Superembedding approach to superstring in $\text{AdS}_5 \times S^5$ superspace

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We review the spinor moving frame formulations and generalized action principle for super-$p$-branes, describe in detail the superembedding approach to superstring in general type IIB supergravity background and present the complete superembedding description of type IIB superstring in the $\text{AdS}_5 \times S^5$ superspace.

This contribution is devoted to the memory of Wolfgang Kummer who untimely left us in 2007. We collaborated with him several years beginning, in 1996, by studying gravity induced on the worldvolume of a brane;\textsuperscript{11} this was one of the pre-Rundall-Sundrum Brane World scenarios (see also\textsuperscript{5}). Search for its supersymmetric generalizations led us to thinking on a new form of D$p$-brane actions\textsuperscript{12} and to studying the super-D9-brane dynamics.\textsuperscript{2} This line was then continued by attacking the problem of supersymmetric Lagrangian description of the interacting superbrane systems\textsuperscript{13} which, in my opinion, still remains open as far as the commonly accepted candidate action for coincident D$p$–branes\textsuperscript{50} does not possess neither supersymmetry nor Lorentz symmetry.

Among the main tools in our studies were embedding and superembedding approaches to bosonic and supersymmetric branes. This is why I decided to chose for my contribution the present manuscript containing a review of the superembedding approach and its specific application for the

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case of superstring in $AdS_5 \times S^5$ superspace (see\textsuperscript{47,48} for Green–Schwarz superstring action in this superspace).

Notice that superstring in $AdS_5 \times S^5$ superspace is often called $AdS_5 \times S^5$ superstring (see\textsuperscript{3} and refs therein). However, in our opinion, this name might produce an erroneous impression that the model is essentially different from the Green–Schwarz (GS) superstring. Such a confusion might be further enlarged by an accent which is made in the literature on the fact that $AdS_5 \times S^5$ superspace (the superspace with bosonic body $AdS_5 \times S^5$) is a coset of $SU(2,2|4)$ supergroup. Although important, this does not change the fact that this ‘$AdS_5 \times S^5$ superstring’ is just a particular case of the GS superstring in a curved superspace\textsuperscript{37}. So is its type IIA counterpart, ‘$AdS_4 \times \mathbb{CP}^3$ superstring’, which attracted recently much attention, but is not a model on a coset of supergroup, just because the type IIA supergravity superspace with the bosonic body $AdS_4 \times \mathbb{CP}^3$ is not a coset.\textsuperscript{35}

Thus we prefer to formulate our problem as superembedding description of the GS superstring model in $AdS_5 \times S^5$ superspace. On one hand, this formalism can be applied to study the N=16 two dimensional supergravity induced on the worldsheet superspace of the superstring moving in AdS superspace. And, in this respect, it is proper for the present volume because two dimensional gravity and supergravity model were always in the center of Wolfgang’s interests, see e.g.\textsuperscript{22,38,44}

On the other hand, the results of this manuscript can be useful in further study of classical and quantum $AdS_5 \times S^5$ superstring, which is of current interest for the applications of AdS/CFT correspondence\textsuperscript{a}.

1. Introduction

The standard GS superstring action\textsuperscript{36} is based on embedding of a bosonic surface $W^2$ in the target superspace $\Sigma^{(D|n)}$ ($D = 3, 4, 6, 10$, $n = 2(D - 2)$ for heterotic and type I and $n = 4(D - 2)$ for type II superstrings). This embedding is described by the bosonic and fermionic coordinate functions

$$W^{p+1} \in \Sigma^{(D|n)} : \quad \hat{Z}^M(\xi) = (\hat{x}^\mu(\xi), \hat{\theta}^\alpha(\xi)),$$

where $p=1$ and $\xi^m = (\tau, \sigma)$ are local coordinates on $W^2$. The more ‘ancient’ Ramond–Neveu–Schwarz (RNS) or spinning string, which becomes equivalent to the GS sigma model on the quantum level and after

\textsuperscript{a}See, for instance,\textsuperscript{21,27} where the $AdS_5 \times S^5$ superstring was used to reveal the mysterious dual superconformal symmetry of the N=4 SYM amplitudes.
For all presently known superbranes the embedding (2) of the worldsheet superspace $W^{(2[1+1])}$ into the spacetime $M^D = \Sigma^{(D|0)}$, described by D bosonic superfields $\hat{X}^{\mu}(\xi, \eta, \bar{\eta}) = \hat{x}^{\mu}(\xi) + i\eta \bar{\psi}^{m}(\xi) + i\bar{\eta} \psi^{m}(\xi) + \cdots$ depending on two bosonic ($\xi^m$) and complex fermionic coordinate $\eta$ (or real fermionic coordinate in the case of heterotic string). There are some known obstacles for extending such a description to supermembrane and other branes.\textsuperscript{25,43}

Following,\textsuperscript{57} the superembedding approach, developed in\textsuperscript{16} for 10D superstrings and 11D supermembrane, and applied in the first studies of dynamics of Dirichlet $p$-branes (Dp–branes) and M-theory 5-brane (M5-brane) in seminal papers\textsuperscript{41} and,\textsuperscript{42} describes strings and branes by embedding of a \textit{worldvolume superspace} $W^{(p+1|n/2)}$ into the target superspace $\Sigma^{(D|n)}$.

Let us denote the $d = p + 1 \leq D$ local bosonic coordinates and $n/2$ fermionic coordinates of $W^{(p+1|n/2)}$ by $\zeta^M = (\xi^m, \eta^q)$. Then the embedding of $W^{(p+1|n/2)}$ into the tangent superspace $\Sigma^{(D|n)}$ with coordinates $Z^M = (x^\mu, \theta^q)$ can be described parametrically by specifying the set of coordinate super-functions, the \textit{worldvolume superfields} $Z^M(\zeta) = \hat{Z}^M(\xi^m, \eta^q)$

$$W^{(p+1|n/2)} \in \Sigma^{(D|n)} : \quad Z^M = \hat{Z}^M(\zeta) = (\hat{x}^\mu(\xi, \eta), \hat{\theta}^q(\xi, \eta)).$$

Here, $\mu = 0, 1, \ldots, (D - 1)$, $\alpha = 1, \ldots, n$, $m = 0, 1, \ldots, p$ and $q = 1, \ldots, \frac{n}{2}$. Notice that the number of fermionic ‘directions’ $\eta^q$ of the worldvolume superspace are usually chosen to be one–half of the number of fermionic dimensions of the target superspace.\textsuperscript{b} This is proper to replace all the $\kappa$–symmetries\textsuperscript{31,36,54} of the standard, Dirac–Nambu–Goto type super-p-brane actions\textsuperscript{241} by the local worldvolume supersymmetry \textsuperscript{c}, thus realizing the idea developed for D=3,4 superparticle in\textsuperscript{57} \textsuperscript{d}.

1.1. \textit{Superembedding equation}

For all presently known superbranes the embedding (2) of their maximal worldvolume superspace $W^{(p+1|\frac{n}{2})}$ into the target superspace $\Sigma^{(D|n)}$ obeys

\textsuperscript{b} For $N = 1 = n = \delta^D_0$ is the number of values of the minimal D–dimensional spinor index; for $D \neq 2 (mod 8)$ this is $n = 2[D/2]$, where is $[D/2]$ is the integer part of $D/2$; and for $D = 2 (mod 8)$ it is $n = 2[D/2]-1$.

\textsuperscript{c} Under the standard super-p-branes we mean supersymmetric extended objects the ground state of which are 1/2 BPS states, i.e. preserve 1/2 of the tangent space supersymmetry reflected by $n/2$ parametric $\kappa$–symmetry of their worldvolume actions. See\textsuperscript{14} as well as\textsuperscript{9,65} for the actions in enlarged (tensorial) superspaces with additional tensorial coordinates (see\textsuperscript{14,30,60} and refs therein) describing the excitations of $k/32$ BPS states, including the $k = 31$ models possessing the properties of BPS preons.\textsuperscript{8}

\textsuperscript{d} See\textsuperscript{58,49} and refs. therein for formulations of superbranes in the worldvolume superspaces with less than $n/2$ fermionic ‘directions’.
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the superembedding equation. For D=3,4 superparticle this was obtained in\(^{57}\) by varying a superfield action (called STV action in ninties). To write its most general and universal form for a super–p–brane in D-dimensional supergravity background, let us denote the supervielbein of the worldvolume superspace \( W^{[p+1]} \) by

\[
e^A = d\zeta^M e_M A(\zeta) = (e^a, e^q), \quad a = 0, 1, \ldots, p, \quad q = 1, \ldots, n/2, \quad (3)
\]

and decompose the pull–back \( \hat{E}^A := E^A(\hat{Z}) = d\hat{Z}^M E_M A(\hat{Z}) \) of the supervielbein of the target superspace, \( E^A = d\hat{Z}^M E_M A(\hat{Z}) = (E^a, E^\alpha) \) \((a = 0, 1, \ldots, (D-1), \alpha = 1, \ldots, n)\), on the basis of (3). In general, such a decomposition reads

\[
\hat{E}^A := E^A(\hat{Z}) = d\hat{Z}^M E_M A(\hat{Z}) = e^b \hat{E}^b_A + e^q \hat{E}^q_A, \quad (4)
\]

where \( \hat{E}^b_A := e^b_M \partial_M \hat{Z}^M E_M A(\hat{Z}) \) and \( \hat{E}^q_A := e^q_M \partial_M \hat{Z}^M E_M A(\hat{Z}) \) are, respectively, bosonic and fermionic components of the pull–back the supervielbein form. The superembedding equations states that the fermionic component of the pull–back of the bosonic supervielbein form vanishes,

\[
[\hat{E}^q_A := \nabla_q \hat{Z}^M E_M A(\hat{Z}) = 0, \quad \nabla_q := e_q^M (\zeta) \partial_M]. \quad (5)
\]

For higher dimensional superbranes of sufficiently large co–dimensions the superembedding equation contains equations of motion among their consequences. This was shown for M2-brane and D=10 type II superstring in,\(^{16}\) for M5-brane in\(^{42}\) and for Dp–branes with \( p \leq 5 \) in\(^{41}\) (the ‘boundary’ \( p \leq 5 \) was established in\(^{28}\)). Hence, in these cases, the description of the classical super-p–brane dynamics by this equation is complete. Moreover, if several types of \( D–\)dimensional p–branes exist, the superembedding equation provides their universal description (see\(^{6}\) for such a universal description of fundamental type IIB superstring and D1–brane and\(^{23}\) for the \( SL(2) \) covariant formulation providing a unified descriptions of all the actions of p–branes related to the Dp-brane by \( SL(2) \) transformations).

On the other hand, this on-shell nature of the superembedding equation prevents from the constructing the complete worldvolume superfield action of the STV type (see\(^{57}\) and\(^{56}\) for the review and further references). A universal although non-standard Lagrangian framework for the superembedding approach is provided by the generalized action principle, proposed in\(^{18}\) for superstrings and \( D = 11 \) supermembrane and in\(^{17}\) for the case of super-Dp–branes. This produces the superembedding equation in its equivalent form

\[
\hat{E}^i(\zeta) := d\hat{Z}^M (\zeta) E_M b_i(\hat{Z}(\zeta)) u^i(\zeta) = 0, \quad (6)
\]
where $u^i_b$ are $(D - p - 1)$ vectors orthogonal to the worldsheet superspace. These moving frame variables or Lorentz harmonics (vector harmonics) will be the subject of the next section.

2. Spinor moving frame formulation and generalized action principle for super-$p$-branes

2.1. Vector harmonics as moving frame adapted to (super)embedding

The standard formulations of superstring,\cite{36} M2-brane (supermembrane)\cite{24} and super-$p$-branes\cite{1} is based on embedding (1) of the bosonic worldvolume $W^{p+1}$ into the tangent superspace $\Sigma^{(D|n)}$.

If the worldvolume $W^{p+1}$ is flat, one always can chose a special Lorentz frame with $p + 1$ vectors being tangential and the remaining $D - p - 1$ vectors - orthogonal to $W^{p+1}$. In general this also can be done, but locally.

It is convenient to use the dual language of the differential forms and to consider the pull–back

$$\hat{E}^a := E^a(Z) = dZ^M(\xi) E_M^a(Z) = d\xi^m \partial_m Z^M E_M^a(Z) =: d\xi^m \hat{E}^a_m \quad (7)$$

of the bosonic supervielbein of the target superspace $E^a := dZ^M E_M^a(Z)$ to the worldvolume $W^{p+1}$ with local coordinates $\xi^m$, $m = 0, 1, \ldots, p$. Only $(p + 1)$ of the $D$ one–forms $\hat{E}^a_m$ may be independent on $W^{p+1}$. This is tantamount to saying that there exist $(D - p - 1)$ linear combinations of $\hat{E}^a_m$ that vanish on $W^{p+1}$. We can express the above statement by the following embedding equation

$$\hat{E}^i(\xi) := \hat{E}^a u^i_a(\xi) = 0 \quad , \quad i = 1, \ldots, (D - p - 1) \quad , \quad (8)$$

where $u^i_a(\xi)$ are some coefficient dependent on the point of $W^{p+1}$. They define $(D - p - 1)$ vectors which are linear independent and orthogonal to the worldvolume $W^{p+1}$. Thus one may chose them orthogonal one to another and normalized (on -1 as the vectors are spacelike and we are working with ‘mostly minus’ metric conventions)

$$u^a_i u^a_j = -\delta^{ij} \quad . \quad (9)$$

One can complete the set of the $(D - p - 1)$ vectors $u^a_i$ orthogonal to the worldvolume by the set of the $(p+1)$ vectors $u^a_b$ tangential to $W^{p+1}$ (also orthogonal among themselves and normalized). Then the $D \times D$ moving
frame matrix constructed from $u^b_a$ and $u^j_a$ obeys $U\eta U^T = \eta$,

$$U^{(\underline{b})} := \begin{pmatrix} u^b_a \\ u^j_a \end{pmatrix}, \quad U^T \eta U = \eta \iff \begin{cases} u^{ca} u^b_a = \eta^{ab}, \\ u^{ca} u^j_a = 0, \\ u^{ai} u^j_a = -\delta^{ij}, \end{cases}$$

by construction, and, hence, belongs to the fundamental representation of the Lorentz group $SO(1,D-1)$,

$$U^{(\underline{b})} := \begin{pmatrix} u^b_a \\ u^j_a \end{pmatrix} \in SO(1,D-1).$$

The splitting of the $D \times D$ matrix $U$ on the $D \times (p+1)$ and $D \times (D-p-1)$ blocks (11) is invariant under the (right multiplication by the matrix from the) $SO(1,p) \otimes SO(D-p-1)$ subgroup of the Lorentz group $SO(1,D-1)$. In the Lorentz harmonic approach of\textsuperscript{19,20} this gauge invariance is usually considered as an identification relation on the set of moving frame variables making possible to consider them as 'homogeneous' coordinate for the coset

$$\left\{ u^b_a, u^j_a \right\} = \frac{SO(1,D-1)}{SO(1,p) \otimes SO(D-p-1)}.$$  \hspace{1cm} (12)

This was the reason to call these moving frame variables Lorentz harmonics,\textsuperscript{4,19} following the spirit of\textsuperscript{34} where the notion of harmonic variables was introduced to construct the unconstrained superfield formulation of the $N = 2$ supersymmetric theories.

Reordering the line of arguments one can start from (12) and notice that $SO(1,D-1)$ group valued moving frame matrix $U$ (11) can be used to define, starting from $\hat{E}^{(\underline{a})}$, another vielbein attached to the worldvolume,

$$\hat{E}^{(\underline{a})} := \hat{E}^{(\underline{a})} u^a = (\hat{E}^a, \hat{E}^i).$$

This vielbein is adapted to the embedding of $W^{p+1}$ into the $D$-dimensional spacetime if the pull–back of $D - p - 1$ 'orthogonal' forms

$$\hat{E}^i := \hat{E}^a u^a_i$$

vanishes, \textit{i.e.} if embedding equation (8) is valid. The $(p+1)$ 'tangential' forms $\hat{E}^a$ defined with the use of the 'parallel' vector harmonics $u^a$ can be used as a vielbein on $W^{p+1}$:

$$\hat{E}^a := \hat{E}^{a} u^a = (\hat{E}_a, \hat{E}_i).$$

\textsuperscript{4}See,\textsuperscript{55,52,45,64,4,33} and\textsuperscript{33} for earlier works.
One says that this vielbein is induced by the embedding.

Now one sees that the superembedding equation (6) is just the straightforward supersymmetric generalization of the above embedding equation (8). However, in contrast to (8), the superembedding equation cannot be derived by imposing a conventional orientation conditions, and in this sense is nontrivial.

2.2. Action of the moving frame formulation

This is the induced vielbein (16) which can be understood as a square root from the induced metric

\[ g_{mn}(\xi) = \hat{E}_m^a \hat{E}_n^a \]  

(17)

provided the embedding equation (8) holds,

\[ E^i = 0 \Rightarrow g_{mn}(\xi) = \hat{E}_m^a \hat{E}_n^a = \hat{E}_m^a \hat{E}_n^a = e_m^a e_n^a . \]  

(18)

As a result, the invariant volume element on \( W^{p+1} \), this is to say the Nambu–Goto term for a (super)–p–brane,

\[ S_{p}^{N−G} := \int d^{p+1}\xi \sqrt{|g|} := \int d^{p+1}\xi \sqrt{|\det(\hat{E}_m^a \hat{E}_n^a)|} . \]  

(19)

can be equivalently presented in terms of \( e^a := \hat{E}^a \) forms,

\[ E^i = 0 \Rightarrow d^{p+1}\xi \sqrt{|g|} = \frac{\epsilon_{a_0...a_p} \hat{E}^{a_0} \wedge ... \wedge \hat{E}^{a_p}}{(p+1)!} =: \hat{E}^{\wedge(p+1)} . \]  

(20)

Now, if one uses \( \hat{E}^{\wedge(p+1)} \) instead of the Nambu–Goto term (20) in the standard super–p–brane action,

\[ S_p^{standard} = S_p^{N−G} + S_p^{WZ} := \int d^{p+1}\xi \sqrt{|g|} − p \int_{W^{p+1}} \hat{B}_{p+1} , \]  

(21)

one arrives at the so–called moving frame or Lorentz harmonic action

\[ S_p = S_p^{LH} + S_p^{WZ} := \int_{W^{p+1}} \hat{E}^{\wedge(p+1)} − p \int_{W^{p+1}} \hat{B}_{p+1} , \]  

(22)

where \( \hat{E}^a = \hat{E}_m^a \), \( u^a \), Eq. (15) and \( u^a \) are \( (p+1) \) orthonormal \( D \)–vectors, \( u^a u^b = \eta^{ab} \) (see (10)). These are the auxiliary variable entering the action without derivatives. The last term of the standard action, \( −p \int \hat{B}_{p+1} \), which
remains in the same form in the spinor moving frame formulation, is the so-called Wess–Zumino (WZ) term. It is given by the integral of the pull-back the worldvolume \( W^{p+1} \) of the gauge \((p+1)\)–superform \( B_{p+1} \) restricted by the superspace constraints imposed on its (super)field strength

\[
H_{p+2} = dB_{p+1} = \propto \Gamma^{(p)}_{\alpha \beta} \wedge E^\alpha \wedge E^\beta + \mathcal{O}(E^{(p+1)}) , \tag{23}
\]

\[
\Gamma^{(p)}_{\alpha \beta} := \frac{1}{p!} E_{a_1} \wedge \ldots \wedge E_{a_p} \Gamma_{a_1, \ldots, a_p, \alpha \beta} . \tag{24}
\]

The relation between coefficient for the first term in the r.h.s. of (23) (replaced by \( \propto \) symbol in our schematic consideration) and the coefficient in front of the WZ term in the action is fixed by the requirement of \( \kappa \)-symmetry.

As it has been noticed above, on the surface of embedding equation (8) the moving frame action (22) coincides with the standard one, Eq.(21),

\[
S_p|_{\hat{E}^i=0} = S_p^{\text{standard}} . \tag{25}
\]

The proof of the classical equivalence will then be completed by showing that the embedding equation follows from the moving frame action (22).

This is indeed the case, the embedding equation appears as a result of varying the auxiliary moving frame variables in the action (22),

\[
\delta_{\epsilon_u} S_p = 0 \quad \Rightarrow \quad \hat{E}^i := \hat{E}^a u^i_a = 0 . \tag{26}
\]

As the harmonics are constrained variables, the variation in Eq. (26) requires some comments.

### 2.2.1. Variations and derivatives of the harmonic variables

Both the spaces of the variations \( \delta u \) of certain variables \( u \) and of the derivatives \( d u \) of such variables can be identified with the elements of the fiber of the tangent bundle over the space of this variables, i.e. with elements of the linear space tangent to the space of the \( u \) variables. In the case of Lorentz harmonics the variables \( u \) are elements of the Lorentz group valued matrix \( U \), Eq. (11) (see also (10)). The space tangent to the Lorentz group is isomorphic to the Lie algebra spanned by antisymmetric \( D \times D \) matrices. This well known fact can be expressed by

\[
dU^{(\omega)}_{\underline{a}} = U_{\underline{a}}^{(\omega)} \Omega_{(\underline{c})}^{(\underline{d})} \equiv U_{\underline{a}}^{(\omega)} \Omega^{\underline{c}}_{\underline{d}} \quad \Leftrightarrow \quad \begin{cases} du^b_{\underline{a}} = u^b_{\underline{c}} \Omega^{\underline{c}}_{\underline{b}} + u^b_{\underline{a}} \Omega^{\underline{a}}_{\underline{b}} , \\ du^i_{\underline{a}} = -u^j_{\underline{a}} \Omega^{ji} + u^i_{\underline{a}} \Omega^i_{\underline{b}} \end{cases} , \tag{27}
\]
which is just an equivalent representation of the definition of the Cartan forms $U^{-1} dU$ for the Lorentz group in which $U^{-1} = \eta^T \eta$,

$$U^2 d\Omega(U) = -\Omega(U)\equiv \left\{ \begin{array}{cc} \Omega^{ab} & \Omega^{ij} \\ -\Omega^{bi} & \Omega^{ij} \end{array} \right\}.$$  

(28)

As far as the harmonics are treated as homogeneous coordinates of the coset $SO(1, D-1)/SO(1,p) \otimes SO(D-p-1)$, the Cartan forms $\Omega^{ab} = -\Omega^{ba}$ and $\Omega^{ij} = -\Omega^{ji}$ have the properties of the connection under the $SO(1,p)$ and $SO(D-p-1)$ local gauge symmetries while the set of one–forms $\Omega^{ai} = u^a_i du^i$ provide the vielbein for the coset $SO(1, p) \otimes SO(D-p-1)$.

One should notice that, in general, the transformation of $SO(1, p)$ and $SO(D-p-1)$ symmetries are local on the coset space itself; however, when harmonics are used to describe a $p$–brane, $U = U(\xi)$, this local $SO(1, p) \otimes SO(D-p-1)$ symmetry of Lorentz harmonic space (or superspace) gives rise to the worldvolume local gauge symmetries with $\xi$–dependent parameters.

The same line of reasoning can be applied to the variations of the Lorentz harmonic variables in some action functional. Formally, the corresponding equation can be derived by using the Lie derivative $L_\delta := i_\delta d + di_\delta$ where the second terms will give zero contributions for a zero–forms so that $\delta u = i_\delta du$.

Applying this simple equation to (27) one finds

$$\begin{align*}
\delta U^i &= U^i_i \delta \Omega \quad \Leftrightarrow \quad \\
\delta u_a^b &= u_a^i \delta \Omega^i_b + u_a^i \delta \Omega^i_b, \\
\delta u_i^a &= -u_i^j \delta \Omega^j_a + u_i^j \delta \Omega^j_a,
\end{align*}$$

(29)

where $i_\delta \Omega(\xi) = -i_\delta \Omega(\xi) = \{i_\delta \Omega^{ab}, i_\delta \Omega^{ij}, i_\delta \Omega^{aj}\}$ are parameters of independent variations which can be identified with the $i_\delta$ contractions of the Cartan forms (hence the notation). Clearly $i_\delta \Omega^{ab} = -i_\delta \Omega^{ba}$ parametrize the worldsheet Lorentz group $SO(1,p)$, $i_\delta \Omega^{ij} = -i_\delta \Omega^{ji}$ parametrize the transformations of the structure group $SO(D-p-1)$ of the normal bundle and $i_\delta \Omega^{bi}$ provides a basis for independent variations of the coset $SO(1, D-1)/SO(1,p) \otimes SO(D-p-1)$.

2.2.2. Lorentz harmonics and generalized Cartan forms for superbrane in curved (super)space

In the curved superspace one has to consider the local Lorentz $SO(1,D-1)$ symmetry and the Cartan forms as defined in (28) or (27) are not covariant. Their covariant counterparts are defined with the use of Lorentz covariant
derivatives \((d + w)\), so that
\[
\begin{aligned}
DU^a_\mu &= du^a_\mu + u^b_\mu \delta e^a_b - u^c_\mu \Omega^b = u^b_\mu \Omega^b, \\
DU^i_\mu &= du^i_\mu + w^{ij} \delta w^j_\mu + u^j_\mu \Omega^i = u^j_\mu \Omega^i,
\end{aligned}
\]  (30)
or
\[
\Omega^{\mu} = U^{\mu}(d + \hat{w})U^{\mu}_\mu \quad \iff \quad \begin{aligned}
\Omega^{\mu} &= u^b_\mu [(d + \hat{w})u^a_b] = \Omega_0^{\mu} + (u \hat{w}u)^{\mu ab}, \\
\Omega^{ij} &= u^a_i [(d + \hat{w})u^b_j] = \Omega_0^{ij} + (u \hat{w}u)^{ij}, \\
\Omega^{ai} &= u^a_i [(d + \hat{w})u^a_i] = \Omega_0^{ai} + (u \hat{w}u)^{ai},
\end{aligned}
\]  (31)
where \(\hat{w} = dZ^M w_M Z^{\hat{a}}(Z)\) is the pull–back of the tangent superspace spin connection \(w_{a \mu} = dZ^M w_M Z^{\hat{a}}(Z)\) and \(\Omega_0^{\mu}, \Omega_0^{ij}, \Omega_0^{ai}\) are the Cartan forms as defined in Eqs. (28). Then \(D\) in (30) is the derivative covariant both with respect to the local Lorentz \(SO(1, D - 1)\) transformations and worldsheet gauge symmetry \(SO(1, p) \otimes SO(D - p - 1)\). This implies that
\[
D\hat{E}^a := d\hat{E}^a - \hat{E}^b \wedge \Omega^b = \hat{T}^a u^a_{\mu} + \hat{E}^a \wedge \Omega^a,
\]  (32)
where \(\Omega^a\) is the generalized Cartan form (31c) and \(\hat{T}^a\) is the pull–back of the superspace torsion \(T^a = DE^a - E^a \wedge u^a_{\mu}\). This is restricted by supergravity constraints
\[
T^a = DE^a := dE^a - E^a_i \wedge u^a_{\mu} = -iE^a \wedge E^b \Gamma^a_{\alpha, b}
\]  (33)
With this in mind one can further specify Eq. (32)
\[
D\hat{E}^a = -i\hat{E}^a \wedge \hat{E}^b \Gamma^a_{\alpha, b} + \hat{E}^a \wedge \Omega^a = \hat{T}^a u^a_{\mu}.
\]  (34)
Using the formal \(i_\delta\) symbol of the previous subsection [extending its definition by \(i_\delta dZ^M := \delta Z^M\) and \(i_\delta (\Omega_q \wedge \Omega_p) = \Omega_q \wedge i_\delta \Omega_p + (-)^p i_\delta \Omega_q \wedge \Omega_p\) for any q- and p-forms \(\Omega_q\) and \(\Omega_p\)] one can write the arbitrary variation of the ‘tangential’ supervielbein \(\hat{E}^a\) (modulo the \(SO(1, p)\) symmetry transformations) in the form
\[
\delta \hat{E}^a = D(i_\delta \hat{E}^a) - 2i \hat{E}^a \Gamma^a_{\alpha, b} \delta_{\alpha, b} + \hat{E}^i \delta \Omega^a - \Omega^a \delta \hat{E}^a,
\]  (35)
where \(i_\delta \Omega^a\) are basic variation of the harmonic variables, Eq. (29), corresponding to the coset \(SO(1, D - 1)/SO(1, p)\) and
\[
\delta \hat{E}^a = \delta \hat{Z}^M E_M (Z) u^a_{\mu} (\xi), \quad i_\delta \hat{E}^i = \delta \hat{Z}^M E_M (Z) u^i_{\mu} (\xi), \quad i_\delta \hat{E}^a = \delta \hat{Z}^M E_M (Z).
\]  (36)
These provide the covariant basis for the variations of the bosonic and fermionic coordinate functions \(\delta \hat{Z}^M\).
2.2.3. Equations of motion of the moving frame action

In the above notation general variation of the action with respect to the coordinate functions and harmonic variables,

\[ \delta S_p = \int_{W^{p+1}} [\hat{E}^{\wedge}_a \wedge \delta \hat{E}^a - p \delta \hat{B}_{p+1}] \]  

(with \( \hat{E}^{\wedge}_a := \frac{1}{p!} \varepsilon_{a_1 \ldots a_p} \hat{E}^{a_1} \wedge \ldots \wedge \hat{E}^{a_p} \)), can be written as

\[ \delta S_p = \int_{W^{p+1}} \hat{E}^{\wedge}_a \wedge [D_i \hat{E}^a + i_{\hat{H}} \hat{E}^a + \hat{E}^{\wedge} i_{\hat{E}} \Omega^{ai} - \Omega^{ai} i_{\hat{E}} \hat{E}^i] - p \int_{W^{p+1}} i_{\hat{H}} \hat{B}_{p+2} . \]  

(39)

Using (33), (23) and the identity \( \hat{E}^{\wedge}_a 1 \ldots \hat{E}^{\wedge} p \hat{\Gamma}^{a_1 \ldots a_p} = \propto \hat{E}^{\wedge}_p \hat{\Gamma} \) with \( \hat{\Gamma} := \frac{1}{p!} \sum_{a_0 \ldots a_p} \varepsilon_{a_0 \ldots a_p} u^0_{a_0} \ldots u^i_{a_p} \hat{\Gamma}_{a_0 \ldots a_p}^{\hat{\Gamma}} \) \( \hat{\Gamma} \)

obeying \( \hat{\Gamma} \hat{\Gamma} = I, \) \( tr \hat{\Gamma} = 0, \) one finds

\[ \delta S_p = \int_{W^{p+1}} \hat{E}^{\wedge}_a \wedge \hat{E}^{\wedge} \hat{\Gamma} \]  

(41)

The second term in (41) contains the basic variations of the harmonic variables and is used to obtain the embedding equation (8). The third term produces the bosonic equations of motion of the \( p \)-brane in the form

\[ \Omega^{ai} = \varepsilon^{a_1 \ldots a_p} H_{a_1 \ldots a_{p+1}} + fermion \ contributions , \]  

(42)

which generalizes the minimal surface equation \( \Omega^{ai} = 0 \) for the case of nonvanishing background flux (see sec. 3.3.4 for more details in \( p=1 \) case). Finally the first term in (41) contains the fermionic variation

\[ i_{\hat{E}} \hat{E}^a := \delta \hat{Z}^M E_M^a (\hat{Z}) \]

and produces the fermionic equation for super-\( p \)-brane

\[ \Psi_{p+1} a(\hat{Z}) := \hat{E}^{\wedge}_a \wedge \hat{E}^{\wedge} \hat{\Gamma} \]  

(43)

\( ^1 \)We do not write explicitly the terms proportional to the ‘tangential’ bosonic variations \( i_{\hat{E}} \hat{E}^a \), denoting them in (41) by \( O(i_{\hat{E}} \hat{E}^a) \), as they do not produce any independent equation. This statement manifests a Noether identity which corresponds to the reparametrization gauge symmetry, i.e. the worldvolume diffeomorphism invariance.
2.3. Irreducible κ-symmetry. Spinor harmonics enter the game

The presence of the projector \((I - \tilde{\Gamma})\) makes half of the fermionic equations (43) to be satisfied identically,

\[
\Psi_{p+1}(\tilde{Z})(I + \tilde{\Gamma}) = 0 .
\] (44)

Eq. (44) is the Noether identity reflecting a fermionic gauge symmetry of the action (22), the κ-symmetry with the basic variations

\[
i_\kappa \tilde{E}^\alpha := \delta_\kappa \tilde{Z}^M(\xi)E_M^\alpha(\tilde{Z}(\xi)) = 0 ,
\]

\[
i_\kappa \tilde{E}_{\dot{\alpha}} := \delta_\kappa \tilde{Z}^M(\xi)E_M^{\dot{\alpha}}(\tilde{Z}(\xi)) = (I + \tilde{\Gamma})^\alpha_{\beta}\kappa^\beta(\xi) .
\] (45)

These are formally the same as the ones for the infinitely reducible κ–symmetry of the standard action (21). However, the presence of additional variables makes the κ–symmetry of the action (22) irreducible in contradistinction to the κ–symmetry of the original action (21).

To see this one should notice that, allowing for additional variables, one can factorize the κ-symmetry projector. Within the Lorenz harmonic approach such a factorization reads

\[
(I^-_{(-)} \tilde{\Gamma})^\alpha_{\beta} = 2v^\ddot{\alpha}_{\dot{\beta}} v^{\dot{\alpha}}_{\ddot{\beta}} ,
\]

\[
(I^+_{(+)})^\alpha_{\beta} = 2v^\ddot{\beta}_{\dot{\alpha}} v^{\dot{\beta}}_{\ddot{\alpha}} ,
\] (46)

where \(\ddot{\alpha}, \dot{\alpha}\) are indices of the spinor representations of \(SO(1,p)\) (the same or different depending on the values of \(D\) and \(p\)) and \(\dot{q}, \ddot{q}\) are indices of the (same or different) representations of \(SO(D - p - 1)\) and

\[
V^{(1)}(\alpha) := \begin{pmatrix} v^{\ddot{\beta}}_{\dot{\alpha}} \\ v^{\dot{\beta}}_{\ddot{\alpha}} \end{pmatrix} \in Spin(1, D - 1) \quad (47)
\]

is the \(Spin(1, D - 1)\)-valued matrix of the spinor moving frame variables or spinor harmonics. These variables are, the ‘square root’ of the vector harmonics (11), (10) in the sense of that the following constraints hold

\[
VT(\omega)V^T = \Gamma(\omega)U(\omega) \quad \Rightarrow \quad \begin{cases}
VT^aV^T = \Gamma^a_{\alpha}\omega^\alpha \\
VT^1V^T = \Gamma^1_{\dot{\alpha}}\omega^\dot{\alpha} \end{cases} .
\] (48)

Eqs. (48) express the well known fact of that the gamma–matrices are Lorentz invariant. An equivalent form of these constraints is given by

\[
(V^{-1})^{\dot{\alpha}}{\ddot{\beta}}V^{-1} = \Gamma^{\dot{\alpha}}_{\ddot{\beta}}U(\omega)^{-1} = \Gamma^a_{\alpha}u^a_{\alpha} - \Gamma^1_{\dot{\alpha}}u^1_{\dot{\alpha}} ,
\] (49)

where \(V^{-1} := V(\omega)^{-1}\) is the matrix inverse to (47),

\[
V^{(\alpha)} := \begin{pmatrix} v^{\ddot{\alpha}}_{\dot{\beta}} \\ v^{\dot{\alpha}}_{\ddot{\beta}} \end{pmatrix} : \quad V^{(\alpha)}V^{(\beta)} = \delta^{(\beta)}_{(\alpha)} = \begin{pmatrix} \delta_{\ddot{\alpha}}{\ddot{\beta}} & 0 \\ 0 & \delta_{\dot{\alpha}}{\dot{\beta}} \end{pmatrix} .
\] (50)
The spinor moving frame variables are also called *spinorial harmonics* because they provide the homogeneous coordinates for the coset of \( \text{Spin}(1, D - 1) \) group doubly covering the coset of Eq. (12),

\[
\{ V_{\alpha}^{(\alpha)} \} := \left\{ \left( v_{\beta}^{\tilde{\alpha} q}, v_{\beta}^{\tilde{\alpha} \tilde{q}} \right) \right\} = \frac{\text{Spin}(1, D - 1)}{\text{Spin}(1, p) \otimes \text{Spin}(D - p - 1)} .
\] (51)

Now, using the factorization (46) one can write the \( \kappa \)-symmetry transformations (45) in the irreducible form

\[
i_\kappa \hat{E}^\alpha := \delta_\kappa \hat{Z}^M E_M^\alpha(\hat{Z}) = 2v^{\tilde{\alpha} \tilde{q}} \kappa^{\tilde{\alpha} \tilde{q}} ,
\] (52)

where the irreducible \( \kappa \)-symmetry parameter is

\[
\kappa^{\tilde{\alpha} \tilde{q}} := v^{\tilde{\alpha} \tilde{q}} \kappa(\xi) .
\] (53)

### 2.4. Spinor moving frame formulation of super-\( p \)-branes

The spinor moving frame formulation of super-\( p \)-brane is described by the moving frame action (22),

\[
S_p = \int_{W^{p+1}} \hat{E}^{(p+1)} - p \int_{W^{p+1}} \hat{B}_{p+1} ,
\] (54)

in which the vector Lorentz harmonics \( u^a_{\underline{L}} \) (moving frame variables), entering the definition of \( \hat{E}^{a} \) in \( \hat{E}^{a} := \frac{1}{(p+1)!} \varepsilon_{a_0...a_p} \hat{E}_{a_0} \wedge ... \wedge \hat{E}_{a_p} \), are composites of spinor harmonics as defined by the gamma–trace parts of the constraints (48),

\[
\hat{E}^a := \hat{E}_{a_{\underline{L}}} , \quad u^a_{\underline{L}} = \frac{1}{n} \text{tr}(\hat{\Gamma}_a V \Gamma^b V^T) ,
\] (55)

where \( n \) the number of values of (minimal) \( D \)-dimensional spinor indices, \( n = \delta_\alpha^a \) (see footnote b). This composite nature does not change the variation of vector harmonics, which are expressed through the Cartan forms as in (29). This is the case because the variation of spinorial harmonics are expressed through the same Cartan forms,

\[
V^{-1} \delta V = \frac{1}{4} i_\delta \Omega^{(\underline{a})} \Gamma_{(\underline{a})} = \frac{1}{4} (U^{-1} \delta U)^{(\underline{a})} \Gamma_{(\underline{a})} .
\] (56)

This reflects the fact that locally the spinorial harmonics carry the same \( D(D - 1)/2 \) degrees of freedom as the vector ones, which is tantamount to stating that the groups \( \text{Spin}(1, D - 1) \) and \( \text{SO}(1, D - 1) \) are locally isomorphic. In this sense moving frame action can always be considered as spinor moving frame action.
2.5. Generalized action principle

Generalized action principle for superbranes gives an extended object counterpart of the rheonomic approach to supergravity. It can be obtained from the spinor moving frame action by the following two steps.

First one replaces all the worldvolume fields dependent on \( W(\xi^{m}\eta^{q}) \) by superfields, depending on the local bosonic and fermionic coordinates, \( \xi^{m} \) and \( \eta^{q} \), of the worldvolume superspace \( W(p+1|n/2) \),

\[
\hat{Z}^M(\xi) \rightarrow \hat{Z}^M(\xi,\eta) \quad V(\xi) \rightarrow V(\xi,\eta) \quad \Rightarrow \quad u_{\xi}^{a}(\xi) \rightarrow u_{\xi,\eta}^{a}(\xi,\eta) .
\tag{57}
\]

Secondly, one replaces the integral over worldvolume \( W(p+1) \) by integral over a surface \( \tilde{W}(p+1) \) of maximal bosonic dimension in the worldvolume superspace. Its embedding into \( W(p+1|n/2) \) can be described by fermionic coordinate functions \( \hat{\eta}^{q}(\xi) \), which provide the counterparts of the Volkov-Akulov Goldstone fermions (these would be \( \theta^{a}(x) \) in our notation),

\[
\tilde{W}(p+1) \in W(p+1|n/2) ; \quad \eta^{q} = \hat{\eta}^{q}(\xi) .
\tag{58}
\]

This is tantamount to saying that the generalized action is given by Eq. (54) with an integral over the bosonic body of the worldvolume superspace \( W(p+1|n/2) \), which is defined by \( \hat{\eta} = 0 \) and is denoted by \( W(p+1) \), and with the Lagrangian form \( \hat{E}^{a}(p+1) - p\hat{B}_{p+1} \) constructed from superfields (57) pulled back to the surface \( \tilde{W}(p+1) \) in the worldvolume superspace, i.e. from

\[
\hat{Z}^M = \hat{Z}^M(\xi,\hat{\eta}(\xi)) \quad V = V(\xi,\hat{\eta}(\xi)) \quad \Rightarrow \quad u_{\xi}^{a} = u_{\xi,\hat{\eta}}^{a}(\xi,\hat{\eta}(\xi)) \quad \hat{E}^{a} := \hat{E}^{a}(\hat{Z}(\xi,\hat{\eta}(\xi))) u_{\xi,\hat{\eta}}^{a}(\xi,\hat{\eta}(\xi)) .
\tag{59}
\]

To resume, the generalized action functional is given by

\[
S_{p} = \int_{\tilde{W}(p+1)} (\hat{E}^{a}(p+1) - p\hat{B}_{p+1}) := \int_{\tilde{W}(p+1)} (\hat{E}^{a}(p+1) - p\hat{B}_{p+1})|_{\eta=\hat{\eta}(\xi)} ,
\tag{61}
\]

where hat (\( \hat{\cdot} \)) implies pull–back to the worldvolume superspace, and also (spinor) moving frame variables are superfields, as in (57). Thus the original moving frame action (54) is a particular case of the generalized action for \( \tilde{W}(p+1) = W(p+1) \), i.e. for \( \hat{\eta}^{q}(x) = 0 \).

The set of equations of motion for this generalized action functional includes a counterpart of (8), but for the superfields pulled back to \( \tilde{W}(p+1) \),

\[
\hat{E}^{a} := \hat{E}^{a}(\hat{Z}(\xi,\hat{\eta}(\xi))) u_{\xi,\hat{\eta}}^{a}(\xi,\hat{\eta}(\xi)) = 0 ,
\tag{62}
\]

and also the dynamical equations of motion (42), (43) but for the superfields pulled back to \( \tilde{W}(p+1) \).
Now the structure of the Lagrangian form guarantees that the action functional is independent on the choice of the surface $\tilde{W}^{p+1}$. The arbitrary changes of this surface, which are described by arbitrary variations of the fermionic functions $\delta \eta^q(\xi)$ are the gauge symmetry of the generalized action functional (61). More details on this symmetry can be found in\textsuperscript{18} as well as in very recent\textsuperscript{26} which uses a 'bottom-up' version of the generalized action principle proposed in.\textsuperscript{40} The consequence of this symmetry 'parametrized' by arbitrary $\delta \eta^q(\xi)$'s is that equations of motion, including (62) are valid on an arbitrary surface $\tilde{W}^{p+1}$ in the worldvolume superspace $W^{(p+1\mid n/2)}$. As the set of such surfaces 'covers' the whole superspace $W^{(p+1\mid n/2)}$, it is natural to assume that the equations are valid in the whole superspace. This implies, in particular, lifting of Eq. (62) to the superembedding equation in its form of Eq. (6),

$$\hat{E}^i = E^b_i(\hat{Z}(\xi, \eta))u^i_b(\xi, \eta) = 0.$$ (63)

This last stage, namely the lifting of the equations valid on an arbitrary surface in superspace to equations on the superspace, is the essence of the rheonomic principle of the group manifold approach to supergravity.\textsuperscript{29,31}

Notice that such a rheonomic lifting does not follow from the action variation, but rather constitutes an additional stage in the procedure of the generalized action principle, which should be made separately after varying the generalized action functional. In particular, the lifted equations written in terms of complete superfields (not pulled back to $M^{p+1}$) should be checked on consistency, and the consistency is not guaranteed. It have to be checked case by case, see\textsuperscript{15} for an example when the consistency does not hold.

The study of the selfconsistency condition for superembedding equation (63) can actually be used to derive equations of motion for D=10 type superstrings, D=11 supermembranes\textsuperscript{16} as well as for M5-brane\textsuperscript{42} and D=10 super-Dp-branes\textsuperscript{41} for $p \leq 5$.\textsuperscript{28} In the next section we will show explicitly how this happens in the case of superstring in general curved type IIB supergravity superspace.

3. Superembedding approach to $D = 10$ Green–Schwarz superstring in type IIB supergravity background.

To discuss the superembedding approach to superstring in a general type IIB supergravity background, we need firstly to discuss the specific features of the stringy spinor moving frame formalism.
3.1. Spinor moving frame action for superstring

The special properties of the stringy (spinor) moving frame variables, i.e. of the $SO(1, \text{D}-1) \otimes SO(1, \text{D}-2)$ harmonics used to describe the D dimensional superstring, comes from the fact that the two dimensional $SO(1,1)$ pseudo rotations of the vectors $u_a^\pm = (u^0_a, u^\#_a)$ (where the symbol $\#$ is used for $(D-1)$-the direction) is reducible and can be split onto the scaling of two light-like vectors $u_a^{++} := u^0_a + u^\#_a$ and $u_a^{--} := u^0_a - u^\#_a$ (the self-dual and anti-self-dual 2–vectors) by mutually inverse factors, $u_a^\pm \mapsto \varepsilon^\pm u_a^\pm$. The constraints on the stringy moving frame variables (vector harmonics)

$$U^{(b)}_a = (u_a^{--}, u_a^{++}, u_a^j) \in SO(1,9)$$

read (cf. (10), (11))

$$U^\dagger \eta U = \eta \iff \begin{cases} u_a^{--} u_a^{--} = 0, & u_a^{++} u_a^{++} = 0, \\ u_a^{--} u_a^{++} = 2, & u_a^{++} u_a^{--} = 0, \\ u_a^{ij} u_a^{ij} = -\delta^{ij} \end{cases}$$

and also imply

$$U \eta U^\dagger = \eta \iff \delta^{ab} = \frac{1}{2} u_a^{++} u_b^{--} + \frac{1}{2} u_a^{--} u_b^{++} - u_a^{ij} u_b^{ij}$$

Then, induced worldvolume supervielbein (55) are

$$\hat{E}^{++} := \hat{E} a^{++}, \quad \hat{E}^{--} := \hat{E} a^{--}$$

and the spinor moving frame action for superstring reads (see\textsuperscript{6})

$$S_{IIB} = \frac{1}{2} \int W^2 \hat{E}^{++} \wedge \hat{E}^{--} - \int W^2 \hat{B}_2,$$

or, using the auxiliary worldvolume vielbein forms $e^{\pm\pm}$ (see\textsuperscript{19}), as

$$S_{IIB} = \frac{1}{2} \int W^2 \left( e^{++} \wedge \hat{E}^{--} - e^{--} \wedge \hat{E}^{++} - e^{++} \wedge e^{--} \right) - \int W^2 \hat{B}_2.$$ 

Indeed, $\delta e^{\pm\pm}$ equations of motion express them through $\dot{E}^{\pm\pm}$ of (67),

$$e^{\pm\pm} = \dot{E}^{\pm\pm} := \hat{E} a^{\pm\pm}.$$ 

Substituting the algebraic equations (70) back to the first order action (69) one arrives at the second order action (68).

As we discussed in Sec. 2.5, the above spinor moving frame action can be used to construct the generalized action.\textsuperscript{18} This is given by formally the same functional (68) (or (69)) with the fields on $W^2$ replaced by the superfields and integration performed about an arbitrary surface $\tilde{W}^2$ in the worldsheet superspace $W^{(2|8+8)}$. The generalized action principle for superstring produces in particular, the superembedding equation (6).\textsuperscript{18}
3.2. Stringy $\text{Spin}(1,9)_{SO(1,1)\otimes SO(8)}$ spinorial harmonics

The D=10 stringy spinor harmonics are collected in $\text{Spin}(1,9)$ matrix

$$V_\alpha^\beta = (v_\alpha^+, v_\alpha^-) \in \text{Spin}(1,9).$$

(71)

The specific of string lays in that the spinor representation of $SO(1,1)$ is one dimensional and is described by sign indices $^+$ and $^-$ of $v_\alpha^-$ and $v_\alpha^+$. For our D=10 case $q$ and $\dot{q}$ are the s- and c-spinorial indices of $SO(8)$.

In the dynamical system with $SO(1,1) \otimes SO(8)$ symmetry, like our superstring described by the action (69) or (68), the harmonics are homogeneous coordinates of the coset $\text{Spin}(1,9)_{SO(1,1)\otimes SO(8)}$,

$$\{V_\alpha^\beta\} = \{(v_\alpha^+, v_\alpha^-)\} = \frac{\text{Spin}(1,9)}{SO(1,1) \otimes SO(8)}.$$  

(72)

The requirement for the matrix $V$ to belong to $\text{Spin}(1,9)$ group (71) is imposed as the (reducible) constraint

$$\sigma_\alpha^\beta \sigma_\gamma^\delta = V \sigma_\alpha^\beta V^T = U_\beta^\alpha \tilde{\sigma}_\alpha^\beta,$$

(73)

which express the Lorentz invariance of the D=10 sigma–matrices $\sigma_\alpha^\beta, \tilde{\sigma}_\alpha^\beta$. These are symmetric, obey $\sigma_\alpha^\beta \sigma_\gamma^\delta + \sigma_\gamma^\delta \sigma_\alpha^\beta = \delta_\alpha^\beta \delta_\gamma^\delta$, and have $\text{Spin}(1,1) \otimes SO(8)$ invariant representation with which the constraints (73a) can be split into the following set of relations

$$u_\alpha^+ \sigma_\alpha^\beta = 2v_\alpha^+ v_\beta^+, \quad u_\alpha^- \sigma_\alpha^\beta = 2v_\alpha^- v_\beta^-, \quad u_\alpha^+ \tilde{\sigma}_\alpha^\beta = 2v_\alpha^+ \dot{v}_\beta^+, \quad u_\alpha^- \tilde{\sigma}_\alpha^\beta = 2v_\alpha^- \dot{v}_\beta^-,$$

(74)

(75)

These imply, in particular, that the spinor harmonics $v_\alpha^+, v_\alpha^-$ can be treated as square roots from the light–like vectors $u_\alpha^+, u_\alpha^-$. The second relations in (74)-(76) are written for inverse harmonics

$$V^\alpha_\beta = (v^-_\beta, v^+_\beta) \in \text{Spin}(1,9).$$

(77)

In the case of $D = 10$ Majorana–Weyl spinor representation, with $\alpha = 1, \ldots, 16$, these cannot be constructed from the ‘original’ spinorial harmonics (72) and are defined by the constraints

$$V^\alpha_\beta V^\beta_\gamma = \delta^\alpha_\gamma.$$  

(78)
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(like e.g. inverse metric in general relativity). Eq. (78) implies

\[ v^{-\alpha} v^+_{\alpha q} = \delta_{pq}, \quad v^{-\alpha} v^-_{\alpha q} = 0, \]
\[ v^+_{\alpha p} v^+_{\alpha q} = 0, \quad v^+_{\alpha p} v^-_{\alpha q} = \delta_{pq}. \]  

(79)

These relations can be used to factorize the projector and to get the irreducible form of the superstring \( \kappa \)-symmetry. They are also necessary to develop the superembedding approach to superstrings.\(^6^{,16}\)

Finally, the split form of Eq. (73b) reads

\[ v^+ q \tilde{\chi}^a i v^+_{\alpha q} = \delta_{pq} u^+_{\alpha} , \quad v^- q \tilde{\chi}^a i v^-_{\alpha q} = \delta_{pq} u^-_{\alpha} , \]
\[ v^+ q \tilde{\chi}^a i v^+_{\alpha q} = \gamma_{\alpha q} u^i_{\alpha} , \quad v^- q \tilde{\chi}^a i v^-_{\alpha q} = -\gamma_{\alpha q} u^i_{\alpha} . \]  

(80)

3.3. Superembedding approach to D=10 superstring in type IIB supergravity background

The starting point is the superembedding equation in its form of Eq. (6),

\[ \hat{E}^i := \hat{E}^{u_1} i (\xi, \eta) = 0. \]  

(83)

This has to be completed by the set of conventional constraints which includes (70) and the relations defining fermionic supervielbein forms

\[ e^{\pm \pm} = \hat{E}^{\pm \pm} := \hat{E}^{\pm \pm} (\alpha), \quad e^{+ q} = \hat{E}^{\alpha_1} v^+_{\alpha q} , \quad e^{- q} = \hat{E}^{\alpha_2} v^-_{\alpha q} . \]  

(84)

(85)

The superembedding equation (83) and the above set of conventional constraints can be collected in the following expressions for the pull-back of the supervielbein of target type IIB superspace,

\[ \hat{E}^{\pm} := \frac{1}{2} e^{++} u^{- -} u^{++} + \frac{1}{2} e^{- -} u^{++} u^{- -} , \]
\[ \hat{E}^{\alpha_1} = e^q v^- - e^q v^- + \lambda^q v^+ + \lambda^q v^+ , \quad \hat{E}^{\alpha_2} = e^{-q} v^+ + e^q v^- + \lambda^q v^- + \lambda^q v^+ . \]  

(86)

(87)

Actually, Eqs. (87) contain a bit more than just Eq. (85): it also states that \( \hat{E}^{\alpha_1} v^-_{\alpha q} = 0 \) and \( \hat{E}^{\alpha_2} v^+_{\alpha q} = 0 \), and this excludes from consideration the case of D1-branes (see\(^6^{,17}\)).

3.3.1. Other conventional constraints

To complete the set of conventional constraints, let us notice that we use the \( SO(1,1) \otimes SO(8) \) connection induced by embedding; this implies that
the complete $SO(1,9) \otimes SO(1,1) \otimes SO(8)$ covariant derivatives of the vector harmonics read

\begin{align}
\begin{cases}
D_{u^{+}} = u_{\alpha}^{i} \Omega^{+i}, \\
D_{u^{-}} = u_{\alpha}^{i} \Omega^{-i}, \\
D_{u^{\pm}} = \frac{1}{2} u_{\alpha}^{i} \Omega^{+i} + \frac{1}{2} u_{\alpha}^{i} \Omega^{-i}.
\end{cases}
\end{align}

(88)

For the spinorial harmonics (72), (77) this connection gives

\begin{align}
D_{v^{+}} & = \frac{1}{2} \Omega^{+i} \gamma_{i}^{\beta} \gamma^{\dot{\beta}}_{p} \\
D_{v^{\alpha}} & = -\frac{1}{2} \Omega^{+i} \nu^{\alpha} \gamma_{i}^{\beta} \gamma^{\dot{\beta}}_{p}, \\
D_{v^{-}} & = \frac{1}{2} \Omega^{-i} \nu^{\alpha} \gamma_{i}^{\beta} \gamma^{\dot{\beta}}_{p} \\
D_{v^{\alpha}} & = -\frac{1}{2} \Omega^{-i} \nu^{\alpha} \gamma_{i}^{\beta} \gamma^{\dot{\beta}}_{p}.
\end{align}

(89)

(90)

The integrability conditions for Eqs. (88) give the curved superspace generalization of the Peterson-Codazzi, Gauss and Ricci equations of the classical XIX-th century surface theory. These read

\begin{align}
D \Omega^{\pm i} & = \hat{R}^{\pm i} := \hat{\Omega}_{ab}^{\pm i} u_{\pm}^{i}, \\
d \Omega^{(0)} & = \frac{1}{4} \hat{R}_{ab}^{\pm i} u_{\pm}^{i} \pm \hat{\Omega}^{\pm i} \wedge \Omega^{\pm i}, \\
\hat{R}^{ij} & = \hat{\Omega}_{ab}^{i} u_{\pm}^{j} - \hat{\Omega}^{\pm i} \wedge \Omega^{\pm j},
\end{align}

(91)

(92)

(93)

where $\Omega^{\pm i}$ are the generalized Cartan forms (see Sec. 2.2.2),

$$
\Omega^{\pm i} := u_{\alpha}^{i} (du_{\alpha}^{i} + \omega_{\alpha}^{i} u_{\alpha}^{i}),
$$

(94)

$\Omega^{(0)} = \frac{1}{4} u^{\pm i} ((d+\dot{w})u^{+i})_{\pm}$ is the $SO(1,1)$ connection (the induced 2d spin connection) and $\hat{R}^{ij} = d \Omega^{ij} - \Omega^{ik} \wedge \Omega^{kj}$ is the curvature of normal bundle with $\Omega^{ij} = u^{\alpha} ((d+\dot{w})u^{i})_{\pm}$; finally, $\hat{\omega}_{ab}^{i}$ is the pull back of the $D = 10$ spin connection superform $\hat{\omega}_{a}^{i} = dZ^{M} w_{M} \hat{u}_{a}^{i}$ and $\hat{R}_{ab}^{i} = (dw - w \wedge w) \hat{u}_{a}^{i}$.  

3.3.2. Torsion constraints

Below we will also need the type IIB torsion constraints,

\begin{align}
T_{a}^{\alpha} & = -i \hat{E}_{a} \wedge \hat{E}_{b}^{\beta} \sigma_{ab}^{\alpha}, \\
T_{a}^{\alpha} & = -E_{a}^{1} \wedge E_{b}^{1} \sigma_{ab}^{\alpha} - i E_{a}^{1} \wedge E_{b}^{\beta} \sigma_{ab}^{\alpha}, \\
T_{a}^{\alpha} & = -E_{a}^{1} \wedge E_{b}^{1} \nabla_{1} e^{-\Phi} + \frac{1}{2} E_{a}^{1} \sigma_{ab} \wedge E_{b}^{1} \sigma_{ab}^{\alpha} \nabla_{1} e^{-\Phi} + \\
& + E_{a}^{1} \wedge E_{b}^{1} \nabla_{1} \nabla_{1}^{\alpha}, \\
T_{a}^{\alpha} & = -E_{a}^{2} \wedge E_{b}^{2} \nabla_{1} e^{-\Phi} + \frac{1}{2} E_{a}^{2} \sigma_{ab} \wedge E_{b}^{2} \sigma_{ab}^{\alpha} \nabla_{1} e^{-\Phi} + \\
& + E_{a}^{2} \wedge E_{b}^{2} \nabla_{1} \nabla_{1}^{\alpha}, \\
T_{a}^{\alpha} & = (E_{a}^{1}, E_{a}^{2}), \quad \alpha = 1, \ldots, 16, \quad \nabla_{1} \nabla_{1}^{\alpha} = \text{diag} \left( \sigma_{a}^{\alpha}, \sigma_{b}^{\alpha} \right).
\end{align}

(95)

(96)

(97)

(98)
The fermionic torsions $T^a_{\beta \alpha} = (T^a_{\beta \alpha}, T^a_{\beta \alpha}^2)$ can be read off from

$$E^\alpha T^\beta_{\alpha \beta} = -\frac{1}{8} \left( H_{\alpha \beta \gamma} \sigma^\alpha \tau_3 + \sigma^\beta \tilde{R}^{(1)} i \tau_2 - \sigma^\gamma \tilde{R}^{(3)} \tau_1 + \frac{1}{2} \sigma^\beta \tilde{R}^{(5)} i \tau_2 \right) \tilde{\sigma}^\gamma.$$  (99)

Here $\tau_3 \sigma^\alpha \tau_3 = \tau_3 \otimes \sigma^\alpha$, etc. and $\tilde{R}^{(2n+1)} = \frac{1}{(2n+1)!} R_{\hat{a}_1 \cdots \hat{a}_{2n+3}} \sigma \hat{a}_1 \cdots \hat{a}_{2n+1} \alpha\beta$ where $R_{\hat{a}_1 \cdots \hat{a}_{2n}}$ are the type IIB RR field strength. Notice that $R_{\hat{a}_1 \cdots \hat{a}_{2n-3}} \in \frac{(2n-3)!}{2^{n-2} n}$, which describes, in particular, the self-duality of the 5-form field strength.

3.3.3. Superstring equations of motion from superembedding

The selfconsistency conditions for the superembedding equations is included in the integrability condition of Eqs. (86),

$${\hat{T}}^a = -i \tilde{E}^\alpha \wedge \tilde{E}^{\beta \alpha} + i \tilde{E}^\beta \wedge \tilde{E}^{\alpha \beta} =$$

$$= \frac{1}{2} D e^{++} u_{\hat{a}}^{--} - \frac{1}{2} D e^{--} u_{\hat{a}}^{++} + \frac{1}{2} u_{\hat{a}}^i (e^{++} \wedge \Omega^{-} + e^{--} \wedge \Omega^{++}).$$  (100)

Using (87) one finds, after some algebra, that the contraction of Eq. (100) with the light–like vectors $u_{\hat{a}}^{\pm}$ determine the worldvolume bosonic torsion,

$$D e^{++} = -2 i e^{+q} \wedge e^{+q} - 4 i e^{++} \wedge e^{-} \chi^q_{++} \chi^{-q},$$

$$D e^{--} = -2 i e^{q-} \wedge e^{q-} - 4 i e^{--} \wedge e^{+} \chi^{-q} \chi^{q-},$$

while the contraction with $u_{\hat{a}}^{i}$ gives the restriction for the covariant Cartan forms (94),

$$e^{++} \wedge \Omega^{-} + e^{--} \wedge \Omega^{++} = -4 i \gamma_{qq} e^{+q} \wedge e^{-} \chi_{q}^{q-} + 4 i \gamma_{qq} e^{-q} \wedge e^{+} \chi_{q}^{-q}.$$  (103)

To proceed further one needs to study the consistency (integability) of the fermionic conventional constraints (87) which read

$$\hat{T}^{a_1} = D e^{+q} v_q^{\alpha-} + e^{+q} \wedge D v_q^{\alpha-} + D e^{\pm} \chi_{q}^{q-} v_q^{\alpha-} + e^{\pm} \wedge D \chi_{q}^{q-} v_q^{\alpha-} +$$

$$+ e^{\pm} \wedge \chi_{q}^{q-} D v_q^{\alpha-},$$  (104)

$$\hat{T}^{a_2} = D e^{-q} v_q^{\alpha+} + e^{-q} \wedge D v_q^{\alpha+} + D e^{\pm} \chi_{q}^{q-} v_q^{\alpha+} + e^{\pm} \wedge D \chi_{q}^{q-} v_q^{\alpha+} +$$

$$+ e^{\pm} \wedge \chi_{q}^{q-} D v_q^{\alpha+}.$$  (105)

The right hand side of these equations can be specified by using Eqs. (89), (90). To specify the l.h.s.’s we need the explicit form of the fermionic torsion constraints for the type IIB superspace, Eqs. (96), (97).
Contracting (104) with \(v_{\alpha q}^+\) and (105) with \(v_{\alpha q}^-\) one finds the fermionic torsion of the worldvolume superspace. These read

\[
D e_{\alpha q}^+ = -e^{p} \wedge e^{p'}(\delta_{\beta q} - \delta_{\beta p} v_{\alpha q}^-)D_{\beta e}e_{-}^{\Phi} + \propto e^{\pm \eta}, \quad (106)
\]

\[
D e_{\alpha q}^- = -e^{-p} \wedge e^{-p'}(\delta_{\beta q} - \delta_{\beta p} v_{\alpha q}^+)(\delta_{\beta p}^+ v_{\alpha q}^+)D_{\beta e}e_{-}^{\Phi} + \propto e^{\pm \eta}, \quad (107)
\]

where \(\propto e^{\pm \eta}\) denotes the contributions from forms containing bosonic
worldvolume supervielbein, which we will not need in this section.

Contracting (104) with \(v_{\alpha q}^-\) one arrives at

\[
0 = -\frac{1}{2}e^{p} \wedge e^{p} \Omega_{-p}^{--} q \hat{\Theta}_{-1}^{\alpha q} - 2ie^{p} \wedge e^{p} \chi_{-}^{-} = -\frac{1}{2}e^{q} \wedge e^{q} \Omega_{-q}^{++} q \hat{\Theta}_{-1}^{\alpha q} - 2ie^{-p} \wedge e^{-p} \chi_{-}^{+} + \propto e^{\pm \eta}. \quad (108)
\]

The similar equation with \(e^{q} \rightarrow e^{-q}\), \(\pm \leftrightarrow \mp\) appears when contracting
(105) with \(v_{\alpha q}^+\). An immediate consequence of these equations are

\[
\chi_{-}^{-} := \hat{E}_{-}^{\alpha q} v_{\alpha q}^- = 0 , \quad \chi_{-}^{+} := \hat{E}_{+}^{\alpha q} v_{\alpha q}^+ = 0 , \quad (109)
\]

\[
\Omega_{-p}^{--} = 0 , \quad \Omega_{p}^{++} = 0 . \quad (110)
\]

Eqs. (109) are the equations of motion for the fermionic degrees of freedom
of superstring. A simple way to be convinced in this is to observe that the
linearized version of (109) can be written in the form similar to the light-
cone gauge fermionic equations of the Green-Schwarz superstring, which
define its 16 fermionic degrees of freedom as two chiral, namely one right-
moving and one left-moving, 8 component fermions,

\[
\partial_{-} \hat{\Theta}_{-1}^{q} = 0 , \quad \hat{\Theta}_{-1}^{q}(\xi, \eta) := \hat{\theta}^{\alpha 1} v_{\alpha q}^- , \quad (111)
\]

\[
\partial_{+} \hat{\Theta}_{+2}^{q} = 0 , \quad \hat{\Theta}_{+2}^{q}(\xi, \eta) := \hat{\theta}^{\alpha 2} v_{\alpha q}^+ . \quad (112)
\]

The above \(\hat{\Theta}_{-1}^{q}, \hat{\Theta}_{+2}^{q}\) correspond to the light–cone gauge fermionic fields,
but defined with the use of moving frame determined by the harmonics.
The other half of the fermionic fields, \(\hat{\Theta}_{+1}^{q} := \hat{\theta}^{\alpha 1} v_{\alpha q}^+\) and \(\hat{\Theta}_{-2}^{q} := \hat{\theta}^{\alpha 2} v_{\alpha q}^-
\)
can be identified with the the fermionic coordinates of \(W^{(2)8+8}\),

\[
\hat{\Theta}_{+1}^{q}(\xi, \eta) = \eta^{+} q , \quad \hat{\Theta}_{-2}^{q}(\xi, \eta) = \eta^{-} q . \quad (113)
\]

These superfield relations imply \(\hat{\Theta}_{+1}^{q}(\xi, 0) = 0, \hat{\Theta}_{-2}^{q}(\xi, 0) = 0,\) and these
equations give a covariant version of the conditions which might be fixed
using the \(\kappa\)–symmetry of the standard Green–Schwarz action.
3.3.4. Bosonic equations of motion and on-shell superembedding of the worldvolume superspace

The fermionic equations of motion (109) simplify the expressions (87) for the pull-back of fermionic supervielbein forms, making them chiral,

\[ \hat{E}^{\alpha 1} = e^{+q} v_+^{-\alpha} + e^{++} \chi_+ v_+^{-\alpha}, \quad \hat{E}^{\alpha 2} = e^{-q} v_+^{+\alpha} + e^{--} \chi_- v_+^{-\alpha}, \]

which means, in particular, left– and right–moving, but also containing the corresponding half of the fermionic coordinates. The bosonic worldsheet torsion (101), (102) and Eq. (103) also simplify,

\[ D e^{++} = -2i e^{+q} \wedge e^{+q}, \quad D e^{--} = -2i e^{-q} \wedge e^{-q}, \]

and Eq. (103) determines the generalized Cartan forms to be

\[ \Omega_{--}^{--} = \Omega_{++}^{++} := K^i, \]

Using the conventional constraints (70), one can write the mean curvature \( \Omega_{--} = \Omega_{++} := K^i \) in the form

\[ K^i := -2D_+ E^{\pm}_{++} u^i_+ = -2D_+ E^{\pm}_{--} u^i_- . \]

Its linearized version reads \( K^i = \Box X^i \), where \( X^i = x^i u^i_+ \), so that one can expect the bosonic equations of motion to appear in the form of conditions for \( K^i \).

This is indeed the case. The bosonic equations of motion appears as \( e^{-q} \wedge e^{+q} \) component of Eq. (108). First one obtains \( K^i \gamma_{\pm q} = -\frac{1}{2} u^{\pm} \hat{H}_{abc} v_{\pm} ^{-} \sigma_{\pm} v_{\pm} ^{+} \). Then, using Eqs. (81), (82), one finds that \( v_{\pm} ^{-} \sigma_{\pm} v_{\pm} ^{+} = \gamma_{\pm q} (u^{--} - u^{+} - u^{--} - u^{+}) \) and arrives at

\[ K^i = \frac{1}{4} \hat{H}^{++++}, \quad \hat{H}^{+++} := \hat{H}_{abc} u^{aa} - u^{bb} + u^{ab}, \]

Thus we have completed the derivation of superstring equations of motion from the superembedding equations.

4. Superembedding description of AdS superstring

4.1. AdS superspace \( AdSS^{(5,5|32)} \) as the solution of type IIB supergravity constraints

The AdS superspace denoted by \( AdSS^{(5,5|32)} \) (see\(^{39}\)) is the D=10 type IIB superspace the bosonic body of which is \( AdS_5 \times S^5 \). This is given by a
solution of the type IIB supergravity constraints (95), (96), (97) with all but five form fluxes equal to zero, this is to say

\[ H_{a_1a_2a_3} = 0, \quad R_{a_1a_2a_3} = 0, \quad C_0 = 0 = \Phi. \quad (120) \]

The nonvanishing five form flux is characterized by a self-dual constant tensor

\[ f_{a_1a_2a_3a_4a_5} = \frac{1}{5!} f^{ab_1b_2b_3b_4b_5} f_{a_1b_1a_2b_2a_3b_3a_4b_4a_5b_5}, \quad (121) \]

\[ df_{a_1a_2a_3a_4a_5} = 0. \quad (122) \]

The torsion and curvature two-forms of the \( AdSS^{(5,5|32)} \) superspace are expressed through this constant tensor and \( \sigma \)-matrices by (see \( ^{10,47} \))

\[ T^a = -i \mathcal{E} \wedge (I \otimes \sigma^a) \mathcal{E} \equiv -i \left( E^{a_1} \wedge E^{b_2} \wedge E^{c_3} \right) \sigma_{a_1b_1c_1} \quad (123) \]

\[ T^a = -\frac{1}{R} E^a \wedge \mathcal{E}^2 f_{ab_1b_2b_3b_4} \left( i \tau_2 \otimes \sigma_{b_1b_2b_3b_4} \right) \gamma^a, \]

\[ \Leftrightarrow \begin{cases} T^{a_1} = -\frac{1}{R} E^{a_1} \wedge E^{b_2} f_{ab_1b_2b_3b_4} \left( \frac{1}{i} \tau_2 \otimes \sigma_{b_1b_2b_3b_4} \right) \gamma^{a_1} \\ T^{a_2} = -\frac{1}{R} E^{a_2} \wedge E^{b_3} f_{ab_1b_2b_3b_4} \left( \frac{1}{i} \tau_2 \otimes \sigma_{b_1b_2b_3b_4} \right) \gamma^{a_2} \end{cases}, \quad (124) \]

\[ R_{ab} = -\frac{1}{2R^2} E^a \wedge E^b - \frac{4i}{R} E^a \wedge \mathcal{E}^2 f_{ab_1b_2b_3b_4} \left( i \tau_2 \otimes \sigma_{b_1b_2b_3b_4} \right) \gamma_{a_1b_1c_1c_2c_3c_4} = -\frac{1}{2R^2} E^a \wedge E^b + \frac{8i}{R} E^a \wedge \mathcal{E}^2 f_{ab_1b_2b_3b_4} \left( \frac{1}{i} \tau_2 \otimes \sigma_{b_1b_2b_3b_4} \right) \gamma_{a_1b_1c_1c_2c_3c_4}. \quad (125) \]

To complete the definition of the \( AdSS^{(5,5|32)} \) superspace we should add that, in a suitable frame one can split the set of bosonic superfield forms as \( E^a = (E^\tilde{a}, E^i) \), with \( \tilde{a} = 0,1,\ldots,4 \) and \( i = 1,\ldots,5 \) and find that the constant self-dual tensor (121) should have the form

\[ f_{\tilde{a}_1\cdots\tilde{a}_5} = \varepsilon_{\tilde{a}_1\cdots\tilde{a}_5}, \quad f_{i_1\cdots i_5} = \varepsilon_{i_1\cdots i_5}, \quad (126) \]

with all other components vanishing. Then \( \tilde{a} = 0,1,\ldots,4 \) is identified as the vector index of the 5d space tangent to \( AdS_5 \), and \( i = 1,\ldots,5 \) as the vector index of the space tangent to \( S^5 \). The constant \( R \) in (124), (125) defines the radius of \( AdS_5 \) or \( S^5 \) (these radii are equal).

Now the superembedding description of the AdS superstring can be obtained by specializing the equations describing superstring in general supergravity background to a particular form of this background given by Eqs. (123), (124), (125), (121), (122). But before turning to this, we describe some properties of constant self-dual tensor \( f^{a_1a_2a_3a_4a_5} := f^{[5]} = \frac{1}{5!} \varepsilon^{[5][5]} f_{[5]} \) as they are seen in stringy spinor moving frame.
4.2. Constant five form flux in stringy moving frame

Below we will mainly use a seemingly SO(1,9) covariant description by Eqs. (123), (124), (125) and (121), (122) so that a big part of our results are applicable for superstring in a generic constant 5-form flux background.

Although the distinction between self-duality and anti-self duality is conventional, the selfduality of the constant flux (121) is singled out by that we use the sigma–matrix representation with self-dual $\sigma^{[5]}_{\alpha\beta} := \tilde{\sigma}^{[5]}_{\alpha\beta} \tilde{\sigma}^{[5]}_{\alpha\beta}$ (which implies anti-selfdual $\tilde{\sigma}^{[5]}_{\alpha\beta} := \tilde{\sigma}^{[5]}_{\alpha\beta} \tilde{\sigma}^{[5]}_{\alpha\beta}$),

$$\sigma^{[5]}_{\alpha\beta} = \frac{1}{5!} \epsilon^{[5][5']} \sigma^{[5']}_{\alpha\beta}, \quad \tilde{\sigma}^{[5]}_{\alpha\beta} = -\frac{1}{5!} \epsilon^{[5][5']} \tilde{\sigma}^{[5']}_{\alpha\beta}. \quad (127)$$

It is convenient to introduce the spinor moving frame components of the constant selfdual tensor (which are not obliged to be constant but rather depend on the coordinate of the superstring worldsheet superspace),

$$f^{ijk} := f^{--++} \tilde{a} = f^{i j k l s i t s i t s i t s i t}, \quad f^{i j k l} = f^{i j k l s i t s i t s i t s}, \quad f^{i j k l s i t} = f^{i j k l s i t s i t s i t s}, \quad f^{i j k l s i t} = f^{i j k l s i t s i t s i t s}. \quad (128)$$

The self-duality equation (121) implies

$$f^{i j k} := f^{--++} \tilde{a} = f^{i j k l s i t s i t s i t s i t}, \quad f^{i j k l} = f^{i j k l s i t s i t s i t s}, \quad f^{i j k l s i t} = f^{i j k l s i t s i t s i t s}, \quad f^{i j k l s i t} = f^{i j k l s i t s i t s i t s}. \quad (129)$$

Now one can prove that, as a consequence of (121) and of the properties (80), (81), (11) of stringy harmonics, the following identities hold

$$f^{--abcd} (v_q^- \sigma_{abcd})^\alpha = 0, \quad f^{++abcd} (v_q^+ \sigma_{abcd})^\alpha = 0, \quad (130)$$

where

$$f^{i j k l s i t s i t s i t s} = u_a^{i j k l s i t s i t s}. \quad (131)$$

Notice by pass that tensors $f^{i j k l s i t s i t s}$ characterize the movement of superstring. For instance, when superstring moves in the AdS part of superspace and is frozen to a point on $S^5$, one can choose the frame in such a way that

$$u_{\tilde{a}}^{\pm} = \delta_{\tilde{a}}^{\pm} u_{\tilde{a}}^{\pm} \text{ and then } f^{i j k l s i t s i t s} = u_{\tilde{a}}^{\pm} \epsilon_{\tilde{a} b l i m n} \epsilon_{\tilde{a} b l i m n} \delta_{\tilde{a} b l i m n} \delta_{\tilde{a} b l i m n} \delta_{\tilde{a} b l i m n}.$$

To prove (130), we observe that (121) and (127) imply $f^{[5]} (\sigma^{[5]}_{\alpha\beta} = 0$ (but $f^{[5]} \tilde{\sigma}^{[5]}_{\alpha\beta} \neq 0$) which implies $f^{abcd} \sigma_{abcd}^\alpha = \frac{1}{5!} \sigma_{\beta\gamma} f^{[5]} \tilde{\sigma}^{[5]}_{\gamma\alpha}$. Multiplying this by $u_{\tilde{a}}^{--} v_q^- \sigma_{abcd}^\alpha = 0$ and $u_{\tilde{a}}^{++} v_q^+ \sigma_{abcd}^\alpha = 0$, one finds that (130) holds due to the identities $u_{\tilde{a}}^{--} (v_q^- \sigma_{abcd})^\alpha = 0$ and $u_{\tilde{a}}^{++} (v_q^+ \sigma_{abcd})^\alpha = 0$, respectively.
Notice that in such a way we cannot prove
\[ f^{--abcd}(\sigma_{abcd}v_q^+)_{\alpha} = 0 , \quad f^{++abcd}(\sigma_{abcd}v_q^+)_{\alpha} = 0 . \] (132)

To show that these are true, let us observe first that, due to (130), we need only to prove the vanishing of their contractions with, respectively, \( v_q^{+\alpha} \) and \( v_q^{-\alpha} \). For these one finds that
\[
\begin{align*}
f^{--abcd}(v_p^+ \sigma_{abcd}v_q^-) & = f^{--i_1i_2i_3i_4}z_{i_1i_2i_3i_4}^{pq} = 0 , \\
f^{++abcd}(v_p^+ \sigma_{abcd}v_q^+) & = f^{++i_1i_2i_3i_4}z_{i_1i_2i_3i_4}^{pq} = 0 ,
\end{align*}
\] (133)

where the last equalities follow from (128), (129) and the anti-self duality (self-duality) of \( \gamma_{i_1i_2i_3i_4} \),

\[
\begin{align*}
\tilde{\gamma}^{ijkl}_{\tilde{q}p} & = - \frac{1}{16} \epsilon^{ijkl'}j'k'l'z_{ijkl}^{\tilde{q}p} , \\
\gamma^{ijkl}_{\tilde{q}p} & = + \frac{1}{16} \epsilon^{ijkl'}j'k'l'\gamma_{ijkl}^{\tilde{q}p} ,
\end{align*}
\] (134)

which correspond to the duality properties (127) of the 10D \( \sigma \) matrices.

To conclude, the only nonvanishing contribution to the expressions \( f^{++[4]}\sigma_{[4]} = f^{++abcd}(\sigma_{abcd})_{[4]} \) is
\[
\begin{align*}
f^{--abcd}(v_q^+ \sigma_{abcd}v_q^+) & = f^{++abcd}(v_q^- \sigma_{abcd}v_q^-) = 4f^{ijkl}\gamma^{ijkl} .
\end{align*}
\] (135)

4.3. Worldsheet superspace geometry of the superstring in AdS superspace

The pull-backs of the AdS supervielbein on the worldsheet superspace have the form of (86) and (114),
\[
\begin{align*}
\hat{E}^a & = \frac{1}{2} e^{++u^{--}-a} + \frac{1}{2} e^{--u^{++}a} , \\
\hat{E}^\alpha & = e^{+q}v^{-\alpha} + e^{++}\chi^{+\alpha}v_q^{+\alpha} , \\
\hat{E}^{a\alpha} & = e^{q}v^{-\alpha} + e^{+\alpha}\chi^{+\alpha}v_q^{+\alpha} .
\end{align*}
\] (136)

As far as the \( H_3 \) flux in AdS superspace is equal to zero, Eq. (120), the string equations of motion (119) read
\[
K^i = 0 ,
\] (138)

so that the generalized Cartan form, which provide the supersymmetric generalization of the second fundamental form for the AdS superstring world-volume superspace, are given by Eqs. (116), (117)
\[
\Omega^{i-} = -4ie^{+q}(\gamma^i_+ \tilde{q})v_q^- + e^{++}\Omega_{++}^{-i} , \\
\Omega^{i+} = -4ie^{-\tilde{q}}(\gamma^i_- \gamma^i_+ \tilde{q})v_q^- + e^{--}\Omega_{--}^{i+} .
\] (139)
The worldvolume superspace torsion forms read (see Eqs. (115), (107))

\[
D e^{++} = -2i e^{+q} \wedge e^{+q}, \quad D e^{-} = -2i e^{-q} \wedge e^{-q},
\]

\[
D e^{+q} = -e^{+} \wedge e^{-} \left( 2i (\chi_{-}^+ \gamma^i)_{p} (\gamma^i \chi_{++})_{q} + \frac{3}{2} f^{ijk} \gamma^j_{pq} \right) + \frac{3}{4} e^{+} \wedge e^{-} \Omega^{++}_{ij} (\gamma^i \chi_{++})_{q},
\]

\[
D e^{-q} = -2i e^{-} \wedge e^{+} \left( 2i (\gamma^i \chi_{++})_{p} (\chi_{-}^+ \gamma^i)_{q} + \frac{3}{2} f^{ijk} \gamma^j_{pq} \right) - \frac{3}{4} e^{+} \wedge e^{-} \Omega^{++}_{ij} (\chi_{-}^+ \gamma^i)_{q}.
\]

To obtain these expressions the identities (130) have to be used; they are also needed to find that $D_{-} \chi_{++}^{-} = D_{-} \chi_{++}^{-} = 0$ and $D_{+} \chi_{-}^{-} = D_{+} \chi_{-}^{-} = 0$ which can be summarized by

\[
D \chi_{++}^{-} = -\frac{1}{2} e^{+q} \Omega_{++}^{-i} + e^{+q} D_{+} \chi_{++}^{-} + \frac{3}{4} f^{ijk} \gamma^j_{pq} \left( \chi_{-}^+ \gamma^i \chi_{++}^{-} \right),
\]

\[
D \chi_{-}^{-} = -\frac{1}{2} e^{-q} \Omega_{-+}^{+} + e^{-q} D_{-} \chi_{-}^{-} + \frac{3}{4} f^{ijk} \gamma^j_{pq} \left( \chi_{-}^+ \gamma^i \chi_{-}^{-} \right).
\]

The pull–back of the Riemann curvature two form of the AdSS superspace to the worldvolume superspace reads

\[
F_{abk} = -\frac{1}{2} e^{+} \wedge e^{-} \left( \frac{1}{R} u^{-} \wedge u^{+} + \frac{3}{2} f^{ijk} (\chi_{-}^+ \gamma^i \chi_{++}^{-}) \right) - \frac{9}{4} e^{-} \wedge e^{+} f^{abc} \gamma^a_{pq} (\chi_{-}^+ \gamma^i \chi_{++}^{-})_{q} + \frac{3}{2} e^{+} \wedge e^{-} f^{++} \left( \gamma^i \chi_{++}^{-} \right)_{q} - \frac{3}{4} e^{+} \wedge e^{-} f^{abc} \gamma^a_{pq} \left( \gamma^i \chi_{++}^{-} \right)_{q}.
\]

In its derivation we have used the following consequences of the properties (74), (75), (76) of spinor moving frame variables (71)

\[
f^{ab[3]} v_p \sigma[3] v_q = -3 f^{abc} \gamma^a_{pq} f^{ab[3]} v_p \sigma[3] v_q = -3 f^{abc} \gamma^a_{pq} v_q,
\]

\[
f^{ab[3]} v_p \sigma[3] v_q = f^{abc} \gamma^a_{pq} v_q.
\]

Substituting Eqs. (145) and (139) into Gauss and Ricci equations (92) and (93), one finds the following expressions for the 2d Riemann curvature two form

\[
d \Omega^{(0)} = e^{+} \wedge e^{-} \left( \frac{1}{4} \Omega^{++} \Omega^{-+} + \frac{3}{2} f^{ijk} (\chi_{-}^+ \gamma^i \chi_{++}^{-}) \right) + \frac{3}{4} e^{+} \wedge e^{-} \Omega^{++} (\gamma^i \chi_{++}^{-})_{q} - \frac{3}{4} e^{+} \wedge e^{-} \Omega^{++} (\chi_{-}^+ \gamma^i)_{q} + \frac{3}{4} e^{+} \wedge e^{-} f^{ijk} \gamma^j_{pq} \left( \chi_{-}^+ \gamma^i \chi_{-}^{-} \right).
\]
and for the curvature of the normal bundle
\[ R^{ij} = e^{++} \wedge e^{--} \left( \Omega^{++}_{--} \Omega^{--}_{++} - \frac{8}{R} f^{ijk_1 k_2 k_3} (\chi^{++}_{--} \gamma^{k_1 k_2 k_3} \chi^{++}_{--}) \right) + \\
+ 4i e^{--} \wedge e^{+q} \left( \Omega^{++}_{--} (\gamma^{j} \chi^{++}_{--})_q - \frac{8}{R} f^{ijkl} (\gamma^{kl} \chi^{++}_{--})_q \right) - \\
- 4i e^{+q} \wedge e^{--} \left( \Omega^{++}_{--} (\chi^{++}_{--} \gamma^{j})_q + \frac{8}{R} f^{ijkl} (\gamma^{kl} \chi^{++}_{--})_q \right) + \\
- 8e^{+q} \wedge e^{--} \left( 2(\gamma^{j} \chi^{++}_{--})_q (\chi^{++}_{--} \gamma^{j})_q + \frac{1}{R} f^{ijk_1 k_2 k_3} \gamma^{k_1 k_2 k_3} \right). \] (148)

The Peterson-Codazzi equations (91) for superstring in $AdS_5 \times S^5$ superspace read
\[ D\Omega^{--} = R^{--} = \frac{8i}{R} e^{++} \wedge e^{--} f^{ijkl} (\chi^{++}_{--} \gamma^{kl} \chi^{++}_{--}) - \\
- \frac{24i}{R} e^{++} \wedge e^{--} f^{ijkl} (\gamma^{kl} \chi^{++}_{--})_q - \frac{8i}{R} e^{+q} \wedge e^{--} f^{ijkl} \gamma^{kl} \]

\[ D\Omega^{++} = R^{++} = - \frac{8i}{R} e^{++} \wedge e^{--} f^{ijkl} (\chi^{++}_{--} \gamma^{kl} \chi^{++}_{--}) + \\
- \frac{24i}{R} e^{--} \wedge e^{+q} f^{ijkl} (\gamma^{kl} \chi^{++}_{--})_q - \frac{8i}{R} e^{+q} \wedge e^{--} f^{ijkl} \gamma^{kl} \] (150).

These do not give us new information after Eqs. (143) and (144) are taken into account.

Thus we have completed the construction of superembedding approach description of the superstring in $AdS^{5} \times S^{5}$ superspace.

5. Conclusion

In this contribution we have presented a brief review of spinor moving frame formulation, generalized action principle and superembedding approach to super-$p$-branes and (in Sec. 3) have elaborated in detail the superembedding approach to superstring in general type IIB supergravity background. On this basis we have given (in Sec. 4) the complete superembedding description of superstring in $AdS_5 \times S^5$ superspace. To our best knowledge, such a description of AdS superstring has not been developed before and we hope that it will be helpful in searching for new exact solutions of the AdS superstring equations which, in its turn, might be useful for further study and applications of the AdS/CFT correspondence.

Acknowledgments. The author thanks José de Azcárraga and Dmitri Sorokin for discussions, useful comments and collaboration on early stages of this work which has been supported by the Basque Foundation for Science, Ikerbasque, and partially by research grants from the Spanish MICI (FIS2008-1980), the INTAS (2006-7928) as well as by the Ukrainian Academy of Sciences and Russian RFFI grant 38/50–2008.
References

1. Achúcarro, A., Evans, J., Townsend, P.K. and Wiltshire, D., Super-p-branes, Phys. Lett. B198, 441 (1987).
2. Akulov, V., Bandos, I., Kummer, W. and Zima, V.G., Nucl. Phys. B527, 61 (1998) [hep-th/9802032].
3. G. Arutyunov and S. Frolov, Foundations of the AdS$\times$S$^5$ Superstring. Part I, arXiv:0901.4937 [hep-th].
4. Bandos, I. A., A superparticle in Lorentz-harmonic superspace, Sov. J. Nucl. Phys. 51, 906 (1990); Multivalued action functionals, Lorentz harmonics, and spin, JETP. Lett. 52, 205 (1990).
5. Bandos, I., String-like description of gravity and possible applications for F-theory, Mod. Phys. Lett. A12, 799 (1997) [hep-th/9608093].
6. Bandos, I.A., Superembedding approach and S-duality: A unified description of superstring and super-D1-brane, Nucl. Phys. B599, 197 (2001) [hep-th/0008249].
7. Bandos, I.A., BPS preons and tensionless super-p-branes in generalized superspace, Phys. Lett. B558, 197 (2003) [hep-th/0208110].
8. Bandos, I.A., de Azcárraga, J.A., Izquierdo, J.M. and Lukierski, J., BPS states in M-theory and twistorial constituents, Phys. Rev. Lett. 86, 4451 (2001) [hep-th/0101113].
9. Bandos, I.A., de Azcárraga, J.A., Picón, M. and Varela, O., D = 11 superstring model with 30 kappa-symmetries and 30/32 BPS states in an extended superspace, Phys. Rev. D69, 085007 (2004). [hep-th/0307106].
10. Bandos, I.A., Ivanov, E., Lukierski, J. and Sorokin, D., On the superconformal flatness of AdS superspaces, JHEP 0206, 040 (2002) [hep-th/0205104].
11. Bandos, I.A. and Kummer, W., $P$–branes, Poisson–sigma–models and embedding approach to $(p+1)$ dimensional gravity, Int. J. Mod. Phys. A14, 4881-4914 (1999) [hep-th/9703099].
12. Bandos, I.A. and Kummer, W., A polynomial first order action for the Dirichlet 3-brane, Phys. Lett. B413, 311 (1997) [Err.-ibid. B420, 405 (1998)] [hep-th/9707110].
13. Bandos, I.A. and Kummer, W., Current density distributions and a supersymmetric action for interacting brane systems, Phys. Lett. B462, 254–264 (1999) [hep-th/9905144]; Superstring ‘ending’ on super-D9-brane: a supersymmetric action functional for the coupled brane system, Nucl. Phys. B565, 291-332 (2000) [hep-th/9906041].
14. Bandos, I. and Lukierski, J., Tensorial central charges and new superparticle models with fundamental spinor coordinates, Mod. Phys. Lett. 14, 1257 (1999) [hep-th/9811022]; New Superparticle Models Outside the HLS Supersymmetry Scheme, Lect. Not. Phys. 539, 195 (2000) [hep-th/9812074].
15. Bandos, I., Pasti, P., Sorokin, D. and Tonin, M., Superbrane Actions and Geometrical Approach, Lect. Notes Phys., 509, 79-91 (1998) [hep-th/9705064].
16. Bandos, I.A., Pasti, P., Sorokin, D.P., Tonin, M. and Volkov, D.V., Superstrings and supermembranes in the doubly supersymmetric geometrical approach, Nucl. Phys. B446, 79 (1995) [hep-th/9501113].
17. Bandos, I.A., Sorokin, D.P. and Tonin, M., Generalized action principle and superfield equations of motion for $D = 10$ D p-branes, Nucl. Phys. B497, 275 (1997) [hep-th/9701127].
18. Bandos, I., Sorokin, D. and Volkov, D.V., On the generalized action principle for superstrings and supermembranes, Phys. Lett. B352, 269-275 (1995) [hep-th/9502141].
19. Bandos, I.A. and Zheltukhin, A.A., Spinor Cartan Moving $N$ Hedron, Lorentz Harmonic Formulations Of Superstrings, And Kappa Symmetry, JETP Lett. 54, 421 (1991); Green-Schwarz superstrings in spinor moving frame formalism, Phys. Lett. B288, 77 (1992); D=10 superstring: Lagrangian and Hamiltonian mechanics in twistor-like Lorentz harmonic formulation, Preprint IC/92/422, Trieste 1992. 81pp., Phys. Part. Nucl. 25, 453 (1994).
20. Bandos, I.A. and Zheltukhin, A.A., Generalization of Newman-Penrose dyads in connection with the action integral for supermembranes in an eleven-dimensional space, JETP Lett. 55 (1992) 81; Eleven-dimensional supermembrane in a spinor moving repere formalism, Int. J. Mod. Phys. A8, 1081 (1993); N=1 super-p-branes in twistor - like Lorentz harmonic formulation, Class. Quant. Grav. 12, 609 (1995) [hep-th/9405113].
21. Beisert, N., Ricci, R., Tseytlin, A.A. and Wolf, M., Dual Superconformal Symmetry from $\text{AdS}_5 \times \text{S}^5$ Superstring Integrability, Phys. Rev. D78, 126004 (2008) [arXiv:0803.3024 [hep-th]].
22. Bergamin, L. and Kummer, W., The complete solution of 2D superfield supergravity from graded Poisson-sigma models and the super pointparticle, Phys. Rev. D68, 104005 (2003) [arXiv:hep-th/0306217].
23. Bergshoeff, E., Howe, P.S., Kerstan, S. and Wulff, L., Kappa-symmetric SL(2,R) covariant D-brane actions, JHEP 0710, 050 (2007) [arXiv:0708.2722 [hep-th]].
24. Bergshoeff, E., Sezgin, E. and Townsend, P.K., Supermembrane and eleven-dimensional supergravity, Phys. Lett. B189, 75 (1987); Ann. Phys.(NY) 185, 330 (1988).
25. Bergshoeff, E., Sezgin, E. and Townsend, P.K., On 'spinning' membrane models, Phys. Lett. B209, 451 (1988).
26. Berkovits, N. and Howe, P.S., The cohomology of superspace, pure spinors and invariant integrals, arXiv:0803.3024 [hep-th].
27. Berkovits, N. and Maldacena, J., Fermionic T-Duality, Dual Superconformal Symmetry, and the Amplitude/Wilson Loop Connection, JHEP 0809, 062 (2008) [arXiv:0807.3196 [hep-th]].
28. Chu, C.S., Howe, P.S., Sezgin, E. and West, P.C., Open superbranes, Phys. Lett. B429, 273 (1998) [hep-th/9803041].
29. Castellani, L. D’Auria, R. and Fré, P. Supergravity and superstrings: a geometric perspective, V.1.2.3, World Scientific, Singapore 1991.
30. Chryssomalakos, C., de Azcárraga, J. A., Izquierdo, J. M. and Pérez Bueno, J. C., The geometry of branes and extended superspaces, Nucl. Phys. B567, 293 (2000) [hep-th/9904137].
31. de Azcárraga, J. A. and Lukierski, J., Supersymmetric particles with internal symmetries and central charges, Phys. Lett. B113, 170–174 (1982).
32. Delduc, F., Galperin, A., Sokatchev, E., Lorentz harmonic (super)fields and (super)particles, Nucl. Phys. B368, 143-171 (1992).
33. Galperin, A.S., Howe, P.S. and K. S. Stelle, K.S., The Superparticle And The Lorentz Group, Nucl. Phys. B368, 248-280 (1992) [hep-th/9201020].
34. Galperin, A., Ivanov, E., Kalitsyn, S., Ogievetsky, V. and Sokatchev, E., Unconstrained N=2 matter, Yang-Mills and supergravity theories in harmonic superspace, Class. Quant. Grav. 1, 469-498 (1984);
Galperin, A.S., Ivanov, E.A., Ogievetsky, V.I and Sokatchev, E. Harmonic Superspace, (CUP Cambridge, UK) 2001.
35. Gomis, J., Sorokin, D., and Wulff, L., The complete AdS(4) x CP(3) super-space for the type IIA superstring and D-branes, arXiv:0811.1566 [hep-th].
36. Green, M.B. and Schwarz, J.H., Covariant description of superstrings, Phys. Lett. B136, 367 (1984).
37. Grisaru, M.T., Howe, P.S., Mezincescu, L., Nilsson, B. and Townsend, P.K. N=2 Superstrings in a supergravity background, Phys. Lett. B162, 116 (1985).
38. Grumiller, D., Kummer, W., and Vassilevich, D.V., Dilaton gravity in two dimensions, Phys. Rept. 369, 327 (2002) [arXiv:hep-th/0204253].
39. Heslop, P. and Howe, P.S., Chiral superfields in IIB supergravity, Phys. Lett. B502, 259 (2001) [arXiv:hep-th/0008047].
40. Howe, P.S., Raetzel, O. and Sezgin, E., On brane actions and superembeddings, JHEP 9808, 011 (1998).
41. Howe, P.S., and Sezgin, E., Superbranes, Phys. Lett. B390, 133 (1997) [hep-th/9607227].
42. Howe, P.S., and Sezgin, E., D = 11, p = 5, Phys. Lett. B394, 62–66 (1997) [hep-th/9611008].
43. Howe, P. S. and Tucker, R. W., A locally supersymmetric and reparametrization invariant action for a spinning membrane, J. Phys. A10, L155 (1977); Local supersymmetry in (2+1)-dimensions. 2. An action for a spinning membrane, J. Math. Phys. 19, 981 (1978).
44. Kummer, W., General treatment of all 2d covariant models, Rakhiv 1995, Methods in mathematical physics. Procs. 12th Hutsulian Workshop, (Hadronic Pr., 1997) pp. 161-176 [gr-qc/9612016].
45. Kallosh, R. and Rahmanov, M., Gauge algebra and quantization of Type II superstrings, Phys.Lett. B209 (1988) 233; Consistency of covariant quantization of GS String, Phys. Lett. B214 (1988) 549.
46. Kallosh, R. and Rahmfeld, J., The GS string action on AdS7 × S5, Phys. Lett. B443, 143 (1998) [hep-th/9808038].
47. Kallosh, R. and Rajaraman, A., Vacua of M-theory and string theory, Phys. Rev. D58, 125003 (1998) [hep-th/9805041].
48. Metsaev, R.R. and Tseytlin, A.A., Type IIB superstring action in AdS(5) x S(5) background, Nucl. Phys. B533, 109 (1998) [hep-th/9805028]; Type IIB Green-Schwarz superstrings in AdS(5) x S(5) from the supercoset approach, J. Exp. Theor. Phys. 91, 1098 (2000).
49. Matone, M., Mazzucato, L., Oda, I., Sorokin, D. and Tonin, M., The superembedding origin of the Berkovits pure spinor covariant quantization of superstrings, Nucl. Phys. B639, 182 (2002) [hep-th/0206104].
50. Myers, R.C., *Dielectric-branes*, JHEP **9912**, 022 (1999) [hep-th/9910053].

51. Neeman, Y. and Regge, T., *Gravity and supergravity as gauge theories on a group manifold*, Phys. Lett. **B74**, 54(1978); D’Auria, R., Fré, P. and Regge, T., *Graded Lie algebra cohomology and supergravity*, Riv. Nuovo Cim. **3**, f12, 1 (1980);

Regge, T., *The group manifold approach to unified gravity*, in: *Relativity, groups and topology II: Les Houches, Session XL* 1983; B. S. DeWitt and R. Stora eds., Elsevier Sci. Pub. 1984, pp.903–1005, and refs therein.

52. Nissimov, E., Pacheva, S., Solomon, S., *Covariant first and second quantization of the N=2 D = 10 Brink-Schwarz superparticle*, Nucl. Phys. **B296**, 462 (1988); **B299**, 183 (1988); *Covariant canonical quantization of the Green-Schwarz superstring*, Nucl. Phys. **B297**, 349 (1988); **B317**, 344 (1989); *The relation between operator and path integral covariant quantizations of the Green-Schwarz superstring*, Phys. Lett. **B228**, 181 (1989).

53. Schwarz, J.H. and West, P.C., *Symmetries And Transformations Of Chiral N=2 D=10 Supergravity*, Phys. Lett. **B126**, 301 (1983); Schwarz, J.H., *Covariant Field Equations Of Chiral N=2 D=10 Supergravity*, Nucl. Phys. **B226**, 269 (1983).

54. Siegel, W., *Hidden local supersymmetry in the supersymmetric particle action*, Phys. Lett. **B128**, 397 (1983).

55. Sokatchev, E., *Light cone harmonic superspace and its applications*, Phys. Lett. **B169**, 209 (1986); *Harmonic superparticle*, Class. Quantum Grav. **4**, 237 (1987).

56. Sorokin, D.P., *Superbranes and superembeddings*, Phys. Rept. **329**, 1 (2000) [hep-th/9906142].

57. Sorokin, D.P., Tkach, V.I., Volkov, D.V., *Superparticles, twistors and Siegel symmetry*, Preprint KPTI-88-31, Apr 1988, Mod.Phys.Lett. **A4**, 901 (1989).

58. Sorokin, D.P., Tkach, V.I., Volkov, D.V. and Zheltukhin, A.A., *From the superparticle Siegel symmetry to the spinning particle proper time supersymmetry*, Phys. Lett. **B216**, 302 (1989).

59. Uvarov, D. V., *On covariant kappa-symmetry fixing and the relation between the NSR string and the type II GS superstring*, Phys. Lett. **493**, 421 (2000) [hep-th/0006185]; *Nucl. Phys. Proc. Suppl. 102*, 120 (2001) [hep-th/0104235]; *Supertwistor formulation for higher dimensional superstrings*, Class. Quant. Grav. **24**, 5383 (2007) [hep-th/0703051].

60. Van Holten, J.W., and Van Proeyen, A., *N=1 Supersymmetry Algebras In D=2, D=3, D=4 Mod-8*, J. Phys. **A15**, 3763 (1982).

61. Volkov, D.V. and Akulov, V.P., *Possible universal neutrino interaction*, JETP Lett. **16**, 438 (1972); *Is the neutrino a Goldstone particle?* Phys. Lett. **B46**, 109 (1973).

62. Volkov, D.V. and Soroka, V.A., *Higgs effect for Goldstone particles with spin 1/2*, JETP Lett. **18**, 312-314 (1973).

63. Volkov, D.V. and Zheltukhin, A.A., *Extension of the Pernose representation and its use to describe supersymmetric models*, JETP Lett. **48**, 63 (1988); *On the equivalence of the Lagrangians of massless Dirac and supersymmetrical particles*, Lett. Math. Phys. **17**, 141 (1989); *Lagrangians For Massless Parti-
cles And Strings With Local And Global Supersymmetry, *Nucl. Phys.* **B335**, 723 (1990).

64. Wiegmann, P.B., *Multivalued functionals and geometrical approach for quantization of relativistic particles and strings, Nucl. Phys.* **B323**, 311-329 (1989); *Extrinsic geometry of superstrings, Nucl. Phys.* **B323**, 330-336 (1989).

65. Zheltukhin, A.A., Uvarov, D.V., *An inverse Penrose limit and supersymmetry enhancement in the presence of tensor central charges, JHEP* **0208**, 008 (2002) [hep-th/0206214].