Anti-reflection structure for perfect transmission through complex media

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The scattering of waves when they propagate through disordered media is an important limitation for a range of applications, including telecommunications1, biomedical imaging2, seismology3 and material engineering4,5. Wavefront shaping techniques can reduce the effect of wave scattering, even in opaque media, by engineering specific modes—termed open transmission eigenchannels—through which waves are funnelled across a disordered medium without any back reflection6–9. However, with such channels being very scarce, one cannot use them to render an opaque sample perfectly transmitting for any incident light field. Here we show that a randomly disordered medium becomes translucent to all incoming light waves when placing a tailored complementary medium in front of it. To this end, the reflection matrices of the two media surfaces facing each other need to satisfy a matrix generalization of the condition for critical coupling. We implement this protocol both numerically and experimentally for the design of electromagnetic waveguides with several dozen scattering elements placed inside them. The translucent scattering media we introduce here also have the promising property of being able to store incident radiation in their interior for remarkably long times.

Full transmission through disorder

Our concept is illustrated in Fig. 1: the starting point is a fixed disordered medium of thickness $L$, for which the average transmission of a wavefront impinging from the left typically scales as $T \sim \epsilon / L$, where $\epsilon$ is the mean free path. Correspondingly, when placing a second

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Simple expressions now relate the total scattering region to the left (L) and $S_0$ of the fixed disordered medium on the right (R).

The initial disorder configuration of the sample, respectively, and of the corresponding transmission matrices $r_l$ and $r_R$. This scattering matrix is composed of two reflection matrices $S = r_L S_0 r_R S_0 = r_L S_0 r_R r_R S_0 = r_L S_0 r_R r_R = r_L S_0 r_R r_R = r_L r_R$. The critical remaining task is to design a complementary medium with a desired reflection matrix at the design frequency (which we choose to be $\nu_0 = 7$ GHz or $\nu_0 = 11.2$ GHz). Equation (3) requires that the individual values of the average transmission through the complementary medium and the fixed disorder are equal $\tau_L = \tau_R$, such that we start the inverse design with a complementary medium that features a combination of Teflon and metallic scatterers (randomly chosen) with a similar reflection to the fixed disorder. At this initial step, the matrix $t' t$, featuring the global transmission $t$, is also random, as shown in Fig. 2. The distribution of transmission eigenvalues $\tau$ of $t'$ follows a bimodal law for random configurations $\tau_G$, $P(\tau) \sim 1/[(\tau/\tau_G - 1)^3]$ as expected in scattering media (Fig. 2c), with an average transmission $\langle \tau \rangle = 0.40$ averaged both over frequencies and configurations.

In a next step, we gradually optimize the positions of the scatterers in the complementary medium to satisfy the desired matching condition. As the design space for finding an optimal solution for this inverse problem is enormous, it is insufficient to just work with a random search algorithm. Instead, we need an iterative procedure based on the gradient of the objective (the average transmission of the sample) with respect to the system parameters (the positions of the scatterers) $Q$. Procedures of this kind have been implemented in various computational techniques such as for the design of compact and efficient devices in nanophotonics, light confinement in strongly scattering disordered media or for analogue computing using metastructures. Here we introduce a tailor-made approach to calculate this gradient on the basis of the generalized Wirger–Smith (GWS) operator, which has recently been exploited for optimal focusing, micromanipulation and information retrieval in disordered systems. The constituting equation of the GWS operator, $Q_{\psi_0}$ reads

$$r_L^* = r_R.$$  (3)
transformation (Methods). b, Simulation results for the amplitude of the elements of $t$ for the situation when the complementary medium on the left is randomly chosen (left panel) and optimized for full transmission (right panel). c, The bimodal distribution of transmission eigenvalues $t$ for 2,500 random configurations becomes a Dirac distribution $t$ for the optimized complementary medium as $t$ becomes the identity matrix.

$$
\langle u|Q_n|u \rangle = \left| u|^{-1} \int dS \nabla N \right| \left| u \right|^2 \int \cos \phi \sin \phi \psi_n^* \psi \sin \phi \, d\phi.
$$

Experimental results

At the end of the optimization, the transmission $T$ is extremely close to unity (between $T_{\text{sim}} = 0.995$ and $T_{\text{sim}} = 0.999$ depending on the sample) such that the matrix $T(N)$ is practically the identity matrix (Fig. 2b, right panel). In this translucent scattering medium, all transmission eigenchannels are naturally open, $r_n = 1$ for all $n$, as seen in Fig. 2c. For these samples, the length of the complementary medium is the same as the one of the fixed disorder. However, because only equation (3) needs to be satisfied, we can in principle optimize the complementary medium in a much smaller area. For the same scattering strength, we indeed achieve perfect transmission at 7 GHz using a length as small as a single wavelength. Finally, we repeat the same procedure for an initial sample with stronger disorder (20 Teflon cylinders and 6 metallic cylinders) giving $r_n = 0.095$ m at 7 GHz.

The spectra of $T(v)$ shown in Fig. 3a–d all display a pronounced resonance at the design frequency $v = v_0$. We emphasize that the optimized design of the complementary medium is indeed a rare event, as the transmission does not exceed 0.75 in all of the 2,500 random configurations we have sampled in our numerics. We also note that the geometry resulting from our optimization is not a hyperuniform structure14, nor does it rely on a special mirror symmetry31, as in previous attempts at realizing a fully transparent disorder or at enhancing transmission through a barrier; in this sense our approach is broadly applicable (see Methods for more information).

In the waveguide experiment, we implement the numerically obtained solution by placing the scatterers at the calculated positions (Fig. 2a) and by measuring the transmission of microwaves using a set of antennas (Methods). The experimental results obtained in this way nicely reproduce the simulation results with a maximum transmission $T_{\text{exp}}$ between 0.91 and 0.94 at 7 GHz and $T_{\text{exp}} = 0.91$ at 11.2 GHz (Fig. 3e–h). The reduction of $T_{\text{exp}}$ compared to the numerical values is due to the impact of dissipative losses in the waveguide walls and in the scatterers, which increases with the dwell time of transmitted waves and hence with the disorder strength.

To demonstrate the flexibility of our inverse design procedure we use it not only to maximize but also to minimize transmission, and hence maximize reflection through the system with the same number of scatterers. The minimal transmission found numerically (experimentally) now varies between 0.001 and 0.03 (between 0.002 and 0.049) for the samples with $N = 4$ and is as small as $T_{\text{sim}} = 0.03$ ($T_{\text{exp}} = 0.1$) with $N = 7$. Interestingly, we observe that the transmission in these opaque samples can be reduced over a broad frequency range. This observation is explained by considering that the incident waves penetrate here only weakly into the disordered sample (Fig. 4), such that no sharply resonant states are excited whose limited bandwidth would make this a narrow-band effect15.

Enhanced energy storage

A remarkable consequence of our medium design concerns the spatial distribution of the intensity $W(x, y)$ within the medium (Fig. 4a), and with it the dwell time $t_0 = \int dx dy W(x, y)$. As has been shown in previous works15–20, open transmission channels in generic disordered media are associated with both an enhanced penetration depth of incoming radiation and an increased dwell time in the disorder. Now that we have
created a system that features only open transmission eigenchannels, the intriguing question that arises is whether our design also naturally increases both of these quantities for all incoming radiation fields. To verify this explicitly, we show in Fig. 4a the spatial distribution of the intensity and in Fig. 4b the energy density averaged over the cross-section of the waveguide, $W(x) = \langle W(x, y) \rangle$, for a translucent sample, for an unoptimized random disorder and for a fully reflecting sample. The peak of maximum energy density for a translucent medium shifts, on average, towards the middle of the sample, in agreement with the profile of ‘open’ channels in random configurations. For samples with thickness $L \gg \epsilon$, we can therefore estimate that the dwell time is, on average, increased by a factor $nL/12\epsilon$ relative to its value in an unoptimized random configuration, for which $\langle W(\xi) \rangle$ is governed by the diffusion equation and decreases linearly with x. The dwell time in translucent samples indeed falls in the tail of the distribution $P(\tau_h)$ computed for many random configurations (Fig. 4c). By contrast, for a highly reflecting sample, $W(x)$ approaches the profile of closed channels and decays almost exponentially with x, corresponding to a dwell time shorter than $\langle \tau_0 \rangle$.

Conclusion

Our approach is general and can be broadly applied to other complex systems. In the Methods section, we extend our inverse design of perfectly transmitting structures to a multichannel cavity coupling four incoming and four outgoing ports. By optimizing the position of 15 metallic scatterers within the cavity, we achieve an average transmission as large as 0.998 numerically and 0.9 experimentally at the selected frequency. Our results also provide an interesting new perspective for research on reflectionless scattering modes, coherent perfect absorption, wireless power transfer and wavefront correction in biomedical imaging. Whereas these coherent wave effects have recently also been realized experimentally for disordered systems, all of these studies remained restricted to single specific wavefronts whose absence of reflection or perfect absorption were demonstrated. Our results point the way to how to conveniently extend these concepts to multiple incoming wavefronts in parallel. Practically speaking, for the case of a broadband absorber being placed behind a disordered medium, the anti-reflection structure we introduce here would have the interesting effect that the opaqueness of the disorder (appearing as white in the visible spectrum) would get a spectral dip (appearing as black) at the design frequency, at which any incoming wavefront penetrates the disorder and gets absorbed perfectly.

In conclusion, we have demonstrated theoretically and experimentally that a disordered medium can be made fully transmitting to all incoming wavefronts by placing an optimized complementary medium in front of it. As only the reflection matrix of this complementary

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**Fig. 3** | Numerical and experimental results. a–d, Simulated transmission spectra, $T(v) = (\Sigma_{\alpha} \tau_{\alpha}(v))/N$, for anti-reflection structures that are optimized for maximal transmission (blue line) and maximal reflection (black line) at $v_0 = 7$ GHz (a–c) or $v_0 = 11.2$ GHz (d). The shaded pink area indicates the range of values obtained for 2,500 random configurations, with the average being represented by the red dotted line. The fixed disorder consists either of 3 metallic and 17 Teflon cylinders (a,b,d) or 6 metallic and 20 Teflon cylinders (c). e, Whereas the size of the optimization region varies to show the design process’s flexibility. f–h, Corresponding experimental results for 3 metallic and 17 Teflon cylinders (e,f,h) and 6 metallic and 20 Teflon cylinders (g). The pink shaded area and the corresponding average values are estimated here from ten random configurations. Left column, sketches of the fixed (red) and optimized disorders (blue) associated with each transmission spectrum.
medium must be engineered, we envision that thin metasurfaces could be used for this purpose, enabling the creation of tailor-made and potentially time-adaptive anti-reflection structures with fascinating properties for applications in the fields of wireless communications, filtering, energy harvesting and imaging. In the long term, we expect that advances both in computing power and in microfabrication will make our approach applicable to systems with an increasingly large number of modes.

Online content

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Methods

Numerical methods

Numerical modelling and optimization algorithm. The experimental setup is modelled using the two-dimensional scalar Helmholtz equation \( \Delta + k_\rho^2(\mathbf{r})|\psi(\mathbf{r})| = 0 \), where \( \Delta \) denotes the Laplacian in two dimensions, \( \varepsilon(\mathbf{r}) \) is the spatially varying dielectric constant, \( k \) the wavevector and \( \psi \) the \( z \) component of the electrical field. We solve the Helmholtz equation using an open-source finite-element library (NGSolve)\(^{44,45} \). This enables us also to numerically evaluate the objective function we maximize, which reads

\[
f = Trt^T t/N = Tr(1-r^T r)/N, \quad \text{with} \quad r = r_n + t_n(1-\delta_{rn})^{-1},
\]

where \( t(\mathbf{r}) \) is the transmission (reflection) matrix of the whole system for a wave injected from the left lead and \( N \) is the number of transverse electric modes. Using the unitarity of \( S \) we rewrite \( f \) in terms of \( r \). This has the considerable advantage that we only need the measurable reflection matrix of the fixed disorder, \( r_n \), to compute the objective function. The gradient of this objective with respect to the position of the \( n \)th scatterer is given by

\[
\nabla_n f = -2ReTr(r^\dagger \nabla_r r)/N, \quad \text{with} \quad \nabla = \nabla_r + \nabla_l + \nabla_t(1-\delta_{rn})^{-1} \nabla_l.
\]

where \( M = (1-\delta_{rn})^{-1} \) encodes the multiple reflections between left and right disorder. \( r^\dagger \) is the reflection matrix for a state injected from the right. Following from the block structure of the \( S \) matrix the gradient of each block is given by

\[
\nabla_n S = 
\begin{pmatrix}
\nabla_n r_n & \nabla_n r_l^\dagger & \nabla_n r_l^\dagger & \nabla_n r_l^\dagger & \nabla_n r_l^\dagger
\end{pmatrix}
\begin{pmatrix}
Q_{11} & Q_{12} \\
Q_{21} & Q_{22}
\end{pmatrix}
\]

where the matrices \( Q_{\mu}\) are the corresponding blocks of the GWS operator associated to the shift of the \( n \)th scatterer in direction \( \mathbf{r}_n \). We note here that when we use equation (5) as our objective function we need to take into account evanescent coupling between left and right disorder, such that the total transmission is faithfully computed. This can be done either by avoiding evanescent modes altogether (hence the gap between left and right disorder in some configurations) or by using a scattering matrix that includes evanescent channels\(^{46} \). Alternatively, one can directly compute the transmission matrix of the whole system.

The GWS operator, as discussed in the main text, was originally introduced for optimal micromanipulation, as its eigenvalues are directly proportional to the force applied on the target scatterer\(^{27,28} \). This has the interesting consequence that we can compute the GWS operator and therefore the gradient of \( f \) using the electric field (with which we calculate the force) at the scatterers without the need to move any single scatterer—a feature that makes the evaluation of the gradient independent of the number of system parameters. To be more precise, we only have to do 2\( N \) simulations, where \( N \) is the number of modes. We access the matrix elements of \( Q_{\mu} \) by

\[
4(Q_{\mu} \mu) = F(|\mathbf{u}_\mu|^2 + |\mathbf{v}_\mu|^2) - F(|\mathbf{u}_\mu|^2 - |\mathbf{v}_\mu|^2) + iF(|\mathbf{u}_\mu|^2 - |\mathbf{v}_\mu|^2) - iF(|\mathbf{u}_\mu|^2 + |\mathbf{v}_\mu|^2),
\]

where the \( \{|\mathbf{u}_\mu|\} \) are a complete and orthonormal set of incoming scattering states (for example, the different waveguide modes) and \( F(|\mathbf{u}_\mu|) \) denotes the force transferred by the electric field onto the \( n \)th scatterer when we inject a wave in the state \( |\mathbf{u}_\mu| \) into the system. This simple reformulation enables us to simulate the electric field inside the scattering region and then calculate \( Q_{\mu} \). For every single scatterer, with which we can build the gradient of \( f \) without the need to move every single scatterer, simulate the system and calculate the gradient. In the case of a dielectric target the force is calculated by

\[
F(|\omega|) = \frac{Rk^2(t-1)}{2} \int_0^{2\pi} \left| \sin \phi \right|^2 d\phi,
\]

and for a metallic one by

\[
F(|\omega|) = -\frac{R}{2} \int_0^{2\pi} \left| \sin \phi \right|^2 d\phi,
\]

where \( \psi_n \) is the electric field distribution inside the scattering region for a wave injected in the state \( |\mathbf{u}_\mu\rangle \).

The optimization is constrained by the strict condition that no two scatterers can overlap. To enforce this at every step, we use the following algorithm: while \( f < 0.999 \) (that is, the transmission is smaller than 99.9\%), we compute \( Q_{\mu} \) for all scatterers and use it to calculate \( f(t_n + \delta r_n) = S(t_n + \delta r_n) \exp(\i r) Q_{\mu} \) for all possible changes in the position of scatterers. We then pick the direction of largest increase in the total transmission that does not result in an overlap of scatterers. If we cannot find such a change for the given step size, we reduce the step size by a factor of 1.5. Otherwise we check whether \( f(t_n + \delta r_n) - f(t_n) - c|\delta r_n| \nabla f \) holds, with \( c = 0.025 \), is fulfilled. If this is the case we update the geometry, if not we also reduce the step size.

Effects of absorption. The perfectly transmitting media we present in the main text were designed without considering the effects of absorption. Here we show numerically and analytically that adding constant global absorption still results in zero reflection across all input channels, whereas the transmission deviates unavoidably from unity (Extended Data Fig. 1). To be more precise, in Extended Data Fig. 1a we first investigate how the transmission and reflection change when we scan the absorption strength in the geometry featuring 49 scatterers designed at 11.2 GHz. We implement the absorption by adding an imaginary part between \( n_l = 0 \) and \( n_l = 10^{-3} \) to the refractive index everywhere. We find that whereas the transmission decreases from unity immediately, the reflection remains close to zero for a much wider range of absorption strength. We corroborate this result in Extended Data Fig. 1b, in which we show a frequency scan of the transmission with and without absorption that features a pronounced dip in the reflection at 11.2 GHz. In this case we use an imaginary part \( n_l = 1.2 \times 10^{-4} \) as it results in a transmission of \( T_{abs} = 0.9 \), comparable to the one found in the experiment. Also the reflection at the target frequency (11.2 GHz) is \( R_{abs} = 0.0042 \), which is comparable to that found without absorption \( R = 0.0049 \).

We put this empirical observation on more solid ground by considering the globally uniform imaginary part as a complex shift of the frequency to \( \omega = \omega_0/2 \), where \( \omega = 2k_\rho \) is the absorption rate\(^{46} \). Under the assumption of small dissipation we can then expand the scattering matrix into

\[
S_\omega(\omega + \i \alpha/2) = S(\omega) - \frac{\alpha}{2} Q(\omega),
\]

where \( Q \) is the Wigner–Smith time-delay operator and the subscript \( \alpha \) denotes absorption (terms without it are evaluated at zero absorption). Using \( \i t = 1 \) and \( r = 0 \) we find that

\[
t^\prime Q = 1 - \alpha Q_{\mu}, \quad \text{and} \quad r^\prime Q = (\frac{\alpha^2}{4}) \partial_r^\prime \partial_{\mathbf{r}}^\prime.
\]

Here \( Q_{\mu} \) is the time-delay operator for a wave impinging onto the disorder from the left. This shows that the reflection deviates from
zero only in $O(a^2)$, whereas the transmission decreases from unity already in first order, making the zero-reflection medium robust to absorption.

**Analysis of the optimization.** Here we look at the probability of converging to a fully transmitting disorder. To do this, we randomly sample fixed disorders consisting of 3 aluminium and 17 Teflon scatterers. We then choose one random initial configuration for the optimization region for each fixed disorder and start the inverse design process at 7 GHz (four open modes). The results for the dependence of the transmission of the fixed disorder are shown in Extended Data Fig. 2a. We see that higher transmission of the fixed disorder leads to a higher probability that the design process converges for one random initial guess. Note that the statistics are not very precise at low transmission because of the low number of configurations.

To interpret this result, we note that the gradient descent optimization used here deterministically follows the path of the steepest gradient until it reaches a local minimum. As the objective function in the present work is non-convex, different initial conditions lead to different local minima, none of which is guaranteed to be the global minimum. This is, in general, an unavoidable problem in inverse design, but as we demonstrate, we are still able to design configurations with a transmission of practically unity.

Global methods such as genetic algorithms are however no alternative, as they are computationally more expensive (by orders of magnitude), among other problems.

We also examine one such optimization process in detail to show that maximizing the transmission also leads us to fulfill equation (3) of the main text. In Extended Data Fig. 2b we show total transmission and the Frobenius norm of equation (3), $\|\mathbf{r} - \mathbf{r}_0\|_F$, for each step of one optimization process shown in Fig. 2a. As the total transmission monotonically (as required) approaches $T = 0.999$ (the stopping condition), the generalized critical coupling condition is fulfilled better and better.

**Steep angles of incidence.** Here we investigate the role of waveguide modes with very steep angles of incidence. To be more precise, we design a fully transmitting disorder at a frequency of 6.04 GHz. This means that the 4th (highest) mode is barely excited, resulting in an angle between $k$ and the longitudinal direction of about 83°. Despite this steep angle of incidence the average transmission of this disorder is still $T = 0.998$, whereas the transmission of the last mode (with the steepest angle) is $T = 0.999$.

To further illustrate this result, in Extended Data Fig. 3 we show the Poynting vector for an empty waveguide and an optimized disorder, when we either inject the highest order mode or a state $\mathbf{u}$ with a high angle of incidence. We construct this state by numerically optimizing the ratio of $\mathbf{u}^\dagger k_x \mathbf{u}$ and $\mathbf{u}^\dagger k_y \mathbf{u}$. Here, $k_x$ and $k_y$ are the operators that give us the $k$-components for a particular incoming wave state. We clearly see in the empty waveguide that we found a state bouncing up and down on the waveguide walls. In the waveguide filled with a perfectly transmitting disorder this state still has perfect transmission, as the disorder was designed to be perfectly transmitting for all incoming states.

**Experimental inaccuracies.** To understand the small shifts of the transmission peaks found in the experimental measurements (see Fig. 3f for complete transmission and Fig. 3e,f,h for maximal reflection) with respect to the ones found via the numerical optimizations (Fig. 3a,b,d), we perform numerical simulations under more realistic conditions, including absorption or possible perturbations of the numerically optimized configurations in the perfect waveguide featuring full transmission.

As in every experimental setup, uniform absorption is typically present and affects the transmission spectrum. Thus we add a uniform imaginary part of $\alpha_z = 3 \times 10^{-4}$ to the refractive index distribution of the waveguide containing the optimized configuration, and, as shown in Extended Data Fig. 1, this only lowers the transmission peak, but does not shift it noticeably.

We first consider uncertainties in the experimental placement of the scatterers, such as when the experimental scatterer positions are slightly different from the ones obtained from the numerical optimization. We investigate the effect of a small global shift of only the optimized part of the scattering configuration in the negative/positive longitudinal direction, which causes a shift of the transmission peak to lower/higher frequencies, where the peak is typically also lowered (orange dashed/dotted line in Extended Data Fig. 4). As the uncertainties in the scatterer placement might not be global, but rather random, we also investigate the effect of small random displacements of the optimized scatterer positions, which causes peak shifts, lowerings and broadenings depending on the magnitude of the displacements (green solid line in Extended Data Fig. 4).

Because of possible fabrication uncertainties and the skin effect in the metallic waveguide walls, we also consider slightly different waveguide dimensions. Specifically, we study the effect of a slightly wider waveguide (with the scatterers kept transversally in the middle of the waveguide), which lowers the transmission peak and shifts it to lower frequencies (blue solid line in Extended Data Fig. 4).

Moreover, in the experiment, not all scatterers may reach the waveguide’s top plate perfectly. The resulting gap can then cause the waves to scatter off the top edge of the cylindrical scatterers, exciting evanescent modes, which might change the transmission spectrum owing to coupling to the surrounding scatterers. To examine this effect, we perform three-dimensional simulations in which we solve the vectorial Helmholtz equation $\nabla \times \nabla \times \mathbf{E} + k^2 \mathbf{E} = 0$. We find that a small gap (especially for metallic scatterers) results in a lowered transmission peak, which is shifted to higher frequencies (red solid line in Extended Data Fig. 4).

In the experiment, we probably observe a combination of all these effects. Additionally, the experiment can suffer from spurious reflections at the non-perfect absorbers at the waveguide ends, as well as from scattering off the antennas used to inject and measure the waves, which can further affect the transmission spectrum.

**Probability for randomly sampling a fully transmitting disorder.** An interesting question is whether it would also be possible to find a fully transmitting disorder merely by sampling the positions of scatterers randomly, under the constraint that the disorder overall has the same composition and thickness as the sample considered in Fig. 3d. A first hint that this is a hard task is the observation that the maximal transmission of random disorders with the same number of scatterers as the one considered in Fig. 3d at 11.2 GHz is $T = 0.58$. Here we make this observation more quantitative and find that the probability of observing a transmission $T \geq 0.995$ (the one achieved with our inverse designed media), when randomly choosing the position of scatterers, is $6.3 \times 10^{-10}$. We arrive at this conclusion by calculating from the bimodal distribution of transmission eigenvalues (Extended Data Fig. 5) a probability of $1.3 \times 10^{-2}$ of finding a single transmission eigenchannel with transmission $T \geq 0.995$. Because the waveguide we investigate supports seven propagating modes this probability has to be taken to the power of seven (not taking into account the repulsion of eigenvalues, which will reduce this probability even further). We thus arrive at the conclusion that it is nearly impossible to encounter a fully transmitting disorder by chance.

**Comparison to hyperuniform medium.** Here we investigate whether the fully transmitting disorder we designed is hyperuniformly distributed, as hyperuniform media can be transparent, while still being dense enough that transparency is not expected. The criterion for a hyperuniform medium to be transparent is that the structure factor $S(q)$ is defined as...
vanishes in the neighbourhood of \(|\mathbf{q}| = 0\). Here, \(N\) is the number of scatterers at positions \(\mathbf{r}_j\), and \(\mathbf{q}\) denotes the wavevector. To find a hyperuniform medium one minimizes a potential

\[
\Phi(\mathbf{r}_1, \ldots, \mathbf{r}_N) = \sum_{j=1}^{N} \frac{\sin[(2P + 1)\pi(x_j - x_j)/L]}{\sin[\pi(x_j - x_j)/L]} \sin[(2P + 1)\pi(y_j - y_j)/L] \times \frac{\sin(\pi(y_j - y_j)/L)}{\sin(\pi(y_j - y_j)/L)},
\]

where \(P = KL/(4\pi)\) and \(L\) is the length of the quadratic system. The extent of the region where \(S(\mathbf{q})\) vanishes is given by \(K\). In Extended Data Fig. 6a we show the structure factor of a hyperuniform medium with periodic boundary conditions. It is clearly visible that in a region of size \(K\) the structure factor vanishes, resulting in a hyperuniform structure. When we compute, for comparison, the structure factor for the fully transparent medium resulting from our design principle, we find that \(S(\mathbf{q})\) does not vanish around \(|\mathbf{q}| = 0\). This shows that the fully translucent disordered medium resulting from our optimization procedure is not a hyperuniform medium.

**Comparison to mirror medium.** It was recently demonstrated that a disordered slab featuring a mirror symmetry along its transverse axis has an increased transmission compared to a fully random medium\(^{49}\). Here we investigate whether such a symmetry is also (partially) responsible for the full transmission through a disorder. To do so we mirror the fixed disorder we use in Fig. 3b. We then pair up every scatterer of the mirror disorder with one of an optimized or random disorder of the same material, such that the total distance between all pairs is minimal. This distance then tells us how close the disorder is to its mirror-symmetric counterpart. We find (Extended Data Fig. 6) that the optimized disorder is even further from the mirror disorder than the mean of random configurations (by a factor of about 1.3).

**Experimental setup**

We first measure the field transmission coefficients \(t'_{y_2, y_1}\) between seven transmitting and seven receiving antennas at locations \(y_2\) and \(y_1\), respectively. The spacing between each transmitting or receiving antenna is equal to \(W/8\), where \(W\) is the width of the waveguide. The elements \(t_{mn}(v)\) of the transmission matrix \(T(v)\) in the basis of waveguide modes are then reconstructed by means of a two-fold sine transformation:

\[
t_{mn} = \sum_{y_1, y_2} t'(y_2, y_1) /k_0(v)k_{mn}(v) \sin\left(\frac{mn}{W}y_2\right) \sin\left(\frac{mn}{W}y_1\right).
\]

The transverse mode number \(n\) is given by \(k_n = (2\pi/c_0)\sqrt{v^2 - (mh)^2}\). the cut-off frequency is \(v_c = c_0/2h\), \(c_0\) is the speed of light and \(h = 8\) mm is the height of the waveguide. We first verify that the transmission matrix \(t(v)\) for an empty waveguide is diagonal in the basis of waveguide modes. The elements of \(t(v)\) are presented in Extended Data Fig. 7 at \(v = 7\) GHz and \(v = 11.2\) GHz. This confirms that the coupling between our antennas is small and barely affects our results. A strong coupling may indeed result in off-diagonal terms with high amplitude. The transmission of the last mode is seen to be slightly smaller than the transmission of the first modes. This results from losses within the waveguide, as the last mode is associated with a larger time delay.

Our antennas have small penetration depths (3 mm) and are thus weakly coupled to the waveguide as a result of an imperfect impedance matching. The transmission coefficients found from the two-fold sine transformation are not flux normalized. We therefore normalize the transmission for each waveguide mode \(n\), \(T_n(v) = \sum_{\mu}|t_{mn}(v)|^2\) by its value for an empty waveguide, \(T_0^n(v)\). The mean transmission through a disordered waveguide is then estimated by

\[
T(v) = \frac{1}{N} \sum_{n=1}^{N} \frac{T_n(v)}{T_0^n(v)}.
\]

This is equivalent to normalizing the transmission matrix such that

\[
\tilde{T}_{mn}(v) = \frac{t_{mn}(v)}{\sqrt{T_0^n(v)}},
\]

and calculating the average transmission of each mode with \(T(v) = \langle \sum_{n,m} |T_{mn}(v)|^2 \rangle / N\). This enables us to compare the experimental results with the numerical simulations in Fig. 3.

To validate our experimental results, we first compute the distribution of transmission eigenvalues \(\tau\) of \(T(v)\) for waveguides with random disorder. Here the distribution is found from ten random realizations with randomly located scatterers and an averaging over the frequency range 6.6–7.4 GHz (\(N = 4\)). The distribution is presented in Extended Data Fig. 8. As expected from diffusion theory, this distribution is bimodal with two peaks centred on closed channels with \(\tau = 0\) and open channels with \(\tau = 1\). The experimental result is in good agreement with the theoretical law:

\[
P_0(\tau) \propto \frac{1}{\tau^{1/\gamma}}.
\]

However, in line with previous work on this subject\(^7\), we observe the presence of transmission eigenvalues above unity. The bimodal distribution and, more precisely, the peak associated with open channels is indeed very sensitive to experimental noise. A non-unitarity of the scattering matrix due to experimental noise leads to dramatic deviations from theory, with transmission eigenvalues exhibiting coefficients larger than unity\(^7\). These open channels virtually violate the energy conservation due to the noise level. As a result of the spreading of eigenvalues with large transmission, the amplitude of the corresponding peak also decreases. In our case, this noise level comes from the normalization of the elements of the transmission matrix using transmission through an empty sample. In particular, the last waveguide mode with a large angle between \(k\) and the longitudinal direction features a large dwell time for the empty waveguide and is therefore very sensitive to global absorption. In the presence of weak disorder, the outgoing field is mixed in all modes leading to a possible smaller sensitivity to the absorption. As a result, the normalization can lead to transmission eigenvalues larger than unity.

Experimentally, we implement anti-reflection structures by projecting on the waveguide an image of the scatterer positions found numerically using a video projector. The image is calibrated to minimize positioning errors. The cylinders are then placed manually. Small inaccuracies may result from this procedure but the overall agreement between numerical and experimental results is excellent, as seen in Fig. 3.

To further confirm our normalization procedure, in Extended Data Fig. 9 we show the transmission associated to each incoming mode of the waveguide for a sample with complete transmission. The configuration corresponds to Fig. 3g. The transmission matrix \(|\tilde{\tilde{E}}_1|^2\) is seen to be random in Extended Data Fig. 9a as a result of strong mode mixing. Nevertheless, each waveguide mode provides almost perfect transmission at \(v_0 = 7\) GHz (Extended Data Fig. 9b).

**Complete transmission through a multichannel cavity**

To further illustrate the potential of our approach, we consider the case of a multichannel cavity. As shown in Extended Data Fig. 10a, the latter is a quasi-two-dimensional square cavity of length and width \(L = W = 0.205\) m and height \(h = 0.010\) mm. A single vertically polarized
mode can propagate within the cavity below $f = 14.7$ GHz. Two arrays of $N = 4$ coaxial antennas are connected on the left and right interfaces. We carry out measurements of the $N \times N$ transmission matrix $t(\omega)$ between 7.8 and 9 GHz. Because coaxial-to-waveguide transitions are well-matched antennas between 7 and 12 GHz, the transmission coefficients are flux normalized and no post-processing is needed. For an empty cavity, the average transmission $T(\omega)$ fluctuates within the selected frequency range between 0.37 and 0.83 numerically and between 0.3 and 0.7 experimentally as a consequence of absorption within the cavity.

We then gradually optimize the positions of 15 metallic cylinders of radius $r = 3$ mm in numerical simulations to reach complete transmission at $v_0 = 8.4$ GHz. Because this configuration does not enable us to write the complete scattering matrix as a composite expression of the scattering matrix of the empty cavity, the cost function $f = \text{Tr} t^N / N$ is directly estimated in terms of the transmission matrix of the full system. The maximum transmission at the end of the numerical optimization reaches 0.998. We then implement the numerical solution experimentally. The transmission spectrum nicely reproduces the numerical result, with a maximal transmission of 0.9 at $v_0$ (Extended Data Fig. 10b). We observe very good agreement overall between numerical simulations and experimental results, even though maximal transmission is slightly reduced by the inevitable presence of absorption.

Data availability
The data that underlie the plots within this paper and other findings of this study are available from the corresponding authors on reasonable request.

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Author contributions
M.D. proposed the project. Numerical simulations were carried out by M.H and M.K. under the supervision of S.R. Measurements and data evaluation were carried out by C.F. and M.D. M.H., S.R. and M.D. wrote the manuscript with input from all authors.

Competing interests
The authors declare no competing interests.

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Extended Data Fig. 1 | Effects of absorption.  

**a**, Transmission (blue) and reflection (orange) plotted over the imaginary part of the global refractive index for the geometry optimized for 11.2 GHz (solid lines) and an empty waveguide (dashed lines).

**b**, Transmission (blue) and reflection (orange) spectrum of the sample with 49 scatterers. The spectra without absorption are depicted by a solid line, while transmission and reflection curves with added absorption are indicated with dashed lines.
Extended Data Fig. 2 | Analysis of the optimization procedure. a, Probability of successfully designing a fully transmitting medium as a function of the fixed disorder's transmission. Success is defined as a transmission $T \geq 0.999$ (blue line) or $T \geq 0.99$ (orange line). Note that we are using binned data here with the bin edges indicated by the grey dashed lines. The number of inverse design processes in each bin is given by the green line. The total number of configurations is 220. b, Plot of the critical coupling condition’s Frobenius norm, $\|r_L^c - r_R^c\|_F$, versus the total transmission for each step of the optimization process. The progress in the inverse design is marked by the colourmap transition from dark blue to bright yellow.
Extended Data Fig. 3 | Steep angles of incidence. Poynting vector distribution in an empty waveguide (a, b) and a waveguide filled with a fully transmitting disorder (c, d). The incoming wave is either the highest order mode (out of 4) (a, c) or the state optimized for a steep angle of incidence (b, d). The white circles indicate the position of the scatterers.
Extended Data Fig. 4 | Experimental inaccuracies. Transmission spectrum of the optimized sample with 52 scatterers (black solid line), where the vertical dashed line marks the frequency at which the optimization has been performed. Increasing the waveguide width by 1% of the initial width $W$ (blue solid line) causes the peak to shift to lower frequencies, where a global shift $\Delta_{x}^{(\text{opt})}$ in the negative/positive longitudinal direction of half a scatterer radius $r$ of only the optimized scatterers results in a shift to lower/higher frequencies (orange dashed/dotted line). In both cases, the peaks are also lowered owing to the deviation from the optimized configuration. Applying small random displacements $\Delta_{x,y}^{(\text{opt})}$ in the range $[-r/4, r/4]$ in $x$- and $y$-direction to every single scatterer also results in a reduction of the peak height and a shift (green solid line). Performing full vectorial 3D simulations, we also find that using cylindrical scatterers with a height $h_{\text{scat}} = 7.98$ mm smaller than the waveguide height $h = 8$ mm also lowers and shifts the peak to higher frequencies (red solid line).
Extended Data Fig. 5 | Probability for randomly sampling a fully transmitting disorder. Histogram of the transmission eigenvalues of 2,500 random configurations composed of 49 scatterers. Note that we have scanned every sample between 10.7 and 11.7 GHz with a resolution of 501 data points within this frequency window.
Extended Data Fig. 6 | Comparison of our anti-reflection structure to hyperuniform media. Structure factor (a) for a hyperuniform medium and (b) for a fully translucent disorder resulting from our design protocol. The hyperuniform medium consists of 100 scatterers, while our inverse designed medium features 52 scatterers, corresponding to the stronger variant disorder presented in the main text. c, Comparison to mirror media. Histogram of the distance of 2,000 random disorders to a mirror symmetric disorder (see text). A distance of 0 would mean that we have perfectly mirror symmetric medium, while larger distances signify that we move away from mirror symmetry. The orange line shows the distance for the optimized medium.
Extended Data Fig. 7 | Transmission matrix for an empty waveguide. a, b, Experimental intensity of the elements of the transmission $|t_{mn}|^2$ in the basis of waveguide modes for an empty waveguide at 7 and 11.2 GHz. At these frequencies, the waveguide supports $N = 4$ and $N = 7$ modes, respectively.
Extended Data Fig. 8 | Experimental result for the bimodal distribution of transmission eigenvalues. Experimental transmission eigenvalue histogram for a waveguide supporting four modes compared to the bimodal law $P_\delta(\tau)$. The random disorder is composed of 6 aluminum cylinders and 34 Teflon cylinders.
Extended Data Fig. 9 | Complete transmission of individual waveguide modes. a, Experimental intensity of the elements of the transmission $|t_{mn}|^2$ in the basis of waveguide modes at 7 GHz for a sample of complete transmission with 52 scatterers. b, Spectrum of the transmission of each mode through the waveguide (dotted lines). The average transmission for the four modes is represented with the blue line.
Extended Data Fig. 10 | Complete transmission through a multichannel cavity. a, Photography of the cavity. The top plate has been removed to see the interior of the cavity. Four transition-to-coax antennas are placed at the left and right side of the cavity. Measurement of the transmission matrix between these two arrays is carried out with a vector network analyser. Fifteen metallic cylinders are placed at the positions determined numerically for perfect transmission. b, Total transmission \( T_{\nu}(\nu) = \Sigma |r_{mn}(\nu)|^2 \) for the four incoming channels (dashed lines) and the average transmission \( T(\nu) = (\Sigma T_{\nu}(\nu))/N \) over incoming channels (blue line). The placement of the cylinders corresponds to positions optimized numerically for perfect transmission at \( \nu_0 = 8.4 \text{ GHz} \). Deviations from the maximal transmission value 1 are primarily due to absorption in the cavity.