NEW EQUATIONS FOR VACUUM CORRELATORS IN GAUGE THEORIES

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Abstract

Stochastic quantization [1] is applied to derivation of equations, connecting multilocal gauge-invariant correlators in different field theories. They include Abelian Higgs Model, QCD with spinless quarks at $T = 0$ and $T > 0$ and QED, where spin effects are taken into account exactly.

1 Introduction

One of the main problems of the modern quantum field theory is determination of nonperturbative effects in the models with nontrivial vacuum structure, like QCD. To investigate this problem the Method of Vacuum Correlators (MVC) [2-8, 17] (for a review see [5]) was suggested. In the framework of this method one, using Feynman-Schwinger path integral representation, extracts from quark-antiquark Green function all the dependence from vacuum gluonic fields in the form of an averaged Wilson loop. After that, through nonabelian Stokes theorem [3] and cumulant expansion [3,9] the averaged Wilson loop is expressed via an infinite set of irreducible gauge-invariant vacuum correlators (cumulants). Lattice data suggest, that the ensemble of gluonic fluctuations is predominantly Gaussian [5]. That is why, in order to calculate observable quantities (like confining potential [4,5], mass spectra of mesons, baryons and glueballs [5-7], Regge trajectories [7,8], temperature of deconfinement, meson and glueball screening masses at finite temperature [17], etc.) one may with a good accuracy neglect all the cumu-
lants higher than quadratic and to parametrize the latter by two independent Kronecker structures, where the space-time dependence of the corresponding coefficient functions is determined from the lattice data.

However, the nonperturbative stochastic Euclidean QCD vacuum is described by the full set of cumulants, and, therefore, for the completeness of the theory, one needs the equations (derived from the Lagrangian), from which the cumulants may be obtained. Two alternative methods of derivation of such equations, based on stochastic quantization [1], were suggested in [10]. First of them, exploiting Feynman-Schwinger path integral representation, where Langevin time is considered as a proper time, was applied to the $\varphi^3$ theory. However, it is inconvenient for gluodynamics, since it breaks down gauge invariance. That is why, in the latter case an alternative approach, leading to integral-differential Volterra type-II equations, was used. In both cases it was shown, that the perturbative expansion of the obtained equations in the lowest orders reproduced correctly known results of standard Feynman diagrammatic technique. An important point is that in the physical limit, when Langevin time $t$ tends to infinity, the solutions of these equations yield exact correlators of Heisenberg field operators, averaged over the physical vacuum, whose structure may be complicated and unknown, e.g. in QCD.

In this paper we demonstrate, how these two approaches work simultaneously in gauge theories with matter fields, presenting the methods of derivation of equations for gauge-invariant correlators in different models. The sketch of the paper is the following. In section 2 we start with the Abelian Higgs Model (AHM), where classical solutions (Abrikosov-Nielsen-Olesen (ANO) strings) are present. These objects may be naturally included into obtained equations as initial conditions in the Cauchy problem for the Langevin equation, and, thus, we obtain a new method of quantization of classical solutions. This method allows one to derive correlators, containing arbitrary number of quantum fluctuations around classical solutions in any theory. We hope, that it will be especially useful in the case of gauge theories, since the obtained equations are explicitly gauge-invariant. Computation of quantum corrections to an instanton in the framework of this method will be a topic of a separate publication. Another point is that the suggested approach is the first attempt to gauge-invariant quantization of theories with spontaneous symmetry breaking and may be applied to quantization of more complicated theories, like Standard Model. In section 3 we exploit methods
of stochastic quantization of fermionic theories to derive equations for gauge-invariant correlators in QCD with spinless quarks. Then we demonstrate, how the obtained equations may be generalized to the finite temperature case. Finally, in section 4 we use the path integral approach to calculation of the propagator of a spinning particle in the Abelian background gauge field [11,12] to obtain equations in QED, where all spin effects are taken into account exactly. In Conclusion we summarize the main results of the paper and outline possible future developments. In the Appendix we obtain a Green function of the Langevin equation for $F_{\mu\nu}(x, t)$ in the AHM.

## 2 Abelian Higgs Model

In this section we consider AHM, where exists a vacuum of classical solutions (ANO strings) [13]. The Euclidean action of the AHM has the form

$$S = \int dx \left[ (D_\mu \varphi)^* (D_\mu \varphi) + \frac{1}{4} F_{\mu\nu} F_{\mu\nu} - \lambda (\varphi^* \varphi)^2 + m^2 \varphi^* \varphi \right],$$  

(1)

where $D_\mu \varphi = (\partial_\mu - ie A_\mu) \varphi$. Let us look for a solution of equations of motion in the cylindrical coordinates in the form $\vec{A}(\vec{r}) = A_\theta(r)$, $A_4 = 0$, $\varphi = |\varphi| e^{i n \theta}$, where $|\varphi|$ is a function of $r$ only. Then it is easy to check, that such a solution (ANO string along z-axis), corresponding to an $n$-quantum Abrikosov vortex of Ginzburg-Landau model, has the form

$$A_\theta(r) = \begin{cases} \frac{n}{e} - \frac{n}{e_0} K_1(\rho), & r \geq \delta, \\ \frac{n}{2 e \delta} \left(-\frac{1}{2} + \frac{1}{2} \rho - \rho \ln \rho\right), & \xi \ll r \ll \delta, \end{cases}$$

(2)

where $\delta \equiv \frac{\sqrt{\lambda}}{m}$ is a London's depth of penetration of the field $A_\mu$, $\xi \equiv \frac{1}{m}$ is a correlation radius of fluctuations of the Higgs field, $\rho \equiv \frac{r}{\delta}$, $K_1$ is the Macdonald function.

The Langevin equations, following from the action (1), have the form

$$\dot{\varphi} = D^2 \varphi - m^2 \varphi + 2 \lambda \varphi^* \varphi^2 + \eta,$$

(3)

$$\dot{\varphi}^* = (D^2 \varphi)^* - m^2 \varphi^* + 2 \lambda \varphi^* \varphi^2 + \eta^*,$$

(4)

$$\dot{A}_\mu = \partial_\nu F_{\nu\mu} + j_\mu + \eta_\mu,$$

(5)
where \( j_\mu = -ie(\varphi^* \partial_\mu \varphi - (\partial_\mu \varphi^*) \varphi - 2ieA_\mu \varphi^* \varphi) \) is a gauge-invariant current,
\[
< \eta(x, t)\eta^*(x', t') > = 2\delta(x - x')\delta(t - t'),
\]
\[
< \eta_\mu(x, t)\eta_\nu(x', t') > = 2\delta_{\mu\nu}\delta(x - x')\delta(t - t'),
\]
and \(< \ldots >\) means the averaging over both Gaussian stochastic ensembles of \((\eta, \eta^*)\) and \(\eta_\mu\).

Passing from equation (5) to the Langevin equation for \(F_{\mu\nu}(x, t)\) and solving it via Fourier transformation with the initial condition \(A_\mu(x, 0) = A_\mu(x)\), where \(A_r(x) = A_z(x) = A_4(x) = 0\) and \(A_\theta(x)\) is defined through the formula (2), one obtains:
\[
F_{\mu\nu}(x, t) = \int_0^t dt' \int dy \ U_{\mu\nu,\beta}(y - x, t - t')(j_\beta(y, t') + \eta_\beta(y, t')) +
\]
\[
+ \int dy \ U_{\mu\nu,\beta}(y - x, t)A_\beta(y),
\]
where
\[
U_{\mu\nu,\beta}(y, t) = \frac{1}{48\pi^2} \left\{ (\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\nu\alpha}\delta_{\mu\beta}) \left[ \frac{y_\alpha}{t^3}e^{-\frac{y^2}{4t}} - 16\pi^2 \frac{\partial}{\partial y_\alpha} \delta(y) \right] +
\right.
\]
\[
+ \frac{2}{y^2}(\delta_{\nu\beta}y_\mu - \delta_{\mu\beta}y_\nu) \left( \frac{1}{t^2}e^{-\frac{y^2}{4t}} - 16\pi^2 \delta(y) \right) + \frac{1}{y^2}(\delta_{\nu\beta}y_\mu y_\alpha +
\]
\[
+ \delta_{\nu\alpha}y_\mu y_\beta - \delta_{\nu\beta}y_\mu y_\alpha - \delta_{\mu\alpha}y_\nu y_\beta \right) \cdot
\]
\[
\cdot \left[ \frac{y_\alpha}{2t^3}e^{-\frac{y^2}{4t}} + 16\pi^2 \left( \frac{\partial}{\partial y_\alpha} \delta(y) \right) \right] \right\}.
\]

The details of derivation of the Green function (8) are presented in the Appendix.

Using Langevin time as a proper time and solving equation (3) via Feynman-Schwinger path integral representation with the initial condition \(\varphi(x, 0) = \varphi_0(x), |\varphi_0|^2 = \frac{m^2}{2\lambda}\), we have:
\[
\varphi(x, t) = \int_0^t dt' \int dy (Dz)_{xy}K_z(t, t')F_z(t, t')\Phi(x, y)\eta(y, t'),
\]
where
\[
(Dz)_{xy} \equiv \lim_{N \to \infty} \prod_{n=1}^N \frac{d^4z(n)}{(4\pi\varepsilon)^2}, , N\varepsilon = t - t', \ z(\xi = t') = y, \ z(\xi = t) = x,
\]
\[K_z(t, t') \equiv \theta(t - t') \exp \left[ -m^2(t - t') - \frac{\int_{t'}^t \dot{z}^2(\xi) d\xi}{4} \right],\]

\[\mathcal{F}_z(t, t') \equiv \exp \left[ 2\lambda \int_{t'}^t d\xi \left| \varphi(z(\xi), \xi) \right|^2 \right],\]

\[\Phi(x, y) \equiv \exp \left[ ie \int_y^x dz_\mu A_\mu(z(\xi), \xi) \right].\]

and we omitted in (9) the term \(\int dy(Dz)_{xy} K_z(t, 0) \mathcal{F}_z(t, 0) \Phi(x, y) \varphi_0(y)\), because we shall see, that in the correlators (11), (12) (and all the other gauge-invariant quantities) this term produces non-gauge-invariant part of the correlator, which in the physical limit, \(t \to + \infty\), tends to zero.

In the same way one may solve equation (4).

To obtain an infinite system of exact equations for gauge-invariant correlators, containing the fields \(\varphi\) and \(\varphi^*\), one should multiply the right hand sides of equation (9) and the equation for \(\varphi^*\) by \(\eta(\bar{x}, \bar{t}), \eta^*(\bar{x}, \bar{t}), \eta_\mu(\bar{x}, \bar{t}), \varphi(\bar{x}, \bar{t}), \varphi^*(\bar{x}, \bar{t}), j_\mu(\bar{x}, \bar{t}), F_{\mu\nu}(\bar{x}, \bar{t})\) as many times as needed, introducing corresponding parallel transporters in order to preserve gauge invariance, and average both sides of the obtained equations. The right hand sides of these equations may be most easily expressed through their left hand sides by introducing the generating functional

\[Z = \prod_{i=1}^k < \mathcal{F}_{z_i}(t_i, t'_i) W_i(C_i) \exp \left[ \int du dt (I_\mu(u, t) \eta_\mu(u, t) + K_\mu j_\mu + L_{\mu\nu} F_{\mu\nu} + M\eta + N\eta^*) \right] >, \tag{10}\]

where \(k\) is a total number of fields \(\varphi\) and \(\varphi^*\) in the correlator, \(W_i(C_i)\) is an Abelian Wilson loop, and the form of a contour \(C_i\) is clarified below. After that one should apply to this generating functional cumulant expansion [3,9].

For example, let us consider the simplest case \(k = 2\). This is the so-called bilocal approximation [10], when one assumes, that the ensemble of the fields \(\varphi, \varphi^*, j_\mu, F_{\mu\nu}\) and stochastic noise fields is Gaussian, so that all the cumulants, higher than quadratic, are equal to zero. In this case we obtain

\[< \varphi^*(\bar{x}, \bar{t}) \Phi(\bar{x}, x, \tau) \varphi(x, t) > = \]
\[ = 2 \int_0^{\min(t,\bar{t})} dt' \int dy(Dz)_{xy}(D\bar{z})_{\bar{x}y} K_z(t, t') K_{\bar{z}}(\bar{t}, t') < F_z(t, t') F_{\bar{z}}(\bar{t}, t') W_{y\bar{x}xy} >, \]

\[ = 2 \int (Dz)_{x\bar{x}} K_z(t, t') < F_z(t, t') W_{\bar{x}x\bar{x}} >, \tag{12} \]

where \( \Phi(\bar{x}, x, \tau) \) is a parallel transporter between the points \( x \) and \( \bar{x} \), given at the moment \( \tau \) of Langevin time (in particular, one may consider the case, when \( x = \bar{x} \) and \( \Phi(\bar{x}, x, \tau) = 1 \)). Here

\[ W_{y\bar{x}xy} \equiv \exp \left\{ ie \left[ \int_y^y d\bar{z}_\mu A_\mu(\tilde{\xi}, \bar{\xi}) + \int \frac{x}{x} du_\mu A_\mu(u, \tau) + \int \frac{x}{y} dz_\mu A_\mu(z(\xi), \xi) \right] \right\}, \tag{13} \]

\[ W_{\bar{x}x\bar{x}} \equiv \exp \left\{ ie \left[ \int \frac{x}{x} du_\mu A_\mu(u, \tau) + \int \frac{x}{\bar{x}} dz_\mu A_\mu(z(\xi), \xi) \right] \right\}. \tag{14} \]

Formulae (13) and (14) clarify the construction of the contours \( C_1 \) and \( C_2 \) in bilocal generating functional. Applying to it cumulant expansion, varying over \( M \) or \( N \) in the case of equations (12), putting \( I_\mu = K_\mu = L_\mu \nu = M = N = 0 \) and rewriting \( W_{y\bar{x}xy} \) and \( W_{\bar{x}x\bar{x}} \) through Stokes theorem, one obtains in the bilocal approximation:

\[ < F_z(t, t') F_{\bar{z}}(\bar{t}, t') W_{y\bar{x}xy} > = \exp \left[ 2\lambda \int_{t'}^{t} d\xi < | \varphi(z(\xi), \xi) |^2 > + \right. \]

\[ + 2\lambda \int_{t'}^{t} d\bar{\xi} < | \varphi(\bar{z}(\bar{\xi}), \bar{\xi}) |^2 > + \]

\[ + \frac{1}{2} \int_{S_1} d\sigma_{\mu_1 \nu_1}(u_1) \int_{S_1} d\sigma_{\mu_2 \nu_2}(u_2) < F_{\mu_1 \nu_1}(u_1, t_1) F_{\mu_2 \nu_2}(u_2, t_2) > \right], \tag{15} \]

\[ < F_z(t, t') W_{\bar{x}x\bar{x}} > = \exp \left[ 2\lambda \int_{t'}^{t} d\xi < | \varphi(z(\xi), \xi) |^2 > + \right. \]

\[ + \frac{1}{2} \int_{S_2} d\sigma_{\mu_1 \nu_1}(u_1) \int_{S_2} d\sigma_{\mu_2 \nu_2}(u_2) < F_{\mu_1 \nu_1}(u_1, t_1) F_{\mu_2 \nu_2}(u_2, t_2) > \right], \tag{16} \]

where \( S_1 \) and \( S_2 \) are arbitrary surfaces, bounded by the contours \( C_1 \) and \( C_2 \) respectively, \( t_1 \) and \( t_2 \) are some moments of Langevin time.
The solution (7) of the Langevin equation for \( F_{\mu\nu}(x, t) \) yields three more equations for gauge-invariant correlators:

\[
< F_{\mu\nu}(x, t)F_{\lambda\rho}(\bar{x}, \bar{t}) >= \int_0^t dt' \int_0^t dt'' \int dyd\bar{y} U_{\mu\nu,\alpha}(y-x, t-t')U_{\lambda\rho,\beta}(\bar{y}-\bar{x}, \bar{t}-\bar{t}').
\]

\[
\cdot \left[ < j_\alpha(y, t')j_\beta(\bar{y}, \bar{t}') > + < j_\alpha(y, t')\eta_\beta(\bar{y}, \bar{t}') > + < j_\beta(\bar{y}, \bar{t}')\eta_\alpha(y, t') > +
\right.
\]

\[
+ 2\delta_{\alpha\beta}(y-\bar{y})\delta(t'-\bar{t}') +
\]

\[
+ \int dyd\bar{y}U_{\mu\nu,\alpha}(y-x, t)U_{\lambda\rho,\beta}(\bar{y}-\bar{x}, \bar{t})A_\alpha(y)A_\beta(\bar{y}),
\]

(17)

\[
< F_{\mu\nu}(x, t)j_\lambda(\bar{x}, \bar{t}) >= \int_0^t dt' \int dy U_{\mu\nu,\alpha}(y-x, t-t') \left[ < j_\alpha(y, t')j_\lambda(\bar{x}, \bar{t}) > +
\right.
\]

\[
+ < j_\lambda(\bar{x}, \bar{t})\eta_\alpha(y, t') >
\]

(18)

\[
< F_{\mu\nu}(x, t)\eta_\lambda(\bar{x}, \bar{t}) >= \int_0^t dt' \int dy U_{\mu\nu,\alpha}(y-x, t-t') \left[ < j_\alpha(y, t')\eta_\lambda(\bar{x}, \bar{t}) > +
\right.
\]

\[
+ 2\delta_{\alpha\lambda}\delta(y-\bar{x})\delta(t'-\bar{t})
\]

(19)

In order to obtain a complete system of equations, let us pass from the Hamilton gauge to the Schwinger gauge, \( A_\mu(x, t)(x-x_0)_\mu = 0 \) (where \( x_0 \) is an arbitrary point), in which it is easy to express explicitly \( A_\mu(x, t) \) through \( F_{\mu\nu}(x, t) \) and to obtain gauge-invariant equations. One can see, that a gauge function, corresponding to such gauge transformation, should depend on \( \theta \) and \( z \) only. After that we introduce the second generating functional

\[
\Phi_\beta(t) = \exp \left\{ ie \oint_C dx_\mu \left[ \int_{x_0}^x d\xi_\nu \alpha(z, \xi)x F_{\nu\mu}(z, t) + \beta \int_0^t dt' (j_\mu(x, t') + \eta_\mu(x, t')) \right] \right\},
\]

(20)

where \( \beta \) is a c-number, \( \alpha(z, x) \equiv \frac{(z-x_0)_\nu}{(x-x_0)_\nu}, C \) is some fixed closed contour.

Differentiating \( \Phi_\beta(t) \) by \( t \), applying to it cumulant expansion, differentiating once by \( \beta \) and putting then \( \beta \) equal to \((-1)\), we obtain, due to (5), the two last equations of bilocal approximation:

\[
\frac{1}{2} \int_{x_0}^y dz_\lambda \alpha(z, y) \int_{x_0}^u dx_\rho \alpha(x, u) \frac{\partial}{\partial t} < F_{\lambda\nu}(z, t)F_{\rho\mu}(x, t) > -
\]
\[- \int_{x_0}^{y} dz_\alpha(z, y) \int_0^t dt' \frac{\partial}{\partial t'} \left[ < F_{\lambda\nu}(z, t) j_\mu(u, t') > + < F_{\lambda\nu}(z, t) \eta_\mu(u, t') > \right] - \]

\[- \int_{x_0}^{u} dx_\rho \alpha(x, u) \int_0^t dt' \frac{\partial}{\partial t'} \left[ < F_{\rho\mu}(x, t) j_\nu(y, t') > + < F_{\rho\mu}(x, t) \eta_\nu(y, t') > \right] - \]

\[- \int_{x_0}^{y} dz_\alpha(z, y) \left[ < F_{\lambda\nu}(z, t) j_\mu(u, t) > + < F_{\lambda\nu}(z, t) \eta_\mu(u, t) > \right] - \]

\[- \int_{x_0}^{u} dx_\rho \alpha(x, u) \left[ < F_{\rho\mu}(x, t) j_\nu(y, t) > + < F_{\rho\mu}(x, t) \eta_\nu(y, t) > \right] + \]

\[+ \int_0^t dt' \left[ < j_\nu(y, t) j_\mu(u, t') > + < j_\nu(y, t') j_\mu(u, t) > + < j_\nu(y, t) \eta_\mu(u, t') > + \right. \]

\[+ < j_\nu(y, t') \eta_\mu(u, t) > + < j_\mu(u, t) \eta_\nu(y, t') > + < j_\mu(u, t') \eta_\nu(y, t) > \] \}

\[+ \delta_{\mu\nu} \delta(y - u) = \frac{\partial}{\partial u_\rho} \left\{ \int_{x_0}^{y} dz_\alpha(z, y) < F_{\rho\mu}(u, t) F_{\lambda\nu}(z, t) > - \right. \]

\[- \int_0^t dt' \left[ < F_{\rho\mu}(u, t) j_\nu(y, t') > + < F_{\rho\mu}(u, t) \eta_\nu(y, t') > \right] \right\}, \quad (21) \]

\[\int_{x_0}^{y} dz_\alpha(z, y) \frac{\partial}{\partial t} \left[ < F_{\lambda\nu}(z, t) j_\mu(u, t') > + < F_{\lambda\nu}(z, t) \eta_\mu(u, t') > \right] - \]

\[- < j_\nu(y, t) j_\mu(u, t') > - < j_\nu(y, t) \eta_\mu(u, t') > - \]

\[- < j_\mu(u, t') \eta_\nu(y, t) > + 2 \delta_{\mu\nu} \delta(y - u) \delta(t - t') - \]

\[- \frac{\epsilon^2}{2} \oint_C d\nu \oint_C d\omega \sigma \left\{ \int_{x_0}^{y} d\tilde{z}_\lambda \alpha(\tilde{z}, v) \int_{x_0}^{w} d\tilde{x}_\rho \alpha(\tilde{x}, w) \frac{\partial}{\partial t} < F_{\lambda\xi}(\tilde{z}, t) F_{\rho\sigma}(\tilde{x}, t) > - \right. \]
\[- \int_{x_0}^{v} d\tilde{z}_\lambda \alpha(\tilde{z}, v) \int_{0}^{t} \frac{d\tilde{t}}{dt} \partial_{\tilde{t}} \left[ < F_{\lambda\xi}(\tilde{z}, t) j_\sigma(w, \tilde{t}) > + < F_{\lambda\xi}(\tilde{z}, t) \eta_\sigma(w, \tilde{t}) > \right] - \]

\[- \int_{x_0}^{w} d\tilde{x}_\rho \alpha(\tilde{x}, w) \int_{0}^{t} \frac{d\tilde{t}}{dt} \partial_{\tilde{t}} \left[ < F_{\rho\sigma}(\tilde{x}, t) j_\xi(v, \tilde{t}) > + < F_{\rho\sigma}(\tilde{x}, t) \eta_\xi(v, \tilde{t}) > \right] - \]

\[- \int_{x_0}^{v} d\tilde{z}_\lambda \alpha(\tilde{z}, v) \left[ < F_{\lambda\xi}(\tilde{z}, t) j_\sigma(w, t) > + < F_{\lambda\xi}(\tilde{z}, t) \eta_\sigma(w, t) > \right] - \]

\[- \int_{x_0}^{w} d\tilde{x}_\rho \alpha(\tilde{x}, w) \left[ < F_{\rho\sigma}(\tilde{x}, t) j_\xi(v, t) > + < F_{\rho\sigma}(\tilde{x}, t) \eta_\xi(v, t) > \right] + \]

\[+ \int_{0}^{t} d\tilde{t} \left[ < j_\xi(v, \tilde{t}) j_\sigma(w, t) > + < j_\xi(v, t) j_\sigma(w, \tilde{t}) > + < j_\xi(v, \tilde{t}) \eta_\sigma(w, t) > + < j_\xi(v, t) \eta_\sigma(w, \tilde{t}) > + < j_\sigma(w, t) \eta_\xi(v, t) > + < j_\sigma(w, \tilde{t}) \eta_\xi(v, t) > \right] + 2\delta_\xi_\sigma \delta(v - w) \right\} \]

\[
\cdot \left\{ \int_{y_0}^{y} d\zeta \alpha(z, y) \left[ < F_{\zeta\nu}(z, t) j_\mu(u, t') > + < F_{\zeta\nu}(z, t) \eta_\mu(u, t') > \right] - \]

\[- \int_{0}^{t} dt'' \left[ < j_\nu(y, t') j_\mu(u, t'') > + < j_\nu(y, t') \eta_\mu(u, t'') > + < j_\mu(u, t'') \eta_\nu(y, t') > + 2\delta_\mu_\nu \delta(y - u) \delta(t'' - t') \right] \right\} = \]

\[
= \frac{\partial}{\partial u_\lambda} \left[ < F_{\lambda\mu}(u, t) j_\nu(y, t') > + < F_{\lambda\mu}(u, t) \eta_\nu(y, t') > \right], \quad (22) \]

where the terms with space-time derivatives are put to the right hand sides. In derivation of equations (21) and (22) we used the following formula [9]:
\[
< e^A B > = < e^A > \left( < B > + \sum_{n=1}^{\infty} \frac{1}{n!} \ll A^n B \gg \right),
\]
(23)

where \( A \) and \( B \) are two arbitrary operators.

The system of equations (11), (12), (15)-(19), (21), (22) is a complete system of equations of bilocal approximation for gauge-invariant correlators 
\[
< \varphi^*(\bar{x},\bar{t})\Phi(\bar{x},x,\tau)\varphi(x,t) >, \quad < \eta^*(\bar{x},t')\Phi(\bar{x},x,\tau)\varphi(x,t) >,
\]
\[
< \varphi^*(x,t)\Phi(x,\bar{x},\tau)\eta(x,t') >, \quad < F_{\mu\nu}(x,t)F_{\lambda\rho}(\bar{x},\bar{t}) >, \quad < F_{\mu\nu}(x,t)j_\lambda(\bar{x},\bar{t}) >,
\]
\[
< F_{\mu\nu}(x,t)\eta_\lambda(\bar{x},\bar{t}) >, \quad < j_\mu(x,t)j_\nu(\bar{x},\bar{t}) >, \quad < j_\mu(x,t)\eta_\nu(\bar{x},\bar{t}) > .
\]
To obtain equations of higher approximations, one should exploit generating functionals (10) and (20) in the same way, as it was done for bilocal approximation.

Hence, there are two alternative ways of investigation of higher correlators. E.g., for threelocal correlators, the first way is to derive, using the generating functionals (10) and (20), an exact system of equations, where threelocal and bilocal cumulants are considered on the same footing. The other way (based on the additional assumption, that bilocal cumulants are dominant) is to obtain bilocal cumulants from the equations (11), (12), (15)-(19), (21), (22) and then to substitute them into the system of equations for threelocal cumulants.

Note, that ANO strings enter equations of bilocal approximation only through an additional term in equation (17). Without writing explicitly twenty equations of threelocal approximation for correlators 
\[
< \varphi^*\Phi\varphi >, \quad < \eta^*\Phi\varphi >, \quad < \varphi^*\Phi\eta >, \quad < F_{\mu\nu}F_{\lambda\rho} >, \quad < F_{\mu\nu}\eta_\lambda >, \quad < F_{\mu\nu}j_\lambda >, \quad < j_\mu j_\nu >,
\]
\[
< j_\mu \eta_\nu >, \quad < \varphi^*\Phi\eta_\mu >, \quad < \eta^*\Phi\varphi_\mu >, \quad < \varphi^*\Phi\varphi_\mu >, \quad < \varphi^*\Phi\varphi_\mu >,
\]
\[
< \varphi^*\Phi\varphi j_\mu >, \quad < \varphi^*\Phi\eta j_\mu >, \quad < F_{\mu\nu}F_{\lambda\rho}j_\sigma >, \quad < F_{\mu\nu}F_{\lambda\rho}\eta_\sigma >, \quad < F_{\mu\nu}F_{\lambda\rho}F_{\sigma\xi} >,
\]
\[
< j_\mu j_\nu j_\lambda >, \quad < j_\mu j_\nu \eta_\lambda >, \quad < j_\mu \eta_\nu \eta_\lambda > , \quad \text{one can see, that in the correlators} \quad < F_{\mu\nu}F_{\lambda\rho}j_\sigma >, \quad < F_{\mu\nu}F_{\lambda\rho}\eta_\sigma > \quad \text{and} \quad < F_{\mu\nu}F_{\lambda\rho}F_{\sigma\xi} > \quad \text{there are groups of terms,}
\]
consisting of bilocal correlators, which have the coefficients, depending on the ANO strings. Therefore, ANO strings play nontrivial role in the process of formation of the hierarchy of correlators, connecting correlators of different orders.

3 QCD with Spinless Quarks at \( T = 0 \) and \( T > 0 \)

In this section we demonstrate, how one can apply the methods of section 2 to derivation of equations for correlators in QCD with spinless quarks, i.e.
the quarks, for which it is possible to omit the term $\frac{g}{2}\sigma_{\mu\nu}F_{\mu\nu}$ in the quadrized Dirac equation, where $\sigma_{\mu\nu} \equiv \frac{i}{2}[\gamma_\mu, \gamma_\nu]$. Then we generalize this approach to the case of finite temperatures.

The main problem in stochastic quantization of fermions is that the solution of naive Langevin equation, obtained directly from the fermionic action, in the physical limit, $t \rightarrow +\infty$, has an asymptotics $e^{-mt+ipx}$, i.e. badly defined in the chiral limit. To avoid this difficulty, the stochastic process is modified by introducing a kernel into the Langevin equation:

$$\delta S \delta \psi(x,t) \rightarrow \int dy K(x,y) \delta S \delta \psi(y,t)$$

where $K(x,y)$ should be chosen in such a way as to precisely cancel the negative eigenvalues of $\delta S \delta \psi(x,t)$. For example, in [14]

$$K(x,y) = \left(i \hat{D} + m\right) \delta(x-y),$$

and the asymptotics of the solution becomes $e^{-(p^2+m^2)t+ipx}$ as in the bosonic case. Our goal is to choose such a kernel, that ensures explicit gauge invariance of the right hand sides of equations for gauge-invariant vacuum correlators, containing quarks, antiquarks and Grassmann stochastic noises.

In order to get desirable asymptotics of the solutions at $t \rightarrow +\infty$ (not $(-\infty)$), i.e. to deal, as usual, with the retarded Green function of the Langevin equation, we shall use the Euclidean $\gamma$-matrices, multiplied by "i":

$$\vec{\gamma} = \beta \vec{\alpha}, \quad \gamma_4 = i\beta,$$

where $\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$, $\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, (24)

which satisfy anticommutational relations $\{\gamma_\mu, \gamma_\nu\} = -2\delta_{\mu\nu}$.

Let us introduce the kernel $K(x,y,t) = \left(i \hat{D}(x,t) + m\right)\delta(x-y)$ into the naive Langevin equation, obtained from the Euclidean QCD action with $\gamma$-matrices, defined in (24),

$$S = \int dx \left[ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - \bar{\psi}(i \hat{D} - m)\psi \right], \quad \text{where} \quad D_\mu = \partial_\mu - igA_\mu.$$ 

For spinless quarks one gets the following Langevin equations:

$$\dot{\psi} = (D^2 - m^2)\psi + \theta,$$

$$\dot{\bar{\psi}} = (D^2 - m^2)\bar{\psi} + \bar{\theta},$$

where $\bar{\psi} \equiv \psi^+ \gamma_4, \mathcal{D}_\mu \equiv \partial_\mu + igA_\mu$. The correlator of Grassmann noises changes correspondingly:

$$< \theta_\alpha(x,t) \bar{\theta}_\beta(y,t') >_{\theta\bar{\theta}} = 2[i \hat{D}(x,t)\delta(x-y) + m\delta(x-y)]_{\alpha\beta} \delta(t-t'),$$

(27)
where for an arbitrary functional of the $\theta$'s we have the stochastic expectation values:

$$< F[\theta, \bar{\theta}] >_{\theta \bar{\theta}} = \frac{\int D\theta D\bar{\theta} F[\theta, \bar{\theta}] \exp[-\frac{1}{2} \int dx dt \bar{\theta}(x, t)(i \hat{D} + m)^{-1} \theta(x, t)]}{\int D\theta D\bar{\theta} \exp[-\frac{1}{2} \int dx dt \bar{\theta}(x, t)(i \hat{D} + m)^{-1} \theta(x, t)]}.$$  \hspace{1cm} (28)

Solving (25) and (26) via Feynman-Schwinger path integral representation with the initial conditions $\psi(x, 0) = \bar{\psi}(x, 0) = 0$ and using (27) and (23), one obtains in bilocal approximation:

$$< \bar{\psi}_\beta(\bar{x}, \bar{t}) \Phi(\bar{x}, x, \tau) \psi_\alpha(x, t) > =$$

$$= 2 \int_0^{\min(t, \bar{t})} dt' \int d\bar{y}(D\bar{z})_{xy} K_z(\bar{t}, t') \left\{ -i \left[ \frac{\partial}{\partial y} (Dz)_{xy} K_z(t, t') \right]_{y=\bar{y}} + m(Dz)_{xy} K_z(t, t') \right\}_{y=\bar{y}} + W_{\bar{y}xx\bar{y}},$$  \hspace{1cm} (29)

where $< ... >$ means the averaging over the vacua of $\eta^a_\mu$ and $(\theta, \bar{\theta})$, $\eta^a_\mu$ is a Gaussian stochastic noise with the bilocal correlator $< \eta^a_\mu(x, t) \eta^b_\nu(y, t') >_{\eta} = 2 \delta_{\mu\nu} \delta^{ab} \delta(x - y) \delta(t - t')$. Here

$$\Phi(\bar{x}, x, \tau) = P \exp \left[ ig \int_{\bar{x}} \bar{\psi} \bar{A}_\mu(w, \tau) \right],$$

$$W_{\bar{y}xx\bar{y}} = P \exp \left\{ ig \left[ \int_{\bar{x}} \bar{\psi}(\bar{z}(\xi), \bar{\xi}) + \int_{\bar{x}} \bar{\psi} \bar{A}_\mu(w, \tau) + \int_{\bar{y}} \bar{\psi} \bar{A}_\mu(z(\xi), \xi) \right] \right\}.$$  \hspace{1cm} (30)

To perform differentiation in the equation (29), let us extract explicitly the dependence on the point $y$ from the measure of integration and from the kinematical factor $K_z(t, t')$. To this end we pass to the integration over trajectories

$$u(\xi) = z(\xi) + \frac{y - x}{t - t'}(\xi - t') - y.$$  \hspace{1cm} (31)

After that differentiation may be easily performed, and the result has the form
\[
< \bar{\psi}_\beta(\bar{x}, \bar{t}) \Phi(\bar{x}, x, \tau) \psi_\alpha(x, t) > = \\
= 2 \min(t, \bar{t}) \int_0^\infty dt' \int d\bar{y}(Dz)_{\bar{x}\bar{y}} K_\bar{z}(\bar{t}, t') \left[ i \frac{\hat{x} - \hat{y}}{2} e^{\frac{(x-y)^2}{4(t-t')}} \right] \int (Du)_{00} \\
\cdot K_u(t, t') < W_{\bar{y}xx\bar{y}} > + m \int (Dz)_{xy} K_z(t, t')_{y=\bar{y}} < W_{\bar{y}xx\bar{y}} > \right]_{\alpha\beta}. \tag{32}
\]

Here in the first Wilson loop in the right hand side it is implied, that in the last term in (30) \( z_\mu \) is expressed through \( u_\mu \) via (31).

In analogous way one may obtain equations for two other gauge-invariant correlators, containing spinor fields, in bilocal approximation:

\[
< \bar{\psi}_\beta(\bar{x}, \bar{t}) \Phi(\bar{x}, x, \tau) \theta_\alpha(x, t) > \quad \text{and} \quad < \bar{\theta}_\beta(\bar{x}, \bar{t}) \Phi(\bar{x}, x, \tau) \psi_\alpha(x, t) >. \tag{33}
\]

In contrast to the AHM, in the case of QCD it is, of course, impossible to calculate exact Green function of the Langevin equation for \( F^a_{\mu\nu}(x, t) \) (this problem is equivalent to the solution of Yang-Mills equation in the general form). Therefore, in order to obtain all the rest equations for gauge-invariant correlators in bilocal approximation, one should act in the same way as it was done in section 2 for derivation of equations (21) and (22). Using Schwinger gauge and introducing the generating functional

\[
\Phi_\beta = P.exp \left\{ ig \oint_C dx_\mu \left[ \int_{x_0}^x dz_\nu \alpha(z, x) F^\nu_{\mu}(z, t) + \beta \int_0^t dt' (gj_\mu(x, t') + \eta_\mu(x, t')) \right] \right\}, \tag{34}
\]

where \( j^a_\mu \equiv \bar{\psi} \gamma_\mu t^a \psi \) is a quark current, differentiating (34) by \( t \) and applying to it cumulant expansion, one obtains, due to the Langevin equation

\[
\dot{A}^a_\mu = (D_\lambda F^a_{\mu\lambda})^a + gj^a_\mu + \eta^a_\mu,
\]

where \( D_\lambda \) is the adjoint covariant derivative, the first equation. After that one should differentiate it five times by \( \beta \), use the formula (23) and in all the equations put \( \beta \) equal to \((-1)\). Finally, to obtain a minimal closed system of equations of bilocal approximation, one should avoid \( D_\lambda \) for \( \partial_\lambda \) in the right hand sides of these equations. This may be done, using the expressions for higher orders' path-ordered cumulants [3,10] and the formula
\[ tr < \Phi(x_0, u, t)(D_\lambda F_{\lambda \mu}(u, t'))\Phi(u, x_0, t)G_{\mu_1...\mu_n} > = \]
\[ = tr \left\{ \frac{\partial}{\partial u_\lambda} < F_{\lambda \mu}(u, x_0, t)G_{\mu_1...\mu_n} > + ig \int_{x_0}^{u} dx_\rho \alpha(x, u) \cdot \left[ < F_{\lambda \mu}(u, x_0, t)G_{\mu_1...\mu_n} F_{\lambda \rho}(x, x_0, t) > - < F_{\lambda \mu}(u, x_0, t)F_{\lambda \rho}(x, x_0, t)G_{\mu_1...\mu_n} > \right] \right\}, \]

where \( F_{\mu \nu}(x, x_0, t) \equiv \Phi(x_0, x, t)F_{\mu \nu}(x, t)\Phi(x, x_0, t) \), any parallel transporter between \( x_0 \) and any other point goes along straight line, \( G_{\mu_1...\mu_n} \equiv G_{\mu_1...\mu_n}(x_1, t_1, ..., x_n, t_n, x_0, t) \) is, generally speaking, a product of some number of \( F_{\mu \nu}, j_\mu \) and \( \eta_\mu \), which are given in the points \( x_1, ..., x_n \) at the moments \( t_1, ..., t_n \) of Langevin time respectively, and all the parallel transporters between \( x_0 \) and each of these points are given at the same moment \( t \). This method of derivation of the system of equations in the case of pure gluodynamics was used in [10].

After that, substituting \( < F_{\mu \nu}(x, x_0, t)F_{\lambda \rho}(y, x_0, t) > \), obtained from this system of six equations for correlators
\[ < F_{\mu \nu}(x, x_0, t)F_{\lambda \rho}(y, x_0, t) >, < F_{\mu \nu}(x, x_0, t)j_\lambda(y, x_0, t, t') >, \]
\[ < F_{\mu \nu}(x, x_0, t)\eta_\lambda(y, x_0, t, t') >, < j_\mu(x, x_0, t, t')j_\nu(y, x_0, t, t'') >, \]
\[ < j_\mu(x, x_0, t, t')\eta_\nu(y, x_0, t, t'') >, < \eta_\mu(x, x_0, t, t')\eta_\nu(y, x_0, t, t'') >, \] (35)

where
\[ j_\mu(x, x_0, t, t') \equiv \Phi(x_0, x, t)j_\mu(x, t')\Phi(x, x_0, t), \]
\[ \eta_\mu(x, x_0, t, t') \equiv \Phi(x_0, x, t)\eta_\mu(x, t')\Phi(x, x_0, t), \]

into the averaged Wilson loops, expanded up to the second cumulant via nonabelian Stokes theorem in (32) and in the right hand sides of equations for correlators (33), we obtain all the correlators of bilocal approximation.
In order to obtain equations for higher correlators, one should introduce
the analog of generating functional (10) and follow the algorithm, described
in section 2.

To conclude this section, we generalize the method of derivation of the sys-
tem of exact equations, suggested above, to the case of QCD at finite tem-
peratures. It was shown in [15], that the ordinary results of finite temper-
ature field theory may be reproduced in the framework of stochastic quantization,
if one requires the stochastic noise to be periodic for bosons (antiperiodic for
fermions) in the coordinate $x_4$ with the period $\beta \equiv \frac{1}{T}$. Therefore, we supply
equations (25) and (26) with the following conditions:

$$\theta(\bar{x}, x_4, t) = -\theta(\bar{x}, x_4 + \beta, t), \quad \bar{\theta}(\bar{x}, x_4, t) = -\bar{\theta}(\bar{x}, x_4 + \beta, t)$$  \hspace{1cm} (36)

and modify the meaning of the averaging over $(\theta, \bar{\theta})$ from (28) to

$$< F[\theta, \bar{\theta}] >_{\theta\bar{\theta}} = \frac{\int D\theta D\bar{\theta} F[\theta, \bar{\theta}] \exp \left[ -\frac{1}{2} \int \beta_0 dx_4 \int d\bar{x} dt \theta(x, t)(i\hat{D} + m)^{-1}\theta(x, t) \right]}{\int D\theta D\bar{\theta} \exp \left[ -\frac{1}{2} \int \beta_0 dx_4 \int d\bar{x} dt \bar{\theta}(x, t)(i\hat{D} + m)^{-1}\theta(x, t) \right]}.$$

Then it is easy to check, that in the absence of gluonic fields, the solution of
equation (25) with initial condition $\psi(x, 0) = 0$ has the form

$$\psi(x, t) = \int_0^\infty dt' \int dy_4 \int d\bar{y} G(x - y, t - t') \theta(y, t'),$$  \hspace{1cm} (37)

where

$$G(x, t) \equiv \frac{\theta(t)}{\beta} \sum_{n=-\infty}^{+\infty} \int \frac{d\tilde{k}}{(2\pi)^3} e^{i\tilde{k}_n x - (\tilde{k}_n^2 + m^2)t},$$

$$\tilde{k}_n \equiv (\tilde{k}, \omega_n^-), \quad \omega_n^- \equiv \frac{(2n + 1)\pi}{\beta}.$$

Solving equation (26) for a free antiquark with the initial condition $\bar{\psi}(x, 0) = 0$ in analogous way, one gets after some calculations

$$< \bar{\psi}_\beta(x', t') \psi_\alpha(x, t) >_{\theta\bar{\theta}} = \frac{1}{\beta} \sum_{n=-\infty}^{+\infty} \int \frac{d\tilde{k}}{(2\pi)^3} (-\tilde{k}_n + m)_{\alpha\beta} e^{i\tilde{k}_n (x-x')} \cdot \frac{e^{-(\tilde{k}_n^2 + m^2)|t-t'|} - e^{-(\tilde{k}_n^2 + m^2)(t+t')}}{\tilde{k}_n^2 + m^2},$$
that in the physical limit $t = t' \rightarrow +\infty$ yields standard result for the propagator of a free fermion at finite temperature [16].

In the case, when gluonic fields are present, one, solving equation (25) with the initial condition $\psi(x, 0) = 0$ through Feynman-Schwinger path integral representation, obtains instead of (37):

$$
\psi(x, t) = \int_0^t dt' \int dy (Dz)^w_{xy} K_z(t, t') \Phi(x, y) \theta(y, t),
$$

where

$$(Dz)^w_{xy} \equiv \lim_{N \rightarrow \infty} \prod_{l=1}^N \frac{d^4 \zeta(l)}{(4\pi \epsilon)^2} \sum_{n=-\infty}^{+\infty} \frac{d^4 p}{(2\pi)^4} e^{ip_\mu \left( \sum_{k=1}^N \zeta_\mu(k) - (x-y)_\mu - n\beta \delta_\mu 4 \right)},$$

$$
\zeta_\mu(k) \equiv z_\mu(\epsilon k) - z_\mu(\epsilon(k - 1)) \quad [17],
$$

$$
\Phi(x, y) \equiv P \exp \left[ i g \int_y^x dz A_\mu(z(\xi), \xi) \right].
$$

Therefore, one can check, that, for example, equation (32) changes as follows:

$$
< \bar{\psi}_\beta(x, t') \Phi(x, x, \tau) \psi_\alpha(x, t) > = 2 \int_0^{\min(t, \bar{t})} dt' \int dy (Dz)^w_{xy} K_z(t, t') \left[ i \frac{\hat{x} - \hat{\gamma}}{2 t' - t} \frac{(x-y)^2}{4(\epsilon^2 - 1)} \right].
$$

$$
\cdot \int_{u'(t') = u(t) = 0} (Du)^w K_u(t, t') < W_{\bar{y} x x y} > + m \int (Dz)^w_{xy} K_z(t, t')|_{y = \bar{y}} < W_{\bar{y} x x y} >\big|_{\alpha \beta},
$$

where

$$(Du)^w = \lim_{N \rightarrow \infty} \prod_{l=1}^N \frac{d^4 \lambda(l)}{(4\pi \epsilon)^2} \sum_{n=-\infty}^{+\infty} \frac{d^4 p}{(2\pi)^4} e^{ip_\mu \left( \sum_{k=1}^N \lambda_\mu(k) - n\beta \delta_\mu 4 \right)},$$

$$
\lambda(k) \equiv u(\epsilon k) - u(\epsilon(k - 1)).
$$

Solving equations for correlators (35), one should keep in mind, that the stochastic noise fields $\eta_\mu^a(x, t)$ are periodic in the coordinate $x_4$ with the period $\beta$. 

16
4 QED: Including Spin Effects

In this section we shall demonstrate, how one can exactly take into account spin effects in the framework of suggested approach. Let us consider for simplicity the case of QED and, instead of equation (25), solve exact Langevin equation

\[ \dot{\psi} = -(\hat{D}^2 + m^2)\psi + \theta, \]  

where

\[ \hat{D}^2 = -(D^2 + \frac{e}{2} \sigma_{\mu\nu} F_{\mu\nu}) , \]

via Feynman-Schwinger path integral representation. We shall follow the method, suggested in [11,12] for derivation of a propagator of a spinning particle in the external Abelian field. The retarded Green function of equation (40) has the form:

\[ G(x, y, t) = \theta(t) \int (Dz)_{xy} Dp \exp \left\{ i \int_0^t d\tau \left[ p \dot{z} + i(p_\mu - eA_\mu(z, \tau)) z^2 + \frac{e}{2} F_{\mu\nu}(z, \tau) \frac{\delta^2}{\delta p_\mu \delta p_\nu} + im^2 \right] \right\} T \exp \left[ \int_0^t \rho_\lambda(\tau') \gamma_\lambda d\tau' \right] |_{\rho=0} , \]

where \( \rho_\mu \) are Grassmann sources, anticommuting with \( \gamma \)-matrices. Using Wick theorem [18], we get:

\[ T \exp \left[ \int_0^t \rho_\lambda(\tau') \gamma_\lambda d\tau' \right] = \]

\[ = \exp \left[ -\frac{1}{2} \int_0^t d\tau_1 \int_0^t d\tau_2 \rho_\mu(\tau_1) \rho_\mu(\tau_2) \operatorname{sgn}(\tau_1 - \tau_2) \exp \left[ \int_0^t \rho_\lambda(\tau) \gamma_\lambda d\tau \right] \right] = \]

\[ = \int D\xi \exp \left\{ \int_0^t d\tau \left[ \frac{1}{4} \xi_\mu(\tau) \dot{\xi}_\mu(\tau) - i\rho_\mu(\tau) \xi_\mu(\tau) \right] \right\} \cdot \exp \left( i\gamma_\mu \frac{\partial}{\partial \theta_\mu} \right) \exp \left[ -i \int_0^t \rho_\mu(\tau) \theta_\mu d\tau \right] |_{\theta=0} , \]
where $\xi_\mu(\tau)$ are odd trajectories. To simplify this expression, one can pass to the integration over trajectories $\chi(\tau) = \frac{1}{2}(\xi(\tau) + \theta)$ and obtain:

$$G(x, y, t) = \frac{1}{2}\theta(t) \int (Dz)_{xy} \exp \left(i\gamma_\mu \frac{\partial}{\partial \theta_\mu} \right) \int_0^t d\tau \left[ -\frac{\dot{z}^2}{4} - m^2 + \right. $$

$$+ ieA_\mu \dot{z}_\mu - 2ieF_{\mu\nu}\chi_\mu \chi_\nu + \chi_\mu \dot{\chi}_\mu \left. \right] \chi_\mu(0) + \chi_\mu(t) \chi_\mu(0) \biggm|_{\theta_\mu=0}. $$

In order to avoid the restriction to the trajectories, over which we perform the integration, let us pass to the integration over trajectories $\omega(\tau)$, such that $\chi(\tau) = \frac{1}{2} \int_0^\tau \text{sgn}(\tau - \tau') \omega(\tau') d\tau' + \frac{1}{2} \theta$. Noticing, that for an arbitrary function, given at the Grassmann algebra, the following formula takes place:

$$\exp \left(i\gamma_\mu \frac{\partial}{\partial \theta_\mu} \right) f(\theta) \biggm|_{\theta=0} = \left( \frac{\partial}{\partial \zeta} \right) \exp \left(i\zeta_\gamma \gamma_\mu \right) \biggm|_{\zeta=0},$$

one obtains the following expression for $G(x, y, t)$:

$$G(x, y, t) = \frac{1}{2}\theta(t) \int (Dz)_{xy} \exp \left\{ \int_0^t d\tau \left[ -\frac{\dot{z}^2}{4} - m^2 + ieA_\mu(z, \tau) \dot{z}_\mu - \right. \right.$$

$$\left. - \frac{ie}{2} F_{\mu\nu}(z, \tau) \frac{\partial^2}{\partial \zeta_\mu \partial \zeta_\nu} \right] \right \} \int D\omega \exp \left\{ \int_0^t d\tau \left[ -\frac{ie}{2} \int_0^t d\tau' \right. \right.$$

$$\cdot \text{sgn}(\tau - \tau') \omega_\mu(\tau') F_{\mu\nu}(z, \tau) \int_0^t d\tau'' \text{sgn}(\tau - \tau'') \omega_\nu(\tau'') +$$

$$+ \frac{1}{2} \int_0^t d\tau' \text{sgn}(\tau - \tau') \omega_\mu(\tau') \omega_\mu(\tau) - \frac{ie}{2} \int_0^t d\tau' \text{sgn}(\tau - \tau') \omega_\mu(\tau') F_{\mu\nu}(z, \tau) \frac{\partial}{\partial \zeta_\nu} -$$

$$\left. - \frac{ie \frac{\partial}{\partial \zeta_\mu} F_{\mu\nu}(z, \tau) \int_0^t d\tau'' \text{sgn}(\tau - \tau'') \omega_\nu(\tau'') + \frac{1}{2} \frac{\partial}{\partial \zeta_\mu} \omega_\mu(\tau) \right] -$$

$$\left. - \frac{1}{4} \int_0^t d\tau' \int_0^t d\tau'' \omega_\mu(\tau') \text{sgn}(\tau' - \tau'') \omega_\mu(\tau'') \right\} \exp(i\zeta_\mu \gamma_\mu) \biggm|_{\zeta=0}. \quad (41)$$

$\int D\omega...$ may be represented in the form

$$\int D\omega \exp \left[ -\frac{1}{2} \omega_\mu T_{\mu\nu}(z, e) \omega_\nu + I_\mu \omega_\mu \right], \quad 18$$
where

\[ T_{\mu\nu}(z, e) = i e F_{\mu\nu}(z)e + \frac{3}{2} \delta_{\mu\nu}e, \quad I_{\mu} = i e F_{\mu\nu}(z)e \frac{\partial}{\partial \zeta_{\nu}} + \frac{1}{2} \frac{\partial}{\partial \zeta_{\mu}}, \]

and we used the following notation: for example,

\[ \omega \epsilon \omega \equiv \int_{0}^{t} d\tau' \int_{0}^{\tau'} d\tau'' \omega(\tau') \text{sgn}(\tau' - \tau'') \omega(\tau''). \]

Here and everywhere later, where such a notation is used, the Langevin time argument of \( F_{\mu\nu} \) is omitted, since it is integrated over. Then

\[
\int D\omega... = \left[ \frac{\text{det}T(z, e)}{\text{det}T(z, 0)} \right]^{1/2} \exp \left\{ -\frac{1}{2} I_{\mu}[T^{-1}(z, e)]_{\mu\nu} I_{\nu} \right\} =
\]

\[
= \exp \left\{ i \int_{0}^{e} d\tau' \text{tr}[Q_{\mu\nu}(z, e')F_{\mu\nu}(z)] \right\} \exp \left\{ -\frac{1}{2} I_{\mu}R_{\mu\nu}(z, e) I_{\nu} \right\},
\]

where

\[ Q_{\mu\nu}(z, e) \equiv \epsilon [T^{-1}(z, e)]_{\mu\nu}e, \quad R_{\mu\nu}(z, e) \equiv [T^{-1}(z, e)]_{\mu\nu}. \]

Differentiation in (41) with the help of the formula

\[
\exp \left( B_{\mu\nu} \frac{\partial^2}{\partial \zeta_{\mu} \partial \zeta_{\nu}} \right) \exp(i \zeta \gamma_{\lambda})|_{\zeta=0} = 1 - i \sigma_{\mu\nu}B_{\mu\nu} + i \gamma_{5} B_{\mu\nu}B_{\mu\nu}^*,
\]

where

\[ \gamma_{5} \equiv -i \gamma_{1} \gamma_{2} \gamma_{3} \gamma_{4}, \quad ^{*} \text{denotes the dual tensor}, \]

yields the final result:

\[
G(x, y, t) = \theta(t) \frac{\theta(t)}{2} \exp \left\{ i \gamma_{5} \gamma_{\lambda} \gamma_{\lambda} \right\} \exp \left\{ \frac{1}{2} \int_{0}^{t} \left( e(e - 2i) - \frac{1}{4} \right) d\tau \right\} \exp \left\{ \int_{0}^{t} d\tau \left[ -\frac{z^2}{4} - m^2 + \right. \right.
\]

\[
+ ie A_{\mu}(z, \tau) \dot{z}_{\mu} - \frac{e}{2} \sigma_{\mu\nu} F_{\mu\nu}(z, \tau) + \frac{e}{2} \gamma_{5} \int_{0}^{t} d\tau' F_{\mu\nu}(z, \tau) F_{\mu\nu}^*(z, \tau') \right] +
\]

\[
+ i \int_{0}^{e} d\tau' \text{tr}[Q_{\mu\nu}(z, e')F_{\mu\nu}(z)] + \Lambda(z, e) \right\},
\]

where
\[ \Lambda(z, e) = -\frac{i}{2} e^2 \left[ \sigma_{\lambda\rho} F_{\mu\lambda}(z) \epsilon R_{\mu\nu}(z, e) F_{\nu\rho}(z) \epsilon - \gamma_5 (F_{\mu\lambda}(z) \epsilon R_{\mu\nu}(z, e) F_{\nu\rho}(z) \epsilon) \right]. \]

\[ \cdot (F_{\alpha\lambda}(z) \epsilon R_{\alpha\beta}(z, e) F_{\beta\rho}(z) \epsilon)^* \right] - \frac{e}{4} \left[ \sigma_{\lambda\nu} F_{\mu\lambda}(z) \epsilon R_{\mu\nu}(z, e) + \sigma_{\mu\rho} R_{\mu\nu}(z, e) F_{\nu\rho}(z) \epsilon - \gamma_5 (F_{\mu\lambda}(z) \epsilon R_{\mu\nu}(z, e) (F_{\alpha\lambda}(z) \epsilon R_{\alpha\nu}(z, e))^* - \gamma_5 (R_{\mu\nu}(z, e) F_{\nu\rho}(z) \epsilon) \right]. \]

\[ \cdot (R_{\mu\alpha}(z, e) F_{\alpha\rho}(z) \epsilon)^* \right] + \frac{i}{8} \left[ \sigma_{\mu\nu} R_{\mu\nu}(z, e) - \gamma_5 R_{\mu\nu}(z, e) R_{\mu\nu}^*(z, e) \right]. \]

In the same way one may obtain the Green function of the Langevin equation

\[ \dot{\bar{\psi}} = -\bar{\psi} \left( \hat{D}^2 + m^2 \right) + \bar{\theta}, \]

where

\[ \hat{D}^2 = -D^2 + \frac{e}{2} \sigma_{\mu\nu} F_{\mu\nu}. \]

To generalize the equation (32) to the spinor case, one should again, for the group of terms in the functional integral, pass to the integration over trajectories (31). The result has the form:

\[ \langle \bar{\psi}_\beta(x, t_0) \Phi(x, x, \tau) \psi_\alpha(x, t) \rangle = 2 \min(t, \bar{t}) \int_0^{\min(t, \bar{t})} dt' \int d\bar{y} (D\bar{z})_{\bar{x}\bar{y}} K_{\bar{z}}(\bar{t}, t', -e) \cdot \left\{ \frac{i}{2(t' - \bar{t})} (\hat{x} - \hat{\bar{y}}) \gamma_5 e^{(z - \bar{y})^2} \int (Du)_{00} K_u(t, t', \bar{e}) \right\} \cdot \langle \bar{W}_{\bar{y}\bar{x}\bar{y}} \rangle \exp \left\{ \frac{e}{2} \int_{t'}^{t} d\xi \left[ -\sigma_{\mu\nu} F_{\mu\nu}(z, \xi) + \gamma_5 \int_{t'}^{t} d\xi' F_{\mu\nu}(z, \xi) F_{\mu\nu}^*(z, \xi') \right] \right\} + \int_{0}^{e} de' tr \left[ Q_{\mu\nu}(z, e') F_{\mu\nu}(z) \right] + \Lambda(z, e) \right\} \cdot \left\{ \exp \left\{ \frac{e}{2} \int_{t'}^{t} d\xi' \left[ \sigma_{\lambda\rho} \right] \right\} \right\} \right\}. \]
\[ F_{\lambda\rho}(\bar{z}, \bar{\xi}) - \gamma_5 \int_{t'}^t d\xi' F_{\lambda\rho}(\bar{z}, \bar{\xi}) F_{\lambda\rho}^*(\bar{z}, \bar{\xi}') + \]

\[ + i \int_0^{\xi''} d\tau [Q_{\lambda\rho}(\bar{z}, \bar{\xi})] + \Lambda(\bar{z}, -\bar{\xi}) \left\{ \right\}_{\delta\beta} > + \]

\[ + m \int (Dz)_{xy} K_z(t, t', e) |_{y=\bar{y}} < W_y \bar{x}x \left\{ e^{\frac{1}{2} \int d\xi} \left[ -\sigma_{\mu\nu} F_{\mu\nu}(z, \xi) + \right. \right. \]

\[ + \gamma_5 \int_{t'} d\xi' F_{\mu\nu}(z, \xi) F_{\mu\nu}^*(z, \xi') \left. \right] + i \int_0^{\xi''} d\tau [Q_{\mu\nu}(z, \xi') F_{\mu\nu}(z)] + \Lambda(z, e) \left\{ \right\}_{\alpha\gamma} \cdot \]

\[ \cdot \left\{ \exp \left\{ \frac{e}{2} \int_{\bar{t}}^{t} d\xi \left[ -\sigma_{\mu\nu} F_{\mu\nu}(z, \bar{\xi}) - \gamma_5 \int_{\bar{t}'}^{\bar{t}} d\xi' F_{\lambda\rho}(\bar{z}, \bar{\xi}) F_{\lambda\rho}^*(\bar{z}, \bar{\xi}') \right] + \right. \right. \]

\[ + i \int_0^{\xi''} d\tau [Q_{\lambda\rho}(\bar{z}, \bar{\xi})] + \Lambda(\bar{z}, -\bar{\xi}) \left\{ \right\}_{\gamma\beta} > \right\}, \quad (42) \]

where

\[ K_z(t, t', e) \equiv \frac{1}{2} \exp \left[ -\int_{t'}^{t} \frac{d^2}{4} - m^2 (t - t') + \frac{e^2}{2} - i e - \frac{1}{8} \right]. \]

Therefore, we reduced the right hand side of the equation for

\[ < \tilde{\psi}_\beta(\bar{x}, \bar{t}) \Phi(\bar{x}, x, \tau) \psi_\alpha(x, t) > \]

into the path integral over bosonic trajectories from the functional, whose spinor structure is given explicitly. The following steps of derivation of the system of equations for gauge-invariant correlators are similar to the previous two sections and based on the introduction of the corresponding generating functionals, analogous to (10) and (20).

### 5 Conclusion

In this paper, using stochastic quantization method [1], we developed a systematical procedure of derivation of the systems of equations for gauge-invariant vacuum correlators in different field theories. In section 2 on the
example of AHM we demonstrated, how this method may be applied to quantization of classical solutions (ANO strings); the quantization of more complicated objects will be treated elsewhere. The generating functionals (10) and (20), introduced in this section, appropriately modified, may be used to derivation of equations in any other gauge theory with the help of the algorithm, described in sections 2 and 3. This algorithm, was used in section 2, where the minimal set of equations of bilocal approximation was derived, and the role of ANO strings in the higher correlators was discussed. The suggested approach yields the gauge-invariant method of quantization of the theories with spontaneous symmetry breaking; this problem will be investigated in the future publications.

According to the general principa of the Method of Vacuum Correlators, the obtained system of equations is splitted into two parts: the part, containing pure gauge fields' correlators and matter fields' currents and the part, containing matter Green functions, so that the equations of the first subsystem may be solved independently from the second equations, but not vice versa. The connection of this fact with the large–N behaviour of QCD and a new type of the Master field equation, based on the suggested approach, will be treated elsewhere.

The method of derivation of equations in QCD with spinless quarks was generalized at the end of section 3 to the case of finite temperatures, and in section 4 we showed on the example of QED, how one can take into account spin effects exactly.

The equations, obtained in the case of QCD, contain both perturbative and nonperturbative gluonic contributions to the vacuum correlators. The problem of separation of perturbative gluonic contributions in all the terms of cumulant expansion and the regularization of the obtained equations will be the topics of separate publications.

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Appendix

Derivation of the Green function (8)

Below we present explicit derivation of a Green function of the Langevin equation

\[ \dot{F}_{\mu\nu}(x, t) + \partial_\nu \partial_\lambda F_{\lambda\mu}(x, t) - \partial_\mu \partial_\lambda F_{\lambda\nu}(x, t) = f_{\mu\nu}(x, t), \]  
(A.1)

where

\[ f_{\mu\nu}(x, t) = \partial_\mu (j_\nu(x, t) + \eta_\nu(x, t)) - \partial_\nu (j_\mu(x, t) + \eta_\mu(x, t)). \]

Writing equation (A.1) in the momentum representation:

\[ \dot{F}_{\mu\nu}(k, t) + k_\mu k_\lambda F_{\lambda\nu}(k, t) + k_\nu k_\lambda F_{\mu\lambda}(k, t) = f_{\mu\nu}(k, t), \]  
(A.2)

one can check, that the retarded Green function \( G_{\lambda\rho,\alpha\beta}(k, t) \) of equation (A.2) satisfies the following equation:

\[ \left[ 1_{\mu\nu,\lambda\rho} \frac{\partial}{\partial t} + k^2 (1 - P)_{\mu\nu,\lambda\rho} \right] G_{\lambda\rho,\alpha\beta}(k, t) = 1_{\mu\nu,\alpha\beta} \delta(t). \]  
(A.3)

Here we introduced the projectors

\[ 1_{\mu\nu,\lambda\rho} \equiv \frac{1}{2} \left( \delta_{\mu\lambda} \delta_{\nu\rho} - \delta_{\mu\rho} \delta_{\nu\lambda} \right) \quad \text{and} \quad P_{\mu\nu,\lambda\rho} \equiv \frac{1}{2} \left( T_{\mu\lambda} T_{\nu\rho} - T_{\mu\rho} T_{\nu\lambda} \right), \]

where

\[ T_{\mu\nu} \equiv \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}, \]

possessing the following properties:

\[ 1_{\mu\nu,\lambda\rho} = -1_{\nu\mu,\lambda\rho} = -1_{\mu\nu,\rho\lambda} = 1_{\lambda\rho,\mu\nu}, \quad 1_{\mu\nu,\lambda\rho} 1_{\lambda\rho,\alpha\beta} = 1_{\mu\nu,\alpha\beta} \]  
(A.4)

(the same properties hold true for \( P_{\mu\nu,\lambda\rho} \)),

\[ P_{\mu\nu,\lambda\rho} (1 - P)_{\lambda\rho,\alpha\beta} = 0. \]

Using (A.4) and (A.5), one can easily get \( G_{\mu\nu,\alpha\beta}(k, t) \) from equation (A.3):

\[ G_{\mu\nu,\alpha\beta}(k, t) = \theta(t) [(1 - P) e^{-k^2 t} + P]_{\mu\nu,\alpha\beta}. \]
Returning back to the coordinate representation, one obtains, that the solution of equation (A.1) with the initial condition $A(x, 0) = A(x)$ has the form (7), where

$$U_{\mu\nu,\beta}(y, t) = \frac{i}{8\pi^4} \int dk \ e^{iky} k_\alpha \left[ (1 - P)e^{-k^2t} + P \right]_{\mu\nu,\beta\alpha},$$

that after some calculations yields (8).
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