Disjunctive Normal Form Schemes for Heterogeneous Attributed Graphs

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Abstract. Several ‘edge-discovery’ applications over graph-based data models are known to have worst-case quadratic complexity, even if the discovered edges are sparse. One example is the generic link discovery problem between two graphs, which has invited research interest in several communities. Specific versions of this problem include link prediction in social networks, ontology alignment between metadata-rich RDF data, approximate joins, and entity resolution between instance-rich data. As large datasets continue to proliferate, reducing quadratic complexity to make the task practical is an important research problem. Within the entity resolution community, the problem is commonly referred to as blocking. A particular class of learnable blocking schemes is known as Disjunctive Normal Form (DNF) blocking schemes, and has emerged as state-of-the art for homogeneous (i.e. same-schema) tabular data. Despite the promise of these schemes, a formalism or learning framework has not been developed for them when input data instances are generic, attributed graphs possessing both node and edge heterogeneity. With such a development, the complexity-reducing scope of DNF schemes becomes applicable to a variety of problems, including entity resolution and type alignment between heterogeneous RDF graphs, and link prediction in networks represented as attributed graphs. This paper presents a graph-theoretic formalism for DNF schemes, and investigates their learnability in an optimization framework. Experimentally, the DNF schemes learned on pairs of heterogeneous RDF graphs are demonstrated to achieve high complexity-reductions (98.25% across ten RDF test cases) at little cost to coverage, and with high reliability (<2.5% standard deviation). Finally, one extant class of RDF blocking schemes is shown to be a special case of DNF schemes.

Keywords: Heterogeneity, Link Discovery, DNF Schemes, Graph Models, Attributed Graphs, Learnability, Machine Learning, Blocking

1 Introduction

Constrained edge-discovery tasks constitute an important class of problems in communities that rely on graph data models [5]. Examples include link discovery (e.g. entity resolution and class matching) in the Semantic Web [10, 19], and a variety of link prediction tasks in network-oriented communities such as social
media, bioinformatics and advertising [9], [11], [9]. Algorithms attempting to solve such tasks take as input either a single graph or two graphs, and predict a set of edges linking nodes in the graphs. The semantics and constraints of the predicted link depends on the task formulation: when performing entity resolution (ER), for example, the link is expected to have :sameAs semantics indicating that the two linked entities refer to the same underlying entity [10].

A real-world observation about many edge-discovery tasks is that many interesting links are typically sparse in the space of all possible edges, which is quadratic in the number of nodes [16]. Due to their quadratic complexity, one-step algorithms that predict a link by performing expensive computations on each pair of nodes have gradually been superseded by two-step algorithms, especially in the ER community (Section 2). In two-step ER, the first step is typically known as blocking [2]. Using an indexing function known as a blocking scheme, a blocking algorithm clusters approximately similar entities into (possibly overlapping) clusters known as blocks. Only entities sharing a block are candidates for further analysis in the second similarity step. State-of-the-art similarity algorithms in various communities (including ER) are now framed in terms of machine learning, typically as binary classification [1], [5].

This basic two-step framework can also be extended to generic link discovery tasks. As an example, suppose the task is discovery and prediction of co-authorship links between scientists in a social network. Rather than exhaustively evaluate all pairs of (scientist) nodes, we could first index scientists based on a simple condition: the overlap between the keywords used in their papers. Only nodes with sufficient keyword overlap would undergo more expensive computations. On datasets with highly specialized domains, a domain expert might be required to hand-craft an appropriate indexing scheme.

Due to expense of manual expertise, automatic discovery of such indexing schemes, also using machine learning, was motivated as a research problem in the previous decade [13], [2]. Schemes learned in a supervised setting are able to adapt to available training data, precluding the need for manual hand-crafting. Like in any machine learning framework, the expressiveness of such a scheme would depend both on the underlying properties of the class of schemes (the ‘hypothesis’ space) as well as the learning algorithm optimizing over this space [3].

In this paper, we develop a class of schemes known as Disjunctive Normal Form (DNF) schemes for a generic data model called a directed, labeled attributed data graph model (Section 3). The model is designed to be generic enough that several extant graphs of interest, including RDF data and directed, heterogeneous networks, can be expressed as its instances. In related work (Section 2), we describe how the current theory on DNF schemes limits their use to a specific data model (homogeneous tables) and a constrained problem (record deduplication) [13], [2], [7]. Despite their excellent performance in that setting, DNF schemes were never proposed or developed for generic graph models or for sparse edge-discovery tasks (on these graph models) that are less constrained than homogeneous record deduplication (Section 4). In Section 5 we present a
constructive formalism for DNF schemes that can be applied on graphs (Section 5.2), followed by results on the learnability of these schemes (Section 5.3). Specific contributions presented in this paper over current state-of-the-art work in DNF indexing are summarized in Table 1. In Section 6 results on the empirical utility of graph-theoretic DNF schemes are also described. The results show that the DNF schemes can be robustly learned from noisy training data, and achieve high reductions (98.25%), on average, with low standard deviations (2.42%). This makes them well-suited for edge-discovery on Linked Data, which has high volume and dynamicity. Finally, a class of indexing schemes designed for schema-free (i.e. heterogeneous) RDF data, namely Attribute Clustering [15], is shown to be a special case of graph-theoretic DNF schemes.

2 Related Work

Link prediction and entity resolution (ER) were both recognized as important steps in the overall link mining community about a decade ago [5]. In the Semantic Web community, instance matching [10], link discovery [14], [20] and class matching [19] are specific examples of such sparse edge-discovery tasks. Other applications include protein structure prediction (bioinformatics) [9], click-through rate prediction (advertising) [6], social media and network science [11], [17].

The sparsity of positive edges (equivalently known as the class imbalance problem in machine learning [3]) is well-known in several communities [16]. Blocking methods for ER have continued to be extensively researched, with more recent research in the Semantic Web focused on data-driven approaches [15]. This work presents graph-theoretic formalism and learnability results for a specific class of blocking schemes called Disjunctive Normal Form (DNF) blocking schemes that have rapidly emerged as state-of-the-art for deduplicating homogeneous tables [13], [2]. We believe this is due to both their strong theoretical foundations, as well as their recently demonstrated experimental robustness, even with noisy training data [7]. The formalism in prior work is briefly reviewed in Section 5.1.

Blocking, as a preprocessing complexity-reduction step, is not the only avenue for addressing scalability. In networks with no edge labels, or otherwise informative property values, structural features are important for predicting missing links [11]. In large networks, techniques like matrix factorization [12], stochastic optimization [21] and message passing [3] are more important than complexity-reduction techniques. Such techniques are complementary, not competitive, with the DNF schemes proposed herein. We also note that such techniques are orthogonal to the Semantic Web, where edge-labels are given by property URIs. On the Web of Linked Data, the usefulness of labels and property values for link discovery problems is well-known, particularly when the data has loose schema bindings [15]. Such datasets are becoming increasingly common [18]. Results presented in Section 5 demonstrate that, on real-world RDF data, the graph-theoretic DNF schemes presented in this work can significantly reduce complexity in data-driven link discovery applications.
Table 1. Contributions in this paper compared to prior work

| DNF schemes in prior work                      | DNF schemes in this work                                                                 |
|-----------------------------------------------|----------------------------------------------------------------------------------------|
| Specific to homogeneous tables                | Proposed for heterogeneous graphs                                                      |
| Specific to the deduplication task            | Applicable to any sparse edge-discovery task where training data is available           |
| Handling missing values not evident           | Addresses the missing value problem                                                     |
| Entities (i.e. tabular records) must necessarily have the same type | Nodes can have different, even multiple, types (denoted attributes herein). |
| A single tabular instance assumed as input    | Proposed for edge-discovery tasks in both one-graph and two-graph scenarios              |
| Learning as single-step optimization         | Learning as multi-step optimization                                                    |
| No reduction results from extant blocking schemes | Attribute Clustering [15] shown to be a special case (Theorem 2)                        |
| No robustness results                         | Empirically robust to noisy training data                                              |

Fig. 1. Illustration of link discovery, used as the running example throughout this work. Dotted lines (labeled type) and nodes respectively represent attributes and attribute mappings (Section 3).

3 Data Model

The specific data model adopted for this work is a labeled, directed attributed data graph model. Let $\Sigma$ denote a finite alphabet (e.g. Unicode characters). We refer to an instance of this graph model as a data graph (in the spirit of [22]):

**Definition 1. Data graph** A data graph $G$ is a 7-tuple $G = (V, E, l_V, A_V, \Sigma_V, \Sigma_E, \Sigma_A)$, where $V$ is the set of nodes, $E \subseteq V \times V \times \Sigma_E$ is the set of directed, labeled edges, $\Sigma_E \subseteq \Sigma^*$ is a finite set denoted as the edge vocabulary, $l_V$ is a function mapping a node $v \in V$ to a label in $\Sigma_V \subseteq \Sigma^*$, $\Sigma_V$ is a finite set denoted as the node vocabulary, $A_V$ is a partially ordered mapping known as the attribute mapping $A_V : V \to \Sigma_A^*$, and $\Sigma_A \subseteq \Sigma^*$ is a finite attribute vocabulary.

Per Definition 1, let $A_V(v)$ represent the attribute set of node $v \in V$, and let $l_V(v)$ represent the label of node $v$. In slight abuse of notation, we typically
refer to a node by its label. Note that RDF can be expressed graph-theoretically
in terms of the defined model, as an RDF graph is just a directed, labeled graph
with constraints. By way of example, one such intuitive constraint is that there
does not exist an edge \((v_1, v_2, l) \in E\) such that \(l_{V}(v_1)\) is a literal, per some
pre-specified predicate (e.g. isLiteral) that distinguishes literal elements in \(\Sigma_{V}\)
from URI elements. In the same vein, \(\Sigma_{V}\) is constrained so that all non-literal
elements (i.e. for which isLiteral returns False) are necessarily (blank or non-
blank) URI\(^{\text{a}}\) and \(\Sigma_{E}\) only contains URIs.

Example 1. Figure 1 illustrates a data graph that resembles an RDF graph. The
dotted type edge, which is not formally an edge per Definition 1, indicates an
attribute mapping, with attributes represented as dotted ovals. For example,
\(A_{V}(\text{John Doe})\) returns the attribute set \{{Actor, Guitarist}\}. Note also that, per
RDF convention, we have placed the literal “03-01-1980” in a rectangle; Defini-
tion 1 does not actually distinguish between literal and non-literal nodes.

In the context of RDF/OWL data, Example 1 illustrates that attributes in
Definition 1 typically serve the same role as a set of ontological classes. Since
class hierarchies (i.e. super-classes and sub-classes) are prevalent in expressive
ontologies, a node is permitted multiple attributes (i.e. an attribute set). In
the model proposed in [22], only a single attribute per node was permitted, and
edges were necessarily undirected and unlabeled. Such graphs are special cases of
Definition 1 and the findings in that paper\(^{b}\) are complementary to the formalism
presented herein. Note also that all three vocabularies in Definition 1 may be
empty. Essentially, nodes and edges are allowed to be unlabeled and untyped.

Whenever two graphs are indicated, subscripts on the relevant notation will
be used to make a distinction. A specific caveat is the usage of the term attribute.
In graph-theoretic terminology, adopted herein for the sake of generality, nodes
are attributed [22], meaning that an attribute is like an RDF class. This is in
contrast to [15], where an attribute was a set of pairs, with each pair consisting
of an edge-label (i.e. an RDF property URI) and an object value. Finally, note
that the finiteness of the various elements in \(G\) is motivated primarily by real-
world applications on finite data graphs. Technically, permitting the sets to be
countably infinite does not fundamentally alter the subsequent formalism, but
does make it unnecessarily more involved.

4 Problem Formulation

With the data model in place, there are two problem scenarios within the scope
of this paper. The first, denoted as the one-graph scenario, concerns sparse edge-
discovery tasks on a single data graph input. Given a data graph \(G\), let there be an unknown partition of the quadratic space \(V \times V\) into two sets \(P\) (links) and \(N\)

\(^{a}\) \(A_{V}\) returns an empty attribute set for nodes lacking class information. It is partially ordered to enable representation of class hierarchies.

\(^{b}\) Namely, building efficient index data structures for speedy query processing.
Fig. 2. Example of a DNF blocking scheme for the deduplication task on a tabular Restaurants benchmark. The scheme takes as input a pair of tabular records and returns True (otherwise False) if they should be included in the candidate set $C$ (Section 4).

(non-links). We assume a sparsity condition i.e. $|N| = \rho(|N| + |P|)$ ($\rho \approx 1.0$, but is strictly less than 1.0), and an available training set (sampled i.i.d) $T = P_T \cup N_T$, where $P_T \subset P$ and $N_T \subset N$. We denote $\rho$ as the optimal reduction ratio (RR).

The pairwise complexity-reduction problem is to learn a sufficiently expressive scheme that, when executed on $G$, results in a candidate set $C$ of node-pairs such that the empirical RR $(1.0 - |C|/|V|^2)$ is maximized while ensuring that the positive link coverage (the Pairs Completeness or PC), defined as $|C \cap P|/|P|$, is above a minimum pre-specified threshold in expectation. This learning problem is formally expressed as an optimization program in Section 5.3.

Concerning the two-graph scenario, the problem is similarly defined as above, except that all links must be in the set $V_1 \times V_2$, with $V_1, V_2$ being the node sets of the two data graph inputs $G_1, G_2$ respectively. Rather than adopt separate formalisms in Section 5.2 for the two scenarios, we frame the definitions, where relevant, in a way such that (1) two data graphs are never required to be distinct and can therefore be the same graph, and (2) two nodes are always required to be distinct. By maintaining (1) and (2) throughout the construction, the one-graph and two-graph treatments are unified, unlike in prior work on the subject [2], [13], [15]. Theoretically, self-link discovery is also avoided.

Most importantly, the complexity-reduction problem studied in this paper is agnostic to the underlying link specification function (LSF), since the ground-truth partitioning of the quadratic node-pair space (into links and non-links) is unknown (and can be arbitrary). This is in contrast to complexity-reduction systems such as Limes, Silk and Attribute Clustering wherein either the LSF is known [14], [20], or the semantics of LSF (e.g. for ER) is known [15].

5 Disjunctive Normal Form Schemes

5.1 Background: DNF blocking schemes for tabular deduplication

The theory (i.e. formalism and learnability) for an adaptive class of complexity-reduction schemes, called Disjunctive Normal Form (DNF) blocking schemes,
is especially well-developed for the task of homogeneous tabular deduplication [13], [2]. Figure 2 illustrates such a scheme by way of an example. The scheme is given by a Boolean DNF expression that can be arbitrarily complex in principle, although in practice, the complexity of the scheme is curbed by a specified parameter. As the mnemonic notation suggests, the scheme takes a pair of entities (tabular records in this case) as input and returns True if they share a token in their Name column or an integer in their Address column. The predicates that comprise the atoms in the DNF expression are compositions of a function (e.g., CommonToken) and a column (e.g., Name). Given a set of \( g \) such functions (known as general predicates [2]) and a table with \( c \) columns, a legal DNF scheme is expressible over \( gc \) atoms. Given training sets of duplicates and non-duplicates, learning a scheme can be framed in terms of solving an optimization problem over the training sets (Section 5.3) [2], [13].

5.2 Constructive formalism for sparse edge-discovery on data graphs

The basic treatment of DNF blocking schemes in Section 5.1 illustrates that, at the highest level, there are two crucial components to their construction. The first is akin to the feature design phase (of typical machine learning), and corresponds to the choice of predicate functions (e.g., CommonToken in Figure 2). The second is the learning algorithm itself (i.e. choosing and combining the atomic predicates into a complete DNF expression), akin to the parameter estimation (e.g. by applying statistical inference techniques on available training data) phase [3].

In the graph model, these two components are not, in themselves, adequate because of the presence of both node and edge heterogeneity. Node heterogeneity arises because nodes may have different sets of attributes associated with them, while edge heterogeneity arises because of edge labels. Different entities may have different sets of ‘properties’ or edge labels associated with them. A naive adoption of the treatment in Section 5.1 to the graph-theoretic case runs into the missing value problem [4].

To accommodate heterogeneity and missing values at the conceptual level, additional technical machinery is needed. In the rest of this section, we ‘construct’ the formalism by defining some of these concepts and illustrating them using the running example in Figure 1. In keeping with practical constraints and intuitions, we impose finiteness constraints on the relevant definitions.

As in the rest of this paper, we assume an alphabet \( \Sigma \). Given a data graph \( G = (V, E, l_V, A_V, \Sigma_V, \Sigma_E, \Sigma_A) \), recall that each of \( \Sigma_V, \Sigma_E \) and \( \Sigma_A \) is a subset of \( \Sigma^* \). Using standard terminology from formal automata theory, an arbitrary

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3 With a finite set of \( n \) predicates (e.g. CommonToken_Name in Figure 2), there are \( 2^n \) canonical (i.e. arrangement-insensitive) positive DNF formulae. Negated literals are not allowed in blocking constructions.

4 This becomes apparent if each column in the table in Figure 2 is thought of as a ‘property’ or edge label. Every entity is constrained to possess this exact set of properties (the table schema) in the homogeneous tabular deduplication task.
element from $\Sigma^*$ is referred to as a string. An arbitrary element from $\Sigma_V$, $\Sigma_E$ and $\Sigma_A$ is referred to as a node label, edge label and attribute respectively.

With these assumptions, we start by defining shallow and deep extractors, which are the most basic (‘primitive’) units in constructing a DNF scheme:

**Definition 2. Primitive shallow extractor (PSE)** Given a graph $G$ and an alphabet $\Sigma$, a primitive shallow extractor (PSE) $P_s : \Sigma_L \rightarrow 2^{\Sigma^*}$ is defined as a mapping that takes a node label from $\Sigma_L$ as input and returns (‘extracts’) a finite set of strings ($\subset \Sigma^*$) as output.

**Example 2.** An example of a PSE would be tokenizing a string into a set (of tokens) based on standard delimiters. For example, using the delimiter - on the date literal “03-01-1980” in Figure 1, a set \{“03”, “01”, “1980”\} is obtained. A useful practice is to represent such extractors mnemonically (e.g. TokenizeString).

**Definition 3. Primitive deep extractor (PDE)** Given a graph $G$ and an alphabet $\Sigma$, a primitive deep extractor (PDE) $P_d : 2^{\Sigma^*} \rightarrow 2^{\Sigma^*}$ is defined as a mapping that takes a finite set of strings as input and returns (‘extracts’) a finite set of strings as output.

**Example 3.** Continuing from Example 2, an example of a PDE would be AddOneToIntegers. It takes a set as input, and for every integer in the set, parses and increments the integer and adds it back to the set (designed for more robust performance against noisy integer inputs \[2\]). On the input set \{“03”, “01”, “1980”\}, the output would be \{“03”, “01”, “1980”, “04”, “02”, “1981”\}. Another example, designed for text, is to remove stop-words (e.g. the) from the set.

The examples above indicate that the PSEs and PDEs must necessarily be specified by the user. Typically, this is not a bottleneck: authors in several communities have already proposed a wide variety of practical functional classes (e.g. phonetic, token-based, set-based and numeric) \[2\], \[8\], \[11\]. Henceforth, we assume the availability of finite sets $P_s$ and $P_d$ of PSEs and PDEs respectively.

**Definition 4. Feature extraction operator (FEO)** Given a graph $G$, and extractor sets $P_s$ and $P_d$, a feature extraction operator (FEO) is a mapping that takes a node $v \in V$ as input, computes its label $l_V(v)$, and performs a finite, non-empty sequence of extraction operations to output a set of strings.

Given an FEO parameterized by $n$ extractors, it is necessarily the case (per Definitions 2 and 3) that the first extraction, which always exists per Definition 4 is shallow, and the following $n - 1$ extractions (if $n > 1$) are deep.

**Example 4.** Consider a free-text literal “Died on 03-03-1943”. A first step, as discussed in Example 2 is to derive a set of tokens from the literal. Next, as discussed in Example 3 the integers could be supplemented with increments\(^5\) and if a token cannot be parsed as an integer, we design the PDE AddOneToIntegers to ignore it.\(^6\)

\(^5\) Thus, the output set can potentially be smaller (even empty) than the input set.

\(^6\) If a token cannot be parsed as an integer, we design the PDE AddOneToIntegers to ignore it.
but also the stop-word ‘on’ should be removed, and the word ‘died’ should be *stemmed* to its canonical form ‘die’. Functionally, this FEO is represented by the composite mapping `StemWords(RemoveStopWords(AddOneToIntegers(TokenizeString(lv())))))`.

One issue with the definition of an FEO is that it only operates on the label of the node. In RDF graphs, in particular, the label does not contain enough discriminative information. It becomes necessary to seek out information that is one or more edges (i.e. a *trail*) away. Given a graph \( G \), a node \( v \in V \) and a sequence \( s \) of \( n \) edge labels, let a trail \( t \), defined as an alternating sequence of nodes and edges in \( G \), be denoted as being valid if (1) the starting node in \( t \) is \( v \), (2) the subsequence of edges in the trail corresponds exactly to \( s \).

**Example 5.** In Figure 1, the starting node *John Doe* and edge-label sequence (*actedIn*) yields a valid trail: (*John Doe, actedIn, Jurassic Park, 4*). If John Doe acted in multiple movies, there would be multiple valid trails.

In general, given an edge-label sequence and a starting node, a (possibly empty) set of valid trails can be constructed. Let the terminating node in a trail \( t \) be denoted by the symbol \( \text{last}(t) \). In a slight abuse of notation, let the set of all edge-label sequences of length exactly \( n \) be denoted by the symbol \( \Sigma^n_E \). Similarly, let \( T^n \) denote the set of all trails with exactly \( n \) edges. Using these symbols, let \( \text{trails}(v, s) \) represent the mapping that takes a starting node \( v \in V \) and an edge-label sequence \( s \in \Sigma^n_E \) as input, and returns a (possibly empty) set of valid trails \( T^n \subset T^* \), where \( T^* \) is the (countably infinite, in the general case) set of all possible trails in graph \( G \).

**Definition 5.** Trail-sensitive feature extraction operator (t-FEO) Given a graph \( G \) and an FEO \( f \), a *trail-sensitive feature extraction operator* (t-FEO) is a mapping that takes a node \( v \in V \) and a finite sequence \( s \in \Sigma^n_E \) with exactly \( n \geq 0 \) edge labels as input, and for \( n = 0 \), returns \( f(v) \). For \( n > 0 \), the operator constructs the set \( T^n = \text{trails}(v, s) \) and returns (1) the empty set if \( T^n \) is empty, (2) \( \bigcup_{t \in T^n} f(\text{last}(t)) \) if \( T^n \) is non-empty.

Notationally, we denote a t-FEO as being parameterized by FEO \( f \), and with a node \( v \) and finite edge-label sequence \( s \) as inputs.

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7 In cases such as Freebase, the ‘label’ as defined here is usually an opaque URI representing the subject of the entity.
8 The data graph, as defined in Section 3, is not required to be *acyclic*. This is why, in the subsequent formalism, we refer to trails (which may have cyclical subsequences) and not *paths*. For practical purposes, this subtlety applies more to networks, where cycles are common, than to RDF graphs.
9 These symbols assume a graph \( G \). For more than one graph, subscripts will be used to make a distinction.
10 Given a graph \( G \) (context) and a non-negative integer \( n \) (hyperparameter), the t-FEOs represent a class of mappings with one degree of freedom (the parameter \( f \)).
Example 6. Consider a t-FEO parameterized by the FEO \( f \) defined in Example 4 on the data graph in Figure 1. Given the node John_Doe and the simple unit-length edge-label sequence (bornOn) as inputs, the t-FEO returns the same output as in Example 4. On the input Christine_Doe and the same sequence, the t-FEO returns \( \{\} \).

Henceforth, we assume a finite set \( F \leq n \) of t-FEOs (with hyperparameters that do not exceed \( n \)), which can be constructed by bounding \( n \) and using a finite set of FEOs. Definition 5 gracefully handles missing values by returning the empty set when the set \( T \) of valid trails is also empty. Furthermore, allowing an edge-label sequence to be empty \((n = 0)\) enables an FEO in Definition 4 to be cast as a special case of a trail-sensitive FEO in Definition 5.

A t-FEO always operates on a single node, while edge-discovery is a pairwise operation. Given two (not necessarily distinct) t-FEOs from two distinct nodes, either from a single graph (one-graph scenario) or two different graphs (two-graph scenario), parameterized t-FEOs can be applied on the respective nodes to obtain two feature-sets \( Z_1 \) and \( Z_2 \).

A set-based relation can now be used to derive a Boolean value from these two sets. Such a relation takes the two sets as inputs and maps them to True or False based on some condition. While any condition can be used, in theory, the motivation behind developing DNF schemes is to avoid quadratic comparisons, and the relation must be amenable to efficient execution. A specific example of such a relation is the thresholded Jaccard, defined as the condition \( |Z_1 \cap Z_2|/|Z_1 \cup Z_2| > \theta \), where \( \theta \) is a specified threshold. An important, highly effective case in the blocking community is \( \theta = 0 \), as checking for a single element common to the sets becomes sufficient (and inverted indexing techniques become applicable).

The rest of this section assumes this simple case; the case of arbitrary real-valued thresholds is left for future work.

Using a set-based relation \( R \), and the definitions thus far, a trail-sensitive predicate is defined below. Such predicates eventually serve as the atoms in the final DNF construction (similar to the role served by CommonToken_Name in Figure 2).

**Definition 6. Trail-sensitive predicate (t-P)** Given a set-based relation \( R \), two t-FEOs \( f_1 \) and \( f_2 \) and two finite sets \( S_1 \subset \Sigma_{E_1}^* \) and \( S_2 \subset \Sigma_{E_2}^* \) of edge-label sequences, defined respectively on two graphs \( G_1 \) and \( G_2 \), a trail-sensitive predicate (t-P) is a binary relation parameterized as a 5-tuple \((R, f_1, f_2, S_1, S_2)\). A t-P takes as input two distinct nodes \( v_1 \in V_1 \) and \( v_2 \in V_2 \), computes the set \( Z_1 = \bigcup_{s \in S_1} f_1(v_1, s) \) (and similarly, set \( Z_2 \)), and returns \( R(Z_1, Z_2) \).

By bounding any sequence in the edge-label sequence sets \( S_1 \) and \( S_2 \) in the definition above, the set of all trail-sensitive predicates (denoted as the...
predicate universe \( U \) is also finite\(^{12}\). Intuitively, these predicates serve as atoms, which can now be used to construct general DNF expressions.

One issue is that, so far, the attributes of the nodes involved (i.e. node heterogeneities) have been neglected. This issue is addressed by defining an attribution relation below:

**Definition 7. Attribution Relation** Given two (not necessarily distinct) graphs \( G_1 \) and \( G_2 \), an attribution relation is a binary relation defined on the attribute mappings \( A_{V_1} \) and \( A_{V_2} \). Functionally, it takes as input two distinct nodes \( v_1 \in V_1 \) and \( v_2 \in V_2 \), and returns True iff some attribute pair in \( A_{V_1}(v_1) \times A_{V_2}(v_2) \) is in the relation, and returns False otherwise.

**Example 7.** A good (in a data-driven sense) attribution relation for the example in Figure 1 is \( \{ (\text{Actor, Director)}, \ (\text{Guitarist, Guitarist}) \} \). A safer (in a coverage sense) but more coarse-grained (i.e. less discriminative) relation is \( \{ (\text{Person, Person}) \} \). Note that, for either relation, including \( (\text{Movie, Person}) \) in the relation is inappropriate, since discovering only collaborator links is of interest.

Technically, discovering an appropriate attribution relation is within the scope of the multi-step optimization problem outlined in Section 5.3. In practice (for reasons described in that section), the problem is constrained enough for an inexpensive external algorithm (e.g. ontology matching) to be used instead \[^{19}\].

**Definition 8. Attribute-aware DNF scheme** Given a predicate universe \( U \), two (not necessarily distinct) graphs \( G_1 \) and \( G_2 \) and an attribution relation \( A \), an attribute-aware Disjunctive Normal Form (DNF) scheme \( D_A \) is a positive\(^{13}\) DNF expression \( D \) composed of the atoms in \( U \). It takes as input two distinct nodes \( v_1 \in V_1 \) and \( v_2 \in V_2 \), and returns True iff \( D \) is True and either \( A \) is empty or \( A(v_1, v_2) \) is True, and returns False otherwise.

**Definition 9. Composite DNF scheme** A Composite DNF scheme \( C \) is defined as a finite set of attribute-aware DNF schemes that takes as input two distinct nodes \( v_1 \in V_1 \) and \( v_2 \in V_2 \), and returns True (otherwise False) iff there exists a scheme \( D_A \in C \) that returns True for the pair \( (v_1, v_2) \).

**Example 8.** Assuming two attribution relations \( \{ (\text{Actor, Director}) \} \) and \( \{ (\text{Guitarist, Guitarist}) \} \), an attribute-aware DNF scheme could be devised for each of the two relations. If the training data is representative, the two schemes would presumably be different. The composite scheme may be thought of as a ‘committee’ of these two schemes. Given two distinct nodes as input, it returns True iff either one of the attribute-aware schemes returns True, and the corresponding attribution relation is satisfied.

\(^{12}\) \( R \) is presently fixed, and both t-FEOs are necessarily drawn from finite sets per Definition \(^{5}\) and the note following it.

\(^{13}\) That is, negated atoms from \( U \) are not permitted in the construction.
Concerning execution of a given DNF blocking scheme on two (not necessarily distinct) graphs $G_1$ and $G_2$ to derive a highly reduced candidate set of node-pairs (recall the original problem in Section 4), it can be shown that, under practical constraints (e.g. finiteness and boundedness), a near linear-time indexing algorithm can be applied on the graphs using a given scheme. In the Semantic Web, an example of one such algorithm is block purging [15].

5.3 Learnability

Section 5.2 presented a formalism for constructing (composite) DNF schemes on entire graphs. Given graph inputs, and training sets $P_T$ and $N_T$ of links and non-links, we would ideally like to learn a DNF scheme from the training data. This section formally explores the learnability of unconstrained DNF schemes.

As with many learning problems, learning a composite DNF scheme can be framed in terms of solving an optimization problem. We assume as inputs two (not necessarily distinct) graphs $G_1 = (V_1, E_1, l_{V_1}, A_{V_1}, \Sigma_{V_1}, \Sigma_{E_1}, \Sigma_{A_1})$ and $G_2 = (V_2, E_2, l_{V_2}, A_{V_2}, \Sigma_{V_2}, \Sigma_{E_2}, \Sigma_{A_2})$, training sets $P_T$ and $N_T$ and a finite predicate universe $U$. Let $C$, denoted as the hypothesis space, be the set of all composite DNF schemes that can be legally composed on graphs $G_1$ and $G_2$, using the predicate universe $U$. The optimization problem is stated as:

$$\arg\min_{C \in C} |\{(v_1, v_2) \in N_T | C(v_1, v_2)\}|$$

subject to the condition that,

$$|\{(v_1, v_2) \in P_T | C(v_1, v_2)\}| \geq \epsilon |P_T|$$

We denote $\epsilon$ as the minimum Expected Pairs Completeness (mEPC). Intuitively, the optimization program states that an ‘optimal’ composite scheme minimizes the number of negative training examples (the non-links) covered (Eqn. 1), while exceeding a required level of recall (i.e. $\epsilon$) with respect to the positive examples (links), at least in expectation. Note that, like other optimization problems, the problem above can be stated as a decision problem, by asking if a composite scheme exists, such that the fraction of negative examples covered does not exceed (a specified parameter) $\eta$.

The composite scheme $C$ is necessarily a finite set by virtue of $U, \Sigma_{A_1}$ and $\Sigma_{A_2}$ being finite. Intuitively, any solution to Eqns. (1)-(2) may be thought of as a multi-step procedure. First, the attribution relations governing the scope of each attribute-aware DNF scheme in the composite scheme need to be determined. Next, for each such relation, an attribute-aware DNF scheme needs to be learned. In the worst case, the two steps would not be independent: choosing the wrong relations could result in a sub-optimal composite scheme, even if each individual attribute-aware DNF scheme is optimal with respect to the training examples ‘covered’ by its corresponding attribution relation.

\textsuperscript{14} The empirical PC of any scheme $C$ on a given training set is, in fact, the expected PC relative to a full ground-truth, since the training set is sampled i.i.d (Section 4).

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Fig. 3. A tree representation, rooted at the special (see text) node Record-1, of the first record in the table in Figure 2. The graph-theoretic representation of the entire table (with five records) is a forest of five trees, each constructed as illustrated above.

Given this dependency and the expressiveness of DNF schemes, a natural question is if a tractable solver for Eqns. (1)-(2) exists. The following theorem provides strong evidence against such an existence.

**Theorem 1.** The decision version of Eqns. (1)-(2) is NP-hard.

**Proof.** In prior work on DNF blocking scheme learning for homogeneous tabular deduplication [2], a simpler version of the decision problem was shown to be NP-hard, by demonstrating a reduction from a known NP-hard problem (red-blue set covering [4]). To describe the original DNF blocking problem formally, consider as input a table with $m$ columns (‘fields’) and $n$ rows (‘records’), training sets $P_T$ and $N_T$ of record pairs representing duplicates and non-duplicates respectively, and a finite set of $g$ general blocking predicates. Using the $g$ general predicates and the $m$ fields, a predicate universe $U$ of $gm$ specific predicates can be constructed. Returning to the table in Figure 2 as the running example for the purposes of proving the theorem, CommonToken is a general predicate and CommonToken_Name is a specific predicate.

Given the inputs above, real-valued parameters $\eta, \epsilon \in [0, 1]$ and a positive integer $k$, the decision problem is to determine whether a DNF blocking scheme (a positive DNF formula) with at most $k$ literals per conjunct exists, such that (1) at least a fraction $\epsilon$ of the positive training set $P_T$ is covered, and (2) at most a fraction $\eta$ of the negative training set $N_T$ is covered. These two conditions are directly analogous (in the tabular setting) to the decision versions of Eqns. (1)-(2).

A subtle point to note here is that the integer parameter $k$ has a finite range $k \in [1, |U|]$, which depends on $m$ and $g$, since $|U| = gm$. If $k$ falls beyond this range (i.e. $> |U|$), the problem is identical to the one with $k = |U|$, since we allow at most $k$ literals per conjunct in the formulation above, and by definition, a conjunct can have at most $|U|$ (positive) literals.

In their treatment, Bilenko et al. [2] showed that even the relatively simple problem of deciding whether a purely disjunctive scheme ($k = 1$) exists that satisfies conditions (1) and (2) above, and with other inputs and parameters
arbitrarily determined, is NP-hard. Earlier, we mentioned that this was accomplished by poly-time reduction from red-blue set covering \[4\]. It is sufficient, then, to ‘translate’ any arbitrary table into a data graph, such that if\[15\] a disjunctive scheme exists that satisfies the decision version of Eqn. (1) (parameterized by the parameter \(\eta\) input to the original problem), and Eqn. (2), then a disjunctive scheme on the tabular instance also exists such that conditions (1) and (2) are satisfied\[16\].

To map a table to a data graph \(G\), we begin by constructing the set \(V\) of nodes. First, we construct a node corresponding to each cell in the table. We then add \(n\) additional nodes (denoted as **special nodes**), with \(n\) being the number of rows in the table. The node vocabulary \(\Sigma_V\) is the set of all cell values in the table; the node label mapping \(l_V\) is simply a tabular lookup for all the nodes that correspond to cells in the table. For the \(n\) additional nodes, we configure the label to return a special placeholder value; this value will not be used in the rest of the treatment. The edge vocabulary \(\Sigma_E\) is the set of column labels, and only comprises \(m\) unique elements.

The edges themselves are constructed as follows. We permit outgoing edges only from special nodes. Intuitively, we can think of a special node as representing a row in the table. Each outgoing edge joins the special node to a ‘cell’ node, and is appropriately labeled per the column from which the cell node was derived. Figure 3 illustrates this process for the first record in Figure 2. Note that there are exactly \(m\) outgoing edges per special node, and in total, there are \(mn\) edges in the graph. Thus, the data graph may be thought of as a forest of rooted trees, with each tree having depth 2 and exactly \(m\) leaves. The root of the tree is always a special node. Finally, let all nodes in the graph be unattributed. The attribute vocabulary \(\Sigma_A\) is the empty set, and the attribute mapping \(A_V\) is configured to return the empty set for all nodes.

In the graph-theoretic setting, the training set inputs \(P_T\) and \(N_T\) are pairs of special nodes, rather than pairs of records. Since each special node bijectively maps to a record in the original table, this translation is straightforward.

It remains to construct the predicate universe \(U\). From Definition \[9\] each such predicate was parametrized as a 5-tuple \((R, f_1, f_2, S_1, S_2)\), where the symbols have their usual meanings. We fix the set-based relation \(R\) as thresholded Jaccard, with the threshold \(\theta = 0\) as earlier discussed. We also constrain the feature extraction operators (FEOS) to be identical (i.e. \(f_1 = f_2 = f\)). There are exactly \(g\) possible values for \(f\), with \(g\) being the number of general blocking predicates in the original problem formulation. In a similar way, we constrain the edge-label sequence sets to be identical in each predicate \((S_1 = S_2 = S)\); each set is furthermore constrained to contain exactly one edge-label sequence, which itself contains exactly one label from \(\Sigma_E\).

\[\text{It is important to note that this is not bidirectional. We only need to show a reduction from the (known) NP-hard problem, such that an oracle for the reduced problem can be used to determine a solution to the original problem.}\]

\[\text{In fact, the subsequent construction will show a straightforward bijection between two such ‘satisfying’ schemes (tabular and graph-theoretic respectively).}\]
Using the rules above, each predicate has exactly \( gm \) possible parametrizations, and on this account, the predicate universe \( U \) has \( gm \) trail-sensitive predicates. Evidently, each trail-sensitive predicate can be bijectively mapped to the set of specific blocking predicates (in the tabular formulation) earlier described. By way of example, the specific blocking predicate \( \text{CommonToken}_{\text{Name}} \) is bijectively mapped to a trail-sensitive predicate with parameters \( f = \text{CommonToken} \) and \( S = \{ (\text{Name}) \} \).

Because the attribution relation (Definition 7) is necessarily empty in this construction, the composite scheme will only consist of a \textit{single} attribute-aware DNF scheme, constrained to be disjunctive by the parameter \( k \) (maximum number of literals per conjunct). If any such scheme exists that satisfies the decision conditions earlier described, it can be bijectively\(^\text{17}\) mapped to a disjunctive blocking scheme in the tabular domain. We claim that a satisfying attribute-aware scheme, thus mapped, also satisfies the decision conditions in the original (i.e. tabular) problem specification\(^\text{18}\).

Finally, note that we do \textit{not} claim that the decision problem in the theorem is NP-complete, although we \textit{conjecture} that it is. We leave an investigation of this claim for future work; from a practical standpoint, showing NP-hardness suffices for the present purposes of the paper.

Theorem 1 illustrates a natural tradeoff between the \textit{expressiveness} of DNF schemes (when they are not subject to any constraints) and their learning properties. Generally, edge-discovery tasks are rarely unconstrained. For example, if the task is entity resolution in the Semantic Web, a first step is to use ontology alignment to bound the possible attribute relations \(^\text{19}\). In the next step, an approximate attribute-aware DNF scheme learning (for each attribute relation output by the ontology aligner) can be learned. In prior work on DNF schemes, a variety of greedy approximation algorithms have already been proposed for the homogeneous tabular deduplication task, including beam search \(^\text{13}\), greedy set covering \(^\text{2}\), and feature selection \(^\text{7}\). In recent work, we developed and evaluated an approximation algorithm for entity resolution on RDF graphs \(^\text{8}\). The empirical results are discussed in Section 6.

6 Empirical Demonstration

Although the primary developments in this work were theoretical, they were motivated by practical large-scale issues in graph-based ecosystems such as the Semantic Web. Recently, we designed an unsupervised entity resolution (ER) system for \textit{schema-free} (i.e. heterogeneous) RDF data \(^\text{8}\). Using bounded parameters and a set of 28 manually crafted extractors (Definitions 2 and 3), we presented an approximation algorithm to learn DNF schemes from training data.

\(^{17}\) The bijection is simple: substitute each trail-sensitive predicate in the expression with the equivalent specific blocking predicate.

\(^{18}\) A correctness proof of this claim is straightforward; we omit it here.
Table 2. Comparative results of a DNF blocking scheme learner (DNF-BSL) against the Attribute Clustering (AC) baseline

| Test Case    | Proposed DNF-BSL | Attribute Clustering |
|--------------|------------------|---------------------|
|              | PC               | RR | F-score   | PC | RR | F-score   |
| 1 Persons 1  | 100.00%          | 99.75% | **99.88%** | 100.00% | 98.86% | 99.43% |
| 2 Persons 2  | 99.00%          | 99.79% | **99.39%** | 99.75% | 99.02% | 99.38% |
| 3 Restaurants| 100.00%          | 99.73% | **99.87%** | 100.00% | 99.57% | 99.79% |
| 4 Eprints-Rexa| 98.16%          | 99.28% | 98.72% | 99.60% | 99.37% | **99.48%** |
| 5 IM-Similarity| 100.00%        | 98.14% | **99.06%** | 100.00% | 62.79% | 77.14% |
| 6 IIMB-059   | 99.76%          | 93.35% | **96.45%** | 97.33% | 73.09% | 83.49% |
| 7 IIMB-062   | 47.73%          | 98.11% | 64.22% | 97.27% | 90.80% | **83.49%** |
| 8 Libraries  | 97.96%          | 99.99% | 99.96% | 99.99% | 99.87% | **99.93%** |
| 9 Parks      | 95.96%          | 94.41% | **95.18%** | 99.07% | 88.27% | 93.36% |
| 10 Video Game| 98.73%          | 99.96% | 99.34% | 99.72% | 99.85% | **99.79%** |
| **Average**  | **93.73%**      | **98.25%** | **95.11%** | **97.27%** | **91.15%** | **93.53%** |

Note that, because the system was designed to be unsupervised, a heuristics-based component called a training set generator (TSG) was also a part of the system; the training examples used to bootstrap the learning processes in the entire system were output by this TSG. Due to its unsupervised nature, the TSG could make mistakes: in many cases, the precision of the generated training set was well below 80%. This, in turn, imposed a strong robustness requirement on the entire system, especially blocking scheme learning.

To evaluate DNF blocking scheme learning, we gathered a set of ten RDF test cases\(^{19}\) and used a token-based blocking algorithm known as token-based Attribute Clustering (AC) as a baseline\(^{20}\). The AC algorithm was designed for the two-graph scenario mentioned in Section 4. It is non-adaptive; the algorithm uses a pre-defined similarity model to cluster edge-label sets \(\Sigma_{E_1}\) and \(\Sigma_{E_2}\). An example of a similarity model is using instance-based measures (like cosine similarity) on corresponding object-values. Once the clusters are obtained, entities can be assigned to blocks based on whether they share common tokens (or by extension, other features) in at least two object values corresponding to edges that were assigned to the same cluster\(^{15}\).

Table 2 illustrates the experimental results. The metrics used are Pairs Completeness (PC), Reduction Ratio (RR) and their harmonic mean (F-score). PC and RR were earlier defined in Section 4. The results in Table 2 show that, due to its adaptive nature, the DNF approximation algorithm (1) was able to outperform AC on the F-score metric on six out of ten test cases, (2) achieved a mean RR that was over 7.5% higher than that achieved by AC, with a mean

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19 These test cases are detailed in the original journal article where we described the overall unsupervised entity resolution system\(^{8}\).

20 As indicated at the end of Section 3, an ‘attribute’ in\(^{15}\) was defined as a set of edge label-object value pairs associated with an entity (a node in the data graph). Herein, the word was used in the traditional graph-theoretic sense.
loss in PC just below 2.6\%, and (3) had stable RR performance, with 2.42\% standard deviation across all ten test cases, compared with 13.13\% deviation for AC. (3), in particular, shows that adaptive DNF learning is reliable compared to AC, which can be an important concern in real-world linkage scenarios that exhibit *dynamicty*, such as Linked Open Data. Even with noisy training data, the learner continued to exhibit stable RR. The competitive performance with AC shows that the DNF schemes are applicable to *schema-free* data.

There are at least two possible reasons behind the performance numbers in Table 2. First, it could be the case that AC is not as *expressive* as DNF blocking. The following theorem formalizes this intuition:

**Theorem 2.** There exists a finite predicate universe $U$ such that an Attribute Clustering (AC) blocking scheme, as presented in [15] (Algorithm 1), can be expressed as a single attribute-aware DNF scheme (Definition 8) that is a disjunction of all the predicates in $U$.

**Proof.** The Attribute Clustering (AC) algorithm assumes as inputs two distinct RDF graphs that we express as constrained data graphs $G_1 = (V_1, E_1, l_{V_1}, A_{V_1}, \Sigma_{V_1}, \Sigma_{E_1}, \Sigma_{A_1})$ and $G_2 = (V_2, E_2, l_{V_2}, A_{V_2}, \Sigma_{V_2}, \Sigma_{E_2}, \Sigma_{A_2})$. As there is no concept of node attribution in AC, we let $\Sigma_{A_1} = \Sigma_{A_2} = \emptyset$; the attribute mapping (of each graph) always returns the empty set when applied on a node.

As the name of the algorithm suggests, AC operates by first clustering attributes (which are edge-labels in our graph-theoretic framework), and then using the clusters to perform blocking. Formally, a cluster $c$, in the context of [15], is defined as a pair of sub-clusters $(P_1, P_2)$, where $P_1 \subseteq \Sigma_{E_1}$ and $P_2 \subseteq \Sigma_{E_2}$. For notational convenience, we use superscripts to indicate the specific sub-cluster of $c$ (i.e. $c^1$ and $c^2$ respectively refer to $P_1$ and $P_2$). A set $C$ of non-overlapping clusters is defined as a non-empty set $\{c_1, \ldots, c_n\}$ of $n$ clusters, such that for any two distinct clusters $c_i, c_j$ from $C$, it is always the case that $c^1_i \cap c^1_j$ and $c^2_i \cap c^2_j$ are both empty. $C$ is *exhaustive* iff $\bigcup_c c^1_i = \Sigma_{E_1}$ and $\bigcup_c c^2_i = \Sigma_{E_2}$. As a first step, AC uses a pre-defined similarity model that takes two graphs as input and determines a non-overlapping, exhaustive set of clusters on their respective edge-labels. The authors describe several such models, but we note herein that the way such a clustering is performed is irrelevant to the current theorem, as we are only interested in the *representation* of AC schemes.

To formally describe the blocking semantics of AC (once the set of clusters $C$ is obtained), we introduce some additional notation. Let us respectively denote $v_1 \in V_1$ and $e_1 \in \Sigma_{E_1}$ as an *entity* and *edge-label* from graph $G_1$. Given $e_1$, let the set of *outgoing nodes* of $v_1$ be defined as the set of all nodes $v$ such that there exists at least one edge $(v, e_1)$ that is of the form $(v_1, v, e_1)$. Note that there can be multiple outgoing nodes (for a particular node and edge-label). For example, suppose an entity represents a person with two addresses. Both address

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21 Importantly, high, stable RR is essential for *high volume* tasks because RR grows quadratically with the number of nodes, and even small improvements or variations (less than a percent) disproportionately impact candidate set size.
nodes would be included in the set defined above for that entity and an address edge-label.

In a similar vein, we define the set $L_1(v_1,e_1)$ (on graph $G_1$) as the set of all outgoing node labels. This set is constructed by first constructing the set of all outgoing nodes (for inputs $v_1$ and $e_1$) and then replacing each outgoing node with its node-label mapping $l_{V_1}$.

It is straightforward to extend the above notation when the input is a node and a set $P_1$ of edge-labels. We use the symbol $L_1(v_1,P_1)$ to express the result $\bigcup_{e_1 \in P_1} L_1(v_1,e_1)$. Finally, note that the same definitions apply to graph $G_2$; as in the rest of this paper, we use the subscript to disambiguate between the two graphs.

The final construction is of a cluster-specific tokens set, denoted by the symbol $Tokens_1(v_1,c)$, where $c$ is a cluster from the set $C$. This construction makes use of the TokenizeString primitive shallow extractor that we introduced in Example 2. Specifically, we define $Tokens_1(v_1,c)$ by the expression $\bigcup_{l \in L_1(v_1,P_1)} \text{TokenizeString}(l)$. Note that $P_1$ is the sub-cluster $c^1$ of $c$. Tokens can ‘operate’ on an entity from either graph; the specific sub-cluster used in the formulation above depends on the graph (hence, the subscript on $Tokens$).

We now describe the blocking semantics of Attribute Clustering. Two entities $v_1$ and $v_2$ (i.e. a pair from $V_1 \times V_2$) are assigned at least one common block iff $\exists c \in C$ such that $Tokens_1(v_1,c) \cap Tokens_2(v_2,c) \neq \emptyset$. We denote this condition as the blocking condition.

To prove the theorem, we construct a predicate universe $U$ as follows. Recall that every element (a trail-sensitive predicate) in $U$ is parametrized as a five tuple $(R,f_1,f_2,S_1,S_2)$ where the symbols stand for their usual meaning (Definition 6). We fix $R$ as thresholded Jaccard with the threshold $\theta = 0$, and we also fix the feature extraction operators (FEOs) $f_1 = f_2 = f$ as a single-element sequence, namely $(\text{TokenizeString})$. The remaining parameters $S_1$ and $S_2$ can (together) take $|C|$ values. Specifically, we constrain joint assignments to $S_1$ and $S_2$ as two sets $P_1$ and $P_2$, such that $(P_1,P_2) \in C$.

By the construction above, there are exactly $|C|$ trail-sensitive predicates in $U$. By the existence qualifier in the blocking condition defined above (which constitutes the blocking semantics of AC as described in [15]), the semantics of AC are precisely captured by a disjunction of all predicates in $U$. We express this as a single disjunctive attribute-aware DNF scheme (Definition 8), qualified by an empty attribution relation (Definition 7). This completes the proof.

This theorem shows that, given a particular ‘reasonable’ predicate universe, AC does not take into account node attribution (and is hence expressible as a single attribute-aware DNF scheme). On this account, a general (i.e. composite) DNF expression is strictly more expressive.

A second issue is that AC schemes are non-adaptive, and cannot be learned from training data (whether manually or automatically constructed). This im-

\textsuperscript{22} Per Definition 4, each FEO is a non-empty sequence where the first element is always a primitive shallow extractor, followed by a (possibly empty) sequence of primitive deep extractors (PDEs). In this case, there are no PDEs.
plies that its performance may not be as competitive for ‘peculiar’ datasets and domains. On the other hand, DNF schemes, in the formulation presented in this paper, can be learned using approximation techniques from the complexity-theory literature. Finally, unlike AC, which requires access to the entire dataset to formulate its predicates [15], DNF schemes only need access to limited training data. This gives them an advantage of scale in cases where the entire dataset, but not the required fraction of training examples, is too large to fit in memory.

7 Conclusion and Future Work

In this paper, we presented a graph-theoretic construction for DNF schemes, applicable to a directed, labeled attributed data graph model. The presented schemes are functions that are useful for reducing pairwise (i.e. quadratic) complexity in sparse supervised machine learning-based edge-discovery on either a single data graph or between two data graphs. Previously, the DNF schemes had only been proposed for homogeneous tabular deduplication. Table 1 summarizes the technical contributions in this work. An optimization-based framework can be used for learning the schemes. The empirical promise of these schemes (in terms of high volume, dynamicity and stability) was demonstrated in real-world settings against the competitive Attribute Clustering baseline.

Given the general applicability of DNF schemes, there are several (theoretical and practical) avenues for future work; by way of example, we state below a conjecture that we believe to be significant:

**Conjecture** Given a sufficiently expressive collection $\mathcal{R}$ of set-based relations, the class of complexity-reduction methods proposed in the Limes framework [14] can be expressed as DNF schemes using the formalism in Section 5.2.

The conjecture is important because complexity-reduction in Limes (and also Silk) is predicated on a known link specification function (LSF). If true, the conjecture presents a way of studying research questions and tradeoffs that straddle the different assumption sets covered by LSF-aware and LSF-agnostic systems.

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