The Bose-Einstein Correlations
and the strong coupling constant at low energies

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Abstract

It is shown that $\alpha_s(E)$, the strong coupling constant, can be determined in the non-
perturbative regime from Bose-Einstein correlations (BEC). The obtained $\alpha_s(E)$ is in
agreement with the prescriptions dealt with in the Analytic Perturbative Theory approach.
It also extrapolates smoothly to the standard perturbative $\alpha_s(E)$ at higher energies. Our
results indicate that BEC dimension can be considered as an alternative approach to the
short range measure between hadrons.

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1 Introduction

In recent years Bose-Einstein (BEC) and Fermi-Dirac (FDC) correlations have been extensively studied mainly with identical pion pairs produced in lepton-lepton and hadron-hadron reactions, as well as in heavy ion (AA) collisions. In the one dimension (1D) correlation analyzes of pion and hadron pairs it was found that the resulting $R_{1D}$ dimension depends on the particle mass and found to be proportional to $1/\sqrt{m}$ where $M$ is the mass of the correlated particles (See e.g. Ref. [1]). It has been further shown [2] that this $R_{1D}(m)$ behavior can be described in terms of the Heisenberg uncertainty relations and from a general QCD potential considerations.

The two identical particle correlation effect can be measured in terms of the correlation function

$$ C(p_1, p_2) = \frac{\rho(p_1, p_2)}{\rho_0(p_1, p_2)}, $$

where $p_1$ and $p_2$ are the 4-momenta of the two correlated hadrons and $\rho(p_1, p_2)$ is the two particle density function. The $\rho_0(p_1, p_2)$ stands for the two particle density function in the absence of the a BEC (or FDC) effect. This $\rho_0$ is often referred to as the reference sample against which the correlation effect is compared to. The BEC and FDC analyzes had often different experimental backgrounds and have chosen various types of reference $\rho_0(p_1, p_2)$ samples. Thus one has to take this situation in account when judging the correlation results in terms of energy and/or mass dependence.

The function frequently used in the BEC and the FDC studies the evaluation of the $R_{1D}$ are:

$$ C(Q) = 1 + \lambda e^{-Q^2R_{1D}^2} \text{ for bosons and } C(Q) = 1 - \lambda e^{-Q^2R_{1D}^2} \text{ for fermions} $$

These are the Goldhaber parametrizations [3] proposed for a static Gaussian particle source in the plane-wave approach which assumes for the particles emitter a spherical volume with a radial Gaussian distribution. The $\lambda$ factor, also known as the chaoticity parameter, lies within the range of 0 to 1. Due to the fact that the major correlation experiments were carried out with identical bosons we will here focus our discussion no the Bose-Einstein correlations.

It has been noticed already some two decades ago that the $R_{1D}$ extracted from BEC and FDC analyzes of hadron pairs produced in the decay of the $Z^0$ gauge boson suggested a mass dependence roughly proportional to $1/\sqrt{m}$ [2]. This is illustrated in Fig. 1 taken from reference [5] where a compilation of the $R_{1D}$ results was obtained from the $Z^0$ hadronic decay experiments at LEP, The difference between the $R_{1D}$ value at the pion mass to those of the proton and $\Lambda$ baryons is indeed impressive. However presently no significant difference is seen between the $R_{1D}$ of the pions and the K-mesons produced in the $Z^0$ decay. Thus this dimension data, deduced from the $e^+e^-$ prompt interactions, cannot serve for a precise expression for the $R_{1D}$ dependence on energy. For that reason we utilize here the BEC dimension results obtained in $Pb-Pb$ collisions experiments [6].

In this letter we show that the BEC can serve in the evaluation of the strong coupling constant $\alpha_s$ at the non-perturbative region of $E < 1$ GeV. The resulting coupling constant is shown to be in good qualitative agreement with the one obtained from solving the Bethe-Saltpeter equation to determine the effective potential of the quarkonium which in turn is consistent with the $\alpha_s$ deduced from an Analytic Perturbative Theory (APT) prescription.

In section 2 we discuss the mass dependence of the BEC source scale and in section 3 we derive an analytic formula relating the strong coupling constant $\alpha_s$ and the BEC source radius.
Figure 1: A compilation of $R_{1D}$ versus the hadron mass obtained from BEC and FDC analyzes of the $Z^0$ hadronic decays of the LEP experiments [1, 4] taken from Ref. [5]. The solid line and the dotted lines represent respectively Eq. (7) with $\Delta t=10^{-24}$ sec and $\Delta t=(1 \pm 0.5) \times 10^{-24}$ sec.

Finally in section 4 we present numerical values for $\alpha_s$ and show that the BEC derived source dimension corresponds to a strong overlapping hadrons where the BEC source radius is of the order of the distance between them.

2 The mass dependence of the BEC dimension $R_{1D}$

Since the maximum of the BEC enhancement of two identical bosons of mass $m$ occurs when $Q \to 0$, the three vector momentum difference of the bosons approaches zero. Thus we can link the BEC effect to the Heisenberg uncertainty principle [2], namely

$$\Delta p \Delta r = 2\mu vr = mv = \hbar c,$$  \hspace{1cm} (3)

where $\mu$ is the reduced mass of the di-hadron system and $r = \Delta r$ is the distance between them. Here we use for $\Delta p$ the GeV unit while $r$ is given in fm units so that $\hbar c = 0.197$ GeV fm. Thus one obtains:

$$r = \frac{\hbar c}{mv} = \frac{\hbar c}{p}. \hspace{1cm} (4)$$

Simultaneously we also apply the uncertainty relation expressed in terms of time and energy

$$\Delta E \Delta t = \frac{p^2}{m} \Delta t = \hbar,$$  \hspace{1cm} (5)

where the energy and $\Delta t$ are given respectively in GeV and seconds. Thus one has

$$p = \sqrt{\hbar m/\Delta t}. \hspace{1cm} (6)$$
Figure 2: BEC analyzes of hadron pairs emerging from central $Pb-Pb$ collisions at 158/A GeV [6]. The continuous line represents a fit of Eq. (7) to the data of the Plastic Ball detector. The result of the fit where the WA98 data and the Kaon-pair $R_{1D}$ value from the NA44 collaboration were also included is shown by the dashed line.

Inserting this expression for $p$ into Eq. (4) one finally obtains

$$r(m) = \frac{hc}{\sqrt{m}} \frac{\sqrt[2]{\Delta t}}{\hbar} = \frac{c\sqrt{\hbar \Delta t}}{\sqrt{m}}. \quad (7)$$

Comparing values of $r$ in Eq. 7 and experimental data for $R_{1D}$ (see Fig. ??) we are lead to identify $r$ with $R_{1D}$.

As it was mentioned above the $R_{1D}$ values, deduced from the BEC and FDC analyzes of the $Z^0$ hadronic decays are shown in Fig. 1. These results provided the first clue that the $R_{1D}$ may depend on the mass of the two identical correlated particles [2]. As can be seen, the measured $R_{1D}$ values of the pion pairs are located at $\sim 0.6$ fm with the exception of one $\pi^0\pi^0$ result where its $R_{1D}$ value lies significantly lower. The $R_{1D}$ Kaon pairs values are seen to be near to those of the charged pions. No however are the $R_{1D}$ values obtained from the $\Lambda$ hyperon and proton baryon pairs which lie close together in the vicinity of 0.15 fm. The solid line in the figure was calculated from Eq. (7) with $\Delta t = 10^{-24}$ sec representing the strong interactions time scale. The dashed lines are derived from Eq. (7) setting $\Delta t = (1 \pm 0.5) \times 10^{-24}$ sec in order to illustrate the sensitivity of Eq. (7) in its ability to estimate the energy dependence of $R_{1D}$. An alternative way to extract $R_{1D}$ dependence on the energy is to use the BEC results of the boson pairs produced in $Pb-Pb$ collisions.

A clear evidence for the dependence of $R_{1D}$ on the mass of the BEC boson pairs is seen in Fig. 2 that was obtained by the WA98 collaboration [6]. In Figure 2 are shown the BEC dimension deduced from identical correlated boson pairs, including the deuteron pairs, produced in $Pb-Pb$ collisions at the nucleon-nucleon center of mass energy of 158 GeV/A. As can be seen, apart from the proton pair result, the $R_{1D}$ dependence on the mass value is very well described by $A/\sqrt{m}$, with the fitted value of $A = (2.75 \pm 0.04)$ fm GeV$^{1/2}$. According to Eq. (7) one finds that $A = c\sqrt{\hbar \Delta t}$ so that in the $Pb-Pb$ collisions case $\Delta t = (1.28 \pm 0.04) \times 10^{-22}$ sec. Taking for prompt $pp$ collision the representing strong interaction value of $\Delta t = 10^{-24}$ sec one obtains
for $R_{1D}$ versus the mass, in GeV units, the relation

$$R_{1D} = \frac{0.244 \pm 0.005}{\sqrt{m(\text{GeV})}} \, \text{fm}, \quad (8)$$

which is shown by a ±1 s.d. band in Fig. 3 which is consistent with the LEP result.

![Figure 3: The ±1 s.d. $R_{1D}$ band versus the mass energy of the boson pairs produced at a time delay of $\Delta t = 10^{-24}$ sec as deduced from BEC of hadron pairs produced in $Pb-Pb$ collisions at 158 GeV/A [6] with a time delay of $\Delta t = (1.28 \pm 0.04) \times 10^{-22}$ sec (see text).](image)

3 The strong coupling constant and the BEC

The short range interactions between two hadrons can be described in terms of the constituent quark model. This idea dates back to [7] (see also [8]) and was applied to the BEC in [2]. Namely, the short range interaction between hadrons can be described by means of the quark-quark interaction potential [9]

$$V(r) = -(4/3)\alpha_s \hbar c/r + \kappa r. \quad (9)$$

The coupling constant $\alpha_s$ is usually taken as a parameter to be fitted, while the constant $\kappa$, that corresponds to the confinement part of the interaction, is of order of 0.9 GeV/fm [10], while $r$ is the distance between the two hadrons.

We now make use of the virial theorem for the two hadron system, which has the form

$$\langle 2T \rangle = \langle \vec{r} \cdot \vec{\nabla} V(r) \rangle, \quad (10)$$

where $\langle T \rangle$ is the average kinetic energy of the hadrons. Taking into account the spherical symmetry of the potential and the uncertainty relations discussed above, one obtains

$$\frac{(\hbar c)^2}{m} = r^3 \frac{dV}{dr}. \quad (11)$$
This yields straightaway an expression for the strong coupling constant, namely

\[ r^3 \left( \kappa + \frac{4 \alpha_s \hbar c}{3 r^2} \right) - \frac{(\hbar c)^2}{m} = 0 , \quad (12) \]

from which one has that \( \alpha_s \) is equal to

\[ \alpha_s = \frac{3 (\hbar c)^2 - r^3 \kappa m}{4 m r \hbar c} . \quad (13) \]

Inserting \( \hbar c = 0.1973 \) GeV fm one obtains

\[ \alpha_s = \frac{1.267(0.1168 - 3r^3km)}{mr} . \quad (14) \]

To evaluate \( \alpha_s \) we use for the parameter \( \kappa \) the value of 0.18 GeV\(^2\)/0.9 GeV/fm \([10]\) corresponding to the meson Regge trajectory. The variable \( r \) and its mass dependence are taken to be identical to the \( R_{1D} \) dimension given by Eq. (8) which was determined from the analyzes of the BEC and FDC deduced from identical hadron pairs (see also \([2]\)).

### 4 Conclusions.

Our main results are shown in Fig. 4. Since our system in the center of mass energy is non-relativistic, the \( \alpha_s \) that we determine corresponds to an energy scale of \( E \sim m \). The non-perturbative \( \alpha_s \) is calculated via Eq. (14) and is shown in Fig. 4 as a function of energy

![Figure 4](image-url)
by the solid line. The accompanying dotted lines represent the ±1 s.d. limits of the band. For comparison we also show the perturbative $\alpha_s$ curve, labeled by pQCD, which essentially overlaps with the non-perturbative strong coupling in the region of about 2 to 11 GeV. Using our low energy non-perturbative strong coupling we obtain for example $\alpha_s$ at the mass energy of the K-meson and the $\Lambda$-hyperon respectively the following values:

$$\alpha_s(0.494 \text{ GeV}) = 0.451 \pm 0.035 \quad \text{and} \quad \alpha_s(1.115 \text{ GeV}) = 0.392 \pm 0.019$$

The non-perturbative $\alpha_s$ determined here as a function of energy agrees well with the results obtained in [12,13], where effective $\alpha_s$ was obtained by solving the Bethe-Salpeter equation for quarkonium. As a result the strong coupling constant $\alpha_s$ determined here agrees well with the one-loop Analytic Perturbative Theory (APT) approach [14]). In particular we have good agreement with the so called "massive" variation of the APT prescription [14,15]. The latter one approach coincide with the standard APT approach for energy scales of $E > 200 \text{ MeV}$ [16], i.e. above the pion mass. However for small $E$ the strong coupling constant goes down to zero [14,15], exactly as we have in Fig. 4. It is worthwhile to note however, that although our curve is in good agreement with the "massive" APT prescription at small $E$ of order the pion mass, strictly speaking it is questionable if we can apply at the pion scale our estimates, based on simple nonrelativistic quark model. Thus "reasonable" agreement of our result for the pion mass with a particular version of APT prescription deserves further study.

In conclusion the strong coupling constant $\alpha_s(E)$ can be evaluated in the non-perturbative region by the use of the Bose-Einstein and Fermi-Dirac correlations dimension results. The resulting $\alpha_s(E)$ is in good agreement with the so called APT "massive" prescription [14,15] and extrapolates well at the higher energies to the conventional perturbative $\alpha_s(E)$. Our results indicate that the BEC/FDC correlations both for baryons and mesons, correspond to a picture where the two participating hadrons strongly overlap, and the $R_{1D}$ radius, that conventionally characterizes the scale of the BEC/FDC, corresponds to the distance $r$ between the centers of these two correlated particles. Thus our results indicate that these correlations may well serve as an alternative approach for the study of short range correlations between hadrons [17].

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