Flavorful Electroweak Precision Observables in the Standard Model Effective Field Theory

Sally Dawson\textsuperscript{a} and Pier Paolo Giardino\textsuperscript{b}

\textsuperscript{a}Department of Physics, Brookhaven National Laboratory, Upton, N.Y., 11973, U.S.A.
\textsuperscript{b}Instituto Galego de Física de Altas Enerxías, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Galicia, Spain

(Dated: January 26, 2022)

Abstract

Electroweak precision observables (EWPO) measured at the $W$ and $Z$ poles provide stringent limits on possible beyond the Standard Model physics scenarios. In an effective field theory (EFT) framework, the next-to-leading order QCD and electroweak results for EWPO yield indirect limits on possible 4-fermion operators that do not contribute to the observables at tree level. Here we calculate the next-to-leading corrections to EWPO induced by flavor non-universal 4-fermion interactions and find that the extracted limits on EFT coefficients have a strong dependence on the flavor structure of the 4-fermion operators.
I. INTRODUCTION

Historically, measurements of electroweak precision observables (EWPO) at the $W$ and $Z$ poles have provided strong bounds on possible beyond the Standard Model (BSM) physics assumed to occur at some high scale $\Lambda$. Similarly, in the Standard Model effective field theory (SMEFT) approach, comparisons of EWPO with theoretical predictions severely constrain the allowed values of the coefficients of the effective field theory operators\cite{1–6}. These bounds rely on precision calculations of the theoretical expectations both in the Standard Model (SM) and in the SMEFT. SMEFT calculations at next-to-leading order (NLO) QCD can be automated\cite{7}, while a growing number of SMEFT electroweak NLO results exist\cite{1, 8–20}. In a previous work\cite{1}, we computed the $\mathcal{O}(\frac{M_Z^2}{\Lambda^2})$ NLO QCD and electroweak corrections to the SMEFT predictions for EWPO at the $Z$ and $W$ poles. At NLO, a dependence on operators beyond those occurring at tree level in the analysis of EWPO arises and limits can be placed on the corresponding coefficients. Quark and lepton flavor effects were neglected in our earlier study and a $U(3)^5$ global flavor symmetry assumed for all operators involving fermions. Experimental anomalies in $B$ physics, however, suggest that including flavor dependencies in the SMEFT analyses could be relevant\cite{21, 22}. Furthermore, tree level global analyses of weak scale processes have a significant dependence on the flavor assumptions that are made in the fermion sector\cite{23–26}. In this note, we generalize our previous results for EWPO to include flavor effects from the 4-fermion operators involving at least 2 quarks that contribute at one-loop level. We present some interesting numerical consequences of allowing the 4-fermion operators to have an arbitrary flavor structure and compare our results with those from fits to top quark data at the LHC.

II. BASICS

The SMEFT parameterizes new physics through an expansion in higher dimensional operators\cite{27},

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_{k=5}^{\infty} \sum_{\alpha=1}^{n} \frac{C^k_\alpha}{\Lambda^{k-4}} O^k_\alpha,$$

(1)

where the $SU(3) \times SU(2)_L \times U(1)_Y$ invariant operators, $O^k_\alpha$, are constructed from SM fields and all of the effects of the BSM physics reside in the coefficient functions, $C^k_\alpha$. We use the Warsaw basis \cite{28, 29}, assume all coefficients are real, include only dimension-6
operators and do not consider CP violation. Observables are computed consistently to $\mathcal{O}(\frac{M_Z^2}{\Lambda^2})$, corresponding to only one operator insertion at the amplitude level, including all NLO QCD and electroweak contributions, as described in Ref. [1]. At lowest order (LO), only the operators $O_{ll}, O_{\phi WB}, O_{\phi D}, O_{\phi u}, O_{\phi d}, O_{\phi q}^{(3)}, O_{\phi q}^{(1)}, O_{\phi l}^{(3)},$ and $O_{\phi l}^{(1)}$ contribute. Ref. [1] also included the contributions of the operators

$$O_{ed}, O_{ee}, O_{eu}, O_{ld}, O_{te}, O_{lq}^{(1)}, O_{lq}^{(3)}, O_{\phi B}, O_{\phi W}, O_{\Box},$$

$$O_{qe}, O_{uB}, O_{uW}, O_{W}, O_{qd}^{(1)}, O_{qq}^{(3)}, O_{qq}^{(1)}, O_{qu}^{(1)}, O_{ud}^{(1)}, O_{uu}, O_{dd},$$

that first arise at NLO, where the coefficients of the 2-quark and 4-quark operators were assumed to be flavor independent. All renormalization group mixing of the operators was included, as required to obtain a finite NLO result. Contrary to Ref. [1], here we include the effects of arbitrary flavor interactions between quarks in the 4-fermion operators. In doing our computation, we set the SM CKM matrix to be the unit matrix and we assumed that the 2-fermion operators that appear at lowest order are diagonal, but not proportional to unity. For example\(^1\)

$$O_{\phi u,[1]} \neq O_{\phi u,[22]} \neq O_{\phi u,[33]} \quad O_{\phi u,[12]} = O_{\phi u,[13]} = O_{\phi u,[23]} = O_{\phi u,[13]} = 0.$$  

(3)

It is worth noting that setting the SM CKM matrix to 1 is enough to ensure that at LO only the diagonal terms of the 2-fermion operators contribute to all Z-pole observables. At NLO, the analysis becomes more complicated. However, since we are interested only in the contributions induced by the 4-fermion operators, it is straightforward to observe that our choice of taking the SM CKM matrix to be unity affects our results mostly through the renormalization group (RGE) mixing of operators. The only observable where this is not true is the W width where extra terms proportional to the off-diagonal terms of the SM CKM matrix are not included. For these reasons, at the level we are working, we expect the effects on our results for the 4-fermion interactions from our choice of taking the SM CKM matrix to be unity to be small.

\(^1\) After renormalization, we drop the generation indices on the 2-fermion operators. Therefore, the results presented here differ from those of Ref. [1] only for the 4-quark and 2-quark, 2-lepton operators listed in Table I.
TABLE I: Dimension-6 4-fermion operators contributing to $Z$ and $W$ pole observables at NLO QCD and electroweak order [1] where $[r,s,t,p] = 1,2,3$ are quark generation indices and $\sigma^A$ are the Pauli matrices.

Defining the fermion fields as,

\[
q_{L,r} = \begin{pmatrix} u_{L,r} \\ d_{L,r} \end{pmatrix}, \quad u_{R,r}, \quad d_{R,r}, \quad l_{L,r} = \begin{pmatrix} \nu_{L,r} \\ e_{L,r} \end{pmatrix}, \quad e_{R,r},
\]

where $r = 1,2,3$ is a generation index, we considered the operators in Table I, where we drop the lepton flavor indices.

III. RESULTS

The observables we consider are:

\[
M_W, \Gamma_W, \Gamma_Z, \sigma_h, R_t, A_{t,FB}, R_b, R_c, A_{FB,b}, A_{FB,c}, A_b, A_c, A_l.
\]
FIG. 1: Sample diagram containing 4-fermion operators contributing to $Z \rightarrow u\pi$ at NLO in the SMEFT. The fermions in the loop can be any quark or lepton (heavy or light). The red circle represents insertions of the operators of Table I.

The SM results for these observables are quite precisely known and we use the experimental and theoretical SM results shown in Table III of Ref. [1]. The NLO SMEFT results for the observables of Eq. 5 contain one-loop contributions from the dimension-6 operators of Table I and the full electroweak and QCD NLO amplitudes assuming that the flavor interactions are independent of fermion generation are in the supplemental material of Ref. [1]. Here we focus on the effects of the 4-fermion operators for on-shell 2-body $Z$ and $W$ decays such as those shown in Fig. 1 and allow for an arbitrary flavor dependence in the 4-fermion operators. When the internal fermions are top quarks, contributions that are enhanced by factors of $M_t^2/M_Z^2$ arise. Such contributions contribute to $Z \rightarrow b\bar{b}$ generically through the coefficients, $C_{\alpha,[3333]}$, and to $Z \rightarrow f_i\bar{f}_i$ (where $f_i$ is a light fermion) through the coefficients, $C_{\alpha,[33ii]}$, etc. We note that not all combinations of generation indices arise in the NLO calculation of the EWPO. For example, the operator $O_{qq}^{(1)}$ occurs with $i, i' = 1, 2$, (where $i \neq i'$)

$$
C_{qq,[3333]}^{(1)}, C_{qq,[33ii]}^{(1)} = C_{qq,[i33]}^{(1)}, C_{qq,[3i3]}^{(1)} = C_{qq,[33i]}^{(1)}, C_{qq,[iii'i']}^{(1)}, C_{qq,[i'vi'i]}^{(1)}, C_{qq,[ii'i'i']}, C_{qq,[iiii']}^{(1)}.
$$

In our calculation we never encounter operators with more than 2 different flavor indices, due to our choice of flavor structure.

The SMEFT predictions for the observables are

$$
O^{SMEFT,LO}_i = O^{SM,LO}_i + \delta O^{LO}_i(C_j),
O^{SMEFT,NLO}_i = O^{SM,NLO}_i + \delta O^{NLO}_i(C_j).
$$

We present numerical results for the observables of Eq. 5 in the supplemental material attached to this note. In the limit where the $C_{\alpha,[rstp]}$ are independent of the generation
indices, our previous result is recovered.

We perform a $\chi^2$ fit to the EWPO data. As an example, if only the 3\textsuperscript{rd} generation quarks contribute to the 4-fermion operators, the NLO contribution to the $\chi^2$ is\(^2\)

$$
\Delta\chi^2_{[3333]} = -0.45893C_{ud,[3333]}^{(1)} + 1.0712C_{qu,[3333]}^{(1)} - 0.16008C_{qq,[3333]}^{(3)} - 1.7914C_{qq,[3333]}^{(1)} + 0.37024C_{qd,[3333]}^{(1)} + 0.019645C_{dd,[3333]}^{(1)} + \vec{X}^T M_{[3333]} \vec{X},
$$

(8)

with \(\vec{X}^T = (C_{ud,[3333]}^{(1)}, C_{qu,[3333]}^{(1)}, C_{qq,[3333]}^{(3)}, C_{qq,[3333]}^{(1)}, C_{qd,[3333]}^{(1)}, C_{dd,[3333]}^{(1)})\) and

$$
M_{[3333]} = \begin{pmatrix}
0.039703 & -0.38683 & +0.057807 & +0.6469 & -0.060256 & -0.0033991 \\
1.1917 & -0.35618 & -3.9858 & +0.28411 & +0.016559 \\
0.026614 & +0.59564 & -0.042458 & -0.0024745 \\
3.3328 & -0.47512 & -0.027691 & \\
0.022951 & 0.0025794 & 7.2752 \times 10^{-5}
\end{pmatrix}.
$$

(9)

It is apparent that the largest sensitivity in this case is to \(C_{qq,[3333]}^{(1)}\) and to \(C_{qu,[3333]}^{(1)}\).

We show the 95\% confidence level single parameter limits on the 4-fermion operators of Table I with various flavor assumptions in Table II. All of our coefficients are evaluated at a scale \(M_Z\). Column 2 is the result of Ref. [1] where there is no flavor dependence in the 4-fermion coefficients,

Column 2 of Tab. II: \(C_{\alpha,[[3333]} = C_{\alpha,[3333]} = C_{\alpha,[3i3]} = C_{\alpha,[3i3]} = C_{\alpha,[3i3]}\)

\(= C_{\alpha,[i3i]} = C_{\alpha,[i3i]} = C_{\alpha,[i3i]}\).

(10)

It is of interest to consider the scenario where the 3\textsuperscript{rd} generation 4-fermion operators are different from those of generations \(i = 1, 2\). Column 3 assumes that the 4-fermion operators are only generated for the 3\textsuperscript{rd} generation quarks and can be obtained from Eqs. 8 and 9,

Column 3 of Tab. II: \(C_{\alpha,[3333]} \neq 0, C_{\alpha,[333]} = C_{\alpha,[i33]} = C_{\alpha,[3i3]} = C_{\alpha,[3i3]} = 0\)

\(C_{\alpha,[i3i]} = C_{\alpha,[i3i]} = C_{\alpha,[i3i]} = 0\).

(11)

\(^2\) \(\Delta\chi^2_{[3333]}\) must be added to the full SMEFT result including all other operators.
The 4th column assumes that 4–fermions operators where only 1st and 2nd generation quarks appear are zero and the remaining operators are equal,

\[
\text{Column 4 of Tab. II: } C_{\alpha,[3333]} = C_{\alpha,[33ii]} = C_{\alpha,[ii33]} = C_{\alpha,[i33i]} \neq 0
\]
\[
C_{\alpha,[ii'i'i']} = C_{\alpha,[i'i'i']} = C_{\alpha,[iii]} = 0. \tag{12}
\]

Finally, the 5th column assumes that there are no contributions to 4–fermion operators from 3rd generation quarks and that all the coefficients involving 1st and 2nd generation quarks for each operator are equal,

\[
\text{Column 5 of Tab. II: } C_{\alpha,[3333]} = C_{\alpha,[33ii]} = C_{\alpha,[ii33]} = C_{\alpha,[i33i]} = 0,
\]
\[
C_{\alpha,[ii'i'i']} = C_{\alpha,[i'i'i']} = C_{\alpha,[iii]} \neq 0. \tag{13}
\]

It is obvious that the results are extremely sensitive to the flavor assumptions and that there are large cancellations between contributions with massive internal top quarks and the massless fermions. We plot these results in Fig. 2. Figs. 3 and 4 show several 2-parameter fits with various assumptions about the flavor structure of the 4-fermion operators. Again, the numerical values of the fits are depend on the generation structure assumed for the 4-fermion operators and strong correlations are observed.

In Table III, we compare the limits from the single parameter EWPO NLO fits to those obtained from fits to LHC \(t\bar{t}\) data\([30–33]\), using the notation of Ref. [34]. Our NLO fits are
FIG. 3: Fit to EWPO with the only non-zero Wilson coefficients being $C^{(1)}_{lq}$ and $C^{(3)}_{lq}$. The quark generation index $i = 1, 2$.

FIG. 4: Fit to EWPO with the only non-zero Wilson coefficients $C^{(1)}_{qq}$ and $C^{(3)}_{qq}$. The quark generation index $i = 1, 2$ and $i \neq i'$.

only consistent to $\mathcal{O}(\frac{1}{\Lambda^2})$, and extending them to $\mathcal{O}(\frac{1}{\Lambda^4})$ would require double insertions of dimension-6 operators and the inclusion of dimension-8 operators. Comparing the $\mathcal{O}(\frac{1}{\Lambda^2})$ EWPO and the $t\bar{t}$ results, we see that for several operators the EWPO result is comparable or better than the top quark result. We note, however, that most of the power of the $t\bar{t}$ fit is coming at $\mathcal{O}(\frac{1}{\Lambda^4})$. It is amusing to note that the EWPO results for $C^{(1)}_{QQ}$ and $C^{(1)}_{Qt}$ are still relevant even when compared to the $t\bar{t}$ $\mathcal{O}(\frac{1}{\Lambda^4})$ results. The impact of the EWPO NLO data is illustrated graphically in Fig. 5 and suggests that including the NLO EWPO result with non-universal flavor effects in the global fits could have a significant effect.
FIG. 5: Comparison of single parameter limits from loop corrections to EWPO involving 3rd generation 4-fermion interactions with similar limits from LHC tt production[31].

In Fig. 6, we compare the current precision from the EWPO on the 3rd generation operators with that projected from a Tera-Z run at FCC-ee with an assumed integrated luminosity of 150 ab$^{-1}$ ($3 \times 10^{12}$ visible Z’s) and with a Giga-Z run at the ILC with an integrated luminosity of 100 fb$^{-1}$ ($10^9$ Z’s). This figure assumes that current theory uncertainties are halved, assumes the FCC-ee precision of Table II in Ref. [35] and the ILC Giga-Z numbers of Table 9 in Ref. [36]. Due to the polarization, for some observables the projected ILC precision surpasses that of the FCC-ee, despite the smaller assumed luminosity.

IV. CONCLUSIONS

We have included flavor non-universal effects from 4-fermion operators with at least 2 quarks into the NLO electroweak and QCD corrections to the SMEFT predictions for the precision electroweak observables. Our results are presented in a numerical form that can be incorporated in the global fitting programs and suggest that the flavor assumptions on the 4-fermion operators can have a significant effect. In particular we showed that the bounds obtained from EWPO on the $C_{QQ}^{(1)}$ and $C_{Qt}^{(1)}$ operators, that appear in the EWPO only at NLO, are competitive with respect to those obtained from current LHC tt observables. Numerical results are posted at https://quark.phy.bnl.gov/Digital_Data_
FIG. 6: Comparison of single parameter limits from loop corrections to EWPO involving 3rd generation 4− fermion interactions with projected Z pole limits from a Tera-Z program at the FCC-ee[35] and with a Giga-Z ILC run[36].

Archive/dawson/ewpo_22.

Acknowledgements

We thank Susanne Westhoff, Danny van Dyk, and Sebastian Bruggisse for pointing out that including non-trivial flavor dependencies in the SMEFT NLO predictions for the EWPO could be important for the global fits and also for valuable comments on the manuscript. S.D. is supported by the U.S. Department of Energy under Grant Contract de-sc0012704. The work of PPG has received financial support from Xunta de Galicia (Centro singular de investigación de Galicia accreditation 2019-2022), by European Union ERDF, and by “María de Maeztu” Units of Excellence program MDM-2016-0692 and the Spanish Research State Agency.

[1] S. Dawson and P. P. Giardino, “Electroweak and QCD corrections to Z and W pole observables in the standard model EFT,” Phys. Rev. D 101 no. 1, (2020) 013001, arXiv:1909.02000 [hep-ph].
[2] T. Corbett, O. J. P. Eboli, and M. C. Gonzalez-Garcia, “Unitarity Constraints on Dimension-six Operators II: Including Fermionic Operators,” *Phys. Rev.* D96 no. 3, (2017) 035006, arXiv:1705.09294 [hep-ph].

[3] L. Berthier and M. Trott, “Consistent constraints on the Standard Model Effective Field Theory,” *JHEP* 02 (2016) 069, arXiv:1508.05060 [hep-ph].

[4] T. Corbett, A. Helset, A. Martin, and M. Trott, “EWPD in the SMEFT to dimension eight,” *JHEP* 06 (2021) 076, arXiv:2102.02819 [hep-ph].

[5] J. de Blas *et al.*, “Higgs Boson Studies at Future Particle Colliders,” *JHEP* 01 (2020) 139, arXiv:1905.03764 [hep-ph].

[6] S. Dawson, S. Homiller, and S. D. Lane, “Putting standard model EFT fits to work,” *Phys. Rev. D* 102 no. 5, (2020) 055012, arXiv:2007.01296 [hep-ph].

[7] C. Degrande, G. Durieux, F. Maltoni, K. Mimasu, E. Vryonidou, and C. Zhang, “Automated one-loop computations in the standard model effective field theory,” *Phys. Rev. D* 103 no. 9, (2021) 096024, arXiv:2008.11743 [hep-ph].

[8] J. M. Cullen and B. D. Pecjak, “Higgs decay to fermion pairs at NLO in SMEFT,” *JHEP* 11 (2020) 079, arXiv:2007.15238 [hep-ph].

[9] J. M. Cullen, B. D. Pecjak, and D. J. Scott, “NLO corrections to $h \rightarrow b\bar{b}$ decay in SMEFT,” arXiv:1904.06358 [hep-ph].

[10] R. Gauld, B. D. Pecjak, and D. J. Scott, “QCD radiative corrections for $h \rightarrow b\bar{b}$ in the Standard Model Dimension-6 EFT,” *Phys. Rev. D* 94 no. 7, (2016) 074045, arXiv:1607.06354 [hep-ph].

[11] C. Hartmann and M. Trott, “Higgs Decay to Two Photons at One Loop in the Standard Model Effective Field Theory,” *Phys. Rev. Lett.* 115 no. 19, (2015) 191801, arXiv:1507.03568 [hep-ph].

[12] C. Hartmann and M. Trott, “On one-loop corrections in the standard model effective field theory; the $\Gamma(h \rightarrow \gamma\gamma)$ case,” *JHEP* 07 (2015) 151, arXiv:1505.02646 [hep-ph].

[13] S. Dawson and P. P. Giardino, “Electroweak corrections to Higgs boson decays to $\gamma\gamma$ and $W^+W^-$ in standard model EFT,” *Phys. Rev. D* 98 no. 9, (2018) 095005, arXiv:1807.11504 [hep-ph].

[14] A. Dedes, M. Paraskevas, J. Rosiek, K. Suxho, and L. Trifyllis, “The decay $h \rightarrow \gamma\gamma$ in the Standard-Model Effective Field Theory,” *JHEP* 08 (2018) 103, arXiv:1805.00302.
[15] S. Dawson and P. P. Giardino, “Higgs decays to ZZ and Z\gamma in the standard model effective field theory: An NLO analysis,” *Phys. Rev.* **D97** no. 9, (2018) 093003, arXiv:1801.01136 [hep-ph].

[16] A. Dedes, K. Suxho, and L. Trifyllis, “The decay h \rightarrow Z\gamma in the Standard-Model Effective Field Theory,” *JHEP* **06** (2019) 115, arXiv:1903.12046 [hep-ph].

[17] C. Hartmann, W. Shepherd, and M. Trott, “The Z decay width in the SMEFT: yt and \lambda corrections at one loop,” *JHEP* **03** (2017) 060, arXiv:1611.09879 [hep-ph].

[18] R. Boughezal, C.-Y. Chen, F. Petriello, and D. Wiegand, “Top quark decay at next-to-leading order in the Standard Model Effective Field Theory,” arXiv:1907.00997 [hep-ph].

[19] S. Dawson, P. P. Giardino, and A. Ismail, “Standard model EFT and the Drell-Yan process at high energy,” *Phys. Rev. D* **99** no. 3, (2019) 035044, arXiv:1811.12260 [hep-ph].

[20] S. Dawson and P. P. Giardino, “New physics through Drell-Yan standard model EFT measurements at NLO,” *Phys. Rev. D* **104** no. 7, (2021) 073004.

[21] S. Bruggisser, R. Schäfer, D. van Dyk, and S. Westhoff, “The Flavor of UV Physics,” *JHEP* **05** (2021) 257, arXiv:2101.07273 [hep-ph].

[22] L. Alasfar, A. Azatov, J. de Blas, A. Paul, and M. Valli, “B anomalies under the lens of electroweak precision,” *JHEP* **12** (2020) 016, arXiv:2007.04400 [hep-ph].

[23] E. d. S. Almeida, A. Alves, O. J. P. Êboli, and M. C. Gonzalez-Garcia, “Electroweak legacy of the LHC run II,” *Phys. Rev. D* **105** no. 1, (2022) 013006, arXiv:2108.04828 [hep-ph].

[24] S. Bißmann, C. Grunwald, G. Hiller, and K. Kröninger, “Top and Beauty synergies in SMEFT-fits at present and future colliders,” *JHEP* **06** (2021) 010, arXiv:2012.10456 [hep-ph].

[25] SMEFiT Collaboration, J. J. Ethier, G. Magni, F. Maltoni, L. Mantani, E. R. Nocera, J. Rojo, E. Slade, E. Vryonidou, and C. Zhang, “Combined SMEFT interpretation of Higgs, diboson, and top quark data from the LHC,” *JHEP* **11** (2021) 089, arXiv:2105.00006 [hep-ph].

[26] J. Ellis, M. Madigan, K. Mimasu, V. Sanz, and T. You, “Top, Higgs, Diboson and Electroweak Fit to the Standard Model Effective Field Theory,” *JHEP* **04** (2021) 279, arXiv:2012.02779 [hep-ph].
[27] I. Brivio and M. Trott, “The Standard Model as an Effective Field Theory,” *Phys. Rept.* **793** (2019) 1–98, arXiv:1706.08945 [hep-ph].

[28] W. Buchmüller and D. Wyler, “Effective Lagrangian Analysis of New Interactions and Flavor Conservation,” *Nucl. Phys. B* **268** (1986) 621–653.

[29] B. Grzadkowski, M. Iskrzynski, M. Misiak, and J. Rosiek, “Dimension-Six Terms in the Standard Model Lagrangian,” *JHEP* **10** (2010) 085, arXiv:1008.4884 [hep-ph].

[30] J. J. Ethier, G. Magni, F. Maltoni, L. Mantani, E. R. Nocera, J. Rojo, E. Slade, E. Vryonidou, and C. Zhang, “Combined smeft interpretation of higgs, diboson, and top quark data from the lhc,” 2021.

[31] C. Zhang, “Constraining $qqtt$ operators from four-top production: a case for enhanced EFT sensitivity,” *Chin. Phys. C* **42** no. 2, (2018) 023104, arXiv:1708.05928 [hep-ph].

[32] I. Brivio, S. Bruggisser, F. Maltoni, R. Moutafis, T. Plehn, E. Vryonidou, S. Westhoff, and C. Zhang, “O new physics, where art thou? A global search in the top sector,” *JHEP* **02** (2020) 131, arXiv:1910.03606 [hep-ph].

[33] A. Buckley, C. Englert, J. Ferrando, D. J. Miller, L. Moore, M. Russell, and C. D. White, “Global fit of top quark effective theory to data,” *Phys. Rev. D* **92** no. 9, (2015) 091501, arXiv:1506.08845 [hep-ph].

[34] D. Barducci *et al.*, “Interpreting top-quark LHC measurements in the standard-model effective field theory,” arXiv:1802.07237 [hep-ph].

[35] A. Blondel *et al.*, “Standard model theory for the FCC-ee Tera-Z stage,” in *Mini Workshop on Precision EW and QCD Calculations for the FCC Studies: Methods and Techniques*, vol. 3/2019 of *CERN Yellow Reports: Monographs*. CERN, Geneva, 9, 2018. arXiv:1809.01830 [hep-ph].

[36] K. Fujii, C. Grojean, M. E. Peskin, T. Barklow, Y. Gao, S. Kanemura, H. Kim, J. List, M. Nojiri, M. Perelstein, R. Poeschl, J. Reuter, F. Simon, T. Tanabe, J. D. Wells, J. Yu, J. Tian, T. Suehara, M. Vos, G. Wilson, J. Brau, and H. Murayama, “Tests of the standard model at the international linear collider,” 2019.
TABLE II: 95% CL single parameter limits in $TeV^{-2}$ from EWPO in the Warsaw basis. The second column corresponds to the NLO results in our previous paper with no flavor dependence in the quark and lepton sectors, the 3rd column has the only non-zero contributions from the 3rd generation fermions, the 4th column sets coefficients which only involve the 1st and 2nd generation quarks to 0, and the 5th column has equal contributions for each coefficient from operators involving the 1st and 2nd generations, with no contributions from the 3rd generation.
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Operator & EWPO & \(t\bar{t}(1/\Lambda^2)\) & \(t\bar{t}(1/\Lambda^4)\) \\
\hline
\(C^{(1)}_{QQ}/\Lambda^2\) & \(2C^{(3)}_{qq,[\bar{q}q]}/\Lambda^2\) - \(2C^{(3)}_{qq,[q\bar{q}]}/\Lambda^2\) & -1.61, 2.68 & -6.132, 23.281 & -2.229, 2.019 \\
\hline
\(C^{(8)}_{QQ}/\Lambda^2\) & \(8C^{(3)}_{qq,[\bar{q}q]}/\Lambda^2\) & -15.23, 25.41 & -26.471, 57.778 & -6.812, 5.834 \\
\hline
\(C^{(1)}_{Qt}/\Lambda^2\) & \(C^{(1)}_{uq,[\bar{q}u]}/\Lambda^2\) & -2.24, 1.35 & -195, 159 & -1.830, 1.862 \\
\hline
\(C^{(1,4)}_{Qq}/\Lambda^2\) & \(C^{(1)}_{qq,[\bar{q}q]}/\Lambda^2\) + \(3C^{(3)}_{qq,[\bar{q}q]}/\Lambda^2\) & -18.67, 20.19 & -0.273, 0.509 & -0.373, 0.309 \\
\hline
\(C^{(1,1)}_{Qq}/\Lambda^2\) & \(C^{(1)}_{qq,[\bar{q}q]}/\Lambda^2\) + \(1/6 C^{(3)}_{qq,[\bar{q}q]}/\Lambda^2\) & -3.47, 3.36 & -3.603, 0.307 & -0.303, 0.225 \\
\hline
\(C^{(3,8)}_{Qq}/\Lambda^2\) & \(C^{(1)}_{qq,[\bar{q}q]}/\Lambda^2\) - \(C^{(3)}_{qq,[\bar{q}q]}/\Lambda^2\) & -3.03, 3.04 & -1.813, 0.625 & -0.470, 0.439 \\
\hline
\(C^{(3,1)}_{Qq}/\Lambda^2\) & \(C^{(3)}_{qq,[\bar{q}q]}/\Lambda^2\) + \(1/6 C^{(3)}_{qq,[\bar{q}q]}/\Lambda^2\) & -0.32, 0.28 & -0.099, 0.155 & -0.088, 0.166 \\
\hline
\(C^{(1)}_{Qd}/\Lambda^2\) & \(C^{(1)}_{qq,[\bar{q}d]}/\Lambda^2\) & -5.74, 5.52 & -0.784, 2.771 & -0.205, 0.271 \\
\hline
\(C^{(1,1)}_{Qd}/\Lambda^2\) & \(C^{(1)}_{qq,[\bar{q}d]}/\Lambda^2\) + \(1/3 C^{(1)}_{qq,[\bar{q}d]}/\Lambda^2\) & -14.28, 12.33 & -0.774, 0.607 & -0.911, 0.347 \\
\hline
\(C^{(1)}_{Qd}/\Lambda^2\) & \(C^{(1)}_{qq,[\bar{q}d]}/\Lambda^2\) & -1.93, 1.67 & -6.046, 0.424 & -0.380, 0.293 \\
\hline
\(C^{(1)}_{Qd}/\Lambda^2\) & \(C^{(1)}_{qq,[\bar{q}d]}/\Lambda^2\) & -4.40, 4.67 & -0.938, 2.462 & -0.281, 0.371 \\
\hline
\(C^{(1)}_{Qd}/\Lambda^2\) & \(C^{(1)}_{uu,[\bar{q}d]}/\Lambda^2\) & -3.38, 6.50 & -9.504, -0.086 & -0.449, 0.371 \\
\hline
\(C^{(1)}_{Qd}/\Lambda^2\) & \(C^{(1)}_{uu,[\bar{q}d]}/\Lambda^2\) & -8.70, 4.59 & -0.889, 4.59 & -0.332, 0.436 \\
\hline
\end{tabular}
\caption{95\% CL single parameter limits in TeV$^{-2}$ from NLO contributions of 3$^{rd}$ generation 4-fermion operators to EWPO (1$^{st}$ column) to LHC $t\bar{t}$ results $t\bar{t}$ results using the $O(\frac{1}{\Lambda^2})$ and $O(\frac{1}{\Lambda^4})$ expansions[30]. The generation index $i = 1, 2$ and $j = 1, 2, 3$ and the scale $\Lambda = 1$ TeV.}
\end{table}