Dimensionality reduction of the 3D inverted pendulum cylindrical oscillator and applications on sustainable seismic design of bridges

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Abstract
The simplest 3D extension of Housner’s planar rocking model is a rocking (wobbling) cylinder allowed to uplift and roll on its circumference, but constrained not to roll out of its initial position. The model is useful for the description of bridges that use rocking as a seismic isolation technique, in an effort to save material by reducing the design moment and the size of the foundations. This paper shows that describing wobbling motion in terms of displacements rather than rotations is more useful. It unveils that a remarkable property of planar rocking bodies extends to 3D motion: A small and a large wobbling cylinder of the same slenderness will sustain roughly equal top displacement, as long as they are not close to overturning. This allows for using the response of an infinitely large wobbling cylinder of slenderness $\alpha$ as a proxy to compute the response of all cylinders having the same slenderness, irrespectively of their size. Thus, the dimensionality of the problem is reduced by one. Moreover, this paper shows that the median wobbling response to sets of ground motions can be described as an approximate function of only two non-dimensional parameters, namely $(g \tan \alpha/PGA, \ u/PGD)$ or $(g \tan \alpha/PGA, \ uPGA/PGV^2)$ where $u$ is the top displacement of the wobbling body.

KEYWORDS
3D rocking, dimensionality reduction, intensity measures, sustainable bridge design, uplifting, wobbling

1 | INTRODUCTION

In the context of structural engineering, sustainability means using less material and materials that are more environmentally friendly. Contrary to what is often implied, designing a structure to perform better while using less material is not a goal invented during the last two decades: For obvious cost reasons, this is what was considered the reasonable path in construction since man started constructing his own shelter.

Up until the late 20th century structural design used linear methods. This has resulted in the dogma that seismic design of bridges requires that the piers be firmly connected to the ground. This approach leads to an irrational and
non-sustainable design: Oftentimes, up to 50% of the total concrete of the bridge is used for massive pile foundations to take the seismic moment. Hence, relaxing this dogma would drastically reduce both the cost and the environmental footprint of the bridge, in any way the latter is measured.

In his visit to Chile after the May 1960 Earthquake, which was the most powerful earthquake ever recorded, George Housner observed that “several golf-ball-on-a-tee types of elevated water tanks survived the ground shaking despite the appearance of instability.” On the other hand, “much more stable-appearing reinforced-concrete, elevated water tanks were severely damaged.” This is the first recorded modern observation in English literature that structures that are not firmly connected to the ground can behave better than their fixed based counterparts and has motivated several researchers to suggest that allowing a structure to uplift and sustain rocking motion, can be used as a seismic design method. The underlying reason is that uplift works as a mechanical fuse and limits the design forces of both the superstructure and the foundation. An extra proof of the stability of rocking structures is the survival of ancient Greco-Roman temples for more than two millennia in earthquake prone regions. The obvious to the visitor damages to the Parthenon have not been induced by earthquakes but by people.

Several versions of rocking isolation have been proposed. To the authors’ knowledge, the first application of this approach in the 20th century is the so-called “kinematic isolation” that has been used in buildings in the Soviet Union: The ground floor comprises columns that are able to uplift.7 In New Zealand, the South Rangitikei Bridge is designed so that it uplifts along its transverse direction.8 This was the first application of rocking in bridge design.

In parallel to the above-described so-called “unrestrained rocking,” engineers in New Zealand and the US have suggested the “restrained rocking” concept for the design of bridges: Precast columns are connected to the cap-beam via ungrouted tendons. The connections are dry and mild reinforcement is provided for energy dissipation. This creates a flag shaped response and a resilient behavior. The concept is based on the early work of Priestley and Tao,9 Stanton et al.,10 and the PReCast Seismic Structural Systems (PRESSS) project.11,12 Different names have been used for similar concepts: Damage Avoidance Design,13 Controlled Rocking,14–19 Self-Centering System,20–32 Precast Hybrid Systems,33–37 Hybrid Sliding-Rocking System,38–40 Pre- or Post-tensioned Rocking,41–45 and it has recently found its way to practice in New Zealand46 and China.47

The concept of restrained rocking leads to resilient systems, but still, the design moment of the foundation remains large. In an effort to reduce the base moment, using flexible restraining systems or completely avoiding a tendon has been suggested.2,51–59 Even though the same term “rocking” is used for both these systems and the ones of the previous paragraph, they present a remarkable difference: Avoiding the tendons or using restraining systems with a flexibility below a threshold leads to systems of negative post uplift stiffness. Then, unlike rocking systems of positive post-uplift stiffness, there exists no “equivalent elastic system” that can be used for their and the elastic oscillator cannot be used as a proxy for the prediction of their behavior. Therefore, not only a designer of a rocking structure of negative stiffness would not be able to use elastic spectra and would have to resort to time history analysis, but the intensity measures developed for elastic systems would be useless.

To address the above issue, a recent work60 has suggested that the established way of describing the motion of a planar rocking block by its tilt angle is correct but not optimal. Instead, using a displacement-based description uncovers that a large and a small block of the same slenderness angle will sustain roughly equal top displacement, as long as they are both not close to overturning. Reggiani and Vassiliou61 extended the above finding to restrained rocking structures that exhibit negative stiffness. The above findings reduce the dimensionality of the problem: the maximum response of any rocking block can be predicted using an infinitely large block of the same slenderness as a proxy: Size can be dropped out.

The aim of this paper is twofold: (a) It extends the above findings to a 3D rocking model and (b) it identifies ground motion intensity measures suitable for 3D rocking structures.

2 | DESCRIPTION OF THE MODEL

2.1 | Solitary column

Recently, a 3D dynamic model of a wobbling rigid cylindrical column was presented.62 The model was developed to describe the motion of cylinders that are constrained to wobble above their initial position (Figures 1 and 2), without sliding. Therefore, it is not applicable to rocking equipment,63–73 but to structures designed to rock and return to their initial position (such as rocking bridges). Such a constraint can be materialized, for example, by conical constraints.74 The cylindrical column has a total mass \( m \), base radius \( b \) and height \( 2h \). Its semidiagonal is \( R \) and its slenderness is \( \alpha \) (\( \tan \alpha = b/h \)). The assumptions are discussed in ref.62 but are repeated in this paper for reasons of completeness:
The cylinder is considered rigid and homogeneous.
- The supporting plane surface (ground) is considered rigid.
- The contact between the cylinder and the ground is pointwise.
- The cylinder is constrained not to roll-out of its initial position.
- No sliding is allowed, that is, the friction between the cylinder and the supporting plane surface is large.
- The cylinder is always in contact with the support (i.e., it never flies). Therefore, the contact force is always compressive.
- No damping mechanism is included.

Given the above assumptions, the model has only two degrees of freedom: the tilt angle, $\theta$, and the rolling angle, $\phi$. The latter determines the location of the contact point between the cylinder and the supporting plane (Figure 1).

The equations of motion of the model are:

$$\ddot{\theta} = -p^2 \left( \sin (\alpha - \theta) + \cos (\alpha - \theta) \left( \cos \phi \cdot \frac{\dot{u}_x}{g} + \sin \phi \cdot \frac{\dot{u}_y}{g} \right) \right)$$

$$- \left( \frac{15 \sin^2 \alpha - 16 \cos^2 \alpha}{12} \right) \cdot \cos \theta \cdot \sin \theta - \frac{3}{2} \sin^2 \alpha \cdot \sin \theta - \cos \alpha \cdot \sin \alpha \cdot (1 + \cos \theta) + 2 \cos \alpha \cdot \sin \alpha \cdot \cos^2 \theta \right) \cdot \frac{1}{\left( \frac{5}{4} + \frac{1}{12} \cos^2 \alpha \right)} \dot{\phi}^2$$

$$= \left( \frac{1}{12} \left( -15 \sin^2 \alpha + 16 \cos^2 \alpha \right) \cdot \sin^2 \theta + 3 \sin^2 \alpha \left( 1 - \cos \theta \right) + 2 \sin \alpha \cos \alpha \sin \theta \cdot (1 - \cos \theta) \right) R \cdot \dot{\phi} + \left( 3 \sin^2 \alpha \sin \theta + \frac{1}{6} \left( -15 \sin^2 \alpha + 16 \cos^2 \alpha \right) \sin \theta \cos \theta + 2 \cos \alpha \sin \alpha \left( 2 \sin^2 \theta + \cos \theta - 1 \right) \right) R \cdot \dot{\phi} \cdot \dot{\theta} =$$

$$= (\sin \alpha + \sin (\theta - \alpha)) \left( \dot{u}_x \sin \phi - \dot{u}_y \cos \phi \right)$$

(1)
where $p$ is the well-known frequency parameter used in the dynamic model of the 2D rocking block, here associated with the tilting motion of the cylinder:

$$p^2 = \frac{mgR}{I_o} = \frac{12}{15 + \cos^2 \alpha} \frac{g}{R}$$

and:

$$R = \sqrt{h^2 + b^2}$$

### 2.2 3D rocking frame model

The solitary column model can be extended by adding a slab on top of $N$ rocking and wobbling columns (Figure 3). The assumptions discussed in the previous section are now made for both the column-foundation contacts and the column-slab contacts. An example of how these assumptions can be materialized can be found in ref.74

The motion of the slab can be described by 3 DOFs ($u_x$, $u_y$ and $\theta_z$) that correspond to the in-plane rigid body horizontal translations and rotation about the slab vertical axis ($\theta_z$, yaw). The rest of the allowed motions are coupled with the abovementioned ones. As all model elements are assumed to be rigid, yaw coupled with horizontal translations would cause displacement incompatibility (unless there is pure torsion, i.e., pure yaw rotation $\theta_z$). However, yaw of the slab is not expected under perfect conditions: the centers of mass and rigidity of the slab coincide because the re-centering force of every column is directly proportional to its bearing load. Therefore, the yaw DOF is disregarded and it is expected that all columns under the slab undergo identical motion governed by the two-horizontal translation DOFs of the slab. However, experimental evidence74 has shown that since imperfections do exist, some yaw rotation of the slab is observed. Nevertheless, following a statistical approach76 the model has been proven overall conservative,77 mainly because it does not attempt to model any form of energy dissipation. Therefore, it can be used to predict the upper bound of the response of wobbling systems. More refined models that dissipate energy by explicitly modelling friction are more accurate78,79 However, these models are computationally more expensive and cannot be used for the extensive parametric analysis required in this paper.

The deformability of the columns is also disregarded for the sake of simplicity. However, it has been shown that deformable structures80–87 are better candidates for rocking isolation.55

Based on the assumptions presented in Section 2.1, the equations of motion of the system are:

$$\ddot{\varphi} = -\bar{p}^2 \left( \sin(\alpha - \varphi) \left( \frac{\ddot{u_x}}{g} \cos \varphi + \frac{\ddot{u_y}}{g} \sin \varphi \right) + \dot{\varphi} \left( \sin \varphi \sin^2 \alpha (4\gamma_m + \frac{3}{2}) + \cos \alpha \sin \alpha (4\gamma_m + 1)(1 + \cos \varphi - 2\cos^2 \varphi) + \sin \varphi \cos \varphi \left( \frac{4}{3} \cos^2 \alpha - \frac{5}{4} \sin^2 \alpha + 4\gamma_m (\cos^2 \alpha - \sin^2 \alpha) \right) \right) \frac{12}{48\gamma_m + 15 + \cos^2 \alpha}$$

FIGURE 3 Rigid slab supported by four columns. The columns are allowed to uplift, rock and wobble, but not roll out of their original position
\[
\begin{align*}
\left(\frac{4\cos^2\alpha - 5\sin^2\alpha + 4\gamma_m \left(\cos^2\alpha - \sin^2\alpha\right)}{4}ight) \cdot \sin^2 \vartheta + \\
+ \left(3 + 8\gamma_m\right) \left(1 - \cos \vartheta\right) \sin^2 \alpha + 2 \sin \alpha \cos \alpha \sin \theta \left(1 + 4\gamma_m\right) \cdot \left(1 - \cos \vartheta\right) \cdot \dot{\varphi} + \\
+ \left(3 + 8\gamma_m\right) \sin^2 \alpha \sin \theta + \left(\frac{8\cos^2\alpha - 5\sin^2\alpha + 8\gamma_m \left(\cos^2\alpha - \sin^2\alpha\right)}{8}\right) \sin \theta \cos \theta + \\
+ 2 \cos \alpha \sin \alpha \left(1 + 4\gamma_m\right) \left(2 \sin^2 \vartheta + \cos \vartheta - 1\right) \cdot \ddot{\varphi} \cdot \ddot{\theta} = \\
= \left(2\gamma_m + 1\right) \left(\sin \alpha + \sin \left(\theta - \alpha\right)\right) \frac{\left(\ddot{u}_{gx} \sin \varphi - \ddot{u}_{gy} \cos \varphi\right)}{R}
\end{align*}
\]

where,
\[
\hat{p}^2 = \frac{12\left(2\gamma_m + 1\right)}{48\gamma_m + 15 + \cos 2\alpha} \frac{g}{R}
\]

and
\[
\gamma_m = \frac{m_s}{Nm_c}
\]

For \(\gamma_m = 0\) the equations yield the solitary wobbling rigid cylindrical column equations derived in.\(^\text{62}\) It can also be seen that, unlike the case of a planar rocking frame, for which there is a formal and elegant equivalence with a geometrically similar, yet larger, column, in the 3D case there is no “equivalent solitary 3D column.” However, if one keeps only the linear term of Equations (5) and (6) the size of a quasi-equivalent solitary 3D column is:

\[
\hat{R} = \frac{48\gamma_m + 15 + \cos^2\alpha}{\left(2\gamma_m + 1\right) \left(15 + \cos^2\alpha\right)} R
\]

In, ref.\(^\text{75}\) it was shown that the equivalent 3D solitary column can sufficiently describe the 3D rocking frame. Hence, this paper will focus on the solitary column model.

3 \hspace{1cm} \textbf{DISPLACEMENT BASED DESCRIPTION OF THE WOBBLING PROBLEM}

In the derivation of the equation of motion it was convenient to use the tilt angle \(\vartheta\) as one of the two DOFs. This means that the maximum response of a wobbling column to a specific ground motion is a function of two variables: the block semidiagonal \(R\) and slenderness \(\alpha\):

\[
\vartheta_{\text{max}} = f_1(\alpha, R)
\]

For a planar rocking model it has been shown\(^\text{60}\) that a rotation-based description of the problem is correct but not optimal. A displacement-based description is also correct and it uncovers an interesting property of the rocking oscillator: A large and a small rocking object of the same slenderness will sustain roughly the same top displacement, as long as they are both away from overturning.

To extend the above conclusion to three dimensions, Equation (11) gives the top displacement (Figure 2) of a wobbling column, \(u\), as a one to one mapping of \(\vartheta\) to \(u\):

\[
u = 2R \sin \alpha - 2R \sin \left(\alpha - \vartheta\right)
\]

Then, the maximum displacement of a wobbling column to a specific ground motion will be a function of \(\alpha\) and \(R\):

\[
u_{\text{max}} = f_2(\alpha, R)
\]
To numerically compute $u_{\text{max}}$, one needs to compute the response in terms of $\theta$ using Equations (1) and (2) and then use Equation (11) to compute $u_{\text{max}}$. The next sections will show that the size of the wobbling column can be dropped out of Equation (12), thus reducing the dimensionality of the problem to unity.

### 3.1 Equal displacement and equal energy rules

It will be shown that the displacement response of any wobbling block of slenderness $\alpha$ and size $R$, $u_{\text{max},R}$ can be approximated by the response of a block of the same slenderness $\alpha$ yet infinite size, $u_{\text{max},\infty}$. Then, the parameter of interest is

$$\gamma = \frac{u_{\text{max},R}}{u_{\text{max},\infty}}$$  \hspace{1cm} (13)

Two rules that could be used to approximate $\gamma$ are defined:

#### 3.1.1 Equal displacement rule

The finite and the infinite size block have the same maximum displacement, $u_{\text{max},R} = u_{\text{max},\infty}$, that is:

$$\gamma_{\text{ED}} = 1$$  \hspace{1cm} (14)

#### 3.1.2 Equal energy rule

The monotonic loading curves of the finite and infinite size blocks produce the same work (Figure 4). Then,

$$\gamma_{\text{EE}} = \frac{u_{\text{max},R}}{u_{\text{max},\infty}} = \frac{2b}{u_{\text{max},\infty}} - \frac{\sqrt{4b \cdot (b - u_{\text{max},\infty})}}{u_{\text{max},\infty}}$$  \hspace{1cm} (15)

Both rules compute the “displacement demand” on the rocking block of finite size. If the demand is larger than $2b$, this is interpreted as overturning.

The above rules should not be confused with the rules that bear the same name and have been developed for the prediction of the response of elastoplastic systems. Here, it is not some equivalent elastic system that will be used as a proxy, but a wobbling system of infinite size.

Notably, the Equal Displacement rule is an approximation of the Equal Energy rule when $\frac{u_{\text{max},\infty}}{2b} \to 0$. 

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**Figure 4**  Graphic illustration of the Equal Displacement and the Equal Energy rule
TABLE 1  Additional far field ground motions

| ID No. | Earthquake | M   | Year | Name                | Recording station                  | PEER record sequence number |
|-------|------------|-----|------|---------------------|-----------------------------------|----------------------------|
| 51    | Parkfield  | 6.19| 1966 | Temblor pre-1969    |                                   | 33                         |
| 52    | San Fernando | 6.61| 1971 | Lake Hughes #12     |                                   | 71                         |
| 53    | Imperial Valley-06 | 6.53| 1979 | Calexico Fire Station |                                   | 162                        |
| 54    | Mammoth Lakes-01 | 6.06| 1980 | Long Valley Dam (Upr L Abut) |                                   | 231                        |
| 55    | ImperialValley-06 | 6.53| 1981 | CalexicoFireStation  |                                   | 313                        |
| 56    | Coalinga-01 | 6.36| 1983 | Cantua Creek School  |                                   | 322                        |
| 57    | Coalinga-01 | 6.36| 1983 | Parkfield - Fault Zone 14 |                                   | 338                        |
| 58    | Coalinga-01 | 6.19| 1984 | Gilroy Array #4      |                                   | 458                        |
| 59    | New Zealand-02 | 6.6 | 1987 | Matahina Dam         |                                   | 587                        |
| 60    | Whittier Narrows-01 | 5.99| 1987 | Downey - Birchdale   |                                   | 614                        |
| 61    | Whittier Narrows-01 | 5.99| 1987 | Inglewood - Union Oil|                                   | 625                        |
| 62    | Whittier Narrows-01 | 5.99| 1987 | LA - 116th St School |                                   | 626                        |
| 63    | Whittier Narrows-01 | 5.99| 1987 | LB - Orange Ave      |                                   | 645                        |
| 64    | Whittier Narrows-01 | 5.99| 1987 | Santa Fe Springs - E.Joslin |                           | 692                        |
| 65    | Superstition Hills-02 | 6.54| 1987 | Westmorland Fire Sta |                                   | 728                        |
| 66    | Loma Prieta | 6.93| 1989 | UCSC Lick Observatory |                                   | 810                        |
| 67    | Landers      | 7.28| 1992 | Joshua Tree          |                                   | 864                        |
| 68    | Northridge-01 | 6.69| 1994 | Santa Monica City Hall |                                 | 1077                       |
| 69    | Kobe_Japan   | 6.9 | 1995 | Amagasaki            |                                   | 1101                       |
| 70    | Chi-Chi_Taiwan | 7.62| 1999 | TCU095               |                                   | 1524                       |
| 71    | Hector Mine  | 7.13| 1994 | Joshua Tree          |                                   | 1974                       |
| 72    | Manjil_Iran  | 7.37| 1990 | Qazvin               |                                   | 1636                       |

3.2  Ground motion selection

The effectiveness of the rules described in the previous section, will be demonstrated by computing the response to sets of recorded ground motions.

There is no consensus in the engineering community on what ground motions should be used in time history analysis. Several approaches exist including using recorded (scaled or unscaled), artificial, or synthetic excitations. This paper chooses to use the three sets of ground motions as proposed by FEMA P695] (far-field, near-field pulse-like, and near-field no pulse-like) only as a means of illustrating its argument, without taking stance on the debate around ground motion selection. More information on the FEMA P695 ground motions can be found in FEMA. In that sense, any discussion on the influence of the ground-motion selection and scaling process on the results presented in this paper, are beyond its scope.

The original sets proposed by FEMA contain 22, 14, and 14 ground motions, respectively. However, in order to obtain better statistical estimates of the response, the number of ground motions is doubled, to include 22+14+14 extra ground motions selected from the PEER ground motion database. For reasons of results reproducibility, Tables 1–2 list these extra ground motions.

According to FEMA P695 the far field set contains 22 ground motions, including #18, Cape Mendocino, Rio Dell Overpass record. However, this was not available in the PEER ground motion database and, thus, was not included in this study. Therefore, the total number of records used is 43, 28 and 28 - far-field, near-field pulse-like, and near-field no pulse-like, respectively.

3.3  Median displacement spectra

The design of structural systems does not involve a single excitation but a set of excitations that characterize the seismic hazard at a given site. Thus, this paper discusses displacements of wobbling columns of different size not by comparing
TABLE 2  Additional near-field ground motions

| ID No. | M   | Year | Name             | Recording station                      | PEER record sequence number |
|--------|-----|------|------------------|----------------------------------------|-----------------------------|
| Pulse Records Additional Subset | | | | | |
| 73     | 6.61| 1971 | San Fernando     | Pacoima Dam (upper left abut)          | 77                          |
| 74     | 7.35| 1978 | Tabas_Iran       | Tabas                                  | 143                         |
| 75     | 6.93| 1989 | Loma Prieta      | Saratoga - W Valley Coll.              | 803                         |
| 76     | 6.9 | 1995 | Kobe_Japan       | Takarazuka                             | 1119                        |
| 77     | 6.9 | 1995 | Kobe_Japan       | Takatori                               | 1120                        |
| 78     | 6.0 | 2004 | Parkfield 02     | Slack Canyon                          | 4097                        |
| 79     | 6.0 | 2004 | Parkfield 02     | Cholame IE                            | 4098                        |
| 80     | 6.93| 1989 | Loma Prieta      | Los Gatos - Lexington Dam             | 3548                        |
| 81     | 7.01| 1992 | Cape Mendocino   | Bunker Hill FAA                       | 3744                        |
| 82     | 6.6 | 2003 | Bam_Iran         | Bam                                   | 4040                        |
| 83     | 6.63| 2004 | Niigata_Japan    | NIGH11                                 | 4228                        |
| 84     | 7.1 | 1979 | Montenegro_Yugoslavia | Bar-Skupstina Opstine              | 4451                        |
| 85     | 7   | 2010 | Darfield_New Zealand | GDLC                              | 6906                        |
| 86     | 7.2 | 2010 | ElMayor-Cucapah_Mexico | El Centro Array #12       | 8161                        |
| No Pulse Records Additional Subset | | | | | |
| 87     | 6.95| 1940 | Imperial Valley-02 | El Centro Array #9                  | 6                           |
| 88     | 7.35| 1978 | Tabas_Iran       | Dayhook                                | 139                         |
| 89     | 6.54| 1987 | Superstition Hills-02 | Superstition Mtn Camera       | 727                         |
| 90     | 6.9 | 1995 | Kobe_Japan       | Kobe University                       | 1108                        |
| 91     | 6.77| 1988 | Spitak           | Gukasian                              | 730                         |
| 92     | 6.76| 1985 | Nahanni          | Site 3                                | 497                         |
| 93     | 7.14| 1999 | Duzce_Turkey     | Lamont 375                            | 1617                        |
| 94     | 6.61| 2000 | Tottori_Japan    | SMNH01                                 | 3947                        |
| 95     | 6.52| 2003 | San Simeon_CA    | Cambria - Hwy 1 Caltrans Bridge       | 3979                        |
| 96     | 6.52| 2003 | San Simeon_CA    | Templeton - 1-story Hospital          | 4031                        |
| 97     | 6.63| 2004 | Niigata_Japan    | NIG017                                 | 4207                        |
| 98     | 6.63| 2004 | Niigata_Japan    | NIG019                                 | 4209                        |
| 99     | 7.1 | 1979 | Montenegro_Yugoslavia | Ulcinj - Hotel Albatros         | 4457                        |
| 100    | 6.8 | 2007 | Chuetsu-oki_Japan | Kashiwazaki NPP_Unit 1: ground surface | 4894                        |

responses to individual ground motions, but by comparing statistics of the response to ensembles of ground motions. A statistical comparison was also followed, for example, by Riddell and Newmark60 to produce inelastic spectra for yielding structures, or by Chopra31 (section 20.8.3) to evaluate the accuracy of modal pushover analysis.

Six variations of the near-field pulse-like, near-field non-pulse-like, and far-field FEMAP695 sets are studied: (a) scaled so that their PGA is equal to 0.25, 0.5, or 1 g and (b) scaled so that their PGV is equal to 0.25, 0.5, 1 m/s (for the far-field and near-field non-pulse-like sets) and 0.5, 1, and 2 m/s for the pulse-like sets. PGA and PGV are defined as

\[
PGA = \max_t \left( \sqrt{\ddot{u}_{gs}(t)^2 + \ddot{u}_{gs}(t)^2} \right)
\]  

\[
PGV = \max_t \left( \sqrt{\dot{u}_{gs}(t)^2 + \dot{u}_{gs}(t)^2} \right)
\]

Figures 5–7 plot the median \( u_{\text{max}} \) of the rocking oscillator as a function of its slenderness \( \alpha \) for several block sizes (top three rows). They also plot the quantity \( u_{\text{max}}/\gamma_{\text{EE}} \) (bottom three rows), in an effort to evaluate the Equal Energy rule. As
FIGURE 5  Far-field ground motion set. Top three rows: Median displacement spectra. Bottom three rows: Median displacement spectra normalized by \( \gamma_{EE} \)
Near-field pulse-like ground motions

FIGURE 6  Near field pulse like ground motion set. Top three rows: Median displacement spectra. Bottom three rows: Median displacement spectra normalized by $\gamma_{EE}$.
Near-field non-pulse-like ground motions

FIGURE 7  Near field non pulse like ground motion set. Top three rows: Median displacement spectra. Bottom three rows: Median displacement spectra normalized by $\gamma_{EE}$
this paper focuses on bridges and it is the height of the bridge that is usually given, the height \(2h\) and not the semidiagonal \(R\) is chosen as a size parameter. The plots show responses for heights equal to \(2h = 4, 10, 20, 40,\) and \(200\) m. The last value is given only for reasons of mathematical completeness to study the limit case of \(2h \to \infty\). Moreover, it plots the 95\% confidence intervals of the median (over the ground motions of each set) response for the case \(2h = 200\) m. The influence of the ground motion selection on the confidence intervals lies beyond the scope of this paper. The confidence intervals were constructed using the bootstrap method\(^\text{92}\) by taking 10,000 response samples for each value of \(\tan \alpha\). For reasons of plot clarity, each line is plotted only for \(\alpha > \alpha_{\text{crit}}\), where \(\alpha_{\text{crit}}\) is the maximum uplift strength for which there is failure.

Throughout the rest of this paper, such a \(u_{\text{max}} - \tan \alpha\) plot will be called a “spectrum.” This should not be confused with Biot’s linear elastic spectrum.\(^\text{93}\) The term “spectrum” is used to declare a collective representation of the maxima of the responses of the rocking oscillator, which is highly non-linear and not correlated to the elastic oscillator. So, “spectrum” does not necessarily mean “elastic spectrum.”

In terms of median response, Figures 5–7 show that:

(i) Similarly, to the planar case, as long as the system is not close to failure, the displacement only loosely depends on the block size. In most cases the curves for the different sizes fall within the 95\% confidence interval of the curve of the \(2h = 200\) m block. This reduces the dimensionality of the problem by one and Equation 12 becomes:

\[
 u_{\text{max}} \approx f_3(\alpha) \text{ if } u_{\text{max}} < < 2b \]  

Equation (15) suggests that to compute the maximum displacement \(u_{\text{max},R}\) of a rocking block of any size \(R\) and given slenderness \(\alpha\), one has to compute the maximum displacement \(u_{\text{max},\infty}\) of the infinite size block of slenderness \(\alpha\). If \(u_{\text{max},\infty} < < 2b\), then \(u_{\text{max},R} \approx u_{\text{max},\infty}\). A physical explanation for this behavior is offered for the planar rocking case in ref.\(^\text{60}\) Therefore, the response of the infinite size block becomes of high importance, because it can be used to determine the response of a class of rocking blocks, that is, of all the blocks having the same slenderness, irrespectively of their size. The above principle is defined as the “Equal Displacement Rule” for rocking structures. It should not be confused with Newmark’s rule for elastoplastic systems, because it does not resort to a linear elastic system, but to a rocking one of infinite size.

(ii) For displacements that bring the system close to collapse, smaller blocks exhibit larger displacements than larger ones of the same slenderness. The “equal displacement rule” would be unconservative in this region. The Equal Energy rule (bottom three rows of Figures 5–7) is slightly more accurate and more conservative.

(iii) Within each set of motions (far-field, near-field pulse-like, or near-field non-pulse-like), the shape of the spectrum is the same, no matter the intensity of the set. Within each set, increasing the intensity of the ground motion results in the spectra dilating linearly in both \(x\) and \(y\) direction, for example, one can obtain the \(\text{PGA} = 0.5\) g plot by multiplying both the \(x\) and \(y\) coordinates of the \(\text{PGA} = 0.25\) g by a factor of 2.

Observations (i) and (ii) suggest that for a specific set and scaling of the ground motions the slenderness of a block suffices to predict its wobbling response (as long as the block is not close to overturning—which is the rational design approach). This drops out the size parameter from Equation (12).

Observation (iii) forms the basis of the next section.

4 | FURTHER DIMENSIONALITY REDUCTION BY PROPER IDENTIFICATION OF INTENSITY MEASURES

Equation (18) implies that the dimensionality of the problem is 1 for a set of ground motions all scaled to a specific intensity, defined either through \(\text{PGA}\) or through \(\text{PGV}\): The response of a block of slenderness \(\alpha\) and infinite size is a good predictor of the response of all blocks having the same slenderness \(\alpha\), irrespectively of their size. For a given set of ground motions (i.e., far-field, near-field pulse-like, or near-field non-pulse-like, scaled to a specific intensity), in mathematical terms this is written as:

\[
 u_{\text{max}} \approx f_4(\alpha, \text{intensity measures}) \text{ if } u_{\text{max}} < < 2b \]  

Hence, this section of this paper will only focus on the infinite size block.
As the rocking problem is highly non-linear, Equation (19) does not convey any information on the dependence of the response on the intensity measures (e.g., PGA or PGV). However, based on observation (iii) of Section 3.3, it makes sense to attempt plotting non-dimensional spectra. Two different non-dimensionalizations are attempted resulting into the following equations:

\[
\frac{u_{\text{max}}}{\text{PGD}} \approx f_5 \left( \frac{g \tan \alpha}{\text{PGA}} \right) \quad \text{if } u_{\text{max}} < 2b
\]  

\[
\frac{u_{\text{max}} \text{PGA}}{\text{PGV}^2} \approx f_6 \left( \frac{g \tan \alpha}{\text{PGA}} \right) \quad \text{if } u_{\text{max}} < 2b
\]  

Then each ground motion was scaled to the PGA and PGV values discussed in Section 3.3 and \( \alpha \) was varied to compute the response for different values of \( g \tan \alpha / \text{PGA} \). Finally, the maximum response \( u_{\text{max}} \), was normalized with the PGD of the ground motion. Figure 8 plots the median response spectra for each different ground motion set. The remarkable conclusion is that within each set all the plots that represent different scalings of the sets collapse to one master curve. This is confirmed by plotting the 95% Confidence Intervals of the PGA = 0.5 g set. Hence, under the assumption that the ground motions used are representative of all far-field, near-field pulse-like, and near-field non-pulse-like ground motions, PGA and (PGV or PGD) suffice to predict the median response of any wobbling block of given slenderness \( \alpha \). Between the two non-dimensionalizations (Equations 20 and 21) the second one is more useful, as in many recorded

**FIGURE 8** Normalized median displacement response spectra for \( 2h = 200 \text{ m} \)
ground motions PGD is not measured with a reliable manner and strongly depends on the ground motion correction method.

5 | EXAMPLE

This section validates the results of Section 4, based on a preliminary design example of a wobbling bridge having columns of semidiagonal $R = 10$ m, slenderness value $\tan \alpha = 0.2$, and mass ratio $\gamma = 10$. The equivalent solitary wobbling column size is $R = 14.80$ m (Equation 9). The seismic response of the bridge is studied under three sets of ground motions (far-field, near-field pulse-like, near-field non-pulse-like) comprising 10 ground motions each and all scaled to 0.30 g. To avoid an event bias only ground motions of seismic events that are not used for the analyses of Section 4 are considered. In the case of near-field pulse-like ground motions, the PEER database could not provide enough records classified as such, by its own record classification system. Therefore, the set was completed with PEER ground motions classified as pulse records by ref.94 and in some cases recorded at distances larger then 10 km to the fault rupture (maximum distance of 22.37 km for the record of the Chi-Chi, Taiwan-03 event). For reasons of reproducibility, Table 3 lists the selected ground motions. Estimates of the median response is computed based on Figure 8, for $\frac{\gamma \tan \alpha}{PGA} = 0.2/0.3 = 0.67$ and for

$$PGD = \text{median}_{\text{along each set}} \left( \max_t \left( \sqrt{u_{g_x}(t)^2 + u_{g_y}(t)^2} \right) \right)$$ (22)

$$PGV = \text{median}_{\text{along each set}} \left( \max_t \left( \sqrt{\dot{u}_{g_x}(t)^2 + \dot{u}_{g_y}(t)^2} \right) \right)$$ (23)

Two estimates are computed, based on the $u/PGD$ or on the $uPGA/PGV^2$ non-dimensionalizations (i.e., Figure 8 left and right, respectively). Since the spectra were derived for the equivalent solitary wobbling column (Equation 9), the required steps to compute the response of the frame using the spectra are:

a. Compute the semidiagonal of the solitary column $\hat{R}$, that is equivalent to the frame using Equation (9).

b. Read the maximum displacement of the equivalent solitary column, $u_{\text{spec, equiv}}$, from the spectrum.

c. Assuming that the equivalent solitary column and the frame will have roughly the same tilt angle $\theta$, get the displacement of the frame as $u_{\text{spec}} = R/\hat{R} \times u_{\text{spec, equiv}}$, where $\hat{R}$is given by Equation (9) and $R$ is the actual semidiagonal of the frame columns.

Table 4 compares the spectral estimates to the median response for each set obtained with direct integration of the equation of motion (i.e., Equations (5) and (6); not (1) and (2)). The “Equal Displacement Rule,” which is more convenient to use, assumes that $u_{\text{spec}} = u_{50}$. Then, the estimates have an error of less than 30%. For the case of far-field and near-field pulse-like sets, the estimates are conservative, while they underestimate the response for the near-field non-pulse-like set by no more than 13%. In no case did direct integration of motion predict collapse.

6 | CONCLUSIONS

A solitary 3D rocking (wobbling) cylinder was studied. The cylinder is the simplest 3D extension of Hounser's planar rocking model. This paper extended a recent finding from 2D rocking to 3D wobbling, namely that describing wobbling motion in terms of displacements rather than rotations is more convenient: A large and a small wobbling cylindrical body of the same slenderness will experience roughly the same top displacement, as long as they are both not close to overturning—which is a design target anyways. This property is named as “Equal Displacement Rule” for rocking structures. A more refined version of the “Equal Displacement Rule,” namely the “Equal Energy Rule” also correlates the displacement response of blocks with equal slenderness but different sizes. Using either of the rules, one can get an estimate of the displacement of all wobbling cylindrical bodies having slenderness $\alpha$, by computing the wobbling response of a wobbling cylinder having slenderness $\alpha$ and infinite size. This shows the size of a wobbling cylinder does not influence its response—as long as it is not close to overturning. The above observation reduces the dimensionality of the problem by one.
Moreover, this paper showed that the median displacement response of a wobbling cylinder excited by far-field, near-field pulse-like, or near-field non-pulse-like sets of ground motions is a function of only two non-dimensional parameters, namely $g \tan \alpha / PGA$ and $(u / PGD$ or $uPGA/PGV^2$). Therefore, $PGA$ and $(PGV$ or $PGD$) seem appropriate intensity measures for wobbling motion.

The above findings can be used for the design of bridges that are allowed to uplift under strong ground excitations. The uplift limits the design moments of the foundation and can reduce its size. Shake table tests for the quantification of this reduction need to be performed. This is a contribution to more efficient and sustainable use of material, as foundations in earthquake prone regions often comprise up to 50% of the reinforced concrete of a bridge.

### TABLE 3  Ground motion set of the design example

| ID No. | M     | Year | Name             | Recording station         | PEER record sequence number |
|--------|-------|------|------------------|---------------------------|----------------------------|
| Far Field Records                          |       |      |                  |                           |                            |
| 101    | 7.36  | 1952 | Kern County      | Taft Lincoln School       | 15                         |
| 102    | 6.5   | 1954 | Northern Calif-03| Ferndale City Hall        | 20                         |
| 103    | 6.5   | 1968 | Borrego Mtn      | El Centro Array #9        | 36                         |
| 104    | 7.2   | 1980 | Trinidad         | Rio Dell Overpass, W Ground| 282                        |
| 105    | 7.3   | 1986 | Taiwan, SMART 1(45)| SMARTI O02               | 578                        |
| 106    | 7.2   | 1995 | Gulf of Aquaba   | Eilat                     | 1144                       |
| 107    | 6.81  | 1972 | Sitka, Alaska    | Sitka Observatory         | 1626                       |
| 108    | 7.21  | 1976 | Caldiran, Turkey | Maku                      | 1627                       |
| 109    | 7.54  | 1979 | St Elias, Alaska | Icy Bay                   | 1628                       |
| 110    | 6.9   | 2008 | Iwata, Japan     | AKTH19                    | 5495                       |
| Near Field Pulse Records                   |       |      |                  |                           |                            |
| 110    | 5.74  | 1979 | Coyote Lake      | Gilroy Array #6           | 150                        |
| 111    | 5.9   | 1981 | Westmoreland     | Parachute Test Site       | 316                        |
| 112    | 5.77  | 1983 | Coalinga 05      | Oil City                  | 407                        |
| 113    | 5.4   | 1986 | Kalamata, Greece-02| Kalamata (bsmt) (2nd trigger) | 566                        |
| 114    | 5.8   | 1986 | San Salvador     | National Geografical Inst | 569                        |
| 115    | 6.1   | 1997 | Northwest China #3| Jiashi                    | 1752                       |
| 116    | 5     | 2000 | Yountville       | Napa Fire Station         | 1853                       |
| 117    | 6.32  | 1999 | Chi-Chi, Taiwan-03| CHY080                    | 2495                       |
| 118    | 6.3   | 2009 | L’Aquila, Italy  | L’Aquila - Parking        | 4483                       |
| 119    | 6.2   | 2011 | Christchurch, New Zealand| Christchurch Resthaven    | 8123                       |
| Near Field No Pulse Records                |       |      |                  |                           |                            |
| 120    | 6.24  | 1972 | Managua, Nicaragua-01| Managua, ESSO           | 95                         |
| 121    | 6.2   | 1980 | Irpinia, Italy-02| Calitri                   | 300                        |
| 122    | 6.06  | 1986 | N. Palm Springs  | Desert Hot Springs        | 517                        |
| 123    | 6.19  | 1986 | Chalfant Valley-02| Zack Brothers Ranch      | 558                        |
| 124    | 6.2   | 1986 | Kalamata, Greece-01| Kalamata (bsmt)           | 564                        |
| 125    | 6.46  | 1992 | Big Bear-01      | Big Bear Lake - Civic Center| 901                       |
| 126    | 6.4   | 1995 | Dinar, Turkey    | Dinar                     | 1141                       |
| 127    | 6.3   | 2009 | L’Aquila, Italy  | L’Aquila - V. Aterno - Colle Grilli | 4481                      |
| 128    | 6.9   | 2008 | Iwata, Japan     | IWT011                    | 5619                       |
| 129    | 6.2   | 2011 | Christchurch, New Zealand| Papanui High School      | 8118                       |
| 130    | 6.24  | 1972 | Managua, Nicaragua-01| Managua, ESSO           | 95                         |
| 131    | 6.2   | 1980 | Irpinia, Italy-02| Calitri                   | 300                        |
| 132    | 6.06  | 1986 | N. Palm Springs  | Desert Hot Springs        | 517                        |
| 133    | 6.19  | 1986 | Chalfant Valley-02| Zack Brothers Ranch      | 558                        |
TABLE 4 Comparison of the values predicted with spectral and time history analysis

| PGD based spectral estimate | PGA/PGV^2 based spectral estimate | Time history analysis |
|-----------------------------|----------------------------------|----------------------|
| PGD<sub>50</sub>            | u<sub>spect, equiv</sub>         | u<sub>spect</sub>    |
| Far field records           |                                  |                      |
| 0.214                       | 0.479                            | 0.324                |
| Near field pulse like       |                                  |                      |
| 0.95                        | 0.129                            | 0.087                |
| Near field non pulse like   |                                  |                      |
| 0.104                       | 0.237                            | 0.160                |

Abbreviations: PGD<sub>50</sub>, median PGD of the ground motion set; PGV<sub>50</sub>, median PGV of the ground motion set; u<sub>spect, equiv</sub>, Top displacement of the equivalent solitary column (Equation 9); u<sub>spect</sub>, Top displacement of the wobbling frame, computed based on u<sub>spect, equiv</sub>; u<sub>50</sub>, median top displacement of the frame computed with time history analysis. All units are in SI.

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DATA AVAILABILITY STATEMENT
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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REFERENCES
1. Housner GW. The behavior of inverted pendulum structures during earthquakes. Bull Seismol Soc Am. 1963;53(2):403-417.
2. Makris N, Vassiliou MF. Planar rocking response and stability analysis of an array of free-standing columns capped with a freely supported rigid beam. Earthquake Engng Struct Dyn. 2013;42(3):431-449.
3. Cherepinskiy Y. Seismic isolation of buildings with application of the kinematics bases. 13th World Conference on Earthquake Engineering, 2004.
4. Uzdin AM, Doronin FA, DavydoVA GV, Avidon GE, Karlina EA. Performance analysis of seismic-insulating elements with negative stiffness. Soil Mech Found Eng. 2009;46(3):99-107.
5. Bachmann JA, Vassiliou MF, Stojadinić B. Dynamics of rocking podium structures. Earthq Eng Struct Dyn. 2018;46(14):2499-2517.
6. Bachmann JA, Vassiliou MF, Stojadinić B. Rolling and rocking of rigid uplifting structures. Earthq Eng Struct Dyn. 2019;48(14):1556-1574.
7. Ríos-García G, Benavent-Climent A. New rocking column with control of negative stiffness displacement range and its application to RC frames. Eng Struct. 2020;206:110133.
8. Beck JL, Skinner RI. The seismic response of a reinforced concrete bridge pier designed to step. Earthquake Engng Struct Dyn. 1973;2(4):343-358.
9. Priestley MJN, Tao JR. Seismic response of precast prestressed concrete frames with partially debonded tendons. PCI J. 1993;38(1):58-69.
10. Stanton J, Stone WC, Cheok GS. A hybrid reinforced precast frame for seismic regions. PCI J. 1997;42(2):20-23.
11. Nakaki SD, Stanton IF, Sritharan S. Overview of the PRESSS five-story precast test building. PCI J. 1999;44(2):26-39.
12. Priestley MJN, Sritharan SS, Conley JR, Pampanin S. Preliminary results and conclusions from the PRESSS five-story precast concrete test building. PCI J. 1999;44(6):42-67.
13. Mander JB, Cheng CT. Seismic resistance of bridge piers based on damage avoidance design. Technical Report NCEER-97-0014.
14. Palermo A, Pampanin S, Calvi GM. Use of 'Controlled Rocking' in the seismic design of bridges. In Proceedings of 13th World Conference on Earthquake Engineering, 2004.
15. Palermo A, Pampanin S, Marriott D. Design, modeling, and experimental response of seismic resistant bridge piers with posttensioned dissipating connections. J Struct Eng. 2007;133(11):1648-1661.
16. White S, Palermo A. Quasi-static testing of posttensioned noncumulative column-footing connections for bridge piers. J Bridg Eng. 2016;21(6):04016025.
17. Liu R, Palermo A. Quasi-static testing of a 1/3 scale precast concrete bridge utilising a post-tensioned dissipative controlled rocking pier. In Proceedings of 16th World Conference on Earthquake Engineering 2017.
18. Mashal M, Palermo A. Low-damage seismic design for accelerated bridge construction. J Bridg Eng. 2019;24(7):04019066.
19. Liu R, Palermo A. Multi-‘hinge’ hierarchical activation to improve structural robustness of post-tensioned rocking piers. ACI Symp Publ. 2020;341:202-225.
20. Sakai J, Jeong H, Mahin SA. Reinforced concrete bridge columns that re-center following earthquakes. In Proceedings of the 8th U.S. National Conference on Earthquake Engineering. 2006.
21. Restrepo JI, Rahman A. Seismic performances of precast segmental column under bidirectional earthquake motions: shake table test and numerical evaluation. Eng Struc. 2019;187:314-328.
22. Li C, Bi K, Hao H. Seismic behavior of precast segmental column under unidirectional earthquake motions: shake table test and numerical evaluation. Eng Struct. 2017;151:1560-1570.
23. Christopoulos C, Tremblay R, Kim HJ, Lacerte M. Seismic performance of precast segmental column with slip-dominant rocking columns. J Struct Eng. 2016;142(12):04016016.
24. Cheng CT. Shaking table tests of a self-centering hybrid rocking bridge system. Eng Struct. 2008;30(12):3426-3433.
25. Cohagen LS, Pang J, Stanton JF, Eberhard MO. A precast concrete bridge bent designed to re-center after an earthquake. Rep No. WA-RD 684.3/TNW 2008–9.
26. ElGawady MA, Sha’lan A. Seismic behavior of self-centering precast segmental bridge bents. J Bridg Eng. 2011;16(3):328-339.
27. Guerrini G, Restrepo JI, Assari M, Vervelidis A. Seismic behavior of posttensioned self-centering precast concrete dual-shell steel columns. J Struct Eng. 2015;141(4):04014115.
28. Trono W, Jen G, Panagiotou M, Schoettler M, Ostertag CP. Seismic response of a damage-resistant recentering posttensioned-HYFRC bridge column. J Bridg Eng. 2015;20(7):04014096.
29. Kashani MM, Gonzalez-Buelga A, Thayalan RP, Thomas AR, Alexander NA. Experimental investigation of a novel class of self-centering rocking columns. J Sound Vib. 2018;437:308-324.
30. Wang J, Wang Z, Tang Y, Liu T, Zhang J. Cyclic loading test of self-centering precast segmental unbonded posttensioned UHPFRC bridge columns. Bull Earthq Eng. 2018;16(11):5227-5255.
31. Giouvanidis AI, Dimitrakopoulos EG. Seismic performance of rocking frames with flag-shaped hysteretic behavior. J Eng Mech. 2017;143(5):04017008.
32. Xia X, Zhang X, Shi J, Tang J. Seismic isolation of railway bridges using a self-centering pier. Smart Structures and Systems. 2021;27(3):447-455.
33. Billington SL, Yoon JK. Cyclic response of unbonded posttensioned precast columns with ductile fiber-reinforced concrete. J Bridg Eng. 2004;9(4):353-363.
34. Ou YC, Tsai MS, Chang KC, Lee GC. Cyclic behavior of precast segmental concrete bridge columns with high performance or conventional steel reinforcing bars as energy dissipation bars. Earthq Eng Struct Dyn. 2010;39:1181-1198.
35. Motaref S, Saiidi MS, Sanders D. Shake table studies of energy-dissipating segmental rocking bridge columns. J Bridg Eng. 2014;19(2):186-199.
36. Panagiotou M, Trono W, Jen G, Kumar P, Ostertag CP. Experimental seismic response of hybrid fiber-reinforced concrete bridge columns with novel longitudinal reinforcement detailing. J Bridg Eng. 2015;20(7):04014090.
37. Bu ZY, Ou YC, Song JW, Lee GC. Hysteresis modeling of unbonded posttensioned precast segmental bridge columns with circular section based on cyclic loading test. J Bridg Eng. 2016;21(6):04016016.
38. Sideris P, Aref AJ, Filiatrault A. Dynamic performance of a hybrid rocking bridge system. J Struct Eng. 2014;140(6):04014025.
39. Sideris P, Aref AJ, Filiatrault A. Experimental seismic performance of a hybrid sliding-rocking bridge for various specimen configurations and seismic loading conditions. J Bridg Eng. 2015;20(11):04015009.
40. Sideris P, Aref AJ, Filiatrault A. Quasi-static cyclic testing of a large-scale hybrid sliding-rocking segmental column with slip-dominant joints. J Bridg Eng. 2014;19(10):04014036.
41. Thonstad T, Mantawy IM, Stanton JF, Eberhard MO, Sanders DH. Shaking table performance of a new bridge system with posttensioned rocking columns. J Bridg Eng. 2016;21(4):04015079.
42. Thonstad T, Kennedy BJ, Schaefer JA, Eberhard MO, Stanton JF. Cyclic tests of precast posttensioned rocking bridge-column subassemblies. J Struct Eng. 2017;143(9):04017094.
43. Yamashita R, Sanders DH. Seismic performance of precast unbonded prestressed concrete columns. ACI Struct J. 2009;106(6):821-830.
44. Thiers-Moggia R, Malaga-Chuquitaype C. Seismic protection of rocking structures with inerter. Earthquake Engng Struct Dyn. 2019;48(5):528-547.
45. Kalliontzis D, Sritharan S. Seismic behavior of unbonded post-tensioned precast concrete members with thin rubber layers at the jointed connection. PCI Journal. 2021;66(1):60.
46. Routledge PJ, Cowan MJ, Palermo A. Low-damage detailing for bridges—a case study of Wigram-Magdala Bridge. In Proceeding of 2016 NZSEE Conference.
47. Qu H, Li T, Wang Z, Wei H, Shen J, Wang H. Investigation and verification on seismic behavior of precast concrete frame piers used in real bridge structures: experimental and numerical study. Eng Struct. 2018;154:1-9.
48. Reggiani Manzo Natalia, Vassiliou Michalis F., Mouzakis Harris, Badogiannis Efstratios. Shaking table tests of a resilient bridge system with precast reinforced concrete columns equipped with springs. Earthquake Engineering & Structural Dynamics. 2021; https://doi.org/10.1002/eqe.3563
49. Makris N, Vassiliou MF. Dynamics of the rocking frame with vertical restrainers. J Struct Eng. 2015;141(10):04014245.
50. Vassiliou MF, Makris N. Dynamics of the vertically restrained rocking column. J Eng Mech. 2015;141(12):04015049.
51. Makris N, Vassiliou MF. Are some top-heavy structures more stable? J Struct Eng. 2014;140(5):06014001.
52. Dimitrakopoulos EG, Giouvanidis AI. Seismic response analysis of the planar rocking frame. J Eng Mech. 2015;141(7):04015003.
53. Thomaidis IM, Kappos AJ, Camara A. Dynamics and seismic performance of rocking bridges accounting for the abutment-backfill contribution. Earthq Eng Struct Dyn. 2020;49(12):1161-1179.

54. Agaliasos A, Psychari A, Vassiliou MF, Stojadinovic B, Anastasopoulos I. Comparative assessment of two rocking isolation techniques for a motorway overpass bridge. Front Built Environ. 2017;3:1-19.

55. Vassiliou MF, Mackie KR, Stojadinovic B. A finite element model for seismic response analysis of deformable rocking frames. Earthq Eng Struct Dyn. 2016;46(3):447-466.

56. Giouvanidis AI, Dimitrakopoulos EG. Rocking amplification and strong-motion duration. Earthq Eng Struct Dyn. 2018;47(10):2094-2116.

57. Xie Y, Zang J, DesRoches R, Padgett JE. Seismic fragilities of single-column highway bridges with rocking column-footing. Earthq Eng Struct Dyn. 2019;48(7):843-864.

58. Giouvanidis AI, Dong Y. Seismic loss and resilience assessment of single-column rocking bridges. Bull Earthq Eng. 2020;18(9):4481-4513.

59. Chen X, Li J. Seismic fragility analysis for tall pier bridges with rocking foundations. Adv Bridg Eng. 2021;2(1).

60. Reggiani Manzo N, Vassiliou MF. Displacement-based analysis and design of rocking structures. Earthq Eng Struct Dyn. 2019;48(14):1613-1629.

61. Reggiani Manzo N, Vassiliou MF. Simplified analysis of bilinear elastic systems exhibiting negative stiffness behavior. Earthq Eng Struct Dyn. 2021;50(2):580-600.

62. Vassiliou MF, Burger S, Egger M, Bachmann JA, Broccardo M, Stojadinovic B. The three-dimensional behavior of inverted pendulum cylindrical structures during earthquakes. Earthq Eng Struct Dyn. 2017;46(14):2261-2280.

63. Konstantinidis D, Makris N. Experimental and analytical studies on the response of 1/4-scale models of freestanding laboratory equipment subjected to strong earthquake shaking. Bull Earthquake Eng. 2010;8(6):1457-1477.

64. Di Egidio A, Alaggio R, Contento A, Tursini M, Della Loggia E. Experimental characterization of the overturning of three-dimensional square based rigid block. Int J Non Linear Mech. 2015;69:137-145.

65. Wittich CE, Hutchinson TC. Shake table tests of stiff, unattached, asymmetric structures. Earthq Eng Struct Dyn. 2015;44(13):2425-2443.

66. Dar A, Konstantinidis D, El-Dakhakhni WW. Evaluation of ASCE 43-05 seismic design criteria for rocking objects in nuclear facilities. J Struct Eng. 2016;142(11):04016610.

67. Sextos AG, Manolis GD, Ioannisid N, Athanasiou A. Seismically induced uplift effects on nuclear power plants. Part 2: demand on internal equipment. Nucl Eng Des. 2017;318:288-296.

68. Dar A, Konstantinidis D, El-Dakhakhni W. Seismic response of rocking frames with top support eccentricity. Earthq Eng Struct Dyn. 2018;47(12):2496-2518.

69. Voyagaki E, Kloukinas P, Dietz M, et al. Earthquake response of a multiblock nuclear reactor graphite core: experimental model vs simulations. Earthq Eng Struct Dyn. 2018;47(13):2601-2626.

70. Di Sarno L, Magliulo G, Cosenza E. Seismic damage assessment of unanchored nonstructural components taking into account the building response. Struct Saf. 2021;93:102126.

71. Vassiliou MF, Cengiz C, Dietz M, et al. Dataset from the shake table tests of a rocking podium structure. Earthquake Spectra. 2021;8755293020988017.

72. Vassiliou MF. Seismic response of a wobbling 3D frame. Earthq Eng Struct Dyn. 2018;47(5):1212-1228.

73. Bachmann JA, Strand M, Vassiliou MF, Broccardo M, Stojadinovic B. Is rocking motion predictable? Earthquake Engng Struct Dyn. 2018;47(2):535-552.

74. Vassiliou MF, Broccardo M, Cengiz C, et al. Shake table testing of a rocking podium: results of a blind prediction contest. Earthquake Engng Struct Dyn. 2021;50(4):1043-1062.

75. Zhong C, Christopoulos C. Finite element analysis of the seismic shake-table response of a rocking podium structure. Earthquake Engng Struct Dyn. 2021;50(4):1223-1230.

76. Malomo D, Mehrotra A, DeJong MJ. Distinct element modeling of the 3D rocking problem. Int J Non Linear Mech. 2012;47(4):85-98.

77. D’Angela D, Magliulo G, Cosenza E. Seismic damage assessment of unanchored nonstructural components taking into account the building response. Struct Saf. 2021;93:102126.
86. Truniger R, Vassiliou MF, Stojadinović B. An analytical model of a deformable cantilever structure rocking on a rigid surface: experimental validation. Earthquake Engng Struct Dyn. 2015;44(15):2795-2815.
87. Acikgoz S, DeJong MJ. Analytical modelling of multi-mass flexible rocking structures. Earthquake Engng Struct Dyn. 2016;45(13):2103-2122.
88. Makris N, Konstantinidis D. The rocking spectrum and the limitations of practical design methodologies. Earthq Eng Struct Dyn. 2003;32(2):265-289.
89. FEMA P695. “Quantification of building seismic performance factors,” Rep. FEMA P695, Federal Emergency Management Agency, Washington, D.C. 2009.
90. Riddell R, Newmark NM. Statistical Analysis of the Response of Nonlinear Systems Subjected to Earthquakes. University of Illinois Engineering Experiment Station. College of Engineering. University of Illinois at Urbana-Champaign; 1979.
91. Chopra AK. Dynamics of Structures: Theory and Applications to Earthquake Engineering. 4th ed. Pearson, Prentice Hall; 2012.
92. Efron B, Tibshirani RJ. An Introduction to the Bootstrap. CRC Press; 1994.
93. Biot MA, (1932). Vibrations of buildings during earthquake [Chapter II in Ph. D. thesis No. 259, entitled “Transient Oscillations in Elastic System”]. Pasadena, California: Aeronautics Department, California Institute of Technology.
94. Vassiliou MF, Makris N. Estimating time scales and length scales in pulselike earthquake acceleration records with wavelet analysis. Bull Seismol Soc Am. 2011;101(2):596-618.

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