Fault Diagnosis of Rolling Bearing based on Permutation Entropy Optimized Maximum Correlation Kurtosis Deconvolution

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Abstract. An improved MCKD algorithm was proposed as an adaptive parameter selection approach. In this method, the optimal filter length of the MCKD algorithm was determined by the maximum permutation entropy value and the optimal fault period of the MCKD algorithm was determined by the maximum kurtosis value. The determined optimal filter length and optimal fault period were verified by simulation signals of the bearing as well as fault experimental signals of the inner and outer rings. Results demonstrate that in early fault diagnosis, the improved MCKD algorithm based on permutation entropy can extract faint characteristic information of early fault of rolling bearing effectively. It can highlight fault pulse signals which are covered by noises and get more ideal results than the minimum entropy deconvolution algorithm.

1. Introduction
Rolling bearing is one of parts which is the easiest to be failed in a set of mechanical equipment. The running state of rolling bearing can influence performance of the whole machine directly. Local damages or defects of the bearing might cause noises and abnormal vibrations of the equipment, and even may destroy the equipment. Therefore, it is of important significance to study state monitoring and fault diagnosis method of rolling bearing.

Vibration signals of rolling bearing will show nonlinear and non-stationary features upon the occurrence of anomaly or faults, which make linear analytic methods unable to diagnose early fault of the bearing effectively. Frequency information of signals also cannot be extracted effectively, because it is drown in abundant noises. Hence, it is necessary to apply the nonlinear method. In
recent years, researchers have introduced many nonlinear signal feature extraction methods into fault diagnosis fields, such as support vector machine [1], neural network [2, 3], multiscale entropy [4] and fractal theory [5]. These methods have achieved good effects, but they still have some shortages at independent use. For instance, fractal dimension calculation depends on length of data and it is a time-consuming task, which is inappropriate for online monitoring. The calculation of Lyapunov index is slow and it has poor accuracy as well as easy interferences to noises. As a time-series complex method, approximate entropy also relies on data length in calculation and has poor relative uniformity [6].

Permutation entropy is a method proposed by Bandt et al. to detect randomness and dynamics mutation behaviors of time series [7]. It is characteristics of easy and fast calculation, strong resistance to noises, and appropriate for online monitoring. Moreover, it can analyze correlation between nonlinear and non-stationary signals well [8]. Maximum correlated kurtosis deconvolution (MCKD) algorithm takes correlation kurtosis as an evaluation index. With full considerations to period characteristics of all impact components in the signal, the MCKD algorithm targets at maximization of correlation kurtosis. Besides, it eliminates irrelevant component, which is conducive to highlight continuous pulse sequences under the current deconvolution period effectively. The MCKD algorithm can highlight continuous pulse in signals which is covered by strong noises through iteration-based deconvolution operation [9]. Wang et al. [10] pointed out that vibration signals can be viewed as the consequence of resonance response convolution between periodic impact signals caused by fault and mechanical parts. Besides, they reported that deconvolution is an effective method to recover pulses. MCKD algorithm has been proved an effective deconvolution method. Nevertheless, deconvolution results of MCKD algorithm are highly sensitive to the filter length ($L$) and deconvolution period ($T$). In addition, input parameters like filter length and shifting number have to be chosen artificially. The superiority of MCKD algorithm can be highlighted only if accuracy and applicability of these parameters are promised.

Therefore, performances of MCKD algorithm in extracting transmission features and recognizing mechanical fault from noise signals are restricted due to its shortages. Superiority of MCKD algorithm can be highlighted as long as these parameters are accurate and used correctly. Kurtosis is a compromised characteristic variable between the low-order moment which is not sensitive enough and the high-order moment, and it can reflect pulse characteristics in vibration signals well. Permutation entropy reflects complexity of one-dimensional time series and it is highly sensitive to signal changes. Permutation entropy can amplify drift signals of the system well. In this study, the MCKD algorithm was improved by permutation entropy. In the improved MCKD algorithm, the fault period ($T$) and filter length ($L$) were optimized and compared with selected parameters based on artificial experiences to verify feasibility of the improved MCKD algorithm in fault recognition.

2. Improved MCKD algorithm

2.1. MCKD algorithm

For the convenience of analysis, influences of noises $e(n)$ were ignored for temporary. It assumed that the vibration signals which were collected by the sensor after occurrence of fault were
expressed $y(n)$. $h(n)$ refers to response to transmission of $y(n)$ by surrounding environment and paths. $x(n)$ refers to the pulse signal generated by fault. Signal expression in this process is:

$$y(n) = h(n) * x(n) + e(n)$$  \hspace{1cm} (1)

The fault pulse signals contained in signal $y(n)$ are covered, thus decreasing the correlation kurtosis. The nature of MCKD algorithm is to find a FIR filter $f(n)$ to maximize correlation kurtosis of the original pulse signal $x$. In this way, the output signal $y$ can recover fault pulse signal as much as possible:

$$x(n) = f(n) * y(n)$$  \hspace{1cm} (2)

To highlight continuous sharp pulses in deconvolution signals, the improved MCKD algorithm takes the maximization of correlation kurtosis $CK_x(T)$ as the objective function:

$$MCKD_x(T) = \max_f CK_x(T) = \max_f \sum_{n=1}^{N} \left( \prod_{m=0}^{M} y_{n-mT} \right)^2 \over \left( \sum_{n=1}^{N} y_n^2 \right)^{M+1}$$  \hspace{1cm} (3)

where $\vec{f} = (f_1, f_2, \cdots, f_L)^T$, $L$ is the filter length, $T$ is the period of impact signals, and $M$ is the drift number.

Correlation kurtosis is a concept proposed based on kurtosis. When $T$ is 0, the correlation kurtosis is degenerated to kurtosis, but kurtosis is too sensitive to few extremely highlighted pulses. Correlation kurtosis gives full considerations to continuity of pulse components of fault signals and it can measure key indexes of proportion of periodic pulse sequences in signals more accurately compared with kurtosis.

An optimal filter $\vec{f}$ to maximize $CK_x(T)$ was chosen. In other words, the abovementioned optimization problem is equal to solving the following equation[11]:

$$\frac{d}{dx} CK_x(T) = 0, \ k = 1, 2, \cdots, L$$  \hspace{1cm} (4)

The final results of filter are expressed by a matrix:

$$\vec{f} = \frac{[v]}{2[\beta]} (X_y X_y^T)^{-1} \sum_{m=0}^{M} X_{my} \alpha_m$$  \hspace{1cm} (5)

where

$$X_r = \begin{bmatrix} X_{1,r} & X_{2,r} & X_{3,r} & \cdots & X_{N,r} \\ 0 & X_{1,r} & X_{2,r} & \cdots & X_{N,r} \\ 0 & 0 & X_{1,r} & \cdots & X_{N-1,r} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & X_{N-L+r+1} \end{bmatrix}$$  \hspace{1cm} (6)

$$r = \begin{bmatrix} 0 & T & 2T & \cdots & mT \end{bmatrix}$$  \hspace{1cm} (7)
2.2. Permutation entropy algorithm

Permutation entropy (PE) is a measurement of complexity of time series and it is used to detect randomness of time series of signals. PE is highly sensitive to signal changes and it has characteristics of simple calculation, good noise resistance, high stability and high accuracy.

For a N long time series \( \{X(i), i=1, 2, ..., N\} \), a matrix could be gained by phase space reconstruction:

\[
Y = \begin{bmatrix}
    x(1) & x(1+\tau) & ... & x(1+(d-1)\tau) \\
    x(2) & x(2+\tau) & ... & x(2+(d-1)\tau) \\
    x(j) & x(j+\tau) & ... & x(j+(d-1)\tau) \\
    \vdots & \vdots & \ddots & \vdots \\
    x(K) & x(K+\tau) & ... & x(K+(d-1)\tau) 
\end{bmatrix}
\]

where \( d \) is number of embedded dimension, \( \tau \) is delay time, \( K \) is number of reconstructed components, and \( x(j) \) is the components at row \( j \) of the reconstructed matrix.

Each row in a matrix can be used as a reconstructed component and there are a total of \( K \) components. The \( j \)th reconstructed component \( \{x(j), x(j+\tau), ..., x(j+(d-1)\tau)\} \) in the reconstructed matrix can be rearranged in an ascending order:

\[
\{x(i+(j_1-1)\tau) \leq x(i+(j_2-1)\tau) \leq \cdots \leq x(i+(j_d-1)\tau)\}
\]

where \( j_1, j_2, ..., j_d \) is the index of columns where elements of reconstructed components lie in.

If there are equal values in reconstructed components:

\[
\{x(i+(j_p-1)\tau) = x(i+(j_q-1)\tau)\}
\]

Then, a group of symbol sequence can be gained from each row of the matrix \( Y \) which is reconstructed from any one time series:

\[
S(l) = \{j_1, j_2, ..., j_d\}
\]

where \( l = 1, 2, ..., k \) and \( k \leq d! \). There are a total of \( d! \) symbol sequences \( \{j_1, j_2, ..., j_d\} \) mapped in the \( d \)-dimensional space and the symbol sequence \( S(l) \) is one of the arrangements.
Probability of occurrence of each symbol sequence \((P_1, P_2, \ldots, P_k)\) is calculated. At this moment, PE \((H_p)\) of \(k\) symbol sequences of the time series \(X(i)\) can be defined according to the form of information entropy:

\[
H_p(d) = -\sum_{j=1}^{k} P_j \ln(P_j) \tag{14}
\]

When \(P_j = \frac{1}{d!}\), \(H_p(d)\) reaches the maximum \(\ln(d!)\). For the convenience, normalization of \(H_p(d)\) could be implemented by using \(\ln(d!)\):

\[
0 \leq H_p = \frac{H_p}{\ln(d!)} \leq 1 \tag{15}
\]

\(H_p\) value refers to the degree of randomness of the time series \(\{X(i), i=1, 2, \ldots, N\}\). The smaller \(H_p\) value indicates that the time series are more ordered. On the contrary, the higher \(H_p\) value means that the time series are closer to randomness. Changes of \(H_p\) reflect and amplify tiny detail changes of time series.

3. Parameter selection for the theoretical model

3.1 Selection of PE parameters

In the solving process of PE, there are three parameters to be considered and set, which are delay time \((\tau)\), sequence length \((N)\) and number of embedded dimensions \((d)\). If \(d\) is too high, it takes a long time to calculate \(H_p\) and the detailed changes cannot be reflected. If \(d\) is too small, there might be few reconstruction vectors which are inadequate to detect sudden changes of time series accurately.

To investigate influences of delay time \((\tau)\) on \(H_p\), a segment of 1024 long measured fault vibration signal data of inner ring of the bearing was chosen. Changes of the normalized \(H_p\) value with \(d\) are shown in figure 1.

It can be seen from Fig.1 that \(H_p\) value is negatively related with \(d\) under different delay time, but \(H_p\) value changes slightly with \(d\). When \(d=5\), \(d=6\) or \(d=7\), PE can reflect variation characteristics of time series well. In this study, \(d=6, \tau=1\).

It can be seen from figure 2 that \(H_p\) value decreases gradually when data length changes with \(d\). Moreover, \(H_p\) value is positively related with data length. When \(d\leq5\), \(H_p\) values with \(N>256\) changes slightly and are similar with the increase of \(d\). When \(d=6\), \(H_p\) values and their differences decrease gradually with the increase of data length. The \(H_p\) value when \(N=2048\) and the \(H_p\) value when \(N=4096\) present a small difference of less than 0.01. The \(H_p\) value tends to be stable when \(N\) is higher than 1024. Therefore, data length is appropriately to be chosen 2048.

Data length \((N)\) also can influence \(H_p\) value to some extent. \(H_p\) values with data length under different numbers of embedded dimensions are shown in figure 2. Data lengths of vibration signal were set 256, 512, 1024, 2048 and 4096, respectively.
3.2 Adaptive parameter selection of MCKD algorithm

MCKD algorithm has strict requirements on multiple input parameters. In addition to the original sampling rate and original signals, selection of deconvolution period ($T$) and filter length ($L$) can influence performances of MCKD algorithm significantly. As a result, it is necessary to assure accuracy and applicability of these parameters to highlight superiority of MCKD. Influenced by many factors in practical project (e.g. instability of number of revolutions), parameters which were selected according to artificial experiences are not accurate enough. Kurtosis is a dimensionless parameter. Since it is unrelated with rotating speed, size and loads of bearings, kurtosis is particularly sensitive to impact signals and it is applicable to diagnosis of surface damages, especially to early fault. To realize adaptive selection of the best influencing factor combination, this study suggested to screening the appropriate $L$ of the MCKD algorithm through PE and choosing appropriate $T$ based on kurtosis selection. The improved MCKD algorithm can overcome influences of noise components and eliminate interference components unrelated with pulse signals.

Specific steps of the diagnosis are:

1. Collect bearing fault signals under different working conditions.
2. Select the initial filter length $L$ according to artificial experiences.
(3) T is increased from 1 to 300 at a rate of 1 and the MCKD algorithm is operated circularly. Kurtosis value of each deconvolution is calculated to find the period T corresponding to the maximum kurtosis value. This is the best fault cycle.

(4) The best fault cycle T is input into the MCKD algorithm, and L increases from 2 to 300 at a rate of 1. Similarly, the MCKD algorithm is operated circularly to search the filter length L corresponding to the maximum $H_P$ value. This filter length is the best filter length and the updated filter length.

(5) The MCKD algorithm after parameter optimization was applied for preprocessing. The processed signals were further treated by Hilbert envelope demodulation to get the envelope spectrum.

(6) Fault mode of bearing is judged by comparing the theoretical fault characteristic frequency of bearings with the evidently highlighted frequency in the envelope spectrum.

4. Verification analysis

Damages of bearings were tested by a QPZZ-II rotating mechanical vibration analysis and fault diagnosis test table the 6205 bearing was chosen as the testing sample. In this experiment, the fault mode chose the level-1 mild cracks on the inner ring and level-1 mild pitting corrosion on the outer ring (figure 3).

Figure 3. Fault modes of inner and outer ring of bearings: (a) Mild cracks on inner ring; (b) Moderate cracks on inner ring; (c) Severe cracks on inner ring; (d) Mild pitting corrosion on outer ring; (e) Moderate pitting corrosion on outer ring; (f) Severe pitting corrosion on outer ring.

Waveform of fault vibration signal on outer ring of the bearing is shown in figure 4(a). The envelope spectrum of fault vibration signal on outer ring of the bearing was gained through further envelope spectral analysis (figure 4(b)).
Figure 4. Waveform and spectra of outer ring fault signals processed by the improved MCKD algorithm: (a) Waveform of fault on outer ring; (b) Envelope spectrum of fault on outer ring.

It can be seen from figure 4(a) that impact components exist in the original fault signals of the outer ring, but no regular laws are found. It is impossible to make effective fault diagnosis from waveform only. Many resonance frequency bands are observed from the frequency spectrum in figure 4(b), where the transform frequency $f_r=25\text{Hz}$ and the character frequency of outer ring fault $f_e=90\text{Hz}$ are obvious. However, there are still many noise interference frequencies, thus making the frequency multiplication components of character frequency of faults drown rather than displayed.

Now, the optimal fault period was calculated. The theoretical value of the optimal fault period is equal to the actual sampling frequency / theoretical character frequency of faults on outer ring. In this experiment, the actual sampling frequency is $f_s=1024\text{Hz}$ and the theoretical character frequency of faults on outer ring is $f_e=90\text{Hz}$. Therefore, the theoretical value of the optimal fault period is calculated $T=114$.

The optimal fault period of actual signal was chosen by the kurtosis index. The fault period corresponding to the maximum kurtosis is used as the optimal fault period. Results are shown in figure 5(a). The optimal filter length is chosen according to the maximum PE value, which is shown in figure 5(b).

It can be seen from figure 5(a) that the maximum kurtosis is 7.776 and the corresponding fault period is $T=117$, which is taken as the optimal fault period. According to calculations, the theoretical value of the optimal fault period is $T=114$, which is close to the practical value $T=117$ and slightly smaller. This is mainly because in practical sampling process, actual rotating speed of motor is slightly smaller than the rated value due to influences of external factors like voltage. Besides, the actual rotating speed of the motor fluctuates slightly rather than stable. Therefore, it can be known from comparative analysis with theoretical value that kurtosis can reflect the optimal fault period of actual sampling signals effectively and it can improve superiority of MCKD algorithm. The variation trend of PE value with filter length is shown in figure 5 (b). On this basis, the optimal filter length corresponding to the maximum PE value is calculated $L=153$.
Early fault signal of outer ring was analyzed by the improved MCKD algorithm. Results are shown in figure 6.

Waveform of fault signals on outer ring which are processed by the improved MCKD algorithm is shown in figure 6(a). There are evident pulse components which present extremely strong regularity. Further envelope spectral analyses are shown in figure 6(b). Main components of spectral line consist of the characteristic frequency of outer ring fault of bearing with deep groove and its multiplications. However, there are few interference components. These just conform to the theoretical value of outer ring fault of the bearing, thus proving validity and accuracy of the improved MCKD algorithm. Waveform and envelope spectrum of vibration signals of the inner ring fault are shown in figure 7(a) and figure 7(b). Experimental signal analysis results of inner ring fault based on the improved MCKD algorithm are shown in figure 8.
According to analysis of figure 7(a), there are impact components in the inner ring fault diagram of the original signal, but no evident laws have been observed. Besides, there are many interference components, such as noises. Therefore, it is impossible to make effective fault diagnosis from waveform only. It can be seen in figure 7(b) that there are many resonance frequency bands in the frequency spectrum, and the transformation frequency component is covered, without evident amplitude. It is found that the inner ring frequency \( f_e = 130 \text{Hz} \) is obvious in signals, and multiplications of transformation frequency are observed, but no transformation frequency is detected. There are still many noise interference frequencies to cover multiplication components of fault character frequency.

![Waveform and envelope spectrum of inner ring fault signals processed by the improved MCKD algorithm](image)

**Figure 7.** Waveform and envelope spectrum of inner ring fault signals processed by the improved MCKD algorithm: (a) Waveform of inner ring fault; (b) Envelope spectrum of inner ring fault.

![Optimal parameter values](image)

**Figure 8.** Optimal parameter values: (a) The optimal fault period; (b) The optimal filter length

According to analysis of figure 8(a) and figure 8(b), the maximum kurtosis is 181.18 and the maximum PE is 6.399. On this basis, the optimal deconvolution period and the optimal filter length
are $T=78$ and $L=69$, respectively. Later, signals of inner ring fault were processed after the optimal parameter combination is determined. Results are shown in figure 9.

![Figure 9](image_url)

**Figure 9.** Waveform and envelope spectrum of inner ring fault signals processed by the improved MCKD algorithm: (a) Signals processed by the improved MCKD algorithm; (b) Envelope spectrum of signal processed by the improved MCKD algorithm.

According to comparison between figure 9(a) and figure 9(a), impact components in 9(a) are very evident and the rest noise waves are eliminated, which contributes a pure waveform. To further determine fault position, envelope spectral analysis of processed signal was carried out (figure 9(b)). The character frequency of fault $f_e = 130\,\text{Hz}$ and its multiplications are very obvious, which show some error with the theoretical value $f_e = 135\,\text{Hz}$. The character frequency of actual signal is slightly smaller than the theoretical value, which is caused the smaller actual rotating speed of the motor than the rated value as a response to external factors (e.g. voltage). On this basis, it can be determined that there are local damages on inner ring of the bearing. The analysis results conform to actual situations.

To further interpret advantages of the improved MCKD algorithm, inner ring fault signals were analyzed by combining the minimum entropy deconvolution (MED) algorithm and envelope spectrum. Signal processing based on MED algorithm is mainly influenced by filter length. To highlight superiority of the improved MCKD algorithm and strengthen the comparison effect, filter length in the MED algorithm was also set 69 to be consistent with that in MCKD algorithm in figure 16(a). The final analysis results of MED algorithm are shown in figure 10(a).

![Figure 10](image_url)

It can be seen from the comparison between figure 10(a) and figure 16(a), the improved MCKD algorithm has evident and high-amplitude impact components and the spectral line shows extremely strong laws. The filtering effect is quite ideal. Envelope spectral analysis of fault signals processed by MED algorithm was carried out. Analysis results are shown in figure 10 (b). Although character frequency of inner ring fault is observed in figure 10(b), there are high-amplitude frequencies of noise signals in the spectrum compared to that in figure 9(b). Moreover, the number of multiplications is smaller than that in the improved MCKD algorithm. The general effect of MED algorithm is not very ideal. Based on above comparative analysis, the improved MCKD algorithm can highlight fault pulse signals covered by noises and extract character frequency of faint fault successfully. Therefore, it can finish accurate diagnosis on faint fault of bearings.
Figure 10. Inner ring fault signal analysis results based on the combination of MED algorithm and envelope spectrum: (a) Inner ring fault signals processed by MED algorithm; (b) Envelope spectrum of inner fault signals processed by MED algorithm.

5. Conclusions

(1) Faint fault characteristic signals of rolling bearing are easy to be covered by noises produced by working environment and the transmission pat, which makes pulse components difficult to be extracted. It is impossible to diagnose early fault of bearings by using envelope spectrum directly. The improved MCKD algorithm can highlight fault pulse signals which are drown in noises and extract character frequency of faint fault successfully.

(2) In MCKD algorithm, the deconvolution outcome is determined by deconvolution period and filter length. In this algorithm, the deconvolution period and filter length are chosen adaptively according to kurtosis index and PE. These can avoid artificial setting effectively and improve the analysis accuracy. According to the simulation comparison, the improved MCKD algorithm achieves quite ideal optimization effect.

(3) According to fault simulation of bearings as well as analysis on experimental signals of faults on inner and outer rings, the MCKD algorithm based on PE optimization is more ideal to extract early fault than the MED algorithm. This also proves superiority and reliability of the improved MCKD algorithm.

Acknowledgments

This work was supported by National Natural Science Foundation of China (Grant no. 61463021 and 61963018), Natural Science Foundation of Jiangxi Province in China (Grant no. 20181BAB202020), Science and Technology Project in Education Department of Jiangxi Province in China (Grant no GIJ180494)
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