NUCLEAR PHOTOABSORPTION AT PHOTON ENERGIES BETWEEN 300 AND 850 MEV

Michihiro HIRATA*, Nobuhiko KATAGIRI, Kazuyuki OCHI
Department of Physics, Hiroshima University, Higashi-Hiroshima 739, Japan

and

Takashi TAKAKI†
Onomichi University, Onomichi 722-002, Japan

Abstract

We construct the formula for the photonuclear total absorption cross section using the projection method and the unitarity relation. Our treatment is very effective when interference effects in the absorption processes on a nucleon are strong. The disappearance of the peak around the position of the $D_{13}$ resonance in the nuclear photoabsorption can be explained with the cooperative effect of the interference in two-pion production processes, the Fermi motion, the collision broadenings of $\Delta$ and $N^*$, and the pion distortion in the nuclear medium. The change of the interference effect by the medium plays an important role.

* hirata@theo.phys.sci.hiroshima-u.ac.jp
† takaki@onomichi-u.ac.jp
I. INTRODUCTION

The total photonuclear absorption cross section has been measured over broad mass number and in the energy range 300–1200 MeV at Frascati. Especially, it was noted that the excitation peaks around the position of the $D_{13}(1520$ MeV) and $F_{15}(1680$ MeV) resonances disappear and above 600 MeV there is a strong reduction of the absolute value of the cross section per nucleon compared with the data for hydrogen and deuteron. These experimental findings have been confirmed by the contemporary data on the photofission cross section of $^{238}U$ and $^{235}U$ obtained at Mainz up to 800 MeV. There were a couple of theoretical attempts to explain the strong reduction of the cross section. It was necessary for them to assume large values for the collision widths of the $D_{13}$ and $F_{15}$ resonances to explain the above strong reduction. However, it was pointed out in Ref. that such significantly increasing resonance widths were hardly justified. These theoretical analyses must miss some important effects other than the collision broadening.

In the previous letter we pointed out that the disappearance of the peaks around the position of the resonances higher than the $\Delta$ resonance in the nuclear photoabsorption can be explained with the cooperative effect of the interference in two-pion photoproduction processes, the collision broadenings of $\Delta$ and $N^*(1520)$, and the pion distortion in the nuclear medium. Our finding means that the change of the interference effect by the medium plays an important role.

In this paper we present a detailed derivation of the formula for photonuclear total absorption cross sections based on the projection operator technique and apply our method to evaluate them in the extended energy region between 300 and 850 MeV. The experimental total absorption cross sections on a nucleon show that there is a fairly deep valley between the energy region of the $\Delta$ resonance and that of the $N^*$ resonance. On the other hand, the experimental total cross sections on nuclei show that the above valley is almost filled up. Furthermore, it is very interesting to note that the mass number dependence of total cross sections quite vary with the photon energy as follows: $A^{0.8}$ around 300 MeV, $A^{1.7}$ around 850 MeV.
500 MeV, and \( A^{0.65} \) around 750 MeV. We will discuss whether the mass number dependence of total cross sections can be explained by our theoretical treatment and speculate what one can learn from the comparison of our results with the data.

In section 2 we derive the formula for total nuclear photoabsorption cross sections and in section 3, numerical results and discussions are given. Section 4 contains our conclusions.

II. FORMALISM OF TOTAL NUCLEAR PHOTOABSORPTION CROSS SECTION

Total photoabsorption cross section is proportional to the imaginary part of the elastic Compton scattering T-matrix \( T_{\gamma\gamma} \):

\[
\sigma_T = -\frac{2\Omega}{\nu} \text{Im} T_{\gamma\gamma} \\
= i\frac{\Omega}{\nu} (T_{\gamma\gamma} - T_{\gamma\gamma}^+),
\]

which is obtained from the unitarity relation. Here \( \Omega \) is the normalization volume and \( \nu \) is the relative velocity between the photon and nucleus. We will use this relation to derive the formula of the total cross section. The expression for the T-matrix \( T_{\gamma\gamma} \) is constructed using the projection operator technique and focusing on the second resonance energy region where the \( N^*(1520) \) resonance plays an important role and the two-pion photoproduction prevails in addition to the one-pion photoproduction. The \( N^*(1520) \) resonance can decay into both the \( \pi N \) and \( \pi\pi N \) channels, which branching fractions are comparable, and its \( \pi\pi N \) decay occurs through two dominant modes, i.e., \( \pi\Delta \) and \( \rho N \). The two-pion photoproduction takes place mainly through the \( \pi\Delta \) and \( \rho N \) channels. So we include the \( N^*,\pi\Delta \) and \( \rho N \) intermediate states in our formalism explicitly. In order to simplify the formulation, we turn off the background interactions except the \( D_{13} \) channel in the one-pion production process, which will be later added in the formalism.

We use the following projection operators to separate the nuclear Hilbert space into subspaces,

\[
P_\gamma + p + q = 1.
\]
$P_\gamma$ projects onto the space of photon plus nuclear ground state, $p$ onto the spaces of both nuclear ground state and nuclear one particle-hole states except the $P_\gamma$-space and $q$ onto the space of nuclear many particle-hole states. We assume that $P_\gamma$ and $q$ does not couple directly, i.e., $H_{\gamma q} = P_\gamma H q = 0$, and neglect higher order terms of photo-coupling. Here $H$ is the total Hamiltonian of the system. Under such assumptions, the elastic compton scattering T-matrix becomes

$$T_{\gamma\gamma} = H_{\gamma p} \frac{1}{E - \mathcal{H}_{pp}} H_{p\gamma},$$

(2.4)

with

$$\mathcal{H} = H + H \frac{q}{E - H_{qq}} H,$$

(2.5)

where $\mathcal{H}_{pp} = p H p$ and $H_{qq} = q H q$, and $E$ is the total energy of the system. The effective Hamiltonian $\mathcal{H}$ is introduced so as to eliminate the $q$-space. The $p$-space is further divided into the following spaces:

$$p = P + D,$$

(2.6)

with

$$P = P_1 + P_2,$$

(2.7)

$$D = D_1 + D_2,$$

(2.8)

where $P_1$ projects onto the space of both one-pion plus nuclear ground state and one-pion plus nuclear one particle-hole states, $P_2$ onto the space of two-pion plus nuclear particle-hole states, $D_1$ onto the space of one $N^*$ plus nuclear one-hole states and $D_2$ onto the spaces of both $\pi\Delta$ plus nuclear one-hole states and $\rho$ plus nuclear particle-hole states. Since $H_{\gamma q} = 0$, then $\mathcal{H}_{\gamma P_1} = H_{\gamma P_1}$, $\mathcal{H}_{\gamma D_1} = H_{\gamma D_1}$ and $\mathcal{H}_{\gamma D_2} = H_{\gamma D_2}$. Unlike the $\Delta$-hole model in the pion-nucleus scattering, the $D_1$-space is not a doorway between the subspaces $P_\gamma$ and $P_1 + D_2$, since the direct couplings described by $H_{\gamma P_1}$ and $H_{\gamma D_2}$ are non-negligible. $H_{\gamma P_1}$ corresponds to the background term in the $D_{13}$ channel and $H_{\gamma D_2}$ corresponds to the $\Delta$ and $\rho$ Kroll-Ruderman terms. This fact reflects the structure of the elastic compton scattering T-matrix which will be shown later. To simplify the evaluation of the T-matrix of Eq.(2.4),
we must make some approximations for the reaction mechanism of the two-pion production:
(a) \( P_γ \) and \( P_2 \) does not couple directly, i.e., \( H_{γP_2} = 0 \), so that \( H_{γP_2} = 0 \). (b) The transition between the space of \( P_1 + D_1 \) and the space of \( P_2 + D_2 \) proceeds only from the direct coupling of \( D_1 \) and \( D_2 \), so that \( H_{P_1P_2} = H_{P_2D_1} = 0 \), and \( H_{D_1D_2} = H_{D_1D_2} \). In this way, we assume that the \( D_2 \)-space plays the role of the doorway to two-pion states.

Inserting the projection operators of Eqs. (2.6), (2.7) and (2.8) into Eq. (2.4) and using the above-mentioned approximations, we obtain the elastic compton scattering T-matrix given by

\[
T_{γγ} = T_{P_1}^{γγ} + T_{D_1}^{γγ} + T_{D_2}^{γγ},
\]

with

\[
T_{P_1}^{γγ} = H_{γP_1} G_{P_1} H_{P_1γ},
\]

\[
T_{D_1}^{γγ} = \tilde{F}_{D_1} G_{D_1} F_{D_1γ}^+, \]

\[
T_{D_2}^{γγ} = H_{γD_2} G_{D_2} H_{D_2γ},
\]

The Green’s functions in Eqs. (2.10), (2.11) and (2.12) are defined as

\[
G_{P_1} = (E - H_{P_1P_1})^{-1}, \]

\[
G_{D_2} = (E - H_{D_2D_2} - H_{D_2P_2} G_{P_2}^0 H_{P_2D_2} - \Sigma_{D_2})^{-1}, \]

\[
G_{D_1} = (E - H_{D_1D_1} - H_{D_1P_1} G_{P_1}^0 H_{P_1D_1} - \Sigma_{D_1} - H_{D_1D_2} G_{D_2} H_{D_2D_1})^{-1}, \]

where

\[
Σ_{D_1} = H_{D_1q} \frac{1}{E - H_{qq}} H_{qD_1} + H_{D_1P_1} G_{P_1} H_{P_1D_1} - H_{D_1P_1} G_{P_1}^0 H_{P_1D_1}, \]

\[
Σ_{D_2} = H_{D_2q} \frac{1}{E - H_{qq}} H_{qD_2} + H_{D_2P_2} G_{P_2} H_{P_2D_2} - H_{D_2P_2} G_{P_2}^0 H_{P_2D_2}, \]

and

\[
G_{P_1}^0 = (E - H_{P_1P_1})^{-1}, \]

\[
G_{P_2}^0 = (E - H_{P_2P_2})^{-1}. \]
The vertex functions in Eq. (2.11) are defined as

\[ F_{D_1 \gamma} = H_{D_1 \gamma} + \mathcal{H}_{D_1 P_1} G_{P_1} H_{P_1 \gamma} + H_{D_1 D_2} G_{D_2} H_{D_2 \gamma}, \quad (2.20) \]

\[ \tilde{F}_{\gamma D_1} = H_{\gamma D_1} + H_{\gamma P_1} G_{P_1} H_{P_1 \gamma} + H_{\gamma D_2} G_{D_2} H_{D_2 D_1}. \quad (2.21) \]

In our formalism, the background coupling of \( P_\gamma \) and \( P_1 \) described by \( H_{\gamma P_1} \) is included because the vertex correction of the second term in eqs. (2.20) and (2.21) is known to be non-negligible in the elementary process. Since the strength of \( H_{\gamma P_1} \) itself, however, is small, we neglect the process of \( P_\gamma \to P_1 \to q \). Thus Eq. (2.10) is approximately written as

\[ T_{P_\gamma} \approx H_{\gamma P_1} G^0_{P_1} H_{P_\gamma}, \quad (2.22) \]

and Eqs. (2.20) and (2.21) become

\[ F^+_{D_1 \gamma} \approx H_{D_1 \gamma} + H_{D_1 P_1} G^0_{P_1} H_{P_1 \gamma} + H_{D_1 D_2} G_{D_2} H_{D_2 \gamma}, \quad (2.23) \]

\[ \tilde{F}^+_{\gamma D_1} \approx H_{\gamma D_1} + H_{\gamma P_1} G^0_{P_1} H_{P_1 \gamma} + H_{\gamma D_2} G_{D_2} H_{D_2 D_1}. \quad (2.24) \]

Hereafter, we use these approximate forms.

From Eq. (2.9), we find the imaginary part of the T-matrix:

\[ T_{\gamma \gamma} - T^+_{\gamma \gamma} = T^+_{P_1 \gamma} \Delta G^0_{P_1} T_{P_1 \gamma} + T^+_{P_2 \gamma} \Delta G^0_{P_2} T_{P_2 \gamma} \]

\[ + \Omega^+_{D_1 \gamma} (\Sigma_{D_1} - \Sigma^+_{D_1}) \Omega_{D_1 \gamma} + \Omega^+_{D_2 \gamma} (\Sigma_{D_2} - \Sigma^+_{D_2}) \Omega_{D_2 \gamma}, \quad (2.25) \]

with

\[ \Omega_{D_1 \gamma} = G_{D_1} F^+_{D_1 \gamma} \quad (2.26) \]

\[ \Omega_{D_2 \gamma} = G_{D_2} (H_{D_2 \gamma} + H_{D_2 D_1} G_{D_1} F^+_{D_1 \gamma}), \quad (2.27) \]

and

\[ T_{P_1 \gamma} = H_{P_1 \gamma} + H_{P_1 D_1} \Omega_{D_1 \gamma}, \quad (2.28) \]

\[ T_{P_2 \gamma} = H_{P_2 D_2} \Omega_{D_2 \gamma}, \quad (2.29) \]
where $\Delta G \equiv G - G^+$. T-matrices of Eqs. (2.28) and (2.29) describe the one-pion photoproduction and two-pion photoproduction, respectively. The final state interaction, however, are not included in these expressions. Thus the cross sections calculated by Eqs. (2.28) and (2.29) do not exactly correspond to experimental cross sections. In our formalism, the effect of the final state interaction is contained in the third and fourth terms of Eq. (2.25) and therefore there is an ambiguity in partitioning of the nuclear inelastic cross section. However, as far as total cross section is concerned, one can use the equation of (2.25) to estimate it.

In the Green’s function $G_{D_2}$, the operator $H_{D_2P_2}G^0_{P_2}H_{P_2D_2}$ is the free self-energies of the $\Delta$ and $\rho$ meson corrected by the Pauli-blocking effect. In order to clarify the physical content of operators in the Green’s function $G_{D_1}$, we rewrite it as

$$G_{D_1} = (E - H_{D_1D_1} - H_{D_1P_1}G^0_{P_1}H_{P_1D_1} - H_{D_1D_2}G^0_{D_2}H_{D_2D_1} - \Sigma_{D_1} - \Sigma'_{D_1})^{-1}, \quad (2.30)$$

where

$$\Sigma'_{D_1} = H_{D_1D_1}(G_{D_2} - G^0_{D_2})H_{D_2D_1}, \quad (2.31)$$

and

$$G^0_{D_2} = (E - H_{D_2D_2} - H_{D_2P_2}G^0_{P_2}H_{P_2D_2})^{-1}. \quad (2.32)$$

Here the operator $H_{D_1P_1}G^0_{P_1}H_{P_1D_1}$ consists of both the free $N^*$ self-energy due to the one-pion channel corrected by the Pauli-blocking effect and the pion rescattering term arising from the coherent $\pi^0$ production. The operator $H_{D_1D_2}G^0_{D_2}H_{D_2D_1}$ corresponds to the free $N^*$ self-energy due to the two-pion channel corrected by the Pauli-blocking effect. The self-energies $\Sigma_{D_1} + \Sigma'_{D_1}$ and $\Sigma_{D_2}$ in Eqs. (2.14) and (2.30) are complicated many-body operators arising from the $q$-space coupling as well as the $P_1$- and $P_2$-spaces coupling. The latter coupling is related to the final state interaction in the one-pion or two-pion production. In practical calculations, these operators are assumed to be simple one-body operators

$$\Sigma_{D_1} + \Sigma'_{D_1} \cong W^{(1)}_{sp}, \quad (2.33)$$

$$\Sigma_{D_2} \cong W^{(2)}_{sp}, \quad (2.34)$$
which are phenomenologically determined.

Using the expressions of Eq. (2.25) with Eqs. (2.33) and (2.34), the total cross section can be written as

\[
\sigma_T = \frac{\Omega}{\nu} \left[ \sum 2\pi \delta(E - H_{P_1P_1})|TP_1\gamma|^2 + \sum 2\pi \delta(E - H_{P_2P_2})|TP_2\gamma|^2 + \Omega^+ \delta(-2ImW_{sp}^{(1)})\Omega_{D_1\gamma} + \Omega^+ \delta(-2ImW_{sp}^{(2)})\Omega_{D_2\gamma} - \Delta_1 - \Delta_2 \right],
\]

(2.35)

where

\[
\Delta_1 = \sum 2\pi \delta(E - H_{P_2D_2})|H_{P_2D_2}G_{D_2H_{D_2D_1}}\Omega_{D_1\gamma}|^2 - \sum 2\pi \delta(E - H_{P_2P_2})|H_{P_2D_2}G^0_{D_2H_{D_2D_1}}\Omega_{D_1\gamma}|^2,
\]

(2.36)

\[
\Delta_2 = \Omega^+ H_{D_1D_2}G^+_{D_2}(-2ImW_{sp}^{(2)})G_{D_2H_{D_2D_1}}\Omega_{D_1\gamma}.
\]

(2.37)

Here the terms \(\Delta_1\) and \(\Delta_2\) appear due to the introduction of the one-body operator \(W_{sp}^{(1)}\) and are subtracted in order to avoid the double counting of the processes included in \(W_{sp}^{(1)}\). Eq. (2.35) is our starting point to calculate the total cross section. This equation shows that the absorption processes consist of three components, i.e., the quasi-free processes such as the one-pion photoproduction and two-pion photoproduction, and genuine many-body absorption processes arising from the interaction between the resonance (or pion) and the nucleon in a nucleus\(^23\). In actual calculation, the \(\Delta\) excitation and the remaining background processes except the \(D_{13}\) channel in the one-pion photoproduction must be added in the above formula.

Now we turn to discuss how to evaluate the total cross section practically. For the one-pion photoproduction, the \(T\)-matrix is given in terms of the \(\Delta\) and \(N^*\) resonant amplitudes described by the isobar model and the remaining background multipole amplitudes. We employ the model by Ochi et al.\(^{16}\) for the two-pion photoproduction. For simplicity we use the Fermi gas model for a nucleus.
The cross section of one-pion photoproduction off a proton in the nuclear matter is given in the laboratory frame by

\[
\sigma_{\pi} = \frac{1}{v} \frac{3Z}{8\pi(k_f^2)^3} \int_0^{k_f^2} d\vec{p}_1 \int \frac{d\vec{q}}{(2\pi)^3} \frac{d\vec{p}}{(2\pi)^3} (2\pi)^4 \delta^4(k + p_1 - q - p) \\
\times \frac{1}{2} \sum_{\lambda,\nu,\nu'} \sum_{t_N} | < \vec{q}_p \vec{p}_1 t_N | T_{\pi \gamma} | \vec{k} \lambda \vec{p}_1 \nu > |^2 \theta(|\vec{p}| - k_{tN}^f) \frac{M^2}{2k2\omega qE_{\vec{p}_1}E_\vec{p}},
\]

where \( T_{N^*} \) and \( T_\Delta \) represent the \( N^* \) and \( \Delta \) resonance terms, respectively and \( T_B \) is the background term. \( \vec{k}, \vec{p}_1, \vec{q} \) and \( \vec{p} \) are the momenta of the incident photon, target proton, outgoing pion and outgoing nucleon, respectively. \( E_{\vec{p}_1}, \omega_{\vec{q}}, \) and \( E_{\vec{p}} \) are the energies of the target proton, outgoing pion, and outgoing nucleon, respectively. \( Z \) and \( v \) denote the proton number and the relative velocity between the photon and nucleus, respectively. \( k_{tN}^f \) is the Fermi momentum depending on the isospin quantum number \( t_N \). The notation for all other spin-isospin quantum numbers are self-explanatory. The \( N^* \) resonance term is expressed as

\[
T_{N^*} = F_{\pi NN} G_{N^*}(s) \tilde{F}_{P N^*}^+,
\]

\[
G_{N^*}(s) = [\sqrt{s} - (M_{N^*}(s) + \delta M_{N^*}) + i(\Gamma_{N^*}(s) + \Gamma_{N^* sp})/2]^{-1},
\]

where \( \sqrt{s} \) is the total center of mass energy. \( M_{N^*} \) and \( \Gamma_{N^*} \) in the \( N^* \) propagator \( G_{N^*} \) are the mass and the free width of \( N^* \), respectively, which are given so as to describe the energy dependence of the \( \pi N \) \( D_{13} \)-wave scattering amplitude and the branching ratios at the resonance energy. As the medium corrections, we introduce the mass shift \( \delta M_{N^*} \) and spreading width \( \Gamma_{N^* sp} \) due to the collisions between \( N^* \) and other nucleons. \( \tilde{F}_{P N^*}^+ \) and \( F_{\pi NN}^+ \) are the \( \gamma P N^* \) and \( \pi NN^* \) vertex functions, respectively, of which detailed forms are given in Ref. [15], and the former operator corresponds to \( F_{P \gamma}^+ \) in Eq.(2.23) which includes the vertex correction. The \( \Delta \) resonance term \( T_\Delta \) is written in a similar form with \( T_{N^*} \) but is important only at the low energy range less than 500 MeV. The effective \( \gamma P \Delta \) coupling constant including the vertex correction is obtained by the same way used in the fixing of
FIG. 1: Diagrams for the two-pion photoproduction on a nucleon and genuine many-body absorption processes on a nucleus. (a) The $N^* \to \pi \Delta$ contribution. (b) The $N^* \to \rho N$ contribution. (c) The $\Delta$ Kroll-Ruderman term. (d) The $\Delta$ pion-pole term. (e) The $\rho$ Kroll-Ruderman term. (f) The many-body absorption process through the $N^*$. (g) The many-body absorption process through the $\Delta$. (h) The many-body absorption process through the $\pi \Delta$. $A$ is the mass number of the target nucleus.

the $\gamma PN^*$ coupling constant. Here the Born term with the cutoff form factor employed in Ref. [22] is assumed as the background multipole amplitude. The Pauli blocking effect for the $\Delta$ decay into $\pi N$ becomes non-negligible at low energies, since the probability of the nucleon being emitted with a small momentum increases compared with the energy region of the $N^*$ resonance [23, 24]. Thus, we include the Fock term in the $\Delta$ propagator to modify the free $\Delta$ self-energy in addition to the collision width. The background amplitude $T_B$ is evaluated by using the experimental multipole amplitudes [14]. The integration over final particle momenta in Eq.(2.38) is performed by using variables defined in the $\gamma N$ center of mass system.

The cross sections of two-pion photoproduction corresponding to Figs. [1](a),(b),(c),(d) and (e) are calculated using the model of Ochi et al. [16]. The cross section of the two-pion
photoproduction off a proton is given by

\[
\sigma^{2\pi}_p = \frac{1}{v^2 8\pi (k_f^p)^3} \int_0^{k_f^p} d\vec{p}_1 \int \frac{d\vec{q}_1}{(2\pi)^3} \frac{d\vec{q}_2}{(2\pi)^3} \frac{d\vec{p}}{(2\pi)^3} (2\pi)^4 \delta^4(k + p_1 - q_1 - q_2 - p)
\]

\[
\times \frac{1}{2} \sum_{\lambda,\nu,\nu'} \sum_{t_{\pi_1}t_{\pi_2}t_{N}} T_{P_2\gamma}\left|\vec{k}\lambda\vec{p}_1\nu\right|^{2\theta(|\vec{p}| - k_f^p)}
\]

\[
\times \frac{M^2}{2k^2\omega_{\vec{q}_1}2\omega_{\vec{q}_2}E_{\vec{p}_1}E_{\vec{p}_2}},
\]

where \(\vec{q}_1\) and \(\vec{q}_2\) are the momenta of the outgoing pions. The medium-modified \(T_{P_2\gamma}\) matrix is expressed as

\[
T_{P_2\gamma} = T_{\Delta KR} + T_{\Delta PP} + T_{N^*\pi\Delta}^s + T_{N^*\pi\Delta}^d + T_{\rho KR} + T_{N^*\rho N}.
\]

The \(\Delta\) Kroll-Ruderman term is written as

\[
T_{\Delta KR} = F_{\pi N\Delta} G_{\pi\Delta}(s, \vec{p}_\Delta) F^{\Delta KR}.
\]

\[
G_{\pi\Delta}(s, \vec{p}_\Delta) = \left[ \sqrt{s} - \omega_{\pi}(\vec{p}_\Delta) - (M_{\Delta}(s, \vec{p}_\Delta) + \delta M_{\Delta}) + i(\Gamma_{\Delta}(s, \vec{p}_\Delta) + \Gamma_{\Delta sp})/2 \right]
\]

\[
- V_{\pi}(\vec{q}_\pi)]^{-1},
\]

Here \(F_{\pi N\Delta}\) is the \(\pi N\Delta\) vertex function, and \(F^{\Delta KR}\) is the \(\Delta\) Kroll-Ruderman vertex function. The detailed forms of vertex functions are given in Ref. [15]. \(V_{\pi}(\vec{q}_\pi)\) is the pion self-energy due to the distortion. \(\vec{p}_\Delta\) is the \(\gamma N\) center of mass momentum of \(\Delta\) and \(\vec{q}_\pi\) is the outgoing pion momentum. \(M_{\Delta}\) and \(\Gamma_{\Delta}\) in the propagator \(G_{\pi\Delta}\) are the mass and the free width of \(\Delta\), respectively and \(\delta M_{\Delta}\) and \(\Gamma_{\Delta sp}\) are the mass shift and collision width, respectively. Here we assume that the one-body operator \(W^{(2)}_{sp}\) is given by the sum of the pion optical potential and the \(\Delta\) spreading potential. We neglect the medium correction for the \(\rho\) meson since it is far off-shell. The other terms in the r.h.s. of Eq.(2.43) and the correction term \(\Delta_1\) are expressed in a similar way. The detailed forms of free \(T\) matrices are given in Ref. [15].

In addition to the one- and two-pion photoproduction processes, there are three genuine many-body processes which are shown in Figs. (f),(g) and (h). The cross section for Fig. (f) corresponds to the third term of Eq.(2.35) and is given by

\[
\sigma^{p}_{N^*(A-1)} = \frac{1}{v^2 8\pi (k_f^p)^3} \int_0^{k_f^p} d\vec{p}_1 \Gamma_{N^*sp} \frac{1}{2} \sum_{\lambda,\nu,\nu'} G_{N^*}(s) \left|\vec{p}_N\cdot\nu_N^*\right| F^{\Delta KR}_{\gamma p N^*} \left|\vec{k}\lambda\vec{p}_1\nu\right|^2
\]
The cross section for Fig. 1(g) has a similar form with Eq. (2.46). The cross section for Fig. 1(h) corresponds to the fourth term of Eq. (2.35) and is given by

\[
\sigma^{p}_{\pi \Delta(A-1)} = \frac{1}{v} \frac{3Z}{8\pi(k_f^p)^3} \int_{0}^{k_f^p} d^3p_1 \int \frac{d^3q}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} (\Gamma_{\Delta sp} + 2\text{Im} V_{\pi}(\vec{q})) (2\pi)^3 \delta(\vec{k} + \vec{p}_1 - \vec{q} - \vec{p}) \times \frac{1}{2} \sum_{\lambda\nu\Delta} |G_{\pi\Delta}(s, p_\Delta) \langle \bar{q}t_\pi \bar{p}_\nu t_\Delta | F^+_{\pi \Delta} \rangle |\vec{k}\lambda\vec{p}_1\nu > |^2 \frac{1}{2k^2\omega} M E_{\vec{p}_1},
\]  

(2.47)

where \( F^+_{\gamma P \pi \Delta} \) describes the \( \gamma P \rightarrow \pi \Delta \) transition corresponding to Figs. 1(a), (c) and (d).

To evaluate the cross section of Eq. (2.47), one needs to know the momentum dependence of \( \Gamma_{\Delta sp} \) and \( \text{Im} V_{\pi}(\vec{q}) \). The width \( \Gamma_{\Delta sp} \) is assumed to be constant in the kinematical region where the process \( \Delta N \rightarrow NN \) occurs and zero outside this physical region. The second term related to \( \text{Im} V_{\pi}(\vec{q}) \) describes the process that the \( \Delta \) resonance decays into \( \pi N \) while the pion is absorbed by nucleus. In order to include this instability of the \( \Delta \), we replace \( \text{Im} V_{\pi}(\vec{q}) \) with \( \text{Im} \tilde{V}_{\pi}(\vec{q}, E_\Delta) \) which is written as

\[
\text{Im} \tilde{V}_{\pi}(\vec{q}, E_\Delta) = (\frac{-1}{\pi} \text{Im} \frac{1}{D(\sqrt{s_\Delta})}) \text{Im} V_{\pi}(\vec{q}, q_0),
\]  

(2.48)

where \( s_\Delta = E_\Delta^2 - \vec{p}^2 \) and \( q_0 = k + E_{p_1} - E_\Delta \), respectively and \( D(\sqrt{s_\Delta}) \) is the free D-function of the \( \Delta \). Then the expression obtained is integrated over the \( \Delta \) energy \( E_\Delta \) in addition to momenta of \( \vec{p}_1, \vec{q} \) and \( \vec{p} \). Generally, the pion absorbed becomes off-shell, but in our calculation we use one-shell value at \( s_{\pi N} = q_0^2 - \vec{q}^2 \) for \( \text{Im} V_{\pi}(\vec{q}, q_0) \) as an approximation.

The correction term \( \Delta_2 \) in Eq. (2.37) is evaluated in the same way as Eq. (2.47). The cross sections of photoabsorption on a neutron in the nuclear matter are also given in a similar form with those of a proton.

III. NUMERICAL RESULTS AND DISCUSSIONS

Let us start from the comments for photoabsorption reaction off a nucleon because the information of the elementary pion productions is very important to understand the strong damping mechanism of the \( N^* \) resonance. The dominant reactions on a nucleon over the
FIG. 2: The total photoabsorption cross sections on a proton and a neutron. The dash-dotted line is the contribution of the one-pion production obtained by using SM95 amplitudes of Arndt et al.\cite{14}. The dashed line is the contribution of the two-pion production calculated by our model \cite{15, 16}. The solid line is the sum of those contributions. (a) Open circles and triangles (up) represent the data of total photoabsorption cross section on a proton\cite{5, 6}. (b) Triangles (down) represent the data of total photoabsorption cross section on a neutron\cite{7}.

photon energies from 300 to 850 MeV are the one-pion and two-pion photoproduction \cite{13}. At first it is noted that the peak of the $N^*$ resonance energy region shows up clearly by the combined effect of one-pion and two-pion productions as shown in Fig. 2(a) and (b), where the cross section of one-pion production has a small peak around 720 MeV and the cross section of two-pion production starts to grow from 400 MeV and increases up to around 800 MeV. The cross section of one-pion production is calculated by using the amplitudes of Arndt et al. \cite{14} and that of two-pion production is calculated by our model \cite{15, 16}.

We briefly review our model for the two-pion production. For the $\gamma N \rightarrow \pi^+\pi^-N$
FIG. 3: The total cross section for the $\gamma p \to \pi^+\pi^- p$. (a) The solid line is the contributions of the $\Delta$ Kroll-Ruderman and $\Delta$ pion-pole terms ($\pi\Delta$ channel), dashed line is the contributions of the $N^* \to \pi\Delta$ (s and d waves) terms, long dashed line is the contribution of the $N^* \to \rho N$ term, and dash-dotted line is the sum of contributions from $\pi\Delta$ channel and $N^* \to \pi\Delta$ term. (b) The solid line corresponds to the full calculation. Theoretical lines are obtained by using parameter set (III) in our model [16]. Experimental data are taken from Refs. [13, 17, 18, 19].

reaction, four processes expressed by the diagrams (a), (b), (c), and (d) in Fig. 1 are assummed to contribute to this channel. In these processes, the $\Delta$ Kroll-Ruderman ($\Delta$KR) term [Fig.1(c)] and the $\Delta$ pion-pole ($\Delta$PP) term [Fig.1(d)] dominate on the cross section. The $N^*$ contributions alone are small, but the interference among the $N^*$ terms, the $\Delta$KR and $\Delta$PP terms is important as shown in Figs. 3(a) and (b). Because of this, the $N^*$
excitation is regarded as an important ingredient in the two-pion photoproduction. For the $\gamma p \to \pi^+\pi^0n$ and $\gamma n \to \pi^-\pi^0p$ reactions, the $\rho$ meson Kroll-Ruderman ($\rho$KR) term can contribute to these isospin channels in addition to four diagrams appearing in the $\gamma N \to \pi^+\pi^-N$. The $\rho$KR term [Fig. 1(e)] and the $N^*$ terms dominate in this case, and the interference among the $\rho$KR term, $N^*\rho N$ term and the $\Delta$KR term is important and gives
rise to the bump in the excitation curve as shown in Figs. 4(a) and (b). With regards to the 
$\gamma N \rightarrow \pi^0\pi^0N$ reaction, the magnitude of the cross section is underestimated about a factor of $\frac{4}{3}$ in our model. However, the underestimate of this channel does not affect our conclusion as far as the total cross section is concerned, since the cross section of $\gamma N \rightarrow \pi^0\pi^0N$ reaction is smaller than 10 percent of total two-pion production reaction.

What we learned from the elementary processes are the followings.

i) The cross section of one-pion photoproduction has only a small bump for a proton and a shoulder for a neutron in the $N^*$ resonance energy region. Therefore, we can easily make the $N^*$ resonance peak from the one-pion production vanish by introducing a much smaller width due to the collision broadening than those given by Alberico et al.\cite{9} and Kondratyuk et al.\cite{10}.

ii) For the two-pion photoproduction the $N^*$ contribution alone is not large. In order to give rise to the bump in the cross section, the interference between the $N^*$ term and other terms is very important. So, we expect that the delicate balance of the interference is broken in the nuclear medium by the collision broadenings of $\Delta$ and $N^*$, the pion distortion in the $\pi\Delta$ channel and the Fermi motion, and therefore, the bump is strongly suppressed due to cooperative effects of the broadenings and interference.

Now we discuss the total cross section of nuclear photoabsorption. For simplicity we adopt the Fermi gas model for a nucleus, and $k_{avf} = \int d\vec{r}\rho(\vec{r})k_f(\rho)$ as the Fermi momentum in order to take into account the finiteness of nucleus. In our calculation, the total cross section per nucleon is obtained by taking the average of contributions from a proton and neutron in the nuclear matter.

As shown in Fig. 5, the Fermi motion produces strong damping of the cross section around the $N^*$ resonance energy region. However, the small bump still remains and its effect cannot fill up the valley between 380 and 500 MeV. For comparison, we also show the experimental cross section on a free proton with the theoretical curve in Fig. 5. Furthermore, to see the details of the Fermi motion effects, individual contributions for the one-pion and
FIG. 5: The Fermi motion effects for total nuclear photoabsorption cross section on nuclei. The dotted line is the nuclear cross section averaged over the initial nucleon momentum. The thick solid line is the nuclear cross section including the Pauli blocking effect for the final emitted nucleon and intermediate Δ state in addition to the average over the initial nucleon momentum. The dash-dotted and dashed lines are two components of the thick solid line, i.e., the one-pion and two-pion production, respectively. The thin solid line corresponds to the cross section on a free proton calculated by using multipole amplitudes. Experimental data for nuclei are taken from Ref. [3]. The open circles represent data of the total photoabsorption cross section on a proton from Ref. [3].

two-pion productions are presented in Fig. 5. The size of the Pauli blocking effect for the intermediate and final states is found to be small but non-negligible as is seen from the difference between the dash-two-dotted and thick solid lines.

To explain the data, thus, one inevitably needs the other damping mechanisms. As additional medium corrections, we take into account the spreading potentials [12] for the \( N^* \) and Δ resonances, and the pion distortion appearing in the formula derived in previous section. The mass shift and collision width of Δ have been already known in the studies of pion-nucleus scattering using the Δ-hole model [24, 23, 26, 27] where they can be identified.
FIG. 6: The total nuclear photoabsorption cross section on nuclei. The thick solid line is the full calculation. The dash-dotted and dashed lines are the contributions of the one-pion and two-pion production, respectively. The dotted line is the contribution for the processes of Figs. 1(f) and 1(g). The long dashed line is the contribution for the process of Fig. 1(h). Experimental data for nuclei are taken from Ref. [3]. The open circles represent data of the total photoabsorption cross section on a proton from Ref. [3].

as the spreading potential. The spreading potential found in these studies is almost energy independent. We take $\delta M_\Delta = 6 \text{ MeV}$ and $\Gamma_{sp} = 36 \text{ MeV}$. As the pion self-energy, we adopt the pion optical potential used by Arima et al. [28]. As for the mass shift and collision width of $N^*$, there are no established values at present. For simplicity, we assume that $\delta M_{N^*}$ and $\Gamma_{N^*sp}$ are energy independent like $\delta M_\Delta$ and $\Gamma_{sp}$. Then we vary the values so that the total nuclear photoabsorption cross sections from 600 to 800 MeV are reproduced. We found $\delta M_{N^*} = 12 \text{ MeV}$ and $\Gamma_{N^*sp} = 48 \text{ MeV}$.

The total photoabsorption cross sections per nucleon (solid line) calculated with the above parameters are shown in Fig. 6. It is found that the simultaneous inclusion of the
spread potentials for the $N^*$ and $\Delta$ resonances, and the pion distortion gives rise to the complete suppression of the bump around the $N^*$ resonance energy region. To see the detailed contents of our calculations, furthermore, we show each contribution of the absorption processes: the one-pion production (dash-dotted line), the two-pion production (dashed line), the many-body absorption through the $\Delta$-nucleus state and $N^*$-nucleus state (dotted line) corresponding to Figs. 6(f) and (g), and the many-body absorption through the $\pi\Delta$-nucleus state (long dashed line) corresponding to Fig. 6(h). The correction terms $\Delta_1$ and $\Delta_2$ in Eqs.(2.36) and (2.37) are already included in the calculations of the dashed and long dashed lines, respectively. The size of the correction terms is small but non-negligible. For instance, there is about 20 percent effect at 750 MeV for the two-pion production. In the one-pion photoproduction, the bump near the mass of the $N^*$ disappears by the spreading potential for $N^*$. The cross section of the two-pion photoproduction is about 3 times smaller than that of the elementary process by the cooperative effects between the following medium corrections: the spreading potentials for $\Delta$ and $N^*$, the pion distortion, and the change of the interference among the related reaction processes. The cross sections of the other many-body processes are almost flat in the energy range above 600 MeV and small. As a consequence of these effects, the excitation peak around the position of the $N^*$ resonance in the total nuclear photoabsorption cross section disappears differently from the hydrogen.

Our model, however, underestimates cross sections in the valley region between 380 and 500 MeV by about 15 percent ($\sim 45 \, \mu b$). There must be some important processes which give enhancement for the nuclear photoabsorption in the valley region but do not appear in
the photoabsorption off a nucleon. The candidates for such processes are shown in Figs. 7(a) and (b) where the intermediate pion and $\rho$ meson are far off-shell. Two nucleons explicitly contribute to these processes. These contributions are suitable to explain the mass number dependence $A^{1.7}$ of total cross section in the valley region. In Refs. [11, 29] the contribution from Fig. 7(a) is taken into account to increase the cross section of one-pion production.

The cross sections are also underestimated slightly at the $\Delta$ resonance energy around 320 MeV. This missing strength may be due to the coherent $\pi^0$ production mechanism in addition to the above two-nucleon mechanism. The coherent $\pi^0$ production is not included in our calculation using the Fermi gas model. In this energy region, our model has to be extended so as to treat the finiteness of the nucleus more reliably, as was done by Koch et al. [23].

**IV. CONCLUSIONS**

The formula derived by using the projection method for the photonuclear total absorption cross section has been presented. Our method is very effective for the case that the interference effect in the photoabsorption off a nucleon such as two-pion productions is strong.

The disappearance of the peak around the position of the $D_{13}$ resonance in the nuclear photoabsorption can be explained by taking into account the cooperative effect of the interference in the two-pion photoproduction processes, the collision broadening of $\Delta$ and $N^*$, and the pion distortion in the nuclear medium. The change of the interference by the medium plays an important role. The mass shift and collision width of $N^*$ are found to be $\delta M_{N^*} = 12$ MeV and $\Gamma_{N^*} = 48$ MeV, respectively. The collision width obtained is about 6 times smaller than those in Refs. [9, 10]. The total absorption cross sections in our theoretical calculation around 320 MeV are about 5 percent smaller than average experimental cross sections for several nuclei. Furthermore, theoretical total absorption cross sections in the valley region between 380 and 500 MeV are about 15 percent smaller than the average
experimental cross sections. In this energy region it may be necessary to take into account such reaction processes as Figs. 7(a) and (b) involving two nucleons explicitly.

[1] N. Bianchi et al., Phys. Lett. B 229, 219 (1993).
[2] M. Anghinolfi et al., Phys. Rev. C 47, R992 (1993).
[3] N. Bianchi et al., Phys. Lett. B 309, 5 (1993); Phys. Lett. B 325, 333 (1994); Phys. Rev. C 54, 1688 (1996).
[4] T.A. Armstrong et al., Phys. Rev. D 5, 1640 (1972).
[5] M. MacCormick et al., Phys. Rev. C 53, 41 (1996).
[6] T.A. Armstrong et al., Nucl. Phys. B41, 445 (1972).
[7] Th. Frommhold et al., Phys. Lett. B295, 28 (1992).
[8] Th. Frommhold et al., Z. Phys. A350, 249 (1994).
[9] W.M. Alberico, G.Gervino, and A.Lavagno, Phys. Lett. B321, 177 (1994).
[10] L.A. Kondratyuk et al., Nucl. Phys. A579, 453 (1994).
[11] M. Effenberger, A. Hombach, S. Teis, and U. Mosel, Nucl. Phys. A613, 353 (1997).
[12] M. Hirata, K. Ochi, and T. Takaki, Phys. Rev. Lett. 80, 5068 (1998).
[13] A. Braghieri et al., Phys. Lett. B363, 46 (1995).
[14] R.A. Arndt, I.I. Strakovsky, and R.L. Workman, Phys. Rev. C56, 577 (1997) and references therein.
[15] K. Ochi, M. Hirata, and T. Takaki, Phys. Rev. C 56, 1472 (1997).
[16] M. Hirata, K. Ochi, and T. Takaki, Prog. Theor. Phys. 100, 681 (1998).
[17] Aachen-Berlin-Bonn-Hamburg-Heidelberg-München Collaboration, Phys. Rev. 175, 1669 (1968).
[18] A. Piazza et al., Lett. Nuovo Cimento 3, 403 (1970).
[19] F. Carbobara et al., Lett. Nuovo Cimento A 36, 219 (1976).
[20] A. Zabrodin et al., Phys. Rev. C55, 1 (1997).
[21] W. Langgartner et al., Phys. Rev. Lett. 87, 052001-1 (2001).
[22] M. Betz and T.-S.H. Lee, Phys. Rev. C23, 375 (1981).
[23] J.H. Koch, E.J. Moniz, and N. Ohtsuka, Ann. Phys. (N.Y.) 154, 99 (1984).
[24] M. Hirata, F. Lenz, and K. Yazaki, Ann. Phys. (N.Y.) 108, 116 (1977).
[25] M. Hirata, J.H. Koch, F. Lenz, and E.J. Moniz, Ann. Phys. (N.Y.) 120, 205 (1979).
[26] Y. Horikawa, F. Lenz, and M. Thies, Nucl. Phys. A345, 386 (1980).
[27] E. Oset and W. Weise, Nucl. Phys. A 319, 477 (1979); A 329, 365 (1979).
[28] M. Arima, K. Masutani, and R. Seki, Phys. Rev. C 51, 285 (1995).
[29] R.C. Carrasco and E. Oset, Nucl. Phys. A 536, 445 (1992).