Forecast technique for roller bearing life as random value

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Abstract. The article deals with the use of the linearization method to take into account the stochastic nature of factors affecting the life of bearings. In addition, we should pay attention to such factors as the degree of contamination and viscosity of the lubricant, as well as the properties of the material from which the bearing parts are made. The developed method allows estimating the degree of influence of these factors on the bearing life. In order to adapt the graphical dependencies to the physic-probabilistic approach, Lagrange interpolation polynomial is used, which allows obtaining the calculated dependencies and automate the resource calculation. The linearization method makes it possible, if necessary, to simplify the mathematical model, neglecting the arguments that have minimal impact on the calculation result.

In addition to the generally accepted ISO 281 methodology for determining bearing life, there is also a technique recommended by SKF. The traditional approach involves the calculation of the resource based on the dynamic bearing capacity and equivalent load [1]. However, not only these parameters affect the reliability of rolling bearings [2-5]. In contrast to the generally accepted method, SKF company also takes into account such factors as lubrication conditions, the degree of contamination of the bearing and the fatigue load limit for the bearing material [6]. The SKF resource equation in millions of turns looks like

\[ L = a_1 \cdot a_{SKF} \cdot (C/P)^p, \]

where \( a_1 \) - the correction coefficient of reliability, with the reliability of 90% \( a_1 = 1; \)

\( a_{SKF} \) - SKF resource coefficient;

\( C \) - dynamic load capacity, kN;

\( P \) - equivalent dynamic load on the bearing, kN;

\( p = 3 \) for ball bearings, \( p = 10/3 \) for roller bearings.

All the arguments in this equation are actually probabilistic in nature, i.e. cannot be clearly defined by a single value, but take one of the many possible values with a certain probability. Two numerical characteristics are used for a complete description of random variables: expectation (average value) and variance. Accordingly, the function of random arguments, also having a probabilistic nature, is determined, in addition to the mean, by the area of scattering.

The simultaneous consideration of both physical factors affecting the value of a resource and their random nature suggests a physical-probabilistic approach. The linearization method allows determining both numerical characteristics of the function of random arguments [7]. The initial data in
this case are the numerical characteristics of the arguments. According to this method, the average value of the function of random arguments is determined similarly to the deterministic approach according to the following relationship:

\[ m_i = f(m_{x_1}, m_{x_2}, \ldots, m_{x_n}) \]  \hspace{2cm} (2)

where \( m_{x_1}, m_{x_2}, \ldots, m_{x_n} \) is the mathematical expectation of the arguments.

The variance and standard deviation of the function are determined by the following formulas (provided that all arguments are mutually independent):

\[ D_x = \sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \right)^2 D_{x_i}, \]  \hspace{2cm} (3)

\[ \sigma_x = \sqrt{\sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \right)^2 \sigma_{x_i}^2}, \]  \hspace{2cm} (4)

where \( \left( \frac{\partial f}{\partial x_i} \right) = f_x'(m_{x_1}, m_{x_2}, \ldots, m_{x_n}) \) is the derivative of the function with respect to the \( i \) argument;

\( D_{x_i} \) - variance of the \( i \) argument;

\( \sigma_{x_i} \) - standard deviation of the \( i \) argument.

In order to find the value of the \( a_{SKF} \) coefficient, it is supposed to use diagrams that are different for bearings of various types. These diagrams show the dependence of SKF resource coefficient on the pollution parameters, the boundary load and the viscosity of the lubricant. The need to use diagrams makes it impossible to use the linearization method, so we deal with the problem of finding mathematical relationships that correspond to the graphs.

One of the solutions to this problem is the use of Lagrange interpolation polynomial [8]. The coordinates of several reference points are taken as the source data, the number of which determines the interpolation accuracy. The best choice of reference points is a rectangular grid with evenly spaced nodes.

In fact, the diagrams, used to find the coefficient of the \( a_{SKF} \) resource, are graphs of functions of two variables. In the case of using a rectangular grid that includes nine reference points with known coordinates, the desired functions will have the following form

\[ a_{SKF} = f(z,k) = a_1 \cdot z^2 \cdot k^2 + a_2 \cdot z^2 \cdot k + a_3 \cdot z \cdot k^2 + a_4 \cdot z^2 + a_5 \cdot k^2 + a_6 \cdot z \cdot k + a_7 \cdot z + a_8 \cdot k + a_9, \]  \hspace{2cm} (5)

where \( a_i \) are the coefficients of Lagrange polynomial;

\( z = \eta_{c} P_u / P; \)

\( \eta_{c} \) - pollution factor;

\( P_u \) - fatigue boundary load;

\( P \) - equivalent load on the bearing;

\( k = v / v_1 \) - relative viscosity;

\( v \) is the actual viscosity of the lubricant at the operating temperature;

\( v_1 \) is the nominal viscosity.

As a result of interpolation of diagrams using Lagrange polynomial, the values of the coefficients \( a_i \) were obtained for bearings of various types (table 1).

\begin{table}  \hspace{1cm} \textbf{Table 1. The coefficients of Lagrange polynomial.}  \\
\hline
& \( a_1 \) & \( a_2 \) & \( a_3 \) & \( a_4 \) & \( a_5 \) & \( a_6 \) & \( a_7 \) & \( a_8 \) & \( a_9 \) \\
\hline
Standard radial ball bearing SKF & -0.45 & 0.06 & 10.63 & 0.00 & -0.07 & -2.19 & 0.11 & 0.15 & 0.09 \\
\hline
\end{table}
Thus, the dependence for determining the SKF resource coefficient, for example, for standard radial bearings, takes the form

$$a_{SKF} = f(z, k)$$

= \(-0.45 \cdot z^2 \cdot k^2 + 0.06 \cdot z^2 \cdot k + 10.63 \cdot z \cdot k^2 - 0.07 \cdot k^2 - 2.19 \cdot z \cdot k + 0.11 \cdot z + 0.15 \cdot k + 0.09\) \tag{6}

This function allows finding the deterministic value \(a_{SKF}\), i.e. the value corresponding to the fixed, exactly given values of all arguments. In fact, all the arguments, and, therefore, the function itself, are random variables and can take one value or another with some probability.

The use of the linearization method gives the following dependencies for determining the variance of the quantities that affect the value of the \(a_{SKF}\) resource coefficient:

- dispersion of relative viscosity \(k\)

$$D_k = \left(\frac{1}{v_1^2} \cdot D_v + \left(\frac{v}{V_1^2} \right)^2 \cdot D_{v_1}\right),$$

\tag{7}

- dispersion of magnitude \(z\)

$$D_z = \left(\frac{P_u}{P} \right)^2 \cdot D_{v_1} \cdot \left(\frac{\eta_c}{P} \right)^2 \cdot D_{v_1} + \left(\frac{\eta_c}{P^2} \right)^2 \cdot D_P,$$

\tag{8}

If to take specific numerical characteristics of the quantities:

- \(v = 32 \text{ mm}^2 / \text{s}, D_v = 3.2\);
- \(v_1 = 11 \text{ mm}^2 / \text{s}, D_{v_1} = 1.1\);
- \(\eta_c = 0.5, D_{\eta_c} = 0.05\);
- \(P_u = 0.7 \text{ kN}, D_{P_u} = 0.07\);
- \(P = 7 \text{ kN}, D_P = 0.7\);

Then substituting these values into the formulas for the expectation and variance of the values of \(k\) and \(z\), we get:

- \(k = v / v_1 = 2.91; D_k = 0.03 + 0.08 = 0.11\)
- \(z = \eta_c P_u / P = 0.05; D_z = 0.0005 + 0.0004 + 0.00003 = 0.00093\).

As can be seen from the last equation, the effect of the dispersion \(D_P\) on the value of the dispersion \(D_z\) is small, because the formula can be simplified by neglecting the last term:

$$D_z = \left(\frac{P_u}{P} \right)^2 \cdot D_{v_1} \cdot \left(\frac{\eta_c}{P} \right)^2 \cdot D_{v_1},$$

\tag{9}

Similarly, the numerical characteristics for the SKF resource coefficient are found. And the decisive influence on the coefficient dispersion value is exerted by the scatter of the \(z\) value; therefore, the effect of the \(D_k\) value can be neglected. Thus, we obtain the following formula

| SKF Explorer radial ball bearing | 1.51 | -0.55 | 18.01 | 0.04 | -0.10 | -3.58 | 0.18 | 0.15 | 0.09 |
| SKF standard radial roller bearings | -0.40 | 0.11 | 10.55 | -0.01 | -0.06 | -3.60 | 0.25 | 0.07 | 0.09 |
| SKF Explorer radial roller bearings | 5.23 | -0.96 | 6.09 | 0.04 | -0.04 | -1.80 | 0.12 | 0.07 | 0.09 |
| SKF Explorer thrust ball bearing | -0.45 | 0.06 | 10.63 | 0.00 | -0.07 | -2.17 | 0.11 | 0.12 | 0.09 |
| SKF standard thrust roller bearings | 1.12 | -0.37 | -1.74 | 0.03 | 0.00 | 1.09 | -0.09 | 0.02 | 0.10 |
| SKF Explorer thrust roller bearings | 1.56 | -0.34 | -1.05 | 0.02 | 0.00 | 0.50 | -0.04 | 0.03 | 0.10 |
\[ D_{\text{var}} = \left( -0.9 \cdot z \cdot k^2 + 0.12 \cdot z \cdot k + 10.63 \cdot k^3 - 2.19 \cdot k + 0.11 \right)^2 \cdot D_z, \]

(10)

The variance of the standard ball-bearing resource, taking into account the effect of the SKF coefficient and provided that the safety correction factor \( a_1 = 1 \), is found by the formula

\[ D_{\text{var}} = \left( 3 \cdot \frac{a_{\text{SKF}} \cdot C}{P^3} \right)^2 \cdot D_c + \left( -3 \cdot \frac{a_{\text{SKF}} \cdot C}{P^4} \right)^2 \cdot D_P + \left( \left( \frac{C}{P} \right)^3 \right)^2 \cdot D_{\text{var}}, \]

(11)

The dispersion of the SKF coefficient and, consequently, such factors as the contamination of the bearing, the fatigue strength of the material, and the level of equivalent load on the bearing have the greatest influence on the dispersion of the bearing life.

Thus, the use of the linearization method makes it possible not only to determine the numerical characteristics of the function of random variables, but also to estimate the degree of influence of each factor. Difficulties in assessing the life of bearings according to the SKF method were caused only by the presence of a large number of diagrams in this method. In order to adapt it to the physical-probabilistic approach, the Lagrange interpolation polynomial is used to obtain mathematical models corresponding to the graphs.

The linearization method allows analyzing the role of the accuracy of the source data in the accuracy of the resource estimate. On the basis of which it becomes possible to simplify the calculated dependencies if the influence of some arguments on the value of the numerical characteristics of the function is small. On the other hand, the method clearly demonstrates which factors have the greatest impact on the result, and, consequently, which input data should be as accurate as possible for effective resource prediction.

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