Stabilization of the rotational motion of a composite nanosatellite with a mobile module mounted on a rotating platform

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Abstract. The paper considers the dynamics of the spatial motion of a composite nanosatellite with a movable module making an angular deviation relative to the carrier body. Control laws have been developed to stabilize the rotation of the nanosatellite relative to the longitudinal axis.

1. Introduction

One of the urgent tasks of space flight is the study of the spatial motion of spacecraft. In connection with the use of nanosatellites with moving parts, the need for the development of the simplest control schemes for the orientation and stabilization of the rotational motion of the nanosatellite is increasing [1-4]. The paper considers the dynamics of the spatial motion of a composite nanosatellite of variable configuration. The nanosatellite consists of two modules of a carrier body and a moveable module. A rotating platform is installed on the body-carrier, to which a movable module is rigidly attached using flexible rods of variable length. A low-thrust engine is also installed on the carrier body. When the lengths of the flexible rods change, the movable module makes an angular motion relative to the carrier's body, which in turn creates a torque of reactive forces.

By changing the angle of inclination of the movable module, it is possible to control the magnitude and direction of the torque of the reactive forces. By rotating the platform on which the mobile module is installed, it is also possible to control the direction of the torque of the reactive forces (provided that the mobile module has been previously deflected using flexible rods).

In this work, the laws of control of the mobile module were developed based on the feedback principle to stabilize the rotational motion of the composite nanosatellite relative to its longitudinal axis.
The coordinates systems are:
- CXYZ – the coordinate frame with the origin in the mass center, which axes are parallel to the main axes of the main body. The point C is the centre of mass of the complete nanosatellite;
- C1X1Y1Z1 – the frame with the origin in mass center of the main body, which axes are parallel to the main axes of the main body. The point C1 is the mass centre only of the main body;
- C2X2Y2Z2 – the main connected frame of the movable module. The point C2 is the mass centre only of the movable unit.

2. Mathematical model
The mathematical model of the NS can be build based on the law of the change of the angular momentum, written in the moving references frame CXYZ[1-3]:

$$\frac{d\mathbf{K}}{dt} + \mathbf{\omega}_1 \times \mathbf{K} = \mathbf{M}$$

(1)

Where \( \mathbf{K} \) is the angular moment of the NSHC, \( \mathbf{\omega}_1 = [p, q, r]^T \) is the angular velocity of the main body of the NS in projections onto axis CXYZ, \( \mathbf{M} \) is the torque of external and the jet-engine thrust.
The angular moment of the NS represents the sum of the angular momentums of bodies of NS:

\[ \mathbf{K} = \mathbf{K}_1 + (\mathbf{\sigma}_1 \mathbf{K}_2) \]  

(2)

where \( \mathbf{K}_1 \) is angular momentums of the main body calculated in frame \( C_1X_1Y_1Z_1 \), \( \mathbf{K}_2 \) – angular momentums of the movable module calculated in frame \( C_2X_2Y_2Z_2 \), \( \mathbf{\sigma}_1 \) – is the transition matrix in the coordinate system \( C_2X_2Y_2Z_2 \) from the coordinate system \( C_1X_1Y_1Z_1 \):

\[ \mathbf{\sigma}_1 = \begin{bmatrix} \cos \gamma_u & -\sin \gamma_u \cos \alpha_u & \sin \gamma_u \sin \alpha_u \\ \sin \gamma_u & \cos \gamma_u \cos \alpha_u & -\cos \gamma_u \sin \alpha_u \\ 0 & \sin \alpha_u & \cos \alpha_u \end{bmatrix} \]  

(3)

where \( \alpha_u \) is the angle of rotation of the movable module around the \( X_2 \) axis, \( \gamma_u \) is the angle of rotation of the movable module around the \( Z_2 \) axis.

The torque of external and the jet-engine thrust:

\[ \mathbf{M} = \begin{bmatrix} m_1 z_2 \cos(\gamma_u) \sin(\alpha_u)P \\ m_1 z_2 \sin(\gamma_u) \sin(\alpha_u)P \\ m_b + m_u \end{bmatrix} \]  

(4)

where \( m_1 \) is weight of the carrier body, \( m_2 \) is weight of the movable unit, \( z_2 \) is distance from the center of mass of the rotating panel to point \( C_2 \).

The kinematical equations for classical Euler’s angles should be added:

\[ \begin{align*} \psi &= p \sin \varphi + q \cos \varphi \\
\dot{\theta} &= p \cos \varphi - q \sin \varphi \\
\dot{\phi} &= r - \frac{\cos \theta}{\sin \theta} \left( p \sin \varphi + q \cos \varphi \right) \end{align*} \]  

(5)

where \( \psi \) is precession, \( \theta \) is nutation, \( \varphi \) intrinsic rotation.

To simplify the mathematical model, the equations were linearized with respect to the angle of relative deviation \( \alpha_u \).

3. Simulation of the controlled dynamics

Let us consider two laws of control of the mobile module of a composite nanosatellite. The first control law in the absence of rotation of the mobile module around the longitudinal axis \( Z_2 (\gamma = \dot{\gamma} = 0) \):

\[ \alpha_u = -k_\alpha p \]  

(6)

where \( k_\alpha \) is feedback coefficient.

The second control law in the case when the movable module can rotate around the longitudinal axis \( Z_2 \):

\[ \begin{align*} \alpha_u &= -k_\alpha \sqrt{p^2 + q^2} \sin \left( \arcsin \left( \frac{p}{\sqrt{p^2 + q^2}} \right) \right) \\
\gamma_u &= \arcsin \left( \frac{q}{\sqrt{p^2 + q^2}} \right) \end{align*} \]  

(7)

The first control law in the absence of rotation of the mobile module around the longitudinal axis \( Z_2 \):
To compare the two laws of control of the mobile module of a composite nanosatellite, we will carry out two simulations with the same initial conditions and inertial-mass parameters of the nanosatellite. The initial conditions are: $\psi = 0$ [rad]; $\theta = 1.4$ [rad]; $\varphi = 0.785$ [rad]; $\alpha = 0.2$ [rad]; $p = q = r = 0.3$ [rad/s]; $k_\alpha = 0.2$. Inertial mass parameters of the NS: main body mass 2[kg], movable unit mass 1[kg], moments of inertia of the main body: $A_b = B_b = 0.01$ [kg·m²], $C_b = 0.005$ [kg·m²], moments of inertia of the movable unit: $A_u = B_u = C_u = 0.0025$ [kg·m²], jet-engine thrust $P = 2$ [N]. The simulation results are presented in Figures 2-6 (red color shows the simulation results using the control law (6) blue using the control law (7)).
Figure 4. Component of the angular velocity $r$.

Figure 5. Tilt angle of the movable module $\alpha$.

Figure 6. Platform rotation angle $\gamma$. 
As you can see from Figures 2-6, the results when using the control laws (6) and (7), the stabilization time of the rotational motion of the composite nanosatellite is practically the same. Let us carry out a simulation with similar initial conditions for the inertial-mass characteristics, taking $k_\alpha = 0.7$. The simulation results are presented in Figures 2-6 (red color shows the simulation results using the control law (6) blue using the control law (7)).
Figure 9. Component of the angular velocity $r$.

Figure 10. Tilt angle of the movable module $\alpha$.

Figure 11. Platform rotation angle $\gamma$. 
As can be seen from figures 7-11, the control law (8) with an increase in the maximum value of the angle of relative deviation of the movable module becomes more efficient than the control law (7) due to the use of an additional degree of freedom (platform rotation).

4. Conclusion
The dynamics of the rotational motion of a composite nanosatellite with a rotating platform is considered, laws of control of stabilization of the composite nanosatellite rotation relative to the longitudinal axis are developed. The addition of an additional degree of freedom significantly reduces the stabilization time of the rotational motion of the composite nanosatellite, provided that the maximum value of the deflection angle of the mobile module is close to 0.3 radians.

Acknowledgments
The work is supported by the Russian Foundation for Basic Research (#19-08-00571).

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