FORMATION, EVOLUTION, AND STRUCTURE OF FRONTS PRODUCED BY UNSTEADY INJECTION OF HIGHLY MAGNETIZED, RELATIVISTIC FLOWS

AMIR LEVINSON
School of Physics and Astronomy, Tel Aviv University, Tel Aviv 69978, Israel

AND

MAURICE H. P. M. VAN PUTTEN
Mathematics Department, Massachusetts Institute of Technology, Cambridge, MA 02139

Received 1997 March 18; accepted 1997 May 27

ABSTRACT

We study the formation, evolution, and structure of dissipative fronts produced by overtaking collisions of relativistic streams, with emphasis on strongly magnetized flows. The evolution of the system is followed using an analytical approach in the simple-wave regime and numerical simulations in the non-simple-wave regime, until a steady state is reached. The steady state structure of the front is then examined by solving the appropriate jump conditions. The conversion of magnetic energy into kinetic energy is parameterized in terms of the Alfvén 4-velocity inside the front. The implications for γ-ray jets are briefly discussed.

Subject headings: galaxies: jets — MHD — relativity — shock waves

1. INTRODUCTION

Highly magnetized, relativistic outflows are believed to play an important role in a variety of astrophysical systems, e.g., pulsar winds (Kennel & Coroniti 1984; Gallant et al. 1992), Galactic and extragalactic superluminal sources (Burns & Lovelace 1982; Begelman & Li 1994; Levinson & Blandford 1996), and γ-ray bursts (Usos 1992; Waxman 1995; Thompson & Duncan 1993; Smolsky & Usos 1996). Such flows are likely to be unsteady, giving rise to formation of shocks which can lead ultimately to the dissipation of some fraction of the bulk energy, either in the form of thermal energy or through shock acceleration (Blandford & Eichler 1987) or magnetic reconnection, high-energy particles with nonthermal distributions. As demonstrated by Romanova & Lovelace (1997) recently, the creation of dissipative fronts in Poynting flux jets may give rise to γ-ray flares, as is often seen in blazars.

One important difference between unmagnetized and highly magnetized flows is that, in the former case, the sound speed is limited to \( c/\sqrt{3} \), whereas in the latter case the Lorentz factor associated with the speed of propagation of a disturbance—the magnetoacoustic speed—can be much larger. Consequently, the conditions required for the formation of strong shocks are expected to be more stringent in highly magnetized flows. Even when the upstream flow is supermagnetoacoustic, so that strong shocks are created, only a relatively small fraction of the incoming flow energy goes to heat the downstream region; the remainder is used up to slightly compress the magnetic field downstream (Kennel & Coroniti 1984). The situation might be markedly different, however, in the presence of rapid magnetic field dissipation behind the shock.

Motivated by the above considerations, we study in this paper the formation, evolution, and structure of fronts produced by unsteady injection of relativistic streams, with particular emphasis on flows in which the energy flux is dominated by electromagnetic fields (commonly referred to as high-sigma flows).

2. THEORY

2.1. Basic Equations

The stress-energy tensor of a perfectly conducting fluid can be written in terms of \( u^a \), the fluid 4-velocity, and the magnetic field 4-vector \( b^a = F^{ab}u_b \), where \( F^{ab} \) is the dual electromagnetic stress tensor, as

\[
T^{ab} = rh u^a u^b + p^a u^b - b^a b^b .
\]  

(1)

Here \( g_{ab} \) denotes the metric tensor, \( r \) is the fluid rest-mass density, \( h^a = h + b^2/r \), and \( p^a = p + b^2/2 \), where \( p \) is the proper gas pressure, \( h \) is the specific enthalpy, and \( b^2 = b^a b_a \). Under the assumption of ideal MHD, Ohm’s law reduces to the condition \( F^{ab} u^b = 0 \). The MHD equations can then be written in a divergence form as (van Putten 1993)

\[
\nabla_a T^{ab} = 0 ,
\]

\[
\nabla_a (ru^a) = 0 ,
\]

\[
\nabla_a (u^a b^b) + g^{ab} c = 0 ,
\]

(2)

where \( c = b^a u^a = 0 \) is a conserved constraint. Note that \( (4\pi)^{1/2} b_a \) now represents the magnetic field measured in the fluid rest frame, the absolute value of which we denote here by \( B \). The above set of equations must be supplemented by an equation of state for the gas. In regions of adiabatic flow where \( u^a \nabla_a S = 0 \), \( S \) being the entropy, the specific enthalpy is given by \( h = 1 + \int dp/r \), with \( p = p(r) \).

Using equation (1), the energy flux carried by the flow can be written in the form

\[
\mathcal{F} = rh u^a (u^2 + u_\Lambda^2) - rh u^2 v ,
\]

(3)

where \( v \) is the fluid 3-velocity, \( u_\Lambda^2 = b^2/(rh)^{1/2} \) is the Alfvén 4-velocity with respect to the fluid rest frame, and \( u_\Lambda \) is the corresponding Lorentz factor.

2.2. Simple Waves

To simplify the analysis, we shall consider in the following a one-dimensional flow with velocity \( u^a = (\gamma, u, 0, 0) \), a
polytropic equation of state, \( p(r) = K r^\gamma \); and magnetic field perpendicular to the fluid 3-velocity. Using the Maxwell equations together with the continuity equation, it is readily shown that \( B/r \) is conserved along streamlines (de Hoffmann & Teller 1958). Consequently, the proper magnetic pressure can be expressed as, \( p_B = \kappa r^2 \), with \( \kappa = B^2/8\pi r^2 \).

Thus, the problem reduces essentially to that of a one-dimensional hydrodynamic flow with an effective equation of state \( p^* (r) = K r^\gamma + \kappa r^2 \). Next, we define

\[
c_i^2 = \frac{1}{\kappa} \frac{\partial p^*}{\partial r} = \frac{\Gamma K r^{\gamma-1} + 2\kappa r}{1 + \Gamma K r^{\gamma-1}/(\Gamma - 1) + 2\kappa r}.
\]

Equation (4) then yields (van Putten 1991)

\[
\frac{\partial \chi_{\pm}}{\partial t} \pm A_{\pm}(\sigma, \lambda) \frac{\partial \chi_{\pm}}{\partial x} = 0,
\]

with \( \chi_{\pm} = \sigma \pm \lambda \) and \( A_{\pm}(\sigma, \lambda) = A_{\pm}(c_{\pm}, v) \), where \( v = u/\gamma \) is the fluid 3-velocity. Equation (5) implies that \( \chi_{\pm} \) and \( \chi_{-} \) are constant along the characteristics \( dx/\partial t = A_{\pm} \) and \( dx/\partial t = A_{+} \), respectively. Note that \( A_{\pm} \) is the relativistic sum and difference of \( c_{\pm} \) and \( v \). In the case of simple waves, one of the Riemann invariants is a constant. Let us suppose that \( \chi_{-} \) is constant. Then \( A_{+} \) depends only on the remaining invariant, \( \chi_{+} \), and it is clear from equation (5) that \( \chi_{+} \) is constant along straight lines with slopes \( A_{+} \), implying that \( A_{+} \) is simply the speed of propagation of a disturbance in the observer frame. For weak disturbances for which \( v \to 0 \), we obtain \( A_{+} \to c_{e} \), verifying that \( c_{e} \) is indeed the speed of propagation of the disturbance in the fluid frame.

For illustration, we consider unsteady injection of one-dimensional, highly magnetized fluid with the following initial and boundary conditions:

\[
\lambda(x, t) = \lambda_0 = \text{constant}, \quad t \leq 0,
\]
\[
\lambda(x, t) = \lambda_0(t) = \lambda_0 + t/\tau, \quad t > 0.
\]

The general solution is then given by

\[
\lambda(x, t) = \lambda_0(t'), \quad t' = t - x \left( \frac{1}{A(\lambda_0(t'))} \right),
\]

where the subscript plus sign has been dropped to simplify the notation. In order to solve for \( t' \), we must determine \( A(\lambda) \). Since we are merely interested in high-sigma flows, we can, to a good approximation, neglect the gas pressure. This corresponds to the limit \( K = 0 \) in equation (4). We then obtain \( A(\lambda) = \tanh (A_0/2 - \lambda_0/2 + \lambda \lambda_0) \), where \( \lambda \) is the Alfven 4-velocity of the initial flow at \( t = 0 \). On substituting this expression for \( A(\lambda) \) into equation (7), we obtain a transcendental equation for the retarded time \( t' \), which can be solved numerically. The solution thereby obtained describes a steepening wave propagating at a velocity \( v = \tanh (\lambda_0 + \lambda \lambda_0) \), the relativistic sum of \( v_\lambda \) and \( v \). This wave will steepen into a shock after time \( t_a \) at some distance \( x_a \) from the injection point. The point \((t_a, x_a)\) is a pole of the function \( \lambda(x, t) \) and can be found from the equation \( d\lambda/dx = \infty \). The result is

\[
t_a = \frac{\tau}{3} \left( 2\lambda_0 + 2\lambda \lambda_0 \right); \quad x_a = c_\tau \tanh (\lambda_0 + \lambda \lambda_0). \tag{8}
\]

If we associate the flow acceleration time \( \tau \) with the size of the injection region, \( R \) (this might be, for example, the radius of a black hole or the inner boundary of an accretion disk from which a jet is ejected), then in the ultrarelativistic limit equation (8) gives \( x_a = R_4 \gamma_\lambda^2 \gamma_\lambda^5/3 \). Note that in this limit \( \gamma_0 \gamma_\lambda \) is just the Lorentz factor associated with the speed of propagation of the disturbance with respect to the injection frame, so that \( \gamma_0 \gamma_\lambda \) is roughly the distance from the injection point at which two disturbances separated by a time \( \tau \) collide. This constraint on the location at which the front is created may have important implications for the characteristics of the high-energy emission from relativistic jets (Levinson 1996). In the case of the Crab pulsar the above criterion (although the different geometry is expected to somewhat alter this result) implies that for \( R \sim 10^6 \) cm any fluctuations of the pulsar wind are not likely to steepen into traveling shocks upstream of the standing shock (which is produced as a result of the interaction of the pulsar wind with the confining, slowly expanding supernova remnant and is located at a radius of about 0.1 pc), unless \( \gamma_0 \gamma_\lambda < 10^5 \).

2.3. The Evolution and Structure of Adiabatic Fronts

The simple-wave solution breaks down after a shock is formed (i.e., for \( t > t_a \)). To study the evolution of the front in the non–simple-wave regime, we have solved equations (2) numerically. A detailed account of the numerical method used is given in van Putten (1993). To test the code, we have compared the numerical results with the analytic solution obtained above in the simple-wave regime. The agreement is found to be good.

An example is shown in Figure 1. As seen from this example, the collision of the initial and injected fluids leads ultimately to the creation of a front consisting of two shocks moving away from each other, and a contact discontinuity across which the total pressure is continuous. The front propagates at a speed (i.e., the speed of the contact discontinuity) intermediate between that of the initial and injected flows, and expands at a rate proportional to the relative velocity of the two shocks. Consequently, there is a net energy flow into the front, giving rise to a pressure buildup inside. After the injection is terminated, the front reaches a steady state whereby the net energy flow into the front is balanced by adiabatic cooling, owing to the expansion of the front. It is then convenient to explore the steady state structure of the front using the appropriate jump conditions. Now, in the ideal MHD limit the specific magnetic field, \( B/r \), must be continuous across the shock. This will no longer be true in the presence of magnetic field dissipation, which may result, e.g., from magnetic reconnection in the front (this would require magnetic field topology more complicated than that invoked above). A self-consistent treatment of dissipative fronts is beyond the scope of this paper. In order to explore the effect of magnetic field dissipation on the parameters of the shocked fluid, we suppose that some fraction of the magnetic field energy is being converted into particle energy inside the front by some unspecified mechanism. We can then treat the magnetic field behind each shock as a free parameter that lies in the range between zero (although the magnetic field is not likely to
denote quantities inside the front by subscript and quantities leftward (rightward) of the contact discontinuity, and shocks. Let the subscript minus (plus) refer to fluid quantities outside the front. In this frame the jump conditions decouple and reduce essentially to those of two single, independent fluids comoving with the initial flow (ahead of the front). A burst of new outflow from the left is modeled by the initial 4-velocity distribution \(u'(x) = \sinh \{1 + \tanh [10(x_0 - x)/L]\}\), where \(x\) is the coordinate along the outflow, \(x_0\) represents the injection point, and \(L\) is the length of the displayed computational grid. The initial density \((\rho = 1)\), pressure \((P = 0.025)\), and Alfvén 4-velocity \([u_A = 0.5\text{ in (a)}\) and \(u_A = 1\text{ in (b)}\)] are taken to be uniform. As seen, the higher Alfvén velocity in (b) results in a broader front. Note the gradual steepening of the leading shock front, which is more delayed in (b). The left shock becomes steady at a very early stage in both examples shown.

No. 1, 1997 FRONTS PRODUCED BY INJECTION OF RELATIVISTIC FLOWS 71

Fig. 1.—Three epochs are shown in two examples of a one-dimensional simulation of the formation of a relativistically hot, magnetized front in which the magnetic field is everywhere transverse to the fluid flow. Displayed are the temperature, the specific magnetic field, and the fluid 3-velocity in the reference frame comoving with the initial flow (i.e., ahead of the shock). The problem can be most easily solved in the rest frame of the initial flow (ahead of the front). A burst of new outflow from the left is modeled by the initial 4-velocity distribution \(u'(x) = \sinh \{1 + \tanh [10(x_0 - x)/L]\}\), where \(x\) is the coordinate along the outflow, \(x_0\) represents the injection point, and \(L\) is the length of the displayed computational grid.

The jump conditions at the right (+) and left (−) shocks can then be written in the front frame as

\[
\begin{align*}
    r_f^+ &= \gamma_f^+(1 - v_\perp/v_{\perp \pm})r_f^\pm, \\
    r_f^+ h_f^+ \gamma_f^+(1 - v_\perp/v_{\perp \pm}) - p_f^+ &= r_f^+ h_f^\pm - p_f^\pm, \\
    r_f^+ h_f^+ \gamma_f^+(v_\perp - v_{\perp \pm}) + p_f^+ &= p_f^\pm. 
\end{align*}
\]

Here the \(v_{\perp \pm}\) denotes the shock 3-velocity, the \(v_\perp\) are the 3-velocities of the fluids outside the front (i.e., ahead of the shocks) with respect to the front frame, and the \(\gamma_f\) are the corresponding Lorentz factors. Since we are merely interested in the strong shock limit, we shall restrict our analysis, in what follows, to the ultrarelativistic case, viz., \(\gamma_f \gg 1\). In this limit we can approximate the enthalpy inside the front as \(h_f^+ \approx \frac{\Gamma_f^+}{\Gamma_f^+-1}(p_f^+/r_f^+)\). Assuming that the fluids upstream of the shocks are cold (i.e., \(p_\perp = 0\)), equations (9) yield, for the shock velocities,

\[
u_\perp = \pm \frac{(2 + u_{A,f}^2\Gamma_f^+ - 2)}{2 + \Gamma_f^+ u_{A,f}^2},
\]

and for the front pressure and temperature to the right (left) of the contact discontinuity,

\[
\begin{align*}
    \frac{p_f^+}{r_f^+ c_s^2} &= \frac{4\Gamma_f^+ (\Gamma_f^+-1)(1 + u_{A,f}^2)(1 + u_{A,f}^2\Gamma_f^+)}{(2 + \Gamma_f^+ u_{A,f}^2)(2\Gamma_f^+ - 2 + \Gamma_f^+ u_{A,f}^2)} , \\
    \frac{kT_f^+}{mc^2} &= \frac{p_f^+}{r_f^+ c_s^2} = \frac{2(\Gamma_f^+-1)(1 + u_{A,f}^2)}{2 + \Gamma_f^+ u_{A,f}^2} .
\end{align*}
\]

Note that the shock velocities given by equation (10) are independent of the magnetic field outside the front.

In the ideal MHD limit, the requirement that the specific...
magnetic field should be continuous across the shock implies that the Alfvén 4-velocities ahead of and behind the shocks are related by
\[ \Gamma_{f\pm} u_{A\pm}^f + [2(\Gamma_{f\pm} - 1) + (\Gamma_{f\pm} - 4)u_{A\pm}^f]u_{A\pm}^f - 2u_{A\pm}^f = 0, \quad (12) \]
where equations (10) and (11) have been used in deriving equation (12). In the weakly \((u_{A\pm} < 1)\) and strongly \((u_{A\pm} \gg 1)\) magnetized cases, equation (12) simplifies to \((u_{A\pm}^f/u_{A\pm})^2 = 1/(\Gamma_{f\pm} - 1)\) and \((u_{A\pm}^f/u_{A\pm})^2 = (4 - \Gamma_{f\pm})/\Gamma_{f\pm}\), respectively. On substituting the latter results into equations (10) and (11), we recover the shock velocity and downstream temperature of a perfectly conducting fluid, obtained earlier by, e.g., Gallant (1992): \(u_{A\pm} = \pm(\Gamma_{f\pm} - 1)\), \(kT_{f\pm} = (\Gamma_{f\pm} - 1)mc^2\gamma_{f\pm}\) in the weakly magnetized case, and \((u_{A\pm}^f/u_{A\pm})^2 = (4 - \Gamma_{f\pm})/(4 - \Gamma_{f\pm})\), \(kT_{f\pm} = [(2\Gamma_{f\pm} - 2)/(4 - \Gamma_{f\pm})]mc^2\gamma_{f\pm}\) in the highly magnetized case. We see that in the strong shock limit considered here, a perfectly conducting front is heated to relativistic temperatures for any value of the magnetic field. However, while in the case of weakly magnetized streams all of the bulk energy per particle incident into the front as measured in the front frame, \(\gamma_{f\pm} mc^2\), is converted into random energy [the average energy per particle in a relativistic gas is \(kT/f(\Gamma_{f\pm} - 1)\)], in the highly magnetized case only a small fraction of the total bulk energy carried by the flow outside the front, specifically the fraction carried by the particles \(~\gamma_{A\pm}^2\) (see eq. [3]), is thermalized; the remainder of the energy is used up to compress the magnetic field inside the front. Consequently, collisions of highly magnetized flows in the ideal MHD limit lead to the formation of magnetically dominated fronts.

The situation could be markedly different if the front is highly dissipative. Then \(u_{A\pm}^f\) can deviate substantially from the maximum value given by equation (12). As seen from equation (11), the energy per particle, \(kT_{f\pm}/(\Gamma_{f\pm} - 1)\), increases as \(u_{A\pm}^f\) decreases, ultimately approaching \(mc^2\gamma_{A\pm}\) in the limit of rapid magnetic field dissipation, i.e., \(u_{A\pm}^f \ll 1\). (We have assumed that the front is adiabatic. It is conceivable that it will become radiative at temperatures lower than that derived above if the synchrotron or inverse Compton cooling time is sufficiently short. We shall not consider this possibility in this paper.) The shock velocities approach \(v_{A\pm} = \pm(\Gamma_{f\pm} - 1)\) in this limit.

Now, the velocity of the contact discontinuity and, hence, the velocities upstream of each shock with respect to the front frame, \(u_{\pm}\), are unknown a priori. The requirement that the total pressure inside the front be continuous across the contact discontinuity surface yields, in the strong shock limit, the relation
\[ \gamma_{\pm} \simeq \eta \gamma_{\pm}^2 ; \]
\[ \eta^2 = \frac{r_+ \Gamma_{f+}(1 + u_{A+}^2)(2 + \Gamma_{f+}u_{A+}^2)(2 + \Gamma_{f+}u_{A+}^2)}{r_{+} \Gamma_{f+}(1 + u_{A+}^2)(1 + u_{A+}^2)(2 + \Gamma_{f+}u_{A+}^2)} . \quad (13) \]
Note that in the case of a symmetric front, which prevails when \(r_+ = r_+, \quad p_+ = p_+, \quad \) and \(u_{A-} = u_{A+}, \) equation (13) gives \(\eta = 1\), as of course it should.

To complete the solution, we transform to the frame in which the fluid is injected (hereafter referred to as the injection frame); this could be, e.g., the rest frame of a star or a black hole from which a jet has emerged, and in which the velocities of the initial and injected flows, denoted here by \(u_{\pm}^i\) and \(u_{\pm}^f\), are specified. The proper mass density, pressure, and magnetic field are of course relativistic invariants and have the same values in the injection frame as in the front frame. We consider the case \(u_{\pm}^i \gg u_{\pm}^f \geq 0\), namely, both the initial and injected fluids propagate in the same direction. On transforming to the injection frame, we obtain
\[ \gamma_{\pm} \simeq \gamma_{\pm}^2 \gamma_{\pm} - u_{\pm}^2 u_{\pm} , \quad (14) \]
with \(u_{\pm}\) being the front 4-velocity with respect to the injection frame. Equations (13) and (14) can be solved now to yield \(\gamma_{\pm}\) and \(\gamma_{\pm}^2\). One finds
\[ \gamma_{\pm} = \frac{\eta u_{\pm}^2 - u_{\pm}^2}{\sqrt{[(\eta u_{\pm}^2 - u_{\pm}^2)^2 - (\eta \gamma_{\pm}^2 - \gamma_{\pm}^2)^2]}} , \quad (15) \]
For an initial fluid at rest \((u_{\pm}^i = 0)\) the above solution simplifies to \(\gamma_{\pm} \simeq (\gamma_{\pm}^2/2\eta)^{1/2}\), and for relativistic initial fluid to \(\gamma_{\pm} \simeq (\gamma_{\pm}^2/4\eta \gamma_{\pm}^2)^{1/2}\). On substituting the latter results into equations (10) and (11), we obtain an expression for the front quantities in terms of \(u_{\pm}\). Evidently, for roughly symmetric fronts \(\eta\) of order unity the front temperature scales roughly as \((\gamma_{\pm}^2/4\eta)^{1/2}\). In the asymmetric case, one side of the front will be heated to much higher temperatures than the other. Heat conduction across the contact discontinuity surface may then become important if the magnetic field is not completely transverse there.

3. DISCUSSION

A highly conducting front produced by overtaking collisions of strongly magnetized, relativistic flows should be magnetically dominated and is not an efficient converter of bulk energy into random energy (i.e., thermal energy or nonthermal distributions of accelerated particles). Much higher temperatures can be attained in the presence of rapid magnetic field dissipation inside the front, which may arise from magnetic reconnection or plasma instabilities near the shocks. A fraction of the dissipated field energy is likely to be radiated away in the form of electromagnetic emission with relatively high efficiency, owing to synchrotron and inverse Compton cooling of the energetic particles. The latter process can be very effective in the presence of background radiation with roughly isotropic distribution, as is believed to be the case for Galactic and extragalactic jets (Dermer & Schlickeiser 1993; Sikora, Begelman, & Rees 1994; Ghisellini & Madau 1996; Levinson & Blandford 1996). The radiative friction may also provide a means for dissipating the magnetic field. Such fronts may explain the transient emission observed in many of these sources (Romanova & Lovelace 1997). However, in order to be an efficient radiator, the front must be created at not too large a distance from the putative black hole, depending on the intensity and spectrum of the external radiation, thereby imposing a constraint on the magnetosonic speed (see discussion following eq. [8]). In the case of streams consisting of electron-positron plasma, the front may become loaded with \(e^\pm\) pairs as a result of effective pair production. Frequent creation of such fronts may lead to a transition from magnetic to particle-dominated jets (Levinson 1996), as seems to be suggested by observations (Blandford & Levin-
son 1995). Again, the location at which the front is created might be important; the fronts must form above the cooling radius, which depends upon the intensity and spectrum of the background radiation, in order to avoid catastrophic X-ray production and the disruption of the jet.

We thank R. D. Blandford for useful comments. A. L. acknowledges support from NSF grants AST 91-19475 and AST 93-15375, an Alon Fellowship, and a TAU Research Authority grant.

REFERENCES

Begelman, M. C., & Li, Z.-Y. 1994, ApJ, 426, 269
Blandford R. D., & Eichler, D. 1987, Phys. Rep., 154, 1
Blandford R. D., & Levinson, A. 1995, ApJ, 441, 79
Burns, M. L., & Lovelace R. V. E. 1982, ApJ, 262, 87
de Hoffmann, F., & Teller, E. 1958, Phys. Rev., 80, 692
Dermer, C., & Schlickeiser, R. 1993, ApJ, 416, 458
Gallant, Y. A., et al. 1992, ApJ, 391, 73
Ghisellini, G., & Madau, P. 1996, MNRAS, 280, 67
Kennel, C. F., & Coroniti, F. V. 1984, ApJ, 283, 694
Levinson, A. 1996, ApJ, 467, 546
Levinson, A., & Blandford, R. D. 1996, ApJ, 456, L29
Romanova, M. M., & Lovelace, R. V. E. 1997, ApJ, 475, 97
Smolsky, M. V., & Usov, V. V. 1996, ApJ, 461, 858
Thompson, C., & Duncan, R. C. 1993, ApJ, 408, 194
van Putten, M. H. P. H. 1991, Commun. Math. Phys., 141, 67
Waxman, E. 1995, Phys. Rev. Lett., 75, 386