DEEP OPTICAL IMAGES OF MALIN 1 REVEAL NEW FEATURES

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ABSTRACT

We present Megacam deep optical images (g and r) of Malin 1 obtained with the 6.5 m Magellan/Clay telescope, detecting structures down to \( \sim 28 \) B mag arcsec\(^{-2} \). In order to enhance galaxy features buried in the noise, we use a noise reduction filter based on the total generalized variation regularizer. This method allows us to detect and resolve very faint morphological features, including spiral arms, with a high visual contrast. For the first time, we can appreciate an optical image of Malin 1 and its morphology in full view. The images provide unprecedented detail, compared to those obtained in the past with photographic plates and CCD, including Hubble Space Telescope imaging. We detect two peculiar features in the disk/spiral arms. The analysis suggests that the first one is a possible background galaxy, and the second is an apparent stream without a clear nature, but could be related to the claimed past interaction between Malin 1 and the galaxy SDSSJ123708.91+142253.2. Malin 1 exhibits features suggesting the presence of stellar associations and clumps of molecular gas, not seen before with such a clarity. Using these images, we obtain a diameter for Malin 1 of 160 kpc, \( \sim 50 \) kpc larger than previous estimates. A simple analysis shows that the observed spiral arms reach very low luminosity and mass surface densities, to levels much lower than the corresponding values for the Milky Way.

Key words: galaxies: general – galaxies: spiral – techniques: image processing

Supporting material: data behind figure

1. INTRODUCTION

One of the main factors precluding the observability of the extragalactic universe is the limitation in surface brightness (SB) by different surveys. In the seventies, different authors showed that the universe is populated by low surface brightness galaxies (LSBGs) having much lower SB than the dark night sky (Disney 1976; Bothun et al. 1987). Dalcanton et al. (1997) showed that one can expect about 4 galaxies deg\(^{-2} \) between the range 23–25 mag arcsec\(^{-2} \) using data from CCD drift scans from the eighties. In the nineties and already entering in the new millennium, there were no surveys with significantly fainter surface brightness limits. Some progress on the statistical significance of the population of LSBGs has been made in recent years because of the higher volumes sampled, thanks to massive surveys like the SDSS and others, but still barely reaching \( \sim 23.0 \) mag arcsec\(^{-2} \). On the other hand, and quite unexpectedly, van Dokkum et al. (2015) and Kodis et al. (2015) reported on the discovery of a dozen Milky Way (MW) sized, passively evolving, ultra diffuse galaxies in the Coma cluster. This population is very likely dark matter dominated and thus represents a challenge to the current theories of galaxy formation. Without the presence of large fractions of dark matter in LSBGs, it is very difficult to prevent the rapid destruction of low-density galaxies within a massive cluster like Coma (Toomre & Toomre 1972; Moore et al. 1999; McGee & Balogh 2010).

Thus, there are still many unsolved issues concerning the nature of LSBGs. One of these issues is the nature of the class of the so-called giant LSBGs: large-format spiral galaxies with a similar or larger size than the MW and very low SB. Big spiral galaxies exhibiting similar morphologies of the grand design spiral galaxies like M31 and the MW, but with much lower stellar density, as described in Sprayberry et al. (1995), Impy et al. (1996), Galaz et al. (2002, 2011), and references therein. The best and more extreme example in this category is Malin 1, a disk/spiral active galaxy (Impy & Bothun 1989; Barth 2007) with a disk SB of \( \geq 24 \) B mag arcsec\(^{-2} \), an uncertain inclination (the H\( \alpha \) data indicate that this galaxy has an inclination of about 50\(^\circ\)), and a barred inner disk (Barth 2007). In addition to this faint SB, what makes Malin 1 exceptional is its size: about 110 kpc diameter in H\( \alpha \) and presumably a similar or larger diameter in the optical, making this galaxy the largest one in the universe detected so far (\( \sim 6.5 \) times the MW diameter).

Malin 1 was serendipitously discovered by Bothun et al. (1987) in photographic plates. It is close to NGC 7145 in the Virgo cluster and near a bright star. This last feature prevents one from obtaining deep optical imaging easily without contaminating the FOV with scattered light, imposing additional difficulties for observing its morphology. Malin 1 has been studied so far in H\( \alpha \) (Pickering et al. 1997; Lelli et al. 2010) and in the optical (Moore & Parker 2006; Barth 2007). It is from these studies that its huge size was measured. Also, the H\( \alpha \) data allow us to estimate a H\( \alpha \) mass of \( \sim 10^{10} M_\odot \), and a dynamical mass of \( \sim 10^{12} M_\odot \), which makes Malin 1 an enormous dark matter reservoir (Seigar 2008). The galaxy has been also an interesting target for Spitzer (Rahman et al. 2007).
and for sub-millimeter observations (Das et al. 2006), for searching both warm and cold dust, as well as for detecting emission from molecular clouds. These observational efforts subsequently failed, which means that Malin 1 has a very low gas density and/or low dust content. These observational facts place Malin 1 at the bottom end of the Krumholz diagram (Krumholz & McKee 2005), which relates the stellar formation efficiency and the gas surface density. In this diagram, Malin 1 is certainly one of the galaxies that has the lowest stellar formation efficiency with one of the lowest molecular gas densities.

In spite of the wealth of information for Malin 1, including Hubble Space Telescope (HST) imaging (O’Neil et al. 2000) and ground-based observations (Moore & Parker 2006; Reshetnikov et al. 2010), optical images usually lack good quality and are not deep enough to detect features fainter than 24 B mag arcsec$^{-2}$. For example, HST observations of Malin 1 (O’Neil et al. 2000) are about 15 minute exposures, reaching data basically for the bulge of the galaxy, but extremely shallow images for other structures, including the disk and the presumed spiral arms. Moore & Parker (2006) built deep imaging of Malin 1 co-adding R images from 63 UKST photographic films digitally scanned by the superCOSMOS machine, covering the astonishing area of 36 deg$^2$. Reshetnikov et al. (2010) obtained spectroscopic observations for the central part of Malin 1 to prove that Malin 1B, a galaxy located 14 kpc from the Malin 1 center, is interacting with Malin 1. However, they do not provide optical imaging. Also relevant for this discussion is the work by Barth (2007), which claims that Malin 1 is a normal galaxy (with a normal disk), embedded in a larger, more diffuse low SB disk. However, in his analysis, Barth uses the same O’Neil et al. (2000) HST images, which are relatively shallow ones. In summary, no deep, high-quality CCD images are available for Malin 1.

In this Letter, we present deep optical images taken with Megacam at the 6.5 m Magellan/Clay telescope. We exposed for about 4.5 hr in r and g, reaching $\sim$28 B mag arcsec$^{-2}$, which allows us to observe for the first time features with unprecedented detail. We perform a careful reduction, and we use a novel image processing in order to enhance data practically buried into the noise.

The paper is organized as follow. In Section 2, we present the observations and key steps in the data reduction, and in Section 3, the main results and discussion. We conclude in Section 4. Throughout this paper we use $H_0 = 71$ km s$^{-1}$ Mpc$^{-1}$; $\Omega_m = 0.27$; $\Omega_\Lambda = 0.73$.

2. OBSERVATIONS AND DATA REDUCTION

Observations were performed at the 6.5 m Magellan/Clay telescope using Megacam with filters r and g, on the night of 2014 April 25. The night was dark, moonless, a necessary restriction for low SB targets. The night was also photometric with a median seeing of 0.8 arcsec. Exposure times were 4.8 hr in r and 4.2 hr in g (individual images of 10 minutes). After cosmic-ray rejection, individual images were flat-field and illumination corrected, then stacked, and properly combined (see left panels of Figure 1). The flat-field and illumination corrections are critical steps. Pixel-to-pixel variations along the images, as well as illumination inhomogeneities can introduce mid- and large-scale image artifacts that can be erroneously attributed to physical features of the target. In addition to the low SB of Malin 1, the field where the galaxy is located makes it difficult to obtain deep images. First, a bright star of 8.5 mag (BD+15 2483) is located 4.7 arcmin SW of Malin 1. Second, the galaxy NGC4571 is located 6.6 arcmin south of Malin 1. These two sources generate scattered light and sky brightness that contaminate the already faint structure of Malin 1. To combine different frames, synthetic background models were extracted. These models allow us to remove large-scale components preventing a proper match between frames. Frame matching, image registration, image combination, and all further post-processing steps were done using PixInsight, co-developed by one of the authors of this Letter (Consejero et al. 2015). The average sky SB was 20.2 r mag arcsec$^{-2}$, and 21.8 g mag arcsec$^{-2}$, equivalent to $\sim$22.5 B mag arcsec$^{-2}$, using the filter transformations from Jordi et al. (2006).

2.1. Noise Treatment and Multiscale Processing

Given the extremely low SB nature of Malin 1 disk/spiral arms, after the basic image processing described before (calibration and image combining), we perform a more sophisticated procedure in order to enhance the signal. For this task we applied a noise reduction filter based on the total generalized variation regularizator (TGV; Bredies et al. 2010). This is similar to an anisotropic diffusion filter, promoting piecewise smooth images instead of piecewise constant images as is common with these filters. This denoising filter was applied to both r and g images. These were then integrated into a false color image, assigning r data to the red channel, g data to the blue channel, and using a weighted average to create a synthetic green channel. Later, the images were de-linearized by applying a histogram transform that uses the rational interpolation mid-tones transfer function described by Schlick (1994). This provides more contrast to low signal features than a standard gamma transform. To enhance and isolate the structures of large diffuse objects, pinpoint sources (i.e., stars) were removed from the images using an inpainting algorithm based on TGV. This separation between features is needed because stars are very contrasted objects, and their signal dominates in the wavelet domain. Direct filtering leads to Gibbs or ringing artifacts or could enhance structures that correspond to regions of different stellar density, instead of enhancing the smooth background signal. Once those features are isolated, local contrast is enhanced using the starlet transform (also known as à trous Wavelets; Stark et al. 2010). Basically, the image is decomposed into layers with a redundant decomposition, isolating structures at certain characteristic sizes. Wavelet coefficients are modified with multiplicative factors to enhance features at each layer. Finally, small- and large-scale objects were combined again to achieve the images presented in the right panels of Figure 1. All the processing steps were performed on a per-channel basis, and no information was transferred through them. To allow better visual inspection of the image, an inverted and monochrome version is also presented (bottom panels of Figure 1 and Figures 2 and 3).

2.2. Photometric Calibrations and SB

In order to assess our SB limit, it is necessary to photometrically calibrate the images. We have observed Landolt standards and used the photcal package within IRAF to obtain the photometric solutions. Then, knowing the pixel
size \((0.16 \text{ arcsec pix}^{-1})\), we estimate that the SB we reach in our flat-corrected and combined images at \(4\sigma\) is \(\sim 26.5 \text{ mag arcsec}^{-2}\) in the \(r\) band and \(\sim 27.5 \text{ mag arcsec}^{-2}\) in the \(g\) band, as observed in regions R3 and R6, labeled in Figure 3. These values translate to \(\sim 28 \text{ B mag arcsec}^{-2}\). This is \(\sim 3-4 \text{ mag arcsec}^{-2}\) fainter than the typical SB reached by surveys and other works to date (Moore & Parker 2006).

3. RESULTS AND DISCUSSION

Figure 1 shows several panels where Malin 1 is presented. Left panels represent the stacked and combined images. Right panels show the images with the additional TGV treatment. In the top panels, the combined RGB images of the field are presented (see Section 2.1). In the bottom panels, the inverted and monochrome version of the same image is shown. Some morphological features are clearly seen, especially in the right panels. Some of them observed for the first time.

3.1. Bulge, Spiral Arms, and Stellar Density

Figure 1 shows clear images of a face-on spiral galaxy presenting a bright inner region that reproduces the image from Barth (2007). In Figure 2, we compare with the \(HST/WFPC2\) image from Barth (2007). The inner region of Malin 1 appears much redder than any other structure visible in the galaxy.

Figure 1. Processed images of Malin 1. Left panels: (top) RGB image generated using the stacked and combined \(g\) and \(r\) images, totaling about 4.5 hr exposure time. (bottom) The same as the top panel but a monochrome and inverted version of the same image. Right panels: the same as the left panels, but after applying the total generalized variation regularizator (TGV), and starlet transform for noise reduction and multiscale processing. The enhancement of the spiral arms and other features discussed in the text are apparent after the TGV processing, comparing the left and right panels. The upper left arrow indicates the galaxy SDSSJ123708.91 +142253.2, which is claimed to have interacted 1 Gyr ago with Malin 1 (Reshetnikov et al. 2010). The near center arrow points to a feature discussed in the text. The data used to create this figure are available.

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Figures 1 and 2 show that Malin 1 exhibits well-formed spiral arms. This demonstrates that in ultra-faint disk LSBGs the spiral structure can be well developed to very faint SB limits. In order to quantify the density contrast of the spiral arms, we can compare the estimated density of these structures with the stellar density in the solar neighborhood. The MW disk at the solar vicinity has a stellar mass surface density of \( \sim 49 M_\odot \text{pc}^{-2} \) (Flynn et al. 2006). From the same author, the luminosity surface density is \( 25.6 L_\odot \text{pc}^{-2} \), or 23.5 mag arcsec\(^{-2} \) in the \( B \) band. Converting the observed SB of \( \sim 28 B \text{mag arcsec}^{-2} \) in luminosity surface density in solar units, we obtain \( \sim 0.41 L_\odot \text{pc}^{-2} \), i.e., about 62 times lower than the surface luminosity density in the solar neighborhood indicated above.\(^8\) Using a mass–luminosity for our local volume of \( (M/L)_B = 1.4 \) (Flynn et al. 2006), we obtain a stellar mass column density of \( \sim 0.57 M_\odot \text{pc}^{-2} \), again, 62 times smaller than the 35.5 \( M_\odot \text{pc}^{-2} \) in the local volume (see also Moni Bidin et al. 2012). Note that we do not correct these values for the effect of inclination on SB since our data suggest that Malin 1 is almost face-on.

As shown in all figures we observe for the first time in full view, Malin 1 is far from having a featureless low SB disk. It presents rich features, suggesting stellar formation regions and well-defined spiral arms. We emphasize that these are well-developed structures with an extremely low SB, which translates to extremely diffuse stellar density (as shown above). The data then suggest that morphological structures as observed here can be long lived in this kind of LSBG and could imply large concentrations of dark matter (Seigar et al. 2014). The formation of spiral arms observed to such low stellar densities is another issue. This could be explained by the action of density waves in the matter distribution. However, such a theory is still in discussion by some authors (Choi et al. 2005). Or perhaps a better explanation for the spiral structure of Malin 1 is tidal driving or even shear driving (Dwarkadas & Balbus 1996), which is more consistent with the presumed past encounter between Malin 1 and the galaxy SDSSJ123708.91+142253.2, shown in the panels of Figure 1 by the upper left arrow.

\(^8\) We have used \( M_\odot : B = 5.45 \) mag and an SB for the solar neighborhood of 27.02 mag arcsec\(^{-2} \).
3.2. Streams and Other Features

We observe two diffuse and cigar-like features embedded in the disk of Malin 1 and seem to cross it radially. The shorter one labeled as A in Figure 3 is likely the disk of a galaxy with a bulge located at R.A. (J2000.0) 12:36:59 and decl. (J2000) +14:19:44. The SDSS database indicates that this object has a photometric redshift of 0.19 ± 0.05, suggesting that this galaxy is in the background, i.e., about 30,000 km s^{-1} away from Malin 1. Its morphology, undisturbed by the presence of Malin 1, seems to confirm that its redshift estimate is correct. Nevertheless, this value should be confirmed spectroscopically in order to ensure this galaxy is not interacting with Malin 1. The other, larger structure, labeled as B in Figure 3 and also apparent in Figures 1 and 2, does not present any bulge and its nature is unclear. We do not discard that we may be observing the jet of the active galactic nucleus of Malin 1, classified as a LINER from spectral data by Barth (2007) and as a high-excitation Seyfert galaxy by Impey & Bothun (1989). The other possibility, and perhaps the most likely, is that this feature is part of the claimed past interaction between Malin 1 and the galaxy SDSSJ123708.91+142253.2 approximately 1 Gyr ago, located at ∼350 kpc from Malin 1 (Reshetnikov et al. 2010; indicated by upper arrows in Figure 1). We note that this stream extends up to ∼200 kpc from the center of Malin 1. We do not discard the relationship of these features with the presence of Malin 1B (see Figure 3), as discussed in Reshetnikov et al. (2010).

3.3. Stellar Formation Regions?

The presence of several diffuse regions in the spiral arms exhibiting morphologies similar to stellar formation regions and/or clumps of gas is apparent. Examples of these clumps are labeled as R1, R2, R3, R4, R5, and R6 in Figure 3. Three of these regions (R1, R2, and R3) are the same as those observed by Braine et al. (2000) with the IRAM 30 m telescope for detecting CO, unsuccessfully, though. These three regions were selected by Braine et al. (2000) after original observations by Bothun et al. (1987). This is the first time that such regions are so well defined in optical images.

3.4. Optical Diameter of Malin 1 and Spiral Arm Thickness

One of the most impressive features of Malin 1 is its large diameter. Bothun et al. (1987, 1997) measured an optical diameter of 110 kpc, almost 6.5 times larger than the diameter of the MW, making Malin 1 the largest disk/spiral galaxy in the universe so far. Considering a redshift of 0.082, a CCD (of the MW, making Malin 1 the largest disk diameter of 110 kpc, almost 6.5 times larger than the diameter.

Nevertheless, this value should be confirmed spectroscopically in order to ensure this galaxy is not interacting with Malin 1. The other, larger structure, labeled as B in Figure 3 and also apparent in Figures 1 and 2, does not present any bulge and its nature is unclear. We do not discard that we may be observing the jet of the active galactic nucleus of Malin 1, classified as a LINER from spectral data by Barth (2007) and as a high-excitation Seyfert galaxy by Impey & Bothun (1989). The other possibility, and perhaps the most likely, is that this feature is part of the claimed past interaction between Malin 1 and the galaxy SDSSJ123708.91+142253.2 approximately 1 Gyr ago, located at ∼350 kpc from Malin 1 (Reshetnikov et al. 2010; indicated by upper arrows in Figure 1). We note that this stream extends up to ∼200 kpc from the center of Malin 1. We do not discard the relationship of these features with the presence of Malin 1B (see Figure 3), as discussed in Reshetnikov et al. (2010).

4. CONCLUSIONS

In this Letter, we have obtained deep optical images of Malin 1, the faintest and largest giant LSBG so far, to unprecedented depths. The galaxy is composed of a faint bright region, well differentiated from the outer part composed of spiral arms. The color difference between the inner part and the spiral arms also appears extreme, with the former much redder than the spiral arms. We clearly see spiral arms and other structures down to an SB ∼28 B mag arcsec^{-2}. Comparing the resulting luminosity and stellar surface density of Malin 1 spiral arms with that derived for the disk of our Galaxy, we obtain ∼0.41 L_{⊙} pc^{-2} and ∼0.57 M_{⊙} pc^{-2}, i.e., 62 times lower than those of corresponding MW values. And yet, the galaxy exhibits a textbook and gigantic spiral structure, to these very low density values. This is certainly the most astonishing result of this work. How spiral arms can form and remain long lived to these very low density contrasts is beyond the scope of this Letter. The deep g and r images reveal details of other features marginally observed in the past. In particular, we observe two conspicuous features with cigar-like morphology crossing the spiral arms: one is very likely a background galaxy, and the other is a fainter structure pointing to the galaxy SDSSJ123708.91+142253.2, claimed by Reshetnikov et al. (2010) to have interacted 1 Gyr ago with Malin 1. Our imaging does not allow us to draw conclusions about the nature of this structure. Other observed features seem to be stellar formation regions or gas clumps, some of them already observed by Braine et al. (2000) with the IRAM 30 m telescope in order to detect CO, unsuccessfully, however. Our data also confirm that Malin 1 is even larger than previously reported, reaching 160 kpc diameter, i.e., more than 6.5 times the diameter of the MW. The fundamental result is that Malin 1 appears, at last, in full view in optical bands, allowing us to see features that were not revealed before in such detail and contrast.

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