Gauge corrections and FI-term in 5D KK theories

D.M. Ghilencea, S. Groot Nibbelink, H.P. Nilles

Physikalisches Institut der Universität Bonn, Nussallee 12, 53115 Bonn, Germany.

Abstract

In the context of a five dimensional $\mathbb{N}=1$ Kaluza Klein model compactified on $S_1/\mathbb{Z}_2 \times \mathbb{Z}_2'$ we compute the one-loop gauge corrections to the self energy of the (zero-mode) scalar field. The result is quadratically divergent due to the appearance of a Fayet-Iliopoulos term.
1 Introduction

There has recently been growing interest in the phenomenological aspects of large additional (compact) space dimensions in the context of Kaluza-Klein models. Various models in this direction have been built and their starting point is in general the assumption of the existence of a 5 dimensional N=1 supersymmetric model compactified on $M^4 \times S^1/\mathbb{Z}_2$ or on $M^4 \times S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$. While the details of these models are rather involved and model dependent, an interesting generic feature emerges, that the potential and the Higgs mass in one-loop order are ultraviolet insensitive as far as Yukawa contribution is concerned. It is not clear to what extent this property may survive to higher orders, since the (one loop) couplings of the 4 dimensional theory have in general a dependence on the high scale worse than that (of logarithmic type) of the minimal supersymmetric standard model (MSSM). This dependence may in turn be re-introduced in the expressions of the potential and the Higgs mass beyond one-loop order. Therefore high scale sensitivity may be restored via the couplings of the theory due to the extra dimension affecting their “running”. This sensitivity may be further affected by the non-renormalizable character of these models.

Of this class of Kaluza Klein models, our attention was drawn to that of reference [4] which is a very interesting construction providing an extension of the standard model to a 5 dimensional N=1 supersymmetric theory compactified on an orbifold $M^4 \times S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$. All Standard Model (SM) states have associated Kaluza Klein states with respect to the extra dimension. Even though the 5 dimensional model is supersymmetric (before compactification), it has the distinctive feature that its low energy particle spectrum contains only one light Higgs doublet, that corresponding to the SM. In this respect the model is more similar to the standard model than to its minimal supersymmetric extension (MSSM).

It is well known that in the SM model the Higgs sector suffers from an extreme fine-tuning due to the quadratic divergence of the mass parameter in the Higgs potential. In the MSSM model all quadratic divergences are however absent. The reason for this is two-fold: the model is supersymmetric and the only quadratic divergent term is the Fayet-Iliopoulos (FI)-term. However, this divergence is proportional to the sum of the hyper-charges of all massless complex scalars. In the MSSM this sum is zero, because it is equal to the sum of hyper charges of the chiral fermions by supersymmetry. This vanishes so that the mixed gravitational-gauge anomaly does not arise. Hence, the MSSM does not contain quadratic divergences.

From this perspective, one would like to address the situation of the model [4], since it apparently has supersymmetric features as it is obtained by compactifying a $N = 1$ supersymmetric theory in 5 dimensions on the orbifold $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2'$. As a result the Kaluza-Klein (KK) towers fall into multiplets of this supersymmetry. However, the low energy spectrum (of massless states) which is precisely that of the standard model has only one Higgs state. It is thus not clear to what extent the properties of $N = 1$ in 5 dimensions manifest themselves after the compactification, to protect the model from divergences.

Not all properties of the initial $N = 1$ theory on 5 dimensional Minkowski space remain after compactification on an orbifold. For example, this compactification makes it possible that a chiral spectrum exists in the effective 4 dimensional theory, while the uncompactified version is necessarily non-chiral. In this way not only can a chiral spectrum arise, but also the possibility of anomalies is opened up. In an accompanying paper [6] anomalies in such theories will be investigated. Not

---

4Here we disregard the renormalization of the vacuum energy or cosmological constant term, as it is not clear to what extent it fits in a theory that does not take quantum gravity into account.
surprisingly, it is found that only the exactly massless chiral fermions in the effective 4 dimensional theory can contribute to it. This is of course in perfect agreement with the index theorems that are behind the anomalies \[7, 8, 9\].

The renormalization of the FI-term, the tadpole of the auxiliary field in the vector multiplet, only depends on massless states at one loop \[10\] and is therefore very similar. However, for the calculation of the FI contribution we have to rely heavily on a regularization prescription. Each scalar mode gives a quadratically divergent contribution.

In the present paper we investigate the possible role of a FI-term in the model of ref. \[4\]. Therefore we discuss \(N = 1\) supersymmetric gauge theory coupled to a hypermultiplet in 5 dimensions and its compactification on the orbifold \(S^1/\mathbb{Z}_2 \times \mathbb{Z}'_2\). We observe that a FI-term at the boundary is allowed by all the symmetries of the theory. We then compute the one loop contribution to the FI-terms using the method set up in ref. \[17\]. We find a quadratic divergence of the FI-term, which in turn induces a quadratic divergence for the Higgs mass term.

## 2 Five-dimensional vector and hyper multiplet

In this section we give the 5 dimensional \(N = 1\) action that will be the basis of our later discussion and describe its compactification on \(M^4 \times S^1/\mathbb{Z}_2 \times \mathbb{Z}'_2\). Before that we briefly motivate our gauge choice for the Abelian gauge field.

The Lagrangian for a 5 dimensional Abelian gauge field \(A_M\) with field strength \(F_{MN}\) reads

\[
L_G = -\frac{1}{4} F_{MN} F^{MN},
\]

(1)

Since in the 5 dimensional theory the 5th dimension is compact while the others are not, the theory has only 4 dimensional Lorentz invariance. This implies that also for the gauge fixing term 5 dimensional Lorentz invariance may be broken. We take

\[
L_{G.F.} = -\frac{1}{2} (\alpha_4 \partial_\mu A^\mu + \alpha_5 \partial_5 A^5)^2,
\]

(2)

with \(\alpha_4\) and \(\alpha_5\) real gauge fixing parameters. (If we take \(\alpha_4 = \alpha_5\) we recover the original 5 dimensional gauge fixing condition.) These gauge fixing terms can be obtained by integrating the gauge fixing \(\alpha_4 \partial_\mu A^\mu + \alpha_5 \partial_5 A^5 = \omega\) in the path integral over a Gaussian distribution \(\int D\omega \exp i \int d^5x \frac{1}{2} \omega^2\). As usual, for an Abelian theory with such a gauge fixing term, the ghost sector decouples from the physical part of the theory. In momentum space this leads to the form

\[
L_{G tot} = -\frac{1}{2} (A^*_\mu A^\mu_5 - (1 - \alpha_4^2)p^\mu p^\nu - (1 - \alpha_4 \alpha_5) p^\mu p_5^\nu) \left( A^\mu_5, A^\nu_5 \right),
\]

(3)

with \(p^2 = p_4^2 + p_5^2\) is the sum of the 4 and 5 dimensional momentum squared. Notice that if we take \(\alpha_5 \alpha_4 = 1\), we find that the gauge field \(A_5\) decouples from the scalar \(A_5\). In addition we set \(\alpha_5 = 1\), thus the propagator for \(A_5\) is simply proportional to \(1/p^2\). In this case \(\alpha_4 = \alpha_5 = 1\), we thus obtained the (generalized) Feynman gauge. For the one-loop calculation that is performed later, this gauge will be used for simplicity.

The \(N = 1\) supersymmetric 5 dimensional Lagrangian \(L = L_V + L_H\) describes a vector and a hyper multiplet. The on-shell form of this Lagrangian can be found in \[11, 12, 2\]; the off-shell
formulation is given in [13] (for the vector multiplet) and [14, 15]. The components of the vector multiplet $V = (A_M, \lambda_i, \Phi, D^a)$ are, apart from the vector field $A_M$, a symplectic Majorana gaugino $\lambda$, a real scalar field $\Phi$ and an $SU_R(2)$ iso-triplet of auxiliary scalars $D^a$. Also the gaugino transforms as a doublet under $SU_R(2)$. Including the gauge-fixing term in the Feynman gauge, their action is given by

$$L_V = -\frac{1}{4} F^{a}_{MN} - \frac{1}{2} (\partial_M A^M)^2 - \bar{\lambda} \gamma^a \lambda - \frac{1}{2} (\partial_M \Phi)^2 + \frac{1}{2} (D^a)^2. \hspace{1cm} (4)$$

The hyper multiplet $H = (h^i_\alpha, \zeta, F^i_a)$ consists of a bi-doublet of $SU(2) \times SU_R(2)$ scalars $h^i_\alpha$, a hyperino $\zeta_\alpha$ and a bi-doublet auxiliary scalars $F^i_a$. This hyper multiplet couples to $A_M$ as it carries hyper charge; the derivatives have become covariant derivatives with gauge coupling constant $\tilde{g}$ in 5 dimensions. This leads to the Lagrangian

$$L_H = -|D_M h^i_\alpha|^2 - \bar{\tilde{\zeta}}^\alpha (D \bar{g} \gamma^5 \Phi) \zeta_\alpha + |F^i_a|^2 - (i \sqrt{2} \tilde{g} h^i_\alpha \bar{\lambda}^{\dot{\alpha}} \zeta_\alpha + \text{h.c.}) - \tilde{g}^2 h^i_\alpha \Phi^2 h^i_\alpha - \tilde{g} D^a h^i_\alpha \sigma_a h^i_\alpha. \hspace{1cm} (5)$$

Let us consider now the situation that arises when these fields are placed on the orbifold $S^1/\mathbb{Z}_2 \times \mathbb{Z}_2$. This orbifold is defined by the periodicity $x_5 \to x_5 + 2\pi R$ and the two parities $x_5 \to -x_5$ and $x_5 \to \pi R - x_5$. By performing similar analyses as discussed in [13, 14], one finds that both the gaugino and the hyperino have to transform under the parities in a non-trivial way. An allowed (though not unique) choice is given by

$$\lambda(-x_5) = a_R \gamma^5 \lambda(x_5), \hspace{1cm} \lambda(\pi R - x_5) = -a_R \gamma^5 \lambda(x_5),$$

$$\zeta(-x_5) = a \gamma^5 \zeta(x_5), \hspace{1cm} \zeta(\pi R - x_5) = -a \gamma^5 \zeta(x_5),$$

where $a$ and $a_R$ are elements of the algebras of $SU(2)$ and $SU_R(2)$, respectively. This means, for example, that $a_R = a_R \sigma$, with $\sigma$ the vector of Pauli matrices. In ref. [14] this is done with $a = a_R = \sigma_3$. One then finds the parity assignments for the Kaluza-Klein modes (see also Appendix A)

| fields $h^\alpha_n$, $h^{\alpha-\alpha}_n$ | $\zeta^\alpha_n$ | $A^a_{M}$ | $A^a_{5}$ | $\lambda^\alpha_n$ | $\Phi_{\alpha}$ | $D^{\parallel}_n$ | $\tilde{D}^\perp_n$ |
|---------------------------------------------|-----------------|----------|----------|-----------------|-------------|--------------|--------------|
| parities                                    | --              | + +      | $\alpha - \alpha$ | + +            | $i - i$     | --           | + +          |
| modes $n$                                   | $\geq 1$ $\geq 0$ | $\geq 0$ | $\geq 0$ $\geq 1$ $\geq 0$ | $\geq 0$ $\geq 1$ $\geq 0$ | $\geq 0$ $\geq 1$ $\geq 0$ |

with $\alpha, i = \pm$. Of the three auxiliary scalars, that form a triplet $\tilde{D}$ under $SU_R(2)$ of the $N = 1$ vector multiplet, two are odd under both parities $\tilde{D}^\perp = (1 - a_R a_R^T) \tilde{D}$, while the other one $D^\parallel = a_R a_R^T \tilde{D}$ is even. The resulting Feynman rules for these KK fields can be derived from the Lagrangians by using products rules for the mode functions and their orthonormality properties [21].

3 The Fayet-Iliopoulos term

It is well-known that in a (unbroken) supersymmetric field theory in 4 dimensions the FI-term is either quadratically divergent or vanishes at one loop. The diagram of the FI-contribution to the selfenergy of a scalar\footnote{This can be the zero mode or any of the KK excitations.} is given by,
where the dotted line corresponds to the auxiliary field $D^{||}$ of the Abelian gauge multiplet in 4 dimensions. We investigate what happens to the FI-term in the effective field theory coming from 5 dimensions with a mass spectrum of the complex scalars of the hyper multiplet on $S^1/\mathbb{Z}_2 \times \mathbb{Z}_{2}'$, as discussed in appendix A. We take the charges of these scalars such that $q_n^{++} = -q_n^{--} = 1$ because they belong to complex conjugate representations. Formally, the expression for the one loop contribution to the FI term reads

$$
\xi = \sum_{n,\alpha} g q_n^{\alpha\alpha} \int \frac{d^4 p_4}{(2\pi)^4} \frac{1}{p_4^2 + (m_n^{\alpha\alpha})^2 + m^2},
$$

(7)

where $m_n^{\alpha\alpha} = 2n/R$ and the sum for $\alpha = +$ is over $n \geq 0$, while for $\alpha = -$ over $n > 0$. Here the 4 dimensional gauge coupling $g$ is related to the 5 dimensional gauge coupling $\tilde{g}$ by $g^2 = \frac{4}{\pi R} \tilde{g}^2$. We have introduced an IR regulator mass $m$, which is needed to turn the sum into a contour integral by complex function analysis [17]. To obtain this expression we made the following observations. Because the components of the vector multiplet are all 5 dimensional fields, for any $n$-point function we have 5 dimensional momentum and parity conservation. This means in particular that there is no momentum flow into the tadpole and that the parity of the tadpole is $++$. Therefore only the zero mode of $D^{||}$ is relevant here. Since only scalars $h$ run around in the loop, their parity is either $++$ or $--$. The vertices are then $\delta_{nn'} + \delta_{n0} \delta_{n'0}$ and $-\delta_{nn'}$ for $++$ and $--$ parity scalars, respectively, because of the orthonormality relation (21) and their charges. Similarly, for the propagators we have

$$
\Delta^{++}_{nn'} = \frac{1}{p_4^2 + (2n/R)^2} (\delta_{nn'} - \frac{1}{2} \delta_{n0} \delta_{n'0}), \quad \Delta^{--}_{nn'} = \frac{1}{p_4^2 + (2n/R)^2} \delta_{nn'}.
$$

(8)

Notice that the different normalization of the propagator the massless mode is compensated by the normalization of the vertex, so that we can simply sum all contributions with the same weight.

As the expression (7) contains a potentially divergent sum and integral, it can only be defined unambiguously using some regularization. We use dimensional regularization for both the integral and the sum, as was discussed in ref. [17]. The relevant pole functions $\mathcal{P}^{++}$ and $\mathcal{P}^{--}$ for the momentum spectra of $\phi_n^{++}$ and $\phi_n^{--}$ are given in eq. (22) of appendix A. According to the dimensional regularization procedure, we obtain

$$
\xi = g \int \frac{d^4 p_4}{(2\pi)^4} \int d^5 p_5 \left\{ \frac{\mathcal{P}^{++}(p_5)}{p_4^2 + p_5^2 + m^2} - \frac{\mathcal{P}^{--}(p_5)}{p_4^2 + p_5^2 + m^2} \right\}.
$$

(9)

Since usual the complex dimensions $D_4$ and $D_5$ are chosen such that the whole expression is convergent and then analytically continued. This procedure is summarized in appendix B. Substituting the expressions of the pole functions (22), gives exactly the same result as the regulated FI term for one massless complex scalar:

$$
\xi = g \int \frac{d^4 p_4}{(2\pi)^4} \int d^5 p_5 \frac{1}{p_4^2 + p_5^2 + m^2} = g \int \frac{d^4 p_4}{(2\pi)^4} \frac{1}{p_4^2 + m^2}.
$$

(10)
Since the result (10) behaves like a zero-mode particle contribution we can safely take \( D_5 = 1 \) and remove the contour integration, to obtain exactly the 4 dimensional expression. This shows that the FI term at one loop is simply given by the contribution of the massless complex scalar, putting \( m \to 0 \). This result holds for any finite \( R \), since it is independent of the radius \( R \) of the compact dimension. Therefore, we conclude that it is also true in the limit \( R \to \infty \). This signals that the boundary condition of the orbifolding is not removed in this decompactification limit.

We have performed a similar calculation for the case in which we have complex scalar states that have odd wave functions (\( \phi^+_{\mu} \) and \( \phi^-_{\mu} \)). However, since in that case all fermionic states have nonvanishing masses, their contribution to a FI term is zero.

For this Fayet-Iliopoulos contribution, an auxiliary field tadpole counter term has to be introduced. Such a counter term of course has to be consistent with the symmetries of the theory. On both branes we have at most \( N = 1 \) supersymmetry because the other supersymmetry has the wrong parity to exist on that brane. Therefore, on the branes \( D \)-terms can be added for the auxiliary fields that do not vanish. As can be seen from the table in section 2 the only auxiliary field that exists on either brane is \( D_\parallel \).

### 4 Other gauge corrections to the self energy

In the previous section we have only focused on the correction to the tadpole of the component of the auxiliary field \( D_\parallel \) of the vector multiplet. This result gives in turn a contribution to the renormalization of the scalar masses. In this section we calculate other gauge coupling corrections to the self energy of scalar fields. Here we restrict ourselves to the corrections to the self energy of the zero mode scalars only. The relevant diagrams are given below

![Diagrams](image-url)

Here a wavy line stands for a gauge field \( (A_\mu, A_5) \), a wavy line with an arrow a gaugino \( \lambda \), a line with an arrow a hyperino \( \zeta \), a dashed line a real scalar \( \Phi \) and a dotted line is an auxiliary field \( D^a \). On the orbifold \( S^1/Z_2 \times Z_2' \) these fields are classified as even or odd under both parities, see table in section 2.

We use dimensional reduction [18] to treat the fermions, i.e. the fermionic traces are computed in 5 dimensions, the resulting sum and integrals are dimensionally regulated [17]. We denote the gauge correction (except the FI tadpole) to the self energy by \( \Sigma_G \) and write the integrant as the sum of five terms, corresponding each to the diagrams given above

\[
-i \Sigma_G = -\frac{1}{2\pi i} \int \frac{d^{D_4} p_1}{(2\pi)^{D_4}} \int_\Theta d^{D_5} p_5 \left( I + I + II + III + IV + V \right). \tag{11}
\]

If the hyper multiplet is a doublet under \( SU_L(2) \) this result is multiplied by 2.
Using the Feynman rules resulting from the Lagrangians (4), (5) and the table given in section 2 we find

\[
I = -4g^2 \frac{1}{p^2} P^{++} - g^2 \frac{1}{p^2} P^{--},
\]

\[
II = +g^2 \frac{p_1^2}{(p^2)^2} P^{++} + g^2 \frac{p_5^2}{(p^2)^2} P^{--},
\]

\[
III = -g^2 \frac{1}{p^2} P^{--},
\]

\[
IV = -g^2 \frac{1}{p^2} P^{++} - 2g^2 \frac{1}{p^2} P^{--},
\]

\[
V = 4g^2 \frac{1}{p^2} (P^{++} + P^{--}).
\]

Here the factor \( \frac{1}{2} \) is a sign that the fermion states have become chiral. The pole-functions (22) are used to rewrite the sums over the KK momentum as contour integrals.

To identify the 5 dimensional, 4 dimensional and finite contributions of these diagrams, we can use the expressions for the pole functions given in (26) and (28) for \( \text{Im} \ p_5 > 0 \) and \( \text{Im} \ p_5 < 0 \), respectively. The purely 5 dimensional divergences for \( \text{Im} \ p_5 > 0 \) (and for \( \text{Im} \ p_5 < 0 \) for the same reason) vanishes:

\[
(I + \ldots + V)_{5D} = -\frac{g^2}{2} \frac{1}{p^2} \left( \frac{i}{2} \pi R \right) \left[ 4 + 1 - \frac{p_1^2}{p^2} - \frac{p_5^2}{p^2} + 1 + 3 - 8 \right] = 0.
\]

This cancellation is reminiscent of the full \( N = 1 \) supersymmetry in 5 dimensions in the uncompactified theory. (If one takes the more general gauge choice \( \alpha_4 = \alpha_5 \neq 1 \), this contribution does not vanish anymore.)

The purely 4 dimensional divergences of these diagrams vanish also. For the integrant we obtain

\[
(I + \ldots + V)_{4D} = -\frac{g^2}{2} \frac{1}{p^2} \left( 4 - 1 - \frac{p_1^2}{p^2} + \frac{p_5^2}{p^2} - 1 + 1 - 2 \right) \frac{1}{p_5} = -\frac{g^2}{2} \frac{1}{p^2} \frac{p_5}{p^2}.
\]

In the corresponding integral we set \( D_5 = 1 \), because this contribution does not contain an infinite sum. Furthermore, it can be written as a contour around the real axis, using the reverse steps as in (23). However, since there is no pole of \( p_5 \) on the real axis, this contribution vanishes identically. (The same result is obtained for the more general gauge fixing \( \alpha_4 = \alpha_5 \neq 1 \).)

Since we do not find any quadratic divergences here, it is clear that it is not possible to cancel the quadratic contribution of the FI-term (10) to the scalar mass. This situation is similar to the case of \( N = 1 \) supersymmetric models in 4 dimensions.

Finally, we can identify the finite contributions. For \( \text{Im} \ p_5 > 0 \), their integrants read

\[
(I + \ldots + V)_{\text{finite}} = -\frac{g^2}{2} \frac{1}{p^2} \left[ 4 + 1 - \frac{p_1^2}{p^2} - \frac{p_5^2}{p^2} + 1 + 3 \right] (-\rho_-(p_5)) - 8\rho_+(p_5)
\]

\[
= +4g^2 \frac{1}{p^2} \left[ \rho_-(p_5) + \rho_+(p_5) \right].
\]

For \( \text{Im} \ p_5 < 0 \), we find

\[
(I + \ldots + V)_{\text{finite}} = -4g^2 \frac{1}{p^2} \left[ \rho_-(p_5) + \rho_-(p_5) \right].
\]

Since this is a finite contribution, we remove the regulators: \( D_5 = 1 \) and \( D_4 = 4 \) and we obtain (using (27))

\[
-i\Sigma_G = i \frac{7g^2}{16\pi^2} \left( \frac{2}{R} \right)^2 \zeta(3).
\]
5 Conclusion

We have discussed gauge corrections to the mass parameter in the scalar potential in the effective 4 dimensional field theory, obtained from a $N = 1$ supersymmetric field theory in 5 dimensions. In particular, we have seen that the tadpole contribution to a component of the auxiliary field is quadratically divergent and proportional to the sum of massless scalar fields. This result was obtained by using dimensional regularization on both Kaluza-Klein sum and 4 dimensional momentum integral separately. Using the properties of the pole functions associated with the orbifold, it is not difficult to identify the 5 and 4 dimensional divergent contributions and the additional, finite parts. For the tadpole of the auxiliary field it turned out that only a purely 4 dimensional divergence is left.

This situation is very similar to that of 4 dimensional supersymmetric field theories, where the FI-term gives an identical result. This contribution cannot be removed: even if we take $R \to \infty$ we find the same expression due to the massless modes. In this sense, the behavior is very similar to anomalies, where only the zero-mode fermions contribute [6].

Apart from this Fayet-Iliopoulos contribution, we have calculated other gauge corrections to the mass parameter in the effective potential. We have shown explicitly that both the possible 5 and 4 dimensional divergences cancel, leaving a finite contribution. Therefore, the quadratically divergent FI contribution cannot be compensated. Thus, only in models where the sum of the hyper charges of the massless complex scalars is zero can the quadratic divergence to the FI-term be absent.

In the $S^1/Z_2 \times Z'_2$ model under consideration we have therefore shown unambiguously that FI-term and Higgs mass are quadratically divergent at one loop. For a somewhat complementary discussion of this issue see ref. [3].

Acknowledgments

The authors thank A. Dedes for helpful discussions. This work is supported by priority grant 1096 of the Deutsche Forschungsgemeinschaft and European Commission RTN programmes HPRN-CT-2000-00131 and HPRN-CT-2000-00148.

Appendix

A Mode functions $S^1/Z_2 \times Z'_2$

The mode functions $\phi_n^{\alpha\beta}(x_5)$ can now be even or odd under either of these $Z_2 \times Z'_2$ symmetries:

\[ Z_2 : \phi_n^{\alpha\beta}(-x_5) = \alpha \phi_n^{\alpha\beta}(x_5), \quad Z'_2 : \phi_n^{\alpha\beta}(\pi R - x_5) = \beta \phi_n^{\alpha\beta}(x_5), \]  

(18)

with $\alpha, \beta = \pm$ as eigenvalues. Their real representations are

\[ \phi_n^{++}(x_5) = \cos \frac{2nx_5}{R}, \quad n \geq 0, \quad \phi_n^{--}(x_5) = \sin \frac{2nx_5}{R}, \quad n > 0, \]

\[ \phi_n^{+-}(x_5) = \cos \frac{(2n+1)x_5}{R}, \quad n \geq 0, \quad \phi_n^{-+}(x_5) = \sin \frac{(2n+1)x_5}{R}, \quad n \geq 0. \]  

(19)
From this it is easy to see that under differentiation
\[
\frac{\partial}{\partial x_5} \phi_{n}^{\alpha \alpha}(x_5) = -\alpha \frac{2n}{R} \phi_{n}^{-\alpha - \alpha}(x_5), \quad \frac{\partial^2}{\partial x_5^2} \phi_{n}^{\alpha \alpha}(x_5) = -\frac{4n^2}{R^2} \phi_{n}^{\alpha \alpha}(x_5),
\]
(20)
\[
\frac{\partial}{\partial x_5} \phi_{n}^{-\alpha - \alpha}(x_5) = -\alpha \frac{2n+1}{R} \phi_{n}^{-\alpha \alpha}(x_5), \quad \frac{\partial^2}{\partial x_5^2} \phi_{n}^{-\alpha - \alpha}(x_5) = -\frac{(2n+1)^2}{R^2} \phi_{n}^{-\alpha - \alpha}(x_5).
\]
The Laplacian \( \frac{\partial^2}{\partial x_5^2} \) gives rise to the KK masses in the effective field theory. The orthonormality of the mode functions \( \phi_{n}^{\alpha \beta} \) takes the form
\[
\frac{4}{\pi R} \int_{0}^{\frac{1}{2} \pi R} dx_5 \phi_{n}^{\alpha \beta}(x_5) \phi_{n'}^{\alpha' \beta'}(x_5) = \delta^{\alpha \alpha'} \delta^{\beta \beta'} (\delta_{nn'} + \delta^{\alpha+1 \beta+1} \delta_{n0} \delta_{0'n}).
\]
(21)

**B Dimensional regularization of the orbifold**

The pole functions, that can be obtained by similar arguments as presented in [17],
\[
\mathcal{P}^{\alpha \alpha} = \frac{1}{2} \left( \frac{\alpha}{p_5} + \frac{1}{2} \frac{\pi R}{\tan \frac{\pi R}{2} p_5} \right), \quad \mathcal{P}^{\alpha - \alpha} = -\frac{1}{2} \frac{\pi R}{\cot \frac{\pi R}{2} p_5},
\]
(22)
can be used to turn sums into contour integrals. For example, for a convergent sum we have
\[
\sum_{n \geq 0} f \left( \frac{2n}{R} \right) = -\frac{1}{2 \pi i} \int_{\mathcal{C}} dp_5 \mathcal{P}^{++}(p_5) f(p_5) = \frac{1}{2 \pi i} \int_{\mathcal{C}} dp_5 \mathcal{P}^{++}(p_5) f(p_5).
\]
(23)
The contour \( \mathcal{C} \) consists of two lines along the real axis (that are infinitesimally near to it) and closed at \( \pm \) infinity. The contour \( \mathcal{C} \) contains the full complex plane, except the real axis, and is anti-clockwise oriented.

Combinations of sum and integrals of a function \( f(p_4, p_5) \) can be regulated by
\[
\int d^4 p_4 \sum_{n \in \mathbb{N}} f(p_4, m_n) \to \frac{1}{2 \pi i} \int_{\mathcal{C}} d^D p_5 \mathcal{P}(p_5) f(p_4, p_5) \equiv \frac{1}{2 \pi i} \int_{\mathcal{C}} dp_5 \int_{0}^{\infty} dp_5 \mathcal{R}_4(p_4) \mathcal{R}_5(p_5) \mathcal{P}(p_5) f(p_4, p_5).
\]
(24)
We have introduced the regulator functions \( \mathcal{R}_4(p_4) \) and \( \mathcal{R}_5(p_5) \) for the 4 dimensional and 5 dimensional integrations, given by
\[
\mathcal{R}_4(p_4) = \frac{2 \pi \frac{1}{2} (D_4)}{\Gamma \left( \frac{1}{2} D_4 \right)} p_4^3 \left( \frac{p_4}{\mu_4} \right)^{D_4-4}, \quad \mathcal{R}_5(p_5) = \frac{\pi \frac{1}{2} (D_5)}{\Gamma \left( \frac{1}{2} D_5 \right)} \left( \frac{p_5}{\mu_5} \right)^{D_5-1},
\]
(25)
respectively. Here \( D_4, D_5 \) are the complex extended dimensions for the Minkowski and compact space, respectively. The parameters \( \mu_4, \mu_5 \) are renormalization scales.

For \( \text{Im} \ p_5 > 0 \), the pole functions may be written as
\[
\mathcal{P}^{\alpha \alpha} = \frac{1}{2} \left( -\frac{i}{2} \pi R + \frac{\alpha}{p_5} - \rho_-(p_5) \right), \quad \mathcal{P}^{\alpha - \alpha} = \frac{1}{2} \left( -\frac{i}{2} \pi R + \rho_+(p_5) \right),
\]
(26)
with
\[ \rho_\alpha(p_5) = \frac{i\pi R e^{i\pi R p_5}}{1 + \alpha e^{i\pi R p_5}}. \] (27)

For \( \text{Im } p_5 < 0 \) one obtains
\[ P^{\alpha\alpha} = \frac{1}{2} \left( \frac{i}{2} \pi R + \frac{\alpha}{p_5} + \rho_-(-p_5) \right), \quad P^{\alpha-\alpha} = \frac{1}{2} \left( \frac{i}{2} \pi R - \rho_+(p_5) \right). \] (28)

When these functions are integrated, using dimensional regularization for both the 4 dimensional integral and the KK sum, over the half plane contours for which either \( \text{Im } p_5 > 0 \) or \( \text{Im } p_5 < 0 \), then we can identify their UV behavior. The first terms in (28) may give rise to genuine 5 dimensional divergences, the \( \alpha/p_5 \) term accounts for 4 dimensional divergences, while the remaining terms give the finite contributions.

References

[1] I. Antoniadis, S. Dimopoulos, A. Pomarol and M. Quiros, Nucl. Phys. B 544 (1999) 503 [hep-ph/9810410].
[2] A. Delgado, A. Pomarol and M. Quiros, Phys. Rev. D 60 (1999) 095008 [hep-ph/9812489].
[3] N. Arkani-Hamed, L. Hall, Y. Nomura, D. Smith and N. Weiner, [hep-ph/0102090].
[4] R. Barbieri, L.J. Hall and Y. Nomura, Phys. Rev. D 63 (2001) 105007 [hep-ph/0011311].
[5] A. Delgado and M. Quiros, [hep-ph/0103058].
[6] S. Groot Nibbelink and H.P. Nilles, work in progress.
[7] L. Alvarez-Gaume, S. Della Pietra and G. Moore, Annals Phys. 163 (1985) 288.
[8] S. Coleman and B. Grossman, Nucl. Phys. B 203 (1982) 205.
[9] L. Alvarez-Gaume, HUTP-85/A092 Lectures given at Int. School on Mathematical Physics, Erice, Italy, Jul 1-14, 1985.
[10] W. Fischler, H. P. Nilles, J. Polchinski, S. Raby and L. Susskind, Phys. Rev. Lett. 47 (1981) 757.
[11] M. Gunaydin, G. Sierra and P. K. Townsend, Nucl. Phys. B 253 (1985) 573.
    M. Gunaydin, G. Sierra and P. K. Townsend, Nucl. Phys. B 242 (1984) 244.
[12] A. Pomarol and M. Quiros, Phys. Lett. B 438 (1998) 255 [hep-ph/9806263].
[13] E. A. Mirabelli and M. E. Peskin, Phys. Rev. D 58 (1998) 065002 [hep-th/9712214].
[14] B. de Wit, P. G. Lauwers and A. Van Proeyen, Nucl. Phys. B 255 (1985) 569.
[15] M. Zucker, Off-shell supergravity in five dimensions and supersymmetric brane world scenarios, PhD thesis Bonn University, BONN-IR-2000-10, ISSN-017208741.
    M. Zucker, JHEP 0008 (2000) 016 [hep-th/9909144].
[16] E. Bergshoeff, R. Kallosh and A. Van Proeyen, JHEP 0010 (2000) 033 [hep-th/0007044].
[17] S. Groot Nibbelink, [hep-th/0108185].
[18] D. M. Capper, D. R. Jones and P. van Nieuwenhuizen, Nucl. Phys. B 167 (1980) 479.
[19] D. Ghilencea, H.P. Nilles and S. Stieberger, [hep-th/0108183].