Charge current in ferromagnet - triplet superconductor junctions

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Abstract

We calculate the tunneling conductance spectra of a ferromagnetic metal / insulator / triplet superconductor from the reflection amplitudes using the Blonder-Tinkham-Klapwijk (BTK) formula. For the triplet superconductor, we assume one special $p$-wave order parameter, having line nodes, and two two dimensional $f$-wave order parameters with line nodes, breaking the time reversal symmetry. Also we examine nodeless pairing potentials. The evolution of the spectra with the exchange potential depends solely on the topology of the gap. The weak Andreev reflection within the ferromagnet results in the suppression of the tunneling conductance and eliminates the resonances due to the anisotropy of the pairing potential. The tunneling spectra splits asymmetrically with respect to $E = 0$ under the influence of an external magnetic field. The results can be used to distinguish between the possible candidate pairing states of the superconductor Sr$_2$RuO$_4$. 
I. INTRODUCTION

The recent discovery of superconductivity in Sr$_2$RuO$_4$ has attracted much theoretical and experimental interest \[1\]. The time reversal symmetry is broken for the superconductor Sr$_2$RuO$_4$, and the magnetic field is spontaneously induced as shown by $\mu$SR experiment. \[2\]. The Knight-shift shows no change when passing through the superconducting state and is a clear indication for spin triplet pairing state, with a $d$-vector aligned to the $z$-axis. \[3\]. In addition the band structure calculations \[4\] and de Haas van Alphen measurements \[3\] show little dispersion along $k_z$ which is consistent with a two dimensional basis function on a cylindrical Fermi surface. Furthermore, the presence of a large residual density of states of quasiparticles inside the superconducting gap is evident from the linear temperature dependence of the nuclear spin lattice relaxation rate $1/T_1$ of $^{101}$Ru bellow 0.4K. \[6\] Also specific heat measurements support the scenario of line nodes within the gap as in the high $T_c$ cuprate superconductors \[7\].

In the tunneling experiments between normal metals and superconductors Andreev reflection process take place \[8,9\]. In the Andreev reflection process an electron incident, in the barrier with an energy bellow the gap can not drain off into the superconductor. It is instead reflected as a hole and a Cooper pair is transferred into the superconductor. In anisotropic high $T_c$ superconductors due to the sign change of the pair potential that the transmitted quasiparticles feel, zero energy states are formed, which are detected as peaks in the conductance spectra at $E = 0$. \[10\] Also in the presence of an imaginary $s$-wave component, which breaks the time reversal symmetry, the zero energy peak is shifted to the amplitude of the subdominant component. \[11,12\]

The properties of the Andreev reflection are modified in the presence of an exchange field as in ferromagnet / insulator / superconductor junctions, since the retro-reflection of the Andreev reflection is broken in the ferromagnet. This phenomenon has been clarified both in $s$-wave and $d$-wave junctions and interesting aspects of the Andreev reflection have been revealed. \[13–16\] Also the properties of ferromagnet / insulator / triplet superconductor
junctions have been studied where two types of pairing potentials are assumed for the triplet superconductor, i.e. the unitary and the non unitary with $E_u$ symmetry. In the unitary case the conductance within the gap is reduced with the exchange interaction while in the non unitary it is not much influenced since the latter pairing state conserves spin.

In order to identify the pairing state of Sr$_2$RuO$_4$ the Bogoliubov-de Gennes (BdG) equations have been used to calculate the quasiparticle bound state wave function around non-magnetic impurities in unconventional superconductors. The characteristic patterns were distinguished for two proposed order parameters, (i) $E_u$, and (ii) $B_{1g} \times E_u$.

In this paper we will use the BdG equations to calculate the tunneling conductance of ferromagnet / triplet superconductor contacts, with a barrier of arbitrary strength between them, in terms of the probability amplitudes of Andreev and normal reflection. For the triplet superconductor we shall assume three possible pairing states of two dimensional order parameters, having line nodes within the RuO$_2$ plane, which break the time reversal symmetry. The first two are the 2D $f$-wave states proposed by Hasegawa et al, having $B_{1g} \times E_u$ and $B_{2g} \times E_u$ symmetry respectively. The other one is called nodal $p$-wave state and has been proposed by Dahm et al, where the pairing potential has the form $d(k) = \Delta_0 \hat{z}(\sin(k_xa) + i \sin(k_ya))$, with $k_xa = \pi \cos \theta$, $k_ya = \pi \sin \theta$. This pairing symmetry has nodes as in the $B_{2g} \times E_u$ case. Also we will consider two nodeless pairing states. One is the isotropic $p$-wave state and the other is the nodeless $p$-wave state initially proposed by K. Miyake and O. Narikiyo, both breaking the time reversal symmetry. Generally the tunneling conductance is suppressed with the increase of the exchange interaction and the peaks are removed. This is due to the suppression of the Andreev reflection in the ferromagnet. For the nodal pairing states the linear dependence of the tunneling conductance with $E$ is not much influenced. For the nodeless cases, the normalized conductance develops a constant value within the gap, which is suppressed as the exchange field gets larger. When the ferromagnet is a normal metal, the magnetic field splits the tunneling spectrum symmetrically around $E = 0$. The exchange field eliminates the negative branch of the tunneling spectra in the half metallic ferromagnetic limit.
II. THEORY OF TUNNELING EFFECT

For spin-triplet superconductors the wave functions describing the quasiparticles \( \hat{\Psi}(\mathbf{r}) \) are four-spinors in Nambu (particle-hole \( \otimes \) spin) space. Their particle and hole components are determined by the solutions of the BdG equations \[23,24\]

\[
E \hat{\Psi}(\mathbf{r}) = \int d\mathbf{r'} \hat{H}(\mathbf{r}, \mathbf{r'}) \hat{\Psi}(\mathbf{r'}),
\]

where,

\[
\hat{\Psi}(\mathbf{r}) = \begin{pmatrix}
    u_\uparrow(\mathbf{r}) \\
    u_\downarrow(\mathbf{r}) \\
    v_\uparrow(\mathbf{r}) \\
    v_\downarrow(\mathbf{r})
\end{pmatrix},
\]

\[
\hat{H}(\mathbf{r}, \mathbf{r'}) = \begin{pmatrix}
    \hat{H}_e & \hat{\Delta} \\
    \hat{\Delta}^* & -\hat{H}_e
\end{pmatrix}.
\]

\( \hat{\Delta} \) is the \( 2 \times 2 \) triplet pairing matrix with elements of the form \( \Delta_{ss'}(\mathbf{r}, \mathbf{r'}) \), and the spin index \( s = \uparrow, \text{ or } s = \downarrow. \) \( \hat{H}_e = H_e(\mathbf{r'})\delta(\mathbf{r} - \mathbf{r'})\hat{\sigma}_0, \) where \( \hat{\sigma}_0 \) is the \( 2 \times 2 \) unit matrix, and \( H_e(\mathbf{r}) \) is the single-particle Hamiltonian which is given by \( H_e(\mathbf{r}) = -\hbar^2 \nabla^2 / 2m_e + V(\mathbf{r}) - E_F, E \) is the energy measured from the Fermi energy \( E_F. \) For the pairing states we examine in Sec. III, the spin-up and spin-down components decouple. We will consider only triplet pairing states where \( \Delta_{\uparrow\uparrow}(\mathbf{r}, \mathbf{r'}) = \Delta_{\downarrow\downarrow}(\mathbf{r}, \mathbf{r'}) = 0, \) while \( \Delta_{\uparrow\downarrow}(\mathbf{r}, \mathbf{r'}) = \Delta_{\downarrow\uparrow}(\mathbf{r}, \mathbf{r'}). \) In that case the cooper pairs have zero spin projection. The spin dependent BdG equations are decoupled into two independent sets of (two component) equations, one for the spin up electron, spin down hole quasiparticle \( (u_\uparrow(\mathbf{r}), v_\downarrow(\mathbf{r})), \) and the other for \( (u_\downarrow(\mathbf{r}), v_\uparrow(\mathbf{r})). \) The corresponding BdG equations for spin index \( s(\overline{s}) = \uparrow (\downarrow) \text{ or } s(\overline{s}) = \downarrow (\uparrow), \) read \[25\]

\[
Eu_s(\mathbf{r}) = (H_e(\mathbf{r}) - \rho U(\mathbf{r}))u_s(\mathbf{r}) + \int d\mathbf{r'} \Delta_s(\mathbf{s}, \mathbf{x})v_{\overline{s}}(\mathbf{r'})
\]

\[
Ev_{\overline{s}}(\mathbf{r}) = -(H_e^*(\mathbf{r}) + \rho U(\mathbf{r}))v_{\overline{s}}(\mathbf{r}) + \int d\mathbf{r'} \Delta^*_{\overline{s}}(\mathbf{s}, \mathbf{x})u_s(\mathbf{r'}),
\]

4
where $U(r)$ is the exchange potential, $\rho$ is $1(-1)$ for up(down) spins. $\Delta_{s\sigma}(s, x)$ is the matrix element of the pair potential, after a transformation from the position coordinates $r, r'$ to the center of mass coordinate $x = (r + r')/2$ and the relative vector $s = r - r'$. After Fourier transformation the pair potential depends on the related wave vector $k$ and $x$. In the weak coupling limit $k$ is fixed on the Fermi surface ($|k| = k_F$), and only its direction $\theta$ is variable. After applying the quasi-classical approximation, i.e.

$$
\left( \begin{array}{c}
\overline{\pi}_s(r) \\
\overline{\pi}_\sigma(r)
\end{array} \right) = e^{-i k \cdot r} \left( \begin{array}{c}
u_s(r) \\
\nu_\sigma(r)
\end{array} \right),
$$

(5)

so that the fast oscillating part, of the wave function is divided out, the BdG equations are reduced to the Andreev equations [9]

$$
E \overline{\pi}_s(r) = -i \hbar^2 / m k_F x \frac{d}{dx} \overline{\pi}_s(x) + \Delta_{s\sigma}(\theta, r) \overline{\pi}_\sigma(x),
$$

$$
E \overline{\pi}_\sigma(r) = i \hbar^2 / m k_F x \frac{d}{dx} \overline{\pi}_\sigma(x) + \Delta_{s\sigma}(\theta, r) \overline{\pi}_s(x),
$$

(6)

where the quantities $\overline{\pi}_s(r)$ and $\overline{\pi}_\sigma(r)$ are electron-like and hole-like quasiparticles with spin index $s$, and $\sigma$ respectively.

We consider the ferromagnet / insulator / superconductor junction shown in Fig. 1. The geometry of the problem has the following limitations. The particles move in the $xy$-plane and the boundary between the ferromagnet ($x < 0$) and superconductor ($x > 0$) is the $yz$-plane at $x = 0$. The insulator is modeled by a delta function, located at $x = 0$, of the form $V \delta(x)$. The temperature is fixed to 0 K. We take both the pair potential and the exchange energy as a step function i.e. $\Delta_{s\sigma}(\theta, r) = \Theta(x) \Delta_{s\sigma}(\theta)$, $U(r) = \Theta(-x)U$. For the geometry shown in Fig. 1, Eqs. (6) take the form

$$
E \overline{\pi}_s(x) = -i \hbar^2 / m k_F x \frac{d}{dx} \overline{\pi}_s(x) + \Delta_{s\sigma}(\theta) \overline{\pi}_\sigma(x),
$$

$$
E \overline{\pi}_\sigma(x) = i \hbar^2 / m k_F x \frac{d}{dx} \overline{\pi}_\sigma(x) + \Delta_{s\sigma}(\theta) \overline{\pi}_s(x),
$$

(7)

When a beam of electrons is incident from the ferromagnet to the insulator, with an angle $\theta$, the general solution of Eqs. (7), is the two component wave function $\Psi_I = (u_{\uparrow}[x], v_{\downarrow}[x])$ which for $x < 0$ is written as

$$ \text{5} $$
\[\Psi_I = \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{iq\parallel I x \cos \theta} + a_{\parallel I} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{iq\perp I x \cos \theta_A} + b_{\parallel I} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-iq\parallel I x \cos \theta},\]

where \(a_{\parallel I}, b_{\parallel I}\) are the amplitudes for Andreev and normal reflection for spin up(down) quasiparticles, and \(q_{\parallel I} = \sqrt{\frac{2m}{\hbar^2}} (E_F \pm U)\) is the wave vector of quasiparticles in the ferromagnet for up (down) spin. The wave vector of the electron-like, hole-like quasiparticles is approximated by \(k_s = \sqrt{\frac{2mE_F}{\hbar^2}}\). Since the translational symmetry holds in the \(y\)-axis direction, the momenta parallel to the interface is conserved, i.e. \(q_t \sin \theta = q_\perp \sin \theta_A = k_s \sin \theta_s\).

Note that \(\theta\) is different than \(\theta_A\) since the retroreflection of the Andreev reflection is broken. Using the matching conditions of the wave function at \(x = 0\), \(\Psi_I(0) = \Psi_{II}(0)\) and \(\Psi'_{II}(0) - \Psi_I(0) = (2mV/\hbar^2)\Psi_I(0)\), the Andreev and normal reflection amplitudes \(a_{\parallel I}, b_{\parallel I}\) for the spin up(down) quasiparticles are obtained

\[a_{\parallel I} = \frac{4n_+ \lambda_1}{(-1 - \lambda_1 - iz_{\parallel I})(-1 - \lambda_2 + iz_{\parallel I}) + (1 - \lambda_1 - iz_{\parallel I})(1 + \lambda_2 - iz_{\parallel I})n_+n_- \phi_+ \phi_+^*},\]

\[b_{\parallel I} = \frac{(-1 - \lambda_2 + iz_{\parallel I})(-1 - \lambda_1 + iz_{\parallel I}) + (-1 - \lambda_2 - iz_{\parallel I})(-1 - \lambda_1 + iz_{\parallel I})n_+n_- \phi_+ \phi_+^*}{(-1 - \lambda_1 - iz_{\parallel I})(-1 - \lambda_2 + iz_{\parallel I}) + (1 - \lambda_1 - iz_{\parallel I})(1 + \lambda_2 - iz_{\parallel I})n_+n_- \phi_+ \phi_+^*},\]

where \(z_0 = \frac{mV}{\hbar^2 k_s}\), \(z_{\parallel I} = \frac{2z_0}{\cos \theta_A}\), \(\lambda_1 = \frac{\cos \theta q_{\parallel I}}{\cos \theta_A k_s}\), \(\lambda_2 = \frac{\cos \theta_A q_{\parallel I}}{k_s}\). The BCS coherence factors are given by

\[u_\pm^2 = [1 + \sqrt{E^2 - |\Delta_\pm(\theta)|^2/E}] / 2,\]

\[v_\pm^2 = [1 - \sqrt{E^2 - |\Delta_\pm(\theta)|^2/E}] / 2,\]

and \(n_\pm = v_\pm / u_\pm\). The internal phase coming from the energy gap is given by \(\phi_\pm = [\Delta_\pm(\theta)/|\Delta_\pm(\theta)|]\), where \(\Delta_\pm(\theta) = \Delta(\theta)\) (\(\Delta_\pm(\theta) = \Delta(\pi - \theta)\), is the pair potential experienced by the transmitted electron-like (hole-like) quasiparticle. \(\Delta(\theta) = \Delta_{\parallel \perp}(\theta) = \Delta_{\perp \parallel}(\theta)\), since the cooper pairs have zero spin projection i.e. \(d \parallel \hat{z}\).
When \( \theta > \sin^{-1}\left(\frac{\lambda_2}{\lambda_1}\right) \equiv \theta_{c1} \) total reflection occurs and the spin and charge current vanishes. When \( \theta_{c1} > \theta > \sin^{-1}\left(\frac{\lambda_2}{\lambda_1}\right) \equiv \theta_{c2} \), although the transmitted quasiparticles in the superconductor do propagate, the Andreev reflected quasiparticles do not propagate. This process is called virtual Andreev reflection (VAR process). In this case the spin and charge current do not vanish since a finite amplitude of the Andreev reflection still exists. \[15\]

According to the BTK formula the conductance for the charge current of the junction, \( \sigma_{q\uparrow\downarrow}(E, \theta) \), for up(down) spin quasiparticles, is expressed in terms of the probability amplitudes \( a_{\uparrow\downarrow}, b_{\uparrow\downarrow} \) as \[8,15\]

\[
\sigma_{q\uparrow\downarrow}(E, \theta) = \text{Re} \left[ 1 + \frac{\lambda_2}{\lambda_1} |a_{\uparrow\downarrow}|^2 - |b_{\uparrow\downarrow}|^2 \right].
\] (13)

The tunneling conductance, normalized by that in the normal state is given by

\[
\sigma_q(E) = \sigma_{q\uparrow}(E) + \sigma_{q\downarrow}(E),
\] (14)

\[
\sigma_{q\uparrow\downarrow}(E) = \frac{1}{R_N} \int_{-\pi/2}^{\pi/2} d\theta \cos \theta \sigma_{q\uparrow\downarrow}(E, \theta) P_{\uparrow\downarrow} q_{\uparrow\downarrow},
\] (15)

where

\[
R_N = \int_{-\pi/2}^{\pi/2} d\theta \cos \theta [\sigma_{N\uparrow}(\theta) P_{\uparrow} q_{\uparrow} + \sigma_{N\downarrow}(\theta) P_{\downarrow} q_{\downarrow}],
\] (16)

\[
\sigma_{N\uparrow\downarrow}(\theta) = \frac{4\lambda_1}{(1 + \lambda_1)^2 + z_{\uparrow\downarrow}^2},
\] (17)

where \( P_{\uparrow\downarrow} = (E_F \pm U)/2E_F \) is the polarization for up(down) spin. In the \( z_0 = 0 \) limit the interface is regarded as a weak link, showing metallic behavior while for large \( z_0 \) values the interface becomes insulating.

**III. POSSIBLE SPIN TRIPLET PAIRING STATES**

For the spin triplet pairing state the Cooper pairs have spin 1 degree of freedom. The gap function is a \( 2 \times 2 \) symmetric matrix which in the spin space can be written as
$$\hat{\Delta}(k) = i\sigma_y (d(k) \cdot \hat{\sigma}), \quad (18)$$

where $\hat{\sigma}$ denotes the Pauli matrices and $d(k)$ is a vectorial function which is odd in $k$. The $d$ vector defines the axis along which the Cooper pairs have zero spin projection. In the following we will take $d \parallel \hat{z}$. In that case $\Delta_{\uparrow\uparrow} = \Delta_{\downarrow\downarrow} = 0$, while $\Delta_{\uparrow\downarrow} = \Delta_{\downarrow\uparrow} = \Delta(\theta)$. The energy spectrum of the quasiparticles consist of two branches which are identical for unitary pairing states, i.e. $\hat{\Delta}^\dagger(k)\hat{\Delta}(k)$ is proportional to the unit matrix, and distinct for non unitary states. The non-unitary states have been ruled out for Sr$_2$RuO$_4$ by the vary small residual value of the specific heat at zero temperature [7]. In this paper we will examine only the case of unitary pairing states. As an example we consider the state

$$\hat{\Delta}(\theta) = \Delta_0 \begin{pmatrix} \Delta_{\uparrow\uparrow}(\theta) & \Delta_{\uparrow\downarrow}(\theta) \\ \Delta_{\downarrow\uparrow}(\theta) & \Delta_{\downarrow\downarrow}(\theta) \end{pmatrix}. \quad (19)$$

a) For the isotropic $p$-wave pairing state $\Delta_{\uparrow\downarrow}(\theta) = \Delta_{\downarrow\uparrow}(\theta) = \Delta_0 \exp(i(\theta - \beta))$, and $\Delta_{\uparrow\uparrow}(\theta) = \Delta_{\downarrow\downarrow}(\theta) = 0$, $\beta$ denotes the angle between the normal to the interface and the $x$-axis of the crystal. This is an opposite spin pairing state, with a gap of constant modulus for both spin parts on the Fermi surface. In the following we will consider the cases where the matrix element $\Delta_{\uparrow\downarrow}(\theta)$ (expressed as $\Delta(\theta)$), of Eq. 13 has the following $\theta$ dependences.

b) In case of a $p$-wave superconductor, proposed by K. Miyake, and O. Narikiyo [21]

$$\Delta(\theta) = \frac{\Delta_0}{S_M} \left[ \sin(k_x a) + i \sin(k_y a) \right], \quad (20)$$

with $k_x a = R\pi \cos(\theta - \beta)$, and $k_y a = R\pi \sin(\theta - \beta)$, $S_M = \sqrt{2} \sin \frac{\pi}{\sqrt{2}} = 1.125$, and $R = 0.9$. This state does not has nodes.

We consider also three pairing symmetries for Sr$_2$RuO$_4$ with line nodes.

c) In the first 2D $f$-wave state $B_{1g} \times E_u$

$$\Delta(\theta) = \Delta_0 \cos 2(\theta - \beta) \left[ \cos(\theta - \beta) + i \sin(\theta - \beta) \right]. \quad (21)$$

This state has nodes at the same points as in the $d_{x^2-y^2}$-wave case.

d) For the second 2D $f$-wave state $B_{2g} \times E_u$
\[ \Delta(\theta) = \Delta_0 \sin 2(\theta - \beta)[\cos(\theta - \beta) + i \sin(\theta - \beta)]. \] (22)

This state has nodes at 0, \(\pi/2\), \(\pi\), \(3\pi/2\), and has also been studied by Graf and Balatsky \[22\].

e) In case of a nodal \(p\)-wave superconductor

\[ \Delta(\theta) = \frac{\Delta_0}{s_M} [\sin(k_x a) + i \sin(k_y a)], \] (23)

with \(k_x a = \pi \cos(\theta - \beta)\), and \(k_y a = \pi \sin(\theta - \beta)\). We use here the same normalization proposed by Dahm et al \[20\] \(s_M = \sqrt{2} \sin \frac{\pi}{\sqrt{2}} = 1.125\), where the Fermi wave vector is chosen as \(k_F a = \pi\), in order to have a node in \(\Delta(\theta)\). This state has nodes as in the \(B_{2g} \times E_u\) state. The corresponding nodeless form was initially proposed by K. Miyake, and O. Narikiyo \[21\] and is considered as a separate case.

### IV. Tunneling Conductance Characteristics

In Figs. 2 - 6 we plot the tunneling conductance \(\sigma_q(E)\) for different values of the exchange interaction \(x = U/E_F\) (a) \(z_0 = 0, \beta = 0\), (b) \(z_0 = 2.5, \beta = 0\), (c) \(z_0 = 2.5, \beta = \pi/4\). The pairing symmetry of the superconductor is \(B_{1g} \times E_u\) in Fig. 4, \(B_{2g} \times E_u\) in Fig. 5, nodal \(p\)-wave, in Fig. 6, \(p\)-wave, proposed by K. Miyake, and O. Narikiyo \[21\] in Fig. 7, and isotropic \(p\)-wave in Fig. 8. When the ferromagnet is a normal metal i.e. \(x = 0\) the results of \[26\] are reproduced. For \(z_0 = 0\), the subgap conductance is suppressed, with the increase of \(x\), as in the case of a \(d_{x^2-y^2}\)-wave superconductor. \[15\]

In the case of normal metal / insulator / triplet superconductor junction the peaks inside the gap, are connected to bound states, which are formed due to the sign change that the transmitted quasiparticles feel, for fixed \(\beta\) at discrete values of \(\theta\). The conductance peaks occurs at these energies where an increased number of bound states is formed. \[26\]. For unitary pairing states the spins of the incident electron and the Andreev reflected hole are opposite and since the spin up and spin down quasiparticles have equal wave vectors, no spin effects are involved in the Andreev reflection. This is not true, when the normal metal is replaced by a ferromagnet. In that case the spin-up and spin-down wave vectors are
not equal and the spin affects the Andreev reflection. The Andreev reflected hole decays in the ferromagnet and the interference with the reflected electron is weak. Moreover the transmitted quasiparticles experience weakly the sign change of the pair potential which is the reason for the formation of the conductance peak. Due to this the conductance peaks are suppressed. This is seen in Fig. 2 b(c), $z_0 = 2.5$ and $\beta = 0(\pi/4)$, for the $B_{1g} \times E_u$ case. Quantitatively the suppression of the conductance peaks in the ferromagnet / insulator / triplet superconductor junction can be seen if we calculate the magnitude of the Andreev reflected amplitude as a function of the exchange field when a bound state is formed. Then the amplitude would decay to zero with the increase of the exchange field. This calculation has been done in the case of a ferromagnet / insulator / time reversal symmetry broken superconductor junction where simple arguments have been derived to connect the suppression of the Andreev reflection as $x$ increases with the reduction of the conductance peaks. [27]

Also in the metallic limit ($x = 0$) for all the pairing states we see the presence of a large residual density of states within the energy gap as a signature of unconventional pairing symmetry with higher than two angular momentum. This is modified by the presence of the ferromagnet, where the increase of the exchange field suppresses the density of states within the gap. Also in the metallic limit the conductance increases linearly with $E$, which is consistent with the presence on line nodes in the pairing potential. This linear form of the spectra remains unchanged when increasing $x$. Generally the evolution of the conductance spectra with the increase of $x$, for the three pairing symmetries with line nodes depends strongly on the position of the nodes in the pairing potential. In the $d_{x^2-y^2}$-wave case the peak at $E = 0$ and $\beta = \pi/4$, due to the sign change of the pairing potential for the transmitted quasiparticles, is largely reduced with the increase of $x$ [15]. In the case of $B_{1g} \times E_u$-wave state for $x = 0$, the pairing potential is more complicated and the sign change occurs at discreet values of $\theta$, for fixed $\beta$. As a result bound states are formed within the gap, and also the position of the conductance peaks depends on the orientation angle $\beta$, as seen in Fig. 2. Also the spectra for angle $\beta$ in the $B_{1g} \times E_u$ case is identical to the
spectra of $B_{2g} \times E_u$ in Fig. 3 for angle $\pi/4 - \beta$, since the nodes for the two symmetries differ by $\pi/4$. The evolution with $x$ of the conductance spectra for $z_0 = 0$, is different in the $B_{1g} \times E_u$, $B_{2g} \times E_u$ cases. It develops a dip at $E = 0$ in the $B_{1g} \times E_u$, while it has a peak in the $B_{2g} \times E_u$ case, at $E = 0$. The nodal $p$-wave case in Fig. 4 has the same nodal structure as the $B_{2g} \times E_u$ case and we see that the spectra for these two candidates are similar.

For nodeless pairing states a subdip or a full gap opens in the tunneling spectra. This is seen in Fig. 3, for the pairing state proposed by K. Miyake, and O. Narikiyo (MN) [21]. The spectra is similar to the nodal $p$-wave case, except that in the (MN) case a subgap opens in the tunneling spectra for certain junction orientation. In this region for $z_0 = 0$ the tunneling conductance is equal to 2, when $x = 0$ and has a constant value for $x > 0$. The tiny subgap is an indication of nodeless pairing state. For the isotropic $p$-wave case the tunneling spectra changes with $z_0$, as can be seen in Fig. 3, but not with the boundary orientation $\beta$. The spectra is nodeless, and for $z_0 = 0$, the conductance is $\sigma_q(E) = 2$, within the energy gap, for $x = 0$. Similar results have been obtained in Ref. [17], where the tunneling conductance of a ferromagnet / triplet superconductor interface is calculated, for both unitary and non-unitary pairing state, having $E_u$ symmetry. Generally in all pairing states the reduction of the subgap conductance with the exchange field is symmetric since the density of states modulation within the subgap is not induced by spin dependent effects.

V. MAGNETIC FIELD EFFECTS

In this section we describe the effect of the external magnetic field $H$ in the spectra for different values of the exchange field $x$. We will see that since the effect of the magnetic field depends on the spin, the evolution of the tunneling spectra with $x$ is asymmetric. The tunneling conductance is given by

$$\sigma_q(E) = \sigma_{q\uparrow}(E - \mu_B H) + \sigma_{q\downarrow}(E + \mu_B H).$$

(24)

In Fig. 7a,b,c the tunneling conductance $\sigma_q(E)$ is plotted for fixed magnetic field $\mu_B H/\Delta_0 = 1$, and barrier strength $z_0 = 2.5$, for different values of the exchange interaction $x$. The
pairing symmetry of the superconductor is $B_{1g} \times E_u$, $B_{2g} \times E_u$, nodal $p$-wave, respectively. The same information is plotted in Fig. 8 a,b, for the nodeless pairing states, isotropic $p$-wave, and $p$-wave, proposed by K. Miyake, and O. Narikiyo, [21] respectively. The orientation of the superconductor is chosen as $\beta = 0$. In the absence of the exchange interaction ($x = 0$) the magnetic field splits symmetrically the tunneling spectrum. The amplitude of the splitting depends linearly on the magnetic field $H$. The main effect of the polarization is the imbalance in the peak heights for $E$ positive and negative. The ratio of the peaks for positive and negative energy is proportional to the exchange field of the material. For $E < 0$ the pattern is suppressed linearly with the increase of the $x$, while for $E > 0$ the tunneling conductance spectra initially increases with $x$ and then decreases. Also the conclusions of the previous section for the nodal (nodeless) form of the tunneling spectra are still valid in the presence of a magnetic field.

VI. CONCLUSIONS

We calculated the tunneling conductance in ferromagnet / insulator / triplet superconductor junctions, using the BTK formalism. We assumed pairing potentials with nodes such as the nodal $p$-wave and two 2D $f$-wave states with line nodes, breaking the time reversal symmetry. Also we examined two nodeless pairing states, the $p$-wave proposed by K. Miyake, and O. Narikiyo [21], and the isotropic $p$-wave. The linear variation of the conductance with $E$ is an indication of line nodes and is not influenced much when the exchange interaction increases. On the other hand the large residual density of states within the gap is reduced with the increase of $x$, and the peaks due to the formation of bound states are removed due to the suppression of the Andreev reflection. The evolution of the spectra with $x$ depends on the position of the nodes and is different in the three pairing states with line nodes. In the case of nodeless pairing states the tunneling conductance develops a subgap or a full gap where $\sigma_q(E)$ has a constant value within. The exchange interaction suppresses the conductance within the gap, and can be considered as a measure of the polarization of
the material. The magnetic field splits linearly the tunneling spectra and in the half metallic ferromagnetic limit $x = 1$, eliminates the negative branch of the spectrum. These features can be used to distinguish between the candidate pairing symmetry states of Sr$_2$RuO$_4$. 
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FIG. 1. The geometry of the ferromagnet / insulator / triplet superconductor interface. The pairing state is unitary with zero spin projection. The vertical line along the $y$-axis represents the insulator. The arrows illustrate the transmission and reflection processes at the interface. $\theta$ is the angle of the incident electron and the normal, $\theta_A$ is the angle of the reflected hole and the normal, and $\theta_s$ is the angle of the transmitted quasiparticle and the normal. Note that $\theta$ is not equal to $\theta_A$ since the retroreflection of the Andreev process is lost.
FIG. 2. Normalized tunneling conductance $\sigma_q(E)$ as a function of $E/\Delta_0$ for $x = 0$ (solid line), $x = 0.4$ (dotted line), $x = 0.8$ (dashed line), and $x = 0.999$ (long dashed line), for different orientations (a) $Z=0$, $\beta = 0$, (b) $Z = 2.5$, $\beta = 0$, (c) $Z = 2.5$, $\beta = \pi/4$. The pairing symmetry of the superconductor is $B_{1g} \times E_u$. 

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FIG. 3. The same as in Fig. 2. The pairing symmetry of the superconductor is $B_{2g} \times E_u$. 
FIG. 4. The same as in Fig. 2. The pairing symmetry of the superconductor is nodal $p$-wave.
FIG. 5. The same as in Fig. 2. The pairing symmetry of the superconductor is $p$-wave proposed by M.N.
FIG. 6. Normalized tunneling conductance $\sigma_q(E)$ as a function of $E/\Delta_0$ for $x = 0$ (solid line), $x = 0.4$ (dotted line), $x = 0.8$ (dashed line), and $x = 0.999$ (long dashed line), for different orientations (a) $z_0 = 0$, $\beta = 0$, (b) $z_0 = 2.5$, $\beta = 0$. The pairing symmetry of the superconductor is isotropic $p$-wave.
FIG. 7. Normalized tunneling conductance $\sigma_q(E)$ as a function of $E/\Delta_0$ for $x = 0$ (solid line), $x = 0.2$ (dotted line), $x = 0.4$ (dashed line), and $x = 0.999$ (long dashed line), for $z_0 = 2.5$, $\beta = 0.0$, for different nodal pairing states (a) $B_{1g} \times E_u$, (b) $B_{2g} \times E_u$, (c) nodal $p$-wave.
FIG. 8. Normalized tunneling conductance $\sigma_q(E)$ as a function of $E/\Delta_0$ for $x = 0$ (solid line), $x = 0.2$ (dotted line), $x = 0.4$ (dashed line), and $x = 0.999$ (long dashed line), for $z_0 = 2.5$, $\beta = 0.0$, for different nodeless pairing states (a) isotropic $p$-wave, (b) nodeless $p$-wave proposed by M.N.