Vortex and glass bead interaction in nematic liquid crystal layer

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Abstract. An homeotropically aligned liquid crystal layer driven by external electric or magnetic fields exhibit an intricate network of defects called umbilics or vortexes. Here, we report an experimental characterization of vortexes-glass bead interaction in nematic liquid crystal layer. The glass spheres, embedded in the liquid crystal, are found to be of two type: some of them attractive, with different strength of attraction; and most of them non-attractive. The attractive glass spheres pull the vortexes so that the distance decays approximately with the square root of time. The glass bead sphere induces an anisotropic force on the vortex. This force can be well approximate by an inverse power law of the distance between vortex and glass bead with exponent 3/4. The vortexes are attracted to a polar region of the attractive glass bead.

1. Introduction
Nematic liquid crystal layers with negative anisotropy and homeotropic anchoring under the influence of an external voltage can exhibit complex spatiotemporal dynamics textures [1]. The critical voltage from which one observes this intriguing dynamics is usually denominated as the Fréedericksz voltage [1, 2, 3]. Above the Fréederickz voltage the liquid crystal molecules change their orientation. This phenomenon is a result of the imbalance between the intrinsic elastic torque and electric torque resulting from the applied field. Indeed, the electric field forces the molecules to reorient in orthogonal directions. In opposition, the elastic coupling tends to restore the molecules in the electric field direction. Hence, the molecules can be oriented with different direction with the similar angle with respect to the electric field, generating an overall highly nonuniform texture.

To monitor this texture, one can consider crossed linear polarizers. Figure 1 shows the typical Schlieren-like texture, where the dark brushes account for directions where the optical axis, to be precise the averaged direction of the molecule in the plane of the cell, parallel or orthogonal to either the polarizer or the analyzer. The dark brushes meet at the defects, umbilical defects [1, 2]. Around this defect, when one circles clockwise, the molecules rotate clockwise or anti-clockwise. From the mathematical point of view, these defects correspond to topological singularities of the averaged 2-D projection of the molecules onto the plane of the cell, which usually are named vortexes [4, 5]. Defects with molecules rotating in clockwise (anti-clockwise) sense are termed positive (negative) charge vortexes. Figure 1 depicts the umbilical defects with different topological charge. Indeed, the the spatiotemporal dynamics of the texture is governed by the dynamics of the vortexes. In the last decades the vortex dynamics have been study theoretical
and experimentally in details [4, 5, 6, 7]. The vortex dynamics are characterized by attraction (repulsion) between opposite (equal) charged vortexes. When the vortexes with opposite charge collide, they are annihilate and disappear [4, 6, 8]. An unified theory of these defects is based on the Ginzburg-Landau equation with real coefficients [4, 5, 9]. Based on this description, one can show that the pair interaction law of vortex is governed by a force, inversely proportional to the distance between the vortexes at a dominant order. A thorough study of the vortex interaction is performed in Ref. [8].

![Figure 1](image)

**Figure 1.** Gas of defects obtained from a biased uniform homeotropically aligned nematic liquid crystal layer. The dashed circumferences account for opposite topological charges. The optical micrograph is obtained using an Olympus BX51 microscope with crossed linear polarizers.

To study experimentally the umbilical defects dynamics, one can consider a nematic liquid crystal layer with negative anisotropy dielectric constant and homeotropic anchoring. To keep the thickness of the cell and avoid cell plates buckling, often one can use randomly distributed glass spheres between cell plates. These spheres are mono-disperse and have approximately the same diameter as the thickness of cell. It could be expected that the presence of glass beads does not affect the vortex dynamics. However, recently it has been shown that these glass beads change the average defects dynamics [10].

In this manuscript, the vortex-glass bead interaction is experimentally characterized. The glass bead have been found to be attractive or non-attractive. We observed that not all glass beads are attractive, at least for the sample and the regions of the sample so far observed, there are more non-attractive beads than attractive ones. The attractive glass bead approximatively pull the vortexes so that the distance decays with the square root of time. Hence, the glass bead induces a force on the vortex proportional to an inverse power law of the distance between them with exponent 3/4. This dynamics is not isotropic, that is, the vortexes are attracted and
annihilated to a polar region of the sphere.

2. Experimental Setup

We have considered a cell, 5B100A150UT180, manufactured by Instec. The cell is filled by capillarity with MLC $-6608$ nematic liquid crystal that has negative anisotropy. The inner walls of this cell are chemically treated so that the molecules of liquid crystal are orthogonally aligned to the cell plates. This configuration is usually named homeotropic anchoring [1]. The glass plates of the cell have 15 $\mu m$ of separation between them. To keep the thickness of the inner gap between the glass plates fixed, the cell is filled with monodisperse glass beads (clear/transparent ceramics) which are randomly distributed in the sample. The sample of liquid crystal is subjected to an external electric field in the vertical direction ($z$-axis), using a low frequency (100kHz) sinusoidal voltage (9V) produced by function generator (Agilent 33521A) with a high voltage amplifier (Tabor Electronics 9200).

![Sketch of the setup. The microscope, here represented by the objective, and accommodates a circular polarizer, bottom quarter-wave plate (QWP) and polarizer, and a circular analyzer, top polarizer and quarter-wave plate. The image of the cell is recorded via a CCD camera. The inset sketches the glass bead and a configuration of liquid crystal directors.](image)

Figure 2. Sketch of the setup. The microscope, here represented by the objective, and accommodates a circular polarizer, bottom quarter-wave plate (QWP) and polarizer, and a circular analyzer, top polarizer and quarter-wave plate. The image of the cell is recorded via a CCD camera. The inset sketches the glass bead and a configuration of liquid crystal directors.

The imaging system used, see Figure 2, is a modified Motic 310 microscope equipped with a circular polarizer (bottom linear polarizer and quarter-wave plate) and a circular analyzer (top quarter-wave plate and polarizer). The light from the microscope condenser, that illuminate the cell mounted on the microscope stage, is filtered with a band-pass filter since the QWPs are designed to operate at $\lambda = 532nm$. Then, the texture of the sample is captured by a charge-coupled device (CCD camera). The images obtained by the CCD camera are recorded and then
analyzed with an image processing and analysis software Fiji (ImageJ)[12]. The position of the glass beads and the vortexes are extracted for further analysis.

3. Results

When a low frequency voltage with a sufficiently large amplitude greater than the Freédericksz voltage is applied to the nematic liquid crystal layer—which was previously described—the creation of vortexes is observed. Due to the presence of the glass beads in the sample, this process occurs in an inhomogeneous medium. The dynamics of vortexes is strongly affected by the glass beads [10]. We observe that there are attractive glass beads. Indeed, the attractive spheres pull the near vortexes. Figure 3 shows a typical observed vortex dynamics.

In the monitored region, there are 39 glass beads, only five of them are attractive. The density of glass non-attractive beads and attractive ones are 27 beads/mm$^2$ and 4 beads/mm$^2$, respectively. Hence, the proportion of attractive glass beads is 15%. Initially, a large number of vortexes are created, then the vortexes with opposite topological charges are attracted to each
Figure 4. Vortex glass bead interaction. From the left to the right is presented a temporal sequence of snapshots. Initially, an attractive glass bead pulls a vortex. When a vortex collides with the attractive glass bead, it disappears (cf. right panel).

other and eventually annihilate and those with same charges are repel each other. As result of the vortex interaction, the density of vortex decreases with time. In this regime of dilute gas of vortexes, some of the defects are attracted by some glass beads. When a vortex collides with the attractive glass bead, it disappears. Figure 4 illustrates the vortex and glass bead interaction, showing a sequence of snapshots. To characterize this process we have considered a region around an attractive bead embedded in the nematic liquid crystal layer, region that is magnified 20 times. Notice that the vortex trajectory is not radial. In addition, the typical time of collision between vortexes is of the order 2.33 s. On the other hand, the typical time of collision between a vortex and glass bead is of the order of 13.0 s.

To illustrate the non attractive glass beads, we have identified a pair of glass beads, one attractive and one unattractive (see Fig. 5). In this figure is illustrated how a sphere attracts three vortexes. Note that also this attractive glass bead modifies the dynamics between vortexes. The non attractive glass bead presumably weakly repels the vortexes, however experimentally we have not been able to verify this property. The above assertion is based on the fact that vortexes motion close to this glass bead have never been observed in our case. Even, the vortexes that are observed close to this sphere move away. A detailed study of this repulsive phenomenon is in progress. In the next section, we will present a detailed study of the vortex attraction towards a glass bead.

Figure 5. Attractive and non-attractive glass beads. From the left to the right is presented a temporal sequence of snapshots. The attractive sphere pulls three vortex. The vortexes that which are pulled by the attractive glass bead are highlighted with blue circular lines.
3.1. Experimental analysis of vortex and glass bead interaction

![Graphs](image)

**Figure 6.** Interaction dynamics of vortex and glass bead. Points and continuous curves in the left panels, respectively, account for the experimental evolution of the distance between vortex and the glass bead as function of time for three experimental realizations and fitted curves using formula (2) with their respective adjustment R-square. Points and continuous curves in the right panels, respectively, stand for the experimental evolution and fit curves using formula (1) of the force as function of the vortex radial position ($r$) with respect to the attractive glass bead, where 1 pixel =0.254 $\mu$m and 1 frame= 0.06 s.
To characterize the interaction dynamics of vortex and glass bead, we have measured and monitored the distance between them. This distance was obtained by processing the glass beads and the vortexes positions, the latter determined from the analysis of a sequence of frames in the acquired video (fixed frame rate) using Fiji (ImageJ).

To carry on with this process, first an attractive spheres was identified via testing. Subsequently, an electric voltage is applied to the liquid crystal sample and the evolution of the vortexes attracted by the glass bead is studied. Later on, the external electrical voltage is turned off and the system is allowed to relax at equilibrium for a period of about 10 s. All this procedure is repeated several times to achieve an adequate statistical description. Left panels of figure 6 shows the experimental evolution of the distance between vortex and the glass bead as function of time for different experimental realizations.

The measurements are represented by the points cloud. Clearly from these measurements, one observes that the distance as a function of time has a well define evolution law. It is well-known that the vortex-pair interaction is characterized by an over-damped force that goes as the inverse of the distance between the vortexes [7]. Namely, the distance between opposite vortexes, \(d(t)\), decreases as the square root of time, \(d(t) = \kappa \sqrt{t_0 - t}\) where \(t_0\) accounts for collision time and \(\kappa\) is an appropriated dimensional parameter. It is worthy to note that vortexes are characterized by the induced molecular distortion around vortex position [1]. Likewise, glass beads can generate molecular distortion in their own surrounding [11]. This molecular distortion generates interaction with the vortexes. Therefore, one would expect the attractive glass bead and the umbilical vortex interaction to be an over-damped force, \(f(r)\), which is inversely proportional to a power of the distance, \(r(t)\), between vortex and glass bead, that is,

\[
\dot{r} = f(r) = -\frac{\gamma}{r^n},
\]

where \(\gamma\) is a dimension parameter and \(n\) is a real positive number. After straightforward calculations, one gets

\[
r(t) = A(b - t)^{\frac{1}{n}},
\]

where \(b\) accounts for collision time, \(A \equiv ((1 + n)\gamma)^{1/(1+n)}\) is a dimensional parameter, and \(c \equiv 1/(1 + n)\).

In order to determine \(A, b\) and \(c\) from the experimental data, we will use formula (2) as a fit law. The continuous curves in left panels of figure 6 are the best fits of the experimental data for different realization. Note that there is a quite fair agreement between the experimental data and the fit law, formula (2). Note that the main source of fluctuations in the data is the detection algorithm of the vortexes.

Table 1 summarizes the fit parameters, using formula (2), of the vortex-glass bead interaction for 14 experimental realizations. We note that there is a trend between \(A\) and \(c\). Indeed, when the \(c\) coefficient increases the \(A\) coefficient decreases. The \(b\) coefficient is related with the collision time of the vortex and the glass bead. The collision time is of the order of 13 seconds.

The above analyzes assume that the interaction is of a central nature. However, the trajectories of the vortexes to the glass bead are not radial.

Figure 7 shows the trajectories of different vortexes that move towards to glass bead, and the vortex glass bead interaction appears to be anisotropic. Experimentally, we observe that the vortexes collide with the glass bead in certain well define region which we termed as south pole. Probably, this anisotropic behavior is due to the fact that the deformation around the glass bead is not cylindrically symmetric, which could generate a deformation equivalent to a dipole. However, we have not detected vortexes that collide with the bead at its north pole. The detailed structure of the interaction is an open problem, and its understanding is in progress. Likewise, when the liquid crystal samples are not subjected to an external voltage and between to circular crossed polarizers, we observe that attractive and non-attractive glass beads exhibit
Table 1. Fit values, using formula (2), of the vortex-glass bead interaction for 14 experimental realizations.

| cases | A   | b    | c    | $R^2$ |
|-------|-----|------|------|-------|
| I     | 30.6| 1027.0| 0.26 | 0.98  |
| II    | 22.2| 29.2 | 0.29 | 0.99  |
| III   | 16.5| 74.1 | 0.35 | 0.99  |
| IV    | 8.8 | 64.8 | 0.49 | 0.99  |
| V     | 8.5 | 79.1 | 0.53 | 0.99  |
| VI    | 7.4 | 127.8| 0.54 | 0.99  |
| VII   | 8.0 | 71.3 | 0.57 | 0.99  |
| VIII  | 6.5 | 124.6| 0.58 | 0.99  |
| IX    | 5.8 | 307.3| 0.59 | 0.99  |
| X     | 5.0 | 108.7| 0.63 | 0.99  |
| XI    | 5.1 | 82.5 | 0.67 | 0.99  |
| XII   | 2.1 | 678.8| 0.76 | 0.99  |
| XIII  | 1.8 | 109.7| 0.81 | 0.99  |
| XIV   | 0.4 | 369.6| 0.91 | 0.99  |
| Average| 9.19| 232.46| 0.57| 0.99 |

an asymmetrical and symmetric halo of light, respectively. The inset of Fig. 7 depicts the typical observed halo of light.

Figure 7. Anisotropic interaction between vortex and glass bead. Different colors account for different vortex trajectories. Inset accounts for the liquid crystal sample without external voltage and between to circular crossed polarizers. The attractive (glass bead in the middle) and non-attractive glass beads (glass beads outside) exhibit an asymmetrical and symmetric halo of light, respectively.
3.2. Vortex-glass bead force

Although the interaction is not isotropic, as a first approximation it will be considered isotropic. From the experimental data on the evolution of the distance between the vortex and the glass bead, one can deduce an evolution law using formula (2), see left panels of Fig. 6. Similarly, from the experimental data and fit curve one can reconstruct the interaction force between the vortex and the glass bead.

The points and the continuous curves in right panels of figure (6) show, respectively, the vortex speed ($\dot{r}$) as function of the distance ($r$) obtained from experimental data and theoretical fit, formula (1) for different experimental realization. From figure 6, one can infer that as the distance ($r$) increases, the interaction force is more abrupt. From this figure we can conclude that the force of interaction as function of the distance between the vortex and the glass bead is an inverse power law in good approximation.

To understand how the force changes as a function of time, from the fit expression of the temporal evolution of the distance between vortex and glass bead, formula (2), we can rebuild the interaction force as function of time,

$$f(t) = \dot{r} = -Ac(b - t)^{c-1}.$$  \hspace{1cm} (3)

Rewriting this expression $f(t) = -C_1(b - t)^{(c-1)}$, where $C_1 = cA$. Considering the average value $c$ from table 1, $\langle c \rangle = 0.57$, we obtain the average force of vortex-glass bead

$$\langle f(t) \rangle = \frac{-C_1}{(b - t)^{0.43}}.$$  \hspace{1cm} (4)

Hence, the attractive glass bead spheres pull the vortexes so that the distance decays approximately as the inverse of the square root of the time to collision. On the other hand, the interaction force as a function of distance has the form given by formula (1), where average exponent is $\langle n \rangle = 0.75$. Therefore from these results, we conclude that when the distance between vortex pair and the distance between vortex and glass sphere are the same, the interaction between vortex and glass bead ($\approx 1/r^{3/4}$) is greater than the vortex pair interaction ($\approx 1/r$). However, we note that after the vortexes are created, the distance between vortex pair is less than distance between vortex and glass bead, hence, the main interaction is between vortex pair. Subsequently, when the remaining number of vortexes is reduced (diluted gas), they begin to interact with the attractive glass beads. This scenario is the one observed experimentally (cf. Fig. 3).

4. Conclusions and Remarks

Liquid crystal driven by external electric and magnetic fields exhibit an intricate network topological defects. These types of defects have attracted the attention of physicists, mathematicians, and engineers for their fundamental properties as macroscopic topological particles and for their potential applications in particular in communications, image processing and quantum computation [4, 5]. Here, we have reported an experimental characterization of vortex-glass bead interaction in nematic liquid crystal layer. The glass bead spheres can be classified in two type from the interaction point of view: attractive and non-attractive.

The attractive glass bead spheres pull the vortexes so that the distance decays as the square root of the time to collision. The glass bead sphere induces an anisotropic force on the vortex. This force can be well approximate by an inverse power law of the distance between vortex and glass bead with exponent $3/4$. The vortexes are attracted to a polar region of the attractive glass bead. After the vortexes are created the main interaction is mediated by vortex pair interaction. Subsequently, when a reduced number of vortexes is left (diluted limit), they begin to interact with the attractive glass beads. This scenario is the one observed experimentally.
Still unknown the origin of these attractive force between vortex and glass bead. Certainly, the presence of glass bead generate deformation around itself. This deformation is responsible of vortex-glass bead interaction, and its characterization is in progress. Likewise, in the present study we have neglected the effect of distant vortexes (vortex pair interaction). In addition, mean field theory is not valid as consequence of long range interaction between vortexes. Hence, the presence of distant vortex can play a role. A systematic study of this issue is in progress. The role of non-attractive glass beads has not been performed, they are probably of repulsive type.

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