Contour detection in images with speckle by using the bi-geometric structure

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Abstract. The speckle produces severe degradations in digital images, which could be introduced by technological devices during the acquisition, capture or transmission of the images. This perturbation affects, for instance, medical and satellite images. Recently, some methods defined in the bi-geometric structure have presented significant improvements in the processing of image with speckle. In this work, we review some aspects of the bi-geometric structure including a discussion about some of the applications found in the literature focused in the detection of contours in grey scale images with speckle.

1. Introduction

The speckle corresponds to a physical phenomenon caused by the construction of some images [1]. Specifically, the image is constructed by the return wave, which can present interferences caused by the geometry and the proper characteristics of the surface studied [2]. Such interferences become an unwanted signal producing random variations of the brightness, intensity or color in the image. The multiplicative noise appears, mainly, in medical applications (US, OCT, among others) and satellite images (SAR) [3–6]. This kind of noise in ultrasound images has been documented since the early 1970s [7–9], and the extraction of relevant information in images with multiplicative noise becomes an arduous task due to the variations in intensity and distortions caused [10]. In the literature can be found several works whose objective has been to develop techniques to increase the quality and moderate the degradation in image with speckle, for instance see [11–13].

The ultrasound image is an useful and massive diagnostic tool in non-invasive medicine. This device has a low cost of use and it is continuously improved. However, the speckle presence in medical image results in diagnostic errors reason that emerges the need of apply more sophisticated and expensive visualization techniques to decrease diagnostic errors [14]. Since, tissue texture and organs morphology provide additional information that complement the medical diagnosis, the exhesive filtering of speackle could produce the loss of important information. Therefore, the image filtering techniques must be more precise still to avoid the loss of valuable information maintaining the image characteristics that are of medical interest [15]. In effect, in [16] was noted that the speckle could be useful by the information that it contains. Such is the case of studies to extract the borders of the carotid artery plaque in images of intravascular
ultrasound (IVUS), in which the speckle is useful [17]. Also, in some images, the directionality of the speckle is also considered when estimating the texture of the area of interest [18].

The multiplicative noise is modelled as a non-linear perturbation of an image in the additive structure of the Newtonian calculus [19]. Because in the bi-geometric structure this model is linear, the introduction of the multiplicative operations in the image processing techniques produces substantial improvements.

Our main objective is to review some of the improvements obtained as consequence of applying the bi-geometric structure in the treatment of images with speckle, including the recall of some principles involved in the multiplicative noise production, in ultrasound images.

This article is organized as follows: in section 2, a brief revision of speckle in ultrasound is presented. Next, in section 3, we summarize some of elements and properties of Bi-geometric structure. Then, section 4 illustrates some applications of bi-geometric structure in the treatment of images with speckle. Finally, in section 5, we present the conclusions and perspectives.

2. Speckle in ultrasound images

In general terms, ultrasound is a sound wave with a frequency that exceeds 20 kHz. Energy is transported and propagated through various media such as a pulsating pressure wave. It is described by a number of wave parameters such as the pressure density, propagation direction, and displacement of particles. If particles displacement is parallel to the propagation direction, then the wave is called a longitudinal or compression wave. If it is perpendicular to the direction of propagation, it is a transverse wave. The interaction of the ultrasound waves with the tissue is subject to the laws of geometric optics, including reflection, refraction, dispersion, diffraction, interference, and absorption, and all these reduce the ultrasound beam intensity [14]. The quality of a ultrasound image depends on the resolution, which is defined as the smallest distance between two points that can be represented as different, and it depends on the frequency and shape of the wave beam.

The frequency depends on the application domain, for instance, for blood close to human skin are used frequencies greater than 7.5 MHz, while frequencies of 1 to 5 MHz are used for arteries that are deeper in the human body or in transcranial applications. We remark that in the ultrasound application there is a trade-off between a higher resolution of ultrasound images at smaller depths and images of lower resolution at greater depths.

The speckle pattern is visible as the typical points of light and darkness of the composite image. Such a noise is produced by the destructive interference of scattered ultrasound waves from different sites, and this noise in images has become an important research topic [7–9]. It is considered that if a fixed rigid object is explored twice in exactly the same conditions, then identical mottling patterns are obtained, and despite its random appearance, multiplicative noise is not random in the same way that electrical noise.

In Figure 1 a tissue can be modeled as a sound absorbers that disperser that contains medians, which scatter sound waves [20]. These dispersers arise from inhomogeneities and structures approximately equal to or smaller in size than the ultrasound wavelength, such as tissue parenchyma, where there are changes in acoustic impedance on a microscopic level within the tissue. Particles of tissue that are relatively small in relation to wavelength (i.e. blood cells), and particles with different impedances that are very close to each other, cause scattering or mottling. The absorption of ultrasound tissue is an additional factor for scattering and refraction, responsible for the loss of impulse energy. The process of energy loss involving absorption, reflection and dispersion are known as attenuation, which increases with depth and frequency. Due to a higher frequency of ultrasound results in an increase in absorption, the consequence is a decreasing in the visualization.

In the homogeneous tissue, the distribution of the dispersers throughout the 3D space is assumed to be isotropic. As shown Figure 1, we distinguish random (or diffuse) dispersers and
structural (or specular) dispersers. Fuzzy scatters are assumed to be distributed evenly over space. Diffuse scattering occurs when there is a series of dispersers with a random phase inside the resolution cell of the ultrasound wave. This random nature of the location of the dispersers results in the statistical nature of the echo signals and, therefore, also of the resulting speckle pattern. Consequently, a statistical approach to its analysis emerges naturally. Furthermore, the absorption of ultrasound is caused by the relaxation phenomenon of biological macromolecules that transfer mechanical energy into heat [21]. The total attenuation is, therefore, the result of absorption and dispersion, which at the same time depend on the frequency, so it is concluded that the attenuation increases with frequency (Figure 1). The diagram in Figure 2 explain the signal pass from the transducer to the screen inside the ultrasound device.

3. Mathematical preliminaries
Let be $\mathbb{R}^+$ the set of positive real numbers. Given a pair $(a, b) \in \mathbb{R}^+ \times \mathbb{R}^+$ we define,

$$a \cdot b = ab \quad \text{and} \quad a * b = a^{\ln(b)},$$

where $\cdot$ corresponds to the product of real numbers and $*$ is the exponenciación [22–25]. Let us define $\varphi$ as follows,

$$\left\{ \begin{array}{ll}
\varphi : & \mathbb{R} \rightarrow \mathbb{R}^+ \\
& x \rightarrow e^x
\end{array} \right.$$

Since $(\mathbb{R}, +, \cdot)$ and $(\mathbb{R}^+, \cdot, *)$ are fields, the function $\varphi$ is continuous, linear, bijective and differentiable [26]. Therefore, $(\mathbb{R}, +, \cdot)$ and $(\mathbb{R}^+, \cdot, *)$ are isomorph. Such equivalence permit us to identify any algebraic expression or operator defined in $(\mathbb{R}, +, \cdot)$ with its isomorph image defined in $(\mathbb{R}^+, \cdot, *)$ (see Table 1, Figure 3 and Figure 4). The Taylor’s expansion for a functions $M$-differentiable in the bi-geometric structure is given by,

$$f(x) = \prod_{k=0}^{M} \left( f^{(k)}(a) \right)^{ \frac{1}{k!} (x-a)^k \left( f^{(M+1)}(\xi) \right)^{ \frac{1}{(M+1)!} (x-a)^{M+1} } }$$

where $f(x) \approx f(a)f^+(a)^{x-a}$ is the linear multiplicative approximation. In Figure 5 Taylor’s expansion exemplified the differences between multiplicative and additive structure (see Figure 6). An image in the gray scale can be represented as a function $I$ defined by

**Figure 1.** The most popular model of tissue adopted in ultrasound images [14, Figure 1.4 pp.11].

**Figure 2.** Signal processing inside ultrasound scanner [14, Figure 1.6 pp.13].
\[
\begin{align*}
    & I : \mathbb{R} \times \mathbb{R} \to [0, 255] \\
    & (i, j) \mapsto I(i, j)
\end{align*}
\]

where \((i, j)\) denotes the pixel position and \(I(i, j)\) is the intensity in such pixel. Let us denote by \(\eta(i, j)\) some contamination in \((i, j)\). Thus, we consider that the noised image can be modelled as follows: \(I_{\eta}(i, j) = I_0(i, j) \cdot \eta(i, j)\). Therefore, the treatment of an image with speckle has as a main objective to obtain the original image \(I_0\) from the contaminated image \(I_{\eta}\).

### Table 1. Some properties of the bi-geometric structure.

| Operation         | \((\mathbb{R}, +, \cdot)\) | \((\mathbb{R}^+, \cdot, \ast)\) |
|-------------------|-----------------------------|----------------------------------|
| Addition          | \(a + b\)                  | \(a \cdot b\)                   |
| Product           | \(a \cdot b\)              | \(a \ast b = a^{\ln b}\)        |
| Exponentiation    | \(a^b\)                    | \(a^{\ln b-1}a\)                |
| Differentiation   | \(\lim_{h \to 0} (f(x + h) - f(x))/h\) | \(\lim_{h \to 1} (f(h \cdot x)/f(x))^{1/\ln(h)}\) |
| Integration       | \(\int f(x)dx\)            | \(\exp \int (\ln | f |)dx\)      |
| Finite differences| \((f(x + \Delta x) - f(x))/\Delta x\) | \((f(\Delta x \cdot x)/f(x))^{1/\ln(\Delta x)}\) |

**Figure 3.** The commutative diagram for differentiation.

**Figure 4.** The commutative diagram for integration.

**Figure 5.** A sigmoid function \(f\) with first order Taylor expansions standard \(f_s\) and multiplicative \(f_m\).

**Figure 6.** The Gaussian function \(f\) with second order standard Taylor expansion denoted by \(f_s\). The second order multiplicative Taylor expansion is exact.
4. Bi-geometrical structure in image treatment

In [27] the potential use of bi-geometrical structure in the analysis of biomedical images is illustrated, exemplifying with the Lagrangian analysis of the myocardium variety, images of analysis tensor and diffusion. An application that allows the processing of linear and non-linear signals is presented in [28]. This is achieved by introducing exponential bases within the bi-geometric Calculus that allows the direct processing to reveal the unknown adjustment parameters through the method of least squares in its bi-geometric version. The Figure 7 and Figure 8 presents the improvements obtained.

Figure 7. Data approximated by Gaussian representation and multiplicative least squares.

Figure 8. Performance function: (a) Low noise level ($\sigma = 0.06$), (b) Medium noise level ($\sigma = 0.12$), (c) High noise level ($\sigma = 0.2$), (d) Continuous noise level ($\sigma = 0, \cdots , 0.2$).

In image treatment, in [29,30] some contour detectors are presented based on the bi-geometric structure. Precisely, in [30] an operator is introduced that consists of a multiplicative gradient. Such operator is constructed from transforming the additive operative into multiplicative operative, and through an objective comparison, its performances in the contour detection in images with speckle is measured. The results obtained show substantial improvements for image with high noise levels. The operator introduced corresponds to the multiplicative norm of the gradient, which is given by,

$$
\| \nabla f(i,j) \| = \exp \left( \sqrt{\ln^2 \left( \frac{f(i+1,j)}{f(i,j)} \right) + \ln^2 \left( \frac{f(i,j+1)}{f(i,j)} \right) } \right).
$$

The results obtained in [30] are illustrated in Figure 8. Also, an anisotropic diffusion technique for speckle filtering, in which such multiplicative gradient is applied as an edge detector, specifically in the anisotropy function. The method presented improvements in speckle removal and conservation of edge properties, which can be used to filter medical images of optical coherence tomography, ultrasound and other types of images degraded by speckle, as acquired in Synthetic Aperture Radar imaging systems (SAR). The performance of the modified operator in speckle removal is tested through objective comparison [31]. Specifically, the multiplicative Equation (3) is adapted and denoted by $\| \nabla f_\sigma(i,j) \|$ for $f_\sigma(i,j)$ a smooth version of the image. The partial differential equation introduced for the image treatment has the form,
\begin{equation}
\frac{\partial f(i,j)}{\partial t} = \frac{\partial}{\partial v} \left[ g^v \left( \| \nabla f \| \right) f_v(i,j) \right] + \frac{\partial}{\partial u} \left[ g^u \left( \| \nabla f \| \right) f_u(i,j) \right],
\end{equation}

with \( g^{v(u)} \left( \| \nabla f \| \right) = \exp \left[ - \left( \frac{\| \nabla f \|}{K_v(u)} \right)^2 \right] \). The multiplicative gradient is introduced to improve the border treatment applied to different threshold of diffusion \( (k_u > K_u) \) favouring the noise filtering in zones where there are contours along the axis of maximum homogeneity \( u \). The values of the luminance function are known for each pixel and the multiplicative gradient can be calculated analytically. The numerical scheme requires the use of sub-pixels. The diffusive threshold \( K_v(u) \) is established for each iteration equal to the Median Absolute Deviation (MAD) of the norm of the multiplicative, or equivalently, \( k_u = 1.4286 \cdot \text{median} \left[ \| \nabla f \| \right] - \text{median} \left[ \| \nabla f \| \right] \] \), where \( k_u = \beta K_v(u) \) with \( (\beta \geq 1) \). It has also been presented in [32], an operator that achieves the segmentation of ultrasound images with speckle, which presents a faster and more accurate performance. Such operator mix Canny’s algorithm and a new definition of multiplicative gradient to generate the initial contour map, which is subsequently optimized by following an contour path. From the objective comparison doing in [32], it is concluded that the proposed operator has a higher processing speed in real time with an performance improvement in the contour detection at least in a 75%. Visually you can see the results of the operator proposed in Figure 9 and Figure 10.

![Figure 9. The original medical image of real ultrasound of the ovary.](image)

![Figure 10. The extraction of the resulting edge in ultrasound images with different methods (a) Fuzzy, (b) ADM-NMS, (c) Canny’s algorithm, (d) Multiplicative Gradient [30], (e) Operator introduced in [32].](image)

Recently, in [33] was introduced the concept of bi-geometric wave transform. In addition, [34] presented a method of directional diffusion to eliminate the noise of polarimetric synthetic aperture radar images (PolSAR) that are affected by speckle. The method is developed by using the formalism of partial differential equations (PDE) and employs a function of contour detection that takes as arguments the multiplicative gradient norm and local adaptive thresholds. The method for filtering SAR data corresponds to an application of the method introduced in [31] using the multiplicative gradient proposed in [30], to detect contours and some important points in the PolSAR data.

5. Conclusion
The introduction of the bi-geometric structure in several operators shows a high efficiency evidenced by the objective comparison with the original operators. We conclude that there is not a general rule to adapt operators applied to images with an additive noise to the treatment of image with speckle by employing the bi-geometric structure, due to, the adapted methods only present modifications in some parts of them where the multiplicative context makes sense. The bi-geometric structure should be implemented according to the needs and characteristics of the image with speckle and the treatment objectives. Therefore, to find how it is more conveniently introduce the multiplicative structure is still the challenge. A remarkable fact is that the main challenge consists of locating precisely the places in which the bi-geometric structure should be introduced to adapt the processing at images with speckle and the correct determination of such
places could make the difference in producing improve results. In the case of the multiplicative gradient and its adapted versions, which have been applied in different context, improvements are produced in the performance of different operators and algorithms.

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