Inverse synthetic aperture radar (ISAR) is a long-distance and subsequent ISAR imaging processing [10–12]. Since only if the high-resolution imaging system that can observe the non-high-quality and real-time performance are two crucial indicators adjustments [13–15]. The range profile alignment is essential to knowledge. It takes the advantage of the symmetric accumulation manner to offset the phase errors and optimise the utilising the relative motion between the radar and the target [1, 2].

The ISAR imaging technique has been widely studied and developed for decades. It utilises frequency diversity in the range dimension and angular diversity in the azimuth direction during the coherent processing interval (CPI), and then obtains both the high-range resolution and the high cross-range resolution [3, 4]. The ISAR imaging plays an important role in both military and civilian applications. Especially, it is normally used for target recognition and classification, since the ISAR image is the essential line of sight (RLOS) can be accurately described by two-order dynamic parameters.

The range profile alignment is the prerequisite step for the subsequent imaging processing. Due to the non-cooperative characteristic of the target, the unknown moving parameters are required to be well estimated. Generally, the translational motion of the target can be accurately described by two-order dynamic parameters. However, the most parameter estimation methods can only estimate the one-order parameter, while the common high-order estimation methods require priori knowledge and are complex to implement. Aiming at this issue, the authors propose a high-order symmetric accumulated cross-correlation method to realise the rapid and accurate estimation of the motion parameters with no requirement of priori knowledge. It takes the advantage of the symmetric accumulation manner to offset the phase errors and optimise the computational complexity simultaneously, and then formulates the estimation to solve the least-square problem. Experimental results verify that the proposed method shows distinct advantages on achieving the high-accuracy and low-complexity parameter estimation, which is highly conductive to realise the high-quality motion compensation for real-time ISAR imaging.

1 Introduction

Inverse synthetic aperture radar (ISAR) is a long-distance and high-resolution imaging system that can observe the non-cooperative moving target in all-time weather conditions, by utilising the relative motion between the radar and the target [1, 2]. The ISAR imaging technique has been widely studied and developed for decades. It utilises frequency diversity in the range dimension and angular diversity in the azimuth direction during the coherent processing interval (CPI), and then obtains both the high-range resolution and the high cross-range resolution [3, 4]. The ISAR imaging plays an important role in both military and civilian applications. Especially, it is normally used for target recognition and classification, since the ISAR image is the essential line of sight (RLOS) can be accurately described by two-order dynamic parameters.

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At all events, since the movement of the non-cooperative target relative to radar is unknown, it is the core of the motion compensation to realise the fast and accurate parameter estimation of the moving target. For the manoeuvring target, its radial movements are conventionally modelled as a Taylor expansion described by high-order dynamic parameters (i.e. velocity, acceleration etc.) [16]. With regard to the most real applications, the relative motion between the radar and the target along the radial line of sight (RLOS) can be accurately described by two-order dynamic parameters.

Various parameter estimation methods from many researchers have been suggested to promote the motion compensation. Common blind estimation algorithms can be classified into two types: one is based on strong scattering points of the target [17, 18], but according to the analysis of practical test data, it is difficult to stably track interested points during the whole dwelling time. So the application of such algorithms is not very wide in practice. The other is based on the cross-correlation processing of adjacent echoes, which achieve the range alignment by using the similarity of range profiles. Classic methods include the global range alignment algorithm [19], the cross-correlation method (CCM) [20], the accumulated CCM (ACCM) [21] and so on. The ACCM is the basic method of this type, it assumes small and approximately constant range shifts between the adjacent range profiles. However, in real-world scenarios, many practical factors may result in random noise to echo signals; for instance, the target with build-in disturbance components or the case under an extremely bad flying situation. In these cases, the sharply fluctuated and multi-peaked profiles will increase aligned errors. More than that, the aligned error of different range profiles may be transferred and accumulated during the cross-correlation processing. The ACCM is the most popular technique for one-order parameter estimation and is widely used in real motion compensation, because it is easy to implement and shows a good balance between the accuracy and the computational cost. It uses the accumulation concept to enhance the stable and strong frequency components in the profile of each echo signal and to suppress the fast varying disturbances, which greatly reduces the aligned error and improves the estimation performance. Nevertheless, accumulated phase errors are inevitably generated during the accumulation processing by the ACCM method, especially in a harsh signal-to-noise ratio (SNR) environment. To solve this, in our previous work [22], we proposed the symmetric manner instead of the unidirectional accumulation.

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Abstract: The inverse synthetic aperture radar (ISAR) technique is an important tool for target recognition and classification; thus, the high-quality and real-time performance are two essential indicators for ISAR imaging. Based on the classical range-Doppler (RD) principle, the motion compensation is the prerequisite step for the subsequent imaging processing. Due to the non-cooperative characteristic of the target, the unknown moving parameters are required to be well estimated. Generally, the translational motion of the target can be accurately described by two-order dynamic parameters. However, the most parameter estimation methods can only estimate the one-order parameter, while the common high-order estimation methods require priori knowledge and are complex to implement. Aiming at this issue, the authors propose a high-order symmetric accumulated cross-correlation method to realise the rapid and accurate estimation of the motion parameters with no requirement of priori knowledge. It takes the advantage of the symmetric accumulation manner to offset the phase errors and optimise the computational complexity simultaneously, and then formulates the estimation to solve the least-square problem. Experimental results verify that the proposed method shows distinct advantages on achieving the high-accuracy and low-complexity parameter estimation, which is highly conductive to realise the high-quality motion compensation for real-time ISAR imaging.

1 Introduction

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manner in ACCM to offset the positive and the negative accumulated phase errors. However, most of the existing CCMs are extensively studied to achieve the one-order parameter estimation and motion compensation, while the estimation of high-order dynamic parameters based on the cross-correlation processing has not been widely developed.

Some high-order motion compensation methods based on calculating the entropy value have been presented to achieve high-accuracy motion compensation of the manoeuvring target [23–25]. The representative one is the minimum entropy method (MEM) [26] and many other improved versions are derived from the MEM. These types of methods are effective on estimating the high-order parameters, but there are two shortcomings. On one hand, priori knowledge is generally required when employing this kind of method. That is to say, the radar system must first transmit a narrow-band signal to initially determine the estimated range for the target moving parameters, and then transmits the wide-band signal to achieve the high-resolution imaging. This undoubtedly puts a burden to the radar system, and it is hard to track fast moving targets. What is worse is that if the true values of the parameters were not included in the preset parameter searching intervals, the motion compensation is completely failed. On the other hand, the estimation precision of these methods depends on the searching step size. That is, the smaller step size is preset, the higher precision can be obtained. However, the smaller step size inevitably leads to a large increase in computational complexity, which is not conducive to real-time processing.

Aiming at achieving the high-accuracy and low-complexity motion compensation for ISAR imaging, we propose a high-order parameter estimation method (i.e. HSACM, high-order symmetric accumulated cross-correlation method) in this paper. The proposed method is a type of CCM, and is derived from the widely used ACCM. Compared with other popular motion compensation methods, the main contributions and the advantages of the proposed method are shown as follows.

First, the proposed method utilises the symmetric accumulation instead of the unidirectional accumulation manner in ACCM to generate cross-correlations. This symmetric manner can effectively offset the positive and negative phase errors, which helps to improve both the range alignment and phase adjustment.

Moreover, the computational cost of the proposed method is optimised by half due to the symmetric characteristic of the cross-correlations, which is greatly benefited for the real-time imaging processing. The determination of the optimal accumulation number and the discussion of the optimisation degree are presented in detail in this work.

Thirdly, the proposed method formulates the parameter estimation to solve the least-square (LS) [27, 28] problem, thus to realise the high-order parameter estimation. What makes sense is that the proposed method makes up for the defect of the CCMs in estimating the high-order parameters.

Furthermore, the most advantage of the proposed method relative to the MEM is that it is a blind processing method. That is to say there is no requirement of priori knowledge, such as the estimating ranges of the parameters.

To sum up, the proposed method is expected to well meet the high-accuracy and low-complexity requirements for motion compensation in the real-time ISAR imaging, which makes promotive sense for the high-quality and rapid target recognition and classification.

2 Signal model

Based on the synthetic aperture principle and the two-dimensional (2D) turntable model [20], the ISAR image is constructed by collecting the scattered field for different frequencies and look angles. That is the received data are collected in the spatial-frequency domains, and then projected onto the XOY plane, as shown in Fig. 1.

As the convention of radar imaging, the observing target is often considered as an aggregation of strong scattering points, and the phase centre O is generally selected as the geometry centre of the target. Then in the geometric ISAR system model, we assume that the X-axis, in the direction of RLOS, to be along the radial direction and the Y-axis in the azimuthal direction. \( P(k, y_k) \) is one of the \( K \) scattering points on the target, with \( A_k \) \((k = 1, \ldots, K)\) representing the backward scattering amplitude. Then the baseband echo signal can be described as

\[
 s(t) = \sum_{k=1}^{K} A_k \exp\left(-j \frac{4\pi f_1}{c} R(t) \right)
\]

where \( f_1 \) is the transmitted carrier frequency, \( c \) refers to the speed of light in the vacuum, and \( R(t) \) denotes the instantaneous range from the scattering point \( P_k \) to the radar. Denote \( T_o \) as the observing duration, then \( 0 \leq t \leq T_o \).

For the far-field targets and the small-angle observation (at most 5–6° generally), \( R(t) \) can approximately be calculated by

\[
 R(t) \approx R_o + x_k - y_k \theta_i (2)
\]

where \( R_o \) is the instantaneous range from the phase centre of the target to the radar and can be eliminated via the translational motion compensation, and \( \theta_i \) is the instantaneous rotation angle.

Without loss of generality, the translational movement of the target can be precisely described in the form of Taylor series as follows:

\[
 R_i = R_o + \Delta r = R_o + \sum_{i=1}^{I} \frac{1}{i!} \sum_{j=1}^{I} v_i^j \Delta t^j
\]

where \( \Delta r \) is the range shift introduced by the translational motion of the target. \( v_i \) represents the \( i \)-th order translational moving parameter, for instance \( v_i \) is the radial velocity and \( v_1 \) is the radial acceleration. \( I \) is the on-demand highest order for describing the radial motion. In fact, for the most real-world scenarios, the target motion can be realistically described by two-order dynamics. For
the convenience of discussion, the initial distance $R_0$ is ignored during the following derivation process.

In practical processing, the successive baseband echo signal is often sampled along the fast-time (range) and the slow-time (cross-range/azimuth) dimensions to represent the relative discrete forms of the range direction and the azimuth direction. Assuming that the step frequency (SF) radar transmits a sequence of $N$ bursts with the transmitting interval $T_B$ and the bandwidth $B$. Each burst consists of $M$ narrow frequency band pulses with the pulse repetition interval $T_s$. During the CPI, the rotated angle of the target is $\Omega$, then the resolution of the system can be expressed as

$$\delta_t = \frac{c}{2B}$$

$$\delta_{\Omega} = \frac{\lambda_c}{2\Omega}$$

(4)

where $\lambda_c$ is the wavelength corresponding to the carrier frequency. $\delta_t$ and $\delta_{\Omega}$ represent the range resolution and the cross-range resolution, respectively.

As the observing time is generally sampled as $t(m,n) = (m - 1)T_s + (n - 1)T_B$ by $m = 0, \ldots, M - 1$ and $n = 0, \ldots, N - 1$, then during the fast-time period, the instantaneous carrier frequency is $f_m = f_r + (m - 1)\Delta f$, where $f_r$ is the reference frequency, $\Delta f$ is the frequency step, and during the slow-time period, the rotated angle in each burst is denoted as $\theta_t$. Thus, the discrete echo data can be arranged into a 2D data array in the size of $M \times N$: (see (5)). Since the value of $T_s$ is very small and can be ignored, $\Delta r_s$ is regarded as constant in an azimuth bin and can be calculated by

$$\Delta r_s = \sum\frac{1}{\sum T_B[(n-1)T_B]}$$

(6)

Then the discrete form of the range echo can be expressed as

$$s_k(m) = \sum A_k \exp\left(-\frac{4\pi}{c} \cdot (x_k + \Delta r_s)\right)$$

(7)

It can be seen that from (5) and (7), only if the phase term including $\Delta r_s$ is eliminated, the Fourier transform pairs $f_m \sim x_k$ and $\theta_t \sim x_k$ can be well employed to stand out the coordinates of each scattering point. Specifically, in the case of uniform rotation, the ISAR image can be obtained by performing 2D inverse Fourier transform to the data after translational motion compensation, as shown in (8):

$$\text{ISAR}(X, Y) = \int_{\Omega} \int_{\Theta} \sum A_k \cdot \exp\left(\frac{4\pi}{c}(-X - x_k)\right) \cdot \exp\left(\frac{4\pi}{\lambda_c}(Y - y_k)\right) \cdot \Delta f \cdot d\theta$$

$$= \frac{4\pi \Omega}{\lambda_c} \sum A_k \sin\left[\frac{2\pi B}{c}(X - x_k)\right] \cdot \frac{2\pi}{\lambda_c}(Y - y_k)$$

(8)

Therefore, the translational motion compensation based on parameters estimation for $v_1, \ldots, v_j$ is the prerequisite for further imaging processing. For the most real cases, the radial velocity and the transmitting interval are sampled along the fast-time (range) and the slow-time (cross-range) dimensions to represent the relative discrete forms of the range direction and the azimuth direction. Assuming that the step frequency (SF) radar transmits a sequence of $N$ bursts with the transmitting interval $T_B$ and the bandwidth $B$. Each burst consists of $M$ narrow frequency band pulses with the pulse repetition interval $T_s$. During the CPI, the rotated angle of the target is $\Omega$, then the resolution of the system can be expressed as

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where $\lambda_c$ is the wavelength corresponding to the carrier frequency. $\delta_t$ and $\delta_{\Omega}$ represent the range resolution and the cross-range resolution, respectively.

3 Motion compensation method

In this section, we present a HSACM to achieve the high-accuracy parameter estimation and motion compensation for ISAR imaging. The HSACM adopts the symmetric accumulation manner to generate cross-correlations and uses the symmetric characteristic to optimise the computational complexity. After that the parameter estimation is formulated as the LS problem and then high-order parameters can be figured out by solving the LS problem.

3.1 Symmetric accumulation

Based on the principle of the one-order parameter estimation methods, such as the CCM and the ACCM, the so-called ‘range-walk’ (i.e. range shift) can be obtained by the cross-correlation processing to different range profiles, and then the moving parameter of the target can be estimated by solving the relation between the range shift and the observation interval. In this paper, we expand the linear range migration introduced by the one-order parameter estimation to the two-order range migration, thus the corresponding ‘range curvature’ is supposed to be collected for estimating the high-order parameters. Then similar to the one-order case, the prior step is to generate a set of cross-correlations for figuring out the unknown parameters from the echo data.

To improve the performance of the cross-correlation processing, we firstly proposed the symmetric accumulation manner in our previous work [22], which calculates the cross-correlation by accumulating processing.

In the actual discrete data processing, the cross-correlation can be fast calculated via computing the convolution [29]. By denoting the range profile of the $q$th echo as $RP_q$, we can obtain the cross-correlation as

$$C_{0q} = \left|\text{IFFT}(\text{FFT}(RP_q) \cdot \text{FFT}(RP_0))\right|$$

(9)

where $RP_0$ represents the reference range profile and the first range profile $RP_q$ is usually selected as the reference. The FFT($\cdot$) and IFFT ($\cdot$) denote the one-dimensional (1D) fast Fourier transform and the inverse fast Fourier transform of a vector, respectively, and the $(\cdot^*)$ denotes the complex conjugate of a matrix. Each $C_{0q}$ is a vector of length $M$, the location of the peak value indicates the range shift between the two involved range profiles.

Accordingly, the symmetric accumulated cross-correlation is expressed as

$$S_{\text{ACK}}^{\text{eff}} = \sum_{q=0}^{Q-1}\left[\text{IFFT}(\text{FFT}(RP_q) \cdot \text{FFT}(RP_0)^*) + \text{IFFT}(\text{FFT}(RP_q^*) \cdot \text{FFT}(RP_0^*))\right]$$

(10)

$$= \sum_{q=0}^{Q-1}(C_{0q,RP} + C_{0q,RP^*})$$

(5)
where \( Q \) is the symmetric accumulation number. For the confirmation of the optimal symmetric accumulation number \( Q \), we define an accumulation factor as
\[
\gamma = \frac{N}{2Q \cdot (L/L_d)} = \frac{1}{\rho}
\]
(11)
where \( \rho = 2Q/N \) represents the accumulation ratio, \( L/L_d \) expresses the required sampling number in the cross-range for the target in size of \( L_d \), and \( L/L_d \) represents the cross-range resolution. Practically, taking the C-band ISAR system with metre-level resolution (e.g. \( \delta_r = 0.375\text{m} \)) as an example, then for most observation targets with the size of tens of metres, the general case can be represented by \( \gamma = 64 \).

On one hand, the larger the accumulated length, the better the stable echo components can be enhanced, and the better accuracy can be achieved for parameter estimation. However, on the other hand, the more range profiles involved in the accumulation processing (especially those far from the reference one), the larger range shifts and rotated angle are generated, which may result in the migration through resolution cell [30]. Therefore, the accumulation number should correspond to the sampling intervals of one or half of a cross-range resolution unit. That is to say the accumulation factor \( \gamma \) should be in the range of 0.5–1. Therefore, the optimal symmetric accumulation number for the general case can be determined by
\[
Q = \frac{N}{256\gamma} \quad \gamma = 0.5 - 1
\]
(12)

### 3.2 Complexity optimisation

Due to the symmetric characteristic, the corresponding optimisation can be carried out to reduce the computational complexity. To discuss the complexity optimisation degree, we set \( Q = 4 \) as a numerical example. Fig. 2 shows the optimised symmetric accumulation structure for generating the cross-correlations. As shown in area II in this figure, there are some cross-correlation pairs in the form of \( C_{ij} \) and \( C_{ji} \). That is the great feature of the symmetric processing. According to the commutativity property of the convolution operation, we can derive that \( C_{ij} = C_{ji} \). Therefore, the number involved in convolution in area II in Fig. 2 can be reduced by half when we calculate the accumulated cross-correlations. Therefore, area II is also called as the ’symmetric area’.

Now, we conduct the quantitative analysis of the optimisation degree. Table 1 lists the required convolution operation number and the index expression of the cross-correlations mapping to the subareas in Fig. 2, where \( U = N - 2(Q - 1) \) denotes the length of the symmetric area, \( \Pi_{\text{Up}} \) and \( \Pi_{\text{Down}} \) correspond to the upper and the lower triangular parts in the symmetric area, and \( \Pi_{\text{Diag}} \) indicates the diagonal elements in the rectangular area. Then the reduced number of the convolution computation can be calculated as
\[
N_{\Pi} = \frac{U^2 - Q^2 - U + Q}{2}
\]
(13)
while the total number of the convolution operations of generating the symmetric accumulation cross-correlations without optimisation is given by
\[
N_{\text{con}} = U^2
\]
(14)
Thus, the complexity optimisation degree can be calculated as
\[
O_{\gamma} = \frac{N_{\Pi}}{N_{\text{con}}} = \frac{1}{2} + \frac{U - Q(Q - 1)}{U^2}
\]
(15)
As the symmetric accumulation number \( Q \) is far smaller than \( N \) (i.e. \( Q << N \) and \( Q << U \)), then the last term in (15) is approximately equal to zero and the optimisation degree is approximately up to 50%. Actually, compared to the conventional accumulation manner in the classical ACCM, the computational complexity is indeed reduced by half for the same accumulation length.

For the numerical case of \( N = 256 \), according to (12), we set \( \gamma = 0.5 \), then \( Q = 4 \), and the convolution operations can be reduced from 62,500 to 31,381 times by taking the advantage of the optimisation.

### 3.3 LS problem formulation

Based on the obtained cross-correlations of the range profiles, the differential range shifts between the different observing intervals can be calculated via the sample statistical method. After that the parameters of the target can be estimated by formulating the high-order parameter estimation as the LS problem. The range shift between the nth range profile and the first range profile as \( \Delta r_{ni} \), according to (7), the mathematical relationship between the motion parameters and the range shifts can be derived: (see (16)). By rewriting the above three matrices as \( \Gamma \in \mathbb{R}^{N - 1 \times \min(I)} \), \( V \in \mathbb{R}^I \), and \( \Lambda \in \mathbb{R}^{N - 1} \), then (16) can be simplified as
\[
\Gamma V = A
\]
(17)
where \( \Gamma \) represents the time interval matrix and \( A \) represents the differential range shifts matrix. These two matrices are known data. The parameter matrix \( V \), including \( I \) orders of the motion parameters, needs to be estimated. Then the parameter estimation can be formulated as the LS problem
\[
\min_{V \in \mathbb{R}^I} \| \Gamma V - A \|_2
\]
(18)
Since in real application scenarios, the target motion can be precisely described by two-order dynamics, i.e. \( I \ll N \), thus \( \Gamma \) is always a column full rank matrix. Therefore, the LS problem can be solved by calculating the left pseudo inverse matrix of \( \Gamma \) as
\[
V = \Gamma^+ A = (\Gamma^T \Gamma)^{-1} \Gamma^T A
\]
(19)

![Fig. 2 Complexity-optimised structure for generating the cross-correlations by utilising the symmetric accumulation manner](image-url)
where \((\cdot)\) denotes the pseudo inverse matrix of a matrix and \((\cdot)^{-1}\) denotes the inverse matrix of a invertible matrix. \((\cdot)^{H}\) denotes the Hermitian transpose of a matrix. As \(I^H = I^T\), its Hermitian transpose coincides with its transpose (i.e. \(I^{H} \approx I^{T}\)). By solving this LS problem, the high-order parameter estimation can be achieved.

### 3.4 High-order motion compensation via HSACM

As is known that both the range alignment and the phase adjustment are expected to be implemented during the motion compensation procedure. In general, the phase adjustment is accomplished by using a constant phase factor to eliminate the phase shift caused by the range shift. Actually, the maximum point of the correlation function of the range profiles corresponds exactly to this phase shift. That is to say, by performing the cross-correlation processing on the complex profile of the echo signal, both the range shift and the phase shift can be included in the complex compensation factor. In addition, due to the symmetric accumulation manner of the HSACM, the accumulated phase errors can be well offset simultaneously.

Therefore, the high-order motion compensation for ISAR imaging can be fast implemented via the HSACM as shown in Fig. 3, and the main procedure is simply listed as follows:

**Step 1:** Sample the baseband echo signal to form the data array \(E_i\) in size of \(M \times N\).

**Step 2:** Perform the range compression on \(E_i\) by applying 1D FFT along the pulses such that a total of \(N\) range profile vectors are obtained (each \(R\) is of length \(M\)).

**Step 3:** Confirm an appropriate symmetric accumulation number \(Q\) according to (12).

**Step 4:** Obtain the collection of the necessary cross-correlation set \(\{C_i\}\) according to the optimised scheme in Table 1.

**Step 5:** Obtain the collection of the symmetric accumulated cross-correlation set \(\{SACR_i\}\) according to (10), where the index \(n\) ranges from \(Q\) to \((N - Q + 1)\).

**Step 6:** Collect all the locations corresponding to the peak values of the obtained \(\{SACR_i\}\), smooth out the outburst ones by the robust locally weighted scatterplot smoothing (LOWESS) method [31], and obtain the average range shifts \(\langle \Delta r_{in} \rangle\).

**Step 7:** Form the time interval matrix \(T\), the range shift matrix \(A\), and the parameter matrix \(V\), and then construct LS expression as shown in (18).

**Step 8:** Solve the LS problem by (19) to obtain the radial motion parameters of the target.

**Step 9:** Perform the translational motion compensation via multiplying \(E_i\) by the following compensation factor:

\[
\hat{E}_i = \exp \left\{ \frac{4\pi f_1}{c} \sum_{j=1}^{I} \frac{1}{T^n_j} v_i \right\}
\]

(20)

where \(v_i \in V\) and \(i = 1, \ldots, I\). \(I\) is practically set as 2.

After that the subsequent ISAR imaging processing such as the azimuth compression can be implemented to get the 2D ISAR image. It is worth mentioning that the parameter estimation via the HSACM is based on the similarity of the envelope of the range profiles, and there is no priori knowledge about the translation motion is required. Hence, the HSACM is a kind of blind processing. Furthermore, compared with those method involved in searching for the matched value (e.g. the MEM), the estimating accuracy of the HSACM is independent of the searching size.

### 4 Experimental analysis and verification

In this section, the numerical simulation and real data experiments are designed to evaluate the performance of the proposed method. To objectively compare the real-time performance of the proposed method with other methods, all the experiments are executed under the same run-time environment. The system configurations of the computer are as follows:

- Intel Core i5-8500 CPU @ 3.00 GHz.
- 8 GB DDR4 RAM @ 1330 MHz.
- 128 GB solid state drive with 6.0 Gb/s SATA interface.
- 1 TB hard disk drive with 7200 RPM.
- 64-bit Windows 10 operating system.

Besides, the ACCM and the MEM are selected as the representative one-order and two-order comparable methods, respectively, in the following experiment. Several experiments are designed to illustrate the deficiency of existing popular methods and to investigate the performance of the proposed method.

![Flowchart of the motion compensation via the HSACM](image-url)
4.1 Numerical simulation

All the simulated experiments in this section are based on the hypothetical target as constructed in Fig. 4. The radar system transmits the SF signal. During the CPI, the radar transmits 128 bursts and each burst consists of 128 narrow band pulses. Other parameters of the radar system are as follows: the carrier frequency is 10 GHz, the bandwidth is 128 MHz, the pulse repetition frequency is 20 kHz. The radial moving parameters of the target are set to be \( v_1 = 30 \text{ m/s} \) and \( v_2 = -2 \text{ m/s}^2 \). The rotating angular speed is \( \omega_1 = 0.03 \text{ rad/s} \).

• **Test 1:** Verification of the advantage of the symmetric accumulation manner.

First, to validate the advantage of the symmetric accumulation manner, Fig. 5 shows the comparing result of the range shifts of the cross-correlations corresponding to different generating manners. To clearly distinguish the waveforms from each other, the illustrated index is selected at intervals of 40. Fig. 5a shows the cross-correlation waveforms generated by the elementary calculation manner in CCM, Fig. 5b shows the result via the conventional accumulation manner in ACCM, Fig. 5c is the waveforms via the symmetric accumulation manner in the proposed method. Fig. 5d visually shows the vibration curves of the range shifts generated by these three methods, in which the ‘smoothed RP shifts’ represents the smoothed range shifts of the actual range curvature and the ‘LOWESS’ smoothing mode is utilised to lower the polynomial order.

It can be seen that the cross-correlation curves in Fig. 5a are very noisy and multi-peaks; moreover, the range shifts present irregular intervals. This obviously introduces difficulty for finding...
the peaks and brings large errors to range alignment. Accordingly, the range shift curves shown in Fig. 5d reveal a sharp disturbance. By utilising the accumulation concept, the curves in Fig. 5b look more uniform. However, multi-peaks still exist obviously, which are mainly caused by the accumulated phase errors. Then by utilising the symmetric accumulation manner, the generated cross-correlation curves in Fig. 5e present much better performance on suppressing noise and reducing the phase errors. It is obvious that the set of curves is well conductive to find their peak portions and implement the range alignment. Consequently, the curves shown in Fig. 5f indicate that the range shifts corresponding to the symmetric accumulation manner produce the smallest disturbance. Therefore, the symmetric accumulation manner is verified to be effective and advanced on decreasing range alignment errors.

• Test 2: Performance analysis of the HSACM.

The above test indicates that the proposed symmetric accumulation manner is effective on offsetting the phase errors. Next, we quantitatively discuss the performance of the proposed method from the perspectives of the estimation accuracy and the computational cost.

Fig. 6 shows the normalised mean square errors (NMSE) of the estimated parameters by different methods, where the NMSE of a set of samples for the parameter \( X \) is defined as

\[
NMSE = 10\log \left( \frac{\sum_{i=1}^{N_{\text{ml}}} (X_{\text{est},i} - X_{\text{real}})^2}{N_{\text{ml}}X_{\text{real}}^2} \right)
\]

\[
NMSE = 20\log \left( \frac{\text{RMSE}}{X_{\text{real}}} \right)
\]

where \( N_{\text{ml}} \) is the Monte-Carlo number for the simulation, \( X_{\text{est},i} \) is the observed value for each estimation, \( X_{\text{real}} \) is the real value of the parameter, and RMSE is the root mean square error. It can be observed that, the NMSE is the normalisation of the RMSE in essence.

Since the CCM and the ACCM are the most popular one-order parameter estimation methods, and the MEM is the representative high-order parameter estimation method, these three methods are selected for comparing experiments. Fig. 6a presents the comparing result of the one-order parameter (velocity) via different motion compensation methods and Fig. 6b presents the high-order parameter (acceleration) estimation performance. In this simulation, we set the accumulation number as 4 (\( \gamma = 0.5 \)) and 2 (\( \gamma = 1 \)) for ACCM, and the symmetric accumulation number as \( Q = 2 \) (\( \gamma = 0.5 \)) and \( Q = 1 \) (\( \gamma = 1 \)) for the HSACM to meet the equivalent accumulation length. As for the MEM, the searching range of the velocity is \([v_1 - 10, v_1 + 10]\) and the acceleration is \([v_2 - 5, v_2 + 5]\). For the low-precision configuration, the searching step sizes are set to be 1 m/s and 0.1 m/s^2; and for the high-precision configuration, the searching step sizes are 0.1 m/s and 0.05 m/s^2. The Monte-Carlo number is 200.

Meanwhile, as the computational cost is the key affecting factor for real-time performance, Fig. 7 depicts their computational cost curves correspondingly. By comparing the detailed part of these three figures, the following conclusions can be derived:

(i) In general, for estimating the one-order parameter, the proposed method performs better than the MEM, ACCM, and the CCM, and produces the smallest errors; while for estimating the high-order parameter, the proposed method and the MEM perform almost the same.

(ii) Due to being limited by the searching step size, the NMSE curves of the MEM no longer keep going down as the SNR increases. That is the precision of the MEM is set in advance, while the propose method can maximise the precision in a good SNR environment.

(iii) For the ACCM and the HSACM, the performance corresponding to \( \gamma = 0.5 \) is better than \( \gamma = 1 \), which verifies that the more echoes involved in the accumulation, the better accuracy is achieved.

(iv) As for the computational complexity, the cost of the MEM is much larger than the CCMs. Due to the optimisation of the symmetric accumulation, the complexity of the HSACM is only half of the ACCM.

• Test 3: Application of the HSACM in ISAR imaging.

Next, we perform the simulation for the application of the proposed method in motion compensation and ISAR imaging. In this test, the target is supposed to move with high-order dynamic parameters, and we compare the performance of the following four cases.

(a) Only one-order parameter estimation is performed by the ACCM. That is, the velocity of the target is estimated, but the acceleration cannot be estimated.

(b) High-order but low-precision parameter estimation is performed by the MEM with the searching step size of 1 m/s and 0.1 m/s^2.

(c) High-order and high-precision parameter estimation is performed by the MEM with the searching step size of 0.1 m/s and 0.05 m/s^2.

(d) High-order parameter estimation is performed by the proposed method (HSACM) with the accumulation factor \( \gamma = 0.5 \).

Fig. 8 shows the range alignment results by different motion compensation methods and Fig. 9 shows the corresponding ISAR images of the simulated target. The SNR is −5 dB. The performances (including the image entropy, run time, and the estimated values) are listed in Table 2.

As is well known that if a successful motion compensation practice is applied, no (or minimal) range shifts between consecutive burst intervals can be expected. Fig. 8a is the range

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**Fig. 6** Comparison of NMSEs of different methods
(a) NMSE of the one-order parameter (velocity), (b) NMSE of the high-order parameter (acceleration)

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compression spectrogram that is compensated by the one-order ACCM. Due to the lack of the high-order motion parameter estimation, the range profiles are not well aligned. Consequently, the image shown in Fig. 9a is severely blurred. Figs. 8b and c are the results by the MEM with low and high precision, respectively. As to the low-precision configuration, the image shows obvious
ghost because of the insufficient precision. The last two sub-figures in Figs. 8 and 9 indicate that the high-precision MEM performs as good as the proposed method. According to the data listed in Table 2, the entropy values of the high-precision MEM and the proposed method are nearly equal, which means the quality of the obtained images is almost the same. However, the computation cost of the MEM is hundreds of times higher than the proposed method. Even for the low-precision MEM, the cost is ten times of the HSACM.

To sum up, compared with other popular methods based on the correlation of the echo data, such as the CCM and the ACCM, the HSACM can realise the high-order and higher precision parameter estimation, and its computation cost is only half of the ACCM (with the same accumulation length). Compared with the most popular high-order method, such as the MEM, the HSACM has no requirement of priori knowledge. What is more, it only spends quite small computation cost but can achieve similar performance.

4.2 Real data

In this subsection, the real plane experiment is performed by using the measured data of the Yak42. The parameters are as follows: the Chirp radar transmits 256 pulses with the PRF = 400 HZ, each pulse is sampled by 256 points, the carrier frequency is 5.52 GHz, the bandwidth is 400 MHz, the translational velocity is set to be 200 m/s, and the acceleration is 5 m/s². In the case of the manoeuvring moving, the range profiles are misaligned as shown in Fig. 10a. Accordingly, Fig. 10b shows the obtained image via the traditional RD approach. It can be seen that, since the scattering points in a range bin cannot be distinguished via the cross-range compression, the image is visibly blurred. Figs. 10c and d show the ISAR images after implementing the high-order motion compensation by MEM and HSACM, separately. The searching ranges for the MEM are set to be [200, 240] m/s and [0, 10] m/s², and the symmetric accumulating number of the HSACM is 4 (γ = 0.5).

Table 3 lists the entropy values of the images and the computational costs of this experiment. From the view of intuitive appearances and the entropy values, it can be seen that Figs. 10c and d present similar quality, and the proposed method shows better performance on noise suppression. However, under the same experimental environment, the MEM spends 10-s level running time to get the image, while the proposed method only demands less than one-tenth of a second. Not to mention that, the priori estimation knowledge is required by the MEM.

It can be concluded that the proposed method is verified to achieve the high-quality and low-complexity motion compensation for ISAR imaging, without requirement of the priori estimation knowledge. There is no doubt that under a more advanced operating environment, the proposed method is greatly conductive to real-time imaging and target recognition.

| Method (precision) | Entropy | Run time, s | Estimated values, m/s, m/s² |
|-------------------|---------|-------------|-----------------------------|
| ACCM (γ = 0.5)    | 3.5334  | 0.0468      | (28.69, −)                  |
| MEM (1, 0.1)      | 3.3529  | 0.4309      | (26, −2.3)                  |
| MEM (0.1, 0.05)   | 2.6864  | 7.3808      | (29.8, −1.90)               |
| proposed (γ = 0.5)| 2.6668  | 0.0484      | (29.78, −1.95)              |

Fig. 9 ISAR image results of the simulated model
(a) ISAR image after one-order motion compensation via ACCM, (b) ISAR image after high-order motion compensation via MEM (low precision), (c) ISAR image after high-order motion compensation via MEM (high precision), (d) ISAR image after high-order motion compensation via HSACM.

Table 2 Imaging quality and cost of the simulated target

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5 Conclusion

In this paper, an optimised and improved CCM, i.e. HSACM, is proposed to realise the high-order parameter estimation for ISAR motion compensation. The proposed method uses the symmetric accumulation manner instead of the common accumulation manner, by which, the negative and positive phase errors can be well offset during the accumulation processing. Additionally, by taking the advantage of the symmetric characteristic of the cross-correlations, the computational cost of the proposed method is optimised by half. After that the parameter estimation is formulated into the LS problem to solve the high-order parameters of the target. Then the high-accuracy motion compensation and real-time imaging can be implemented. Theoretical analyses and experimental simulation confirm that the proposed method has the advantages of high estimation accuracy and low computational complexity compared with other comparable methods. Specifically, compared with other popular methods based on the correlation of the echo data, such as the CCM and the ACCM, the proposed method can realise the high-order and higher precision parameter estimation, but only yields half of the computation cost of the ACCM; while compared with the most popular high-order method, such as the MEM, the proposed method has no requirement of the priori knowledge. What is more, it only spends 1% the computation cost of the MEM, but can achieve the same quality of estimation performance. Therefore, the proposed method is well conductive to the high-quality and real-time ISAR imaging, and facilitates the high-quality and rapid target recognition and classification.

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