Natural Supergravity Inflation

Jennifer A. Adams

Department of Theoretical Physics, Uppsala University,
Box 803, S-75108 Uppsala, Sweden

Graham G. Ross & Subir Sarkar *

Theoretical Physics, University of Oxford,
1 Keble Road, Oxford OX1 3NP, U.K.

(submitted August 15, revised October 28, 1996; to appear in Phys. Lett. B)

Abstract

We identify a new mechanism in supergravity theories which leads to successful inflation without any need for fine tuning. The simplest model yields a spectrum of density fluctuations tilted away from scale-invariance and negligible gravitational waves. We demonstrate that this is consistent with the observed large-scale structure for a cold dark matter dominated, critical density universe. The model can be tested through measurements of microwave background anisotropy on small angular scales.

98.80.Cq, 04.65.+e, 98.65.Dx, 98.70.Vc

*PPARC Advanced Fellow

hep-ph/9608336
I. INTRODUCTION

Field theoretic models of cosmological inflation are generically required to provide a very flat potential for the ‘inflaton’ field, the large and approximately constant vacuum energy of which drives an exponential increase of the scale-factor and is then converted to radiation when the inflaton settles into its global minimum [1]. In building such a model care must be taken to avoid the hierarchy problem which arises because the fundamental interactions, in particular gravity, will communicate any such high scale of new physics to all sectors of the theory, driving the electroweak breaking scale far above its observed value. The only known way to avoid this problem is through the introduction of supersymmetry which can protect the low energy scales from such radiative corrections to all orders [2]. The local version of supersymmetry — supergravity — incorporates gravity and has therefore been extensively studied in attempts to construct a unified description of all the fundamental interactions, the most ambitious of which is the superstring.

However supergravity inflationary models [3] suffer from their own ‘hierarchy’ problem. The large cosmological constant during inflation breaks supersymmetry, giving all scalar fields a soft mass of order the Hubble parameter [4]. The resulting curvature of the inflaton potential is typically too large to allow a sufficiently long period of inflation to occur [3]. There have been many suggested solutions to this problem but most of them are deemed to be ad-hoc or unworkable [4] (although inflation driven by a D-term [7,8] may be viable). In continuation of this discussion and our previous work [9], we wish to propose a new mechanism leading to successful inflation in the low-energy effective supergravity theory following from the superstring. We demonstrate that in a wide class of such theories, the equations of motion have an infra-red fixed point at which successful inflation can occur, even for minimal kinetic terms, along a F-flat direction. Moreover the resulting inflationary potential has a very specific structure, allowing precise predictions for the generated perturbations — both scalar and tensor. We compute these in detail for a cold-dark matter dominated critical density universe, including non-linear evolutionary effects, and compare with the results of the APM galaxy survey [10]. We find reasonable agreement with the data without having to invoke a component of hot dark matter. The expected power spectrum of the angular anisotropy in the cosmic microwave background (CMB) is also calculated using a Boltzmann code and normalized to the COBE observations [11] on large angular scales. Ongoing and future observations on small angular scales [12] will thus provide a definitive test of the model.

II. REQUIREMENTS OF THE INFLATIONARY POTENTIAL

We begin by briefly reviewing the necessary ingredients for successful inflation with a scalar potential \( V(\phi) \). Essentially all model generating an exponential increase of the cosmological scale-factor \( a \) satisfy the ‘slow-roll’ conditions [13]

\[
\dot{\phi} \simeq -\frac{V'}{3H}, \quad \epsilon \equiv \frac{M^2}{2} \left( \frac{V'}{V} \right)^2 \lesssim 1, \quad |\eta| \equiv \left| M^2 \frac{V''}{V} \right| \lesssim 1, \quad (1)
\]

where \( H \simeq \sqrt{V/3M^2} \) is the Hubble parameter during inflation, and the normalized Planck mass \( M \equiv M_{Pl}/\sqrt{8\pi} \simeq 2.44 \times 10^{18} \) GeV. Inflation ends (i.e. \( \ddot{a} \) drops through zero) when \( \epsilon, |\eta| \simeq 1 \). The spectrum of adiabatic scalar perturbations is [13]

\[
\delta^2_{\text{HI}}(k) = \frac{1}{150\pi^2} \frac{V}{M^4} \frac{1}{\epsilon_*}, \quad (2)
\]
where $\ast$ denotes the epoch at which a scale of wavenumber $k$ crosses the ‘horizon’ $H^{-1}$ (more correctly, Hubble radius) during inflation, i.e. when $aH = k$. The CMB anisotropy measured by COBE [11] allows a determination of the fluctuation amplitude at the scale, $k_{\text{COBE}}^{-1} \sim H_0^{-1} \simeq 3000 h^{-1}$ Mpc, corresponding roughly to the size of the presently observable universe, where $h \equiv H_0/100 \, \text{km} \, \text{sec}^{-1} \, \text{Mpc}^{-1}$ is the present Hubble parameter. The number of e-folds before the end of inflation when this scale crosses the Hubble radius is

$$N_{\text{COBE}} \equiv N_\ast (k_{\text{COBE}}) \simeq 51 + \ln \left( \frac{k_{\text{COBE}}^{-1}}{3000 h^{-1} \, \text{Mpc}} \right) + \ln \left( \frac{V_\ast}{3 \times 10^{14} \, \text{GeV}} \right) + \ln \left( \frac{V_\ast}{V_{\text{end}}} \right)$$

$$- \frac{1}{3} \ln \left( \frac{V_{\text{end}}}{3 \times 10^{14} \, \text{GeV}} \right) + \frac{1}{3} \ln \left( \frac{T_{\text{reheat}}}{10^5 \, \text{GeV}} \right),$$

(3)

where we have indicated the numerical values anticipated for the various energy scales in our model. (Note that $N_{\text{COBE}}$ is smaller than the usually quoted [13] value of 62 because the reheat temperature must be low enough to suppress the production of unstable gravitinos which can disrupt primordial nucleosynthesis [2].) The COBE observations sample CMB multipoles upto $l \sim 20$, where the $l^{\text{th}}$ multipole probes scales around $k^{-1} \sim 6000 h^{-1}$ Mpc/l. The low multipoles, in particular the quadrupole, are entirely due to the Sachs-Wolfe effect on super-horizon scales ($k^{-1} > k_{\text{eq}}^{-1} \simeq 1800 h^{-1}$ Mpc) at CMB decoupling and thus a direct measure of the primordial perturbations. However the high multipoles are (increasingly) sensitive to the composition of the dark matter which determines how the primordial spectrum is modified through the growth of the perturbations on scales smaller than the horizon at the epoch of matter-radiation equality, i.e. for $k^{-1} < k_{\text{eq}}^{-1} \simeq 80 h^{-1}$ Mpc. Thus the normalisation of the spectrum (2) to the COBE data is sensitive to its $k$ dependence and also on whether there is a contribution from gravitational waves to the CMB anisotropy. The 4-year COBE data is fitted by a scale-free spectrum, $\delta_H^2 \sim k^{n-1}$, $n = 1.2 \pm 0.3$, with $Q_{\text{rms}} = 15.3^{+3.8}_{-2.2} \mu \text{K}$ [11]. For a scale-invariant ($n = 1$) spectrum, $Q_{\text{rms}} = 18 \pm 1.6 \mu \text{K}$, so assuming that there are no gravitational waves, the amplitude for a $\Omega = 1$ CDM universe is $\delta_H = 1.94 \pm 0.14 \times 10^{-5}$ [11]. Using eq.(2), the vacuum energy at this epoch is then given by

$$V_{\text{COBE}} \simeq (6.7 \times 10^{16} \, \text{GeV})^4 \epsilon_{\text{COBE}},$$

(4)

showing that the inflationary scale is far below the Planck scale [13]. A similar limit obtains, viz. $V_{\text{COBE}} \lesssim (4.9 \times 10^{16} \, \text{GeV})^4$, if the observed anisotropy is instead ascribed entirely to gravitational waves, the amplitude of which, in ratio to the scalar perturbations, is just [13]

$$r = 12.4 \, \epsilon.$$

(5)

Thus it is legitimate to study inflation in the context of an effective field theory. We will consider the class of models in which the evolution of the inflaton potential is dominated by a single power at the point where the observed density fluctuations are produced, as is the case in all of the supergravity models so far considered. The potential then has the form

$$V \sim \Lambda^4 \left[ 1 + c_n \left( \frac{\phi}{M} \right)^n \right].$$

(6)

In the usual model of chaotic inflation [16], one has $\phi/M \gg 1$ so the first term on the rhs is negligible and $\epsilon$ and $\eta$ are small because they are proportional to $(\phi/M)^{-2}$. The alternative possibility [17] is for $\phi$ to have an initial vacuum expectation value (vev) much smaller than the Planck scale during
inflation, in which case the smallness of $V'$ and $V''$, and hence $\epsilon$ and $\eta$, results from the relative smallness of the second term on the rhs.

It is convenient to introduce a general formalism capable of describing both cases by expanding the (slowly varying) potential about the value $\phi^*$ in inflaton field space at which the observed density fluctuations are produced. Writing $\phi = \tilde{\phi} + \phi^*$ (in units of $M$) we have

$$V(\phi) = \Lambda^4 \left[ 1 + c_1 \tilde{\phi} + c_2 \tilde{\phi}^2 + c_3 \tilde{\phi}^3 + c_4 \tilde{\phi}^4 + \ldots \right].$$  \hspace{1cm} (7)

Here we have factored out the overall scale of inflation $\Lambda$, which we have seen must be small relative to the Planck scale $M$. The constraints on the parameters in the potential following from the slow-roll conditions (1) are therefore

$$c_1 \ll 1, \quad c_2 \ll 1, \quad c_3 \tilde{\phi} \ll 1, \quad c_4 \tilde{\phi}^2 \ll 1, \ldots$$  \hspace{1cm} (8)

We now examine whether these conditions can be naturally satisfied in supergravity theories.

### III. NATURAL SUPERGRAVITY INFLATION

In supersymmetric theories with a single supersymmetry generator ($N = 1$), complex scalar fields are the lowest components, $\phi^a$, of chiral superfields $\Phi^a$ which contain chiral fermions, $\psi^a$, as their other component. (We will take $\Phi^a$ to be left-handed chiral superfields so that $\psi^a$ are left-handed fermions.) Masses for fields will be generated by spontaneous symmetry breaking so that the only fundamental mass scale is the Planck scale, $M$. (This is aesthetically attractive and also what follows if the underlying theory generating the effective low-energy supergravity theory follows from the superstring.) The $N = 1$ supergravity theory describing the interaction of the chiral superfields is specified by the Kähler potential

$$G(\Phi, \Phi^\dagger) = d(\Phi, \Phi^\dagger) + \ln |f(\Phi)|^2,$$  \hspace{1cm} (9)

which yields the scalar potential

$$V = e^{d/M^2} \left[ F_A^\dagger (d_B)^{-1} F_B - 3 |f|^2 / M^2 \right] + D - \text{terms},$$  \hspace{1cm} (10)

where

$$F_A = \frac{\partial f}{\partial \Phi^A} + \left( \frac{\partial d}{\partial \Phi^A} \right) \frac{f}{M^2}, \quad (d_B)^{-1} \equiv \left( \frac{\partial^2 d}{\partial \Phi^A \partial \Phi_B} \right)^{-1}.$$  \hspace{1cm} (11)

Here the function $d$ sets the form of the kinetic energy terms of the theory

$$L_{\text{kin}} = \frac{\partial^2 d}{\partial \phi_{A} \partial \phi_{\mu B}} \partial_{\mu} \phi_{A} \partial^{\mu} \phi_{\mu B},$$  \hspace{1cm} (12)

while the superpotential $f$ determines the non-gauge interactions of the theory. For canonical kinetic energy terms, $d = \sum_A \phi_A^\dagger \phi^A$, the potential takes the relatively simple form

$$V = \exp \left( \sum_A \phi_A^\dagger \phi^A \left[ \sum_B \left| \frac{\partial f}{\partial \phi_B} \right|^2 - 3 |f|^2 \right] \right).$$  \hspace{1cm} (13)
In order for there to be a period of inflation, it is necessary for at least one of the terms \( \frac{\partial f}{\partial \phi} \) to be non-zero. However, these are precisely the order parameters for supersymmetry so this corresponds to supersymmetry breaking during inflation. While there are several possible mechanisms for such breaking, it suffices for the purposes of this discussion to simply assume that one of the terms has nonvanishing value \( \Lambda^4 \), where \( \Lambda \) denotes the supersymmetry breaking scale. Now expansion of the exponential in eq. (13) shows that \( c_2 = 1 \) and \( c_4 = 1 \) in eq. (10), in conflict with the requirements for successful inflation (8). It is seen that the problem arises due to the presence of the overall factor involving the exponential in the potential (13). The same structure typically occurs even for more general kinetic terms (see eq. 10) and it is this that has led to the conclusion that inflation is difficult to achieve within the context of supergravity.

In ref. [9] we suggested that in theories with moduli the problem is easily avoided. Moduli are fields in superstring theories which, in the absence of supersymmetry breaking, have no potential. The moduli vevs serve to determine the fundamental couplings of the theory and for the moduli of interest here they appear in the superpotential only in combination with non-moduli fields, serving to determine the latter’s couplings in terms of their vevs. We argued earlier that the quadratic terms in the potential involving the non-moduli fields such as the inflaton would be absent for special values of these vevs and, since the resultant potential would drive inflation, just this desired configuration would come to dominate the final state of the universe [9]. In this paper we demonstrate that it is not even necessary to invoke such an ‘anthropic principle’ because there is a quasi-fixed point in the evolution of the moduli. This ensures, for initial values in the basin of attraction of the fixed point, that the cancellation of the quadratic terms applies, ensuring that condition (8) is satisfied.

Although the moduli have a flat potential in the absence of supersymmetry breaking, once supersymmetry is broken they may acquire a potential through the moduli dependence of the \( d \) function in the scalar potential (10). This is potentially disastrous for the mechanism discussed above because such a potential would drive the moduli vevs away from the value needed to cancel the quadratic inflaton term. However the kinetic term often has a larger symmetry than the full Lagrangian; for example the canonical form has an \( SU(2) \) symmetry where \( N \) is the total number of chiral fields. In this case there will be many moduli left massless even when supersymmetry is broken because they will be (pseudo) Goldstone modes associated with the spontaneous breaking of this symmetry. These moduli can play the role discussed above eliminating the quadratic term in the inflaton potential.

The mechanism we propose applies to a large class of models, the only condition being that the kinetic term does indeed have a symmetry leading to pseudo-Goldstone modes. However it is instructive to construct a definite model to illustrate the idea in detail. Consider a simple case with just two moduli \( \nu_{1,2} \). The canonical kinetic term has \( g_\nu = A^\dagger A \) where \( A^\dagger = (\nu_1, \nu_2) \). This clearly has an \( SU(2) \) symmetry. Since moduli are not determined before supersymmetry breaking, they may have Planck scale vevs so it is not sufficient to keep only the lowest order (canonical) terms. Thus we generalize the \( \nu \) kinetic term, allowing for a general functional dependence on \( A^\dagger A \), so \( g_\nu = g_\nu(A^\dagger A) \), which preserves the \( SU(2) \) symmetry. When supersymmetry breaking is switched on, the moduli will develop a potential which fixes the vev \( \chi \) of \( A^\dagger A \) at a minimum of \( g \) but leaves the ratio of the vevs of \( \nu_1 \) to \( \nu_2 \) undetermined. We now introduce a chiral matter multiplet \( \phi \)

\[ ^1 \text{Another model where a Goldstone mode has been similarly employed is ‘natural’ inflation [18].} \]
which will contain the inflaton. Unlike the moduli it may have couplings in the superpotential which can keep it in thermal equilibrium at temperatures below the Planck scale. (For example a superpotential of the form $f = \rho \phi^3$ will generate a term $\rho^2 |\phi|^2 T^2$ in the effective potential at high temperature, which would force $\langle \phi \rangle / M < T$.) Although such an initial condition is not essential to our argument, it does simplify the discussion, so we take the initial conditions, at a temperature $T \approx \Lambda$, the scale of the putative inflationary potential, to be $\langle \phi \rangle / M \ll 1$, $\langle \nu \rangle / M \sim 1$. Using this we expand the $\phi$ dependence of the kinetic function $d$ keeping only the low-order terms:

$$d(\phi, \phi^\dagger, \nu, \nu^\dagger) = g_\nu(A^\dagger A) + \phi^\dagger \phi h(A^\dagger A) + \kappa(\nu_1^2 \phi^2 + \nu_1^4 \phi^4) + \ldots$$

where $h$ is an unknown function and $\kappa$ is a constant. The last term on the rhs above involves only chiral or antichiral fields separately and can be absorbed in the superpotential. In writing this term we have assumed there is a $U(1)$ symmetry under which $\nu_1$ and $\phi$ have opposite charges while $\nu_2$ is a singlet. Thus the larger $SU(2)$ symmetry of the kinetic term is broken by interactions, the gauge interactions and couplings in the superpotential. The third term on the rhs above leads, via eq.(14), to a term in the potential proportional to Re($\nu_1^2 \phi^2$). This is the first term sensitive to the ratio of the vevs of $\nu_1$ to $\nu_2$.

Consider now the potential following from this kinetic function. One combination of the moduli will be driven rapidly to a minimum of $g_\nu(A^\dagger A)$ due to the term $\Lambda^4 g_\nu(A^\dagger A)$. Writing $\nu_i = [\tilde{\nu}_i + \langle \nu_i \rangle] e^{i \theta_i /\langle \nu_i \rangle}$, the combination $\nu = \sqrt{\tilde{\nu}_1^2 + \tilde{\nu}_2^2}$ is seen to acquire a mass of $\mathcal{O}(\Lambda^2 / M)$ through this term, while the other three components (pseudo-Goldstone modes) remain massless. In studying the inflationary possibilities of the potential we are only interested in “slow” modes, viz. those fields with masses much less than $\Lambda^2 / M$ which satisfy the inflationary constraints (8). Thus we will henceforth ignore the “fast” mode $\nu$. We turn now to the second and third terms of eq.(14). Setting $A^\dagger A$ to its vev $\chi$, we have for the fields involving $\phi$

$$d_\phi = h(\chi)|\phi|^2 \left[ 1 + \lambda (\tilde{\nu}_1 + \langle \nu_1 \rangle)^2 \cos \left( \frac{2 \theta_1}{\langle \nu_1 \rangle} - \frac{2 \theta_2}{\langle \nu_2 \rangle} \right) \right],$$

where $\lambda = 2 \kappa / h(\chi)$ and we have expanded $\phi$ about the point $\langle \phi \rangle$ as $\phi = [\tilde{\phi} + \langle \phi \rangle] e^{i \theta /\langle \phi \rangle}$, $\nu_i = \tilde{\nu}_i + \langle \nu_i \rangle$. Expanding the exponent in eq.(10) now yields the leading potential term for $\phi$ of the form $V_\phi = d_\phi \Lambda^4$. The important point is that for specific values of $\langle \nu_1 \rangle$ and $\langle \nu_2 \rangle$, the term proportional to $|\phi|^2$ in eq.(13) may vanish, offering the possibility of a potential for $|\phi|$ which satisfies eq.(8). We will expand eq.(13) in the neighbourhood of this point and show that for a large range of initial values the fields will be driven to values such that inflation does occur.

Consider first the possible slow modes of relevance to an inflationary era. These are $|\bar{\phi}|$ and the non-$\nu$ component of $\tilde{\nu}_1$ and $\tilde{\nu}_2$. On the other hand the phase $\theta \propto \langle \phi \rangle$ is a fast field because, for the small $\langle \phi \rangle$ of interest here, it is dominated by the last term and has a large mass. This is readily seen by expanding the cosine in the potential leading to the term $V_{\theta_\phi} = \lambda \Lambda^4 h(\chi) |\phi|^2 \langle \nu_1 \rangle^2 \theta^2 \sim \lambda \Lambda^4 h(\chi) \langle \nu_1 \rangle^2 |\phi|^2 \left( \frac{\phi \theta_1}{\langle \nu_1 \rangle} - \theta \phi \right)^2$, with a piece unsuppressed by the small vev $\langle \phi \rangle$. Thus the phase $\theta$ will flow rapidly to the minimum $\lambda \cos \theta = -|\lambda|$. Having identified the fast and slow variables, we now expand the potential in the latter only, about the vev $\langle \nu_1 \rangle, \langle \nu_2 \rangle$ for which the

\[\text{In general, the ‘thermal constraint’ is irrelevant to chaotic inflationary models wherein the initial conditions are taken to be random, as is appropriate for singlet fields.}\]
quadratic term in \( d_\phi \) vanishes, i.e. for \( \langle \nu_1 \rangle, \langle \nu_2 \rangle \) satisfying \( 1 - |\langle \nu_2^2 \rangle| = 0 \). This gives \( V(|\phi|, \varphi) = \Lambda^4 |\beta| \phi^2 \varphi \), where \( \varphi \) is the slow component of \( \tilde{\nu}_1, \tilde{\nu}_2 \) in the neighbourhood of the expansion point and \( \beta \) is a constant of order unity. We wish to study the evolution of these fields for a range of initial conditions. For this we need to know the equation of motion, which in turn requires the form of the kinetic terms following from eq.(12). For small \( \langle \phi \rangle \), the dominant term in \( d \) giving the \( \tilde{\phi} \) kinetic energy is just \( \tilde{\phi}_1^2 \), as the \( \phi \) kinetic energy term is canonical. (Note that this applies even though the quadratic \( |\tilde{\phi}|^2 \) term in the potential has been cancelled. The reason is that the term responsible for the cancellation in the potential is a \( \phi^2 \) term in \( d \) and this does not affect the kinetic term at all! This is the underlying mechanism that evades the problems highlighted in ref. 7.) For small oscillations in \( \varphi \), the kinetic function can also be expanded with leading term, \( \varphi^4 \), giving canonical kinetic energy for this field too. Thus the equations of motion for \( |\tilde{\phi}| \) and \( \varphi \) are both of the canonical form.

The example presented above assumes that the kinetic term has a larger symmetry than the full Lagrangian. However it is straightforward to construct other examples in which the potential discussed above follows from a symmetry of the full theory. We illustrate this in a model with a single modulus field, \( \nu \), by the choice of kinetic function

\[
d(\phi, \phi^\dagger, \nu, \nu^\dagger) = g_\nu(|\nu|^2) + \phi^\dagger \phi h(|\nu|^2) + \kappa[\ln(\nu)\phi^2 + \ln(\nu^\dagger)\phi^\dagger^2] + \ldots
\]

Here we have written the most general form of \( d \) up to terms quadratic in \( \phi \), consistent with a \( Z_2 \) symmetry under which \( \phi \) is odd, and a \( U(1) \) \( R \)-symmetry under which \( \nu \) transforms non-trivially as \( \nu \to e^{2i\kappa} \nu \) and the superspace co-ordinates transform as \( \theta \to e^{-i\kappa} \). In this case, the pseudo-Goldstone modes associated with the first two terms of eq.(16) are the phases of \( \nu \) and \( \phi \). The latter is a fast variable but the former is a slow variable and generates a \( |\phi|^2 \) term in the potential via \( \kappa \Lambda^4 [\ln(\nu)|\phi|^2 + \ln(\nu^\dagger)|\phi|^\dagger^2] \to -|\kappa| \Lambda^4 |\phi|^2 [(|\ln|\nu||)^2 + \beta^2/(|\nu|^2)]^{1/2} \). Here we have allowed the fast variable to determine the overall sign of the term as before. Again a choice of the phase, \( \theta_\nu \), will cancel the \( |\phi|^2 \) term and expanding about this point leads to the same potential as above.

We hope these two examples have illustrated how the cancellation of the quadratic term can occur in a wide class of models. Now we consider the evolution of the fields for various initial conditions. The field potential is of the form

\[
V(|\tilde{\phi}|, \varphi) = \Lambda^4 \left( 1 + \beta |\tilde{\phi}|^2 \varphi + \gamma |\tilde{\phi}|^3 + \delta |\tilde{\phi}|^4 + \ldots \right)
\]

where we have added further terms in the expansion of \( V \). The cubic term may arise from a cubic term in the superpotential [4]; this is allowed by the symmetries we have been discussing if the additional \((U(1))\) symmetry of the \( \phi \) field in an \( R \)-symmetry. (Alternatively there may be another modulus with \( U(1) \) charge such that a cubic term can appear in the kinetic function \( d \).) If the cubic term is not present, then the quartic term, which is always allowed in the kinetic term by the \( SU(2) \) and \( U(1) \) symmetries of the model, will dominate. Note that the parameters \( \beta, \gamma \) and \( \delta \) are all naturally of order unity. (The parameter \( \beta \) may be chosen to be positive by definition while the parameter \( \gamma \) should be negative if it is to lead to an inflationary potential.)

We are interested in initial conditions which lead, through thermal effects or otherwise, to \( |\tilde{\phi}| \) being small but there is nothing which constrains the initial conditions of \( \varphi \). However we note that the potential (17) has an infrared fixed point with \( \tilde{\phi} = \varphi = 0 \). Consequently, any initial value of \( \tilde{\phi} \) and \( \varphi \) will be driven there if they are within the domain of attraction, given (for positive \( \beta \)) by

\[
\varphi \geq \frac{3|\gamma|}{2\beta} \left[ 1 + \left\{ 1 + \frac{4}{9} \left( \frac{\beta}{|\gamma|} \right)^2 \right\}^{1/2} \right] |\tilde{\phi}|.
\]
Therefore, without any fine tuning of the initial conditions (beyond the condition that the fields lie in this domain of attraction), the fields are driven to fixed values and the potential becomes a constant, driving a period of inflation. This fixed point corresponds to a point of inflection in the potential which is unstable with respect to small perturbations. Thus inflation is naturally terminated by a mechanism which we believe has not been discussed earlier. The equations of motion for $\varphi$ and $|\tilde{\varphi}|$ are

$$\ddot{\varphi} + 3H \dot{\varphi} = -\beta |\tilde{\varphi}|^2, \quad |\ddot{\tilde{\varphi}}| + 3H |\dot{\tilde{\varphi}}| = -\beta \varphi |\tilde{\varphi}| + 3|\gamma| |\tilde{\varphi}|^2,$$

so while $\varphi$ is positive, the fields are driven to the fixed point and inflation begins. However if $\varphi$ should fluctuate and become negative the fields will be driven away from the fixed point thus ending inflation. (For $\beta$ negative, the reverse would be the case.) Now the fluctuations of $\varphi$ are of order the Hawking temperature of the De Sitter vacuum, $T_H = H/2\pi$, thus once $\varphi$ is driven (from the positive direction) to be of $O(T_H)$, fluctuations will lead to it becoming negative and end inflation. The initial conditions for this stage are $\varphi, |\tilde{\varphi}| \sim H$; thereafter, as we see from eq.(19), $|\tilde{\varphi}|$ will grow more rapidly than $\varphi$ and the cubic term in the potential will soon dominate.

We have argued that the potential of the form (17) arises naturally in supergravity models with moduli such as may be expected from the superstring. There are two distinctive features of this potential which ensure that, after the transition to positive $\varphi$, there will be an inflationary period yielding density fluctuations of the magnitude observed. The first is that this potential has a very small gradient in the neighbourhood of the origin in field space so it generates a long period of slow-roll inflation during which quantum fluctuations are naturally small. The second feature is that the full potential, including higher order terms, is governed by an overall scale, $\Lambda$. The reason is that the potential arises from the $d$ term of eq.(9) which, in the absence of supersymmetry breaking, gives rise to the kinetic term and thus does not contribute to the potential, vanishing when derivatives are set to zero. Thus the potential is proportional to the (fourth power of the) overall supersymmetry breaking scale, $\Lambda$. This scale is expected to be of $O(10^{14})$ GeV \[^{[9]}\] and, in conjunction with the small slope, correctly yields the required magnitude of density fluctuations.

**IV. IMPLICATIONS FOR LARGE-SCALE STRUCTURE AND CMB ANISOTROPY**

The inflationary period following from a potential of the form (17) with no quadratic term (and $\gamma = -4$) has been closely studied earlier \[^{[8]}\]. The field value when perturbations of a given scale cross the Hubble radius is obtained by integrating the equation of motion (19) back from the end of inflation, which occurs at $\tilde{\varphi}_{\text{end}} \approx M/6|\gamma|$ when $\epsilon = 1$. Thus $\tilde{\varphi}_* \approx M/3|\gamma|[N_*(k) + 2]$ and using eq.(8) we find a logarithmic (squared) deviation from scale invariance for the scalar perturbations,

$$\delta^2 H(k) = \frac{9\gamma^2}{75\pi^2 M^4} \Lambda^4 [N_*(k) + 2]^4.$$

This corresponds to a ‘tilted’ spectrum, $\delta^2 H(k) \propto k^{n-1}$, with

$$n(k) = 1 + 2\eta - 6\epsilon \approx \frac{N_*(k) - 2}{N_*(k) + 2},$$

i.e. $n \approx 0.92$ for $N_* = 51$ corresponding to the scales probed by COBE \[^{[8]}\]. We emphasize that a leading cubic term in the potential gives the maximal departure from scale-invariance. The
slope of the potential is tiny, \( \epsilon = 1/18 \gamma^2 (N_\star + 2)^4 \simeq 7.0 \times 10^{-9} \gamma^{-2} \), but its curvature is not: \( \eta = -2/(N_\star + 2) \simeq -0.038 \). Consequently, although the spectrum is tilted, the gravitational wave background \( \gtrsim 3 \) is negligible. Furthermore the tilt would be greater if \( N_\star \) is smaller, for example if there is a second epoch of ‘thermal inflation’ when the scale-factor inflates by \( \sim 20 \) e-folds \( \ref{20} \) so that the value of \( N_\star \) appropriate to COBE is 31 rather than 51, and \( n \simeq 0.88 \). We normalize the spectrum \( \ref{20} \) to the CMB anisotropy using the expression for the (ensemble-averaged) quadrupole \( \ref{12} \),

\[
\frac{(Q_{\text{rms}})^2}{T_0^2} = \frac{5C_2}{4\pi} = \frac{5}{4} \int_0^\infty \frac{dk}{k} j_2^2 \left( \frac{2k}{H_0} \right) \delta_H^2(k) ,
\]

where \( j_2 \) is the second-order spherical Bessel function. According to the COBE data \( \ref{11,13} \), \( Q_{\text{rms}} \simeq 20 \pm 2 \mu \text{K} \) for \( n \simeq 0.9 \) which fixes the inflationary scale to be

\[
\frac{\Lambda}{M} \simeq 2.8 \pm 0.14 \times 10^{-4} |\gamma|^{-1/2} ,
\]

consistent with general considerations of supersymmetry breaking during inflation \( \ref{9} \).

The spectrum of the (dimensionless) rms mass fluctuations after matter domination (per unit logarithmic interval of \( k \)) is given by \( \ref{21} \)

\[
\Delta^2(k) \equiv \frac{k^3P(k)}{2\pi^2} = \delta_H^2(k) T^2(k) \left( \frac{k}{aH} \right)^4 ,
\]

where \( P(k) \) is the usual power spectrum and the ‘transfer function’ \( T(k) \) takes into account that linear perturbations grow at different rates depending on the relation between their wavelengths, the Jeans length and the Hubble radius. For CDM we use the parametrization \( \ref{21} \),

\[
T(k) = \left[ 1 + \left\{ ak + (b k)^{3/2} + (c k)^2 \right\}^\nu \right]^{-1/\nu}
\]

with \( a = 6.4 \Gamma^{-1} h^{-1} \text{Mpc}, b = 3 \Gamma^{-1} h^{-1} \text{Mpc}, c = 1.7 T^{-1} h^{-1} \text{Mpc} \) and \( \nu = 1.13 \), where the ‘shape parameter’ is \( \Gamma \simeq \Omega h e^{-21N} \ref{22} \). For ‘standard’ CDM, \( h = 0.5 \) and \( \Omega_N = 0.05 \ref{21} \). However, observational uncertainties still permit the Hubble parameter to be as low as 0.4 \( \ref{23} \) and the nucleon density parameter \( \Omega_N \) may be as high as \( \sim 0.033 h^{-2} \), taking into account the recent upward revision of the \( ^4 \text{He} \) mass fraction \( \ref{24} \). We show \( P(k) \) for \( \Omega_N = 0.05, 0.1 \) and \( h = 0.4, 0.5 \) in figure \( \ref{1} \), having taken account of non-linear gravitational effects at small scales using the prescriptions of ref. \( \ref{25} \) (PD) and ref. \( \ref{26} \) (BG). The tilt in the primordial spectrum which increases logarithmically with decreasing scales allows a good fit to the data points obtained \( \ref{10} \) from the angular correlation function of APM galaxies, if the Hubble parameter (nucleon density) are taken to be at the lower (upper) end of the allowed range. (We have not allowed for the evolution of clustering in the APM data which is estimated to systematically raise the data points by 14% for an unbiased \( \Omega = 1 \) CDM universe \( \ref{30} \).) We have shown the data separately for the 4 zones of the APM survey to illustrate that there are large errors \( \ref{10} \) for \( k \lesssim 0.1 h \text{Mpc}^{-1} \); thus the apparent discrepancy here requires further investigation \( \ref{29} \). However at small scales, the data sets agree well and reveal the expected characteristic “shoulder” due to non-linear evolution which is reproduced by our model. Other studies of tilted spectra \( \ref{27,28} \) focussed on the linear evolution and/or used a compendium \( \ref{22} \) of data from different surveys (having different systematic biases) rather than one set of high quality data. We conclude that the problem with the excess power on small scales in the COBE-normalized
standard CDM model \(^{31}\) is naturally alleviated in supergravity inflation, as anticipated earlier \(^{1,2}\), with no need for a component of hot dark matter.

We also quote some averaged quantities of observational interest for this model. A common measure of large-scale clustering is the variance, \(\sigma^2(R)\), of the density field smoothed over a sphere of radius \(R\), usually taken to be \(8\,h^{-1}\) Mpc, given in terms of the matter density spectrum by

\[
\sigma^2(R) = \frac{1}{H_0^2} \int_0^\infty W(kR) \delta^2_H(k) T^2(k) k^3\,dk ,
\]

where a ‘top hat’ smoothing function, \(W(kR) = 3 \left[ \frac{\sin(kR)}{(kR)^3} - \frac{\cos(kR)}{(kR)^2} \right]\), has been used. As seen from figure 2, the observational value of \(\sigma(8h^{-1}\text{Mpc}) = 0.60_{-0.15}^{+0.19}\) (95\% c.l.), inferred from the abundances of rich clusters of galaxies \(^{31,32}\) favours high tilt, high \(\Omega_N\) and low \(h\). For the two models shown in figure 1 we find,

\[
\sigma(8h^{-1}\text{Mpc}) = 0.78\pm0.08 \left( N_{\text{COBE}} = 51, \ \Omega_N = 0.05, \ h = 0.4 \right) ,
\]

\[
= 0.75\pm0.08 \left( N_{\text{COBE}} = 31, \ \Omega_N = 0.1, \ h = 0.5 \right) .
\]

Another interesting quantity is the smoothed peculiar velocity field or ‘bulk flow’,

\[
\sigma_v^2(R) = \frac{1}{H_0^2} \int_0^\infty W^2(kR) e^{-\left(12h^{-1}k\right)^2} \delta^2_H(k) T^2(k) k\,dk ,
\]

where, for direct comparison with observations, we have applied an additional gaussian smoothing on \(12h^{-1}\text{Mpc}\). With the same parameters as above,

\[
\sigma_v(40h^{-1}\text{Mpc}) = 383\pm38 \text{ km sec}^{-1} \left( N_{\text{COBE}} = 51, \ \Omega_N = 0.05, \ h = 0.4 \right) ,
\]

\[
= 320\pm32 \text{ km sec}^{-1} \left( N_{\text{COBE}} = 31, \ \Omega_N = 0.1, \ h = 0.5 \right) .
\]

An unambiguous test of the model is the predicted CMB anisotropy. To compute this accurately requires numerical solution of the coupled linearized Boltzmann, Einstein and fluid equations for the perturbation in the photon phase space distribution. We use the COSMICS computer code \(^{35}\) developed to calculate the angular power spectrum using the primordial scalar fluctuation spectrum \(^{20}\). These programmes include a careful treatment of the hydrogen recombination and the decoupling of the matter and radiation, a full treatment of Thompson scattering, and a full computation of all relativistic shear stresses of photons and neutrinos. The first 1000 multipoles are plotted in figure 3, taking \(\Omega_N = 0.05, 0.1\), along with a compendium of recent observational data \(^{12}\), and the prediction of standard CDM is shown for comparison. The height of the first ‘Doppler peak’ is preferentially boosted for the higher value of \(\Omega_N\) and this is favoured by the CMB observations in conjunction with the large-scale structure data, as has been noted independently \(^{36}\). For a given value of \(\Omega_N\) the effect of the spectral tilt is to suppress the heights of all Doppler peaks. Although present ground-based observations are inconclusive, this prediction will be definitively tested by the forthcoming satellite-borne experiments, MAP and COBRAS/SAMBA.

**Acknowledgements**: We are grateful to George Efstathiou and especially to Enrique Gaztanaga, for providing the APM data, and for many stimulating discussions. We thank David Lyth for motivating us to clarify and extend our previous work and Ed Copeland for helpful comments. This research was supported by the EC Theoretical Astroparticle Network CHRX-CT93-0120 (DG12 COMA).
REFERENCES

[1] A.D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood Academic Press, 1990).
[2] J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton University Press, 1993).
[3] For a review, see K.A. Olive, Phys. Rep. 190 (1990) 307.
[4] M. Dine, W. Fischler and D. Nemechansky, Phys. Lett. 136B (1984) 169; G.D. Coughlan *et al*, Phys. Lett. 140B (1984) 44.
[5] E.J. Copeland *et al*, Phys. Rev. D 49 (1994) 6410.
[6] For a review, see D.H. Lyth, Preprint LANCS-TH/9614 [hep-ph/9609431].
[7] E.D. Stewart, Phys. Rev. D 51 (1995) 6847.
[8] P. Binétruy and G. Dvali, Preprint CERN-TH/96-149 [hep-ph/9606342]; E. Halyo, Phys. Lett. B 387 (1996) 43.
[9] G.G. Ross and S. Sarkar, Nucl. Phys. B 461 (1996) 597.
[10] C.M. Baugh and G.P. Efstathiou, Mon. Not. R. Astr. Soc. 265 (1993) 145.
[11] C.L. Bennett *et al* (COBE collab.), Astrophys. J. 464 (1996) L1.
[12] For a review, see D. Scott and G.F. Smoot, Phys. Rev. D 54 (1996) 118.
[13] For a review, see A.R. Liddle and D.H. Lyth, Phys. Rep. 231 (1993) 1.
[14] For a review, see S. Sarkar, Preprint OUTP-95-16P [hep-ph/9602260].
[15] E.F. Bunn and M. White, Eprint astro-ph/9607060.
[16] A.D. Linde, Phys. Lett. 132B (1983) 317.
[17] A.D. Linde, Phys. Lett. 129B (1983) 177.
[18] F.C. Adams *et al*, Phys. Rev. D 47 (1993) 426.
[19] B.A. Ovrut and P.J. Steinhardt, Phys. Lett. 133B (1983) 161.
[20] D.H. Lyth and E.D. Stewart, Phys. Rev. Lett. 75 (1995) 201; Phys. Rev. D 53 (1996) 1784.
[21] G.P. Efstathiou, *Physics of the Early Universe*, eds. J.A. Peacock *et al* (SUSSP Publications, 1990) p 361.
[22] J.A. Peacock and S.J. Dodds, Mon. Not. R. Astr. Soc. 267 (1994) 1020.
[23] For a review, see C.J. Hogan, Phys. Rev. D 54 (1996) 112.
[24] P.J. Kernan and S. Sarkar, Phys. Rev. D 54 (1996) R3681.
[25] J.A. Peacock and S.J. Dodds, Mon. Not. R. Astr. Soc. 280 (1996) L19.
[26] B. Jain, H.J. Mo and S.D.M. White, Mon. Not. R. Astr. Soc. 276 (1996) L25; C.M. Baugh and E. Gaztaña, Eprint astro-ph/9601085.
[27] M. White *et al*, Mon. Not. R. Astr. Soc. 276 (1995) L69.
[28] A. Liddle *et al*, Mon. Not. R. Astr. Soc. 281 (1996) 531.
[29] J.A. Adams, G.G. Ross and S. Sarkar, in preparation.
[30] E. Gaztaña, Astrophys. J. 454 (1995) 561.
[31] S.D.M. White, G.P. Efstathiou and C.S. Frenk, Mon. Not. R. Astr. Soc. 262 (1993) 1023.
[32] S. Sarkar, *Proc. Intern. EPS Conf. on High Energy Physics, Brussels*, ed. J. Lemoine *et al* (World Scientific, 1996) p.95.
[33] P. Viana and A. Liddle, Mon. Not. R. Astr. Soc. 281 (1996) 323.
[34] A. Dekel, Annu. Rev. Astron. Astrophys. 32 (1994) 371.
[35] E. Bertschinger, Eprint astro-ph/9506070 [http://arcturus.mit.edu/cosmics/].
[36] M. White *et al*, Preprint SUSSEX-AST 96/5-2 [astro-ph/9605057].
FIG. 1. Predicted power spectrum of density fluctuations in cold dark matter compared with data from the APM survey. The dotted line shows the linear spectrum, and the dashed lines the non-linear evolution according to two different prescriptions. The spectra are normalized to COBE adopting (a) $N_{\text{COBE}} = 51$, $\Omega_N = 0.05$, $h = 0.4$, and (b) $N_{\text{COBE}} = 31$, $\Omega_N = 0.1$, $h = 0.5$. 
FIG. 2. Predicted value of the variance of the density field smoothed over a sphere of radius $8\,h^{-1}\text{Mpc}$ for (a) $N_{\text{COBE}} = 51$ and (b) $N_{\text{COBE}} = 31$, as a function of the Hubble parameter and the nucleon density parameter. The region within the marked contours is consistent with the observational limits (horizontal planes) inferred from rich clusters of galaxies.
FIG. 3. Predicted angular power spectrum of CMB anisotropy normalized to COBE and compared with current data, adopting (a) $\Omega_N = 0.05$, and (b) $\Omega_N = 0.1$, both with $h = 0.5$. The standard scale-invariant spectrum (full line) is compared with the tilted spectra from supergravity inflation for $N_{\text{COBE}} = 51$ (dashed line) and $N_{\text{COBE}} = 31$ (dotted line).