Mapping Quantitative Trait Loci Underlying Function-Valued Traits Using Functional Principal Component Analysis and Multi-Trait Mapping

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ABSTRACT We previously proposed a simple regression-based method to map quantitative trait loci underlying function-valued phenotypes. In order to better handle the case of noisy phenotype measurements and accommodate the correlation structure among time points, we propose an alternative approach that maintains much of the simplicity and speed of the regression-based method. We overcome noisy measurements by replacing the observed data with a smooth approximation. We then apply functional principal component analysis, replacing the smoothed phenotype data with a small number of principal components. Quantitative trait locus mapping is applied to these dimension-reduced data, either with a multi-trait method or by considering the traits individually and then taking the average or maximum LOD score across traits. We apply these approaches to root gravitropism data on Arabidopsis recombinant inbred lines and further investigate their performance in computer simulations. Our methods have been implemented in the R package, funqtl.

Technology developments have enabled the automated acquisition of numerous phenotypes, included function-valued traits, such as phenotypes measured over time. High-dimensional phenotype data are increasingly considered as part of efforts to map the genetic loci (quantitative trait loci, QTL) that influence quantitative traits.

Numerous methods are available for QTL mapping with function-valued traits. Ma et al. (2002) considered parametric models such as the logistic growth model, \( g(t) = \frac{a}{1 + be^{-rt}} \). The high-dimensional phenotype is reduced to a few parameters. This works well if the parametric model is approximately correct but in many cases the correct functional form is not clear. Yang et al. (2009) proposed a nonparametric functional QTL mapping method, with a selected number of basis functions to fit the function-valued phenotype. Min et al. (2011), Sillanpää et al. (2012), and Li and Sillanpää (2013) extended this method for multiple-QTL models. Min et al. (2011) used Markov chain Monte Carlo (MCMC), Sillanpää et al. (2012) used hierarchical modeling, and Li and Sillanpää (2013) used a multivariate regression method. Xiong et al. (2011) proposed an additional nonparametric functional mapping method based on estimating equations. An important barrier to these methods is the long computation time required for the analysis.

In Kwak et al. (2014), we proposed two simple regression-based methods to map QTL underlying function-valued phenotypes, based on the results from the individual analysis of the phenotypes at each time point. With the SLOD score, we take the average of the LOD scores across time points, and with the MLOD score, we take the maximum. These approaches are fast to compute, work well when the trait data are smooth, and provide results that are easily interpreted. Another important advantage is the ability to consider multiple QTL, which can improve power and enable the separation of linked QTL. However, the approaches do not work as well when the trait data are not smooth, and they do not take account of the correlation among time points.

In the present paper, we describe methods to overcome these weaknesses. First, we replace the observed trait data with a smooth approximation. Second, we apply functional principal component analysis (PCA) as a dimension-reduction technique, and replace the smoothed phenotype data with a small number of principal components (PCs).
QTL analysis is then performed on these dimension-reduced data. We consider either the multivariate QTL mapping method of Knott and Haley (2000), or use the SLOD or MLOD scores, as in Kwak et al. (2014). These methods all have analogs for multiple-QTL models, by extending the penalized LOD scores of Broman and Speed (2002) and Manichaikul et al. (2009).

We illustrate these methods by application to the root gravitropism data of Moore et al. (2013), measured by automated image analysis over a time course of 8 hr across a population of Arabidopsis thaliana recombinant inbred lines (RIL). We further investigate the performance of these approaches in computer simulations.

METHODS

We will focus on the case of recombinant inbred lines, with genotypes AA or BB. Consider \( n \) lines, with function-valued phenotypes measured at \( T \) discrete time points \( (t_1, \ldots, t_T) \).

**Smoothing**

We first smooth the phenotype data for each individual. Let \( y_i(t_i) \) denote the observed phenotype for individual \( i \) at time \( t_i \). We assume underlying smooth curves \( x_i(t) \), with \( y_i(t_i) = x_i(t_i) + e_i(t_i) \).

We approximate the functional form of \( x_i(t) \) as \( x_i(t) = \sum_{k=1}^{K} c_{ik} \phi_k(t) \) for a set of basis functions \( \phi_1(t), \ldots, \phi_K(t) \), where the number of basis functions, \( K \), is generally much smaller than the number of time points, \( T \). There are many possible choices of basis functions. We are using B-splines (Ramsay and Silverman 2005, pp. 49–53). Xiong et al. (2011) and Li and Sillanpää (2013) also used B-splines for QTL analysis with functional traits.

Define the \( K \) by \( T \) matrix \( \Phi \), with \( \Phi_{jk} = \phi_k(t_j) \), for \( k = 1, \ldots, K \) and \( j = 1, \ldots, T \). We estimate the coefficients, \( c_{ik} \) by minimizing the least squares criterion (Ramsay and Silverman 2005):

\[
\text{SMSE}(y|c) = \sum_{i=1}^{n} \sum_{j=1}^{T} \left[ y_i(t_j) - \sum_{k=1}^{K} c_{ik} \phi_k(t_j) \right]^2 = (y - \Phi c)'(y - \Phi c).
\]

This gives the solution \( \hat{c} = (\Phi'\Phi)^{-1}\Phi' y \). We then obtain the smoothed phenotypes \( \hat{y} = \Phi \hat{c} \), which are used in all subsequent analyses.

A key issue is the choice of the number of basis functions. We use tenfold cross-validation and choose the number of basis functions that minimizes the estimated sum of squared errors.

**Functional principal component analysis**

Having replaced the phenotype data with a smooth approximation, \( \hat{y}_j(t) \), we use functional PCA (Ramsay and Silverman 2005) to reduce the dimensionality with little loss of information.

In functional PCA, we seek a sequence of orthonormal functions, \( \psi_j(t) \), that take the role of PCs but for functional data. By orthonormal, we mean \( \langle \psi_j(t), \psi_k(t) \rangle = \int \psi_j(t)\psi_k(t) dt = 0 \), for all \( j \neq k \), and \( \|\psi_j(t)\|^2 = \langle \psi_j(t), \psi_j(t) \rangle = 1 \) for all \( j \).

With our smoothed functional data, we first find the function \( \psi_j(t) \) that maximizes \( \sum_j \langle \dot{\hat{y}}_j(t), \psi_1(t) \rangle^2 = \sum_j \langle \dot{\hat{y}}_j(t) \psi_1(t) dt \rangle^2 \), subject to the constraint \( \|\psi_1(t)\| = 1 \). The computations make use of the B-spline basis representation of the \( \dot{\hat{y}}_j(t) \).

Conceptually, the procedure then proceeds inductively. Having identified \( \psi_1, \ldots, \psi_j \), we choose \( \psi_{j+1} \) that maximizes \( \sum_j \langle \dot{\hat{y}}_j(t), \psi_1(t) \rangle^2 \) subject to the constraint that \( \psi_1, \ldots, \psi_{j+1} \) are orthonormal.

We focus on a small number, \( p \), of functional PCs that explain 99% of the data variation and consider the coefficients \( (\dot{\hat{y}}_j(t), \psi_1(t)) \), as derived traits.

**Single-QTL analysis**

Having smoothed and dimension-reduced the phenotype data, we then use the \( p \) PCs as derived traits for QTL analysis, using one of three methods. First, we apply the multivariate QTL mapping method of Knott and Haley (2000). Our second and third approaches are to analyze the \( p \) derived traits individually and take the SLOD or MLOD score, respectively, at each putative QTL position, as in Kwak et al. (2014).

**HKLOD score**

In the method of Knott and Haley (2000), we take the \( n \times p \) matrix of derived phenotypes and scan the genome, and, at each position, \( \lambda \), we fit a multivariate regression model with a single QTL. The basic model is \( Y = X\beta + E \), where \( Y \) is the \( n \times p \) matrix of phenotypes, \( X \) is an \( n \times 2 \) matrix of QTL genotype probabilities, and \( E \) is a \( 2 \times p \) matrix of QTL effects. The rows of the \( n \times p \) matrix of errors, \( E \), are assumed to be independent and identically distributed draws from a multivariate normal distribution.

Figure 1 The SLOD, MLOD, EE (Wald) and EE(Residual), HKLOD, SL, and ML curves for the root tip angle data. A red horizontal line indicates the calculated 5% permutation-based threshold.
The maximum likelihood estimate of the coefficients is $\hat{\beta} = (X'X)^{-1}X'Y$, the same as if the traits were analyzed separately. A key component of the likelihood is the matrix of sums of squares and cross-products of residuals, $RSS = (Y-X\hat{\beta})'(Y-X\hat{\beta})$, and particularly its determinant, $|RSS|$. The log$_{10}$ likelihood ratio comparing the model with a single QTL at position $\lambda$, to the null model of no QTL, is

$$\text{HKLOD} = \frac{n}{2} \log_{10} \left( \frac{|RSS_0|}{|RSS(\lambda)|} \right),$$

where $|RSS_0|$ is for the null model with no QTL, and $|RSS(\lambda)|$ is for the model with a single QTL at $\lambda$.

**SL and ML scores:** As further approaches, we apply the method of Kwak et al. (2014) to the $p$-derived traits. We perform a genome scan by Haley-Knott regression (Haley and Knott 1992) with each trait separately, giving LOD$_j(\lambda)$ for trait $j$ at position $\lambda$.

Our second criterion is to take the average LOD score across traits:

$$\text{SL}(\lambda) = \frac{1}{p} \sum_{j=1}^{p} \text{LOD}_j(\lambda).$$

We call this the SL score, to distinguish it from the SLOD of Kwak et al. (2014), calculated using the original phenotype data.

Our third criterion is to take the maximum LOD score,

$$\text{ML}(\lambda) = \max_j \text{LOD}_j(\lambda).$$

We call this the ML score, to distinguish it from the MLOD score of Kwak et al. (2014).

**Multiple-QTL analysis**

As in Kwak et al. (2014), we use the penalized LOD score criterion of Broman and Speed (2002) to extend each of the LOD-type statistics defined above, for use with multiple-QTL models. The penalized LOD score is $p\text{LOD}(\gamma) = \text{LOD}(\gamma) - T|\gamma|$, where $\gamma$ denotes a multiple-QTL model with strictly additive QTL, and $|\gamma|$ is the number of QTL in the model.

**Figure 2** HKLOD, SL, and ML profiles for a multiple-QTL model with the root tip angle data set.
model $g$. $T$ is a penalty on model size, chosen as the $1-\alpha$ quantile of the genome-wide maximum LOD score under the null hypothesis of no QTL, derived from a permutation test (Churchill and Doerge 1994). We may replace the LOD score in the above equation with any of the HKLOD, SL and ML scores.

To search the space of models, we use the stepwise model search algorithm of Broman and Speed (2002): we use forward selection up to a model of fixed size (e.g., ten QTL), followed by backward elimination to the null model. The selected model $\hat{g}$ is that which maximizes the penalized LOD score criterion, among all models visited.

The selected model is of the form $Z = Q \beta + \varepsilon$, where $Z$ contains the derived traits (the coefficients from the functional PCA) and $Q$ contains an intercept column and genotype probabilities at the inferred QTL. The derived phenotypes, $\hat{Z}$, are linearly related to the smoothed phenotypes, $\hat{Y}$, by the equation $\hat{Y} = \hat{Z} \Psi$, where $\Psi$ is a matrix with $(i,j)$th element $\psi_j(t_i)$. Thus we have $\hat{Y} = \hat{Z} \Psi = Q (\beta \Psi) + \varepsilon$, and so $\hat{\beta} \Psi$ are the estimated QTL effects, translated back to the time domain (see Figure 3, below).

**APPLICATION**

As an illustration of our approaches, we considered data from Moore et al. (2013) on gravitropism in Arabidopsis RIL, Cape Verde Islands (Cvi) × Landsberg erecta (Ler). For each of 162 RIL, 8–20 replicate seeds per line were germinated and then rotated 90 degrees, to change the orientation of gravity. The growth of the seedlings was captured on video, over the course of 8 hr, and a number of phenotypes were derived by automated image analysis.

We focus on the angle of the root tip, in degrees, over time (averaged across replicates within a RIL), and consider only the first of two replicate data sets examined in Moore et al. (2013). There is genotype data at 234 markers on five chromosomes; the function-valued root tip angle trait was measured at 241 time points (every 2 min for 8 hr).

The data are available at the QTL Archive, which is now part of the Mouse Phenome Database, as the Moore1b data set: http://phenome.jax.org/db/q?rtn=projects/projdet&reqprojid=282.

**Single-QTL analysis**

We first performed genome scans with a single-QTL model by multiple methods: SLOD and MLOD from Kwak et al. (2014), EE(Wald) and EE (Residual) from Xiong et al. (2011), and the HKLOD, SL, and ML methods described above (and using four PCs). We used a permutation test (Churchill and Doerge 1994) with 1000 permutation replicates to estimate 5% significance thresholds, which are shown in Supporting Information, Table S1.

The results are shown in Figure 1. The SLOD method (Figure 1A) gave similar results to the EE(Residual) method (Figure 1D), with significant evidence for QTL on chromosomes 1, 4, and 5. The MLOD method (Figure 1B) also showed evidence for a QTL on chromosome 3. The EE(Wald), HKLOD, and SL methods (Figure 1, C, E, and F) all gave similar results, with significant evidence for QTL on each chromosome. The ML method (Figure 1G) is different, with significant evidence for QTL on chromosomes 1, 2, and 4.

**Multiple-QTL analysis**

We further applied multiple-QTL analysis, extending the HKLOD, SL, and ML methods to use the penalized LOD score criterion of Broman and Speed (2002) for function-valued traits. We focused on additive QTL models, and used the 5% permutation-based significance thresholds (Table S1) as the penalties.

The penalized-HKLOD and penalized-SL criteria each indicated a five-QTL model, with a QTL on each chromosome. The inferred positions of the QTL showed only slight differences. The penalized-SL criterion indicated a two-QTL model, with QTL on chromosomes 1 and 4.

LOD profiles for these models are displayed in Figure 2. These curves, which visualize both the evidence and localization of each
QTL in the context of a multiple-QTL model, are calculated following an approach developed by Zeng et al. (2000): the position of each QTL was varied one at a time, and, at each location for a given QTL, we derived a LOD-type score comparing the multiple-QTL model with the QTL under consideration at a particular position and the locations of all other QTL fixed, to the model with the given QTL omitted. For the SL (or ML) method, the profile is calculated for the four derived traits, individually, and then the SL (or ML) profiles are obtained by averaging (or maximizing) across traits. For the HKLOD method, the profiles are calculated using the multivariate LOD test statistic.

In Kwak et al. (2014), we applied, to these same data, the analogous multiple-QTL modeling approach, with the penalized-SLOD and penalized-MLOD criteria (that is, using the original phenotype data, without the smoothing and dimension-reduction steps). The result (see Figure 3 in Kwak et al. 2014) with the penalized-SLOD criterion was the same two-QTL model identified by ML (Figure 2C), while the penalized-MLOD criterion gave a three-QTL model, with a QTL on chromosome 3, similar to that inferred by single-QTL analysis with MLOD (Figure 1B).

The estimated QTL effects, translated back to the time domain, are displayed in Figure 3. The red curves are for the five-QTL model identified with the penalized-HKLOD and penalized-SL criteria. The blue dashed curves are for the two-QTL model identified with the penalized-ML criterion. The estimated QTL effects in panels B–F are for the difference between the Cvi allele and the Ler allele.

The effects of the QTL on chromosomes 1 and 4 are approximately the same, whether or not the chromosome 2, 3, and 5 QTL are included in the model. The chromosome 1 QTL has greatest effect at later time points, while the chromosome 4 QTL has greatest effect earlier and over a wider interval of time. For both QTL, the Cvi allele increases the root tip angle phenotype.

In summary, the HKLOD and SL methods gave similar results. For these data, the HKLOD and SL methods indicate evidence for a QTL on each chromosome. The results suggest that these approaches, with the initial dimension-reduction via functional PCA, may have higher power to detect QTL, as the multiple-QTL analysis with SLOD and MLOD, without dimension-reduction, indicated fewer QTL.

**SIMULATIONS**

In order to investigate the performance of our proposed approaches and compare them to existing methods, we performed computer simulation studies with both single-QTL models and models with multiple QTL. For the simulations with a single-QTL model, we compared the
HKLOD, SL, and ML methods to the SLOD and MLOD methods of Kwak et al. (2014) and the estimating equation approaches of Xiong et al. (2011). For the simulations with multiple QTL, we omitted the methods of Xiong et al. (2011), as they have not yet been implemented for multiple QTL.

**Single-QTL models**

To compare approaches in the context of a single QTL, we considered the simulation setting described in Yap et al. (2009), though exploring a range of QTL effects.

We simulated an intercross with sample sizes of 100, 200, or 400, and a single chromosome of length 100 cM, with six equally spaced markers and with a QTL at 32 cM. The associated phenotypes were sampled from a multivariate normal distribution with the mean curve following a logistic function,

$$g(t) = \frac{a}{1 + be^{-rt}}.$$  

The AA genotype had $a = 29$, $b = 7$, $r = 0.7$; the AB genotype had $a = 28.5$, $b = 6.5$, $r = 0.73$; and the BB genotype had $a = 27.5$, $b = 5$, $r = 0.75$. The shape of the growth curve with this parameter was shown in Figure S3 of Kwak et al. (2014). The phenotype data were simulated at ten time points.

The residual error was assumed to follow a multivariate normal distribution with covariance structure $\Sigma$. The constant $c$ controls the overall error variance, and $\Sigma$ was chosen to have one of three forms: (1) auto-regressive with $\sigma^2 = 3$, $\rho = 0.6$, (2) equicorrelated with $\sigma^2 = 3$, $\rho = 0.5$, or (3) an “unstructured” covariance matrix, as given in Yap et al. (2009) (also shown in Table S2 of Kwak et al. 2014).

The parameter $c$ was given a range of values, which define the percent phenotypic variance explained by the QTL (the heritability). The effect of the QTL varies with time; we took the mean heritability across time as an overall summary. For the auto-regressive and equicorrelated covariance structures, we used $c = 1, 2, 3, 6$; for the unstructured covariance matrix, we took $c = 0.5, 1, 2, 3$.

For each of 10,000 simulation replicates, we applied the previous SLOD and MLOD methods of Kwak et al. (2014), the EE(Wald) and EE (Residual) methods of Xiong et al. (2011), and the HKLOD, SL, and ML methods proposed here. For all seven approaches, we fit a three-parameter QTL model (that is, allowing for dominance).

The estimated power to detect the QTL as a function of heritability due to the QTL, for $n = 100, 200, 400$ and for the three different covariance structures, is shown in Figure 4. With the autoregressive covariance structure, all methods worked well, though HKLOD had noticeably lower power. With the equicorrelated variance structure, EE(Wald), SL and ML methods had higher power than the other four methods. The
Followed a cubic polynomial, the Application criterion, for the root tip angle data of Moore QTL model modeled after that estimated, with the penalized-MLOD same setting as in Kwak and penalized-ML criteria in the context of multiple QTL, we used the considerably longer computation time; on the other hand, it provided than the SLOD/MLOD methods. The EE(Wald) method requires con- siderably longer computation time, and the ability to consider multiple-QTL models. The EE(Wald) method continued to perform best.

Table 1 shows the average computation time for SLOD/MLOD, EE (Wald), EE(Residual), HKLOD and SL/ML methods. For the single-QTL simulations, the computation time for the SLOD/MLOD and HKLOD methods are similar, while the SL/ML methods are somewhat slower. This is because, in the simulation data set, the phenotype was measured at ten time points. In the application, with 241 time points, the functional PCA-based methods (HKLOD and SL/ML) are faster than the SLOD/MLOD methods. The EE(Wald) method requires considerably longer computation time; on the other hand, it provided the highest power to detect QTL.

**Multiple-QTL models**

To investigate the performance of the penalized-HKLOD, penalized-SL and penalized-ML criteria in the context of multiple QTL, we used the same setting as in Kwak et al. (2014). We simulated data from a three-QTL model modeled after that estimated, with the penalized-MLD criterion, for the root tip angle data of Moore et al. (2013) considered in the Application section.

We assumed that the mean curve for the root tip angle phenotype followed a cubic polynomial, \( y = a + bt + ct^2 + dt^3 \), and assumed that the effect of each QTL also followed such a cubic polynomial. The four parameters for a given individual were drawn from a multivariate normal distribution with mean defined by the QTL genotypes and variance matrix estimated from the root tip angle data. Details on the parameter values used in the simulations appear in Kwak et al. (2014).

Table 1 Average computation time for each method, in the single-QTL simulations and in the application

| Method          | Simulations | Application |
|-----------------|-------------|-------------|
| SLOD/MLOD       | 0.011       | 0.96        |
| EE(Wald)        | 0.853       | 224.4       |
| EE(Residual)    | 0.030       | 3.97        |
| HKLOD           | 0.013       | 0.21        |
| SL/ML           | 0.114       | 0.21        |

HKLOD method also worked reasonably well, but the EE(Residual), SLOD and MLOD methods had low power. In the unstructured variance setting, EE(Wald), MLOD, SL and ML methods worked better than the other three methods. EE(Residual) did not work well in this setting.

The precision of QTL mapping, measured by the root mean square (RMS) error in the estimated QTL position, is displayed in Figure S1. Performance, in terms of precision, corresponds quite closely to performance in terms of power: when power is high, the RMS error of the estimated QTL position is low, and vice versa.

A possible weakness of the SLOD and MLOD approaches was that they do not make use of the function-valued form of the phenotypes. The methods may further suffer lower power in the case of noisy phenotypes. The methods proposed in this paper use smoothing to handle the measurement error. We repeated the same simulations in Kwak et al. (2014) with \( n = 200 \), adding independent, normally distributed errors (with standard deviation 1 or 2) at each time point.

The estimated power to detect the QTL as a function of heritability due to the QTL, for added noise with SD = 0, 1, 2 and the three different covariance structures, is shown in Figure 5. The power of the SLOD and MLOD were greatly affected by the introduction of noise. However, the SL and ML methods, which used both smoothing and dimension-reduction, work well in the presence of noise. The EE(Wald) method performed well in this case, as well. The EE(Residual) method did not work well compared to the other six methods. Overall, the EE(Wald) method continued to perform best.

Table 2 Power to detect QTL, for a three-QTL model modeled after the data of Moore et al. (2013)

| QTL Position | Power |
|--------------|-------|
| chr cM location | HKLOD | SL | ML | SLOD | MLOD |
| n = 162      | 1     | 61 | 82 | 86 | 65 | 89 | 54 |
| n = 324      | 1     | 61 | 100 | 100 | 95 | 40 | 77 |

Normally distributed measurement error (with mean 0 and variance 1) was added to the phenotype at each time point for each individual. Phenotypes are taken at 241 equally spaced time points in the interval of 0 to 1. We considered two sample sizes: \( n = 162 \) [as in the Moore et al. (2013) data] and twice that, \( n = 324 \).

We performed 100 simulation replicates. For each replicate, we applied a stepwise model selection approach with each of the penalized-HKLOD, penalized-SL, penalized-ML, penalized-SLOD, and penalized-MLOD criteria. The simulation results are shown in Table 2.

The penalized-SL and penalized-HKLOD criteria had higher power to detect all three QTL. In particular, the power to detect the QTL on chromosome 3 was greatly increased, in comparison to the penalized-SLOD and penalized-MLOD criteria.

**DISCUSSION**

We have described two techniques for improving the regression-based methods of Kwak et al. (2014) for QTL mapping with function-valued phenotypes: smoothing and dimensional-reduction. Smoothing leads to better performance in the case of noisy phenotype measurements (Li and Sillanpää 2013), and dimension-reduction improves power. The particular methods we used (smoothing via B-splines, and dimensional reduction via functional PCA) are not the only possibilities, but they are natural choices widely used in functional data analysis (Ramsay and Silverman 2005).

Following smoothing and dimensional-reduction, we applied QTL analysis to the small number of derived traits, either by analyzing the traits individually and then combining the log likelihoods, as in Kwak et al. (2014), or by applying the multivariate QTL mapping method of Knott and Haley (2000). The latter approach cannot be applied directly to the original phenotypes, due to the large number of time points at which the traits were measured, but it can work well with the dimension-reduced derived traits.

Key advantages of our proposed methods include speed of computation, and the ability to consider multiple-QTL models. The EE(Wald) method of Xiong et al. (2011), based on estimating equations, was seen to be most powerful for QTL detection in our simulation study, but it is orders of magnitude slower and has not yet been implemented for multiple-QTL models.

Many other methods have been developed for QTL mapping with function-valued traits. However, most focus on single-QTL models (e.g., Ma et al. 2002; Yang et al. 2009; Yap et al. 2009). Bayesian methods for multiple-QTL mapping with function-valued traits have been proposed (Min et al. 2011; Sillanpää et al. 2012; Li and Sillanpää 2013), but these methods are computationally intensive, and software is not available.

In considering multiple-QTL models, we have focused on strictly additive QTL. Manichaikul et al. (2009) extended the work of Broman and Speed (2002) by considering pairwise interactions among QTL. Our approaches may be similarly extended to handle interactions.
The enormous recent growth in capabilities for high-throughput phenotyping, including images and time series, particularly in plants (see, for example, Cabrera-Bosquet et al. 2012; Araus and Cairns 2014; Ghanem et al. 2015), is accompanied by a growth in interest in the genetic analysis of such phenotype data. Speed of computation will be particularly important in the analysis of such high-dimensional data, as will the joint consideration of multiple loci. The methods we have proposed can meet many of these challenges.

Our efforts on this problem were inspired by the data of Moore et al. (2013), in which the phenotype was measured at a large number of time points. While our approaches do not require such a high density of time points, we expect that the use of smoothing and functional PCA will be most suitable in the case of at least eight to ten time points. In the case of a smaller number of time points, we would recommend the direct use of multivariate QTL analysis, or of the SLOD and MLOD methods of Kwak et al. (2014).

We implemented our methods as a package, funqtl, for the general statistical software R (R Core Team 2015). Our package makes use of the fda package (Ramsay et al. 2014) for smoothing and functional PCA. It is available at https://github.com/ikwak2/funqtl.

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