The fate of nonlinear perturbations near the QCD critical point

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The impact of the QCD critical point on the propagation of nonlinear waves has been studied. The effects have been investigated within the scope of second-order causal dissipative hydrodynamics by incorporating the critical point into the equation of state, and the scaling behaviour of transport coefficients and of thermodynamic response functions. Near the critical point, the nonlinear waves are found to be significantly damped which may result in the disappearance of the Mach cone effects of the away side jet. Such damping may lead to enhancement in the fluctuations of elliptic and higher flow coefficients. Therefore, the disappearance of Mach cone effects and the enhancement of fluctuations in flow harmonics in the event-by-event analysis may be considered as signals of the critical endpoint.

I. Introduction: Relativistic Heavy Ion Collider Experiment (RHIC-E) is an excellent tool to explore the rich phase structure of Quantum Chromodynamics (QCD) under extreme conditions of temperature (T) and baryonic chemical potential (µ). One of the main features of the QCD phase diagram is the presence of the Critical Endpoint (CEP), where the first order phase transition from quark matter to hadronic matter terminates [1–5] and the transition becomes a crossover [6–8]. The search for the elusive CEP is one of the active areas of contemporary research [9]. Its exact location in the QCD phase diagram is not known from the first principle (lattice QCD based) calculation due to the well known sign problem for spin half particles. The theoretical prediction of the position of the CEP based on effective models is still ambiguous as its location depends on the parameters of the models used. On the experimental front, the beam energy scan (BES) program is being run with fine-tuning of the centre of mass energy (√s), such that if the quark-gluon plasma (QGP) passes through the CEP, the effects should be reflected on the particle spectra. However, for experimental detection of the CEP, theoretical investigations are required to understand its imprints on the data.

In this regard, mainly fluctuations of various thermodynamic quantities are studied [10], as the CEP is characterised by large fluctuations [11] resulting from the diverging nature of the correlation length (ξ) [12, 13] (see [14, 15] for RHIC-E related studies). However, apart from the studies of the fluctuations, the prediction on the fate of perturbations and their consequences on various experimental observables are significantly important for the detection of CEP. In RHIC-E, partons (quarks and gluons) are produced with a wide range of transverse momentum (pT). Partons with relatively lower pT, on subsequent scattering, produce a locally thermalized hot medium of QGP, whereas, the high pT partons do not contribute in the medium formation, but they pass through the medium as jets with associated radiated partons. While propagating, these jets produce disturbances in the medium through interaction. The partons, especially the supersonic ones, can produce perturbations in the medium leading to nonlinear waves. Apart from the disturbances generated by the jets, quantum fluctuations leading to inhomogeneity in the medium may serve as perturbations in the hydrodynamically evolving medium. The hydrodynamic response of the medium to such perturbations, are reflected on the spectra of the produced hadrons at the freeze-out hypersurface and on other penetrating particles like photons and lepton pairs emitted throughout the evolution history of the fireball. Specifically, the appearance of two maxima at Δφ = π ± 1.2 radian in the quenched away side jet or the double-hump in the correlation function of the jet, the structure is explained as the effect of the Mach cone produced due to hydrodynamic response to the perturbation created by jets [16]. The momentum anisotropy, which is quantified as flow harmonics of the produced particle is attributed to the hydrodynamic response of the QGP to the initial geometry. During the expansion and cooling, when the system makes a transition to the hadronic phase, these anisotropies get transmitted into hadronic momentum spectra through momentum conservation.

The damping of the Mach cone is connected to the hydrodynamic response. Any change in the nature of QGP medium and in the nature of the transition from QGP to hadron e.g. presence of the CEP will affect the response. In general, the hydrodynamic response can be treated as either linear or nonlinear depending on the magnitude of the perturbations. A small disturbance is treated as a linear perturbation [17–22] whereas a relatively large disturbance (of the order of unperturbed value) should be treated as nonlinear perturbation [23–26]. The development of the momentum anisotropy can be accounted

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mostly as a linear response to the initial eccentricity with a small contribution from nonlinear effects \cite{27-29}. Moreover, the produced Mach front has been found to travel as a shock front \cite{30}. The propagation of this shock front is controlled by the hydrodynamic response which may be linear or nonlinear. Due to the large amplitude of this shock front, one expects the perturbation to be nonlinear in nature \cite{31}. In Refs. \cite{32-34} the presence of Mach cone effect on the away side jet in two particles and three-particle correlations have been explained by using the non-dissipative property of the nonlinear waves.

The fate of nonlinear waves in QGP has been studied by Fogaca et al. \cite{25,26} by using Navier-Stokes (NS) and Israel-Stewart (IS) \cite{35} hydrodynamics with the inclusion of shear viscous effects only. They found that despite the presence of shear viscous effects the nonlinear waves survive. The nonlinear perturbations induced by jets have been considered to be responsible for broadening of the away side jet \cite{25}. The effects of linear response have already been investigated and found to be suppressed near the CEP \cite{20-22}. In this regard, however, it will be pertinent to investigate the fate of the nonlinear perturbations in presence of the CEP. This is particularly important because physical processes like the deposition of energy by jets can be very large which may ignite nonlinear effects. In such cases, the nonlinear wave may play a dominant role in the formation of the Mach-cone. Therefore, the study of the effects of nonlinear perturbations in QGP under the influence of the CEP is crucial.

The understanding of the broadening of the away side jet and the fate of conical flow or double-hump in the correlation function will be useful for the critical point search. In fact, the fate of the nonlinear perturbations due to the presence of CEP has not been addressed till date to the best of our knowledge. In this work, we address the propagation of nonlinear waves in the presence of the CEP. To study the role of the CEP on the nonlinear perturbations, the inclusion of all the relevant dissipative coefficients are crucial as some of these are known to diverge near the CEP \cite{36-40}. However, the equations governing the evolution of the nonlinear perturbations in the presence of transport coefficients \ie shear viscosity ($\eta$), bulk viscosity ($\zeta$) and thermal conductivity ($\kappa$) are not readily available within the scope of the second-order causal fluid dynamics and hence, we deduce these equations in the present work.

II. Non-linear wave equations: The nonlinear evolution equations for perturbation of hydrodynamic fields in the QGP can be derived from the relativistic viscous hydrodynamic equations. In NS hydrodynamics the dissipative flux is assumed to be proportional to the first-order gradient of the hydrodynamic fields which leads to acasual and unstable solutions. Though there are frame stabilized first-order relativistic hydrodynamics \cite{41-43}, it is not simultaneously casual and stable for a non-conformal system \ie a system with quasi-particles with non-zero mass \cite{44}. Therefore, for a consistent hydrodynamic description of QGP, second-order casual dissipative hydrodynamics of IS is considered as appropriate.

Here, we use the second-order theory \ie IS hydrodynamics, which respects causality and provides stable solutions. In general, there are two choices of frames of reference which are mostly used, namely Landau-Lifshitz (LL) \cite{45} and Eckart frames \cite{46}. The LL frame represents a local rest frame where the energy dissipation is zero but the net charge dissipation is non-zero. Whereas, the Eckart frame represents a local rest frame where the net charge dissipation is vanishing but the energy dissipation is non-vanishing. We adopt the Eckart frame of reference \cite{46} here. The energy-momentum tensor (EMT) and particle current can be written in the Eckart frame as:

$$ T^{\mu\nu} = eu^\mu u^\nu - p \Delta^{\mu\nu} + \Delta T^{\mu\nu}; \quad J^\mu = nu^\mu \quad (1) $$

where, $e$, $p$ and $u^\mu$ are respectively the local energy density, pressure and fluid four velocity. $\Delta T^{\mu\nu}$ is the dissipative part of the EMT. We choose the Minkowski metric as: $g^{\mu\nu} = (1,−1,−1,−1)$ with $u^\mu u_\mu = 1$ and $u^\mu \partial_\mu u_\nu = 0$. The projection operator normal to the four velocity is defined as $\Delta^{\mu\nu} = g^{\mu\nu} − u^\mu u^\nu$, such that $\Delta^{\mu\nu} u_\nu = 0$ and $\Delta^{\mu\nu} \Delta_{\mu\nu} = 3$. The symmetric traceless projection normal to the fluid four velocity is defined as $\Delta_{\mu\beta} = \frac{1}{2}(\Delta^{\mu\alpha} \Delta_{\alpha\beta} + \Delta^{\beta\alpha} \Delta_{\alpha\mu} − \frac{3}{2} \Delta^{\mu\nu} \Delta_{\nu\alpha} \delta_{\alpha\beta})$.

The dissipative part of EMT can be written in terms of scalar, vector and tensor as:

$$ \Delta T^{\mu\nu} = −\Pi \Delta^{\mu\nu} + u^\mu q^\nu + u^\nu q^\mu + \pi^{\mu\nu} \quad (2) $$

Therefore, the EMT in Eckart frame is:

$$ T^{\mu\nu} = eu^\mu u^\nu − (p + \Pi) \Delta^{\mu\nu} + u^\mu q^\nu + u^\nu q^\mu + \pi^{\mu\nu} \quad (3) $$

The vector and tensor forms of dissipation are considered to be non-existent in the frame of fluid velocity, such that $u_\nu q^\nu = 0, u_\mu \pi^{\mu\nu} = 0$. The scalar dissipation ($\Pi$) corresponds to the non-equilibrium pressure perturbation which can not be related to energy density through the equation of state (EoS). The scalar dissipation is related to volume expansion of fluid that triggers small non-equilibrium perturbation which drives the system to a new state of equilibrium leading to dissipative correction. Energy density, pressure, and the dissipative fluxes are related to EMT by $u_\mu T^{\mu\nu} u_\nu = e, q_\alpha = u_\mu T^{\mu\nu} \Delta_\nu \alpha = u_\mu \Delta T^{\mu\nu} \Delta_\nu \alpha, p + \Pi = −\frac{1}{3} \Delta_{\mu\nu} T^{\mu\nu}, \Pi = −\frac{1}{3} \Delta_{\mu\nu} \Delta T^{\mu\nu}, u_\nu \Delta T^{\mu\nu} = q^\nu$. The conservation of energymomentum and the net charge (net baryon number here) density are given by the following equations:

$$ \partial_\mu T^{\mu\nu} = 0; \quad \partial_\mu J^\mu = 0 \quad (4) $$

The dissipative fluxes \cite{35} are given by

$$ \Pi = −\frac{1}{3} \zeta (\partial_\mu u^\mu + \beta_0 D^\mu \Pi − \tilde{\alpha}_0 \partial_\mu \rho^\mu) $$

$$ \pi^{\lambda\mu} = −2\eta \Lambda^{\lambda\mu} \Delta^{\nu\beta} \{\partial_\nu u_\beta + \beta_2 D \pi_{\alpha\beta} − \tilde{\alpha}_1 \partial_\alpha q_\beta\} $$

$$ q^{\lambda} = −\kappa T \Lambda^{\lambda\mu} \left[\partial_\mu T + D u_\mu + \beta_1 D q_\mu − \tilde{\alpha}_0 \partial_\mu \Pi − \tilde{\alpha}_1 \partial_\nu \pi^\nu\right] \quad (5) $$
where \( D \equiv u^a\partial_a \) is the co-moving derivative. In the local rest frame (LRF) \( DII = \Pi \), representing the time derivative. The relaxation times for the bulk pressure (\( \tau_{\Pi} \)), the heat flux (\( \tau_q \)) and the shear tensor (\( \tau_s \)) are given by \[ \tau_{\Pi} = \zeta\alpha_0, \quad \tau_q = k_B T \beta_1, \quad \text{and} \quad \tau_s = 2\eta\beta_2. \] The relaxation lengths which couple the heat flux and bulk pressure (\( l_{\Pi q}, l_{q\Pi} \)), the heat flux and shear tensor (\( l_{\Pi s}, l_{s\Pi} \)) are defined as: \[ l_{\Pi q} = \zeta\alpha_0, \quad l_{q\Pi} = k_B T \alpha_0, \quad l_{\Pi s} = k_B T \alpha_1, \quad \text{and} \quad l_{s\Pi} = 2\eta\alpha_1. \] The quantities, \( \alpha_0, \alpha_1, \beta_1, \beta_2 \) are called the relaxation coefficients, and \( \alpha_0, \alpha_1 \) are coupling coefficients. The quantities \( \alpha_0, \alpha_1, \beta_1, \beta_2 \) in Eq. (5) are related to relaxation and coupling coefficients as \( \alpha_0 - \alpha_1 = \alpha_1 - \alpha_1 = \beta_1 - \beta_0 = -[(\epsilon + p)]^{-1} \) \[ \text{[21, 35].} \] The above hydrodynamic equations from IS theory is used to derive the nonlinear equations in \((1 + 1)D \) \[ \text{[26].} \] To achieve this we have adopted the Reductive Perturbative Method (RPM) \[ \text{[48–50],} \] with “stretched co-ordinates” defined as,

\[
X = \frac{\sigma^{1/2}}{L}(x - c_s t) \quad \text{and} \quad Y = \frac{\sigma^{3/2}}{L}(c_s t)
\]

where, \( L \) is the characteristic length, \( c_s \) is the speed of sound and \( \sigma \) is the expansion parameter. Therefore, we have

\[
\frac{\partial}{\partial x} = \frac{\sigma^{1/2}}{L} \frac{\partial}{\partial X} \quad \text{and} \quad \frac{\partial}{\partial t} = -c_s \frac{\sigma^{1/2}}{L} \frac{\partial}{\partial X} + c_s \frac{\sigma^{3/2}}{L} \frac{\partial}{\partial Y}
\]

The coordinate \( X \) is measured from the frame of propagating sound waves, whereas, the \( Y \) represents a fast-moving coordinate. The RPM technique is devised to preserve the structural form of the parent equation in different orders of \( \sigma \) i.e Breaking wave, Burger’s, Korteweg–De Vries(KdV) equation etc. Now we do the simultaneous series expansion of hydrodynamic quantities in powers of \( \sigma \). For \( \epsilon \), the series appears as:

\[
\hat{\epsilon} = \frac{\epsilon}{\epsilon_0} = 1 + \sigma \epsilon_1 + \sigma^2 \epsilon_2 + \sigma^3 \epsilon_3 + \ldots
\]

Here, we keep terms up to \( \sigma^3 \). We derive the required equations by collecting the terms corresponding to the different orders of \( \sigma \). Finally we revert from \((X, Y) \rightarrow (t, x)\) to get the following equations for the perturbation in \( \hat{\epsilon} \) as:

\[
\frac{\partial \hat{\epsilon}_1}{\partial t} + \left[ 1 + (1 - c_s^2) \frac{\epsilon_0}{\epsilon_0 + p_0}\right] \hat{c}_s \frac{\partial \hat{\epsilon}_1}{\partial x} - \frac{1}{2(\epsilon_0 + p_0)}(\zeta + \frac{4}{3}\eta) \frac{\partial^2 \hat{\epsilon}_1}{\partial x^2} = 0,
\]

and

\[
\frac{\partial \hat{\epsilon}_2}{\partial t} + \left[ S_1 \frac{\partial \hat{\epsilon}_2}{\partial x} + S_2 \frac{\partial \hat{\epsilon}_1}{\partial x} + S_3 \frac{\partial^2 \hat{\epsilon}_1}{\partial x^2} + S_4 \frac{\partial^2 \hat{\epsilon}_1}{\partial x^2} + S_5 \frac{\partial^2 \hat{\epsilon}_2}{\partial x^2} = 0
\]

where the coefficients, \( S_i \)'s for \( i = 1 \) to 5 are given by,

\[
S_1 = \frac{1}{\epsilon_0 + p_0} \left[ c_s (\epsilon_0 (1 - \hat{\epsilon}_1 (c_s^2 - 1)) + p_0) \right]; \quad S_2 = \frac{1}{\epsilon_0 + p_0} \left[ \epsilon_0 c_s (c_s^2 - 1) (\epsilon_0 ((2c_s^2 + 1)\hat{\epsilon}_1^2 - \hat{\epsilon}_2) - p_0 \hat{\epsilon}_2) \right];
\]

\[
S_3 = \frac{1}{12(\epsilon_0 + p_0)^2} \left[ \epsilon_0 c_s (3c_s^2 (7\zeta + 8\eta) + 3\zeta + 4\eta) \right]; \quad S_4 = \frac{1}{72c_s c_v (\epsilon_0 + p_0)^2} \left[ 4c_v c_s^2 (3\kappa \epsilon_0 (3\alpha_0 \zeta + 4\alpha_1 \eta)) + 3\kappa T (3\zeta + 4\eta) + 3\kappa T p_0 (3\alpha_0 \zeta + 4\alpha_1 \eta) + (\epsilon_0 + p_0)(9\beta_2 \zeta^2 + 16\beta_2 \eta^2) \right] - c_v (3\zeta + 4\eta)^2 + 12c_s (\epsilon_0 + p_0) \left[ \epsilon_0 (3\alpha_0 \zeta + 4\alpha_1 \eta) + 3\zeta + 4\eta + p_0 (3\alpha_0 \zeta + 4\alpha_1 \eta) \right]; \quad S_5 = \frac{3\zeta + 4\eta}{6(p_0 + \epsilon_0)}
\]

with, \( \hat{\epsilon}_1 = \sigma \epsilon_1 \) and \( \hat{\epsilon}_2 = \sigma^2 \epsilon_2 \). We observe that Eq. 9 does not contain any effect of the second-order theory (IS) as earlier observed in Refs. \[ \text{[25, 26]} \] with conformal EoS. This equation can be derived from the NS theory as well. However, the second equation contains the second-order effects via relaxation and coupling coefficients of IS theory. We take into account all the transport coefficients to provide a general equation for the propagation of the nonlinear waves. The dispersive terms in Eq. (10) indicate that the combined effects of shear and bulk viscosities act against the effect of thermal conductivity.

This might dilute the diffusion of nonlinear waves in a dissimilar way.

III. Results and Discussions: In this section we discuss the effects of the CEP on the propagation of nonlinear waves. We assume that the CEP is located at \( (T_c, \mu_c) = (154, 367) \text{ MeV} \), where \( \mu_c \) and \( T_c \) are the critical values of baryonic chemical potential and temperature respectively. The initial profile of the
perturbations is taken as:

$$\dot{\epsilon}_i = A_i \left[ \text{sech}\left( \frac{x - 10}{B_i} \right) \right]^2$$

(12)

where $i = 1$ and $2$ stand for first and second order perturbations respectively with $A_i$ and $B_i$ determining the height and width of the initial profile. The effects of the CEP have been taken into consideration through the EoS and the scaling behaviour of transport coefficients and of the thermodynamic response functions. The CEP in QGP-hadron transition belongs to the same universality class as that of the 3D Ising model. The construction of EoS with the CEP have been studied in Refs.\[51–54\] (for details we refer to these references). The procedure discussed in Refs.\[51, 52\] has been followed to construct the EoS with the CEP and used in Refs.\[20, 21\] to study linear perturbation. The same EoS has been used in the present work for the evolution of nonlinear perturbation. The critical behaviour of various transport coefficients and response functions are taken from Refs.\[22, 39, 54\].

Fig.1 shows the propagation of a nonlinear wave through the medium when the system is formed away from the CEP. It is found that the nonlinear wave survives with reduced amplitude despite the presence of dissipative effects introduced via the non-zero values of transport coefficients. In contrast, the nonlinear wave is substantially dissipated near the CEP as evident from the results depicted in Fig. 2. It has been observed that the dissipative feature remains unchanged with the variation of the position of the CEP along the transition line in the QCD phase diagram. This clearly indicates that the nonlinear perturbations will provide detectable effects of the CEP. It is to be noted that the speed of the nonlinear wave is non-zero in contrast to linear waves\[20\]. This is due to the amplitude-dependent propagation speed of the nonlinear waves. The attenuation of the perturbation is smaller for the second-order correction (\(\dot{\epsilon}_2\)). Furthermore, the second-order perturbation travels a bit faster than the linear one. This is clear from the results displayed in Figs.1 and Fig.2 which can be understood from Eq.(10), where the second-order correction contains the third-order derivatives of first-order perturbation. This is similar to the dispersive term in the KdV equation responsible for height preserving solitonic behaviour\[55\]. Therefore, dispersive terms compete with the diffusive terms to weakening the damping effect. The relaxation effects on the dissipative fluxes incorporated in IS theory (Eq.(10)) makes the dissipation slower in comparison to the NS theory (Eq.(9)). It is also found that the diverging nature of thermal conductivity near the CEP\[39\] dominates over the shear and bulk viscous effects as the the degree of divergence of \(\kappa\) is stronger than the other transport coefficients.

Therefore, near the CEP the waves with large amplitudes created in the medium by the energetic particles or jets, will be highly suppressed. It has been seen earlier that the linear waves are fully halted near the CEP. We find similar suppression of nonlinear waves due to the presence of the CEP. Although the propagation speed of nonlinear waves are amplitude dependent, they are highly suppressed near the CEP irrespective of their amplitude. This has important consequences in detecting the CEP.

It has been predicted earlier\[20–22\] that the formation of Mach cone is prevented near the CEP for linear perturbation. Whether a similar effect is observed for the nonlinear waves, is an interesting question to address. This is crucial because nonlinear effects are found to be shape and height-preserving in comparison to linear propagation. In this work, we find that even the nonlinear perturbations will not be able to retain the Mach cone effects if it hits the CEP. These findings can be used to detect the CEP by looking into the suppression of Mach cone in two-particle correlation\[32\]. Due to the suppression of nonlinear waves, the broadening effect of localized waves will also vanish \[25\] along with the Mach cone effect.

The flow harmonics play a crucial role in characterizing the medium formed in RHIC-E. It was shown in Refs.\[56, 57\] that some of the flow harmonics will collapse at the CEP. The present work has crucial consequences on the flow harmonics. Based on linear analysis it has been argued in Refs.\[20–22\] that \(v_2\) will be reduced near the CEP. The same conclusion can be drawn for the nonlinear waves too in presence of the CEP. If the initial spatial shape of the system formed in HICs is highly distorted azimuthally, which may be the case for off-central collisions, the fluid dynamical response to the initial eccentricity may be nonlinear. The present work suggests that \(v_2\) and higher harmonics will be suppressed near the CEP due to the absorption of the sound wave, even for highly off-central collisions.

In a recent work\[58\], it is found that the path to the critical point is heavily influenced by far from equilibrium initial conditions, where viscous effects lead to dramatically different \((T, \mu)\) trajectories. This means that the trajectory of the system in the QCD phase diagram will be event-dependent. Therefore, it will be useful to analyze the flow harmonics on event-by-event basis. If a particular event evolve through the CEP, then the rms (root mean square) value of flow harmonics will be suppressed because of the absorption of the sound wave. On the contrary, if an event evolves through a trajectory that is away from the CEP, then the flow harmonics will survive. Hence, the presence of the CEP will cause large event-by-event fluctuations of flow harmonics.

We emphasize that the nonlinear perturbations damped substantially if the system passes through the CEP. In absence of the critical point the nonlinear perturbation survives although the medium is dissipative with non-zero \(\eta, \zeta\) and \(\chi\). This indicates that the formation of Mach cone will be prohibited in the presence of the CEP. Therefore, the conclusion of the present theoretical investigation is that the Mach cones disappear in the presence of the CEP. The effects of Mach cone manifest as a double hump in the two-particle correlation in the low momentum domain of associated particles i.e. the CEP plays a unique role in suppressing the double hump in
the two-particle correlation contrary to the other mechanisms: e.g. (i) deflection of away side jets, (ii) Cherenkov radiation and (iii) radiation of gluons which produce the double hump. These mechanisms have the ability to obscure the suppression due to the CEP by creating double hump.

The observation [59, 60] of the dip in the azimuthal distribution of two particle correlation at $\Delta\phi = \phi - \phi_{\text{trig}} = \pi$ accompanied by two local peaks on either side of $\Delta\phi = \pi$ for transverse momentum range, $0.15 < p_T^{\text{assoc}} < 4$ GeV, is attributed to the physical processes mentioned above. Therefore, we contrast the effect of the CEP to the following mechanisms. (i) Deflection of the away side jets by strong asymmetric flow in non-central collisions and third flow harmonics due to initial state fluctuations (Refs. [32, 61, 62] for further details) lead to peaks of the away side jet on the either side of $\Delta\phi = \pi$. However, if the system passes through the CEP, the flow will be highly suppressed and hence, the deflection too will be strongly reduced. (ii) Cherenkov radiation [63, 64] is characterized by strong momentum dependence of the cone angle ($\sim 1/\mu$ where $\mu$ is the refractive index of the medium). This process is unlikely to be responsible for the double hump because of the lack of observed momentum dependence of the location of the double peaks of associated particles. (iii) The radiation of gluons by the away side jet will deviate it from propagating at an angle $180^\circ$ with respect to the near side (trigger) jet. However, the quantitative prediction of the Mach cone positions studied through three-particle correlation [65, 66] and the momentum independence of the location of the double hump indicate that the double hump may originate from Mach cone effects. The vanishing of the Mach-cone like structure in particle correlation will therefore, indicate the existence of the CEP.

In this study, the propagation of perturbation is studied in a static background. In presence of expansion, the system will cool and move towards the phase transition line with changing transport coefficients. However, during this cooling, if the trajectory goes away from the CEP, then the perturbations will still survive. But if it passes near the CEP, the perturbative effects will be mostly washed out and no effect will survive at the latter stage. Therefore, it is expected that the results obtained in the static situation may not vary with the inclusion of expansion if the system passes through the CEP.

We have investigated the hydrodynamic propagation of perturbations in the system without considering the fluctuations [67, 68] originating from the CEP itself, as they do not create any angular pattern as jets produce in correlations. Although the non-equilibrium fluctuations due to the CEP is not taken into the account here for obtaining the non-linear wave equations, the enhancement of thermodynamic fluctuations near the critical point which affects the hydrodynamic response is inherently taken into account through the EoS and other
thermodynamic quantities via the critical exponents.

**IV. Summary and Conclusion:** In summary, we have investigated the response of the QCD critical point to the nonlinear perturbations within the scope of second-order IS hydrodynamics. The effects of the CEP on the propagation of nonlinear waves have been taken into accounts through the EoS, critical behavior of the transport coefficients and of thermodynamic response functions. We have derived relevant equations governing the propagation of nonlinear waves within the purview of second-order causal hydrodynamics by taking into account the non-zero values of $\eta$, $\zeta$ and $\kappa$ in contrast to earlier works where the effects of $\zeta$ and $\kappa$ were ignored. In the presence of the CEP $\zeta$ and $\kappa$ play important roles as they diverge near the CEP and hence can not be ignored. It is found that, similar to the linear perturbation, the nonlinear perturbations too get suppressed near the CEP. The diverging nature of thermal conductivity near the CEP plays the most dominant role in the suppression of nonlinear waves. The nonlinear effects make the perturbation to travel a bit faster than the linear one. The presence of the CEP will be resulting in the vanishing of Mach cone effects (or away side double-peak structure) and the broadening of the two and three-particle correlation. The suppression or collapse of elliptic flow will also indicate the existence of the CEP. This may lead to the large event-by-event fluctuation of flow harmonics between two events with and without the CEP. Therefore, the vanishing Mach cone effects (or away side double-peak structure) on the away side jet and the enhancement of fluctuation of flow harmonics in event-by-event analysis accompanied by suppressed flow harmonics could be considered as signals of the CEP.

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