Impact of axial loads on the sound directivity of angular contact ceramic ball bearings

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Abstract. This paper mainly focuses on the sound directivity of the angular contact ceramic ball bearing applied to the ceramic motorized spindle. The nonlinear dynamic model is proposed with consideration of axial load, nonlinear hydrodynamic forces and interaction between balls and cage. Then the sub-source decomposition method is introduced for the establishment of an acoustic radiation model. Sound pressure levels along the circumferential direction are used as evaluation indexes of sound directivity, and the sound directivity with axial loads at different rotation speed are analyzed. Results indicate that the sound directivity tends to be obvious with the increase of rotation speed, and the axial loads have significant influence on the deviation angles of directivity performance. This work provides insights into monitoring the acoustic performance of ceramic bearings, and can guide further research and development.

1. Introduction

As the key components of mechanical systems, bearings are required to have high precision and reliability [1]. Ceramic ball bearings take ceramic as the material of rolling elements and has been proved of higher turning accuracy and thermal stability over traditional steel ball bearings. Therefore, the ceramic ball bearings are widely applied in research fields such as aeronautics and navigation. However, with the increase of rotation speed, the excessive radiation noise affects the entire acoustic performance, and hinders the further improvement of working speed. The radiation noise results from the friction and impact between the components of the bearing, and has great impact on the surrounding acoustic environment [2]. Sound directivity is an important evaluation index of acoustic performance and varies greatly with the working parameters. Researches on the sound directivity can help get the accurate sound radiation performance and lead to the noise reduction.

Ceramic ball bearings applied on mechanical systems are mainly angular contact bearings, which work under axial and radial loads. Axial loads are widely used to eliminate deflections due to external loads and prevent skidding. However, excessive axial loads may lead to large deformations on the rolling elements and raceways, and bring about extra friction and noise [3]. The deformations on the steel raceways and ceramic balls are different because of different stiffness and rigidity. The influence of axial loads on the sound directivity is complex, hence the theoretical analysis of the impact of axial loads on the sound directivity of ceramic bearings is quite rare as far as the author’s knowledge [4-6].
Many researchers have conducted researches on the dynamic model of angular contact bearings. In 1995, Chapman J J proposed an alternative theoretical model of angular contact ball bearing, and validated the results by an experimental investigation [7]. As one of the following influential studies, Cao Y Z and Altintas Y presents a model of spindle bearing and machine tool system, and used Hertzian contact theory to calculate the contact force and displacement of bearing. The centrifugal force and gyroscopic moment were also taken into consideration to get the dynamic response of the bearing-spindle system [8]. It has been proved that the vibration of rolling elements and rings are the main sources of noise radiation at low rotation speed, and the noise radiated by the cage is relatively slight because of the flexibility of the material. Wang Y L focused on the ball skidding and proposed an analytical model to investigate the contact balls and raceways, cage and lubricant. It was pointed out that the internal load influenced the skidding behavior significantly and appropriate loads should be determined to reduce skidding [9]. Zhao C J and Yu X K analyzed the relationship between the gyroscopic torque and friction coefficient between rolling element and groove and built a quasi-statics model of axial load for calculation. The variation of the outer contact angle with axial force was obtained and the ratio of contact forces with the increase of axial forces was also studied to get the influence situation of axial load on the dynamic characteristics [10]. Kerst S proposed a semi-analytical dynamic model addressing the flexibility of the outer ring. The results showed that the flexibility of outer ring had a significant effect on the deformation of the raceway, thus affecting the dynamic characteristics. The model was proved to be of low computation cost and high accuracy [11]. Gunduz A made an investigation of the vibration response of angular contact ball bearings with respect to bearing preload, and proved that the axial preload affects the dynamic characteristics by changing the diagonal and off-diagonal elements of the stiffness matrix [12]. However, with the increase of rotation speed, the oil whirl tends to be obvious and the cage runs more irregularly. The cage has to be taken as a sub-source at high rotation speed accordingly. Deng S E conducted several studies on the impact of lubricant traction coefficient on cage’s dynamic characteristics in high-speed angular contact bearing and the stability of cage under various working conditions were assessed by the slip ratio of cage [13-15]. Cao H R, Niu L K et al also stressed in their review that the cage dynamic response was easily influenced with the change of working conditions and the contact between cage and other elements could vary greatly [16].

As are displayed above, the researchers have conducted detailed research on the nonlinear dynamic response of the elements in angular contact bearing, but refused to go a step further to study the acoustic characteristics. It has been figured out in previous work that the acoustic performance of the bearing can be obtained by the sub-source decomposition method and the acoustic model has been proved of consistency with the experiment result. As one of the key acoustic characteristics, the sound directivity shows the circumferential distribution of the radiation noise and is affected by the working parameters which may bring about nonlinear and unstable factors. The studies on the trends of directivity with the working conditions or structural parameters are quite rare to the authors’ knowledge. In this paper, the nonlinear dynamic model of ceramic bearing is proposed and the sub-source decomposition method is introduced for the calculation of radiation noise. The variation of sound directivity with axial loads and rotation speed are analysed and discussed afterwards.

2. Ceramic ball bearing model

In this paper, the outer ring is fixed to simulate the actual working condition and the inner ring rotates under stable speed. Assuming that the mass centers of the components coincide with the geometrical centers and the balls are of the same diameters. Here we neglect the impact of surface roughness of the raceways and the dynamic model of the ceramic ball bearing is shown in figure 1.

Multiple coordinate systems are applied in the analysis of ceramic ball bearing as shown in figure 1. Coordinate system \( O_0XYZ \) is fixed, and coordinate systems \( O_iX_iY_iZ_i \), \( O_cX_cY_cZ_c \) are used to describe the dynamic response of inner ring and cage, respectively. \( O_t \) and \( O_c \) show the center of curvatures of inner and outer raceway. \( Q_0 \) and \( Q_cj \) show the contact forces given by the inner ring and outer ring, and \( Q_{cj} \) shows the impact force between the \( j \)th ball and the cage. \( Q_{cij}, Q_{cyj} \) and \( Q_{sz} \) are
decomposition components along axes \( O_X, O_Y \) and \( O_Z \). \( \alpha_i \) and \( \alpha_{ij} \) show the contact angles between the \( j \)th ball and inner/outer raceway, respectively. \( F_{Ry} \) shows the frictional forces between the \( j \)th ball and inner ring and \( F_{Roj} \) shows that between the \( j \)th ball and outer ring. Subscripts \( \xi \) and \( \eta \) show the forces in plane \( YOZ \) and \( XOZ \), respectively. \( P_{Roj} \) and \( P_{Ryo} \) are frictional forces between the \( j \)th ball and cage. \( F_c \) is the axial load, and \( F_Z \) shows the hydrodynamic force acting on the cage, with \( F_{cz} \) and \( F_{cy} \) the decomposition components along axes \( O_Y \) and \( O_Z \), respectively. \( T_{ij} \) is the traction force contact surface between the \( j \)th ball and inner raceway. \( \phi_i \) and \( \phi_j \) are phase angles of the \( j \)th ball in coordinate systems \( O_iY_iZ_i \) and \( O_YiZ_i \). \( m_j \) is the mass of the \( j \)th ball. When the bearing is running, and axial load \( F_z \) is applied on the inner ring, the contact forces bring about deformations on the elements.

![Figure 1. Dynamic model of ceramic ball bearing](image)

Assuming that the deformations of the balls are negligible compared with the deformations of the rings owing to the great stiffness of the ceramic balls, and the contact forces can be expressed as:

\[
\begin{align*}
Q_{oj} &= K_{oj} \cdot \delta_{oj}^{1.5} \\
Q_{oy} &= K_{oy} \cdot \delta_{oy}^{1.5}
\end{align*}
\]

(1)

where \( K_{oj} \) and \( K_{oy} \) are the contact stiffness between the \( j \)th ball and the outer/inner raceway, respectively, and \( \delta_{oj} \) and \( \delta_{oy} \) show the deformation of the corresponding raceway. The inner ring is in contact with the balls and the dynamic differential equations can be described as:

\[
F_x = \sum_{j=1}^{N} \left( Q_y \sin \alpha_j - F_{Ryo} \cos \alpha_y \right) = m_i \ddot{x}_i
\]

(2)

\[
F_y + \sum_{j=1}^{N} \left[ \left( Q_y \cos \alpha_j + F_{Ryo} \sin \alpha_y \right) \cos \phi_j + \left( T_{ij} - F_{Ryo} \right) \sin \phi_j \right] = m_i \ddot{y}_i
\]

(3)

\[
F_z - \sum_{j=1}^{N} \left[ \left( Q_y \cos \alpha_j + F_{Ryo} \sin \alpha_y \right) \sin \phi_j - \left( T_{ij} - F_{Ryo} \right) \cos \phi_j \right] = m_i \ddot{z}_i
\]

(4)

\[
M_y = \sum_{j=1}^{N} \left[ \left( Q_y \sin \alpha_j - F_{Ryo} \cos \alpha_y \right) \sin \phi_j + \frac{D_y}{2} f_i T_{ij} \sin \alpha_j \cos \phi_j \right] = I_{y} \dot{\omega}_y - (I_{iz} - I_{ix}) \dot{\alpha}_z \alpha_z
\]

(5)

\[
M_z = \sum_{j=1}^{N} \left[ \left( Q_y \sin \alpha_j - F_{Ryo} \cos \alpha_y \right) \cos \phi_j - \frac{D_y}{2} f_i T_{ij} \sin \alpha_j \sin \phi_j \right] = I_{z} \dot{\omega}_z - (I_{ix} - I_{iy}) \dot{\alpha}_x \alpha_x
\]

(6)
where: $m_i$ shows the mass of inner ring, $\ddot{x}_i$, $\ddot{y}_i$, $\ddot{z}_i$ are the accelerations along axes $OX_i$, $OY_i$, and $OZ_i$, respectively. $F_{ij}$, $F_{zy}$, $M_z$, $M_c$ are external loads and $D_w$ is the ball diameter. $f_i$ is the inner ring raceway curvature radius coefficient. $I_{Ox}$, $I_{Oy}$, $I_{Oz}$ are the moments of inertia of inner ring, and $\omega_{Ox}$, $\omega_{Oy}$, $\omega_{Oz}$ are the angular velocities. $r_y$ is the rolling radius and can be expressed as:

$$r_y = 0.5d_m - 0.5D_w g_i \cos \alpha_y$$  \hspace{1cm} (7)

where: $d_m$ is the pitch diameter of the bearing, and $g_i$ is the inner ring raceway curvature radius coefficient.

The contact between balls and cage lead to impact and friction noise and the dynamic equations of the cage can be shown as:

$$\sum_{j=1}^{N} \left( P_{Ryj} + Q_{cyj} \right) = m_c \ddot{x}_c$$  \hspace{1cm} (8)

$$\sum_{j=1}^{N} \left[ F_{cy} - (Q_{cyj} - P_{Ryj} \cos \phi_j) \right] = m_c \ddot{y}_c$$  \hspace{1cm} (9)

$$\sum_{j=1}^{N} \left( Q_{cyj} + P_{Ryj} \sin \phi_j - F_{czj} \right) = m_c \ddot{z}_c$$  \hspace{1cm} (10)

$$\sum_{j=1}^{N} \left( P_{Ryj} \cos \phi_j - \sqrt{Q_{cyj}^2 + Q_{czj}^2} \frac{d_m}{2} \right) + M_{ex} = I_{ex} \ddot{\omega}_{ex} - (I_{cy} - I_{cz}) \omega_{cy} \omega_{cz}$$  \hspace{1cm} (11)

$$\sum_{j=1}^{N} \left( P_{Ryj} + Q_{cyj} \right) \frac{d_m}{2} \sin \phi_j = I_{cy} \ddot{\omega}_{cy} - (I_{cx} - I_{cz}) \omega_{cx} \omega_{cz}$$  \hspace{1cm} (12)

$$\sum_{j=1}^{N} \left( P_{Ryj} + Q_{cyj} \right) \frac{d_m}{2} \cos \phi_j = I_{cz} \ddot{\omega}_{cz} - (I_{cx} - I_{cy}) \omega_{cx} \omega_{cy}$$  \hspace{1cm} (13)

where $m_c$ is the mass of the cage, $\ddot{x}_c$, $\ddot{y}_c$, $\ddot{z}_c$ are the accelerations of the cage along axes $OX_c$, $OY_c$, and $OZ_c$, respectively. $M_{ex}$ is the external load, and $I_{cx}$, $I_{cy}$, $I_{cz}$ are moments of inertia of the cage. $\omega_{cx}$, $\omega_{cy}$, $\omega_{cz}$ are angular velocities of the cage, and $\dot{\omega}_{cx}$, $\dot{\omega}_{cy}$, $\dot{\omega}_{cz}$ are angular accelerations.

The balls are in contact with the cage, inner ring and outer ring, and are the main sources of the radiation noise. The dynamic differential equations of the $j$th ball can be expressed as:

$$F_{bx} + F_{Ryj} \cos \alpha_{oj} - F_{Ryz} \cos \alpha_{oj} + F_{Qyj} \sin \alpha_{oj} - Q_{Qyj} \sin \alpha_{oj} + Q_{cyj} + P_{Ryj} = m_b \ddot{x}_{bj}$$  \hspace{1cm} (14)

$$F_{by} + m_b g \cos \phi_j + Q_{czj} \cos \phi_j + Q_{cyj} \sin \phi_j + F_{Ryz} - F_{Ryj} \sin \phi_j = m_b \ddot{y}_{bj}$$  \hspace{1cm} (15)

$$F_{bz} + Q_{yj} \cos \alpha_{oj} - Q_{Qyj} \cos \alpha_{oj} + F_{Ryj} \sin \alpha_{oj} - F_{Ryzj} \sin \alpha_{oj} - m_b g \sin \phi_j - P_{Ryj} = m_b \ddot{z}_{bj}$$  \hspace{1cm} (16)

$$\left( T_{cijk} - T_{Ryij} + P_{Ryj} \right) \frac{D_w}{2} = I_{b} \omega_{byj} + J_s \dot{\omega}_{yj}$$  \hspace{1cm} (17)

$$\left( F_{Ryzj} + F_{Ryj} + P_{Ryj} \right) \frac{D_w}{2} = I_{b} \omega_{byj} + J_s \dot{\omega}_{yj} + I_b \omega_{byj} \dot{\theta}_{byj}$$  \hspace{1cm} (18)

$$P_{Ryj} \frac{D_w}{2} = I_b \omega_{byj} - I_s \omega_{yj} \dot{\theta}_{byj} + J_s \omega_{yj}$$  \hspace{1cm} (19)

where $m_b$ is the mass of the $j$th ball, $\ddot{x}_{byj}$, $\ddot{y}_{byj}$, $\ddot{z}_{byj}$ are the accelerations of the ball, and $F_{bx}$, $F_{by}$, $F_{bz}$ are the hydrodynamic forces acting on the ball in different directions. $\dot{\omega}_{yj}$, $\dot{\omega}_{yj}$, $\dot{\omega}_{zj}$ are angular
accelerations of $x_{bj}$, $y_{bj}$ and $z_{bj}$, respectively. $\omega_{xbj}$, $\omega_{ybj}$, $\omega_{zbj}$ are angular velocities of the $j$th ball in coordinate $\{O; X, Y, Z\}$, $\dot{\theta}_{bj}$ is the orbit speed of the $j$th ball in coordinate $\{O; X, Y, Z\}$.

3. Numerical simulations

Assuming that the bearing works with perfect lubrication and ignore the interference during rotation. The rotation speed is set from 15000r min$^{-1}$ to 30000r min$^{-1}$, and bearing works under stable rotation speed without fluctuation. The axial loads are set to vary from 1000N to 3000N. The cage is made of phenolic resin, and the balls are made of silicon nitride ceramics. The bearing has the same structural parameters with SCHAEFFLER HCB7009-C-T-P4S, and the parameters are shown in table 1.

### Table 1. Major parameters of the bearing.

| Item                          | Value |
|-------------------------------|-------|
| Bearing outside diameter(mm)  | 75    |
| Bearing width(mm)             | 16    |
| Initial contact angle(degree) | 15    |
| Cage outside diameter(mm)     | 65    |
| Pocket diameter(mm)           | 10    |
| Cage width(mm)                | 14    |
| Ball diameter(mm)             | 9.5   |
| Ball number                   | 15    |
| Inner ring bore diameter(mm)  | 45    |

The inner ring, the cage and the balls can be regarded as sub-sources of the radiation noise and the radiation noise can be obtained by a sub-source decomposition method derived from previous work in ref. [2]. The bearing runs with a horizontal axis, and the radial loads are ignored.

4. Results and discussion

4.1. Sound directivity performance with different axial loads

The field points are distributed in a circle on a plane 10mm from the $YOZ$ plane and the diameter of the circle is set to be 200mm. The calculation step is set as 6 degree and therefore there are 60 field points on the circumference. Here we take the 12 o’clock direction as 0 degree, and the other points arranged clockwise. The bearing is set to run at 30000 r min$^{-1}$ in clockwise direction, and the axial loads are 1000N, 1500N, 2000N, 2500N and 3000N. The sound pressure levels (SPL) at the field points with different axial loads are shown in figure 2.

![Figure 2. Sound pressure levels with different axial loads.](image)
Here we define $SPL_{\text{max}}$ and $SPL_{\text{min}}$ as the maximum and minimum sound pressure level along the circumference, and $\phi_m$ as the directivity angle where it arrives at $SPL_{\text{max}}$, then the variation trends of $SPL_{\text{max}}$ and $\phi_m$ are shown in figure 3.

![Figure 3](image_url)

Figure 3. Variation trends of $SPL_{\text{max}}$ and $\phi_m$ at $\omega=30000$ r min$^{-1}$.

In figure 2, the solid line, dashed line, dotted line, dot-dashed line and double dot-dashed line show the circumferential distributions of radiation noise with $F_a=1000N$, $1500N$, $2000N$, $2500N$ and $3000N$, respectively. In figure 3, the line with squares shows the variation trends of $SPL_{\text{max}}$, while the line with circles shows the variation trends of $\phi_m$. It can be seen that the radiation noise at a fixed axial load varies little from $300^\circ$ to $84^\circ$, but shows an obvious directivity in the lower semicircle at around $204^\circ$. The overall radiation noise shows an uptrend with the increase of axial load except a decline at $F_a=1500N$, which indicates that $F_a=1500N$ is more applicable to the rotation speed of $\omega=30000$ r min$^{-1}$.

When the axial load continues to increase, $SPL_{\text{max}}$ tends to increase and $\phi_m$ stays still which indicates that the friction and impact raise with the axial load, but the place where the largest deformation took place remains unchanged. Furthermore, the $SPL$ curve tends to be more smooth which indicates that the friction and impact at the directivity angle increase more obviously and thus show a clearer directivity.

### 4.2. Sound directivity performance at different rotation speed

The circumferential distributions vary with the axial load, and the effect of rotation speed also needs to be taken into consideration. $F_a=1500N$ has already been proved to be the optimized value at $\omega=30000$ r min$^{-1}$ in the former study, and therefore here we take $F_a=1500N$ as a fixed axial load and the rotation speed is set as a variable. The arrangement of field points is the same with that in section 4.1 and the sound pressure levels at $\omega=15000$ r min$^{-1}$, $20000$ r min$^{-1}$, $25000$ r min$^{-1}$, $30000$ r min$^{-1}$ are shown in figure 4 and the variation trend of $SPL_{\text{max}}$ and $\phi_m$ are shown in figure 5.

In figure 4, the solid line, dashed line, dotted line and dot-dashed line show the sound pressure distribution at $\omega=15000$ r min$^{-1}$, $20000$ r min$^{-1}$, $25000$ r min$^{-1}$ and $30000$ r min$^{-1}$, respectively. In figure 5, the lines with rectangles and circles show the variation of $SPL_{\text{max}}$ and $\phi_m$, respectively. It can be seen from figure 4 that the radiation noise has an overall rise with the increase of rotation speed and the gap between $SPL_{\text{max}}$ and $SPL_{\text{min}}$ also has a great disparity with the increase of rotation speed. The $SPL$ curve turns from circumferential average to showing obvious directivity which is due to the aggravation of friction and impact. Similar with that in figure 2, the curves in figure 4 vary little in circumference from $300^\circ$ to $84^\circ$ and the changed amplitudes between curves are also not very large in this section. In the lower semicircles, the curves show clearer directivity with the increase of rotation speed.
speed, and there seem to be small differences between the directivity angles. According to figure 5, \( \phi_m \) appear at right below the bearing, but slightly skewed to the direction of rotation and the skewed angle increases with the increase of rotation speed (from 0° to 24°). This is mainly caused by the deviation of load zone. As is shown in figure 1, there are eccentricities between \( O, \ O_i \) and \( O_c \). The centrifugal forces brought by the eccentricities grow rapidly with the increase of rotation speed and the angles of resultant forces are changed accordingly, thus leading to the deviation of directivity angles.

Figure 4. Sound pressure levels at different rotation speed.

Figure 5. Variation trends of \( SPL_{\text{max}} \) and \( \phi_m \) at \( F_a=1500N \).

Compared with figure 2 and figure 3, it can be seen that the rotation speed acts differently on the directivity of the radiation noise. The \( SPL \) curves at \( \omega=30000 \text{ r min}^{-1} \) have almost the same tendencies, and the shapes tend to be smooth with the increase of axial load, while the curves with \( F_a=1500N \) have almost the same smoothness with different directivity angles. As the nonlinear dynamic model has universal applicability, the findings can be extended to the series of ceramic ball angular contact bearings for the further analysis of sound directivity. As is illustrated in section 2, an obvious sound directivity will lead to fault or even breakdown in the directivity angle owing to severe wear. It can be concluded that the overall radiation noise raises with the rotation speed at any fixed axial load,
however, a specific axial load at a fixed rotation speed can be obtained to minimize the radiation noise and make the directivity less obvious.

5. Conclusion
In this paper, the dynamic model of the ceramic ball bearing is established, and the sound directivity of ceramic ball angular contact bearings with different axial loads and at different rotation speed are analysed. In terms of SPL amplitudes, it is found that the radiation noise varies greatly with the change of axial load and rotation speed. The SPL distribution tends to be stable and even in the upper semicircles and shows obvious directivity in the lower ones. When the axial load increases at a given rotation speed, the directivity generally turns obvious, but the directivity angle stays unchanged. Similarly, when the rotation speed rises with a fixed axial load, the directivity becomes clearer with the directivity angle skewed to a large angle towards the running direction. The radiation noise of ceramic ball bearings increases with the rotation speed and an optimized axial load can be selected at each rotation speed to reduce the radiation noise and make the directivity less obvious. The investigation provides detailed information of the acoustic performance of ceramic ball bearings, thus putting forward the optimization direction of further development.

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References
[1] Cao H R, Niu L K, Xi S T and Chen X F 2018 Mechanical model development of rolling bearing- motor systems: A review Mech. Syst. Signal. Pr. 102 37
[2] Bai X T, Wu Y H, Zhang K, Chen C Z and Yan H P 2017 Radiation noise of the bearing applied to the motorized spindle based on the sub-source decomposition method J. Sound. Vib. 410 35
[3] Xu T, Xu G H, Zhang Q, Hua C, Tan H H, Zhang S C and Luo A L 2013 A preload analytical method for ball bearings utilising bearing skidding criterion Tribol. Int. 67 44
[4] Niu L K, Cao H R and Xiong X Y 2017 Dynamic modelling and vibration response simulations of angular contact balls with ball defects considering the three- dimensional motion of balls Tribol. Int. 109 26
[5] Bizarre L, Nonato F and Cavalca K L 2018 Formulation of five degrees of freedom ball bearing model accounting for the nonlinear stiffness and damping of elastohydro- dynamic point contacts Mech. Mach. Theory. 124 179
[6] Kumar S, Goyal D and Dhami S S 2018 Statistical and frequency analysis of acoustical signals for condition monitoring of ball bearing Mater. Today. Proc. 5 5186
[7] Chapman J J 1995 Angular Contact Ball Bearing Dynamics, an Experimental and Theoretical Investigation Tribol. Ser. 30 435
[8] Cao Y Z and Altinas Y 2007 Modeling of spindle-bearing and machine tool systems for virtual simulation of milling operations Int. J. Mach. Tool. Manu. 47 1342
[9] Wang Y L, Wang W Z, Zhang S G and Zhao Z Q 2015 Investigation of skidding in angular contact ball bearings under high speed Tribol. Int. 92 404
[10] Zhao C J, Yu X K, Huang Q X, Ge S D and Gao X 2015 Analysis on the load characteristics and coefficient of friction of angular contact ball bearing at high speed Tribol. Int. 87 50
[11] Kerst S, Shyrokau B and Holweg E 2018 A semi-analytical bearing model considering outer race flexibility for model based bearing load monitoring Mech. Syst. Signal. Pr. 104 384
[12] Gunduz A, Dreyer J T and Singh R 2012 Effect of bearing preloads on the modal characteristics of a shaft-bearing assembly: Experiments on double row angular contact ball bearings Mech. Syst. Signal. Pr. 31 176
[13] Deng S E, Li X L, Wang J G and Teng H F 2011 Frictional torque characteristic of angular contact ball bearings J. Mech. Eng. 47 114 (In Chinese)

[14] Zhang W H, Deng S E, Chen G D and Cui Y C 2016 Study on the impact of roller convexity excursion of high-speed cylindrical roller bearing on roller's dynamic characteristics Mech. Mach. Theory. 103 21

[15] Zhang W H, Deng S E, Chen G D and Cui Y C 2017 Impact of lubricant traction coefficient on cage’s dynamic characteristics in high-speed angular contact ball bearing Chinese. J. Aeronaut. 30 827

[16] Wang H, Han Q K and Zhou D N 2017 Nonlinear dynamic modelling of rotor system supported by angular contact ball bearings Mech. Syst. Signal. Pr. 85 16