TAM, we also can control the position of the focus point arbitrarily.

We consider a tunable unit cell, which consists of a square epoxy frame with a circle cavity and an elliptical resonator in the center of the cavity, as shown in Fig. 1(a). The length of the frame and the radius of the cavity are \( p = 0.4p \), respectively. The cavity is ﬁlled up with water, so the resonator is rotatable. The resonator is designed with three components31 to provide dipolar resonance.28 The resonator is composed of an elliptical epoxy shell (with semi-minor axis \( R_0 = 0.3p \)) and semi-major axis \( R_3 = 0.35p \), an elliptical rubber layer (with semi-minor axis \( r_1 = 0.25p \)) and semi-major axis \( r_2 = 0.3p \), and a circle steel core (with radius \( r_0 = 0.18p \)). The rotational angle of the resonator is \( \theta \), which is the only variable parameter in our system and can continuously change from 0 to 90 degrees. The used material parameters are: \( p = 1180 \text{ kg m}^{-3}, \quad \lambda_e = 4.4 \times 10^9 \text{ N m}^{-2}, \quad \mu_e = 1.6 \times 10^9 \text{ N m}^{-2} \) for epoxy; \( p = 1000 \text{ kg m}^{-3} \) and \( c_\nu = 1490 \text{ m s}^{-1} \) for water; \( \rho_\text{w} = 980 \text{ kg m}^{-3}, \quad \lambda_c = 1.96 \times 10^9 \text{ N m}^{-2}, \quad \mu_c = 5.5 \times 10^7 \text{ N m}^{-2} \) for rubber; \( \rho_\text{c} = 7900 \text{ kg m}^{-3}, \quad \lambda_c = 1 \times 10^{11} \text{ N m}^{-2}, \quad \mu_c = 8.1 \times 10^{10} \text{ N m}^{-2} \) for steel. Here, \( p \) is the mass density, \( \lambda \) and \( \mu \) are the Lamé constants, and \( c \) is the speed of sound. An elastic metamaterial is constructed by the tunable unit cells (with \( \theta = 0^\circ \)) periodically arranged in a square lattice. We calculate the band structure with the ﬁnite element method (using COMSOL Multiphysics software) and plot the lowest seven bands in Fig. 1(b).

By carefully checking the patterns of eigenstates, we ﬁnd three ﬂat bands induced by rotational resonances (denoted by blue hollow circles around \( p/\lambda = 0.026, p/\lambda = 0.047 \) and \( p/\lambda = 0.059 \), and the other four bands (highlighted by red solid lines) are induced by a non-degenerate dipolar resonance.28 The eigenstates of the eigenmodes A and B on the ﬁfth and third bands (with normalized wavelengths \( p/\lambda_A = 0.071 \) and \( p/\lambda_B = 0.052 \)) on the Brillouin zone boundary along the \( \Gamma \) direction are plotted in Fig. 1(c) and 1(d), respectively. The movements of modes A and B are mainly along the semi-major and semi-minor axis of the ellipse, respectively. These two modes with movement perpendicular to each other are a pair eigenmodes of a non-degenerate dipolar resonance. The corresponding wavelengths of mode A and B are \( \lambda_A = 14.1p \) and \( \lambda_B = 19.2p \), respectively. The modes A and B represent the...
longitudinal and transversal modes of a dipolar resonance, respectively, due to their wavelengths. We let one layer of the elastic metasurface lay at the bottom of water and study the phase properties for reflected waves. The calculation region is shown in Fig. 2(a). As the mass density and compression stiffness of water are not so different from solid materials, the solid unit can no longer act as a rigid structure. The interactions make the actual phase change provided by every unit of a metasurface different from that predicted by the periodic unit. In this work, two air voids (with length $d = 0.02p$ and the thickness $h = 0.96p$), which are more real than the rigid boundary, are added on each side of the unit to reduce the interaction between the neighboring units. The distance between the air void and the side boundary is $w = 0.01p$. The used material parameters are $\rho_w = 1.29 \text{ kg m}^{-3}$ and $c_w = 340 \text{ m s}^{-1}$ for air. We let plane pressure waves normally incident from water and calculate the reflected phase. The results are plotted in Fig. 2(b), where the phase change $\varphi$ is as a function of the rotational angle $\theta$ and incident wavelength $\lambda$. In general, the phase change $\varphi$ is trivial and insensitive to the rotated angle because of long wavelengths, which are more than 10 times the unit size. However, large phase changes are found in a range between $p/\lambda = 0.045$ and $p/\lambda = 0.08$. We choose point C, which has an incident wavelength of $\lambda_C$ the same as that of mode A, and plot the excited velocity field in Fig. 2(c). The vibration is mainly along the semi-major axis, and the pattern of point C is almost the same as that of mode A. We can see that the longitudinal mode of the dipolar resonance is excited by the incident acoustic wave, and the large phase change around the point C is induced by the longitudinal mode. Here, the elliptical resonator is vertically placed and the rotational angle is $\theta = 0^\circ$. If the elliptical resonator is horizontally placed (corresponding to $\theta = 90^\circ$), the longitudinal mode cannot be excited due to symmetry, and thus the phase change will reduce to trivial proportions. As a result, the phase change $\varphi$ is controlled by the rotational angle $\theta$. When $\theta$ gradually increases from $\theta = 0^\circ$ to $\theta = 90^\circ$, the phase change $\varphi$ can roughly cover a range of $2\pi$. Based on this phase property, we can design an AM consisting of identical units, and the rotational angle $\theta$ is a new degree of freedom to control the phase change. A continuously tunable rotational angle brings a flexible phase regulation, resulting in a continuous TAM.

Similar phase properties can be found at the incident wavelength of $\lambda_B$. When the elliptical resonator is horizontally placed (with $\theta = 90^\circ$), the transverse mode of the dipolar resonant can be excited. Point D marked in Fig. 2(b) has an incident wavelength of $\lambda_B$ the same as that of mode B. The excited velocity field of point D is plotted in Fig. 2(d). The vibrations are mainly along the semi-minor axis, where the pattern is the same as that of mode B. In contrast to the longitudinal mode, the transverse mode can induce large phase change at $\theta = 90^\circ$ but trivial phase change at $\theta = 0^\circ$. The two (longitudinal and transverse) resonant modes of a non-degenerate dipolar resonance have different resonant wavelengths, which make the dependent relationship between phase change and rotational angle exist at a broad wavelength range. In a range roughly between $p/\lambda = 0.045$ and $p/\lambda = 0.08$, the phase changes are controlled by the rotational angle. Not all wavelengths or the phase change $\varphi$ can cover a full $2\pi$ span. The refined wavelengths are in a region between $p/\lambda_1 = 0.049$ and $p/\lambda_2 = 0.059$ (corresponding to wavelengths $\lambda_1 = 20.4p$ and $\lambda_2 = 16.9p$) as denoted in Fig. 2(b) by two white dashed lines. In this region, the phase change $\varphi$ can cover a full $2\pi$ span, and thus we can build a continuously broad-band TAM.

As shown in Fig. 3(a), a TAM is composed of 120 identical tunable unit cells as studied in Fig. 2(a). We let pressure waves incline incident (with incident angle $\alpha$) from the water and manipulate the reflected wavefront by using TAM. In this work, we demonstrate that the TAM can focus (as an example) the reflected waves and manipulate the focus position by adjusting the rotational angle in every unit cell.
when the incident wave come from different directions and
with different wavelengths.

Based on the generalized Snell’s law (GSL), \[ \frac{\sin \theta}{\sin \theta'} = \frac{n_1}{n_2}, \]
the reflected pressure field is determined by the phase profile \( \varphi(x) \) provided by the TAM. A focusing effect requires a hyperbolic function of \( \varphi(x) \). The expression is

\[ \varphi(x) = \frac{2\pi}{\lambda} \left( \sqrt{(x-x_0)^2 + f^2} - f - x \sin \alpha \right). \tag{1} \]

Here, we fix the focal length of \( f = 30\mu m \) at \( \alpha_0 = 0^\circ \), and the phase profile \( \varphi(x) \) can be calculated with given parameters of incident angle \( \alpha \) and wavelength \( \lambda \). Following the relationship between phase change and rotational angle \( \varphi(\theta) \), which has been calculated in Fig. 2(b), the rotational angle distributions of the unit cells \( \theta(x) \) can be achieved to satisfy the required phase profile \( \varphi(x) \). For example, in the case of \( \alpha = 0^\circ \) and \( \lambda = \lambda_0 \), the functions of \( \varphi(\theta) \) and \( \varphi(x) \) can be obtained from Fig. 2(b) (the lower white dashed line) and Eq. (1); the results are plotted in Figs. 3(b) and 3(c) by the red thick lines. Then, the rotational angle distributions \( \theta(x) \) are obtained from \( \varphi(\theta) \) and \( \varphi(x) \), and the result is plotted at the bottom of Fig. 3(d). The corresponding reflected pressure field is shown in Fig. 3(d). The pressure pattern exhibits a focal point at about \( y = 30.6\mu m \), which is in good agreement with the prediction \( (f = 30\mu m) \). When the incident angle changes to \( \alpha = 60^\circ \), the acoustic focusing should fail, as shown in Fig. 3(i) as a reference, if the geometry of TAM is fixed as CAMs. Here, we show that the TAM can adapt to the change of incident direction by adjusting the rotational angle distributions. In the case of \( \alpha = 60^\circ \) and \( \lambda = \lambda_0 \), the functions of \( \varphi(\theta) \) and \( \varphi(x) \) are recalculated and plotted in Figs. 3(b) and 3(c) by the blue thin lines, respectively. Accordingly, the obtained \( \theta(x) \) as well as the pressure field are shown in Fig. 3(e). The focal point is kept at \( y = 27.4\mu m \), which is slightly different to the designed position. Good focusing effects can be achieved by TAM when the incident angle changes within a range \(-60^\circ \leq \alpha \leq 60^\circ \). When the incident wavelength changes to \( \lambda = \lambda_2 \), the TAM can also keep the focus position within a range \(-60^\circ \leq \alpha \leq 60^\circ \).

Figures 3(f) and 3(g) show results corresponding to the cases of normal (\( \alpha = 0^\circ \)) and incline (\( \alpha = 60^\circ \)) incidences at the wavelength of \( \lambda = \lambda_2 \), respectively. The rotational angle distribution \( \theta(x) \) is obtained in a similar way, and the pressure field also shows good focusing effects. Figures 3(j) and 3(k) are the results obtained from the AM with fixed geometry.

Next, we show the manipulation of the focus position. Without loss of generality, we set the incident angle \( \alpha = 60^\circ \) and wavelength of \( \lambda = \lambda_1 \) as constants. The manipulation of vertical and horizontal position can be achieved by the change in focal length and shift rotational angle distributions, respectively. Different focal lengths \( f \) lead to different phase profiles \( \varphi(x) \). The rotational angle distributions \( \theta(x) \) can be obtain from the \( \varphi(\theta) \) studied in Fig. 3(b) and a new function of \( \varphi(x) \) from Eq. (1). Figures 4(a), 4(b) and 4(c) show the pressure fields with focal points at \( y = 8.38\mu m, y = 24.9\mu m \) and \( y = 55.6\mu m \), which roughly agree with the designed focal lengths of \( f = 10\mu m, f = 30\mu m \) and \( f = 60\mu m \), respectively. Figures 4(d)–4(f) show that the focus position shift to the left side when we shift the rotational angle distribution. The focusing effect becomes weaker because of the edge effect. In principle, an infinite long TAM can smoothly move the whole pressure field along the horizontal direction. If we simultaneously change the focal length and shift the rotational angle distribution, we can manipulate the focus position in a wide range.

In summary, we propose a TAM composed of identical anisotropic resonant units, which can induce the non-
degenerate dipolar resonance, causing an evident phase change. With anisotropic resonant units, we can control phase changes with a new degree of freedom, the polarization change. With anisotropic resonant units, we can control degenerate dipolar resonance, causing an evident phase shift.

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