Transitory Interporosity Flow in Shale/Tight Oil Reservoirs: Model and Application

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ABSTRACT: The dual-porosity model has been used widely to describe the fracture network in well test or numerical simulation due to the high computational efficiency. The shape factor, which can be used to determine the capability of mass transfer between the matrix and fracture, is the core of the dual-porosity model. However, the conventional shape factor, which is usually obtained under pseudo-steady state assumption, has certain limitation in characterization of the mass transfer efficiency in a shale/tight reservoir. In this study, a new transient interporosity flow model has been established by considering the influence of nonlinear flow, stress sensitivity, and fracture pressure depletion. To solve this new model, a finite difference and Newton iteration method was applied. According to the Duhamel principle, the solution for time-dependent fracture pressure boundary condition has been obtained. The solution has been verified by using the fine-grid finite element method. Then, the influence of nonlinear flow, stress sensitivity, and fracture pressure depletion on shape factor and interporosity flow rate has been studied. The study results show that constant shape factors are not suitable for unconventional reservoirs, and the interporosity flow in the shale/tight reservoir is controlled by multiple factors. The new model can be used in test interpretation and numerical simulation, and also provides a new approach for the optimization of the perforation cluster number.

1. INTRODUCTION

Shale/tight reservoirs are characterized by low permeability and low porosity, and multicluster fracturing with a horizontal well is effective for the development of such reservoirs. Stimulated reservoir volume (SRV) is formed after fracturing. There are many methods to characterize the SRV zone, such as the unstructured perpendicular bisection (PEBI) grid, discrete fracture networks (DFN), and embedded-discrete-fracture model (EDFM). However, in well test models or numerical simulation models, the dual-porosity model is still widely used to compromise between accuracy and computational efficiency. To overcome the high in situ stress and horizontal stress difference and to improve the complexity of the fracture, multicluster fracture with tight cutting has been used in recent years. The flow between the perforation tunnel and matrix can also be characterized by the dual-porosity model, which can provide the possibility to optimize the number of perforation clusters.

The concept of the dual-porosity model was first proposed by Barenblatt et al. They divided the fractured reservoirs into two flow systems, namely, the matrix system and the natural fracture system. The following transfer function connects these two flow systems:

Table 1. Constant Shape Factor by Various Researchers

| investigator(s) | time | method | 1D | 2D | 3D |
|----------------|------|--------|----|----|----|
| Warren and Root | 1963 | geometrical | 12 | 32 | 60 |
| Kazemi et al. | 1976 | numerical | 4 | 8 | 12 |
| Thomas et al. | 1983 | numerical | | | 25 |
| Ueda et al. | 1989 | numerical | 8 | 24 |
| Coats | 1989 | numerical | 8 | 16 | 24 |
| de Swaan | 1990 | analytical | 15 | 60 |
| Kazemi and Gilman | 1993 | analytical | 9.87 | 19.74 | 29.61 |
| Zimmerman et al. | 1993 | analytical | 9.87 | 18.17 | 29.61 |
| Lim and Aziz | 1995 | analytical | 9.87 | 18.17 | 25.67 |
| Quintard and Whitaker | 1996 | averaging | 12 | 28.45 | 49.58 |
| Bourbiaux et al. | 1999 | numerical | | | 20 |
| Noetinger et al. | 2000 | random walk | 11.5 | 27.1 |
| Sarda et al. | 2002 | numerical | 8 | 24 | 48 |
| Rasmussen and Civan | 2003 | analytical | 9.87 | 18.17 | 25.67 |

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By extracting the coordinate systems by using Laplace transform and Duhamel principle. Their study results reveal that the value of the shape factor decreased and tended to be constant when the flow state gradually reached the pseudo-steady state. The stabilized values of the shape factors were the same as the values obtained by Kazemi and Gilman.

To overcome the shortcomings of the pseudo-steady state method, an unsteady-state diffusion equation was then used by several researchers, leading to a time-dependent shape factor. The research on the time-dependent shape factor was first started by Chang. By solving the three-dimensional pressure diffusion equations under unsteady-state conditions, a series of shape factors at constant fracture pressure or constant flow rate were obtained. He proposed that the value of the shape factor decreased and tended to be constant when the flow state gradually reached the pseudo-steady state. The stabilized values of the shape factors were the same as the values obtained by Kazemi and Gilman. To obtain an approximate analytical solution, Lim and Aziz used an exponential function to solve the pressure diffusion equation without using the pseudo-steady state assumption. By extracting the first term of the infinite summation series, they obtained the constant shape factors. Rangel-German and Kovscek proposed a piecewise function for cubic matrix blocks based on the imbibition experiment. A power function was used to control the unsteady state. Hassanzadeh and Pooalidi-Darvish solved pressure diffusion equations under different fracture boundary conditions and coordinate systems by using Laplace transform and Duhamel’s principle. Their study results reveal that the value of the shape factor is influenced by the pressure depletion regime of fractures. Based on their work, Ranjbar et al. obtained the shape factor of compressible fluid under different pressure depletion regimes for one-dimensional conditions. He et al. further obtained the constant and time-dependent shape factors by considering the influence of tortuosity and threshold pressure based on Lim and Aziz’s work. Considering the stress sensitivity of the tight reservoir, Wang et al. established an unsteady interporosity model and obtained the time-dependent shape factor. Liu et al. designed some experiments to investigate the influence of stress sensitivity on interporosity flow. Rostami et al. have calculated the shape factors for multidimensional irregular bodies in a systematic approach by using finite-grid simulation. Abbasi et al. obtained the time-dependent shape factor by considering the influence of the quadratic pressure gradient, the heterogeneous matrix, and the pressure-dependent rock properties. In addition, scholars have studied the influence of condensation, capillary imbibition process, gravity drainage, and nonisothermal process on the interporosity flow, and the corresponding time-dependent shape factors have been obtained.

Although scholars mentioned above have carried out a detailed studies about the shape factor, the existing shape factors ignored the unique seepage mechanisms of shale/tight oil reservoirs, such as nonlinear flow and stress sensitivity. Additionally, the fracture pressure was mostly considered as a constant in previous studies. To overcome these shortcomings, a new transient interporosity flow model has been established by considering the influence of nonlinear flow, stress sensitivity, and time-dependent fracture pressure boundary conditions. The model was solved by using finite difference and Newton iteration method. By using the Duhamel principle, the solution of time-dependent fracture pressure boundary condition was obtained. Then, sensitivity analysis of the shape factor and the interporosity flow rate was conducted. Finally, the new model was used in well test interpretation and perforation cluster number optimization.

### 2. METHODOLOGY

The transfer function can also be expressed as a form of Darcy’s law:

$$q = \frac{\kappa_m V}{\mu} (\bar{p}_m - \bar{p}_f)$$  \hspace{1cm} (1)

where $q$ is the interporosity rate, $10^{-6}$ $m^3/s$; $\kappa$ is the shape factor, $m^{-2}$; $V$ is the volume of the matrix block, $m^3$; $k_m$ is the matrix permeability, $10^{-3}$ $m^3/s$; $\mu$ is the fluid viscosity, $mPa/s$; $\bar{p}_m$ is the volumetric average matrix pressure, MPa; $p_f$ is the fracture pressure, MPa.

During early research, the shape factor was commonly obtained based on the pseudo-steady state assumption, leading to a constant shape factor. The shape factors obtained by different researchers are summarized in Table 1. However, the constant shape factor has some defects in characterization of the transient flow features.

To overcome the shortcomings of the pseudo-steady state method, an unsteady-state diffusion equation was then used by several researchers, leading to a time-dependent shape factor. The research on the time-dependent shape factor was first started by Chang. By solving the three-dimensional pressure diffusion equations under unsteady-state conditions, a series of shape factors at constant fracture pressure or constant flow rate were obtained. He proposed that the value of the shape factor decreased and tended to be constant when the flow state gradually reached the pseudo-steady state. The stabilized values of the shape factors were the same as the values obtained by Kazemi and Gilman. To obtain an approximate analytical solution, Lim and Aziz used an exponential function to solve the pressure diffusion equation without using the pseudo-steady state assumption. By extracting the first term of the infinite summation series, they obtained the constant shape factors. Rangel-German and Kovscek proposed a piecewise function for cubic matrix blocks based on the imbibition experiment. A power function was used to control the unsteady state. Hassanzadeh and Pooalidi-Darvish solved pressure diffusion equations under different fracture boundary conditions and coordinate systems by using Laplace transform and Duhamel’s principle. Their study results reveal that the value of the shape factor is influenced by the pressure depletion regime of fractures. Based on their work, Ranjbar et al. obtained the shape factor of compressible fluid under different pressure depletion regimes for one-dimensional conditions. He et al. further obtained the constant and time-dependent shape factors by considering the influence of tortuosity and threshold pressure based on Lim and Aziz’s work. Considering the stress sensitivity of the tight reservoir, Wang et al. established an unsteady interporosity model and obtained the time-dependent shape factor. Liu et al. designed some experiments to investigate the influence of stress sensitivity on interporosity flow. Rostami et al. have calculated the shape factors for multidimensional irregular bodies in a systematic approach by using finite-grid simulation. Abbasi et al. obtained the time-dependent shape factor by considering the influence of the quadratic pressure gradient, the heterogeneous matrix, and the pressure-dependent rock properties. In addition, scholars have studied the influence of condensation, capillary imbibition process, gravity drainage, and nonisothermal process on the interporosity flow, and the corresponding time-dependent shape factors have been obtained.

Although scholars mentioned above have carried out a detailed studies about the shape factor, the existing shape factors ignored the unique seepage mechanisms of shale/tight oil reservoirs, such as nonlinear flow and stress sensitivity. Additionally, the fracture pressure was mostly considered as a constant in previous studies. To overcome these shortcomings, a new transient interporosity flow model has been established by considering the influence of nonlinear flow, stress sensitivity, and time-dependent fracture pressure boundary conditions. The model was solved by using finite difference and Newton iteration method. By using the Duhamel principle, the solution of time-dependent fracture pressure boundary condition was obtained. Then, sensitivity analysis of the shape factor and the interporosity flow rate was conducted. Finally, the new model was used in well test interpretation and perforation cluster number optimization.

## 2. METHODOLOGY

The transfer function can also be expressed as a form of Darcy’s law:

$$q = \frac{\kappa_m A}{\mu} (\bar{p}_m - \bar{p}_f)$$  \hspace{1cm} (2)

where $A$ is the cross-sectional area of the matrix block, $m^2$; $\Delta L$ is the characteristic length which is defined as the distance between $p_f$ and $\bar{p}_m$.  

We get the following equation by combining eqs 1 and 2:

$$\sigma = \frac{A}{V \Delta L}$$  \hspace{1cm} (3)

As shown in eq 3, the shape factor is the ratio of the cross-sectional area for fluid transfer to the characteristic flow distance under unit volume, and it is a parameter related to several geometric factors. $A/V$ can be used to reflect the geometric features of the matrix block. $1/\Delta L$ can be used to control the fluid exchange between the matrix and fracture.

As seen in Figure 1, at the initial moment, the pressure in the matrix is the initial reservoir pressure. The location of the average matrix pressure $\bar{p}_m$ is at the fracture surface, and $\Delta L = 0$. With the decreasing of $\bar{p}_m$, the location of $\bar{p}_m$ moves toward the center of the matrix block, and the value of $\Delta L$ increases, which leads to a decrease of the shape factor $\sigma$. It can be seen from the symmetry that there is no fluid flow in the center of the matrix. The pseudo-steady state is reached when the location of $\bar{p}_m$ reaches the center of the matrix block. At this time ($t_c$), $\Delta L$ is equal to $L_c$ and the value of $\sigma$ becomes constant.

### Figure 1. Schematic diagram of the average matrix pressure.

$$q = \frac{\sigma k_m V}{\mu} (\bar{p}_m - \bar{p}_f)$$  \hspace{1cm} (1)
The fluid volume of interporosity flow from the matrix to fracture is equal to the expanded volume of the fluid in the matrix due to the pressure drop, according to the law of conservation of mass. Hence, the interporosity flow rate can be expressed as:

\[ q = -V_p m c_i \frac{\partial \bar{p}_{m}}{\partial t} \]  
\[ (4) \]

Combining eqs 1 and 4, we get the following equation:

\[ \sigma = -\frac{1}{\eta (\bar{p}_{m} - \bar{p}_{i})} \frac{\partial \bar{p}_{m}}{\partial t} \]  
\[ (5) \]

where \( \eta = k_m/(\phi_m \mu c_i) \) is the hydraulic diffusivity, \( m^2/s \).

Equation 5 can be used to calculate the value of the time-dependent shape factor when the relationship between the average matrix pressure and time is obtained.

Nonlinear flow in a shale/tight reservoir is caused by the boundary layer.\(^3\) To characterize the nonlinear flow in shale/tight formation, Huang et al.\(^36,39\) established a new seepage model based on the capillary bundle model and the fractal theory. The thickness of the boundary layer \( \delta \) can be described by exponential function \( \delta = \delta_0 + ae^{-b \cdot \bar{p}} \). According to Huang’s study, the motion equation in the matrix is:

\[ v_m = \frac{k_m}{\mu} (1 - \delta_0 M - aM \mu^{-b \cdot \bar{p}_{m}}) \frac{\partial \bar{p}_{m}}{\partial t} \]  
\[ (6) \]

where \( v_m \) is the fluid’s velocity in the matrix, \( m/s \); \( \mu \) is the viscosity, \( mPa\cdot s \); \( k_m \) is the matrix permeability, \( mD \); \( \partial \bar{p}_{m} \) is the pressure gradient in the matrix, \( MPa/m; M = 4(3 - D_i + D_T)/[\gamma_{max}(2 - D_i + D_T)] \) is the nonlinear coefficient; \( \delta_0 \) is the thickness of stable layer; \( a \) and \( b \) are the boundary layer coefficients; \( D_i \) is the pore fractal dimension; \( D_T \) is the tortuosity fractal dimension; \( \gamma_{max} \) is the maximum pore radius of the reservoir, \( \mu \cdot \delta_0 \cdot a \), and \( b \) can be obtained by the nonlinear flow experiment. \( \gamma_{max} \cdot D_i \) and \( D_T \) can be obtained by mercury intrusion porosimetry.

To account for the influence of effective stress on matrix permeability, a stress-dependent permeability is used. According to previous research studies,\(^30\) there is an exponential relationship between permeability and pressure, which is given as:

\[ k_m = k_0 e^{-\gamma(p_i - p_{\infty})} \]  
\[ (7) \]

where \( k_0 \) is the initial permeability of the matrix, \( 10^{-3} \mu m^2 \); \( \gamma \) is the permeability modulus, \( MPa^{-1} \); \( p_i \) is the initial pressure of the reservoir, \( MPa \); \( p_{\infty} \) is the matrix pressure, \( MPa \).

During the development of shale/tight reservoirs, the fracture pressure is not constant and decreases over time. The research conducted by Ranjbar et al. shows that the exponential decline of fracture pressure is more in line with the field conditions. We assumed that the initial fracture pressure equals the initial pressure \( p_i \) and decreases exponentially with time:

\[ p_i(t) = p_{\infty} + (p_i - p_{\infty}) e^{-\alpha t} \]  
\[ (8) \]

where \( p_{\infty} = p_i \rightarrow \infty \); \( \alpha \) is the decline constant, \( s^{-1} \).

3. MATHEMATICAL MODEL

The matrix–fracture system is shown in Figure 2. On both sides of the matrix, there are two parallel fractures. The matrix pressure satisfies the following equation:

\[ \frac{\partial}{\partial x} \left[ k_m e^{-\gamma(p_i - p_{\infty})} (1 - \delta_0 M - aMe^{-b \cdot \bar{p}_{m}}) \frac{\partial \bar{p}_{m}}{\partial x} \right] = \frac{\partial \bar{p}_{m}}{\partial t} \]  
\[ (9) \]

where \( c_i \) is the total compressibility, \( MPa^{-1} \); \( \phi_m \) is the porosity of the matrix.

The abovementioned function can be simplified as:

Table 2. Definition of Dimensionless Variables

| name | expression |
|------|------------|
| dimensionless pressure | \( P_D = \frac{p - p_i}{p_{\infty} - p_i} \) |
| dimensionless time | \( t_D = \frac{t}{t_i} \) |
| dimensionless distance | \( x_D = \frac{x}{x_i} \) |
| dimensionless permeability modulus | \( \gamma_D = \gamma(p_i - p_{\infty}) \) |
| dimensionless decline constant | \( k = \frac{c_i^2}{\mu \phi_m} \) |
| dimensionless interporosity flow rate | \( q_D = \frac{\mu c_i^2}{k \phi_m (p_{\infty} - p_i)^3} \) |
Initially, the matrix pressure equals the initial pressure:

$$p_{m,1} = p_i$$  

(11)

Because the unit is symmetrical, there is no fluid flow in the middle of the matrix:

$$\frac{\partial p_{m}}{\partial x} \bigg|_{x=0} = 0$$  

(12)

The matrix pressure equals to the fracture pressure at the interface between matrix and fracture:

$$p_{m} \big|_{x=L_i} = p_f$$  

(13)

To facilitate the model development and solution, the dimensionless variables are defined in Table 2.  

With the substitutions of these dimensionless parameters, the dimensionless model can be obtained:

$$\begin{align*}
\alpha \frac{\partial^2 p_{mD}}{\partial x_D^2} + \beta \frac{\partial p_{mD}}{\partial x_D} &= \frac{\partial p_{mD}}{\partial t_D}
\end{align*}$$

(14)

$$\begin{align*}
p_{mD,1} &= 0 \\
\frac{\partial p_{mD}}{\partial x_D} \bigg|_{x_D=0} &= 0 \\
p_{mD} \big|_{x_D=1} &= 1 - e^{-x_D}
\end{align*}$$

where $\alpha$ and $\beta$ are the dimensionless nonlinear coefficients,

$$\alpha = \left[ 1 - M\delta_0 \left( 1 - b \frac{\partial \phi_{mD}}{\partial \phi_{mD}} \right) a e^{-\gamma\phi_{mD}} \right]$$

\(\text{and}\)

$$\beta = -\frac{\partial \phi_{mD}}{\partial \phi_{mD}} \left( 1 - M\delta_0 - a e^{-\gamma\phi_{mD}} \right)$$

Because the outer boundary condition is not constant, the equivalent model was solved first. Then, based on Duhamel's principle, the solution of the model with the time-dependent boundary condition was obtained.

The equivalent model is:

$$\begin{align*}
\alpha \frac{\partial^2 p_{mD}}{\partial x_D^2} + \beta \frac{\partial p_{mD}}{\partial x_D} &= \frac{\partial p_{mD}}{\partial t_D}
\end{align*}$$

(15)

$$\begin{align*}
p_{mD,1} &= 0 \\
\frac{\partial p_{mD}}{\partial x_D} \bigg|_{x_D=0} &= 0 \\
p_{mD} \big|_{x_D=1} &= 1
\end{align*}$$

The model is differentially discretized and then numerically solved with the Newton iteration method.

The pressure diffusivity equation can be written in the following finite difference format:

$$a_i p_{mD,i+1} + b_i p_{mD,i} + c_i p_{mD,i+1} = d_i$$  

(16)

where $a_i = \frac{\alpha \Delta x_D}{\Delta t_D}, b_i = \frac{\beta \Delta x_D}{\Delta t_D}, c_i = \frac{\gamma \phi_{mD}}{\Delta x_D}$.

The solution under the condition of exponentially declining fracture pressure can be obtained by using Duhamel's principle:

$$p_{mD,Depletion} = \int_{0}^{t_f} [1 - e^{-\xi(t_f-\tau)}] \frac{\partial p_{mD}(x_D, \tau)}{\partial \tau} d\tau$$  

(21)

Then the average matrix pressure $\bar{p}_{mD,Depletion}$ can be obtained by integrating of $p_{mD,Depletion}$ over bulk volume of the matrix block:
The dimensionless shape factor $\sigma H_m^2$ and the dimensionless interporosity flow rate $q_D$ can be obtained by nondimensionalizing eqs 4 and 5:

$$\sigma H_m^2 = -\frac{4}{(\bar{p}_{mD} - P_D)} \frac{\partial \bar{p}_{mD}}{\partial t_D}$$

and

$$q_D = \frac{\partial \bar{p}_{mD}}{\partial t_D}$$

Finally, $\sigma H_m^2$ and $q_D$ can be obtained by substituting $\bar{p}_{mD}$ to eqs 23 and 24.

4. VALIDATION

To validate this model, a numerical model has been established by using the fine-grid finite element method (FEM), as shown in Figure 4. The parameters used for comparison are: $L_c = 10\text{ m}$, $p_i = 20\text{ MPa}$, $p_{\infty} = 10\text{ MPa}$, $k_0 = 0.1\text{ mD}$, $\phi_m = 0.1$, $\mu = 1\text{ mPa}\cdot\text{s}$, $\epsilon_t = 4 \times 10^{-4}\text{ MPa}^{-1}$, $\delta_0 = 0.01\text{ mm}$, $a = 0.2\text{ mm}$, $b = 3\text{ MPa}^{-1}$, $m_r = 0.01\text{ MPa}^{-1}$, and $\kappa = 1$. Then we compared the average matrix pressure calculated by this model with the results obtained by the fine-grid finite element simulation. As illustrated in Figure 5, the new model’s solution is consistent with the results of the finite element simulation. In addition, a relatively simple case without considering any mechanism was conducted and a comparison was made with previous research studies. The stabilized value of the time-dependent shape factor calculated by the new model is in good accordance with the shape factor obtained by researchers such as Kazemi and Gilman and Lim and Aziz (Figure 6). Therefore, the solution in this study is accurate and reliable.

5. RESULTS AND DISCUSSION

5.1. Sensitivity Analysis. To better understand the influence of nonlinear flow, stress sensitivity, and fracture pressure depletion on the dimensionless shape factor ($\sigma H_m^2$), five factors have been selected for sensitivity analysis, which are $\gamma$, $\delta_0$, $a$, $b$, and $\kappa$.

As shown in Figure 7, the larger the boundary layer coefficient $a$ is, the smaller the dimensionless shape factor is. The influence of coefficient $a$ is not obvious at first, but increases with time. This is due to the fact that the pressure gradient in the matrix is the largest at the beginning, which weakens the influence of the coefficient $a$ to the greatest extent. As the interporosity flow proceeds, the average matrix pressure $\bar{p}_m$ and the pressure gradient $\frac{dp_m}{dx}$ decreases, resulting in a more significant influence of the coefficient $a$.

It can be seen from Figure 8 that the dimensionless shape factor increases with the increase of coefficient $b$. $b$ is the...
coefficient directly acting on the pressure gradient. The larger the value of coefficient \( b \) is, the greater the influence of the pressure gradient on the thickness of the boundary layer and the weaker the influence of nonlinear flow.

The influence of the thickness of stable layer \( \delta_0 \) is the most significant and continuous through the entire flow stage, as shown in Figure 9. The larger the \( \delta_0 \) is, the smaller the dimensionless shape factor is.

Figure 8. Comparison of the dimensionless shape factor for different \( b \).

Figure 9. Comparison of the dimensionless shape factor for different \( \delta_0 \).

Figure 10. Comparison of the dimensionless shape factor for different \( \gamma_m \).

Figure 11. Comparison of the dimensionless shape factor for different \( \kappa \).

Figure 12. Comparison of the dimensionless shape factor under different conditions.

Table 3. Experimental Design

| level | \( \delta_0 \) (\( \mu \text{m} \)) | \( a \) (\( \mu \text{m} \)) | \( b \) (MPa\(^{-1}\)·m) | \( \gamma_m \) (MPa\(^{-1}\)) | \( \kappa \) |
|-------|-----------------|-----------------|-----------------|-----------------|--------|
| 1     | 0.01            | 0.1             | 2               | 0.01            | 1      |
| 2     | 0.01            | 0.1             | 2               | 0.03            | 10     |
| 3     | 0.01            | 0.2             | 4               | 0.01            | 1      |
| 4     | 0.01            | 0.2             | 4               | 0.03            | 10     |
| 5     | 0.03            | 0.1             | 4               | 0.01            | 10     |
| 6     | 0.03            | 0.1             | 4               | 0.03            | 1      |
| 7     | 0.03            | 0.2             | 2               | 0.01            | 10     |
| 8     | 0.03            | 0.2             | 2               | 0.03            | 1      |

Figure 10 indicates that the stress sensitivity affects the early stage \( (t_0 \leq 0.2) \) of the interporosity flow, which is different from the influence of nonlinear flow. This is consistent with the variation of matrix permeability with stress.

The dimensionless shape factor calculated under different dimensionless decline constant \( \kappa \) is shown in Figure 11. The larger the decline coefficient, the faster the decline speed of the fracture pressure and the smaller the value of \( \sigma H_{nm}^0 \). What is more, with increasing of \( \kappa \), the decline speed of \( \sigma H_{nm}^0 \) becomes faster and the transient stage becomes shorter.

5.2. Control Mechanisms of Multiple Factors. To further evaluate the comprehensive influence of nonlinear flow, stress
sensitivity, and fracture pressure depletion on the dimensionless shape factor ($\sigma H^2_m$) and the dimensionless interporosity flow rate ($q_{D}$), two different levels are taken for each factor. The following eight schemes have been designed according to the orthogonal table $L_8(4^{12} \times 2^{4})$, which is shown in Table 3.

The dimensionless shape factors of Schemes 1−8 are shown in Figure 12. Due to the least influence of nonlinear flow and stress sensitivity, and the slowest rate of fracture pressure depletion, Scheme 3 has the highest shape factor. On the contrary, Scheme 7 has the smallest shape factor. The value of the shape factor is found to be less when the influence of nonlinear flow and stress sensitivity is higher. The value of the shape factor is larger because the fracture pressure depletion rate is smaller. In addition, the unsteady stage becomes longer after considering the influence of nonlinear flow, stress sensitivity, and fracture pressure depletion. Constant shape factors (refs 7, 8, 13) will greatly underestimate the rate of the interporosity flow at the initial stage. The stabilized value of Schemes 1−8 is between Kazemi and Warren and Root’s constant shape factors.

6. APPLICATION

6.1. Well Test Interpretation. We have established a trilinear flow model in previous research studies.42,43 Figure 14 is the schematic diagram of the trilinear flow model. It is assumed that all fractures are equally spaced along the horizontal well,
Table 4. Basic Parameters and Interpretation Results of Well CARD-1

| parameters                        | values | parameters                        | values |
|-----------------------------------|--------|-----------------------------------|--------|
| lateral length \( L_e \) (m)      | 1180   | fracture permeability of region 2 in \( x \) direction \( k_{2x} \) (mD) | 4.5    |
| number of fractures \( N_f \)     | 10     | fracture permeability of region 2 in \( y \) direction \( k_{2y} \) (mD) | 5      |
| fracture spacing \( 2y_i \) (mD)  | 130    | fracture permeability of region 1 \( k_{1} \) (mD) | 5800   |
| bottom hole pressure \( p_{bf} \) (MPa) | 1.8    | porosity of region 2 \( \phi_2 \) | 0.2    |
| initial pressure \( p_i \) (MPa)  | 13.9   | porosity of region 1 \( \phi_1 \) | 0.25   |
| reservoir thickness \( \bar{h} \) (m) | 5      | fractal dimension of fractures of region 2 \( D_{fr} \) | 1.91   |
| matrix permeability \( k_{mf} \) (mD) | 0.28   | conductivity index of fractures of region 2 \( \Theta_{1y} \) | 0.25   |
| matrix porosity \( \phi_m \)      | 0.12   | permeability modulus of region 1 \( f_1 \) (MPa\(^{-1}\)) | 0.030  |
| length of fractures \( x_i \) (m) | 130    | permeability modulus of region 2 in \( x \) direction \( f_{2x} \) (MPa\(^{-1}\)) | 0.015  |
| fracture aperture \( w_i \) (m)   | 0.01   | permeability modulus of region 2 in \( y \) direction \( f_{2y} \) (MPa\(^{-1}\)) | 0.025  |

The time-dependent shape factor was used in the trilinear flow model and production data of CARD-1 in Pembina Cardium was selected for matching and interpreting. The result of the well test interpretation is shown in Figure 15 and Table 4. The first eight parameters with an asterisk in the table are known parameters, and the others are interpretation parameters.

The result shows that the permeability in the \( y \) direction of the SRV is greater than the permeability in the \( x \) direction. In addition, the artificial fractures have the highest stress sensitivity, and the stress sensitivity of the SRV in the \( x \) direction is the weakest. This indicates that the \( y \) direction is the main seepage direction in the SRV zone. To illustrate the influence of transient interporosity flow, the constant shape factor was also used to perform the matching while other parameters remain unchanged. The pseudo-steady state flow model will underestimate the production at the initial stage. The matching accuracy can be improved by changing other parameters, while the results will become unreasonable to some extent.

6.2. Optimization of the Number of the Perforation Cluster. Assuming that the fracture spacing is 60 m, the distribution of 2–7 perforation clusters in a fracturing unit is shown in Figure 16. The black line represents a horizontal wellbore, and the black dotted line is the artificial fracture. The red line is the position of the perforation cluster, and the blue dotted line is the no-flow boundary. The fracturing unit can be divided into two parts, namely, the matrix between perforation clusters and the matrix outside perforation clusters. The matrix length of different cluster numbers is listed in Table 5.

The pressure and interporosity rate of the matrix with a length of 3.75–20 m were calculated based on the new model. The

Figure 16. Schematic diagram of distribution of multicluster perforation.

Figure 17. Type curve for optimization of the perforation cluster number.

Table 5. Matrix Length of the Fracturing Unit with Different Cluster Numbers

| perforation cluster number | matrix length in the cluster (m) | matrix length out of the cluster (m) | composition of the fracturing unit |
|----------------------------|----------------------------------|--------------------------------------|----------------------------------|
| 2                         | 20                               | 10                                   | 20 × 2 + 10 × 2                  |
| 3                         | 15                               | 7.5                                  | 15 × 2 + 7.5 × 4                 |
| 4                         | 12                               | 6                                    | 12 × 2 + 6 × 6                   |
| 5                         | 10                               | 5                                    | 10 × 2 + 5 × 8                   |
| 6                         | 8.5                              | 4.25                                 | 8.5 × 2 + 4.25 × 10              |
| 7                         | 7.5                              | 3.75                                 | 7.5 × 2 + 3.75 × 12              |

Table 6. Parameters for Calculation

| parameters                        | values | parameters                        | values |
|-----------------------------------|--------|-----------------------------------|--------|
| matrix permeability (mD)          | 0.001  | matrix porosity (%)               | 5      |
| viscosity (mPa·s)                 | 5      | total compressibility (MPa\(^{-1}\)) | 5 × 10\(^{-4}\) |
| final fracture pressure (MPa)     | 10     | initial matrix pressure (MPa)      | 30     |
| reservoir thickness (m)           | 20     | perforation cluster length (m)    | 2      |

with the same properties. According to symmetry, the basic unit can be obtained, and regions 1–3 represent the hydraulic fractures, the stimulated reservoirs, and the outer reservoirs.
relevant parameters are listed in Table 6. The cumulative production of each matrix was obtained by integrating the interporosity rate. Then, the cumulative production of the unit with different perforation cluster numbers was obtained. The time for pressure diffusion to the no-flow boundary was obtained by analyzing the pressure feature of the grid where the nonflow boundary is located. Finally, the type curve for optimization of the perforation cluster number was drawn, as shown in Figure 17.

The cumulative production increased with the increasing in the perforation cluster number, but the growth rate slowed down gradually. The shorter the production time, the greater the difference in cumulative production under different perforation cluster numbers. However, the difference gradually decreased with the increase in the production time. In addition, the time for matrix pressure diffusion to the no-flow boundary between perforation clusters \((t_{fl})\) was different from the time for that outside perforation clusters \((t_{fo})\). When the number of perforation clusters is small, the difference between \(t_{fl}\) and \(t_{fo}\) is great, leading to an unbalanced utilization of the fracturing unit. On the contrary, there will be mutual interference between perforation clusters in a very short time when the number of perforation clusters is large. Therefore, there is an optimal value for the perforation cluster number. For example, in Figure 17, the optimal perforation cluster number is 4 or 5, which can balance production and interference. It is important to note that this only provides a new idea for optimization of the perforation cluster number. It is necessary to consider comprehensively the influence of multiple factors during the optimization process, such as in situ stress and fracturing parameters.

7. CONCLUSIONS
In this study, a transient interporosity flow model has been established. In this model, the nonlinear flow and stress sensitivity of the shale/tight oil reservoirs have been taken into account. Moreover, the influence of fracture pressure depletion has also been taken into account. Finite difference, Newton iteration method, and Duhamel principle have been used to solve the new model. The study results show that the fluid flow between matrix and fracture in a shale/tight reservoir is controlled by multiple factors. The interporosity flow rate at the initial stage might be underestimated when constant shape factors are used. The nonlinear flow and stress sensitivity have an obvious influence on the interporosity flow. When the influence of nonlinear flow and stress sensitivity increases, the value of the shape factor and interporosity flow rate decrease. The shape factor becomes larger and the nonsteady state becomes longer after considering the influence of fracture pressure depletion. In addition, the interporosity rate will rise first to reach equilibrium and then decrease when the decline constant is very small. The new model can accurately characterize the interporosity flow in shale/tight reservoirs, which has important implications for well test interpretation, numerical simulation, and even optimization of the perforation cluster number.

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**Notes**
The authors declare no competing financial interest.

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