Mathematical analysis for phase noise characterization of pulse modulation signal

Cuiling Peng 1, Xiaolong Chen 1*, Huiting Huan 1, Fushun Nian1,2, Baoguo Yang2

1School of Electro-Mechanical Engineering, Xidian University, Xi’an 710071, China.
2Science and Technology on Electronic Test & Measurement Laboratory, Qingdao 266555, China
* E-mail: xlchen@mail.xidian.edu.cn

Abstract. Pulse modulation is of great importance in radar system. The performance of pulse modulated oscillating signal is usually characterized by its phase noise. The phase noise of pulse modulated signal can be hardly measured by phase demodulation. The correlation between the carrier and modulated signal needs to be established for the purpose of noise characterization. In this work, a mathematical form of phase noise of pulse modulated signal was proposed in the frequency domain. After pulse modulation, phase noise distribution of the oscillator shifts and superimposes upon the harmonics of the modulating pulse. Experiments were carried out to verify the phase noise property. The obtained results show that by referring to the phase noise of carrier, the phase noise measured by instrument is in good agreement with the mathematical prediction. The phase noise of pulse width modulated signal can be well depicted by the proposed mathematical form.

1. Introduction
The measurement of phase noise of continuous oscillating source has reached high accuracy. However, in some systems, such as in the process of radar system testing, it is necessary to test the oscillating source performance of the transmitted signal which is burst pulse modulated signal [1]. When the high frequency signal from the oscillator is modulated by pulse width modulation (PWM), the frequency spectrum is transformed into discrete with equal intervals with respect to pulse repetition frequency (PRF) [2]. The phase noise measurement of pulse modulated signals is of practical significance for many systems [3]. However, because pulse modulated signal is obtained by modulation of the continuous wave signal the phase noise of a PWM signal is more difficult to be measured in the time domain. In the frequency domain, it is known that the spectrum of PWM signal is the convolution of the carrier and the pulse spectrum, the spectrum of the carrier is moved to that of the PWM signal, and the phase noise sideband also changes [4]. As a fact that the interval between each spectral line of the pulse signal is smaller than that of the phase noise sideband, the phase noise sideband overplots each other because of convolution [5]. So, the phase noise characteristics of pulse modulated signals should be connected with the continuous wave signal, despite the discrepancy they have in their expressions.

Power loss, periodic stray and direct current (DC) bias are three main problems associated with the analysis of phase noise of a PWM signal. They bring complications to a standard measurement scheme [6]. The phase noise can be represented by the single sideband power spectrum of the signal. The power loss of PWM signal can induce power decent such that the power of PWM signal is lower than that of the carrier [7]. Periodic stray can affect the phase noise of the measured PWM signal seriously.
and cause the deterioration of phase noise. Because of the existence of periodical DC bias, phase cannot be locked by the phase detector [8]. These three problems need to be considered when establishing the correlation of phase noise between the carrier and the PWM signal. In this paper, the mathematical expression of the phase noise of PWM signal was proposed based on the known carrier phase noise, the relationship between phase noise the carrier and PWM signal was verified.

2. Theoretical background
The theoretical mathematical expression of the burst pulse modulated signal was is established as follows. The single sideband phase noise of the carrier before modulation is labelled as $L_c(f)$, and the phase noise of PWM signal is $L_{\text{pulse}}(f)$. In the two terms, $f$ represents the carrier frequency offset. The phase noise of the PWM signal before modulation is

$$L_{\text{pulse}}(f) = L_c(f)$$

(1)

After pulse modulation, the harmonic spectrum of phase noise is overlapped to the main spectrum, which results in periodic stray. In this case, the phase noise can be expressed as

$$L_{\text{pulse}}(f) = L_c(f)(nPRF \pm f)$$

(2)

The power of PWM signal is lower than that of the carrier, and the decreases of PWM signal power can lead to the deterioration of the measured phase noise. And the power spectrum of DC bias generated by the pulse signal is superimposed at the frequency of zero frequency offset. When measuring phase noise, only single sideband power spectrum of pulse modulated signal is considered. So the DC bias makes no contribution to the final results, the mathematical expression of phase noise of PWM signal is

$$L_{\text{pulse}}(f) = 10\log \left( \frac{\tau}{T} L_c(f) + \sum_{n=1}^{\infty} \left( \frac{\tau}{T} \sin^2 \left( \frac{n\pi f}{T} \right) L_c(nPRF \pm f) \right) \right)$$

(3)

Where $\tau$ is the pulse width and $T$ is the period of signal.

A number of experiments were conducted to verify the correctness of the mathematical form.

3. Experiments and verification
Three experiments were designed. The carrier signals were modulated by pulse signals with different pulse repetition frequencies and duty cycles. The carrier was a sinusoidal wave of 600 MHz generated by a signal generator. The phase noise of the carrier was measured by phase noise analyzer, as shown in Figure 1. Then, the carrier was modulated by an ideal pulse signal with 1 kHz PRF and 10% duty cycle, and the measured phase noise of the PWM signal was shown in Figure 2.
Because of pulse modulation, the phase noise of carrier signal moves to the fundamental and harmonic frequencies. The phase noise at the fundamental and harmonic frequencies of the pulse modulated signal is mixed with each other for the frequency offset greater than the 1/2PRF band, and thus is impossible to be measured by the mathematical model. The phase noise measured by mathematical model can only be compared with the phase noise simulated by experiment in the frequency offset of 1/2 PRF band. In this experiment, the PRF of the pulse signal is 1 kHz, so the phase noise curve of the PWM signal with frequency offset less than 500 Hz is compared.

By introducing the phase noise of the carrier in the mathematical expression of PWM signal, the phase noise of the PWM signal can be calculated. Figure 3 shows the comparison between the measured phase noise and the phase noise calculated by the mathematical expression.

![Figure 3. Comparison of phase noise of PWM signal measured by instrument and expression.](image)

It can be seen from the figure that the two phase noise curves are very close to each other in all curves. The phase noise curve measured by the mathematical expression are in good agreement with the phase noise curve of the PWM signal obtained by the phase noise measuring instrument.

In the second experiment and the third experiment, the modulation signal changed to an ideal rectangular pulse signal with 1 kHz PRF and 50% duty cycle, and then with 10 kHz PRF and 50% duty cycle respectively.

Data from the three groups of experiments were summarized in Table 1, 2 and 3. Regarding the measured phase noise data as the true value, the absolute error between the phase noise of PWM signal measured by instrument and expression was calculated.

**Table 1. The absolute errors of the first experiment with 1 kHz PRF and 10% duty cycle.**

| Frequency (Hz) | 1    | 5    | 10   | 50   | 100  | 200  | 500  |
|---------------|------|------|------|------|------|------|------|
| The phase noise from experiment (dBc/Hz) | -91.50 | -104.92 | -108.68 | -120.50 | -126.67 | -134.11 | -140.57 |
| The phase noise from calculation (dBc/Hz) | -89.57 | -103.28 | -108.58 | -121.44 | -127.38 | -133.62 | -138.98 |
| The absolute error (dBc/Hz) | 1.93  | 1.64  | 0.10  | 0.94  | 0.71  | 0.49  | 1.59  |
### Table 2. The absolute errors of the second experiment with 1 kHz PRF and 50% duty cycle.

| Frequency (Hz) | 1     | 5     | 10    | 50    | 100   | 200   | 500   |
|---------------|-------|-------|-------|-------|-------|-------|-------|
| The phase noise from experiment (dBc/Hz) | -84.51 | -97.93 | -101.69 | -113.51 | -119.68 | -127.12 | -133.58 |
| The phase noise from calculation (dBc/Hz)  | -82.80 | -96.49 | -101.55 | -114.95 | -120.79 | -126.82 | -134.19 |
| The absolute error (dBc/Hz) | 1.71 | 1.44 | 0.14 | 1.44 | 1.11 | 0.30 | 0.61 |

### Table 3. The absolute errors of the third experiment with 10 kHz PRF and 50% duty cycle.

| Frequency (Hz) | 1     | 5     | 10    | 50    | 100   | 200   | 500   |
|---------------|-------|-------|-------|-------|-------|-------|-------|
| The phase noise from experiment (dBc/Hz) | -84.51 | -101.69 | -113.42 | -119.74 | -136.22 | -140.20 | -143.33 |
| The phase noise from calculation (dBc/Hz)  | -82.58 | -102.00 | -114.95 | -120.34 | -134.65 | -139.62 | -143.24 |
| The absolute error (dBc/Hz) | 1.93 | 0.31 | 1.53 | 0.60 | 1.57 | 0.58 | 0.09 |

From the data in Table 1, 2 and 3, it can be seen that the largest error between instrumental measurement and mathematical interpretation are 1.93 dBc/Hz, 1.71 dBc/Hz and 1.93 dBc/Hz respectively. The comparatively small error indicates that mathematical expression is able to explain the phase noise spectrum of a pulse modulated signal.

### 4. Conclusion

In this work, a mathematical form of phase noise of pulse modulated signal was proposed to analyze the noise characterization. The correlation between the carrier and modulated signal was illustrated. The phase noise from calculation and experiment was compared. Numerical results show the phase noise depicted by the proposed mathematical form is accurate compared with the instrumental measurements. Furthermore, the correlation between the carrier and modulated signal can be used to analyse the phase noise property.

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