A Reconstruction of Quintessence Dark Energy

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Abstract

A quintessence scalar potential is obtained by a parametric reconstruction from the observed data. The model in fact is found to generalize a double exponential potential and is found to be consistent with the distance modulus data versus the redshift parameter.

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1 Introduction

Consistent with the indication given by the observations in the late nineties [1], subsequent observations [2] have confirmed that the universe at present is undergoing an accelerated phase of expansion. The observations also indicate that the alleged acceleration is rather a recent phenomenon [3]. The component of the matter sector, responsible for this acceleration, still eludes any observational detection or even a sound theoretical prediction. A cosmological constant certainly does very well in explaining this dynamics of the universe, but it has the huge discrepancy between the observed value and the theoretically predicted one. A scalar field, called the quintessence field, can certainly drive the acceleration but no scalar potential has a firm theoretical motivation. There are excellent reviews that describe the suitability and problems of various dark energy models[4]. A modification of General Relativity (GR) is also looked at, leaving the matter sector intact, but as GR describes the local astronomy so efficiently, the search for the dark energy is still very much on so as to keep the gravitation theory unaltered.

One of the attempts towards finding a quintessence field is a "reconstruction", i.e., building up the model from the observational data. This kind of the "reverse way" of finding a scalar potential has been there for a long time in the literature[5]. The idea is to assume a particular evolution scenario, may be consistent with the observation, and then to fix the matter field giving rise to that. In the context of dark energy, this was utilised by Starobinsky[6] who used density perturbation and by Huterer and Turner where the data of distance measurement [7] were invoked.
In the absence of a clear indication in favour of any particular potential, a large amount of work has already been done in this direction. Reconstruction of a dark energy potential normally involves the search for equation of state parameter for scalar field, \( w_{DE} = \frac{p_\phi}{\rho_\phi} \), where \( p_\phi \) and \( \rho_\phi \) are the contribution of the scalar field to the pressure and density sector respectively. A review on the initial attempts in this direction can be found in [8]. A reconstruction of \( w_{DE} \) may be done in two ways. One is to choose a form of \( w_{DE} = w_{DE}(z) \) where \( z \) is the redshift and evaluate the parameters in \( w_{DE} \) with the help of the observational data [9]. The other is to directly find the functional form of \( w_{DE}(z) \) from the data. Recently Holsclaw et al [10] discussed exhaustively the second kind of reconstruction, a 'non-parametric' one, as an inverse statistical problem. Sahlén et al also discussed a 'direct' or non-parametric reconstruction of quintessence potential [11]. The time evolution of \( w_{DE} \) had been reconstructed very recently with non-parametric Bayesian method by Crittenden et al [12]. Pan and Alam investigated the usefulness of various cosmological parameters in selecting or rejecting different reconstructed dark energy models [13]. In the context of other theories of gravity, e.g. the scalar tensor theories of gravity, reconstruction of dark energy has also been discussed [14].

The present work deals with the former, the parametric approach for the reconstruction of the quintessence potential. A one parameter family of \( w_{DE} = w_{DE}(z) \) is chosen, and the parameter is evaluated based on the data. This approach has its weakness (see [10] for discussion), but it looks elegant and is almost analytical. We find a simple potential and also a complete set of solution to the cosmological model. In section 2 the reconstruction is presented and in section 3 the results are discussed.

## 2 Reconstruction of the scalar field potential from the equation of state of the scalar field

The field equations for a spatially flat FRW universe with cold dark matter, given by a pressureless fluid and a scalar field are

\[
3H^2 = \rho_m + \rho_\phi, \tag{1}
\]

\[
2\dot{H} + 3H^2 = -p_\phi, \tag{2}
\]

where \( H \) is the Hubble parameter given by \( H = \frac{\dot{a}}{a} \) (\( a \) being the scale factor), \( \rho_m \) is the matter energy density and \( \rho_\phi \) and \( p_\phi \) are the contributions of the scalar field to the energy density and pressure sector respectively. The latter two are given by

\[
\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \tag{3}
\]

\[
p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi), \tag{4}
\]

where \( V(\phi) \) is the scalar potential. An overhead dot here indicates a differentiation with respect to the cosmic time \( t \). The units are so chosen that \( 8\pi G = 1 \). The pressureless cold dark matter satisfies its own conservation equation which leads to

\[
\rho_m = \rho_{m0}(1 + z)^3, \tag{5}
\]
Figure 1: Plots of deceleration parameter (left panel) and equation of state parameter (right panel) of dark energy for the present model with $\Omega_{m0} = 0.27$ and the model parameter $\alpha = 0.50^{+0.50}_{-0.40}$. The central dark line corresponds to $\alpha = 0.50$, while $\alpha = 1.0$ and $0.1$ mark the upper and the lower bounds of the spread respectively.

where $z$ is the redshift parameter defined as $1 + z = \frac{a_0}{a}$ where $a$ is the scale factor and $a_0$ is its present value. This $\rho_{m0}$ is the present value of the dark matter density. With equation (1),(2) and (5) the wave equation for the scalar field

$$\square \phi + \frac{dV}{d\phi} = 0,$$

is a consequence of the Bianchi identity and does not lead to an independent equation. From equation (1) and (2), the equation of state parameter $w_{DE}$ can be written as

$$w_{DE} = \frac{-2\dot{H} + 3H^2}{3H^2 - \rho_m}.$$  \hfill (7)

One can replace the argument $'t'$ by the redshift $z$ in this equation. With the aid of equation (5), the equation (7) would look like

$$2(1 + z)H\frac{dH}{dz} = 3(1 + w_{DE})H^2 - \rho_{m0}(1 + z)^3 w_{DE}.$$  \hfill (8)

As we have three unknown quantities $a$, $\phi$ and $V(\phi)$ against only two equations, namely equation (1) and (2) to solve for them, we can choose an ansatz so as to close the system of equations. In what follows, a one parameter equation of state parameter for the purpose, given by

$$w_{DE}(z) = -\frac{3}{(1 + z)^3 + (\frac{2}{\alpha})^3},$$  \hfill (9)

where $\alpha$ is a constant parameter, is chosen. The reason for choosing this kind of $w_{DE}$ is that for high $z$, i.e. at the early stage of evolution, $w_{DE}$ is almost zero so that it is hardly distinguishable
Figure 2: The left panel is the scalar field potential $V(\phi)$ and right one is showing the evolution of the potential with redshift $z$ with $\Omega_{m0} = 0.27$ and the model parameter $\alpha = 0.50^{+0.50}_{-0.40}$. The upper and lower bounds for the left panel are for $\alpha = 0.10$ and 1.00 respectively while the role is reversed for the right panel. The dark line inside the shaded region is for $\alpha = 0.50$.

from a pressureless fluid, but gradually decreases to more and more negative value to yield an increasing negative pressure. With equation (9), one can integrate equation (8) to yield

$$H^2(z) = H_0^2 \left[ \frac{(\alpha + 3\Omega_{m0})}{(\alpha + 3)} \left( (1 + z)^3 + \left( \frac{3}{\alpha} \right) \right) - \frac{3\Omega_{m0}}{\alpha} \right],$$

(10)

where $H_0$ is the present value of Hubble parameter and $\Omega_{m0}$ is the present density parameter given by $\Omega_{m0} = \frac{\rho_{m0}}{3H_0^2}$. The deceleration parameter $q$, defined as $(-\frac{a\ddot{a}}{a^2})$, can be written, in terms of $z$, as

$$q(z) = -1 + \frac{3(\alpha + 3\Omega_{m0})}{2(\alpha + 3)} \frac{H_0^2(1 + z)^3}{H^2(z)}.$$

(11)

This is a two parameter expression for $q(z)$. With a knowledge of $q_0$ ($q$ at $z = 0$) and $\Omega_{m0}$ from observations, the value of the parameter $\alpha$ can be estimated. Now from equation (1) and (2) one can write (using the expression for $\rho_\phi$ and $p_\phi$)

$$\dot{H} = -\frac{\rho_m}{2} - \frac{\dot{\phi}^2}{2},$$

(12)

which would look like

$$(1 + z)^2 H^2 \left( \frac{d\phi}{dz} \right)^2 = 2(1 + z)H \frac{dH}{dz} - 3H_0^2\Omega_{m0}(1 + z)^3,$$

(13)

if $z$ is used as the argument instead of $t$. This can be integrated to yield (using equation (10))

$$\phi(z) = \frac{2}{3} \sqrt{\frac{3(1 - \Omega_{m0})}{\alpha + 3\Omega_{m0}}} \ln \left[ 2(\alpha + 3\Omega_{m0})(1+z)^{\frac{\alpha}{2}} + 2\sqrt{(\alpha + 3\Omega_{m0})^2(1+z)^3 + 3(1 - \Omega_{m0})(\alpha + 3\Omega_{m0})} \right].$$

(14)
Figure 3: This is showing the evolution of matter density $\rho_m(z)$ (red curve) and scalar field energy density $\rho_\phi(z)$ (gray colored distribution) for the present model with $\Omega_{m0} = 0.27$ and the model parameter $\alpha = 0.50^{+0.50}_{-0.40}$. For $\rho_\phi(z)$, the upper and the lower bounds for the shaded region are for $\alpha = 1.0$ and $0.10$ respectively while the central line is for $\alpha = 0.50$.

An addition of the field equations (1) and (2) will now yield

$$V(z) = \frac{3\alpha H_0^2(1 - \Omega_{m0})(1 + z)^3}{2(\alpha + 3)} + \frac{9H_0^2(1 - \Omega_{m0})}{(\alpha + 3)}.$$  \hspace{1cm} (15)

In this expression $z$ can be replaced by $\phi$ using equation (14) to obtain the potential as a function of $\phi$,

$$V(\phi) = \frac{3H_0^2(1 - \Omega_{m0})exp(\Phi)}{128(\alpha + 3)(\alpha + 3\Omega_{m0})^2} + \frac{27H_0^2(1 - \Omega_{m0})^2exp(-\Phi)}{2(\alpha + 3)} + \frac{9H_0^2(1 - \Omega_{m0})(3\alpha + \alpha\Omega_{m0} + 12\Omega_{m0})}{4\alpha(\alpha + 3)(\alpha + 3\Omega_{m0})},$$ \hspace{1cm} (16)

where $\Phi = 3\sqrt{\frac{\alpha + 3\Omega_{m0}}{3\alpha(1 - \Omega_{m0})}}\phi$.

The value of the parameter $\alpha$ is estimated to be $0.5$ for $\Omega_{m0} = 0.27$ if the transition of $q(z)$ from a positive to a negative value takes place close to $z = 0.5$, which is within the observational limits. The plots of $q(z)$ and $w_{DE}(z)$ against $z$ are shown in the figure 1, with $\Omega_{m0} = 0.27$ and $\alpha = 0.5$. The plots also show the evolutions of these variables for a range of values of $\alpha$. The scalar field potential is plotted against the scalar field $\phi$ as well as against $z$ in figure 2. With $\Omega_{m0} = 0.27$ and the aforesaid range of $\alpha$, the potential is found to be changing with $z$ at a slower rate at the present epoch compared to the past. So this is more like a freezing potential rather than a thawing one [16]. And the dark matter density $\rho_m(z)$ as well as dark energy density $\rho_\phi(z)$ against $z$ are given in figure 3. The dark energy overcomes the dark matter density in the range between $z = 0.4$ to $0.6$. For $\alpha = 0.5$, the crossing is at around $z = 0.55$.

The distance modulus $\mu_B(z)$, which is the difference between the apparent magnitude $m_B$ and the absolute magnitude $M_B$ of (B-band) the observed supernova, is given by

$$\mu_B(z) = m_B - M_B = 5log_{10}\left(\frac{d_L}{1Mpc}\right) + 25,$$  \hspace{1cm} (17)
where $d_L$ is the luminosity distance. The luminosity distance $d_L$ is related to the Hubble parameter $H(z)$ for a spatially flat universe as

$$d_L(z) = c(1 + z) \int_0^z \frac{dz'}{H(z')}.$$  \hspace{1cm} (18)

where $c$ is the speed of light. Using (18) in (17) one can write

$$\mu_B(z) = 5log_{10} \left[ c(1 + z) \int_0^z \frac{dz'}{H(z')} \right] + 25.$$  \hspace{1cm} (19)

Using equation (10) with relevant value of $\Omega_{m0} = 0.27$ and $\alpha = 0.50^{+0.50}_{-0.40}$, $\mu_B(z)$ can be computed from equation (19). From figure 4 it will be clear that the reconstructed model reproduces the observed values quite effectively(observed values taken from N. Suzuki et. al. [17]). The value of $\mu_B(z)$ appears to be not too sensitive to the value of $\alpha$ if it is between 0.1 and 1.0.
3 Discussion

The present work gives a quintessence model reconstructed from an equation of state parameter $w_{DE}$, which is chosen as a one parameter function of $z$. The value of the parameter ‘$\alpha$’ can be fixed from the observed data. A value consistent with the present value of the deceleration parameter is chosen. The model is completely consistent with the data set for the distance modulus $\mu_z(B)$.

The corresponding potential is found, which comes out to be a double exponential potential. A similar potential had already been discussed by Sen and Sethi [18]. So in a way, the potential obtained in the present work in the form $V(\phi) = V_1 e^{\lambda \phi} + V_2 e^{-\lambda \phi} + V_0$ where $v_1, V_2, V_0$ and $\lambda$ are constant, is a generalization of the potential given by Sen and Sethi, where $V_1 = V_2$.

Certainly the model is plagued with the non-existence of a firm theoretical reason which essentially warrants the potential, but this is the generic drawback of any quintessence field. The advantage of the model is that it is analytical, only the parameter ‘$\alpha$’ is chosen from a relevant value of $q$ the deceleration parameter.

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