The Littlest Higgs Model with T-Parity
Facing CP-Violation in $B_s - \bar{B}_s$ Mixing

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Abstract

The non-minimal flavour violating interactions of mirror quarks and new heavy
gauge bosons in the Littlest Higgs model with T-parity (LHT) give rise to naturally
large CP-violating effects in the $B_s$ system. In view of a large new CP phase in
$B_s - \bar{B}_s$ mixing hinted by the CDF and DØ data and the recent UTfit analysis, we
update our 2006 analysis of particle-antiparticle mixing and rare $K$ and $B$ decays
in the LHT model, using the most recent values of a number of input parameters
and performing a more careful error analysis. We find that the CP-asymmetry $S_{\psi\phi}$ can easily reach values $\sim 0.15 - 0.20$, compared to the SM value $\sim 0.04$, while higher values are rather unlikely though not excluded. Large enhancements are also
possible in the branching ratios for $K_L \to \pi^0 \nu\bar{\nu}$, $K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \ell^+\ell^-$
with much more modest effects in $B_{s,d} \to \mu^+\mu^-$. We perform a detailed study of
correlations between the latter decays and $S_{\psi\phi}$ as well as of the correlation between
$S_{\psi\phi}$ and $S_{\psi K_S}$. We also point out that the possible tension between $\varepsilon_K$ and the
tree level CKM determination recently hinted by various analyses can easily be
resolved in the LHT model.
Note added

An additional contribution to the Z penguin in the Littlest Higgs model with T-parity has been pointed out in [1,2], which has been overlooked in the present analysis. This contribution leads to the cancellation of the left-over quadratic divergence in the calculation of some rare decay amplitudes. Instead of presenting separate errata to the present work and our papers [3–6] partially affected by this omission, we have presented a corrected and updated analysis of flavour changing neutral current processes in the Littlest Higgs model with T-parity in [7].
1 Introduction

One of the most attractive solutions to the so-called little hierarchy problem that affects the Standard Model (SM) is provided by Little Higgs models \cite{8,9}. They are perturbatively computable up to $\sim 10 \text{ TeV}$ and have a rather small number of parameters, although their predictivity can be weakened by a certain sensitivity to the unknown ultraviolet (UV) completion of the theory. In these models, in contrast to supersymmetry, the problematic quadratic divergences to the Higgs mass are cancelled by loop contributions of new particles with the same spin-statistics of the SM ones and with masses around 1 TeV.

The basic idea of Little Higgs models \cite{10} is that the Higgs is naturally light as it is identified with a Nambu-Goldstone boson (NGB) of a spontaneously broken global symmetry. While an exact NGB would have only derivative interactions, gauge and Yukawa interactions of the Higgs have to be incorporated. This can be done without generating quadratically divergent one-loop contributions to the Higgs mass, through the so-called collective symmetry breaking. The collective symmetry breaking has the peculiarity of generating the Higgs mass only when two or more couplings in the Lagrangian are non-vanishing, thus avoiding one-loop quadratic divergences.

The most economical, in matter content, Little Higgs model is the Littlest Higgs (LH) model \cite{11}, where the global group $SU(5)$ is spontaneously broken into $SO(5)$ at the scale $f \sim O(1 \text{ TeV})$ and the electroweak sector of the SM is embedded in an $SU(5)/SO(5)$ non-linear sigma model. Gauge and Yukawa Higgs interactions are introduced by gauging the subgroup of $SU(5)$: $[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2$. In the LH model, the new particles appearing at the TeV scale are the heavy gauge bosons ($W^\pm_H, Z_H, A_H$), the heavy top ($T$) and the scalar triplet $\Phi$.

In the LH model, the custodial $SU(2)$ symmetry, of fundamental importance for electroweak precision studies, is unfortunately broken already at tree level, implying that the relevant scale of New Physics (NP), $f$, must be at least $(2 - 3)$ TeV in order to be consistent with electroweak precision data \cite{12,18}. As a consequence, the original motivation of solving the little hierarchy problem is partly lost. Moreover, the contributions of the new particles to FCNC processes turn out to be at most $(10 - 20)\%$ \cite{19,23}, which will not be easy to distinguish from the SM due to experimental and theoretical uncertainties. In particular, detailed analyses of particle-antiparticle mixing and of rare $K$ and $B$ decays in the LH model have been published in \cite{19,23}.

More promising and more interesting from the point of view of FCNC processes is the Littlest Higgs model with a discrete symmetry called T-parity \cite{24,26} under which all new particles listed above, except $T_+$, are odd and do not contribute to processes with external SM quarks (T-even) at tree level. As a consequence, the NP scale $f$ can...
be lowered down to 1 TeV and even below it, without violating electroweak precision constraints [27,28].

A consistent and phenomenologically viable Littlest Higgs model with T-parity (LHT) requires the introduction of three doublets of "mirror quarks" and three doublets of "mirror leptons" which are odd under T-parity, transform vectorially under $SU(2)_L$ and can be given a large mass. Moreover, there is an additional heavy $T_-$ quark that is odd under T-parity [26].

Mirror fermions are characterised by new flavour violating interactions with SM fermions and heavy gauge bosons, which involve in the quark sector two new unitary mixing matrices, called $V_{Hu}$ and $V_{Hd}$, analogous to the CKM matrix [29,30]. $V_{Hd}$ contains 3 angles, like $V_{CKM}$, but 3 (non-Majorana) phases [31], i.e. two additional phases relative to the SM mixing matrices, that cannot be rotated away in this case.

Because of these new mixing matrices, the LHT model does not belong to the Minimal Flavour Violation (MFV) class of models [32–36] and significant effects in flavour observables are possible. In particular, the mirror fermion effects on FCNC observables in the quark sector have been studied in detail in [3,5,6,37–39], while an extensive analysis of lepton flavour violation has been performed in [4,40].

The beauty of the LHT model, when compared with other models with non-minimal flavour violating interactions, like the general MSSM and Randall-Sundrum [41,42] scenarios, is a relatively small number of new parameters and the fact that the local operators involved are the same as in the SM. Therefore, the non-perturbative uncertainties present in certain quantities already in the SM are the same in the LHT model, and the departures from the SM are entirely due to short distance physics that can be calculated within perturbation theory. In stating this we are aware of the fact that we deal here with an effective field theory whose ultraviolet completion has not been specified, with the consequence that at a certain level of accuracy one has to worry about the effects coming from the cut-off scale $\Lambda \sim 4\pi f$.

During the last two years several changes in the input parameters involved in our LHT analysis have taken place, and most recently some hints for large CP-violating effects in $b \to s$ transitions with $\Delta F = 2$ have been pointed out. More explicitly:

1. The value of $|V_{ub}|$ has been visibly lowered so that presently the tree level determination of the CKM matrix parameters is, in contrast to the situation in 2006, fully compatible with the size of the mixing-induced CP-asymmetry $S_{\psi K_S}$, although a small negative phase $\varphi_{B_d}$, defined through

$$S_{\psi K_S} = \sin(2\beta + 2\varphi_{B_d}),$$

(1.1)

cannot be excluded. Here $\beta$ is the true angle of the unitarity triangle.
2. The value of the running top quark mass $m_t(m_t)$ has decreased by roughly 1 GeV.

3. In the last two years, the lattice calculations of the parameter $\hat{B}_K$, that governs the CP-violating parameter $\varepsilon_K$, have been performed including for the first time the effects of dynamical quarks [43–45]. An average based on the unquenched results and on the scale dependence suggested by quenched studies reads $\hat{B}_K = 0.75 \pm 0.07$ [46] that is lower than the central value used in our 2006 analysis. In addition the analyses in [44–45] favour values for $\hat{B}_K$ in the ballpark of 0.70, whose implications on possible NP effects in $\varepsilon_K$ have been discussed in [47–48].

4. Most interestingly the extracted value of $S_{\psi\phi}$ from the CDF [49] and DØ [50] data turns out to be much larger than the SM value $S_{\psi\phi} \simeq 0.04$. By combining the CDF and DØ data, in fact, the UTfit collaboration finds [51]

\begin{equation}
0.32 \leq S_{\psi\phi} \leq 0.87 \quad (95\% \text{ C.L.})
\end{equation}

This latter result is certainly of interest for the LHT model as large values of this asymmetry have been found to be allowed by our analysis in [38]. Moreover the three effects 1.–3. significantly decrease the value of $\varepsilon_K$ in the SM so that some small contributions from NP in the $K$ system are welcome in order to reproduce the experimental value of $\varepsilon_K$.

All these findings motivate a new analysis of particle-antiparticle mixing and of rare $K$ and $B$ decays. In the present paper we update the most interesting results of our analyses in [3,38] addressing, in particular, the following questions:

- How large values of $S_{\psi\phi}$ can be obtained in the LHT model consistently with all other available data on FCNC processes?
- What would be the impact on the LHT predictions, if any, of a low value for $\hat{B}_K \simeq 0.70$ as hinted by the recent lattice determinations [44–45]?
- What are the LHT upper bounds on the branching ratios of the rare decays $K_L \to \pi^0 \nu\bar{\nu}$, $K^+ \to \pi^+ \nu\bar{\nu}$, $K_L \to \pi^0 \ell^+\ell^-$ and $B_s \to \mu^+\mu^-$ as functions of $S_{\psi\phi}$?
- How strong is in the LHT model the possible violation of the “golden” MFV relations between CP-violation in $B_d$-mixing and in rare $K$ decays and between $B_d$ and $B_s$ observables [52–53], and how does it depend on $S_{\psi\phi}$?

Our paper is organised as follows. In Section 2 we summarise very briefly the main ingredients of the LHT model, referring frequently to [38] and [3], where a more detailed description and all analytical expressions for the observables considered in our numerical analysis can be found. In Section 3 the main section of the present paper, we answer the questions posed above. We summarise and conclude in Section 4.
2 The LHT Model

A detailed description of the LHT model can be found for instance in\cite{[3,54]}. Here we just want to briefly review the particle content and the flavour structure of the LHT model.

2.1 Gauge Boson Sector

The T-even electroweak gauge boson sector \cite{[11]} consists only of SM electroweak gauge bosons

\[ W_L^\pm, \quad Z_L, \quad A_L, \quad (2.1) \]

with masses given to lowest order in \( v/f \) by

\[ M_{W_L} = \frac{g v}{2}, \quad M_{Z_L} = \frac{M_{W_L}}{\cos \theta_W}, \quad M_{A_L} = 0, \quad (2.2) \]

where \( \theta_W \) is the weak mixing angle. T-parity ensures that the second relation in (2.2) is satisfied at tree level to all orders in \( v/f \).

The T-odd gauge boson sector \cite{[11]} consists of the three heavy “partners” of the SM gauge bosons in (2.1):

\[ W_H^\pm, \quad Z_H, \quad A_H, \quad (2.3) \]

with masses given to lowest order in \( v/f \) by

\[ M_{W_H} = g f, \quad M_{Z_H} = g f, \quad M_{A_H} = \frac{g f}{\sqrt{5}}, \quad (2.4) \]

that satisfy the relation

\[ M_{A_H} = \frac{\tan \theta_W}{\sqrt{5}} M_{W_H} \simeq \frac{M_{W_H}}{4.1}. \quad (2.5) \]

2.2 Fermion Sector

The T-even fermion sector \cite{[11]} consists of the SM quarks and leptons and a colour triplet heavy quark \( T_+ \) that is, to leading order in \( v/f \), singlet under \( SU(2)_L \) and has the mass

\[ m_{T_+} = \frac{f}{v} \frac{m_t}{\sqrt{x_L(1-x_L)}}, \quad \text{with} \quad x_L = \frac{\lambda_1^2}{\lambda_1^2 + \lambda_2^2}. \quad (2.6) \]

Here \( \lambda_1 \) is the Yukawa coupling in the \((t, T_+)\) sector and \( \lambda_2 \) parameterises the mass term of \( T_+ \).

The T-odd fermion sector \cite{[26]} consists first of all of three generations of mirror quarks and leptons with vectorial couplings under \( SU(2)_L \). In this paper only mirror quarks are
relevant. We will denote them by
\[
\begin{pmatrix}
u^1_H \\ d^1_H 
\end{pmatrix}, \quad \begin{pmatrix}
u^2_H \\ d^2_H 
\end{pmatrix}, \quad \begin{pmatrix}
u^3_H \\ d^3_H 
\end{pmatrix},
\]
(2.7)
with their masses satisfying to first order in \(v/f\)
\[
m_{H1}^u = m_{H1}^d, \quad m_{H2}^u = m_{H2}^d, \quad m_{H3}^u = m_{H3}^d.
\]
(2.8)

The T-odd fermion sector contains also a T-odd heavy quark \(T_\pm\) which, however, does not enter our analysis [3,37,38].

For completeness we mention that also a Higgs triplet \(\Phi\) belongs to the T-odd sector. The charged Higgs \(\phi^\pm\), as well as the neutral Higgses \(\phi^0, \phi^P\), are relevant in principle for the decays considered here, but their effects turn out to be of higher order in \(v/f\) [3,38], and consequently similarly to \(T_\pm\) will not enter our analysis.

### 2.3 Weak Mixing in the Mirror Sector

As discussed in detail in [3,31,37,38], one of the important ingredients of the mirror quark sector is the existence of two CKM-like unitary mixing matrices \(V_{Hu}\) and \(V_{Hd}\), that satisfy
\[
V_{Hu}^\dagger V_{Hd} = V_{CKM}.
\]
(2.9)

These mirror mixing matrices parameterise flavour violating interactions between SM fermions and mirror fermions that are mediated by the heavy gauge bosons \(W_H, Z_H\) and \(A_H\). The notation indicates the type of light fermion that is involved in the interaction, i.e. if it is of up- or down-type.

Following [31] we will parameterise \(V_{Hd}\) generalising the usual CKM parameterisation, as a product of three rotations, and introducing a complex phase in each of them, thus obtaining
\[
V_{Hd} = \begin{pmatrix}
c_{12}c_{13} & c_{12}d_{13}e^{-i\delta_{12}} & s_{12}d_{13}e^{-i\delta_{12}} \\
s_{12}c_{13} & c_{12d_{13}} & s_{12}d_{13}e^{-i\delta_{12}} \\
s_{12}d_{13} & -s_{12}d_{13}e^{i\delta_{12}} & c_{12}d_{13}e^{i\delta_{12}}
\end{pmatrix}
\]
(2.10)

As in the case of the CKM matrix the angles \(\theta_{ij}^d\) can all be made to lie in the first quadrant with \(0 \leq \delta_{12}^d, \delta_{23}^d, \delta_{13}^d < 2\pi\). The matrix \(V_{Hu}\) is then determined through \(V_{Hu} = V_{Hd}V_{CKM}^\dagger\).
\[ |V_{us}| = 0.2261(15) \]
\[ |V_{ub}| = 3.8(4) \times 10^{-3} \]
\[ |V_{cb}| = 4.1(1) \times 10^{-2} \]
\[ \lambda = |V_{us}| = 0.2261(15) \]
\[ G_F = 1.16637 \times 10^{-5} \text{GeV}^{-2} \]
\[ M_W = 80.425 \text{GeV} \]
\[ \alpha(M_Z) = 1/127.9 \]
\[ \sin^2 \theta_W = 0.23122 \]
\[ m_K^0 = 497.648 \text{MeV} \]
\[ m_{B_s} = 5366.4 \text{MeV} \]
\[ m_{B_d} = 5279.5 \text{MeV} \]
\[ m_{B_s} = 5279.5 \text{MeV} \]
\[ m_{B_d} = 245(25) \text{MeV} \]
\[ \Delta M_K = 0.5292(9) \times 10^{-2} \text{ps}^{-1} \]
\[ \Delta M_d = 0.507(5) \text{ps}^{-1} \]
\[ \Delta M_s = 17.77(12) \text{ps}^{-1} \]
\[ S_{\psi K_S} = 0.681(25) \]
\[ \bar{m}_c = 1.30(5) \text{GeV} \]
\[ \bar{m}_t = 162.7(13) \text{GeV} \]
\[ f_K = 156(1) \text{MeV} \]
\[ f_{B_s} = 245(25) \text{MeV} \]
\[ f_{B_d} = 200(20) \text{MeV} \]
\[ B_K = 0.75(7) \]
\[ \hat{B}_{B_s} = 1.22(12) \]
\[ \hat{B}_{B_d} = 1.22(12) \]
\[ \hat{B}_{B_s}/\hat{B}_{B_d} = 1.00(3) \]
\[ \hat{B}_{B_s} \sqrt{\hat{B}_{B_s}} = 270(30) \text{MeV} \]
\[ \hat{B}_{B_d} \sqrt{\hat{B}_{B_d}} = 225(25) \text{MeV} \]
\[ \lambda = 80(20)^\circ \]
\[ \gamma = 80(20)^\circ \]
\[ \bar{m}_c = 1.30(5) \text{GeV} \]
\[ \bar{m}_t = 162.7(13) \text{GeV} \]
\[ f_K = 156(1) \text{MeV} \]
\[ f_{B_s} = 245(25) \text{MeV} \]
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\[ \bar{m}_c = 1.30(5) \text{GeV} \]
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\[ \hat{B}_{B_s} \sqrt{\hat{B}_{B_s}} = 270(30) \text{MeV} \]
\[ \hat{B}_{B_d} \sqrt{\hat{B}_{B_d}} = 225(25) \text{MeV} \]

Table 1: Values of the experimental and theoretical quantities used as input parameters.

3 Numerical Analysis

3.1 Preliminaries

In our previous analyses of flavour physics observables \([3, 5, 6, 38]\), we simplified the numerical analysis by setting all input parameters to their central values and allowing instead \(\Delta M_K\), \(\varepsilon_K\), \(\Delta M_d\), \(\Delta M_s\)/\(\Delta M_d\) and \(S_{\psi K_S}\) to differ from their experimental values by \(\pm 50\%\), \(\pm 40\%\), \(\pm 40\%\), \(\pm 40\%\), \(\pm 20\%\) and \(\pm 8\%\), respectively. While this simplifying assumption was justified in order to determine the size of possible NP effects in observables that have not been observed so far, an improved error analysis is now required in order to be able to draw more accurate remarks, in view of the recent significant improvements both of the experimental constraints on the NP phase \(\varphi_{B_s}\) in the \(B_s\) system \([49–51]\) and of the lattice determinations of the non-perturbative parameter \(\hat{B}_K\) \([43–45]\). Therefore, in what follows, we will take all input parameters to be flatly distributed within their 1\(\sigma\) ranges indicated in Table 1. At the same time we require the observables \(\varepsilon_K\), \(\Delta M_d\), \(\Delta M_s\) and \(S_{\psi K_S}\), resulting from SM and LHT contributions, to lie within their experimental 1\(\sigma\) ranges. In the case of \(\Delta M_K\) where the theoretical uncertainty is large due to unknown long-distance contributions, we allow the generated value to lie within \(\pm 30\%\) of the experimental central value. All formulae for the observables
discussed in the present paper can be found in [3, 38].

In [3, 38] several benchmark scenarios for the structure of the $V_{Hd}$ matrix have been discussed. Here we confine our discussion only to the general scan, where we perform, as in [3], a scan over the mirror fermion masses and the $V_{Hd}$ parameters, with the NP scale $f$ fixed to 1 TeV. To achieve this, we generate a large number of points where all mirror fermion masses are varied in the interval $[300\text{GeV}, 1\text{TeV}]$, all angles in the interval $[0, \pi/2]$, all phases between 0 and $2\pi$ and all SM input parameters are varied in their 1$\sigma$ ranges. Our plots show a sample of 15000 points that fulfil all experimental constraints. We do not specifically filter for “interesting” points, therefore the point density gives us an idea of how likely it is for the LHT model to generate a certain effect. To have a global view of the most general LHT effects, we have allowed here the phases $\delta_{d12}^d$ and $\delta_{d23}^d$ to differ from zero. Qualitatively their effect is not significant, although they can help in achieving very large effects in certain observables. In [3] we found that there exist some sets of masses and $V_{Hd}$ parameters where the NP effects turn out to be spectacular in both $B$ and $K$ systems. As we will see below, the present analysis, where a more accurate treatment of the uncertainties has been performed with respect to [3, 38], still permits large departures from the SM expectations.

### 3.2 Results

In all the plots presented here, we show only points consistent with the experimental 1$\sigma$ ranges of $\Delta F = 2$ data, in particular with $\varepsilon_K$ and $S_{\psi K_S}$. We represent the SM predictions and the LHT T-even contributions by black and light-blue dots, respectively.

In Fig. 1 we show the CP-asymmetry $S_{\psi K_S}$ versus $S_{\psi \phi}$. We observe that the SM prediction for $S_{\psi K_S}$ (black dot), obtained from the tree-level unitarity triangle analysis with the input parameters given by the central values in Table 1, is around $1.5\sigma$ higher than the data. On the other hand, when one considers the uncertainties on the input parameters, the SM prediction for $S_{\psi K_S}$ is found to be in good agreement with the data, showing that the tension between the data on $V_{ub}$ and $S_{\psi K_S}$, previously known as the “$\sin 2\beta$ problem” [38, 63–65], has disappeared. In order to illustrate this, we show for the SM prediction for $S_{\psi K_S}$ also the error bar originating from the uncertainty in $V_{ub}$, which turns out to be the dominant one. In the LHT model we find that sizable both positive and negative values of $S_{\psi \phi}$ relative to the SM value $(S_{\psi \phi})_{\text{SM}} \simeq 0.04$ are possible, and values as high as $\sim 0.15 - 0.20$ can easily be reached. While higher values are rather unlikely, they are not excluded at present. In addition a reversal of the sign of $S_{\psi \phi}$ appears to be unlikely, in accordance with recent data. On the other hand, basically all values for $S_{\psi \phi}$ found in the LHT model are outside the range [1.2], and consequently the confirmation of this result would put the LHT model into difficulties.
Figure 1: $S_{\psi K_S}$ as a function of $S_{\psi \phi}$. The black dot represents the tree-level SM prediction (see text), whose uncertainty is dominated by the error on $V_{ub}$, shown as error bar. The grey horizontal band displays the experimental $1\sigma$ range for $S_{\psi K_S}$, while the range given in [1,2] is shown by the light-grey vertical band.

Figure 2: The $B_s^0 - \bar{B}_s^0$ phase $\varphi_{B_s}$ as a function of the $B_d^0 - \bar{B}_d^0$ phase $\varphi_{B_d}$. 
Figure 3: The semileptonic CP-asymmetry $A_{SL}^\delta$ normalised to its SM central value as a function of $S_{\psi\phi}$.

In Fig. 2 we show the allowed points in the $(\varphi_{B_d}, \varphi_{B_s})$ plane. We note, again, that $\varphi_{B_d} < 0$ is preferred to fit the $S_{\psi K_s}$ data, while $\varphi_{B_s}$ appears rather symmetric around zero, with $\varphi_{B_s} < 0$ favoured by the recent CDF and DØ data [49–51], being $S_{\psi\phi} = \sin(2|\beta_s| - 2\varphi_{B_s})$. (3.1)

We observe, in particular, that in the LHT model $|\varphi_{B_s}| < 10^\circ$ which is significantly lower than $|\varphi_{B_s}| \approx 20^\circ$ corresponding to the central value of (1.2). Compared to our results in [3, 38], this improved error analysis and the modified input parameters do not significantly lower the maximal NP effects in $S_{\psi\phi}$, but make values $S_{\psi\phi} \gtrsim 0.2$ more unlikely.

In addition, we analysed possible correlations between the NP phase $\varphi_{B_s}$ and the LHT contributions to $\varepsilon_K$, where we found no relevant correlation. Therefore, the confirmation of a low value of $\hat{B}_K \approx 0.70$ hinted by recent lattice determinations [44] would not significantly modify our present conclusions. Indeed, as the contributions from the T-even sector always enhance $\varepsilon_K$ with respect to its SM value [38], the LHT model would be welcome to cure a possible tension between $\varepsilon_K$ and the tree level determined $\sin 2\beta_{true}$, recently hinted in [47, 48].

In Fig. 3 we show the correlation between $A_{SL}^\delta$ normalised to its SM central value versus $S_{\psi\phi}$. This plot is similar to the one in [38], but again huge enhancements of $A_{SL}^\delta$ and $S_{\psi\phi}$ are less likely than found in our 2006 analysis.

In Fig. 4 we show the correlation between $Br(K^+ \rightarrow \pi^+\nu\bar{\nu})$ and $Br(K_L \rightarrow \pi^0\nu\bar{\nu})$. The experimental $1\sigma$ range for $Br(K^+ \rightarrow \pi^+\nu\bar{\nu})$ [66, 67] and the model-independent Grossman-Nir (GN) bound [68] are also shown. We observe that the two branches of
Figure 4: The branching ratio $\text{Br}(K_L \to \pi^0 \nu \bar{\nu})$ as a function of $\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})$.

Figure 5: The branching ratio $\text{Br}(K_L \to \pi^0 \nu \bar{\nu})$ as a function of $S_{\psi\phi}$.

possible points identified in [3] still appear. The first one is parallel to the GN-bound and leads to possible large enhancements of $\text{Br}(K_L \to \pi^0 \nu \bar{\nu})$ up to values as high as $3 \cdot 10^{-10}$, being perfectly consistent with the measured value for $\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})$. The second branch corresponds to $\text{Br}(K_L \to \pi^0 \nu \bar{\nu})$ being rather close to its SM prediction, while $\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})$ is allowed to vary in the range $[1 \cdot 10^{-11}, 5 \cdot 10^{-10}]$, however values above $4 \cdot 10^{-10}$ are experimentally disfavoured. We also note, in accordance with our previous findings and in contrast to the SM relation $\text{Br}(K_L \to \pi^0 \nu \bar{\nu}) \simeq \text{Br}(K^+ \to \pi^+ \nu \bar{\nu})/3$, that in the LHT model $\text{Br}(K_L \to \pi^0 \nu \bar{\nu})$ can exceed $\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})$. In order to determine how these enhancements would be reduced in the case of a higher NP scale, we have performed an analysis with $f = 3$ TeV, finding that $\text{Br}(K_L \to \pi^0 \nu \bar{\nu})$ can be at most enhanced up to $7 \cdot 10^{-11}$, i.e. roughly a factor 5 less relative to the $f = 1$ TeV case.

In Figs. 5 and 6 we show the correlation between $S_{\psi\phi}$ and $\text{Br}(K_L \to \pi^0 \nu \bar{\nu})$ and...
Figure 6: The branching ratio $Br(K^+ \to \pi^+ \nu \bar{\nu})$ as a function of $S_{\psi \phi}$.

$Br(K^+ \to \pi^+ \nu \bar{\nu})$, respectively. We observe that large simultaneous enhancements of $S_{\psi \phi}$ and $Br(K_L \to \pi^0 \nu \bar{\nu})$ are rather unlikely, although for $S_{\psi \phi} \simeq 0.2$ a factor 3 enhancement of $Br(K_L \to \pi^0 \nu \bar{\nu})$ with respect to its SM value $3 \cdot 10^{-11}$ is possible. For higher values of $S_{\psi \phi}$, $Br(K_L \to \pi^0 \nu \bar{\nu})$ is expected to be SM-like in the LHT model. Consequently, a precise measurement of $S_{\psi \phi}$ will have an important impact on the allowed range for $Br(K_L \to \pi^0 \nu \bar{\nu})$. Similarly, for $S_{\psi \phi} \gtrsim 0.2$, the largest enhancements of $Br(K^+ \to \pi^+ \nu \bar{\nu})$ are not allowed, but values as high as $3 \cdot 10^{-10}$ are still possible.

Our new results for the correlation of $K_L \to \pi^0 \mu^+ \mu^-$ with $K_L \to \pi^0 e^+ e^-$ are very similar to those found in [3], therefore, here we just mention that $Br(K_L \to \pi^0 \mu^+ \mu^-)$ and $Br(K_L \to \pi^0 e^+ e^-)$ can be enhanced up to $3 \cdot 10^{-11}$ and $7 \cdot 10^{-11}$, respectively.

Then, we consider two theoretically clean ratios that are equal to unity in the SM and in MFV models, the so-called “golden” MFV relations [52, 53, 69]. Their deviation from one, therefore, would signal an evident effect of NP beyond MFV. As we discuss below, significant deviations are allowed in the LHT model and could be measured in the near future.

In Fig. 7 we show, as a function of $S_{\psi \phi}$, the ratio $\sin 2\beta_X^K / S_{\psi K_S}$, where $\beta_X^K$ denotes the angle $\beta$ determined from the $K \to \pi \nu \bar{\nu}$ system. In the SM and in MFV models $S_{\psi K_S}$ and $\sin 2\beta_X^K$ both provide a direct measurement of $\sin 2\beta$ and their ratio is then equal to one. Beyond MFV, instead, NP phases can affect the two determinations in a different way. As we can see from Fig. 7 in the LHT model, an enhancement of 50% and an even stronger suppression are allowed. Moreover, a sign inversion is less likely but not excluded. For $S_{\psi \phi} \gtrsim 0.1$, very strong suppressions are unlikely, while $\sim 50\%$ effects are still possible.

Another interesting parameter that would clearly reveal the presence of NP beyond
constrained MFV (CMFV) \cite{32,33,65} is the ratio \( r \) defined by \cite{53}

\[
\frac{Br(B_s \to \mu^+ \mu^-)}{Br(B_d \to \mu^+ \mu^-)} = \frac{\hat{B}_{B_s} \tau(B_s) \Delta M_s}{\hat{B}_{B_d} \tau(B_d) \Delta M_d} r.
\] (3.2)

This correlation relates the ratios \( Br(B_s \to \mu^+ \mu^-)/Br(B_d \to \mu^+ \mu^-) \) and \( \Delta M_s/\Delta M_d \). In the SM and in CMFV models it is valid with \( r = 1 \), while a value of \( r \) different from one would signal a NP effect beyond CMFV. As we can see from Fig. 8 in the LHT model, values in the range \([0.6, 1.3]\) are possible and are more probable if \( S_{\psi\phi} \) is close to the SM value.

Finally, we show in Fig. 9 the ratio \( Br(B_s \to \mu^+ \mu^-)/Br(B_s \to \mu^+ \mu^-)_{SM} \) as a function of \( S_{\psi\phi} \). A deviation from one would represent a NP signal that could be measured in future experiments. As we can see from Fig. 9 in the LHT model the branching ratio \( Br(B_s \to \mu^+ \mu^-) \) tends to be larger than the SM value, mainly due to the T-even contribution (denoted by the light-blue point in the plot). Large (\( \gtrsim 30\% \)) enhancements, however, are found to be very unlikely. In addition it is interesting to note that large NP effects in \( S_{\psi\phi} \) coincide with non-vanishing T-odd contributions to \( Br(B_s \to \mu^+ \mu^-) \), leading to either an additional enhancement or a suppression compensating the T-even effect \( Br(B_s \to \mu^+ \mu^-)_{T\text{-even}} \approx 1.2 Br(B_s \to \mu^+ \mu^-)_{SM} \), where

\[
Br(B_s \to \mu^+ \mu^-)_{SM} = (3.61 \pm 0.39) \cdot 10^{-9},
\] (3.3)

that we update here for completeness. In total, in the LHT model values as high as \( Br(B_s \to \mu^+ \mu^-) \approx 5.5 \cdot 10^{-9} \) can be reached.
Figure 8: The ratio \( r \), defined in (3.2), as a function of \( S_{\psi\phi} \).

Figure 9: The ratio \( Br(B_s \to \mu^+\mu^-)/Br(B_s \to \mu^+\mu^-)_{SM} \) as a function of \( S_{\psi\phi} \).
4 Conclusions

In the present paper we have reanalysed in the LHT model the most interesting FCNC observables in the quark sector, paying particular attention to the possible implications of a large value of $S_{\psi\phi}$, as hinted by the analysis of [51]. Our main findings are as follows:

1. The CP-asymmetry $S_{\psi\phi}$ in the LHT model can reach values up to $S_{\psi\phi} \simeq 0.4$, although values above 0.2 appear to be unlikely. In addition we find no significant correlation of $S_{\psi\phi}$ with the NP contribution to $\varepsilon_K$. In particular a possible tension between $\varepsilon_K$ and the tree level $\sin 2\beta_{\text{true}}$ can easily be accommodated in the LHT model.

2. For values $S_{\psi\phi} \gtrsim 0.2$ large enhancements of the rare decay branching ratios $Br(K^+ \to \pi^+\nu\bar{\nu})$, $Br(K_L \to \pi^0\nu\bar{\nu})$ and $Br(K_L \to \pi^0\ell^+\ell^-)$ are rather improbable. However, our general scan shows that with $S_{\psi\psi} \simeq 0.2$, $Br(K^+ \to \pi^+\nu\bar{\nu})$, $Br(K_L \to \pi^0\nu\bar{\nu})$ and $Br(K_L \to \pi^0\ell^+\ell^-)$ can still be enhanced by factors of 3, 3 and 1.5, respectively.

3. We also find a strong correlation between $A_s^{\psi\phi}$ and $S_{\psi\phi}$, such that $A_s^{\psi\phi}$ can largely deviate from the SM prediction.

4. The MFV “golden” relations $\sin 2\beta_K^X/S_{\psi K} = 1$ and $r = 1$ in (3.2) can be significantly violated. Deviations of 50% and 30%, respectively, turn out to be likely in the LHT model.

5. The branching ratio $Br(B_s \to \mu^+\mu^-)$ can be enhanced in the LHT model by at most 40% relative to the SM. A $Br(B_s \to \mu^+\mu^-)$ measurement above $6 \cdot 10^{-9}$, therefore, would put the LHT model in difficulties.

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Note

A version of this paper with full resolution plots is available at http://users.physik.tu-muenchen.de/mblanke/LHT-2008.pdf
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