$L^p - L^q$ BOUNDEDNESS OF BERGMAN-TYPE OPERATORS
OVER THE SIEGEL UPPER HALF-SPACE

CONGWEN LIU, JIAJIA SI, AND PENGYAN HU

ABSTRACT. We characterize the $L^p - L^q$ boundedness of Bergman-type operators over the Siegel upper half-space. This extends a recent result of Cheng et. al. (Trans. Amer. Math. Soc. 369:8643–8662, 2017) to higher dimensions.

1. INTRODUCTION

Let $\mathbb{C}_+ := \{z \in \mathbb{C} : \text{Im} z > 0\}$ be the upper half-plane. For $\alpha > 0$, consider the integral operator

$$T_\alpha f(z) = \int_{\mathbb{C}_+} \frac{f(w)}{(z - \overline{w})^\alpha} dA(w), \quad z \in \mathbb{C}_+.$$ 

where $dA$ is the Lebesgue measure on $\mathbb{C}_+$.

Very recently, Cheng, Fang, Wang and Yu [2] characterized the $L^p - L^q$ boundedness of $T_\alpha$ as follows.

**Theorem A** ([2, Theorem 5]). Let $\alpha > 0$ and $1 \leq p, q \leq \infty$.

(i) If $\alpha > 2$ then $T_\alpha : L^p(\mathbb{C}_+) \rightarrow L^q(\mathbb{C}_+)$ is unbounded for any $1 \leq p, q \leq \infty$.

(ii) If $0 < \alpha \leq 2$, then $T_\alpha : L^p(\mathbb{C}_+) \rightarrow L^q(\mathbb{C}_+)$ is bounded if and only if $p, q$ satisfy

$$1 < p < \frac{2}{2 - \alpha} \quad \text{and} \quad \frac{1}{q} = \frac{1}{p} + \frac{\alpha}{2} - 1.$$ 

The purpose of this note is to extend the above result to the several complex variables setting.

We fix a positive integer $n$ throughout this paper and let $\mathbb{C}^n = \mathbb{C} \times \cdots \times \mathbb{C}$ denote the $n$-dimensional complex Euclidean space. For $z \in \mathbb{C}^n$, we use the notation

$$z = (z', z_n), \quad \text{where} \quad z' = (z_1, \ldots, z_{n-1}) \in \mathbb{C}^{n-1} \text{ and } z_n \in \mathbb{C}^1.$$ 

The Siegel upper half-space in $\mathbb{C}^n$ is the set

$$\mathcal{U} := \{z \in \mathbb{C}^n : \text{Im} z_n > |z'|^2\}.$$ 

This domain is biholomorphically equivalent to the unit ball of $\mathbb{C}^n$, and its boundary $\partial \mathcal{U} := \{z \in \mathbb{C}^n : \text{Im} z_n = |z'|^2\}$ is the standard representation of the Heisenberg group $\mathbb{H}^{n-1}$. See [4] Chapters 9–10 and [7] Chapter XII.

1991 Mathematics Subject Classification. Primary 32A35, 47G10; Secondary 32A26, 30E20.

Key words and phrases. Siegel upper half-space; Bergman type operators; $L^p - L^q$ boundedness.

The first author was supported by the National Natural Science Foundation of China grants 11571333, 11471301.
For real parameter $\alpha > 0$, we consider the integral operator

$$T_\alpha f(z) := \int_U \frac{f(w)}{\rho(z, w)^\alpha} dV(w), \quad z \in U,$$

where

$$\rho(z, w) := \frac{i}{2} (\bar{w}_n - z_n) - \langle z', w' \rangle$$

with

$$\langle z', w' \rangle := z_1 w_1 + \cdots + z_{n-1} \bar{w}_{n-1},$$

and $dV$ is the Lebesgue measure on $\mathbb{C}^n$. These operators are modelled on the Bergman projection on $U$. Recall that the Bergman projection $P$ on $U$ is given by

$$P f(z) = \frac{n!}{4\pi^n} T_{n+1} f(z) = \frac{n!}{4\pi^n} \int_U \frac{f(w)}{\rho(z, w)^{n+1}} dV(w), \quad z \in U.$$

Our main result is the following

**Theorem 1.1.** Let $\alpha > 0$ and $1 \leq p, q \leq \infty$.

(i) If $\alpha > n + 1$ then $T_\alpha$ is unbounded from $L^p(U)$ to $L^q(U)$ for any $1 \leq p, q \leq \infty$.

(ii) If $0 < \alpha \leq n + 1$, then $T_\alpha : L^p(U) \to L^q(U)$ is bounded if and only if $p, q$ satisfy

$$1 < p < \frac{n + 1}{n + 1 - \alpha} \quad \text{and} \quad \frac{1}{q} = \frac{1}{p} + \frac{\alpha}{n + 1} - 1.$$

**Remark.** Our proof also shows that Theorem 1.1 remains true if $T_\alpha$ is replaced by the integral operator

$$S_\alpha f(z) := \int_U \frac{f(w)}{|\rho(z, w)|^\alpha} dV(w), \quad z \in U.$$

2. Preliminaries

We begin by recalling the definition of the Heisenberg group and some basic facts which can be found in [7, Chapter XII]. The Heisenberg group $\mathbb{H}^{n-1}$ is the set

$$\mathbb{C}^{n-1} \times \mathbb{R} = \{ [\zeta, t] : \zeta \in \mathbb{C}^{n-1}, t \in \mathbb{R} \},$$

endowed with the group operation

$$[\zeta, t] \cdot [\eta, s] = [\zeta + \eta, t + s + 2\text{Im}(\zeta, \eta)],$$

where $\langle \zeta, \eta \rangle := \zeta_1 \bar{\eta}_1 + \cdots + \zeta_{n-1} \bar{\eta}_{n-1}$. To each element $h = [\zeta, t]$ of $\mathbb{H}^{n-1}$, we associate the following (holomorphic) affine self-mapping of $U$:

$$h : (z', z_n) \mapsto (z' + \zeta, z_n + t + 2i(z', \zeta) + i|\zeta|^2).$$

These mappings are simply transitive on the boundary $\partial U$ of $U$, so we can identify the Heisenberg group with $\partial U$ via its action on the origin

$$\mathbb{H}^{n-1} \ni [\zeta, t] \mapsto (\zeta, t + i|\zeta|^2) \in \partial U.$$

Also, it is easy to check that

$$\rho(h(z), h(w)) = \rho(z, w)$$

for any $z, w \in U$ and any $h \in \mathbb{H}^{n-1}$. 

Lemma 2.1. For any fixed \( w \in U \) and any \( R > 0 \), we have
\[
|\{ z \in U : |\rho(z, w)| < R \} | \leq \frac{2^{n+1}\pi^n}{(n-1)!} R^{n+1}.
\]
Here and in the sequel, \( |E| = V(E) \) denotes the Lebesgue measure of \( E \subset \mathbb{C}^n \).

Proof. We first note that
\[
|\{ z \in U : |\rho(z, w)| < R \} | = |\{ z \in U : |\rho(z, h(w))| < R \} |
\]
for any \( w \in U \) and any \( h \in \mathbb{H}^{n-1} \). Indeed, by (2.2),
\[
\int_{|\rho(z, w)| < R, \ \text{Im} \ z_n > |z'|^2} dV(z) = \int_{|\rho(z, h(w))| < R, \ \text{Im} \ z_n > |z'|^2} dV(z),
\]
where the last equality follows by the change of variables \( z \mapsto h(z) \) in the integral.

Now let \( h = [-w', 0] \), then \( h(w) = (0', w_n - i|w'|^2) \) and
\[
|\rho(z, h(w))| = \frac{i}{2} (\overline{w}_n + i|w'|^2 - z_n).
\]
Also, the inequalities \( |z_n - \overline{w}_n - i|w'|^2| < 2R \) and \( \text{Im} \ z_n > |z'|^2 \) imply that \( |z'|^2 \leq 2R \).
Hence
\[
|\{ z \in U : |\rho(z, h(w))| < R \} | = \int_{|z_n - \overline{w}_n - i|w'|^2| < 2R, \ \text{Im} \ z_n > |z'|^2} dV(z) \leq \int_{|z_n - \overline{w}_n - i|w'|^2| < 2R, \ |z'| \leq \sqrt{2R}} dV(z) = \pi (2R)^2 \cdot \frac{\pi^{n-1}}{(n-1)!} (2R)^{n-1} = \frac{2^{n+1}\pi^n}{(n-1)!} R^{n+1}.
\]
Together with (2.4), this completes the proof. \( \square \)

For \( 0 < p < \infty \), the weak-\( L^p \) space \( L^{p,\infty}(U) \) is defined as the set of all measurable functions \( f \) such that
\[
\|f\|_{L^{p,\infty}} := \sup_{\lambda > 0} \lambda \cdot |\{ z \in U : |f(z)| > \lambda \}|^{1/p}
\]
is finite.

We need the following variant of Schur’s test, which is a special case of Lemma 1.11.17 in [8, p.181].

Lemma 2.2 (Weak-type Schur’s test). Let \( 1 < p < q < \infty \) and \( 1 < r < \infty \) be such that \( \frac{1}{q} = \frac{1}{p} + \frac{1}{r} - 1 \). Suppose that \( Q(z, w) \) is a measurable function on \( U \times U \) and \( T \) is the associated integral operator
\[
Tf(z) = \int_U Q(z, w) f(w) \, dV(w), \quad z \in U.
\]
If there exists a positive constant \( C \) such that
\[
\|Q(\cdot, w)\|_{L^{r,\infty}} \leq C
\]
for almost every \( w \in U \) and
\[
\|Q(z, \cdot)\|_{L^r} \leq C
\]
for almost every \( z \in U \), then \( T \) is bounded from \( L^p(U) \) to \( L^q(U) \).

We denote by \( H^\infty(U) \) the space of bounded holomorphic functions on \( U \), and by \( L^2_a(U) \) the closed subspace of \( L^2(U) \) consisting of holomorphic functions on \( U \). The orthogonal projection from \( L^2(U) \) onto \( L^2_a(U) \), known as the Bergman projection, can be expressed as an integral operator:
\[
P f(z) = \int_U f(w) K(z, w) dV(w),
\]
with the Bergman kernel
\[
K(z, w) := \frac{n!}{4\pi^n} \frac{1}{\rho(z, w)^{n+1}}, \quad (z, w) \in U \times U.
\]
For \( z \in U \), we put \( K_z(\cdot) := K(\cdot, z) \) and \( k_z := K_z/\|K_z\|_2 \). The Berezin transform on \( U \) is given by
\[
B f(z) := \langle k_z, f(z) k_z \rangle = \frac{1}{\sqrt{K(z, z)}} \int_U f(w) \frac{\rho(z, w)^{n+1}}{\rho(z, w)^{2n+2}} dV(w), \quad z \in U,
\]
where \( \rho(z) := \rho(z, z) = \text{Im} z_n - |z|^2 \).

**Lemma 2.3.** If \( f \in H^\infty(U) \) then \( B f = f \).

**Proof.** Since \( f \in H^\infty(U) \), for each fixed \( z \in U \), \( f k_z \in L^2_a(U) \). By the reproducing property of \( K_z \),
\[
B f(z) = \frac{1}{\sqrt{K(z, z)}} (f k_z, K_z) = \frac{1}{\sqrt{K(z, z)}} f(z) k_z(z) = f(z).
\]
\( \square \)

We end this section by recalling two formulas from [5].

**Lemma 2.4 ([5 Key Lemma]).** Suppose that \( r, s > 0 \), \( t > -1 \) and \( r + s - t > n + 1 \). Then
\[
\int_U \frac{\rho(w)^t}{\rho(z, w)^r \rho(w, u)^s} dV(w) = \frac{C_1(r, s, t)}{\rho(z, u)^{r+s-t-n-1}}
\]
holds for all \( z, u \in U \), where
\[
C_1(r, s, t) := \frac{4\pi^n \Gamma(1+t) \Gamma(r+s-t-n-1)}{\Gamma(r) \Gamma(s)}.
\]

**Lemma 2.5 ([5, Lemma 5]).** Let \( s, t \in \mathbb{R} \). Then we have
\[
\int_U \frac{\rho(w)^t}{\rho(z, w)^s} dV(w) = \begin{cases} 
C_2(s, t), & \text{if } t > -1 \text{ and } s - t > n + 1 \\
\rho(z)^{s-t-n-1}, & \text{if } t < -1 \text{ and } s - t > n + 1 \\
+\infty, & \text{otherwise}
\end{cases}
\]
for all \( z \in U \), where
\[
C_2(s, t) := \frac{4\pi^n \Gamma(1+t) \Gamma(s-t-n-1)}{\Gamma^2(s/2)}.
\]
3. Proof of Theorem 1.1: Part (i)

We begin with the following lemma.

**Lemma 3.1.** If $T_\alpha : L^p(\mathcal{U}) \to L^q(\mathcal{U})$ is bounded, then $p$ and $q$ must be related by

$$
\frac{1}{q} = \frac{1}{p} + \frac{\alpha}{n+1} - 1.
$$

**Proof.** Suppose that $T_\alpha : L^p(\mathcal{U}) \to L^q(\mathcal{U})$ is bounded, that is, there is a positive constant $C = C(p, q, n, \alpha)$ such that

$$
\|T_\alpha f\|_q \leq C\|f\|_p
$$

for all $f \in L^p(\mathcal{U})$.

Fix a function $f \in L^p(\mathcal{U})$, say $f(z) = |\rho(z, i)|^{-n-2}$, where $i = (0', i)$. For $t > 0$, we define the dilation $t \circ z$ by

$$
t \circ z = (t^2'z_n), \quad z = (z', z_n) \in \mathcal{U}.
$$

It is obvious that the dilations map $\mathcal{U}$ to $\mathcal{U}$. Now we consider the dilations $\delta^t(f)$ of $f$ given by

$$
\delta^t(f)(z) := f(t \circ z), \quad z \in \mathcal{U}.
$$

It is easy to verify that

$$
\|\delta^t(f)\|_p = t^{\frac{2(n+1)}{p}}\|f\|_p.
$$

Note that $\rho(t \circ z, t \circ w) = t^2\rho(z, w)$ holds for any $z, w \in \mathcal{U}$ and any $t > 0$. Making the change of variables $u = t \circ w$ in the integral defining $T_\alpha(\delta^t(f))$, we see that

$$
T_\alpha(\delta^t(f))(z) = \int_{\mathcal{U}} \frac{f(u)}{t^{-2\alpha}\rho(t \circ z, u)\alpha}t^{-2(n+1)} dV(u)
$$

and hence

$$
\|T_\alpha(\delta^t(f))\|_q = t^{2(\alpha - n - 1 - \frac{n+1}{p})}\|T_\alpha f\|_q.
$$

Replacing $f$ by $\delta^t(f)$ in (3.2) and using (3.3) and (3.4), we obtain

$$
t^{2(\alpha - n - 1 - \frac{n+1}{p})}\|T_\alpha f\|_q \leq C t^{\frac{-2(n+1)}{p}}\|f\|_p.
$$

Suppose now that $\frac{1}{q} < \frac{1}{p} + \frac{\alpha}{n+1} - 1$. We can write (3.5) as

$$
\|T_\alpha f\|_q \leq C t^{2(n+1)(\frac{1}{q} - \frac{1}{p} - \frac{\alpha}{n+1} + 1)}\|f\|_p
$$

and let $t \to \infty$ to obtain that $T_\alpha f = 0$, a contradiction. Similarly, if $\frac{1}{q} > \frac{1}{p} + \frac{\alpha}{n+1} - 1$, we could write (3.5) as

$$
t^{2(n+1)(\frac{1}{q} + \frac{\alpha}{n+1} - 1 - \frac{1}{p})}\|T_\alpha f\|_q \leq C\|f\|_p
$$

and let $t \to 0$ to obtain that $\|f\|_p = \infty$, again a contradiction. It follows that (3.1) must necessarily hold.

Now we turn to the proof of Part (i) of Theorem 1.1. We argue by contradiction. Suppose that $\alpha > n + 1$ and $T_\alpha : L^p(\mathcal{U}) \to L^q(\mathcal{U})$ is bounded. By Lemma 3.1, $p$ and $q$ must be related by

$$
\frac{1}{q} = \frac{1}{p} + \frac{\alpha}{n+1} - 1.
$$
Let $N$ be a positive integer such that $N > n + 1$ and consider the function

$$f_N(z) := \rho(z, i)^{-N}, \quad z \in \mathcal{U}.$$ 

By Lemma 2.5, we see that $f_N \in L^p(\mathcal{U})$ and

$$\|f_N\|_p = \left\{ \frac{4\pi^n \Gamma(pN - n - 1)}{\Gamma^2(pN/2)} \right\}^{1/p}.$$

Using Legendre’s duplication formula (see [1, p.26, (2.3.1)])

$$\Gamma(2x) = \frac{2^{2x-1}}{\sqrt{\pi}} \Gamma(x) \Gamma\left(x + \frac{1}{2}\right), \quad x \neq 0, -1, -2, \ldots$$

and the asymptotic formula for the Gamma function ([1, p.22, (2.1.9)])

$$\Gamma(x + a) \Gamma(x) = x^a \left[ 1 + O\left(x^{-1}\right) \right], \quad x \to +\infty,$$

we see that

$$\|f_N\|_p \sim 2^N \cdot N^{-\frac{\alpha}{q} - \frac{1}{2p}}, \quad \text{as } N \to \infty.$$

Also, by Lemma 2.4 we have

$$(T_\alpha f_N)(z) = \int_{\mathcal{U}} \frac{dV(w)}{\rho(z, w)^{\alpha} \rho(w, 1)^N} = \frac{4\pi^n \Gamma(N + \alpha - n - 1)}{\Gamma(N) \Gamma(\alpha)} \rho(z, i)^{n+1-\alpha-N}.$$ 

Thus, by Lemma 2.5 we obtain

$$\|T_\alpha f_N\|_q = \left\{ \frac{4\pi^n \Gamma(N + \alpha - n - 1)}{\Gamma(N) \Gamma(\alpha)} \right\} \left\{ \frac{4\pi^n \Gamma(q(N + \alpha - n - 1) - n - 1)}{\Gamma^2(q(N + \alpha - n - 1)/2)} \right\}^{1/q}.$$

Again, by (3.7) and (3.8), we get

$$\|T_\alpha f_N\|_p \sim 2^N \cdot N^{-\frac{\alpha}{q} - \frac{1}{2p} + \alpha - n - 1}, \quad \text{as } N \to \infty.$$ 

Since there exists a positive constant $C$ such that $\|T_\alpha f_N\|_q \leq C \|f_N\|_p$ for all $N > n + 1$, we can find another positive constant $C'$, independent of $N$, such that

$$2^N \cdot N^{-\frac{\alpha}{q} - \frac{1}{2p} + \frac{\alpha - n - 1}{q}} \leq C' \cdot 2^N \cdot N^{-\frac{\alpha}{q} - \frac{1}{2p}}$$

for all $N > n + 1$. Keeping (3.10) in mind, we see that

$$-\frac{n}{q} - \frac{1}{2q} + \alpha - n - 1 + \frac{n}{p} + \frac{1}{2p} = 12 \left( \frac{\alpha}{n + 1} - 1 \right),$$

and hence (3.11) can be rewritten as

$$N^{\frac{1}{q} - \frac{1}{2q} + \frac{\alpha - n - 1}{q}} \leq C'$$

for all $N > n + 1$. But this is impossible, since $\alpha > n + 1$. 
4. Proof of Theorem 1.1. Part (ii)

4.1. Necessity. As we have already shown in Lemma 3.1, if $T_\alpha$ is bounded from $L^p(U)$ to $L^q(U)$ then $p$ and $q$ satisfy

$$\frac{1}{q} = \frac{1}{p} + \frac{\alpha}{n+1} - 1.$$ 

It remains to show that $T_\alpha$ is unbounded in the endpoint cases $(p, q) = (1, \frac{n+1}{n+1-\alpha})$ and $(p, q) = (n+1, \infty)$. We only consider the former case, since, once this case is done, the other case follows by duality.

Consider the function

$$f(z) := \frac{1}{|\rho(z, i)|^{2n+2}}, \quad z \in U.$$ 

Then $f \in L^1(U)$, by Lemma 2.5.

It is clear that, for any fixed $z \in U$, the function $g_z(\cdot) := \rho(\cdot, z)^{-\alpha}$ is in $H^\infty$. Hence $B\rho g_z = g_z$, in view of Lemma 2.3. In particular, $B\rho g_z(i) = g_z(i)$, that is,

$$(4.1) \quad \frac{n!}{4\pi^n} \int_U \rho(w, z)^{-\alpha} \frac{\rho(i, i)^{n+1}}{|\rho(i, w)|^{2n+2}} dV(w) = \rho(i, z)^{-\alpha}.$$

Taking the complex conjugate of both sides of (4.1), we obtain

$$(T_\alpha f)(z) = \frac{4\pi^n}{n!} B\rho g_z(i) = \frac{4\pi^n}{n!} g_z(i) = \frac{4\pi^n}{n!} \rho(z, i)^{-\alpha}, \quad z \in U.$$ 

Hence, according to Lemma 2.3

$$\|T_\alpha f\|_{L^{\frac{n+1}{n+1-\alpha}}} = \left(\frac{4\pi^n}{n!}\right)^{\frac{n+1}{n}} \int_U |\rho(z, i)|^{n+1} = +\infty.$$ 

This show that $T_\alpha$ does not send $L^1(U)$ into $L^{\frac{n+1}{n+1-\alpha}}(U)$.

4.2. Sufficiency. The case $\alpha = n+1$ is well-known (see for instance [3, Lemma 2.8]).

Suppose now that $0 < \alpha < n+1$ and (1.1) hold. Put $Q_\alpha(z, w) = \rho(z, w)^{-\alpha}$.

For any fixed $w \in U$, by Lemma 2.4 we have

$$\|Q_\alpha(\cdot, w)\|_{L^{\frac{n+1}{n+1-\alpha}}, \infty} = \sup_{\lambda > 0} \lambda \cdot \left\{z \in U : |Q_\alpha(z, w)| > \lambda\right\}^{\frac{1}{n+1-\alpha}}$$

$$= \sup_{\lambda > 0} \lambda \cdot \left\{z \in U : |\rho(z, w)| < \lambda^{-\frac{1}{\alpha}}\right\}^{\frac{1}{n+1-\alpha}}$$

$$\leq \sup_{\lambda > 0} \lambda \cdot \left(\frac{2n+1}{(n-1)!}\right)^{\frac{n+1}{n}} \frac{\lambda^{-\frac{n+1}{\alpha}}}{\lambda^{-\frac{1}{\alpha}}}$$

$$= \left(\frac{2n+1}{(n-1)!}\right)^{\frac{n+1}{n}} \frac{n+1}{\alpha}$$

for all $w \in U$. By symmetry,

$$\|Q_\alpha(z, \cdot)\|_{L^{\frac{n+1}{n+1-\alpha}}, \infty} \leq \left(\frac{2n+1}{(n-1)!}\right)^{\frac{n+1}{n}} \frac{n+1}{\alpha}$$

for all $z \in U$. Therefore, $T_\alpha$ is bounded from $L^p(U)$ to $L^q(U)$, by Lemma 2.2.
C. LIU, J. SI, AND P. HU

REFERENCES

[1] R. Beals and R. Wong, Special Functions: A Graduate Text. Cambridge Studies in Advanced Mathematics, 126. Cambridge University Press, Cambridge, 2010.

[2] G. Cheng, X. Fang, Z. Wang and J. Yu. The hyper-singular cousin of the Bergman projection. Trans. Amer. Math. Soc. 369 (2017), no. 12, 8643–8662.

[3] R. R. Coifman and R. Rochberg, Representation theorem for holomorphic and harmonic functions in $L^p$, pp. 12-66, Astérisque, 77, Soc. Math. France, Paris, 1980.

[4] S. G. Krantz, Explorations in Harmonic Analysis: with Applications to Complex Function Theory and the Heisenberg Group. Birkhäuser Boston, Inc., Boston, MA, 2009.

[5] C. Liu, Y. Liu, P. Hu and L. Zhou. A class of integral operators over the Siegel upper half-space, arXiv e-print (arXiv:1701.04074), 2017.

[6] W. Rudin, Function Theory in the Unit Ball of $\mathbb{C}^n$, Reprint of the 1980 Edition, Springer, Berlin 2008.

[7] E. M. Stein, Harmonic Analysis: Real-Variable Methods, Orthogonality, and Oscillatory Integrals. Princeton University Press, Princeton, NJ, 1993.

[8] T. Tao, An Epsilon of Room, I: Real Analysis. Pages from Year Three of a Mathematical Blog. Graduate Studies in Mathematics, 117. American Mathematical Society, Providence, RI, 2010.

[9] K. Zhu, Operator Theory in Function Spaces. Second edition. Mathematical Surveys and Monographs, 138. American Mathematical Society, Providence, RI, 2007.

School of Mathematical Sciences, University of Science and Technology of China, Hefei, Anhui 230026, People’s Republic of China., and, Wu Wen-Tsun Key Laboratory of Mathematics, USTC, Chinese Academy of Sciences

E-mail address: cwlui@ustc.edu.cn

School of Mathematical Sciences, University of Science and Technology of China, Hefei, Anhui 230026, People’s Republic of China.

E-mail address: sijiajia@mail.ustc.edu.cn

College of Mathematics and Statistics, Shenzhen University, Shenzhen, Guangdong 518060, People’s Republic of China.

E-mail address: pyhu@szu.edu.cn