Condensate losses and oscillations induced by Rydberg atoms

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Abstract
We numerically analyze the impact of a single Rydberg electron onto a Bose–Einstein condensate. Both S- and D-Rydberg states are studied. The radial size of S- and D-states are comparable, hence the only difference is due to the angular dependence of the wavefunctions.

Additionally, we investigate the mechanical effect in which the Rydberg atoms force the condensate to oscillate. Our numerical analysis is based on the classical fields approximation.

Finally, we compare numerical results to experimental data.

Keywords: Rydberg atoms, Bose–Einstein condensation, condensate oscillations

(Some figures may appear in colour only in the online journal)

Introduction
Effects related to the interaction of a highly excited electron in a Rydberg atom with neutral atoms have been extensively studied for many decades [1]. First seminal experiments were performed by Amaldi and Segrè [2, 3], who studied Rydberg absorption lines for principal quantum number $n \leq 30$ of sodium atoms immersed in hydrogen background gas, with the hydrogen atoms playing the role of perturbers for the Rydberg electron. The important finding of these experiments was the shift of the absorption lines proportional to the density of the background gas. The high pressure of the perturbing gas (up to one atmosphere) resulted in a large number of perturbing atoms (of the order of $10^9$) being in the volume of the Rydberg electron wavefunction. Surprisingly, the shift of the absorption lines can be both negative or positive, depending on the kind of perturbing gas. The explanation of this observation was given by Fermi who described the interaction between the Rydberg electron and the atom introducing the pseudopotential [4, 5].

This concept of applying the pseudopotential to the Rydberg electron–atom interaction [6] led to the prediction of ultralong-range Rydberg molecules [7], which consist of a Rydberg atom and at least one ground state atom bound to the Rydberg electron by low-energy scattering. Rb₂ Rydberg molecules were experimentally realized in [8]. Since then, diatomic homonuclear Rydberg molecules were realized using various atoms and Rydberg states like Rb in D-states [9, 10] and P-states [11]; Cs in S-states [12] and P-states [13]; Sr in S-states [14]. Recently, ultralong-range Rydberg molecules bound by mixed singlet and triplet scattering have been predicted [15] and experimentally realized in [13, 16, 17]. Coherent creation and breaking of molecular bonds has been shown in [18]. The scattering resonance present for e-Rb p-wave scattering [19] led to the observation of butterfly state Rydberg molecules [20] and Rydberg molecules bound by quantum reflection [21]. Trilobite molecules possessing large permanent electric dipole moment has been observed in [22, 23]. Polyatomic Rydberg molecules with up to four ground state perturbers located inside the Rydberg electron orbit has been observed in [24].

A Rydberg electron can interact with even thousands of ground state atoms at high densities. An experiment studying...
the coupling of the Rydberg electron to a Bose–Einstein condensate was performed in [25]. Large principal quantum numbers of the Rydberg states were considered allowing these states to have the size of a few micrometers, comparable to the size of the condensate. Hence, the Rydberg electron was interacting with a large (as in [2]) number of condensate atoms being within the extent of its orbit. As opposed to the conditions of the experiment of [2], the Rydberg electron now interacts with the condensed atoms what makes a huge difference. The coupling of a single Rydberg electron to the condensate is surprisingly strong in comparison to the analogous coupling of the ionic impurity. The electron can excite the condensate atoms leading to the significant depletion of the condensate. The electron can as well induce the collective oscillations of the whole condensate [25]. Recently, Rydberg spectroscopy of a single Rydberg atom excited in a BEC revealed the impact of the p-wave shape resonance onto the spectral profile [26]. Furthermore, ultracold chemical reactions of a single Rydberg atom excited in a BEC were studied in [27].

In [28] we have developed a stochastic model which describes the process of a creation of Rydberg atoms in the Bose–Einstein condensate and the following interaction of Rydberg electron with the condensed atoms. Our simulations [28] show the agreement with the experimental results in the part regarding the condensate losses due to thermal depletion in the case of S-Rydberg states [25]. Here, we extend our model to the D-Rydberg states and study also the condensate oscillations caused by the creation of a sequence of Rydberg atoms in the condensate.

**Theoretical model**

The single electron of a Rydberg atom polarizes near-by atoms. The atoms become electric dipoles. The interaction between an electric dipole and a charge is of short range type \( \sim 1/r^4 \). To describe the interaction between the electron and the surrounding ground state atoms we use a pseudopotential [4, 5]. In the simplest case only s-wave partial wave is considered and this potential takes the following form

\[
V_{\text{Ryd}}(\vec{r}') = \frac{2\pi \hbar^2 a(k')}{m_e} |\Psi_{\text{Ryd}}(\vec{r}')|^2 ,
\]

(1)

where \( \Psi_{\text{Ryd}}(\vec{r}') \) is the Rydberg electron wavefunction, \( m_e \) is the electron mass, \( \vec{r}' \) is the position of electron with respect to the center of Rydberg atom and \( a(k') \) denotes the electron–atom s-wave scattering length.

\[
a(k') = a_e + \frac{\hbar^2 \alpha}{6 \varepsilon^2 a_0^3 m_e} k',
\]

(2)

where \( a_0 \) is Bohr radius, \( a_e \) is the zero-energy scattering length, \( \alpha \) is the polarizability, \( \varepsilon \) is the elementary charge and \( k' \) is the electron wave number.

In a more sophisticated approach also p-wave partial wave is taken into account [29]. Now, the Rydberg potential takes the following form

\[
V_{\text{Ryd}}(\vec{r}') = 2\pi \hbar^2 S_{1-T}(k')|\Psi_{\text{Ryd}}(\vec{r}')|^2 + 6\pi \hbar^2 S_{1}(k')|\nabla \Psi_{\text{Ryd}}(\vec{r}')|^2 ,
\]

(3)

where \( S_{1-T}(k') = -\tan \delta_{1-T}(k')/k^{2l+1} \) and \( \delta_{1-T}(k') \) are the scattering phase shifts. \( S \) and \( T \) refer to the singlet and triplet channels, respectively. Here we are interested in the triplet case. The values of the scattering phase shifts are taken from [30].

There is also the second charge coming from the nucleus. We neglect this ingredient because a reduced mass of the neutral atom and the positive ion is about five orders of magnitude larger. This way the interaction energy is much smaller than in case of the electron.

To describe the effect of the Rydberg electron on the condensate, the pseudopotential is introduced as an additional term in the Gross-Pitaevskii equation

\[
i\hbar \frac{\partial}{\partial t} \Psi(\vec{r}, t) = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{trap}}(\vec{r}) + g|\Psi(\vec{r}, t)|^2 + f(t) V_{\text{Ryd}}(\vec{r} - \vec{R}) \right] \Psi(\vec{r}, t),
\]

(4)

where \( \Psi(\vec{r}, t) \) is the amplitude of the classical field representing the atomic operator, \( g = 4\pi\hbar^2 a_n/m_a \) is the coupling constant for neutral atoms and \( f(t) \) is a function which takes the values 1 or 0, depending on whether the Rydberg atom is present or not. The scattering \( a_n \) and the mass \( m_a \) correspond to \( ^{87}\text{Rb} \) atom and are set to 5.313\,\text{nm} and \( 1.44 \times 10^{-25} \text{kg} \) respectively in our simulations. The frequencies of the trapping potential \( V_{\text{trap}} = m_a(\omega_0^2 x^2 + y^2) + \omega_z^2 z^2/2 \) are \( \omega_0 = 2\pi \times 81.7 \text{Hz} \) and \( \omega_z = 2\pi \times 22.4 \text{Hz} \). It is important to appropriately model the excitation process in order to reproduce the experimental findings. The excitation process depends both on time and position.

To find the position where the Rydberg atom is excited we use the following procedure. We choose a position of a possible excitation according to the density distribution of neutral atoms \( \rho(\vec{R}, t) = |\Psi(\vec{R}, t)|^2 \). Then we draw a random number between 0 and 1 and compare it to the excitation probability \( \rho(\vec{R}, t) \) to determine if an excitation indeed takes place. The probability to find an atom at position \( \vec{R} \) in the Rydberg state is given by

\[
\rho(\vec{R}, t) = \frac{\Omega^2}{\Omega^2(\vec{R}, t)} \sin^2 [\Omega(\vec{R}, t)t/2],
\]

(5)

where \( \Omega_0 \) is the single atom Rabi frequency in vacuum. In the presence of neutral atoms the Rydberg level gets shifted and a local Rabi frequency is given by

\[
\Omega(\vec{R}, t) = \sqrt{\Omega_0^2 + \Delta^2(\vec{R}, t)} ,
\]

(6)

where \( \Delta(\vec{R}, t) \) is a local detuning. This quantity reads

\[
\Delta(\vec{R}, t) = \Delta \omega_L - \delta E(\vec{R}, t)/\hbar ,
\]

(7)

where \( \Delta \omega_L \) is the laser detuning from the vacuum Rydberg level, \( \delta E(\vec{R}, t) \) is the shift of the Rydberg level caused by the neutral atoms. This shift, called the pressure effect [2, 4],
takes the following form
\[ \delta E(\vec{r}, t) = \int V_{\text{Ryd}}(\vec{r} - \vec{R}) \rho(\vec{r}, t) \, d^3r. \]
(8)

As it was already mentioned we draw a random number between 0 and 1 and compare it to the excitation probability \( p(\vec{R}, t) \). If this random number is smaller than \( p(\vec{R}, t) \), \( f(t) \) changes its value from 0 to 1. If not, another position is picked randomly and the procedure is repeated until either an excitation occurs or the number of trials reaches \( N \), the number of atoms in the sample. If the latter happens no Rydberg excitation occurs during this time step and we advance by a time-step of \( \Delta t = 1/T \). Stating it differently, we propagate the condensate wave function by the time-step \( \Delta t \). So \( \Delta t \) is the time-step used in our numerical procedure. The evolution is performed using the split operator method and the kinetic part of the evolution is done in the momentum space by the Fast Fourier Transform. Once created, the condensate and the kinetic part of the evolution is done in the momentum space by the Fast Fourier Transform. Once created, the condensate wave function

\[ \rho(\vec{r}, t) = \frac{1}{N} \int d\vec{y} \, \Psi(\vec{y}, y, z, t) \, \Psi^*(\vec{y}', y', z', t). \]
(13)

After spectral decomposition \([34]\), the fraction of the condensed atoms is given by the dominant eigenvalue. The amount of remaining atoms measures the losses.

To summarize the theoretical model we show a flow chart of our numerical procedure in figure 1.

**Condensate losses**

One of the main outcomes of article \([25]\) are resonance lines appearing in the relative BEC atom number after a series of Rydberg atoms was excited. Here, we present a measurement of the BEC losses for the Rydberg atoms in three different states, namely for \( (n = 110, l = 0, m_l = 0) \), \( (n = 106, l = 2, m_l = 0) \) and \( (n = 106, l = 2, m_l = 2) \). In the further discussion we will call them 110S, 106D0 and 106D2. Details on the experimental setup can be found in \([25, 35]\). The experimental sequence was as follows. First, a BEC of \(^8\text{Rb}\) atoms were created. The number of atoms in the condensate was about \( 1 \times 10^6 \) plus some amount of atoms in the thermal cloud. Then a series of Rydberg atoms was excited by two photon excitation process. To avoid an unnecessary shaking of condensate the red laser was switched on by ramping adiabatically from zero to its maximum value in 24 ms. Then the power of the red laser was kept at its maximum for additional 14 ms. Finally, the condensate was released from the trap and time of flight (TOF) imaging was performed after 50 ms. From these images both the condensate losses and the condensate oscillations were extracted.

Experimental results of losses are shown in figure 2 as black disks (110S), red squares (106D0), and green triangles (106D2). In all cases the Rabi frequency \( \Omega_R = 408 \) Hz. Each point is an average result of twenty repetitions of the experiment.

In case of 110S and 106D0 the experimental lines are quite symmetric. In the last case the line is clearly asymmetric. Despite this fact we do the Gaussian fits and we extract the minimum value, the full width at half magnitude (FWHM) and the position of each line. The results are collected in table 1.

Before we will try to compare the theoretical results with the experiment we would like to discuss how the parameters of our model influence the shape of the resonance lines. There
are three parameters we are going to alter, namely: the scattering rate $\Gamma$, the Rabi frequency $W_R$ and the lifetime $\tau$.

Simulations show that the condensate fraction goes down when the scattering rate grows. This result is easy to understand. By increasing the scattering rate we increase the frequency we try to excite the Rydberg atom. This way more Rydberg atoms get excited. This leads directly to the increase of losses, i.e., the reduction of the condensate fraction. While the losses are affected, the line width and the line position remain almost unchanged.

Similar analysis is done for different values of the Rabi frequency. The increase of the Rabi frequency leads to the increase of losses. This is caused by increase of the probability to excite the Rydberg atom. This fact is obvious from the formula (5) written for short times which is the case in our simulations. The probability is then proportional to $W^4R^2/2$. Two other parameters, the FWHM and the line position, remain almost unaffected as in the previous case.

The lifetime of the Rydberg state influences the resonance lines in the strongest way. Together with growth of the lifetime both losses and FWHM are growing. The increase in losses is very strong. Surprisingly this increase in losses is accompanied by the reduction of the number of Rydberg atoms created during the excitation sequence. So, the longer the lifetime, the smaller the number of Rydberg atoms is created during the same time. The longer the lifetime, the longer the total time when Rydberg atoms interact with the neutral atoms and shorter the total time when there is no Rydberg atom in the system. The lines positions are almost unaffected.

We try to recover the experimental resonance lines using our procedure. The initial number of atoms $N$ in the unperturbed condensate is $1.2 \times 10^5$ in our simulations. Then we prepare a cloud of atoms at the initial temperature containing $1 \times 10^5$ of atoms in the condensate and $0.2 \times 10^5$ of atoms in the thermal cloud and follow the experimental procedure described earlier in this section. If we use directly the experimental values of parameters only the S$^1$10 state line agrees with the experiment in terms of the maximum losses. In the case of D$^106$0 state and D$^106$2 state losses are too high. So we tune the scattering rate and the lifetime to get the maximum losses close to the experimental one. We always use the Rabi frequency $\Omega_R = 408$ Hz. So if we chose the scattering rate $\Gamma = 5$ MHz, the lifetime $\tau = 4.8 \mu s$ for the 110S state, $\Gamma = 1.25$ MHz, $\tau = 3.2 \mu s$ for the 106D0 state and $\Gamma = 1.25$ MHz, $\tau = 3.2 \mu s$ for the 106D2 state we are close to the experimental maximum losses. The chosen very

| State   | $n_0$     | FWHM (MHz) | $\Gamma_0$ (MHz) |
|---------|-----------|------------|------------------|
| 110S    | 0.565 ± 0.018 | 11.9 ± 0.6 | −7.3 ± 0.2 |
| 106D0   | 0.598 ± 0.012 | 10.0 ± 0.4 | −7.83 ± 0.15 |
| 106D2   | 0.685 ± 0.015 | 13.6 ± 0.9 | −9.2 ± 0.3 |
The theoretical state is shown as black, the 106D0 state is shown as red and 106D2 state is shown as green respectively.

**Figure 3.** The comparison between the experiment and the theory in terms of the relative BEC atom number. Points show experimental values whereas lines theoretical one. The 110S state is shown in black, the 106D0 state is shown as red and 106D2 state is shown as green respectively.

**Table 2.** The values of parameters of the Gaussian fits for three considered states.

| State   | $n_0$     | FWHM (MHz) | $\Gamma_0$ (MHz) |
|---------|-----------|------------|------------------|
| 110S    | 0.56 ± 0.02 | 7.4 ± 0.5 | -7.1 ± 0.2      |
| 106D0   | 0.575 ± 0.018 | 6.5 ± 0.3 | -6.44 ± 0.13    |
| 106D2   | 0.635 ± 0.017 | 6.4 ± 0.4 | -6.22 ± 0.15    |

long experimental sequence minimizes the influence of the red laser light onto the many-body dynamics of the BEC with respect to the work presented in [25, 28]. However, at the same time, there are complex chemical processes which could take place on such long time scale [27, 36]. Given the limitations in our knowledge about the exact processes, we use a simplified description introducing an effective lifetime as the first approach, which results in a qualitative agreement with the experimental data.

The comparison between the experiment and the theory is shown in figure 3. The experimental results are shown as the black disks, the red squares and the green triangles for 110S, 106D0 and 106D2 state, respectively. The theoretical results are drawn as lines. The condensate fraction at maximum losses, the line width and the line position are extracted using a Gaussian fits for three parameters. The theoretical width is smaller than the experimental result. We have checked that the width of the line can be increased by increasing the number of atoms. Alternatively, the discrepancy on the left side of the losses curve could be originating from the p-wave shape resonance [26, 27].

**Condensate oscillations**

After the series of Rydberg atoms is excited the remaining condensate undergoes oscillations. So there is a mechanical effect visible. This effect depends on the state of Rydberg atom which is used. To describe this oscillations quantitatively we measure the width of the condensate in the radial direction as a function of time. Now, before the width measurement is done, the cloud of atoms evolves in the trap by an additional hold time $t_h$. This measurement is done for the detuning equal $-8.5$ MHz which is on average close to the detuning leading to the maximum losses for each state. The measurement is done by fitting the following function to the radial or axial cuts of the measured column densities

$$d(\alpha) = p_1 \left(1 - \frac{(\alpha - p_3)^2}{p_5^2}\right)^{3/2},$$

where $\alpha$ is the radial or axial coordinate respectively, $p_1$ is the peak column density, $p_3$ is the average position of the condensate and $p_5$ is the condensate width. From these fits we get $p_3$ for the radial or axial direction. Then we calculate the width normalized to the unperturbed width and call this quantity the radial oscillation strength or the axial oscillation strength respectively. This procedure is repeated for growing the hold time $t_h$. In the experiment the radial oscillation strength is measured and this quantity versus the hold time $t_h$ is shown in figure 4.

Then to extract the amplitude of oscillations and the frequency we fit the sinusoidal function to this data

$$A \sin(\omega t + \phi) + B,$$

where $A$ is the amplitude of oscillations, $\omega$ is the frequency of oscillations, $\phi$ is the phase shift and $B$ is the average strength of oscillations. The values of the quantities we are interested in are shown in table 3.
Table 3. The relative amplitude and the frequency of oscillations for different states measured in the experiment (look figure 4).

| State   | A/B (%) | ω(2πHz) |
|---------|---------|---------|
| 110S    | 16.5 ± 1.1 | 33.3 ± 0.8 |
| 106D0   | 14.3 ± 2.1  | 31.2 ± 1.6  |
| 106D2   | 12.1 ± 1.1  | 32.4 ± 1.1  |

Figure 5. The theoretical axial oscillation strength after the excitation sequence for Γ = −8.0 MHz. The black disks, the red squares and the green triangles correspond to the 110S state, the 106D0 state and the 106D2 state respectively.

Table 4. The relative amplitude and the frequency of oscillations for different states coming from the theory (look figure 5).

| State   | A/B (%) | ω(2πHz) |
|---------|---------|---------|
| 110S    | 6.1 ± 0.3  | 31.8 ± 0.6  |
| 106D0   | 7.8 ± 0.6   | 33.7 ± 0.7   |
| 106D2   | 6.3 ± 0.5   | 32.7 ± 0.7   |

The oscillation frequencies are close to the slow quadrupole oscillation frequency $\omega \approx \sqrt{5/2} \omega_c = 2\pi 34.8$ Hz. The oscillation amplitude is the strongest for the 110S and smallest for the 106D2 state with the 106D0 state in between.

We repeat the experimental procedure using our theoretical description. Then we measure the condensate width in axial direction. This step is different than in the experiment. The width measurement in the experiment was done after the TOF expansion and the radial width was bigger than the axial one and therefore more suitable to perform measurements. In the theoretical description we measure the width of the condensate inside the harmonic trap. The axial width is larger and more suitable to perform the width measurement. In this case the oscillations are measured for the detuning equals −8.0 MHz which roughly corresponds to the maximum losses in the theoretical description. The strength of oscillations is shown in figure 5.

The parameters of the sinusoidal fit are collected in table 4.

The main observation is that our theory can capture the mechanical effect. The frequency of oscillations agrees very well with the experiment. This means that both in the experiment and in our theory the same oscillation mode is excited independently of which Rydberg state is excited. The phase of oscillations is very close to the experimental value as well. Unfortunately the oscillation amplitude is roughly two times smaller than measured. There are at least two reasons why this discrepancy may appear. Firstly, in the experiment the amplitude is calculated with respect to the reference amplitude which comes from runs where the red laser is detuned from resonance. The presence of the red laser introduces some small oscillations and affects in some way the final result. In the theoretical description we always divide the calculated amplitude by the unperturbed amplitude. Secondly we measure the axial width inside the trap instead of the radial width after TOF. Both effects can affect the final result and hence it is rather expected that we do not see full quantitative agreement. It has been shown in [25] that the mechanical effect of the Rydberg atom on the BEC strongly depends on the principal quantum number $n$ of the excited Rydberg state. Here, we show that many-body dynamics is essentially the same for two angular momenta states $S$ and $D$, which confirms the general applicability of the wavefunction imaging method in a BEC [28].

Conclusions

We have analyzed numerically the impact of the excitation of a sequence of Rydberg atoms on the Bose–Einstein condensate. We found that, by creating Rydberg atoms, the condensate is heated. Hence, the number of condensed atoms decreases. After the cloud of atoms is released from the trap, the thermal component is gone and the condensate losses can be measured. We calculated the final (i.e., when the excitation of Rydberg atoms is stopped) condensate fraction both for $S$- and $D$-Rydberg states. In fact, within the stochastic model we have developed we are able to obtain the full resonance curve. We find that the position of the center of the resonance line is determined solely by the total number of atoms in the sample whereas its width and depth strongly depend on the vacuum Rabi frequency, the scattering rate, and the lifetime of the Rydberg atom, i.e., on the parameters of our stochastic model. The scattering rate is calculated within the simplified semiclassical approach and the lifetime of the Rydberg atom is known from the measurement. We compared our numerical results to experimental data. There is a good agreement in the case of $S$-states. The good agreement for $D$-states is achieved by use of an effective lifetime of the Rydberg atom. We also investigated the mechanical effect of Rydberg atoms on the condensate. We found the oscillations of the size of the condensate. The frequencies of these oscillations agree very well with experimentally measured while the amplitudes remain in qualitative agreement.

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References

[1] Beigman I and Lebedev V 1995 Phys. Rep. 250 95
[2] Amaldi E and Segrè E 1934 Nature 133 141
[3] Amaldi E and Segrè E 1934 Nuovo Cimento 11 145
[4] Fermi E 1934 Nuovo Cimento 11 157
[5] Fermi E 1936 Ric. Sci. 7 13 translated by G M Temmer in Enrico Fermi, Collected Papers vol I, University of Chicago press, 1962
[6] Omont A 1977 J. Phys. France 38 1343
[7] Greene C H, Dickinson A S and Sadeghpour H R 2000 Phys. Rev. Lett. 85 2458
[8] Bendkowsky V, Butscher B, Nipper J, Shaffer J P, Löw R and Pfau T 2009 Nature 458 1005
[9] Krupp A T, Gaj A, Balewski J B, Ilzhöfer P, Hofferberth S, Löw R, Pfau T, Kurz M and Schmelcher P 2014 Phys. Rev. Lett. 112 143008
[10] Anderson D A, Miller S A and Raithel G 2014 Phys. Rev. Lett. 112 163201
[11] Bellos M A, Carollo R, Banerjee J, Eyler E E, Gould P L and Stwalley W C 2013 Phys. Rev. Lett. 111 053001
[12] Tallant J, Rittenhouse S T, Booth D, Sadeghpour H R and Shaffer J P 2012 Phys. Rev. Lett. 109 173202
[13] Sämannhaussen H, Merkt F and Deiglmayr J 2015 Phys. Rev. Lett. 114 133201
[14] DeSalvo B J, Aman J A, Dunning F B, Killian T C, Sadeghpour H R, Yoshida S and Burgdörfer J 2015 Phys. Rev. A 92 031403
[15] Anderson D A, Miller S A and Raithel G 2014 Phys. Rev. A 90 062518
[16] Böttcher F, Gaj A, Westphal K M, Schlagmüller M, Kleinbach K S, Löw R, Liebisch T C, Pfau T and Hofferberth S 2016 Phys. Rev. A 93 032512
[17] Niederprüm T, Thomas O, Eichert T and Ott H 2016 Phys. Rev. Lett. 117 123002
[18] Butscher B, Nipper J, Balewski J B, Kukota L, Bendkowsky V, Löw R and Pfau T 2010 Nat. Phys. 6 970
[19] Hamilton E L, Greene C H and Sadeghpour H R 2002 J. Phys. B: At. Mol. Opt. Phys. 35 L199
[20] Niederprüm T, Thomas O, Eichert T, Lippe C, Pérez-Ríos J, Greene C H and Ott H 2016 Nat. Commun. 7 12820
[21] Bendkowsky V et al 2010 Phys. Rev. Lett. 105 163201
[22] Li W et al 2011 Science 334 1110
[23] Booth D, Rittenhouse S T, Yang J, Sadeghpour H R and Shaffer J P 2015 Science 348 99
[24] Gaj A, Krupp A T, Balewski J B, Löw R, Hofferberth S and Pfau T 2014 Nat. Commun. 5 4546
[25] Balewski J B, Krupp A T, Gaj A, Peter D, Büchler H P, Löw R, Hofferberth S and Pfau T 2013 Nature 502 664
[26] Schlagmüller M et al 2016 Phys. Rev. Lett. 116 053001
[27] Schlagmüller M et al 2016 Phys. Rev. X 6 031020
[28] Karpiuk T, Brewczyk M, Rzążewski K, Gaj A, Balewski J B, Krupp A T, Schlagmüller M, Löw R, Hofferberth S and Pfau T 2015 New J. Phys. 17 053046
[29] Omont A 1977 J. Phys. France 38 1343
[30] Fabrikant I I 1986 J. Phys. B: At. Mol. Opt. Phys. 19 1527
[31] Góral K, Gajda M and Rzążewski K 2001 Opt. Express 8 92
[32] Brewczyk M, Gajda M and Rzążewski K 2007 J. Phys. B: At. Mol. Opt. Phys. 40 R1
[33] Karpiuk T, Brewczyk M, Gajda M and Rzążewski K 2010 Phys. Rev. A 81 013629
[34] Penrose O and Onsager L 1956 Phys. Rev. 104 576
[35] Löw R, Weimer H, Nipper J, Balewski J B, Butscher B, Büchler H P and Pfau T 2012 J. Phys. B: At. Mol. Opt. Phys. 45 113001
[36] Niederprüm T, Thomas O, Manthey T, Weber T M and Ott H 2015 Phys. Rev. Lett. 115 013003