Guaranteeing Positive Secrecy Capacity with Finite-Rate Feedback using Artificial Noise

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Abstract—While the impact of finite-rate feedback on the capacity of fading channels has been extensively studied in the literature, not much attention has been paid to this problem under secrecy constraint. In this work, we study the ergodic secret capacity of a multiple-input multiple-output multiple-antenna-eavesdropper (MIMOME) wiretap channel with quantized channel state information (CSI) at the transmitter and perfect CSI at the legitimate receiver, under the assumption that only the statistics of eavesdropper CSI is known at the transmitter. We refine the analysis of the random vector quantization (RVQ) based artificial noise (AN) scheme in [1], where a heuristic upper bound on the secrecy rate loss, when compared to the perfect CSI case, was given. We propose a lower bound on the ergodic secrecy capacity. We show that the lower bound and the secrecy capacity with perfect CSI coincide asymptotically as the number of feedback bits and the AN power go to infinity. For practical applications, we propose a very efficient quantization codebook construction method for the two transmit antennas case.

Index Terms—artificial noise, secret capacity, physical layer security, wiretap channel.

I. INTRODUCTION

Complexity-based cryptographic technologies (e.g., AES [2]) have traditionally been used to provide a secure gateway for communications and data exchanges at the network layer. The security is achieved if an eavesdropper (Eve) without the key cannot decipher the message in a reasonable amount of time. This premise becomes controversial with the rapid developments of computing devices (e.g., quantum computer).

In contrast, physical layer security (PLS) does not depend on a specific computational model and can provide security even when Eve has unlimited computing power. Wyner [3] and later Csiszár and Körner [4] proposed the wiretap channel model as a basic framework for PLS. Wyner has shown that for discrete memoryless channels, if Eve intercepts a degraded version of the intended receiver’s (Bob’s) signal, a prescribed degree of data confidentiality could be simultaneously attained by channel coding without any secret key. The associated notion of secrecy capacity was introduced to characterize the maximum transmission rate from the transmitter (Alice) to Bob, below which Eve is unable to obtain any information.

Wyner’s wiretap channel model has been extended to fading channel [5]. Gaussian broadcast channel [6], multiple-input single-output multiple-antenna-eavesdropper (MISOME) channel [7], and multiple-input multiple-output multiple-antenna-eavesdropper (MIMOME) channel [8]. All these works rely on the perfect knowledge of Bob’s channel state information (CSI) at Alice to compute the secrecy capacity and enable secure encoding. In particular, Eve’s CSI is also assumed to be known at Alice in [5], [6], [8], although the CSI of a passive Eve is very hard to be unveiled at Alice. It is more reasonable to assume that Alice only knows the statistics of Eve’s channel. Even the assumption of perfect knowing Bob’s CSI is not realistic. In practice, Bob can only provide Alice with a quantized version of his CSI via a rate constrained feedback channel (i.e., finite-rate feedback).

In this work, we are interested in the secrecy capacity conditioned on the quantized CSI of Bob’s channel and the statistics of Eve’s channel. While the impact of finite-rate feedback on the capacity of fading channels has been extensively studied (see [9]–[13]), not much attention has been given to this problem under secrecy constraint. In [14], assuming that Alice only knows the statistics of Eve’s channel, the authors derived lower and upper bounds on the ergodic secrecy capacity for a single-input single-output single-antenna-eavesdropper (SISOSE) system with finite-rate feedback of Bob’s CSI. In the MIMOME scenario, the artificial noise (AN) scheme has been shown to guarantee positive secrecy capacity without knowing Eve’s CSI in [15]. Alice is assumed to have perfect knowledge of Bob’s eigenchannel vectors. This assumption allows her to align artificial noise within the null space of a MIMO channel between Alice and Bob, so that only Eve’s equivocation is enhanced. In [1], the authors show that if only quantized CSI is available at Alice, the artificial noise will leak into Bob’s channel, causing a decrease in the achievable secrecy rate. A heuristic upper bound on the secrecy rate loss (compared to the perfect CSI case) is proposed in [1] Eq. 34).

The main contribution of this paper is to provide a lower bound on the ergodic secrecy capacity for the AN scheme with quantized CSI, valid for any number of Alice/Bob/Eve antennas, as well as for any Bob/Eve SNR regimes. Following the work in [1], we use the random vector quantization (RVQ) scheme in [2]. Namely, given B feedback bits, Bob quantizes his eigenchannel matrix to one of $N = 2^B$ random unitary matrices and feeds back the corresponding index. We first show that RVQ is asymptotically optimal for security purpose, i.e., the secrecy capacity/rate loss compared to the perfect CSI case converges to 0 as $B \to \infty$. This result implies that the heuristic bound in [1] Eq. 34] is not tight, since it reduces to a positive constant as $B \to \infty$. To refine the analysis in [1],
we establish a tighter upper bound on the secrecy rate loss, which leads to an explicit lower bound on the ergodic secrecy capacity. We further show that the lower bound and the secrecy capacity with perfect CSI coincide asymptotically as \( B \) and the AN power go to infinity. This allows us to provide a sufficient condition guaranteeing positive secrecy capacity.

From a practical point of view, it is often desirable to use a deterministic quantization codebook rather than a random one. The problem of derandomizing RVQ codebooks is related to discretizing the complex Grassmannian manifold. Since the optimal constructions are possible only in very special cases, deterministic codebooks are mostly generated by computer search. Interestingly, the case of codebook design with two transmit antennas is equivalent to quantizing a real sphere. According to this fact, we propose a very efficient codebook construction method for the two-antenna case. Simulation results demonstrate that near-RVQ performance is achieved by a moderate number of feedback bits.

The paper is organized as follows: Section II presents the system model, followed by the analysis of secrecy capacity with finite-rate feedback in Section III. Section IV provides the deterministic quantization codebook construction method for the two-antenna case. Conclusions are drawn in Section V.

Proofs of the theorems are given in Appendix.

\textbf{Notation:} Matrices and column vectors are denoted by upper and lowercase boldface letters, and the Hermitian transpose, inverse, pseudoinverse of a matrix \( B \) by \( B^H \), \( B^{-1} \), and \( B^† \), respectively. \( |B| \) denotes the determinant of \( B \). Let the random variables \( \{X_n\} \) and \( X^* \) be defined on the same probability space. We write \( X_n \xrightarrow{a.s.} X \) if \( X_n \) converges to \( X \) almost surely or with probability one. \( I_n \) denotes the identity matrix of size \( n \). An \( m \times n \) null matrix is denoted by \( 0_{m \times n} \). A circularly symmetric complex Gaussian random variable \( x \) with variance \( \sigma^2 \) is defined as \( x \sim \mathcal{CN}(0, \sigma^2) \). The real, complex, integer and real, complex, integer number vectors are denoted by \( \mathbb{R}, \mathbb{C}, \mathbb{Z} \), and \( \mathbb{Z}[i] \), respectively. \( I(x; y) \) represents the mutual information of two random variables \( x \) and \( y \). We use the standard asymptotic notation \( f(x) = O(g(x)) \) when \( \limsup_{x \to \infty} |f(x)/g(x)| < \infty \). \( [x] \) rounds to the closest integer, while \( \lfloor x \rfloor \) to the closest integer smaller than or equal to \( x \) and \( \lceil x \rceil \) to the closest integer larger than or equal to \( x \). A central Wishart matrix \( A \in \mathbb{C}^{m \times m} \) with \( n \) degrees of freedom and covariance matrix \( \Sigma \), is defined as \( A \sim W_m(n, \Sigma) \). Trace of a square matrix \( B \) is denoted by \( \operatorname{Tr}(B) \). We write \( \hat{=} \) for equality in definition.

\section{II. System Model}

We consider secure communications over a three-terminal system, including a transmitter (Alice), the intended receiver (Bob), and an unauthorized receiver (Eve), equipped with \( N_A \), \( N_B \), and \( N_E \) antennas, respectively. The signal vectors received by Bob and Eve are

\[ z = Hx + n_B, \quad y = Gx + n_E, \]

where \( x \in \mathbb{C}^{N_A} \) is the transmit signal vector, \( H \in \mathbb{C}^{N_B \times N_A} \) and \( G \in \mathbb{C}^{N_E \times N_A} \) are the respective channel matrices between Alice to Bob and Alice to Eve, and \( n_B, n_E \) are AWGN vectors with i.i.d. entries \( \sim \mathcal{CN}(0, \sigma_B^2) \) and \( \mathcal{CN}(0, \sigma_E^2) \). We assume that the entries of \( H \) and \( G \) are i.i.d. complex random variables \( \sim \mathcal{CN}(0, 1) \).

Without loss of generality, we normalize Bob’s channel noise variance to one, i.e.,

\[ \sigma_B^2 = 1. \tag{3} \]

In this paper, we assume that Bob knows its own channel matrix \( H \) instantaneously and Eve knows both its own channel matrix \( G \) and the main channel \( H \), instantaneously, whereas Alice is only aware of the statistics of \( H \) and \( G \). There is also an error-free public feedback channel with limited capacity from Bob to Alice that can be tracked by Eve. In our setting, the feedback is exclusively used to send the index of the codeword in a quantization codebook that describes the main channel state information \( H \). The quantization codebook is assumed to be known \textit{a priori} to Alice, Bob and Eve.

\subsection{A. Artificial Noise Scheme with Perfect CSI}

The original AN scheme assumes \( N_B < N_A \), in order to ensure that \( H \) has a non-trivial null space with an orthonormal basis \( Z = \{0\} \in \mathbb{C}^{N_A \times (N_A - N_B)} \) (such that \( HZ = 0_{N_B \times (N_A - N_B)} \)). Let \( H = U \Sigma V^H \) be the singular value decomposition (SVD) of \( H \), where \( U \in \mathbb{C}^{N_A \times N_A} \) and \( V \in \mathbb{C}^{N_A \times N_A} \) are unitary matrices. Then, we can write the unitary matrix \( V \) as

\[ V = [\tilde{V}, Z], \tag{4} \]

where the \( N_B \) columns of \( \tilde{V} \in \mathbb{C}^{N_A \times N_B} \) span the orthogonal complement subspace to the null space spanned by the columns of \( Z \in \mathbb{C}^{N_A \times (N_A - N_B)} \).

With unlimited feedback (i.e., perfect CSI), Alice has perfect knowledge of the preceding matrix \( V \), and transmits

\[ x = \tilde{V}u + Zv = V \begin{bmatrix} u \\ v \end{bmatrix}, \tag{5} \]

where \( u \in \mathbb{C}^{N_A} \) is the information vector and \( v \in \mathbb{C}^{(N_A - N_B)} \) is the “artificial noise”. For the purpose of evaluating the achievable secrecy rate, both \( u \) and \( v \) are assumed to be circular symmetric Gaussian random vectors with i.i.d. complex entries \( \sim \mathcal{CN}(0, \sigma_u^2) \) and \( \mathcal{CN}(0, \sigma_v^2) \), respectively. In \cite{17}, we have shown that Gaussian input alphabets asymptotically achieves the secrecy capacity as \( \sigma_v^2 \to \infty \).

Equations \( (1) \) and \( (3) \) can then be rewritten as

\[ z = H\tilde{V}u + HZv + n_B = H\tilde{V}u + n_B, \tag{6} \]

\[ y = GV\tilde{V}u + GZv + n_E, \tag{7} \]

to show that with unlimited feedback, the artificial noise only degrades Eve’s channel, resulting in increased secrecy capacity (compared to the non-AN case).

\subsection{B. Artificial Noise Scheme with Quantized CSI}

In \cite{1}, the authors analyzed the impact of finite-rate feedback on the secrecy rate achievable by the AN scheme. To quantize the matrix \( \tilde{V} \) in \( (4) \), the random vector quantization
(RVQ) scheme in [9] is used. Given $B$ feedback bits per fading channel, Bob specifies $\hat{V}$ from a random quantization codebook

$$V = \{V_i, 1 \leq i \leq 2^B\},$$

where the entries are independent $N_A \times N_B$ random unitary matrices, i.e., $V_iHV_i = \mathbb{I}_{N_B}$. The codebook $V$ is known a priori to both Alice, Bob and Eve. Bob selects the $V_j$ that minimize the chordal distance between $V_j$ and $\hat{V}$ [11]:

$$\hat{V}_j = \min_{V_j \in V} d^2 \left( \hat{V}_j, \tilde{V} \right),$$

where

$$d \left( \hat{V}_j, \tilde{V} \right) = N_B - \text{Tr} \left( \tilde{V}^H \hat{V}_j \tilde{V} \hat{V}_j^H \tilde{V} \right).$$

Note that $\text{Tr} (A)$ denotes the trace of the square matrix $A$. And then, Bob relays the corresponding index $j$ back to Alice.

Alice generates the precoding matrix from $V_j$ as follows. Let $\tilde{v}_1, ..., \tilde{v}_{N_B}$ be the columns of $\tilde{V}_j$, and $v_1, ..., v_{N_A - N_B}$ be the standard basis vectors. Alice applies the Gram-Schmidt algorithm to the matrix

$$\tilde{v}_1, ..., \tilde{v}_{N_B}, v_1, ..., v_{N_A - N_B}$$

to generate the remaining orthonormal basis vectors spanning the orthogonal complement space to the one generated by the columns of $\tilde{V}_j$. This provides Alice with a unitary matrix

$$\hat{V} = [\hat{V}_j, \tilde{V}] \in \mathbb{C}^{N_A \times N_A},$$

that can be used to precode $u$ and $v$ as in (5).

Since $\tilde{Z} \neq \tilde{Z}$, the interference term $H\hat{V}v$ cannot be nulled at Bob. Therefore, equations (6) and (7) reduce to

$$z = H\hat{V}_j u + H\tilde{V} v + n_B,$$

$$y = G\hat{V}_j u + G\tilde{V} v + n_E,$$

and show that with finite rate feedback (i.e., quantized CSI), some of the artificial noise will inevitably leak into the main channel from Alice to Bob, causing degradation in the secrecy capacity (compared to the unlimited feedback case).

C. Assumptions and Notations

The analysis in [11], [15] are based on the assumption of $N_E < N_A$. Clearly, this assumption is not always realistic. In this work, we remove this assumption and evaluate the secrecy capacity for any number of Eve antennas.

Since $\hat{V}$ in (11) is a unitary matrix, the total transmission power can be written as

$$||x||^2 = \begin{bmatrix} u \\ v \end{bmatrix}^H \hat{V}^H \hat{V} \begin{bmatrix} u \\ v \end{bmatrix} = ||u||^2 + ||v||^2.$$  

Then the average transmit power constraint $P$ is

$$P = E(||x||^2) = P_u + P_v,$$

where

$$P_u = E(||u||^2) = \sigma^2_u N_B,$$

$$P_v = E(||v||^2) = \sigma^2_v (N_A - N_B),$$

are fixed by the power allocation scheme that selects the power balance between $\sigma^2_u$ and $\sigma^2_v$.

We define Bob’s and Eve’s SNRs as

- $\text{SNR}_B \triangleq \frac{\sigma^2_u}{\sigma^2_B}$
- $\text{SNR}_E \triangleq \frac{\sigma^2_v}{\sigma^2_E}$

To simplify our notation, we define three system parameters:

- $\alpha \triangleq \frac{\sigma^2_u}{\sigma^2_B} = \text{SNR}_B$
- $\beta \triangleq \frac{\sigma^2_v}{\sigma^2_E} (\text{AN power allocation})$
- $\gamma \triangleq \frac{\sigma^2_v}{\sigma^2_E} (\text{Eve-to-Bob noise-power ratio})$

Note that $\text{SNR}_B = \alpha \gamma$. If $\gamma > 1$, we say Eve has a degraded channel. Since we have normalized $\sigma^2_E$ to one, we can rewrite (16) as

$$P_u = \alpha \gamma N_B$$

$$P_v = \alpha \beta \gamma (N_A - N_B)$$

D. Instantaneous and Ergodic Secrecy Capacities

We recall from [8] the definition of instantaneous secrecy capacity for MIMO channel:

$$C_S \triangleq \max_{p(u)} \{I(u;z) - I(u;y)\},$$

where the maximum is taken over all possible input distributions $p(u)$. We remark that $C_S$ is a function of $H$ and $G$, which are embedded in $z$ and $y$. To average out the channel randomness, we further define the ergodic secrecy capacity, as in [15]

$$E(C_S) \triangleq \max_{p(u)} \{I(u;z|H) - I(u;y|H, G)\},$$

where $I(X;Y|Z) \triangleq E_Z [I(X;Y) | Z]$, following the notation in [18].

Since closed form expressions for $C_S$ and $E(C_S)$ are not always available, we often consider the corresponding secrecy rates, given by

$$R_S \triangleq I(u;z) - I(u;y),$$

$$E(R_S) \triangleq I(u;z|H) - I(u;y|H, G),$$

assuming Gaussian input alphabets, i.e., $v$ and $u$ are mutually independent Gaussian vectors with i.i.d. complex entries $N_C(0, \sigma^2_v)$ and $N_C(0, \sigma^2_u)$, respectively.

From (6), (17) and (19), the achievable secrecy rate with perfect CSI can be written as

$$R_S = \log |I_{N_a} + \alpha \gamma HHH^T| + \log |I_{N_e} + \alpha \beta (GZ)(GZ)^H|$$

$$- \log |I_{N_e} + \alpha (GV)(GV)^H + \alpha \beta (GZ)(GZ)^H|. $$

The closed-form expression of $E(R_S)$ can be found in [17, Th. 1]. It is shown that $E(R_S) \rightarrow E(C_S)$ as the AN power $P_v \rightarrow \infty$ in [17, Th. 3].

From (12), (13) and (19), the achievable secrecy rate with quantized CSI can be written as

$$R_{S,Q} = \log |I_{N_a} + \alpha \gamma (H\tilde{V}_j)(H\tilde{V}_j)^H + \alpha \beta (H\tilde{Z})(H\tilde{Z})^H|$$

$$- \log |I_{N_e} + \alpha \beta \gamma (H\tilde{Z})(H\tilde{Z})^H|$$

$$- \log |I_{N_e} + \alpha \beta (G\tilde{V})(G\tilde{V})^H + \alpha \beta (G\tilde{Z})(G\tilde{Z})^H|$$

$$- \log |I_{N_e} + \alpha \beta (G\tilde{Z})(G\tilde{Z})^H|. $$

(22)
E. Open Problems and Motivations

Using [19 Eq. 2, pp. 56], it is simple to show that
\[ E(R_S) \geq E(R_{S,Q}). \] (23)

In [11], the ergodic secrecy rate loss is defined by:
\[ E(\Delta R_S) \triangleq E(R_S) - E(R_{S,Q}). \] (24)

A heuristic upper bound was proposed in [11 Eq. 34]:
\[ E(\Delta R_S) \lesssim N_B \log \left( \frac{N_B + \alpha \beta \gamma N_A D \left( N_A, N_B, 2^B \right)}{N_B - D \left( N_A, N_B, 2^B \right)} \right) + N_B \log \left( 1 + \frac{1}{\alpha \gamma (N_A - N_B)} \right) \triangleq \text{UB}_{\text{heuristic}} \] (25)

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and \( d(\cdot, \cdot) \) is given in [10].

However, (25) is insufficient to characterize the impact of quantized CSI on the secrecy rate achievable by the AN scheme. To see this, we first provide the following theorem.

**Theorem 1:** For the RVQ-based AN scheme, as \( B \to \infty \),
\[ \mathbf{\hat{V}} \to \mathbf{V}, \] (27)

where \( \mathbf{V} \) is given in (4) and \( \hat{\mathbf{V}} \) is given in [11].

**Proof:** See Appendix A.

Theorem [11 shows the RVQ scheme is asymptotically optimal for large \( B \), i.e., the secrecy capacity/rate loss (compared to the perfect CSI case) converges to zero. In contrast, as \( B \to \infty \), \( \text{UB}_{\text{heuristic}} \) in (25) reduces to a positive constant:
\[ \text{UB}_{\text{heuristic}} \to N_B \log \left( 1 + \frac{1}{\alpha \gamma (N_A - N_B)} \right), \] (28)

since \( D \left( N_A, N_B, 2^B \right) \to 0 \) as \( B \to \infty \) [11]. Hence, the heuristic bound in (25) is not tight.

**Remark 1:** The ergodic secrecy capacity with quantized CSI, denoted by \( E(C_{S,Q}) \), is lower bounded by
\[ E(C_{S,Q}) \geq E(R_{S,Q}) = E(R_S) - E(\Delta R_S). \] (29)

Using the closed-form expression of \( E(R_S) \) given in [11 Th. 1], we are motivated to establish a tighter upper bound on \( E(\Delta R_S) \), which allows us to obtain a lower bound on \( E(C_{S,Q}) \).

III. Secrecy Capacity with Quantized CSI

In this section, we wish to determine the secrecy capacity with RVQ scheme. A tight upper bound on the ergodic secrecy rate loss and a lower bound on the ergodic secrecy capacity are provided in Theorem 2 and Theorem 4, respectively. In Theorem 5, we show that the lower bound and the secrecy capacity with perfect/quantized CSI coincide asymptotically as \( B \) and \( P_t \) go to infinity. This provides a sufficient condition guaranteeing positive secrecy capacity.

To describe our result, we first recall the following function from [20]:
\[ \Theta(m, n, x) \triangleq e^{-1/x} \sum_{k=0}^{m-1} \sum_{i=0}^{k} 2 \left( \frac{(-1)^i (2l)! (n - m + i)!}{2^{k-l} i! l! (n - m + l)!} \right) \left( \frac{2(k-l)}{k-l} \right)^{l-1} \sum_{j=0}^{n-m+i} x^{-j} \Gamma(-j, 1/x), \] (30)

where \( \binom{n}{b} = a! / (a-b)!b! \) is the binomial coefficient, \( n \geq m \) are positive integers, and \( \Gamma(a, b) \) is the incomplete Gamma function
\[ \Gamma(a, b) = \int_b^\infty x^{a-1} e^{-x} dx, \] (31)

We further define
\[ N_{\min} \triangleq \min \{ N_E, N_A - N_B \}, \] (32)
\[ N_{\max} \triangleq \max \{ N_E, N_A - N_B \}, \] (33)
\[ N_{\min} \triangleq \min \{ N_E, N_A \}, \] (34)
\[ N_{\max} \triangleq \max \{ N_E, N_A \}. \] (35)

Finally, we define a set of \( N_A \) power ratios \( \{ \theta_i \}^N_A \), where
\[ \theta_i \triangleq \left\{ \begin{array}{ll}
\frac{\alpha}{\alpha \beta} & 1 \leq i \leq N_B \\
1 & N_B + 1 \leq i \leq N_A
\end{array} \right. \] (36)

We recall from [11 Th. 4) that
\[ \mu \left( N_A, N_B, 2^B \right) \leq D \left( N_A, N_B, 2^B \right) \] (37)

where \( D(\cdot, \cdot) \) is given in (25) and
\[ \eta(n, p, K) = \frac{\Gamma \left( \frac{1}{p(n-p)} \right) (K c(n, p))}{p(n-p)} - \frac{1}{p(n-p)}, \] (38)

\[ \mu(n, p, K) = \frac{p(n-p)}{p(n-p) + 1} (K c(n, p)) \] (39)

\[ c(n, p) = \left\{ \begin{array}{ll}
\frac{1}{\Gamma(p(n-p) + 1)} \prod_{i=1}^{p} \Gamma(n-i + 1), & n \geq 2p \\
\frac{1}{\Gamma(p(n-p) + 1)} \prod_{i=1}^{n-p} \Gamma(n-p-i + 1), & n \leq 2p
\end{array} \right. \] (40)

for any \( 0 < \zeta < 1 \). Note that \( \Gamma(\alpha) \) is the Gamma function.

A. Bounds on Ergodic/Instantaneous Secrecy Rate Loss

We first consider the ergodic secrecy rate loss \( E(\Delta R_S) \).

**Theorem 2:** Let \( \theta_{\min} = \min \{ \alpha \gamma, \alpha \beta \gamma \} \). We have
\[ E(\Delta R_S) \leq \Theta(N_B, N_A, \alpha \gamma) - \Theta(N_B, N_A, \theta_{\min}) \] (41)

where \( \Theta(\cdot, \cdot, \cdot) \) is given in (30) and \( \eta(\cdot, \cdot, \cdot) \) is given in (38).

**Proof:** See Appendix B.
Theorem 2 gives a tight upper bound on $E(\Delta R_S)$, for any number of Alice/Bob/Eve antennas, as well as for any Bob/Eve SNR regimes. Different from (28), if $\beta \geq 1$, as $B \to \infty$, $UB \to 0$, (42)

which is consistent with Theorem 1.

Example 1: Let us apply Theorem 2 to the analysis of a RVQ-based AN scheme with $\beta = 1, \alpha \gamma = 1, N_A = 4$ and $N_B = 2$. The numerical result in Fig. 1 shows that the proposed upper bound in (41) is much tighter than the heuristic one in (25), and captures the behavior of $E(\Delta R_S)$.

We then study the distribution of instantaneous secrecy rate loss, defined by

$$\Delta R_S \triangleq R_S - R_{S,Q}. \quad (43)$$

Here, we consider the large system limit as $N_A$ and $B \to \infty$ with fixed ratio $B/N_A$. An interesting case that leads to a closed-form bound can be found when $N_B = N_E = 1$.

Theorem 3: If $N_B = N_E = 1$, as $N_A$ and $B \to \infty$ with $B/N_A \to B$,

$$\Delta R_S \overset{\text{a.s.}}{=} \log (1 + P/\beta) + \log \left(1 + 2^{-B} P\right) - \log \left(1 + P + \frac{1 - \beta}{\beta} \left(1 - 2^{-B} P\right)\right). \quad (44)$$

Proof: See Appendix C.

Theorem 3 provides a closed-form asymptotic expression for $\Delta R_S$ when $N_B = N_E = 1$. Hence, the ergodic secrecy rate loss also converges to the same constant, as stated in the following corollary.

Corollary 1: Under the same assumptions of Theorem 3

$$E(\Delta R_S) \to \log (1 + P/\beta) + \log \left(1 + 2^{-B} P\right) - \log \left(1 + P + \frac{1 - \beta}{\beta} \left(1 - 2^{-B} P\right)\right). \quad (45)$$

Proof: The proof is straightforward.

Example 2: The numerical result in Fig. 2 shows that (45) is very accurate even for finite $N_A$ and $B$.

**B. A Lower Bound on Ergodic Secrecy Capacity**

A lower bound on $E(C_{S,Q})$ can be derived using the results from [29], Theorem 2 and [17] Th. 1.

**Theorem 4:**

$$E(C_{S,Q}) \geq \Theta(N_{\min}, N_{\max}, \alpha \beta, \eta) - \Theta(N_B, N_A, \theta_{\min})$$

$$\bar{\Delta} \equiv \frac{N_A N_B}{N_B} \frac{\eta}{N_A, N_B, 2^B} \lesssim C_{L,B,Q}. \quad (46)$$

where $\Theta(\cdot, \cdot, \cdot)$ is given in (30), $\gamma(\cdot, \cdot, \cdot)$ is given in (33) and

$$\Omega = \left\{ \begin{array}{ll} K \sum_{k=1}^{N_{\min}} \det (\mathbf{R}(k)), & \beta \neq 1 \\ \Theta(N_{\min}, N_{\max}, \alpha), & \beta = 1 \end{array} \right. \quad (47)$$

$$K = \left( -1 \right)^{N_A - N_{\min}} \prod_{i=1}^{\beta} \frac{\mu_i^{m_i} N_E}{\Gamma_{N_{\min}}(N_E)} \prod_{j=1}^{\beta} \frac{\Gamma_{m_j}(m_j)}{\Gamma_{m_j}(m_j + 1)} \prod_{i < j} \left( \theta_{ij}^{-1} \right)^{N_A} \Gamma_{k}(n) = n! \quad (48)$$

and $\mu_1 > \mu_2$ are the two distinct eigenvalues of the matrix $\mathbf{R}(k)$, with corresponding multiplicities $m_1$ and $m_2$ such that $m_1 + m_2 = N_A$. The matrix $\mathbf{R}(k)$ has elements

$$r_{i,j}^{(k)} = \begin{cases} (\mu_{e_1})^{N_A-j-d_i} (N_A-j)! / (N_A-j-d_i)! , & 1 \leq j \leq N_A \\ (1-d_i) / (\mu_{e_1}), & 1 \leq j \leq N_{\min}, j \neq k \\ (-1)^d_i \varphi(i,j) / (\mu_{e_1} \varphi(i,j) + 1), & \text{otherwise} \end{cases} \quad (49)$$

where

$$e_i = \begin{cases} 1 & 1 \leq i \leq m_1 \\ 2 & m_1 + 1 \leq i \leq N_A \end{cases}$$

$$d_i = \sum_{k=1}^{e_i} m_k - i.$$
have been used to prove new results on secrecy capacity with quantized CSI. The methods of constructing random unitary matrices $V_t$ in (8) can be found in [21]. In practice, it is often desirable that the quantization codebook is deterministic. The problem of derandomizing RVQ codebooks is typically referred to as Grassmannian subspace packing [9, 10]. Despite of a few special cases (e.g., $B \leq 4$ [13]), analytical codebook design in general remains an intricate task. In this section, we propose a very efficient quantization codebook construction method for the case of $N_A = 2$ and $N_B = 1$.

According to [13 Eq. (20)], the codeword $\tilde{V}_i$ can be expressed as

$$\tilde{V}_i(\omega, \phi) = \begin{bmatrix} \cos \omega e^{i\phi} \sin \omega \end{bmatrix},$$

which fully describes the complex Grassmannian manifold $G_{2,1}$ by setting $0 \leq \phi \leq 2\pi$ and $0 \leq \omega \leq \pi/2$. Let $(\hat{\omega}, \hat{\phi})$ be spherical coordinates parameterizing the unit sphere $S^2$, where $0 \leq \phi \leq 2\pi$ and $0 \leq \omega \leq \pi$. In [13 Lemma 1], the authors further show that the map

$$S^2 \rightarrow G_{2,1},$$

$$(\hat{\omega}, \hat{\phi}) \mapsto \tilde{V}_i(\hat{\omega}/2, \hat{\phi})$$

is an isomorphism. In other words, the sampling problem on $G_{2,1}$ can analogically be addressed on the real sphere $S^2$.

The method of sampling points uniformly from $S^2$ is provided in [22]. In details, one can parameterize $(x, y, z) \in S^2$ using spherical coordinates $(\hat{\omega}, \hat{\phi})$:

$$x = \sin \hat{\omega} \cos \hat{\phi},$$
$$y = \sin \hat{\omega} \sin \hat{\phi},$$
$$z = \cos \hat{\omega}.$$

The area element of $S^2$ is given by

$$dS = \sin \hat{\omega} d\hat{\omega} d\hat{\phi} = -d(\cos \hat{\omega}) d\hat{\phi}.$$ (55)

Hence, to obtain a uniform distribution over $S^2$, one has to pick $\phi \in [0, 2\pi]$ and $t \in [-1, 1]$ uniformly and compute $\hat{\omega}$ by:

$$\hat{\omega} = \arccos t.$$ (56)

In this way $\cos \hat{\omega} = t$ will be uniformly distributed in $[-1, 1]$.

Based on above analysis, we give a straightforward method for codebook construction:

$$\hat{Y} = \left\{ \hat{V}_{t,i} = \begin{bmatrix} \cos(0.5 \arccos t_i) \\
\sin(0.5 \arccos t_i) \end{bmatrix} \right| i = 1, \ldots, 2^B \right\},$$

where

$$t_i = -1 + 2 \left\lfloor \frac{i/2^{[B/2]} - 1}{2^{[B/2]}} \right\rfloor$$

$$\phi_i = \frac{2\pi (i \mod 2^{[B/2]})}{2^{[B/2]}}.$$ (59)

Note that $[x]$ rounds to the closest integer smaller than or equal to $x$, while $\lfloor x \rfloor$ to the closest integer larger than or equal to $x$.

Using the deterministic codebook in (57) can save storage space on Alice, since she can generate the target codeword $\hat{V}_{t,i}$ directly without the knowledge of the whole codebook $\hat{Y}$. We remark that the proposed codebook construction is valid.

**IV. IMPLEMENTATION USING A DETERMINISTIC CODEBOOK**

In the previous section, random quantization codebooks have been used to prove new results on secrecy capacity with quantized CSI. The methods of constructing random unitary matrices $V_t$ in (8) can be found in [21]. In practice, it is often desirable that the quantization codebook is deterministic. The problem of derandomizing RVQ codebooks is typically referred to as Grassmannian subspace packing [9, 10]. Despite of a few special cases (e.g., $B \leq 4$ [13]), analytical codebook design in general remains an intricate task. In this section, we propose a very efficient quantization codebook construction method for the case of $N_A = 2$ and $N_B = 1$.

According to [13 Eq. (20)], the codeword $\tilde{V}_i$ can be expressed as

$$\tilde{V}_i(\omega, \phi) = \begin{bmatrix} \cos \omega e^{i\phi} \sin \omega \end{bmatrix},$$

which fully describes the complex Grassmannian manifold $G_{2,1}$ by setting $0 \leq \phi \leq 2\pi$ and $0 \leq \omega \leq \pi/2$. Let $(\hat{\omega}, \hat{\phi})$ be spherical coordinates parameterizing the unit sphere $S^2$, where $0 \leq \phi \leq 2\pi$ and $0 \leq \omega \leq \pi$. In [13 Lemma 1], the authors further show that the map

$$S^2 \rightarrow G_{2,1},$$

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is an isomorphism. In other words, the sampling problem on $G_{2,1}$ can analogically be addressed on the real sphere $S^2$.

The method of sampling points uniformly from $S^2$ is provided in [22]. In details, one can parameterize $(x, y, z) \in S^2$ using spherical coordinates $(\hat{\omega}, \hat{\phi})$:

$$x = \sin \hat{\omega} \cos \hat{\phi},$$
$$y = \sin \hat{\omega} \sin \hat{\phi},$$
$$z = \cos \hat{\omega}.$$

The area element of $S^2$ is given by

$$dS = \sin \hat{\omega} d\hat{\omega} d\hat{\phi} = -d(\cos \hat{\omega}) d\hat{\phi}.$$ (55)

Hence, to obtain a uniform distribution over $S^2$, one has to pick $\phi \in [0, 2\pi]$ and $t \in [-1, 1]$ uniformly and compute $\hat{\omega}$ by:

$$\hat{\omega} = \arccos t.$$ (56)

In this way $\cos \hat{\omega} = t$ will be uniformly distributed in $[-1, 1]$.

Based on above analysis, we give a straightforward method for codebook construction:

$$\hat{Y} = \left\{ \hat{V}_{t,i} = \begin{bmatrix} \cos(0.5 \arccos t_i) \\
\sin(0.5 \arccos t_i) \end{bmatrix} \right| i = 1, \ldots, 2^B \right\},$$

where

$$t_i = -1 + 2 \left\lfloor \frac{i/2^{[B/2]} - 1}{2^{[B/2]}} \right\rfloor$$

$$\phi_i = \frac{2\pi (i \mod 2^{[B/2]})}{2^{[B/2]}}.$$ (59)

Note that $[x]$ rounds to the closest integer smaller than or equal to $x$, while $\lfloor x \rfloor$ to the closest integer larger than or equal to $x$.

Using the deterministic codebook in (57) can save storage space on Alice, since she can generate the target codeword $\hat{V}_{t,i}$ directly without the knowledge of the whole codebook $\hat{Y}$. We remark that the proposed codebook construction is valid.
for any $B$. This is different from the construction scheme in [13] Sec. VI-A], which is only possible for the case of $B \leq 4$.

Example 4: Fig. 4 examines the performance of the proposed codebook construction with $\beta = 2$, $\gamma = 1$, $P = 10$, $N_A = 2$, and $N_B = N_E = 1$. When $B \leq 4$, it is seen that the performance of codebook $\tilde{V}$ in (57) is indistinguishable from the optimal one in [13] Sec. VI-A]. When $B \geq 8$, the proposed codebook provides the same performance as the random one in (8).

V. CONCLUSIONS

In this work, we have discussed the problem of guaranteeing positive secrecy capacity for MIMOME channel with the quantized CSI of Bob’s channel and the statistics of Eve’s channel. We analyzed the RVQ-based AN scheme and provided a lower bound on the ergodic secrecy capacity. We proved that a positive secrecy capacity is always achievable by Gaussian input alphabets when $N_E \leq N_A - N_B$, and the number of feedback bits $B$ and the artificial noise power $P_b$ are large enough. We also proposed an efficient implementation of discretizing the RVQ codebook which exhibits similar performance to that of random codebook.

APPENDIX

A. Proof of Theorem 1

According to [12], as $B \to \infty$, the RVQ operation in (9) can guarantee

$$\tilde{V}_j \to \tilde{V}. \quad (60)$$

We then check the matrix $\tilde{Z}$ generated by Alice. The SVD decomposition of $H$ can be written as

$$H = U \Lambda \tilde{V}^H. \quad (61)$$

From (60) and (61), as $B \to \infty$, we have

$$H \tilde{Z} = U \Lambda \tilde{V}^H \tilde{Z} \to U \Lambda \tilde{V}^H 0 = 0_{N_A \times (N_A - N_B)}, \quad (62)$$

which means

$$\tilde{Z} \to \text{null}(H). \quad (63)$$

From (11), (60) and (63), we have $\tilde{V} \to V$ as $B \to \infty$. \hfill \blacksquare

B. Proof of Theorem 2

Using [19] Eq. 12, pp. 55], we have

$$\begin{align*}
&\left| I_{N_b} + \alpha \gamma (H \tilde{V}_j)(H \tilde{V}_j)^H + \alpha \beta \gamma (H \tilde{Z})(H \tilde{Z})^H \right| \\
&\geq \left| I_{N_b} + \theta_{\min}(H \tilde{V}_j)(H \tilde{V}_j)^H + \theta_{\min}(H \tilde{Z})(H \tilde{Z})^H \right| \\
&= \left| I_{N_b} + \theta_{\min} HH^H \right|, \quad (64)
\end{align*}$$

where $\theta_{\min} = \min \{\alpha \gamma, \alpha \beta \gamma\}$.

Since the unitary matrix $\tilde{V} = [\tilde{v}_j, \tilde{z}]$ is independent of $G$ and its realization is known to Alice, $G \tilde{V}_j \in \mathbb{C}^{N_E \times N_b}$ and $G \tilde{Z} \in \mathbb{C}^{N_E \times (N_A - N_B)}$ are mutually independent complex Gaussian random matrices with i.i.d. entries [23] Th. 1]. We can write

$$E \left( \log \left| \frac{I_{N_b} + \alpha (G \tilde{V}_j)(G \tilde{V}_j)^H + \alpha \beta (G \tilde{Z})(G \tilde{Z})^H}{I_{N_b} + \alpha \beta (G \tilde{Z})(G \tilde{Z})^H} \right| \right) \quad (65)$$

as the average of a function of $N_E \times N_A$ i.i.d complex Gaussian random variables $\sim N_{C}(0,1)$.

Similarly, with unlimited feedback, we have

$$E \left( \log \left| \frac{I_{N_b} + \alpha (G \tilde{V}_j)(G \tilde{V}_j)^H + \alpha \beta (G \tilde{Z})(G \tilde{Z})^H}{I_{N_b} + \alpha \beta (G \tilde{Z})(G \tilde{Z})^H} \right| \right) \quad (66)$$

as the average of a function of $N_E \times N_A$ i.i.d complex Gaussian random variables $\sim N_{C}(0,1)$.

From (65) and (66), we have

$$E \left( \log \left| \frac{I_{N_b} + \alpha (G \tilde{V}_j)(G \tilde{V}_j)^H + \alpha \beta (G \tilde{Z})(G \tilde{Z})^H}{I_{N_b} + \alpha \beta (G \tilde{Z})(G \tilde{Z})^H} \right| \right) \quad (67)$$

From (21), (23), (64) and (67), $E(\Delta R_S)$ can be upper bounded by

$$E(\Delta R_S) \leq E \left( \log \left| I_{N_b} + \alpha \gamma HH^H \right| \right) - E \left( \log \left| I_{N_b} + \theta_{\min} HH^H \right| \right) + E \left( \log \left| I_{N_b} + \alpha \beta \gamma (H \tilde{Z})(H \tilde{Z})^H \right| \right). \quad (68)$$

We then estimate the third term in (68). Let $\lambda_1, \ldots, \lambda_{N_A}$ be the eigenvalues of $HH^H$. We have

$$HH^H = \tilde{V} \Lambda \tilde{V}^H \text{ and } \Lambda = \text{diag} ([\lambda_1, \ldots, \lambda_{N_A}]). \quad (69)$$

Recalling the fact that for a Wishart matrix, its eigenvalues and eigenvectors are independent. Therefore $\tilde{V}$ and $\Lambda$ are independent. This allows us to bound the third term in (68).
Note that $\mathbf{GZ}$ (or $\mathbf{GZ}$) is a complex Gaussian random vector with i.i.d. entries $\left[ \mathbb{G} \right]$. By substituting (73) and (74) into (72), we obtain (44).
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