Electric Charge Quantization

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ABSTRACT

Experimentally it has been known for a long time that the electric charges of the observed particles appear to be quantized. An approach to understanding electric charge quantization that can be used for gauge theories with explicit $U(1)$ factors – such as the standard model and its variants – is pedagogically reviewed and discussed in this article. This approach uses the allowed invariances of the Lagrangian and their associated anomaly cancellation equations. We demonstrate that charge

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may be de-quantized in the three-generation standard model with massless neutrinos, because differences in family-lepton–numbers are anomaly-free. We also review the relevant experimental limits. Our approach to charge quantization suggests that the minimal standard model should be extended so that family-lepton–number differences are explicitly broken. We briefly discuss some candidate extensions (e.g. the minimal standard model augmented by Majorana right-handed neutrinos).

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1. Introduction

The electric charges of all the known elementary particles are integer multiples (to within experimental errors) of the d-quark charge. Thus, experimentally, it seems that electric charge is quantized. A theoretical understanding of electric charge quantization is important if we hope to understand the interactions of elementary particles. Of course, understanding elementary particle interactions is one of the major goals of theoretical physics.

It is not known for certain why electric charge is quantized. There have been many suggestions over the years, including higher dimensions [1], magnetic monopoles [2] and grand unified theories [3]. All of these suggestions are highly speculative and very difficult to test experimentally. Recently, a less speculative approach to electric charge quantization has emerged. The purpose of this article to review this approach to understanding the phenomenon of electric charge quantization [4].

2. The Standard Model

The starting point is the minimal standard model (MSM), as defined by its Lagrangian, which is the synthesis of much work over the last few decades. It appears that the three non-gravitational forces – the strong, weak and electromagnetic – can be successfully described by a Yang-Mills theory with the gauge group $G_{SM}$ where

$$G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y.$$ (1)

Under this gauge group, the quarks and leptons of each generation transform as

$$Q_L \sim (3, 2, 1/3), \quad u_R \sim (3, 1, 4/3), \quad d_R \sim (3, 1, -2/3)$$

$$f_L \sim (1, 2, -1), \quad e_R \sim (1, 1, -2).$$ (2)

There is also a Higgs doublet $\phi$ which can be defined through the Yukawa Lagrangian $\mathcal{L}_{Yuk}$ where

$$\mathcal{L}_{Yuk} = \lambda_1 \bar{f}_L \phi e_R + \lambda_2 \bar{Q}_L \phi d_R + \lambda_3 \bar{Q}_L \phi^c u_R + \text{H.c.}$$ (3)

Note that there is an implicit summation over fermion generations in this equation. The spin-0 multiplet $\phi$ transforms under $G_{SM}$ as

$$\phi \sim (1, 2, 1),$$ (4)
and $\phi^c \equiv i\tau_2 \phi^*$. 

The Higgs doublet $\phi$ is assumed to be responsible for the fermion and gauge boson masses. This is achieved by having the Higgs doublet gain a nonzero vacuum expectation value (VEV), thus spontaneously breaking $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ to $SU(3)_c \otimes U(1)_Q$. The gauge group $U(1)_Q$ describes the electromagnetic force. The generator, $Q$, of this Lie group is the linear combination of $Y$ and the generators $I_i (i = 1, 2, 3)$ of $SU(2)_L$ which is not broken by the VEV of $\phi$. By the $SU(2)_L \otimes U(1)_Y$ symmetry of the Lagrangian, we are free to work in the basis in which the VEV of $\phi, \langle \phi \rangle$ has the form

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ u \end{pmatrix}$$

(5)

and $u$ is a real number. Denoting the hypercharge (i.e. the $Y$ charge) of the Higgs doublet as $y_\phi$, we see that the linear combination

$$Q \equiv c (I_3 + \frac{Y}{2y_\phi})$$

(6)

annihilates the VEV given in Eq.(5), for all values of the constant $c$. Thus, in order to understand the electric charges values of the quarks and leptons we need to understand each of the $U(1)_Y$ assignments of the quarks and leptons [as can be seen from Eq.(2), there are five of these values to understand per generation] and also the value for the $U(1)_Y$ charge of the Higgs doublet, since its value determines which linear combination of $I_3$ and $Y$ is the electric charge generator.

The overall normalization constant $c$ in Eq.(6) has no independent physical meaning in the standard model. Also, the overall normalization of $Y$ has no independent physical meaning. This is because of a rescaling degree of freedom. A $U(1)_Y$ gauge theory is invariant under the rescaling

$$Y \rightarrow \eta Y, \quad g \rightarrow g/\eta.$$ 

(7)

The value of $\eta$ is not physically observable, so we are always free to change the overall normalization of $Y$ in Eqs.(2) and (4). For simplicity, we will use this degree of freedom up by fixing the Higgs boson hypercharge $y_\phi$ to be $+1$ in the forthcoming discussion [note that this value for $y_\phi$ obeys convention as per Eq.(4)]. Similarly, we are free to choose the normalization constant $c$ for the electric charge generator $Q$. It is conventional to set $c$ equal to one. The result of this is that we are free to choose the electric charge generator $Q$ to be given by

$$Q = I_3 + Y/2$$

(8)

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without loss of generality. It is important to clearly understand the origin of Eq.(8). The primary consideration that went into the above derivation is that the electromagnetic group is defined to be the $U(1)$ invariance left exact after electroweak symmetry breaking. This told us the key fact that $Q$ was some linear combination of $I_3$ and $Y$. Having realised this simple but crucial point, we then made the (trivial) observation that we could conventionally choose this linear combination to be as given in Eq.(8) by normalizing the $U(1)$ generators appropriately. The moral of the story is that the contemptibly familiar relation $Q = I_3 + Y/2$ has semantic value; it is not just convention.

So, after all of this, we find that there are five unknowns per generation to be determined: the quark and lepton hypercharges. We will denote these through the transformation laws,

$$Q_{mL} \sim (3, 2, y_{m1}), \ u_{mR} \sim (3, 1, y_{m2}), \ d_{mR} \sim (3, 1, y_{m3}), \ f_{mL} \sim (1, 2, y_{m4}), \ e_{mR} \sim (1, 1, y_{m5}),$$

(9)

where $m = 1, \ldots, n_G$ is the generation index. Observe that we do not intend to make any a priori assumptions about whether corresponding multiplets in different generations have the same weak hypercharges or not. If our goal is to understand weak hypercharges, then we should make as few a priori assumptions as possible about them.

Now, we stated at the outset that our approach to electric-charge quantization would begin with the study of the MSM Lagrangian. Clearly the generation dependence of the hypercharges in Eq.(9) will be severely constrained by the requirement of family mixing in the Yukawa Lagrangian. However, for pedagogical reasons it is best to analyse this issue one step at a time. We therefore now specialise to the unrealistic case of just a single quark and lepton generation.

3. Toy Model #1

Consider the MSM Lagrangian truncated to include only a single fermion family. This defines our toy model #1. Our task is to see how many of the hypercharges $y_{1,2,3,4,5}$ can be determined (we will omit the redundant generation index for this one generation Toy Model). There are constraints on these hypercharge values which come from requiring the Lagrangian to be invariant under the gauge symmetry [5] (by this we mean in particular, that the Lagrangian must have a $U(1)$ invariance.
since the gauge group is assumed to have a $U(1)$ factor). We will refer to these as classical constraints. The only part of the Lagrangian which restricts the possible values of the hypercharges is the Yukawa Lagrangian in Eq.(3). These Yukawa terms imply the conditions

$$y_1 = y_3 + 1, \ y_1 = y_2 - 1, \ y_4 = y_5 + 1.$$  \hfill (10)

Thus out of the five original unknown hypercharges, only two are left undetermined by the classical constraints. We will take these as $y_1$ and $y_4$.

In the case of the MSM, this is as far as the Lagrangian can take us. However, because we are dealing with a quantum gauge field theory, we can also use the quantum consistency requirement of gauge anomaly cancellation [6]. This requirement may be justified by either demanding that gauge invariance not be broken by quantum effects, or by requiring that the standard proof of renormalizability hold. Gauge anomalies arise from fermionic triangle diagrams with gauge bosons on the external lines. Their amplitudes are proportional to

$$\text{Tr}[T^a \{T^b, T^c\}] \equiv A d^{abc},$$  \hfill (11)

where $A$ is a representation-dependent anomaly-coefficient, and $d^{abc}$ is a set of numbers characteristic of the group. In this equation, $T^a$, $T^b$ and $T^c$ denote the generators in the appropriate representations of the Lie algebra of the gauge group. Theories are anomalous if the anomaly coefficient does not vanish when it is summed over the chiral fermions of the theory (the left- and right-handed fermions enter with a relative minus sign).

There are two anomaly equations in the MSM which are independent of the classical constraints. The first of these arises when two of the external lines in the triangle graph are from $SU(2)_L$ gauge bosons with the third being $U(1)_Y$ [7]. The second triangle anomaly arises when all three external lines are $U(1)_Y$ gauge bosons. These two types of triangle anomalies are denoted respectively as $[SU(2)_L]^2 U(1)_Y$ and $[U(1)_Y]^3$. Evaluating these anomalies by using Eqs.(9-11) leads to the equations

$$y_1 = -\frac{y_4}{3} \quad \text{and} \quad y_4 = -1$$  \hfill (12)

from $[SU(2)_L]^2 U(1)_Y$ and $[U(1)_Y]^3$ anomaly cancellation respectively. Thus for the Toy Model case of the MSM restricted to only one generation we see that the consistency conditions of gauge invariance of the Yukawa Lagrangian and anomaly cancellation uniquely imply that electric charge is quantized. Not only is it quantized, but the quarks and leptons have the correct electric charges.
4. Toy Model #2

To facilitate a greater understanding of why electric charge is quantized in this one generation model, we consider the addition of a gauge singlet fermion (the putative right-handed neutrino) to the above one generation model:

\[ \nu_R \sim (1,1,y_6). \]  

The Yukawa Lagrangian of Eq.(3) is assumed to contain the additional term

\[ \Delta L_{yuk} = \lambda_4 \bar{f}_L \phi^c \nu_R + \text{H.c.} \]  

By following the same steps as in the previous case we might think that electric charge will still be quantized, since we have one extra equation [arising from the classical gauge invariance of Eq.(14)] for the one extra parameter \( y_6 \). However, it turns out that the \( [U(1)_Y]^3 \) gauge anomaly now does not give an independent constraint. Thus in this case we have only five equations for six unknowns, and the hypercharges [and hence electric charges through Eq.(8)] depend on a free continuous parameter, \( \epsilon \), in the following way [5]:

\[ Q_L \sim \left( 3,2,\frac{1}{3} - \frac{\epsilon}{3} \right), \quad u_R \sim \left( 3,1,\frac{4}{3} - \frac{\epsilon}{3} \right), \quad d_R \sim \left( 3,1,-\frac{2}{3} - \frac{\epsilon}{3} \right), \]

\[ f_L \sim (1,2,-1+\epsilon), \quad e_R \sim (1,1,-2+\epsilon), \quad \nu_R \sim (1,1,\epsilon). \]

There is a very easy way to understand the origin of the charge dequantization evident in Eq.(15). If one studies this equation, then it is clear that the non-standard hypercharge (i.e. the \( \epsilon \) bit) is proportional to \( B - L \), where \( B \) and \( L \) are the usual baryon and lepton number global \( U(1) \) symmetries [8]. The combination \( B - L \) is an anomaly free global symmetry which is gaugeable. By “anomaly free”, we mean that \( [U(1)_{B-L}]^3, G_{SM}[U(1)_{B-L}]^2 \) and \( G_{SM}^2U(1)_{B-L} \) anomalies cancel [9]. The symmetry \( U(1)_{B-L} \) is also independent of the symmetries \( G_{SM} \). Thus, a priori, there is nothing to stop us from gauging any combination

\[ Y \equiv Y_{SM} + \epsilon(B - L) \]

instead of just \( Y_{SM} \) (where \( Y_{SM} \) is the standard hypercharge generator). In the example of Toy Model #1 above, where the standard model is restricted to one generation, the Lagrangian contains only one gaugeable \( U(1) \) symmetry and this is
precisely $U(1)_{Y_{SM}}$. Thus the consistency of that theory requires hypercharge (and hence electric charge) to be quantized to their standard values.

We have thus come to quite a general principle:

*If a Lagrangian contains global symmetries which are anomaly-free (and hence gaugeable) and independent of the standard hypercharge $Y$, then that Lagrangian does not yield electric charge quantization* [5, 10, 11].

5. Minimal Standard Model with Three Generations

Having discussed the toy model case of the standard model restricted to one generation, we move on to the more interesting case of the realistic three generation minimal standard model [12, 4]. From the principle annunciated above, it is clear that to analyze charge quantization in this theory we have to find all of its anomaly-free global $U(1)$ symmetries.

The minimal standard model Lagrangian with three generations has four global $U(1)$ symmetries. These are generated by electron-lepton number ($L_e$), muon-lepton number ($L_\mu$), tau-lepton number ($L_\tau$) and baryon number ($B$). To work out which, if any, combinations of these global symmetries are anomaly free, we start by considering the most general linear combination $L'$ where

$$L' = \alpha L_e + \beta L_\mu + \gamma L_\tau + \delta B.$$  

(17)

The anomaly constraints are

\[
[U(1)_{L'}]^3 \Rightarrow \alpha^3 + \beta^3 + \gamma^3 = 0
\]

\[
[SU(2)_L]^2 U(1)_{L'} \Rightarrow 3\delta + \alpha + \beta + \gamma = 0.
\]

(18)

One can check that all other gauge anomaly equations are not independent of the above equations. Note that we have only two equations for the four parameters $\alpha$, $\beta$, $\gamma$ and $\delta$. So we see that there are an infinite number of gaugeable global $U(1)$ symmetries in the standard model. This infinite class can be parameterized as

$$L' = \alpha L_e + \beta L_\mu + (-\alpha^3 - \beta^3)^{1/3} L_\tau + \frac{[-\alpha - \beta - (-\alpha^3 - \beta^3)^{1/3}]}{3} B$$  

(19)

This leads to hypercharge (and hence electric charge) dequantization through the non-standard formula,

$$Y = Y_{SM} + L' \Rightarrow Q = Q_{SM} + L'/2.$$  

(20)
where $L'$ depends on two continuous free parameters ($\alpha$ and $\beta$), and $Q_{SM} \equiv I_3 + Y_{SM}/2$ is standard electric charge. Equation (20) is the analogue in the three generation MSM of the Toy Model #2 result given by Eq.(16). Note that an $\epsilon$ parameter in Eq.(20) (multiplying $L'$) would be redundant given our definition of $L'$ here.

If one believes in quantum gravity, then one may also wish to impose the requirement that the mixed gauge-gravitational anomaly cancel \cite{13, 14}. This requirement is equivalent to

$$Tr \ L' = 0$$

which gives the constraint

$$\alpha + \beta + \gamma = 0.$$  \hspace{1cm} (22)

This equation together with Eq.(18) implies that $\delta = 0$, and either $\alpha$ or $\beta$ or $\gamma$ is equal to zero. In the first case,

$$L' = L_\mu - L_\tau,$$  \hspace{1cm} (23)

while the second case corresponds to

$$L' = L_e - L_\tau,$$  \hspace{1cm} (24)

and the last case corresponds to

$$L' = L_e - L_\mu.$$  \hspace{1cm} (25)

So, with the extra imposition that the mixed gauge-gravitational anomaly cancel, we see that electric charge may be dequantized through \cite{12, 4}

$$Q = I_3 + \frac{Y_{SM} + \epsilon L'}{2} = Q_{SM} + \epsilon L'/2,$$  \hspace{1cm} (26)

where $L' = L_i - L_j, (i, j = e, \mu, \tau; i \neq j)$ [as given in Eqs.(23-25)] and $\epsilon$ is an arbitrary parameter.

So we conclude that the three generation minimal standard model is not a straightforward generalization of the one generation Toy Model #1 as regards electric charge quantization. Electric charge may be dequantized in the three generation case but not in the one generation case. Furthermore, this dequantization can only occur in the ways specified by Eq.(26) [or Eq.(20) if the mixed gauge gravitational anomaly cancellation is not assumed].
6. Extensions to the Standard Model

So we have seen that the minimal standard model with three generations does not have electric charge quantization. Neither does the one generation case when a gauge singlet, $\nu_R$, is added to the fermion spectrum. To obtain electric charge quantization, one can modify the Lagrangian so that the anomaly free global symmetries inherent in these models are absent. For example, the Lagrangian of the one generation model with a gauge singlet neutrino can be modified by adding the Majorana mass term $\mathcal{L}_{Maj}$ where

$$\mathcal{L}_{Maj} = M\bar{\nu}_R(\nu_R)^c.$$  \hfill (27)

Since this term breaks the gaugeable global symmetry $U(1)_{B-L}$ [and it doesn’t break $U(1)_{Y_{SM}}$] the resulting model will necessarily have electric charge quantized correctly \cite{10}. Note that the three generation standard model with Dirac neutrinos has charge dequantization via $B-L$ only, just like the one generation Toy Model, because the family-lepton number symmetries are in general explicitly broken. The three generation case can also be extended by giving Majorana masses to the right-handed neutrinos in order to ensure charge quantization. The above extension of the three generation minimal standard model (featuring three generations of Majorana right-handed neutrinos) is our first example of new physics which is specifically motivated by our approach to electric charge quantization.

In physics there are often many solutions to a given problem, and the charge quantization problem is no exception. In the following we will mention a few other ways in which the three generation minimal standard model can be extended to achieve electric charge quantization.

The simplest and most obvious way to modify the three generation minimal standard model is to add one right-handed neutrino singlet with or without a Majorana mass term. One can show explicitly \cite{15, 16} by writing down the charged current interactions between the charged leptons and neutrinos that the individual lepton numbers–$L_{e,\mu,\tau}$–are no longer in general conserved. This then leaves standard hypercharge as the only remaining anomaly free $U(1)$ symmetry in the model and hence electric charge quantization results. There are many other extensions of the lepton and neutrino sector in particular which lead to electric charge quantization \cite{17}.

Instead of extending the fermion sector of the minimal standard model, we can extend the Higgs sector in a way that breaks those unwanted gaugeable global symmetries. The simplest extension to the Higgs sector which can do this is to add
another Higgs doublet (the two-Higgs doublet model). In this case [16], one finds that the diagonalization of the charged lepton mass matrix no longer simultaneously diagonalizes the corresponding Higgs-fermion interactions. In fact there will now be Higgs-induced flavour changing neutral processes (FCNPs). [It is usual to introduce some discrete symmetries if one wants the physical Higgs particles to be less than about a TeV [18] otherwise these FCNPs will be too large to be compatible with experiment. However, from a charge quantization point of view it is desirable that these discrete symmetries not be imposed, since we require generation mixing interactions to break the gaugeable global symmetries in Eqs.(23-25).]

Another solution, which does not involve mass terms and which is relevant to the case where charge dequantization ensues through gaugeable $B-L$, can be obtained by enlarging the gauge group so that $U(1)_{B-L}$ is no longer anomaly free [19].

The examples above illustrate our contention that the methodology we have presented for the understanding of electric charge quantization serves as a concrete heuristic guide to model building. Indeed, we have quoted some specific extensions of the MSM (e.g. the addition of Majorana right-handed neutrinos) that can be motivated by our charge quantization analysis. Of course it should be emphasised that the only unique, model-independent prediction that one can make from this electric charge quantization argument is that the global symmetries $L_e - L_\mu$, $L_\mu - L_\tau$ and $L_e - L_\tau$ must be broken.

Furthermore, it is important to note that the method encourages a “bottom-up” approach to model building, whereby the successful low-energy theory (the three generation MSM) is altered in small and experimentally testable ways. This is to be contrasted with the traditional approaches to the problem (GUTs, monopoles and higher dimensions) which introduce quite speculative pieces of new physics at high and experimentally inaccessible energies.

The critical reader may at this stage comment that our approach is not in principle completely falsifiable either, because the parameters defining the global symmetry breaking terms can be arbitrarily small and still serve their purpose. For instance, the Majorana mass in Eq.(27) can always be made small enough to evade experimental bounds. Since charge quantization is exact no matter how small the Majorana mass is, we seem to have an unfalsifiable principle. However, small symmetry breaking parameters just swap one problem with another. If the terms are absent and charge is dequantized, the smallness of the parameter $\epsilon$ is a mystery. If the terms are present but the coefficients small, their smallness is the mystery. Thus our philosophy successfully answers this important criticism; the symmetry breaking parameters must have “natural” values.
Before we conclude this section we would like to mention some other issues. Firstly, the result that the standard model restricted to one generation (Toy Model #1) has electric charge quantized correctly is useful for constructing alternatives to the standard model. If the standard model emerges as an effective theory at some scale, then provided all of the non-standard fermions have $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ invariant masses, charge quantization for the standard fermions must follow (provided we restrict ourselves to one generation). This is because fermions with masses that are invariant under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ do not contribute to the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ anomaly equations, so that the Toy Model #1 result that electric charge is quantized correctly will hold. Examples where this result has been used can be found in Ref.[20].

Also, in all the examples discussed so far we have assumed that symmetry breaking is due to elementary scalar bosons developing nonzero VEVs. It may be that some other symmetry breaking scenario occurs in nature. For example, technicolour is one such possibility. One can of course analyse models of this kind using the classical constraints and anomaly conditions as was done for models with elementary scalars [21].

7. Experimental Constraints on Charge Dequantization

In the preceding section we reviewed some small extensions to the standard model which ensure electric charge quantization. It is now up to experiment to discover whether or not any of these extensions is actually realised in nature. An alternative possibility is that electric charge is dequantized. If this is so, then we know that the parameter $\epsilon$ is constrained to be very small. In this section we will discuss the various constraints one can derive on $\epsilon$.

Let us first consider the standard model with massive Dirac neutrinos. As the discussion above concerning Toy Model #2 demonstrated, charge dequantization ensues in this case via

$$Q = Q_{SM} + \epsilon(B - L)/2.$$  \hspace{1cm} (28)

Note that this is true for any number of generations. This result implies that both quarks and leptons have non-standard electric charges.

Since the $B - L$ value of the hydrogen atom is zero, electrical neutrality for hydrogen is maintained even in the face of this form of charge dequantization. However, neutrons are no longer electrically neutral, so atoms in general will now be charged.
Another unusual feature of this scenario is charged neutrinos \([5]\). Experiments on the neutrality of neutrons \([22]\) yield the stringent bound \([5]\)

\[
|\epsilon| < 10^{-21}.
\]  

(29)

Such tiny values for \(\epsilon\) raise a serious theoretical issue if charge is in fact dequantized in nature. Why should nature choose such a small value, when the theoretical consistency of the theory allows any value? This type of naturalness puzzle suggests that it is more likely for nature to have chosen an extension of the standard model which guarantees charge quantization, rather than opting for dequantization. However, we should be circumspect in treating theoretical prejudices such as this as inviolate principles; only experiment can provide the ultimate answer.

What are the bounds on charge dequantization in the three generation minimal standard model \([23]\)? Recall that in this case the electric charge formula is given by Eq.\((26)\), and therefore only leptons have non-standard charges (given the adopted normalization for \(Q\) and the aforementioned insistence on mixed gauge-gravitational anomaly cancellation). Also, charge dequantization is generation-dependent, with one lepton family retaining canonical charges. For this model, the experimental constraints depend critically on whether the first generation leptons are chosen to have standard charges (the \(L_\mu - L_\tau\) case) or not (the \(L_e - L_\mu, \tau\) cases).

Consider first dequantization via either \(L_e - L_\mu\) or \(L_e - L_\tau\). The hydrogen atom, and ordinary matter in general, now acquires a nonzero charge. Direct experimental measurements on atomic neutrality yield the bounds \([23, 24]\)

\[
|\epsilon| < 10^{-17} - 10^{-21}.
\]  

(30)

More stringent, but less rigorous, bounds may be derived by considering the effects of a charged planet earth. The most severe example comes from requiring that the radial electric field near the earth’s surface be less than about 100 V/m, yielding \(|\epsilon| < 10^{-27}\) \([23, 25]\). Unfortunately this bound assumes that the number of protons in the earth equals the number of electrons, so it cannot be taken as rigorous. (One can also derive bounds on dequantization via \(B - L\) from the effects of a charged earth \([5]\). The bounds in this case are more rigorous because the neutron is charged, and so an overall charge for the earth does not arise from a delicate cancellation between proton and electron charges.)

The \(L_\mu - L_\tau\) case is more interesting, because the bounds are many orders of magnitude smaller. Ordinary matter is neutral in this model because first generation quarks and leptons have their standard charges. The first bound we will quote comes
from determining the maximum allowed charge difference between the electron and the muon. This is derived by considering the 1-loop photonic contribution to the anomalous magnetic moments of these two particles. Due to the celebrated precision of the agreement between the standard theoretical predictions for these quantities and their measured values, the constraint $|\epsilon| < 10^{-6}$ must be imposed in order to preserve this success. A better bound is obtained by considering the effects of charged muon-neutrinos in $\nu_\mu$-$e$ scattering experiments. By requiring the new photon exchange contribution to this process to be smaller than the experimental uncertainty on the cross-section, the bound

$$|\epsilon| < 10^{-9}$$

(31)

can be derived [23]. As far as we are aware, this is the best constraint one can derive from terrestrial experiments.

If we allow ourselves to also consider astrophysical and cosmological phenomena, somewhat more stringent bounds can be obtained. Since $\nu_\mu$ and $\nu_\tau$ are charged and massless, massive plasmon states in red giant stars can decay into charged neutrino-antineutrino pairs, which subsequently escape the star and so carry off energy. By requiring that the rate of energy loss per unit volume due to these unorthodox processes not exceed the nuclear energy generation rate per unit volume, the bound $|\epsilon| < 10^{-14}$ is obtained [23, 26]. A cosmological constraint is obtained by noting that photons will acquire a nonzero thermal electric mass from interactions with the charged relic neutrino background plasma [23]. The induced electric mass for photons will result in an effective long-distance violation of Gauss’ Law for classical electric fields. The best experimental test of Gauss’ Law [27] yields the bound $|\epsilon| < 10^{-12}$. Both of the bounds quoted in this paragraph rely on the correctness of standard astrophysics and cosmology for their veracity. Of course, since it is impossible to test astrophysical and cosmological models in nearly as much detail as the standard model of particle physics, we can never be sure that we really understand stellar objects and the universe properly. We would therefore have much more confidence in the rigor of the bounds quoted in the previous paragraph than those in the present paragraph.

8. Concluding Remarks

Before concluding this review, we would like to mention some other work that has been motivated by electric charge quantization: The two simplest $Z'$ models one
can construct are immediately evident if one understands our approach to charge quantization. If no right-handed neutrinos exist, then $Z'$ bosons coupling to family-lepton number differences can exist. The detailed phenomenology of these models has recently been considered in the literature [28]. If, on the other hand, neutrinos have Dirac masses then a $Z'$ boson coupling to $B - L$ can be introduced [29]. Both of these $Z'$ models are simple, because no exotic fermions need to be introduced in order to cancel the gauge anomalies (unless you consider right-handed neutrinos to be exotic).

The discussion of charge dequantization has motivated some work on charge nonconservation [25, 30]. Charge dequantization may occur as a result of charge nonconservation. Usually the constraints on the photon mass are so tight as to rule out the possibility of observing charge dequantization if charge non-conservation is the only source of charge dequantization in the model. However, it has been shown [30] that there are models which have observable charge dequantization arising from electric charge nonconservation.

In conclusion then, we have reviewed recent work on a simple, falsifiable, bottom-up approach to electric charge quantization. We have demonstrated how the consistency of the three generation minimal standard model fails to ensure charge quantization. Several small extensions of the minimal standard model constructed to deliver exact charge quantization were then discussed, and constraints on the unorthodox prospect of charge dequantization were presented.

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[7] Note that the anomaly cancellation equations are the same whether or not spontaneous gauge symmetry breaking occurs. This allows us to work in the weak eigenstate basis here.

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[9] Note that for a $U(1)$ gauge symmetry to be anomaly-free, we must require that the $[U(1)]^3$ anomaly cancel. A confusion in this regard can arise from the fact that an
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