Approximation-Based Event-Triggered Control Against Unknown Injection Data in Full States and Actuator of Uncertain Lower-Triangular Nonlinear Systems

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This work was supported in part by the National Research Foundation of Korea (NRF) Grant funded by the Korean Government under Grant NRF-2019R1A2C1004898, and in part by the Human Resources Development of the Korea Institute of Energy Technology Evaluation and Planning (KETEP) Grant funded by the Korean Government Ministry of Trade, Industry and Energy under Grant 20174030201810.

ABSTRACT This paper addresses an approximation-based adaptive event-triggered control problem against unknown injection data in full state measurements and an actuator of systems with unknown strict-feedback nonlinearities. It is assumed that full state variables measured for state-feedback control are corrupted by unknown injection data that denote cyber attacks or fault signals, and all system nonlinearities are unknown. Owing to the corrupted state feedback information, error surfaces using exactly measured state variables become unknown during the recursive control design procedure for strict-feedback nonlinear systems. Thus, they cannot be used to implement the adaptive event-triggered controller. To address this problem, an approximation-based adaptive recursive event-triggered control design using the corrupted state variables is established to ensure that error surfaces using exactly measured state variables converge to an adjustable neighborhood of the origin in the Lyapunov sense. The adaptive controller and its event-triggering law using corrupted states are designed under uncertain injection data where the adaptive injection data compensators using the neural networks are constructed to deal with the unknown injection data effects. The stability of the closed-loop systems and the exclusion of Zeno behavior are analyzed.

INDEX TERMS Event-triggered control, corrupted full state measurements, unknown injection data, dynamic surface design, unknown strict-feedback nonlinearities.

I. INTRODUCTION

The development of information and communication technology has stimulated the tight interaction of control systems and cyber components [1], [2]. Under the network-based control environment, the exact transmission of measured state variables of physical systems is an important problem for ensuring the performance of controllers in the network. During the network transmission of the state information for the feedback control, unexpected time-varying injection data can be added to the state information measured by sensors. The sources of the injection data in the resilient control studies are largely divided into two categories: (i) measurement faults and (ii) adversarial cyber attacks. In the first category, the sensor faults influence the measurement information for the feedback control. Many studies have addressed the resilient control problems of linear and nonlinear systems with measurement faults [3]–[6]. Especially, some limited studies have appeared for the recursive control design of lower-triangular nonlinear systems with measurement faults where backstepping [7] and dynamic surface designs [8], [9] have been used to construct the resilient control systems. In [10], [11], the output functions were regarded as sensor faults, and output-feedback control approaches were presented for nonlinear systems. In [12], an output-feedback stabilizer design problem using a dual-domination approach was addressed for lower-trianglar nonlinear systems with unknown measurement sensitivity. In [13], an adaptive
compensation method of sensor failure was proposed for parametric strict-feedback systems. In [14], a fuzzy adaptive sensor fault compensation approach was studied for nonlinear strict-feedback systems. In [15], an adaptive control method was presented for nonlinear systems with time-varying sensor sensitivities. However, the existing works [10]–[15] for lower-triangular nonlinear systems considered only an output measurement fault. That is, they cannot be applied to state-feedback recursive control design and stability analysis problems in the presence of full state measurement faults because corrupted state variables, instead of exactly measured state variables, should be used to implement virtual and actual controllers. In the second category, the measured state information is stealthily monitored by adversarial attackers and deception attack signals for attackers’ desire are maliciously injected into the measured states. Thus, the deception attack signals may depend on the state variables. For known cyber attacks, some resilient control approaches have been presented for linear and nonlinear systems [16]–[21]. To consider unknown cyber attack signals in state variables and an actuator, some adaptive control studies have recently been investigated. In [22], sensor attacks of uncertain linear systems were considered to design adaptive control architectures. An adaptive control methodology was presented in [23] where the time-varying parameters in deception attacks were estimated via the projection algorithm. In [24], an adaptive resilient control approach using Nussbaum functions was studied in the presence of unknown control direction. However, these research results [22]–[24] for compensating for unknown deception attacks in the adaptive control framework are only available for linear systems. They cannot be applied to the recursive design problem against unknown injection data in full state measurements of lower-triangular nonlinear systems.

Adaptive control problems of nonlinear lower-triangular systems with uncertainties unmatched to a control input have received great attention in the control design field because of their application to many practical systems such as robot manipulators, biological systems, power systems, flight systems, and traffic control systems. Thus, adaptive recursive control design strategies have been actively developed to deal with unmatched parametric or nonparametric uncertainties. During the early stages of the research, the adaptive technique was combined with the recursive designs to estimate unmatched parametric uncertainties online (see [25]–[31] and references therein). Function approximation techniques using neural networks or fuzzy logic systems have been applied to design approximation-based adaptive control systems, in an attempt to deal with unmatched nonparametric uncertainties (i.e., nonlinear uncertainties) (see [32]–[39] and references therein). In addition, distributed adaptive control approaches have been presented for uncertain multi-agent nonlinear systems in the strict-feedback form [40]–[44]. In these studies, unknown nonlinear functions derived from the recursive control design steps were estimated via radial basis function neural networks (RBFNNs) or fuzzy logic systems.

The uncorrupted state feedback information was used as the input for these function approximators. If the state feedback information is corrupted by unknown injection data, the corrupted state feedback information should be used as the input of the function approximators in the adaptive control framework. This problem still persists in the full-state-feedback-based adaptive control field of strict-feedback nonlinear systems.

The event-triggered control technique is particularly popular in the network-based control field of linear and nonlinear systems, for ensuring the efficient use of the network bandwidth [45]–[48]. In event-triggered control, a control input is updated when a triggering law is satisfied. Thus, the computational and communicational resources required for implementing the controller can be conserved. Therefore, event-triggered control problems have been studied for systems with nonlinearities unmatched to the control input. In [49]–[51], adaptive event-triggered control designs were developed for lower-triangular nonlinear systems with parametric uncertainties. In [52], the finite-time stabilization problem of uncertain nonlinear systems was addressed in the event-triggered control framework. In [53]–[56], approximation-based adaptive control techniques for estimating unknown nonlinear uncertainties were combined with event-triggered control designs. Despite these efforts, no studies have been reported thus far on the event-triggered control problem against unknown injection data in full state measurements and an actuator of lower-triangular nonlinear systems.

On the basis of the above discussion, the main difficulties in designing an adaptive event-triggered resilient control scheme against the aforementioned unknown injection data are as follows.

(D1) Because full state measurements for feedback control are corrupted by unknown injection data, the corrupted state variables can be only used in the adaptive control scheme using backstepping or dynamic surface techniques and thus the error surfaces using the exactly measured state variables for the recursive design are unknown for the control design. Accordingly, the first difficulty is how to design an adaptive controller using the corrupted full state variables to ensure the convergence of the error surfaces using the exactly measured state variables for achieving the control objective in Lyapunov-based recursive stability analysis.

(D2) Since the exactly measured state variables are not available, the second difficulty is how to design the triggering law using corrupted state variables in the adaptive event-triggered control framework. Furthermore, the stability of the closed-loop system and the exclusion of Zeno behavior should be analyzed by the triggering law using the corrupted state variables.

The objective of this paper is to propose a remedy for difficulties (D1) and (D2), that is, to establish an adaptive resilient event-triggered control design strategy using corrupted full state measurements for uncertain nonlinear strict-feedback systems with unknown injection data in full
state measurements and an actuator. It is assumed that full state measurements corrupted by unknown injection data are available for feedback, and system nonlinearities are unknown. To overcome difficulties (D1) and (D2), auxiliary signals using corrupted full state variables are designed, and an approximation-based adaptive event-triggered controller and its triggering law are recursively constructed using the auxiliary signals. For the proposed control scheme, dynamic injection data compensators using RBFNNs are designed to compensate for unknown injection data effects. Although the proposed adaptive event-triggered controller is based on corrupted full state variables, it is shown that the convergence of the error surfaces using the exactly measured state variables is ensured for achieving the control objective in the Lyapunov-based stability sense. Finally, simulation examples including a practical application are given for testing the validity of the proposed theoretical result.

The rest of this paper is outlined as follows. An adaptive resilient event-triggered control problem of uncertain nonlinear strict-feedback systems with unknown injection data in full state measurements and an actuator is formulated in Section 2. The proposed event-triggered control design using corrupted state variables and its stability analysis are presented in Sections 3. Simulation studies are given in Section 4. Finally, we conclude in Section 5.

II. PROBLEM FORMULATION

Let us consider the following uncertain nonlinear strict-feedback systems with unknown injection data in full state measurements and an actuator

\[
\begin{align*}
\dot{x}_k &= x_{k+1} + h_k(\bar{x}_i) \\
\dot{x}_n &= u + \kappa_n(t, \bar{x}_n) + h_n(\bar{x}_n) \\
x_{i,a} &= x_i + \kappa_{i,a}(t, x_i)
\end{align*}
\]

where \( k = 1, \ldots, n - 1, i = 1, \ldots, n \), \( \bar{x}_i = [x_{i,1}, x_{i,2}, \ldots, x_{i,n}]^T \in \mathbb{R}^n \) are state variable vectors, \( h_i(\bar{x}_i) : \mathbb{R}^n \mapsto \mathbb{R}^n \) are unknown \( C^1 \) nonlinear functions, \( \kappa_n \) and \( \kappa_{i,a} \) denote injection data in the actuator and the \( i \)th state measurement, respectively, and \( u \) is the corrupted state variables, and \( u \in \mathbb{R} \) is an event-triggered control input that is intermittently updated by a triggering law to be designed later. The injection data are represented by \( \kappa_{i,a}(t, x_i(t)) = \xi_i(t) \delta_i(x_i(t)) \) where \( \xi_i \) and \( \xi \) are unknown time-varying signals and \( \delta \) is a continuous nonlinear function.

**Assumption 1:** Instead of the exactly measured state variables \( x_i \), the corrupted state variables \( x_{i,a} \) are available only for the feedback control design.

**Assumption 2:** [23], [24] There exist unknown constants \( \zeta_i, \zeta_{i,d} \), and \( \bar{\zeta} \) such that \( |\zeta_i| \leq \bar{\zeta}, |\zeta_i| \leq \bar{\zeta}_{i,d} \), and \( |\bar{\zeta}| \leq \bar{\bar{\zeta}} \).

**Assumption 3:** [23] The time-varying signals \( \zeta_i \) satisfy \( \zeta_i + 1 \neq 0 \) and the sign of \( \zeta_i + 1 \) is assumed to be positive.

**Problem 1:** Consider system (1). Our problem is to design an adaptive resilient event-triggered control law \( u \) using corrupted state variables so that system (1) is stabilized in the presence of unknown strict-feedback nonlinearities and unknown injection data in full state measurements and an actuator while all other signals remain bounded.

**Remark 1** (The following statements are noted): (i) In contrast to the existing resilient control results [23], [24] for linear systems, uncertain nonlinear strict-feedback systems are considered in this paper. A recursive resilient event-triggered control design strategy using the corrupted state variables is established in the presence of unknown injection data in full state measurements and an actuator;

(ii) Assumption 3 is reasonable for ensuring the controllability of the system (1) with injection data in full state measurements [23]. This implies the existence of a nominal solution for Problem 1;

(iii) Contrary to the existing recursive designs [10]–[15] against output measurement faults, this paper considers full state variables corrupted by unknown injection data (i.e., \( x_{i,d} = x_i + \kappa_{i,a}(t, x_i) \) in (1)) and the adaptive event-triggered control problem using the corrupted state variables. Thus, Problem 1 is the first trial in the adaptive control branch of uncertain lower-triangular nonlinear systems.

**Remark 2:** System (1) in the strict-feedback form can represent many nonlinear practical applications such as aircraft wing rock models, jet engines, flight systems, biochemical processes, and flexible-joint robots [7]. The state variables measured for the network-based feedback control of these practical systems can be corrupted by sensor faults or cyber attacks. The resilient event-triggered control strategy proposed in this paper can then be applied to these practical control problems.

III. MAIN RESULTS

A. FUNCTION APPROXIMATION USING RADIAL BASIS FUNCTION NEURAL NETWORKS

In this paper, RBFNNs are employed to approximate unknown continuous nonlinear functions to be specified in the adaptive event-triggered control design procedure. Consider continuous real-valued nonlinear functions \( \Psi(y_i) : \mathbb{R}^{q_i} \mapsto \mathbb{R} \) where \( i = 1, \ldots, n \). RBFNNs can approximate \( \Psi(y_i) \) over a compact set \( \Pi_{\gamma_i} \subset \mathbb{R}^{q_i} \) as [57], [58]

\[
\Psi(y_i) = \theta_i^T G_i(y_i) + \epsilon_i(y_i)
\]

where \( i = 1, \ldots, n \), \( \gamma_i \) is the input vector, \( \epsilon_i \) is the approximation error, \( \theta_i \in \mathbb{R}^{l_i} \) is the optimal weighting vector defined as \( \theta_i = \arg \min_{\theta_i} \| y_i - \hat{G}_i(y_i) \| \); \( \hat{G}_i \) is an estimate of \( G_i(y_i) \), and \( G_i(y_i) = [g_{i,1}(y_i), \ldots, g_{i,l_i}(y_i)]^T \in \mathbb{R}^{l_i} \); Gaussian functions \( g_{i,j}(y_i), j = 1, \ldots, l_i \), are defined as

\[
g_{i,j}(y_i) = \exp \left[ \frac{-\| y_i - a_{i,j} \|^2}{r_{i,j}^2} \right]
\]

with the center of the receptive field \( a_{i,j} \in \mathbb{R}^{q_i} \) and the width of the Gaussian function \( r_{i,j} \in \mathbb{R} \).

**Assumption 4:** [59] \( \theta_i \) and \( \epsilon_i \) are bounded as \( \| \theta_i \| \leq \tilde{\theta}_i \) and \( \| \epsilon_i \| \leq \tilde{\epsilon}_i \), respectively, where \( i = 1, \ldots, n \), \( \theta_i > 0 \) and \( \tilde{\epsilon}_i > 0 \) are unknown constants.
Lemma 1: [59] It holds that $\|G_i(\gamma_i)\| \leq \tilde{G}_i$ where $i = 1, \ldots, n$ and $\tilde{G}_i$ is a constant.

B. RECURSIVE ADAPTIVE EVENT-TRIGGERED CONTROL DESIGN

In this section, a recursive design strategy using the corrupted state feedback information is presented for the adaptive event-triggered control of system (1).

Let us consider the following coordinate transformation on the basis of the dynamic surface design technique

$$
e_1 = x_1$$  \hspace{1cm} (4)  

$$
e_k = x_k - \hat{v}_k$$  \hspace{1cm} (5)  

$$
\rho_k = \tilde{v}_k - v_k$$  \hspace{1cm} (6)

where $k = 2, \ldots, n, e_1$ and $e_k$ are error surfaces using exactly measured state variables $x_1$ and $x_k$, $\rho_k$ and $\tilde{v}_k$ denote boundary layer errors and virtual control signals, respectively, and $v_k$ are signals provided by the first-order low-pass filters

$$
\tau_k \dot{v}_k + \tilde{v}_k = v_k, \quad \tilde{v}_k(0) = v_k(0)
$$  \hspace{1cm} (7)

with small time constants $\tau_k > 0$. Since the exactly measured state variables $x_i, i = 1, \ldots, n$, are corrupted by unknown injection data $\delta_i$, the error surfaces $e_i, i = 1, \ldots, n$, are not available for designing the virtual control laws $v_k$ and the actual control law $u$. Thus, we propose auxiliary signals using corrupted state variables $x_{i,a}, i = 1, \ldots, n$, as follows:

$$
\sigma_1 = x_{1,a}
$$  \hspace{1cm} (8)

$$
\sigma_k = x_{k,a} - \tilde{v}_k - \zeta_k \tilde{v}_k
$$  \hspace{1cm} (9)

where $k = 2, \ldots, n$, and $\zeta_k$ is the estimate of $\zeta_k$ which is provided by the injection data compensator to be designed later.

Remark 3: It is well known that the convergence of the error surfaces $e_i$ should be analyzed for stable control design based on the Lyapunov stability theorem in conventional dynamic surface control where $i = 1, \ldots, n$. However, the error surfaces $e_i$ are unknown for our control design because the exactly measured state variables $x_i$ are corrupted during the feedback procedure. Thus, we present a recursive event-triggered control design using the available auxiliary signals $\sigma_i$, instead of the unknown error surfaces $e_i$, to ensure the convergence of the error surfaces $e_i$ in the Lyapunov sense.

For the recursive design based on the Lyapunov stability analysis, (8) and (9) can be rewritten as

$$
\sigma_1 = \frac{e_1}{\phi_1}
$$  \hspace{1cm} (10)

$$
\sigma_k = e_k + \zeta_k x_k - \hat{\zeta}_k \tilde{v}_k + \zeta_k \tilde{v}_k - \tilde{\zeta}_k \tilde{v}_k
$$  \hspace{1cm} (11)

where $k = 2, \ldots, n, \hat{\zeta}_k = \zeta_k - \hat{\zeta}_k$, and $\phi_k = 1/(1 + \zeta_k)$. From Assumption 3, (10) and (11) are well defined. In addition, from Assumption 2, there exist unknown positive constants $\phi_1, \hat{\phi}_1$, and $\phi_{i,a}$ such that $\phi_i < |\phi_i| \leq \hat{\phi}_i$ and $|\phi_i| \leq \phi_{i,a}$ where $i = 1, \ldots, n$.

From now on, the approximation-based adaptive event-triggered stabilizer using the auxiliary signals $\sigma_i$ is recursively designed. The control design procedure is based on the Lyapunov stability theorem [7]. This theorem provides a design methodology to deal with the adaptive control design and stability analysis, simultaneously. In the first design step, a Lyapunov function $V_1$ consisting of the error surface $e_1$ and the weight estimation error $\hat{\theta}_1$ for RBFNN is chosen to analyze the convergence of the error surface $e_1$ and the boundedness of $\hat{\theta}_1$. In the $k$th design step, a Lyapunov function $V_k$ consisting of the error surface $e_k$, weight estimation error $\hat{\theta}_k$ for RBFNN, and compensation error $\hat{\zeta}_k$ for the injection data is selected to analyze the convergence of the error surface $e_k$ and the boundedness of $\hat{\theta}_k$ and $\hat{\zeta}_k$ where $k = 2, \ldots, n$.

**Step 1:** Differentiating the first error surface $e_1$ with respect to time yields

$$
\dot{e}_1 = x_2 + h_1(x_1).
$$  \hspace{1cm} (12)

Consider the Lyapunov function $V_1 = e_1^2/(2\phi_1) + \hat{\theta}_1^\top \hat{\theta}_1/(2\lambda_1)$ where $\theta_1 = \theta_1 - \hat{\theta}_1$. Its time derivative is

$$
\dot{V}_1 = -\frac{\phi_1}{2\phi_1} e_1^2 + 1

\frac{1}{\phi_1} e_1 (e_2 + \rho_2 + v_2 + h_1(x_1)) - \frac{1}{\lambda_1} \hat{\theta}_1^\top \hat{\theta}_1

\leq -\frac{1}{\phi_1} e_1 (e_2 + \rho_2 + v_2 + \phi_1) - \frac{1}{\lambda_1} \hat{\theta}_1^\top \hat{\theta}_1
$$  \hspace{1cm} (13)

where $\phi_1 = h_1(x_1) + (\hat{\phi}_{1,a}/(2\phi_1)) e_1$.

Then, there exists a continuous nonlinear function $\Psi_1(\gamma_1)$ such that

$$
\frac{1}{\phi_1} e_1 \phi_1 \Psi_1(\gamma_1) \leq \frac{1}{\phi_1} e_1 \Psi_1(\gamma_1)
$$  \hspace{1cm} (14)

where $\Psi_1(\gamma_1) = h_1(x_1) + (\hat{\phi}_{1,a}/(2\phi_1)) e_1$ and $\gamma_1 = e_1$. From (2), employing the RBFNN $\theta_1^\top G_1(\gamma_1)$ to estimate $\Psi_1(\gamma_1)$ yields

$$
\Psi_1(\gamma_1) = \theta_1^\top G_1(\gamma_1) + e_1
$$  \hspace{1cm} (15)

Based on the auxiliary signal $\sigma_1$, the first virtual control law is derived as

$$
v_2 = -(\beta_1 + m_1) \sigma_1 - \hat{\theta}_1^\top G_1(\gamma_1, a)
$$  \hspace{1cm} (16)

$$
\dot{\hat{\theta}}_1 = \lambda_1 \sigma_1 \sigma_1(\gamma_1, a) - \lambda_1 \sigma_1 \sigma_1(\gamma_1, a)
$$  \hspace{1cm} (17)

where $\beta_1 > 0, m_1 > 0, \lambda_1 > 0$, and $\sigma_1 > 0$ are design constants, and $\gamma_1, a = e_{1,a}$ with $e_{1,a} = x_{1,a}$.

Then, substituting (14) and (15) into (13) gives

$$
\dot{V}_1 \leq \frac{1}{\phi_1} e_1 (e_2 + \rho_2 + v_2 + \phi_1) - \frac{1}{\lambda_1} \hat{\theta}_1^\top \hat{\theta}_1
$$  \hspace{1cm} (18)

From (10), it holds that $e_1 = \phi_1 \sigma_1$. Using (16) and $e_1 = \phi_1 \sigma_1$, (18) becomes

$$
\dot{V}_1 \leq \frac{1}{\phi_1} e_1 (e_2 + \rho_2 + \sigma_1(\beta_1 + m_1) \sigma_1 + \hat{\theta}_1^\top G_1(\gamma_1, a))

+ \hat{\theta}_1^\top \phi_1 + e_1 - \frac{1}{\lambda_1} \hat{\theta}_1^\top \hat{\theta}_1
$$  \hspace{1cm} (19)
where \( \psi_1 = G_1(\gamma_1) - G_1(\gamma_{1,a}) \). Adding and subtracting the term \((\beta_1/\phi_1^2)e_1^2\) to (19), and using \( e_1 = \phi_1 \sigma_1 \) and (17) yield

\[
\dot{V}_1 \leq \frac{1}{\phi_1} e_1 (e_2 + \rho_2) + \sigma_1 \dot{\theta}_1^T \dot{\theta}_1 + \sigma_1 \dot{\theta}_1^T \dot{\theta}_1
\]

(20)

where \( \sigma_1 = \theta_1^T \psi_1 + \epsilon_1 \).

**Step** \( k = 2, \ldots, n - 1 \): The time derivative of the \( k \)th error surface (5) is given by

\[
\dot{e}_k = x_{k+1} + h_k(\xi_k) - \hat{v}_k
\]

(21)

where \( k = 2, \ldots, n - 1 \).

Consider the Lyapunov function \( V_k = e_k^2/(2\phi_k) + \bar{z}_k^2/2 + \hat{\theta}_k^T \hat{\theta}_k/(2\lambda_k) \) where \( k = 2, \ldots, n - 1, \xi_k = \zeta_k - \zeta_k, \) and \( \hat{\theta}_k = \theta_k - \hat{\theta}_k \). Its time derivative along (6) and (21) is represented by

\[
\dot{V}_k = -\frac{\phi_k}{2\phi_k^2} e_k^2 + \frac{1}{\phi_k} e_k (e_{k+1} + \rho_{k+1} + v_k + \hat{v}_k + h_k(\xi_k))
\]

\[
+ \bar{z}_k (\zeta_k - \hat{z}_k) - \frac{1}{\lambda_k} \bar{z}_k^T \bar{z}_k
\]

\[
\leq \frac{1}{\phi_k} e_k (e_{k+1} + \rho_{k+1} + v_k + \hat{v}_k + \psi_k) - \frac{1}{\phi_k} e_{k-1} - e_k
\]

\[
+ \frac{1}{\phi_k} e_k (\beta_k + \mu_k \hat{v}_k \zeta_k + \hat{z}_k (\zeta_k - \hat{z}_k) - \frac{1}{\lambda_k} \bar{z}_k^T \bar{z}_k)
\]

(22)

where \( \phi_k = h_k(x_k) + (\phi_{k,d}/(2\phi_k))e_k + (\phi_{k,v}/(\phi_k - 1))e_{k-1} - (\beta_k + \mu_k \hat{v}_k \zeta_k; \beta_k > 0 \) and \( \mu_k > 0 \) are constants.

From the boundedness of \( \zeta_k, \phi_k \) and \( \phi_{k-1} \), there exists a continuous function \( \Psi_k(\gamma_k) \) such that

\[
\frac{1}{\phi_k} e_k \Psi_k \leq \frac{1}{\phi_k} e_k \Psi_k(\gamma_k) + \alpha_k
\]

(23)

where \( \alpha_k > 0 \) is a constant and \( \gamma_k = \Xi_k^T e_k, e_{k-1}, e_{k,v}, \hat{v}_k, b_k \Xi_k^T \) with a constant \( b_k > 0 \). By employing the RBFNN \( \theta_k^T G_k(\gamma_k) \) to estimate \( \Psi_k(\gamma_k) \), we have

\[
\Psi_k(\gamma_k) = \theta_k^T G_k(\gamma_k) + \epsilon_k
\]

(24)

An adaptive injection data compensator is presented as

\[
\dot{\zeta}_k = \bar{\theta}_k^T G_k(\gamma_k,a) \hat{v}_k - (\beta_k - m_k - (\beta_k + \mu_k) \hat{v}_k^2) \sigma_k - (\beta_k + \mu_k) \hat{v}_k^2 \zeta_k - \alpha_k \zeta_k
\]

(25)

where \( k = 2, \ldots, n, \beta_k > 0, m_k > 0, \) and \( \alpha_k > 0 \) are design parameters, \( \gamma_k,a = [\Xi_k,a, e_{k-1,a}, e_{k,a}, \hat{v}_k, b_k \Xi_k,a]^T, \) \( x_{k,a} = [x_{k,a}, \ldots, x_{k,a}]^T, e_{k-1,a} = x_{k-1,a} - \hat{v}_{k-1} \) with \( \hat{v}_1 = 0.0 \) and \( \hat{v}_{k-1} = x_{k-1,a} - \hat{v}_{k-1} \).

The \( k \)th virtual control law, using the auxiliary signal \( \sigma_k \), is designed as

\[
v_{k+1} = -(\beta_k + m_k + (\beta_k + \mu_k) \sigma_k) \sigma_k
\]

\[
+ \bar{\theta}_k^T G_k(\gamma_k,a) + \frac{\sigma_k - \hat{v}_k}{\tau_k}
\]

(26)

\[
\dot{\theta}_k = \lambda_k \sigma_k \sigma_k G_k(\gamma_k,a) - \lambda_k \sigma_k \bar{\theta}_k
\]

(27)

where \( \lambda_k > 0 \) and \( \sigma_k > 0 \) are design constants.

Using \( e_k = \phi_k (\sigma_k - \hat{v}_k) \) in (11) and \( v_{k+1} \) in (26), (22) becomes

\[
\dot{V}_k \leq \frac{1}{\phi_k} e_k (e_{k+1} + \rho_{k+1}) - \frac{\bar{e}_k^2}{\phi_k} + \frac{e_k^2}{\phi_k}
\]

\[
+ (\sigma_k - \hat{v}_k)(-m_k \sigma_k + \hat{v}_k \sigma_k)
\]

\[
+ \hat{\theta}_k^T G_k(\gamma_k,a) + \theta_k^T \psi_k + \epsilon_k + \mu_k \hat{z}_k \hat{v}_k - \frac{1}{\lambda_k} \bar{z}_k^T \bar{z}_k
\]

\[
+ \hat{z}_k \hat{v}_k (\hat{z}_k - \hat{z}_k)
\]

\[
= \frac{1}{\phi_k} e_k (e_{k+1} + \rho_{k+1})
\]

\[
+ \sigma_k [-m_k \sigma_k + (\beta_k + \mu_k) \hat{v}_k - \bar{\theta}_k^T G_k(\gamma_k,a) + \epsilon_k]
\]

\[
- \hat{z}_k \hat{v}_k [(-m_k \hat{v}_k + \beta_k + \mu_k) \hat{v}_k - \bar{\theta}_k^T G_k(\gamma_k,a) + \epsilon_k]
\]

\[
- \frac{1}{\phi_k} e_{k-1} - e_k + \frac{1}{\phi_k} e_k (\beta_k + \mu_k) \hat{v}_k \zeta_k
\]

\[
- \hat{\theta}_k^T \hat{\theta}_k + \alpha_k
\]

(28)

where \( \psi_k = G_k(\gamma_k) - G_k(\gamma_{k,a}) \) and \( \zeta_k = \theta_k^T \psi_k + \epsilon_k \).

Then, from \( \hat{z}_k \hat{v}_k = \sigma_k - (e_k/\phi_k) \), we have

\[
(\beta_k + \mu_k) \hat{v}_k \zeta_k = (\beta_k + \mu_k) \hat{v}_k \zeta_k \left( \sigma_k - \frac{e_k}{\phi_k} \right)
\]

\[
\leq (\beta_k + \mu_k) \sigma_k^2 + \frac{\sigma_k^2}{4}
\]

\[
- \frac{1}{\phi_k} e_k (\beta_k + \mu_k) \hat{v}_k \zeta_k
\]

(29)

Substituting (25), (27), and (29) into (28) yields

\[
\dot{V}_k \leq \frac{1}{\phi_k} e_k (e_{k+1} + \rho_{k+1}) + \sigma_k [-m_k \sigma_k + \zeta_k]
\]

\[
- \hat{z}_k \hat{v}_k [\theta_k^T G_k(\gamma_k,a) + \epsilon_k]
\]

\[
- \frac{1}{\phi_k} e_{k-1} + \hat{z}_k \hat{v}_k + \hat{z}_k \hat{v}_k (\hat{z}_k + \alpha_k) + \sigma_k \hat{\theta}_k \hat{\theta}_k
\]

\[
+ \frac{\hat{z}_k^2}{4}
\]

(30)
Step n: Consider the Lyapunov function $V_n = e_n^T(2\phi_n) + \bar{\xi}_n^2/2 + \tilde{\theta}_n^T \tilde{\theta}_n/(2\lambda_n)$ with $\xi_n = \hat{\xi}_n - \xi_n$ and $\tilde{\theta}_n = \theta_n - \tilde{\theta}_n$. From $\dot{e}_n = u + \xi \phi(\xi_n) + h_{\bar{n}}(\xi_n) - \bar{v}_n$, the time derivative of $\dot{V}_n$ is given by

$$
\dot{V}_n \leq \frac{1}{\phi_n} e_n(u - \bar{v}_n + \varphi_n) - \frac{1}{\phi_n} e_{n-1}e_n + \frac{1}{\phi_n} e_n(\beta_n + \iota_n)\bar{v}_n\xi_n + \bar{\xi}_n(\xi_n - \bar{\xi}_n) - \frac{1}{\lambda_n} \tilde{\theta}_n^T \tilde{\theta}_n
$$

where $\varphi_n = h_{\bar{n}}(\xi_n) + \xi \phi(\xi_n) + (\phi_{n,d}/(2\phi_n))e_n + (\phi_{n,b}/\phi_n)e_{n-1} - (\beta_n + \iota_n)\bar{v}_n\xi_n$; $\beta_n > 0$ and $\iota_n > 0$ are constants.

Due to the boundedness of $\varphi_n, \varphi_{n-1}, \xi_n$, and $\xi$, there exists a continuous function $\Psi_n(\gamma_n)$ such that

$$
\frac{1}{\phi_n} e_n \varphi_n \leq \frac{1}{\phi_n} e_n \Psi_n(\gamma_n) + a_n
$$

where $a_n > 0$ is a constant and $\gamma_n = [\bar{\xi}_n^T, e_n, e_{n-1}, \bar{v}_n, b_n]^T$ with a constant $b_n > 0$. By employing the RBFNN $\tilde{\theta}_n^T G_n(\gamma_n)$ to estimate $\Psi_n(\gamma_n)$, we have

$$
\Psi_n(\gamma_n) = \tilde{\theta}_n^T G_n(\gamma_n) + e_n.
$$

The approximation-based adaptive event-triggered control law is presented as

$$
u(t) = \bar{u}(t), \quad t \in [t_1, t_1+1)
$$

(34)

$$
t_{l+1} = \text{inf}(t > t_l | S_n(t) \geq m_{u,1}|\sigma_n(t) + m_{u,2})
$$

(35)

where for $l \in \mathbb{Z}^+$, $t_l$ denotes the update time of the controller, $S_n = \bar{u}(t) - u(t)$ is the measurement error due to the triggering, $\sigma_n$ is defined as $\sigma_n = x_{n,a} - \bar{v}_n - \tilde{\xi}_n \bar{v}_n$ in (9), and $m_{u,1}$ and $m_{u,2}$ are positive design constants. (34) and (35) imply that the control law $u$ is set to $\bar{u}(t_l)$ for $t \in [t_l, t_{l+1})$, and when the triggering law (35) is satisfied, its value is updated at $t_{l+1}$. Here, the signal $\bar{u}$ with the auxiliary signal $\sigma_n$ and the injection data compensator $\tilde{\xi}_n$ is designed as follows:

$$
\bar{u} = -(\beta_n + m_n + (\beta_n + \iota_n)^2 \tilde{\xi}_n^2)\sigma_n - \tilde{\theta}_n^T G_n(\gamma_{n,a}) + \frac{\bar{v}_n - \bar{v}_n}{\iota_n}
$$

(36)

$$
\dot{\tilde{\xi}}_n = \tilde{\theta}_n^T G_n(\gamma_{n,a}) \bar{v}_n - (\beta_n - m_n) - (\beta_n + \iota_n)^2 \tilde{\xi}_n^2 \bar{v}_n\sigma_n - (\beta_n + \iota_n)\tilde{\xi}_n^2 \bar{v}_n\sigma_n
$$

(37)

$$
\dot{\sigma}_n = x_{n,a} - \bar{v}_n - \tilde{\xi}_n \bar{v}_n, \quad \gamma_{n,a} = [\bar{\xi}_{n,a}^T, e_{n-1,a}, e_{n,a}, \bar{v}_n, b_n]^T, \quad \bar{\xi}_{n,a} = [\bar{\xi}_{n,a}^T, x_{n,a} - \bar{v}_n, m_n] > 0 \text{ and } a_n > 0 \text{ are design parameters,}\quad \lambda_n > 0 \text{ and } \sigma_n > 0 \text{ are design constants.}
$$

Using $e_n = \phi_n(\sigma_n - \tilde{\xi}_n \bar{v}_n)$ in (11), $S_n = \bar{u}(t) - u(t)$, and (36), (31) becomes

$$
\dot{V}_n \leq \sigma_n(-\beta_n + m_n + (\beta_n + \iota_n)^2 \tilde{\xi}_n^2)\sigma_n + \tilde{\theta}_n^T G_n(\gamma_{n,a}) + \varphi_n
$$

$$
- \tilde{\xi}_n \bar{v}_n(-\beta_n + m_n + (\beta_n + \iota_n)^2 \tilde{\xi}_n^2)\sigma_n + \tilde{\theta}_n^T G_n(\gamma_{n,a})
$$

$$
+ \tilde{\xi}_n(\sigma_n - \tilde{\xi}_n) - \frac{\tilde{\xi}_n^2}{\phi_n} + \beta_n(\sigma_n^2 - 2\tilde{\xi}_n \bar{v}_n \sigma_n) + (\beta_n + \iota_n)^2 \tilde{\xi}_n^2 \bar{v}_n^2
$$

(38)

Owing to $S_n(t_l) = 0$ for $l \in \mathbb{Z}^+$ and $\sigma_n(t) = \tilde{\xi}_n \bar{v}_n + (\nu_n/\phi_n)$, we have

$$
- \frac{1}{\phi_n} e_n \varphi_n \leq \frac{1}{\phi_n} |e_n| m_{u,1}|\sigma_n(t)| + m_{u,2}
$$

$$
\leq \frac{m_{u,1}}{\phi_n} e_n^2 + \frac{m_{u,1}}{\phi_n} |e_n| \tilde{\xi}_n \bar{v}_n |m_{u,2}| e_n
$$

$$
\leq 2m_{u,1} \frac{e_n^2}{\phi_n^2} + \frac{m_{u,1}^2}{\phi_n^2} \tilde{\xi}_n^2 \bar{v}_n^2 + \frac{\lambda_n}{\phi_n^2} e_n^2 + \frac{m_{u,2}^2}{4\lambda_n}
$$

(40)

where $\lambda_n > 0$ is a constant.

Using (40) and $(\beta_n + \iota_n)\tilde{\xi}_n^2 \bar{v}_n^2 = (\beta_n + \iota_n) \tilde{\xi}_n^2 \bar{v}_n (\tilde{\xi}_n - \tilde{\xi}_n)$, we obtain

$$
\dot{V}_n \leq \sigma_n(-m_n \sigma_n - (\beta_n + \iota_n)^2 \tilde{\xi}_n^2\sigma_n + \tilde{\theta}_n^T G_n(\gamma_{n,a}) + \varphi_n)
$$

$$
- \tilde{\xi}_n \bar{v}_n[(\beta_n + \iota_n)^2 \tilde{\xi}_n^2 \sigma_n + (\beta_n + \iota_n)^2 \tilde{\xi}_n^2 \bar{v}_n^2 \sigma_n]
$$

$$
+ (\beta_n + \iota_n)\tilde{\xi}_n \bar{v}_n \sigma_n + \tilde{\theta}_n^T G_n(\gamma_{n,a}) + \varphi_n
$$

$$
- (\beta_n - 2m_{u,1}) \frac{e_n^2}{\phi_n^2} + (\beta_n + \iota_n) \tilde{\xi}_n^2 \bar{v}_n^2
$$

$$
- \left(\frac{\iota_n - m_{u,1}}{4}\right) \tilde{\xi}_n^2 \bar{v}_n^2 - \frac{1}{\phi_n} e_n \varphi_n
$$

(41)

Substituting (38), (37), and (42) into (41) yields

$$
\dot{V}_n \leq \sigma_n(-m_n \sigma_n + \varphi_n) - \tilde{\xi}_n \bar{v}_n[\tilde{\theta}_n^T G_n(\gamma_{n,a}) + \varphi_n]
$$

$$
- (\beta_n - 2m_{u,1}) \frac{e_n^2}{\phi_n^2} - \left(\frac{\iota_n - m_{u,1}}{4}\right) \tilde{\xi}_n^2 \bar{v}_n^2
$$

$$
- \frac{1}{\phi_n} e_n \varphi_n
$$

(43)

Remark 4: Compared with the existing event-triggered control results [53]–[55], this paper considers the event-triggered control problem against unknown injection data in full state measurements and an actuator. Moreover, the adaptive controller (34) and its event-triggering law (35) are designed using the auxiliary signal $\sigma_n$ with the corrupted state variable $x_{n,a}$ and the injection data compensation $\tilde{\xi}_n$. 

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Although the corrupted state variables are used in the proposed adaptive event-triggered control scheme, the convergence of the error surfaces \( e_i \) using the exactly measured state variables \( x_i \), the boundedness of all closed-loop signals, and the exclusion of Zeno behavior are successfully analyzed in the following section.

### C. STABILITY ANALYSIS

For the stability analysis, the dynamics of the boundary layer \( \tilde{e}_k \) are considered as

\[
\tilde{e}_{k+1} = \frac{-\rho_{k+1}}{\tau_{k+1}} + \eta_{k+1} \tilde{e}_{k+1} + \rho_2 \ldots \rho_{k+1} \tilde{\theta}_{k}, \tilde{\xi}_k, \Delta_k \)
\]

where \( k = 1, \ldots, n - 1 \), \( \tilde{e}_{k+1} = [e_1, \ldots, e_{k+1}]^T \), \( \tilde{\theta}_{k} = [\tilde{\theta}_1, \ldots, \tilde{\theta}_k]^T \), \( \tilde{\xi}_k = [\tilde{\xi}_1, \ldots, \tilde{\xi}_k]^T \) with \( \tilde{\xi}_1 = 0 \), \( \Delta_k = [\xi_1, \ldots, \xi_k, \tilde{\xi}_k, \ldots, \tilde{\xi}_k]^T \), and \( \eta_2 = (\beta_1 + m_1)\tilde{\theta}_{1} + \tilde{\theta}_{1}^T G_1 + \tilde{\theta}_{1}^T (\partial G_1 / \partial \gamma_1) \tilde{\gamma}_1, a + \eta_1 = (\beta_{k+1} + m_{k+1}) + (\beta_{k+1} + \eta_{k+1})^2 \tilde{\theta}_{k+1} - 2(\beta_{k+1} + \eta_{k+1})^2 \tilde{\theta}_{k+1} \).

A total Lyapunov function \( V \) is selected as \( V = \sum_{i=1}^{n} \frac{1}{2} \tau_{i+1}^2 \).

**Theorem 1:** Consider uncertain nonlinear strict-feedback systems (1) with unknown injection data in full state measurements and an actuator. The proposed adaptive event-triggered controller (34)-(38) with an event-triggering condition (35) achieves that for any initial conditions satisfying \( V(0) \leq \mu \) with a constant \( \mu > 0 \),

(i) all signals of the closed-loop system are semi-globally uniformly ultimately bounded;

(ii) the error surfaces \( e_i, i = 1, \ldots, n \), converge to an adjustable neighborhood of the origin;

(iii) the minimum inter-event time \( \tau_k \) > 0 satisfying \( |t_{i+1} - t_i| \geq \tau_l \) for \( l \in \mathbb{Z}^+ \) exists.

**Proof:** The time derivative of \( V \) using (20), (30), and (43) is

\[
\dot{V} \leq \sum_{i=1}^{n} \left( -\frac{\beta_i}{\phi_i} \frac{e_i^2}{2} - m_i \sigma_i^2 + \sigma_i \tilde{\xi}_i + \sigma_i \tilde{\theta}_i \right) + \sum_{i=2}^{n} \left( -\zeta_i \tilde{\xi}_i \chi_i - \zeta_i \tilde{\theta}_i \right) + \sum_{i=1}^{n-1} \left( -\frac{\rho_{i+1}}{\tau_{i+1}} + \tilde{\eta}_{i+1} \rho_{i+1} \right) + \sum_{i=2}^{n} \left( \frac{m_{i+1} \beta_i}{4 \mu^2} + \sum_{i=2}^{n} \frac{m_{i+1} \tilde{\theta}_i}{4 \mu^2} \right)
\]

where \( \tilde{\eta}_{i+1} = \eta_{i+1} + (1/\phi_i) e_i \) and \( \chi_i = \theta_i^T G_i (\gamma_{ia}) + \xi_i \).

Consider the sets \( A_1 := \{ \sum_{j=1}^{i} (e_j^2 / \phi_j + \theta_j^T \tilde{\theta}_j / \phi_j) \} \) and \( B_1 := \{ \sum_{j=1}^{i} (\tilde{\xi}_j^2 + \tilde{\theta}_j^2) \} \) where \( i = 1, \ldots, n \) and \( d_i = \sum_{j=1}^{i} (\tilde{\xi}_j^2 + \tilde{\theta}_j^2) \) are constants and \( \dim(A_1) \leq \dim(A_2) \). Since \( A_1 \times B_1 \) is compact in \( \mathbb{R}^{\dim(A_1)} \) and \( \mathbb{R}^{\dim(B_1)} \), respectively, \( A_1 \times B_1 \) is also compact in \( \mathbb{R}^{\dim(A_1) + \dim(B_1)} \). \( \tilde{\eta}_{i+1} \geq \eta_{i+1} \) on \( A_1 \times B_1 \) where \( q_{i+1} \geq 0 \) are constants and \( \dim(A_1) \) denotes the dimension of the set \( A_1 \).

From Assumption 4 and Lemma 1, there exist constants \( \xi_i > 0 \) and \( \tilde{\xi}_i > 0 \) such that \( |\xi_i| \leq \xi_i \) and \( |\tilde{\xi}_i| \leq \tilde{\xi}_i \).

Using \( \sigma_i \tilde{\xi}_i = m_i \sigma_i^2 + \tilde{\xi}_i^2 / (4m_i) \), \( -\tilde{\xi}_i \chi_i \tilde{\theta}_i \leq \tilde{\xi}_i^2 / (4m_i) \) with a constant \( \tilde{\xi}_i > 0 \), \( \tilde{\xi}_i \chi_i \tilde{\theta}_i \leq -(1/2) \tilde{\theta}_i^T \tilde{\theta}_i + (1/2) \tilde{\theta}_i \theta_i \).

Remark 5: From the proof of Theorem 1, the stabilization performance can be improved by reducing the compact set \( \Omega \).

From this standpoint, the design parameters can be selected. The choice of the design parameters is only a sufficient condition for Theorem 1. The guidelines for the choice of the design parameters are as follows.
(i) Increasing the design parameters $\beta_i$, $i = 1, \ldots, n$ helps to increasing of the convergence rate of the error surfaces $e_i$ (i.e., increasing $c_1$), and thus $\Omega$ can be reduced.

(ii) As the design parameters $m_i$, $i = 1, \ldots, n$ and $t_j$, $j = 2, \ldots, n$ increase, $c_0$ is reduced. Therefore, $\Omega$ can be reduced.

(iii) As the design parameters $\lambda_i$ denoting the tuning rate increase and the design parameters $\sigma_i$ for the $\sigma$-modification are fixed as small values, the tuning speed of the adaptive parameters $\hat{h}_i$ can be increased where $i = 1, \ldots, n$.

(iv) According to the limited resources of the network, the number of released data during the transient and steady-state responses can be adjusted by choosing the design parameters $m_{u,1}$ and $m_{u,2}$ in the event-triggering law (35).

Remark 6: The control gain functions and the event-triggered state variables are not considered in the strict-feedback nonlinear systems (1). The problems caused by the control gain functions reported in [60] and the event-triggered state variables reported in [61] should be reformulated under the state feedback information corrupted by unknown injection data. Thus, the proposed approach cannot be applied in a straightforward manner to these problems. Future work can explore the extension of the proposed approach to complex systems in the strict-feedback form.

IV. SIMULATION RESULTS

Two simulation studies including a wing rock model with ailerons are presented to illustrate the validity of the proposed theoretical result against unknown injection data in full states and an actuator. In these simulations, the triggering law (35) is checked periodically with the sampling time 1 ms.

Example 1: Consider the following second-order nonlinear system with unknown injection data in full state measurements and an actuator:

$$\begin{align*}
\dot{x}_1 &= x_2 + h_1(x_1) \\
\dot{x}_2 &= u + \kappa_\alpha(t, \hat{x}_2) + h_2(x_2) \\
x_{i,a} &= x_i + \kappa_{i,s}(t, x_i)
\end{align*}$$  (48)

where $i = 1, 2$, $h_1 = x_1^2 \sin(x_1) + x_2$, and $h_2 = x_1 x_2 + x_3 \cos(x_1)$. Then, the injection data are given by $\kappa_{1,s} = (0.5 + \cos(1.2t))x_1$, $\kappa_{2,s} = (0.5 + \sin(1.8t))x_2$, and $\kappa_{\alpha} = x_1 x_2 \cos(t)$. It is assumed that the inherent nonlinearities $h_1$ and $h_2$, and the injection data $\kappa_{1,s}$, $\kappa_{2,s}$, and $\kappa_{\alpha}$ are unknown. The initial values are set to $x_1(0) = 1.5$ and $x_2(0) = -1$ where $i = 1, 2$.

The design parameters are selected as $\beta_1 = \beta_2 = 1$, $m_1 = 1$, $m_2 = 4$, $t_1 = 0.1$, $t_2 = 0.001$, $\lambda_1 = \lambda_2 = 0.02$, $\sigma_1 = \sigma_2 = 0.001$, $\sigma_2 = 0.5$, $m_{u,1} = 0.4$, and $m_{u,2} = 0.04$.

Fig. 1 shows the stabilization result and the event-triggered control input. Fig. 2 displays the outputs of RBFNNs and the injection data compensator. The inter-execution times and the cumulative number of events are displayed in Fig. 3. The total triggering number is 163 and thus only 1.087% of the total 15000 sampled data are used to implement the proposed adaptive controller. Thus, the control efforts can be reduced using the event-triggering laws. Although the corrupted state variables are only used in the controller and its event-triggering law, the proposed recursive event-triggered design against unknown injection data in full state measurements and an actuator achieves the stabilization of uncertain nonlinear strict-feedback systems.

Example 2: The stabilization problem of the wing rock model with ailerons in the presence of unknown injection data.
in full state measurements and an actuator is considered in this example. The model dynamics is described by

\[
\begin{align*}
\dot{x}_1 &= x_2 + h_1(x_1) \\
\dot{x}_2 &= x_3 + h_2(\bar{x}_2) \\
\dot{x}_3 &= u + \kappa_a(t, x_3) + h_3(\bar{x}_3) \\
\end{align*}
\]

where \( i = 1, 2, 3 \), \( x_1 \) is the roll angle, \( x_2 \) is the roll rate, \( x_3 \) is the aileron deflection angle, \( h_1 = 0 \), \( h_2(\bar{x}_2) \) is a regressor vector, \( \kappa_a(t, \bar{x}_3) \) is the input injection signal, and \( h_3(\bar{x}_3) \) is a function of the injection signal.
$h_2 = p_1x_1 + p_2x_2 + p_3|x_1|x_2 + p_4|x_2|x_2 + psx_1^2$, and $h_3 = -x_3$. Here, the model parameters of the delta wing for a 25$^\circ$ angle of attack are set to $p_1 = -0.01859521$, $p_2 = 0.015162375$, $p_3 = -0.06245153$, $p_4 = 0.00954708$, and $ps = 0.02145291$. The unknown injection data $k_{1,i} = (1 + \cos(t))x_1$, $k_{2,i} = (1 + \sin(t))x_2$, $k_{3,i} = (1 + \sin(t))x_3$, and $k_\alpha = (x_1^2 + x_3^2)\cos(t)$ influence the system (49). It is assumed that $h_2$, $h_3$, $k_{1,i}$, $k_{2,i}$, $i = 1, 2, 3$, and $k_\alpha$ are unknown for the adaptive event-triggered control design. In the uncontrolled system (49) with $u = 0$, the roll angle $x_1$ is divergent for the large initial condition [62]. Thus, the large $x_3$ is divergent for the large initial condition [62]. Therefore, only 1.62% of the 15000 sampled data are used to achieve good stabilization performance against unknown injection data in full state measurements and an actuator, and unknown system nonlinearities.

V. CONCLUSION

An adaptive event-triggered control approach has been proposed for uncertain nonlinear strict-feedback systems with unknown injection data in full state measurements and an actuator. The approximation-based adaptive controller and its event-triggering condition have been constructed using state variables corrupted by unknown injection data in full state measurements. To this end, auxiliary signals using corrupted state variables and dynamic injection data compensators using neural networks have been designed to compensate for unknown injection data effects. It has been shown that the proposed event-triggered control scheme using corrupted state variables ensures the convergence of the error surfaces using exactly measured state variables. Further extension to the decentralized resilient event-triggered control problem of interconnected nonlinear systems with unknown injection data is conceivable. This remains as a meaningful subject for future research.

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