New Super Calogero Models and OSp(4|2) Superconformal Mechanics

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Abstract

We report on the new approach to constructing superconformal extensions of the Calogero-type systems with an arbitrary number of involved particles. It is based upon the superfield gauging of non-abelian isometries of some supersymmetric matrix models. Among its applications, we focus on the new \( \mathcal{N}=4 \) superconformal system yielding the U(2) spin Calogero model in the bosonic sector, and the one-particle case of this system, which is a new OSp(4|2) superconformal mechanics with non-dynamical U(2) spin variables. The characteristic feature of these models is that the strength of the conformal inverse-square potential is quantized.

1 Motivations and contents

The conformal Calogero model \( \Pi \) describes \( n \) identical particles interacting pairwise through an inverse-square potential

\[
V_C = \sum_{a \neq b} \frac{g}{(x_a - x_b)^2}, \quad a, b = 1, ..., n. \tag{1}
\]

It is a nice example of integrable \( d = 1 \) system. This simplest \( (A_{n-1}) \) Calogero model has some integrable generalizations, both conformal and non-conformal \([1, 2]\).

As for superconformal extensions of the Calogero models (s-C models in what follows), the basic facts about them can be shortly summarized as follows:

- \( \mathcal{N} = 2 \) superextension of the model \( \Pi \) and its some generalizations for any \( n \) was given by Freedman and Mende in 1990 \([3]\) (see also \([4]\) for \( \mathcal{N} = 1 \) extensions).

\(^1\)Talk presented by E. Ivanov at the XIII International Conference “Symmetry Methods in Physics”, Dubna, July 6-9, 2009.
First attempts toward $\mathcal{N} = 4$ extensions were undertaken by Wyllard in 2000 [5]. Further progress was achieved in refs. [6] - [10].

Until recently, $\mathcal{N}=4$ s-C models generalizing (11) for a generic $n$ were not constructed.

At the same time, s-C systems are of great interest from various points of view. In 1999, Gibbons and Townsend [11] suggested that $\mathcal{N} = 4$ s-C models might provide a microscopic description of the extreme Reissner-Nordström black hole in the near-horizon limit and, even more, be one of the faces of the hypothetical M-theory. Also, this sort of models can bear a tight relation to AdS/CFT and brane stuff (M-theory, strings, etc), quantum Hall effect (see, e.g., [2], [12] - [14]), etc. One-particle prototype of s-C systems is the superconformal mechanics. The first $\mathcal{N} = 2$ and $\mathcal{N} = 4$ variants of the latter were constructed and studied by Akulov and Pashnev in 1983 [15], Fubini and Rabinovici in 1984 [16] and Ivanov et al in 1989 [17]. These models attract a lot of attention mainly because of their intimate relationships to the description of the black-hole type solutions of supergravity (see e.g. [18, 19]).

Recently, we suggested a universal approach to s-C models for an arbitrary number $n$ of interacting particles, including the $\mathcal{N}=4$ models [20]. It is based on the superfield gauging of non-abelian isometries of some supersymmetric matrix models along the line of ref. [21].

This new approach is based upon the following two primary principles:

- U($n$) gauge invariance for $n$-particle s-C models;
- $\mathcal{N}$-extended superconformal symmetry.

The models constructed in this way display the following salient features:

- Their bosonic sector is:
  - the standard $A_{n-1}$ Calogero model for $\mathcal{N}=1$ and $\mathcal{N}=2$ cases,
  - a new variant of the U(2)-spin Calogero model [2] in the $\mathcal{N}=4$ case;

- In the $\mathcal{N}=2$ case there arise new superconformal extensions (different from those of Freedman and Mende);

- In the $\mathcal{N}=4$ case the gauge approach directly yields $\text{OSp}(4|2)$ as the superconformal group, but the general $\mathcal{N}=4, d=1$ superconformal group $D(2, 1; \alpha)$ can be incorporated as well;

- The center-of-mass coordinate in the $\mathcal{N}=4$ case is not decoupled, and it acquires a conformal potential on shell. So a new model of $\mathcal{N}=4$ superconformal mechanics emerges in the $n=1$ limit [22, 23] (see also [24]).

In the present talk we give a brief account of this gauge approach, with the main focus on the $\mathcal{N} = 4$ super Calogero model and the new $\mathcal{N}=4$ superconformal mechanics just mentioned.
2 Bosonic Calogero as a gauge matrix model

The nice interpretation of the model (1) as a gauge model was given in [12, 13].

The starting point of this approach is the U(n), d=1 matrix gauge theory which involves:

- an hermitian $n \times n$-matrix field $X^b_a(t), (X^b_a = X^a_b)$, $(a, b = 1, \ldots, n)$;
- a complex U(n)-spinor field $Z_a(t), \bar{Z}^a = \bar{Z}^a_++ \bar{Z}^a_-$;
- $n^2$ non–propagating U(n) “gauge fields” $A^b_a(t), (A^b_a) = A^b_a_+$.

The invariant action is written as [12]:

$$S_0 = \int dt \left[ \text{Tr} (\nabla X \nabla X) + \frac{i}{2}(\bar{Z} \nabla Z - \nabla \bar{Z} Z) + c \text{Tr} A \right],$$

where

$$\nabla X = \dot{X} + i[A, X], \quad \nabla Z = \dot{Z} + iAZ.$$ 

It respects the following invariances:

• The $d = 1$ conformal SO(1, 2) invariance realized by the transformations:

$$\delta t = a, \quad \delta \dot{t} = 0, \quad \delta X^b_a = \frac{1}{2} \dot{a} X^b_a, \quad \delta Z_a = 0, \quad \delta A^b_a = -\dot{a} A^b_a.$$

• The invariance under the local U(n) transformations:

$$X \rightarrow gXg^+, \quad Z \rightarrow gZ, \quad A \rightarrow gAg^+ + i\dot{g}g^+,$$

where $g(\tau) \in \text{U(n)}$.

Using this gauge U(n) freedom, one can impose the following gauge conditions:

$$X^b_a = x^b_a \delta^a_a X^b, \quad Z^a = Z_a.$$ (3)

As the next step, one makes use of the algebraic equations of motion

$$\delta A^a_a : (Z_a)^2 = c, \quad \delta A^a_b (\text{for } a \neq b) : A^a_a = \frac{Z_a Z_b}{2(x_a - x_b)^2}.$$ 

Substituting the expression for $A^a_a$ back into the gauge-fixed form of the action (2), one recovers the standard Calogero action

$$S_0 = \int dt \left[ \sum_a \dot{x}^a_x x^a_x - \sum_{a \neq b} \frac{c^2}{4(x_a - x_b)^2} \right].$$

Our approach to supersymmetric extensions of the Calogero models is just supersymmetrization of the above gauge approach, with the fields $X, Z, A$ substituted by the appropriate $d = 1$ superfields.
3 $\mathcal{N} = 1$ superconformal Calogero system

We start with a brief account of the $\mathcal{N} = 1, d = 1$ supersymmetric version.

The point of departure in this case is the one-dimensional $\mathcal{N} = 1$ supersymmetric $U(n)$ gauge theory which involves:

- an even matrix superfield $X^a(t, \theta), (X)^\dagger = X$,
- an even $U(n)$-spinor superfield $Z_a(t, \theta), \bar{Z}^a(t, \theta) = (Z_a)^\dagger$,
- an odd gauge connection $A^b_a(t, \theta), (A)^\dagger = -A$.

The invariant action is written as an integral over the $\mathcal{N} = 1, d = 1$ superspace:

$$S_1 = -i \int dt d\theta \left[ \text{Tr} \left( \nabla_t X D X + cA \right) + \frac{i}{2}(\bar{Z} D Z - D \bar{Z} Z) \right], \quad (4)$$

where

$$D X = DX + i[A, X], \quad \nabla_t X = -i DX, \quad D Z = D Z + iA Z,$$

$$D = \partial_\theta + i\theta \partial_t, \quad \{D, D\} = 2i \partial_t.$$

The action (4) possesses $\mathcal{N} = 1$ superconformal $\text{OSp}(1|2)$ invariance:

$$\delta t = -i \eta \theta t, \quad \delta \theta = \eta t,$$

$$\delta X = -i \eta \theta X, \quad \delta A = i \eta \theta A, \quad \delta Z = 0,$$

and gauge $U(n)$ invariance:

$$X' = e^{i\tau} X e^{-i\tau}, \quad Z' = e^{i\tau} Z, \quad A' = e^{i\tau} A e^{-i\tau} - i e^{i\tau} D e^{-i\tau},$$

where $\tau^b_a(t, \theta) \in u(n)$ is an hermitian matrix parameter.

One can choose WZ gauge for the spinor connection:

$$A = i\theta A(t). \quad (5)$$

After integrating over $\theta$ s in the gauge-fixed form of (4) and eliminating auxiliary fields, one obtains:

$$S_1 = S_0 + S^\Psi, \quad S^\Psi_1 = -i \text{Tr} \int dt \Psi \nabla \Psi, \quad (6)$$

where $\Psi = -i D X|$ and $\nabla \Psi = \dot{\Psi} + i[A, \Psi]$. The bosonic part $S_0$ of $S_1$ in (6) is just the “gauge-unfixed” Calogero action (2).

After integrating out non-propagating gauge fields $A^b_a$ from the total action $S_1$ and fixing the residual $U(n)$ gauge freedom of the WZ gauge (5) in the same way as in (3), we obtain an $\mathcal{N} = 1$ superconformal action which contains $n$ bosonic fields $x_a$ with the standard conformal Calogero potential (1) accompanied by interactions with $n^2$ physical fermionic fields $\psi^a_b$.  

4
4 $\mathcal{N} = 4$ superconformal Calogero

$\mathcal{N}=4, d=1$ models are naturally formulated in the $d = 1$ version of harmonic superspace (HSS) \[25\]:

$$(t, \theta^i, \bar{\theta}^\dot{i}, u^\pm_i), \quad i, k = 1, 2.$$  

Bosonic SU(2)-doublets $u^\pm_i$ are harmonic coordinates, with the basic relation $u^i_+ u^-_i = 1$. The main feature of HSS is the presence of harmonic analytic subspace in it (an analog of chiral superspace), closed under the action of $\mathcal{N}=4$ supersymmetry:

$$(\zeta, u) = (t_A, \theta^+, \bar{\theta}^+, u^\pm_i),$$

$$t_A = t - i(\theta^+ \bar{\theta}^- + \theta^- \bar{\theta}^+), \quad \theta^\pm = \theta^i u^\pm_i, \quad \bar{\theta}^\pm = \bar{\theta}^\dot{i} u^\pm_i.$$  

The integration measures in the full HSS and its analytic subspace are defined, respectively, as:

$$\mu_H = dudtd^4\theta, \quad \mu^{(-2)}_A = dud\zeta^{(-2)}.$$  

The $\mathcal{N} = 4, d = 1$ supergauge theory which generalizes the bosonic and $\mathcal{N}=1$ examples described above involves the following superfields:

- The hermitian matrix superfields $\mathcal{X} = (\mathcal{X}_a^0)$ (multiplets $(1, 4, 3)$):

  $$\mathcal{D}^{++}\mathcal{X} = 0, \quad \mathcal{D}^+\mathcal{D}^-\mathcal{X} = 0, \quad (\mathcal{D}^+\bar{\mathcal{D}}^- + \bar{\mathcal{D}}^+\mathcal{D}^-)\mathcal{X} = 0;$$

- Analytic superfields $\tilde{Z}^+(\zeta, u)$ (multiplets $(4, 4, 0)$):

  $$\mathcal{D}^{++}\tilde{Z}^+ = 0, \quad \mathcal{D}^+\mathcal{Z}^+ = 0, \quad \bar{\mathcal{D}}^+\mathcal{Z}^+ = 0;$$

- The analytic gauge matrix connection $V^{++}(\zeta, u)$. It specifies the gauge-covariant derivatives (harmonic and spinor):

  $$\mathcal{D}^{++}\tilde{Z}^+ = (\mathcal{D}^{++} + iV^{++})\tilde{Z}^+, \quad \mathcal{D}^{++}\mathcal{X} = \mathcal{D}^{++}\mathcal{X} + i[V^{++}, \mathcal{X}], \text{ etc.}$$

The $\mathcal{N}=4$ superconformal action is a sum of three terms:

$$S_4 = S_X + S_{WZ} + S_{FI},$$  

where

$$S_X = -\frac{1}{2} \int \mu_H \text{Tr}(\mathcal{X}^2), \quad S_{WZ} = \frac{i}{2} \int \mu^{(-2)}_A \mathcal{V}_0 \tilde{Z}^+ \mathcal{Z}^+, \quad S_{FI} = \frac{i}{2} c \int \mu^{(-2)}_A \text{Tr} V^{++}.$$  

and $\mathcal{V}_0(\zeta, u)$ is a real analytic superfield, which is related to $X_0 \equiv \text{Tr}(\mathcal{X})$ by the integral transform

$$X_0(t, \theta^i, \bar{\theta}^\dot{i}) = \int du_0 \mathcal{V}_0(t_A, \theta^+, \bar{\theta}^+, u^\pm_i) \bigg|_{\theta^\pm = \theta^i u^\pm_i, \bar{\theta}^\pm = \bar{\theta}^\dot{i} u^\pm_i},$$

$$\mathcal{V}_0' = \mathcal{V}_0 + \mathcal{D}^{++}\mathcal{A}^{--}.$$  

The action (7) respects the following set of invariances:
• $\mathcal{N}=4$ superconformal invariance under the supergroup $D(2,1;\alpha = -1/2) \simeq \text{OSp}(4|2)$:

$$
\delta X = -\Lambda_0 X, \quad \delta Z^+ = \Lambda Z^+, \quad \delta V^{++} = 0, \quad \delta \mathcal{V}_0 = -2\Lambda \mathcal{V}_0,
$$

$$
\Lambda = 2i\alpha(\bar{\eta}^- \theta^+ - \eta^- \bar{\theta}^+), \quad \Lambda_0 = 2\Lambda - D^- D^{++} \Lambda;
$$

• Gauge $U(n)$ invariance:

$$
X' = e^{i\lambda}Xe^{-i\lambda}, \quad Z^{'\prime} = e^{i\lambda}Z^{'}, \quad V^{+'} = e^{i\lambda}V^{++}e^{-i\lambda} - i e^{i\lambda}(D^{++}e^{-i\lambda}),
$$

where $\lambda^b_a(\zeta, u^\pm) \in u(n)$ is the ‘hermitian’ analytic matrix parameter, $\bar{\lambda} = \lambda$.

Like in the $\mathcal{N}=1$ case, using the $U(n)$ gauge freedom we can choose the WZ gauge for $V^{++}$:

$$
V^{++} = -2i \theta^+ \bar{\theta}^+ A(t_A).
$$

In this gauge:

$$
S_4 = S_b + S_f,
$$

$$
S_b = \int dt \left[\text{Tr} (\nabla X \nabla X + c A) + \frac{i}{2} x_0 (\bar{Z}_k \nabla Z^k - \nabla Z_k Z^k) + \frac{n}{8} (\bar{Z}^i Z^k)(\bar{Z}_i Z_k)\right],
$$

$$
S_f = -i \text{Tr} \int dt \left(\bar{\Psi}_k \nabla \Psi^k - \nabla \bar{\Psi}_k \Psi^k\right) - \int dt \frac{\bar{\Psi}_0 \bar{\Psi}_0 (\bar{Z}_i Z_k)}{X_0}.
$$

Here $X = X(t_A) + \theta^- \Psi^i(t_A) u_i^+ + \bar{\theta}^- \bar{\Psi}^i(t_A) u_i^+ + \ldots$, $X_0 \equiv \text{Tr}(X)$, $\Psi_0^i \equiv \text{Tr}(\Psi^i)$, $\bar{\Psi}_0^i \equiv \text{Tr}(\bar{\Psi}^i)$, $Z^+ = Z^i(t_A) u_i^+ + \ldots$.

Let us study the bosonic limit of the action (14). To pass to this limit, one needs:

• to impose the gauge $X_a^b = 0$, $a \neq b$;

• to eliminate $A^b_a$, $a \neq b$, by their algebraic equations of motion;

• to pass to the new fields $Z^a_i = (X_0)^{1/2} Z^i_a$ (in what follows, we shall omit primes).

As a result, we obtain the following bosonic action

$$
S_b = \int dt \left\{\sum_a \dot{x}_a \dot{x}_a + \frac{i}{2} \sum_a (\dot{Z}^a_k Z^k_a - \dot{Z}_k^a Z^k_a) + \sum_{a \neq b} \frac{\text{Tr}(S_a S_b) - n \text{Tr}(\dot{S} \dot{S})}{2(X_0)^2}\right\},
$$

where

$$
(S_a)^{\prime}_i \equiv \ddot{Z}^a_i Z^a_i, \quad (\dot{S})_{i}^{\prime} \equiv \sum_a [(S_a)^{\prime}_i - \frac{1}{2} \delta^{\prime}_i (S_a)_{k}^{k}] .
$$

The fields $Z^a_i$ are subject to the constraints

$$
\ddot{Z}^a_i Z^a_i = c \quad \forall a .
$$

6
Since the fields $Z^a_k, \bar{Z}^a_k$ are described by the Lagrangian of the first order in the time derivative, we are led quantify them by the Dirac quantization method:

$$i \frac{1}{2} \int dt \sum_a (\dot{Z}^a_k \dot{Z}_a^k - \dot{\bar{Z}}^a_k \bar{Z}_a^k) \quad \Rightarrow \quad [\bar{Z}^a_i, Z^b_j]_D = i \delta^a_b \delta^i_j.$$  

Now it is easy to check that the quantities $S_a$ defined in (14) form, for each $a$, $u(2)$ algebras

$$[(S_a)^i_j, (S_b)^l_k]_D = i \delta^a_b \delta^i_j \{ \delta^l_k (S_a)^i_j - \delta^i_k (S_a)^l_j \}.$$  

Modulo center-of-mass conformal potential, the bosonic action (13) can be written as

$$S'_b = \int dt \left\{ \sum_a \dot{x}_a \dot{x}_a + \sum_{a \neq b} \frac{\text{Tr}(S_a S_b)}{4(x_a - x_b)^2} \right\}. \quad (16)$$

It is none other than the action of integrable $U(2)$-spin Calogero model [2].

5 OSp(4|2) superconformal mechanics

The $n = 1$ case of the $\mathcal{N}=4$ Calogero model, as distinct from the $\mathcal{N}=1$ and $\mathcal{N}=2$ models, already yields a conformal inverse-square potential at the component level (for the center-of-mass coordinate) and so yields a non-trivial $\mathcal{N}=4$ superconformal mechanics model [22].

The corresponding superfield action is

$$S = S_X + S_{FI} + S_{WZ}, \quad (17)$$

where

$$S_X = -\frac{1}{2} \int \mu_H X^2, \quad S_{FI} = \frac{i}{2} \mathcal{C} \int \mu_A^{(-2)} V^{++}, \quad S_{WZ} = \frac{1}{2} \int \mu_A^{(-2)} V \bar{Z}^+ Z^+, \quad D^{++} X = 0, \quad D^+ D^- X = \bar{D}^+ \bar{D}^- X = (D^+ \bar{D}^- + \bar{D}^+ D^-) X = 0,$$

$$D^{++} \bar{Z}^+ \equiv (D^{++} + i V^{++}) \bar{Z}^+ = 0, \quad D^+ \bar{Z}^+ \equiv (D^{++} - i V^{++}) \bar{Z}^+ = 0, \quad \delta V^{++} = -D^{++} \lambda, \quad \delta Z^+ = i \lambda \bar{Z}^+, \quad \delta \bar{Z}^+ = D^{++} \lambda^- . \quad (18)$$

The actions $S_{FI}$ and $S_{WZ}$ are invariant under the general $\mathcal{N}=4$ superconformal group $D(2,1; \alpha)$ for arbitrary $\alpha$, while $S_X$ - only with respect to $D(2,1; \alpha = -1/2) \sim OSp(4|2)$, so the total action is $OSp(4|2)$-superconformal.

The action (17) can be generalized to any $\alpha$, but at cost of nonlinear action for $X$. So we limit our presentation here to the case of $\alpha = -1/2$.

The off-shell component content of the model is $(1, 4, 3) \oplus (4, 4, 0) \oplus V^{++} = (1, 4, 3) \oplus (3, 4, 1)$. So it is reducible. The on-shell content can be most clearly revealed in the WZ gauge $V^{++} = -2i \theta^+ \bar{\theta}^+ A(t)$. After eliminating auxiliary fields from the total action, the latter takes the form

$$S = S_b + S_f, \quad (19)$$
where
\[
S_b = \int dt \left[ \dot{x} + \frac{i}{2} (\bar{z}_k z^k - \dot{\bar{z}}_k z^k) - \frac{(\bar{z}_k z^k)^2}{16x^2} - A (\bar{z}_k z^k - c) \right],
\]
\[
S_f = -i \int dt \left( \bar{\psi}_k \dot{\psi}^k - \dot{\bar{\psi}}_k \psi^k \right) - \int dt \frac{\dot{\psi}^i \bar{\psi}^j z^i (\bar{z}_k)^j}{x^2}.
\]
(20)

Varying with respect to \(A(t)\) as a Lagrange multiplier yields the constraint
\[
\bar{z}_k z^k = c.
\]
(21)

After properly fixing the residual U(1) gauge invariance, one can solve this constraint in terms of two independent fields \(\gamma(t)\) and \(\alpha(t)\) as
\[
z^1 = \kappa \cos \frac{\gamma}{2} e^{i \alpha/2}, \quad z^2 = \kappa \sin \frac{\gamma}{2} e^{-i \alpha/2}, \quad \kappa^2 = c.
\]
(22)

Then the bosonic action takes the form
\[
S_b = \int dt \left[ \dot{x} \dot{x} - \frac{c^2}{16x^2} - \frac{c}{2} \cos \gamma \dot{\alpha} \right].
\]
(23)

It is a sum of the standard conformal mechanics action for the variable \(x(t)\), and the \(S^2\) Wess-Zumino term for the angular variables \(\gamma(t)\) and \(\alpha(t)\), of the first-order in time derivative. Taken separately, this term provides an example of Chern-Simons mechanics [26, 27]. The variables \(\gamma(t)\) and \(\alpha(t)\) (or \(z^k\) and \(\bar{z}_k\) in the manifestly SU(2) covariant formulation) become spin degrees of freedom ("target harmonics") upon quantization.

We quantize by Dirac procedure,
\[
[X, P] = i, \quad [Z^i, \bar{Z}_j] = \delta^i_j, \quad \{\Psi^i, \bar{\Psi}^j\} = -\frac{1}{2} \delta^i_j,
\]
\[
P = \frac{1}{i} \partial/\partial X, \quad \bar{Z}_i = v^+_i, \quad Z^i = \partial/\partial v^+_i, \quad \Psi^i = \psi^i, \quad \bar{\Psi}^i = -\frac{1}{2} \partial/\partial \psi^i.
\]
(24)

The wave function is subject to the constraint
\[
D^0 \Phi = \bar{Z}_i Z^i \Phi = v^+_i \frac{\partial}{\partial v^+_i} \Phi = c \Phi,
\]
(25)

whence
\[
\Phi = A^{(c)}_1 + \psi^i B^{(c)}_i + \psi^i \psi_j A^{(c)}_2,
\]
(26)
and the component fields are collections of the SU(2) irreps with the isospins \(c/2, (c + 1)/2, (c - 1)/2\) and \(c/2\), respectively (the component fields depend on \(X\)).

The constant \(c\) gets quantized, \(c \in \mathbb{Z}\), as a consequence of requiring the wave function [26] with the constraint [25] to be single-valued. The same phenomenon takes place in the general \(n\)-particle \(\mathcal{N}=4\) Calogero system due to the constraints [13] which, after quantization, become analogs of [25]. This is also in agreement with the analogous
arguments in the topological Chern–Simons quantum mechanics \cite{27}, which are based upon the path integral quantization\textsuperscript{2}.

The quantum Hamiltonian is given by the expression

$$H = \frac{1}{4} \left( p^2 + \frac{l(l+1)}{X^2} \right), \quad l = (c/2, (c+1)/2, (c-1)/2, c/2).$$ \hspace{1cm} (27)

The basic distinguishing feature of the new superconformal mechanics model is that the bosonic sector of the space of its quantum states (with fermionic states neglected) is a direct product of the space of states of the standard conformal mechanics (parametrized by $X$) and a fuzzy sphere \cite{29} (parametrized by $Z_i, \bar{Z}_i$). The full wave functions are irreps of SU(2). The whole space of states (with fermions), shows no any product structure.

One can ask what is the brane analog of this new superconformal mechanics via AdS\textsubscript{2}/CFT\textsubscript{1} correspondence. The preliminary answer is that it is some superparticle evolving on AdS\textsubscript{2} and coupled to the external magnetic charge via WZ term.

6 Summary and outlook

Let us summarize the results presented in the Talk and outline directions of the further studies.

- We proposed a new gauge approach to the construction of superconformal Calogero-type systems. The characteristic features of this approach are the presence of auxiliary supermultiplets with WZ type actions, the built-in superconformal invariance and the emergence of the Calogero coupling constant as a strength of the FI term of the U(1) gauge (super)field. This strength is quantized.

- We used the U($n$) gauging and obtained superextensions of the $A_{n-1}$ Calogero model. Superextensions of other conformal Calogero models could be obtained by choosing other gauge groups.

- The $\mathcal{N}=4$ action presented is invariant under $D(2, 1; -1/2) \cong$ OSp(4|2). It can be easily generalized to an arbitrary $\alpha$. \textsuperscript{3}

- We constructed a new $\mathcal{N}=4$ superconformal mechanics with the OSp(4|2) invariance as the extreme $n=1$ case of our $\mathcal{N}=4$ Calogero system. After quantization it yields a fuzzy sphere in the bosonic sector.

As the mainstream directions of the future work we would like to distinguish (i) construction of the full quantum version of the new $\mathcal{N}=4$ super Calogero model for any

\textsuperscript{2}Actually, in the considered gauge approach to Calogero-type models the constant $c$ is quantized in all cases, including the purely bosonic one, due to its appearance as a strength of the term $c\int dt \text{Tr}A$ in the total component actions \cite{12} (see also a recent paper \cite{28}).

\textsuperscript{3}For the particular case of $n=1$ (superconformal mechanics) such a generalization was given in a recent paper \cite{23}. For $\alpha \neq 0$, the general $n$ particle action was given in \cite{20}.
number of particles (as well as of the new $\mathcal{N} = 1$ and $\mathcal{N} = 2$ extensions); (ii) elucidating
the relationships of this $\mathcal{N} = 4$ super Calogero system to the $\mathcal{N} = 4$ Calogero-type systems
considered in [5] - [10] and to the black-hole stuff; (iii) analysis of possible integrability
properties of the new super Calogero systems (searching for their Lax pair representation,
etc).

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