Wick quantisation of a symplectic manifold
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The notion of the Wick star-product is covariantly introduced for a general symplectic manifold equipped with two transverse polarisations. Along the lines of Fedosov method, the explicit procedure is given to construct the Wick symbols on the manifold. The cohomological obstruction is identified to the equivalence between the Wick star-product and the Fedosov one. In particular in the Kähler case, the Wick star-product is shown to be equivalent the Weyl one, iff the manifold is a Calabi-Yau one.

1. INTRODUCTION

The progress of the quantum field theory was always related, directly or indirectly, with in-depth study of the quantisation methods. Nowadays we observe an explosive development in the deformation quantisation theory. In particular, for the symplectic manifolds Fedosov has suggested a simple method to construct a manifestly covariant star-product \cite{Fedosov}. The Fedosov quantisation can be thought of as a direct generalisation of the Weyl symbols known for the linear symplectic spaces. The Wick symbol is supposed to be more appropriate for the field theory quantisation, being underlaid by the notion of the creation and annihilation operators. Usually, the Fock space construction is known for the free fields only, whereas the nonlinearity is considered as a small perturbation to a linear background. This artificial disintegration of the theory into the “free” (linear) part and the (nonlinear) “interaction” is not always adequate to the problem, because it may break fundamental symmetries of the model. The extended list of examples is provided to this problem by sigma models where the nonlinear metric can’t be naturally represented as a constant “free” part plus “interaction”. From this standpoint, the covariant Wick symbol construction acquires a primary importance for applying the deformation quantisation schemes in the field theory.

In general, the Wick and the Weyl quantisations are not equivalent to each other in the field theory even in the linear approximation because of possible quantum divergencies in the formal transformation from one symbol to another. The nontrivial geometry of the phase space may result in the cohomological obstruction to the equivalence between different symbols (even for the system with finite number degrees of freedom, i.e. without any divergencies). The explicit construction for the Wick symbol seems to be very important to this end as well as an efficient criterion of identifying this symbol among the other ones.

The classification of the star-products on a symplectic manifold has been studied in several works \cite{Vassiliev}, \cite{Hochschild}, \cite{Kontsevich}, \cite{Sarnak}, \cite{Shoikhet}. Each equivalence class of the star-products has been shown to correspond to formal power series (in $\hbar$) taking values in the second De Rham cohomologies. This result seems to be most transparent in the context of the Fedosov method \cite{Fedosov}, where the space of series in the second De Rham cohomologies naturally appears as a moduli space of the flat Fedosov connections of an auxiliary symbol bundle. Thus the equivalence class of the connection defines the equivalence class of the star-product and vice versa.

In this talk we discuss covariant construction for the Wick-type symbol \cite{Vassiliev} and identify its equivalence class as a De Rham class of a certain 2-form. In the Kähler case, the Wick and Weyl symbols turn out to be equivalent to each other iff the manifold is the Calabi-Yau one.

In Sec 2 we briefly describe the Wick type symbol construction \cite{Vassiliev} based on the Fedosov approach \cite{Fedosov}. In Sec 3 we study the equivalence

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between Wick and Weyl symbols. Conclusion is devoted to some open questions.

2. FEDOSOV STAR-PRODUCT OF THE WICK TYPE

Let \((M, \{\cdot, \cdot\})\) be a 2n-dimensional Poisson manifold with Poisson bracket

\[\{a, b\} = \omega^{ij} \partial_i a \partial_j b, \quad a, b \in C^\infty(M).\] (1)

Below we suppose that \(\omega^{ij}\) is nondegenerate, so that inverse matrix \(\omega_{ij}\) defines the symplectic 2-form \(\omega = \omega_{ij} dx^i \wedge dx^j\), which is closed in virtue of Jacobi identity for (1). The commutative algebra of smooth functions \(C^\infty(M)\) assigned by the Poisson bracket is called Poisson algebra of classical observables.

The aim of deformation quantisation program is to define a new multiplication operation \(\ast\) (named a star-product) depending on \(\hbar\), which would be one-parametric associative deformation of the ordinary point-wise function multiplication. More precisely, let \(C^\infty(M)[[\hbar]]\) be the space of formal series

\[a = a(x, \hbar) = \sum_{n=0}^{\infty} \hbar^n a_n(x),\]

where \(a_n(x) \in C^\infty(M)\). This space is regarded as the space of quantum observables. Then the star-product of two quantum observables is given by

\[a \ast b = \sum_{n=0}^{\infty} \hbar^n D_n(a, b),\]

where the following conditions are assumed to be satisfied:

a) \(D_n\) are bi-differential operators on \(C^\infty(M)\);

b) \(D_0(a, b) = ab;\)

c) \(D_1(a, b) - D_1(b, a) = -i\{a, b\}\).

The deformation quantisation, being so defined, has a natural “gauge group” acting on the set of all star-products. Two star-products \(\ast\) and \(\hat{\ast}\) are called equivalent if there exists an isomorphism of algebras

\[B : (C^\infty(M)[[\hbar]], \ast) \to (C^\infty(M)[[\hbar]], \hat{\ast})\] (2)
given by a formal differential operator \(B = 1 + \hbar B_1 + \hbar^2 B_2 + \ldots\).

Given a Poisson manifold \((M, \{\cdot, \cdot\})\), the problem of deformation quantisation is to describe all the star-products satisfying axioms a), b), c) up to the equivalence (2).

As we have shown in [1], the most natural way to define the Wick-type star-product is to introduce a complex-valued metric structure \(g^{ij}\) subject to certain conditions. To represent these conditions in a compact form, let us combine the Poisson bi-vector and the metric into the tensor field

\[\Pi^{ij}(x) = \omega^{ij}(x) + g^{ij}(x).\] (3)

We also define a form \(\Pi\) with the lower indices as \(\Pi_{ij} = \omega_{in}\omega_{jm}\Pi^{nm}\). The aforementioned conditions can be written as

i) \(\text{rank}(\Pi_{ij}) = \frac{1}{2}\dim M = n\)

ii) \( (\Pi^{in}\partial_n\Pi^{jk} - \Pi^{jn}\partial_n\Pi^{ik})\Pi_{kn} = 0 \) (4)

These conditions guarantee the existence of two transverse Lagrangian polarisations on \(M\) associated with left and right kernel distributions of the form \(\Pi\). The integrability condition ii) for these polarisations is also equivalent to existence of a torsion-free Levi-Civita connection preserving \(\Pi\). In what follows we will refer to \(\Pi\) as the Wick tensor.

The most notable examples of the manifolds admitting Wick structure are provided by the Kähler manifolds. In this case \(\Pi^I = -\Pi\), and the left (right) kernel distribution of \(\Pi\) generates the holomorphic (anti-holomorphic) polarisation. The case of a real metric \(g\) satisfying rels. (3) corresponds to the so-called para-Kähler geometry [3].

The Wick-type deformation quantisation can be defined by replacing the axiom c) with a stronger one

\[c') \ D_1(a, b) = -\frac{1}{2} \Pi^{ij}(x)\partial_i a \partial_j b.\] (5)

In the paper [3] the modification of the Fedosov method was proposed to produce the \(\ast\)-product satisfying the axioms of Wick-type quantisation. In so doing the Kähler and para-Kähler geometries are naturally related with the algebras of genuine Wick- and \(qp\)-symbols respectively.

Let us now outline the main steps of our construction. First, note that Wick tensor \(\Pi\) on
$M$ determines a constant Wick structure on each tangent space $T_x M$, which can be used to quantise $T_x M$ by means of standard Wick product. More precisely it is expressed by

**Definition.** The formal algebra $W_x$ associated to $T_x M$ is an associative algebra with a unit over $\mathbb{C}$, whose elements are formal power series in the deformation parameter $\hbar$ with coefficients being formal polynomials on $T_x M$:

$$a(y, \hbar) = \sum_{n,m \geq 0} \hbar^n a_{ni_1...i_n} y^{i_1} ... y^{i_m},$$

where $y$’s are linear coordinates on $T_x M$. The product of elements $a, b \in W_x$ is defined by the Wick rule

$$a \circ b = \exp \left( \frac{i \hbar}{2} \Pi^{ij} \frac{\partial}{\partial y^i} \frac{\partial}{\partial z^j} \right) a(y, \hbar) b(z, \hbar)|_{z=y}. \quad (6)$$

Taking a union of algebras $W_x$, $x \in M$, we obtain a bundle $W$ of formal Wick algebras, whose sections are formal functions

$$a(x, y, \hbar) = \sum_{n,m \geq 0} \hbar^n a_{ni_1...i_m} y^{i_1} ... y^{i_m},$$

where $a_{ni_1...i_m}$ are components of symmetric covariant tensors on $M$. The $\circ$-product can naturally be extended to the space $W \otimes \Lambda$ of $W$-valued differential forms by means of the usual exterior product of the scalar forms from $\Lambda$. The general element from $W \otimes \Lambda$ reads

$$a(x, y, dx, \hbar) = \sum_{k+p,q \geq 0} \hbar^k \delta a_{ak...i_1...i_n,j_1...j_q}(x) \times y^{i_1} ... y^{i_p} dx^{j_1} \wedge ... \wedge dx^{j_q}. \quad (7)$$

There are two useful gradings $\deg_1$ and $\deg_2$ defined on the homogeneous elements $h, y, dx$ as follows: $\deg_1(y^i) = 1$, $\deg_1(\hbar) = 2$, $\deg_2(dx^i) = 1$, and all other gradings are vanishing. One may see that $W \otimes \Lambda$ is the bi-graded associative algebra with respect to the gradings $\deg_1$, $\deg_2$. The commutator of two homogeneous forms from $W \otimes \Lambda$ is defined as

$$[a, b] = a \circ b - (-1)^{\deg_2(a)\deg_2(b)} b \circ a.$$

Introduce the Fedosov operators $\delta$ and $\delta^{-1}$ on $W \otimes \Lambda$, defined as follows. For a homogeneous element $a$ with $\deg_1(a) = p$ and $\deg_2(a) = q$ we put

$$\delta a = dx^k \wedge \frac{\partial a}{\partial y^k} = \frac{i}{\hbar} [\omega_{ij} y^j dx^i, a], \quad (8)$$

$$\delta^{-1} a = \begin{cases} \frac{1}{p+q} y^i i(\frac{\partial}{\partial y^k})a, & \text{if } p+q \neq 0 \\ 0, & \text{otherwise}, \end{cases} \quad (9)$$

Note that both the operators are nilpotent, but only $\delta$ is the (inner) antiderivation of $\circ$-product. The properties of these operators are very similar to those for the usual exterior differential and codifferential, in particular they satisfy to an analogue of the Hodge-De Rham decomposition:

$$a = \sigma(a) + \delta \delta^{-1} a + \delta^{-1} \delta a, \quad (10)$$

where $\sigma(a) = a(x,0,0,\hbar)$ denotes the canonical projection onto $C^\infty(M)[[\hbar]]$.

Now let $\nabla$ be a torsion-free connection respecting Wick tensor $\Pi^{ij}$. It induces the covariant derivative on $W \otimes \Lambda$:

$$\nabla a = dx^i \wedge \left( \frac{\partial a}{\partial x^i} - y^j \Gamma^k_{ij}(x) \frac{\partial a}{\partial y^k} \right), \quad (11)$$

$\Gamma^k_{ij}$ being the Christoffel symbols.

Following Fedosov, consider more general connection of the form:

$$D = \nabla - \delta + \frac{1}{i\hbar} [r, \cdot], \quad (12)$$

where $r = r_i(x, y, \hbar) dx^i \in W \otimes \Lambda^1$. Clearly, $D$ is a derivation of $\circ$-product, i.e.

$$D(a \circ b) = Da \circ b + (-1)^{\deg_2(a)} a \circ Db. \quad (13)$$

A simple calculations show that

$$D^2 a = \frac{1}{i\hbar} [\Omega, a], \quad \forall a \in W \otimes \Lambda, \quad (14)$$

$$\Omega = -\frac{1}{2} \omega + R - \delta r + \nabla r + \frac{1}{i\hbar} r \circ r. \quad (15)$$

Here $R = \frac{4}{3} R_{ijkl} y^j y^l dx^k \wedge dx^i$ and $R_{ijkl} = \omega_{in} R^n_{jkl}$ is the curvature tensor of $\nabla$. A connection of the form $[12]$ is called Abelian or flat if $\Omega$ does not depend on $y$’s. In this case $D^2 = 0$. 

Using the “Hodge-De Rham decomposition” \((\Pi)\), the Bianchi identity \(\nabla R = 0\) and the symmetry property of the curvature tensor,

\[
\omega_{ij}, R^n_{ijkl} = \omega_{ji} R^n_{iklj}, \quad g_{ij} R^n_{ijkl} = -g_{ij} R^n_{iklj},
\]

one may prove the following counterparts of Fedosov’s theorems.

**Theorem 1.** Let \(\nabla\) be any torsion-free connection respecting symplectic form \(\omega = \omega_{ij} dx^i \wedge dx^j\). There is a unique Abelian connection \(D\) of the form \((\ref{extension})\) for which

\[
\delta^{-1} r = 0, \quad \Omega = \frac{1}{2} \omega
\]

and the expansion \((\ref{extension})\) for \(r\) involves elements of \(deg \geq 3\).

Denote by \(W_D\) the subspace of all parallel sections in \(W\) with respect to a flat Fedosov connection \((\ref{extension})\). Due to rel.\((\ref{flat})\) the space \(W_D\) is an associative subalgebra. The next theorem establishes isomorphism between \(W_D\) and \(C^\infty[[h]]\), which induces a star-product on \(C^\infty[[h]]\).

**Theorem 2.** For any formal function \(a \in C^\infty(M)\) there is a unique section \(\tilde{a} \in W_D\) such that \(\sigma(\tilde{a}) = a\). The pull-back of the \(\circ\)-product via \(\sigma\) induces the Wick-type star-product on \(M\):

\[
a \circ b = \sigma((\sigma^{-1} a) \circ (\sigma^{-1} b)), \quad \forall a, b \in C^\infty(M)\] (13)

The elements \(r \in W \otimes \Lambda^1\) and \(\tilde{a} \in W\) mentioned in the Theorems can be efficiently constructed by iterating a pair of coupled equations

\[
r = \delta^{-1}(\nabla r + \frac{1}{ih} r \circ r),
\]

\[
\tilde{a} = a + \delta^{-1}(\nabla \tilde{a} + \frac{1}{ih} [r, \tilde{a}])
\]

with respect to the first degree.

As one may see, the rank condition imposed on \(\Pi^g\) \((\ref{extension})\) is not so essential for the construction of an associative \(\ast\)-product obeying \((\ref{extension})\). The only fact we have used here is the existence of a torsion-free connection preserving \(\Pi\) and the reversibility of the Poisson bi-vector \(\omega^{ij}\). The construction would work, for example, with a degenerate \(g\), when \(g = 0\) it reduces to the Fedosov quantisation.

3. THE QUESTION OF EQUIVALENCE

The rich geometry of symplectic manifolds equipped by the metric structure \((\ref{extension})\) offers at least two different schemes for their quantisation: the Fedosov quantisation, which exploits only antisymmetric part of the Wick form \(\Pi \ref{wick}\), and the deformation quantisation involving the entire form \(\Pi\) to meet the condition \((\ref{extension})\). The question is whether these two quantisations are actually different or an equivalence transform may be found to establish a global isomorphism between both algebras of quantum observables. Below we formulate the necessary and sufficient conditions for such an isomorphism to exist. As in the general case, the obstruction for equivalence of two star-products lies in the second De Rham cohomology of symplectic manifold and we identify a certain 2-form as its representative.

In order to distinguish Wick-type star product from the Weyl one, all the constructions related to the former product will be attributed by the additional symbol \(g\) (pointing on non-zero symmetric part \(g\) in \(\Pi\)). In particular, through this section the fibre-wise multiplication \((\ref{extension})\) will be denoted by \(\circ\), while \(\circ\) will be reserved for the Fedosov \(\circ\)-product \((\ref{extension})\) resulting from \((\ref{extension})\) if put \(g = 0\).

First we note that fibre-wise \(\circ\) and \(\circ_g\) products are equivalent in the following sense:

\[
a \circ_g b = G^{-1}(G a \circ G b), \quad \forall a, b \in W
\]

where the formally invertible operator \(G\) reads as

\[
G = \exp \left(-\frac{ih}{4} g^{ij} \frac{\partial}{\partial y^i} \frac{\partial}{\partial y^j}\right).
\]

The operator \(G\) has following properties:

\[
\nabla G = G \nabla, \quad \delta G = G \delta
\]

Using the automorphism \(G\) we can define a new Abelian connection \(\tilde{D} = GDgG^{-1}\), which in virtue of rels. \((\ref{structure})\) can be written as

\[
\tilde{D} = \nabla - \delta + \frac{1}{ih} [\tilde{r}, \cdot], \quad \tilde{r} = Gr_g.
\]

Here the brackets \([\cdot, \cdot]\) stand for \(\circ\)-commutator. The elements \(\tilde{r}\) and \(r\) satisfy the equations

\[
\tilde{r} = \delta^{-1}(GR + \nabla \tilde{r} - \delta r + \frac{1}{ih} r \circ \tilde{r})
\] (21)
where \( \Delta r \) equation (27) resulting from the identity 

\[ M \]  

a so-called (23), (24) show that the second star product 

identity for the curvature tensor. In fact, rels. 

The 2-form \( \Omega \) is closed in virtue of the Bianchi 

Thus we have two star-products * and \( \bar{*} \) corresponding to the pair of Abelian connections \( D \) and \( \bar{D} \). Since \( D \neq \bar{D} \), in general, the action of the fibre-wise isomorphism \( G \) establishing the equivalence between \( \star \) and \( \sigma \)-products (and hence between the star products \( \star \) and \( \bar{*} \)) is not automatically followed by the equality \( \star = \bar{*} \). Indeed, evaluating lowest orders in \( \hbar \) we get:

\[
a \star b = ab + \frac{i\hbar}{2} \omega^{ij} \nabla_i a \nabla_j b - \frac{\hbar^2}{4} \omega^{ik} \omega^{lj} \nabla_i \nabla_j a \nabla_k \nabla_l b + O(\hbar^3),
\]

\[
\bar{a} \bar{*} b = ab + \frac{i\hbar}{2} \bar{\omega}^{ij} \nabla_i a \nabla_j b - \frac{\hbar^2}{4} \bar{\omega}^{ik} \omega^{lj} \nabla_i \nabla_j a \nabla_k \nabla_l b + O(\hbar^3),
\]

where

\[
\bar{\omega}^{ij} = \omega^{ij} + \hbar \omega^i_j, \quad \omega^i_j = \omega^{ik} \Omega_{kl} \omega^{lj},
\]

\[ \Omega = \frac{1}{4} (GR - R) = \frac{i}{2} R_{ijkl} g^{ij} dx^k \wedge dx^l \]

The 2-form \( \Omega \) is closed in virtue of the Bianchi identity for the curvature tensor. In fact, rels. (23), (24) show that the second star product \( \bar{*} \) is a so-called 1-differentiable deformation \( \bar{a} \) of \( \star \). This deformation is known to be trivial iff the 2-form \( \Omega \) is exact \( [3] \). Now supposing \( \Omega = d\psi \) let us try to establish an equivalence between \( \star \) and \( \bar{*} \) by means of a fibre-wise conjugation automorphism

\[
a \rightarrow U \circ a \circ U^{-1},
\]

where \( U \) is an invertible element of \( W \). The element \( U \) is so chosen that transformation (25) turns \( D \) to \( \bar{D} \). This is equivalent to

\[ D(U \circ a \circ U^{-1}) = U \circ (\bar{D} a) \circ U^{-1}, \quad \forall a \in W \]  

(26)

The last condition means that

\[ U^{-1} \circ DU = \frac{1}{i\hbar} \Delta r + \frac{1}{i\hbar} \psi, \]

(27)

where \( \Delta r = \bar{r} - r \) and \( \psi \) is a globally defined 1-form on \( M \). The compatibility condition for equation (27) resulting from the identity \( D^2 = 0 \) requires that

\[ D \Delta r + \frac{1}{i\hbar} \Delta r \circ \Delta r + d\psi = 0 \]

(28)

The analogous relation is obtained if we subtract (21) from (22)

\[ D \Delta r + \frac{1}{i\hbar} \Delta r \circ \Delta r + \Omega = 0 \]

(29)

Comparing (28) with (29) we conclude that the compatibility condition holds provided \( \Omega \) is exact. Now rewrite (27) in the form

\[ \delta U = \nabla U + \frac{1}{i\hbar} [r, U] - \frac{1}{i\hbar} U \circ (\Delta r + \psi) \]

(30)

and apply the operator \( \delta^{-1} \) to both sides of the equation. Using the Hodge-De Rham decomposition (10) and taking \( \sigma(U) = 1 \), we get

\[ U = 1 + \delta^{-1}(\nabla U + \frac{1}{i\hbar} [r, U] - \frac{1}{i\hbar} U \circ (\Delta r + \psi)) \]

(31)

The iterations of the last equation yield a unique solution for (31) provided the compatibility condition (23) is fulfilled. Starting from 1, this solution defines an invertible element of \( W \). Then the equivalence transform \( B : (C^\infty(M)[[\hbar]], \star) \rightarrow (C^\infty(M)[[\hbar]], \bar{*}) \) we are looking for is defined as the sequence of maps

\[ Ba(x) = (U \circ G(\sigma_\hbar^{-1}(a)) \circ U^{-1})|_{y=0}, \]

(32)

so that

\[ a \star b = B^{-1}((Ba) \star (Bb)) \]

(33)

Thus we have proved the following

**Theorem 3.** The obstruction to equivalence between Weyl and Wick type deformation quantisations lies in the second De Rham cohomology \( H^2(M) \). The quantisations are equivalent iff the 2-form \( R_{ijkl} g^{ij} dx^k \wedge dx^l \) is exact.

For the anti-Hermitian matrix \( \Pi \) the 2-form \( \Omega \) is nothing but the Ricci form of the \( \bar{\Pi} \) and is known to depend only on the complex structure of the manifold \( M \). Since, for example, \( c_1(CP^n) \neq 0 \) and for any \( \bar{\Pi} \), the Weyl and Wick quantisations on \( CP^n \) are not equivalent for any \( \Pi \). More generally, the \( \bar{\Pi} \) manifolds with the vanishing first Chern class are known as the
Calabi-Yau ones. These manifolds have been intensively studied by physicists in the context of string compactification problem during the last decades. The above Theorem allows one to characterize these manifolds as those Kähler manifolds on which Weyl and Wick quantisations are equivalent to each other. This observation might be crucial for the consistent string compactification on the Calabi-Yau in the presence of non-constant background B-field [12], [13].

4. CONCLUDING REMARKS

In this talk we have explained a method to covariantly construct Wick-type symbol for the system whose phase space is equipped with both the symplectic structure and the metric one, interrelated by the conditions (3), (4). We have also described the relationship between Weyl and Wick symbols and the cohomological obstructions to their equivalence. Let us briefly discuss how this construction could be used in quantising field theories. First, the quantisation scheme should be extended to the constrained Hamiltonian systems, as the phase space of strings and gauge fields is subject to constraints. A step has been done in this direction in the paper [8], where the Weyl deformation quantisation is worked out for the Dirac brackets. Now this should be combined with the Wick structure of the constrained phase space. Second, it should be understood what could be taken as a natural symmetric phase space tensor \( g_{ij} \) for the constrained systems like strings in curved space-time. A possible way of finding this structure is to induce it from an enveloping linear phase space (where one usually has an obvious Wick structure) to the nonlinear constrained surface. It is not quite obvious, however, that the constrained surface would be equipped in this way by an integrable Wick structure. If the integrability condition (ii) (3) was not automatically satisfied for the induced Wick structure, one may hope to overcome this obstacle by introducing an appropriate torsion. This can probably be done in the way of the recent paper [14] where our construction of the Wick-type star-product is generalised to the case of the almost Kähler manifold.

Acknowledgments. We are grateful to I.A. Batalin, A. Borowiec, M. Grigoriev, A. Karabegov, R. Marnelius for valuable discussions related to the topic of this talk. The work is partially supported by the RFBR grant No 00-02-17-956. VAD is supported by grant for Support of Scientific Schools N 00-15-96557, SLL and AAS acknowledge the support from the grant E-00-33-184 from Russian Ministry of Education.

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