Dominant Spin-Flip Effects for the Hadronic Produced $J/\psi$

Polarization at TEVATRON

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Abstract

Dominant spin-flip effects for the direct and prompt $J/\psi$ polarizations at TEVATRON run II with collision energy 1.96 TeV and rapidity cut $|y^{J/\psi}| < 0.6$, have been systematically studied, especially, the spin-flip effect for the transition of $(c\bar{c})_8^{[3S_1]}$ into $J/\psi$ has been carefully discussed. It is found that the spin-flip effect shall always dilute the $J/\psi$ polarization, and with a suitable choice of the parameters $a_{0,1}$ and $c_{0,1,2}$, the $J/\psi$ polarization puzzle can be solved to a certain degree. At large transverse momentum $p_t$, $\alpha$ for the prompt $J/\psi$ is reduced by $\sim 50\%$ for $f_0 = v^2$ and by $\sim 80\%$ for $f_0 = 1$. We also study the indirect $J/\psi$ polarization from the $b$-decays, which however is slightly affected by the same spin-flip effect and then shall provide a better platform to determine the color-octet matrix elements.

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I. INTRODUCTION

Within the non-relativistic QCD (NRQCD) framework [1], the hadronic production of \( J/\psi \) is dominated by the gluon fragmentation in which a gluon fragments into a color-octet state \((c\bar{c})_8[^3S_1]\). And if the spin-symmetry hold for charm quarks, as is usually adopted in the literature, then the formed \( J/\psi \) shall always show large transverse polarization at sufficiently large \( p_t \). But this prediction contradicts with the measured at TEVATRON [2, 3]. This is the well-known \( J/\psi \) polarization puzzle. Recently, the next-to-leading order (NLO) analysis of the \( J/\psi \) polarization have been done by Refs. [4, 5], with \((c\bar{c})\) pair in \( J/\psi \) Fock expansion being in color-singlet state \([^3S_1]_1\), color-octets \([^3S_1]_8\) and \([^1S_0]_8\) respectively. It is found that even with those NLO corrections, the \( J/\psi \) polarization puzzle still can not be solved. This implies that by doing the NLO calculations, one can make the perturbative QCD results more convergent on \( \alpha_s \) and then more reliable, but one can not change the fact that the produced \( J/\psi \) is largely transversely polarized.

There are many suggestions to solve such polarization puzzle, e.g. the effects of the initial gluon off-shellness may provide a possible explanation for such \( J/\psi \) spin alignment in \( p\bar{p} \) collision [6], which depends heavily on the unintegrated parton distribution of the initial gluon(s). While the spin-flip interaction provides another effective way to solve the \( J/\psi \) polarization puzzle as suggested by Ref. [7]. By taking the spin-flip interactions into account, it has been argued in Ref. [7] that the direct \( J/\psi \) and \( \psi' \) polarizations can be diluted to a certain degree. And an analysis for the prompt \( \psi' \) production with proper ranges for the newly introduced parameters \( a_{0,1} \) and \( c_{1,2} \) has been presented there. In the present paper, we shall make a systematical discussion on how the spin-flip interaction affects the polarization of the direct, the prompt and the indirect produced \( J/\psi \) respectively, which is much more involved than that of \( \psi' \), since we need to consider contributions from the higher charmonium states accordingly. And then we shall make a comparison with the newly obtained prompt \( J/\psi \) data by TEVATRON CDF collaboration [3].

Assuming the produced \( J/\psi \) is measured with the momentum \( p \), we define the rest frame of \( J/\psi \) by a Lorentz boost from its moving frame. The produced \( J/\psi \) is polarized with the polarization vector \( \epsilon^* \) in the rest frame. Within the framework of NRQCD factorization [1], after decomposing Dirac- and color- indices, the contribution from the channel through the
color-octet \((c\bar{c})_8[^3S_1]\) to the differential cross section can be generally written as:

\[
\frac{d\sigma}{d\Omega}[(c\bar{c})_8[^3S_1]] = H_{ij} \cdot T_{ij}(\epsilon, \epsilon^*, \hat{p}),
\]

where \(\hat{p} = \frac{p}{|p|}\), \(H_{ij}\) is the 3 \times 3 spin density matrix for producing \((c\bar{c})_8[^3S_1]\) and \(T_{ij}\) is the spin density matrix for the transition of \((c\bar{c})_8[^3S_1]\) into a polarized \(J/\psi\), which can be decomposed as:

\[
T_{ij}(\epsilon, \epsilon^*, \hat{p}) = \delta_{ij} (\epsilon \cdot \epsilon^* a_0 + \epsilon \cdot \hat{p} \cdot \epsilon^* \cdot \hat{p} a_1) + (\epsilon_i \epsilon_j^* + \epsilon_j \epsilon_i^*) c_0 \\
+ \left[ (\epsilon_i \hat{p}_j + \epsilon_j \hat{p}_i) \epsilon^* \cdot \hat{p} + (\epsilon \leftrightarrow \epsilon^*) \right] c_1 + \hat{p}_i \hat{p}_j \epsilon \cdot \epsilon^* c_2,
\]

where \(a_{0,1}\) and \(c_{0,1,2}\) are un-determined, non-perturbative but universal parameters. By taking the heavy quark spin symmetry, only \(c_0 = \langle 0|O_{J/\psi}[^3S_1]|0 \rangle / 6\) is non-zero, which implies that \(J/\psi\) will have the same spin as the color-octet \((c\bar{c})\) pair. In the present, we shall make a detailed discussion on the direct \(J/\psi\) production through \(p\bar{p} \rightarrow J/\psi[n] + X\), where \(n\) stands for the intermediate \((c\bar{c})\)-charmonium states up to \(v^4\) corrections, i.e. \(n = (^3S_1)_1\), \((^3S_1)_8\), \((^1S_0)_8\) or \((^3P_J)_8\). Further more, the hard subprocess of the hadronic process is \(ab \rightarrow J/\psi[n] + X\), where \(ab = gg, gq, g\bar{q}\) and \(q\bar{q}\) respectively. Based on these processes, the relative importance of the undetermined parameters \(a_{0,1}\) and \(c_{0,1,2}\) shall be discussed. The prompt \(J/\psi\) polarization shall also be discussed, whose signal includes \(J/\psi\) meson that comes from decays of the higher charmonium states \(\chi_{c1}, \chi_{c2}\) and \(\psi'\). Further more, we shall take the same spin-flip effect to study the indirect \(J/\psi\) production from \(b\)-decays, i.e. \(b \rightarrow J/\psi + X\).

The paper is organized as follows. In Sec. II, we present the calculation technology for the \(J/\psi\) production. Numerical results and discussions are presented in Sec. III, where the results for the direct, the prompt and the indirect \(J/\psi\) polarization shall be presented. The final section is reserved for a summary.

**II. CALCULATION TECHNOLOGY**

We adopt the same calculation technology as pointed out in Ref. [7] to calculate the direct \(J/\psi\) production, and for self-consistency, we present the calculation procedure in more detail.

It is more convenient to transform the spin density matrix \(T_{ij}\) into a covariant form. For such purpose, we introduce a Lorentz boost matrix \(L^\mu_\alpha\), whose components can be written
as 

\[ L^0_i = \frac{p_i}{M}, \quad L^j_i = \delta_{ij} - \frac{p_ip_j}{p^2} + \left( \frac{p_ip_j}{p^2} \right) \frac{E_p}{M}. \]  

(3)

With the help of such boost matrix, one can transform a purely space-like four-vector, such as \( p \), from the rest frame of \( J/\psi \) where the components of \( p \) are \((M, 0)\) to the frame in which its components are \( p_\mu = (E_p, \mathbf{p}) \), \( E_p = \sqrt{M^2 + p^2} \). By applying \((L^\mu_i)(L^\nu_j)\) to both sides of Eq.(2), and noting the fact that we obtain

\[ g_{\mu\nu}L^\mu_i L^\nu_j = -\delta_{ij}, \quad L^\mu_i L^\nu_i = -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}, \]  

(4)

we obtain

\[ T_{\mu\nu}(\epsilon, \epsilon^*, \hat{\rho}) = \left[ -g_{\mu\nu} + \frac{p_\mu p_\nu}{M^2} \right] \left[ \epsilon \cdot \epsilon^*(-a_0) + \epsilon \cdot \hat{\rho} \cdot \epsilon^* \cdot \hat{\rho} a_1 \right] + \left[ \epsilon_\mu \epsilon^*_\nu + \epsilon_\nu \epsilon^*_\mu \right] c_0 + \left[ (\epsilon_\mu \hat{\rho}_\nu + \epsilon_\nu \hat{\rho}_\mu) \epsilon \cdot \hat{\rho} + (\epsilon \leftrightarrow \epsilon^*) \right] (c_1) + (\hat{\rho}_\mu \hat{\rho}_\nu \epsilon \cdot \epsilon^*)(c_2) + \cdots, \]  

(5)

where in the \( J/\psi \) rest frame, \( \epsilon^\mu = L^\mu_i \epsilon_i = (0, \epsilon) \) and \( \hat{\rho}_\mu = L^\mu_i \hat{\rho}_i = (0, \hat{\rho}). \) Further more, for a particular polarization state \( \lambda = (0, \pm 1) \), \( T_{\mu\nu}(\epsilon, \epsilon^*, \hat{\rho}) \) changes to \( T_{\mu\nu}(\epsilon(\lambda), \epsilon^*(\lambda), \hat{\rho}). \)

A convenient measure of \( J/\psi \) polarization is the variable \( \alpha = (1 - 3\xi)/(1 + \xi) \), where \( \xi = \sigma_L/(\sigma_L + \sigma_T) \), \( \sigma_L \) and \( \sigma_T \) stand for the longitudinal and the transverse components of the direct hadronic cross section respectively. As for the indirect \( J/\psi \) production from \( b \) decays, \( \xi = \Gamma_L/(\Gamma_L + \Gamma_T) \) with \( \Gamma_L \) and \( \Gamma_T \) stand for the longitudinal and the transverse components of the indirect decay width respectively.

With the help of Eq.(5), we can write the total differential cross section for the direct \( J/\psi \) production through the process \( pp \to J/\psi^\lambda[n]+X \) as:

\[ d\sigma_\lambda(pp \to J/\psi^\lambda[n]X) = \sum_{ab} \int dx_adx_b f_{a/p}(x_a)f_{b/\bar{p}}(x_b)d\hat{\sigma}_{\mu\nu}[n, ab]T_{\mu\nu}(\epsilon(\lambda), \epsilon^*(\lambda), \hat{\rho}), \]  

(6)

where the differential cross sections for the hard subprocesses, \( a(k_1)+b(k_2) \to J/\psi[n](p)+X \), can be written in the following factorization form,

\[ \frac{d\hat{\sigma}_{\mu\nu}[n, ab]}{dt} = A_{ab}[n]g_{\mu\nu} + B_{ab}[n]k_{1\mu}k_{1\nu} + C_{ab}[n]k_{2\mu}k_{2\nu} + D_{ab}[n]\frac{1}{2}[k_{1\mu}k_{2\nu} + k_{2\mu}k_{1\nu}]. \]  

(7)

\( n \) stands for the \((c\bar{c})\)-charmonium state up to \( v^4 \) corrections, i.e. \( n = (3S_1)_1, (3S_1)_8, (1S_0)_8 \) and \((3P_J)_8\) respectively. \( k_1 \) and \( k_2 \) are the momenta of the initial partons and \( ab = gg, gq, g\bar{q}, q\bar{q} \). The coefficients \( A_{ab}[n], B_{ab}[n], C_{ab}[n] \) and \( D_{ab}[n] \) can be read from Refs.[8, 10].

\(^1\) We have calculated all these channels and found a good agreement with those in Refs.[8, 10].
calculate the longitudinal cross section, we adopt the covariant form of the $J/\psi$ longitudinal polarization vector,

$$\epsilon_L(p)_\mu = \frac{pQ}{\sqrt{(p-Q)^2-M^2Q^2}} \left( \frac{p_\mu - MQ_\mu}{pQ} \right),$$

where $Q = p_p + p_\bar{p}$ is the sum of the initial hadron momenta. In Refs.\[11, 12\], the fragmentation effect has also been resummed to the leading logarithms $[\alpha_s \ln p_t^2/(2m_c)^2]^{\alpha}$ accuracy for the production of the color-octet $^3S_1$ state with the help of the Altarelli-Parisi evolution equation. By taking the fragmentation effect into account, the value of $\alpha$ shall be further suppressed \[11\]. In this paper, we shall concentrate our attention on the spin-flip interactions and will not take this effect into consideration.

Furthermore, the hadronic differential cross section can be simplified so as to obtain $J/\psi$ $p_t$ distribution:

$$E_p \frac{d^3 \sigma}{d^3 p}(p\bar{p} \rightarrow J/\psi[n]\lambda X) = \sum_{ab} \int dx_a dx_b f_{a/p}(x_a) f_{b/\bar{p}}(x_b) \frac{\hat{s} \hat{\sigma}[n, ab]}{\pi} \delta(\hat{s} + \hat{t} + \hat{u} - M^2),$$

(8)

where $\hat{s} = \hat{s}_\mu [n, ab] T_{\mu\nu}(\epsilon, \epsilon^*, \hat{p})$, and for a particular polarization state of $J/\psi$,

$$\frac{d\sigma_L}{dp_t}(p\bar{p} \rightarrow J/\psi[n]\lambda \lambda X) = \sum_{ab} \int dy dx_a f_{a/p}(x_a) f_{b/\bar{p}}(x_b) \frac{2p_t}{x_a x_b^2 (x_a - M^2/M_T^2)} \times \frac{d\hat{\sigma}_{\mu\nu}[n, ab]}{dt} T_{\mu\nu}(\epsilon(\lambda), \epsilon^*(\lambda), \hat{p}),$$

(9)

where

$$x'_b = \frac{1}{\sqrt{S}} \frac{x_a \sqrt{SM_T e^{-y} - M^2}}{x_a \sqrt{S - M_T e^y}},$$

(10)

with $M_T^2 = \sqrt{p_t^2 + M^2}$, $S = (p_p + p_\bar{p})^2$ and $y$ stands for the rapidity of $J/\psi$.

Secondly, we present the formulae for the prompt $J/\psi$ polarization. Theoretical predictions of the polarization of prompt $J/\psi$ are complicated by the fact that the prompt signal includes $J/\psi$ mesons that also come from decays of the higher charmonium states $\chi_{cJ}$ ($J = 0, 1, 2$), and $\psi'$. The unpolarized differential cross-section $d\sigma_{\text{tot}}^{\text{prompt } J/\psi}/dp_t$ is simply obtained by adding the unpolarized cross-sections of the various direct-production processes multiplied with the appropriate branching fractions. While the longitudinal differential cross-section $d\sigma_{L}^{\text{prompt } J/\psi}/dp_t$ is much more involved, which equals

$$\frac{d\sigma_L^{\text{prompt } J/\psi}}{dp_t} = \frac{d\sigma_L^{\text{direct } J/\psi}}{dp_t} + \frac{d\sigma_L^{\chi_{cJ}}}{dp_t} + \frac{d\sigma_L^{\psi'}}{dp_t} + \frac{d\sigma_L^{\psi' - \chi_{cJ}}}{dp_t},$$

(11)

where following the discussion in Refs.\[12, 13\], we have

$$\sigma_{L}^{\text{direct } J/\psi} = \sum_n \hat{\sigma}_L(n) \langle \mathcal{O}^{J/\psi}(n) \rangle,$$

(12)
where \( \sigma \) elements accordingly, and the summation in the first equation is over \( n = (3S_1)_1, (3S_1)_8, (1S_0)_8 \) and \( (3P_J)_8 \). The direct \( \psi' \) production \( \sigma_L^{\psi'} = \sum_n \sigma_L(n) \langle O^{\psi'}(n) \rangle \) can be obtained from that of \( J/\psi \) by changing the \( J/\psi \) matrix elements to \( \psi' \) matrix elements. The cross-section for \( n = (c\bar{c})_1(3P_J) \) is much more involved and we put some necessary formulae in the APPENDIX A.

Finally, by taking the spin-flip effect into consideration, we recalculate the \( J/\psi \) production from the \( b \)-decay process, \( b \rightarrow J/\psi[n] + X \). Following the same procedure of Ref.14, the total unpolarized cross-section can be written as

\[
\Gamma_{\text{tot}}(b \rightarrow J/\psi + X) = \frac{G_F^2}{144\pi} |V_{cb}|^2 m_c m_b^3 \left( 1 - \frac{4m_c^2}{m_b^2} \right)^2 \times \left[ a \left( 1 + \frac{8m_c^2}{m_b^2} \right) + b \right],
\]

where

\[
a = \left( 3C_+ + C_- \right)^2 \times \left[ \frac{2c_0 + 3a_0 + a_1}{2m_c^2 + 8m_c^2} \left[ 4c_1 + 3c_2 \right] + \frac{\langle O_{8/\psi}^{l/\psi}(3P_0) \rangle}{m_c^4} \right] + (2C_+ - C_-)^2 \frac{\langle O_{8/\psi}^{l/\psi}(3S_1) \rangle}{3m_c^2},
\]

\[
b = 3(C_+ + C_-) \frac{\langle O_{8/\psi}^{l/\psi}(1S_0) \rangle}{2m_c^2}.
\]
The Wilson coefficients $C_+(m_b) = 0.868$ and $C_-(m_b) = 1.329$ \cite{15}. In Eq. (16), upon summing over the light quarks $s$ and $d$, we have used $|V_{cb} V_{cs}^*|^2 + |V_{cb} V_{cd}^*|^2 \approx |V_{cb}|^2$. We have taken the initial $b$-quark to be unpolarized. Here $\langle O^{J/\psi}_{n}\rangle \equiv \langle 0|O^{J/\psi}_{n}|0\rangle$ are NRQCD $J/\psi$ production matrix elements. If neglecting the spin-flip effects, we return to the same results derived by Ref. \cite{14}. As for the longitudinal cross-section from $b$ decays, we obtain

$$
\Gamma_L(b \to J/\psi + X) = \frac{G_F^2}{864\pi} \left( \frac{m_b^2 - 4m_c^2}{m_b^3m_c} \right)|V_{cb}|^2 \left\{ 2(2C_+ - C_-)^2m_b^2\langle O^{J/\psi}_{1}(3S_1)\rangle + 9(m_b^2 + 8m_c^2)(C_+ + C_-)^2\left[ a_0 + a_1 + \frac{m_b^2}{m_b^2 + 8m_c^2}(2c_0 + 4c_1 + c_2) \right] + 3m_b^2(C_+ + C_-)^2\langle O^{J/\psi}_{8}(1S_0)\rangle + 72m_c^2(C_+ + C_-)^2\langle O^{J/\psi}_{8}(3P_0)\rangle \right\}.
$$

In these equations, we have adopted the relation $\langle O^{J/\psi}_{8}(3P_0)\rangle = (2J + 1)\langle O^{J/\psi}_{8}(3P_0)\rangle$ \cite{1}.

### III. NUMERICAL RESULTS AND DISCUSSIONS

As for numerical calculation, we adopt the matrix elements derived by Ref. \cite{12}, which are shown in Table I. To be consistent, the parton distribution function is chosen to be CTEQ5L \cite{16} and the value of $\alpha_s$ is evaluated from the one-loop formula using the corresponding value in CTEQ5L for $\Lambda_{QCD}$, $m_c = 1.5 GeV$ and $M_{J/\psi} = 2m_c$. Both the factorization scale and the renormalization scale are taken to be the transverse mass of $J/\psi$, i.e. $\mu_f = \mu_r = \sqrt{M^2 + p_T^2}$. The collision center of mass (C.M.) energy is 1.96 TeV. The branching ratio of $J/\psi \to \mu^+\mu^-$ is $\beta = (5.93 \pm 0.06) \times 10^{-2}$ \cite{17}. As for the charmonium, we take $v^2 = 0.30$.

There are two types of power counting rules for the undetermined parameters $a_{0,1}$ and $c_{0,1,2}$ \cite{7, 18}, i.e. the first type is

$$
a_0 \sim v^2, \quad a_1 \sim v, \quad c_0, c_1, c_2 \sim v^3
$$

and the second type is

$$
a_0 \sim a_1, \quad a_1 \sim c_0, \quad c_1 \sim c_2 \sim \mathcal{O}(1).
$$

| $\langle O^{J/\psi}_{1}(3S_1)\rangle$ | $\langle O^{J/\psi}_{8}(3S_1)\rangle$ | $M_{3,4}$ | $\langle O^{J/\psi}_{1}(3S_1)\rangle$ | $\langle O^{J/\psi}_{8}(3S_1)\rangle$ | $M_{3,5}$ | $\langle O^{J/\psi}_{8}(3P_0)\rangle$ | $\langle O^{J/\psi}_{8}(3S_1)\rangle$ |
|---|---|---|---|---|---|---|---|
| 1.4 ± 0.1 | 3.9 ± 0.7 | 6.6 ± 0.7 | 6.7 ± 0.7 | 3.7 ± 0.9 | 0.78 ± 0.36 | 9.1 ± 1.3 | 1.9 ± 0.2 |
| GeV$^3$ | 10$^{-3}$GeV$^3$ | 10$^{-2}$GeV$^3$ | 10$^{-1}$GeV$^3$ | 10$^{-3}$GeV$^3$ | 10$^{-2}$GeV$^3$ | 10$^{-2}$GeV$^5$ | 10$^{-3}$GeV$^3$ |

**TABLE I:** Adopted NRQCD matrix elements from Ref.\cite{12}.
FIG. 1: \(R[gg, (3S_1)_s]\)-distributions defined by Eq. (23), where the left diagram is derived by summing over all the polarizations and the right diagram is only for the longitudinal polarization of \(J/\psi\). For the right diagram, the curves for \(a_0\) and \(a_1\) are coincide with each other.

Since the value of \(v\) is generally not small for the case of charmonium, those higher \(v\)-suppressed terms can have a significant impact on theoretical predictions even for the first type of power counting rule.

A. relative importance among the different terms in \(T_{\mu\nu}(\epsilon, \epsilon^*, \hat{p})\)

This subsection is served to show the relative importance among the different terms in \(T_{\mu\nu}(\epsilon, \epsilon^*, \hat{p})\). In order to show clearly the contributions from each parts of \(T_{\mu\nu}(\epsilon, \epsilon^*, \hat{p})\), we rewrite \(\frac{d\sigma[ab, n]}{dp_t}\) as \((n = (3S_1)_s)\),

\[
\frac{d\sigma[ab, n]}{dp_t} = \frac{d\sigma^0[ab, n]}{dp_t} c_0 + \frac{d\sigma^1[ab, n]}{dp_t} c_1 + \frac{d\sigma^2[ab, n]}{dp_t} c_2 + \frac{d\sigma^{a_0}[ab, n]}{dp_t} a_0 + \frac{d\sigma^{a_1}[ab, n]}{dp_t} a_1, \tag{22}
\]

where \(\frac{d\sigma^{c_0}[ab, n]}{dp_t}\) stands for the \(p_t\)-distribution for \(c_1 = c_2 = a_0 = a_1 = 0\) with an overall factor \(c_0\) being contracted out, and etc.. To show the relative importance among different distributions, we define the ratio,

\[
R^{c_1}[ab, n] = \frac{\frac{d\sigma^1[ab, n]}{dp_t}}{\frac{d\sigma^0[ab, n]}{dp_t}}, \quad R^{c_2}[ab, n] = \frac{\frac{d\sigma^2[ab, n]}{dp_t}}{\frac{d\sigma^0[ab, n]}{dp_t}}, \quad R^{a_0}[ab, n] = \frac{\frac{d\sigma^{a_0}[ab, n]}{dp_t}}{\frac{d\sigma^0[ab, n]}{dp_t}}, \quad R^{a_1}[ab, n] = \frac{\frac{d\sigma^{a_1}[ab, n]}{dp_t}}{\frac{d\sigma^0[ab, n]}{dp_t}}, \tag{23}
\]

where \(ab = gg, gq, g\bar{q}\) and \(q\bar{q}\).
The $R$-distributions for the dominant gluon-gluon fusion mechanism ($ab = gg$) are shown in Fig.(1), where the left diagram is derived by summing over all the polarizations and the right diagram is only for the longitudinal polarization of $J/\psi$. The left diagram of Fig.(1) shows that when summing over all the polarization vectors of $J/\psi$, the weights of $a_0$ and $a_1$ shall always at the same order of that of $c_0$, and more explicitly $R^a[gg, (^3S_1)_8] \equiv 3/2$ and $R^{a_1}[gg, (^3S_1)_8] \equiv 1/2$; while those of $c_1$ and $c_2$ drop down quickly with the increment of $p_t (\mathcal{O}(1/p^2_t))$. The right diagram of Fig.(1) shows that for only the longitudinal part, the weights of $a_0$ and $a_1$ increase quickly in comparison with that of $c_0$ with the increment of $p_t \mathcal{O}(p^2_t)$; while the weights of $c_1$ and $c_2$ have the same order of that of $c_0$, or more explicitly $R^{c_1}[gg, (^3S_1)_8] \equiv 2$ and $R^{c_2}[gg, (^3S_1)_8] \equiv 1/2$.

B. a simple discussion on the color-octet $^3S_1$ matrix element under the spin-flip interaction

Summing over the polarizations on both sides of Eq.(2), we obtain a new matrix element for the color-octet $^3S_1$ state,

$$
\langle 0| O^J/\psi_8 (^3S_1)|0 \rangle' = 6c_0 \left[ 1 + \frac{3a_0}{2c_0} + \frac{a_1}{2c_0} + \frac{2c_1}{3c_0} + \frac{c_2}{2c_0} \right].
$$

(24)

In principle, the value of the new matrix element $\langle 0| O^J/\psi_8 (^3S_1)|0 \rangle'$ defined in Eq.(24) is different from that of the usual $\langle 0| O^J/\psi_8 (^3S_1)|0 \rangle$, since it involves an extra gauge link in the definition and it also takes the spin-flip interaction into account. By taking into the spin-flip interaction and the extra gauge links, the matrix element $\langle 0| O^J/\psi_8 (^3S_1)|0 \rangle'$ contains more non-perturbative parameters to be determined, i.e. $a_0$, $a_1$, $c_0$, $c_1$ and $c_2$. In the following, we shall discuss the spin-flip effects based on the two counting rules (20) and (21), and to predict the polarization, we make the ansatz that $a_1 = c_1 = c_2$ and introduce two parameters $f_0 = a_0/c_0$ and $f_1 = a_1/c_0$, we obtain

$$
\langle O^J/\psi_8 (^3S_1) \rangle' = 6c_0 \left( 1 + \frac{3}{2} f_0 + \frac{5}{3} f_1 \right).
$$

(25)

$f_1 = v f_0$ for the power counting rule (20), $f_1 = f_0$ for the power counting rule (21) respectively. By setting $f_0 = f_1 = 0$, we return to the result without taking into account the spin-flip interaction, which leads to $c_0 = \langle 0| O^J/\psi_8 (^3S_1)|0 \rangle / 6$.

In Fig.(2), we show the $p_t$-distributions with different value of $f_0$ and $f_1 = v f_0$ for the direct production of $J/\psi$ through $(c\bar{c})_8[^3S_1]$ at TEVATRON run II $(\sqrt{s} = 1.96TeV)$, where
FIG. 2: $p_t$-distributions for the direct production of $J/\psi$ through $(c\bar{c})_8[^3S_1]$ at C.M. energy $\sqrt{s} = 1.96$ TeV. Left diagram is derived by summing over all the polarizations and the right diagram is only for the longitudinal polarization of $J/\psi$. $\beta$ stands for the branching ratio $J/\psi \rightarrow \mu^+\mu^-$. In the calculation, the contributions from all the considered subprocesses have been summed up, and rapidity cut $|y^{J/\psi}| < 0.6$ is adopted.

The left diagram is derived by summing over all the polarizations and the right diagram is only for the longitudinal polarization of $J/\psi$. In the calculation, the contributions from all the considered subprocesses (with $ab = gg, gq, g\bar{q}, q\bar{q}$) have been summed up, and the rapidity cut $|y^{J/\psi}| < 0.6$ is adopted. Fig. 2 shows that by considering the spin-symmetry-breaking factors $a_{0,1}$ and $c_{1,2}$, the $p_t$-distributions derived by summing over all the polarizations of $J/\psi$ shall not be affected too much in comparison with the case of not considering the spin flip interactions (with $f_0 = f_1 = 0$). While for the longitudinal $p_t$-distributions, the spin-symmetry-breaking factors $a_{0,1}$ and $c_{1,2}$ might be important in the large $p_t$ regions, i.e. they can raise the longitudinal contributions to a certain degree. So such spin symmetry breaking corrections cannot be ignored safely as has been done in Ref. [19], since as shown in the left diagram of Fig. 2, they can change the fraction of the longitudinal part dramatically.

To show this point more clearly, we present the differential cross-section formulae for $J/\psi$ production channel through $g + g \rightarrow J/\psi(^3S_1)_8 + g$ in APPENDIX B. It can be found that $c_0$ comes into contributions at $O(1/p_t^2)$ for the longitudinal differential cross section. This is the reason why by taking the spin-symmetry, $a_0 = a_1 = c_1 = c_2 = 0$, the longitudinal contributions should be neglected at large $p_t$ regions, i.e. the $J/\psi$ is transverse polarized at large $p_t$ regions. And it is clear that longitudinal differential cross section will not be
suppressed by $1/p_{\perp}^2$, if one takes spin-flip interaction into account, i.e., if those coefficients beside $c_0$, especially $a_0$ and $a_1$, are not zero.

In the above, we have shown that the spin-symmetry-breaking factors $a_0$, $a_1$, $c_1$ and $c_2$ might be important in the large $p_t$ regions for the production through $(\bar{c}c)s[^3S_1]$, i.e. they can raise the longitudinal contributions to a certain degree. In the large $p_t$ regions, it is the $(\bar{c}c)s[^3S_1]$ production channel that yields the transverse polarization, while the $(\bar{c}c)s[^1S_0]$ and $(\bar{c}c)s[^3P_J]$ channels both yield unpolarized quarkonia in this limit. So, the large transverse polarization might be considerably changed by the spin-symmetry-breaking factors $a_0$, $a_1$, $c_1$ and $c_2$, and then the large discrepancy between the theoretical prediction and the experimental data might be compensated.

C. Numerical results of $\alpha$ for direct $J/\psi$ production

The polarization is predicted with the parameter $\alpha$ as a function of $p_{\perp}$, which is defined as:

$$\alpha = \left( \frac{d\sigma_{\text{tot}}}{dp_{\perp}} - 3 \frac{d\sigma_L}{dp_{\perp}} \right) / \left( \frac{d\sigma_{\text{tot}}}{dp_{\perp}} + \frac{d\sigma_L}{dp_{\perp}} \right) = 1 - 3\xi \frac{1 + \xi}{1 + \xi},$$

(26)

where $\xi = \frac{d\sigma_L}{dp_{\perp}} / \frac{d\sigma_{\text{tot}}}{dp_{\perp}}$. If $\alpha = 1$, the produced $\psi$ is transversely polarized. If $\alpha = -1$ the produced $\psi$ is longitudinally polarized.

After summing over all the above mentioned channels, we represent the longitudinal polarization fraction $\xi$ for the direct $J/\psi$ production as

$$\xi(f_0, f_1) = \frac{\frac{d\sigma^{[^3S_1]}_{\text{tot}}}{dp_{\perp}} \langle O^{J/\psi[^3S_1]} \rangle + \frac{d\sigma^{[^3S_1]}_8}{dp_{\perp}} \langle O^{J/\psi[^3S_1]}_8 \rangle}{\frac{d\sigma^{[^1S_0]}_{\text{tot}}}{dp_{\perp}} \langle O^{J/\psi[^1S_0]} \rangle + \frac{d\sigma^{[^1S_0]}_8}{dp_{\perp}} \langle O^{J/\psi[^1S_0]}_8 \rangle + \frac{d\sigma^{[^3P_J]}_{\text{tot}}}{dp_{\perp}} \langle O^{J/\psi[^3P_J]} \rangle} \frac{M_r}{M_{r J/\psi}},$$

(27)

where $\frac{d\sigma^{[^S_1]}_{\perp}}{dp_{\perp}}$ stands for the differential cross section without the matrix element $\langle O^{J/\psi[^3S_1]} \rangle$ for the case of $J/\psi$ through color-singlet $[^3S_1]$-charmonium state, and so on. $M_{r J/\psi} = \langle O^{J/\psi[^1S_0]}_8 \rangle + r \langle O^{J/\psi[^3P_0]}_8 \rangle / m_c^2$ with $r = 3.4^{[12]}$, and the parameter $x = \langle O^{J/\psi[^1S_0]}_8 \rangle / M_{r J/\psi}$, whose center value is 1/2 and can be varied between 0 to 1.

2 We have also calculated the spin-flip effects for $(\bar{c}c)s[^3S_1]$, which is quite small in comparison to those of $(\bar{c}c)s[^1S_0]$, so we shall not take these spin-flip effects into consideration.
FIG. 3: $\alpha$ for the direct $J/\psi$ production. Left diagram is for $f_1 = v f_0$ and the right diagram is for $f_1 = f_0$. The upper shaded band is for $f_0 = 0$, the higher middle yellow band is for $f_0 = v^4$, the lower middle green band is for $f_0 = v^2$ and the lowest band is for $f_0 = 1$, where the uncertainties of the matrix elements are also included. The upper edge of each band is for $x = 1$ and the lower edge for $x = 0$. The rapidity cut $|y_{J/\psi}| < 0.6$ is adopted. The experimental data is from Ref.[3].

In Fig.(3), we present the value of $\alpha$ as a function of $p_t$ for the direct $J/\psi$ hadronic production, where typical values for $f_0$ and $f_1$ are adopted and the uncertainties from the matrix elements are also included. From the figure the produced $J/\psi$ will be dominantly with transverse polarization at large $p_t$, if one does not take the spin-flip interaction into account, i.e., $f_0 = f_1 = 0$. Increasing $f_0$ and $f_1$ from 0 to 1, $\alpha$ will be decreased accordingly. And if one takes $f_0$ and $f_1$ at the order of 1, $\alpha$ shall be close to 0, which implies that the $J/\psi$ is unpolarized.

D. Numerical results of $\alpha$ for the prompt $J/\psi$ production

In calculating $\alpha$ for the prompt $J/\psi$ production, we need to know the non-perturbative matrix elements $\langle O_1^{\chi c J}(3P_J) \rangle$ and $\langle O_8^{\chi c J}(3S_1) \rangle$. And for their values we adopt the following relations:

$$\langle O_1^{\chi c J}(3P_J) \rangle = (2J + 1) \langle O_1^{\chi c o}(3P_0) \rangle \quad (28)$$

$$\langle O_8^{\chi c J}(3S_1) \rangle = (2J + 1) \langle O_8^{\chi c o}(3S_1) \rangle \quad (29)$$

$\langle O_1^{\chi c o}(3P_0) \rangle = (9.1 \pm 1.3) \times 10^{-2} \text{ GeV}^5$ and $\langle O_8^{\chi c o}(3S_0) \rangle = (1.9 \pm 0.2) \times 10^{-3} \text{ GeV}^3$. As for the relevant branching fractions listed in Eqs.(13,14,15), we adopt the values from Ref.[17], we adopt the values from Ref.[17],
FIG. 4: $\alpha$ for the prompt $J/\psi$ production. Left diagram is for $f_1 = v f_0$ and the right diagram is for $f_1 = f_0$. The upper shaded band is for $f_0 = 0$, the higher middle yellow band is for $f_0 = v^4$, the lower middle green band is for $f_0 = v^2$ and the lowest band is for $f_0 = 1$, where the uncertainties of the matrix elements are also included. The upper edge of each band is for $x = 1$ and the lower edge for $x = 0$. The rapidity cut $|y^{J/\psi}| < 0.6$ is adopted. The experimental data on the prompt $J/\psi$ is from Ref. [3]. Note the upper shaded band is close to Braaten’s results [12].

i.e. $B(\chi_{c0} \rightarrow J/\psi + \gamma) = (1.30 \pm 0.11)\%$, $B(\chi_{c1} \rightarrow J/\psi + \gamma) = (35.6 \pm 1.9)\%$, $B(\chi_{c2} \rightarrow J/\psi + \gamma) = (20.2 \pm 1.0)\%$, $B(\psi' \rightarrow J/\psi + X) = (56.1 \pm 0.9)\%$, $B(\psi' \rightarrow \chi_{c0} + \gamma) = (9.2 \pm 0.4)\%$, $B(\psi' \rightarrow \chi_{c1} + \gamma) = (8.7 \pm 0.4)\%$, $B(\psi' \rightarrow \chi_{c2} + \gamma) = (2.6 \pm 0.4)\%$. As for the direct production $\sigma^{\text{direct}}_{J/\psi}$ and $\sigma^{\text{direct}}_{\psi'}$, only the spin-flip effects in the channel of $(c\bar{c})[3^3S_1]$ are sizable. So we shall only consider the spin-flip effects in the direct production $\sigma^{\text{direct}}_{J/\psi}$ and $\sigma^{\text{direct}}_{\psi'}$.

In Fig. (4), we present the value of $\alpha$ as a function of $p_t$ for the prompt $J/\psi$ production, where typical values for $f_0$ and $f_1$ are adopted and the uncertainties from the matrix elements are also included. The experimental data on the prompt $J/\psi$ is from the TEVATRON CDF collaboration [3]. It can be found that the results for $f_0 = f_1 = 0$, i.e. without taking the spin-flipping effects into account, is consistent with Braaten’s results [12]. It is clear that under the proper spin-flip interaction, $\alpha$ can be more closer to the experimental data than that without these interactions. At large $p_t$, $\alpha$ is reduced by $\sim 50\%$ for $f_0 = v^2$ and by $\sim 80\%$ for $f_0 = 1$. 

3
E. Numerical results of $\alpha_B$ for indirect $J/\psi$ production through $b \rightarrow J/\psi + X$

By varying the matrix elements within the region of TABII we calculate the numerical results of $\alpha_B$ versus $x$ for the indirect $J/\psi$ production through $B \rightarrow J/\psi + X$, where $x = \langle O^{J/\psi}(1S_0) \rangle/M_{3,4}^{J/\psi}$. It is found that the predicted $J/\psi$ polarization parameter $\alpha_B$ depends weaker on $f_0$ and $f_1$ than the case of direct and prompt $J/\psi$ production. More explicitly under the case of $f_1 = vf_0$, the range of $\alpha_B$ is shifted from $[-0.100, 0.193]$ to $[-0.123, 0.175]$ by varying $f_0$ from 0 to 1, while under the case of $f_1 = f_0$, the range of $\alpha_B$ is shifted from $[-0.100, 0.193]$ to $[-0.143, 0.163]$ by varying $f_0$ from 0 to 1. On the other hand, it dependents heavily on the matrix elements $\langle O^{J/\psi}(1S_0) \rangle$ and $M_{3,4}^{J/\psi}$. To be consistent with $\alpha_B$ derived in literature, e.g. $\alpha_B = -0.13 \pm 0.01$ for $J/\psi$ events with $p_T(J/\psi) > 4 GeV$ by CDF group[20], a larger $x$ that approaches 1 should be taken, as is implicitly adopted by Ref.[14]. Since the spin-flip effect is weaker in this indirect $J/\psi$ production, a more precise measurement of it can predict more precise matrix elements. As a cross check, we have found that without considering the spin-flip effect and by varying the involved matrix elements within the same uncertainty region derived by Ref.[14], we can obtain the same allowable region for $\alpha_B$, i.e. $\alpha_B \in [-0.33, 0.05]$. 

FIG. 5: The predicted $\alpha_B$ for $b \rightarrow J/\psi + X$ as a function of the unknown parameter $x$. In the left diagram $f_1$ is taken as $v f_0$, while the right one is for $f_1 = f_0$. 
IV. SUMMARY

It is noted that the Υ(nS) production is somewhat different from the case of J/ψ and ψ′. Ref. [23] shows that the NLO correction plus the LO results for the color-singlet of Υ(nS) can explain both the total unpolarized and the polarized Υ(nS) production cross sections. Then there is no need to consider the contributions from the color-octet transitions for Υ(nS) production, or in another words, the color-octet components give negligible contributions to the Υ(nS) production. For the J/ψ or ψ′ production, the NLO correction plus the LO results for the color-singlet production can also lead to longitudinal polarized J/ψ or ψ′ [4]. However, it is well-known that with the color-singlet contribution only, one can not explain the unpolarized J/ψ or ψ′ production cross section. And by taking the color-octet c ¯c components into consideration, one can well explain the total unpolarized cross section of J/ψ or ψ′ [24], which is regarded as a great triumph of NRQCD. Within the NRQCD framework, if keeping the spin symmetry for the charm quark, Refs. [4, 5] show that the J/ψ polarization puzzle can not be solved even by including the NLO corrections. Hence to well explain both the unpolarized and longitudinal cross sections of J/ψ or ψ′ is much more involved than the case of Υ(nS).

We have shown that the spin-flip interaction can have a significant impact on the transition of a color-octet 3S1 c ¯c pair into J/ψ. Such impact can be parameterized by introducing new parameters in the transition matrix T, i.e. a0,1 and c0,1,2. If the heavy quark spin symmetry holds, the matrix has only one parameter c0. The newly introduced parameters a0,1 and c1,2 are power suppressed in comparison to that of c0 in principle. However the charm quark is not heavy enough, or v is not small enough, these new parameters are not small in comparison with c0, or even they can be at the same size of c0. Numerically, it is found that these parameters can significantly reduce the polarization parameter α of J/ψ. More explicitly, we have calculated the direct J/ψ polarization, the prompt J/ψ polarization and the indirect J/ψ polarization from the b decays.

Without the spin-flip effect, we return to the same results with those in literature under the same parameters. While by taking the spin-flip interaction into consideration, the predicted α is more close to those measured at TEVATRON. At large pt, α for the prompt J/ψ is reduced by ~ 50% for f0 = v2 and by ~ 80% for f0 = 1. Then such spin-flip interaction as have been argued by several authors may provide a suitable way to solve the J/ψ po-
larization puzzle at TEVATRON. Since the NLO correction shall provide a large $K$ factor of the total cross section (ratio of NLO to LO), e.g. $K \sim 2$ for color-singlet $(c\bar{c})_1[3S_1]$ and $K \sim 1$ for color-octet $(c\bar{c})_8[3S_0]$ and $(c\bar{c})_8[3S_1]$ [4, 5], and because the NLO can increase the transverse distributions of produced $J/\psi$ more than the total distributions, then the value of $\alpha$ can be further lowered by including NLO results into our present calculation. Furthermore, it has been argued that the production of $J/\psi$ associated with a $c\bar{c}$ quark pair might also help to dilute the $J/\psi$ polarization [22]. Such analysis is out of the range of the present paper, which is much more involved since it involves a NLO calculation of these processes with spin-flip effects being under consideration and a newly systematical determination of the color-octet matrix elements. Moreover, we have found that the predicted indirect $J/\psi$ polarization parameter $\alpha_B$ depends weaker on the spin-flip effects than the case of direct and prompt $J/\psi$ production. And then a more precise measure of the indirect $J/\psi$ polarization from the $b$ decays can be adopted to predict more precise color-octet matrix elements, which can inversely improve our estimations on the direct and prompt $J/\psi$ production.

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V. EXPAND $\frac{d\hat{\sigma}^{(8)}[p, g]}{dt}$ IN THE LARGE $p_t$ LIMIT

A. Formulae for the production of $J/\psi$ through the channel of $(c\bar{c})_1[3P_2]$

For the case of $n = (c\bar{c})_1[3P_2]$, the differential cross-section takes the form:

$$d\sigma_{\lambda}(p\bar{p} \rightarrow J/\psi^{\lambda}(n)X) = \sum_{ab} \int dx_a dx_b f_{a/p}(x_a)f_{\bar{b}/\bar{p}}(x_b)d\hat{\sigma}_{\mu\nu\rho\sigma}[n, ab]c_{\lambda}^{\mu\nu\rho\sigma},$$  \hspace{1cm} (30)

where we have

$$\frac{d\hat{\sigma}_{\mu\nu\rho\sigma}[n, ab]}{dt} = A_{ab}[n]g_{\mu\nu}g_{\rho\sigma} + B_{ab}[n]g_{\mu\nu}k_{1\nu}k_{1\sigma} + C_{ab}[n]g_{\mu\nu}k_{2\nu}k_{2\sigma} + D_{ab}[n]g_{\mu\nu}k_{1\nu}k_{2\sigma}$$
\[ + E_{ab[n]} k_1 \rho k_1 \rho k_1 \rho k_1 \rho + F_{ab[n]} k_2 \rho k_2 \rho k_2 \rho + G_{ab[n]} k_1 \rho k_1 \rho k_1 \rho \\
+ H_{ab[n]} k_2 \rho k_2 \rho k_2 \rho + I_{ab[n]} k_1 \rho k_1 \rho k_1 \rho + J_{ab[n]} k_1 \rho k_1 \rho k_1 \rho, \tag{31} \]

where
\[
\rho_{\mu \nu \rho \sigma} = \sum_{\lambda=-2}^{2} \epsilon_{\mu \nu \lambda} \epsilon_{\rho \sigma} (\lambda) = \frac{1}{2} (\rho_{\mu \nu} \rho_{\rho \sigma} + \rho_{\mu \sigma} \rho_{\rho \nu}) - \frac{1}{3} \delta_{\mu \rho} \delta_{\nu \sigma} \tag{32} \]

and
\[
\rho_{0 \mu \nu \rho \sigma} = \epsilon_{\mu \nu \lambda}(0) \epsilon_{\rho \sigma}(0) = \frac{1}{6} (2 \rho_{0 \mu} - \rho_{1 \mu}) (2 \rho_{0 \sigma} - \rho_{1 \sigma}) \tag{33} \]
\[
\rho_{1 \mu \nu \rho \sigma} = \sum_{|\lambda|=1} \epsilon_{\mu \nu \lambda}(\lambda) \epsilon_{\rho \sigma}(\lambda) = \frac{1}{2} (\rho_{1 \mu} \rho_{1 \sigma} + \rho_{0 \mu} \rho_{1 \sigma} + \rho_{0 \mu} \rho_{1 \sigma} + \rho_{0 \mu} \rho_{1 \sigma}) \tag{34} \]
\[
\rho_{2 \mu \nu \rho \sigma} = \sum_{|\lambda|=2} \epsilon_{\mu \nu \lambda}(\lambda) \epsilon_{\rho \sigma}(\lambda) = \frac{1}{2} (\rho_{1 \mu} \rho_{1 \sigma} + \rho_{1 \mu} \rho_{1 \sigma} - \rho_{0 \mu} \rho_{1 \sigma}) \tag{35} \]

and
\[
\rho_{\mu \nu} = \sum_{\lambda=-1}^{1} \epsilon_{\mu \nu \lambda}(\lambda) \epsilon^{\nu}(\lambda) = -g_{\mu \nu} + \frac{P_{\mu} P_{\nu}}{M^2} \tag{36} \]
\[
\rho_{0 \mu \nu} = Z_{\mu} Z_{\nu}, \quad \epsilon_{\mu}(0) = Z_{\mu} = \frac{(P \cdot Q / M) P_{\mu} - M Q_{\mu}}{\sqrt{(P \cdot Q)^2 - M^2 Q^2}} \tag{37} \]
\[
\rho_{1 \mu \nu} = \sum_{|\lambda|} \epsilon_{\mu \nu \lambda}(\lambda) \epsilon^{\nu}(\lambda) = \rho_{\mu \nu} - \rho_{0 \mu \nu}. \tag{38} \]

\(k_1\) and \(k_2\) are the momenta of the initial state partons, \(ab = gg, gq, gq\) and \(qq\). \(P\) and \(Q\) are the momenta of the bound state and the total four-momentum of the colliding hadrons respectively. All coefficients, \(A_{ab[n]}, B_{ab[n]}, C_{ab[n]}\) and etc. can be read from Refs.\[9, 11, 21].

**B. Formulae for the \(p_t\)-expansion of the dominant gluon-gluon fusion subprocess**

As for the dominant subprocess: \(g(p_1) + g(p_2) \rightarrow J/\psi((\psi S)_{s})(p_3) + g(p_4)\), in the laboratory Frame, we have
\[
p_1 = \sqrt{S} \left( x_a, 0, 0, x_a \right), \quad p_2 = \sqrt{S} \left( x_b, 0, 0, -x_b \right), \quad p_3 = (M_T \cosh(y), p_3^x, p_3^y, M_T \sinh(y)), \tag{39} \]
where \(p_i = (p_i^x, p_i^y, p_i^z, p_i^\tau)\), \(y\) is the rapidity of \(J/\psi\), \(M_T^2 = M^2 + p_T^2\), and we have
\[
\dot{s} = (p_1 + p_2)^2 = x_a x_b S, \\
\dot{t} = (p_1 - p_3)^2 = M^2 - \sqrt{S} M T x_a [\cosh(y) - \sinh(y)], \\
\dot{u} = (p_1 - p_4)^2 = M^2 - \sqrt{S} M T x_b [\cosh(y) + \sinh(y)], \tag{40} \]
with $S$ the square of C.M. energy for the hadronic collider. Using the above formulae, we obtain the $p_t$-expansion for the dominant gluon-gluon fusion subprocess in the large $p_t$ limit:

$$\frac{d\sigma([3S_1]_8, gg)}{dt} = \frac{3e^{2y}f_1\pi^2\alpha_s^3}{2M^3S^2x_ax_b(x_a + e^{2y}x_b)^2} \left[ \frac{1}{p_t} \frac{3e^{2y}f_1\pi^2\alpha_s^3(M^2(x_a - e^{2y}x_b)^2 - Sx_ax_b(x_a + e^{2y}x_b)^2)}{2M^3S\pi x_ax^2(x_a + e^{2y}x_b)^3} \right] - \\
\frac{1}{p_t^2} \left[ \frac{36(1 + e^{2y})M^3S^3x^3x^3(x_a + e^{2y}x_b)^4}{27e^{12y}(f_1 - f_2)S^2x_a^2x_b^6 + 2e^{10y}x_b^3(f_1x_a(100M^4 - 81M^2Sx_ax_b + 27S^2x_a(2x_a - x_b)x_b^2) + 27f_2Sx_b^2(2x_a + x_b)) - 2e^{2y}x_a^3(f_1x_a(-100M^4 + 81M^2Sx_ax_b + 27S^2x_a^2(x_a - 2x_b)x_b) + 27f_2Sx_a^2(M^2x_a - Sx_b^2(x_a + 2x_b)) + e^{8y}x_a^8x_b^2(f_1(-8M^4(31x_a - 50x_b) - 324M^2Sx_ax_b(x_a + x_b) + 27S^2x_a^2x_b^2(10x_a^2 + 8x_a x_b - x_b^2)) + 27f_2Sx_b^2(-4M^2x_b + Sx_a(6x_a^2 + 8x_a x_b + x_b^2))) + e^{4y}x_a^8x_b.}

\quad (f_1(8M^4(50x_a - 31x_b) - 324M^2Sx_ax_b(x_a + x_b) - 27S^2x_a^2x_b(x_a^2 - 8x_a x_b - 10x_b^2)) + 27f_2Sx_b^2(Sx_b(x_a^2 + 8x_a x_b + 6x_b^2) - 4M^2x_a)) + 2e^{6y}x_a^6x_b(-27f_2Sx_ax_b(M^2(x_a^2 + x_b^2) - 2Sx_a x_b(x_a^2 + 3x_a x_b + x_b^2)) + f_1(-81M^2Sx_ax_b(x_a^2 + 4x_a x_b + x_b^2) + 54S^2x_a^2x_b^2(x_a^2 + 5x_a x_b + x_b^2) + M^4(25x_a^2 - 62x_a x_b + 25x_b^2))) \right] + O\left(\frac{1}{p_t^3}\right),

(41)

where $f_1 = 3a_0 + a_1 + 2c_0$ and $f_2 = 4c_1 + 3c_2$. It can be found that $f_2$ comes into contribution at least at $O(1/p_t^2)$ in comparison to $f_1$. While for longitudinal distribution of $J/\psi$, we only need to make the change: $f_i \rightarrow g_i$ ($i = 1, 2$) with $g_1 = a_0 + a_1$, $g_2 = 2c_0 + 4c_1 + c_2$, i.e.

$$\frac{d\sigma_L([3S_1]_8, gg)}{dt} = \frac{d\sigma([3S_1]_8, gg)}{dt} \cdot (f_1 \rightarrow g_1; f_2 \rightarrow g_2).

(42)

One may also find that for the longitudinal part, $c_0$ comes into contributions at $O(1/p_t^2)$ in comparison to the total summed results, this is the reason why by taking the spin-symmetry, i.e. $a_0 = a_1 = c_1 = c_2 = 0$, the longitudinal contributions should be neglected at large $p_t$.
regions, i.e. the $J/\psi$ is transverse polarized at large $p_t$ regions.

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