Application of vortex three-phase separators for improving the reliability of pump and compressor stations of hydrocarbon processing plants

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Abstract. The article is devoted to the description of the theoretical foundations of the work of a new type of vortex three-phase separators for hydrocarbon processing plants. The expediency of using vortex flows in the technology of three-phase separation to improve the reliability of pumping and compressor stations is substantiated. The principle of operation of separators with a variable cross-sectional area of the working space and an inter-ring drain of the liquid is presented. A theoretical description of the hydrodynamic conditions for the separation of multiphase flows is given. A scheme for the installation of hydrocarbon feedstock processing using a three-phase vortex separator is proposed.

1. Introduction
The majority of the largest oil and gas fields in the stages of primary oil separation, hydrocarbon gas preparation, transportation and compression face problems of qualitative separation of liquid and gaseous hydrocarbon feedstocks. An additional problem is the presence of solid inclusions in the hydrocarbon feedstock. This fact significantly narrows the range of equipment for further transportation of hydrocarbon raw materials and its compression.

Currently three-phase separators [1-5] are used, which are characterized by sufficient efficiency, however:
- they have large mass-dimensional characteristics;
- the design of the separator involves a complex internal arrangement (cyclone elements, inertial and filter sections, partitions, etc.), which leads to hydraulic resistance increasing;
- when the composition of raw materials changes (which can occur over time), structural change in the internal elements of the separator is necessary.

The actual problem of chemical engineering in this industry is the creation of new types of separators with relatively simple construction, which can work on different types of raw materials (they do not need to be modified in a constructive way, further operation of the separator is possible due to adjustment of process parameters).

The introduction of new methods of intensification of heat and mass exchange processes will increase the specific productivity of equipment and ensure high indicators of products quality. The use of new technologies makes it possible to improve the mass-dimensional characteristics of separation equipment and to increase the productivity and efficiency of the separation process.
Authors [6-9] classify the methods of heat-mass transfer processes intensification into two categories:

1. Active methods: mechanical action on the surface of heat-mass transfer (rotation, surface vibration, mixing etc.); the impact on the flow by electric, magnetic or acoustic field, pressure pulsations; blowing or suctioning of working environment through porous surfaces etc.

2. Passive methods, that are based on influence principle on the flows by the surface form of individual structural elements of equipment: application of inserted intensifiers (swirling flow units), ribbing surfaces, increasing surface area of heat-mass transfer from the working environment with less heat-mass transfer coefficients; intensification of heat-mass transfer during phase transformations (surface treatment, use of surface tension effect, drop condensation), additive in liquid solid particles or gas bubbles etc.

![Figure 1. Schematic diagram of a three-phase separator.](image)

Among the passive methods of heat-mass transfer processes intensification the method of inserted intensifiers application, that create vortex flow in the workspace of device, is one of the most promising.

Using of vortex flow as a method of intensifying heat exchangers or heat-mass transfer processes has spread in processes of combustion [10], absorption [11], rectification [12], in turbines [13], ejecting devices [14], reactors [15], granulators [16,17] (theoretical description and experimental study of certain aspects of granulation in vortex granulators also is considered in series of papers describing the hydrodynamics of flow motion [18, 19] and the thermodynamic conditions of granulators operation [20-23], processes of granules classification and separation [24-26], ecological aspects of block developing of vortex recycling modules for production wastes [27,28]) etc., but the use of this method in separation process is not widespread.

In this paper, we offer a description and theoretical basis of three-phase vortex separators operation (Figure 1).

2. Physical model of flows movement in vortex separator
In the separator workspace it is necessary to allocate three areas of gas flow movement in height (Figure 2). In each of the areas the intensity of gas flow movement is determined by velocity components and preferred direction of total velocity.
Figure 2. The main zones of gas flow movement in vortex separator in height of working space: I – zone of preferred gas flow vortex movement; II – zone of combined vortex and upward gas flow movement; III – zone of preferred seating upward gas flow movement.

Figure 3. The phenomenon of disperse phase movement in the dense ring at some distance S from the wall of separator.

During the study of disperse phase trajectories we have found that in zone I in the intensive initial gas flow twisting we see the phenomenon of disperse phase movement in dense ring at some distance S from the wall of separator (Figure 3). This phenomenon can be explained by the presence in the vortex separator workspace significant pressure difference at the center and the periphery (Figure 4). This pressure difference is caused by the opposition to the centrifugal force. Further this phenomenon can be used to avoid contact of disperse phase with the wall of separator.
Figure 4. Pressure field in working space of vortex separator.

The feature of separator, that is investigated, is a variable height of cross sectional area of workspace. Devices with the constant cross-sectional area don’t provide the full process of classification and separation of disperse phase with a different mass (density, size) in working volume. This can be explained by the fact, that in the workspace of vortex separator consistency upward gas flow rate remains, that corresponds to the velocity of disperse phase. To carry out classifications processes in device with a constant cross-sectional area is possible with the introduction of gas to the unit in several streams with a different injection height marks. This method of classification is quite energy intensive and is not widely used.

Using separators with variable cross-sectional area of working chamber allows to do classification process of polydisperse system of wide fractional composition. Through the creation different hydrodynamic gas flow movement conditions at different height marks of working volume of device it becomes possible to classify of disperse phase to the required number of factions and to carry out separation of small particles (Figure 5).

Figure 5. Distribution of disperse phase sizes and mass in the working volume of vortex separator.

3. Mathematical model of flows movement in vortex separator

Gas flow

The immediate modeling of turbulent flows by quantitative decision of Navier-Stokes equations, recorded for instant velocities, today are unsolved. However, for specific tasks solving it becomes possible application of this fundamental equation of hydrodynamics when performing of number of conditions:

- working with time-averaged value of the velocity in modeling of turbulent flows;
- selection the coordinate system, which is most satisfying for the specific case and facilitates the recording of equations.

For the simulation the solving of these equations is comfortable when using numerical methods (DEM – Discrete Element Method), defining discrete solution instead of continuous set of values in desired location (cell, grid node) of space (at stationary mode of flow motion). For maximum accuracy of solution it is chosen a way of discrete values representing, which on this occasion sample corresponds analogues of algebraic equations. As a result the mathematical problem solution of differential or integral equations can be reduced to the problem of solving the algebraic equations.

In practice, various models of numerical solution of classical hydrodynamic equations for turbulent flows, which in one way or another way are successfully used in different cases and have their advantages and disadvantages [29]: Direct numerical simulation (DNS), Reynolds Averaged Navier-Stokes (RANS), Large Eddy Simulation (LES).

Given that, in practice, interest is usually deals not with the instantaneous, but the average in time velocity values for the mathematical description of gas flow turbulent swirling motion the Reynolds equation as a modification of Navier-Stokes equations is used [24]:

\[
\frac{\partial}{\partial t}(\rho V_i) + \frac{\partial}{\partial q_j}(\rho V_i V_j) = -\frac{\partial p}{\partial q_i} + \frac{\partial}{\partial q_j}\left(\mu \left(\frac{\partial V_i}{\partial q_j} + \frac{\partial V_j}{\partial q_i}\right)\right) + f_i
\]

where \(V\) – time averaged velocity values; \(\overline{V}\) – components of velocities pulsation; \(\mu\) – turbulent viscosity coefficient; \(t\) – time; \(\rho\) – gas density; \(p\) – pressure; \(f_i\) – variable, describing the action of mass forces; \(q_j\) – coordinate axes (in the case of hydrodynamic modeling in working volume of separator it is more advisable to use curvilinear coordinate system), \(i, j = 1 \ldots 3\); for the cylindrical coordinate system (Figure 6 (a)) index «1» – axial direction (z), index «2» – radial direction (r), index «3» – circular direction (φ) (Figure 6 (b)).

![Figure 6. Vortex separator workspace scheme and coordinate system.](image)

The Reynolds equations system is complemented with flow continuity equation

\[
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial q_j}(\rho V_j) = 0.
\]

The main advantage of description and solving method of hydrodynamics problems, based on the numerical solution of complete equations Reynolds, is precision and versatility.
For Reynolds equation (1) Boussinesq hypothesis is used [30]. According to this hypothesis, the members with velocity pulsations \( \rho \bar{V} V_j' \) in equation (3) associated with the averaged flow characteristics of such relation:

\[
\rho \bar{V} V_j' = -\mu \left( \frac{\partial \bar{V}_i}{\partial q_j} + \frac{\partial \bar{V}_j}{\partial q_i} \right) + \frac{2}{3} \rho \delta_{ij} k
\]

(3)

where \( k = 0.5 \left( \bar{V} V_j' \right) \) – kinetic energy of turbulence, \( \delta_{ij} = 1 \) when \( i = j \), \( \delta_{i} = 0 \) and \( i \neq j \).

The Reynolds equations system is elliptic. It is used to calculate the trends in those cases where flow characteristics at an arbitrary point area depend on the structure of flow both above and downstream, i.e. when the dominant direction of the fluid is absent or weakly expressed. Ellipticity of equations system means, that to address it it is necessary to set the boundary conditions for all variables in all borders of settlement area.

If axisymmetrical flow modeling equation of motion (1) and flow continuity (2) is significantly simplified. For curved (cylindrical) coordinate system they are follows (with the introduction of equation (1) Reynolds number \( Re = V_0 D / \nu \) – where characteristic parameters, \( D \) – diameter of input section of calculation area; \( V_0 \) – average expenditure rate in the input section; \( \nu \) – kinematic viscosity):

– Reynolds equations projected on the axial direction \( q_1 \):

\[
\begin{align*}
\frac{V_1}{H_1} \frac{\partial V_1}{\partial q_1} &+ \frac{V_2}{H_2} \frac{\partial V_2}{\partial q_2} - \frac{V_2^2}{H_1 H_2} \frac{\partial H_2}{\partial q_1} - \frac{V_1^2}{H_1 H_3} \frac{\partial H_3}{\partial q_1} + \frac{1}{H_2} \frac{\partial \left( V_1 V_2 \right)}{\partial q_2} + \frac{1}{H_1} \frac{\partial \left( V_1 V_2' \right)}{\partial q_1} + \frac{2}{3} \rho \delta_{ij} k \\
&+ \frac{1}{H_1} \frac{\partial \left( V_1 V_1' \right)}{\partial q_1} + \frac{1}{H_2} \frac{\partial \left( H_1 H_2 H_3 \right)}{\partial q_1} - \frac{V_1}{H_1 H_2 H_3} \frac{\partial H_2}{\partial q_1} - \frac{V_2}{H_1 H_2 H_3} \frac{\partial H_3}{\partial q_1} - \frac{1}{H_1 H_2 H_3} \frac{\partial \left( H_1 H_2 H_3 \right)}{\partial q_1} = -1 \frac{\partial p}{\partial q_1}
\end{align*}
\]

(4)

– Reynolds equations in projection on the radial direction \( q_2 \):

\[
\begin{align*}
\frac{V_1}{H_1} \frac{\partial V_1}{\partial q_1} &+ \frac{V_2}{H_2} \frac{\partial V_2}{\partial q_2} - \frac{V_2^2}{H_1 H_2} \frac{\partial H_2}{\partial q_1} - \frac{V_1^2}{H_1 H_3} \frac{\partial H_3}{\partial q_1} + \frac{1}{H_2} \frac{\partial \left( V_1 V_2 \right)}{\partial q_2} + \frac{1}{H_1} \frac{\partial \left( V_1 V_2' \right)}{\partial q_1} + \frac{2}{3} \rho \delta_{ij} k \\
&+ \frac{1}{H_2} \frac{\partial \left( V_2 V_2' \right)}{\partial q_2} + \frac{1}{H_1} \frac{\partial \left( H_1 H_2 H_3 \right)}{\partial q_2} - \frac{V_1}{H_1 H_2 H_3} \frac{\partial H_2}{\partial q_2} - \frac{V_2}{H_1 H_2 H_3} \frac{\partial H_3}{\partial q_2} - \frac{1}{H_1 H_2 H_3} \frac{\partial \left( H_1 H_2 H_3 \right)}{\partial q_2} = -1 \frac{\partial p}{\partial q_2}
\end{align*}
\]

(5)
Reynolds equations projected on circular direction \( q_2 \): \[
\frac{V_1}{H_1} \frac{\partial V_1}{\partial q_1} + \frac{V_2}{H_2} \frac{\partial V_2}{\partial q_2} + \frac{V_3}{H_3} \frac{\partial H_3}{\partial q_3} + \frac{1}{H_2} \frac{\partial \left( V_2 V_3^* \right)}{\partial q_2} + \frac{1}{H_3} \frac{\partial \left( V_3^* V_1 \right)}{\partial q_3} + 1 \frac{\partial \left( \frac{V_2}{V_3^*} \frac{\partial H_3}{\partial q_2} \right)}{H_2 H_3 H_3^*} = 0
\]

continuity equation:
\[
\frac{1}{H_1 H_2 H_3^*} \left( \frac{V_1}{H_1} \frac{\partial (H_2 H_3)}{\partial q_1} + \frac{V_2}{H_2} \frac{\partial (H_1 H_3)}{\partial q_2} + \frac{1}{H_3} \frac{\partial \left( \frac{V_2}{V_1} \frac{\partial (H_1 H_3)}{\partial q_3} \right)}{H_1 H_2 H_3} \right) = 0
\]

where \( H_1, H_2, H_3 \) – Lame coefficient [31].

Further simplification of equations system (4)-(7) for gas phase vortex flow simulation in the separator workspace is possible using the following assumptions [32]:
- presence of flow dominant direction, along which the axial component of gas flow velocity is everywhere positive and far exceeds the radial;
- component of gas flow velocity in the axial direction varies considerably slower than in the radial;
- velocity and pressure in every gas flow elementary volume depends only on the downstream conditions and don’t depend on upstream conditions.

These assumptions allow to conduct analysis of components orders in equations (4)-(7) and discard those, that provide significant impact on the result of the calculation.

After accounting assumptions for axisymmetrical gas flow equation (4) – (7) will be written as
\[
\frac{V_1}{H_1} \frac{\partial V_1}{\partial q_1} + \frac{V_2}{H_2} \frac{\partial V_2}{\partial q_2} - \frac{V_3^2}{H_3} \frac{\partial H_3}{\partial q_3} + \frac{1}{H_2} \frac{\partial \left( V_2 V_3^* \right)}{\partial q_2} + \frac{1}{H_3} \frac{\partial \left( V_3^* V_1 \right)}{\partial q_3} + 1 \frac{\partial \left( \frac{V_2}{V_3^*} \frac{\partial H_3}{\partial q_2} \right)}{H_2 H_3 H_3^*} = 0
\]

\[
\frac{1}{H_1 H_2 H_3^*} \left( \frac{V_1}{H_1} \frac{\partial (H_2 H_3)}{\partial q_1} + \frac{V_2}{H_2} \frac{\partial (H_1 H_3)}{\partial q_2} + \frac{1}{H_3} \frac{\partial \left( \frac{V_2}{V_1} \frac{\partial (H_1 H_3)}{\partial q_3} \right)}{H_1 H_2 H_3} \right) = 0
\]

This system of equations is closed with the equation of sustainability costs:
\[ Q_2 - \text{coordinate } q_2 \text{ on the wall of the working volume of the vortex separator.} \]

\[ \int_{0}^{V_1} H_2 H_3 dq_2 = \text{const.} \quad (12) \]

where \( Q_2 \) – coordinate \( q_2 \) on the wall of the working volume of the vortex separator.

**Figure 7.** Construction of calculation grid.

The resulting system of equations (8) - (12) is parabolic in nature, and its decision based on the method proposed by Patankar and Spalding [33] and realized in the process SIMPLE (Semi-Implicit Method for Pressure-Linked Equations) and its modifications.

Numerical solutions of equations of mathematical models implemented in one marching passage from the input section to the initial working volume using finite volume elements of finite-element approach. Before calculating the estimated net constructed (Figure 7), and unknown values of velocity and pressure are found in the nodes of the grid.

For computer simulation and visualization of research results in this study it was used complex ANSYS CFX, which allows to export obtained using program «Conical channel».

**Disperse phase (solid particles, liquid)**

This approach is used to simulate two-phase flows in which the substance of one of the phases is presented in the form of dispersed particles, and the volume fraction, occupied by these particles, is small (up to 10% of the total volume). The substance that forms the main phase is assumed to be a continuous medium, and its flow is modeled by the Navier-Stokes (or Reynolds) and continuity equations. The substance present in the flow in the form of discrete particles does not form a continuous medium, the individual particles interact with the main phase flow and with each other discretely. To model the motion of the particles of the scattered phase, the Lagrange approach is used, i.e. the movement of separately particles of the scattered phase is monitored under the action of forces on the side of the main phase flow.

The forces acting on the dispersed phase are caused by the difference in the velocity of the disperse phase and the flow velocity of the main phase, as well as the displacement of the main phase by the dispersed phase. The equation of motion of such a particle was derived in [10] and has the form:

\[ m_p \frac{dv_p}{dt} = 3\pi \mu dC_{cor} (v_f - v_p) + \frac{\pi d^3}{6} \rho_f \frac{dv_f}{dt} + \frac{\pi d^3}{12} \rho_f \left( \frac{dv_f}{dt} - \frac{dv_p}{dt} \right) + \frac{F_e}{6} \left( \rho_p - \rho_f \right) \hat{\omega} \times (\hat{\omega} \times \hat{r}) - \frac{\pi d^3}{3} \rho_p \left( \hat{\omega} \times v_p \right) \quad (13) \]
Here \( m_p \) - particle’s mass, \( d \) - particle’s diameter, \( v \) - velocity, \( \mu \) - dynamic viscosity of the main phase substance, \( C_{cor} \) - its viscous drag coefficient; \( \omega \) - angular velocity of rotation, \( r \) - radius vector (when considering motion in the relative frame of reference). The index \( p \) refers to the disperse phase, index \( f \) - to the substance of the main phase.

The left-hand side of equation (13) is the sum of all the forces acting on the particle, expressed in terms of the mass and acceleration of this particle. The first element on the right-hand side expresses the deceleration of the particle as a result of viscous friction against the flux of the main phase according to the Stokes law. The second element is the force applied to the particle, due to the pressure drop in the main phase surrounding the particle caused by the acceleration of the main phase flow. The third term is the force required to accelerate the weight of the main phase in the volume displaced by the particle. These two elements must be taken into account when the density of the main phase exceeds the particle density, for example, when considering air bubbles in a liquid flow. The fourth element \((Fe)\) is an external force acting directly on a particle, for example, gravity or the strength of an electric field. The last two elements are the centrifugal force and the Coriolis force, which take place only when considering motion in a relative frame of reference. In addition, sometimes it is necessary to take into account some additional forces on the right-hand side of (13) (for example, if there is a significant temperature difference in the flow).

Equation (13) is a first-order differential equation in which the only unknown quantity is the particle velocity \( v_p \), and the argument is the time \( t \). The velocity of the substance of the main phase \( v_f \) is assumed to be known throughout the space points. As initial data, in addition to the size and properties of the particle, its position at the initial instant of time is given. It is also indicated what should occur when a particle collides with a wall or with another particle. To carry out the calculation, the terms containing \( v_p \) are transferred to the left-hand side of equation (15). The velocity and position of the particle at each subsequent moment of time is determined by numerical integration with respect to time with some step \( \Delta t \) of all the remaining terms of equation (15).

The coefficient of viscous resistance \( C_{cor} \) at moderate Reynolds numbers \( 0.01 < \text{Re}_p < 260 \) can be calculated, for example, using formulas

\[
C_{cor} = \begin{cases} 
1 + 0.1315 \left( \text{Re}_p \right)^{0.82 - 0.05\alpha} & \text{npu} \quad \text{Re}_p < 20 \\
1 + 0.1935 \left( \text{Re}_p \right)^{0.6305} & \text{npu} \quad \text{Re}_p \geq 20 
\end{cases}
\]

where \( \text{Re}_p = \rho_f \left| v_f - v_p \right| d / \mu \), \( \alpha = \log \text{Re}_p \).

4. Conclusions
The proposed design of the vortex separator makes it possible to achieve:
- prolongation of equipment service life;
- reducing the cost of equipment as a result of improving its mass-size characteristics
- reducing the downtime during the modernization of equipment
- reduction of operating and capital costs.

On Figure 8 the basic schemes for the preparation of hydrocarbon gas before its further processing and the compressor section are shown; at the primary separation stage the vortex separator is used. The use of a vortex separator makes it possible to eliminate filtering equipment from the plant and to ensure a better quality of separation of the multicomponent mixture.
Figure 8. Schematic diagram of the unit for the preparation of hydrocarbon gas (a) and booster compressor station (b): 1 - initial mixture (gas-liquid-solid); 2 - hydrocarbon gas; 3 - liquid hydrocarbons + water; 4 - mechanical impurities; VS – vortex separator; A – absorber; D – desorber; T – tank; H – heat exchanger; P – pump; GS – gas separator; OS – oil separator; GEC – gas engine compressor.
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