Neutron Skin Thickness of $^{90}$Zr Determined By Charge Exchange Reactions

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Charge exchange spin-dipole (SD) excitations of $^{90}$Zr are studied by the $^{90}$Zr($p,n$) and $^{90}$Zr($n,p$) reactions at 300 MeV. A multipole decomposition technique is employed to obtain the SD strength distributions in the cross section spectra. For the first time, a model-independent SD sum rule value is obtained: $148 \pm 12$ fm$^2$. The neutron skin thickness of $^{90}$Zr is determined to be $0.07 \pm 0.04$ fm from the SD sum rule value.

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Proton and neutron distributions are among the most fundamental properties of nuclei. Proton distributions are precisely known from the charge distributions determined by electron scattering[1]. On the other hand, our knowledge of neutron distributions, which have been studied mainly by hadron-nucleus scattering, is limited because descriptions of strong interactions in nuclei are highly model-dependent[2]. Reliable neutron distributions will improve the understanding of the nucleus and nuclear matter[3,4,5,6,7]. Recent theoretical studies using the Skyrme Hartree-Fock (HF) and relativistic mean-field models[3,4,5,6,7,8,9] have shown that the neutron skin thickness, defined as the difference between the root mean square (rms) radii of the proton and neutron distributions, imposes a strict constraint on the neutron matter equation of state, which is an important ingredient in studies of neutron stars[5,6,9]. It is also known that the neutron skin thickness is strongly correlated with the nuclear symmetry energy[3,4,9,10]. Reliable neutron distributions are also needed for analyses of atomic parity violation experiments[10,11] and of pionic states in nuclei[12].

Several attempts have been made to determine neutron distributions[2,13,14,15,16,17]. Ray et al. analyzed proton elastic scattering on several nuclei at 800 MeV using impulse approximation and obtained a neutron thickness of $0.09 \pm 0.07$ fm for $^{90}$Zr[15]. The cross sections for excitation of isovector giant dipole resonances with alpha scatterings were measured at KVI and neutron skin thicknesses of $^{116,124}$Sn and $^{208}$Pb were obtained[17] with uncertainties of $\pm 0.12$ fm. Trzcińska et al. measured the strong-interaction effects on antiprotonic x-rays on various targets ranging from $^{16}$O to $^{238}$U and obtained a neutron skin thickness of $0.09$ fm for $^{90}$Zr with a statistical error of $\pm 0.02$ fm[16]. Unfortunately, these analyses are model-dependent. Parity violation electron scattering is a promising tool for observing the neutron distributions cleanly[17,18], although no data are available so far.

An alternative method for determining the neutron rms radius is provided by the model-independent sum rule strength of charge exchange spin-dipole (SD) excitations[13]. The operators for SD transitions are defined by

$$\hat{S}_\pm = \sum_{im\mu} t_\pm^i m_\mu Y_1^{\mu}(\hat{r}_i)$$

(1)

with the isospin operators $t_3 = t_z$, $t_\pm = t_x \pm it_y$. The model-independent sum rule is derived as

$$S_- - S_+ = \frac{9}{4\pi} (N\langle r^2 \rangle_n - Z\langle r^2 \rangle_p),$$

(2)

where $S_\pm$ are the total SD strengths. The mean square radii of the neutron and proton distributions are denoted as $\langle r^2 \rangle_n$ and $\langle r^2 \rangle_p$, respectively. Thus, the rms radius of the neutron distribution $\sqrt{\langle r^2 \rangle_n}$ or the neutron skin thickness $\delta_{np} = \sqrt{\langle r^2 \rangle_n} - \sqrt{\langle r^2 \rangle_p}$ can be derived from Eq. (2) by using the rms radius of the proton distribution $\sqrt{\langle r^2 \rangle_p}$ obtained from the charge radius if the sum rule value ($S_- - S_+$) is obtained experimentally.

To obtain the SD sum rule value, Krasznahorkay et al. measured the $(^3$He, $t$) reaction on tin isotopes at 450 MeV[21]. The cross section spectra at $\theta = 1^\circ$ were analyzed by a peak fitting technique assuming the Lorentzian shape, and the $S_-$ value was obtained. Since there was no $(n,p)$-type measurement, they determined the $S_+$ value by assuming the energy-weighted sum rule in a simple model where the unperturbed particle-hole (ph) energies are degenerate at a certain energy[17,20]. The overall normalization of the sum rule value was done by using the calculated result of neutron skin thickness of $^{120}$Sn. To determine the sum rule value model-independently, however, one needs to perform $(n,p)$-type experiments as well as to identify the $\Delta L = 1$ cross sections in the observed spectra. Then, one needs to relate the $\Delta L = 1$ cross sections to the SD strengths.

This paper represents the first attempt to extract the model-independent sum rule value from both $(p,n)$ and $(n,p)$ reactions on $^{90}$Zr. The dipole components of the cross section spectra are identified by multipole decomposition (MD) analysis[21] of the $^{90}$Zr($p,n$)[22] and $^{90}$Zr($n,p$)[23] data at 300 MeV. It should be noted that at 300 MeV, the spin-flip cross sections are large, while...
the distortion effects are minimal. Thus, the characteristic shapes of the angular distributions for each angular momentum transfer \(\Delta L\) are most distinct. The contribution of the non-spin dipole component is small due to the energy dependence of the effective interaction.

The MD analyses were performed on the \((p, n)\) and \((n, p)\) excitation energy spectra in a consistent manner to obtain various \(\Delta L\) components of the cross section. We use here the extracted \(\Delta L = 1\) cross section spectra given in Fig. 2 of Ref. \[23\]. The SD strengths are obtained by assuming a proportionality relation similar to that established for Gamow-Teller excitation. The proportionality relation between \(B(\text{SD})\) and the \(\Delta L = 1\) component of the cross section, \(\sigma_{\Delta L=1,\pm}(q, \omega)\), is given by

\[
\sigma_{\Delta L=1,\pm}(q, \omega) = \sigma_{\text{SD\pm}}(q, \omega)B(\text{SD\pm}),
\]

where \(\sigma_{\text{SD\pm}}(q, \omega)\) is the SD cross section per \(B(\text{SD\pm})\) and depends on the momentum transfer \(q\) and the energy transfer \(\omega\). The \(\sigma_{\Delta L=1}(q, \omega)\) data for the \((p, n)\) and \((n, p)\) channels were taken from the result of MD analysis at 4.6° and 4–5°, respectively. The \(\sigma_{\Delta L=1}(q, \omega)\) spectra at these angles are most sensitive to the SD cross sections since the corresponding momentum transfers are \(q = 0.3\)–0.4 fm\(^{-1}\) and thus, the \(\Delta L = 1\) cross sections take on maximum values.

Distorted wave impulse approximation (DWIA) calculations are used to obtain \(\sigma_{\text{SD\pm}}(q, \omega)\). The DWIA calculations are performed with the computer code DW81 \[27\] for the \(0^-, 1^-,\) and \(2^-\) transitions. The one-body transition densities are calculated from pure \(1p1h\) configurations. All 1\(\hbar\omega\) configurations were examined. The optical model potential (OMP) parameters are taken from Ref. \[21\] and \[22\]. The effective \(NN\) interaction is taken from the \(t\)-matrix parameterization of the free \(NN\) interaction by Franey and Love at 325 MeV \[24\]. The radial wave functions are generated from a Woods-Saxon potential \[29\], adjusting the depth of the central potential to reproduce the binding energies. Details of the calculations are found in Refs. \[21\]–\[23\].

The calculated SD\(\pm\) unit cross sections at \(\theta = 4.6^\circ\) and \(\omega = 0\) MeV are 0.28 \(\pm\) 0.03, 0.24 \(\pm\) 0.06, and 0.29 \(\pm\) 0.05 mb/sr/fm\(^2\) for the \(0^-, 1^-,\) and \(2^-\) transitions, respectively. The uncertainty indicated for each multipole is the dependence of the cross sections on the \(ph\) configurations. Gaarde et al. pointed out that the tensor term of the effective \(NN\) interaction affects the proportionality among the multipole in the study of SD excitations in the \(^{12}\text{C}(p, n)\) reaction at 160 MeV. At 300 MeV, however, the ratio of the amplitude of the isovector tensor interaction to that of the isoscalar spin (\(\sigma\tau\)) interaction at \(q = 0.3\)–0.4 fm\(^{-1}\) is smaller \[27\] so that the calculated unit cross sections are close to one another. To check the validity of the above calculations with pure \(1p1h\) configurations, the calculations of SD unit cross sections are performed by using the transition densities for several states obtained by the HF+RPA (random phase approximation) calculations \[31\]. The calculated unit cross sections are found to be consistent with the above values.

Since the main subject of this study is the total SD strength, rather than the individual strength of each transition, we use the averaged unit cross section of \(\sigma_{\text{SD\pm}}(4.6^\circ, 0\) MeV) = 0.27 mb/sr/fm\(^2\) in the analysis. Similarly, the unit cross section in the SD\(\pm\) channel is estimated to be \(\sigma_{\text{SD\pm}}(4–5^\circ, 0\) MeV) = 0.26 mb/sr/fm\(^2\). The unit cross sections are calculated at each energy bin of the cross section histogram. The systematic uncertainty in \(\sigma_{\text{SD\pm}}(q, \omega)\) due to the input parameters for DWIA calculations has been evaluated by using harmonic oscillator radial wave functions and by other sets of OMP \[24\]–\[29\]. The systematic uncertainty thus estimated is 14%.

The \(\frac{d\sigma}{dE}\) distributions obtained by using Eq. \(\text{(6)}\) are shown in Fig. 1. The horizontal axis in the \(\frac{d\sigma}{dE}\) spectrum is the excitation energy of the residual \(^{90}\text{Zr}\) nucleus. The SD\(\pm\) strength spectrum shows a dominant resonance structure centered at \(E_x = 20\) MeV and the strength extends to \(\sim 50\) MeV excitation. The \(\frac{d\sigma}{dE}\) distribution is shifted by \(+17\) MeV to account for the Coulomb displacement energy \[28\] and the nuclear mass difference. The SD\(\pm\) strength distribution forms a broad bump centered at 28 MeV (or 11 MeV in \(^{90}\text{Y}\)) with a width of \(\sim 15\) MeV.

Drożdż et al. studied the SD\(\pm\) strengths with the RPA model including the coupling between \(1p1h\) and \(2p2h\) states, though the calculation is not self-consistent, using a phenomenological Woods-Saxon potential to obtain the ground state wave function \[32\]. They found that the mixing of \(2p2h\) states results in a large asymmetric spread in the strength of the SD resonances, with
FIG. 2: Integrated charge exchange SD strengths. The upper panel shows the $S_-$ and $S_+$ spectra. The lower panel shows the $S_- - S_+$ spectrum.

about 30% of the total strength shifted to excitation energies above 28 MeV [32]. The curve in Fig. 1 shows the predicted strength distribution. This calculation gives a reasonable description of the strengths above 20 MeV although it overestimates the strengths at lower excitation energies below 15 MeV. To discuss the details of SD strength distributions and their relations to the neutron skin thickness, the self-consistent HF+RPA models should be employed where the same two-body interaction is used throughout the calculations.

The integrated SD strength,

$$S_\pm = \int_0^{E_x} \frac{dB(SD_\pm)}{dE} dE,$$  

(4)

is plotted in Fig. 2. While both integrated SD strengths increase steadily, the sum rule value, $S_- - S_+$, remains almost constant in the excitation energy range of 30–50 MeV, where it lies within $149 \pm 5$ fm$^2$. Since the MD analysis in the $(n, p)$ channel is unstable above an excitation energy of 40 MeV in Fig. 2, we integrate the strengths up to 40 MeV. The $S_- - S_+$ strength is $247 \pm 4$(stat.)$\pm 12$(MD) fm$^2$, where the statistical uncertainty and the uncertainty in the MD analysis are given.

The corresponding $S_+$ value is $98 \pm 4$(stat.)$\pm 5$(MD) fm$^2$, integrated to an excitation energy of 23 MeV in $^{90}$Y (40 MeV in Fig. 2). The sum rule value yields $S_- - S_+ = 148 \pm 6$(stat.)$\pm 7$(syst.)$\pm 7$(MD) fm$^2$, where the systematic uncertainty of the normalization in the cross section data (5%) is also included. The 14% uncertainty in the SD unit cross section is not included.

If this sum rule value is interpreted in terms of Eq. 2, we obtain $N(r^2)_n - Z(r^2)_p = 207 \pm 17$ fm$^2$, where the statistical and systematic uncertainties are combined in quadrature with the uncertainty in MD analysis. The rms radius of the proton matter in $^{90}$Zr is estimated to be 4.19 fm after correcting for the effect of the proton form factor from the charge radius [2]. The neutron skin thickness and the rms radius of the neutron distribution calculated from Eq. 2 are $\delta_{np} = 0.07 \pm 0.04$ fm and $\sqrt{\langle r^2 \rangle}_n = 4.26 \pm 0.04$ fm, respectively. The $\delta_{np}$ value obtained in the present study is consistent with that obtained from the analysis of proton elastic scattering (0.09$\pm$0.07 fm) [13] but with a smaller uncertainty. Our results also agree with the neutron skin thickness determined by antiprotonic x-ray measurements by Trzcińska et al. (0.09$\pm$0.02 fm) [16], though their analysis contains some assumptions whose uncertainties are not specified.

We note that the accuracy of $\sqrt{\langle r^2 \rangle}_n$ in this study is 1% level, which is the same as the goal of the parity violation experiment of electron scattering at Jefferson Laboratory [18]. To improve the reliability of the present analysis, the SD unit cross sections should be studied in the mass region around 90 by measuring the cross sections of the SD transitions whose strengths are known from decay measurements.

In summary, a consistent MD analysis of the $(p, n)$ and $(n, p)$ reaction data from $^{90}$Zr has been performed and SD strength distributions of both channels are obtained experimentally for the first time. The integrated strengths obtained experimentally are $S_- = 247 \pm 4$(stat.)$\pm 4$(syst.)$\pm 12$(MD) fm$^2$ and $S_+ = 98 \pm 4$(stat.)$\pm 4$(syst.)$\pm 5$(MD) fm$^2$, up to 40 MeV and 29 MeV, respectively. By using the two experimental sum rule values, the model-independent formula [2] yields a $N(r^2)_n - Z(r^2)_p$ value of $207 \pm 17$ fm$^2$, which corresponds to a neutron skin thickness of $\delta_{np} = 0.07 \pm 0.04$ fm. The method used in this work is applicable to heavy and medium heavy nuclei on which both the $(p, n)$ and the $(n, p)$ measurements are possible.

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