N-body simulations, halo mass functions, and halo density profile in $f(T)$ gravity

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We perform N-body simulations for $f(T)$ gravity using the ME-Gadget code, in order to investigate for the first time the structure formation process in detail. Focusing on the power-law model, and considering the model-parameter to be consistent within 1σ with all other cosmological datasets (such as SNIa, BAO, CMB, CC), we show that there are clear observational differences between ΛCDM cosmology and $f(T)$ gravity, due to the modifications brought about the latter in the Hubble function evolution and the effective Newton’s constant. We extract the matter density distribution, matter power spectrum, counts-in-cells, halo mass function and excess surface density (ESD) around low density positions (LDPs) at present time. Concerning the matter power spectrum we find a difference from ΛCDM scenario, which is attributed to about 2/3 to the different expansion and to about 1/3 to the effective gravitational constant. Additionally, we find a difference in the cells, which is significantly larger than the Poisson error, which may be distinguishable with weak-lensing reconstructed mass maps. Moreover, we show that there are different massive halos with mass \( M > 10^{14} M_\odot/h \), which may be distinguishable with statistical measurements of cluster number counting, and we find that the ESD around LDPs is mildly different. In conclusion, high-lighting possible smoking guns, we show that large scale structure can indeed lead us to distinguish General Relativity and ΛCDM cosmology from $f(T)$ gravity.

I. INTRODUCTION

Since the discovery of late-time cosmic acceleration at the end of last century [1–3], a number of observations of various kinds [4–7] have confirmed its existence. The simplest paradigm to explain most of these observations is the ΛCDM one, in which the source of acceleration is the cosmological constant \( \Lambda \) within the context of general relativity (GR) and with the additional presence of the Cold Dark Matter (CDM) sector [8]. However, the possibility of a dynamical nature, as well as the need to describe also the initial (inflationary) accelerating phase [9], lead to two ways of modification of the above concordance model. The first direction is to introduce the concept of dark energy within general relativity [10, 11], while the second way is to construct modifications of gravity which can offer the new degrees of freedom required to describe dark matter [12, 13]. Note that the second direction has the advantage of being closer to a quantum description of gravity [14] as well as alleviating the possible tension [15] between local Hubble constant observations [16] and Cosmic Microwave Background (CMB) estimations [17].

There are many ways that one can follow in order to construct modified and extended theories of gravity. The most used one is to start from the Einstein-Hilbert Lagrangian, namely from the curvature formulation of gravity, and extend it in various ways, resulting to \( f(R) \) gravity [18–21], \( f(G) \) gravity [22, 23], Lovelock gravity [24], Weyl theory [25] Galileon gravity [26, 27], etc. An interesting alternative is to start from the equivalent, torsional formulation of gravity [28], and extend it in many ways, such as in \( f(T) \) gravity [29–31], in \( f(T, G) \) gravity [32], in symmetric teleparallel gravity [33], etc. The cosmological applications of \( f(T) \) gravity have been proven to be very interesting [28, 34, 35] and the theory has also been confronted to solar-system observations [36–38], gravitational waves data [39–41], as well as cosmological data from Supernovae type Ia data (SNIa), Cosmic Microwave Background (CMB), Baryonic Acoustic Oscillations (BAO), and \( f_{\sigma 8} \) observations [42–53].

Since General Relativity (GR) has passed very precise tests in the Solar System [54], any modified gravity should be very close to GR in these scales. Therefore, the screening mechanism is introduced to provide the link between deviations from GR at cosmological scales and local consistency with GR. In particular, screening mechanisms [55] are prevalent extensions of GR over the last decade, and they utilize non-linear dynamics to introduce new scalar fields that couple to gravity (these scalar fields can behave very differently between solar-system and cosmological scales).

In order to confront modified gravity theories with various kinds of observational data, we need to extract their precise predictions, from cosmic expansion to structure formation. N-body simulation has been proved to be a powerful tool to study large scale structure, and its appli-
cation to various modified theories of gravity has undergone rapid developments through the past decade [56]. Additionally, with the application of large galaxy surveys, weak lensing [57] has been proven a strong method to probe the matter distribution of the large structure, while it also provides a way to test modified gravity theories. Within this field, cosmic voids [58] have obtained much attention [59] due to the fact that they are less affected by baryonic physics and nonlinear evolution compared to dark matter halos.

In this work we are interested in performing N-body simulations in the case of \( f(T) \) gravity. For this purpose we use the ME-Gadget code [60, 61]. Furthermore, we will analyze the ESD around low density positions in \( f(T) \) gravity and we will extract the halo mass functions. The paper is organized as follows: In Section II we briefly provide the cosmological equations of \( f(T) \) gravity at the background and perturbation levels. In Section III we introduce the ME-Gadget code [60] and we describe the simulation steps we utilize. The results of our simulations are presented and discussed in Section IV. Finally, Section V is devoted to summary and conclusions.

II. \( f(T) \) GRAVITY AND COSMOLOGY

Let us briefly present \( f(T) \) gravity and apply it in a cosmological framework. We start from teleparallel gravity, in which one uses torsion instead of curvature to describe gravity, and where for convenience one uses the tetrad fields \( e^A_{\mu} \), namely the four orthonormal vectors in the tangent space, as the dynamical variables (Greek and Latin indices are respectively used for the coordinate and tangent space). Note that the metric is related to the tetrad field through \( g_{\mu\nu} = e^A_{\mu}e^B_{\nu} \), with \( g_{AB} = \text{diag}(1, -1, -1, -1) \). One introduces the Weitzenböck connection as \( \hat{\Gamma}^{\lambda}_{\mu\nu} = e^A_{\lambda}(\partial_{\mu}e^A_{\nu} - \partial_{\nu}e^A_{\mu}) \), and thus the corresponding torsion tensor is

\[
T^{\lambda}_{\mu\nu} \equiv \hat{\Gamma}^{\lambda}_{\mu\nu} - \hat{\Gamma}^{\lambda}_{\nu\mu} = e^A_{\lambda}(\partial_{\mu}e^A_{\nu} - \partial_{\nu}e^A_{\mu}),
\]

while the torsion scalar \( T \) reads as

\[
T = \frac{1}{4} T^{\rho\mu\nu}T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu}T_{\nu\rho\mu} - T_{\mu\nu}T^{\mu\nu}.
\]

Since the Ricci scalar \( R \) corresponding to the torsionless Levi-Civita connection, and \( T \), differ by only a boundary term, their corresponding use as Lagrangians leads to equivalent theories, namely to GR and the teleparallel equivalent of general relativity (TEGR), respectively.

One can now construct \( f(T) \) gravity by extending the action of TEGR, writing the action

\[
S = \int d^4x \frac{e}{16\pi G} [T + f(T) + L_m + L_r],
\]

where \( e = \det (e^A_{\mu}) = \sqrt{-g} \), and \( L_m \) and \( L_r \) represent the Lagrangians of matter and radiation sector respectively. In order to proceed to the cosmological application, we impose the flat Friedmann-Robertson-Walker (FRW) metric

\[
ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j,
\]

where \( a(t) \) is the scale factor. The corresponding tetrad is \( e^A_{\mu} = \text{diag}(1, a, a, a) \), and one can find that \( T = -6H^2 \).

Hence, the modified Friedmann equations in \( f(T) \) cosmology can be extracted as

\[
H^2 = \frac{8\pi G}{3}(\rho_m + \rho_r) - \frac{f(T)}{6} + \frac{Tf_T}{3},
\]

\[
\dot{H} = -4\pi G(\rho_m + P_m + \rho_r + P_r)\frac{1}{1 + f_T + 2Tf_T},
\]

with \( f_T \equiv \partial f/\partial T \), \( T_{TT} \equiv \partial^2 f/\partial T^2 \), and where \( \rho_m, \rho_r \) and \( P_m, P_r \) represent the energy densities and pressures for matter and radiation respectively. Therefore, the extra terms of gravitational origin can be considered as an effective dark-energy component.

We can express the background cosmological equations in a more convenient way, by introducing the function \( E(z) \) as

\[
E^2(z) \equiv \frac{H^2(z)}{H_0^2} = \frac{T(z)}{T_0},
\]

where the subscript “0” denotes the value of a quantity at present time. Then, the first Friedmann equation becomes

\[
E^2(z, r) = \Omega_{m0}(1 + z)^3 + \Omega_{r0}(1 + z)^4 + \Omega_{F0}y(z, r),
\]

with

\[
y(z, r) = \frac{1}{T_0\Omega_{F0}} [f - 2Tf_T],
\]

and where \( \Omega_{F0} \equiv 1 - \Omega_{m0} - \Omega_{r0} \).

Let us now examine the linear matter perturbation level. In particular, the linear growth rate under the \( f(T) \) gravity, is determined by the equation [44, 51, 52]

\[
d'' + \delta' \left( \frac{H'}{H} + \frac{3}{a} \right) - \frac{G_{\text{eff}}}{G} \delta \left( \frac{3\Omega_0 H_0^2}{2H^2 a^3} \right) = 0,
\]

where \( \delta \equiv \delta_m/\rho_m \) is the matter overdensity and with \( \delta' \equiv d\delta/da \) and \( \delta'' \equiv d^2\delta/da^2 \). We mention that in the above expression the effect of \( f(T) \) gravity is quantified into the effective Newtonian constant, which according to the Poisson equations is found to be [44, 51, 62, 63]:

\[
G_{\text{eff}} = \frac{G}{1 + f_T}.
\]

In this work we focus on the widely-used power-law \( f(T) \) model [64], namely:

\[
f(T) = T + \alpha(-T)^k,
\]
with $b$ the free model parameter, and where the parameter $\alpha$ is related to $\Omega_{F0}$ through
\[
\alpha = \left(6H_0^2\right)^{1-b}\Omega_{F0}\frac{2b}{2b-1}.
\] (13)
Thus, the effective Newton’s constant in this specific model becomes:
\[
G_{\text{eff}}(z) = \frac{G_N}{1 + \frac{\Omega_{F0}}{(1-2b)H_0^2(1+z)}}.
\] (14)
Finally, we mention that this model recovers $\Lambda$CDM scenario for $b = 0$, i.e. $T + f(T) = T - 2\Lambda$, with $\Lambda = 3\Omega_{F0}H_0^2$ and $\Omega_{F0} = \Omega_{A0}$. Moreover, cosmological data form various origins lead to the constraint $b \in [-0.29, 0.26]$ and $\Omega_{m0} \in [0.24, 0.308]$ [42–51].

### III. N-BODY SIMULATIONS

In this section we present the method of N-body simulations, in order to apply it to the case of the power-law $f(T)$ gravity. Screening mechanism is a widely adopted mechanism for modified gravity theories [65] in order to be consistent with local GR tests. There are several different N-body simulation codes implementing this mechanism for modified gravity [66–68]. In the following we will use the ME-Gadget code [60, 61] in order to test the effect of screening phenomenologically, using a periodic box of size 512 Mpc/h and 512$^3$ particles.

The ME-Gadget code is a modified version of the public available N-body simulation code Gadget2 [69]. The original modification algorithm was proposed in Baldi et al. [70] for coupled dark energy models. ME-Gadget was designed to use tabulated text files to include the change of $H(z)$ and $G_{\text{eff}}(z,k)$, which makes it perfect to study the structure formation of $f(T)$ gravity model. The pre-initial condition is prepared with CCVT algorithm [71] and the initial condition is calculated with second-order Lagrangian Perturbation Theory using a modified version of 2LPTic code [72]. Finally, the halos are identified using Amiga Halo Finder [73].

We impose a free parameter $k_{\text{screen}}$, and therefore for scales $k < k_{\text{screen}}$ we adopt the $G_{\text{eff}}(z)$ arising from $f(T)$ gravity, namely expression (11), while for scales $k > k_{\text{screen}}$ we adopt the usual Newtonian gravitational constant. We choose three $k_{\text{screen}}$ values for the simulations, namely 0.025, 0.05 and 0.1. Concerning the $f(T)$ gravity model parameter $b$, without loss of generality we consider the extreme values according to cosmological constraints, i.e. $b = 0.20579 \equiv b_1$ and $b = -0.2371 \equiv b_2$, and for completeness we consider also the value $b = 0 \equiv b_0$ which corresponds to $\Lambda$CDM cosmology. Finally we set $\Omega_{m0} = 0.2917, h = 0.7324$ [17].

| $b_0$ | $k_{\text{screen}}$ |
|-------|-----------------|
| 0.0   | 0.025           |
| 0.20579 | 0.05           |
| 0.2371 | 0.1            |

TABLE I: Model abbreviations corresponding to different simulations.

In summary, in our simulations we have two free parameters, $b$ and $k_{\text{screen}}$, and the corresponding models are presented in Table I.
the deviation is not more than 3%, and the largest difference is realized at scale factors around \( a \sim 0.5 \) or around redshift \( z = 1 \). On the other hand, both at high redshift and redshift close to 0, the difference is negligible. Additionally, in Fig. 2 we present the evolution of the effective gravitational constant \( G_{\text{eff}} \) as a function of the scale factor. As we observe, \( G_{\text{eff}} \) starts to deviate from \( G \) more than 5% around \( a = 0.5 \), too. Furthermore, the difference is increasing as \( a \) approaches the present value \( a = 1 \). Since structure formation process is cumulative over time, we deduce that the most significant difference of all observables is expected to be measured at \( a = 1 \), i.e at redshift \( z = 0 \).

In summary, we perform N-body simulations and our purpose is to identify which observable may be possible to quantify the effect of \( f(T) \) gravity.

IV. RESULTS

In this section we present the obtained results of N-body simulations in detail. In particular, we will examine the matter density distribution and the matter power spectrum, the redshift-space distortion effect, the
halo mass function, the halo density profile, and the low-density position (LDP) lensing.

A. Matter density distribution

For a straightforward comparison of the simulation, we use the Pylians python library \(^3\) to calculate the dark-matter over-density \(\delta \equiv \delta \rho_m/\rho_m\). We choose a 20 Mpc/h thick slice along the z-axis at \(a = 1\) for illustration, and in Fig. 3 we show the logarithmic dark-matter over-density distribution, focusing on the results for \(k_{\text{screen}} = 0.05\). Since we use the same initial condition for each model, their structures are almost the same.

The first column depicts the ΛCDM scenario, the second column the b1 model, and the third column shows the b2 model. In the first row, we present the density distribution over the whole simulation box. Moreover, in the second row we zoom-in for one over-dense region, while in the third row we show the neighboring low-dense region. Comparing to the ΛCDM scenario, we see that there have been faster (slower) cosmic expansion and smaller (larger) gravitational constant in b1(b2) model. Therefore, it is expected that the structure growth in b1(b2) case should be less (more) strong than that in the ΛCDM paradigm. Qualitatively, the difference can be seen from the density distribution, which motivates us to observe the quantitative difference in more detail.

B. Matter power spectrum

To quantitatively compare the difference between b0, b1 and b2 models, we calculate their matter power spectrum at \(a = 1\), and we present the results in Fig. 4. The power spectrum is the correlation within a certain density field in k-space. We can clearly notice two phenomena in this figure. Firstly, the matter power spectrum in b1(b2) is smaller (larger) than that of ΛCDM scenario, and secondly there is a sharp jump of the matter power spectrum ratio at \(k = 0.025, 0.5, 0.1 \text{hMpc}^{-1}\), which is exactly related to the screening scale we have set for the simulations. The difference of matter power spectrum at all scales is caused by the different expansion history and the sharp jump, provided the understanding of the effects of the effective gravitational constant. By looking at the ratio of matter power spectrum at scales smaller than the screening scale \((k < k_{\text{screen}})\), we can understand that quantitatively the change of \(H(z)\) results to a \(~5\%\) power-spectrum increase for b2 model and to a \(~10\%\) power-spectrum decrease for b1 model. By looking at the ratio of matter power spectrum at scales larger than the screening scale \((k > k_{\text{screen}})\), the change of \(G_{\text{eff}}(z)\) results to an additional \(~2.5\%) power spectrum increase for b2 and \(~5\%) power spectrum decrease for b1 model. The percent-level difference is actually distinguishable, nevertheless the problem is that the value of the power spectrum is also determined by \(\sigma_8\), which is degenerate with the value of \(\delta\).

Finally, in Fig. 5 we present the comparison between the simulated nonlinear power spectrum and the theoretical linear power spectrum, under the \(H(z)\) and \(G_{\text{eff}}(z)\) evolution depicted in figures 1 and 2 of Section II. This is in agreement with our prediction that the growth of the density field at large scales is consistent between theory and the simulation results. The error bar in the figure is the root-mean-square of power spectrums, derived from several smaller boxes located at different places of the corresponding simulation boxes.

C. Redshift-space distortion effect

As it is known, in observations the distribution of galaxies is different from the real space, due to the peculiar velocity of each galaxy, which is the so-called redshift-space distortion (RSD) effect. Since we cannot calculate the percentage of the measured velocity that arises from the Hubble flow or from the peculiar velocity, the measured distance becomes inaccurate at an amount of \(\Delta D\), since

\[
D = \frac{v}{H_0} = \frac{v_{\text{Hubble}} + V_{\text{pec}}}{H_0} = D_{\text{real}} + \Delta D. \quad (15)
\]

\(^3\) https://github.com/franciscovillaescusa/Pylians
In Fig. 6 we depict the 3D power spectrum. As we observe, the power spectrum is stronger in redshift space than in real space at large scales, but it is weaker at small scales. Furthermore, at small scales the scatter of the velocity of galaxies is quite large, which adds on the cosmological redshift along the line of sight and causes a wider distribution of the redshift, and this elongates the distribution of galaxies along the line of sight and causes a weaker correlation between galaxies, which is the so-called “Finger of God” effect. Since at large scales the outside galaxies are still within the gravitational field and fall towards the center, then for the side near the observer the redshift will be larger. However, for the other side it will be smaller, and then the distribution of these galaxies will seem to be pressed and the correlation will be stronger between them.

In the case of b1 model, since the growth of the density field is weaker than b0, the scatter of the dark-matter particles is smaller, and as it shown in the lower panel of Fig. 6, the ratio of power spectrum between simulations with and without RSD effect is weaker for b1 than b0, while it is stronger for b2 than b0. According to Fig. 5 and Fig. 6, we could tell that with RSD effect considered, the deviations between b0, b1 and b2 models are still not so different from those in the real-space power spectrum.

Finally, in Fig. 7 we also plot the power spectrum in 2D to directly show the distribution in Fourier space. As we observe, the “Finger of God” effect is clearly shown in the lower panel of the figure, and the differences between them are small.

### D. Counts-in-cells

Apart from the correlation of densities of each model, we can additionally compare the simulation particle number counts-in-cells among different models. The number of cells counted by over-density is shown in Fig. 8. Firstly, we calculate the over-density in $512^3$ cells for each model, and then we take the ratio between $b1(b2)$ and $b0$ models. The error bars represent the Poisson error, which

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4 https://github.com/franciscovillaescusa/Pylians
is estimated as $1/\sqrt{N}$. As expected, with larger structure growth in b2 model comparing to b0 (i.e. ΛCDM) model, there are less cells with $1 + \delta < 10$ and more cells with $1 + \delta > 10$. On the contrary, since the structure growth in b1 model is suppressed, there are more cells with $1 + \delta < 10$ and less cells with $1 + \delta > 10$. This difference indicates that with mass map reconstructed from weak-lensing convergence map, it is possible to distinguish $f(T)$ gravity from ΛCDM scenario. We also expect that the difference can be identified in the halo mass function. And this is what we will describe in the following.

E. Halo mass function

The dark-matter halo is the basic feature of large scale structure. Galaxies formed inside dark matter halos and sub halos. The statistics of dark matter halos has been used for a long time to investigate cosmological scenarios. The halo mass function is the measurement of the abundance of halos in different mass ranges.

In Fig. 9 we present the ratio of number of halos in different mass ranges, resulted from our simulations. As we observe, there are clearly more massive halos in b2 model than in b0 one (i.e. ΛCDM scenario), and less massive halos in b1 model than b0 one, which is consistent with the other observables we have discussed above. The number difference is very clear for halos with mass larger than $10^{14} M_{\odot}/h$, since clusters lie in such massive dark matter halos. The difference of halo mass function can lead to different predictions for cluster number counting, which can be compared to observations. The distinguishable difference of halo mass function indicates that cluster number counting can also be used to constrain $f(T)$ gravity. This is one of the main results of the present work.

After calculating the linear growth rate, it is straightforward to calculate the halo mass function using EPS (Extended Press-Schechter) theory:

$$n(M) = \sqrt{\frac{2}{\pi}} \frac{\bar{\rho}}{\sigma(M)} \frac{d \ln \sigma(M)}{d \ln M} \exp \left[ -\frac{\delta_c^2}{2\sigma^2(M)} \right],$$

where $\delta_c$ is the critical matter contrast of around 1.69. The result of the simulation is shown with dashed lines in Fig. 9. As we can see, the halo mass function difference measured from simulations is not as much as that using EPS theory. It is possibly caused by the screening effect, in this figure, and the difference is given under $k_{\text{screen}}=0.05$, implying that the effect of $G_{\text{eff}}$ has been screened for these halos. On the other hand, the EPS theory using linear power-spectrum calculation, it does take $G_{\text{eff}}$ into account but not the screening, and hence the difference shown in EPS theory can be seen as an upper limit. This feature also explains why at high-mass end we can see larger differences, while at low-mass end there is almost no difference.

F. Halo density profile

In this subsection, we analyse the density profile of dark-matter halos at present time $z=0$. In particular, we measure the density profile for three different mass bins, namely $10^{13} - 10^{13.1} M_{\odot}/h$, $10^{13.5} - 10^{13.55} M_{\odot}/h$, and $10^{14} - 10^{14.25} M_{\odot}/h$. We pick up the sub-halos for each mass bin, and then we count the particle numbers in each radial bin ranges from $0.01 r_{200}$ to $r_{200}$ kpc/h logarithmically ($r_{200}$ is the virial radius within which the mean density is 200 times the cosmic matter density $\rho_m(z)$ at each redshift $z$).

In our analysis we adopt the Navarro-Frenk-White (NFW) functional form [75], which has been an excellent description for the ΛCDM halo profile over a wide
mass range, given by
\[ \rho(r) = \frac{\rho_s}{r_s(1 + \frac{r}{r_s})^2}, \tag{17} \]
where \(\rho(r)\) is the spherically averaged density at radius \(r\). There are two parameters in this model: \(r_s\) is the scale radius and \(\rho_s\) is the characteristic density. In Fig. 10 we present the obtained results. As we can see, the halo profile of b1 and b2 models of \(f(T)\) gravity, as well as of b0 ΛCDM cosmology, is similar in all three mass bins. Hence, we deduce that the impact of \(f(T)\) gravity is negligible within the halo scale.

G. Low density position (LDP) lensing

Gravitational lensing is a well-studied field in cosmology that can lead to the reconstruction of the matter along the line of sight. Since voids are less affected by the nonlinear structure formation and other physical processes, differences between dark energy and modified gravity models for cosmic acceleration might be more sensitive in cosmic voids [76]. On the other hand, due to the screening mechanism, the difference between modified gravity and ΛCDM scenario is expected to be suppressed in small-scale high-density regions, such as halos. These features make voids a suitable place to test modified gravity.

The excess surface density (ESD) is related to the average background tangential shear signals through
\[ \Delta \Sigma(R) = \Sigma(< R) - \Sigma(R) = \gamma_t \Sigma_{\text{crit}}(z_l, z_s), \tag{18} \]
where
\[ \Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_s}{D_l D_t}, \tag{19} \]
and with \(D_s, D_l\) and \(D_t\) the angular diameter distance of the source, lens and the distance between them, respectively. In the simulations we can simply measure the ESD around voids.

We use the LDP defined in [77] to find the voids. In the simulation box at present time \(a = 1\), we exclude all regions of 500kpc/h around the most massive 7000 halos. For the left regions, we choose the grid points spaced 200kpc/h each as the LDP positions. By stacking the simulation particles in cylinders with 10Mpc/h thickness around these LDPs, we measure the ESD signal. The result is shown in Fig. 11. Again, this is consistent with the measurements discussed in the previous subsections, and the voids are deeper in b2 model while they are more shallow in b1 model. The difference is mildly distinguishable at \(R > 3\text{Mpc}/\text{h}\).

However, we have to mention here that the difference of matter distribution is not the only contribution to the weak lensing signal. Since the photons are not affected by the gravitational field in the same way as massive particles in \(f(T)\) gravity, there will be some additional modification of the predictions. In the analysis of Chen et al. [78] it is shown that such additional modification plays an important role in galaxy-galaxy lensing. Thus, it would also be essential for us to include it in the LDP lensing. We would need to perform a more detailed analysis in order to acquire it comparing to observations. Nevertheless, our simulation results here provided a self-consistent prediction of matter distribution, which forms the missing brick.

V. SUMMARY

In this work, for the first time, we performed N-body simulations for \(f(T)\) gravity using ME-Gadget code, in order to investigate in detail the structure formation process. We focused on the most widely used power-law model, and concerning the model-parameter \(b\) we considered the extreme values arising from cosmological observations, as well as the value \(b = 0\) which reproduces ΛCDM scenario.

Our analysis shows that there are clear observational differences between ΛCDM paradigm and \(f(T)\) gravity in the phenomenology of structure formation. In partic-
ular, the effect of $f(T)$ gravity is twofold: Firstly, due to the modifications in the Friedmann equations, the Hubble function (shown in Fig. 1) deviates from the one of ΛCDM cosmology, mainly around $z = 1$, even for model-parameter values that are consistent within $1\sigma$ with all other cosmological datasets (SNIa, BAO, CMB, CC etc). Secondly, since in $f(T)$ gravity the effective Newton’s constant (shown in Fig. 2) deviates from the standard one, this will alter the Jeans equation and thus the growth of structure comparing to ΛCDM scenario. These two effects determine the deviations of the N-body simulations of $f(T)$ gravity from ΛCDM paradigm.

As we saw, changing the model-parameter $b$, we obtain a faster expansion with smaller $G_{\text{eff}}$ for $b = b_1 = 0.20579$, and a slower expansion with larger $G_{\text{eff}}$ for $b = b_2 = -0.2371$. When $b = b_0 = 0$ the model reproduces GR and ΛCDM cosmology. In comparison to ΛCDM model, the structure growth in $b_1$ model is slower, while the structure growth in $b_2$ model is faster. In particular, we have extracted the matter density distribution, matter power spectrum, counts-in-cells, halo mass function and excess surface density (ESD) around LDPs at present time $a = 1$. Clearly, the simulation results are consistent with our expectations. The comparison results are summarized as follows:

- The matter power spectrum in $b_1(b_2)$ model is smaller(larger) than ΛCDM scenario, specifically the differences are around $5\% \sim 7.5\%$ for $b_1$ and $10\% \sim 15\%$ for $b_2$. In particular, we found that the contribution from the different expansion history is responsible for about $2/3$ of the matter power spectrum difference, while the effective gravitational constant is responsible for the remaining $1/3$. Up to our knowledge, this is the first time where we acquire quantitative information on the strength of the two effects of the $f(T)$ modification in the structure formation.

- There are more (less) cells with $1 + \delta < 10$ in $b_1(b_2)$ model, and less (more) cells with $1 + \delta > 10$ in $b_1(b_2)$ model. The difference is way larger than the Poisson error, which may be distinguishable with weak lensing reconstructed mass maps.

- There are less (more) massive halos with mass $M > 10^{14} M_\odot/h$ in $b_1(b_2)$ model (the difference in halo mass function is around $\sim 40\%$ at high mass end), and the difference is larger than the Poisson error, which may be distinguishable with statistical measurement of cluster number counting.

- The ESD around LDPs is mildly different, nevertheless a further analysis of light-bending contribution is necessary in order to obtain it compared to weak lensing observations.

In conclusion, large scale structure can indeed lead us to distinguish $f(T)$ gravity from General Relativity and ΛCDM cosmology, even for ranges of the mode parameter that are consistent within $1\sigma$ with cosmological observations. The detailed simulations of this work have set the framework for a quantitative discrimination and of testing General Relativity, and have high-lighted some possible smoking guns of torsional modified gravity.

One could try to use the whole analysis in order to extract constraints on the parameters of various $f(T)$ models, which are expected to be significantly stronger than those arising from the usual cosmological datasets. Additionally, it would be both necessary and interesting to orient the investigation towards the possible alleviation of the $H_0$ and $\sigma_8$ tensions. These detailed studies lie beyond the scope of the present work and are left for future projects.
Acknowledgments

This work was inspired by the discussions during the second HOUYI Workshop for Non-standard Cosmological Models in Kunming, China, 2019. We are grateful to Sheng-Feng Yan for helpful communications. The research grants from the China Manned Space Project with no. CMS-CSST-2021-A03. Yi-Fu Cai is supported by the NSFRC (11961131007, 11653002). The computation resources of this work are provided by the Gravity Supercomputer at the Department of Astronomy, Shanghai Jiao Tong University.

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