Pressure, Resistance, and Current Activation of Anisotropic Compressible Hall States

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Thermodynamic and electric properties of anisotropic compressible Hall states at higher Landau levels are studied using a mean field theory on the von Neumann lattice basis. It is shown that resistances agree with the recent experiments of anisotropic compressible states and the states have negative pressure. As implications, the collapse phenomena of the integer quantum Hall effect are discussed.

Keywords: IQHE, Anisotropic Hall gas, Negative pressure, Current activation

In the quantum Hall systems, kinetic energy is quenched and the one particle energies are discrete with enormous degeneracy. Its degeneracy per area and level spacing are proportional to the magnetic field. Electron interaction could change the electron system to have completely different properties. It is quite interesting if compressible gas states that have continuous one particle energies are formed. If it happens, pressure and compressibility are expected to become negative because the starting kinetic energy which gives positive contributions to these thermodynamic quantities in normal electron gas is frozen by the magnetic field. In this paper we apply Hartree-Fock method and find compressible mean field states. Their physical properties are shown to agree with recent experiments of anisotropic compressible states at higher Landau levels\[4, 5\]. Strikingly we find that the gas has in fact unusual electronic resistances to the whole system at finite temperature. It is quite interesting if compressible gas states that are formed in bulk region of finite realistic systems. The strip gives unusual electronic resistances to the strip of compressible states. Hence the gas tends to shrink, and strip of compressible states is formed in bulk region of finite realistic systems. It is shown to exist and to play an important role in metrology.

We use von Neumann lattice representation for the Landau levels, which treats two dimensional spaces symmetrically and are useful for investigating interaction effects and deriving exact relations. Formation of compressible states are studied using a mean field theory and their physical properties and implications are analyzed. Electron operators \(a_l(p)\) have momentum, \(p\), which is defined in the magnetic Brillouin zone \(|p| < \pi/a\) \((a = \sqrt{2\hbar eB}/m)\) and Landau level index, \(l\), which determines the energy eigenvalues \(E_l = (\hbar eB/m)(l + 1/2)\). We use the unit of \(\hbar = e = a = 1\) for simplicity. The total Hamiltonian of neutral system \(H = H_0 + H_1\) is given by

\[
H_0 = \sum E_l a_l^\dagger(p) a_l(p) \tag{1}
\]

\[
H_1 = \int_{k \neq 0} \frac{d^2 k}{(2\pi)^2} \rho(k) \frac{V(k)}{2} \rho(-k) \tag{2}
\]

where \(V(k) = 2\pi q^2 / k\) and the density operator \(\rho(k)\) is given by

\[
\int_{BZ} \frac{d^2 p}{(2\pi)^2} \sum_{l'w} a_{l'}^\dagger(p) a_l(p - k) \langle f_l | e^{ik\cdot\xi} | f_{l'} \rangle \times e^{-ik_x(2p-k)_y} / 4\pi. \tag{3}
\]

Technical details are given in Ref.\[4\]. We obtain anisotropic self-consistent solutions for two point functions and the mean field Hamiltonian in the \(l\) th Landau level defined from Eqs.\(1\) and \(2\),

\[
\langle a_{l'}^\dagger(p') a_l(p) \rangle = \theta(\mu - \epsilon(p)) \delta_{l,l'} \delta(2\pi)^2 \delta(p' - p), \tag{4}
\]

\[
H_{\text{mean}} = \int_{BZ} \frac{d^2 p}{(2\pi)^2} \epsilon(p) a_l^\dagger(p) a_l(p) + (a_l, a_l^\dagger) \text{ independent terms}. \tag{5}
\]
It would be reasonable to study an anisotropic solution, because this matches with the Hall bar geometry. The phase factor in Eq. (3) is a characteristic factor in the strong magnetic field and plays important roles. In the present translationally invariant mean field solution, this phase factor cancels in Eq. (5). The momentum is a good quantum number and the Fermi sea is shown in Fig. (1). This solution breaks translational invariance in the momentum, $p_y$, and is invariant under translation in the momentum, $p_x$. The $p_x$ direction is like integer Hall state which has the energy gap of Landau levels but in $p_y$ direction there is no energy gap. Density modulation of long wave length along $p_y$ direction stabilizes the system, but those along $p_x$ direction does not. Due to the phase factor, the density is uniform in $y$-direction but is periodic in $x$-direction.

We study infinite systems first. The above mean field solution has the energy per particle shown in Fig. (2). Pressure and compressibility are computed from these energies and given in Figs. (2) and (3). We see that both values are negative. Since charge neutrality is assumed from the beginning and positive charge does not move in the real material, negative pressure does not lead to instability. We will study the implications of negative pressure later. Energy per particle and pressure of an interacting two dimensional gas without magnetic field are also shown (gas) at density= $\nu'/a^2$ for $B = 6T$ in GaAs.

We compute resistances of the compressible mean field states. From Fig.(1), in $p_x$ direction the empty level is in the next Landau level and the system has energy gap. Hence longitudinal conductance $\sigma_{xx}$ vanishes. The electron system is like one dimensional system and $\sigma_{yy}$ is given by that of Landauer formula, $e^2/h$. The Hall conductance is given by the topological formula [3].

$$\sigma_{xy} = \frac{e^2}{h} \frac{1}{24\pi^2} \int \text{tr}(\tilde{S}(p)d\tilde{S}^{-1}(p))^3. \quad (6)$$

where $\tilde{S}(p)$ is the propagator in the current basis in which the standard Ward-Takahashi identity is satisfied. The propagator for the compressible states are written by using the momentum dependent energy $e(p)$ as, $S^{-1}(p)_{ll'} = \{p_0 - (E_l + e(p))\} \delta_{ll'}$. The Hall conductance becomes,

$$\sigma_{xy} = \frac{e^2}{h} (n + \nu'). \quad (7)$$

The values of $\sigma_{ij}$ agree with the recent experiments of anisotropic compressible states at $\nu = n + 1/2$ [3].

In the real experiments, are used the semiconductors of finite sizes. They have impurities and confining potentials. Owing to random impurities, one particle states are localized and have finite spatial extensions except those at the center of the Landau level. They contribute to transport if their localization lengths are same order as or larger than the
system’s sizes. There are three system’s sizes, length of the system $L_s$, width at Hall probe region $L_w1$, and width at potential probe region $L_w2$. Corresponding to three system’s sizes, three energies in which localization lengths agree with the system’s sizes are defined. In the center of Landau levels, wave functions are extended. We consider the energy regions between the center of the $l$-th Landau level $E_l$ and the boundary of the $l$-th Landau level with the $l+1$-th Landau level. Let the energies, $E_s$, $E_{w1}$, and $E_{w2}$ be the energy values where the localization lengths, $\lambda(E)$, agree with the three system’s sizes, $\lambda(E_s) = L_s$, $\lambda(E_{w1}) = L_{w1}$, $\lambda(E_{w2}) = L_{w2}$. States in the energy range $E < E_s$ are extended states, states in the energy range $E_s < E < E_{w1}$ bridge from one edge to the other edge at the potential probe region and at the Hall probe region, states in the energy range $E_{w1} < E < E_{w2}$ bridge from one edge to the other edge at the potential probe region. Finally states in the energy range $E_{w2} < E$ are localized. We call these energy regions the extended state region, collapse region, pre-collapse region or dissipative quantum Hall regime, and localized state region or quantum Hall regime, respectively. Reasons why we use particular names for the last two regions will become clear later. The system’s lengths for a typical Hall bar and energy regions are shown in Fig. (4).

Compressible states from degenerate Landau levels are extended and cover whole spatial area from one end to another end and have negative pressure and compressibility. Positive charges are composed of ions and do not move in the materials. Hence once the negative charges shrink, Coulomb energy makes the energy of the system higher. At one point, pressure effect and the Coulomb force from positive charge ions are balanced. The shape is determined by minimizing the total energy. If the filling of the compressible gas is around half filling, the effects of Coulomb interaction are negligible. But if the filling is slightly away from integer, the negative pressure effects are strong and strip is formed. The width of strip is determined from the balance between negative pressure and Coulomb interaction of electrons with neutralizing background charge.

Wave functions in collapse regime and dissipative quantum Hall regime bridge from one edge to another edge and are of stripe shape. Coulomb interaction due to positive charge ions makes them shrink further. Hence wave functions are modified slightly. Thus the strip is formed in the bulk regions.

The strip is located in the bulk region and its wave functions vanish at source and drain regions. The current does not flow through compressible gas strip at zero temperature even though states have energies of Fermi energy. At finite temperature, the current is activated into the strip from the current carrying extended states at lower Landau levels. The electric current in the strip is induced from the higher order effect shown in Fig. (5), and is given by

$$J = \frac{e^2}{h} (\frac{e}{\beta}) e^{-\beta \Delta E} F_1(\beta e E_H/\gamma),$$

where $1/\beta$ is temperature, $\Delta E$ is the energy gap, $E_H$ is Hall electric field, and $1/\gamma$ is an enhancement factor due to localized states. $F_1$ is a function which does not vary so largely and is calculated numerically. The energy gap $\Delta E$ in Eq. (8) is given by $\Delta E = E_F - E_s - eE_H/2\gamma$ in the systems of finite inject current [1].

The longitudinal resistance and Hall resistance are computed and are summarized in Table.(1). The Fermi energy varies with magnetic field, electron density, or injected current. By tuning them, we are able to adjust the Fermi energy in dissipative quantum Hall regime or quantum Hall regime. The Hall resistance is quantized exactly as inverse of integer multiple of $e^2/h$ in both regions. In the former regime the longitudinal resistance does not vanish but behaves like activation type with finite energy gap. The heating from dissipation determines the temperature at strip regions $(1/\beta_s)$ and the energy gap depends on Hall electric field in lower Landau levels. In the latter regime, the longitudinal resistance vanishes at zero temperature and becomes activation type at finite temperature. The energy gap is determined from the difference between extended state’s energy and the Fermi energy.

![Fig. 4. (a) Schematic view of a Hall bar. (b) Density of state (DOS) for the $l$-th Landau level.](image)

![Fig. 5. Feynmann diagram for the induced current.](image)
Table 1 Resistances are summarized in each regions. \( \Delta \rho_{xy} \) is a deviation from the quantized value and \( 1/\beta_s \) is a temperature at strip regions.

|                  | collapse | pre-collapse | quantum Hall |
|------------------|-----------|--------------|--------------|
| \( \Delta \rho_{xy} \) | \( \propto e^{-\beta_s \Delta E} \) | 0 | 0 |
| \( \rho_{xx} \)    | \( \propto e^{-\beta_s \Delta E} \) | \( \propto e^{-\beta_s \Delta E} \) | 0 |

In summary, it is shown that anisotropic states have unusual properties and the dissipative quantum Hall regime exists at finite temperature. Collapse phenomena observed by Kawaji et al. are shown to be understandable theoretically.

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