Feynman Graphs from D-Particle Dynamics

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Abstract

It is argued that quantum propagation of D-particles in the limit $\alpha' \to 0$ can represent the "joining-splitting" processes of Feynman graphs of a certain field theory in the light-cone frame. So basically it provides the possibility to define a field theory by its Feynman graphs. The application of this observation to define M-theory by an energetic expansion approach is discussed.

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Perturbation string theory is defined as sum over different topologies of the string world-sheet with weights proportional to the action of string. The perturbation series can be presented as sum over different graphs which describe the "joining-splitting" (JS) processes of strings as events in space-time. In fact these graphs are those which should be obtained as Feynman graphs of a string field theory. In spite of the absence of a string field theory some limits of these graphs, especially those produced by small strings ($\alpha' \to 0$) correspond to well known particle field theories namely various (super) gravity and gauge theories. So basically one expects to have a similar interpretation for the particle field theory graphs as events representing JS processes of particles in space-time. This expectation will be motivated more when one notes that all of the above achievements of string theory to produce different field theories is only at a first quantised description of strings.

Recently through the developments of the understanding of M-theory as a M(atrix) model [1], a description of perturbative string theory was achieved based on a non-Abelian gauge theory living on a cylinder which sometimes can be interpreted as the world-sheet of free strings. This description was mentioned in [2, 3] and more concreted in [4]. This picture have been checked in bosonic and super string theory in [5] and [6] respectively. Also in [7] (see also [8]) it was argued that configurations with different lengths of strings and their JS processes are corresponded to various backgrounds of world-sheet (gauge) fields. So by such description based on gauge theory world-sheet for JS processes of strings it is tempting to find a similar one for D-particles.

It is argued in this letter that different JS processes of D-particles have the potential to produce and be corresponded to certain particle field theory graphs and their related amplitudes, but in the light-cone frame. So it provides the possibility to define a particle field theory by its Feynman graphs, those generated by JS processes of D-particles.
Discussions on the question “what field theory?” are presented based on M(atrix) theory interpretation of D-particles as super-gravitons in the light-cone frame.

Dp-branes are p dimensional objects which are defined as (hyper)surfaces which can trap the ends of strings \[\mathbb{F}]. One of the most interesting aspects of D-brane dynamics appears in their coincident limit. In the case of coinciding \(N\) Dp-branes in a (super)string theory, their dynamics is captured by a dimensionally reduced \(U(N)\) (S)YM theory from \((9)25+1\) to \(p+1\) dimensions of Dp-brane world-volume \[\mathbb{F}].

In case of D-particle \(p = 0\), the above dynamics reduces to quantum mechanics of matrices because only time exists in the world-line. The bosonic part of the corresponding Lagrangian is \[\mathbb{F}\]

\[
L = m_0 \text{Tr} \left( \frac{1}{2} D_t X_i^2 + \frac{1}{(2\pi\alpha')^2} [X_i, X_j]^2 \right),
\]

where \(\frac{1}{2\pi\alpha'}\) and \(m_0 = (\sqrt{\alpha'} g_s)^{-1}\) are the string tension and the mass of D-particles, with \(g_s\) as string coupling. Here \(D_t = \partial_t - i A_0\) acts as covariant derivative in the 0+1 dimensional gauge theory.

For \(N\) D-particles \(X\)'s are in adjoint representation of \(U(N)\) and have the usual expansion \(X_i = x_{ia} T_a\), \(a = 1, \ldots, N^2\).

Firstly let us search for D-particles in the above Lagrangian:

For each direction \(i\) there are \(N^2\) variables and not \(N\) which one expects for \(N\) particles. Although there is an ansatz for the equations of motion which restricts the \(T_{(a)}\) basis to its \(N\) dimensional Cartan subalgebra. This ansatz causes vanishing the potential and one finds the equations of motion for \(N\) free particles. In this case the \(U(N)\) symmetry is broken to \(U(1)^N\) and the interpretation of \(N\) remaining variables as the classical (relative) positions of \(N\) particles is meaningful. The centre of mass of D-particles is represented by the trace of the \(X\) matrices and easily can be seen that the center of mass momentum is conserved.

In the case of unbroken gauge symmetry, the \(N^2 - N\) non-Cartan elements have a stringy interpretation, governing the dynamics of low lying oscillations of strings stretched between D-particles. Although the gauge transformations mix the entries of matrices and the interpretation of positions for D-particles remains obscure \[\mathbb{F}\], but even in this case the centre of mass is meaningful and the ambiguity about positions only comes back to the relative positions of D-particles.

Let us concentrate on the limit \(\alpha' \to 0\) \[\mathbb{F}\]. In this limit to have a finite energy one has

\[
[X_i, X_j] = 0, \quad \forall \ i, j,
\]

and consequently vanishing the potential term in the action. So D-particles do not interact and the action reduces to the action of \(N\) free particles \[\mathbb{F}\]

\[
S = \int dt \sum_{a=1}^{N} \frac{1}{2} m_0 \dot{x}_a^2.
\]

\[\text{Footnotes:}
\footnote{1}{Here this limit is analogous of the limit \(g_s \to 0\) in \[\mathbb{F}\].}
\footnote{2}{Repeatedly we forget \(A_0\) even in path integrals.}
\]

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But the above observation fails in the times which D-particles arrive each other. When two D-particles come very near each other two eigenvalues of $X_i$ matrices will be equal and this make the possibility that the corresponding off-diagonal elements take non-zero values. In fact this is the same story of gauge symmetry restoration. In summary one may deduce that in the limit $\alpha' \to 0$ D-particles do not interact with each other except for when they coincide.

For two coincident D-particles one may write the corresponding Lagrangian and Hamiltonian as

$$S = \int dt \left( \frac{1}{2} (2m_0) \dot{X}^2 + L_{\text{int}}(\dot{x}_a, \dot{\hat{x}}_a) \right), \quad a = 1, 2, 3,$$

$$H = \frac{p^2}{2(2m_0)} + H_{\text{int}}(\dot{x}_a, \dot{\hat{p}}_a),$$

which $X$ and $P$ are the position and momentum of the center of mass and $\hat{x}_a$’s and $\hat{p}_a$’s are the non-Abelian (not only non-Cartan) positions and momentums. $L_{\text{int}}$ and $H_{\text{int}}$ are responsible for interactions.

**Amplitudes in $\alpha' \to 0$ limit:**

Take the probability amplitude corresponded to detecting two D-particles in positions $x_1$ and $x_2$ at time $t_i$ and in $x_3$ and $x_4$ at time $t_f$, presented by path integral as

$$< x_3, x_4; t_f|x_1, x_2; t_i > = \int e^{-S}. \quad (6)$$

In the limit $\alpha' \to 0$ in that parts of paths which D-particles are not coincident, only the diagonal matrices have contribution to the path integral. This is because of large value of action in the exponential. So the action in the path integral reduces to the action of two free D-particles for non-coincident paths, i.e. $(x_1, x_2)$ till $X_1$ in Fig.1. Accordingly one may write, Fig.1 \footnote{3Here as the same which one does in field theory we have dropped the dis-connected graphs. Also in Fig.1 many other graphs with different number of loops could be drawn. We will come back to them later.},

$$< x_3, x_4; t_f|x_1, x_2; t_i > = \left[ \int e^{-S} \right]_{\alpha' \to 0} = \int_{t_i}^{t_f} (dT_1 dT_2) \int_{-\infty}^{\infty} (dX_1 dX_2)$$

\[ \text{Figure 1: A typical tree path.} \]
\[
\times \left(K_{m_0}(X_1, T_1; x_1, t_i)K_{m_0}(X_1, T_1; x_2, t_i)\right)
\times \left(K_{2m_0}(X_2, T_2; X_1, T_1)K_{int}(\hat{x}_2a, T_2; \hat{x}_1a, T_1)\right)
\times \left(K_{m_0}(x_3, t_f; X_2, T_2)K_{m_0}(x_4, t_f; X_2, T_2)\right),
\]
which \(K_m(y_2, t_2; y_1, t_1)\) is the non-relativistic propagator of a free particle with mass \(m\) between \((y_1, t_1)\) and \((y_2, t_2)\), and \(K_{int}\) is the corresponding propagator for the non-Abelian path integrations. In the above relation \(\int dT_1 dT_2 dX_1 dX_2\) is for a summation over different JS times and points.

By translating the above to momentum space using \(E_k = \frac{p_k^2}{2m_0}, k = 1, 2, 3, 4\)

\[
< p_3, p_4; t_f|p_1, p_2; t_i > \sim e^{i(E_3 + E_4)t_f - i(E_1 + E_2)t_i}
\times \int \prod_{k=1}^{4} d^4x_k e^{i(p_1x_1 + p_2x_2 - p_3x_3 - p_4x_4)} \langle x_3, x_4; t_f|x_1, x_2; t_i >, \tag{8}
\]
and doing integrations one finds (for \(t_i = -\infty\) and \(t_f = \infty\)) \(^4\)

\[
< p_3, p_4; \infty|p_1, p_2; -\infty > \sim \delta^{(d)}(p_1 + p_2 - p_3 - p_4)\delta(E_1 + E_2 - E_3 - E_4)
\times \lim_{\epsilon \rightarrow 0^+} \sum_n C_n \frac{1}{E - \frac{q^2}{2m_0} - E_n + i\epsilon}, \tag{9}
\]
where \(q = \vec{p}_1 + \vec{p}_2\) and \(E = E_1 + E_2\). Now by recalling the energy-momentum relation in the light-cone frame for a particle with mass \(M\)

\[
E \equiv P_+ = \frac{\vec{p}^2 + M^2}{2P_-},
\]
one sees that the fraction in the sum of (9) is the "light-cone" propagator \(^3\) of a particle by identifications \(^5\)

\[
P_- = 2m_0, \quad M_n^2 = 4m_0 E_n. \tag{10}
\]

\(^4\) We use in \(d\) dimensions the representations

\[
K_m(y_2, t_2; y_1, t_1) = \theta(t_2 - t_1) \frac{1}{(2\pi)^d} \int d^d p \exp \left(\frac{ip \cdot (y_2 - y_1) - ip^2(t_2 - t_1)}{2m}\right),
\]

\[
K_{int}(\hat{x}_2a, T_2; \hat{x}_1a, T_1) = \int d\hat{x}_a e^{-S_{int}[T_2, T_1]} = \sum_n <\hat{x}_{2a}|n > < n|\hat{x}_{1a} > e^{-iE_n(T_2 - T_1)},
\]
where \(E_n\)'s are the eigenvalues of \(H_{int}\) of (3) and \(\theta(t_2 - t_1)\) is the step function.

\(^5\) In the supersymmetric case of M(atrix) theory there is an easy answer about the least value of \(E_n\)'s, i.e. \(E_0\). D-particles can make marginal bound states \((E_0 = 0)\) and so: \(M_0 = 0\). It is an important ingredient when one wants to identify the above graphs with those ones which come from supergravity in light-cone frame.
The first relation learns to us that each D-particle has light-cone momentum equal to $m_0$ to have the light-cone momentum for two of them $2m_0$ in the time interval $[T_1, T_2]$ in Fig.1.

Now one sees that (9) is the same expression which one writes (in momentum space) as tree diagram contribution to 4-point function of a field theory but in the light-cone frame [13], with exchanging masses as $M_n$’s.

In a more covariant form one may write for (9)

$$<< p_3^\mu, p_4^\mu; \infty|p_1^\mu, p_2^\mu; -\infty > \sim \delta(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4)\delta(p_{1+} + p_{2+} - p_{3+} - p_{4+})$$

$$\times \lim_{\epsilon \to 0^+} \sum_n C_n \frac{i4m_0}{q^\mu q^\mu - M_n^2 + i\epsilon},$$

where $q^\mu = p_1^\mu + p_2^\mu$ and

$$V^\mu \equiv \left(\frac{V_+ + V_-}{\sqrt{2}}, \frac{V_+ - V_-}{\sqrt{2}}, \vec{V}\right), \mu = 0, d + 1, 1, 2, ..., d,$$

$$p_{k^+} \equiv \frac{\vec{p}_k^0}{2m_0}, \quad p_{k^-} \equiv m_0, \quad k = 1, 2, 3, 4.$$

The longitudinal momentum conservation trivially is satisfied. Besides because of conservation of this momentum one can not expect so-called $t$-channel processes.

**Loop diagrams:**

As is apparent loop-ed paths have contributions to the path integral (9). In fact the graph in Fig.1 is the first connected graph. One can justify that generically the probability amplitudes associated to the loop graphs is the same which one writes in field theory Fig.2.

There is an important difference between tree and loop graphs related to their contributions to the path integral. Because of free moving of D-particles in times which D-particles are not coincident there is no classical solution of equations of motion which can be (topologically) corresponded with a graph with loop(s). But it is not the case for tree diagrams, i.e. there are classical solutions which are corresponded with tree paths. So the dominant contribution to the path integral is due to the tree diagrams; the diagrams which are corresponded to classical solutions or some deviations around them. This also indicates a possible well defined perturbative expansion for the graphs, because graphs with more vertices have less contribution to the amplitudes.
Field theory of Feynman graphs:
What field theory is corresponded to the above graphs? Because of the large number kinds of graphs the answer seems highly hard and may have only a definite answer for some subsets of graphs. The analogue case in string theory as mentioned in the beginning of this letter is the subset of graphs corresponded with the limit $\alpha' \to 0$.

A candidate can be guessed by M(atrix) theory approach to M-theory \[1\]. M-theory and type IIA string theory both have supergravities in their low energy limit, but in different dimensions (11 and 10 respectively). Also at present we have a gauge theoretic description of stringy "Feynman Graphs" \[2, 3, 4\]. Since the 10 dimensional (IIA) supergravity is corresponded to stringy "Feynman Graphs" in the limit $\alpha' \to 0$, so one may expect to have a similar program for D-particle "Feynman Graphs" and 11 dimensional supergravity. So (after adding sufficient supersymmetry and extracting tensorial structures) one should show that a few first terms in expansions like (9) or (11) are the same which come from 11D supergravity in the light-cone frame \[6\]. One may summerize this discussion in Fig.3.

Evidence for the above guess needs more graph-ology.

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$^6$I feel a close connection between this discussion and the formula (2.6) of \[14\].
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