Linear Optics Simulation of Quantum Non-Markovian Dynamics

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The simulation of open quantum dynamics has recently allowed the direct investigation of the features of system-environment interaction and of their consequences on the evolution of a quantum system. Such interaction threatens the quantum properties of the system, spoiling them and causing the phenomenon of decoherence. Sometimes however a coherent exchange of information takes place between system and environment, giving rise to memory effects and thus making the dynamics of the system becomes non-Markovian. Here we report the experimental realisation of a non-Markovian process where system and environment are coupled through a simulated transverse Ising model. By engineering the evolution in a photonic quantum simulator, we demonstrate the role played by system-environment correlations in the emergence of memory effects.

The simulation of quantum processes is a key goal for the grand programme aiming at grounding quantum technologies as the way to explore complex phenomena that are inaccessible through standard, classical calculators. Some interesting steps have been performed in this direction: simple condensed matter and chemical processes have been implemented on controllable quantum simulators. The relativistic motion and scattering of a particle in the presence of a linear potential has been demonstrated in a trapped-ion quantum simulator that opens up the possibility to the study of quantum field theories. The quantum Ising model has been experimentally analysed under the perspective of universal digital quantum simulators and very recently scaled up to hundreds of particles.

This scenario has recently been extended to open quantum evolutions, marking the possibility to investigate important features of the way a quantum system interacts with its environment, including the so-called sudden death of entanglement induced by a memoryless environment. This interaction can destroy the most genuine quantum properties of the system, or involve exchange of coherence between system and environment, giving rise to memory effects and thus making the dynamics “non-Markovian”. Characterizing non-Markovian evolutions is currently at the centre of extensive theoretical and experimental efforts. Here we demonstrate experimentally the (non-)Markovianity of a process where system and environment are coupled through a simulated transverse Ising model. By engineering the evolution in a fully controlled photonic quantum simulator, we assess and demonstrate the role that system-environment correlations have in the emergence of memory effects.

The paradigmatic description of quantum open dynamics involves a physical system $S$ evolving freely according to the Hamiltonian $H_S$ and embedded in an environment $E$ (whose free dynamics is ruled by $H_E$). System and environment interact via the Hamiltonian $H_{SE}$, which we assume to be time-independent for easiness of description. While the dynamics of the joint state $\rho_{SE}$ is closed and governed by the unitary operator $U_t = e^{-iH_{SE}t}$, the state of $S$, which is typically the only object to be accessible directly, is given by the reduced density matrix $\rho_S(t) = \text{Tr}_E[\rho_{SE}(t)]$. For factorized initial states $\rho_{SE}(0) = \rho_S(0) \otimes \rho_E(0)$ and moving to an interaction frame defined by the free Hamiltonian of the total system, such reduced evolution can be recast in the operator-sum picture $\rho_S(t) = \sum_{\mu} K_{\mu}(t) \rho_S(0) K_{\mu}^\dagger$, where $\{K_{\mu}\}$ is the set of trace-preserving, non-unitary Kraus operators of $S$ that are responsible for effects such as the loss of populations and coherence from the state of the system.

The roots of such decoherence mechanism have long been studied, together with their intimate connection to the so-called measurement problem and the implications of the collapse of the wave function. Numerous experimentally oriented techniques have been proposed to counteract decoherence. Yet, a complementary viewpoint can be taken, where the possibility to engineer structured environments and tailored $S-E$ couplings is seen as a resource to achieve longer coherence times of the system, prepare entangled states, perform quantum
entanglement increase, the dynamics is necessarily non-Markovian. For certain time-windows there is a kick-back from tonically in time, the dynamics of on the modifications induced by the emergence of non-Markovianity in the evolution of Methods.

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Some of them have been recently used in order to characterize, both quantification of the degree of non-Markovianity of a process26–29. evolutions, the formulation of criteria for the emergence of non-Markovianity of a non-Markovian dynamics computation, and realise quantum memories. This calls for the exploitation of memory-effects typical of a non-Markovian dynamics as a useful tool for the processing of a quantum state.

Unfortunately, a satisfactory understanding of non-Markovianity is yet to be reached, which motivates the recent and intense efforts performed towards the rigorous characterisation of non-Markovian evolutions, the formulation of criteria for the emergence of non-Markovian features, and the proposition of factual measures for the quantification of the degree of non-Markovianity of a process26–29. Some of them have been recently used in order to characterize, both theoretically and experimentally, the character of system-environment interactions for few- and many-body quantum systems26–29. Here we use a photonic setup to simulate a system-environment coupling ruled by a transverse Ising spin model (see Supplementary Material). Such interaction gives rise to the non-Markovian evolution of S, as witnessed by the non-monotonic behavior of its entanglement with an ancilla A that is shielded from the environmental effects. Our goal is the experimental investigation of the fundamental connection between system-environment correlations and non-Markovianity. The simulator that we propose allows for the implementation of various free evolutions of S and E, as well as the adjustment of their mutual coupling, thus making possible the transition from deeply non-Markovian dynamics, all the way down to a fully forgetful regime.

Results

Experimental setup and theoretical model. The simulator is experimentally realised using different degrees of freedom of the photonic device shown in Fig. 1, which consists of the concatenation of two Mach-Zehnder interferometers and a single Sagnac loop. The information carriers are two photons (referred hereafter as “high” and “low”): the system S is embodied by the polarisation of the low photon (which can be either horizontal $|H\rangle = |0\rangle$ or vertical $|V\rangle = |1\rangle$), while the environment E is encoded in the longitudinal momentum degree of freedom (the path) of the same photon (which will be right $|r\rangle = |0\rangle$ or left $|l\rangle = |1\rangle$). The ancilla A is embodied by the polarisation of the high photon. High and low photons are emitted by the source of polarization-entangled described in the Methods.

The ancilla is the key tool for our goals. Indeed, to investigate the emergence of non-Markovianity in the evolution of S due to its interaction with E, we use the method proposed in Ref. 27: we focus on the modifications induced by the S-E coupling on a prepared entangled state of S and A. If the S-A entanglement decays monotonically in time, the dynamics of S is fully Markovian. Differently, if for certain time-windows there is a kick-back from E that makes such entanglement increase, the dynamics is necessarily non-Markovian.

In fact, if the local action of the environment is no longer represented by a continuous family of completely positive maps, the S-A entanglement is no longer constrained to decrease monotonically. This is evidence of the flow-back of coherence on the system and results in an increment of the S-A entanglement.

As in other digital quantum simulators, the dynamics is approximated by a stroboscopic sequence of quantum gates. Conceptually, the simulation consists of the forward evolution of S over discrete time slices according to a Trotter-Suzuki decomposition of the total time propagator (see Supplementary Material). This approach is known to be effective for quantum simulation and is implemented here by the sequence of operations shown in Fig. 2a). The free evolution of the environment is accounted for by the Hadamard gate $\mathcal{H}_E = (\sigma_x^E + i\sigma_z^E)/\sqrt{2}$ ($\sigma_m^E$ is the $m = x, y, z$ Pauli matrix of qubit $j = S, E$ and $\mathcal{H}_E$ is the Hadamard gate of qubit $j = S, E$), while $\mathcal{H}_S \times \sigma_3^E$. The S-E interaction is engineered by implementing the controlled-rotation $G^{E_S} = |0\rangle\langle 0| \otimes I^S + |1\rangle\langle 1| \otimes \mathcal{R}_S^E (\phi)$, which rotates the system according to the general single-qubit operation $\mathcal{R}_s^E (\phi) = \cos \phi I - i \sin \phi \sigma_z^E$, depending on the state of the environment. This class of conditional operations is obtained from Hamiltonian generators of the two-qubit transverse Ising form (see Supplementary Material), which motivates our choice and is thus the class of system-environment interactions that is simulated in this work. Experimentally, we fix $\phi = \pi/4$ so as to implement a conditional-$\sigma_z^E\mathcal{H}_E$ gate. By using the identity $\sigma_x^E = \mathcal{H}_S \sigma_x^E \mathcal{H}_S$, it is straightforward to prove that this gate is locally equivalent (via $\sigma_z^E$) to the composition of a controlled Hadamard and a controlled anti-Z (i.e. a gate applying $\sigma_z^E$ only when E is in $|0\rangle$), namely $G^{E_S} = CH_{1E} C\mathcal{Z}_{1S} = |0\rangle\langle 0| \otimes I^S + |1\rangle\langle 1| \otimes \mathcal{H}_S$. The extra $\sigma_z^E$ needed to make the two conditional gates equivalent can thus be absorbed in $\mathcal{H}_S$, which thus becomes $\mathcal{H}_S^3$.

This set of gates is experimentally realised in our photonic simulator as sketched in Fig. 2b). The $\mathcal{H}_S^3$ gate is implemented by means of a beam splitter (BS) in conditions of temporal and spatial indistinguishability of the optical modes. This scheme allows to evolve the input modes $|r\rangle$ and $|\ell\rangle$ at each step as

$$|r\rangle \rightarrow \frac{1}{\sqrt{2}} (|r\rangle + e^{i\phi}|\ell\rangle), \quad |\ell\rangle \rightarrow \frac{1}{\sqrt{2}} (|r\rangle - e^{i\phi}|\ell\rangle),$$

(1)

where the phases $\phi_i$ are varied by rotating thin glass plates intercepting one of the optical modes entering the BS. In order to perform a more general rotation of $E$ it will be necessary to unbalance the output modes (using intensity attenuators) making the probability of occurrence of $|r\rangle$ and $|\ell\rangle$ unequal.

The controlled gate $CH_{1E}$ is realised by placing a half-wave plate (HWP) on one of the output modes with optical axis at 22.5°.
with respect to the vertical direction. The temporal delay introduced by this waveplate is compensated by another HWP on the opposite output mode with optical axis set at 0° with respect to the vertical direction. This implements the C Z gate, as shown in Fig. 2b). In order to assess non-Markovianity, ancilla and system are prepared in the maximally entangled state \[ \left| \phi^+ \right>_{SA} = \left( |HH \rangle + |VV \rangle \right)/\sqrt{2}. \] This two-qubit Bell state is engineered using the polarization entanglement source of Ref. 32 (see Methods). The environment is initialised in \[ \left| \chi \right>_{E}^k = \left( |x \rangle + \sqrt{1-x^2} |\ell \rangle \right)/(x \in \mathbb{R} \text{ is set by the transmissivity of } BS_1 \text{ [cf. Fig. 1] and can be varied by placing an intensity attenuator on one of the two output modes}), \] a state endowed with quantum coherence as it is key due to the conditional nature of the dynamics that we simulate. The stability, modular structure and long coherence time of our interferometric setting allows for the repeated iteration of each block of gates \[ g^S \otimes \mathbb{1}. \]

As A does not evolve with the environment, the polarisation state of the high photon (depicted in red in Fig. 1) is immediately detected and only the low photon (in yellow in Fig. 1) goes through each sequence of gates at the various steps. A standard optical setup for the performance of quantum state tomography (QST)\(^{33}\) is used to reconstruct the state of the S-A system after the evolution. The radiation is collected by using an integrated system composed of GRaded InDex (GRIN) lens and single-mode fibre\(^34\), and is then detected by single-photon counters. For each step of the simulated dynamics, we measure the state of the environment by projecting the polarisation qubits on the states \[ |HH \rangle, |HV \rangle, |VH \rangle, |VV \rangle. \] Finally, we trace out the degrees of freedom of the S-A system by summing up the corresponding counts measured for every single projection needed for the implementation of single-qubit QST. The Pauli operators for E, generated after the first passage through the BS\(_1\), are measured using BS\(_2\). The same procedure is followed to perform the QST of the environmental state at each step. The state of the S-A system is reconstructed in a similar way, by summing up the counts collected after projecting E onto \[ |\text{r} \rangle \text{ and } |\ell \rangle. \]

**Non-Markovian simulation.** Figure 3 summarises the results obtained by running through the evolution of the overall system. In order to quantify entanglement we use the entanglement of formation EOF(SA)\(^{35}\) between S and A, which is operatively linked to the cost of engineering a given state by means of Bell-state resources (see the Methods section). We are also interested in the correlations shared by the environment with the rest of the system, hence we evaluate the von Neumann entropy of the environment, defined as \[ S(E) = -\text{Tr}[\rho_E \log_2(\rho_E)], \] which under the assumption of pure total states quantifies the entanglement in the partition AS against E. Figure 3a) shows the experimental entanglement of formation at each time step (black-square points) against the results of a theoretical model (red line) that, including all the most relevant sources of imperfections, deviates from the ideal picture sketched above. First, the BS\(_2\) are not entirely polarisation-insensitive: for BS\(_2\), and H (V) polarisation the reflectivity over transmissivity ratio \[ R/T = 42/58 \text{ (55/55)} \] whereas for BS\(_1\), \[ R/T = 45/55 \text{ (55/45)} \]. Second, although the desired input state \[ |\phi^+ \rangle_S \] is created with high fidelity (\( \approx 93\% \)), see Supplementary Material), the entangled-state source generates spurious \[ |HV \rangle \text{ and } |VH \rangle \] components, accounting for about 5% of the total state, which reduce the initial system–environment entanglement to about 0.8. The inclusion of such imperfections makes the agreement between theory and experimental data very good up to the fourth step of our simulation, showing at least one revival of the S-A entanglement and thus witnessing the non-Markovian nature of the evolution\(^37\). The fifth experimental point is significantly far from the theoretical model (red line) that, including all the most relevant sources of imperfections, deviates from the ideal picture sketched above. Future work should be devoted to increase the probability of detecting the entangled state by using a more perfect source of entangled pairs. We have verified that their values are not completely polarisation independent. This determines a slight difference for the four contributions \[ |HH \rangle_{SA}, |HV \rangle_{SA}, |VH \rangle_{SA}, |VV \rangle_{SA}. \] The fifth experimental point is significantly far from the theoretical behaviour because of the off-diagonal setting of the phases \[ \phi_i \] \( i = 1, 2, 3 \). In fact, we have verified that their values are not completely polarisation independent. This determines a slight difference for the four contributions between the ideal picture sketched above and the experimental data.
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Discussion

By simulating a non-trivial two-qubit coupling model, we have
demonstrated the non-Markovianity of the evolution induced on S
by a dynamical environment and a system-environment interaction
allowing for kick-back of coherence. By adopting a witness that
makes use of the effects that non-Markovianity has on entanglement,
we have explored experimentally the link between the emergence of
non-Markovianity and system-environment correlations. The next
step in this endeavour will be the experimental proof that non-
Markovianity can be used as a resource for the advantageous proces-
sing of information\textsuperscript{46,47}, such as the preparation of interesting states\textsuperscript{48},
along the lines of previous studies on state engineering and informa-
tion manipulation through Markovian processes\textsuperscript{55-57}. Our setup
will be particularly well suited for this task, in light of the effective control
over both system and environment that can be engineered both in space (thanks to the modular nature of the setting) and in time.

Methods

Source of entangled states. The black box shown in Fig. 1 of the main text represents
the source used for the generation of the hyperentangled state encoded in the
polarization and path degrees of freedom of the photons produced by spontaneous
parametric down-conversion. Here polarization entanglement, corresponding to the state $|\phi^+\rangle = \frac{1}{\sqrt{2}} (|HH\rangle + |VV\rangle)$, was realized by spatial and temporal superposition of photon pair emissions occurring with equal probability, back and forth, from a single barium borate (BBO) type-I crystal under double excitation of a vertically polarized UV CW pump beam. A suitable rotation of the polarization on one of the two possible emission directions of the photons was then applied. The source that we have used is such that the two photons, belonging to the degenerate BBO emission cone, are also path entangled, as explained in Ref. 32. For our purposes we need to select a pair of correlated directions, belonging to the emission cone surface, along
which two photons travel, obtaining in this way the high and low photons shown in red and yellow in Fig. 1.

Entanglement of formation. For two qubits, the entanglement of formation is given
by\textsuperscript{13} $\text{EOF} = \sum_{i} \lambda_i C_i^2$, where $C_i = \text{max}(0,1 - \lambda_i - \lambda_j - \lambda_k)$ for $i, j, k \in \{1, 2, 3\}$, and $\lambda_i$ is the set of eigenvalues (arranged in non-
increasing order) of the Hermitian matrix $R = \sqrt{\rho^*} R \sqrt{\rho}$ where
$\rho = (\sigma_x \otimes \sigma_x) \rho^* (\sigma_x \otimes \sigma_x)$ and $\rho^*$ is the conjugate of the density matrix.

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**Acknowledgments**

This work was supported by EU Project CHISTERA-QUASAR, PRIN 2009 and FIRB-Futuro in ricerca HYTEQ, the EU under a Marie Curie IEF Fellowship (L.M.), and the UK EPSRC (M.P.) under a Career Acceleration Fellowship and a grant of the “New Directions for Research Leaders” initiative (EP/G004579/1).

**Author contributions**

M.P. proposed the original idea, A.C. and P.M. designed the experimental setup. A.C., C.G. and P.M. performed the experiment, L.M. and M.P. analysed and interpreted the data. All authors contributed to the writing of the manuscript.

**Additional information**

Supplementary information accompanies this paper at http://www.nature.com/scientificreports/.

**Competing financial interests:** The authors declare no competing financial interests.

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**How to cite this article:** Chiuri, A., Greganti, C., Mazzola, L., Paternostro, M. & Mataloni, P. Linear Optics Simulation of Quantum Non-Markovian Dynamics. *Sci. Rep.* **2**, 968; DOI:10.1038/srep00968 (2012).