Studying joint spectral intensity of spontaneous four-wave mixing in optical nanofibers

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Abstract. Joint spectral intensity of the biphoton field generated via spontaneous four-wave mixing in an optical nanofiber was studied both experimentally and theoretically. The measured two-photon frequency distribution agrees well with the theoretically expected one that is calculated taking into account the spatially inhomogeneous profile of the taper. It is shown that measuring joint spectral intensity of the biphoton field makes it possible to determine the nanofiber radius with high accuracy.

1. Introduction
Sources of single-photon and two-photon states are an important part of the implementation of optical quantum technologies. Such devices form a basic ingredient for quantum communication and optical quantum computing, which makes them a topic of active research [1–5]. One of the promising approaches to the problem is to use nonlinear optical effects such as spontaneous four-wave mixing (SFWM) in a third-order nonlinear optical medium [1,2]. SFWM can be observed in various systems like microring resonators, photonic crystal fibers and micro-/nanofibers (MNF). In particular, MNFs seem to be very promising nonlinear materials that have a number of unique features due to the small mode diameter, significant evanescent field, small weight and size [6]. Compared to standard optical fibers, MNFs provide higher nonlinearity, which allows one to reduce the fiber length. At the same time, adiabatic fiber tapers are characterized by very small losses so that MNFs also perfectly match optical communication lines.

The generation of correlated two-photon and heralded single-photon states via SFWM in a MNF has been studied recently in [7–11]. In the present work, we continue the measurements of the joint spectral intensity (JSI) of the biphoton field presented in [11], which is one of the most important characteristics of the SFWM-based quantum light sources. We show that measuring JSI of the biphoton field makes it possible to determine the radius of a MNF with high accuracy.

2. Basic results
SFWM is an optical nonlinear process, which is observed during the interaction of a (usually strong) pump field with a third-order nonlinear material, whereby two photons of the pump field are occasionally annihilated and simultaneously two photons, which are usually called signal and idler, are created. This process satisfies the following energy and phase-matching conditions:

\[ 2\omega_p = \omega_s + \omega_i, \quad 2k_p = k_s + k_i, \]  
(1)
where $\omega_n$ and $k_n$ are the frequency and wave-vector, respectively, of the pump $(n = p)$, signal $(n = s)$ and idler $(n = i)$ photons.

A two-photon (biphoton) state of the SFWM field generated in a MNF can be calculated in the first-order perturbation theory of quantum mechanics [11, 12]. Under this approach, the state vector of the biphoton field can be written as follows:

$$|\Psi\rangle = |0\rangle_s|0\rangle_i + A \int d\omega_s d\omega_i F(\omega_s, \omega_i) a^\dagger(\omega_s) a^\dagger(\omega_i)|0\rangle_s|0\rangle_i,$$

where $|0\rangle_s|0\rangle_i$ is the vacuum state of the signal and idler modes of the electromagnetic field, $A$ is a constant proportional to the cubic susceptibility of the fiber material, a$^\dagger(\omega_n)$ are signal $(n = s)$ and idler $(n = i)$ photon creation operators in the fundamental spatial mode of the MNF with the frequency $\omega_n$, and $F(\omega_s, \omega_i)$ is the joint spectral amplitude (JSA) of the biphoton field.

The latter is proportional to the convolution of the pump field spectral amplitude and phase-matching function, and can be calculated in a MNF with a variable cross section as described in [11].

The efficiency of generating pairs of photons at certain frequencies of the signal and idler fields is proportional to the joint spectral intensity (JSI) of the biphoton field $|F(\omega_s, \omega_i)|^2$. This function is nothing more than the joint frequency distribution of the generated photons, which can be measured by registering the coincidence count rate between the signal and idle channels. JSI reveals the most important spectral properties of the biphoton field. In particular, it describes frequency correlations between the signal and idler fields, which is important for heralded preparation of single-photon states. In general case, JSI of the biphoton field is not represented as the product of the spectral distribution functions corresponding to the signal and idle fields, which corresponds to an entangled biphoton state in the frequency domain. As a result, each of the generated photons proves to be in a mixed state. However, under certain conditions JSI and even JSA can be factorized [13], which makes it possible to generate pure single-photon states. In the context of SFWM in non-uniform MNFs, this possibility was demonstrated theoretically in [14].

In the present experiment, we used a picosecond pulsed pump laser operating at a wavelength of 1062 nm (100 mW average power, 100 ps pulse width, 17 MHz repetition rate), the radiation of which was fed into a nanofiber (890 ± 12) nm in diameter, which was made from a standard single-mode fiber by heating and stretching as described in [11]. The diagram of the experimental setup is shown in figure 1. In the process of SFWM, correlated pairs of photons were created at wavelengths of about 1310 nm (idler field) and 880 nm (signal field). At the exit of the nanofiber, four notch filters were installed to cut off the pump radiation. A dichroic mirror was used to separate the signal and idler photons. Idler photons passed through the dichroic mirror were directed to an IR detector through a coarse wavelength division multiplexing (CWDM) unit (20 nm channel bandwidth). Signal photons reflected from the dichroic mirror were directed to a diffraction grating (100 lines/mm), after being reflected from which they were directed to a visible light detector. With such spectral selection in both channels the observed photon flux was about 4 pairs per second, and the measured coincidence-to accidental ratio (CAR) was about 4.5.

The main objective of the experiment is to measure JSI of the biphoton field via measuring the coincidence count rate as a function of the signal and idler frequencies determined by the transmission wavelength of the CWDM unit and the angle of the diffraction grating. Switching channels in CDWM made it possible to scan the wavelength in the range of 1270 – 1610 nm with a step of 20 nm, while turning the diffraction grating made it possible to cover the range of 840 – 920 nm with a step of 1.5 nm. The result of measuring JSI is shown in figure 2.

To compare the experimental results with the theory, we take advantage of the formalism developed in [11]. First, we calculated the effective refractive index for all the interacting fields...
Figure 1. Schematic of the experimental setup for studying SFWM in optical MNFs. PL is the pump laser, LF is the laser line filter, NF is the notch filter, DM is the dichroic mirror, DG is the diffraction grating, FC is the free space to fiber coupler, SPD is the single-photon detector, and CC is the coincidence counter.

Figure 2. Measured (left) and calculated (right) JSI for a given MNF (pump, idler, and signal) using commercial software (Comsol Multiphysics). Due to the small diameter of the MNF, all the fields correspond to the fundamental mode of the tapered fiber. An example of the calculated spatial field distribution is shown in figure 3. To this end, we used Sellmeier equations characterizing the fiber material from [15]. Second, we calculated the phase-matching function taking into account the measured profile of the fabricated MNFs [11]. Finally, multiplication of the spectral pump function and phase-matching function gives the JSA of the biphoton field. The resulting JSI is shown in figure 2. In doing so, the nanofiber waist radius was fitted so that the maximum values of the experimentally measured and theoretically calculated JSIs coincide. The theoretical calculation error was determined using the Rayleigh criterion. As a result, the MNF diameter is estimated to be $(887 \pm 3)\text{ nm}$, which coincides with the diameter measured with a scanning electron microscope $(890 \pm 12)\text{ nm}$ [11]. Thus, the accuracy of MNF diameter estimation proves to be about $3\text{ nm}$.

3. Conclusion
In this work, we have studied the joint spectral intensity of the biphoton field generated via SFWM in a tapered fiber. The measured joint spectral intensity agrees well with the theoretically
expected distribution that is calculated taking into account the spatially inhomogeneous profile of the taper. It can be concluded that the measurement of the joint spectral intensity of the biphoton field makes it possible to determine its radius with high accuracy.

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5. References

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