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Size-dependent frequency of simply supported elastic ultra-thin films with surface effect under periodic vibration

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Abstract

Although surface effects play an important role in the mechanical properties of ultra-thin films, the nonlinear vibrations of ultra-thin films influenced by surface effects have not been fully understood. This paper develops an analytical framework for studying the nonlinear vibrations of simply supported ultra-thin films with surface effects. The framework is based on the modified Kirchhoff plate theory. The surface stress effects are treated by the Gurtin–Murdoch surface elasticity model and the motion equations include the effects of curvature and classical inertia. The dimensionless frequency of forcibly vibrated ultra-thin films with a simple support and surface effects is explicitly deduced through a series of perturbation procedure. Finally, the surface effects are evaluated in two numerical examples. In these demonstrations, the surface effects significantly influenced the dimensionless frequency when the film thickness reduced to one micrometer or less.

1. Introduction

Beams, plates and shells with rectangular geometry are key structural elements, and they have attracted an increasing share of attention owing to their singular structures, excellent properties, and potential applications as the general building blocks of microelectromechanical systems (MEMSs) and nanoelectromechanical systems (NEMSs). For example, they are used in radio-frequency switches, microscaled pumps, and electrostatic actuators [1–6]. Their bending, buckling and vibration of many MEMS/NEMS structures involving ultra-thin films (nano-scale thick) depend on their absolute geometrical parameters in the scale of submicron [7–13].

Surfaces effects caused by the atoms at or near the free surface of an ultra-thin film have different equilibrium requirements as compared to those within the bulk of material due to different environmental conditions. These surface effects of thin films are usually neglected in classical plate elasticity theory as the classical continuum theories neglect the size-depend property. Based on the classical continuum theories, the stress at a point is a function of strains at that point; thus, the elastic responses of a structure can be linear responses.

However, surface effects are demonstrated to be significant at high surface-to-bulk ratios [14–20], so they largely contribute to the elastic responses of ultra-thin films. In fact, as a type of nanostructure, ultra-thin film indeed has size-dependent properties and nonlinear mechanical properties. Modeling the size dependence of such nonlinear responses has attracted the interest of many researchers [21–27]. Experimental results indicate that nanoplates or free-standing films either become stiffer or softer than their bulk counterparts through surface reconstruction [28–32]. Also, atomistic simulations have shown that the elastic constants of ultra-thin films with surface effects can be larger or smaller than those of their bulk counterparts [33–35]. For example, Gurtin and Murdoch [36–38] formulated a generic continuum model of surface elasticity, which treats a solid surface as a two-dimensional elastic membrane with different material constants adhering without slipping to the underlying bulk material. When the surface constitutive constants are properly set, the continuum model incorporating surface elasticity can accurately predict the elastic responses of thin films, as can atomistic modeling [39–42]. Given the complex nonlinear response of ultra-thin films due to factors such as surface effect,
nonlinear vibration of different ultra-thin film structures like plates, beams and shells have attracted a wide research attention in recent years.

The present study investigates a nonlinear size-dependent vibration model of a simply supported elastic thin film subjected to harmonic forces and surface effects. The governing equations with the surface effect include membrane forces and bending moments incorporating the curvature term, planar inertia term, and rotational inertia as described in our previous study [22]. By incorporating the transverse periodic load, the model simultaneously considers the influences of the nonlinear strain terms, harmonic forces, and dynamic factor on the dimensionless frequency of the thin film. To verify the importance of the size-dependent surface effect on the dimensionless frequency, this study numerically investigates two thin films composed of different materials. Each film is less than one micrometer thick.

2. Problem formulation

Consider a rectangular thin film of dimensions a along the x axis and b along the y axis under a transverse load $P_3 = e^{i\omega P} \sin \pi x \sin \pi y$, as shown in figure 1. The four edges of the film are simply supported and free of boundary stresses. The simply supported plate problem is solved in this paper to demonstrate the new proposed formulation. This simply supported plate problem has been widely used in many studies, where ultra-thin film problems are studied. The boundary conditions along these edges are written as follows:

$$
\begin{align*}
    u_x &= w_{,xx} = 0 \quad \text{at } x = 0, a \\
    u_y &= w_{,yy} = 0 \quad \text{at } x = 0, b,
\end{align*}
$$

This paper assumes the Cartesian coordinate system $x_i (i = 1, 2, 3)$ in which the $x_3$ plane coincides with the undeformed midplane of the film. The upper surface $S^+$ and lower surface $S^-$ of the film are defined by $x_3 = \pm h/2$, respectively. The usual summation convention for repeated indices is adopted, with Latin indices ranging from 1 to 3 and Greek indices taking values of 1 and 2. The subscript or superscript 0 denotes a stress or displacement parameter of the midplane. A comma represents differentiation with respect to the suffix index. The resultant forces and moments, denoted as $N_{ab}^*$ and $M_{ab}^*$, respectively, satisfy the following governing equation (2). The upper and lower surface stresses satisfy equilibrium relations (Gurtin and Murdoch, 1978), which employ in the strain energy and kinetic energy. The material in the interior of the film obeys the usual three-dimensional Hooke’s law, which employ in the strain energy and kinetic energy. The generalized Lagrangian function of strain energy, kinetic energy and work done by external force is variational for displacement by using Hamilton’s principle. The following governing equations then derived in our previous study [22]

$$
\begin{align*}
    N_{\alpha\beta,\gamma}^{\gamma\nu} - \bar{I}u_\nu^{\bar{\alpha}} &= 0, \\
    M_{\alpha\beta,\gamma}^{\gamma\nu} + u_{\alpha\beta\gamma}N_{\alpha\beta} + P_3 + Iu_{\delta\gamma\alpha}u_\alpha - \bar{I}u_\gamma + f\bar{I}u_{\delta\gamma\alpha} &= 0.
\end{align*}
$$

where $I = \int_{-h/2}^{h/2} \rho \, dx_3$ and $J = \int_{-h/2}^{h/2} \rho \, x_3^2 \, dx_3$. The size-dependent membrane forces $N_{\alpha\beta}^*$ and bending moments $M_{ab}^*$ include contributions from surface stresses and are written as...
where \( l_1, l_2, l_3 \) and \( l_4 \) have dimensions of length and are defined as \( l_1 = 4(1 + \nu)(\mu_0 - \tau_0)/E \), \( l_2 = 2(1 + \nu)(\lambda_0 + \tau_0)/E \nu \), \( l_3 = 2(1 + \nu)(2\mu_0 - \tau_0)/E \), and \( l_4 = 2(1 + \nu)\tau_0/E \). Later, we will introduce \( l_5 = h\nu/6(1 - \nu) \). The other terms in equation (3) are the curvature \( u_{\alpha\gamma}/N_{\alpha\beta} \) and the classical, planar, and rotational inertia terms \( J\nu\), \( \bar{u}_n^0 \), and \( J\bar{u}_{n\alpha\beta} \), respectively.

If the surface effect is neglected (i.e., the elastic material constants at the surface \( \tau_0 \to 0, \lambda_0 \to 0 \), and \( \mu_0 \to 0 \)), equation (3) reduces to the expressions of membrane forces and bending moments in classical plate theory.

To simplify the problem, we ignore the small contributions from the higher-order planar inertia \( u_{\alpha\beta}^0 \) and rotational inertia \( J\bar{u}_{n\alpha\beta} \). The motion equations are then expressed as

\[
\begin{align*}
N^*_{\alpha\beta\gamma\delta} &= 0 \\
M^*_{\alpha\beta\gamma\delta} + u_{\alpha\beta\gamma\delta}N_{\alpha\beta} &= P_3 - if_3 = 0
\end{align*}
\]

in which

\[
\begin{align*}
N^*_{\alpha\beta\gamma\delta} &= 2\tau_0(\delta_{\alpha\beta} + u_{\alpha\beta}^0) + D_1 \left[ (1 - \nu) \left( 1 + \frac{h_i}{h} \right) e_{\alpha\beta}^0 + \nu \left( 1 + \frac{h_i}{h} \right) e_{\gamma\delta}^0 \right] \\
M^*_{\alpha\beta\gamma\delta} &= -D_2 \left[ (1 - \nu) \left( 1 + \frac{3h_i}{h} \right) u_{\alpha\beta\gamma\delta}^0 + \nu \left( 1 + \frac{3h_i}{h} \right) u_{\gamma\delta\alpha\beta}^0 \right]
\end{align*}
\]

where \( D_1 = Eh/(1 - \nu^2) \) and \( D_2 = Eh^3/12(1 - \nu^2) \). The following dimensionless governing equation are obtained from equation (3):

\[
\begin{align*}
m_1 \dddot{U} + m_2 \dddot{V} + m_3 (V \dddot{\eta} + \dddot{\xi} \dot{\eta}) + m_4 \dddot{W} \dddot{\xi} + m_5 \dddot{U} \dddot{\xi} = 0 \\
m_1 \dddot{V} + m_2 \dddot{U} + m_3 (U \dddot{\eta} + \dddot{\xi} \dot{\eta}) + m_4 \dddot{W} \dddot{\eta} + m_5 \dddot{U} \dddot{\eta} = 0 \\
-\frac{k_1^2}{12} (k_1 + k_2) (U \dddot{\xi} + 2V \dddot{\eta} + 2W \dddot{\xi}) + [(k_1 + k_2) \dddot{W} \dddot{\xi} + k_2 \dddot{W} \dddot{\eta}] (U \dddot{\xi} + \frac{1}{2} \dddot{W} \dddot{\eta}) \\
+ (k_1 \dddot{V} \dddot{\xi} + k_2 \dddot{V} \dddot{\eta}) (V \dddot{\eta} + \frac{1}{2} \dddot{W} \dddot{\eta}) + k_1 (U \dddot{\xi} + V \dddot{\xi} + \dddot{W} \dddot{\eta}) \\
+ 2k_1 [U \dddot{W} \dddot{\xi} + V \dddot{W} \dddot{\eta} + \dddot{W} \dddot{\xi} + \dddot{W} \dddot{\eta} + (U \dddot{\eta} + V \dddot{\xi}) \dddot{\xi}] - \frac{k_2^2}{12} \dddot{W} \\
- \frac{\ddot{U}^2}{\partial \xi^2} + P_3 = 0
\end{align*}
\]

where \( k_1 = (1 - \nu) (1 + l_1/h) \), \( k_2 = \nu (1 + l_2/h) \), \( m_1 = D_1(k_1 + k_2 + 2k_3) \), \( m_2 = D_1(k_1 + 2k_3) \), \( m_3 = D_2(k_1 + k_2 + 2k_3) \), \( m_4 = D_1(k_1 + 2k_3) \), \( m_5 = D_2(k_1 + 2k_3) \), \( \xi = x/a \), \( \eta = y/a \), \( \dddot{\xi} = c_p \dot{t} \sqrt{\delta/2a} \), \( \delta = h/a \), \( \dddot{U} = \dddot{u}_n^0/a \), \( \dddot{V} = \dddot{u}_n^0/a \), \( \dddot{W} = \dddot{u}_n^0/a \), and \( P_3 = P_3(1 - \nu^2)/E \). The bending-wave speed of plate \( c_p \) is expressed as \( c_p = E/\rho (1 - \nu^2) \). The dimensionless boundary conditions of the simplified support film are written as

\[
\begin{align*}
U &= \dddot{W} = \dddot{U} = \dddot{W} = 0 \quad at \quad \xi = 0, 1 \\
V &= \dddot{W} = \dddot{W} = 0 \quad at \quad \eta = 0, 1/a
\end{align*}
\]

The dimensionless governing equations (6)–(8) with the surface effect can be solved by a perturbation method. The dimensionless planar displacements \( U \) and \( \dddot{V} \) and the deflection \( \dddot{W} \) are expressed as a series of a small parameter \( \delta \) that depends on the scale of the film and is independent of the mode of vibration. The midplane displacements \( u_n^0 \) and \( u_\delta^0 \), which are ignored in classical linear plate theory, are functions of the higher-order even terms of \( \delta \) while the deflection \( \dddot{W} \) is a function of the higher-order odd terms of \( \delta \). The perturbation series of the dimensionless planar displacements \( U \) and \( \dddot{V} \) and the deflection \( \dddot{W} \) are thus written as

\[
\begin{align*}
U &= U_2(\xi, \eta, \dddot{\xi}) \delta^2 + U_4(\xi, \eta, \dddot{\xi}) \delta^4 + \cdots \\
V &= V_2(\xi, \eta, \dddot{\xi}) \delta^2 + V_4(\xi, \eta, \dddot{\xi}) \delta^4 + \cdots \\
\dddot{W} &= \dddot{W}_2(\xi, \eta, \dddot{\xi}) \delta + \dddot{W}_3(\xi, \eta, \dddot{\xi}) \delta^3 + \cdots
\end{align*}
\]

Substituting the perturbation series (10) into the dimensionless governing equations (6)–(8) and considering the lowest order term of \( \delta \), the governing equations become
\[ m_1 U_{2,\xi\xi} + m_2 U_{2,\eta\eta} + m_3 (V_{2,\xi\xi} + \bar{w}_{1,\xi\eta} \bar{w}_{1,\eta\xi}) + m_4 \bar{w}_{1,\xi \xi} \bar{w}_{1,\xi \xi} + m_5 \bar{w}_{1,\xi \eta} \bar{w}_{1,\eta \eta} = 0 \]  
\[ m_1 V_{2,\xi\xi} + m_2 V_{2,\eta\eta} + m_3 (U_{2,\xi\xi} + \bar{w}_{1,\xi\xi} \bar{w}_{1,\eta\eta}) + m_4 \bar{w}_{1,\xi\eta} \bar{w}_{1,\eta\xi} = 0 \] 
\[ \frac{1}{12} (k_1 + k_2) (\bar{w}_{1,\xi\xi\xi} + 2 \bar{w}_{1,\xi\eta\eta} + \bar{w}_{1,\eta\eta\eta}) + [(k_1 + k_2) \bar{w}_{1,\xi\xi} + k_2 \bar{w}_{1,\xi\eta}] \left( U_{2,\xi\xi} + \frac{1}{2} \bar{w}_{1,\xi\xi} \right) \] 
\[ + (k_1 \bar{w}_{1,\xi\xi} + (k_1 + k_2) \bar{w}_{1,\xi\eta})(V_{2,\xi\xi} + \frac{1}{2} \bar{w}_{1,\xi\xi}) + k_4 (U_{2,\eta\eta} + V_{2,\eta\eta} + \bar{w}_{1,\eta\eta}) \] 
\[ + 2k_5 [U_{2,\xi\xi} \bar{w}_{1,\xi\xi} + V_{2,\eta\eta} \bar{w}_{1,\eta\eta} + \bar{w}_{1,\xi\xi} \bar{w}_{1,\eta\eta} + (U_{2,\xi\xi} + V_{2,\xi\xi}) \bar{w}_{1,\xi\xi}] - \frac{1}{12} \partial^2 \bar{w}_{1,\xi\xi} + P_s^0 = 0 \] 

where \( P_s^0 = P_s^0/\delta^3 \). Similarly, the boundary conditions with a simple support are written as 
\[ U_2 = \bar{w}_1 = \bar{w}_{1,\xi\xi} = 0 \quad \text{at} \quad \xi = 0, 1 \] 
\[ V_2 = \bar{w}_1 = \bar{w}_{1,\eta\eta} = 0 \quad \text{at} \quad \eta = 0, \frac{1}{\lambda} \] 

Solving the governing equations (11)–(13) under the boundary conditions (14), we obtain the following separate forms:
\[ U_2 = \frac{\pi W_m^* q_1}{16} \sin 2\pi \eta \cos 2\pi \xi + q_2 \sin 2\pi \xi \] 
\[ V_2 = \frac{\pi W_m^* q_3}{16} \sin 2\pi \eta \cos 2\pi \xi + q_4 \sin 2\pi \xi \] 
\[ w_1 = W_m T(t) \sin \pi \xi \sin \pi \eta \] 

where \( W_m = w_m/h \) is the dimensionless maximum deflection and 
\[ q_1 = -m_4 (m_1 + m_2 - m_3) + (m_2 - m_3 + m_1 \lambda^2) (m_3 + m_5) \lambda^2, \] 
\[ q_2 = m_3 \lambda^2 - (m_2 + m_5) \lambda^2 (m_2 \lambda^2 + m_1) \] 
\[ q_3 = \frac{m_3}{m_5} \lambda^2 - (m_2 + m_5) \lambda^2 (m_2 \lambda^2 + m_1) \] 
\[ q_4 = \frac{m_3}{m_5} \lambda^2 - (m_2 + m_5) \lambda^2 (m_2 \lambda^2 + m_1) \] 

There is a coupling term \( \cdot \) between the basic-order and higher-order modes. Substituting (15) into (13) and ignoring the effect of higher-order modes, the nonlinear second-order nonsingular ordinary differential equation is approximated as 
\[ T_{,\eta\eta} + \omega_0^2 T + \alpha^2 T^2 = Qe^{i\xi} \] 

where \( \omega_0^2 = \pi^2 [24l_2 (1 + \lambda^2) + (k_1 + k_2) \pi^2 (2 + \lambda^2)] \), \( \alpha^2 = \frac{3}{2} (k_1 + k_2) \pi^2 W_m^* (1 + \lambda^2), \) and \( Q = 12P_s^0 \).

The function \( T(t) \) is solved as 
\[ T(t) = e^{i\xi} + \frac{\alpha^2 W_m^2}{32 (\omega_0^2 + \frac{3}{4} \alpha^2 W_m^2 - Q W_m^* \omega_0^2)} e^{i\xi} \] 

The dimensionless frequency is related to the maximum deflection as follows: 
\[ \frac{\omega}{\omega_0} = \left[ 1 + \frac{3}{4} \left( \frac{\alpha}{\omega_0} W_m \right)^2 - \frac{Q W_m^* \omega_0^2}{W_m^* \omega_0^2} \right]^{\frac{1}{2}} \] 

If the surface effect of the film is ignored, then \( \tau_0 \to 0, \lambda_0 \to 0 \) and \( \mu_0 \to 0 \). The relationship (18) between the dimensionless frequency \( \omega/\omega_0 \) and maximal deflection \( W_m \) is then independent of the absolute size of the film.

3. Case study

To analyze the relation between the frequency and surface effect, we demonstrate the solution (17) in numerical illustrations based on the calculated results of two sets of material parameters given by Gurtin and Murdoch [38].

Set I includes the parameters of a glass substrate with a 100-nm-thick iron coating deposited on its surface (Material 1):


\[ E = 5.625 \times 10^{10} \text{ N m}^{-2}, \quad \nu = 0.25, \quad \rho = 3 \times 10^3 \text{ kg m}^{-3}, \quad \lambda_0 = 7 \times 10^3 \text{ N m}^{-1}, \]
\[ \mu_0 = 8 \times 10^3 \text{ N m}^{-1}, \quad \tau_0 = 110 \text{ N m}^{-1}, \quad \rho_0 = 7 \times 10^{-4} \text{ kg m}^{-2}, \]  

(19)

and

Set II includes the parameters of a freshly cleaved surface (Material 2):
\[ E = 17.73 \times 10^{10} \text{ N m}^{-2}, \quad \nu = 0.27, \quad \rho = 7 \times 10^4 \text{ kg m}^{-3}, \]
\[ \lambda_0 = -8 \text{ N m}^{-1}, \quad \mu_0 = 2.5 \text{ N m}^{-1}, \quad \tau_0 = 1.7 \text{ N m}^{-1}, \quad \rho_0 = 7 \times 10^{-6} \text{ kg m}^{-2}, \]  

(20)

Note that coating may produce residual stress in ultra-thin film, while no residual stress is considered in Material I in this study as we focus on the surface effect due to different geometry features such as film thickness and other inherent properties such as film density and elastic modulus; thus, the size-dependent frequency of ultra-thin film is analyzed considering surface effect under forced vibration.

4. Results and discussion

Figures 2 and 3 plot the dimensionless frequency versus thickness curves of Material I with the same length–width ratio (\( \lambda = 3 \)) at maximal deflections of \( W_m = 1 \) and \( W_m = 2 \), respectively.

When the surface effect was neglected, the dimensionless frequency \( \omega/\omega_0 \) was independent of the film thickness. After incorporating the surface effect (i.e., after considering the contributions of the surface material constants \( \tau_0 \) and \( \lambda_0 \) in the constants \( \omega_0 \) and \( \alpha \) in equation (17)), the dimensionless frequency increases with the
film thickness exponentially and gradually stabilizes when the film thickness approaches up to $10^{-5}$. The results also show that the dimensionless frequency predicted by the model is constant and independent to thin film thickness without considering the surface effect. When the film was less than approximately $10^{-6}$ m thick, its frequency deviated significantly from that calculated in this paper with considering the surface effect. In addition, the dimensionless frequency increases significantly when the maximal deflection increases from 1 to 2, as shown in figures 2 and 3.

Figures 4 and 5 plot the dimensionless frequency versus thickness curves of Material II with the same length–width ratio ($\lambda = 3$) at maximal deflections of $W_m = 1$ and $W_m = 2$, respectively. Similar to Material I, the dimensionless frequency increases exponentially with increasing thickness of the film, and it gradually stabilizes when the film thickness approaches up to $10^{-5}$. When the surface effect was ignored (i.e., the surface material constants involved in $\omega_0$ and $\alpha$ were set to zero), the frequency was again independent of the film thickness. When the film thickness reduced to approximately $10^{-7}$ m, the forced vibrational frequency deviated significantly from that predicted by the model without the surface effect.

Also, the comparison between figure 2 and figure 4 shows that, at the same size-dependent vibrational frequency, Material II needs less film thickness than material I. This is because surface-to-bulk constants ratio of material II is less than Material I. This observation indicates that the surface effect is more significant when the thin film has a larger thickness and larger surface-to-bulk constants ratio.

For both Material I and Material II, the dimensionless frequency exhibits the same trends and is extremely sensitive to film thickness regardless of maximal deflections. Consistently, the frequency depends not only on the film thickness but also on the maximal deflection caused by the harmonic force. To summarize, the different
surface-to-bulk constants ratios between Material I and Material II cause different size-dependent nonlinear properties of ultra-thin film.

5. Conclusion

We propose a large-deflection model for studying the nonlinear responses of ultra-thin (nano-scale thickness) elastic films. The modified model incorporates the surface stresses derived from surface elastic theory. The dimensionless frequency due to the surface effect was derived using the perturbation method. The exact frequency of a simply supported thin film includes additional terms associated with the surface stresses, which are omitted in the conventional solutions without surface stresses. The numerical results clearly showed the thickness dependence of the forced vibrations of nano-scale elements. This size-dependent response is more obvious when the thin film has a larger surface-to-bulk constants ratio.

The analytical solutions of the present model can guide the analysis and design of ultra-thin film MEMS and NEMS structures with linear constitutive relations on surfaces incorporating nonlinear strain terms and the surface stress effect. To more precisely predict the responses of ultra-thin films, the nonlinear surface relations should be reliably determined.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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