Quantum field theory and time machines

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Abstract

We analyze the “F-locality condition” (proposed by Kay to be a mathematical implementation of a philosophical bias related to the equivalence principle, we call it the “GH-equivalence principle”), which is often used to build a generalization of quantum field theory to non-globally hyperbolic spacetimes. In particular we argue that the theorem proved by Kay, Radzikowski, and Wald to the effect that time machines with compactly generated Cauchy horizons are incompatible with the F-locality condition actually does not support the “chronology protection conjecture”, but rather testifies that the F-locality condition must be modified or abandoned. We also show that this condition imposes a severe restriction on the geometry of the world (it is just this restriction that comes into conflict with the existence of a time machine), which does not follow from the above mentioned philosophical bias. So, one need not sacrifice the GH-equivalence principle to “emend” the F-locality condition. As an example we consider a particular modification, the “MF-locality condition”. The theory obtained by replacing the F-locality condition with the MF-locality condition possesses a few attractive features. One of them is that it is consistent with both locality and the existence of time machines.

04.62.+v, 04.20.Gz
I. INTRODUCTION

In recent years much progress has been achieved toward the development of a rigorous and meaningful quantum field theory in curved background (semiclassical gravity). In particular, in the framework of the “algebraic approach” (see [1] and references there) for globally hyperbolic spacetimes a complete and self-consistent description was constructed of the real scalar field obeying the Klein-Gordon equation

\[(\Box - m^2)\phi = 0.\]  

(1)

However, there are non-globally hyperbolic spacetimes [e.g., the Kerr black hole or spacetimes with a conical singularity (those are universes containing a cosmic string)] quantum effects in which are of obvious interest. So it would be desirable to have a theory applicable to such spacetimes as well. Unfortunately, global hyperbolicity plays a crucial role in the above mentioned theory, which therefore cannot be straightforwardly extended to the general case. The desired generalization has not been constructed so far, but a few “reasonable candidates for minimal necessary conditions” [1] were considered, that is “statements which begin with the phrase ‘Whatever else a quantum field theory (on a given non-globally hyperbolic spacetime) consists of, it should at least involve . . . ’” [1]. The best-studied candidate necessary condition is the “F-locality condition” proposed by Kay [2]. Its importance is in that it turns out to be quite restrictive. In particular, a theorem was recently proved by Kay, Radzikowski, and Wald, which says, roughly speaking, that the F-locality condition cannot hold in a spacetime containing a time machine with the compactly generated Cauchy horizon [1].

The present paper is devoted to the problem of how the F-locality condition can be amended. The necessity of the amendments [revealed by the Kay-Radzikowski-Wald (KRW) theorem] stems from the fact that

One cannot just forbid time machines!

It is about six years now that a mechanism which could “protect causality” [3] against time machines is actively looked for. The driving force for this search is apparently the idea that the existence of a time machine would defy the usual notion of free will. And this would be the case indeed if we found a paradox (like that usually called “the grandfather paradox”). For, suppose we found such a system and such its initial (that is fixed to the past of the time machine creation) state that the equations governing its evolution have no solution due to the nontrivial causal structure of the spacetime. We know that the system being prepared in this state must evolve according just to those equations (to change them we must have confessed that we overlooked some effects, which would have implied that we simply built an improper model, but not found a paradox) and at the same time we know that they have no solution. So we have to conclude that such an initial state somehow cannot be realized, that is “...if there are closed timelike lines to the future of a given spacelike hypersurface, the set of possible initial data for classical matter on that hypersurface ...[is] heavily constrained compared with the same local interactions were embedded in a chronology-respecting spacetime” [4]. The dislike for such a contradiction with “a simple notion of free will” [4] was so strong that Rama and Sen in [6], Visser in [7], and in fact Hawking and Ellis in [3] proposed just to postulate the impossibility of time
machines. Also a postulate prohibiting time machines is implicitly contained (as is shown by the KRW theorem) in Kay’s F-locality condition (from now on by a “time machine” we mean exclusively a time machine with the compactly generated Cauchy horizon). The irony of the situation is that, while no paradoxes have been found so far [8], such postulates in the absence of a mechanism that could enforce them lead to precisely the same constraints on one’s will. Indeed, we know that there are initial conditions on the metric and the fields such that, when they are fixed at a spacelike surface [3], the Einstein equations coupled with the equations of motion for these fields lead to the formation of a time machine. So if a postulate forbids time machines we only can conclude either that (1) there are some (e. g., quantum) effects which we have overlooked and which being taken into consideration always change the equations of motion so that the time machine does not form, or that (2) such initial conditions are somehow forbidden. Both possibilities were considered in the literature.

(1). A popular idea was that the vacuum polarization near a would-be Cauchy horizon (when it is compactly generated) becomes so strong that its back-reaction on the metric prevents the formation of the horizon. This idea, however, has never been embodied in specific results. The vacuum polarization in spacetimes with a time machine was evaluated for a few simplest cases [3, 9, 10] and it turned out that sometimes it diverges on the Cauchy horizon and sometimes it does not (in the perfect analogy with, say, the Minkowski space). So it is unlikely that this effect could always protect causality.

(2). It is possible that initial data leading to the formation of a time machine are forbidden not by a restriction on our will, but simply by the fact that they require some unrealizable conditions. It was shown [3], for example, that to create a time machine of a non-cosmological nature (that is evolving from a non-compact Cauchy surface) one has to violate the Weak energy condition (WEC) and a number of restrictions were found on such violations (see, e. g., [11]). None of them, however, has been able to rule time machines out. Moreover, recently a classical scenario for WEC violations was proposed [12].

Thus causality remains still unprotected and any postulate prohibiting time machines without adducing a mechanism that enforces this prohibition raises the alternative of rejecting either the postulate, or the idea that whether one can perform an experiment does not depend on whether causality still holds somewhere in the future.

In the case of the F-locality condition the alternatives at first glance seem equally unattractive since this condition is based on the GH-equivalence principle (see Section [11]). However a closer inspection shows that the F-locality condition does contain a strong arbitrary requirement (in Section [14] we discuss this fundamental point in great detail). So one can reconcile the GH-equivalence principle with quantum field theory in spacetimes with a time machine by just abandoning this requirement. In doing so one still can use the GH-equivalence principle in the theory. It is only necessary to find its another mathematical

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1 Actually, even on a part of the surface \( t = 0 \) of an “almost Minkowskian” space (cf. [3]).

2 There are also papers where (for non simply connected time machines) different results based on the “method of images” are obtained and discussed. This method, however, involves manipulations with incurably ill-defined entities and generally allows one to obtain almost any result one wants (see [10] for a detailed discussion).
implementation. As an example we consider in Section V the “MF-locality condition”. An important point is that while expressing the GH-equivalence principle (and seemingly doing it more adequately than the F-locality condition), it does not forbid time machines. From this we conclude in particular that, contrary to what was claimed in [1] and in a number of succeeding papers, the KRW theorem does not at all “provide strong evidence in support of Hawking’s chronology protection conjecture”. It rather rules out the F-locality condition.

II. GEOMETRICAL PRELIMINARIES

An important role in our discussion will be played by the notion of global hyperbolicity. Globally hyperbolic (GH) spacetimes most adequately meet the concept of a “good” or “usual” spacetime (the Minkowski spacetime, for example, is GH).

Definition 1. A subset $N$ of a spacetime $(M, g)$ is called globally hyperbolic if strong causality holds in $N$ and for any points $p, q \in N$ the set $J^+(p) \cap J^-(q)$ is compact and lies in $N$.

Whether or not a neighborhood $N \subset M$ is GH is not determined exclusively by its geometry. Due to the requirement that $(J^+(p) \cap J^-(q)) \subset N$ it may happen that $N$ is not GH even though $(N, g|_N)$ is GH when it is regarded as a spacetime in its own right. So to describe the geometrical properties of a neighborhood proper we introduce a new notion:

Definition 2. We call a subset $N$ of a spacetime $(M, g)$ intrinsically globally hyperbolic if $(N, g|_N)$ is a globally hyperbolic (GH) spacetime.

Clearly, whether a neighborhood $N$ is an intrinsically GH neighborhood (IGHN) does not depend on the geometry of $M - N$ (in contrast to whether it is a GHN). To avoid confusion, note that our notion of “global hyperbolicity” is that of [5] and differs from that in [1,2]. The latter corresponds to our “intrinsic global hyperbolicity”. For later use let us list a few obvious properties of (intrinsically) globally hyperbolic neighborhoods [(I)GHNs]:

1. An intersection of two (I)GHNs is an (I)GHN;
2. Any GHN is an IGHN and an IGHN is a GHN iff it is causally convex (that is iff no causal curve leaving the IGHN returns in it).

Thus, intrinsic global hyperbolicity is a weaker condition than global hyperbolicity. In particular,

1. For any point $P \in M$ and any its neighborhood $V$ there exists an IGHN $N : P \in N \subset V$, while such a GHN exists if and only if strong causality holds in $P$.

Property (GH2) enables us to construct a simple and useful example of a connected IGH but not a GH subset of the (3-dimensional) Minkowski space.

3. Connected IGH neighborhoods were called locally causal in [13].

4. The existence of such a neighborhood was mentioned in [2] with reference to Penrose.
FIGURES

Example: A “bad” set. Let $V$ be the cube $\{x_k \in (-4, 4)\}$. Consider the strip $S \subset V$ (see Fig. 1) given by the system

$$ x_0 = \varphi/2, \quad \varphi \in [-\pi, \pi], \quad \rho \in [1, 2], $$

where $\rho, \varphi$ are the polar coordinates on the plane $x_1, x_2$. There are causally connected points on $S$ and, in particular, there are points connected by null geodesics lying in $V$ (or null related in $V$, in terms of [1, 2]). A simple calculation based on the fact that

$$ A \text{ is spacelike whenever } |A_0/A_i| < 1 $$

shows, however, that

$$ v_1, v_2 \in S, \quad v_1 \neq v_2, \quad v_1 \leq v_2 \quad \Rightarrow \quad \varphi(v_2) - \varphi(v_1) > \varphi_0 > \pi. $$

So a causal curve can connect two points in $S$ only if one of them lies above the plane $\Phi \equiv \{v| x_0(v) = 0\}$ and the other below $\Phi$. Hence,

(a) All causal curves connecting points of $S$ intersect the plane $\Phi$.  

Similarly, by simple though tiresome considerations one can show that

(b) There is a closed set $\Theta \subset \Phi$ such that $S \cap \Theta = \emptyset$ and none of the causal curves from $S$ to $S$ intersects $\Psi \equiv \Phi - \Theta$. 

(For example, we can choose $\Psi \equiv \{v \in \Phi| \rho(v) \in (0.8, 2.2), |\varphi(v)| < 0.1\}$.) Consider now $S$ as a subset of the spacetime $M' \equiv M - \Theta$. Properties (a, b) ensure that $S$ is a (closed) achronal set and hence by Prop. 6.6.3 of [3] the interior $B$ of its Cauchy domain in $M'$ is a GH subset of $M'$. Thus by (GH2) $B$ is an IGHN and not a GHN.
Note that we have used the fact that $M$ is the Minkowski space only in stating (3). It can be easily seen, however, that within any neighborhood in any spacetime coordinates $x_i$ can be found such that (3) holds in the cube $\{x_k \in (-4, 4)\}$. So (being generalized to the 4-dimensional case) this example proves the following proposition:

**Proposition 1.** For any point $p$ and any its neighborhood $V$ such a connected IGHN $B \subset V$ of $p$ and such a pair of null related in $V$ points $r, q \in B$ exist that $r$ and $q$ are not connected by any causal curve lying in $B$.

### III. F-LOCALITY

The algebraic approach to quantum field theory (below we cite only some basic points that have to do with F-locality, for details see [1] and references therein) is based on the notion of the “field algebra”, which is a $\ast$-algebra with identity $I$ generated by polynomials in “smeared fields” $\phi(f)$, where $f$ ranges over the space $\mathcal{D}(M)$ of smooth real valued functions compactly supported on $M$. The smeared fields $\phi(f)$ are just some abstract objects (informally they can be understood as $\phi(f) = \int_M \phi(x)f(x)\sqrt{-g}d^4x$, where $\phi(x)$ is the “field at a point” operator of the (non-rigorous) conventional QFT). A field algebra is defined by the relations (for all $f, h \in \mathcal{D}(M)$ and for all pairs of real numbers $a, b$):

$$\phi(f) = \phi(f)^\ast, \hspace{1em} \phi(af + bh) = a\phi(f) + b\phi(h), \hspace{1em} \phi((\Box - m^2)f) = 0$$

(4)

(defining a “pre-field algebra”) and a relation fixing commutators $[\phi(f), \phi(h)]$, which we discuss in the following subsections.

Given a field algebra one can proceed to build a complete quantum theory of the free scalar field by introducing the notion of states, postulating some properties for “physically realistic” states and prescriptions for evaluating physical quantities (such as the renormalized expectation value of the stress-energy tensor) for these states. We will not go into this “second level” [4] of the theory.

#### A. The globally hyperbolic case

**Definition 3.** Let $\mathcal{E}$ be a subset of $\mathcal{D}(M) \times \mathcal{D}(M)$ and let $\triangle$ be a functional on pairs $f, h$, where $f, h \in \mathcal{D}(M)$ and $(f, h) \in \mathcal{E}$. We shall call $\triangle$ a bidistribution on $\mathcal{E}$ if it is separately linear and continuous (with respect to topology of $\mathcal{D}(M)$) in either variable.

To fix a commutator relation for the field algebra consider the Klein-Gordon equation (1) given on an IGHN $U$. Let $\triangle$ be its bidistributional solution, that is, a bidistribution on $\mathcal{D}(U) \times \mathcal{D}(U)$ satisfying $\triangle((\Box - m^2)f, h) = \triangle(f, (\Box - m^2)h) = 0$ for all $f, h \in \mathcal{D}(U)$. Among all such solutions there is a preferred one:

**Definition 4.** Let $\triangle^{A,R}_U$ be the fundamental solutions of the inhomogeneous Klein-Gordon equation on a neighborhood $U$ satisfying the property:

$$\triangle^{A,R}_U(f, g) = 0 \hspace{1em} \text{whenever} \hspace{1em} \text{supp } f \cap J^\pm(\text{supp } g, U) = \emptyset.$$  

(5)

Then we call a bidistributional solution of the homogeneous Klein-Gordon equation $\triangle_U \equiv \triangle^A_U - \triangle^R_U$ the advanced minus retarded solution on $U$.
It turns out that for any IGHN $U$, $\triangle_U$ exists and is unique. So we complete the definition of a “field algebra” by adding to (1) the following commutator relation:

$$[\phi(f), \phi(h)] = i\triangle_M(f, h)I. \quad (6)$$

Which of the bidistributional solutions of equation (1) is the advanced minus retarded solution for a given region $U$ is completely determined by the causal structure of $U$. This allows one to prove the following important fact [2]:

**The F-locality property (Form I).** Every point $p$ in a GH spacetime $M$ has an intrinsically globally hyperbolic neighborhood $U_p$ such that for all $f, h \in \mathcal{D}(U_p)$, relation (6) holds with $\triangle_M$ replaced by $\triangle_{U_p}$.

We can also reformulate the F-locality property in a slightly different form by “gluing” all these $\triangle_{U_p}$ into a single bidistribution $\triangle^F$.

Let $\triangle$ be a bidistribution on $\mathcal{E} \supset \mathcal{D}(U) \times \mathcal{D}(U)$. It induces a bidistribution $\triangle|_U$ on $\mathcal{D}(U) \times \mathcal{D}(U)$ by the rule

$$\forall f, h \in \mathcal{D}(U) \quad \triangle|_U(f, h) \equiv \triangle(f, h).$$

**Definition 5.** We shall call $U$ and $\triangle$ matching if $U$ is a connected IGHN and

$$\triangle|_U = \triangle_U.$$

**The F-locality property (Form II).** There are such an open covering of a GH spacetime $M$ by IGHNs $\{U_\alpha\}$ and such a bidistribution $\triangle^F$ on $\mathcal{E}_U$ that

\begin{align*}
(\text{Pr}_1) \quad & \triangle^F \text{ matches any } U_\alpha \\
(\text{Pr}_2) \quad & \text{When } (f, h) \in \mathcal{E}_U, \text{ relation (6) holds with } \triangle_M \text{ replaced by } \triangle^F.
\end{align*}

Here and subsequently if $\{U_\alpha\}$ is a set of neighborhoods in $M$ we write $\mathcal{E}_U$ for $\bigcup_\alpha [\mathcal{D}(U_\alpha) \times \mathcal{D}(U_\alpha)]$.

### B. The non-globally hyperbolic case

To build a field algebra in a non-globally hyperbolic spacetime we can start with a “pre-field algebra” (4). Then, however, we meet a problem with the commutator relation since $\triangle_M$ is (uniquely) defined only for GH spacetimes and there are no obviously preferred solutions of (1) any longer. So, we need some new postulate and Kay proposed [2] to infer such a postulate from the equivalence principle, which as applied to our situation he formulated as follows.

**The GH-equivalence principle.** On an arbitrary spacetime, the laws in the small should coincide with the “usual laws for quantum field theory on globally hyperbolic spacetimes”.

From this principle he postulated in a sufficiently small neighborhood of a point in an arbitrary spacetime what holds by itself in a GH spacetime. Namely, he requires:
The F-locality condition (Form I). Every point \( p \) in \( M \) should have an intrinsically globally hyperbolic neighborhood \( U_p \) such that, for all \( f, h \in D(U_p) \), relation (3) holds with \( \Delta_M \) replaced by \( \Delta_{U_p} \).

It is implied that a spacetime for which there is no field algebra satisfying this condition (a “non-F-quantum compatible” spacetime) cannot arise as an approximate description of a state of quantum gravity and must thus be considered as unphysical.

To reveal the logical structure of the F-locality condition we reformulate it analogously to the F-locality property.

The F-locality condition (Form II). There should be such an open covering of a spacetime \( M \) by IGHNs \( \{ U_\alpha \} \) and a bidistribution \( \Delta^F \) on \( \mathcal{E}_U \) that

(Con₁) \( \Delta^F \) matches any \( U_\alpha \);

(Con₂) When \( (f, h) \in \mathcal{E}_U \), relation (3) holds with \( \Delta_M \) replaced by \( \Delta^F \).

An important difference between these two parts of the F-locality condition is that (Con₂) just specifies what algebra we take to be the “field algebra”, while (Con₁) is a nontrivial requirement placed from the outset upon the spacetime. It is significant that the proof of the KRW-theorem rests upon (Con₁).

The F-locality condition clearly does not fix all commutators. The value of \([\phi(f), \phi(h)]\) remains still undefined for \( f, h \) whose supports do not belong to a common \( U_\alpha \). It is more important, however, to find out whether this uncertainty extends to arbitrarily small regions. Indeed, to find such local quantities as \( \langle T_{\mu\nu} \rangle(p) \) or, say, \( \langle \phi^2 \rangle (p) \) it would be enough to know all commutators \([\phi(f), \phi(h)]\) with functions \( f, h \) both supported in a small neighborhood \( V \) of \( p \). This leads us to the following question: Is it true for at least some open covering \( \{ V_\alpha \} \) that

\[
\forall (f, h) \in \mathcal{E}_V \quad \Delta^{F'}(f, h) = \Delta^F(f, h)
\]

whenever both \( \Delta^F \) and \( \Delta^{F'} \) satisfy (Con₁) (with possibly different \( \{ U_\alpha \} \))? It turns out that the answer is negative even in the simplest case. Indeed, if \( M \) is the Minkowski space and \( \Delta^F \) is a solution of (5) satisfying (Con₁), then so is \( \Delta^{F'} \):

\[
\Delta^{F'}(f, g) \equiv \Delta^F(f', g), \quad \text{where } f'(x^\mu) \equiv f(x^\mu) + f(x^\mu + a^\mu)
\]

and where by \( a^\mu \) we denote an arbitrary constant spacelike vector field. Clearly, for any \( \{ V_\alpha \} \) we can find an \( a^\mu \) such that (7) breaks down.

So the F-locality condition was proposed only as a necessary condition which is to be supplemented with conditions of “the second level” to obtain a complete theory.

IV. THE PARADOX AND ITS RESOLUTION

The F-locality condition (or (Con₁) to be more specific) includes actually a postulate forbidding time machines. This follows from the Kay-Radzikowski-Wald theorem:
The KRW theorem. If a spacetime has a time machine with the compactly generated Cauchy horizon, then there is no extension to $M$ of the usual field algebra on the initial globally hyperbolic region $D$ which satisfies the F-locality condition.

Here by “the usual field algebra” an algebra is meant where for $f, h \in \mathcal{D}(D)$ relation (8) holds with $\Delta_M$ replaced by $\Delta_D$ (for the proof of the theorem and the precise definition of $D$ see [1].)

As is discussed in the Introduction, postulating causality without adducing a “protecting” mechanism, one comes up against a contradiction with the usual notion of free will, which can be regarded as a paradox.

Such a situation (when a paradox arises from postulating in the general case a condition harmless in the GH case) is in no way strange or new.

Example: Classical pointlike particles. Consider a system of elastic classical balls. As long as one studies only GH spacetimes one sees that the following property holds\(^5\)

“The property of balls conservation”. Any Cauchy surface intersects the same number of the world lines of the balls.

Going to arbitrary spacetimes, one finds that the evolution of a system of balls is no longer uniquely fixed by what fixes it in the GH case. To overcome this problem (in the perfect analogy with the F-locality condition) one could adopt the following postulate\(^6\) (note that in the general case it is just a postulate, that is an extraneous (global) condition and not a consequence of any other local principles accepted in the model):

“The condition of balls conservation”. Any partial Cauchy surface should intersect the same number of the world lines of the balls.

Then one would find [8] that there are “non-classical compatible” spacetimes (e. g., the Deutsch-Politzer space) that are spacetimes in which initial data (i. e., data at some partial Cauchy surface) exist incompatible with the “Postulate of balls conservation”. This fact constitutes an (apparent, see [8]) paradox and so one could claim that the existence of such paradoxes suggests that time machines are prohibited [8]. On the other hand, as we discussed above, it seems more reasonable to look for contradictions which we ourselves could introduce in the model in the process of constructing. In doing so we would interpret the “non-classical compatibility” of the Deutsch-Politzer spacetime as evidence not against the realizability of this spacetime, but rather against the postulate. Indeed, abandoning this postulate we resolve the paradox (and thus permit time machines) while causing no harm to any known physics [8].

\(^5\)A model describing such a system can be found in [8]. A specific mathematical meaning is assigned there to the words “a world line of a ball”, etc. The property then can be proven within this model.

\(^6\)Such an approach was really developed in a number of works (e. g. see [1, 4, 14]).
The above example suggests that to avoid the difficulties connected with forbidding the time machine, which we discussed in the Introduction, it would be natural just to abandon the F-locality condition. The problem, however, is that while we can easily abandon “the postulate of balls conservation”, the F-locality condition seems to be based on the philosophical bias resembling the equivalence principle, which is something one would not like to reject. So, in the remainder of the Section we show that the F-locality condition contains actually an arbitrary (i.e., not implied by the GH-equivalence principle or any other respectable physical principle) global requirement and therefore can be rejected or modified without regret.

Proposition 2. For any $\Delta$ and any neighborhood $V$ there exists a connected IGHN $B \subset V$ that does not match $\Delta$.

Proof. Without loss of generality (see (GH3)) $V$ may be thought of as being an IGHN. So either $V$ itself is the desired neighborhood or $\Delta|_{V}$ is the advanced minus retarded solution $\Delta_{V}$ on $V$. In the latter case we can simply adapt the proof of the KRW theorem $[1]$ for our needs. Namely, let $B$ be the set from Prop. $[1]$ and $r, q$ the points appearing there. To obtain a contradiction suppose that $B$ matches $\Delta$ and hence matches $\Delta_{V} = \Delta_{V}$ also. This would mean, by definition, that

$$(\Delta_{V})|_{B} = \Delta_{B}, \quad (8)$$

but $\Delta_{B}(r, q) = 0$ since $r$ and $q$ are not causally connected in $B$, while $(\Delta_{V})|_{B}$ is singular at the pair $r, q$ (see $[1]$ for the proof) since both of these points belong to $V$ and are null connected in it. Contradiction.

Thus we see that even if a spacetime is globally hyperbolic there are two families of IGHNs for any its point: causally convex (and thus GH) sets $\{G_{\alpha}\}$ (let us call them “good”) and those containing null related points that are intrinsically non-causally connected (we shall call them “bad” and denote by $\{B_{\beta}\}$). Both families include “arbitrarily small” sets (i.e., for any neighborhood $V$ one can find both a “good” ($G_{\alpha_{0}}$) and a “bad” ($B_{\beta_{0}}$) subsets of $V$). Irrespective of what meaning one assigns to the term “the laws in the small”, it seems reasonable to assume that they are the same for $B_{\beta_{0}}$ and $G_{\alpha_{0}}$. The more it is so as an observer cannot determine (by geometrical means) whether a neighborhood is “good” or “bad” without leaving it. We have seen that the “good” sets match the commutator function, while the “bad” ones do not. So it follows that the identity of physics in two sets does not imply that they both match the same bidistribution. Correspondingly, the fact that the laws in a small region coincide with any other laws does not imply that it (or any its subset) matches the commutator function on a bigger region. So the requirement $[\text{Con}1]$ that a point should have a neighborhood matching a global commutator function is not an expression of the GH-equivalence principle, but is rather an extraneous condition. It is also an essentially global condition. Indeed, for any point one always can find a bidistribution matching some IGHN of the point and so the main idea of $[\text{Con}1]$ is that such a bidistribution should exist globally.

We see thus that indeed the F-locality condition needs emendations, since while leading to possible paradoxes it contains a strong nonjustified requirement.
V. MODIFIED F-LOCALITY

In this Section we formulate and discuss a candidate necessary condition alternative to the F-locality condition. Being an implementation of the GH-equivalence principle (coupled with the locality principle, see below), it nevertheless does not forbid any causal structure whatsoever. Thus a theory based on this condition is free from the paradoxes discussed above, which provides further evidence in favor of the idea that the existence of time machines is inconsistent not with the equivalence principle, but only with its inadequate implementation.

Consider a commutator $\left[ \phi(x), \phi(y) \right]$. Physically this commutator describes the process in which a particle created from vacuum in $x$ annihilates in $y$. So when we require [as we did in (6)] that the commutator function should vanish for non-causally connected $x$ and $y$ we just implement the (most fundamental) idea that an event can affect only those events that are connected with it by causal curves or, in other words, that particles (or information in any other form) cannot propagate faster than light. The very same idea (called locality, or causality, or local causality depending on the formulation and application) suggests that if the conditions are fixed in $J^+(x) \cap J^-(y)$ (that is, in all points where a non-tachionic particle propagating from $x$ to $y$ can find itself), then $\left[ \phi(x), \phi(y) \right]$ is thereby also fixed. Thus, from locality it seems natural to require as a necessary condition that the field algebra in a globally hyperbolic neighborhood $G$ does not ‘feel’ whether or not there is something outside $G$ [recall that for any $x, y \in G$ any point $z \in M - G$ lies off $J^+(x) \cap J^-(y)$]. We can then construct a field algebra (at least on a part of $M$, see below) by adopting the following modification of the F-locality condition:

**The MF-locality condition.** If $\{G_\alpha\}$ is the collection of all globally hyperbolic subsets of a spacetime $M$, then for all $(f, h) \in \mathcal{E}_G$ relation (3) should hold with $\Delta_M$ replaced by $\Delta_{MF}$ defined to be a bidistribution on $\mathcal{E}_G$ matching each $G_\alpha$.

(In other words, we require that $\phi(f)$ and $\phi(h)$ with $f$ and $h$ supported on a common GHN $G_\alpha$ should commute as if there were no ambient space $M - G_\alpha$ at all.) This condition obviously holds in a GH spacetime, where $\Delta_{MF} = \Delta_M$.

Remark. In discussing the field algebra we operate with such ‘non-local’ (by their very nature) entities as commutators $\left[ \phi(x), \phi(y) \right]$, where $x$ and $y$ can be wide apart. No wonder that relevant statements are also formulated in ‘non-local’ terms. In particular, both the MF-locality and the F-locality conditions distinguish some classes of IGHNs of a point from the others. In the former case those are the causally convex neighborhoods and in the latter case the distinguished class is not specified, but its existence is postulated. But to learn whether or not a given IGHN belongs to the distinguished class we have to consider how it is embedded in the ambient space and to take into account properties of this space [e. g. to check whether or not a set $V$ is causally convex one must consider the whole $J^+(V)$]. In this connection we emphasize that the MF-locality condition is not a non-local postulate (much less a postulate contradicting locality). That is, it does not require that a spacetime,

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7 Generally $\Delta_{MF}$ is not the same as $\Delta_F$. This follows directly from the non-uniqueness of $\Delta_F$ shown at the end of Sect. Ill.
or a field algebra, possess any non-local properties. On the contrary, we found out what locality requires in a specific situation (it is the description of this situation that necessitates non-local terms as we argued above) and chose these requirements as a postulate of the theory.

The MF-locality condition differs from the F-locality condition in that

A. Some IGHN are replaced by each GHN and

B. A condition is imposed only on the field algebra, but not on the geometry of the background spacetime.

Correspondingly, two important consequences take place:

A. As we discussed in Section II the F-locality condition does not uniquely fix the commutator function. Neither does the MF-locality condition. The situation has improved, however, in that now we can fix at least the ultraviolet behavior of the commutator function in the region $G \equiv \bigcup \alpha G_\alpha$ where strong causality holds.

Proposition 3. For any spacetime $M$ with non-empty $G$, $\triangle^{MF}$ exists and is unique.

Proof. Consider two GHNs $G_i$ and $G_k$. By (GH1) their intersection is also a GHN (we can thus denote it $G_j$) and points in $G_j$ are causally related if and only if they are causally related in $G_i$ (and thus also in $G_k$). So (see [2])

$$(\triangle_{G_i})|_{G_j} = \triangle_{G_j} = (\triangle_{G_k})|_{G_j},$$

which means that we can define $\triangle^{MF}$ by the equation

$$\triangle^{MF}|_{G_{\alpha}} \equiv \triangle_{G_{\alpha}}.$$ (9)

This guarantees that $\triangle^{MF}$ is a desired bidistribution matching all $G_\alpha$. At the same time, any functional matching them must satisfy (9), which proves the uniqueness.

B. As we discussed above, the F-locality condition is global in nature. Either $\triangle^F$ does not exist on $M$ and we must exclude the whole $M$ from consideration or $\triangle^F$ exists and then no region is distinguished in this sense. The situation differs greatly if we postulate the MF-locality condition instead. On the one hand, any spacetime is allowed now (since $\triangle^{MF}$ always exists (see Prop. 3); there are no “non-MF-compatible spaces”) and, on the other hand, different parts of a spacetime now have different status. Namely, each point $p \in G$ has a neighborhood $U_p$ such that $\triangle^{MF}(f, h)$ is determined by the MF-locality condition at least for $f, h \in D_{U_p}$. So one can develop the theory as we mentioned in the beginning of Section II and eventually find $\langle T_{\mu\nu} \rangle(p)$. But this cannot be done at this stage for a point

$^8$Note that the same situation takes place in the globally hyperbolic case. The postulate (3) also may seem non-local since the condition defining $\triangle_M$ contains [see (3)] a ‘non-local’ part $\text{supp} f \cap J^\pm(\text{supp} g, U) = \emptyset$. 

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in \((M - G)\), where no field algebra is fixed. Thus the surface \(\partial G\) separates the area of the present version of semiclassical gravity from terra incognita\(^9\).

The role of \(\partial G\) is especially important in the time machine theory since it is \(\partial G\) where the divergence of the stress-energy is expected by many authors. So it should be stressed that physically there is nothing particular in points of \(\partial G\) (including the “base points”; see [1]). In perfect analogy with coordinate singularities in general relativity, \(\partial G\) does not correspond to any physical entity and the fact that we cannot find the energy density in a point of \(\partial G\) means not that it is singular or ill-defined here but simply that we do not know how to do this.

Remark. The MF-locality condition was proposed in this paper primarily to clarify the relation between causality violations and the GH-equivalence principle. However, the uniqueness proved above and the simplicity of the underlying physical assumption suggest that perhaps it deserves a more serious consideration as a possible basis for constructing semiclassical gravity in non-globally hyperbolic spacetimes. Then it would be interesting to find out whether the theory proposed by Yurtsever [13] (which does not, at least explicitly, appeal to any locality principle) is consistent with it.

\(^{9}\)In this respect \(\partial G\) is similar to Visser’s “reliability boundary” [7]. The main difference is that the latter conceptually bounds the region where semiclassical gravity breaks down because of quantum gravity corrections.
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