Quantum “states” are neither states of Nature nor states of knowledge; they are objective probability measures. An objective probability measure is the formal expression of an objective indefiniteness, such as the positional indefiniteness that contributes to “fluff out” matter. An objective indefiniteness entails that the values of certain observables are extrinsic (possessed only because they are indicated) rather than intrinsic (indicated because they are possessed). This dependence on value-indicating facts is not a dependence on anything external to the quantum world. The latter constitutes a free-standing reality that owes nothing to observers, information, or our interventions into the course of Nature. The core interpretational issue does not concern the kind of reality that quantum states possess—ontological or epistemic—but the kind of reality that the spatial and temporal referents of quantum states possess. What stands in the way of even the correct identification of the central issue is our habit of projecting into the quantum world the detached, intrinsically differentiated spatiotemporal background of classical physics.

1 OVERVIEW

The title of this article echoes that of two well-known articles by John Bell [1]. Bell objected to the special role played by measurements in standard formulations of quantum mechanics (QM):

Why this aversion to “being” and insistence on “finding”? To restrict quantum mechanics to be exclusively about piddling laboratory operations is to betray the great enterprise. A serious formulation will not exclude the big world outside the laboratory. [1]

David Mermin gave vent to the same feeling:

Why should the scope of physics be restricted to the artificial contrivances we are forced to resort to in our efforts to probe the world? Why should a fundamental theory have to take its meaning from a notion of “measurement”
external to the theory itself? Should not the meaning of “measurement” emerge from the theory, rather than the other way around? Should not physics be able to make statements about the unmeasured, unprepared world? 

In a response to Mermin I explained QM’s inevitable reference to measurements in a way that does not imply a restriction of the scope of physics to “piddling laboratory operations.” So understood, measurements are not external to the theory, the special status of measurements does emerge from the theory itself, physics is able to make statements about an unmeasured world, even though there is no unmeasured world. Obviously two distinct notions of “measurement” are involved, and our failure to pry them apart is what is largely responsible for our failure to make sense of QM.

Bell was right; “measurement” is a bad word. It is a bad way of expressing the extrinsic nature of such properties as positions or orientations. One of the reasons why Bell thought that “measurement” was a bad word is that it makes us “think of the result as referring to some preexisting property of the object in question” [1, original emphasis]. In other words, it makes us think of properties as intrinsic (indicated because they are possessed) rather than as extrinsic (possessed because they are indicated). The value of a quantum-mechanical observable such as position or a spin component is possessed only if, and only to the extent that, it is indicated by, or inferable from, a fact—an actual event or state of affairs. No property is a possessed property unless it is an indicated property. The relation between possessed positions and position-indicating events is obviously best studied in a suitably equipped laboratory. But it is just as obvious that this does not imply that position-indicating events occur only inside laboratories.

In Sec. 2 I will argue that QM, as a theory of the big world outside the laboratory, entails a radically new way of thinking about the world’s spatial aspect, in which the extrinsic nature of positions plays a crucial role. The special status of position-indicating facts emerges from the theory itself, provided that this is taken seriously as an ontological theory.

It is much easier, however, to construe QM as an epistemic theory, inasmuch as this saves us from the necessity of learning a new way of thinking about space. Conversely, if we persist in viewing the quantum world against the self-existent and intrinsically differentiated spatial background of classical physics, it seems impossible to avoid the conclusion that

[t]here is no quantum world. There is only an abstract quantum physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature. [4]

In his response to Bell, Rudolf Peierls likewise defended the view that

the most fundamental statement of quantum mechanics is that the wave function or, more generally the density matrix, represents our knowledge of the system we are trying to describe. [5, original emphasis]
Since the beginning of time (in about 1926) it has been argued that quantum theory is about knowledge or, as they now say, about information. There are various reasons for this attitude. For one thing, it is safe. I can’t prove it wrong, as little as I can disprove the claim that the world was created yesterday with all our memories in place. (In this case we would have knowledge of a nonexistent past, rather than knowledge of a nonexistent world.) Inasmuch as it is irrefutable, this attitude is also unscientific, at least according to Karl Popper’s definition of “science.” For another thing, it seems obvious. If the properties of the quantum world are extrinsic (that is, if they “dangle” from, or supervene on, something), and if the quantum world is coextensive with the physical world, then from what “hook” can they “dangle”? The obvious answer is, from us, from what we perceive, or from what we know.

But is it true that the quantum world is coextensive with the physical world? A. Peres and W.H. Zurek have forcefully argued against it: “[A]lthough it can describe anything, a quantum description cannot include everything” original emphasis. Yet this by itself does not save us from the conclusion that the wave function represents knowledge; it rather appears to confirm it. Bell demanded that “the theory should be fully formulated in mathematical terms, with nothing left to the discretion of the theoretical physicist.” What appears to be the case, instead, is that the division of the world into “system” and “apparatus” is both arbitrary and inevitable. This suggests that the “full elision of the subject” cannot be achieved: We, the creators of the theory, are unable to entirely withdraw from what the theory describes. Because the description carries our signature—the “shifty split” between system and apparatus—it must not be mistaken for the thing described.

There can be no question...without changing the axioms...of getting rid of the shifty split. Sometimes some authors of “quantum measurement” theories seem to be trying to do just that. It is like a snake trying to swallow itself by the tail. It can be done...up to a point. But it becomes uncomfortable for the spectators even before it becomes painful for the snake.

Bell’s simile of a snake swallowing its own tail is appropriate. Position-indicating positions, such as the position of a detector or a pointer needle, are extrinsic, too; they too need to be indicated in order to be possessed. This seems to send us chasing the ultimate property-defining facts in never-ending circles. But only seems. In truth there is a perfectly natural and perfectly objective way of dividing the physical world into a macroscopic and a quantum domain. The divide is not left to the discretion of the physicist. It emerges from the theory. The “full elision of the subject” can be achieved. But not without a radical change in our conception of physical space. Not if we insist on projecting the self-existent and intrinsically differentiated spatial background of classical physics into the physical world. What makes that exercise painful to watch is the use of an inappropriate concept of space. This is the subject of Sec. 3.
In Bell’s view \( [1] \), QM, as taught in most textbooks, is only for all practical purposes (FAPP) consistent with the ideal of an objective description in which nothing is left to the discretion of theoretical physicists. A classic case in point, quoted by Bell, is Kurt Gottfried’s justification of the substitution of the mixture

\[
\hat{\rho} = \sum_{n} |c_n|^2 |\psi_n\rangle\langle\psi_n|
\]

for the pure state

\[
\rho = \sum_{m} \sum_{n} c_m^* c_n |\psi_m\rangle\langle\psi_n|
\]

“[W]e are free to replace \( \rho \) by \( \hat{\rho} \) after the measurement, safe in the knowledge that the error will never be found” \([14]\). In the same way Asher Peres dispose of the difference between \( \rho \) and \( \hat{\rho} \) by arguing that if it were ever observed, it would be considered as a statistical quirk or an experimental error \([15]\). If safety from the consequences of errors in a description of the world is what justifies the description then obviously the description is valid only FAPP. Bernard d’Espagnat \([13, 16, 17]\) concludes from this that standard QM, although consistent with a weak notion of objectivity based on intersubjective agreement, is inconsistent with a strongly objective description free from traces left by its creators. Recently Christopher Fuchs and Asher Peres \([18]\) have argued along similar lines that there exists no free-standing reality independent of our interventions into the course of Nature. There only exists an “effective reality” (their shudder quotes) that QM “produces in some regimes of our experience.” This “forms the ground for all our other quantum predictions simply because it is the part of nature that is effectively detached from the effect of our experimental interventions” \([19]\).

In point of fact, the “‘effective reality’” of Fuchs and Peres (my shudder quotes around theirs) is a perfectly free-standing reality, in the sense that the totality of macroscopic positions and the properties these indicate in no wise depend on observers, their information, their arbitrary decisions, or their interventions into the course of Nature. What this reality is effectively (rather than absolutely) detached from is property-indicating facts. Positions are extrinsic. While macroscopic objects are no exception, their indicated positions are so correlated that they are consistent with definite trajectories. For this reason the positions of macroscopic objects are effectively detached from the facts by which they are indicated. They can therefore consistently be regarded as intrinsic, or self-indicating, or factual per se, and hence as capable of indicating properties, including each other.

This conclusion is valid not just FAPP but strictly. It is true that arguments based on decoherence \([20]\) and environment-induced superselection \([21]\) do not lead to strictly vanishing off-diagonal terms, and therefore cannot justify the substitution of \( \hat{\rho} \) for \( \rho \) except FAPP. But it is irrelevant. What matters is not that the probability of a departure from a classical trajectory is so low that it is unlikely that one will ever be observed. What matters is that none is ever indicated. This forms part of the definition of “macroscopic object” given in Sec. 3, where it is also shown that macroscopic objects exist.
To settle the question of what the laws of QM are about, we must be clear what the laws of QM are. As I explained in a previous article [3], they are concise encapsulations of statistical correlations—diachronic correlations between facts concerning the properties of, or the values of observables on, the same system at different times, and synchronic correlations between facts concerning the properties of different systems, or the values of (locally measurable) observables, in spacelike separation. Quantum “states,” accordingly, are probability measures. They are uniquely determined ways of assigning probabilities in the presence of an objective indefiniteness, such as the objective positional indefiniteness that is essential for the stability of extended material objects. The proper formal expression of an objective indefiniteness is to make counterfactual probability assignments, and the search for a suitable probability algorithm leads straight to a unique density operator and the well-known trace rule. If you were looking for a duck and you found something that walks like a duck and that quacks like a duck, you would think that you have found a duck. Yet when it comes to quantum “states,” many physicists are willing to suspend this sensible maxim and attempt to construe an obvious probability algorithm as if it represented an actual state of affairs. I am in complete agreement with Fuchs and Peres when they criticize such attempts [18]:

Attributing reality to quantum states leads to a host of ‘quantum paradoxes.’
These are due solely to an incorrect interpretation of quantum theory... The time dependence of the wave function does not represent the evolution of a physical system.

Fuchs and Peres overshoot the mark when they claim that quantum theory (as against the quantum state of a system) does not describe physical reality, that the time dependence of the wave function gives the evolution of probabilities, and that quantum states are states of knowledge. These claims are rooted in the same misconception that makes other physicists look for ways of construing the density operator as describing an actual state of affairs. They are different symptoms of the same illness, diagnosed in Sec. 5 as the endemic misconception that quantum states evolve in a detached, intrinsically differentiated temporal background. If I were to vote on the worst words in the vocabulary of theoretical physics, I would vote for the mutual implicates “state” and “evolution.” Eliminating these words (along with the misconceptions they connote) rids us not only of the spurious “measurement problem”—Why are there two modes of evolution?—but also of the false disjunction of possible interpretations of quantum states into ontological and epistemic ones. Quantum states are neither states of Nature nor states of knowledge. They are generic probability measures.

But does not the presence of nontrivial probabilities in a fundamental theory by itself imply that QM is about knowledge? Not any more than the presence of mathematical laws in any physical theory does. If QM is about knowledge then so is classical physics. Contrariwise, if classical physics is a theory from which a free-standing reality can be distilled, as Fuchs and Peres affirm, then QM too is such a theory. The statistical laws of
QM are as objective as the deterministic laws of classical physics. This, along with the question of the completeness of QM, is discussed in Sec. 6.

The concluding section is cast in the form of a comment on a recent review article by F. Laloë [22].

2 SPACE AND THE QUANTUM WORLD

If it is to make sense as a theory of the big world outside the laboratory, QM needs a radically new way of thinking about the world’s spatial aspect [3, 23, 24]. Space cannot be something that exists by itself and that is intrinsically differentiated or divided. Here is why: On the one hand, QM tells us that an electron can go through two slits without going through either slit in particular and without being divided by the slits [25]. (The present article is concerned with the interpretation of standard QM unadulterated with, e.g., Bohmian trajectories [26] or nonlinear modifications of the dynamics [27].) On the other hand, for reasons that are neurophysiological rather than physical [24, 28, 29], we tend to think of space as something that exists by itself, independently of its material “content,” and that is intrinsically divided into distinct, separate regions. Yet if this were the case, it would affect every material object. No particle could go through $L\&R$—the regions $L$ and $R$ defined by the slits considered as one region—without going through a particular slit and without being divided into two distinct parts. Therefore $L$ and $R$ cannot be self-existent and intrinsically distinct “parts of space.” A fortiori, what we conceive of as the “parts of space” cannot be real and distinct per se. A “part of space” has a contingent reality, in the sense that it may exist for one material object at one time and not exist for another object at the same time or for the same object at another time.

This calls for a criterion, and the criterion that suggests itself to me is this: A region of space $V$ is real for an object $O$ at a time $T$ if—and only if—two conditions are satisfied: (i) $V$ must exist as an intrinsic property of a macroscopic object $M$ (to be rigorously defined in the next section), and (ii) the proposition “$O$ is in $V$ at the time $T$”—symbolically, $O\rightarrow V(T)$—must have a truth value. [In this symbolic expression “$O$” stands for the position of $O$, and “$V$” stands for a region that is realized (made real) by being an intrinsic property of $M$. The lack of a truth value means that the proposition is neither true nor false but meaningless.] And the sufficient and necessary condition for the existence of a truth value is that one is indicated.

Some comments are in order. Crucial to the proposed interpretational scheme is the extrinsic nature of the values of quantum observables. Being essentially a probability algorithm, QM presupposes events (specifically, value-indicating events) to which probabilities can be assigned. In agreement with Niels Bohr I maintain that a value-indicating event or state of affairs is not only sufficient but also necessary for the existence of a value. But it does not have to be the click of a laboratory counter or the deflection of a pointer needle. Any actual event or state of affairs from which the possession of a property or
a value, by a system or an observable, can in principle be inferred, is sufficient for the existence of that property or value [3].

To avoid a potential misunderstanding, let us assume that the presence of $O$ in $V$ is indicated. Then the absence of $O$ from the complement $V'$ of $V$ is also indicated. If nothing indicates the presence of $O$ in any region $W \subset V$ then condition (ii) is not met for $W$—such a region does not exist for $O$. The absence of $O$ from any region $U \subset V'$, on the other hand, seems to be inferable from the absence of $O$ from $V'$, so that the proposition $O \rightarrow U(T)$ seems to have a truth value. Suppose it has a truth value but condition (i) is not satisfied. Then this truth value—the truth or falsity of $O \rightarrow U(T)$—concerns something that exists in our imagination, rather than something that is the case in the real world. If $O \rightarrow U(T)$ or its negation is to be a meaningful statement about the physical world, the region $U$ must exist in the physical world, and for this it must be intrinsically possessed (that is, possessed by a macroscopic object, as defined in the following section). This is why the region referred to by condition (ii) is stipulated to be an intrinsically possessed region. If $U$ is not realized as an intrinsic property of a macroscopic object, the property of being in $U$ is not available for attribution to $O$. It does not form part of physical reality. Hence its possession by $O$ cannot be indicated, and it cannot be real for $O$.

Nobody is likely to assert that red, or a smile, can exist without a red object or a smiling face (the smile of the Cheshire cat notwithstanding). Yet, for the neurophysiological reasons alluded to, we tend to think that positions (points or regions) can exist even when they are not properties of material objects. This way of thinking appears to me to be as inconsistent with QM (as a theory about the big world outside the laboratory) as absolute simultaneity is with special relativity. In the world according to QM [23], no position exists unless it is possessed, and no position is possessed unless its possession is indicated [30], even though no position can be indicated unless it is possessed by a macroscopic object. (The resolution of this apparent vicious circle will concern us in the following section.)

When dealing with extrinsic properties a clear distinction has to be made between the times of property- or value-indicating events and the times at which the indicated properties or values are possessed. This allows for an innocuous kind of advanced (backward-in-time) influence, inasmuch as the value-indicating event may happen after the time at which the indicated value is possessed [3, 31].

In the context of a two-slit experiment the relevant events are the emission of an electron $e$ in front of the slit plate $S$ and its detection behind $S$. Together with the geometry of the setup they warrant the truth of $e \rightarrow L \& R(T)$, while (in the absence of any further matter of fact about the electron’s whereabouts in the interim) the propositions $e \rightarrow L(T)$ and $e \rightarrow R(T)$ lack truth values. Hence whenever it was that the electron went through $S$, $L \& R$ was real for the electron at that time, while neither $L$ nor $R$ were real for it.

But when was that? What is the value of $T$ in the true proposition
e→L&R(T)? Just as any material object is spatially localized only to the indicated extent, so an event like the electron’s passage through $S$ is temporally localized only to the indicated extent. Not only the values of quantum observables are extrinsic but also the times at which they are possessed. The extent to which $T$ is indicated is defined by the respective times of emission and detection, $T_e$ and $T_d$. Just as the electron’s position at $T$ is the entire region $L&R$, so $T$ is the entire interval $[T_e,T_d]$. Nothing warrants the inference that the electron passed $S$ at some particular time during this interval. All that QM permits us to say in this case is counterfactual and probabilistic: Imagine the interval $[T_e,T_d]$ divided into intervals $T_i$. If there were an event from which the electron’s passage through $S$ during a particular interval $T_i$ could be inferred, it would indicate the interval during which the electron went through $S$ with probability $p_i$. This set of counterfactuals is the proper formal expression of an objective temporal indefiniteness.

3 A FREE-STANDING REALITY

Even position-indicating positions are extrinsic; they too need to be indicated in order to be possessed. To avoid a vicious regress one must show that some positions can consistently be thought of as intrinsic, as self-indicating, or as factual per se. Here is how this may be done.

There are objects whose indicated positions are so correlated that every one of them is consistent with every prediction that is based on (i) previous indicated positions and (ii) a classical law of motion (except, of course, when their indicated positions themselves serve to indicate unpredictable values). If we take this characterization as a definition of “macroscopic object” then what needs to be shown is that such objects exist. Note that this definition does not require that the probability of finding a macroscopic object where classically it could not be, is strictly 0. What it requires is that there be no position-indicating fact that is inconsistent with predictions based on a classical law of motion and earlier position-indicating facts.

The departure of an object $O$ from a classical trajectory can be indicated only if there are detectors whose positions are sharper than $O$’s position, or whose position probability distributions are narrower than that of $O$. One of the things QM tells us is that the relative position of two objects cannot be exact. Some relative positions are sharper than others. Some objects have the sharpest positions in existence. For such objects the probability of a position-indicating event that is inconsistent with a classical trajectory is necessarily very low. It is therefore certain that among such objects there will be macroscopic objects.

Since no object has an exact position, it might be argued that even for a macroscopic object $M$ there always exists a small enough region $V$ such that $M→V$ lacks a truth value. But this is an error. Macroscopic objects have the sharpest positions in existence. There isn’t any object that has a (significantly) sharper position. A fortiori, there isn’t
any object for which $V$ is real. But a region exists only if it is real for at least one material object. It follows that there exists no sufficiently small region $V$ such that $M \rightarrow V$ lacks a truth value. Such a region may exist in our imagination but it does not exist in the physical world.

Now recall why positions are extrinsic: The proposition $O \rightarrow V$ may or may not have a truth value. One therefore needs a criterion for the existence of a truth value: A truth value must be indicated. On the other hand, one doesn’t need a criterion for the existence of a truth value if for every existing region $V$ the proposition $M \rightarrow V$ has a truth value. Since this condition is satisfied by macroscopic objects, the positions of such objects can consistently be regarded as intrinsic, or factual per se.

The laws of classical mechanics define a set of nomologically possible worlds. They do not single out the actual world. While exactly one possible world has the mysterious quality of being real, classical mechanics cannot tell us which one it is. It has no symbol for the property of being real; reality is not a classical observable. QM on the other hand seems to possess a criterion of reality: To be is to be indicated. Yet being indicated is not a physical observable either. There is no projector or subspace that represents the property of being indicated. Instead QM gives us a choice of two rules: To calculate the probability of a state of affairs $A$ that can come about in several ways, add the probabilities/amplitudes associated with the alternatives if something/nothing indicates the alternative taken. (We also add probabilities if the various ways in which $A$ can come about are so correlated with a set of possible, mutually exclusive events that the occurrence of one of these events would indicate the actual way in which $A$ comes about.) QM seems to leave it to our discretion whether or not the alternative is indicated.

Let us take a closer look at what seems to be left to our discretion. An electron’s probability of being detected at a particular location behind the slit plate depends on two amplitudes, one for each slit. We add the absolute squares of these amplitudes, rather than the amplitudes themselves, if the individual regions $L$ and $R$ are real for the electron $e$, or if the propositions $e \rightarrow L$ and $e \rightarrow R$ have truth values, or if their truth values are indicated. The reality, for the electron, of a particular region—say, $R$—requires two things: $R$ must exist (that is, it must be real for at least one object), and the electron’s presence in or absence from $R$ must be indicated. A detector—in the broadest sense of the word—serves both purposes: It warrants the reality of $R$, and it indicates the truth value of $e \rightarrow R$. QM thus presupposes detectors but seems to leave it to our discretion to decide what is, and what is not, a detector.

The problem of granting detector status is essentially the problem of identifying positions that can be consistently regarded as factual per se. This must not be confused with the spurious problem of the “emergence” of facts. Factuality is not a physical observable; it cannot be measured. The notion that physics ought to account for the actuality of the actual world or the factuality of facts is therefore misconceived. A theory that describes the actual world without fully determining it cannot possibly do this, and even if we had
a theory that determined a unique world it could not account for the existence of that world, inasmuch as it could not account for its own truth. The actual world does not exist by courtesy of laws that describe it. To “explain why events occur” or “how it is that probabilities become facts” is as impossible as to explain why there is anything rather than nothing.

QM concerns the transmittal of factuality from self-existent, property-indicating positions to indicated properties. The philosophically correct term is supervenience: Indicated properties supervene on indicating properties. Factuality is always presupposed, not only because QM assigns probabilities on the basis of facts but also because it assigns them to properties indicated by facts. Pointers do deflect, as certainly as the world exists, and just as inexplicably. What is inexplicable is not only why the pointer deflects to one side rather than the other but also why it deflects at all. QM honors the impossibility of explaining the latter by assigning probabilities on the condition that the pointer deflects. (The probability that the observable $Q$ has the value $q$ is the product of two probabilities: The probability that any one of the possible values of $Q$ is indicated, and the probability that the indicated value is $v$, given that a value is indicated. The probabilities QM allows us to calculate are exclusively of the latter type.)

For the same reason that classical mechanics does not determine which possible world corresponds to the actual one, QM does not determine which possible outcome corresponds to the actual one. (“Possible” here means “consistent with the laws of physics.”) To expect otherwise is to ignore the fundamental indefiniteness that finds expression in the statistical laws of QM. A macroscopic pointer, however, is exempt from this indefiniteness, inasmuch as its own indicated positions before and after a value-indicating deflection are predictably correlated and therefore effectively intrinsic. This makes the deflection an unpredictable change from one actual state of affairs to another. It is therefore impossible for a macroscopic pointer to “catch” the indefiniteness of a quantum-mechanical observable. Indefiniteness is not contagious; factuality is.

Indefiniteness means that certain propositions lack truth values, and this, we have seen, entails the supervenience of indicated properties on indicating properties. There is a dependent actuality, and this entails the existence of an independent actuality. What seems to be left to our discretion is the attribution of this independent actuality. But only seems, for QM itself furnishes the sufficient and necessary criterion of self-existence. The positions of macroscopic objects—and only these—can consistently be considered factual per se.

The crux of the matter is that space is not something that exists by itself and is intrinsically differentiated. Regions of space exist only as properties of matter, to the extent that propositions of the form $O \rightarrow V$ possess truth values, and a region $V$ is real only for those objects $O$ for which $O \rightarrow V$ possesses a truth value. Since no exact position is ever indicated, infinitesimal regions of space do not exist. Nor do sharply bounded regions. The extent to which the world is differentiated spacewise is both relative (for
some objects greater, for some objects less) and finite. For a macroscopic object the world possesses the highest degree of spatial differentiation, and the space over which its position is “smeared out” is never probed. This space is undifferentiated; it contains no smaller regions. We may imagine smaller regions, but they don’t exist in the physical world.

It is therefore unnecessary to treat the position of a macroscopic object as a probability distribution. Probability of what? Nothing in the realm of fact indicates a departure from a definite trajectory. The fuzziness of the trajectory only exists in relation to an imaginary spatial background that is more differentiated than the actual world. We do need to acknowledge that even the positions of macroscopic objects exist only because they are indicated—by the positions of macroscopic objects. Each macroscopic position presupposes the others; none exists without the others; each has the value that is indicated by the others. But the entire system of macroscopic positions is self-contained. Together with all the properties that supervene on it, it constitutes a free-standing reality—a quantum world that does not depend on observers, their information, their arbitrary decisions, or their interventions in “the course of Nature.” The relevant dependence is the mutual dependence of the positions of macroscopic objects and the supervenience, on these, of all possessed properties. This is an internal dependence—it does not presuppose anything external to the quantum world.

4 QUANTUM “STATES” ARE PROBABILITY MEASURES

A typical but didactically disastrous approach to QM begins with the observation that in classical physics the state of a system is represented by a point \( P \) in some phase space, and that the system’s possessed properties are represented by the subsets containing \( P \). The question then is, what are the quantum-mechanical counterparts to \( P \) and the subsets containing \( P \), respectively, \textit{qua representations of an actual state of affairs and possessed properties}? Once we accept this as a valid question, we are on a wild-goose chase. It would be better to begin with the observation that in classical physics a system is associated with a probability measure, that this is represented by a point \( P \) in some phase space, that observable properties are represented by subsets, and that the probability of finding a property is 1 if the corresponding set contains \( P \); otherwise it is 0. The question then is, what are the quantum-mechanical counterparts to \( P \) and the subsets containing \( P \), respectively, \textit{qua representations of a probability measure and observable properties}?

Classical probability measures assign trivial probabilities: either 0 or 1. This permits us to treat \( P \) without further ado as representing an actual state of affairs connoting a set of possessed properties. Quantal probability measures generally assign nontrivial probabilities, and this forbids us to treat them as representing states connoting possessed properties. With one exception: For a macroscopic object \( M \) there exists no region \( V \)
such that the proposition $M \to V$ lacks a truth value. (In this proposition, recall, “$M$” stands for the position of $M$.) The position probability measure associated with $M$ assigns trivial probabilities to all regions of space that can legitimately be said to exist in the physical world. This permits us to represent the probability measure associated with $M$ by a point in a phase space, and to treat this point as representing a state connoting a set of possessed properties. If this point is fuzzy, it is so only relative to a spatial background that is more differentiated than the physical world.

Whence the nontrivial probabilities? It is well known that the objective indefiniteness of relative positions is crucial for the stability of extended material objects \[34\]. Together with the exclusion principle it “fluffs out” matter. The proper way of dealing with objectively indefinite values is to make counterfactual probability assignments \[3, 35\]. If a quantity is said to have an “indefinite value,” what is really intended is that it does not have a value (inasmuch as no value is indicated) but that it would have a value if one were indicated, and that at least two possible values are associated with positive probabilities. Nontrivial probabilities thus are a consequence of the objective positional indefiniteness to which extended matter owes its stability.

To find the quantum counterpart to $P$, we have to make room for nontrivial probabilities, and the obvious way to do this is to replace the subsets of a phase space by the subspaces of a vector space, or by the corresponding projectors. (Why this vector space has to be complex will be discussed in a separate article.) An atomic probability measure will then be a 1-dimensional subspace $U$ instead of a 0-dimensional subset, probability 1 will be assigned to properties that contain $U$, probability 0 will be assigned to properties that are orthogonal to $U$, and the remaining properties will be associated with nontrivial probabilities. These probabilities are uniquely determined by Gleason’s theorem \[36, 37\] for vector spaces with at least 3 dimensions, if three postulates equivalent to the following hold \[38\]:

(A1) Attributions of the form “System $S$ has property $u$” are represented by projectors in a complex vector space $\mathcal{V}(S)$.

(A2) Commuting projectors in $\mathcal{V}(S)$ correspond to attributions the truth values of which can be indicated together regardless of the truth value of each attribution.

(A3) If $P_u$ and $P_v$ are orthogonal projectors (representing attributions of $u$ and $v$, respectively), the probability associated with their sum $P_{uv} = P_u + P_v$, which is itself a projector (representing the attribution of the property spanned by $u$ and $v$), is the sum of the probabilities associated with $P_u$ and $P_v$.

A few remarks: (i) The possession of the property spanned by $u$ and $v$ is indicated if the possession of either $u$ or $v$ is indicated, but it can also be indicated if neither the possession of $u$ nor the possession of $v$ is indicated. For instance, the electron’s presence in $L\&R$ can be indicated when neither its presence in $L$ nor its presence in $R$ is indicated.
(ii) Suppose that the respective attributions of $u$, $v$, and $w$ are represented by 1-dimensional projectors in a 3-dimensional vector space, that $P_w$ is orthogonal to both $P_u$ and $P_v$, and that $P_u$ is neither the same as nor orthogonal to $P_u$. Then $P_{uw}$ and $P_{vw}$ do not commute even though the corresponding attributions can both be true. However, if either of them is false, the other attribution cannot have a truth value. Hence the clause “regardless of the truth value of either attribution” in the second postulate, which is not explicit in the formulation given by Peres [38].

(iii) The third postulate is the classical sum rule for the probability of “either $A$ or $B$,” $p(A \lor B) = p(A) + p(B)$, which holds for a set of mutually exclusive events provided that one of them happens. Since probabilities are assigned on the proviso that a value be indicated, this condition is always satisfied.

A recent generalization of Gleason’s theorem by P. Busch and by Caves et al. [39, 40] makes the theorem applicable to 2-dimensional vector spaces as well. So does work by D.I. Fivel [42]. The thrust of the theorem is that the nontrivial probabilities that are entailed by the objective indefiniteness of relative positions are uniquely determined by what is most inappropriately called a quantum “state.” The search for a suitable probability algorithm leads to a unique density operator. What you seek is what you get. We sought a probability measure and we got a probability measure. Quantum “states” are probability measures.

5 AGAINST “EVOLUTION”

While surveying several good textbooks Bell stumbled at various invocations of that infamous postulate according to which the quantum state of a system either “jumps” from $\rho$ to $\hat{\rho}$ or “collapses” into an eigenstate of the observable that is being measured. Something is evidently wrong with this kind of narrative, but exactly what? For Bell the root of all evil is the special status granted to measurements by the authors of those books. While under “normal” circumstances quantum states evolve deterministically, measurements are said to cause them to change discontinuously. Yet the elimination of all references to measurements qua “piddling laboratory operations” does not solve the problem posed by the existence of two kinds of evolution. Measurement outcomes qua property-indicating facts are an integral part of QM. The properties of the quantum world supervene on the facts; they need to be indicated in order to exist. The root of the problem is not the extrinsic nature of the values of quantum-mechanical observables but the notion that a quantum state is something that can evolve.

Fuchs and Peres stress that

the time dependence of the wavefunction does not represent the evolution of a physical system. It only gives the evolution of our probabilities for the outcomes of potential experiments on that system. [18]
The wave function being solely a “compendium of probabilities,” what holds for the time
dependence of probabilities must also hold for the time dependence of the quantum states
that define them. If probabilities evolve, so do quantum states. On this account, quantum
states are things that exist and change in time, for only such things can be said to evolve.

By denying that the evolution of a quantum state represents the evolution of a physical
system, Fuchs and Peres avoid the various “quantum paradoxes” entailed by realistic
construals of quantum states. However, the notion that probabilities (and hence quantum
states) nevertheless evolve entails inconsistencies of its own. For instance, the authors
state that “no wavefunction exists either before or after we conduct an experiment.”
This is correct. The time on which a wave function depends is the time at which an
observable has an indicated value, either in the actual world or counterfactually, on the
false assumption (but nevertheless on the assumption) that a value is indicated. Hence
no wave function “exists” at a time that is not the time of an actual or counterfactual
measurement. But this is clearly inconsistent with the notion that quantum states evolve.

Again, Fuchs and Peres stress that “collapse is something that happens in our descrip-
tion of the system, not to the system itself.” But if collapse does not happen to the system,
a “description” of the system in which it happens cannot be a (correct, valid, acceptable)
description! This distinction between knowledge or a description on the one hand and
the thing known or described on the other has always baffled me. How can there be an
abstract quantum physical description” if “[t]here is no quantum world” answering
that description? What does such a description describe? Knowledge? If so, knowledge
about what? I fear that the commendable objective of these authors—to deflate realistic
construals of the quantum state—will not be achieved by the self-contradictory and
philosophically defunct stratagem of driving a wedge between knowledge and the known.
What we need is a correct description, and once we have it, we can stop talking about
our description and instead talk about the thing described.

If collapse does not happen to the system, it cannot happen in a valid description
of the system. So what makes it seem to happen? Nothing but the erroneous notion
that quantum states (or the probabilities they assign) evolve. What evolves, evolves
in relation to a detached, intrinsically differentiated temporal background. An evolving
quantum state, if it existed, would be a function of a succession of self-existing instants.
It would exist at every “moment of time.” Yet, so Fuchs and Peres assure us, no wave
function exists at a time that is not the time of an actual (or counterfactual) measurement.
Nor, as a matter of fact, does a wave function exist at the time of a measurement, for the
probability for something to happen at a certain time is not something that exists at that
time, any more than the probability for something to be found in a region $V$ is something
that exists in $V$. Probabilities (and hence quantum states) do not exist at any time. A
fortiori they are not things that evolve, whether deterministically or discontinuously.

The question, therefore, is not whether quantum states are states of Nature or states
of knowledge. This question only arises if quantum “states” are conceived as states—
something that exists in time and evolves. If quantum “states” were states, Fuchs and Peres would be right: They could only be states of knowledge. The idea that quantum states are states of knowledge is therefore a direct consequence of the notion that quantum states evolve. Quantum realists and the proponents of epistemic interpretations are equally wrong, inasmuch as they both take for granted that the temporal referent of a quantum state is a detached, intrinsically differentiated temporal background. The question is not whether \( \rho(x, t) \) represents a state of this or that kind—ontological or epistemic. The question is what the true referents of \( t \) and \( x \) are if they do not refer to a detached, intrinsically differentiated spatiotemporal background. The real issue is not the kind of reality that quantum states possess but the kind of reality that the spatial and temporal referents of quantum states possess. Where space is concerned we already know the answer: a relative and contingent kind. It will come as no surprise that the same holds true for time.

A quantum state is a generic probability measure. It depends on a set of indicated properties, which constitute the factual basis on which probabilities are assigned. The generic measure defines specific measures, which depend on specific observables and on the time \( t \) of measurement. In the case of a position measurement on \( O \), the specific measure \( p(V_i, t) \) further depends on a partition of space into regions \( V_i \). Such a region exists for \( O \) if and only if the proposition “\( O \) is in \( V_i \)” has a truth value. This is the case if and only if a truth value is indicated, and this is possible only if the region is realized (made real) by being an intrinsic property of a macroscopic detector.

The regions \( V_i \) thus are not self-existent and intrinsically distinct “parts of space.” They are realized by macroscopic detectors. There is no such thing as a detached, intrinsically differentiated spatial background. Space consists of the indicated spatial properties of material objects, and these supervene on the self-indicating spatial properties of macroscopic objects. By the same token, the time on which \( p(V_i, t) \) depends is not a self-existent “moment of time.” There is no such thing as a detached, intrinsically differentiated temporal background. Times, too, are extrinsic. Time consists of the times that are indicated by macroscopic clocks (that is, by the positions of macroscopic objects that are suitable for indicating time). Times exist to the extent that they are indicated, and a time \( t \) exists for a system \( S \) if and only if it is the time of a “measurement” (that is, the indicated time at which an observable on \( S \) possesses an indicated value).

Thus when Fuchs and Peres say that “no wavefunction exists either before or after we conduct an experiment,” which suggests that a wave function exists at the time of measurement, what they really mean (or ought to mean) is that the only times that exist for a quantum system are the indicated times at which the system has indicated properties. This is why it makes no sense to speak of an “evolution.” The word “evolution” is appropriate neither for the time dependence of a generic probability measure that is based on a “preparation” (that is, on a given set of property-indicating facts), nor for the replacement of a generic probability measure based on one set of facts by a generic
6 QUANTUM MECHANICS IS COMPLETE

QM is about probabilities. Until 1926, when this was first understood by Max Born, all probabilities known in physics were concessions to our ignorance of some of the relevant data, and therefore essentially subjective. This is no longer the case. Born probabilities are assigned by fundamental physical laws, on the basis of particular sets of facts, in a way that does not involve anyone’s knowledge or ignorance of the relevant facts. A clear distinction has to be maintained between physical laws and the uses we make of them. We may use them to make statistical predictions on the basis of whatever facts we were able to gather. In doing so we single out a particular set of facts. We can make use only of known facts. But the laws of QM do not single out any particular set of facts. They assign nontrivial probabilities to the possible values of any observable and on the basis of any set of relevant facts. These probability assignments are fundamental physical laws. They are as objective as the indefiniteness of which they are the formal expression, and they are objective in the same sense in which the laws of classical physical would be objective if they were true. QM provides as much a model of a free-standing world as classical physics did. The only difference is that the classical model is deterministic while the quantal model incorporates an objective indefiniteness in the form of an objective probability algorithm.

Why are we not satisfied with this? Probably because we harbor the illusion that a deterministic description provides a complete explanation. That this is an illusion was pointed out two and a half centuries ago by Immanuel Kant. Newton knew that he had described gravity but was unable to explain it. Kant made it clear why:

That the possibility of fundamental forces should be made conceivable is a completely impossible demand; for they are called fundamental forces precisely because they cannot be derived from any other force, i.e. they cannot be conceived. [43]

This is as true today as it was in Kant’s time. A fundamental physical theory is, by virtue of being fundamental, an inexplicable description of correlations among the goings-on in the world, regardless of whether these are instantaneous or retarded, local or nonlocal, deterministic or probabilistic. The question, therefore, is not whether QM affords a complete explanation but only whether it affords a complete description. And that it does. A description that includes everything that is indicated is complete. If anything is incomplete it is the physical world itself, but this is incomplete only in relation to an imaginary spatiotemporal background that is more differentiated than the physical world.

The question of why the world answers the description that it does is beyond the reach of physics. So is the question of why it does answer a description that owes so much
to logic, probability theory, and pure mathematics, and comparatively little to empirical input. “The most incomprehensible thing about the world is that it is comprehensible,” Einstein remarked [44]. Other physicists have voiced similar astonishment. To countless philosophers since Plato the comprehensibility of the world has suggested an affinity of our minds with the Power or Principle responsible for the existence of the physical world. I won’t contradict anyone who feels the necessity of conceiving of such a Power or Principle. But I would insist that this necessity did not arise with the discovery of QM; it arose when philosophers first began to think about the reality of concepts [45].

7 CONCLUSION

In the opening paragraph of a recent review article by F. Laloë [22] two interpretational issues are mentioned by way of example. One of them is the question, does $|\psi\rangle$ “describe the physical reality itself, or only some partial knowledge that we might have of this reality?” Further on much the same question is identified as one of the key issues:

To what extent should we consider that the wave function describes a physical system itself (realistic interpretation), or rather that it contains only the information that we may have on it (positivistic interpretation)…?

This false disjunction leaves no room for a successful resolution of the interpretational issues raised by QM. It has its roots in an illegitimate projection of an imaginary spatiotemporal background into the physical world. The wave function itself neither describes a physical system nor does it contain only the information that we may have. An objective probability measure, it is the proper expression of an objective indefiniteness. Its failure to describe the state of the system is not a shortcoming but a consequence of the nonexistence of a detached temporal background that would allow us to assign a state to the system at every “moment of time.” The temporal referent of a quantum state (in the Schrödinger picture) is not a self-existent instant but the indicated time of possession of an indicated property. The spatiotemporal properties of the world supervene on the domain of facts, which is constituted by the positions of macroscopic objects.

The second part of Laloë’s article, entitled “Difficulties, Paradoxes,” begins as follows:

We have seen that, in most cases, the wave function evolves gently, in a perfectly predictable and continuous way, according to the Schrödinger equation; in some cases only (as soon as a measurement is performed), unpredictable changes take place, according to the postulate of wave packet reduction.

Once quantum state evolution is taken for granted one is confronted with two distinct modes of evolution. This has not only led to a host of inane paradoxes but also elicited such inappropriate responses as the claim that “state reductions… are nothing but mental processes” [46]. If quantum states did evolve, if there were such things as state reductions,
or if state reductions were processes, then I suppose they would have to be regarded as mental processes. But none of the above antecedents is true. Although the literature is infested with statements suggesting the contrary (for instance, “coherent superpositions tend to constantly propagate toward the environment” [22]), the time dependence of the density matrix has nothing to do with any process, whether physical or mental. “Logically it is... clear that this problem will never be solved by invoking any process that is entirely contained in the linear Schrödinger equation,” Laloë writes. It would be more appropriate to say that this problem will never be solved by any process. Nor is any process contained in the Schrödinger equation. Unless we learn to talk about probability measures in a language that is suitable for probability measures, I fear that making sense of QM will remain a distant goal.

But perhaps the situation is not all that bleak. The first interpretation discussed by Laloë, a sort of common ground he calls the “correlation interpretation” (CORI), emphasizes the correlations between “successive results of experiments.” Significantly, in this interpretation “no conflict of postulates takes place... no paradox can be expressed in terms of correlations.” The trouble starts when the CORI is conjoined with a detached, intrinsically differentiated spatiotemporal background. This is what has engendered all those interpretations whose

general purpose always remains the same: to solve the problems and questions
that are associated with the coexistence of two postulates for the evolution of the state vector. [22]

Objections to the CORI are of two kinds. Those of the first kind are either misconceived or readily parried. The others can be defused by going beyond the CORI without invoking a detached spatiotemporal background. One objection, directed against the perceived undue emphasis on experiments, is easily rebutted by drawing the appropriate distinction between experiments and property-indicating facts. The emphasis on the latter—the supervenience of properties on facts—is a direct consequence of an objective indefiniteness in the physical world. Another perceived shortcoming—that the CORI does not tell us “which sequence [of possible measurement results] is realized in a particular experiment”—is not a shortcoming at all but a consequence of this objective indefiniteness.

The objection that the CORI “shows no interest whatsoever in questions related to physical reality as something ‘in itself’” is of the second kind. (So is the objection that “the boundary between the measured system and the environment of the measuring devices is flexible,” which has been dealt with in Sec. 3.) Nobody can be forced to show interest in such questions. Unfortunately some proponents of the CORI go further by declaring that QM is inconsistent with a free-standing physical reality. What QM is actually inconsistent with is the notion of a detached, intrinsically differentiated spatiotemporal background. Once this is rejected, a consistent conception of a free-standing
quantum world is perfectly possible, as this article attempted to demonstrate. The quantum world consists of the positions of macroscopic objects and of whatever is indicated by them. The dependence that has proved such a stumbling block to understanding QM is not a dependence on anything external to the quantum world but (i) an internal, mutual dependence of the positions of macroscopic objects and (ii) the supervenience on these positions of the remaining properties of the quantum world, which is a direct consequence of the indefiniteness of the quantum world.

A possible objection to the emphasis on positions has been met by Laloë himself (if one reads “property-indicating facts” for “experiments” and “indicated” for “measured”):

One can easily convince oneself that... what is measured in all experiments is basically the positions of particles or objects (pointers, etc.), while momenta are only indirectly measured.

One final question: Why does the present article, including its title, single out epistemic interpretations, considering that realistic interpretations of quantum states are even farther off the mark? The answer is that the absurd consequences of the latter interpretations have been adequately highlighted by the proponents of the former, while the proponents of the latter are unable to adequately highlight the errors of the former inasmuch as they share them.

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The view that no position exists unless it is possessed accords with relationalism, the doctrine that space (and time) must be thought of as a family of spatial (and temporal) relations holding among the material constituents of the universe. The view that no position is possessed unless its possession is indicated goes beyond previous versions of relationalism (such as that advocated by Leibniz), which treat spatial relations as intrinsic.

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