Study of $\bar{B}_s^0 \rightarrow (D_s^{(*)}, D_{s}^{*+}) l^- \bar{\nu}_l$ decays in the pQCD factorization approach

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Abstract

The $\bar{B}_s^0 \rightarrow D_s^{(*)} l^- \bar{\nu}_l$ semileptonic decays were calculated in the framework of the standard model (SM) by employing the perturbative QCD (pQCD) factorization approach. We defined four ratios of the branching ratios of the considered decays $R(D_s^{(*)})$ and $R_{D_s}^{l,\tau}$. From the numerical results and phenomenological analysis we found that: (a) The pQCD predictions for the branching ratios $Br(\bar{B}_s^0 \rightarrow D_s^{(*)} l^- \bar{\nu}_l)$ generally agree well with the previous theoretical predictions; (b) For the four ratios, the pQCD predictions are $R(D_s) = 0.392 \pm 0.022$, $R(D_s^*) = 0.302 \pm 0.011$, $R_{D_s}^{l} = 0.448^{+0.058}_{-0.041}$ and $R_{D_s}^{\tau} = 0.582^{+0.071}_{-0.045}$, which show a very good $SU(3)_F$ flavor symmetry with the corresponding ratios for $B \rightarrow D^{(*)} l^- \bar{\nu}_l$ decays; and (c) we strongly suggest the measurements of the new ratios $R(D_s^{(*)})$ and $R_{D_s}^{l,\tau}$ in the forthcoming Super-B experiments.

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I. INTRODUCTION

In Ref. [1], the BaBar collaboration reported a combined $3.4\sigma$ deviation of their measured ratios $R(D)$ and $R(D^*)$ from the standard model (SM) predictions. The measured values are [1]

$$R(D) = 0.440 \pm 0.072, \quad R(D^*) = 0.332 \pm 0.030,$$

while the SM predictions obtained by using the traditional heavy quark effective theory (HQET) [2, 3] to evaluate the form factors of $B \to D, D^*$ transitions are the following [4]:

$$R(D)^{\text{SM}} = 0.296(16), \quad R(D^*)^{\text{SM}} = 0.252(3).$$

which are indeed much smaller than those measured values as given in Eq. (1). This $R(D^{(*)})$ anomaly has invoked intensive studies about the semileptonic $B \to D^{(*)}l^-\bar{\nu}_l$ decays$[5–12]$ in the framework of the SM by employing the different mechanisms or methods, but they all failed to interpret the data.

Motivated by the great difference between the theoretical predictions and the BaBar’s measurements about the ratios $R(D^*)$, we calculated the ratios $R(D^*)$ by employing the perturbative QCD (pQCD) factorization approach [13] to evaluate the $B \to (D, D^*)$ transition form factors, and then found numerically that [14]

$$R(D) = 0.430^{+0.021}_{-0.026}, \quad R(D^*) = 0.301 \pm 0.013.$$  (3)

These pQCD predictions agree very well with the BaBar’s measurements.

Among the various $B/B_s$ semileptonic decays, the $\bar{B}^0_s \to D^{(*)}_s l^-\bar{\nu}_l$ decays are closely related with those $B \to D^{(*)} l^-\bar{\nu}_l$ decays through the $SU(3)_F$ flavor symmetry: they are all controlled by the same $b \to c l^-\bar{\nu}_l$ transitions at the quark level, but with a different spectator quark, from the $\bar{s}$ quark to the $\bar{u}$ or $\bar{d}$ quark: i.e.

$$\bar{B}^0_s \to D^{(*)}_s l^-\bar{\nu}_l \quad \longleftrightarrow \quad B^- / \bar{B}^0 \to D^{(*)} l^-\bar{\nu}_l$$  (4)

In the limit of $SU(3)$ flavor symmetry, these two kinds of decays should have very similar properties. It is therefore very interesting to make a systematic study for the $\bar{B}^0_s \to D^{(*)}_s l^-\bar{\nu}_l$ decays, and most importantly measure them in the forthcoming Super-B experiments, even if the LHCb can not do the job due to the escape of the neutrinos.

In this paper, we will study $\bar{B}^0_s \to D^{(*)}_s l^-\bar{\nu}_l$ decays by employing the pQCD factorization approach. Analogous to $R(D^{(*)})$ for $B \to D^{(*)}$ transitions, we here define the similar ratios of the branching ratios in the form of:

$$R(D_s) \equiv \frac{\mathcal{B}(\bar{B}^0_s \to D^+_s l^-\bar{\nu}_l)}{\mathcal{B}(\bar{B}^0_s \to D^+_s l^-\bar{\nu}_l)}, \quad R(D^*_s) \equiv \frac{\mathcal{B}(\bar{B}^0_s \to D^{*_s} l^-\bar{\nu}_l)}{\mathcal{B}(\bar{B}^0_s \to D^{*_s} l^-\bar{\nu}_l)},$$  (5)

where $l^- = (e^-, \mu^-)$, which measures the mass effects of the heavy $\tau$ and light $e^-$ or $\mu^-$ leptons. Following Ref. [14], furthermore, we here also define other two ratios in the form of

$$R^l_{D_s} \equiv \frac{\sum_{l=e,\mu} \mathcal{B}(\bar{B}^0_s \to D^+_s l^-\bar{\nu}_l)}{\sum_{l=e,\mu} \mathcal{B}(\bar{B}^0_s \to D^{*_s} l^-\bar{\nu}_l)}, \quad R^\tau_{D_s} \equiv \frac{\mathcal{B}(\bar{B}^0_s \to D^{*_s} l^-\bar{\nu}_l)}{\mathcal{B}(\bar{B}^0_s \to D^{*_s} l^-\bar{\nu}_l)}.$$  (6)

It is easy to see that these two ratios reveals the effects induced by the different form factors of $B_s \to D_s$ and $B_s \to D^*_s$ transitions, and can also be measured in the future Super-B experiments.
Theoretically, the semileptonic $B_s \to (D_s, D^*_s)\ell \bar{\nu}$ decays have been studied frequently in the framework of the SM. The branching ratios of these decay modes have been studied, for example, in terms of the constituent quark meson (CQM) model [15], in the framework of the QCD sum rules (QCDSRs) [16], in the light cone sum rules (LCSRs) or the covariant light-front quark model (CLFQM) [17, 18]. In Ref. [19, 20], $B^0_s \to (D^-_s, D^*_s^-)$ transition form factors are estimated by using the method based on an instantaneous approximated Mandelstam formulation (IAMF) and the instantaneous Bethe-Salpeter equation, or the relativistic quark model. The numerical predictions as presented in all these mentioned works [15–20] will be listed in Table III of this paper for the purpose of comparisons.

On the experiments side, the semileptonic $B_s \to (D_s, D^*_s)\ell \bar{\nu}$ decays have not been measured up to now. In LHCb experiments, it can not be measured too since the neutrino is inaccessible there. But in the forthcoming Super-B experiments, these semileptonic $B_s$ decays with a neutrino as one of the final state lepton can be measured precisely.

In the pQCD factorization approach, the lowest order Feynman diagrams for $\bar{B}^{0}_s \to D^{(*)}_s \ell \bar{\nu}_l$ decays are displayed in Fig. 1, where the leptonic pairs come from the $b$-quark’s weak decay through charged current. The study for the semileptonic decays $\bar{B}^{0}_s \to (D^+_s, D^*_s^+)\ell \bar{\nu}_l$ can certainly be a great help for us to understand the BaBar’s measurements for $R(D^{(*)})$.

The paper is organized as follows: In Sec. II, we firstly give a short review for the kinematics of the $\bar{B}^{0}_s \to D^{(*)}_s \ell \bar{\nu}_l$ decays, and then we make a pQCD calculation for the form factors $F_{0,1}(q^2)$, $V(q^2)$ and $A_{0,1,2}(q^2)$ for $\bar{B}^{0}_s \to D^{(*)}_s$ transitions, and present the formulaes for the differential decay rates of the considered decay modes. In Sec. III, we will present the pQCD predictions for the branching ratios of all considered decays, as well as the ratios $R(D^{(*)})$ and $R^{D_s^{(*)}}$ and make a comparative study with those currently known theoretical predictions. The final section contains the conclusions and a short summary.

II. THEORETICAL FRAMEWORK

A. Kinematics and the wave functions

In the $B_s$ meson rest frame, we define the $B_s$ meson momentum $P_1$, the $D_s^{(*)}$ momentum $P_2$ and the polarization vectors $\epsilon$ of the $D^*_s$ in the light-cone coordinates as[21]

$$P_1 = \frac{m_{B_s}}{\sqrt{2}} (1, 1, 0_\perp), \quad P_2 = \frac{r m_{B_s}}{\sqrt{2}} (\eta^+, \eta^-, 0_\perp),$$

$$\epsilon_L = \frac{1}{\sqrt{2}} (\eta^+, -\eta^-, 0_\perp), \quad \epsilon_T = (0, 0, 1), \quad (7)$$

FIG. 1. The typical Feynman diagrams for the semileptonic decays $\bar{B}^0_s \to (D^+_s, D^*_s^+)\ell \bar{\nu}_l$ in the pQCD approach.
with the ratio \( r = m_{D_s}/m_{B_s} \) or \( r = m_{D_{s}^{(*)}}/m_{B_s} \). The factors \( \eta^{\pm} = \eta \pm \sqrt{\eta^2 - 1} \) is defined in terms of the parameter \( \eta = \frac{1}{2 \mu} \left[ 1 + r^2 - \frac{q^2}{m_{B_s}^2} \right] \) as in Ref. [21], while \( \epsilon_L \) and \( \epsilon_T \) denotes the longitudinal and transverse polarization of the \( D_s^{(*)} \) meson, respectively. The momenta of the spectator quarks in \( B_s^{0} \) and \( D_{s}^{(*)} \) mesons are parameterized as

\[
k_1 = (0, x_1 \frac{m_{B_s}}{\sqrt{2}}, k_{1\perp}), \quad k_2 = \frac{m_{B_s}}{\sqrt{2}}(x_2 r \eta^+, x_2 r \eta^-, k_{2\perp}).
\]  

For the \( B_s \) meson wave function, we make use of the same parameterizations as in Refs. [22, 23] and we adopt the B-meson distribution amplitude widely used in the pQCD approach

\[
\phi_{B_s}(x, b) = N_{B_s} x^2 (1 - x)^2 \exp \left[ -\frac{M_{B_s}^2 x^2}{2 \omega_{B_s}^2} - \frac{1}{2} (\omega_{B_s} b)^2 \right],
\]  

where the normalization factor \( N_{B_s} \) depends on the values of the parameter \( \omega_{B_s} \) and decay constant \( f_{B_s} \) and defined through the normalization relation [23]. For \( \omega_{B_s} \), one usually take \( \omega_{B_s} = 0.50 \pm 0.05 \) GeV for \( B_s^{0} \) meson.

For the pseudoscalar meson \( D_s \), the wave function is in the form of

\[
\Phi_{D_s}(p_2, x) = \frac{i}{\sqrt{6}} \gamma_5 (\bar{p}_2 + m_{D_s}) \Phi_{D_s}(x).
\]  

For the vector \( D_{s}^{(*)} \) meson, we take the wave function as follows,

\[
\Phi_{D_{s}^{(*)}}(p_2, x) = -\frac{i}{\sqrt{6}} \left[ f_L (\bar{p}_2 + m_{D_{s}^{(*)}}) \phi_{D_{s}^{(*)}}^L(x) + f_T (\bar{p}_2 + m_{D_{s}^{(*)}}) \phi_{D_{s}^{(*)}}^T(x) \right].
\]  

For the distribution amplitudes of \( D_s \) and \( D_{s}^{(*)} \) meson, we adopt the same one as defined in Ref. [24]

\[
\phi_{D_s}(x) = \frac{3 f_{D_s}}{\sqrt{6}} x (1 - x) \left[ 1 + C_{D_s} (1 - 2x) \right] \exp \left[ -\frac{\omega_{D_s}^2 b^2}{2} \right],
\]

\[
\phi_{D_{s}^{(*)}}(x) = \frac{3 f_{D_{s}^{(*)}}}{\sqrt{6}} x (1 - x) \left[ 1 + C_{D_{s}^{(*)}} (1 - 2x) \right] \exp \left[ -\frac{\omega_{D_{s}^{(*)}}^2 b^2}{2} \right].
\]

From the heavy quark limit, we here assume that

\[
f_{D_{s}^{(*)}}^L = f_{D_{s}^{(*)}}^T = f_{D_{s}}, \quad \phi_{D_{s}^{(*)}}^L = \phi_{D_{s}^{(*)}}^T = \phi_{D_s}.
\]  

For the \( D_{s}^{(*)} \) mesons we also set \( C_{D_s} = C_{D_{s}^{(*)}} = 0.5, \omega_{D_s} = \omega_{D_{s}^{(*)}} = 0.1 \) GeV [24] in the calculations. Of course, the three distribution amplitudes \( \phi_{D_s}^L, \phi_{D_s}^T, \) and \( \phi_{D_s} \) should be different according to the general expectations, but we currently do not have other possible choices. The approximations as given in Eq. (14) may be too simple, even poor.

### B. Form factors and differential decay rates

The form factors of the \( B_s^{0} \rightarrow D_s \) transition induced by vector currents are defined as[23]:

\[
\langle D_s(p_2)|\bar{c}(0)\gamma_\mu b(0)|B_s^{0}(p_1)\rangle = F_+(q^2) \left[ (p_1 + p_2)_\mu - \frac{m_{B_s}^2 - m_{D_s}^2}{q^2} q_\mu \right] + F_0(q^2) \frac{m_{B_s}^2 - m_{D_s}^2}{q^2} q_\mu,
\]  

where \( q^2 = (p_1 - p_2)^2 \) is the four-momentum transfer to the final state, \( p_1 \) and \( p_2 \) are the four-momentum of the initial and final states, respectively.
where \( q = p_1 - p_2 \) is the lepton-pair momentum. In order to cancel the poles at \( q^2 = 0 \), \( F_+(0) \) should be equal to \( F_0(0) \). For the sake of the calculation, it is convenient to define the auxiliary form factors \( f_1(q^2) \) and \( f_2(q^2) \),

\[
\langle D_s(p_2)|\bar{c}(0)\gamma_{\mu}b(0)|\bar{B}_s^0(p_1)\rangle = f_1(q^2)p_{1\mu} + f_2(q^2)p_{2\mu},
\]

where the form factors \( f_1(q^2) \) and \( f_2(q^2) \) are related to \( F_+(q^2) \) and \( F_0(q^2) \) through the relation,

\[
\begin{align*}
F_+(q^2) &= \frac{1}{2} \left[ f_1(q^2) + f_2(q^2) \right], \\
F_0(q^2) &= \frac{1}{2} f_1(q^2) \left[ 1 + \frac{q^2}{m_{B_s}^2 - m_{D_s}^2} \right] + \frac{1}{2} f_2(q^2) \left[ 1 - \frac{q^2}{m_{B_s}^2 - m_{D_s}^2} \right].
\end{align*}
\]

For the \( \bar{B}_s^0 \to D_s^+l^-\bar{\nu}_l \) decays, by analytical calculations in the pQCD approach we find the \( \bar{B}_s^0 \to D_s \) form factors \( f_1(q^2) \) and \( f_2(q^2) \) as following:

\[
\begin{align*}
f_1(q^2) &= 8\pi m_{B_s}^2 C_F \int dx_1dx_2 \int b_1db_1b_2db_2\phi_{B_s}(x_1, b_1)\phi_{D_s}(x_2, b_2) \\
&\times \left\{ [2r(1-rx_2)] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \cdot \exp[-S_{ab}(t_1)] \\
&+ \left[ 2r(2r_c-r) + x_1 r(-2 + 2\eta + \sqrt{\eta^2 - 1} - \frac{2\eta}{\sqrt{\eta^2 - 1}} + \frac{\eta^2}{\sqrt{\eta^2 - 1}}) \right] \\
&\quad \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \cdot \exp[-S_{ab}(t_2)] \right\},
\end{align*}
\]

\[
\begin{align*}
f_2(q^2) &= 8\pi m_{B_s}^2 C_F \int dx_1dx_2 \int b_1db_1b_2db_2\phi_{B_s}(x_1, b_1)\phi_{D_s}(x_2, b_2) \\
&\times \left\{ [2 - 4x_2r(1-\eta)] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_3) \cdot \exp[-S_{ab}(t_1)] \\
&+ \left[ 4r - 2r_c - x_1 + \frac{x_1}{2}(2 - \eta) \right] \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \cdot \exp[-S_{ab}(t_2)] \right\},
\end{align*}
\]

where \( C_F = 4/3 \) is a color factor, \( r = m_{D_s}/m_{B_s}, \ r_c = m_c/m_{D_s} \) with the \( m_c \) is the mass of c-quark. The hard functions \( h(x_1, b_1) \), the hard scales \( t_{1,2} \) and the Sudakov factors \( S_{ab} \) will be given in the Appendix A.

For the charged current \( \bar{B}_s^0 \to D_s^+l^-\bar{\nu}_l \) decays, the quark level transitions are the \( b \to cl^-\bar{\nu}_l \) with the effective Hamiltonian:

\[
\mathcal{H}_{\text{eff}}(b \to cl\bar{\nu}_l) = \frac{G_F}{\sqrt{2}} V_{cb} \bar{c}\gamma_{\mu}(1 - \gamma_5)b \cdot \bar{\nu}_l \gamma^\mu(1 - \gamma_5)\nu_l,
\]

where \( G_F = 1.16637 \times 10^{-5} \text{GeV}^{-2} \) is the Fermi-coupling constant. With the two form factors \( F_+(q^2) \) and \( F_0(q^2) \), we can write down the differential decay rate of the decay mode \( \bar{B}_s^0 \to D_s^+l^-\bar{\nu}_l \) as[25]:

\[
\frac{d\Gamma(b \to cl\bar{\nu}_l)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_{B_s}^3} \left( 1 - \frac{m_l^2}{q^2} \right)^2 \frac{\lambda^{1/2}(q^2)}{2q^2} \cdot \left\{ 3m_l^2 (m_{B_s}^2 - m_{D_s}^2)^2 |F_0(q^2)|^2 \\
+ (m_l^2 + 2q^2) \lambda(q^2)|F_+(q^2)|^2 \right\},
\]
where $m_l$ is the mass of the leptons $e^-, \mu^-$ or $\tau^-$. For the cases of $l^- = (e^-, \mu^-)$, the corresponding mass terms $m_l^2$ could be neglected, the Eq. (21) then becomes very simple,

$$\frac{d\Gamma(b \to cl\bar{n}_l)}{dq^2} = \frac{G_F^2}{192\pi^3 m_B^2} \lambda^{3/2}(q^2)|V_{cb}|^2|F_+(q^2)|^2,$$

(22)

where $\lambda(q^2) = (m_{B_s}^2 + m_{D_s}^2 - q^2)^2 - 4m_{B_s}^2m_{D_s}^2$ is the phase space factor.

For the $B_s^0 \to D_s^*$ transitions, the hadronic matrix elements of the vector and axial-vector currents are described by the four QCD form factors $V(q^2)$ and $A_{0,1,2}(q^2)$ via [26]:

$$\langle D_s(p_2, e^*)|c(0)\gamma_\mu b(0)|\bar{B}_s^0(p_1)\rangle = \frac{2iV(q^2)}{m_{B_s} + m_{D_s}^*}\epsilon^{*\mu}\epsilon_{\mu}\langle p_1|P^0_{12}\rangle,$$

$$\langle D_s(p_2, e^*)|c(0)\gamma_\mu\gamma_5 b(0)|\bar{B}_s^0(p_1)\rangle = 2m_{D_s}A_0(q^2)\frac{\epsilon^* q}{q^2}q_\mu + (m_{B_s} + m_{D_s}^*)A_1(q^2)\left(\epsilon^* - \frac{\epsilon^* q}{q^2}q_\mu\right) - A_2(q^2)\frac{\epsilon^* q}{m_{B_s} + m_{D_s}^*}\left[(p_1 + p_2)_\mu - \frac{m_{B_s}^2 - m_{D_s}^2}{q^2}q_\mu\right].$$

(23)

In the pQCD approach, we find the form factors $V(q^2)$ and $A_{0,1,2}(q^2)$ for $B_s^0 \to D_s^*$ transition are of the form:

$$V(q^2) = 8\pi m_{B_s}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1)\phi_{D_s}^T(x_2, b_2) \cdot (1 + r) \times \left\{ [1 - r x_2] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \cdot \exp[-S_{ab}(t_1)] + \left[ r + \frac{x_1}{2\sqrt{\eta^2 - 1}} \right] \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \cdot \exp[-S_{ab}(t_2)] \right\},$$

(24)

$$A_0(q^2) = 8\pi m_{B_s}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1)\phi_{D_s}^F(x_2, b_2) \times \left\{ [1 + r - r x_2(2 + r - 2\eta)] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \cdot \exp[-S_{ab}(t_1)] + \left[ r^2 + r_c \frac{x_1}{2} + \frac{\eta x_1}{2\sqrt{\eta^2 - 1}} \right] + \frac{r x_1}{2\sqrt{\eta^2 - 1}} (1 - 2\eta(\eta + \sqrt{\eta^2 - 1})) \right\] \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \cdot \exp[-S_{ab}(t_2)] \right\},$$

(25)

$$A_1(q^2) = 8\pi m_{B_s}^2 C_F \int dx_1 dx_2 \int b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1)\phi_{D_s}^T(x_2, b_2) \cdot \frac{r}{1 + r} \times \left\{ 2[1 + \eta - 2r x_2 + r\eta x_2] \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \cdot \exp[-S_{ab}(t_1)] + [2r_c + 2\eta r - x_1] \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \cdot \exp[-S_{ab}(t_2)] \right\}.$$

(26)
\[ A_2(q^2) = \frac{(1 + r)^2(\eta - r)}{2r(\eta^2 - 1)} \cdot A_1(q^2) - 8\pi m_{B_s}^2 C_F \int dx_1 dx_2 \int b_1 db_1 db_2 \phi_{B_s}(x_1, b_1) \]
\[ \cdot \phi_{D_s^*}(x_2, b_2) \cdot \frac{1 + r}{\eta^2 - 1} \times \left\{ [(1 + \eta)(1 - r) - \eta x_2(1 - 2r + \eta(2 + r - 2\eta))] \right\} \]
\[ \cdot h_1(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_1) \cdot \exp[-S_{ab}(t_1)] \]
\[ + \left[ r + r_c(\eta - r) - \eta r^2 + r x_1 \eta^2 - \frac{x_1}{2}(\eta + r) + x_1(\eta r - \frac{1}{2})\sqrt{\eta^2 - 1} \right] \]
\[ \cdot h_2(x_1, x_2, b_1, b_2) \cdot \alpha_s(t_2) \cdot \exp[-S_{ab}(t_2)] \right\}, \quad (27) \]

where \( r = m_{D_s^*}/m_{B_s} \), while \( C_F \) and \( r_c \) is the same as in Eqs.(18,19). And the hard function \( h(x_i, b_i) \), the hard scales \( t_{1,2} \) and Sudakov factor \( S_{ab} \) are given in the Appendix A.

For \( \bar{B}_s^0 \rightarrow D_s^{*+} l^- \bar{\nu}_l \) decays, the corresponding differential decay widths can be written as [27]:

\[
\frac{d\Gamma_L(\bar{B}_s^0 \rightarrow D_s^{*+} l^- \bar{\nu}_l)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_{B_s}^3} \left( 1 - \frac{m_q^2}{q^2} \right)^2 \frac{\lambda^{1/2}(q^2)}{2q^2} \cdot \left\{ 3m_s^2 \lambda(q^2) A_0^2(q^2) \right. \\
+ \left. \left( m_{B_s}^2 - m^2 - q^2 \right)(m_{B_s} + m) A_1(q^2) - \frac{\lambda(q^2)}{m_{B_s} + m} A_2(q^2) \right\}^2, \quad (28) \]

\[
\frac{d\Gamma_\pm(\bar{B}_s^0 \rightarrow D_s^{*+} l^- \bar{\nu}_l)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 m_{B_s}^3} \left( 1 - \frac{m_q^2}{q^2} \right)^2 \frac{\lambda^{3/2}(q^2)}{2} \times \left\{ (m_s^2 + 2q^2) \left[ \frac{V(q^2)}{m_{B_s} + m} + \frac{(m_{B_s} + m) A_1(q^2)}{\sqrt{\lambda(q^2)}} \right]^2 \right\}, \quad (29) \]

where \( m = m_{D_s^*} \), and \( \lambda(q^2) = (m_{B_s}^2 + m_{D_s^*}^2 - q^2)^2 - 4m_{B_s}^2 m_{D_s^*}^2 \) is the phase space factor. The combined transverse and total differential decay widths are defined as:

\[
\frac{d\Gamma_T}{dq^2} = \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_\pm}{dq^2}, \quad \frac{d\Gamma}{dq^2} = \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_T}{dq^2}. \quad (30) \]

III. NUMERICAL RESULTS AND DISCUSSIONS

In the numerical calculations we use the following input parameters (here masses and decay constants are in units of GeV)[23, 28]

\[
m_{D_s^*} = 1.969, \quad m_{D_s^{*+}} = 2.112, \quad m_{B_s} = 5.367, \quad m_c = 1.35 \pm 0.03, \]
\[
m_r = 1.777, \quad |V_{cb}| = (39.54 \pm 0.89) \times 10^{-3}, \quad \Lambda_{MS}^4 = 0.287, \]
\[
f_{B_s} = 0.24 \pm 0.02, \quad f_{D_s} = 0.274, \quad \tau_{B_s^0} = 1.497 \times 10^{-12} s, \quad (31) \]

while \( f_{D_s^*} = f_{D_s}\sqrt{m_{D_s}/m_{D_s^*}} \).
A. The form factors in the pQCD approach

For the considered semileptonic decays, the differential decay rates strongly depend on the value and the shape of the relevant form factors $F_{0,+}(q^2)$, $V(q^2)$ and $A_{0,1,2}(q^2)$. The evaluation of these form factors play the key role in such works. The two well-known traditional methods of evaluating the form factors are the QCD sum rule for the low $q^2$ region and the Lattice QCD for the high $q^2$ region of $q^2 \approx q^2_{\text{max}}$.

In the pQCD factorization approach [13, 21, 29], one can also calculate the form factors perturbatively in the lower $q^2$ region [13]. For $B$ to light meson (such as $K, \pi, \rho, \eta^{(')}, \eta^{(')}, \eta^{(')}$ etc) transitions, the values of the relevant form factors have been evaluated successfully by employing the pQCD factorization approach for example in Refs.[13, 22–24, 26, 30, 31]. The pQCD predictions for the form factors obtained in these papers agree very well with those obtained from the QCD sum rule.

For $B \to D^{(*)}$ transitions, one usually use the heavy quark effective theory (HQET) to evaluate the form factors at the lower $q^2$ region, and consider the lattice QCD results at the higher $q^2$ region with $q^2 \approx q^2_{\text{max}}$. In Ref. [14], we evaluated the form factors for $B \to (D, D^*)$ transitions in the lower $q^2$ region and obtained the pQCD predictions for the ratios $R(D)$ and $R(D^*)$ being consistent with those measured by BaBar Collaboration. In this paper, by using the expressions as given in Eqs.(18-19,24-27) and the definitions in Eq. (17), we calculate the values of the form factors $F_{0}(q^2), F_{+}(q^2), V(q^2)$ and $A_{0,1,2}(q^2)$ for given value of $q^2$ in the lower $q^2$ region: $0 \leq q^2 \leq m^2$. For the form factors in the larger $q^2$ region, we make an extrapolation for them from the lower $q^2$ region to larger $q^2$ region: $m^2 < q^2 \leq q^2_{\text{max}}$ with $q^2_{\text{max}} = (m_{B_{s}}-m_{D_s})^2$ for $B_{s} \to D_{s}$ ($B \to D_s$) transition. In this work we make the extrapolation by using the formula as given in Ref. [27]

$$F(q^2) = \frac{F(0)}{1 - a \frac{q^2}{m_{H_{bs}}^2} + b \frac{q^2}{m_{H_{bs}}^2}}$$

where $F$ stands for the form factors $F_{0,+}, V, A_{0,1,2}$. The parameters $a$ and $b$ in above equation are determined by the fitting to the pQCD predicted values of the form factors at the sixteen points in the lower $q^2$ region, as illustrated explicitly in Fig.2 for the case of $F_{0,+}(q^2)$.

In Table I, we list the pQCD predictions for the form factors $F_{0,+}(q^2), V(q^2)$ and $A_{0,1,2}(q^2)$ and the corresponding parametrization constants “$a$” and “$b$” in Eq. (32) for $\bar{B}^0 \to (D_s, D^*_s)$ transitions at the scale $q^2 = 0$ and $q^2 = m^2$. The theoretical error of the form factors as shown in Table I is the total error: a combination of the three major theoretical errors from the uncertainties of the parameter $\omega_{B_{s}} = 0.50 \pm 0.05$ GeV, $f_{B_{s}} = 0.24 \pm 0.02$ GeV and $m_{c} = 1.35 \pm 0.03$ GeV. For the parametrization constants “$a$” and “$b$”, they do not depend on the variation of $f_{B_{s}}$, and the total errors are a combination of the two theoretical errors from the uncertainties of $\omega_{B_{s}}$ and $m_{c}$.

B. Differential decay widths and branching ratios

From the differential decay rates as given in Eqs. (21,22,28-30), it is straightforward to make the integration over the range of $m^2 \leq q^2 \leq (m_{B_{s}} - m)^2$ with $m = (m_{D_s}, m_{D^*_s})$. The pQCD predictions for the branching ratios of the semileptonic decays $\bar{B}^0 \to D^+_s \tau^- \bar{\nu}_\tau$ and $\bar{B}^0 \to D^+_s l^- \bar{\nu}_l$ are the following:

$$B(\bar{B}^0 \to D^+_s \tau^- \bar{\nu}_\tau) = (0.84^{+0.34}_{-0.23}(\omega_{B_{s}})^{+0.15}_{-0.13}(f_{B_{s}})^{+0.04}_{-0.01}(V_{cb})^{+0.03}_{-0.02}(m_{c}))\%,$$
$$B(\bar{B}^0 \to D^+_s l^- \bar{\nu}_l) = (2.13^{+0.37}_{-0.34}(\omega_{B_{s}})^{+0.07}_{-0.07}(f_{B_{s}})^{+0.10}_{-0.10}(V_{cb})^{+0.07}_{-0.07}(m_{c}))\%,$$

(33)
TABLE I. The pQCD predictions for the form factors $F_{0,\pm}(q^2)$, $V(q^2)$ and $A_{0,1,2}(q^2)$ at the scale $q^2 = 0$ and $q^2 = m_T^2$. The parametrization constants “a” and “b” are also listed in last two columns.

|       | $F(0)$  | $F(m_T^2)$ | $a$    | $b$    |
|-------|---------|------------|--------|--------|
| $F_{0}^{B_s \rightarrow D_s}$ | $0.55^{+0.15}_{-0.12}$ | $0.67^{+0.17}_{-0.13}$ | $1.69^{+0.06}_{-0.10}$ | $0.78^{+0.23}_{-0.32}$ |
| $F_{0}^{B_s^0 \rightarrow D_s^0}$ | $0.55^{+0.15}_{-0.12}$ | $0.74^{+0.19}_{-0.15}$ | $2.44^{+0.05}_{-0.08}$ | $1.70^{+0.18}_{-0.33}$ |
| $V_{0}^{B_s \rightarrow D_s}$ | $0.62^{+0.15}_{-0.12}$ | $0.83^{+0.18}_{-0.15}$ | $2.48^{+0.12}_{-0.16}$ | $1.66^{+0.12}_{-0.12}$ |
| $V_{0}^{B_s^0 \rightarrow D_s^0}$ | $0.47^{+0.11}_{-0.09}$ | $0.63^{+0.13}_{-0.11}$ | $2.49^{+0.09}_{-0.12}$ | $1.74^{+0.06}_{-0.00}$ |
| $A_{0}^{B_s \rightarrow D_s}$ | $0.49^{+0.12}_{-0.10}$ | $0.60^{+0.13}_{-0.11}$ | $1.64^{+0.09}_{-0.15}$ | $0.59^{+0.06}_{-0.13}$ |
| $A_{0}^{B_s^0 \rightarrow D_s^0}$ | $0.52^{+0.13}_{-0.10}$ | $0.67^{+0.15}_{-0.12}$ | $2.33^{+0.09}_{-0.16}$ | $1.81^{+0.00}_{-0.20}$ |

FIG. 2. The $q^2$-dependence of the form factors $F_{0,\pm}(q^2)$ in the pQCD approach for the case of $B_s \rightarrow D_s$ transition. The dots in (a) and (b) refer to the pQCD predictions for each given value of $q^2$ in the range of $0 \leq q^2 \leq m_T^2$.

\[
\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^{\mp} \tau^- \bar{\nu}_\tau) = (1.44^{+0.43}_{-0.34}(\omega_{B_s})^{+0.25}_{-0.23}(f_{B_s})^{+0.07}_{-0.06}(V_{cb})^{+0.07}_{-0.07}(m_c))\%,
\]

\[
\mathcal{B}(\bar{B}_s^0 \rightarrow D_s^{\mp} l^- \bar{\nu}_l) = (4.76^{+1.65}_{-1.25}(\omega_{B_s})^{+0.83}_{-0.76}(f_{B_s})^{+0.22}_{-0.21}(V_{cb})^{+0.18}_{-0.17}(m_c))\%,
\]

where the four major theoretical errors come from the uncertainties of the input parameters $\omega_{B_s} = 0.50 \pm 0.05$ GeV, $f_{B_s} = 0.24 \pm 0.02$ GeV, $|V_{cb}| = (39.54 \pm 0.89) \times 10^{-3}$ and $m_c = 1.35 \pm 0.03$ GeV.

In Figs. 3 and 4, we show the $q^2$-dependence of the theoretical predictions for the differential decay rates $d\Gamma/dq^2$ for $\bar{B}_s^0 \rightarrow D_s^{\mp} l^- \bar{\nu}_l$ and $\bar{B}_s^0 \rightarrow D_s^{\mp} l^- \bar{\nu}_l$ decays calculated by using the pQCD factorization approach.

In Table II, the pQCD predictions for the branching ratios of the considered decay modes are listed in column two where the theoretical errors from different sources have been added in quadrature. As a comparison, we also show the theoretical predictions as given previously in Refs. [15–20]. One can see from the numerical results as shown in Table II that:

(i) For the branching ratios, although the previous theoretical predictions basically agree within a factor of 2, they are rather different from each other. The reason is that these results
The theoretical predictions are given in Refs. \([15\text{--}20]\) and are listed as a comparison.

\[ |\Gamma| \text{ (in units of } 10^{-2}) \]

\[ \begin{array}{c|ccccccc}
\text{Channel} & \text{pQCD} & \text{CQM[15]} & \text{QCDSRs[16]} & \text{[17, 18]} & \text{IAMF[19]} & \text{RQM[20]} \\
\hline
\bar{B}_s^0 \to D_s^+ \tau^- \bar{\nu}_\tau & 0.84^{+0.38}_{-0.28} & -- & -- & 0.33^{+0.14}_{-0.11} & 0.47 - 0.55 & 0.62 \pm 0.05 \\
\bar{B}_s^0 \to D_s^+ l^- \bar{\nu}_l & 2.13^{+1.12}_{-0.77} & 2.73 - 3.00 & 2.8 - 3.8 & 1.0^{+0.4}_{-0.3} & 1.4 - 1.7 & 2.1 \pm 0.2 \\
\bar{B}_s^0 \to D_s^+ \tau^- \bar{\nu}_\tau & 1.44^{+0.51}_{-0.42} & -- & -- & 1.3^{+0.2}_{-0.1} & 1.2 - 1.3 & 1.3 \pm 0.1 \\
\bar{B}_s^0 \to D_s^+ l^- \bar{\nu}_l & 4.76^{+1.87}_{-1.49} & 7.49 - 7.66 & 1.89 - 6.61 & 5.2 \pm 0.6 & 5.1 - 5.8 & 5.3 \pm 0.5 \\
\end{array} \]

\( |\Gamma| \text{ (in units of } 10^{-2}) \)
C. The pQCD predictions for $R(X)$-ratios

It is straightforward to calculate the four $R(X)$ ratios of the branching ratios for $\bar{B}_s^0 \to D_s^{(*)} l \bar{\nu}_l$ decays by using the definitions as made previously in Eqs. (5,6), the corresponding pQCD predictions are listed in Table III. As a comparison, the pQCD predictions for the corresponding $R(X)$ ratios for $B \to D^{(*)} l \bar{\nu}_l$ decays as given in Ref. [14] are also shown in Table III. The errors of the pQCD predictions as given in Table III are the combination of the major theoretical errors come from the uncertainties of $\omega_{B_s} = 0.50 \pm 0.05$ GeV ($\omega_B = 0.40 \pm 0.04$ GeV) and $m_c = 1.35 \pm 0.03$ GeV, while those induced by the variations of $f_{B_s} (f_B)$ and $|V_{cb}|$ are canceled completely in the pQCD predictions for the $R(X)$-ratios.

| $R(D_s)$ | $R(D_s^*)$ | $R_{D_s}^{l\tau}$ | $R_{D_s}^{l\tau}$ |
|----------|------------|-------------------|------------------|
| 0.392 ± 0.022 | 0.302 ± 0.011 | 0.448^{+0.068}_{-0.041} | 0.582^{+0.074}_{-0.045} |
| $R(D)$ | $R(D^*)$ | $R_{D_s}^{l\tau}$ | $R_{D_s}^{l\tau}$ |
| 0.430^{+0.021}_{-0.020} | 0.301 ± 0.013 | 0.450^{+0.064}_{-0.051} | 0.642^{+0.084}_{-0.070} |

From the numerical results as listed in Table III one can see the following points:

(i) The theoretical errors of the pQCD predictions for the R(X) ratios are less than 13%, much smaller than those for the branching ratios. These ratios could be measured at the forthcoming Super-B experiments.

(ii) The ratio $R(D_s)$ and $R(D_s^*)$ are defined here in the same way as the ratios $R(D^{(*)})$ in Refs. [1,4]. These ratios generally measure the mass effects of heavy $m_\tau$ against the light $m_e$ or $m_\mu$.

(iii) The new ratio $R_{D_s}^{l\tau}$ will measure the effects induced by the variations of the form factors for $\bar{B}_s^0 \to D_s$ and $\bar{B}_s^0 \to D_{s}^{(*)}$ transitions. In other words, the new ratios $R_{D_s}^{l\tau}$ may be more sensitive to the QCD dynamics which controls the $\bar{B}_s^0 \to D_{s}^{(*)}$ transitions than the ratios $R(D_{s}^{(*)})$.

(iv) In the limit of $SU(3)_F$ flavor symmetry, the four ratios defined for $\bar{B}_s^0 \to D_{s}^{(*)} l \bar{\nu}_l$ decays should be very similar with the corresponding ones for $B \to D^{(*)} l \bar{\nu}_l$ decays. The pQCD predictions as listed in Table III do support this expectation. The breaking of $SU(3)_F$ flavor symmetry is less than 10%.

(v) At present, only the ratio $R(D)$ and $R(D^*)$ have been measured by Belle and BaBar [1,32–35]. In order to check if the BaBar’s anomaly of $R(D^{(*)})$ do exist or not for $\bar{B}_s^0 \to D_{s}^{(*)} l \bar{\nu}_l$ decays, and to test the $SU(3)_F$ flavor symmetry among $\bar{B}_s^0 \to D_{s}^{(*)} l \bar{\nu}_l$ and $B \to D^{(*)} l \bar{\nu}_l$ decays, we strongly suggest the forthcoming Super-B experiments to measure these four new ratios $R(D_{s}^{(*)})$ and $R_{D_s}^{l\tau}$. 

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IV. SUMMARY AND CONCLUSIONS

In this paper, we studied the semileptonic decays $\bar{B}_s^0 \to D_s^{(*)+} l^- \bar{\nu}_l$ in the framework of the SM by employing the pQCD factorization approach. We calculated the branching ratios $B(\bar{B}_s^0 \to D_s^{(*)+} l^- \bar{\nu}_l)$ and the four ratios of the branching ratios: $R(D_s)$, $R(D_s^*)$, $R_{D_s}^l$, and $R_{D_s}^r$. From the numerical results and phenomenological analysis we found that

(i) For the branching ratios $Br(\bar{B}_s^0 \to D_s^{(*)+} l^- \bar{\nu}_l)$, the pQCD predictions generally agree well with previous results obtained by using different methods to evaluate the relevant form factors.

(ii) For the four new ratios $R(D_s^{(*)})$ and $R_{D_s}^{l,r}$, the pQCD predictions are

$$
R(D_s) = 0.392 \pm 0.022, \quad R(D_s^*) = 0.302 \pm 0.011,
R_{D_s}^l = 0.448^{+0.058}_{-0.041}, \quad R_{D_s}^r = 0.582^{+0.071}_{-0.045}.
$$

(iii) The pQCD predictions do support the $SU(3)_F$ flavor symmetry between $\bar{B}_s^0 \to D_s^{(*)+} l^- \bar{\nu}_l$ and $B \to D^{(*)} l^- \bar{\nu}_l$ decay modes, while the breaking of $SU(3)_F$ flavor symmetry is less than 10%.

(iv) Based on our analysis, we strongly suggest the forthcoming Super-B experiments to measure the new ratios $R(D_s^{(*)})$ and $R_{D_s}^{l,r}$.

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Appendix A: Relevant functions

The threshold resummation factor $S_\ell(x)$ is adopted from [26]:

$$
S_\ell = \frac{2^{1+2c} \Gamma(3/2 + c)}{\sqrt{\pi} \Gamma(1 + c)} [x(1 - x)]^c,
$$

(A1)

and we here set the parameter $c = 0.3$. The hard functions $h_1$ and $h_2$ come form the Fourier transform and can be written as

$$
h_1(x_1, x_2, b_1, b_2) = K_0(\beta_1 b_1)[\theta(b_1 - b_2)I_0(\alpha_1 b_2)K_0(\alpha_1 b_1) + \theta(b_2 - b_1)I_0(\alpha_1 b_1)K_0(\alpha_1 b_2)]S_\ell(x_2),
$$

(A2)

$$
h_2(x_1, x_2, b_1, b_2) = K_0(\beta_2 b_1)[\theta(b_1 - b_2)I_0(\alpha_2 b_2)K_0(\alpha_2 b_1) + \theta(b_2 - b_1)I_0(\alpha_2 b_1)K_0(\alpha_2 b_2)]S_\ell(x_2),
$$

(A3)

where $K_0$ and $I_0$ are modified Bessel functions, and

$$
\alpha_1 = m_{B_s} \sqrt{x_2 r \eta^+}, \quad \alpha_2 = m_{B_s} \sqrt{x_1 r \eta^+ - r^2 + r_c^2}, \quad \beta_1 = \beta_2 = m_{B_s} \sqrt{x_1 x_2 r \eta^+},
$$

(A4)
where $r = m_{D(*)}/m_{B_s}$, $r_c = m_c/m_{B_s}$.

The factor $\exp[-S_{ab}(t)]$ contains the Sudakov logarithmic corrections and the renormalization group evolution effects of both the wave functions and the hard scattering amplitude with $S_{ab}(t) = S_B(t) + S_M(t)$ [26, 30],

$$S_B(t) = s \left( x_1 \frac{m_{B_s}}{\sqrt{2}}, b_1 \right) + \frac{5}{3} \int_{1/b_1}^t \frac{d\mu}{\mu} \gamma_q(\alpha_s(\bar{\mu})), \quad (A5)$$

$$S_M(t) = s \left( x_2 \frac{m_{B_s}}{\sqrt{2}} r_0, b_2 \right) + s \left( (1 - x_2) \frac{m_{B_s}}{\sqrt{2}} r_0, b_2 \right) + 2 \int_{1/b_2}^t \frac{d\mu}{\mu} \gamma_q(\alpha_s(\bar{\mu})), \quad (A6)$$

with the quark anomalous dimension $\gamma_q = -\alpha_s/\pi$. The explicit expressions of the functions $s(Q, b)$ can be found for example in Appendix A of Ref. [30]. The hard scales $t_i$ in Eqs. (19-27) are chosen as the largest scale of the virtuality of the internal particles in the hard $b$-quark decay diagram,

$$t_1 = \max \{ m_{B_s} \sqrt{x_2 r_0}, 1/b_1, 1/b_2 \},$$

$$t_2 = \max \{ m_{B_s} \sqrt{x_1 r_0} - r^2 + r_c^2, 1/b_1, 1/b_2 \}. \quad (A7)$$

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