A coupled thermo-mechanical damage model for fired clay bricks based on the unified strength theory

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ABSTRACT

On the rise request for long-lasting materials, clay materials are in between the well-nigh minerals exploited by production and ecological fields in the making of fired bricks. Clay incessantly expounded to high temperature reacts differently at ambient temperature which critically touches its longevity.

In present study, a coupled thermo-mechanical damage model of clay is established. In this model, the Unified Strength Theory (UST) criterion is used as the failure criterion based on the Weibull distribution and the continuous damage theory. The proposed model is validated by uniaxial compression experiment of high-temperature clay. The variation of the two distribution factors (m and W0) in the combined TM damage relation with temperature is analysed. The results show that the damage evolvement speed of the clay shows a curving form getting closed to one as the temperature rises, indicating that the temperature can delay the development of cumulative damage. The damage fundamental modelling discussed is in accord with the testings curves at the various phases of yielding and pre-apex force. This study leads to an enhanced understanding of high temperature clay mechanics and affords the fundament to heighten clay bricks resourcefulness lastingness.

1. Introduction

Clay results from specific geologic conditions such as soil horizons, continental and marine sediments, geothermal fields, volcanic deposits and weathering rock formations. These processes involve physical disaggregation and chemical decomposition given various properties to the clay. This leads to different types of clay that affect their use in natural and industrial applications. Therefore, clay being very common, abundant and inexpensive is among the raw materials used for building and maintaining infrastructure.

The damaging and collapse progress of clay accustomed in constructions need to be considered through the assess of its physical and mechanical properties under high-temperature as suggested by Xiaoli et al. (2017) for rocks.

Several authors suggested that the random defects in material are manifestations of material damage and consistent with its natural characteristics (Cao and Li, 2008; Wang et al., 2019; Weiji et al., 2020). Many researchers have established a statistical damage constitutive model of materials such as rock based on continuum mechanics, damage mechanics and material mechanics (Krajcinovic and Silva 1982, Zhi-Liang et al., 2007) with the assumption that the rock is made up of micro-units; each microunit contains abundant micro-damage, and the microunit is seen as a point particle in continuum mechanics (Wen-gui et al., 1998; Cao et al., 2007; Li et al., 2012; Weiji et al., 2020).

Thus, a damage variable built on statistical damage fundamental relationship of rock draws considerable concern to indicate the damage stage. For that reason, several distributions functions, in particular, the Weibull distribution, the lognormal distribution, the power distribution and the maximum entropy distribution, are most often exploited as statistical functions which depend on the strength of the microunit of rock to establish a damage constitutive model (Deng and Gu 2011; Lin et al., 2019; Yu, 2004; Ying et al., 2019).

Because of these numerous utilizations (Xiaoli et al., 2017, Zaobao et al., 2018; Thomas et al. 2018, 2019), the determination of the mechanical performance of rock becomes a necessity whereby significantly researches outline the failure stages consisting of extension and joining of

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sparked microfractures (damage) and frictional sliding of fracture planes (plasticity) (Blair and Cook 1992; Liu et al., 2004; Zhi-Liang et al., 2007; Thomas et al., 2019).

The decisive matter in rock mechanics is the query of thermic damaging fundamental modelling giving way to the ruin and collapse evolution and stress-strain relation of rock under thermal load (Xiaoli et al., 2017).

Statistics physics is recognized as the joint between continuum mechanics, damage mechanics and material mechanics, playing a decisive role in developing rock constitutive laws (Zhi-Liang et al., 2007).

In a sense, many statistical fundamental laws proposed by Xiaoli et al. (2017), Xu and Karakus (2018), Lin et al. (2019) for rock material highlight the distribution parameter of the constituent robustness primarily introduced by Xiaoli et al. (2017) in the Hoek-Brown strength criterion. In the same time, the effect of intermediate principal stress and hydrostatic pressure appropriate for its uncomplicated configuration and extensively practice in geotechnical engineering are incorporated thru the Drucker–Prager criterion by Lin et al. (2019) and Xu and Karakus (2018).

As Xiaoli et al. (2017) indicated, the Drucker-Prager (Cao et al. 2005, 2011; Wei, et al., 2020), Mohr –Coulomb (Cao et al., 2008; Li et al., 2012; Wang et al., 2013; Wei, et al., 2020) and the Hoek–Brown (H–B) strength criterion and its modified configurations strength criterion have been considerably exploited to infer constituting laws of rock damaging, with acceptable results (Ma et al., 2013; Priest 2012; Zhang et al., 2013).

A twin-shear unified yield criterion for materials was formulated on the basis of the idea of twin-shear stress by Mahong Yu. The twin-shear unified yield criterion describes prior criteria, and considers the Tresca's yield criterion and the Mohr-Coulomb's criterion as its only special cases precisely (Yu and He, 1992; 2004). The twin-shear unified criterion has been studied in some engineering applications and adopted in many

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**Table 1. Chemical and mineral compositions of clay.**

| Element | SiO₂ | Al₂O₃ | Fe₂O₃ | TiO₂ | MnO | MgO | CaO | Na₂O | K₂O | P₂O₅ | LOI | Somme | SiO₂/Al₂O₃ |
|---------|------|-------|-------|------|-----|-----|-----|------|-----|------|-----|--------|------------|
| wt%     | 61.14| 31.09 | 0.74  | 1.25 | 0.01| 0.00| 0.09| 0.00 | 0.07| 0.06 | 5.36| 99.81  | 1.97       |

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**Figure 1.** Experimental plan flow chart.

**Figure 2.** Complete Stress – strain curves of clay F0 under different heating temperatures.
Figure 3. TM Damage values for different values of the effect of the intermediate principal shear stress a) $b = -1/3$, b) $b = 5/4$, c) $b = 7/16$, d) $b = 5/16$, e) $b = 1/4$, f) $b = 3/8$.

Figure 4. Normalized W0 with temperature a) $0 \leq \theta \leq \theta_\alpha$, b) $\theta_\alpha \leq \theta \leq 60^\circ$.
textbooks of ‘the mechanics of materials’ in China (Xiaorong and Xiao-
mei, 2011).
Hitherto, no mathematical fundamental equation of clay thermal
damage depending on the unified strength theory criterion has been
established. As a result, the thermo-mechanical (TM) coupling damage
constitutive model of clay developed from the UST criterion established,
and the proposed model validated by experimental data are presented in
this paper.

2. Experimental investigation

2.1. Clay samples and specimens preparation

The clay used to perform experimental studies comes from Nsimalen a
locality (F0) around Yaoundé (Cameroon). It was chosen because of its
subject of valorization applications by the Local Materials Promotion
Authority (MIPROMALO), a public research institute dealing with the
promotion of the use of local materials in Cameroon. From the Casa-
grande plasticity chart (Casagrande, 1948), F0 has an acceptable plas-
ticity. Table 1 shows the chemical and mineralogical compositions of the
clay obtained in a previous study.

The test briquettes were made as typical cubes of $4 \times 4 \times 4$ cm ac-
cording to ASTM C67 standard, using about 100 g of clay mixed with
convenient amount of water to achieve good workability. The obtained
blocks were first dried at ambient temperature for 72 h and placed in an
oven at 105 °C for 24 h to complete removal of hydrate, before being
fired in a muffle furnace up to 1050 °C. The heating rate was of 3 °C/min
during the first step up to 200 °C, then 4 °C/min up to 1050 °C, to ensure
proper heat diffusion within the material.

2.2. Temperature exposure procedure

The thermal exposure was adapted from Xiao-Ting et al., 2009. The
following hypotheses were made for the variation of thermal exposure
into five phases:

- As the fired brick are left under atmosphere, rehydration of the
  sample is possible and what effect on the mechanical properties when
  this water is released? That is why 95 °C is considered
- The resurgence of OH links which forms new phases because of
  exposure to humidity and air and have an effect on the structures (200
  °C)
- Cases of fires, and phases reorganization for mullite and other com-
  ponents (550, 700 and 950 °C)

Therefore, five temperatures were chosen: 95, 200, 550, 700 and 950
°C. The specimens placed in the furnace are heated to the end tempera-
ture at a rate of 4 °C/min and left for a 1 h – stay to establish a stable
temperature profile in the samples. The final cooking temperature is
chosen in accordance to MIPROMALO (Xiaoyong and Fanjie, 2011)
prescription for standard fired bricks using Nsimalen clay.

As the furnace is stopped, they are naturally cooled to room tem-
perature and equilibrated to the ambient temperature then removed and
stored at ambient prior to the compressive test. The reference sample is
an unfired specimen dried for 24 h in an oven at 105 °C to constant
weight and kept at the laboratory temperature (29 ± 2 °C).

For each temperature, a series of five specimens were used and the
mean value from five tests was considered. Figure 1 summarizes the
experimental investigation.

2.3. Compressive test

The tests were carried out on a universal testing machine of 2000kN
to ASTM C67-80a standard. The tests were performed at a constant
displacement rate of 5 mm/min to gain the complete stress – strain
curves. A uniformly distributed and constantly increasing load was
applied up to specimen failure, on lateral faces described by the length
and the thickness of the brick (4 $\times$ 4 $\times$ 4 cm).

In Figure 2, the compressive stress – strain curves at different tem-
peratures are given. From the various stress – strain curves formulations,
four domains are observable:

- a linear behaviour for all the temperature in the range of strain
deformation lower than 0.02;
- a brittle behaviour at the pre peak strength for a temperature range of
  29 $\leq$ $T$ $\leq$ 550 °C;
- a ductile behaviour after the peak for 700 and 950 °C;
- a non linear deformation for 700 and 950 °C.
3. Obtaining a combined thermal-mechanical damage advance relationship

In this section, the different components of the total damage are launched and considering the Unified Strength Theory (UST) failure criterion, a combined thermo-mechanical damage relation is achieved.

3.1. Thermic damaging ($\Phi_T$)

Higher temperatures enduce thermal stress in rocks thereby increasing micro-fractures. This steer to mechanical characteristic deterioration within the rocks (Xiaoli et al., 2017).

The increase of temperature gives way to a large decrease of the elastic modulus thus making damage to the rock. Therefore, the elastic modulus is chosen to define the damage variable, which is used to describe the temperature effect on the mechanical properties of the clay material (Liu and Xu 2000). Using the concept of thermal damage in rocks, we have the equations below (Hueckel et al., 1994; Martin and Chandler, 1994; Liu and Xu 2000; Heap and Faulkner 2008; Mao et al., 2009; Gautam et al., 2015; Shi and Xu 2015; Xiaoli et al., 2017; Xu and Karakus (2018); Xiongbin and Haibo 2019):

$$\Phi_T = \frac{1}{C_0} \frac{E}{E_0}$$

$$0 \leq \Phi_T \leq 1$$

where:

- $\Phi_T$ represents the thermic damaging of clay sample after exposure to elevated temperatures,

Figure 6. Test data and fitted models curves at various temperatures for $b = 0$. a) 29 °C, b) 95 °C, c) 200 °C, d) 550 °C, e) 700 °C, f) 950 °C.
- $E_T$ represents the elastic modulus of clay sample after exposure to high temperature,
- $E_0$ represents for the elastic modulus of clay sample at ambient temperature.

Unfortunately, the relation is inappropriate for clay material subjected to elevated temperatures due to the behavior mentioned by S. Russo (2012), Bidoung et al., 2016. Therefore a new relation is proposed which complies Eq. (1)

$$\Phi_T = \frac{1}{C_0} \frac{E_T}{E_0}$$

with

$$0 \leq \mu \leq 1$$

(4)

3.2. Mechanical damaging ($\Phi_M$)

Krajcinovic and Silva (1982) proposed a simple statistical model for uniaxial response of a gradually damaging structure which emphasized on the formulation of a simplest possible model allowing the analytical prediction of the general experimentally detected trends of the material behavior. He mentioned that the strength distribution commonly identified with the structural failure applications is the Weibull distribution (Weibull, 1939).

Many researchers (Zhi-Liang et al., 2007; Cao et al., 2007; Cao et al., 2010; Xiang et al., 2012; Shi and Xu 2015; Tao et al., 2017; Xu and Karakus (2018); Ying et al., 2019; Xiongbin and Haibo, 2019; Zhang and

![Figure 7. Test data and fitted models curves at various temperatures for b = 1/5. a) 29 °C, b) 95 °C, c) 200 °C, d) 550 °C, e) 700 °C, f) 950 °C.](image-url)
Xuexue 2019; Peiwu et al., 2019; Weiji et al., 2020) have used the Weibull distribution of the rupture strengths to establish the damage law. Exploitation of Weibull distribution by Xu and Karakus (2018) in Eq. (5) below:

$$h(W) = \frac{m}{W_0} \left( \frac{W}{W_0} \right)^{m-1} \exp \left[ - \left( \frac{W}{W_0} \right)^m \right]$$  \hspace{1cm} (5)

where:
- $h(W)$ is the submicroscopic constituent force of the distribution function.
- $W$ is a Weibull variable that designates the force level of the submicroscopic constituent.
- $W_0$ the scale factor related to the submicroscopic constituent force and $m$ (the shape factor defining the level of uniformity) are the statistical variable used in the Weibull distribution.

Also the mechanical damaging $\Phi_M$ (Xu and Karakus, 2018) can be written as:

$$\Phi_M = \int_0^W h(W)dW = 1 - \exp \left[ - \left( \frac{W}{W_0} \right)^m \right]$$ \hspace{1cm} (6)

In line with Lemaitre’s principle (1988), the clay constitutive equation is given as:
σ_{ij}' = \sigma_i / (1 - \Phi_M)

(7)

where \sigma_i is the initial stress, \sigma_i' is the final stress, and \Phi_M is the mechanical damage variable assessing the extent of material degradation such that,

\[
\begin{align*}
\Phi_M &= 0, \text{ undestructive condition} \\
0 &\leq \Phi_M \leq 1, \text{ various stages of degradation} \\
\Phi_M &= 1, \text{ complete degradation}
\end{align*}
\]

Due to residuary strength integration in the inception of rock damage constitutive equation (Xu and Karakus, 2018 and Lin et al., 2019), an adjustment factor \( \zeta \), fulfilling \( 0 \leq \zeta \leq 1 \) is provided Gao et al. (2018). Eq. (7) turns to:

\[
\sigma_{ij}' = \frac{\sigma_i}{1 - \zeta \Phi_M}
\]

(8)

According to the Hooke's law, the mechanical strain can be written as:

\[
\varepsilon_{ij} = \varepsilon^0 = \frac{\sigma_i}{E_0} = \frac{\sigma_i}{E_0[1 - \zeta \Phi_M]}
\]

(9)

Figure 9. Test data and fitted models curves at various temperatures for \( b = -1/3 \). a) 29 °C, b) 95 °C, c) 200 °C, d) 550 °C, e) 700 °C, f) 950 °C.
Combining Eq. (8) and Eq. (9) we have:

\[ \sigma_{ij} = (1 - \zeta \Phi_M) E_0 (\epsilon_{ij} - \epsilon^h) \]  

(10)

where:
- \( \Phi \) is the overall damage variable.
- \( \Phi_T \) is the thermic damaging variable.

Eq. (6) and Eq. (10) give:

\[ \sigma_{ij} = E_0 (\epsilon_{ij} - \epsilon^h) \left( 1 - \zeta + \zeta \exp \left[ - \left( \frac{W}{W_0} \right)^m \right] \right) \]  

(11)

3.3. Combined thermo-mechanical damage relationship

The combined thermomechanical damage is given as follow, using the equivalent strain principle and the overall damaging variable,

\[ \Phi = \Phi_T + \Phi_M - \Phi_T \Phi_M \]  

(12)

Substituting \( \Phi_M \) in Eq. (11) by the overall damage variable in Eq. (12), the damage fundamental modelling of clay after thermal exposure is:

\[ \sigma_{ij} = E_0 (\epsilon_{ij} - \epsilon^h) (1 - \Phi_T)(1 - \Phi_M) \]  

(13)

Combining Eq. (3), Eq. (6) and Eq. (13), the damage fundamental modelling after exposure to high temperature is:

\[ \sigma_{ij} = \mu E_T (\epsilon_{ij} - \epsilon^h) \left( 1 - \zeta + \zeta \exp \left[ - \left( \frac{W}{W_0} \right)^m \right] \right) \]  

(14)
where: \( \sigma_i \) is the uniaxial compressive strength at ambient temperature, \( I_1 \) is the first invariant of the stress tensor, \( J_2 \) is the second invariant of the stress deviator, \( \theta \) is the stress angle corresponding to the twin-shear parameter of shear stress \( (Yu \, 2004) \), and \( m \) and \( b \) are the ratio of tensile stress on compressive stress and the yield criterion parameter that represents the effect of the intermediate principal shear stress on the yield materials respectively.

Eq. (15) into Eq. (6), the mechanical damaging \( \Phi_M \) is given as:

\[
\Phi_M = \begin{cases} 
1 - \exp \left( \frac{\alpha_0 + b_0 \sqrt{J_2 \sin \theta + c_0 \sqrt{J_2 \cos \theta - \sigma_i}}}{W_0} \right) \exp (\theta - \sigma_i), & 0 \leq \theta < \theta_b \\
1 - \exp \left( \frac{\alpha_0 + b_0 \sqrt{J_2 \sin \theta + c_0 \sqrt{J_2 \cos \theta - \sigma_i}}}{W_0} \right), & \theta_b \leq \theta \leq 60^\circ 
\end{cases}
\]

Then, the overall combined thermomechanical damage (\( \Phi \)) can be outlined as:

\[
\Phi = \begin{cases} 
1 - \frac{E_T}{E_0} \exp \left( \frac{\alpha_0 + b_0 \sqrt{J_2 \sin \theta + c_0 \sqrt{J_2 \cos \theta - \sigma_i}}}{W_0} \right), & 0 \leq \theta < \theta_b \\
1 - \frac{E_T}{E_0} \exp \left( \frac{\alpha_0 + b_0 \sqrt{J_2 \sin \theta + c_0 \sqrt{J_2 \cos \theta - \sigma_i}}}{W_0} \right), & \theta_b \leq \theta \leq 60^\circ 
\end{cases}
\]

Differentiating Eq. (11) we have:

\[
\frac{\partial \sigma_{i}}{\partial \varepsilon_{i}} = \frac{\partial}{\partial \varepsilon_{i}} \ln \left( \frac{W}{W_0} \right)^{-m} E_T \left( \varepsilon_i - \varepsilon^0 \right)^2 E_T \left( \varepsilon_i - \varepsilon^0 \right)^{3} \left( \frac{W}{W_0} \right)^{-m-1} \left( \frac{\partial W}{\partial \varepsilon_{i}} \right) = 0
\]

Re-arranging Eq. (18) gives:

\[
1 + (m-1) \left( \varepsilon_i - \varepsilon^0 \right) \frac{W}{W_0}^{m-1} \left( \frac{\partial W}{\partial \varepsilon_{i}} \right) = 0
\]

Referring to stress – strain characteristics curves for clay, Eq. (11) satisfies the subsequent spatial requirement of the apex point of the curve:

\[
\begin{align*}
\frac{d\varepsilon_{i}}{d\varepsilon_{i}} &= 0 \\
\varepsilon_{i} &= \varepsilon_{p} \\
\sigma_{i} &= \sigma_{p}
\end{align*}
\]

Therefore:

\[
\sigma_{p} = E_0 \left( \varepsilon_{p} - \varepsilon^0 \right) \left( 1 - \exp \left( - \frac{W}{W_0} \right)^{m} \right)
\]

Therefore:

\[
\mu W_{m-1}^{m-1} \left( \frac{\partial W}{\partial \varepsilon_{i}} \right) = \frac{E_0 \left( \varepsilon_{p} - \varepsilon^0 \right)}{\left( \varepsilon_{p} - \varepsilon^0 \right)} \left( \frac{\sigma_{p}}{E_0 \left( \varepsilon_{p} - \varepsilon^0 \right) - 1 + \varepsilon^0} \right) \frac{W}{W_0}
\]

\( F_p \) being the force of the submicroscopic constituent when the stress reaches the apex value.

Writing the two invariants with effective stresses gives:

\[
\begin{align*}
I_1 &= \sigma_1 + \sigma_2 + \sigma_3 \\
J_2 &= \frac{1}{6} \left( (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right)
\end{align*}
\]

For uniaxial compressive test, Eq. (22) becomes:

\[
\begin{align*}
I_1 &= \sigma_1 \\
J_2 &= \frac{1}{3} (\sigma_1)^2
\end{align*}
\]

Differentiating the failure criterion enables us to show that:

\[
\frac{\partial W}{\partial \varepsilon_i} = \frac{\alpha_0 + b_0 \sqrt{J_2 \sin \theta + c_0 \sqrt{J_2 \cos \theta - \sigma_i}}}{W_0} \left( \frac{E_T}{E_0} \right)^2 \left( \frac{\sigma_{p}}{E_0 \left( \varepsilon_{p} - \varepsilon^0 \right)} - 1 + \varepsilon^0 \right) \left( \frac{W}{W_0} \right)^{-m-1} \left( \frac{\partial W}{\partial \varepsilon_{i}} \right) = 0
\]

Solving Eq. (21), \( m \) and \( W_0 \) can be calculated as:

\[
\begin{align*}
\frac{m = \frac{W_0 \sigma_{p}}{E_0 \left( \varepsilon_{p} - \varepsilon^0 \right)}}{1 + \varepsilon^0} \frac{\left( \frac{\sigma_{p}}{E_0 \left( \varepsilon_{p} - \varepsilon^0 \right)} - 1 + \varepsilon^0 \right)}{\left( \frac{W}{W_0} \right)^{m-1}} \left( \frac{\partial W}{\partial \varepsilon_{i}} \right) = 0
\end{align*}
\]

\[
\begin{align*}
W_0 &= \frac{W_{p} \sigma_{p}}{E_0 \left( \varepsilon_{p} - \varepsilon^0 \right)} \left( \frac{\sigma_{p}}{E_0 \left( \varepsilon_{p} - \varepsilon^0 \right)} - 1 + \varepsilon^0 \right) \left( \frac{W}{W_0} \right)^{-m-1} \left( \frac{\partial W}{\partial \varepsilon_{i}} \right) = 0
\end{align*}
\]

Combining Eq. (14) and Eq. (25), the constitutive model of clay under thermal loading conditions is improved.

4. Damage evolution investigation

In this section, the practicability of the damage evolution equation and damage fundamental equation will be looked into. This will be done by comparing theoretical results with test results related to the uniaxial compression test of the clay by varying temperatures and the physical model parameters.

4.1. Damage developing with temperature

The curves of the overall combined thermomechanical damage (\( \Phi \)) are shown (Figure 3). In each curve we have presented the two cases related to \( \theta \) the stress angle corresponding to the twin-shear parameter of shear stress: \( 0 \leq \theta \leq \theta_b \) and \( \theta_b \leq \theta \leq 60^\circ \).

The rock damaging progress may be partitioned in three phases (Peiwu et al., 2019; Xiongbin and Haibo, 2019; Alfarah et al., 2017; Xu and Karakus, 2018; Cao et al., 2007). These phases encompass: (1) the inception phase, (2) the expedient expansion phase, (3) the ultimate consolidation phase. Unfortunately, the overall combined thermomechanical damage of clay does not show the same damage process at different temperatures mostly at the opening phase.
Different values of \( \beta \), the effect of the intermediate principal shear stress on the yield materials, have been examined:

\[
b = \left\{ \begin{array}{l} -1/3, 0, 1/3, 5/3, 7/3, 1, 5/3 \\
2/3, 4/3, 8/3, 10/3, 2, 5/3 \\
\end{array} \right. \]

We noticed three groups of trends of the curves change with the temperature with the shape and behaviors are similar. At ambient temperature, the overall combined thermomechanical damage curve fluctuates with the temperature. This indicates instability of the clay rendering it very brittle. Therefore, the plasticity will also fluctuate. The parameter \( F_0 \) shows a quasi-logistic increase with the rise of temperature while the parameter \( n \) abruptly decreases and then asymptotically approaches zero with the rise of the temperature. The effect of the intermediate principal shear stress on the yield materials and the twin-shear parameter of shear stress have no effect on both parameters.

The following relationships have been suggested:

\[
\begin{align*}
F_0 &= \frac{29.788}{1 + \exp \left( -\left( -19, 26 + 30, 76 \frac{T}{1000} \right) \right)}, \quad R^2 = 0.9385; \quad RMSE = 4.1869; \quad 0 \leq \theta \leq \theta_0 \\
F_0 &= \frac{29.796}{1 + \exp \left( -\left( -19, 29 + 30, 82 \frac{T}{1000} \right) \right)}, \quad R^2 = 0.9883; \quad RMSE = 2.064; \quad \theta_b \leq \theta \leq 60^\circ
\end{align*}
\]

For the parameter \( n \)

\[
\begin{align*}
n &= -0.064 tan \left( -2, 680 \frac{T}{1000} + 1, 7125 \right); \quad R^2 = 0.9044; \quad RMSE = 0.2196; \quad 0 \leq \theta \leq \theta_b \\
n &= -0.00122 tan \left( -0.055 \frac{T}{1000} + 1, 5736 \right); \quad R^2 = 0.8830; \quad RMSE = 0.234; \quad \theta_b \leq \theta \leq 60^\circ
\end{align*}
\]

asymptotically approaches one.

- \( \beta = -1/3, 7/16, 5/4 \)

In this group, for the following temperature 95, 200 and 550 °C, the overall combined thermomechanical damage curve has a constant value of 0.925; 0.9 and less than 0.95 respectively then suddenly increases approaching the value \( \Phi = 1 \) with temperature although, it is observed that the overall combined thermomechanical damage curve of 200 °C has the lowest value. For the temperatures 700 and 950 °C, an abrupt increase is observed until \( \Phi = 1 \) followed by an abrupt decrease closed to 0.95. The different cases of the stress angle corresponding to the twin-shear parameter of shear stress match in at the various temperatures suggesting that only case can considered for further analysis.

- \( \beta = 5/16 \)

The difference with the earlier case lies in the increase of the overall combined thermomechanical damage curve at 550 °C which goes beyond 0.95 and shows a slight decrease followed by an increase.

- \( \beta = 1/4, 3/8 \)

In this case, the overall combined thermomechanical damage curve at 550 °C suddenly increases until a peak followed by a downward sloping line until 0.85 and 0.9 respectively. A particular attention should be pay to the last case and at temperature 550 °C.

4.2. Physical significance of modeling factors

Parameters \( n \) and \( F_0 \) are the brittleness and the macroscopic average force of rock material respectively (Ying et al., 2019). Figures 4 and 5 show the variation of the two distribution parameters with temperature.

It can be seen that \( n \) which is comprised between 0.75 and 1, fluctuates with the temperature. This indicates instability of the clay material. Therefore, the plasticity will also fluctuate. The parameter \( F_0 \) shows a quasi-logistic increase with the rise of temperature while the parameter \( n \) abruptly decreases and then asymptotically approaches zero with the rise of the temperature. The effect of the intermediate principal shear stress on the yield materials and the twin-shear parameter of shear stress have no effect on both parameters.

5. Modeling corroboration and analysis

The curves fitting efficiency is reviewed by means of the root mean square error (RMSE) and the coefficient of determination \( R^2 \). \( R^2 \) values combining two variables while RMSE obtains for each sample the standard deviation error between regressed and actual values. Ideally, \( R^2 = 1 \) and \( RMSE = 0 \) (Ying et al., 2019).

The following values of the yield criterion parameter \( \beta \) which represents the effect of the intermediate principal shear stress on the yield of materials have been used:

\[
b = \left\{ \begin{array}{l} -1, 0, 1/3, 5/3, 7/3, 1, 5/3 \\
2, 4/3, 8/3, 10/3, 2, 5/3 \\
\end{array} \right. \]

Because the stress-strain and total coupled TM damage (D) have similar shapes at different values, only the curves of stress-strain, total coupled TM damage (D) at the following values \( b = \left\{ \begin{array}{l} -1, 0, 1/3, 5/3, 7/3, 1, 5/3 \\
2, 4/3, 8/3, 10/3, 2, 5/3 \\
\end{array} \right. \}

are shown in Figures 6, 7, 8, 9, and 10.

Figure 6, Figure 7 and Figure 8 present the experiments and model fitting for common failure criteria:
- \( \beta = 0 \) Mohr – Coulomb failure criterion
- \( \beta = 1/4 \) Approximation of the extended von Mises yields criterion, or Drucker-Prager criterion
- \( \beta = 1 \) Twin shear yield criterion

It can be observed in Figure 6, Figure 7 and Figure 8 that there is no good agreement between the experiments and the model. That observation is in line with what Yu (2004) mentioned for geomaterials and for Drucker – Prager criterion which cannot match with the practices.

Figures 9 and 10 show a good agreement between the experiments and the model. That complies what Yu (2004) observations for geomaterials for convex case (Figure 10) where \( 1/3 < \beta \leq 1 \) and non-convex case (Figure 9). It is also observed that the piecewise stress angle matches except for the following cases \( b = 1/4 \) and \( b = 1 \) at 700 and 950 °C.
suggested as mentioned earlier the possibility of using one case instead of both.

6. Conclusion

In this study, a combined TM damage modeling for clay is presented on the basis of the statistical damage theory. It investigates the damage characteristics and the stress – strain behaviors of clay under the thermal condition. The UST criterion is introduced as a random variable of the Weibull distribution. The thermal damage caused by temperature variations is defined according to the evolution of elastic modulus. Then, the mechanical damage caused by loading is derived on the basis of Weibull distribution which considers the residual strength of clay in post-peak stage. As conclusions:

- The correlation between mechanical factors and temperature are ascertained. The physical sense of the factors in the damage fundamental modeling is screened underpinned the previous matching.
- The relationship between the parameters of theoretical model and the characteristic values of clay failure is determined; the model parameters have clearly physical meaning, which enhanced the adaptability of the model.
- Built on the empirical data, the combined TM damage modeling is corroborated. The fulfillment is righteousness with $R^2 > 0.90$ arguing that the modeling is suitable to depict the damage evolvement of clay at assorted temperatures.
- The submitted modeling is foreseen to other geomaterials with thermal damage ascendency. Therefore, in future studies, the plasticity should be taken into account and used to capture TM behavior of clays and other geomaterials.

Declarations

Author contribution statement

Mpunou L.A.: Conceived and designed the experiments; Wrote the paper.
Bidoung J.C.: Analyzed and interpreted the data.
Sonia Metekong J.V.: Performed the experiments.
Meva’a J.R.L.: Contributed reagents, materials, analysis tools or data.

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Data availability statement

The authors do not have permission to share data.

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The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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