On the Spectral Efficiency Limits of an OAM-based Multiplexing Scheme

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Abstract—As reported in several recent publications, a spatial multiplexing involving the transmission of orthogonal waves carrying Orbital Angular Momentum (OAM) is unable to provide improvements in spectral efficiency with respect to the conventional techniques. With this work we want to emphasize how such consideration can be easily derived from the Shannon capacity formula. Taking as a reference the performance of a multiplexing scheme based on the higher-order channel singular modes, we analyze the spectral efficiency of an OAM multi-mode transmission between antenna arrays. Our approach clearly indicates that the two techniques offer the same on-axis performance. Conversely, small misalignments in the arrays positions strongly affect the OAM scheme, highlighting the greater robustness of a traditional multiplexing method in the context of radio communications.

Index Terms—array synthesis; Orbital Angular Momentum (OAM); spectral efficiency; uniform circular arrays.

I. INTRODUCTION

A result of the growing spread of broadband services, the recent years have witnessed a strong increase in the demand for spectral resources within the context of wireless communications. To address this problem, the technological evolution in telecommunication has led to the development of new techniques employing different modulation schemes, polarization and spatial/temporal diversity in order to exploit the electromagnetic spectrum more efficiently. In addition to the above mentioned methods, it has recently been proposed a new approach based on the use of waves carrying Orbital Angular Momentum (OAM), with relevant applications both in optics [1]–[3] and at the radio frequencies [4]–[6].

OAM beams are well-known solutions to the Helmholtz equation, characterized by the presence of a phase singularity along the propagation axis, which determines a central intensity null and twisted wavefronts. Mathematically, such phase structure is described by a screw dislocation of the form $e^{i\ell \varphi}$, where $\varphi$ is the azimuthal angle, while the index $\ell \in \mathbb{Z}$, related to the orbital angular momentum content of the beam, indicates the number of twists performed by the phase profile around the central point of zero intensity [7].

The orthogonality among OAM modes with different $\ell$ index led to consider the possibility of exploiting the peculiar phase distribution of such beams for conveying multiplex communication subchannels at the same frequency [5], [6]. Although this idea sounds innovative from a physical point of view, it has been shown that an OAM-based transmission can be considered as a particular case of spatial multiplexing [8]–[10]. Within this framework, the over-quadratic power decay in the central region has been pointed out as the real drawback to the use of OAM beams [11]; however, it should be noted that such behaviour also affects all the higher-order free-space modes which are generally used in spatial multiplexing. Rather, a more relevant issue lies in the strong sensitivity of the OAM orthogonality to misalignments, which seems to restrict this kind of transmissions to the on-axis line-of-sight (LOS) communications only.

In this letter we provide an in-depth analysis on the limits of OAM radio transmissions by proposing a fair comparison between two multiplexing techniques, one based on OAM modes with index $\ell \neq 0$ (OAM Mux) and the other on the channel higher-order singular modes (CM Mux). In a scenario involving antenna arrays, the Shannon spectral efficiency is computed in the two cases as a function of the propagation distance for both on-axis and off-axis configurations; then, a more realistic example is implemented with the use of some commonly employed modulation schemes and the corresponding $C/N$ reference values under Gaussian channel assumption.

II. CHANNEL MATRIX

Let’s consider a communication link made of two facing antenna arrays. For an incident electromagnetic beam generated by $N_T$ transmitting antennas, the circuit voltage induced on a $p$th receiving element can be expressed by [12]:

$$V_p = \frac{i \kappa \eta}{R} \sum_{n=1}^{N_T} \frac{e^{-i k R_{nt}}}{4 \pi R_{nt}} V_n^T \mathbf{h}_n^T \cdot \mathbf{h}_p^R + w_p, \quad (1)$$

where:

$$\mathbf{E}_p = \frac{i \kappa \eta}{R} \sum_{n=1}^{N_T} \frac{e^{-i k R_{nt}}}{4 \pi R_{nt}} V_n^T \mathbf{h}_n^T$$

represents the electric field evaluated at the spatial location of the $p$th antenna. In the above expressions, $k = 2\pi/\lambda$ is the modulus of the wave vector, being $\lambda$ the wavelength, $\eta$ is the vacuum impedance, $R$ is the resistance of the radiators and $V_n^T = V_0 \mathbf{e}_n^T$ is the voltage supply relative to the $n$th transmitting antenna, being $\mathbf{e}_n^T \in \mathbb{C}^{N_T}$ a unity-normalized vector of input coefficients and $V_0$ a voltage constant term associated to the input power $P_{in}$ through the following expression:

$$P_{in} = \frac{V_0^2}{2 R} \sum_{n=1}^{N_T} |\mathbf{e}_n^T|^2 = \frac{V_0^2}{2 R}. \quad (3)$$

Furthermore, $R_{np}$ is the modulus of the vector connecting the $n$th transmitting element to the $p$th receiving one, $\mathbf{h}_n^R$ and $\mathbf{h}_p^R$.
receiving array in the absence of noise can be defined as:

\[ P_{\text{out}} = \frac{1}{8R} \left( \sum_{p=1}^{N_R} \sum_{n=1}^{N_T} H_{pn} V_{n}^* \right)^2, \]  

where the coefficients \( \xi_p^k \) belong to a unitary-normalized vector \( \xi^k \in \mathbb{C}^{N_R} \) of ideal beamforming weights introduced at the receiver.

To find the transmit and receive vectors \( \xi^T \) and \( \xi^R \) defining the channel modes of a communication system, it is necessary to resort to the singular value decomposition (SVD) of the channel matrix, given by:

\[ H = U \Sigma V^\dagger, \]  

III. OAM MODES

In the radio domain, OAM beams can be generated by means of an array synthesis process. Starting from given requirements on the array structure, this method allows to find the elements excitations that best reproduce a certain electromagnetic field by solving a linear inverse problem. Among all the possible array geometries, the most natural layout for the production of OAM radiation with index \( \ell_T \) is that of a \( N \)-element UCA with \( N > 2|\ell_T| \), which exploits the beams circular symmetry minimizing the number of employed antennas. In this case the input excitations are simply given by:

\[ \xi_n^T = \frac{1}{\sqrt{N}} \exp \left[ i \ell_T 2\pi \left( \frac{n - 1}{N} \right) \right]. \]
so that the phase difference between each couple of subsequent elements results $2\pi \ell_T / N$.

When considering a communication channel made of two facing UCAs, the helical phase structure of the incoming OAM beam must be properly compensated in order to avoid a null on-axis power transfer response. To this end, by exploiting the reciprocity theorem and the intrinsic orthogonality of OAM beams, the receiving weight coefficients:

$$\ell_p^k = \frac{1}{\sqrt{N}} \exp \left[ -i\ell_R 2\pi \left( \frac{p - 1}{N} \right) \right], \quad (9)$$

that must be applied to a $N$-element receiving UCA in order to maximize the on-axis reception, are obtained by imposing $\ell_R = \ell_T$.

Fig. 3 shows the intensity and phase electric field profiles of some OAM modes with different $\ell$ index, generated by the transmitting array of Fig. 1 at $f = 584$ MHz and $R = 1$ m. It should be noted that the choice of considering circularly polarized crossed dipoles ensures the circular symmetry and thus the OAM orthogonality to be fully preserved.

IV. SHANNON CAPACITY CURVES

Making use of the Shannon capacity formula \[15\], the total spectral efficiency associated to a mode-division multiplexed transmission over a band-limited Gaussian channel can be expressed by:

$$\eta = \sum_{i=1}^{Q} \log_2 \left[ 1 + \frac{P_{\text{out}}^{\text{ii}}}{N_0 + \sum_{j=1,j\neq i}^{Q} P_{\text{out}}^{\text{ii}}} \right], \quad (10)$$

where $Q$ indicates the number of transmitted modes, $P_{\text{out}}^{\text{ii}}$ is the properly received power relative to the $i$th subchannel, $N_0$ is the thermal Gaussian noise in the considered bandwidth, while $P_{\text{out}}^{\text{ii}}$ are noise power contributions representing the crosstalk among the different modes. As a result, each power ratio in (10) can be intended as the signal-to-interference-plus-noise ratio (SINR) of the $i$th subchannel.

In order to provide a comparison in terms of spectral efficiency between two different multiplexing schemes, we considered as a common framework the LOS link depicted in Fig. 1 and we set the total input power to a fixed value. Then we evaluated the total spectral efficiency (10) relative to the simultaneous transmission of the second and the third channel modes (CM Mux), on one hand, and of two OAM beams with index $\ell = \pm 1$ (OAM Mux), on the other, as a function of the link distance. The total input power has been equally divided in both cases between the two considered subchannels and the power contributions $P_{\text{out}}$ have been estimated from \[5\] by means of the MATLAB® software \[16\]. Given the $i$th subchannel, $P_{\text{out}}^{\text{ii}}$ in (10) represents the power associated to a mode-matched reception, while the $P_{\text{out}}^{\text{ii}}$ noise contribution provides the intercepted power from the transmitted mode $j \neq i$ (or $\ell_T \neq \ell_R$ in the OAM case), which is negligible only in the absence of misalignments for the modes orthogonality. As we can see from Fig. 4, the two multiplexing methods show the same performance in the on-axis case, i.e., when the arrays are facing each other perfectly. Conversely, as shown in Fig. 5 when a small angular shift in the arrays position is introduced, the CM Mux strongly outperforms the OAM-based scheme, highlighting the greater sensitivity to misalignments of the latter method. It should be noted that in both the reported figures the considered arrays have a radius $R = 1$ m, the emitted radiation is circularly polarized and the frequency is fixed at 584 MHz with a channel bandwidth of 7.61 MHz. Moreover, it is important to emphasize that, even in presence of misalignments, the beamforming coefficients used to determine the $P_{\text{out}}$ contributions in the CM Mux case descend from the SVD of the channel matrix relative to the perfectly aligned arrays, ensuring a fair comparison between the two multiplexing methods.
The performance of the OAM-based multiplexing has been studied on two UCAs composed by crossed dipoles. The performance of the OAM-based multiplexing has been compared with that of a scheme exploiting the first two higher order singular modes of the considered channel. We have proven how the latter method shows a better behaviour in the presence of misalignments, while the two techniques share the same performance in the on-axis configuration. Our results clearly demonstrate that, apart from the simplicity of the generating methods, no benefit can be expected from the use of the OAM-based multiplexing in the considered LOS scenario.

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