Axial-vector $f_1(1285) - f_1(1420)$ mixing and $B_s \rightarrow J/\psi(f_1(1285), f_1(1420))$ decays

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Inspired by the very recent measurement of $B_s \rightarrow J/\psi f_1(1285)$ reported by the LHCb Collaboration and encouraged by the good agreement between the perturbative QCD predictions and experimental data for $B \rightarrow J/\psi V$ decays, we investigate the $B_s \rightarrow J/\psi f_1(1285)$ and $B_s \rightarrow J/\psi f_1(1420)$ decays for the first time by employing the perturbative QCD (pQCD) approach, in which the $^1S_0 P_1$ states $f_1(1285)$ and $f_1(1420)$ are the mixture of flavor singlet $f_1$ and octet $f_8$ or quark-flavor states $f_{1q}$ and $f_{1s}$ as the $\eta - \eta'$ mixing in the pseudoscalar sector. We show that the pQCD predictions for the branching ratio of $B_s \rightarrow J/\psi f_1(1285)$ agree well with the data within errors for the mixing angle $\theta_{3P_1} \approx 20^\circ (\phi_{3P_1} \approx 15^\circ)$ between $f_1(f_{1q})$ and $f_8(f_{1s})$ states. Furthermore, the branching ratio of $B_s \rightarrow J/\psi f_1(1420)$ and the large transverse polarization fractions in these two considered channels are also predicted for tests by the LHC measurements and the forthcoming Super-B factory data. Based on the decay rates for the $B_s \rightarrow J/\psi f_1(1285)$ and $B_s \rightarrow J/\psi f_1(1420)$ modes predicted in the pQCD approach, the extracted angle for $f_1(1285) - f_1(1420)$ mixing is basically consistent with the available measurements, lattice QCD analysis, and phenomenological fits within still large theoretical errors.

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Very recently, the LHCb Collaboration located at CERN reported the first observation of $B_s \rightarrow J/\psi f_1(1285)$...
$J/\psi f_1(1285)$ decay with the branching ratio [1],

$$Br(B_s \to J/\psi f_1(1285))_{\text{Exp.}} = (7.14 \pm 0.99^{+0.83}_{-0.91} \pm 0.41) \times 10^{-5},$$  (1)

Of course, the accuracy of the above data is expected to be improved rapidly with the future LHCb and Super-B experiments. By combining also the first measurement of $B_d \to J/\psi f_1(1285)$ channel, the $f_1(1285)$ mixing angle between strange and non-strange components of its wave function in the $q\bar{q}$ structure model is determined to be $\pm (24.0^{+3.1+0.6}_{-2.6-0.8})^\circ$ [1] for the first time.

It is well known that, in the quark model, the $f_1(1285)$ is a $p$-wave axial-vector meson with $J^{PC} = 1^{++}$, which is believed to mix with the partner $f_1(1420)$ [2, 3] just like the "$\eta - \eta'$" mixing in the pseudoscalar sector. Up to now, many discussions have been presented on the mixing angle $\theta_{3P_1}$ or $\phi_{3P_1}$ of $f_1(1285) - f_1(1420)$ mixing corresponding to two mixing schemes, the singlet-octet (SO) basis and the quark-flavor (QF) basis [4–15]. One of the most important reasons is that the mixing angle $\theta_{3P_1}$ or $\phi_{3P_1}$ can be utilized to pin down the angle $\theta_{K_1}$ of axial-vector $K_1(1270) - K_1(1400)$ mixing system [13], which is a very special mixing between two distinct types of axial-vector states $K_{1A}(1^3P_1)$ and $K_{1B}(1^1P_1)$. The axial-vector $f_1(1285) - f_1(1420)$ mixing can be written in the SO basis following the common convention [16],

$$\begin{pmatrix} f_1(1285) \\ f_1(1420) \end{pmatrix} = \begin{pmatrix} \cos \theta_{3P_1} & \sin \theta_{3P_1} \\ -\sin \theta_{3P_1} & \cos \theta_{3P_1} \end{pmatrix} \begin{pmatrix} f_1 \\ f_8 \end{pmatrix},$$  (2)

with the SO states $f_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$ and $f_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$. While, in the QF basis, the $f_1(1285) - f_1(1420)$ mixing can be written into the following pattern [16],

$$\begin{cases} f_1(1285) = \cos \phi_{3P_1} f_{1q} + \sin \phi_{3P_1} f_{1s} \\ f_1(1420) = \sin \phi_{3P_1} f_{1q} - \cos \phi_{3P_1} f_{1s} \end{cases}$$  (3)

with the QF states $f_{1q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $f_{1s} = s\bar{s}$. The QF mixing angle $\phi_{3P_1}$ is related to the SO mixing angle $\theta_{3P_1}$ by the relation $\phi_{3P_1} = \theta_i - \theta_{3P_1}$, where $\theta_i$ is the "ideal" mixing angle with $\theta_i = 35.3^\circ$. Therefore, $\phi_{3P_1}$ measures the deviation from ideal mixing.

Though the $f_1(1285)$ mixing angle has been preliminarily determined through the $B_{d/s} \to J/\psi f_1(1285)$ decays in the QF basis by the LHCb Collaboration, it is necessary to point out that the assumption of exact SU(3) flavor symmetry on the decay amplitudes of $B_{d/s} \to J/\psi f_1(1285)$ has been adopted [1]. In fact, at the theoretical aspect, the contributing components in the above mentioned $B_d \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1285)$ decays at the quark level should be the QF states $f_{1q}$ and $f_{1s}$ respectively, whose behavior may be different from each other because of
FIG. 1. (Color online) Leading quark-level Feynman diagrams for the $B_s \rightarrow J/\psi f_1(1285)$ (left) and $B_s \rightarrow J/\psi f_1(1420)$ (right) decays. In the former (latter) decay, the coefficient for the component $s\bar{s}$ is $\sin \phi_3 P_1 (-\cos \phi_3 P_1)$ in the quark-flavor basis and $\cos \theta_3 P_1 \sqrt{3} - \sqrt{2} \sin \theta_3 P_1 \sqrt{3} - \sqrt{2} \cos \theta_3 P_1 \sqrt{3}$ in the singlet-octet basis.

the real existence of SU(3) breaking symmetry for $f_{1q}$ and $f_{1s}$. Consequently, the resultant mixing angles may considerably shift away from the expected values.

It may be very interesting to study the mixing angle of $f_1(1285) - f_1(1420)$ mixing through the same components at the quark level, for example, the $f_1(1285)$ and $f_1(1420)$ mesons are produced through their strange components in the $B_s$ meson decays, as easily seen in Fig. 1. Following this way, the mixing angle $\phi_3 P_1$ of $f_1(1285) - f_1(1420)$ mixing in the QF basis could be extracted more directly and cleanly through the following ratio of $Br(B_s \rightarrow J/\psi f_1(1285))$ over $Br(B_s \rightarrow J/\psi f_1(1420))$,

$$R^{QF}_s \equiv \frac{Br(B_s \rightarrow J/\psi f_1(1285))}{Br(B_s \rightarrow J/\psi f_1(1420))} = \frac{\Phi_{f_1(1285)} \cdot |\sin \phi_3 P_1 \cdot A(B_s \rightarrow J/\psi f_1)|^2}{\Phi_{f_1(1420)} \cdot |-\cos \phi_3 P_1 \cdot A(B_s \rightarrow J/\psi f_1)|^2} \equiv \frac{\Phi_{f_1(1285)}}{\Phi_{f_1(1420)}} \cdot \tan^2 \phi_3 P_1 ,$$

(4)

where $\Phi_{f_1(1285)}$ and $\Phi_{f_1(1420)}$ are the phase space factors for $B_s \rightarrow J/\psi f_1(1285)$ and $B_s \rightarrow J/\psi f_1(1420)$ decays, respectively. Once the measurements for the decay rates of these two channels are available, one could extract the mixing angle $\phi_3 P_1$ through Eq. (4) directly. In view of the equivalence for $f_1(1285) - f_1(1420)$ mixing between QF basis and SO basis, the above ratio given in Eq. (4) can also be expressed in the SO basis as follows,

$$R^{SO}_s \equiv \frac{Br(B_s \rightarrow J/\psi f_1(1285))}{Br(B_s \rightarrow J/\psi f_1(1420))} = \frac{\Phi_{f_1(1285)} \cdot \left|\frac{\cos \theta_3 P_1}{\sqrt{3}} \cdot A(B_s \rightarrow J/\psi f_1) - 2 \cdot \frac{\sin \theta_3 P_1}{\sqrt{6}} \cdot A(B_s \rightarrow J/\psi f_8)\right|^2}{\Phi_{f_1(1420)} \cdot \left|-\frac{\sin \theta_3 P_1}{\sqrt{3}} \cdot A(B_s \rightarrow J/\psi f_1) - 2 \cdot \frac{\cos \theta_3 P_1}{\sqrt{6}} \cdot A(B_s \rightarrow J/\psi f_8)\right|^2} ,$$

(5)
which can also be used to extract the mixing angle $\theta_{3P_1}$ approximately based on the assumption [5] that $\mathcal{A}(B_s \to J/\psi f_1) \approx \mathcal{A}(B_s \to J/\psi f_8)$ \footnote{Actually, as presented in Ref. [7], the two SO states $f_1$ and $f_8$ have the similar hadronic parameters, which can also be seen from the similarity of the relevant input parameters in Eq. (6) and the related phenomenological discussions.}, then extract out the mixing angle $\phi_{3P_1}$ via the relation $\phi_{3P_1} = \theta_{3P_1} - \phi_{3P_1}$.

And what’s more, decays of $B$ mesons into final states containing the $J/\psi$ charmonium state play a special role in studies of CP violation physics [17]. As discussed in the literature [7–10], the behavior of $1^3P_1$ axial-vector meson is similar with that of the vector meson. It is naturally expected that the $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decays can serve as the alternative channels to reduce the errors in the determination of the $B_s - \bar{B}_s$ mixing phase $\phi_s$ effectively.

We will therefore investigate the $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decays in the perturbative QCD (pQCD) approach [18–20] with the aforementioned two mixing schemes. Because of the similar behavior between the $1^3P_1$ axial-vector mesons and the vector mesons and the global agreement between the theoretical predictions in the pQCD approach and the presently existing experimental data for the $B \to J/\psi V$ decays [21], we can therefore derive the decay amplitudes for the $B_s \to J/\psi f_1$ and $J/\psi f_8$ decays or $B_s \to J/\psi f_{1s}$ decay at next-to-leading order of the strong coupling constant $\alpha_s$ straightforwardly by substituting the kinematic variables and distribution amplitudes of $\phi$ in the $B_s \to J/\psi \phi$ mode to those of $f_1$ and $f_8$ or $f_{1s}$ in the considered decays, apart from an overall minus sign that arising from the definitions of the wave functions for axial-vector and vector mesons. As the same distribution amplitudes of $\omega$ and $\rho$ in the vector mesons just with different decay constants, we assume that the QF state $f_{1q}$ is same as $a_1(1260)$ with the decay constant $f_{1q} = 0.193^{+0.043}_{-0.038}$ GeV [22]. For the $f_{1s}$ state, for the sake of simplicity, we adopt the same distribution amplitude as $f_1$ with decay constant $f_{1s} = 0.230 \pm 0.009$ GeV [22]. In fact, we have confirmed that the CP-averaged branching ratios just vary $3\%$ for the change of the distribution amplitude of $f_1$ into that of $f_8$.

The following input parameters, such as the QCD scale (in units of GeV), masses (GeV), decay constants (GeV) and $B_s$ meson lifetime (ps) as given in Refs. [7, 11, 16, 22], will be used in the numerical calculations:

\[
\begin{align*}
A_{MS}^{(f=4)} & = 0.287, & m_W & = 80.41, & m_b & = 4.8, & m_{B_s} & = 5.37; \\
m_{J/\psi} & = 3.097, & m_{f_1} & = 1.28, & m_{f_8} & = 1.29, & m_c & = 1.50; \\
f_{J/\psi} & = 0.405, & f_{B_s} & = 0.23, & f_{f_1} & = 0.245, & f_{f_8} & = 0.239; \\
\tau_{B_s} & = 1.497, & f_{f_{1s}} & = 0.230, & \theta_{3P_1} & = 20^\circ, & \phi_{3P_1} & = 15.3^\circ. \\
\end{align*}
\]
For the mixing angle of $f_1(1285) - f_1(1420)$ system, in this work, we adopt the recently updated value $\theta_{sP_1} \approx 20^\circ$, i.e., $\phi_{sP_1} \approx 15.3^\circ$ [11] to calculate the physical quantities in these two $B_s$ decays. For the Cabibbo-Kobayashi-Maskawa matrix elements, we adopt the Wolfenstein parametrization up to corrections of $\mathcal{O}(\lambda^5)$ and the updated parameters $A = 0.811, \lambda = 0.22535, \bar{\rho} = 0.131^{+0.026}_{-0.013}$, and $\bar{q} = 0.345^{+0.013}_{-0.014}$ [16].

The theoretical predictions in the pQCD approach for the CP-averaged branching ratios of the $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decays within errors in the standard model with two mixing schemes are the following,

- In the QF basis

\[
Br(B_s \to J/\psi f_1(1285)) = 7.70^{+2.30}_{-1.74}(\omega_B)^{+1.05}_{-0.99}(f_M)^{+3.33}_{-2.50}(a_t)^{+1.22}_{-1.25}(m_c)^{+4.38}_{-3.45}(\phi_{sP_1})^{+0.22}_{-0.30}(a_t) \\
= \left[7.70^{+6.18}_{-4.88}\right] \times 10^{-5}, \tag{7}
\]

\[
Br(B_s \to J/\psi f_1(1420)) = 0.97^{+0.30}_{-0.21}(\omega_B)^{+0.14}_{-0.12}(f_M)^{+0.42}_{-0.31}(a_t)^{+0.17}_{-0.15}(m_c)^{+0.04}_{-0.04}(\phi_{sP_1})^{+0.04}_{-0.04}(a_t) \\
= \left[0.97^{+0.56}_{-0.42}\right] \times 10^{-3}; \tag{8}
\]

- In the SO basis

\[
Br(B_s \to J/\psi f_1(1285)) = 8.71^{+2.59}_{-1.99}(\omega_B)^{+2.46}_{-2.23}(f_M)^{+9.26}_{-5.40}(a_t)^{+1.25}_{-1.34}(m_c)^{+4.96}_{-3.91}(\theta_{sP_1})^{+0.23}_{-0.34}(a_t) \\
= \left[8.71^{+11.17}_{-7.44}\right] \times 10^{-5}, \tag{9}
\]

\[
Br(B_s \to J/\psi f_1(1420)) = 1.06^{+0.32}_{-0.23}(\omega_B)^{+0.16}_{-0.14}(f_M)^{+0.31}_{-0.25}(a_t)^{+0.19}_{-0.18}(m_c)^{+0.04}_{-0.04}(\theta_{sP_1})^{+0.04}_{-0.04}(a_t) \\
= \left[1.06^{+0.51}_{-0.41}\right] \times 10^{-3}; \tag{10}
\]

where the total errors are obtained by adding the various errors from different sources in quadrature. The individual theoretical errors are induced by the variation of the shape parameter $\omega_B = 0.50 \pm 0.05$ GeV [23] for the $B_s$ meson wave function, of the $J/\psi$ meson decay constant $f_{J/\psi} = 0.405 \pm 0.014$ GeV [24, 25] and the $f_1(f_8)$ state decay constant $f_{f_1} = 0.245 \pm 0.013(f_{f_8} = 0.239 \pm 0.013)$ GeV [7] or the $f_{1s}$ state decay constant $f_{f_{1s}} = 0.230 \pm 0.009$ GeV [22], of the Gegenbauer moments $a_2 = -0.04 \pm 0.03$ and $a_4 = -1.06 \pm 0.36 (a_2^0 = -0.07 \pm 0.04$ and $a_4^0 = -1.11 \pm 0.31)$ for the $f_1(f_8)$ distribution amplitudes [7], of the charm quark mass $m_c = 1.50 \pm 0.15$ GeV, and of the mixing angle $\phi_{sP_1} = (15.3 \pm 4)^\circ$ or $\theta_{sP_1} = (20 \pm 4)^\circ$ in the QF or SO basis [11], respectively. Moreover, in this work, as displayed in the above equations, the
higher order contributions are also simply investigated by exploring the variation of the hard scale $t_{\text{max}}$, i.e., from $0.8t$ to $1.2t$ (not changing $1/b_i, i = 1, 2, 3$), in the hard kernel, which have been counted into one of the source of theoretical uncertainties. It is found that the higher order corrections to these considered $B_s \rightarrow J/\psi f_1(1285)$ and $B_s \rightarrow J/\psi f_1(1420)$ decays are finite as the naive expectation. It is worthwhile to stress that the variation of the Cabibbo-Kobayashi-Maskawa parameters has almost no effects to the CP-averaged branching ratios and polarization fractions of these considered $B_s \rightarrow J/\psi f_1(1285)$ and $B_s \rightarrow J/\psi f_1(1420)$ decays in the pQCD approach and thus will be neglected in the numerical results as shown in Eqs. (7)-(10) and Table I.

From the numerical results predicted in the pQCD approach, it is interesting to find that the CP-averaged branching ratio, $(8.71^{+11.17}_{-7.44}) \times 10^{-5}((7.70^{+6.18}_{-4.88}) \times 10^{-5}$), of $B_s \rightarrow J/\psi f_1(1285)$ in the SO(QF) basis agrees well with the available data $(7.14^{+1.30}_{-1.41}) \times 10^{-5}$ [1] within the theoretical errors. Meanwhile, we also make the prediction theoretically on the CP-averaged branching ratio of $B_s \rightarrow J/\psi f_1(1420)$ channel as $(1.06^{+0.51}_{-0.41}) \times 10^{-3}((0.97^{+0.56}_{-0.42}) \times 10^{-3})$, which is very similar to that of $B_s \rightarrow J/\psi \phi$ decay and can be easily accessed and tested at the running LHCb and forthcoming Super-B experiments in the near future. The slightly larger results for the decay rates in the SO basis than those in the QF basis are mainly due to the larger decay constants of $f_1$ and $f_8$ than that of $f_{1s}$, which can be clearly seen in Eq. (6). When the very recently measured value of the mixing angle $\phi_{3P_1} = 24^\circ$ [1] is adopted, we get $18.29 \times 10^{-5}(20.71 \times 10^{-5})$ for the $B_s \rightarrow J/\psi f_1(1285)$ decay rate and $0.87 \times 10^{-3}(0.95 \times 10^{-3})$ for the $B_s \rightarrow J/\psi f_1(1420)$ decay rate, respectively, in the QF (SO) basis. One can find that the pQCD predictions for the $B_s \rightarrow J/\psi f_1(1285)$ decay rate in both mixing schemes with the measured mixing angle significantly exceed the experimental measurements. Moreover, according to the theoretical predictions in the pQCD approach, one can see that the decay rate for $B_s \rightarrow J/\psi f_1(1285)$ is more sensitive to the mixing angle $\theta_{3P_1}(\phi_{3P_1})$ than that for $B_s \rightarrow J/\psi f_1(1420)$, since the $f_1(1285)$ meson is dominated by the $u\bar{u} + d\bar{d}$ component while the $f_1(1420)$ meson is determined by the $s\bar{s}$ component.

With the help of Eq. (4), by combining the decay rate of $B_s \rightarrow J/\psi f_1(1285)$ as given in Eq. (1) and $\tan^2 \phi = 0.1970 \pm 0.053^{+0.014}_{-0.012}$ [1], one can deduce the branching ratio of $B_s \rightarrow J/\psi f_1(1420)$ mode as $(3.42^{+1.15}_{-1.16}) \times 10^{-4}$. Once the measurements at relevant experiments eventually confirm this value, it may imply the large exotic gluonic component hidden in the $f_1(1420)$ meson just like that in the pseudoscalar $\eta'$ meson of the $\eta - \eta'$ mixing system, which would need further studies in the future, although there are now no any signals observed at the experiments.

Based on the above theoretical predictions for the CP-averaged branching ratios of $B_s \rightarrow
$J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decays in the pQCD approach, the ratios of the decay rates between these two modes can be obtained straightforwardly as follows

$$R_{s}^{\text{QF;th.}} \equiv \frac{Br(B_s \to J/\psi f_1(1285))}{Br(B_s \to J/\psi f_1(1420))} = 0.079^{+0.078}_{-0.061}, \quad (11)$$

and

$$R_{s}^{\text{SO;th.}} \equiv \frac{Br(B_s \to J/\psi f_1(1285))}{Br(B_s \to J/\psi f_1(1420))} = 0.082^{+0.113}_{-0.077}, \quad (12)$$

where we have kept the masses of $f_1(1285)$ and $f_1(1420)$ mesons in the phase space factors for the $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decay rates. The good consistency between these two ratios $R_{s}^{\text{QF;th.}}$ and $R_{s}^{\text{SO;th.}}$ verify the equivalence of the QF basis and SO basis for the $f_1(1285) - f_1(1420)$ mixing in the pQCD calculations. Therefore, one can extract the mixing angle $\phi_{3 F_1}$ from the ratio of the branching ratios for $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ modes in the SO basis theoretically. The mixing angles for the $f_1(1285) - f_1(1420)$ system extracted through Eq. (4) are $\phi_{3 F_1} = (15.3^{+13.8}_{-12.1})^\circ$ in the QF basis and $\phi_{3 F_1} = (15.5^{+17.3}_{-14.2})^\circ$ in the SO basis, respectively. Here, we should point out that the errors induced by the variation of the input mixing angle are not considered in the extraction of the QF mixing angle $\phi_{3 F_1}$. The tiny deviation between the central values of these two QF mixing angles arises from the very small differences between the decay amplitudes $A(B_s \to J/\psi f_1)$ and $A(B_s \to J/\psi f_8)$ in the SO basis. Moreover, within the still large theoretical uncertainties from the non-perturbative inputs in the pQCD approach, our extracted mixing angle $\phi_{3 F_1}$ is basically in agreement with the earlier determination $(15^{+5}_{-10})^\circ$ by Mark-II detector at SLAC [4], with the updated Lattice QCD analysis $(21 \pm 5)^\circ$ [14], and with preliminary $(24.0^{+3.2}_{-2.7})^\circ$ reported by the LHCb Collaboration [1]. Strictly speaking, the non-perturbative inputs for the involved hadrons need stringent constraints from the experimental measurements, which then makes the relevant predictions theoretically reliable and comparable to the data. Of course, it needs to clarify that the definite mixing angle demands enough data samples collected from various processes.

We have also computed the CP-averaged polarization fractions for $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decay modes in the pQCD approach. The numerical results for the polarization fractions are presented in Table I, in which various errors induced by the input parameters have been added in quadrature.

From the theoretical predictions for the CP-averaged polarization fractions of the considered $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decays in the pQCD approach as shown in Table I,
TABLE I. The theoretical predictions for the CP-averaged polarization fractions of $B_s \to J/\psi f_1(1285)$ and $J/\psi f_1(1420)$ decays in the pQCD approach with different mixing schemes.

| Decay modes | QF basis (%) | SO basis (%) | Experimental data |
|-------------|--------------|--------------|-------------------|
| $B_s \to J/\psi f_1(1285)$ | 34.3$^{+14.7}_{-9.9}$ ($L$) | 36.3$^{+37.2}_{-17.7}$ ($L$) | - |
|              | 40.7$^{+6.2}_{-8.7}$ ($||$) | 39.8$^{+10.4}_{-22.5}$ ($||$) | - |
|              | 24.9$^{+3.8}_{-5.8}$ ($\perp$) | 23.9$^{+7.3}_{-15.5}$ ($\perp$) | - |
| $B_s \to J/\psi f_1(1420)$ | 34.7$^{+14.3}_{-10.0}$ ($L$) | 33.9$^{+9.8}_{-8.5}$ ($L$) | - |
|              | 42.5$^{+6.7}_{-9.0}$ ($||$) | 42.7$^{+5.9}_{-6.3}$ ($||$) | - |
|              | 22.8$^{+3.5}_{-5.3}$ ($\perp$) | 23.4$^{+2.7}_{-3.5}$ ($\perp$) | - |

one can easily find the analogous fractions on three polarizations are obtained between these two modes in each of the mixing schemes. Furthermore, the CP-averaged polarization fractions in each of the decay modes are very close to each other in both schemes. Another point is that, in the pQCD approach, the transverse polarization contributions dominate these two decays in the QF basis and the longitudinal polarization fractions are $(24.4 \sim 49.0\%)$ for $B_s \to J/\psi f_1(1285)$ decay and $(24.7 \sim 49.0\%)$ for $B_s \to J/\psi f_1(1420)$ decay (See Table I), respectively, which seems slightly different from that for $B_s \to J/\psi \phi$ channel [21]. Meanwhile, as can be seen in Table I, the polarization fractions calculated in the SO basis indicate that $B_s \to J/\psi f_1(1285)$ decay possibly has a little large longitudinal contributions when the large theoretical errors induced by the less constrained hadronic parameters are taken into account. The above theoretical predictions for the CP-averaged polarization fractions and the related phenomenology in both mixing schemes can be tested by the near future experiments at LHCb and Super-B.

In summary, motivated by the very recent LHCb measurement on the $B_s \to J/\psi f_1(1285)$ decay and encouraged by the good agreement between the pQCD predictions and the available data for the $B \to J/\psi V$ decays, we studied the $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decays for the first time within the framework of the pQCD approach by including higher order QCD corrections. We made the first pQCD evaluation for the CP-averaged branching ratios for the considered $B_s \to J/\psi f_1(1285)$ and $B_s \to J/\psi f_1(1420)$ decays. The results arising from a smaller angle $\phi^3_{P_1} \approx 15^\circ$ turn out to be well consistent with the current measurements within
theoretical errors. By employing the ratio of the decay rates for the considered two modes, we extracted the mixing angle $\phi_{3P_3}$ of $f_1(1285) - f_1(1420)$ mixing system as $\phi_{3P_3} = (15.3^{+13.8}_{-12.1})^\circ$ and $(15.5^{+17.3}_{-14.2})^\circ$, which are basically consistent with currently available measurements or estimations within still large theoretical errors. Furthermore, the large transverse polarization fractions for these two decay modes are also predicted for tests by the LHCb and the forthcoming Super-B experiments.

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