ENERGY INJECTION IN GAMMA-RAY BURST AFTERGLOW MODELS

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ABSTRACT

We extend the standard fireball model, widely used to interpret gamma-ray burst (GRB) afterglow light curves, to include energy injections and apply the model to the afterglow light curves of GRB 990510, GRB 000301C, and GRB 010222. We show that discrete energy injections can cause temporal variations in the optical light curves, and we present fits to the light curves of GRB 000301C as an example. A continuous injection may be required to interpret other bursts, such as GRB 010222. The extended model accounts reasonably well for the observations in all bands ranging from X-rays to radio wavelengths. In some cases the radio light curves indicate that additional model ingredients may be needed.

Subject headings: gamma rays: bursts — gamma rays: theory — radiation mechanisms: nonthermal — shock waves

1. INTRODUCTION

After a quarter of a century of modest progress, a breakthrough in gamma-ray burst (GRB) research came in 1997, when X-ray and optical afterglow emission was first detected from GRB 970228 (van Paradijs et al. 1997; Costa et al. 1997), showing a redshift of \( z = 0.695 \). This confirmed the cosmological origin of GRBs, and to date cosmological redshifts have been obtained for about 50 afterglows.\(^1\)

Initially, afterglow light curves were often sparsely sampled and had a natural explanation in the standard fireball model (see, e.g., Piran 2005 for a recent review). Many showed a smooth power-law decay, followed by a steepening 1–2 days after the burst. This temporal behavior has been interpreted as being due to a relativistically expanding collimated outflow (e.g., Rhoads 1999, hereafter R99) or more recently to a structured jet viewed to a relativistically expanding collimated outflow (e.g., Rhoads 2003; Coe et al. 2004, 2005). As continuous energy injection into the fireball sustains higher fluxes for longer periods of time, this resulting in slowly decaying light curves, without requiring \( p < 2 \). The cause of the slow decay is thus shifted from the electron population to the properties of the source itself. This extension of the standard model was, however, not used to interpret actual data until later (e.g., Nakar & Piran 2003; Björnsson et al. 2002, 2004). As continuous energy injection can sustain slower light-curve decay rates with \( p > 2 \), it is important to explore fully the properties and light-curve signatures of such models.

In addition to the above quoted references, energy injection in the context of GRBs has previously been considered by, e.g., Panaitescu et al. (1998), who give an example of how the afterglow of GRB 970508 may be explained by refreshed shocks, although a spherically symmetric model may also work. Cohen & Piran (1999) explore the self-similarity of spherical blast waves with continuous energy injection, while Dai & Lu (2000, 2001) consider a model in which a pulsar is assumed to provide the additional energy, a variant of which was also considered by Zhang & Mészáros (2001). Kumar & Piran (2000) explore a toy model of the interaction of two colliding shells and estimate the flux contribution from both the forward and the reverse shocks. Then Zhang & Mészáros (2002) consider the interaction of two colliding shells in detail and determine a criteria for such collisions to be mild or violent. Panaitescu (2005) considers additional energy injection in interpreting observations of several GRBs. Finally, Granot et al. (2003) and Huang et al. (2006) conclude that discrete energy injections are needed to properly account for additional energy injection in the GRB context.

\(^1\) See http://www.mpe.mpg.de/~jcg/grbgen.html for the most recent list.
for the optical afterglow of GRB 030329. All of these studies adopt different degrees of approximations on various model ingredients.

In recent months observations by the Swift satellite have shown that early light-curve behavior may not be as smooth as previously thought. The origin of these variations may be due to the transition from the prompt phase to the afterglow phase or to a strong initial reverse shock (Panaitescu et al. 2006). The slower decaying portions of the light curves may be due to continuous energy injection (Nousek et al. 2006; Granot & Kumar 2006). In particular, GRB 050319 (Cusumano et al. 2006) and GRB 050315 have shown this kind of a behavior (see Chincarini et al. 2005 for the latter burst and further examples).

In this paper we present a detailed analysis of relativistic fireballs with either discrete or continuous energy injection. We show that injection has interesting observational consequences, in addition to the slower decay, and provides clear signatures in the light curves. An example of these effects has already been presented by Björnsson et al. (2004), where a discussion of the polarization properties can also be found. The clearest of these signatures are bumps in the light curves due to refreshed shocks, which may in fact have been observed in, e.g., GRB 021004 and GRB 030329. These signatures provide additional information on the hydrodynamic properties of the fireball. We give examples of fits to three GRB afterglows and show that even with the additional energy injection, there remains some discrepancy between the model and the data, especially at radio wavelengths. In our approach we try to keep the treatment as self-consistent as possible and improve on several of the approximations of some of the previous work.

The paper is structured as follows. In § 2 we review the effects that energy injections have on the dynamics of the fireball and outline our approach to the problem. In § 3 we present approximations for simple modeling of the dynamical evolution. In § 4 we discuss the ingredients in the radiative component of our model, and in § 5 we give examples of bursts that may be interpreted with this modified model. Finally, in § 6 we discuss our results and conclude the paper.

2. THE DYNAMICS

We begin this section by considering energy and momentum conservation. We follow the approach of RM98 and SM00 and assume that energy is released over a period that is short compared to the duration of the afterglow. We then specialize the method to three cases: an instantaneous energy injection, § 2.1, for easy comparison with R99 (see also Paczyński & Rhoads 1993); several discrete injection events, § 2.2; and finally continuous injection with a power-law distribution of the Lorentz factors, in § 2.3. We assume the dynamical evolution to be adiabatic in all cases.

Assume that the total ejected mass with initial Lorentz factor higher than \( \Gamma \) can be written as \( M(\geq \Gamma) = M_0 + \Delta M \), where \( M_0 \) is the mass ejected with the highest Lorentz factor \( \Gamma_0 \), and \( \Delta M = \int_{\Gamma_0}^{\Gamma} (dM/d\Gamma)d\Gamma \), where \( dM/d\Gamma \) is a function specifying the distribution of ejected mass with Lorentz factor \( \Gamma \). We prefer to use \( x = \Gamma/\Gamma_0 \) as a variable and introduce a dimensionless function \( F(x) \), defined by \( F(x) = M(\geq \Gamma)/M_0 \), in the interval \( 1 \geq x \geq \Gamma_m/\Gamma_0 \), where \( \Gamma_m \) is the minimum Lorentz factor occurring in the ejected mass. We then have \( dF/dx = (\Gamma_0/M_0) \cdot dM/d\Gamma \). Note that in general \( dF/dx \leq 0 \), since in realistic scenarios we must have \( \Delta M \geq 0 \). Note also that \( F(1) = 1 \). The original fireball scenario with an instantaneous energy release is equivalent to \( dF/dx = 0 \).

The energy of a mass element \( dM \) is \( dE = \Gamma dMe^2 \), where \( e \) is the velocity of light, and the total energy of the ejecta with Lorentz factor greater than \( \Gamma \) is therefore given by

\[
E(\geq \Gamma) = E_0 + \Delta E = E_0 + \int_{\Gamma_0}^{\Gamma} \frac{dE}{d\Gamma} d\Gamma = E_0 \left( 1 + \int_{1}^{x} \frac{dF}{dx} \, dx \right),
\]

where \( E_0 = \Gamma_0 M_0 e^2 \). Integrating by parts, we find that

\[
E = E_0 \left[ x F(x) + \int_{x}^{1} F(x) \, dx \right].
\]

The expanding shell decelerates by sweeping up the ambient medium. Assuming that the instantaneous total mass of the swept-up medium is \( M_\infty \), the total energy of the shell can be expressed as \( E = \Gamma \{ \gamma_0 M(\geq \Gamma) + M_\infty e^2 \} \), where \( \gamma_0 \) is the average internal Lorentz factor of the particles in the expanding shell.2 Using this and equation (2), we find that the conservation of energy may be written as

\[
\Gamma \gamma_0 [ F(x) + f ] = \Gamma_0 [ x F(x) + I_1 ] + f,
\]

with \( f = M_\infty/M_0 \) and

\[
I_1 = \int_{1}^{x} F(x) \, dx.
\]

Note that the swept-up mass ratio, \( f \), increases as \( \Gamma \) decreases.

In the preceding equations we have assumed that the slower moving mass shells ejected by the central engine catch up with the decelerating shock front as soon as the Lorentz factors of the two coincide. This is a simplification, as the speed difference between the ejected mass elements will cause a time delay. The dynamical evolution, however, is not affected by this simplifying assumption, although it should be kept in mind when considering reverse shocks originating from shells colliding with the shock front. A method for including this time delay will be discussed in § 2.2.

Before impact with the expanding shock front, each mass element has momentum \( dP = \Gamma dMe^2 \), where \( \beta e \) is the velocity of the mass element in the burster frame. Hence, the total momentum before impact is

\[
P = P_0 + \int_{\Gamma_0}^{\Gamma} dP = P_0 - M_0 e \int_{x}^{1} \left( \Gamma_0^2 x^2 - 1 \right)^{1/2} \frac{dF}{dx} \, dx,
\]

with \( P_0 = \Gamma_0 M_0 \beta_0 e \). As the momentum of the hot shell is \( P = \Gamma \gamma_0 [ M(\geq \Gamma) + M_\infty \beta e ] \), momentum conservation results in

\[
\Gamma \gamma_0 / \beta [ F(x) + f ] = \Gamma_0 \beta_0 - I_2,
\]

where

\[
I_2 = \int_{x}^{1} \left( \Gamma_0^2 x^2 - 1 \right)^{1/2} \frac{dF}{dx} \, dx.
\]

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2 To be fully consistent, one would also need to distinguish between the internal Lorentz factors of the ejected shells and that of the shock front. This distinction does not, however, change the final results as the total energy of the shell cancels out in the derivation of eq. (8).
Dividing equation (6) by equation (3), we obtain an expression for the dimensionless velocity of the expanding fireball:
\[
\beta = \frac{\Gamma_0\beta_0 - t_2}{\Gamma_0[xF(x) + I_1] + f}.
\]  
(8)

We then find that the Lorentz factor is given by
\[
\Gamma = \frac{1}{\sqrt{1 - \beta^2}} = \frac{\Gamma_0[xF(x) + I_1] + f}{\sqrt{\left(\Gamma_0[xF(x) + I_1] + f\right)^2 - \Gamma_0\beta_0 - t_2^2}}.
\]  
(9)

This is an implicit equation for \(\Gamma\) and describes the evolution of the Lorentz factor with radius (or time), once the distribution of the injected Lorentz factors, \(F(x)\), has been specified along with the initial conditions of the fireball (e.g., \(\Gamma_0\) and \(M_0\)). A description of the swept-up mass, \(t\), is also required (see eq. [10]). Equation (9) is the key equation for obtaining the dynamical evolution of the fireball and has to be solved numerically in all realistic cases. Since we take the evolution to be adiabatic, the dynamics can be solved for separately and the radiative properties computed afterward.

For a complete description of the fireball evolution we must solve four differential equations (R99)
\[
\begin{align*}
\frac{dr}{dt_b} &= \beta c, \\
\frac{dt'}{dt_b} &= \frac{1}{\Gamma'}, \\
\frac{dt}{dt_b} &= \frac{1 + z}{\Gamma'^2(1 + \beta)}, \\
\frac{df}{dr} &= \frac{\Omega \rho}{M_0},
\end{align*}
\]  
(10)

Here \(r\) is the radial coordinate in the burster frame and \(t_0, t', \) and \(t\) are the burster frame time, the comoving frame time, and the observer frame time, respectively. The solid angle into which the energy is beamed is estimated by \(\Omega = 2\pi\left(1 - \cos\theta_0 + c_z t'/(ct_b)\right)\), where \(\theta_0\) is the initial opening angle of the collimated outflow and \(c_z \approx c/\sqrt{3}\) is the sound speed in the comoving frame, assumed to be relativistic. The mass density of the external medium is \(\rho\), which in general is a function of radius.

2.1. Instantaneous Energy Injection

In the standard fireball scenario it is assumed that the energy is released instantaneously, with \(x = 1\) and \(F(x) = 1\). We then find that \(x F(x) + I_1 = 1; I_2 = 0\) and equation (9) reduces to
\[
\Gamma = \frac{\Gamma_0 + f}{\sqrt{(\Gamma_0 + f)^2 - (\Gamma_0^2 - 1)}} = \frac{\Gamma_0 + f}{\sqrt{1 + 2\Gamma_0 f + f^2}},
\]  
(11)

which agrees with the result of Paczynski & Rhoads (1993) and is the starting point of the R99 analysis (see his eq. [4]). Some authors have based their analysis of GRB afterglows on this equation (e.g., Salmonson 2003; Bianco & Ruffini 2005).

GRB 990510 is an example of a burst in which the afterglow may be readily interpreted with a single instantaneous energy release event. We present our fit to the observations in § 5.1.

2.2. Discrete Energy Injection

Assume that in addition to the initial injection \(M_0\), the central engine expels several shells with masses \(M_i\) and Lorentz factors \(\Gamma_i\). Converting to dimensionless variables, \(x_i = \Gamma_i/\Gamma_0\) and \(m_i = M_i/M_0\), we have
\[
F(x) = 1 + \sum_i m_i H(x_i - x),
\]  
(12)

where
\[
H(x_i - x) = \begin{cases} 
0, & \text{for } x_i < x \\
1, & \text{for } x_i \geq x
\end{cases}
\]  
(13)

is the Heaviside step function. We then obtain
\[
\Gamma = \Gamma_0 \left[1 + \sum x_i m_i H(x_i - x)\right] + f \times \left(\left[\Gamma_0 \left[1 + \sum x_i m_i H(x_i - x)\right] + f\right]^2 - \left[\Gamma_0\beta_0 + \sum m_i \sqrt{\Gamma_0^2 x_i^2 - 1} H(x_i - x)\right]^{-1/2}\right).
\]  
(14)

Because the Heaviside function is zero until \(x_i \geq x\), it is clear from this equation that the dynamical evolution between injections is similar to the instantaneous case, as the evolution of \(\Gamma\) depends only on \(f\) in the same way.

When a shell collides with the shock front, a reverse shock may propagate back through the shell if the collision is violent (Zhang & Mészáros 2002). Depending on the circumstances, radiation from the reverse shock may make a substantial contribution to the afterglow light (Nakar & Piran 2004). According to Zhang & Mészáros (2002), the likelihood of getting a reverse shock depends on the relative energy and speed between the shells and the width of the incoming shell. As the energy is known and we neglect the dynamics in the shell collision, only the relative speed is needed. Assuming that only the shock front slows down due to interaction with the surrounding medium and that the trailing shell travels with constant speed, shells ejected with the average Lorentz factor of the shock, \(\Gamma_s\), will have reached the shock front at \(r\) in time \(t\). Replacing \(x_i\) with \(x_a = \Gamma_a/\Gamma_0\) in equation (9) is then sufficient to account for the delay. As shown by Kumar & Piran (2000), the Lorentz factor of the incoming shell is \(\Gamma_a \approx 2\Gamma\) in a constant density environment for most of the evolution. Combined with the results from Zhang & Mészáros (2002; see their Fig. 3), this indicates that for a reverse shock to result, the energy of the incoming shell must be at least twice the energy of the shock front.

We emphasize that in our numerical calculations we assume the shell collisions to be instantaneous, and we neglect the effects of the interaction on the light curves. These effects are expected to be small, considering that the relative Lorentz factor of the collision is \(\Gamma_r \approx 1.25\) (Kumar & Piran 2000). We estimate that the shells would have to be at least several hundred light seconds thick for the effects of the interaction to be measurable with current instruments; the shells would also be strongly smoothed because of the equal arrival time surface (EATS).

An example of an afterglow that may be interpreted with discrete energy injections is that of GRB 021004. Björnsson et al.
while in the laterally spreading case we adopt (see R99)
\[
\frac{df}{dr} \approx \frac{\pi}{M_0} \rho(c_s t')^2.
\] (18)

To obtain a full solution of the problem and the light-curve behavior, these need to be consistently included in the calculation.

Approximate solutions applicable in the instantaneous case have been discussed in detail by, e.g., R99 and Salmonson (2003). We emphasize that the approximations can give useful insight into the properties of the fireball, but they should only be expected to be valid during limited time intervals in the afterglow evolution. This is partly due to the validity of the underlying assumptions in those limited intervals and partly due to the influence of the EATS on the evolution (see § 4.1). Another factor is the spectral energy distribution (SED), which is not really a set of broken power laws (see the Appendix) but has an intrinsic curvature, and so do the light curves. In the following we first give a brief discussion of the discrete case, but then concentrate on the case of continuous energy injection.

3.1. Discrete Injections

If the energy is released in several discrete events (as discussed in § 2.2), the evolution after a shell impact can be approximated with the evolution following a single release event with the appropriate initial parameters. Of these, only the energy is relevant, being equal to the total energy that has accumulated in the shock front after the impact. Integration over the EATS then smooths out the transition.

As mentioned in § 2.2, the relative speed between the impacting shell and the decelerating shock front is an important factor in evaluating the reverse shock emission arising from the collision. The speed of the incoming shell equals the average speed of the shock front at the time of collision, and to estimate it we need to know how the radius of the fireball, \( r \), evolves with time in the burster frame, \( t_b \). Using \( \Gamma \approx (\Gamma_0/2f)^{1/2} \), appropriate for a single release (see R99), and equations (17) and (10), it is easy to show that in the confined case the speed of the incoming shell is \( \Gamma_0 \approx (4 - g)^{1/2} \Gamma \). Although this should only be valid early on in the evolution, numerical calculations indicate that it is in fact appropriate during most of the evolution.

3.2. Continuous Energy Injection

All results derived in this section assume that the shock speed, \( \beta \approx 1 \). We first note that, as \( f \) is the swept-up mass in units of the initial ejected mass, it is small compared to the other terms in equation (16) for a substantial fraction of the initial fireball evolution, but will dominate at later times.

Early in the evolution of the fireball, when \( x \) is close to 1, \( Z < \Gamma_0 \) and we recover equation (4) in R99, and an identical initial fireball evolution. The most interesting case, however, is when \( Z/\Gamma_0 > 1 \). From the definition of \( Z \) we see that this occurs when \( x = \Gamma/\Gamma_0 < [\sqrt{(2s-1)}/(s-1)] \), i.e., when \( x < \frac{3}{4} \) for \( s = 2 \) and for \( x < \frac{1}{2} \), when \( s = 3 \). In this case equation (9) simplifies to
\[
\Gamma \approx \frac{\Gamma_0 + f + Z}{\sqrt{1 + 2f^2}}.
\] (19)

When both \( \Gamma_0 \) and \( f \) can be neglected compared to \( Z \), we have that \( \Gamma \approx (Z/2f)^{1/2} \). Dividing by \( \Gamma_0 \) and solving for \( x = \Gamma/\Gamma_0 \), we find
\[
x \approx \left( \frac{1}{2\Gamma_0 s - 1} \right)^{1/(s+1)} f^{-1/(s+1)}.
\] (20)
Note that in the limit, \( s \to 1 \), we recover the power-law behavior of the instantaneous injection case, where \( x \propto f^{-1/2} \) (R99).

**Confined outflow.**—Here we adopt equation (17), with \( \Omega_0 \approx \pi \theta_0^2 \), as the solid angle into which the beam propagates. Inserting into equation (20), we find that

\[
x \approx \left[ \frac{1}{2\Gamma_0} \frac{s}{s-1} \frac{(3-g)M_0}{\Omega_0 \rho} \right]^{1/(s+1)} r^{-3/(s+1)} \propto r^{-(3-g)/(s+1)}.
\]

(21)

As in R99, we also evaluate the relevant times. From \( dt'/dr = 1/(c\Gamma_0\alpha) \), we find by integration that the time in the comoving frame is

\[
t' \propto \left( \frac{s+1}{s+4-g} \right) r^{(s+4-g)/(s+1)}.
\]

(22)

Similarly, from \( dt/dt_0 = (1+z)/(2c\Gamma_0^2\Delta) \), the time in the observer frame is given by

\[
t \propto \left( \frac{s+1}{s+7-2g} \right) r^{(s+7-2g)/(s+1)}.
\]

(23)

Inverting this result and inserting into equation (21), we find that \( \Gamma \propto r^{-(3-g)/(s+7-2g)} \), in agreement with the SM00 result. Also note that if we set \( s = 1 \) and \( g = 0 \), we recover the results of R99.

**Laterally spreading outflow.**—The treatment is similar to that of R99. The \( g = 2 \) case cannot, however, be solved analytically due to the coupling of the variables. We therefore only consider the case \( g = 0 \) in this section (constant density ISM). The general case has to be solved numerically.

We start from equation (18), divide by \( dt'/dr \), and integrate to obtain

\[
f^{(2+s)/(1+s)} \propto \frac{P\rho^2}{3} (r^3 - c_1),
\]

(24)

in agreement with equation (14) of R99 if we set \( s = 1 \). Here \( c_1 \) is a constant of integration to be determined by the initial conditions. It becomes negligible when \( c_1, r' \gg \theta_0 r \).

The exponential behavior discussed by R99 is seen to follow from equation (24). If \( s = 1 \) (formally equivalent to the instantaneous case), then equation (24) shows that \( f \propto r^2 \), so that \( dt'/dr \propto t' \), and the time in the comoving frame increases exponentially with \( r \). Therefore, \( f \) also increases exponentially with \( r \).

On the other hand, if \( s > 1 \), the behavior becomes a pure power law, \( f \propto r^{(3/(1+s))(s+2)} \) and \( t' \propto t^{(2+s)/(s-1)} \), which shows that the exponential regime is a special case. It follows that \( f \propto r^{3/(1+s)-(s-1)} \), \( r \propto t^{(s-1)/(5+s)} \), and \( \Gamma \propto r^{3/(5+s)} \). The physical reason for this change in behavior is that the dynamical evolution is now determined by the power-law energy injection rather than the lateral spreading. We note that in general, if the incoming shell has the same opening angle as the initial \( \theta_0 \), the resulting decay will be a mixture of a power-law decay for the part of the outflow within \( \theta_0 \) and an exponential for that part of the flow that is outside of \( \theta_0 \).

By assumption, the dynamical evolution is independent of the radiative properties of the fireball. To proceed, a prescription of the radiative processes is required. We adopt the standard synchrotron emission mechanism (see, e.g., Piran 2005 for a review), which is widely assumed to be the source of radiation in GRBs (e.g., Tavani 1996; Mészáros & Rees 1997; Sari et al. 1998). The details of the radiative component of our modeling are given in § 4 and in the Appendix.

### Table 1

| Exponent | \( \nu_m < \nu < \nu_c \) | \( \nu > \nu_c \) |
|----------|--------------------------|------------------|
| \( \alpha \) | \( 3(s - s + 4\beta)/(s + 5) \) | \( 2(1 - s + 6\beta)/(s + 5) \) |
| \( \beta \) | \( (p - 1)/2 \) | \( p/2 \) |

To explore the approximate light-curve behavior, \( F_{\nu} \propto \nu^{-\alpha} \), we need to evaluate the characteristic frequencies \( \nu_m \) and \( \nu_c \), and the flux \( F_{\nu} \) at the former (see the Appendix for definitions of these quantities).

For the confined case, the temporal behavior of the characteristic frequencies, the \( F_{\nu_c} \) and the light-curve decay indices, \( \alpha \), agree with those previously derived for the forward shock by SM00 (see Table 1 in their paper).

For the laterally spreading case we find, on the other hand,

\[
\Gamma \propto r^{3/(s+5)},
\]

(25)

\[
r \propto t^{(s-1)/(5+s)},
\]

(26)

\[
\nu_m \propto r^{12/(s+5)},
\]

(27)

\[
\nu_c \propto r^{-2(s-1)/(s+5)},
\]

(28)

\[
F_m \propto r^{3(s-9)/(s+5)}.
\]

(29)

The corresponding light-curve exponents are given in Table 1. (Note that if we let \( s = 1 \), we recover the corresponding results for the instantaneous case.) From the indices in Table 1 and the results of SM00 we see that if \( \nu_m < \nu < \nu_c \), the steepening in the light curve when the observer starts to see the entire outflow surface is \( \Delta \alpha = \alpha_2 - \alpha_1 = 24(1 + \beta)/[s(5 + s)] \), with \( \beta = (p - 1)/2 \). For parameters typically inferred in afterglows (\( p = 2 \) and \( \beta = 0.75 \)) and taking \( s = 2 \) as an example, we see that \( \Delta \alpha \sim 0.3 \). When \( \nu > \nu_c \) the result is \( \Delta \alpha = 24(1 + \beta)/[s(5 + s)] \), with \( \beta = p/2 \) in this case. For typical parameters we now find \( \Delta \alpha \sim 0.85 \). As the steepening of the light curve is similar in the two cases, the magnitude of the break does not indicate which spectral region the observing frequency is located in. Clearly, a more detailed fit to the observations is needed. Inferring \( p < 2 \) from a standard model fit may be taken as a strong indication that a continuous injection is involved.

4. LIGHT CURVES AND SPECTRA

As is customary in GRB fireball models and was discussed in the previous section, we assume the radiation to be of synchrotron origin. We adopt the approach of Wu et al. (2004), although their method of calculating the radiation flux by integrating the synchrotron emissivity over the electron energy distribution is too computer intensive for our purposes. Instead, we use a broken power-law approximation, as in Wijers & Galama (1999), smoothly joined together in a manner similar to Granot & Sari (2002). The detailed expressions are summarized in the Appendix.

Similarly to Wu et al. (2004), we calculate the synchrotron self-absorption coefficient directly, and therefore the self absorption frequency is not included in our smoothly joined power-law approximation. We also include inverse Compton scattering, but neglect higher orders, as these are suppressed by the Klein-Nishina effect (Panaitescu & Kumar 2000). The Compton \( Y \) parameter is calculated from

\[
Y = \frac{4}{3} \tau_c \int_{\chi_c} \gamma^2 N_\gamma(\gamma) d\gamma,
\]

(30)
where $N_e$ is the normalized electron distribution and $\tau_e$ is the optical depth to electron scattering. The modified cooling Lorentz factor is found from the implicit equation

$$\gamma_{c,IC} = \frac{\gamma_c}{1 + Y},$$

where $\gamma_c$ is the usual cooling Lorentz factor (see eq. [A4]). The Comptonized emission component is then found from the synchrotron spectrum using the scalings, $\nu_{em} = 2\gamma_c^2 \nu_{ic}, \quad \nu_c = 2\gamma_c^2 \nu_{ic,IC},$ and $P_{max} = \tau_e P_{max}$ (Sari & Esin 2001).

4.1. The Emitting Region

In the standard afterglow model the fireball is described by the self-similar formulation of Blandford & McKee (1976, hereafter BM76). Analytical solutions, however, are only available if the ambient medium has a density profile of the form $n \propto r^{-9}$. Some authors (Nakar et al. 2003; Lazzati et al. 2002) have adopted these solutions in models with modest density variations. Using their approach, we are able to reproduce their results only if we assume that the shock thickness follows the BM76 solution, $\Delta \propto r/\Gamma^2$. Care must be exercised here, since in this approximation the number of radiating particles may not be estimated correctly. To clarify, let us consider a fireball in a constant density ISM. Using the Blandford & McKee (1976) solution for the density behind the shock front and integrating over the swept-up volume, we find that the number of particles contained behind the shock is

$$N_p = \frac{4\pi n^3}{3} + O(\Gamma^{-2}),$$

where $r$ is the radius of the swept up volume, and $n$ is the (constant) ISM particle density. This result is appropriate for a homogeneous medium but leads to over- or underestimates, depending on the sign of the density gradient, in the number of radiating particles when used for variable ISM densities. In such cases it leads to incorrect flux estimates. We also remind the reader that density variations do affect the evolution of $\Gamma$, which has to be calculated self-consistently to obtain correct fluxes.

Instead of using the BM76 solution directly, we make the simplifying assumption that the shock front is homogeneous in the comoving frame. Using particle conservation and the jump conditions, we find that the shock thickness in the burster frame is

$$\Delta = \frac{M_c}{2\pi m_p r^2 (1 - \cos \theta) \Gamma^{n'}},$$

appropriate for all density profiles. Here $m_p$ is the proton mass and $n' = 4n/3$. $M_c$ is the density in the comoving frame. Note that when $(1 - \cos \theta) = 2$ and $M_c = 4\pi r^2 n/3$, we recover the BM76 shock thickness. Although this is apparently a crude estimate, Granot et al. (1999) showed that the difference between the calculated flux levels in such a thin shell approximation and the full solution are small.

In all our flux calculations, we follow the evolution of the EATS. The total flux arriving at the observer at a given time, $t$, is obtained by integrating over the EATS for a given observer frequency, $\nu$ (Granot et al. (1999):

$$F(\nu, \Delta t) = \frac{1 + z}{2d_l} \int_0^\theta \int_0^r P'(\nu', \nu_0, r) \frac{r^2 dr d\cos \theta}{\Gamma^2 (1 - \beta \cos \theta)^2}$$

where $P'$ is the radiation power density in the comoving frame (see also the Appendix), $\nu'$ is the frequency in the comoving frame, and

$$t_{em} = \frac{t}{1 + z} + \frac{r \cos \theta}{c}$$

is the time of emission in the burster frame. Also, $d_l$ is the luminosity distance, which depends on the cosmology. In our numerical calculations we assume a standard cosmological model, with $\Omega_L = 0.7$, $\Omega_m = 0.3$, and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

4.2. Energy Injection Effects on Radiation Properties

To understand the effects energy injections have on the radiation properties of the afterglow, it is best to start by considering the effects of a single injection. Since we neglect the dynamics of shell collisions, the Lorentz factor of the leading shell increases instantaneously as soon as another shell catches up with it. The subsequent evolution will then quickly relax to a solution that is similar to the one obtained if the total energy of both shells were injected instantaneously. As the energy of the shock front increases instantaneously, so does its luminosity. However, the increase in flux received by the observer will not be instantaneous but gradual, because the integration over the EATS smooths out the transition. The timescale of this gradual brightening depends highly on both the Lorentz factor and the jet opening angle (Piran 2005). Energy injection in wider and slower moving jets will cause the flux to change on longer timescales and thus create smoother bumps in the light curves. Shell interactions considered in more detail than in this work would also contribute to the smoothing.

5. EXAMPLES OF DATA MODELING

In this section we apply our model to observations and give three examples of how well our numerical results fit afterglow data. We use reduced $\chi^2$ deviation to assess the goodness of the fit. As our model extends to the entire electromagnetic spectrum, an unweighted $\chi^2$ is not adequate for our purposes. The number of points in the optical light curves often exceeds that in other wave bands by a factor of 10 or more, which tends to decrease the quality of the fit at radio and X-ray wavelengths. To correct for this, we weight the contribution from each band with the square root of the number of data points in that band. We then normalize the result so that if the contribution from each point to the unweighted $\chi^2$ is unity, the weighted $\chi^2$ equals the total number of data points. The function we use to check the goodness of the fit is then

$$\chi^2_{\text{dof}} = \frac{1}{(N_{dp} - N_{\text{par}})} \sum_{i=1}^{N_d} \frac{1}{\sqrt{N_i}} \sum_{j=1}^{N_i} \left( \frac{\log(F_i(t_j)) - \log(F_i(t_j)^{\text{fit}})}{\log(1 + \Delta F_i(t_j)/F_i(t_j))} \right)^2,$$

where $N_{dp}$ is the total number of data points, $N_i$ is the number of light curves, $N_{\text{par}}$ is the number of data points for light curve $i$, $N_{\text{par}}$ is the number of parameters, $F_i(t_j)$ is the flux in light curve $i$, measured at time $t_j$ with the error $\Delta F_i(t_j)$, and $F_i(t_j)^{\text{fit}}$ is the flux from our model light curve $i$ at time $t_j$. Dividing by the number of degrees of freedom (dof) in the first factor of the equation provides the reduced $\chi^2$, which we denote by $\chi^2_{\text{dof}}$. The second factor is the normalization, and $1/\sqrt{N_i}$ in the sum over light curves is the weight.
the goodness-of-fit function, equivalent to a linear fit in magnitudes. Note that the fit is done for log $s$ as the evolution strategy (Runarsson & Yao 2000). The essence of our purposes. Instead, we employ a global search method known normally distributed search distribution is set. These points are new points within their Gaussian search distribution. Initially, the search distribution reaches the entire search space. The evolutionary process is repeated for a number of iterations. The search distribution reaches the entire search space. The evolutionary process is repeated for a number of iterations until the search distribution’s standard deviation is below a prespecified limit.

In our fits we did not allow the $g$-parameter (the ambient density profile) to vary, but assumed it to be fixed at either $g = 0$ (homogeneous ISM) or $g = 2$ (wind). In the following sections we set $g = 0$, as this provides a better fit in all three cases considered here. In addition, the angle that the line of sight makes with the jet axis (the viewing angle) is assumed to be zero. This is because it mainly affects the polarization properties of the afterglow that are not considered here.

The complex landscape of the goodness function, caused by the nonlinearity and complexity of the underlying model, makes it difficult to find a unique parameter set that is a well-defined global minimum of the goodness function. Degeneracy of the parameters can cause several minima to be considered as candidates for the global minimum. The $1 \sigma$ errors we quote in our parameter determinations reflect the statistical uncertainty of the model parameters as determined from the measurements and do not include estimates accounting for uncertainties in the model (see also Panaitescu 2001 for a similar discussion). The results of our fits for three GRB afterglows are given in Table 2. Several comments on the results are in order. As the afterglow light curves are weakly dependent on $\Gamma_0$, it is not constrained by the data. We only quote lower limits on it. The electron energy slope, $p$, is constrained both by the spectral slopes and the afterglow slopes and is the most constrained model parameter. In the case of GRB 000301C we only quote an upper limit on $\epsilon_B$, as it is poorly constrained, mostly due to the lack of X-ray data. Finally, we note that the $1 \sigma$ errors quoted in the table are generally not symmetric around the best-fit values.

5.1. GRB 990510

GRB 990510 has well-resolved afterglow light curves in many wavelengths and shows a smooth achromatic break during the second day of the afterglow (Harrison et al. 1999; Kuulkers et al. 2000; Stanek et al. 1999; Holland et al. 2000). This afterglow is a good example of smooth light curves and can be interpreted within the standard fireball model (Panaitescu & Kumar 2001b). It is therefore an ideal burst to test the accuracy of our model and to compare its results to those obtained by others.

Figure 1 shows the afterglow light curves in radio, optical, and X-rays. The fit resulted in $\chi^2_{\text{dof}} = 2.6$, quite an acceptable fit given the simplifying assumptions for the underlying dynamics and radiation and the fact we are fitting the entire electromagnetic spectrum with a single model. The best-fit parameters are given in Table 2. Most of them are similar to the parameters found by Panaitescu & Kumar (2001b), within a factor of 2 or 3. The order-of-magnitude difference in $\epsilon_e$, the fixed fraction of the total internal energy assumed to be in the electron population, stems from our different definition of $\gamma_m$. Although not shown here, the slope of the light curves varies continuously from $\alpha = 0.6$ at 0.1 days to about $\alpha = 2.2$ at 30 days. A rapid steepening sets in at 1 day after the burst, but there is no sharp transition from a pre-to a postbreak slope. We note that for this burst, $\theta \approx 1/\Gamma$ at 0.7 days, just before the steepening of the light curve.

GRB 990510 was the first burst in which a polarized afterglow was detected (Covino et al. 1999). As there was only one positive detection, it does not constrain the model very much, although it does further strengthen the case for a synchrotron origin of the afterglow emission. This polarization measurement is easily accounted for by an off-axis viewing angle. It was not included in the fit presented here.

5.2. GRB 000301C

The afterglow of GRB 000301C is well time resolved at optical and radio wavelengths and was observed in the near-infrared and at millimeter wavelengths, although no X-ray observations have been published. A short-lived brightening was observed in the optical light curves at about 3.5 days, followed by a steep achromatic break around 7 days after the burst (Sagar et al. 2000; Berger et al. 2000). It has been suggested that this bump may be caused by microlensing (Garnavich et al. 2000). Garnavich et al.’s $\chi^2$ value of the fit is reduced by half by introducing the lens effect.

Here we fit the afterglow light curves with our model containing one additional energy injection, $E_I$ at $t_I = 1.44$ days postburst. This is shown in Figure 2, where a host contribution of $40 \pm 20 \mu Jy$ at 8.5 GHz has been subtracted from the data. The data in the figure is corrected for Galactic extinction of $E(B-V) = 0.053$ (Schlegel et al. 1998) and an intrinsic SMC-like extinction of $A_B = 0.1$ using the profile of Pei (1992). This value for the intrinsic extinction was found by Jensen et al. (2001), who fitted a power-law spectrum with various extinction curves to the measured SEDs.

Fitting the data with a model with just an initial instantaneous energy injection results in $\chi^2_{\text{dof}} = 11$ (Fig. 2, dotted curves). Adding the energy injection $E_I$, reduces this by a factor of 3, to $\chi^2_{\text{dof}} = 3.8$, and gives the parameters presented in Table 2. Most of the reduction in the $\chi^2_{\text{dof}}$ is due to the optical light curves, as

| Parameter       | 990510 | 000301C | 010222 |
|-----------------|--------|---------|--------|
| $\chi^2_{\text{dof}}$ | 2.6    | 3.8     | 4.6    |
| $E_{\text{dof}}$ (10$^{50}$ ergs) | 1.8    | 52      | 17     |
| $E_0$ (10$^{50}$ ergs) | 1.8$^{+0.9}_{-0.3}$ | 16 $\pm$ 4 | 1.3$^{+0.2}_{-0.4}$ |
| $\Gamma_0$      | $>$300 | $>$500  | $>$800 |
| $n_0$ (cm$^{-3}$) | 4.1$^{+0.3}_{-0.3}$ | 0.18$^{+0.13}_{-0.12}$ | 3.4$^{+0.3}_{-0.4}$ |
| $\theta_0$      | (1.2$^{+0.3}_{-0.3}$) | 1.1$^{+0.3}_{-0.2}$ | 1.1$^{+0.2}_{-0.3}$ |
| $p$             | 2.08$^{+0.14}_{-0.07}$ | 2.43 $\pm$ 0.16 | 2.22 $\pm$ 0.05 |
| $\epsilon_{\text{e}}$ | 0.25$^{+0.02}_{-0.01}$ | 0.15$^{+0.01}_{-0.04}$ | 0.11$^{+0.02}_{-0.06}$ |
| $\epsilon_{\text{B}}$, 10$^{-4}$ | 3.5$^{+0.2}_{-0.1}$ | $<$4.0 | 4.5$^{+0.2}_{-0.3}$ |
| $s$             | \ldots | \ldots | 1.47$^{+0.10}_{-0.08}$ |
| $\Gamma_\infty$ | \ldots | \ldots | 62$^{+10}_{-12}$ |
| $E_I$ ($E_0$)    | \ldots | 2.1 $\pm$ 0.8 | \ldots |
| $t_I$ (days)     | \ldots | 1.4$^{+0.1}_{-0.2}$ | \ldots |

Note:—The errors quoted are the $1 \sigma$ estimates.

[Table 2: Values of Parameters Obtained by Fitting the Three Afterglows]
Fig. 1.—Afterglow light curves of GRB 990510 fitted with a model with an instantaneous energy release. The multiband fit is reasonably good, resulting in $\chi^2_{\text{red}} = 2.6$ with the model parameters given in Table 2. The X-ray data are from Kuulkers et al. (2000), the optical data from Harrison et al. (1999) and Stanek et al. (1999), and the radio data from Harrison et al. (1999). The optical data are corrected for Galactic extinction of $E(B-V) = 0.2$ (Schlegel et al. 1998).

Fig. 2.—Sample of the available afterglow light curves of GRB 000301C fitted with a jet profile and an additional energy injection, $E_1 = 2.1E_0$ at $t_1 = 1.4$ days after the burst (solid curves). The fit has $\chi^2_{\text{red}} = 3.8$ using all available data and is a reasonably good fit. The energy injection allows for both the sharp break in the optical/NIR bands and the smooth decline of the radio light-curve. The bump around 4 days seen in the R band causes some of the contribution to $\chi^2_{\text{red}}$ along with the radio data. It rises and decays too sharply to be fitted with an energy injection. The model parameters are given in Table 2. The dotted curves show the best-fit model without additional energy injection, resulting in $\chi^2_{\text{red}} = 11$. See the text for further discussion. Optical and near-infrared data are from Jensen et al. (2001), Rhoads & Fruchter (2001), Bhargavi & Cowsik (2000), Maseetti et al. (2000), and Sagar et al. (2000). Radio and millimeter data are from Berger et al. (2000), Smith et al. (2001), and Frail et al. (2003). All optical data are corrected for Galactic extinction of $E(B-V) = 0.053$ (Schlegel et al. 1998) and intrinsic SMC-like extinction of $A_g = 0.1$ using the profile of Pei (1992). A host contribution of $40 \pm 20 \mu$Jy has been subtracted from the 8.46 GHz data.
the energy injection allows for a fairly constant emission at 8.5 GHz and the sharp steepening of the optical light curves at approximately 6–7 days (Fig. 2, solid curves). The remaining contribution to the $\gamma/C_{31}$ is due to the bump at 3.5 days in the optical light curves, which rises too sharply to be fitted with an injection event. This is because of the EATS and the homogeneity of the emitting region in our model, making the flux increase too smooth to fit the sharp rise and decline observed. This sharp bump may be due to an injection that is local in the sense that it only affects a small patch of the radiating shock front. We also note that the best fit is obtained by having the energy injection at $t_1 = 1.4$, i.e., just before the first optical data points, rather than at times just before the light-curve bump. Such an early injection does fit the flux levels in the various bands better than an injection that is constrained to lie within the time window of the observations.

Our model parameter values agree with those of Panaitescu (2001) within a factor of 2 or 3, except for quantities that are defined differently (such as $e_c$). Given the differences in approach and the different level of details in his work and ours, these differences are rather minor.

Finally, although energy injection that is uniform across the forward shock front is unable to account for the bump in the light curves at 3.5 days, other explanations besides microlensing may apply. These could be density inhomogeneities in the ambient medium or an inhomogeneous injection that energizes the emitting surface only partly (patchy shell).  

5.3. GRB 010222

GRB 010222 had well time resolved afterglow light curves at most wavelengths and can be interpreted with the standard broken power law in the optical bands. The shallow slopes of the light curves require a hard electron energy distribution ($p < 2$), if interpreted within the standard model. An unknown host extinction may influence the SED slope, which in turn may lead to an inconsistent determination of $p$, between the spectra and the light curves.

Björnsson et al. (2002) found that a continuous energy injection in an otherwise standard fireball model could explain both the light curves and SEDs without a hard electron energy distribution. We follow their suggestion and use a continuous energy injection model to fit the afterglow data. The result is shown in Figure 3, where the data points have been corrected for Galactic extinction of $E(B-V) = 0.018$ (Schlegel et al. 1998) and intrinsic SMC-like extinction of $A_B = 0.1$, using the profile of Pei (1992), which was also adopted by Galama et al. (2003). The host of GRB 010222 is a bright starburst galaxy (Frail et al. 2002) and where available, the host contribution has been subtracted from the light curves.

The fit resulted in $\chi^2_{\text{red}} = 4.6$ with the model parameters presented in Table 2. The fit is not particularly good, mainly because the model gives too high a flux at the radio wavelengths. Both the optical and the X-ray data are well accounted for. We note that the value of $\Gamma_m$ is rather high (Table 2), and the continuous energy injection ceases at approximately 0.1 day, which along with the jet geometry ($\Gamma \approx 1/\theta$ at about 3 days), accounts for the steepening in the light curves. If we fit the data without the continuous energy injection, we obtain $\chi^2_{\text{red}} = 5.8$ instead of 4.6, with the additional contribution mainly due to a worse fit at radio wavelengths. The model parameters, however, are similar in both cases: a narrow and energetic jet with a low magnetic field strength in a low-density environment. The continuous energy injection thus better accounts for the radio light curves, but
otherwise the evolution in both cases is similar. We are therefore able to fit the optical data reasonably well within the standard model and still have $p > 2$.

6. DISCUSSION

We have presented a detailed description of a relativistic fireball afterglow model with discrete and continuous energy injection. We have given explicit examples of fits to observations for both types of injections and shown how they can provide a better fit to afterglow measurements than the standard model with single initial energy release.

Although not of direct concern in this paper, we note that even polarization measurements can in some cases be accounted for with the extended model (Björnsson et al. 2004). However, interpreting afterglow polarimetry is not straightforward. First, polarization measurements are difficult, as the polarization level in general seems to be small in afterglows. The data may therefore be sensitive to polarized contributions from other sources such as the host galaxy, which, however, is most likely to be constant. Second, our model is sensitive to the geometry of the outflow and the assumed homogeneity of the emission region. The expanding shock front is unlikely to be homogeneous during its entire evolution, especially with the added energy injections. This is because an injected shell will not expand laterally in exactly the same way as the original shock front. The transverse size of the incoming shell will then be smaller, and its energy deposition in the shock front will be concentrated in the contact zone between the two. Accounting for the effects of the EATS could then lead to a rise in the level of polarization shortly after the injection and even abrupt changes in the polarization angle if the center of the colliding shell does not coincide with the geometrical center of the front.

The origin of the additional shells is most likely the same as the origin of the shells producing the GRB in internal shocks. It is highly unlikely that the matter ejected from the central engine always collides and collects into one shell in the GRB event. The remaining shells will then be the ones to refresh the external shock later on in the evolution of the fireball. This might even be true of most bursts, but it goes unnoticed in many cases, since the energy of the colliding shells must be of the same order or higher as the total accumulated energy of the front if their interaction is to produce a measurable brightening.

When fitting GRB afterglow light curves with the standard fireball model, it is quite common to use power-law approximations with a light-curve break due to jet structure. The slope of the light curve, $\alpha$, before and after the break is then used to determine the electron energy distribution index, $p$, which is then compared to the value of $p$ inferred from the SED of the burst (see, e.g., Sari et al. 1998). These results must, however, be carefully considered, as the value of $\alpha$ depends on $s$ as well (see Table 1) and may not be smoothly varying between two fixed asymptotes, although that seems to be the general case. This is especially true when the characteristic frequencies of the synchrotron spectrum, $\nu_m$ and $\nu_c$, are near the observing frequency, where the SED does not follow a power law.

The light-curve break time has traditionally been used to infer the jet opening angle from the assumption that it occurs when $\theta \sim 1/\Gamma$. This can lead to erroneous angle estimates, because energy injections can delay the appearance of the light-curve break. An example of this is GRB 021004 (Björnsson et al. 2004). As a result, the opening angle will be overestimated and the energy corrected for beaming may thus not be correct (Johannesson et al. 2006). Studies based on such energy estimates (e.g., Frail et al. 2001; Bloom et al. 2003; Ghirlanda et al. 2004) should include error analysis that accounts for this “bias.”

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APPENDIX

SYNCHROTRON RADIATION MODEL

We approximate the synchrotron radiation power density, $P'$, in the comoving frame (primed quantities) with a smoothly joined power law at the characteristic frequencies. This takes the form

$$P'(\nu') = P'_{\text{max, f}} \left[ \left( \frac{\nu'}{\nu_c'} \right)^{(1/3)(-\kappa_1)} + \left( \frac{\nu'}{\nu_m'} \right)^{(-1/2)(-\kappa_1)} \right]^{-1/\kappa_1} \left[ 1 + \left( \frac{\nu'}{\nu_m'} \right)^{(-1/2+2p/2)(\kappa_2)} \right]^{-1/\kappa_2}$$

(A1)

for the fast-cooling regime, $\nu'_c < \nu'_m$, and

$$P'(\nu') = P'_{\text{max, s}} \left[ \left( \frac{\nu_m'}{\nu_c'} \right)^{(1/3)(-\kappa_3)} + \left( \frac{\nu_m'}{\nu_m'} \right)^{|-(p-1)/2|(-\kappa_3)} \right]^{-1/\kappa_3} \left[ 1 + \left( \frac{\nu_m'}{\nu_c'} \right)^{|-(p-1)/2+p/2|(-\kappa_4)} \right]^{-1/\kappa_4}$$

(A2)

for the slow-cooling regime, $\nu'_m < \nu'_c$. Here $p$ is the electron energy distribution index and $\nu'_m$ and $\nu'_c$ are the characteristic synchrotron frequencies corresponding, respectively, to the minimum Lorentz factor of the distribution,

$$\gamma_m = \frac{p - 2}{p - 1} \left[ \epsilon \frac{m_p}{m_e} (\Gamma - 1) + 1 \right]$$

(A3)
and the Lorentz factor above which electrons are radiative,

\[
\gamma_c = \frac{6\pi m_e c}{\sigma_T(1 + \beta)B'^2 t}.
\]  

(A4)

Here \(m_p\) and \(m_e\) are the mass of the proton and the electron, respectively, \(\varepsilon_e\) is the fraction of thermal energy in the electrons compared to the total thermal energy of the fireball, \(\sigma_T\) is the Thompson scattering cross section. Furthermore, \(\beta\) is the dimensionless speed of the shock front and \(B' = [\varepsilon_B 8\pi (1 - n')^{1/2}]\) is the magnetic field strength, with \(\varepsilon_B\) the fraction of magnetic field energy compared to the total thermal energy of the fireball, and \(n' = 4\Gamma n\) is the density in the comoving frame. The characteristic frequencies are given by

\[
\nu'_i = \chi_p \frac{3\gamma_i^2 B'}{4\pi m_e c},
\]  

where \(i\) is either \(m\) or \(c\). The maximum values for the power density are given by

\[
P_{\text{max}, f} = \phi_p \frac{2.234 e^3 n' B'}{m_e c^2},
\]

(A6)

\[
P_{\text{max}, s} = \phi_p \frac{11.17(p - 1) e^3 n' B'}{3p - 1 m_e c^2},
\]

(A7)

where \(e\) is the elementary charge and the parameters \(\phi_p\) and \(\chi_p\) are introduced here, as in Wijers & Galama (1999), to account for an isotropic distribution of angles between the electron velocity and the magnetic field. Comparing the results from these approximations with the more accurate calculations using the approach from Wu et al. (2004), we find the following \(p\)-dependence of the exponents and coefficients in the fast cooling regime to be

\[
\kappa_1 = 2.37 - 0.3p,
\]

(A8)

\[
\kappa_2 = 14.7 - 8.68p + 1.4p^2,
\]

(A9)

\[
\phi_p = 1.89 - 0.935p + 0.17p^2,
\]

(A10)

\[
\chi_p = 0.06 + 0.28p.
\]

(A11)

In the slow-cooling regime we find similarly:

\[
\kappa_3 = 6.94 - 3.844p + 0.62p^2,
\]

(A12)

\[
\kappa_4 = 3.5 - 0.2p,
\]

(A13)

\[
\phi_p = 0.54 + 0.08p,
\]

(A14)

\[
\chi_p = 0.455 + 0.08p.
\]

(A15)

The difference between these approximations and the more accurate calculations is generally less than 10%, unless \(\gamma_c\) is close to \(\gamma_m\). Then the difference can be up to 40%. However, for most parameter sets the characteristic frequencies are close only very early in the afterglow evolution, and this is therefore not a concern in the later evolution. Even then the difference is mostly smoothed out because of the integration over the EATS. We find the difference between the two solutions to be less than 10% for most realistic cases. We also remind the reader that the electron energy distribution is an approximation, and little is known about the detailed microphysics of the shocked material.

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