A Multi-objective Tracking Task Assignment Algorithm Based on Game Theory

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Abstract. Aiming at the relationship between competition and cooperation among sensors in multi-target tracking task allocation, this paper proposes a game theory-based method to allocate sensor resources for multi-target tracking tasks. The dissertation first establishes a target tracking model and designs a game theory payment function based on the model. Then a multi-sensor multi-target tracking task allocation game model is designed. Finally, a multi-target tracking task allocation algorithm based on game theory is given. Compared with other allocation algorithms, the average tracking error of the proposed allocation algorithm is reduced from 68.5630m to 19.7601m through experiments.

Keywords: multi-target tracking; multi-sensor; game theory; payoff function; game model.

1. Introduction
Under the strong support of today’s advanced science and technology, modern war has developed into the form of information. Both the enemy and the enemy can carry out multi-level and multi-directional attacks. In the process of combat, it is necessary to provide the positions of multiple enemy fighters in real time, so as to monitor the battlefield space and early warn and intercept the enemy fighters. Therefore, the problem of multi-target tracking task allocation is brought forward. According to the theory of information fusion [1, 2], the more sensors are allocated to the same target, the more information of the target can be obtained, and then the tracking performance of the system to the target can be improved. However, the number of sensors in the battlefield environment is often limited. If one target obtains more sensors, sensor resources of other targets will be reduced [3, 4]. How to allocate sensor resources to multiple targets to meet the tracking performance requirements has become one of the research hotspots.

There are usually some objective functions to measure the tracking performance of sensors in multi-sensor multi-target tracking task allocation method. According to these objective functions, the optimal allocation scheme for a specific target can be calculated under the condition of balancing the tracking performance of other targets. It is a very effective method to solve the problem of competition and cooperation with game theory [5]. In order to solve the problem of allocating appropriate sensor resources to each target to meet the requirements of target tracking performance when the number of
sensors is limited, this paper uses Interacting Multiple Model and Unscented Particle Filter (IMMUPF) to establish the target tracking model, and then combines game theory to propose a multi-sensor multi-target tracking task allocation algorithm based on game theory.

2. Game theory and filtering algorithm

2.1. Concept of game theory

Game theory is not only a new branch of modern mathematics, but also an important subject of operational research. Game theory considers the prediction behavior and actual behavior of the individual in the game, and studies their optimization strategies.

In game theory, there are usually three elements: player, strategy and payment. The following describes the three concepts in game theory under the framework of multi-objective tracking task:

1) Player is the main body of decision-making in a game. The responsibility of players is to choose the appropriate decision to maximize their own interests through the analysis of the game situation. In the multi-target tracking framework, each target has an agent associated, which is responsible for negotiating with other agents to obtain more or better sensor resources. The players are the agents associated with each target;

2) Strategy is the solution given by participants to a certain problem. In this paper, it mainly refers to the sensor resource allocation scheme given by the agents (i.e. participants) of each target under the condition of meeting their own tracking performance requirements;

3) Payoff is the benefit of the strategy adopted by the participants. In this paper, payment refers to the improvement of tracking performance obtained by the final sensor allocation scheme selected by each agent. Therefore, it is necessary to establish the target tracking performance measurement standard, which is related to the filtering algorithm of target tracking model. This paper adopts Interacting Multiple Model and Unscented Particle Filter (IMMUPF) algorithm according to the characteristics of actual battlefield distribution.

2.2. Target tracking model

The classic BP neural network is a forward neural network with three or more layers, including input layer, several hidden layers and output layer. There is no connection between the neurons in the same layer, but the neurons between layers are connected with each other by weight and threshold. Neural network can be trained to achieve better results by adjusting the weight threshold [6]. The structure of BP neural network is shown in Figure 1.

The main idea of the IMMUPF algorithm used in this paper is that: firstly, the observation value of the sensor to the target is input interacted by Interacting Multiple Model (IMM), then filtered by Unscented Kalman Filter [7] (UPF) algorithm, and finally output interacted, so that the target tracking can be completed through repeated recursion.

Suppose the number of particles in each group is \( N \), the number of IMM models is \( M \). \( U = \{ u_{i,i}, i=1,2,...,M \} \) and \( T = \{ \pi_{i,j}, i,j=1,2,...,M \} \) are the probability matrix and transition probability matrix of the model, respectively.

Step 1: Input interaction. Let \( \hat{x}_{ik} \) be the state in model \( i \), and \( \hat{P}_{ik} \) be the covariance.

\[
\mathbf{x}_{n,i,k} = \sum_{i=1}^{M} U_{i,j,k} \hat{x}_{j,i,k}
\]

\[
\mathbf{P}_{n,i,k} = \sum_{i=1}^{M} U_{i,j,k} \left( \hat{P}_{j,k} + (\hat{x}_{j,k} - \overline{x}_{j,k})(\hat{x}_{j,k} - \overline{x}_{j,k})^T \right)
\]

Where \( U_{i,j,k} = \frac{1}{c_{j}} \pi_{j,i,k} \).
\[ \tau_j = \sum_{i=1}^{n} \pi_{ij}u_{ij}, i, j = 1, 2, \ldots, m \]

**Step2:** Model matching UPF filtering. Use the UPF of the model \( m \) to filter \( \mathbf{x}_{n, j|j} \) and \( \mathbf{P}_{n, j|j} \) obtained in the previous step to obtain the filtered estimated value \( \mathbf{x}_{n, j|j+1} \), the covariance \( \mathbf{P}_{n, j|j+1} \) and the likelihood function \( \Lambda_{n, j|j+1} \).

**Step3:** Model probability update.

\[ u_{n, j|j+1} = \frac{1}{c} \sum_{j=1}^{m} \Lambda_{n, j|j+1} \tau_j \quad (3) \]

Where \( c = \sum_{j=1}^{m} \Lambda_{n, j|j+1} \).

**Step4:** Output the mixed estimation result of the model.

\[ \hat{x}_{j+1|j+1} = \sum_{j=1}^{m} u_{n, j|j+1} \hat{x}_{j|j+1} \quad (4) \]

\[ \hat{P}_{n,j|j+1} = \sum_{j=1}^{m} u_{n, j|j+1} \left[ \hat{P}_{j|j+1} + (\hat{x}_{j|j+1} - \mathbf{x}_{j|j+1})(\mathbf{x}_{j|j+1} - \mathbf{x}_{j|j+1})^{\top} \right] \quad (5) \]

2.3. Game payoff calculation

Based on the target tracking filtering algorithm in the previous section, when there are multiple sensors tracking a target, the preliminary results of each sensor's IMMUPF filtering of the target need to be fused (without mixing estimation), which can be calculated as follows:

\[ x_{n,j|j} = P_{n,j|j}^{-1} \left[ P_{n,j|j}^{-1} x_{n,j|j-1} + \sum_{j=1}^{m} (P_{n,j|j}^{-1} - (P_{n,j|j}^{-1})^{-1} x_{n,j|j-1}) \right] \quad (6) \]

\[ P_{n,j|j}^{-1} = P_{n,j|j-1}^{-1} + \sum_{j=1}^{m} ((P_{n,j|j})^{-1} - (P_{n,j|j})^{-1})^{-1} \quad (7) \]

Where \( x_{n,j|j} \) and \( P_{n,j|j}^{-1} \) are the estimation results of the model. It can be seen from the above formula that \( [(P_{n,j|j})^{-1} - (P_{n,j|j-1})^{-1}] \) represents the covariance influence of sensor \( j \) on the estimation of the target, or the contribution of sensor \( j \) to the estimation of the target state. The larger the value, the larger \( P_{n,j|j}^{-1} \). That is, the smaller the covariance \( P_{n,j|j} \), the smaller the error of the final result. Therefore, this value reflects the information gain of sensor \( j \) to the target to a certain extent, and it can be used as a payoff for the game. Define \( g(k) \) as the information gain of the sensor set \( S \) to a target at time \( k \), and:

\[ g(k) = \sum_{j=1}^{m} [(P_{n,j|j})^{-1} - (P_{n,j|j-1})^{-1}] \quad (8) \]

It can be obtained from the above formula that when the number of sensors is increased or a higher performance sensor is replaced, \( g(k) \) increases, that is:

\[ S_1 \leq S_2 \Rightarrow \sum_{j=1}^{m} [(P_{n,j|j})^{-1} - (P_{n,j|j-1})^{-1}] \leq \sum_{j=1}^{m} [(P_{n,j|j})^{-1} - (P_{n,j|j-1})^{-1}] \quad (9) \]
3. Multi-target tracking task allocation method based on game theory

3.1. Game process

In order to complete the game of sensor resource allocation among multiple targets, the game can be regarded as a dynamic game. Each target has an associated agent responsible for negotiating with other agents to obtain more or better sensor resources for its associated target. Suppose that at a certain moment, the motion state of a target suddenly changes drastically, and more or better sensor resources are needed to track it. At this time, the agent of the target will initiate negotiations to obtain more or better resources for itself. Suppose the initial sensor resources owned by the two agents participating in the game (referred to as agent 1 and agent 2) are \((B_1, B_2)\), and the sensor resources owned by the game after the game are \((A_1, A_2)\), and \((A_1 \cup A_2) = (B_1 \cup B_2)\). If the two parties reach an agreement and the negotiation ends at time \(k\), the two parties' payment in this game is defined as:

\[
U_1(A_k, k) = \sum_{k=1}^{\Delta k} \left[ \frac{\sum_1 \left( (P_{a,b})^I - (P_{a,b})^I \right)^2}{\Delta k + 1} + \right] \sum_{k=1}^{\Delta k} \left[ \frac{\sum_1 \left( (P_{a,b})^I - (P_{a,b})^I \right)^2}{\Delta k + 1} \right]
\]

(10)

\[
U_2(A_k, k) = \sum_{k=1}^{\Delta k} \left[ \frac{\sum_1 \left( (P_{a,b})^I - (P_{a,b})^I \right)^2}{\Delta k + 1} + \right] \sum_{k=1}^{\Delta k} \left[ \frac{\sum_1 \left( (P_{a,b})^I - (P_{a,b})^I \right)^2}{\Delta k + 1} \right]
\]

(11)

Where \(k\) is the time spent in negotiation, and the payments \(U_1(A_k, k)\) and \(U_2(A_k, k)\) of both parties represent the average gain during the negotiation. Suppose now that target 2 requires higher tracking performance, so agent 2 initiates a negotiation, and agent 1 passively accepts the negotiation, and must take part of its own interests to participate in the negotiation. Agent 2 hopes to reach an agreement as soon as possible in the negotiation. It can choose to withdraw from the negotiation to force Agent 1 to give more resources. Once Agent 2 exits the negotiation, Agent 1 will not be able to use the sensor resources it owns, and it needs to initiate another negotiation. Therefore, considering its own interests, Agent 1 must accept part of the request made by Agent 2 to prevent it from withdrawing from the negotiation. The payment of both parties when agent 2 opts out of the negotiation is defined as follows:

\[
U_1(O_k, k) = \frac{\Delta k \cdot \sum_1 \left( (P_{a,b})^I - (P_{a,b})^I \right)^2}{\Delta k + K + 1}
\]

(12)

\[
U_2(O_k, k) = \frac{(\Delta k + K) \sum_1 \left( (P_{a,b})^I - (P_{a,b})^I \right)^2}{\Delta k + K + 1}
\]

(13)

Where \(K \geq 0\) is the exit parameter.

\[
P_{poss}(k) = \{A = (A_1, A_2) | U_2(A_1, k) > U_2(O, k)\}
\]

(14)

\[
U_1(A_1(k), k) = \max_{A \in P_{poss}(k)} U_1(A_1(k), A_2(k) \in P_{poss}(k))
\]

(15)

In formula (14), \(P_{poss}(k)\) is the distribution plan that the agent 2 can get a higher payment at time \(k\) than withdrawing from the negotiation. In formula (15), \(A_1(k)\) is the distribution plan with the highest payment for agent 1 in \(P_{poss}(k)\).
According to the above, at time k, agent 2 actively initiates the negotiation, and the game process that agent 1 passively accepts is as follows:

**Step1:** Agent 2 proposes an alternative distribution plan $P_{pos}(k)$ to Agent 1, and Agent 1 chooses the plan $A_j(k)$ that makes the highest payment in $P_{pos}(k)$. At this time, $A_j(k)$ makes Agent 2 pay the lowest, and Agent 2 does not accept.

**Step2:** Then, agent 1 proposes $A^* \in C(k)$, and there is $U_i(A^*, k) = \max_{A \in C(k)} (U_i(A, k))$. This scheme not only makes the payment of agent 1 higher than the highest payment it can get at the next moment, but also maximizes the payment of agent 2.

**Step3:** Agent 2 accepts $A^*$ and goes to Step5.

**Step4:** When $C(k) = \emptyset$, agent 1 recommends that agent 2 adopt $A$. In order to protect the long-term benefits of both parties, Agent 2 accepts.

**Step5:** The game ends

3.2. Algorithm implementation

Based on the above sections, the steps of the multi-target tracking task allocation algorithm based on game theory are as follows:

**Step 1** IMMUPF preliminary estimation

The observation value of the sensor performs input interaction and UPF filtering to obtain the estimation $\hat{x}_{mk, k+1}$ and $P_{mk, k+1}$, where $m$ represents the model number, and the estimation of target $i$ by sensor $j$ under model $m$ is denoted as $x_{m,j,i}$. The expected covariance levels of each target defined as $\Delta^m_{j,i}$, and the difference is as:

$$\Delta^m_{j,i} = \left| \mathbf{P}^m_{i,j} - \sum_{k=1}^{n} \left( \mathbf{P}^m_{i,j} - \mathbf{P}^m_{0,j} \right) \right|$$

2) Calculate the expected covariance levels of each target defined as $\| \mathbf{P}^m_i \|$, and the difference is as:
\[ E(k) = ||r_x - r_x^e|| \]  

(17)

3) The target agent of \( \max[E(k)] \) initiates a negotiation with the target agent of \( \min[E(k)] \). The rules are as described in 1.3. After the negotiation, the two agents will get a new resource allocation plan \( (A_i, A_j) \)

4) Calculate new allocation plan \( E(k) \), if \( \max[E(k)] \approx \min[E(k)] \) or \( \max[E(k)] \approx 0 \), skip to 5, otherwise skip to 3;

5) End the negotiation and output the final distribution plan \( (A_i, A_j,...,A_j) \).

**Step 3 State integration**

According to the allocation plan in step 2, the estimation of the sensors obtained by fusing each target by equation (6) and equation (7) can obtain more accurate \( x_m, x_m^k \).

**Step 4 Update IMMUPF model update and state mixture estimation**

4. Simulation experiment

Assuming that the motion state of the target is represented as \( x_{k+1} = f_x \cdot x_k + w_k \), where \( f_x \) includes three states: \( f_1 \) is a uniform straight line state, \( f_2 \) is a right turning state, angular velocity is \( \omega = -3^\circ / s \), and \( f_3 \) is a left turning state, angular velocity is \( \omega = 3^\circ / s \). There are 3 targets, the movement process is as follows, target 1: 1~20s maintain a constant speed state, 21~40s right turn state, 41~60s enter a constant speed state, 61~80s left turn state, 81~100s maintain a constant speed state; Target 2: 1~100s is a constant speed state with loud noise; Target 3: 1~30s is a constant speed state, 31~50s is a left turn state, 51~70s is a right turn state, and 71~100s is a constant speed state. The initial parameters of the 3 targets are:

\[
\begin{align*}
  x_1 &= [0.55km, 2.10km, 0.06, 0.04km/s] \\
  x_2 &= [0.80km, 0.45km, 0.04km/s, 0.01km/s] \\
  x_3 &= [6.55km, 2.45km, 0.06km/s, 0.05km/s]
\end{align*}
\]

Suppose there are 9 sensors, S1, S2, S3, S4, S5 and S6 measure distance and azimuth, S7, S8 and S9 measure distance and speed, the position of the sensor is shown in Table 1, and the target motion track is shown in Figure 1. Table 2 shows the distribution results at different times under the same conditions using distribution algorithms based on Game Theory (GT) and Non-dominated Sorting Genetic Algorithm (NSGA-II).

![Fig. 1 Schematic diagram of target trajectory](image-url)
Figure 2 is the average error of the target position estimation at different times using the two algorithms. It can be seen from the figure that after the first task assignment at 2s, the tracking performance of NSGA-II and GT is not much different, but the second assignment at 20s After that, the error of the NSGA-II algorithm increased significantly, and it gradually decreased after the third reallocation of resources in 40s, but error level was relatively high throughout the process, and the error of the GT algorithm proposed in this paper has been kept relatively high. Low level, its average tracking error is reduced from 68.5630m to 19.7601m.

5. Conclusion
In the multi-target tracking task allocation problem, each target wants to get more sensors to improve its tracking performance. However, if one target gets more sensors, the sensor resources of other targets will decrease. In the case of limited sensor resources, this paper proposes a multi-target tracking task allocation algorithm based on game theory. The algorithm can adaptively allocate sensor resources to each target, so that it has higher tracking performance as a whole. Finally, the comparison with other allocation algorithms shows that the algorithm proposed in this paper has good performance in multi-target tracking task allocation problem.

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