Solving Simultaneous Target Assignment and Path Planning Efficiently with Time-Independent Execution

Keisuke Okumura, Xavier Défago
School of Computing, Tokyo Institute of Technology
Tokyo, Japan
{okumura.k, defago}@coord.c.titech.ac.jp

Abstract
Real-time planning for a combined problem of target assignment and path planning for multiple agents, also known as the unlabeled version of Multi-Agent Path Finding (MAPF), is crucial for high-level coordination in multi-agent systems, e.g., pattern formation by robot swarms. This paper studies two aspects of unlabeled-MAPF: (1) offline scenario: solving large instances by centralized approaches with small computation time, and (2) online scenario: executing unlabeled-MAPF despite timing uncertainties of real robots. For this purpose, we propose TSWAP, a novel sub-optimal complete algorithm, which takes an arbitrary initial target assignment and then repeats one-timestep path planning with target swapping. TSWAP can adapt to both offline and online scenarios. We empirically demonstrate that Offline TSWAP is highly scalable; providing near-optimal solutions while reducing runtime by orders of magnitude compared to existing approaches. In addition, we present the benefits of Online TSWAP, such as delay tolerance, through real-robot demos.

1 Introduction
Target assignment and path planning for multiple agents, i.e., deciding where to go and how to go, are fundamental problems to achieve high-level coordination in multi-agent systems. This composite problem has attractive applications such as automated warehouses (Wurman, D’Andrea, and Mountz 2008), robot soccer (MacAlpine, Price, and Stone 2015), pattern formation of robot swarms (Turpin et al. 2014; Hönig et al. 2018b), a robot display (Alonso-Mora et al. 2012), to name just a few. These applications typically require real-time planning, i.e., planners have a limited time for deliberation until deadlines.

The problem above is a non-trivial composition of two fundamental problems: 1) target assignment is well-studied (Gerkey and Matarić 2004) with well-known efficient algorithms, such as the Hungarian algorithm (Kuhn 1955); 2) path planning, also known as Multi-Agent Path Finding (MAPF) (Stern et al. 2019), has been extensively studied in recent years. Given a graph, a set of agents, their initial locations, and their targets, a solution of MAPF maps collision-free paths to agents. This “labeled” MAPF regards targets as being assigned to each agent. This paper studies the “unlabeled” version of MAPF (unlabeled-MAPF) which considers agents and targets to be distinct, and hence requires to assign a target to each agent. In both labeled or unlabeled cases, the main objective is to minimize makespan, i.e., the maximum arrival time of agents.

Paradoxically, finding makespan-optimal solutions for unlabeled-MAPF is easier than for MAPF which is known to be NP-hard (Yu and LaValle 2013b; Ma et al. 2016). Indeed, unlabeled-MAPF has a polynomial-time optimal algorithm based on a reduction to maximum flow (Yu and LaValle 2013a); however, the size of the flow network is quadratic to the size of the original graph, making practical problems in large graphs (e.g., 500 × 500 grid) still challenging. Despite its importance, unlabeled-MAPF has received little attention compared to conventional MAPF, for which many scalable sub-optimal solvers have been developed (Surynek 2009; Wang and Botea 2011; de Wilde, ter Mors, and Witteveen 2013; Okumura et al. 2019).

The first objective of this paper is thus to propose a centralized approach to solve large unlabeled-MAPF instances with sufficiently good quality in small computation time. We present Offline TSWAP, a sub-optimal complete algorithm. Specifically, Offline TSWAP uses arbitrary assignment algorithms, then repeats one-timestep path planning with target swapping until all agents have reached targets.

We further extend TSWAP to an online version, aiming at executing unlabeled-MAPF despite timing uncertainties of real robots; the second objective of this paper. In practice, plan execution on robots is subject to timing uncertainties (e.g., kinematic constraints, unexpected delays, friction, clock drift). Even worse, the potential for unexpected interference increases with the number of agents because agents’ actions usually depend on each other’s; hence perfect on-time execution is unlikely to be expected.

To overcome this problem, we propose Online TSWAP, an online version of TSWAP based on the concept of time-independent planning (Okumura, Tamura, and Défago 2021). In other words, it abandons all timing assumptions (e.g., synchronization, traveling time, rotation time, delay probabilities) and regards the whole system as a transition system that changes its configuration according to atomic actions of agents. Regardless of movement timings, TSWAP ensures that all targets are eventually reached.

Our main contribution is proposing TSWAP to solve or execute unlabeled-MAPF, specifically; (1) offline scenario: we propose a novel algorithm and empirically demon-
strate that TSWAP is scalable and can yield near makespan-optimal solutions while reducing runtime by orders of magnitude in most cases when compared to the polynomial-time optimal algorithm (Yu and LaValle 2013a). Furthermore, TSWAP also yields good solutions with respect to sum-of-costs, another commonly used metric in MAPF studies. (2) online scenario: we formulate an online time-independent problem and propose a complete algorithm. We show the benefits of TSWAP, such as time independence and delay tolerance, through real-robot demos. Incidentally, (3) we also present efficient assignment algorithms with lazy evaluation of distances, assuming to use with TSWAP.

The paper is structured as follows. Section 2 summarizes related work about unlabeled-MAPF and time-independent execution methods. Section 3 formalizes offline and online time-independent problems of unlabeled-MAPF. Section 4 presents Offline TSWAP and its theoretical analysis. Section 5 presents assignment algorithms with lazy evaluation. Section 6 presents empirical results of offline planning. Section 7 presents Online TSWAP. Section 8 presents robot demos of online planning. Section 9 concludes the paper. The technical appendix, code, and video are available on https://kei18.github.io/tswap.

2 Related Work

2.1 Target Assignment and Path Planning

The unlabeled-MAPF problem, also known as anonymous MAPF, consists of two sub-problems: (1) target assignment, more generally, task allocation, and (2) path planning. The multi-robot task allocation problems are a mature field (Gerkey and Mataric 2004). Path planning for multiple agents, embodied as MAPF, has been actively studied in recent years (Stern et al. 2019). We focus on reviews of related studies covering both aspects.

Unlike conventional MAPF, unlabeled-MAPF is always solvable (Kornhauser, Miller, and Spirakis 1984; Yu and LaValle 2013a; Adler et al. 2015; Ma et al. 2016). Among them, TSWAP relates to the analysis presented by Yu and LaValle (2013a) because both approaches use target swapping. Their analysis relies on optimal linear assignment whereas TSWAP works for any assignments. They also showed that unlabeled-MAPF has a Pareto optimal structure for makespan and sum-of-costs metrics (summation of traveling time of each agent; see the next section), i.e., there is an instance for which it is impossible to optimize both metrics simultaneously. Furthermore, they present a polynomial-time makespan-optimal algorithm, in contrast with conventional MAPF being known to be NP-hard (Yu and LaValle 2013b; Ma et al. 2016).

The combined target assignment and path finding (TAPF) problem (Ma and Koenig 2016), also called as colored MAPF (Barták, Ivanová, and Svancara 2021), generalizes both MAPF and unlabeled-MAPF by partitioning the agents into teams. The paper proposes a makespan-optimal algorithm for TAPF that combines Conflict-based Search (CBS) (Sharon et al. 2015), a popular optimal MAPF algorithm, with an optimal algorithm for unlabeled-MAPF. Hönig et al. (2018a) studied a sum-of-costs optimal algorithm for TAPF by extending CBS. They also proposed a bounded sub-optimal algorithm, called ECBS-TA: we compare TSWAP with ECBS-TA in the experiment. There is a study (Wagner, Choset, and Ayanian 2012) using another optimal MAPF algorithm (Wagner and Choset 2015) to solve the joint problem of target assignment and path planning.

The multi-agent pickup and delivery (MAPD) problem (Ma et al. 2017), motivated by applications in automated warehouses (Wurman, D’Andrea, and Mountz 2008), aims at making agents convey packages and has to solve target assignment and path planning jointly. Many approaches to MAPD have been proposed, e.g., (Ma et al. 2017; Liu et al. 2019; Okumura et al. 2019). Although MAPD is a problem different from unlabeled-MAPF, TSWAP is similar to an MAPD algorithm TPTS (Ma et al. 2017) in the sense that both algorithms swap assigned targets adaptively. One difference though is that, unlike TSWAP, TPTS sets additional conditions about start and target locations.

MAPF is a kind of pearl motion problem, in which objects are moved on a graph one-at-a-time, like a sliding tile puzzle. The unlabeled version of pearl motion has also been studied (Kornhauser, Miller, and Spirakis 1984; Calinescu, Dumitrescu, and Pach 2008; Goraly and Hassin 2010). However, in unlabeled-MAPF, agents can move simultaneously; different from those studies, TSWAP explicitly assumes this fact, resulting in practical outcomes.

Pattern formation of multiple agents (Oh, Park, and Ahn 2015) is one of the motivating examples of unlabeled-MAPF. Various approaches have been studied, e.g., (Alonso-Mora et al. 2011; Wang and Rubenstein 2020). We highlight two studies closely related to ours as follows. SCRAM (MacAlpine, Price, and Stone 2015) is a target assignment algorithm considering collisions and works only in open space without obstacles; hence its applications are limited. The assignment algorithm in this paper (Alg. 2) uses a scheme similar to SCRAM but differs in its use of lazy evaluation. Turpin et al. (2014) proposed a method that first solves the lexicographic bottleneck assignment (Burkard and Rendl 1991) then plans trajectories on graphs. To avoid collisions, the method uses the delay offset about when agents start moving, resulting in a longer makespan. TSWAP avoids using such offsets by swapping targets on demand.

2.2 Execution without Timing Assumptions

Ma, Kumar, and Koenig (2017) studied robust execution policies using offline MAPF plans as input, but assuming that agents might be delayed during the execution of timed schedules. The proposed Minimum Communication Policies (MCPs) make agents preserve two types of temporal dependencies: internal events within one agent and order relation of visiting a node. Regardless of delays, MCPs make all agents reach their destinations without conflicts. We later use MCPs for robot demos as a comparison of Online TSWAP.

Okumura, Tamura, and Défago (2021) studied time-independent planning to execute MAPF by modeling the whole system as a transition system that changes configurations according to atomic actions of agents. The online
3 Problem Definition and Terminologies

Unlabeled-MAPF Instance A problem instance of unlabeled-MAPF is defined by a connected undirected graph $G = (V, E)$, a set of agents $A = \{a_1, \ldots, a_n\}$, a set of distinct initial locations $S = \{s_1, \ldots, s_n\}$ and distinct target locations $T = \{g_1, \ldots, g_m\}$, where $|T| \leq |A|$.

Offline Problem Given an unlabeled-MAPF instance, let $\pi_i[t] \in V$ denote the location of an agent $a_i$ at discrete time $t \in \mathbb{N}$. At each timestep $t$, $a_i$ can move to an adjacent node, or can stay at its current location, i.e., $\pi_i[t+1] \in Neigh(\pi_i[t]) \cup \{\pi_i[t]\}$, where $Neigh(v)$ is the set of nodes adjacent to $v \in V$. Agents must avoid two types of conflicts: (1) vertex conflict: $\pi_i[t] \neq \pi_j[t]$, and, (2) swap conflict: $\pi_i[t] \neq \pi_j[t+1] \lor \pi_i[t+1] \neq \pi_j[t]$. A solution is a set of paths $\{\pi_1, \ldots, \pi_n\}$ such that a subset of agents occupies all targets at a certain timestep $T$. More precisely, assign a path $\pi_i = (\pi_i[0], \pi_i[1], \ldots, \pi_i[T])$ to each agent such that $\pi_i[0] = s_i$ and there exists an agent $a_j$ with $\pi_j[T] = g_k$ for all $g_k \in T$.

We consider four metrics to rate solutions:
- $\text{makespan:}$ the first timestep when all targets are occupied, i.e., $T$.
- $\text{sum-of-costs:}$ $\sum_i T_i$ where $T_i$ is the minimum timestep such that $\pi_i[T_i] = \pi_i[T_i+1] = \ldots = \pi_i[T]$.
- $\text{maximum-moves:}$ the maximum of how many times each agent moves to adjacent nodes.
- $\text{sum-of-moves:}$ the summation of moves of each agent.

Online Time-Independent Problem An execution schedule is defined by infinite sequence $E = (a_1, a_2, a_3, \ldots)$ defining the order in which each agent is activated and can move one step.

Given an unlabeled-MAPF instance, a situation where all agents are at their initial locations, and an execution schedule $E$, an agent $a_i$ can move to an adjacent node if (1) it is $a_i$’s turn in $E$ and (2) the node is unoccupied by others. $E$ is called fair when all agents appear infinitely often in $E$. Termination is a configuration where all targets are occupied by a subset of agents simultaneously. An algorithm is called complete when termination is achieved within a finite number of activations for any fair execution schedules.

Given an execution schedule, we rate the efficiency of agents’ behaviors according to two metrics: maximum-moves and sum-of-moves. Their definitions are the same as for the offline problem.

Remarks for Online Problem Since any complete algorithms must deal with any fair schedules, they inherently assume timing uncertainties. For simplicity, we assume that at most one agent is activated at any time, hence the execution is determined by a sequence over the agents. There is no loss of generality as long as an agent can atomically reserve its next node before each move. Note that we do not formally define sum-of-costs and makespan for the online problem since they should be measured according to actual time.

Algorithm 1: Offline TSWAP

input: unlabeled-MAPF instance
output: plan $\pi$
1: get an initial assignment $\mathcal{A}$; a set of pairs $s \in S$ and $g \in T$
2: $a_i.v, a_i.g \leftarrow (s_i, g) \in \mathcal{A}$ : for each agent $a_i \in A$
3: $t \leftarrow 0$ (timestep)
4: while $\exists a \in A, a.v \neq a.g$ do
5: for $a \in A$ do
6: if $a.v = a.g$ then continue
7: $u \leftarrow \text{nextNode}(a.v, a.g)$
8: if $\exists b \in A$ s.t. $b.v = u$ then
9: if $u = b.g$ then
10: swap targets of $a.b$: $a.g \leftarrow b.g, b.g \leftarrow a.g$
11: else if detect deadlock for $A' \subseteq A \land a \in A'$ then
12: rotate targets of $A'$
13: end if
14: else
15: $a.v \leftarrow u$
16: end if
17: end for
18: $t \leftarrow t + 1$
19: $\pi_i[t] \leftarrow a_i.v$ : for each agent $a_i \in A$
20: end while

Other Assumptions and Notations For simplicity, we assume $|T| = |A|$ unless explicitly mentioned. We denote the diameter of $G$ by $diam(G)$, and its maximum degree by $\Delta(G)$. Let $dist(u, v)$ denote the shortest path length from $u \in V$ to $v \in V$. We assume the existence of admissible heuristics $h(u, v)$ for computing the shortest path length in constant time, i.e., $h(u, v) \leq dist(u, v)$, e.g., the Manhattan distance. This paper uses a simplified notation of the asymptotic complexity like $O(V)$ rather than $O(|V|)$.

4 Offline TSWAP

This section presents Offline TSWAP, a sub-optimal path planning algorithm for the offline problem, which is complete for any initial target assignments. Here, an assignment is a set of pairs $s \in S$ and $g \in T$ such that all agents have a distinct target. Most proofs are deferred to the Appendix.

4.1 Algorithm Description

TSWAP assumes that an initial assignment is given externally, then mainly determines how to go but not only. This is because the initial assignment is potentially an unsolvable MAPF instance (e.g., see $t = 0$ at path planning in Fig. [1]); we cannot apply MAPF solvers directly to design a complete unlabeled-MAPF algorithm for arbitrary assignments. The algorithm must consider swapping targets as necessary.

Algorithm [4] generates a solution $\pi$ by moving agents incrementally towards their targets following the shortest paths, using the following function:

$$\text{nextNode}(u, v) := \arg\min_{w \in Neigh(u) \setminus \{u\}} dist(v, w)$$

nextNode is assumed to be deterministic; tie-break between nodes having the same scores is done deterministically.

Each agent $a$ has two variables: $a.v$ is the current location, and $a.g$ is the current target. They are initialized
by the initial assignment [Lines 12]. After that and until all agents reach their targets, one-timestep planning is repeated as follows [Lines 4–20]. If $a$ is on its target $a.g$, it stays there ($a.v = a.g$). Otherwise, $a$ attempts a move to the nearest neighbor of $a.v$ towards $a.g$, call it $a'$ [Line 7]. When $u$ is occupied by another agent, the algorithm either performs target swapping or deadlock resolution [Lines 8–14]. Here, a deadlock is defined as follows. A set of agents $A' = \{a_{i_1}, a_{i_2}, a_{i_3}, \ldots, a_{i_j}\}$ is in a deadlock when $\text{nextNode}(a_{i_1}.v, a_{i_1}.g) = a_{i_2}.v \wedge \text{nextNode}(a_{i_2}.v, a_{i_2}.g) = a_{i_3}.v \wedge \ldots \wedge \text{nextNode}(a_{i_j}.v, a_{i_j}.g) = a_{i_1}.v$. When detecting a deadlock for $A'$, the algorithm “rotates” targets: $a_{i_1}.g \leftarrow a_{i_2}.g, a_{i_2}.g \leftarrow a_{i_3}.g, a_{i_3}.g \leftarrow a_{i_1}.g, \ldots$ [Line 12]. The detection incrementally checks whether the next location of each agent is occupied by another agent and concurrently checks the existence of a loop.

Figure 1 shows an example of TSWAP, together with target assignment by Alg. 2 introduced in Sec. 5.

4.2 Theoretical Analysis

Theorem 1. Offline TSWAP (Algorithm 1) is complete for the offline problem.

Proof. Let $\Pi(u, u') \subset V$ be a set of nodes in a shortest path from $u \in V$ to $u' \in V$, identified by $\text{nextNode}$, except for $u$ and $u'$. Consider the following potential function in Alg. 1

\[
\phi = \sum_{a \in A} \{\text{dist}(a.v, a.g) + \{b \mid b \in A, b.g \in \Pi(a.v, a.g)\}\}
\]

Observe that $\phi = 0$ means that the problem is solved. Furthermore, for each iteration of Lines 4–20, $\phi$ is non-increasing. We now proof that $\phi$ decreases for each iteration only when $\phi > 0$ by contradiction.

Suppose that $\phi(\neq 0)$ does not differ from the last iteration. Since $\phi \neq 0$, there are agents not on their targets. Let them be $B \subseteq A$. First, there is no swap operation by Line 10 otherwise, the second term of $\phi$ must decrease. Second, all agents in $B$ do not move; otherwise, the first term of $\phi$ must decrease. Furthermore, for an agent $a \in B$, $\text{nextNode}(a.v, a.g)$, let denote this as $a.u$, must be occupied by another agent $b \in B$; otherwise, $a$ moves to $a.u$. This is the same for $b$, i.e., there is an agent $c \in B$ such that $c.v = b.u$. By induction, this sequence of agents must form a deadlock somewhere; however, by deadlock detection and resolution in Lines 11–12 both the first and second terms of $\phi$ must decrease. Hence, this is a contradiction.

Any assignment algorithms can be applied to TSWAP because Theorem 1 does not rely on initial assignments. Furthermore, TSWAP can easily adapt to unlabeled-MAPF instances with $|A| > |T|$ by assigning agents without targets to any non-target locations.

Proposition 1. Offline TSWAP has upper bounds of:

- makespan: $O(A \cdot \text{diam}(G))$
- sum-of-costs: $O(A^2 \cdot \text{diam}(G))$
- maximum-moves: $O(A \cdot \text{diam}(G))$
- sum-of-moves: $O(A \cdot \text{diam}(G))$

Compared to sum-of-costs, the upper bound on sum-of-moves is significantly reduced because it ignores all “wait” actions. The bound on sum-of-moves is tight in some scenarios, such as a line graph with all agents starting on one end and all targets on the opposite end. In general, the upper bound on makespan is greatly overestimated. We see empirically later that TSWAP yields near-optimal solutions for makespan depending on initial assignments.

Proposition 2. Assume that the time complexity of nextNode and the deadlock resolution [Lines 7–12] in Alg. 1 are $\alpha$ and $\beta$, respectively. The time complexity of Offline TSWAP excluding Line 7 is $O(A^2 \cdot \text{diam}(G) \cdot (\alpha + \beta))$.

From Proposition 2, TSWAP has an advantage in large fields compared to the time complexity $O(AV^2)$ of the makespan-optimal algorithm [Yu and LaValle 2013] with a natural assumption that $E = O(V)$. 

Figure 1: Example of Offline TSWAP with Alg. 2 as an assignment algorithm. An unlabeled-MAPF instance is shown at the top. The target assignment is illustrated in the middle, using a bipartite graph $B$. The assignment $M$ is denoted by red lines. The right part, with annotations of costs, corresponds to solving initial location and a target then updating the matching. The left part, with red lines, corresponds to finding the bottle-neck cost [Lines 7–13], i.e., incrementally adding pairs of an initial location and a target then updating the matching. The right part, with annotations of costs, corresponds to solving the minimum cost maximum matching problem [Line 16]. Two edges are added from the last situation due to Line 12. Offline TSWAP (Alg. 1) is illustrated at the bottom. $a_3$, $a_2$, and $a_1$ repeat one-step planning. Current locations of agents, i.e., $a.v$, are shown within nodes. Arrows represent the targets, i.e., $a.g$. A target swapping happens between $a_2$ and $a_3$ at $t = 0$ [Line 10]. Note that we artificially assign $s_2$ to $g_3$ to show the example of target swapping.
Algorithm 2: Bottleneck Assignment

input: unlabeled-MAPF instance
output: $M$: assignment, a set of pairs $s \in S$ and $g \in T$
1. initialize $M$; Let $B$ be a bipartite graph $(S, T, \emptyset)$
2. $Q$: priority queue of tuple
   $s \in S, g \in T$, real distance, and estimated distance
   in increasing order of distance
   (use real one if exists, otherwise use estimated one)
3. $Q$.push($(s, g, \bot, h(s, g)))$: for each pair $s \in S, g \in T$
4. while $Q \neq \emptyset$
5. $(s, g, d, \Delta) \leftarrow Q$.pop()
6. if $d = \bot$ then
7. $Q$.push($(s, g, \text{dist}(s, g), \Delta))$: continue
8. end if
9. add a new edge $(s, g)$ to $B$
10. update $M$ by finding an augmenting path on $B$
11. if $|M| = |T|$ then
12. optional: add all $(s', g', d', \Delta) \in Q$ to $B$ s.t. $d' = d$
13. break
14. end if
15. end while
16. optional: $M \leftarrow$ minimum cost maximum matching on $B$

Algorithm 3: Greedy Assignment with Refinement

input: unlabeled-MAPF instance
output: $M$: assignment, a set of pairs $s \in S$ and $g \in T$
1. initialize $M$; initialize queue $U$ by $A$
2. while $U \neq \emptyset$
3. $a_i \leftarrow U$.pop()
4. while true do
5. $g \leftarrow$ the non-evaluated nearest target from $s_i$
6. if $B(s_j, g) \in M$ then
7. add $(s_i, g)$ to $M$; break
8. else if $\exists(s_j, g) \in M$ and $\text{dist}(s_i, g) < \text{dist}(s_j, g)$ then
9. replace $(s_j, g) \in M$ by $(s_i, g)$; $U$.push($a_i$); break
10. end if
11. end while
12. end while
13. while $M$ is updated in the last iteration do
14. $(s_i, g_i) \leftarrow \text{argmax}_{(s,g) \in M} \text{dist}(s_i, g_i)$
15. for $(s_j, g_j) \in M$ do
16. if $\text{h}(s_j, g_i) \geq \text{cswap} \text{ then continue }$ (for lazy evaluation)
17. $\text{cswap} \leftarrow \max(\text{dist}(s_j, g_i), \text{dist}(s_i, g_j))$
18. if $\text{cswap} < \text{cswap}$ then swap $g_i$ and $g_j$ of $M$; break
19. end for
20. end while

5 Target Assignment with Lazy Evaluation

Target assignment determines where to go; takes an unlabeled-MAPF instance as input, then returns an assignment, a set of pairs of an initial location and a target, as output. An initial assignment is crucial for TSWAP. Ideal assignment algorithms are quick, scalable, and with reasonable quality for solution metrics (e.g., makespan). It is possible to apply conventional assignment algorithms [Kuhn 1955, [Kuhn 1955]]. However, costs (i.e., distances) for each start-target pair are unknown initially, which is typically computed via breadth-first search with time complexity $O(A(V + E))$. This would be a non-negligible overhead. We thus present two examples that efficiently solve target assignment with lazy evaluation, which avoids exhaustive distance evaluation.

5.1 Bottleneck Assignment

Algorithm 2 aims to minimize makespan by solving the bottleneck assignment problem [Gross 1959, [Gross 1959]], i.e., assign each agent to one target while minimizing the maximum cost, regarding distances between initial locations and targets as costs.

The algorithm incrementally adds pairs of initial location and target to a bipartite graph $B$ [Line 9], in increasing order of their distances using a priority queue $Q$. $B$ is initialized as $(S, T, \emptyset)$ [Line 1]. This iteration continues until all targets are matched to initial locations, i.e., agents [Line 11]. At each iteration, the maximum bipartite matching problem on $B$ is solved [Line 10]. In general, the Hopcroft-Karp algorithm [Hopcroft and Karp 1973, [Hopcroft and Karp 1973]] efficiently solves this problem in $O(\sqrt{V \cdot E})$ runtime for any bipartite graph $(V, E)$, but we use the reduction to the maximum flow problem and the Ford-Fulkerson algorithm [Ford and Fulkerson 1956, [Ford and Fulkerson 1956]].

The basic concept of this algorithm is finding repeatedly an augmenting path, i.e., a path from source to sink with available capacity on all edges in the path, then making the flow along that path. Such paths are found, e.g., via depth-first or breadth-first search. Here, finding a single augmenting path in $O(E)$ runtime is sufficient to update the matching because the number of matched pairs increases at most once for each adding.

The algorithm uses lazy evaluation of real distance [Line 7]. We use the priority queue $Q$ [Line 2], and admissible heuristics $h$ [Line 2], then evaluate the real distance as needed. $\bot$ denotes that the corresponding real distance has not been evaluated yet. The lazy evaluation contributes to speedup of the target assignment, as we will see later.

The algorithm optionally solves the minimum cost maximum matching problem [Line 16], aiming at improving the sum-of-costs metric. The problem can be solved by reducing to the minimum cost maximum flow problem then using the successive shortest path algorithm [Ahuja, Magnanti, and Orlin 1993, [Ahuja, Magnanti, and Orlin 1993]]. Note that when finding the bottleneck cost, all edges in $Q$ with their costs equal to the bottleneck cost are added to $B$ to improve the sum-of-costs metric of the assignment [Line 12]. This operation includes lazy evaluation similar to the main loop [Line 4–15]. We denote the corresponding algorithm as Alg. 2.

**Proposition 3.** The time complexity of Algorithm 2 is $O(\max(\Delta(V + E), A^2))$.

5.2 Greedy Assignment with Refinement

Algorithm 3 aims at finding a reasonable assignment for makespan as quickly as possible, which uses:

- greedy assignment [Lines 1–12]: assigns one target to one agent step by step, while allowing reassignment if a better assignment will be expected [Lines 8–10].
- iterative refinement [Lines 13–20]: swaps targets of two agents until no improvements are detected.
• lazy evaluation of distances for start-target pairs, implemented by pausing the breadth-first search as soon as the query start-target pair is in the search tree.

This algorithm is expected to run in a very short time;

**Proposition 4.** The time complexity of Algorithm 3 is $O(A(V + E))$.

Algorithm 3 presents the refinement for makespan but the refinement for sum-of-costs is straightforward (see Alg. 5 in the Appendix).

### 6 Evaluation of Offline Planning

The experiments aim at demonstrating that Offline TSWAP is efficient, i.e., it returns near-optimal solutions within a short time and scales well, depending on initial assignments. In particular, this section has three aspects: (1) illustrating the effect of initial assignments including the proposed assignment algorithms, (2) comparing with the makespan-optimal polynomial-time algorithm (Yu and LaValle 2013a), (3) assessing another metric, sum-of-costs. We carefully picked up several 4-connected grids from MAPF benchmarks (Stern et al. 2019) as a graph $G$, shown in Fig. 2 they are common in MAPF studies. The simulator was developed in C++ and the experiments were run on a laptop with Intel Core i9 2.3 GHz CPU and 16 GB RAM. For each setting, we created 50 instances while randomly generating starts and targets. Implementation details of (Yu and LaValle 2013a) are described in the Appendix. Throughout this section, the runtime evaluation of TSWAP includes both target assignment and path planning.

Figure 2: Used maps. $|V|$ is shown with parentheses.

#### 6.1 Effect of Initial Target Assignment

The first part evaluates the effect of initial assignments on TSWAP while varying $|A|$. We tested Alg. 2 (bottleneck; minimizing maximum distance), Alg. 3 (with min-cost maximum matching), Alg. 3 (greedy with refinement for makespan), Alg. 3 (for sum-of-costs), naive greedy assignment (Avis 1983), and optimal linear assignment (minimizing total distances) solved by the successive shortest path algorithm (Ahuja, Magnanti, and Orlin 1993). Note that these assignment algorithms do not consider inter-agent collisions. The last two used distances for start-target pairs obtained by the breadth-first search as costs. To assess the effect of lazy evaluation, we also tested the adapted version of Alg. 2 and Alg. 3 without lazy evaluation, denoted as Alg. 2 and Alg. 3.

Table 1 summarizes the results on random-64-64-20. In summary, Alg. 2 contributes to finding good solutions for makespan. Algorithm 2 significantly improves sum-of-costs. Algorithm 3 and Alg. 5 are blazing fast while solution qualities outperform those of the naive greedy assignment. The lazy evaluation speeds up each assignment algorithm. The optimal linear assignment requires time because its time complexity is $O(A^3)$.

#### 6.2 Makespan-optimal Algorithm vs. TSWAP

This part is further divided into two: (1) assessing scalability for both $G$ and $A$, and (2) testing the solvers in large graphs. Figure 3 and Table 2 summarize the results.

Figure 3 (left) displays the average makespan and runtime of “quadrupling” the size of $G$, i.e., those of random-64-64-20, regarding the results of random-32-32-20 as a baseline. Figure 3 (right) shows dense situations ($|A| \geq |V|$). The main observations are: (1) TSWAP quickly yields near-optimal solutions in non-dense situations with either Alg. 2 or Alg. 3. (2) The runtime of TSWAP remains small when enlarging $G$ while the optimal algorithm increases dramatically. (3) As agents increase, the runtime of TSWAP with Alg. 2 quickly increases compared to the optimal algorithm, because Alg. 2 is quartic on $|A|$ (see Prop. 3). Meanwhile, TSWAP with Alg. 3 immediately yields solutions even with a few thousand agents. Note that, as the number of agents increases, the optimal makespan decreases because we set initial locations and targets randomly.

### Table 1: The results of TSWAP with different assignments in random-64-64-20.

| $|A|$ | metric | Alg. 2 | Alg. 3 | Alg. 3 | Alg. 3 | Alg. 3 | Alg. 5 | greedy linear |
|------|--------|--------|--------|--------|--------|--------|--------|--------------|
|      | runtime (ms) | 10 | 11 | 12 | 12 | 12 | 23 | 23 | 23 |
| 110  | sum-of-costs | 1079 | 937 | 1139 | 1136 | 958 | 1196 | 940 | 940 |
| 500  | runtime (ms) | 72 | 174 | 213 | 15 | 78 | 22 | 77 | 602 |
|      | sum-of-costs | 2595 | 2169 | 2169 | 2878 | 2880 | 2564 | 4653 | 2429 |
| 1000 | runtime (ms) | 335 | 757 | 882 | 28 | 233 | 58 | 221 | 3811 |
|      | sum-of-costs | 3591 | 2922 | 2922 | 4020 | 4043 | 3695 | 7571 | 3491 |
| 2000 | runtime (ms) | 1453 | 3035 | 3205 | 66 | 784 | 188 | 725 | 32944 |
|      | sum-of-costs | 5200 | 5244 | 5465 | 12292 | 5122 |

Figure 3: The average makespan and runtime. Alg. 2 (TSWAP-2) and Alg. 3 (TSWAP-3) were used in TSWAP. “Flow” is the optimal algorithm. left: $|A| \in \{30, 70, 110\}$. right: $|A| \in \{500, 1000, 1500, 2000\}$ on random-64-64-20.
Table 2: The results in large graphs. “Flow” is the optimal algorithm. The scores are averages over instances that were solved by all solvers. The sub-optimality is for makespan, dividing the makespan of TSW AP by the optimal scores.

| map   | 30 agents | 70 agents | 110 agents |
|-------|-----------|-----------|------------|
|       | runtime (sec) | success rate(%) | sub-optimality |
|       | Flow | Alg. 2 | Alg. 3 | Flow | Alg. 2 | Alg. 3 | Flow | Alg. 2 | Alg. 3 |
| lak30ld | 100 | 26.2 | 0.0 | 0.0 | 100 | 100 | 100 | 1.001 | 1.001 |
|        | 500 | 60.6 | 0.6 | 0.1 | 100 | 100 | 100 | 1.099 | 1.022 |
|        | 1000 | 54.7 | 3.5 | 0.2 | 100 | 100 | 100 | 1.064 | 1.073 |
|        | 2000 | 56.3 | 25.4 | 0.4 | 100 | 100 | 100 | 1.340 | 1.358 |
| dens20ld | 100 | 46.0 | 0.0 | 0.0 | 100 | 100 | 100 | 1.000 | 1.052 |
|        | 500 | 67.5 | 0.3 | 0.1 | 100 | 100 | 100 | 1.003 | 1.118 |
|        | 1000 | 82.3 | 1.6 | 0.2 | 98 | 100 | 100 | 1.014 | 1.097 |
|        | 2000 | 89.8 | 9.3 | 0.4 | 100 | 100 | 100 | 1.043 | 1.169 |
| brc202d | 100 | 141.9 | 0.1 | 0.1 | 60 | 100 | 100 | 1.000 | 1.001 |
|        | 500 | 214.7 | 0.7 | 0.3 | 48 | 100 | 100 | 1.001 | 1.003 |
|        | 1000 | 238.5 | 2.7 | 0.6 | 42 | 100 | 100 | 1.002 | 1.007 |
|        | 2000 | 230.2 | 14.8 | 1.1 | 16 | 100 | 100 | 1.021 | 1.026 |

Figure 4: The results for the sum-of-costs metric. We used random-32-32-20. The average scores are plotted. We also show scatter plots of 50 instances by transparent points.

Table 2 shows the results on large graphs with a timeout of 5 min. The optimal algorithm took time to return solutions or sometimes failed before the timeout, whereas TSWAP succeeded in all cases in a comparatively very short time, thus highlighting the need for sub-optimal algorithms of unlabeled-MAPF. In addition, TSWAP yields high-quality solutions for the makespan.

### 6.3 Sum-of-costs Metric

We next evaluate the sum-of-costs metric. As a baseline, we used ECBS-TA [Hönig et al. 2018], which yields bounded sub-optimal solutions with respect to the sum-of-costs. The implementation of ECBS-TA was obtained from the authors. We used random-32-32-20 with 30, 70, and 110 agents. The sub-optimality of ECBS-TA was set to 1.3, which was adjusted to solve problems within the acceptable time (5 min). TSWAP used Alg. 2. Note that we preliminary confirmed that ECBS-TA in denser situations failed to return solutions within a reasonable time.

Figure 4 shows that TSWAP yields solutions with acceptable quality while reducing computation time (lower, vertical axis) by orders of magnitude compared to the others; the quality of sum-of-costs (horizontal axis) is competitive with ECBS-TA, with makespan quality close to optimal (upper, vertical axis). TSWAP is significantly faster than ECBS-TA because, unlike ECBS-TA, TSWAP uses a one-shot target assignment and a simple path planning process.

### 7 Online TSWAP

TSWAP is not limited to offline planning and can also adapt to online planning with timing uncertainties. Algorithm 4 presents Online TSWAP to solve the online time-independent problem. Before execution, TSWAP assigns targets to each agent [Lines 1–2]. During execution, the online version runs the procedure of the offline version for one agent [Line 3]. If the virtual location (i.e., a,v) is updated, then let the agent actually moves there [Line 4].

**Theorem 2.** Online TSWAP (Algorithm 4) is complete for the online time-independent problem.

**Proof.** The most part is the same as for the offline version (Theorem 1). The problem assumes a fair execution schedule, therefore, \( \phi \) must decrease within the sufficiently long period during which all agents are activated at least once; otherwise, \( \phi = 0 \).

**Proposition 5.** Regardless of execution schedules, Online TSWAP has upper bounds of:

- sum-of-moves: \( O(A \cdot \text{diam}(G)) \)
- sum-of-costs: \( O(A \cdot \text{diam}(G)) \)

### 8 Demos of Online Planning

This section rates Online TSWAP (using Alg. 2) with real robots. The video is available at https://kei18.github.io/tswap.

**Platform** We used toio robots (https://toio.io/) to implement TSWAP. The robots, connected to a computer via Bluetooth, evolve on a specific playmat and are controllable by instructions of absolute coordinates.

**Usage** The robots were controlled in a centralized style, described as follows. We created a virtual grid on the playmat; the robots followed the grid. A central server (a laptop) managed the locations of all robots and issued the instructions (i.e., where to go) to each robot step-by-step. The instructions were periodically issued to each robot (per 50 ms) but they were issued asynchronously between robots. The code was written in Node.js.
9 Conclusion

This paper presented a novel algorithm TSWAP to solve or execute unlabeled-MAPF; simultaneous target assignment and path planning problems for indistinguishable agents. TSWAP is complete for both offline and online problems regardless of initial assignments. We empirically demonstrated that it can solve large offline instances with acceptable solution quality in a very short time, depending on assignment algorithms. It can also work in online situations with timing uncertainties as shown in our robot demos.

Future directions are: (1) applying TSWAP to other situations, e.g., lifelong scenarios, and (2) decentralized execution with only local interactions by Online TSWAP.

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Appendix

A Proofs

Proposition 1. Offline TSWAP has upper bounds of:

- makespan: $O(A \cdot \text{diam}(G))$
- sum-of-costs: $O(A^2 \cdot \text{diam}(G))$
- maximum-moves: $O(A \cdot \text{diam}(G))$
- sum-of-moves: $O(A \cdot \text{diam}(G))$

Proof. The potential function $\phi$ in the proof of Theorem 1 is $O(A \cdot \text{diam}(G))$; this is the makespan upper bound. Note that the second term of $\phi$ is bounded by $\text{diam}(G)$ because II assumes the shortest paths on $G$. That of sum-of-costs is trivially obtained by multiplying $|\psi|$. Similarly, $|\psi|$ is non-increasing. $\psi$ becomes zero when the problem is solved. $\psi$ is decremented when a vertex is updated by a vertex by Line 15, i.e., each “move” action decrements $\psi$. As a result, $\psi$ eventually reaches zero. Since $\psi = O(A \cdot \text{diam}(G))$, we derive the upper bound of sum-of-moves. This bound works also for maximum-moves.

Proposition 2. Assume that the time complexity of \texttt{nextNode} and the deadlock resolution (Lines 11, 12) in Alg. 4 are $O(\alpha)$ and $O(\beta)$, respectively. The time complexity of Offline TSWAP excluding Line 7 is $O(A^2 \cdot \text{diam}(G) \cdot (\alpha + \beta))$.

Proof. According to Proposition 1, the makespan is $O(A \cdot \text{diam}(G))$, i.e., the repetition number of Line 4-20. Each operation in Lines 4-7 is constant except for Line 7, $O(\alpha)$, and Lines 11, 12, $O(\beta)$. Those operations repeat exactly $|A|$ times for each timestep, thus deriving the statement.

Proposition 3. The time complexity of Algorithm 2 is $O(O\text{max}(V+E), A^4)$.

Proof. Consider the worst case, i.e., all start-target pairs are evaluated and contained in $B$. The number of vertices and edges of $B$ is $2|A|$ and $|A|^2$, respectively. Then, Line 16 is $O(A^3)$ by the successive shortest path algorithm because its time complexity is $O(f(E' + V' \log V'))$ where $f$ is the maximum flow size and $V'$ and $E'$ represent the network. The total operations of $dist$ becomes running the breadth-first search $|A|$ times, therefore, $O(A(V+E))$. Operations for a priority queue $Q$ are both $O(\log n)$ for extracting and inserting, where $n$ is the length of the queue. Thus, the runtime of Line 3 is $O(A^2 \log A)$. The queue operations in Lines 4-15 require $O(A^2 \log A)$. Line 10 finds a single augmenting path and this is linear for the number of edges in $B$, thus, its complexity is $1 + 2 + \cdots + |A|^2 = O(A^4)$. As a result, the complexity of Alg. 2 is:

- $O(A(V+E))$ finding shortest path
- $O(A^3)$ min-cost maximum matching
- $+O(A^2 \log A)$ queue operations
- $+O(A^4)$ update matching

Proposition 4. Regardless of execution schedules, Online TSWAP has upper bounds of:

- maximum-moves: $O(|A| \cdot \text{diam}(G))$
- sum-of-moves: $O(|A| \cdot \text{diam}(G))$

Proof. The same proof of Proposition 1 is applied.

Proposition 5. The time complexity of Algorithm 3 is $O(A(V+E))$.

Proof. In the worst case, the algorithm requires $O(A(V+E))$ in total to evaluate all distances of start-target pairs by running the breadth-first search $|A|$ times. The operations in Lines 5, 10 repeat at most $|A|^2$ times because each target for each agent is evaluated at most once. Lines 13-20 repeat at most $|\text{diam}(G)|$ times because each iteration must reduce the maximum cost of the assignment $M$. Both Line 14 and Lines 15-19 are $O(A)$. As a result, the algorithm is $O(A(V+E) + A^2 + A \cdot \text{diam}(G))$, which equals to $O(A(V+E))$.

We additionally present the correctness of Alg. 3.

Proposition 6. Algorithm 3 is correct; returns a distinct target for each agent.

Proof. We focus on the initial assignment phase [Lines 1-12] because the refinement phase [Lines 13-20] only swaps targets for the existing assignment $M$ and terminates within finite iterations (see Prop. 5). Trivially, each assignment operation never assigns one target to more than one agents. Observe that $|U| + |M| = |A|$ is invariant. Thus, an output of the algorithm is correct.

B Refinement for Sum-of-costs

The sum-of-costs version of Alg. 3 is presented in Alg. 5.

Proposition 7. The time complexity of Algorithm 3 is $O(A(V+E) + A^3 \cdot \text{diam}(G))$.

Proof. Lines 2-10 repeat at most $|A| \cdot \text{diam}(G)$. Each iteration requires $O(A^2)$. Together with the proof of Proposition 3 we derive the statement.
Algorithm 5: Refinement for Sum-of-costs

input: unlabeled-MAPF instance
output: $M$: assignment, a set of pairs $s \in S$ and $g \in T$
1: execute Lines 1-9 of Alg. 4
2: while $M$ is updated in the last iteration do
3: for $(s_i, s_i) \in M$, $(s_j, g_j) \in M$, $i \neq j$ do
4: $c_{\text{swap}} \leftarrow \text{dist}(s_i, s_i) + \text{dist}(s_j, g_j)$
5: if $h(s_j, g_j) + h(s_i, g_i) \geq c_{\text{swap}}$ then continue
6: $c_{\text{swap}} \leftarrow \text{dist}(s_i, g_j) + \text{dist}(s_j, g_i)$
7: if $c_{\text{swap}} < c_{\text{swap}}$ then swap $g_i$ and $g_j$ of $M$; break
8: end for
9: end while

Table 3 presents further details of Table 1 and an additional result of another map lak303d. The main observations are the same as described in Sec. 6.

Table 3: The detailed results of TSWAP with different assignment algorithms. We also display 95% confidence intervals of the mean, on which bold characters are based.

| A | metric | Alg. 2 | Alg. 2 | Alg. 2 | Alg. 3 | Alg. 3 | greedy | linear |
|---|-------|--------|--------|--------|--------|--------|--------|--------|
| runtime (ms) | 110 | 17 (17.18) | 17 (17.18) | 17 (17.17) | 4 (4.4) | 12 (12.12) | 4 (4.4) | 12 (12.12) | 23 (22.23) |
| sum-of-costs | 1079 (1038.1119) | 937 (890.972) | 937 (890.972) | 1139 (1094.1184) | 1186 (1104.1188) | 958 (919.995) | 936 (1302.1470) | 940 (900.980) |
| runtime (ms) | 500 | 17 (17.18) | 17 (17.18) | 17 (17.17) | 4 (4.4) | 12 (12.12) | 4 (4.4) | 12 (12.12) | 23 (22.23) |
| sum-of-costs | 2595 (2497.2690) | 2169 (2084.2252) | 2169 (2083.2251) | 2878 (2761.2993) | 2880 (2749.2969) | 2546 (2423.2635) | 4653 (4394.4088) | 429 (2306.2550) |
| runtime (ms) | 2000 | 17 (17.18) | 17 (17.18) | 17 (17.17) | 4 (4.4) | 12 (12.12) | 4 (4.4) | 12 (12.12) | 23 (22.23) |
| sum-of-costs | 4760 (4471.4862) | 3469 (3331.3620) | 3469 (3330.3622) | 5220 (4955.5435) | 5244 (5001.5471) | 5465 (5122.5796) | 12292 (11357.1315) | 5122 (4750.5479) |

D Implementation of Offline TSWAP

We use a priority queue with re-insert operations for agents instead of a simple list for Line 7 because the ordering of a list affects results. To observe this, consider a line graph with two adjacent agents on the left side. Their targets are on the right side. With a simple list implementation, when the left agent plans prior to the right one, the left agent has to wait for one timestep until the right agent has moved. In the reverse case, this wait action never happens. Therefore, we avoid such wasteful wait actions by using the priority queue with re-insert operations.

E Implementation of the Polynomial-Time MakeSpan-Optimal Algorithm

In our experiment, we used the polynomial-time make-span-optimal algorithm [Yu and LaValle 2013]. This algorithm has several techniques to improve the runtime performance. All of them are straightforward, however, their quantitative evaluation has not been performed to our knowledge. Thus, we evaluated them and selected the best one for each experimental setting. This section describes the details.
This technique is used in (Ma et al. 2016; Liu et al. 2019). Swap conflicts can be easily converted to plans without conflicts.

E.1 Preliminaries — Algorithm Description

Given a timestep \( T \), a decision problem of whether an unlabeled-MAPF instance has a solution with makespan \( T \) can be solved in polynomial time. This is achieved by a reduction to maximum flow problems on a large graph called time expanded network (Yu and LaValle 2013a). Let denote \( N_T \) be the time expanded network for makespan \( T \). To clarify the context, we use “vertices” for the network \( N_T \) and “nodes” for the original graph \( G \).

For each timestep \( 0 \leq t < T \) and each node \( v \in V \), the network \( N_T \) has two vertices \( v^t_{\text{in}} \) and \( v^t_{\text{out}} \). In addition, there are two special vertices \( \text{source} \) and \( \text{sink} \) to convert the unlabeled-MAPF instance to the maximum flow problem. \( N_T \) has five types of edges with a unit capacity. The intuitions are the following.

- \((v^t_{\text{in}}, v^t_{\text{out}})\): An agent can stay at \( v \) during \([t, t+1]\).
- \((u^t, v^t_{\text{out}})\) if \((u, v) \in E\): An agent can move from \( u \) to \( v \) during \([t, t+1]\).
- \((v^t_{\text{out}}, v^{t+1}_{\text{in}})\): Prevent vertex conflicts.
- \((\text{source}, v^t_{\text{in}})\) if \( v \in S\): Initial locations.
- \((v^{T-1}_{\text{out}}, \text{sink})\) if \( v \in G\): Targets.

We show an example of time expanded networks in Fig. 7 with the maximum flows. Once the maximum flow with size equals to \(|A|\) is obtained, the solution for the unlabeled-MAPF instance is easily obtained from the flow.

Since many polynomial-time maximum flow algorithms exist, the maximum flow problem for time expanded networks can be solved in polynomial-time. For instance, the time complexity of the Ford-Fulkerson algorithm (Ford and Fulkerson 1956), a major algorithm for the maximum flow problem, is \( O(fE') \) where \( f \) is the maximum flow size and \( E' \) denotes edges in the network; the running time in \( N_T \) is \( O(NT) \) with a natural assumption of \( E = O(V) \). According to (Yu and LaValle 2013a), \( T = A + V - 2 \) in the worst case, thus, the time complexity is \( O(NT^2) \).

Using the above scheme, the remaining problem is to find an optimal \( T \). This phase has many design choices. The typical one is incremental search (i.e., \( T = 1, 2, 3, 4 \ldots \)).

E.2 Techniques

This part introduces three effective techniques to speedup the optimal algorithm. We assume that the Fold-Fulkerson algorithm is used to find the maximal flow. The first two techniques are about finding an optimal makespan \( T \). The last one is for reducing the search effort of the maximum flow; this is new in MAPF literature.

Lower Bound Starting the search for \( T \) from makespan lower bound is reduced to the computational effort because the number of solving the maximum flow problems is reduced. A naive approach to obtain the bound is computing \( \max \{ \min_j h(\pi_t(0), g_j) \} \). A tighter bound is obtained by solving the bottleneck assignment problem (Gross 1959), i.e., assigning each agent to one target while minimizing the maximum cost, regarding distances between initial locations and targets as costs. This bound is easily obtained by an adaptive version of Alg. 2.

Pruning of Redundant Vertices During the search of augmenting paths, vertices that never reach the sink can be pruned. We highlight such vertices by bold lines in Fig. 7. The pruning is realized by two processes.

- Preprocessing: Before searching optimal makespans, calculate the minimum distance to reach one of the targets from each node \( v \in V \). Let denote this distance \( \lambda(v) \), e.g., \( \lambda(u) = 2 \) in Fig. 7. This is computed by an one-shot breadth-first search from all targets; its time complexity is \( O(V + E) \), i.e., the overhead of the preprocessing.
- Pruning: During the search of augmenting paths, \( v^t_{\text{out}} \) such that \( t + \lambda(v) \geq T \) is avoided from expanding as successors. This also prevents from expanding \( v^{T+1}_{\text{out}} \).

Pruning reduces search time of the maximum flow algorithm without affecting its correctness and optimality. This concept to flow network can be seen in (Yu and LaValle 2016), while similar concepts can be seen in other reduction-based approaches to labeled MAPF, e.g., SAT-based (Surynek et al. 2016) and ASP-based (Gomez, Hernandez, and Baier 2020).

Reuse of Past Flows Consider the incremental search of optimal makespan and expanding the network from \( N_T \) to \( N_{T+1} \). The Ford-Fulkerson algorithm iteratively finds a augmenting path until no such path exists. Thus, a reduction of the iterations is expected to reduce computation time.

A feasible flow of \( N_{T+1} \) with size equal to the maximum flow of \( N_T \) can be obtained immediately without search. To

\[ 3 \text{We slightly change the structure of the network in the original paper to make the network slim, i.e., removing internal two vertices for preventing swap conflicts. In the unlabeled setting, plans with swap conflicts can be easily converted to plans without conflicts. This technique is used in (Ma et al. 2016; Liu et al. 2019).} \]
see this, let $v_{\text{out}}^{T-1}$ be a vertex used in the maximum flow of $N_T$. Let this flow extending for $N_{T+1}$ by using $v_{\text{out}}^{T-1}$, $v_{\text{in}}^T$, $v_{\text{out}}^T$, and the sink. In Fig. 7, we show the example of $N_2$ highlighted by a blue dotted line started from $w_{\text{out}}^0$. This new flow is trivially feasible in $N_2$; in general, it is feasible in $N_{T+1}$. As a result, the Ford-Fulkerson algorithm in $N_2$ only needs to find one augmenting path (green), rather than two. Hence, the reuse of the past flow contributes to reducing the iterations of the Ford-Fulkerson algorithm.

E.3 Evaluation of Techniques

We evaluated the three techniques using a 4-connected grid random-64-64-20, shown in Fig. 2, while changing the number of agents. The simulator and the experimental environment were the same as Section 6. All instances were created by choosing randomly initial locations and targets.

The average runtime over 50 instances is shown in Fig. 8. We additionally show a single run of the maximum flow algorithm with optimal makespan, unknown before experiments (green bars). Since all combinations yield optimal solutions, the smaller runtime is better.

As for the technique of the lower bounds, we tested two: the conservative one obtained by $\max_i \min_j h(\pi_i(0), g_j)$ (without checkmarks at “LB”), or, the aggressive one obtained by solving the bottleneck assignment problem using Alg. 2 (with checkmarks). The runtime includes computing the bounds. The aggressive one has an advantage when the number of agents is small; however, as increasing, solving the assignment problem itself takes time then it loses the advantage. Rather, the conservative one scores smaller runtime.

The other two techniques surely contribute to reducing runtime. Notably, the best runtimes with the proposed techniques (blue) do not differ or are faster from those given the optimal makespan (green).

E.4 Implementations in the Experiments

Following the above result, in our experiments, the optimal algorithm used the techniques of the aggressive “LB”, “Prn”, and “Re” except for $|A| \geq 1000$; in this case, it used the conservative “LB” instead of the aggressive one because Alg. 2 becomes costly. brc202d is an exception; we used aggressive “LB” even when $|A| \geq 1000$. Since the map is too large, conservative “LB” more often failed to find solutions within a time limit.

Additional References

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