Theoretical investigation on an oscillating buoy WEC-floating breakwater integrated system

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Abstract

Based on the concept of cost sharing, an oscillating buoy wave energy converter-floating breakwater integrated system is proposed, in which the surge or pitch motion of the front-pontoon equipped with Power Take-off (PTO) system is allowed. In this study, the methods of eigenfunction matching and variables separation are applied to establish a theoretical model on hydrodynamic performance of the proposed integrated system. Under the condition of optimal PTO damping, the effects of the front-pontoon width, draft and pontoon spacing on the capture width ratio, reflection coefficient and transmission coefficient are investigated, respectively. It is found that the integrated system with surging front-pontoon performs better on the wave energy conversion than that with pitching one, especially in the high frequency region. The peaks of capture width ratio $C_w$ are associated with the gap resonance and the increase of front-pontoon motion induced by the ‘hydrodynamic constructive effect’.

1 INTRODUCTION

A variety of technologies and concepts of wave energy conversion have been proposed in recent decades [1, 2]. However, limited by the variability and complexity of ocean waves, only a few wave energy converters (WECs) achieved the full-scale prototype test, or, reached the full-scale demonstration stage up to now. According to working principle, WECs can be classified as oscillating buoy (OB), oscillating water column (OWC) and overtopping devices [1]. Among which, the OB WECs as offshore devices are capable to harvest more plentiful wave energy in deep sea. The typical single-body OB WEC is composed of an oscillating buoy, Power Take-off system and mooring system, by which wave energy is extracted from wave-induced buoy motions. However, offshore devices mean that more difficulties, such as installation, maintenance and placement of electricity storage component, need to be solved. Therefore, it is of practical significance to improve the configurations of single-body OB WECs.

Massive studies for OB WECs have been performed since the first oil crisis happened. Early theoretical research on the hydrodynamic characteristics of OB WECs is mostly based on linear potential theory. Evans [3] and Mei [4] derived the optimum conversion efficiency of 2D WECs for one degree of freedom (DOF), and they gave the optimizing criteria that the WEC needs to be adjusted to resonance and the PTO damping must equal to the hydrodynamic radiation damping. Mei [4], Budal [5] and Newman [6] further discussed the optimum conversion efficiency of 3D WECs. Based on those definitions and theoretical studies on OB WECs, many researchers explored the effects of buoy shapes and PTO system on the hydrodynamic performance, such as Mavrakos & Katsaounis [7] and Zhang et al. [8]. To resolve installation problems in deep water or large tidal region, two-body or multi-body systems are proposed, which were theoretically studied by Falnes [9] in detail. One representative study was undertaken by Yeung et al. [10–12], who proposed a dual coaxial-cylinder WEC and investigated systematically the effects of geometric parameters, outer shape...
and PTO system on its energy conversion ability based on the theoretical derivations and experimental tests. Commonly, recent works on the two-body systems are focused on adaptability and survivability in complex and variable ocean environment [13–15]. In addition, some other OB WECs extracting wave energy from their relative rotation were proposed, such as Pelamis devices [16], PS FrogMk5 [17], Oyster [18] and so on.

However, compared with other renewable energy (such as solar, tidal and wind energy) technologies, wave energy technologies are still immature and demand relatively higher manufacturing, constructing and installing costs. At present, one promising solution to reduce budget is to integrate stand-alone devices into breakwaters, which significantly promotes the commercialization process of wave energy exploitation. In fact, the integrated system is not only beneficial for cost and space sharing, but have the potential to enhance the energy conversion ability of WECs [19]. According to categories of WECs, those integrated systems mainly consists of OWC WEC-breakwater and OB WEC-breakwater integrated systems.

Ojima et al. [20] first proposed an OWC WEC-fixed caisson breakwater hybrid system, then Takahashi [21] and Raju & Neelamini [22] discussed the reliability and stability of the integrated system thoroughly. An alternative OWC WEC device with an additional seaward wall, namely U-shaped OWC device, integrated into the caisson was proposed and studied by Bocciotti [23], Arena et al. [24] and Malara & Arena [25]. Martins-rivas & Mei [26] and Zheng et al. [27] established analytical models based on linear potential theory to investigate the hydrodynamic performance of a cylindrical OWC embedded into the straight coast/breakwater and found that the coastal effect can significantly enhance the power output performance of the OWC WEC. Moreover, an OWC device-floating breakwater integrated system was proposed by Neelamani et al. [28], who discussed the wave reflection, transmission and pneumatic efficiency of this system under the tidal effect in the deep water. He et al. [29] proposed a pile-supported OWC-breakwater integrated system and analytically investigated its hydrodynamic performance. Also, a floating box-type breakwaters with dual pneumatic chambers was reported by He et al. [30–32].

With regard to the OB WEC-breakwater, Ning et al. [33, 34] proposed a heaving floating box breakwater-WEC integrated system, and theoretically investigated the effects of geometrical parameters and PTO damping on the reflection coefficient, transmission coefficient and capture width ratio. They reported that both the effective coastal protection and wave energy extraction functions were achieved by the proposed integrated system. Successively, Ning et al. [35, 36] extended the integrated system into a dual-pontoon one whose theoretical maximum hydrodynamic efficiency can reach up to 0.8 and the effective frequency bandwidth (i.e., transmission coefficient \( K_T < 0.5 \), capture width ratio \( C_w > 0.2 \)) is relatively wide in comparison with the previous single-pontoon system. More recently, Konispoliatis et al. [37] developed an analytical model to investigate the hydrodynamics of array of vertical axisymmetric floaters with arbitrary bottom in front of a vertical breakwater. To deal with the effect of rear breakwater, the image method was adopted so that the hydrodynamic problem can be transferred to one without the presence of breakwater. Meanwhile, the boundary approximation method was employed to discretize the floaters bottom. Their numerical results were focused on the effects of array configurations and bottom shapes of floaters on the wave exciting force and hydrodynamic coefficients. A parallel investigation was carried out by Konispoliatis & Mavrakos [38], demonstrating that the existence of vertical breakwater always exerts constructive effect on the energy conversion ability of the proposed system, regardless of incident wave angle, array configuration or inner spacing between floaters.

To the authors’ knowledge, most of the previous studies on WEC-breakwater integrated system were focused on OWC WEC-breakwater and heaving OB WEC-breakwater system. On the other hand, compared with bottom-mounted breakwater, the floating breakwater performs strong adaptability to weak geological conditions and reduces engineering cost in deep water and under large tide condition. Therefore, in this article, the OB WEC-floating breakwater integrated system is further investigated as an extension work of previous research by Ning et al. [36, 39]. In present model, the integrated system consists of dual rectangular pontoons and the seaward one is allowed to oscillate in surge or pitch mode. A theoretical model is developed based on linear potential flow theory to mainly investigate the energy conversion performance of the integrated system under different geometrical parameters. The objective of this study is to explore the variation of energy conversion efficiency with wavenumber or geometrical parameters in principle.

This work is organized as follows: Section 2 formulates the diffraction and radiation problems. Section 3 compares the present numerical results with theoretical relations given in published literatures. And then Section 4 displays and discusses the effects of front-pontoon width, draft and pontoon spacing on the hydrodynamic performance of the proposed system systematically. Finally, Section 5 draws the key conclusions.

## 2 | MATHEMATICAL MODEL

The schematic diagram of the OB WEC-floating breakwater integrated system is shown in Figure 1. The front rectangular pontoon is allowed to surge or pitch. It is considered that the integrated system dimension along the wave-crest line (y-axis, i.e., perpendicular to the wave propagation direction) is much...
longer than the wavelength, thus a two-dimensional Cartesian coordinate \((Ox\bar{z})\) system is employed, whose origin \((O)\) is set on the undisturbed free surface, \(x\)-axis positive in the propagation direction of incident wave, and \(\bar{z}\)-axis positive upwards. \(d\) is the water depth; \(B_n, T_n\) and \(H_n\) denote the width, draft and height of the front-pontoon respectively; \(D\) is the spacing between pontoon 1 and 2. The PTO system is applied to extract the wave energy from surge or pitch motion of the front-pontoon.

It is assumed that the fluid is inviscid, incompressible and the flow is irrotational, so the fluid field can be described by the velocity potential \(\Phi\). Also, all time-dependent variables are assumed to be harmonic,

\[
\Phi (x, \bar{z}, t) = \text{Re} \left[ \phi (x, \bar{z}) e^{-i\omega t} \right]
\]  

(1)

where \(\text{Re}\) denotes the real part of a complex number, \(t\) the time, \(i = \sqrt{-1}\) the imaginary unit, and \(\omega\) the angular frequency. \(\phi\) is complex velocity potential satisfying the Laplace equation,

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial \bar{z}^2} = 0
\]  

(2)

The total velocity potential can be decomposed into three components for the linear system,

\[
\phi = \phi_1 + \phi_D + \phi_m
\]  

(3)

where \(\phi_1\) is the incident velocity potential, \(\phi_D\) the diffraction velocity potential; \(\phi_m\) is the radiation velocity potential due to the motion of the front-pontoon in \(m\) mode. \(m = 1\) and 3 represent surge and pitch modes, respectively. The incident velocity potential can be expressed as follow,

\[
\phi_1 = \frac{i g A}{\omega} \cosh \left( k (z + d) \right) e^{ikx}
\]  

(4)

where \(A\) is the incident wave amplitude, and \(g\) is the gravity acceleration. \(k\) is the wavenumber satisfying the dispersion relation \(\omega^2 = gk \tanh (kd)\).

The corresponding boundary conditions of all velocity potentials can be described as,

\[
\left\{ \begin{array}{l}
\frac{\partial \phi_m}{\partial x} - \frac{\omega^2}{g} \phi_m = 0 \quad (z = 0, x < \delta_{i1} \text{ or } x > \delta_{i2}) \\
\frac{\partial \phi_m}{\partial \bar{z}} = 0 \quad (z = -d) \\
\frac{\partial \phi_m}{\partial \bar{z}} = i \omega \delta_{i1} (x - \delta_{i0}) \delta_{i3,m} - \frac{\partial \phi_1}{\partial \bar{z}} \delta_{i3,m} \quad (z = -T_i, x < x_{i1} < x < \delta_{i2}, \ i = 1, 2) \\
\frac{\partial \phi_m}{\partial x} = -i \omega \delta_{i1} \left[ \delta_{i3,m} (z - \delta_{i0}) \delta_{i3,m} - \frac{\partial \phi_1}{\partial x} \delta_{i3,m} \right] \quad ( -T_i < z < 0, x = x_{i1} \text{ or } x = x_{i2}, \ i = 1, 2) \\
\lim_{x \to \pm \infty} \left\{ \frac{\partial}{\partial x} + ik \right\} \phi_m = 0 \\
\end{array} \right.
\]  

(5)

in which \(\delta_{i,j}\) and \(\delta_{i,k}\) \((i = 1\) and 2\) denote the horizontal coordinate values of right and left boundaries of the \(i\)-th pontoon, respectively. \(\delta_{1}\) and \(\delta_{2}\) are the complex motion response amplitude in surge and pitch modes. \(\mathbf{c}_0^i = (\delta_{i0}, \delta_{i3})\) is the rotational centre of the front-pontoon. To simplify and unify the boundary conditions, \(\phi_j\) (i.e., when \(m = 0\)) is introduced to represent the diffraction velocity potential \(\phi_D\),

\[
\delta_{m} = \left\{ \begin{array}{c}
1 \quad p = q \\
0 \quad p \neq q
\end{array} \right.
\]  

(6)

The analytical expressions of all velocity potentials are given by the method of variables separation in each subdomain plotted in Figure 1, namely I, II, III, IV and V. Then, the eigenfunction matching method is employed to determine the unknown coefficients in analytical expressions. To be specific, those expressions are matched on the interfaces of adjacent subdomains based on the continuous conditions of the velocity potential and the horizontal velocity [36, 39]. Hence, the explicit forms of all velocity potentials are obtained. The horizontal exciting force and the rotational exciting moment can then be calculated by,

\[
F_m = i \omega \rho \int \left[ \phi_1 + \phi_D \right] n_m ds
\]  

(7)

where \(F_m\) denotes the generalized wave exciting force of the front-pontoon in \(m\) mode, \(\rho\) the fluid density, \(\delta\) the wetted surface of the front-pontoon, and \(n_m\) is the generalized normal direction. Note \(n_1 = n_x\) and \(n_3 = (\bar{z} - \delta_{i0}) n_x - (x - \delta_{i0}) n_{\bar{z}}\) and \(n = n_x + n_{\bar{z}} k\) is the unit normal vector on the pontoon surface pointing into the body.

The generalized wave force resulting from radiation potential due to unit-surge or unit-pitch motion is decomposed into two parts related to the added mass (proportional to the front-pontoon motion acceleration) and the radiation damping (proportional to the front-pontoon motion velocity), which can be calculated by,

\[
\mu_m = \rho \int \text{Re} \left[ \phi_m \right] n_m ds
\]  

(8)

\[
\lambda_m = \rho \omega \int \text{Im} \left[ \phi_m \right] n_m ds
\]  

(9)

where \(\mu_m = -i \omega \phi_m \delta_{i3,3}; \mu_m\) and \(\lambda_m\) are the added mass and the radiation damping of the front-pontoon in \(m\) mode induced by a unit-amplitude motion of the front-pontoon in \(m\) mode, respectively.

Finally, the equation of motion for the integrated system can be written as,

\[
(-\omega^2 (M_m + \mu_m) - i \omega (\lambda_m + \lambda_{m,PTO}) + K_m) \xi_m = F_m
\]  

(10)

where \(M_m\) and \(K_m\) are the mass and stiffness of the integrated system in \(m\) mode \((m = 1, 3)\), respectively; \(\lambda_{m,PTO}\) is the PTO
damping in \( m \) mode. In this study, \( \lambda_{PTO}^{m} \) is selected to be the optimal PTO damping [36, 39],

\[
\frac{\partial C_w}{\partial \lambda_{PTO}^{m}} = 0 \Rightarrow \lambda_{PTO}^{m} = \sqrt{(K_m/\omega - \omega (M_m + \mu_m))^2 + (\lambda_m)^2}
\]

\[M_1 = \rho B_1 T_1, \ K_1 = \rho g A_1\]

\[M_3 = I_{11} + I_{33} \ K_3 = \rho g \left( I_{11}^{*} + I_{33}^{*} \right) - \rho B_1 T_1 \left( \zeta - \zeta_0 \right) \]

\[(11)\]

where \( I_{11}^{*} \) and \( I_{33}^{*} \) the rotational inertia of the first pontoon about \( x = x_0 \) and \( z = z_0 \), respectively, \( \rho_1, A_1 = B_1 \) and \( V_1 = B_1 T_1 \) the density of the first pontoon, water line and section area of underwater part of the first pontoon, respectively, \( I_{A_1}^{11} \) the quadratic moment of inertia of \( A_1 \) about \( x = x_0 \), \( IV_1^{3} \) the quadratic moment of inertia of \( V_1 \) about \( z = z_0 \). \( Z_c = (x_c, z_c) \) the centroid of the first pontoon.

The power \( P_{\text{cap}} \) absorbed by the integrated system in \( m \) mode is,

\[
P_{\text{cap}} = \frac{1}{2} \omega^2 \lambda_{PTO}^{m} |\xi_m|^2
\]

(12)

According to the linear wave theory, the averaged energy flux per unit width \( P_{\text{inc}} \) in the incident wave can be expressed as,

\[
P_{\text{inc}} = \frac{1}{4} \frac{\rho g A^2 \omega}{k^2} \left( 1 + \frac{2kd}{\sinh 2kd} \right)
\]

(13)

The capture width ratio \( C_w \) (i.e., energy conversion efficiency) is defined as a ratio of the captured wave energy against the incident wave energy per unit width,

\[
C_w = \frac{P_{\text{cap}}}{P_{\text{inc}}}
\]

(14)

The reflection coefficient \( K_R \) and transmission coefficient \( K_T \) of the breakwater can be calculated by,

\[
K_R = \frac{\phi_D + \phi_m}{\phi_1} \bigg|_{\zeta = 0, x = -\infty}
\]

\[
K_T = \frac{\phi_1 + \phi_D + \phi_m}{\phi_1} \bigg|_{\zeta = 0, x = -\infty}
\]

(15)

(16)

For \( K_T < 0.5 \), a floating breakwater is basically considered satisfactory for shore protection [32, 40]. A WEC is regarded as being in the effective working state [33, 36] for capture width ratio \( C_w > 0.2 \) taking the energy losses in real engineering application. Therefore, the effective frequency bandwidth \( (K_T < 0.5, \ C_w > 0.2) \) is used to assess the overall performance of the proposed system. From the perspective of structural safety, this paper also defines the dimensionless free-surface elevation at the centre between pontoon 1 and 2 as,

\[
A_\epsilon = \frac{A_{\text{outer}}}{A}
\]

(17)

3 | MODEL VALIDATION

To validate the proposed analytical model on the diffraction potential, radiation potential and the energy conversion performance, three examples are carried out. With regard to the diffraction potential, Kreisel [41] derived that the transmission coefficient in diffraction potential field is independent of the direction of the incident wave even though the body may not possess any symmetry at all. So the present model can be validated by comparing the transmission coefficient between two cases, that is, (1) an incident wave from left to right (2) an incident wave from right to left. Figure 2(a) shows the comparisons of the present model on \( K_T \) in diffraction potential field, in which the geometrical parameters are set as \( B_1/d = 0.2, B_2/d = 0.6, T_1/d = 0.125, T_2/d = 0.25 \) and 0.5, \( D/d = 0.2 \). It can be seen that the calculated results of two incident directions

\[
\begin{align*}
\text{FIGURE 2} & \quad \text{Variations of the transmission coefficient for the dual-pontoon versus the dimensionless wavenumber } kd. \text{ Case 1 representing the incident wave from left to right, Case 2 representing the incident wave from right to left.}
\end{align*}
\]
are in good agreement, indicating that the present model can correctly solve the diffraction problem. In addition, Figure 2(b) shows the comparisons of the present dual-pontoon system on $K_T$ when the first pontoon is allowed to surge to capture energy with the optimal PTO damping. Interestingly, $K_T$ is still irrelevant to the incident wave direction whose proofs can be seen in [42]. However, $K_T$ is different from the cases that the pontoons are both fixed due to the energy absorption of the moving front pontoon.

Furthermore, the Haskind Relation [43] is used to verify the correctness of the radiation potential due to the surge or pitch motion of front-pontoon. Figure 3(a) shows the variation of generalized wave exciting force with the dimensionless wavenumber $kd$. The geometrical parameters are $B_1/d = 10^{-4}$, $B_2/d = 0.6$, $T_1/d = T_2/d = 0.125$, $D/d = 0.2$, $H_1/T_1 = H_2/T_2 = 2$, and $C_0 = [(\gamma_1 + \gamma_2)/2, -T_1]$. To illustrate that present model is still reliable for the integrated system with relatively small pontoon widths, the front pontoon $B_1/d$ is set as $10^{-4}$. Additionally, the power output associated with the surge and pitch motions is tested against the energy conservation relation of $K_R^2 + K_T^2 + C_w = 1$. Figure 3(b) shows the variation of $K_R$, $K_T$, $C_w$ and $K_R^2 + K_T^2 + C_w$ with $kd$ for $B_1/d = 0.3$, $B_2/d = 0.6$, $T_1/d = 0.125$, $T_2/d = 0.25$, $D/d = 0.2$. As shown in Figure 3(a,b), the present results fully satisfy those theoretical relations. Overall, the reliability of the present analytical model is well validated.

4 | NUMERICAL RESULTS AND DISCUSSIONS

The present study is mainly focused on the effects of the front-pontoon width, draft and pontoon spacing on the wave energy conversion for the integrated system with surging or pitching front-pontoon. In fact, it is found in this study that the hydrodynamic performances of the system with pitching and surging front-pontoon are similar to each other, which can be explained by the basically same physical mechanisms. Therefore, with regard to the pitching front-pontoon, this paper briefly describes the difference on hydrodynamic performance between pitch and surge motions. The study is carried out under the conditions of $B_2/d = 0.6$, $T_2/d = 0.25$, $H_1/T_1 = H_2/T_2 = 2$, $C_0 = [(\gamma_1 + \gamma_2)/2, -T_1]$, $d = 10$ m and $g = 9.81$ m/s². Note that $\xi_1$ and $\xi_3$ are altered to respectively represent the dimensionless $[\xi_1]/[\text{rad}]$ and $[\xi_3]/[\text{m}]$ in order to simplify the expression. $\xi_1$ and $\xi_3$ are named as surging and pitching RAO (response amplitude operator of the front-pontoon), respectively.

4.1 | System with the surging front-pontoon

4.1.1 | Effects of the front-pontoon width

For the surging front-pontoon, the effects of the width of front-pontoon on the hydrodynamic performance of the integrated system are firstly studied for different $B_1/d = 0.2$, 0.3, 0.4 and 0.6 in this subsection, maintaining $T_1/d = 0.125$ and $D/d = 0.2$. Figure 4 shows the variations of the reflection coefficient $K_R$, transmission coefficient $K_T$ and capture width ratio $C_w$ with dimensionless wavenumber $kd$ for various widths of the front-pontoon. Also, $K_R$ and $K_T$ of the fixed isolated single pontoon ($B_2/d = 0.6$, $T_2/d = 0.25$) are plotted in Figure 4 to compare with the proposed integrated system with the WEC. The shape of the curve of $K_R$ (or $K_T$, $C_w$) with $kd$ for four front-pontoon widths are generally similar. It can be seen from Figure 4(a) that $K_R$ decreases considerably when $kd > 2$ (relatively high frequency region) due to the existence of the surging front-pontoon (i.e., WEC), in which the reflection coefficient presents an obvious trough region near $kd = 3$ and the trough is shifted to the low frequency region with the increase of $B_1$. In Figure 4(e), $C_w$ increases firstly to the peak $C_w \approx 1$ near $kd = 3$ and then decreases with the increase of $kd$. Additionally, the variation of $C_w$ with the width $B_1$ shows similar frequency shift phenomenon to the trough of $K_R$. Associated with that the reflection coefficient of isolated single pontoon does not present the decreasing trend, it is considered that the obvious trough region of $K_R$ is because a certain amount of wave energy is extracted by the front-pontoon. In term of wave transmission $K_T$, with the increase of $B_1$, it slightly decreases in the range of $kd < 2$ and $kd > 4$, but increases in $2 < kd < 3$. Moreover, compared with
FIGURE 4 Variations of reflection coefficient $K_R$, transmission coefficient $K_T$ and capture width ratio $C_w$ versus the dimensionless wavenumber $kd$ for various widths of the front-pontoon

The capture width ratio is close to zero in long wave region $kd < 2$ (i.e. low frequency region) as shown in Figure 4(c), demonstrating that the integrated system as a WEC mainly works in the intermediate and short waves. The reason is that most of the incident wave energy can transmit across the integrated system due to the strong transmission ability of long waves. As a result, the transmission wave amplitude is approximately equal to the incident wave amplitude. Thus, as illustrated in Figure 5(b), the wave exciting forces on the left and right sides of front-pontoon almost counterbalance (i.e. $F_1 \approx 0$) in the range of $kd < 2$, which results in $C_w \approx 0$. In addition, it can be seen from Figure 4(c) that the peak value of $C_w$ slightly increases with the decrease of $B_1$, and the effective bandwidth of $C_w$ is significantly widened in the high frequency region of $kd > 5$. As the width $B_1$ increases, the wave exciting force basically maintains constant in the high frequency region as displayed in Figure 5(b), but the front-pontoon mass $M_1$ increase linearly. Thus, the surging RAO $\xi_1$ of a smallwidth pontoon is larger than that of a large one in high frequency region, leading to a relative wider effective bandwidth of $C_w$.

4.1.2 Effects of the front-pontoon draft

Both the width ratio of the front-pontoon $B_1/d$ and pontoon spacing ratio $D/d$ are set as 0.2 to examine the effects of
FIGURE 5 Variations of wave amplitude $A_c$ between pontoons, wave exciting forces $F_1$, added mass $\mu_1$, radiation damping $\lambda_1$ and surging $\mathcal{R}AO$ of the front-pontoon $\xi_1$ versus the dimensionless wavenumber $kd$ for various widths of the front-pontoon $T_1$. It can be seen from Figure 6(a) that the variation of $K_R$ with $kd$ is similar to that mentioned above in Section 4.1.1. In particular, $K_R$ shows two local troughs and an obvious local peak in the range of $2 < kd < 5$, which is different from that in Figure 4(a) where only one trough was observed. Also, the local peak and first trough (near $kd = 2 \sim 3.5$) of $K_R$ both increase and shift to low frequency region with the increase of $T_1$, while the second trough occurring at about $kd = 4$ varies little with $T_1$.

Two local peaks (corresponding to the two local troughs of $K_R$) and a local trough (corresponding to the local peak of $K_R$) occur on the curves of $C_w$ in the frequency range of $2 < kd < 5$. The explanation for this phenomenon is that the peak of $A_c$ is gradually away from that of $\xi_1$ with the increase of $T_1$. Figure 7 shows the distributions of the wave amplitude $A_c$, surging $\xi_1$ and optimal PTO damping $\lambda_1$ with $kd$. As shown in Figures 7(a,b), the peaks of $A_c$ and $\xi_1$ are near $kd = 2.5$ and 4, respectively. With increasing the draft $T_1$, the frequency left-shift of the peak of $A_c$ due to the increase of water mass between pontoons is obviously greater than that of the peak of $\xi_1$, which means that the affected regions of the gap resonance and $\xi_1$ peak are gradually separated with the increase of $T_1$. The integrated system hence performs the weak energy conversion ability in the frequency region between the frequencies of $A_c$ and $\xi_1$ peaks so that the local trough occurs on the curves of $C_w$ and two local peaks appear. Obviously, the first local peak (or trough) of $C_w$ (or $K_R$) is dominated by gap resonance and the second local peak is related to the peak of $\xi_1$. But it should be noted that the gap resonance inevitably affects the second local peak of $C_w$ and the peak of $\xi_1$ affects the first one, which is the reason why frequencies of the local peaks of $C_w$ are not exactly the same as those of the peaks of $A_c$ or $\xi_1$. Additionally, the effect of the draft $T_1$ on $\xi_1$ is very limited in frequency region $2 < kd < 3$ as shown in Figure 7(b), while its effect on $C_w$ is apparent as illustrated in Figure 6(c). The fact is that both the value and frequency of the peak of optimal
PTO damping $\lambda_{PTO}$ are affected by the gap resonance. In view of structural safety, although the surge motion causes the increase of $A_c$ in the range of $kd > 3$, $A_c$ is less than 2, and it should be noted that the maximum wave amplitude $A_c$ of the present system is significantly decreased at gap resonance frequency in comparison with ‘fixed dual pontoons’ due to the fact that some wave energy is captured by the surging front-pontoon. Overall, the proposed system can better satisfy the requirements of engineering safety.

It is found that the two local peaks of $C_w$ (or the corresponding local troughs of $K_R$) are related to the zero points of the sum of front-pontoon mass $M_1$ and added mass $\mu_1$. Figure 8 shows the variations of $G(\omega)$ with $kd$ for four different front-pontoon drafts, in which $G(\omega) = M_1 + \mu_1$. There are just two zero points in $G(\omega)$ and the variations of them with the draft $T_1$ are similar to the two local troughs of $K_R$ respectively. Table 1 gives precise frequencies corresponding to zero points of $G(\omega)$.
and local troughs of \( K_R \). Whether the first or second trough, there is a little deviation between the corresponding frequencies to \( G(\omega) = 0 \) and to the local trough of \( K_R \), especially the small drafts \( T_1 \). The deviation decreases with the increase of \( T_1 \), which is due to the weaker interaction between the gap resonance and \( \xi_1 \) peak. Overall, \( G(\omega) \) can be employed to predict the local troughs of \( K_R \) and local peaks of \( C_w \) well.

### 4.1.3 Effects of the pontoon spacing

The effects of the pontoon spacing \( D \) on the hydrodynamic performance are also studied under the conditions of \( B_1/d = 0.2 \) and \( T_1/d = 0.125 \). Figure 9 gives the variations of the reflection coefficient \( K_R \), transmission coefficient \( K_T \) and capture width ratio \( C_w \) versus the dimensionless wavenumber \( kd \) for various spacing \( D \) between pontoons.
width ratio $C_w$ versus dimensionless wavenumber $kd$ for various pontoon spacing $D/d = 0.1, 0.15, 0.2$ and $0.3$. As shown in Figure 9(a), the $K_R$ curves show a similar oscillation phenomenon to that in Figure 6(a), in which $K_R$ is considerably small in the intermediate frequency region and presents two local troughs and a local peak in general. With the increase of $D$, the local peak gradually decreases and even disappears for $D/d = 0.3$. Consequently, the two troughs merge into a single one. It can also be seen that all extreme points of $K_R$ are obviously shifted to the low frequency region as $D$ increases. $C_w$ presents the corresponding local peaks and trough as given in Figure 9(c). Moreover, the effective frequency bandwidth of $C_w$ decreases with the increase of $D$. With regard to the wave transmission, $K_T$ slightly increases with the increasing $D$ in the range of $1 < kd < 2.5$, and then presents an opposite variation trend as shown in Figure 9(b). Generally, the effect of pontoon spacing on wave transmission is very limited.

The decrease of the local peak of $K_R$ (or the increase of the local trough value of $C_w$) is due to the fact that the peak frequency regions of $A_c$ and $\xi_1$ are close to each other. The variations of wave amplitude $A_c$ between pontoons and surging RAO $\xi_1$ with wavenumber $kd$ are plotted in Figure 10 for various spacing $D$. It can be found that the peak of $A_c$ and $\xi_1$ are both shifted to the low frequency region with the increase of $D$, in which frequency-shift distance of $A_c$ is only 1.38 (i.e. the frequency difference of $A_c$ peaks between $D/d = 0.1$ and $0.3$), while that of $\xi_1$ is 2.97. It implies that the gap resonance frequency and peak frequency of $\xi_1$ gradually approach with the increasing $D$ (though the peak frequency region of $A_c$ and $\xi_1$ does not overlap fully), thus the wave energy conversion ability in the frequency region between the peaks of $A_c$ and $\xi_1$ is enhanced, i.e., the local trough on the curve of $C_w$ fades away. Similarly, the frequencies corresponding to local troughs of $K_R$ and zero points of $G(\omega)$ for various spacing $D$ are listed in Table 2. It can be seen that difference between the corresponding frequencies to $G(\omega) = 0$ and to the local trough of $K_R$ slightly increases with the enhancement of interaction between gap resonance and $\xi_1$ peak as $D$ increases. Unfavourably, for the largest spacing $D/d = 0.3$, zero points are not available in the curve of $G(\omega)$ in Figure 11, meaning that $G(\omega)$ is not applicable to predict the occurrence of the local troughs of $K_R$ or local peaks of $C_w$ for large pontoon spacing $D/d$. 

### Table 2

| $D/d$ | 0.10 | 0.15 | 0.20 | 0.30 |
|-------|------|------|------|------|
| Troughs of $K_R$ | 3.92 | 3.52 | 3.376 | 3.056 |
| $G(\omega) = 0$ | 3.92 | 3.536 | 3.416 | N/A |

The first local trough of $K_R$

| $D/d$ | 0.10 | 0.15 | 0.20 | 0.30 |
|-------|------|------|------|------|
| Troughs of $K_R$ | 6.376 | 4.928 | 3.88 | 3.056 |
| $G(\omega) = 0$ | 6.376 | 4.928 | 3.872 | N/A |

The second local trough of $K_R$
4.2 System with the pitching front-pontoon

As mentioned above, the hydrodynamic performance of the integrated system with pitching front-pontoon is similar to that with surging one due to that both the two motions are anti-symmetric. Thus, this section only shows the effects of front-pontoon draft to discuss some extraordinary phenomena in the integrated system with pitching front-pontoon. The variations of the reflection coefficient $K_R$, transmission coefficient $K_T$ and capture width ratio $C_w$ are depicted in Figure 12 for front-pontoon draft $T_1/d = 0.1, 0.125, 0.2$ and $0.25$, and keeping $B_1/d = 0.5$ and $D/d = 0.2$.

As shown in Figures 12, $K_R$ increases and $K_T$ decreases with $kd$ in the low frequency region (i.e. $kd < 2$) generally. For $kd > 2$, since a large amount of wave energy is absorbed by the WEC in the intermediate frequency region, $C_w$ reaches up to its main peak near $kd = 3$ and $K_R$ shows a trough region. The main peak of $C_w$ is shifted to the low frequency region and its peak value decreases with the increase of $T_1$, and the effective frequency region of $C_w$ becomes narrow at the same time. Furthermore, a sharp peak (hereafter named as sub-peak) occurs on the curve of $C_w$ in the range of $kd < 2$ for a relatively small draft $T_1$ as shown in Figure 12(c). $K_R$ and $K_T$ also display similar extreme points at the same frequency points as illustrated in Figures 12(a,b). To clarify the variations of the main peak of $C_w$ and the occurrence of sub-peak, Figure 13 gives the variations of wave amplitude $A_c$ between pontoons and pitching RAO $\xi_3$ with $kd$ for the corresponding drafts $T_1$. It can be seen that both the main peaks of $A_c$ and $\xi_3$ are shifted to the low frequency region with the increase of $T_1$. Just as discussed in Section 4.1, the main peak of $C_w$ thus shifts to the low frequency region. In addition, $\xi_3$ decreases obviously as $T_1$ increase in the range of $kd > 2.8$ as shown in Figure 13(b). Meanwhile, the peak shift distance of $\xi_3$ is larger than that of $A_c$, which causes peak frequency regions of $A_c$ and $\xi_3$ further overlapping each other. The two aspects jointly lead the main peak of $C_w$ and the effective frequency bandwidth of $C_w$ to decrease with the increase of $T_1$.

It can be found that $\xi_3$ presents a sharp peak at the frequency corresponding to the sub-peak of $C_w$. The frequency is equal to the resonance frequency which can be calculated by $\omega_{res} = (K_1/(M_1+\mu_1))^{1/2}$. Therefore, it is confirmed that the occurrence of the sub-peak is due to the pitch-motion resonance. The restore moment decreases and the rotational inertia increases with the increase of draft $T_1$, which leads to a decreasing pitching resonance frequency for $T_1/d = 0.1, 0.125$ and $0.15$ as shown in Figure 12(c). However, according to the formula of restore moment $K_1 = \rho g (B_1^2 - 6 T_1^2)$, the restore moment is positive only when $B_1 > \sqrt{6} T_1$, so there is no sharp sub-peak on the curves of $C_w$ for $T_1/d = 0.2$ and $0.25$. With regard to the wave attenuation ability, the effect of $T_1$ on $K_T$ is concentrated in intermediate frequency region $2 < kd < 5$ as shown in Figure 12(b). Because of the influence of gap resonance, $K_T$ increases with increase of $T_1$ in the range of $2 < kd < 2.5$. However, the pitch motion of front-pontoon considerably decreases with increase of $T_1$ in $kd > 3$ as depicted by Figure 13(b), leading to the decrease of radiation waves, so $K_T$ gradually decreases.

![Figure 12](image12.png)

**FIGURE 12** Variations of reflection coefficient $K_R$, transmission coefficient $K_T$ and capture width ratio $C_w$ versus the dimensionless wavenumber $kd$ for various drafts of the front-pontoon

5 CONCLUSIONS

The fixed floating breakwater with a movable front-pontoon is proposed as an OB WEC-breakwater integrated system. The front-pontoon is equipped PTO system to extract wave energy...
from its surge or pitch motion. Based on linear potential flow theory, the energy conversion performance of the integrated system is investigated by methods of eigenfunction matching and variables separation. In this paper, the effects of the front-pontoon width, draft and pontoon spacing on the reflection coefficient, transmission coefficient and capture width ratio are presented and discussed. Key conclusions are summarized as follows:

1. There are many similarities on the hydrodynamic performance between the integrated system with pitching front-pontoon and that with surging one. Their effective working frequency are both in the range of $kd > 2$ and the peaks of reflection coefficient, transmission coefficient and capture width ratio are presented and discussed. Key conclusions are summarized as follows:

2. With the increase of the front-pontoon width, the effective frequency bandwidth gradually decreases and the peak of $C_w$ is shifted to the low frequency region. It is found that the occurrence of peak of $C_w$ is due to the gap resonance and the increase of front-pontoon motion resulted from ‘hydrodynamic constructive effect’. Consequently, it is obvious that the frequency shift phenomenon of $C_w$ peak is because the frequencies corresponding to peaks of $A_c$ and $\xi_1$ are both shifted to the low frequency region with the increase of $B_1$.

3. As the draft of front-pontoon increases, two local troughs and a clear local peak appear on the curve of reflection coefficient, in which the first local trough and the peak of $K_R$ increase and shift to the low frequency region, while the value and occurrence frequency of the second trough are almost kept constant. The capture width ratio presents the corresponding extreme points to $K_R$. That is because the frequency shift distance of gap resonance is relatively large, but that of peak of $\xi_1$ is small with the increase of $T_1$. In addition, it is found that $F(\omega) = M_1 + \mu_1 = 0$ can be adopted to predict the occurrence frequencies of local peaks of $C_w$ (or local troughs of $K_R$) in $2 < kd < 5$.

4. Since both the frequencies to peaks of $A_c$ and $\xi_1$ are obviously shifted to the low frequency region with the increasing pontoon spacing $D$, the extreme points on the curve of $C_w$ also shows the same shift phenomenon. Moreover, the peaks of $A_c$ and $\xi_1$ are close to each other with the increase of $D$, causing the two local peaks of $C_w$ merging and the effective frequency bandwidth of $C_w$ decreasing.

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