A Direct Inspection of the Displacement Current Using the Phase Measurement

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After J. C. Maxwell brought forward the concept of displacement currents, H. R. Hertz and other scholars verified the existence of electromagnetic waves in experimental, and then confirmed indirectly the concepitive correctness of displacement currents. During the recent years, along with the evolution of electronic measurement technologies, the researchers are attempting to validate directly the amplitude and orientation of displacement currents in experimental. The paper proposes and achieves one phase measurement experiment to scrutinize the orientation of displacement currents. The study indicates that the existing measurement technology is capable of inspecting directly the amplitude and orientation of displacement currents. The test results do not locate on the predicted range of classical electromagnetic theory presently. The displacement current may not be treatable similar to the conductive current to a certain extent. This conclusion enriches the understanding to the property of displacement currents.

I. INTRODUCTION

On the basis of summarizing the previous scholars’ study related with the electromagnetic phenomena in 18th and 19th centuries, J. C. Maxwell put forward the theoretical concept of displacement currents [1] in 1865. This vital physics concept may be too ahead of its time, so that Maxwell himself seemed to lack chances to validate the theoretical concept of displacement currents in the experiments.

H. R. Hertz discovered firstly the electromagnetic waves by means of the induction-balance devices in 1889, thereby confirmed indirectly the concepitive validity of displacement currents. Subsequently other scholars [2, 3] repeated and modified the Hertz’s experiment, but none of them verified directly the orientation of displacement currents. Along with the recent development of various measurement technologies, the scholars originate to make an attempt at scrutinizing directly the accuracy of displacement currents.

In 2008, A. Chakrabarty etc [4] proposed one experiment scheme, and attempted to measure the amplitude of magnetic effects induced by displacement currents, in order to verify directly the validity of the displacement current. On the basis of some scholars’ researching, the paper brings forward and achieves one experimental proposal via the existing phase measurement method in the measurement technology. The experiment is capable of inspecting directly the amplitude and orientation of displacement currents, by means of measuring the phase value of the induced electromotive force which produced from the magnetic effect of displacement currents.

In the processing of phase experiments, it is found that there exist many interference factors to impact the phase of displacement currents. Thereby it requires the experiment prototype to be as simple and compact as possible, and reduces the extra experimental components and accessorial measurement parts, and then eliminates the interference effects disturbing the phase accuracy. In the phase experiment, the displacement currents are measured and scrutinized under some groups of different conditions, and the test results are analyzed theoretically. The test results in the paper laid the groundwork for further study of displacement currents.

II. DISPLACEMENT CURRENTS IN MAXWELL’S EQUATIONS

A. Displacement Currents

In the Maxwell’s classical electromagnetic theory described with the vector terminology, the basis vector of the 3-dimensional vector space is \( \{ \hat{j}_1, \hat{j}_2, \hat{j}_3 \} \), and its co-ordinate is \( r_k \). In the electromagnetic field, the scalar potential and the vectorial potential are \( \phi \) and \( A \) respectively, while the electromagnetic strength \( (E, B) \) can be defined as follows,

\[
E = -\partial A/\partial t - \nabla \phi , \quad B = \nabla \times A ,
\]

where \( E \) is the electric intensity, \( B \) is the magnetic flux density, the operator \( \nabla = \Sigma j_k \partial/\partial r_k \). \( t \) is the time, \( j_k^2 = 1, k = 1, 2, 3 \).

In the classical electromagnetic theory, the Maxwell’s equations are,

\[
\nabla \cdot B = 0 , \quad \nabla \cdot E = \rho/\varepsilon , \quad \nabla \times E = -\partial B/\partial t , \quad \nabla \times B = \partial E/\partial (c^2 t) + \mu A ,
\]

where \( \rho \) and \( \varepsilon \) are the charge density and electric permittivity, and \( \mu \) is the magnetic permeability.
where \( \rho \) and \( j \) are the electric charge density and current density respectively. \( \varepsilon \) and \( \mu \) are the dielectric permittivity and the magnetic permeability respectively. \( c \) is the speed of light.

In the above, the term \( (\varepsilon \partial E/\partial t) \) is the displacement current. From H. R. Hertz [5], many scholars including J. C. Bose [6] and G. Marconi [7] etc validated indirectly the displacement current and the relevant effects by means of various experimental methods [8]. And the results reveal that the displacement current should not be treatable similar to the conductive current [9] to a certain extent. During the recent years, A. Chakrabarty etc are making an attempt at verifying directly the amplitude of magnetic effects induced by the displacement current. They brought forward one experimental proposal of the parallel-plate capacitor. And it inspires the paper to apply the phase measurement method to inspect directly the displacement currents.

### B. Induced Electromotive Force

In the parallel-plate air dielectric capacitor (Fig.1), it appeals to apply the alternating voltages across the capacitor plates, in order to yield the induced electromotive force large enough to be capable of measuring. In the capacitor plates, the alternating voltage with the frequency \( f \) is,

\[
V_1 = V_{10} \exp(j\omega t) ,
\]

and it forms the electric field \( E_1 \) between the plates, while its electric displacement \( D_1 \) is,

\[
D_1 = \varepsilon_0 E_1 = i_z (V_{10}/d) \exp(j\omega t) ,
\]

where \( \varepsilon_0 \) is the air dielectric coefficient, \( d \) is the interval between two plates, \( \omega = 2\pi f \). \( i_z \) is the unit vector of the normal of plate. \( j \) is imaginary unit, and \( j^2 = -1 \).

The displacement current in the capacitor is,

\[
J_d = \partial D_1 / \partial t = i_z (j\omega \varepsilon_0 V_{10}/d) \exp(j\omega t) ,
\]

The displacement current is along the direction of \( i_z \), and the magnetic field \( H_\varphi \) produced by \( J_d \) surrounds the direction of \( i_z \). According to the Eq.(5) when \( j = 0 \),

\[
\oint H \cdot dl = I ,
\]

where the closed curvilinear integral of the magnetic field \( H \) is \( 2\pi r H_\varphi \), the displacement current \( I \) in the round cross-section with the radius \( r \) is \( \pi r^2 J_d \), with \( J_d = i_z J_d \) . \( a \) is the radius of plate, with \( r \leq a \). \( dl \) is the oriented line.

The above deduces the annular magnetic field,

\[
H_\varphi = J_d r/2 \quad (r < a) .
\]

The annular magnetic field \( H_\varphi \) produced by the displacement current induces the magnetic flux, \( \Psi = \int \mu_0 H_\varphi h dr \), in the open loop (Fig.2),

\[
\Psi = -(j\omega \mu_0 \varepsilon_0 a^2 h V_{10}/(4d)) \exp(j\omega t) .
\]

According to the Faraday’s law, the induced electromotive force is \( V_2 = -\partial \Psi / \partial t \), and

\[
V_2 = (\pi^2 a^2 f^2 V_{10}/(c^2 d)) \exp(j\omega t) ,
\]

where \( c^2 = 1/(\mu_0 \varepsilon_0) \), and \( h \) is the height of rectangular single-turn open loop.

Comparing Eq.(6) with Eq.(12) reveals that, the phase difference between the induced electromotive force \( V_2 \) and the applying voltage \( V_1 \) is 0 (or \( 2\pi \)). That is, two of them are in phase in the classical electromagnetic theory.

### III. PHASE VALUE MEASUREMENT

#### A. Experimental Scheme

Due to the magnitude of displacement currents produced by the low-frequency signal is too tiny to measure,
it appeals to apply radio-frequency signals in the experiment to yield steadier measurement signals of displacement currents. However the radio frequency signal is quite vulnerable to be disturbed by the interference factors. For instance, when the measurement equipments come near the radio frequency circuits or measure the radio frequency signals, they may be likely to disturb the signal transmission of radio frequencies. Therefore the experiment devices must be chosen as simple as possible, in order to diminish the latent interferences to impact on the radio frequency signals and their phases.

The phase measurement precision of displacement currents may be influenced by various factors. It requires the experiment model to be as simple as possible in order to survey the phase exactly. In the experiment, it is helpful to improve the phase measurement precision by means of decreasing the unwanted test components and auxiliary measurement parts. Meanwhile it should be chosen the chip fixed resistors with the excellent high frequency features and be avoided to be connected with the electrical inductances as well as the extra capacitors. As a consequent, the paper chooses one simple and compact experiment model.

In the experiment, the interval of two plates is \( d = a \), and the height of the rectangle single-turn open loop is \( h = d/2 \). The applying voltage across the capacitor plates is \( V_1 = V_{10}\sin(\omega t + \varphi_1) \), while the outputting induced electromotive force is \( V_2 = V_{20}\sin(\omega t + \varphi_2) \). If the inputting power is low enough, the outputting voltage will be on the verge of the error range. Thereby the signal source should be adopted either the function generator or the signal generator combined with the power amplifier, while the inputting voltage is \( V_{10} = 10\text{ V} \).

When the radius is \( a = 0.1\text{m} \), there is one constant, \( k_1 = V_{20}/(V_{10} f^2) \), depending solely on the geometrical structure of the capacitor and of the single-turn open loop,

\[
k_1 = (\pi^2 h a^2)/(c^2 d) = 55 \times 10^{-20}.
\]  

If \( V_{20} \) is in millivolt, \( V_{10} \) is in volt, and \( f \) is in MHz, the above can be rewritten as,

\[
k_2 = k_1 \times 10^{15} = V_{20}(\text{mv})/(V_{10}(\text{v}) f^2(\text{MHz}))
\]

further the above can be magnified to constant \( A \), in order to calculate and compare expediently,

\[
A = k_1 \times 10^5 = 55 .
\]  

In the experiment, the measured results from the oscillograph and the network analyzer both reveal that the measured value of \( A \) is not one constant within a tiny error range, but is one curve with sharp undulating amplitudes, and its function value is \( A_m \). The phase difference of the outputting induced electromotive force \( V_2 \) with the inputting applying voltage \( V_1 \) is not one constant, and is one varied curve along with the increasing of frequencies. The results relate to some interference factors, and it is necessary to establish the judgment standard to select the phase values.

The judgment standard to select the phase values includes three parts: (a) there exists one constant \( A \) depended solely on the geometrical structure of the capacitor, to diminish the disturbance of the most measurement processes; (b) the wave shapes of outputting signals must be similar to that of the inputting signals and must be regular without distorting, to decrease the phase errors; (c) it is adequate to consider the impedance matching for whole test system, to cut down the disturbance of the stationary waves produced by the signal transmission to impact on the amplitudes.

### B. Phase Value Correction

In the phase measurement system, the coaxial line for radio frequency signals is chosen to accord with MIL-C-17-F, and the joint accords with MIL-C-39012. The length of coaxial line \( L_1 \) and \( L_2 \) are both 0.55m, and that of \( L_3 \) is 0.33m. Meanwhile each resistor is chosen as...
TABLE I: The frequency range and the phase difference of two cross points between the measured curve $A_m$ with the theoretical value $A = 55$.

| Resistors $R_2 = R_3$ (Ω) | The first cross point | The second cross point |
|--------------------------|-----------------------|------------------------|
|                          | Frequency range (MHz) | Phase difference $\Delta \varphi_1 (°)$ | Frequency range (MHz) | Phase difference $\Delta \varphi_2 (°)$ |
| 50                       | 70 ~ 71               | −159                   | 75 ~ 76               | 117                   |
| 100                      | 69 ~ 70               | −158                   | 76 ~ 77               | 103                   |
| 150                      | 69 ~ 70               | −166                   | 77 ~ 78               | 91                    |
| 200                      | 69 ~ 70               | −172                   | 78 ~ 79               | 93                    |
| 250                      | 68 ~ 69               | −176                   | 79 ~ 80               | 92                    |
| 300                      | 68 ~ 69               | −181                   | 79 ~ 80               | 87                    |

![Graph](image1.png)

FIG. 5: The measured curve of the phase value $\varphi$ within the frequency range 60MHz-80MHz (before phase value correcting, $R_1 = 50\Omega$, $R_2 = R_3 = 50\Omega$).

![Graph](image2.png)

FIG. 6: The measured phase graph around the frequency 70.3MHz (before phase value correcting, $R_1 = 50\Omega$, $R_2 = R_3 = 50\Omega$).

the chip fixed resistors with the excellent high frequency features.

When the resistors $R_2 = R_3 = 0$, the two plates of the capacitor will be connected parallel the resistor $R_1 = 50\Omega$, in order to match the load impedance of the capacitor with the characteristic impedance of coaxial line ($50\Omega$), and then to dominate the most of the peak-to-peak value $V_{PP10}$ of the applying voltage $V_1$ to be confined to the span 10V-20V. This is a rational choice. Meanwhile the choice can output the induced electromotive force $V_2$ with the regular sine wave shape in the frequency range 1MHz-200MHz, in order to diminish the distortion of transmission signals.

However there is not the cross point between the measured curve $A_m$ and the theoretical line $A = 55$ in the case of $R_2 = R_3 = 0$ and $\alpha = 0.1m$, and then the phase experiment cannot inspect any theoretical prediction. Thereby the experiment has to choose the condition, $R_2 = R_3 \neq 0$, to acquire the corresponding cross points and their phases. In the theory of radio frequency circuits, it should begin to calculate the influences of the impedance matching impacting on the power transmissions, when the signal frequency achieves the range 100MHz-200MHz. It means that the test conditions, $R_2 = R_3 \neq 0$, have only one tiny influence on the power transmissions, when the signal frequency is lower than 80MHz.

Because the coaxial lines $L_1$ and $L_2$ have the same one length, the phase difference is equals to zero when same one signal inputs into two lines. The measured phase difference includes mainly the phase difference caused by the coaxial line $L_3 = 0.53m$. The phase correction formula for the coaxial line $L_3$ is, $\Delta \varphi = 360° \times (L_3/\lambda) = 90.86° \times f/100$. Herein the wavelength is $\lambda = c/f$, $c'$ is the speed of electromagnetic waves in the coaxial line and is about $2.1 \times 10^8$ m/s, the signal frequency is in MHz.

IV. EXPERIMENTAL RESULTS

A. Experiment A

In the following several groups of experiments, the capacitor plates are the brass with the radius 100mm, and the measured frequency range is within 1MHz-80MHz. In case the signal frequency is lower, the function value $A_m$ within the frequency range 1MHz-40MHz is quite high and is deviated severely from the theoretical value $A = 55$. When the signal frequency approaches the frequency range 60MHz-80MHz, the function value $A_m$ has the chance to be able to close to the theoretical value $A$.
The measured frequency range is 40MHz-80MHz when the serial-connected resistor, R2 = R3, is 50, 100, 150, 200, 250, 300, 350Ω respectively (a = 0.15m).

Similarly, there are other measured results, when the R2 (= R3) is 100Ω, 150Ω, 200Ω, 250Ω, 300Ω respectively (Table 1). The above measured results state that the phase difference \( \phi \) of cross points between the measured curve \( A_m \) and the theoretical value \( A = 55 \) is close to \(-\pi \) or \( \pi/2 \). Those groups of the measured result \( \phi \) are deviated severely from the theoretical value \( 2\pi \) of the Maxwell's classical electromagnetic theory.

B. Experiment B

In the following several groups of experiments, the capacitor plates are the brass with the radius 150mm, and the measured frequency range is within 40MHz-80MHz. The measured results reveal that each curve \( A_m \) has two cross points with the theoretical value \( A = 123.75 \) (Figs.9 and 10). After the phase correcting, the phase differences of cross points are located on the span from \(-3\pi/4 \) to \(-5\pi/4 \) (Table 2).

The above measured results state that the phase difference \( \phi \) of cross points between the measured curve \( A_m \) and the theoretical value \( A = 123.75 \) is close to \( -\pi \). Those groups of the measured result \( \phi \) are deviated severely from the theoretical value \( 2\pi \) of the Maxwell's classical electromagnetic theory.

V. DISCUSSIONS

In the classical electromagnetic theory, it has not been concluded the direct inspection of displacement currents, especially for the phase inspection of magnetic effects produced by the displacement current, for the related experiments will be affected still by quite a number of interfering factors. Therefore the paper proposes and implements an experimental proposal, to inspect the orientation of displacement currents.
TABLE II: The frequency range and the phase difference of two cross points between the measured curve $A_m$ with the theoretical value $A = 123.75$.

| Resistors $R_2 = R_3$ (Ω) | The first cross point | The second cross point |
|---------------------------|-----------------------|------------------------|
|                           | Frequency range (MHz) | Phase difference $\Delta \varphi_1(°)$ | Frequency range (MHz) | Phase difference $\Delta \varphi_2(°)$ |
| 150                       | 68 $\sim$ 69         | $-135$                  | 76 $\sim$ 77         | $-195$                  |
| 200                       | 69 $\sim$ 70         | $-144$                  | 77 $\sim$ 78         | $-195$                  |
| 250                       | 66 $\sim$ 67         | $-151$                  | 77 $\sim$ 78         | $-207$                  |
| 300                       | 63 $\sim$ 64         | $-183$                  | 78 $\sim$ 79         | $-201$                  |
| 350                       | 60 $\sim$ 61         | $-201$                  | 78 $\sim$ 79         | $-213$                  |

The interfering factors impacting on the phase values include mainly, the joints among the coaxial lines and the test apparatus; the connection lines among the coaxial lines and the capacitor; the lengths of the coaxial lines; the resistor type of the serial-connected and parallel-connected resistors; the inductance and the capacitance of parasitic elements; the geometric structure of the capacitor; the interfering of the environment high-frequency signals; the brackets of the capacitor plates; the frequency shift of the outputted signals related to the inputting signals; the conducting impact of conduction currents on the capacitor plates; the impact of bias voltages between the two plates; the impact of the capacitive resistance in the low-frequency range; the impact of the inductance resistance in the high-frequency range; the geometric shape of the rectangular single-turn open loop.

The above interfering factors will impact on the measured phases on several levels. The paper calculates mainly following factors and their phase corrections.

A. Radius of Capacitor Plate

In the lower testing frequency range (1MHz-20MHz), the measured curve value $A_m$ is deviated badly from the theoretical value $A = 55$, due to the noisy interferences coming from environments, testing apparatus, parasitic elements, and capacitive resistances etc.

Particularly prominent is that the cross point between the measured curve value $A_m$ with the theoretical value $A = 13.75$ will be disappeared totally in 7 groups of testing data within the frequency range 1MHz-80MHz (Fig.11), when the plate radius is decreased to 0.05m. Moreover the amplitude of outputting induced electromotive force is on the verge of the error range when the serial-connected resistor is 350Ω. This means that it may be more difficult to measure directly the displacement current in smaller dimensional ranges. Thereby it is necessary to adopt the plate with a larger radius in future testing to decrease interfering levels of noises.

Comparing the test results of Experiments A and B finds that the phase value of the first cross point between the measured curve value $A_m$ with the theoretical value $A = 123.75$ is still located on the value around -180°, and the phase value of the second cross point is moved gradually to the value around -180° from that around -270°, when the plate radius is increased from 0.1m to 0.15m.
B. Resistor

The resistor will exert an important influence on the phase difference. The chip fixed resistor is chosen as the serial-parallel connected resistor in the experiment, for its better high-frequency following performance in comparison with the film resistor and the metal film resistor etc. With the value increasing of the serial-connected resistors $R_2$ and $R_3$, the measured curves $A_m$ are descended gradually (Figs.3 and 9), while the measured phase curves $\varphi$ are narrowed (Fig.4) or shifted to the right gradually (Fig.10).

C. Frequency Shifting

The capacitor exerts one weaker impediment to the higher frequency sine signals, and one stronger impediment to the lower frequency sine signals. This feature causes the single-turn open loop to impact the stronger aberrational influences on the electric field between the parallel plates within lower frequency range.

Due to the interfering influences of the coaxial line material, the capacitor plate material, and the capacitor geometrical structure etc, there are a few frequency shifting between the outputting sine signals and the inputting sine signals. That is, two signals are not the same frequency within all frequency ranges. The frequency shifting causes the phase values displayed on the oscillograph stay always on the fluctuant states. Sometimes the phase value recorded with the visual inspection has a little departure from that in the saved measured graph. This bias is quite tiny by comparison with the final result, and its effect is able to be neglected often.

D. Capacitor Fixation

The parallelism of capacitor plates as well as the fringe burr of plates has an influence on the distributing of electric fields between two plates. Along with the decreasing of plate’s radius, it will be acute the interfering effect of the open loop impacting on the electric fields between the plates. The capacitor plates are fixed by the plastic ruler or the wood strip, and are avoided to use the metal material bracket to diminish the distortion degree of electric fields.

E. Circuitry Connection

The experiment must keep ensuring same voltage for two plates of the capacitor. Otherwise the voltage bias between the plates will sway the measured results. The azimuth angle of the connecting point of the capacitor plate in connection with the inputting signal line will impact the transporting path of conduction currents on the plates. Further it produces the magnetic fields to interfere the magnetic flux in the rectangular open loop and its outputting induced electromotive force. When the azimuth angle $\alpha = \pm 90^\circ$, the interfering influences of magnetic fields produced by the conduction currents on the capacitor plates will be lesser comparatively.

VI. CONCLUSION

In the range of classical electromagnetic theory, it has not been finished the direct inspection of displacement currents yet. Based on previous research on displacement currents inspection, the paper proposes and fulfills an experiment proposal adopting the phase measurement method, to scrutinize directly the amplitude and orientation of displacement currents. It should be noted that the study for the phase measurement of displacement currents has examined only some simple cases, of which the radii of capacitor plates are 0.05m, 0.10m, and 0.15m respectively and the frequency range is only within 1MHz-80MHz. Despite its preliminary character, this study can clearly indicate that each measured phase value, of which the cross points of the measured curve $A_m$ with the theoretical value $A$, does not locate on the predictive range of the classical electromagnetic theory described with the vector terminology. It is undoubted that the above results enrich the understanding to the physical features of displacement currents.

During the measuring processing of phase values, it is found that there are many interfering factors for the measured phase values. Thereby the phase measuring experiment will adopt one compact and simple scheme, use the coaxial lines and joints according with some certain standards to diminish the phase errors, decrease the extra measurement components to obtain accurately the measured phase values, and correct the final phase values relevantly. For the future experimental study, the research will concentrate on only the adopting large radii of plates to achieve precise measured phase values.

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