Fast and robust 3D electrical capacitance tomography

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Abstract
Electrical capacitance tomography (ECT) is well established for 2D imaging of multiphase flow. Increasingly, ECT is now being used to image in 3D. In this paper we examine the challenges of 3D image reconstruction using simulations of a 24 electrode (4 planes of 6 electrodes each) sensor. In particular, we demonstrate that the changes in capacitance in a 3D sensor can be as much as an order of magnitude less than in a 2D sensor and that the condition number for 3D imaging can be of the order of $10^6$. We show that the condition number for 3D imaging with this sensor is dominated by the contributions from the most widely separated electrodes. If these pairs of electrodes are eliminated the condition number can be reduced by up to four orders of magnitude. Interestingly, although cross-plane measurements are essential for accurate reconstruction of 3D images, measurements from the widely separated electrodes are shown to have little influence on the quality of images that can be reconstructed, even in the noise-free case. This finding leads us to propose a new sampling strategy for 3D ECT in which only those capacitance measurements from nearby electrodes are included. This sampling approach leads to a reduction in acquisition time for each ECT data set by 40%, with no degradation in image quality and increased robustness to noise. We demonstrate our findings using experimental measurements on a 3D sensor.

Keywords: electrical capacitance tomography, volume imaging, multiphase flow, rapid sampling

(Some figures may appear in colour only in the online journal)

1. Introduction

Industrial processes rely heavily on multiphase flow phenomena. To understand and optimize the performance of an industrial process, it is essential that engineers have access to detailed, accurate measurements of the distribution of phases within the process. Electrical capacitance tomography (ECT) is an established technique for non-invasively monitoring industrial processes, e.g. settling, oil and gas flow, or fluidization [1–6]. The advantages of ECT are that it is fast, scalable and relatively cheap. Traditionally, ECT has been used to obtain two-dimensional (2D) maps of the distribution of material in a process. In recent years there has been significant interest in extending the technology to enable three-dimensional (3D) imaging [7–16]. Three-dimensional ECT, pioneered by Warsito, Marashdeh and Fan at Ohio State University [8], is appealing as it can help to minimize the fringe field effect [17–19] inherent to ECT, which causes blurring of the objects along the length of the sensor [20]. However, the downside of 3D ECT is that it increases the challenges associated with image reconstruction. In this paper, we examine the sensitivity matrix and capacitance measurements using a four plane 3D ECT sensor and demonstrate that the acquisition time of 3D ECT can potentially be reduced without compromising the accuracy of the image reconstruction by measuring only a sub-set of all the possible combinations of capacitance measurements.

ECT is inherently ill-posed, with the number of permittivity values to be estimated vastly exceeding the
number of measurements [21]. The number of independent measurements that can be performed with ECT is given by
\[ m = N(N - 1)/2, \]
where \( N \) is the number of electrodes in the sensor. Thus, in a typical 2D sensor with 12 electrodes, 66 independent measurements can be made. The resolution of an image reconstructed with ECT varies, but is typically \( 64 \times 64 \) pixels. Thus, the problem is underspecified by a factor of approximately 50 (assuming the sensor geometry is circular). This problem is exacerbated in 3D ECT, owing to the need to resolve the permittivity distribution in the third dimension. For this reason the number of permittivity values in the cross-section in 3D ECT is often reduced, however even if the in-plane resolution were halved, the total number of permittivity values to be recovered increases by a factor 8, assuming cubic volume elements (voxels). To overcome this, the number of electrodes has been increased [11, 15, 16]. However, this leads to an increase in the measurement time for each image. Furthermore, the problem is still strongly ill-posed and becomes increasingly ill-conditioned [22, 23].

Image reconstruction in ECT has been performed using linear back projection (LBP) or various iterative algorithms [21]. These algorithms overcome the ill-posedness of the problem by introducing some regularization function or smoothing to the reconstructed image, which result in robust reconstructions but at the cost of loss of spatial resolution. 3D ECT is more strongly ill-posed and ill-conditioned leading to various more advanced algorithms being proposed [7], [10]. However, despite these advances, the ill-conditioning of the image reconstruction problem still poses a significant challenge.

The ill-conditioning in ECT arises because large changes in the permittivity, only result in small changes in some of the measured capacitances. Hence, small changes in capacitance can cause large changes in the reconstructed permittivity distribution, meaning the reconstruction is very sensitive to noise. In this paper, we examine three factors that might contribute to the reduced image quality and stability of image reconstruction when performing 3D ECT measurements: (1) the variation in the signal-to-noise for different electrode pairs, (2) the conditioning of the 3D problem, and (3) the accuracy of the linearization approximation. We examine these factors using both numerical and experimental analysis of a 3D ECT system. On the basis of these results we define a new approach for performing 3D ECT measurements and demonstrate that our approach leads to a more stable image reconstruction and reduced imaging time.

2. Background

The background of ECT has been well covered before [4, 8, 16, 18, 24], herein only a brief summary of the most relevant material is presented. In ECT, an array of \( N \) electrodes is placed around the outside of a vessel or pipe. An ac voltage is applied across a pair of electrodes and the resulting capacitance is measured [25, 26]. The capacitance for the \( i \)th pair of electrodes, \( C_i \), is a function of the permittivity distribution, as given by
\[
C_i = -\frac{1}{V} \int \int_S \varepsilon(x, y, z) \nabla \phi(x, y, z) \, dS_i,
\]
where \( \varepsilon(x, y, z) \) is the 3D permittivity distribution, \( V \) is the potential difference between the two electrodes, \( \phi(x, y, z) \) is the 3D potential field distribution and \( S_i \) is the electrode surface of the \( i \)th electrode. The potential field varies smoothly throughout the sensor and both the potential and permittivity will be affected by changes in the distribution of permittivity within the sensor. For these reasons, ECT is regarded as a soft field technique and equation (1) describes a nonlinear system. Therefore, to reconstruct the permittivity distribution it is necessary to account for changes in both the permittivity and potential. This can be approximated by assuming that the changes in permittivity, i.e. \( \Delta \varepsilon \), are small and that the map of permittivity can be discretized. Following [21], a perturbation analysis on equation (1) for small discrete changes gives
\[
\Delta C_i = \sum_{j=1}^{n} s_{ij} \Delta \varepsilon_j,
\]
where \( s_{ij} = \frac{\partial C_i}{\partial \varepsilon_j} \) are the elements of a matrix describing the change in the capacitance between the \( i \)th pair of electrodes obtained when there is a small change in the permittivity \( \Delta \varepsilon_j \) in the \( j \)th cell. The elements of the permittivity vector \( \varepsilon_j \) correspond to different spatial locations (\( j \)). By measuring the capacitance between all pairs of electrodes, a set of linear equations can be obtained relating the capacitance and the distribution of permittivity. It is then possible to obtain a map of the permittivity distribution in the material contained within the electrodes by solving the inverse problem described by the set of equations (2).

Capacitance values for neighbouring electrodes are much greater than for distant electrodes. For this reason, image reconstruction is normally performed using the normalized capacitance values obtained from
\[
\lambda_i = \frac{C_i - C_i^{\text{low}}}{C_i^{\text{high}} - C_i^{\text{low}}},
\]
where \( C_i^{\text{high}} \) and \( C_i^{\text{low}} \) are the capacitance between the \( i \)th pair of electrodes when measured with the sensor filled with a material of high permittivity and low permittivity, respectively. Following this process, all normalized capacitance values, \( \lambda_i \), should be approximately between 0 and 1. Thus, the ‘signal’ that determines the accuracy of the reconstructed image in ECT is really the change in the capacitance, and not the raw capacitance itself. We characterize the change in capacitance by defining the dynamic range (\( D_i \)),
\[
D_i = C_i^{\text{high}} - C_i^{\text{low}}.
\]
In this work, images are reconstructed using the LBP estimate with the normalized capacitance values and the normalized sensitivity matrix. The LBP solution is obtained by approximating the solution to the set of equations (2), according to
\[
g = S^T \lambda,
\]
where \( g \) is the estimated image of the normalized permittivity distribution, \( S^T \) is the transpose of the normalized sensitivity matrix and \( \lambda \) is the vector of normalized capacitance measurements.
3. Experiment

The ECT sensor used in this work is shown in figure 1 and consisted of four planes of electrodes with six electrodes on each plane. The inner diameter of the sensor was 50 mm and the size of each electrode is 25 mm × 25 mm. The total length of the sensor is 175 mm, the length of the sensing volume is 125 mm. There are two grounded ring electrodes on the ends of the pipe; the length of each ring electrode is 25 mm. The electrodes in each plane were rotated by 60° relative to the plane above to provide a homogeneous 3D sensitivity distribution [8]. The electrodes and shielding were manufactured out of copper tape on a poly methyl methacrylate pipe (permittivity ε ≈ 3.2). SMA connectors were used to connect to the electrodes and BNC connectors were used to connect to the data acquisition system. For comparison we also use an 8-electrode 2D sensor on a similar pipe of inner diameter 50 mm. The electrodes of the 2D sensor have similar area to those used in the 3D sensor.

The data acquisition system was an Industrial Tomography Systems (ITS) M3C ECT system with 24-channels and an excitation frequency of 1 MHz. The noise in the M3C system is approximately Gaussian with a standard deviation of 1 fF, which corresponds to a signal-to-noise ratio of 60 dB if calculated on the basis of the absolute capacitance values, or a peak SNR of 40 dB on the basis of the change in capacitance. The data sampling frequency is 7 Hz when taking all 268 measurements for the 24-electrode sensor. Poppy seeds (ε ≈ 3.2 [27]) are used as the high permittivity solids phase (high calibration) and air is used as the low permittivity material (low calibration). The packing factor of the solids is 0.38. The sensor was modelled in Comsol Multiphysics to calculate the sensitivity matrix and simulate capacitance measurements. The permittivity and sensitivity distributions are discretized into a 3D volume consisting of 32 × 32 × 64 voxels, each of size 1.56 mm × 1.56 mm × 1.95 mm, in the x-, y- and z-directions, respectively; the z-direction is aligned along the axis of the pipe. Permittivity values in the cells outside the boundary of the cylindrical sensor are set to zero. Thus, the total number of voxels for which a permittivity is estimated is 32 000.

4. Results

In this section we present an analysis of capacitance measurements using simulations of a standard 3D electrode design. First, we examine three effects: (1) differences in the accuracy of the measurements required in 2D and 3D sensor geometries, (2) the effect of particular electrode pairs on the condition number of the linearized capacitance problem, (3) the effect of nonlinearity in Poisson’s equation for 3D sensors. Second, we show the effect of each of the above factors on 3D ECT reconstructions. We demonstrate our findings using experimental measurements on a 3D sensor.

4.1. Comparison of 2D and 3D sensors

Figure 2 shows the dynamic range of the raw capacitance measurements, Dc, between the empty and filled permittivity distributions in both simulation and experiment for the 3D sensor. The change in capacitance in the simulation and experiment are in good agreement. The largest change in capacitance is observed for those electrodes in the same plane, where the change in capacitance is typically ∼100 fF.

The smallest change in capacitance is for electrodes that are separated by one or more planes of electrodes; for these electrodes the change in capacitance is typically in the range 0.3–3 fF. Thus, as the distance between electrodes increases, the capacitance values decrease, as expected. Electrodes that are more than one plane apart yield a change in capacitance approximately 300 times smaller than electrodes on the same plane. It is interesting to compare the range of these changes in capacitance, to those that would be observed in the 2D sensor geometry. For a 2D system with eight electrodes of similar area to those used in our 3D sensor, the change in capacitance would typically be in the range 50–20 fF. Thus, the maximum change
in capacitance in the 2D electrode is slightly smaller than in the 3D electrode owing to the larger number of electrodes in each cross-section in the 2D sensor, however the minimum change in capacitance is much greater. In other words, the range of change in capacitance is only a factor of 2–3 for a typical 2D sensor, compared with two orders of magnitude for our 3D sensor geometry.

Our M3C ECT system is capable of measuring the capacitance with an uncertainty (standard deviation) of $\sim 1 \text{ fF}$. Since the observed change in capacitance for electrodes that are more than one plane apart is less than the uncertainty of the measurement, these measurements will provide limited information during image reconstruction. The effect of noise on the measured capacitance data is illustrated in figure 3 by considering the normalized capacitance obtained from data generated using the linearized model, equation (2), but with Gaussian noise of standard deviation 1 fF added to each capacitance value to approximate the noise in the measured data and the high and low permittivity calibration measurements. Normalized capacitance values corresponding to electrodes on the same plane or in neighbouring planes are all approximately in the range 0 to 1, as expected since the simulated data are obtained from the linearized model. However, normalized capacitance values corresponding to electrodes separated by at least one plane of electrodes are scattered between $-4$ and $+4$. Values outside the range 0 to 1 correspond to capacitance data that is strongly influenced by noise. The error in these measurements arises from the small change in capacitance between electrodes that are widely separated, as shown in figure 2. The effect of these noisy measurements on the image reconstruction problem will now be considered.

4.2. Conditioning of problem

Figure 4 shows examples of the sensitivity maps for electrodes on the opposite side of the sensor and located in the top plane, and one, two or three planes below the activated electrode. In these figures, negative values in the sensitivity matrix are encoded using the green-black colour scale, with positive values in the red-black colour scale. A logarithmic colour scale has been used for both positive and negative values to enable characterization of the full range of sensitivity variations. As indicated in figure 4, capacitances that are in the same plane, or the plane below are of similar magnitude, with a change in normalized permittivity in one voxel corresponding to an increase in normalized capacitance of about $1 \times 10^{-4}$ for voxels located in the centre of the sensor. By contrast, the change in normalized capacitance for electrodes between planes 1 and 3 is at most $1 \times 10^{-5}$ and for those separated by two planes the difference is even smaller. These results indicate that there is a significant decrease in sensitivity as the separation between electrodes increases, as expected [18].
Figure 4. Sensitivity maps for an electrode in plane 1 and an electrode situated on the opposite side of the sensor and (a) in plane 1, (b) in plane 2, (c) in plane 3 and (d) in plane 4. The sensitivity maps are shown on a logarithmic colour scale with green colours indicating negative sensitivity coefficients and red colours indicating positive sensitivity coefficients.

Thus, the sensitivity profile becomes ‘flatter’ or more uniform for electrodes that are widely separated. This flattening of the sensitivity profile is further illustrated by considering the change in the standard deviation of the sensitivity map for each electrode pair. The standard deviations of the sensitivity maps shown in figure 4 are $223 \times 10^{-6}$, $96 \times 10^{-6}$, $18 \times 10^{-6}$ and $2 \times 10^{-6}$, where the pairing is to an electrode on the opposite side of the activated electrode in the same plane, one plane below, two planes below or three planes below, respectively. Thus, the variation in the sensitivity map is two orders of magnitude smaller for distant electrode pairs than it is for nearby electrode pairs.

The sensitivity distribution can also be characterized by the variation in the normalized sensitivity as a function of axial position (see [8]). Figure 5 shows a plot of the axial variation in the normalized sensitivity along the centre line of the sensor. The sensor geometry shows a good variation in sensitivity with axial position and the only ‘dead zones’ occur at the end of the sensor, as desired. It is interesting to compare the sensitivity profiles for widely separated electrodes and those that are close together. In figure 5, the sensitivity profile for electrodes in the same plane or adjacent planes are shown in black, whilst those for electrodes separated by more than one plane are shown in red. It is clear that the widely separated electrodes show relatively little variation in the axial distribution of the sensitivity profile. Further, even if these measurements were excluded, the ‘dead zone’ regions would be confined to the ends of the sensor where no cross-plane measurements are possible.

The flattening of the sensitivity maps affects the ease with which the inverse solution may be calculated. The sensitivity of the inverse problem to noise can be described by the condition number of the sensitivity matrix. For our sensor geometry, the condition number for the sensitivity matrix when using full sampling is $11 \times 10^6$. This condition number is very large and indicates that the problem will be very sensitive to noise. The condition number can be somewhat reduced by normalizing the sensitivity matrix by rows, i.e. normalizing the sensitivity of each voxel as a function of the different electrode pairings [21]. In this case, for full sampling, the condition number decreases by an order of magnitude to $\sim 1 \times 10^6$. However, even this reduced condition number is still large and suggests that the problem will be very sensitive to noise. This sensitivity to noise essentially arises from the small variations in the sensitivity maps for well-separated electrodes.

The condition number for the problem can be significantly improved by eliminating the measurements between the most distant electrodes. If all measurements corresponding to electrodes that are separated by more than one plane are removed, the condition number for the sensitivity matrix is
nonlinear forward model and (equation (1). In this section we examine whether the relationship between these parameters, as described by changes in permittivity. In reality, there is a nonlinear assumption of a linear model relating the capacitance to ECT reconstructions are normally performed under the section 4.1. Therefore, the effect of this change in the separated electrodes will have the greatest noise, as discussed of ECT, the normalized capacitance for the most widely assumes equivalent noise for each measurement. In the case number to characterize the sensitivity of the inverse problem are excluded from the analysis. The use of the condition number to characterize the sensitivity of the inverse problem assumes equivalent noise for each measurement. In the case of ECT, the normalized capacitance for the most widely separated electrodes will have the greatest noise, as discussed in section 4.1. Therefore, the effect of this change in the condition number is likely to be even more pronounced than expected.

4.3. Nonlinearity of ECT problem

ECT reconstructions are normally performed under the assumption of a linear model relating the capacitance to changes in permittivity. In reality, there is a nonlinear relationship between these parameters, as described by equation (1). In this section we examine whether the nonlinearity of the forward model has a greater influence when considering electrodes that are spaced a long distance from each other than nearby electrodes. To test this hypothesis we consider the changes in capacitance that would be obtained when using the linear forward model (i.e. \( \lambda = S_0 \)) and the nonlinear forward model (i.e. as defined by equation (1)) to simulate the capacitance.

Figure 6 shows the normalized capacitance obtained when simulating a pipe \( \frac{1}{4} \) filled with a high permittivity material (\( \varepsilon = 3.2 \)) using both the linear and nonlinear forward model. The approximation of the linearized forward model is evident with differences of up to 0.2 between the normalized capacitance from the full nonlinear forward model and the linearized model. Interestingly, the relative error in the normalized capacitance is largest for electrodes that are separated by one plane or more. These results indicate that the nonlinearity may amplify errors arising from the low signal-to-noise ratio of capacitance measurements for electrodes separated by one plane or more. We investigate the effect of nonlinearity and noise on the quality of reconstructed images in the next section.

4.4. Image reconstruction

On the basis of the results presented in sections 4.1–4.3, we define two sampling strategies: (1) conventional, full sampling whereby the capacitance between all independent combinations of electrodes is measured and (2) a sampling scheme whereby only the independent measurements between electrodes in the same plane or neighbouring planes are measured (henceforth we refer to this scheme as the rapid sampling scheme). Figure 7 shows isosurface renderings of the 3D permittivity distribution obtained when using either full sampling or our rapid sampling scheme to reconstruct images from capacitance data generated using the nonlinear forward model in the absence of noise. Figure 8 shows slices through the centre of the 3D image reconstructions shown in figure 7. Figures 7 and 8 show that the images reconstructed using both sampling schemes are essentially equivalent. Indeed the image of the two spheres (figure 7(a)) shows improved demarcation of the two spheres when using the rapid sampling scheme, indicating that the rapid sampling scheme may even slightly improve image quality. Any improvement in image quality here likely arises from a reduction in the error in the nonlinearity of the forward model, since these images contain no noise. The observed improvement is only minor indicating that nonlinear effects are only slightly more significant for well-separated electrodes than electrodes that are in neighbouring planes. These results demonstrate that the measurements from the well-separated electrodes contain relatively little information about the details of the permittivity distribution, as was evident in the essentially flat sensitivity maps shown in figure 4. The results shown in figures 7 and 8 confirm that the image acquisition time can be reduced by almost a factor of 2 without adversely affecting the image quality by excluding measurements of the capacitance between electrodes separated by one plane or more.

Figure 9 shows the same slices obtained from the same systems as shown in figure 8, but with additional Gaussian

Figure 6. Normalized capacitance values obtained from a simulation of a 1/4 filled distribution in the absence of noise using (●) the full nonlinear forward model and (○) the linearized forward model. Capacitances are grouped according to the separation between the electrodes: (a) electrodes in the same plane, (b) electrodes in adjacent planes and (c) electrodes separated by at least one plane.
noise with a standard deviation of 1 fF added to the capacitance data prior to image reconstruction, to match the experimental peak SNR level of \(\sim 40\) dB. The images obtained using full sampling show significant artefacts including difficulty in detecting one of the two spheres (figure 9(a)), a spurious region of low permittivity above the single jet (figure 9(b)), and an area of increased permittivity above the particles in the 1/4 filled system (figure 9(c)). The results shown in figure 9 demonstrate the significant influence that the noise has on measurements from electrodes separated by one or more planes. In the past, the effect of noise has been overcome through the use of a regularized reconstruction [8]. Regularization makes the inverse problem more robust but requires accurate knowledge of the regularization function and in some cases may introduce additional smoothing to the reconstructed image. The rapid sampling approach provides an alternative means of overcoming the effect of noise on the reconstruction. The rapid sampling approach excludes the measurements that cause the problem to be poorly conditioned and is therefore significantly more robust to noise even when using a simple algorithm such as LBP, as shown in figures 9(d)–(f). Indeed, the results shown in figures 8 and 9 show that the rapid sampling approach can achieve essentially equivalent reconstructions from both noise-free and noisy measurements, at least up to a noise level of 40 dB.

The simulation results were verified by experimental measurements of similar phantom objects using a home-built 3D sensor. Reconstructions using both the full sampling and rapid sampling schemes are presented in figure 10. The results are in good agreement with the simulated results shown in figure 9, where Gaussian noise was added at a similar magnitude to that seen experimentally. The rapid sampling
images provide at least as good quality reconstructions of the phantoms as the full sampling data.

Since ECT is a soft field technique, the quality of images of phantoms reconstructed using ECT is going to depend on the location of the phantom within the sensor. We investigate the sensitivity of the image reconstruction to the different sampling approaches when the location of a single sphere of diameter 16 mm is placed in different positions within the sensor. The sampling approaches tested were full sampling, rapid sampling, and sampling using only the in-plane measurements. The recovered volume of the sphere is given in figure 11 for each of these sampling schemes. The volume of the sphere...
was calculated from the sum of the image when reconstructed using the LBP reconstruction but with the values constrained to be between 0 and 1, where 0 represents air and 1 represents the solid phase. Data are shown when images are reconstructed using electrode pairs that are located (♦) only in the same plane, (∗) in the same or neighbouring planes, and (●) from all possible electrode pairs.

The experimental and numerical results presented in this paper suggest that acquiring measurements from widely separated electrodes in a 3D sensor is often not beneficial, and may even be detrimental to achieving a high quality 3D reconstruction of the permittivity distribution. Electrodes that are widely separated are only weakly sensitive to changes in the permittivity. Hence changes in the capacitance can lead to large errors in the reconstruction of the permittivity distribution. This problem may be exacerbated by the effect of electrodes that are grounded in between the pair of electrodes to be measured, e.g. if taking measurements between the electrodes on plane 1 and 3, the electrodes on plane 2 will be grounded. These grounded electrodes may restrict the electric field from the excitation electrode to the detection electrode, and hence lead to a reduction in the magnitude of the potential distribution.

5. Conclusions

This paper presents an analysis of 3D ECT sensors and examines whether it is possible to reduce the acquisition time without adversely affecting the image quality. In a typical ECT measurement, one measurement is acquired for each pair of electrodes in the sensor, which can be time consuming when large numbers of electrodes are used as is typical in 3D imaging. In this paper, we demonstrate that the change in capacitance for well-separated electrodes in a 3D sensor can fall below the noise threshold for the ECT instrument. Thus, whilst some cross-plane measurements are essential for accurate 3D ECT reconstruction, measurements from some well-separated electrodes may make the inverse problem more challenging whilst providing little benefit to the reconstruction. For the sensor geometry used here, measurements from well-separated electrodes caused the condition number to increase by up to four orders of magnitude. Interestingly, these well-separated measurements can be ignored during reconstruction with little effect on the quality of the reconstructed image. For the sensor design used here, which consisted of four planes with six electrodes per plane, we find that high quality images can be reconstructed using only the in-plane and neighbouring plane measurements. Eliminating all other electrode pairings from the acquisition process could reduce the acquisition time for 3D imaging by 40%, without compromising image quality. The findings presented here are independent of the speed of the acquisition hardware; the approach presented could reduce the acquisition time of even the fastest ECT systems available today up to 40%, though we note that systems with a greater signal-to-noise ratio may be able to extract useful information from more distant electrodes than the system used here. Furthermore, reducing the ill-conditioning of the inverse problem may reduce the need for (strong) regularization during image reconstruction, potentially improving spatial resolution and reducing image reconstruction time. Thus, in practice, researchers will need to determine which measurements to include by balancing the accuracy of their measurement hardware, need for reducing the data acquisition time, and their confidence in the prior knowledge used during regularization for their particular sensor geometry and application.

The approach developed in this paper is also of relevance when it comes to designing suitable arrangements of electrodes in 3D sensors. The analysis presented demonstrates that certain pairs of electrodes in this sensor contain less useful information for reconstructing the permittivity distribution than other pairs. Indeed, electrodes that are separated by one or more planes do not provide useful or robust measurements. The methodology developed here also provides a means for quantitatively...
comparing different sensor geometries to determine which is most likely to lead to high quality, robust 3D ECT images.

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References

[1] Du B, Warsito W and Fan L 2006 Imaging the choking transition in gas-solid systems using electrical capacitance tomography Industr. Eng. Chem. Res. 45 5384–95
[2] Wang H G, Senior P R, Mann R and Yang W Q 2009 Online measurement and control of solids moisture in fluidised bed dryers Chem. Eng. Sci. 64 2893–902
[3] Wang F, Marashdeh Q, Fan L-S and Warsito W 2010 Electrical capacitance volume tomography: design and applications Sensors 10 1890–917
[4] Dyakowski T, Jeanmeure L F C and Jaworski A J 2000 Applications of electrical tomography for gas–solids and liquid–solids flows—a review Powder Technol. 112 174–92
[5] Makkawi Y T and Wright P C 2002 Fluidization regimes in a conventional fluidized bed characterized by means of electrical capacitance tomography Chem. Eng. Sci. 57 2411–37
[6] Reinecke N and Mewes D 1997 Multielectrode capacitance sensors for the visualization of transient two-phase flows Exp. Therm. Fluid Sci. 15 253–66
[7] Warsito W and Fan L-S 2005 Dynamics of spiral bubble plume motion in the entrance region of bubble columns and three-phase fluidized beds using 3D ECT Chem. Eng. Sci. 60 6073–84
[8] Warsito W, Marashdeh Q and Fan L 2007 Electrical capacitance volume tomography IEEE Sensors J. 7 525–35
[9] Wajman R, Banasiak R, Mazurkiewicz L, Dyakowski T and Sankowski D 2006 Spatial imaging with 3D capacitance measurements Meas. Sci. Technol. 17 2113–8
[10] Banasiak R and Soleimani M 2010 Shape based reconstruction of electrical capacitance tomography NETD & E Int. 43 241–9
[11] Rimpiäinen V, Heikkilä L M and Vauhkonen M 2012 Moisture distribution and hydrodynamics of wet granules during fluidized-bed drying characterized with volumetric electrical capacitance tomography Chem. Eng. Sci. 75 220–34
[12] Banasiak R, Wajman R, Betiu J and Soleimani M 2009 Feasibility study of dielectric permittivity inspection using a 3D capacitance CT method NETD & E Int. 42 316–22
[13] Nurge M A 2007 Electrical capacitance volume tomography with high contrast dielectrics using a cuboid sensor geometry Meas. Sci. Technol. 18 1511–20
[14] Romanosky R, Seachman S and Marashdeh Q 2011 Development and implementation of 3-D, high-speed tomography for imaging large-scale, cold-flow circulating fluidized bed NETL Project Report pp 3–4
[15] Soleimani M, Mitchell C N and Banasiak R 2009 Four-dimensional electrical capacitance tomography imaging using experimental data Prog. Electromagn. Res. 90 171–86
[16] Weber J M and Mei J S 2013 Bubbling fluidized bed characterization using electrical capacitance volume tomography (ECVT) Powder Technol. 242 40–50
[17] Yan H, Shao F Q, Xu H and Wang S 1999 Three-dimensional analysis of electrical capacitance tomography sensing field Meas. Sci. Technol. 10 717–25
[18] Yang W 2010 Design of electrical capacitance tomography sensors Meas. Sci. Technol. 21 042001
[19] Marashdeh Q, Fan L, Du B and Warsito W 2008 Electrical capacitance tomography—a perspective Industr. Eng. Chem. Res. 47 3708–19
[20] Chandrasekera T C, Wang A, Holland D J, Marashdeh Q, Pore M, Wang F, Sederman A J, Fan L S, Gladden L F and Dennis J S 2012 A comparison of magnetic resonance imaging and electrical capacitance tomography: an air jet through a bed of particles Powder Technol. 227 86–95
[21] Yang W and Peng L 2003 Image reconstruction algorithms for electrical capacitance tomography Meas. Sci. Technol. 14 R1–13
[22] Wang H G, Li Y and Yang W Q 2006 Demonstration of true 3D image reconstruction capacitance tomography 5th Int. Symp. on Measurement Techniques for Multiphase Flows (Macau, China, 10–13 Dec.) pp 2381–91
[23] Soleimani M, Wang H G, Li Y and Yang W Q 2007 A comparative study of 3D electrical capacitance tomography Int. J. Inform. Syst. Sci. 3 292–306
[24] Makkawi Y T and Wright P C 2004 Electrical capacitance tomography for conventional fluidized bed measurements—remarks on the measuring technique Powder Technol. 148 142–57
[25] Yang W Q, Scott A L and Gambio J C 2003 Analysis of the effect of stray capacitance on an AC-based capacitance tomography transducer IEEE Trans. Instrum. Meas. 52 1647–81
[26] Yang W Q and York T A 1999 New AC-based capacitance tomography system Proc. Inst. Electr. Eng. Sci. Meas. Technol. 146 47–53
[27] Sacilik K and Colak A 2005 Dielectric properties of opium poppy seed Tarım Bilimleri Derg. (J. Agric. Sci.) 11 104–9