Abstract: This paper analyzes the unified performance of energy detection (ED) of spectrum sensing (SS) over generalized fading channels in cognitive radio (CR) networks. The detective performance of SS schemes will be obviously affected by fading channel between communication nodes, and ED has the advantages of fast implementation, no requirement of priori received information and low complexity, so it is meaningful to investigate ED that is performed over fading channels such as Nakagami-\(m\) channel and Rice channel, or generalized fading channels such as \(\kappa-\mu\) fading distribution and \(\eta-\mu\) fading distribution. The \(\alpha-\kappa-\mu\) fading distribution is a generalized fading model that represents the nonlinear and small-scale variation of fading channels. The probability density function (p.d.f.) of instantaneous signal-to-ratio (SNR) of \(\alpha-\kappa-\mu\) distribution is derived from the envelope p.d.f. to evaluate energy efficiency for sensing systems. Next, the probability of detection model with Marcum-\(Q\) function has been derived and the close-form detective expressions with moment generating function (MGF) method are deduced to achieve sensing communications over generalized fading channels. Furthermore, novel and exact closed-form analytic expressions for average area under the receiver operating characteristic (AUC) also have been deduced to analyze the performance characteristics of ED over \(\alpha-\kappa-\mu\) fading channels. Besides, cooperative spectrum sensing (CSS) with diversity reception has been applied to improve the detection accuracy and mitigate the shadowed fading features with OR-rule. At last, the results show that the detection capacity can be evidently affected by \(\alpha-\kappa-\mu\) fading conditions, but appropriate channel parameters will improve sensing performance. On the other hand, the established ED-fading pattern is approved by simulations under diversity combination with CSS at the receiving end and it can significantly enhance the detection performance of proposed algorithms.

Keywords: spectrum sensing (SS); energy detection (ED); fading channels; \(\alpha-\kappa-\mu\) distribution; receiver diversity

I. INTRODUCTION

Nowadays, as wireless services rapidly develop and spectrum wideband has been applied extensively, spectrum scarcity has been extremely serious because of sustained growth of data rate applications in wireless communications, based on this, cognitive radio (CR) networks have been proposed to effectively manage spectrum resources and efficiently make full use of limited spectrum bands [1-5]. Spectrum sensing (SS) is the most important part of CR technologies which enables secondary users (SUs) better access the allocated wideband of primary users (PUs) [6-8]. There are mainly three categories spectrum sensing techniques, which include matched filter detection, cyclostationary feature detection and energy detection (ED). The matched filter detection technique that is normally implemented in the digital domain needs exact bandwidth and modulation type
transmission information, but it requires the minimum possible number of samples since match filter uses the optimal processing [9, 10]. Similarly, statistical characteristics of the transmitted signals are applied to cyclostationary feature detection to improve the probability of detection model [11]. Moreover, ED is the most popular non-coherent signal detection algorithm and it has the advantages in low-complexity, implementation simplicity and detection without a priori knowledge [12], it employs ED radiometer at the receiver to compare the received energy value with fixed threshold and determine the state of PU is absent or present instantaneously. Therefore, considering the detection performance of ED algorithm at the real time is a highly significant research work [12-15].

Urkowitz firstly adopted binary hypothesis-testing signal detection over a flat band-limited Gaussian noise channel by deriving the probability of detection $P_d$ and false alarm $P_f$ which follow the central chi-square and non-central chi-square distribution respectively [16]. Next, Kostylev and Alouini et al. considered ED model over conventional fading channels and even multi-channel fading conditions such as Rayleigh, Nakagami-$q$ and Nakagami-$m$ [17-19]. Since then, ED algorithm have been widely used in corresponding fading scenarios with diversity combining, for example, maximal ratio combining (MRC), selection combining (SC) and equal gain combining (EGC) etc [20-23]. Numerous studies on the basis of ED are investigated to achieve SS over different communication circumstances, in Ref. [24] universal close-form expressions of average probability of ED with cooperative sensing (CSS) are derived over generalized multipath fading channels to improve the efficiency and usefulness of SS. In Ref. [25] novel expressions have been derived over extended generalized-$K$ composite fading channels to provide a unified model for wireless communication channel statistics. Likewise, exact closed-form expressions of detection pattern over $N^*$Rayleigh channels are derived with Meijer $G$-function and Marcum-$Q$ function, then it is extended to the case of square-law selection (SLS) to achieve better performance than conventional detection over Rayleigh fading conditions.

On the other hand, as wireless radio propagation is affected by multipath and shadowing effects simultaneously, the adequate fading expressions are needed to represent the fundamental characteristics for composite statistical patterns [26]. Hence, the generalized fading channels $\kappa-\mu$ and $\eta-\mu$ which describe the line-of-sight (LOS) and non line-of-sight (NLOS) communication conditions are proposed to provide accurate representation of radio propagation [27]. Next, the fading distributions $\alpha-\kappa-\mu$ and $\alpha-\eta-\mu$ are derived to represent the non-linear LOS and NLOS small-scale variation of the fading signals, then a complex general and comprehensive complex fading model $\alpha-\eta-\kappa-\mu$ is put forward to account for short-term propagation phenomena by employing joint phase-envelop method to represent the device-to-device communications, vehicle-to-vehicle communications and indoor-to-outdoor propagation in 5G [28]. Moreover, severe fading channels and composite shadowed models such as $\kappa-\mu$ extreme distribution, $\kappa-\mu$Gamma, $\eta-\mu$Gamma and $K$-distribution have been developed to measure the practical communication channels in the complex environments [29-31].

The $\alpha-\kappa-\mu$ distribution is a very general and flexible fading model which contains more fading parameters to describe the nonlinear and LOS small-scale transmission condition. Specially, it contains $\alpha-\mu$ when $\kappa$ is approached to 0 and $\kappa-\mu$ distribution when $\alpha$ is
2. Furthermore, the special classical distributions such as Rayleigh, Rice, Nakagami-\(m\), Weibull and One-sided Gaussian distribution can be obtained by estimating the fading parameters \(\alpha\), \(\kappa\) and \(\mu\) [32-35]. It obviously shows that the \(\alpha\)-\(\kappa\)-\(\mu\) fading model is more effective and practical than \(\alpha\)-\(\mu\), \(\kappa\)-\(\mu\), Rayleigh, Rice, Nakagami-\(m\) and Weibull distributions. However, although a large number of papers have been devoted to study the sensing performance over generalized fading channels, no studies are related to achieve SS over generalized non-linear LOS fading channels in the open technical literatures, in addition how to complete and improve the performance evaluations of detection algorithm is a key problem to be solved. Motivated by above, due to the merits of ED, in this paper we consider ED algorithm over \(\alpha\)-\(\kappa\)-\(\mu\) generalized fading channels to analyze the generic unified detection model, the contributions of this paper are summarized as follows:

1) The \(\alpha\)-\(\kappa\)-\(\mu\) LOS fading channels model have been derived under instantaneous SNR environment to represent the non-linear small-scale variation of the fading signals, which could be used for investigating the real-time and short-range sensing features in SS under severe fading communications.

2) The novel and unified exact close-form expressions of detection models are derived for different fading scenarios to achieve the sensing communications over \(\alpha\)-\(\kappa\)-\(\mu\) generalized fading channels, to the best of the author’s knowledge, the moment generating function (MGF) method with probability density function-based (PDF-based) approach is firstly utilized to deduce the close-form detective sensing expressions over \(\alpha\)-\(\kappa\)-\(\mu\) fading models towards different nonlinear fading conditions.

3) The novel close-form asymptotic expressions for area under the receiver operating characteristics curve (AUC) and average AUC (\(\overline{\text{AUC}}\)) are derived for revealing and quantifying the relationship between the behavior of ED with the variations of the involved fading parameters algebraically that is not only limited by integer values, but also for non-integer variables, in order to characterize the further receiver operating characteristic curves (ROC), although the exact computation of AUC and \(\overline{\text{AUC}}\) are hard to derive.

4) CSS under diversity reception cases such as MRC, SLC (square law combining) and SLS have been considered to mitigate the shadowed fading features and improve the corresponding detection probability theoretically and practically.

The remaining of this paper is organized as follows. In section II the ED system models are presented. In section III the p.d.f. of \(\alpha\)-\(\kappa\)-\(\mu\) fading models are derived under instantaneous SNR condition. In Section IV the close-form detection expressions have been derived to achieve sensing communications over generalized fading channels. In Section V the \(\overline{\text{AUC}}\) performance of ED which is implemented over \(\alpha\)-\(\kappa\)-\(\mu\) fading models have been analyzed. CSS with diversity reception is given in Section VI. Simulation results are presented in Section VII and conclusions are provided in Section VIII.

**II. SYSTEM MODEL**

![ED system model](Fig1)

The ED model can be assumed to be the binary hypothesis-testing problem in Eq. (1) to determine the absence or presence of unknown wireless signal (H0: signal is absent; H1: signal is present) [12],

\[
\begin{align*}
H_0 : y(t) &= n(t) \\
H_1 : y(t) &= h \cdot s(t) + n(t)
\end{align*}
\]  

(1)

where \(y(t)\) is the received signal, \(n(t)\) denotes
zero-mean complex additive white Gaussian noise (AWGN), \( h \) is the wireless channel gain, \( s(t) \) denotes the transmitted primary signal. From Fig.1 the ED expression can be obtained,

\[
Y = \frac{1}{m} \sum_{i=1}^{m} \frac{y_i^2(t)}{\sigma_i^2} \tag{2}
\]

where \( m \) is the sampling number of received signal, \( \sigma_i^2 \) is the AWGN for received signal \( y_i(t) \). By defining the time bandwidth product as \( u=TW \), \( T \) denotes the time interval and \( W \) denotes the single-sided signal bandwidth. The probability of detection \( P_d \) and the probability of false alarm \( P_f \) can be expressed as [14],

\[
P_d = P_r (y > \lambda | H_1) = Q_u (\sqrt{2\gamma}, \sqrt{\lambda}) \tag{3}
\]

\[
P_f = P_r (y > \lambda | H_0) = \frac{\Gamma(u, \frac{\lambda}{2})}{\Gamma(u)} \tag{4}
\]

where \( Q_u(a,b) \) is the \( u \)-th order generalized Marcum-Q function, \( \Gamma(\cdot) \) is the Gamma function which is defined by the integral

\[
\Gamma(x) = \int_{0}^{\infty} t^{x-1} e^{-t} dt \quad \text{and} \quad \Gamma(\cdot) \text{ is the incomplete Gamma function which is defined by the integral}
\]

\[
\Gamma(a,x) = \int_{x}^{\infty} t^{a-1} e^{-t} dt . \quad \gamma \text{ is the signal-to-ratio (SNR) and } \lambda \text{ is the ED threshold } [14]. \quad \text{Correspondingly, the cumulative density function (c.d.f.) of the binary hypothesis-testing in Eq. (1) can be obtained,}
\]

\[
Y \sim \begin{cases} 
X^2_m; & H_0 \\
X^2_m(2\gamma); & H_1
\end{cases} \tag{5}
\]

From Eq. (2) the probability density function (p.d.f.) of \( y(t) \) can be expressed as,

\[
f_y(y) = \begin{cases} 
\frac{1}{2\pi} \frac{1}{\Gamma(\frac{u}{2})} y^{\frac{u-1}{2}} e^{-\frac{y}{2}}; & H_0 \\
\frac{1}{2} \left(\frac{y}{2}\right)^{\frac{u-1}{2}} e^{\frac{y}{2}} I_{u-1}(\sqrt{2\gamma}); & H_1
\end{cases} \tag{6}
\]

where \( I_{u-1}(\cdot) \) is the first kind modified Bessel function with the order \( u-1 \).

### III. THE \( \alpha\-\kappa\-\mu \) FADING MODELS

The \( \alpha\-\kappa\-\mu \) fading distribution is a general fading distribution that can represent the small-scale fading characteristics and describe the received signal which is propagated in the non-linear LOS condition. For the normalized \( \alpha\-\kappa\-\mu \) distribution with envelope \( R \), the normalized envelope \( r=\sqrt{E(R^2)} \) \( (E(\cdot) \text{ is the expectation}) \) is the root mean square value and \( y=\rho^2 \), the envelope p.d.f. can be shown as [26],

\[
f_{\rho}^{\alpha\-\kappa\-\mu}(\rho) = \frac{\alpha \mu^\frac{1+\mu}{2}}{\mu^{\frac{1+\mu}{2}}} \frac{\Gamma(\frac{1+\mu}{2})}{\Gamma(\frac{1+\mu}{2})} \exp(-\kappa \rho^2 + \kappa \rho^2) \tag{7}
\]

\[
I_{\mu-1}(2\mu, \sqrt{\kappa(1+\kappa)})
\]

where parameter \( \alpha \) indicates the nonlinear characteristics of the propagation medium, variable \( \kappa \) represents the ratio between the total power of the dominant components and the total power of the scattered waves, parameter \( \mu \) is related to the number of multipath waves. For the fading signal with the power \( w=\rho^2 \) and the normalized power is \( w/E(w) \), the power probability density function can be expressed as,

\[
f_{\rho}^{\alpha\-\kappa\-\mu}(w) = \frac{\alpha \mu^\frac{1+\mu}{2}}{\mu^{\frac{1+\mu}{2}}} \frac{\Gamma(\frac{1+\mu}{2})}{\Gamma(\frac{1+\mu}{2})} \exp(-\frac{w}{\rho^2} + \frac{\kappa \rho^2}{\rho^2}) \cdot I_{\mu-1}(2\mu, \sqrt{\kappa(1+\kappa)}) \frac{w^\frac{a}{2}}{w^\frac{a}{2}} \tag{8}
\]

Then the p.d.f. of the instantaneous SNR \( \gamma \) over the \( \alpha\-\kappa\-\mu \) fading channels can be derived
from [Eq. (2), Eq. (6), Eq. (7) and Eq. (8), 27],
\[ f_{\gamma}^{a-k-\mu}(\gamma) = A_{l} \cdot \frac{\gamma^{\frac{a}{2}}}{\gamma^{\frac{a}{2}+\frac{k}{2}}} \exp(-\mu \frac{\gamma^{\frac{a}{2}}}{\gamma^{\frac{a}{2}}}) \cdot \frac{I_{\nu-1}(2\mu\sqrt{(1+\kappa)}\frac{\gamma^{\frac{a}{2}}}{\gamma^{\frac{a}{2}}})}{{\gamma^{\frac{a}{2}}}} \]  
(9)
where
\[ A_{l} = \frac{\alpha\mu\kappa}{{\gamma^{\frac{a}{2}}}^{l}} (1+\kappa) \frac{\gamma^{\frac{a}{2}}}{\gamma^{\frac{a}{2}}} \]  
(10)

IV. SPECTRUM SENSING OVER FADING CHANNELS

Evaluating the detection probability \( P_{d} \) of ED algorithm with Eq. (3) and Marcum-Q function from Ref. [15] and Ref. [16] for single sensing node,
\[ \mathcal{Q}_{ED}^{\alpha-\kappa-\mu} = \sum_{i=0}^{\infty} \frac{\exp(-\gamma^{\frac{a}{2}})\Gamma(l+u,\frac{\lambda}{2})}{\Gamma(l+1)\Gamma(l+u)} \]  
(11)

It is recalled that the average probability of detection over \( \alpha-\kappa-\mu \) generalized fading channels can be obtained as,
\[ P_{d}^{a-k-\mu} = \int_{0}^{\infty} P_{d}^{ED} \cdot f_{\gamma}^{a-k-\mu}(\gamma) d\gamma \]  
(12)
where \( P_{d}^{ED} \) denotes the probability of detection, \( f_{\gamma}^{a-k-\mu}(\gamma) \) is the p.d.f. of the \( \alpha-\kappa-\mu \) distribution under instantaneous SNR. Then by substituting Eq. (9) and Eq. (11), the average probability of detection \( P_{d}^{a-k-\mu} \) can be expressed as,
\[ P_{d}^{a-k-\mu} = \int_{0}^{\infty} \frac{\exp(-\gamma^{\frac{a}{2}})\Gamma(l+u,\frac{\lambda}{2})}{\Gamma(l+1)\Gamma(l+u)} \cdot I_{\nu-1}(2\mu\sqrt{(1+\kappa)}\frac{\gamma^{\frac{a}{2}}}{\gamma^{\frac{a}{2}}}) d\gamma \]  
(13)

Adopting the moment generating function (MGF) algorithm, the p.d.f. of MGF under the instantaneous SNR over \( \alpha-\kappa-\mu \) fading channels is shown as [36],
\[ \phi_{\gamma}^{a-k-\mu}(s) = E^{a-k-\mu}(e^{-sy}) \]  
(14)
where \( E(.) \) denotes the expectation. Deducing the \( n \)-th derivative of the MGF as,
\[ \frac{\delta^{(n)} \phi_{\gamma}^{a-k-\mu}(s)}{\delta s^{n}} = \left\{ \frac{-1}{\lambda^{n}} \right\} \frac{\delta^{(n)} E^{a-k-\mu}(e^{-sy})}{\delta s^{n}} \]  
(15)
Likewise, with Eq. (15) the generic expression \( P_{d}^{a-k-\mu} \) can be obtained in another way,
\[ P_{d}^{a-k-\mu} = \sum_{l=0}^{\infty} \frac{(\frac{l}{\lambda})^{n}}{\Gamma(l+1)\Gamma(l+u)} \cdot \frac{\delta^{(n)} \phi_{\gamma}^{a-k-\mu}(s)}{\delta s^{n}} \bigg|_{s=1} \]  
(16)

Moreover, the closed-form expression over \( \alpha-\kappa-\mu \) fading channels can be evaluated as,
\[ \phi_{\gamma}^{(a,k,\mu)}(s) = \int_{0}^{\infty} (-\gamma)^{a} e^{\frac{-\lambda}{\mu}} A_{l} \cdot \frac{\gamma^{a}}{\lambda^{a}} \cdot \frac{\exp(-\mu \gamma^{\frac{a}{2}} - \kappa \mu \gamma^{\frac{a}{2}}) \cdot I_{\nu-1}(2\mu\sqrt{(1+\kappa)}\gamma^{\frac{a}{2}})}{\gamma^{\frac{a}{2}}} d\gamma \]  
(17)

The modified Bessel function of the first kind with the order \( \nu \) in Eq. (17) can be simplified with [Eq. (8.445), 37]
\[ I_{\nu}(z) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+1)\Gamma(v+k+1)} \frac{z^{k}}{2^{v}} \]  
(18)
Then Eq. (17) can be derived by using infinite series representation,
\[ \phi_{\gamma}^{(a,k,\mu)}(s) = (-1)^{a} A_{l} \cdot \sum_{k=0}^{\infty} \frac{\kappa^{k} \gamma^{\frac{a}{2}}}{\gamma^{\frac{a}{2}}} \frac{\mu^{\frac{a}{2}}}{\gamma^{\frac{a}{2}}} \frac{2^{\frac{a}{2}}}{\gamma^{\frac{a}{2}}} \frac{\exp(-sy - \frac{\mu}{\gamma^{\frac{a}{2}}} \gamma^{\frac{a}{2}} - \frac{\kappa \mu}{\gamma^{\frac{a}{2}}} \gamma^{\frac{a}{2}})}{\gamma^{\frac{a}{2}}} \]  
(19)
Evaluating the integral in Eq. (19) with extended incomplete Gamma function in Ref. [38] and Ref. [39],
\[ \Gamma(a, b) = \int_0^\infty e^{-x}x^{a-1}dx \]
(20)

Next, the closed-form expression of Eq. (19) can be simplified as,
\[ \phi_{\alpha-k-\mu}(s) = (-1)^r A_r \cdot \frac{\Gamma(k+1)\mu^{2k+\mu-1}}{\alpha^2} \sum_{i=0}^{\infty} \frac{\kappa^i}{(\kappa+1)\gamma^i s^{\mu}} . \]
(21)

Furthermore the Eq. (21) can be evaluated with [Theorem 3.1, 39] as,
\[ \phi_{\alpha-k-\mu}(s) = (-1)^r A_r \cdot \frac{\Gamma(k+1)\mu^{2k+\mu-1}}{\alpha^2} \sum_{i=0}^{\infty} \frac{\kappa^i}{(\kappa+1)\gamma^i s^{\mu}} . \]

where
\[ A_r = \frac{\mu^2}{\kappa^2} \frac{\Gamma(k+1)\mu^{2k+\mu-1}}{\alpha^2} \frac{\mu^2}{\gamma^2 s^{\mu+\kappa}} \]
(23)

The close-form expression of the FOX-H function can be expressed as,
\[ H^{\alpha-k-\mu}(\cdot) \]
5.1 AUC under instantaneous SNR condition

The AUC measurement is usually used for characterizing the performance of ED [40], health care field tests [41] and machine learning algorithms [42] such as plotting \( P_d \) versus \( P_f \) (ROC) or missed detection probability \( P_m \) (complementary ROC). It varies between 0.5 and 1, and it can be considered comprehensively by analyzing \( P_d \) and \( P_f \) which represents the probability of more correct choosing, so it makes sense that deriving the close-form expressions for the AUC of ED under appropriate scenarios to quantify the detection performance like Ref. [43-46]. Here we introduce the ED threshold \( \lambda \) varies from 0 to \( \infty \) to analyze the capability of energy detector. When the instantaneous SNR value is \( \gamma \), the AUC can be shown as [46],
\[ AUC(\gamma) = \int_0^\infty P_d(\gamma, \lambda) dP_f(u, \lambda) \]
(25)

Taking the derivative with \( P_f(u, \lambda) \), Eq. (25) can be written as,
\[ AUC(\gamma) = -\int_0^\infty P_f(u, \lambda) \frac{\partial P_f(u, \lambda)}{\partial \lambda} d\lambda \]
(26)

With Eq. (3), Eq. (4) and [Eq. (12), 46], Eq. (26) can be derived as,
\[ AUC(\gamma) = 1 - \sum_{i=0}^{\infty} \sum_{u=0}^\infty \left( \frac{u+1}{u+1} \right)^{\gamma' \exp(-\frac{\gamma}{2})} d\gamma \]
(27)

5.2 \( \overline{AUC} \) over \( \alpha-k-\mu \) fading channels

The average AUC (\( \overline{AUC} \)) can be investigated with the p.d.f. of generalized fading models for the instantaneous SNR distribution to indicate the properties of the fading channels, therefore, \( \overline{AUC} \) can be shown as,
\[ \overline{AUC} = \int_0^\infty AUC(\gamma) \cdot f_{\alpha-k-\mu}(\gamma) d\gamma \]
(28)

where
\[
\left(\begin{array}{c}
a \\ b
\end{array}\right) = \frac{a!}{b!(a-b)!}
\] (29)

From Ref. [21], Eq. (28) can be derived with the help of Eq. (25)-(Eq. 27),
\[
\bar{AUC} = 1 - \sum_{i=0}^{n} \sum_{l=0}^{i} \left( \frac{1}{2^{l+i+1}} \cdot i! \right) \int_{0}^{\gamma} e^{-\gamma/2} \frac{\gamma^l}{l!} d\gamma
\] (30)

Expanding Eq. (34) by the Binomial theorem [37],
\[
B_i = \sum_{i=0}^{\infty} \sum_{n=0}^{\infty} \sum_{u=0}^{\infty} \left( \frac{\gamma}{2} \right)^u \frac{n!}{w^n} \left( \frac{\mu + k\mu}{\gamma^2} \right)^w d\gamma
\] (35)

Next, the integral in Eq. (35) can be simplified as,
\[
B_i = \sum_{n=0}^{\infty} \sum_{u=0}^{\infty} \left( \frac{\gamma}{2} \right)^u \frac{n!}{w^n} \left( \frac{\mu + k\mu}{\gamma^2} \right)^w
\] (36)

Lastly, the exact close-form expression can be obtained by substituting Eq. (36) into Eq. (33), then Eq. (33) can be simplified as,
\[
\bar{AUC} = 1 - \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \sum_{u=0}^{\infty} \left( \frac{1}{2^{l+i+1}} \cdot i! \right) \int_{0}^{\gamma} e^{-\gamma/2} \frac{\gamma^l}{l!} d\gamma
\] (37)

where
\[
A_i = \frac{\alpha\mu^w(1+\kappa)^\mu}{\exp(\kappa\mu) - \frac{\alpha\mu}{\gamma}}
\] (38)

VI. ED WITH RECEIVER DIVERSITY OVER \(\alpha-\kappa-\mu\) FADING CHANNELS

6.1 Upper bound of detection performance with MRC
It makes sense that implementing ED with diversity over fading channels at the receiver can increase the receiver SNR for the $L$ diversity paths which are normally supposed to independent and identically distribution (i.i.d.) [36].

For MRC, at the receiving end the total output SNR will be combined by all diversity branches and the collected data is added up to the reserved energy detector. Although MRC requires prior channel knowledge of the signal, in practice it estimates the upper bound on the performance of ED [11]. The total instantaneous SNR of MRC is given by,

$$\gamma_{MRC} = \frac{L}{i=1} \gamma_i$$

(39)

where $L$ denotes the number of antennas for each SU, the SNR of $i$-th receiver branch is defined as $\gamma_i$.

When the number of diversity branches at the receiver is $L$, the average probability of detection with MRC can be computed by substituting Eq. (39) and Eq. (22) in Eq. (13),

$$P_{d,MRC}^{\alpha-\kappa-\mu} = \frac{\sum_{i=0}^{L} (-1)^i \Gamma(l + u, \frac{L}{2})}{\Gamma(l + 1) \Gamma(l + u)} \delta^{(i)} \frac{\phi_{\text{MGF}}^{\alpha-\kappa-\mu}}{\sum_{i=0}^{L} \gamma_i} (s)$$

(40)

where the $\phi_{\text{MGF}}^{\alpha-\kappa-\mu}(s)$ denotes the MGF with the total SNR of diversity branches. Using [Eq. (24), 21], the MGF based on MRC scheme can be evaluated as,

$$\phi_{\text{MGF}}^{\alpha-\kappa-\mu}(s) = \prod_{i=1}^{L} \phi_{\gamma_i}^{\alpha-\kappa-\mu}(s)$$

(41)

On the other hand, it is worth noting that the Leibniz’s rule can be obtained when the number of product terms of functions is two [Eq. (0.42), 37],

$$d^{(n)}(u \cdot v)(x) = \sum_{i=0}^{n} \binom{n}{i} d^{(i)}u(x) d^{(n-i)}v(x)$$

(42)

Deriving the $n$-order Leibniz’s rule with the aid of [Eq. (26), 21], the $n$-th derivative of (41) can be deduced by means of Eq. (14), Eq. (15) and Eq. (42),

$$\phi_{\gamma_i}^{(n-k-\mu)}(s) = \sum_{n=0}^{\infty} \sum_{n_1=0}^{n} \ldots \sum_{n_L=0}^{n} \binom{n}{n_1} \ldots \binom{n_{L-1}}{n_L} (n_{L-1})$$

(43)

Furthermore, from Eq. (22), Eq. (24), Eq. (40) and Eq. (43), the close-form detective expressions of ED over $\alpha-\kappa-\mu$ fading channels are given by,

$$\begin{align*}
\Phi_{d,MRC}^{\alpha-\kappa-\mu} &= \sum_{i=0}^{L} (-1)^i \Gamma(l + u, \frac{L}{2}) \\
\phi_{\gamma_i}^{(n-k-\mu)}(s) &= \phi_{\gamma_i}^{(n_{L-1})}(s) \cdots \phi_{\gamma_i}^{(n_{L-1})}(s) (\phi_{\gamma_i}^{(n_{L-1})}(s)) \\
\end{align*}$$

(44)

where

$$\phi_{\gamma_i}^{(n-k-\mu)}(s) = (-1)^n A_i \cdot A_i$$

$$\sum_{i=0}^{L} \frac{\kappa^k (k + 1)^{\mu_{\gamma_i}}}{\Gamma(k + 1) \Gamma(k + \mu)} s^{k-\mu}$$

$$H_{0,2,4}^{\alpha-\kappa-\mu} = \frac{\mu}{\gamma^{\alpha-\kappa-\mu}} + \frac{\kappa_{\mu}}{\gamma^{\alpha-\kappa-\mu}} (0,1), (n + \alpha \frac{2(2k + 2\mu)}{4}, -\frac{\alpha}{2})$$

(45)

6.2 Cooperative sensing with diversity reception SLC and SLS

Square Law Combining (SLC): Similar to MRC, the SLC scheme requires the received signals are integrated and squared, further the output signals are summed together [45]. The decision model follows a central chi-square distribution with $2Lu$ degrees of freedom if the binary hypothesis testing is $H_{0,2}$ or a non-central chi-square distribution with $2Lu$ degrees of freedom under binary hypothesis testing $H_1$. The total received SNR $\gamma_{SLC}$ is equal to combined instantaneous SNR $\gamma_{2MRC}$ under MRC, besides the time bandwidth product $u$ is replaced by $Lu$ for Eq. (44) to represent the detection capacity over $\alpha-\kappa-\mu$ fading channels. When the number of
diversity branches is $L$, the detection probability can be shown as,

$$P_{d,SLS} = \sum_{l=0}^{n} \frac{(-1)^l \Gamma(l + Lu, \frac{L}{2})}{\Gamma(l + 1) \Gamma(l + Lu)} \cdot \sum_{n_l=0}^{n} \sum_{n_{l-1}=0}^{n_l} \ldots \sum_{n_1=0}^{n_L} \left( \frac{l}{n_l} \right) \ldots \left( \frac{n_{l-1}}{n_{l-2}} \right),$$

where

$$\gamma_{SLC} = \gamma_{MRC}$$

Square Law selection (SLS): In SLS scheme the maximum decision statistics $\gamma_{SLS}$ ($\gamma_{SLS} = \max\{y_1, y_2, \ldots, y_L\}$) is selected to calculate the average probability of detection of ED over fading channels [47]. The detection probability can be expressed as,

$$P_{d,SLS} = 1 - \prod_{i=1}^{L} (1 - P_{d,SLS})$$

where

$$P_{d,SLS} = (-1)^x \frac{\alpha \mu \kappa}{2} \frac{\kappa\mu}{2} \exp(\kappa \mu) \sum_{k=0}^{\infty} \frac{\kappa^k (\kappa + 1)^k \mu^{2k}}{\Gamma(k + 1) \Gamma(k + \mu) \Gamma(\mu + k)^k},$$

$$H_{0,2} = \frac{\mu}{\gamma^2 s^2} + \frac{\kappa \mu}{\gamma^2 s^2} \left[ (0, 1), (n + \frac{\alpha}{2}, \frac{2k + 2 \mu}{2}) \right]$$

Cooperative spectrum sensing (CSS): CSS have been proposed to help the shadowed SUs detect the occupied wideband of PUs and improve the sensing capability for severe multipath fading, in which firstly SUs send their own decisions to the fusion center (FC) respectively, next FC makes the global decision by combining the received information to determine the absence or presence of PU [24]. In view of the above, the CSS decision rule follows,

$$D = \sum_{i=1}^{N} D_i = \left\{ \begin{array}{ll}
< n, H_0 \\
\geq n, H_1
\end{array} \right.$$

where $D$ is the sum of sensed decisions with hard decision for sending “1-bit” under $H_1$ or “0-bit” under $H_0$, $N$ is the number of collaborative users. From Eq. (50) the AND-rule is corresponding to the case of $n = N$ and OR-rule is corresponding to the case of $n = 1$, otherwise it represents the “n-out-of-N” collaborative voting rule for $n \neq 1$ and $n \neq N$. In order to evaluate the detection probability $P_{d,CSS}$ and the false-alarm probability $P_{f,CSS}$ of CSS with SLC and SLS for the OR-rule, it can be respectively determined by,

$$P_{d,CSS,\alpha-\kappa-\mu} = 1 - \prod_{i=1}^{N} (1 - P_{d,SLS})$$

$$P_{f,CSS,\alpha-\kappa-\mu} = 1 - \prod_{i=1}^{N} (1 - P_{d,SLS})$$

VII. NUMERICAL SIMULATION AND ANALYSIS

In this section, numerical simulation and analysis for the behavior of ED have been provided to reveal the crucial impact on the performance of SS over $\alpha$-\kappa-\mu fading channels with software packages as MATLAB and MATHEMATICA [48]. The corresponding performances of Section IV-Sec VI are quantified in the following figures under different fading scenarios to show the various numerical features of severe shadowing conditions.

Fig.2 illustrates the average detection probability $P_{d,\alpha-\kappa-\mu}$ versus average SNR $\bar{\gamma}$ over $\alpha$-\kappa-\mu fading channels for different channel parameters $\kappa$ and $\mu$ under constant nonlinear coefficient $\alpha$, when $u = 2$ and $P_f$
=0.01. It can be observed that although nonlinearity coefficient of channel condition is low, the raise of \( \kappa \) and \( \mu \) can improve the detection capacity of ED because the higher ratio between the total power of the dominant components and the total power of the scattered waves and more related variable of multipath clusters will lead to more received power of dominant components, when \( \alpha=1.35 \), \( \kappa=0.7-1.0 \) and \( \mu=0.7-1.0 \). Besides the \( \bar{P}^{\alpha-\kappa-\mu} \) also improves substantially as \( \bar{\gamma} \) raises for low value of constant \( \alpha \), and if \( \kappa \) and \( \mu \) are constant, higher \( \alpha \) will lead to higher average detection probability. Furthermore, if \( \kappa \) and \( \mu \) are invariable, the detection probability improves more quickly for higher \( \alpha \) as \( \bar{\gamma} \) increases, for example, when \( \alpha=1.35-1.75 \), \( \kappa=1.0 \) and \( \mu=1.0 \).

Furthermore, it can be seen that under low values of \( \alpha \) small variations of \( \mu \) will evidently improve the sensing performance of ED, comparing with the values change for \( \kappa \). It has been demonstrated that the effects of related variable of multipath clusters are more critical than the ratio of the total dominant components power to the total scattered waves power if the value of \( \alpha \) is low.

![Fig.2](image-url) **Fig.2** Average probability of detection versus average SNR (dB) with \( u=2 \) and \( P_f=0.01 \)

On the other hand, Fig.3 and Fig.4 jointly show the \( \text{AUC} \) value against \( \bar{\gamma} \) for different variables of \( \alpha \), \( \kappa \) and \( \mu \) to present corresponding performance characteristics of ED like Ref [26, 32-35].

Fig.3 depicts \( \text{AUC} \) versus \( \bar{\gamma} \) with \( u=2 \) for low linearity coefficient. If \( \kappa=0.7 \), \( \mu=0.7 \) and when \( \alpha=1.5 \) is twice more than \( \alpha=0.7 \), the case for \( \alpha=1.5 \) exceeds the case for \( \alpha=0.7 \) obviously as \( \bar{\gamma} \) increases when \( \bar{\gamma} \geq 2 \text{dB} \).

![Fig.3](image-url) **Fig.3** Average AUC versus average SNR (dB) over \( \alpha-\kappa-\mu \) fading channels with \( u=2 \) for low values of \( \alpha \)

Fig.4 illustrates \( \text{AUC} \) against average SNR with time bandwidth product \( u=2 \) for higher nonlinear coefficient compared to \( \alpha \) in Fig.3. Similarly better average SNR condition is corresponding to better ROC curve performance for ED-based SS scheme. However, although under high values of \( \alpha \), it is also can be seen that higher \( \alpha \) will lead to better detection capability, the variations of other channel parameters \( \kappa \) and \( \mu \) could not significantly alter the \( \text{AUC} \) as parameter \( \alpha \) increases. Besides, it can be obtained from Fig.3 and Fig.4 at the same time that higher \( \alpha \) corresponds to better simulation results when average SNR is greater than 3 dB, and better simulation for higher \( \kappa \) and \( \mu \) with the same value of \( \alpha \) when average SNR is less than 3 dB.
The average probability of detection versus average SNR (dB) with \( u=2 \) for high values of \( \alpha \).

In Fig. 5 the upper bound of detection probability for MRC have been analyzed theoretically with \( u=2 \) and \( P_f=0.01 \) when channel parameters are assumed as follows: \( \alpha=1.35, \kappa=1.0 \) and \( \mu=1.0 \). It is shown that the performance of detection is proportional to the average SNR and the number of diversity branch. Furthermore as the diversity branch increases, the average probability of detection can be obviously raised. For example, despite of low \( P_f \), if \( L=2 \), \( P_f=0.767 \) which is compared to \( P_f=0.519 \) for \( L=1 \) when average SNR is 8 dB. Likewise, \( P_f=0.835 \) for \( L=3 \) and it far exceeds the detection probability \( (P_f=0.617) \) for \( L=2 \) when average SNR is 4 dB.

On the other side, Fig. 6 and Fig. 7 show that CSS with diversity combining can availabley improve the sensing performance of ED over \( \alpha-\kappa-\mu \) fading channels for \( u=2, \alpha=1.35, \kappa=1.0 \) and \( \mu=1.0 \). The average probability of detection versus average SNR with the number of collaborative users \( N=1,2 \) and \( N=2,3 \) are analyzed respectively to present the relationship between the detection capability and average instantaneous SNR for various cooperative users. It can be indicated that under low nonlinear environmental model, in practical sensing course the diversity technique and CSS improve the detection performance in common, although the \( P_f \) for SLC can be expressed as \( 1-(1-\Gamma(u,\lambda/2)/\Gamma(\lambda))^{N} \). Furthermore, it can be inferred that SLC has better effects for detection probability than SLS under the same channel condition, and the improvement capability for diversity combining is almost similar to CSS.
VIII. CONCLUSIONS

This paper investigates the SS based on ED that is implemented over $\alpha$-$k$-$\mu$ fading channels to reveal the relationship between the performance of ED and nonlinear LOS fading channels. The $\alpha$-$k$-$\mu$ fading model under instantaneous SNR condition have been derived to achieve SS communications and the novel unified close-form expressions of ED over fading channels have been deduced to show essential sensing probability with PDF-based and MGF algorithm. In addition, novel exact close-form expressions of average AUC have been derived to quantify the behavior of ED under different values of nonlinear coefficient and other crucial fading parameters. Next, it is demonstrated that diversity technique and CSS can jointly improve the detection performance under severe fading conditions and upper bound with MRC have been inferred to evaluate the sensing performance theoretically. Generally speaking, the sufficient results that are derived above can be completely used to quantify the performance of SS over nonlinear LOS fading scenarios, and it can radically improve the energy efficiency for CR systems in wireless communications.

References

[1] J. Mitola, G. Q. Maguire. Cognitive radio: making software radios more personal[J]. IEEE Personal Communications, 1999, 6(4): 13-18.

[2] S. Haykin. Cognitive radio: brain-empowered wireless communications[J]. IEEE Journal on Selected Areas in Communications, 2005, 23(2): 201-220.

[3] T. Yucek, H. Arslan. A survey of spectrum sensing algorithms for cognitive radio applications[J]. IEEE Communications Surveys and Tutorials, 2009, 11(1): 116-130.

[4] P. C. Sofotasios, A. Bagheri, T. A. Tsiftsis, et al. A Comprehensive Framework for Spectrum Sensing in Non-Linear and Generalized Fading Conditions[J]. IEEE Transactions on Vehicular Technology, 2017.

[5] D. Bera, I. Chakrabarti, S. S. Pathak, et al. Another Look in the Analysis of Cooperative Spectrum Sensing over Nakagami-m Fading Channels[J]. IEEE Transactions on Wireless Communications, 2017, 16(2): 856-871.

[6] S. K. Sharma, T. E. Bogale, S. Chatzinotas, et al. Cognitive Radio Techniques Under Practical Imperfections: A Survey[J]. IEEE Communications Surveys and Tutorials, 2015, 17(4): 1858-1884.

[7] Lu L, Zhou X, Onunkwo U, et al. Ten years of research in spectrum sensing and sharing in cognitive radio[J]. EURASIP Journal on Wireless Communications and Networking, 2012(1): 28.

[8] Akyildiz I. F., Lo, B. F., Balakrishnan R. Cooperative spectrum sensing in cognitive radio networks: A survey[J]. Physical Communication, 2011, 4(1): 40-62.

[9] D. Cabric, A. Tkachenko, R. W. Brodersen. Spectrum Sensing Measurements of Pilot, Energy, and Collaborative Detection[C]. Proceedings of the 2006 Military Communications conference (MILCOM): 2006, Washington. DC, USA, 2016: 1-7.

[10] A. Sahai, D. Cabric. A tutorial on spectrum sensing: Fundamental limits and practical challenges[J]. IEEE DySPAN, 2005: 77-79.

[11] A. Al Hammadi, O. Alhussein, P. C. Sofotasios, et al. Unified Analysis of Cooperative Spectrum Sensing Over Composite and Generalized Fading Channels[J]. IEEE Transactions on Vehicular Technology, 2016, 65(9): 6949-6961.

[12] E. Chatziantoniou, B. Allen, V. Velisavljevic, et al. Energy Detection Based Spectrum Sensing Over Two-Wave With Diffuse Power Fading Channels[J]. IEEE Transactions on Vehicular Technology, 2017, 66(1): 868-874.

[13] M. Zheng, L. Chen, W. Liang, et al. Energy-Efficiency Maximization for Cooperative Spectrum Sensing in Cognitive Sensor Networks[J]. IEEE Transactions on Green Communications and Networking, 2017, 1(1): 29-39.

[14] A. Bagheri, P. C. Sofotasios, T. A. Tsiftsis, et al. AUC study of energy detection based spectrum sensing over $\eta$-$\mu$ and $\alpha$-$\mu$ fading channels[C]. Proceedings of 2015 IEEE International Conference on Communications (ICC): 2015, London, England, 2015: 1410-1415.

[15] A. Bagheri, P. C. Sofotasios, T. A. Tsiftsis, et al. Energy detection based spectrum sensing over enriched multipath fading channels[C]. Proceedings of 2016 IEEE Wireless Communications and Networking Conference: 2016, Doha, Qatar, 2016: 1-6.

[16] Urkowitz H. Energy detection of unknown deterministic signals[J]. Proc IEEE, 1967, 55(4): 523-531.

[17] V. I. Kostylev. Energy detection of a signal with random amplitude[C]. Proceedings of 2002 IEEE
International Conference on Communications. Conference Proceedings (ICC): 2002, New York, USA, 2002: 1606-1610.

[18] Herath S. P, Rajatheva N., Tellumbura C.. On the energy detection of unknown deterministic signal over Nakagami channels with selection combining[JC]: Proceedings of 2009 Canadian Conference on Electrical and Computer Engineering (CCECE): 2009, St. John’s, Canada, 2009: 745-749.

[19] F. F. Digham, M. S. Alouini, M. K. Simon. On the Energy Detection of Unknown Signals Over Fading Channels[J]. IEEE Transactions on Communications, 2007, 55(1): 21-24.

[20] B. Li, M. Sun, X. Li, et al. Energy Detection Based Spectrum Sensing for Cognitive Radios Over Time-Frequency Doubly Selective Fading Channels[J]. IEEE Transactions on Signal Processing, 2015, 63(2): 402-417.

[21] A. Bagheri, P. C. Sofotasios, T. A. Tsiftsis, et al. Spectrum sensing in generalized multipath fading conditions using square-law combining[J]: Proceedings of 2015 IEEE International Conference on Communications (ICC): 2015, London, England, 2015: 7528-7533.

[22] P. C. Sofotasios, E. Rebeiz, L. Zhang, et al. Energy Detection Based Spectrum Sensing Over κ-μ and extreme κ-μ fading channels[J]. IEEE Transactions on Vehicular Technology, 2013, 62(3): 1031-1040.

[23] R. K. Mallik, R. D. Murch, Y. Li. Channel Magnitude Based Energy Detection with Receive Diversity for Multi-Level Amplitude-Shift Keying in Rayleigh Fading[J]. IEEE Transactions on Communications, 2017.

[24] L. Mohjazi, D. Dawoud, P. Sofotasios, et al. Unified analysis of cooperative spectrum sensing over generalized multipath fading channels[JC]: Proceedings of 2015 IEEE 26th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC): 2015, Hong Kong, CHINA, 2015: 370-375.

[25] H. R. Alhennawi, M. H. Ismail, H. A. M. Mourad. Performance evaluation of energy detection over extended generalised-κ composite fading channels[J]. Electronics Letters, 2014, 50(22): 1643-1645.

[26] P. C. Sofotasios, S. Freear. The α - κ - μ/gamma distribution: A generalized non-linear multipath/ shadowing fading model[JC]: Proceedings of 2011 Annual IEEE India Conference: 2011, Hyderabad, India, 2011: 1-6.

[27] M. D. Yacoub. The κ-μ distribution and the η-μ distribution[J]. IEEE Antennas and Propagation Magazine, 2007, 49(1): 68-81.

[28] M. D. Yacoub. The α-η-κ-μ Fading Model[J]. IEEE Transactions on Antennas and Propagation, 2016, 64(8): 3597-3610.

[29] P. C. Sofotasios, S. Freear. The η-μ/gamma composite fading model[JC]: Proceedings of 2010 IEEE International Conference on Wireless Information Technology and Systems: 2010, Honolulu, USA, 2010: 1-4.

[30] P. C. Sofotasios, S. Freear. The κ-μ/gamma composite fading model [JC]: Proceedings of 2010 IEEE International Conference on Wireless Information Technology and Systems: 2010, Honolulu, USA, 2010: 1-4.

[31] P. C. Sofotasios, S. Freear. The κ-μ Extreme/ Gamma Distribution: A Physical Composite Fading Model[JC]: Proceedings of 2011 IEEE Wireless Communications and Networking Conference: 2011, Cancun, Mexico, 2011: 1398-1401.

[32] G. Fraidenrach, M. D. Yacoub. The α-η-μ and α-κ-μ Fading Distributions[JC]: Proceedings of 2006 IEEE Ninth International Symposium on Spread Spectrum Techniques and Applications: 2006 Manaus-Amazon, Brazil, 2006: 16-20.

[33] Souza R. A. A. D., Ribeiro A. M. O., Guimarães D. A.. On the Efficient Generation of α-κ-μ and α-η-μ White Samples with Applications[JC]. International Journal of Antennas and Propagation, 2015.

[34] Li B., Hou J., Li X., et al. Deep Sensing for Space-Time Doubly Selective Channels: When a Primary User Is Mobile and the Channel Is Flat Rayleigh Fading[J]. IEEE Transaction on Signal Processing, 2016, 64(13): 3362-3375.

[35] P. C. Sofotasios, S. Freear. The α-κ-μ Extreme distribution: Characterizing non-linear severe fading conditions[JC]: Proceedings of 2011 Australasian Telecommunication Networks and Applications Conference (ATNAC): 2011, Melbourne, Australia, 2011: 1-4.

[36] A. Annamalai, O. Olabiyi, S. Alam, et al. Unified analysis of energy detection of unknown signals over generalized fading channels[JC]: Proceedings of 2011 7th International Wireless Communications and Mobile Computing Conference: 2011, Istanbul, Turkey, 2011: 636-641.

[37] I. S. Gradshteyn, I. M. Ryzhik. Table of Integrals, Series, and Products, 7th ed. New York: Academic, 2007.

[38] I. E. Atawi, O. S. Badameh, M. S. Aloqlah, et al. Energy-detection based spectrum-sensing in cognitive radio networks over multipath/shadowed fading channels[J]. 2015 Wireless Telecommunications Symposium (WTS): 2015, New York, USA, 2015: 1-6.

[39] Chaudhry M. A., Zubair S. M.. Extended incomplete gamma functions with applications[JC]. Journal of mathematical analysis and applications, 2002, 274(2): 725-745.

[40] J. P. Egan. Signal Detection Theory and ROC Analysis. NewYork: Academic Press, 1975.
[41] J. A. Swets, R. M. Dawes, J. Monahan. Better decisions through science[J]. Scientific American, 2000, 283(4): 82–87.
[42] F. J. Provost, T. Fawcett. Robust classification for imprecise environments[J]. Machine Learning, 2001, 42(3): 203–231.
[43] J. H. Shapiro. Bounds on the area under the ROC curve[J]. Journal of the Optical Society of America A, 1998, 16(1): 53-57.
[44] S. Atapattu, C. Tellambura, H. Jiang. Analysis of area under the ROC curve of energy detection[J]. IEEE Transactions on Wireless Communications, 2010, 9(3): 1216-1225.
[45] A. Bagheri, P. C. Sofotasios, T. A. Tsiftsis, et al. Area under ROC curve of energy detection over generalized fading channels. Proceedings of 2015 IEEE 26th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC): 2015, Hong Kong, China, 2015: 656-661.
[46] S. Alam, O. Olabiyi, O. Odejide. A performance study of energy detection for dual-hop transmission with fixed gain relays: Area under the ROC Curve (AUC) approach [C]// Proceedings of 2011 IEEE 22nd International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC): 2011, Toronto, Canada, 2011: 1840-1844.
[47] P. C. Sofotasios, L. Mohjazi, S. Muhaidat, et al. Energy Detection of Unknown Signals Over Cascaded Fading Channels[J]. IEEE Antennas and Wireless Propagation Letters, 2016, 15: 135-138.
[48] Wolfram, The wolfram functions site. [Online]. Available: http://functions.wolfram.com