Influence of detector motion on discrimination between photon polarizations

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Abstract. We investigate the discrimination between photon polarizations when measured by moving detectors. Both unambiguous and minimum-error discriminations are considered, and we analyze the the optimal successful (correct) probability as a function of the apparatus’ velocity. The Holevo bound for polarization discrimination is also discussed and explicit calculation shows that the Holevo bound and the optimal successful (correct) probability for unambiguous (minimum-error) discrimination simultaneously increase or decrease.

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1. Introduction

As a recent development, the possibility of discrimination between quantum states can be potentially useful for many applications in quantum computation and quantum communication. In this problem, a quantum state is chosen from a set of known states but we do not know which and want to determine the actual states. If the states in the set are not orthogonal, it cannot be successfully identified with unit probability because of the non-cloning theorem. Two basic strategies have been introduced to achieve the state discrimination, one of which is the minimum-error discrimination [1, 2, 3, 4, 5, 6, 7] and the other is the unambiguous discrimination [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. In the minimum-error discrimination, errors are permitted and the optimum measurement is required such that the probability of error is minimum. In the unambiguous discrimination not errors but inconclusive results are permitted, and in the optimum strategy the probability of failure is a minimum.

Recently, testing quantum mechanics for large space distances and eventually in implementing quantum information protocols in global scales attract a lot of interest [20, 21, 22, 23]. Photon is an ideal physical object in quantum communications. Because of the present limits on the use of fiber optics in long distance communications, the most feasible alternative may be free-space transmission using satellites and ground stations. And then theoretical studies on the influence of the detector velocity are demanded by using satellites in quantum information experiments. In this paper, we address this issue by considering the discrimination between two photon polarizations when the measurements performed in different inertial frames are allowed. Pure polarization states for two monochromatic photons with different momenta can unambiguously distinguished in moving frames, while the polarizations of two non-monochromatic photons cannot. Following the proposals in Ref. [24], the effective reduced density matrix for the polarizations can be defined and calculated in moving frames, and the polarization states can be distinguished with minimum error.

The organization of the paper is as follows. In Sec. 2, we give a brief description of basis transformation under the Lorentz boost. In Sec. 3, we discuss the discrimination between two pure polarizations of two monochromatic photons in moving frames. How to calculate the effect reduced density matrix for photon polarizations is discussed in Sec. 4, and numeric results are shown for the minimum-error discrimination between polarizations of two non-monochromatic photons. In Sec. 5, we compare the Holevo bound and polarization discrimination. Finally, the paper is ended with a short discussion in Sec. 6.

2. Relativistic state transformations for photons

To give the state transformation in different frames, we should first discuss the photon basis states. We define the standard vector $|\tilde{k}, \sigma\rangle$, where $\tilde{k} = (1, 0, 0, 1)$, as follows

$$P^\mu|\tilde{k}, \sigma\rangle = k^\mu|\tilde{k}, \sigma\rangle,$$
Influence of detector motion on discrimination between photon polarizations

\[ J_\sigma |\vec{k}, \sigma\rangle = \sigma |\vec{k}, \sigma\rangle, \]  
and for photons \( \sigma = \pm 1 \). The momentum-helicity eigenstates can be generated from the standard vector \( |\vec{k}, \sigma\rangle \),

\[ |k, \sigma\rangle = U(L_k)|\vec{k}, \sigma\rangle, \]  
where \( k = L_k \vec{k} \) is a four-component null vector, \( k^2 = 0 \) and the helicity is denoted by \( \sigma \). The choice of Lorentz transformation \( L_k \) is not unique \([25]\) and in the present paper we set

\[ L_k = R(\hat{k})L_z(k_0), \]  
where \( L_z(k_0) \) is a pure Lorentz boost along \( z \) axis taking \( \vec{k} \) to \( k_0 \vec{k} \) and \( R(\hat{k}) \) denotes a rotation taking the vector \( (1,0,0,1) \) to the vector \( (1,\hat{k}) \). In polar coordinate, \( \hat{k} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \), and \( R(\hat{k}) \) can be chosen as

\[ R(\hat{k}) = R_z(\phi)R_y(\theta). \]  
The carrier space \( \mathcal{H} \) of the irreducible representation of the Poincaré group for photons is spanned by the momentum-helicity eigenstates \( |k, \sigma\rangle \) and the basis are normalized by

\[ \langle k, \sigma|k', \sigma' \rangle = (2\pi)^3(2k_0)^3\delta_{\sigma\sigma'}\delta^{(3)}(k - k'). \]  

A Lorentz boost \( \Lambda \) will induce a unitary operator \( U(\Lambda) \) on the Hilbert space \( \mathcal{H} \)

\[ U(\Lambda)|k, \sigma\rangle = U(L_{\Lambda k})U(W(\Lambda, k))|\vec{k}, \sigma\rangle, \]  
where the Wigner rotation \( W(\Lambda, k) = L_{\Lambda k}^{-1}\Lambda L_k \) is an element in the little group which leaves \( \vec{k} \) invariant. For massless particles, the little group is the \( E(2) \) group and in Eq. (6) \( W(\Lambda, k) \) is just a rotation or translation in the \( x-y \) plane. Since the helicity is not affected by translations, only a rotation by an angle \( \Theta(\Lambda, k) \) is left, and then

\[ U(\Lambda)|k, \sigma\rangle = e^{-i\Theta(\Lambda, k)}|\Lambda k, \sigma\rangle. \]  
The angle \( \Theta(\Lambda, k) \) is explicitly given in Ref. \([25]\),

\[ \Theta(\Lambda, k) = \begin{cases} 
0 & : \Lambda = L_z(k_0) \\
0 & : \Lambda = R_z(\gamma), \vec{k} \neq \hat{z} \\
\gamma & : \Lambda = R_z(\gamma), \vec{k} = \hat{z} \\
\arg(B + iA) & : \Lambda = R_y(\gamma) 
\end{cases} \]  
for different Lorentz transforms and momenta, where

\[ A = \sin \gamma \sin \phi, \quad B = \sin \gamma \cos \theta + \cos \gamma \sin \theta. \]  

Since all the Lorentz boosts can be constructed by \( L_z, R_y \) and \( R_x \), and

\[ W(\Lambda'\Lambda, k) = W(\Lambda', \Lambda k)W(\Lambda, k), \]  
\( \Theta(\Lambda, p) \) for all \( \Lambda \) any momentum \( k \) can be calculated from Eq. (8).

Now, we can use the two helicity states as a basis for the polarization states. The four-vectors of helicity states corresponding to momentum \( k \) are given by

\[ \epsilon_\lambda^\pm_k = R(\hat{k})\epsilon_\lambda^\pm, \]
where $\epsilon^\pm$ is the helicity vectors corresponding to the standard basis states $|\tilde{k}, \sigma\rangle$ and $R(\hat{k})$ is the rotation taking the standard space direction $(0, 0, 1)$ to $\hat{k}$, given in Eq. (4). The polarization state $|\alpha(k)\rangle$ for a photon with momentum $k$ can be expressed as

$$|\alpha(k)\rangle = \alpha_+(k)|\epsilon_+^k\rangle + \alpha_-(k)|\epsilon_-^k\rangle,$$

with $|\alpha_+(k)|^2 + |\alpha_-(k)|^2 = 1$. And a generic one-photon state is given by a wave-package [28]

$$|\Psi\rangle = \int d\mu(k)f(k)|k, \alpha(k)\rangle,$$

normalized by $\int d\mu(k)|f(k)|^2 = 1$ with the Lorentz-invariant measure

$$d\mu(k) = \frac{d^3k}{(2\pi)^32k^0}.$$

According to Eq. (8), the transformation for the polarization under Lorentz boost $\Lambda$ is

$$D(\Lambda)|\alpha(k)\rangle = R(\Lambda\hat{k})R_\Sigma(\Theta(\Lambda, k))R(\hat{k})^{-1}|\alpha(k)\rangle.$$

When the Lorentz boost is along the $z$ axis, it can be simplified as

$$D(\Lambda)|\alpha(k)\rangle = R(\Lambda\hat{k})R(\hat{k})^{-1}|\alpha(k)\rangle.$$

### 3. Unambiguous discrimination in moving frames

The polarized photon is an essential tool for both quantum communication and quantum computation. In quantum communication, optical fibers are usually used, and the photons may be absorbed or depolarized owing to the fiber’s imperfections. In some cases, such as communication with space stations, the photons must propagate and the beam then has a diffraction angle. These mean that the photons are usually not monochromatic and have a momentum distribution in quantum communication. In this section, for simplicity, we discuss the idea case, where the photons are monochromatic. It will be shown that the photon polarizations can be unambiguously distinguished in this case. In the next section, we will discuss a more realistic case, where the photons are non-monochromatic.

Let us assume that Alice prepares a single photon in one of the two polarization states $|\alpha_1\rangle$ and $|\alpha_2\rangle$ with equal probabilities. Besides the polarization freedom, the photon also has momentum freedom, and we assume that the two polarizations have two momenta $|k_1\rangle$ and $|k_2\rangle$, respectively. In this case, the photon corresponds to plane wave pulse. The receiver Bob tries to unambiguously distinguish the two polarization states. We consider the effect of Bob’s motion relative to Alice, with a constant velocity $v$. For convenience, we restrict that $v$ is along $k_1$. A coordinate system in Alice’s rest frame can be selected such that $k_1 = (1, 0, 0, 1)$, $k_2 = (1, \sin \vartheta, 0, \cos \vartheta)$ and $v = (0, 0, v)$. And we suppose the receiver Bob has an infinite flat detector parallel to $x$-$y$ plane. In this paper, $c = 1$ and we choose the zero-components of the momenta to be unit, because according to Eq. (8) and (15), the transformations for polarizations are independent on
the magnitude of the momentum \( \mathbf{k} \). The Lorentz transformation yields momenta’s new components in Bob’s rest frame,
\[
\begin{align*}
  k'_1 &= (\gamma(1 - v), 0, 0, \gamma(1 - v)), \\
  k'_2 &= (\gamma(1 - v \cos \vartheta), \sin \vartheta, 0, \gamma(\cos \vartheta - v)),
\end{align*}
\]
with \( \gamma = (1 - v^2)^{-1/2} \). New unit vectors of momenta in Bob’s rest frame are \( \hat{k}'_1 = (0, 0, 1) \) and \( \hat{k}'_2 = (\sin \vartheta', 0, \cos \vartheta') \), where
\[
\sin \vartheta' = \frac{\sin \vartheta}{\gamma(1 - v \cos \vartheta)}.
\]

According to Eq. (15), the new polarization states are
\[
|\alpha'_1\rangle = |\alpha_1\rangle, \quad |\alpha'_2\rangle = R(\hat{k}'_2)R(\hat{k}'_2)^{-1}|\alpha_2\rangle
\]

To unambiguously distinguish the two polarizations, the POVM detection operators for the optimum discrimination should be given. Let the elements of POVM be \( \Pi_1 \), corresponding to unambiguously detecting \( |\alpha'_1\rangle \), \( \Pi_2 \), corresponding to unambiguously detecting \( |\alpha'_2\rangle \) and \( \Pi_0 \), corresponding to inconclusive result. The condition of no errors requires that
\[
\Pi_1|\alpha'_2\rangle = 0, \quad \Pi_2|\alpha'_1\rangle = 0,
\]
and in addition, because the POVM exhausts all possibilities, it is implied that
\[
\Pi_0 = I - \Pi_1 - \Pi_2.
\]

The probabilities of successfully identifying the two polarization states is
\[
P = \frac{1}{2}\langle \alpha'_1 | \Pi_1 | \alpha'_1 \rangle + \frac{1}{2}\langle \alpha'_2 | \Pi_2 | \alpha'_1 \rangle.
\]

The optimal POVM detection operators satisfying Eqs. (19) and (20) are given in Ref. [19]
\[
\Pi_1 = \frac{2}{3}|\alpha'^{\perp}_1\rangle\langle \alpha'^{\perp}_1 |, \quad \Pi_2 = \frac{2}{3}|\alpha'^{\perp}_2\rangle\langle \alpha'^{\perp}_2 |, \quad \Pi_0 = I - \Pi_1 - \Pi_2,
\]
and the optimal successful probability is
\[
P_{\text{opt}} = 1 - |\langle \alpha'_1 | \alpha'_2 \rangle| = 1 - |\langle \alpha_1 | R(\hat{k}'_2)R(\hat{k}'_2)^{-1} |\alpha_2 \rangle|.
\]

\( |\alpha'^{\perp}_1\rangle \) and \( |\alpha'^{\perp}_2\rangle \) are called reciprocal basis [19] which lie in the space spanned by \( |\alpha'_1\rangle \) and \( |\alpha'_2\rangle \), defined as
\[
\langle \alpha'^{\perp}_i | \alpha'_j \rangle = t_i \delta_{ij}.
\]

Eq. (23) shows that the optimum successful probability \( P_{\text{opt}} \) is dependent on the velocity \( v \). To make it more obvious, we consider an example Alice prepares \( |\alpha_1\rangle = |\epsilon^+_{k_1}\rangle \) and \( |\alpha_2\rangle = |\epsilon^-_{k_2}\rangle \) with \( k_1 = (0, 0, 1) \) and \( k_2 = (\cos \vartheta, 0, \sin \vartheta) \),
\[
|\epsilon^+_{k_1}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ i \\ 0 \end{pmatrix}, \quad |\epsilon^-_{k_2}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \cos \vartheta \\ -i \\ -\sin \vartheta \end{pmatrix}
\]
Influence of detector motion on discrimination between photon polarizations

Figure 1. The optimal successful possibility $P_{opt}$ in moving frames as a function of $\vartheta$ and the relative velocity $v$.

and then in Bob’s rest frame

$$|\alpha'_1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ i \end{pmatrix}, \quad |\alpha'_2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \cos \vartheta' \\ -i \end{pmatrix},$$

(26)

and

$$\langle \alpha'_1 | \alpha'_2 \rangle = \frac{1}{2} (\cos \vartheta' - 1).$$

(27)

We see from Eq. (27) that the overlap between the two polarization states changes for different observers. This conclusion is reasonable since the transformation for photon polarization in different frames is dependent on the photon’s momentum, which is shown in Eq. (14). Finally, the optimal successful possibilities are given as

$$P_{opt}(\vartheta, v) = \frac{(1 + \cos \vartheta)(1 - v)}{2(1 - v \cos \vartheta)}.$$

(28)

The results are shown in Fig. 1. When $\vartheta = 0$ or $k_1 = k_2$, the optimum probabilities $P_{opt}$ are the same in different frames and there is no influence on the discrimination between the two polarizations, because the transformations for the two polarization states are the same. However, if $k_1 \neq k_2$, the optimal probability of unambiguous discrimination between the two polarization states is sensitive to Bob’s relative velocity to Alice. It is obvious in Fig. 1 that $P_{opt}$ drops as $v$ increase. When Bob moves toward the opposite direction of $z$ axis, with the magnitude of the velocity large enough ($v \rightarrow -1$), $P_{opt}$ can become arbitrarily close to 1, which means the two polarization states can almost be perfectly distinguished even if Alice prepares two nonorthogonal polarization states. This shows how important the detector motion can be for polarization measurements when the velocity is high enough.
4. Minimum-error discrimination in moving frames

As mentioned before, the photons in quantum communication are usually non-monochromatic, and in this section, we will discuss the more realistic case. Assume that Alice prepares a single photon in one of the two helicity states. In the long-range propagation of the polarized photon, due to the imperfections of fiber or the diffraction in the free space, the photon may have a momentum distribution, and the corresponding one-photon states read

\[ |\Psi_\pm\rangle = \int d\mu(\mathbf{k}) f(\mathbf{k}) |\epsilon^\pm_{\mathbf{k}}\rangle, \]  

where \( f(\mathbf{k}) \) represents the momentum distribution. For simplify, we suppose that the momenta have a Gaussian distribution and the dispersion is restricted to \( x - y \) plane,

\[ |f(\mathbf{k})|^2 = \frac{1}{N} \exp\left(-\frac{k_r^2}{2\sigma^2}\right) \delta(k_3 - k_0), \]  

where \( k_r = \sqrt{k_1^2 + k_2^2} \), and \( N \) is a normalization factor. To discriminate between the two polarization states, we should first calculate the reduced density matrix for the polarizations. Following the proposals in Ref. \cite{24}, a longitudinal (unphysical) part of a polarization state \( |\alpha_{\mathbf{k}}\rangle \) can be defined as \( \epsilon'_{\mathbf{k}} = \mathbf{k} \). A polarization state along the \( x \) axis is

\[ |\hat{x}\rangle = x_+(\mathbf{k})|\epsilon^+_k\rangle + x_-(\mathbf{k})|\epsilon^-_k\rangle + x_t(\mathbf{k})|\epsilon^t_k\rangle. \]  

Here, \( x_\pm(\mathbf{k}) = \epsilon^\pm_{\mathbf{k}} \cdot \hat{x} = (\cos \theta \cos \phi \pm i \sin \phi)/\sqrt{2} \), and \( x_t(\mathbf{k}) = \hat{x} \cdot \mathbf{k} = \sin \theta \cos \phi \). The transverse part of \( |\hat{x}\rangle \) is

\[ |b_x(\mathbf{k})\rangle = x_+(\mathbf{k})|\epsilon^+_k\rangle + x_-(\mathbf{k})|\epsilon^-_k\rangle, \]  

and similarly \( |b_y(\mathbf{k})\rangle \) and \( |b_z(\mathbf{k})\rangle \) can be obtained. Then, we can define

\[ E_{mn} = \int d\mu(\mathbf{k}) |\mathbf{k}, b_m(\mathbf{k})\rangle \langle \mathbf{k}, b_n(\mathbf{k})|, \quad m, n = x, y, z. \]  

Then, the effective reduced density matrix for polarization of a one-photon state \( |\Psi\rangle \) can be expressed as

\[ \rho_{mn} = \langle \Psi | E_{mn} | \Psi \rangle = \int d\mu(\mathbf{k}) |f(\mathbf{k})|^2 \langle \alpha(\mathbf{k})|b_m(\mathbf{k})\rangle \langle b_n(\mathbf{k})|\alpha(\mathbf{k})\rangle. \]  

According to Eq. \( (15) \), in a moving frame with relative velocity \( \mathbf{v} = (0, 0, v) \), the reduced polarization density matrix can be obtained

\[ \rho'_{mn} = \int d\mu(\mathbf{k}) |f(\mathbf{k})|^2 \langle R(\Lambda \hat{\mathbf{k}}) R(\hat{\mathbf{k}})^{-1}\alpha(\mathbf{k})|b_m(\mathbf{k})\rangle \times \langle b_n(\mathbf{k})|R(\Lambda \hat{\mathbf{k}}) R(\hat{\mathbf{k}})^{-1}\alpha(\mathbf{k})\rangle \]  

Using Eq. \( (34) \), we can calculate the reduced density matrix \( \rho_{\pm} \) for \( |\Psi_\pm\rangle \) in Bob’s frame. And at this point, the above unambiguous state discrimination is not appropriate for this case because the two spaces supported by \( \rho_+ \) and \( \rho_- \) are the same. But we can still distinguish them with the minimum-error strategy. For minimum-error
discrimination inconclusive results do not occur, so that \( \Pi_0 = 0 \) and we require that the probability of errors in the discrimination procedure is a minimum.

The error probability can be expressed as

\[
P_E = \frac{1}{2} \text{Tr}(\rho_+ \Pi_-) + \frac{1}{2} \text{Tr}(\rho_- \Pi_+) = \frac{1}{2} + \frac{1}{2} \text{Tr}[(\rho_- - \rho_+) \Pi_+].
\]

Introducing the operator \( \Omega = \rho_- - \rho_+ = \sum_k \omega_k |\phi_kangle \langle \phi_k| \), and it is obvious that the minimum of the error probability is obtained when \( \Pi_+ \) is the projector onto those eigenstates \( |\phi_k\rangle \) of \( \Omega \) that belong to negative eigenvalues \( \omega_k \). The optimum detection operators therefore read

\[
\Pi^\text{opt}_+ = \sum_{k < k_0} |\phi_k\rangle \langle \phi_k|, \quad \Pi^\text{opt}_- = \sum_{k \geq k_0} |\phi_k\rangle \langle \phi_k|,
\]

where \( \omega_k < 0 \) for \( 1 \leq k < k_0 \) and \( \omega_k \geq 0 \) for \( k \geq k_0 \). Clearly, the optimal minimum-error measurement for discriminating between two quantum states is a von Neumann measurement. The resulting minimum-error probability is [1]

\[
P_E = \frac{1}{2} - \frac{1}{4} \text{Tr}|\rho_+ - \rho_-|.
\]

For any operator \( O \), the operator \( |O| \) is defined as \( (O^\dagger O)^{1/2} \).

Next, we perform a numerical investigation of Eq. (37). Fig. 2 shows the minimum-error probability of discrimination between \( \rho_+ \) and \( \rho_- \) vs Bob’s velocity \( v \) relative to Alice, for \( W = 0.01, W = 0.5, \) and \( W = 1 \), respectively. Here, \( W \) is the wave packet width, and \( W = \sigma/k_0 \). We see that when Bob moves along the opposite direction of \( z \) axis, \( P_E \) drops as \( v \) decreases. However, there is a maximal value for \( P_E \) when Bob moves along the same direction of \( z \) axis. This can be explained as follows. When \( v = 0 \), the polar angles \( \theta \) for different momenta satisfy \( 0 \leq \theta \leq \pi/2 \). \( \theta \) drops as \( v \) decreases and this helps to diminish \( P_E \). And \( \theta \) goes up as \( v \) increases, and this will enlarge the minimum error probability \( P_E \) before \( \theta \) exceeds \( \pi/2 \). With \( v \) large enough, \( \theta \) will exceed \( \pi/2 \), this will lead to an opposite effect that helps to diminish \( P_E \) again. Thus, we can define a “critical point” for the velocity \( v \), where the minimum error probability gets the maximal value. And the “critical point” for \( v \) vs \( W \) is shown in Fig. 3. The “critical point” decreases as \( W \) goes up.

5. Holevo bound and quantum state discrimination

Distinguishing quantum states is like gaining information. Alice has a classic information source encoded in quantum states \( \rho_1, \rho_2, \ldots, \rho_n \), sent to Bob with probabilities \( \{p_1, p_2, \ldots, p_n\} \), and Bob tries to determinate the states to obtain the information. The higher the successful probability is, the more information Bob gets. A good measure of how much information Bob can obtain is the accessible information. The upper bound of accessible information called Holevo bound, is defined as follows [29],

\[
\chi = S(\rho) - \sum_i p_i S(\rho_i),
\]

\[
\text{(38)}
\]
Influence of detector motion on discrimination between photon polarizations

Figure 2. The resulting minimum error probabilities $P_E$ for distinguishing $\rho_+$ and $\rho_-$ in Bob’s frame as a function of the relative velocity $v$ between Alice and Bob. Data is shown for $W = 0.01$, $W = 0.5$ and $W = 1$, where $W = \sigma/k_0$.

Figure 3. The “critical point” for $v$ as a function of wave packet width $W$.

where $\rho = \sum_i p_i \rho_i$ and $S(\rho) = - \text{Tr}\rho \log \rho$ is the von Neumann entropy for $\rho$. Since the von Neumann entropy for photon polarization is not a relativistic scalar, which is similar to that for massive particles [30, 31], the Holevo bound is not invariant in different frames. This is the reason why the discrimination between photon polarization is influenced by detector motion. We compare the Holevo bound and photon polarization discrimination in the following.

For the unambiguous discrimination between $|e^+_{k_1}\rangle$ and $|e^-_{k_2}\rangle$ above, it is easy to
Influence of detector motion on discrimination between photon polarizations

Figure 4. The Holevo bound $\chi(\cos \vartheta')$ and successful possibility $P(\cos \vartheta')$ are plotted as a function of $\cos \vartheta'$. Both $\chi(\cos \vartheta')$ and $P(\cos \vartheta')$ increase as $\cos \vartheta'$ increasing.

Figure 5. The Holevo $\chi(\rho_+, \rho_-)$ and optimal correct probability $P(\rho_+, \rho_-)$ are plotted as a function of velocity $v$, (a) for $W = 0.5$ and (b) for $W = 1$. $\chi(\rho_+, \rho_-)$ and $P(\rho_+, \rho_-)$ simultaneously decrease or increase as $v$ changes.

obtain the Holevo bounds in moving frames. Simple calculation yields

$$
\chi(\cos \vartheta') = -\frac{1 + \cos \vartheta'}{4} \log \frac{1 + \cos \vartheta'}{4} - \frac{3 - \cos \vartheta'}{4} \log \frac{3 - \cos \vartheta'}{4},
$$

where $\cos \vartheta'$ corresponds to the relative velocity $v$. The Holevo bound $\chi(\cos \vartheta')$ and the successful probability $P(\cos \vartheta')$ are shown in Fig. 4. Both $\chi(\cos \vartheta')$ and $P(\vartheta')$ increase as $\cos \vartheta'$ goes up.

For the minimum-error discrimination between $\rho_+$ and $\rho_-$ that we mentioned earlier, the Holevo bound can be numerically given according to Eqs. (34) and (38). The
Holevo bound $\chi(\rho_+,\rho_-)$ and optimal correct possibility $P(\rho_+,\rho_-)$ are shown in Fig. 5, for $W = 0.5$ and $W = 1$, respectively. When Bob moves along the opposite direction of z axis, $v < 0$, both $\chi(\rho_+,\rho_-)$ and $P(\rho_+,\rho_-)$ simultaneously increase as $v$ decreases. When Bob move along the same direction of z axis, $\chi(\rho_+,\rho_-)$ and $P(\rho_+,\rho_-)$ decrease as the magnitude of $v$ increases, and both reach the minimum values at the “critical point” of $v$. After the velocity $v$ exceeds the “critical point”, $\chi(\rho_+,\rho_-)$ and $P(\rho_+,\rho_-)$ increase again with $v$ rising.

The two examples above reveal that, both the Holevo bound and the optimal successful (or correct) probability simultaneously decrease or increase as $v$ changes. The discrimination between photon polarizations is influenced by the measurement apparatus velocity.

6. Conclusions and discussions

In summary, we investigate the influence of detector velocity on discrimination between photon polarizations. The successful (correct) probability for unambiguous (minimum-error discrimination) is dependent on the apparatus velocity $v$ relative to the emitter. For some cases, there are “critical points” for the apparatus velocity at which the correct probabilities to distinguish the polarizations reach the maximal values. The Holevo bound and polarization discrimination are also compared in the present work, and we discover that they simultaneously decrease or increase in different frames.

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