Algorithms of Determination of the Boundaries Shaded Seabottom Areas

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Abstract. The problem of determining the seabottom relief using the side scan sonar is considered in a fluctuating ocean. Effective algorithms have been developed for the determining of the shaded seabottom areas and taking into account different movement properties of the sonar. To show the efficiency of algorithms, a numerical experiment was carried out.

1. Introduction

The problem of mapping the ocean floor using the side-scan sonars is topical and quite promising in nowadays (see e.g. [1-4]). The functioning of the sonar is based on periodic emission of the pulsed sound signals and detection of the echo signals reflected from the distant segments of the seabottom. An acoustic image is formed on both sides of the vehicle when the sonar's antenna is moving.

The study of the problems of mapping the seabed, the main purpose of which is to determine the deviation of the height of the bottom surface from a certain predetermined level, is devoted a number of works [5-9]. There are large deviations of the seabed relief or volume scattering in the ocean [10]. Usually, the volume scattering influence on the total signal in the near-bottom layer or at long distances due to increasing of diffusers number, covered by the receive antenna pattern. It should be noted that the problem was considered only under the condition of visibility of each point of the seabed from the antenna carrier.

However, zones of invisible areas of the bottom surface arise when simulating the acoustic sounding of the seabed by the side-scan sonar. In this paper, algorithm for determining invisible seabed areas in the case of a stationary source and a receiver are described. Also, this algorithm is generalized for a point isotropic source moving along the $r_2$ axis and emitting impulse messages at regular intervals.

2. Static observer

2.1. Problem Formulation

A new problem formulation is studied for a static source and a receiver. Therefore the problem for determining of shaded areas on the seabottom could be considered in $\mathbb{R}^2$ space. Echo signals are propagated in the medium.
which represents the top semi-space, bounded by the surface \( \gamma' = \partial G' = \{ y \in \mathbb{R}^2: y_2 = -l + u(y_1) \} \), interpreted as an ocean seabed. The function \( u \in C^4(-\infty, +\infty) \) describes some deviations of a bottom surface from the average level \(-l\). The source and the receiver (observer) are located in the origin. Geometry of the problem is presented in Fig. 1.

![Figure 1. Geometry of the problem in \( \mathbb{R}^2 \).](image)

Denote \( \hat{\gamma}' = \{ y_i \in \mathbb{R}^2: y_i = (y_i, -l + u(y_i)) \} \), where \( i \in \mathbb{N}_0, y_i = ih \). \( h \) denotes the sampling step. Therefore, vector \( k = (-y_i, l - u(y_i)) \) exists for any point \( y \in \hat{\gamma}' \) and connects the point at the seabottom with the position of the antenna carrier. Let \( n = (u'(y_i), -1) \) define a vector of an external normal of the surface \( \gamma' \). Hence, an inner product is \( (n \cdot k) = -y_i u'(y_i) - l + u(y_i) \). The solution of the problem authors search in the following form

\[
\hat{u}(y_i) = \begin{cases} 
  u(y_i), & \text{if } y_i \text{ is visible by the observer}, \\
  0, & \text{otherwise}.
\end{cases}
\]

It is worth to note that in section 1, \( n \) and \( k \) were taken as unit vectors, but here and further authors consider only the sign of inner product \( (n \cdot k) \) which is independent on vector length.

2.2. Algorithm #1

**Definition 1** (Invisibility criteria). The point \( y_i \) is invisible by observer if \( (n \cdot k) > 0 \).

Each point \( y_i \) is verified by the invisibility criteria. If criteria are not satisfied, we let then \( \hat{u}(y_i) = u(y_i) \). Otherwise, we construct a beam in the direction \( k \) from the point \( y_{i,0} \) in which invisibility criteria is valid. All points \( y_i \) are invisible while the beam does not intersect \( \gamma' \). The construction of the algorithm is presented below.

1. \( y_{i,0} = 0 \)
2. FOR \( i = 0, 1, \ldots, N \) DO
   (a) \( \hat{u}(y_i) = u(y_i) \)
   (b) \( (n \cdot k) = -y_i u'(y_i) - l + u(y_i) \)
   (c) IF \( (n \cdot k) > 0 \) THEN
      (i) \( y_{i,0} = y_i \)
      (ii) REPEAT
         (A) \( u(y_i) = 0 \)
         (B) \( \text{INC}(i) \)
         (C) \( y_i = ih \)
For numerical experiment, authors set $l = 5$. The function describing a seabottom has the following form

$$u(y) = e^{\left(\frac{(y-100)^2}{10^2}\right)} - e^{\left(\frac{(y-200)^2}{10^2}\right)}.$$  

(1)

For the calculation of a derivative $u'(y) = \left(\frac{u(y_1 + h) - u(y_1)}{h}\right)$ authors use the finite differential approximation.

![Graph](image)

**Figure 2.** The algorithm #1. Functions: (a) $\tilde{u}$; (b) $|\tilde{u} - u|$.

The results of the algorithm #1 are shown in Fig. 2. The right side of the first pick and most inner points of the second cavity are invisible by the observer. Thus, a series of numerical experiments showed efficiency of the algorithm #1.

3. Semi-static observer

3.1. **Problem Formulation**

After two algorithms for simplified model, we return to the $\mathbb{R}^3$ problem. Further, we consider the case of a narrow directivity pattern of the receiving antenna [10]. Actually, in this case a sonar receives a
signal on the plane which orthogonal to the motion of vehicle only, i.e. for each sounding interval $j$ the problem is transformed to $\mathbb{R}^2$ with constant $r_2 = Vt_j$.

Denote the discrete set of the seabottom by $\mathcal{Y}' = \{ y_{i,j} \in \mathbb{R}^2 : y_{i,j} = (y_i, Vt_j - l + u(y_i, Vt_j)) \}$, where $i = 0, N$, $j = 0, M$. Hence, for each point $y_{i,j} \in \mathcal{Y}'$ we can construct a vector $k = (-y_i, 0, 0 - u(y_i, Vt_j))$, which connects point $y_{i,j}$ on the seabottom to the observer located in $z = (0, Vt_j, 0)$.

Let $\mathbf{n} = (u_y(y_{i,j}), u_z(y_{i,j}), -1)$ denote the exterior normal for $\mathcal{Y}'$ in $y_{i,j}$. Therefore, the inner product has the following form

$$\langle \mathbf{n} \cdot \mathbf{k} \rangle = y_i u_x(y_{i,j}) (Vt_j) - l + u(y_i, Vt_j)$$

(3)

Geometry of the problem is presented in Fig. 3.

![Figure 3. Geometry of the problem in $\mathbb{R}^3$.](image)

Thus, we present the solution of the problem in the following form

$$\hat{u}(y_i) = \begin{cases} u(y_i, Vt_j), & \text{if } y_{i,j} \text{ is visible} \\ -2l, & \text{otherwise.} \end{cases}$$

The "otherwise" condition changed to $-2l$ for better visualization results.

3.2. Algorithm

The idea of the algorithm #2 is based on the algorithm #1. Actually, authors add a new cycle for $j$ which corresponds to sounding intervals. The construction of the algorithm #2 is presented below.

FOR $j = 0, 1, ..., M$ DO

(1) $t_j = j \Delta t$

(2) $y_i = 0$

(3) FOR $i = 0, 1, ..., N$ DO

(a) $\hat{u}(y_i, Vt_j) = u(y_i, Vt_j)$

(b) COMPUTE $\langle \mathbf{n} \cdot \mathbf{k} \rangle$ AS (14)

(c) IF $\langle \mathbf{n} \cdot \mathbf{k} \rangle > 0$ THEN

(i) $y_{ij} = y_i$

(ii) REPEAT

(A) $\hat{u}(y_i, Vt_j) = -2l$

(B) INC($i$)
In numerical experiments for the function, describing the seabottom relief authors use

\[
(y) \quad y_i = ih
\]

\[
(iii) \quad \text{UNTIL} \left( l - u(y, V_t) \right) f y_i < \left( l - u(y, V_t) \right) / y_q
\]

\[
(d) \quad y_i = ih
\]

In numerical experiments for the function, describing the seabottom relief authors use

\[
u(y_1, y_2) = 3e^{-\frac{(y_1-100)^2+(y_2-20)^2}{10^2}} - 3e^{-\frac{(y_1-200)^2+(y_2-20)^2}{10^2}}.
\]

The first term in Eq. (4) corresponds to peak of the seabottom relief, the second one corresponds to the cavity. Visualization of Eq. (4) is shown in Fig. 4. White color corresponds to the level +3, black is for the level −3. All values, which are not in [−3; 3], have a projection to the boundary levels.

![Figure 4. The seabottom relief satisfying Eq. (4).](image)

![Figure 5. The algorithm #2. Input data: \( u \) satisfies Eq. (4) and \( l = 5 \).](image)

![Figure 6. The algorithm #2. Input data: \( u \) satisfies Eq. (4) and \( l = 10 \).](image)

Figg. 5 and 6 show results of the algorithm #2 for the vehicle altitude \( l = 5 \) and \( l = 10 \), respectively. The monoblack zones correspond to shaded area on the seabottom, i.e. the part of \( \gamma \) which is invisible by the observer due to peak and cavity on the seabottom relief.

Therefore, the algorithm #1 is successfully modified to algorithm #2.

4. Conclusions

Moreover, each of three algorithms of determining shaded areas on the sea bottom could be used depending on properties of the observer. The numerical experiment showing efficiency of the algorithms is done. The algorithm #1 is satisfied to the static observer. The algorithms #2 is satisfied to the semi-static sonar, i.e. assuming that during one sounding interval the observer is stopped. The results of this work could be used as continuous research of the [11].

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