Semi-Supervised Density Peaks Clustering Based on Constraint Projection

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ABSTRACT

Clustering by fast searching and finding density peaks (DPC) method can rapidly identify the centers of clusters which have relatively high densities and high distances according to a decision graph. Various methods have been introduced to extend the DPC model over the past five years. DPC was originally presented as an unsupervised learning algorithm, and the thought of adding some prior information to DPC emerges as an alternative approach for improving its performance. It is extravagant to collect labeled data in real applications, and annotation of class labels is a nontrivial work, while pairwise constraint information is easier to get. Furthermore, the class label information can be converted into pairwise constraint information. Thus, we can take full advantage of pairwise constraints (or prior information) as much as possible. So this paper presents a new semi-supervised density peaks clustering algorithm (SSDPC) that uses constraint projection, which is flexible in loosening a few constraints over the learning stage. In the first stage, instances involving instance-level constraints and the remaining instances are concurrently projected to a lower dimensional data space led by the pairwise constraints, where viewing the distribution of data instances more clearly is available. Subsequently, traditional DPC is executed on the new lower dimensional dataset. Lastly, a few datasets from the Microsoft Research Asia Multimedia (MSRA-MM) image and UCI machine learning repository datasets are adopted in the experimental validation. The experimental results demonstrate that the proposed SSDPC achieves better performance than other three semi-supervised clustering algorithms.

1. INTRODUCTION

The swift advancement in information technology has caused a massive increase in the amount of data generated. As storage becomes more affordable, we will continue to witness rapid growth. How much of this data makes sense to a naive user remains a challenge to researchers in fields such as data mining. As much of the data is not labeled, annotated, or captioned. One tool that has been used by the research communities to attempt to organize such mixed data is clustering. Clustering is grouping of objects into classes according to objects’ similarities [1]. These classes exhibit close intra-class similarity and wide interclass difference, and much of this process is performed in an unsupervised way [2], where clustering in its original form takes mixed data that is unlabeled and attempts to categorize it [3].

Clustering is a useful technique because when datasets are apportioned into groups based on data homogeneity, we can hence give labels to such small groups. The process can adapt to changes and classify groups using suitable features [1]. Features describing each object are all the algorithm has available to separate the objects. Clustering can pinpoint sparse and dense areas in object space, and we can easily observe the entire distribution shapes and find any correlations in data marks. It can also be a preprocessing input to other algorithms, like classification and feature subset selection, which can conveniently work on the identified groups.

The most common approach in the use of clustering algorithms is with unlabeled data and without a tutor; it is therefore unsupervised. The input is simply a collection of data instances to be clustered based on some conceptual relationship. Unsupervised clustering can be greatly improved by some supervision, and numerous algorithms have been recommended to enhance the quality of clustering by employing supervision.

The clustering algorithms are not given any information at all, yet the experimenter in the real-world application domain may have some clues regarding the dataset that could be useful to the algorithm, for example, where to place each instance within the partition. Traditional unsupervised clustering algorithms cannot benefit from such information when it does exit [2]. We therefore are interested in clustering that requires some minimal involvement by the users and we refer to it as semi-supervised clustering.

Such supervision could be to place constraints either to revise the cost function or to master the distance and similarity measures [4]. This type of supervision is generally known as semi-supervised learning, it involves acquiring knowledge from labeled and at the same time unlabeled data by computers and natural systems like human beings [5–7]. This method is better than unsupervised and supervised learning as long as it offers improved performance and
accuracy [5]. The system of semi-supervised learning works both for classification and clustering [8], hence the capability of unsupervised clusters can improve with little amounts of supervision by way of labels on the data or constraints [2]. Studies on semi-supervised clustering show that it is much more effective than unsupervised classification techniques [9–13].

Semi-supervised learning has been researched under models such as graph-based methods, mixture models, self-training, multi-view learning and co-training. The recent algorithms of semi-supervised clustering generalizes to two types: constraint and distance based. Constraint-based processes operated by users providing labels or constraints to control the algorithms to even more correct data separation [4,14]. This is achieved by tuning the objective function for gauging clustering such that it satisfies the constraints [4,15]. The constraints can be enforced through the clustering steps [2], or the clustering can be initialized and then constrained based on labeled examples [16]. Constraint-based semi-supervised clustering could, for example, use pairwise constraints, where two instances are grouped in the same or different bundles [4].

Pairwise constraint is a type of supervision information that specifies whether two instances of data locate in the same group (a must-link [ML]) or separate groups (a cannot-link [CL]). The use of pairwise constraints from a practical perspective is often obvious selection in certain applications, meaning they are gathered spontaneously alongside unlabeled data [17,18]. For example, the co-occurring information of Interacting Proteins. The dataset can be taken as the obvious ML constraints in gene data clustering [18,19]. Notably, semi-supervised learning methods have unlocked access to the use of constraints in clustering, and consequently, with selection of active and effective pairwise constraints, clustering can be improved by specifying similarities between pairs of instances [14]. In fact, semi-supervised clustering can be referred as constrained clustering, since the supervision information is provided by the pairwise constraints [18]. So one has to consider the semi-supervised clustering problem as one where it is known that by varying the degree of inevitability, some sample pairs are (or are not) in the same class [20]. For label-breeding algorithms, the available labels are disseminated to unlabeled points whereas the available labels are changed to pairwise constraints, in constrained-clustering algorithms, then a controlled cut is made as a tradeoff between the cut cost minimization and the constraint satisfaction maximization [21]. The use of partly labeled information as pairwise constraints has been investigated by Nguyen and Caruana [22].

The density peaks clustering (DPC) algorithm is a clustering method for finding the clusters centers quickly. However, DPC was originally introduced as an unsupervised learning algorithm. As mentioned above, obtaining constraints from a dataset is achievable. Motivated by the DPC algorithm, the idea of adding some prior information to DPC emerges as an alternative means to improve its performance. In this paper, we introduce pairwise constraints to DPC and construct a framework of semi-supervised density peaks clustering (SSDPC). Pairwise constraint information is applied to guide projecting the original data. The data points then can be observed more obviously in the lower dimensional data space, and making use of some domain knowledge will enhance the clustering effect.

The rest of the paper is arranged as follows: Section 2 reports related works. Section 3 introduces constraint projection and the proposed SSDPC algorithm. Section 4 is the experimental procedures and results. Thereafter, the conclusions is drawn in Section 5.

2. RELATED WORKS

2.1. Clustering by Fast Search and Find of Density Peaks

Rodriguez and Laio proposed an advanced clustering algorithm by identifying density peaks (DPC) [23]. The concept of the DPC algorithm is simply based on distinguishing cluster centers from their neighbors by higher densities and comparatively large distances from other points with higher densities. The method uses the local density \( \rho \) and the distance \( \delta \) of point \( x \) to locate class centers.

These assumptions involve features of cluster centers: in other words, cluster centers have neighbors with lower local densities and they are placed comparatively far from the points with high densities. According to [23], for a given dataset \( X = [x_1, x_2, ..., x_n] \) where \( n \) is the quantity of data points, then the distance matrix between \( x_1 \) and \( x_j \) is constructed depend on the Euclidean distance of the data points.

\[
d(x_i, x_j) = \| x_i - x_j \| 
\]

Here \( \| . \| \) denotes a 2-norm and in the next steps, we can compute the local density \( \rho \), and distance \( \delta \) of point \( x_i \) as follows:

\[
\rho_i = \sum_{j \neq i} \chi(d(x_i, x_j) - d_c) 
\]

Note that \( d_c \) is the cutoff distance. \( d_c \) is the neighborhood range of data point \( i \) and \( \rho_i \). It specifies the quantity of points adjacent to the data point \( x_i \). However, the implementation code that was presented has another value of \( \rho_i \), given by the Gaussian Kernel function.

\[
\rho_i = \sum_{j \neq i} \exp \left( - \left( \frac{d_{ij}}{d_c} \right)^2 \right) 
\]

In Eq. (3), \( d_{ij} \) is a controllable parameter to handle the weight degradation rate and \( d_c \) is called the soft threshold while Eq. (2) represents the hard threshold. The lowest distance between the data point \( x_i \) and any other point with higher density \( \delta_i \) is obtained by the equation below.

\[
\delta_i = \begin{cases} 
\min_{j: \rho_j > \rho_i} (d_{ij}), & \text{if } \exists j, \text{s.t. } \rho_j > \rho_i \\
\max(d_{ij}), & \text{otherwise}
\end{cases}
\]

Cluster centers are considered to be points with high \( \rho \), and \( \delta_i \), and these points are known as the peaks. These cluster center points have higher densities than other points. Any other point therefore belongs in the same cluster like its nearest neighbor peak. Once every cluster center is spotted, the algorithm attaches the rest of the points to the same cluster like their nearest neighbor with larger density. A plot of the distance \( \delta_i \) and the local density \( \rho_i \) of all the points helps in making decisions on \( d_c \) and the cluster centers. Sometimes producing an estimate based on the density peaks is very difficult in situations when the quantity of clusters is up to the quantity of objects in the dataset. In the case of sparse data points, the number of peaks is obscure, so plotting \( \rho_i \times \delta_i \) arranged
in a decreasing order is used to choose the number of clusters. It is
denoted as \( \gamma \) [23]. The distance \( \delta \) is given by \( \max(d_i) \) for the points
with the high density. It is clear the points that possess local or
global maximum density have substantial \( \delta \). There can exist some
characteristics where for example, (1) a point with low \( \rho \) and low
\( \delta \) implies that the point is placed in the boundary of a cluster, (2)
a point with high \( \rho \) and low \( \delta \) means the point is near to a cluster
center, (3) a point with low \( \rho \) and high \( \delta \) specifies the point is dis-
tant from any clusters and can be an outlier or noise. Therefore only
the points having both high \( \rho \) and large \( \delta \) are candidates of cluster
centers.

The DPC algorithm described above indicates the process for a sin-
gle step; in other words, no iterations are involved and there are a
few parameters to initialize. Accordingly a variety of extended DPC
algorithms are demonstrated in recent studies [24–32], in which part
of the shortcomings in DPC algorithm are upgraded.

2.2. Pairwise Constraint

Pairwise constraint is an exemplary approach of using prior infor-
mation of datasets as injecting labels into clustering, typically in the
manner of ML and CL pairwise constraints. A given pair of data
points in ML specifies they should in the same group, while they
are in CL shows the pair of data points are from different groups.
Abounding of semi-supervised clustering techniques exploit pair-
wise constraints as prior knowledge in [18,33–38].

For a dataset \( X \), the pairwise constraints (ML set and CL) set are
expressed as follows:

1. \( ML = \{(x_i, x_j) | l_i = l_j, \forall i, j\} \) (data points \( x_i \) and \( x_j \) are placed in
   the same group);

2. \( CL = \{(x_i, x_j) | l_i \neq l_j, \forall i, j\} \) (data points \( x_i \) and \( x_j \) are placed in
different groups).

where \( l \) is the label of data point \( x \).

Pairwise constraint is symmetric,

\((x_i, x_j) \in ML \Rightarrow (x_j, x_i) \in ML\)
\((x_i, x_j) \in CL \Rightarrow (x_j, x_i) \in CL\)

It is also transitive,

\((x_i, x_j) \in ML \& (x_j, x_k) \in ML \Rightarrow (x_i, x_k) \in ML\)
\((x_i, x_j) \in CL \& (x_j, x_k) \in CL \Rightarrow (x_i, x_k) \in CL\)

3. CONSTRAINT PROJECTION FOR SSDPC

3.1. Proposed Constraint Projection Model

Given a dataset \( X = \{x_1, x_2, ..., x_n | \forall x_i \in \mathbb{R}^q\} \), \( p \) is the dimen-
sion of attributes of data point \( x_i \). The label of each data point
\( l_i \in \{c_1, c_2, ..., c_h\}, l = 1, 2, ..., n, c_i \) is the category symbol of cluster
\( C_p \in \{1, 2, ..., h\} \), \( h \) indicates there are \( h \) clusters in the dataset.
For a data point, the information in each dimension represents
different attributes. Considering the importance of each attribute
is not equivalent, this requires selection of the key attributes.
Constraint projection implies choosing the crucial \( q \) dimensions,
which are used to reduce the \( p \)-dimensional original data to \( q \)-
dimensional data by determining a projective matrix \( W_{pq} = [w_{11}, w_{22}, ..., w_{pq}] \), \( w^Tw = I \). The data points in lower dimensional
space will be represented as \( z_i = W^Tx_i \). The data after projection
must maintain the relationship between pairwise constraints of the
original data. That is, points involving the ML set must be close,
while points involving the CL set must be distant.

The definition of the objective function is to maximize \( J(W) \).

\[
J(W) = \frac{1}{2n_c} \sum_{(x_i, x_j) \in CL} \| W^T x_i - W^T x_j \|^2
- \frac{d}{2n_m} \sum_{(x_i, x_j) \in ML} \| W^T x_i - W^T x_j \|^2
\]

where \( n_c \) is the cardinal number of the cannot-link set CL and \( n_m \) is
the cardinal number of the must-link set ML. \( d \) is a scaling para-
meter, whose role is to counteract the contributions of distances in CL
set and ML set. The parameter \( d \) can be estimated by Eq. (6)

\[
d = \frac{1}{n_c} \sum_{(x_i, x_j) \in CL} \| x_i - x_j \|^2
- \frac{1}{n_m} \sum_{(x_i, x_j) \in ML} \| x_i - x_j \|^2
\]

As we usually use only a portion of the pairwise constraint infor-
mation, it is not necessary to calculate all pairwise distances in the
constraint sets CL and ML, but just the distances of the portion of
constraints that are utilized.

3.2. Inference

The purpose of maximizing \( J(W) \) is to obtain a solution set of vec-
tors \( W_{pq} = [w_{11}, w_{22}, ..., w_{pq}] \). The problem involving the maximiza-
tion of \( J(W) \) is a typical Eigen-vector/value problem.

\[
J(W) = \frac{1}{2n_c} \sum_{(x_i, x_j) \in CL} \| W^T x_i - W^T x_j \|^2
- \frac{d}{2n_m} \sum_{(x_i, x_j) \in ML} \| W^T x_i - W^T x_j \|^2
\]

\[
= \frac{1}{2n_c} \sum_{(x_i, x_j) \in CL} \left( W^T (x_i - x_j) \right)^T \left( W^T (x_i - x_j) \right)
- \frac{d}{2n_m} \sum_{(x_i, x_j) \in ML} \left( W^T (x_i - x_j) \right)^T \left( W^T (x_i - x_j) \right)
\]

\[
= \frac{1}{2n_c} \sum_{(x_i, x_j) \in CL} \text{trace} \left( W^T (x_i - x_j) \cdot \left( W^T (x_i - x_j) \right)^T \right)
- \frac{d}{2n_m} \sum_{(x_i, x_j) \in ML} \text{trace} \left( W^T (x_i - x_j) \cdot \left( W^T (x_i - x_j) \right)^T \right)
\]

\[
= \text{trace} \left( W^T \left( \frac{1}{2n_c} \sum_{(x_i, x_j) \in CL} (x_i - x_j)^T (x_i - x_j) \right) \right)
- \frac{d}{2n_m} \sum_{(x_i, x_j) \in ML} (x_i - x_j)^T (x_i - x_j) W
\]
For simplicity, $H_C$ and $H_M$ are defined as follows:

$$H_C = \frac{1}{2n_c} \sum_{(x_i, x_j) \in CL} (x_i - x_j)(x_i - x_j)^T$$  \hspace{1cm} (8)$$

$$H_M = \frac{1}{2n_m} \sum_{(x_i, x_j) \in ML} (x_i - x_j)(x_i - x_j)^T$$  \hspace{1cm} (9)$$

Then Eq. (7) can be edited as

$$f(W) = \text{trace}(W^T(H_C - dH_M)W)$$  \hspace{1cm} (10)$$

Eq. (10) indicates that we can solve the problem in Eq. (5) by computing the top $q$ eigenvalues of $H_C - dH_M$ and the corresponding eigenvectors. Suppose the first $q$ eigenvalues of $H_C - dH_M$ are $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_q$, and $w_1, w_2, \ldots, w_q$ are corresponding eigenvectors. The projection solution can be expressed as $W = [w_1, w_2, \ldots, w_q]$.

Denote $\Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_q)$, Eq. (10) then will be reformulated as

$$\text{trace}(W^T(H_C - dH_M)W) = \text{trace}(\Lambda) = \sum_{j=1}^{q} \lambda_j$$  \hspace{1cm} (11)$$

From Eq. (11), it is clear that Eq. (7) will achieve the maximum value when the set of eigenvalues $(\lambda_1, \lambda_2, \ldots, \lambda_q)$ includes all the nonpositive eigenvalues of $H_C - dH_M$. Note that $\lambda_j = 0$ will not contribute to maximizing Eq. (11). In order to avoid losing too much of the characteristic information, all the nonnegative eigenvalues $\lambda$ are chosen. From this process, $H_C$ and $H_M$ are positive semi-definite.

### 3.3. Semi-Supervised Density Peaks Clustering Algorithm

Figure 1 shows the flow of the presented SSDPC.

![Algorithm Flow](image)

**Figure 1.** Depiction of the semi-supervised density peaks clustering (SSDPC) algorithm. Steps 1–6 involve the constraint projection of the original dataset $X$ led by pairwise constraints, and steps 7–8 are clustering processes.

Based on the depiction in Figure 1, the SSDPC algorithm is expressed as follows:

From the process of SSDPC algorithm, we can get complexity of SSDPC model. The time complexity is $O(n^3)$, the space complexity is $O(n^2)$, $n$ is the number of instances in the dataset.

### SSDPC algorithm:

**Input:** Dataset $\{x_i\}_{i=1}^{n} \in \mathbb{R}^d$, $x_i$ is the data point.
1. Establish the cannot-link set $CL$ and must-link set $ML$ according to the part of existing labels or other field information.
2. Calculate matrix $H_C - dH_M$ according to Eqs. (8) and (9).
3. Compute all the eigenvalues $\lambda$ and corresponding eigenvectors $w$ of matrix $H_C - dH_M$.
4. In order to maximize the objective function $f(W)$ in Eq. (5), all the non-negative eigenvalues and corresponding eigenvectors are selected.
5. Use the selected eigenvectors to set up the projection matrix $W = [w_1, w_2, \ldots, w_q]$.
6. Construct the new dataset $Z$ depending on $Z = W^TX$.
7. Run the standard density peaks clustering (DPC) algorithm on the projected dataset $Z$. Clustering procedures are completed.

**Output:** Clusters $C_1, \ldots, C_k$. $C_k = \{i | x_i \in C_k\}$.

### 4. EXPERIMENTAL STUDIES

In this section, we describe the experiment settings and performance comparisons for our proposed algorithm SSDPC.

### 4.1. Datasets Setup

In this part, we investigate the performance of the SSDPC algorithm on a variety of datasets. In the experiments, 17 datasets are acquired from Microsoft Research Asia Multimedia (MSRA-MM) image datasets [39] and three datasets are picked up from the UCI machine learning repository [40]. The details of these 20 datasets are summarized in Table 1.

The MSRA-MM dataset was assembled from a commercial search engine with more than 1 million images and 20 thousand videos. The purpose of MSRA-MM is to promote research in the field of multimedia information retrieval and related areas. Seventeen image datasets are chosen for the assignment of semi-supervised density peaks clustering (DPC). Each image dataset embraces practically 1,000 instances with nearly 900 features.

UCI Machine Learning Repository currently maintains 507 datasets as a service to the machine learning communities for experimental studies of machine learning methods. New datasets are constantly supported by researchers’ donations from all over the world. The most popular UCI datasets are Iris, Cancer, Wine, Breast, Heart Disease, Bank Marketing, Adult, Car Evaluation, Forest Fires, Wisconsin (Diagnostic), Human Activity Recognition Using Smartphones, Wine Quality, Poker Hand, and Abalone. In this empirical analysis, we pick credit approval, vertebral column, and congressional voting records from the UCI datasets. The particulars of each dataset, including classes, the quantity of instances, and features are tabulated in Table 1.

### 4.2. Experiments

To evaluate how the proposed SSDPC algorithm performs, three other contemporary semi-supervised learning algorithms, constrained k-means clustering with background knowledge (Copkmeans) [2], constrained 1-spectral clustering (COSC) [41], and semi-supervised clustering based on affinity propagation (SemAP) [42] are also implemented. All experiments are organized on a
computer with Intel (R) Core (TM) i5-4460M CPU 3.20 GHz and 4.00 GB RAM, running Matlab R2010b. Each algorithm experiment is repeated 10 times on the 20 datasets.

The crucial modification of Cop-kmeans is to make sure none of the specified constraints are violated when updating cluster assignments. Cop-kmeans allocate each point $x_i$ to the closest cluster $C_k$ as long as violate-constraints $(x_i, C_k, ML, CL)$ is false. The definition of violate-constraints (data point $x$, cluster $C$, must-link constraints $ML$, cannot-link constraints $CL$) is: (1) For any $(x, x_p) \in ML$: If $x_p \notin C$, return true. (2) For any $(x, x_p) \in CL$: If $x_p \in C$, return true. (3) Else, return false. COSC is able to guarantee all the given constraints are satisfied in the field of constraint spectral clustering. must-link constraints in $ML$ are integrated via sparsification, while cannot-link constraints should be considered for the bi-partitioning scheme. In Semi-AP, the similarity matrix $[S(i, k)]_{max}$ is modified depending on pairwise constraints with several principles: (1) $(x_i, x_j) \in ML$ ⇒ $s(i, j) = 0$ & $s(j, i) = 0$. In case data point $k$ conforms to $(x_i, x_j) \notin ML$, $(x_i, x_j) \notin ML$ & $s(i, j) = -\infty$ & $s(j, i) = -\infty$. (2) If point $k$ is connected to data points $i$ and $j$ indvidually, $(x_i, x_j) \notin ML \cup CL$ ⇒ $s(i, j) = max(s(k, j), s(j, k)) + s(i, k))$. (4) $(x_i, x_j) \notin ML \cup CL$ & $(x_i, x_j) \notin CL$ & $(x_i, x_j) \notin ML$ ⇒ $s(i, j) = -\infty$ & $s(j, i) = -\infty$ or $(x_i, x_j) \notin ML \cup CL$ & $(x_i, x_j) \notin ML$ & $(x_i, x_j) \notin CL$ ⇒ $s(i, j) = -\infty$ & $s(j, i) = -\infty$. Then, $CL = (x_i, x_j) \cup CL$, and an exemplar is obtained for each data point. They also make some adjustments to the affinity propagation cluster result for the condition when a constraint is violated.

Parameters for Cop-kmeans, COSC, and Semi-AP are all determined as described in the original studies [2,41,42].

The proposed algorithm SSDPC is implemented in accordance with the depiction in Figure 1. In this case, 10% of pairwise constraints information are applied to construct the ML and CL sets. The cutoff distance $d_C$ satisfies the average number of neighbors is 2% of the total numbers of instances in the dataset. In comparison with Cop-kmeans, COSC and Semi-AP, SSDPC is more flexible in following the constraints we got.

For the evaluation measurement, we apply micro-averaged-precision (MAP) [43] to evaluate the accuracy of each clustering result. MAP is defined as follows:

$$MAP = \frac{\sum_b a_h}{\sum_b (a_h + b_h)}$$

where $h$ is the quantity of clusters, $a_h$ is the quantity of instances accurately allotted to cluster $C_h$, and $b_h$ is the number of instances incorrectly allotted to cluster $C_h$. Generally, $0 \leq MAP \leq 1$, so the larger the value of MAP, the better the quality of the clustering algorithm.

### 4.3. Results and Discussion

In this part, results of our experiments are provided. Table 2 shows the accuracy results of the different algorithms on 20 datasets. Records are tabulated in terms of averaged mean accuracies and standard deviations over 10 repetitions of the experiments. The mean accuracies of the SSDPC algorithm are higher than the other three semi-supervised learning algorithms (Cop-kmeans, Semi-SC, Semi-AP) on the 20 different datasets, and the highest accuracy value is for the congressional voting records dataset, which reaches 0.8805. The proposed SSDPC algorithm also has the highest average accuracy for all datasets with an average MAP value of 0.5797.

Table 1 | Characteristics, classes, the quantity of instances, and features in each dataset.

| Dataset                        | Characteristic | Classes | Instances | Features |
|--------------------------------|----------------|---------|-----------|----------|
| Ambulances                     | Real           | 3       | 930       | 892      |
| Aquarium                       | Real           | 3       | 922       | 892      |
| Balloon                        | Real           | 3       | 830       | 892      |
| Bed                            | Real           | 3       | 888       | 892      |
| Birthdaycake                   | Real           | 3       | 932       | 892      |
| Blog                           | Real           | 3       | 943       | 892      |
| Boat                           | Real           | 3       | 857       | 892      |
| Bonsai                         | Real           | 3       | 867       | 892      |
| Bread                          | Real           | 3       | 885       | 892      |
| Building                       | Real           | 3       | 882       | 892      |
| Bus                            | Real           | 3       | 911       | 892      |
| Butterflytattoo                | Real           | 3       | 910       | 892      |
| Cactus                         | Real           | 3       | 738       | 892      |
| Credit approval                | Mixed          | 2       | 690       | 15       |
| Vertebral column               | Real           | 2       | 310       | 6        |
| Vista                          | Real           | 3       | 799       | 899      |
| Vistawallpaper                | Real           | 3       | 799       | 899      |
| Voituretuning                  | Real           | 3       | 879       | 899      |
| Congressional voting records   | Discrete        | 2       | 453       | 16       |

To provide a robust comparison among the four algorithms, we carry out a $1 \times n$ comparison by way of the Friedman Aligned test [44]. The presented method SSDPC is the control algorithm. Table 3 reflects the aligned ranks and the aligned results in parentheses for the four algorithms and 20 datasets. From this table, SSDPC ranks first with an average rank value of 10.85. Cop-kmeans ranks second with average rank value of 38.35. Semi-AP ranks third with average rank value of 48.5, and COSC ranks last with average rank value of 64.3. The purpose of the Friedman Aligned Rank test is to analysis if the gauged sum of aligned ranks are notably different
Table 2 | Performance of different algorithms in terms of accuracy on 20 datasets (mean ± std%).

| Datasets              | Cop-kmeans | COSC     | Semi-AP  | SSDPC   | Total |
|-----------------------|------------|----------|----------|---------|-------|
| Ambulances            | 0.4512±0.0435 | 0.3967±0.0371 | 0.4261±0.0271 | 0.5970±0.0536 |       |
| Aquarium              | 0.4344±0.0493 | 0.3989±0.0560 | 0.4055±0.0270 | 0.6505±0.0731 |       |
| Balloon               | 0.4536±0.0416 | 0.3796±0.0277 | 0.4589±0.0452 | 0.5530±0.0409 |       |
| Bed                   | 0.4468±0.0592 | 0.4314±0.0822 | 0.4337±0.0418 | 0.5680±0.0702 |       |
| Birthdaycake          | 0.5004±0.0264 | 0.3820±0.0354 | 0.4923±0.0257 | 0.5599±0.0514 |       |
| Blog                  | 0.4467±0.0590 | 0.4340±0.0680 | 0.4060±0.0351 | 0.5883±0.0891 |       |
| Boat                  | 0.4288±0.0585 | 0.4260±0.0571 | 0.4097±0.0347 | 0.5718±0.0819 |       |
| Bonsai                | 0.3978±0.0393 | 0.3973±0.0406 | 0.4321±0.0334 | 0.5608±0.0924 |       |
| Bread                 | 0.4660±0.0225 | 0.4546±0.0801 | 0.4460±0.0513 | 0.5786±0.0615 |       |
| Bugat                 | 0.4418±0.0600 | 0.4206±0.0568 | 0.4011±0.0388 | 0.5898±0.0814 |       |
| Building              | 0.5568±0.0608 | 0.4617±0.0920 | 0.4651±0.0619 | 0.6422±0.0709 |       |
| Bus                   | 0.4700±0.0384 | 0.4163±0.0777 | 0.4293±0.0359 | 0.5522±0.0675 |       |
| Butterflytattoo       | 0.6070±0.0572 | 0.3737±0.0204 | 0.5680±0.0235 | 0.6347±0.0634 |       |
| Cactus                | 0.4868±0.0753 | 0.4133±0.0631 | 0.4771±0.0574 | 0.5929±0.0679 |       |
| Credit approval       | 0.6162±0.0142 | 0.5868±0.0180 | 0.5619±0.0007 | 0.6446±0.0268 |       |
| Vertebral column      | 0.6955±0.0216 | 0.6448±0.0652 | 0.6697±0.0436 | 0.7177±0.0671 |       |
| Vista                 | 0.4426±0.0410 | 0.3930±0.0234 | 0.4464±0.0291 | 0.5601±0.0834 |       |
| Vistawallpaper        | 0.4451±0.0326 | 0.4126±0.0814 | 0.4606±0.0207 | 0.5603±0.0652 |       |
| Voituretuning         | 0.4207±0.0288 | 0.4354±0.1005 | 0.4243±0.0365 | 0.5994±0.0643 |       |
| Congressional voting records | 0.8678±0.0141 | 0.5460±0.0311 | 0.8619±0.0138 | 0.8805±0.0184 |       |
| Average               | 0.5038 | 0.4402 | 0.4839 | 0.5797 |

COSC, constrained 1-spectral clustering; SSDPC, semi-supervised density peaks clustering; AP, affinity propagation.

from the total average aligned rank \( R_j = \frac{1}{4} \sum_{i=1}^{4} \hat{R}_i / 4 = 810 \) expected under the null hypothesis:

\[
\sum_{j=1}^{4} \hat{R}_j^2 = 767^2 + 1286^2 + 970^2 + 217^2 = 3,230,074
\]

\[
\sum_{j=1}^{20} \hat{R}_j^2 = 177^2 + 202^2 + 154^2 \ldots + 132^2 = 529,658
\]

In the above two formulas, \( \hat{R}_j \) is the total aligned rank of the \( j \)th algorithm and \( \hat{R}_i \) denotes the total aligned rank of the \( i \)th dataset.

Accordingly, the Friedman Aligned Rank test statistic is determined as

\[
T = \frac{(4 - 1)[3,230,074 - (4 \cdot 20^2)/4 \cdot (4 \cdot 20 + 1)^2]}{[4 \cdot 20 \cdot (4 \cdot 20 + 1)/2 \cdot (4 \cdot 20 + 1) / 6]} - (1/4)529,658 = 43.82
\]

With four algorithms and 20 datasets, \( T \) is distributed in accordance with the chi-square distribution with \( (4 - 1) = 3 \) degrees of freedom. The \( p \)-value estimated by operating the \( \chi^2(3) \) distribution is \( 1.64807 \times 10^{-9} \), which indicates the null hypothesis is repudiated at a notable level of significance. The \( p \)-value is much smaller.
of thumb given by Rodriguez and Laio [23] is to choose
the results show the semi-DPC algorithm is feasible. However, the
we extend DPC algorithm to a semi-supervised learning algorithm,
efficient method when it was proposed in 2014 [23]. In this work,
the proposed SSDPC algorithm in analyzing different kinds of data
As various data is achievable in daily life, for instance, signal lights
data, railway track circuit data, diagnostic data, and so on. Applying
the proposed SSDPC algorithm in analyzing different kinds of data
is practicable. As we know, clustering by using density peaks is an
efficient method when it was proposed in 2014 [23]. In this work,
we extend DPC algorithm to a semi-supervised learning algorithm,
the results show the semi-DPC algorithm is feasible. However, the
algorithm is still robust with the value of cutoff distance \(d_c\). A rule
of thumb given by Rodriguez and Laio [23] is to choose \(d_c\) in a way
that gives the average of neighbors between 1 and 2 % of the overall
quantity of points in the dataset. We are going to do some research
on how to choose an optimal distance \(d_c\) for a dataset in order to
achieve better clustering performance. In general, extending a clus-
tering method or introducing a new clustering approach is to get a
valuable understanding of datasets we have.

5. CONCLUSION AND PROSPECTS

In this paper, we present a new SSDPC using pairwise con-
straints, as it is simpler to obtain pairwise constraint informa-
tion than to acquire class tags (or labels). We consider applying
pairwise constraint knowledge to project the original data onto
a well-preserved lower dimensional space, which forces the dis-
tance between instances in ML pairs ) to be decreased and the distance
between instances in CL to be increased, resulting in a
clearer observation of the instances. Pairwise constraints can be vi-
olated; it is not necessary for the clustering process to always satisfy
constraints. Consequently, the novel SSDPC algorithm is a flexible
semi-supervised clustering technique. Experiments over 20 datasets
display that our SSDPC approach performs better than three other
semi-supervised clustering methods.

As various data is achievable in daily life, for instance, signal lights
data, railway track circuit data, diagnostic data, and so on. Applying
the proposed SSDPC algorithm in analyzing different kinds of data
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CONFLICTS OF INTEREST

The authors declare they have no conflicts of interest.

AUTHORS’ CONTRIBUTIONS

All authors contributed to this study. All authors read and approved
the final manuscript.

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