MAGNETIC REYNOLDS NUMBER DEPENDENCE OF RECONNECTION RATE AND FLOW STRUCTURE OF THE SELF-SIMILAR EVOLUTION MODEL OF FAST MAGNETIC RECONNECTION

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Received 2005 June 24; accepted 2005 September 26

ABSTRACT

This paper investigates the magnetic Reynolds number dependence of the “self-similar evolution model” (previously reported by Nitta and coworkers) of fast magnetic reconnection. I focused my attention on the flow structure inside and around the reconnection outflow, which is essential for determining the entire reconnection system. The outflow consists of several regions divided by discontinuities, for example, shocks, and it can be treated by a shock-tube approximation. By solving the junction conditions (e.g., the Rankine-Hugoniot conditions), one obtains the structure of the reconnection outflow. Magnetic reconnection in most astrophysical problems is characterized by a huge dynamic range of its expansion ($\sim 10^7$ for typical solar flares) into a free space that is free from any influence of external circumstances. Such evolution results in a spontaneous self-similar expansion that is controlled by two intrinsic parameters: the plasma-$\beta$ and the magnetic Reynolds number. The plasma-$\beta$ dependence was investigated in our previous paper. This paper newly clarifies the relation between the reconnection rate and the inflow structure just outside the Petschek-like slow shock: as the magnetic Reynolds number increases, strongly converging inflow toward the Petschek-like slow shock forms, which significantly reduces the reconnection rate. 

Subject headings: Earth — ISM: magnetic fields — MHD — Sun: flares

1. INTRODUCTION

Magnetic reconnection is one of several probable candidates for a very powerful energy conversion mechanism in magnetized plasma systems, for example, solar flares and geomagnetospheric substorms. Most believe that magnetic reconnection is also a universal mechanism for recent energetic topics, for example, accretion disk X-ray emission of young stellar objects (YSOs; see Koyama et al. 1994; Hayashi et al. 1999) and Galactic ridge X-ray emissions (GRXEs; see Koyama et al. 1986; Tanuma et al. 1999).

There are many phenomenological studies of magnetic reconnection, mainly by numerical simulation researchers, but essential open questions still remain, regarding not only the microscopic physics of the anomalous resistivity but also the macroscopic magnetohydrodynamical (MHD) structure. In particular, understanding of the coupling process between different scales, i.e., how the microscopic-scale process relating to the origin of anomalous resistivity ($\sim 10^9$ m for solar corona) governs the macroscopic-scale process in which enormous magnetic energy releases ($\sim 10^7$ m for typical flare loop), is very important but still an open question. One may think it is very curious for an extremely small diffusion region to govern the entire huge reconnection system (e.g., the idea of the “fractal diffusion region”; see Shibata & Tanuma 2001). The aim of this series of papers is to clarify the relationship and causality between these two different scales in a regime of MHD (see § 4.4). In this work, our attention is focused on the magnetic Reynolds number dependence of the flow structure and the reconnection rate.

We consider a magnetic energy conversion in an antiparallel magnetic equilibrium configuration called a “current sheet system.” In this case, the electric resistivity plays an important role: it breaks the frozen-in condition and leads to magnetic diffusion. The magnetic diffusion can release the magnetic energy by Ohmic dissipation. However, since the Spitzer resistivity in most astrophysical plasma systems is considerably small, the magnetic diffusion speed should be very small. This means that magnetic diffusion alone is not sufficient for a quick energy conversion, like in solar flares. We here introduce an essential nondimensional parameter called the “magnetic Reynolds number” $R_{\text{m}}$, which is defined as the ratio of the Alfvén wave transit timescale across the system to the magnetic diffusion timescale. Most astrophysical plasma is highly conductive (e.g., $R_{\text{m}} \sim 10^{14}$ for typical solar corona plasma); thus, energy conversion by Ohmic dissipation is very slow (the energy conversion timescale is of the order of $10^8$ yr: this is too long to explain solar flares).

It is important that, even in this case, the processes involving magnetic reconnection can release magnetic energy at a very large power. The most successful reconnection model for astrophysical applications is the Petschek model (see Petschek 1964). Although the reconnection of magnetic field lines itself is caused by nonuniform magnetic diffusion, the main energy conversion results from the propagation of the X-shaped slow shocks in the Petschek model. Consequently, the Petschek model can convert the magnetic energy in a very short time (the wave propagation timescale, estimated to be of the order of $10^7$ s for typical solar flares) almost independent of the magnetic Reynolds number. Such a powerful process, in which the magnetic diffusion timescale does not determine the entire energy conversion, is called “fast reconnection.” As discussed above, the Petschek model might be the most significant model to explain actual reconnection phenomena, but the author finds this model to be deficient as a real astrophysical reconnection model. The following part of the introduction is focused on this point.

We must note that many cases of actual magnetic reconnection in astrophysical systems usually grow over a huge dynamic range in the spatial dimension. For example, the initial scale of the reconnection system can be defined by the initial current sheet thickness, but this is too small to be observed in typical solar flares. We do not have any convincing estimate of the scale,
but if we estimate it to be of the order of the ion Larmor radius, it is extremely small ($\sim 10^9$ m in the solar corona). Finally, the reconnection system develops to a scale of the order of the initial curvature radius of the magnetic field lines ($\sim 10^7$ m $\sim 1.5\%$ of the solar radius for typical solar flares). The dynamic range of the spatial scale is obviously huge ($\sim 10^7$ for solar flares). For geomagnetospheric substorms, their dynamic range of growth is also large ($\sim 10^4$ for substorms). Such a very wide dynamic range of growth suggests that the evolution of the magnetic reconnection should be treated as a development in “free space” and that the external circumstances do not affect the evolutionary process of a magnetic reconnection, at least at the expanding stage just after the onset of reconnection.

Even if a system is not completely free from the influence of the external circumstances, we can approximately treat it as a spontaneously evolving system if the evolution timescale (which is estimated as the Alfvén transit time for the final system scale because the spontaneous expansion speed of the reconnection system is equivalent to the fast-mode propagation speed; see Nitta et al. [2001], hereafter Paper I) is much smaller than the timescale imposed by the external circumstances (e.g., the convection timescale). For typical substorms, the evolution timescale of the reconnection is of the order of seconds or minutes, while the convection timescale to compress the current sheet is of the order of hours. In such cases, the external circumstances simply play the role of triggering the onset of reconnection, and the evolution itself is approximately free from the influence of the external circumstances. Although, of course, the exceptional cases in which the external circumstances intrinsically influence the evolution can arise for the particular situation (e.g., the cases that very fast convection drives the reconnection), the author believes that it is worthwhile to establish a new model applicable to the cases that are free from the external influences.

However, no reconnection model evolving in a free space has been studied. We should note that most of the previous theoretical (e.g., Petschek 1964; Vasyl’ianas 1975; Priest & Forbes 1986) and numerical works on reconnection treated it as a boundary problem strongly influenced by external circumstances. Our interest is focused on a spontaneous evolutionary process of magnetic reconnection in a free space as a macroscopic instability in a current sheet system. This evolutionary reconnection model can explain a coupling process between different scales, i.e., a microscopic scale represented by the ion Larmor radius and a macroscopic scale represented by the scale of external circumstances.

The author and collaborators studied the problem of two-dimensional MHD reconnection in a series of papers. Nonlinear evolution of spontaneous reconnection in a free space is numerically simulated (two-dimensional MHD simulations with artificially enhanced localized resistivity) in Paper I. The result of our numerical simulation clearly shows a self-similar expansion of the reconnection system. Owing to technical reasons (restriction of CPU time, memory, etc.), we could confirm the stable expansion of the reconnection system only in a dynamic range of $10^2-10^3$ in spatial scale of the expansion. This dynamic range is obviously insufficient, because the actual dynamic range seems to be $10^4-10^7$, as discussed above. In order to supplement the lack of the dynamic range, we studied the inflow region of the reconnection system semi-analytically by the so-called Grad-Shafranov approach (Nitta et al. 2002, hereafter Paper II) with a boundary condition along the Petschek-like X-shaped slow shock (Nitta 2004, hereafter Paper III). We obtained a self-similarly expanding solution that is fairly consistent with our numerical result in Paper I. Therefore, we can conclude that the self-similar solution of an expanding reconnection system is stable with any ideal MHD modes and will continue to expand until the system scale becomes comparable to the typical scale of the external environment, for example, the diameter of a magnetic flux loop. In Paper III, we studied a structure of the reconnection outflow that provides the boundary condition along the Petschek-like slow shock by a shock-tube approximation. This boundary condition is very important because it determines the structure of the inflow region (see Paper II).

The spontaneously expanding reconnection system has only two parameters: the plasma-$\beta$ value and the magnetic Reynolds number. We clarified the plasma-$\beta$ dependence for a very wide dynamic range of plasma-$\beta$ in Paper III (a brief summary is in §2). This enables us to apply our self-similar reconnection model to a wide variety of phenomena that may have different plasma-$\beta$. In this paper, we study the magnetic Reynolds number dependence of the reconnection system. While most believe that a spatially localized anomalous resistivity is important for realizing a fast reconnection (e.g., Yokoyama & Shibata 1994), we do not yet have any established model for the mechanism or a convincing estimated value of the anomalous resistivity (Coppi & Friedland 1971; Ji et al. 1998; Shinohara et al. 1998). Thus, it is important to study the magnetic Reynolds number dependence of the properties of the reconnection system. We have obtained the following results.

We define the effective magnetic Reynolds number as $\text{Re}_m \equiv V_{A0}/V_{\text{diff}}$, where $V_{A0}$ is the Alfvén speed at the asymptotic region (the region far from the current sheet) and $V_{\text{diff}}$ is the actual magnetic diffusion speed at the diffusion region.

1. Strongly converging inflow.—As $\text{Re}_m$ increases, the convergence of the inflow toward the Petschek-like slow shock becomes significant. This leads the inflow velocity and the magnetic field line at the inflow region to be parallel to each other.

We here define the reconnection rate $R^*$ as the nondimensional reconnection electric filed (the ratio to the product of $V_{A0}$ and $B_0$ [the magnetic field strength at the asymptotic uniform region]).

2. Reconnection rate.—As a result of the converging inflow, the reconnection electric field decreases as $\text{Re}_m$ increases. Finally, we find $R^* \propto 1/\text{Re}_m$ at the large magnetic Reynolds number limit.

This paper is organized as follows. We state a model of the reconnection outflow as a kind of shock-tube problem in §2. The basic equations for the outflow structure and the numerical procedure to solve the equations are also listed. By solving the basic equations, we obtain the flow structure and the reconnection rate. The results are shown in §3. We discuss the properties of these results in §4.

2. NUMERICAL SCHEME

2.1. Concept

As we mentioned in Paper III, the reconnection outflow in our spontaneous model has a structure involving several discontinuities, like a shock-tube problem. We solve the reconnection outflow region by approximating the problem as a kind of shock-tube problem (see details in §2.2). In Paper III, we assumed a nonconverging inflow toward the slow shock approximating the original Petschek model and discussed the structure of the outflow region as a function of the ambient plasma-$\beta$ value. Consequently, we obtain the plasma-$\beta$ dependence of the spontaneous inhalation speed (see Fig. 1) and the reconnection rate (see Fig. 2). These figures are essentially the
can find any unknown variable as a function of \( V_0 \) can be treated as an initial guess for the procedure in the fast shock and the no–fast shock regimes. This figure is almost the same as Fig. 2 of Nitta 2004, but the following points are different: the much higher converging precision and the precise treatment for junction between the fast shock regime and the no–fast shock regime.

In this paper we study the magnetic Reynolds number dependence of the outflow structure by the above shock-tube approximation in the regime of ideal (nonresistive) MHD. One may find this curious, because we cannot treat the magnetic Reynolds number in the regime of ideal MHD as in this work. We should note, however, that if we can evaluate the reconnection electric field \( E_z \) (note that \( E_z \) is uniform in the vicinity of the reconnection point from Ampere’s law because this region is almost stationary in our self-similar model) and the magnetic field \( B_z^\ast \) around the diffusion region from the shock-tube problem, we can estimate the inflow speed \( v_{\text{in}}^\ast (\equiv E_z/B_z^\ast) \) toward the diffusion region. This inflow speed is in balance with the magnetic diffusion speed \( v_{\text{diff}}^\ast \) at the diffusion region. Treating the Alfvén speed \( v_{\text{A0}} \) at the asymptotic region to be a normalization value as in Paper III, the inverse of the inflow speed toward the diffusion region shows the effective magnetic Reynolds number \( R_{\text{m}}^\ast (\equiv v_{\text{A0}}/v_{\text{diff}}^\ast) \). Thus, we can treat the magnetic Reynolds number in the context of ideal MHD. We stress again that our attention is focused on the magnetic Reynolds number dependence of the reconnection system, i.e., the outflow structure and the reconnection rate.

### 2.2. Model

We present a schematic picture of the reconnection outflow in our self-similar evolution model (see Fig. 3). The coordinates are defined as follows: the \( x \)-axis is parallel to both the current sheet and the initial antiparallel magnetic field, the \( y \)-axis is perpendicular to the current sheet, and the \( z \)-axis is parallel to the current sheet but perpendicular to the initial magnetic field (hence, \( \beta_z = 0 \) in this two-dimensional problem). The essence of the model is quite the same as the model in Paper III. A similar structure of the reconnection outflow is shown in Figure 9 of Ugai (1999) as a simulation result. Because of symmetry with respect to the \( x \)- and \( y \)-axes, we treat only the region \( x, y > 0 \). The outflow is composed of two different plasmas that have different origins. These two plasmas are in contact at the point \( x = x_c \) (the contact discontinuity). A plasmoid forms around the contact discontinuity. The rear-half region \( (x < x_c \) the connection jet) is filled with the connected plasma coming from outside the current sheet (the inflow region). The front-half region \( (x > x_c \) the plasmoid) is filled with the original current sheet plasma.

The entire outflow is surrounded by a slow shock that has a complicated “crab-hand” shape (see Abe & Hoshino 2001). The Petschek-like \( X \)-shaped slow shock (an oblique shock) is elongated from the diffusion region with a slight opening angle \( \theta \). There is a reverse fast shock (an almost perpendicular shock) inside the reconnection jet \((x = x_f)\). In front of the plasmoid, a forward \( V \)-shaped slow shock (an oblique shock) forms. The opening angle and the locus (the crossing point with the \( x \)-axis) are \( \phi \) and \( y_\ast \), respectively.
The entire structure, including the several discontinuities, is analogous to the one-dimensional shock-tube problem. The forward shock (a V-shaped slow shock) and the reverse shock (a fast shock) are formed by the collision of the reconnection jet and the original current sheet plasma, and they propagate in both directions. Between these two shocks, the contact discontinuity forms. We approximate this reconnection outflow as a quasi–one-dimensional problem in order to solve it analytically. Such an approximation may be valid near the x-axis, because the system is symmetric with respect to the y-axis.

We focus our attention on the quasi–one-dimensional problem along the inflow stream line and the reconnection outflow. We treat the L-shaped region x, y = finite ≫ D if the inflow stream line is parallel to the y-axis, where D is the initial current sheet thickness. Note that this region apparently coincides with the x- and y-axes in the self-similar stage, which is a very late stage from the onset when we observe the evolution in a zoom-out coordinate system (see Paper I). If the inflow stream lines are inclined owing to their convergence, we consider the modified L-shaped region along the inflow stream line (see SSL of Fig. 7 below) instead of the y-axis. Each region between two neighboring discontinuities is approximated as uniform. We note that the upstream region p just above the X-shaped slow shock that the region between the slow shock and the separatrix field line (X-shaped field line reaching the X-point) is also approximated as uniform. In this region, each reconnect field line has an almost straight shape and crosses the X-shaped slow shock, while each field line has a hyperbolic shape in the region above the separatrix field line. In Paper III, we suppose a non-converging inflow in the region p for the spontaneous inflow, but if inhalation speed is reduced when Re_m increases, it may result in an alteration of the inflow structure. Hence, we should factor the possibility of convergence or divergence of the inflow in the region p into our problem (see below and § 2.4).

Region 3 between the contact discontinuity and the V-shaped slow shock looks to be nonuniform. Since we do not know how to solve the two-dimensional structure of this region analytically, we approximate this region as uniform.

The quantities denoting the initial uniform equilibrium at the asymptotic region are gas pressure $P_0$, mass density $\rho_0$, and magnetic field strength $B_0$. The plasma-$\beta$ value at the asymptotic region is defined as $\beta_0 \equiv P_0 / [B_0^2 (2\mu)]$, where $\mu$ is the magnetic permeability of vacuum. In the rest of this paper, we use the normalization of physical quantities as in Paper II. We define units for each dimension as follows: unit of velocity $V_{A0} \equiv B_0 / (\mu \rho_0)^{1/2}$ (Alfvén speed at the asymptotic region); unit of length $V_{A0} t$, where $t$ is the time from the onset of reconnection; unit of mass density $\rho_0$; unit of magnetic field $B_0$; and unit of pressure $(\beta_0/2) \rho_0 V_{A0}^2$.

We set the quasi–one-dimensional shock-tube-like problem as follows. The system has 22 unknown quantities: $P_{in}, \rho_{in}, v_{in}, B_{in}, \theta, P_1, \rho_1, v_1, B_1, v_x, P_2, \rho_2, v_2, B_2, v_x, \rho_3, v_3, B_3, B_3, \phi$, and $x_\perp$ where $P_{in}, \rho_{in}, v_{in}, B_0$, denote the pressure, density, velocity, and magnetic field, respectively (note that $x_\perp = v_3 / v_2$ because no mass flux passes through the contact discontinuity). The suffixes $i, j, 1, 2, 3, x\parallel$, and $x\perp$ denote the vector components. The quantities $\theta, x_\parallel, \phi$, and $x_\perp$ denote the inclinations of the X-shaped slow shock, the locus of the fast shock, the inclination of the V-shaped slow shock, and the locus of the V-shaped slow shock (crossing point with x-axis), respectively.

These unknowns should be related to each other via conditions coming from the integrated form of conservation laws (i.e., the Rankine-Hugoniot [R-H] conditions) or other relations. The set of relations is listed in the next subsection.

There is an essential difference from the model in Paper III. In Paper III, we assumed (or imposed) the condition that the transverse component of the inflow velocity vanishes ($v_{yp} = 0$) and solved the vertical component $v_{yp}$ of the inflow velocity (and other unknowns). We here treat $v_{yp}$ as an artificially controllable variable, and its value is given by hand. The change of $v_{yp}$ may induce a change of the strength of the fast-mode rarefaction. Since the inflow is induced by the fast-mode rarefaction, this change may result in a change of $v_{yp}$. Hence, we must solve $v_{yp}$ as a new unknown quantity.

2.3. Basic Equations

There is no essential difference in the basic equations for the outflow structure from the equations used in Paper III, except that we must prepare the equations denoting the oblique inflow case $v_{yp} \neq 0$. In most cases, we must simply add several terms involving $v_{yp}$ (e.g., in eqs. [3], [5], [6], [7], [13], and [22] in Paper III).

In equation (A1) (the frozen-in condition at the region p), a physical insight to modify it is required. In the oblique ($v_{yp} \neq 0$) inflow case, we should note that the frozen-in condition requires that the ratio of the perpendicular component $B_3$ of the magnetic field with respect to the inflow velocity versus the mass density holds to be a conservative quantity. If we can a priori obtain the stream line configuration all over the inflow region, we can state the exact modified equation for the frozen-in condition; however, we never know it in the context of this quasi–one-dimensional shock-tube approximation. Hence, we need an approximating model of the inflow stream line configuration: we here simply approximate it to be straight. Thus, we obtain equation (A1) in the Appendix.

According to the above model of the reconnection outflow, we obtain the following 22 equations for 22 unknown quantities (detailed forms of each equation are listed in the Appendix):

1. relations between pre-X-shock and asymptotic region
   (a) Frozen-in condition (eq. [A1]);
   (b) Polytropic relation (eq. [A2]);
2. R-H jump conditions at X-shock slow
   (a) Pressure jump (eq. [A3]);
   (b) Density jump (eq. [A4]);
   (c) Velocity jump (parallel component; eq. [A5]);
   (d) Velocity jump (perpendicular component; eq. [A6]);
   (e) Magnetic field jump (parallel component; eq. [A7]);
   (f) Magnetic field jump (perpendicular component; eq. [A8]);
3. R-H jump conditions at reverse fast shock
   (a) Pressure jump (eq. [A9]);
   (b) Density jump (eq. [A10]);
   (c) Velocity jump (eq. [A11]);
   (d) Magnetic field jump (eq. [A12]);
4. magnetic flux conservation at X-point (eq. [A13]);
5. force balance at contact discontinuity (eq. [A14]);
6. R-H jump conditions at forward V-shock slow
   (a) Pressure jump (eq. [A15]);
   (b) Density jump (eq. [A16]);
   (c) Velocity jump (parallel component; eq. [A17]);
   (d) Velocity jump (perpendicular component; eq. [A18]);
   (e) Magnetic field jump (parallel component; eq. [A19]);
   (f) Magnetic field jump (perpendicular component; eq. [A20]);
7. boundary Condition at the tip of the outflow (eq. [A21]);
8. magnetic flux conservation all over the outflow (eq. [A22]).

We can solve the majority of these equations by hand, and by substituting the solutions into other equations, we can reduce equations. Finally, 10 equations (eqs. [A1], [A3], [A4], [A5], [A6], [A7], [A11], [A14], [A21], and [A22]) remain complicated nonlinear coupled equations for 10 unknowns: \( \rho_p, v_{yp}, B_{xp}, B_{yp}, \theta, P_1, \rho_1, x_p, \phi, \) and \( x_r \).

### 2.4. Numerical Procedure

We solve these 10 coupled equations by an iterative method (the Newton-Raphson method). In order to find the well-converging solution of the nonlinear coupled equations, a precise initial guess of the unknowns is required. In general, this step to find an appropriate initial guess is the core difficulty (this is comparable to “treasure hunting in ten-dimensional space” with an incomplete treasure map). Fortunately, this most difficult step had already been cleared in Paper III for the case \( v_{yp} = 0 \).

We can start from the solution of the case \( v_{yp} = 0 \) for an arbitrary value of \( \beta_0 \). Our interest is focused on low-\( \beta \) cases that typically appear in astrophysical applications. We demonstrate the numerical procedure for the case \( \beta_0 = 0.01 \) as the reference case, because this is a typical value in the solar corona and geomagnetosphere. In Paper III, we have already had the solution in the spontaneous inhalation case \( (v_{yp} = 0) \) for \( \beta_0 = 0.01 \).

First, we start with the solution of the case \( v_{yp} = -v_{insp}(0.01) = -0.06398 \) [where \( v_{insp}(0.01) \) is the spontaneous inhalation speed for \( \beta_0 = 0.01 \) as the initial guess for the case \( v_{yp} = -v_{insp}(0.01) + \Delta v_y \), where \( \Delta v_y \) is the increment of \( v_{yp} \).]

Once we find a converged solution of the Newton-Raphson procedure for the case \( v_{yp} = -v_{insp}(0.01) + \Delta v_y \), we treat it as the initial guess for the case \( v_{yp} = -v_{insp}(0.01) + 2\Delta v_y \), and we have successively obtained the solutions for different \( v_{yp} \). Of course, \( \Delta v_y \) should be a small enough value to keep good convergence of the Newton-Raphson method. From several trials, we carefully adopt the following two cases of the increment: \( \Delta v_y = 10^{-6} \) and \( 10^{-7} \). We have checked that the results of these two cases coincide reasonably well with each other.

We obtain the result that, as \( |v_{yp}| \) decreases, the strength of the reverse fast shock decreases. At a critical value of \( |v_{yp}| \), the reverse fast shock no longer forms (the density jump ratio and the pressure jump ratio reduce to unity). At this point we exchange the numerical code to another one for the case of no reverse fast shock forms (see the last part of the Appendix), and we can continue the calculation. The above procedure is valid for any value of \( \beta_0 \).

### 3. RESULT

We have investigated the response of the reconnection outflow structure to a variation of \( v_{yp} \) (the \( y \)-component of the inflow velocity at the pre–slow shock region [region p] normalized by the ambient Alfvén speed \( V_{A0} \)). Usually, it shows the reconnection rate if the transverse component \( B_{yp} \) of the magnetic field is almost the same as its ambient value \( B_0 \) and \( x \)-component \( v_{xp} \) of the inflow velocity is negligible (\( |v_{xp}| \ll |v_{yp}| \)). We must note, however, that if a significant converging inflow arises or the transverse magnetic component reduces from its ambient value, we need an alternative estimate of the reconnection rate. Actually, a significant converging inflow and decrease of the transverse magnetic component occur when \( |v_{yp}| \) decreases (see Fig. 4).

Thus, we introduce the effective reconnection rate

\[
R' \equiv -v_{yp} B_{xp} + v_{yp} B_{yp},
\]

which denotes the reconnection electric field normalized by \( V_{A0} B_0 \).

The magnetic Reynolds number \( \text{Re}_m \) is defined as the ratio of the ambient Alfvén speed to the magnetic diffusion speed. We should note that the shock-tube approximation adopted here is within the regime of ideal MHD, so we cannot treat the diffusion region itself in this scheme. Although one may think that we cannot argue the diffusion speed in this scheme at all, actually we can estimate it by a simple approximation as follows. In the vicinity of the reconnection point, the system is approximately in a steady state in the self-similarly expanding solution. This fact implies, from Ampere’s law, that the electric field \( E_z'' \equiv -v_{yp} B_{xp} + v_{yp} B_{yp} \) is approximately uniform around the diffusion region. The diffusion speed \( v_{inf}'' \) must be balanced with the local inflow speed \( v_{inf}'' \) in the stationary state, and \( v_{inf}'' \) is estimated as \( v_{inf}'' = E_z'' / B'_y = (v_{yp} B_{xp} + v_{yp} B_{yp}) / B'_y \), where \( B'_y \) is the magnetic field just outside the diffusion region. We suppose \( B'_y \sim B_{yp} \) in order to approximately satisfy the magnetic flux conservation.

Thus, we obtain the effective diffusion speed

\[
v_{inf}'' = v_{inf}'' \sim \left( -v_{yp} B_{xp} + v_{yp} B_{yp} \right) / B_{xp},
\]

and define the effective magnetic Reynolds number

\[
\text{Re}_m'' \equiv 1 / v_{inf}'' = B_{xp} / \left( -v_{yp} B_{xp} + v_{yp} B_{yp} \right).
\]

Note that \( V_{A0} = 1 \) in our normalization. Hence, we obtain

\[
R' = B_{xp} / \text{Re}_m''.
\]

It is very important to clarify the relation between \( R' \) and \( \text{Re}_m'' \). For this purpose, we treat \( \text{Re}_m'' \) as a control parameter instead of \( v_{yp} \). The effective magnetic Reynolds number \( \text{Re}_m'' \) dependence
the decrement of the reconnection rate \( R \) because the increase of \( \beta_0 \). Although the magnetic Reynolds number \( \text{Re}_m^* \) increases from the case \( \beta_0 = 0 \), the reconnection rate \( R^* \) decreases because the directions of the inflow velocity and the inflow magnetic field tend to be parallel as the converging speed \( v_{yp} \) increases (see Fig. 4).

Converging inflow—The converging speed \( v_{yp} \) (note that \( v_{yp} \) \( \neq 0 \) in our result, which denotes a converging inflow) of the spontaneously expanding reconnection system in a free space is investigated for the cases \( \beta_0 = 0.01 \) and 0.2. We select the \( y \)-component \( v_{yp} \) of the inflow velocity toward the slow shock as the artificially controllable variable for our numerical parameter survey. Starting at the solution with its spontaneous inhalation speed, \( v_{yp} = v_{yp}(\beta_0) \) toward the slow shock, and we gradually decrease its absolute value with a finite decrement (see \$ 2.4 \) and numerically solve the coupled equations listed in \$ 2.3 \) by the Newton-Raphson method for each value of \( v_{yp} \). Thus, we obtain a series of solutions for the outflow structure. Note that \( \text{Re}_m^* \) is a monotonically increasing function of \( v_{yp} \) in our result (see Fig. 4), so we adopt \( \text{Re}_m^* \) as the control parameter in the following discussion. The results are as follows:

1. Converging inflow—The converging speed \( v_{yp} \) (note that \( v_{yp} \) \( \neq 0 \) in our result, which denotes a converging inflow) of the inflow drastically increases as the magnetic Reynolds number \( \text{Re}_m^* \) increases (see Fig. 4). The directions of the inflow velocity and the magnetic field tend to be parallel as the converging speed increases (see Fig. 5).

2. Magnetic Reynolds number dependence of reconnection rate.—As \( \text{Re}_m^* \) increases, first the reconnection rate \( R^* \) increases owing to the increase of \( B_{yp} \) (see Figs. 4 and 6), and then \( R^* \) decreases because the directions of the inflow velocity and the magnetic field tend to be parallel (see Fig. 5). This causes a reduction of the reconnection electric field. Thus, the reconnection rate \( R^* \) decreases as \( R^* \propto 1/\text{Re}_m^* \) because \( B_{yp} \sim 1 \) in the large \( \text{Re}_m^* \) region (see Figs. 4 and 6).

4. DISCUSSION

4.1. Channel Flow Structure in the Inflow

For large magnetic Reynolds number (say, \( \text{Re}_m^* \geq 20 \)), a significantly converging inflow toward the slow shock arises. It can grow to be almost parallel to the magnetic field lines as \( \text{Re}_m^* \) increases. In such a case, the inflow toward the diffusion region must be very slow because the reconnection rate (note \( R^* \equiv |v_p \times B_p| \)) is very small, even though the Petschek-like slow shock still strongly intakes the plasma in order to satisfy the Rankine-Hugoniot condition. This means that the inflow forms a channel flow structure: relatively large speed converging inflow toward the slow shock and very slow inflow toward the diffusion region (see Fig. 7).

It is possible to observationally estimate the reconnection rate from the inflow Alfvén Mach number (see Yokoyama et al. 2001; Isobe et al. 2005). If we can obtain the Alfvén speed and the inflow speed perpendicular to the inflow magnetic field by observation (i.e., if we observe a motion of a bright filament-like structure toward the X-point, presuming it shows a field line motion, we can estimate the normal speed of the field line motion), we can estimate the reconnection rate as a good approximation. We must note, however, that the inflow Alfvén Mach number itself does not denote the reconnection rate if the inflow is inclined with respect to the magnetic field lines. Such a situation may occur when the flow is significantly converging (see Fig. 7).

4. DISCUSSION

4.1. Channel Flow Structure in the Inflow

For large magnetic Reynolds number (say, \( \text{Re}_m^* \geq 20 \)), a significantly converging inflow toward the slow shock arises. It can grow to be almost parallel to the magnetic field lines as \( \text{Re}_m^* \) increases. In such a case, the inflow toward the diffusion region must be very slow because the reconnection rate (note \( R^* \equiv |v_p \times B_p| \)) is very small, even though the Petschek-like slow shock still strongly intakes the plasma in order to satisfy the Rankine-Hugoniot condition. This means that the inflow forms a channel flow structure: relatively large speed converging inflow toward the slow shock and very slow inflow toward the diffusion region (see Fig. 7).

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In this case, the reconnection rate might be rather small, even if the inflow speed is very large. We must note that, in order to estimate the reconnection rate, we need not only the inflow Alfvén Mach number but also the direction of the magnetic field relative to the inflow velocity.

4.2. The Role of the Diffusion Region

In a very short period (of the order of the Alfvén transit time over the current sheet thickness) just after the onset of the reconnection, the evolution strongly depends on the resistivity model. Such information propagates by MHD waves. In the self-similar stage, since the effect of the resistivity model remains only in a thin layer in the vicinity of the FRWF, it does not influence the entire structure of our self-similar solution. When the induced inflow evolves sufficiently and a Petschek-like slow shock forms, the slow shock begins to play major role in spontaneous structure formation. The region around the diffusion region with a finite spatial scale seems also to be determined by the slow shock and tends to a stationary state as time proceeds. The resistive region plays only passive roles in this stage. These dynamics had been partially clarified by our numerical simulation (see Nitta et al. 2001). The discussion in this paper aims to clarify the spontaneously determined structure led by the shock-tube problem in the region described by ideal MHD. The following is just a conjecture in the present state, but I believe that it can be clarified in our near future works.

In the self-similar stage, the size of the diffusion region (similar to the current sheet thickness or the ion Larmor radius) is infinitesimally small compared with the entire system scale. Hence, most of the inflow does not pass through the diffusion region but passes through the Petschek-like slow shock to form the reconnection outflow. In such situation, it may be trivial that the entire structure should be determined not by the diffusion region, but by the Petschek-like slow shock. The sole yet significant role of the diffusion region is to restrict the diffusion speed of the magnetic field lines. This effect of the diffusion region is clarified by this paper (see §§3 and 4.1).

In my conjecture, the structure of the diffusion region in the self-similar stage is passively determined in a circumstance of the surrounding nonresistive region that is determined by the shock-tube problem involving the Petschek-like slow shock, as is treated in this paper. In this case, the environmental circumstance that is determined by the shock-tube problem will play the role of a kind of boundary condition to determine the diffusion region. If we focus our attention on the diffusion region, this situation is very similar to the case of the “driven reconnection,” which is also determined by the external boundary condition.

This intuition should be verified by solving the diffusion region under some matching condition with the environmental ideal MHD region. We must realize, however, that the structure of the diffusion region cannot be described by MHD but by some particle scheme, because the scale of the intended diffusion region is of the order of the ion Larmor radius. This is obviously outside the limits of MHD approximation. To inquire further into the matter would lead us into that specialized area of the particle reconnection, and such a digression would undoubtedly obscure the outline of our argument in the regime of the MHD reconnection.

4.3. Petschek-Type Slow Shock Formation

In the shock-tube approximation adopted in this paper, we assume the steady formation of the Petschek-type X-shaped slow shock as a presupposition of the problem since we are interested in the fast reconnection regime.

One may think this is curious, because the entire system is strongly time-dependent. Although we cannot find any definite answer at the present time, we offers the following conjecture. First, we must realize that the central region with a finite spatial scale around the diffusion region is in a steady state in the self-similar stage even the entire system is self-similarly expanding. In this case, the central region will be controlled by the surrounding stationary circumstance, which is determined by the shock-tube problem (see §4.2). In naive intuition, we may expect a resultant stationary diffusion region and steady slow shock formation in the central region. The further study of this problem lies outside the scope of this paper, as discussed in §4.2.

We must note that the assumption of steady slow shock formation might not be appropriate for the case with a very small reconnection rate. The converging speed $\nu_{eq}$ drastically increases as the magnetic Reynolds number $Re\_m$ increases (see Fig. 4), and it exceeds unity at $Re\_m \sim 50$. This is very curious. As long as the inflow is spontaneously induced by the reconnection system, its speed may not exceed unity for the case of low $\beta$, because the Alfvén speed is the maximum speed when magnetic energy is completely converted to the bulk kinetic energy. This curious result might result from our presupposition that a Petschek-type slow shock always forms: in order to keep forming the slow shock, the Rankine-Hugoniot condition requires unrealistically high speed inflow.

In actual reconnection, if the resistivity is very small and hence the magnetic Reynolds number becomes large (say, $Re\_m > 50$), the Petschek-type slow shock may not form. Thus, reconnection may change its type to another one with no slow shock (e.g., the Sweet-Parker type; Sweet 1958; Parker 1963) or an X-O-X type one (see §4.6). For Sweet-Parker reconnection, we cannot adopt a shock-tube approximation, and it is beyond the scope of this paper.

4.4. Coupling between Microscopic and Macroscopic Processes

The size of the diffusion region seems to be extremely small if we estimate it to be of the order of ion Larmor radius ($\sim 10^7$ m for typical solar corona). From a naive intuition, it is very curious that such a very small diffusion region governs the huge entire reconnection system (e.g., the radius of magnetic flux tube, say, $\sim 10^7$ m for typical solar flares). Shibata & Tanuma (2001) proposed a new idea of “fractal reconnection,” in which an effective diffusion region has a fractal structure in which total scale of the diffusion region is not so small and is composed of spatially self-similar small structures. Our self-similar evolution model, however, gives an alternative picture for this problem as follows.

The self-similar reconnection model grows in a very wide dynamic range of its spatial scale. It may be triggered when the system scale is of the order of ion Larmor radius, which is the scale of the current sheet thickness to drive an anomalous resistivity. The self-similar solution can approximate the actual phenomenon when the reconnection system scale is much larger than ion Larmor radius (say, $10^5$ m). This self-similar solution can describe the system until the expanding system scale reaches an environmental proper scale that is imposed by external circumstances. Hence, we must note that the self-similar reconnection model describes the coupling process between the microscopic scale and the macroscopic scale.

 Needless to say, the major energy release is determined by the largest structure of the reconnection system. There are two different points of view about how the largest structure is determined. In most of the previous works, it is treated to be governed.
by external circumstances through the boundary condition (the externally “driven” reconnection; see, e.g., Vasyliunas 1975; Priest & Forbes 1986; Sato & Hayashi 1979). In this scheme, the external circumstances directly determine the large-scale structure independent of the microscopic process of the diffusion region. In another scheme, the large-scale structure is determined by intrinsic causes that are properly included in the system itself, so we call it the “spontaneous” reconnection. Ugai & Tsuda (1977) first showed a spontaneous model: the resistivity model denoting microscopic processes of the anomalous resistivity determines the entire system. Our self-similar evolution model is an advanced spontaneous reconnection model extended to the time-dependent reconnection that may be appropriate in astrophysical applications.

The evolution just after the onset of reconnection will depend on detail properties of the resistivity model, i.e., size and shape of the resistive region, value and distribution of the resistivity inside the diffusion region, etc. Anyway, once reconnection starts, its size expands in the fast-mode rarefaction wave (FRW) propagation speed. When the system scale (i.e., the scale of the wave front FRWF of the FRW) becomes much larger than the initial scale (the initial current sheet thickness ~ ion Larmor radius), the system tends to be described by our self-similar solution. After this stage, the reconnection will proceed without limit in a self-similar way. The self-similar solution depends only on two parameters (the plasma-β value and the effective magnetic Reynolds number Re_m) and is independent of other properties. For example, the effects of above-mentioned detailed properties of the resistivity model are contracted within a very narrow region in the vicinity of the FRWF. As the self-similar expansion proceeds, the influence of detailed properties is quickly diminished. Because the self-similar solution well approximates the system as long as its scale is much larger than the initial current sheet thickness (~10^6 m), our self-similar solution is valid after the system scale becomes, say, 10^7 m. This is very small compared with the final system scale (~10^7 m). After that stage, the structure does not change anymore but simply expands self-similarly. Thus, we should realize that the self-similar solution that is determined when the system scale is very small (<10^2 m) governs much larger structure in a later stage by the manner of self-similar expansion. Even when the system scale reaches some external scale, the properties of the self-similar solution hold for very long period (roughly several hundred times the fast-mode transit time; see § 4.5 of Paper I).

4.5. Steady vs. Time-dependent and Driven vs. Spontaneous

In “time-dependent” reconnection models starting from an equilibrium state, a plasmoid forms around the contact discontinuity between the reconnection jet and the initial current sheet plasma. This plasmoid is a result of the interaction between these two different plasmas. In our “spontaneous” model in a free space, the entire system is self-consistently determined from this interaction (see Nitta et al. 2002 and Nitta 2004). Thus, we do not need any external boundary condition in order to determine the resultant reconnection system.

On the other hand, in “steady” models (see Vasyliunas 1975 or Priest & Forbes 1986), the information of this interaction is completely lost. Thus, we need to impose the external boundary condition in order to determine the reconnection system. This is called a “driven” reconnection. A similar discussion about the difference between steady and time-dependent reconnection is in Ugai & Zheng (2005).

Needless to say, the answer to the question of which type is relevant depends on the circumstance of the problem. The author has discussed the relation between the spontaneous model and the driven model in Paper III (see § 5.6 there) in detail. I here emphasize again that these two different models are not opposite subjects, but models for different phases of the entire evolutionary process of the astrophysical reconnection. In my opinion, the driven phase follows the spontaneous phase in usual astrophysical problems as follows.

A large-scale plasma convection (e.g., convection of flux tubes in the solar plasma) will form a current sheet system and store an enormous magnetic energy into the current sheet system. In usual cases of convection with the speed at very small Alfvén Mach number, it will need a very long time compared with the Alfvén transit timescale for the energy storage (e.g., in typical solar flares, it needs several days ~10^5 s). As a result of current sheet thinning, once a well-localized anomalous resistivity is switched on in the current sheet, it triggers the onset of a fast reconnection. The evolution quickly moves to the self-similar expansion. The reconnection system expands at the speed of the FRW propagation, which is estimated as the ambient Alfvén speed (much faster than the convection speed). Hence, the evolution just after the onset is spontaneous, because the timescale of the expansion is much smaller (~10^2 s) than the timescale imposed by the external circumstance (~10^5 s). When the FRW reaches the ambient circumstance, the evolution will start to be influenced by the external boundary condition, but the central energy conversion region holds as the spontaneous state during almost 100 times the Alfvén transit timescale (see § 4.5 of Nitta et al. 2001). After the central region is altered by the external boundary condition, the system moves to the driven phase at last.

4.6. Possibility of Another New Type of Solution

Many people believe that Petschek-type fast reconnection can turn on under the existence of the locally enhanced anomalous resistivity (Biskamp 1986; Scholer 1989; Yokoyama & Shibata 1994). We have not been able to identify the mechanism of the anomalous resistivity yet, but some kind of current-driven microscopic instability should be the origin of the anomalous resistivity. In a naive picture, such a current-driven anomalous resistivity may arise at the locus where the current sheet is strongly compressed.

In this paper, we considered only the case that the diffusion region is fixed around the origin of the coordinate. If we part from this assumption, we may realize another possibility. From the discussion of the channel flow structure (see §§ 4.1), we can speculate the following new type of solutions. In the channel flow, the inflow speed toward the slow shock is much larger than that toward the diffusion region. This nonuniform inflow leads a nonuniform compression of the field reversal region. Since the compression outside the diffusion region is much stronger owing to a strong inhalation by the slow shock, the reconnection point may move outside the present diffusion region. A bipolar reconnection outflow will be ejected from the new X-point. One outflow toward the direction apart from the previous diffusion region, another toward the previous diffusion region. Thus, an O-point will form around the original diffusion region. Consequently, a typical X-O-X structure will form. We must note that this X-O-X structure is not a result of the tearing-mode instability but of the nonuniform inflow speed led by the Petschek-like slow shock.

The existence of this new solution shows the possibility of the bifurcation of the solution. If the magnetic Reynolds number Re_m is small enough (say, <20), the Petschek-like X-point structure will be stable because no channel flow structure forms. When
Re, increases and exceeds a critical value (roughly several tens), a remarkable channel flow structure will take place, and the solution will bifurcate into two different types of solutions: the single X-point solution and the X-O-X solution.

A similar discussion about nonuniqueness of the reconnection solutions was shown in Hameiri (1979). He applied his model to flare phenomena as abrupt jumps to different branches of the solution. The answer to the question, in our case, of which solution will bifurcate into two different types of solutions: the single X-point solution and the X-O-X solution. The solution to the X-O-X structure appears in our preliminary result in a hysteresis behavior. An indication of the transition of the time variation of the magnetic Reynolds number. It may result in a hysteresis behavior. An indication of the transition of the solution to the X-O-X structure appears in our preliminary numerical simulation. The details will be discussed in our following papers.

We must realize, however, that we cannot completely avoid these kinds of ambiguity arising from the nonuniqueness of the resistivity model as long as our study is restricted within the regime of MHDs, because the resistivity cannot be determined from the MHD equations. Unfortunately, we have not had any convincing microscopic model of anomalous resistivity. I believe that these facts do not reduce the importance of studying the MHD reconnection; on the contrary, we must study MHD reconnection further as an elementary process in astrophysical plasma before applying to complicated global problems because we have not established complete astrophysical MHD reconnection model yet.

The author thanks Atsuhiro Nishida (SOKENDAI), Kazunari Shibata (Kyoto Univ.), Takaaki Yokoyama (Tokyo Univ.), Takahiro Kudoh (National Astronomical Observatory of Japan), and Syuniti Tanuma (Kyoto Univ.) for fruitful scientific discussion and comments. Thanks also to Tetsuyuki Yukawa (SOKENDAI) for the comments on numerical procedure. Advice from Naoko Kato (SOKENDAI) improved the English usage.

APPENDIX

EQUATIONS FOR STRUCTURE OF THE RECONNECTION OUTFLOW

The structure of the reconnection outflow is determined by the following equations. We assume that the region between the asymptotic region and the preshock region is filled with nonresistive plasma. Hence, the magnetic flux is frozen into the induced inflow. We also assume a polytropic variation in the induced inflow because there is no violent process in the fast-mode rarefaction. Thus, we impose the frozen-in condition

$$\frac{\sqrt{B_0^2 - B_{x_0}^2 v_{x_0}^2 / (v_{x_0}^2 + v_{y_0}^2)}}{\rho_0} = \frac{\sqrt{B_0^2 + B_{x_0}^2 - (B_{x_0} v_{x_0} + B_{y_0} v_{y_0})^2 / (v_{x_0}^2 + v_{y_0}^2)}}{\rho_p}$$

(A1)

and the polytropic relation

$$P_0 \rho_0^{-\gamma} = P_p \rho_p^{\gamma},$$

(A2)

where $\gamma$ is the specific heat ratio. We assume $\gamma = 5/3$ (monatomic ideal gas).

There are several discontinuities in the reconnection outflow, i.e., X-shaped slow shock, reverse fast shock, contact discontinuity, and forward V-shaped slow shock (see Fig. 3). We set jump conditions for both sides of each discontinuity: (1) X-shaped slow shock Rankine-Hugoniot (R-H) jump conditions and (2) the pressure jump,

$$\frac{P_1}{P_p} = 1 + \frac{\gamma}{c_{sp}^2} (-\sin \theta v_{x_0} + \cos \theta v_{y_0})^2 (X - 1) \left( \frac{1}{X} - \frac{(\cos \theta B_{x_0} + \sin \theta B_{y_0})^2}{2} \left\{ \frac{-2V_{Ap}^2 X + (-\sin \theta v_{x_0} + \cos \theta v_{y_0})^2 (X + 1)}{(-\sin \theta v_{x_0} + \cos \theta v_{y_0})^2 - X V_{Ap}^2} \mu \rho_p \right\} \right),$$

(A3)

where

$$c_{sp} = \sqrt{\gamma P_p / \rho_p},$$

$$V_{Ap} = \sqrt{\left(-\sin \theta B_{x_0} + \cos \theta B_{y_0}\right)^2 / (\mu \rho_p)},$$

$\mu$ is the magnetic permeability of vacuum and $X$ is the compression ratio.

Then the density jump is

$$\frac{\rho_1}{\rho_p} = X,$$

(A4)

The velocity (parallel) jump is

$$\frac{\cos \theta v_1 - v_0}{\sin \theta v_1 - v_0} = \frac{(-\sin \theta v_{x_0} + \cos \theta v_{y_0})^2 - V_{Ap}^2}{(-\sin \theta v_{x_0} + \cos \theta v_{y_0} - X V_{Ap}^2)}.$$
where \( v_0 = (v_p B_{xp} - B_{xp} v_{xp})/(B_{xp} \sin \theta - B_{xp} \cos \theta) \) is the shift speed of the de Hoffmann-Teller coordinate, the velocity (perpendicular) jump is

\[
\frac{-\sin \theta v_1}{-\sin \theta v_{xp} + \cos \theta v_{yp}} = \frac{1}{X},
\]

(A6)

the magnetic field (parallel) jump is

\[
\frac{\sin \theta B_1}{\cos \theta B_{xp} + \sin \theta B_{yp}} = \frac{\left[(-\sin \theta v_{xp} + \cos \theta v_{yp})^2 - V_{Axp}^2\right] X}{\left[(-\sin \theta v_{xp} + \cos \theta v_{yp})^2 - XV_{Axp}^2\right]},
\]

(A7)

and the magnetic field (perpendicular) jump is

\[
\frac{\cos \theta B_1}{-\sin \theta B_{xp} + \cos \theta B_{yp}} = 1.
\]

(A8)

The compression ratio \( X \) is defined by the following equation (third-order algebraic equation for \( X \)),

\[
\left\{ (-\sin \theta B_{xp} + \cos \theta B_{yp}) \left\{ (\cos \theta B_{xp} + \sin \theta B_{yp})^2 (\gamma - 1) \rho_p (-\sin \theta v_{xp} + \cos \theta v_{yp})^2 \right. \right. \\
+ \left. \left. (-\sin \theta B_{xp} + \cos \theta B_{yp})^2 [2 \gamma P_p - \rho_p (-\sin \theta v_{xp} + \cos \theta v_{yp})^2 + \gamma \rho_p (-\sin \theta v_{xp} + \cos \theta v_{yp})^2] \right\} \right\} X^3 \\
+ \left\{ -\rho_p (-\sin \theta v_{xp} + \cos \theta v_{yp})^2 \left\{ (-\sin \theta B_{xp} + \cos \theta B_{yp})^4 (\gamma - 1) + (\cos \theta B_{xp} + \sin \theta B_{yp})^2 (\gamma - 2) \mu \rho_p (-\sin \theta v_{xp} + \cos \theta v_{yp})^2 \right. \right. \\
+ \left. \left. (-\sin \theta B_{xp} + \cos \theta B_{yp})^2 \left\{ (\cos \theta B_{xp} + \sin \theta B_{yp})^2 (\gamma + 1) + 2 \mu [2 \gamma P_p - \rho_p (-\sin \theta v_{xp} + \cos \theta v_{yp})^2 + \gamma \rho_p (-\sin \theta v_{xp} + \cos \theta v_{yp})^2] \right\} \right\} \right\} X^2 \\
+ \left\{ \mu \rho_p (-\sin \theta v_{xp} + \cos \theta v_{yp})^4 \left\{ (\cos \theta B_{xp} + \sin \theta B_{yp})^2 \gamma + 2 (-\sin \theta B_{xp} + \cos \theta B_{yp})^2 (\gamma + 1) \right. \right. \\
+ \left. \left. \mu [2 \gamma P_p - \rho_p (-\sin \theta v_{xp} + \cos \theta v_{yp})^2 + \gamma \rho_p (-\sin \theta v_{xp} + \cos \theta v_{yp})^2] \right\} \right\} \right\} X \\
+ \left\{ -\gamma + 1 \mu \rho_p (-\sin \theta v_{xp} + \cos \theta v_{yp})^6 \right\} = 0.
\]

This equation has three roots. We must choose a real positive root larger than unity.

The reverse-fast shock R-H conditions are as follows. The pressure jump is

\[
\frac{P_2}{P_1} = \zeta_f,
\]

(A9)

where

\[
\zeta_f = \gamma M_{ff}^2 (1 - 1/\xi_f) + \left(1 - \xi_f^2\right)/\beta_1 + 1,
\]

with

\[
\xi_f = \left[-l + \sqrt{l^2 + 2/\beta_1 (2 - \gamma)(\gamma + 1) M_{ff}^2} \right]/\left[2/\beta_1 (2 - \gamma)\right],
\]

\[
l = \gamma (1/\beta_1 + 1) + (\gamma - 1) \gamma M_{ff}^2/2,
\]

\[
\beta_1 = P_1/(B_{ff}^2/(2 \mu)),
\]

\[
M_{ff} = (v_1 - x_f V_\Lambda)/c_f,
\]

\[
c_f = \sqrt{\gamma P_1/\rho_1},
\]

and

\[
V_\Lambda = B_0/\sqrt{\mu \rho_0}.
\]
The velocity jump is
\[ \frac{\rho_2}{\rho_1} = \xi_f. \]  
\[ \text{(A10)} \]

The velocity jump gives
\[ \frac{v_1 - x_f}{v_2 - x_f} = \xi_f. \]  
\[ \text{(A11)} \]

The magnetic field jump (eq. [A12])
\[ \frac{B_2}{B_1} = \xi_f. \]  
\[ \text{(A12)} \]

We must impose local magnetic flux conservation on both sides of region \( p \) and region \( 1 \). Magnetic flux conservation at the X-point gives
\[ -v_1 p B_{yp} + v_1 p B_{xp} + v_1 B_1 = 0. \]  
\[ \text{(A13)} \]

Force balance at the contact discontinuity is
\[ P_2 + \frac{B_2^2}{2\mu} = P_3 + \frac{B_2^2}{2\mu}. \]  
\[ \text{(A14)} \]

The V-shaped forward slow shock R-H conditions are as follows. The pressure jump is
\[ P_3 = P_0 \left[ 1 + \frac{\gamma c_{s0}}{c_{s0}} (x_s \sin \phi)^2 (X_h - 1) \left( \frac{1}{X_h} - \frac{(B_0 \cos \phi)^2}{2} \left\{ \frac{-2V_{A0}^2 x_s + (x_s \sin \phi)^2 (X_h + 1)}{(x_s \sin \phi)^2 - X_h V_{A0}^2 \mu_0} \right\} \right) \right], \]  
\[ \text{(A15)} \]

where \( c_{s0} = (\gamma P_0/\rho_0)^{1/2} \), \( V_{A0} = [(-\sin \theta B_0^2)/(\mu_0)]^{1/2} \), and \( X_h \) is the compression ratio. The density jump is
\[ \frac{\rho_2}{\rho_1} = X_h. \]  
\[ \text{(A16)} \]

The velocity (parallel) jump gives
\[ \frac{\cos \phi (v_1 x_s - x_s V_{A0}) + \sin \phi v_{13}}{V_{A0} x_s \cos \phi} = \frac{\frac{v_{m0x}^2}{V_{A0}^2 x_s} - X_h \frac{V_{A0}^2}{V_{A0x}^2}}{\frac{v_{m0x}^2}{V_{A0}^2 x_s} - X_h \frac{V_{A0}^2}{V_{A0x}^2}}, \]  
\[ \text{(A17)} \]

where \( v_{m0x} = x_s \sin \phi \) and \( V_{A0} = B_0/(\mu_0)^{1/2} \). The velocity (perpendicular) jump condition gives
\[ \frac{-\sin \phi (v_1 x_3 - x_3 V_{A0}) + \cos \phi v_{13}}{V_{A0} x_s \sin \phi} = \frac{1}{X_h}. \]  
\[ \text{(A18)} \]

The magnetic field (parallel) jump is
\[ \frac{\cos \phi B_{13} + \sin \phi B_{y3}}{B_0 \cos \phi} = \frac{(\frac{v_{m0x}^2}{V_{A0}^2 x_s} - X_h \frac{V_{A0}^2}{V_{A0x}^2}) X_h}{\frac{v_{m0x}^2}{V_{A0}^2 x_s} - X_h \frac{V_{A0}^2}{V_{A0x}^2}}. \]  
\[ \text{(A19)} \]

The magnetic field (perpendicular) jump is
\[ \frac{-\sin \phi B_{13} + \cos \phi B_{y3}}{-B_0 \sin \phi} = 1. \]  
\[ \text{(A20)} \]

The compression ratio \( X_h \) is a solution of the following third-order algebraic equation:
\[ ((B_0 \sin \phi)^2 \left\{ ((B_0 \sin \phi)^2 (\gamma - 1)\rho_0 (x_s \sin \phi)^2 + (B_0 \sin \phi)^2 \left[ 2\gamma P_0 - \rho_0 (x_s \sin \phi)^2 + \gamma \rho_0 (x_s \sin \phi)^2 \right] \right\}) X_h^3 \]
\[ + \left\{ -\rho_0 (x_s \sin \phi)^2 \left[ (B_0 \sin \phi)^2 (\gamma + 1) + (B_0 \cos \phi)^2 (\gamma - 2) \mu_0 (x_s \sin \phi)^2 \right] \right\} X_h^2 \]
\[ + (B_0 \sin \phi)^2 \left\{ (B_0 \cos \phi)^2 (\gamma + 1) + \left( 2\mu_0 [2\gamma P_0 - \rho_0 (x_s \sin \phi)^2 + \gamma \rho_0 (x_s \sin \phi)^2] \right) \right\} X_h \]
\[ + (\mu_0^2 (x_s \sin \phi)^4 \left( (B_0 \cos \phi)^2 \gamma + 2 (B_0 \sin \phi)^2 (\gamma + 1) + \mu [2\gamma P_0 - \rho_0 (x_s \sin \phi)^2 + \gamma \rho_0 (x_s \sin \phi)^2] \right) \} X_h \]
\[ + \left[ (\gamma + 1) \mu \rho_0^2 (x_s \sin \phi)^6 \right] = 0. \]

We must choose a real positive root larger than unity.
The tip of the reconnection outflow touches the FRWF (see Fig. 3); hence, \( A'_f = 0 \) at this point. This condition reduces to the following boundary condition at the tip of the outflow:

\[
-B_{33}(1-x_c) + B_{33}(1-x_c) \tan \phi - x_c \tan \theta - B_0(1-x_c) \tan \phi = 0. \tag{A21}
\]

From magnetic flux conservation, the injected magnetic flux must be redistributed in the reconnection jet. This leads to the following equation for magnetic flux conservation:

\[
-B_{3p}v_{3p} + v_{3p}B_{3p} + \left[ x_f B_1 + (x_c - x_f)B_2 \right] = 0. \tag{A22}
\]

We assume the following trivial relations:

Definition from the contact discontinuity

\[ x_c = v_2 = v_{33}. \]

We can solve equation (A2) for \( P_{3p} \), equation (A8) for \( B_1 \), equation (A9) for \( P_2 \), equation (A10) for \( \rho_2 \), equation (A12) for \( B_2 \), equation (A13) for \( v_{3p} \), equation (A15) for \( P_3 \), equation (A16) for \( \rho_3 \), equation (A17) and equation (A18) for \( v_{33} \) and \( v_{33} \), equation (A19) and equation (A20) for \( B_{33} \) and \( B_{33} \) by hand, then substitute them into other nine equations (eqs. [A1], [A3], [A4], [A5], [A6], [A7], [A11], [A14], [A21], and [A22]) for the following unknowns: \( P_{3p}, v_{3p}, B_{3p}, B_{3p}, \theta, P_1, \rho_1, x_f, \phi, \) and \( x_c \). The only parameter included in this problem is the plasma-\( \beta \) value at the asymptotic region. By using a Newton-Raphson routine, with an initial guess for these unknowns, we obtain converged solutions. The procedure to obtain the series of converged solutions is discussed in \( \S \). 2.3 in detail.

As \( |v_{3p}| \) decreases, the strength of the reverse fast shock reduces, and then the pressure jump \( c_f \) and density jump \( c_f \) simultaneously become unity at the critical value of \( |v_{3p}| \). At that point, the reverse fast shock vanishes. We convert the coupled equations to another set compatible to the situation with no fast shock; i.e., several equations reform to the following equations:

- equation (A9) is replaced by\( p_2 = p_1 \),
- equation (A10) is replaced by \( \rho_2 = \rho_1 \),
- equation (A11) is replaced by \( v_1 = v_2 \),
- equation (A12) is replaced by \( B_1 = B_2 \),

and equation (A22) becomes equivalent to equation (A13) and removed from the set of coupled equations. The number of coupled equations reduces to 21. This is consistent with the locus \( x_f \) of the fast shock no longer being included in the set of unknowns, and the total number of unknowns becomes 21.

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