Confinement transition in the QED$_3$-Gross-Neveu-XY universality class

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The coupling between fermionic matter and gauge fields plays a fundamental role in our understanding of nature, while at the same time posing a challenging problem for theoretical modeling. In this situation, controlled information can be gained by combining different complementary approaches. Here, we study a confinement transition in a system of $N_f$ flavors of interacting Dirac fermions charged under a U(1) gauge field in 2+1 dimensions. Using Quantum Monte Carlo simulations, we investigate a lattice model that exhibits a continuous transition at zero temperature between a gapless deconfined phase, described by three-dimensional quantum electrodynamics, and a gapped confined phase, in which the system develops valence-bond-solid order. We argue that the quantum critical point is in the universality class of the QED$_3$-Gross-Neveu-XY model. We study this field theory within a $1/N_f$ expansion in fixed dimension as well as a renormalization group analysis in $4-\epsilon$ space-time dimensions. The consistency between numerical and analytical results is revealed from large to intermediate flavor number.

I. INTRODUCTION

The coupling between fermionic matter and gauge fields is of fundamental importance in both high-energy and condensed-matter physics. In the latter context, gauge fields can emerge as a consequence of fractionalization in quantum materials. Prominent examples include spin liquids [1, 2] and deconfined quantum critical points [3–8] in frustrated magnets. As such states are characterized by topological order, it is often difficult to identify in an experiment or a simulation the relevant low-energy excitations and their characteristic properties.

In many cases, however, it is possible to tune the system by nonthermal external parameters, such as pressure or magnetic field, through a low-temperature transition between an exotic phase with topological order and a conventionally ordered phase. If this quantum phase transition is continuous, the presence of fractionalized low-energy excitations on the topologically ordered side of the transition leaves characteristic fingerprints on the pertinent quantum critical behavior. These are in principle measurable and can be used to identify the topological order. To take advantage of this fact, it is decisive to gain a comprehensive understanding of the quantum universality classes that involve fluctuating gauge fields.

Recent work on a system of fermions coupled to a discrete $\mathbb{Z}_2$ gauge field in 2+1 dimensions has demonstrated the existence of such a quantum transition between a deconfined phase with gapless fermionic excitations at weak coupling and a symmetry-broken confined phase with gapped fermionic excitations at strong coupling [9–13]. Systems of gapless fermions coupled to gauge fields with continuous gauge groups are of significant current interest as well. An important example is (2+1)-dimensional quantum electrodynamics (QED$_3$) with U(1) gauge group, both in its compact and noncompact version [14–30]. This theory is a natural candidate for the low-energy description of gapless U(1) spin liquids, which may be realizable in certain planar magnets [31–37].

Using Quantum Monte Carlo (QMC) simulations, the phase diagram of a lattice model of fermions coupled to a compact U(1) gauge field has recently been mapped out by some of the present authors [38, 39]. The model has a free parameter that can be used to drive a transition from a U(1) deconfined (U1D) phase with nodal dispersion of the fermionic excitations to confined phases in which spin and/or lattice symmetries are spontaneously broken, see Fig. 1. The U1D phase represents a lattice realization of QED$_3$, while the confined phases describe different conventionally ordered phases.

Meanwhile, several analytical works, such as renormalization group (RG) and large-$N_f$ calculations, have made quantitative predictions for the universal behavior of quantum critical fermion systems coupled to gauge fields [40–48]. However, while there has been tremendous progress in the case of the ungauged (2+1)-dimensional Gross-Neveu-like transitions [49–63], the situation with fluctuating gauge fields remains significantly less clear. In particular, a quantitative comparison of universal properties between analytical predictions from the continuum quantum field theory and measurements in corresponding lattice simulations has so far not been possible. Such a comparison is the goal of this work.

The remainder of the paper is organized as follows: In Sec. II, we review the lattice model studied in Ref. [38] and present additional evidence for a continuous transition between deconfined and confined phases. The continuum field theory that we propose to describe the universal behavior of the confinement transition is discussed in Sec. III. Section IV
contains the comparison of the field-theory predictions with the numerical results. Conclusions are drawn in Sec. V.

II. LATTICE MODEL AND QMC RESULTS

We simulate a square-lattice model described by the action

\[ S = \int_0^\beta d\tau (L_F + L_\phi) \]

\[ L_F = \sum_{\langle ij \rangle, \alpha} \bar{\psi}_i^{\alpha} \left[ (\partial_\tau - \mu) \delta_{ij} - t e^{i\phi_1} \right] \psi_j^{\alpha} + \text{h.c.,} \]

\[ L_\phi = \frac{4}{J N_f \Delta \tau^2} \sum_{\langle ij \rangle} \left[ 1 - \cos \left( \phi_1 (\tau + 1) - \phi_1 (\tau) \right) \right] \]

\[ + \frac{1}{2} K N_f \sum_{\square} \cos (\text{curl} \phi), \]

where \( \psi_\alpha \) and \( \bar{\psi}_\alpha \) denote fermion creation and annihilation operators at lattice site \( i \) and flavor index \( \alpha = 1, \ldots, N_f \). The compact U(1) gauge field lives on nearest-neighbor bonds \( \langle ij \rangle \) and is denoted by \( \phi_1 \). The nearest-neighbor fermion hopping amplitude \( t \) is modulated with phase \( \phi_1 \), thereby inserting magnetic flux in each plaquette. The expression \( \text{curl} \phi \) sums the gauge fields in each elemental plaquette \( \square \) and the coupling \( K > 0 \) stabilizes a flux in each plaquette, see Fig. 1(a).

In the \( J \to 0 \) limit, the gauge field has no imaginary time dynamics. The ground state in this limit is characterized by gapless Dirac excitations, as both \( L_F \) and the flux term in \( L_\phi \) with \( K > 0 \) favor a flux in each plaquette. The Brillouin zone contains two Dirac cones per flavor and the number of (irreducible) Dirac fermions is \( 2 N_f \).

Turning on a small finite \( J \) allows the U(1) gauge field to fluctuate. This model has been studied by sign-problem-free QMC simulations [38]. For \( N_f = 2, 4, 6, 8 \), the low-temperature phase for small \( J \) has been found to be characterized by a gapless spectrum and emergent scale invariance. This phase represents a lattice realization of QED3 with deconfined excitations and as such has been dubbed U1D. In particular, for small \( J \), the simulations suggested no evidence for confining monopole proliferation [39].

Upon increasing \( J \) beyond a certain threshold, however, the fermions acquire a mass, monopoles of the compact gauge field start to proliferate, and the fermions exhibit a confining potential, see Fig. 1(b). For \( N_f = 4, 6, 8 \), the confined phase is described by a valence bond solid (VBS), characterized by an ordered array of spin singlets, thereby breaking lattice translation and rotation symmetries spontaneously.

To improve the understanding of the U1D-VBS transition, we have carried out additional QMC simulations near the transition point and on larger lattices. Technical details of the simulations, including the update scheme, the determination of the phase diagram, and the calculation of dynamical properties near the transition, are given in Refs. [38, 39]. Fig. 2 shows the numerical data of VBS correlation ratios [64]

\[ r_{\text{VBS}} = 1 - \frac{\chi_D (\bar{M} + \delta \bar{q})}{\chi_D (\bar{M})}, \]

where \( \bar{M} = (\pi, 0), \delta \bar{q} = (\frac{2 \pi}{7}, 0) \), for different flavor numbers \( N_f \) in the vicinity of the U1D-VBS phase transition. In the above equation, \( \chi_D \) denotes the VBS structure factor, given as

\[ \chi_D (\hat{k}) = \frac{1}{L^2} \sum_{ij} \langle (D_i D_j) - \langle D_i \rangle \langle D_j \rangle \rangle e^{i\hat{k} \cdot (r_i - r_j)}, \]

where the dimer operator \( D_i = \sum_{\alpha \beta} S^\alpha_\beta (i) S^\beta_\alpha (i + \hat{x}) \) is defined along the \( \hat{x} \) direction, and the \( S^\alpha_\beta \) is the spin operator defined as \( S^\alpha_\beta (i) = \psi_\alpha^\dagger (i) \psi_\beta (i) - \frac{1}{\sqrt{2}} \delta_{\alpha\beta} \sum_\gamma \psi_\gamma^\dagger (i) \psi_\gamma (i) \). In the thermodynamic limit, \( r_{\text{VBS}} \) is zero (one) in the U1D disordered (VBS ordered) phase. The curves on different lattices suggest a single crossing point in the thermodynamic limit, implying the existence of a continuous quantum phase transition.

More evidence for a continuous phase transition comes from the measurement of the spin and dimer correlation functions \( C_S (r) \) and \( C_D (r) \). Technical details concerning these measurements are deferred to Appendix A. Fig. 3 shows the
FIG. 2. Correlation ratio of VBS order parameter across the U1D-VBS phase transition with linear system size up to \( L = 20 \), for (a) \( N_f = 4 \), (b) \( N_f = 6 \), and (c) \( N_f = 8 \).

FIG. 3. Real space decay of spin (a,c,e) and dimer (b,d,f) correlation function for \( N_f = 4 \), 6, 8, at the U1D-VBS transition point. The scaling dimension is measured by fitting the slope in the log-log plot. The gray points in (b), (d) and (f) are data points after deducting a background, more details are given in Appendix A.

real-space decay of the spin and dimer correlators at the crossing points of the correlation ratios for different flavor numbers \( N_f \). Instead of the usual exponential behavior, the correlators show a power-law decay with characteristic exponents \( 2\Delta_{AFM} \) and \( 2\Delta_{VBS} \). This constitutes further strong evidence for quantum criticality beyond the findings previously presented in Ref. [38].

The VBS order parameter has a discrete \( \mathbb{Z}_2 \) rotation symmetry, associated with the four possible VBS ordering patterns on the square lattice. At criticality, such a \( \mathbb{Z}_2 \) symmetry is usually expected to get enhanced to a continuous \( U(1) \approx O(2) \) rotation symmetry [65, 66]. This suggests that the low-energy description of the quantum critical point has to include three different types of excitations: \( 2N_f \) flavors of (irreducible) gapless Dirac fermions \( \psi \) and \( \bar{\psi} \), coupled to a \( U(1) \) gauge field \( a_\mu \), and a real two-component scalar field \( \phi = (\phi^1, \phi^2) \) serving as an order parameter for spontaneous O(2) symmetry breaking. The minimal theory that is consistent with these requirements is the QED\(_3\)-Gross-Neveu-XY (QED\(_3\)-GN-XY) model proposed in Ref. [38]. The purpose of the rest of this paper is to confirm that the numerical data is indeed consistent with this proposal.

III. CONTINUUM FIELD THEORY

In this section, we discuss the critical behavior of the QED\(_3\)-GN-XY model within two complementary analytical approaches: a \( 1/N_f \) expansion in fixed dimension and a RG analysis in \( D = 4 - \epsilon \) space-time dimensions. The Lagrangian of the QED\(_3\)-GN-XY model in \( D = 3 \) reads

\[
\mathcal{L} = \bar{\psi}(i\gamma_\mu(\partial_\mu - ia_\mu))\psi_\mu + \frac{1}{2g^2}(r - \partial_\mu)^2\phi^2 + \lambda(\phi^2)^2 + \frac{1}{2}\epsilon_{\mu
u\rho}a_\mu a_\nu a_\rho, \tag{5}
\]

where \( \psi \) and \( \bar{\psi} = \psi^\dagger \gamma_0 \) are two-component spinors, \( (\phi^\dagger) = (\phi^1, \phi^2) \), \( a = 1, 2 \), is the VBS order parameter, and the \( 2 \times 2 \) Dirac matrices satisfy the Euclidean Clifford algebra \( \{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}I_2, \mu, \nu = 0, 1, 2 \). In the Yukawa vertex \( \mu^\dagger \otimes I_2 \otimes I_{N_f/2} \), the \( 2 \times 2 \) Pauli matrices \( (\mu^\dagger, \mu) \) connect the two Dirac nodes, \( I_2 \) acts on spin indices, and \( I_{N_f/2} \) acts on the additional flavor indices; see Ref. [34] for details. Thus, the indices \( i \) and \( j \) run from 1 to \( 2N_f \), and we assume \( N_f \) even. \( a_\mu \) denotes the \( U(1) \) gauge field and \( \epsilon_{\mu\nu\rho} \) is the totally antisymmetric tensor. The parameters \( c, g \), and \( \lambda \) denote the charge, Yukawa, and quartic scalar couplings. The parameter \( r \) can be used to tune through the U1D-VBS transition.

For \( r > 0 \), the scalar field \( \phi^a \) can be integrated out and the theory describes the U1D phase with enhanced SU(2\( N_f \)) symmetry. Note that \( N_f = 2 \) then corresponds to the staggered-flux state on the square lattice [32], while \( N_f = 4 \) corresponds to the \( \pi \)-flux state [35, 67]. For \( r < 0 \), the fermions are fully gapped, since \( \gamma_0\mu^\dagger \) anticommutes with the Hamiltonian \( \mathcal{H}(\rho) = \gamma_0\rho_1\mu_1 \), and the gauge field confines due to the prolif-

\( \Delta_{AFM} \) and \( \Delta_{VBS} \) will be defined as scaling dimensions of AFM and VBS order parameters in Sec. III.
eration of monopoles [14]. This represents the VBS ordered phase. The critical point is given by $r = 0$.

Besides the universal exponents that describe the critical behavior in the vicinity of the quantum phase transition, we aim at computing the correlators of the staggered magnetization

$$O_{\text{AFM}}^\alpha(\vec{r}) \sim (-1)^{r_x + r_y} S_x(\vec{r}) \sim \hat{\psi}_i (\mu^\alpha \otimes \sigma^\alpha \otimes \mathbb{1}_{N_f/2}) \psi_j,$$

where $\alpha = x, y, z$ denotes the spin components and $\vec{r} = r_x \hat{x} + r_y \hat{y}$ the lattice site, and the staggered dimer operator

$$O_{\text{VBS}}^\alpha(\vec{r}) \sim \left((-1)^{r_x + r_y} \hat{S}_x(\vec{r}) \otimes \hat{S}_y(\vec{r} + \hat{y}) - (-1)^{r'_x + r'_y} \hat{S}_x(\vec{r}) \cdot \hat{S}_y(\vec{r} + \hat{y})\right)^\alpha
\sim \hat{\psi}_i (\mu^\alpha \otimes \mathbb{1}_2 \otimes \mathbb{1}_{N_f/2}) \psi_j$$

with $\alpha = x, y$ [34]. At criticality, they are given by power laws

$$\langle O_{\text{AFM}}^\alpha(\vec{r}) O_{\text{AFM}}^\alpha(0) \rangle \propto \frac{1}{r^{2\Delta_{\text{AFM}}}},$$

$$\langle O_{\text{VBS}}^\alpha(\vec{r}) O_{\text{VBS}}^\alpha(0) \rangle \propto \frac{1}{r^{2\Delta_{\text{VBS}}}},$$

where $\Delta_{\text{AFM}}$ and $\Delta_{\text{VBS}}$ are the scaling dimensions of the operators $O_{\text{AFM}}^\alpha$ and $O_{\text{VBS}}^\alpha$.

### A. Large-$N_f$ expansion

In the limit of large fermion flavor number $N_f$, the universal critical exponents and the scaling dimensions of the fermion bilinears can be computed analytically. We start by discussing the scaling dimensions $\Delta_{\text{AFM}}$ and $\Delta_{\text{VBS}}$. The diagrams that contribute at order $O(1/N_f)$ are shown in Figs. 4 and 5. We denote that at this order of the $1/N_f$ expansion, mixed diagrams that involve both scalar and gauge field propagators are finite and therefore do not contribute to the scaling dimensions. This can be understood as a consequence of the Ward identity associated with the U(1) gauge symmetry, in analogy to the QED$_3$-Gross-Neveu-Ising case [43, 44].

The gauge-field contributions to the scaling dimensions (Fig. 4) are well known [27, 32, 34, 44].

$$\Delta_{\text{VBS}}\big|_{\text{QED}_3} = \Delta_{\text{AFM}}\big|_{\text{QED}_3} = 2 - \frac{32}{3\pi^2 N_f} + O(1/N_f^2).$$

We note that Aslamazov-Larkin diagrams (not shown) vanish due to $\text{tr}(\mu^\alpha) = \text{tr}(\mu^\beta) = 0$ [44]. Both operators have exactly the same scaling dimensions as a consequence of the enhanced SU(2) symmetry for $r > 0$ in the U1D phase [32, 38]. However, at criticality, $r = 0$, this symmetry is explicitly broken and the scalar-field fluctuations lead to different values for $\Delta_{\text{VBS}}$ and $\Delta_{\text{AFM}}$. Evaluation of the diagrams in Fig. 5 gives the scalar contributions as

$$\Delta_{\text{VBS}}^{\text{GN-XY}} = 2 + \frac{4}{3\pi^2 N_f} + O(1/N_f^3),$$

$$\Delta_{\text{AFM}}^{\text{GN-XY}} = 2 - \frac{8}{3\pi^2 N_f} + O(1/N_f^3).$$

Since $\Delta_{\text{VBS}}$ is related to the scaling dimension of $\phi$ via $\Delta_{\phi} = (D - 2 + \eta_\phi)/2 = 3 - \Delta_{\text{VBS}}$, we obtain the order-parameter anomalous dimension of the pure Gross-Neveu-XY model (i.e., without the coupling to a gauge field) as

$$\eta_\phi^{\text{GN-XY}} = 1 - \frac{8}{3\pi^2 N_f} + O(1/N_f^3),$$

which is consistent with the previous calculation [68].

Summing both gauge-field and scalar-field contributions yields the scaling dimensions in the QED$_3$-GN-XY model as

$$\Delta_{\text{VBS}}^{\text{QED}_3-\text{GN-XY}} = 2 - \frac{28}{3\pi^2 N_f} + O(1/N_f^3),$$

$$\Delta_{\text{AFM}}^{\text{QED}_3-\text{GN-XY}} = 2 - \frac{40}{3\pi^2 N_f} + O(1/N_f^3).$$

The order-parameter anomalous dimensions then is

$$\eta_\phi^{\text{QED}_3-\text{GN-XY}} = 1 + \frac{56}{3\pi^2 N_f} + O(1/N_f^3).$$

We note that the VBS fermion bilinear correlator decays faster at the U1D-VBS transition than in the U1D phase itself, while the Néel fermion bilinear decays slower. The correlation-length exponent $\nu$ can be obtained within an analogous calculation to that of Ref. [44] for the QED$_3$-Gross-Neveu-Ising case. We find

$$\nu^{-1}\big|_{\text{QED}_3-\text{GN-XY}} = 1 - \frac{80}{3\pi^2 N_f} + O(1/N_f^3),$$

in agreement with the recent calculation presented in Ref. [69].

In order to make contact with the $4 - \epsilon$ expansion result discussed in the next subsection, it is instructive to generalize Eqs. (15), (16) to general space-time dimension $2 < D < 4$. Using the integration formulae derived in Refs. [43, 68], we
obtain
\[
\eta_\theta\big|_{\text{QED}_3,\text{GN-XY}} = 4 - D + \frac{4(D^2 - 2D)\Gamma(D - 1)}{D^2(2D - 2)\Gamma(-D/2)\Gamma(D/2)^3} \frac{1}{N_f} + O(1/N_f^2) \tag{17}
\]
and
\[
\nu^{-1}\big|_{\text{QED}_3,\text{GN-XY}} = D - 2 + \frac{2(D^2 - 2D + 2)\Gamma(D)}{D(D - 2)\Gamma(1 - D/2)\Gamma(D/2)^3} \frac{1}{N_f} + O(1/N_f^2). \tag{18}
\]

Further expanding the above equations in \( \epsilon = 4 - D \) leads to
\[
\eta_\theta\big|_{\text{QED}_3,\text{GN-XY}} = \epsilon + \frac{7\epsilon}{2N_f} - \frac{9\epsilon^2}{8N_f} + O(\epsilon^3, 1/N_f^2),
\]
\[
\nu^{-1}\big|_{\text{QED}_3,\text{GN-XY}} = 2 - \epsilon - \frac{15\epsilon}{2N_f} + \frac{41\epsilon^2}{8N_f} + O(\epsilon^3, 1/N_f^2),
\]
which, as we show below, agrees with the leading-order result that we obtain within the \( 4 - \epsilon \) expansion. This constitutes another nontrivial cross-check of our calculations.

The above analysis neglects the compactness of the U(1) gauge field in our lattice model. A compact gauge field admits instanton events, which correspond to insertion of magnetic flux in a localized region of space-time. In fact, the monopole operators, which perform this flux insertion, are relevant when the fermions are gapped out, leading to confinement of charges in the VBS phase [14, 37]. In the gapless U(1) phase, the scaling dimension \( \Delta_{q=1/2} \) of the least irrelevant monopole operator with minimal magnetic charge \( q = 1/2 \) (corresponding to \( 2\pi \) flux insertion) can be computed by employing the operator-state correspondence, which implies that \( \Delta_{q=1/2} \) is given by the ground-state energy of the corresponding theory on a sphere in the presence of a magnetic monopole [70]. For pure QED3, this computation gives
\[
\Delta_{q=1/2}\big|_{\text{QED}_3} = c_{\text{QED}_3} \cdot 2N_f + O(1/N_f^0), \tag{21}
\]
with \( c_{\text{QED}_3} = 0.265 \ldots \) (The correction at order \( O(1/N_f^0) \) has been computed in [71].) We note that \( \Delta_{q=1/2} \ll N_f \) in the large-\( N_f \) limit. The technical reason for this property is that the fermions can be integrated out in this limit, such that \( \Delta_{q=1/2} \) is simply the ground-state energy of \( N_f \) free Dirac fermions in the presence of a classical background gauge field. At large enough \( N_f \), monopoles are therefore irrelevant and the ground state of QED3 hence describes a stable U(1) phase with conformal invariance.\(^2\) The same argument applies to the QED3-GN-XY model. Hence,
\[
\Delta_{q=1/2}\big|_{\text{QED}_3,\text{GN-XY}} = c_{\text{QED}_3,\text{GN-XY}} \cdot 2N_f + O(1/N_f^0), \tag{22}
\]

with an unknown numerical constant \( c_{\text{QED}_3,\text{GN-XY}} > 0 \). Its value can in principle be computed in a way similar to Refs. [47, 48]. For our purposes, however, it is sufficient to note that \( \Delta_{q=1/2} > D = 3 \) for large enough \( N_f \), such that all monopoles operators are irrelevant and the critical theory indeed describes a stable RG fixed point with a unique infrared relevant direction (corresponding to the tuning parameter of the continuous transition).

B. RG analysis in \( D = 4 - \epsilon \) dimensions

We now demonstrate the existence of this fixed point beyond the \( 1/N_f \) expansion within a standard RG analysis. The QED3-GN-XY theory can be straightforwardly generalized to general space-time dimension \( 2 < D < 4 \) by replacing the \( 2N_f \) flavors two-component Dirac fermions by \( N_f \) flavors of four-component fermions [72]. In agreement with the QED3-Gross-Neveu-Ising case [40-42], all couplings become simultaneously marginal in four space-time dimensions. This allows to set up an \( \epsilon \) expansion around four space-time dimensions in the same way as in the standard Wilson-Fisher \( \phi^4 \) theory. Integrating over the \( D \)-dimensional momentum shell \( \Lambda/b < \sqrt{q^2 + \omega^2} < \Lambda \) with cutoff \( \Lambda \) and \( b > 1 \) causes the couplings to flow according to the equations
\[
\frac{de^2}{d\ln b} = \epsilon e^2 - \frac{4}{3}N_f e^4, \tag{23}
\]
\[
\frac{dg^2}{d\ln b} = \epsilon g^2 - (N_f + 1)g^4 + 6\epsilon g^2, \tag{24}
\]
\[
\frac{d\lambda}{d\ln b} = \epsilon \lambda - 2Ng^2(\lambda - g^2) - 5\lambda^2, \tag{25}
\]
where we have rescaled \( (e^2, g^2, \lambda) \mapsto (e^2, g^2, \lambda)/(4\pi^2\Lambda^2) \) and assumed that the theory is tuned to the critical point. Note that these flow equations agree with those of the continuum description of the Kekulé transition on the honeycomb lattice in the presence of a U(1) gauge field [73]. The flow admits an infrared stable RG fixed point at
\[
(e^2, g^2, \lambda) = \left( \frac{3}{4N_f}, \frac{2N_f}{2N_f(N_f + 1)}, \frac{C(N_f) - 8 - N_f}{10(N_f + 1)} \right) \epsilon + O(\epsilon^2), \tag{26}
\]
with \( C(N_f) = \sqrt{N_f^2 + 56N_f + 424 + 810/N_f} \).

The wave function renormalization function of the two-component scalar field reads
\[
\gamma_\theta(g^2) = N_f g^2. \tag{27}
\]
This expression is formally identical to the corresponding equation in the non-gauged Gross-Neveu-XY model, because the XY scalar field is not charged. At the fixed point, we obtain the order parameter anomalous dimension as
\[
\eta_\theta = \gamma_\theta(g^2) = \frac{2N_f + 9}{2(N_f + 1)} \epsilon + O(\epsilon^2). \tag{28}
\]

\(^2\) For small \( N_f \), the scaling dimension \( \Delta_{q=1/2} \) may drop below \( D = 3 \), such that monopoles would become relevant. The critical value of \( N_f \), below which this happens, cannot be computed in a controlled way within the \( 1/N_f \) expansion.
Further expanding this result for large $N_f$ leads to
\[ \eta_\phi = \epsilon + \frac{7 \epsilon}{2N_f} - \frac{7 \epsilon}{2N_f^2} + O(\epsilon^2, 1/N_f^3), \] (29)
in agreement with Eq. (19). The correlation-length exponent $\nu$ can be obtained from the flow of the tuning parameter,
\[ \frac{dr}{d \ln b} = (2 - 2\lambda - N_t g^2) r + 2\lambda - 2N_t g^2, \]
(30)

implying
\[ \nu^{-1} = 2 - \frac{2C(N_t) + 8N_t + 29}{10(N_t + 1)} \epsilon + O(\epsilon^2). \] (31)

To next-to-leading order in $1/N_t$, we obtain
\[ \nu^{-1} = 2 - \epsilon - \frac{15\epsilon}{2N_t} + \frac{87\epsilon}{2N_t^2} + O(\epsilon^2, 1/N_t^3), \] (32)
in agreement with Eq. (20), as announced above.

IV. EVIDENCE FOR QED$_2$-GN-XY UNIVERSALITY

We now discuss three possible universality classes that could in principle describe the continuous U1D-VBS transition observed in the lattice model. Among these three scenarios, we will see that the numerical data is consistent only with the QED$_2$-GN-XY universality class.

The transition between the U1D disordered phase and the VBS ordered phase is detected via a real two-component order parameter. Hence, if the U1D-VBS transition were a standard quantum transition within the Landau paradigm, quantum-to-classical mapping would apply, and one would expect the transition to be in the classical 3D XY universality class. Within Scenario 1 (Scen1), we thus assume the critical exponents $[74, 75]$

Scen1: $\nu^{-1}^{\text{QED}_1-\text{GN-XY}} = 1.48864(22), \quad \eta_\phi^{\text{QED}_1-\text{GN-XY}} = 0.0385(7).$ (33)

However, it is well known that the presence of gapless fermions at the transition can change the universality class $[76].$ Previous studies in systems with Dirac fermions (but without a fluctuating gauge field) coupled to a $O(2)$ order parameter have put forward the Gross-Neveu-XY universality class $[52, 77-83].$ This describes our Scenario 2 (Scen2), with exponents

Scen2: $\nu^{-1}^{\text{QED}_1-\text{GN-XY}} = 1 - \frac{16}{3\pi^2 N_t} + O(1/N_t^2), \quad \eta_\phi^{\text{QED}_1-\text{GN-XY}} = 1 - \frac{8}{3\pi^2 N_t} + O(1/N_t^2), \quad \Delta_{\text{VBS}}^{\text{QED}_1-\text{GN-XY}} = 2 + \frac{4}{3\pi^2 N_t} + O(1/N_t^2), \quad \Delta_{\text{AFM}}^{\text{QED}_1-\text{GN-XY}} = 2 - \frac{8}{3\pi^2 N_t} + O(1/N_t^2).$ (34-37)

Note that both $\nu^{-1}^{\text{QED}_2-\text{GN-XY}} < 2$ and $\Delta_{\text{AFM}}^{\text{QED}_2-\text{GN-XY}} < 2.$

In the vicinity of the quantum critical point at coupling $J = J_c$, the dimensionless correlation ratio $r_{\text{VBS}}$ should obey the finite-size scaling law
\[ r_{\text{VBS}}(J, L) = b((J/J_c - 1)L^{1/\nu}) \] (42)
with scaling function $b(x)$ and linear system size $L$. Fig. 6 shows the correlation ratios $r_{\text{VBS}}$ as function of the rescaled coordinate $x := (J/J_c - 1)L^{1/\nu}$ for $N_t = 4, 6, 8$, using the predictions for $1/\nu$ within the three scenarios proposed above, as well as using fitted values. Due to the limited system sizes that are available within tenable simulation times, the data collapses are not perfect in all cases. However, for all flavor numbers checked, the data collapses using the standard XY prediction are significantly worse than for the other two scenarios. This rules out Scenario 1, revealing that the transition evades the usual quantum-to-classical mapping. The differences between the qualities of the data collapses in Scenario 2 and 3 are less significant, in particular for $N_t = 8$. The reason for this behavior is that the critical exponents in these two scenarios approach the same limiting value for large $N_t$, $\lim_{N_t \to \infty} (1/\nu) = 1$. Nevertheless, for $N_t = 4$ and 6, we note that the spread of values of $r_{\text{VBS}}$ within the ranges of $x$ shown is significantly larger within Scenario 3 than for the other two scenarios. We interpret this as first evidence for QED$_2$-GN-XY universality (Scenario 3).

To clearly rule out the pure Gross-Neveu-XY universality class (Scenario 2), we have performed fits to the dimer $C_D(r)$ and spin $C_S(r)$ correlation functions, assuming power-law behaviors
\[ C_D(r) \propto \frac{1}{r^{2\Delta_{\text{VBS}}}}, \quad C_S(r) \propto \frac{1}{r^{2\Delta_{\text{AFM}}}}, \] (43)
with exponents $\Delta_{\text{VBS}}$ and $\Delta_{\text{AFM}}$. The fits are shown as straight lines in the log-log plots of Fig. 3, and we plot the extracted exponents in Fig. 7 in comparison with the theoretical predictions for Gross-Neveu-XY universality (Scenario 2) and QED$_2$-GN-XY universality (Scenario 3). While the errors are
large (e.g., larger than the theoretically predicted difference between VBS and AFM scaling dimensions), the result that both $\Delta_{\text{AFM}}$ and $\Delta_{\text{VBS}}$ are below 2 for all considered values of $N_f$ is significant. This rules out Scenario 2, suggesting that the transition observed in the simulation is indeed described by a new universality class in which the fluctuating U(1) gauge field plays a significant role.

V. CONCLUSIONS

In conclusion, we have provided evidence for the existence of an unconventional quantum critical point between a deconfined phase, characterized by gapless fermionic degrees of freedom coupled to a U(1) gauge field, and a conventionally ordered phase with spontaneously broken XY symmetry, in which the fermions acquire a band gap and the gauge field confines. The critical point can hence be understood as an example of a continuous confinement transition in $2 + 1$ space-time dimensions.

Using a finite-size scaling analysis of our QMC data, we have demonstrated that this transition evades the usual quantum-to-classical mapping. We have also shown that the numerical data is inconsistent with the pure Gross-Neveu-XY universality class. Instead, the data is consistent with our predictions for a novel universality class, dubbed QED$_3$-GN-XY, which includes besides the fluctuating fermionic degrees of freedom also the effects of the coupling to the U(1) gauge field.

We have shown that the corresponding QED$_3$-GN-XY field theory can be studied within a large-$N_f$ expansion in fixed dimension, or alternatively within an $\epsilon$ expansion in $D = 4 - \epsilon$ space-time dimensions. Using these expansion schemes, we have computed estimates for the characteristic exponents that describe the QED$_3$-GN-XY universality class, including the order-parameter anomalous dimension $\eta_\Delta$ and the correlation-length exponent $\nu$, as well as the scaling dimensions of the spin and dimer correlation functions. The accuracy of these estimates increases for larger flavor number, and should be reasonable for our simulations of the cases $N_f = 6$ and 8. (E.g., in the pure Gross-Neveu-XY universality class, the accuracy of the leading-order $1/N_f$ results is expected to be of...
the order of $\leq 3\%$ for $N_t = 8$ [52, 55].) For $N_t = 4$, the corrections may be more significant. This could be one of the reasons for the reduced quality of the data collapse in this case. Improving on this would require to extend our analytical calculations to the next order in the large-$N_t$ expansion (e.g., along the lines of the calculation in Ref. [43] for the QED$_2$-Gross-Neveu-Ising case) and/or to a higher order in the $4 - \epsilon$ expansion (e.g., along the lines of Refs. [41, 42]). On the numerical side, independent quantitative predictions for the critical exponents and universal scaling dimensions require simulations on significantly larger lattices. In this respect, the scaling dimensions of the dimer and spin correlators would be particularly interesting to measure. While these operators have the same scaling dimensions in the U1D phase as a consequence of an emergent SU(2$N_t$) symmetry [34, 38], this symmetry is not present at the U1D-VBS quantum critical point, leading to different scaling dimensions. At the transition, the $1/N_t$ expansion predicts that the dimer correlator decays faster than the spin correlator; however, for the values of $N_t$ considered here, the difference is small and beyond our numerical resolution. Testing this prediction in a refined simulation represents an excellent direction for future work.

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Appendix A: Measurement of correlation functions

We measure connected correlation functions in real space. The spin correlator is defined as

$$C_S(r) = \sum_{\alpha \beta} \langle S_\alpha^z(r) S_\beta^z(0) \rangle - \sum_{\alpha \beta} \langle S_\alpha^z(r) \rangle \langle S_\beta^z(0) \rangle,$$  \hspace{0.5cm} (A1)

and the dimer correlator is

$$C_D(r) = \langle D(r) D(0) \rangle - \langle D(r) \rangle \langle D(0) \rangle.$$ \hspace{0.5cm} (A2)

For the spin correlator, the background is zero, $\langle S_\alpha^z(r) \rangle = 0$, which is a consequence of the unbroken SU(2) spin symmetry. However, for the dimer correlator, the background may be finite, $\langle D(r) \rangle \neq 0$, and must be measured numerically. Near the quantum critical point and for large distances $r$, the difference between the two terms in the right-hand-side of Eq. A2 may become significantly smaller than their individual magnitudes, which complicates the numerical analysis. To obtain an accurate estimate of the decay of $C_D(r)$, we compare the naively measured observables with corrected values, for which a systematic background given by $C_D(r)$ at $r = r_{\text{max}} = L/2$, is deducted. The deducted backgrounds are

$$C_D(r_{\text{max}}) \approx \begin{cases} 5.1 \times 10^{-4} & \text{for } N_t = 4, \\ 1.5 \times 10^{-3} & \text{for } N_t = 6, \\ 1.9 \times 10^{-2} & \text{for } N_t = 8. \end{cases} \hspace{0.5cm} (A3)$$

The resulting corrected values for the connected dimer correlator are shown as gray data points in Fig. 3(b,d,f).

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