Tunable single-photon multi-channel quantum router based on an optomechanical system

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Abstract
Routing of photons plays a key role in optical communication networks and quantum networks. Although the quantum routing of signals has been investigated for various systems, both in theory and experiment, the general form of a quantum router with multi-output terminals still needs to be explored. Here, we propose an experimentally accessible tunable single-photon multi-channel routing scheme using an optomechanics cavity which is Coulomb coupled to a nanomechanical resonator. The router can extract single photons from the coherent input signal and directly modulate them into three different output channels. More importantly, the two output signal frequencies can be selected by adjusting the Coulomb coupling strength. For application purposes, we justify that there is insignificant influence from the vacuum and thermal noises on the performance of the router under cryogenic conditions. Our proposal may pave a new avenue towards multi-channel routers and quantum networks.

Keywords: single-photon router, quantum router, nanomechanical resonators

(Some figures may appear in colour only in the online journal)
as in different quantum router systems, cavity QED systems [10], circuit QED systems [3], optomechanical systems [4], pure linear optical system [11] and \(\Lambda\)-type three-level system [12–14]. The essence lying at the core is the realization of the strong coupling between the photons and photons or photons and phonons [19–29], but these methods require high-pump-laser powers due to the very weak optical nonlinearity. To the best of our knowledge, the quantum routers demonstrated in most experiments and theoretical proposals have only one output terminal, except for only a few proposals in [12–18]. However, the two output ports are composed with two photonic crystal cavities [12] or two coupled-resonator waveguides [13, 14] whose two output channels have maximal probabilities of unity and no more than 1/2, respectively. [15] extended the method in [13] to N output ports. Thus, an ideal quantum router with multi-channels based on only one cavity with an extremely high probability still needs to be explored. In the following, we propose a novel scheme, via Coulomb coupling interaction, in which one can realize three output signals in unity probability with just one input signal. Our proposal is different from previous methods in [12–18], because we are able to directly modulate the coherent input signal into three different output ports.

In this paper, we theoretically propose a scheme for a single-photon quantum router with three output ports based on an optomechanical cavity which is Coulomb coupled to a nanomechanical resonator (NMR). More importantly, the two output signal frequencies can be selected by adjusting Coulomb coupling strength. The thermal noise could be more critical in deteriorating the performance of the single-photon router. So, we also demonstrate that the vacuum and thermal noise can be insignificant for the optical performance of the single-photon router at temperature of the order of 20 mK.

2. Model setup and the solutions

The model for realizing single-photon routing is sketched in figure 1, where a partially transparent nanomechanical mirror (NMM) charged with a bias voltage \(V_1\) is placed at the center of the Fabry–Perot cavity formed by two fixed mirrors with finite identical transmission [30]. The whole cavity length is \(L\). The cavity field is driven by a strong coupling field at frequency \(\omega_t\) from the left-hand mirror. Further, the field in a single-photon Fock state at frequency \(\omega_p\) is incident on the cavity through the left-hand mirror. Besides the radiation pressure force coupling the NMM to the cavity mode, the charged NMM is also subject to Coulomb force to the charged NMR with the bias gate voltage \(-V_2\) near the cavity. That is to say, the auxiliary NMR provides Coulomb coupling to the NMM and adds an extra level to the optomechanical system. \(q_1\) and \(q_2\) represent the small displacements of the NMM and the NMR around their equilibrium positions, and \(r_0\) is the distance between their equilibrium position. Then the Hamiltonian of the whole system can be written as

\[
H_{\text{whole}} = \hbar \omega_t c^\dagger c + \sum_{j=1}^{2} \left( \frac{p_j^2}{2m_j} + \frac{1}{2} m_j \omega_j^2 q_j^2 \right) - \hbar g (c^\dagger q_1 + H.C.) + \hbar \epsilon_i (c^\dagger e^{-i\omega_{\ell} t} - H.C.),
\]

where the first term is the single-mode cavity field with frequency \(\omega_c\) and annihilation (creation) operator \(c\) \((c^\dagger)\). The second term describes the vibration of the charged NMM (NMR) with frequency \(\omega_1\) \((\omega_2)\) and effective mass \(m_1\) \((m_2)\) and \(p_1\) \((p_2)\) and \(q_1\) \((q_2)\) are the momentum and the position operator of NMM (NMR), respectively [4]. The third term describes the radiation pressure coupling between the cavity field and the NMM, with \(g = \frac{\lambda}{2}\) being the coupling strength [30]. The last term describes the interaction between the cavity field and the input fields. The pump field strength, \(\epsilon_i\), depends on the power \(\nu\) of coupling field, \(\epsilon_i = \sqrt{2\kappa \nu / \omega_t}\) with \(\kappa\) being the cavity decay rate.

The fourth term \(H_I\) in equation (1) presents the Coulomb coupling between the charged NMM and NMR [31], where the NMM and NMR take the charges \(C_1 V_1\) and \(-C_2 V_2\), where \(C_1\) and \(C_2\) are the capacitance of the gates, respectively. The interaction energy between the NMM and NMR is given by

\[
H_I = \frac{e_0 c_0}{4\pi\epsilon_0} \frac{C_1 V_1 C_2 V_2}{r_0^4},
\]

In the case of \(q_1, q_2 \leq r_0\), with the second order expansion, the above Hamiltonian is rewritten as

\[
H_I = \frac{e_0 c_0}{4\pi\epsilon_0} \frac{C_1 V_1 C_2 V_2}{r_0^4} \left[ 1 - \frac{r_0^2 q_1^2}{2} + \left( \frac{r_0^2 q_2^2}{2} \right)^2 \right],
\]

where the linear term may be absorbed into the definition of the equilibrium positions, and the quadratic term includes a renormalization of the oscillation frequency for both the NMM and NMR. It implies a reduced form \(H_I = \hbar \lambda q_1 q_2\), where \(\lambda = \frac{C_1 V_1 C_2 V_2}{2\hbar c_0 \epsilon_0}\) [31].

In a frame rotating with the frequency \(\omega_{\ell}\) of the pump field, the Hamiltonian of the total system equation (1) can be rewritten as,
where $\Delta_c = \omega_c - \omega_f$. Note that the NMM and the NMR are coupled to the thermal surrounding at the temperature $T$, which results in the mechanical damping rate $\gamma_1$ and $\gamma_2$, and thermal noise force $\xi_1$ and $\xi_2$ with a frequency–domain correlation [4],

$$\langle \xi_\tau(\omega)\xi_\tau(\Omega) \rangle = 2\pi \hbar \gamma_c m_r \omega [1 + \coth(\frac{\hbar \omega}{2k_B T})] \delta(\omega + \Omega),$$  

(3)

where $\tau = 1$ or 2 and $k_B$ is the Boltzmann constant. In addition, the cavity field $c$ is coupled to the input quantum fields $c_{in}$ and $d_{in}$. If there are no photons in the right, then $d_{in}$ would be the vacuum field. Let $2\kappa$ be the rate at which photons leak from each of the cavity mirrors. The output fields can be written as

$$x_{out}(\omega) = \sqrt{2\kappa}c(\omega) - x_m(\omega), \ x = c, d.$$  

(4)

These couplings are included in the standard way by writing quantum Langevin equations for the cavity field operators. Putting together all the quantum fields, thermal fluctuations and the Heisenberg equations from the Hamiltonian (2), we can obtain the working quantum Langevin equations:

$$q_1 = \frac{p_1}{m_1}, \quad q_2 = \frac{p_2}{m_2},$$

$$\dot{c} = -[2\kappa i(\Delta_c - \hbar g_{11})c + \epsilon_1 + \sqrt{2}\kappa c_{in} + \sqrt{2}\kappa d_{in},$$

$$\dot{p}_1 = -m_1 \omega_1^2 q_1 - \hbar \lambda q_2 + h g_c c - \gamma_1 p_1 + \xi_1,$$

$$\dot{p}_2 = -m_2 \omega_2^2 q_2 - \hbar \lambda q_1 - \gamma_2 p_2 + \xi_2.$$  

(5)

The quantum Langevin equations (5) can be solved after all operators are linearized as their steady-state mean values and small fluctuations:

$$q_{r} = q_{rs} + \delta q_{r}, \quad p_{r} = p_{rs} + \delta p_{r}, \quad c = c_s + \delta c,$$  

(6)

where $\delta q_{rs}, \delta p_{rs}, \delta c$ being the small fluctuations around the corresponding steady values and $\tau = 1, 2$. After substituting equation (6) into equation (5), ignoring the second-order small terms, and introducing the Fourier transforms $f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f(\omega)e^{-i\omega t}d\omega$, $f^*(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} f^*(-\omega)e^{-i\omega t}d\omega$. We can get the steady values

$$p_{1s} = p_{2s} = 0, \quad q_{1s} = \frac{h g_c |c_s|^2}{m_1 \omega_1^2 - \hbar \lambda |c_s|^2},$$

$$q_{2s} = -m_2 \omega_2^2 c_s, \quad \xi_1 = \frac{\epsilon_1}{2\kappa} + i\Delta_c,$$  

(7)

with $\Delta = \Delta_c - \hbar g_{11}$, and the solution of $\delta c$ [32],

$$\delta c = E(\omega)c_{in}(\omega) + F(\omega)c^*_{in}(-\omega) + E(\omega)d_{in}(\omega) + F(\omega)d^*_{in}(-\omega) + V_1(\omega)\xi_1(\omega) + V_2(\omega)\xi_2(\omega),$$  

(8)

in which

$$E(\omega) = \sqrt{2\kappa} \frac{1}{2\kappa + i(\Delta - \omega)} + \frac{i g^2 |c_s|^2(2\kappa i + \Delta + \omega) m_r (\omega^2 + i\omega \gamma_2 - \omega_2^2)}{d(\omega)},$$

$$F(\omega) = iv \frac{\sqrt{2\kappa} g^2 |c_s|^2(2\kappa i - \Delta + \omega) m_r (\omega^2 + i\omega \gamma_2 - \omega_2^2)}{d(\omega)},$$

$$V_1(\omega) = \frac{g |c_s|^2 (2\kappa i + \omega^2 - \Delta^2)}{D(\omega)},$$

$$V_2(\omega) = \frac{g \hbar \lambda |c_s|^2 (2\kappa i + \omega^2 - \Delta^2)}{D(\omega)},$$  

(9)

with

$$D(\omega) = (2\kappa i + \omega^2 - \Delta^2 + h^2 \lambda^2 [\Delta^2 + (2\kappa i + \omega^2)] + m_r (\omega^2 + i\omega \gamma_2 - \omega_2^2) [2|c_s|^2 g^2 \hbar \Delta + (\Delta - 2\kappa i - \omega) \times (2\kappa i + \omega i + m_r (\omega^2 + i\omega \gamma_2 - \omega_2^2))].$$  

(10)

Here, we define the spectrum of the field via $|c^*(t)|^2 = 2\pi |S_c(\omega)|^2 \delta(\omega + \Omega), (\omega) = 2\pi |S_\text{in}(\omega)|^2 \delta(\omega + \Omega)$. The incoming vacuum field $d_{in}$ is characterized by $|d(\omega)|^2 = 2\pi \delta(\omega + \Omega)$ with $S_{\text{in}}(\omega) = 0$. From equations (4) and (8), we can obtain the spectrum of the output fields,

$$S_{\text{out}}(\omega) = R(\omega)S_{\text{in}} + S^{(v)}(\omega) + S^{(\pi)}(\omega) + S^{(\pi)}(\omega),$$

(11)

where

$$R(\omega) = |\sqrt{2\kappa}E(\omega) - 1|^2, \quad T(\omega) = |\sqrt{2\kappa}E(\omega)|^2, \quad S^{(v)}(\omega) = 2\kappa |V_1(\omega)|^2 \hbar \gamma_1 m_1 (\omega)[1 + \coth(-\frac{\hbar \omega}{2k_B T})],$$

$$S^{(\pi)}(\omega) = 2\kappa |V_2(\omega)|^2 \hbar \gamma_2 m_2 (\omega)[1 + \coth(-\frac{\hbar \omega}{2k_B T})],$$

$$S^{(\pi)}(\omega) = 4\kappa |F(\omega)|^2.$$  

(12)

When there is no Coulomb coupling $\lambda$ (i.e. $\lambda = 0$) between the NMM and the NMR, equation (11) can be reduced to equation (10) in [4]. However, different from the output field in [4] involving a single output channel for a quantum router, there are three output channels with different frequencies due to Coulomb interaction. Furthermore, the frequencies of two output channels can be selected by adjusting the Coulomb coupling strength $\lambda$.

3. Quantum routing for single photons

In equation (11), $R(\omega)$ and $T(\omega)$ are the contributions arising from the presence of single photons in the input field. $S^{(v)}(\omega)$ is the contribution from the incoming vacuum field. The $S^{(\pi)}(\omega)$
and $S_T^f(\omega)$ are contributions from the fluctuation of the NMM and the NMR, respectively. Equation (11) shows that even if there is no incoming photon, the output signal is generated via quantum and thermal noises. For the purpose of achieving a single-photon router, the key quantities are $R(\omega)$ and $T(\omega)$. Further, we also demonstrate that the performance of the single-photon router should not be deteriorated by the quantum and thermal noise terms $S^{(v)}(\omega)$ and $S_T^f(\omega)$ ($\tau = 1, 2$).

To demonstrate the routing functions of our hybrid optomechanics system, we first investigate the reflection $R(\omega)$ and transmission spectrum $T(\omega)$. For illustration of the numerical results, we choose realistically reasonable parameters from a recent experiment [30] on the observation of the strong dispersive coupling between a cavity and the NMM. The wavelength of the pump field $\lambda_l = 1054$ nm, $L = 6.7$ cm, $\omega_1 = \omega_2 = 2\pi \times 134 \times 10^3$ Hz, $Q_1 = Q_2 = 1.1 \times 10^6$, $m_1 = m_2 = 40$ ng, $\kappa = \omega_1/10$, $\varphi_l = 2$ $\mu$W, and $\lambda = 3 \times 10^{15}$ Hz nm$^{-2}$.

![Figure 2](image1.png)

(Upper panel) The reflection spectrum $R(\omega)$ and (lower panel) the transmission spectrum $T(\omega)$ of the single photon as a function of normalized frequency $\omega/\omega_1$ when the Coulomb interaction is turned off (red dashed line) or turned on (blue solid line). $\lambda_l = 1054$ nm, $L = 6.7$ cm, $\omega_1 = \omega_2 = 2\pi \times 134 \times 10^3$ Hz, $Q_1 = Q_2 = 1.1 \times 10^6$, $m_1 = m_2 = 40$ ng, $\kappa = \omega_1/10$, $\varphi_l = 2$ $\mu$W, and $\lambda = 3 \times 10^{15}$ Hz nm$^{-2}$.

![Figure 3](image2.png)

The transmission spectrum $T(\omega)$ of the single photon as a function of normalized frequency $\omega/\omega_1$ with different $\lambda$ (in units of $\lambda_0$, $\lambda_0 = 10^{15}$ Hz nm$^{-2}$). Other parameters take the same values as in Figure 2.

![Figure 4](image3.png)

(Upper panel) The vacuum noise spectrum $S^{(v)}(\omega)$ as a function of normalized frequency $\omega/\omega_1$. (Lower panel) The thermal noise spectrum $S_T^{(v)}(\omega)$ (red dot-dashed line) and $S_T^f(\omega)$ (blue dashed line) as a function of normalized frequency $\omega/\omega_1$. $T = 20$ mK and the other parameters take the same values as in Figure 2.
are fixed, the equilibrium position is decided by the Coulomb interaction. With an increase of the Coulomb interaction, the NMM will acquire a larger displacement to provide a larger strain for compensating the increase of the Coulomb interaction, while the radiation pressure in the optomechanical system increases. The spectrum of the bandwidth of these peaks/dips becomes greater for the larger displacement of the NMM. We can find \( R(\omega_1 \pm \omega_0) \approx 1 \) and \( T(\omega_1 \pm \omega_0) \approx 0 \). That is to say, the single photons are completely transmitted through the cavity to the right output port at frequency \( \omega = \omega_1 \); meanwhile, the single photons are completely reflected to the left two output ports with different frequencies \( \omega_1 \pm \omega_0 \) and \( \omega_1 - \omega_0 \).

Then, we can describe the working process of the single-photon multi-output router. When we turn off the Coulomb coupling, the single photons are complete reflected through the cavity to the left output port at frequency \( \omega = \omega_1 \) (i.e., \( R(\omega_1) \approx 1, \quad T(\omega_1) \approx 0 \)). However, when we turn on the Coulomb coupling, the single photons with different frequencies are completely transmitted to the right output port at frequency \( \omega = \omega_1 \) (i.e., \( R(\omega_1) \approx 0, \quad T(\omega_1) \approx 1 \)); at the same time, they are completely reflected to the left two outputs at different frequencies \( \omega = \omega_1 + \omega_0 \) and \( \omega = \omega_1 - \omega_0 \) (i.e., \( T(\omega_1 \pm \omega_0) \approx 1 \) and \( T(\omega_1 \pm \omega_0) \approx 0 \)). Figure 3 shows the transmission spectrum of the single-photon router with different Coulomb coupling strengths. From figure 3, we find that the different output frequencies can be selected by adjusting the Coulomb coupling strength.

Next, we discuss the effects of the quantum and thermal noise on the reflection and transmission spectra of single photons. From figure 4, the contribution of the vacuum noise is about 2.5% and is thus insignificant. The thermal noise could be more critical in deteriorating the performance of the single-photon router. Clearly, to beat the effects of thermal noise, the number of photons in the probe pulse has to be much bigger than the thermal noise photons. However, if we work with NMM and NMR temperatures like 20 mK, then the thermal noise term is insignificant, as shown in figure 4.

### 4. Conclusions

We have proposed an experimentally accessible tunable single-photon multi-channel routing scheme based on the hybrid optomechanical system which consists of a high-quality Fabry–Perot cavity and a charged NMR. Our proposal is essentially different from previous methods in [12–18], because we are able to directly modulate the coherent input single-photon signals into three different output ports with unit probability. More important, the two output signal frequencies can be selected by adjusting the Coulomb coupling strength. We also demonstrate that vacuum and thermal noise will be insignificant for the optical performance of the single-photon router at temperature of the order of 20 mK. We hope that the quantum routing function predicted in this paper can be observed in some experiments. Our proposal may have paved a new avenue towards multi-channel router and quantum networks.

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