Measurements of $D$ and $D_s$ decay constants at CLEO

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Using CLEO data collected at 3370 MeV and 4170 MeV, we determine $f_{D^+} = (205.8 \pm 8.5 \pm 2.5) \text{ MeV}$ and an interim preliminary value of $f_{D_s^+} = (267.9 \pm 8.2 \pm 3.9) \text{ MeV}$, where both results are radiatively corrected. They agree with the recent most precise unquenched Lattice-QCD calculation for the $D^+$, but do not for the $D_s^+$. Several consequences are discussed.

1. Introduction

Leptonic decay $D_{(s)}^+ \rightarrow \ell^+ \nu$ is described by the annihilation of the initial quark-antiquark pair into a virtual $W^+$ that materializes as a pair (Fig. 1).

![Decay diagram for $D_{(s)}^+ \rightarrow \ell^+ \nu$.](image)

The decay rate is given by

$$
\Gamma(D_{(s)}^+ \rightarrow \ell^+ \nu) = \frac{G_F^2 f_{D_{(s)}^+} m_{\ell}^2 m_{D_{(s)}^+}}{8\pi} \left(1 - \frac{m_{\ell}^2}{m_{D_{(s)}^+}^2}\right) |V_{cd(s)}|^2,
$$

where $f_{D_{(s)}^+}$ is decay constant [1], related to the overlap of the heavy and light quark wave-function at zero spatial separation, $G_F$ is the Fermi constant, $m_{D_{(s)}^+}$ is the $D_{(s)}^+$ mass, $m_{\ell}$ is the final state charged-lepton mass, and $V_{cd(s)}$ is a CKM matrix element, taken as $V_{ud} = V_{ts} = 0.2256$ and $V_{cs} = V_{us} = 0.9742$. Thus, measurement of purely leptonic decays allows a determination of the decay constant $f_{D_{(s)}^+}$.

Meson decay constants in the $B$ system are used to translate measurements of $B\bar{B}$ mixing to CKM matrix elements. Currently, it is not possible to determine $f_B$ accurately from leptonic $B$ decays, so theoretical calculations of $f_B$ are used. Since the $B^0$ meson does not have $\ell^+\nu$ decays, it will never be possible to determine $f_B$, experimentally, thus theory must be relied on. If calculations disagree on $D$ mesons, they may be questionable on $B$ mesons. If, on the other hand new physics is present, we need understand how it effects SM based predictions of the $B$ decay constants. Decay constants can be calculated using lattice quantum-chromodynamics (LQCD). Recent calculation from Follana et al. using an unquenched LQCD predicts $f_{D^+} = (207 \pm 4) \text{ MeV}$ and $f_{D_s^+} = (241 \pm 3) \text{ MeV}$ [2].

We use the reactions $e^+e^- \rightarrow D^-D^+$, and $e^+e^- \rightarrow D_s^-D_s^+$ or $D_s^-D_s^+$. The $D^+$ is studied at 3770 MeV using 818 pb$^{-1}$ of data [3]. And the $D_s^+$ is studied at 4170 MeV, using 424 pb$^{-1}$ for $\mu^+\nu$ and $\tau^+\nu$, $\tau^+ \rightarrow \pi^+\nu$, and 300 pb$^{-1}$ for $\tau^+\nu$, $\tau^+ \rightarrow e^+\nu\bar{\nu}$. (Eventually CLEO will present results using 600 pb$^{-1}$.)

2. $D^+ \rightarrow \ell^+\nu$

We fully reconstruct $D^-$ [4] as a tag and examine the properties of the other $D^+$, which can be found even if there is a missing neutrino in the final state. This method is called as “double tag” technique. To reconstruct $D^-$
tags we require that the tag candidates have a measured energy consistent with the beam energy, and have a “beam constrained mass”, \( m_{BC} \), consistent with the \( D^- \) nominal mass [5], where
\[
m_{BC} = \sqrt{E_{beam}^2 - (\sum p_i)^2},
\]
\( E_{beam} \) is the beam energy and \( i \) runs over all the final state particles three-momenta. Fig. 2 shows the \( m_{BC} \) distribution summed over all the decay modes we use for tagging. Selecting events in the mass peak we count 460,055±787 signal events over a background of 89,472 events.

![Figure 2: The beam-constrained mass distributions summed over \( D^- \) decay candidates in the final states: \( K^+\pi^-\pi^- \), \( K^+\pi^-\pi^-\pi^0 \), \( KS\pi^- \), \( KS\pi^-\pi^-\pi^+ \), \( KS\pi^-\pi^-\pi^0 \) and \( K^+K^-\pi^- \).](image)

We then search for signal events with one and only one additional track with opposite sign of charge to the tag, not identified as kaon. The track must make an angle > 25.8° with respect to the beam line (90% of the solid angle), and in addition we require that there not be any photon detected in the calorimeter with energy greater than 250 MeV. The latter selection can highly suppress \( D^+ \rightarrow \pi^+\pi^0 \) background. We separate these events into two cases, where case (i) refers to muon candidates depositing < 300 MeV, characteristic of 98.8% of muons and case (ii) is for candidates depositing > 300 MeV, characteristic of 45% of the pions, those that happen to interact in the calorimeter and deposit significant energy.

We look for \( D^+ \rightarrow \mu^+\nu \) by computing the square of the missing mass
\[
MM^2 = (E_{beam} - E_{\mu^+})^2 - (p_{D^-} - p_{\mu^+})^2,
\]
where \( p_{D^-} \) is the three-momentum of the fully reconstructed \( D^- \), and \( E_{\mu^+}(p_{\mu^+}) \) is the energy (momentum) of the \( \mu^+ \) candidate. The signal peaks at zero for \( \mu^+\nu \) and is smeared toward more positive values for \( \tau^+\nu, \tau^+ \rightarrow \pi^+\nu \).

The fit to the case (i) MM\(^2\) distribution shown in Fig. 3 contains separate shapes for signal, \( \pi^+\pi^0 \), \( K^0\pi^+ \), \( \tau^+\nu \) (\( \tau^+ \rightarrow \pi^+\nu \)), and a background shape describing three-body decays. Here we assume the SM ratio of 2.65 for the ratio of the \( \tau^+\nu/\mu^+\nu \) component and constrain the area ratio of these components to the product of 2.65 with \( B(\tau^+ \rightarrow \pi^+\nu) = (10.90±0.07)\% \) [5] and the 55% probability that the pion deposits <300 MeV in the calorimeter. The \( \pi^+\pi^0 \) background are fixed at 9.2 events obtained from Monte Carlo (MC) simulation. The normalizations of the signal, \( K^0\pi^+ \), and 3-body background are allowed to float. The \( K^0\pi^+ \) shape is obtained from double tag events of \( D^0 \rightarrow K^-\pi^+, D^0 \rightarrow K^+\pi^- \) where we ignore one kaon to calculate the MM\(^2\). All other shapes are obtained from the MC simulation.

The fit yields 149.7±12.0 \( \mu^+\nu \) signal events and 25.8 \( \tau^+\nu, \tau^+ \rightarrow \pi^+\nu \) events (for the entire MM\(^2\) range). We also perform the fit allowing the \( \tau^+\nu, \tau^+ \rightarrow \pi^+\nu \) component to float. Then we find 153.9±13.5 \( \mu^+\nu \) events and 13.5±15.3 \( \tau^+\nu, \tau^+ \rightarrow \pi^+\nu \) events, compared with the 25.8 we expect in the SM. Performing the fit in this manner gives a result
Figure 3: Fit to the $M M^2$ for case (i). The points with error bars show the data. The black (dashed) curve centered at zero shows the signal $\mu^+\nu$ events. The dot-dashed (red) curve that peaks around 0.05 GeV$^2$ shows the $D^+ \to \tau^+\nu$, $\tau^+ \to \pi^+\nu$ component. The solid (blue) Gaussian shaped curve centered on the pion-mass squared shows the residual $\pi^+\pi^0$ component. The dashed (purple) curve that falls to zero around 0.03 GeV$^2$ is the sum of all the other background components, except the $K^0\pi^+$ tail which is shown by the long-dashed (green) curve that peaks up at 0.25 GeV$^2$. The solid (black) curve is the sum of all the components.

that is independent of the SM expectation of the $D^+ \to \tau^+\nu$ rate. To extract a branching fraction, in either case, we subtract off 2.4 ± 1.0 events found from simulations and other studies to be additional backgrounds, not taken into account by the fit.

We find $B(D^+ \to \mu^+\nu) = (3.82 \pm 0.32 \pm 0.09) \times 10^{-4}$. The decay constant $f_{D^+}$ is then obtained from Eq. (1) using $1040 \pm 7$ fs as the $D^+$ lifetime [5] and 0.2256 as $|V_{cd}|$. Our final result, including radiative corrections is

$$f_{D^+} = (205.8 \pm 8.5 \pm 2.5) \text{ MeV}.$$ (3)

3. $D_s^+ \to \ell^+\nu$

To reconstruct the tag, the difference here than the $D^+$ case is that we need to include an additional photon from $D_s^-$ in the tag. The $D_s^-$ tags we reconstructed can either from directly produced $D_s^-$ mesons or those that result from the decay of $D_s^+$ mesons. We calculate the missing mass squared $MM^2$ recoiling against the photon and the $D_s^-$ tag,

$$MM^2 = (E_{CM} - E_{D_s} - E_\gamma)^2 - (p_{CM} - p_{D_s} - p_\gamma)^2,$$ (4)

here $E_{CM}$ ($p_{CM}$) is the center-of-mass energy (momentum), $E_{D_s}$ ($p_{D_s}$) is the energy (momentum) of the fully reconstructed $D_s^-$ tag, and $E_\gamma$ ($p_\gamma$) is the energy (momentum) of the additional photon. We determine number of tags by simultaneously fit to the invariant mass ($M_{D_s}$) and $MM^2$, shown in Fig. 3. The signal is fit to a sum of two Gaussians for the $M_{D_s}$ and a Crystal Ball function for the $MM^2$. The tail parameters of the Crystal Ball function are obtained from fully reconstructed $D_s^+D_s^-$ events. The background has two components: either comes from the background under the invariant mass peak (fake $D_s$), or is due to random photon combinations. The former background is modeled by linear function for the mass and 5th order Chebyshev polynomial for the $MM^2$. Since the
latter background is from a true $D_s$, its mass distribution has the same shape as that from the signal, while its MM$^2$ is modeled by another 5th order Chebyshev polynomial function. The total number of single tags is $30848 \pm 695 \pm 925$ in the invariant mass signal region ($\pm 17.5$ MeV from the nominal $D_s$ mass) and MM$^2 \in [3.782, 4.0]$ GeV$^2$.

![Graph showing invariant mass distribution](image)

Figure 4: Distributions of the invariant mass (left) in MM$^2 \in [3.5, 4.25]$ GeV$^2$ and MM$^2$ (right) in the invariant mass $|M_{D_s} - 1968.3$ MeV$| < 17.5$ MeV in the final states: $K^+ K^- \pi^-$, $K_S K^- \eta \pi^-$; $\eta \rightarrow \gamma \gamma$, $\eta' \rightarrow \pi^+ \pi^- \eta$, $\eta \rightarrow \gamma \gamma$, $\phi \rho$; $\phi \rightarrow K^+ K^-$, $\rho^- \rightarrow \pi^- \pi^0$, $\pi^+ \pi^- \pi^0$, $K^{*+} K^{*0}$, $K^{*-} \rightarrow K^0_S \pi^-$, $K^{*0} \rightarrow K^+ \pi^-$, $\eta \pi^-$; $\eta \rightarrow \pi^+ \pi^- \gamma$. Data (point) and fit function (red curve) are shown. The dashed (blue) curve corresponds to the fake $D_s$ background, and the dashed (green) curve to the random photon background.

Similarly as the $D^+$, we reconstruct the signal side. We veto events with an extra neutral energy cluster $> 300$ MeV (it is $250$ MeV in the $D^+$ case). It is highly effective in reducing backgrounds, especially for $D^{+} \rightarrow \pi^+ \pi^0$, $\eta \pi^+$ and the processes $D^{(*)/\bar{D}^{(*)}}$. The missing mass squared, MM$^2$, evaluated by taking into account the observed $\mu^+$, $D_s^-$, and photon should peak at zero;

$$MM^2 = (E_{CM} - E_{D_s} - E_{\gamma} - E_{\mu})^2 - (p_{CM} - p_{D_s} - p_{\gamma} - p_{\mu})^2. \quad (5)$$

We also make use of a set of kinematical constraints and fit each event to two hypotheses one of which is that the $D_s^-$ tag is the daughter of a $D_s^*$ and the other that the $D_s^*$ decays into $\gamma D_s^+$, with the $D_s^+$ subsequently decaying into $\mu^+ \nu$. In addition, we constrain the invariant mass of the $D_s^-$ tag to the known $D_s$ mass. This gives us a total of 7 constraints. The missing neutrino four-vector needs to be determined, so we are left with a three-constraint fit. We perform a standard iterative fit minimizing $\chi^2$. We choose the fitted MM$^2$ from the hypothesis giving the smaller $\chi^2$. The MM$^2$ distribution from data is show in Fig. 5. After fixing the ratio of $\tau^+ \nu / \mu^+ \nu$ to the SM value we find $f_{D_s^+} = (268.2 \pm 9.6 \pm 4.4)$ MeV.

We can also use the decay mode $\tau^+ \rightarrow e^+ \nu \pi^0$. This result has already been published. [6] The technique here is to use only three tagging modes: $\phi \pi^-$, $K^- K^{*0}$ and $K^0_S K^-$, to ensure that the tags are extremely clean. Then events with an identified $e^+$ and no other charged tracks are selected. Any energy not associated with the tag decay products is tabulated. Those events with small extra energy below 400 MeV are mostly pure $D_s^+ \rightarrow \tau^+ \nu$ events. After correcting for efficiencies and residual backgrounds we find $f_{D_s^+} = (273 \pm 16 \pm 8)$ MeV.

4. Conclusions

The preliminary CLEO average is $f_{D_s^+} = (267.9 \pm 8.2 \pm 3.9)$ MeV (radiatively corrected). Averaging in the Belle result [7] $f_{D_s^+} = (269.6 \pm 8.3)$ MeV, which differs from the Follana et al. calculation [2] by 3.2 standard deviations, while the result for $f_{D^+} = (205.8 \pm 8.5 \pm 2.5)$ MeV is in good agreement. This discrepancy could be due to physics beyond the standard model [8], or systematic uncertainties that are not understood in the LQCD calculation or the experimental measurements, or unlikely statistical fluctuations in the experimental measurements or the LQCD
Figure 5: The MM$^2$ distribution. The dashed (grey) Gaussian shaped curve peaked near zero is the $\mu^+\nu$ component, while the dashed (purple) curve that rises sharply from zero and then flattens out shows the $\tau^+\nu$ component. The two lines are background components. The solid curve shows the sum.

calculation. Fits to the CKM matrix parameters use theoretical predictions of $f_{B_s}/B_d$. As similar calculations are used for $f_{B_s}/B_d$, we need to be concerned with them.

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