MODELLING PRIMARY RECRYSTALLIZATION IN fcc AND bcc METALS BY ORIENTED NUCLEATION AND GROWTH WITH THE STATISTICAL COMPROMISE MODEL

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Statistical models of primary recrystallization are described. Particularly the superposition model and the compromise model must be distinguished. Both models are able to consider oriented nucleation, orientation dependence of driving force, and misorientation dependence of boundary mobility. The superposition model requires abundant nucleation, whereas the compromise model corresponds to sparse nucleation. In order to model the two-step recrystallization observed in low carbon steel the compromise model was generalized for the case of inhomogeneous (two-step) recrystallization. The compromise model was applied to the recrystallization textures of fcc-copper type and brass-type metals as well as to bcc-metals, i.e. low carbon steel. All these textures are well represented by the model on the basis of experimentally established input parameters, i.e. the growth laws $40^\circ(111)$, $27^\circ + 84^\circ(110)$ and $\alpha(111)$ respectively for fcc- and bcc-metals.

KEY WORDS: Primary recrystallization, Nucleation, Sparse, Abundant, Superposition model, Compromise model, fcc-metals, bcc-metals, Misorientation, Mobility, Cube texture, Two-step recrystal-
lization.

INTRODUCTION

Primary recrystallization is defined by nucleation and subsequent growth of new grains into the deformed matrix. For both these processes three characteristic dimensions are important, Fig. 1:

- the size of microbands $d^M \sim 0.1 \mu m$,
- the size of deformation bands $d^D \sim 1 \mu m$,
- the size of recrystallized grains $d^R \sim 10 \mu m$.

(For general surveys see e.g. Margolin, 1966; Schulze, 1966; Cotteril and Mould, 1976; Haessner, 1978; Gottstein, 1984; Humphreys and Hatherly, 1995.)

Microbands may be considered as the basic microstructural units. Their walls have high dislocation density. They carry the main part of deformation energy $P$. It is assumed that the size $d^M$ depends on the orientation $g^D$ of the deformed material.

Deformation bands are groups of microbands having only small orientation differences. They may be considered as the “grains” of the deformed structure, meaning that such a grain has a (nearly) unique orientation $g^D$.

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Recrystallized grains have the orientations $g^R$. According to the definition of primary recrystallization each grain develops out of one nucleus. Hence, there is a one-to-one correspondence between nuclei and recrystallized grains. The size $d^R$ of recrystallized grains thus also defines the location distribution of the nuclei. It also determines the process of impingement of the growing grains in the later stages of recrystallization.

The deformation texture is defined by the volume fraction of the deformed material having the orientation $g^D$

$$\frac{dV(g^D)}{dg^D} = f^D(g^D); \quad g^D = \{\varphi_1, \phi, \varphi_2\}^D,$$

(1)

where $V$ is the total sample volume.

The recrystallization texture is defined by the volume fraction of recrystallized grains having the orientation $g^R$. The recrystallization texture changes with time $t$

$$\frac{dV(g^R)}{dg^R} = x^R(t) \cdot f^R(g^R, t); \quad g^R = \{\varphi_1, \phi, \varphi_2\}^R,$$

(2)

where $x^R(t)$ is the total recrystallized volume fraction which can be approximated by the Johnson, Mehl, Avrami equation

$$x^R(t) = 1 - e^{-b \cdot t^m}.$$

(3)

In a narrower sense only the final texture, after recrystallization is complete, is called the recrystallization texture

$$f^R(g^R) = f^R(g^R, t^R); \quad x^R(t^R) = 1.$$

(4)
This is reached at the time $t = t^R$. After that, there are no more changes of $f^R(g^R)$. In this approximation it is assumed that recrystallization texture formation $f^R(g^R)$ and recrystallization kinetics $x^R(t)$ are two independent factors. This assumption is correct as long as the growing grains do not yet impinge. It does, however, no more hold exactly when impingement starts. A more correct treatment then must include the statistical correlation between nucleation sites $\bar{x}$, nucleation times $t_0$ and the orientations $g^R$. As long as no good information about these quantities is available, it is at least a reasonable assumption to take kinetics and texture as two independent factors as in Eq. (2).

Nucleation is defined as any process which finally gives rise to a recrystallized grain. Hence, strictly speaking it can only be defined by "back-extrapolation" of each recrystallized grain to its starting point. It must be assumed that a recrystallized grain starts its existence as an instability in the deformed microstructure. Hence, a nucleation site $\bar{x}$ can only be defined within a certain volume $dV$ centered at $\bar{x}$, Fig. 2. Below the size $d^M$ of microbands the term "nucleus" has no meaning except that the whole volume $dV$ is the "source area" of a later nucleus. Above this size any microstructural feature will be identified (later on) as a nucleus by the definition that it finally survives and grows into a grain. A nucleus is thus characterized by its location $\bar{x}$, the time $t_0$ at which it reaches the "status" of a nucleus and its orientation $g^R$. The time $t < t_0$ may be called "incubation" time.

The orientation distribution of nuclei appearing at the time $t_0$ is described by

$$\frac{dN(g^R)/N}{dg^R \cdot dt_0} = f^N(g^R, t_0),$$

(5)

Figure 2  Areas in a deformed matrix which may grow, later on, into real nuclei (schematic).
where $N$ is the total number of nuclei, i.e. (per definition) the total number of recrystallized grains after recrystallization is finished. Nucleation sites $\bar{x}$ are assumed to be statistically distributed (this may comprise the statistical preference for certain distinguished sites). The distances $\Delta x$ between the nucleation sites have a certain frequency distribution

$$\frac{dN}{d|\Delta x|} = L(|\Delta x|); \quad \Delta x = \bar{x}_n - \bar{x}_m,$$

which specifies the average distance $\langle |\Delta x| \rangle$ between nuclei. Then the following situations may be distinguished (see Fig. 1):

- $\langle |\Delta x| \rangle < d^M$ (excluded per definition),
- $d^M < \langle |\Delta x| \rangle < d^D$, abundant nucleation,
- $\langle |\Delta x| \rangle \gg d^D$, sparse nucleation.

_Growth_ of a nucleus, to become a recrystallized grain, is driven by the deformation energy $P$, the main part of which is localized in the walls of the microbands, Figs. 1 and 2. This energy pulls at the grain boundary which moves under this pull with a mobility $m(\Delta g, h^D)$ depending on the misorientation $\Delta g$ between growing grain ($g^R$) and deformed grain ($g^D$) as well as on the orientation of the normal direction $h^D$ of the boundary, Fig. 3.

![Figure 3](image-url) _Figure 3_ Direction dependence of mobility and averaged mobility $\bar{m}(\Delta g)$ depending only on the misorientation $\Delta g$ (schematic).
If we take into account that a nucleus must have a closed surface it is evident that for the same misorientation $\Delta g$ between growing grain and matrix many different orientations of the normal direction of the boundary must occur. Hence, often only the average $\overline{m}(\Delta g)$ of the complete mobility function $m(\Delta g, h^D)$ is finally considered, Fig. 3. Then the local growth velocity can be expressed in the form

$$W(\Delta g, g^D) = \overline{m}(\Delta g) \cdot P(g^D),$$

where $P(g^D)$ is the driving force depending on the (local) orientation of the deformed matrix.

MODELS OF RECRYSTALLIZATION TEXTURE FORMATION

Recrystallization and the formation of recrystallization textures may be considered on different size levels. These are particularly:

The atomistic level. On this level the position of each atom must be considered as a function of time.

The dislocation level. On this level the crystal lattice is already "averaged out" into a continuum, but each dislocation is considered individually during the whole recrystallization process.

The microband level. This level is characterized by the complete microstructure described by size, shape, arrangement and orientation of all microbands as well as dislocation densities in the band walls. On this level nucleation and growth are not yet considered as different mechanisms.

The deformation band level. On this level deformation bands as the "deformed grains" are being considered. They are characterized by their size, shape, arrangement, orientation and deformation energy. On this level nucleation must be considered as a separate process. Hence, only from this level on, recrystallization is being considered in the classical concept of nucleation and growth. On this level the deformed microstructure is still taken completely into account and the development of microstructure during the whole recrystallization process is considered.

The statistical level. On this level statistical averages of the microstructural features are being considered. From this level on the results are assumed to be statistically valid for a whole group of statistically equivalent samples whereas on all levels before that, they are, strictly speaking, only valid for one considered sample the starting microstructure of which must be known in all details.

STATISTICAL RECRYSTALLIZATION MODELS

On the statistical level two limiting models may be considered (see e.g. Bunge, 1966).

The superposition model

In this model it is assumed that each nucleus grows only into one deformation band (deformed grain) with the orientation $g^D$, Fig. 1. According to Fig. 1 this situation is encountered for:
Abundant nucleation throughout the whole recrystallization process. The initial stage of sparse nucleation for any recrystallized grain as long as it has not yet left its "native" deformation band.

In this model each texture component of the deformed material with the orientation $g^D$ recrystallizes on its own. (It must be mentioned that "texture component" in this sense means only one crystal orientation and does not include its equivalents according to any possibly existing sample symmetry.) Hence, the recrystallization texture corresponding to the orientation $g^D$ of deformed grains may be expressed in the form

$$f_{g^R}^D(g^R, t) = G(n) \cdot W(\Delta g, g^D)^n \cdot \int_{t_0=0}^t N(t_0) \cdot f^N(g^R, t_0) \cdot (t - t_0)^n \cdot I(x^R(t)) \cdot dt_0,$$  \hspace{1cm} (9)

where $G(n)$ is a geometrical factor depending on the "dimensionality" $n$ of grain growth which may be one-, two-, or three-dimensional. Accordingly it is

$$G(n) = \begin{cases} 1 & \text{for } n = 1, \\ 2\pi & \text{for } n = 2, \\ 4\pi/3 & \text{for } n = 3. \end{cases}$$  \hspace{1cm} (10)

The factor $I(x^R(t))$ describes impingement of grains. It depends on the already recrystallized volume fraction $x^R$, Eq. (2). In a first approximation we may put

$$I(x^R) \approx 1 - x^R.$$  \hspace{1cm} (11)

Thereby it is assumed that impingement is independent of crystal orientation $g^R$ of the growing grain and also independent of the growth rate $W$. The factor describes the fraction of the surface of the growing grain which continues to grow (with the same growth rate as before) after other parts of it have been stopped by impingement. As an approximation we assume here that this happens to fast- and slow-growing grains in the same way. The total recrystallization texture is the "superposition", i.e. the integral over all partial recrystallization textures. With Eq. (8) it can be written

$$f^R(g^R, t) = G(n) \cdot \iint_{g^D} \bar{m}(\Delta g)^n \cdot P(g^D) P(g^D)^n \cdot f^D(g^D) \cdot dg$$

$$\times \int_{t_0=0}^t N(t_0) \cdot f^N(g^R, t_0) \cdot (t - t_0)^n \cdot I(x^R(t)) \cdot dt_0.$$  \hspace{1cm} (12)

For the final recrystallization texture after $t = t^R$ one obtains

$$f^R(g^R) = G \cdot f^N(g^R) \cdot f^{\text{trans}}(g^R),$$  \hspace{1cm} (13)
where

\[
\bar{f}^N(g^R) = \int_{t_0=0}^t N(t_0) \cdot f^N(g^R, t_0) \cdot (t - t_0)^n \cdot I(x^R(t)) \cdot \mathrm{d}t_0
\]

is a nucleation dependent factor and

\[
f^{\text{trans}}(g^R) = \int g^D \bar{m}(\Delta g)^n \cdot \bar{f}^D(g^D) \cdot \mathrm{d}g^D
\]

is the “transformed” deformation texture. Thereby the product

\[
\bar{f}^D(g^D) = P(g^D)^n \cdot f^D(g^D)
\]

may be considered as some “effective” deformation texture (but only in a formal sense). This model has often been considered for the simplified cases

\[
\bar{f}^N(g^R) \equiv 1; \quad P(g^D) \equiv 1,
\]

i.e. only for selective growth and without taking the orientation dependence of deformation energy into account. In this sense Eq. (15) has been called “texture transformation without variant selection” (Bunge et al., 1984). It can be applied for recrystallization as well as for phase transformation.

The compromise model

In this model (Schmidt and Lücke, 1979) it is assumed that a nucleus must grow simultaneously and successively during its lifetime into all crystal orientations of the deformed material, Fig. 1. Its average growth velocity is then obtained by some average of the local growth velocities over the deformation texture. Strictly speaking the times which a grain boundary spends in any orientation \(g^D\) of the deformed material must be averaged, i.e. the inverse growth velocities must be averaged, Fig. 4 (Bunge and Plege, 1986 a,b). Hence, in this model the average growth rate is given by

\[
\frac{1}{\bar{W}(g^R)} = \int g^D \frac{f^D(g^D)}{\bar{m}(\Delta g) \cdot P(g^D)} \cdot \mathrm{d}g^D.
\]

From a formal point of view, and in order to compare the compromise model with the superposition model, the two factors depending on \(g^D\) in Eq. (18) can be combined into one, which may then also be called an “effective” texture

\[
\tilde{f}(g^D) = \frac{f^D(g^D)}{P(g^D)},
\]

which is, however, different from that defined in Eq. (16).
The recrystallization texture can then be expressed in the form

$$ f^R(g^R, t) = G \cdot \overline{W}(g^R)^n \cdot \int_{t_0=0}^{t} N(t_0) \cdot f^N(g^R, t_0) \cdot (t - t_0)^n \cdot I(x^R(t)) \cdot dt_0. $$

(20)

The final recrystallization texture then has the form

$$ f^R(g^R) = G \cdot \overline{W}(g^R)^n \cdot \tilde{f}^N(g^R), $$

(21)

where $\tilde{f}^N(g^R)$ is the "nucleation factor" of Eq. (14).
According to Fig. 1 the compromise model must be assumed to be valid for the case of
- sparse nucleation after the nucleus has outgrown its “native” microband.

With the sizes of \(d^M\), \(d^D\) and \(d^R\) mentioned above this leads to the situation illustrated in Fig. 5. Thereby it is assumed that the local growth rate during growth only into one deformation band will be higher than the compromise growth rate.

A recrystallization operator can be defined formally. It expresses the recrystallization texture as the application of the recrystallization operator to the deformation texture

\[
f^R(g^R) = R(\bar{m}, P, \hat{f}^N) \otimes f^D(g^D).
\]  

(22)

It is then important whether the operator is distributive (linear) or non-distributive (non-linear) (see e.g. Bunge, 1996):

\[
R \otimes [f_A + f_B] \neq R \otimes f_A + R \otimes f_B.
\]  

(23)

Figure 5 Growth rates of nuclei or grains in the different regions (schematic).
For the superposition model the recrystallization operator $R^{\text{sup}}$ is defined by Eq. (12) whereas the one for the compromise model $R^{\text{comp}}$ is defined by Eqs. (18) and (20). One easily sees that:

$$R^{\text{sup}}(\bar{m}, P, \bar{f}^N), \text{ linear},$$
$$R^{\text{comp}}(\bar{m}, P, \bar{f}^N), \text{ non-linear}. \quad (24)$$

Hence, in the compromise model it is difficult to guess the result for realistic deformation textures $f^D$ (as well as for the other parameters $\bar{m}, P, \bar{f}^N$) until precise numerical calculations have been done for any considered set of input parameters.

The inhomogeneous compromise model
This model is a generalization of the homogeneous compromise model (Köhler, 1996). In this case it is assumed that the deformed structure consists of several partial regions having their own partial deformation textures and recrystallizing under their own conditions. This can be expressed by several partial deformation textures as well as several partial recrystallization operators and it leads to as many partial recrystallization textures

$$f^R_i(g^R) = R_i(\bar{m}, P, \bar{f}^N) \otimes f^D_i(g^D). \quad (25)$$

The total textures (deformation and recrystallization) are the weighted sums of the partials

$$f^R(g^R) = \sum_{i=1}^{\nu_i} f^R_i(g^R), \quad f^D(g^D) = \sum_{i=1}^{\nu_i} f^D_i(g^D), \quad \sum \nu_i = 1. \quad (26)$$

Formation of Quasi-Equilibrium Structure
Nucleation and subsequent growth of the formed nuclei, terminated by impingement, leads to grain shapes described by Voronoi cells. These cells are confined by grain boundaries which are not in mechanical equilibrium in the grain edges and corners. Hence, immediately after impingement some grain boundary movement must be assumed in the direction of minimizing the total grain boundary energy. The driving force for that process pertains throughout continuous grain growth or secondary recrystallization which may (or may not) follow primary recrystallization. Also the mobility of the individual segment of the grain boundaries will be approximately the same throughout continuous grain growth. The total changes of the grain structure will, however, slow down considerably after some near mechanical equilibrium has been reached by the grain boundaries. This means that the immediate mechanical adjustment of Voronoi cells into near-equilibrium grain shapes will proceed much faster than subsequent continuous grain coarsening (but it obeys similar principles). The models of primary recrystallization described above lead to the original Voronoi cells. The adjustment from these to near-equilibrium grains requires grain boundaries to move through some volume fraction of the material in the order of $\sim 10\%$. Thereby
one crystal orientation is replaced by that of the neighbouring grain. Hence, a texture change in the same order of magnitude may be assumed. The above mentioned recrystallization texture models are thus uncertain in this order of magnitude assuming that we usually measure the recrystallization texture after mechanical adjustment. This is illustrated schematically in Fig. 6.

**Superposition Model versus Compromise Model**

Considering the typical dimensions shown in Fig. 5, it must be concluded that in most cases recrystallized grains are much bigger than the deformation bands. Hence, in the greatest part of their lifes they encounter the situation of the compromise model. When they outgrow their “native” deformation band they have, as an average, only reached 1/1000 of their final volume. It may then be meaningful to redefine “nucleation” in a slightly modified sense and call a “nucleus” (for the compromise model) a small grain which has just left its “native” deformation band. Some “nuclei” in this generalized sense are to be seen in Fig. 1 in the upper right. In this sense it may be assumed that at least in deformed polycrystalline materials the compromise model should be the appropriate one (Bunge, 1966).

The growth rates according to both models are compared (schematically) in Fig. 7 (see e.g. Bunge, 1966). It is seen that the compromise model gives highest growth rates in the overlapping regions and not in the maxima of the individual (local) growth rates. This makes it difficult to estimate qualitatively the actual average growth rate unless it has been calculated quantitatively.

![Figure 6](image-url)  
*Figure 6* Texture development in the ranges of primary recrystallization, mechanical adjustment and continuous grain coarsening (schematic).
NUCLEATION VERSUS GROWTH RATE

In both statistical models described above the recrystallization texture $f^R(g)$ is expressed as the product of two factors, i.e. a nucleation factor $f^N(g)$ and a growth rate factor $f^G(g)$ (i.e. the growth rate of the volume)

$$f^R(g) = f^N(g) \cdot f^G(g).$$  (27)
Several situations may be distinguished which are illustrated in Fig. 8:

- Nucleation is orientation-independent:
  \[ f^N(g) \equiv 1. \quad (28) \]
  Then the recrystallization texture is given by the average growth rate
  \[ f^R(g) = f^G(g), \quad \text{oriented growth.} \quad (29) \]

- Growth rate is orientation-independent:
  \[ f^G \equiv 1. \quad (30) \]

\[ \begin{array}{ccc}
\text{Oriented} & \text{Oriented} & \text{Mixed} \\
\text{Growth} & \text{Nucleation} & \text{Mode} \\
\end{array} \]

\[ \begin{array}{ccc}
f^N & f^G & f^R \\
\text{gi gi gi} & \text{gi gi gi} & \text{gi gi gi} \\
\end{array} \]

Figure 8  Contribution of oriented nucleation \( f^N \) and oriented growth \( f^G \) to the final recrystallization texture \( f^R \) (defined by volume) (schematic).
Then the recrystallization texture is given by the orientation distribution function of the nuclei

\[ f^R(g) = f^N(g), \quad \text{oriented nucleation.} \]  

(31)

- Both factors contribute to the development of a particular texture component \( g_i \).
- This latter situation must be assumed to be realistic since experimental investigations of the same recrystallization texture often have given evidence for oriented nucleation as well as for oriented growth. The most prominent example is the cube texture.
- This same situation must also be expected if the nucleation process itself is physically based on a growth process (in small dimensions) which has the same orientation dependence as the following growth process (in large dimensions).

Also the opposite situation must be considered, i.e. that one of the two factors is zero.

**Figure 9** The growth of grains is suppressed if one of the two factors, nucleation \( f^N \) or growth \( f^G \), is zero (schematic).
This is illustrated in Fig. 9:

- A fast growing orientation does not lead to a texture component because there are no nuclei in this orientation.
- Nuclei are being formed but they cannot grow. It is, however, difficult to specify clearly what this situation really means since we have defined earlier a "nucleus" by the fact that it finally grows into a grain. Hence, this situation can only be accepted if we have a definition of the term "nucleus" independent of its final ability to grow.

There is no reason to assume that recrystallization texture formation in a particular sample must necessarily follow only one of the cases described in Figs. 8 and 9. Hence, the complete situation may be as illustrated in Fig. 10. It is:

\[ f^N \]

\[ f^G \]

\[ f^R \]

\( g_1 \) \( g_2 \) \( g_3 \) \( g_4 \) \( g_5 \)

**Figure 10** A real recrystallization texture may contain components formed according to nucleation, to growth or to both (schematic).
• $g_1 =$ nucleation component,
• $g_5 =$ growth component,
• $g_3 =$ mixed-mode component,
• $g_2 =$ suppressed by lack of nuclei,
• $g_4 =$ suppressed by zero growth rate.

If only the recrystallization texture $f^R(g^R)$, defined in Eq. (4), is considered then the different situations of Figs. 8 and 9 cannot be distinguished. We may, however, consider additionally the "texture by number"

$$\frac{dN(g)}{N} = f^N(g).$$

(32)

If we keep the definition that a nucleus is defined by the fact that it actually forms a recrystallized grain then $f^N(g)$ of Eq. (32) is identical with $f^N(g)$ of Eq. (27).

The texture by number can be measured by a "spot-counting" technique (Bunge, 1967; Bleck and Bunge, 1981), particularly by using automated area detector methods

![Graph](image)

**Figure 11** The texture by volume and the texture by number in recrystallized Al-wires corresponding to the situation of oriented nucleation.
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(Bunge and Klein, 1996). As an example, Fig. 11 shows the recrystallization texture of an Al-wire measured by volume and by number respectively. It is seen that in this case both textures are identical, i.e. in this case oriented nucleation must be assumed.

Textures of both types, i.e. by number and by volume, are simultaneously available by orientation imaging techniques in the electron microscope (Adams et al., 1993; Schwarzer et al., 1996).

In order to estimate the influence of nucleation and growth on the final recrystallization texture Eq. (27) in more detail the following parameters must be considered, Fig. 12:

- The anisotropy ratio

\[ A = \frac{f_{\text{max}}(g)}{f_{\text{min}}(g)} \]  \hspace{1cm} (33)

of both factors in Eq. (27).

- The widths \( b \) of both functions.

- The degree \( n \) with which nucleation and (linear) growth rate enter the expression of the recrystallization texture, i.e. linear or by the third power respectively.

- The relative accuracy of texture measurement

\[ \frac{f_{\text{min}}(g)}{f_{\text{max}}(g)} \geq 0.02 \sim 2\% \]  \hspace{1cm} (34)

according to the present state of the experimental techniques.

Figure 12  Definition of anisotropy ratio and width of both texture forming factors \( f^N \) and \( f^G \) (schematic).
The values \( f_{\text{max}} \) of both functions in Eq. (27), i.e. the nucleation frequency and the growth rate have been measured in some cases. Also they may be estimated with appropriate theoretical models.

The values \( f_{\text{min}} \) of both functions are, however, difficult to estimate:

- Experimental investigations of these values are probably not better than \( \sim 10\% \) of the maximum values \( f_{\text{max}} \).
- Theoretical models usually are only concerned with \( f_{\text{max}} \) and not with \( f_{\text{min}} \). Hence, they assume explicitly (or tacitly) \( f_{\text{min}} = 0 \), which may, however, not be realistic.

Hence, assuming anisotropy ratios of 1:10 for the density of nuclei and the linear growth rate the latter one leads to an anisotropy ratio 1:10^3 of the volume fraction in the recrystallization texture. If this is multiplied or divided by the factor 10 assumed for the nuclei distribution then the differences stay below the experimental accuracy of texture measurement as is illustrated in Fig. 13.

![Diagram](https://via.placeholder.com/500)

**Figure 13** The influence of linear growth rate and nucleation factor varying by 1:10 on the recrystallization texture. Also the 2% limit of experimental accuracy is shown (schematic).
Hence, it can be concluded that an existing anisotropy of growth rate will probably dominate recrystallization texture formation.

Only if the anisotropy of the average growth rate is small (due to the particular features of the deformation texture and the local growth rate) then an existing anisotropy of nuclei distribution may really dominate the recrystallization texture.

APPLICATIONS OF THE COMPROMISE MODEL

The essential part of the compromise model is Eq. (18), and in it the mobility $\bar{m}(\Delta g)$ is the critical factor. It is known from single crystal experiments that certain misorientations $\Delta g_i$ have higher mobilities than others. The complete functions $\bar{m}(\Delta g)$ in the whole misorientation space have, however, never been measured. Hence, model assumptions must be made for $\bar{m}(\Delta g)$. We assume that a mobility function has maximum values at some (experimentally found) misorientations $\Delta g_i$ and about them a Gaussian spread with the spread widths $\omega_i$. From mathematical reasons it must be $\bar{m} \neq 0$. Hence, we put

$$\bar{m}(\Delta g) = m_0 + \sum_i \rho_i \cdot e^{-\left(\omega_\ell/\omega_i\right)^2}, \quad (35)$$

where

$$\omega = |\Delta g \cdot \Delta g_i^{-1}| \quad (36)$$

is the orientation distance from the preferred growth misorientation $\Delta g_i$, and

$$\Delta g = g^R \cdot \left(g^D\right)^{-1} \quad (37)$$

is the misorientation between growing grain and deformed matrix. The functions $f^D(g^D), f^R(g^R), f^N(g^R)$ are treated in terms of series expansions (Bunge, 1982)

$$f(g) = \sum_{\lambda=0}^L \sum_{\mu=1}^{M(\lambda)} \sum_{\nu=1}^{N(\lambda)} C^\mu_{\lambda} \tilde{T}^{\mu\nu}_\lambda (g). \quad (38)$$

These functions must obey crystal symmetry (e.g. cubic) and sample symmetry (e.g. orthorhombic sheet symmetry). The function $\bar{m}(\Delta g)$ is also expressed in terms of a series expansion

$$\bar{m}(\Delta g) = \sum_{\lambda=0}^L \sum_{\mu=1}^{M(\lambda)} \sum_{\sigma=1}^{M(\lambda)} C^\sigma_{\lambda} \cdot \tilde{T}^{\sigma\mu}_\lambda (\Delta g), \quad (39)$$

which has double cubic symmetry.
Linear operations with orientation functions particularly the execution of the integral Eq. (15) are easily expressed in terms of series coefficients (Bunge, 1982).

Non-linear operations such as calculation of inverse functions Eq. (18), the $n$th power, as well as products of functions are more easily executed numerically going through the Euler space in appropriate steps (Köhler et al., 1992).

The orientation sensitivity of the model is mainly determined by the “sharpness” of the function $\tilde{m}(\Delta g)$ expressed in the values $\omega_i$ which are in the order of $\sim 5^\circ$ (FWHM $\sim 10^\circ$). Hence, it was necessary to carry out the operations in the Euler space with steps of $\sim 5^\circ$ or smaller. The series expansion degree had to be at least $L = 34$. Also, it turned out that the value $m_0$ influenced the sharpness of the obtained recrystallization textures (Köhler and Bunge, 1996). For qualitative results $m_0 = 0.1$ was sufficient. Better quantitative results required lower values e.g. $m_0 \sim 0.01$ (which in turn requires higher accuracy of the numerical calculations).

Recrystallization Textures in fcc-Metals

For fcc-metals the Liebmann et al. (1956) growth rate is assumed to be applicable in a wide range. It is

$$\Delta g \sim 40^\circ (111); \quad \omega \sim 12^\circ.$$  \hspace{1cm} (40)

This growth rate is being assumed (with only slight variations) for high and low stacking fault energy metals.

Copper-Type Textures

The rolling texture of the copper type is characterized by an orientation tube extended between the copper and brass orientation and passing through the S-orientation. It may thus be rationalized by the components

$$\text{Copper} = \{112\} (111), v_C, \omega_C,$$

$$\text{Brass} = \{110\} (112), v_B, \omega_B,$$

$$S = \{123\} (634), v_S, \omega_S.$$  \hspace{1cm} (41)

Hence, this texture can be “parametrized” by the volume fractions $v_i$ and spread widths $\omega_i$ of these three components. With the assumptions

$$\tilde{f}^N(g^R) \equiv 1; \quad P(g^D) \equiv 1$$  \hspace{1cm} (42)

the model produces a very strongly preferred growth rate in the cube orientation, Fig. 14 (Bunge and Köhler et al., 1992).

Also the modification of the parameters $v_i, \omega_i$ (Eq. (41)) according to modified rolling conditions leads to the experimentally observed shifts of the cube texture in the sense of rotations about the normal and rolling direction respectively, Fig. 15.
Figure 14  The cube recrystallization texture obtained with the 40° (111) growth law from the copper deformation texture. Modelling parameters are: $v_S = 0.50$, $v_C = 0.25$, $v_B = 0.25$, $\omega_S = \omega_C = \omega_B = 10^\circ$, $\omega = 12.5^\circ$, $n_0 = 0.01$.

Figure 15  Shift of the cube orientation by varying the volume fractions of the copper, brass, and S-component in the copper deformation texture. Modelling parameters are: (a) $v_S = 0.25$, $v_C = 0.50$, $v_B = 0.25$ (nearly cube), (b) $v_S = 0.25$, $v_C = 0.25$, $v_B = 0.50$ (rotated about $ND$), (c) $v_S = 0.33$, $v_C = 0.33$, $v_B = 0.33$ (rotated about $RD$), $\Delta g$, $\omega$ and $n_0$ as in Fig. 6.
The cube orientation turned out to be a very strongly preferred growth orientation becoming sharper and sharper with decreasing spread widths of the deformation texture, Fig. 16. Also this is in good agreement with experimental results.

**Brass-Type Textures**

The rolling texture of brass is characterized by the brass orientation as the major one and a smaller Goss component

\[
\text{Brass} = \{111\} \langle 112 \rangle, v_B, \omega_B, \\
\text{Goss} = \{011\} \langle 110 \rangle, v_G, \omega_G. 
\]  

(43)
With the same assumptions for $\tilde{f}^N$ and $P$, Eq. (41), the model produces high growth rates in the brass-recrystallization orientation $\{236\}{(385)}$ (Köhler et al., 1992). This compares very well with the experimental brass recrystallization texture, Fig. 17. Other "compromise" growth orientations, i.e. $\{130\}{(001)}$ and $\{112\}{(110)}$, assumed according to qualitative estimations of the model (Schmidt and Lücke, 1979), turn out to be much worse compromises, i.e. they have much lower compromise mobilities. Hence, the quantitative model is in better agreement with the experimental results than it seemed only by qualitative considerations.

Recrystallization Textures in bcc-Metals

For bcc ARMCO-iron the recrystallization process was found to be inhomogeneous (Kern, 1981; Kern et al., 1986; Plutka and Hougardy, 1991; Plutka, 1991). The deformed microstructure shows at least two types of regions which recrystallize in a different way. These regions are distinguished by their respective textures and by the deformation energy. By metallographic inspection they have respectively "rough" and "smooth" appearance:

- Rough-regions, $\gamma$-fibre, $\{111\}{(hkl)}$, $P(g^D) = \text{high}$,
- Smooth-regions, $\alpha$-fibre, $\{hkl\}{(110)}$, $P(g^D) = \text{low}$.

Hence, the inhomogeneous model Eqs. (25) and (26) had to be applied. Within the two regions, $P(g^D)$ was not further distinguished.

In the rough regions classical nucleation was observed. According to hypotheses in the literature the orientation distribution of the nuclei was assumed between random and with orientations of the deformed matrix. Hence, the nucleation distribution was written in the form

$$\tilde{f}^N(g_R)^{\text{rough}} = \nu + (1 - \nu) \cdot f^D(g_R).$$

These nuclei grow in three dimensions within the rough regions. Hence, in this part we assume $n = 3$.

![Figure 17](image-url)
In the smooth regions no classical nucleation was found. Rather the recrystallized grains of the rough regions serve as nuclei. They grow one dimensionally (from the boundaries) into the smooth regions. Hence, we put

\[ \tilde{f}^N(g^R)_{\text{smooth}} = f^R(g^R)_{\text{rough}}, \quad n = 1. \quad (46) \]

This is illustrated in Fig. 18.

The growth law for bcc-metals according to Eq. (35) was measured by Ibe and Lücke (1966, 1968):

\[ \Delta g_1 = 27^\circ(110), \quad \rho_1 = 0.9, \quad \omega_1 = 12^\circ, \]
\[ \Delta g_2 = 84^\circ(110), \quad \rho_2 = 0.1, \quad \omega_2 = 6^\circ. \quad (47) \]

But also other growth laws may be assumed. These are particularly \( \langle 111 \rangle \) rotations

\[ \Delta g = \alpha\langle 111 \rangle, \quad 20^\circ \leq \alpha \leq 60^\circ. \quad (48) \]

![Figure 18](image.png)

**Figure 18** The two-step model of recrystallization (schematic): (a) rough and smooth areas, (b) nucleation in the rough areas, (c) the rough areas are completely recrystallized, (d) growth of the grains from the rough areas into the smooth ones.
either with single values $\alpha$ or with a whole range of $\alpha$ (i.e. the CAD-model, Pumphrey, 1972). In Fig. 19 the experimental rolling and recrystallization textures of a low carbon steel are shown.

In Fig. 20 the deformation texture is split into the two parts. The splitting allows some degree of arbitrariness. In the work by Kern (1981) the splitting was done

![Figure 19](image1.png)

*Figure 19* Experimental rolling and recrystallization texture of a low carbon steel.

![Deformation Texture](image2.png)

*Figure 20* Splitting of the deformation texture into the textures of the rough and smooth areas, $v_1 = v_2 = 0.5$. 

| $\varphi_1 = 0^\circ$ | $\varphi_1 = 45^\circ$ | $\varphi_1 = 90^\circ$ | $\varphi_2 = 45^\circ$ |
|-----------------------|------------------------|------------------------|------------------------|
| Rolling               | Recrystallization       |                         |                        |
according to microscopical observations. It is assumed that the different behavior results from different stored energy in the two different regions. Hence, quantitative modelling of the splitting can be based on the Taylor factor (Köhler, 1996). Figure 20 corresponds to a most plausible choice obtained in this way with equal volume fractions $v_1 = v_2 = 0.5$ of rough and smooth regions.

The mobility function $\bar{m}(\Delta g)$ had to be varied in a wide range according, on the one hand, to experimentally not very well established values and, on the other hand, to the mentioned high orientation sensitivity of the model particular to this function. Figure 21 shows two particular mobility functions which turned out to give results nearest to the experimental ones. Finally, Fig. 22 compares the experimental recrystallization texture with the best fitting model calculations. It is seen that the agreement between experimental results and the modelled texture is reasonably good.

**Figure 21** Different mobility functions $\bar{m}(\Delta g)$ showing also the asymmetric unit in the misorientation space: (a) $27^\circ + 84^\circ (110)$, (b) $\alpha (111)$. 
CONCLUSIONS

It has been shown that for many cases the statistical compromise model of recrystallization texture formation is a reasonable approximation. On the one hand, it takes the most important and experimentally established features of primary recrystallization into account, i.e.

- orientation distribution of nuclei,
- sparse nucleation,
- misorientation dependence of boundary mobility,
- orientation dependence of driving force.

On the other hand, the model can still be implemented by a computer code and can be thoroughly investigated by numerical calculations based on experimental data. Where the experimental data are insufficient they can be "reasonably" interpolated and the results can be compared with the experiments. This allows to find "reasonable" ranges for the unknown input parameters. Based on the available experimental data the model can explain the cube recrystallization texture of high stacking fault fcc-metals as well as the major variants of it as a function of varying

Figure 22  Experimental recrystallization texture of low carbon steel compared with three different model calculations.
deformation texture. It also explains the brass recrystallization texture with the same principles following from the brass type deformation textures. These two very prominent types of recrystallization textures can be understood on the basis of 40° (111) preferred growth alone. Furthermore, it turns out that the growth rate factor in these cases has a very strong maximum which remains stable even under modifications introduced by the nucleation factor and orientation dependence of the driving force. This is plausible since the average growth rate $\overline{W}$ enters Eq. (20) in the third power whereas the orientation dependence of the nucleation factor comes in only in the first power.

Based on the experimental data the two-stage model had to be introduced (following, however, the same principles as above for each of the two regions in the deformation texture) in order to understand recrystallization texture formation in bcc-metals (in fact low-carbon steel). In this case it turned out that non-random orientation distribution of nuclei had to be taken into account, in accordance with experimentally corroborated assumptions given in the literature. Also for this material the experimentally observed growth law $27° + 84° (110)$ leads to correct results. Because of the two-component nature of this growth law, compared with the one-component 40° (111) law for the fcc-metals, the growth compromise is much less pronounced here. Hence, it can be subsequently influenced by the orientation dependence of nucleation as well as driving force. It turned out that an equally good agreement between experimental and modelled recrystallization texture can be obtained with the completely different $\alpha (111)$ growth law. In fact the results which the model gives with these two growth laws are so similar that, on the basis of the presently available experimental data, the model cannot discriminate these two growth laws.

Summarizing the results of all numerical calculations carried out thus far, it can be concluded that the statistical compromise model is able to explain the most prominent texture types in cubic metals. It is capable of being further specified with more detailed experimental data becoming available.

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