\textbf{Abstract}

\textbf{$B_s - \bar{B}_s$ Mixing constraints on FCNC and a non-universal $Z'$}

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\begin{abstract}

$B_s - \bar{B}_s$ mixing has been measured recently by D0 and CDF. The range predicted in the standard model is consistent with data. However, the standard model central values for $\Delta M_{B_s}$ and also $\Delta M_{B_d}$ are away from the data which may be indications of new physics. Using the observed values of $\Delta M_{B_s}$ and $\Delta M_{B_d}$ we study general constraints on flavor changing $Z'$ interactions. In models with non-universal $Z'$ couplings we find that significant enhancements over the standard model are still possible in the rare decay modes $B \rightarrow X_s \tau^+ \tau^-$, $B \rightarrow X_s \nu \bar{\nu}$ and $B_s \rightarrow \tau^+ \tau^-$. Tree level $Z'$ contributions to $K \rightarrow \pi \nu \bar{\nu}$ are now constrained to be very small, but one loop effects can still enhance the standard model rate by a factor of two.
\end{abstract}

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I. INTRODUCTION

The D0 and CDF collaborations have recently measured $B_s - \bar{B}_s$ mixing with the result

\[ \begin{align*}
D0 : & \quad 17 \text{ ps}^{-1} < \Delta M_{B_s}^{\text{exp}} < 21 \text{ ps}^{-1}, \\
CDF : & \quad \Delta M_{B_s}^{\text{exp}} = (17.33^{+0.42}_{-0.21} \pm 0.07) \text{ ps}^{-1}. \end{align*} \]

This last measurement is sufficiently precise to place new constraints on tree-level flavor changing neutral currents. In this paper we explore the consequences of these constraints in models with an additional gauge boson, a $Z'$. We first discuss general constraints and then specialize to the case of non-universal $Z'$ models. For our numbers we will use the CKMfitter [3] average including these measurements,

\[ \Delta M_{B_s}^{\text{exp}} = (17.34^{+0.49}_{-0.20}) \text{ ps}^{-1}. \] (2)

This mass difference is related to the $B_s - \bar{B}_s$ mixing parameter $M_{B_s}^{12}$ by $\Delta M_{B_s} = 2|M_{B_s}^{12}|$, if the lifetime difference is neglected. In the Standard Model, $M_{B_s}^{12}$ arises from the so called “box” diagram and is given by

\[ \begin{align*}
M_{B_s}^{12,\text{SM}} &= \frac{G_F^2}{12\pi^2} \eta_B m_{B_s} \xi^2 f_{B_d}^2 B_{B_d} m_W^2 S(x_t)(V_{ts}V_{tb}^*)^2, \\
S(x) &= \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3 \ln x}{2(1-x)^3}, \end{align*} \] (3)

where $x_t = m_t^2/m_W^2$, $\eta_B = 0.551 \pm 0.007$ is a QCD correction. The hadronic parameters $f_{B_d} = (0.191 \pm 0.027) \text{ GeV}$, $B_{B_d} = 1.37 \pm 0.14$ and $\xi = f_{B_s} \sqrt{B_s}/f_{B_d} \sqrt{B_d} = 1.24 \pm 0.04 \pm 0.06$ are obtained from lattice calculations [3]. This value of $f_{B_d}$ is in reasonable agreement with the recently observed branching ratio $B(B_d \to \tau \nu_\tau) = (1.06^{+0.3}_{-0.28} (\text{sat})^{+0.18}_{-0.16} (\text{syst}) \times 10^{-4} \ [6].$

To quantify the uncertainty in the input parameters to Eq. (3), we use the latest result from the CKMfitter overall fit (excluding the measurement) [3],

\[ (\Delta M_{B_s})_{\text{SM}} = 21.7^{+13.1}_{-9.1} \text{ ps}^{-1} \] (4)

where the errors indicate the $3\sigma$ range. Notice that the central value of this prediction is slightly higher than the measured mass difference, although the predicted and measured ranges are in good agreement.
This agreement between the SM prediction and the data places stringent constraints on new physics that will become more severe as the theoretical uncertainty is reduced. There are many models beyond the SM containing additional flavor changing sources which can be constrained by recent data on $\Delta M_{B_s}$ [5]. We concentrate here on the impact of the measured $\Delta M_{B_s}$ on non-universal $Z'$ models that are motivated by the apparent anomaly in the measurement of $A_{FB}^b$ at LEP [7,8,9]. These models are variations of left-right models in which the right-handed interactions single out the third generation with enhanced couplings to the $Z'$.

II. GENERIC BOUNDS ON NEW PHYSICS

We begin by considering a generic new physics contribution to $M^{B_s,N}_{12}$ that is real, and add it to the standard model 3-$\sigma$ range. By requiring this to overlap with the 3-$\sigma$ range for the measured $\Delta M_{B_s}$ we extract the allowed range for new physics. Normalized to the central value of the measured $\Delta M_{B_s}$ we find

$$\delta_{B_s} \equiv \frac{2M^{B_s,N}_{12}}{\Delta M_{B_s}^{exp}} \sim (-3 \text{ to } -1.7) \text{ or } (-1 \text{ to } 0.27),$$

as seen in Figure (1).}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{Fig1.png}
\caption{$\Delta M_{B_s}$ as a function of $\delta_{B_s}$ for a new physics contribution assumed to be real. We show a shaded band obtained by adding the new physics to the 3-$\sigma$ standard model range. The horizontal band corresponds to the 3-$\sigma$ experimental range.}
\end{figure}

Because the central value of the SM prediction is already larger than the measured mass difference, there is little room for a new contribution in phase with the standard model. A larger range is allowed for a new physics contribution opposite in sign to the SM. More
generally, $\delta_{B_s}$ is complex and we refer to cases in which the real part of $\delta_{B_s}$ has the same sign as (opposite sign to) the SM as having constructive (destructive) interference with the SM.

For a new physics contribution that is complex, we require that $2|M_{12}^{B_s,N} + M_{12}^{B_s,SM}|$ reproduce the measured mass difference. Once again we allow a 3-$\sigma$ range in both the SM prediction and the measurement. In Figure 2 we show the allowed ranges for $Re(\delta_{B_s})$ and $Im(\delta_{B_s})$ for two cases. In the first case we use the central value of the SM prediction, $\Delta M_{B_s} = 21.7$ $ps^{-1}$ whereas in the second case we use the full 3-$\sigma$ range $\Delta M_{B_s} = 12.6$ $ps^{-1}$ to $34.8$ $ps^{-1}$. We see again that there is very little room for a constructive new physics contribution.

FIG. 2: Constraints on $Re(\delta_{B_s})$ and $Im(\delta_{B_s})$ for a complex new physics contribution to $M_{12}^{B_s}$. The shaded regions indicate where the prediction falls within the 3-$\sigma$ experimental range on $\Delta M_{B_s}$ when we use (a) the central value of the SM prediction; and (b) the 3-$\sigma$ range for the SM prediction.

III. CONSTRAINTS ON GENERIC $Z'$ MODELS

We now restrict ourselves to the case where the new physics contributions originate in the exchange of $Z'$ bosons with flavor changing couplings. The flavor changing $Z'$ couplings can be written in general as

$$\mathcal{L} = \frac{g}{2c_W} \bar{q}_i \gamma^\mu (a_{ij} P_L + b_{ij} P_R) q_j Z'_\mu.$$  \hfill (6)

A tree-level exchange of the $Z'$ generates the effective Lagrangian responsible for neutral meson mixing (and in particular $B_s - \bar{B}_s$ mixing) at the $M_{Z'}$ scale,

$$\mathcal{L}_{Z'} = -\frac{G_F}{\sqrt{2}} \frac{M_Z^2}{M_{Z'}^2} [a_{ij}^2 O_{LL} + b_{ij}^2 O_{RR} + 2a_{ij}b_{ij} O_{LR}].$$  \hfill (7)
where the operators are given by
\[ O_{LL} = \bar{q}_i \gamma^\mu P_L q_j \bar{q}_i \gamma^\mu P_L q_j, \quad O_{RR} = \bar{q}_i \gamma^\mu P_R q_j \bar{q}_i \gamma^\mu P_R q_j, \]
\[ O_{LR} = \bar{q}_i \gamma^\mu P_L q_j \bar{q}_i \gamma^\mu P_R q_j, \quad \tilde{O}_{LR} = \bar{q}_i P_L q_j \bar{q}_i P_R q_j. \tag{8} \]

The operator \( \tilde{O}_{LR} \) does not appear directly in \( Z' \) exchange, but is induced by renormalization through mixing with \( O_{LR} \). Starting from an effective Lagrangian at the high energy scale \( m \) given by
\[ \mathcal{L} = a_{LL}(m)O_{LL} + a_{RR}(m)O_{RR} + a_{LR}(m)O_{LR} + \tilde{a}_{LR}(m)\tilde{O}_{LR}, \tag{9} \]
we find at a low energy scale \( \mu = m_b \) relevant to \( B_s \) mixing,
\[ \mathcal{L} = a_{LL}(\mu)O_{LL} + a_{RR}(\mu)O_{RR} + a_{LR}(\mu)O_{LR} + \tilde{a}_{LR}(\mu)\tilde{O}_{LR}. \tag{10} \]
At leading order in QCD RG running, the coefficients are \[ [10, 11] \]
\[
\begin{align*}
a_{LL}(\mu) &= a_{LL}(m)\eta_{LL}(\mu), & a_{RR}(\mu) &= a_{RR}(m)\eta_{RR}(\mu), \\
a_{LR}(\mu) &= a_{LR}(m)\eta_{LR}(\mu), & \tilde{a}_{LR}(\mu) &= \tilde{a}_{LR}(m)\tilde{\eta}_{LR}(\mu) + \frac{2}{3}a_{LR}(m)(\eta_{LR} - \tilde{\eta}_{LR}), \\
\eta_{LL}(\mu) &= (\eta_m)^{6/23}, & \eta_{RR}(\mu) &= (\eta_m)^{6/23}, \\
\eta_{LR}(\mu) &= (\eta_m)^{3/23}, & \tilde{\eta}_{LR}(\mu) &= (\eta_m)^{-24/23}. \tag{11} \end{align*}
\]
where \( \eta_m \equiv \alpha_s(m)/\alpha_s(\mu) \).

From the low energy effective Lagrangian one obtains the mass difference in terms of the “bag factors”,
\[ M_{12}^{P,Z'} = -\frac{1}{3} f_P^2 m_P B_p \left[ a_{LL}(\mu) + a_{RR}(\mu) + a_{LR}(\mu)(-\frac{3}{4} + \frac{\epsilon}{2}) + \tilde{a}_{LR}(\mu)(\frac{1}{8} - \frac{3\epsilon}{4}) \right], \tag{12} \]
where \( B_P = B_{LL} = B_{RR} = B_{LR} \) is the ratio between the matrix element \( \langle P|\bar{q}\gamma^\alpha\gamma_5 b\gamma_\mu\gamma_5 |P \rangle \) and its value in factorization. Similarly, \( \epsilon \) is defined as \( \epsilon = (\tilde{B}_{LR}/B_{LL})(m_P^2/(m_i + m_j)^2) \) where \( \tilde{B}_{LR} \) is the ratio between the matrix element \( \langle P|\bar{q}\gamma_5 b\gamma_5 |P \rangle \) and its value in factorization. We will use \( \epsilon = 1 \) for our numerical results.

With all this we finally obtain the new physics contribution to \( M_{12} \) from \( Z' \) exchange,
\[ M_{12}^{P,Z'} = \frac{G_F}{\sqrt{2}} \frac{m_Z^2}{m_{Z'}} \eta_{Z'}^{6/23} \frac{1}{3} f_P^2 M_P B_P \left( a_{ij}^2 + b_{ij}^2 \right) + \eta_{Z'}^{-3/23} \frac{1}{2} a_{ij} b_{ij} (2\epsilon - 3) + \frac{2}{3} (\eta_{Z'}^{-3/23} - \eta_{Z'}^{-30/23}) \frac{1}{4} a_{ij} b_{ij} (1 - 6\epsilon) \), \tag{13} \]
The mass difference $\Delta M^P$ is then obtained by adding the SM and new physics contributions,

$$\Delta M^P = 2 |M_{12}^{P,SM} + M_{12}^{P,Z'}|.$$  \hfill (14)

As mentioned before, the new physics contribution can be constructive or destructive with respect to the SM. In the case of $B_s$ mixing, if $a_{sb}$ and $b_{sb}$ are real relative to the CKM matrix element $V_{tb}^* V_{ts}$, the new contributions are constructive. We show in Figure 3 the allowed region for the parameters $a_{sb}$, and $b_{sb}$ (assumed to be real). The shaded region is obtained for a SM contribution allowed to vary in its 3-$\sigma$ range. For emphasis, the darker shaded region corresponds to $(\Delta M_{B_s})_{SM} = 12.6$ ps$^{-1}$ indicating the lower end of the theory range. Notice that for the central value of the SM prediction, $(\Delta M_{B_s})_{SM} = 21.7$ ps$^{-1}$, there is no room for constructive new physics.

![FIG. 3: Constraints on the flavor changing parameters $\hat{a}_{sb} \equiv (m_Z/m_{Z'}) a_{sb} \times 10^3$ and $\hat{b}_{sb} \equiv (m_Z/m_{Z'}) b_{sb} \times 10^3$. The shaded region is allowed when the SM contribution varies in its 3-$\sigma$ range. The darker shaded region corresponds to $(\Delta M_{B_s})_{SM} = 12.6$ ps$^{-1}$.](image)

A similar exercise can be done for $B_d$ mixing with the 3-$\sigma$ SM range from Ref. [3] as well as the HFAG experimental average [4]

$$\langle \Delta M_{B_d} \rangle_{SM} = 0.394^{+0.361}_{-0.162} \text{ ps}^{-1}$$
$$\langle \Delta M_{B_d} \rangle_{exp} = (0.507 \pm 0.004) \text{ ps}^{-1}.$$ \hfill (15)

Assuming that the new physics is real it will also interfere constructively with the SM.
Taking the central value of the SM prediction and requiring the total $\Delta M_{B_d}$ to fall within the 3-$\sigma$ experimental range leads to the allowed region shown in Figure 4.

![Figure 4](image-url)\[FIG. 4: Constraints on the flavor changing parameters $\hat{a}_{db} \equiv (m_Z/m_{Z'}) a_{db} \times 10^3$ and $\hat{b}_{db} \equiv (m_Z/m_{Z'}) b_{db} \times 10^3$. The shaded region indicates the 3-$\sigma$ experimental range on $\Delta M_{B_d}$ corresponding to the central value of the SM prediction.]

IV. A NON-UNIVERSAL Z' MODEL AND FLAVOR CHANGING PARAMETERS

We now apply the above constraints to a $Z'$ model with non-universal couplings. The non-universal $Z'$ models we consider here have been discussed in Ref. [7, 8, 9] motivated by the apparent anomaly in the measurement of $A_{FB}^b$ at LEP [12, 13]. The models are variations of left-right models in which the right-handed interactions single out the third generation giving enhanced $Z'$ couplings to the $b$ and $t$ quarks and to the $\tau$ and $\nu_\tau$ leptons. In general the models contain tree-level flavor changing neutral currents as well. We find that the new measurement of $\Delta M_{B_s}$ can place stringent constraints on some of the parameters of the model, but that there is still room for a substantial enhancement in the modes $B \to X_s \tau^+ \tau^-(\nu_\tau \bar{\nu}_\tau)$, $B_s \to \tau^+ \tau^-$, and also $K \to \pi \nu_\tau \bar{\nu}_\tau$. We briefly review the relevant aspects of the models and refer the reader to Ref. [7, 8, 9] for details.

In these models the first two generations are chosen to have the same transformation properties as in the standard model with $U(1)_Y$ replaced by $U(1)_{B-L}$,

$$Q_L = (3, 2, 1)(1/3), \quad U_R = (3, 1, 1)(4/3), \quad D_R = (3, 1, 1)(-2/3),$$
\[ L_L = (1, 2, 1)(-1), \quad E_R = (1, 1, 1)(-2). \] (16)

The numbers in the first parenthesis are the \( SU(3) \), \( SU(2)_L \) and \( SU(2)_R \) group representations respectively, and the number in the second parenthesis is the \( U(1) \) charge. For the first two generations the \( U(1) \) charge is the same as the \( U(1)_Y \) charge in the SM and for the third generation it is the usual \( U(1)_{B-L} \) charge of LR models. The third generation is chosen to transform differently,

\[
Q_L(3) = (3, 2, 1)(1/3), \quad Q_R(3) = (3, 1, 2)(1/3), \\
L_L(3) = (1, 2, 1)(-1), \quad L_R = (1, 1, 2)(-1).
\] (17)

The correct symmetry breaking and mass generation of particles can be induced by the vacuum expectation values of three Higgs representations: \( H_R = (1, 1, 2)(-1) \), whose non-zero vacuum expectation value (vev) \( v_R \) breaks the group down to \( SU(3) \times SU(2) \times U(1) \); and the two Higgs multiplets, \( H_L = (1, 2, 1)(-1) \) and \( \phi = (1, 2, 2)(0) \), which break the symmetry to \( SU(3) \times U(1)_em \).

The models contain flavor changing neutral currents at tree level that contribute to \( B_s \) mixing and other related flavor changing decays, the relevant interactions are [8],

\[
\mathcal{L}_Z = \frac{g}{2} \tan \theta_W (\tan \theta_R + \cot \theta_R) (\sin \xi_Z Z_\mu + \cos \xi_Z Z'_\mu) \\
\times \left( \bar{d}_{Ri} \gamma^\mu V_{R\bar{d}} \gamma^\nu V_{Rd} d_{Rj} - \bar{u}_{Ri} \gamma^\mu V_{R\bar{u}} \gamma^\nu V_{Ru} u_{Rj} + \bar{\tau}_{Ri} \gamma^\mu \tau_{Rj} - \bar{\nu}_{Ri} \gamma^\mu \nu_{Rj} \right)
\] (18)

In this expression \( g \) is the usual \( SU_L(2) \) gauge coupling, \( \theta_W \) the usual electroweak angle, \( \theta_R \) parametrizes the relative strength of the right-handed interactions, \( \xi_Z \) is the \( Z-Z' \) mixing angle and \( V_{R\bar{u}}^{u,d} \) are the unitary matrices that rotate the right-handed up-(down)-type quarks from the weak eigenstate basis to the mass eigenstate basis.

The relative strength of left- and right-handed interactions is determined by the parameter \( \cot \theta_R \). In the limit in which this parameter is large, the new right-handed interactions affect predominantly the third generation. It was found in Ref. [8] that the measurement of \( g_{R\tau} \) at LEP [13] implies a small \( \cot \theta_R \xi_Z \leq 10^{-3} \) if the new interaction affects the third generation leptons as well as the quarks. It is possible to construct models in which the third generation lepton couplings are not enhanced. Here we consider models in which they are enhanced but in which the \( Z - Z' \) mixing is negligible.

In Ref. [8], the process \( e^+e^- \to b\bar{b} \) at LEP-II was used to obtain a lower bound for the mass of the new \( Z' \) gauge boson for a given \( \cot \theta_R \). For our present purpose that bound can
be approximated by the relation

$$\cot \theta_R \tan \theta_W \left( \frac{M_W}{M_{Z'}} \right) \sim 1.$$  \hspace{1cm} (19)

Within this framework there are two potentially large sources of FCNC. The first one, through the coupling \( \bar{d}_i \gamma_\mu P_R d_j Z'^\mu \) which occurs at tree level and which also receives large one-loop corrections (enhanced by \( \cot \theta_R \)). There is a second operator responsible for FCNC which has the form \( \bar{d}_i \gamma_\mu P_L d_j Z'^\mu \). This operator first occurs at one-loop with a finite coefficient that is enhanced by \( \cot \theta_R \), and is present even when there are no FCNC at tree-level. Because it is enhanced by \( \cot \theta_R \), it can contribute to a low energy FCNC process at the same level as the ordinary electroweak penguins mediated by the \( Z \) boson even though \( M_{Z'} >> M_Z \). It can be written as [9]

$$L_{eff} = \frac{g^3}{16\pi^2} \tan \theta_W \cot \theta_R V^*_t V_j I(\lambda_t, \lambda_H) \bar{d}_i \gamma_\mu P_L d_j Z'^\mu,$$  \hspace{1cm} (20)

where \( I(\lambda_t, \lambda_H) (\lambda_i = m_i^2/m_W^2) \) is the corresponding Inami-Lim type function. With a Higgs mass in the range of a few hundred GeV, this function varies between a few and about 20 [9].

When a third generation lepton pair is attached to the \( Z' \) in Eq. (20), a second factor \( \cot \theta_R \) is introduced which compensates for the small \( M_Z/M_{Z'} \) ratio and makes this mechanism comparable to the standard \( Z \) penguin as follows from Eq. (19).

Collecting the above FCNC interactions, we find that for large \( \cot \theta_R \), \( Z' \) exchange will produce the following effective flavor changing parameters,

$$a_{ij} = \frac{\alpha}{2\pi \sin^2 \theta_W} I(\lambda_t, \lambda_H) \cos \theta_W \tan \theta_W \cot \theta_R V^*_t V_{ij},$$

$$b_{ij} = \cos \theta_W \tan \theta_W \cot \theta_R \cos \xi_Z V^d_{bi} V^d_{bj}.$$  \hspace{1cm} (21)

Here \( \cos \xi_Z = 1 \) since we are working in the limit of no \( Z - Z' \) mixing.

It is interesting to find that the one loop generated \( a_{ij} \) can have a significant contribution to \( B_s \) mixing. For example, with \( \cot \theta_R \tan \theta_W (m_W/m_Z') = 1 \) and \( b_{bs} = 0 \), Figure (3) shows that the range \( 0.0012 \lesssim (m_Z/m_{Z'})|a_{bs}| \sim 0.0015 \) added to the lowest bound of the SM reproduces the measured \( B_s \) mass difference. This range implies

$$5.5 \lesssim I(\lambda_t, \lambda_H) \left| \frac{V^*_t V_{ts}}{0.04} \right| \lesssim 6.5$$  \hspace{1cm} (22)

This range is accessible in our models with reasonable parameters for the Higgs mass and \( \cot \theta_R \) (See Figure 3 in Ref. [9]). Interestingly, this is the same range in which \( K^+ \to \pi^+ \nu \nu \)
reproduces the branching ratio measured by E787 and E949 (which requires \( I(\lambda_t, \lambda_H) = 5.54 \)).

If we take the value, \( I(\lambda_t, \lambda_H) = 5.54 \), then \( (m_Z/m_{Z'})|a_{bs}| \sim 0.0012 \) and from Figure (3) we see that the following range of values is allowed for real \( b_{bs} \)

\[
0.0005 \sim |V_{Rbb}^d V_{Rbs}^d| \sim 0.0009. \tag{23}
\]

Allowing \( V_{bb}^d V_{bs}^d \) to be complex, we find an allowed range shown in Figures (5) and (6). In Figure (5) we use \( \Delta M_{B_s}^{SM} = 12.6 \) ps\(^{-1} \), at the low end of the theoretical range. The region shaded in light gray corresponds to \( a_{bs} = 0 \), whereas the region in dark gray corresponds to \( a_{bs} = -0.0012 \). If we take \( a_{bs} = -0.0012 \) but use the central value of the SM range then we obtain Figure (6). Notice that in this case there are no solutions for real values of \( b_{bs} \) (or \( \zeta \)). New physics in this case is only allowed with a large CP violating phase.

\[ \text{FIG. 5: Constraints on the flavor changing parameter } \zeta \equiv V_{bb}^d V_{bs}^d \text{ with } \Delta M_{B_s}^{SM} = 12.6 \text{ ps}^{-1}. \text{ The shaded regions correspond to the new physics contribution necessary to match the 3-}\sigma \Delta M_{B_s} \text{ range for } a_{bs} = 0 \text{ (light gray) and } a_{bs} = -0.0012 \text{ (dark gray).} \]

obtain Figure (6). Notice that in this case there are no solutions for real values of \( b_{bs} \) (or \( \zeta \)). New physics in this case is only allowed with a large CP violating phase.

\[ \text{V. } B \to X_s \tau^+ \tau^- (\nu \bar{\nu}), \ B_s \to \tau^+ \tau^- \text{ AND } K \to \pi \nu \bar{\nu} \]

We now show that the constraints on the flavor changing parameters from \( \Delta M_{B_s} \) still allow for a substantial enhancement in \( b \to s \tau^+ \tau^- (\nu \bar{\nu}) \) transitions. In the large cot \( \theta_R \) limit, a \( Z' \) exchange at tree level leads to an effective interaction

\[
\mathcal{L} = \frac{g^2 \tan^2 \theta_W \cot^2 \theta_R}{4M_{Z'}^2} V_{Rbb}^d V_{Rbs}^d \bar{s}_\gamma \gamma \mu_P R b (\bar{\nu}_\tau \gamma^\mu P_R \nu_\tau - \bar{\tau} \gamma^\mu P_R \tau) + h.c., \tag{24}
\]
and at one loop level to

\[ \mathcal{L} = \frac{g^2 \tan^2 \theta_W \cot^2 \theta_R}{4 M_{Z'}^2} \frac{g^2}{8 \pi^2} V_{ts}^* V_{tb} I(\lambda_t, \lambda_H) \bar{s} \gamma_\mu P_L b \left( \bar{\nu}_\tau \gamma^\mu P_R \nu_\tau - \bar{\tau} \gamma^\mu P_R \tau \right) + \text{h.c.} \]  

(25)

The corresponding transitions in the SM are mediated by the effective Hamiltonian

\[ \tilde{H}_{\text{eff}} = \frac{G_F}{\sqrt{2} \pi \sin^2 \theta_W} V_{ts}^* V_{tb} \bar{s} \gamma_\mu P_L b \left[ X(x_t) \sum_\ell \bar{\nu}_\ell \gamma^\mu P_L \nu_\ell - Y(x_t) \bar{\tau} \gamma^\mu P_L \tau \right] + \text{h.c.}, \]

(26)

where \( x_t = \frac{m_t^2}{M_W^2} \) and the Inami-Lim functions \( X(x_t) \) and \( Y(x_t) \) are approximately equal to 1.6 and 1.06 respectively [14].

Comparing the tree-level \( Z' \) exchange and SM contributions, we have

\[ \frac{\Gamma_{\text{new}}(B \to X_s \nu \bar{\nu})}{\Gamma_{\text{SM}}(B \to X_s \nu \bar{\nu})} \approx 1130 \cot^4 \theta_R \tan^4 \theta_W \left( \frac{M_W}{M_{Z'}} \right)^4 \left| \frac{V_{ts}^* V_{tb}}{V_{ts}^* V_{tb}} \right|^2, \]

\[ \frac{\Gamma_{\text{new}}(B_s \to \tau \bar{\tau})}{\Gamma_{\text{SM}}(B_s \to \tau \bar{\tau})} \approx 7730 \cot^4 \theta_R \tan^4 \theta_W \left( \frac{M_W}{M_{Z'}} \right)^4 \left| \frac{V_{ts}^* V_{tb}}{V_{ts}^* V_{tb}} \right|^2. \]

(27)

In the SM, with \( |V_{ts}^* V_{tb}| \approx 0.04 \) these branching ratios are predicted to be \( B(B \to X_s \nu \bar{\nu}) = 4 \times 10^{-5} \) and \( B(B_s \to \tau \bar{\tau}) = 1.1 \times 10^{-6} \).

For \( B \to X_s \tau^+ \tau^- \) it is easier to compare the new contribution to the semileptonic decay,

\[ \frac{\Gamma_{\text{new}}(B \to X_s \tau^+ \tau^-)}{\Gamma_{\text{SM}}(B \to X_s \tau^+ \tau^-)} \approx 0.06 \cot^4 \theta_R \tan^4 \theta_W \left( \frac{M_W}{M_{Z'}} \right)^4 \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \]

(28)
The short distance contributions to $B \rightarrow X_s \tau^+ \tau^-$ within the SM have been estimated to be $B(B \rightarrow X_s \tau^+ \tau^-) = 3.2 \times 10^{-7}$.

Using the constraint $|V_{Rbb}^d V_{Rbs}^d| \lesssim 3.5 \times 10^{-3}$ from $\Delta M_{B_s}$ with $\cot \theta_R \tan \theta_W (m_W/m_{Z'}) \approx 1$ (see Figure 5), we find the following upper bounds for these decays

$$B(B \rightarrow X_s \tau^+ \tau^-) \leq 4.4 \times 10^{-5},$$
$$B(B \rightarrow X_s \nu \bar{\nu}) \leq 3.7 \times 10^{-4},$$
$$B(B_s \rightarrow \tau \tau) \leq 6.3 \times 10^{-5}. \quad (29)$$

These upper bounds are larger than the respective SM predictions by factors of about 2 over the SM that is required to reproduce the measured central value $B(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (1.47^{+1.30}_{-0.80}) \times 10^{-10}$ by E787 and E949 [16, 17]. The tree-level $Z'$ contribution compared with the SM is given by

$$\frac{\Gamma_{\text{tree}}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{\Gamma_{\text{SM}}(K^+ \rightarrow \pi^+ \nu \bar{\nu})} \approx 1130 \cot^4 \theta_R \tan^4 \theta_W \left( \frac{m_W}{m_{Z'}} \right)^4 \left| \frac{V_{Rbs}^d V_{Rbd}^d}{V_{ts}^* V_{td}} \right|^2. \quad (30)$$

The parameters involved are different than those in $B_s$ mixing. They can be related when the matrix $V_{Rij}^d$ is almost diagonal. In that case $V_{Rbb}^d \approx 1$, and $|V_{Rbb}^d V_{Rbs}^d| \approx |V_{Rbs}^d| \lesssim 3.5 \times 10^{-3}$. A similar analysis for $\Delta M_{B_D}$ using Figure 4 leads to $|V_{Rbd}^d| \lesssim 2.5 \times 10^{-4}$. Combining these two results one obtains

$$\left| \frac{V_{Rbs}^d V_{Rbd}^d}{V_{ts}^* V_{td}} \right|^2 \lesssim 7 \times 10^{-6} \quad (31)$$

With these numbers, the tree-level $Z'$ exchange contributions to $B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, $B(B_d \rightarrow X_d \tau^+ \tau^- (\nu \bar{\nu}))$ and to $B(B_d \rightarrow \tau^+ \tau^- (\nu \bar{\nu}))$ are much smaller than their SM counterparts.

The situation is different for the one loop level $Z'$ flavor changing interaction Eq. (20). Here we have

$$\frac{\Gamma_{\text{loop}}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{\Gamma_{\text{SM}}(K^+ \rightarrow \pi^+ \nu \bar{\nu})} \approx \frac{1}{12} \cot^4 \theta_R \tan^4 \theta_W \left( \frac{m_W}{m_{Z'}} \right)^4 \left| I(\lambda_t, \lambda_H) X(x_t) \right|^2. \quad (32)$$
The total contribution to the rate is simply the sum of the SM and the one loop $Z'$ exchange since they have the same CKM factor and the same sign. With $I(\lambda_t, \lambda_H) = 5.54$, we obtain the central value of $1.47 \times 10^{-10}$ by E787 and E949, and as we saw in Eq. (22), this value is allowed by $\Delta M_{B_s}$. A similar situation occurs for the CP violating decay $K_L \rightarrow \pi^0 \nu \bar{\nu}$ where

$$\Gamma(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \Gamma(K_L \rightarrow \pi^0 \nu \bar{\nu})_{SM} \frac{\Gamma(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{\Gamma(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{SM}}. \quad (33)$$

VI. CP VIOLATION

A complex new physics contribution to $M_{12}^{B_s}$ can have significant effects on CP violation in $B_s$ decays [5]. We briefly comment on the effects on two experimental observables, the dilepton and the time dependent CP asymmetries $a$ and $A_{TCP}$ defined as

$$a = \frac{N^{++} - N^{--}}{N^{++} + N^{--}} = \frac{|q_{B_s}/q_{B_s}|^2 |A|^4 - |q_{B_s}/p_{B_s}|^2 |\bar{A}|^4}{|q_{B_s}/q_{B_s}|^2 |A|^4 + |q_{B_s}/p_{B_s}|^2 |\bar{A}|^4},$$

$$A_{TCP} = 2e^{\frac{\Delta \Gamma_{B_s}}{2} A_f \cos(\Delta M_{B_s} t) + S_f \sin(\Delta M_{B_s} t)} \frac{1}{1 + e^{\Delta \Gamma_{B_s} t} - A_f^{\Delta t} (1 - e^{\Delta \Gamma_{B_s} t})}, \quad (34)$$

where $N^{ii}$ is proportional to $\Gamma(b\bar{b} \rightarrow t\bar{t}X)$ and $A$ and $\bar{A}$ are the decay amplitudes for $B \rightarrow t^+ \nu X$ and $\bar{B} \rightarrow \bar{t} \bar{\nu} \bar{X}$. $\Delta \Gamma$ is the lifetime difference between the heavy and light states $B_s^L$ and $B_s^H$. The other quantities are defined as

$$A_f = \frac{|A(f)|^2 - |\bar{A}(f)|^2}{|A(f)|^2 + |A(f)|^2}, \quad S_f = -2 \frac{\text{Im}((q_{B_s}/p_{B_s}) \bar{A}(f) A^*(f))}{|A(f)|^2 + |A(f)|^2},$$

$$A_f^{\Delta t} = 2 \frac{\text{Re}((q_{B_s}/p_{B_s}) \bar{A}(f) A^*(f))}{|A(f)|^2 + |A(f)|^2}, \quad |A_f|^2 + |S_f|^2 + |A_f^{\Delta t}|^2 = 1. \quad (35)$$

Here $A(f)$ and $\bar{A}(f)$ are decay amplitudes for $B_s$ and $\bar{B}_s$ decay into CP eigen-states $f$. In terms of the $B_s$ mixing parameters

$$\frac{q_{B_s}}{p_{B_s}} = \sqrt{\frac{M_{12}^{B_s} - i \Gamma_{12}^{B_s}/2}{M_{12}^{B_s} - i \Gamma_{12}^{B_s}/2}}, \quad (36)$$

Assuming that CP violation in $A$ and $\bar{A}$ is small, $|A| = |\bar{A}|$, and

$$a = \frac{\text{Im}(\Gamma_{12}^{B_s}/M_{12}^{B_s})}{1 + |\Gamma_{12}^{B_s}/M_{12}^{B_s}|^2/4}. \quad (37)$$

In the SM one has [18]

$$\frac{2\Gamma_{12}^{B_s,SM}}{\Gamma_{B_s}} = -\frac{f_{B_s}^2}{(230\text{MeV})^2} (0.007 B_{B_s} + 0.132 \epsilon - 0.078) \approx -0.11. \quad (38)$$

13
This number is consistent with the 95% CL HFAG experimental upper bound of 
\( \Delta \Gamma(B_s)/\Gamma(B_s) < 0.54 \) \[4\].

Since \( \Gamma_{12}^{B_s} = |\Gamma_{12}^{B_s}|e^{i\alpha_s} \) arises from loop contributions involving light quarks, we do not expect a significant new physics contribution. In the following we will use the SM value above to estimate this quantity. In terms of the phase of \( M_{12}^{B_s} = |M_{12}^{B_s}|e^{i\alpha_s+i\theta_s} \), we obtain

\[ a \approx 0.004 \sin \theta_s. \] (39)

The phase \( \theta_s \) is not constrained by the measurement of \( \Delta M_{B_s} \), as we saw it can take any value from 0 to \( 2\pi \). Therefore the asymmetry \( a \) can vary in the range from -0.004 to 0.004. \[20\]

There may also be large effects in the time dependent CP asymmetry. The time dependent CP asymmetries in B decays have been shown to provide crucial information about CP violation in \( B_d \) decays. For \( B_s \) decays, the “Gold Plated” mode to study CP violation is \( B_s \rightarrow \psi \phi \). In the SM, \( S_f \) is about 0.038 and \( A_f \) is also very small. But with new physics, \( S_f \) can be much larger (since \( S_f = \sin(2\theta_s) \)) even if there is no CP violating phase in \( A(f) \) and \( \bar{A}(\bar{f}) \). Future experiments should test CP violation in the \( B_s \) sector \[19\].

There is another special aspect of time dependent CP violation in \( B_s \) decays due to the fact that \( \Delta \Gamma \) is not equal to zero \[19\]. If \( \Delta \Gamma = 0 \), which is a very good approximation for \( B_d \) decays, it is not possible to measure \( A_{f,\Delta \Gamma} \), and one cannot check the last equation in Eq. (35). Assuming again that CP violation in the decay amplitudes is small, we have

\[ a_{TCP} = A_{TCP}(\Delta \Gamma_{B_s}) - A_{TCP}(0) \] 

In Figure (7), we show \( a_{TCP} = A_{TCP}(\Delta \Gamma_{B_s}) - A_{TCP}(0) \) as a function of \( t \). We have chosen two values \( 2\pi/3 \) and \( \pi/5 \) for \( \theta_s \) for illustration. We can see that at a few percent level, there are differences compared with the \( \Delta \Gamma = 0 \) case such difference may be tested at LHCb.

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[1] V. Abazov [D0 Collaboration], arXiv:hep-ex/0603029.

[2] Preliminary result from http://www-cdf.fnal.gov/physics/new/bottom/060406.blessed-Bsmix/
FIG. 7: $a_{TCP}$ as a function of $t$ (ps) for two values of $\theta_s$; $\theta_s = 2\pi/3$ shown as a solid line, and $\theta_s = \pi/5$ shown as a dashed line.

[3] J. Charles et al. [CKMfitter Group], Eur. Phys. J. C 41, 1 (2005) arXiv:hep-ph/0406184. Updated results used in this paper are from http://ckmfitter.in2p3.fr/ “Results as of FPCP 2006, Vancouver, Canada.

[4] E. Barberio et al. [The Heavy Flavor Averaging Group], arXiv:hep-ex/0603003.

[5] Z. Ligeti, M. Papucci and G. Perez, arXiv:hep-ph/0604112 Y. Grossman, Y. Nir and G. Raz, arXiv:hep-ph/0605028 P. Ball and R. Fleischer, arXiv:hep-ph/0604249 K. Cheung, C. W. Chiang, N. G. Deshpande and J. Jiang, arXiv:hep-ph/0604223 M. Blanke, A. J. Buras, D. Guadagnoli and C. Tarantino, arXiv:hep-ph/0604057.

[6] K. Ikado et al., arXiv:hep-ex/0604018.

[7] X. G. He and G. Valencia, Phys. Rev. D 66, 013004 (2002) [Erratum-ibid. D 66, 079901 (2002)] arXiv:hep-ph/0203036.

[8] X. G. He and G. Valencia, Phys. Rev. D 68, 033011 (2003) arXiv:hep-ph/0304215.

[9] X. G. He and G. Valencia, Phys. Rev. D 70, 053003 (2004) arXiv:hep-ph/0404229.

[10] G. Ecker and W. Grimus, Z. Phys. C 30, 293 (1986).

[11] V. Barger, C. W. Chiang, P. Langacker and H. S. Lee, Phys. Lett. B 580, 186 (2004) arXiv:hep-ph/0310073.

[12] M. S. Chanowitz, Phys. Rev. Lett. 87, 231802 (2001) arXiv:hep-ph/0104024.

[13] D. Abbaneo et al. [ALEPH Collaboration], arXiv:hep-ex/0112021.

[14] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996) arXiv:hep-ph/9512380.
[15] J. L. Hewett, Phys. Rev. D 53, 4964 (1996) [arXiv:hep-ph/9506289].

[16] S. Adler et al. [E787 Collaboration], Phys. Rev. Lett. 88, 041803 (2002) [arXiv:hep-ex/0111091].

[17] A. V. Artamonov [E949 Collaboration], arXiv:hep-ex/0403036.

[18] M. Beneke, G. Buchalla, C. Greub, A. Lenz and U. Nierste, Phys. Lett. B 459, 631 (1999) [arXiv:hep-ph/9808385].

[19] P. Ball et al., arXiv:hep-ph/0003238.

[20] It can also reach 2% if $\Delta \Gamma(B_s)/\Gamma(B_s)$ is close to its experimental upper bound.