Higgs–Inflaton Mixing and Vacuum Stability

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Abstract

The quartic and trilinear Higgs field couplings to an additional real scalar are renormalizable, gauge and Lorentz invariant. Thus, on general grounds, one expects such couplings between the Higgs and an inflaton in quantum field theory. In particular, the (often omitted) trilinear coupling is motivated by the need for reheating the Universe after inflation, whereby the inflaton decays into the Standard Model (SM) particles. Such a coupling necessarily leads to the Higgs–inflaton mixing, which could stabilize the electroweak vacuum by increasing the Higgs self–coupling. We find that the inflationary constraints on the trilinear coupling are weak such that the Higgs–inflaton mixing up to order one is allowed, making it accessible to colliders. This entails an exciting possibility of a direct inflaton search at the LHC.
I. INTRODUCTION

The current data favor metastability of the electroweak (EW) vacuum, although the result is very sensitive to the top quark mass [1–4]. Assuming that our vacuum is indeed metastable, we face a number of cosmological challenges including why the Universe has chosen an energetically disfavored state and why it stayed there during inflation despite quantum fluctuations [5, 6]. Minimal solutions to these puzzles require modification of the Higgs potential during inflation only [6], although introduction of a single extra scalar is sufficient to make the electroweak vacuum completely stable [7, 8].

In this Letter, we suggest another minimal option which does not employ any extra fields beyond the usual inflaton. We show that the Higgs mixing with an inflaton can lead to a stable EW vacuum. A trilinear Higgs–inflaton coupling always leads to such a mixing and it is generally present in realistic models describing the reheating stage correctly [9]. We find that cosmological constraints on this coupling are weak and an order one mixing is possible. In this case, the model is effectively described by a single mass scale of the EW size making it particularly interesting for direct LHC searches.

A. THE SET-UP

In quantum field theory, one should include all the couplings that are (up to) dimension–4, gauge and Lorentz invariant. Thus, on general grounds, we expect a quartic $H^\dagger H\phi^2$ and a trilinear $H^\dagger H\phi$ interaction between the Higgs field and an inflaton $\phi$. The presence of the trilinear term can be motivated by the need for reheating the Universe after inflation: the inflaton transfers (at least in part) its energy to the SM particles through decay and the relevant interactions generate the $H^\dagger H\phi$ term at loop level [9]. It can only be forbidden if the inflaton is assumed to be stable, for instance, due to the $\phi \to -\phi$ symmetry, and constitutes part of dark matter [10]. However, it is not clear whether this symmetry remains exact in quantum gravity.

Apart from the renormalizable QFT interactions, the Higgs dynamics are affected by its coupling to gravity. Although gravity is non–renormalizable, one may focus on the coupling of lowest dimension $H^\dagger H \hat{R}$ [11], with $\hat{R}$ being the scalar curvature, assuming that the effective field theory expansion applies. In any case, such a coupling is generated radiatively [12].
Thus, on general grounds, we expect the following leading interactions between the Higgs and an inflaton/gravity (see also [13]),

\[-L_{h\phi} = \lambda_{h\phi} H \phi^2 + 2\sigma H^\dagger H \phi,\]
\[-L_{hR} = \xi_h H^\dagger H \hat{R}.\]  

(1)

It is interesting that this setup necessarily leads to the mixing between the Higgs and the inflaton. This is required by the \(H^\dagger H \phi\) term with \(H\) developing a vacuum expectation value.

Including an analogous \(\phi\) coupling to gravity and all renormalizable \(\phi\)–self-interactions, we obtain the following Jordan frame action:

\[S = \int d^4x \sqrt{-\hat{g}} \left[ \frac{1}{2} \hat{R} - \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu h \partial_\nu h - V(\phi, h) \right],\]

(2)

where we have set \(M_{\text{Pl}} = 1\) and used the unitary gauge \(H = (0, h/\sqrt{2})^T\). The frame function \(\Omega^2\) and the potential \(V(\phi, h)\) are given by

\[\Omega^2 = 1 + \xi_\phi \phi^2 + \xi_h h^2,\]
\[V(\phi, h) = \frac{\lambda_h}{4} h^4 - \frac{\mu_h}{2} h^2 + \frac{\lambda_{h\phi}}{2} h^2 \phi^2 + \sigma h^2 \dot{\phi} + \frac{\lambda_\phi}{4} \phi^4 + b_3 \frac{\phi^3}{3} - \frac{\mu_\phi}{2} \phi^2 + b_1 \phi,\]

(3)

where we have eliminated the \(\phi \hat{R}\) term by field redefinition of \(\phi\). We take \(\lambda_\phi > 0, \xi_\phi \gg |\xi_h|, 1\) as well as \(\lambda_h > 0\) at the inflation scale, which we justify later by the Higgs–inflaton mixing. Further, we assume that all the dimensionful parameters are far below the Planck scale. In a particularly interesting case of a single mass scale, these parameters are of electroweak size.

II. INFLATION

In what follows, we consider a representative inflation model which fits the PLANCK data [14] very well. That is, we assume that inflation is driven by the non–minimal \(\phi\) coupling to gravity \(\xi_\phi \phi^2 \hat{R}\) with \(\xi_\phi \phi^2 \gg 1\), in analogy with the “Higgs inflation” model [15]. The transition to the Einstein frame, where the curvature–dependent term becomes the usual \(R/2\), is achieved by the metric rescaling [16]

\[\hat{g}^{\mu\nu} = \Omega^2 \hat{g}^{\mu\nu}.\]

(4)
This induces non–canonical kinetic terms for the scalars. Since $|\xi_h| \ll \xi_\phi$, $h \ll \phi$ and the dimensionful quantities are far below the Planck scale, during inflation one can neglect all the terms apart from $\lambda_\phi \phi^4$ and $\xi_\phi \phi^2$. For the canonically normalized variable $\chi$ satisfying \[ \frac{d\chi}{d\phi} = \sqrt{\frac{1 + \xi_\phi (1 + 6 \xi_\phi) \phi^2}{1 + \xi_\phi \phi^2}}, \] one finds $\chi \simeq \sqrt{\frac{3}{2}} \ln \xi_\phi \phi^2$ in our regime and the potential is given by \[ U(\chi) \simeq \frac{\lambda_\phi}{4 \xi_\phi^2} \left(1 - e^{-\sqrt{\frac{3}{2}} \chi}\right)^2, \] where $U \equiv V/\Omega^4$. At $\chi \gg 1$, it is exponentially close to a flat potential and thus supports inflation. The CMB normalization \[17\] requires $\lambda_\phi/\xi_\phi^2 \simeq 0.5 \times 10^{-9}$. A further constraint on these parameters comes from unitarity considerations. The unitarity cutoff scale of our theory is given by $\xi_\phi^{-1}$ at which higher dimension operators cannot be ignored \[18\] \[19\], while the energy density during inflation is of order $\lambda_\phi/\xi_\phi^2$. Requiring $\xi_\phi^{-4} \simeq \lambda_\phi/\xi_\phi^2$, one finds $\lambda_\phi \xi_\phi^2 \lesssim 1$. Combining this with the CMB normalization constraint, we get \[ \lambda_\phi(\Lambda_I) \lesssim 2 \times 10^{-5} \] and $\xi_\phi(\Lambda_I) \lesssim 2 \times 10^2$, where $\Lambda_I$ is the inflation scale which can be taken to be $U^{1/4} \sim (\lambda_\phi/\xi_\phi^2)^{1/4}[\Gamma]$. This may be a somewhat conservative bound \[21\]. We further impose the condition that the radiative corrections to the inflaton potential, e.g. in the Coleman–Weinberg form, be small (see, for example, \[6\]). This gives approximately $\lambda_{h\phi}^2/16\pi^2 \ll \lambda_\phi$ restricting $\lambda_{h\phi}$ to be below $10^{-2}$ at the inflation scale. On the other hand, the Coleman–Weinberg correction induced by the trilinear $\phi h^2$ term is negligible: it is suppressed by $(\sigma/\phi)^2$ which is vanishingly small in the range of interest.

During inflation, the Higgs field is a spectator. For $\lambda_h > 0$ and $\lambda_{h\phi}$ in the range of interest, it is a heavy field at the inflation scale, with mass of order $\sqrt{\lambda_{h\phi}/\xi_\phi} \gg H_I$, stabilized at the origin \[22\]. Since the inflationary dynamics are dictated by the quartic couplings, the Higgs–inflaton mixing is completely negligible at this stage.

The inflationary predictions of the model are in excellent agreement with the PLANCK data. In particular, the scalar spectral index is predicted to be $n_s \simeq 0.97$ and the tensor-to-scalar ratio is $r \simeq 3 \times 10^{-3} \re^\[15\]$. The latter is within the range of detectability by future CMB

\[1\] For the renormalization group running of the couplings, we take $\Lambda_I \sim M_{Pl}$ to simplify numerical computations.
missions [23]. Note that, unlike the Higgs inflation scenario, our model is free of significant radiative corrections.

III. PREHEATING AND REHEATING

During inflation the $\chi$ field slowly rolls towards smaller values, while the Higgs is anchored at the origin by the inflaton–induced effective mass. When $\chi$ reaches the critical value $\chi = \chi_{\text{end}} \simeq \sqrt{3/2} \ln (1 + 2/\sqrt{3})$ [24], the slow-roll ends and $\chi$ rolls fast to the minimum of the potential where it oscillates with a decaying amplitude.

In terms of the original variable $\phi$, inflation ends at $\phi \sim 1/\sqrt{\xi_{\phi}}$. As its amplitude decreases further, the relevant for preheating regimes are described by the canonically normalized inflaton $\chi$ via the relation

$$\chi \simeq \begin{cases} 
\phi & \text{for } \phi^2 \ll \frac{1}{6\xi_{\phi}} , \\
 \pm \sqrt{\frac{3}{2}} \xi_{\phi} \phi^2 & \text{for } \frac{1}{6\xi_{\phi}} \ll \phi^2 \ll \frac{1}{\xi_{\phi}} .
\end{cases}$$

(8)

In these regimes, the potential is $U(\chi) = \frac{1}{4} \lambda_{\phi} \chi^4$ and $U(\chi) = \frac{\lambda_{\phi}}{6\xi_{\phi}^2} \chi^2$, respectively. The inflaton starts oscillating in the quadratic potential with the effective mass–squared $\mu^2 = \frac{\lambda_{\phi}}{3\xi_{\phi}^2}$, while its amplitude decreases as $(\mu t)^{-1}$. Thus, after $\mu t \sim \mathcal{O}(6\xi_{\phi})$ the system enters the quartic regime and the inflaton becomes massless (at the classical level). At this stage, the Universe quickly becomes radiation–dominated [25] although that does not imply thermal equilibrium. In particular, as shown in the left panel of Fig. 1, the equation of state approaches that of radiation, $p = w \rho$ with $w = 1/3$, where $p$ and $\rho$ are the pressure and the energy density, respectively. This is known as the prethermalization phase [26].

The time it takes for the system to reach chemical and finally thermal equilibrium depends rather sensitively on the input parameters. The Higgs quanta can efficiently be produced via parametric resonance [27] due to the $h^2 \phi^2$ coupling (Fig. 1, right). If the resonance stays active long enough, chemical equilibrium between the Higgs and inflaton fields sets in earlier. For a substantial $\lambda_h \sim 1$, however, the resonance is shut off by the backreaction effects which induce an extra contribution to the Higgs mass–squared $\sim \lambda_h \langle h^2 \rangle$. In this case, the Higgs quanta are produced through perturbative scattering and thermal equilibrium is reached much later.

The lower bound on the reheating temperature can be estimated by equating the perturbative interaction rate with the Hubble rate in the radiation–dominated phase. The
FIG. 1. Left: Evolution of the equation of state for representative values of $\lambda_h(\Lambda_I)$ (lattice simulation). Here $\mu = \sqrt{\lambda_\phi/3\xi^2}$, $\xi(\Lambda_I) = 10^2$, $\lambda_h(\Lambda_I) = 10^{-3}$ and at late times $w = p/\rho$ approaches $1/3$. Right: Ratio between the Higgs and the inflaton energy densities. For $\lambda_h(\Lambda_I) = 10^{-2}$, the Higgs quanta are produced efficiently through parametric resonance.

scattering is expected to be dominated by the $\phi^2h^2$–interaction, which gives $T_{\text{reh}} \gtrsim O(\lambda_{h\phi}^2)$. For typical coupling values, this results in $T_{\text{reh}} \sim 10^{12}$ GeV.

As the Universe expands and the temperature drops below the inflaton mass, the inflaton undergoes the usual “freeze–out”. Owing to the trilinear $\phi h^2$ interaction, it will quickly decay either into Higgs pairs or light particles (at 1–loop). We emphasize that the trilinear term plays a crucial role for consistency of the model: the stable inflaton relics would “overclose” the Universe since the $\phi$–annihilation cross section is too small to be consistent with the dark matter relic abundance. The latter requires larger couplings, $\lambda_{h\phi} \sim 10^{-1} – 1$ [28].

IV. VACUUM STABILITY AND LOW ENERGY CONSTRAINTS

Our next step is to analyze constraints on the model imposed by vacuum stability. In this low energy analysis, the dimensionful parameters play a crucial role.

Presently, the curvature is so small that the distinction between the Jordan and Einstein frames becomes immaterial. Thus, we may focus entirely on the potential $V(\phi, h)$ of Eq. (3), treating $\phi$ and $h$ as canonically normalized scalars. In general, both the Higgs and the inflaton develop vacuum expectation values (VEVs) at the minimum of the potential, $v \equiv \langle h \rangle$ and $u \equiv \langle \phi \rangle$. It is convenient, however, to redefine the inflaton field $\phi' = \phi – u$ such that $\langle \phi' \rangle = 0$. In terms of the primed field, the potential retains the same form [3] if we define
the primed *dimensionful* parameters as \([29, 30]\)

\[
\begin{align*}
    b'_3 &= b_3 + 3\lambda_\phi u, \\
    \mu'_\phi &= \mu_\phi^2 - 3\lambda_\phi u^2 - 2b_3u, \\
    b'_1 &= b_1 + \lambda_\phi u^3 + b_3 u^2 - \mu_\phi^2 u \\
    \sigma' &= \sigma + \lambda_{h\phi} u, \\
    \mu'_{h'} &= \mu_h^2 - \lambda_{h\phi} u^2 - 2\sigma u.
\end{align*}
\]  

(Note that the dimensionless couplings are not affected by this redefinition. At the electroweak minimum \((\langle h \rangle, \langle \phi' \rangle) = (v, 0)\), the Higgs and the inflaton mix such that the mass eigenstates \(h_1, h_2\) are given by

\[
\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} h - v \\ \phi' \end{pmatrix}.
\]  

The masses \(m_{1,2}\) of \(h_{1,2}\) and the mixing angle \(\theta\) are related to the input parameters by

\[
\begin{align*}
2\lambda_h v^2 &= m_1^2 \cos^2 \theta + m_2^2 \sin^2 \theta, \\
\lambda_{h\phi} v^2 - \mu_\phi^2 &= m_1^2 \sin^2 \theta + m_2^2 \cos^2 \theta, \\
\sigma' v &= \frac{\sin 2\theta}{4} (m_1^2 - m_2^2).
\end{align*}
\]  

If we identify the observed 125 GeV Higgs–like boson with \(h_1\), for \(m_2 > m_1\) the first relation in (11) implies that the Higgs self–coupling \(\lambda_h\) is greater than that in the SM (obtained by setting \(\theta = 0\)). This correction can stabilize the Higgs potential at large field values such that \(\lambda_h\) would never turn negative.

It is important to note that a substantial mixing angle \(\theta\) implies that \(m_2\) cannot be arbitrarily large. Indeed, if \(m_2\) is far above the weak scale, the first relation in (11) makes \(\lambda_h\) non–perturbative. In fact, if we require our model to be valid from the electroweak to the Planck (or unitarity) scale, all the mass parameters are confined to the electroweak/TeV scale.

Our next step is to identify parameter regions in which our model remains perturbative up to the Planck scale and the electroweak vacuum remains global. To do that, we use the
renormalization group (RG) equations

\begin{align*}
16\pi^2 \frac{d\lambda_h}{dt} &= 24\lambda_h^2 - 6y_t^4 + \frac{3}{8} (2g^4 + (g^2 + g'^2)^2) + (12y_t^2 - 9g^2 - 3g'^2)\lambda_h + 2\lambda_h^2, \\
16\pi^2 \frac{d\lambda_{h\phi}}{dt} &= 8\lambda_h^2 + 12\lambda_h\lambda_{h\phi} - \frac{3}{2} (3g^2 + g'^2)\lambda_h\phi + 6\lambda_t^2\lambda_{h\phi} + 6\lambda_{h\phi}\lambda_{h\phi}, \\
16\pi^2 \frac{d\lambda_\phi}{dt} &= 8\lambda_{h\phi}^2 + 18\lambda_\phi^2, \\
16\pi^2 \frac{d\sigma}{dt} &= \sigma \left( 12\lambda_h + 8\lambda_{h\phi} - \frac{3g^2}{2} - \frac{9g'^2}{2} + 6y_t^2 \right) + 2\lambda_{h\phi}b_3, \\
16\pi^2 \frac{db_3}{dt} &= 24\sigma\lambda_{h\phi} + 18\lambda_\phi b_3, \\
16\pi^2 \frac{dy_t}{dt} &= y_t \left( \frac{9}{2} g_t^2 - \frac{17}{12} g'^2 - \frac{9}{4} g^2 - 8g_3^2 \right), \\
16\pi^2 \frac{dg_i}{dt} &= c_i g_i^3 \quad \text{with} \quad (c_1, c_2, c_3) = (41/6, -19/6, -7),
\end{align*}

where $t = \ln \mu$ with $\mu$ being the energy scale and $g_i = (g', g, g_3)$ denote the gauge couplings.

As the input values at the top quark mass scale $M_t$, we use $g(M_t) = 0.64, \ g'(M_t) = 0.35, \ g_3(M_t) = 1.16$ and $y_t(M_t) = 0.93$. Here we omit the RG equations for $\mu_i^2$ and $b_1$, which are unimportant for a potential analysis at large field values (although taken into account numerically).

Our results are presented in Fig. 2. The left panel shows parameter space allowed by perturbativity and positivity of $\lambda_h$ at all scales up to $M_{Pl}$. This is analogous to the analysis of [32] for a $Z_2$–symmetric scalar potential. Here we have cut $|\sin \theta|$ at 0.3 which is the upper bound imposed by the Higgs coupling measurements [33]. (Almost all of the white region with $m_2 > 300$ GeV is also consistent with the LHC and electroweak constraints [32, 34].) We conclude that electroweak to TeV values of $\sigma'$ and $m_2$ can lead to a stable Higgs potential.

The right panel shows the $\{\lambda_{h\phi}, b_3\}$ parameter region in which the electroweak vacuum is the global minimum of the scalar potential. The left part of the panel is excluded by the stability constraint on the running couplings,

$$\lambda_{h\phi}(\mu) > -\sqrt{\lambda_h(\mu)\lambda_\phi(\mu)},$$

which ensures that there is no unbounded from below direction at large field values. Relatively large $|\lambda_{h\phi}| \gtrsim 2 \times 10^{-3}$ lead to a significant RG contribution to $\lambda_\phi$ thus violating the

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2 We have computed these equations analytically and verified the result with **SARAH** [38].
FIG. 2. **Left:** Values of $\sin \theta$ and $m_2$ consistent with Higgs potential stability and perturbativity up to $M_{Pl}$ (white region). Also displayed are the curves of constant $\sigma'$. Here $\lambda_{h\phi} = 10^{-3}, \lambda_\phi = 10^{-5}$ at the EW scale and negative $\sin \theta$ are obtained by flipping the sign of $\sigma'$. **Right:** The $\{\lambda_{h\phi}, b_3'\}$ parameter region (in GeV) in which the electroweak vacuum is a global minimum. The other EW scale parameters are fixed to be $m_2 = 600$ GeV, $\sin \theta = 0.144, \lambda_\phi = 10^{-5}$ corresponding to $\sigma' = -100$ GeV.

unitarity constraint (7) at the high scale. This excludes the rightmost part of the panel. In the upper and lower shaded regions, there exist further minima of the scalar potential at large $\phi' \sim -b_3' / \lambda_\phi$ which are deeper than the electroweak one. We exclude these regions to be conservative although thermal and inflationary effects may stabilize the fields at smaller values in the Early Universe.

We find that for $u$ up to 10 TeV, the numerical difference between $\sigma$ and $\sigma'$ is negligible. In particular, $\sigma \gg \lambda_{h\phi} u$ and according to Eq. (11) the Higgs–inflaton mixing is governed entirely by the trilinear $\sigma$–term.

We also note that for negative values of $\lambda_{h\phi}$, the field that drives inflation is a combination of $\phi$ with a small admixture of $h$ [22]. The Early Universe dynamics develops along the lines discussed above except the reheating process is expected to be more efficient due to the Higgs interactions.

Our analysis shows that there are exciting prospects for the LHC new physics searches. First of all, the Higgs–inflaton mixing manifests itself as a universal reduction in the Higgs couplings to gauge bosons and fermions. Deviations at a few percent level can be detected in the high luminosity LHC phase [35]. Furthermore, the mostly–inflaton state $h_2$ can be found...
directly as a heavy Higgs–like resonance. This is facilitated by the decay $h_2 \to h_1 h_1$ which makes $m_2$ in the TeV range with $|\sin \theta| \sim 10^{-1}$ accessible to LHC searches [30, 32, 36, 37].

V. CONCLUSIONS

We have studied the minimal option of stabilizing the EW vacuum via the Higgs–inflaton mixing, where inflation is driven by a non–minimal scalar coupling to curvature. In the presence of the trilinear Higgs–inflaton interaction, such a mixing is inevitable and can significantly increase the Higgs self–coupling. We find that this scenario is cosmologically viable and fits the PLANCK data very well. The model is particularly attractive when it is described by a single (TeV) mass scale, in which case the mixing angle is substantial. This opens up an exciting avenue for a direct inflaton search at the LHC.

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