Digital Particle Image Velocimetry: Partial Image Error (PIE)

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Abstract. This paper quantifies the errors due to partial imaging of seeding particles which occur at the edges of interrogation regions in Digital Particle Image Velocimetry (DPIV). Hitherto, in the scientific literature the effect of these partial images has been assumed to be negligible. The results show that the error is significant even at a commonly used interrogation region size of 32 x 32 pixels. If correlation of interrogation region sizes of 16 x 16 pixels and smaller is attempted, the error which occurs can preclude meaningful results being obtained. In order to reduce the error normalisation of the correlation peak values is necessary. The paper introduces Normalisation by Signal Strength (NSS) as the preferred means of normalisation for optimum accuracy. In addition, it is shown that NSS increases the dynamic range of DPIV.

1. Introduction

Digital Particle Image Velocimetry (DPIV) is an important non-intrusive, whole-field measurement technique, which provides quantitative measurement and instantaneous snapshots of velocity fields over a two-dimensional region within a flow. With recent advancements in laser technology and digital cameras time-resolved DPIV systems, capable of producing velocity maps at kHz rates, have become commercially available. Time-resolved DPIV is now capable of providing high spatial and temporal resolution velocity field data, which can provide turbulence statistics for the refinement of CFD codes. It is important that inherent random errors associated with time-resolved DPIV measurements are quantified before comparisons with code predictions are attempted.

In a DPIV experiment images of seeding particles from a light sheet within the flow are captured twice, separated by a small time interval. These flow images are then subdivided into interrogation regions and corresponding regions are cross-correlated typically using a fast Fourier Transform algorithm (FFT) in the frequency domain. This produces a correlation peak whose position corresponds to the average particle image displacement within the region. A curve fitting routine (typically three-point Gaussian) is used to estimate the position of the centre of the peak to sub-pixel accuracy. The accuracy of this sub-pixel measurement is a function of the Signal to Noise Ratio (SNR) defined by the ratio of the height of the correlation peak to the average background noise level. This ratio is affected by several factors including velocity gradients across the interrogation region, seeding image density, inadequate pixel resolution, particle image size, electronic imaging noise, uncorrelated particle images, irregular shaped images and

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intensity variations both across and between image snapshots. The effect of these parameters has been well documented in the literature [1] and a carefully designed experiment can reduce their error to an acceptable level.

It is important to quantify the inherent error which occurs in a time resolved DPIV experiment due to the random positioning of seeding particle images between subsequent measurements. For an actual displacement of \( d_a \) we define a mean-displacement \( d_m \) and a mean-bias error \( d_b \) over \( N \) realisations by:

\[
d_m = \frac{1}{N} \sum_{i=1}^{N} d_i \iff d_b = d_m - d_a
\]

where \( d_i \) is the displacement value from a single measurement.

In addition, the random root mean square (rms) error, \( \sigma \) is defined by:

\[
\sigma = \left[ \frac{1}{N} \sum_{i=1}^{N} (d_i - d_m)^2 \right]^{1/2}
\]

In this way, the inherent measurement error in calculating the average velocity from \( d_m \) and the turbulence intensity from \( \sigma \) for a time-resolved DPIV experiment can be quantified before comparison with a CFD code prediction is attempted.

2. Processing Methods

2.1. Interrogation Region Correlation

For equal-size interrogation regions (typically 32 x 32 pixels) image displacement is calculated by cross-correlation and the technique favoured by most commercial DPIV systems is correlation in the frequency domain using the fast Fourier transform (FFT). The latter is used to reduce computation time. For two images \( I_1(i, j) \) and \( I_2(i, j) \) from the same interrogation region their cross-correlation \( \otimes \) can be written:

\[
I_1(i, j) \otimes I_2(i, j) \iff FFT^{-1} \left( \hat{I}_1(\xi, \eta) \times \hat{I}_2^*(\xi, \eta) \right)
\]

where \( \hat{I}_1(\xi, \eta) \) denotes the Fourier transform of \( I_1(i, j) \) and \( \hat{I}_2^*(\xi, \eta) \) represents the complex conjugate of the Fourier transform of \( I_2(i, j) \). Pixel position is denoted by \( (i, j) \).

For equal or unequal sized interrogation regions the cross-correlation \( R(m, n) \) defined by:

\[
R(m, n) = \sum_i \sum_j I_1(i, j) I_2(i + m, j + n)
\]

can be computed directly in the spatial domain. Computation of the cross-correlation defined by equation (3) suffers from ‘wrap-around’ effects due to the assumed periodicity of the signal when the FFT algorithm is used. These errors can be minimised by the use of zero-padding [2]. If zero-padding is used then to a very good approximation the correlation computed by the FFT method defined by equation (3) equates to \( R(m, n) \) define by equation (4) [3].
2.2. Correlation Peak Location and Sub-Pixel Interpolation

Once the correlation field has been calculated the position of the highest peak (the signal peak) determines the average displacement of the seeding particle images within the interrogation region to an accuracy of ± 0.5 pixels. It is usual to use a curve fitting routine around the peak value to determine displacement to sub-pixel accuracy. The most common routine is the Gaussian and if the pixel location of the signal peak is given by \((x_s, y_s)\) the actual peak centre position \((x_{pk}, y_{pk})\) is given by [4]:

\[
x_{pk} = x_s + \frac{[\log R(x_s - 1, y_s) - \log R(x_s + 1, y_s)]}{2[\log R(x_s - 1, y_s) + \log R(x_s + 1, y_s) - 2\log R(x_s, y_s)]}
\]

\[
y_{pk} = y_s + \frac{[\log R(x_s, y_s - 1) - \log R(x_s, y_s + 1)]}{2[\log R(x_s, y_s - 1) + \log R(x_s, y_s + 1) - 2\log R(x_s, y_s)]}
\]

It is clear from equation (5) that the relative values of the peak at \((x_s, y_s)\) and its four immediate surrounding neighbours are very important. They must be unbiased before the curve fitting routine is applied. Further to this, for the same velocity measurement, changes in the relative values between measurements (in a time-resolved DPIV experiment) must be minimised to reduce the rms error defined by equation (2).

3. The Experimental Model

In order to quantify the inherent errors due to changes in position of seeding particle images between measurements in a time-resolved DPIV experiment a theoretical model of the latter was developed.

To a good approximation an individual particle image can be modelled as a two-dimensional Gaussian intensity profile, \(I(x, y)\) given by [5]:

\[
I(x, y) = I_o \exp\left[\frac{-(x-x_o)^2-(y-y_o)^2}{d_i^2}\right]
\]

where the centre of the particle image is located at \((x_o, y_o)\) with a peak intensity of \(I_o\). The particle image diameter, \(d_i\), is defined as the \(e^{-\frac{1}{2}}\) intensity value of the Gaussian function.

Using this definition, high-resolution Gaussian profile particle images were randomly located in a 3200 x 3200 pixels region as shown in figure 1. This high-resolution image was then digitised to produce a pixelated version of the particle images in a 32 x 32 pixels interrogation region \(I(x_p, y_p)\) as shown in figure 2. The digitisation process is based on a 8-bit imaging device (256 grey scale), similar to those commonly used in a commercially available DPIV imaging system. By digitising the high-resolution image to create a pixelated interrogation region, it is possible to locate the centre of the Gaussian particle images to an accuracy of 0.01 pixel. A 32 x 32 pixels interrogation region was chosen in which particle images were randomly distributed. This interrogation region size is commonly used in DPIV correlation analysis. In order to avoid confusing errors from poor signal to noise ratio (SNR) and inadequate pixel resolution with those due to changes in image spatial position, 22 particle images with a particle image diameter of 2.8 pixels were randomly distributed [5].
In what follows we will examine the inherent errors for a uniform displacement of particle images. Having established the first interrogation region image \( I_1(x_p, y_p) \), the second image \( I_2(x_p, y_p) \) was calculated, where each particle image was shifted in the x-direction to simulate uniform particle image displacement, which extends from sub-pixel to integer pixel values.

We use the model to simulate a time-resolved DPIV experiment in which a sequence of (first and second) exposure realisations of the same interrogation region are provided. Each realisation contains the same true displacement value between exposures but in each case the spatial position of the seeding particle images is varied randomly as would be the case in a real experiment.

### 4. Model Predictions

In order to understand the change in SNR which occurs as a function of particle image displacement, average correlation noise fields were calculated over 300 realisations. In each case a correlation field was calculated in the frequency domain using the FFT method defined by equation (3) and in the spatial domain \( R(m,n) \) using equation (4). Two statistically independent image distributions (of 22 particle images) were correlated in each case. Figures 3 and 4 show the average correlation noise fields for \( R(m,n) \) and FFT respectively.
The ‘pyramid’ noise floor in figure 3 is due to the decrease in the area of overlap which occurs as each correlation value is calculated. When a signal peak is added to this noise floor the peak centre value is biased towards smaller values. This negative mean-bias is well reported in the literature and can be avoided by use of an appropriate weighting function [1] or by subtraction of the mean image intensity in the interrogation regions before correlation is attempted [6]. It is important that this negative mean-bias is corrected before a curve fitting routine is used to obtain sub-pixel displacement accuracy.

The average noise floor which occurs when the FFT method is used is essentially flat in nature. To the authors’ knowledge this has not been previously reported in the literature and is due to the wrap-around effects referred to in Section 2.1.

For increasing particle image displacement the maximum value of the signal correlation peak is reduced since proportionally more correlated images are lost from the second exposure of the interrogation region. Consequently it can be anticipated that when the correlation is computed by $R(m, n)$ the SNR will be maintained (since the noise level decreases) in contrast to the use of FFT where the SNR will decrease (due to the flat noise floor).

The mean bias error over a range of displacements up to 10 pixels averaged over 300 realisations is shown in figure 5 and figure 6. These results were produced from correlation of two equal sized 32 x 32 pixels interrogation regions using $R(m, n)$ and FFT. Figure 7 and figure 8 shows the pixel and percentage rms error, respectively, computed by both methods using equation 2.
5. Discussion of Model Results

The negative mean bias errors shown in figure 5 are anticipated using $R(m, n)$ because of the ‘pyramid’ noise floor shown in figure 3. However, the FFT results are a surprise since the noise floor using this method does not (on average) bias the results due to its flat noise floor illustrated in figure 4. It is interesting to consider the equivalent percentage errors when two equal sized 16 x 16 pixels interrogation regions are correlated by these methods. The results are shown in figure 9 and figure 10. The latter raise serious concerns about experimental accuracy when the FFT method, used in most commercially available PIV systems, is used to compute correlation fields from small interrogation region sizes.
In what follows we show that the negative mean-bias errors are dominated by the effect of partial images at the boundary of the interrogation region. The presence of these partial images has been recognised previously [7] but their effect on measurement error has previously been assumed to be negligible. In fact, they are a dominant source of error when equal sized regions of 32 x 32 pixels or less are correlated. Partial image error (PIE) will now be described in detail.

6. Partial Image Error (PIE)

When equal-sized interrogation regions are correlated the signal peak is inherently skewed towards lower values of displacement due to the partial images at the edge of the region. This is best explained in a simplified example shown in figure 11 where two particle images in \( I_1(i,j) \) are displaced by two pixels in the x direction between exposures so that one of the images in the second exposure \( I_2(i,j) \) contains a partial image at the boundary.

The corresponding (one-dimensional) correlation signal peak for \( R(m,n) \) and FFT is shown in figure 12. The value to the left of the correlation peak value is larger than that to the right and this inherent bias, due to the partial image, is always present in the signal peak.

Further to this when a partial image occurs in the first exposure and is correlated with a whole image in the second exposure the effect is still to skew the correlation signal peak centre to lower values. This inherent bias explains why even in the presence of a flat noise floor (as with the FFT method) the negative mean-bias error shown in figure 5 is produced. It is important to note that window shifting [6] in order to preserve SNR does not avoid this error since partial images always occur in the first exposure.
If the second interrogation region is made larger so that partial images from the first exposure are always correlated with ‘whole’ images in the second exposure then this bias can be both positive and negative. In this case, on average over many realisations there is no mean-bias of the signal peak but there are still consequences for the rms errors which will be discussed in the next section where normalisation of the correlation peak values in order to reduce partial image error is recommended.

7. PIE Reduction: Normalisation by Signal Strength (NSS)
PIE can be significantly reduced if the correlation field values are normalised before the Gaussian curve fitting routine is used. It is important to note that each calculated value in the correlation field is not only specific to an area \( A \) of overlap between \( I_1(i, j) \) and \( I_2(i, j) \) but also to the number and magnitude of pixel intensity values included in this area. We define a Signal Strength (SS) in the overlap area by:
Accordingly, the correlation field Normalised by Signal Strength (NSS) is defined as:

\[
SS = \left[ \sum_{i,j,A} I_1^2(i,j) \sum_{i,j,A} I_2^2(i,j) \right]^{1/2}
\]

If the correlation field for the partial image example shown in figure 11 is computed using NSS the asymmetry of the correlation peak is corrected. This result is shown in figure 13.

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\[
NSS = \frac{R(m,n)}{SS} = \sum_i \sum_j I_1(i,j)I_2(i-m, j-n) \left[ \sum_{i,j,A} I_1^2(i,j) \sum_{i,j,A} I_2^2(i,j) \right]^{1/2}
\]

Figure 13. Correlation values for NSS.

Normalisation by Signal Strength inherently removes the mean-bias produced by the ‘pyramid’ noise floor using \( R(m,n) \) and on-average produces the flat noise floor shown in figure 14 (over 300 realisations). It is now possible to compare the change in SNR for increasing particle image displacements between FFT and NSS. The results are presented in figure 15, where it is shown that the constant SNR of NSS effectively increases the dynamic range of DPIV.
It is important to note that NSS has the effect of ‘smoothing’ the correlation field. This is because it inherently compensates for changes in particle image seeding density per unit area across the interrogation region. In this way NSS is to be preferred to normalisation by overlap area [1] which is used to remove the negative mean-bias due to the pyramid noise floor (Section 4 and figure 3). The latter is equivalent to assuming a constant particle image seeding density per unit area which does not occur in practice. The ‘smoothing’ of the correlation field produced by NSS can be anticipated to significantly reduce the rms error due to changes in particle image spatial position over sequential measurements in time-resolved DPIV.

7.1. Experimental Tests
In order to test the effectiveness of NSS in practice a carefully controlled experiment was conducted. A glass slide containing randomly distributed particles sprayed onto the surface was accurately displaced by means of a piezo activator. The particles were Aluminium Oxide and were illuminated by a Nd:YAG laser from a commercially available DPIV system. Results from an average of 300 realisations over a range of
displacements for mean-bias and rms errors are shown in figure 16 and figure 17 respectively. These were obtained by correlating 16 x 16 pixels interrogation regions. The experimental image size (2.8 pixels) and seeding image density (22 in a 32 x 32 pixels region) were chosen to be representative of a real experiment and to avoid errors from other sources.

![Figure 16. Mean bias error.](image1)

![Figure 17. rms error.](image2)

It is clear from these results that the use of NSS to avoid PIE is important when correlation of a small sized interrogation region is attempted. The performance of FFT is seen to fail beyond a displacement of 7 pixels and this is due to the deterioration in SNR. The enhanced performance of NSS compared to FFT for the RMS error is due to the ‘smoothing’ of the correlation field mentioned earlier.

8. Conclusions

A common method to enhance signal to noise in DPIV in the presence of velocity gradients is to reduce the size of the interrogation region size. Further to this window-shifting together with a serial reduction in window size has also become common practice to increase the spatial resolution of velocity measurements. This paper has shown that for correlation of equal-sized interrogation regions of 32 x 32 pixels and smaller, particular attention must be paid due to the error caused by partial imaging of seeding particles, PIE. For equal sized regions the error causes velocities to be measured which are too low. When DPIV is used in a time-resolved mode with FFT processing errors due to PIE can preclude a reliable measurement being obtained. e.g. A 16 x 16 pixels interrogation region produces an inherent negative mean bias and rms error of 7% and 9% respectively for a displacement of one pixel. This paper has shown for the first time that the negative mean bias normally associated with non-uniform weighting of the correlation routine in the literature is, in fact, dominated by the effect of partial images.

PIE can be reduced if the correlation peak is normalised by the signal strength before sub-pixel interpolation with a Gaussian curve fit is attempted. Without correlation peak normalisation the use of FFT in the frequency domain to compute the correlation of small equal-sized interrogation regions can produce catastrophic errors. In addition NSS has been shown to extend the dynamic range of DPIV in comparison with the FFT method favoured in commercially available DPIV systems.
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