Z–Z′ mixing and oblique corrections
in an SU(3) × U(1) model

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We address the effects of the new physics predicted by the SU(3)_L × U(1)_X model
on the precision electroweak measurements. We consider both Z–Z′ mixing and
one-loop oblique corrections, using a combination of neutral gauge boson mixing
parameters and the parameters S and T. At tree level, we obtain strong limits on
the Z–Z′ mixing angle, −0.0006 < θ < 0.0042 and find M_{Z′} > 490GeV (both at 90%
C.L.). The radiative corrections lead to T > 0 if the new Higgs are heavy, which
bounds the Higgs masses to be less than a few TeV. S can have either sign depending
on the Higgs mass spectrum. Future experiments may soon place strong restrictions
on this model, thus making it eminently testable.
I. INTRODUCTION

Recently a model based on the gauge group SU(3)_L × U(1)_X has been proposed as a possible explanation of the family replication question [1]. By matching the gauge coupling constants at the electroweak scale [2], the mass of the new heavy neutral gauge boson, Z', is bounded to be less than 2.2 TeV and the mass upper bound for the new charged gauge bosons, Y^{±±} and Y^±, is 435 GeV [3]. Since Y^{++} and Y^+ carry two units of lepton number, they are called dileptons. Unlike most extensions of the standard model, in which the masses of the new gauge bosons are not bounded from above, this model would be either realized or ruled out in the future high energy colliders such as the superconducting supercollider and the next linear collider.

The new Z', by mixing with the standard model neutral gauge boson Z, modifies the neutral current parameters as well as the ρ-parameter [4]. The dileptons, Y^{±±} and Y^±, and the new charged Higgs, H^{±±} and H^±, on the other hand, do not participate directly in the precision LEP experiments [5] nor the neutrino scattering experiments [6]. Instead, they only enter radiatively, mainly via their oblique corrections to the W^± and Z propagators [7,8,9,10,11,12]. Nevertheless, such radiative corrections may be comparable to the tree level corrections due to the Z–Z' mixing. Thus we treat both cases in the following.

If the masses of the dileptons are degenerate, we may expect the oblique corrections to vanish. However, the mass degeneracy is lifted when SU(2)_L × U(1)_Y breaks into U(1)_Q; thus the mass squared splitting would be on the order of M_W^2. As a result, the oblique corrections to the parameters S and T [7] are expected to be on the order of (1/π)(M_W^2/M_{Y^{++}}^2), where M_{Y^{++}} is the mass of Y^{±±}. In addition, oblique corrections due to the new heavy charged Higgs, H^{±±} and H^±, are induced by the small mixing between Higgs multiplets. The contributions have the general form (1/π)(M_W^2/M_{Y^{++}}^2)(m_H^2/M_{Y^{++}}^2), where m_H is the mass of the new charged Higgs. Hence, the heavy charged Higgs contributions would be important even when the dilepton mass splitting is small.

Our analysis in this paper concentrates on both tree level and one-loop oblique corrections to the standard model due to the new physics of the SU(3)_L × U(1)_X model. For the dileptons and the new Higgs, which only contribute radiatively, we use the S, T and U parameters. However the effects of the Z', which enters at tree level, cannot be fully incorporated into this formalism, and may instead be parametrized by a Z–Z' mixing angle as well as the
mass of the heavy \( Z_2 \). We thus use five parameters to describe the new physics: the two \( Z' \) parameters and the three oblique ones. Starting with a discussion of tree level mixing, we perform a five parameter fit to experimental data to put strong limits on the \( Z-Z' \) mixing angle. We then discuss the consequences of the fit on the other particles by carrying out a complete one-loop calculation of \( S \) and \( T \) for dilepton gauge bosons and the new Higgs bosons. The new quarks, which are SU(2) singlets, do not contribute.

II. TREE LEVEL MIXING

We first outline the model, following the notation given in [2]. The fermions transform under SU(3)\(_c\) \( \times \) SU(3)\(_L\) \( \times \) U(1)\(_X\) according to

\[
\psi_{1,2,3} = \begin{pmatrix} e \\ \nu_e \\ \mu \\ \nu_\mu \\ \tau \\ \nu_\tau \\ \ell^c \\ \ell^c \\ \ell^c \end{pmatrix} : (1, 3^*, 0) , \quad (2.1a)
\]

\[
Q_{1,2} = \begin{pmatrix} u \\ c \\ d \\ s \\ D \\ S \end{pmatrix} : (3, 3, -\frac{1}{3}) , \quad (2.1b)
\]

\[
Q_3 = \begin{pmatrix} t \\ b \\ T \end{pmatrix} : (3, 3^*, \frac{2}{3}) , \quad (2.1c)
\]

\[
d^c, s^c, b^c : (3^*, 1, \frac{1}{3}) , \quad (2.1d)
\]

\[
u^c, c^c, t^c : (3^*, 1, -\frac{2}{3}) , \quad (2.1e)
\]

\[
D^c, S^c : (3^*, 1, \frac{4}{3}) , \quad (2.1f)
\]

\[
T^c : (3^*, 1, -\frac{5}{3}) . \quad (2.1g)
\]

where \( D, S \) and \( T \) are new quarks with charges \(-4/3, -4/3\) and \(5/3\) respectively. The minimal Higgs multiplets required for the symmetry breaking hierarchy and fermion masses are given by

\[
\Phi = \begin{pmatrix} \phi^{++} \\ \phi^+ \\ \phi^0 \end{pmatrix} : (1, 3, 1) , \quad (2.2a)
\]
\[ \Delta = \begin{pmatrix} \Delta^+_1 \\ \Delta^0 \\ \Delta^-_2 \end{pmatrix} : (1, 3, 0) , \] 

(2.2b)

\[ \Delta' = \begin{pmatrix} \Delta'^+_1 \\ \Delta'^0 \\ \Delta'^-_2 \end{pmatrix} : (1, 3, -1) , \] 

(2.2c)

and

\[ \eta = \begin{pmatrix} \eta^{++}_1 \\ \eta^+_1 / \sqrt{2} \\ \eta^0 / \sqrt{2} \\ \eta^- / \sqrt{2} \\ \eta^-_1 / \sqrt{2} \\ \eta^{--} \end{pmatrix} : (1, 6, 0) . \] 

(2.2d)

The non-zero vacuum expectation value (VEV) of \( \phi^0 \), \( u / \sqrt{2} \), breaks SU(3)\(_L\) \( \times \) U(1)\(_X\) into SU(2)\(_L\) \( \times \) U(1)\(_Y\). The SU(2) components of \( \Delta \) and \( \Delta' \) then behave like the ordinary Higgs doublets of a two-Higgs standard model. The sextet, \( \eta \), is required to obtain a realistic lepton mass spectrum. For simplicity, we will assume its VEVs are zero. As SU(3)\(_L\) \( \times \) U(1)\(_X\) is broken into SU(2)\(_L\) \( \times \) U(1)\(_Y\), the sextet will decompose into an SU(2) triplet, an SU(2) doublet and a charged SU(2) singlet. We will also assume that the mass splitting of these scalars within their multiplets is small; hence their contributions to \( S \) and \( T \) will be negligible.

As SU(2)\(_L\) \( \times \) U(1)\(_Y\) is broken by the VEVs of \( \Delta^0 \) and \( \Delta'^0 \), \( u / \sqrt{2} \) and \( v' / \sqrt{2} \), they will provide masses for the standard model gauge bosons, \( W^\pm \) and \( Z \). The VEVs also induce \( Z-Z' \) mixing as well as the mass splitting of \( Y^{\pm\pm} \) and \( Y^{\pm} \). Hence we obtain the masses for the charged gauge bosons,

\[ M^2_W = \frac{1}{4} g^2 (u^2 + v'^2) , \] 

(2.3a)

\[ M^2_{Y^\pm} = \frac{1}{4} g^2 (u^2 + v^2) , \] 

(2.3b)

and

\[ M^2_{Y^{++}} = \frac{1}{4} g^2 (u^2 + v'^2) , \] 

(2.3c)

and the mass-squared matrix for \( \{ Z, Z' \} \)

\[ \mathcal{M}^2 = \begin{pmatrix} M^2_Z \\ M^2_{ZZ'} \\ M^2_{Z'} \end{pmatrix} , \] 

(2.4)
with

$$M_Z^2 = \frac{1}{4} \frac{g^2}{\cos^2 \theta_W} (v^2 + v'^2), \quad (2.5a)$$

$$M_{Z'}^2 = \frac{1}{3} g^2 \left[ \frac{\cos^2 \theta_W}{1 - 4 \sin^2 \theta_W} u^2 + \frac{1 - 4 \sin^2 \theta_W}{4 \cos^2 \theta_W} v^2 \right. \right.$$ \[\left. + \frac{(1 + 2 \sin^2 \theta_W)^2}{4 \cos^2 \theta_W (1 - 4 \sin^2 \theta_W)} v'^2 \right], \quad (2.5b)

$$M_{ZZ'}^2 = \frac{1}{4 \sqrt{3}} g^2 \left[ \frac{\sqrt{1 - 4 \sin^2 \theta_W}}{\cos^2 \theta_W} v^2 - \frac{1 + 2 \sin^2 \theta_W}{\cos^2 \theta_W \sqrt{1 - 4 \sin^2 \theta_W}} v'^2 \right]. \quad (2.5c)$$

The mass eigenstates are

$$Z_1 = \cos \theta \ Z - \sin \theta \ Z', \quad (2.6a)$$

and

$$Z_2 = \sin \theta \ Z + \cos \theta \ Z', \quad (2.6b)$$

where the mixing angle is given by

$$\tan^2 \theta = \frac{M_Z^2 - M_{Z_1}^2}{M_{Z_2}^2 - M_Z^2}. \quad (2.7)$$

with $M_{Z_1}$ and $M_{Z_2}$ being the masses for $Z_1$ and $Z_2$. Here, $Z_1$ corresponds to the standard model neutral gauge boson and $Z_2$ corresponds to the additional neutral gauge boson. For small mixing, we find $\theta \approx M_{ZZ'}^2/M_{Z_2}^2 \ll 1$.

Since $M_{Z_1}$ has been precisely determined by the LEP experiments, the new contributions are parametrized by the two $Z'$ parameters, $M_{Z_2}$ and $\theta$. The structure of the minimal Higgs sector gives additional constraints on the allowed region of $(M_{Z_2}, \theta)$ parameter space, and forces $\theta \ll 1$ for $M_{Z_2} \gg M_{Z_1}$. However, we will not make use of this constraint so as to allow for extended Higgs sectors.

While we have only been discussing tree level relations so far, it is important to include both the standard model and new radiative corrections as well. We take the oblique corrections into account by using the starred functions of Kennedy and Lynn [13]. Following [1], the effect of new heavy particles on the starred functions may be expressed in terms of $S$, $T$ and $U$. The effects of the tree level $Z-Z'$ mixing and the presence of the new $Z_2$ gauge boson can then be expressed as shifts of the starred functions. We ignore effects due to the combination of both mixing and radiative corrections, as they are suppressed.
In order to perform a fit to experiment, we need to express the SU(3)\(_L\) × U(1)\(_X\) model predictions in terms of both tree level \(Z'\) parameters, \((M_{Z_2}, \theta)\), and one-loop parameters, \((S, T, U)\). This is most easily done by first calculating the standard model observables with the addition of \(S, T\) and \(U\) and then shifting the results by the tree level parameters. We consider both (i) \(Z\)-pole experiments which are sensitive to the mixing only and (ii) low energy experiments which are sensitive to both mixing and the presence of the \(Z_2\). The experimental values that we use for the five parameter fit, along with the standard model predictions (for reference values of \(m_t = 150\text{GeV}\) and \(m_H = 1000\text{GeV}\)), are given in table I. For the \(Z\)-pole data, \(M_W/M_Z\) and \(Q_W(Cs)\), we use the values given in Ref. [14], while \(g_L^2\) and \(g_R^2\) are given in Ref. [15]. We find it convenient to approximate the top quark and standard model Higgs mass dependence through shifts in \(S, T\) and \(U\).

The new contributions to the measurable quantities due to the presence of the \(Z_2\) and \(Z-Z'\) mixing are given in the appendix. For the \((S, T, U)\) dependence of the observables, we use the results given in Ref. [7]. The result of the fit in the \((M_{Z_2}, \theta)\) plane (with \(S, T\) and \(U\) unrestricted) is presented in Fig. 1 and indicates that \(Z-Z'\) mixing is highly restricted. This is partially due to the large couplings of the \(Z'\) to quarks. At 90% C.L., we find \(-0.0006 < \theta < 0.0042\) and \(M_{Z_2} > 490\text{GeV}\). Note the latter restriction is comparable to that obtained from tree level FCNC considerations in the quark sector.

Although not used in the fit, the minimal Higgs sector leads to further restrictions on the \(Z_2\) mass and mixing. The constraint on the \(Z-Z'\) mixing is shown by the dotted line in Fig. 1. Due to the symmetry breaking hierarchy, \(u \gg v, v'\), the dilepton and \(Z_2\) masses are related. Using the limit \(M_{Y^+} > 300\text{GeV}\) from polarized muon decay [16,17], we find \(M_{Z_2} > 1.4\text{TeV}\), as indicated on the figure. Because of the upper bound on SU(3)\(_L\) × U(1)\(_X\) unification, \(M_{Z_2}\) must be below 2.2\text{TeV}, thus giving a narrow window for the allowed \(Z_2\) mass.

The presence of the \(Z_2\) gauge boson affects the fit in the \(S-T\) plane as shown in Fig. 2. We see that the tree level mixing may appear as effective contributions to \(S\) and \(T\). The dominant effect is to give a positive contribution to \(T\) due to the downshift in the \(Z_1\) mass. The large region of negative \(T\) corresponds to high \(Z_2\) mass and small mixing. Imposing an upper bound on \(M_{Z_2}\) will affect the fit in this region. At 90% C.L. we find

\[-1.34 \leq S \leq 0.28, \quad -3.07 \leq T \leq 0.45,\]  

(2.8)
keeping in mind that the errors are nongaussian. Although the definitions of $S$, $T$ and $U$ are model independent, these numbers are valid only for the SU(3)$_L \times$ U(1)$_X$ model due to the tree level effects. We use these results in the next section to constrain the new charged Higgs masses.

III. RADIATIVE CORRECTIONS

The radiative corrections arising from the dileptons and the new heavy Higgs are process independent and may be parametrized by $S$, $T$ and $U$. Following the notation of [7], we define

$$S = 16\pi \left[ \Pi'_{33}(0) - \Pi'_{3Q}(0) \right],$$

$$T = \frac{4\pi}{\sin^2\theta_W M_W^2} \left[ \Pi_{11}(0) - \Pi_{33}(0) \right],$$

$$U = 16\pi \left[ \Pi'_{11}(0) - \Pi'_{33}(0) \right].$$

In the above, the vacuum polarizations, $\Pi(q^2)$, and their derivatives with respect to $q^2$, $\Pi'(q^2)$, include only new physics beyond the standard model. Implicit in this parametrization is the assumption that the scale of new physics is much larger than $M_Z$.

The SU(3)$_L \times$ U(1)$_X$ model predicts three classes of new particles: the new quarks $D$, $S$ and $T$, new gauge bosons, $Y^{\pm\pm}$, $Y^{\pm}$ and $Z'$ and new Higgs scalars. Since the new quarks are SU(2) singlets, they do not enter into the oblique corrections which are only sensitive to SU(2) electroweak physics. Similarly, $Z'$ will not contribute except through $Z-Z'$ mixing which was addressed in the previous section. Thus in the limit of small mixing, only dileptons and new Higgs particles will contribute radiatively to $S$ and $T$ (in addition to the deviations of the top quark and standard model Higgs masses from their reference values). Because of spontaneous symmetry breaking, we must examine the new gauge and Higgs sector simultaneously.

In order to simplify the analysis of the Higgs sector, we assume that the sextet $\eta$ does not acquire a VEV. As a result it can be treated separately from the dileptons, and we now focus on the three SU(3) triplet Higgs, (2.2a–c). These three Higgs contain a total of 18 states of which 8 are “eaten up” by the Higgs mechanism to give masses to the various gauge bosons. Ignoring $Z-Z'$ mixing, the SU(2) doublets coming from $\Delta$ and $\Delta'$ form a standard
two-Higgs model with \( \tan \beta = v'/v \) and five physical Higgs particles, \( h^\pm, a^0 \) and \( h^0_{1,2} \) [18]. The remaining 5 Higgs are given by

\[
H^{\pm \pm} = \sin \alpha_+ \phi^{\pm \pm} + \cos \alpha_+ \Delta^{\pm \pm}, \\
H^\pm = \sin \alpha_+ \phi^{\pm} + \cos \alpha_+ \Delta^+_2, \\
H^0 = \sqrt{2} \text{Re} \phi^0, 
\]

(3.2a)

(3.2b)

(3.2c)

where we have defined the ratio of VEVs as \( \tan \alpha_+ = v'/v' \) and \( \tan \alpha_+ = v/v \). These two VEV angles and \( \tan \beta \) are not independent, but are related by \( \tan \beta = \tan \alpha_+ / \tan \alpha_+ \).

Orthogonal to these states are the would be Goldstone bosons

\[
\pi^{\pm \pm} = \cos \alpha_+ \phi^{\pm \pm} - \sin \alpha_+ \Delta^{\pm \pm}, \\
\pi^\pm = \cos \alpha_+ \phi^\pm - \sin \alpha_+ \Delta^+_2, \\
\pi^0 = \sqrt{2} \text{Im} \phi^0, 
\]

(3.3a)

(3.3b)

(3.3c)

corresponding to \( Y^{\pm \pm}, Y^\pm \) and \( Z' \) respectively. Again we have assumed the \( Z-Z' \) mixing is not important for one-loop oblique corrections.

Since the two-Higgs model has already been considered in detail (see for example Ref. [19,20]), we will only focus on the dileptons and additional Higgs. Assuming the symmetry breaking hierarchy \( u \gg \{v, v'\} \), we see that \( \{\tan \alpha_+, \tan \alpha_+\} \ll 1 \) so that \( H^{\pm \pm} \) and \( H^\pm \) are mostly SU(2) singlets, and the would be Goldstone bosons giving masses to the dilepton doublet (\( Y^{\pm +}, Y^{+} \)) are mostly contained in the \( \Phi \) doublet (\( \phi^{\pm +}, \phi^+ \)). Although the mixings between the SU(2) singlet and doublet scalars are small, the oblique corrections can be important as their contributions are not protected by the custodial symmetry.

Let us first consider only the contributions from the dilepton gauge bosons (\( Y^{\pm +}, Y^{+} \)) which corresponds to the limit \( \{\tan \alpha_+, \tan \alpha_+\} \to 0 \). In this limit, the new Higgs, (3.2a–c), are all SU(2) singlets and only the dilepton doublet contributes to \( S, T \) and \( U \). We find

\[
S = -\frac{9}{4\pi} \ln \frac{M_{Y^{++}}^2}{M_{Y^{++}}^2}, \\
T = \frac{3}{16\pi \sin^2 \theta_W M_W^2} F(M_{Y^{++}}^2, M_{Y^{++}}^2), \\
U = -\frac{1}{4\pi} \left[ -\frac{19 M_{Y^{++}}^4 - 26 M_{Y^{++}}^2 M_{Y^{++}}^2 + 19 M_{Y^{++}}^4}{3 (M_{Y^{++}}^2 - M_{Y^{++}}^2)^2} \\
+ \frac{3 M_{Y^{++}}^6 - M_{Y^{++}}^4 M_{Y^{++}}^2 - M_{Y^{++}}^2 M_{Y^{++}}^4 + 3 M_{Y^{++}}^6}{(M_{Y^{++}}^2 - M_{Y^{++}}^2)^3} \ln \frac{M_{Y^{++}}^2}{M_{Y^{++}}^2} \right],
\]

(3.4a)

(3.4b)

(3.4c)
where \( F \) is defined by

\[
F(M_1^2, M_2^2) = M_1^2 + M_2^2 - 2 \frac{M_1^2 M_2^2}{M_1^2 - M_2^2} \ln \frac{M_1^2}{M_2^2}.
\] (3.5)

Since \( F(M_1^2, M_2^2) \geq 0 \) and vanishes only when the masses are degenerate, we see that \( T \geq 0 \) and parametrizes the size of custodial SU(2) breaking. \( S \) vanishes when the dileptons are degenerate, but can pick up either sign when the masses are split. While \( U \) does not play as important a role in confronting experiment \[7\], we note that the dilepton doublet gives \( U \leq 0 \). This result is the opposite of that found for a chiral fermion doublet where \( U \) is non-negative.

A complete calculation of \( S \) and \( T \) must take into account the mixing between the SU(2) singlet and doublet Higgs. This is especially important in light of the upper limit on the SU(3)_L \times U(1)_X breaking scale which puts a non-zero lower bound on the mixing. Because of the mixing, the dileptons and physical Higgs combine in their contributions. For \( S \), we find the full result

\[
S = -\frac{1}{\pi} \left[ \frac{2}{3} \sin^2 \alpha_{++} - \frac{1}{3} \sin^2 \alpha_+ + \frac{9}{4} \ln \frac{M_{Y^+}^2}{M_{Y^{++}}^2} \right. \\
+ \frac{1}{4} \sin^2 \alpha_+ \ln \frac{m_{H^+}^2}{M_{Y^{++}}^2} - \frac{1}{4} \sin^2 \alpha_+ \ln \frac{m_{H}^2}{M_{Y^{++}}^2} \\
- \sin^2 \alpha_+ \cos^2 \alpha_+ G \left( \frac{m_{H^+}^2}{M_{Y^{++}}^2} \right) - \sin^2 \alpha_+ \cos^2 \alpha_+ G \left( \frac{m_{H}^2}{M_{Y^{++}}^2} \right) \left].
\] (3.6)

The function \( G \) is defined by

\[
G(x) = \frac{7x^2 - 38x - 29}{36(x - 1)^2} + \frac{x^3 - 3x^2 + 21x + 1}{12(x - 1)^3} \ln x,
\] (3.7)

and vanishes when \( x = 1 \). \( G \) is positive when the Higgs are heavier than the dileptons and is usually negative when they are lighter. We see that the Higgs corrections always enter with a factor of either \( \sin \alpha_{++} \) or \( \sin \alpha_+ \) and arise because of the mixing of scalars with different hypercharges. As a result, \( S \) reduces to Eqn. (3.4a) in the limit when the Higgs do not mix.

Turning to \( T \), we find that it has the general form

\[
T = \frac{3}{16\pi \sin^2 \theta_W M_W^2} \left[ F(M_{Y^+}^2, M_{Y^{++}}^2) + \sin^2 \alpha_+ \cos^2 \alpha_+ F(m_{H^+}^2, M_{Y^+}^2) + \sin^2 \alpha_+ \cos^2 \alpha_+ F(m_{H}^2, M_{Y^{++}}^2) - \sin^2 \alpha_+ \cos^2 \alpha_+ [F(m_{H^+}^2, M_{Y^{++}}^2) - F(M_{Y^+}^2, M_{Y^{++}}^2)] \right]
\]
\[-\cos^2 \alpha_+ \sin^2 \alpha_+ [F(m_{+}^2, M_{Y^+}^2) - F(M_{Y^+}^2, M_{Y^+}^2)]
+ \frac{1}{3} \sin^2 \alpha_+ \sin^2 \alpha_+ [F(m_{H^+}^2, m_{H^+}^2) - F(M_{Y^+}^2, M_{Y^+}^2)]
+ \frac{4}{3} \sin^2 \alpha_+ (\sin^2 \alpha_+ - \sin^2 \alpha_+ + \sin^2 \alpha_+ - \sin^2 \alpha_+)(m_{H^+}^2 - M_{Y^+}^2)
+ \frac{4}{3} \sin^2 \alpha_+ (\sin^2 \alpha_+ - \sin^2 \alpha_+)(m_{H^+}^2 - M_{Y^+}^2)] . \quad (3.8)

In deriving this, we had to use the relation \(\cos^2 \alpha_+ M_{Y^+}^2 = \cos^2 \alpha_+ M_{Y^+}^2\) implied by the definitions of \(\tan \alpha_+\) and \(\tan \alpha_+\). Again, the Higgs corrections come in only through their small mixing into an SU(2) doublet. We find that \(T\) is positive in most of parameter space and becomes large when the Higgs or dilepton masses are split greatly, thus breaking custodial SU(2). A similar calculation for \(U\) is straightforward, but since experimental constraints on \(U\) are not as strong, we do not present it here.

The full expressions for \(S\) and \(T\) depend on four unknown parameters of the new physics — the two dilepton masses and the two new Higgs masses (the VEV angles are determined completely from the dilepton masses). In order to understand the general behavior of these radiative corrections, we now turn to three interesting cases: (a) the dileptons are degenerate in mass, \(M_{Y^+} = M_{Y^+}\); (b) the dileptons are maximally split in mass, \(\sin^2 \alpha_+ = 0\); and (c) the Higgs and dilepton masses are related by \(m_{H^+} = M_{Y^+}\) and \(m_{H^+} = M_{Y^+}\).

(a) \(M_{Y^+} = M_{Y^+}\). In order to give identical masses to \(Y^+\) and \(Y^\pm\), the VEVs, \(v\) and \(v'\) must be equal. As a result, \(\tan \beta = 1\) and \(\sin^2 \alpha_+ = \sin^2 \alpha_+ = M_{W}^2/2M_{Y^+}^2\). From Eqn. (3.6), we find for \(S\)

\[
S = \frac{1}{2 \pi} \frac{M_{W}^2}{M_{Y^+}^2} \left[ -\frac{1}{3} + \frac{1}{4} \ln \frac{m_{H^+}^2}{m_{H^+}^2} + \cos^2 \alpha_+ \left( G \left( \frac{m_{H^+}^2}{M_{Y^+}^2} \right) + G \left( \frac{m_{H^+}^2}{M_{Y^+}^2} \right) \right) \right] . \quad (3.9)
\]

Note that even when all masses are degenerate, \(S\) takes on a non-zero result. In this case, we see that the singlet–doublet mixing in the scalar sector gives rise to a negative \(S\) \([21,22]\). For large Higgs mass splittings, the second term in (3.9) dominates, and \(S\) is positive for \(m_{H^+} \gg m_{H^+}\) and negative for \(m_{H^+} \ll m_{H^+}\). From the fit in the previous section, (2.8), we see that \(m_{H^+} \lesssim m_{H^+}\) is favored.

For \(T\), we find the simple result

\[
T = \frac{1}{16 \pi \sin^2 \theta_W M_{W}^2} \sin^4 \alpha_+ F(m_{H^+}^2, m_{H^+}^2) , \quad (3.10)
\]

which gives the bounds
\[ 0 \leq T \leq \frac{1}{64\pi \sin^2 \theta_W} \frac{M_W^2}{M_{Y^{++}}^2} \max(m_{H^{+}}, m_{H^{++}}) \frac{M_{Y^{++}}^2}{M_{Y^{++}}^2}. \] (3.11)

The lower limit corresponds to Higgs mass degeneracy and the upper limit to large mass splitting. From Eqn. (2.8), we obtain the upper bound for the heavier Higgs, namely

\[ \max(m_{H^{+}}, m_{H^{++}}) \leq 7.0 \text{TeV}, \] for \( M_{Y^{++}} \leq 350 \text{GeV}. \)

(b) \( \sin^2 \alpha^{++} = 0 \). Due to the VEV structure, the mass splitting of the dileptons is restricted by the condition \( |M_{Y^{++}}^2 - M_{Y^{++}}^2| \leq M_W^2 \). The limiting case \( M_{Y^{++}} = M_{Y^{++}} + M_W^2 \) can be realized by \( v \gg v' \) or \( \sin^2 \alpha^{++} \to 0 \). In this case, the doubly charged Higgs, which is \( \Delta^{\pm \pm} \), is a pure SU(2) singlet and is not involved in the oblique corrections.

The parameter \( T \) is then given by

\[ T = \begin{cases} \frac{1}{16\pi \sin^2 \theta_W} \frac{M_W^2}{M_{Y^{++}}^2} \frac{m_{H^{+}}^2}{M_{Y^{++}}^2}, & \text{for } \frac{m_{H^{+}}^2}{M_{Y^{++}}^2} \gg 1, \\ \frac{3}{16\pi \sin^2 \theta_W} \frac{M_W^2}{M_{Y^{++}}^2}, & \text{for } \frac{m_{H^{+}}^2}{M_{Y^{++}}^2} \ll 1. \end{cases} \] (3.12)

We see that \( T \) can be negative if \( m_{H^{+}}^2 \ll M_{Y^{++}}^2 \). However, it is negligible unless the dileptons are extremely light. On the other hand, \( T \) is always positive for heavy Higgs, \( m_{H^{+}}^2 \gg M_{Y^{++}}^2 \).

Although the Higgs contributions are induced by the small mixing, namely \( \sin^2 \alpha_{+} = M_W^2 / M_{Y^{++}}^2 \), we obtain a stringent bound for the Higgs mass, \( m_{H^{+}} \leq 3.5 \text{TeV} \), for \( M_{Y^{++}} \leq 350 \text{GeV} \). If we take the other limit \( v' \gg v \), then \( \sin^2 \alpha_{+} \to 0 \). By the same token, we find \( m_{H^{++}} \leq 3.5 \text{TeV} \). Combining this with the case for \( v = v' \) in part (a), we expect the new charged Higgs to be lighter than a few TeV. Using both limits and the restriction on the Higgs mass, we also find \( |S| \lesssim 0.3 \) provided all new particles are heavier than \( M_W \).

(c) \( m_{H^{+}} = M_{Y^{+}} \) and \( m_{H^{++}} = M_{Y^{++}} \). Both expressions for \( S \) and \( T \) simplify considerably when the Higgs masses are equal to the dilepton doublet masses. Since the symmetry breaking hierarchy ensures that the mass splittings for the dileptons and the Higgs bosons are small, we find

\[ -\frac{23}{12\pi} \frac{M_W^2}{M_{Y^{++}}^2} \leq S \leq \frac{19}{12\pi} \frac{M_W^2}{M_{Y^{++}}^2}, \] (3.13a)

\[ 0 \leq T \leq \frac{1}{16\pi \sin^2 \theta_W} \frac{M_W^2}{M_{Y^{++}}^2}. \] (3.13b)

For \( M_{Y^{++}} \geq 250 \text{GeV} \), we obtain \(-0.06 \leq S \leq 0.05\) and \( 0 \leq T \leq 0.009 \) as expected for a small mass splitting.

When the \( \eta \) sextet is taken into account, it introduces 12 additional physical Higgs fields. In this case the mixing between scalars in different SU(2) multiplets becomes more
complicated. Nevertheless, our conclusions that $S$ can pick up corrections due to the mixing of scalars with different hypercharge and that $T$ measures the mass splitting between scalars still hold. Without any fine tuning in the Higgs sector, we expect all physical Higgs to be lighter than a few TeV.

IV. CONCLUSIONS

To summarize, we have examined both tree level $Z-Z'$ mixing and one-loop oblique effects induced by the new charged gauge bosons and Higgs bosons in the SU(3)$_L \times$ U(1)$_X$ model. The precision experiments constrain the mixing angle to be in the range $-0.0006 < \theta < 0.0042$ and gives $M_{Z_2} > 490$GeV. Additional indirect lower bounds can be placed on the $Z_2$ mass from both FCNC considerations and from the $Z'$-dilepton mass relation. The latter gives the strongest limit and, along with the upper bound on the SU(3)$_L \times$ U(1)$_X$ scale highly restricts the neutral gauge sector of the model, giving $1.4 < M_{Z_2} < 2.2$TeV.

Constraints on the new Higgs bosons are obtained from examination of the one-loop radiative corrections using the parameters $S$ and $T$. The parameter $T$ can be negative for very light charged Higgs and is positive for heavy Higgs. Hence we obtain an upper bound for the new charged Higgs masses, namely $m_{H^{++}}, m_{H^+} \leq$ a few TeV. The Higgs sector places strong constraints on the mass splitting between the singly and doubly charged members of the dilepton doublet. Hence no restrictions can be placed on the dilepton masses past that coming from the Higgs structure. Nevertheless, other experiments, in particular polarized muon decay [16], strongly restrict the dilepton spectrum.

We note that in this model, it is possible to obtain (small) negative values of $S$ and $T$. This result is quite general and occurs because of scalar mixing. In order to obtain a negative $T$, there has to be mixing between different SU(2) multiplets (in this case singlets and doublets). Mixing of states with different hypercharge also allows negative $S$ for the case when all masses are degenerate. These observations have also been made in Ref. [22].

As the precision electroweak parameters are measured to higher accuracy, we can start placing more stringent bounds on the new physics predicted by this SU(3)$_L \times$ U(1)$_X$ model. When the top quark mass is determined, it will remove much uncertainty in the standard model contributions to $S$ and $T$; the parameters then become much more sensitive to truly new physics. Because the masses of the new particles are already tightly constrained, both
direct and indirect experiments at future colliders may soon realize or rule out this model.

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TABLE I. The experimentally measured values $^{14,15}$, and standard model predictions $^7$ (for $m_t = 150\text{GeV}$ and $m_H = 1000\text{GeV}$) used in the fit.

| Quantity          | experimental value     | standard model |
|-------------------|------------------------|----------------|
| $M_Z$ (GeV)       | $91.187 \pm 0.007$     | input          |
| $\Gamma_Z$ (GeV)  | $2.491 \pm 0.007$      | 2.484          |
| $R = \Gamma_{\text{had}}/\Gamma_{\ell\ell}$ | $20.87 \pm 0.07$   | 20.78          |
| $\Gamma_{bb}$ (MeV) | $373 \pm 9$         | 377.9          |
| $A_{FB}(\mu)$    | $0.0152 \pm 0.0027$   | 0.0126         |
| $A_{pol}(\tau)$  | $0.140 \pm 0.018$     | 0.1297         |
| $A_e(P_{\tau})$  | $0.134 \pm 0.030$     | 0.1297         |
| $A_{FB}(b)$       | $0.093 \pm 0.012$     | 0.0848         |
| $A_{LR}$          | $0.100 \pm 0.044$     | 0.1297         |
| $M_W/M_Z$         | $0.8789 \pm 0.0030$   | 0.8787         |
| $Q_W(Cs)$         | $-71.04 \pm 1.81$     | $-73.31$       |
| $g_L^2$           | $0.2990 \pm 0.0042$   | 0.3001         |
| $g_R^2$           | $0.0321 \pm 0.0034$   | 0.0302         |
APPENDIX

In the electroweak sector, we can choose three independent precisely measured parameters, $\alpha$, $G_F$ and $M_{Z_1}$, from which in principle we can predict all the outcome of experiments in the SU(3)$_L \times$ U(1)$_X$ theory. Due to the presence of an additional gauge boson, $Z'$, and the corresponding $Z$–$Z'$ mixing, the results of the standard model predictions, which are written in terms of the starred functions [13], need to be modified. If we neglect the effects due to any combinations of both the $Z'$ parameters and the standard model radiative corrections, the results can be expressed as shifts with respect to the starred functions.

For convenience, we can define a parameter, $s_0^2$, which is given by

$$s_0^2 (1 - s_0^2) = \frac{\pi \alpha(M_{Z_1})}{\sqrt{2} G_F M_{Z_1}^2}.$$  \hfill (A1)

From the present data, $s_0^2 = 0.23146 \pm 0.00034$ is precisely known. Because of the $Z$–$Z'$ mixing, the mass of the $Z_1$ is shifted by a factor

$$\frac{\delta M_{Z_1}}{M_{Z_1}} = -\frac{1}{2} \frac{M_{Z_2}^2 \theta^2}{M_{Z_1}^2}.$$  \hfill (A2)

Hence, we obtain

$$\frac{M_W/M_{Z_1}}{M_{W*}/M_{Z*}} = 1 + \frac{1}{2} \frac{1 - s_0^2}{1 - 2 s_0^2} \frac{M_{Z_2}^2 \theta^2}{M_{Z_1}^2}.$$  \hfill (A3)

(i) $Z$–pole physics. The gauge interaction of the light neutral gauge boson, $Z_1$, is given by

$$\mathcal{L} = \frac{e_s}{c_s s_s} \sqrt{Z_1(f)} Z_{1\mu} \left[ J_3^\mu(f) - Q(f)s_{\text{eff}}^2(f) J_V^\mu \right],$$  \hfill (A4)

$$\delta s_{\text{eff}}^2(f) = s_{\text{eff}}^2(f) - s_s^2 = \left[ a^f + b^f \frac{Q(f)}{T_3(f)} \right] \theta,$$  \hfill (A5)

and

$$\delta Z_{Z}(f) = \frac{4b^f}{T_3(f)} \theta + \frac{M_{Z_2}^2}{M_{Z_1}^2} \theta^2,$$  \hfill (A6)

where $a^f$ and $b^f$, given in Ref. [2], are the vector and axial vector coupling coefficients for the $Z'$. Therefore, the partial width for the $Z$–boson relative to the standard model prediction is given by
\[ \Gamma(Z \to f \bar{f}) = 1 + \frac{\delta Z_Z(f)}{Z_Z} - 2Q(f) \frac{g_V(f)}{g^2_V(f)}(f) + g^2_A(f) \delta s^2_{eff}(f) , \]  

(A7)

where \( g_V(f)_* = \frac{1}{2}T_3(f) - \frac{1}{2}Q(f)s^2 \) and \( g_A(f)_* = -\frac{1}{2}T_3(f) \). For \( \Gamma(Z \to b \bar{b})_* \), we also include the vertex correction due to the top-quark.

By the same token, we can express the polarization asymmetry of fermion \( f \) as

\[ \frac{A_{pol}(f)}{A_{pol}(f)_*} = 1 - \delta A(f) , \]  

(A8)

with

\[ \delta A(f) = \frac{Q(f) g^2_A(f)_* - g^2_V(f)_*}{g_V(f)_* g^2_A(f)_* + g^2_V(f)_*} \delta s^2_{eff}(f) . \]  

(A9)

Hence we obtain

\[ \frac{A_{pol}(\tau)}{A_{pol}(\tau)_*} = 1 - \delta A(l) \]  

(A10)

\[ \frac{A_{FB}(\mu)}{A_{FB}(\mu)_*} = 1 - 2\delta A(l) \]  

(A11)

\[ \frac{A_{FB}(b)}{A_{FB}(b)_*} = 1 - \delta A(b) - \delta A(l) . \]  

(A12)

(ii) low energy experiments. The low energy interaction Hamiltonian is given by

\[ \frac{4G_F}{\sqrt{2}} \left( 1 + \frac{M^2_{Z_2}}{M^2_{Z_1}} \theta^2 \right) \left[ J_\mu J^\mu - 2\theta J'_\mu J^\mu + \frac{M^2_{Z_1}}{M^2_{Z_2}} J'_\mu J'^\mu \right] . \]  

(A13)

Therefore the effective left- and right-handed coupling coefficients for neutrino scattering are modified to be

\[ \epsilon_\lambda(q) = g^0_\lambda q_*(1 + \frac{M^2_{Z_2}}{M^2_{Z_1}} \theta^2 - 4\theta a^\nu) - (\theta - 4a^\nu \frac{M^2_{Z_1}}{M^2_{Z_2}})(a^q + \eta(\lambda)b^\nu) . \]  

(A14)

where \( \eta(\lambda) = 1 \) and \( -1 \) for \( \lambda = R \) and \( L \) respectively. Hence we obtain

\[ \frac{g^2_\lambda}{g^2_{\lambda_*}} = \frac{\epsilon_\lambda(u)^2 + \epsilon_\lambda(d)^2}{g^0_\lambda u^2_*(u) + g^0_\lambda d^2_*(d)} \]

\[ = 1 + 2(\frac{M^2_{Z_2}}{M^2_{Z_1}} \theta^2 - 4\theta a^\nu) \]

\[ -2\frac{g^2_\lambda u_*(a^u + \eta(\lambda)b^u) + g^2_\lambda d_*(a^d + \eta(\lambda)b^d)}{g^0_\lambda u^2_*(u) + g^0_\lambda d^2_*(d)} (\theta - 4a^\nu \frac{M^2_{Z_1}}{M^2_{Z_2}}) . \]  

(A15)

For atomic parity violation, the weak charge is given by

\[ 16 \]
\[
\frac{Q_W}{Q_{W^*}} = 1 + \frac{M_{Z_2}^2}{M_{Z_1}^2} \theta^2 + \frac{\delta C_1(u)(2Z + N) + \delta C_1(d)(Z + 2N)}{g_{A}^0(e)_* g_{U}^0(u)_*(2Z + N) + g_{A}^0(e)_* g_{V}^0(d)_*(Z + 2N)}, \tag{A16}
\]

where

\[
\delta C_1(q) = -\theta (g_{A}^0(e)_* a^q + b^e g_{V}^0(q)_*) + \frac{M_{Z_1}^2}{M_{Z_2}^2} b^f b^q. \tag{A17}
\]

The quantities \(g_{R,L}^0\) and \(g_{V,A}^0\) in Eqs. (A14)–(A17) are evaluated at zero energy.
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FIGURES

FIG. 1. 90% C.L. allowed region in $(M_{Z_2}, \theta)$ parameter space. The dotted lines indicate the constraints from the minimal Higgs structure. Also included are the FCNC bound of Ref. \cite{3}, the lower bound from the $Z'$–dilepton mass relation, and the upper bound on $SU(3)_L \times U(1)_X$ unification.

FIG. 2. Best fit point (cross) and 90% C.L. contour in the $S$–$T$ plane for the $SU(3)_L \times U(1)_X$ model (solid line). For comparison, the model independent (oblique parameters only) fit to the same data is also shown (dotted line). $S = T = 0$ corresponds to the reference point $m_t = 150\text{GeV}$ and $m_H = 1000\text{GeV}$. $U$ is always taken as a free parameter.