Properties of phoneme $N$-grams across the world’s language families

ABSTRACT
In this article, we investigate the properties of phoneme $N$-grams across half of the world’s languages. We investigate if the sizes of three different $N$-gram distributions of the world’s language families obey a power law. Further, the $N$-gram distributions of language families parallel the sizes of the families, which seem to obey a power law distribution. The correlation between $N$-gram distributions and language family sizes improves with increasing values of $N$. We applied statistical tests, originally given by physicists, to test the hypothesis of power law fit to twelve different datasets. The study also raises some new questions about the use of $N$-gram distributions in linguistic research, which we answer by running a statistical test.

KEYWORDS: N-grams, ASJP, language families.
1 Introduction and related work

1.1 Power laws

Many real-life phenomena such as word-type frequencies in a corpus, degrees of nodes in a network representation of the internet, the number of species in a genus of mammals and populations of cities follow a power law distribution. Power law distributions seem ubiquitous in nature and many other phenomena are also claimed to obey a power law (Clauset et al., 2009). Computational linguists will typically have come across power laws in a form popularly known as Zipf’s law (Zipf, 1935). Zipf’s law is stated as $f \propto r^{-1}$, where $f$ and $r$ is the frequency and rank of a word type $x$. This is a special case of the power law with the probability density function $p(x)$ defined as $p(x) = C x^{-\alpha}$ where $\alpha = 1$. $\alpha$ is the scaling parameter ($\alpha > 1$) and $C$ is the normalizing constant. If $p(x)$ is lower-bounded at $x_{min}$ then the power law assumes the form of $p(x) = (\alpha - 1)x^{\alpha-1}x^{-a}$.

Power law is just one member of a larger class of distributions called large number of rare events (LNRE) distributions (Baayen, 1991). As pointed by Evert and Baroni (2007), LNRE distributions have applications in NLP/CL. LNRE distributions can be used to predict the total vocabulary size from a smaller sample. We now turn to a discussion of some recent work in computational historical linguistics where power laws play a central role in the argumentation.

There are about 7000 languages in the world (Lewis, 2009; Hammarström, 2010), forming more than 120 families according to the Ethnologue, whereas more than 400 are listed by Hammarström (2010). A language family is a group of related languages (or a single language when there are no known related languages, such as Basque) descended from a common ancestor (Campbell and Poser, 2008). Each of these language families is assigned a tree structure in at least two classifications (Lewis, 2009; Hammarström, 2010). The size of a language family is defined as the number of related languages included in the family. Wichmann (2005) observes that the frequency-rank plot of the sizes of language families (as defined in Ethnologue) seems to follow a power law. Figure 1a (reproduced from Wichmann 2005) is plotted on a log-log scale and shows a slight deviation from the regression line in the region of 50 and $\geq 100$. Figure 1b shows the frequency-rank plot for Hammarström’s classification.

There is a slight deviation from the strict adherence to the straight line in Figure 1b. However, the goodness-of-fit $r^2$ is in the range of 0.957 and 0.98 in both the classifications. Looking into the closely related field of linguistic typology, Maslova (2008) proposes that meta-typological distributions obey power law. A meta-typological distribution is defined as the number of languages having a particular linguistic feature value, such as a particular word order or a phoneme inventory of a particular size (e.g., small, medium, or large). In response, Cysouw (2010) proposes that the distribution is actually exponential masquerading as a power law.

1.2 Testing power laws

The scaling parameter $\alpha_{sp}$, $\alpha$ – estimated using a spreadsheet package – in Figures 1a and 1b is 1.905 and 1.36 respectively. Apart from the high $r^2$ value, there seems to be no independent

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1 $C$ is calculated by solving for $\int_{x_{min}}^{\infty} C x^{-\alpha} = 1$ for $\forall x \in \mathbb{R}$. In this paper, $x$ takes up integer values only.

2 Hammarström (2010) uses cardinal size to indicate family size.

3 In this context, frequency denotes the family size.

4 The data for these experiments is derived from the World Atlas of Language Structures (WALS; Haspelmath et al., 2011). The data is generated by random selection of a linguistic feature value and counting the number of languages for that value.
A statistical test for the support of a power law. However, a recent paper by Clauset et al. (2009) revisited this topic and proposed a number of statistical tests for validating power law models. The authors provide a maximum likelihood estimate of the two parameters, $x_{\text{min}}$ and $\alpha$ and a method for computing the statistical significance score of the estimates. Further, they test the superiority of the power law with respect to candidate distributions, listed in Table 1.

In a recent paper, Jäger (2012) applied the statistical tests of Clauset et al. (2009) to test the fit of the power law model to global linguistic datasets such as frequency of color terms, phonological templates for selected basic vocabulary items, and meta-typological distributions. Jäger also applied a series of statistical tests to Maslova’s data and showed that a power law with exponential cutoff describes the data better than a power-law model.

| name                                      | probability density function $p(x)$ | # of parameters ($m$) |
|-------------------------------------------|-------------------------------------|-----------------------|
| power law (PL)                            | $(\alpha - 1)x_{\text{min}}^{\alpha - 1}x^{-\alpha}$ | 1                     |
| power law with exponential cutoff (PLWC)  | $\frac{\lambda^{1-\alpha}}{\Gamma(1-\alpha, \lambda x_{\text{min}})} x^{-\alpha}e^{-\lambda x}$ | 2                     |
| log-normal (LN)                           | $\sqrt{\frac{2}{\pi \sigma^2}} \left[ \text{erfc} \left( \frac{\ln x_{\text{min}} - \mu}{\sqrt{2 \sigma}} \right) \right]^{-1} \frac{1}{x} \exp \left[ -\frac{(\ln x - \mu)^2}{2\sigma^2} \right]$ | 2                     |
| exponential (exp)                         | $\beta \lambda e^{\lambda x_{\text{min}}^\beta} x^{\beta - 1} e^{-\lambda x^\beta}$ | 2                     |
| stretched exponential (str exp)           | $\frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-x/\theta}$ | 2                     |
| gamma ($\Gamma$)                          |                                      |                       |

Table 1: List of various candidate distributions and the number of parameters in each model. These distributions are popularly referred to as “heavy-tail” distributions.

The standard method for testing a power law hypothesis consists of plotting a frequency-rank plot on a log-log scale and applying a linear regression. The linear regression boils down to determining the parameters of $\log p(x) = c + \alpha \log(x)$. However, Clauset et al. (2009) warn against this. Further, they demonstrate that the value of estimated $\alpha$ differs largely from that derived from the regression analysis. The validity of the power law is tested through the following steps:

- Estimate $\alpha_{sp}$ and $r^2$ using a spreadsheet package by plotting the frequency-rank plot of
the data on a log-log scale.

- Estimate $\alpha_{est}$ and $x_{min}$ based on the maximum likelihood criterion ($L$).
- The preference of a power law to rest of the candidate distributions is tested through a likelihood ratio test (Dunning, 1993) under a significance criterion of $p \leq 0.1$.
- The absolute goodness-of-fit of a model is computed using the Akaike Information criterion which is defined as in $AIC = 2m - 2\log(L)$ (1).

Jäger simplifies the computation of $\alpha$ for discrete data by assuming a continuous approximation of the power-law model and fixing $x_{min}$ at 0.5. This assumption implies that all data points in a dataset completely fit a power-law model. However, it can always be the case that only a part of the dataset follows a power-law model. As a necessary diversion, it is useful to know the computation of $x_{min}$. The scaling parameter $\alpha$ is estimated by successive removal of the lowest value of $x$. The fitted distribution is then compared to the empirical distribution through a Kolmogorov-Smirnov statistic ($D$). The value of $x$ which minimizes $D$ is chosen as $x_{min}$.

In this paper, we find that the rank plot of the size of phoneme $N$-grams for 45 language families seems to obey a power law distribution as given in Figure 2a. This finding is in parallel to that of Wichmann (2005). By applying the statistical procedure mentioned above, we attempt to establish whether the family sizes given in three different language classifications actually obey a power law. Subsequently we test if the phoneme $N$-grams also obey a power law model. We describe the database in the next section.

2 Database

In this section, we describe the global linguistic database (depicted in Figure 2a). A consortium of international scholars known as ASJP (Automated Similarity Judgment Program; Wichmann et al. 2011) have collected reduced word lists – 40 items from the original 200 item Swadesh word lists (Swadesh, 1955), selected for maximal diachronic stability – for more than half of the world’s languages and embarked on an ambitious program for investigating automated classification of the world’s languages. The ASJP database in many cases includes more than one word list for different varieties of a language (identified through its ISO 693-3 code). A word list is included into the database if it has attestations of at least 28 of the 40 items (70%). Only language families with at least 4 members are included in our experiments. This leaves a dataset with 45 language families representing 3151 languages (or 4524 word lists) of the world. The names of the language families are as defined in the Ethnologue. A word list might include known borrowings marked as such and these are not used in our experiments. The words in the ASJP database are transcribed using a reduced phonetic transcription known as ASJP code consisting of 34 consonants, 7 vowels and a symbol for nasalization, and two other ‘modifiers’, which are used to indicate that preceding symbols combine as single segments. All click sounds are reduced to a single click symbol and distinctions such as tones, vowel length, and stress are ignored. The computation of a phoneme $N$-gram profile for a language family is described in section 3. The frequency-rank plot of the language families in the current sample is shown in Figure 2b. The regression shows a $r^2$ value of 0.85 which is quite high.
3 Experiments and results

All the word lists belonging to a single language family are merged together. Recall that the ASJP database can include more than one word list – representing different varieties – for a language. All the consecutive symbol sequences of length varying from 1–5 are extracted and the size of the N-gram profile is defined as the total number of unique 1–N-grams obtained through this procedure. Thus, a 3-gram profile consists of all the phoneme 1-, 2- and 3-grams. The size of the 3-, 4- and 5-gram profiles for each of the language families as defined in the Ethnologue is given in Table 2. In effect, an N-gram profile is the sum of all the n-gram types leading up to N. As evident from right panel of Figure 3, each of the N-gram profiles, \( N \geq 3 \), seem to follow a power law.

When a power law regression is applied to each of the frequency-rank plots, the goodness-of-fit \( r^2 \) is 0.91, 0.96 and 0.97 for 3-grams, 4-grams and 5-grams respectively. The \( r^2 \) value of both 1-grams and 2-grams is quite low, only 0.49 and 0.73 when compared to the \( r^2 \) value of the number of languages, 0.85. We also plot the frequency-rank plots for each n-gram type in the left panel of Figure 3. The \( r^2 \) values are quite high and are 0.84, 0.93 and 0.94 respectively. The \( r^2 \) scores in Figure 1a for 4–5 grams are very high and fall within the range of the correlation of 0.957 (with language family size), reported by Wichmann 2005.

As we have shown above, the correlation of N-gram distribution to language family size improves with increasing \( N \) (for \( N = 1 \) – 5). This is a kind of behavior familiar from corpus studies of word distributions (Baayen, 2001), where closed-class items – typically function words – yield distributions similar to the 1-grams (phonemes) in this study, whereas open-class words display typical power-law behavior for all corpus sizes, just like the 3–5-grams in this study. We take this as an indication that we are on the right track, investigating a genuine linguistic phenomenon. We also test if the genus size across the world’s languages displays a power-law like behavior. A genus (pl. genera) is a language classification unit which contains related languages and is estimated to be 3000–3500 years old. The genus level was originally introduced by Dryer (2000). We use the genus information given in ASJP database. The current dataset has 538 genera and 5315 word lists. Table 3 shows the results of the application of the
Table 2: The number of languages (NOL), 3-gram, 4-gram and 5-gram profiles for 45 language families.

| Language family       | NOL | 3-gram | 4-gram | 5-gram | Language family       | NOL | 3-gram | 4-gram | 5-gram |
|-----------------------|-----|--------|--------|--------|-----------------------|-----|--------|--------|--------|
| Austronesian          | 692 | 9808   | 26527  | 40561  | Macro-Ge              | 20  | 1813   | 2684   | 3180   |
| Niger-Congo           | 615 | 13085  | 27766  | 36987  | Sepik                 | 22  | 1677   | 2623   | 3144   |
| Trans-NewGuinea       | 275 | 6492   | 17051  | 25634  | Tai-Kadai             | 42  | 2036   | 2723   | 3077   |
| Afro-Asiatic          | 201 | 8456   | 17403  | 22403  | Chibchan              | 16  | 1522   | 2484   | 3035   |
| Australian            | 104 | 3690   | 9143   | 14401  | WestPapuan            | 14  | 1244   | 2203   | 2806   |
| Indo-European         | 139 | 6320   | 11252  | 13896  | EasternTrans-Fly      | 4   | 1300   | 2166   | 2729   |
| Nilo-Saharan          | 123 | 5224   | 10046  | 12891  | Dravidian             | 24  | 1376   | 2238   | 2674   |
| Sino-Tibetan          | 137 | 5753   | 9386   | 11043  | LakesPlain            | 17  | 1142   | 2010   | 2518   |
| Arawakan              | 39  | 2626   | 5148   | 7021   | Border                | 8   | 1132   | 1922   | 2467   |
| Austro-Asiatic        | 82  | 3370   | 5552   | 6608   | South-CentralPapuan   | 6   | 1074   | 1878   | 2464   |
| Oto-Manguean          | 69  | 3522   | 5607   | 6579   | Penutian              | 14  | 1326   | 2017   | 2384   |
| Uto-Aztecane          | 43  | 2318   | 4395   | 5873   | Panoan                | 15  | 1192   | 1915   | 2288   |
| Altaic                | 46  | 2634   | 4304   | 5248   | Wototoan              | 7   | 1185   | 1847   | 2264   |
| Salishan              | 16  | 2492   | 3944   | 4903   | Hokan                 | 14  | 1253   | 1864   | 2192   |
| Algic                 | 19  | 1922   | 3466   | 4643   | Quechuan              | 22  | 1101   | 1734   | 2093   |
| Tupi                  | 46  | 2250   | 3722   | 4619   | Siouan                | 11  | 1131   | 1674   | 1952   |
| Torricelli            | 21  | 2011   | 3518   | 4523   | Na-Dene               | 15  | 1225   | 1637   | 1810   |
| Mayan                 | 48  | 2083   | 3485   | 4386   | Hmong-Mien            | 15  | 1246   | 1563   | 1717   |
| Tucanoan              | 18  | 1880   | 3162   | 3979   | Totonacan             | 10  | 679    | 1139   | 1510   |
| Ramu-LowerSepik       | 14  | 1491   | 2738   | 3676   | Khoisan               | 12  | 995    | 1265   | 1377   |
| Carib                 | 20  | 1662   | 2868   | 3649   | Sko                   | 12  | 775    | 1068   | 1179   |
| NorthCaucasian        | 29  | 2180   | 3158   | 3537   | Mixe-Zoque            | 12  | 625    | 897    | 1028   |
| Uralic                | 23  | 1896   | 2818   | 3284   |                       |     |        |        |        |

Judging by AIC, none of the classification unit datasets follow a power-law distribution. It is important to notice that the $\alpha_{est}$ widely differs from $\alpha_{sp}$. As demonstrated by Clauset et al. (2009), there can be a large difference when estimating $\alpha$ for small datasets of size $\leq 50$. Only the $2-5$-gram profiles follow a power-law with cutoff model ascertained by the lowest AIC value. Interestingly, $n_{tail}$ values are highest for $n = 3$ followed by 4 and 5. The value of $\alpha$ for a power law is typically between 2 and 3. The values of $\alpha_{est}$ for $N$-gram profile also lie between 2 and 3.

The AIC values in Table 3 suggest that the power-law with cutoff is a better model than power-law for for $N$-gram profiles. We assess this superiority through a likelihood ratio test. The results
Table 3: The first three rows correspond to the language size data of Hammarström (2010), ASJP data (from Table 2) and genera sizes. Columns 2–5 correspond to the estimated parameters in a power law. Column 5 shows the AIC value for a power law. Column 6 shows the $\alpha_{sp}$ estimated by a standard spreadsheet package. The remaining columns correspond to the AIC values for the other candidate distributions. The last four rows show the fit of each $n$-gram profile leading up to 5 and are indicated by a †. For each dataset, the best fit model is indicated in bold. Here, the common factor 2 is not included in the AIC computation. All the above results are computed using the power-law python package (Alstott, 2012).

Table 4: The table shows the results of the likelihood-ratio test for comparing the power law with the rest of candidate distributions. The – sign indicates the test favoring the candidate model than the power law model. Each number is the $p$-value and the significance is indicated in bold.

are given in Table 4. The results suggest that the PLWC is a better fit than PL at a significance criterion $p \leq 0.1$. Interestingly, none of the family-size datasets are genuinely power-lawish. They seem to belong to other “heavy-tailed” distributions. Incidentally, Hammarström’s dataset – covering more than 7000 languages – fits better to a stretched exponential model than a power-law distribution.

Even though this study shows that phoneme $N$-gram profiles closely mirror the power-law-with-cutoff behavior, it raises more questions than it answers about the use of $N$-gram distributions in linguistic research, such as:

Q. Is the $N$-gram distribution an effect strictly connected with genetic relatedness among the languages, or simply an effect of the number of languages in a group (regardless of
whether they are related or not)?

A. We answer this question through the following procedure:

1. For a family size \( s \), make a random sample of languages of size \( s \).
2. Compute the \( N \)-gram profiles.

Repeat steps 1 – 2 for all family sizes and plot the \( N \)-gram profile sizes. The results are shown in Figure 4a. All the \( r^2 \) values are in the range of 0.68 to 0.75. This experiment suggests that the \( N \)-gram distribution is related to genetic relatedness and not an effect of a sample size.

Q. If the effect is genetic, can the size of the family be predicted from \( N \)-gram profiles of smaller samples than the full family? (This could be very useful.)

A. We answer this question through the following procedure:

1. For a family of size \( s \), create a random language sample of size \( i \), where \( 1 \leq i \leq s \).
2. Compute the \( N \)-gram profiles for each random sample.
3. Repeat steps 1 – 2 for 10 iterations and compute the average size of a \( N \)-gram profile for each \( i \).

Repeat the steps 1 – 3 for all \( i \). The results of this experiment for \( s = 104 \) (Australian family) is shown in Figure 4b. Figure 4b shows the plot for the average number of \( N \)-gram types vs. the size of language sample. All \( N \)-gram curves (except 1 and 2) seems to be increasing monotonically and not stabilizing after a particular sample size. Only 1-grams and 2-grams tend to stabilize with respect to sample size. The \( N \)-gram curves for other language families also follow the same trend. These results suggest that the \( N \)-grams (\( N \geq 3 \)) of smaller samples cannot be used to predict the full family size.

4 Conclusion

In this paper, we tested if the language units of the three classifications obey a power law. We find that the three datasets are not well modeled by a power law model. We then tested if the \( N \)-gram profiles follow a power law and observed that they actually follow a power-law with cutoff distribution. Finally, we posed two questions about the utility of \( N \)-grams for historical linguistics and found that \( N \)-grams do not pass the test.
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