Geometric and static analysis of the historical truss of the medieval truss Church of St. Martin in Dolní Újezd

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Abstract. The late-Romanesque St. Martin’s Church in Dolní Újezd was probably built in the mid-13th century. The nave has a wooden joist ceiling whose beams serve as tie beams too. The Gothic vaults above the presbytery date from 30s of the 15th century. The truss of the church is dendro-dated to 1420 - 1421. Some members of a truss structure are dendro-dated to 1823 - 1824, when the repair was undertaken. The examined truss is a rafter-collar tie structure with collar beams and pedestal struts. It has a characteristic shape and may be typologically classified as a structure of the earlier period. This type of a truss structure was typically built in this region and represents one of the features occurring in the particular social and culture circles at the time. Consequently, a numerical model of the roof structure was created in order to perform a static analysis of the roof structure in accordance with present standards. With regard to the structural analysis results, it can be said that the original design of the structure based on geometrical and proportional principles satisfy the reliability conditions defined by current European standards for structural design.

1. Introduction
Analysis of proportional conception is based on historical-structural, archival, and dendrochronological research of the object as a whole. The method of geometrical and proportional analysis of the concept of the roof is based on acquiring information from period literature and from previous roof analyses. The proportions of historic trusses were analyzed mainly at roof constructions from Slovakia and Czech republic. The object of research is analyzing the proportion of historical constructions and relation into the floor plan of building and layout of the building solution. The spatial arrangement of the historical building and structures is also built on the concept of proportional relationships. The roof height is proportionally and geometrically derived from the ground plan of the building. These principles were also identified in other historical trusses in Slovakia and Bohemia, which we analyzed (e.g. trusses in churches in Belá Dulice, Abramová, Okolicna, Klimkovice, Vlčovice, the church of St. Catherine in Banská Štiavnica and others [3, 4, 6]).

2. History of St. Martin’s Church in Dolní Újezd
The church (figure 1) was probably built in the mid-13th century. It has a late-Romanesque core - only the tower and west nave survived to this day. At the end of the 14th century was built a new presbytery on the eastern part in high-level crafts realization. The old presbytery was demolished and the new one was interconnected with a late-Romanesque nave, which can be seen in the irregular floorplan. At the time of the Hussite Wars, like many other churches in this region, the church was destroyed by the armies. Soon after was built today’s truss dated from 1420 - 1421d. Later, probably in the 30s of the 15th century, the presbytery was newly vaulted with a brick vault and stone ribs.
Further modifications were made after 1533 - an addition of a northern antechapel, knocking out of windows, southern entrance and several small modifications that did not affect the overall appearance of the church, [1, 2] (figure 1).

![Figure 1. The researched object – The St. Martin’s Church in Dolní Újezd.](image1)

3. Truss construction

The church has a gable roof with a single ridge above the nave and sanctuary with a polygonal roof-end above the polygonal end of the sanctuary. Above the sanctuary, there is a baroque polygonal Sanctus bell turret. The truss above the nave has a rafter-collar construction with two levels of collar beams and heel struts. The truss is longitudinally tied by a central frame that is divided into two parts by a Sanctus bell turret. The truss contains three types of ties. The first type consists of rafters, two levels of collar beams, and a central post. The central post of the first collar beam, into which it is halved, is jointed with the second collar beam using the straight halving; the top of a post is also halved into the rafters. The post is cross-braced by the heel struts into the first level of the collar beams. The heel struts are halved into the tie beam and rafters. The collar beams are dovetailed into rafters. The second type, unlike the first one, does not contain a central post with heel struts. The cross-bracing is made by a saltire cross dovetailed into the tie beam and rafters below the level of the second collar beam. The third type is similar to the second one, except to saltire crosses at the level of the first collar beam which is replaced by heel struts dovetailed into the collar beam and rafters. The truss is longitudinally braced using span pieces located on the upper and lower collar beams, with saltire crosses halved and dovetailed into the posts and span pieces (figures 2 and 3).

![Figure 2. The inner view of the truss.](image2)
4. Geometric concepts and proportional relationships within the truss

In the geometric analysis of historical trusses on sacral buildings, we encounter various ratios such as arithmetic, geometric, harmonic and with the Golden Ratio. When determining the length and width of the church, we can also find musical ratios.

We have found that a principle is very often used to determine the basic dimensions of the truss, which we denoted it as „n plus k“, where \( k = 1, 2, 3 \). By this method, the height of the truss is derived from its width so that the width of the truss is divided into \( 2n \) of units \( x \) and the truss height is then \( (n + k) \) of units \( x \). In the St Martin’s Church in the village of Dolní Újezd was used this method for \( k = 3 \). The geometric analysis of the truss shows in the St Martin’s Church that the cross-sections of the truss above the nave are derived from the base square ABCD (figure 4). The length of the side \( AB \) of the square is \( nx \), where in this case \( x \) is unit of measure and \( n = 4 \) („\( n \) plus 3“). Using the principle described before, the height of the truss is \( (n + 3) x = 7x \). It is easy to realize that the height of the truss is 7/4 of the side AB of square ABCD, and the ratio of the truss width to its height is 8:7. The truss above the nave contains three different but quite similar in type of ties A,B and C. In the ties (figure 3.), the position of a lower collar beam is defined as 5/8 of the length of the side BC of the basic square ABCD. The upper collar beam is placed at the height 9/8 of the length of the side BC of the base square ABCD in all ties (figure 4). Hence, the ratio of the distance of a tie beam from a lower collar beam to the distance of both collar beams is 5:4. The all ties have the same positioned of a heel strut on the tie beam and on a rafter. The heel strut is attached to the tie beam at a distance \( x \) from the rafter heel (\( x \) represents 1/8 of the truss width) and is attached to the rafter at the height 3/8 of the length of the side BC of the square ABCD (figure 4).

Figure 3. Longitudal truss section.

Figure 4. A cross-section.  
Figure 5. B cross-section.  
Figure 6. C cross-section.
Now we are dealing with the individual cross-sectional relationships of the truss above the ship and their differences. The cross-sectional A has a post that connects only the both collar beams and does not proceed to the tie beam. The heel struts of a post are horizontally positioned on a lower collar beam at a distance 1/6 of a length of the side AB. The heel struts of the post are positioned vertically on the post and the distance from the anchorage to the tie beam is 11/12 of the length of the side BC of the square ABCD (figure 4).

In the cross-sectional B was using a saltire cross. From the rafter heel, the strut is placed on the tie beam at a distance 5/8 of the length of the side AB. The strut mount on the rafter is where the side CD of the square ABCD intersect to the rafter (figure 5). The cross-sectional C is a modified the cross-sectional B so that the struts are shortened and are only down to the lower collar beam (figure 6).

5. Static analysis of the cross trusses

In order to verify the static behaviour of the truss, a computational model of the load-bearing structure was created in the FEM software SCIA Engineer [9]. The geometry of numerical model is related to the roof structure’s geometrical analysis presented hereinbefore. Each of the cross trusses were modelled separately by means of planar models using beam elements. The softwood of grade C24 according to [10] was applied in the models, which corresponds to the applied type of wood. Geometrical schemes of the numerical models are presented in figure 7.

All the member connections are modelled as hinge joints with axial rigid connection and with capability of initial slip of 1 mm in the axial direction of member in order to consider an influence of gaps, cracks and geometry imperfections, occurring in historical carpentry joints. The effect of local weakening of the cross sections by carpentry joints is neglected in the models.

The individual load-bearing structures were loaded according to European standards [11, 12, 13] by permanent loads (self-weight and weight of roofing) and variable loads (wind actions only). With regard to the roof pitch angle (about 60°), the snow load was not applied on the roof. The self-weight load was generated by the FEM software. The combinations of load cases were generated according to the standard STN EN 1990 [11].

The results of numerical analysis are presented by the values of maximum (tension) and minimum (compression) normal stresses as well as displacements in the major structural members of the trusses, calculated for the decisive load combinations. Envelopes of the maximum and minimum normal stresses, respectively, in the major cross truss members are presented in figure 8. Envelopes of the vertical deflections caused by characteristic load combinations are presented in figure 9.
Based on the results of numerical analysis it can be stated out that the normal stresses in all truss members do not exceed the design values of bending and compression strength equal to 16.60 MPa and 14.53 MPa, respectively, determined according to STN EN 1995-1-1 [14], assuming the timber strength class C24, modification factor \( k_{mod} = 0.9 \) and partial safety factor \( \gamma_M = 1.3 \). The possible buckling effects can be neglected, regarding the low share of compression forces due to normal forces (caused mainly by permanent loads). Similarly, the local vertical deformations of the major cross truss members do not exceed the standard [14] limit values (for example, in the case of rafters, it is equal to \( L/200 = 8220/200 = 41 \) mm).

6. Conclusion
We discovered the proportional principle used in the truss of The St. Martin’s Church in Dolní Újezd. A truss with a similar proportional system and basic measure used is located in the village of the St. Catherine church in Banská Štiavnica [15], but its age (1655d) and typology are different. Therefore, the ratio of a half-width and a height of the truss is 4:7 [8]

The research results point to a rational approach to the design of roof trusses using geometric and proportional principles. Unlike now, an approach to the planning of the structure service life was significantly different, looked through a number of other generations. The results showed that the roof structure safely satisfy (with reserve about 58%) the reliability conditions defined by current European standards for structural design, both in terms of the ultimate limit states and serviceability limit states. The reserve extends the service life in relation to the potential long-term damage to the life of roof truss constructions and enables their rehabilitation.
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References
[1] Líbal D 2001 *Katalog gotické architektury v České republice do husitských válek* (Praha: Unicornis)
[2] Václavík F 2000 Nové poznatky ze stavební historie kostela sv. Martina v Dolním Újezdě okr. Svitavy *Výroční zpráva za rok 2000 V Pardubicích: Památkový ústav v Pardubicích* pp 201-203
[3] Krušinský P, Capková E and Gocál J 2015 Comparison of two medieval trusses from the viewpoint of geometric and statistic *Advanced Materials Research* **1122** p 243-248
[4] Krušinský P, Capková E, Gocál J and Holešová M 2015 Geometric and static analysis of the historical trusses in roman catholic church of the holy Kozma and Damian in the Abramová village *Civil and environmental engineering: Scientific technical journal* **11** (2) pp 136-41
[5] Krušinský P, Capková E, Augustinková L and Korenková R 2016 Proportions - disposition relationship analysis of a historical truss in a rural house in Vápenná Village Czech Republic *Civil and Environmental Engineering* **12** (2) pp 111–5 (ISSN 2199-6512, DOI: https://doi.org/10.1515/cee-2016-0015)
[6] Gocál J, Krušinský P, Capková E and Kekeliak M 2014 Geometric and static analysis of the historical truss in village Belá Dulice In: *Advanced materials research* **969** p 199-207
[7] Struhár A 1977 *Geometric harmony of historical architecture in Slovakia* (Bratislava: Pallas)
[8] Makýš O 2014 *Včasnohistorické a ranostredoveké drevené a drevozemné pamiatky, hypotetické rekonštrukcie a konzervácia* (Renesans) p 189
[9] SciaEngineer 2010 Software for structural analysis. http://nemetschek-scia.com/sk
[10] Standard STN EN 338 2004 Structural timber. Strength classes (Bratislava: SÚTN)
[11] Standard STN EN 1990 2009 Eurocode. Basis of structural design (Bratislava: SÚTN)
[12] Standard STN EN 1991-1-1 Eurocode 1. Actions on structures. Part 1-1: General actions. Densities, self-weight, imposed loads for buildings 2007 (Bratislava: SÚTN)
[13] Standard STN EN 1991-1-4 Eurocode 1. Actions on structures. Part 1-4: General actions - Wind actions 2007 (Bratislava: SÚTN)
[14] Standard STN EN 1995-1-1 + A1 Eurocode 5. Design of timber structures. Part 1-1: General. Common rules and rules for buildings 2008 (Bratislava: SÚTN)
[15] Krušinský P, Holešová M and Ŏurian K 2018 Proportional analysis of a transversal bond of the historic truss in the Gothic Roman-catholic Church of St Catherine in Banská Štiavnica dated to the mid-17th century *SGEM 2018 conference proceedings* **5 Ancience Science**, Issue 2.3. Sofia: STEF92 Technology Ltd. p 179-84