Analysis of Simulated Ultrasonic Echo Signals Based on Fractal Theory

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Abstract. Ultrasonic signals can be thought of as one-dimensional time series, which is difficult to describe in traditional Euclidean space. Fractal theory and fractal dimension can describe the internal self-affine or self-similar characteristics in different state spaces to reveal the working mechanism of complex systems. This paper introduces fractal theory into the analysis of simulated ultrasonic echo signals. The fractal scale-free interval of simulated ultrasonic signal is determined by grid method. The effects of center frequency, bandwidth factor and signal-to-noise ratio of simulated ultrasonic signal on the box dimension are analyzed.

1. Introduction

Fractal Theory is a science that reveals the unity of order and disorder, the unity of certainty and randomness in nonlinear systems. Order, stability, balance and certainty are ideal conditions under certain assumptions, while natural world is a disordered, non-stationary, random and complex system including a large number of nonlinear processes[1]. However, there are often certain rules behind these seemingly random and complex phenomena. Fractal is a frontier discipline that studies the inherent self-similar or self-affine law of these complex natural phenomena. It breaks through the shackles of people's thinking in the Euclidean space, brings people into the fractal space, and depicts the complex phenomena in the natural world from the perspective of geometric morphology.

The fractal behaviour of ultrasonic testing signals is studied to provide new ideas and methods for the extraction of effective features of ultrasonic signals. This paper is to explore and discover the fractal characteristics and laws of simulated ultrasonic echo signals.

2. Basic theory of fractal

Fractal geometry takes the irregular and complex phenomena that are common in nature as the research object, and describes the general structure of matte or irregular sets and functions that cannot be described by Euclidean geometry and calculus methods. Fractal theory was first proposed in the 1970s. After more than 40 years of development, it has been applied to almost every field of the natural sciences and social sciences. It has achieved fruitful results and has become a frontier research topic in many international disciplines.

The English word fractal is derived from the Latin fractus, introduced by American mathematician B. Mandelbrot in 1975. B. Mandelbrot originally wanted to use fractals to describe a large class of complex and irregular geometric objects that cannot be described by traditional European geometry in nature, such as curved coastlines, undulating mountains, rough sections, randomly changing images, etc.
Fractal geometry is an irregular shape geometry, but this irregularity or roughness is hierarchical, that is, it can be observed at different levels or scales. In fact, the abstraction of irregular geometry more accurately meshes the natural world than the regular geometry of smooth planes in classical geometry.

Fractal dimension and integer dimension in the traditional sense are at two different levels. When we look at a fractal system, we will see singularity and complexity according to traditional experience. But when we understand the fractal system with the concept of fractal dimension, the result is affirmative and ordinary. Systems with fractal features are complex systems whose complexity can be described by fractal dimensions to some extent. From the perspective of measure theory, fractal dimension describes the ability of the system to fill space, characterizes the disorder of the system, and is the most basic feature of complex systems.

The mathematical dimension is not a simple and understandable concept. Mathematicians have developed more than ten different dimensions, such as topological dimension, box dimension, capacity dimension, information dimension, correlation dimension, generalized dimension, Lyapunov dimension, etc. Box dimension is the most widely used, and it is also a dimension that is relatively easy to understand and implement[2-6].

3. Box dimension of ultrasonic echo signals

Ultrasonic echo testing adopts pulse reflection method based on vertical incidence theory. The characteristics of the propagation medium and the physical properties of the reflector can be obtained by studying the ultrasonic reflection wave. In order to extract these information related to the characteristics of the reflector, it is necessary to establish a model of the ultrasonic signal. The ultrasonic signal mainly consists of transmitted wave, bottom echo, defect echo, and noise signal.

In the pulse reflection ultrasonic testing system, ultrasonic emission pulses are usually generated by sudden discharge of capacitors precharged to high voltage. The signal waveform is a single-frequency carrier pulse signal, which can be expressed by equation (1).

\[
s(t) = \begin{cases} 
A \cdot \cos(2\pi f_0 t) & -\frac{t_c}{2} < t < t - nT < \frac{t_c}{2} \\
0 & \text{other}
\end{cases}
\]

(1)

Where, \(p(t)\) is the pulse signal whose amplitude, pulse width and period are \(A\), \(t_c\) and \(T\). \(f_0\) represents the natural oscillation frequency of the piezoelectric wafer and is also the working frequency of the transducer. \(n\) is an integer.

The generation mechanism of the bottom echo, the defect echo and the material scattering wave signal are very similar. All of them are reflected and scattered signals when the transmitted waves are encountered with obstacles in the specimen. This paper assumes that these signals all satisfy the Gaussian envelope distribution for signal modeling. In broadband narrow-pulse ultrasonic testing, the ultrasonic pulse signal is usually a broadband signal modulated by the center frequency of the transducer[7]. The mathematical model of the ultrasonic echo can be established by equation (2).

\[
s(t) = A \cdot \exp[-\alpha(t - \tau)^2] \cdot \cos[2\pi f_0 (t - \tau) + \phi_0]
\]

(2)

As in equation (2), \(A\) is the amplitude of the reflected echo, which contains the energy loss caused by attenuation when ultrasonic wave propagates in the medium. \(\alpha\) is a positive constant named bandwidth factor, which determines the bandwidth of the ultrasonic echo signal. As \(\alpha\) gets larger, the duration of waveform gets shorter. \(\tau\), \(f_0\), \(\phi_0\) are arrival time of the echo, center frequency of the ultrasonic emission pulse and initial phase. It can be seen that the ultrasonic echo signal received by the system is a Gaussian envelope pulse, which is a signal modulated by the center frequency of the transducer.

It is only necessary to judge whether there is a scale-free interval to determine whether a curve has fractal feature. In this paper, the box dimension is used to describe fractal feature, and the grid method proposed by Grassberger. \(P\) is used to obtain the scale-free interval.
Before analyzing the fractal characteristics of signals, the time domain waveform of the signal should be preprocessed, that is, scale normalization. For a time domain waveform \( \{ (t, f(t)): 0 \leq t \leq N \} \) consisting of \( N \) sampling points, the waveform is first translated \( \min(f(t)) \) in the positive direction of the \( Y \) axis so that the waveform is in the first quadrant of the coordinate system, and then \( \max(f(t)) - \min(f(t)) \) is divided into \( N-1 \) equal parts, so that each aliquot is equally spaced from the sample point. Figure 1 shows the time domain waveform of the signal to be analyzed, and figure 2 shows the scale normalization of the waveform. After the ultrasonic echo signal is preprocessed, it does not affect the shape and complexity of the waveform.

![Figure 1. Time domain waveform of the signal.](image1)

![Figure 2. Scale normalization of the signal.](image2)

Let the pre-processed ultrasonic echo signal have a graph \( F \) in the \( XOY \) plane, and square grid with the length of \( \delta \) in the plane is made. Then the number \( N_{\delta}(F) \) of squares where \( F \) intersects the grid is the number of boxes of graph \( F \) in the scale \( \delta \), indicating the irregularity and complexity of the defect waveform. For realistic irregular fractals, if the double logarithmic curve \( \log_{10} N_{\delta}(F) - \log_{10} \delta \) maintains a nearly constant slope within a scale range \([\delta_1, \delta_2]\), that is, the signal maintains self-similarity, and \([\delta_1, \delta_2]\) is a scale-free interval.

The value of the scale \( \delta \) should not be less than 1 and not greater than the length of the signal. Each scale corresponds to a box number \( N_{\delta}(F) \). When the scale is taken to a certain value, the logarithm of the scale and its corresponding box number no longer maintain a linear relationship, that is, the signal no longer has fractal characteristics. Table 1 lists the number of boxes with scales from 1 to 30 for a simulated ultrasonic signal with a signal-to-noise ratio of 30 dB.

| \( \delta \) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| \( N_{\delta}(F) \) | 3926 | 1961 | 1308 | 972 | 772 | 634 | 540 | 473 | 435 | 381 | 336 | 310 | 279 | 255 | 226 |

| \( \delta \) | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| \( N_{\delta}(F) \) | 206 | 190 | 153 | 202 | 128 | 168 | 166 | 152 | 143 | 135 | 124 | 105 | 99 | 110 | 92 |

It can be seen the good downward trend is no longer maintained when \( \delta \) is equal to 19 from table 1, that is, the ultrasonic signal is a non-strict random fractal. The corresponding double logarithmic values \( \log_{10} \delta \) and \( \log_{10} N_{\delta}(F) \) were calculated according to table 1, as in table 2. It can be seen that \( \log_{10} N_{\delta}(F) - \log_{10} \delta \) no longer have a linear relationship when \( \log_{10} \delta \) is taken to 4.2479. Therefore, \( \log_{10} \delta \) has self-similarity in \([0, 4.1699]\) and the scale-free interval of the signal is \([1, 18]\).

| \( \log_{10} \delta \) | 0.0000 | 1.0000 | 1.5850 | 2.0000 | 2.3219 | 2.5850 | 2.8074 | 3.0000 | 3.1699 | 3.3219 |
|---|---|---|---|---|---|---|---|---|---|---|
| \( \log_{10} N_{\delta}(F) \) | 11.9388 | 10.9374 | 10.3531 | 9.9248 | 9.5925 | 9.3083 | 9.0768 | 8.8857 | 8.7649 | 8.5736 |

| \( \log_{10} \delta \) | 3.4594 | 3.5850 | 3.7004 | 3.8074 | 3.9069 | 4.0000 | 4.0875 | 4.1699 | 4.2479 | 4.3219 |
|---|---|---|---|---|---|---|---|---|---|---|
| \( \log_{10} N_{\delta}(F) \) | 8.3923 | 8.2761 | 8.1241 | 7.9944 | 7.8202 | 7.6865 | 7.5699 | 7.2574 | 7.6582 | 7.0000 |
Figure 3 and figure 4 show the scale interval log-log graph and scale-free interval log-log graph of the signal. In the scale-free interval, a straight line is fitted by the least square method, and the opposite of the slope is box dimension, which is 1.4023. The box dimension of the signal can be calculated to be 1.0762 after noise reduction. That is to say, the box dimension of the signal after noise reduction is reduced, which is related to the fact that the signal is clear and simple after noise reduction.

Figure 3. Scale interval log-log graph. Figure 4. Scale-free interval log-log graph.

For the ultrasonic echo signal model of equation (2), the bandwidth factor $\alpha$ and the center frequency $f_0$ are two main parameters that can be adjusted. The change of these two parameters will affect the echo shape and also cause the box dimension to change accordingly.

Figure 5 shows the effect of the bandwidth factor $\alpha$ on the box dimension. When the bandwidth factor becomes larger, the box dimension has a tendency to decrease. And when the bandwidth factor is small, the decreasing trend is more obvious. This can be matched with the transducer used in the actual ultrasonic testing. The bandwidth of the narrow pulse transducer is relatively large, the box dimension of the echo signal is relatively small, and the waveform is relatively simple.

As in figure 6, the change of the center frequency of echo signal also has a certain impact on the box dimension with the same bandwidth factor. The same rule exists when the bandwidth factor takes different values, that is, the box dimension increases approximately linearly with the increase of the center frequency.

By adding noise to the simulated signal, it is found that with the other parameters unchanged, as the signal-to-noise ratio decreases, the box dimension increases. The existence of noise undoubtedly increases the complexity and irregularity of the signal, so the box dimension of the low SNR signal is relatively large, which also shows the signal denoising is very important in actual ultrasonic testing. It is often impossible or difficult to find the rule from original signals with noise, but the useful information hidden by noise will be displayed immediately after proper noise reduction.
Figure 5. Effect of bandwidth factor $\alpha$ on box dimension.

Figure 6. Effect of center frequency $f_0$ on box dimension.
4. Conclusions
In this paper, fractal theory was applied to analyze the simulated ultrasonic signal quantitatively, and the relationship between the box dimension and the simulated ultrasonic signal is established by taking the box dimension as characteristic parameter. The results show that the box dimension can be used to describe the complexity of the signal, and is closely related to the center frequency, bandwidth factor and signal-to-noise ratio of the simulated ultrasonic signal. The results can be used as reference in practical ultrasonic testing.

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