Numerical analysis of the leaflet elasticity effect on the flow in the model of a venous valve

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Abstract. The paper presents the results of the numerical study of the flow in a symmetric model of the venous valve with different leaflet elasticity. The focus is on the analysis of the velocity field and the stagnant region. The stagnant region behind the valve is of great practical interest for phlebologists in terms of the potential for the formation of blood clots. The constructed simplified model of the venous valve has given a good qualitative agreement with the clinical ultrasound data on the position of the valve leaflet and the stagnant zone behind the valve.

1. Objectives
Venous hemodynamics is studied to improve the methods of diagnosis and treatment of venous diseases and life-threatening thrombotic complications \cite{1-5}. It is known that ageing makes venous valve leaflets more rigid, thereby preventing the valve from normal functioning. Numerical simulation provides a lot of information about the flow in the venous valve and helps to analyze the effect of the leaflet elasticity on the stagnant zones behind the valve, where thrombus is likely to form. The study \cite{2} aims at evaluating the effect of the leaflet elasticity on the venous valve hemodynamics. The lowest fluid shear stress is observed in the sinus zone, and the base zone of the sinus side of the leaflet has the highest tissue stress. The leaflet elasticity increases the tissue stress observed in a very low fluid shear region. The study \cite{5} aims to evaluate the sensitivity of the valve dynamics to the elastic modulus of the valve leaflets. Authors show that cumulative total fluxes over time and backflow are lower when increasing the valve elastic modulus. Thus, with a flexible valve, more blood can flow from the lower to the upper part of the body, but there may be a higher backflow. These results show that the valve dynamics is very sensible to the elastic modulus of the valve leaflets, and that the flexibility of the leaflets plays a central role in the valve mechanism. Previously in our study \cite{6} a two-dimensional computational model of the popliteal venous valve was constructed and the effect of the gap width between the valve leaflets (venous valve incompetence) on the valve reverse flow (reflux) was investigated.

The purpose of this work is to study the effect of the popliteal venous valve leaflets elasticity on the stagnant zones formation.
2. Methods
This work investigates a 2D symmetric model of a valve in the popliteal vein with insufficiency based on clinical measurements (Fig. 1). The vein radius is \( R = 5 \text{ mm} \) and the leaflet thickness is \( h = 0.4 \text{ mm} \). An abnormal venous valve with insufficiency was modeled with the gap width between leaflets \( \delta = (R-L) \) equalled by \( 0.06R \).

![Computational domain geometry of the venous valve model.](image1)

![Variation in the mean flow velocity during the cycle (T = 1.7 s, \( V_{b\text{ max}} = 0.07 \text{ m/s} \)).](image2)

A uniform velocity profile and a variation in the mean flow velocity over the cycle time, obtained from the Doppler measurements, were imposed at the inlet boundary (Fig.2). The cycle period is \( T = 1.7 \text{ s} \). The maximum mean flow velocity is \( V_{b\text{ max}} = 0.07 \text{ m/s} \). Constant pressure was set at the outlet boundary. A rheological Newtonian model with density \( \rho_f = 1060 \text{ kg/m}^3 \) and viscosity \( \mu_f = 0.004 \text{ Pa·s} \) was implemented. The flow was laminar with Reynolds number \( \text{Re} = \rho_f V_{b\text{ max}} R / \mu_f = 90 \) and Womersley number \( Wo = R \sqrt{2 \pi \rho_f / (T \mu_f)} = 5 \). The approximation of rigid walls was used as their movement is insignificant for the popliteal vein. The valve leaflets were assumed to be isotropic, linear elastic with density \( \rho_s = 1200 \text{ kg/m}^3 \), Poisson ratio \( \nu_s = 0.45 \) and elastic modulus \( E \) of the valve leaflets ranged from 0.2 to 2 MPa [2].

In this study, ANSYS Fluent and Mechanical solvers were used for the fluid and structure modeling, respectively.

The governing equations are formulated using the Arbitrary Lagrangian-Eulerian (ALE) approach for modeling unsteady, incompressible fluid flow on a deforming mesh. In the ALE formulation, the continuity equation is written as

\[
\nabla \cdot \mathbf{u}_i = 0,
\]

where \( \mathbf{u}_i \) is the fluid velocity. For mesh velocity \( \mathbf{u}_m \), a convective term \((\mathbf{u}_i - \mathbf{u}_m)\) is introduced, such that that the momentum equation in ALE formulation may be given

\[
\frac{\partial \mathbf{u}_i}{\partial t} + (\mathbf{u}_i - \mathbf{u}_m) \nabla \mathbf{u}_j = -\frac{1}{\rho_f} \nabla \mathbf{p} + \frac{1}{\rho_f} \nabla^2 \mu_f \mathbf{u}_i, \quad i, j = 1, 2
\]

where \( \mathbf{p} \) is the pressure and \( \mu_f \) is the fluid dynamic viscosity.

The Structure motion is described by the Lagrangian formulation. Solid with density \( \rho_s \) has the displacement \( \mathbf{d}_s \) which is given by

\[
\rho_s \frac{\partial^2 \mathbf{d}_s}{\partial t^2} = \nabla \cdot (S \cdot F^T) + \mathbf{f}_b,
\]

where \( \mathbf{f}_b \) is the body force and \( F \) is the deformation gradient tensor given by \( F = I + \nabla \mathbf{d}_s^T \),

where \( I \) is the identity and \( S \) is the Piola-Kirchoff stress tensor.

The Green-Lagrangian strain tensor \( \mathbf{G} \) given by
is related to $S$ by the following relation:

$$ S = 2\mu, G + \lambda, tr(G)I , $$

where $tr$ is the tensor trace and $\lambda$, and $\mu$, are elastic material Lamé constants. In ANSYS Structural, the elastic modulus $E$ and the Poisson ratio $\nu$, are specified as inputs and are related to $\lambda$ and $\mu$, as

$$ \lambda = \frac{E\nu}{(1+\nu)(1-2\nu)} , \mu = \frac{E}{2(1+\nu)} . $$

At the interface $\Gamma$, the following conditions were satisfied:

$$ d_s^T = d_f^T , u_s^T = u_f^T , T_s^T = -T_f^T , $$

where $T^T$ is the traction force at the interface, which is the sum of the pressure and viscous forces.

As the structure moves inside the fluid domain, the fluid mesh deforms consequently (Fig.3). In this study, the smoothing and remeshing mesh update methods were used. The computational domain was discretized using an unstructured triangular mesh. The mesh around the leaflet was refined. Computations were carried out only with a step of 0.01 s. As a result of the grid independence test, the fluid mesh with 80,000 elements and the solid mesh with 8500 elements were chosen for unsteady computations.

Figure 3. Computational meshes at two time instance of the flow rate increase (a) and decrease (c) phases (E = 0.2 MPa)

3. Results

Figure 4 shows the axial velocity fields and streamlines at different time instances of the cycle for the venous valve model with elastic modulus $E = 0.2$ MPa. The maximum velocity is observed between the leaflets of the valve, the minimum - in the sinus (expansion behind the valve). At the beginning of the flow rate increase phase, the valve continues to close during a short period of time ($t / T < 0.1$) and the flow still retains the vortex structure, which remains from the previous flow rate decrease phase. As the flow rate increases, the valve opens, and the streamlines straighten. This flow structure remains the same in the flow decrease phase until the valve fully opens, corresponding to the time instance $t/T = 0.6$. During the valve closing phase, the flow becomes more complex due to the presence of vortex structures. A large vortex is formed in the sinus with the center in the vicinity of the leaflet tip. The form of the leaflet also changes during the cycle. During the opening and closing phases, the leaflet has an arched shape, and the maximum curvature is observed during the valve closing phase. During a short period of time ($t / T = 0.5 - 0.7$) the valve leaflets have a double curved shape.

Figure 5 shows the variation in the transverse displacement of the leaflet tip $r' - \delta$ ($r$ is counted from the axis of the vein) over time for different elastic modulus of the leaflet. The maximum of the displacement curve is shifted relative to the mean flow velocity curve by $\Delta t / T = 0.35$ for the valve with elastic modulus $E = 0.2$ MPa. The valve is fully closed ($r' / R = 0.19$) in the flow rate increase phase and
fully opened \((r'/R = 0.46)\) in the flow rate decrease phase. With leaflet stiffening, the maximum of the displacement curve shifts closer to the maximum of the mean flow curve.

Figure 4. Axial velocity field and streamlines at four time instances of the cycle \((E = 0.2 \text{ MPa})\).

Figure 6 shows the dependence of the oscillations amplitude of the leaflet tip \(\Delta r\) on the elastic modulus. The dependence is a power function: a rapid amplitude decrease for elastic modulus from 0.2 to 1 MPa (3 times) is replaced by its slow decrease for elastic modulus from 1 to 2 MPa (1.5 times).

Figure 5. Transverse displacement of the leaflet tip (colored lines) during the cycle (mean flow velocity is shown with a black line).

Figure 6. Oscillations amplitude of the leaflet tip \(\Delta r\) and stagnant zone area \(S\) versus elastic modulus (solid and dashed lines show approximations by power functions)
Figure 7 shows a comparison of clinical and numerical data on the position of the valve leaflet and the stagnant zone of the venous valves with different elasticity at two time instances of the cycle. In the ultrasound image (B-mode of the scanner), the wall and leaflet of the venous valve are displayed in white, and the zone of the intense flow - in black. According to the clinical and numerical data, the stagnant zone is observed in the sinus, closely to the leaflet, during the valve opening phase. In the stagnant zone fluid rotates at a relatively low velocity (less than 1 cm / s) and red blood cells form agglomerations. In the ultrasound image of Fig. 7, the stagnant zone can be seen as a gray "cloud" at the bottom of the leaflet (circled in red). This zone in venous hemodynamics is called sludge [7,8]. In the valve closing phase, the stagnant zone almost disappears. Figure 6 shows the dependence of the stagnant zone area at the time instance of the fully open valve S on the elastic modulus. A rapid increase in the stagnant zone area for a small elastic modulus (E < 1 MPa) is replaced by its slow growth at a large elastic modulus (E > 1 MPa).

Figure 7. Comparison of the numerical and registered positions of the leaflet and the stagnant zone in the venous valves with different elasticity at two time instances of the cycle: fully open (left) and closing phase (right). The stagnation zone is shown in gray. In the ultrasound image, the borders of the leaflet are marked with a yellow line, and the stagnation zone at the bottom of the leaflet is gray (E = 0.2 MPa (a), 1 MPa (b)).

Conclusions

The described symmetrical two-dimensional model of a venous valve with insufficiency has a good qualitative agreement with the clinical ultrasound data on the position and size of the stagnant zone behind the leaflet. The stagnant zone in the venous valves is of great practical interest for phlebologists due to potential thrombosis.
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