Transitory Response Characteristics Analysis of High-Power Piezoelectric Transducers

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Highlights:

What are the main findings?

• Derive the characteristics of transient equations under different excitations, and conclude the time constant is the key to the stability of the transducers.
• Applying a vector reduction method to analyze a series RLC transient matching scheme, which can change the electromechanical damping and improve the transient response time of the system.

What is the implication of the main finding?

• A matching scheme combined with full-bridge inverter, transformer and series LC, is adopted to improve the transient response characteristic, which can provide suitable electrical damping without causing energy losses.

Abstract: To improve suitability in applications with high dynamic performance requirements, the transient response characteristics of high-power piezoelectric transducers should be studied quantitatively. This paper proposes the vector reduction method to solve the complex transient equations and obtains a transient matching scheme clarifying the mechanism of electrical matching resistance on electromechanical damping. A matching scheme with a combination of full-bridge inverter, transformer and series LC circuit is designed and validated, which can provide suitable electrical damping without causing energy losses. Consequently, the experiment verifies the transient properties of the proposed scheme. For a typical piezoelectric cutting transducer with 100.8 ms response time, our scheme is verified to have high dynamic performance within frequency response time of 5.5 ms and vibration response time of 15.0 ms.

Keywords: transient response characteristic; high-power piezoelectric transducer; vector reduction method; transient matching scheme

1. Introduction

High-power piezoelectric transducers have the advantages of low energy consumption, wide application range and high processing efficiency, which are widely used in food, medicine, plastic, composite materials and metal processing [1–3]. Owing to the particular mechanical, thermal and cavitation effects of the high-power piezoelectric transducers, researchers are trying to use them to assist or replace traditional methods in more and more processing and manufacturing fields. However, the control of a high-power piezoelectric system is not an easy task, as it has stricter dynamic performance requirements. For example, in high-power piezoelectric cutting and welding systems, the practical application requires a dynamic response time of 20 to 100 ms (milliseconds); the shorter the better. However, for high-power piezoelectric transducers, the large size of the mechanical structure leads to high modal mass, so that the transient response time will be longer than 100 ms, which takes a serious limitation to the dynamic control [4,5].
In studies of signal ultrasound and piezoelectric transducer, researchers [6] have optimized the dynamic response of piezoelectric vibrators from the perspective of variable excitation. However, these optimization methods either rely on a large amount of data simulation and experimental verification, or complex algorithms for driving control are required, which are not suitable for practical application processes. In order to characterize the transient process of the piezoelectric transducer system accurately, Chang [7] used circuit equations to analyze the vibration transient response equation of the piezoelectric transducer system excited by the AC (Analog Circuit) constant voltage source. The results show that the transient response is inverse exponential. Moreover, there is a constant time which is proportional to the equivalent mass and inversely proportional to the equivalent damping. The transient response at the excitation of the AC constant voltage source is shown in Figure 1. Niederberger [8] analyzed the effect to optimize the transient characteristics in a piezoelectric damping system with three different methods using passive LR circuit method, negative capacitive shunt method and synchronous inductive damping method. The analysis result is shown in Figure 2. However, as the analytical relationship is very complex, these studies only calculated the simplified matching features, and the specific analytical relationship between the matching circuit and the transient response remains to be further studied.

Figure 1. Transient response at the excitation AC constant voltage source.

Figure 2. Comparison of amplitude–frequency and phase–frequency with and without LR matching.
Later, Niederberger [9] found if the transient equations can be further analyzed, especially the analysis of the amplitude and phase-frequency transients under non-resonance and variable excitation, which will play a key role in guiding the control design. Under the guidance of this concept, the optimal control algorithms are proposed by Wang [10] and Ghenna [11]. The admittance circle method proposed by the former needs to be calculated based on the steady-state signal, while the vector method proposed by the latter needs to wait for the amplitude to be established before frequency tracking. Although these control algorithms have strong reference significance, they all failed to achieve better dynamic performance. Di [12] proposed a parallel resonant frequency tracking method based on on-line measurement of static capacitance, but the initial frequency deviation in the experiment is about 20 times the bandwidth, and the large frequency deviation means that the number of trackings increases, so the frequency tracking needs to spend a lot of time. This effect will be superimposed with the transient response, which increases the difficulty of control.

For high-power piezoelectric systems, the frequency deviation is up to a hundred times the bandwidth and time constants exceed one hundred milliseconds. The above problems become extremely fatal in application. In research and actual products, a common method is adopting frequency sweeping to find the approximate frequency of the working point when starting up or failing to start due to changes in operating conditions. However, over-reliance on frequency sweeping, on the one hand will reduce the processing efficiency and increase the difficulty of operation; on the other hand, the influence of load changes during operation can also lead to significant frequency drift, such as insufficient dynamic tracking performance, which may cause the transducer piezo stack to crack or the generator to sustain electrical damage.

The characteristic of transient response has a restrictive effect on the dynamic control for high-power piezoelectric transducers. This paper focuses on the theoretical analysis and transient matching scheme design of the high-power piezoelectric transducers. Section 2 analyzes the transient equation of piezoelectric transducer and elaborates that the piezoelectric transducer tends to be a steady state with a specific time constant regardless of voltage excitation or current excitation. In Section 3, a vector reduction method is applied to analyze a series RLC (resistance inductance capacitance) transient matching scheme, which can change the electromechanical damping and improve the transient response time of the system. However, this matching method will cause great resistance loss and energy loss. In Section 4, the drive scheme of full-bridge inverter combined with transformer and matching circuit is proposed to achieve electromechanical damping without resistance loss, and the analytical relationship of the equivalent damping is deduced. In Section 5, experiments are used to verify the accuracy of the theoretical analysis. The proposed scheme and method can improve the transient characteristics of the system and meet the stability requirements in startup, tracking and amplitude rising for high-power piezoelectric transducers.

2. Transient Equation for Piezoelectric Transducers

As a forced vibrating body, the mechanical properties of the high-power piezoelectric transducer system obey the Butterworth model and can be characterized by an RLC series circuit [13]. The dielectric property is characterized by \( C_0 \) and \( R_d \). \( C_0 \) is the static capacitance and \( R_d \) is the equivalent polarization loss resistance. The series \( R_1 \), \( L_1 \) and \( C_1 \) characterize the modal damping, mass and stiffness, respectively. The electrical model and the equivalent circuit model of a high-power piezoelectric system are shown in Figure 3.

When the excitation of the piezoelectric transducer changes, its vibration state has a transient response process. The typical transient response are zero state response and zero input response, and the difference between voltage excitation response and current excitation response needs to be considered.
2.1. Analysis of Transient Equation for Voltage Excitation Mode

Assuming that the piezoelectric transducer is excited by a sinusoidal voltage expressed as $u_T(t) = u_{in}(t) = U_{in} \sin(\omega t), t > 0$, according to

$$I_1(t) = U_{in} \frac{1}{R_1 + sL_1 + 1/sC_1}$$

(1)

The characteristic equation can be obtained:

$$s^2L_1 + sR_1 + 1/C_1 = 0$$

(2)

From the circuit topology relationship, the initial zero state $i_1(0^+) = 0, \frac{di_1}{dt}(0^+) = 0$ can be deduced, and the variation relationship of the mechanical branch current $i_1(t)$ with time can be obtained:

$$i_1(t) = I_1(\sin(\omega t + \varphi) - e^{-\frac{t}{\tau}} \sin(\omega s t + \varphi))$$

(3)

Among them, since $\frac{1}{1/C_1} \gg \frac{R_1^2}{4L_1^2}$ in the derivation process, the item $\frac{R_1^2}{4L_1^2}$ is simplified [7]:

$$I_1 = U_{in} \left| \frac{1}{R_1 + j\omega L_1 + 1/j\omega C_1} \right|$$

(4)

$$\varphi = \text{angle}(R_1 + j\omega L_1 + 1/j\omega C_1)$$

(5)

$$\tau = \frac{2L_1}{R}$$

(6)

Under this excitation, the system takes $f_s$ as the resonant frequency, and the resonant state satisfies:

$$i_1(t) = \frac{U_{in}}{R_1} (1 - e^{-t/\tau}) \sin(\omega s t)$$

(7)

2.2. Analysis of Transient Equation of Current Excitation Mode

When the piezoelectric transducer is excited by a sinusoidal current expressed as $i_T(t) = i_{in}(t) = I_{in} \sin(\omega t), t > 0$, according to the static state

$$I_1(t) = I_{in} \frac{1/sC_0}{R_1 + sL_1 + 1/sC_1 + 1/sC_0}$$

(8)

Figure 3. (a) The electrical model and (b) the equivalent circuit model of a high-power piezoelectric system.
The characteristic equation can be obtained:

\[
\frac{R_1 + sL_1 + 1/sC_1 + 1/sC_0}{1/sC_0} = 0
\]  

(9)

From the circuit topology relationship, the initial zero state \(i_1(0+) = 0, \frac{di_1}{dt}(0+) = 0\) can be deduced, and the variation relationship of the mechanical branch current \(i_1(t)\) with time can be obtained:

\[
i_1(t) = -C_0 U'_1 \left[ \omega \cos(\omega t + \theta) - e^{-t/\tau} \left( -\frac{1}{\tau} \sin(\omega_p t + \theta) + \omega_p \cos(\omega_p t + \theta) \right) \right]
\]  

(10)

where

\[
U'_1 = I_{in} \left| \frac{1}{1/R'_1 + j\omega C'_1 + 1/j\omega L'_1} \right|
\]  

(11)

\[
\theta = \text{angle}(1/R'_1 + j\omega C'_1 + 1/j\omega L'_1)
\]  

(12)

\[
\tau = \frac{2L_1}{R}
\]  

(13)

According to the relationship \(i_1(t) = -C_0 \frac{du'_1(t)}{dt}\), the transient equation of the voltage division \(u'_1(t)\) in the series equivalent circuit model can be obtained:

\[
u'_1(t) = U'_1 \left[ \sin(\omega t + \theta) - e^{-t/\tau} \sin(\omega_p t + \theta) \right]
\]  

(14)

2.3. Transient Characteristics of High-Power Piezoelectric Transducer

It can be seen from the above theory that the time constant \(\tau\) of the piezoelectric transducer system is \(2L_1/R_1\), whether it is voltage-driven or current-driven. For example, in high-power piezoelectric transducer systems with high modal quality, the \(\tau\) is usually larger. Especially under no-load conditions for piezoelectric cutting and welding systems, the \(\tau\) can reach more than 100 ms, which will have a limiting effect on the dynamic response with a target time of 20–100 ms. Moreover, when the excitation deviates from the system resonance state, that is, when the voltage excitation frequency deviates from the series resonant frequency or the current excitation frequency deviates from the parallel resonant frequency, the amplitude of the piezoelectric transducer system will vibrate, and the maximum amplitude overshoot can reach twice the steady state amplitude. The shock could cause fatal damage to a high-power piezoelectric transducer system.

On the other hand, it is worth noting that the zero input corresponding to the voltage excitation is a short circuit, while the current excitation corresponds to an open circuit. When the state of the excitation mode changes and the voltage excitation mode changes to an open circuit, a piezoelectric transducer system will generate transient impulse voltage, as mentioned in the literature [8]. Therefore, the parallel resonance frequency excitation should be adopted in this paper, no matter from the view of transient state or steady state.

3. Transient Characteristic Analysis Based on Electrical Matching

To change the transient characteristics of the system, the transient response time of the system can be improved by changing the electromechanical damping. The tandem RLC scheme is a proven approach. According to the above analysis, the influence of electrical matching on the transient characteristics of the piezoelectric transducer system operating at the parallel resonant frequency is mainly studied.
3.1. Series RLC Matching Inferred by Vector Reduction Method

First, reduce the order of the transient equation represented by the trigonometric function using the magnitude vector, and express the transient Equation (15) as a vector.

\[ U'_1(t) = I_{in}R'_1'(1 - e^{-t/\tau}) \]  

(16)

At any time, the current excitation \( I_{in}(t) \) can be divided into two parts. One part keeps \( U'_1(t) \) unchanged, and the other part makes it change. According to this relationship, the vector reduced order differential equation under current excitation can be derived.

\[ I_{in}(t) = \frac{1}{R'_1} \left( U'_1(t) + \tau \frac{dU'_1(t)}{dt} \right) \]  

(17)

Next, according to the target response function, the excitation signal function is reversed. Assuming that the system is still an exponential response function, but the response time is less than \( \tau \) (response time), it can be expressed as:

\[ U'_1(t) = U'_1(1 - e^{-t/\tau'}) \]  

(18)

where \( \tau' \) is the new response time, then the response stimulus can be obtained as:

\[ I_{in}(t) = \frac{U'_1}{R'_1} \left[ 1 + \left( \frac{\tau}{\tau'} - 1 \right)e^{-t/\tau'} \right] \]  

(19)

Furthermore, both the amplitude response \( U'_1(t) \) and the current excitation \( I_{in}(t) \) have a common time-varying parameter \( e^{-t/\tau'} \), so consider converting the system to a constant voltage excitation mode through matching resistors according to Ohm’s law. The ideal form of this match is:

\[ U_{in} = U'_1(t) + I_{in}(t)R_m \]  

(20)

Then the ideal response equation is:

\[ U'_1 = \frac{R'_1}{R'_1 + R_m} U_{in} e^{-t/\tau'} \]  

(21)

where

\[ \tau' = \tau \frac{R_m}{R'_1 + R_m} \]  

(22)

Considering that the electrostatic capacitance \( C_0 \) will be included in the transducer circuit model, it is also necessary to cancel the capacitive reactance of the electrostatic capacitance by matching. Here, the series LC is considered for matching, so it is necessary to satisfy the relationship:

\[ \omega_pL_m = \frac{1}{\omega_pC_m} + \frac{1}{\omega_pC_0} \]  

(23)

After matching, the real system has one more series second-order RLC branch composed of matching inductance \( L_m \), electrostatic capacitance \( C_0 \) and matching resistance \( R_m \), compared with the ideal Formula (23). Its transient differential equation is expressed as:

\[ U_{in} - U'_1(t) = R_m \left( I_{in}(t) + \tau_m \frac{dI_{in}(t)}{dt} \right) \]  

(24)

where the time constant is:

\[ \tau_m = \frac{2L_m}{R_m} \]  

(25)

It is the ideal situation when \( \tau_m \ll \tau \). At this time, the transient response process of this branch can be ignored and can be equivalent to a separate \( R_m \). The whole system is
approximate to the ideal Equation (19). With the \( R_m \) decreases, the system will be affected by the transient characteristics of the matching circuit LC since \( L_m \) can only take \( 1/\omega^2 \) \( C_0 \).

Combining the vector reduced-order transient differential Equations (17) and (24), it can be seen that the amplitude transient response envelope equation of the piezoelectric transducer system after matching is a second-order response, in which the critical damping is:

\[
R_m = \frac{2}{B_0} \sqrt{\frac{L_m}{L_1}}
\]

(26)

It is worth noting that the needed analyzing system is a fourth-order system after matching, the solution of the analytical solution for a fourth-order differential equation is extremely complicated, and the vector reduction method is the key to solving the analytical solution. In the following, the numerical method will be adopted to solve the complete fourth-order equation and verify the analysis result of the vector reduction method.

3.2. Ferrari Method Verification

For the original fourth-order electrical equation, it can be expressed as:

\[
I_1(t) = U_{in} \frac{Z_0}{Z_{in}(Z_0 + Z_1)}
\]

(27)

where

\[
Z_0 = \frac{1}{sC_0}; Z_1 = R_1 + sL_1 + \frac{1}{sC_1}
\]

(28)

\[
Z_{in} = R_m + sL_m + \frac{Z_0Z_1}{Z_0 + Z_1}
\]

(29)

The characteristic equation can be obtained as:

\[
as^4 + bs^3 + cs^2 + ds + e = 0
\]

(30)

where

\[
a = L_mL_1C_0
\]

\[
b = L_mR_1C_0 + R_mL_1C_0
\]

\[
c = L_mC_0\left(\frac{1}{C_0} + \frac{1}{C_1}\right) + L_1C_0\frac{1}{C_m} + C_0R_mR_1 + L_1
\]

\[
d = R_1C_0\frac{1}{C_m} + C_0R_m\left(\frac{1}{C_0} + \frac{1}{C_1}\right) + R_1
\]

\[
e = \frac{C_0}{C_m}\left(\frac{1}{C_0} + \frac{1}{C_1}\right) + \frac{1}{C_1}
\]

(31)

For complex fourth-order equations, this paper uses the Ferrari method to solve, first calculate the three parameters \( P, Q, D \):

\[
\begin{align*}
P &= c^2 + 12ae - 3bd \\
Q &= 27a^2 + 2c^2 + 27b^2e - 72ace - 9bd \\
D &= \sqrt{Q^2 - P^3}
\end{align*}
\]

(32)

Next, calculate the three parameters \( u, v \) and \( w \):

\[
u = \max(\sqrt{Q - D}, \sqrt{Q + D})
\]

(33)

\[
v = \begin{cases}
\frac{P}{u}, u \neq 0 \\
0, u = 0
\end{cases}
\]

(34)

\[
w = -\frac{1}{2} + \frac{\sqrt{3}}{2}i
\]

(35)
Let the value of \( k \) be 1, 2, 3. Among the obtained \((m,S,T)\) values, take the group with the largest \(|m|\). If all three \(|m|\) are zero, choose \((m,S,T)\) as:

\[
\begin{align*}
    m &= 0 \\
    S &= b^2 - \frac{8}{3} \mu c \\
    T &= 0
\end{align*}
\] (36)

The equation solution can be finally expressed as:

\[
x_n = \frac{-b + (-1)^\left\lfloor \frac{n}{2} \right\rfloor m + \left(-1\right)^{n+1} \sqrt{S + (-1)^\left\lfloor \frac{n}{2} \right\rfloor T}}{4d}, \quad n = 1, 2, 3, 4
\] (37)

According to the solution of the characteristic equation, the general solution of the system can be expressed as:

\[
i_1(t) = A_1 e^{x_1 t} + A_2 e^{x_2 t} + A_3 e^{x_3 t} + A_4 e^{x_4 t} + c
\] (38)

Assuming that \( u_{in}(t) = U_{in} \sin(2\pi f_p t) \), the initial zero state can be deduced from the circuit topology \( i_1(t)(0+) = 0, \frac{d^2i_1(t)}{dt^2}(0+) = 0, \frac{d^3i_1(t)}{dt^3}(0+) = 0, \frac{d^4i_1(t)}{dt^4}(0+) = 0 \). Special solution equations can be obtained:

\[
\begin{align*}
    A_1 + A_2 + A_3 + A_4 &= I_1 \\
    x_1 A_1 + x_2 A_2 + x_3 A_3 + x_4 A_4 &= 0 \\
    x_1^2 A_1 + x_2^2 A_2 + x_3^2 A_3 + x_4^2 A_4 &= (2\pi f_p)^2 I_1 \\
    x_1^3 A_1 + x_2^3 A_2 + x_3^3 A_3 + x_4^3 A_4 &= 0
\end{align*}
\] (39)

where

\[
I_1 = 2\pi f_p C_0 \frac{U_{in} R'_1}{R'_1 + R_m}
\] (40)

Then we obtain:

\[
i_1(t) = A_1 e^{x_1 t} + A_2 e^{x_2 t} + A_3 e^{x_3 t} + A_4 e^{x_4 t} + c
\] (41)

Take a typical piezoelectric cutting system parameter as the piezoelectric transducer system. Determine the numerical value of the matching inductance \( L_m \) to be 7.47 mH, the matching capacitor \( C_m \) is 14.81 nF; the excitation voltage \( U_{in} \) is 1 V; the excitation frequency is 20201.72 Hz. Numerical solution of the transient equation for calculating the mechanical branch current \( i_1(t) \) of a piezoelectric transducer system by the Ferrari method under different values of \( R_m \).

When \( R_m = 10 \, \Omega \)

\[
\begin{align*}
    A_1 &= 6.0052 \times 10^{-4} - 5.8828 \times 10^{-5} \\
    A_2 &= 6.0052 \times 10^{-4} + 5.8828 \times 10^{-5} \\
    A_3 &= 6.0022 \times 10^{-4} + 5.8828 \times 10^{-5} \\
    A_4 &= 6.0022 \times 10^{-4} - 5.8828 \times 10^{-5}
\end{align*}
\] (42)

\[
\begin{align*}
    x_1 &= -339.58 + 1.3035 \times 10^5 \\
    x_2 &= -339.58 - 1.3035 \times 10^5 \\
    x_3 &= -339.74 + 1.2341 \times 10^5 \\
    x_4 &= -339.74 - 1.2341 \times 10^5
\end{align*}
\] (43)

\[
c = -I_1 \cos \omega_p t = -0.0024 \cos 126931.15 t
\] (44)
When $R_m = 2700 \, \Omega$

\[
\begin{align*}
A_1 &= 0.0010 \\
A_2 &= 0.0010 \\
A_3 &= 1.8051 \times 10^{-7} - 1.4571 \times 10^{-20} \\
A_4 &= -1.0712 \times 10^{-6} - 1.1153 \times 10^{-19}
\end{align*}
\]  
(45)

\[
\begin{align*}
x_1 &= -77.080 - 1.2693j \times 10^5 \\
x_2 &= -77.080 + 1.2693j \times 10^5 \\
x_3 &= -3.0943 \times 10^5 \\
x_4 &= -5.1912 \times 10^4
\end{align*}
\]  
(46)

\[
c = -I_1 \cos \omega_p t = -0.0021 \cos 126931.15 t
\]  
(47)

It can be seen that the characteristic roots of the equations under-damped and over-damped state in the RLC match are significantly different. The former is two pairs of conjugate complex roots, and the latter is a pair of conjugate complex roots plus a pair of unequal real roots.

Next, the transient analytical value of the mechanical branch current $i_1(t)$ of the piezoelectric transducer system calculated by the Ferrari method compared with the vector reduction method. Take $R_m = 10 \, \Omega$ (only consider dielectric loss and line loss) and $R_m = 104.7 \, \Omega$ (critical damping) for example showing in Figure 4. The results show that the vector reduction method can analyze the transient vibration of the piezoelectric transducer system accurately, and confirm that the piezoelectric vibrator with under-damped matching will generate amplitude fluctuation similar to the beat phenomenon even if it is excited at the parallel resonant frequency transiently.

![Figure 4. Comparison of the Ferrari method and the reduction analysis for underdamped (a) and critically damped (b).](image)

For further verification, take the two typical overdamping cases of $R_m = 2.7 \, \text{k}\Omega$ and $R_m = R'_1 (18.27 \, \text{k}\Omega)$, the comparison between the simplified analytical value and the full analytical value calculated by the Ferrari method is shown in Figure 5.
For further verification, take the two typical overdamping cases of $R_m=2.7\,k\Omega$ and $R_m=R'_1 (18.27\,k\Omega)$, the comparison between the simplified analytical value and the full analytical value calculated by the Ferrari method is shown in Figure 5.

Figure 5. The waveform comparison of the Ferrari method, the reduction analysis and the simplified analysis in two typical over-damping situations, including the details of the start-up and near steady state.

The results show that the simplified analytical value coincide with the complete analytical value of the Ferrari method basically, and the near-steady-state value almost completely coincides, but there is a slight deviation in several vibration cycles at the moment of startup, and the deviation increases with a decrease of $R_m$. This result proves the accuracy of the transient simplified analytical formula proposed in this paper in the case of typical overdamped matching.

However, for high-power piezoelectric transducer systems, the matching resistance required to achieve critical damping will consume ten times the energy of the sum of electrical transmission and dielectric losses. Moreover, from the perspective of transient inrush current, the system hopes to control the inrush current within ten times the steady-state current value, so the matching resistance $R_m$ needs to reach at least 10% of $R'_1$, which means it needs to be more than 1.83 kΩ. The resulting high resistance loss is unbearable in high-power applications. Therefore, this research needs a solution to replace the physical resistor, which can play the role of electromechanical damping without causing resistive loss.
4. Transient Characteristic with Full-Bridge Inverter Scheme

As known, when the full-bridge inverter works in the full-bridge mode, it is equivalent to a constant voltage source with adjustable output resistance from the perspective of the power supply. Therefore, the full-bridge inverter scheme is proposed to explore the transient characteristics and optimization mechanism. The proposed scheme contains a rectifier bridge, a full-bridge inverter, an LC matching circuit and a transformer, as shown in Figure 6.

![Figure 6. The proposed electrical scheme.](image_url)

The commercial power (220 V, 50 Hz) is rectified into the DC power $U_{DC}$, and then inverted to the AC power in ultrasonic frequency. A series LC matching circuit is used for DC isolating and harmonic filtering. More importantly, the specific configuration of $L_m$ and $C_m$ is also related to impedance matching and vibration excitation, which are analyzed in the next sector. Since a transformer ratio arm bridge (TRAB) is easy to intergrade with little impact on the electrical circuit [14], it is adopted to detect the partial voltage $U_1'$ online. A tap is drawn from the secondary side of the transformer with the coil turns satisfying $n_2 \gg n_3$, and a detection capacitor $C_d$ is connected to the circuit. According to the electrical theory, the bridge parameters are configured as $n_2 = 155$, $n_3 = 4$ and $C_d = 752 \, nF$, and the effective value of $U_h$ is about 30 V at 100% amplitude, which makes this a key signal with strong anti-electrical interference ability.

4.1. Transient Analysis of Full-Bridge Mode

From the transient equation in the case of the low duty cycle of parallel resonant frequency, it can be shown that:

$$U'_1(t) + \tau \frac{dU'_1(t)}{dt} = \frac{2d^2}{k^2L_m f_p} R'_1 (kU_{DC} - U'_1(t)) \frac{2kU_{DC}}{kU_{DC} + U'_1(t)}$$  \hspace{1cm} (48)

If it is regarded as the RL matching mode, and according to Equation (20), it can be calculated as:

$$R_m = \frac{k^2L_m f_p kU_{DC} + U'_1(t)}{2d^2}$$  \hspace{1cm} (49)

This result shows that the equivalent $R_m$ of this scheme is a non-constant value, which is the product of two parts: the fundamental term $\frac{k^2L_m f_p}{2d^2}$ is only related to the duty cycle after the matching circuit is determined by the ratio $d$; the coefficient term $\frac{kU_{DC} + U'_1(t)}{2dU_{DC}}$ is related to the value of $U'_1(t)$, $U'_1(t)$ is 0 at the initial moment of startup, and the coefficient term is 0.5. The coefficient term gradually increases as $U'_1(t)$ increases, and the coefficient term is close to 1 when $U'_1(t)$ approaches the theoretical maximum value $kU_{DC}$. This nonlinear differential equation is difficult to analyze further, but its form is relatively simple. Equations (48) and (49) can be iterated by a computer to obtain a numerical solution with high precision.
According to Ohm’s law, the relationship for estimating the transient response time in steady state can be simplified as:

\[ kU_{DC} = U'_1 + I_T R_m = I_T (R'_1 + R_m) \]  

(50)

For ideal RL matching, the time constant is:

\[ \tau' = \frac{R_m}{R'_1 + R_m} \tau \]  

(51)

Combining the above two formulas we can get the relationship:

\[ \tau' = \frac{kU_{DC} - U'_1}{kU_{DC}} \tau \]  

(52)

Therefore, for the steady-state target amplitude, its transient time is mainly determined by the transformation ratio \( k \) under proper matching. Considering factors such as mains voltage fluctuation, critical matching damping and transient impulse current, \( U'_1 = 0.75 kU_{DC} \), \( k = 159/22 \) can be obtained under 100% amplitude condition. It is worth noting that in a startup transient, \( R_m \) will gradually increase and reaches its maximum value in steady state as \( U'_1(t) \) increases with time. Therefore, if \( U'_1(t) \) can reach \( 1 - e^{-1} \) times, the steady-state value is defined as the transient response time, it must be smaller than the time constant calculated by Equation (22). This paper proposes that the transient response time is estimated by taking the coefficient term \( \frac{kU_{DC} - U'_1}{kU_{DC}} \tau \) to be 0.5, that is \( R_m = \frac{k^2 L_m f_p}{4d^2} \). The amplitude transient response simplified formula is obtained as:

\[ U'_1(t) = \frac{-(1 + 2D) + \sqrt{1 + 12D + 4D^2}}{2} kU_{DC} (1 - e^{-\frac{t}{\tau'}}) \]  

(53)

\( \tau' \) is calculated as:

\[ \tau' = \frac{k^2 L_m f_p}{4d^2 R'_1 + k^2 L_m f_p} \tau \]  

(54)

Therefore, it can be estimated that the transient response time of this matching parameter under the 100% amplitude condition is \( \tau' = 0.143 \tau \).

4.2. Simulation Analysis

MATLAB/Simulink (MathWorks, Natick, MA, USA, 2017b) is used to simulate the improved effect by proposed full-bridge inverter scheme. The simulation conditions are as follows: The piezoelectric transducer system uses the piezoelectric cutting system, its parameters are shown in Table 1, the duty cycle of the drive gate signal takes the design typical value of 11.5%, and the \( U_{DC} \) takes 311 V. The variation curve of the mechanical branch current \( i_1(t) \) is measured in the simulation. The results are compared with the iterative numerical calculation results (Equations (17) and (24)) and the simplified analytical results (Equations (17) and (24)), as shown in Figure 7. The results show that the iterative numerical calculation results and the simplified analytical results are all close to the actual curve.

**Table 1. Parameters in the proposed model of the high-power piezoelectric systems.**

|                | \( I_1 \) (Arms) | \( C_0 \) (nF) | \( C_1 \) (nF) | \( L_1 \) (mH) | \( R_m \) (Ω) | \( R_A \) (Ω) |
|----------------|------------------|----------------|----------------|----------------|---------------|---------------|
| 10-inch Piezoelectric Cutter | 3.00             | 18.93          | 0.1308         | 477.8          | 9.48          | 6.34          |
Therefore, it can be estimated that the transient response time of this matching parameter under the 100% amplitude condition is 143 ms, which is the smallest at the initial moment, and it decreases with the increase of the amplitude. Compared with the inverse exponential curve, the actual amplitude response curve has a higher initial slope, but decays faster, as shown in Figure 7. In this system, the moment is around a double time constant when the two basically intersect. This paper believes that the transient response time is not a particularly sensitive parameter for such a system including closed-loop control, and a certain estimation error should be allowed. Therefore, this paper proposes to use the simplified analytical formula for estimation.

On the other hand, the iterative value rises slightly slower than the simulated current, and the steady-state value is slightly lower. This may be due to the approximate time-frequency transformation when extracting the excitation current and the error caused by ignoring the capacitor voltage.

4.3. Discussion on Transient Effect of Full-Bridge Inverter Scheme

The analysis in this section proves that when the full-bridge inverter works in full-bridge mode, the combined action of the series LC matching circuit and the matching inverter can significantly improve the transient response capability of the piezoelectric transducer system. From the view of matching design, the higher the ratio of piezoelectric partial voltage to the equivalent excitation voltage, the shorter the transient response time; from the view of control, the greater the duty cycle of the inverter drive signal, the shorter the transient response time. This improvement is attributed to the fact that the inverter operating in full-bridge mode acts as both a constant voltage source and a matching resistor. Especially, this scheme provides sufficient electromechanical damping without causing additional resistive losses.

5. Experimental Verification

A high-power piezoelectric cutting transducer with a 10-inch cutter head (parameters are described in Table 1, the time constant is 100.8 ms) is used to perform experiments. System frame diagram is shown in Figure 8a. A 20 kHz 2 kW ultrasonic generator is developed independently, as shown in Figure 8b. This device includes rectifier bridge, matching circuit, inverter and control system. The control system adopts the Cortex M4
The microcontroller (micro control unit, MCU) STM32F407 (STMicroelectronics, Naples, Italy and Grenoble, France) with fast serial calculation speed and abundant peripheral resources. The device is able to support a high-power piezoelectric transducer system of 19–21 kHz, with frequency tracking capability. It supports constant mechanical amplitude control of the piezoelectric transducer system, and the amplitude adjustment range is from 20% to 100%. In terms of dynamic response performance, the device supports direct start within a frequency deviation range of ±300 Hz. The waveform of the signal is observed by a TDS 2024B oscilloscope (sampling period is 0.04 μs, and the resolution of each channel is 0.04 times the division value, Tektronix, Beaverton, OR, USA). Install the piezoelectric transducer system in a pneumatic thruster (produced by NICLE ultrasonic company, Wuxi, China). The experimental setup is shown in Figure 8c, which verifies the transient properties of the proposed scheme.

(a)

(b)

(c)

**Figure 8.** Experimental setup. (a) System frame diagram. (b) Controller. (c) Experiment frame.

### 5.1. Signal Transient Reliability Verification

Before the experiment, first set a constant driving signal with a duty cycle of 9.09% and the driving frequency of 20,201.00 Hz, then record the amplitude $U_1'$ and phase $\theta$ of each vibration period, as shown in Figure 9. It can be seen that the two signals are rapidly established after startup, and the random errors of amplitude and phase are less than 1% and 0.1° respectively. All the results show that the random error of the signal obtained by the signal scheme is small, which can support single-cycle sampling and achieve the expected experiment requirements.
time is slightly smaller than the theoretical value. The driving frequency is set to the parallel resonance frequency, the results (shown in green lines) in the figure. The steady-state value correctly and has good control system −

Figure 10. Comparison of piezoelectric the partial voltage signal and the measured vibration displacement.

5.2. Transient Characteristic Improvement Verification

It is necessary to verify the real-time performance of the detection signal and the dynamic amplitude control capability of the system. In this experiment, the control system sets the initial target amplitude as 100%, 350 ms later decreases to 60%, and 700 ms later increases to 80%. Record the change of the detected amplitude during this period, and compare it with the amplitude of the welding head measured by the laser displacement sensor. The experimental results are shown in Figure 10. The results show that the amplitude recorded by the system is consistent with the measured displacement of the laser (red line). It also indicates that the system detects the amplitude correctly and has good real-time performance.

Next, the transient properties of the proposed scheme are verified by the experiment. In the experiment, the driving frequency is set to the parallel resonance frequency, the amplitude response process under different duty cycles \( d \) is recorded, the transient response time is defined \( U'_b \) reaching \( 1 - e^{-1} \) times the steady-state value, and the response time is measured. The change of \( d \) is shown by the blue line in Figure 11. Those also include the numerical results of the nonlinear differential equation iterations (shown in red lines) and simplified analytical results (shown in green lines) in the figure. The results show that the
simplified experimental results are close to the theoretical values, which confirms that the scheme implementation is basically consistent with the theory. Among them, the measured response time is slightly smaller than the theoretical value. The reason may be due to the influence of the parasitic capacitance in circuit driver module. There is a slight leakage current in the high-resistance region, which slightly reduces the equivalent resistance in the scheme. This experiment proves that the scheme proposed in this paper can effectively shorten the vibration response time of the piezoelectric transducer and supports the use of simplified analytical results for transient matching estimation in engineering.

Figure 11. The transient response time under different duty cycle.

In the end, the dynamic performance of the proposed system will be evaluated under different load conditions, and the method proposed in this paper is compared with the conventional phase method.

In no-load conditions, the target amplitude is set to 100%. The driving frequency, phase $\theta$, and detection amplitude are recorded as shown in Figure 12. The figure includes the time consumed by each control stage, the startup time is the initialization stage in the control, which is set to a fixed 2 ms. The tracking time is the initial frequency tracking stage in the control, from startup to frequency tracking completion, it takes 5.5 ms in total. After the frequency tracking, the amplitude starts to rise and gradually reaches a plateau, the measured rise time is about 8 ms. Compared with the phase method, the proposed method has a shorter response time and better amplitude control capability.

Immerse the cutter head in the water to simulate a highly variable load condition. When the cutter head vibrates, the resistance of the water increases the resistance to about 700 $\Omega$, and the viscosity of the water is mechanically and electromechanically coupled with the piezoelectric transducer system through contact, which reduces the equivalent stiffness and the parallel resonance frequency. With the increase of the amplitude, the resistance gradually decreased to 200–250 $\Omega$ at 100% amplitude and fluctuated violently in this range with the generation and collapse of cavitation bubbles. The experimental results are shown in Figure 13. The results show that the response time of the piezoelectric transducer system has been shortened under high load. For example, frequency tracking time is about 4.8ms when the resistance is 700 $\Omega$. It takes longer to establish equilibrium than in no-load conditions, due to the fact that the high load reduces the phase frequency and greatly reduces its tracking speed in the non-saturated condition. The steady-state amplitude has periodic oscillation within $\pm$5%, and the oscillation frequency is about
100 Hz. The reason is inferred to be the influence of the rectifier bridge. The amplitude settling time is below 15.0 ms. All these results demonstrate the proposed method has advantages in transient response compared with the phase method.

![Graph 1](image1.png)

**Figure 12.** Comparison with the phase method under no-load condition.

![Graph 2](image2.png)

**Figure 13.** Comparison with the phase method under high and varying load condition.

6. Conclusions

This paper makes an in-depth analysis of the transient characteristics of high-power piezoelectric transducer systems, expounds on the influence mechanism of electrical matching on the transient characteristics, and proposes a transient optimized electrical drive scheme. First, it is found that the piezoelectric transducer system, whether it is voltage excitation or current excitation, tends to a steady state with a specific time constant. The transient response time is below 15.0 ms. All these results demonstrate the proposed method has advantages in transient response compared with the phase method.
excitation or current excitation, tends to a steady state with a specific time constant. A flapping-like phenomenon occurs when the voltage excitation frequency deviates from the series resonance frequency or the current excitation frequency deviates from the parallel resonance frequency. Then, a vector reduction method is proposed to derive a series RLC transient matching method that can change the electromechanical damping to improve the transient response time, and the accuracy of the analysis method is verified by the Ferrari method. Furthermore, in order to reduce the resistive loss, a switching power supply driven in full-bridge mode is proposed to achieve electrical damping matching without generating resistive losses. The analytical relationship of the equivalent damping is deduced, and it is pointed out that the matching transformer ratio is the key factor affecting the transient characteristics of the proposed scheme. Finally, the reliability of the optimization of transient characteristics is verified by experiments. The experiments also evaluate the dynamic performance of the system. For a piezoelectric cutting system with the time constant of 100.8 ms, the tracking time of the proposed system (driving frequency $\pm 300$ Hz around the parallel resonance frequency) under no-load and high-loads are less than 5.5 ms, the amplitude settling time are less than 15.0 ms, which has great advantages for high dynamic applications. All these results verify that the proposed scheme and analysis method can effectively shorten the vibration response time of the piezoelectric transducer system and provide a simplified analytical process for transient matching in engineering applications.

The innovation of this study is that the series RLC matching scheme is derived by applying the vector reduced-order differentiation method and verified with the Ferrari method. The analysis proves the transient response boosting mechanism of the electric drive scheme in full-bridge mode. While improving the transient response of resonance excitation, the real-time performance and startup stability of the system are improved, and the resistance loss and power loss are reduced.

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