Unsteadiness in hypersonic leading-edge separation

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Abstract
Hypersonic leading-edge separation is studied toward understanding the varying shock-related unsteadiness with freestream Reynolds number \(1.66 \times 10^5 \leq \text{Re}_D \leq 5.85 \times 10^5\) in the newly constructed hypersonic Ludwieg tunnel (HLT) at a freestream design Mach number of \(M_\infty = 6.0\). An axisymmetric flat-face cylinder of base body diameter \(D = 35\) mm is fitted with axial protrusions of different fineness \((d/D = 0.1, 0.2, 0.26, 0.34\) at \(L/D = 1.4\)) and slenderness \((L/D = 0.7, 1, 1.4, 1.9\) at \(d/D = 0.2\)) ratio to induce a wide range of leading-edge separation intensities. Qualitative and quantitative assessments are made using schlieren imaging, planar laser Rayleigh scattering, and unsteady pressure measurements. A well-known to-and-fro shock motion called pulsation and a flapping shock-shear layer oscillation is observed as \(\text{Re}_D\) changes. A shorter protrusion length \((L/D = 0.7)\), associated with pulsation, produces a pressure loading four orders higher than the cases with longer protrusion lengths associated with flapping. There exists a critical separation length \((L/D \geq 1.4)\) beyond which the separated shear layer trips to turbulence and introduces fluctuations in the recirculation region as \(\text{Re}_D\) increases. The intensity of the separated turbulent shear layer is dampened by an order of magnitude, provided the reattachment angle is shallow by increasing the fineness ratio to \(d/D \geq 0.34\). There also exists a set of critical geometrical parameters \((L/D = 1, d/D = 0.2)\) for which the unsteady modes (pulsation and flapping) switch between themselves during successive runs, probably due to upstream fluctuations. From the modal analysis of the Rayleigh scattering images, the first four dominant modes that drive flapping are identified as translatory flapping, sinuous flapping, and large and small-scale shedding.

1 Introduction

High-speed vehicles flying at hypersonic speed are often exposed to severe pressure loading and aerodynamic heating due to a bow shock ahead of them. Therefore, continuous effort is being made to mitigate these challenges. Various ways of active and passive control techniques (Ahmed and Qin 2011) have been adopted, such as energy deposition, opposing jets, cavities, axial protrusions, and combinations of them. Using axial protrusions as spikes (Maull 1960; Kenworthy 1978) is considered the simplest. The passive control axial protrusion module has been used in various missiles and air intakes (Venukumar et al. 2006; Kulkarni and Reddy 2008) of high-speed flying vehicles (Sekar et al. 2020; Devaraj et al. 2020) owing to its low drag benefits. The protrusion tip can take various shapes, such as a sharp tip, hemisphere, aerodisk, circular, elliptical, etc. Typically, the incorporation of an axial protrusion weakens the strength of the oblique shock by pushing it away from the object through shock-shock and shock-shear layer interactions. Nevertheless, the leading-edge separation on the protrusion causes severe shock and shear layer interactions and leads to global unsteadiness. Figure 1 shows typical flow features present during the leading-edge separation. The formation of a leading-edge shock is usually evident at the protrusion’s tip, followed by the formation of a recirculation region due to an adverse pressure gradient. The recirculating zone is accompanied by a shock and a separated shear layer formation. Further, a reattachment shock and a series of expansion fans are formed near the model shoulder, thereby turning the flow parallel to the model.

Different forms of unsteadiness occur depending on the freestream conditions and the ratio between the protrusion’s length and diameter to the base body diameter \((L/D, d/D)\). It can have strong pressure fluctuations as in the case of pulsation or moderately weak pressure fluctuations as in...
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oscillation/flapping (Feszty et al. 2004b, a; Sugarno et al. 2022). Typically, in a pulsation mode of unsteadiness, the emanated oblique shock from the separation point on the protrusion undergoes continuous shape change from conical to hemispherical. During the shape transformation, the shock moves to-and-fro along the protrusion. In flapping, local lateral movement of the shock and shear layer is seen. Understanding the dependence of this unsteadiness on the problem parameters is of primary importance in reducing the fluid-structure interactions. Since the inception of axial protrusions on aerodynamic bodies, experimental and numerical studies have been conducted to describe the driving potential behind sustaining the pulsating and flapping modes of unsteadiness. Several theories have also been proposed to assess the flow characteristics under these conditions. Protrusions of different geometrical features produce different flow physics (Panaras and Drikakis 2009). Recently, Sahoo et al. (2020); Sahoo et al. (2021) studied some of the underlying physical aspects involving shock-related unsteadiness at supersonic speeds. However, information related to hypersonic speeds, particularly for a wide range of Reynolds numbers ($Re_D$), is limited (Ahmed and Qin 2011). Another challenge is the design and construction of facilities suited for studying such flow problems.

A hypersonic Ludwieg tunnel can be used to generate hypersonic flow with different $Re_D$ at a specified $M_\infty$, as it is easy to set up, maintain, and control. It comprises a long storage tube followed by a fast-acting valve, convergent-divergent (C-D) nozzle, and dump tank. Detailed operational procedure of the tunnel is described in the Section IA of supplementary material. Ludwieg tunnels can generate cold flows corresponding to a low enthalpy (0.3 to 0.7 MJ/kg) and a relatively low flight velocity (0.7 to 2 km/s). This facility’s typical test duration lies in a few tens to a few hundreds of milliseconds. Ludwieg tunnel has been extensively used for boundary layer transition (Juliano et al. 2008), skin friction measurement (Schülein 2004), shock boundary layer interaction (Neet and Austin 2020), flow over external bodies (Labuda et al. 2020), and so on. As an advancement, high enthalpy flows can also be generated in this facility with the implementation of heated driver gas (Segal 2011) and using a free-piston (Chung et al. 2018).

The current experimental campaign is aimed to fill the knowledge gap about hypersonic flow around axial protrusions. In particular, the flow field around different geometries at various $Re_D$ and the associated dominant modes responsible for global unsteadiness are assessed through experimentation in the newly-built Hypersonic Ludwieg Tunnel (HLT). Current investigation focuses on assessing the instabilities associated with sharp protrusions of different $[L/D]$ at various $Re_D$ conditions at $M_\infty = 6$. Attempts are made to understand the source of unsteadiness and quantify the fluctuations spatiotemporally through high-speed flow visualizations and unsteady pressure measurements. The key objectives for the present study include the following:

- To design, develop, and calibrate the hypersonic Ludwieg tunnel to study the leading-edge separation problem at $M_\infty = 6$ for a wide range of Reynolds numbers based on the freestream conditions and the base body diameter ($Re_D$).
- To experimentally generate and vary the leading-edge separation intensity in hypersonic flow using axial protrusions of different lengths ($L/D$) and diameters ($d/D$) from a cylindrical flat-face forebody having a base diameter of $D$. 

Fig. 1 Schematic showing the time-averaged hypersonic leading-edge separated flow over a generic flat-face forebody with an axial protrusion undergoing flapping type of unsteadiness. Prominent flow attributes include a leading-edge shock, a reattachment shock, a separated shear layer, a boundary layer, and an expansion fan. A streamline passing through the system of shocks and expansion fans is added. The governing parameters of the problem include the freestream Reynolds number ($Re_D$) and the geometrical parameters of the protrusion (length-$L$ and diameter-$D$).
To estimate the nature of unsteadiness pertaining to the shock and shear layer oscillations through space-time ($x - t$) and space-frequency ($x - f$) diagrams constructed from the high-speed schlieren images.

- Quantification of the pressure loading ($\kappa$), fluctuation intensity ($\zeta$) and unsteady spectra for protrusion having different geometrical parameters at different Re$_D$.
- To identify the dominant spatiotemporal modes responsible for the global unsteadiness through the Proper Orthogonal Decomposition (POD) of the high-speed planar laser Rayleigh scattering (PLRS) images.

The rest of the manuscript is arranged as follows. Details about the experimental methodology are given in Sect. 2. Uncertainty in the measurements is quantified in Sect. 3. Vital results from the qualitative and quantitative measures and their analysis are provided in Sect. 4. In Sect. 5, the significant findings of the present analysis are listed.

**2 Experimental methodology**

**2.1 Facility description**

An impulse facility based on the Ludwieg tube operation is proposed to generate hypersonic flow for a short duration (Russell and Tong 1973; Li et al. 2022). A typical layout of the experimental facility along with the optical diagnostics is shown in Fig. 2. The Ludwieg tube is 3 m long and 22 mm in diameter (inner). The tube is arranged in a ‘U’-tube configuration to save some floor space (Schrijer and Bannink 2010). The tube is wrapped by heater pads which can maintain a surface temperature up to 500 K. At one end of the tube, a commercial fast-acting valve (ISTA® KB-20-10) with a valve opening time of < 1 ms is connected. The tube can be pressurized to the desired fill pressure of $p_0$, ranging between 3 to 10 bar, by using the ports in the fast-acting valve itself. Computer-controlled pneumatic actuators and pressure regulators operate the fast-acting valve. During the fast-acting valve operation, there is a total pressure drop of about 20% in $p_0$. Hence, a small tube is connected after the valve to measure the actual flow stagnation properties (total pressure and temperature, $p_{01}$ and $T_{01}$). A convergent-divergent (C-D) nozzle is attached
next, whose interior is mirror-polished to have a smooth wall surface with a roughness radius of about 0.1 µm. The nozzle is designed to produce an exit Mach number of $M_D = 6.0$. However, after manufacturing, the nozzle’s divergent section is measured to have a semi-cone angle of $7^\circ$ with a throat and exit diameter of 10.4 mm and 76.2 mm, resulting in an actual exit Mach number of $M_e = 6.01$. Throughout the manuscript, for the ease of flow representation, the design Mach number of $M_\infty = 6.0$ is used. The nozzle expands freely into a closed chamber of volume 0.1 m$^3$ and 0.5 m long. The chamber also possesses four hermetically sealed optical windows (each 100 mm in diameter) closer to the nozzle exit section and wiring ports for instrumentation downstream. The chamber can be kept to a vacuum level of 0.05 mbar using a rotary vacuum pump (Busch® RH0021B-Zebra) while monitoring the vacuum pressure level ($p_v$). At the nozzle exit, the unit freestream Reynolds number ($Re = \frac{u_\infty L}{\nu_\infty}, L = 1$ m) is changed by varying the $p_0$ in the Ludwieg tube. A wide range of $Re$ between $4.75 \times 10^6$ to $1.8 \times 10^7$ is achieved while changing $p_0$ between 3 to 10 bar with dry air as the test gas. More details on the freestream conditions from the isentropic relations for $M_\infty = 6$ and air as the ideal gas ($\gamma = 1.4$) are given in Table 1. The tunnel is fully automated and operated autonomously for safety and efficiency. The NI (National Instruments)-based multi-functional I/O data card monitors the necessary pressure ratio between the vacuum tank and the Ludwieg tube. Once the desired value is reached, a trigger is sent to actuate the fast-acting valve to initiate the flow. The freestream Mach number calibration is important to size and place the testing model in the exhausting free-jet from the C-D nozzle. The calibration is done through both experiments and computations in a complimentary manner and is extensively discussed in the supplementary material (see Sect. S3).

### 2.2 Flow quality and duration

Quantifying the freestream fluctuations and effective run-time is essential before the actual experiments. In Fig. 3, stagnation pressure ($p_0$) and freestream pitot pressure ($p_{02}$) variation (see Fig. 2 for $p_{01}$ and $p_{02}$ measurement locations) between the successive runs over a cycle of the expansion process in the Ludwieg tube for a particular flow condition ($[p_0]/p_a = 4, [p_{01}]/p_a = 3.27, Re_D = 2.34 \times 10^5$) are shown. The signals are unfiltered with a ground noise fluctuation intensity corresponding to $\sigma_p = \pm 0.2\%$. A constant $p_0$ is experienced over an effective run-time of 12.5 ms. Theoretically, based on the Ludwieg tube length ($L$) and the speed of sound at the local temperature ($a_0$), the expected run-time is calculated to be $t_\text{e} = [2L/a_0] = [2 \times 3/340] \sim 17.7$ ms. However, the transient process due to the valve opening and flow settling in the stagnation chamber consumes a part of the flow time. The signal in Fig. 3a is also found to be repeatable between the runs with the standard deviation of $\sigma_p = \pm 0.6\%$ achieved during the effective run-time. Similar events are also observed in the $p_{02}$ measurements at the freestream (Fig. 3b), except for the fact that the fluctuation intensity is amplified to $\sigma_p = \pm 2\%$. One of the primary reasons for the increment is the laminar-to-turbulent transition of the boundary layer inside the C-D nozzle, thereby introducing freestream fluctuations through weak waves emanation into the core flow (Schneider 1992; Juliano et al. 2008).

| Parameters                              | Values   |
|-----------------------------------------|----------|
| Total pressure in the driver supplied ($p_0 \times 10^5$, Pa) | 3 4 5 6 7 8 9 10 |
| Total pressure realized in the flow ($p_0 \times 10^7$, Pa) | 2.31 3.25 4.14 4.80 5.70 6.48 7.37 8.13 |
| Freestream pressure ($p_{01}$, Pa)     | 152.64 205.84 262.21 304.01 361.02 410.42 466.79 514.92 |
| Freestream density ($\rho_{\infty}$, kg/m$^3$) | 0.015 0.02 0.025 0.03 0.034 0.039 0.044 0.049 |
| Freestream kinematic viscosity ($v_{\infty}$, $10^{-5}$, m$^2$/s) | 14.7 10.9 8.6 7.4 6.2 5.5 4.8 4.4 |
| Freestream Reynolds number ($Re_D \times 10^5, D = 35$ mm) | 1.66 2.34 2.98 3.45 4.09 4.66 5.29 5.85 |
| Equivalent altitude ($h, \text{km}$) | 25.2 23.4 21.6 20.6 19.6 18.8 18.0 17.2 |
| Total enthalpy in the driver ($h_{01}$, MJ/kg) | 0.3 |
| Total temperature in the driver ($T_{01}$, K) | 300 |
| Freestream temperature ($T_{\infty}$, K) | 36.58 |
| Freestream velocity ($u_{\infty}$, m/s) | 727.46 |
| Freestream design Mach number ($M_\infty$) | 6.0 |

The freestream values are calculated based on the isentropic relations assuming ideal gas flow (air, $\gamma = 1.4$) for the considered design $M_{0\infty}$. The only parameter that is varied between the experiments is $p_0$ at a constant $T_{0\infty}$; thereby, the freestream $Re_D$ can be changed. $Re_D$ is defined based on the model’s base body diameter $D = 35$ mm. To non-dimensionalize pressure and time, the atmospheric pressure of $p_a = 1 \times 10^5$ Pa and a reference time of $t_0 = 1$ ms are used wherever needed.
2.3 Model description and instrumentation

Leading-edge separation is induced by using a flat-face circular cylinder with an axial protrusion, as shown in Fig. 4. For the experimentation, a flat-face circular cylinder with a base body diameter of $D = 35$ mm is made of perspex. Drilling is done at two different diameters ($2D/3$ and $D/3$) on the forebody’s flat face to mount the unsteady pressure transducers. The protrusion is introduced in the form of a circular cylinder with a sharp tip at one end, whose length ($L$) and diameter ($d$) are varied as $[L/D] = [0.7, 1, 1.4, 1.9]$ and $[d/D] = [0.1, 0.2, 0.26, 0.34]$ in four steps, respectively. The protrusion is made of stainless steel material with the sharp tip having a radius of 0.1 mm and a $10^\circ$ half-cone angle. The machined surface generally has a roughness of 3 to 5 $\mu$m. The machining tolerance is kept at 25 to 30 $\mu$m. The model’s tip with the protrusion is always placed 5 mm away from the nozzle exit plane.

2.4 Measurement methodology

Time-varying pressure data in the Ludwieg tube and the vacuum tank are acquired using transducers having a 1 ms response time (Omega® PX309-150GV, and Omega® Vacuum PX409-015VV). Mean, and fluctuating pressures in the stagnation chamber are measured with confidence at two diametrically opposite locations using Meggitt® 8530CM65-100 and PCB® 113B28 transducers at 50 kHz. The same PCB® 113B28 transducer is also used on the flat-face forebody model to calculate the pressure loading ($\bar{p}$) and fluctuation intensity ($\sqrt{\bar{p}^2}$) at 50 kHz. All the sensors require a supply voltage of 5-10 V to excite the sensing head, and the response is registered in millivolt. A suitable signal conditioning box is used to amplify the signals recorded in the NI data acquisition cards. A ‘Type K’ thermocouple capable of measuring 70-700 K with 1 ms response time is used to monitor the flow temperature in the Ludwieg tube, stagnation chamber, and vacuum tank.

A ‘Z-type’ schlieren imaging setup (Settles 2001) is used to resolve the density gradients in the transverse direction ($\partial \rho / \partial y$) by placing the knife-edge at the focal point in the horizontal direction. Figure 2 shows a typical schlieren imaging setup. The plane and parabolic concave (focal length of 1.5 m) mirrors used in the schlieren system are 150 mm in diameter. For imaging purposes, a high-speed monochromatic Phantom® v211 camera is used with an external lens arrangement for projection and focusing. Two sets of light sources are used in the imaging process. The first light source from Cavitar® is used to acquire an image at a high resolution of 1280 $\times$ 800 pixels and a low frame rate of 1 kHz. The light exposure time is 150 ns, and the images are obtained at 0.1 mm/pixel to capture flow structure details instantly. The next is a continuous light source from Thor Labs®, which is utilized for low-resolution imaging at 256 $\times$ 128 pixels and at a high repetition rate of 50 kHz. The frame exposure is 2 $\mu$s, and the imaging resolution is 0.4 mm/pixel to quantify the dynamics of the shock and the separated shear layer. A typical schlieren image done using type-I imaging parameters is given in Fig. 5-I. The moderately quiet and noisy recirculation regions are distinctly observed in a typical body with a protrusion at two extreme $\text{Re}_D$ to highlight the laminar and turbulent separated shear.
layer’s role. Other common features like the separation and reattachment shocks are also shown.

Measurements in a particular streamwise plane (at $z/D = 0$) is possible using planar laser Rayleigh scattering or PLRS (Do et al. 2010; Zhang and Lee 2016). Carbon dioxide gas is mixed with the compressed air in the Ludwieg tube by 10% of the desired fill pressure ($p_0$). After the fast-acting valve actuation, the gas expands in the C-D nozzle causing the static temperature to drop significantly (to $\sim 36.6$ K). Gaseous carbon dioxide undergoes deposition-type phase change (gas to solid) to form ice particles of size $< 0.5 \, \mu m$. The particles’ size is within the incident laser light source’s wavelength, thereby scattering the light in the Rayleigh regime (uniform scattering everywhere). The scattering visualizes the vapour cloud’s distribution in the freestream and helps in imaging the shock and the separated shear layer. A 10 W continuous 532 nm laser having a 5 mm beam diameter from CNI® laser (MGL-V-532-10W) is used as the illuminating laser light source. LaVision’s® sheet and collimating optics along with their laser-guiding arm are used to steer the light and convert it into a sheet of uniform width ($< 50$ mm). The scattered light intensity is captured using a Nikon® 40 mm Micro lens (f/1.8) using the same Phantom® high-speed camera. The high-resolution imaging is digitized at 1 kHz, whereas the high-repetition rate imaging is sampled at 50 kHz. Throughout the manuscript, for both the visualizations, the former imaging condition is called ‘type-I’ imaging, and the latter is ‘type-II’ imaging. In general, a series of post-processing routines must be carried out on the acquired Rayleigh scattering images to interpret the results meaningfully. Imaging routines in similar planar laser scattering experiments as described in Karthick et al. (2017) and Rao et al. (2020) are proven to be useful in compressible flow fields, and they are adopted in the present investigations. A typical PLRS imaging arrangement is explained in Fig. S2.1 of the supplementary. A representative PLRS image obtained using type-II imaging parameters is given in Fig. 5-II for the two extreme $\text{Re}_D$ observed in the experimental campaign. The presence of a laminar separated shear layer with K-H (Kelvin-Helmholtz) instabilities in the transition zone for the lowest $\text{Re}_D$ case and a fully turbulent separated shear layer at the highest $\text{Re}_D$ case can also be seen in Fig. 5-II.
Uncertainty in experimental measurements is inevitable. Experiments in the Hypersonic Ludwieg Tunnel are repeated five times in each set of conditions. The considered geometry with an axial protrusion is described by \([L/D] = 1.4\), and \([d/D] = 0.2\). Each experiment is done at the lowest (a) and the highest (b) \(Re_D\) achieved at a constant freestream Mach number of \(M_{\infty} = 6.0\). Key flow features in the schlieren and the Rayleigh scattering imaging: 1. leading-edge separation shock; 2. separated shear layer; 3. recirculation bubble; 4. reattachment shock; 5. K-H instabilities; 6. moderately quiet recirculation zone; 7. laminar part of the shear layer; 8. noisy recirculation zone; 9. turbulent part of the shear layer; 10. a series of expansion fans.

3 Uncertainty

Uncertainty in experimental measurements is inevitable. Experiments in the Hypersonic Ludwieg Tunnel are repeated five times in each set of conditions. The deviation in the stagnation pressure \((p_{01})\) between successive runs is bounded within \(\pm 0.2\%\). Asides, the total uncertainty arising from shot repeatability, acquisition methods, data conversion, storage, and other sensitive external factors are considered as suggested by the recommendations in Coleman and Steele (2009). Point measurements, including the pressure and temperature acquisitions in the Ludwieg tube and vacuum tank, contain 3% uncertainty. The spatial range across the unsteady pressure logging depends on the sensor diameter, which in this case is about 6 mm. The frequency resolution of the resulting signal is identified to be 80 Hz. Schlieren imaging poses line-of-sight light integration despite its ability to resolve shock motion. Spatial resolution is within 0.1 mm and 0.4 mm for type-I and type-II imaging, respectively. Rayleigh scattering, the planar measurement technique, is only limited by the probing laser sheet thickness. In the present experiment, the sheet thickness is calculated to be \(\sim 1\) mm. Spectral resolution for type-II imaging resides within 80 Hz for schlieren imaging and 88 Hz for Rayleigh scattering. Spectra from type-I imaging are seldom used in the manuscript owing to the low sample rate in both visualization techniques.
Fig. 6 Normalized operator-based time-averaged image (R) showing the mean characteristics of the fluctuating flow field, where owing to the flow symmetry and brevity, the top and bottom halves are only represented for the cases of Re_D = 1.66 × 10^5 and Re_D = 5.85 × 10^5 while varying (I) [L/D] and (II) [D/d]. See the supplementary for viewing the corresponding video files.

4 Results and discussions

In this section, qualitative and quantitative measurements are provided, and the findings are discussed.

4.1 Qualitative imaging

Two visualization methods are adopted in qualitative imaging: Schlieren imaging and Planar Laser Rayleigh Scattering.

4.1.1 Schlieren imaging

The schlieren images are obtained using type-II imaging (see Sect. 2.4) for each variation in [L/D] and [D/d] as shown in Fig. 6. In Fig. 6 I and II, operator-based time-averaged images are given to qualitatively draw some conclusions on the intensity of the shock-related unsteadiness. The top and bottom halves of the image represent the flow field at the lowest and the highest Re_D, respectively. Schlieren images resolve the density gradients along the transverse direction by cutting the focal point using a horizontal knife edge. The obtained image is processed through a mathematical routine to bring out the desired features of the flow field with clarity as described by the below equation, where I corresponds to the normalized light intensity.

\[ R = \| \bar{I} - I_{\text{ref}} \| \].

The aftermath of the pulsation form of unsteadiness is representatively visualized in the operator-based time-averaged image through the system of smeared shocks hanging around the protrusion’s tip and forebody shoulder from Fig. 6 I-a for [L/D, d/D] = [0.7, 0.2]. The dynamics of pulsation event is also better understood through the supplementary video carrying a series of instantaneous schlieren images. The shock travel to and fro between the protrusion’s tip and the forebody at a fixed frequency. The pulsation mode of oscillation is persistent throughout the Re_D cases under discussion. The image also agrees with a similar pulsating flow field observed at supersonic Mach numbers in the experiments of Sahoo et al. (2021). On the other hand, all the remaining cases exhibit a ‘flapping’ form of shock oscillation. During flapping, the separated shear layer and the shock move inwards and outwards, with the separation point being the hinge. Flapping is severe with a higher amplitude of shock oscillations, especially at the highest Re_D (5.85 × 10^5). The least Re_D case (1.66 × 10^5) exhibits only a feeble flapping which is even difficult to notice. Visually, for all the [L/D] and [D/d] variations, the images look sharper and have good contrast for the highest Re_D. The presence of a larger density in the freestream at the highest Re_D (see Table 1) is probably the reason for such a difference in the image contrast between the cases. From the qualitative analysis on time-averaged images for [L/D] variations (Fig. 6-I a-d), flapping is seen for [L/D] ≥ 1. However, the flapping intensity seems to be reduced as [L/D] increases. Likewise, for [d/D] variations (Fig. 6-II e-h), flapping intensity decreases with increments in [d/D].

The lowest Re_D case (1.66 × 10^5) possesses a laminar separated shear layer for most of it, with the incoming flow also being laminar. Hence, the regions of shade seen at the low Re_D case are minimal. It also means that the resulting fluctuations or shock-related unsteadiness are not significantly pronounced. On the other hand, for the highest Re_D
case \( (5.85 \times 10^5) \), the shaded zones are more prominently seen, especially around the separated shear layer and shock. The reason is attributed to the unsteadiness in the separated shock and shear layer motion arising from the laminar to turbulent transition in the separated shear layer. From the high-resolution type-I imaging shown in Fig. 5 I-b, the ‘wavy-motion’ populates the recirculation zone, which is confined by the forebody, separated shear layer, and protrusion. It is suspected that these waves travelling between the forebody and the separation point might excite the separated shear layer to flap vigorously. However, aside from these images, there is no further experimental evidence to support the postulate.

### 4.1.2 Planar laser Rayleigh scattering: PLRS

Distinguishing pulsation and flapping is convenient in schlieren imaging itself. However, schlieren imaging is a line-of-sight integrated technique that does not allow capturing some flow features, such as small-scale structures or the transition point on the separated shear layer, whose behaviour dictates the extent of shock-related unsteadiness. PLRS imaging helps in understanding the unsteadiness in detail. Normalized instantaneous Rayleigh scattering images (see also the supplementary video) obtained using type-II imaging parameters are given in the top half of Fig. 7 for variations in \([L/D]\) and \([d/D]\) at the highest \(Re_D\) of \(5.85 \times 10^5\). The bottom half contains the operator-based time-averaged image \((R)\) shown in an inverted colour, displaying the mean characteristics of the fluctuating flow field. The two-dashed vertical lines in each image denote the locations at which the intensity profiles are investigated to deduce the separated shock and shear layer properties as described in Sect. 4.2.3. See the supplementary for viewing the corresponding video files.

![Fig. 7](image-url) Normalized instantaneous Rayleigh scattering images taken at an arbitrary time-stamp during the effective run-time for different cases: (I, a-d) show \([L/D] = [0.7, 1.4, 1.9]\) variations at a constant \([d/D] = 0.2\), and (II, a-d) show \([d/D] = [0.1, 0.2, 0.26, 0.34]\) variations at a constant \([L/D] = 1.4\). Images are only provided for the highest \(Re_D\) encountered in the experiments: \(Re_D = 5.85 \times 10^5\). In each image, the bottom half represents the normalized operator-based time-averaged image \((R)\) shown in an inverted colour, displaying the mean characteristics of the fluctuating flow field. The two-dashed vertical lines in each image denote the locations at which the intensity profiles are investigated to deduce the separated shock and shear layer properties as described in Sect. 4.2.3. See the supplementary for viewing the corresponding video files.
The extent and nature of unsteady shock oscillations and
driving frequencies are examined quantitatively using $x-t$ and $x-f$ plots. The variations in the aforementioned quan-
tities are studied for different $[L/D]$, $[d/D]$, and Re$_D$ cases. Firstly, a series of $x-t$ plots are constructed by stacking
the light intensity variation obtained from the instantaneous schlieren images over the effective run-time as shown in
Fig. 8. Intensity variation along a particular line profile of $[y/D] = 0.25$ in a given time-instant helps in identifying
the separated shear layer (bright patch, $x/D \sim 0.7$) and shock (dark patch, $x/D \sim 0.8$) as shown in Fig. 8a. Stacking
the intensity profiles along $[y/D] = 0.25$ obtained at various times during the effective run-time are shown in Fig. 8b.
Such a procedure creates an image containing the motion of the separation shock and shear layer with good contrast.

Performing a fast Fourier transform on the $x-t$ plots results in $x-f$ plots that reveal spectral contents of those time-
varying events as shown in Fig. 8c. Examples of both $x-t$ and $x-f$ plots are given for two particular cases in Fig. 8 b-e
for brevity. For some of the cases, the $x-t$ plots reveal the wave pattern being broadband with a small amplitude like in
the case of $[L/D, d/D] = [1.4, 0.2]$ at Re$_D = 1.66 \times 10^5$ (Fig. 8b). The corresponding $x-f$ plot also confirms the
presence of broadened spectra (Fig. 8c). The wave patterns sometimes look like saw-tooth as in the $x-t$ plot (Fig. 8d)
of $[L/D, d/D, Re_D] = [1.4, 0.26, 3.45 \times 10^5]$ whose $x-f$ plot (Fig. 8e) reveals the presence of a discrete dominant non-
dimensional frequency.

A series of $x-t$ plots is given in Fig. 9 for a wide range of variations in $[L/D]$, $[d/D]$, and Re$_D$. From the initial
assessment of the $x-t$ plots (Fig. 9-I a-h), the case of $[L/D, d/D] = [0.7, 0.2]$ (shortest) produces pulsating mode
of unsteadiness as observed from the qualitative schlieren (Fig. 6 I-a) and the Rayleigh scattering (Fig. 7-Ia). The pulsa-
ting shock produces a high-frequency sinuous wave pattern as a trace. The extent of shock oscillation or the ampli-
tude of the sine wave is stretched over the entire protrusion length of $0 \leq [x/D] \leq 0.7$. From the corresponding $x-f$
analysis, the dominant pulsation frequency is computed to be $[fD/u_\infty] \sim 0.22$ and it remains unchanged with Re$_D$. The
pulsating frequency also agrees with the empirical relation proposed by Kenworthy (1978) as given in Eqn. 2. In the
empirical relation, the independence of the pulsating fre-
quency on the Re$_D$ can be also seen. The non-dimensional experimental frequency $([fD/u_\infty] = 0.216)$ is having a devia-
tion of $\pm 6.4\%$ from the empirical relation $([fD/u_\infty] = 0.203)$ as given in Eqn. 2.

$$
\left[ \frac{fD}{u_\infty} \right] = 0.25 - \left( \frac{L}{D} \right) 0.067.
$$

While changing the $[L/D]$ to 1, an interesting flow feature is observed for repeated experiments on the $x-t$ and $x-f$
plots as shown in Fig. 10. The pattern of shock unsteadiness changes from pulsation to flapping and vice versa between
runs, irrespective of Re$_D$. Although the upstream and the downstream conditions are kept almost constant (based on
the instruments’ limitations), the variation of flow patterns is inevitable. To understand the cause for such a change
between the two forms of shock unsteadiness, $x-t$ and $x-f$ plots are constructed at a specific Re$_D$ of $1.66 \times 10^5$. The
$x-t$ plots are drawn from the beginning of the test till the end of the effective run-time. However, the $x-f$
plots are drawn based on the signal obtained only during the effective run-time, where a self-sustained shock oscillation
is observed. While reviewing the $x-t$ plots (Fig. 10), the wave patterns seen during the transient portion of the run-
time are observed to be different. A self-sustained pulsating shock oscillation between $0 \leq [x/D] \leq 1$ is seen if the
fluctuations in the transient portion are severe (Fig. 10a). Whereas, a self-sustained flapping shock oscillation between
$0.1 \leq [x/D] \leq 0.6$ is seen for a mild fluctuation in the transient zone (Fig. 10b). We suspect that the observation of
different fluctuation types during the transient period is probably due to the slightest non-repeatable movement of the
fast-acting valve during its opening time of $\sim 1$ ms. The $x-f$ plot for pulsation and flapping, however, reveal the same
non-dimensional frequency at $([fD/u_\infty]) \sim 0.2$ and its harmonics. Thus, it can be inferred that the initial disturbance
levels seen in the transient zone do not affect the non-
dimensional frequency pertaining to the shock oscillation.
Contrary to the aforementioned case, it has to be noted that the other cases of \([L/D, d/D]\) indeed offer a repeatable unsteady signature for a wide range of \(Re_D\). They are not affected by the smallest imperfections in the fast-acting valve operation. It is just the aforementioned case that is sensitive to the upstream disturbance levels. We consider only the self-sustained flapping motion for the rest of the discussion pertaining to the \([L/D, d/D] = [1, 0.2]\) case.

Going back to Fig. 9 II-IV, for cases of \([L/D] \geq 1\), the shock signature looks more like a saw-tooth wave indicating flapping unsteadiness with a considerably lower frequency than the pulsating shock’s case, especially for moderate Reynolds numbers. In general, for \([L/D, d/D] = [1, 0.2]\) (Fig. 9 II), the dominant non-dimensionalized frequency varies a bit between 0.18 ≤ \([fD/u_\infty]\) ≤ 0.19 in an arbitrary manner with \(Re_D\). While increasing the \([L/D]\), as in the case of \([L/D, d/D] = [1.4, 0.2]\) (Fig. 9 III), no noticeable fluctuations are seen in the shock motion for 1.66 × 10^5 ≤ \(Re_D\) ≤ 2.34 × 10^5. However, a transition in shock motion (from a broad low-amplitude signal to a discrete saw-tooth signal of considerable amplitude) is observed from \(Re_D \geq 2.98 \times 10^5\) with the dominant non-dimensionalized frequency varying between 0.13 ≤ \([fD/u_\infty]\) ≤ 0.15. The formation of the laminar...
Fig. 9 A series of $x$–$t$ plots showing the shock and separated shear layer motion over a particular region of an effective run-time for a wide range of variations in $[L/D]$ (I-IV), $[d/D]$ (V-VIII), and $Re_D$ (a–h). The $x$–$t$ plots are constructed from the instantaneous schlieren images obtained using type-II imaging. In each $x$–$t$ plot, the dominant non-dimensionalized frequency due to the discrete shock/shear layer oscillation obtained from the respective $x$–$f$ diagram (fast Fourier transform analysis of the $x$–$t$ plot) is written at the top-right.
Fig. 9 (continued)
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separated shear layer at lower \( \text{Re}_{D} \) with low-amplitude transient growth (see Fig. 5 I-a and II-a) is suspected to be responsible for the less intense broadband signal. The shear layer characteristics change at higher \( \text{Re}_{D} \) (see Fig. 5 I-b and II-b), where the separated shear layer undergoes laminar-to-turbulent transition leading to an intense discrete flapping signal. For the case of \([L/D], [d/D]\) = [1.9, 0.2] (Fig. 9 IV), the separated shear layer has more space between the separation and reattachment point owing to the long protrusion as seen in Fig. 6I-d. At lower \( \text{Re}_{D} \), the separated shear layer begins to transition from laminar-to-turbulent with sufficiently high amplitude (see the top-half of Fig. 6 I-d). The transient structures grow as they convect and impinge on the forebody’s shoulder. We suspect that the strong disturbances are introduced into the recirculation region and thereby excite the shear layer at the upstream point to introduce flapping, shown as a saw-tooth wave in the \( x - t \) plot. Moreover, the separated shear layer undergoes a transition from a laminar to a turbulent state for \( \text{Re}_{D} \geq 4.66 \times 10^5 \) (see the bottom half of Fig. 6 I-d). Such an event leads to observing a broadened signal with a significant amplitude in the \( x - t \) plot. During the discrete flapping motion, the dominant non-dimensionalized frequency varies between \( 0.13 \leq [fD/u_\infty] \leq 0.14 \).

While varying \([d/D]\) (Fig. 9 V-VIII), as in the case of \([L/D], [d/D]\) = [1.4, 0.1], a distinct saw-tooth wave pattern of shock oscillation is observed irrespective of \( \text{Re}_{D} \). However, as the \([d/D]\) increases, particularly for \([L/D], [d/D]\) = [1.4, 0.2], initial set of \( \text{Re}_{D} \) cases \((1.66 \times 10^5 \leq \text{Re}_{D} \leq 2.34 \times 10^5)\) exhibit broadened low-amplitude oscillations and later transforms in to a discrete saw-tooth wave at higher \( \text{Re}_{D} \), as explained in the previous paragraph. Pushing the \([d/D]\) further only delays the occurrence of discrete flapping to higher \( \text{Re}_{D} \). For example, in the case of \([L/D], [d/D]\) = [1.4, 0.26], discrete waveform is seen for \( \text{Re}_{D} \geq 2.98 \times 10^5 \) with \( 0.12 \leq [fD/u_\infty] \leq 0.14 \). Moreover, for \([L/D], [d/D]\) = [1.4, 0.34], the transition occurs at \( \text{Re}_{D} \geq 3.45 \times 10^5 \) with \( 0.12 \leq [fD/u_\infty] \leq 0.13 \).
In summary, flapping is not observed if the shear layer is laminar from the point of separation to the reattachment; instead, a broadband signal with a low amplitude is seen. Similarly, in the case of the entire separated shear layer being turbulent, we do not see flapping but only a broadband oscillation signal with higher amplitude. Flapping is only observed when the separated laminar shear layer begins to transition downstream of the separation point and upstream of the reattachment point.

### 4.2.2 Lateral extent of shock/shear layer oscillation

Aside from the frequency of the wave pattern left behind by the shock motion, the lateral extent of the shock motion along the $x$-direction is also a good measure of the intensity of flapping. It has to be emphasized that the flapping motion is traced both from the shock and the separated shear layer as they move in relation and stay close to each other. The $x$ – $t$ diagrams shown in Fig. 9 are subjected to a series of image processing routines to extract the extent of shock motion. Firstly, the light intensity variation along the $x$-direction in the $x$ – $t$ plot is averaged over time. Then, the values are normalized between 0 to 1 and are plotted in solid red line as shown in Fig. 8 b and d. Local extrema are seen in the time-averaged intensity profile in the region of interest where the shock motion is relevant (between $0.5 \leq \Delta x_0/D \leq 1$). The distance between the local two peaks (the minimum and the maximum) is used as an apparent measure ($\Delta x_0/D$) to quantify the lateral extent of shock motion. It has to be noted that the actual lateral width is almost twice that of the apparent width from the image processing. All the $x$ – $t$ plots shown in Fig. 9 are subjected to these processing routines and the corresponding $\Delta x_0/D$ is calculated. Later, these values are plotted to see the variations with respect to $L/D$ and $d/D$ over a range of $Re_D$ as shown in Fig. 11 a, b.

While monitoring the changes in $\Delta x_0/D$ by varying $L/D$ and $Re_D$ (see Fig. 11a), a constant value of $\Delta x_0/D \sim 0.65$ is observed over the entire range of $Re_D$, for the pulsating case ($L/D, d/D = [0.7, 0.2]$). While increasing the $L/D$ to 1, the value of $\Delta x_0/D$ is considerably lower than the previous case and it remains almost a constant ($\Delta x_0/D \sim 0.23$) over different $Re_D$s. Further increasing the length to $L/D = 1.4$, values of $\Delta x_0/D$ is even lesser for laminar range of $Re_D$ with $\Delta x_0/D \sim 0.1$. However, after the critical $Re_D$ of $2.98 \times 10^5$, the values of $\Delta x_0/D$ increases drastically to about $\Delta x_0/D \sim 0.5$ for the rest of the $Re_D$s under consideration. While analyzing the respective $x$ – $t$ plots (Fig. 9-III a-h) in the critical $Re_D$ range, one can see the shock/shear layer motion pattern transforming from a broadened low-amplitude wave to a high-amplitude saw-tooth wave. For the longest $L/D$ of 1.9, values of $\Delta x_0/D$ is observed to be very high ($\Delta x_0/D \sim 0.35$) for the first two laminar $Re_D$. While changing from lower to higher $Re_D$, $\Delta x_0/D$ values are observed to be decreasing and asymptotically almost becomes a constant at $\Delta x_0/D \sim 0.2$. The reason behind the asymptotic trend can once again be explained through the time-averaged schlieren images in Fig. 6 and the $x$ – $t$ plots in Fig. 9. For the longer $L/D$ case (see Fig. 6I-d), the separated shear layer travels to a longer distance from the separation point to the forebody’s shoulder. While doing so, the separated shear layer transitions from a laminar to a turbulent state, especially at higher $Re_D$. As the shear layer is only transitional at lower $Re_D$, the transient structures are comparatively larger, leading to the observation of saw-tooth wave with high amplitude in $\Delta x_0/D$ in the $x$ – $t$ plots (see Fig. 9 IV a). On the other hand, when the shear layer is fully turbulent, only small-scale structures are seen in the $x$ – $t$ plots (see Fig. 9 IV h), which results in a broadband oscillation spanning over a small axial extent. Thus, the $\Delta x_0/D$ values decrease as $Re_D$ increases.

While changing the $[d/D]$, variations of $\Delta x_0/D$ with $Re_D$ are plotted in Fig. 11b. For the thinnest protrusion ($d/D = 0.1$), globally the values of $\Delta x_0/D$ decrease as $0.4 \leq \Delta x_0/D \leq 0.57$ with $Re_D$. The thinnest protrusion case exhibits $\Delta x_0/D$ equivalent to that of the shortest protrusion case at least at the lower $Re_D$ range. Watching the instantaneous schlieren image video given in the supplementary, the case under consideration exhibits a combination of both pulsating and flapping oscillations. The trace of shock oscillation intensifies at low $Re_D$ and progressively decays with increments in $Re_D$. We suspect that the formation of the recirculation bubble (see Fig. 5) with a comparatively larger volume with respect to the other cases of $[d/D]$, is responsible for the trend in $\Delta x_0/D$. The next $[d/D]$ case ($[L/D, d/D] = [1.4, 0.2]$) was explained in the previous discussion. For the next $[d/D]$ variation at 0.26 and also for 0.34, the values of $\Delta x_0/D$ are observed to be at $\sim 0.1$ until the critical $Re_D$ of $2.98 \times 10^5$. Above it, the values of $\Delta x_0/D$ increase moderately to $\sim 0.2$, unlike the previous case ($[L/D, d/D] = [1.4, 0.2]$). One of the reasons for the mild jump in $\Delta x_0/D$ is due to the thickness of the protrusion, causing a reduction in the recirculation bubble’s volume (see Fig. 5). The thicker protrusion makes the separated shear layer travel to a comparatively shorter length between the separation point and the forebody’s shoulder. Moreover, the separated shear layer’s deflection angle decreases considerably and alters the properties of the structures shed in the shear layer as $Re_D$ increases.

### 4.2.3 Apparent separated shear layer properties

Rayleigh scattering images shown in Fig. 7 are subjected to certain image processing routines to extract
the apparent separated shock and shear layer properties. During the apparent shear layer properties calculation, the estimation of shear layer and separation shock angles along with the shear layer growth rate help in understanding the extent of unsteadiness arising due to the dynamics between the shear layer and the separation shock for different geometrical parameters. The image processing algorithms utilize the operator-based time-averaged image from the Rayleigh scattering (bottom half of Fig. 7). The processing routines are demonstrated in Fig. 12. One example of the operator-based time-averaged image is shown in Fig. 12a. Grabbing the time-varying light intensity profiles along the \( y \)-direction at two different \( x \)-locations is required for getting shock and shear layer properties. Intensity profiles at two arbitrary locations are depicted in Fig. 12a as solid red-color lines. For the \([L/D]\) cases, 20% and 30% of the total \([L/D]\) are considered, whereas for the \([d/D]\) variations, 15% and 35% of the total \([L/D]\) are chosen. The corresponding intensity profile grabbing locations for each case is marked as a dashed red-colour line in the bottom half of Fig. 7. The reason behind the selection of different criteria for \([L/D]\) and \([d/D]\) cases is to ensure that both lines are within the turbulent part of the shear layer. Quantities like shear layer deflection angle (\( \theta \)), shock angle (\( \beta \)), and shear layer growth rate (\( \delta/x \)) are computed. The aforementioned shear layer properties are derived from a planar visualization image. Hence, the adopted processing routines yield only the apparent values. It has to be noted the procedure remains consistent across the whole \([L/D]\) and \([d/D]\) variations.

The \( y \)-position of the local peaks seem to be growing linearly along the \( x \)-direction (Fig. 12a). The above observation agrees with the linear slope observed in Fig. 7. The manner through which the apparent values are extracted is depicted particularly in Fig. 12b. The first peak is identified in light intensity profile by searching it from \([y/D]\) > 0. Linearly connecting the peak \( x-y \) locations

### Table 2

| Cases | \([d/D] = 0.2\) | \([d/D] = 0.1\) | \([d/D] = 0.2\) | \([d/D] = 0.26\) | \([d/D] = 0.34\) |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \([L/D] = 0.7\) | 37.9 | 24.6 | 15.9 | 23.1 | 25.1 | 25.2 |
| \([L/D] = 1\) | 41.6 | 30.3 | 24.9 | 28.0 | 29.9 | 30.8 | 30.5 |
| \([L/D] = 1.4\) | 0.35 | 0.18 | 0.05 | 0.04 | 0.08 | 0.05 | 0.06 |
| \([L/D] = 1.9\) | \(\ast\) | \(\ast\) | \(\ast\) | \(\ast\) | \(\ast\) | \(\ast\) | \(\ast\) |
from both profiles results in calculating the line’s slope, which corresponds to the shear layer deflection angle ($\theta$). Similarly, the second peak identification helps mark the apparent separation shock angle ($\beta$). A half-width distance along the $y$-direction from the first peak identification in each profile is computed and represented as $\delta_1$ and $\delta_2$, respectively. The growth rate of the shear layer is approximated as,

$$\left[ \frac{\delta}{x} \right] = \frac{|\delta_1 - \delta_2|}{|x_1 - x_2|}. \quad (3)$$

In Eqn. 3, $x_1$ and $x_2$ corresponds to the location where $\delta_1$ and $\delta_2$ are measured. All the computed values are tabulated in Table 2 for the highest $Re_D$. In the tabulation, properties for $[L/D, d/D] = [0.7, 0.2]$ are omitted as the shock pulsates, and apparent shear layer properties cannot be extracted.

From Table 2, we observe that the shear layer properties decrease as $[L/D]$ increases. As the separation point moves upstream for longer $[L/D]$ cases and the necessity for the separated shear layer to leave the forebody’s shoulder tangentially, $\theta$ should be reduced. Moreover, the point of laminar-to-turbulent transition in the separated shear layer is shifted downstream. In turn, such an event drastically reduces the shear layer growth rate ($\delta/x$) by $\sim 87\%$ between $[L/D] = 1$ and 1.9. In contrast, there is no distinguishable trend in the apparent shear layer properties while varying $[d/D]$. Most values are very close to each other and fall within the imaging uncertainty ($\pm 5\%$ in $\theta$ and $\beta$). Hence, for most of the discussion, the apparent shear layer properties can be considered almost a constant.

### 4.2.4 Pressure loading and fluctuation intensity

The unsteady pressure transducer response located at a radial location of $2D/3$ (Fig. 4) is utilized to measure the pressure loading ($\zeta$) and fluctuation intensity ($\kappa$). The definitions of these are given in the equation below.

$$\zeta = \frac{\bar{p}}{p_\infty}, \quad (4)$$

$$\kappa = \sqrt{\frac{\bar{p}^2}{p}}. \quad (5)$$

Variation in $\zeta$ and $\kappa$ on the forebody for different $[L/D]$ and $[d/D]$ cases at various $Re_D$ are plotted in Fig. 13. Values of $\zeta$ are obtained by taking the ratio of the average pressure ($\bar{p}$) during the useful run time and freestream static pressure ($p_\infty$) as shown in Eqn. 4. The variations of $\zeta$ for $[L/D]$ and $[d/D]$ cases are plotted in Fig. 13 a, b, respectively. For the pulsating case ($[L/D, d/D] = [0.7, 0.2]$), values of $\zeta$ lies almost a constant for the entire range of $Re_D$ at $\zeta \sim 32$. While increasing the $[L/D]$ to higher values, $\zeta$ is also observed to remain almost constant over the entire $Re_D$. Among the flapping cases for $[L/D] \geq 1$ and $[d/D] = 0.2$, the monotonic drop in $\zeta$ with increasing $[L/D]$ at the highest $Re_D (5.85 \times 10^5)$ can also be correlated with the trend of the apparent shear layer growth rate described in Table 2. With decrement in $[\delta/x]$ the local extent of unsteadiness closer to the forebody reduces, thereby imposing lesser $\zeta$. At $[L/D] = 1.0$, the global $\zeta$ value remains around 9, which is $\sim 72\%$ smaller than the pulsating case. A further rise in $[L/D]$ to 1.4 and 1.9 results in the drop of $\zeta$ to 5 and 3, respectively. These values correspond to a total drop of $\sim 84\%$ and $\sim 91\%$ in global $\zeta$ relative to the pulsating case. While changing the $[d/D]$ at $[L/D] = 1.4$ across different $Re_D$, the values of $\zeta$ remain almost constant at 4. Thus, to reduce the mean pressure loading across a wide range of $Re_D$, the protrusion length needs to be increased at least above $[L/D] = 1$.

Values of $\kappa$ are calculated by taking the ratio of root-mean-square of the pressure fluctuations and the mean pressure during the effective run-time, as shown in Eqn. 5. A typical plot of $\kappa$ between different $[L/D]$, $[d/D]$ and $Re_D$ is given in Fig. 13 c-d. For increasing $Re_D$, values of $\kappa$ are gradually reduced for most of the $[L/D]$ cases, except for
$[L/D] = 1.4$. The reduction is attributed to the stabilizing effect of the shear layer turbulence, as explained in the previous section. For the pulsating flow ($[L/D, d/D] = [0.7, 0.2]$), $\kappa$ varies between 0.8 to 0.7. While changing the $[L/D]$ to 1.0, these values are bounded between $0.5 \leq \kappa \leq 0.65$, which are considerably less than the previous case by $\sim 24\%$. For $[L/D] = 1.4$, transition in the shock oscillation happens at a critical $Re_D = 2.98 \times 10^5$ as shown in the $x-t$ plots (Fig. 9-III a-h). Irrespective of the two types of physical shock motion as seen in the $x-t$ plots (saw-tooth and broadband wave), the unsteady pressure sensor placed on the model shoulder detects fluctuations at the same level. Hence, the $\kappa$ value remains constant. The trend observed in $[L/D] = 1.4$ is also seen in the cases of $[d/D] = [0.2, 0.26, 0.34]$ for the same reasons; however, with $\kappa$ remaining around 0.5. While considering the longest protrusion case ($L/D = 1.9$), values of $\kappa$ are observed to be higher as equivalent to the cases of pulsation across the entire $Re_D$, owing to the turbulent separated shear layer impinging on the forebody’s shoulder. The fluctuations in the turbulent shear layer are comparatively higher for the longest protrusion case as the separated shear layer travels farther between the point of separation and reattachment. For the thinnest protrusion ($d/D = 0.1$, see Fig. 13d), the values of $\kappa$ are bounded between $0.6 \leq \kappa \leq 0.82$. As there is no transition in the pattern of shock motion across the $Re_D$, a drop in $\kappa$ is seen as before in the previous cases of $[L/D]$ variations. In summary, while varying $[L/D]$, a critical length exists up to which the $\kappa$ value decreases. In the present case, the corresponding $[L/D]$ is 1.4, and the corresponding $\kappa$ is 0.5. On the other hand, while increasing the $[d/D]$, $\kappa$ values continue to drop. Hence, a thick protrusion is beneficial.

### 4.2.5 Unsteady spectra

Asides from $\zeta$ and $\kappa$, computing the spectra from the unsteady pressure signals reveals the dominant unsteady components, if any. A series of spectra for different variations in $[L/D]$, $[d/D]$, and $Re_D$ is given in Fig. 14. Effect of $[L/D]$ and $Re_D$ changes are particularly seen in Fig. 14-I. For the shortest protrusion ($[L/D, d/D] = [0.7, 0.2]$), see Fig. 14 I-a), a discrete non-dimensional frequency of $[d/U_{\infty}] \sim 0.22$ is seen for all the $Re_D$ under consideration. The value indeed matches with the $x-t$ plots shown in Fig. 9-I a-h. Noticeable variation is seen in the non-dimensional power content as the $Re_D$ changes. It increases.
monotonically with $Re_D$ from 100 to 1200 (an increment of 12 times). The ramming of high freestream pressure on the forebody at high $Re_D$ is the reason behind the monotonic variation in the non-dimensional power content.

As the $L/D$ increases to 1, the non-dimensional frequency decreases to $[fD/\infty] \sim 0.19$. In the earlier discussion of $x-t$ plots (Fig. 9), we have already shown that the shock motion of the pulsation form of unsteadiness is packed with high-frequency sinuous wave movements, whereas the flapping one contains considerably a lower frequency with a saw-tooth wave. The non-dimensional power of the discrete non-dimensional frequency varies between 5 to 20 (an increment of 4 times) across different $Re_D$. For $[L/D]=1.4$, until the critical $Re_D$ of $2.98 \times 10^5$, no distinguishable non-dimensional power is visible in the spectra. The presence of low-amplitude broad oscillations seen from the $x-t$ plots (Fig. 9-III a-b) concur that the unsteady pressure is completely driven by the oscillation of shock and shear layer. Across the critical $Re_D$, a discrete non-dimensional frequency owing to flapping is seen with $[fD/\infty] \sim 0.15$. The non-dimensional power is seen to vary between 5 to 15 (an increment by three times) across the $Re_D$. For the longest protrusion $[L/D]=1.9$, at lower $Re_D$, weak non-dimensional spectra are seen with a non-dimensional power lesser than 0.1. However, as $Re_D$ increases, a discrete non-dimensional frequency begins to appear ($[fD/\infty] \sim 0.14$) and gradually turns broader. The impingement of turbulent structures from the separated shear layer on the forebody leads to the detection of an unsteady pressure signal with broader spectral contents. In summary, between the cases of pulsation ($[L/D], [d/D]=[0.7, 0.2]$) and flapping ($[L/D], [d/D]=[1, 0.2]$), a two-order reduction in the non-dimensional power is seen. Among the cases of flapping ($[L/D]=[1, 1.4, 1.9]$ at $[d/D]=0.2$), the non-dimensional power reduces by almost two-order.

While changing the $[d/D]$ across $Re_D$ as shown in Fig. 14-II, a similar pattern as seen in the $[L/D]$ cases are observed. The non-dimensional power content reduces by an order while tripling the $[d/D]$. The non-dimensional frequency, however, varies in a nonlinear manner. Initially $[fD/\infty] \sim 0.14$ for $[d/D]=0.1$. While increasing the $[d/D]$ to 0.2, $[fD/\infty]$ raises to $\sim 0.15$. From there onwards, any further increment in $[d/D]$ (from 0.2 to 0.34) leads to an almost linear decay in the non-dimensional spectra from $[fD/\infty]=0.15$ to 0.13. The intermittent rise in $[fD/\infty]$ can be attributed to the formation of the separation point upstream to the expansion corner on the protrusion. Another noticeable feature in the unsteady pressure spectra is the presence of harmonics across all the cases of $[d/D]$. We suspect a strong resonating behaviour in the shock/shear layer

![Fig. 14](image-url) Variation of unsteady pressure power spectra for different $Re_D$ emphasizing the effects of (I, a-d) protrusion length-$L$ and (II, a–d) protrusion diameter $d$. The unsteady pressure spectra summarized here are taken at the $x_2$ location on the model forebody as shown in Fig. 4. The power spectra $[G_{xx}(f)]$ is normalized by the square of a reference pressure, $p_0=1.45$ mPa. The average $[fD/\infty]$ observed across the different $Re_D$ is written in each plot.
oscillation, resulting in the harmonics across the \( [d/D] \) variations at \( [L/D] = 1.4 \).

### 4.2.6 Modal analysis

Modal analysis (Taira et al. 2017) like the Proper Orthogonal Decomposition (POD) reveals the dominant energetic contents that constitute the flow field both spatially and temporally. High-speed images obtained through several visualization techniques are subjected to POD analysis to extract the underlying dominant modes. In the present discussion, Rayleigh scattering images from type-II imaging, as shown in Fig. 7, are subjected to POD analysis. Rayleigh scattering images contain the distribution of a passive scalar across the flow domain, indicating variations in local density. In the leading-edge separation problem, unsteadiness at hypersonic speed is driven primarily by the shock and shear layer. Those components are well marked in a particular plane in terms of density variations using the Rayleigh scattering technique. Analyzing the images using the POD technique is called a snapshot POD. Accordingly, the images are stacked as a column matrix, and a covariance matrix is sought. Later, the matrix is decomposed as an eigenvalue problem, and the respective eigenvalues and eigenmodes are extracted. The eigenvalues and the eigenmodes are then ranked in descending order of energy contents to identify the dominant spatial modes. The time coefficient matrix is later constructed using a series of algebraic operations, the spectra of which reveal the dominant temporal contents in the flow. The review of Taira et al. (2017) shows more details on the POD techniques.

The cumulative energy contents reveal the presence of the dominant modes across different \( [L/D] \)s and \( [d/D] \)s as shown in Fig. 15. As the shortest protrusion case \( ([L/D], [d/D]) = (0.7, 0.2) \) is driven by the dominant three phases of a typical pulsation cycle, the cumulative energy contents rise quickly across a few modes. Half of the total flow energy is represented by the first three modes. As the \( [L/D] \) varies as \( [L/D] = [1, 1.4, 1.9] \), the total number of modes required to represent half of the flow total energy varies as \( k = [12, 18, 10] \), respectively. The requirement of many modes to represent 50% of the total flow energy arises due to flow turbulence as in jets or separated flows. In these, the flow is dominated by the coupling of many events, unlike in the case of discrete pulsation. However, the leading mode \( (k = 1) \), especially for the longest protrusion \( ([L/D] = 1.9) \), shows a drastic jump in energy to 25%. The suspected reason is attributed to the interaction of the turbulent separated shear layer with the forebody during flapping with a pronounced fluctuation intensity.

While varying the \( [d/D] \), the thinnest protrusion contains higher energy in the leading modes. It progressively decays with the increase in \( [d/D] \) until 0.26. Above \( [d/D] = 0.26 \), an energy increment is seen in the leading modes. The separation point, in this case, is observed to be very close to the protrusion tip, as can be seen using Fig. 7 II-d. The presence of a downstream expansion corner and the varying recirculation zone might be attributed to the deviant behaviour of the leading modes. It is also the reason behind the trend change in the total number of modes \( (k = [8, 18, 20, 14]) \) representing half of the flow total energy, while changing \( [d/D] \) as \( [d/D] = [0.1, 0.2, 0.26, 0.34] \).

![Fig. 15](image_url) Cumulative energy distribution from the Proper Orthogonal Decomposition (POD) of the Rayleigh scattering images using type-II imaging for variations in a \( [L/D] \) and b \( [d/D] \) across all the modes at \( Re_D = 5.85 \times 10^5 \). The cases corresponding to the pulsating and flapping type of shock/shear layer unsteadiness are marked for clarity.
Identifying the leading spatial modes requires careful investigation and tuning of how an image is acquired. In the recent work of Rao and Karthick (2019), the effect of frame rate and image exposure on the leading modes are explained in the case of a screeching supersonic jet. While decomposing the images of flapping unsteadiness, the leading modes in all the \([L/D]\) and \([d/d]\) cases look the same. However, the order of appearance of the secondary spatial modes and the corresponding energy contents are different. In Fig. 16 a-c, the first four dominant spatial modes are shown for three different cases: one uncommon case from each \([L/D]\) and \([d/d]\), and the common case between them, for brevity. The first mode indicates the presence of a separated shock and shear layer in alternating colours. The colour map (red-white-blue) suggests a correlation value between -1 to 1. Hence, the mode can be interpreted as events in the shock and shear layer region happening in out-of-phase. Physically, from the time-averaged or instantaneous Rayleigh scattering image (Fig. 7, see also the supplementary video), the variations in shock \((\beta)\) and shear layer \((\theta)\) angle can be seen. One can show that for a given upstream Mach number \((M)\), the values of \(\beta - \theta\) decrease when \(\theta\) increases and vice versa using the \(\theta - \beta - M\) relations for the conical flow (Sims 1964). It can also be seen evidently from the shear layer properties tabulated in Table 2. During flapping, values of \(\theta\) continuously change and using the \(x - t\) plots and unsteady pressure monitors one can obtain the dominant spectra for such variations (Fig. 9 and Fig. 14). Thus, the first spatial mode can be interpreted as the coupled shock and shear layer mode that drives flapping about the separation point or translatory flapping mode. The leading mode for the represented three cases contains 32.5%, 15.6%, and 20.6% of the total flow energy. The corresponding temporal mode spectra shown in Fig. 16 d-f for \(k = 1\), reveal the dominant non-dimensional frequency lying close to the values as identified in Fig. 9 and Fig. 14. The only contrast is the broadband of spectra seen in the temporal plots of Fig. 16 e-f which can be attributed to the coupling of the leading mode with a few noisy trailing modes in that specific case. However, only the leading mode’s dominant frequency is considered for comparison.

The secondary modes are only composed of the separated shear layer, particularly the sinuous flapping of the shear layer and the structures that are shed along its length. These convecting structures sometimes appear in mode pairs as seen in the compressible jet or shear flows (Rao and Karthick 2019; Rao et al. 2020; Nanda et al. 2021). For the upcoming discussion, the best-represented mode pairs are discussed in succession. In the selected \([L/D]\) and \([d/d]\) cases, the second mode is always sinuous flapping. The energy contents in the second mode are small compared to the leading mode (only in the range of 4% to 7%). The flapping spatial mode is identified by the presence of two distinct alternate colour patches in the shear layer. The third and fourth spatial modes contain the coherent structures present in the separated shear layer after it trips to a turbulent state. The third mode mainly contains large-scale structures, while the fourth mode has smaller structures. These structures’ energy contents are even smaller than the leading mode. Across the considered cases, the third and the fourth mode have \(\sim 3-4%\) and \(\sim 2-3%\) energy contents, respectively. The respective temporal spectra shown in Fig. 16 d-f do not contain any significant amplitude compared to the leading modes, and they are mostly broadband.

To ensure the existence of the obtained spatial modes (translatory flapping, sinuous flapping, or shedding), a separate modelling exercise is done using synthetic image sequences. Rayleigh scattering images are used as references to construct these synthetic images by representing most of the flow physics encountered in the leading-edge separation. The oblique separation shock is modelled as a line with a higher slope than the separated shear layer. The shear layer is modelled as a line with amplifying sine wave whose wave number decreases along its length. A discrete frequency is given to simulate the flapping oscillation about the separation point and to create a travelling waveform along the shear layer. The equations responsible for the spatiotemporal modelling to simulate these synthetic images are given below. Within the spatial points of \(0 \leq x/D \leq 3\) and temporal points of \(1 \leq t/f_{s} \leq 25\) (where \(f_{s} = 40\ kHz\), the oblique shock is modelled as,

\[
y_{1} = 2h(1 - 0.8h)x, \tag{6}
\]

where \(h\) varies as,

\[
h = b_{1} + \left[ \frac{\Omega - \text{min}(\Omega)}{\text{max}(\Omega) - \text{min}(\Omega)} \right] (b_{2} - b_{1}),
\]

\[
\Omega = \sin(2\pi f_{s} t).
\]

The shear layer angle, along with flapping and convection of transitional waves along its length, are modelled as,

\[
y_{2} = hx + a_{1}e^{-a_{2}/x} \cos \left( \frac{a_{3}}{x^{a_{4}} + a_{5}} - 2\pi f_{s} t \right) ...
\]

\[
+ a_{6}e^{-a_{7}/x} \sin \left( \frac{a_{8}}{x^{a_{9}} + a_{10}} + 2\pi f_{s} t \right). \tag{7}
\]

In Eqn. 7, the first term represents the separated shear angle, the second term dictates the sinuous flapping about the shear layer, and the last term indicates the convection of instability or transitional waves along the shear layer. A synthetic image is prepared as shown in Fig. 17-1 (a-d) by shading the zones above Eqn. 6 as moderate grey colour to represent the low-density passive scalar seeding in the freestream. Similarly,
the zone bounded between Eqn. 6 and 7 is shaded in light gray color to represent the density jump across the separated shock. The region below Eqn. 7 is coloured as dark grey as it is a recirculation zone. The overall colour scheme is selected to represent a typical Rayleigh scattering image. The constants values represented in Eqns. 6-7 can be varied slightly to change the order of the resulting secondary modes. The constants considered for the present exercise are given as: $a_1 = 0.5$, $a_2 = 10$, $a_3 = 0.1$, $a_4 = 0$, $a_5 = 0$, $a_6 = 1$, $a_7 = 20$, $a_8 = 2000$, $a_9 = 1.2$, $a_{10} = 20$, $b_1 = 0.35$, $b_2 = 0.38$, $f_1 = 350$, $f_2 = 1000$, and $f_3 = 3000$. The time-resolved synthetic images are stitched together and represented in the supplementary video to visualize the simulated shock and shear layer oscillation. The leading and secondary spatiotemporal modes are obtained using Eqns. 6-7 as shown in Fig. 17. The simple mathematical modelling of the shock and shear layer oscillations seems to represent the actual modes from the Rayleigh scattering images as shown in Fig. 16. Thus, the exercise with the synthetic image analysis sheds light on the physics involved and gives confidence to the experimental modal analysis findings. A typical schematic shown in Fig. 18 summarizes the realized spatial modes both from the actual Rayleigh scattering experiments and the synthetic images encountered in the leading-edge separation problems at hypersonic speed.

**5 Conclusions**

Experiments to understand the leading-edge separation in hypersonic flows, particularly at $M_{\infty} = 6$, are done using the newly commissioned Hypersonic Ludwieg Tunnel (HLT)
at Technion for a range of Reynolds numbers between $1.66 \times 10^5 \leq \text{Re}_D \leq 5.85 \times 10^5$. Leading-edge separation is induced using an axial sharp-tip protrusion attached to a flat-face cylindrical forebody of diameter $D = 35$ mm. The length and diameter of the axial protrusion are varied as $[L/D] = [0.7, 1.0, 1.4, 1.9]$ and $[d/D] = [0.1, 0.2, 0.26, 0.34]$, respectively. Two visualization techniques: schlieren imaging (density gradients along the transverse direction) and Rayleigh scattering, are employed to understand the inherent flow structures responsible for the unsteadiness. Unsteady pressure transducers are utilized to extract the total pressure loading (mean pressure, $\zeta$), fluctuation intensity (root-mean-square of the pressure fluctuations, $\kappa$), and the associated spectral contents for the different cases. Following are a few major observations from the present study.

- **Effects of protrusion length**: Two forms of shock and shear layer unsteadiness are seen as $[L/D]$ increases: pulsation (well-known and understood) for the shortest protrusion and flapping (lesser-known and unclear) for the rest of the longer protrusion cases. Pulsation is characterized by a high-frequency shock motion along the protrusion length. Flapping has a reduced frequency with diagonally transverse oscillations of the separated shear layer toward and away from the protrusion. Two-orders reduction in the power of unsteady pressure spectra is seen between pulsation and flapping. Among the flapping cases, as the protrusion length increases, the power reduces by almost two orders. At the highest $\text{Re}_D$, as $[L/D]$ increases, the shock angle ($\beta$), shear layer angle ($\theta$), and shear layer growth rate ($\delta/x$) monotonically...

Fig. 17 (I) a–d Synthetic representation of the Rayleigh scattering image instants showing the leading-edge separation in a hypersonic flow. The corresponding video is also available in the supplementary. (II) a–d Some vital energetic spatial modes from the POD analysis depicting translatory flapping, sinuous flapping, and shedding structures are given along with the corresponding energy contents. (III) $a–d$ Line plot showing the variations in the non-dimensional temporal spectra across the different modes.

Fig. 18 A series of schematics representing the interpretation of the first four dominant spatial modes observed from the POD analysis of the Rayleigh scattering images and the synthetic images as shown in Figs. 16 and 17, respectively.
decrease. Mean, and RMS (root-mean-square) pressures are at their highest for the shortest protrusion due to pulsation. In the flapping cases, as the $[L/D]$ decreases, mean pressure decreases by 70-80% as $\theta$ and $\beta$ decrease. The RMS pressure for the flapping cases decreases until the longest protrusion, where the values are equivalent to the pulsation case. The reason is attributed to the transition to turbulence of the separated shear layer.

- **Effects of protrusion diameter**: All the cases of $[d/D]$ variations exhibit a flapping form of unsteadiness. Shock and shear layer properties like $\beta$, $\theta$, and $\delta/x$ at the highest $Re_D$ remain almost constant as $[d/D]$ increases. The mean pressure remains almost invariant with $[d/D]$ changes. The RMS pressure monotonically decreases as $[d/D]$ increases. In the unsteady pressure spectra, the power reduces by almost an order as $[d/D]$ increases by three times. The decreasing recirculation bubble volume while increasing the $[d/D]$ for a constant $[L/D]$ is suspected of stabilizing the fluctuations in the separated shear layer along with the upstream movement of the separation point ahead of the expansion corner.

- **Effects of Reynolds number**: The strong pulsation type of shock and shear layer oscillations is seen consistently across the $Re_D$ for the shortest protrusion. However, the power spectra of the unsteady pressure monotonically increase while the dominant frequency remains unchanged. As $[L/D]$ increases, a low or high amplitude broadband oscillation is seen owing to the presence of a laminar or turbulent shear layer. The critical $Re_D$ at which the shear layer characteristics change are observed to shift as $[d/D]$ increases. The mean pressure for the $[L/D]$ and $[d/D]$ cases remains almost constant across the different $Re_D$s. The RMS pressure for the $[L/D]$ and $[d/D]$ cases either stay constant or decays slightly as $Re_D$ increases.

- **Modal analysis**: Proper Orthogonal Decomposition of the Rayleigh scattering images reveals the presence of four driving modes in the flapping unsteadiness: translatory flapping, sinuous flapping, large-scale shedding, and small-scale shedding. The energy contents of these modes vary based upon the intensity of flapping across $[L/D]$ and $[d/D]$ cases. However, the leading mode remains the same in all of them, with some rank changes between the secondary modes.

### 6 Supplemental material

The manuscript contains the following videos as supplementary in addition to few texts and figures: 1. video of the high-speed schlieren imaging using type-II imaging, 2. video of the high-speed Rayleigh scattering imaging using type-II imaging, and 3. video of the synthetic Rayleigh scattering imaging.

#### Supplementary information

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#### Declarations

#### Ethical approval

Not applicable.

#### Conflict of interest

The authors declare that the they have no competing interests as defined by Springer, or other interests that might be perceived to influence the results and/or discussion reported in this paper.

#### Availability of data and materials

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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