Simulation of Pull-in mechanism of plate actuator considering damping

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Abstract. Based on electrostatic micro-actuator, energy method is used to analyses its static and dynamic Pull-in mechanism. In addition, the model of electrostatic micro-actuator perforated-plate is used for example analysis to derive its dynamic Pull-in parameter without and with considering squeeze damping effect. By comparing the parameter values from MATLAB calculation and ANSYS simulation under the two circumstances mentioned above, the effects of damping ratio on Pull-in parameter can be obtained. The results show that, the dynamic Pull-in displacement is more than the static Pull-in displacement. And with the existence of squeeze damping effect, Pull-in voltage will increase. When the damping ratio reaches a certain extent, the dynamic Pull-in voltage value will be as large as the static Pull-in voltage value.

1. Introduction

Micro-actuator is the core part of MEMS, which can not only provide power for micro system, but also becomes the operation and execution unit of micro system. The electrostatic actuator is one of the most widely used actuators. In electrostatic actuators, the Pull-in [1] phenomenon, which limits the travel range of the actuator, is an important factor including static and dynamic.

In practical applications, some devices need to avoid pull-in phenomena, such as pressure sensors [2]. However, the pull-in phenomenon can also be exploited in some applications, such as MEMS switches [3]. Usually, in the design of MEMS, in order to accelerate the release or reduce the damping, the release hole or damping hole will be processed on the plate [4], so as to reduce the impact of squeeze damping. Taking the RF MEMS switch for example, the perforated structure with etched holes is used to reduce the damping in the driving process [5].

The Pull-in phenomenon in MEMS actuators has been a research hotspot. Nowadays, the static pull-in parameters of perforated plate have been studied in detail [6]. Therefore, the example analysis in this paper is mainly based on the structure of perforated plate to study the dynamic pull-in phenomenon. By comparing the theoretical calculation results with the simulation results, the influence of damping on the pull-in parameters is explored.

2. Static Pull-in mechanism analysis

The simplified structure of a parallel plate electrostatic actuator with a single degree of freedom can help describe the pull-in phenomenon, as shown in Figure 1. The actuator is composed of an upper electrode plate (movable) and a lower electrode plate (fixed), and the mass of upper plate is \( m \), the opposite area of the two plates is \( A \). The upper plate is suspended over a linear spring with an elastic
coefficient of k, and the initial clearance between the upper plate and the lower plate is \( g_0 \). Apply a voltage \( V \) to the upper electrode and therefore the two electrodes form a capacitor. \( C(x) \) represents the capacitance value when the plate displacement is \( x \). Using this structure, the static Pull-in phenomenon of the micro-actuator is analyzed.

**Figure 1.** Simplified structure diagram of parallel plate electrostatic actuator.

In the static analysis system, the effect of inertia and damping is ignored, and the moving plate will not oscillate. Applying a fixed voltage to two plates, under the action of the electric field force, the movable plate moves downward, and the distance between the two plates becomes smaller. The spring deforms. The movable plate is subject to the spring restoring force at the same time. Under the action of the electric field force and the spring restoring force, the system reaches equilibrium and the plate is in a static state. When the voltage increases to a certain value, the electrode plate will move to a critical equilibrium position. At this time, if the voltage continues to increase or a small disturbance is given to the system, the system will become unstable and the movable plate will be immediately attracted to the fixed plate. The corresponding critical point is the static Pull-in point, where the voltage is the static Pull-in voltage \([7]\). The following static pull-in analysis of the actuator is carried out by energy method:

2.1. Energy analysis

The energy equation is established for the system, and the following expression can be obtained:

\[
W = W_{\text{elec}} - W_{\text{elas}} = \frac{1}{2} C(x) V^2 - \frac{1}{2} k x^2
\]  

When calculating the first and second derivatives of the formula (1) respectively. At the same time, under the static condition, the capacitance between the two plates ignoring the edge effect is:

\[
C(x) = \frac{\varepsilon A}{g_0 - x}
\]

Among them \( \varepsilon \) is the dielectric constant.

From the above analysis, it can be seen that for the static pull-in phenomenon, \( x_{\text{pi}} = \frac{1}{3} g_0 \) and \( V_{\text{pi}} = \frac{\sqrt{6k g_0^2}}{27\varepsilon A} \).

3. Mechanism analysis on dynamic Pull-in

In the dynamic analysis system, the step voltage signal is applied between the two plates, and the energy is injected into the system instantaneously. The movable plate is displaced and moves to the maximum displacement in the first cycle. Due to the existence of inertia, if the pull-in point is not reached, the movable plate will do periodic oscillatory motion with constant amplitude (ignoring damping) or periodic oscillatory motion with attenuation (considering damping). If the pull-in position is reached, the movable plate will be immediately attracted to the fixed plate when it reaches the maximum displacement.

Electrostatic micro-actuators, after loading step voltage signal, need a certain amount of energy in an instant into the system for the pull-in phenomenon. Taking the advantage of the mechanical system of energy storage and conversion and the energy method, the energy change of the whole system is
utilized to study the Pull-in features of the perforated plate electrostatic micro-actuators after applying step voltage signal.

The expression of step voltage signal is:

\[ v(t) = V_0 U(t) \] \hspace{2cm} (2)

Where, \( U(t) \) is the unit step function, \( V_0 \) is the amplitude, and Figure 2 is the step voltage signal. In the following analysis of the dynamic pull-in phenomenon, the applied voltage value refers to the amplitude of the step voltage.

![Step voltage signal](image)

**Figure 2.** Step voltage signal.

Before the voltage is applied, the whole system is in a static state. After the voltage applied, energy is injected into the system instantaneously. The movable plate moves to generate kinetic energy, and the spring deforms to generate elastic potential energy. Hence, the energy expression for the whole system [8] is:

\[ E_{in} = E_{\text{kin}} + E_{\text{poten}} + E_{\text{damp}} \] \hspace{2cm} (3)

Where, \( E_{in} \) is the energy injected into the system, \( E_{\text{kin}} \) is the kinetic energy of the system, \( E_{\text{poten}} \) is the potential energy of the system, and \( E_{\text{damp}} \) is the energy consumed by damping.

3.1. Theoretical calculation of ignoring damping

In a vacuum, the damping is negligible, which is \( E_{\text{damp}} = 0 \). And when the movable plate reaches the maximum displacement, all energy is converted into elastic potential energy, that is \( E_{\text{kin}} = 0 \). According to the relationship between elastic resilience \( F_{\text{elas}} \) and electrostatic attraction \( F_{\text{elec}} \) is \( F_{\text{elec}} = F_{\text{elas}} \).

Equation (3) can be formulated as \( W = \int_0^x \frac{k_0 x (x_0 - x)^2}{2} dx = \int_0^x k x. \) Carry on integral processing, the amplitude of the step voltage applied \( V_0 \) is related to the displacement of movable plate in the following equation (4):

\[ V_0 = \sqrt{\frac{k g_0 x (x_0 - x)}{k \varepsilon A}} \] \hspace{2cm} (4)

When the movable plate reaches the maximum displacement, the energy injected into the system is transformed into elastic potential energy. The work done by electrostatic energy is zero, i.e. \( \frac{\partial W}{\partial x} = \frac{\partial V_0}{\partial x} = 0 \). When the voltage amplitude increases to a certain value, the maximum displacement reached by the movable plate is just the critical equilibrium position. At this time, only a small disturbance can make the system collapse. From the above analysis, it can be seen that for the dynamic pull-in phenomenon, \( x_{\text{spi}} = \frac{1}{2} g_0 \) and \( V_{\text{spi}} = \sqrt{\frac{k g_0^3}{4 k \varepsilon A}} \).

3.2. Theoretical calculation considering damping

In the actual MEMS application, when two parallel plates are moving relative to each other, the gas between them will be squeezed and show a damping effect, which is called squeeze damping. The squeeze damping has a great influence on the dynamic characteristics of MEMS structure. Shiqiao
Gao pointed out that the larger the damping is, the smaller the system quality factor will be. Sometimes this relationship can be used to reduce the damping effect on the system by increasing the quality factor of the micro-mechanical structure [9].

This section mainly studies the characteristics of the electrostatic micro-actuator under the influence of damping. The nonlinear vibration equation considering damping effect is (5).

\[
m \frac{d^2x}{dt^2} + \mu \frac{dx}{dt} + kx = -\frac{\varepsilon AV_0^2}{2(g_0 - x)^2}
\]

Among them, \(m\) is the mass of movable plate. In order to consider the pull-in instability under damping condition, a new variable natural frequency \(\omega_0\) is introduced to associate with the damping ratio \(\xi\) and the damping coefficient \(\mu\) [6]. The damping ratio \(\xi\) is the proportion between the damping coefficient \(\mu\) and the critical damping coefficient \(\mu_c\). Where, \(\mu = 2\xi m \omega_0\) and \(\omega_0 = \sqrt{\frac{k}{m}}\) are the relations between variables. Formula (5) can be expressed as

\[
\frac{d^2x}{dt^2} + 2\xi \omega_0 \frac{dx}{dt} + \omega_0^2 x = -\frac{\varepsilon AV_0^2}{2m(g_0 - x)^2}
\]

Therefore, this is a nonlinear ordinary differential equation, and it is difficult to obtain an exact analytical expression. Therefore, the theoretical value of considering damping in the following part will use MATLAB and adopt Runge-Kutta method [10] to solve the differential equation (6). And it is compared with the simulation value of ANSYS, so as to explore the influence of damping on the dynamic characteristics of the system.

4. Example analysis

4.1. Simulation structure

Figure 3 illustrates the structure of the perforated plate electrostatic micro-actuator fixed on both ends. The movable plate with holes in the middle is the upper electrode, and the fixed plate without deformation is the lower electrode. The four legs supporting the structure are \(L_{\text{leg}}\) in length and \(W_{\text{leg}}\) in width. With the driving voltage applied between the upper and lower driving electrodes, under the action of electrostatic force, the fixed beam deforms and flexes downward [10]. As the voltage increases, the deflection of the beam increases. When the voltage exceeds the critical voltage, the upper electrode will be attracted to the lower electrode, thus resulting in the pull-in phenomenon.

![Figure 3](image)

**Figure 3.** Structure of static micro-actuator with perforated plates fixed at both ends. (a) 3D figure. (b) Planform

![Figure 4](image)

**Figure 4.** Simplified model of perforated plate electrostatic micro-actuator.
When the dynamic performance of the perforated plate micro-actuator structure is analyzed, the electrostatic driving beam can be assumed as a spring resonance system, as shown in Figure 4. Plate B is a fixed plate, and plate A is a movable plate, with the distance of \( g_0 \) between them. The length of the perforated plate is \( l \), \( w \) for its width and \( h \) for its height. The hole is a square with the side length \( a \). The distance between two hole edges is defined as \( d \), and \( r \) is described as the distance between the hole edge and the plate edge.

4.2. Simulation settings
This section mainly adopts the commercial FEM software ANSYS to carry on the dynamic simulation for the perforated plate micro actuator. Simulations for perforated plates whose parallel plate and ribbon efficiency are 72.73\%, 63.64\% and 54.55\% respectively, and the ribbon efficiency \( \alpha \) is \( d/\) (hole pitch), \( \alpha \) is the ratio between the space of hole - hole edge and the space of hole - hole center [11]. The top view of the simulation model is shown in Figure 5.

![Figure 5. Top view of simulation model.](image)
(a)Parallel plate. (b) \( \alpha = 72.73\% \). (c)\( \alpha = 63.64\% \). (d)\( \alpha = 54.55\% \)

The basic structure parameters and other simulation parameters of the device are as follows: the movable plate is 50 um long in \( l \), 50 um wide in \( w \), 2 um thick in \( h \); the hole side length \( a \) is 5um, and the initial spacing \( g_0 \) between the movable plate and the fixed plate is 2 um; dielectric constant in vacuum \( \varepsilon_0 \) is \( 8.854 \times 10^{-12} \) F/m, and the relative dielectric constant of air \( \varepsilon_r \) is 1; Young's modulus \( E \) of movable plate is 70GPa and material density \( \rho = 2700 \) kg/m\(^3\). According to the elastic coefficient of fixed beam [11] \( k = 4EW_{\text{leg}} \left( \frac{h}{d_{\text{leg}}} \right)^3 \), the equivalent spring elasticity coefficient \( k \) can be obtained as 35.84 N/m.

ANSYS simulation involves the coupling between the electrostatic field and the structure field. The problem of parameter exchange between the electrostatic field and the structure field can be solved by coupling analysis on the electrostatic structure. Therefore, the TRANS126 unit is created on the contact surface of the two fields [12] in the process of analysis. TRANS126 represents a conversion unit that converts energy from an electrostatic field to a structural field, fully coupling the electromechanical field.

4.3. Simulation results
Figure 6 is the simulation diagram of applying 45V voltage between parallel plates under different damping ratios.

From the Figure 6, for the same specifications of the micro-actuators, the total energy of the system dissipates continuously in the process of oscillation due to the existence of the squeeze film damping. With the increasing damping ratio, the biggest displacement of the movable plate in the first phase will decrease, the maximum of kinetic energy and potential energy also will continue to decrease. Therefore, the critical Pull - in voltage value will change due to the existence of the squeeze film damping.

Figure 7 shows the influence of damping ratio on pull-in voltage for movable plates with different strip efficiencies. From the Figure 7, the theoretical value of pull-in voltage solved by Runge-Kutta method is basically consistent with the simulation value of ANSYS. Moreover, no matter in which structure, the squeeze damping effect will make the pull-in voltage increase when the damping is neglected. When the damping ratio is less than 0.4, the dynamic pull-in instability voltage required by
the micro actuator will increase with the increase of the damping ratio. When the damping ratio is greater than 0.4, the dynamic pull-in voltage is consistent with its static pull-in voltage.

![Figure 6](image)

**Figure 6.** Oscillation curve of parallel plates at 45V with different damping ratios.

![Figure 7](image)

**Figure 7.** The relation between damping ratio and pull-in voltage.

(a) Parallel plate. (b) $\alpha = 72.73\%$. (c) $\alpha = 63.64\%$. (d) $\alpha = 54.55\%$

5. **Conclusions**

In this paper, energy method is used to analyses the Pull-in mechanism of the fixed-fixed perforated-plate electrostatic micro-actuator. Energy analysis method and ANSYS simulation are used to obtain
the dynamic Pull-in parameters. By comparing the parameter values without and with considering squeeze damping effect, the study about effects of damping ratio on dynamic Pull-in phenomenon is obtained. The results show that, the static Pull-in displacement is \(\frac{1}{5} g_0\); the dynamic Pull-in displacement is \(\frac{1}{7} g_0\), and with the existence of squeeze damping effect, Pull-in voltage will increase. When the damping ratio is greater than 0.4, the dynamic Pull-in voltage will increase to static Pull-in voltage.

By our research, we can calculate the Pull-in position and Pull-in voltage when the Pull-in phenomenon occurs under the condition of knowing the specific size of the actuator. In this way, we can take advantage of or avoid the Pull-in phenomenon more effectively. At present, the research on parallel plate Pull-in phenomenon has been relatively mature. However, the perforated plate micro-actuator has the advantage of reducing damping effect compared with parallel plate, so it is necessary to study the Pull-in parameter of perforated plate. Therefore, by calculating the consistency between theoretical calculation and practical application of perforated plate, we can better control the error under the condition of small-sized devices, thus ensuring more accurate estimation of some parameters of Pull-in phenomenon.

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