Absorption of dilaton partial waves by D3-branes

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Abstract

We calculate the leading term in the low-energy absorption cross section for an arbitrary partial wave of the dilaton field by a stack of many coincident D3-branes. We find that it precisely reproduces the semiclassical absorption cross section of a 3-brane geometry, including all numerical factors. The crucial ingredient in making the correspondence is the identification of the precise operators on the D3-brane world-volume which couple to the dilaton field and all its derivatives. The needed operators are related through T-duality and the IIA/M-theory correspondence to the recently determined M(atrix) theory expressions for multipole moments of the 11D supercurrent. These operators have a characteristic symmetrized trace structure which plays a key combinatorial role in the analysis for the higher partial waves. The results presented here give new evidence for an infinite family of non-renormalization theorems which are believed to exist for two-point functions in $\mathcal{N} = 4$ gauge theory in four dimensions.
1 Introduction

Black $p$-brane solutions of type II supergravity carrying Ramond-Ramond (RR) charges have been known since the early 90’s [1, 2]. The string frame metric and dilaton backgrounds in such solutions may be expressed in the following simple form:

$$ds^2 = H^{-\frac{7}{2}}(r) \left[ -dt^2 + \sum_{i=1}^{p}(dx^i)^2 \right] + H^{\frac{1}{2}}(r) \left[ dr^2 + r^2 d\Omega^2_{8-p} \right],$$

(1)

$$e^\Phi = H^{(3-p)/4}(r),$$

where

$$H(r) = 1 + \frac{R^{7-p}}{r^{7-p}}.$$

The importance of these solutions was not fully appreciated until Polchinski realized that the Dirichlet $p$-brane is the elementary object in string theory that couples to the $(p+1)$-form RR potential [3]. This made it clear that the $p$-brane solutions of [1] describe the classical fields created by a large number of coincident $Dp$-branes. Since the low-energy world-volume dynamics of $N$ parallel $Dp$-branes is governed by maximally supersymmetric $U(N)$ gauge theory [4], this suggests a relation between such a gauge theory in $p+1$ dimensions and type II string theory in the background of the classical $p$-brane solution.

Among early hints that this relationship between supersymmetric gauge theory and string theory is exact was the calculation of the dilaton absorption cross section by threebranes [5]. The threebrane solution is of particular interest because it is the only non-singular solution of the form [1]. Furthermore, the low-energy dynamics of coincident $D3$-branes is described by $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, which is an attractive theory because of its exact conformal invariance. A related fact is that the dilaton background is constant, so that the dilaton fluctuation satisfies the minimally coupled scalar equation

$$\partial_\mu \left( \sqrt{-g} g^{\mu\nu} \partial_\nu \phi \right) = 0.$$  

(2)

In [3] this equation was solved for incident $s$-waves of low-energy $\omega$. The leading term in the absorption cross section was calculated to be

$$\sigma_{SUGRA} = \frac{\pi^4}{8} \omega^3 R^8.$$  

(3)

This result was compared to a corresponding calculation in the SYM theory, where the dilaton couples to the operator $\frac{T_3}{4} \text{Tr} (F^2 + \cdots)$ ($T_3$ is the $D3$-brane tension). At weak coupling the leading order absorption process is for the dilaton to turn into a pair of gluons

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1 This calculation was in turn motivated by similar calculations in the D1-D5 system [3, 4, 5, 6]. There such studies are more difficult, however, due to the complexity of the world volume dynamics of intersecting $D$-branes.
on the world-volume. The rate for this process was calculated in the 3-brane gauge theory and was found to be [3]

$$\sigma = \frac{\kappa^2 \omega^3 N^2}{32\pi},$$

(4)

Remarkably, this is equal to (3) after we take into account the relation

$$R^4 = \frac{\kappa}{2\pi^{3/2}} N,$$

(5)

which can be found by equating the tensions of the black 3-brane and $N$ D3-branes [9]. This equality of the low-energy cross sections raises the hope of an exact relation between SYM theory and gravity. There seems to be a puzzle, however, because the gravitational calculation becomes reliable in the weak curvature limit where $g_{YM}^2 N \to \infty$ while the SYM calculation was carried out to leading order in $g_{YM}^2 N$. In [10] this puzzle was resolved by arguing that all higher order corrections in the coupling vanish due to supersymmetric non-renormalization theorems. (This theorem was made explicit for the absorption cross section of gravitons calculated in [11], which is related to the 2-point function of the stress-energy tensor of the gauge theory.) Thus, the agreement of s-wave cross sections found in [5, 11] is actually necessary if SYM theory and gravity are exactly related. This agreement is one of the pieces of evidence in favor of the exact AdS/CFT correspondence between the threebrane throat and the $\mathcal{N} = 4$ SYM theory formulated in [12, 13, 14].

An immediate question is whether this agreement persists for the absorption of higher partial waves. In [5] it was suggested that the operator responsible for absorption of the $l$th partial wave of the dilaton should be of the form

$$\frac{T_3}{4l!} \text{Tr} (F_{ab} F^{ab} X(i_1 \cdots i_l))_{\text{Traceless}}.$$  

(6)

It is not hard to show that the operator (6) leads to a SYM cross section which scales in the same way with respect to $N$ and $\omega$ as the cross section computed from the semiclassical gravity theory [3]. In [11] an attempt was made to compare the constant factors in these cross sections, but the results seemed discouraging: the SYM answer seemed to grow with $l$ much faster than the gravity answer.

In this paper we resolve this problem and show that the gravity and the SYM cross sections are in exact agreement for all $l$. This is the first example of such a match occurring for all partial waves. Higher partial wave absorption processes were considered for the D1 + D5 system in [13, 16, 17]; because the world-volume theory of the branes is not as well understood in that case, however, it is not yet possible to make a precise numerical comparison between the supergravity and D-brane predictions for the absorption cross section.

In order to do an exact absorption calculation in the D3-brane gauge theory we need to know the precise operators in the world-volume super Yang-Mills theory which couple linearly to the bulk dilaton field and its derivatives. These operators will contain terms of the form (3), but this is not a complete description of the operators we need. There is an ordering ambiguity in (3) when $N > 1$. There are also additional fermion terms which must
be considered for $l > 0$. Analogous operators to those we need were recently computed in \cite{18}, where recent results on the M(atrix) theory form of the supercurrent in DLCQ M-theory \cite{19,20,21} were used to find the operators in the world volume theory of a system of D0-branes coupling to weak IIA background fields. By T-dualizing the results of \cite{18} in three directions, we can determine the desired D3-brane operators and use them to precisely compute the absorption cross sections. Let us consider the $l = 1$ partial wave as an example. In \cite{11} it was claimed that the operator $\frac{T_3}{4} \text{Tr} F_{ab} F^{ab} X^i$ gives a SYM absorption cross section which agrees with the classical result. We have found, however, that when ordering effects are taken into consideration this term accounts for only $1/2$ of the classical absorption cross section for the leading terms in a large $N$ expansion. Luckily, the results in \cite{18} indicate that there is another operator contributing at the same order:

$$T_3 \frac{1}{16} \text{Tr} (F_{jk} \Theta \Gamma^{[jkl]} \Theta - F_{ab} \Theta \Gamma^{[ab]} \Theta),$$

where $F, \Theta$ and the matrices $\Gamma$ are written in 10D notation with $a, b \in \{0,1,2,3\}$ and $i, j, k \in \{4,\ldots,9\}$. This operator accounts for the other half of the classical cross section and restores the agreement for the $l = 1$ partial wave.

For $l > 1$, a two-fermion operator of the form of (4) must again be included in the cross section calculation. In addition, there are quartic terms in the fermions which appear at $l = 2$. Although the four-fermion terms in the relevant D0-brane and Matrix theory operators have not been calculated, it is possible to fix these terms by using supersymmetry and our knowledge of the bosonic terms. The operators we need are essentially the same ones that correspond to Kaluza-Klein modes of the dilaton in the correspondence between $AdS_5 \times S^5$ and $\mathcal{N} = 4$ SYM theory, and can be found by acting with 4 supercharges on the superconformal chiral primary fields

$$O^{cp}_{(l+2)} \sim \text{Tr} (X^{(i_1} \ldots X^{i_{l+2}}))_{\text{Traceless}}.$$

Since we only act with 4 supersymmetry transformations, we encounter at most 4 fermion fields. From our knowledge of the bosonic and two-fermion components of the operators, it is possible to find the proper combination of supersymmetry operators which give the unique four-fermion extension of the lower order components compatible with supersymmetry.

A key feature of the operators found in the study of Matrix theory supercurrents is the symmetrized trace structure which dictates that all traces should be averaged over orderings of the $N \times N$ matrices $F^{\mu\nu}, X^i, \Theta$ and $D\Theta$. It was suggested some time ago by Tseytlin \cite{22} that the symmetrized trace is the correct way to extend the abelian Born-Infeld action to a nonabelian theory. While for the full nonabelian Born-Infeld action this remains a conjecture, we emphasize that for the operators we are interested in here this structure has been deduced from an explicit calculation in the Matrix theory context. As further evidence for this structure, in \cite{18} it was shown that the symmetrized trace gives rise to nontrivial combinatorial factors which allow the D0-brane action in weakly curved backgrounds to satisfy the geodesic length condition suggested by Douglas in \cite{23}. In the present paper
we find that the symmetrized trace structure and the correct counting of graphs according to 't Hooft’s large $N$ limit are crucial in achieving exact agreement between the D-brane absorption calculation and the semiclassical results for $l > 0$.

In Section 2 we review the semiclassical calculation of the higher partial wave absorption cross sections originally found in \cite{11}. The complete construction of the world-volume operators in the D3-brane theory is presented in Section 3. In Section 4 we calculate the 2-point functions of these operators to leading order in $g^2_{YM}N$ and convert these results into absorption cross sections, finding exact agreement with the semiclassical calculations for all $l$. Since the semiclassical calculations are valid for $g^2_{YM}N \rightarrow \infty$, this is evidence in favor of non-renormalization theorems protecting the 2-point functions of all operators constructed in section 3. In Section 5 we present a discussion of our results and conclude.

2 Semiclassical absorption calculation

In this section we review the semiclassical calculation of the absorption cross section for an arbitrary partial wave of the dilaton in the extremal 3-brane background. The results of this calculation were originally given in \cite{11}.

The semiclassical approach to computing the absorption cross section for a field propagating in a black hole background geometry was pioneered in the thesis of Unruh \cite{24}. In recent times, this method has been used to study the absorption cross section for fields in the 5D black hole geometry produced by a D1 + D5 system \cite{7, 8} and in the 7D black hole geometry produced by multiple D3-branes \cite{5, 11}. The first step in a calculation of this type is to determine the field equation for a fixed partial wave of the field of interest. This wave equation can usually be solved approximately in certain regimes of the radial parameter $r$. These approximate solutions are then matched between regions and a solution is chosen which satisfies the boundary condition that there is no outgoing flux at the horizon. The absorption coefficient is then given by the ratio of the inward flux at the horizon over the inward flux at $r = \infty$. Application of the standard optical theorem from quantum mechanics gives the absorption cross section in terms of the absorption coefficient. It is often necessary to obtain approximate solutions in three distinct regions of the parameter $r$. A particularly nice feature of the minimally coupled scalars in the 3-brane background is that only two regions are necessary, which simplifies some aspects of the story.

We now outline the application of this method to the dilaton in the 3-brane background, following \cite{3, 11}. The wave equation for the $l$th partial wave of a dilaton mode with frequency $\omega$ in the background \cite{11} with $p = 3$ is

$$\left[ \frac{1}{\rho^5} \frac{d}{d\rho} \rho^5 \frac{d}{d\rho} + 1 + \frac{\omega^4 R^4}{\rho^4} - \frac{l(l + 4)}{\rho^2} \right] \phi^{(l)}(\rho) = 0$$

(9)

where $\rho = \omega r$. The absorption process we are interested in thus corresponds to quantum mechanical tunneling through a centrifugal potential barrier in the reduced one-dimensional...
system. In both the large $\rho \gg (\omega R)^2$ and small $\rho \ll 1$ regimes, (9) reduces to a Bessel equation. The solution for large $\rho$ is

$$\phi^{(l)}(\rho) = A\rho^{-2}J_{l+2}(\rho) + B\rho^{-2}N_{l+2}(\rho)$$

(10)

where $A, B$ are undetermined constants. The solution in the regime $\rho \ll 1$ is

$$\phi^{(l)}(\rho) = i(\omega R)^4\rho^{-2} \left[ J_{l+2} \left( \frac{\omega^2 R^2}{\rho} \right) + iN_{l+2} \left( \frac{\omega^2 R^2}{\rho} \right) \right]$$

(11)

where the overall normalization has been fixed to an arbitrary constant and the relative coefficients $J_{l+2} + iN_{l+2}$ are fixed by the condition that the flux at $\rho \to 0$ describes a purely incoming wave. In the overlap region $(\omega R)^2 \ll \rho \ll 1$ we can use the asymptotic forms for the Bessel function to find from (11)

$$\phi^{(l)}(\rho) \sim \frac{2^{l+2}(l+1)!\rho^l}{\pi(\omega R)^{2l}} + \text{subleading}$$

This determines the coefficients $A, B$ in (10) to be

$$A = \frac{4^{l+2}(l+1)!(l+2)!}{\pi(\omega R)^{2l}}, \quad B = 0$$

The absorption coefficient is then given by the ratio of the incoming flux at $\rho = 0$ over the incoming flux at $\rho = \infty$

$$P_l = \frac{(\omega R)^{4l+8}\pi^2}{4^{2l+3}(l+1)!^2[(l+2)!]^2}$$

(12)

The optical theorem in 7 space-time dimensions relates the absorption cross section $\sigma_s^l$ to the absorption coefficient through [17]

$$\sigma_s^l = \frac{8\pi^2}{3\omega^5}(l+1)(l+2)^2(l+3)P_l.$$  

(13)

Combining this with (12) we find that the semiclassical result for the leading order contribution to the total absorption cross section is

$$\sigma_s^l = \frac{\pi^4}{24} \frac{(l+3)(l+1)}{[(l+1)!]^4} \left( \frac{\omega R}{2} \right)^{4l} \omega^3 R^8.$$  

(14)

Replacing $R^4$ through (5) this can be rewritten as

$$\sigma_s^l = \frac{N^{l+2}k^{l+2}\omega^{4l+3}(l+3)}{3 \cdot 2^{5l+5} \pi^{5l/2+1} [(l+1)!]^3}.$$  

(15)

The semiclassical result (15) is the leading term in the absorption cross section for an arbitrary partial wave $l$; this is the result which we will reproduce from the D-brane point of view in the remainder of the paper. Although the method we have just outlined gives the
correct answer at leading order in $\omega R$ for each partial wave $l$, there are subleading corrections to this result which may also be of interest. These subleading corrections were determined by Gubser and Hashimoto in [25]. In that paper, it is shown that the wave equation (9) is equivalent to Mathieu’s modified differential equation
\[
\left[ \frac{\partial}{\partial z^2} + 2q \cosh 2z - a \right] \psi(z) = 0.
\]

The exact solution of this equation is known as a power series in $q = \omega R$. This exact solution is used in [25] to write the complete expansion for the absorption probability of the $l$th partial wave
\[
P_l = \frac{4\pi^2}{[(l+1)!]^2[(l+2)!]^2} \left( \frac{\omega R}{2} \right)^{8+4l} \sum_{0\leq k\leq n} b_{n,k} (\omega R)^{4n}(\ln \omega \gamma R/2)^k
\]
where $b_{n,k}$ are computable coefficients with $b_{0,0} = 1$ and $\ln \gamma$ is Euler’s constant. In the final section of this paper we briefly discuss the possibility of extending the results in this paper to include some of these higher order corrections.

3 Coupling of the dilaton to the world volume theory

In this section we determine how the type IIB dilaton field couples to the world-volume theory on the branes.

The world volume theory of $N$ D3 branes is the $D = 4$, $\mathcal{N} = 4$ supersymmetric Yang-Mills theory with gauge group $U(N)$. This theory may be obtained as the dimensional reduction of $D = 10$ super Yang-Mills theory, and throughout this work, we will use $D = 10$ language, writing operators in terms of 32 component Majorana-Weyl spinors and $32 \times 32$ gamma matrices. From a four dimensional perspective, these gamma matrices contain not only the four $D = 4$ gamma matrices, but also Clebsch-Gordon coefficients relating the $6$ representation of the $R$ symmetry group $SU(4)$ (equivalent to the fundamental representation of the $SO(6)$ manifest in the $D = 10$ language) to the representation $4 \oplus \bar{4}$ carried by a fermion bilinear in the $D = 4$ language.

The coupling of the type IIB dilaton field to the world volume theory of $N$ D3 branes is in principle given by the non-abelian Born-Infeld action which sums all planar string diagrams describing interactions between the lightest string fields on the D-brane world-volume and in the bulk. The complete form of this action is not known, although it has been proposed that the background independent bosonic terms are those obtained by T-duality from a 9-brane action obtained from the abelian version by symmetrizing all traces [22]. For the purposes of this paper we only require terms linear in a weak background supergravity field. Such terms have recently been found in [18] for a system of many D0-branes in a weak background field. The result for D0-branes is derived using a proposal for the linear terms in the general background Matrix theory action motivated by the structure of the linearized supergravity currents in Matrix theory [24, 21].
The results of [18] can be carried over to the D3-brane system by T-dualizing on a 3-torus and taking the limit of infinite torus volume in the IIB theory. For the case of the dilaton field, the complete set of couplings is given by [20]

\[ S_\phi = T_3 \int d^4x \sum_{n=0}^{\infty} \frac{1}{n!} \{ \partial_{i_1} \cdots \partial_{i_n} \phi(x,0) \} \left\{ \frac{1}{6} T^{i_1 j_1 \cdots l_n j_n} - \frac{1}{3} T^{a_1 a_2 \cdots (l_1 \cdots l_n)} - \frac{1}{3} T^{+ j_1 \cdots (l_1 \cdots l_n)} \right\} \]

\[ \equiv T_3 \int d^4x \sum_{n=0}^{\infty} \frac{1}{n!} \{ \partial_{i_1} \cdots \partial_{i_n} \phi(x,0) \} A^{i_1 \cdots l_n} \]  \hspace{1cm} (17)

where \( T^{\mu \nu (l_1 \cdots l_n)} \) are T-dualized versions of the Matrix theory expressions for the multipole moments of the \( D = 11 \) DLCQ supergravity stress-energy tensor that were shown to appear coupled to the background metric in the action for Matrix theory in a general background. Here, the index \( \hat{a} \) runs from 1 to 3 while the remaining indices are \( SO(6) \) indices running from 4 to 9. Explicit expressions for the \( T \)'s were determined in [20, 18] by comparing the one-loop Matrix theory interaction potential between two arbitrary objects with the tree level supergravity result. Using those results, we may write

\[ A^{i_1 \cdots l_n} = \text{STr} \left( \{ \frac{1}{4} F_{ab} F_{ab} - \frac{1}{4} F_{ij} F_{ij} + \frac{1}{4} \bar{\Theta} \Gamma^i D_i \Theta \} X^{l_1} \cdots X^{l_n} \right) + A_{f}^{i_1 \cdots l_n} \]

Here STr denotes an average of all possible orderings of the expressions \( F, X, \Theta \) and \( D \Theta \) in the trace. The terms \( A_{f}^{i_1 \cdots l_n} \) are a set of additional terms involving fermion fields which appear for \( n > 0 \). For \( n = 1 \), the explicit expression may be determined from the results of [18] and is

\[ A_{f}^i = -\frac{1}{16} \text{STr} \left( F_{ab} \bar{\Theta} \Gamma^{[a[b]} \Theta - F_{ij} \bar{\Theta} \Gamma^{[ij]} \Theta \right) \]

The terms for \( n > 1 \) could be determined by extending the Matrix theory calculation in [18] to higher orders in \( 1/r \), but we will determine them more efficiently below from the purely bosonic terms using supersymmetry and a connection to the AdS/CFT correspondence.

For a dilaton field \( \phi(x^a, x^i) \), the \( l \)th partial wave is precisely the part whose expansion in transverse coordinates \( x^i \) is in the \( l \) index symmetric traceless representation of \( SO(6) \). To rearrange the terms in the action which couple to a particular dilaton partial wave we may rearrange the terms in (17) as

\[ S_\phi = T_3 \int d^4x \left\{ \phi(x,0) A + \frac{1}{12} \{ (\partial_i)^2 \phi(x,0) \} A^{kk} + \cdots \right\} \]

\[ + \{ (\partial_i \phi(x,0)) \} A^i + \frac{1}{16} \{ (\partial_i)^2 \partial_k \phi(x,0) \} \delta^{ij} A^{kk} + \cdots \]

\[ + \frac{1}{2} \{ \partial_i \partial_j \phi(x,0) \} (A^{ij} - \frac{1}{6} \delta^{ij} A^{kk}) + \cdots \]

\[ + \cdots \]

Here and throughout the rest of this work, indices \( a, b, \ldots = 0, 1, 2, 3 \) are world-volume indices on the brane while \( i, j, \ldots \) and \( p, q, \ldots \) are transverse indices running from 4 to 9. Also, \( (i_1 \cdots i_n) \) and \( [i_n \cdots i_n] \) denote averaged symmetrization and antisymmetrization respectively. All quantities are to be interpreted as their dimensional reduction from \( D = 10 \), so for example \( F_{ij} \equiv i[X^i, X^j] \) and \( D_i \Theta \equiv i[X_i, \Theta] \).
Here, the first line gives the coupling of the $s$-wave part of the dilaton, the second line gives the coupling of the $l = 1$ part, and so forth. For each $l$, the leading low-energy cross section will come only from the terms with $l$ derivatives on $\phi$, since additional derivatives on $\phi$ will result in additional powers of $\omega$. Therefore, we define operators $^{[1]}$

$$\mathcal{O}^{k_1 \cdots k_n} = A^{k_1 \cdots k_n} - \{\text{traces}\}$$

such that the low-energy contribution to the $l$-wave absorption cross section will be determined by the term

$$S_l = T_3 \int d^4x \frac{1}{n!} \{\partial_{k_1} \cdots \partial_{k_n} \phi(x, 0)\} \mathcal{O}^{k_1 \cdots k_l}$$

(18)

To determine the remaining fermionic terms in the operators $\mathcal{O}$, we now make a connection with the AdS/CFT correspondence for D3 branes.

In the correspondence between large $N D = 4, N = 4$ super Yang-Mills theory and type IIB supergravity on $AdS^5 \times S^5$ $^{[12, 13, 14]}$, gauge theory operators corresponding to the complete spectrum of Kaluza-Klein modes of the supergravity fields have been found. These operators lie in short multiplets of the superconformal group and may be obtained by acting with various combinations of the $D = 4$ supercharges $Q$ and $\bar{Q}$ (up to four of each) on the chiral primary operators

$$\mathcal{O}_{cp}^n = \text{Tr} (X^{p_1} \cdots X^{p_n}) C_{i_1 \cdots i_n}^{p_1 \cdots p_n}$$

(19)

In $^{[27, 28]}$ it was conjectured that the gauge theory operators coupling to the various supergravity modes may also be determined by expanding the Born-Infeld action for a D3 brane about the AdS background. In the case of the dilaton field, apart from some power of $r/R$, the operator determined in this way is exactly the same as the operator which couples to the dilaton in the Born-Infeld action expanded about flat space, since the dilaton does not mix with any other fields in either picture. Hence, the operator we are interested in should be obtainable by taking a supersymmetry variation on the chiral primary fields above.

More precisely, it may be seen from the analysis in $^{[29]}$ and $^{[30]}$ (the table in $^{[31]}$ is useful in relating the results in these papers to the 4D theory) that the particle corresponding to the $l$th partial wave of the dilaton couples to an operator obtained by applying four supercharges of the same chirality to the primary operator in (19) with $n = l + 2$. From the $D = 10$ point of view, both $Q$ and $\bar{Q}$ are contained in the Majorana-Weyl supercharge $Q_\alpha$, so the operator coupling to the $l$th partial wave of the dilaton field is contained in the operator

$$Q_\alpha Q_\beta Q_\gamma Q_\delta \text{Tr} (X^{p_1} \cdots X^{p_{l+2}}) C_{p_1 \cdots p_{l+2}}^{i_1 \cdots i_l}$$

(20)

We use conventions in which the $D = 10$ supersymmetry transformation rules are$^{4}$

$$Q_\alpha A^\mu = i(\Gamma^0 \Gamma^\mu)_{\alpha\beta} \Theta_\beta$$

$^3$Here, $C_{i_1 \cdots i_n}^{p_1 \cdots p_n}$, whose explicit form is given in the appendix, is a combination of delta functions which picks off the symmetric traceless part of any operator with $l$ $SO(6)$ indices.

$^4$we include explicitly the projection operator $P = (1 + \Gamma^{11})/2$ in terms of which the Weyl condition is $P_{\alpha\beta} Q_\beta = Q_\alpha$
\[ Q_\alpha \Theta_\beta = \frac{i}{2} (\Gamma^{[\mu\nu]} P)_{\beta\alpha} F_{\mu\nu} \]

The operator we are interested in is a Lorentz scalar and a traceless \( l \)-index symmetric tensor of \( SO(6) \), so our desired operator is actually obtained from the above expression by contracting the extra indices with a combination of 10D gamma matrices of the form

\[ A^{\alpha\gamma\delta}_{\alpha\beta\gamma\delta} \].

In principle, the linear combinations of terms in (20) in which we are interested can be determined from group theory, using the results of [29]. These terms can also be isolated by performing a component expansion of polynomials in superfields as in [32]. We find it easier in practice to simply find a combination of gamma matrices which correctly reproduce the bosonic and two-fermion terms described above. This is achieved by contracting (20) with \[ A_{\alpha\beta\gamma\delta} = \frac{1}{3 \cdot 2 \cdot 1.0 \cdot (l + 2)(l + 1)} \left( \left\{ \Gamma^{[\alpha\beta\gamma]} \Gamma^{0} \right\}_{\alpha\beta} \left\{ \Gamma^{[\gamma\delta]} \Gamma^{0} \right\}_{\gamma\delta} - \left\{ \Gamma^{[i\alpha\beta]} \Gamma^{0} \right\}_{[\alpha\beta} \left\{ \Gamma^{[\gamma\delta]} \Gamma^{0} \right\}_{\gamma\delta] \right) \]

This gives us the complete operator coupling to the the \( l \)th partial wave of the dilaton, which may now be computed to be

\[ O^{i_1 \cdots i_l} = \left\{ \frac{1}{4} \text{Str} \left( \{ F_{ab} F_{ab} - F_{ij} F_{ij} + \bar{\Theta} \Gamma^i D_i \Theta \} X^{p_1} \cdots X^{p_l} \right) - \frac{l}{16} \text{Str} \left( \{ F_{ab} \bar{\Theta} \Gamma^{[abp_1]} \Theta - F_{ij} \bar{\Theta} \Gamma^{[ijp_1]} \Theta \} X^{p_2} \cdots X^{p_l} \right) + \frac{l(l - 1)}{768} \text{Str} \left( \{ \bar{\Theta} \Gamma^{[abp_1]} \Theta \bar{\Theta} \Gamma^{[abp_2]} \Theta - \bar{\Theta} \Gamma^{[ijp_1]} \Theta \bar{\Theta} \Gamma^{[ijp_2]} \Theta \} X^{p_3} \cdots X^{p_l} \right) \right\} C^{i_1 \cdots i_l}_{p_1 \cdots p_l} \]

Terms in the first line arise from the four supersymmetry generators acting on either one or two of the \( X \)'s in (19) and appear for any \( l \). In writing these terms, we have used the equations of motion, written compactly using \( D = 10 \) indices as

\[ \Gamma^\mu D_\mu \Theta = 0, \quad D_\mu F_{\mu\nu} = i \bar{\Theta} \Gamma^\nu \Theta \]

to rewrite terms in a form with no world-volume derivatives acting on \( F \) or \( \Theta \) (recall that \( D_i \Theta \equiv i [X^i, \Theta] \)). Terms in the second line result when the \( Q \)'s are spread over three separate \( X \)'s and appear for \( l \geq 1 \). Finally, the four fermion terms come when each supersymmetry generator acts on a different \( X \) and therefore appear only for \( l \geq 2 \).

For each of the dilaton partial waves, we have now determined the complete form of the non-abelian operators \( O \) which determine the leading term in the low-energy expansion of the absorption cross section.

\footnote{The antisymmetrization in fermion indices is required since we are trying to reproduce the action of four \( D = 4 \) supercharges of like chirality, which anticommute. This restricts to antisymmetric matrices for which \{\( \Gamma^{[\mu\nu\lambda\gamma]} \Gamma^{0} \}_{\alpha\beta} \) form a basis when sandwiched between Majorana Weyl spinors.}
4 World volume absorption

In this section, we use the operator determined in the previous section to calculate the cross section for absorption of the \( l \)th partial wave of the dilaton field by a set of \( N \) coincident parallel D3 branes.

The most obvious way to proceed, and the method originally used in [5] to show agreement between the world volume and supergravity approaches for the s-wave absorption, is to treat the dilaton as a time dependent perturbation in the world-volume theory and calculate the transition amplitude to each possible set of final particles on the brane, summing over the various contributions in the usual way to obtain a cross section. However, as explained in [10], it turns out that there is a simpler method exploiting the fact that the cross section arising from a given operator is simply related to the two-point function of that operator on the brane. For a canonically normalized scalar coupling to the brane through an interaction

\[
S = \int d^4x \phi(x,0)\mathcal{O}(x)
\]

the precise relation is given by

\[
\sigma = \frac{1}{2\omega} \text{Disc } \Pi(p) \bigg|_{-p^2=\omega^2+i\epsilon}^{p^2=\omega^2+i\epsilon} \quad (22)
\]

Here, \( \omega \) is the energy of the particle, and

\[
\Pi(p) = \int d^4xe^{ip\cdot x} \langle \mathcal{O}(x)\mathcal{O}(0) \rangle
\]

which depends only on \( s = p^2 \). To evaluate (22) we extend \( \Pi \) to complex values of \( s \) and compute the discontinuity of \( \Pi \) across the real axis at \( s = \omega^2 \). This method has the advantage that it is not necessary to determine all of the distinct final particle states or sum over the polarizations, which would be rather complicated for large values of \( N \) and \( l \).

We now use this method to calculate the absorption cross section for each partial wave of the dilaton field. We assume that the dilaton is normally incident on the brane in the 9 direction so that

\[
\phi(x) = e^{i\omega(x^9-t)}
\]

From (18), we see that the absorption cross section for the \( l \)th partial wave is determined by the two-point function of the operator

\[
\mathcal{O}_l = T_3 \frac{\omega^l}{l!} \mathcal{O}^{99\ldots9}
\]

From (21), we note that \( \mathcal{O}_l \) has terms involving \( l+2 \) or more fields, so the leading contribution to \( \langle \mathcal{O}_l(x)\mathcal{O}_l(0) \rangle \) will be an \( l + 1 \) loop planar diagram with each field in the operator at \( x \) contracted with a field in the operator at 0. We can ignore all contributions from operators
containing commutators $F_{ij}$ and $D_i \Theta$ since these contain more than $l + 2$ fields and will come in at higher order in $(g_\gamma^2 M N)$. The terms which do contribute are a bosonic term
\[
O_{l}^{\text{bos}} \equiv \frac{T_3 \omega^l}{4l!} \text{STr} \left( F_{ab} F_{ab} X^{p_1} \ldots X^{p_l} C_{p_1 \ldots p_l}^{\bar{\Theta}} \right),
\]
a two fermion term
\[
O_{l}^{2\Theta} \equiv -\frac{T_3 \omega^l}{16(l-1)!} \text{STr} \left( F_{ab} \bar{\Theta} \Gamma^{[abp_1]} \Theta X^{p_2} \ldots X^{p_l} C_{p_1 \ldots p_l}^{\bar{\Theta}} \right),
\]
and a four fermion term,
\[
O_{l}^{4\Theta} \equiv \frac{T_3 \omega^l}{768(l-2)!} \text{STr} \left( \{ \Theta \Gamma^{[abp_1]} \Theta \bar{\Theta} \Gamma^{[abp_2]} \Theta - \Theta \Gamma^{[ijp_1]} \Theta \bar{\Theta} \Gamma^{[ijp_2]} \Theta \} X^{p_3} \ldots X^{p_l} C_{p_1 \ldots p_l}^{\bar{\Theta}} \right).
\]
The complete two point function is the sum of the two-point functions of each of these operators since there are no cross terms at leading order.

**Propagators**

To evaluate the two-point functions at leading order, all we need to know are the propagators of the various fields. In $D = 10$ language, choosing a gauge fixing term which enforces the Feynman gauge, the quadratic action which determines the propagators is simply
\[
S = T_3 \int d^4x \text{Tr} \left( -\frac{1}{2} A_b(\partial_a)^2 A_b - \frac{1}{2} X_i(\partial_a)^2 X_i - \frac{1}{2} \bar{\Theta} \Gamma^a \partial_a \Theta \right)
\]
In terms of the scalar propagator
\[
\Delta(x-y) \equiv \frac{1}{4\pi^2 |x-y|^2}
\]
the propagators for the various fields are
\[
\langle X_{i}^{kl}(x) X_{j}^{mn}(y) \rangle = \frac{1}{T_3} \delta_{ij} \delta^{km} \delta^{ln} \Delta(x-y)
\]
\[
\langle A_{a}^{kl}(x) A_{b}^{mn}(y) \rangle = \frac{1}{T_3} \delta_{ab} \delta^{kn} \delta^{lm} \Delta(x-y)
\]
\[
\langle \Theta_{a}^{kl}(x) \Theta_{b}^{mn}(y) \rangle = \frac{1}{T_3} (P \Gamma^a \Gamma^b)_{\alpha\beta} \delta^{kn} \delta^{lm} \Delta(x-y)
\]
Note that the projection matrix $P$, defined above, appears in the fermion propagator, since half of the components of each spinor are zero. From the gauge field propagator, we also have to leading order in $1/x$ that
\[
\langle F_{a}^{kl}(x) F_{c}^{mn}(y) \rangle = \frac{4}{T_3} \delta^{kn} \delta^{lm} \partial^{[a} \delta_{b]} \delta_{[c} \partial_{d]} \Delta(x-y)
\]

\[\text{In these expressions, the indices } k, l, m, n \text{ are } U(N) \text{ indices.}\]
Bosonic contribution

We first compute the two-point function of the bosonic operator. We have

\[
\Pi_l^{\text{bos}}(x) = \langle \mathcal{O}_l^{\text{bos}}(x) \mathcal{O}_l^{\text{bos}}(0) \rangle
\]

\[
= \frac{T_3^2 \omega x^{2l}}{16(\eta)^2} C_l^{\vec{p}} C_l^{\vec{q}} \Pi_{\text{bos}} \langle \text{STr} (F_{ab} F_{cd} X^{p_1} \cdots X^{p_l})_x \text{STr} (F_{ab} F_{cd} X^{q_1} \cdots X^{q_l})_0 \rangle
\]

Note that since the \(X\)'s in each symmetrized trace contract with a totally symmetric tensor \(C_l^{\vec{p}} \equiv C_l^{p_1 \cdots p_l}\), we need only average over the \((l+1)\) orderings of operators in which one \(F\) is fixed in the first position by cyclicity of the trace and the other runs over positions 2 through \(l+2\). By Wick’s theorem, the correlator for each ordering of operators in the two symmetrized traces is evaluated by summing over all possible contractions matching the operators in the first trace to those in the second trace. However, only those contractions which match up the operators in reverse cyclic order contribute with the maximal power of \(N\), namely \(N^{l+2}\). For each of the \((l+1)\) orderings of operators in the first trace there will be exactly such 2 contractions with the sum of operators in the second trace.

\[
\Pi_l^{\text{bos}}(x) = \frac{T_3^2 \omega x^{2l}}{16(\eta)^2} C_l^{\vec{p}} C_l^{\vec{q}} 2N^{l+2}
\]

\[
\quad \times \langle F_{ab}(x) F_{cd}(0) \rangle \langle F_{ab}(x) F_{cd}(0) \rangle \langle X^{p_1}(x) X^{q_1}(0) \rangle \cdots \langle X^{p_l}(x) X^{q_l}(0) \rangle
\]

\[
= \frac{T_3^{-l} \omega x^{2l} N^{l+2}}{l!(l+1)!} C_{p_1 \cdots p_l}^{\vec{p}} C_{p_1 \cdots p_l}^{\vec{q}} \Delta(x) \left( \partial_a \partial_b \Delta(x) \partial_a \partial_b \Delta(x) + \frac{1}{2} \partial^2 \Delta(x) \partial^2 \Delta(x) \right)
\]

In the second line, we have already evaluated the contractions of \(U(N)\) delta functions to give \(N^{l+2}\), so the correlators there have the values of \(U(1)\) correlators. In the last line, the term involving \(\partial^2 \Delta(x) \propto \delta(x)\) will give a constant contribution to \(\Pi(p)\) so we can ignore it for the purposes of computing the discontinuity. The evaluation of \(C_{p_1 \cdots p_l}^{\vec{p}} C_{p_1 \cdots p_l}^{\vec{q}}\) is described in detail in the appendix. The simple result is that

\[
C_{p_1 \cdots p_l}^{\vec{p}} C_{p_1 \cdots p_l}^{\vec{q}} = \frac{(l+2)(l+3)}{3 \cdot 2^{l+1}}
\]

Thus, we find

\[
\Pi_l^{\text{bos}}(x) = \frac{k^l \omega x^{2l} N^{l+2}(l+2)(l+3)}{2^{3l+1} \pi^{l+4} l!(l+1)! |x|^{2l+8}}
\]

(23)

where we have substituted \(T_3 = \sqrt{\pi}/\kappa\). Using the result (see, for example [23]) that

\[
\text{Disc} \left( \int d^4x e^{ip \cdot x} \right) \bigg|_{-p^2 = \omega^2 + i\epsilon}^{p^2 = \omega^2 - i\epsilon} = \frac{2 \pi^2 i \omega^{2m}}{4^{m} m!(m + 1)!}
\]

These may come from two different contractions with the same operator for orderings such as \(\text{Tr} (F X F X)\) or a single contraction with two different operators for orderings which are not invariant under a cyclic shift by \((l+2)/2\) positions.
we may now use \( \left[ 22 \right] \) to give our final result for the cross section arising from \( \mathcal{O}_l^{bos} \) as

\[
\sigma^l_{\text{bos}} = \frac{2\kappa^2}{2\ell \omega} \text{Disc} \Pi^l_{\text{bos}}(p) \left| -p^2 = \omega^2 + i\epsilon \right| -p^2 = \omega^2 - i\epsilon \\
= \frac{N^l + 2\kappa^l + 2\omega^{l+3}}{2^{5l+4} \pi^{5l/2 + 1} l! ((l + 1)!)^2 (l + 2)!} \\
= \frac{6}{(l + 2)(l + 3)} \sigma^l_s
\]

(24)

where \( \sigma^l_s \) is the cross section \( \left[ 15 \right] \) computed from classical supergravity. Recalling that the two and four fermion operators only contribute for \( l \geq 1 \), we see that we have reproduced the agreement for the \( l = 0 \) case originally found in \( \left[ 4 \right] \). For \( l > 0 \), where we expect additional contributions from the other operators, our result is safely less than \( \sigma^l_s \), so we do not find the problem encountered in \( \left[ 4 \right] \).

Two-fermion contribution

We now calculate the two-point function of the two fermion operator \( \mathcal{O}_l^{2\Theta} \) to determine its contribution to the cross section. The calculation is similar to the bosonic two-point function so we will be brief. We have

\[
\Pi^l_{2\Theta}(x) = \langle \mathcal{O}_l^{2\Theta}(x) \mathcal{O}_l^{2\Theta}(0) \rangle \\
= \frac{T^2_3 \omega^{2l}}{16^2 ((l - 1)!)^2} C^\delta_p C^\delta_q \left( \Gamma^0 \Gamma^{[\alpha \beta \gamma \delta]} \right)_{\alpha \beta} (\Gamma^0 \Gamma^{[cd\delta \gamma]})_{\gamma \delta} \\
\times \langle \text{STr} (F_{ab} \Theta \Theta \times \cdots \times \times 0) \rangle \text{STr} (F_{cd} \Theta \Theta \times \times \times \times 0)
\]

Here, for each of the \( l(l + 1) \) orderings in the first symmetrized trace, there are two terms in the second symmetrized trace (related by switching the \( \Theta \)'s) which have the correct ordering to give a non-vanishing set of contractions with the maximal power of \( N \). Again, all contributions are identical, due to the symmetry of \( C^\delta_p \) and the antisymmetry in the fermionic indices of \( (\Gamma^0 \Gamma^{[\alpha \beta \gamma \delta]})_{\alpha \beta} \), so we get a factor \( 2/l(l + 1) \) from the symmetrizations and find

\[
\Pi^l_{2\Theta}(x) = \frac{T^2_3 \omega^{2l}}{16^2 ((l - 1)!)^2} C^\delta_p C^\delta_q \frac{2N^{l+2}}{l(l + 1)} (\Gamma^0 \Gamma^{[\alpha \beta \gamma \delta]})_{\alpha \beta} (\Gamma^0 \Gamma^{[cd\delta \gamma]})_{\gamma \delta} \\
\times \langle F_{ab}(x) F_{cd}(0) \rangle \langle \Theta_{\alpha}(x) \Theta_{\beta}(0) \rangle \langle \Theta_{\gamma}(x) \Theta_{\delta}(0) \rangle \langle X_{p_1}^2 X_{q_1}^2 \rangle \cdots \langle X_{p_l}^2 X_{q_l}^2 \rangle \\
= \frac{T^{-1}_3 \omega^{2l} N^{l+2}}{2^{5l} (l + 1)!} C^\delta_{p_{1+2 \cdots n}} C^\delta_{q_{1+2 \cdots n}} \Delta^{l-1} (x) \partial_{\alpha \beta \delta \gamma} (c) \partial_{\delta \gamma} (x) \partial_{x} \Delta (x) \partial_{f} \Delta (x) \\
\times \text{Tr} (\Gamma^{[\alpha \beta \gamma \delta]} P \Gamma^{[cd\delta \gamma]} P \Gamma^{[e]})
\]

The projection matrices in the trace\( \left[ 4 \right] \) just serve to reduce its value by 1/2, and we may

\footnote{We include an extra factor of \( 2\kappa^2 \) relative to the formula \( \left[ 22 \right] \) since the dilaton field is not canonically normalized due to the factor of \( 1/2\kappa^2 \) in front of the supergravity action.}

\footnote{For multiple \( P \)'s in a trace, as long as they are all separated by an even number of \( \Gamma \) matrices, we may bring them together into a single \( P \) since \( P \) commutes with any pair of \( \Gamma \)'s and \( P^2 = P \). Then, since \( P = (1 + \Gamma_{11})/2 \) we will just get a factor of \( 1/2 \) unless there are at least 10 other \( \Gamma \)'s with distinct indices.}
evaluate the trace over the remaining gamma matrices by the usual rules to find
\[
\text{Tr } (\Gamma^{[abp_1]} P \Gamma^{e} \Gamma^{[cdq_1]} P \Gamma^f) \rightarrow \delta_{p_1 q_1} (128 \delta_{ef} \delta_{be} \delta_{ac} + 32 \delta_{ef} \delta_{cb} \delta_{da})
\]

Note that the two sides of this expression are not equal, but equivalent when appearing in the two-point function above, since we have used the antisymmetry of the index pairs \([ab]\) and \([cd]\) and the symmetry of index pair \((ef)\) in order to simplify the trace. Inserting this trace into the expression above and simplifying, we find that
\[
\Pi_l^{2\Theta}(x) = l \cdot \Pi_l^{\text{bos}}(x)
\]
where \(\Pi_l^{\text{bos}}\) is the bosonic two-point function given in (23). We therefore see immediately from (24) that the contribution of the two-fermion operator to the cross section is
\[
\sigma_{2\Theta}^l = \frac{6l}{(l + 2)(l + 3)} \sigma_s^l
\]
The total cross section so far,
\[
\frac{6(l + 1)}{(l + 2)(l + 3)} \sigma_s^l
\]
agrees with the classical result for \(l = 0\) and \(l = 1\), and is less than the supergravity result for \(l \geq 2\), consistent with the fact that the four fermion operator is only present for \(l \geq 2\).

**Four fermion contribution**

Finally, we calculate the contribution to the cross section from the four fermion operator which appears for \(l > 1\). In this case, the operator \(O_l^{4\Theta}\) has two pieces, so we have to calculate the two-point function of each of the pieces as well as a cross term. These three correlators differ only in the indices on the \(\Gamma\) matrices, so we may write them together as

\[
\Pi_l^{4\Theta}(x) = \langle O_l^{4\Theta}(x) O_l^{4\Theta}(0) \rangle
= \frac{T_3^2 \omega^{2l}}{9 \cdot 2^{16}((l - 2)!)^2} C_p^\alpha C_q^\beta
\times \langle \text{STr } (\Theta_\alpha \Theta_\beta \Theta_\gamma \Theta_\delta X^{p_1} \cdots X^{p_l}) \rangle_2
\times \langle \text{STr } (\Theta_\alpha \Theta_\beta \Theta_\gamma \Theta_\delta X^{q_1} \cdots X^{q_l}) \rangle_0
\times \left\{ \langle (\Gamma^0 \Gamma^{[abp_1]})_{\alpha \beta} (\Gamma^0 \Gamma^{[cdq_1]})_{\gamma \delta} \rangle \delta_{\beta \gamma} \delta_{\delta \gamma}
- 2(\Gamma^0 \Gamma^{[abp_1]})_{\alpha \beta} (\Gamma^0 \Gamma^{[cdq_1]})_{\gamma \delta} \rangle \delta_{\beta \gamma} \delta_{\delta \gamma}
- 2(\Gamma^0 \Gamma^{[abp_1]})_{\alpha \beta} (\Gamma^0 \Gamma^{[cdq_1]})_{\gamma \delta} \rangle \delta_{\beta \gamma} \delta_{\delta \gamma}
\right\}
\]

This time, each of the \((l + 1)!/(l - 1)!\) orderings in the first symmetrized trace may couple in a single way to 24 different terms in the second trace (related by permuting the \(\Theta\)'s), but this time not all of the contributions are equivalent. When we contract the fermion propagators
with the $\Gamma$ matrices above, 1/3 of the terms give two traces over $\Gamma$’s while the remaining 2/3 give a single trace. The result is

$$\Pi^4_{l}(x) = \frac{T_3^{-1} \omega^{2l} N_i^{l+2}}{9 \cdot 2^{16} ((l - 2)!)^2} C_{p_1p_2p_3...p_l} C_{q_1q_2q_3...q_l} \Delta^{l-2}(x) \partial_p \Delta(x) \partial_q \Delta(x) \partial_r \Delta(x)$$

$$\times \frac{8}{(l + 1)!l(l - 1)} \{ (\text{Tr Tr}^{aa} - 2 \text{Tr}^{aa}) + (\text{Tr Tr}^{ii} - 2 \text{Tr}^{ii}) - 2(\text{Tr Tr}^{ai} - 2 \text{Tr}^{ai}) \}^{p_1p_2q_1q_2efgh}$$

Here, the traces are defined as

$$\text{Tr Tr}^{aa} \equiv \text{Tr} (P\Gamma^a \Gamma^{[ab]} P\Gamma \Gamma^{[cdq]} \Gamma \text{Tr} (P\Gamma^g \Gamma^{[ab]} P\Gamma^h \Gamma^{[cdq]})$$

$$\text{Tr}^{aa} \equiv \text{Tr} (P\Gamma^a \Gamma^{[ab]} P\Gamma \Gamma^{[cdq]} \Gamma P\Gamma^g \Gamma^{[ab]} P\Gamma^h \Gamma^{[cdq]}$$

$$\text{Tr Tr}^{ii} \equiv \text{Tr} (P\Gamma^i \Gamma^{[ij]} P\Gamma \Gamma^{[klq]} \Gamma \text{Tr} (P\Gamma^g \Gamma^{[ij]} P\Gamma^h \Gamma^{[klq]}$$

$$\text{Tr}^{ii} \equiv \text{Tr} (P\Gamma^i \Gamma^{[ij]} P\Gamma \Gamma^{[klq]} \Gamma P\Gamma^g \Gamma^{[ij]} P\Gamma^h \Gamma^{[klq]}$$

$$\text{Tr Tr}^{ai} \equiv \text{Tr} (P\Gamma^a \Gamma^{[ab]} P\Gamma \Gamma^{[ij]} \Gamma \text{Tr} (P\Gamma^g \Gamma^{[ab]} P\Gamma^h \Gamma^{[ij]}$$

$$\text{Tr}^{ai} \equiv \text{Tr} (P\Gamma^a \Gamma^{[ab]} P\Gamma \Gamma^{[ij]} \Gamma P\Gamma^g \Gamma^{[ab]} P\Gamma^h \Gamma^{[ij]}$$

$$\rightarrow 0$$

$$\rightarrow 9 \cdot 2^9 \delta_{p_1q_1} \delta_{p_2q_2} \delta_{ef} \delta_{gh}$$

Again, the evaluation of the traces is simplified using the fact that the sets of indices $(efgh)$, $(p_1p_2)$, and $(q_1q_2)$ are symmetric in the expression to which the traces are contracted. The rest of the evaluation is straightforward and in terms of the bosonic two-point function, we find

$$\Pi^4_{l}(x) = \frac{l(l - 1)}{6} \cdot \Pi^{\text{bos}}_{l}(x)$$

From (24), we may immediately read off the final contribution to the cross section to be

$$\sigma_{4\Theta}^{l} = \frac{l(l - 1)}{(l + 2)(l + 3)} \sigma^{l}_{s}$$

Combining the bosonic, two-fermion, and four-fermion contributions, we find the total cross section from the world volume calculation to be

$$\sigma^{l}_{tot} = \sigma^{l}_{\text{bos}} + \sigma^{l}_{2\Theta} + \sigma^{l}_{4\Theta}$$

$$= \frac{6l}{(l + 2)(l + 3)} \sigma^{l}_{s} + \frac{6l}{(l + 2)(l + 3)} \sigma^{l}_{s} + \frac{l(l - 1)}{(l + 2)(l + 3)} \sigma^{l}_{s}$$

$$= \sigma^{l}_{s}$$
Thus, for all values of $l$, the total low-energy cross section for absorption of the $l$th partial wave of the dilaton by $N$ coincident D3-branes is exactly the same when computed in the world volume theory as when computed in classical supergravity.

## 5 Conclusions

In this paper we used the world-volume theory of many parallel D3-branes to exactly reproduce the semiclassical absorption cross section of an arbitrary higher partial wave of the dilaton field. This is the first time that such a correspondence has been made precise for the absorption of higher partial waves by any D-brane black hole configuration. This result provides additional evidence for the conjectured exact correspondence between the world-volume theory of $N$ D3-branes and type IIB string theory on $AdS_5 \times S^5$ \cite{12, 13, 14}. The fact that arbitrary partial waves on the sphere $S^5$ are accurately described in the D-brane gauge theory suggests a number of interesting directions for further research. In particular, these results indicate that incoming wave packets of the supergravity fields can be localized on the sphere in the asymptotic regime. It would be interesting to study in more detail the behavior of such localized wave packets in the D-brane gauge theory.

In performing the calculation in this paper, it was necessary to have an exact formulation of the coupling of the D-brane world-volume fields to the background supergravity fields. We were able to precisely fix the operators on the D3-brane world-volume which couple linearly to derivatives of the background dilaton field by utilizing recent results for similar operators in M(atrix) theory and the related D0-brane theory in type IIA. It has been suggested in various contexts that the AdS/CFT correspondence and the M-theory/Matrix correspondence are in some sense equivalent \cite{33, 34, 35, 36, 37, 38}. The fact that similar operator structures appear coupling to background fields in the two theories may help to make this relationship more precise. Certainly, the symmetrized trace structure which appears in the supergravity operators found in \cite{19, 20, 18} plays a key combinatorial role in exact calculations in both theories, as seen in \cite{20} and the present paper. A fruitful direction for further progress may be to use results from one of these correspondences in deriving new information about the other, as we have here used Matrix theory results to obtain new information in the D3-brane context.

The exact correspondence between the semiclassical gravity calculation, which we expect to be valid for large $g_{YM}^2 N$, and the super Yang-Mills calculation, which is an expansion to leading order in $g_{YM}^2 N$, indicates that there is a non-renormalization theorem for the two-point functions of all the operators $\mathcal{O}_l$ coupling to $l$th partial waves of the dilaton. Such a non-renormalization theorem was proven in \cite{11} for the two-point function of the stress tensor, and in \cite{39} for the two-point function of the R-symmetry current. These operators lie in the same $p = 2$ representation of the superconformal algebra $SU(2, 2|4)$ as the operator $\mathcal{O}_0$ corresponding to the s-wave of the dilaton. From general arguments based on supersymmetry \cite{40} it is believed that all two-point functions of operators in this representation are related by
supersymmetry so that the non-renormalization of the s-wave absorption amplitude is implied by the non-renormalization theorems proven in [10, 39]. For the operators coupling to the higher partial waves there is as yet no analogous non-renormalization theorem, although it is widely believed that all two and three-point functions of operators in short representations of the superconformal algebra are protected by non-renormalization theorems. Evidence for such non-renormalization theorems was given in [11], where it was shown that the free field calculation of the 3-point functions of the chiral primary operators \( \mathcal{O}_p^{cp} \) in (19) agrees with the predictions of supergravity through the AdS/CFT correspondence. This calculation was somewhat different in spirit from ours, though, because in [11] the overall normalization of operators was left undetermined and only appropriate ratios of correlators were shown to agree between weak and strong coupling. The advantage of using the absorption cross sections to calculate two-point functions is that the overall normalization of operators is completely fixed by comparing the coupling of the throat region of the threebrane geometry to the bulk region and the corresponding coupling of the D3-branes to the bulk fields.

Perturbative evidence for the non-renormalization theorems was given in [12], where it was shown that the first perturbative correction to the two- and three-point functions of all the chiral primaries vanishes for all \( p \). We expect similar results to hold for the descendant operators that we have constructed. In such calculations it will be important to use the complete vertex operators (21), including the parts that contain more than \( 2 + l \) fields. Our results provide a strong piece of evidence for the existence of non-renormalization theorems for two-point functions; it would be very nice, however, to have a more direct demonstration of these theorems and a better understanding of why they occur.

The existence of an infinite family of non-renormalization theorems for the two-point functions of short operators in the 4D super Yang-Mills theory seems to be related to a similar infinite family of non-renormalization theorems in the one-dimensional matrix quantum mechanics theory underlying Matrix theory. One piece of evidence for the conjecture that Matrix theory describes light-front supergravity is the agreement between the leading \( v^4/r^7 \) term in the 1D super Yang-Mills effective potential describing the interaction between a pair of D0-branes and the long-range supergravity effective potential between a pair of gravitons with longitudinal momentum \([13, 14]\). This agreement arises due to a non-renormalization theorem in the matrix quantum mechanics theory \([15, 16]\). In \([20, 18]\) it is shown that all linearized supergravity interactions are correctly reproduced by one-loop terms in the matrix quantum mechanics theory, suggesting an infinite family of non-renormalization theorems for terms of the form \( F^4X^l/r^{7+l} \). It seems likely that similar non-renormalization theorems occur in the effective action of the 4D \( \mathcal{N} = 4 \) gauge theory, generalizing the non-renormalization theorem proven in \([17, 18]\) for \( F^4 \) terms. It is unlikely, though, that there are similar theorems for operators involving higher powers of \( F \) than \( F^4 \) because such operators do not belong to short multiplets (in the AdS/CFT correspondence such operators are assumed to couple to massive string modes). Note, however, that at least for the \( SU(2) \) Matrix theory there appears to be a non-renormalization theorem for the \( v^6 \) terms \([19, 50]\).
Just as non-renormalization of two-point functions in the 4D gauge theory seems to correspond with the non-renormalization theorems in matrix theory associated with linearized gravity interactions, there is evidence for non-renormalization of three-point functions in the gauge theory \[21, 22, 11, 12, 23\] as well as for 3-body interactions in Matrix theory \[30, 24\]. A possible explanation for the non-renormalization of 2-point and 3-point functions in the \(D = 4\) theory is given in \[55\]. In both the AdS/CFT and matrix theory contexts, however, it appears that there are no non-renormalization theorems for four-point interactions. In the AdS/CFT correspondence there are corrections to supergravity 4-point functions coming from explicit \(O(\alpha'^3)\) corrections present in the string action \[56, 57, 55\]. Analogous logarithmic corrections have been found in the 4-point functions of the super Yang-Mills theory \[58\]. Similarly, it does not seem to be possible to extend the existing Matrix theory non-renormalization theorems to 4-body interactions \[59, 60\]. In the case of Matrix theory, the absence of such non-renormalization theorems at higher order would imply that agreement between matrix quantum mechanics and supergravity might only be achieved through subtleties in the large \(N\) limit. The suspected non-renormalization theorems for 2- and 3-point interactions, which are as yet poorly understood, form another interesting point of contact between Matrix theory and the AdS/CFT correspondence. It may be possible to use the correspondence we have exploited in this paper between operators in the two theories to achieve a better understanding of the structure in supersymmetric gauge theory responsible for these non-renormalization theorems.

In this paper we have only considered the leading term in the absorption cross section for each partial wave \(l\). It would be very interesting to study whether any of the higher order terms in the semiclassical absorption result \([14]\) can be reproduced from the D3-brane gauge theory. It was suggested in \([14]\) that the nonabelian Born-Infeld action might give rise to this entire series of terms. It is argued in \([23]\), however, that string corrections to the NBI action will be needed to make the correspondence precise beyond leading order. This would not be too surprising, as we have no reason to believe that the subleading operators coupling to the higher partial waves of the dilaton will not be renormalized. In the past we have encountered many surprising agreements, however, so it would be very interesting to extend the analysis to subleading terms and to see if any further structure of the absorption cross section can be understood on the D3-brane side.

### A Properties of the symmetric traceless tensor

The symmetric traceless tensor \(C_{\mu_1 \cdots \mu_n}^{i_1 \cdots i_n}\) is a combination of delta functions which projects onto the symmetric traceless part of any object with \(l\) \(SO(6)\) indices. For example,

\[
\text{Tr} \left( X^p X^q X^l \right) C_{\mu_1 \cdots \mu_n}^{i_1 \cdots i_n} = \text{STr} \left( X^i X^j X^k - \frac{3}{8} \delta^{ij} X^k X^l X^l \right)
\]
The precise definition of $C$ is

$$C_{p_1 \cdots p_n}^{i_1 \cdots i_n} = \sum_{k=0}^{\lfloor n/2 \rfloor} a_k^n \delta(p_1 p_2 \cdots \delta_{p_{2k-1} p_{2k}} \delta_{i_{2k+1}} \cdots \delta_{i_{2k+2k+1}} \cdots \delta_{i_{2k+1} i_{2k+1}})$$  \hspace{1cm} (25)$$

By tracing over $i_1$ and $i_2$ and requiring that the result vanishes, we obtain a recursion relation for the coefficients $a_k^n$ which may be solved to give

$$a_k^n = \left(-\frac{1}{4}\right)^k \binom{n-k+1}{k+1} \frac{k+1}{n+1}$$

To evaluate the two-point functions in section 3, it is necessary to determine the value of

$$c_n \equiv C_{p_1 \cdots p_n}^{i_1 \cdots i_n} C_{p_1 \cdots p_n}^{i_1 \cdots i_n}$$

We do this by noting that on the unit 5-sphere, we have

$$C_{p_1 \cdots p_n}^{i_1 \cdots i_n} x^{p_1} \cdots x^{p_n} = \frac{1}{2^n(n+1)} C_n^2(x^9)$$  \hspace{1cm} (26)$$

where $C_n^\lambda$ are the Gegenbauer polynomials, defined by

$$\sum_{n=0}^\infty C_n^\lambda(x) \alpha^n = (1 - 2x \alpha + \alpha^2)^{-\lambda}$$

which play the same role for $S^{2\lambda+1}$ as the Legendre polynomials play for $S^2$. That is, they are the subset of spherical harmonics which arise in the expansion of a function of only a single coordinate (in our case, the plane wave $e^{i \omega x^9}$). The Gegenbauer polynomials thus obey an orthogonality relation, given by

$$\int_{-1}^{1} dx C_m^\lambda(x) C_n^\lambda(x) (1 - x)^{\lambda-1/2} = \delta_{mn} \frac{\pi 2^{1-2\lambda} \Gamma(n + 2\lambda)}{n!(\lambda + n)(\Gamma(\lambda))^2}$$  \hspace{1cm} (27)$$

To determine $c_n$, we square both sides of (26) and integrate over the unit five-sphere, evaluating the right side using the orthogonality relation (27), and the left side using the relation

$$\int_{S^5} x^{i_1} \cdots x^{i_{2m}} = \frac{\omega_5}{2^{m-1}(m+2)!} \times \{\text{Sum of all index contractions}\}$$

where $\omega_5$ is the area of a unit 5-sphere. The result is

$$\frac{\omega_5 n!}{2^{n-1}(n+2)!} C_{p_1 \cdots p_n}^{i_1 \cdots i_n} C_{p_1 \cdots p_n}^{i_1 \cdots i_n} = \left(\frac{1}{2^n(n+1)}\right)^2 \omega_4 \int_{-1}^{1} dx (1 - x^2)^{3/2} C_n^2(x) C_n^2(x)$$

$$= \left(\frac{1}{2^n(n+1)}\right)^2 \frac{\pi (n+3)!}{8n!(n+2)!}$$

and using $\omega_4 = 8\pi^2/3$, $\omega_5 = \pi^3$ our final result is

$$\sum_{p_i=4}^{9} C_{p_1 \cdots p_n}^{i_1 \cdots i_n} C_{p_1 \cdots p_n}^{i_1 \cdots i_n} = \frac{(n+3)(n+2)}{3 \cdot 2^{n+1}}$$  \hspace{1cm} (28)$$

This expression is used in each of the two-point function evaluations to obtain a closed form for the final cross section.
Acknowledgments

We would like to thank Steve Gubser and Samir Mathur for helpful conversations. WT would also like to thank the students in the MIT special topics class 8.872, spring 1999. IRK is grateful to the Physics Department of the Universities of Torino and Milano for hospitality during the final stages of this project. The work of IRK is supported in part by the NSF grant PHY-9802484 and in part by the James S. McDonnell Foundation Grant No. 91-48. The work of WT is supported in part by the A. P. Sloan Foundation and in part by the DOE through contract #DE-FC02-94ER40818. The work of MVR is supported in part by the Natural Sciences and Engineering Research Council of Canada (NSERC).

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