Energy harvesting and vibration reduction analysis on cantilever piezoelectric double vibration absorber (CPDVA) mechanism

W Hendrowati, H L Guntur and A A A Daman
Dynamic System and Vibration Laboratory, Department of Mechanical Engineering, Institut Teknologi Sepuluh Nopember (ITS), Surabaya, Indonesia
wiwiek@me.its.ac.id

Abstract. The CPDVA (Cantilever Piezoelectric Double Vibration Absorber) is a mechanism that combines the working principles of DVA and energy harvesting to reduce vibration while generating electrical energy. The working principle of CPDVA uses two masses as an absorber of the vibration system ie main absorber mass and piezoelectric mass. The CPDVA mechanism is designed to get the vibration reduction value at the first natural frequency (36.544rad/s) of system. The simulation results show the optimum value can be achieved if the design of CPDVA mechanism using piezoelectric as much as 4000 pieces. Because, by using 4000 piezoelectric on the CPDVA mechanism, the highest reduction value and electrical power can be achieved.

1. Introduction
Dynamic vibration absorber (DVA) is widely used passive vibration control device. When a mass-spring system, referred to as primary system, is subjected to a harmonic excitation at a constant frequency, its steady state response can be suppressed by attaching a secondary mass-spring system or DVA. However, a DVA consisting only of mass and spring has a narrow operation region and its performance deteriorates significantly when exciting frequency varies. The key design parameters of a damped DVA are its tuning parameter and damping ratio [1]. Pachpute [2], Kefu [3] and Daman [4] have found the response of the system without DVA and with damped DVA. From the experimental results, the system that produced the maximum reduction was Dynamic Vibration Absorber (DVA) with the optimum ratio of mass ratio (μ) = 0.2, the damping ratio (ζ) = 0.125 and the tuning ratio (f) = 0.7 to 0.8.

Research on energy harvesting and simultaneous vibration control device are less reported. The device is used to transfer the primary structural vibrations to the secondary structure, which is DVA. A properly designed DVA absorb all the vibration energy that was input to the primary structure and itself undergoes large vibrations. The two main vibration based energy harvesting technologies are electromagnetic and piezoelectric. The electromagnetic harvester generates power from changes in magnetic field induced due to motion of the host structure [5, 6]. Piezoelectric energy harvester generates power from strain in piezoelectric materials attached to the host, which vibrates in response to external excitations [7, 8, 9]. Patel's research [10] investigated the use of cantilever piezoelectric to determine the effect of piezoelectric length and mass on natural frequency and its deflection, as the deflection affects the generated electrical energy. Liao [11] investigated the effects of placement on
damping of various vibration modes, and the effects of patch size on optimal placement. For small piezoelectric patches, the optimal location is very close to the location of the overall maximum bending moment.

G.A. Hassaan [12] conducted a DVA system research to reduce vibration in the main system and serve as a harvester of electrical energy. It is found that the optimum natural frequency of absorber-harvester with mass ratio between 0.05 to 0.45 and damping ratio between 0.1 to 0.4. W. Similarly, Mineto’s research [13], de Silva [14] which states that the greatest electrical energy occurs at a natural frequency.

Rahma Efendy [15] examined the application of the mechanism of CPVA (Cantilever Piezoelectric Vibration Absorber) using a simulation method. The results of the simulation show that the generated power and the highest CPVA reduction percentage is 3.52E-7 watt and 20.36% at natural frequency. Besides Wiwiek Hendrowati [16] also examined piezoelectric applications in the form of Multilayer Piezoelectric to harvest the kinetic energy of shock absorber. By pairing the Multilayer Piezoelectric Vibration Energy Harvesting (ML PZT VEH) mechanism to shock absorber, the shock absorber performance is undisturbed and the wasted energy can be utilized into electrical energy, i.e. 6.23 volts and 1.6 m Watt.

This paper discusses the analysis of vibration reduction and the electrical energy generated by CPDVA mechanism. The CPDVA (Cantilever Piezoelectric Double Vibration Absorber) is a mechanism that combines the working principles of DVA and energy harvesting to reduce vibration while generating electrical energy. The design of this mechanism consists of mass, spring and piezoelectric. It is said that the double vibration absorber is because the working principle of DVA uses two masses as an absorber of the vibration system.

2. Methodology
In this paper, a vibration simulator (a system without CPDVA) is modeled with the main mass $M_1$, springs $K_1$ and dampers $C_1$, which is driven by a motor rotating at certain frequency $\omega$ and amplitude $X_0$, as shown in figure 1. Then the input in the form of displacement is forwarded to the main mass $M_1$ by spring exciter $K_0$, so that the main mass obtains a harmonic force and will oscillate with amplitude $X_1$.

![Figure 1. Dynamic model of vibration system without CPDVA](image)

From the free body diagram of the system without CPDVA, the mathematical equations are obtained based on Newton's law as follows:

$$\sum F = m_1 \ddot{x}_1$$

$$m_1 \ddot{x}_1 + Fc_1 + Fk_1 - Fc_0 - Fk_0 = 0$$

$$m_1 \ddot{x}_1 + (c_1 + c_0)x_1 + (k_1 + k_0)x_1 = c_0 \dot{x}_0 + c_0x_0$$

(1)

In figure 2, CPDVA is added to the main system, consisting of main absorber (absorber mass $M_3$, spring $K_3$ and damper $C_3$) and piezoelectric cantilever: piezoelectric mass $M_{2eq}$, spring $K_{2eq}$ and
damper \( C_{2eq} \). This CPDVA serves as a vibration damper and an electric generator. Where, Piezoelectric is mounted on the absorber frame and the main absorber.

When the main mass oscillates with an amplitude of \( X_1 \), the absorber frame and the main absorber move with an amplitude of \( X_2 \). The relative movement between the absorber frame and the main absorber will maintain a maximum piezoelectric deflection of 6mm. While the excessive kinetic energy will be absorbed by the main absorber that moves with an amplitude of \( X_3 \). The dynamic model of systems with CPDVA can be seen in figure 2.

\[ \Sigma F = m_1 \ddot{x}_1 \]
\[ m_1 \ddot{x}_1 + Fc_1 + Fk_1 + Fc_{2eq} + Fk_{2eq} + Fe = Fc_0 + Fk_0 \]
\[ m_1 \ddot{x}_1 + (c_1 + c_0 + c_{2eq}) \dot{x}_1 + (k_1 + k_0 + k_{2eq}) x_1 - c_{2eq} \dot{x}_2 - k_{2eq} x_2 + Fe = c_0 \dot{x}_0 + k_0 x_0 \]

where

\( \dot{x}_1 = \text{amount of piezoelectric cantilever} \)

The equation of translational motion of the piezoelectric mass and absorber mass are seen in equation

\[ \Sigma F = m_{2eq} \ddot{x}_2 \]
\[ m_{2eq} \ddot{x}_2 + Fc_3 + Fk_3 - Fc_{2eq} - Fk_{2eq} - Fe = 0 \]
\[ m_{2eq} \ddot{x}_2 + (c_3 + c_{2eq}) \dot{x}_2 + (k_3 + k_{2eq}) x_2 - c_{3} x_3 - c_{2eq} \dot{x}_1 - k_{2eq} x_1 - Fe = 0 \]

\[ \Sigma F = m_3 \ddot{x}_3 \]
\[ m_3 \ddot{x}_3 - Fc_3 \dot{x}_3 - Fk_3 = 0 \]
\[ m_3 \ddot{x}_3 + c_3 \dot{x}_3 + k_3 x_3 - c_3 x_2 - k_3 x_2 = 0 \]
Natural Frequency Analysis

To obtain natural frequency of system, a fundamental analysis is done. In the analysis, the damping constant and the external force are assumed to be zero (extremely small). The value is changed by substituting \( \ddot{x} = -\lambda x \) and \( \lambda = \omega^2 \), so the above equation becomes:

\[
\begin{bmatrix}
-m_1\omega^2 + (k_0 + k_1 + k_{2eq} + n\Gamma k_v) & -(k_{2eq} + n\Gamma k_v) & 0 \\
-(k_{2eq} + n\Gamma k_v) & -m_{2eq}\omega^2 + (k_{2eq} + k_3 + n\Gamma k_v) & -k_3 \\
0 & -k_3 & -m_3\omega^2 + k_3
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]

(5)

The deflection that occurs in piezoelectric will produce electrical voltage which is proportional to the deflection, according to equation 7.

\[
V_p = \frac{3d_{31}E_p w_p t_p}{4C_p} x_p = k_3 x_p
\]

(6)

\[
\Gamma = \sqrt{\frac{k_{31}k_p C_p}{}}
\]

(7)

Table 1. Parameters of Cantilever Piezoelectric

| Parameter                          | Symbol | Value        | unit  |
|-----------------------------------|--------|--------------|-------|
| Piezoelectric mass                | \( m_p \) | 6 \times 10^{-4} | kg    |
| Piezoelectric Spring Coefficient  | \( k_p \) | 0.575 | N/m   |
| Piezoelectric capacitance         | \( C_p \) | 244 \times 10^{-10} | F     |
| Piezoelectric thickness           | \( t_p \) | 1 \times 10^{-4} | m     |
| Piezoelectric Width               | \( w_p \) | 6 \times 10^{-3} | m     |
| Piezoelectric length              | \( l_p \) | 12 \times 10^{-3} | m     |
| Piezoelectric strain constant     | \( d_{31} \) | 1.1 \times 10^{-10} | C/N   |
| Electromechanical Coupling Factor | \( k_{31} \) | 12 | %     |
| Modulus Young                     | \( E \) | 3 \times 10^9 | N/m²  |

3. Analysis And Discussion

A vibration system that is given an excitation force with a certain amplitude and frequency will vibrate. Systems without CPDVA has a natural frequency of 36.544rad/s. And after adding CPDVA to the vibration system, the system has 3 natural frequencies of 31,056rad/s, 46,735rad/s, and 92.12rad/s. The additional CPDVA to the vibration system aims to reduce vibration at the first natural frequency of the system. Figure 3 shows the main mass bode diagram without CPDVA and with CPDVA.

Figure 4 presents, the comparison of the displacement respon of each operating frequency of the system without and with CPDVA. On system without CPDVA, at a frequency of 36,544 rad /s, the main mass has the highest amplitude compared to the amplitude at other frequencies, i.e 31,056rad/s and 46,735rad/s, because the first natural frequency of the system is 36,544 rad / s. While on a system with CPDVA, at this frequency, the main mass has the lowest amplitude compared to the amplitude at other frequencies. This shows that the additional CPDVA can reduce the main system vibration at its first natural frequency.
Figure 3. Bode diagram of the system without and with CPDVA

Figure 4. Comparison of the displacement response from the main mass

Figure 5. The displacements response of the main mass, with and without CPDVA at various operating frequencies. The biggest reduction occurs in the first natural frequency of the system, i.e. at 36.44 rad/s.

The percentage of reduction in displacement response from the main mass can be calculated as follows

\[
\%red = \frac{RMS\ without\ CPDVA - RMS\ with\ CPDVA}{RMS\ without\ CPDVA}
\]  

Figure 6. The relation of the frequency variation to the reduction is simulated and the result is presented in figure 6.
From the simulation results, it can be seen that CPDVA is able to reduce the main system vibration with the percentage of reduction value above 0%. The percentage of reduction value below 0% indicates that, the additional CPDVA increases the vibration amplitude of the main mass. Thus, the main mass vibration amplitude that can be reduced is at the frequency interval between 32.91 rad/s to 44.59 rad/s. While the operating frequency is avoided around the frequency of 31,056 rad/s and frequency of 46,735 rad/s because it is in the lowest valley of the graph.

The mechanism of CPDVA which reduces vibration from the main mass will use its energy to drive piezoelectric in the CPDVA mechanism. The response of Piezoelectric is the deflection of the piezoelectric which will produce electrical energy, as in Figure 7.

Figure 8 shows the electrical power that can be produced by the CPDVA mechanism with variations in operating frequency and simulated with number of piezolectric 500 pieces and amplitude of 0.02 m.
Figure 8. The generated Electrical power from piezoelectric against operating frequency

Figure 9 shows that increasing in the value of reduction will decrease in the generated power. So, the system with CPDVA does not have the optimum frequency to get the largest reduction and power generation value. However, from this graph, it can be concluded that the biggest reduction value is occur at operating frequency is 36,544 rad/s.

Figure 9. Percentage of reduction and power generation against the operating frequency

The increase in the amplitude of the excitation force to the system only cause an increase in response of the main mass without CPDVA or with CPDVA. Likewise, increasing the excitation force amplitude will not shift the natural frequency value without or with CPDVA. So that the percentage value of the resulting reduction is also relatively the same. This can be seen in figures 10 and 11

Figure 10. Bode diagram of the main mass with amplitude variations without CPDVA
However, increasing the value of amplitude will increase the deflection of piezoelectric and as a result, the generated power will also increase at each working frequency, as shown in figure 12.

The amount of piezoelectric in the CPDVA mechanism will affect the natural frequency. Increasing the amount of piezoelectric will increase the piezoelectric stiffness so that the natural frequency will be achieved faster. This will also affect the percentage of the reduction value. In addition, if more piezoelectric is used, it will increase the generated voltage by the piezoelectric, as shown in Figures 13 and 14.
Figure 14. Graph of piezoelectric voltage responses to variations in piezoelectric quantities

From Figure 15, it is shown in the graph that the optimum value is between the reduction value and the electrical power generated by the CPDVA mechanism. The optimum value of the CPDVA mechanism is obtained when using piezoelectric as much as 4000 pieces. The simulation results show that by using 4000 piezoelectric, the highest reduction value and the electrical power produced is also greater.

Figure 15. The optimum value of reduction and generated power of the CPDVA mechanism

4. Conclusion
In this investigation, the design of CPDVA mechanism was presented. The CPDVA mechanism has the biggest reduction value at operating frequency is 36,544 rad/s, but increasing the excitation force amplitude does not change the natural frequency value without or with CPDVA. The optimum value of the CPDVA mechanism design is obtained when using piezoelectric as much as 4000 pieces. Because, by using 4000 piezoelectric on the CPDVA mechanism, the highest reduction value and electrical power can be achieved.

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