Physical and mathematical modeling and formation of the optical signal transducer on the basis of gradient lithium niobate

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Abstract. The present research focuses on modeling and analyzing a PPLN-transducer as a logical element, taking into account the heterogeneity in dispersion of group velocities along its length, and aims at the creation of a gradient PPLN. Within the framework of the research, a model has been built and logical elements AND, OR, XOR have been simulated on the example of periodically poled waveguide structure of lithium niobate with gradient composition.

1. Introduction

Modern trends in communication system development stem from the principle of building all-optical networks which include optically controlled switches, remote power optical amplifiers, multiplexers, and demultiplexers. According to theoretical and experimental research on commutation and control of optical signals, nonlinear photonic devices (such as PPLN) demonstrate high jitter tolerance and, consequently, lower bit-error ratio and greater transmission capacity [1].

Present-day model studies of optical PPLN-based switches feature information velocities up to 200 Gbit/s while devices based on silicon matrices demonstrate error ratio lower than $10^{-12}$ only with velocities no higher than 12.5 Gbit/s with frequency-division multiplexing of 80 GHz [2].

A further advance in the efficiency of PPLN may be achieved through gradient doping of the main components, which allows for greater temperature stability with higher pump signals [3].

The current study aims to build and analyze a model of PPLN-transducer as a logical element with regard to the heterogeneity in dispersion of group velocities along its length, and creating a gradient PPLN.

Currently, the search for new methods of building systems for information processing and conversion is conducted in several directions, connected with the construction of optical logic elements and optoelectronic processors of active and passive action used for multiscale information conversion in laser devices. The so-called PPLN-structures with four-wave mixing are among the devices which can perform the functions of several types of optical logic gates.

Implementation of optical logic gates using PPLN-transducer with waveguide channel is carried out according to the following principle: two independent flows of data in the form of amplitude-modulated light intensity at the wavelengths of $\lambda_{SA}$ and $\lambda_{SB}$ enter corresponding input ports A and B. Then, along with continuous optical pumping at the wavelength of $\lambda_p$, they enter PPLN-waveguide
where sum and difference frequency generation occurs in the conditions of quasi-phase-matching. The output signal of PPLN-transducer comes from A and B output ports at the wavelengths of $\lambda_{SA}$ and $\lambda_{SB}$ correspondingly. In the processes of sum and difference frequency generation (SFG-DFG), one photon of the input channel A at the wavelength of $\lambda_{SA}$ and another photon from the input channel B at the wavelength of $\lambda_{SB}$ are converted by SFG-interaction into a sum frequency photon at the wavelength of $\lambda_{SF}$ (SF output port). At the same time the sum frequency photon is converted to an idle-wave photon at the wavelength of $\lambda_i$ through difference frequency generation (DFG) and by interacting with the pump photon at the wavelength of $\lambda_p$, and goes to the output port I.

However, in SFG-DFG processes the mismatch parameter is constant only when the composition of the crystal does not vary along the direction of the interaction. The suggested method of growing gradient crystals [4] assumes that the concentration of main components (such as Li) and impurities in the composition of the crystal can be changed along the boule length by using the critical values of mass crystallization rate found on the basis of the method for studying the concentration relaxation of the melt. Such change can provide the necessary length of the transducer to ensure vacancy compensation in the main components (Li) during the growth of the monocrystal and necessary concentration gradient of Li (Figure 1, delta $C_{Li}$). The concentration gradient of the main components along the length of the transducer will affect the dependence of the group velocities of the signals and their dispersions on the length of the transducer linearly or parabolically. Assuming that we have the dependency of the refraction ratio on the composition of LiNbO$_3$ crystal defined by Sellmeier equation [5], we can now theoretically evaluate to what extent the gradient composition of PPLN-transducer may influence the efficiency of the logical elements.

2.  Research method

The present paper deals with several types of concentration gradients of the main components (constant concentration of Li along the PPLN-device, and increasing/decreasing concentration of Li in the form of a semi-parabola (Figure 1) (featured by concentration profiles 1 (black line), 2 (red line) and 3 (green line) correspondingly)). The vertical axis in Figure 1 shows the deviation of Li concentration from the optimal one for the given period of domains succession without any gradient along the PPLN. To simulate signal transmission through the PPLN structure an equation system deriving from Maxwell equations was used:

$$\frac{\partial A_{SA}}{\partial z} + \beta_{1SA} \frac{\partial A_{SA}}{\partial t} + \frac{i}{2} \beta_{2SA} \frac{\partial^2 A_{SA}}{\partial t^2} = i\omega_{SA} k_{SFG} A_{SB} A_{SF} \exp(i\Delta k_{SFG} z),$$

(1)

$$\frac{\partial A_{SB}}{\partial z} + \beta_{1SB} \frac{\partial A_{SB}}{\partial t} + \frac{i}{2} \beta_{2SB} \frac{\partial^2 A_{SB}}{\partial t^2} = i\omega_{SB} k_{SFG} A_{SA} A_{SF} \exp(i\Delta k_{SFG} z),$$

(2)

$$\frac{\partial A_{SF}}{\partial z} + \beta_{1SF} \frac{\partial A_{SA}}{\partial t} + \frac{i}{2} \beta_{2SF} \frac{\partial^2 A_{SF}}{\partial t^2} = i\omega_{SF} k_{SFG} A_{SA} A_{SB} \exp(-i\Delta k_{SFG} z) + i\omega_{k} k_{\Delta k_{DFG}} A_{p} A_{SF} \exp(-i\Delta k_{DFG} z),$$

(3)

$$\frac{\partial A_{li}}{\partial z} + \beta_{1li} \frac{\partial A_{li}}{\partial t} + \frac{i}{2} \beta_{2li} \frac{\partial^2 A_{li}}{\partial t^2} = i\omega_{li} k_{k} A_{p} A_{SF} \exp(i\Delta k_{DFG} z),$$

(4)

$$k_{SFG} = d_{eff} \left( \frac{2\mu_0}{c n_{SA} n_{SB} n_{SF} A_{eff}} \right)^{1/2},$$

(6)

$$k_{DFG} = d_{eff} \left( \frac{2\mu_0}{c n_{p} n_{SF} A_{eff}} \right)^{1/2},$$

(7)

$$\Delta k_{SFG} = k_{SF} - k_{SA} - k_{SB} - \frac{2\pi}{\lambda},$$

(8)

$$\Delta k_{DFG} = k_{SF} - k_{p} - k_{i} - \frac{2\pi}{\lambda},$$

(9)
where \( A_{SA}, A_{SB}, A_{P}, A_{SF} \) and \( A_i \) are the complex amplitudes of the intensity of the light-waves of the signals at the wavelengths of \( \lambda_{SA}, \lambda_{SB}, \lambda_{P}, \lambda_{SF}, \lambda_i \) going through the input ports A and B, optical pumping port PPLN, and output ports A, B, SF, I; \( \beta_{ij} \) are the first- and second-order derivatives of the propagation constant of light \( k_j \) with respect to angular frequency \( \omega_j \); \( k_{SFG} \) and \( k_{DFG} \) are the coupling coefficients for SFG and DFG processes; \( d_{eff} \) is effective nonlinear coefficient; \( A_{eff} \) is effective interaction aperture; \( n_j \) is refraction ratio for the frequency of \( \omega_j \); \( \varepsilon_0 \) is permittivity; \( c \) is the speed of light in vacuum; \( \Delta k_{SFG} \) and \( \Delta k_{DFG} \) are phase mismatch parameters in SFG/DFG processes; \( \Lambda \) - the period of domain alternation in PPLN.

For the calculation, two independent pseudo-random bit frequencies entering input ports A and B were used. In these sequences, the presence of an impulse corresponded to unit bit, and its absence – to zero bit. The impulses had the shape of a hyperbolic secant and the width of 5 ps. The length of PPLN varied up to 50 mm, effective aperture of the waveguide was 50 µm\(^2\), the period of domains succession was 18.8 µm, so that to create quasi-phase-matching for the SFG wavelength of 772 µm. Nonlinear coefficient \( d_{eff} \) is 17.2 pm/V. The central wavelengths were 1550 µm for channel A and 1538 µm for channel B. Pump wavelength was 1555 µm (third transmission window), the wavelength of the idle wave generated during DFG process was 1533.2 µm. Peak output signal strengths for signals A and B and the pump signal are 100*\( \lambda_{SA}/\lambda_{SB} \) mW and 1000 mW correspondingly.

![Figure 1. Structural scheme of Li concentration gradient in PPLN-based logical elements: 1 (black line) – constant concentration profile of Li; 2 (red line) – concentration profile of Li in the form of an upward semi-parabola; 3 (green line) – concentration profile of Li in the form of a downward semi-parabola.](image)

3. Results and discussion
The result of the simulation of optical impulses passing through gradient PPLN-structures is featured in Figure 2. It can be seen that the decrease in amplitude of A and B channels at the output with matching impulses at the input is a significant value and allows to determine logic levels.

Figure 2 shows the results of nonlinear interaction of A and B signals, the pump signal, the signal at the wavelength corresponding to SFG and the signal at the idle wavelength. It can be seen that Li concentration gradient along the length of PPLN changes the outcome of the interaction, and, in general terms, the most effective interaction is demonstrated when the condition of quasi-phase-matching is fulfilled along the whole length of PPLN. It is though worth noting that the result of performing logical functions for the input channels A and B is dependent on the type of Li gradient coefficient along the length of PPLN. The result of the logical operation was evaluated either in accordance with the decrease in the input signal amplitude or in accordance with the increase of the signal amplitude at the wavelength corresponding to that of the SFG or the idle wave. The decrease of the input signal amplitude and the increase in the amplitudes of SF and I channels is more prominent when Li coefficient gradient along the length of PPLN has the form of upward semi-parabola.

The calculation conducted allows us to make a truth table for logical elements based on PPLN-structure. Output A stands for the modulated light intensity at the wavelength of \( \lambda_{SA} \) (i.e. optical carrier \( \omega_{SA} \)), output B stands for the modulated light intensity at the wavelength of \( \lambda_{SB} \) (i.e. optical carrier ...
\( \omega_{SB} \), output I stands for the modulated light intensity at the wavelength of \( \lambda_I \) (i.e. optical carrier \( \omega_I \)), output SF stands for the modulated light intensity at the wavelength of \( \lambda_{SF} \) (i.e. optical carrier \( \omega_{SF} \)).

The table 1 shows that at the outputs I and SF logical multiplication of the signals from input A and input B is carried out. The difference of the outputs I and SF lies in the fact that the optical carrier frequency SF is the sum of the optical carriers of A and B, and the optical carrier frequency I is found from the expression for difference frequency generation (DFG-process):

\[
\omega_I = \omega_{SF} - \omega_P
\]

where \( \omega_P \) is the optical pump frequency of PPLN.

\[\text{Figure 2. Temporal view of the impulses at the input and output of the transducer: 1 – for the constant Li concentration gradient along the length of PPLN; 2 – for Li concentration gradient in the form of an upward semi-parabola; 3 – for Li concentration gradient in the form of a downward semi-parabola.}\]

| Table 1. Truth table for the structural scheme of logical elements. |
|---------------------------------------------------------------|
| Input A | Output A | Input B | Output B | Output I | Output SF |
|---------|----------|---------|----------|----------|-----------|
| 0       | 0        | 0       | 0        | 0        | 0         |
| 0       | 0        | 1       | 1        | 0        | 0         |
| 1       | 1        | 0       | 0        | 0        | 0         |
| 1       | 0        | 1       | 0        | 1        | 1         |

It can be as well seen from the table, that at the output A the \( A \cdot B \) logical function is performed, and at the output B, it is the \( B \cdot \bar{A} \) logical function.

Besides, the expression for Q-factor was used to evaluate the efficiency of the optical logic element based on graded crystals:

\[ Q = 20 \log \left( \frac{\mu_1 - \mu_0}{\sigma_1 + \sigma_0} \right) \]

where \( \mu_1 \) and \( \mu_0 \) are the power thresholds for logical one and logical zero, \( \sigma \) is the rms deviation, and the extinction coefficient is:

\[ ER = 10 \log \left( \frac{\mu_1}{\mu_0} \right). \]
The efficiency of the PPLN-based optical logic element was evaluated for mutual changes in the wavelengths at the inputs A and B while the sum frequency value \(\omega_{\text{SF}}\) remained unchanged (Figure 3). It can be seen that, for graded PPLN-elements, the decrease of the Q-factor causes the pass band at the level of 15 dB to become 30 \(\mu\)m wider to the right of the pump wavelength (which makes it 60 \(\mu\)m for the whole band) as compared to the idealized situation of zero mismatch between SFG and DFG. The same results can be obtained for the extinction coefficient \(\text{ER}\) for different Li gradients along the length of PPLN. The evaluation of the performance of the logical functions for the input signals A and B has demonstrated that the result is determined by the direction of Li gradient along the length of PPLN. The performance of logical functions may be different in the same PPLN made of the same graded crystal depending on the direction of signal distribution along the concentration gradient of Li. This can be viewed as an additional asset for controlling optical signals interaction.

**Figure 3.** The dependency of Q-factor and extinction coefficient \(\text{ER}\) on the wavelength of channel A with different types of Li coefficient gradient along the length of PPLN: 1 – for the constant Li concentration gradient along the length of PPLN; 2 – for Li concentration gradient in the form of an upward semi-parabola; 3 – for Li concentration gradient in the form of a downward semi-parabola.

4. **Experiment**

The theoretical research on the efficiency of the logical element provided a starting point for building a prototype model of such an element by growing crystals of lithium niobate with a concentration gradient of the main components. Plates oriented perpendicular to the optical axis were cut from the LiNbO\(_3\) crystal. The plates were polished and prepared for applying a layer of photoresist through a photomask. The thickness of the photoresist layer was determined by the Linnik interferometer and amounted to 1.5 \(\mu\)m. After applying a photoresist, a potential difference of 8 to 15 kV/mm was applied to opposite sides of the plate through an electrolyte layer.

To visualize the obtained periodically poled structures, the method of chemical etching in a mixture of nitric and hydrofluoric acids was used. After making a periodically poled LiNbO\(_3\) gradient plate to increase the efficiency of the implementation of the logic functions for the input signals, a surface waveguide was produced by the proton exchange method (Figure 4).

**Figure 4.** View of the obtained surface waveguide in PPLN with Li concentration (on the left is the top view, on the right is the output radiation of the visible range).
After the formation of the waveguide, high-voltage polling was performed on the surface of the lithium gradient niobate. For this, the value of the necessary potential for the polarization of domains through the deposited masking layer was established (Figure 5). By smoothly increasing the voltage applied to the lithium niobate plate through the electrolyte, a range of voltage values was observed at which current spikes were recorded in the circuit, indicating a change in the direction of the polarization vector of ferroelectric domains. A wide range of voltage values indicates a gradient in the composition of lithium niobate along the plate. Figure 5 shows a typical picture of the distribution of domains with a uniform step of the composition gradient over the main components in the crystal plate. Domains were visualized after etching in a mixture of hydrofluoric and nitric acid when heated.

5. Conclusion
A model of an optical logic gate based on a periodically poled waveguide structure in lithium niobate with Li coefficient gradient in the composition of PPLN-transducer has been built. Elements implementing the function of logical multiplication of A and B input signals have been modelled. Decay coefficients of the signals at the outputs A and B (Q-factor, extinction coefficient) during the implementation of the $A \cdot B$ and $B \cdot A$ logical functions were calculated for different directions of Li coefficient gradient in the composition of the PPLN-transducer. With Li concentration gradient along the length of PPLN (gradient of the dispersion of group velocities) the peak values of the Q-factor and extinction coefficient in the third transmission window become lower than without such gradient. At the same time, the transmission band for the logical elements in such a device becomes 60 μm wider at the level of 15 dB, depending on the direction of the distribution of signals along the gradient.

Based on the results of modeling optical logic elements, experimental prototypes of optical transducers from lithium niobate with a concentration gradient of the main components were created using photolithography, high-voltage polling, and proton exchange.

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