PITCH ANGLE RESTRICTIONS IN LATE-TYPE SPIRAL GALAXIES BASED ON CHAOTIC AND ORDERED ORBITAL BEHAVIOR

A. Pérez-Villegas\(^1\), B. Pichardo\(^1\), E. Moreno\(^1\), A. Peimbert\(^1\), and H. M. Velázquez\(^2\)

\(^1\) Instituto de Astronomía, Universidad Nacional Autónoma de México, A.P. 70-264, 04510 México D.F., Mexico; barbara@astroscu.unam.mx
\(^2\) Observatorio Astronómico Nacional, Universidad Nacional Autónoma de México, Apdo. Postal 877, 22800 Ensenada, Mexico

Received 2011 March 28; accepted 2011 December 8; published 2012 January 5

ABSTRACT

We built models for low bulge mass spiral galaxies (late type as defined by the Hubble classification) using a three-dimensional self-gravitating model for spiral arms, and analyzed the orbital dynamics as a function of pitch angle, ranging from 10\(^\circ\) to 60\(^\circ\). Indirectly testing orbital self-consistency, we search for the main periodic orbits and studied the density response. For pitch angles up to approximately \(\sim 20\)^\circ, the response closely supports the potential readily permitting the presence of long-lasting spiral structures. The density response tends to “avoid” larger pitch angles in the potential by keeping smaller pitch angles in the corresponding response. Spiral arms with pitch angles larger than \(\sim 20\)^\circ would not be long-lasting structures but would rather be transient. On the other hand, from an extensive orbital study in phase space, we also find that for late-type galaxies with pitch angles larger than \(\sim 50\)^\circ, chaos becomes pervasive, destroying the ordered phase space surrounding the main stable periodic orbits and even destroying them. This result is in good agreement with observations of late-type galaxies, where the maximum observed pitch angle is \(\sim 50\)^\circ.

Key words: chaos – galaxies: evolution – galaxies: kinematics and dynamics – galaxies: spiral – galaxies: structure

Online-only material: color figure

1. INTRODUCTION

It is assumed that the Hubble sequence reveals a strong correlation between the morphology of galaxies and their formation process. The study of this relation is a very active field where the merger hypothesis plays a fundamental role; however, it is very unlikely that all trends observed in this sequence can be explained solely by this hypothesis. In particular, the long-term galactic dynamics of non-interacting galaxies is also determined by their inner structure. These highly nonlinear stellar systems are prone to exhibit the coexistence of an astonishing dichotomy, an exquisite order together with counterintuitive complex regions of chaos, where non-axisymmetric features such as spiral arms, bars, etc., play a key part.

Regarding ordered motion, smooth and weak long-lasting large-scale structures such as spiral arms (in general, low mass and/or low pitch angle), when modeled, present a relatively simple orbital structure made of families of quasi-periodic orbits surrounding the main periodic orbits that sculpt spiral arms. The density response is also smooth and coincides nicely with the imposed potential in the case of low pitch angle spiral arms, for example. There are indications that even chaotic orbits could reinforce observed morphological features in this case (Kaufmann & Contopoulos 1996; Patsis et al. 1997a; Harsoula et al. 2011). However, orbital analysis on dynamical models suggests that chaotic motion plays a significant role (Contopoulos 1983, 1995; Contopoulos et al. 1987; Grosbøl 2003) in spirals.

Indeed, large-scale structures are not expected to emerge on systems built out of pure chaos, that is, as long as chaos does not become pervasive, large-scale structures of disks are practically unaffected. Recently, there has been an interesting discussion about the possible chaotic nature of the spiral structure (Patsis 2006; Voglis et al. 2006a, 2006b; Romero-Gómez et al. 2007; Contopoulos & Patsis 2008; Patsis et al. 2009).

Although the best-known part of the Hubble classification regarding pitch angles categorizes galaxies assuming that late types possess the most open spiral arms (largest pitch angles), this is just the envelope of the classification. Late-type spirals actually present a large scatter in this parameter (Kennicutt 1981; Ma et al. 2000), ranging from about \(10\)^\circ to \(50\)^\circ. In particular, late-type spirals (Sb to Scd) are better fitted with strong spirals (Contopoulos & Grosbøl 1986; Patsis et al. 1991, 1997b, and references therein), meaning they are far from being a slight perturbation that can be reproduced by the Lin & Shu (1967) spiral arm potential with a cosine function, a solution of the tight winding approximation (TWA).

We present a first result of a detailed orbital study on models of late-type spiral galaxies as defined by Hubble (1926). In particular, this work is devoted to one of the structural parameters of spiral arms: the pitch angle. In this study some restrictions are imposed theoretically on their steady or transient nature, and on their maximum pitch angle.

This Letter is organized as follows. In Section 2, the three-dimensional galactic potential used to compute orbits is briefly described. In Section 3, we present our results with periodic orbit analysis, density response, and phase space studies. Finally, in Section 4 we present our conclusions.

2. METHODOLOGY AND NUMERICAL IMPLEMENTATION

A common method to study the effect of spiral arms on stellar dynamics uses elegant but simple two-dimensional bisymmetric local approximations such as cosine functions (TWA based). In this scheme, spiral arms are assumed to be smooth self-consistent perturbations to the axisymmetric potential. Cosine potentials to represent spiral arms are in several cases taken beyond its self-consistent validity regime by imposing large pitch angles and/or large amplitudes for the spiral arms. However, in this regime, other methods to indirectly test self-consistency of steady potentials, such as the construction of periodic orbits, are applicable. Details of a non-axisymmetric potential are not negligible when we are talking about a global
The Astrophysical Journal Letters, 745:L14 (5pp), 2012 January 20

Pérez-Villegas et al.

Table 1

| Parameter                      | Value                     | Reference |
|--------------------------------|---------------------------|-----------|
| **Spiral Arms**                |                           |           |
| Locus                          | Logarithmic               | 1, 10     |
| Arms number                    | 2                         | 2         |
| Pitch angle                    | $10^\circ$–60$^\circ$     | 3, 8      |
| $M_{\text{Spiral}}/M_{\text{disk}}$ | 3%                        |           |
| Scale length                   | Disk based: 3 kpc         | 4, 5      |
| Radial force contrast          | 5%–10%                    | 6         |
| Pattern speed ($\Omega_{sp}$)  | $-20$ (clockwise) km s$^{-1}$ kpc$^{-1}$ | 1, 7 |
| ILR position                   | 2.03 kpc                  |           |
| Corotation position            | 8.63 kpc                  |           |
| Inner limit                    | 2.03 kpc                  | ~ILR position based |
| External limit                 | 8.63 kpc                  | ~Corotation position based |

| **Axisymmetric Components**    |                           |           |
| Disk mass/halo mass            | 0.1 (up to 100 kpc halo radius) | 4, 9    |
| Bulge mass/disk mass           | 0.2                       | 5, 9      |
| Rotation curve ($V_{\text{max}}$) | 170 km s$^{-1}$          | 8         |
| Disk mass                      | $5.10 \times 10^{10} M_\odot$ | 4     |
| Bulge mass                     | $1.02 \times 10^{10} M_\odot$ | $M_B/M_D$ based |
| Halo mass                      | $4.85 \times 10^{10} M_\odot$ | $M_D/M_H$ based |
| Disk scale length              | 3 kpc                     | 4, 5      |

References. (1) Grosbol & Patsis 1998; (2) Drimmel 2000; (3) Kennicutt 1981; (4) Pizagno et al. 2005; (5) Weinzirl et al. 2009; (6) Contopoulos 2007; (7) Patsis et al. 1991; Fathi et al. 2009; (8) Ma et al. 2000; Brosche 1971; Sofue & Rubin 2001; (9) Block et al. 2002; (10) Pichardo et al. 2003.

We employ the spiral arm potential formed by oblate spheroids called PERLAS (Spiral arms potential formed by oblate spheroids) from Pichardo et al. (2003) to represent their three-dimensional density-based nature. This three-dimensional steady two-armed self-gravitating potential is more realistic in the sense that it considers the force exerted by the whole spiral arm structure, sculpting much more complicated shapes for the gravitational potential and gravitational force than a simple two-dimensional cosine function. This intrinsic difference gives rise to significant deviations on orbital dynamics when compared to a cosine potential. Comparisons of the model with observations and with other models have already been published (Pichardo et al. 2003; Martos et al. 2004; Antoja et al. 2009, 2011).

The corresponding parameters used to produce models for late-type spiral galaxies are presented in Table 1. Spiral arm self-consistency has been tested through the reinforcement of the spiral potential by stellar orbits (Patsis et al. 1991; Pichardo et al. 2003). The total mass of the spiral arms in our model is 3% of the disk mass, which represents conservative (low-mass) spiral arms for late-type disk galaxies.

![Figure 1](qtmax.png)

Figure 1. $(Q_T)_{\text{max}}$ represents the maximum value of the parameter $Q_{T}(R)$ (maximum relative torques) vs. pitch angle of the spiral arm model.
observational parameters on late spirals: mainly rotation curves, mass ratios (between the components: bulge, disk, and halo), and scale lengths. In Table 1, we present the employed axisymmetric and non-axisymmetric (spiral arms) parameters.

3. RESULTS

In the case of long-lasting steady potentials, self-consistency can be tested indirectly through the construction of periodic orbits. The existence of periodic orbits supporting a large-scale structure such as spiral arms increases the probability of maintaining long-lasting large-scale structures. We present a periodic orbit study in Figure 2. In order to quantify the support of periodic orbits to the spiral arm potential, we follow the method of Contopoulos & Grosbøl (1986) to obtain the density response to the given spiral perturbation. This method assumes that the stars with orbits trapped around an unperturbed circular orbit, and with the sense of rotation of the spiral perturbation, are also trapped around the corresponding central periodic orbit in the presence of the perturbation. In this manner, we computed a series of central periodic orbits and found the density response along their extension using the conservation of mass flux between any two successive orbits. We found the position of the density response maxima (filled squares in Figure 2) along each periodic orbit, and thus the positions of the response maxima on the galactic plane are known. These positions are compared with the center of the imposed spiral arm potential, i.e., the spiral locus (open squares in Figure 2).

For smaller pitch angles \(i \lesssim 20^\circ\), the density response maxima support closely the spiral arm potential, making the existence of long-lasting spiral arms more probable. On the other hand, for spiral arms with pitch angles larger than \(\sim 20^\circ\), the response maxima precede systematically the spiral arm potential, i.e., the response produces spiral arms with much smaller pitch angles than the spiral arms potential. The response “avoids” open long-lasting spiral arms. Beyond \(20^\circ\) of pitch angle for the spiral potential, the density response has almost the same pitch angle (approximately \(18^\circ\) to \(22^\circ\)) independently of the potential. Long-lasting spirals are not supported anymore, spiral arms in this case may be rather transient.
Figure 3. Phase space diagrams with $E_J = [-1080, -1010]$, in units of $10^2$ km$^2$ s$^{-2}$. From upper to bottom panels of the diagram, pitch angles go from 30° to 50°.

In Figure 3, we present a $3 \times 3$ phase space diagram mosaic to show the main results. These nine panels show Poincaré diagrams with different Jacobi energy families running from $E_J = -1080$ to $-1010 \times 10^2$ km$^2$ s$^{-2}$, covering the total extension of the spiral arms, as we go from 30° (upper line of diagrams) to 50° (bottom line of diagrams). The left part of each diagram represents prograde orbits in the reference system of the spiral arms.

Observations of spiral galaxies show pitch angles up to $\sim 50°$ (Ma et al. 2000). Despite periodic orbits not supporting spiral arms, the existence of very open spirals, could indicate a probable transient nature. However, even then, some ordered orbits are expected to support these large-scale structures for short periods.

We have studied the effect of increasing the pitch angle in a typical late-type spiral galaxy model. With this study we search for a limit for the pitch angle, for which chaos becomes pervasive and destroys all ordered orbits in the relevant spiral arm region. At a pitch angle of 20° or less, the majority of orbits are ordered and simple, periodic orbits support spiral arms up to close to corotation. As we increase the pitch angle, at 30° (first line of diagrams from the top of Figure 3), the orbital behavior becomes much more complex, presenting resonant islands and the onset of chaos is clear, surrounding the stable periodic orbits, yet supporting them in a contained region. For 40° (second line of diagrams), chaos becomes pervasive, compromising the available phase space around the stable periodic orbits. For 50° (bottom line of diagrams), the chaotic region covers almost all regular prograde orbits, approaching closely to the main periodic orbits. For pitch angles beyond $\sim 50°$ chaos destroys periodic orbits.

In all cases reported in this work, the radial position of corotation was kept fixed (see Table 1). However, taking into account earlier work by Contopoulos & Grosbol (1986) and Patsis et al. (1991), among others, we produced several experiments taking the position of corotation (to 16.5 kpc by reducing the spiral arms angular speed from 20 to 10 km s$^{-1}$ kpc$^{-1}$) considerably far from the end of the spiral arms to check whether it is relevant to the observed chaotic behavior. Although a large fraction of chaos is indeed produced close to the corotation resonance, we found that even with corotation far from the end of the spiral arms, a fraction of chaos is generated toward larger pitch angles, although not enough to compromise ordered regions around the periodic orbits. We then produced experiments where we fixed the end of the spiral arms close to the corotation position, and changed only one parameter: the pitch angle. We found then that the chaos produced in corotation becomes much stronger as we increase the pitch angle. Chaos seems to be a combination between the effect of the corotation resonance and the pitch angle.
4. CONCLUSIONS

With the use of an axisymmetric fixed model to simulate a typical late-type galaxy as a background, we superposed a bisymmetric steady spiral arm potential (PERLAS) and studied the evolution of orbital behavior in the plane of the disk, as we change the pitch angle ranging from 10° to 60°, in order to set some structural restrictions on the spiral arms, based on orbital dynamics. Observed galaxies classified as late-type spirals present a wide scatter in pitch angles, going from ∼10° to 50°. With this family of models, we have carried out an exhaustive orbital study (order and chaos) with periodic orbits and with phase space diagrams.

In this Letter, we present the first restriction relative to the pitch angle. In the case of ordered motion with periodic orbits, a limit in the pitch angle of the density response is found at approximately 20°, up to which the density response reinforces the spiral arm potential at all radii, i.e., with a more long-lasting nature. Beyond this limit, the density response “avoids” following the spiral arm potential, producing pitch angles much smaller than the background spiral potential. Spiral arms beyond this limit might be better explained as transient structures.

A second restriction is obtained from phase space studies of ordered and chaotic behavior. Going from pitch angles of 10°, where order reigns, to more than 50°, where chaos becomes pervasive. With this orbital study we are able to set a limit for the maximum pitch angle before the system becomes completely chaotic. This limit closely coincides with the observational maximum pitch angle of spirals (∼50°), suggesting a possible relation between the structural characteristics of the galaxy and chaos.

It is a pleasure to acknowledge Panos Patsis and the anonymous referee for enlightening comments that considerably improved this work. We thank PAPIIT through grants IN110711-2, IN119306, and IN-109509.

REFERENCES

Allen, C., & Santillán, A. 1991, RevMexAA, 22, 256
Antoja, T., Romero-Gómez, M., Figueras, F., et al. 2011, MNRAS, 418, 1423
Antoja, T., Valenzuela, O., Pichardo, B., et al. 2009, ApJ, 700, L78
Block, D. L., Bournaud, F., Combes, F., Puera, I., & Buta, R. 2002, A&A, 394, L35
Brosche, P. 1971, A&A, 13, 293
Buta, R., & Block, D. L. 2001, ApJ, 550, 243
Buta, R., Vasylyev, S., Salo, H., & Laurikainen, E. 2005, AJ, 130, 523
Combes, F., & Sanders, R. H. 1981, A&A, 96, 164
Contopoulos, G. 1983, A&A, 117, 89
Contopoulos, G. 1995, NYASA, 751, 112
Contopoulos, G. 2007, in Conference 2007, Chaos in Astronomy, ed. G. Contopoulos & P. A. Patsis (New York: Springer), 3
Contopoulos, G., & Grosbol, P. 1986, A&A, 155, 11
Contopoulos, G., & Patsis, P. A. (ed.) 2008, Chaos in Astronomy (Berlin: Springer)
Contopoulos, G., Varvoglis, H., & Barbansis, B. 1987, A&A, 172, 55
Drimmel, R. 2000, A&A, 358, L13
Fathi, K., Beckman, J. E., Pino-Ferrer, N., et al. 2009, ApJ, 704, 1657
Grosbol, P. 2003, Galaxies and Chaos, ed. G. Contopoulos & N. Voglis (Lecture Notes in Physics, Vol. 626; Berlin: Springer-Verlag), 201
Grosbol, P. J., & Patsis, P. A. 1998, A&A, 336, 840
Harsoula, M., Kalapotharakos, C., & Contopoulos, G. 2011, MNRAS, 411, 1111
Hubble, E. P. 1926, ApJ, 64, 321
Kalapotharakos, C., Patsis, P. A., & Grosbol, P. 2010, MNRAS, 403, 83
Kaufmann, D. E., & Contopoulos, G. 1996, A&A, 309, 381
Kenneicutt, R. 1981, AJ, 86, 1847
Laurikainen, E., & Salo, H. 2002, MNRAS, 337, 1118
Lin, C. C., & Shu, F. H. 1967, in IAU Symp. 31, Radio Astronomy and the Galactic System, ed. H. van Woerden (London: Academic), 313
Ma, J., Zhao, J.-L., Zhang, F.-P., & Peng, Q.-H. 2000, Chin. Astron. Astrophys., 24, 435
Marts, M., Hernandez, X., Yáñez, M., Moreno, E., & Pichardo, B. 2004, MNRAS, 350, 47
Patsis, P. A. 2006, MNRAS, 369, L56
Patsis, P. A., Athanassoula, E., & Quillen, A. C. 1997a, ApJ, 483, 731
Patsis, P. A., Contopoulos, G., & Grosbol, P. 1991, A&A, 243, 373
Patsis, P. A., Grosbol, P., & Hotelis, N. 1997b, A&A, 323, 762
Patsis, P. A., Kaufmann, D. E., Gottesman, S. T., & Boonyasait, V. 2009, MNRAS, 394, 142
Pichardo, B., Martos, M., Moreno, E., & Espesate, J. 2003, ApJ, 582, 230
Pizagno, J., Prada, F., Weinberg, D. H., et al. 2005, ApJ, 633, 844
Romero-Gómez, M., Masdemont, J. J., Athanassoula, E., & García-Gómez, C. 2007, A&A, 472, 63
Sofue, Y., & Rubin, V. 2001, ARA&A, 39, 137
Voglis, N., Stavropoulos, I., & Kalapotharakos, C. 2006a, MNRAS, 372, 901
Voglis, N., Tsoutsis, P., & Efthymiopoulos, C. 2006b, MNRAS, 373, 280
Vorobyov, E. I. 2006, MNRAS, 370, 1046
Weinzirl, T., Jogee, S., Khochfar, S., Burkert, A., & Kormendy, J. 2009, ApJ, 696, 411