Lagrangian approach in spin-oscillations problem

P.V. Pyshkin, A. I. Kopeliovich, A.V. Yanovsky

1 B.Verkin Institute for Low Temperature Physics & Engineering, National Academy of Sciences of Ukraine, 47 Lenin Ave, 61103, Kharkov, Ukraine

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Lagrangian of electronic liquid in magneto-inhomogeneous micro-conductor has been constructed. Corresponding Euler-Lagrange equation has been solved. It was shown that described system has eigenmodes of spin polarization and total electric current oscillations. Suggested approach allows to study spin dynamics in open-circuit which contains capacitance and/or inductivity.

Key words: spin transport, spintronics

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At present, spintronics [1] is one of the most active areas of investigation in solid state physics. This interest is due, above all, to practical applications of spin-transport effects in modern microelectronics (for example, applications of the giant magnetoresistance [2] phenomenon). In most publications on spin transport research, a stationary situation is assumed, for instance, the transmission of direct current through the magnetic inhomogeneous layered conductor [3]. But specific spin-transport effects are also possible in a non-stationary regime. For example, in [4] AC spin-valve has been investigated. Another example is the effect of the “spin pendulum” [5]: the oscillations of the total current and the spin polarization in an inhomogeneous magnetic closed conductor provided the hydrodynamic transport of electrons. The consideration in [5] is based on the solutions of the spin hydrodynamics equations, which in turn were obtained by standard Chapman-Enskog method of quasi-classical kinetic equation solution. A description of “spin-pendulum” oscillations from more general principles is of independent interest as well as it can contribute to a better understanding of spin dynamics in solid state systems. In this article, we have developed classical Lagrange approach to the “spin pendulum” oscillations of a spin electron liquid. It allows us to study the effects of capacitance and inductivity on the “spin pendulum” oscillations.

The system which is investigated is a small-size conductor (closed or open circuit). We suppose that length of the conductor is much greater than square root of its cross section and all physical values depend only on distance \( x \) along the conductor (cross section of the conductor \( s \) does not depend on \( x \)). Let us consider a magnetically inhomogeneous conductor, when a value of the conductor’s magnetization depends on \( x \). That is in term of a spin of conducting electrons, the spin polarization of an electron density in such a conductor depends only on \( x \) in the equilibrium state:

\[
P(x) = \left( n_{\uparrow}(x) - n_{\downarrow}(x) \right) / \left( n_{\uparrow}(x) + n_{\downarrow}(x) \right),
\]

where \( P \) is a spin polarization and \( n_{\uparrow}(x) \) and \( n_{\downarrow}(x) \) are equilibrium densities of electrons with spin ”up” and ”down” (Fig. 1).

In particular, such a system can be realized by a spatial inhomogeneous doping with magnetic impurities and placing the conductor in a magnetic field. Here we consider the magnetization direction that is collinear along the conductor. A behavior of the conducting electrons can be described by several macroscopic parameters: \( v(x, t) \) – the drift velocity which we consider is common for spin-up and spin-down electrons (this is due hydrodynamic transport regime which we also consider), \( n_{\uparrow}(x, t) = n_{\uparrow 0}(x) + \delta n_{\uparrow}(x, t) \) – electron densities (\( \delta n_{\uparrow}(x, t) \) – non equilibrium perturbation of electron densities), \( \mu_{\uparrow}(x, t) \) – chemical potential for spin-up and spin-down electrons. We can express \( \mu_{\uparrow}(x, t) \) as sum of the equilibrium value \( \mu_0 \) and perturbation \( \delta \mu_{\uparrow}(x, t) \) of the equilibrium chemical potential. We suppose that all sizes of the system are more than electronic screening length, and characteristic frequency is
Figure 1. Magnetically inhomogeneous closed conductor (“spin pendulum”) with constant cross section $s$. Various sized black arrows denote spatial inhomogeneity of equilibrium spin polarization.

Figure 2. Illustration of origin of potential energy, which is connected with non-equilibrium spins. Grey fill area corresponds to occupied electron states (at zero temperature).

less than plasmonic frequency, so we can assume that condition of electro-neutrality in the conductor is: $\delta n_\uparrow(x, t) + \delta n_\downarrow(x, t) = 0$. In the case of small perturbation ($|\delta \mu_{\uparrow}(x, t)| << \mu_0$), the electro-neutrality condition can be expressed as follows:

$$\delta \mu_\uparrow N_\uparrow + \delta \mu_\downarrow N_\downarrow = 0,$$

where $N_{\uparrow\downarrow}$ are equilibrium densities of states at Fermi level.

Neglecting the spin-flip processes we can write the continuity equation for electrons of each spin $\sigma$

$$\frac{\partial \delta n_\sigma}{\partial t} + \frac{\partial(n_0 \nu)}{\partial x} = 0.$$  

Let us describe the system we study by a classical Lagrange function $L = T - U$ [6], where $T$ is the kinetic energy and $U$ is the potential energy of conducting electrons. The kinetic energy is connected with drift of electrons:

$$T = \int_V n \frac{m \nu^2}{2} dV = \frac{s j^2}{2} \int_W \frac{m}{n(x)} dx,$$

where $j = n \nu$ is total electronic current (due to electro-neutrality $\partial j/\partial x = 0$), $m$ is an electron mass, $V$ is a total volume of the conductor, $n = n_1 + n_1$, $s = Const$ is the cross section of the conductor, $W$ is length of the conductor. The potential energy $U$ is connected with non-equilibrium spin density i.e. deviation from equilibrium value of chemical potential $\delta \mu_\sigma$:
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\[ U = \int \left\{ \int_{\mu_0}^{\mu_0+\delta \mu_i} \epsilon N_i \, d\epsilon + \int_{\mu_0}^{\mu_0+\delta \mu_i} \epsilon N_i \, d\epsilon \right\} \, dV, \]  \hspace{1cm} (3)

where \( \epsilon \) - electron energy. In main approximation \( N_i \) does not depend on energy \( \epsilon \) (but it can depends on coordinate \( x \)). There is an explanatory drawing at Fig. 2 which contains schematic illustrations of equilibrium (a) and non-equilibrium (b) spin states which are corresponds to potential energy. Using electro-neutrality condition (1) and by integration (2) over energy we can transform (3) to the following expression:

\[ U = \frac{s}{2} \int \left[ N_i \delta \mu_i^2 + N_i \delta \mu_i^2 \right] \, dx. \] \hspace{1cm} (4)

Using the fact that a drift velocity \( v = j/n \) is common for two spins we can transform (2) to:

\[ \frac{\partial \delta n_\sigma}{\partial t} + \frac{\partial}{\partial x} \left( n_0 \sigma n \right) = 0. \] \hspace{1cm} (5)

It is easy to integrate (5) over time (note that \( n_0 \) and \( n \) does not depend on time):

\[ \delta \mu_\sigma(x, t) = -\left( \frac{1}{N_\sigma(x)} \right) \left( \frac{\partial}{\partial x} \left( n_0 \sigma(x) \right) \right) \int_j^{t} j(t') \, dt'. \] \hspace{1cm} (6)

Based on the form of (6) let us introduce generic variable:

\[ q(t) = \int_j^{t} j(t') \, dt', \dot{q}(t) = j(t). \]

Now we can write Lagrange function in term of \( q(t) \):

\[ \mathcal{L} = \frac{s \dot{q}^2}{2} \int \frac{m}{n} \, dx - \frac{s q^2}{2} \int \frac{1}{N^*} \left( \frac{d}{dx} \frac{n_0}{n} \right)^2 \, dx. \] \hspace{1cm} (7)

Using common Lagrange equation

\[ \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial q} \]

and substituting (7) into (8), we obtain harmonic oscillator equation with frequency \( \omega \)

\[ \ddot{q} + \omega^2 q = 0, \quad \omega^2 = \left[ \int \frac{1}{N^*} \left( \frac{d}{dx} \frac{n_0}{n} \right)^2 \, dx \right] \left[ \int \frac{m}{n} \, dx \right]^{-1}, \] \hspace{1cm} (9)

where \( N^* = N_0 + N_1 \). Expression (9) which determines spin oscillations frequency is the same as obtained in [5]. Notice that our approach in this work does not require closure of the conductor as in [5]. Of course, closed conductor is one of the methodology to satisfy electro-neutrality condition (1), but now we can investigate a more general situation when we have a circuit which consists of magnetically inhomogeneous part, capacity and inductor (Fig. 3). Electro-neutrality of the open-circuit is reached due to the presence of the capacity.

To take into account presence of the capacity and inductor in the magneto-inhomogeneous conductor we must add additional terms into the Lagrangian. The kinetic energy must be supplemented by a term relevant to the inductor energy \( \frac{e^2}{2} L (s q)^2 / 2c^2 \) and potential energy must be supplemented by term relevant to capacity energy \( \frac{e^2}{2} (s q)^2 / 2C \), where \( L \) and \( C \) are inductivity and capacitance of the conductor, \( e \) is electron charge, \( c \) is speed of light. Using (5) with the new Lagrangian we obtain frequency of oscillations of our system:

\[ \omega^2 = \left[ \int \frac{1}{N^*} \left( \frac{d}{dx} \frac{n_0}{n} \right)^2 \, dx + \frac{e^2 s}{2C} \right] \left[ \int \frac{m}{n} \, dx + \frac{e^2 s L}{2c^2} \right]^{-1}. \] \hspace{1cm} (10)
Figure 3. Modification of “spin pendulum” – magnetically inhomogeneous conductor with capacitor plates which can collect electric charge. Inductivity of the conductor is taken into account.

It is easy to see that if we neglect the spin inhomogeneous and inertial terms, the expression (10) grades into well known Thomson’s oscillation formula for LC circuit.

As can be seen from above, we propose and describe the new device – “extended” oscillatory circuit, which frequency depends on magnetic properties of the conductor. It can be seen from the result that external magnetic field can be used to vary the oscillatory frequency of the device (by changing spatial distribution of the equilibrium spin density $n_{\uparrow\downarrow}$). Due to the presence of a capacity in the device, it can be easy used as a part of external circuit by a capacitive coupling. The described above resonant properties will appear as a peak of the impedance dependence on the frequency [4,6]. It can be seen from (4) that the potential energy is proportional to a square of the perturbation of a chemical potentials (non-equilibrium additions to spin densities) and to the volume $\Delta V$ in which non-equilibrium densities are enclosed $U \propto \delta \mu / \sigma \Delta V$. Thus, spin diffusion (which increases $\Delta V$, decreases $\delta \mu$, and doesn’t change $\delta \mu / \sigma \Delta V$) must be considered as a dissipative process. Therefore we emphasize that the expressions (9) and (10) are valid when $\nu_{\text{relax}} \ll \omega \ll \nu_{\text{ee}}, \nu_{\text{ee}}$, where $\nu_{\text{relax}}$ is the biggest efficient frequency of relaxation processes (frequency of the collisions with a momentum loss, frequency of the spin-flip processes, characteristic diffusion frequency $\nu_{\text{ee}}l_{ee}^2 / l_{tr}^2$, where $l_{ee}$ - electron free path length, $l_{tr}$ - characteristic length of equilibrium spatial spin inhomogeneity), $\nu_{\text{ee}}$ is the frequency of electron-electron collisions, $\nu_{\text{p}}$ is the plasmonic frequency.

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References

1. Zutic, J. Fabian, and S. Das Sarma, Rev. Mod. Phys. 76, 323 (2004); doi:10.1103/RevModPhys.76.323
2. M.N. Baibich , J.M. Broto , A. Fert , F. Nguyen Van Dau , F. Petroff , P. Etienne, G. Creuzet , A. Friederich and J. Chazelas, Phys. Rev. Lett. 61, 2472 (1988). doi:10.1103/PhysRevLett.61.2472
3. T. Valet and A.Fert, Phys. Rev. B 48, 7099 (1993) doi:10.1103/PhysRevB.48.7099
4. Denis Kochan, Martin Gmitra, and Jaroslav Fabian, Phys. Rev. Lett. 107, 176604 (2011) doi:10.1103/PhysRevLett.107.176604
5. R. N. Gurzhi, A. N. Kalinenko, A. I. Kopeliovich, P. V. Pyshkin, and A. V. Yanovsky, Phys. Rev. B 73, 153204 (2006) doi:10.1103/PhysRevB.73.153204
6. L.D. Landau & E.M. Lifshitz Mechanics ( Volume 1 of A Course of Theoretical Physics ), Pergamon Press, 1969
7. P.V. Pyshkin, Low Temp. Phys. 36, 1071 (2011) doi:10.1063/1.3536341