The Noncommutative U(N) Kalb-Ramond Theory

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We present the noncommutative extension of the U(N) Cremmer-Scherk-Kalb-Ramond theory, displaying its differential form and gauge structures. The Seiberg-Witten map of the model is also constructed up to 0(θ²).

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1. INTRODUCTION

The first ideas about space-time noncommutativity were formulated by Heisenberg in the thirties [1], although the first published work on the subject appeared in 1947 [2], introducing a possible framework for avoiding the characteristic singularities of quantum field theories. Although the original motivations have been eclipsed by the renormalization program, recently the interest on noncommutative theories has grown up in large scale, mainly associating the space-time noncommutativity with results coming from string theory. In this approach noncommutativity can be found in the study of perturbative strings in the presence of the branes in a constant background magnetic field [3, 4, 5]. For a review on the subject, where several interesting features of noncommutative field theories can be commented, see for instance [6] and references therein. One of the important ingredients of noncommutative gauge field theories is the Seiberg-Witten map connecting field variables which transform under a noncommutative gauge structure with ordinary field variables transforming under an ordinary gauge structure [7, 8, 9, 10]. This map seems to be essential for the construction of a phenomenological viable noncommutative description of Nature [11, 12].

Interesting models of gauge theories that have not had their noncommutative extensions very explored are those constructed with the aid of antisymmetric tensor fields. The antisymmetric Kalb-Ramond tensor field has been first introduced within a string theory [13, 14, 15], but the so called Cremmer-Scherk-Kalb-Ramond (CSKR) model appears in a great variety of scenarios, including supersymmetric theories [16, 17, 18], cosmology [19] and cosmic strings [20]. Other interesting points are related to the rank of the CSKR model gauge structure and its consequences under quantization, including the possibility of mass for gauge theories without spoiling gauge invariance [21, 22, 23].

In the present work, we propose a noncommutative generalization of the CSKR theory. We show that its covariant description, with the aid of differential forms, can be extended in order to incorporate Moyal products, characteristic of noncommutative field theories. It is also possible to show that there exits an underlying commutative gauge invariant theory and a suitable Seiberg-Witten map linking the noncommutative model and its ordinary counterpart.

The outline of this paper is as follows: in Section 2, we start by presenting the ordinary U(N) CSKR model in terms of differential forms. After that we show that it is possible to deform the form structure in order to incorporate Moyal products. This essentially permits the construction of the noncommutative extension of the model. In Section 3, the appropriate Seiberg-Witten map is derived. It takes into account not only the usual Yang-Mills sector but also the gauge sector which arises when one considers the invariance associated with the 1-form gauge parameters. We reserve Section 4 for some concluding remarks.

2. THE NONCOMMUTATIVE U(N) CSKR MODEL

To fix notations and conventions, let us start with a brief review of the non Abelian commutative Kalb-Ramond theory. We will follow a notation close to the one found in Ref. [23]. After that we will study the corresponding noncommutative theory.

Let \( a = a^a_{\mu} T^a dx^\mu \) represent a one-form connection taking values in the \( U(N) \) algebra in the fundamental representation. We assume that

\[
[T^a, T^b] = i f^{abc} T^c \\
\{T^a, T^b\} = d^{abc} T^c \\
tr(T^a T^b) = \frac{1}{2} \delta^{ab} \tag{2.1}
\]

On any \( u(N) \) valued p-form \( \alpha \) it is possible to define the exterior covariant derivative

\[
D \alpha = d \alpha - i \alpha \wedge \alpha + i (-1)^p \alpha \wedge a \tag{2.2}
\]

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It follows the first Bianchi identity
\[ DD\alpha = i[\alpha, f] \] (2.3)
where we have defined the curvature two-form
\[ f = da - ia \wedge a \] (2.4)
and the wedge product is implicit in the commutator. For completeness, we also note the second Bianchi identity
\[ Df = 0 \] (2.5)
The gauge sector of the $U(N)$ Yang-Mills theory, with this notation, is described by the action
\[ S = Tr \int f \wedge f \] (2.6)
where the symbol $\circ$ here denotes the space-time dual. Action (2.6) is invariant under the gauge transformation
\[ \delta a = D\alpha \] (2.7)
since under (2.4) $f$ transforms as
\[ \delta f = i[\alpha, f] \] (2.8)
and the invariance of (2.6) is achieved due to the cyclic property of the trace operation. In the above expressions we have used $\delta$ to represent ordinary gauge variations. We will let $\delta$ represent the corresponding noncommutative gauge variations.

To describe the CSKR model, besides the connection $a$, we need a two-form gauge field $b = \frac{1}{4}b_{\mu\nu}dx^\mu \wedge dx^\nu$ and a compensating one-form field $\omega = \omega_\mu dx^\mu$, both taking values in $\mathfrak{u}(N)$. The 3-form field strength associated with $b$ is defined as
\[ g = Db \] (2.9)
Now the CSKR action
\[ S = Tr \int \left[ f \wedge f - \tilde{g} \wedge \tilde{g} + 2mf \wedge \tilde{b} \right] \] (2.10)
shows itself to be invariant under the set of gauge transformations
\[ \begin{align*}
\delta a &= D\alpha \\
\delta b &= D\xi + i[\alpha, b] \\
\delta \omega &= i[\alpha, \omega] - \xi
\end{align*} \] (2.11)
where we have used in (2.10) the collective two-form
\[ \tilde{b} = b + D\omega \] (2.12)
and the modified field strength
\[ \tilde{g} = Db \] (2.13)
We observe from (2.11) and the definitions above that $b$ and $\omega$ transform not only as Yang-Mills tensors but also present an additional transformation related to the one-form gauge parameter $\xi = \xi_\mu dx^\mu$. The quantities $\tilde{b}$ and $\tilde{g}$, however, transform only as Yang-Mills tensors, in the same way as $f$ in (2.8). This fact permits the gauge invariance of (2.10). Actually, in the Abelian case, the compensating one-form $\omega$ is not necessary, and the corresponding theory is gauge invariant, although reducible [21, 22, 23].

All of the transformations defined above close in an algebra, defined by the parameters composition rule
\[ \begin{align*}
\alpha_3 &= i[\alpha_2, \alpha_1] \\
\xi_3 &= i[\xi_2, \alpha_1] - i[\xi_1, \alpha_2]
\end{align*} \] (2.14)
when the commutation of two successive gauge transformations is applied to any one of the fields appearing in the theory, here generically represented by $y$:
\[ [\delta_1, \delta_2]y = \delta_3y \] (2.15)

The field-antifield quantization of the model described above was studied in Ref. [23].

Let us now pass to consider the noncommutative version of this theory. As already commented, the basic procedure to construct the noncommutative extension of some theory consists in deforming ordinary products to noncommutative Moyal products. For any two fields $\Phi_1(x)$ and $\Phi_2(x)$, we define their Moyal product as
\[ \Phi_1(x) \star \Phi_2(x) = \exp \left( \frac{i}{2} \theta_{\mu\nu} \partial_\mu \partial_\nu \right) \Phi_1(x) \Phi_2(y)|_{x=y} \] (2.16)
where $\theta_{\mu\nu}$ is assumed to be a real, constant and antisymmetric quantity which characterizes the noncommutativity of the theory. These products are associative and cyclic under the integral sign, if adequate boundary conditions are assumed. As it is well known [19, 15, 17, 24], the $U(N)$ group elements are also deformed by such a product in the sense that their construction by exponentiation involves Moyal products. Also the group multiplication is defined as a Moyal product. In this way the symmetry structure of the noncommutative $U(N)$ theory is not the same as the corresponding commutative one and the group closure property is only achieved if the algebra
generators close not only under commutations but also under anticommutation. This essentially constitutes the reason for choosing \( U(N) \) in place of \( SU(N) \) as a symmetry group of this noncommutative gauge theory, although other possibilities can be considered \([1]\). Similar deformations can also be implemented in the differential forms structure. In this way, the exterior product is modified in order to accommodate the Moyal structure with the formal replacement \( \wedge \rightarrow \wedge \). In a coordinate basis, this modification is trivial and consists in introducing Moyal star products in place of the ordinary ones between the forms components, keeping the wedge product between the form basis. Here we will restrict ourselves to this situation. Similar procedures can also be implemented in the definition of the exterior covariant derivative. The noncommutative version of the exterior covariant derivative of a p-form \( \Lambda \) is given by

\[
D\Lambda = d\Lambda - iA \wedge \Lambda + i(-1)^p\Lambda \wedge A \tag{2.17}
\]

Definition \([2.4]\) is in the same way trivially deformed to

\[
F = dA - iA \wedge A
\]

\[
= \frac{1}{2}((\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)T^a - i[A^a_\mu T^a_\nu, A^b_\nu T^b_\mu])dx^\mu \wedge dx^\nu
\]

\[
= \frac{1}{2}F_{\mu\nu}dx^\mu \wedge dx^\nu \tag{2.18}
\]

where \( A = A^a_\mu T^a dx^\mu \) now represents the noncommutative 1-form connection. The above expression shows the rule played by the noncommutative wedge product \( \wedge \). As can be observed, \( F \) involves both structure functions defined in \( \text{(2.1)} \). Actually, \( F_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \frac{i}{2}f^{abc}\{A^b_\mu, A^a_\nu\} - \frac{i}{2}e^{abc}[A^b_\mu, A^a_\nu] \). Other expressions follow the same rules. The Bianchi identities are now written as

\[
DDA = i[A \wedge F]
\]

\[
DF = 0 \tag{2.19}
\]

To construct the noncommutative version of the model described above, in place of the ordinary quantities \( b \) and \( \omega \) we define the noncommutative forms \( B = \frac{1}{2}B_{\mu\nu}dx^\mu \wedge dx^\nu \) and \( \Omega = \Omega_\mu dx^\mu \). Also in place of \( g \), \( \dot{g} \) and \( \dot{\Omega} \) defined in \( \text{(2.10), (2.12) and (2.13)} \), we introduce the corresponding noncommutative forms

\[
G = DB
\]

\[
\dot{B} = B + D\Omega
\]

\[
\dot{G} = D\dot{B} \tag{2.20}
\]

These quantities present the following set of gauge transformations:

\[
\delta A = D\epsilon
\]

\[
\delta B = D\Xi + i[\epsilon \wedge B] \tag{2.21}
\]

\[
\delta G = i[\Xi \wedge F] + i[\epsilon \wedge G]
\]

\[
\delta \Omega = i[\epsilon \wedge \Omega] - \Xi
\]

\[
\delta F = i[\epsilon \wedge F]
\]

\[
\delta \dot{B} = i[\epsilon \wedge \dot{B}]
\]

\[
\delta \dot{G} = i[\epsilon \wedge \dot{G}]
\]

and, as can be verified, the noncommutative extension of action \( \text{(2.10)} \),

\[
S = Tr \int (F \wedge F - \dot{G} \wedge \dot{G} + 2mF \wedge \dot{B}) \tag{2.22}
\]

is gauge invariant under \( \text{(2.21)} \) due to cyclic properties of the Moyal product under the integral sign. Due to the boundary conditions we are adopting, it is irrelevant to use in the above expression \( \wedge \) or \( \wedge \).

The transformations \( \text{(2.21)} \) close in an algebra

\[
[i_1, i_2]Y = i_3 Y \tag{2.23}
\]

with the composition rule given by

\[
\epsilon_3 = i[\epsilon_2 \wedge \epsilon_1]
\]

\[
\Xi_3 = i[\Xi_2 \wedge \epsilon_1] - i[\Xi_1 \wedge \epsilon_2] \tag{2.24}
\]

in place of \( \text{(2.1)} \) and \( \text{(2.5)} \).

As one can observe, the noncommutative extension of the CSKR theory has been constructed without difficulties. Of course its quantum version would show all those characteristic points associated with noncommutative field theories and their non planar diagrammatic expansions \([3]\). We will not consider these points in this work. In what follows let us study the Seiberg-Witten map of the model, which presents some interesting features.

### 3. The Seiberg-Witten Map

Accordingly to what we have been discussing in the last section, let us represent the noncommutative field variables by capital letters, here generically denoted by \( Y \) and the corresponding ordinary ones by small letters generically written as \( y \). Also accordingly to the notations of the last section, their gauge transformations are respectively represented by \( \delta Y \) and \( \delta y \). The basic idea in the construction of the Seiberg-Witten map is to obtain the gauge transformations \( \delta Y \) of the noncommutative variables departing from the gauge structure of the ordinary theory, with variables transforming accordingly to \( \delta y \). This is equivalent to solve the equation

\[
\delta Y = \delta y[y] \tag{3.1}
\]
which is done by using expansions in powers of the noncommutative parameter $\theta$ and assuming that up to $0(\theta^2)$ terms, $Y \to y$. This map is non trivial when the noncommutative parameters $\epsilon$ and $\Xi$ are considered as functions of the commutative parameters $\alpha$ and $\xi$ as well as of the ordinary fields $y$. In this case, the fundamental expressions implicit in (2.23) reduce to

$$
\left[ \delta_1, \delta_2 \right] A[y] = D\xi_3[y]
$$

$$
\left[ \delta_1, \delta_2 \right] \Omega[y] = i[\xi_3[y], \Omega] - \Xi_3[y]
$$

$$
\left[ \delta_1, \delta_2 \right] B[y] = i[\xi_3[y], B] + D\Xi_3[y]
$$

(3.2)

where now, in place of (2.24), we get

$$
\epsilon_3[y] = \bar{\delta}_1 \epsilon_2[y] - \bar{\delta}_2 \epsilon_1[y] + i \{ \epsilon_2[y], \epsilon_1[y] \}
$$

$$
\Xi_3[y] = \bar{\delta}_1 \Xi_2 - \bar{\delta}_2 \Xi_1 + i \{ \epsilon_2, \Xi_1 \} - i \{ \epsilon_1, \Xi_2 \}
$$

(3.3)

due to the dependence of the parameters in the fields. Indices 1, 2 and 3 represent the dependence of $\epsilon$ and $\Xi$ in $\alpha_i$ and $\xi_i$, $i = 1, 2, 3$. For instance, $\epsilon_3[y] \equiv \epsilon(\alpha_3, \xi_3, y)$. Related quantities such as $F, \hat{B}$ or $\hat{G}$ also follow similar rules related to the closure of the algebra.

The first of equations (3.2) is not new in the literature (3.4), what does not occur with the second one, as far as we know. They will be important for the results that we will derive. As in the pure Yang-Mills case, the gauge transformation 0-form parameter $\epsilon$ can be expanded to first order in $\theta^{\mu\nu}$ as $\epsilon[y] = \alpha + \epsilon^{(1)}[y]$. In the same way, the 1-form parameter $\Xi$ is expanded as $\Xi[y] = \xi + \Xi^{(1)}[y]$, to first order in $\theta$. From (3.4) and the above expansions we deduce that

$$
\bar{\delta}_1 \epsilon_1^{(1)} - \bar{\delta}_2 \epsilon_2^{(1)} - i[\alpha_1, \epsilon_1^{(1)}] + i[\alpha_2, \epsilon_2^{(1)}] - \epsilon_3^{(1)} = -\frac{1}{2} \theta^{\mu\nu} \{ \partial_\mu \alpha_1, \partial_\nu \alpha_2 \}
$$

(3.4)

and also that

$$
\bar{\delta}_1 \Xi_1^{(1)} - \bar{\delta}_2 \Xi_2^{(1)} + i[\alpha_2, \Xi_1^{(1)}] - i[\alpha_1, \Xi_2^{(1)}] - \Xi_3^{(1)} = -i \{ \epsilon_1^{(1)}, \xi_1 \} + i \{ \epsilon_2^{(1)}, \xi_2 \}
$$

$$
+ \frac{1}{2} \theta^{\rho\sigma} \left( \{ \partial_\rho \alpha_2, \partial_\sigma \xi_1 \} - \{ \partial_\rho \alpha_1, \partial_\sigma \xi_2 \} \right)
$$

(3.5)

The general solution of (3.4) given by (3.6) is

$$
\epsilon^{(1)} = \frac{\theta^{\alpha\beta}}{4} \{ \partial_\alpha \alpha, a_\beta \} + \lambda \theta^{\alpha\beta} \{ \partial_\alpha \alpha, a_\beta \}
$$

where the term in $\lambda$ is the solution of the homogeneous part of (3.4). Now it is possible to show that

$$
\Xi^{(1)} = \frac{1}{2} \theta^{\alpha\beta} \{ \partial_\alpha \alpha, a_\beta \} + i \lambda \theta^{\alpha\beta} \{ \partial_\alpha \alpha, a_\beta \}
$$

(3.7)

solves (3.5) when one uses (3.6) for $\epsilon^{(1)}$. Once we expand the fields to first order in $\theta$, this is: $A = a + A^{(1)}$, $\Omega = \omega + \Omega^{(1)}$ and $B = b + B^{(1)}$, we can rewrite the corresponding gauge transformations appearing in (2.24) as

$$
\bar{\delta} A^{(1)} - i[\alpha, A^{(1)}] = -\frac{1}{2} \theta^{\alpha\beta} \{ \partial_\alpha \alpha, \partial_\beta a \} + D \epsilon^{(1)}
$$

$$
\bar{\delta} \Omega^{(1)} - i[\alpha, \Omega^{(1)}] = -\frac{1}{2} \theta^{\alpha\beta} \{ \partial_\alpha \alpha, \partial_\beta \omega \} + i \{ \epsilon^{(1)}, \omega \} - \Xi^{(1)}
$$

$$
\bar{\delta} B^{(1)} - i[\alpha, B^{(1)}] = -\frac{1}{2} \theta^{\alpha\beta} \left( \{ \partial_\alpha \alpha, \partial_\beta a \} - \{ \partial_\alpha \alpha, \partial_\beta \xi \} \right)
$$

$$
+ i \{ \epsilon^{(1)}, b \} - i \{ \xi, A^{(1)} \} + D \Xi^{(1)}
$$

(3.8)

where $D \epsilon^{(1)}$ and $D \Xi^{(1)}$ represent now ordinary covariant derivatives as defined in (2.22). After inserting (3.6) into the first of equations (3.8), it is possible to find the general solution of the Seiberg-Witten map for the connection $A^{(1)}$

$$
A = a - \frac{1}{4} \theta^{\alpha\beta} \{ a_\alpha, 2 \partial_\beta a - D_\beta \omega \}
$$

$$
+ \theta^{\alpha\beta} D \left( \sigma f_{\alpha\beta} + \lambda \frac{1}{2} [a_\alpha, a_\beta] \right) + O(\theta^2)
$$

(3.9)

where $\lambda$ appears in (3.9) and $\sigma$ is a second parameter associated with the homogeneous part of the first of equations (3.8). Observe that the indices associated with the noncommutativity appear explicitly, what be expected since the Lorentz invariance is broken by the Moyal structure. The covariance associated with the form structure is however kept. This means that it is not necessary to write the forms components in equations (3.9) since they do not mix with the noncommutative structure. Theses features will also appear in the following.

It is possible to show from (3.6), (3.7) and the second one of equations (3.8) that

$$
\Omega^{(1)}[y] = -\frac{1}{4} \theta^{\alpha\beta} \{ a_\alpha, (\partial_\beta - D_\beta) \omega \}
$$

$$
+ i \frac{1}{2} \lambda \theta^{\alpha\beta} [a_\alpha, \omega]
$$

(3.10)

is the desired solution for the compensating 1-form field. To solve the third equation in (3.8) for $\hat{B}^{(1)}$, we need to consider the already derived expressions for $\epsilon^{(1)}$, $\Xi^{(1)}$ and $A^{(1)}$ given above. As can be inferred from them, it is not an easy task to achieve a complete solution for $\hat{B}^{(1)}$ following this route.

However, if we consider the equation defining the gauge variation for $\hat{B}$ in (3.8), we find a much simpler mapping equation given by

$$
\bar{\delta} \hat{B}^{(1)} - i[\alpha, \hat{B}^{(1)}] = -\frac{1}{2} \theta^{\alpha\beta} \{ \partial_\alpha \alpha, \partial_\beta \hat{b} \}
$$

$$
+ i \{ \epsilon^{(1)}, \hat{b} \}
$$

(3.11)
whose general solution, when one keeps the form covariance in the sense discussed above, is given by

\[
\hat{B}^{(1)} = -\frac{1}{4} \theta^{\alpha\beta} \left\{ a_{\alpha}, (\partial_{\beta} + D_{\beta})\hat{b} \right\} \\
+ \theta^{\alpha\beta} \left( \rho [\hat{b}, f_{\alpha\beta}] - i \frac{\chi}{2} \left[ \hat{b}, [a_{\alpha}, a_{\beta}] \right] \right) \tag{3.12}
\]

where \( \rho \) is a new parameter associated with the homogeneous part of \( \hat{A}^{(1)} \). Now remembering that \( \hat{B} = B + D\Omega \) from (2.22), one can verify that

\[
B^{(1)} = \hat{B}^{(1)} - D\Omega^{(1)} + i \{ A^{(1)}, \omega \} \\
- \frac{1}{2} \theta^{\alpha\beta} [\partial_\alpha a, \partial_\beta \omega] \tag{3.13}
\]

Inserting \( \hat{B}^{(1)} \) and the expressions for \( \Omega^{(1)} \) and \( A^{(1)} \) given in (3.10) and (3.9) in the above expression, it is simple to obtain the complete expression for \( B^{(1)} \).

For completeness, we note also that

\[
F^{(1)} = \frac{1}{2} \theta^{\alpha\beta} \left\{ D a_{\alpha} - \partial_\alpha a, D a_{\beta} - \partial_\beta a \right\} \\
- \frac{1}{2} \{ a_{\alpha}, (\partial_{\beta} + D_{\beta}) f \} + i \sigma [f_{\alpha\beta}, f] \\
- \frac{i}{2} \lambda [f, [a_{\alpha}, a_{\beta}]] \tag{3.14}
\]

and

\[
\hat{G}^{(1)} = D\hat{B}^{(1)} - i [A^{(1)}, \hat{b}] + \frac{1}{2} \theta^{\alpha\beta} \{ \partial_\alpha \hat{b}, \partial_\beta a \} \tag{3.15}
\]

Now the action (2.22) is mapped to

\[
S = 2 Tr \int d^4x \left( \frac{1}{2} f \wedge f - \frac{1}{2} \hat{g} \wedge \hat{g} + mf \wedge \hat{b} + f \wedge F^{(1)} \right) \\
- \hat{g} \wedge \hat{G}^{(1)} + mf \wedge \hat{B}^{(1)} + mF^{(1)} \wedge \hat{b} \tag{3.16}
\]

4. CONCLUSIONS

We have studied in this work a noncommutative formulation of the U(N) Cremmer-Scherk-Kalb-Ramond theory. The gauge structure has been considered in detail, as well as a noncommutative differential form structure appropriated to describe the model. It was also constructed the Seiberg-Witten map between this noncommutative gauge theory and its ordinary counterpart, in first order in the noncommutative parameter \( \theta \). In this last subject, not only the vector gauge sector has been considered, but also the rank two gauge algebra associated with the antisymmetric tensor gauge fields. We observe that it has not been explored here some characteristic features usually associated with the ordinary CSKR model such as the effective topological mass generation for the vectorial sector, since they constitute direct generalizations of the ordinary case. Specific features associated with noncommutative quantum field theories such as renormalizability, broken of unitarity, presence of anomalies or mixing of ultraviolet and infrared divergencies have been left for consideration in a future work.

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