Application of the envelope method to synthesis of control in the differential game “pursuit-evasion”

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Abstract. The report is devoted to the applications of the envelope method to the synthesis of control in the differential game "pursuit-evasion". It is proved that optimal trajectories can be represented as envelopes of a parametric family of singular curves, called instantaneous solutions within this approach, and that control can be found on this family. It is shown that, in the general case, the synthesized game control law will contain the maximum permissible control values, current values of phase coordinates, parameters of tangents and their derivatives.

1. Introduction.
The report considers the case when the players' movements are described by systems of the same type of differential equations with generally nonlinear right-hand sides, but with linearly entering control. The players' control is limited to a closed set of maximum permissible values. As the terminal optimality criterion, the Euclidean measure of the residuals of the coordinates of the pursuer and the pursued at the end of the game (miss) is taken. The minimax formulation of the deterministic problem assumes that the goal of the pursuer is to minimize the miss, and of the pursued – to maximize it. All currently known approaches to solving antagonistic differential games (the maximum principle or dynamic programming, the theory of positional differential games, etc. [1-4]) are associated with solving a two-point boundary value problem, or bypassing it in one way or another. In this sense, the present report is no exception and is devoted to the application of the envelope method to solving differential games of the "pursuit-evasion" type.

2. Main part
2.1. The optimal control and the envelop method
Synthesis of control in the differential game “pursuit – evasion” is a generalization of the optimal control problem in the case of two persons. In this setting, the optimal control and trajectories are found after solving the corresponding boundary value problem. For nonlinear systems, its solution is difficult. In addition, numerical methods are largely influenced by inevitable rounding errors, which in some cases do not allow obtaining a result. To optimize the control of dynamic systems, an approach was developed based on the application of the envelope method [5,6]. It is based on the fact that the phase trajectory of a dynamical system is the envelope of a family of surfaces (in particular cases of singular curves) reconstructed from each of its points. This is the basis of the graphical-analytical...
method of integration by V.P. Vetchinkin [7], when the trajectory is represented by the envelope of a family of circles of instantaneous radius of curvature.

It should be noted that at each specific point of the phase trajectory, the velocity vector and the vector of the generalized momentum are tangent to it. Therefore, there is a possibility of synthesizing control on a family of singular curves. In [5,6], a method for constructing singular curves called instantaneous solutions is presented, and a proof of the possibility of finding a control within a given terminal optimality criterion can be found on a family of instant solutions both on the boundary of an admissible set of controls and inside it.

Let us pay attention to the fact that the obtained control law has adaptive properties and is constructed according to the principle of feedback in relation to the parameters determined on the family of instantaneous solutions. The adaptation is carried out according to the parameters calculated at each moment of time on instantaneous solutions and the corresponding derivatives.

2.2. The differential game “pursuit – evasion” and the envelop method

The above ideas formed the basis of the control synthesis method in the differential game “pursuit – evasion”. For concretization, the method of positional differential games was used [4]. Within the framework of this approach, the reachable domains of the pursued and pursuer were constructed. In the reduced space of instantaneous solutions, the reachability regions were involutes to curves with the maximum permissible control values. Due to the uniformity of the mathematical models of the players and the properties of the involutes, the extreme aiming point, as the most distant from the pursuer’s attainability boundary, was on the tangent to the curves with the maximum permissible value of the pursuer and pursued control.

Thus, the optimal trajectories of the pursuer and the pursued consist of a section with the maximum permissible control value and a section with special control. The tangent parameters and their derivatives determine the structure of the special control.

Since the sign function of the tangent to the curves with the maximum permissible value of control corresponds to the sign function of the line of sight, then, according to the Yu.B. Germeier principle of information redefinition [8], further reduction of the space of instantaneous solutions is possible, i.e. further simplification of building instant solutions.

In the general case, the synthesized game control law will contain the maximum allowal control values, current values of phase coordinates, parameters of tangents and their derivatives.

The effectiveness of the proposed approach was evaluated in comparison with the classical solution of the two-point boundary value problem by the method of successive approximations. Numerical solutions have shown that the envelope method gives a sevenfold reduction in the time for solving the boundary value problem.

3. Conclusion

The studies have shown that the application of the envelope method simplifies the synthesis of control in the differential game "pursuit-evasion" and the interpretation of the results. This approach can be further applied to games of several persons, to solutions of coalition, hierarchical and cooperative games.

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