Vacuum Energy Estimates in Quantum Gravity and the

Wheeler-DeWitt Equation

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Abstract

In the Wheeler-DeWitt framework, by a gauge fixing procedure, we set up a scheme to recover a Schrödinger type equation, living in the orbits space with the lapse function as evolution parameter. By means of the associated stationary equation, we have the possibility of calculating quantum corrections to classical quantities. The Schwarzschild wormhole case is discussed as an example of application.

I. INTRODUCTION

Any attempt to apply the fundamentals of Quantum Mechanics to a complicated theory such as gravity is of central importance; as regards this problem it is enough thinking of any kind of physical situation whatsoever to convince ourselves that the Quantum Principles constitute an intrinsic property of nature. However a satisfactory model of Quantum Gravity is still far from realization. Nevertheless, the basic principles that lead to a quantization scheme for theories such as, e.g., quantum electrodynamics, are very encouraging; in particular the Feynman sum over histories is a powerful method to deal with quantities like two point correlation functions, propagators, etc . . . . Although the sum over histories maintains
a covariant form, we encounter two problems with this approach (we omit renormalization and the unboundedness from below problems):

a) Due to the integration over $4D$, one looses any meaning for time.

b) If we decide to develop our formalism in the Euclidean time, some topologies suggest that the manifold has to be compact or compactified; usually this means that every result has to be understood from a thermodynamical point of view, i.e., at equilibrium: when time ceases to have a meaning.

In both cases, the result is the disappearance of the dynamics. No time evolution can be recognized or extracted. On the other side, the canonical procedure, via the hamiltonian formulation, introduces a natural splitting between space and time, described by the lapse and shift functions, denoted as

\[(N, N^i) = N^\mu.\]  \hspace{1cm} (1)

respectively. In terms of these functions, the line element becomes

\[ds^2 = g_{\mu\nu}(x) \, dx^\mu \, dx^\nu = (-N^2 + N_i N^i) \, dt^2 + 2N_j dtdx^j + g_{ij} dx^i dx^j.\]  \hspace{1cm} (2)

Unfortunately, when we make this choice, every dynamical quantity is constrained to zero, both at the classical and at the quantum level. The quantum constraint for the lapse function is known as Wheeler-DeWitt equation (WDW) and is represented by the equation:

\[\mathcal{H}\Psi = 0.\]  \hspace{1cm} (3)

Once again, any time information is entangled inside the formalism. Accordingly, any possible energy definition is meaningless within this framework. Since a basic feature of a Quantum Theory is the existence of a Ground State and this is related to our capability of defining and calculating energy, one sees the necessity of a Schrödinger equation. Actually, within the WDW framework, it is possible to recover an internal time by a temporary suspension of the constraint and it is well known from Quantum Cosmology that this procedure
gives rise to a Schrödinger equation. In practice, to see how things work, we introduce a
gauge fixing that foliates spacetime. But it is well known that the consequence of a gauge
fixing is the appearance of the Faddeev-Popov determinant and of an integration over the
volume group. The volume group integration represents the sum over all possible lapses
and this restores the gauge invariance. Anyway, this is not enough, because we have a very
large variety of configurations available from the 3-space, and for this reason, we focus our
attention on wormhole physics, in the sense of ref. [3]. To examine how wormholes affect the
ground state energy configuration, we suggest that matter be excluded from this picture:
although this representation looks unreal, because matter is intimately related to gravity,
it seems to be important understanding the effect of pure gravity on some primordial con-
figurations of the Universe. To this aim, it has been suggested that Wheeler’s spacetime
foam could be a good candidate for the gravitational ground state [3]. It is clear, now,
how wormholes can enter into the vacuum state: spacetime foam can be approximated by
a multi-wormholes configuration. However, in this paper we begin with the analysis of a
single wormhole to understand what kind of consequences on the energy defined on every
space-time slice we can observe. But, how can we probe vacuum? Despite the covariant
philosophy, we use the Hamiltonian in its unconstrained version. The plan of the paper is
the following: in section [II], we introduce the path integral representation; in section [III], we
propose the wave functional needed to study the possible ground state; in section [IV], we
show the properties of the variational calculation, in section [V], we summarize and conclude.

II. PATH INTEGRAL REPRESENTATION

If we take a glance to (3), we see that the path integral expression

\[
\int \mathcal{D}g_{\mu\nu} \exp \left( i S_g \left[ g_{\mu\nu} \right] \right) \quad \text{(Lorentzian)}
\]

\[
\int \mathcal{D}g_{\mu\nu} \exp \left( -I_g \left[ g_{\mu\nu} \right] \right) \quad \text{(Euclidean)}
\]

(4)
generates solutions to the WDW equation and the momentum constraints
\nonumber H \Psi = 0 \quad \mathcal{H} \Psi = 0. \hspace{1cm} (5)

In (4), \( S_g [g_{\mu \nu}] \) \( I_g [g_{\mu \nu}] \) is the Lorentzian (Euclidean) Einstein-Hilbert action for gravity that is stationary under variations of the metric vanishing on the boundary

\begin{align}
S_g [g_{\mu \nu}] &= \frac{1}{16 \pi G} \int_M d^4x \sqrt{-g} R + \frac{1}{8 \pi G} \int_{\partial M} d^3x \sqrt{h} K \quad \text{(Lorentzian)} \nonumber \hspace{1cm} (6) \\
I_g [g_{\mu \nu}] &= -\frac{1}{16 \pi G} \int_M d^4x \sqrt{\bar{g}} R - \frac{1}{8 \pi G} \int_{\partial M} d^3x \sqrt{h} K \quad \text{(Euclidean)}.
\end{align}

Consider, now, expression (4), in its lorentzian form: the wave function constructed from the path integral is represented, explicitly, by [4]:

\[ \Psi[\bar{g}_{ij}] = \int_\gamma D N \int D g_{ij} D \pi^{ij} \Delta [g_{ij}, \pi^{ij}, N] \delta (\dot{N} - \chi (g_{ij}, \pi^{ij}, N)) \exp (i S [g_{ij}, \pi^{ij}, N]), \] \hspace{1cm} (7)

where \( S \) is the action, \( N \) is the lapse function and \( \chi \) is an arbitrary function entering the gauge-fixing condition in the argument of the delta function. \( \Delta \) is the associated Faddeev-Popov determinant. Equation (4) defines an invariant class of paths to integrate over. Suppose to consider the gauge \( \chi = 0 = \dot{N} \), then \( \Delta \) may be shown to equal a constant and after integrating over the momenta \( \pi^{ij} \), the wave function becomes

\[ \Psi [\bar{g}_{ij}] = \int_\gamma D N \int D g_{ij} \exp (i S [g_{ij}, N]); \] \hspace{1cm} (8)

the integral over \( g \) defines a wave function \( \Psi [\bar{g}_{ij}, N] \) which satisfies the familiar Schrödinger equation

\[ \mathcal{H} \Psi = i \frac{\partial \Psi}{\partial N}. \] \hspace{1cm} (9)

For completeness, we report eq.(4) in the Euclidean signature

\[ \mathcal{H} \Psi = -\frac{\partial \Psi}{\partial N}. \] \hspace{1cm} (10)

The remaining integral is over the constant value of \( N \)

\[ \Psi [\bar{g}_{ij}] = \int_\gamma dN \Psi [g_{ij}, N]. \] \hspace{1cm} (11)
\( \Psi \left[ \tilde{g}_{ij} \right] \), defined by (11), will satisfy the WDW equation, if the range is chosen to be from \(-\infty \) to \(+\infty \), or if the \( N \) contour is closed. The action takes the form

\[
S \left[ g_{ij}, N \right] = \frac{1}{16\pi G} \int_0^1 dt N \int d^3x \sqrt{g} \left[ K_{ij} K^{ij} - K^2 - \frac{3}{2} R \right],
\]

where \( K_{ij} = \frac{1}{2N} \left( N_{ij} + N_{ji} - g_{ij,0} \right) \) is called the second fundamental form and \( K = g^{ij} K_{ij} \) is the trace associated to \( K_{ij} \). The range of the \( t \) integration may be taken from 0 to 1, by shifting \( t \) and by scaling the lapse function. Consider the WDW operator introduced in (3) or in its unconstrained form (9), whose expression is

\[
\mathcal{H} = G^{ijkl} \frac{\pi^{ij} \pi^{kl}}{\sqrt{g}} - \sqrt{g} R,
\]

where

\[
G^{ijkl} = g_{ik} g_{jl} + g_{il} g_{jk} - 2 g_{ij} g_{kl}
\]
is called the inverse supermetric. Since the scalar curvature behaves as a potential and since we are interested to the Ground State problem, we perform an expansion of the 3-metric around a fixed background: in particular, let us choose to work with stationary spherical symmetric metrics. If we consider the expansion up to the second order, (13) becomes a harmonic oscillator. Therefore, after some algebraic manipulations we obtain

\[
\mathcal{H} = G^{ijkl} \frac{\pi^{ij} \pi^{kl}}{\sqrt{g}} - \sqrt{g} G^{ijkl} h_{\alpha\beta} O_{\alpha\beta}^{jk} h_{\gamma\delta},
\]

where \( O_{\alpha\beta}^{jk} \) is an operator containing second order derivatives and

\[
G^{ijkl} = g^{ik} g^{jl} + g^{il} g^{jk} - 2 g^{ij} g^{kl}
\]
is called the supermetric (an explicit expression and derivation of (14) being discussed in [1]). This implies that the wave function, satisfying the approximated problem, has to be changed too. By defining

\[
\Psi \left[ \tilde{g}_{ij} \right],
\]

Since we are working with static configurations, all contributions coming from the temporal part, i.e. from the Kinetic term, have to be integrated out from the space, but not from the path integral.
\[ \omega = \frac{1}{16\pi G} \int d^3 x \sqrt{g} \left[ K_{ij} K^{ij} - K^2 \right] = \frac{1}{16\pi G} \int d^3 x \sqrt{g} \left[ G^{ijkl} K_{ij} K_{kl} \right], \] (15)

and taking

\[ K_{ij} = -\frac{1}{2N} \hat{g}_{ij} \simeq -\frac{1}{2N} \hat{h}_{ij}, \] (16)

we obtain

\[ \omega = \frac{1}{16\pi G} \int d^3 x \sqrt{g} \left[ G^{ijkl} \frac{1}{4N^2} \hat{h}_{ij} \hat{h}_{kl} \right]. \] (17)

In (16), the compatibility with the choice of the background metric is evident. We recall, in fact, that the spherical symmetric metrics (s.s.m.) have their (non-zero) elements on the principal diagonal, and it seems natural that along non principal directions something will fluctuate. This choice is completely in agreement with the homogeneous minisuperspace framework. As to eq. (17), we can see quantum fluctuations entering it, and this means that \( \omega \) is a functional of \( h \); actually, when we consider the operator appearing in (14), it is immediate to recognize that, if we solve the following eigenvalue problem

\[ O^a_j h^i_a = \lambda h^i_j, \] (18)

a basis of eigenfunctions can be used to express the fields: so that the integration in (17) is justified.

On the previous grounds, the wave function

\[ \int \mathcal{D} g_{ij} \exp \left( i \left( \int_0^1 dt N \omega - \frac{1}{16\pi G} \int_0^1 dt N \int d^3 x \sqrt{g} \left( \frac{3}{2} R \right) \right) \right) \] (19)

becomes

\[ \exp iN \omega \int \mathcal{D} g_{ij} \exp -i \left( \frac{1}{16\pi G} \int d^3 x \sqrt{g} \left( \frac{3}{2} R \right) \right) = \exp iN \omega \Phi (N, g_{ij}). \] (20)

2The operator appearing in (14) is formally the same that we find when a one loop expansion is performed around a certain fixed background in 4D. Nevertheless, in this approach, the associated equation (18) is defined in the three dimensional subspace of the initial covariant space.
However, by considering rescaled fields with respect to the lapse function, we obtain

\[ \exp iN\omega \Phi (N, g_{ij}) = \exp iN\omega \Phi \left( g'_{ij} \right), \]

where \( g'_{ij} = g_{ij}/\sqrt{N} \).

**Remark** - Actually, equation (14) together with eq.(18) contain a substructure that comes up when a convenient decomposition is made with the help of ultralocal metric. Indeed, ultralocal metric is a more general kind of supermetric and within such a framework we can distinguish three types of operators describing spin 2, spin 1 and spin 0 fields; each of these components is orthogonal to the others in the sense of ultralocality [1].

**III. THE SCHRÖDINGER EQUATION: ANOTHER POINT OF VIEW**

Instead of attacking the problem of the solutions of the WDW equation and consequently of the associated Schrödinger equation, we observe that

\[ \mathcal{H}\Psi [\tilde{g}_{ij}] = 0 = \int_\gamma dN \left( i\partial\Psi [g_{ij}, N] / \partial N \right). \]  

(22)

is not forbidden by (3), provided the integration contour is invariant under reparametrization: and this is true for closed contours or infinite end points. While the r.h.s. of (22) vanishes because of boundary conditions, the l.h.s. of the same equation needs a specification: if we have a look at (22), we immediately see that, by defining

\[ \Psi [\bar{g}_{ij}] := \int_\gamma dN\Psi [g_{ij}, N], \]

(23)

we have formally

\[ \mathcal{H}\Psi [\bar{g}_{ij}, N] = i\frac{\partial\Psi [g_{ij}, N]}{\partial N}, \]

(24)

which is a Schrödinger equation; and the invariance under time reparametrization survives if and only if the sum over all lapses of \( \Psi [\bar{g}_{ij}, N] \) recovers the WDW equation. Suppose, now, to separate the variables
\[ \Psi [\bar{g}_{ij}, N] = F(N) \Phi (g_{ij}) ; \quad (25) \]

then (24) splits into:

\[ \mathcal{H} \Phi [g_{ij}] = \omega \Phi [g_{ij}] \]

\[ i \frac{dF[N]}{dN} = \omega F[N] \]

and (26) becomes

\[ \Psi [\bar{g}_{ij}, N] = \exp -iN\omega \Phi (g_{ij}) , \]

which is eq.(21), up to a phase factor. Taking advantage of this temporary separation between lapse and three-fields, one immediately sees a procedure for calculating the spectrum of \( \mathcal{H} \) at fixed \( N \). That problem was studied in [2]. Our present purpose is understanding the vacuum structure, and in section II an assumption about the ground state was made by us. In other words, instead of solving directly the stationary equation, we adopt a variational procedure with gaussian wave functional as trial functional.

**IV. THE GAUSSIAN WAVE FUNCTIONALS: AN APPLICATION TO THE SCHWARZSCHILD WORMHOLE**

Let us recall the basic rules on gaussian wave functionals.

The action of the operator \( h_{ij} \) on \( \Phi ) = \Phi \left[ h_{ij} \left( \vec{x} \right) \right] \) is realized by

\[ h_{ij} (x) |\Phi \rangle = h_{ij} (x) \Phi \left[ h_{ij} \left( \vec{x} \right) \right] . \]

(27)

The action of the operator \( \pi_{ij} \) on \( \Phi \), in general, is

\[ \pi_{ij} (x) |\Phi \rangle = -i \frac{\delta}{\delta h_{ij} (x)} \Phi \left[ h_{ij} \left( \vec{x} \right) \right] . \]

(28)

The inner product is defined by the functional integration:

\[ \langle \Phi_1 | \Phi_2 \rangle = \int \left[ Dh_{ij} (x) \right] \Phi_1^* \left\{ h_{ij} \right\} \Phi_2 \left\{ h_{kl} \right\} , \]

(29)
and the energy eigenstates satisfy the Schrödinger equation:

$$\int d^3x \mathcal{H} \left\{ -i \frac{\delta}{\delta h_{ij}(x)} , h_{ij}(x) \right\} \Phi \left\{ h_{ij}(\mathbf{x}) \right\} = \omega \Phi \left\{ h_{ij}(\mathbf{x}) \right\},$$

(30)

where $\mathcal{H} \left\{ -i \frac{\delta}{\delta h_{ij}(x)} , h_{ij}(x) \right\}$ is the Hamiltonian density. Instead of solving (30), which is of course impossible, we can formulate the same problem by means of a variational principle. We demand that

$$\frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} = \int \left[ \mathcal{D} g_{ij}(x) \right] \int d^3x \Phi^* \left\{ g_{ij}(x) \right\} \mathcal{H} \Phi \left\{ g_{kl}(y) \right\}$$

(31)

be stationary against arbitrary variations of $\Phi \left\{ h_{ij}(\mathbf{x}) \right\}$. The form of $\langle \Phi | H | \Phi \rangle$ can be computed as follows. We define normalized mean values

$$\bar{g}_{ij}(x) = \frac{\int \left[ \mathcal{D} g_{ij}(x) \right] \int d^3x g_{ij}(x) \Phi \left\{ g_{ij}(x) \right\} |^2}{\int \left[ \mathcal{D} g_{ij}(x) \right] | \Phi \left\{ g_{ij} \right\} |^2},$$

(32)

$$\bar{g}_{ij}(x) \bar{g}_{kl}(x) + K_{ijkl}(\mathbf{x}, \mathbf{y}) = \frac{\int \left[ \mathcal{D} g_{ij}(x) \right] \int d^3x g_{ij}(x) g_{kl}(y) \Phi \left\{ g_{ij}(x) \right\} |^2}{\int \left[ \mathcal{D} g_{ij}(x) \right] | \Phi \left\{ g_{ij} \right\} |^2}. $$

(33)

It follows that

$$\int \left[ \mathcal{D} h_{ij}(x) \right] (g_{ij}(x) - \bar{g}_{ij}(x)) \Phi \left\{ g_{ij}(x) \right\} |^2 = 0$$

by translation invariance of the measure

$$\int \left[ \mathcal{D} h_{ij}(x) \right] h_{ij}(x) \Phi \left\{ g_{ij}(x) + \bar{g}_{ij}(x) \right\} |^2 = 0$$

$$\implies \int \left[ \mathcal{D} h_{ij}(x) \right] h_{ij}(x) \Phi \left\{ h_{ij}(x) \right\} |^2 = 0$$

(34)

and (33) becomes

$$f \left[ \mathcal{D} h_{ij}(x) \right] \int d^3x h_{ij}(x) h_{kl}(y) \Phi \left\{ h_{ij}(x) \right\} |^2 =$$

$$K_{ijkl}(\mathbf{x}, \mathbf{y}) \int \left[ \mathcal{D} h_{ij}(x) \right] | \Phi \left\{ h_{ij} \right\} |^2$$

(35)

Rather than applying the variational principle arbitrarily, the gaussian Ansatz is used, according to which in the beginning of this calculus one has to replace the previous general formulas by
\[ \Phi_\alpha [h_{ij} (\vec{x})] = \mathcal{N} \exp \left\{ \frac{-N}{4l_p^2} \left( (g - \bar{g}) K^{-1} (g - \bar{g}) \right)_{x,y} + \ldots \right\}. \]  

(36)

The notation \( \langle \cdot , \cdot \rangle_{x,y} \) means that we are integrating over \( x \) and \( y \) in the volume and considering only the physical fields are considered, that is, traceless and divergenceless fields (spin 2). Actually, in (36) we should have to consider the rescaled fields with respect to the lapse function, like in (21). With this choice and with formulae (34, 35), the one loop-like Hamiltonian can be written as

\[ \omega^\perp = \frac{1}{4l_p^2} \int_{\mathcal{M}} d^3 x \sqrt{g} G^{ijkl}_\alpha \left[ K^{-1 \perp} (x, x)_{ijkl} + (\Delta_2)^a_j K^\perp (x, x)_{iakl} \right] \]  

(37)

where the first term in square brackets comes from the kinetic part, while the second comes from the expansion of the \( ^3R \) up to second order; \( G^{ijkl}_\alpha \) is called the ultralocal metric which describes the WDW metric with the choice \( \alpha = 1 \) and \( l_p \) is the Planck length. The Green function \( K^\perp (x, x)_{iakl} \) can be represented as

\[ K^\perp (x, x)_{iakl} := \sum_N \frac{h^\perp_{ia} (x) h^\perp_{kl} (y)}{2\lambda_N (p)}, \]  

(38)

where \( h^\perp_{ia} (x) \) are the eigenfunctions relative to (18), and \( \lambda_N (p) \) are infinite variational parameters. In formula (37) we have written the Spin 2 contribution to the energy density alone; expressions like (37) exist for the Spin 1 and Spin 0 terms of \( \mathcal{H} \). As an application we consider the energy evaluations coming from the Schwarzschild and Flat background. If

\[ g_{ij} = g_{ij}^{(S)} + h_{ij} \]  

(39)

\[ g_{ij} = g_{ij}^{(F)} + h_{ij}, \]

then the stationary Schrödinger equations are respectively

\[ \mathcal{H}_S \Phi (h_{ij}) = \omega_S \Phi (h_{ij}) \]  

(40)

for the curved background, and

\[ \mathcal{H}_F \Phi (h_{ij}) = \omega_F \Phi (h_{ij}) \]  

(41)
for the flat one. Due to the ultraviolet divergences, it is better to calculate the difference between (40) and (41):
\[
(\mathcal{H}_S - \mathcal{H}_F) \Phi (h_{ij}) = (\omega_S - \omega_F) \Phi (h_{ij}).
\] (42)

Since we are performing a difference between two different manifolds and this operation is ill defined, the same evaluation can be done by approaching flat space in the limit of $M$ tending to zero. Then the result, up to quadratic order, is:
\[
\omega_S - \omega_F = \Delta \omega (M) = -\frac{V}{2\pi^2} \left( \frac{3MG}{r_0^3} \right)^2 \frac{1}{16} \ln \left( \frac{\Lambda^2}{3MG r_0} \right),
\] (43)
where $M$ is the Schwarzschild mass parameter, $V$ is the volume of the system, $\Lambda$ is the UV cut-off and $r_0$ represents the closest radius reachable compatible with quantum effects. In terms of the wormhole volume eq.(43) becomes
\[
\Delta \omega (M) = -\frac{V}{2\pi^2} \left( \frac{4\pi MG}{V_S} \right)^2 \frac{1}{16} \ln \left( \frac{V_S \Lambda^2}{4\pi MG} \right).
\] (44)

V. CONCLUSION, OPEN PROBLEMS AND OUTLOOKS

Although the framework presented in section III and IV need more rigorous arguments, equations (26) and (21) define a calculation scheme useful for the energy estimates. Indeed, what we have built is a calculation framework useful to estimate the contribution of the three field quantum fluctuation around a classical solution of the Einstein equations, even though, when we turn to the Hamiltonian, the lapse is more general. It is important to point out that in previous formulae no mention has been made to the possibility of having contributions from the asymptotic energy or $\text{ADM}$ mass, which in the particular case of the Schwarzschild background is identified with the parameter $M$ entering the line element expression. Since $E_{\text{ADM}}$ corresponds to the total mass, which is conjugate to the time separation at spatial infinity, a problem of matching with the time appears into eq.(3) . Indeed, by defining an appropriate asymptotic wavefunctional $\Psi = \Psi [T,^3G]$, which obeys the Schrödinger equation.

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we can observe a difference with respect to the wave functional used throughout the paper, which obeys eq. (45). According to ref. [6], a possibility is

\[ N \to T \quad \text{when} \quad r \to \infty. \]  

(46)

Then the complete wavefunction assumes the form

\[ \Psi[\bar{g}_{ij}] := \int_{\gamma} dN \Psi[g_{ij}, N] = \int_{\gamma} dN e^{-i(N\omega+TM)} \Phi(g_{ij}) \]  

(47)

and (44) becomes

\[ M - \frac{V}{2\pi^2} \left( \frac{4\pi MG}{V_s} \right)^2 16 \ln \left( \frac{V_s\Lambda^2}{4\pi MG} \right). \]  

(48)

Throughout this paper, apart a reparametrization invariance condition, no mention has been made about the lapse boundary conditions. The reason is that a part of this scheme is independent from the boundary choices: in fact, the average calculation performed with the help of the variational approach involves only the spatial part, and only at the end of this operation one recovers invariance by integrating over all possible lapses. At this point a conjecture on how thermal problems can be described by pure dynamics can be made. In particular, we could ask ourselves how the constraint equation (WDW) affects the Hawking temperature and in which way one can extract the characteristic relation \( T_{eq} = \frac{1}{8\pi M} \).

Let us consider this calculation scheme: because the eigenvalue obtained by the variational procedure is independent of the lapse, we could, in principle, compare the results (44) of the Lorentzian and Euclidean formulations; but it is well known that the introduction of an Euclidean time involves a temperature in the Schwarzschild sector and usually this is identified with the Hawking temperature to avoid the conical singularity. But in the Lorentzian signature we do not need introducing neither a temperature nor an imaginary time, which means that we can understand what is happening in this particular thermalization process. Another open issue is the appearing of a discrete spectrum in the Schwarzschild as well.
as in the Schwarzschild-DeSitter sector [5], together which the continuum one. Since the discrete spectrum was obtained by means of an Euclidean path integral, the usual interpretation of the associated eigenvalues is the possibility of having instability from hot flat space. From the point of view of this paper, it could be interesting to verify the existence of a discrete spectrum in this situation and to perform a comparison between the Euclidean and Lorentzian sectors [7].

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