Cosmology of Bifundamental Fields

Tanmay Vachaspati  
Institute for Advanced Study, Princeton, NJ 08540  
CERCA, Department of Physics, Case Western Reserve University, Cleveland, OH 44106-7079

If a field theory contains gauged, non-Abelian, bi-fundamental fields i.e. fields that are charged under two separate non-Abelian gauge groups, the transition from a deconfined phase to a hadronic phase may be frustrated. Similar frustration may occur in non-Abelian gauge models containing matter only in higher dimensional representations e.g. models with pure glue, or if ordinary quarks are confined by two flux tubes, as implied in the triangular configuration of baryons within QCD. In a cosmological setting, such models can lead to the formation of a web of confining electric flux tubes that can potentially have observational signatures.

Many current theories of the fundamental interactions, contain bi-fundamental fields (e.g. $[\mathbf{1}, \mathbf{2}]$) that transform non-trivially under two separate non-Abelian symmetry groups. The bi-fundamental nature arises in string theory models because a string has two ends, each of which is confined to a brane. The string state bridging the two branes acts like a field that is charged under the symmetry groups corresponding to each of the two branes. Thus it is a bi-fundamental field. Here we will explore possible cosmological implications of such models. More specifically, we consider a field theory with symmetry group

$$[SU(N) \times SU(M)] \times (SM) \quad (1)$$

where SM refers to the Standard Model groups and the factors within square brackets will be referred to as the “hidden sector”. Interesting cosmological considerations arise if we further assume

- There is matter that transforms non-trivially under both $SU(N)$ and $SU(M)$.
- $SU(N)$ and $SU(M)$ are both confining.
- Matter which is a singlet under either $SU(N)$ or $SU(M)$ (but not both) and in the fundamental representation of the other group does not exist or is very heavy compared to the confinement scale.

The class of models above can be trivially generalized to the case of more symmetry factors in the hidden sector. Our considerations also extend to models of the kind

$$[SU(N)] \times (SM) \quad (2)$$

provided

- There exist matter fields transforming in the adjoint (or higher dimensional) representations of $SU(N)$.
- The $SU(N)$ factor is confining.
- Fields in the fundamental representation of the $SU(N)$ factor are not present or are very heavy compared to the confinement scale.

To be concrete, we will mostly discuss models of the type in Eq. (1). Though it should be noted that models with only glue fall in the class in Eq. (2). Also, if each quark is connected to two confining flux tubes, as in the triangle model of baryons (e.g. $[3]$), then QCD will fall into this class.

The simplest case to consider is with $N = M = 2$ but we shall use $N = M = 3$ for illustrative purposes. In the flux tube picture of confinement, each bi-fundamental is attached to two electric flux tubes, one to confine the $SU(N)$ flux and the other to confine the $SU(M)$ flux. Further, since there are no states in the fundamental representation, the flux tubes cannot break. The same holds true in the model in Eq. (2), where each particle is confined by two (or more) flux tubes since the particle is assumed to be in a higher than minimal representation and the absence of fundamentals means that the flux tubes cannot break.

The low energy excitations of the hidden sector of these theories include states similar to mesons and baryons in the standard model. (We will continue to use standard model terminology, e.g. hadrons, baryons, mesons etc., to refer to objects in the hidden sector when there is no confusion.) The hadrons are clusters of bi-fundamentals that are singlets of both $SU(N)$ and $SU(M)$. In the picture of particles and confining flux tubes they can be represented as in Fig. 1 where we show both the “Y” configuration and the “triangular” configuration for the baryons. If $N = M = 2$, the hadrons correspond to closed loops of string beaded with bi-fundamental particles.

Now let us consider the deconfined to confined phase transition. Our experience with the corresponding dual picture, where particles are replaced by magnetic monopoles and confining flux tubes by magnetic strings, suggests that such a transition is not possible. Instead, particles and strings form an infinite cosmic web as schematically represented in Fig. 2. (In the case of $N = M = 2$, the web is replaced by a set of infinite strings with bi-fundamental beads on them.) The system of confining flux tubes percolates, much as cosmic strings percolate at a phase transition, putting almost
all of the energy of the cosmic string network in infinite strings [4]. In other words, the transition from deconfined bi-fundamentals to a hadronic phase is “frustrated”. To make the transition to a purely hadronic phase, the flux tubes have to find very particular bi-fundamental particles to connect to, so that the entire web can break up into hadrons. There are many more ways to connect the bi-fundamentals so that the structure is that of a web. Even though the lowest energy state contains only baryons and mesons, the lowest energy state is also one of very low entropy and is hard to arrive at. The larger $N$ and $M$ are, the greater is the frustration, and the denser is the network.

A direct way to see that a web, and not a gas of baryons and mesons, is the likely outcome, is to note in Fig. 2 that vertices form where 3 flux tubes come together. The bi-fundamental particles are simply junctions between two different types of flux tubes and may be ignored for the purpose of the web structure. Then the flux tubes form a network that is just like a network of $Z_3$ strings. Simulations of $Z_3$ string network formation show that $> 90\%$ of the string is in one infinite network [5]. It may also be possible to study the formation of a web in the dual picture where magnetic monopoles carrying several different non-Abelian charges get confined by strings due to the breaking of large non-Abelian symmetries. Such a possibility is discussed in the context of grand unified models in [6].

A second view of the frustration is in terms of the competition between energy and entropy. In a system containing strings without junctions, the low temperature equilibrium state consists of a distribution of closed loops, which are the analogs of hadrons. At high temperature it is known to be favorable to put all the string energy into infinite strings. The temperature at which long strings become favored is called the Hagedorn temperature [7] and occurs at the temperature where the entropy contribution to the free energy becomes more important than the Boltzmann suppression [8, 9, 10, 11]. The Hagedorn transition has also been found for strings with junctions [12]. Note that the Hagedorn picture is based on a system in thermal equilibrium. It is a separate question as to whether interactions occur sufficiently rapidly that they can maintain equilibrium. At a phase transition, the interactions become too slow to maintain equilibrium and it becomes possible for topological defects and, in our case, a cosmic web to survive as a remnant. This is similar to the freeze-out of heavy particles that is so crucial to the existence of cosmic dark matter.

The Hagedorn picture has been directly confirmed in studies of $U(1)$ cosmic string formation [13] (also see [20]) where a $U(1)$ system in equilibrium at high temperature is gradually cooled down resulting in the production of infinite structures. Similarly in the case of bi-fundamentals, as the universe cools, it may become energetically favored for the system to break up into baryons and mesons but, before this can happen, the web falls out of equilibrium and the transition to the hadronic phase is frustrated. The formation of a web will occur even if the number density of bi-fundamental particles is very low, as low as one per horizon volume.

It is an interesting question if a web can form in the total absence of bi-fundamentals. Then the model only contains gluons and, as noted above, falls into the class in Eq. 2. The system at high temperature will contain gauge particles of the non-Abelian groups, $SU(N)$ for example. As the system is cooled below the deconfining temperature, all the gauge particle must get confined by two flux tubes each since they are in the adjoint representation. Once more we expect an infinite $Z_N$ web, not a gas of glueballs, to form.

In the QCD literature, there is discussion of whether baryons are better modelled by quarks connected by con-
fining strings that are in a Y configuration or a triangular (Δ) configuration. Fig. 1 can be used to illustrate these configurations if we remove one type of confining flux tube e.g. the dashed lines. A crucial difference between the Y and Δ configurations is that in the former case each quark is connected to one flux tube, whereas in the latter case each quark is connected to two flux tubes. Just as for the bi-fundamentals, if during the QCD phase transition, each quark is connected to two flux tubes, we expect hadronization to be frustrated because the strings will percolate and form infinite structures. Only a fraction of quarks will then end up as hadrons. The rest will form beads on a network of QCD cosmic strings that contain color electric fields. The structure of the QCD string network will then be like the network of light strings.

Going back to bi-fundamental models, despite the conclusive evidence that an infinite network forms in the dual (magnetic) model, there is no proof that such networks must exist in the electric sector. Hence it is useful to review the ingredients that lead to the existence of the infinite (magnetic) network. In the magnetic sector, the vacuum expectation value of an order parameter spontaneously breaks a symmetry. The order parameter lies on a “vacuum manifold” and the choice of point on the vacuum manifold is uncorrelated beyond a certain distance. The finite correlation length of the order parameter also follows generally from causality arguments [10] and implies that any topological structures (e.g. strings) that are formed are oriented in random fashion beyond a certain distance. Hence, a string is approximately described by a random walk [4]. Now random walks in three spatial dimensions are known not to close and so the strings typically formed during a phase transition are infinite in extent. Infinite strings correspond to infinite networks when the strings can have junctions. In the electric sector too, we are discussing a phase transition from the deconfined to confined phase, and the phase transition must be described by an order parameter. It seems reasonable to assume that the order parameter lies on some non-trivial manifold. Causality implies that the order parameter takes on uncorrelated values beyond a certain distance and hence any strings that are formed are also randomly oriented. Then, just as in the magnetic sector, we expect infinite networks to be present in the electric sector too. The situation appears similar to that in string theory where it is possible to have cosmic networks in both magnetic and electric sectors [21, 22].

Once a web freezes out, it can only relax due to the usual dynamical factors – tension in the strings, cosmic expansion, interactions of strings, and annihilation of particle-antiparticle. Flux tubes belonging to the same symmetry group intercommute on intersection, while flux tubes of different type will pass through each other. If the tension in one kind of string is larger than the other, the dynamics will cause the larger tension strings to shrink while stretching out the lower tension strings. Then the network will evolve toward a web of just one kind of string [14, 17]. In Fig. 2 if the strings shown by dashed lines have larger tension, they shrink and bring together 3 bi-fundamentals to form a singlet of (say) SU(M) which is now a vertex for a Z3 network of light strings.

The energy density in the network as compared to that in other standard model particles is an important quantity. Work on cosmic string networks [13, 14, 15, 17, 18] suggests that if there are efficient energy loss mechanisms, it is reasonable to expect that the energy density of the web will scale with cosmic expansion i.e. the fraction of energy density in the web remains constant in a given cosmology. (The constant may change in transitions such as from radiation to matter to dark energy domination.) Possible energy loss channels for the web include production of mesons, baryons, closed string configurations (“glueballs”), and gravitational waves. If there are interactions between the hidden sector and the standard model particles, the web could also decay into photons and other light standard model particles.

We now summarize the properties of the cosmic web following Ref. 13. Cosmological consequences of a frozen network of strings have also been investigated in Refs. 23, 24.

The canonical scenario is that the bi-fundamentals do not carry any charges other than the confined SU(N) and SU(M) charges. Then there is no efficient energy loss mechanism for the web and the cosmological evolution is that of a fluid with equation of state parameter, w, that takes into account the energy-momentum for both the strings and the bi-fundamental particles. The pressure, $P_{\text{web}}$, and energy density of the web, $\rho_{\text{web}}$, are related by

$$P_{\text{web}} = \frac{\rho_{\text{web}}}{3} \left[ \beta + (1 - \beta) (2\langle v_s^2 \rangle - 1) \right] \equiv \gamma \rho_{\text{web}} \quad (3)$$

where $\beta \in (0, 1)$ is the fraction of the web energy in (relativistic) bi-fundamentals and $\langle v_s^2 \rangle$ is the average squared velocity of the strings. Since $0 < \langle v_s^2 \rangle < 1$, the equation of state parameter is constrained by $-1/3 < \gamma < 1/3$. Recent studies of string networks indicate $2\langle v_s^2 \rangle - 1 < 0$ [25].

The cosmological evolution follows from solving the Friedmann-Robertson-Walker equations with four components to the energy density: radiation, matter, web, and cosmological constant (or dark energy), with equation of state parameters $w = 1/3, 0, \gamma$, and $-1$ respectively. The energy densities in the components decay with scale factor as $a^{-4}, a^{-3}, a^{-3(1+\gamma)}$, and $a^0$. The radiation density decays the fastest. Assuming $\gamma < 0$, which will happen if the web is dominated by strings that
are not too relativistic, the matter density decays faster than the web density, while the cosmological constant energy density does not decay. If we start in a radiation dominated universe, depending on the initial energy density of the web, evolution can lead to early web domination in conflict with observation, or to a recent epoch of web domination that may be cosmologically acceptable. These possibilities are shown schematically in Fig. 3.

To determine the initial energy density of the web, we note that confinement will set in when the separation of bi-fundamental particles becomes of order of the inverse confinement scale: \( n^{-1/3} \sim \Lambda^{-1} \) where \( n \) is the number density of bi-fundamentals and \( \Lambda \) the confinement scale. So the energy density in the web at the confinement scale is \( \rho_w(t_c) \sim m\Lambda^2 + \mu\Lambda^2 \) where \( m \) is the mass of a bi-fundamental and the string tension \( \mu \sim \Lambda^2 \). If the bi-fundamental mass is less than the confinement scale \( (m < \Lambda) \), the energy density in the network is dominated by strings and \( \rho_w(t_c) \sim \Lambda^4 \). The cosmic temperature at formation is \( T_c \sim \Lambda \) and the critical cosmic energy density \( \sim \Lambda^4 \). Hence the web initially contains an \( O(1) \) fraction of the cosmic energy density. If \( m > \Lambda \), the bi-fundamentals become non-relativistic at some high temperature and start red-shifting like pressureless matter. When the annihilation cross-section drops below the Hubble expansion rate, they freeze-out of equilibrium. The exact freeze-out density depends on the exact interactions but, in addition, freeze-out may be more complicated than for standard dark matter because once the separation between bi-fundamentals grows to \( \Lambda^{-1} \), the bi-fundamentals get confined by flux tubes.

Another scenario considered in [12] corresponds to one in which the bi-fundamentals also carry \( U(1) \) gauge charges, effectively making them “tri-fundamentals”. Then the web has an efficient energy loss mechanism due to radiation of gauge quanta and the web does not come to dominate the matter density and instead scales at a fixed ratio to the matter energy density.

The web scaling hypothesis implies that the average distance between strings in the web grows linearly with cosmic time, \( d = \chi t \), where \( \chi \) is a constant. If we assume a thermal density of bi-fundamentals such that the separation is the confinement scale \( \Lambda^{-1} \), as in the \( m < \Lambda \) case discussed above, we have \( \chi = 1/(\Lambda t_c) \) where \( t_c \) is the epoch at which the confinement transition is supposed to occur. The cosmic temperature is also given by \( \Lambda: T(t_c) \approx \Lambda \). Therefore \( t_c = T_P/\Lambda^2 \) where \( T_P \) is the Planck temperature. Hence \( \chi = \Lambda/T_P \).

Assuming that the web energy is dominated by non-relativistic strings, the web energy density is \( \rho_{\text{web}} \sim \mu/d^2 \). The cosmic critical energy density is \( \bar{\rho} = 3/(8\pi G \Lambda^2) \) and so the fraction of cosmic energy density in the web is constant at the value

\[
\Omega_{\text{web}} \sim \frac{8\pi G \mu}{3\chi^2} \sim 1
\]  

This result is sensitive to the assumed value of \( \chi \) but it is interesting to see that a cosmologically significant amount of energy density may reside in a bi-fundamental web.

The scenario where a tangled web of strings plays the role of the observed dark energy has been considered in [23, 24]. Current observations indicate an equation of state parameter \( w \lesssim -0.8 \) which is well outside the range for the web equation-of-state parameter. This implies that the web cannot explain all of the observed cosmological acceleration but it may still be a component of the cosmic energy density. As shown in Fig. 3 the web could be the dominant energy component after last scattering and before cosmological constant domination i.e. at cosmological redshifts larger than a few. In this scenario, large-scale structure growth would slow down during the string-dominated epoch and this constrains the redshift at which the web can start dominating, \( z_{\text{sd}} \lesssim 1 \), so that there is sufficient time available to obtain non-linear structures from density perturbations \( \delta \rho/\rho \sim 10^{-3} \) at last scattering (\( z \approx 1000 \)).

The web does not affect the dynamics of standard model particles through particle interactions since it lies in the hidden sector of particle physics. The exception is that the web still has gravitational interactions with the cosmological medium. In particular, in Ref. [26] it was pointed out that the interaction of a tangled network of strings with black holes at the centers of galaxies would displace the black holes and may possibly be used to constrain the energy density in the web.
Another possibility is that a dense web may be a candidate for the dark matter. It should be noted that the strings in the web themselves are expected to be relativistic but the coarse grained, root-mean-squared velocity of the web may still be non-relativistic, in which case the web could be a candidate for cold dark matter ("confined cold dark matter"). The gravitational clustering properties of the web deserve further investigation.

Finally, for completeness, we consider the case when a heavy fundamental field is present. Then a confining string can break due to the nucleation of a pair of fundamental particles. We may expect the whole network to break up into hadrons on a time scale set by the nucleation rate of fundamental particles on strings. If this time scale is shorter than the Hubble time, the network will break up; if it is larger, we may expect the web to survive. The breaking process is essentially that of Schwinger pair production and the breaking rate per unit time per unit length of string is [24, 28]

$$\frac{d\Gamma}{dt} = C \frac{\mu}{2\pi} e^{\frac{-\pi m_f^2}{\mu}}$$

where $m_f$ is the mass of the fundamental and $C \sim 1$. With Hubble expansion, the length of string in a Hubble volume grows and eventually the breaking rate becomes faster than the Hubble rate. The network decays at cosmic time $t_*$ given by,

$$H(t_*) = l(t_*) \frac{d\Gamma}{dl} = t_* \frac{\Lambda^2}{2\pi} e^{\frac{-\pi m_f^2}{\Lambda^2}}$$

where $l(t_*)$ is the total length of string in a Hubble volume at $t_*$ and $\mu = \Lambda^2$. Further, with $H(t_*) \approx 1/t_*$, we get

$$t_* \sim \frac{\Lambda^{-1}}{e^{\pi m_f^2/(2\Lambda^2)}}$$

Depending on parameters, interesting cosmological scenarios are possible. For example, there may be a period of string domination followed by the rapid break-up of the web, leading to the production of other hidden sector particles. It may also happen that the web breaks up relatively early, producing hadrons that are out of equilibrium and whose energy density then redshifts like matter. In this scenario, there is the danger that the bi-fundamental hadrons will start to dominate the universe too early, as in the old cosmological magnetic monopole over-abundance problem [24].

To conclude, we have shown that bi-fundamental fields that occur in current high energy physics models may lead to a network of (electric) cosmic strings. Depending on the string tension and the density of the network, the web may be probed and constrained by observations. For example, in some cases, the network may dominate the cosmological energy density during the early universe or affect galactic dynamics. If the network energy density scales due to the emission of quanta, these could be non-gravitational signatures of the web. Such consequences can lead to constraints on model building. Further work on the evolution of string networks within the context of specific models is needed to further explore the cosmological consequences of field theories containing bi-fundamental fields.

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