Thomson backscattering in combined two laser and a magnetic fields

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Abstract – The Thomson backscattering of an electron moving in combined fields is studied by a dynamically assisted mechanism. The combined fields are composed of two co-propagating laser fields and a magnetic field, where the first laser field is strong and low-frequency while the second is weak and high-frequency, relatively. The dependence of the fundamental frequency of emission on the ratio of the incident laser high-to-low frequency is presented and the spectrum of backscattering is obtained. It is found that, with a magnetic field, the peak of the spectrum and the corresponding radiation frequency are significantly larger in the case of two lasers than that in the case of only one laser. They are also improved obviously as the frequency of the weak laser field. Another finding is the nonlinear correlation between the emission intensity of the backscattering and the intensity of the weak laser field. These results provide a new possibility to adjust and control the spectrum by changing the ratios of frequency and intensity of the two laser fields.

Introduction. – The Thomson backscattering is widely used to create x-rays and γ-rays. During the past twenty years, several studies about the Thomson backscattering have revealed how the spectra depend on the laser field [1–3] and external magnetic field [4,5].

At the beginning of this century, He et al. showed that the spectrum of the Thomson scattering does not occur at integer multiples of the laser frequency and they got a simple scaling law for the spectrum of the backscattered radiation of an electron in a linearly polarized plane wave [6,7]. Actually the perspective of the laser-magnetic resonance acceleration [8–12] has shown that the relativistic cyclotron motion will occur in transverse direction under an external magnetic field. There are a lot of advantages in the electron resonance acceleration in a laser field assisted by the external magnetic field [13].

It is demonstrated that, in the plane of the electron cyclotron orbit, the electron will emit radiation at high harmonics of the cyclotron frequency [14]. Furthermore, the cyclotron resonance condition is met when the cyclotron frequency approaches the laser frequency, and the helically periodic motion of electrons would result in the fact that the fundamental frequency of harmonic emission is neither the laser frequency nor the cyclotron frequency for the backscattered Thomson spectra [6].

The cyclotron resonance motion in the combined laser and magnetic fields will affect the frequency of emission, therefore, in our previous work, Thomson scattering in combined fields with a circularly [15] or elliptically [16] polarized laser field and a strong magnetic field has been studied in detail. The scale invariance and scaling law for the laser intensity and initial axial momentum are found [15]. Another interesting finding is a new way to produce THz emission by using the Thomson backscattering [16].

In the past two years, strong nonlinear characteristics of the Thomson scattering have been reported. With nonlinearity, the shape of the backscattering spectrum can be controlled by proper laser chirping. The results allow the prediction of the spectral form and narrowing bandwidth together with high efficiency [17,18].
The case of the Thomson scattering using two counterclockwise propagating laser pulses has also been reported [19]. It is found that the radiation power of the electron's Thomson scattering can be significantly enhanced in comparison with that obtained by only using one of these two pulses, both in the case of linear and circular polarization. Moreover, the Compton scattering using two different wavelength lasers has been considered [20]. It has been proposed that it can provide a new way for controlling scattered photon energy distributions. The electron trajectory and photon energy have been calculated under different ratios of the two lasers' intensity. And the best ratio of intensity can be found through comparison.

It is worth saying a few words about the fact that when the second laser field is introduced, the radiation spectra exhibit very complicated phenomena and the radiation is enhanced greatly. The idea is very similar to the dynamically assisted mechanism in the study of emission from the position of an electron (with mass $m$ and charge $-e$) moving in combined laser and magnetic fields. The magnetic field $B_0$ is parallel to the laser propagating direction ($z$-direction). The phase of the laser field is denoted as $\eta = \omega_0 t - k \cdot r$, where $\omega_0$ is the laser frequency, $t$ is the time, $k$ and $r$ are the wave vector and electron displacement vector, respectively.

We assume that the phase has an initial value $\eta_0 = -z_0$ at $t = 0$. Similarly to the techniques in [15], we obtain the $m$-th spectrum of the harmonic radiation in unit of erg/s per unit solid angle for backscattering as follows:

$$\frac{d^2 I_m}{d\Omega dt} = \frac{e^2}{4\pi^2 c^2} \left( m\omega_0 \right)^2 \left( |F_{mx}|^2 + |F_{my}|^2 \right),$$

(1)

where

$$F_{mx,my} = \omega_1 \int_{\eta_0}^{\eta_0 + T} d\eta p_{x,y}(\eta) \exp[i m \omega_1 (\eta + 2z)],$$

(2)

from which one can see why the electron's transverse momenta $p_x$ and $p_y$ and longitudinal displacement $z$ are important here. The constant of motion is obtained as $z = \gamma z - p_z = \gamma_0 - p_{z0}$, where $\gamma$ is the electron relativistic factor derived from electron velocity, $p_z$ is the $z$ component of the electron relativistic momentum and $\gamma_0$, $p_{z0}$ are their initial values. The fundamental frequency $\omega_1$ in dimensionless form for backscattering reads now

$$\omega_1 = \frac{2\pi}{T + 2z_0},$$

(3)

where $T$ is the period of the electronic periodic motion and $z_0 = (0, 0, z_0)$ is the drift distance.

The laser field is composed of two plane waves $A_1$ and $A_2$. The ratio of the intensity is $\lambda$ and the ratio of the frequency is $\mu$. Since the laser propagation and external magnetic field are along the $z$-direction, the combinational total vector potential of combined fields can be written as

$$A = A_1 + A_2 + A_3 = \frac{A_0}{\sqrt{1 + \alpha^2}} (-\sin \eta \hat{j} + \alpha \cos \eta \hat{j}) + \frac{\lambda A_0}{\sqrt{1 + \alpha^2}} (-\sin \mu \hat{j} + \alpha \cos \mu \hat{j}) + B_0 \hat{j},$$

(4)

where $\alpha$ stands for the ellipticity of the elliptically polarized plane wave. By the way it is noted that since there is no scalar potential, two gauges, the Coulomb one or the Lorentz one, are equivalent to each other.

We normalize time by $1/\omega_0$, velocity by $c$, distance by $k_0^{-1} = c/\omega_0$, and momentum by $mc$. We obtain the equations for the momentum of the electron in the following

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forms:

\[
\frac{d^2 \omega_z}{d \eta^2} + \omega_z^2 \omega_x = \left( \omega_b + \frac{1}{\sqrt{1 + \alpha^2}} \right) a \sin \eta + \left( \omega_b + \frac{\mu}{\sqrt{1 + \alpha^2}} \right) a \mu \lambda \sin (\mu \eta), \tag{5}
\]

\[
\frac{d^2 \omega_y}{d \eta^2} + \omega_y^2 \omega_y = -\left( \omega_b + \frac{1}{\sqrt{1 + \alpha^2}} \right) a \cos \eta - \left( \omega_b + \frac{\mu}{\sqrt{1 + \alpha^2}} \right) a \mu \lambda \cos (\mu \eta), \tag{6}
\]

where \( \omega_b = b/\varsigma \) with \( a = eA_1/mc^2 \) and \( b = eB_0/m\omega_0c \) being the normalized vector potential and magnetic field, respectively.

The transverse momentum of the electron are obtained and the longitudinal momentum can also be obtained with the help of \( p_z = \frac{\rho^2 + p_z^2 + 1 - c^2}{2c} \). Furthermore, assuming that at \( t = 0 \) the electron is static and located at \( x = 0 \), \( y = 0 \) and \( z = z_0 \), so that \( \eta = \eta_0 = -2z_0 \). Moreover, the momentum and longitudinal displacement of the electron, \( z_0 \), would be easily obtained from the trajectory equations of the electron \( dr/d\eta = p/\varsigma \). These expressions are not presented here since the formulae are lengthy for a general elliptical case. We just point out a fact that \( p_x \) and \( p_y \) are linearly superimposed, while \( z \) is nonlinear, which will then lead to a great influence on the following study about the backscattering.

From now on for the simple and convenience calculations and discussions, we just consider the circular case, \( i.e., \), ellipticity \( \alpha = 1 \). In this case, the required momentum and displacement are expressed as

\[
p_x = na \{ \sin \eta - \sin [\omega_b \eta - (\omega_b - 1) \eta_n] \} + la \{ \sin \mu \eta - \sin [\omega_b \eta - (\omega_b - \mu) \eta_n] \}, \tag{7}
\]

\[
p_y = na \{ \cos [\omega_b \eta - (\omega_b - 1) \eta_n] \} - \cos \eta + la \{ \cos [\omega_b \eta - (\omega_b - \mu) \eta_n] \}, \tag{8}
\]

\[
z(\eta) = \left( \frac{na}{\varsigma} \right)^2 \left\{ (\eta - \eta_n) - \frac{1}{\omega_b - 1} \sin [(\omega_b - 1)(\eta - \eta_n)] \right\} + \left( \frac{la}{\varsigma} \right)^2 \left\{ (\eta - \eta_n) - \frac{1}{\omega_b - \mu} \sin [(\omega_b - \mu)(\eta - \eta_n)] \right\} + \frac{1 - c^2}{2\varsigma^2} \right\} (\eta - \eta_n). \tag{9}
\]

The trajectory equations of the electron can be obtained with the help of \( p_z = \frac{\rho^2 + p_z^2 + 1 - c^2}{2c} \) and \( dr/d\eta = p/\varsigma \). Then the drift distance can be expressed as

\[
z_0 = \frac{2n \pi}{\varsigma^2} \left\{ n^2 a^2 + l^2 a^2 + \frac{3}{2} - \frac{c^2}{2} \right\}, \tag{10}
\]

where \( n = \frac{1}{\omega_b - 1} \) and \( l = \frac{\omega_\lambda \mu}{(n+1)/\mu - n} \).

Substituting eqs. (7)–(10) into eqs. (1)–(3), we can calculate the backscattering spectra, on which the following numerical results are based.

By the similar discussion about the period \( T = 2\pi n \) as in refs. [15,16], and keeping in mind that \( r_0 = (0,0, z_0) \) in eq. (3), then we have the fundamental frequency

\[
\omega_1 = \frac{c^2/n}{1 + 2a^2n^2 + 2a^2\lambda^2} = \frac{c^2/n}{1 + 2a^2n^2 + 2a^2 \left( \frac{n^2}{r(1+2\mu^2)} \right)^2}, \tag{11}
\]

Obviously this fundamental frequency depends on parameters \( a, n, p_z, \lambda, \) and \( \mu \).

In order to qualitatively and semi-quantitatively analyze the typical features of the Thomson backscattering spectrum, we can make an appropriate estimation of the emission intensity. For simplicity we only consider the case of \( p_z = 0, i.e., \varsigma = 1 \). Now the analytical expression of the radiation spectrum is approximated as

\[
\frac{d^2 I_m}{d \Omega} \approx \frac{e^2}{2\pi^2 c} (\omega_m)^2 (\pi n_1 a m_n)^2 J_0^2 (z_m), \tag{12}
\]

where \( z_m = m \omega_1 a_2 z_0^2 = \frac{2m \nu_n}{n+2n^2 \nu^2 + 2n^2 c^2} \) is defined to simplify the formula of the radiation spectrum. The asymptotic form of the Bessel functions, \( J_\nu(x) \sim \sqrt{\frac{2}{\pi x}} \cos(x - \frac{x^2}{2} - \frac{\nu^2}{4}) \) when \( x \gg |\nu^2 - 1/4| \), which is obviously met since \( z_m \sim m \gg 1/4 \), makes us simplify eq. (12) further as

\[
\frac{d^2 I_m}{d \Omega} \approx \frac{e^2}{2\pi^2 c} (\omega_m)^2 (\pi n_1 a m_n)^2 J_0^2 (z_m) \approx \frac{e^2}{4\pi^2 c} y_1 y_2, \tag{13}
\]

with

\[
y_1 = \frac{m \nu_1^{3/2}}{2n}, \tag{14}
\]

and

\[
y_2 = 1 + \sin(2z_m). \tag{15}
\]

Obviously \( y_1 \) is a power function depending on parameters \( a, n, \lambda, \) and \( \mu, \) and \( y_2 \) is an oscillation one.
Fig. 2: The backscattering spectrum (normalized by $\frac{e^2}{4\pi^2c}$) for different laser cases: (a) $\lambda = 0$; (b) $\lambda = 0.2$, $\mu = 8$; (c) $\lambda = 0.2$, $\mu = 15$ and (d) $\lambda = 0.5$, $\mu = 8$. The other parameters are $a = 1$, $n = 5$ and $p_{z0} = 0$.

Numerical results and analysis. – For the ignorance of the radiation backreaction [15], as mentioned in the Introduction, the chosen parameters need to satisfy $n^4a^5 < 10^7$ in the case of $p_{z0} = 0$ and $p_{z0}^2n^4a^5 < 10^6$ in the case of $p_{z0} \neq 0$. So in the following all chosen parameters are compelled to satisfy these requirements.

From eq. (9) we see that the displacement of the electron in the z-direction is nonlinear. It greatly influences the backscattering radiation intensity in eq. (2). While we have examined and calculated the velocity, the acceleration and also the trajectory of an electron, these results are not presented with figures in this letter because of the limited scope. However, it should be pointed out that in strong magnetic field, the axial motion in two lasers is much larger than that only in their linear addition by $A_1$ and $A_2$. These results confirm that the axial focusing effect on the spiral movement of electrons is enhanced by a larger magnetic field as well the strong nonlinear effect via relativistic charged particle dynamics.

Since the fundamental frequency $\omega_1$ depends on parameters $a$, $n$, $p_{z0}$, $\lambda$, and $\mu$, we can briefly discuss the relation between these parameters and the fundamental frequency. First from eq. (11), it is expected that $\omega_1$ decreases as $a$, $n$, $p_{z0}$, $\lambda$. In fact our numerical calculations have confirmed these conclusions. However, $\mu$ can lead to the resonance case where $\mu_{res} = \omega_0 = 1 + 1/n$ which makes $l$ very large so that $\omega_1$ would be reduced drastically. This interesting nontrivial results are shown in fig. 1. Indeed in the resonance case of the second laser frequency matching the cyclotron frequency, the fundamental frequency is decreased drastically, which is very favourable to the THz regime.

Now let us turn to see the influence of two lasers on the Thomson backscattering spectrum. In fig. 2, the backscattering spectra (normalized by $\frac{e^2}{4\pi^2c}$) in four cases are shown. For simplicity the initial axial momentum $p_{z0} = 0$ is considered and $a = 1$, $n = 5$. Other parameters are: (a) $\lambda = 0$; (b) $\lambda = 0.2$ and $\mu = 8$; (c) $\lambda = 0.2$ and $\mu = 15$; (d) $\lambda = 0.5$ and $\mu = 8$. Obviously, fig. 2(a) is the spectrum which corresponds to the case of only $A_1$. Compared to fig. 2(a), the remaining three plots show that the frequency and intensity of the spectrum tend to increase in the case of $A_1 + A_2$, see figs. 2(b), (c) and (d).

By comparing fig. 2(c) with (b), one can see that the emission spectrum moves to a higher-frequency regime while the high-frequency emission intensity is enhanced and the low-frequency emission is reduced as the $A_2$ field frequency increases. When the ratio of the frequency of
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two lasers $\mu$ increases by less than two times (from 8 to 15), however, the values of the harmonic orders $m$ changes from 318 to 2964, and the frequency where the main-peak emission locates is increased by about ten times from $\omega/\omega_0 \sim 1$ to $\sim 12$. On the other hand the main-peak intensity is also increased by about two times. It provides an alternative way to modulate and control the frequency and intensity of backscattering spectra by adjusting the second weak laser field frequency in addition to the first strong laser field.

Comparing fig. 2(d) with (b), when the intensity of $A_2$ increases and other parameters are fixed, it is not obvious that the spectra move to a high-frequency regime as well as the main-peak frequency shift, while the main-peak intensity is increased by about two times. It provides an alternative way to modulate and control the frequency and intensity of backscattering spectra by adjusting the second weak laser field frequency in addition to the first strong laser field.

It is worthy to note that, in our studied two-laser case, the main peak of the radiation spectra no longer appears at the fundamental frequency like in the one-laser case of fig. 2(a). With different parameters, the main peak will appear in the low-frequency (fig. 2(b)), high-frequency (fig. 2(c)) or intermediate (fig. 2(d)) region of the radiation spectrum. It also implies that the addition of $A_2$ to $A_1$ will change the shape of the spectrum nonlinearly.

The dependence of the main-peak frequency (black line with squares) in the spectra and the corresponding emission intensity (blue line with circles) on the ratio of two lasers’ frequency, $\mu$, are shown in fig. 3. The parameters are chosen as $p_{z0} = 0$, $\lambda = 0.2$ and (a) $a = 1$, $n = 5$; (b) $a = 1$, $n = 10$; (c) $a = 2$, $n = 5$ and (d) $a = 3$, $n = 5$. It is found that the frequency at peak, $\omega_{\text{peak}} = m\omega_1$, and the corresponding intensity both tend to increase with $\mu$. In addition, this increasing tendency is not a linear relation but has an oscillatory increasing behavior. The peak is probably in the low-frequency region or the high-frequency region of the radiation spectrum rather than only in that of fundamental frequency, as mentioned above and observed in fig. 2. It reflects indeed the nonlinear effects of two lasers on the Thomson backscattering.

Similarly we can also see the dependence of the main-peak frequency (black line with squares) of the spectra and the corresponding emission intensity (blue line with circles) on the ratio of two lasers’ intensity, $\lambda$, which are shown in fig. 4. Now $p_{z0} = 0$, $\mu = 5$ and $n = 5$ are fixed but the different laser intensity of $A_1$ is given as $a = 1$, 2, 3, 4 for (a)–(d), respectively. It is found that as the intensity of $A_1$ increases, obviously the dependence of the main-peak frequency on $\lambda$ seems to increase also in an oscillatory way. However, the corresponding peak intensity changes with $\lambda$ either ascendant or descendant, which exhibits such a high nonlinear characteristic that there is no obvious global tendency.

Here we maybe have a simple but qualitative discussion on the above results. In fact as is shown in eqs. (13)–(15),
Fig. 4: The main-peak frequency of spectra (black line with square symbols) and the corresponding main-peak intensity normalized by $e^2/4\pi^2c$ (blue line with circle symbols) vs. two-lasers intensity ratio, $\lambda$, for different cases of the first laser field intensity: (a) $a = 1$; (b) $a = 2$; (c) $a = 3$ and (d) $a = 4$. The other parameters are $n = 5, \mu = 5$ and $p_{z0} = 0$.

$y_2$ oscillates as $\mu$ or $\lambda$ increases, so we pay attention to the value of $y_1$. When $\mu$ increases, in particular in most cases with $\mu \gg \mu_{res} = 1 + 1/n$, the parameters $l$ and $\omega_1$ almost do not change with $\mu$, see fig. 1. Therefore, as $\omega_{peak} = m\omega_1$ increases as $m$, the radiation intensity increases, too, but in an oscillatory way. On the other hand, when $\lambda$ increases, the three parameters $l$, $\omega_1$ and $m$ are all changed, which affect the $y_1$ in a complex nonlinear way. So the strong nonlinear characteristics of the intensity are exhibited in fig. 4. Although they do not show a strict relation, we can adjust the backscattering spectra to what we need by adjusting the appropriate intensity ratio of two lasers. For example, in fig. 4(b), by slightly increasing the intensity of the $A_2$ from 0.5 to 0.6, the backscattering spectrum increases by about two times in the peak intensity, however, the corresponding peak frequency is shifted little.

**Conclusions and discussions.** – In this letter, we study the Thomson backscattering of an electron in combined two lasers and constant magnetic fields with a dynamically assisted mechanism. The emission fundamental frequency and backscattering spectra are obtained and then analyzed. We have revealed the effects of some factors on the Thomson backscattering.

In summary, the nonlinear effects of the axial motion of the electron would result in the nontrivial behavior on the Thomson backscattering spectra. When the second laser frequency matching the cyclotron frequency, the decreased fundamental frequency is favourable to the THz regime. Moreover, by changing the frequency or the intensity ratio of high-frequency to low-frequency lasers, the main-peak frequency of the backscattering spectrum and the intensity of the peak intensity exhibit strong nonlinear characteristics.

It should be pointed out that the dynamically assisted mechanism is studied extensively in the research of pair production in strong background fields, however, it is the first time that it is applicable to the Thomson scattering study presented here. The enhanced spectra intensity, the high nonlinear characteristics as well as the controllability of the emitted spectral bandwidth are evident by this mechanism.

On the other hand, the present research topic has still some open problems. For example, we only discuss backscattering under two laser fields and a magnetic field, while scattering in other directions is interesting and important, as we considered before with respect to forward and full-angle scattering under a single laser field [25,26]. Another very interesting and invaluable point is to consider the more realistic laser with profile and electron bunch with initial distribution. Moreover, the concrete interference structure due to two fields’ nonlinear addition of the corresponding electron’s displacement, which appears first in the phase part of the Thomson scattering formula, i.e., the exponent function part of eq. (2) and

\[ \phi_{peak} = m\omega_1 \]
then the modulus as in eq. (1), and how this interference affects the final spectra are still open questions. However, these complicated and abundant topics exceed the scope of present letter and certainly need to be researched in detail in the future.

Anyway our results presented here provide a new possibility to adjust or/and control the Thomson spectra for the THz or/and x-ray light source. Obviously, the production of either THz or x-rays depends on the fundamental frequency which is strongly associated to the electrons’ initial longitudinal momentum, two lasers’ parameters as well as the applied magnetic field.

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