Sagnac interferometer and the quantum nature of gravity

Chiara Marletto\textsuperscript{1,2,3} and Vlatko Vedral\textsuperscript{1,2,3}

\textsuperscript{1} Clarendon Laboratory, University of Oxford, Parks Road, Oxford OX1 3PU, United Kingdom
\textsuperscript{2} Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, Singapore 117543
\textsuperscript{3} Department of Physics, National University of Singapore, 2 Science Drive 3, Singapore 117542

Abstract

We use a quantum variant of the Sagnac interferometer to argue for the quantum nature of gravity assuming the equivalence principle, which we formulate in its quantum version. We first present an original derivation of the phase acquired in the conventional Sagnac matter-wave interferometer, within the Hamiltonian formalism. Then we modify the interferometer in two crucial respects. The interfering matter wave is interfered along two different distances from the centre and the interferometer is prepared in a superposition of two different angular velocities. We argue that if the radial and angular degrees of freedom of the matter wave become entangled through this experiment, then, via the equivalence principle, the gravitational field must be non-classical.

Einstein’s ‘happiest thought’ of his life was that a falling observer is actually an inertial one, i.e. that if we were placed in a falling elevator we would not experience gravity, [1]. This led Einstein to formulate the equivalence principle according to which acceleration is locally indistinguishable from gravity. Once that is established, it is clear that gravity can no longer be a scalar, as it happens to be in Newtonian physics. One number associated to every point in space (and every instant of time) is insufficient to capture all the relevant aspects of gravity since an accelerated observer, standing somewhere away from the centre on a rotating disk, experiences not just the centrifugal force (pushing her away from the centre), but also the Coriolis force (therefore one needs at least two numbers associated with every point on the disk). When this argument is extended to generally accelerated frames, it turns out one needs a two component tensor, known as the metric tensor $g_{\mu\nu}$, to describe fully the gravitational field.

We have previously used two massive particle interferometers to argue that if the two particles only couple through the gravitational field, any entanglement generated between them would be evidence of non-classicality of the gravitational field, [2, 3]. Now we will show that, if we also assume the equivalence principle to hold at this scale, we can reach the same conclusion using a single particle matter wave Sagnac interferometer, but with a twist that the interferometer has to be prepared in a superposition of two different angular frequencies. This argument is of interest for three reasons. The first one is that unlike in [2, 3], here there is only one interfering mass (but we can still use the same general assumption expressed in [4, 5]). The superposition of geometries is created by the disk spinning in a superposition of two angular frequencies. The second point is that the entanglement created is between different degrees of freedom of one and the same particle. The third, is that we use the equivalence principle. Hereinafter, we will take the equivalence principle in the presence of superposed spacetime configurations to mean that in each branch of the superposition gravity and acceleration are locally indistinguishable. There is currently a wide debate about how to formulate the equivalence principle in quantum theory [6–9], but here we assume this particular version of the principle and explore its consequences.

Under the same assumptions of the proposals in [2, 3], assuming also the equivalence principle, we can reason as follows. At each point $x$, consider two metrics: $g_{\mu\nu}$ and $\tilde{g}_{\mu\nu}$. The former corresponds to there being the Sagnac acceleration (and zero stress-energy tensor); the other corresponding to there being the mass and energy distributions that would generate an equivalent gravitational field on a test mass at point $x$. The equivalence principle says that the two metrics are locally (at $x$) indistinguishable from each other. Now suppose that for every point $x$, we show that the metric $g_{\mu\nu}$ is able to create entanglement between the relevant degrees of freedom.

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of the mass. That means that $g_{\mu \nu}$ must be non-classical, according to the general argument in [3, 5]. Then, we use the equivalence principle to argue that point by point $\tilde{g}_{\mu \nu}$ must be non-classical too. If it were not, then the equivalence principle would be false. For it would be false that the two metrics are locally equivalent: one would be a c-number, the other a q-number. Hence, assuming the equivalence principle between gravity and acceleration, the non-classicality of spacetime geometry must also imply the non-classicality of gravity.

In the regime explored in this paper, just as in the of the proposals in [2, 3], we can assume that the amplitudes computed in a particular coordinate system are invariant under coordinate system transformations, because the latter are approximately unitary in this regime and effectively equivalent to gauge transformations in the standard quantum field theory picture. This theoretical assumption is well corroborated by the neutron interferometry of the Colella-Overhauser-Werner (COW) and related experiments, [10, 11]. Also, we can neglect the corrections necessary to take into account effects, such as Unruh’s and related effects, [12], that become important only at higher accelerations.

Let us first review the standard Sagnac matter wave interferometer, [13], but emphasising in the derivation of the interference the analogous nature of acceleration and gravity. The metric on a rotating disk is given by:

$$g_{\mu \nu} = \begin{pmatrix}
-1 + \frac{\Omega^2 r^2}{c^2} & 0 & \Omega r^2 & 0 \\
0 & 1 & 0 & 0 \\
\Omega r^2 & 0 & r^2 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}$$

where $\Omega$ is the angular frequency of the disk, $r$ is the coordinate labelling the distance from the centre and $c$ is the speed of light. We can write the metric in the linear regime away from the flat Minkowski metric as $g_{\mu \nu} = \delta_{\mu \nu} + h_{\mu \nu}$, where $h_{\mu \nu}$ is the deviation from the Minkowski metric $\delta_{\mu \nu}$. In this notation, the relevant linear element will be $h_{00} = \frac{\Omega^2 r^2}{c^2}$. We will now start the particle at some angle, a distance $r$ away, and coherently split it to take two different paths (in a superposition) along the positive and negative directions (i.e. clockwise and anti-clockwise).

The phase difference obtained along two paths that meet after competing a full circle is given by

$$\Delta \phi = \frac{T_0 h_{00}}{\hbar} = \frac{mc^2}{\hbar} \times \frac{\Omega^2 r^2}{c^2} \times \frac{2\pi}{\Omega} = \frac{2m}{\hbar} \times \Omega \times A$$

(1)

where $A = \pi r^2$ is the area enclosed by the interfering matter wave. Here we have simply used the fact that the evolution of the system is given by $e^{-i\mathcal{H}t}$ where, in the linear regime, the Hamiltonian is given by $H = -\frac{1}{2} T_0 h_{00}$. To the best of our knowledge, our derivation here is original, even though the result is well known.

As a side remark, it is fruitful to think of this phase generation via a formal analogy between the Sagnac effect and the Aharonov–Bohm effect, where the role of the $B$ field would be played here by $\Omega$, [14]. We can then think of the angular velocity as an effective curvature (even though there is no curvature in this metric), corresponding to the actual curvature that is responsible for the gravitational field in General Relativity. Indeed, in the analogy between linear quantum gravity and the quantised electromagnetic field, the magnetic field corresponds to the curvature, while the metric plays the role of the electromagnetic vector potential.

So far we have only described a single particle superposed a fixed distance away from the centre of the rotating disk. Imagine now that the particle is superposed across two distances, $r_1$ and $r_2$, and that at each it completes interference (by being in a clockwise and anti-clockwise superposition—see figure 1).

Then at the end of the interference the state would be

$$e^{i\frac{\pi}{2} \times \Omega \times A |r_1\rangle} + e^{i\frac{\pi}{2} \times \Omega \times A |r_2\rangle}.
$$

(2)

The detectable phase is then the difference between the two, i.e. $\frac{2m}{\hbar} \times \Omega \times (A_2 - A_1)$. However, imagine further that the disk is spinning in a state that is a superposition of two frequencies $\Omega_1$ and $\Omega_2$ (just like an electron in an atom being in a superposition of two different angular momenta). Then the total state at the end of each complete interference is

$$e^{i\frac{\pi}{2} \times \Omega_1 \times A |r_1\rangle} |\Omega_1\rangle + e^{i\frac{\pi}{2} \times \Omega_2 \times A |r_1\rangle} |\Omega_2\rangle + e^{i\frac{\pi}{2} \times \Omega_1 \times A |r_2\rangle} |\Omega_1\rangle + e^{i\frac{\pi}{2} \times \Omega_2 \times A |r_2\rangle} |\Omega_2\rangle$$

(3)

In our case, the state $|r\rangle |\Omega\rangle$ describes the radial and angular state of the particle. The state is initially clearly a product state of the two degrees of freedom (radial and angular, being in a superposition of each), $|r_1\rangle + |r_2\rangle) |\Omega_1\rangle + |\Omega_2\rangle$). However, for a carefully chosen radii and angular frequencies, this state can (after one circle) become maximally entangled of the form $|r_1\rangle (|\Omega_1\rangle + |\Omega_2\rangle) + |r_2\rangle (|\Omega_1\rangle - |\Omega_2\rangle)$. It is then possible to proceed to detection by for instance choosing the angular frequencies as commensurable, i.e. $\frac{\Omega_1}{\Omega_2} = \frac{p}{q}$ (with $p$ and $q$ integers).
1. Discussion

As explained, the key assumptions of our argument are: (1) that the equivalence principle holds; (2) that coherent, stable superpositions of given positions and angular frequencies are possible; (3) that the coupling between the spatial and the angular degrees of freedom satisfies linearity. Specifically, via the equivalence principle, we infer that, if detected, the relevant entanglement is an indirect witness of non-classicality of gravity. Note that we have neglected the gravitational self-interaction, which is a well-motivated assumption if the object undergoing the interferometry is of small mass, \cite{15}. Note also that the observed phase does not contain Newton’s gravitational constant, but it can still be used as a witness of non-classicality assuming the equivalence principle to hold locally point by point along the trajectory of the superposed particle. It is an indirect witness of non-classicality of gravity. In our previously proposed experiment \cite{3}, the path entanglement between two interfering masses was proven to be a witness of the fact that the mediating gravitational field is non-classical. In the present experiment, both degrees of freedom that become entangled belong to the same particle. In this sense, it is at first hard to see what the mediator is. This becomes clear by considering that the spinning disk generates a curvature in space: each spinning frequency corresponds to a different curvature and therefore to a different gravitational field. Therefore, witnessing entanglement in this situation would, too, imply that the field has to be quantum, if one assumes that acceleration and gravitation are equivalent and that the equivalence principle satisfies linearity of quantum theory (meaning that it holds in each branch of the superposition). In order to explain the entanglement one has to resort to the superposition of two different spacetime geometries, one for each frequency, (similarly to what described in \cite{16}). This is an important example of the generality of our argument for the non-classicality of gravity as a mediator of entanglement, where it becomes clear that the locality assumption is to be intended more generally than in the context of a propagator between spatially separated locations (see also the general assumptions in \cite{4, 5}).

A final thought-provoking comment on the testability of this idea: it might seem that the experiment involving a superposition of different rotations and different spatial locations is difficult to perform in practice. First, consider that the complexity of the proposals in \cite{2, 3} is greatly reduced by the fact that there is only one
mass. In addition, this experiment could be much closer to realisation than it seems. If we assume that the particle is an electron in e.g. the Hydrogen atom, and that this electron is superposed across two different principal quantum numbers and across two different orbital quantum numbers, we have exactly the scenario we need: the entanglement could be created between these degrees of freedom of the electron, if such superpositions were possible. For example, the atomic states $j = 1, m = 1$ and $j = 1, m = -1$ can be used to implement a long lived quantum bit. Namely, the electron in this state has one unit of angular momentum, i.e. $m_\omega = 1$, where $m_\omega$, $v_\omega$, $r_\omega$ are the mass, velocity and the radius of the electron respectively, but the two $m = \pm 1$ states rotate in the opposite directions, thus providing each two different angular frequencies as in our analysis. If we start with an equal superposition of two ‘counter-rotating’ states, $|j = 1, m = 1\rangle + |j = 1, m = -1\rangle$, we would expect that after one half period of rotation the state would evolve into

$$|j = 1, m = 1\rangle + e^{i2m_\omega \Omega A / h} |j = 1, m = -1\rangle$$  (4)

where $\Omega$ is the electron’s angular frequency and $A = \pi r_\omega^2$ the corresponding area described by its orbit. The numbers are encouraging; given that $\delta \phi \approx m_\omega \Omega A / h \approx 1$, we already have conditions in nature for the maximum entanglement to be created via the mechanism we suggested. If the equivalence principle holds in the form we have envisaged, then already some of the phases acquired in the ordinary atomic physics situation would contain a proof of the quantum nature of gravity.

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ORCID iDs

Chiara Marletto  𝖳 HTTPS://ORCID.ORG/0000-0002-2690-4433

References

[1] Pais A 1992 Subtle is the Lord: The Science and Life of Albert Einstein (Oxford: Oxford University Press)
[2] Bose S et al 2017 Phys. Rev. Lett. 119 240401
[3] Marletto C and Vedral V 2017 Phys. Rev. Lett. 119 240402
[4] Marletto C and Vedral V 2017 Npj Quantum Information 3 29
[5] Marletto C and Vedral V 2020 Phys. Rev. D 102 086012
[6] Anastopoulos C and Hu B L 2018 Class. Quant. Grav. 35 035011
[7] Hardy I 2019 arXiv:1903.01289
[8] Zych M and Brukner C 2018 Nat. Phys. 14 1027–31
[9] Rosi G et al 2017 Nat. Commun. 8 15529
[10] Colella R, Overhauser A W and Werner S A 1975 Phys. Rev. Lett. 34 1472–4
[11] Werner S A, Staudenmann J L and Colella R 1979 Phys. Rev. Lett. 42 1103
[12] Alting P and Fuentes I 2012 Class. Quantum Grav. 29 086012
[13] Sagnac G 1913 Comptes Rendus 157 708
[14] Rizzi G and Ruggiero M L 2003 Relativity in Rotating Frames ed G Rizzi and M L Ruggiero (Dordrecht: Kluwer Academic Publishers)
[15] Howl R, Vedral V, Naik D, Christodoulou M, Rovelli C and Iyer A 2021 PRX Quantum 2 010325
[16] Christodoulou M and Rovelli C 2019 Phys. Rev. B 792 64–8