Open Strings on the Neveu - Schwarz Pentabranes

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Abstract

We analyze the propagation of open and unoriented strings on the Neveu-Schwarz pentabranes (N5-branes) along the lines of a similar analysis for the SU(2) WZNW models. We discuss the two classes of open descendants of the diagonal models and a series of $\mathbb{Z}_2$ projected models which exist only for even values of the level $k$ and correspond to branes at D-type orbifold singularities. The resulting configurations of branes and planes are T-dual to those relevant to the study of dualities in super Yang-Mills theories. The association of Chan-Paton factors to D-brane multiplicities is possible in the semi-classical limit $k \to \infty$, but due to strong curvature effects is unclear for finite $k$. We show that the introduction of a magnetic field implies a twist of the SU(2) current algebra in the open-string sector leading to spacetime supersymmetry breaking.

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1 Introduction

Starting from the initial proposal \[1\] of interpreting open string theories as world-sheet orbifolds of left-right symmetric closed string theories, open descendents of (Type II) models have been systematically constructed \[2\]. The worldsheet consistency conditions have been further refined \[3, 4, 5\] and a large class of open descendents of rational Conformal Field Theories (CFT) have been completely understood \[6, 7\]. It was not until the advent of D-branes \[8\] that the above procedure, now termed “orientifold”, has received so much attention and interest. In the past two years D-brane techniques have lead to the (re)discovery \[9\] of peculiar $N = (1, 0)$ supersymmetric Type I vacuum configurations in $d = 6$ with various numbers of tensor multiplets and Chan-Paton (CP) symmetry breaking/enhancement initially derived in \[2\] at rational points of their moduli spaces. New Type I vacua in $d = 6$ have been derived as open descendents \[10\] of Gepner models and a chiral model in $d = 4$ has been found as the open descendant of the Type IIB theory on the Z-orbifold \[11\]. Other Type I vacua in $d = 4$ with or without D5-branes have been constructed \[12\] as “orientifolds” of Type IIB compactifications on six-dimensional abelian orbifolds.

More recently, D-branes \[13, 14\] and orientifolds \[15\] have proved to be a powerful alternative to the “geometric engineering” \[16\] of supersymmetric Yang-Mills (SYM) theories. In this context the dynamics of D-branes and orientifold planes (O-planes) in the background of NS pentabranes (N5-branes) \[17\] seems to provide a geometrical interpretation of dualities in some SYM theories \[18\]. A unifying picture emerged from the proposal \[19\] that the relevant configurations of N5-branes and D-branes be interpreted as a single M5-brane (M-theory 5-brane) wrapped around a Riemann surface. Contrary to the vacuum configurations with D-branes invading all the non-compact spacetime dimensions, the configurations of D-branes and O-planes relevant to the study of SYM theories allow for the R-R charge to leak out at infinity and the tadpole consistency conditions are not to be imposed \[20\]. Moreover, in the cases that we discuss there are no massless closed-string states and it is meaningless to cancel massive tadpoles, even for branes in compact spaces.
In this paper we apply the by now standard procedure for open-string descendants of rational conformal field theories (RCFT) to the study of open and unoriented string propagation in the presence of N5-branes. After reviewing some known facts about N5-branes [17], we determine the correct parametrization of the spectrum of the open and unoriented strings in the cases of factorized diagonal models and for a series of nonfactorized $Z_2$ projected models that exist only for even level $k$ [21]. The scaling of the various amplitudes (Annulus, Möbius strip, Klein bottle) for large $k$ allows one to identify the relevant configuration of D-branes and O-planes in the N5-brane background. We argue that tadpole cancellation needs not to be imposed and discuss the association of CP charges to D-brane multiplicities in the limit $k \to \infty$. We also argue that due to strong curvature effects the distinction between different kinds of branes becomes unclear for finite $k$. Some useful formulae for the open-descendants of the $SU(2)$ WZNW models [6, 7] are collected in the Appendix. Finally we show that the addition of a magnetic field induces a twisting of the $SU(2)$ current algebra in the open sectors that implies a breaking of spacetime supersymmetry [22].

Similar issues have been considered recently in [23] but we find their analysis incomplete and to some extent inconsistent with the non-abelian structure of the fusion algebra for $k > 1$.

2 N5-branes, D-branes and O-planes

String solitons with NS-NS magnetic charge correspond to extended objects with a 5 + 1-dimensional worldvolume, i.e. pentabranes or briefly N5-branes [17]. For Type II superstrings, the background is completely characterized by setting to zero all the R-R fields and taking

$$
\begin{align*}
\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2\phi}(dr^2 + r^2 ds_3^2) \\
e^{-2\phi} &= e^{-2\phi_o}(1 + \frac{k}{r^2}) \\
H &= dB = *de^{-2\phi} = -kd\Omega_3
\end{align*}
$$

(2.1)
where the indices $\mu, \nu = 0, 1, \ldots, 5$ are tangent to the N5-brane, while $ds^2_3$ and $d\Omega_3$ are the line and volume elements on $S^3$, respectively. The geometry of the space transverse to the N5-brane is that of a semiwormhole with the size of the throat fixed by the axionic charge $k$ (the number of coincident N5-branes). In the throat region ($r \to 0$) the dilaton diverges and the N5-brane background admits an exact CFT description as the tensor product of a $SU(2)$ WZNW model at level $k$ times a Feigin-Fuchs boson $X_4$ with background charge $Q = \sqrt{2/(k + 2)}$. After including the fermionic partners $\{\psi^i, \psi'^4\}$, the world-sheet theory gains an extended $N = (4, 4)$ superconformal symmetry which guarantees the absence of both perturbative and non-perturbative corrections in $\alpha'$ [17]. The modular invariant spectrum of the closed string excitations around the semi-wormhole background for even values of $k$ has been worked out in [21], where also other classes of 4-d backgrounds with exact $N = (4, 4)$ superconformal symmetry have been constructed.

Many other exact 4-d backgrounds (corresponding to generalized hyper-Kähler manifolds) and their Buscher T-duals [24] have been analyzed in [23] in relation to non-compact Calabi-Yau manifolds and axionic instantons. Stringy ALE instantons were thoroughly analyzed in [26]. More recently, string dualities in $d = 6$ have been given support by the observation that Type IIA (B) in the presence of $k$ coincident N5-branes is equivalent (Buscher T-dual) to Type IIB (A) superstring on an ALE space $(R^4/\Gamma_k)$ [27] at vanishing B-field [28]. The crucial observation is that the 6-d effective field theory of the symmetric N5-branes displays $N = (2, 0)$, respectively $N = (1, 1)$, supersymmetry for the Type IIA, respectively Type IIB, superstring [17]. For the Type IIB N5-brane one expects an $N = (1, 1)$ vector multiplet whose four scalar components are the collective coordinates for the translation of the N5-brane in the transverse 4-d space. This fits with the conjectured $SL(2, Z)$ U-duality of the 10-d Type IIB superstring which relates the N5-brane to the D5-branes, the world-volume degrees of freedom of the latter being massless open-string excitations in an $N = (1, 1)$ vector multiplet. On the contrary, the Type IIA N5-brane requires an $N = (2, 0)$ tensor multiplet with 5 scalars, that are very suggestive of an 11-d interpretation in terms of M-theory. Indeed the Type IIA N5-branes are conjectured to arise from M5-branes [24]. M5-branes that are wrapped around the eleventh dimension
give rise to D4-branes.

Using U-duality one can show that to a given configuration of parallel N5-branes one can add D4-branes terminating on them and (compatibly with the surviving supersymmetry) a collection of transverse D6-branes \[14, 15\]. Denoting by \(x^0, x^1, x^2, x^3, x^4, x^5\) the coordinates tangent to the N5-brane world-volume, the D4-brane worldvolume may be taken along the directions \(x^0, x^1, x^2, x^3, x^6\) (the last direction is “compactified” either on a segment or on a circle), while the D6-brane worldvolume may be taken along the directions \(x^0, x^1, x^2, x^3, x^7, x^8, x^9\). Denoting by \(N_4\) and \(N_6\) the number of D4-branes and D6-brane respectively, the effective gauge theory \[30, 31\] on the non-compact directions of the world-volume of the D4-brane \((x^0, x^1, x^2, x^3)\) is \(N = 2\) SYM with gauge group contained in \(U(N_4)\) and a “flavor” symmetry contained in \(U(N_6)\) \[14\]. One can also add Op-planes parallel to the Dp-branes and break the gauge/flavor symmetry to orthogonal or symplectic groups \[15\]. Performing a T-duality on the directions \(x^4\) and \(x^5\) the D4-(D6-) branes turn into D6- (D8-) branes \[8\].

The microscopic string excitations of BPS configuration of D-branes and O-planes in the background of \(k\) coincident N5-branes can be explicitly determined as open-string descendants of the rational CFT that describes the throat region. After discussing the case of N5-branes in a priori flat spacetime we will also discuss the open-descendants of the configurations studied in \[21, 32\], that we interpret as N5-branes at D-type singularities \[33, 20\]. Separating the N5-branes or rotating some of them \[14 \ 15\] in order to obtain \(N = 1\) configurations seems to be out of the reach of our simple CFT analysis so far and remains a challenge for future work on the subject.

### 3 From Spheres to Semi-Wormholes

After an anomalous chiral transformation, the fermionic partners \(\psi^i\) of the bosonic coordinates \(X^i\) decouple from the \(SU(2)\) currents \(J^i\). The prize one has to pay is a finite renormalization of the level \(k \rightarrow k - 2\) \[17\]. The only interacting degrees of freedom for the N5-brane CFT are the bosonic coordinates of the \(SU(2)\) group manifold, \(S^3\). Open
string propagation on $S^3$ has been considered in connection to 2-d charged black holes \cite{34}. The problem was thoroughly addressed along the lines of \cite{2} and completely solved for the $A$, $D_{even}$ and $E$ series in \cite{3} and for the $D_{odd}$ series in \cite{7}. Some relevant formulae are collected in the Appendix. The central charge of the Virasoro algebra of the $SU(2)$ WZNW models at level $k$ is $c = 3k/(k + 2)$. The conformal weights of the integrable unitary representations are

$$h_{i}^{(k)} = \frac{I(I + 1)}{k + 2}, \quad (3.1)$$

with isospin $I$ in the range $I = 0, \ldots, k/2$. The generalized character formula is given by \cite{35}

$$\chi_{I}^{(k)}(\tau, z, u) = \text{Tr}_{\mathcal{H}_{I}^{(k)}} q^{L_0 - \frac{c(k)}{24}} e^{2\pi i z J^{(3)}_{0}} = e^{2\pi i k u} q^{h_{i}^{(k)} - \frac{c(k)}{24}} \sum_{n} \frac{q^{(k+2)n^2 + (2I+1)n} \sin[\pi z (2I + 1 + 2n(k + 2))] \sin(\pi z) \prod_{n=1}^{\infty} (1 - q^n)(1 - e^{2\pi i z q^n})(1 - e^{-2\pi i z q^n})}{sin(\pi z) \prod_{n=1}^{\infty} (1 - q^n)(1 - e^{2\pi i z q^n})(1 - e^{-2\pi i z q^n})} \quad (3.2)$$

The unoriented projection can be chosen to preserve the diagonal $SU(2)$ subalgebra of the $SU(2)_{L} \times SU(2)_{R}$ current algebra. Corresponding to the two geometrical involutions on the $SU(2)$ group manifold $g \rightarrow -g^{-1}$ and $g \rightarrow g^{-1}$, there are two different unoriented projections (Klein bottle amplitudes) of the parent torus partition function. Notice that the two involutions have the same action on the integer isospin representations. Thus the two projections differ only for the cases of diagonal ($A$) and $D_{odd}$ modular invariants. On the half-integer isospin representations the first one corresponds to keeping the singlets of the diagonal $SU(2)$, while the second one corresponds to removing them from the spectrum.

A similar analysis can be carried over directly to the open-descendants of the Type IIA superstring in the background of $k$ N5-branes. One simply has to combine the $SU(2)$ characters (3.2) with the contributions of the FF boson $X^4$, the flat bosonic coordinates $X^\mu$ and the fermions $\{\psi^\mu, \psi^i, \psi^4\}$. The latter contribution is easily expressed in terms of $\theta$-functions or better, for the purpose of reading off the spectrum, in terms of the characters of the four integrable representations of $SO(2n)$ at level one $\{O_{2n}, V_{2n}, S_{2n}, C_{2n}\}$. The FF boson and each non-compact coordinate give the standard contribution $(\sqrt{\tau_2} |\eta|^2)^{-1}$. The discrete states of the FF boson are a set of zero measure and do not contribute to
the partition function [21, 32]. They however play a crucial role in the computation of correlation functions where they act as screening operators for the background charge. It would be interesting to see whether they can be related to the collective coordinates of the N5-brane.

For N5-branes embedded in a flat space-time the torus partition function for the Type IIA superstring assumes a factorized form

\[ T = (V_8 - S_8)(\bar{V}_8 - \bar{C}_8) \sum_{ab} I_{ab} \chi_a \bar{\chi}_b \]  

(3.3)

where \( I_{ab} \) is one of the \( A-D-E \) modular invariant combination of \( SU(2) \) characters\(^4\) that we label by the dimension of the corresponding representation \( a = 2I + 1 \). The unoriented projection is similar to the pure \( SU(2) \) case (see the Appendix). Note that the partition function (3.3) is effectively left-right symmetric, since the left-right interchange in the \( SU(2) \) factor, corresponding to \( g \rightarrow \pm g^{-1} \), has to be combined with a flip of the chirality of the spinors \( S_8 \leftrightarrow \bar{C}_8 \) [15]. This is consistent with the introduction of Dp-branes with \( p \) even.

Since the volume of the throat of the wormhole is quantized in units of \( k \), one can use the overall factor of \( k \) to trace the scaling of the open and unoriented amplitudes with the volume. Let us focus on the two descendants of the \( A \)-series. One can analyze the large \( k \) behavior of the partition functions given in the Appendix, keeping only the states that become massless in this limit (that is the states with \( a < \sqrt{k} \)). The Klein bottle expression for real CP charges is independent of \( k \) for large \( k \) plus subleading terms. This we interpret as an indication that the unorientifold introduces D6-branes and O6-planes in this case. The resulting CP group is a product of orthogonal and symplectic factors, namely \( \prod_{a \ odd} SO(n_a) \times \prod_{b \ even} Sp(n_b) \) or \( \prod_{a \ odd} Sp(n_a) \times \prod_{b \ even} SO(n_b) \) depending on the overall sign in (A.13). For complex CP charges, one finds amplitudes that up to subleading terms behave as \( k \) for large \( k \). This we interpret as an indication that the unorientifold

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\(^3\) Were it not for the addition of D-branes and O-planes, one would have had to truncate the spectrum as in [21, 32] via a generalized GOS projection that correlate the internal isospin to one of the two transverse Lorentz spins.

\(^4\) The three exceptional cases corresponding to the \( E \) modular invariants do not have however a clear geometrical interpretation.
introduces D8-branes and O8-planes in this case. The resulting CP group is a product of unitary factors $U(n_a)$ times (for even level $k$ only) an $SO(n_{\rho})$ or $Sp(n_{\rho})$ (where $\rho = k/2+1$) factor. Depending on the choice of the nonvanishing CP charges, the annulus partition functions of both classes of models either grow linearly or are independent of $k$.

If some of the non-compact directions $x^1 \ldots x^5$ are compactified, by T-duality transformations one can generate from the D6- and D8-branes any configuration of D$p$- and D$(p+2)$-branes with $p$ in the range 1 \ldots 6, where even(odd) $p$ correspond to the descendents of the Type IIA(B) superstring.

4 Pentabranes at orbifold singularities

Another possibility is to put the N5-branes at orbifold singularities \[20\]. In the CFT approach, a simple way to achieve this for even values of the level $k$ is to entangle the fermion boundary conditions with the $Z_2$ action on the $SU(2)$ currents that generates the D-type modular invariant from the diagonal A-type modular invariant. The resulting configuration admits a geometrical interpretation in terms of N5-branes at D-type orbifold singularities of $R^4$. For the Type II superstring the spectrum is encoded in the modular invariant torus partition function of \[21\], that can be conveniently re-expressed in terms of the “supercharacters”\[2\]

\[
Q_o = V_4O_4 - S_4S_4, \quad Q_v = O_4V_4 - C_4C_4 \\
Q_s = O_4C_4 - S_4O_4, \quad Q_c = V_4S_4 - C_4V_4
\]

The introduction of D-branes breaks the accidental $SO(4)$ symmetry to the $SO(3) \equiv SU(2)$ automorphism of the spacetime superalgebra. The characters of the internal $SO(4)$ would then break according to

\[
O_4 = O_3O_1 + V_3V_1, \quad V_4 = V_3O_1 + O_3V_1, \quad S_4 = \sigma_3\sigma_1 = C_4
\]

where $\{O_1, V_1, \sigma_1\}$ are the characters of the Ising model associated to the FF fermion $\psi^4$. After resolving the ambiguity in the modular transformation matrix $S$ one however
recovers the above characters \{Q_o, Q_v, Q_s, Q_c\}. Neglecting the (modular invariant) contributions of the bosonic coordinates as well as of the FF boson, the closed-string untwisted sector reads

\[
T_u = \frac{1}{2} \left( |Q_o + Q_v|^2 \left( \sum_{\text{odd } a} |\chi_a|^2 + \sum_{\text{even } a} |\chi_a|^2 \right) + |Q_o - Q_v|^2 \left( \sum_{\text{odd } a} |\chi_a|^2 - \sum_{\text{even } a} |\chi_a|^2 \right) \right)
\]

(4.3)

where \(a\) is the shifted weight \(a = 2I + 1 (\leq k + 1)\). Thus odd/even \(a\) corresponds to integer/halfinteger isospin \(I\). A modular S transformation displays the twisted sector

\[
T_t = \frac{1}{2} \left( |Q_s + Q_c|^2 \left( \sum_{\text{odd } a} \chi_a \bar{\chi}_{k+2-a} + \sum_{\text{even } a} \chi_a \bar{\chi}_{k+2-a} \right) + (-)^{k/2} |Q_s - Q_c|^2 \left( \sum_{\text{odd } a} \chi_a \bar{\chi}_{k+2-a} - \sum_{\text{even } a} \chi_a \bar{\chi}_{k+2-a} \right) \right)
\]

(4.4)

The spectrum consists of the \(N = (2,0)\) supergravity multiplet and five \(N = (2,0)\) tensor multiplets with mass shifted from zero by the non-trivial dilaton background \[21\], i.e. \(m^2 = Q^2/8 = 1/4(k + 2)\). The twisted sector gives rise only to massive excitations except for the case \(k = 2\), where however the semiclassical analysis is inappropriate and the model has to be interpreted in terms of non-critical \(N = 2\) strings \[21\]. For the sake of a geometrical interpretation we will stick to the semiclassical regime of very large \(k\).

There is a striking similarity between the modular invariant torus amplitude for the propagation of Type II superstrings in the wormhole geometry and the \(SU(2)_{4p+2}\) WZW model with a \(D_{\text{odd}}\) type modular invariant. In particular only the fields corresponding to the characters \(Q_o\chi_a, Q_v\chi_a\), with odd \(a\), \(Q_s\chi_\rho\) and \(Q_c\chi_\rho\) may flow in the “tube” channel and can enter in the direct channel Klein bottle amplitudes

\[
K_\pm = (Q_o + Q_v) \sum_{\text{odd } a=1}^{k+1} \chi_a \pm (Q_s + Q_c)\chi_\rho
\]

(4.5)

where \(\rho = k/2 + 1\). The overall sign of the projection in the unoriented closed-string spectrum is the only freedom left by imposing the consistency conditions.

In order to determine the correct CP charge assignments one has to determine first the relevant boundary states. The transverse channel annulus amplitude can then be written
in the form
\[ \tilde{A} = \sum_j \tilde{\chi}_j \left[ \sum_\alpha B_j^{(\alpha)} n_\alpha \frac{C_\alpha^i}{\sqrt{C_v^j}} \right]^2, \tag{4.6} \]
where \( C_b^\alpha \) and \( C_v^j \) and are the normalizations of the boundary and bulk 2-point functions, while \( B_j^{(\alpha)} \) are the reflection coefficients of the fields in the \( j^{th} \) sector of the spectrum from the \( \alpha^{th} \) type of boundary. With an appropriate choice of the normalizations \( C_b \) and \( C_v \) the reflection coefficients \( B_i^{(\alpha)} \) satisfy
\[ B_i^{(\alpha)} B_j^{(\alpha)} = \sum_k \sigma_{ij}^{k} N_{ij}^{k} B_k^{(\alpha)} \tag{4.7} \]
where \( N_{ij}^{k} \) are the fusion rule coefficients and \( \sigma_{ij}^{k} \) are just signs \[ \text{[7]} \]. In the case at hand, the non trivial part of the CFT comes from the coupling of the WZW model and the supercharacters \( Q_x \), with \( x = o, v, s, c \). Thus in (4.6) \( \tilde{\chi}_j = Q_x \chi_a \) and the index \( j \) corresponds to the pair \((x,a)\). The signs \( \sigma \) are symmetric in all pairs of indices and are given by
\[ \sigma_{ax,by,dz} = \begin{cases} (-1)^{\frac{d+1}{2}} & \text{if both } x \text{ and } y \text{ are equal to } s \text{ or } c \\ +1 & \text{else} \end{cases} \tag{4.8} \]
Note that for levels \( k = 4p + 2 \) there is an equivalent definition independent of the values of \( x, y \) and \( z \)
\[ \sigma_{ax,by,dz} = \sigma_{a,b,d} = \begin{cases} (-1)^{\frac{d+1}{2}} & \text{if both } a \text{ and } b \text{ are even} \\ +1 & \text{else} \end{cases} \tag{4.9} \]
thus for these values of \( k \) one can directly use the results of \[ \text{[7]} \] for the \( SU(2) \) model. For levels \( k = 4p \) however one must solve explicitly the system (4.7) for the reflection coefficients. The construction is simplified by the observation that the subset of fields of integer isospin is common both to the diagonal \( (A) \) and the \( D \) model and the corresponding reflection coefficients coincide, so can be read from \( \text{(A.16)} \). The remaining reflection coefficients are then easily determined from (4.7,4.8).

After a modular transformation \( S \) from the direct Klein bottle projections (1.3) we obtain for the transverse Klein bottle amplitudes
\[ \tilde{K}_\pm = \sqrt{\frac{T}{\rho}} \sum_{a \text{ odd}} \frac{\chi_a}{\sin \left( \frac{a\pi}{k+2} \right)} \left\{ Q_o \left[ \sqrt{2} (-1)^{\frac{s+1}{2}} \sin \left( \frac{a\pi (\rho \pm 1)}{2(k+2)} \right) \right]^2 + \right. \]
\[ \left. Q_v \left[ \sqrt{2} (-1)^{\frac{s+1}{2}} \sin \left( \frac{a\pi (\rho \mp 1)}{2(k+2)} \right) \right]^2 \right\} \tag{4.10} \]
Note that for $k = 4p + 2$ this expression is a linear combination of the real and complex Klein bottle amplitudes in the pure $SU(2)$ WZW model \((A.25,A.26)\). In fact this is valid for all the amplitudes below, thus the following expressions are an alternative representation of the CP charge assignments obtained in [7].

The corresponding transverse channel annulus amplitudes are

$$
\bar{A}_\pm = \sqrt{\frac{1}{\rho}} \sum_{a \ odd} \frac{\chi_a}{\sin(\alpha \pi)} \times
\begin{aligned}
&Q_o \left[ \sqrt{2} \sum_{a=1}^{k/2} (n^{(1)}_a n^{(2)}_a) \sin \left( \frac{\alpha \pi}{k + 2} \right) + \left( -1 \right)^{\frac{a^2 - 1}{8}} \left( n^{(1)}_\rho + \tilde{n}^{(1)}_\rho + n^{(2)}_\rho + \tilde{n}^{(2)}_\rho \right)^2 \right] + \\
&Q_v \left[ \sqrt{2} \sum_{a=1}^{k/2} (n^{(1)}_a - n^{(2)}_a) \sin \left( \frac{\alpha \pi}{k + 2} \right) + \left( -1 \right)^{\frac{a^2 - 1}{8}} \left( n^{(1)}_\rho + \tilde{n}^{(1)}_\rho - n^{(2)}_\rho - \tilde{n}^{(2)}_\rho \right)^2 \right] \pm \\
&\pm (-1)^p \sqrt{\rho} \chi_\rho \left[ \frac{1}{\sqrt{2}} \left( n^{(1)}_\rho - \tilde{n}^{(1)}_\rho + n^{(2)}_\rho - \tilde{n}^{(2)}_\rho \right)^2 + \\
&Q_c \left[ \frac{1}{\sqrt{2}} \left( n^{(1)}_\rho - \tilde{n}^{(1)}_\rho - n^{(2)}_\rho + \tilde{n}^{(2)}_\rho \right)^2 \right] \right] 
\end{aligned}
$$

(4.11)

where $n^{(1)}$ and $n^{(2)}$ are two sets of CP charges. The charges $n_\alpha$ correspond to boundary states that are linear combinations of the ones in the diagonal case, while $n_\rho$ and $\tilde{n}_\rho$ correspond to a splitting of one diagonal boundary state thus giving rise to multiplicities that have no counterpart in the closed sector. After a modular $S$ transformation we obtain for the Annulus partition function

$$
A_\pm = (Q_o + Q_v) \sum_a \chi_a \left[ \sum_{a,\beta=1}^{k/2} A_{a\beta} (n^{(1)}_a n^{(1)}_\beta + n^{(2)}_a n^{(2)}_\beta) + \\
\sum_{\beta=1}^{k/2} A_{a\beta} (n^{(1)}_\rho n^{(1)}_\beta + \tilde{n}^{(1)}_\rho n^{(1)}_\beta + n^{(2)}_\rho n^{(2)}_\beta + \tilde{n}^{(2)}_\rho n^{(2)}_\beta) \right] + \\
(Q_s + Q_c) \sum_a \chi_a \left[ \sum_{a,\beta=1}^{k/2} A_{a\beta} (2n^{(1)}_a n^{(2)}_\beta) + \\
\sum_{\beta=1}^{k/2} A_{a\beta} (n^{(1)}_\rho n^{(2)}_\beta + \tilde{n}^{(1)}_\rho n^{(2)}_\beta + n^{(2)}_\rho n^{(1)}_\beta + \tilde{n}^{(2)}_\rho n^{(1)}_\beta) \right] + \\
Q_o \sum_{a \ odd} \chi_a \left[ \frac{1 + \epsilon^{k,a}}{2} \left( (n^{(1)}_\rho)^2 + (\tilde{n}^{(1)}_\rho)^2 + (n^{(2)}_\rho)^2 + (\tilde{n}^{(2)}_\rho)^2 \right) + \\
\frac{1 + \epsilon^{k,a}}{2} (2n^{(1)}_\rho \tilde{n}^{(1)}_\rho + 2n^{(2)}_\rho \tilde{n}^{(2)}_\rho) \right] 
$$

(4.12)
\[ Q_v \sum_{a \text{ odd}} \chi_a \left[ \frac{1 + \varepsilon^{k,a}}{2} \left( (n^{(1)}_\rho)^2 + (\tilde{n}^{(1)}_\rho)^2 + (n^{(2)}_\rho)^2 + (\tilde{n}^{(2)}_\rho)^2 \right) + \frac{1 - \varepsilon^{k,a}}{2} \left( 2n^{(1)}_\rho \tilde{n}^{(1)}_\rho + 2n^{(2)}_\rho \tilde{n}^{(2)}_\rho \right) \right] + Q_s \sum_{a \text{ odd}} \chi_a \left[ \frac{1 + \varepsilon^{k,a}}{2} \left( 2n^{(1)}_\rho n^{(2)}_\rho + 2\tilde{n}^{(1)}_\rho \tilde{n}^{(2)}_\rho \right) + \frac{1 - \varepsilon^{k,a}}{2} \left( 2n^{(1)}_\rho n^{(2)}_\rho + 2\tilde{n}^{(1)}_\rho \tilde{n}^{(2)}_\rho \right) \right] + Q_c \sum_{a \text{ odd}} \chi_a \left[ \frac{1 + \varepsilon^{k,a}}{2} \left( 2n^{(1)}_\rho n^{(2)}_\rho + 2\tilde{n}^{(1)}_\rho \tilde{n}^{(2)}_\rho \right) + \frac{1 - \varepsilon^{k,a}}{2} \left( 2n^{(1)}_\rho n^{(2)}_\rho + 2\tilde{n}^{(1)}_\rho \tilde{n}^{(2)}_\rho \right) \right] \]

where \[ \varepsilon^{k,a} = (-1)^{\frac{k}{2} + 1 + \frac{a-1}{2}} \], and

\[ A_{a\alpha\beta} = 2 \sum_{b \text{ odd}} \frac{S_{ab}S_{ab}S_{\beta b}}{S_{1b}}. \]  

For the \( +(-) \) case and even(odd) \( k/2 \) one has to use pairs of complex charges \( \tilde{n}^{(i)}_\rho = \bar{n}^{(i)}_\rho = n^{(i)}_\rho \), while the other parametrization is real. Note that the coefficients \( A_{a\alpha\beta} \) are NOT proportional to the standard \( SU(2) \) fusion rules since the sum goes only over the odd values of \( b \). Nevertheless all \( A_{a\alpha\beta} \) take nonnegative integer values (0,1 and 2 to be precise). Eq. (4.13) gives the simplest (and best understood) example of the general construction of [36]. There are again two distinct limits for large \( k \) of these annulus amplitudes. This can be demonstrated by considering the leading terms in the amplitude proportional to \( n^2_\alpha \). In particular the limit of the contributions with \( \alpha \) small with respect to \( \sqrt{k} \) is independent of \( k \) (since e.g. \( A_{a11} = \delta_{a1} \)), while the limit of the contributions with \( \alpha \) close to \( k/2 \) grows linearly with \( k \) (since all \( A_{a\frac{k}{2} \frac{k}{2}} \) are nonvanishing). This we again interpret as an indication of the presence of D6-branes in the former and of D8-branes in the latter case. It is important to note that in all cases the Klein bottle amplitudes grow linearly with \( k \) in the limit \( k \to \infty \). This we interpret as an indication that there are always \( O8 \)-planes present. The subleading terms that correspond to \( O6 \)-planes are not easy to trace. In the cases when the annulus and Klein bottle amplitudes have different large \( k \) limits it is impossible (and in fact not necessary) to impose a cancellation of the (massive!) tadpoles.

Since there are no tadpole conditions, the transverse Möbius amplitude is only defined up to an overall sign and is obtained in the standard way from the transverse annulus and transverse Klein bottle amplitudes where we have made explicit also the relevant signs of
the reflection coefficients. After a modular $P^\dagger (= P$ in the case at hand) transformation for the direct channel we get

$$M_\pm = \left\{ \hat{Q}_a \sum_{a \text{ odd}} \hat{\chi}_a \left[ \sum_{a=1}^{k/2} (n^{(1)}_a - n^{(2)}_a) M_{aa}^\pm + (n^{(1)}_\rho + \tilde{n}^{(1)}_\rho - n^{(2)}_\rho - \tilde{n}^{(2)}_\rho) \frac{1 \pm \varepsilon_{k,a}}{2} \right] + \hat{Q}_v \sum_{a \text{ odd}} \hat{\chi}_a \left[ \sum_{a=1}^{k/2} (n^{(1)}_a + n^{(2)}_a) M_{aa}^\pm + (n^{(1)}_\rho + \tilde{n}^{(1)}_\rho + n^{(2)}_\rho + \tilde{n}^{(2)}_\rho) \frac{1 \mp \varepsilon_{k,a}}{2} \right] \right\} (4.14)$$

where the coefficients $M_{aa}^\pm$ are given by

$$M_{aa}^\pm = \frac{2}{\sqrt{P}} \sum_{b \text{ odd}} (-1)^{\frac{b^2 - 1}{8}} \sin \left( \frac{b(\rho \pm 1)\pi}{2(k+2)} \right) \frac{S_{ab} P_{ab}}{S_{1b}} (4.15)$$

It is instructive to verify that $M_{\pm}$ give a consistent symmetrization of the above annulus partition functions. In particular one can demonstrate that:

- $A_{aaa} = 0$ for all even $a$ and all $\alpha \leq k/2$;
- $A_{aaa} = M_{aa}^\pm \pmod{2}$ for all odd $a$ and all $\alpha \leq k/2$.

The resulting open-string spectrum in principle contains tachyons coming from the combinations of $Q_s$ with $\chi_a$ with small $a$, i.e. $a < \sqrt{k+2}$. In order to remove them it is sufficient to put to zero either one of the two sets of CP multiplicities, say $n^{(2)}_\alpha = 0$ for all $\alpha$. An explicit analysis of some small $k$ examples indicates however that this choice is not necessary. Of particular interest are the models with height equal to a square integer $k + 2 = \ell^2$ for even $\ell \geq 4$, since their open spectra generically contain also massless states.

Indeed the conformal weight of the lowest lying states in $Q_s \chi_\ell$ is exactly $1/2$. As an example let us consider the simplest such case corresponding to $k = 14$, where there are 9 inequivalent choices of the non-vanishing CP multiplicities that remove the tachyons from the spectrum while keeping the massless open string state. We shall present only one of these solutions that generalizes for all (even) $\ell$, namely if one chooses all $n^{(1)}_\alpha = 0$ with $\alpha = 1, \ldots, k/2$, but $n^{(1)}_\rho$ and $\tilde{n}^{(1)}_\rho$ are nonvanishing, to remove the tachyons it is sufficient to put to zero only the charges $n^{(2)}_\alpha$ with $\alpha \geq \ell(\ell - 2)/2 + 2$ as well as $n^{(2)}_\rho$ and $\tilde{n}^{(2)}_\rho$. It is not clear whether the presence of massless chiral fermions in the open-string spectrum

\[\text{5As already noted the case } k = 2, \text{ which corresponds to } \ell = 2, \text{ is degenerate and contains massless states also in the closed spectrum.}\]
implies any anomaly cancellation condition in a background where both the gravity and the vector multiplets are massive for any finite $k$.

The resulting CP group is a product of orthogonal and symplectic factors times (only in the $+(-)$ case and even(odd) $k/2$) a unitary $U(n_\rho)$ factor. Due to the absence of tadpole conditions (only massive states appear in the transverse spectrum) the values of the non-vanishing CP multiplicities remain totally undetermined. Moreover their correspondence with different Dp-branes multiplicities is far from being obvious for finite values of the level $k$, due to strong curvature effects.

As a side remark, notice that our derivation of the partition functions started from the transverse channel expressions and required a detailed knowledge of the 2d structure constants to determine the signs (4.8). It should be stressed however that there is an alternative derivation of these partition functions that uses only the fusion rules and the modular transformation matrices. Namely, one has to solve the polynomial relations [7] for the integer valued direct channel expressions $A_{\alpha\beta}$, $M_{\alpha}$ and $K_{a}$, the fusion rules $N_{abc}$ and the integers $Y_{abc}$ (A.8). The importance of this second approach, although technically more complicated in the case at hand, comes from the fact that it can be used for essentially any left-right symmetric 2d CFT.

5 Adding a Magnetic Field

As in toroidal and orbifold compactifications of open strings, the N5-brane backgrounds allow for the introduction of a constant magnetic field. This corresponds to the insertion on the boundary of an operator of the form [22]

$$B^i = J^i + \frac{i}{2} \epsilon^{ijk} \psi^j \psi^k$$

(5.1)

This boundary deformation of the rational CFT is integrable and one can express the open-string spectrum in terms of the characters (3.2) with $z$ related to the magnetic field and the charges of the open-string states $q_i$ by [22]

$$z = \frac{1}{\pi} (\arctg(q_1 B) + \arctg(q_2 B))$$

(5.2)
Using the $SU(2)$ symmetry one can always choose $\mathcal{B}$ pointing in the third direction. From the modular $S$-transformation \[^{[35]}\]

$$\chi_{a}^{(k)}\left(-\frac{1}{\tau},-\frac{z}{\tau},u-\frac{z^{2}}{2\tau}\right) = \sum_{b} S_{ab} \chi_{b}^{(k)}(\tau, z, u) \quad (5.3)$$

one immediately deduces the Casimir energy and the shift of the modes of the currents $J_{n}^{(\pm)} \rightarrow J_{n \pm z}^{(\pm)}$. Notice that, since the modes of $J^{(3)}$ are unaffected, the current algebra is preserved

$$\left[J_{n+\pm z}^{(+)}, J_{m-\pm z}^{(-)}\right] = 2J_{n+m}^{(3)} + k\delta_{n+m}$$

$$\left[J_{n}^{(3)}, J_{m \pm z}^{(\pm)}\right] = \pm J_{n+m \pm z}^{(\pm)} \quad (5.4)$$

$$\left[J_{n}^{(3)}, J_{m}^{(3)}\right] = \frac{k}{2}\delta_{n+m}$$

thus the introduction of the magnetic field simply amounts to a modulation of the boundary reflection coefficients. By world-sheet supersymmetry considerations the modes of the fermions get an opposite shift. Indeed the total $N = 1$ supercurrent, that couples to the worldsheet gravitino,

$$G = J^{i}\psi_{i} + i\partial X^{4}\psi_{4} + \frac{i}{3!} \epsilon^{ijk}\psi_{i}\psi_{j}\psi_{k} + Q\partial\psi_{4} \quad (5.5)$$

forbids a twist of $\psi_{4}$ (and similarly of $X^{4}$) due to the presence of the background charge. The twisting of only two currents and two fermions leads to an explicit breaking of the spacetime supersymmetry. Since the curvature of the spin connection with torsion is self-dual one may ask if there is a possibility of adding a self-dual field-strength, \textit{i.e.} an instanton like gauge field in order to make the background supersymmetric. A possibility of this kind is suggested by the standard embedding in the heterotic version of the N5-brane. This issue deserves further study and it may have interesting applications to other curved backgrounds such as orbifolds, Gepner models and fermionic models. The final goal would be to address the issue of consistency of magnetized D-branes inside Calabi-Yau manifolds \[^{[37]}\].
6 Conclusions

In the curved spacetime models at hand there are no tadpole conditions to be imposed, since there are no massless states in the closed-string spectrum. Indeed, the contribution of the FF boson shifts all masses by \( Q^2/8 = 1/4(k + 2) \) thus all closed-string states become massive. Moreover, the distinction between the different Dp-branes is smeared out for any finite value of the level \( k \). In fact only the limit \( k \to \infty \) which corresponds to flat spacetime allows for a simple geometrical interpretation.

The systematic construction of open string models elaborated in [4], further improved in [3, 4] and finally completely established in [7] has proven to be quite a powerful tool for analyzing D-branes and O-planes in non-trivial backgrounds, that is backgrounds that are not simply related by orbifolds to free field theories and have non-abelian fusion rules. Indeed the dynamics of the D-branes and O-planes in the background of \( k \) coincident N5-branes is almost completely captured by the open string descendants of the \( SU(2) \) WZNW models [3, 4]. The wormhole geometry however is only the simplest instance of a large class of generalized hyper-Kähler manifolds that due to the enhanced \( N = (4, 4) \) superconformal symmetry correspond to exact superstring backgrounds [25]. The open string descendants of the parent Type II superstring vacua seem at reach as well as the issues of Buscher’s T-dualities in this non-compact backgrounds with isometries [24].

Finally, much in the same way as in [32], this curved background may be used as a consistent string IR regulator in order to extract the stringy loop correction to the low-energy effective Type I superstring lagrangian. For \( N = 1 \) Type I vacua with D5-branes, heterotic - Type I duality should map this corrections into non-perturbative corrections to the heterotic string effective lagrangian. Elaborating a systematic approach to the study of D-branes and their open-string excitations in non trivial backgrounds may thus provide a powerful tool for investigating new \( N = 1 \) dualities, that generalize the simple instances so far considered both in the superstring and in the gauge theory setting.
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A Appendix

In this Appendix we collect some formulae concerning the open descendants of SU(2) WZNW models [3, 4].

The central charge of the Virasoro algebra for the current algebra at level \( k \) is \( c = 3k/(k+2) \) while the conformal weights of the integrable unitary representations with isospin \( I \) in the range \( I = 0, \ldots, k/2 \) are

\[
h^{(k)}_I = \frac{I(I+1)}{k+2}. \tag{A.1}\]

The character formula is obtained from (3.2) for \( z = u = 0 \)

\[
\chi^{(k)}_I(\tau, 0, 0) = \text{Tr}_{^H_i}q^{L_0 - c/24} = q^{h^{(k)}_I - \frac{c}{24}} \sum_n q^{(k+2)n^2 + (2I+1)n}(2I + 1 + 2n(k+2)) \prod_{n=1}^{\infty} (1 - q^n)^3. \tag{A.2}\]

It is convenient to label states and characters in terms of the dimension \( a = 2I + 1 \) of the corresponding highest weight representations of SU(2). The modular matrices in the above basis are [35]

\[
S_{ab} = \sqrt{\frac{2}{k+2}} \sin \left( \frac{\pi ab}{k+2} \right), \tag{A.3}
\]

and

\[
T_{ab} = \delta_{ab} e^{i\pi \left( \frac{a^2}{2(k+2)} - \frac{1}{3} \right)}. \tag{A.4}\]

The charge conjugation matrix is equal to the identity \( C = S^2 = (ST)^3 = 1 \). The modular transformation between the direct and the transverse channel of the Möbius strip is induced by \( P = T^{1/2}ST^2ST^{-1/2} \) that acts on hatted characters [2]

\[
\hat{\chi}_h(i\tau_2 + 1/2) = e^{-i\pi(h-c)/24} \chi_h(i\tau_2 + 1/2) = T^{-1/2} \chi_h(i\tau_2 + 1/2) \tag{A.5}\]

and in general satisfies \( P^2 = C \). For the SU(2) WZNW model \( P \) is represented by

\[
P_{ab} = \frac{2}{\sqrt{k+2}} \sin \left( \frac{\pi ab}{2(k+2)} \right) (E_kE_{a+b} + O_kO_{a+b}), \tag{A.6}\]

where \( E_n \) and \( O_n \) are projectors on even and odd \( n \) respectively. The fusion rule coefficients are given by the Verlinde formula [38]

\[
N^{e}_a^b c = \sum_{d=1}^{k+1} \frac{S_{ad}S_{bd}S_{cd}^\dagger}{S_{1d}} = \begin{cases} 1 & \text{if } |a-b| + 1 \leq c \leq \min(a + b - 1, 2k - a - b + 3) \\ 0 & \text{else} \end{cases}, \tag{A.7}\]

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It turns out to be convenient to introduce also the integer (!) coefficients \[6\]

\[ Y_{ab}^c = \sum_{d=1}^{k+1} \frac{S_{ad} P_{bd} P_{cd}^\dagger}{S_{1d}}. \] (A.8)

Cappelli, Itzykson and Zuber [39] have shown that the modular invariant torus partition functions are in one to one correspondence with the \(A - D - E\) simply laced simple Lie algebras. At any level \(k\) there is a diagonal modular invariant denoted by \(A_{k+1}\) that reads

\[ Z^{\{A_{k+1}\}} = \sum_{a=1}^{k+1} |\chi_a|^2. \] (A.9)

For \(k = 4p + 2\), there is also a permutation modular invariant, denoted by \(D_{2p+1}\),

\[ Z^{\{D_{2p+1}\}} = \sum_{\text{odd } a=1}^{4p-1} |\chi_a|^2 + |\chi_{2p+2}|^2 + \sum_{\text{even } a=2}^{2p} (\bar{\chi}_a \chi_{4p+4-a} + \bar{\chi}_{4p+4-a} \chi_a). \] (A.10)

The \(D_{\text{even}}\) series (present for level \(k = 4p\)) and the three \(E\) cases correspond to extended chiral algebras.

The unoriented projection can be chosen to preserve the diagonal \(SU(2)\) subalgebra of the \(SU(2)_L \times SU(2)_R\) current algebra symmetry. Corresponding to the two geometrical involution on the \(SU(2)\) group manifold, \(i.e. g \rightarrow -g^{-1}\) and \(g \rightarrow g^{-1}\), there are two different unoriented projections (Klein bottle amplitudes) of the parent torus partition function [34]. The action of the two involutions coincides on the integer isospin representations, while on the half-integer isospin representations the first one corresponds to keeping the singlets of the diagonal \(SU(2)\), and the second one corresponds to removing them from the spectrum.

We shall label the two choices by an index \(r\) and \(c\) in order to streamline their relation to real and complex CP charge assignments. The diagonal \(A_{k+1}\) models allow for the introduction of \(k+1\) CP multiplicities or equivalently \(k+1\) independent boundary states that are in one to one correspondence with the integrable \(SU(2)\) representations. For the \(D_{2p+1}\) models there is a reduction of the independent boundary states and only \(k/2 + 2\) CP charges can be introduced [4].

For the \(A\)-series with real CP charges the Klein (K), annulus (A) and Möbius strip
(M) direct ("loop") channel amplitudes read

\[ K_r^{(A_{k+1})} = \sum_{a=1}^{k+1} Y_{a11}^a \chi_a = \sum_{a=1}^{k+1} (-1)^{a-1} \chi_a , \]  

\[ A_r^{(A_{k+1})} = \sum_{a,b,c=1} N_{ab}^c \chi_c n^a n^b , \]  

\[ M_r^{(A_{k+1})} = \pm \sum_{a,b=1}^{k+1} Y_{a1}^b \chi_b n^a = \pm \sum_{a,b=1}^{k+1} (-1)^{a-1} (1)^{k+1} N_{ab}^b \chi_b n^a . \]  

A modular transformation yields the transverse ("tree") channel amplitudes

\[ \tilde{K}_r^{(A_{k+1})} = \sum_a \left( \frac{P_{1a}}{\sqrt{S_{1a}}} \right)^2 \chi_a , \]  

\[ \tilde{A}_r^{(A_{k+1})} = \sum_a \left( \sum_b \frac{S_{ab}^b}{\sqrt{S_{1a}}} \right)^2 \chi_a , \]  

\[ \tilde{M}_r^{(A_{k+1})} = \pm \sum_a \left( \sum_b \frac{P_{1a} S_{ab}^b}{S_{1a}} \right) \tilde{\chi}_a . \]

For the \( A \)-series with complex CP charges the various the direct channel partition functions read

\[ K_c^{(A_{k+1})} = \sum_{a=1}^{k+1} Y_{a,k+1,1}^a \chi_a = \sum_{a=1}^{k+1} \chi_a , \]  

\[ A_c^{(A_{k+1})} = \sum_{a,b,d=1} N_{ab}^d \chi_{k+2-a} n^a n^b , \]  

\[ M_c^{(A_{k+1})} = \pm \sum_{a,b=1}^{k+1} Y_{a,k+1}^b \chi_b n^a = \pm \sum_{a,b=1}^{k+1} N_{ab}^b \chi_{k+2-b} n^a . \]

The transverse channel amplitudes are then given by

\[ \tilde{K}_c^{(A_{k+1})} = \sum_a \left( \frac{P_{k+1,a}}{\sqrt{S_{1a}}} \right)^2 \chi_a , \]  

\[ \tilde{A}_c^{(A_{k+1})} = \sum_a (-1)^{a-1} \left( \sum_b \frac{S_{ab}^b}{\sqrt{S_{1a}}} \right)^2 \chi_a , \]  

\[ \tilde{M}_c^{(A_{k+1})} = \pm \sum_a \left( \sum_b \frac{P_{k+1,a} S_{ab}^b}{S_{1a}} \right) \tilde{\chi}_a . \]

Notice that positivity of the transverse channel requires the numerical identifications

\[ n_{k+2-a} = \bar{n}_a = n_a. \]

For the \( D_{2p+1} \)-series, at level \( k = 4p + 2 \), the two choices for the Klein bottle are

\[ K_r^{(D_{2p+1})} = \left( \sum_{a=1}^{k+1} \chi_a - \chi_{k/2+1} \right) , \]  

\[ \tilde{K}_r^{(D_{2p+1})} = \left( \sum_{a=1}^{k+1} \chi_a + \chi_{k/2+1} \right) , \]  

\[ A_r^{(D_{2p+1})} = \left( \sum_{a=1}^{k+1} \chi_a \right) , \]  

\[ M_r^{(D_{2p+1})} = \left( \sum_{a=1}^{k+1} - \chi_a \right) . \]
and
\[ K_{c}^{\{D_{2p+1}\}} = \left( \sum_{\text{odd } a=1}^{k+1} \chi_{a} + \chi_{k/2+1} \right) . \]  
\text{(A.24)}

Correspondingly, in the transverse channel one finds
\[ \tilde{K}_{r}^{\{D_{2p+1}\}} = \sum_{a=1}^{k+1} \left( (-1)^{a^2-a} \frac{P_{k/2,a}}{\sqrt{S_{1a}}} \right)^2 \chi_{a} , \]  
\text{(A.25)}

and
\[ \tilde{K}_{c}^{\{D_{2p+1}\}} = \sum_{a=1}^{k+1} \left( (-1)^{a^2-a} \frac{P_{k/2+2,a}}{\sqrt{S_{1a}}} \right)^2 \chi_{a} . \]  
\text{(A.26)}

The annulus and Möbius strip partition functions can be expressed either in terms of \( k + 1 \) \textit{linearly dependent} pseudo-charges \( \nu_{a} \) as in \([7]\) or can be read off the expressions presented in Section 4.

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