QCD Corrections to Decay Distributions of Neutral Higgs Bosons with (In)definite CP Parity

W. Bernreuther, A. Brandenburg*, and M. Flesch†

Institut für Theoretische Physik, RWTH Aachen, 52056 Aachen, Germany

(March 26, 2022)

Abstract

We compute the order $\alpha_s$ QCD corrections to the density matrix for the decay of a neutral Higgs boson $\varphi$ with (in)definite CP parity into a quark antiquark pair, respectively the QED corrections for the decay into a pair of charged leptons. We classify and calculate single spin asymmetries and spin-spin correlations which are generated by the scalar and pseudoscalar Yukawa couplings. These spin effects can be traced in $\varphi \rightarrow \tau^- \tau^+$ and, for heavy Higgs bosons, in $\varphi \rightarrow t\bar{t}$. We also calculate resulting correlations among the final states and estimate, for the respective decay modes, the number of events needed to measure the Yukawa couplings with these correlations at the $3\sigma$ level.

PACS number(s): 11.30.Er, 12.60.Fr, 14.80.Cp

*supported by Deutsche Forschungsgemeinschaft.

†supported by BMBF contract 057AC9EP.
I. INTRODUCTION

Many extensions of the Standard Model (SM) involve more than one scalar field multiplet and thus predict the existence of more than just one Higgs boson. In particular, quite a number of these models – including the 2 Higgs doublet extensions of the SM \[1-4\] – allow for the violation of CP symmetry by the scalar self interactions. This type of CP violation is of great interest for scenarios that attempt to explain the baryon asymmetry of the universe \[3\].

A manifestation of CP violation in the scalar sector would be the existence of neutral Higgs bosons \(\phi\) of undefined CP parity, i.e., Higgs bosons having both scalar and pseudoscalar couplings to quarks and leptons. If neutral Higgs boson(s) should be discovered in the future, one would eventually like to know its (their) P and CP quantum numbers. In this context several proposals and theoretical studies have been made in the literature. For instance if \(\phi\) has both scalar and pseudoscalar Yukawa couplings then a CP-violating spin-spin correlation is induced already at tree level in the decays of \(\phi\) to fermion-antifermion pairs. This spin-spin correlation could be traced in \(\phi \rightarrow \tau^-\tau^+\) and \(\phi \rightarrow t\bar{t}\) \[3\]. (For related proposals, see \[7-13\].) The modes \(\phi \rightarrow W^+W^-, ZZ\) could also be employed to infer the parity and CP properties \[14\] of \(\phi\) \[11,13-18\]. Other reactions that may be used for this purpose are associated \(t\bar{t}\phi\) production \[19,20\], \(\phi\) production by high energetic photon photon collisions \[11,21,22\], and by high energetic \(\mu^-\mu^+\) annihilation \[23,24\].

In this article we calculate the order \(\alpha_s\) QCD corrections to the density matrix for the decay of a neutral Higgs boson \(\phi\) with arbitrary scalar and pseudoscalar Yukawa couplings into a quark antiquark pair. (QCD corrections to the decay width of a scalar and/or pseudoscalar \(\phi \rightarrow q\bar{q}\) were computed in \[22,31\].) We classify and calculate single spin asymmetries and spin-spin correlations which are generated by the scalar and/or pseudoscalar Yukawa couplings. These spin effects can be traced for heavy Higgs bosons in \(\phi \rightarrow t\bar{t}\)
because top quarks decay before they hadronize and because they auto-analyze their spins through their parity-violating weak decays. We show that there are two correlations among the final states from $t\bar{t}$ decay whose combined use allows to investigate the CP property of a neutral $\phi$.

We then apply these results to the decay $\phi \to \tau^-\tau^+$ and its QED corrections. We analyze correlations for $\tau$ decay modes that have the best $\tau$ spin analyzer quality. Finally we estimate, for the respective $\phi$ decay modes, the number of events needed to measure the top and $\tau$ Yukawa couplings with these correlations at the 3$\sigma$ level.

II. THE DECAY $\phi \to f\bar{f} X$

Let us briefly recapitulate the salient features of the simplest models that predict neutral Higgs bosons with undefined CP parity; these are 2 Higgs doublet extensions of the SM with natural flavor conservation at the tree level (see, e.g., [4]). Explicit CP violation in the Higgs potential leads to three physical neutral boson states with scalar and pseudoscalar couplings to fermions. In the following $\phi$ denotes one of these bosons. The Yukawa interactions with a quark or lepton field $f$ read:

$$\mathcal{L}_Y = -\frac{m_f}{v} \bar{f}(a_f + i\gamma_5 \tilde{a}_f)f \phi,$$  \hspace{1cm} (II.1)

where $v = (\sqrt{2}G_F)^{-1/2}$ and $G_F$ is Fermi’s constant, $m_f$ is the fermion mass, $a_f$ and $\tilde{a}_f$ are scalar and pseudoscalar coupling constants, respectively. If $a_f \tilde{a}_f \neq 0$ then $\mathcal{L}_Y$ is CP-violating.

For the reaction $\phi \to f\bar{f}X$ the spin density matrix of the $f\bar{f}$ subsystem is defined in the $f\bar{f}$ center of mass system by:

$$R_{\alpha_1\alpha_2, \beta_1\beta_2}(k) = \sum_X \langle f(k_1, \alpha_1), \bar{f}(k_2, \beta_1), X | T | \phi(q) \rangle \langle f(k_1, \alpha_2), \bar{f}(k_2, \beta_2), X | T | \phi(q) \rangle^* .$$  \hspace{1cm} (II.2)
Here \( k = k_1 \) is the \( f \) momentum in the \( f \bar{f} \) c.m. frame and \( \alpha \) and \( \beta \) are spin indices. The sum in (II.2) is taken over all discrete and continuous degrees of freedom of the unobserved part \( X \) of the final state. The matrix (II.2) can be decomposed in the spin spaces of \( f \) and \( \bar{f} \) as follows:

\[
R = A \mathbb{I} \otimes \mathbb{I} + B^+_i \sigma^i \otimes \mathbb{I} + B^-_i \mathbb{I} \otimes \sigma^i + C_{ij} \sigma^i \otimes \sigma^j .
\]

(II.3)

The first (second) factor in the tensor products of the \( 2 \times 2 \) unit matrix \( \mathbb{I} \) and of the Pauli matrices \( \sigma^i \) refers to the \( f(\bar{f}) \) spin space. Because of rotational invariance, the functions \( B^\pm_i \) and \( C_{ij} \) can be further decomposed:

\[
B^\pm_i = b^\pm_i \hat{k}_i ,
\]

\[
C_{ij} = c_1 \delta_{ij} + c_2 \hat{k}_i \hat{k}_j + c_3 \epsilon_{ijl} \hat{k}_l ,
\]

(II.4)

with \( \hat{k} = k/|k| \) and coefficients \( A, b^\pm \) and \( c_i \), which only depend on the masses and couplings. These coefficients can all be separately measured by using suitable observables built from the spin operators of \( f \) and \( \bar{f} \). The trace of \( R \), \( \text{Tr}\{R\} = 4A \) is a Lorentz scalar, and is related to the decay rate \( \Gamma(\varphi \to f\bar{f}X) \) by:

\[
\Gamma = \frac{1}{2m_\varphi} \frac{\beta}{8\pi} \text{Tr}\{R\} ,
\]

(II.5)

where \( \beta = (1 - 4m_f^2/m_\varphi^2)^{1/2} \) is the velocity of the fermion in the \( \varphi \) rest frame. One may construct a complete set of observables that determine the other coefficients:

\[
\mathcal{O}_0 = \hat{k} \cdot (s_1 + s_2) ,
\]

\[
\mathcal{O}_1 = \hat{k} \cdot (s_1 - s_2) ,
\]

\[
\mathcal{O}_2 = \hat{k} \cdot (s_1 \times s_2) ,
\]

\[
\mathcal{O}_3 = s_1 \cdot s_2 ,
\]

\[
\mathcal{O}_4 = (\hat{k} \cdot s_1) (\hat{k} \cdot s_2) ,
\]

(II.6)
where $s_1 = \frac{1}{2} \sigma \otimes \mathbb{1}$ and $s_2 = \frac{1}{2} \mathbb{1} \otimes \sigma$ are the spin operators of $f$ and $\bar{f}$. The expectation values of the observables are given by:
\[
\langle O_i \rangle = \frac{\text{Tr}\{R \cdot O_i\}}{\text{Tr}\{R\}}. \tag{II.7}
\]
These expectation values are trivially related to the coefficients of the density matrix (II.2):
\[
\langle O_0 \rangle = \frac{2(b_+ + b_-)}{4A},
\]
\[
\langle O_1 \rangle = \frac{2(b_+ - b_-)}{4A},
\]
\[
\langle O_2 \rangle = \frac{2c_3}{4A},
\]
\[
\langle O_3 \rangle = \frac{3c_1 + c_2}{4A},
\]
\[
\langle O_4 \rangle = \frac{c_1 + c_2}{4A}. \tag{II.8}
\]

In Table 1 we give a list of the transformation properties of the coefficients under discrete symmetries. The T and CPT transformation properties hold up to non-hermitian contributions to the decay amplitude. This table tells us that the CP-invariance relation $b_+ = b_-$ can only be violated if CP is not conserved (i.e., $a_f \bar{a}_f \neq 0$) and if absorptive parts of the amplitude are taken into account. In fact, the functions $b_{\pm}$ are zero to leading order for model (II.1). Since the Born decay matrix is hermitian, we get from CPT invariance the relation $b_{\pm} = b_\mp$. Further the interaction (II.1) is C-conserving, which implies $b_{\pm} = -b_\mp$ and hence $b_{\pm} = 0$. On the other hand the CP-invariance relation $c_3 = 0$ can be – according to Table 1 – violated and is, in fact, violated by (II.1) already at tree level.

We will now discuss some general properties of the observables (II.6). Since the observable $O_0$ is C-odd, its expectation value is zero at tree level and at arbitrary order in the QCD or QED couplings. CP-violating absorptive parts generated by one and higher loop corrections induce a nonzero difference $b_+ - b_-$ and therefore a nonzero expectation value of the observable $O_1$. The CP-violating triple correlation $\langle O_2 \rangle$ is generated by (II.1) already at tree level and receives at the next order a contribution from the CP-violating
dispersive part of the one-loop decay amplitude. The other two observables $\mathcal{O}_{3,4}$ measure CP-conserving spin-spin correlations.

We will now discuss the decay of a Higgs boson into top quark pairs, $f = t$ (Later we will also consider the case $f = \tau$). We have calculated the density matrix $R$ to order $\alpha_s$. We use the following notation:

$$R = R_0 + \frac{\alpha_s}{\pi} R_1 + \mathcal{O}\left(\frac{\alpha_s^2}{\pi^2}\right),$$

$$\Gamma = \Gamma_0 + \frac{\alpha_s}{\pi} \Gamma_1 + \mathcal{O}\left(\frac{\alpha_s^2}{\pi^2}\right),$$

$$\langle \mathcal{O}_i \rangle = \langle \mathcal{O}_i \rangle_0 + \frac{\alpha_s}{\pi} \langle \mathcal{O}_i \rangle_1 + \mathcal{O}\left(\frac{\alpha_s^2}{\pi^2}\right), \tag{II.9}$$

where

$$\langle \mathcal{O}_i \rangle_0 = \frac{\text{Tr}\{R_0 \cdot \mathcal{O}_i\}}{\text{Tr}\{R_0\}},$$

$$\langle \mathcal{O}_i \rangle_1 = \frac{\text{Tr}\{R_1 \cdot \mathcal{O}_i\}}{\text{Tr}\{R_0\}} - \langle \mathcal{O}_i \rangle_0 \Gamma_1 \Gamma_0. \tag{II.10}$$

At Born level the rate and the expectation values are found to be:

$$\Gamma_0 = \frac{N_C m_t^2}{8\pi} m_\varphi^2 \beta (a_t^2 \beta^2 + \bar{a}_t^2),$$

$$\langle \mathcal{O}_1 \rangle_0 = 0,$$

$$\langle \mathcal{O}_2 \rangle_0 = \frac{-a_t \bar{a}_t \beta}{a_t^2 \beta^2 + \bar{a}_t^2},$$

$$\langle \mathcal{O}_3 \rangle_0 = \frac{a_t^2 \beta^2 - 3\bar{a}_t^2}{4(a_t^2 \beta^2 + \bar{a}_t^2)},$$

$$\langle \mathcal{O}_4 \rangle_0 = -\frac{1}{4}, \tag{II.11}$$

where $N_C = 3$. At tree level the $t\bar{t}$-system is in a pure state. This can also be explicitly checked using (II.11). Radiative corrections to the coefficients will in general lead to mixed states.

The calculation of the QCD corrections to the results (II.11) proceeds as follows: We calculate separately the virtual and real corrections to order $\alpha_s$. All singularities are treated
by dimensional regularization. Soft (IR) singularities appear both in the virtual and real corrections; they cancel explicitly in the sum of the two in accordance with the Kinoshita-Lee-Nauenberg theorem. We note that the necessary integrations over the hard gluon momentum can be done analytically for all coefficients of the density matrix. In order to write the results for $\Gamma_1$ and $\text{Tr}\{R_1 \cdot O_1\}$ entering (II.9) and (II.10) in a compact form, we define $\omega \equiv (1 - \beta)/(1 + \beta)$. Further $C_F = 4/3$, and $\text{Li}_2(x)$ denotes the dilogarithmic integral. In the results below the top quark mass is defined in the on-shell scheme. For the order $\alpha_s$ contribution to the decay rate we find:

$$\Gamma_1 = \frac{m_t^2 N_C C_F}{v^2 4\pi} m_\varphi \left\{ (a_t^2 \beta^2 + \tilde{a}_t^2) \left[ (-2\beta + \ln(\omega)(1 + \beta^2)) \left( \frac{\ln(1 + \omega)}{2} + \ln(1 - \omega) \right) \right. \right.
$$

$$\left. + (1 + \beta^2) \left( \frac{\text{Li}_2(\omega^2)}{2} + \text{Li}_2(\omega) \right) + \left( \frac{13\beta^4 + 48\beta^3 - 34\beta^2 - 3}{32\beta^2} \ln(\omega) - 6\beta(1 - 7\beta^2) \right) \right\} + 3 \left( 1 - \beta^2 \right) \left( \frac{\beta^4 + 6\beta^2 + 1}{32\beta^2} \ln(\omega) + 2\beta(1 + \beta^2) \tilde{a}_t^2 \right). \quad (\text{II.12})$$

The result (II.12) reduces to the standard model result for $a_t = 1$, $\tilde{a}_t = 0$ and agrees with the one given in [25]. In Fig.1a-c we plot the decay rate $\varphi \rightarrow t\bar{t}$ for different values of $a_t$ and $\tilde{a}_t$ as a function of the Higgs mass for $m_t = 175$ GeV. We include the 1-loop running of $\alpha_s$ with five active flavours and $\Lambda_{QCD} = 200$ MeV. In all plots the dashed line represents the Born result and the full line is the result to order $\alpha_s$. Fig 1a and Fig 1c depict the well known cases of a scalar and pseudoscalar Higgs boson, respectively. For illustrating the case of a Higgs boson with indefinite CP parity we choose $a_t = (2/3)^{1/2}$ and $\tilde{a}_t = (1/3)^{1/2}$. In the two Higgs doublet model this corresponds to $v_1 = v_2$ and maximal mixing of the three neutral Higgs bosons with definite CP parity (see, e.g. [31]). For a given Higgs mass a nonzero pseudoscalar coupling enhances the rate. Note that in general the QCD corrections are important.

We now turn to the other observables defined in (II.6). The QCD corrections to their expectation values are determined by (cf (II.10)):
\[ \text{Tr}\{ R_1 \cdot O_1 \} = \frac{m_t^2}{v^2} N_C C_F 2\pi m_\phi^2 (1 - \beta^2) a_t \bar{a}_t, \quad (\text{II.13}) \]

\[
\text{Tr}\{ R_1 \cdot O_2 \} = \frac{m_t^2}{v^2} N_C C_F m_\phi^2 a_t \bar{a}_t \left\{ 2 \left( 1 + \beta^2 \right) \left[ \text{Li}_2(\omega^2) - 4\text{Li}_2(\omega) \right] + 8\beta \ln(1 - \omega) + 2 \left[ \ln(\omega) \left( 1 + \beta^2 \right) - 2\beta \right] \ln(1 + \omega) - 2\beta \left( 1 + 2\beta^2 \right) - \ln^2(\omega) (1 - \beta)^2 + \ln(\omega) \left( 1 - 2\beta - 3\beta^2 \right) \right\}, \quad (\text{II.14})
\]

\[
\text{Tr}\{ R_1 \cdot O_3 \} = \frac{m_t^2}{v^2} N_C C_F m_\phi^2 \beta^3 \left\{ \left( a_t^2 \beta^2 - 3\bar{a}_t^2 \right) \left[ 48 (1 + \beta^2) \left( \text{Li}_2(\omega^2) + 2\text{Li}_2(\omega) \right) - 96 (1 - \beta^2) \text{Li}_2(\omega^2) + 48\beta^2 \left( \ln(\omega)(1 + \beta^2) - 2\beta \right) \left( 2 \ln(1 - \omega) + \ln(1 + \omega) \right) + 8(1 - \beta^2) \left( 3\ln(\omega) \left( \ln(\omega) - 4\ln(1 + \omega) \right) - \pi^2 \right) + 6 \left( 23 + 15\beta^2 \right) \beta + 3\ln(\omega) \left( 7(1 - \beta^2)^2 - 24\beta^2 + 48\beta^3 + 16 \right) \right] + \bar{a}_t^2 \left[ 48 (1 - \beta^2)(3 + \beta^2) \left( 2\text{Li}_2(\omega) - \text{Li}_2(\omega^2) - 2 \ln(\omega) \ln(1 + \omega) \right) + 6\beta \left( 69 - 4\beta^2 - 9\beta^4 \right) + 8 \left( \pi^2 - 3\ln^2(\omega) \right) (1 + \beta^2)^2 - 4) + 3\ln(\omega) \left( 69 - 9\beta^6 - 43\beta^2 + 15\beta^4 \right) \right] \right\}, \quad (\text{II.15})
\]

\[
\text{Tr}\{ R_1 \cdot O_4 \} = \frac{m_t^2}{v^2} N_C C_F \frac{m_\phi^2}{\beta^3} \left\{ \left[ -24\beta^2 \left( 1 + \beta^2 \right) \left( \text{Li}_2(\omega^2) + 4\text{Li}_2(\omega) \right) - 48\text{Li}_2(\omega^2) + 48\beta \left( 1 + \beta^2 \right) \left( \ln(\omega + 1) + 2 \ln(1 - \omega) \right) + 12 \ln^2(\omega) (1 - \beta)^2 (1 + \beta)^2 - 24 \ln(\omega) \left( 2(\beta^2 + 2\beta^4 + 1) \ln(1 - \omega) + (3 + \beta^4) \ln(\omega + 1) \right) - 30\beta \left( 1 + \beta^2 \right) - 4\pi^2 (1 - \beta^2) \left( 1 + 2\beta^2 \right) + 3\ln(\omega) \left( 10\beta^2 - \beta^4 + 15 - 24\beta(1 + \beta^2) \right) \left( a_t^2 + \bar{a}_t^2 \right) + 24 \left( 1 - \beta^2 \right) \left( \left[ 1 + \beta^2 \right] \left( \text{Li}_2(\omega^2) + 2\text{Li}_2(\omega) \right) + \left( \ln(\omega + 1) + 2 \ln(1 - \omega) \right) \times \left( \ln(\omega)(1 + \beta^2) - 2\beta \right) - \ln(\omega) \left( 1 - \beta^2 \right) + 3\beta \left( 1 + \ln(\omega) \right) \right) \right] \right\}. \quad (\text{II.16})
\]

The strikingly simple result obtained from (II.13) for \( \langle O_1 \rangle \) is plotted in Fig. 2, again for \( a_t = (2/3)^{1/2} \) and \( \bar{a}_t = (1/3)^{1/2} \). Remember that a nonzero value for this correlation signals CP violation induced by absorptive parts. Close to the \( t\bar{t} \) threshold the effect is of the order
of 20%. Note that close to threshold the infrared singularities due to Coulomb gluons should be resummed and hence our result will be substantially modified by higher order corrections which we do not consider here. For Higgs masses around 400 GeV the correlation drops to about 10%. To exhibit the dependence of the expectation values on the unknown model parameters $a_t$ and $\tilde{a}_t$ we assume for the moment $a_t\tilde{a}_t \geq 0$ and define

$$r_t = \frac{\tilde{a}_t}{a_t + \tilde{a}_t},$$

(II.17)

which takes values $0 \leq r_t \leq 1$. It is easy to see that the expectation values of the observables (II.6) depend only on $r_t$ and not separately on $a_t$ and $\tilde{a}_t$. In Fig. 3 this dependence is shown for $\langle O_1 \rangle$ and a Higgs mass $m_\varphi = 400$ GeV. Around $r_t = 0.3$ the value of $\langle O_1 \rangle$ is about 11%. The case $a_t\tilde{a}_t \leq 0$ may be analysed in complete analogy by defining $\tilde{r}_t = \tilde{a}_t/(\tilde{a}_t - a_t)$. Clearly, both cases can be distinguished by measuring the sign of $\langle O_1 \rangle$.

Fig. 4 shows the CP-violating triple correlation $\langle O_2 \rangle$ defined in (II.7) for the same set of parameters as used in Fig. 2. This correlation is nonzero at tree level and the QCD corrections are tiny. In contrast to $\langle O_1 \rangle$, it vanishes for $m_\varphi \to 2m_t$. For a given Higgs mass the correlation may reach values of $\pm 0.5$. The dependence of $\langle O_2 \rangle$ on $r_t$ is shown in Fig. 5, again for $m_\varphi = 400$ GeV. We note that for all $r_t$ the order $\alpha_s$ corrections are again very small.

We now discuss the results for the CP-even observables. For a scalar (pseudoscalar) Higgs particle, the expectation value $\langle O_3 \rangle$ is $1/4 (-3/4)$ at tree level. QCD corrections to these results are smaller than 1% for all Higgs masses below 500 GeV and vanish for $m_\varphi = 2m_t$. The expectation value $\langle O_3 \rangle$ is quite sensitive to the value of $r_t$, as shown in Fig. 6.

Finally, the QCD corrections to the expectation value $\langle O_4 \rangle$, which is $-1/4$ for all values of $a_t$ and $\tilde{a}_t$, are at most 2% for $m_\varphi < 500$ GeV. The dependence of these corrections on $r_t$ is very weak as demonstrated in Fig. 7. This completes our discussion of the order $\alpha_s$.
corrections to the $t\bar{t}$ density matrix (II.2).

For a Higgs boson with a mass smaller than $2m_W$ the main decay modes are $\varphi \to bb, c\bar{c}$ and $\varphi \to \tau^-\tau^+$. Because of hadronization effects, a spin analysis is difficult for $b$ and $c$ quark pairs – but it is feasible for $\varphi \to \tau^-\tau^+$. We illustrate this below for the case of a Higgs boson with $m_\varphi = 100$ GeV decaying into a pair of $\tau$ leptons. The QED result (again in the on-shell scheme) follows from the formulae (II.9)–(II.17) by replacing

$$N_C \rightarrow 1, \quad C_F \rightarrow 1, \quad \alpha_s \rightarrow \alpha, \quad m_t \rightarrow m_\tau, \quad a_t \rightarrow a_\tau, \quad \tilde{a}_t \rightarrow \tilde{a}_\tau, \quad r_t \rightarrow r_\tau. \quad (II.18)$$

The partial decay rate $\Gamma(\varphi \to \tau^-\tau^+)$ to order $\alpha$ is 0.202 MeV, which is 2.4% smaller than the Born rate. This result is practically the same for all the three choices of the model parameters $a_\tau$ and $\tilde{a}_\tau$ which were also used before in the case of $\varphi \to t\bar{t}$. Because of the factors $m^2_\tau/m^2_\varphi \sim O(10^{-3})$ and $\alpha(m_\varphi) \approx 1/128$, the correlation $\langle O_1 \rangle$ is tiny and is therefore not a useful tool for analyzing the CP nature of a light Higgs boson. In contrast, the triple correlation $\langle O_2 \rangle$ takes the large value $-0.47$ for $a_\tau = (2/3)^{1/2}$ and $\tilde{a}_\tau = (1/3)^{1/2}$. Its dependence on $r_\tau$ is shown in Fig. 8. The QED corrections to this result are below 1%. As in the QCD case, the CP-even $\tau^-\tau^+$ spin-spin correlation $\langle O_3 \rangle$ is also sensitive to the value of $r_\tau$. This is shown in Fig. 9. Finally, the QED corrections to the value $\langle O_4 \rangle = -1/4$ are below 0.3%.

III. CP ASYMMETRIES IN ANGULAR CORRELATIONS OF THE FINAL STATES

In this section we investigate the prospects to determine the CP parity of the Higgs boson decaying into $t\bar{t}$ or $\tau^-\tau^+$ pairs by measuring suitable angular correlations between the final state particles into which the fermions decay. As we have shown in the last section, the radiative corrections to normalized observables are of the order of a few percent. From now on we will therefore neglect these corrections and perform a leading order analysis of
the CP asymmetries. In this approximation the rest frame of the Higgs boson is identical to the $f \bar{f}$ c.m. system. In other words, the relation $\mathbf{k}_f = -\mathbf{k}_\bar{f}$ holds and will be used in constructing the CP asymmetries below. It is straightforward to generalize the correlations $\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3$ defined below to data samples where $f$ and $\bar{f}$ are not antiparallel.

We first discuss the case of a Higgs boson heavy enough to decay into $t\bar{t}$ pairs. We define a sample $\mathcal{A}$ containing events where the top quark decays semileptonically and the top antiquark decays hadronically,

$$\mathcal{A} : \begin{cases} t \to \ell^+ + \nu_\ell + b \\ \bar{t} \to W^- + \bar{b} \to q + \bar{q}' + \bar{b} \end{cases} \tag{III.1}$$

The sample $\bar{\mathcal{A}}$ is defined by the charge conjugated decay channels of the $t\bar{t}$ pairs,

$$\bar{\mathcal{A}} : \begin{cases} t \to W^+ + b \to q + \bar{q}' + b \\ \bar{t} \to \ell^- + \bar{\nu}_\ell + \bar{b} \end{cases} \tag{III.2}$$

Each of these samples contains a fraction of $2/9$ of all $t\bar{t}$ pairs. If we dismiss $\tau$ leptons as spin analyzers for the top quark, the remaining fraction is $4/27$.

The above decay modes are especially suited to study correlations that result from top spin-momentum correlations. From the hadronic decays of the $t$ ($\bar{t}$) the momentum of the $t$ ($\bar{t}$) may be reconstructed on an event by event basis and the accompanying lepton of the $\bar{t}$ ($t$) decay serves as an excellent spin analyzer. The knowledge of the top momentum allows a reconstruction of the top rest system. In our approximation we have $\mathbf{k}_t = -\mathbf{k}_\bar{t}$, which means that also the $t$ and $\bar{t}$ rest system is known in the case of (III.1) and (III.2), respectively. We may therefore define the CP-odd correlation

$$\mathcal{E}_1 = \langle \hat{\mathbf{k}}_t \cdot \hat{\mathbf{p}}_{\ell^+} \rangle_{\mathcal{A}} + \langle \hat{\mathbf{k}}_t \cdot \hat{\mathbf{p}}_{\ell^-} \rangle_{\bar{\mathcal{A}}} \tag{III.3}$$

where $\hat{\mathbf{p}}_{\ell^+}$ is the flight direction of $\ell^+$ in the top quark rest system, and $\hat{\mathbf{p}}_{\ell^-}$ is the flight direction of $\ell^-$ in the top antiquark rest system. (In the more general case one should
replace $\hat{p}_{\ell+}^* \rightarrow \hat{p}_{\ell+}, \hat{p}_{\ell-}$ defined in the $\varphi$ rest frame.) To measure $\mathcal{E}_1$, the corresponding expectation values are taken separately with respect to the two samples $\mathcal{A}$ and $\bar{\mathcal{A}}$. The correlation (III.3) traces the CP-odd spin asymmetry $\langle \mathcal{O}_1 \rangle$.

We further define the following triple correlation:

$$\mathcal{E}_2 = \langle \hat{k}_t \cdot (\hat{p}_{\ell+}^* \times \hat{p}_{\ell-}^*) \rangle_{\mathcal{A}} - \langle \hat{k}_t \cdot (\hat{p}_{\ell+}^* \times \hat{p}_{\ell-}^*) \rangle_{\bar{\mathcal{A}}} ,$$

(III.4)

where $\hat{p}_{\ell+}^*$ is the unit momentum of the $\bar{b}$ in the $\bar{t}$ rest system and $\hat{p}_{\ell-}^*$ is measured in the $t$ rest system. The triple correlation (III.4) probes the CP violating spin-spin correlation $\langle \mathcal{O}_2 \rangle$.

The calculation of the correlations (III.3), (III.4) involves a trace over the spin spaces of $t$ and $\bar{t}$ of the form $\text{Tr}\{ R \rho_t \otimes \rho_{\bar{t}} \}$. The decay density matrix $\rho_t$ for semileptonic $t$ decays, which is needed for expectation values taken with respect to sample $\mathcal{A}$, is given by

$$\rho_t(t \rightarrow \ell^+ \nu \bar{b}) = \frac{6x_+(1-x_+)}{1-3\kappa^2 + 2\kappa^3} \left[ 1 + \hat{p}_{\ell+}^* \cdot \sigma_t \right] dx_+ \frac{d\Omega_{\ell+}}{4\pi} ,$$

(III.5)

where $\kappa = m_W^2/m_t^2$ and $x_+ = 2E_{\ell+}^*/m_t \in [\kappa, 1]$ is the scaled energy of the lepton. Further we need the decay density matrix for hadronic $\bar{t}$ decays, in which the $\bar{b}$ analyzes the top antiquark spin. It reads:

$$\rho_{\bar{t}}(\bar{t} \rightarrow W^- \bar{b} \rightarrow q + \bar{q}' + \bar{b}) = \left[ 1 + \frac{1 - 2\kappa}{1 + 2\kappa} \hat{p}_{\bar{b}}^* \cdot \sigma_{\bar{t}} \right] d\Omega_{\bar{b}} \frac{d\Omega_{\bar{t}}}{4\pi} \cdot$$

(III.6)

The corresponding decay density matrices for sample $\bar{\mathcal{A}}$ can be obtained by replacing $x_+ \rightarrow x_-, \hat{p}_{\ell+}^* \rightarrow -\hat{p}_{\ell-}^*$, $d\Omega_{\ell+} \rightarrow d\Omega_{\ell-}$, $\sigma_t \rightarrow \sigma_{\bar{t}}$ in (III.3), and replacing $\hat{p}_{\bar{b}}^* \rightarrow -\hat{p}_{\bar{b}}^*$, $\sigma_{\bar{t}} \rightarrow \sigma_t$, $d\Omega_{\bar{b}} \rightarrow d\Omega_b$, $\sigma_{\bar{t}} \rightarrow \sigma_t$ in (III.6). Using these decay matrices, we get for the correlations $\mathcal{E}_{1,2}$:

$$\mathcal{E}_1 = \frac{2}{3} \langle \mathcal{O}_1 \rangle ,$$

$$\mathcal{E}_2 = \frac{8}{9} \cdot \frac{1 - 2\kappa}{1 + 2\kappa} \langle \mathcal{O}_2 \rangle .$$

(III.7)

The factor $(1 - 2\kappa)/(1 + 2\kappa) \approx 0.41$ is the spin analyzer quality of the $b(\bar{b})$. The statistical sensitivities of these correlations on the unknown couplings $a_t$ and $\tilde{a}_t$ are determined by the mean square fluctuations of the observables. We have
\[ \langle (\hat{k}_t \cdot \hat{p}_{\ell_+}^*)^2 \rangle_A = \langle (\hat{k}_t \cdot \hat{p}_{\ell_-}^*)^2 \rangle_A = \frac{1}{3}, \]
\[ \langle (\hat{k}_t \cdot (\hat{p}_{\ell_+}^* \times \hat{p}_{\ell_-}^*)^2 \rangle_A = \langle (\hat{k}_t \cdot (\hat{p}_{\ell_-}^* \times \hat{p}_{\ell_+}^*)^2 \rangle_A = \frac{2}{9}. \] (III.8)

In order to decide whether or not the Higgs boson is a CP eigenstate, it is sufficient to observe a nonzero value of \( E_1 \) or \( E_2 \), which implies a nonzero \( \langle O_1 \rangle \) or \( \langle O_2 \rangle \), respectively.

The number \( N^{(1,2)}_{tt} \) of \( \varphi \to t\bar{t} \) events that are needed to establish a nonzero correlation \( \langle O_{1,2} \rangle \) with three standard deviation (s.d.) significance are given by:

\[ N^{(1)}_{tt} = 9 \cdot \frac{27}{4} \cdot \frac{3}{2} \cdot \frac{1}{\langle O_1 \rangle^2}, \]
\[ N^{(2)}_{tt} = 9 \cdot \frac{27}{4} \cdot \frac{9}{16} \cdot \left( \frac{1 + 2\kappa}{1 - 2\kappa} \right)^2 \cdot \frac{1}{\langle O_2 \rangle^2}. \] (III.9)

Here we took into account only electrons and muons in the samples \( \mathcal{A} \) and \( \bar{\mathcal{A}} \). In Fig. 10 we plot these numbers as a function of \( r_t \) defined in (II.17) for \( m_\varphi = 400 \text{ GeV} \) (again assuming without loss of generality \( a_t \bar{a}_t \geq 0 \)). For a given number of events we can read off from the figure the interval for \( r_t \) which leads to a nonzero correlation with 3 s.d. significance. For example, values \( 0.18 \lesssim r_t \lesssim 0.52 \) would generate a nonzero \( E_2 \) with 3 s.d. significance in a data sample of \( N_{tt} = 1500 \) events. If no effect is seen in this sample, this interval for \( r_t \) is excluded.

The above CP studies may be complemented by considering the correlation

\[ \mathcal{E}_3 = \langle \hat{p}_{\ell_+}^* \cdot \hat{p}_{\ell_-}^* \rangle_A + \langle \hat{p}_{\ell_-}^* \cdot \hat{p}_{\ell_+}^* \rangle_{\bar{A}}. \] (III.10)

which is related to the CP-even spin-spin correlation \( \langle O_3 \rangle \) by

\[ \mathcal{E}_3 = \frac{8}{9} \cdot \frac{1 - 2\kappa}{1 + 2\kappa} \cdot \langle O_3 \rangle. \] (III.11)

From \( \langle (\hat{p}_{\ell_+}^* \cdot \hat{p}_{\ell_-}^*)^2 \rangle_A = \langle (\hat{p}_{\ell_-}^* \cdot \hat{p}_{\ell_+}^*)^2 \rangle_{\bar{A}} = 1/3 \) we get

\[ N^{(3)}_{tt} = 9 \cdot \frac{27}{4} \cdot \frac{27}{32} \cdot \left( \frac{1 + 2\kappa}{1 - 2\kappa} \right)^2 \cdot \frac{1}{\langle O_3 \rangle^2}. \] (III.12)
This number is also plotted (dotted line) as a function of \( r_t \) in Fig. 10. A simultaneous measurement of \( \mathcal{E}_2 \) and \( \mathcal{E}_3 \) with \( N_{\ell\ell} = 1500 \) would have a 3 s.d. sensitivity to values of \( r_t \) between 0.18 and 1.

Higgs boson production processes where the \( \varphi \) rest system can be reconstructed include \( e^+e^- \to Z\varphi \) and \( e^+e^- \to e^+e^-\varphi \) at a future linear collider \[37\] and, if realizable, \( \mu^+\mu^- \to \varphi \). For the \( W^+W^- \) fusion process \( e^+e^- \to \nu_e\bar{\nu}_e\varphi \) and for \( \varphi \) production at the LHC the above observables should be used with momenta obtained in the laboratory frame \[3\]. This decreases the sensitivity of the observables as compared to above.

Essentially the same analysis as above can be carried out for the case \( \varphi \to \tau^-\tau^+ \). This mode is of interest for a quantum number analysis of Higgs bosons with mass \( m_\varphi < 2m_W \). For a wide range of model parameters one typically expects in this case the branching ratio of the \( \varphi \to \tau^-\tau^+ \) mode to be about 8 percent. For analyzing the \( \tau \) spin several decay modes can be used. We discuss here only the decays \( \tau^\pm \to \pi^\mp\nu_\tau(\bar{\nu}_\tau) \) and \( \tau^\pm \to \rho^\mp\nu_\tau(\bar{\nu}_\tau) \). The decay density matrices are of the form

\[
\rho_{\tau^\pm}(\tau^\pm \to B^\mp\nu_\tau(\bar{\nu}_\tau)) = \left[1 \pm c_B\hat{p}_{B^\mp}^* \cdot \sigma_{\tau^\mp}\right] \frac{d\Omega_{B^\mp}}{4\pi} \quad (B = \pi, \rho),
\]

with \( c_\pi = 1 \), \( c_\rho = 0.456 \) and \( \hat{p}_{B^\mp}^* \) is the direction of flight of \( B^\mp \) in the \( \tau^\mp \) rest system. The mode \( \tau \to a_1 \), respectively \( \tau \to 3\pi \) can also be taken into account, see \[32\]. We will assume in the following that the \( \tau^\mp \) flight directions can be reconstructed for the above decays. This should be possible with some effort, as was done in the CP studies in \( \tau^-\tau^+ \) production at LEP \[33, 34\] (see also \[30\]). Since the CP-odd spin asymmetry \( \langle \mathcal{O}_1 \rangle \) is tiny for \( \varphi \to \tau^-\tau^+ \), we only discuss the CP-violating spin-spin correlation \( \langle \mathcal{O}_2 \rangle \) and the CP-even quantity \( \langle \mathcal{O}_3 \rangle \). Table 2 summarizes our results for the different decay modes of the \( \tau^-\tau^+ \) pairs and the corresponding correlations of the final state momenta which are sensitive to \( \langle \mathcal{O}_2 \rangle \) and \( \langle \mathcal{O}_3 \rangle \).

To constrain the unknown parameter \( r_\tau \), we combine all these decay channels. The
number of $\varphi \rightarrow \tau^-\tau^+$ events to get a 3 s.d. significance for a nonzero $\langle O_2 \rangle$ is:

$$N^{(2)}_{\tau^-\tau^+} = 9 \cdot \frac{9}{8} \cdot \frac{1}{\langle O_2 \rangle^2} \cdot \frac{1}{0.013 + 0.056 c_\rho^2 + 0.064 c_\rho^4}. \quad \text{(III.14)}$$

$N^{(3)}_{\tau^-\tau^+}$ is obtained from (III.14) by the replacements $9/8 \rightarrow 27/16$ and $\langle O_2 \rangle \rightarrow \langle O_3 \rangle$. Both numbers are plotted in Fig. 11 as a function of $r_\tau$. According to Fig. 11 about 2000 $\varphi \rightarrow \tau^-\tau^+$ events would be needed to establish $r_\tau = \tilde{a}_\tau/(a_\tau + \tilde{a}_\tau) > 0.36$ also at the 3$\sigma$ level.

**IV. CONCLUSIONS**

The analysis of the decays $\varphi \rightarrow f\bar{f}$ made above shows that the spin-spin correlations $O_2$ and $O_3$, respectively the correlations $E_2$ and $E_3$ are useful tools for determining the CP properties of neutral Higgs bosons. If there is CP violation in the Higgs sector it could be established with these observables in a direct way. For a light Higgs boson with mass $m_\varphi < 2m_W$ the decay $\varphi \rightarrow \tau^-\tau^+$ offers a good possibility to determine whether or not $\varphi$ is a CP eigenstate. We found that with 2000 $\varphi \rightarrow \tau^-\tau^+$ events the combined use of $E_2$ and $E_3$ yields a sensitivity at the 3$\sigma$ level to a range of scalar and pseudoscalar Yukawa couplings corresponding to the ratio $r_\tau = \tilde{a}_\tau/(a_\tau + \tilde{a}_\tau) > 0.36$. The sensitivity can be improved by using also other $\tau$ decay modes than those considered above.

For the case of $\varphi \rightarrow t\bar{t}$ we showed that the order $\alpha_s$ QCD corrections to the single spin asymmetry and to the spin-spin correlations are small. Again, a sizeable pseudoscalar component of $\varphi$ can be traced with the combined use of the correlations $E_2$ and $E_3$. We found that $r_t = \tilde{a}_t/(a_t + \tilde{a}_t) > 0.18$ could be established as a 3$\sigma$ effect with about 1500 reconstructed $\varphi \rightarrow t\bar{t}$ events.

If Higgs boson(s) will be found these quantum number analyses should eventually be feasible with appropriately tuned new high luminosity colliders.
Table 1: Transformation properties of the structure functions under discrete symmetry transformations.

| C | P | CP | T (ImT = 0) | CPT (ImT = 0) |
|---|---|----|-------------|---------------|
| A | A | A | A | A |
| $b_\pm$ | $-b_\mp$ | $-b_\mp$ | $b_\pm$ | $b_\mp$ |
| $c_1$ | $c_1$ | $c_1$ | $c_1$ | $c_1$ |
| $c_2$ | $c_2$ | $c_2$ | $c_2$ | $c_2$ |
| $c_3$ | $-c_3$ | $-c_3$ | $-c_3$ | $c_3$ |

Table 2: Different decay modes of the $\tau^-\tau^+$ pairs and the corresponding correlations of the final state momenta which are sensitive to $\langle \mathcal{O}_2 \rangle$ and $\langle \mathcal{O}_3 \rangle$.
REFERENCES

[1] T.D. Lee, Phys. Rev. D 8, 1226 (1973); Phys. Rep. C 9, 143 (1974).

[2] G.C. Branco and M.N. Rebelo, Phys. Lett. B 160, 117 (1985).

[3] J. Liu and L. Wolfenstein, Nucl. Phys. B 289, 1 (1987).

[4] S. Weinberg, Phys. Rev. D 42, 860 (1990).

[5] A.G. Cohen, D.B. Kaplan, and A.E. Nelson, Annu. Rev. Nucl. Part. Sci. 43, 27 (1993).

[6] W. Bernreuther and A. Brandenburg, Phys. Lett. B 314, 104 (1993); Phys. Rev. D 49, 4481 (1994).

[7] J.R. Dell’Aquila and C.A. Nelson, Nucl. Phys. B 320, 61 (1989).

[8] A. Skjold and P. Osland, Phys. Lett. B 311, 261 (1993); Phys. Lett. B 329, 305 (1994); Nucl. Phys. B 453, 3 (1995).

[9] D. Chang, W.Y. Keung, and I. Phillips, Phys. Rev. D 48, 3225 (1993).

[10] X.G. He, J.P. Ma, and B.H.J. McKellar, Phys. Rev. D 49, 4548 (1994); Mod. Phys. Lett. A 9, 205 (1994).

[11] M. Kremer, J. Kühn, M. Stong, and P. Zerwas, Z. Phys. C 64, 21 (1994).

[12] T. Arens, U. Gieseler, and L.M. Sehgal, Phys. Lett. B 339, 127 (1994).

[13] B. Grzadkowski and J.F. Gunion, Phys. Lett. B 350, 218 (1995).

[14] In extensions of the SM such as 2 doublet extensions CP violation in these modes is not a tree-level but at best a one-loop effect and, hence, quite small.

[15] A. Soni and R.M. Xu, Phys. Rev. D 48, 5259 (1993).
[16] V. Barger, K. Cheung, A. Djouadi, B. Kniehl, and P. Zerwas, Phys. Rev. D 49, 79 (1994).

[17] T. Arens and L.M. Sehgal, Z. Phys. C 66, 89 (1995).

[18] W.N. Cottingham and I.B. Whittingham, Phys. Rev. D 52, 539 (1995).

[19] S. Bar-Shalom et al., Phys. Rev. D 53, 1162 (1996).

[20] J.F. Gunion, B. Grzadkowski, and X.G. He, Phys. Rev. Lett. 77, 5172 (1996).

[21] B. Grzadkowski and J.F. Gunion, Phys. Lett. B 294, 361 (1992).

[22] H. Anlauf, W. Bernreuther, and A. Brandenburg, Phys. Rev. D 52, 3803 (1995); ibid. D 53, 1725 (1996) (E).

[23] D. Atwood and A. Soni, Phys. Rev. D 52, 6271 (1995).

[24] A. Pilaftsis, Phys. Rev. Lett. 77, 4996 (1996).

[25] E. Braaten and J.P. Leveille, Phys. Rev. D 22, 715 (1980).

[26] N. Saka, Phys. Rev. D 22, 2220 (1980); T. Inami and T. Kubota, Nucl. Phys. B 179, 17 (1981).

[27] S.G. Gorishnii, A.L. Kataev, and S.A. Larin, Sov. J. Nucl. Phys. 40, 329 (1984).

[28] K. G. Chetyrkin and A. Kwiatkowski, Nucl. Phys. B 461, 3 (1996).

[29] A. Djouadi and P. Gambino, Phys. Rev. D 51, 218 (1995).

[30] For a review, see B. A. Kniehl, preprint hep-ph/9610500, to appear in Proceedings of the 28th International Conference on High Energy Physics, Warsaw, Poland, 25-31 July 1996, edited by A. Wroblewski.

[31] W. Bernreuther, T. Schröder, and T.N. Pham, Phys. Lett. B 279, 389 (1992).
[32] W. Bernreuther, O. Nachtmann, and P. Overmann, Phys. Rev. D 48, 78 (1993).

[33] R. Akers et al. (OPAL collab.), Z. Phys. C 66, 31 (1995).

[34] D. Buskulic et al. (ALEPH collab.), Phys. Lett. B 346, 371 (1995);
    D. Buskulic et al. (ALEPH collab.), paper submitted to the XXVIII Int. Conf. on High
    Energy Physics, Warsaw (1996).

[35] For a review, see N. Wermes, Bonn preprint BONN-HE-96-10, to appear in the Proceedings of the TAU96 workshop, Estes Park, Colorado (1996).

[36] J. H. Kühn, Phys. Lett. B 313, 458 (1993).

[37] See, e.g., Proceedings of the Linear Collider Workshop on $e^+ e^-$ Collisions at TeV Energies: The Physics Potential, edited by P. Zerwas, DESY orange report DESY 96-123D, Hamburg (1996).
Figure 1a: Decay rate $\varphi \rightarrow t\bar{t}$ in GeV as a function of the Higgs mass for $m_t = 175$ GeV, $a_t = 1$ and $\tilde{a}_t = 0$. The dashed line represents the Born result and the full line is the result to order $\alpha_s$. 
Figure 1b: Decay rate $\varphi \to t\bar{t}$ in GeV as a function of the Higgs mass for $m_t = 175$ GeV, $a_t = (2/3)^{1/2}$ and $\tilde{a}_t = (1/3)^{1/2}$. The dashed line represents the Born result and the full line is the result to order $\alpha_s$. 
Figure 1c: Decay rate $\phi \to t\bar{t}$ in GeV as a function of the Higgs mass for $m_t = 175$ GeV, $a_t = 0$ and $\tilde{a}_t = 1$. The dashed line represents the Born result and the full line is the result to order $\alpha_s$. 
Figure 2: Expectation value $\langle O_1 \rangle$ as a function of the Higgs mass for $m_t = 175$ GeV, $a_t = (2/3)^{1/2}$ and $\tilde{a}_t = (1/3)^{1/2}$. The dashed line represents the Born result and the full line is the result to order $\alpha_s$. 
Figure 3: Expectation value $\langle O_1 \rangle$ as a function of $r_t = \tilde{a}_t/(a_t + \tilde{a}_t)$ for fixed Higgs mass $m_\phi = 400$ GeV and $m_t = 175$ GeV. The dashed line represents the Born result and the full line is the result to order $\alpha_s$. 
Figure 4: Expectation value $\langle O_2 \rangle$ as a function of the Higgs mass for $m_t = 175$ GeV, $a_t = (2/3)^{1/2}$ and $\tilde{a}_t = (1/3)^{1/2}$. The dashed line represents the Born result and the full line is the result to order $\alpha_s$. 
Figure 5: Expectation value $\langle O_2 \rangle$ as a function of $r_t = \tilde{a}_t / (a_t + \tilde{a}_t)$ for fixed Higgs mass $m_\phi = 400$ GeV and $m_t = 175$ GeV. The dashed line represents the Born result and the full line is the result to order $\alpha_s$. 
Figure 6: Expectation value $\langle O_3 \rangle$ as a function of $r_t = \tilde{a}_t/(a_t + \tilde{a}_t)$ for fixed Higgs mass $m_\phi = 400$ GeV and $m_t = 175$ GeV. The dashed line represents the Born result and the full line is the result to order $\alpha_s$. 
Figure 7: Expectation value $\langle \mathcal{O}_4 \rangle$ as a function of $r_t = \tilde{a}_t/(a_t + \tilde{a}_t)$ for fixed Higgs mass $m_\phi = 400 \text{ GeV}$ and $m_t = 175 \text{ GeV}$. The dashed line represents the Born result and the full line is the result to order $\alpha_s$. 
Figure 8: Expectation value $\langle O_2 \rangle$ for $\varphi \rightarrow \tau^- \tau^+$ as a function of $r_\tau = \tilde{a}_\tau/(a_\tau + \tilde{a}_\tau)$ for fixed Higgs mass $m_\varphi = 100$ GeV. The dashed line represents the Born result and the full line is the result to order $\alpha$. 
Figure 9: Expectation value $\langle O_3 \rangle$ for $\varphi \to \tau^- \tau^+$ as a function of $r_\tau = \tilde{a}_\tau / (a_\tau + \tilde{a}_\tau)$ for fixed Higgs mass $m_\varphi = 100$ GeV. The dashed line represents the Born result and the full line is the result to order $\alpha$. 
Figure 10: Number of events $\varphi \rightarrow t\bar{t}$ to establish a nonzero correlation $\langle O_{1,2,3} \rangle$ (with 3 s.d. significance) as a function of $r_t = \tilde{a}_t/(a_t + \tilde{a}_t)$ for fixed Higgs mass $m_\varphi = 400$ GeV and $m_t = 175$ GeV. The dashed line represents the result for $N_{t\bar{t}}^{(1)}$, the full line is the result for $N_{t\bar{t}}^{(2)}$ and the dotted line is the result for $N_{t\bar{t}}^{(3)}$. 
Figure 11: Number of events $\phi \to \tau^+\tau^-$ to establish a nonzero correlation $\langle O_{2,3} \rangle$ (with 3 s.d. significance) as a function of $r_\tau = \tilde{a}_\tau/(a_\tau + \tilde{a}_\tau)$ for fixed Higgs mass $m_\phi = 100$ GeV. The full line is the result for $N_{2}^{(2)}$ and the dotted line is the result for $N_{3}^{(3)}$. 
