Quantization of Higher Spin Superfields
In the Anti–De Sitter Superspace

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Abstract

Lagrangian quantization of the free superfield gauge theories of higher massless superspins is performed both in the anti-de Sitter and flat superspaces. For the models under consideration the straightforward application of the BV procedure for quantization of reducible theories leads to immense calculations. In order to avoid the difficulty, a simplification of this procedure in reduction coordinates is considered. The contribution to the effective action is shown to be independent on the gauge structure of the classical formulations dually equivalent to each other. It is the same both for the actions with finite and infinite stages of reducibility.
Recently superfield gauge formulations have been constructed in the $N = 1, D = 4$ flat superspace for arbitrary massless multiplets of higher superspins [1,2], and their anti-de Sitter (AdS) counterparts have been found [3]. Thus the old problem of finding off-shell superfield realizations for the massless unitary representations of the Poincaré superalgebra has been completely solved and the results have been naturally extended in the AdS superspace. Previously such realizations were known only for the multiplets of lower superspins $s \leq 3/2$ where the choice $s = 3/2$ corresponds to linearized supergravity. The actions obtained in Refs.[1–3] constitute the manifestly supersymmetric formulations of the known free higher-spin theories given in the papers [4–8] in the form with an implicit supersymmetry [6,9,10]. The formulations of Ref.[3] are explicitly invariant under the action of the AdS superalgebra $osp(1, 4)$ and realize on-shell massless higher-spin representations of this superalgebra.

Some years ago there was a considerable progress [10–14] towards the solution of the famous higher-spin problem. In particular it was shown that consistent gravitational interactions of massless higher-spin fields exist, at least in the first nontrivial order, but turn out to be nonanalytic in the cosmological constant. In this respect the AdS formulations of Ref.[3] represent the necessary step for development of a superfield approach to the higher-spin problem.

It was mentioned in Ref.[3] that the obtained gauge models are reducible according to the terminology of Lagrangian quantization [15]. In the flat superspace the stage of reducibility is finite or infinite depending on the superspin and the type of the formulation. It means that there exists an analogy between some of the obtained models and the Green-Schwarz superstring theory. The remarkable feature of the formulations in the AdS superspace is that there is always a covariant replacement of gauge parameters which converts the infinite stage of reducibility to the finite one. This arises interest in the problem of quantization. For the superfield supergravity (the first stage of reducibility) the question of quantization was discussed in Refs.[16,17].

Quantization of the theories under consideration opens a possibility to set up the problem of calculating effective action. The effective action corresponding to lower-spin fields in the AdS space was investigated in a number of papers (see e.g. [18–25]). As to higher-spin field contributions to effective action in AdS space, they have not been considered so far. Clearly this problem is closely related to that of the higher-spin field propagators in the AdS space (see Ref.[25] where such a problem was studied from the supersymmetric point of view). We expect that our research allows to develop a completely superfield approach to the problem of effective action induced by higher-spin superfields in the AdS superspace.

In the present letter we perform the Lagrangian quantization for the formulations of Refs.[1–3]. To investigate the question the BV method [15] can be used, but its straightforward application for these models leads to immense nonlocal calculations. In order to avoid this difficulty we describe some special simplification of the BV procedure in reduction coordinates for a general quadratic action. In our case these coordinates are given by transversal irreducible superfields (ISes). The path integrals over the space of such superfields are expressed in terms of those over unconstrained superfields and chiral scalars. It allows us to receive a neat natural result and explain why in the case at hand the infinite stage of reducibility does not lead to obstacles in quantization.

1Completely superfield approach to effective action of lower spin superfields has been developed in early papers [28,29].
2. The key feature of the formulations of Ref. [3] is the use of transversally and longitudinally linear superfields. A complex symmetric superfield $\Gamma(s-1,s-1)$ satisfying the constraint

$$D^\beta_\alpha(s-1)\dot{\beta}(s-2) = 0 \Leftrightarrow (\not{D}^2 - 2(s+1)\mu)\Gamma_{\alpha(s-1)}\dot{\alpha}(s-1) = 0$$

is called transversally linear. A complex symmetric superfield $\Gamma(s-1,s-1)$ satisfying the constraint

$$\not{D}(\dot{\alpha}, G_{\alpha(s-1)}\dot{\alpha}(s-1)) = 0 \Leftrightarrow (\not{D}^2 + 2(s-1)\mu)G_{\alpha(s-1)}\dot{\alpha}(s-1) = 0$$

(the symmetrization all over the dotted indices is indicated) is called longitudinally linear. In quantization superfields $\Gamma, G$ are to be expressed in terms of unconstrained superfields $\Psi(s-1, s)$ and $\Psi(s-1, s-2)$ by the rule

$$\Gamma_{\alpha(s-1)}\dot{\alpha}(s-1) = \not{D}^\beta\Psi_{\alpha(s-1)}\dot{\beta}(s-1)$$

$$G_{\alpha(s-1)}\dot{\alpha}(s-1) = \not{D}(\dot{\alpha}, \Psi_{\alpha(s-1)}\dot{\alpha}(s-1)).$$

Being expressed in these terms the actions of the transversal and longitudinal formulations of half-integer superspin $s+1/2$ look like

$$S_{s+1/2}^{1} = \left. \frac{1}{L} \right| d^8z E^{-1} \left\{ \frac{1}{8} H^{\alpha(s)}\dot{\alpha}(s) D^\beta (\not{D}^2 - 4\mu) D\beta H_{\alpha(s)}\dot{\alpha}(s) 
- \frac{1}{8} s \mu \mu H^{\alpha(s)}\dot{\alpha}(s) H_{\alpha(s)}\dot{\alpha}(s) + [H^{\alpha(s)}\dot{\alpha}(s) D_{\alpha(s-1)} D_{\alpha} D^\beta \Psi_{\alpha(s-1)}\dot{\beta}(s-1)] 
- \frac{s+1}{8} \Psi^{\alpha(s-1)}\dot{\alpha}(s-1) D^\gamma D^\alpha \Psi_{\alpha(s-1)}\dot{\gamma}(s-1) + c.c. + 2 \Psi^{\alpha(s-1)}\dot{\alpha}(s) D_{\alpha(s-1)} D_{\alpha} \Psi_{\alpha(s-1)}\dot{\alpha}(s-1) \right\}$$

$$S_{s+1/2}^{2} = \left. \frac{1}{L} \right| d^8z E^{-1} \left\{ \frac{1}{8} H^{\alpha(s)}\dot{\alpha}(s) D^\beta (\not{D}^2 - 4\mu) D\beta H_{\alpha(s)}\dot{\alpha}(s) 
- \frac{1}{8} \frac{s}{2s+1} [D_{\beta}, \not{D}_{\beta}] H^{\beta\alpha(s-1)}\dot{\alpha}(s-1) \not{D}^\gamma [D^\gamma \not{D}_\gamma] H_{\gamma\alpha(s-1)}\dot{\gamma}(s-1) 
- \frac{s}{2} \not{D}^\gamma \not{D}_{\gamma\alpha(s-1)}\dot{\gamma}(s-1) D_{\beta} D_{\beta} H^{\beta\alpha(s-1)}\dot{\alpha}(s-1) 
+ \frac{2s}{2s+1} \not{D}_{\gamma\alpha(s-1)}\dot{\gamma}(s-1) D_{\alpha(s-1)} D_{\alpha(s-1)} \Psi_{\alpha(s-1)}\dot{\alpha}(s-2) 
- \frac{s+1}{2s+1} \Psi^{\alpha(s-1)}\dot{\alpha}(s-2) D_{\alpha(s-1)} D_{\alpha(s-1)} D_{\alpha(s-1)} \Psi_{\alpha(s-1)}\dot{\alpha}(s-1) + c.c. \right\}$$

Here $d^8z E^{-1}$ is the super AdS invariant measure, $\mu$ is the curvature of the AdS superspace, $\not{D}^2 = D_{\alpha} D^\alpha$ and c.c. stands for complex conjugation.

The gauge structure of the actions (5,6) is as follows

$$\delta H_{\alpha(s)}\dot{\alpha}(s) = \not{D}(\dot{\alpha}, L^0_{\alpha(s)}\dot{\alpha}(s)) - D_{\alpha(s-1)} L^0_{\alpha(s)}\dot{\alpha}(s)$$

$$\delta L^k_{\alpha(s)}(s-k-1) - D_{\alpha(s-1)} L^{k-1}_{\alpha(s)}(s-k-2), \quad k = 0, \ldots, s-2$$

$$\delta L^{s-1}_{\alpha(s)} = L^s_{\alpha(s)}$$

$$\delta L^s_{\alpha(s)} = 0$$

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Our notations coincide mainly with those adopted in [26,27], in particular $\not{D}_A = (D_\alpha, D_{\alpha}, \not{D})$ are the super AdS covariant derivatives. All superfields in this letter are symmetric in their undotted and dotted indices separately, the number of indices being just indicated in parentheses: $\Psi(k,l) \equiv \Psi_{\alpha(s-1)}\dot{\alpha}(s-1) \equiv \Psi_{\alpha_1 s-2} \equiv \Psi_{\alpha(s-1)}\dot{\alpha}(s-1)$.
Let us split each index into two subsets by the rule 

\[ \delta \Psi_{\alpha(s-1)} \delta(s) = -\frac{s}{2} \beta \beta L^0_{\alpha(s-1)} \delta(s) + \bar{D}^0 e^0_{\alpha(s-1)} \delta(s) \]  

(10)

\[ \delta e^k_{\alpha(s-1)} \delta(s+k+1) = \bar{D}^k e^k_{\alpha(s-1)} \delta(s+k+1) \delta(s), \quad k = 0, 1, \ldots, \infty \]  

(11)

\[ \delta \Psi_{\alpha(s-1)} \delta(s-2) = -\frac{1}{2} \beta \beta L^0_{\beta\alpha(s-1)} \delta(s-2) + i(s-1) \beta \beta L^0_{\beta\alpha(s-1)} \delta(s-2) \]  

\[ + \bar{D}^0 (\delta s^0_{\alpha(s-1)} \delta(s-3)) \]  

(12)

Here \( L^k(s, s-k-1), k = 0, \ldots, s-1; \epsilon^k(s-1, s+k+1), k = 0, \ldots, \infty; \epsilon^k(s-1, s-k-3), k = 0, \ldots, s-3 \) are complex gauge parameters and \( L^0_{\alpha(s)}; \epsilon^0_{\alpha(s)} \) are chiral superfields. Eqs.(5–15) give the initial data for the quantization of the formulations under consideration. The gauge structure (7–11) of the transversal formulation (5) has infinite stage of reducibility while the gauge structure (7–9,12–15) of the longitudinal formulation (6) (dually equivalent to the former [3]) has finite stage of reducibility s [15].

We remark that both the formulations of the theories of integer superspin have infinite chains of reducibility transformations (similar to (11)) already for the parameters \( L^k \) instead of (8). There exists a possibility in the AdS superspace to convert the infinite stage theory to the finite stage one by the change of gauge parameters [3]. But this operation has no correct flat limit.

3. In quantization of the formulations (5,6) the main technical difficulty is to bring the operator in the resultant action in path integral to a diagonal form. Already in the case of the supergravity \( s = 3/2 \) this requires the use of a nonlocal gauge [16] which for higher s turns into that containing finite series in powers of \( \Box^{-1} \) up to the 8th order. Then so-called catalist ghosts are to be introduced [16]. For higher \( s \) each catalist requires several catalists of their own and so on up to \( s \)th generation, that leads to an extremely formidable construction. Here we suggest the way out of this difficulty.

Consider a quadratic action and its reducible gauge structure (in this section we shall consider all the dynamical variables \( \phi^i \) of the theory to be bosonic)

\[ s[\phi] = s_{ij} \phi^i \phi^j, \quad i = 1, \ldots, n \]  

(16)

\[ s_{ij} Z^j_{\alpha_0} = 0, \quad \alpha_0 = 1, \ldots, m_0 \]  

(17)

\[ Z^j_{\alpha_k+1} Z^j_{\alpha_k+1} = 0, \quad \alpha_k = 1, \ldots, m_k; \alpha_{-1} = i. \]  

(18)

Let us split each index into two subsets by the rule 

\[ i = \{a, \mu_0\}, \quad a = 1, \ldots, \text{rank } s_{ij} \]  

(19)

\[ \alpha_k = \{\mu_k, \mu_{k+1}\}, \quad \mu_k = 1, \ldots, \text{rank } Z^j_{\alpha_k} \]  

(20)
i.e. \( a \) is running through the number of true physical degrees of freedom, \( \mu_0 \) is running through the number of true gauge invariances and so on. Now consider the homogeneous tensor transformation

\[
\phi^i = V^i_a \phi^a + V^i_{\mu_0} \phi^{\mu_0} \tag{21}
\]

\[
c_k^{\alpha_k} = V^{\alpha_k}_{k \mu_k} c_k^{\mu_k} + V^{\alpha_k}_{k \mu_k+1} c_k^{\mu_k+1} \tag{22}
\]
satisfying the conditions

\[
s_{ij} V^i_{\mu_0} = 0, \quad Z^{\alpha_k-1} V^\alpha_{k \alpha_k+1} = 0. \tag{23}
\]

Then without going into the details we state that in the new coordinates from the r.h.s. of (21,22) the whole BV action is given effectively by the following

\[
S = \phi^a s_{ab} \phi^b + \sum_k \tilde{c}_k \mu_k \tilde{Z}^\mu_k \nu_k c_k^\nu_k \tag{24}
\]

where \( \tilde{c}_k, c_k \) are the Faddeev-Popov ghosts and their analogues for higher stages and

\[
\tilde{s}_{ab} = V^a_i s_{ij} V^j_b, \quad \tilde{Z}^{\mu_k}_{\nu_k} = \left(V^{-1}_{k-1}\right)^{\mu_k}_{\alpha_k-1} Z^{\alpha_k-1} V^\alpha_{k \alpha_k+1} \tag{25}
\]

When passing to the action (24) in path integral, the Jacobian of the replacement (21,22) has to be accounted. It is important to note here that at the \( k \)th stage except the ghost \( c_k^{\alpha_k} \) of the minimal sector all the fields were introduced in [15] by pairs ghost + Lagrangian multiplier with the opposite statistic. Hence the Jacobians for every pair cancel and only one Jacobian of the change (22) remains at the fixed stage.

4. The key point in our construction is the use of some off-shell irreducible superfields (ISes) to construct the decomposition (21,22) for the quantization of the longitudinal formulation (6-9,12-15). We use the abbreviation "IS" to name the transversally linear and antilinear simultaneously superfield \( \zeta(k,l) \)

\[
(\tilde{D}^2 - 2(l + 2)\mu)\zeta(k,l) = (D^2 - 2(k + 2)\mu)\zeta(k,l) = 0 \tag{26}
\]

(see also (1)), and chiral superfield \( \sigma_{\alpha(k)} \)

\[
\tilde{D}_\alpha \sigma_{\alpha(k)} = 0. \tag{27}
\]

In the case \( k = l \) the additional reality condition is admissible (such a real IS we shall denote by \( \rho \))

\[
\rho(k,k) = \tilde{\rho}(k,k) \tag{28}
\]

The superfields (26,27) describe irreducible complex representations of the super AdS algebra upon specifying the value of the quadratic Casimir

\[
Q = -\frac{1}{2} D_{\alpha\dot{\alpha}} D^{\alpha\dot{\alpha}} + \frac{1}{4} (\mu D^2 + \tilde{\mu} \tilde{D}^2) - \mu \tilde{\mu} (M^{\alpha\beta} M_{\alpha\beta} + \tilde{M}_{\dot{\alpha}\dot{\beta}} \tilde{M}^{\dot{\alpha}\dot{\beta}}), \quad [Q, D_\alpha] = 0. \tag{29}
\]
Now we define a parametrization of the superfields of the theory (6-9,12-15), separating explicitly transversal and longitudinal parts of the complex ones:

$$H_{\alpha(s)} = \sum_{k=0}^{s} D_{(\alpha_1 \cdots \alpha_{s+k} \alpha_{s+k+1} \cdots \alpha_s)} \rho_{\alpha_{s-k+1} \cdots \alpha_s} \hat{\alpha}_{s-k+1} \cdots \hat{\alpha}_s$$

$$+ \sum_{k=0}^{s-1} [D_{(\alpha_1 \cdots \alpha_{s-k} \alpha_{s-k+1} \cdots \alpha_s)} \rho_{\alpha_{s-k+1} \cdots \alpha_s} \hat{\alpha}_{s-k+1} \cdots \hat{\alpha}_s]$$

$$+ \sum_{k=1}^{s} \tilde{D}_{(\alpha_1 \cdots \alpha_{s-k} \alpha_{s-k+1} \cdots \alpha_s)} \{ \tilde{D}_{\hat{\alpha}_{s-k+1} \cdots \hat{\alpha}_s} \} \sigma + c.c.$$

(30)

$$\Psi(s-1,s-2) = \Psi^\perp(s-1,s-2) + \Psi^\parallel(s-1,s-2) \quad \text{(31.a)}$$

where

$$\Psi^\perp_{\alpha(s-1)\hat{\alpha}(s-2)} = \sum_{k=0}^{s-2} \tilde{D}^\perp D_{(\alpha_1 \cdots \alpha_{s-k} \alpha_{s-k+1} \cdots \alpha_s)} \tilde{\zeta}_{s-k} \hat{\alpha}_{s-k+1} \cdots \hat{\alpha}_s$$

$$+ \sum_{k=1}^{s-1} \frac{1}{\mu} (\tilde{D}^2 + 2(s-2)\mu) D_{(\alpha_1 \cdots \alpha_{s-k} \alpha_{s-k+1} \cdots \alpha_s)} \tilde{\zeta}_{s-k} \hat{\alpha}_{s-k+1} \cdots \hat{\alpha}_s$$

(31.b)

$$\Psi^\parallel_{\alpha(s-1)\hat{\alpha}(s-2)} = \sum_{k=1}^{s-2} \tilde{D} D_{(\alpha_1 \cdots \alpha_{s-k} \alpha_{s-k+1} \cdots \alpha_s)} \tilde{\zeta}_{s-k} \hat{\alpha}_{s-k+1} \cdots \hat{\alpha}_s$$

$$+ \sum_{k=1}^{s-2} \frac{1}{\mu} (\tilde{D}^2 - 2s\mu) D_{(\alpha_1 \cdots \alpha_{s-k} \alpha_{s-k+1} \cdots \alpha_s)} \tilde{\zeta}_{s-k} \hat{\alpha}_{s-k+1} \cdots \hat{\alpha}_s$$

(31.c)

and similarly in more condensed notations

$$L^p(s,s-p-1) = L^p(\alpha_1 \cdots \alpha_k) \hat{\alpha}_{s-k+1} \cdots \hat{\alpha}_s$$

$$+ L^{p||} (m^p(k-1,k-p-1), m^p(k,k-p-1), \mu^p_{\alpha(p+1)})$$

$$j = p, \ldots, s-1, \quad k = p+1, \ldots, s-1$$

(32)

$$e^p(s-1,s-p-3) = e^p(\alpha_1 \cdots \alpha_k, \hat{\alpha}_{s-k+1} \cdots \hat{\alpha}_s)$$

$$+ e^{p||} (a^p(k+1,k-p-2), a^p(k,k-p-2), \alpha^p_{\alpha(p+2)})$$

$$j = p+1, \ldots, s-2, \quad k = p+2, \ldots, s-2$$

(33)

In Eqs.(30-33) $\zeta$, $z$, $m$, $n$ and $a$ are subjected to the conditions (26), $\rho$ satisfies (26,28) and $\sigma$, $\phi$, $\mu$, $\nu$, $\alpha$ are chiral (27). Note that, owing to the constraints (1,26), ISes with different total number of indices cannot be mixed in a quadratic action that leads to a diagonalization of the operator in the BV action (24).

In the above parametrization the path integral for the BV action implies an integration over the ISes (26). We define the measure on the space of such superfields by the relation

$$\int d\zeta e^{i\zeta^2} = 1 \quad \text{(34)}$$
Note that the expansions (30–33) have the general structure
\[
\Phi(k, l) = \zeta_0(k, l) + \sum_I \zeta_I(k_I, l_I)
\]
with \(k_I + l_I < k + l\). One can show that for an arbitrary operator \(A\) of the form
\[
\prod_i (Q - q_i \mu \bar{\mu})
\]
the relation holds (under the integration over the superspace):
\[
\Phi A \Phi = \zeta_0 A \zeta_0 + \sum_I \zeta_I B_I A \zeta_I
\]
where \(B_I\) have the similar form (36). Eq.(37) enables us to express Gaussian integrals over the ISes (26)
\[
\int d\zeta e^{\zeta A \zeta}
\]
in terms of the integrals over the unconstrained and chiral superfields which are usually defined [27,31]. This can be done by induction, the first step being the integration over chiral superfields. Then the integral over \(\zeta_0\) from (35) can be expressed via the integrals over the unconstrained superfield \(\Phi\) and the ISes \(\zeta_I\) with lower total number of indices:
\[
\int d\zeta_0(k, l) e^{\zeta_0 A \zeta_0} = \int d\Phi e^{\Phi A \Phi} \left[ \prod_I \int d\zeta_I e^{\zeta_I A \zeta_I} \right]^{-1}.
\]
The Jacobian of the change (35) takes the following form:
\[
J = \left[ \prod_I \int d\zeta_I e^{\zeta_I B_I \zeta_I} \right]^{-1}
\]
Note that owing to Eq.(39) we can consider the Jacobian for each IS separately.

5. Let us continue with a description of the contributions to the effective action. Consider, for example, the sector of the ISes \(\rho(k, k), \rho'(k, k), \) and \(\zeta(k, k)\) from (30,31) being transformed with the parameter \(z^0(k, k)\) from (40) for \(k = 0, \ldots, s - 2\). To satisfy the conditions (23) one should pass to the gauge invariant combination \(\zeta'(k, k)\),
\[
\zeta(k, k) \rightarrow \zeta'(k, k) = \zeta(k, k) + \frac{s + k + 1}{s} [Q - (s(s + 1) + k(k + 2))\mu \bar{\mu}] \left\{ \frac{i}{2} \rho(k, k) + \rho'(k, k) \right\}
\]
the Jacobian of the change (40) being unit. One can derive from Eqs.(37,39) at \(A = 1\), that the relevant contributions to the Jacobian of the change (21,22), \(k = 0\) and to the action \(\phi^a \overline{s}_{ab} \phi^b\) in (24) are determined by the relations
\[
H(s, s)^2 \sim \sum_k \rho(k, k) \Omega(s - k, k, k) \rho(k, k) + \sum_k \rho'(k, k) \Omega(s - k, k, k) \rho'(k, k)
\]
\[ \Psi(s - 1, s - 2)^2 \sim \sum_k \zeta(k, k) \Omega(s - k - 1, k, k) \zeta(k, k) \]

\[ (L^0(s, s - 1))^2 \sim \sum_k \xi^0(k, k) \Omega(s - k, k) \xi^0(k, k) \]

\[ S^I \sim \sum_k \zeta'(k, k) \Omega(s - k - 1, k, k) \zeta'(k, k) \]

\[ \Omega(m, k, l) = \prod_{j=1}^{m} \left[ Q - \frac{\mu \bar{\mu}}{2} ((j + 1)(j + k + l) - (k + l)(k + l + 1)) \right] \]

where only the sector at issue is extracted and numerical factors are omitted. As the contributions from (43,44) enter with the sign opposite to that for (41,42), the overall cancelation occurs. Then the transformation law for the ISes \( \rho, \rho' \) with the parameter \( z^0 \) does not contain derivatives, so the correspondent block in \( \tilde{Z}_\nu^\mu \) from (24) has unit determinant. The analogous cancelation takes place in the sector of the ISes \( \zeta'(k, k - 1), \zeta(k, k - 1), k = 1, \ldots, s - 1, \) (38,39).

In fact, after all cancellations the remained contributions are given in our scheme by the following sectors of Eq.(30) only. The gauge invariant IS \( \rho(s, s) \) enters the action (24) and the purely gauge IS \( \zeta'(s, s - 1) \) contributes to the Jacobian of the change (30). The contributions prove to be proportional, giving the summary result

\[ -\frac{1}{2} \text{Tr}_{\rho(s,s)} \ln[Q - s(2s + 3)\mu \bar{\mu}] + \frac{1}{4} \text{Tr}_{\zeta(s,s-1)} \ln[Q - s(2s + 3)\mu \bar{\mu}] \]

where \( \text{Tr}_{\rho(s,s)} \) and \( \text{Tr}_{\zeta(s,s-1)} \) denote the trace of the operator in the space of IS \( \rho(s, s) \) and \( \zeta(s, s - 1) \) respectively. The result is natural firstly because \( Q = s(2s + 3)\mu \bar{\mu} \) is exactly the value of the Casimir in the massless representation of the super AdS algebra with the superspin \( s + 1/2 \). Secondly Eq.(46) gives effectively the trace of the operator in the space of an irreducible representation. This can be established by counting the dimensions of the ISes \( \rho(s, s) \) and \( \zeta(s, s - 1) \). In terms of the dimension of a complex scalar field, \( \text{dim} \zeta(k, l) = 4(k + l + 2), \text{dim} \rho(s, s) = 2(2s + 1) \). So Eq.(46) means the trace over the space with the dimension

\[ \text{dim} \rho(s, s) = \frac{1}{2} \text{dim} \zeta(s, s - 1) = 2 \]

that is exactly the dimension of the massless representation of the AdS superalgebra [30].

All the contributions from the rest of ghost sector mutually cancel. In some more detail the ISes \( a, \alpha \) (33) and \( m, \mu \) (32,33) are purely gauge superfields being transformed under (8,9,13–15) through the parameters \( z, \phi \) (32) and certain linear combinations \( l, \lambda \) of the parameters \( m, \mu \) (32) and \( n, \nu \) (33) without derivatives. The Jacobian of the change \( \{ n, \nu \} \rightarrow \{ l, \lambda \} \) is unit. The Jacobian of the change (33,34) corresponding to the ISes \( a, \alpha, m, \mu \) is cancelled by that corresponding to the ISes \( z, \phi, n, \nu \). This ensures that every ghost stage in (22,24) gives zero contribution to the effective action. Thus even if the theory had the infinite stage of reducibility the ghost contribution in the effective action would be equal to zero.

The latter phenomenon works for the transversal formulation (5) with the gauge structure (7–11) of the infinite stage of reducibility and enables us to quantize this formulation. It turns out that after a series of cancellations the remained contributions are the same (46). This proves the quantum equivalence of the formulations (5,6) which are classically dual to each other [1–3]. Since we have used the expression (3) for the variable \( \Gamma(s - 1, s - 1) \), the considered quantization admits the flat limit.
Thus we have accomplished the lagrangian quantization of the known superfield formulations of massless theories of an arbitrary half-integer superspin. The result (46) has the sense of the trace in the physical subspace of the logarithm of the massless Casimir \( Q - s(2s + 3)\mu\bar{\mu} \). The reducibility iterations in the ghost sector gives zero contributions no matter is the stage finite or not. Hence the theories of integer superspins can be quantized in similar fashion.

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