Noise compensation in a Fabry-Perot-based displacement sensor operating at picometer-level resolution

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Abstract. An approach for compensating the influence of the interrogator noises on the readings of the extrinsic Fabry-Perot interferometric displacement sensor is proposed. With the use of a reference interferometer and special processing, the sensor resolution was about twofold improved and comprised 12-25 pm instead of 15-46 pm for a wide range of EFPI baselines.

1. Introduction
During the last two decades a great progress in manufacture and implementation of the fiber optic sensors based on the extrinsic Fabry-Perot interferometers (EFPI) [1] has been achieved by the academic institutions and commercial companies. Such sensors demonstrate a great dynamic measurement range and a high resolution [2], [3]. Sensors of a great variety of physical quantities have been designed and implemented. The most commonly used EFPI baseline demodulation approaches are the white-light interferometry, using a scanning readout interferometer [4], [5] and wavelength-domain interferometry, in which the measurement and further processing of the interrogated interferometer spectral function is used to measure the interferometer OPD [6]–[9]. The best attained baseline resolution was 14-15 picometers [10], estimated as doubled standard deviation of the measured baseline fluctuations. The analysis of the noise sources and the light propagation inside the EFPI cavity was made in a paper [10], dedicated to resolution limits analysis for single EFPI sensors.

An idea of compensating hardware noises in interferometric sensing systems had been proposed long ago, mainly dedicated to reducing the laser frequency noise influence. In a paper [11] the idea of noise reduction with a compensating interferometer was realized for extrinsic Fabry-Perot sensors, interrogated by means of the wavelength-domain interferometry. As a result, the sensor resolution was tenfold improved (from 700 nm to 70 nm).

However, the approach proposed in [11] is suitable only for suppression of the fluctuations, induced by environmental influences and isn’t able to suppress OPD noises, induced by fluctuations in the acquisition hardware. The goal of the current paper is therefore to develop an approach of OPD resolution enhancement by means of reducing the influence of hardware noises on the OPD readings. This will provide an ability to either use cheaper interrogation units with poorer characteristics or to improve the resolution of high end displacement sensors.
Throughout this paper we consider the case of registering the spectrum of the light reflected from the sensor, which is the most common case for both single-sensor and multiplexed systems. The spectral function of a low-finesse Fabry-Perot interferometer is a sum of quasi-static and quasi-harmonic components, and is expressed as \( S_{FP}(L, \lambda) = S_0(L, \lambda) + S_M S(L, \lambda) \), where (1):
\[
S(L, \lambda) = \cos[4\pi nL/\lambda + \gamma_R(L, \lambda)],
\]
\[
S_M = 2\sqrt{R_1 R_2} \cdot \eta(L), \quad \eta(L) = \frac{\left(\pi n_0^2\right)^2}{L^2 \lambda^2 + \left(\pi n_0^2\right)^2},
\]
\[
\gamma_R(L, \lambda) = \psi_R + \varphi_R = \text{atan}\left[\frac{L}{z_R} \cdot \left(4L^2/z_R^2 + 3\right)\right] + \phi_R,
\]
and an approximation of Gaussian profile was applied to the fiber mode and the beam inside the interferometer. \( \eta(L) \) is a coupling coefficient of a light beam, irradiated by a fiber mode and travelled a distance \( 2L \) back to a fiber mode [10]; \( L \) is the interferometer baseline; \( n_0 \) is an effective radius of a mode at the output of the first fiber; \( \lambda \) is the light wavelength; \( n \) is the refractive index of the media between the mirrors; \( z_R = \pi n_0^2/\lambda \) – Rayleigh length of the beam; the argument additive \( \gamma_R(L, \lambda) \) contains a phase shift \( \psi_R \), induced by the light diffraction inside the cavity, and a phase \( \phi_R \), induced by the mirrors (typically for dielectric mirrors \( \phi_R=\pi \)).

2. Noise mechanisms

An extensive study of single EFPI displacement sensors resolution limits with wavelength-scanning interrogation was done in [10]. It was shown that the main noise sources are:

1. Absolute wavelength scale shift \( \Delta \lambda_0 \), determined by fluctuations of the triggering of the scanning start, \( \sigma_{\Delta\lambda} = \text{stdev}\{\Delta \lambda_0\} \).
2. Jitter of the wavelength points \( \delta \lambda_i \), caused by the fluctuations of the signal sampling moments, \( \sigma_{\delta\lambda} = \text{stdev}\{\delta \lambda_i\} \).
3. Additive noises \( \delta S_i \), produced by the photo registering units, by the light source intensity noises, etc. \( \sigma_S = \text{stdev}\{\delta S_i\} \).

These mechanisms will result in distortion of the registered interferometer spectral function \( S'(\lambda) \). Therefore, the spectrum approximation procedure gives an erroneous result, denoted throughout the paper as \( L_0 \). When considering a vector of consequently measured baseline values, its standard deviation \( \sigma_{L_0} \) can be calculated. Generally, \( \sigma_{L_0} \) is used as a quantitative characteristic of a sensor resolution, which is approximated as \( 2 \sigma_{L_0} \).

The first mechanism provides the shift of the measured interferometer spectrum, inherently shifting the displacement sensor readings as follows
\[
\delta L \approx - \Delta \lambda_0 / L_0 \lambda_0.
\]
The jitter of spectral points during interrogation produces the distortion of the measured spectral function \( S'(\lambda) \). This distortion can be interpreted as an additive noise with some variance. The resulting signal-to-noise ratio \( \text{SNR}_{JIT} \) can easily be estimated by simple trigonometric derivations [10]:
\[
\text{SNR}_{JIT} = \frac{S_M^2 / 2}{\left(\sigma_{\delta\lambda} S_M 4\pi n L_0 / \lambda_0^3\right)^2} = \frac{2\lambda_0^4}{\left(8\pi n L_0 \sigma_{\delta\lambda}\right)^2}.
\]
Considering the third mechanism, one has to take into account that generally the noise variance can depend on the mean optical power $P_0$, incident to the photodetector (shot noise level and laser intensity noise influence are strongly related to the mean power level). The dependency can be adequately approximated by a power function as follows

$$\sigma_s = a P_0^b,$$

which, for an EFPI, produces the signal-to-noise ratio of the spectrum, written as follows

$$\text{SNR}_s = \frac{S_{m0}^2/2}{\sigma_s^2} = \frac{2 P_0^{2-2b}}{a^2} \frac{R_1 R_2^*}{(R_1 + R_2)^{2b}},$$

$P_0$ is the optical power irradiated by the light source; $R_{1,2} = R_{1,2} \eta$ are effective mirrors reflectivities, taking into account light losses due to divergence of a non-guided beam inside the cavity. The parameters $a$ and $b$ must be obtained explicitly for a given experimental setup.

The influence of the additive noises on the standard deviation of the baseline measurement can be found either by numeric simulation, or analytically using the Cramer-Rao bound [12]. It can be shown that for the approximation approach [3] the corresponding relation can be written as

$$\sigma_s(SNR) = C \cdot SNR^{-1/2},$$

where $C$ is within the interval $(9 ÷ 11) \cdot 10^{-4} \mu m^{-1}$ for different parameters of the baseline measurement approach [3].

Finally, the expression for the baseline standard deviation can be obtained by combining expressions (5) and (9) and taking into account the variance summation rule:

$$\sigma_L = \sqrt{\frac{L}{\lambda_0}^2 \cdot \sigma_{\Delta \lambda}^2 + \frac{C^2}{\text{SNR}_s} + \frac{C^2}{\text{SNR}_{\text{JIT}}}}.$$ (10)

Substituting (2), (6) and (8) to (10), one will be able to obtain the final explicit expression, which isn’t done due to excessive bulkiness.

3. Baseline noises statistics

Throughout the paper the following noise compensation scheme will be assumed: two interferometers with similar parameters are used for the measurement, one of them is exposed to the target perturbation (which is the actual measurand) and will be referred to as sensing or signal interferometer, and the second one is isolated from any environmental changes and will be referred to as reference interferometer. The interferometer baselines will be denoted $L_s$ and $L_r$, respectively. Both these interferometers will be assumed to be interrogated by the same tunable laser, while their spectral functions will be registered by independent (although, similar) photodetectors.

Considering the impact of individual spectral points jitter, one has to take into account that it is indirect in nature: jitter itself produces equivalent additive noises, which, in turn, affect the accuracy of the approximation procedure [3]. If the sensing and the reference interferometers have equal baselines $L_s \approx L_r$, the shapes of their spectral functions will be the same, resulting in identical noise patterns, produced by the jitter. This will provide equal baseline measurement errors for the both interferometers and $C_{\text{IF}} = 1$. Recall the properties of the spectral function $S(\lambda)$, particularly that it is quasi-harmonic with period close to $\lambda_0/2$, one can expect secondary maximums of correlation in cases $|L_s - L_r| = \lambda_0/2$ and $|L_s - L_r| = \lambda_0/4$ (when interferometers spectral functions are nearly inverted). On the other hand, for arbitrarily different baselines $L_s$ and $L_r$, the spectral functions will be uncorrelated,
producing sufficiently different noise patterns, which will result in uncorrelated baseline noises. The dependency of the produced baseline noises correlation on the difference of interferometer baselines can be found by means of the numeric calculations.

Considering the additive noises (the third mechanism), one needs to take into account their two main sources – laser intensity noises, equal for the both interferometers and the photodetector noises, which are produced by different devices, and hence, are uncorrelated, producing uncorrelated errors. Laser intensity noises, being the same for the measurement and reference interferometers, will produce identical errors in case of close baseline values \( L_S \approx L_R \) and \( |L_S - L_R| = \lambda_0/2 \). In case of \( |L_S - L_R| = \lambda_0/4 \) the influence of laser intensity noises will be inverse for signal and reference interferometers and will produce anti-correlation \( C_{BF}(\lambda_0/4) = -1 \). For arbitrarily different baselines the baseline fluctuations will be uncorrelated, \( C_{BF} = 0 \). Dependency \( C_{BF}(L_S - L_R) \) for a particular setup will depend on the relation of the above mentioned noise sources.

In order to study the exact dependency of the signal and reference interferometers’ baseline noises correlation \( C_{BF} \) on the baseline difference, a numeric simulation has been performed. Two spectral functions \( S(L_{S,R}, \lambda) \) were calculated according to (1), the difference \( \delta L = L_S - L_R \) was varied within the interval \( 0 \div 0.8 \mu m \), the values \( L_{S,R} \) themselves were around 200 \( \mu m \). The rest parameters were chosen close to the experimental: wavelength scanning range \( 1510 \div 1590 \) nm, interval between spectral points \( \Delta = 4 \) pm, resulting in \( M = 20001 \) points in spectrum; wavelength jitter stdev \( \sigma_{\delta\lambda} = 1 \) pm; scale shift stdev \( \sigma_{\Delta\lambda} \approx 0.05 \) pm; laser output optical power \( P_0 \approx 0.06 \) mW; RIN in full frequency band 60 dB (in case of \( \sim 1 \) MHz photodetector band this corresponds to practical 120dB/\( \sqrt{\text{Hz}} \) RIN spectral density); photodetector noise equivalent power (NEP) 30nW in the full frequency band (3∙10^{-11} W/\( \sqrt{\text{Hz}} \) for \( \sim 1 \) MHz photodetector band). For all baseline combinations \( L_S, L_R \) an ensemble of \( N=100 \) realizations of noises were calculated for better statistical validity of the simulation results. The correlation coefficients \( C_{BF} \) of the resulting vectors of the baseline fluctuations were calculated. In figure 1 the simulated dependency \( C_{BF}(\delta L) \) is shown as a solid line.

![Figure 1](image)

**Figure 1.** Dependencies of the baseline fluctuations correlation coefficient on the baseline difference for all noise mechanisms: modelling (line) and experimental (points).

From this dependence it can be predicted that for baselines difference less than 50 nm the correlation is still enough for noise compensation.

### 4. Experimental noise compensation

In order to support the theoretical results, an experimental study of EFPI displacement sensor resolution was carried out. Spectral measurements were performed using the optical sensor interrogator NI PXIe 4844, utilizing a tunable laser with SMF-28 single-mode fiber output. Spectrometer parameters are the following: scanning range \([1.51; 1.59]\) \( \mu m \); wavelength jitter stdev \( \sigma_{\delta\lambda} = 1 \) pm; optical power \( P_0 \approx 0.06 \) mW; scale shift stdev \( \sigma_{\Delta\lambda} \approx 0.05 \) pm; spectral points stepping \( \Delta = 4 \) pm; number of spectral points \( M = 20001 \); spectrum acquisition rate about 1 Hz, spectrum acquisition time 0.035 s.
The sensing and reference interferometers were formed by the two ends of SMF-28 fiber packaged with PC connectors, fixed in a standard mating sleeve. The air gaps $L_S$ and $L_R$ between the fiber ends was varied from $\sim 100 \, \mu m$ up to $\sim 800 \, \mu m$ with steps $\sim 100 \, \mu m$ by the use of Standa 7TF2 translation stages. The experimental setup is schematically shown in figure 2.

The aim of the first performed experiment was to verify the correlation properties of the baseline fluctuations, predicted by means of numeric modeling. As in the modeling, the baselines of the both interferometers were set to $\sim 200 \, \mu m$, the signal interferometer baseline was fixed, while the reference interferometer baseline was scanned with step $\sim 100 \, nm$. In figure 1 the experimental relation $C_{BF}(\delta L)$ (dots) is compared with the simulated dependency $C_{BF}(\delta L)$ (solid line). Not exact correspondence of simulated and experimental dependencies can be due to some discrepancy of experimental setup parameters and those implied in the simulation. Also, a lower level of correlation is observed in experiment, therefore, limiting the admissible baseline discrepancy to somewhat 10 nm.

After that the noise compensation possibilities were tested. In order to compensate the parasitic baseline fluctuations, the baselines of both the signal and reference interferometers $L_S$ and $L_R$ were measured, after that the deviations of the reference interferometer baseline from the initial value were subtracted from the sensing interferometer baseline values:

$$L_{SC}(t) = L_S(t) - [L_R(t) - L_R(0)].$$

The baselines of both interferometers were set nearly equal with accuracy better than 1 nm. For each baseline value the measurements were performed for about 5 minutes, resulting in 300 measured points, with respect to them the standard deviation was calculated. Then the baselines were changed and measurements were repeated for another $L_S$ and $L_R$ values. The dependencies of standard deviations of a single sensor readings $L_S$ and noise-suppressed double sensor readings $L_{SC}$ on the baseline value are shown in figure 3. Estimation of the measured baseline standard deviation calculated according to (10) is also presented as a solid curve for reference.

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**Figure 2.** Experimental setup.

**Figure 3.** Dependencies of measured baselines fluctuations’ standard deviations on the mean baseline value.
As can be seen in figure 3, in the performed experiment the effort of the noise compensation exceeded 1.5 for baselines greater than 400 µm and reached 2 for $L_S \approx L_R \approx 700$ µm.

5. Conclusion
In the current paper an original approach for eliminating the influence of the interrogating unit noises on the measured value of interferometer baseline is proposed. Parasitic baseline fluctuations, induced by unideal interrogator operation are estimated by means of additional reference interferometer, isolated from the environment, which had the same parameters as the sensing interferometer. After that the fluctuations of the reference interferometer measured baseline were subtracted from the measured baseline of the signal interferometer. As a result, the standard deviation of the signal interferometer measured baseline value was about twofold decreased.

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