Estimating the static shear stiffness of joints by wave attenuation

Hua Li¹,*, Gheibi Amin², Hedayat Ahmadreza³, Jianhui Deng¹

1 State Key Laboratory of Hydraulics and Mountain River Engineering, Sichuan University, Chengdu, China
2 Colorado School of Mines, Golden, CO, USA

* Corresponding author: Email: huali@scu.edu.cn

Abstract: Recent studies on the shear behavior of joints demonstrated that the joint seismic stiffness value measured using seismic wave propagation is different from the static stiffness values determined by static stress-displacement measurements. Several experimental studies on mated fractures quantified this difference and revealed that the seismic stiffness is almost two to eight times larger than the static one. However, the underlying physical mechanisms responsible for this discrepancy are still poorly understood. In this study, the difference between the seismic and static shear stiffnesses was attributed to the frequency dependence of joint stiffness. A velocity discontinuity model composed of Hooke and Newton elements was used to interpret experimentally collected shear waves transmitted through a rock joint to predict the static shear stiffness at different strain levels. Specifically, the rate-independent stiffness of this model was associated with joint deformation responses at low frequencies, providing an estimation of the static stiffness. Existing experimental results support that the excess stress model enables the estimation of the values of static stiffnesses of rough joints, as indicated by a simple shear wave transmission experiment. Moreover, the results indicate that strain rate effects and frequency effects are intrinsically related.

1. Introduction

Many experimental studies have found that the dynamic and seismic stiffnesses of a rock joint is typically larger than the static stiffness due to strain rate effects[1-4]. Cai[5] conducted a series of laboratory tests and demonstrated that the seismic compressive stiffness of joints was approximately 2–3 times higher than the dynamic stiffness, and the dynamic stiffness was 1.2–2 times larger than static/quasi-static stiffness. A similar trend was observed by Pyrak-Nolte, Myer[6], who found that the value of joint seismic stiffness measured using seismic/ultrasonic wave propagation was 4–8 times larger than the static values determined by static stress–strain measurements.

A simple explanation for the difference is the frequency-dependent joint stiffness (Pyrak-Nolte and Nolte, 1992), i.e., joint stiffness is smaller at low frequencies than at high frequencies. The theoretical description of this frequency dependency is usually associated with the joint contact model (e.g., elastic, elastoplastic, and viscoplastic models). In general, the seismic stiffness of rough joints is usually calculated based on the frequency-domain analysis using displacement discontinuity model (DDM)[6-8], where the joint is simplified as a spring with a constant stiffness. With this model, the best fit between the predicted and measured spectra of transmitted/reflected waves can be worked out in a simple trial-and-error process. The stiffness obtained is the average value of stiffness in the frequency domain. Other types of models such as the Kelvin model and Maxwell model are rate-dependent models, which have
been successfully used in wave propagation to provide a more flexible stiffness–frequency relationship than DDM does[9-12]. Notably, the Hooke–Hooke/Newton (H-H/N) model (Figure 1) shows the advantage of quantifying the frequency dependence of joint stiffness[12]. The upper and lower boundaries of the stiffness in the frequency domain determined by the H-H/N have the potential to determine the dynamic and static joint stiffnesses, respectively[12].

This study aims to examine the method for determining the static shear stiffness of joints through a simple shear wave transmission experiment. The H-H/N model is used as a prediction model in the frequency analysis, and the experimental and predicted stiffness values are compared to assess the prediction accuracy.

2. H-H/N model
This velocity discontinuity model was proposed by Li, Gheibi[12], who combined the Kelvin and the Maxwell models to reflect the frequency dependence of joint stiffness. A sketch of the H-H/M model is shown in Figure 1.

![Figure 1. Sketch of the H-H/N model][1]

The global stiffness $k$ is expressed as

$$k = \sqrt{\frac{k_0^2 k_1^2 + k_0^2 \eta^2 \omega^2}{(k_0 + k_1)^2 + \eta^2 \omega^2}}$$  

where $k_0$ and $k_1$ are the stiffnesses of the Hooke elements; $\eta$ is the specific viscosity of the Newton element; and $\omega$ is the angular frequency.

Thus, the low-frequency stiffness (i.e., the lower boundary value) that gives the approximate value of joint static stiffness is

$$k_{(\omega \rightarrow 0)} = k_{\text{static}} = \frac{k_0 k_1}{k_0 + k_1}$$  

The transmission coefficient for a joint represented by the H-H/N model is derived as

$$T(\omega)_{H-H/N} = \frac{2\sqrt{k_0^2 k_1^2 + k_0^2 \eta^2 \omega^2}}{2\sqrt{k_0^2 k_1^2 + k_0^2 \eta^2 \omega^2} - i\omega Z \sqrt{(k_0 + k_1)^2 + \eta^2 \omega^2}}$$

where $Z$ is the seismic impedance as the product of phase velocity and density of the intact rock. Eq. 3 determines the wave propagation across an H-H/N joint. The value of $\eta$ can be determined by the wave signals at the residual strength stage, where the Maxwell model is applicable. More details regarding this method were discussed by Li, Gheibi[12]. The values of $k_1$ and $k_0$ can be determined by frequency analysis, i.e., predicted spectral amplitudes of transmitted waves are compared with the measured ones to provide a best-fitting transmission coefficient in the frequency domain. This concept was originally introduced by Pyrak-Nolte, Myer[6].
3. Experimental tests

To investigate the stiffness changes during shearing, a series of direct shear tests under constant normal load conditions were conducted on tension-fractured sandstone joints (Figure 2a). The specimen preparation method was introduced by Hedayat, Pyrak-Nolte[13]. The fully mated rough surfaces were obtained by using the Brazilian technique. The nominal size of the joint specimens is 102 mm in length, 102 mm in width, and 54 mm in thickness. The uniaxial compressive strength and Young’s modulus of the sandstone material are 69.7 MPa and 15.65 GPa, respectively. The rock joint roughness was evaluated based on the 2D joint profiles that were extracted along the shearing direction, including the steepest slopes. The value of the roughness coefficient is determined by the root mean square of the first deviation of the profiles ($Z_2$). For convenience, joint specimens used in this study are labeled as J-$Z_2$.

![Figure 2](image)

**Figure 2.** (a) Sandstone joint specimens and (b) the schematic of the experimental setup, including transducer holder plates and shear wave transducers (2S, 4S, 5S, and 8S)

![Figure 3](image)

**Figure 3.** (a) Instrumented direct shear apparatus that allows precise ultrasonic shear wave measurements during shearing; (b) thin layer of textured paint sprayed prior to the test to produce a random pattern of speckles (2S, 4S, 5S, and 8S indicate the places where ultrasonic sensors are aligned).

The direct shear apparatus used in this study was described in detail by Gheibi and Hedayat[14]. This apparatus was instrumented with a set of ultrasonic transducers (Figure 2b) for monitoring the wave propagation across the joints. The shear wave transducers (Panametrics V153) used in this study were broadband with a central frequency of 1 MHz. Shear wave signals in transmission modes were recorded every 1 s during shearing. Digital image correlation (DIC) was used to measure the in-plane displacement field on the outward-facing specimen surface. A charge-coupled camera
(2448 × 2048 square pixels) with a 35 mm focal length (model CF35HA-1) was mounted at a fixed distance from the joint specimen with the optical axis perpendicular to the outward-facing surface. This surface was precoated with textured spray paint to produce a random pattern of speckles. Figure 3 shows this instrumented direct shear apparatus and the pretreated surface of joint specimens.

In the 2D-DIC method, the reference image (undeformed specimen) was divided into small square groups of pixels (subsets), which leads to the generation of a standard virtual grid on the reference image. A similar process was conducted on the images recorded at different levels of shear loads. Then, the grayscale values in the subsets of the deformed images were compared with the reference image to obtain the correlation criterion as a function of vertical and horizontal displacement. The maximum value in the distribution of correlation criterion is determined to evaluate the displacement for that specific subset. The same procedure was repeated for other generated subsets to compute the displacement throughout the surface of the specimen [15, 16].

With the DIC technique, the joint static stiffness, defined by the ratio of stress increment to local shear displacement increment, was calculated. The four fenced-in areas in Figure 3b show the projected wave propagation paths on the prepainted surface. The local shear displacements near the transducers were estimated directly by the relative movement in these four areas.

4. Results and discussion

Figure 4 compares the model predictions and laboratory measurements of the static shear stiffness for the joint specimen J-0.17. Predicted values were determined by assuming the H-H/N model and measured values were calculated based on the DIC data. The H-H/N model shows its ability to predict the static stiffness of joints during shearing, although the agreement between the predicted and measured values is not good enough in the presented study. The maximal relative error, compared with measured data, is around 300% and occurs at the beginning of shearing, while the minimal one is lower than 15% arising before the peak stress. A possible explanation for this mismatch is relevant to the difference of the stress values used in ultrasonic and static calculations. The measured stiffness values were calculated based on the stress on the entire joint area. In contrast, the predicted stiffness values were calculated by the seismic wave propagation, which examines the stress-strain response within a small area determined by the diameter of the ultrasonic transducers. This concept offers an explanation as to why the predicted values from ultrasonic testing are consistently larger than the measured ones throughout the shear process and supports the phenomenon that the relative error decreases with the increasing shear displacement/time.
Figure 4. Variations of static shear stress during shearing for the joint specimen J-0.07 under the normal stress of 9 MPa. The predicted values were determined by assuming the H-H/N model; the measured values were calculated based on the relationship between shear stress and local shear displacement at four different transverse sections (2S[a], 4S[b], 5S[c], and 8S[d]).

5. Conclusions
The main conclusions of this study are as follows:
1. The H-H/N model can approximately predict the value of the static shear stiffness of rough joints through a simple shear wave transmission experiment.
2. The difference between static and seismic joint stiffnesses is attributed to dynamical effects concerning strain rate and strain level. Another explanation refers to the frequency dependence of joint stiffness. This assumption provides an explanation as to why the low-frequency stiffness gives an approximate value of the joint static stiffness.

Acknowledgment
This manuscript was funded by the National Natural Science Foundation of China (U19A2098), the Postdoctoral Research Fund from the Sichuan University (2021SCU12038), and the Research Student Attachment Programme of the Hong Kong Polytechnic University.

Reference
[1] Hencher S R 1981 Dams and earthquake (Thomas Telford Publishing) p 74-89
[2] Kana D D, Fox D J and Hsiung S M 1996 Int. J. Rock Mech. Min. Sci. 33 371-86
[3] Cui Z, Sheng Q and Leng X 2017 Rock Mech. Rock Eng. 50 2695-707
[4] Barton N 1988 Proc. Int. Conf. on Mechanics and Engineering of Rocks (Italy: Torino Politecnica) p17-1
[5] Cai J G 2001 Effects of parallel fractures on wave attenuation in rock (Singapore: Nanyang Technological University, PhD thesis)
[6] Pyrak-Nolte L J, Myer L R and Cook N G W 1990 J. Geophys Res. Solid Earth 95 8617-638
[7] Kendall K and Tabor D 1971 Proc. the Royal Society of London. A. Mathematical and Physical Sciences 323 321-40
[8] Myer L, Hopkins D and Cook N 1985 Proc. 26th U.S. Rock Mechanics Symposium (Rapid City, South Dakota /American Elsevier) p 135
[9] Królikowski J, Szczepak J and Witzczak Z 1989 Ultrasonics 27 45-9
[10] Pyrak-Nolte L J 1996 Int. J. Rock Mech. Min. Sci. 33 787-802
[11] Zhu J B 2011 Geophys. J. Int. 186 1315-30
[12] Li H 2020 *Proc. 54th U.S. Rock Mechanics Symposium* (Golden: Colorado /American Elsevier) p 331
[13] Hedayat A, Pyrak-Nolte L J and Bobet A 2014 *Geotech. Testing J.* **37** 786-99
[14] Gheibi A and Hedayat A 2020 *J. Rock Mech. Geotech. Eng.*
[15] Patel S and Martin C D 2018 *Geotech. Testing J.* **41** 664-74
[16] Shirole D, Hedayat A and Walton G 2020 *J. Geophys Res. Solid Earth.* **125** 2020JB019526