Measurement of $B(D_s^+ \to \mu^+ \nu_\mu)$

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We present a measurement of the branching fraction $B(D_s^+ \rightarrow \mu^+ \nu_\mu)$ using a 548 fb$^{-1}$ data sample collected by the Belle experiment at the KEKB $e^+e^-$ collider. The $D_s$ momentum is determined by reconstruction of the system recoiling against $DK\gamma X$ in events of the type $e^+e^- \rightarrow D_s^+DKX, D_s^+ \rightarrow D_s\gamma$, where $X$ represents additional pions or photons from fragmentation. The full reconstruction method provides high resolution in the neutrino momentum and thus good background separation, equivalent to that reached by experiments at the tau-charm factories. We obtain the branching fraction $B(D_s^+ \rightarrow \mu^+ \nu_\mu) = (6.44 \pm 0.76(\text{stat}) \pm 0.57(\text{syst})) \cdot 10^{-3}$, implying a $D_s$ decay constant of $f_{D_s} = (275 \pm 16(\text{stat}) \pm 12(\text{syst}))$ MeV.

PACS numbers: 13.20.-v, 13.20.Fc
are required to have at least one associated hit in the SVD and an impact parameter with respect to the interaction point of less than 2 cm in the radial direction and less than 4 cm in the beam direction. Tracks are also required to have momenta in the laboratory frame greater than 100 MeV/c. A likelihood ratio for a given track to be a kaon or pion, \( \mathcal{L}(K, \pi) \), is obtained by utilizing specific ionization energy loss measurements in the CDC, light yield measurements from the ACC, and time-of-flight information from the TOF [3]. We require \( \mathcal{L}(K, \pi) > 0.5 \) for kaon candidates. The momentum of the lepton candidates is required to be larger than 500 MeV/c. For electron identification we use position, cluster energy, shower shape in the ECL, combined with track momentum and \( dE/dx \) measurements in the CDC and hits in the ACC. For muon identification, we extrapolate the CDC track to the KLM and compare the measured range and transverse deviation in the KLM with the expected values.

Photons are required to have energies in the laboratory frame of at least 50 - 150 MeV, depending on the detecting part of the ECL. Neutral pion candidates are reconstructed using photon pairs with invariant mass within ±10 MeV/c² of the nominal π° mass. Neutral kaon candidates are reconstructed using charged pion pairs with invariant mass within ±30 MeV/c² of the nominal \( K^0 \) mass.

Charged and neutral tag-side \( D \) mesons are reconstructed in \( D \rightarrow Kn\pi \) decays with \( n = 1,2,3 \) (total branching fraction \( \approx 25\% \)). Mass windows were optimized for each channel separately, and a mass-constrained vertex fit (requiring a confidence level greater than 0.1%) is applied to the \( D \) meson to improve the momentum resolution. \( D_s^* \)-candidates are not directly reconstructed: we construct the mass of the system recoiling against \( D K X \), using the known beam momentum, and require a value within ±150 MeV/c² of the nominal \( D_s^* \) mass [10]. A recoil mass \( M_{\text{rec}}(Y) \) is defined as the magnitude of the four-momentum \( p_{\text{beams}} - p_Y \), for an arbitrary set of reconstructed particles \( Y \). \( p_{\text{beams}} \) is the momentum of the initial e⁺e⁻ system. Since at this point in the reconstruction \( X \) can be any set of remaining pions and photons, there is usually a large number of combinatorial possibilities. It is reduced by requiring the presence of a photon that is consistent with the decay \( D_s^* \rightarrow D_s \gamma \), where the \( D_s \) mass lies within ±150 MeV/c² of its nominal mass [10]. Further selection criteria are applied on the momenta of particles in the \( e^+e^- \) rest frame; for the primary \( K \) meson the momentum should be smaller than 2 GeV/c, for the \( D \) meson larger than 2 GeV/c and for the \( D_s \) meson larger than 3 GeV/c. The energy of the photon from \( D_s^* \rightarrow D_s \gamma \) in the lab frame is required to be larger than 150 MeV, irrespective of its polar angle. To further improve the recoil momentum resolution, inverse mass-constrained vertex fits are then performed for the \( D_s^* \) and \( D_s \), requiring a confidence level greater than 1%. After applying these selection criteria, the average number of combinatorial reconstruction possibilities is approximately 2 per event. The sample is further divided into a right- (RS) and wrong-sign (WS) part. If the primary \( K \) meson is charged, both it and the \( D \) meson are required to have opposite flavor (strangeness or charm respectively) to the \( D_s^* \), to be counted in the right-sign sample; all other combinations are wrong-sign. If the primary \( K \) meson is a \( K^0_S \), the assignment is based on the relative flavor of the \( D \) and \( D_s^* \) mesons alone. The flavor of the \( D_s^* \) is fixed by the total charge of the \( X \), assuming overall charge conservation for the event.

Within this sample of tagged inclusive \( D_s \) decays (named \( D_s \)-tags in the following), decays of the type \( D_s \rightarrow \mu\nu \) are selected by requiring another charged track that is identified as a muon and has the same charge as the \( D_s \) candidate. No additional charged particles are allowed in the event. Remaining photons not used in the described reconstruction are allowed only if their total energy is smaller than 1.0/m GeV, where \( m \) is the number of such particles. After these selections, in almost all cases only one combinatorial reconstruction possibility remains. Figure [11] shows the mass spectra of \( M_{\text{rec}}(DKX\gamma) \) (corresponding to the candidate \( D_s \) mass) and of \( M_{\text{rec}}(DKX\gamma\mu) \) (corresponding to the neutrino candidate mass).

We define \( n_X \) as the number of primary particles in the event, where primary means that the particle is not a daughter of any particle reconstructed in the event. The minimal value for \( n_X \) is three corresponding to an \( e^+e^- \rightarrow D_s^*DKX \) event without any further particles from fragmentation. The upper limit for \( n_X \) is determined by the reconstruction efficiency; Monte Carlo (MC) simulation shows that the number of reconstructed signal events is negligible for \( n_X > 10 \). As the efficiency very sensitively depends on \( n_X \), it is crucial to use MC simulation that correctly reflects the \( n_X \) distribution observed in the data. Unfortunately, the details of fragmentation processes are not very well understood, and standard MC events show notable differences compared to the data. Furthermore, the true (generated) \( n_X^0 \) value differs from the reconstructed \( n_X^R \), as particles can be lost or wrongly assigned. Thus the measured (reconstructed) \( n_X^R \) distribution has to be deconvoluted so that the analysis can be done in bins of \( n_X^R \) to avoid bias in the results.

To extract the number of \( D_s \)-tags as a function of \( n_X^R \) in data, two dimensional simulated distributions in \( n_X^R \) (ranging from 3 to 8) and the recoil mass \( M_{\text{rec}}(DKX\gamma) \) are fitted to the RS and WS data distributions. The signal shapes for different values of \( n_X^R \) (ranging from 3 to 9 [12]) of the signal are modeled with generic MC simulation [13], which has been filtered at the generator level for events of the type \( e^+e^- \rightarrow D_s^*DKX \). The weights of these components, \( w_i^{D_s}, i = 3, \ldots, 8 \), are free parameters in the fit to the data. As a model for the background in the RS sample, the WS data sample is used. The normalization constants between WS and RS (which vary
The shapes of the first half to fit the signal in the second. The resulting weights as function of $n^{T}_{X}$ fit to a constant of $0.990 \pm 0.046$, which agrees well with the expectation of 1. The total number of reconstructed $D_{s}$-tags in data is calculated as

$$N_{D_{s}}^{\text{rec}} = \sum_{i=3}^{8} w_{i}^{D_{s}} N_{MC,i}^{\text{MC}}$$

(2)

where $N_{D_{s}}^{\text{MC},i}$ represents the total number of reconstructed filtered MC events that were generated with $n^{T}_{X} = i$ (regardless of the reconstructed $n^{R}_{X}$) and $w_{i}^{D_{s}}$ the fitted weight of this component.

To fit the number of $D_{s} \rightarrow \mu \nu_{\mu}$ events as a function of $n^{T}_{X}$, two-dimensional histograms in $n^{R}_{X}$ and the recoil mass $M_{\text{rec}}(DKX\gamma\mu)$ are used. The shape of the signal is modeled with signal MC distribution. As MC studies show, the background under the $\mu\nu_{\mu}$ signal peak consists primarily of non-$D_{s}$ decays ($\approx 18\%$ of signal), leptonic $\tau$ decays (where the $\tau$ decays to a muon and two neutrinos, $\approx 7\%$) and semileptonic $D_{s}$ decays (where the additional hadrons have low momenta and remain undetected, $\approx 3.6\%$). Hadronic $D_{s}$ decays (with one hadron misidentified as a muon) are a rather small background component ($\leq 2\%$ of signal). Except for hadronic decays, which are negligible, all backgrounds are common to the $e\nu_{e}$ mode, which is suppressed by a factor of $\mathcal{O}(10^5)$. Thus, the $e\nu_{e}$ sample provides a good model of the $\mu\nu_{\mu}$ background that has to be corrected only for kinematical and efficiency differences. Including this corrected shape in the fit, the total number of fitted $\mu\nu_{\mu}$ events in data is given by

$$N_{\mu\nu}^{\text{rec}} = \sum_{i=3}^{8} w_{i}^{e\nu} N_{\mu\nu}^{\text{MC},i}$$

(3)

where $N_{\mu\nu}^{\text{MC},i}$ represents the total number of reconstructed signal MC events that were generated in the $i$-th bin of $n^{T}_{X}$ (regardless of the reconstructed $n^{R}_{X}$) and $w_{i}^{e\nu}$ is the fitted weight of this component.

The numerical result for $N_{D_{s}}^{\text{rec}}$ is $32100 \pm 870(\text{stat}) \pm 1210(\text{syst})$, that for $N_{\mu\nu}^{\text{rec}}$ is $169 \pm 16(\text{stat}) \pm 8(\text{syst})$. The statistical errors reflect the finite number of data signal candidates. The systematic errors are due to the limited statistics of WS data and MC signal and background samples. The errors were estimated by varying the bin contents of data and MC distributions and repeating the fits. By this procedure the non-negligible correlations among the fitted weights were taken into account.

As the branching fraction of $D_{s} \rightarrow \mu \nu_{\mu}$ used for the generation of MC events is known, the branching fraction in data can be determined using the following formula:

$$B(D_{s} \rightarrow \mu \nu_{\mu}) = \frac{N_{\mu\nu}^{\text{rec}}}{N_{\mu\nu}} = \frac{N_{\mu\nu}}{N_{\mu\nu}^{\text{MC},\text{rec}}} B_{MC}(D_{s} \rightarrow \mu \nu_{\mu})$$

(4)
where \( B_{\text{MC}}(D_s \rightarrow \mu \nu) = 0.51\% \) and \( N_{\mu \nu}^{\text{MC,rec}} \) is the number of reconstructed \( \mu \nu \) events in MC simulation, weighted according to the fit to data, i.e.

\[
N_{\mu \nu}^{\text{MC,rec}} = \sum_{i=3}^{8} w_i^D s_{\mu \nu}^i.
\]

The average efficiency for the reconstruction of \( D_s \rightarrow \mu \nu \) decays, \( \epsilon_{\mu \nu} \), is not needed explicitly for the computation of the branching fraction \([14]\). The final result is:

\[
\mathcal{B}(D_s \rightarrow \mu \nu) \cdot 10^3 = 6.44 \pm 0.76(\text{stat}) \pm 0.57(\text{syst}).
\]

The quoted statistical error reflects the statistical uncertainty of the fitted weights \( w_i^D \) and \( w_i^\mu \nu \), including their correlations. The systematic error combines the contributions due to the statistical uncertainties of data and MC background samples (0.29), the statistical uncertainty of the signal MC distribution (0.41), muon tracking and identification efficiency (0.18) and possible differences in relative rates of individual \( D_s \) decay modes between MC simulation and data (0.19). Since the branching fraction is determined relative to the number of \( D_s \)-tags, the systematic errors in the reconstruction of the tag side cancel. Differences in the neutrino peak resolution between data and simulation have been found to have a negligible effect on the systematic error.

Figure 2 (top) shows the branching fraction determined in bins of \( n_X^T \). The result is stable within errors in \( n_X^T \); note that the errors shown for the \( n_X^T \) bins are correlated. As a cross check, also the branching fraction in a limited range \( n_X^T \leq 6 \) has been determined as \( (6.54 \pm 0.76(\text{stat}) \pm 0.57(\text{syst})) \cdot 10^{-3} \), which agrees well with the result given above. Figure 2 (bottom) shows our result in comparison with the PDG value and recent results from other experiments.

In conclusion, we have studied events of the type \( e^+e^- \rightarrow D_s^+D_s^-K^{\pm}X, D_s^+ \rightarrow D_s^0 \gamma \) with \( X = n\pi(\gamma) \) where the \( D_s^0 \) is identified in the recoil of the remainder of the event. Normalizing to this sample of \( D_s \)-tags, the branching fraction of \( D_s \rightarrow \mu \nu \) was measured to be \( (6.44 \pm 0.76(\text{stat}) \pm 0.57(\text{syst})) \cdot 10^{-3} \), which is in good agreement with the current PDG value of \( (6.1 \pm 1.9) \cdot 10^{-3} \) and also compatible with recent results from BaBar \( (6.74 \pm 1.09) \cdot 10^{-3} \) and CLEO-c \( (5.94 \pm 0.73) \cdot 10^{-3} \). Finally we obtain the decay constant \( f_{D_s} \), using Eqn. 10 (with \( |V_{cs}| = 0.9730 \))

\[
f_{D_s} = (275 \pm 16(\text{stat}) \pm 12(\text{syst})) \text{ MeV}.
\]

A simple average of the decay constants following from the cited measurements has an uncertainty of around 10 MeV. Recently an LQCD calculation of significantly improved precision was performed, with the result \( f_{D_s} = (241 \pm 3) \text{ MeV} \). This value is somewhat lower than the experimental average and the comparison with the experimental results may point to some inconsistency between the two. More precise measurements are needed for a firm comparison and will become possible in the near future at both \( B \) and tau-charm factories.

We thank the KEKB group for excellent operation of the accelerator, the KEK cryogenics group for efficient solenoid operations, and the KEK computer group and the NII for valuable computing and Super-SINET network support. We acknowledge support from MEXT and JSPS (Japan); ARC and DEST (Australia); NSFC and KIP of CAS (contract No. 10575109 and IHEP-U-503, China); DST (India); the BK21 program of MOEHRD, and the CHEP SRC and BR (grant No. R01-2005-000-10089-0) programs of KOSEF (Korea); KBN (contract No. 2P03B 01324, Poland); MES and RFAE (Russia); ARRS(Slovenia); SNSF (Switzerland); NSC and MOE (Taiwan); and DOE (USA).

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