Quasi-Andreev reflection in inhomogeneous Luttinger liquids

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Reflection of charge excitations at the step in the interaction strength in a Luttinger liquid can be of the Andreev type, even the interactions are purely repulsive. The region with stronger repulsion plays the role of a normal metal in a normal-metal/superconductor junction, whereas the region with weaker repulsion plays the role of a superconductor. It is shown that this quasi-Andreev reflection leads to a number of proximity-like effects, including the local enhancement (suppression) of superconducting fluctuations on the quasi-normal (quasi-superconducting) side of the step, significant modification of the local density of states, as well as others. The observable consequences of these proximity effects are analyzed for the case of single- and two-particle tunneling from a normal-metal or superconducting tip into an inhomogeneous Luttinger-liquid wire.

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Recent developments in microfabrication technologies have led to a renaissance in the physics of (quasi) one-dimensional (1D) strongly correlated electronic systems. Much of theoretical work has been devoted to various scattering processes in these systems, such as single- [1] and multiple- [2] impurity scattering, and Umklapp scattering [3]. These processes result in some unique features in observable quantities, e.g., in specific temperature and/or voltage dependences of the conductance, which have been instrumental in the experimental search for strongly correlated effects in 1D systems [4].

The focus of this paper is on another scattering process, which has only recently attracted attention [5, 6], namely, scattering caused by inhomogeneities in the electron-electron interaction strength. Such inhomogeneities should be readily realizable, and sometimes even unavoidable, in 1D systems. For example, one can change the electron density, and therefore the effective interaction strength, by applying a potential to a top gate (shaded); (b) Squeezing the wire inhomogeneously by using additional side gates. As soon as \( |K F w(x)/\pi| = 1 \) for any \( x \), there is only one propagating mode in the wire.

Another advantage of the adiabatic approximation is that a 1D system with variable interaction strength can be described in terms of the inhomogeneous Luttinger-liquid model \([6, 7, 8]\), in which the velocity of collective charge excitations \( v_p \) and the dimensionless parameter \( K_\rho \) vary in space. \( K_\rho \) characterizes the strength and sign of interactions: \( K_\rho < 1 \) for repulsion; \( K_\rho > 1 \) for attraction; \( K_\rho = 1 \) in the absence of the interaction.)

An interface between two regions having different filling factors in a quantum Hall system \([9, 10]\). In what follows, we shall concentrate on the adiabatic case, in which the scale of the inhomogeneity \( W \) is much larger than the Fermi wavelength \( 2\pi/k_F \). In this case, single-particle backscattering from the inhomogeneity may be neglected 11.

FIG. 1. Some ways of producing inhomogeneities in the interactions strength: (a) Creating a depleted (enriched) region by applying a voltage to a top gate (shaded); (b) Squeezing the wire by additional side gates. As soon as \( |K F w(x)/\pi| = 1 \) for any \( x \), there is only one propagating mode in the wire.
be modified in a similar way. We shall then study how both of these effects could manifest themselves in single- and two-particle tunneling into a Luttinger liquid. For the sake of simplicity we shall concentrate on the case of a single kink of zero width (a step). Although kinks usually come in pairs, our calculations are applicable if the separation of the kinks is larger than $L_{c}$. The overall structure of the DOS in the case of a well in $K_{\rho}$ was studied in Ref. [14].

An inhomogeneous Luttinger liquid is described by the Hamiltonian

$$H = \sum_{\mu=\rho,\sigma} \int dx \frac{v_{\mu}(x)}{2} \left[ \frac{\partial_{x} \phi_{\mu}}{K_{\mu}(x)} + K_{\mu}(x) (\partial_{x} \theta_{\mu})^{2} \right],$$

where $\phi_{\mu}$ and $\theta_{\mu}$ are canonically conjugated boson fields, i.e., $[\phi_{\mu}(x), \phi_{\nu}(x')] = i \delta_{\mu \nu} \delta(x - x')$. Unless mentioned specifically, we shall be considering the case of repulsive interactions, when there is no gap in the spin sector.

For a conventional SN interface, Andreev reflection provides a microscopic mechanism for the proximity effect, i.e., for the formation of a superconducting condensate in a region of the N-side adjacent to the interface, and the suppression of the condensate on the S-side. The proximity effect is usually described by the profile of the (singlet) condensate amplitude $\langle \psi_{\uparrow}(x) \psi_{\downarrow}(x) \rangle$, which varies from some non-zero value in the bulk of S to zero in the bulk of N. In our case, both the singlet and triplet condensate amplitudes are equal to zero. However, one can expect quasi-Andreev reflection to modify the correlation functions of singlet and triplet superconducting fluctuations, defined as $F_{s}(x,x') \equiv \langle S(x) S^{\dagger}(x') \rangle$ and $F_{t}(x,x') \equiv \langle T(x) T^{\dagger}(x') \rangle$, where $S(x) = \sum_{\sigma=\uparrow,\downarrow} R_{\sigma} L_{-\sigma}$ and $T(x) = \sum_{\sigma=\uparrow,\downarrow} R_{\sigma} L_{\sigma}$, and where $R_{\sigma}(L_{\sigma})$ are the right- (left-) moving components of fermion fields. In a bosonized form, the equal-time correlators are given by

$$F_{s}(x,x') = \frac{1}{(2\pi\alpha)^{2}} e^{2\pi \left( \Delta \Phi_{s}(x,x',0) + \Delta \Theta_{s}(x,x',0) \right)},$$

$$F_{t}(x,x') = \frac{1}{(2\pi\alpha)^{2}} e^{2\pi \left( \Delta \Phi_{t}(x,x',0) + \Delta \Theta_{t}(x,x',0) \right)},$$

where $\Delta \Phi_{s}(x,x',\tau) \equiv \Phi_{\mu}(x,x',\tau) - \Phi_{\mu}(x,x,0) + \Phi_{\mu}(x',x',0)/2$, with the Matsubara propagator defined as $\Phi_{\mu}(x,x',\tau) \equiv \langle T_{\tau} \phi_{\mu}(x,\tau) \phi_{\mu}(x',\tau) \rangle$ (and similarly for the relation between $\Delta \Theta_{\mu}$, $\Theta_{\mu}$, and $\theta_{\mu}$), and $\alpha$ is a short-distance cut-off. In the presence of an inhomogeneity, the temporal Fourier transform of $\Phi_{\mu}$ satisfies the equation of motion

$$\left\{ \frac{\omega_{n}}{v_{\mu}(x) K_{\mu}(x)} - \partial_{x} \left( \frac{v_{\mu}(x)}{K_{\mu}(x)} \partial_{x} \right) \right\} \Phi_{\mu}(x) = \delta(x - x'), \quad \text{(3)}$$

which, in the case of a step in $v_{\rho}$ and $K_{\rho}$, is supplemented by the condition that $\Phi_{\mu}$ and $(v_{\mu}/K_{\mu}) \partial_{x} \Phi_{\mu}$
are continuous at \( x = 0 \). The equation of motion and the replacement \( K_\mu \rightarrow 1/K_\mu \) in the presence of the \( SU(2) \) symmetry, the spin propagators are given by \( \Phi_\sigma = \Theta_\sigma = \exp(-|\omega_n(x-x')|/v_F)/2|\omega_n| \), whereas \( \Theta_\rho \) for \( x, x' < 0 \) can be written as

\[
\Theta_\rho(x, x', \omega_n) = \frac{e^{-|\omega_n(x-x')|/v_{\rho 1}} + Re^{-|\omega_n(x+x')|/v_{\rho 1}}}{2|\omega_n|K_{\rho 1}} \quad (4)
\]

At zero temperature, the superconducting correlation functions take the following form

\[
F_s(x, x') = F_t(x, x') = f^{(0)}(x - x') \left[ \frac{4xx'}{(x + x')^2} \right]^{2K_{\rho 1}} \quad (5)
\]

where \( f^{(0)}(x - x') \propto |x - x'|^{-(K_{\rho 1} + 1)} \) describes the decay of superconducting fluctuations in a homogeneous Luttinger liquid with parameter \( K_{\rho 1} \). The presence of the inhomogeneity is manifested through the factor in the square brackets in Eq. (3). For the case shown in Fig. 2a, \( R < 0 \) and \( F_{s/f} \) diverges as \( |x|^{-|R|/2K_{\rho 1}} \) for fixed \( x' \) and \( x \rightarrow 0^- \). For \( x, x' > 0 \), the propagator \( \Theta_\rho \) is obtained from Eq. (4) by replacing \( K_{\rho 1} \rightarrow K_{\rho 2}, \quad v_{\rho 1} \rightarrow v_{\rho 2} \). Consequently, \( F_{s/f} \) vanishes as \( |x|^{-|R|/2K_{\rho 2}} \) for fixed \( x' \) and \( x \rightarrow 0^+ \). We thus see that the superconducting fluctuations are enhanced (suppressed) on the QN (QS) side. This is not the only consequence of the inhomogeneity however. As one can show, the charge- and spin-density-wave (CDW and SDW) fluctuations also get modified in a manner opposite to that of the superconducting fluctuations, i.e., the \( 2K_F \) components of CDW and SDW correlation functions vanish (diverge) as \( |x|^{-|R|/2K_{\rho 1}} \) for \( x \rightarrow 0^- \) approaching the step from the QN (QS) side. Last, but not least, the local single-particle DOS at fixed energy \( \epsilon \) diverges as \( |x|^{-\kappa_1} \) for \( x \rightarrow 0^- \), where \( \kappa_1 = \frac{1}{4}|R|(K_{\rho 1} - K_{\rho 1}) \) and vanishes as \( x^{\kappa_2} \) for \( x \rightarrow 0^+ \), where \( \kappa_2 = \frac{1}{4}|R|(K_{\rho 2} - K_{\rho 2}) \); the bulk behavior is restored at distances \( |x| \gg L_\omega \).

The modification of the DOS and various two-particle correlation functions should lead to some observable consequences. In what follows, we consider a particular example, namely, we study how tunneling between a Luttinger liquid and a normal-metal or superconducting tip is modified in the presence of an inhomogeneity. Tunneling from a normal metal tip measures the single-particle DOS of a Luttinger liquid. Tunneling between a normal-metal (including Luttinger-liquid) conductor and a superconductor at biases smaller than the superconducting energy gap measures the pair susceptibility of a normal metal \( \Theta_\rho \), which is related to the correlation function of superconducting fluctuations. The CDW- and SDW-susceptibilities do not enter the tunneling current, as they correspond to electrically neutral excitations.

Suppose that a narrow tunneling tip is scanned along an inhomogeneous Luttinger liquid wire (see Fig.2). We model the tip by a 1D Luttinger liquid, whose parameters \( K_{\rho 0} \) and \( K_{\sigma 0} \) are chosen to be either \( K_{\rho 0} = K_{\sigma 0} = 1 \) (when the tip is in the normal, Fermi-liquid state) or \( K_{\rho 0} = \infty, \quad K_{\sigma 0} = 0 \) (when the tip is in the superconducting state). The dynamics of the fields in the wire at the position of the tip \( x \) is described by a local action obtained from the full action by integrating out the bulk degrees of freedom

\[
S = \frac{1}{2} \sum_{\omega_n} \sum_{\mu = \rho, \sigma} \frac{1}{\mu_\mu(x, x', \omega_n)} |\phi_\mu(x, \omega_n)|^2. \quad (6)
\]

Equivalently, \( S \) can be written in a dual form by making the replacement \( \phi_\mu \rightarrow \theta_\mu \) and \( \Phi_\mu \rightarrow \Theta_\mu \). The tip is described by similar equations involving corresponding (homogeneous) propagators of boson fields. The single-particle tunneling action is given by

\[
S_1 = \frac{\gamma v_F}{\alpha} \int d\tau \cos(\sqrt{\pi} \theta_\rho + A(\tau)) \cos(\sqrt{\pi} \varphi_\sigma) \times \cos(\sqrt{\pi} \varphi_\rho), \quad (7)
\]

where \( \gamma \) is a dimensionless tunneling amplitude, \( \vartheta_\rho \equiv (\theta_\rho - \theta_\rho)/\sqrt{2}, \varphi_\mu \equiv (\phi_\mu - \phi_\rho)/\sqrt{2} \), subindex “0” denotes the boson fields in the tip, and where \( A(\tau) \equiv \int d\tau' V(\tau') \), with \( V(\tau) \) being the bias between the tip and the wire. As is well known, single-particle tunneling action \( S_1 \) generates two-particle tunneling terms under a renormalization group (RG) procedure. For electrons with spin, the two-particle tunneling action was written down by Khveshenko and Rice.

\[
S_2 = \frac{v_F}{\alpha} \int d\tau \left[ \Gamma_s \cos(2\sqrt{\pi} \vartheta_\rho + 2A) \cos(2\sqrt{\pi} \varphi_\sigma) \right. \left. + \Gamma_t \cos(2\sqrt{\pi} \vartheta_\rho + 2A) \cos(2\sqrt{\pi} \varphi_\rho) \right. \left. + \Gamma_{sdw} \cos(2\sqrt{\pi} \varphi_\rho) \cos(2\sqrt{\pi} \varphi_\sigma) \right. \left. + \Gamma_{sdw} \cos(2\sqrt{\pi} \varphi_\rho) \cos(2\sqrt{\pi} \varphi_\sigma) \right]. \quad (8)
\]

The first (last) two terms on the right hand side of Eq. (8) correspond to particle-particle (particle-hole) tunneling. The first term describes the tunneling of a (virtual) singlet Cooper pair from the tip to the wire (and vice versa). Similarly, the second term describes the tunneling of a (virtual) triplet Cooper pair. Finally, the third and fourth term correspond to tunneling processes of electron-hole pairs of CDW- and SDW-type, respectively.

The RG equations for the single- and two-particle tunneling amplitudes can be derived following the conventional procedure. When the tip probes, e.g., the QN part of the wire, the flow equation for \( \gamma \) takes the form

\[
\frac{d \ln \gamma}{d \ln \Lambda} = \frac{1}{2} \left( \kappa + \kappa_1 e^{-2\Lambda|x|} \right), \quad (9)
\]

where \( \Lambda \equiv 1/\alpha \) and
\[ \kappa = \frac{K_{\rho 1} + K_{\sigma 1}^{-1} + K_{\rho 0} + K_{\sigma 0}^{-1} + K_{\sigma 0} + K_{\sigma 0}^{-1}}{4} - \frac{3}{2}. \]  

The solution of Eq. (3) is

\[ \gamma(\Lambda) = \gamma_0 (\Lambda/\Lambda_0)^{\kappa/2} e^{2\kappa_1 (E_1(2\Lambda_0|x|) - E_1(2\Lambda|x|))}, \]  

where \( \Lambda_0 \sim k_F \) and \( E_1(y) = \int_1^\infty dt e^{-yt}/t. \) If the tip is in the normal state (\( K_{\rho 0} = K_{\sigma 0} = 1 \)) then at distances \( x \) from the step satisfying \( \Lambda_0^{-1} \ll |x| \ll 1/\Lambda \) Eq. (11) reduces to

\[ \gamma(\Lambda) \propto \Lambda^{(K_{\rho 1}^{-1} - 1)^{2}/8K_{\rho 1}} (\Lambda|x|)^{-|\sigma_1|/2}. \]  

The (dimensionless) tunneling conductance \( G_1 \sim \gamma^2(\Lambda = eV/\nu_F) \) is thus enhanced in the vicinity of the step and reverts to its bulk value for \( |x| \gg \nu_F/eV. \) Accordingly, \( G_1 \) is suppressed if the tip probes the QS region. The voltage dependence of the conductance can be read off from the energy dependence of the DOS: the first factor in Eq. (11) corresponds to the DOS of a homogeneous Luttinger liquid, while the second one arises due to the step. If the tip is in the superconducting state (\( K_{\rho 0} = \infty, K_{\sigma 0} = 0 \)), \( \gamma \) vanishes for \( \Lambda \to 0 \), reflecting the absence of a single-particle tunneling current between a normal metal and a superconductor at voltages below the superconducting gap.

Next we consider the renormalization of the two-particle tunneling amplitudes. The charge transfer between the tip and the wire is determined only by the particle-particle tunneling processes, and we thus need to consider only the flow equations for \( \Gamma_s \) and \( \Gamma_{\sigma} \), which take the form

\[ \frac{d\Gamma_i}{d\ln \Lambda} = \frac{1}{2} \left( \kappa_i + \frac{R}{K_{\rho 1}} e^{-2\Lambda|x|} \right) \Gamma_i + \lambda_i \gamma^2(\Lambda) \]  

where \( i = s, t \}

\[ \kappa_s = K_{\rho 1}^{-1} + K_{\sigma 1}^{-1} + K_{\sigma 0} - 1; \]

\[ \lambda_s = K_{\rho 1} - K_{\sigma 1}^{-1} + K_{\sigma 0} - K_{\rho 0}^{-1} + K_{\sigma 0}^{-1} - K_{\sigma 0} - c \left( K_{\rho 1}^{-1} + K_{\rho 1} \right); \quad c \sim 1. \]  

The parameters \( \kappa_i \) and \( \lambda_i \) are obtained from \( \kappa_s \) and \( \lambda_s \) by replacing \( K_{\sigma 0} \to K_{\sigma 0}^{-1} \). The first term in Eq. (13) describes the self-generation of two-particle tunneling, the second one represents the generation of both-particle tunneling by single-particle processes. Accordingly, the initial condition for \( \Gamma_t \) is \( \Gamma_t(\Lambda_0) = 0 \) (if the tip is normal) and \( \Gamma(\Lambda_0) = t_0 \neq 0 \) (if the tip is superconducting). Analysis of the former case shows that two-particle tunneling does not significantly modify the tunneling conductance for repulsive interactions: two-particle tunneling is irrelevant and subdominant to single-particle tunneling.

The situation changes dramatically when the tip is superconducting. In this case, single-particle tunneling is forbidden at voltages smaller than the superconducting gap, and the only current flowing from the tip to the wire is the two-particle one. Solving Eq. (13) with \( \gamma(\Lambda) = 0, K_{\rho 0} = \infty, K_{\sigma 0} = 0 \), we get for \( \Gamma_s \)

\[ \Gamma_s(\Lambda) = \Gamma_0 (\Lambda/\Lambda_0)^{q_s/2} e^{2\kappa_1 (E_1(2\Lambda_0|x|) - E_1(2\Lambda|x|))}, \]  

where \( q_s \equiv K_{\rho 1}^{-1} - 1 \), whereas \( \Gamma_t = 0 \) (there is no triplet tunneling into a singlet superconductor). Close to the step, the (dimensionless) tunneling conductance \( G_2 \sim \Gamma_s^2(\Lambda = eV/\nu_F) \) is

\[ G_2 \propto \nu_F^2 V^{q_s} (V|x|)^{-1|R|/K_{\rho 1}}. \]  

In a homogeneous wire (\( R = 0 \)), \( G \propto V^{q_s}. \) Two-particle tunneling is irrelevant for the case of repulsion (\( K_{\rho 1} < 1 \)) and relevant for attraction (\( K_{\rho 1} > 1 \)), the non-interacting case (\( K_{\rho 1} = 1 \)) being marginal. The step enhances (suppresses) the two-particle tunneling for a tip to the left (right) from the step. The enhancement can be so strong that two-particle tunneling becomes relevant, even for repulsion: the criterion of relevancy changes to \( K_{\rho 1} > 1 - |R| \), which can be satisfied for \( K_{\rho 1} < 1 \). However, if the reflection coefficient is given by Eq. (10), this criterion can be satisfied only if the interaction in the second part of the wire is attractive (i.e., \( K_{\rho 2} > 2 - K_{\rho 1} \)).

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The interaction-induced variation of the chemical potential can be added to the single-particle potential, and thus neglected altogether.

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The two-particle tunneling action is not affected by inhomogeneity in the wire. Indeed, the two-particle tunneling terms occur as a result of fusions of fermion fields, which initially were evaluated at different space-time points. Fusion reduces the separation between these points down to the scale of a microscopic cutoff. At such distances, the presence of a smooth (on a microscopic scale) inhomogeneity is irrelevant.