Neutrino masses beyond the minimal seesaw

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Abstract. The simplest possibility to generate small Majorana neutrino masses is the seesaw mechanism. However, the smallness of the observed neutrino masses can also be understood, if neutrino masses are generated by higher-dimensional operators and/or at higher loop level. In this talk recent work on systematic classifications of higher-dimensional and radiative neutrino mass models is summarized. Two particular classes of special genuine loop diagrams, i.e. diagrams which can lead to genuine neutrino mass models only under some specific, well-defined conditions, are also discussed.

1. Introduction
At low energy, all Majorana neutrino mass models can be described effectively by the Weinberg operator [1] or its higher-dimensional variants $O_{d=5+2n} = O_W(H H^\dagger)^n$. Opening the Weinberg operator at tree-level leads to three well-known realizations of the seesaw [2]. However, many models beyond these minimal seesaw mechanisms have been proposed in the literature, with different suppression mechanisms to explain the observed smallness of neutrino masses. In full generality we can write [3]:

$$m_\nu \propto \epsilon \cdot \left( \frac{1}{16\pi^2} \right)^n \cdot \left( \frac{v^4}{\Lambda} \right)^{d-5} \cdot \frac{v^2}{\Lambda}.$$  

(1)

Here, $v$ is the vacuum expectation value of the Higgs, $d$ stands for the dimension of the operator, $n$ is the number of loops at which neutrino masses are generated and $\epsilon$ expresses symbolically possible additional suppression factors. Finally, in addition, small couplings not shown explicitly in Eq. (1) could lead to smaller neutrino masses than naively expected.

There have been several attempts to systematically analyze the generation of Majorana neutrino masses. The authors of [3] have found all 1-loop topologies and diagrams at $d = 5$ level, the corresponding $(d = 5)$ 2-loop study was published in [4]. Neutrino masses have also been studied at $d = 7$. A complete tree-level analysis for $d = 7$ was given in [5], the extension to $d = 7$ 1-loop can be found in [6]. In this talk, we will mainly discuss two recent papers: [7] presented a systematic analysis of $d = 9 – 13$ tree-level neutrino mass models, while [8] gives a complete description of 3-loop realizations of the Weinberg operator. We also comment on special genuine diagrams, not discussed in detail in [4].
| Dimension (d) | 5 | 7 | 9 | 11 | 13 |
|---------------|---|---|---|----|----|
| Topologies    | 1 | 5 | 18 | 92 | 576|
| Diagrams      | 3 | 9 | 66 | 504| 4199|
| Genuine models | 3 | 1 | 2  | 2  | 6  |

**Table 1.** The number of tree-level topologies, diagrams and genuine models as function of the dimension $d$ of the neutrino mass operator.

2. **High-dimensional tree-level models**

Finding all possible topologies and diagrams for generating neutrino masses at tree-level and a given $d$ is a straight-forward but tedious task, because the number of possible topologies grows quickly with $d$ [7], see table 1. Here, topology is used for Feynman diagrams in which fermions and scalars are not identified, while diagrams are descended from topologies by distinguishing the scalar or fermion nature of the lines.

Most of the diagrams at larger dimensions, however, do not lead to interesting models. Already at $d = 7$ one can find examples of diagrams for the inverse seesaw [9] or linear seesaw [10, 11] realizations, but from the classification point of view these are only seesaw type-I like diagrams with additional singlet fields. Many other diagrams are just propagator corrections, etc. Thus, only very few diagrams lead to genuine neutrino mass models. Here, the word genuine is used for models which give automatically the leading order contribution to the neutrino mass at dimension $d$, without the need to introduce additional symmetries beyond those of the SM.

At $d = 7$ there is only one genuine model, it was first described in [12]. At $d = 9$ and $d = 11$ there are two possibilities each; two of them are shown in fig. (1). For the six variations at $d = 13$ see [7].

![Figure 1. Two examples of genuine models at $d = 9$ (left) and $d = 11$ (right).](image)

3. **Radiative models and genuineness**

Classical radiative neutrino mass models are the well-known 1-loop Zee model [13] and the 2-loop Babu-Zee model [14, 15, 16]. At 3-loop, we mention the KNT [17] and the AKS models [18].

Due to lack of space, here we mention only that the systematic 3-loop analysis [8] identified 4367 topologies, from which however only 44 are genuine and 55 are special genuine, see below. Five example models are shown in fig. (2). Note that the 2nd model from the left is the KNT [17], which, different from all other models shown here, requires an additional discrete symmetry in order to avoid a tree-level seesaw contribution to the neutrino mass.
Figure 2. Five examples of 3-loop model diagrams.

The concept of genuineness, mentioned above, becomes much more subtle in the case of loop models. This was first discussed in [3], where it was shown that some particular diagrams with finite loop integrals may lead to loop generation of 3-point vertices, if one adds a $\mathbb{Z}_2$ symmetry and a Majorana fermion to the model in the correct way. In [4] the corresponding diagrams at 2-loop level were called “non-genuine, but finite”. The work of [8], however, introduced two more special cases (class-I and class-II), where diagrams with finite loop integrals can be associated with genuine models.

Two different types of special genuine diagrams have been discussed in [8]. While [8] concerns itself mostly with 3-loop diagrams, here we will discuss this point with 2-loop examples. An example of a class-I special genuine diagram is shown in fig. (3). Note, that this type of diagram has been labeled as ISC-i-2 (“internal scalar correction”) in [4]. Here, we can see that if the loop connecting the scalars $S_A$, $S_B$ and the Higgs is allowed by the SM quantum numbers (or any additional symmetry), then the vertex $S_A S_B H$ on the right can also not be forbidden by symmetry. Thus, in general one would conclude that this diagram is non-genuine, although it leads to a finite loop integral. However, if either $S_A$ or $S_B$ is identified with the SM Higgs, the coupling $HHS_B \equiv 0$ vanishes identically, due to $SU(2)_L$. In this special case, the 2-loop model is genuine.

Figure 3. Example of a special genuine diagram, class-I. Here, the loop on the left can be contracted to the vertex shown on the right, unless either $S_A$ or $S_B$ is identified with the SM Higgs.

Class-II of the special genuine diagrams is even more subtle. Fig. (4) shows an example. Consider first the 1-loop diagram on the left. If the internal fermion $F_3$ is massive, its propagator contains two pieces: $m_F + \frac{1}{k}$. However, if the internal fermion in this diagram is massless, then only the term $\frac{1}{k}$ exists. In that case, in order to form a mass term, another momentum term (derivative) can be added at 2-loop level and the corresponding diagram becomes genuine. The diagram on the right of fig. (4), NG-CLBZ-4 in the notation of [4], shows an example.
Figure 4. To the left: One loop realization of the vertex $\bar{F}_2 F_1 S_1^\ast$. To the right: Example for a special genuine diagram in which the 1-loop generated vertex $\bar{e} L S_{1,2,3/2}$ is proportional to a derivative, due to the presence of the massless SM fermion in the loop.

One can easily show that all 20 “non-genuine, but finite” diagrams, listed in the appendix of [4], can be identified as special genuine diagrams in this way.

4. Summary
In this paper we have briefly summarized high-dimensional tree-level neutrino mass models [7] and the systematic 3-loop [8] analysis. We also discussed special genuine diagrams, not spelled out in detail in [4].

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