A wavelet-NARX Model for SDINS/GPS Integration System

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Abstract. Recently, the Strapdown Inertial Navigation System (SDINS) has been successfully integrated with Global Positioning System (GPS) to obtain reliable navigation solutions. SDINS is based on Micro-Electro Mechanical System (MEMS) sensors such as accelerometer and gyroscope sensors. To improve the overall performance of the integrated navigation system, the Inertial Measurement Unit (IMU) readings is treated through effectively band-limiting the high frequency noise using the Discrete Wavelet Multi-resolution Algorithm (DWMRA) as a first step, while the second step is to enhance the navigation position and velocity through utilizing the Nonlinear Autoregressive model with eXogenous inputs (NARX) to fuse GPS and INS systems. The performance of the proposed integrated navigation system is validated through comparison with other systems. Finally, the obtained results suggest a promising and superior prospect for NARX in the field of navigation for low-cost IMU’s during GPS denied signals, since it outperforms the Conventional Neural Network (CNN) and Extended Kalman Filter (EKF) by 84% and 92%, respectively.

Keywords. IMU, DWMRA, NARX, INS, GPS.

1. Introduction

The convergence of mechanical technology and traditional inertia has brought about a major change in inertia technology. Since the 1980s, the Draper Laboratory (United States) has researched microscopes, accelerometers, and other inertial devices. Micro-Electro Mechanical System (MEMS) technology has been widely used in the manufacturing process. It is used in the development of inertial measurement sensors and especially accelerometers and gyroscopes in small size, low power consumption and cheapness [1]. Strapdown Inertial Navigation System (SDINS) is an independent system that has good concealment and does not depend on external information and also never radiate energy into outer space, which makes it applicable in the airspace, marine or underground. Since SDINS updates data quickly and has the advantages of short-term accuracy and stability with small size and light weight, it can provide powerful navigation data such as location, speed or attitude. Therefore, the SDINS plays an important role in the military and civil navigation fields. Gyroscopes and accelerometers can be used as separate inertial instruments to measure angular velocity, acceleration, or other parameters separately [2].

On the other hand, Global Positioning System (GPS) is a satellite based navigation system that can provide location services and velocity with high-precision and low-cost measurements. GPS is requiring a direct sight of line at least four satellite between the receiver and satellites, when the number of satellites
increased then the accuracy of the receiver calculation is increased. When the time of receiving the signal from the satellite is long, this indicates that the signal was subjected to reflections, which led to its delay, and thus the GPS signal may be lost [3]. Each GPS and INS system has advantages and disadvantages, so the integration between GPS/INS systems results in precise navigation solutions, maximizes their advantages and minimizes their disadvantages. The current paper focuses on developing a successful navigation system through enhancing the short-term noises of the SDINS by de-noising the Inertial Measurement Unit (IMU) readings before mechanization and further processing and reducing long-term errors through the integration between GPS and SDINS systems as depicted in Figure 1.

The above figure shows clearly two types of errors existed in the inertial measurement sensors which are short and long-term errors [4]. Most of the resultant errors that distort the position and velocity are basically sensor errors or random disturbances. These are the remaining errors shown by gyroscopes and accelerometers after SDINS calibration. The dominant sources of error affecting the accuracy of the navigation solution are obtained from SDINS such as direction, scale factor, bias, non-orthogonally and random noise. The accelerometers and gyroscopes cause nine navigation errors. Such as acceleration and angular rate errors lead to progressively increasing errors in the information about position, velocity and attitude. Such navigation errors are due to the process of mathematical integration in the SDINS algorithm. In addition to these navigation errors, the SDINS algorithm also suffers from inaccuracy in reading the acceleration and angular rates sensor induced by earth gravity and rotation. These errors should be handled with care, especially in strapdown systems instead of inertial gimbaled sensors. The GPS navigation system can support the SDINS and avoid such time drift errors. Knowledge of the causes of error helps the machine to cancel its consequences as it is navigating. Nevertheless, only a few of the sensor errors can be adjusted inside a strapdown system. Errors which cannot be calibrated can spread to navigation errors when the navigation system starts. Long alignment times are also needed for these systems. However the most reliable SDINS will become useless if both of these needs are not met. There are various strategies that have been used to integrate both the long-term accuracy systems (i.e. GPS) and
the short-term accuracy systems (i.e. SDINS). Kalman Filter (KF) is considered as one of the most popular estimators that have been widely used to combine SDINS and GPS from short-range, high performance systems with reference systems for long-term stability. Shaghaghian A. and Karimaghaee P. [5] proposed a KF to integrate the SDINS and GPS. The proposed KF is very accurate in minimum variance optimal estimation. However, it is considered as an unlikely integration method for enhancing the main and classic features as steady state error reduction. Moreover, Shaker et al. [6] introduced an Extended Kalman Filter (EKF) as a basic data fusion algorithm; the kinematic constraints of land vehicle navigation (i.e. velocity and height) are presented. Loose coupling has been considered in this integration. However, KF has drawbacks such as its requirement of an accurate linear mathematical model for GPS and INS errors which is varied according to the sensor type to be used, in addition to the prior knowledge required about some parameters such as R and Q matrix. Artificial Intelligence (AI) is another promising technique that is utilized to estimate the SDINS error by combining both GPS and INS systems. Various types of Artificial Neural Network (ANN) have been employed. Guo et al, [7] proposed an algorithm to improve the navigation accuracy of hypersonic vehicle. An error parameter identification method of SDINS based on ANN is proposed. The problem of choosing the optimal number of hidden layers and the number of nodes in each hidden layer is improved by numerical simulation using a gradient Particle Swarm Optimization (PSO). However, the complexity of the intelligent navigator is increased. Moreover, Malleswaran et al [8] employed another type of AI called Radial Basis Function (RBF) in order to overcome the problem of choosing the number of hidden layers that are created dynamically during the training phase, but it is not the correct method to be used in real time. In addition, Hopfield Neural Network (HNN) is used to realize the integration between GPS and INS to estimate the GPS/INS errors [9]. Unfortunately, it requires a very large memory capacity to store the large number of learning parameters. There is another type of AI, called the Adaptive Neuro-Fuzzy Inference System (ANFIS) proposed by Hasan et al, [10] in order to improve the INS error estimation based on Dynamic ANFIS (DANFIS), however, due to limitation of the number of its outputs, ANFIS requires more than one structure. Therefore, six networks have been developed for position and velocity to overcome the limitations of classical intelligent integrated systems. Abdolkarimi et al, [11] introduced an integrated system through developing two different steps: (1) Improving the signal-to-noise ratio (SNR) of the inertial sensor measurements through utilizing the Discrete Wavelet Transform (DWT) technique, and (2) enhancing position and speed accuracy using an Extreme Learning Machine (ELM). ELM is performed faster than Conventional Neural Network (CNN) during training phase when it has one hidden layer but it stuck in training when used in difficult classification or identification problems unless increasing the number of hidden layers to a large number. Most importantly it is very slow in testing phase. While most applications consider the testing consumed time is more critical than training time elapsed. Moreover, due to failure of the most previous works such as KF, EKF, and most of the classical intelligent systems (ANN, RBF, HNN, ANFIS), then, Dai et al, [12] proposed a Recurrent Neural Network (RNN), since its current output depends on the current input and previous output to predict the INS errors. The suggested method accordingly outperforms ELM and EKF by approximately 30% and 60%, respectively. All the above AI used to predict the SDINS error by relying only on the instant values of SDINS data without taking into consideration the past values of the SDINS data. Fortunately, Bai et al, [13] introduced a promising intelligent method to integrate GPS and INS systems for prediction the position of the moving vehicle by developing a Nonlinear AutoRegressive model with eXogenous inputs (NARX) model instead of the previous intelligent systems. Thus, good results are obtained. However, in this paper, a NARX model is presented to estimate and predict the instant SDINS errors depending on the instant and previous IMU measurements and provide a consistent solution for navigation even when losing the GPS signal. This paper is organized as follows: Section 2 describes the concepts of a discrete wavelet multi-resolution analysis (DWMRA) with their filters and thresholding techniques. Section 3 illustrates the terrestrial SDINS algorithm implemented in this work. Section 4 describes the employed Nonlinear AutoRegression model with eXogenous inputs (NARX) model and its structure. Section 5 describes the procedure of the building the proposed intelligent
nominator in detailed steps. The results obtained and discussion based on these results is given in Section 6. Finally, the conclusion drawn based on the evaluation of the proposed intelligent navigator is given in Section 7.

2. Discrete wavelet multi-resolution analysis (DWMRA)

The basic idea of multi-resolution analysis of functions such as signals expands the robust framework to realize the process of wavelet decomposition. The main concept is that of sequential approximation, in addition to the resultant details as one goes from one level to another [14]. The loss of time-domain information when transferring to frequency domain is an undesirable issue in Fourier Transformation (FT). Therefore, the primary useful feature of the wavelet analysis is allowing the user to investigate long-time wavelet periods that require accurate low-frequency information and concise periods where high-frequency information is examined [15]. Hence, the wavelet transformation is very competent to show what other signal analysis techniques hide, such as breaking points, trends, and sharp interruptions. Moreover, it has the ability to compress or de-noise any signal without any degradation of the original signal. Actually, wavelet transformation in the time-domain is determined through the projections of the intended signal into a family of all normalized dilations and translations of the mother wavelet function.

2.1. Discrete wavelet transform (DWT)

Since the inertial measurement readings are considered as discrete time domain, then Discrete Wavelet Transform (DWT) is recommended instead of the Continuous Wavelet Transform (CWT). The two main parts of DWT of a discrete time sequence for a specific signal \( x(n) \) is given as [16]:

\[
C_{s,k} = \left\langle x(n), \Phi_{s,k}(n) \right\rangle = 2^{(s/2)} \sum_n x(n) \Phi_{s,k}(2^s n - k)
\]

\[
d_{s,k} = \left\langle x(n), \Psi_{s,k}(n) \right\rangle = 2^{(s/2)} \sum_n x(n) \Psi_{s,k}(2^s n - k)
\]

Where \( \Phi_{s,k} \) and \( \Psi_{s,k} \) are the scale and wavelet functions, respectively. \( 2^{(s/2)} \Phi_{s,k}(2^s n - k) \) and \( 2^{(s/2)} \Psi_{s,k}(2^s n - k) \) are the scaled and shifted versions of \( \Phi_{s,k} \) and \( \Psi_{s,k} \) respectively. This is established on the values of \( s \) and \( k \) which are the scaling and shifting coefficients. Hence, it attains integer values for various scaling and shifted versions of \( \Phi_{s,k}(n) \), \( \Psi_{s,k}(n) \), \( C_{s,k} \) and \( d_{s,k} \) respectively. The original signal reconstruction \( x(n) \) can be created from the scaled and shifted wavelet function by using equation (3):

\[
x(n) = \sum_n C_{s,k} \Phi_{s,k}(n) + \sum_s \sum_k d_{s,k} \Psi_{s,k}(n)
\]

Actually, the wavelet function \( \Psi_{s,k} \) is not restricted to exponential functions as in the situation of Fourier Transform (FT) or Short-Time Fourier Transform (STFT). However, the only limitation on \( \Psi_{s,k} \) is that it should be short and oscillating (i.e. it should be zero average and deteriorate rapidly on both sides). This condition is required to keep the sum in the DWT equation in a finite form [15].

Because the low-frequency portion of the accelerometers and gyroscopes readings hold the most dynamic behavior of the inertial sensors throughout the static alignment period. Therefore, these inertial readings could be de-noised via Discrete Wavelet Multi-Resolution Analysis (DWMRA) to dismiss the low and high frequencies [14].

DWMRA splits each component of the measurement readings into two parts. These two parts are called approximation and details. The first part is the output of low-pass filter which involves long-term noises,
with also Earth’s gravity and rotational frequency rate components. Actually, these two components stick with one another in a very small frequency range with a low frequency. Therefore, DWMRA are incapable to divide up Earth’s gravity and rotational rate from the inertial measurement readings and hence it will be astonished at the calculation of the SDINS algorithm. The second part (i.e. details) produced by the high-pass filter of the DWMRA which mainly contain unwanted high-frequency noise components of the strapdown INS algorithm in addition to many noise disturbances such as the vehicle vibration. Analysis and synthesis of the inertial measurement readings are conducted through equations (1-3). Since DWT outperforms FT, anywhere the basis function provides stable frequency resolution and no localization in time [15]. In theory, wavelet decomposition analysis can be persistent until reaching to a single frequency of the individual coefficients. Actually, the optimum Level of Decomposition (LOD) is selected based on the nature of the signal according to some appropriate criteria [4]. In this paper, the inertial sensors data rate is 32 Hz. Therefore, five levels of decomposition will set the frequency to 0.5 Hz. The results obtained showed that 5 level of decomposition is sufficient to eliminate high frequency noise from inertial measurement readings.

2.1.1. Allocate an optimum wavelet filter. Discrete wavelet transformation has the advantage of adapting by using various types of filters that vary according to their transactions. After employing various types of filters such as (Symlets, Coiflet, Daubaches, Biorspline). Literature indicate clearly the superiority of some types of wavelet filters against other types to separate high-frequency noise from inertial measurement readings (i.e. accelerations, and angular rates) that reduce the mean square error.

2.1.2. Performance evaluation of using various thresholding technique. Thresholding techniques are performed on the wavelet coefficients and can be categorized into two types called (Soft and Hard) thresholding as illustrated by Burrus et al. [15]. Selecting an optimum threshold technique is crucial to the quality of de-noising process and should be made cautiously. Practically, coefficients smaller than the specified value of the threshold (ThrV) are neglected, or considered as noise exceeds the specified threshold value (ThrV). This paper employs various types of methods in order to select the optimum threshold value (ThrV). These six methods are illustrated in Table 1.

| Table 1. Various methods to select the threshold value. |
|--------------------------------------------------------|
| Method number | Method Name                                      | Matlab Code |
|---------------|--------------------------------------------------|-------------|
| 1st           | $ThrV = \sigma^2 / \sigma$                      | N/A         |
| 2nd           | $ThrV = \sqrt{\frac{(2\sigma^2 \ln N)}{2}}$     | N/A         |
| 3rd           | Steins Unbiased Risk Estimate (SURE)             | “rigrsure”  |
| 4th           | Fixed form threshold                             | “sqtwolog”  |
| 5th           | Mixture of both 3rd and 4th method                | “heursure”  |
| 6th           | Minimax principle                                | “minimaxi”  |

Where: $\sigma^2$ is the noise power for the noisy measurements; $\sigma_x$ is the standard deviation for the detail coefficients; $N$ is number of samples

Soft and Hard thresholding techniques are widely used in wavelet analysis to enhance the de-noising results. Practically, hard thresholding type can maintain the characteristics of the de-noised signals. But, it results in unsmoothing signal. On the other hand, soft thresholding can achieve a very smooth de-noised signal. The soft threshold technique was used to eliminate some of the noise components accompanied with the details portion of the inertial measurement while preserving the features of the original signal through improving the Signal-to-Noise Ratio (SNR). After applying six methods of
thresholding techniques, the results show that the Steins Unbiased Risk Estimation method (SURE) is the lowest Root Mean Square Error (RMSE) with higher SNR. Therefore, optimizing the best base of choice is very important to choose the value of the threshold for wavelet analysis and has a great influence on prediction in the training stage later.

3. Terrestrial strapdown INS system dynamic equations

The Strapdown Inertial Navigation System (SDINS) mechanization algorithm was implemented using MatLab after de-noising the accelerometers and gyroscopes readings via wavelet de-noising. The kinematic differential mechanization equation of the relative quaternion between body and geographic frames [17] is:

\[ \dot{u} = \frac{1}{2} (\Omega_b^h - \Omega_{bn}^h) u \]  

(4)

Where, \( u \) is quaternion matrix while the angular velocity skew-symmetric matrices \( \Omega_b^h \) and \( \Omega_{bn}^h \) are given by:

\[ \Omega_b^h = \begin{bmatrix} 0 & -w_D & w_E & w_N \\ w_D & 0 & -w_N & w_E \\ -w_E & w_N & 0 & w_D \\ -w_N & -w_E & -w_D & 0 \end{bmatrix} \]  

(5)

\[ \Omega_{bn}^h = \begin{bmatrix} 0 & -w_y & -w_p & w_R \\ -w_y & 0 & w_p & w_R \\ w_p & -w_R & 0 & w_y \\ -w_R & -w_p & -w_y & 0 \end{bmatrix} \]  

(6)

Also,

\[ \begin{bmatrix} w_N \\ w_E \\ w_D \end{bmatrix} = \begin{bmatrix} |w_{we} + h| \cos L \\ -L \\ -|w_{we} + h| \sin L \end{bmatrix} \]  

(7)

Where, \( w_N, w_E, w_D \) are the body accelerations in the body coordinates \((x, y, z)\), respectively. \( w_R, w_p, w_y \) are the body angular velocities in the body coordinates \((\text{roll}, \text{pitch}, \text{and yaw})\), respectively \( L, l \), and \( h \) are abbreviation of latitude, longitude, and height as geodetic positions. While \( w_{we} \) is the earth angular velocity \((7.2921 \times 10^{-5} \text{ rad/s})\) while the transformation between body and navigation coordinates can be depicted in terms of quaternion parameters:

\[ C_b^n = \begin{bmatrix} u_0^2 + u_1^2 - u_2^2 - u_3^2 \\ 2(u_0u_2 - u_1u_3) \\ 2(u_0u_3 + u_1u_2) \\ 2(u_0u_1 - u_2u_3) \end{bmatrix} \]  

(8)

\[ R_N = r_e / \left( (1 - e^2 \sin^2(L))^5 \right) \]  

(9)

\[ R_E = r_e / \sqrt{(1 - e^2 \sin^2(L))^3} \]  

(10)

Where, \( V^n = [V_n V_E V_D] \) are the geodetic velocity vector in north, east and down directions, \( R_N, R_E \) are the radii of curvature in the north and east direction, \( r_e \) represent the semi-major axis length of the earth \((6378137.0 \text{ m})\), \( e \) represent the eccentricity which is equal to 0.0818.

The second derivative of the differential equation (12) for both geodetic position and velocities can be
expressed as follows:

\[
\begin{bmatrix}
\dot{V}_N \\
\dot{V}_E \\
\dot{V}_D
\end{bmatrix} = C^b_h \cdot f^b + \begin{bmatrix}
\frac{V_E}{(R_E + h)} \cos L + 2w_\mu \sin L + \frac{V_N V_D}{(R_N + h)} \\
\frac{V_E}{(R_E + h)} \cos L + 2w_\nu \sin L + \frac{V_E V_D}{(R_E + h)} + 2w_\mu V_\rho \cos L \\
\frac{V^2_E}{(R_E + h)} - \frac{V^2_N}{(R_N + h)} - 2w_\mu V_E \cos L + g_e
\end{bmatrix}
\] (12)

Where; \( f^b = [f_x, f_y, f_z] \) which is the specific force outputs in the body coordinate, \( g_e \) is a down direction of gravity force. However, the specified gravity force can be calculated depending on the initial gravity (\( g_0 \)) as follows;

\[
g_0 = 9.780327[1+0.0053024 \sin^2(L)-0.0000058 \sin^2(2L)];
\] (13)

\[
ge_e = g_0 - [3.0877*10^{-6} - 0.0044*10^{-6} \sin^2(L)]^2 h + 0.072*10^{-12} \times h^2;
\] (14)

Therefore, previous equations exemplify the strapdown INS mechanization equation.

4. Artificial neural network

Artificial Neural Networks (ANN) presents an effective way to predicting events in time series. The Artificial Neural Network (ANN) is a nonlinear system consisting of a large number of direct neurons. It has real-time benefits, self-learning, and fault-tolerance capabilities. Therefore, ANN is widely used in the field of integrated navigation. It can be classified into static and dynamic categories of neural networks. The first category, static neural networks are shown in Figure 2-a feed-forward networks have not any feedback connection or delays, and thus the output from the forward feeding connection (inputs) is calculated by the output. The second category, dynamic neural networks are shows in Figure 2-b and they have a feedback connection with delays and they are more efficient than static neural networks. In the dynamic neural networks, the output depended on the instant and previous data. Generally, dynamic neural networks are divided into two types: the first one is a feedback connection time delay networks and the second type is the recurrent or feedback networks [18].

![Figure 2. Types of Neural Network](image)

(a) (b)

Figure 2. Types of Neural Network (a) Static Neural Network and, (b) Dynamic Neural Network [19].
4.1. NARX navigator architecture

A Nonlinear AutoRegressive model with eXogenous inputs (NARX) is a dynamic (recurrent) neural network that contains feedback connections with several layers of the network. It is based on the linear ARX model and it's used in time series modeling. The NARX neural network is used to predict the instant time series value based on the last value of the output rather than the actual measurements [13]. The NARX neural network is most commonly used compared to other neural networks because it is more effective than other networks and is good in learning time series and estimation. Therefore in this paper, the NARX neural network converges better and faster than other networks, so it is used for integration between GPS and INS to estimate the INS error using both instant and past IMU measurements. Historical IMU measurements are useful for better error estimation. The NARX model's structure is similar to the classic model of multi-layer perceptron (MLP). The MLP and NARX models consist of an input layer, a hidden layer and an output layer, but the NARX model feeds the output layer to the input layer as part of the input [20]. The general structure of the NARX neural network is shown in Figure 3.

![Figure 3. The general structure of NARX neural network [20].](image)

NARX is an important type of nonlinear system that can be expressed as shown in equation (15):

\[ y_{(n+1)} = f[y_{(n)}, \ldots, y_{(n-d_y+1)}; u_{(n)}, u_{(n-1)}, \ldots, u_{(n-d_u+1)}] \]  

(15)

Where \( u \) is the target values; \( n \) are the previous predicted values; \( d_y \) and \( d_u \) are the input and output orders; \( y \) is the exogenous variable (\( y_n \) and \( u_n \) denotes the external (exogenous) output and input of network at the time); and \( f \) is a nonlinear function (generally used multilayer perceptron (MLP)).

The performance indicator used in training the NARX model is the mean squared error (MSE), which is one of the typical performance functions as shown in equation (16),
\[ \text{MSE} = \frac{1}{N} \sum_{i=1}^{N} (e_i)^2 = \frac{1}{N} \sum_{i=1}^{N} (t_i - y_i)^2 \]  

(16)

Where \( t_i \) and \( y_i \) represent the target and network output and \( N \) is the number of training samples.

NARX networks have two modes: series-parallel and only parallel. The series parallel mode (open loop) will result in less error than parallel mode (closed loop) can be considered for multi-step-ahead forecasting. The series-parallel architecture is better and faster than parallel architecture, so it has two advantages: firstly, the feedforward input is more accurate; secondly, the resulting network has completely pure feedforward architecture and can be used to train on static back-propagation. In general, the training in series-parallel is better and the efficiency is greater than when using the architecture of the parallel. Figure 4 shows the architectures of NARX neural network [21].

The equations of the two architectures of NARX neural network are expressed as in equations (17) and (18):

\[ \hat{y}(t+1) = \hat{f}(y_{t-1}, y_{t-2}, \ldots, y_{t-n}, u_{t-1}, u_{t-2}, \ldots, u_{t-n}) \]  

(17)

\[ \hat{y}(t+1) = \hat{f}(\hat{y}_{t-1}, \hat{y}_{t-2}, \ldots, \hat{y}_{t-n}, u_{t-1}, u_{t-2}, \ldots, u_{t-n}) \]  

(18)

5. Methodology

The proposed intelligent navigator consists of two main stages as depicted in Figure 5:

**Stage 1:** consists of preprocessing for the IMU readings and stage 2 which consists of training the intelligent navigator. Stage 1 consists of two sub stages, firstly, applying Discrete Wavelet Multi-Resolution Analysis algorithm (DWMRA) for the IMU readings and then using these de-noised readings as inputs to the SDINS mechanization equation to compute the navigation solution (i.e. position, and velocity) in the second sub-stage.

**Stage 2:** uses one type of Artificial Intelligence (AI) called NARX for training process. All stages and sub-stages shown in Figure 5 will be explained later.
Stage 1: Preprocessing analysis Two processes are performed in this stage. Firstly, apply DWMRA to de-noise the IMU readings and secondly, applying SDINS mechanization equation to compute position and velocity.

Stage 1.1: In Fourier Transform (FT) and when transforming to frequency-domain, time-domain information will be lost. Therefore, Short Time Fourier Transform (STFT) was considered in order to overcome this necessity. However, the fixed window width inherit in the STFT caused a major drawback. Thus, Wavelet Transformation (WT) is adopted since it allows the use of long period of time when specific low-frequency information and vice versa are required. Moreover, WT is able to keep important information and shape of the signal that other signal analysis transformation techniques miss, such as break down points, trends, and discontinuities [22].

Figure 5. Schematic design for the Wavelet-NARX model for predicting the SDINS error when GPS signal lost.
Most of the error sources that distort the output of low-cost MEMS based inertial sensors are sensor errors or random disturbances. These residual errors occur in accelerometers and gyroscopes following calibration of SDINS. Alignment, biasing, scale factor, non-orthogonally and long with short term noises have a great effect on the navigation solution. These errors in the accelerations and angular rates measurements lead to increase errors rapidly with time due to the accumulating error through the mathematical integration process in the SDINS algorithm. Therefore, DWMRA algorithm is utilized for removing the high-frequency noises and enhancing the Signal-to-Noise Ratio (SNR) though keeping the useful information such as vehicle movement information. Hence, if the harmful noises such as sensor and vehicle engine vibration noises can be eliminated before SDINS mechanization and data fusion, the resultant navigation solution will provide more accurate results. According to Figure 5, it is necessary to de-noise the inertial measurement readings via implementing DWMRA prior to its mechanization and then to enhance the prediction accuracy of the intelligent navigator.

Practically, sensors readings de-noising using DWMRA consist of three main points: (1) apply wavelet decomposition to the measurement signals, (2) utilizing thresholding technique to details part, (3) reconstruct the signal using the altered details and appropriate approximation. Wavelet decomposition in first step consists of both low and high pass filter with down sampling. This step is repeated on the sub-sampled output of the low-pass filter (i.e. Approximation part) until optimum Level of Decomposition (LOD) is reached. Taking in consideration removing the high-frequency noise with keeping the most useful information, the optimum LOD is decided based on the sampling frequency and the nature of the measurement signal. Wavelet thresholding techniques represent an important issue which is crucial in order to enhance the quality of signal de-noising. There are two types of thresholding: Hard and Soft thresholding techniques, which eliminate the noise level through excluding coefficients that are trivial. Finally, the reconstruction of the de-noised measurements is accomplished through using the summation of the difference for the altered details and the last approximation part.

5.1. Multi-resolution analysis de-noising procedure

In order to de-noise the Inertial Measurement Unit (IMU) readings that can be used in the SDINS mechanization equation to calculate the position and velocity components, a DWMRA must be performed for each acceleration and angular velocity readings as described in Figure 5. The following steps describe the mathematical multi-resolution analysis procedure:

**Step 1:** For each components of the accelerometers and gyroscopes readings, calculate the approximation coefficients at $s^{th}$ resolution level using equation (19). This step is amounting to LPF.

$$x_s(n) = \sum_{k=-\infty}^{\infty} C_{s,k} \Phi_{s,k}(n)$$  \hspace{1cm} (19)

**Step 2:** Extract the approximation part from the approximation coefficient which results from step1 utilizing:

**Step 3:** For each sensor reading compute the details coefficient at various $s^{th}$ resolutions level utilizing equation (20). This step is considered as HPF.

$$g_s(n) = \sum_{k=-\infty}^{\infty} d_{s,k} \Psi_{s,k}(n)$$  \hspace{1cm} (20)

**Step 4:** Extract the detail from the details coefficients which results from step3 utilizing:

**Step 5:** Go back to step1 and apply wavelet decomposition again until reaching the optimum Level of Decomposition (LOD). It should be noted that the following wavelet decomposition should be applied on the results of last wavelet decomposition and so on.
Step 6: Apply the thresholding technique to de-noise the details of the accelerometers and gyroscopes readings.

Step 7: Evaluate the inertial measurements reading of both accelerometers and gyroscopes at several resolution levels, where each level produces a new approximation and details.

Step 8: Reconstruct the de-noised signal for both accelerometers and gyroscopes through adding the last approximation with the summation of differences of all details obtained from the step7 depending on the optimum LOD as follows:

\[
\begin{align*}
S_1^{v} &= A_1^{v} + D_1^{v} \\
S_2^{v} &= A_2^{v} + (D_1^{v} - D_2^{v}) \\
S_3^{v} &= A_3^{v} + (D_1^{v} - D_2^{v} - D_3^{v}) \\
\vdots \\
S_n^{v} &= A_n^{v} + (D_1^{v} - D_2^{v} - \ldots - D_n^{v})
\end{align*}
\]

(21)

Where; \(A_n^{v}\) is the approximation part of \(v^{th}\) level; \(D_n^{v}\) is the details part of \(n^{th}\) level.

The output of the DWMRA algorithm is generally the de-noised acceleration and angular velocity readings which are used to evaluate the SDINS mechanization and as an input to the NARX intelligent navigator as well. Several important points to be considered such as the criteria for optimal LOD selection, the type of filters used, and the thresholding technique chosen to de-noise the details part of the signal.

Stage 1.2: SDINS mechanization equations. Inputs to the SDINS algorithm are de-noised measurements including specific force (\(f^{b}\)) and the angular velocity (\(w^{b}\)) provided by the IMU sensors. While the outputs are the vehicles position, velocity, and attitude. These outputs are computed through solving the differential equations of the SDINS algorithm.

Stage 2: Nonlinear AutoRegressive model with eXogenous inputs (NARX), the training mode is performed in stage 2 through employing NARX as depicted in Figure 5.

5.2. NARX navigator design steps
For more illustration about the input, output, and procedure, the algorithmic steps of NARX are given as:

5.2.1. An input of the NARX model contains:

1. Accelerations and angular rates for three axes with SDINS error as target for training purposes.
2. Number of delay input.

5.2.2. Outputs of the NARX model represents:

1. The predicted SDINS position and velocity error for three axes.

5.2.3. Training procedure:

1. Initialize randomly both weights and biases.
2. Compute the outputs of all hidden layers.
3. Compute the actual output of the output layer.
4. Train the network to the desired accuracy or reaching the maximum number of iterations.

According to the proposed NARX navigator system, the proposed structure will be designed as three main steps as below:
A. Computing the INS error
The proposed structure used NARX neural network to estimate SDINS error. The SDINS error is produced by subtracting SDINS components from GPS components, so the SDINS error was considered as a desired output to the NARX neural network. Any neural network has two phases: the learning phase and the test phase.

B. NARX neural network (learning phase)
In the learning phase, to estimate low-cost MEMS (IMU) sensors only the SDINS sensors will be used to measure motion information, consist of tri-axial acceleration (x, y, and z-axes) and tri-axial angular velocity (north, east, and down directions).
The basic function of the NARX neural network used in this work is feed-forward, back propagation. The number of layers in the neural network is calculated when computational complexity is decreased and a local minimum is avoided after many training experiments have been carried out. The calculation was conducted using raw data from the GPS/SDINS.
Figure 6 shows the learning phase for the NARX networks using both the GPS and SDINS measurements (accelerometer and gyroscope) to create an experimental sample of SDINS error for current and past value of SDINS sample measurements for position and velocity.

Figure 6. NARX training mode when GPS signal is available.
When the GPS is available, the NARX Navigator system has been trained to estimate the SDINS error by computing the desired output (SDINS error) by subtracting the SDINS components from the GPS components for both position and velocity and provide an accuracy navigation solution for the moving vehicle. To reduce the value of MSEs, learning parameters must be modified by comparing actual outputs with desired outputs (SDINS error) which the result is a feedback to the neural network.

C. NARX neural network (testing phase)
In the GPS outages, the testing stage is used to compute the performance of the NARX neural network. After the training phase is finished, the NARX network is ready to perform GPS and SDINS modeling data to estimate the SDINS error for both position and velocity. Figure 7 shows NARX operation in the testing stage when the signal of GPS is lost. It provides a prediction of INS error based on the specific time available in the input of NARX network. Consequently, the inputs of NARX network are the IMU measurements. In other words, when GPS signal is available then NARX model works in updating mode as the signal is available. When a GPS signal is lost then NARX model is switched to prediction mode.

![Figure 7. NARX predicted mode when GPS signal is blocked.](image)

6. Results and discussion
Discrete Wavelet Multi-resolution Analysis (DWMRA) was performed in a pre-processing step and prior to SDINS mechanization. According to the sampling rate therefore five LOD is adequate to remove high-frequency noises from both accelerations and angular rates measurements without removing any useful information of the vehicle dynamics. Extensive attempts to select the optimum wavelet parameters such as (Wavelet filter type, LOD, and threshold type) are taken into consideration carefully. Depending on the type of IMU measurement used, then Db3 wavelet filter is chosen as the ideal filter for accelerations and
angular rates measurements. Figure 8 shows the effectiveness of applying DWMRA to remove most of high-frequency noise components from the short-term measurements of the motion dynamics of the moving vehicle for improving the Signal-to-Noise Ratio (SNR) for the acceleration and angular rates compared to raw measurements in all directions. Figure 9 shows clearly the improvement in SNR for the de-noised measurements of the IMU compared to the raw measurements. However, these de-noised IMU measurements are applied as inputs to the proposed intelligent navigator which aims to produce more accurate estimate of SDINS errors during the prediction phase. In this work, the performance is evaluated based on the Root Mean Square Error (RMSE) function. Throughout a GPS satellite signal is available, the NARX model works in training mode as the GPS signal is available while the prediction mode is activated whenever the signal is blocked to produce a prediction of SDINS error based on the outputs of the IMU readings to get corrected navigation information. The RMSE described in equation (22) is utilized in this paper since it is widely used in literature to compare the actual output of the NARX model to the desired output (i.e. reference data) in order to measure the accuracy and evaluate the performance during the GPS unavailability.

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_d - y_a)^2}$$  

(22)

Where $N$ is the number of samples; $y_d$ and $y_a$ are the desired and actual outputs of NARX model.

**Figure 8.** Specific accelerations and angular rates after de-noising via wavelet for all components.

The results of this work are shown in Figure 9 utilizing NARX model during GPS blockage.
Figure 9. SDINS error (desired) and the predicted NARX output (actual) for x-axis, y-axis, and z-axis in positions and North, East, and Down directions for velocity.
It can be noticed that the RMSE between the actual and the desired outputs for position along $x$, $y$, and $z$ axes and velocity components in north, east and down directions are improved compared to previous works as illustrated in Table 2. The results obtained in this work may open the door for future research investigation to take into consideration various motion situation for generalizing the proposed navigator model.

Table 2. Comparison between the proposed method and other methods.

|                      | RMSE          |
|----------------------|---------------|
|                      | X  | Y  | Z  | N  | E  | D  |
| H-infinity [23]      | 0.232 | 0.298 | 0.222 | N/A | N/A | N/A |
| EKF [24]             | 6.51 | 7.1 | 1.99 | 0.15 | 0.18 | 0.09 |
| Dynamic ANFIS [10]   | 1.05 | 0.93 | 0.65 | 1.25/2 | 1.13/2 | 0.78/2 |
| ELM-wavelet [11]     | 2.96 | 2.49 | 3.68 | N/A | N/A | N/A |
| ANSCFIS-Wavelet [25] | 2.64 | 0.61 | 0.71 | N/A | N/A | N/A |
| LSTM [26]            | 2.86 | 2.67 | N/A | N/A | N/A | N/A |
| NARX [13]            | 1.03*10^-5 | 2.2*10^-5 | N/A | N/A | N/A | N/A |
| LPF (SG) [28]        | 6.526 | 4.244 | 5.506 | N/A | N/A | N/A |
| KF [7]               | 0.3736 | 0.4744 | 0.5516 | 0.0111 | 0.0100 | 0.0105 |
| Savitzky Golay Filters | 5.3423 | 3.4727 | 12.6362 | 0.5236 | 0.5012 | 0.6379 |
| IIIDNN [29]          | 0.5267 | 0.5428 | 0.5098 | N/A | N/A | N/A |
| RBFNN [30]           | 1.9186 | 1.9105 | 1.6285 | N/A | N/A | N/A |
| Proposed Wavelet NARX | 0.2125 | 0.3016 | 0.2902 | 0.1098 | 0.1581 | 0.1396 |

7. Conclusions
This paper presents both data pre-processing in terms of data filtering and data fusion based on various conditions such as stochastic noise, compound error attributes of the low cost MEMS sensors. Firstly, de-noising algorithm based on DWMRA has been introduced to filter out the short-term high frequency noises and improve the SNR in raw IMU measurement. The results indicate that thresholding type has a vital role in improving the noise removal process and should be carefully selected as well as the type and arrangement of wavelet functions. Secondly, extracting the best features of NARX to fuse data from both SDINS and GPS systems. It has been confirmed that the proposed intelligent navigator is very effective in modeling a high complex nonlinear relationships with high level of uncertainty in the measurements readings. The NARX navigator keeps learning as the reference GPS signal is available in order to update the navigator data-base and start to predict SDINS error when the signal of GPS is loss in order to keep the latest variations in the SDINS error over the time. Approximately, a considerable improvement has been gained compared previous related work.

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