RESCATTERING CONTRIBUTIONS TO RARE $B$-MESON DECAYS

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Several $B$ and $B_s$ decays have been observed which have been cited as evidence for exchange ($E$), penguin annihilation ($PA$) and annihilation ($A$) processes, such as $\bar{b}d \to \bar{u}u$, $\bar{b}s \to \bar{u}u$ and $\bar{b}u \to W^* \to \bar{c}s$, respectively. These amplitudes are normally thought to be suppressed, as they involve the spectator quark in the weak interaction and thus should be proportional to the $B$-meson decay constant $f_B$. However, as pointed out a number of years ago, they can also be generated by rescattering from processes whose amplitudes do not involve $f_B$, such as color-favored tree amplitudes. In this paper we investigate a number of processes such as $B^0 \to K^+K^-$, $B_s \to \pi^+\pi^-$, and $B^+ \to D^+_s\phi$, and identify promising states from which they can be generated by rescattering. We find that $E$ and $PA$-type processes are characterized respectively by amplitudes ranging from 5% to 10% and from 15% to 20% with respect to the largest amplitude from which they can rescatter. Based on this regularity, using approximate flavor SU(3) symmetry in some cases and time-reversal invariance in others, we predict the branching fractions for a large number of as-yet-unseen $B$ and $B_s$ decays in an extensive range from order $10^{-9}$ to $10^{-4}$.

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I Introduction

The decays of $B$ mesons to two-body final states provide rich data for determining parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix which is thought to describe the observed violations of CP symmetry. These processes also yield valuable tests of the SU(3) flavor symmetry obeyed by final-state $u$, $d$, and $s$ quarks. Following early SU(3) analyses of $B$ decays [1, 2, 3], a hierarchy of invariant amplitudes was established, based on a convenient graphical language [4]. Dominant amplitudes were found to be color-favored tree ($T$) followed by color-suppressed tree ($C$) and penguin ($P$). These three amplitudes
Figure 1: Graphical representation of invariant amplitudes describing $B$-meson decays. (a) Color-favored tree ($T$); (b) Color-suppressed tree ($C$); (c) Penguin ($P$); (d) Exchange ($E$); (e) Annihilation ($A$); (f) Penguin annihilation ($PA$). Dashed lines indicate $W$ exchanges; $\times$ denotes a penguin $\bar{b} \to \bar{d}$ or $\bar{b} \to \bar{s}$ insertion.

involve only the decaying $\bar{b}$ quark in the initial $B$ meson and hence are approximately independent of the light “spectator” quark. Amplitudes considerably suppressed in comparison with them, all of which require participation of the spectator quark, are exchange ($E$), annihilation ($A$), and penguin annihilation ($PA$). All six amplitudes are illustrated in Fig. 1.

As pointed out a number of years ago [5, 6], effects of the amplitudes $E$, $A$, and $PA$ can also be generated by rescattering from processes whose amplitudes [color-favored tree ($T$), color-suppressed tree ($C$), or penguin ($P$)] do not involve $f_B$. Since then, both electron-positron and hadron collisions have yielded a wealth of information on many suppressed processes, such as new limits on the branching fraction for $B^0 \to K^+K^-$ [7, 8] and observation of the decays $B_s \to \pi^+\pi^-$ [7, 9] and $B^+ \to D_s^+\phi$ [10]. In the present paper we study such processes systematically, identifying promising intermediate states contributing to rescattering. We find that the suppressed processes have typical $E$ amplitudes ranging from 5% to 10% of the largest amplitude contributing to rescattering, while $PA$ amplitudes are somewhat larger. Based on this regularity, and using relations based on U-spin or on time-reversal, we predict the branching fractions for a large number of as-yet-unseen $B$ and $B_s$ decays.

Calculations of $E$, $A$ and $PA$-type amplitudes in QCD factorization are quite challenging. In $B$ decays with one charmed meson in the final state these amplitudes involve unknown matrix elements of non-local four-quark operators [11], while $E/A/PA$ amplitudes for charmless decays depend on divergent integrals [12]. Refs. [13, 14, 15] and a few references quoted therein have presented model-dependent attempts to calculate $E$, $A$ and $PA$ amplitudes within QCD.

In Section II we outline our strategy for evaluating rescattering contributions to suppressed $E$, $A$, and $PA$ amplitudes. In Section III we use current data to obtain ranges of ratios characterizing the suppression of these amplitudes relative to relevant $T, C$ and
Figure 2: Rescattering contributions. (a) To $B^0 \rightarrow K^+K^-$; initial tree ($T$) amplitude, $\rho^+\rho^-$ intermediate state contributing to exchange ($E$) amplitude. (b) To $B_s \rightarrow \pi^+\pi^-$; initial penguin ($P$) amplitude, $K^+K^-$ intermediate state contributing to penguin annihilation ($PA$) amplitude.

Figure 3: Rescattering contributions to $B^+ \rightarrow D^+_s \phi$ from a $D^{*0}K^{*+}$ intermediate state whose amplitude is of the color-suppressed tree ($C$) form.

In $B$ decays, whose average multiplicity is quite large, a given final state can be generated by rescattering from any number of intermediate states, many of which have not

### II E, A and PA amplitudes from rescattering

The manner in which a suppressed amplitude is generated by rescattering can be illustrated by some examples. Fig. 2(a) depicts the contribution to an exchange ($E$) amplitude for $B^0 \rightarrow K^+K^-$ from the $\rho^+\rho^-$ intermediate state, where the initial amplitude is of the tree ($T$) form. Fig. 2(b) describes a penguin annihilation ($PA$) amplitude for $B_s \rightarrow \pi^+\pi^-$ obtaining a contribution from a $K^+K^-$ intermediate state, where the initial amplitude is of the penguin ($P$) form. Fig. 3 shows the contribution of a $D^{*0}K^{*+}$ intermediate state [initial amplitude of the color-suppressed tree ($C$) form] to an annihilation ($A$) amplitude for $B^+ \rightarrow D^+_s \phi$. Finally, Fig. 4 shows the contribution of a $D^+_s D^-_s$ intermediate state (from $T$) to a penguin annihilation ($PA$) amplitude in $B_s \rightarrow \pi^+\pi^-$. We then apply these ratios in Section IV to predict branching ratios for a number of $B$ and $B_s$ decays. Section V highlights predictions based on flavor SU(3) and time-reversal invariance, while Section VI concludes.
Figure 4: Rescattering contributions to $B_s \to \pi^+ \pi^-$ from a $D_s^+ D_s^-$ intermediate state whose amplitude is of the color-favored tree ($T$) form.

Table I: $PP$, $PV$, and $VV$ intermediate states contributing to $B \to PP, PV$ decays. Other states are forbidden to contribute by parity conservation in the strong interactions.

| Final State | Contributing intermediate state(s) |
|-------------|-----------------------------------|
| $PP$        | $PP, (VV)_{L=0,2}$                |
| $PV$        | $PV, (VV)_{L=1}$                  |

yet been observed. Even if they were seen, it would not be clear with what relative phases their contributions should be added together. We do expect (quasi) two-body intermediate states to dominate, because rescattering from three-body or higher multiplicity states to two-body final states is expected to be greatly suppressed. For instance, while momenta are fixed for decays to two particles, they fill the plane of the Dalitz plot for three-body decays. We assume that rescattering is dominated by light-quark exchange. Rescattering due to heavy charm-quark exchange, depicted in Fig.4, is highly suppressed and will be mentioned briefly at the end of Section III.

In order to circumvent the shortcoming due to having several contributing states, we identify the (quasi) two-body intermediate state with the largest branching fraction, whose $T$, $C$, or $P$ amplitude we compare with the $E$, $A$, or $PA$ amplitude of the suppressed process. For several such processes, we find that the ratio $|E/T|$ lies within a narrow range of values between 0.05 and 0.10 while $|PA/P|$ is between 0.15 and 0.20. Finding no experimental evidence for a nonzero $|A/T|$, we will assume that this ratio takes values in the same range as $|E/T|$. The values of these three ratios are then used to predict branching fractions for a large number of the suppressed processes originally identified in Ref. [5, 6].

For simplicity we limit our consideration to intermediate states with two pseudoscalar mesons ($PP$), one pseudoscalar and one vector ($PV$), and two vector ($VV$) mesons. The states contributing to $PP$ and $PV$ final states are summarized in Table I.
Table II: \(E/A\)-type decays of nonstrange \(B\) mesons to two pseudoscalars, and \(T\)-type decays to intermediate states contributing to these decays by rescattering. Measured \(E/A/PA\) decays (first line or two lines in each subtable), along with possible contributing rescattering decays (subsequent lines). The branching ratios for all measured decays are given, along with the value of \(R\) found for each rescattering decay, assuming that it is dominant. The ratio of amplitudes probed in each \(E/A/PA\) decay is given at the top of the “ratio” column. For entries which are flagged with letters [such as (a)] further details are given in the text.

\[
\begin{array}{cccccc}
\text{CKM} & \text{Decay} & \text{Type} & \text{Int. State} & \text{BR} & \text{Ratio } R \\
V_{cb}V_{ud} & B^0 \to D_s^- K^+ & E & (2.31 \pm 0.24) \times 10^{-5} & \text{[17]} & |E/T| \\
 & & & (2.68 \pm 0.13) \times 10^{-3} & 0.09 \pm 0.01 & \\
 & & & D^- \pi^+ & & \\
 & & & D^{*-} \rho^+ & 0.96 \cdot (6.8 \pm 0.9) \times 10^{-3} \text{ (a)} & 0.06 \pm 0.01 \\
V_{cb}V_{cd} & B^0 \to D^0 \bar{D}^0 & E & < 4.3 \times 10^{-5} & |E/T| & \\
 & B^0 \to D_s^+ D_s^- & E & < 3.6 \times 10^{-5} & |E/T| & \\
 & & & (2.11 \pm 0.31) \times 10^{-4} & 0.4 & \\
 & & & D_s^+ D_s^- & (7.0\pm 0.8) \times 10^{-4} \text{ (b)} & < 0.2 \\
V_{ub}V_{ud} & B^0 \to K^+ K^- & E & < 2 \times 10^{-6} & |E/T| & \\
 & & & \pi^+ \pi^- & (5.15 \pm 0.22) \times 10^{-6} & 0.2 & \\
 & & & \rho^+ \rho^- & (2.42 \pm 0.31) \times 10^{-5} \text{ (c)} & < 0.1 \\
V_{ub}V_{cs} & B^+ \to D^+ K^0 & A & < 2.9 \times 10^{-6} & |A/T| & \\
 & & & D_s^+ \pi^0 \text{ (d)} & (1.6 \pm 0.5) \times 10^{-5} & < 0.3 \\
V_{ub}V_{cd} & B^0 \to D_s^+ K^- & E & -- & -- & \\
 & & & D^+ \pi^- & (7.8 \pm 1.4) \times 10^{-7} & -- \\
V_{ub}V_{cd} & B^+ \to D_s^+ K^0 & A & -- & -- & \\
 & & & D^+ \pi^0 \text{ (d)} & < 8 \times 10^{-4} & -- \\
\end{array}
\]

### III Relative magnitude of suppressed amplitudes

We begin by reviewing the status of the suppressed decays discussed in Ref. [5, 6]. Table II lists the \(PP\) decays of nonstrange \(B\) mesons with examples of contributing intermediate states. Table III lists the corresponding final and intermediate states for \(B_s\) decays. We note four isospin relations between \(B_s\) decay amplitudes to charged and neutral mesons,

\[
A(B_s \to D^+ D^-) = -A(B_s \to D^0 \bar{D}^0) \quad (\Delta I = 0), \\
A(B_s \to \pi^+ \pi^-) = -\sqrt{2} A(B_s \to \pi^0 \pi^0) \quad (\Delta I = 0), \\
A(B_s \to D^+ \pi^-) = -\sqrt{2} A(B_s \to D^0 \pi^0) \quad (\Delta I = 1/2), \\
A(B_s \to D^- \pi^+) = -\sqrt{2} A(B_s \to \bar{D}^0 \pi^0) \quad (\Delta I = 1/2).
\]

One can also list a number of nonstrange \(B\) decays through \(E\) and \(A\) amplitudes to \(PV\) final states. (No such \(B_s\) decays have been reported yet.) These are given in Table IV with examples of contributing non-suppressed intermediate states. All branching ratios quoted in Tables II, III and IV are taken from the Particle Data Group [16], unless otherwise indicated. Finally, one can consider suppressed \(B \to VV\) decays by replacing both pseudoscalars in Tables II and III by vector mesons.
Table III: $E/PA$-type decays of $B_s$ mesons to two pseudoscalars, and $T$-type decays to intermediate states contributing to these decays by rescattering. Information organized as in Table II.

| CKM     | Decay                  | Type | Int. state | BR                        | Ratio R |
|---------|------------------------|------|------------|---------------------------|---------|
| $V_{cb}V_{cs}$ | $B_s \to D^+D^-$     | $E$  |            | $(5.3 \pm 0.9) \times 10^{-3}$ | $|E/T|$  |
|         | $B_s \to D^0\bar{D}^0$ | $E$  |            | $(1.60 \pm 0.29) \times 10^{-2}$ | $|E/T|$  |
| $V_{cb}V_{cs}$ | $B_s \to \pi^+\pi^-$ | $PA$ |            | $(0.73 \pm 0.14) \times 10^{-6}$ | $|PA/P|$ |
|         | $B_s \to \pi^0\pi^0$  | $PA/\sqrt{2}$ |            | $(2.45 \pm 0.18) \times 10^{-5}$ | $0.17 \pm 0.02$ |
|         |                        |      |            | $(1.7 \pm 0.5) \times 10^{-5}$ | $0.21 \pm 0.04$ |
| $V_{cb}V_{us}$ | $B_s \to D^-\pi^+$   | $E$  |            | $D_s^-K^+$                  | $|E/T|$  |
|         | $B_s \to D^0\pi^+$    | $E/\sqrt{2}$ |            | $D_s^+K^-$                  | $|E/T|$  |
| $V_{ub}V_{cs}$ | $B_s \to D^+\pi^-$   | $E$  |            | $D_s^+D_s^-$ $(5.3 \pm 0.9) \times 10^{-3}$ | $0.012 \pm 0.002$ |
|         | $B_s \to D^0\pi^0$   | $E/\sqrt{2}$ |            | $D_s^+D_s^-$ $(1.60 \pm 0.29) \times 10^{-2}$ | $0.007 \pm 0.002$ |
Table IV: E/A-type decays of nonstrange B mesons to PV final states, and T or C-type decays to intermediate states contributing to these decays by rescattering. Information organized as in Table III.

| CKM | Decay | Type | Int. State | BR | Ratio R |
|-----|-------|------|------------|----|---------|
| $V_{cb}^{*} V_{ud}$ | $B^{0} \rightarrow D_s^{-} K^{+}$ | $E$ | $(2.19 \pm 0.30) \times 10^{-5}$ | $|E/T|$ | |
| | $B^{0} \rightarrow D_{s}^{-} K^{*+}$ | $E$ | $(3.5 \pm 1.0) \times 10^{-5}$ | $|E/T|$ | |
| | $D^{*-} \pi^{+}$ | | $(2.76 \pm 0.13) \times 10^{-3}$ | 0.09 $\pm$ 0.01 | |
| | $D^{*-} \rho^{+}$ | | $(7.8 \pm 1.3) \times 10^{-3}$ | 0.05 $\pm$ 0.01 | |
| | $D^{*-} \rho^{0}$ | | $(2.7 \pm 0.4) \times 10^{-4}$ | (h) | |
| $V_{cb}^{*} V_{cd}$ | $B^{0} \rightarrow D^{0*} D^{0}, D^{0} D^{0*}$ | $E$ | < $2.9 \times 10^{-4}$ | $|E/T|$ | |
| | $B^{0} \rightarrow D^{0*} D^{0*}$ | $E$ | < $1.3 \times 10^{-4}$ | $|E/T|$ | |
| | $D^{*+} D^{-}$ | | $(6.1 \pm 1.5) \times 10^{-4}$ | < 0.5 | |
| $V_{ub}^{*} V_{ud}$ | $B^{0} \rightarrow K^{* \pm} K^{\mp}$ | $E$ | | | |
| | $\rho^{\pm} \tau^{\mp}$ | | $(2.30 \pm 0.23) \times 10^{-5}$ | | |
| $V_{ub}^{*} V_{cs}$ | $B^{+} \rightarrow D^{+} K^{00}$ | $A$ | < $1.8 \times 10^{-6}$ | $|A/T(C)|$ | |
| | $B^{-} \rightarrow D^{*+} K^{0}$ | $A$ | < $9.0 \times 10^{-6}$ | $|A/T(C)|$ | |
| | $D_{s}^{*+} \pi^{0} (T)$ | | < $2.6 \times 10^{-4}$ | | |
| | $D_{s}^{+} \rho^{0} (T)$ | | < $3.0 \times 10^{-4}$ | | |
| | $D^{0} K^{*+} (C)$ | | $\sim 1 \times 10^{-5}$ | (i,j) | < 0.4 |
| $V_{ub}^{*} V_{cs}$ | $B^{+} \rightarrow D_{s}^{+} \phi$ | $A$ | $(1.87^{+1.70}_{-0.82}) \times 10^{-6}$ | $|A/T(C)|$ | |
| | $D^{0} K^{*+} (C)$ | | $\sim 1 \times 10^{-5}$ | (i,j) | $\sim 0.4 \pm 0.1$ |
| | $D_{s}^{+} \omega$ (T) | | < $4 \times 10^{-4}$ | (k) | (l) |
| $V_{ub}^{*} V_{cd}$ | $B^{0} \rightarrow K^{-} D_{s}^{+}$ | $E$ | | | |
| | $D^{+} \rho^{-}$ | | | | |
| | $D^{*+} \pi^{-}$ | | | | |
| $V_{ub}^{*} V_{cd}$ | $B^{0} \rightarrow \overline{K}^{0} D_{s}^{+}$ | $A$ | < $4.4 \times 10^{-6}$ | $|A/T|$ | |
| | $D^{+} \rho^{0}$ | | | | |
| | $D^{*+} \pi^{0}$ | | | | |

As noted in the introduction, rescattering can occur via many intermediate states. We can identify at most a few of them, but there will always be one with the largest branching fraction. We can use that one to calculate a “typical” ratio of the suppressed amplitude to the largest unsuppressed one. We then have to assume that the effect of many intermediate states (whether constructive, incoherent, or destructive) is roughly the same for all cases. With this in mind, we calculate the amplitude ratio for all measured E/A/PA-type decays, assuming a single intermediate state, that with the largest branching fraction.

Amplitudes are evaluated as square roots of branching fractions, with phase-space differences ignored. We consider only suppressed amplitudes and amplitudes for intermediate states which share the same CKM factor. Thus, for instance, in the amplitude $E$ for $B^{0} \rightarrow K^{+} K^{-}$ involving $V_{ub}^{*} V_{ud}$ we ignore a rescattering contribution from $B^{0} \rightarrow D^{+} D^{-}$ involving $V_{cb}^{*} V_{cd}$. For the amplitude ratio (i.e., the rescattering suppression factor), for which CKM factors associated with the decays in the numerator and denominator cancel, we use
the symbol $R \equiv |[E/A/PA]/[T/C/P]|$.

For $B \to PP E/A/PA$-type decays, we give the $PP$ and $VV$ intermediate states; for $B \to PV/VV E/A/PA$ decays, we give the $PV$, $VP$ and $VV$ intermediate states. Now, for the $VV$ intermediate states, Ref. [10] gives the branching ratio for the decay to all three helicity states. However, only two (one of these − those with positive (negative) parity − contribute to rescattering to $PP$ ($PV/VP$) final states. Thus, the effective rescattering branching ratio is probably smaller than that given in the tables, and the value of $R$ larger. For many $B \to VV$ decays, the polarization fractions have been measured. This allows us to modify the total branching ratios appropriately, which we do where possible.

In Tables II, III and IV we list all measured $E/A/PA$-type decays, along with the value of $R$ obtained from individual decays into intermediate rescattering states. Some of the quoted branching ratios of the latter processes require some details which we give now.

(a) The helicity amplitudes for $B^0 \to D^{*-}\rho^+$ were measured in Ref. [18], with the result $|H_0| = 0.941$, $|H_\parallel| = 0.27$, $|H_\perp| = 0.21$. The fraction of decays with positive parity is thus $f_+ = |(H_0^2 + |H_\parallel|^2)/(H_0^2 + |H_\parallel|^2 + |H_\perp|^2)| = 0.96$. This indicates that the rescattering of $B^0 \to D^{*-}\rho^+$ contributes significantly to $B^0 \to D^-K^+$. On the other hand, the fraction of decays with negative parity is 0.04, so that there is little rescattering contribution to $B^0 \to D^-K^*$.

(b) The fraction of $B^0 \to D^{(*)+}D^{*-}$ decays with positive parity is $f_+ = 0.850 \pm 0.025$ [16]. We quote $f_+ B(B^0 \to D^{(*)+}D^{*-}) = (0.850 \pm 0.025)/(8.2 \pm 0.9) \times 10^{-3} = (7.0 \pm 0.8) \times 10^{-3}$.

(c) $B^0 \to \rho^+\rho^-$ is dominated by longitudinal polarizations, $f_L = |H_0|^2/(|H_0|^2 + |H_\parallel|^2 + |H_\perp|^2) = 0.977^{+0.028}_{-0.024}$ [16].

(d) Decay amplitude is given by $T/\sqrt{2}$.

(e) We are assuming that the fraction of $B_s \to D^{(*)+}_s D^{(*)-}_s$ decays with positive parity is the same as in $B^0 \to D^{(*)+}D^{*-}$. This assumption is supported by a calculation based on the heavy-quark expansion and factorization [19]. We quote $f_+ B(B_s \to D^{(*)+}_s D^{(*)-}_s) = (0.850 \pm 0.025)(1.88 \pm 0.34) \times 10^{-2} = (1.60 \pm 0.29) \times 10^{-2}$.

(f) The helicity amplitudes for $B_s \to K^{*0}\bar{K}^{*0}$ were measured in Ref. [20], leading to $f_+ = 0.62 \pm 0.12$. We quote $f_+ B(B_s \to K^{*0}\bar{K}^{*0}) = (0.62 \pm 0.12)(2.8 \pm 0.7) \times 10^{-5} = (1.7 \pm 0.5) \times 10^{-5}$, $f_- B(B_s \to K^{*0}\bar{K}^{*0}) = (1.1 \pm 0.4) \times 10^{-5}$.

(g) Using untagged $B_s$ decays only the charge-averaged branching ratio has been measured, $B(B_s \to D^{(*)}_s K^{(*)}_s) = (2.9 \pm 0.6) \times 10^{-4}$ [16].

(h) We quote $f_- B(B^0 \to D^{*-}\rho^+) = 0.04 \cdot (6.8 \pm 0.9) \times 10^{-3} = (2.7 \pm 0.4) \times 10^{-4}$.

(i) The decays $B^+ \to D^{(*)0}K^{(*)+}$ have not been measured, but the decays $B^+ \to \bar{D}^{(*)0} \bar{K}^{(*)+}$ have: $B(B^+ \to \bar{D}^{(*)0}K^{(*)+}) = (5.3 \pm 0.4) \times 10^{-4}$, $B(B^+ \to \bar{D}^{(*)0}K^{(*)+}) = (4.20 \pm 0.34) \times 10^{-4}$, $B(B^+ \to \bar{D}^{(*)0} \bar{K}^{(*)+}) = (8.1 \pm 1.4) \times 10^{-4}$. BaBar has found that $r_B \equiv |A(B^+ \to D^{(*)0}K^{(*)+})/\sqrt{2}| 10^{-4}$, $B(B^+ \to D^{(*)0}K^{(*)+}) = 0.31 \pm 0.07$ [21]. This gives $B(B^+ \to D^{(*)0}K^{(*)+}) = (31 \pm 0.07)^2 \cdot (5.3 \pm 0.4) \times 10^{-4} = (5.1 \pm 2.3) \times 10^{-5}$.

(j) The isospin triangle relation, $A(B^0 \to D^{(*)0}K^{(*)0}) = A(B^+ \to D^{(*)0}K^{(*)+}) + A(B^- \to D^{(*)-}K^{(*)0})$, shown in Ref. [22] implies that $r_B$ is smaller by at least one $\sigma$ than its above-mentioned central value. With the experimental limits [16] $B(B^0 \to D^{(*)0}K^{(*)0}) < 1.1 \times 10^{-5}$ and $B(B^+ \to D^{(*)+}K^{(*)0}) < 1.8 \times 10^{-6}$ [10], we have (in units of $10^{-3}$) $|A(B^0 \to D^{(*)0}K^{(*)0})| < 3.3$ and $|A(B^+ \to D^{(*)+}K^{(*)0})| < 1.3$. But taking $B(B^+ \to D^{(*)0}K^{(*)+}) = (5.1 \pm 2.3) \times 10^{-5}$ yields
$|A(B^+ \to D^0 K^{*+})| = 7.1 \pm 1.6$; for the central value of this last branching ratio, the triangle does not close. It closes only if the branching ratio for $B^+ \to D^0 K^{*+}$ is at least $1.5\sigma$ below its central value. This, and independent supporting evidence discussed in the next point below, suggest that a likely value of $\mathcal{B}(B^+ \to D^0 K^{*+})$ is around $1 \times 10^{-5}$, corresponding to $r_B \approx 0.15$. This value is consistent with a value $r_B = 0.115 \pm 0.045$ obtained in Ref. [28] by a global fit to CKM parameters.

(k) A potential rescattering state contributing to $B^+ \to D_s^+ \phi$ is $D_s^+ \omega$, for which one has a rather old upper bound $\mathcal{B}(B^+ \to D_s^+ \omega) < 4 \times 10^{-4}$ [24]. An order of magnitude stronger upper bound, $\mathcal{B}(B^+ \to D_s^+ \omega) \lesssim 1.2 \times 10^{-5}$, is obtained if one assumes $\mathcal{B}(B^+ \to D_s^+ \omega) \approx \mathcal{B}(B^+ \to D_s^+ \rho^0)$ using an isospin relation, $\mathcal{B}(B^+ \to D_s^+ \rho^0) = \mathcal{B}(B^0 \to D_s^+ \rho^-)/2 < 1.2 \times 10^{-5}$ [16]. We note that while $B^+ \to D^0 K^{*+}$ is due to a color-suppressed amplitude $C$, $B^+ \to D_s^+ \omega$ involves a color-favored tree amplitude $T/\sqrt{2}$ which is usually expected to be larger than $C$. Recalling our discussion of $B^+ \to D^0 K^{*+}$ in point (j) we are led to conclude that both $\mathcal{B}(B^+ \to D^0 K^{*+})$ and $\mathcal{B}(B^+ \to D_s^+ \omega)$ are most likely around $1 \times 10^{-5}$. Improved measurements of $\mathcal{B}(B^+ \to D_s^+ \omega)$ and $\mathcal{B}(B^+ \to D_s^+ \rho^0)$ (a potential dominant rescattering contributor to $B^+ \to D^0 K^{*0}$ and $B^+ \to D^+ K^0$), using the BaBar, Belle and LHCb high statistics data, are of great importance.

(l) The rescattering contribution of $B^+ \to D_s^+ \omega$ to $B^+ \to D_s^+ \phi$ due to $\omega-\phi$ mixing is OZI-suppressed [25]. It is given by $\mathcal{B}(B^+ \to D_s^+ \phi)_{\omega-\phi} = \mathcal{B}(B^+ \to D_s^+ \omega) \delta^2$. Here $\delta$ is the $\omega-\phi$ mixing angle, $\delta = -3.34^\circ$ or $\delta(m = m_\phi) = -4.64^\circ$ in mass-independent or mass-dependent analyses [26]. Assuming $\mathcal{B}(B^+ \to D_s^+ \omega) \sim 1 \times 10^{-5}$ as argued above and taking a mass-dependent $\delta$, one finds $\mathcal{B}(B^+ \to D_s^+ \phi)_{\omega-\phi} \sim 0.7 \times 10^{-7}$. This is only a tiny fraction of the measured value of $\mathcal{B}(B^+ \to D_s^+ \phi)$.

The information on ratios $R$ given in the last columns of Tables II, III and IV can be summarized as follows:

- The ratio $|E/T|$, obtained from $\mathcal{B}(B^0 \to D^- K^+)$, $\mathcal{B}(B^0 \to D^0 \pi^+)$ and all their $VP$ analogues, lies in the narrow range $|E/T| = 0.05 - 0.1$. This range describes well contributions of rescattering in $B^0 \to D^- \pi^+, D^- \rho^+ \to D^- K^+$ and $B^0 \to D^- \pi^+, D^- \rho^+ \to D^- K^+$, $D^- K^{*+}$. The different angular momenta involved in these decays do not seem to affect much the value of $R$. A number of other decay modes involving $(dd) \to (ss)$ rescattering are expected to have values of $R$ in the same range.

- The ratio $|A/T|$ cannot be extracted from $\mathcal{B}(B^+ \to D_s^+ \phi)$ and $\mathcal{B}(B^+ \to D_s^+ \omega)$ because rescattering from $D_s^+ \omega$ to $D_s^+ \phi$ is OZI-suppressed. This seems like a singular case, in which we are unable to identify a dominant intermediate state contributing to rescattering. A less likely interpretation for the branching ratio $B^+ \to D_s^+ \phi$ is that physics beyond the CKM framework is at work.

- The value of $|PA/P|$, obtained from $\mathcal{B}(B_s \to \pi^+ \pi^-)$ and $\mathcal{B}(B_s \to K^+ K^-)$, is near $0.2$, about twice the value of $|E/T|$. In the last subtable in Table III we also obtain a value for a ratio $|PA/T|$, where $T$ is a color-favored tree amplitude determined by $\mathcal{B}(B_s \to D_s^{(s)^0} D_s^{(s)^-})$. This very small ratio of order $0.01$, corresponding to $D_s^{(s)^0} D_s^{(s)^-} \to \pi^+ \pi^-$ rescattering, is suppressed by requiring two quark-antiquark rescatterings as shown in Fig. IV. Some portion of the suppression may be due to the exchange of the heavy charm quark in rescattering.
IV Predictions based on ranges of $R$

With the above-mentioned ranges of $R$ we can now predict the branching ratios for other $E/A/PA$ decays. We will use the value $|E/T| = 0.07 \pm 0.02$ and will assume the same range for $|A/T|$ in cases where one may identify a potentially dominant $T$-type decay contributing by rescattering to an $A$-type decay. Finally, the value $|PA/P| = 0.17 \pm 0.02$ extracted from $\mathcal{B}(B_s \to \pi^+\pi^-)$ and $\mathcal{B}(B_s \to K^+K^-)$ will be used to predict branching ratios for $B_s$ decays into other pairs of unflavored mesons. The central values and uncertainties in the three ratios are chosen to describe ranges for these parameters. Thus the errors in predicted branching ratios, obtained by adding in quadrature these uncertainties and experimental errors in branching ratios, are not statistical. Rather, under our assumptions, they give reasonable ranges for a large number of branching ratios of decay modes which have not yet been observed.

Using the above values for the ratios $|E/T|$, $|A/T|$ and $|PA/P|$, we obtain predictions for $B$ and $B_s$ decay branching ratios. Results for $B, B_s \to PP$ and $B, B_s \to VP$ are presented in Tables [V] and [VI] respectively. Predictions appear in the first one or two lines in each subtable, while the last line in each subtable quotes the corresponding largest measured branching ratio for a process of type $T$ or $P$. Entries in the last subtable of Table [V] and in all but the second subtable in Table [VI] refer to CP-averaged branching ratios which are measured using untagged $B^0$ and $B_s$ decays. Our prediction for $\mathcal{B}(B^+ \to D^+K^0)$ in Table [V] can test our assumption $|A/T| = 0.07 \pm 0.02$.

Table V: Predictions for branching ratios of $B$ and $B_s$ decays to two pseudoscalar mesons. $E/A/PA$ decays appear in the first line or two lines in each subtable, while corresponding rescattering decay with largest branching ratio is given in the last line of each subtable. Entries in the last subtable refer to CP-averaged branching ratios.

| CKM factor | Decay | Measured BR | Predicted BR |
|------------|-------|-------------|--------------|
| $V_{cd}^*V_{cd}$ | $B^0 \to D^0 D^0$ | $< 4.3 \times 10^{-5}$ | $(3.4 \pm 2.0) \times 10^{-6}$ |
| | $B^0 \to D^+_s D^-_s$ | $< 3.6 \times 10^{-5}$ | $(3.4 \pm 2.0) \times 10^{-6}$ |
| | $B^0 \to D^{*+} D^{*-}$ | $(7.0 \pm 0.8) \times 10^{-4}$ | |
| $V_{ud}^*V_{ud}$ | $B^0 \to K^+ K^-$ | $2 \times 10^{-5}$ | $(1.2 \pm 0.7) \times 10^{-4}$ |
| | $B^0 \to \rho^+ \rho^-$ | $(2.42 \pm 0.31) \times 10^{-5}$ | |
| $V_{ub}^*V_{cd}$ | $B^0 \to D^+_s K^-$ | $< 3.8 \times 10^{-9}$ | |
| | $B^0 \to D^+ \pi^-$ | $(7.8 \pm 1.4) \times 10^{-7}$ | |
| $V_{ub}^*V_{cs}$ | $B^+ \to D^+_s K^0$ | $< 2.9 \times 10^{-5}$ | $(1.6 \pm 1.0) \times 10^{-4}$ |
| | $B^+ \to D^+_s \pi^0$ | $(1.6 \pm 0.5) \times 10^{-5}$ | |
| $V_{cd}^*V_{cs}$ | $B_s \to D^+ D^-$ | $< 7.8 \times 10^{-5}$ | $(7.8 \pm 4.7) \times 10^{-5}$ |
| | $B_s \to D^0 \bar{D}^0$ | $< 7.8 \times 10^{-5}$ | $(7.8 \pm 4.7) \times 10^{-5}$ |
| | $B_s \to D^{*+} D^{*-}_s$ | $(1.60 \pm 0.29) \times 10^{-2}$ | |
| $V_{cb}^*V_{us}, V_{ub}^*V_{cs}$ | $B_s \to D^{\pm} \pi^\mp$ | $< 1.4 \times 10^{-6}$ | $(1.4 \pm 0.9) \times 10^{-6}$ |
| | $B_s \to D^0 \pi^0, \bar{D}^0 \pi^0$ | $< 1.4 \times 10^{-6}$ | $(0.7 \pm 0.4) \times 10^{-6}$ |
| | $B_s \to D^{*\pm} K^\mp$ | $(2.9 \pm 0.6) \times 10^{-4}$ | |
Table VI: Predictions for branching ratios of $B$ and $B_s$ decays to vector and pseudoscalar mesons organized as in Table [V]. Entries in all but the second subtable refer to CP-averaged branching ratios.

| CKM factor | Decay | Measured BR | Predicted BR |
|------------|-------|-------------|--------------|
| $V_{cs}^*$ | $B_s \rightarrow D_s^0 D_s^0$ | $-\lambda$ | $(6.1 \pm 3.6) \times 10^{-5}$ |
| | $B_s \rightarrow D_s^0 \bar{D}_s^0, D_s^0 \bar{D}_s^0$ | $-\lambda$ | $(6.1 \pm 3.6) \times 10^{-5}$ |
| | $B_s \rightarrow D_s^0 D_s^0$ | $(1.24 \pm 0.21) \times 10^{-2}$ |
| $V_{cb}^*$ | $B_s \rightarrow \rho^- \pi^-$ | $-\lambda$ | $(3.1 \pm 1.4) \times 10^{-7}$ |
| | $B_s \rightarrow \rho^+ \pi^+$ | $-\lambda$ | $(3.1 \pm 1.4) \times 10^{-7}$ |
| | $B_s \rightarrow K^0 \bar{K}^0$ | $(1.1 \pm 0.4) \times 10^{-5}$ |
| $V_{cd}$ | $B^0 \rightarrow D_s^0 D_s^0$ | $<1.3 \times 10^{-4}$ | $(3.0 \pm 1.9) \times 10^{-6}$ |
| | $B^0 \rightarrow D_s^+ \bar{D}_s^0, D_s^0 \bar{D}_s^0$ | $<2.9 \times 10^{-4}$ | $(3.0 \pm 1.9) \times 10^{-6}$ |
| | $B^0 \rightarrow D_s^0 D_s^0$ | $(6.1 \pm 1.5) \times 10^{-4}$ |
| $V_{ub}^*$ | $B^0 \rightarrow K^{*+} K^-$ | $-\lambda$ | $(1.1 \pm 0.7) \times 10^{-7}$ |
| | $B^0 \rightarrow \rho^+ \pi^+$ | $(2.30 \pm 0.23) \times 10^{-5}$ |

V  Predictions based on flavor SU(3) or time-reversal

In Eqs. (11) we have presented four isospin relations in pairs of $E$ and $PA$-type $B_s$ decay amplitudes, leading to relations between corresponding decay branching ratios. Other relations among $E$ and $A$-type $B$ and $B_s$ decay amplitudes follow in the limit of flavor SU(3) symmetry. Two subgroups of SU(3), U-spin and V-spin of which $(d, s)$ and $(u, s)$ are fundamental doublet representations, are useful in deriving these relations. We will focus our attention on relations for decays into two pseudoscalar mesons, discussing in certain cases also relations for $B, B_s \rightarrow VP$ and $B, B_s \rightarrow VV$.

In the V-spin symmetry limit, applying $u \leftrightarrow s$ reflection, one has

$$A(B^0 \rightarrow D_s^0 D_s^0) = A(B^0 \rightarrow D_s^0 D_s^0),$$

as assumed in Tables [II] and [V]. Thus, in the V-spin symmetry limit the two corresponding branching ratios are predicted to be equal.

Using approximate symmetry of strong interactions under U-spin reflection, $d \leftrightarrow s$, and considering the U-spin structure of the effective weak Hamiltonian and of initial and final states, we find:

$$A(B_s \rightarrow D^- \pi^+) = \lambda A(B^0 \rightarrow D_s^- K^+),$$
$$-\lambda A(B_s \rightarrow D^+ D^-) = A(B^0 \rightarrow D_s^+ D_s^-),$$
$$-\lambda A(B_s \rightarrow D^+ \pi^-) = A(B^0 \rightarrow D_s^+ K^-),$$

and

$$A(B^+ \rightarrow D_s^+ K^0) = -\lambda A(B^+ \rightarrow D^+ K^0).$$

Here $\lambda \equiv V_{us}/V_{ud} \approx -V_{cd}/V_{cs} = 0.231 \ [16]$. Given the value of $B(B^0 \rightarrow D_s^- K^+)$ in Table [II] the first of Eqs. (3) leads to predicting $B(B_s \rightarrow D^- \pi^+)$ in the U-spin symmetry limit,

$$B(B_s \rightarrow D^- \pi^+) \approx (1.23 \pm 0.13) \times 10^{-6}.$$
This value is in agreement with the prediction for the CP-averaged branching ratio quoted in Table IV which involves a larger uncertainty. In a similar manner one has U-spin relations for corresponding \( B, B_s \rightarrow VP \) decays, such as

\[
A(B_s \rightarrow D_s^- K^+) = \lambda A(B^0 \rightarrow D_s^- K^+) , \\
A(B_s \rightarrow D^- \rho^+) = \lambda A(B^0 \rightarrow D_s^- \rho^+) .
\] (6)

Taking branching ratios quoted in Table IV we obtain

\[
\mathcal{B}(B_s \rightarrow D_s^- \pi^+) = (1.2 \pm 0.2) \times 10^{-6} , \\
\mathcal{B}(B_s \rightarrow D^- \rho^+) = (1.9 \pm 0.5) \times 10^{-6} .
\] (7)

These predictions add to those already given in Table VI.

We will now show that the predictions obtained in Section IV, assuming a dominant rescattering contribution in \( E \)-type decays, are consistent with U-spin relations such as Eqs. (3) and (6). We will use the fact that final states on the left-hand side of these equations are rescattering states contributing to corresponding amplitudes on the right-hand side, while final states on the right-hand side contribute as rescattering states to amplitudes on the left-hand side.

Let us focus, for instance, on the first U-spin relation in Eqs. (3) between two \( E \)-type amplitudes. We will show now that this relation may be derived using our assumption of dominant rescattering states for which two respective \( T \)-type amplitudes are related to each other by U-spin. We are assuming that \( B^0 \rightarrow D_s^- K^+ \) is dominated by a positive-parity \( D_s^- \rho^+ \) rescattering state,

\[
|A(B^0 \rightarrow D_s^- K^+)| = |A(B^0 \rightarrow [D_s^- \rho^+]_+)||A([D_s^- \rho^+]_+ \rightarrow D_s^- K^+)| ,
\] (8)

where

\[
|A(B^0 \rightarrow [D_s^- \rho^+]_+)| \equiv \sqrt{|A(B^0 \rightarrow [D_s^- \rho^+]_0)|^2 + |A(B^0 \rightarrow [D_s^- \rho^+]_1)|^2} .
\] (9)

Similarly one obtains

\[
|A(B_s \rightarrow D^- \pi^+)| = |A(B_s \rightarrow [D_s^- K^{**}]_+)| |A([D_s^- K^{**}]_+ \rightarrow D^- \pi^+)| ,
\] (10)

where dominance of \( B_s \rightarrow [D_s^- K^{**}]_+ \) over \( B_s \rightarrow D^- K^+ \) is implied by \( |A(B^0 \rightarrow [D_s^- \rho^+]_+)| > |A(B^0 \rightarrow D^- \pi^+)| \) and U-spin symmetry.

Assuming that the rescattering amplitude is invariant under U-spin, \( A([D_s^- K^{**}]_+ \rightarrow D^- \pi^+) = A([D_s^- \rho^+]_+ \rightarrow D_s^- K^+) \), we obtain the first of Eqs. (3) as required,

\[
\frac{|A(B_s \rightarrow D^- \pi^+)|}{|A(B^0 \rightarrow D_s^- K^+)|} = \frac{|A(B_s \rightarrow [D_s^- K^{**}]_+)|}{|A(B^0 \rightarrow [D_s^- \rho^+]_+)|} = \lambda .
\] (11)

The second equality, giving the ratio of two positive parity \( T \)-type amplitudes, follows from the behavior under U-spin reflection of the effective weak Hamiltonian and of initial and final states.

At this point we wish to comment on the definition of the magnitude of the effective rescattering amplitude for positive parity, \( |A([D_s^- \rho^+]_+ \rightarrow D_s^- K^+)| \) in (8), which we have
defined in Table II as the ratio $|E/T| = 0.06 \pm 0.01$ in $B^0 \to D_s^- K^+$. Eq. (3) may be expanded,

$$A(B^0 \to D_s^- K^+) = A(B^0 \to [D^*- \rho^+]_0) A([D^*- \rho^+]_0 \to D_s^- K^+)$$

$$+ A(B^0 \to [D^*- \rho^+]_{\parallel}) A([D^*- \rho^+]_{\parallel} \to D_s^- K^+) \ , \quad (12)$$

where $[D^*- \rho^+]_{0,\parallel}$ are longitudinal and parallel polarization states, and $A([D^*- \rho^+]_{0,\parallel} \to D_s^- K^+)$ are corresponding strong interaction rescattering amplitudes. The $B^0 \to D_s^- K^+$ decay rate is obtained by squaring the above sum and integrating over the angular dependence of the two pairs of final pseudoscalars, $D^0 \pi^-$ (or $D^- \pi^0$) and $\pi^+ \pi^0$. The interference term drops by integration implying (we omit phase-space factors),

$$|A(B^0 \to D_s^- K^+)|^2 = |A(B^0 \to [D^*- \rho^+]_0)|^2 |A([D^*- \rho^+]_0 \to D_s^- K^+)|^2$$

$$+ |A(B^0 \to [D^*- \rho^+]_{\parallel})|^2 |A([D^*- \rho^+]_{\parallel} \to D_s^- K^+)|^2 \ . \quad (13)$$

Comparing this expression for $|A(B^0 \to D_s^- K^+)|^2$ with that given in (5) and (6), we find

$$|A([D^*- \rho^+]_+ \to D_s^- K^+)|^2 = g_0 |A([D^*- \rho^+]_0 \to D_s^- K^+)|^2 + g_{\parallel} |A([D^*- \rho^+]_{\parallel} \to D_s^- K^+)|^2 \quad \text{(14)}$$

where $g_0 = 0.924$ and $g_{\parallel} = 0.076$ are longitudinal and parallel fractions of $B^0 \to D^*+ \rho^+$ decays relative to decays with positive parity. [See comment (a) above.]. That is, the effective rescattering probability for positive parity is given by a weighted average of the two rescattering probabilities for longitudinal and parallel helicity states.

To conclude this section let us show that using merely time-reversal invariance and assuming a dominant intermediate state for rescattering permits predicting a ratio of $E$ and $T$-type amplitudes for one pair of processes in terms of a similar (sometimes given) ratio of another pair of processes. Applying relations similar to (8) and (10) to VV amplitudes for a given helicity $h$, one has

$$A(B^0 \to [D^*_s- K^{**+}]_{h}) = A(B^0 \to [D^*- \rho^+]_{h}) A([D^*- \rho^+]_{h} \to [D^*_s- K^{**+}]_{h}) \ , \quad (15)$$

$$A(B_s \to [D^*- \rho^+]_{h}) = A(B_s \to [D^*_s- K^{**+}]_{h}) A([D^*_s- K^{**+}]_{h} \to [D^*- \rho^+]_{h}) \ .$$

Using time-reversal invariance (neglecting the small $B_s-B^0$ mass-difference),

$$A([D^*_s- K^{**+}]_{h} \to [D^*- \rho^+]_{h})) = A([D^*- \rho^+]_{h} \to [D^*_s- K^{**+}]_{h}) \ , \quad (16)$$

one obtains

$$A(B_s \to [D^*- \rho^+]_{h}) = A(B^0 \to [D^*_s- K^{**+}]_{h}) \ . \quad (17)$$

Thus a similar relation holds also for ratios of square roots of total branching ratios,

$$\frac{B(B_s \to D^*- \rho^+)}{B(B_s \to D_s^- K^+)} = \left( \frac{B(B^0 \to D_s^- K^+)}{B(B^0 \to D^*- \rho^+)} \right) = 0.07^{+0.02}_{-0.01} \ . \quad (18)$$

Here we have used $B(B^0 \to D_s^- K^+)= (3.2^{+1.5}_{-1.3}) \times 10^{-5}$ [16] and the value of $B(B^0 \to D^*+ \rho^+)$ quoted in Table II for the sum of positive and negative parity states. The two ratios of amplitudes in (13), corresponding to values of $|E/T|$ not discussed earlier in our study, lie precisely in the range of $|E/T|$ assumed for all our other predictions.
The relations (11) and (18) have been derived for \( PP \) and \( VV \) final states belonging to a class of the pair \((D^+ - \pi^+, D_s^- K^+)\) appearing in the first of Eqs. (3). Similar amplitude relations can be derived for \( PP, VP \) and \( VV \) final states belonging to classes of states appearing in the other two equations. For instance, the rescattering relations, 

\[
A(B^0 \to D_s^{*+} D_s^-) = A(B^0 \to D_s^{*+} D_s^-) A(D_s^{*+} D_s^+ \to D_s^{*+} D_s^-),
\]

\[
A(B_s \to D_s^{*+} D_s^-) = A(B_s \to D_s^{*+} D_s^+) A(D_s^{*+} D_s^- \to D_s^{*+} D^-),
\]

and time-reversal invariance, 

\[
A(D^{*+} D^- \to D_s^{*+} D_s^-) = A(D^{*+} D_s^- \to D^{*+} D^-),
\]

imply 

\[
\frac{A(B_s \to D_s^{*+} D_s^-)}{A(B_s \to D_s^{*+} D_s^+)} = \frac{A(B^0 \to D_s^{*+} D_s^-)}{A(B^0 \to D_s^{*+} D^-)}.
\]

Similarly, the relations 

\[
A(B^0 \to D_s^{+} K^{*-}) = A(B^0 \to D^{+} \rho^-) A(D^{+} \rho^- \to D_s^{+} K^{*-}),
\]

\[
A(B_s \to D^{+} \rho^-) = A(B_s \to D_s^{+} K^{*-}) A(D_s^{+} K^{*-} \to D^{+} \rho^-),
\]

and invariance of rescattering under time-reversal lead to 

\[
\frac{A(B_s \to D^{+} \rho^-)}{A(B_s \to D_s^{+} K^{*-})} = \frac{A(B^0 \to D_s^{+} K^{*-})}{A(B^0 \to D^{+} \rho^-)}.
\]

While experimental information exists on \( T \)-type amplitudes in the two denominators in (21) (see Table VI), the four numerators in this equation and in (23) representing \( E \)-type amplitudes have not yet been measured. We expect the magnitudes of all four \(|E/T|\) ratios to lie in the range \(0.07 \pm 0.02\).

VI Summary and conclusions

We have shown that some observed \( B \) decays which have been cited as evidence for exchange and annihilation processes can be generated by rescattering from decays whose amplitudes do not involve the spectator quark and hence are not suppressed by powers of \( f_B/m_B \). We have studied a number of processes such as \( B^0 \to K^+ K^- \), \( B_s \to \pi^+ \pi^- \), and \( B^+ \to D_s^{+} \phi \), and have identified promising states from which they can be generated by rescattering. We have found that such decays have typical amplitude ratios ranging from 5% to 20% with respect to the largest amplitude from which they can rescatter.

Using a narrower range between 5% and 10% associated with exchange amplitudes, we have predicted branching fractions, in a vast range from \( \mathcal{O}(10^{-9}) \) to \( \mathcal{O}(10^{-4}) \) for a large number of as-yet-unseen \( B \) and \( B_s \) decay processes. These include \( B^0 \) decays to \( K^+ K^- \), \( K^{*+} K^{*-} \), \( D_s^{(*)+} D_s^{(*)-} \), \( D_s^{(*)0} \bar{D}_s^{(*)0} \), \( D^{(*)+} \bar{D}^{(*)-} \) and \( B_s \) decays to \( D^{(*)+} D^{(*)-} \), \( D_s^{(*)0} \bar{D}_s^{(*)0} \), \( D_s^{*+} \), \( D_s^{*0} \), \( D_s^{(0)} \pi^0 \). Predictions of order a few times \( 10^{-7} \) have also been presented for \( \mathcal{B}(B^+ \to D_s^{+} K^-) \) and \( \mathcal{B}(B_s \to \rho^+ \pi^-), \mathcal{B}(B_s \to \rho^- \pi^+) \), providing tests for the suppression of annihilation and penguin annihilation amplitudes. Other predictions for \( \mathcal{B}(B_s \to D_s^{*+} \pi^-) \) and \( \mathcal{B}(B_s \to D^- \rho^+) \) around \((1 - 2) \cdot 10^{-6}\) have been derived in the limit of U-spin symmetry. Finally, a class of processes has been identified in which time-reversal invariance...
of strong interactions leads to further relations between ratios of exchange amplitudes and
unsuppressed amplitudes.

**Note added:** After the completion of this work we were made aware of an unpublished measurement of $\mathcal{B}(B_s \rightarrow D^+D^-)/\mathcal{B}(B^0 \rightarrow D^+D^-) = 1.00 \pm 0.18 \pm 0.09$. Using the values $\mathcal{B}(B^0 \rightarrow D^+D^-)$ and $f_s\mathcal{B}(B_s \rightarrow D_s^+D_s^-)$ in Tables I and II we calculate for $B_s \rightarrow D^+D^-$ a ratio $|E/T| = 0.11 \pm 0.02$, on the high side of the range which we have assumed.

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