Scalar Quarkonium and the Scalar Glueball

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Valence approximation glueball mass and decay calculations support the identification of \( f_J(1710) \) as the lightest scalar glueball. An alternate glueball candidate is \( f_0(1500) \). I present evidence for the identification of \( f_0(1500) \) as \( s\bar{s} \) quarkonium.

1. Introduction

The identification of \( f_J(1710) \) as the lightest scalar glueball is supported at present by two different sets of calculations.

For the valence approximation to the infinite volume continuum limit of the lightest scalar glueball mass, a calculation on GF1 (4), using 25000 to 30000 gauge configurations, gives \( 1740 \pm 71 \) MeV. An independent calculation by the UKQCD-Wuppertal (2) collaboration, using 1000 to 3000 gauge configurations, gives \( 1625 \pm 94 \) MeV when extrapolated to zero lattice spacing according to Ref. (3). The GF11 and UKQCD-Wuppertal data combined predicted \( 1707 \pm 64 \) MeV. The calculation with larger statistics and the combined result both favor \( f_J(1710) \) as the lightest scalar glueball.

The mass calculations by themselves, however, leave open the possibility that the lightest glueball may have too large a total width to be found in experiment. A valence approximation calculation on GF1 (4) of couplings for glueball decay to all possible pseudoscalar pairs, \( \pi + \pi, K + \bar{K}, \eta + \eta, \) and \( \eta + \eta' \), using a \( 16^3 \times 24 \) lattice at \( \beta = 5.7 \), gets \( 108 \pm 29 \) MeV for the total two-body width. Based on this number, any reasonable guess for the width to multibody states yields a total width small enough for the lightest scalar glueball to be seen easily in experiment. In fact, the predicted total two-body width agrees with the \( f_J(1710) \) width of \( 99 \pm 15 \) MeV of Ref. (3), as do the partial widths to individual channels.

Among established resonances with the quantum numbers to be a scalar glueball, aside from \( f_J(1710) \) all are clearly inconsistent with the mass calculation expect \( f_0(1500) \). The mass of \( f_0(1500) \) is still more than 3 sigma away both from the prediction with larger statistics or from the combined result. Ref. (4) proposes to interpret \( f_0(1500) \) as dominantly composed of \( s\bar{s} \) scalar quarkonium. The interpretation of \( f_0(1500) \) as \( s\bar{s} \) quarkonium, however, encounters three difficulties. First, it appears possible that the gap between \( 1740 \pm 71 \) MeV and \( 1500 \) MeV might simply be an error arising from the valence approximation. Second, \( f_0(1500) \) does not seem to decay mainly into states containing an \( s \) and an \( \bar{s} \) quark (5). Third, the Hamiltonian of full QCD couples quarkonium and glueballs so that \( f_J(1710) \) and \( f_0(1500) \) could both be linear combinations of quarkonium and a glueball, perhaps even half glueball and half quarkonium each.

In the remainder of this article, I show that the pattern of established quarkonium masses, an estimate of the error in valence approximation mass calculations, a calculation of the \( s\bar{s} \) scalar quarkonium mass and a model of quarkonium-glueball mixing help resolve these difficulties and support the interpretation of \( f_J(1710) \) as dominantly a glueball and \( f_0(1500) \) as dominantly \( s\bar{s} \) scalar quarkonium.

2. Comparison with Experiment

The simplest piece of evidence suggesting that \( f_J(1710) \) is mainly a glueball while \( f_0(1500) \) is mainly \( s\bar{s} \) quarkonium is provided by the pattern of masses among established meson resonances.

Figure (4) shows the established \( 0^+, 1^+ \) and \( 2^+ \) resonances and their strange \( 0^+, 1^+ \) and \( 2^+ \) partners, with the omission only of \( f_0(980), a_0(980) \) and \( f_1(1420) \), all irrelevant to glueball spectroscopy. The scalar glueball prediction of
Figure 1. Established $0^{++}$, $1^{++}$ and $2^{++}$ resonances and their strange $0^+$, $1^+$ and $2^+$ partners.

$1740 \pm 71$ MeV is labeled “g”. If $f_J(1710)$ were also omitted, the diagram would include exactly one state for each possible combination of $u$, $d$ and $s$ quark-antiquark pairs with orbital angular momentum 1 and total spin angular momentum 1. For all three values of total angular momentum, the $a_J$ states are isovector combinations of $u$ and $d$ quarks. From their masses and from their decay modes, it is not hard to show that the lower $f_J$ states are dominantly isoscalar combinations of $u$ and $d$ quarks. The $K_J$ states are all combinations of one $s$ and a $u$ or $d$. The higher $f_1$ and $f_2$ states both decay dominantly into states including both an $s$ and an $\overline{s}$. Thus the higher $f_1$ and $f_2$ are primarily $s\overline{s}$.

What about $f_J(1710)$ and $f_0(1500)$? From the discussion of the other states in the picture, it is clear that $f_0(1500)$ is about where it should for an $s\overline{s}$ quark-antiquark meson. The mass of $f_J(1710)$, meanwhile, is far from the region expected for $s\overline{s}$ quarkonium but fits the predicted glueball mass.

### 3. Valence Approximation Errors

Can the disagreement between $1740 \pm 71$ MeV and the mass of $f_0(1500)$ be an error arising from the valence approximation? In the GF11 calculation of eight infinite volume continuum limit hadron masses [7], the largest disagreement with experiment was 6%. Out of eight random variables, one is expected to be above its mean by a standard deviation or more. So 6% is a plausible estimate for the one sigma upper bound on valence approximation errors for light hadron masses. A 6% valence approximation error bound on the glueball mass would be 100 MeV.

A simple argument suggests, however, that as in the case of meson decay constants, full QCD is likely to yield a predicted value higher than the valence approximation prediction and will thus agree with 1500 MeV no better than does the valence approximation result. The valence approximation may be viewed as replacing the momentum and frequency dependent color dielectric constant arising from quark-antiquark vacuum polarization with its low momentum limit. At low momentum, then, the effective color charge appearing in the valence approximation will agree with the low momentum effective charge of the full theory. The valence approximation’s effective color charge at higher momentum can be obtained from the low momentum charge by the Callan-Symanzik equation. As a consequence of the absence of dynamical quark-antiquark vacuum polarization, the color charge in the valence approximation will fall faster with momentum than it does in the full theory, and therefore be smaller than it should be at high momentum. Since the scalar glueball is significantly heavier and significantly smaller in radius than the $\rho$, whose mass is used in the glueball mass calculations to set the color charge at low momentum, the glueball should include more high momentum chromoelectric field than built into the determination of the color charge. This high momentum part of the scalar glueball sees a smaller color charge in the valence approximation than it does in full QCD and therefore contributes less than it should to the glueball’s total energy. Thus the valence approximation scalar glueball mass will lie below the full QCD prediction.

In passing, it is perhaps also useful to consider how far the valence approximation, finite lattice spacing decay couplings are likely to be
from the real world. From the comparison of finite lattice spacing valence approximation hadron masses with their values in the real world, I would expect an error of 15% or less in going to the continuum limit and another 6% or less arising from the valence approximation. The total predicted width for glueball decay to two pseudoscalars should then have an error of less than 50%. A 50% increase in our predicted two-body decay width, combined with any reasonable corresponding guess for multibody decays, gives a total glueball width small enough for the particle to be observed easily.

4. Scalar Quarkonium Mass Calculation

Weonjong Lee and I [8] are in the process of calculating, in the valence approximation, the mass of scalar quark-antiquark bound states as an additional test of the hypothesis that $f_0(1500)$ is mainly $s\bar{s}$ quarkonium. Figure 2 shows the scalar $s\bar{s}$ mass we have obtained at two different values of lattice spacing. The lattice period in both cases is nearly 2.3 fm. The square at zero-lattice-spacing is the continuum limit of the scalar glueball mass, and horizontal lines show the $f_0(1500)$ and $f_0(1710)$ masses, all in units of the $\rho$ mass. It is clear from the two points so far that the valence approximation $s\bar{s}$ mass in the continuum limit with lattice period fixed at 2.3 fm will lie significantly below 1710 MeV and significantly below the predicted scalar glueball mass. It is shown in Ref. [8] that taking the infinite volume limit of the $s\bar{s}$ mass will not change this conclusion. These results tend to support the interpretation of $f_0(1500)$ as an $s\bar{s}$ state, and tend to exclude the possibility that $f_J(1710)$ might be an $s\bar{s}$.

5. Glueball-Quarkonium Mixing Model

I will now consider a simple model of the mixing between the scalar glueball and isosinglet quarkonium states. The result will be that the mixed glueball is more than 75% pure glueball and the mixed quarkonium states are more than 75% pure quarkonium. The mixing is still sufficient, however, to give a strong suppression of $K\bar{K}$ decays from the $f_0(1500)$. An orthogonal model of glueball-quarkonium mixing, which takes $f_0(1500)$ to be primarily a glueball, is discussed in Ref. [9].

To leading order in the valence approximation, with valence quark annihilation turned off, corresponding isotriplet and isosinglet states composed of $u$ and $d$ quarks will be degenerate. For the scalar meson multiplet, the isotriplet state of $u$ and $d$ quarks has a mass of 1450 MeV [10]. An isosinglet mass of 1390 MeV is reported by the Crystal Barrel collaboration [6]. Mark III finds 1430 MeV [11]. The isosinglet-isovector splitting gives a measure of the strength of the valence quark annihilation process for isosinglet scalar mesons. Some part of this splitting will arise from annihilation into a scalar glueball. If we assume the splitting arises entirely from coupling to the scalar glueball and take the lower isosinglet mass, 1390 MeV, we should get an upper estimate on the strength of this coupling. Since scalar meson masses appear to depend relatively weakly on quark mass, let us introduce the further assumption that this valence quark annihilation amplitude is the same for $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$.

The structure of the Hamiltonian coupling together the scalar glueball, the scalar $s\bar{s}$ and the scalar $(u\bar{u} + d\bar{d})$ isosinglet becomes

$$
\begin{pmatrix}
  m_g & z & \sqrt{2}z \\
  z & m_{s\bar{s}} & 0 \\
  \sqrt{2}z & 0 & m_{u\bar{u} + d\bar{d}}
\end{pmatrix}
$$

Here $z$ is the annihilation amplitude for quark-
antiquark into a glueball, \( m_g \) is the glueball mass before mixing with quarkonium, and \( m_{\pi^+}, m_{\pi^-}, m_{\pi^0} \) are quarkonium masses before mixing with the glueball and each other. The four unknowns in this matrix can be determined from four observed masses. Our key assumption, described above, is that the isosinglet \( m_{u^u+d^d} \) before mixing is the same as the observed isotriplet mass, 1450 MeV. In addition, we take the masses of the physical, mixed states with the largest contributions from \( u^u+d^d, s^s \) and the glueball to be, respectively, the observed resonance masses of 1390 MeV, 1500 MeV and 1710 MeV.

Adjusting the parameters in the matrix to give the physical eigenvalues we just specified, the unmixed masses and mixing parameter become

\[
\begin{align*}
m_g &= 1635 \text{ MeV}, \\
m_{\pi^+} &= 1516 \text{ MeV}, \\
z &= 77 \text{ MeV},
\end{align*}
\]

while the physical mixed states as linear combinations of the unmixed states are

\[
\begin{align*}
|1710> &= \\
&\quad 0.87|g> + 0.34|s^s> + 0.36|u^u+d^d>, \\
|1500> &= \\
&\quad -0.19|g> + 0.90|s^s> - 0.40|u^u+d^d>, \\
|1390> &= \\
&\quad -0.46|g> + 0.28|s^s> + 0.84|u^u+d^d>.
\end{align*}
\]

All mixed and unmixed states vectors here are normalized to 1. Measured in probability, the mixed glueball is more than 75% pure glueball and the mixed \( f_0(1500) \) is more than 75% pure \( s^s \). The mixed \( f_0(1390) \) is 71% \( u^u+d^d \) and 8% \( s^s \) for a total of about 79% pure quarkonium.

The negative sign of the contribution of \( |u^u+d^d> \) to \( f_0(1500) \) can lead to interference with the decay amplitude coming from \( |s^s> \) and suppress the width for \( f_0(1500) \) decays to \( K\overline{K} \). Assuming, for example, SU(3) symmetry of the coupling constants for unmixed scalar quarkonium to \( K\overline{K} \), we obtain a rate for the mixed state \( f_0(1500) \) which is about 40% of the rate for unmixed \( |s^s> \) decay to \( K\overline{K} \). The negative sign of the \( |g> \) coefficient will lead to a further suppression if, as simple models suggest, \( |s^s> \) decay to pairs of pseudoscalars has the same sign as glueball decay to pairs of pseudoscalars.

On the other hand, since the glueball probability in \( f_0(1390) \) is nearly six times that in \( f_0(1500) \), we expect the rate for the decay of \( J/\Psi \) to \( \gamma f_0(1390) \) to be significantly larger than the rate for decay to \( \gamma f_0(1500) \). This expectation is supported by Mark III data [1].

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