Coupling of spatially separated carriers in crossed electric and magnetic fields and a possibility of a metastable superconducting state in bilayer systems

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It is shown that in bilayer conducting structures in crossed electric and magnetic fields of a special configuration (the fields should have opposite signs in the adjacent layers) the dependence of the energy of a pair of equally charged carriers on the momentum of the pair has a local minimum. This minimum corresponds to a bound state of the pair. The local minimum is separated from the absolute minimum by a large energy barrier. That provides the stability of the bound state with respect to different scattering processes. If the number of the pairs is macroscopic, a phase transition into a metastable superconducting state may take place.

I. INTRODUCTION

Up to now a lot of mechanisms for the electron pairing in metals and different composite semi-metal structures have been proposed. In all that mechanisms the electron pairing is caused by an attraction between electrons due to an exchange by a quantum of a boson field - phonon, plasmon, magnon or their combination. After the discovery of the high-temperature superconductivity a number of the mechanisms proposed has grown significantly [1,2]. In this paper we show that a principally different mechanism for the binding of the electron pair into a bound state exists. This binding is caused by a special configuration of external electric and magnetic fields, in which the pair has to be situated.

Let us consider a three layer sandwich consisting of two two-dimensional layers with an electron conductivity separated by a dielectric layer of the thickness \( d \). Let the homogeneous parallel to the layers electric fields \( \mathbf{E} = (E, 0, 0) \) and \( \mathbf{E} = (-E, 0, 0) \), and normal to the layers magnetic fields \( \mathbf{H} = (0, 0, H) \) and \( \mathbf{H} = (0, 0, -H) \) are applied to the upper and the lower layer, correspondingly. The Schroedinger equation for the pair of electrons, one of which belongs to the upper layer (layer 1) and the other one - to the lower layer (layer 2), has the form

\[
\begin{align*}
\left\{ \frac{1}{2m_\ast} \left(-i\hbar \nabla_1 + \frac{1}{2} \frac{e}{c} \mathbf{H} \times \mathbf{r}_1 \right)^2 + \frac{1}{2m_\ast} \left(-i\hbar \nabla_2 - \frac{1}{2} \frac{e}{c} \mathbf{H} \times \mathbf{r}_2 \right)^2 \\
+ e\mathbf{E} \cdot (\mathbf{r}_1 - \mathbf{r}_2) + \frac{e^2}{c\sqrt{\left| \mathbf{r}_1 - \mathbf{r}_2 \right|^2 + d^2}} \right\} \Psi(\mathbf{r}_1, \mathbf{r}_2) = \varepsilon \Psi(\mathbf{r}_1, \mathbf{r}_2).
\end{align*}
\]

(1)

In this equation we assume the same effective electron masses \( m_\ast \) in both layers and use the symmetric gauge for the vector potential \( \mathbf{A} = \frac{1}{2} \mathbf{H} \times \mathbf{r} \). The vectors \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \) are two-dimensional ones.

The Hamiltonian (1) differs from the Hamiltonian of an electron-hole pair in homogeneous crossed fields only by the sign of the electron-electron interaction. Therefore, as in the case of the electron-hole pair(compare with [3]), the operator

\[
P = -i\hbar \nabla_1 - i\hbar \nabla_2 - \frac{1}{2} \frac{e}{c} \mathbf{H} \times (\mathbf{r}_1 - \mathbf{r}_2)
\]

(2)

commutes with the Hamiltonian (1) (hence, it conserves in time) and their components commute with each other. It allows to parameterize the energy of the electron pair by the momentum \( \mathbf{p} \) and introduce the dispersion law for the pair.

Using the new variables

\[
\mathbf{R} = \frac{\mathbf{r}_1 + \mathbf{r}_2}{2}, \quad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2,
\]

(3)

and, as in [3], rewriting the wave function in Eq.(1) in the form

\[
\Psi(\mathbf{r}_1, \mathbf{r}_2) = \exp \left\{ i \left( \mathbf{p} + \frac{e}{2c} \mathbf{H} \times \mathbf{r} \right) \cdot \frac{\mathbf{R}}{\hbar} \right\} \Phi(\mathbf{r} - \mathbf{r}_0),
\]

(4)
where

\[ \mathbf{r}_0 = \frac{e}{eH^2} \mathbf{H} \times \mathbf{p}', \quad \mathbf{p}' = \mathbf{p} + \frac{2m_e c}{H^2} \mathbf{H} \times \mathbf{E}, \]  

(5)

one can easily check, that the function \( \Phi(\mathbf{r}) \) should satisfy the equation

\[ \left\{ \frac{\hbar^2}{m_e} \frac{\partial^2}{\partial \mathbf{r}^2} + \frac{e^2 H^2}{4m_e c^2} r^2 + \frac{e^2}{\epsilon \sqrt{|\mathbf{r} + \mathbf{r}_0|^2 + d^2}} + \frac{p^2 - p_r^2}{4m_e} \right\} \Phi(\mathbf{r}) = \varepsilon \Phi(\mathbf{r}). \]  

(6)

In strong magnetic fields \( H \), for which the Larmour frequency \( \omega_c = eH/m_e c \) multiplied by the Plank constant is much large then the Coulomb energy \( e^2/\ell \) (where \( \ell = (\hbar c/\epsilon e H)^{1/2} \) is the magnetic length), in zero order approximation one can neglect the energy of the Coulomb interaction. The solution of Eq. (6), in which the Coulomb energy is omitted, is

\[ \Phi(\mathbf{r}) = \frac{1}{\sqrt{2\pi \ell}} \exp \left( -\frac{1}{2} \frac{\xi}{L_{\|}^{\|}} \right) L_{n}^{m} (\xi) e^{-im\varphi}. \]  

(7)

Here \( \xi = r^2/2\ell^2 \), \( L_{n}^{m} \) is the generalized Laguerre polynomial, \( n, m \), the integer numbers \((n \geq 0)\). The eigenvalue, which corresponds to the eigenfunction \( \Phi(\mathbf{r}) \), minus the quantity \((p^2 - p_r^2)/4m_e\) reads as

\[ \varepsilon_{n,m} = \frac{eH}{m_e c} \hbar \left( 2n + |m| + 1 \right). \]  

(8)

The ground state of the pair is realized at \( n = m = 0 \). The first order correction in the Coulomb interaction is equal to

\[ \delta \varepsilon = \frac{e^2}{2\pi \ell} \int \frac{\exp \left( -\frac{|\mathbf{r} - \mathbf{r}_0|^2}{2\ell^2} \right)}{\epsilon \sqrt{\mathbf{r}^2 + d^2}} d^2 r. \]  

(9)

Below we restrict our consideration by the case of small thickness of the dielectric layer, when the inequality \( d \ll \ell \) is fulfilled. In this case the integrals in (9) can be evaluated analytically. The result is

\[ \delta \varepsilon = \left( \frac{\pi}{2} \right)^{\frac{1}{2}} \frac{e^2}{\ell} \exp \left( -\frac{r_0^2}{4\ell^2} \right) I_0 \left( \frac{r_0^2}{4\ell^2} \right). \]  

(10)

Here \( I_0(z) \) is the modified Bessel function.

If one introduces the electron drift velocity in crossed fields \( \mathbf{u} = e\mathbf{E} \times \mathbf{H}/H^2 \), then, in the ground state \( n = m = 0 \) the total energy of the electron pair is equal to

\[ \varepsilon = \hbar \omega_c + \mathbf{u} \cdot \mathbf{p} - \frac{m_e c^2 E^2}{H^2} + \delta \varepsilon \left( \mathbf{p} - 2m_e \mathbf{u} \right). \]  

(11)

To investigate the dependence of the energy of the pair on its momentum, we take into account, that at the fields \( \mathbf{E} \) and \( \mathbf{H} \) specified the vector \( \mathbf{u} \) has the form \( \mathbf{u} = (0, -\frac{\mathbf{H}}{H}, 0) \). Therefore, we also consider, that the momentum of the pair is equal to \( \mathbf{p} = (0, p, 0) \). Let us assume the field \( E \) is such small, that the condition

\[ \frac{e^2}{\epsilon c \hbar} \gg \frac{E}{H} \]  

(12)

is satisfied.

One can easily find that the function \( \varepsilon(p) \) has a local minimum and a local maximum (both at \( p < 0 \)). If the inequality \( (13) \) is satisfied, the maximum is in the region, where \( r_0(p)/\ell \ll 1 \), while the minimum - in the region \( r_0(p)/\ell \gg 1 \). It allows one to use the well known approximations for the Bessel function \( I_0 \) in that regions. After simple calculations we find the minimum is reached at \( p = p_0 = -\left( e^2 H^2/\epsilon c^2 E \right)^{1/2} \). At this point the energy is equal to

\[ \varepsilon_{\text{min}} = \hbar \omega_c - \frac{m_e c^2 E^2}{H^2} + \frac{e^2}{\epsilon \sqrt{\epsilon c E}}. \]  

(13)
The maximum is reached at $p = p_m \equiv -2(2/\pi)^{1/2}\hbar^2 c E/e^2 \ell H$, and the energy is equal to

$$
\varepsilon_{\text{max}} = \hbar \omega_c - \frac{m_e c^2 E^2}{H^2} + \left( \frac{\pi}{2} \right)^\frac{1}{2} \frac{e^2}{\ell} + \left( \frac{2}{\pi} \right)^\frac{1}{2} \frac{\hbar^2 c}{e^2 \ell} \left( \frac{E}{H} \right)^2.
$$

(14)

The function $\varepsilon(p)$ is plotted in Fig. 1. As it follows from Eq. (2), at the minimum the following relation between $p_0$ and the size of the pair $r_0$ (which is the average distance between electrons in the plane) is fulfilled: $|p_0| = eHr_0/c$.

It means that $r_0 = (e/\epsilon E)^{1/2}$. If the inequality (12) is satisfied, the size of the pair $r_0$ greatly exceeds the magnetic length $\ell$.

The physical reason for the bounding of two particles with the same sign of the charges is the following. The Coulomb forces repulse the particles. The electric fields, directed oppositely in the adjacent layers, try to make the particles stand closer (at the right sign of the fields). Since the kinetic energy of the electrons is quenched by the strong magnetic field, the size of the pair $r_0$ is found from the minimum of the potential energy

$$
U \equiv eEr + \frac{e^2}{er}.
$$

(15)

(at $E \parallel r$). Although this minimum is a local one, but it is separated from the absolute minimum by a barrier of the height of order $e^2/\ell \epsilon$. Due to this reason the electron pairs with the momenta in the vicinity of $p_0$ are stable with respect to their collisions with each other and they cannot overcome the energy barrier. This circumstance allows to put a macroscopic number of the electron pairs into the state with a momentum $p_0$. Since the pairs are bosons, a transition into an unusual superconducting state is possible in a system with pure Coulomb repulsion. The presence of other pairs does not destruct this picture under conditions that the pairs do not overlap. Hence it follows that the density of the pairs $n$ should satisfy the inequality $nr_0^2 \ll 1$. Substituting $r_0$ into that inequality, we arrive to a restriction from below on the value of the electric field $E$. The restriction from above on $E$ follows from (12). As a result, one can easily find that the theory is valid at

$$
\frac{e}{\ell^2} \gg E \gg \frac{en}{\epsilon}.
$$

(16)

In conclusion, we discuss two important questions. How the magnetic structure required can be designed and how to get into the state with the momentum $p_0$?

To answer the first question we note the following. For instance, it was reported in Ref. [4] on the study of properties of an electron gas in a periodic magnetic field. Such a field was induced by dysprosium magnetic stripes sputtered on the surface of the conducting layer. One can expect the required configuration of the magnetic fields can be realized if the stripes with a special orientation of the magnetic moments are sputtered on both conducting layers.

The one of the possibilities for obtaining the state with a large number of the pairs with the momenta lying in the vicinity of $p_0$ is the following. Let us assume that a bilayer system in the crossed fields of the special configuration considered consists of two subsystems separated by a partition with a hole. As it follows from the dispersion law (see Fig. 1), the right moving pairs have the momenta belonging to the branch 2, while the left moving pairs - the momenta belonging to the branches 1 and 3. As a result, an excess number of the branch 2 pairs will be accumulated in the right subsystem. Further interaction of the pairs with a thermostat will lower their energy and put them into states with the momenta close to $p_0$. At the temperature of order $\hbar^2 n/2m^*$ the pairs may condense into a long-living superconducting state under condition that the height of the barrier $\sqrt{\pi/2e^2/\ell \epsilon}$ is large in comparison with the temperature.

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FIG. 1. The dependence of the energy $\varepsilon$ of a spatially separated electron pair on its momentum $\mathbf{p} = (0, p, 0)$ in crossed electric $\mathbf{E} = (\pm E, 0, 0)$ and magnetic $\mathbf{H} = (0, 0, \pm H)$ fields (the upper signs correspond to the layer 1, the lower signs - to the layer 2).