Reciprocal interparticle attraction in complex plasmas with cold ion flows

R Kompaneets\textsuperscript{1,3}, S V Vladimirov\textsuperscript{2}, A V Ivlev\textsuperscript{1} and G Morfill\textsuperscript{1}

\textsuperscript{1} Max-Planck-Institut für extraterrestrische Physik, 85748 Garching, Germany
\textsuperscript{2} School of Physics, The University of Sydney, New South Wales 2006, Australia
E-mail: kompaneets@mpe.mpg.de

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\textbf{Abstract.} The paper investigates, in the framework of the kinetic approach, the linear screening of a point non-absorbing charge in a homogeneous collisionless plasma with cold ion flow. The velocity distribution of ions far from the charge is assumed to be a shifted Maxwellian distribution with infinitesimal thermal velocity and finite flow velocity, and the electron component is considered as a homogeneous neutralizing background. The potential in the direction perpendicular to the flow is found to have an attractive part (i.e. two like charges aligned perpendicular to the flow can attract each other electrostatically). The paper addresses in detail the relevance of the model to the screening of charged dust particles levitated in the plasma–wall transition layer. In particular, by using the Bohm collisionless sheath model the paper shows that in certain cases electrons do not contribute significantly to the screening of dust particles: deep in the non-neutral sheath the electron density is much smaller than the ion density, and in the quasineutral presheath (well above the sheath entrance) kinetic energies of ions are small as compared with the electron temperature. Also, the model of the present paper is shown to be in agreement with existing measurements of the interaction potential between two dust particles aligned perpendicular to the ion flow in the plasma–wall transition layer of a radio-frequency discharge (Konopka \textit{U et al} 2000 \textit{Phys. Rev. Lett.} \textbf{84} 891) and predicts a minimum of the interaction potential to be located outside the distance range in which the measurements were performed.

\textsuperscript{3} Author to whom any correspondence should be addressed.
1. Introduction

In the field of complex plasmas [1]–[3], one of the challenging problems is the problem of obtaining reciprocal attraction between negatively charged dust particles in plasmas. Systems of dust particles interacting via a potential with an attractive part could be used, for example, as models for studying gas-to-liquid transitions and critical point phenomena [4].

One effect that may cause reciprocal attraction between dust particles is the so-called shadowing force [4, 5]. However, the possibility of attraction due to this effect in complex plasmas has still not been demonstrated experimentally.

Attraction between dust particles is possible in the presence of anisotropy of the velocity distribution of ions in the surrounding plasma (e.g. in the presence of ion flow). In the presence of anisotropy of the ion velocity distribution, the screening cloud around a charged dust particle is not spherically symmetric, which may allow attraction in certain directions but leads to non-reciprocal interaction forces: in experiments [6, 7], two dust particles of different sizes/masses were levitated in the plasma–wall transition layer of a radio-frequency (rf) discharge, in the presence of an ion flow directed downward, and the lower particle strongly tended to occupy the position below the upper particle whereas the upper particle almost did not ‘feel’ the lower one.

However, if two identical particles are aligned perpendicular to the flow, their interaction forces in the plane perpendicular to the flow are reciprocal because of the symmetry. Can these reciprocal forces be attractive? (If so, this effect can be used to study 2D systems of particles interacting via a potential with an attractive part.) Konopka et al performed measurements of the interaction potential between two identical dust particles aligned perpendicular to the ion flow.
in the plasma–wall transition layer of an rf discharge and found no attractive part [8, 9]. This, however, does not mean that two like charged dust particles aligned perpendicular to the ion flow cannot ever attract each other, since the measurements of Konopka et al were performed in a limited range of plasma parameters and distances between the particles.

In the present paper, we consider a particular case of the charge screening in plasmas with ion flows—linear screening of a point charge in homogeneous collisionless plasma with cold ion flow in the absence of the electron response. (We consider the ion velocity distribution far from the charge to be a shifted Maxwellian distribution with infinitesimal thermal velocity and finite flow velocity, and we consider the electron component as a homogeneous neutralizing background.) The physics of the charge screening in this case is as follows. Streaming ions are deviated by the charge, which results in the ion density perturbation and hence in the screening of the Coulomb potential of the charge. The characteristic size of the screening cloud is determined by the condition that the time during which ions pass through the screening cloud is about the inverse ion plasma frequency, which means that the characteristic size of the screening cloud is the ratio of the ion flow velocity to the ion plasma frequency. The present paper calculates the spatial distribution of the potential around the charge and, in particular, addresses whether the potential in the direction perpendicular to the flow has an attractive part.

The model assumptions of cold ion flow and absence of the electron response are not simply ad hoc assumptions. In certain cases, these assumptions are reasonable for modelling the screening of charged dust particles levitated in the plasma–wall transition layer. Indeed, the measurements [10] of the ion velocity distributions near one of the electrodes of an rf discharge show that when the gas pressure is small and/or the rf power is large the flow velocity is much larger than the dispersion of velocities. Concerning electrons, in certain cases they do not contribute significantly to the screening of dust particles. For example, in the quasineutral presheath (well above the entrance of the non-neutral sheath) kinetic energies of ions are small as compared with the kinetic energies of electrons, while deep in the non-neutral sheath the electron density is much smaller than the ion density. These issues are discussed in detail in section 5.3. The present paper also provides a quantitative comparison of the derived potential with the aforementioned experiment of Konopka et al.

Our problem is a particular case of the general problem of the charge screening in a streaming Maxwellian plasma (or, in another reference frame, the problem of the screening of a moving charge in a stationary Maxwellian plasma). This general problem is one of the most fundamental and extensively studied problems in plasma physics [11]–[15]. However, to our best knowledge, the limit of infinitesimal thermal velocity and finite relative charge–plasma velocity has not yet been the subject of a detailed investigation (including contour plots of the potential, graph of the potential in the direction perpendicular to the flow, and asymptotic expressions for the potential at large distances). It is also worth noting that we do not have to investigate the stability of the unperturbed system because a Maxwellian plasma is well known to be stable [16].

2. Objective

The objective of the present paper is

1. to investigate the linearized potential of a point non-absorbing charge in a homogeneous collisionless plasma with cold ion flow in the absence of the electron response and
2. to quantitatively compare the derived potential with the experiment [8, 9].
The present paper also addresses the relevance of this model to the screening of charged dust particles levitated in the plasma–wall transition layer.

3. Methods

We consider a non-absorbing point charge $Q$ embedded in homogeneous quasineutral collisionless plasma with ion flow. In the frame of the charge, the velocity distribution of ions far from the charge is $f(v)$. The electron density is assumed to be unperturbed by the embedded charge (i.e. we assume that the characteristic kinetic energy of electrons is much larger than that of ions so that electrons do not contribute significantly to the screening).

In the framework of the linear response formalism (i.e. as long as the first-order perturbation due to the charge is considered), the static potential $\phi$ induced in the plasma by the charge is [11, 17]

$$\phi(r) = \frac{Q}{2\pi^2} \int \frac{\exp(ikr)}{|k|^2[1 + \chi(k)]} \, dk,$$

where $r$ is the radius-vector from the charge to the observer, $\chi(k)$ is the static plasma susceptibility. For an arbitrary velocity distribution $f(v)$, the static susceptibility is [11, 18]

$$\chi(k) = -\frac{4\pi e^2}{m|k|^2} \int k \frac{df(v)}{dv} \, \frac{dv}{kv - i0},$$

where $m$ is the ion mass, $e$ is the elementary charge (all ions are assumed to be singly ionized).

When the velocity distribution $f(v)$ is a shifted Maxwellian distribution,

$$f(v) = \frac{n}{(2\pi v_T^2)^{3/2}} \exp\left(-\frac{|v-u|^2}{2v_T^2}\right),$$

the static susceptibility (2) becomes

$$\chi(k) = \frac{4\pi ne^2}{mv_T^2|k|^2} \frac{1}{\sqrt{2\pi}} \left[ P \int_{-\infty}^{\infty} t \exp(-t^2/2) \, dt - \pi i \frac{k u}{|k|v_T} \exp\left(-\frac{1}{2} \left(\frac{(ku)^2}{|k|v_T^2}\right)\right) \right],$$

where $n$ is the plasma density, $u$ is the ion flow velocity, $v_T$ is the ion thermal velocity, $P$ denotes the principal value.

In the limit $v_T \to 0$, the susceptibility (4) becomes

$$\chi(k) = -\frac{4\pi ne^2}{m(ku - i0)^2}.$$  

Equations (1) and (5) define the potential $\phi$ as a function of (i) the charge $Q$, (ii) the spatial coordinates, and (iii) the length $\lambda$ defined by

$$\lambda = \sqrt{\frac{m|u|^2}{4\pi ne^2}},$$

and no further parameters are required. Note that the length $\lambda$ is the ratio of the ion flow velocity to the ion plasma frequency.
In the present paper, we present the results of the following calculations. Starting from (1) and (5), after some algebra (given in appendix A), we derived a simple exact expression for the potential. Then we obtained a contour plot of the potential by numerically evaluating the derived expression. Furthermore, we investigated analytically the asymptotic behaviour of the potential at large distances: we expressed the potential as a function of $|r|$ and the angle $\theta$ between $r$ and $u$ and then found the first non-zero term in the expansion of the resulting expression in a series of $1/|r|$.

Comparison with the experiment [8, 9]: in the experiment [8, 9], the potential energy of the interaction between two charged dust particles aligned perpendicular to the ion flow was measured as a function of the distance between the particles. The table of the experimental data (i.e. the potential energy versus distance) is provided in [19], while a detailed description of the experimental setup is provided in the original papers of Konopka et al [8, 9]. The procedure of finding the best fit by our expression to the experimental data is completely analogous to the procedure described in [19]: the only difference is that in [19] another theoretical expression for the potential was used to fit the data.

4. Results

We derived the following exact expression for the potential:

$$
\phi(\rho, z) = \begin{cases} 
\frac{Q}{\lambda} \int_0^\infty \frac{t^2 \text{dt}}{1+t^2} J_0 \left( t \frac{\rho}{\lambda} \right) \exp \left( t \frac{z}{\lambda} \right), & \text{for } z \leq 0, \\
\frac{Q}{\lambda} \int_0^\infty \frac{t^2 \text{dt}}{1+t^2} J_0 \left( t \frac{\rho}{\lambda} \right) \exp \left( -t \frac{z}{\lambda} \right) - \frac{2Q}{\lambda} \sin \left( \frac{z}{\lambda} \right) K_0 \left( \frac{\rho}{\lambda} \right), & \text{for } z \geq 0.
\end{cases}
$$

(7)

Here, $\rho$ is the distance from the charge in the plane perpendicular to the ion flow, $z$ is the distance from the charge along the ion flow ($z > 0$ downstream and $z < 0$ upstream), $J_0$ is the zero-order Bessel function of the first kind, $K_0$ is the zero-order modified Bessel function of the second kind.

Note the logarithmic singularity of the potential (7) at $\rho \to 0$, $z > 0$. This singularity disappears when the thermal velocity is finite, which can be confirmed by existing numerical calculations of the potential in the case of a shifted Maxwellian distribution with a finite thermal velocity (see, e.g. figures 5 and 6 of [13]).

Formally, the potential (7) does not vanish for $z \to +\infty$ and finite $\rho$. This is because in the last term of (7) we omitted the factor $\exp(-0 \times z)$ which originates due to the term $-i0$ in (5).

The potential (7) is shown in figure 1. (Before performing the numerical evaluation, we transformed (7) in order to improve the convergence, see appendix B). For $z = 0$ (i.e. in the direction perpendicular to the flow), the potential (7) simplifies to

$$
\phi(\rho, 0) = \frac{Q}{\rho} - \frac{Q}{\lambda} \int_0^{\pi/2} \exp \left( -\frac{\rho}{\lambda} \cos \xi \right) \text{d}\xi.
$$

(8)

The potential (8) is shown in figure 2.

The asymptotic behaviour of the potential (7) at large distances is as follows:

$$
\phi = \frac{Q\lambda^2 (3\cos^2 \theta - 1)}{|r|^3} + O \left( \frac{1}{|r|^5} \right), \quad r \to \infty, \quad \theta \neq 0,
$$

(9)

where, as stated above, $\theta$ is the angle between $r$ and $u$. 

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Figure 1. Contour plot of the linearized electrostatic potential of a point charge in a plasma with cold ion flow in the absence of the electron response. The dimensions of the regions shown are $4\lambda \times 4\lambda$ and $40\lambda \times 40\lambda$, respectively, where $\lambda$ is the ratio of the ion flow velocity to the ion plasma frequency. The charge is in the center of each plot. In the red–yellow–green regions, the potential is of the same sign as the charge. (The potential at infinity is taken to be zero.) In the violet–blue regions, the potential is of the sign opposite to that of the charge. The closer to the red or violet end of the spectrum, the larger the absolute value of the potential is. The black lines are equipotential lines. (Here, the potential difference between different neighbouring equipotential lines is not the same.) The thick black lines are lines where the potential is zero. We can see that (i) downstream of the charge, the potential has an oscillatory structure, (ii) in the direction perpendicular to the ion flow, the potential has an attractive part (i.e. two like charges aligned perpendicular to the ion flow can mutually attract each other).
Figure 2. Potential in the direction perpendicular to the ion flow. The vertical axis is the normalized potential, and the horizontal axis is the normalized distance from the charge. The minimum is located at \( \approx 2.2\lambda \), and its depth is \( \approx 0.039 Q/\lambda \).

Figure 3. Comparison of the measurements of Konopka et al [8] with our model (8) and with the Yukawa potential. The experimental data can be fitted within experimental error both by our model and by the Yukawa potential. We can see that the potential minimum predicted by our model (8) is located slightly outside the distance range in which the measurements were performed.

Comparison with the experiment of [8, 9]: figure 3 shows that (8) fits the data within experimental error. The parameters deduced from the fit are as follows: the dust particle charge is \( \approx 1.45 \times 10^4 \) electrons and the length \( \lambda \) is \( \approx 0.79 \) mm. The normalized chi-squared value (see (5) of [19]) appears to be \( \chi^2 \approx 0.150 \). (The fact that the value of \( \chi^2 \) appears to be much less than unity can be explained by possible overestimation of the experimental uncertainties.) Note that the same experimental data can be also fitted by the Debye–Hückel (Yukawa) potential, as shown in figure 3 and in [8, 9, 19]. The fit by the Debye–Hückel (Yukawa) potential yields the...
dust particle charge \( \approx 1.66 \times 10^4 \) electrons, the screening length \( \approx 0.39 \) mm, and the normalized chi-squared value \( \chi^2 \approx 0.143 \). The model of [19] is also in agreement with these experimental data and gives \( \chi^2 \approx 0.160 \) (see [19] for details).

5. Discussion

5.1. Properties of the potential

The properties of the derived potential (7) can be summarized as follows:

1. The characteristic length of the spatial distribution of the potential around the charge is the ratio of the ion flow velocity to the ion plasma frequency.
2. At small distances \( (r \to 0) \), the potential is of the Coulomb form.
3. At large distances \( (r \to \infty) \), the charge produces a quadrupole field.
4. In the direction perpendicular to the flow (i.e. for \( \theta = \pi/2 \)), the potential has an attractive part (i.e. the charge can attract another charge of the same sign).
5. The potential downstream of the charge (i.e. at \( z > 0 \) and small \( \rho \)) has an oscillatory structure.

5.2. Comparison of our model with other models for screening in plasmas with anisotropic velocity distributions

Montgomery et al [11] demonstrated that, for the susceptibility (2) and an arbitrary anisotropic velocity distribution \( f(v) \), the potential (1) generally falls off as the inverse third power of the distance, at large distances. Our findings for our particular case are in agreement with this general inverse third power law (see (9)).

Cooper [12] considered the particular case where the velocity distribution \( f(v) \) is a weakly shifted Maxwellian distribution (i.e. the flow velocity was assumed to be much smaller than the thermal velocity). His case is opposite to that of ours, since our case can be regarded as the case of strongly shifted Maxwellian distribution. Nevertheless, he also obtained that the potential in the direction perpendicular to the flow has an attractive part.

Existing numerical calculations of the potential for a shifted Maxwellian distribution with small but finite thermal velocity [13, 14] yield an oscillatory structure of the potential and, also, an attractive part of the potential in the direction perpendicular to the flow. This is in full agreement with our findings.

There have been calculations of the potential for a shifted Maxwellian distribution with different upstream, downstream and perpendicular temperatures [20]. These calculations show that the depth of the minimum of the potential in the direction perpendicular to the flow can strongly depend on the ratio of the upstream and downstream temperatures.

Vladimirov and Nambu [21], Vladimirov and Ishihara [22] and Ishihara and Vladimirov [23] studied the screening in a plasma with cold ion flow in the presence of the Boltzmann electron response. They concluded that, if the ion flow velocity is greater than the ion sound velocity (i.e. when \( |u| > \sqrt{T_e/m} \), where \( T_e \) is the electron temperature), then an oscillatory potential is formed inside the Mach cone downstream of the charge. They did not investigate in detail the case of subsonic flow velocities and, in particular, the case of the...
absence of the electron response. Our findings demonstrate that an oscillatory structure of the potential is possible even in the absence of the electron response.

Lampe et al [24] calculated the potential by assuming a shifted Maxwellian distribution of ions (with small but finite thermal velocity) and the Boltzmann response of electrons. For \( |u| = \sqrt{T_e/m} \), they obtained a series of potential extrema located downstream of the charge (see figure 3 of their paper). For \( |u| = 0.5\sqrt{T_e/m} \), at least three potential extrema ‘behind’ the charge are evident in figure 5 of their paper. This shows that an oscillatory structure of the potential is possible not only for supersonic flow velocities.

Maiorov et al [25, 26] performed molecular dynamics 3D simulation of the kinetics of electrons and ions around one or two charges in plasma with a shifted Maxwellian distribution of ions. The ratio of the ion flow velocity to the ion thermal velocity was taken to be large but finite. For one charge, they obtained a potential well located downstream of the charge, but they did not observe an oscillatory structure of the potential. However, their results must be considered with caution, since the simulation time was comparable with the inverse ion plasma frequency.

While the present paper and the aforementioned studies deal with collisionless plasmas, the presence of ion–neutral collisions can modify the results [19, 27]: the potential at large distances can fall off as the inverse second power of the distance (see (8) of [19]).

5.3. Applications to complex (dusty) plasmas: screening/interaction of charged dust particles in the plasma–wall transition layer

In many laboratory experiments in the field of complex plasmas, dust particles of micrometre size are introduced into a weakly ionized low-pressure capacitively coupled rf discharge (see reviews [1]–[3]). The parameters of the discharge are usually as follows: inert gas (usually argon), gas pressure 0.3–50 Pa, gas temperature 300 K, discharge frequency 13.56 MHz, electrode separation 2–4 cm, peak-to-peak voltage between the electrodes 50–500 V. The resulting values of the parameters of the quasineutral plasma in the central region of the discharge are usually as follows: plasma density \( 10^7–10^{10} \text{ cm}^{-3} \) (which corresponds to the ionization fraction of \( 10^{-6}–10^{-7} \)), ion temperature 300 K (thermal equilibrium with neutrals), ions are mostly singly ionized, electron temperature/mean kinetic energy 1–5 eV. The dust particles are usually melamine-formaldehyde (1.5 g cm\(^{-3}\)) microspheres with a diameter of 5–10 \( \mu \text{m} \). They acquire large negative charges (~10\(^4\) e) determined by the balance of collecting free ions and electrons from the plasma. Under gravity conditions, dust particles are levitated in the plasma–wall transition layer near the lower electrode where the (averaged over rf period) vertical electric field acting on charged dust particles is sufficient to compensate for gravity. This electric field causes ions to drift towards the electrode and thus makes their velocity distribution anisotropic. The problem of the screening of charged dust particles in such conditions is one of the fundamental issues in the physics of complex plasmas, because the screening of charged dust particles determines the (electrostatic) interaction forces between them and thus governs their dynamics [17].

The present subsection addresses the relevance of the theoretical problem considered in the present paper to the screening of charged dust particles under the above described conditions.

We start from the discussion of what plasma component—ions or electrons—provides the primary contribution to the screening. This question is under debate: some attribute the screening to electrons [1, 3, 28, 29], some attribute the screening to ions [19, 30], and others.
suggest that both species can provide comparable contributions to the screening [22, 31]. Those who attribute the screening to electrons often justify this by the claim that dust particles are levitated in the non-neutral sheath where the ion flow velocity must, according to the Bohm criterion [32], exceed the Bohm (ion-sound) velocity. What is not taken into account in this ‘argument’ is that, in the non-neutral sheath, the electron density is significantly smaller than the ion density, which leads to the screening of a dust particle in the non-neutral sheath being, in fact, primarily due to ions as will be shown in the following.

We employ the Bohm collisionless direct-current (dc) sheath model [32]. In this model, the dynamics of ions and electrons is described by the following equations:

\[ n_i u_i = n_{i,\infty} u_{i,\infty}, \tag{10} \]
\[ \frac{m_i u_i^2}{2} + e\Phi = \frac{m_i u_{i,\infty}^2}{2}, \tag{11} \]
\[ n_e = n_{e,\infty} \exp\left(\frac{e\Phi}{T_e}\right), \tag{12} \]
\[ \frac{d^2\Phi}{dz^2} = 4\pi(n_i - n_e)e, \tag{13} \]
\[ \Phi \to 0, \quad \frac{d\Phi}{dz} \to 0, \quad \text{at } z \to -\infty, \tag{14} \]

where \( z < 0 \) is the space coordinate (\( z = 0 \) corresponds to the wall), \( n_i = n_i(z) \) and \( n_e = n_e(z) \) are the ion and electron densities, respectively, \( u_i = u_i(z) \) is the ion flow velocity, \( \Phi = \Phi(z) \) is the electrostatic potential, \( T_e \) is the electron temperature, \( n_{\infty} \) is the plasma density at \( z \to -\infty \) (i.e. at the sheath entrance), \( u_{i,\infty} \) is the ion flow velocity at \( z \to -\infty \), \( m_i \) is the ion mass, \( e > 0 \) is the elementary charge (all the ions are assumed to be singly ionized). As can be easily shown from (10)–(14), the sheath is formed (i.e. the system (10)–(14) has a non-trivial solution \( \Phi(z) \neq 0 \)) only if \( u_{i,\infty} \geq \sqrt{T_e/m_i} \). This is the Bohm criterion [32]. Concerning the screening of a dust particle levitated in this sheath, the characteristic ion and electron screening lengths are given by \( [m_i u_i^2/(4\pi e^2)]^{1/2} \) (see (6)) and \( [T_e/(4\pi n_e e^2)]^{1/2} \), respectively, where all parameters must be taken at the position \( z \) where the dust particle is levitated. The squared ratio of these screening lengths,

\[ R = \frac{m_i u_i^2 n_e}{T_e n_i}, \tag{15} \]
determines which plasma component—ions or electrons—provides the primary contribution to the screening. From (10)–(14), we easily derive the parameter \( R \) as a function of the electrostatic potential \( \Phi \),

\[ R = \sqrt{\frac{T_e}{m_i u_{i,\infty}^2}} \left( \frac{m_i u_{i,\infty}^2}{T_e} - \frac{2e\Phi}{T_e} \right)^{3/2} \exp\left(\frac{e\Phi}{T_e}\right). \tag{16} \]

This dependence is shown in figure 4. We can see that near the sheath entrance (i.e. at small \( |\Phi| \)) ions and electrons provide comparable contributions to the screening, while deep in the sheath (i.e. at large \( |\Phi| \)) the screening is primarily due to ions.

It is also worth noting that if ions enter the sheath with the Bohm velocity \( (u_i = \sqrt{T_e/m_i}) \) and a dust particle is levitated above the entrance of the non-neutral sheath (i.e. in the
Figure 4. Graph showing which plasma component—ions or electrons—provides the primary contribution to the screening of a charged dust particle levitated in the Bohm collisionless dc sheath. What is shown is the squared ratio of the ion and electron screening lengths as a function of the normalized electrostatic potential at the levitation position. (The potential at the sheath entrance is taken to be zero.) The solid line corresponds to the case where ions enter the sheath with the Bohm velocity, and the dashed line corresponds to the case where the entrance velocity is two times larger than the Bohm velocity. We can see that (i) near the sheath entrance (i.e. at small $|\Phi|$) ions and electrons provide comparable contributions to the screening, and (ii) deep in the sheath (i.e. at large $|\Phi|$) electrons do not contribute significantly to the screening.

From the above analysis, we draw the following important conclusion: the only place where electrons and ions provide comparable contributions to the screening of a dust particle is the edge between the quasineutral presheath and the non-neutral sheath, whereas both in the presheath and deep in the sheath the screening is primarily due to ions.

Now, we discuss the velocity distributions of ions. Zeuner and Meichsner [10] performed measurements of the ion kinetic energy distributions by sampling ions through an aperture in the quasineutral presheath [32], the characteristic kinetic energy of ions at the levitation position is smaller than that of electrons and hence the screening is again primarily due to ions.

While the above analysis is only valid for the dc plasma–wall transition layer, the rf plasma–wall transition layer is different. In an rf discharge, the rf excitation frequency is usually in between the ion and electron plasma frequencies so that electrons respond to the rf electric field (while ions respond to the time-averaged field only). As a consequence, the edge of the non-neutral sheath oscillates significantly [33, 34]. Hence, a dust particle cannot be always on the sheath edge, which suggests that electrons cannot contribute significantly to the screening. However, this question requires detailed investigation which is beyond the scope of the present paper. It is worth noting in this regard that in the rf plasma–wall transition layer the mean kinetic energy of electrons can significantly increase towards the electrode [33], in which case the model with constant electron temperature does not apply.

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\[ u_{\infty} = 2 \frac{\sqrt{T_e}}{m_i}, \quad u_{\infty} = \frac{\sqrt{T_e}}{m_i} \]
the grounded electrode of an rf discharge. They demonstrated that the shape of the measured distribution was different in two limits:

1. The first limit occurred at small gas pressure and/or large rf power. In this limit, the measured distribution had the form of a narrow peak (the width of the peak was much smaller than the position of the peak). In other words, the mean energy of ions was much larger than the dispersion of energies. The authors of [10] suggested that this limit occurred when the sheath thickness was smaller than the ion–neutral collision length.

2. The second limit occurred at large gas pressure and/or small rf power. In this limit, the authors observed a wide distribution, and the energy corresponding to the maximum of the distribution function was much less than both the mean energy and the dispersion of energies (the latter two were comparable with each other). The authors of [10] suggested that this limit occurred when the sheath thickness was larger than the ion–neutral collision length (i.e. when the ion flow near the electrode was mobility-limited).

Similar observations were made by Olthoff et al [35].

The model considered in the present paper is relevant to the first case, while a recently proposed model of [19] suits the second case. The model of [19] is a kinetic model that takes the ion–neutral collisions into account and defines the zeroth-order state as a homogeneous plasma with the balance of the acceleration of ions in a homogeneous external electric field and collisions of ions with neutrals. Then, the first-order perturbation due to a charged dust particle is self-consistently calculated.

The model of [19] is also relevant to the case where a dust particle is levitated in the quasineutral presheath with mobility-limited ion flow.

The fact that three models—that of [19], that of the present paper and the Debye–Hückel (Yukawa) potential—are in formal agreement with the experiment of Konopka et al can be explained by significant experimental uncertainties, small distance range and three degrees of freedom of the fit by each of the three models. It is not clear which model is more relevant to this particular experiment, since measurements of plasma parameters at the levitation position are lacking.

It is worth noting that the applicability of the model of the present paper is limited by the non-neutrality and hence inhomogeneity of the sheath. (Our model assumes a homogeneous quasineutral plasma.) In the experiment of Konopka et al, the length of inhomogeneity of the electric field in the region of particle levitation was estimated (from the measurements of the resonance frequency of the vertical oscillations of a single particle) to be about 1 mm which is comparable with the deduced screening length. Also, the measured distance between the levitation position and the electrode corresponds to about one and a half periods of the oscillatory structure shown in figure 1 (for the value of $\lambda$ deduced from the fit). Thus, a realistic model including a self-consistent modelling of the plasma–wall transition layer may not yield the oscillatory structure shown in figure 1.

Also, the applicability of the model of the present paper is limited by nonlinear effects. (Our calculations are performed by using the linear perturbation analysis.) The screening is nonlinear in a certain region around a charged dust particle, where the potential energy of ions in the field of the dust particle, $e\phi(r)$, is comparable with their characteristic kinetic energy. For kinetic energy of ions of $\epsilon_i = 2$ eV (in the sheath ions can even have much larger kinetic energies) and dust charge of $Q = 10^4$ electrons, the size of this region is $|Q|e/\epsilon_i \sim 7 \mu m$ which is two orders of magnitude less than the characteristic screening length deduced from
the measurements of Konopka et al. In such conditions, nonlinear effects are unimportant. In other situations, however, nonlinear effects may be important and may significantly change the potential structure shown in figure 1. The region around a dust particle where the screening is nonlinear can be considered as a large charged 'particle' which is linearly screened by the plasma. When the spatial dimensions of this 'particle' are about or larger than the spatial period of the oscillatory structure shown in figure 1, it is reasonable to assume that this oscillatory structure will be less pronounced or even disappear. At the same time, we see no physical reason for disappearance of the attractive part of the potential in the direction perpendicular to the flow.

5.4. Applications to electrorheological complex plasmas

In recent experiments performed onboard the International Space Station [36], dust particles were embedded in an isotropic plasma and then a linearly oscillating external electric field was applied to induce oscillating ion flow. The frequency of the oscillations of the field/flow was in between the ion plasma frequency and the inverse timescale of the dust dynamics. As a result, dust particles did not respond to the oscillations of the field, while ions responded instantaneously. In such conditions, the interaction forces between dust particles are reciprocal, because non-reciprocal forces are averaged out over the period of the oscillations of the ion flow. In this experiment, the oscillating flow was believed to be mobility-limited and the flow velocity was believed to be less than the thermal velocity. For this reason, the (time-averaged) interaction potential between dust particles was believed to have a single potential well in the direction along the flow and no attractive part in the direction perpendicular to the flow (see, e.g. (A.38) of [17]). This assumption was supported by the observed behaviour of dust particles: they arranged themselves into strings aligned along the flow.

Based on the results of the present paper, it is worth noting that in the case of linear oscillations of a cold ion flow we would obtain an interaction potential with an infinite series of minima and maxima in the direction along the flow and one minimum in the direction perpendicular to the flow. This form of the interaction potential may lead to a quite different rheological behaviour of dust particles.

6. Conclusions

We studied the linear screening of a point charge in a homogeneous collisionless plasma with cold ion flow in the absence of the electron response. Our findings can be summarized as follows:

1. The characteristic length of the spatial distribution of the potential around the charge is the ratio of the ion flow velocity to the ion plasma frequency.
2. The potential in the direction perpendicular to the flow has an attractive part (i.e., two like charges can attract each other electrostatically). This finding is very important to the field of complex plasmas, since the problem of reciprocal attraction between like charged dust particles is one of the challenging problems in this field.
3. The potential downstream of the charge has an oscillatory structure.
4. At large distances, the charge produces a quadrupole field.

We compared the derived potential with the experiment of Konopka et al who measured the interaction potential between charged dust particles aligned perpendicular to the ion flow in

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the plasma–wall transition layer of an rf discharge. Although no attraction was found in this experiment, the model of the present paper is an excellent agreement with these measurements and predicts a minimum of the interaction potential to be located outside the distance range in which the measurements were performed. However, the model of [19] and the Debye–Hückel (Yukawa) potential are also in agreement with these measurements, and it is not clear which model is more relevant to this experiment. Measurements in a wider distance range could be very helpful to clarify the issue.

We addressed in detail the applicability of the model of the present paper to the screening of dust particles in the plasma–wall transition layer and found that the applicability depends significantly on the concrete conditions. For example, if dust particles are light enough and are therefore levitated in the presheath with mobility-limited ion flow, the ion flow cannot be considered to be cold and thus the model of the present paper is not suitable. (The model of [19] is relevant to this case.) If dust particles are levitated near the entrance of the non-neutral sheath, the electron contribution to the screening can be significant, in which case our model is inapplicable as well. If dust particles are levitated deep in the non-neutral collisionless sheath, both key model assumptions—the assumption of cold ion flow and the assumption of negligibility of the electron response—are satisfied. Therefore, to observe the attractive part of the potential in the direction perpendicular to the ion flow, we propose to perform experiments with small pressure (e.g. like in experiments [37, 38]), large rf power and heavy dust particles. These measures are necessary to ensure that a collisionless non-neutral sheath is formed and that the particles are levitated deep in this sheath. We estimate the depth and location of the minimum of the interaction potential: for a particle charge of \(2 \times 10^4\) electrons, ion density of \(2 \times 10^8\) cm\(^{-3}\) (at the particle levitation position), and ion kinetic energy of 5 eV (at the particle levitation position), our model gives that the minimum is located at \(\approx 3.7\) mm and that its depth is \(\approx 14\) eV. This is a significant potential well which indeed could be detected in experiments. However, it should be noted that our model assumes a homogeneous quasineutral plasma, which is not valid for the non-neutral sheath. Therefore, although our model predicts an attractive part of the potential in the direction perpendicular to the flow, we cannot guarantee that this attraction can be observed in the non-neutral sheath. Apart from experiments, a more realistic theoretical model including modelling of the sheath would be helpful to clarify the issue.

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Appendix A. Derivation of (7)

We substitute the susceptibility (5) to (1) and employ the cylindrical coordinates:

\[
\phi = \frac{Q}{\pi} \int_0^\infty k_\perp dk_\perp \int_{-\infty}^{\infty} \frac{J_0(k_\perp \rho) \exp(ik_\parallel z) dk_\parallel}{(k_\parallel^2 + k_\perp^2)[1 - 1/(k_\parallel \lambda - i0)^2]}. \tag{A.1}
\]

Then we perform the integration over \(k_\parallel\) analytically by using the residue theorem and the Jordan lemma. (The poles of the integrand in the complex plane are given by \(k_\parallel = \pm i k_\perp\) and...
To simplify the final result, we use the following formula:

\[
\int_0^{\infty} \frac{J_0(at)}{t^2 + b^2} \, dt = K_0(ab), \quad a, b > 0.
\]  

\[ \text{(A.2)} \]

**Appendix B. Transformation of (7) in order to improve the convergence**

By using the formula

\[
\int_0^{\infty} J_0(at) \exp(-bt) \, dt = \frac{1}{\sqrt{a^2 + b^2}}, \quad a, b > 0,
\]

we transform (7) to

\[
\phi(\rho, z) = \begin{cases} 
\frac{Q}{\sqrt{\rho^2 + z^2}} \int_0^{\infty} \frac{dt}{1 + t^2} J_0 \left( \frac{t \rho}{\lambda} \right) \exp \left( \frac{t z}{\lambda} \right), & \text{for } z \leq 0, \\
\frac{Q}{\sqrt{\rho^2 + z^2}} \int_0^{\infty} \frac{dt}{1 + t^2} J_0 \left( \frac{t \rho}{\lambda} \right) \exp \left( -t \frac{z}{\lambda} \right) - 2 \frac{Q}{\lambda} \sin \left( \frac{z}{\lambda} \right) K_0 \left( \frac{\rho}{\lambda} \right), & \text{for } z \geq 0.
\end{cases}
\]

\[ \text{(B.2)} \]

The convergence in (B.2) is better than in (7), particularly at small \( |z| \).

**Appendix C. Derivation of (8)**

For \( z = 0 \), the integrand in (B.2) is an even function of \( t \) so that we can extend the integration over \( t \) to the interval \( (-\infty, \infty) \). Then, we employ the integral representation of the Bessel function,

\[
J_0(x) = \frac{2}{\pi} \Re \int_0^{\pi/2} \exp(ix \cos \xi) \, d\xi, \quad x > 0.
\]

\[ \text{(C.1)} \]

After that, the integration over \( t \) can be performed analytically by using the residue theorem and the Jordan lemma, which gives (8).

**Appendix D. Derivation of (9)**

In (7), we replace \( \rho \) and \( z \) by \( |r| \sin \theta \) and \( |r| \cos \theta \), respectively. Then, we change the variable of integration to \( \tilde{t} = t|r| \). After that, we expand the integrand in a series of \( 1/|r| \). Then, in each term, we employ the integral representation of the Bessel function (C.1). After that, the integration can be easily performed analytically, first over \( \tilde{t} \) and then over \( \xi \). These integrals converge for \( \theta \neq \pi/2 \). Therefore, for \( \theta = \pi/2 \), the asymptotic behaviour of the potential at large distances has to be derived in a different way: in (8), we change the variable of integration to \( \eta = \rho \cos \xi \) and then expand the integrand in a series of \( 1/\rho \). Then the integration over \( \eta \) can be performed analytically. The coefficient in front of \( 1/|r|^3 \) appears to be continuous at \( \theta = \pi/2 \).
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