Accurate Determination of Halo Velocity Bias in Simulations and Its Cosmological Implications

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Received 2018 March 1; revised 2018 May 11; accepted 2018 May 25; published 2018 July 3

Abstract

A long-standing issue in peculiar velocity cosmology is whether the halo/galaxy velocity bias \( b_v = 1 \) at large scale. The resolution of this important issue must resort to high-precision cosmological simulations. However, this is hampered by another long-standing “sampling artifact” problem in volume-weighted velocity measurement. We circumvent this problem with a hybrid approach. We first measure the statistics free of sampling artifacts, then link them to volume-weighted statistics in theory, and finally solve for the velocity bias. \( b_v \) (determined by our method) is not only free of sampling artifacts but also free of cosmic variance. We apply this method to a \( \Lambda \)CDM N-body simulation of \( 3072^3 \) particles and 1200 Mpc/h box size. For the first time, we determine the halo velocity bias to 0.1%–1% accuracy. Our major findings are as follows: (1) \( b_v \neq 1 \) at \( k > 0.1 \) h/Mpc. The deviation from unity \((|b_v - 1|)\) increases with \( k \). Depending on halo mass and redshift, it may reach \( O(0.01) \) at \( k = 0.2 \) h/Mpc and \( O(0.05) \) at \( k \sim 0.3 \) h/Mpc. The discovered \( b_v \neq 1 \) has a statistically significant impact on the structure growth rate measurement by spectroscopic redshift surveys, including DESI, Euclid, and SKA. (2) Both the sign and the amplitude of \( b_v - 1 \) depend on mass and redshift. These results disagree with the peak model prediction in that \( b_v \) has much weaker deviation from unity, varies with redshift, and can be bigger than unity. (3) Most of the mass and redshift dependences can be compressed into a single dependence on the halo density bias. Based on this finding, we provide an approximate two-parameter fitting formula.

Key words: cosmology; observations – dark energy – dark matter – large-scale structure of universe

1. Introduction

A crucial (but often ignored) assumption in cosmology based on large-scale peculiar velocity is that the galaxy/halo velocity bias \( b_v = 1 \) equals unity \((b_v = 1)\). The usual argument is based on the weak equivalence principle (EP). On scales larger than about 10 Mpc, gravity is mainly dictated by the large-scale distribution of dark matter in the universe, instead of individual bound structures such as halos and galaxies. Therefore, the motions of halos and galaxies should faithfully follow this large-scale structure (LSS), leading to \( b_v = 1 \) at \( >10 \) Mpc.

As galaxies/halos are not the dominant sources of gravity over \( >10 \) Mpc scale, they can be treated as test particles. Therefore, their motions should faithfully follow dark matter, and therefore \( b_v = 1 \) at \( >10 \) Mpc scale.

However, this argument overlooks the fact that halos/galaxies only reside at (local) density peaks. Along with the fact that the density gradient is tightly correlated with the velocity field, the seminal BBKS (Bardeen et al. 1986) paper predicted \( \sigma^2_{v,\text{halo}} < \sigma^2_{v,\text{matter}} \). This result was derived using one-point Gaussian statistics. Desjacques & Sheth (2010, hereafter DS10) extended the peak model to two-point statistics. They derived an elegant expression, \( b_v(k) = 1 - R^2 v^2 k^2 < 1 \). The prefactor \( R \) depends on the halo mass \( M \) but not redshift. DS10 predicted significant and redshift-independent deviation of \( b_v \) from unity, even at scales \( >10 \) Mpc. For example, \( b_v \approx 0.93(0.73) \) at \( k = 0.1(0.2) \) h/Mpc for \( 10^{13} M_\odot / h \) protohalos (e.g., Elia et al. 2012). Later theoretical and numerical works (Chan et al. 2012; Elia et al. 2012; Baldauf et al. 2015; Chan 2015) investigated and verified the DS10 prediction. Nonetheless, these works are all for proto-halos defined in the linear and Gaussian initial conditions, instead of real halos that host galaxies in observations. Due to the stochastic relation between proto-halos and halos, and the complexities in halo velocity evolution (e.g., Colberg et al. 2000), ambiguities exist to extrapolate these works to peculiar velocities of real halos/galaxies.

Therefore, despite of decades of effort, the velocity bias remains an unresolved issue. Even worse, it will become a limiting factor of peculiar velocity cosmology in the near future (e.g., Howlett et al. 2017). In particular, stage IV dark energy surveys such as DESI, PFS, Euclid, and SKA, and WFIRST (e.g., Schlegel et al. 2011; Abdalla et al. 2015; Spergel et al. 2015; Amendola et al. 2018; DESI Collaboration et al. 2016) aim to measure the structure growth rate \( f(z) \sigma_8(z) \) to 1% or higher accuracy, through redshift space distortion (RSD). However, what RSD actually measures is the galaxy peculiar velocity and therefore the combination \( b_v \times f\sigma_8 \). Systematic bias in the understanding of \( b_v \) then induces a systematic error in \( f\sigma_8 \).

\[
\frac{\delta (f\sigma_8)}{f\sigma_8} \bigg|_{k,z} \approx -\frac{\delta b_v}{b_v} \bigg|_{k,z}.
\]

\( b_v \) is in principle dependent of scale \( k \). So, the induced systematic error depends on the small scale cut \( k_{\text{max}} \) (namely, we only use the velocity information at \( k < k_{\text{max}} \)). Various cuts have been adopted in peculiar velocity cosmology. DESI
We have tried various approaches to measure the volume-weighted velocity statistics in simulations. We have designed new velocity assignment methods, including the NP method (Zheng et al. 2013) and the Krigeing method (Yu et al. 2015, 2017). We have built a theoretical model of the sampling artifacts (Zhang et al. 2015), verified it in a simulated dark-matter velocity field (Zheng et al. 2015b), and then applied it to correct the sampling artifacts in the halo velocity field (Zheng et al. 2015a). Despite these efforts, we have not yet succeeded in measuring \( b_v \) with 1% accuracy at \( k = 0.1 \) h/Mpc. The \( b_v \) measurement at larger \( k \) is even more challenging.

The current paper presents an exact approach to determine \( b_v \) from simulations. It circumvents the problem of sampling artifacts by a hybrid method. For the first time, we determine \( b_v(k, z) \) to 0.1%–1% at \( k \leq 0.4 \) h/Mpc and \( 0 < z < 2 \), for various halo mass bins. In Section 2, we present our method of determining \( b_v \). We leave further technical details in the Appendix. In Section 3, we describe the simulation used for the velocity bias measurement. In Section 4, we present the determined \( b_v(k) \) for various halo mass and redshift. In Section 5, we discuss its impact on cosmological surveys. We discuss and conclude in Section 6.

2. The Method

The method is hybrid in the sense that it is composed of two steps, direct measurements and theoretical interpretation.

1. Direct measurements are for three quantities, all with negligible sampling artifacts. (1) One is the halo momentum \( p_h = (1 + b_v) v_m \). (2) One is the halo number overdensity \( \delta_h(x) \), for which the sampling artifact is irrelevant. It is the density-weighted velocity, free of sampling artifacts. (3) Another is the matter velocity \( v_m \). In principle, it contains a sampling artifact. Fortunately, it has been suppressed to a negligible level in our simulation. Our simulation has, on average, \( 216 \) simulation particles per \((2.4 \, \text{Mpc}/h)^3\) volume. For such dense sampling, the resulting sampling artifact in the dark-matter velocity power spectrum is 0.02% \( \times (k/0.1 \, \text{Mpc})^2 \) (estimated by Equations (16) and (24) in Zhang et al. 2015). The induced systematic error in the determined \( b_v \) is half of that, 0.01% \( \times (k/0.1 \, \text{Mpc})^2 \). Therefore, the dark-matter velocity field at scales of interest \( k \lesssim 0.4 \) h/Mpc is well sampled and is essentially free of sampling artifacts.

2. These direct measurements are then exactly linked to the volume-weighted statistics in theory, where the only unknown parameter is \( b_v(k) \). We then solve for \( b_v \). The determination of \( b_v \) is then free of sampling artifacts.

Step 1. We first measure the correlation function

\[
\xi_{(1+\delta_h)v_v}(r) \equiv \langle (1 + \delta_h(x_1)) v_v(x_1) \cdot v_v(x_2) \rangle
\]

and its Fourier counterpart \( P_{(1+\delta_h)v_v}(k) \). These measurements are then linked to the following correlation functions

\[
\xi_{(1+\delta_h)v_m}(r) \equiv \langle v_h \cdot v_m \rangle + \langle \delta_h v_h \cdot v_m \rangle
\]

and their power spectra (Fourier components)

\[
P_{(1+\delta_h)v_m}(k) = P_{v_m}(k) + B_{\delta_h v_v}(k).
\]

Here, \( \langle \cdots \rangle \) denotes the volume/ensemble averaging.

5 Unlike the density-weighted velocity statistics, the volume-weighted velocity does not depend on the galaxy density bias. For theoretical modeling of RSD, the volume-weighted velocity statistics is preferred in some approaches (e.g., Kaiser 1987; Scoccimarro 2004; Taruya et al. 2010; Zhang et al. 2013), while the density-weighted statistics is preferred in the distribution function approach (Seljak & McDonald 2011; Okumura et al. 2012) and the streaming model (Peebles 1980; White et al. 2015).
Table 1

Five Sets of Halo Mass Bins

| Set ID | Mass Range | ⟨M⟩ | Nh/10^5 | bh(k < 0.1 h/Mpc) |
|--------|------------|-----|----------|------------------|
| A1(ζ = 0.0) | > 10 | 37.7 | 7.1 | 1.36 |
| ζ = 0.5 | > 10 | 29.8 | 5.5 | 1.89 |
| ζ = 1.0 | > 10 | 23.7 | 3.5 | 2.68 |
| ζ = 2.0 | > 10 | 17.7 | 0.88 | 4.98 |
| A2(ζ = 0.0) | 1–10 | 2.7 | 54.4 | 0.81 |
| ζ = 0.5 | 1–10 | 2.6 | 52.5 | 1.04 |
| ζ = 1.0 | 1–10 | 2.5 | 46.9 | 1.48 |
| ζ = 2.0 | 1–10 | 2.3 | 28.6 | 2.64 |
| A3(ζ = 0.0) | 0.5–1 | 0.70 | 52.9 | 0.70 |
| ζ = 0.5 | 0.5–1 | 0.70 | 54.0 | 0.86 |
| ζ = 1.0 | 0.5–1 | 0.69 | 52.4 | 1.15 |
| ζ = 2.0 | 0.5–1 | 0.69 | 39.7 | 1.97 |
| ζ = 3.0 | 0.5–1 | 0.68 | 21.8 | 3.15 |
| B1(ζ = 0.0) | 0.5–1 | 0.70 | 52.9 | 0.70 |
| B2(ζ = 0.0) | 1–7 | 2.4 | 51.5 | 0.80 |
| B3(ζ = 0.0) | 7–10 | 8.2 | 3.3 | 1.02 |
| B4(ζ = 0.0) | > 10 | 37.7 | 7.1 | 1.36 |
| C1(ζ = 0.0) | 7–10 | 8.2 | 3.3 | 1.02 |
| C2(ζ = 0.5) | 1.2–4.0 | 2.1 | 34.1 | 1.02 |
| C3(ζ = 1.0) | 0.31–0.35 | 0.33 | 20.3 | 1.01 |

Note. The mass unit is 10^12 M_☉/h. ⟨M⟩ is the mean halo mass. Nh is the total number of halos in the corresponding mass bin. bh is the halo density bias at k < 0.1 h/Mpc.

Step 2. We then solve Equation (4) for the velocity bias b_v(κ), defined through

\[ b_v(κ) = \frac{P_{\text{num}}(κ)}{P_{\text{num}}(κ)}. \]  

A key point to pay attention is that b_v(κ) is the only unknown quantity in Equation (4). The proof is presented in the Appendix. Then, comparing the left- and right-hand sides drawn from the same simulation, we determine b_v. For this reason, the determined b_v is essentially free of cosmic variance. In the Appendix, we present the maximum likelihood approach to solve Equation (4) for b_v(κ).

3. The Simulation

The simulation we analyze (J0620) adopts the standard ΛCDM cosmology, with Ω_m = 0.268, Ω_Λ = 0.732, Ω_b = 0.044, σ_8 = 0.83, n_s = 0.96, and h = 0.71. It has box size L_box = 1200 Mpc/h, particle number N_p = 3072^3 and the mass resolution 4.4 × 10^8 M_☉. J0620 is run with a particle-particle-particle-mesh (P3M) code, detailed in Jing et al. (2007). The halo catalog is constructed by the Friends-of-Friends method. The linking length is b = 0.2. In the catalog, gravitationally unbound “halos” have been excluded. The halo center is defined as the mass-weighted center and the halo velocity is defined as the velocity averaged over all member particles. We adopt various mass and redshift bins to calculate the mass and redshift dependence of velocity bias. Table 1 shows details of these bins.

The number density and momentum fields of both halos and dark matter are measured using the NGP method. Namely, (1 + δ_h)ν_ν = ∑_i ν_i^2/h. The sum (∑_i) is over all particles in the given cell. n is the mean number of halos in each cell. We adopt 512^3 grid points. The grid cell size is L_grid = 2.4 Mpc/h. Each cell has 216 simulation particles on average. Therefore, we have an excellent sampling of the dark-matter velocity field above such scale. This allows us to robustly measure the dark-matter velocity field through v_n = ∑_i v_i^2/∑_i, with negligible sampling artifacts. The aliasing effect is also negligible, as we are interested in the scales of k ≲ 0.4 h/Mpc, much smaller than the Nyquist wavenumber k_N = π/L_grid = 1.31 h/Mpc (Jing 2005).

Figure 1 shows the directly measured P_1 + δ_h v_n for M > 10^{13} M_☉/h and 10^{12} M_☉/h < M < 10^{13} M_☉/h halos at z = 0. For comparison, we also show P_1 + δ_h v_n. The three are almost identical to each other until k ≥ 0.2 h/Mpc. These terms have two contributions. The contribution from v · v dominates at k ≤ 0.3 h/Mpc. The contribution from (v · v) becomes significant at k ≥ 0.2 h/Mpc and becomes dominant at k ≥ 0.3 h/Mpc. These results already imply b_v ∼ 1 at k ≤ 0.1 h/Mpc. The exact determination of b_v(κ) requires us to solve Equation (4) using the method described in the Appendix.

4. The Velocity Bias

We obtain the best-fit value and the associated error of b_v(κ) over the k ranges of (0, 0.05), (0.05, 0.1), (0.1, 0.15), (0.15, 0.20), (0.20, 0.25), (0.25, 0.3), (0.3, 0.4), and (0.4, ∞). As explained earlier, the determined b_v is essentially free of cosmic variance, as it is obtained by comparing the halo and dark-matter fields in the same simulation box. The only source of noise is shot noise in the halo distribution. The large number of halos (∼10^{5–7}) then enables us to determine b_v(k ≤ 0.4 h/Mpc) with 0.1%–1% statistical error. Such accuracy also enables us to detect any significant deviation of b_v from unity.

Figures 2–4 show the redshift dependence of velocity bias for three mass bins (halo sets A1, A2, A3, respectively). Figure 5 shows the mass dependence of velocity bias at z = 0 (set B). Furthermore, Table 2 lists the velocity bias at selected ranges of k.
The velocity bias of halo set A1 ($M > 10^{13} h^{-1} M_{\odot}$) at $z = 0$, 0.5, 1.0, 2.0. The velocity bias decreases with increasing redshift. For these halos, $b_v < 1$ at all redshifts. Notice that for clarity, we shift the $z > 0$ data points horizontally. The result invalidates the usual assumption of $b_v = 1$ in peculiar velocity cosmology. $|b_v - 1|$ is much weaker than in the peak model prediction. It also shows significant redshift evolution, in contrast to the peak model prediction.

**Table 2**

| Set ID | $(b_v - 1) \times 100$ | $(b_v - 1) \times 100$ | $(b_v - 1) \times 100$ |
|--------|------------------------|------------------------|------------------------|
| A1($z = 0$) | 0.03 ± 0.13 | −0.05 ± 0.38 | −0.14 ± 0.93 |
| $z = 0.5$ | −0.02 ± 0.16 | −0.29 ± 0.32 | −1.01 ± 0.63 |
| $z = 1.0$ | −0.04 ± 0.27 | −0.40 ± 0.55 | −2.57 ± 0.97 |
| $z = 2.0$ | −0.31 ± 0.43 | −1.46 ± 0.82 | −6.90 ± 1.32 |
| A2($z = 0$) | 0.04 ± 0.06 | 0.37 ± 0.12 | 1.21 ± 0.33 |
| $z = 0.5$ | 0.07 ± 0.06 | 0.27 ± 0.14 | 0.81 ± 0.19 |
| $z = 1.0$ | 0.04 ± 0.06 | 0.06 ± 0.08 | 0.26 ± 0.20 |
| $z = 2.0$ | −0.05 ± 0.08 | −0.37 ± 0.18 | −1.25 ± 0.13 |
| A3($z = 0$) | −0.01 ± 0.06 | 0.40 ± 0.15 | 1.50 ± 0.19 |
| $z = 0.5$ | 0.05 ± 0.04 | 0.35 ± 0.13 | 1.06 ± 0.21 |
| $z = 1.0$ | 0.04 ± 0.05 | 0.22 ± 0.15 | 0.60 ± 0.17 |
| $z = 2.0$ | −0.04 ± 0.07 | −0.08 ± 0.15 | −0.31 ± 0.15 |
| $z = 3.0$ | −0.12 ± 0.10 | −0.60 ± 0.22 | −1.58 ± 0.26 |
| B1($z = 0$) | −0.01 ± 0.06 | 0.40 ± 0.15 | 1.50 ± 0.19 |
| B2($z = 0$) | 0.04 ± 0.07 | 0.33 ± 0.14 | 1.22 ± 0.24 |
| B3($z = 0$) | −0.01 ± 0.28 | 0.43 ± 0.50 | 0.57 ± 1.03 |
| B4($z = 0$) | 0.03 ± 0.13 | −0.05 ± 0.38 | −0.14 ± 0.93 |
| C1($z = 0$) | −0.01 ± 0.06 | 0.43 ± 0.50 | 0.57 ± 1.03 |
| C2($z = 0.5$) | 0.06 ± 0.07 | 0.34 ± 0.17 | 0.88 ± 0.38 |
| C3($z = 1.0$) | 0.04 ± 0.07 | 0.29 ± 0.22 | 0.66 ± 0.35 |

**Note.** We discover a statistically significant deviation of $b_v$ from unity at $k > 0.1 h/Mpc$. $|b_v - 1|$ increases with $k$ and may reach $\mathcal{O}(10\%)$ at $k \sim 0.3 h/Mpc$.

We detect a statistically significant deviation of $b_v$ from unity at $k > 0.1 h/Mpc$. This invalidates the assumption of $b_v = 1$ commonly adopted in peculiar velocity cosmology. It will significantly impact RSD cosmology of stage IV dark energy projects. The deviation $b_v - 1$ shows rich behavior in $k$, halo mass $M$, and redshift $z$. Nonetheless, it shows significant difference to the peak model prediction and poses a new question to the halo peculiar velocity theory. Major findings are discussed below.

**4.1. The $k$ Dependence**

$b_v(k) - 1$ can have either a negative or positive sign. This challenges the peak mode, which predicts a negative sign. The sign of $b_v - 1$ does not vary with $k$. Furthermore, $|b_v(k) - 1|$ increases with $k$, and roughly speaking $b_v(k) - 1 \propto k^2$. (1) At $k \leq 0.1 h/Mpc$, the deviation is very weak. Over all of the halo mass and redshift investigated, the deviation is 0.3% or less and it is statistically insignificant. $|b_v - 1|$ is orders of magnitude weaker than the peak model prediction on proto-halos. It means that we cannot simply extrapolate the predictions on proto-halo velocity to real halo velocity. (2) At $0.1 \leq k \leq 0.2 h/Mpc$, $b_v$ may show a statistically significant deviation from unity. Depending on the type of halos, the deviation may reach 1%. As discussed later, despite its weakness, it will become a significant source of systematic error for DESI RSD cosmology. (3) At $0.2 \leq k \leq 0.4 h/Mpc$, some halo samples show $\mathcal{O}(10\%)$ deviation from unity. One task of RSD cosmology is to extract cosmological information deep into the nonlinear regime of $k \leq 0.5 h/Mpc$ (e.g., the cosmic vision dark energy report; Dodelson et al. 2016). The existence of significant deviation of $b_v$ from unity at this regime is a challenge to this task.

**4.2. The Mass and Redshift Dependence**

$b_v$ increases with increasing redshift (Figures 2–4) and decreasing mass (Figure 5). As a consequence, the sign of $b_v - 1$ depends on halo mass and redshift. For example, $b_v - 1$ is always negative for halos of $M > 10^{13} h^{-1} M_{\odot}$ at all redshifts (Figure 2). But it changes from positive at $z = 0$ to negative at $z = 2$ for less massive halos (Figures 3 and 4). Another consequence is that $b_v - 1$ has strong dependence on the halo mass and redshift. For example, for $M > 10^{13} h^{-1} M_{\odot}$,
halos at $0.25 < k < 0.3 \ Mpc$ and $z = 1(2)$, $b_v - 1 = -0.026 \pm 0.069$. But for halos of $10^{12} < M < 10^{13} M_{\odot}/h$, $b_v - 1 \sim 0.3\%$ at $z = 1$. Figure 5 compares $b_v$ of various mass at $z = 0$. Now the biggest deviation from unity happens for the least massive halos.

To translate the above results into an impact on peculiar velocity cosmology, we need specifications of galaxy surveys, because different surveys probe different galaxies in different halos and different redshifts. Here, we just present a qualitative description on halos that may be probed by various surveys. Later we will quantify the impact of velocity bias to some of these surveys in Section 5. (1) $10^{13} M_{\odot}$ halos at $z < 1$ may be probed by LRGs in DESI (e.g., Guo et al. 2015; DESI Collaboration et al. 2016). Galaxies in the TAIPAN redshift and peculiar velocity survey are also expected to reside in these halos, but at $z \sim 0$ (Howlett et al. 2017). (2) For smaller halos ($\leq 10^{13} M_{\odot}$), 21 cm surveys may be capable of detecting them. SKA are capable of detecting billions of 21 cm emitting galaxies residing in these halos, given its sensitivity in H1 mass (Abdalla & Rawlings 2005; Yang & Zhang 2011) and the observationally constrained H1 mass-halo mass relation (e.g., Guo et al. 2017). SKA is also able to indirectly detect them through the intensity mapping. (3) For halos of $M < 10^{12} M_{\odot}$, a large fraction of ELGs in surveys such as DESI and PFS may reside in these halos (Favole et al. 2016; Gonzalez-Perez et al. 2018). DESI can probe them at $0.6 < z < 1.6$ while PFS can probe them to higher redshift ($z < 2.4$). 21 cm intensity mapping surveys such as CHIME (Bandura et al. 2014) and Tianlai (Xu et al. 2015) are also sensitive to these halos, although they may not be able to detect individual galaxies.

### 4.3. The Dependence on the Halo Density Bias

An interesting observation is that the mass and redshift dependence of $b_v$ may be absorbed into a single dependence on the halo density bias $b_h$. This can be seen by first checking $b_v$ in Table 1 and then comparing $b_v$ of halos with similar $b_h$. More explicitly, halos of set C (Table 1) at different redshifts are chosen to have identical density bias ($\approx 1$). Table 2 shows that they have roughly the same velocity bias. This motivates us to propose the following fitting formula,

$$b_v(k|M, z) \approx 1 - [c_0 + c_1(b_h(M, z) - 1)]k^2.$$  \hfill (6)

Here, $k \equiv k/(\text{Mpc}/h)$. This is basically the Taylor expansion of $b_v(k)$ around $(0, 0, 0)$. The isotropy of the velocity bias ($b_v(k) = b_v(k)$) requires that terms of odd power in $k$ such as $k^3$ and $k^2 k^2 k^2$ ($m, n, l = 1, 2, 3$) vanish in the Taylor expansion. Therefore, the leading order term is $k^2$. We find that $c_0 = -0.138 \pm 0.01$ and $c_1 = 0.186 \pm 0.007$ (Figure 6). Small error in $c_1$ shows that the dependence on $b_h$ is statistically significant. We caution that this fitting formula is only approximate, as it ignores dependence beyond $b_h$ and ignores higher-order $k$ dependence (e.g., $k^3$). Nevertheless, it is
sufficient to describe the excessive dependence of the velocity bias on halo property and scale (Figure 6). Another caveat in this fitting formula is the implicit assumption that all of the cosmological dependences are encoded in the cosmological dependence of $b_b$, and therefore $c_{0,1}$ do not depend on cosmology. This is motivated by the primary dependence of $b_v$ on $b_b$. If valid, we are then able to use this fitting formula for other cosmologies such as the Planck 2015 cosmology (Planck Collaboration et al. 2016), whose $\Omega_m$ is 13% larger. Future works will use simulations of different cosmologies to investigate this assumption.

5. Implications for Peculiar Velocity Surveys

We discuss two implications of velocity bias on cosmology. The first is that it may bias the structure growth rate measurement in spectroscopic redshift surveys. The second is that it may open a window of testing the EP at cosmological scales.

5.1. Impact on Structure Growth Rate Constraint

A major task of cosmological surveys is to constrain the structure growth rate through peculiar velocity. The velocity bias, if ignored or modeled inappropriately, will become a source of systematic error. Whether it is of statistical significance depends on surveys and galaxy types. The low-redshift TAIPAN survey aims to measure the peculiar velocities of $\sim 10^4$ galaxies. It will constrain $f_{\sigma_8}$ at $z \sim 0$ with $\sim 10\%$ accuracy, using information at $k < 0.2 \ h/\text{Mpc}$ (Howlett et al. 2017). The target galaxies (with $M \sim 10^3 M_\odot/h$) have $|b_v - 1| \ll 1\%$ at $k < 0.2 \ h/\text{Mpc}$ (Figure 2 and Table 2). Therefore, the usual assumption of $b_v = 1$ results in negligible systematic error and can be adopted safely.

On the other hand, the spectroscopic redshift survey DESI can measure $f_{\sigma_8}$ to 1% level over a number of redshift bins, resulting in an overall statistical error of 0.4% (DESI Collaboration et al. 2016). Figure 7 plots the predicted $b_v$ of various galaxies in DESI. We estimate $b_v$ using the fitting formula of Equation (6). The density biases of various galaxy types are adopted as $b_{\text{LRG}} = 1.7/D(z)$, $b_{\text{ELG}} = 0.84/D(z)$, and $b_{\text{QSO}} = 1.2/D(z)$ (DESI Collaboration et al. 2016). Here, $D(z)$ is the linear density growth rate, and it is normalized as unity at $z = 0$. For DESI, we are no longer able to approximate $b_v = 1$, otherwise systematic error at $1\sigma$ level can be induced.

For PFS ELGs, the predicted $b_v$ at $z < 1.6$ is similar to that of DESI ELGs. For clarity, we neglect them in Figure 7. We only show the results at $z = 1.8(2.2)$, where we adopt $b_{\text{ELG}} = 1.62(1.78)$ (the PFS SSP proposal). If only using information at $k \leq 0.2 \ h/\text{Mpc}$, the systematic error in $f_{\sigma_8}$ caused by ignoring $b_v = 1$ is less than 0.4%. Due to a factor of 10 smaller sky coverage than DESI, the overall statistical error of $f_{\sigma_8}$ constrained by PFS RSD to $k = 0.2 \ h/\text{Mpc}$ is expected to be $\sim 1\%$. Therefore, the impact of velocity bias on PFS RSD is subdominant, and we can simply approximate $b_v = 1$.

However, due to the higher number density of PFS galaxies, it can measure RSD to smaller scales and has the potential to further reduce the statistical error in $f_{\sigma_8}$. Figure 7 shows that, when we push to $k_{\text{max}} = 0.3 \ h/\text{Mpc}$, we may no longer adopt $b_v = 1$.

The proposed SKA HI survey has the capability of detecting $\sim 10^8$ 21 cm emitting galaxies to $z \lesssim 2$ over 30000 deg$^2$ (Abdalla et al. 2015). The statistical error in $f_{\sigma_8}$ is $\sim 0.3\%$ for each $\Delta z \sim 0.1$ redshift bins over $0.4 < z < 1.3$. If assuming $b_v = 1$, the induced systematic error will overwhelm the statistical error. This will also be true for the proposed stage V survey of measuring a billion spectra of LSST galaxies (Dodelson et al. 2016). The situation for Euclid may fall between DESI and SKA.

5.2. A Cosmological Test of the Equivalence Principle

Velocity bias also provides a new test of $\Lambda$CDM cosmology. Observationally, we are not able to measure the velocity bias (with respect to dark matter) directly. However, we are able to measure the ratio of velocity bias between two tracers of the LSS. Furthermore, if the two tracers overlap in space, the measured ratio will be free of cosmic variance (McDonald & Seljak 2009). Our result predicts that in $\Lambda$CDM, the velocity ratio of two tracers is

\[
\frac{b_{v,1}(k)}{b_{v,2}(k)} - 1 \approx b_{v,1}(k) - b_{v,2}(k)
\]

\[
\approx -0.19\% \left( \frac{k}{0.1 \ h/\text{Mpc}} \right)^2 \times (b_{h,1} - b_{h,2}).
\]

The first approximation holds, because $b_v = 1$ at leading order. This weak deviation from unity is a genuine consequence of the EP. Therefore, if 1% or large deviation at $k < 0.2 \ h/\text{Mpc}$ is detected, it may be a smoking gun of EP violation and therefore modifications of general relativity (e.g., Hui et al. 2009). We will further investigate this issue in future works.
6. Discussions and Conclusions

We invent a novel method to determine the volume-weighted halo velocity bias \( b_{v} \). This method is free of the long-standing sampling artifact problem, which has hindered the accurate determination of velocity bias. We apply this method to a 3072\(^3\) particle simulation and measure \( b_{v} \) to \( k \sim 0.4\) h/Mpc with better than \( 1\% \) accuracy. Our findings confront both the \( b_{v} = 1 \) standard assumption in peculiar velocity data analysis and the peak model prediction. (1) There exists a statistically significant deviation of \( b_{v} \) from unity at \( k > 0.1\) h/Mpc. Depending on halo mass, redshift, \( b_{v} - 1 \) may reach \( \mathcal{O}(1\%) \) at \( k \sim 0.2\) h/Mpc and \( \mathcal{O}(10\%) \) at \( k \sim 0.4\) h/Mpc. If ignored, this velocity bias will become a significant source of systematic error in RSD cosmology of DESI. Its impacts on SKA H1 galaxy survey and Euclid are even stronger. (2) However, \( b_{v} - 1 \) is a factor of \( \sim 10 \) smaller than the prediction of peak model. Furthermore, its mass and redshift dependence do not agree with the peak model prediction. \( b_{v} \) varies with redshift, while the peak model predicts the opposite. \( b_{v} \) of less massive halos can be larger than unity, while the peak model always predicts \( b_{v} < 1 \). The peak model is based on proto-halo statistics. Therefore, we must consider the mismatch between proto-halos and real halos, and the displacement of halos from the corresponding proto-halos, to improve the theoretical understanding of velocity bias. Another issue to consider is the displacement of halos from their initial positions (Lagrangian positions) to their present positions (Eulerian positions). This affects the velocity correlation, which is defined in Eulerian space. It is expected to make \( b_{v} \) larger than the peak model prediction, and make it increase toward \( z = 0 \). We also expect that the environment in which halos reside (e.g., filaments or clusters) may play a role in the halo velocity bias. For example, the infall velocity within filaments may be responsible or partly responsible for the \( b_{v} > 1 \) behavior of less massive halos.

There are many issues to resolve in further investigations. For example, because the velocity bias depends on the density bias, would it also depend on the halo formation time? Or more generally, besides the density bias, what could affect the velocity bias? How does it depend on parameters within the standard cosmology? How does it behave in modified gravity cosmology? Also, to robustly predict its impact on RSD cosmology, we need to generate realistic mocks for target surveys and measure the velocity bias of LRGs, ELGs, 21 cm emitting galaxies, etc.

This work was supported by the National Science Foundation of China (11621303, 11433001, 11653003, 11320101002, 11533006), National Basic Research Program of China (2015CB85701, 2015CB857003).

Appendix

The Algorithm to Solve for the Scale-dependent Velocity Bias

Here, we describe in detail the procedure of solving for the scale-dependent \( b_{v}(k) \). The key is the statement in Section 2, that \( b_{v}(k) \) is the only unknown quantity in Equation (4), where all other quantities are provided by the same simulation. The proof is as follows. In Fourier space, we can decompose \( \mathbf{v}_{s}(k) = b_{v}(k)\mathbf{v}_{m}(k) + v_{h}^{S}(k) \). The first term is completely correlated with the density velocity field. In contrast, the second term \( v_{h}^{S} \) is the stochastic component of halo velocity. It is uncorrelated to the density velocity field at the two-point level. Namely, \( \langle v^{S}(k) \cdot v_{h}^{S}(k) \rangle = 0 \). One can verify that the above condition leads to Equation (5) as the definition of \( b_{v}(k) \). The decomposition above is therefore uniquely fixed. Clearly, \( v_{h}^{S} \) does not contribute to \( P_{v_{h}v_{m}} \). Furthermore, it does not contribute to \( B_{\delta_{\delta}}(k) \). The direction of \( \mathbf{v}_{h}(x) \) is uncorrelated with \( \delta_{\delta}(x) \) due to the statistical isotropy of the universe. This holds whether the halo density bias is scale dependent or nonlocal; otherwise, the statistical isotropy will be violated. The direction of \( \mathbf{v}_{h}(x) \) is also uncorrelated with \( v_{m}(x) \), by definition. Averaging over its direction, we have \( \langle \delta_{\delta}(x) v_{h}^{S}(x) \cdot v_{m}(x) \rangle = 0 \). Therefore, \( v_{h}^{S} \) does not contribute to the right-hand side of Equation (4). Because we know \( \delta_{\delta}(x) \) and \( v_{m}(x) \) from the same simulation, \( b_{v}(k) \) is all we need to fix the right-hand side of Equation (4).

We are then able to determine \( b_{v}(k) \) uniquely. The only complexity in determining \( b_{v}(k) \) is the nonlocal dependence of the right-hand side of Equation (4) on \( b_{v}(k) \). It is caused by \( B_{\delta_{\delta}}(k) \), in which \( b_{v}(k') \) also contributes. Here, we present the maximum likelihood solution to \( b_{v}(k) \).

We bin the unknown \( b_{v}(k) \) into a number of \( k \) bins, each with central value \( k_{a} \) and bin width \( \Delta k_{a} \). \( b_{v}(k) = \sum b_{v}(k_{a}) W_{v_{m}}(k_{a}) \). \( W_{v_{m}}(k_{a}) = 1 \) if \( k_{a} - \Delta k_{a}/2 < k \leq k_{a} + \Delta k_{a}/2 \), and zero otherwise. \( b_{v}(k) \) is the averaged value of \( b_{v} \) in the range \( k_{a} - \Delta k_{a}/2 < k \leq k_{a} + \Delta k_{a}/2 \). The power spectrum \( B_{\delta_{\delta}}(k) = \sum b_{v}(k_{a}) B_{\delta_{\delta}}(k_{a}) \). Here, \( B_{\delta_{\delta}}(k_{a}) = B_{\delta_{\delta}}(k_{a}) \) in which we replace \( v_{h}(k') \) with \( v_{m}(k_{a}) \). The calculation of \( B_{\delta_{\delta}}(k) \) is done with several FFTs. First, we obtain \( v_{m}(k_{a}) \) from the simulated \( v_{m}(x) \). We then inverse Fourier transform \( v_{m}(k_{a}) W_{v_{m}}(k_{a}) \) and denote it as \( v_{m}(x) \). Finally, we Fourier transform \( \delta_{\delta}(x) v_{m}(x) \), multiply it by \( v_{h}^{S}(k_{a}) \), and obtain \( b_{v}(k_{a}) \). The estimated/modelled power spectrum is then

\[
\hat{P}_{\hat{v}_{\hat{v}}}(k_{a}) = \sum_{\alpha} b_{v}(k_{a}) P_{v_{m}v_{m}}(k_{a} + B_{\alpha}(k))
\]

\[
\sum_{\alpha} b_{v}(k_{a}) S_{\alpha}(k_{a}) S_{\alpha}(k_{a}) \equiv W_{v_{m}}(k_{a}) P_{v_{m}v_{m}}(k_{a})
\]

\[
+ B_{\alpha}(k_{a}).
\]  

Figure 8 shows \( S_{\alpha} \) for halos in the range \( 10^{12} < M/(M_{\odot}/h) < 10^{13} \). For small \( \alpha \) (small \( k_{a} \)), \( S_{\alpha} \) is dominated by the first term and is very close to a step function in the \( k \) space. But when \( k \) increases, the contribution from \( B_{\alpha} \) becomes nonnegligible. Tails beyond the \( k \) range \( [k_{a} - \Delta k_{a}/2, k_{a} + \Delta k_{a}/2] \) develop.

Both \( P_{v_{m}v_{m}}(k) \) and \( B_{\alpha}(k) \) are measured from the same simulation, and the only set of unknown parameters are \( b_{v} \). Both sides are drawn from the same simulation; therefore, the determined \( b_{v} \) will be essentially free of cosmic variance. The only relevant statistical error in determining \( b_{v} \) then arises from shot noise in the halo distribution. This allows us to write down the likelihood function \( \mathcal{L} \propto \exp(-\chi^{2}/2) \) straightforwardly, with

\[
\chi^{2} = \sum_{k} \frac{(P_{\delta_{\delta}}(k_{a}) - \hat{P}_{\delta_{\delta}}(k_{a}))^{2}}{\sigma_{k}^{2}}.
\]

\( \sigma_{k} \) is the rms fluctuation of the data \( P_{\delta_{\delta}}(k_{a}) \) caused by the finite number of halos. We estimate it by splitting halos in a given mass bin into eight nonoverlapping sub-samples by randomly selecting among these halos. We measure the
dispersion between these sub-samples, divide it by \sqrt{8}$, and obtain $\sigma_k$. Due to the shot noise origin, the errors are uncorrelated over different $k$.

Because $P_{1+b_{v}v_{x}v_{y}}$ is a linear combination of $b_{\alpha}$ (Equation (8)), the likelihood function $L$ of $b_{\alpha}$ is a multivariate Gaussian distribution. The best fit of $b_{\alpha}$ is given by the following linear equations,

$$\frac{\partial \chi^2}{\partial b_{\alpha}} = 0 \Rightarrow b_{\alpha} = \frac{\sum_{k} S_{\alpha}(k) S_{\beta}(k)}{\sum_{k} P_{1+b_{v}v_{x}v_{y}}(k) S_{\beta}(k)}.$$  \hspace{1cm} (10)

The solution (best-fit value) is

$$b = F^{-1}D, \text{ with } F_{\alpha\beta} \equiv \frac{\sum_{k} S_{\alpha}(k) S_{\beta}(k)}{\sum_{k} P_{1+b_{v}v_{x}v_{y}}(k) S_{\beta}(k)} \text{ and }$$

$$D_{\alpha} = \frac{\sum_{k} P_{1+b_{v}v_{x}v_{y}}(k) S_{\alpha}(k)}{\sum_{k} P_{1+b_{v}v_{x}v_{y}}(k) S_{\beta}(k)}.$$  \hspace{1cm} (11)

The error covariance matrix $E_{\alpha\beta} \equiv \langle \delta b_{\alpha} \delta b_{\beta} \rangle$ is given by

$$E_{\alpha\beta} = (F^{-1})_{\alpha\beta}.$$  \hspace{1cm} (12)

The matrices $F$ and $E$ are shown in Figure 8. Due to overlap of $S_{\alpha}$ and $S_{\beta}$ ($\beta \neq \alpha$) in the $k$ space, $F$ has non-diagonal elements. They result in a correlated error in the determined $b_{\alpha}$, quantified by $E_{\alpha\beta} 
eq 0$. The correlation is stronger for pairs of larger $k_{\alpha}$ and $k_{\beta}$.

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