TEST OF QCD ANALYTIC PREDICTIONS FOR
GLUON AND QUARK JET DIFFERENCES

J. WILLIAM GARY
Department of Physics, University of California,
Riverside, California 92521, USA
e-mail: bill.gary@cern.ch, william.gary@ucr.edu

1 Introduction and analysis method

Differences between the properties of gluon and quark jets have been convincingly
established by experiments operating at the e+e− collider LEP at CERN. The most
conclusive results have been from so-called Y events, which are three jet events in
which the angle between the highest energy jet and each of the two lower energy
jets is about 150°. Table 1 summarizes the results for the mean charged particle
multiplicity ratio between gluon and quark jets, r_{ch}, found using Y events at LEP. A
60% variation in the value of r_{ch} is observed if the Cone or JADE-E0 jet finders are
used to identify the three jet events instead of the k_{T} (“Durham”) jet finder.

QCD analytic predictions exist for the ratio of the mean particle multiplicity
between gluon and quark jets. A quantitative test of these predictions has not been
possible, however, because of differences between the theoretical and experimental
definitions of the jets and event samples. The analytic calculations employ definitions
of the event samples and jets which are entirely inclusive. For the calculations, two
samples of events are chosen: a sample of gluon-gluon gg events produced from a color
singlet point source is used to define the gluon jet properties and a sample of quark-
antiquark q\overline{q} events produced under the same circumstances is used to define
the quark jet properties. The gluon and quark jet characteristics are given by inclusive
sums over the particles in these two samples. Thus, the theoretical results are not
restricted to three-jet events and do not employ a jet finder to assign particles to the
jets, in contrast to the Y event studies and other previous experimental studies of
 gluon and quark jet differences at high energy colliders.

The experimental difficulty in obtaining a jet definition corresponding to the the-
etorical one lies in the gluon jet sample, since gg production from a point source does
not occur naturally in e+e− annihilations. In contrast, the q\overline{q} sample employed by
the theory is the inclusive e+e− multihadronic one and so is readily available. In ref. 9,
a method was proposed for LEP experiments to identify gluon jets using an inclusive
definition similar to that used for the analytic calculations. The method is based on
rare events of the type e+e− → q\overline{q} g_{incl}, in which the q and \overline{q} are identified quark jets
which appear in the same hemisphere of an event. The quantity g_{incl}, taken to be the
 gluon jet, is defined by the sum of all particles observed in the hemisphere opposite to
that containing the q and \overline{q}. In the limit that the q and \overline{q} are collinear, the gluon jet
g_{incl} is produced under the same conditions as the gluon jets in gg events from a color

---

*Talk given at the XXVI International Symposium on Multiparticle Dynamics, Faro, PORTU-
GAL, September 1-5, 1996.*
singlet point source. The jets \( g_{incl} \) therefore correspond closely to single gluon jets in gg events, defined by dividing the gg events in half using the plane perpendicular to the principal event axis.

Although gg events from a color singlet point source are not produced in \( e^+e^- \) annihilations, they may be generated using QCD Monte Carlo programs such as the Herwig and Jetset parton shower models. To illustrate that the definition of gluon jets \( g_{incl} \) from the \( e^+e^- \) events corresponds closely to that employed by theory from the gg events, the Monte Carlo is used to calculate the mean charged particle multiplicity in gg events, \( \langle n_{ch.} \rangle_{gg\ events} \), and the results are divided by two so that they correspond to a single hemisphere: these results are shown as a function of the jet energy \( E_{jet} = E_{c.m.}/2 \) by the solid and dashed lines in Fig. 1(a). Shown in comparison, by the cross and diamond symbols, are the results for \( g_{incl} \) jets from \( e^+e^- \) events generated using Herwig and Jetset, respectively. The results for the gg and \( g_{incl} \) samples are seen to agree well over the entire jet energy range from 5 GeV to 5 000 GeV, i.e. \( \langle n_{ch.} \rangle_{g_{incl}} \approx \frac{1}{2} \langle n_{ch.} \rangle_{gg\ events} \) for both Herwig and Jetset. This establishes that the properties of the \( g_{incl} \) sample derived from the \( e^+e^- \) data do indeed correspond closely to those of gg events generated from a color singlet point source, confirming the results of ref. 9.

Below, the results of a recent study which uses this analysis method are presented. The data sample employed is that of the OPAL detector at LEP. Details of the event selection are presented in ref. 12. In total, 278 events are selected for the final gluon jet \( g_{incl} \) event sample. With the final cuts, Jetset with detector simulation and the same selection criteria as are applied to the data predicts that both jets in the tagged hemisphere are quark jets with \((83.0 \pm 1.7)\%\) probability, where the uncertainty is statistical: this is the estimated purity of the \( g_{incl} \) gluon jet sample. The mean energy of the gluon jets, \( \langle E \rangle_{g_{incl}} \), is determined to be \( \langle E \rangle_{g_{incl}} = 39.2 \pm 0.3 \) (stat.) \( \pm 1.8 \) (syst.) GeV. The uds quark jets in the study are defined inclusively using particles observed in event hemispheres opposite to those containing identified uds jets. The selection criteria are described in ref. 12. In total, 28 007 uds hemispheres are tagged. The estimated uds purity of the sample, obtained by treating Jetset events with detector simulation in the same manner as the data, is \((93.2 \pm 0.2)\%\), where the uncertainty is statistical. The energy of the uds jets is given by the beam energy, 45.6 GeV.

### 2 Results

The mean charged particle multiplicity value measured for the gluon jets is \( \langle n_{ch.} \rangle_{g_{incl}} = 14.63 \pm 0.38 \) (stat.) \( \pm 0.59 \) (syst.). The corresponding result for the uds jets is \( \langle n_{ch.} \rangle_{uds\ hemis.} = 10.05 \pm 0.04 \) (stat.) \( \pm 0.23 \) (syst.). Before forming the ratio between the gluon and uds jet measurements, it is necessary to account for the different energies of the two samples: the gluon jets have a mean energy of 39.2 GeV while the uds jets have a mean
energy of 45.6 GeV. To account for the larger energy of the uds jets, the QCD analytic formula for the evolution of the mean event multiplicity in $e^+e^-$ annihilations is employed. Assuming $n_f=5$, the QCD evolution formula predicts the mean multiplicity in 78.4 GeV events to be $(6.2 \pm 0.4)$% smaller than in 91.2 GeV events, where the uncertainty results from the maximum variation found by using the jet energies (39.2 and 45.6 GeV) rather than the event energies, $n_f=3$ rather than $n_f=5$, and varying the value of $\alpha_S$ within its allowed range. Applying a multiplicative correction of $0.938 \pm 0.004$ to the 45.6 GeV uds jet measurement presented above yields $\langle n_{ch}\rangle_{uds\ hems.}^{39.2\ GeV}=9.43 \pm 0.06\ (stat.) \pm 0.22\ (syst.)$ for the mean multiplicity of 39.2 GeV uds jets.

The result for the multiplicity ratio $r_{ch.}$ between 39 GeV gluon and quark jets is therefore found to be:

$$r_{ch.} \equiv \frac{\langle n_{ch.}\rangle_{g_{incl.}}}{\langle n_{ch.}\rangle_{uds\ hems.}^{39.2\ GeV}} = 1.552 \pm 0.041\ (stat.) \pm 0.061\ (syst.)$$

(1)

The systematic uncertainty is discussed in ref. It is to be noted that one of the systematic variations consists in using the Cone or JADE-E0 jet finders to tag quark jets for the $g_{incl.}$ identification, rather than the $k_T$ jet finder: only a small change of $\Delta r_{ch.}\approx0.02$ occurs as a result of this variation. Thus, the result for $r_{ch.}$ changes by only about 4% if different jet finders are employed for the analysis, in contrast to the $Y$ events for which a difference of about 60% is observed (Table I). This
emphasizes that the result (1) is only weakly dependent on a jet finding algorithm. This measurement of $r_{ch}$ is shown by the solid point in Fig. 1(b). The experimental statistical uncertainty is indicated by the small horizontal bars.

Various analytic results exist for the ratio $r \equiv \langle n \rangle_{\text{gluon}} / \langle n \rangle_{\text{quark}}$ of the mean number of partons in a gluon jet to that in a quark jet. The original results, valid to leading order, predict $r$ to be $r = \frac{C_A}{C_F} = 9/4$. Later, higher order corrections valid to the next-to-next-to-leading order were found to reduce this result by about 10%. These results do not incorporate energy conservation into the quark and gluon branching processes. Recently, $r$ has been calculated including not only the next-to-next-to-leading order terms but also energy conservation, and is found to be reduced yet further in magnitude. Momentum conservation is not included in this latter calculation, however: therefore energy-momentum conservation is only approximate. The analytic results for $r$, valid to leading order (l.o.), to next-to-next-to leading order (n.n.l.o.), and including approximate energy-momentum conservation (n.n.l.o., E-cons.) are shown in Fig. 1(b) as a function of $E_{\text{jet}} = \frac{E_{\text{c.m.}}}{2}$. For the evaluation of the strong coupling constant, $\alpha_S$, the values $n_f=5$ and $\Lambda_{\text{MS}}^{(n_f=5)} = 0.209$ GeV have been used, where $n_f$ is the number of active quark flavors. The results for the two n.n.l.o. calculations are shown as bands: the upper edges of the bands show the results if the energy scale used to evaluate $\alpha_S$ is taken to be $E_{\text{c.m.}}$, while the lower edges show the results if this energy scale is taken to be $E_{\text{c.m.}}/4$. The widths of the bands therefore indicate the level of uncertainty associated with the ambiguity of the energy scale at which to evaluate $\alpha_S$. The predictions of Herwig and Jetset at both the parton and hadron levels are included in Fig. 1(b). The width of the bands for the Monte Carlo results shows the variation which occurs if the shower cutoff parameters are allowed to vary within their allowed ranges.

For jet energies of 39 GeV, the n.n.l.o. calculation which incorporates energy conservation predicts values of $r$ between 1.83 (if $n_f=3$ and the energy scale of $\alpha_S$ is $E_{\text{c.m.}}$) and 1.64 (if $n_f=5$ and the energy scale is $E_{\text{c.m.}}/4$): this last value is only slightly above the measured result given above in relation (1). The analytic result is valid for quarks and gluons while the measurement refers to charged hadrons. Jetset predicts a hadronization correction for $r_{ch}$, defined by the ratio of the parton to hadron level curves in Fig. 1(b) (for $E_{\text{jet}} = 39$ GeV), of 0.91. The corresponding prediction from Herwig is 1.02. In this sense, the hadronization correction can be estimated to be about unity and to have an uncertainty of about 10%. Given the ambiguities of the energy scale at which to evaluate $\alpha_S$, of the number of active flavors $n_f$, of the hadronization correction, and due to the approximate nature by which energy-momentum conservation is included, is it seen that the analytic calculation of Dremin, Hwa and Nechitailo is in general agreement with the measurement. In contrast, the analytic results which do not incorporate energy conservation are seen to be in clear disagreement with this measurement, even considering the theoretical ambiguities.

3 Summary

The first quantitative test of analytic predictions for the ratio of the mean particle multiplicity between gluon and quark jets has been presented. The technique that allows this test is the identification of gluon jets in an inclusive manner, by all the particles observed in the event hemisphere opposite to a hemisphere containing a tagged quark and antiquark jet. The resulting definition is in close correspondence to the definition of gluon jets in analytic calculations, for the first time in the analysis of high energy $e^+e^-$ data. In this study, the gluon jet measurement, valid for 39 GeV
jets, is compared to the corresponding measurement from light quark (uds) jets, which has also been defined inclusively.

The result for the ratio $r_{ch}$, the mean charged particle multiplicity of gluon jets divided by the corresponding value for uds quark jets, is $r_{ch} = 1.552 \pm 0.041 \text{ (stat.)} \pm 0.061 \text{ (syst.)}$. This result is substantially smaller than the predictions of analytic calculations which do not include energy conservation in the parton branchings. A recent analytic calculation$^8$, which incorporates approximate energy-momentum conservation predicts a parton level multiplicity difference between 39 GeV gluon and quark jets in the range from about 1.64 to 1.83, depending on the choice for the energy scale $Q$ at which the strong coupling constant $\alpha_s(Q)$ is evaluated and on the number of active quark flavors, $n_f$. This latter prediction is in overall agreement with the measurement, given the uncertainties due to the approximate nature of energy-momentum conservation in the calculation, missing higher order terms, the energy scale, the number of active flavors $n_f$, and hadronization.

4 References

1. See, for example, J. W. Gary, “Quark and Gluon Jet Differences from Symmetric Events at LEP”, Proceedings of the XXV International Symposium on Multiparticle Dynamics, Stará Lesná Slovakia; September 12-16, 1995, Eds: D. Bruncko, L. Sandor and J. Urban, World Scientific;
J. Fuster and S. Martí, “Charged Particle Production in the Fragmentation of Quark and Gluon Jets”, Proceedings of the International Europhysics Conference on High Energy Physics, Brussels, Belgium, July 27-August 2, 1995, Eds. J. Lemomne, C. Vander Velde and F. Verbeure, World Scientific.
2. OPAL Collaboration, M. Z. Akrawy et al., Phys. Lett. B261 (1991) 334.
3. ALEPH Collaboration, D. Buskulic et al., CERN-PPE/95-184.
4. DELPHI Collaboration, P. Abreu et al., Z. Phys. C70 (1996) 179.
5. OPAL Collaboration, R. Akers et al., Z. Phys. C68 (1995) 179.
6. S.J. Brodsky and J. Gunion, Phys. Rev. Lett. 37 (1976) 402;
K. Konishi, A. Ukawa and G. Veneziano, Phys. Lett. B78 (1978) 243.
7. A. H. Mueller, Nucl. Phys. B241 (1984) 141;
J. B. Gaffney and A.H. Mueller, Nucl. Phys. B250 (1985) 109;
E. D. Malaza and B.R. Webber, Nucl. Phys. B267 (1986) 702.
8. I. M. Dremin and R. C. Hwa, Phys. Lett. B324 (1994) 477;
I. M. Dremin and V. A. Nechitailo, Modern Phys. Lett. A9 (1994) 1471.
9. J. W. Gary, Phys. Rev. D49 (1994) 4503.
10. G. Marchesini, B.R. Webber et al., Comp. Phys. Comm. 67 (1992) 465.
11. T. Sjöstrand, Comp. Phys. Comm. 82 (1994) 74;
T. Sjöstrand, CERN-TH.7112/93 (revised August 1995).
12. OPAL Collaboration, G. Alexander et al., Phys. Lett. B388 (1996) 659.
13. A. H. Mueller, Nucl. Phys. B213 (1983) 85;
A. H. Mueller, Nucl. Phys. B228 (1983) 351;
B. R. Webber, Phys. Lett. B143 (1984) 501.
14. L. Montanet et al., Phys. Rev. D50 (1994) 1173 and 1995 off-year partial update for the 1996 edition available on the PDG WWW pages (URL: http://pdg.lbl.gov/).