The effect of many sources on the genuine multiparticle correlations

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Abstract

We report on a study aimed to explore the dependence of the genuine multiparticle correlations on the number of sources when the influence of other possible factors during multihadron production is avoided. The analysis utilised the normalised cumulants calculated in three-dimensional phase space of the reaction $e^+e^- \rightarrow Z^0 \rightarrow \text{hadrons}$ using a large Monte Carlo sample. The multi-sources reactions were simulated by overlaying a few independent single $e^+e^-$ annihilation events. It was found that as the number of sources $S$ increases, the cumulants do not change significantly their structure, but those of an order $q > 2$ decrease fast in their magnitude. This reduction in the one-dimensional rapidity cumulants can be understood in terms of combinatorial considerations of source mixing which dilutes the correlations by a factor of about $1/S^{q-1}$. The diminishing of the genuine correlations is consistent with recent cumulant measurements in hadron and nucleus induced reactions and should also be relevant to other dynamical correlations like the Bose-Einstein one, in $e^+e^- \rightarrow W^+W^- \rightarrow \text{hadrons}$ and in nucleus-nucleus reactions.

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1 Introduction

During the last decade an increase interest has been shown for the genuine multiparticle correlations in multihadron final states of hadronic, $e^+e^−$ and other reactions [1]. Recently OPAL, in its study of $e^+e^−$ annihilations on the $Z^0$ mass, has established the existence of strong genuine multihadron correlations up to the fifth order [2]. In hadron-hadron, like proton-proton, collisions the correlations of more than three particles have also been observed [3, 4]. In contrast to this situation, in heavy ion collisions, at low energies and/or in reactions of light nuclei, genuine correlations are found to have non-zero values only up to the third order [5]. Furthermore it has been found out that in general these correlations become weaker as the reaction average multiplicity increases. In nucleus-nucleus collisions at high energies, of tens and hundreds GeV per nucleon, the two-particle correlations are the only one that survive [3, 7–11].

This correlation dependence on the average multiplicity is very similar to the one observed in the investigations of multiparticle dynamical fluctuations, i.e. variation of many particle bunches in restricted phase space regions [1]. In these studies, known as intermittency analyses, the average multiplicity dependence has been proposed to be the consequence of a mixing of several independent emission sources [8, 10, 11]. As a result, the dynamical fluctuations in nucleus-nucleus collisions are already well accounted for by two-particle correlations [7, 12], whereas in hadron-hadron interactions [4, 13] and in $e^+e^−$ annihilations [2] higher order genuine correlations do exist.

An analogous situation is also observed in Bose-Einstein correlations (BEC) where identical bosons are correlated when they emerge from the interaction in nearby phase space. A genuine three-pion BEC has been detected in hadron-hadron reactions [14] and found to be even more pronounced in $e^+e^−$ annihilations [15]. In contrast to these, no genuine three-pion BEC were found in nucleus-nucleus collisions where the three-body correlations were well reproduced in terms of two-particle BEC [16]. Since the intermittency phenomenon and BEC seem to be closely related [1], the dependence of many sources on the strength of the BEC cannot be excluded. The superposition of emitters may also be a reason for the suppression of BEC of hadrons produced from W-boson pairs in $e^+e^−$ annihilations at LEP2 energies where the overlapping of hadrons affect the accuracy of the W mass measurements [17].

All this, as well as the obvious intrinsic interest in the genuine correlations which carry most of the dynamics of the hadron production process, points to the need of dedicated studies aimed to investigate the correlation dependence on the number of emission sources. Here we propose to study this dependence by grouping several $e^+e^− \rightarrow hadrons$ events to represent a multi-emission sources of particles. To obtain significant results, even when only few sources are considered, one needs a very high statistics, like that which can be supplied by a Monte Carlo (MC) generated sample, to minimise the calculation error and thus be sensitive to the correlations dependence on the number of sources. Another advantage of using MC generated events is in its possibility to generate a multihadron sample free from contamination of other processes like, for example, $e^+e^− \rightarrow \tau^+\tau^− \rightarrow hadrons$.

In this Letter we present the results of a MC study on the effect of several emission sources on the genuine higher order multiparticle correlations. The study was based on a generated sample of about $5 \times 10^6$ events of the reaction $e^+e^− \rightarrow Z^0 \rightarrow hadrons$ which passed a full simulation of the OPAL detector at LEP and did reproduce rather well the measured genuine
high order correlations present in the OPAL hadronic $Z^0$ decay data \[2\]. Moreover, the $e^+e^- \rightarrow Z^0 \rightarrow \text{hadrons}$ annihilations should represent well the one emission source situation in contrast to events produced in hadron-hadron interactions. Our analysis on the dependence of many sources on the genuine correlations was thus carried out in a way that avoided effects of other reaction features, like the multiplicity which was discussed recently in connection with two-particle BEC analyses in \[18\] and \[19\]. In as much that final state interactions between hadrons coming from different sources can be neglected, our correlation study based on $e^+e^- \rightarrow Z^0 \rightarrow \text{hadrons}$ annihilations may be extended to other types of reactions since the hadronisation process is believed to rest on a common basis \[20\].

2 The analysis method

The analysis is based on a generated sample of hadronic $Z^0$ decays using the Jetset 7.4 MC program \[21\] including a full simulation of the OPAL detector at LEP \[22\]. The MC sample also included initial-state radiation and effects of finite lifetimes. The parameters of the program were tuned to yield a good description of the measured event shape and single particle distributions \[23\].

The selection criteria for multihadron events used here are identical to the ones previously utilised by OPAL in their recent data analysis of multiparticle correlations \[4\]. In particular, selected events were required to have at least five charged tracks each having at least 20 measured points in the jet chamber where the first point had to be closer than 40 cm from the beam axis. The cosine of the polar angle of the event sphericity axis with respect to the beam direction was required to be less than 0.7 to ensure the event to be within the volume of the detector. The sphericity axis was calculated by using all accepted tracks and electromagnetic and hadronic calorimeter clusters.

To simulate several emission sources we did overlay several $e^+e^- \rightarrow Z^0 \rightarrow \text{hadrons}$ generated events and analysed the correlations between pions as if they were created in a single event. The kinematic variables are defined within each generated $e^+e^-$ event with respect to its own sphericity axis. For correlation analyses of variables like rapidity this procedure is equivalent to the one where the events are rotate to a common sphericity axis. This simulates multi-sources’ events with hadrons, here all taken to be pions, emerging from a common emitter. To note is that in this procedure the average event multiplicity is directly proportional to the number of sources. In our analysis each generated event was used only once, and hence required a very large MC event statistics.

To extract the genuine dynamical $q$-particle correlations, we used bin-averaged normalised factorial cumulant moments, or cumulants, first proposed in Ref. \[24\] as a tool for search for genuine multiparticle correlation, \[K_q = \frac{1}{M} \sum_{m=1}^{M} \left\{ \int_{\delta y} dy \left( \frac{C_q(y_1, \ldots, y_q)}{\int_{\delta y} dy \rho_1(y)} \right)^q \right\}, \]

where $C_q(y_1, \ldots, y_q)$ are the $q$-particle correlation functions given by the inclusive $q$-particle density distributions $\rho_q(y_1, \ldots, y_q)$ in terms of cluster expansion, e.g.,
\[C_3(y_1, y_2, y_3) = \rho_3(y_1, y_2, y_3) - \sum_{(3)} \rho_1(y_1)\rho_2(y_2, y_3) + 2 \rho_1(y_1)\rho_1(y_2)\rho_1(y_3). \] (2)

Here, \(M\) is the number of equal bins of a width \(\delta y\) into which the event phase-space is divided and the subscript \((3)\) denotes the number of permutations. For simplicity, we show all formulae in one-dimensional (e.g., rapidity) phase space.

The feature of the \(C_q\)-functions is that they vanish whenever there are no genuine correlations, i.e., the correlations are due to those existing in lower orders. The correlations extracted are of a dynamical nature since the cumulants share with normalised factorial moments (the intermittency analysis tool) the property of statistical noise suppression.

In this paper, we computed the cumulants as they were used in experimental studies, in particular, we used the form applied in Ref. [2], namely,

\[K_q = \frac{N_q \cdot \sum_{m=1}^{M} k_q^{(m)}}{\sum_{m=1}^{M} N_m (N_m - 1) \cdots (N_m - q + 1)}. \] (3)

Here, the factors \(k_q^{(m)}\) are the unnormalised factorial cumulant moments, or the Mueller moments [25], calculated for the \(m\)th bin. These factors represent the correlation functions \(C_q\) integrated over the bin and \(N_m\) is the number of particles in the \(m\)th bin summed over all the \(N\) events. The definition (3) takes into account the non-uniform shape of the single-particle distribution and the bias when the cumulants are computed at small bins.

The cumulant calculations were performed in the three-dimensional phase space of the kinematic variables commonly utilised in this kind of studies [1], namely:

- The rapidity, \(y = \ln \left(\sqrt{E + p_\parallel}/\sqrt{E - p_\parallel}\right)\), with \(E\) and \(p_\parallel\) being the energy and longitudinal momentum of the hadron in the interval \(-2.0 \leq y \leq 2.0\);
- The transverse momentum in the interval \(0.09 \leq p_T \leq 2.0\) GeV/c;
- The azimuthal angle, \(0 \leq \phi < 2\pi\), calculated with respect to the eigenvector of the momentum tensor having the smallest eigenvalue, in the plane perpendicular to the sphericity axis.

These variables are defined with respect to the sphericity axis, in a way and within the intervals similar to those used in a recent OPAL analysis [3] and in other cumulant studies [4].

3 Genuine correlations and the number of sources

3.1 Monte Carlo studies

In Fig. 1, we compare the MC based cumulants of orders \(q = 2, 3\) and 4 calculated from a single \(e^+e^- \to Z^0 \to \text{hadrons}\) source (solid symbols) with those obtained by overlaying seven such events representing seven hadronic sources (open symbols). The calculations were performed in one-dimensional rapidity, in two-dimensional rapidity vs. azimuthal angle subspaces and in...
three-dimensional phase space of rapidity, azimuthal angle and transverse momentum.

The following observations can clearly be made from Fig. 1.

- The existence of a dynamical component, i.e. rise of the cumulants with increasing number of bins $M$, is seen to be present both in the single source as well as in the case of many sources. Although the slopes of this scaling behaviour are smaller for several sources than for a single source, they are still strongly present. It is also evident that the character of the scaling is kept as the number of sources increases e.g., for one source as well as for seven sources the one-dimensional (rapidity) cumulants level off at the same $M$ value. No such saturation exists for the one and seven sources cumulants in the two and three dimensions.

- The genuine dynamical correlations, measured by the cumulants, significantly decrease with the increase of the number of sources. This decrease is stronger for higher order correlations namely, whereas the two-particle cumulants suffer a reduction of an order of magnitude as the number of sources increases from one to seven, the four-particle cumulants diminish by three or four orders of magnitude.

- The hierarchy of the $K_q$ cumulants is reversed as the number of sources increases. The cumulants derived from the single-source events increase with increasing $q$-order so that $K_2^{(1)} < K_3^{(1)} < K_4^{(1)}$, whereas the hierarchy in the cumulants calculated for seven sources is reversed namely, $K_2^{(7)} > K_3^{(7)} > K_4^{(7)}$. In addition, the multi-sources cumulants of order $q > 2$ have almost the same reduced value namely, $K_3^{(7)} \approx K_4^{(7)} \lesssim O(0.1)$. This last feature does not change as the dimension increases.

- The overall dominant feature of the analysis results is the diminishing value of the higher order cumulants as the sources number increases leaving the $K_2$ to be the dominant genuine multiparticle correlations.

The observed diminished correlations is further illustrated in Fig. 2 where the $M$-averaged one dimensional (rapidity) $K_q^{\text{av}}$ cumulants, are plotted with their errors against the number of sources. The cumulants were averaged over the $M \geq 10$ region where they are seen in Fig. 1 to approach an almost constant value. The values of these $K_q^{\text{av}}$ are also listed in Table 1 together with the OPAL measured data cumulants [2] of single $e^+e^- \rightarrow Z^0 \rightarrow \text{hadrons}$ events. These data cumulants for $q \leq 3$ are seen to agree with those derived from the MC sample. The one source MC based $q = 4$ cumulant lies lower than the measured data value but is still consistent within errors.

It is obvious from Fig. 2 that the $M$-averaged rapidity cumulants of order $q > 2$ decrease fast with the increase of the number of sources. Already for two sources the hierarchy changes and the two-particle correlations visibly dominate over the higher order ones. At higher number of sources the dominant role of the two-particle correlations is even more pronounced.

### 3.2 Correlation dilution due to source mixing

Within the procedure adopted here for the simulation of multi-source events, it is clear that if a genuine correlation exists it can only be detected in groups of $q$ pions emerging from the very
same source. In those \( q \)-group combinations which emerge from at least two sources, genuine correlations should not be present. This means that for \( K_q \) cumulants that are calculated over all possible \( q \)-pion groups, the higher the number of sources the more diluted will be the signal for genuine correlations.

For a given \( q \)-order the genuine correlation dilution factor is thus:

\[
R_q = \frac{P^G}{P^G + P^{NG}},
\]

where \( P^G \) denotes the number of \( q \)-particle groups, e.g., pairs or triplets of pions, which emerge from the same source. The term \( P^{NG} \) stands for the number of all possible combinations of \( q \)-particle groups which emerge from at least two sources.

Since all sources are produced in the same reaction and at the same energy, they do have an identical average charged multiplicity. For the estimation of \( R_q \) we assume that all the \( S \) sources have the same fixed charged multiplicity \( n \). In this case one has \( P^G = S \binom{n}{q} \), and the dilution factors at \( q = 2, 3 \) and 4 are given by

\[
\begin{align*}
R_2 &= \frac{\binom{n}{2}}{\binom{n}{2} + n^2 \binom{s}{2}} \xrightarrow{n \gg q} \frac{1}{S}, \\
R_3 &= \frac{\binom{n}{3}}{\binom{n}{3} + n^3 \binom{s}{3} + 2n \binom{n}{2} \binom{s}{2}} \xrightarrow{n \gg 2} \frac{1}{S^2}, \\
R_4 &= \frac{\binom{n}{4}}{\binom{n}{4} + n^4 \binom{s}{4} + 3n^2 \binom{n}{2} \binom{s}{3} + 2n \binom{n}{2} \binom{s}{2} + 2n \binom{n}{3} \binom{s}{2}} \xrightarrow{n \gg 3} \frac{1}{S^3},
\end{align*}
\]

where the denominators include the number of all possible \( q \)-particle combinations in \( S \) sources of charged multiplicity \( n \). This dilution factor dependence on the number of sources can also be derived in terms of cumulants \[10\].

From these \( R_q \) relations one can show that as long as \( n \gg q \) one obtains a general expression for the dilution factor,

\[
R_q \xrightarrow{n \gg q} \frac{1}{S^{q-1}}.
\]

To compare the dilution factors \( R_q \) with our correlation results shown in Fig. 2, they do have to be multiplied by \( K_q^{av(1)} \) which is a measure of the genuine \( q \)-order correlation present in a single sources. The solid lines shown in Fig. 2 thus represent the diluted cumulants \( K_q^{av} = K_q^{av(1)} \times R_q \). The striped areas in which the lines are embedded are the allowed regions when \( q \) is not neglected with respect to the multiplicity \( n \). The agreement between the cumulant calculations and the dilution factors predictions is really remarkable for \( q = 2 \) and 3 and certainly is still well within the rather large errors of the \( q = 4 \) cumulants.

For the order \( q = 2 \) one can relax the fixed charged multiplicity assumption and allow them to be different and still retain the \( R_q \approx 1/S^{q-1} \) relation as long as the multiplicity distribution
is of a Poisson nature. This however is not the case for orders higher than 2. Nevertheless for order \( q = 3 \) the relation \( R_3 = 1/S^2 \), derived from the fixed multiplicity assumption, is still valid as it describes well the \( K_{3}^{\text{av}} \) values up to at least thirteen sources (see Fig. 2). The large cumulants’ errors associated with the \( q = 4 \) order prohibits to judge how accurate is the \( R_4 = 1/S^3 \) relation.

An additional interesting and useful application of the relation \( R_q \approx 1/S^{q-1} \) is that it offers a method to estimate the average number of sources \( \langle S \rangle \) via the cumulant averaged values over the large \( M \) region of two sequential \( q \)-orders through the ratio,

\[
\langle S \rangle \approx \frac{K_{q+1}^{\text{av}(1)}}{K_q^{\text{av}(1)}} \times \frac{K_q^{\text{av}}}{K_{q+1}^{\text{av}}}. \tag{7}
\]

### 3.3 Comparison with hadron and nucleus induced reactions

As is already mentioned in the introduction, the genuine correlations measured in \( e^+e^- \) annihilations \cite{2} are found to be weaker in hadronic interactions \cite{3,4,5,13} and even more so in nuclear collisions \cite{3,6-9}. In nucleus-nucleus collisions at ultra-relativistic energies only the second-order correlations were so far detected \cite{3,6-9}.

In Table 2 we list the results obtained by several experiments \cite{3,6,8,9} on the \( M \)-averaged rapidity cumulant values for \( q = 2 \) and 3. These average values were taken over the \( M \)-region where the cumulants are seen in the published figures to reach a constant level. The hadronic reactions and their cumulants values are ordered according to the reported mean charged multiplicity, from the lowest value to the highest one.

Table 2 shows that in hadron including nucleus induced reactions the two-particle correlations decrease rather fast as the mean multiplicity increases. However the three-particle correlations are found to be essentially non-existing even at moderately small mean multiplicity. Notwithstanding the possibility that production of hadrons in \( e^+e^- \) annihilation may well be simpler than in hadron induced reactions, it may nevertheless be instructive to relate our findings to the measured correlation data listed in Table 2. In as much that the mean multiplicity increases with the number of sources, the decrease in the two-particle correlations and the absence of three-pion correlations in nucleus induced reactions is consistent with our findings which demonstrated the dilution of the correlations with increased number of sources. A quantitative comparison between our findings and the correlations in nucleus-nucleus and hadron induced reactions is hard mainly because of the large errors associated with the average cumulant values. In particular the application of relation (7) is prohibited because most of the \( K_3^{\text{av}} \) are consistent within errors with zero.

Recently the two-pion BEC have been studied \cite{19} in \( pp \) reaction at centre of mass energy of 630 GeV as a function of multiplicity by using the normalised cumulants method similar to the one used here. In that analysis it has been found that the correlations of the cumulants of the like-sign pions as well as the opposite-sign pions decrease with the multiplicity \( n_{ch} \). From our analysis we expect the pair correlation to decrease as \( 1/S \), where \( S \) is the number of sources. This indicates that indeed the multiplicity is at least partially proportional to \( S \). The BEC dependence has also been investigated in the framework of the totally coherent emission picture \cite{20} and in the quantum optical approach \cite{18} where the conclusions were that these
correlations are weaker as the multiplicity increases.

4 Summary and conclusions

To investigate the effect of many emission sources on the genuine correlations in multihadron final state we adopted a procedure which should minimise the confusion introduced by other variables like charged multiplicity. For the genuine correlations measurement we utilised the normalised cumulant method. To simulate the situation of many sources event we did overlay Monte Carlo generated hadronic $Z^0$ events treating them as one event. This Jetset7.4 MC sample of some five million events, tuned to the OPAL data taken at LEP1 on the $Z^0$ mass, has previously described rather well the measured correlations in the $e^+e^- \rightarrow Z^0 \rightarrow hadrons$ data.

The results obtained here show that the cumulants, obtained from a single-source events and from events of many sources, almost do not change their structure with the decrease of the width of phase-space bins. This means that the scaling is preserved although larger slopes are seen to be in the case of one source compared with those for several sources. However, when the number of sources increases, the cumulants of order higher than two are suppressed and diminish to zero due to source mixing. The two-particle cumulants are also somewhat reduced in their value but they are way above the higher order ones and they are seen to completely dominate when the source number $S$ exceeds the value ten.

The one-dimensional (rapidity) correlations are very well reproduced by assuming that genuine correlation of the order $q$ can only be present when all the $q$ hadrons are emerging from the same source. Therefore the dilution of the genuine correlation signal is proportional to the ratio of the probability that the $q$ hadrons will come from the same source. From simple combinatorial considerations this probability is approximately equal to $1/S^{q-1}$. Thus a measurement of the one-dimensional correlation for two sequential orders renders the number of sources.

The genuine correlations measured in hadron and nucleus induced reactions do follow qualitatively the findings of our work. In particular in nucleus-nucleus reactions, where many sources are expected to contribute to the final hadronic state, the $q > 2$ orders are very small and indeed consistent with zero. The $q = 2$ order is still present but it is also getting smaller as the atomic number of the nuclei increases. The general decrease of the second order cumulants with the increase of multiplicity re-confirms the belief that the higher the multiplicity the larger the number of sources. Our results may also be useful for the understanding of other types of measured correlations like the Bose-Einstein interference of two and more identical bosons. It has been previously pointed out [27] that in the absence of final state interactions the BEC of the $e^+e^- \rightarrow W^+W^- \rightarrow hadrons$ will be half of that of the $Z^0$ decay to hadrons. From our study it follows that the two-particle BEC, or any other correlations, in the two-source reaction $e^+e^- \rightarrow W^+W^- \rightarrow hadrons$ should be reduced by a factor two as compared to that of the hadrons emerging from one W-boson.

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References

[1] E.A. De Wolf, I.M. Dremin and W. Kittel, Phys. Reports 270 (1996) 1.
[2] OPAL Collab., G. Abbiendi et al., Eur. Phys. J. C 11 (1999) 239.
[3] P. Carruthers, H. Eggers and I. Sarcevic, Phys. Rev. C 44 (1991) 1629.
[4] EHS/NA22 Collab., N. Agababyan et al., Z. Phys. C 59 (1993) 405; M. Charlet, Ph.D. Thesis, Multiparticle Correlations in $\pi^+ p$ and $K^+ p$ interactions at 250 GeV/c, (Nijmegen Univ., 1994), unpublished.
[5] EHS/NA22 Collab., N.M. Agababyan et al., Phys. Lett. B 332 (1994) 458; E665 Collab., M.R. Adams et al., Phys. Lett. B 335 (1994) 535.
[6] P.L. Jain and G. Singh, Nucl. Phys. A 596 (1996) 700.
[7] EMU01 Collab., M.I. Adamovich et al., Nucl. Phys. B 388 (1992) 3.
[8] EMU01 Collab., M.I. Adamovich et al., Phys. Rev. D 47 (1993) 3726.
[9] P.L. Jain, A. Mukhopadhyay, G. Singh, Z. Phys. C 58 (1993) 1.
[10] P. Lipa and B. Buschbeck, Phys. Lett. B 223 (1989) 465.
[11] S. Barshay, Z. Phys. C 47 (1990) 199; D. Seibert, Phys. Rev. D 41 (1990) 3381.
[12] EMU01 Collab., M.I. Adamovich et al., Phys. Lett. B 263 (1991) 539, Phys. Lett. B 407 (1997) 92; E802 Collab., T. Abbott et al., Phys. Rev. C 52 (1995) 2663.
[13] P. Carruthers, H. Eggers and I. Sarcevic, Phys. Lett. B 254 (1991) 258.
[14] EHS/NA22 Collab., N.M. Agababyan et al., Z. Phys. C 68 (1995) 229.
[15] DELPHI Collab., P. Abreu et al., Phys. Lett. B 355 (1995) 415; OPAL Collab., K. Ackerstaff et al., Eur. Phys. J. C 7 (1999) 379.
[16] NA44 Collab., H. Bøggild et al., Phys. Lett. B 455 (1999) 77.
[17] See e.g., W. Kittel, talk given at the QCD and Hadronic Interactions session of the 34 Rencontres de Moriond (Les Arcs, France, Mar. 1999), [hep-ph/9905394] and references therein.
[18] N. Suzuki and M. Biyajima, Phys. Rev. C 60 (1999) 034903.
[19] B. Buschbeck, H.C. Eggers and P. Lipa, Phys. Lett. B 481 (2000) 187.
[20] I.M. Dremin, Physics-Uspekhi 37 (1994) 715; V.A. Khoze and W. Ochs, Int. J. Mod. Phys. A 12 (1997) 2949.
[21] T. Sjöstrand, Comp. Phys. Comm. 82 (1994) 74.
[22] J. Allison et al., Nucl. Instr. Meth. A 317 (1992) 47.
[23] OPAL Collab., G. Alexander et al., Z. Phys. C 69 (1996) 543.
[24] P. Carruthers and I. Sarcevic, Phys. Rev. Lett. 63 (1989) 1562;  
E.A. De Wolf, Acta Phys. Pol. B 21 (1990) 611.

[25] M.G. Kendall and A. Stuart, The Advanced Theory of Statistics (C. Griffin & Co., London,  
1969), Vol. 1;  
A.H. Mueller, Phys. Rev. D 4 (1971) 150.

[26] M. Biyajima et al., Phys. Lett. B 386 (1996) 297.

[27] S.V. Chekanov, E.A. De Wolf, W. Kittel, Eur. Phys. J. C 6 (1999) 403.
Table 1: The Monte Carlo $M$ averaged rapidity $K_q^\text{av}$ cumulants obtained in the $e^+e^- \to Z^0 \to \text{hadrons}$ reaction compared with those obtained in a recent OPAL measurement of single data events. The averages were taken over the $M$ region where the cumulants approached a constant value (see Fig. 1).

| No. of sources | $K_q^\text{av} (M \geq 10)$ | Sample |
|----------------|-----------------------------|--------|
|                | $q = 2$                     | $q = 3$| $q = 4$ | |
|                | 0.45 ± 0.01                 | 0.67 ± 0.04 | 1.36 ± 0.21 | Data [2] |
| 1              | 0.486 ± 0.002               | 0.632 ± 0.017 | 0.950 ± 0.225 | MC |
| 2              | 0.260 ± 0.001               | 0.171 ± 0.006 | 0.147 ± 0.034 | ” |
| 4              | 0.124 ± 0.001               | 0.041 ± 0.003 | 0.016 ± 0.012 | ” |
| 7              | 0.072 ± 0.001               | 0.013 ± 0.003 | 0.003 ± 0.008 | ” |
| 13             | 0.037 ± 0.001               | 0.004 ± 0.003 | 0.001 ± 0.009 | ” |

Table 2: The $M$ averaged rapidity cumulants $K_q^\text{av}$, of orders $q = 2$ and 3 measured in several hadronic reactions. The cumulant values were averaged over the $M$ regions where they were seen to approach a constant value. These quoted values were estimated from the relevant published figures given in the references listed in the table.

| $\langle n_{\text{ch}} \rangle$ | $K_q^\text{av}$ | Reaction | Beam energy (GeV) | Ref. |
|----------------|----------------|-----------|-------------------|------|
| $\sim 8$      | 0.32 ± 0.02   | 0.26 ± 0.12 | $\pi p$           | 250  | [3] * |
| 21.1           | 0.21 ± 0.04   | 0.14 ± 0.18 | $pEm$             | 200  | [3] * |
| $> 50$         | 0.34 ± 0.02   | 0.12 ± 0.03 | $AuEm$            | 10.6A| [3] |
| 73.3           | 0.21 ± 0.03   | 0.05 ± 0.07 | $SiEm$            | 14.5A| [3, 4] |
| 81.1           | 0.20 ± 0.05   | 0.00 ± 0.12 | $OEm$             | 60A  | [4] |
| 154.9          | 0.11 ± 0.05   | 0.01 ± 0.18 | $OEm$             | 200A | [3] * |
| 216.1          | 0.25 ± 0.05   | 0.08 ± 0.10 | $SEm$             | 200A | [3] |
| 272.6          | 0.09 ± 0.05   | 0.00 ± 0.18 | $SEm$             | 200A | [3] * |
| 289.8          | 0.11 ± 0.08   | 0.02 ± 0.10 | $SAu$             | 200A | [3] ** |
| 355.0          | 0.08 ± 0.05   | 0.00 ± 0.08 | $SAu$             | 200A | [3] * |
| 383.9          | 0.07 ± 0.04   | 0.00 ± 0.05 | $SAu$             | 200A | [3] *** |

* Calculations are based on the measured factorial moments.
** Semicentral collisions.
*** Central collisions.
Figure 1: The MC predicted cumulants of order $q = 2$, 3 and 4 as a function of $M^{1/D}$, where $M$ is the number of bins of the $D$-dimensional subspaces of the phase space of rapidity ($y$), azimuthal angle ($\Phi$), and transverse momentum ($p_T$). The solid symbols represent the cumulants for a single source, while the open symbols are the cumulants values of seven sources.
Figure 2: The dependence of the averaged rapidity cumulants $K_{q}^{av}$ of order $q = 2, 3$ and $4$ on the number of $e^+e^-$ sources. The cumulants were averaged over the $M$-range where they approached a constant value (see Fig. 1). The lines represent the expected dilution according to Eq. (6) where $q$ is neglected in comparison to the multiplicity $n$. The striped areas are the allowed regions when $q$ is not neglected with respect to $n$ (see text).