Gate sequence for continuous variable one-way quantum computation

Xiaolong Su, Shuhong Hao, Xiaowei Deng, Lingyu Ma, Meihong Wang, Xiaojun Xie and Kunchi Peng
State Key Laboratory of Quantum Optics and Quantum Optics Devices, Institute of Opto-Electronics, Shanxi University, Taiyuan, 030006, People’s Republic of China

Measurement-based one-way quantum computation (QC) using cluster states as resources provides an efficient model to perform computation and information processing of quantum codes. Arbitrary Gaussian QC can be implemented by sufficiently long single-mode and two-mode gate sequences. However, continuous variable (CV) gate sequences have not been realized so far due to an absence of cluster states larger than four submodes. Here we present the first CV gate sequence consisting of a single-mode squeezing gate and a two-mode controlled-phase gate based on a six-mode cluster state. The quantum property of this gate sequence is confirmed by the fidelities and the quantum entanglement of two output modes, which depend on both the squeezing and controlled-phase gates. The experiment demonstrates the feasibility of implementing Gaussian QC by means of accessible gate sequences.

Measurement-based one-way QC performs computation by measurement and classical feedback on a multipartite cluster entangled state [1]. One-way QC was first demonstrated using four-photon cluster states [2, 3]. Besides photonic systems [2, 3], other QC systems with discrete quantum variables, such as ionic [4, 5], superconducting [6, 7], and solid state [8, 9] systems, have been investigated.

In parallel, the theoretical and experimental explorations on one-way CVQC have also been developed [19–30]. In contrast to the probabilistic generation of photonic qubits in most cases, CV cluster states are produced in an unconditional fashion and thus the one-way QC with CV cluster states can be implemented deterministically [27–34]. Although individual single-mode and two-mode logical gates towards implementing multimode Gaussian transformation in a one-way CVQC fashion have been experimentally demonstrated by using four-mode cluster entangled states of light [27, 28], CV gate sequences consisting of different single logical elements, which are necessary for realizing practical QC, have not been reported up to now. It is now important to investigate the gate sequences for QC as sufficiently large cluster states have recently been prepared, including eight-photon [35, 36], eight-quantum mode [37, 38] and up to 10,000-qumode [39] optical cluster states.

Here, we design and experimentally accomplish a CV gate sequence, in which a single-mode squeezing gate and a two-mode controlled-phase (CZ) gate are cascaded. A vacuum optical input signal is first squeezed by the squeezing gate and successively the squeezed output enters the CZ gate as one of its two inputs. A vacuum state or a squeezed state of light produced by an off-line nondegenerate optical parametric amplifier (NOPA) is used for the other CZ gate input. The experimental result shows that after two independent input states are transformed by the gate sequence, the two output states produced are entangled and their fidelities are better than that obtained by using coherent states as resources. Our experiments also prove that the entanglement degree and the fidelity depend simultaneously on two cascaded logical gates, which manifests the sequence feature of the presented system. Besides the gate sequences only involving multimode Gaussian transformation, for demonstrating universal one-way CVQC, at least a non-Gaussian operation is required [19]. Many theoretical protocols and schemes on the universal CVQC have been proposed [19, 20].

Results

Preparation of six-mode CV cluster states

CV cluster states are defined as [21, 22]

\[ \hat{\rho}_a - \sum_{b \in N_a} \hat{x}_b \equiv \hat{\delta}_a \to 0, \quad a \in G. \]  

(1)

In the limit of infinite squeezing, the N-mode cluster states are a simultaneous zero eigenstate of the N linear combinations in Eq. (1). Here the amplitude (\( \hat{x} \)) and phase (\( \hat{\rho} \)) quadratures of an optical mode \( \hat{a} \) are defined as \( \hat{x} = (\hat{a} + \hat{a}^\dagger)/2 \) and \( \hat{\rho} = (\hat{a} - \hat{a}^\dagger)/2i \). The modes \( a \in G \) denote the vertices of the graph \( G \), while the modes \( b \in N_a \) are the nearest neighbors of mode \( a \). One time measurement on cluster state can not destroy the entanglement totally, which means that cluster state has a strong property of entanglement persistence.

A general way to build CV cluster state is to implement an appropriate unitary transformation (\( U \)) on a series of input \( \hat{p} \)-squeezed states, \( \hat{a}_i = e^{+\tau \hat{x}_l^{(0)}} + i e^{-\tau \hat{p}_l^{(0)}} \), where \( \tau \) is the squeezing parameter, \( \hat{x}_l^{(0)} \) and \( \hat{p}_l^{(0)} \) represent the quadratures of a vacuum state, which has a noise variance \( \langle \Delta^2 \hat{x}_l^{(0)} \rangle = \langle \Delta^2 \hat{p}_l^{(0)} \rangle = 1/4 \). According to a general linear optics transformation \( \hat{x}_k = \sum_{l} U_{kl} \hat{a}_l \), the output modes can be obtained [22]. The transformation matrix \( U \) can be decomposed into a network of beam-splitters, which corresponds to the experimental

*Electronic address: kcpeng@sxu.edu.cn
system for generating required CV cluster state. We
designed the beam-splitter network of producing CV six-
mode linear cluster state with three NOPAs, as shown in
Fig. 1. A NOPA can simultaneously generate a \( \hat{x} \)-
squeezed state and a \( \hat{p} \)-squeezed state \[ \text{[40]} \]. The three
\( \hat{x} \)-squeezed states and three \( \hat{p} \)-squeezed states prepared
by the three NOPAs, are denoted by \( \hat{a}_1, \hat{a}_3, \hat{a}_5, \hat{a}_m =
e^{-r \hat{x}_m(0)} + ie^{+r \hat{p}_m(0)}, (m = 1, 3, 5) \) for \( \hat{x} \)-squeezed states,
and \( \hat{a}_2, \hat{a}_4, \hat{a}_6, \hat{a}_n = e^{+r \hat{x}_n(0)} + ie^{-r \hat{p}_n(0)} \) for \( \hat{p} \)-
squeezed states, respectively. Here we have assumed that
all squeezed states produced by the three NOPAs have
identical squeezing degree due to the equality of their
configuration (see Methods). We deduce the transfor-
mation matrix for generating CV six-mode linear cluster
state using three \( \hat{x} \)-squeezed states and three \( \hat{p} \)-squeezed states
as input states, which is given by \[ \text{[22]} \]

\[
U_6 = \begin{pmatrix}
\frac{i}{\sqrt{2}} & \frac{i}{\sqrt{3}} & -\sqrt{\frac{2}{26}} & -\sqrt{\frac{3}{26}} & 0 & 0 \\
-\frac{i}{\sqrt{2}} & \frac{i}{\sqrt{3}} & i \sqrt{\frac{2}{26}} & i \sqrt{\frac{3}{26}} & 0 & 0 \\
0 & 2 \sqrt{\frac{3}{26}} & \sqrt{\frac{2}{26}} & 0 & 0 & 0 \\
0 & 0 & -i \sqrt{\frac{6}{13}} & 2i \sqrt{\frac{2}{26}} & \frac{1}{\sqrt{3}} & -\frac{i}{\sqrt{3}} \\
0 & 0 & i \sqrt{\frac{2}{26}} & -i \sqrt{\frac{2}{26}} & \frac{1}{\sqrt{3}} & -\frac{i}{\sqrt{3}} \\
\end{pmatrix}
\]  \tag{2}

The matrix can be decomposed into

\[
U_6 = F_1 I_2 (1) F_3 F_4 I_6 (1) B_{56} (1/2) F_5 B_{12} (1/2) B_{45} (2/3) F_3 B_{24} (2/3) F_2 B_{34} (4/13),
\]

where \( F_k \) denotes the Fourier transformation of mode \( k \),
which corresponds to a 90° rotation in the phase space;
\( B_k^T (T_j) \) stands for the linearly optical transformation
on the \( j \)th beam-splitter with the transmission of \( T_j \)
(\( j = 1, \ldots, 5 \)), where \( (B_k^T)_{kk} = \sqrt{1 - T_j}, (B_k^T)_{ld} = \sqrt{T_j},
(B_k^T)_{lk} = \pm \sqrt{T_j}, \) and \( (B_k^T)_{ll} = \mp \sqrt{1 - T_j} \) are elements
of beam-splitter matrix. \( I_x (1) = e^{i \pi} \) corresponds to
a 180° rotation of mode \( k \) in phase space. When
the transmissions of the five beam-splitters are chosen as
\( T_1 = 4/13, T_2 = T_3 = 2/3, T_1 = T_5 = 1/2 \), the six
output optical modes construct a six-mode CV linear
cluster state. The corresponding excess noise terms for
each mode of the CV six-mode linear cluster state are
respectively denoted by

\[
\begin{align*}
\hat{\delta}_1 &= \sqrt{2} e^{-r \hat{x}_1(0)}, \\
\hat{\delta}_2 &= \sqrt{3} e^{-r \hat{p}_2(0)}, \\
\hat{\delta}_3 &= \frac{1}{\sqrt{2}} e^{-r \hat{x}_1(0)} + \frac{\sqrt{13}}{6} e^{-r \hat{p}_4(0)} + \frac{1}{\sqrt{3}} e^{-r \hat{x}_5(0)}, \\
\hat{\delta}_4 &= \frac{1}{\sqrt{2}} e^{-r \hat{p}_2(0)} - \frac{\sqrt{13}}{6} e^{-r \hat{x}_3(0)} + \frac{1}{\sqrt{2}} e^{-r \hat{p}_6(0)}, \\
\hat{\delta}_5 &= \sqrt{3} e^{-r \hat{p}_3(0)}, \\
\hat{\delta}_6 &= \sqrt{2} e^{-r \hat{p}_5(0)}. 
\end{align*}
\]  \tag{3}

Obviously, in the ideal case with infinite squeezing \( r \rightarrow
\infty \), these excess noises will vanish and the better the
squeezing, the smaller the noise term.

Fig. 2 shows the experimental results of six-mode
CV cluster state. Red and black lines correspond
to shot-noise-level (SNL) and quantum correlation
noise, respectively. The measured noise are
\( \langle \Delta^2(\hat{p}_1 - \hat{x}_2) \rangle = -4.04 \pm 0.09 \) dB, \( \langle \Delta^2(\hat{p}_2 - \hat{x}_1 + \hat{x}_3) \rangle = -4.22 \pm 0.10 \) dB, \( \langle \Delta^2(\hat{p}_3 - \hat{x}_2 - \hat{x}_4) \rangle = -3.80 \pm 0.10 \) dB,
$$\langle \Delta^2(\hat{p}_1 - \hat{x}_2) \rangle + \langle \Delta^2(\hat{p}_2 - \hat{x}_1 - \hat{x}_3) \rangle < 1,$$
$$\langle \Delta^2(\hat{p}_3 - \hat{x}_2 - \hat{x}_4) \rangle + \langle \Delta^2(\hat{p}_4 - \hat{x}_3 - \hat{x}_5) \rangle < 1,$$
$$\langle \Delta^2(\hat{p}_5 - \hat{x}_4 - \hat{x}_6) \rangle + \langle \Delta^2(\hat{p}_6 - \hat{x}_5) \rangle < 1.$$

Substituting the measured quantum noise (Fig. 2) into Eqs. (4)-(8), we can calculate the combinations of the correlation variances, which are 0.48, 0.59, 0.63, 0.62 and 0.50, respectively. Since all these values are smaller than the boundary of 1, the prepared six quantum modes satisfy the inseparability criteria and form a six-mode cluster entangled state.

Configuration of the gate sequence

As shown in Fig. 3a, we demonstrate the gate sequence of a squeezing gate and a CZ gate using a six-mode cluster state as ancillary state. First, we perform squeezing gate on input mode \(\alpha\) (target mode) to produce a phase squeezed state. Then a CZ gate is performed on the output mode from the squeezing gate and the other input mode \(\beta\) (control mode) coming from outside of the sequence.

The single-mode squeezing gate is expressed by \(S(r_s) = e^{i r_s \hat{x}}\). The input-output relation of the single-mode squeezing gate is \(\hat{\xi}_j' = S\hat{\xi}_j\), where \(\hat{\xi}_j = (\hat{x}_j, \hat{p}_j)^T\) and

$$S = \begin{pmatrix} e^{r_s} & 0 \\ 0 & e^{-r_s} \end{pmatrix}$$

represents the squeezing operation on phase quadrature. The CV CZ gate corresponds to the unitary operator \(C_{Zjk} = e^{2i\hat{x}_j\hat{x}_k}\) with the input-output relation,

$$\hat{\xi}_j' = \begin{pmatrix} 1 & C \\ C & 1 \end{pmatrix} \hat{\xi}_j,$$

where \(\hat{\xi}_j = (\hat{x}_j, \hat{p}_j, \hat{x}_k, \hat{p}_k)^T\),

$$C = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},$$

and \(I\) is a \(2 \times 2\) identity matrix.

The transformation matrix of the gate sequence is given by

$$U = \begin{pmatrix} 1 & C \\ C & 1 \end{pmatrix} \cdot \begin{pmatrix} S & 0 \\ 0 & I \end{pmatrix}.$$
C5 through classical feedforward circuits. In the operation of gate sequence, the measured observables are
\[\hat{x}_{d1} = \frac{\cos \theta_1 (\hat{x}_a - \hat{x}_1) + \sin \theta_1 (\hat{p}_a - \hat{p}_1)}{\sqrt{2}}, \quad \hat{x}_{d2} = \frac{\cos \theta_2 (\hat{x}_a + \hat{x}_1) + \sin \theta_2 (\hat{p}_a + \hat{p}_1)}{\sqrt{2}}, \quad \hat{p}_{d2}, \quad \hat{p}_{d3}, \quad \hat{x}_{d3} = (\hat{x}_\beta - \hat{p}_0)/\sqrt{2}, \quad \text{and} \quad \hat{p}_{d4} = (\hat{p}_\beta - \hat{x}_6)/\sqrt{2}.\]
The measurement angle \(\theta_1\) and \(\theta_2\) in the homodyne detection for \(\hat{x}_{d1}\) and \(\hat{x}_{d2}\) determine the squeezing operation according to the transformation matrix
\[S = \begin{pmatrix} \cot \theta_2 & 0 \\ 0 & \tan \theta_2 \end{pmatrix}, \quad (14)\]
if we choose \(\theta_2 = -\theta_1\). \(S\) corresponds to a single-mode amplitude and phase squeezing gate when \(\cot \theta_2 = e^{-r_\pi}\) and \(e^{r_\pi}\), respectively. The measurement angles \((\theta_1, \theta_2)\) for 0 dB, −3 dB, −6 dB, −9 dB and −12 dB squeezing are \((-45.00^\circ, 45.00^\circ), \quad (-35.30^\circ, \quad 35.30^\circ), \quad (-26.62^\circ, \quad 26.62^\circ), \quad (-19.54^\circ, \quad 19.54^\circ),\) and \((-14.10^\circ, \quad 14.10^\circ)\), respectively. These measurement results are feedforwarded to the amplitude and phase quadratures of modes C4 and C5 via electro-optical modulators (EOM), respectively. The corresponding feedforward operation is expressed by \(\hat{X}_{C4}(f_1)\hat{Z}_{C4}(f_2)\hat{X}_{C5}(f_3)\hat{Z}_{C5}(f_4)\), where \(\hat{X}_{\ell}(s) = e^{-2i\alpha_\ell s}\) and \(\hat{Z}_{\ell}(s) = e^{2i\alpha_\ell s}\), and phase displacement amplifiers, respectively, \(f_1 = -\alpha_3 + \frac{\csc \delta_3}{\sqrt{2}} \hat{x}_{d1} + \frac{\csc \delta_3}{\sqrt{2}} \hat{x}_{d2}, \quad f_2 = -\alpha_2 + \frac{\csc \delta_2}{\sqrt{2}} \hat{x}_{d1} + \frac{\csc \delta_2}{\sqrt{2}} \hat{x}_{d2}, \quad f_3 = \frac{\csc \delta_3}{\sqrt{2}} \hat{x}_{d1} + \frac{\csc \delta_3}{\sqrt{2}} \hat{x}_{d2}, \quad f_4 = -\alpha_3 + \frac{\csc \delta_3}{\sqrt{2}} \hat{x}_{d1} + \frac{\csc \delta_3}{\sqrt{2}} \hat{x}_{d2} \). The output modes are detected by two homodyne detection systems. The quadrature components of the output modes for the gate sequence are given by
\[
\begin{pmatrix} \hat{x}_\mu \\ \hat{p}_\mu \\ \hat{x}_\nu \\ \hat{p}_\nu \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \cot \theta_2 & 0 & 0 & 0 \\ 0 & \tan \theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cot \theta_2 & 0 & 0 & 0 \\ \csc \theta_2 & \csc \theta_2 & 0 & 0 \\ 0 & \csc \theta_2 & \csc \theta_2 & 0 \\ 0 & 0 & \csc \theta_2 & \csc \theta_2 \end{pmatrix} \begin{pmatrix} \hat{x}_1 \\ \hat{p}_1 \\ \hat{x}_2 \\ \hat{p}_2 \end{pmatrix} + \begin{pmatrix} \delta_1 - \delta_3 \\ \delta_2 \delta_2 - \delta_2 \delta_3 \\ \delta_5 - \delta_5 - \delta_1 \end{pmatrix}, \quad (15)
\]
where
\[G = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}, \quad (16)\]
is the feedforward gain factor. From Eqs. (14) and (15) we obtain
\[
\begin{align*}
\hat{x}_\mu &= \hat{x}_\alpha e^{r_\pi} + \hat{\delta}_1 - \hat{\delta}_3, \\
\hat{p}_\mu &= \hat{p}_\alpha e^{r_\pi} + \hat{x}_\beta - \hat{\delta}_2 + \hat{\delta}_4 - \hat{\delta}_6, \\
\hat{x}_\nu &= \hat{x}_\beta - \hat{\delta}_6, \\
\hat{p}_\nu &= \hat{p}_\beta + \hat{x}_\alpha e^{r_\pi} + \hat{\delta}_1 + \hat{\delta}_5 - \hat{\delta}_3.
\end{align*}
\]
After a vacuum signal \((\alpha)\) passes through the squeezing gate, its phase \((\hat{p}_\alpha)\) and amplitude \((\hat{x}_\alpha)\) are squeezed

\[
\langle \Delta^2(g\hat{p}_\mu - \hat{x}_\nu) \rangle + \langle \Delta^2(g\hat{p}_\nu - \hat{x}_\mu) \rangle < g, \quad (18)
\]
where \(g\) is the optimal gain factor which makes the left side of the inequality minimum. By calculating the minimal value of equation (16), the optimal gain is obtained
\[
g = \frac{e^{2r_\beta} \left(3 + 2e^{2r} + e^{2r + 2r_\beta} + e^{2r} \cot^2 \theta_2\right)}{e^{2r} + 8e^{2r_\beta} + e^{2r + 4r_\beta} + e^{2r + 2r_\beta} \left(\cot^2 \theta_2 + \tan^2 \theta_2\right)}, \quad (19)
\]
where \(r_\beta\) is the squeezing parameter of the input mode \(\beta\). Fig. 4 shows the dependence of entanglement degree between the two output states of the gate sequence on squeezing degree of the squeezing gate with an input vacuum mode \(\alpha\) for three different \(\beta\) states (blue: vacuum state, red: −4 dB phase squeezed state, yellow: −12 dB phase squeezed state). The entanglement degree is

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{Dependence of entanglement on squeezing of the squeezing gate. The dependence of the entanglement between the gate sequence output modes on the squeezing of the squeezing gate is plotted here. The blue, red and yellow lines correspond to the input mode \(\beta\) being a vacuum state, −4 dB and −12 dB phase squeezed state, respectively. The solid and dashed lines correspond to −4 dB and −6 dB squeezing of the six-mode cluster state, respectively. The black dots represent the experimental data. Error bars represent ± one standard deviation and are obtained based on the statistics of the measured noise variances.}
\end{figure}
quantified by \( E = \langle \Delta^2(g\hat{p}_\mu - \hat{x}_\nu) \rangle + \langle \Delta^2(g\hat{p}_\nu - \hat{x}_\mu) \rangle - g \). When \( E < 0 \), the entanglement exists and the smaller the \( E \), the stronger the entanglement. The solid and dashed lines correspond to \(-4\) and \(-6\) dB squeezing of the six-mode cluster state, respectively. All traces are plotted according to the real loss in our experimental system (see Methods). We can see that the entanglement degree between the output states \((\mu\) and \(\nu)\) not only depends on the operation of \(\text{CZ}\) gate, but also is controlled by the squeezing operation of the squeezing gate. For a given \(\beta\) state, when the squeezing of the squeezing gate increases the entanglement degree of output states increases. On the other hand, the operation of \(\text{CZ}\) gate also depends on the feature of \(\beta\) state. The phase squeezing of \(\beta\) state will improve the entanglement degree of the output modes. Of course, the largest influence to the capacity of the gate sequence comes from the quality of the resource state. When the squeezing of the six-mode cluster state increases from \(-4\) dB (solid lines) to \(-6\) dB (dashed lines) the entanglement degree of output states will be significantly improved. Since \(-4\) dB cluster squeezing is available in our experiment, the obtained maximal entanglement degree is only \(-0.005\) for the case of using two vacuum states to be \(\alpha\) and \(\beta\) (solid blue line). If \(\beta\) is a phase squeezed state (red and yellow lines) the entanglement will increase for the same squeezing degree of the squeezing gate and an identical resource state. The experimentally measured data points obtained under different measurement angles of the squeezing gate are marked by little black dots with error bars, which shows that the experimental results are in reasonable agreement with the theoretical expectation. The experimentally measured entanglement degrees, fidelities, and corresponding optimal gain factors are listed in table 1. The entanglement degree of output modes depends on both operations of two cascaded gates, which clearly shows that the final results are decided by the gate sequence.

Fig. 5 shows the measured noise variances of the quadratures \((\hat{x}_\mu, \hat{p}_\mu, \hat{x}_\nu, \text{ and } \hat{p}_\nu)\) of the output modes \((\mu\) and \(\nu)\) with a vacuum mode \(\alpha\) and a phase squeezed state \(\beta\) with squeezing of \(-4\) dB as two inputs of the gate sequence, where \(-12\) dB squeezing is implemented on input \(\alpha\). In the ideal case with the cluster state of infinite squeezing (yellow lines), the noise powers of \(\hat{x}_\mu\) and \(\hat{x}_\nu\) are \(12\) and \(4\) dB above the SNL (black lines), which correspond to the anti-squeezing noises resulting from the squeezing gate (12 dB) and the input phase squeezed state (4 dB), respectively. The noise powers of \(\hat{p}_\mu\) and \(\hat{p}_\nu\) are \(4.1\) and \(12.1\) dB above the SNL due to the effect of \(\text{CZ}\) gate (see equation (17)). The blue and red lines stand for the output noises without and with the cluster state as ancillary state, respectively. The blue lines are obtained by using a coherent light of identical intensity to replace each of cluster submodes. In this case, the measured values of \(\hat{x}_\mu, \hat{p}_\mu, \hat{x}_\nu, \text{ and } \hat{p}_\nu\) are \(12.75 \pm 0.07, 9.05 \pm 0.09, 7.76 \pm 0.08\) and \(13.07 \pm 0.09\) dB above the SNL, respectively. The noise variances of \(\hat{x}_\mu, \hat{p}_\mu, \hat{x}_\nu, \text{ and } \hat{p}_\nu\) measured with the existence of the cluster state (red lines) are \(12.34 \pm 0.11, 7.60 \pm 0.13, 6.84 \pm 0.12\) and \(12.55 \pm 0.13\) dB above the SNL, respectively, all of which are lower than the corresponding values without using the cluster resource.

In order to further verify the general input-output relation of the gate sequence, we employ a coherent state with a 15 dB modulation signal as input state (Fig. 6a-d). Fig. 6a shows the noise powers of quadratures of the output modes \(\mu\) and \(\nu\) when input modes \(\alpha\) and \(\beta\) are a coherent state with nonzero amplitude of 15 dB (\(\hat{x}_\alpha\)-coherent) and a vacuum state, respectively. The measured noise variance of \(\hat{x}_\mu\) (red line) is \(27.01 \pm 0.13\) dB above SNL (black lines) that is because 12 dB anti-squeezing noise resulting from squeezing gate is added on the 15 dB input amplitude of \(\hat{x}_\alpha\). In the ideal case (yellow lines), the noise variance of \(\hat{p}_\mu\) is a little higher than SNL since \(\hat{x}_\nu\) is added on the squeezed noise of \(-12\) dB, and the noise variance of \(\hat{x}_\nu\) is at the level of SNL. The measured noise powers of \(\hat{p}_\mu\) and \(\hat{p}_\nu\) are \(4.43 \pm 0.16\) and \(2.68 \pm 0.18\) dB above the SNL because of the effect of excess noises coming from the imperfect entanglement of the cluster state. The measured noise variance of \(\hat{p}_\nu\) is \(27.02 \pm 0.11\) dB above the SNL because the amplitude on \(\hat{x}_\nu\) is added to \(\hat{p}_\nu\), which satisfy the input-output relation of the \(\text{CZ}\) logic gate in equation (17). Fig. 6b shows the noise powers of output modes when a coherent state with a modulation signal of 15 dB on \(\hat{p}_\alpha\) (\(\hat{p}_\alpha\)-coherent) and a vacuum state are used for the input states \(\alpha\) and \(\beta\), respectively. The measured noise power (red lines) of \(\hat{x}_\mu\)
| Data point | g | E   | $F_\mu$   | $F_\nu$   |
|------------|---|-----|-----------|-----------|
| a          | 0.72 | 0.112 ± 0.026 | 0.832 ± 0.011 | 0.873 ± 0.013 |
| b          | 0.81 | 0.053 ± 0.033 | 0.882 ± 0.011 | 0.902 ± 0.014 |
| c          | 0.87 | 0.023 ± 0.026 | 0.905 ± 0.009 | 0.942 ± 0.012 |
| d          | 0.92 | 0.004 ± 0.027 | 0.888 ± 0.009 | 0.951 ± 0.011 |
| e          | 0.95 | −0.005 ± 0.024 | 0.886 ± 0.012 | 0.956 ± 0.009 |
| f          | 0.83 | 0.040 ± 0.026 | 0.860 ± 0.013 | 0.854 ± 0.013 |
| g          | 0.90 | −0.033 ± 0.029 | 0.903 ± 0.014 | 0.891 ± 0.013 |
| h          | 0.94 | −0.085 ± 0.024 | 0.922 ± 0.009 | 0.934 ± 0.009 |
| i          | 0.96 | −0.103 ± 0.031 | 0.932 ± 0.011 | 0.950 ± 0.010 |
| j          | 0.98 | −0.124 ± 0.022 | 0.923 ± 0.006 | 0.947 ± 0.006 |

$a$-$e$: $\alpha$ and $\beta$ are vacuum state, squeezing of the squeezing gate are 0, $-3$, $-6$, $-9$ and $-12$ dB, respectively. $f$-$j$: $\alpha$ is a vacuum state, $\beta$ is a $-4$ dB phase squeezed state, squeezing of the squeezing gate are 0, $-3$, $-6$, $-9$ and $-12$ dB, respectively.

![FIG. 6](image_url)

**FIG. 6:** Experimentally measured noise powers of the output modes with coherent inputs. a-d: $\hat{x}_\alpha$, $\hat{p}_\alpha$, $\hat{x}_\beta$, and $\hat{p}_\beta$-coherent state as input, respectively. Black lines are SNL, red lines are output noise power with cluster state as ancillary state, yellow lines are ideal output noise power. Squeezing degree of the squeezing gate is $-12$ dB. Measurement frequency is 2 MHz. The spectrum analyzer resolution bandwidth is 30 kHz, and the video bandwidth is 100 Hz. The noise power of $\hat{p}_\nu$ is $12.34 \pm 0.17$ dB above the corresponding SNL (black line) because of the effect of anti-squeezing noise resulting from the squeezing gate. The noise power of $\hat{p}_\mu$ and $\hat{x}_\nu$ (red lines) are $6.72 \pm 0.12$ dB and $2.68 \pm 0.12$ dB above the corresponding SNL, respectively. The noise power of $\hat{p}_\nu$ is $12.68 \pm 0.14$ dB above the SNL because the noise of $\hat{x}_\mu$ is added on $\hat{p}_\nu$ [see equation (17)]. Figs. 6e-d are the noise powers of output modes when the input is the coherent state with the modulation signal of 15 dB on $\hat{x}_\beta$, and $\hat{p}_\beta$ ($\hat{x}_\beta$-coherent and $\hat{p}_\beta$-coherent), respectively. It is
FIG. 7: Measured quantum correlation variances. This figure shows the measured quantum correlation variances of the output modes of the gate sequence with a vacuum state and a phase squeezed state as inputs. a and b are noise power of \( \Delta^2(g_{\mu} - \bar{x}_\mu) \) and \( \Delta^2(g_{\nu} - \bar{x}_\nu) \), respectively. Black lines and red lines are SNL and quantum correlation noise, respectively. Squeezing degree of the squeezing gate is -12 dB. Measurement frequency is 2 MHz, the spectrum analyzer resolution bandwidth is 30 kHz, and the video bandwidth is 100 Hz.

obvious that the measured noise powers of output modes satisfy the input-output relation of the gate sequence in equation (17).

Fig. 7 shows the measured correlation noise variances of the output modes with a vacuum mode (\( \alpha \)) and a -4 dB phase squeezed mode (\( \beta \)) as the inputs of the sequence, where -12 dB squeezing is implemented on input \( \alpha \). The measured noise variance (red lines) of \( \Delta^2(g_{\mu} - \bar{x}_\mu) \) (a) and \( \Delta^2(g_{\nu} - \bar{x}_\nu) \) (b) are 0.53 ± 0.11 and 0.65 ± 0.11 dB below the corresponding SNL (black lines), respectively. The entanglement is quantified by

\[
\langle \Delta^2(g_{\mu} - \bar{x}_\mu) \rangle + \langle \Delta^2(g_{\nu} - \bar{x}_\nu) \rangle = 0.856 \pm 0.022. \quad (20)
\]

For our experimental system the calculated optimal gain factor \( g = 0.98 \). The total correlation variances in the left side of equation (18) is smaller than \( g \) and thus satisfies the inseparability criteria, which confirms the quantum entanglement between the two output modes (\( \mu \) and \( \nu \)) from the gate sequence.

We also use fidelity \( F = \left\{ \text{Tr}[\sqrt{\rho_1}\sqrt{\rho_2}\sqrt{\rho_1}]^{1/2} \right\}^2 \), which denotes the overlap between the experimentally obtained output state \( \rho_2 \) and the ideal output state \( \rho_1 \), to quantify the performance of the gate sequence. The fidelity for two Gaussian states \( \rho_1 \) and \( \rho_2 \) with covariance matrices \( A_j \) and mean amplitudes \( \alpha_j \equiv (\alpha_{jx}, \alpha_{jp}) \) (\( j = 1, 2 \)) is expressed by \[43\] \[44\]

\[
F = \frac{2}{\sqrt{\Delta + \sigma} - \sqrt{\sigma}} \exp[-\epsilon^T(A_1 + A_2)^{-1} \epsilon], \quad (21)
\]

where \( \Delta = \text{det}(A_1 + A_2) \), \( \sigma = (\text{det} A_1 - 1)(\text{det} A_2 - 1) \), \( \epsilon = \alpha_2 - \alpha_1 \), \( A_1 \) and \( A_2 \) for the ideal (\( \hat{\rho}_1 \)) and experimental (\( \hat{\rho}_2 \)) output states, respectively. The covariance matrices \( A_j \) (\( j = 1, 2 \)) for target mode are given by

\[
A_{\mu 1} = 4 \begin{bmatrix} \langle \Delta^2 \hat{x}_\mu \rangle & 0 \\ 0 & \langle \Delta^2 \hat{p}_\mu \rangle \end{bmatrix}, \quad (22)
\]

\[
A_{\mu 2} = 4 \begin{bmatrix} \langle \Delta^2 \hat{x}_\mu \rangle & 0 \\ 0 & \langle \Delta^2 \hat{p}_\mu \rangle \end{bmatrix}, \quad (23)
\]

\[
A_{\nu 1} = 4 \begin{bmatrix} \langle \Delta^2 \hat{x}_\nu \rangle & 0 \\ 0 & \langle \Delta^2 \hat{p}_\nu \rangle \end{bmatrix}, \quad (24)
\]

\[
A_{\nu 2} = 4 \begin{bmatrix} \langle \Delta^2 \hat{x}_\nu \rangle & 0 \\ 0 & \langle \Delta^2 \hat{p}_\nu \rangle \end{bmatrix}. \quad (25)
\]

The coefficient “4” comes from the normalization of SNL. Since the noise of a vacuum state is defined as 1/4 in the text, while in the fidelity formula the vacuum noise is normalized to “1”, so a coefficient “4” appears in the expressions of covariance matrices. Similarly, we can write out the covariance matrices for the control mode.

In case of infinite squeezing, both fidelities for the target and control states, \( F_\mu \) and \( F_\nu \), equal to 1, which can be calculated from equation (6) with \( r \to \infty \).

Fig. 8 is the fidelities of the output modes \( \mu \) and \( \nu \) as the function of squeezing degree of the squeezing gate based on the experimental data for two different input state (\( a \): vacuum state, \( b \): a -4 dB phase squeezed state). In Fig. 8, black and red lines correspond to fidelity of output modes \( \mu \) and \( \nu \), respectively. Dashed lines are obtained at the case without the use of cluster resource (using the coherent states in the same intensity to substitute the cluster states in Fig. 1) and solid lines are completed under the case using cluster resource state. Obviously, when the cluster state is applied, the fidelity of the output states is higher than that obtained at the case using the coherent state, which is usually named as the classical limit in quantum optics. The experimentally measured data (see table 1) are marked in Fig. 8a and 8b which are in reasonable agreement with the theoretical calculation.

Discussion

We demonstrated a gate sequence in one-way QC fashion by applying a six-mode CV cluster state as quantum resource. The quantum feature of the gate sequence is verified quantitatively by both the inseparability criterion of two-mode entanglement and the fidelities of output states. The entanglement degree of two output modes depends on two cascaded gates, simultaneously,
FIG. 8: Dependence of fidelity on squeezing of the squeezing gate. a and b are the fidelity for the input mode $\beta$ as a vacuum state and a $-4$ dB phase squeezed state, respectively. The solid and dashed lines correspond to fidelity with and without cluster resource, respectively. Black and red lines present the fidelity of output modes $\mu$ and $\nu$, respectively. Error bars represent $\pm$ one standard deviation and are obtained based on the statistics of the measured noise variances.

FIG. 9: The schematic of reverse blind CVQC. The quantum server prepares squeezed states and sends them to anyone of clients in demand. Client prepares CV cluster state with linear optics and then implements quantum computation via measurements and feedforwards on the prepared cluster state, which exhibits the sequence character of the system. Although in our experiment only two gates are linked together, the scheme can be easily extended to construct any large QC gate sequence with a number of gates.

Today, quantum computers have become a physical reality and are continuing to be developed. One-way QC based on quantum teleportation [23, 24] is able to implement secure information processing and accomplish the unbreakable quantum coding [43, 46]. On the other hand, the large cluster states can be prepared only by linear optical systems if appropriate squeezed states are available. Thus, one-way quantum computers consisting of this type of gate sequences can be operated in a reverse blind CVQC model to realize the secure QC network (Fig. 9), in which only a server owns the ability of preparing quantum states (such as squeezed states) and all remote clients ask the server to send them necessary squeezed states through a quantum channel [47]. Then, clients produce the cluster states using linear optical transformation and perform arbitrary CV one-way Gaussian QC by means of classical measurements and feedforwards on the prepared cluster state at their stations. In this way, the server and any eavesdroppers never know what clients want to do, thus the security of the blind QC is guaranteed by no-signaling principle [48]. The presented gate sequence for one-way Gaussian CVQC is an essentially experimental exploring toward developing universal QC and practical quantum networks.

Methods

Experimental details. The three $\hat{x}$-squeezed and three $\hat{p}$-squeezed states are produced by three NOPAs. These NOPAs are pumped by a common laser source, which is a continuous wave intracavity frequency-doubled and frequency-stabilized Nd:YAP/LBO(Nd-doped YAlO$_3$ perorokite/lithium triborate) [49]. The output fundamental wave at 1080 nm wavelength from the laser is used for the injected signals of NOPAs and the local oscillators of the homodyne detectors (HDs). The output second-harmonic wave at 540 nm wavelength serves as the pump field of the four NOPAs, in which a pair of signal and idler modes with the identical frequency at 1080 nm and the orthogonal polarizations are generated through an intracavity frequency-down-conversion process [50]. Each of NOPAs consists of an $\alpha$-cut type-II KTP crystal and a concave mirror [50]. The front face of the KTP was coated to be used for the input coupler and the concave mirror serves as the output coupler of the squeezed states, which is mounted on a piezoelectric transducer for locking actively the cavity length of NOPA on resonance with the injected signal at 1080 nm. The transmissions of the input coupler at 540 nm and 1080 nm are 99.8% and 0.04%, respectively. The
transmissions of the output coupler at 540 nm and 1080 nm are 0.5% and 5.2%, respectively. The fineses of the NOPA for 540 nm and 1080 nm are 3 and 117, respectively. In our experiment, all NOPAs are operated at the parametric deamplification situation, i.e. the phase difference between the pump field and the injected signal is \((2n + 1)\pi\) \((n\) is an integer). Under this condition, the coupled modes at +45° and -45° polarization directions are the quadrature-amplitude and the quadrature-phase squeezed states, respectively \([31, 40]\).

Three NOPAs are locked individually by means of Pound-Drever-Hall (PDH) method with a phase modulation of 56 MHz on 1080 nm laser beam \([51]\). In the experiment, the relative phase \((2n+1)\pi\) locking is completed with a lock-in amplifier, where a signal around 15 kHz is modulated on the pump light by the piezo-electric transducer (PZT) mounted on a reflection mirror which is placed in the optical path of the pump laser and then the error signal is fed back to the other PZT which is mounted on a mirror placed in the optical path of the injected beam. In the beam-splitter network used to prepare six-mode cluster states, the relative phase between two incident beams on T1 and T4 is phase-locked to zero, and that on T2, T3 and T5 is phase-locked to \(\pi/2\). The zero phase difference (T1 and T4) between two interfered beams on a beam-splitter is locked by a lock-in amplifier. The \(\pi/2\) phase difference (T2, T3 and T5) is locked by DC locking technique, where the photocurrent signal of the interference fringe is fed back to the PZT mounted on a mirror which is placed before the beam-splitter. In the homodyne detection system, zero phase difference for the measurement of quadrature-amplitude is locked by PDH technique with a phase modulation of 10.9 MHz on local oscillator beam. The \(\pi/2\) phase for the measurement of quadrature-phase is locked with DC locking technique too.

The transmission efficiency of an optical beam from NOPA to a homodyne detector is around 96%. The quantum efficiency of a photodiode (FD500W-1064, Fermions) used in the homodyne detection system is 95%. The interference efficiency on a beam-splitter is about 99%. The phase fluctuation of the phase locking system is about 2-3°.

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