Sterile Particles from the Flavor Gauge Model of Masses

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The existence of right-handed neutrinos follows from theoretical consistence of the recently suggested electroweak symmetry breaking model, based on dynamical flavor gauge symmetry breaking. Only finite number of versions of the model exists. They differ by the number and the flavor structure of the right-handed neutrinos. We choose for inspection one of them, the non-minimal version with right-handed neutrinos in sextet flavor representation, and at some points we compare it with the minimal version. We show that a Majorana pairing of the sextet right-handed neutrinos is responsible for the flavor symmetry breaking, and the seesaw pattern of the neutrino mass matrix naturally arises. The dynamically generated neutrino mass matrix spontaneously breaks the lepton number and the chiral sterile symmetry of the right-handed neutrino sector. As a result spectrum of majorons, neutrino composites, manifests. We study main characteristics of both massive sterile neutrinos and majorons which show their relevance as dark matter candidates.

PACS numbers: 11.30.Hv, 12.60.Cn, 14.60.St, 14.80.Va
Keywords: Dynamical mass generation; flavor gauge symmetry; right-handed neutrino condensation; sterile neutrinos; majorons

I. INTRODUCTION

With recent significant improvement of quality of cosmological, astrophysical and neutrino observations, the neutrino sector of particle spectrum is just becoming increasingly powerful tool to discriminate among various models of the electroweak symmetry breaking. Recently suggested model [1–3] of dynamically generated masses turns out to be extremely difficult to be approved or disproved through direct computation of its mass spectrum. Nevertheless, the model provides clear and clean predictions about the structure of the right-handed neutrino sector, about its global symmetries, and about majorons, the composite Nambu–Goldstone scalars, the consequences of the spontaneous breaking of the global symmetries.

The right-handed neutrinos are often proposed to exist for their power to explain straightforwardly the observed neutrino masses and, especially, to explain why the neutrinos are so light via the see-saw mechanism [4–6]. Since they are Standard Model singlets they do not produce color and electroweak gauge anomaly. Therefore, if they are not charged with respect to some new gauge force, their number is not constrained. In models where the family or flavor SU(3)F index is gauged [7–14] the number of all fields that feel the new flavor gauge force has to be balanced so that the flavor gauge anomaly cancels. The overall contribution of observed electroweakly charged fermions to the flavor gauge anomaly does not vanish. Therefore additional fields, the chromodynamically and electroweakly neutral right-handed neutrinos, are needed in a specific number [15].

The flavor gauge model studied in this paper intends not to postulate any further new dynamics and leaves whole responsibility for the electroweak symmetry breaking on the gauge flavor SU(3)F dynamics. In order to make sense the gauge flavor dynamics is strong, asymptotically free, self-breaking, and non-confining, i.e., non-vector-like1.

The flavor gauge symmetry breaking, the cause for the mass generation, is so far only assumed in the model. It is supposed to be achieved neither by the vacuum expectation value of a scalar field, nor by the confining strong gauge dynamics, but it is the strong flavor gauge dynamics itself that self-breaks. As there is no similar effect observed in nature, as the effort to put the chiral theories on a lattice fails, and as the solution of the equations of the model are painfully unattainable, it is not clear whether the self-breaking mechanism is possible at all. In general, it is not clear whether a chiral gauge dynamics alone could dynamically generate self-energies by which it breaks its own gauge symmetry. On the other hand this scenario pioneered by [16] has not been disqualified yet.

Fortunately, the gauge symmetries tight the model so much that there is no room for fine-tuning and firm predictions arise. Theoretical consistence of the flavor gauge model predicts the number of right-handed neutrinos with only little ambiguity. The model admits right-handed neutrinos only in selected flavor representation settings whose number is finite and not large. Throughout the paper we bring more or less heuristic arguments for that there is only one right-handed neutrino setting that defines phenomenologically viable and preferred version of the model.

The preferred version of the model is non-minimal in the sense that it contains the right-handed neutrinos in

1 Non-vector-like gauge theory arises from gauging not only vector currents, like in QCD, but also axial-vector currents.
a flavor sextet representation. It provides appealing features: (i) It is chiral, i.e., the only mass scale comes from the dimensional transmutation of the running flavor coupling constant. (ii) It is essentially non-vector-like. (iii) The sextet right-handed neutrino Majorana pairing leads naturally to the see-saw pattern of the neutrino mass matrix. (iv) It provides light sterile neutrinos, in addition to the three electroweak neutrinos what can be of particular interest with respect to the dark matter [17–19]. (v) The dynamically generated neutrino mass requires the existence of the right-handed neutrinos and the sterility symmetry breaking. In section VI we conclude.

The aim of the paper is to point out aspects of the sterile sector of the non-minimal version of the flavor gauge model, and document both theoretically and phenomenologically why others are not so preferable.

The paper is organized as follows. In section II we investigate the right-handed neutrino structure of the flavor gauge model: After a brief recapitulation of the model we summarize all viable versions of the model. We argue why we choose the non-minimal version for the rest of the paper. In section III we write the flavor structures of self-energies should be understood via the flavor representations of right-handed neutrinos. Due to the flavor setting the mass hierarchy among different charges can be achieved. The mass hierarchy among generations then has completely different origin. It follows from the fact that the flavor symmetry is completely broken providing distinct eigenvalues of the self-energies.

A. Flavor gauge model

The basis of the model is that the chiral electroweak symmetry is broken dynamically by chirality changing fermion self-energies $\Sigma(p^2)$ generated by the strong flavor dynamics. The flavor structure of the self-energies $\Sigma(p^2)$ is crucial for it should reflect the hierarchical pattern of fermion masses.

The model is defined by the flavor setting of electroweakly charged Weyl fermions. There are two distinct cases, case I and case II, see Tab. I. The ultimate discrimination among them can be made after the successful solution of mass equations is found, or after full structure of the neutrino sector is revealed.

The purpose of this setting is to distinguish self-energy matrices for fermions of various charges, as generally $\Sigma^{3\times3} \neq \Sigma^{2\times2} \neq \Sigma^{3\times5} \neq \Sigma^{5\times5}$. (This idea was also pursued in the class of extended technicolor models [23].)

In order to achieve the exclusivity of the $u$-type quarks, whose observed mass spectrum is significantly heavier, we prefer the case I. Their self-energy is of type $\Sigma^{3\times3}$ as distinct from $d$-type and $e$-type fermion self-energies which are of type $\Sigma^{5\times5}$ and $\Sigma^{3\times3}$. Neutrino self-energies are distinguished from others by their Majorana components and by possible higher flavor representation settings of right-handed neutrinos. Due to the flavor setting the mass hierarchy among different charges can be achieved. The mass hierarchy among generations then has completely different origin. It follows from the fact that the flavor symmetry is completely broken providing distinct eigenvalues of the self-energies.

The characteristic fermion flavor setting plays also another important role. It makes the flavor gauge dynamics non-vector-like, what distinguishes it from QCD and makes it non-confining.

As well as the QCD, the flavor gauge dynamics is asymptotically free, i.e., the effective flavor gauge coupling constant $\bar{h}(q^2)$ in perturbative regime runs accord-

\[ q_L, u_R, d_R, f_L, e_R \mid N \]

|   | case I | case II |
|---|--------|---------|
| $q_L$ | $3$   | $3$     |
| $u_R$ | $3$   | $3$     |
| $d_R$ | $3$   | $3$     |
| $f_L$ | $3$   | $3$     |
| $e_R$ | $3$   | $5$     |

TABLE I: Two possible flavor settings of electroweakly charged fermions. The number $N$ tells how many flavor triplets are necessary to cancel the flavor gauge anomaly. The notation is obvious: $q_L = (u_L, d_L)^T$, $\ell_L = (\nu_L, e_L)^T$, $u = (u, c, t)$, $d = (d, s, b)$, $\nu = (\nu_e, \nu_\mu, \nu_\tau)$, and $e = (e, \mu, \tau)$.

II. RIGHT-HANDED NEUTRINO FIELDS

The quantum flavor gauge dynamics of the model requires the existence of the right-handed neutrinos and restricts severely their number. As a main result of this section we list the finite number of all acceptable flavor settings that are anomaly and asymptotically free, and do not provide the perturbative infrared fixed point. These three properties are necessary for a viability of the model. Later, we rather heuristically argue that some settings are more preferable than others.

First we briefly recapitulate the model.

\[ \Sigma^{3\times3}_u \neq \Sigma^{2\times2} \neq \Sigma^{3\times5} \neq \Sigma^{5\times5} \]

\[ \Sigma^{3\times3}_d \neq \Sigma^{3\times5} \]

\[ \Sigma^{3\times3}_e \neq \Sigma^{5\times5} \]

\[ \Sigma^{3\times3} \neq \Sigma^{5\times5} \neq \Sigma^{3\times3} \neq \Sigma^{5\times5} \]

\[ \Sigma^{3\times3} \neq \Sigma^{2\times2} \neq \Sigma^{3\times5} \neq \Sigma^{5\times5} \]

\[ \Sigma^{3\times3} \neq \Sigma^{2\times2} \neq \Sigma^{3\times5} \neq \Sigma^{5\times5} \]

\[ \Sigma^{3\times3} \neq \Sigma^{2\times2} \neq \Sigma^{3\times5} \neq \Sigma^{5\times5} \]

\[ \Sigma^{3\times3} \neq \Sigma^{2\times2} \neq \Sigma^{3\times5} \neq \Sigma^{5\times5} \]
ing to
\[
\frac{\bar{h}^2(q^2)}{4\pi} = \frac{4\pi}{(11 - \frac{1}{3}N_{\text{EW}} - \frac{4}{3}\eta_{\nu R}) \ln q^2/\Lambda_F^2},
\]
where \(N_{\text{EW}} = 15\) is the number of electroweakly charged flavor triplets and \(\eta_{\nu R}\) is the right-handed neutrino contribution to the coefficient of the flavor \(\beta\)-function. Well above the scale of the flavor gauge dynamics, \(\Lambda_F\), everything is weakly coupled and symmetric. Decreasing the energy scale the effective flavor gauge coupling increases till it surpasses its critical value at the energy scale around \(\Lambda_F\). Because of its non-vector-like nature the flavor symmetry itself does not survive anymore and is spontaneously broken [16, 24]. The flavor gauge bosons acquire masses of order of the flavor symmetry breaking scale \(\Lambda_F\). (For details see [3].)

The flavor gauge bosons have to be enormously heavy in order to suppress the processes with flavor changing neutral currents, giving a lower bound for their mass to be more than \(10^6\) GeV [25]. But in order to make the axion, which is naturally present in the model, invisible it is better to assume that the quark self-energies are formed at the scale in the so called axion window \(10^{10}\) GeV \(<\Lambda_F < 10^{12}\) GeV [26]. We will see that in the non-minimal versions of the model the right-handed neutrino Majorana self-energy should be generated at even much higher scale.

The 'would-be' Nambu–Goldstone bosons of the broken electroweak symmetry, which are composites of fermions, manifest themselves as the longitudinal components of the electroweak gauge bosons, producing their masses. The electroweak gauge boson masses are therefore directly, though non-trivially linked to the masses of electroweakly charged fermions. Therefore we expect \(M_{W,Z}\) being proportional rather to \(m_t\) and not to some electroweak scale \(\Lambda_{\text{EW}}\), which in fact does not exist in this model.

B. Constraints on the number of right-handed neutrino fields

1. Anomaly freedom

The model would suffer from the flavor gauge anomaly unless the proper number of right-handed neutrino fields is added into the model. They are needed to compensate the non-zero flavor anomaly contribution of electroweakly charged fermions. In Tab. I the number \(N\) indicates that 3 (5) additional triplets of right-handed neutrinos make the flavor gauge dynamics anomaly free.

Adding of triplets is not the only possibility. Specially balanced settings including higher representations, sextet, octet, or decuplet, etc., lead to the anomaly free models too. Constructing the non-minimal versions of the model, notice that a pair of complex multiplet and its conjugate, as well as real representation multiplet do not contribute to the anomaly.

2. Asymptotic freedom

On the other hand, we should not add too many right-handed neutrinos in order not to destroy the asymptotic freedom of the flavor dynamics. Within the one-loop approximation of the \(\beta\)-function, the \(\eta_{\nu R}\) coefficient is constrained as
\[
\eta_{\nu R} \approx \frac{1}{2}N_{3}^{\nu R} + \frac{5}{2}N_{6}^{\nu R} + \frac{3}{2}N_{8}^{\nu R} + \frac{15}{2}N_{10}^{\nu R} + \ldots < 9,
\]
where \(N_{\nu R}\) is the number of right-handed neutrino multiplets of a given representation \(R\) and \(\overline{R}\). The inequality (2) leaves us to combine only lower dimensional multiplets, \(3, \overline{3}, 6, \overline{6}, \) and \(8\).

3. Absence of the perturbative infrared fixed point

Even more stringent limit comes from demand not to produce too small, i.e., sub-critical, perturbative infrared fixed point, say \(\alpha_F^{IR} < 0.5\), where \(\alpha_F \equiv \frac{\alpha_{\text{QCD}}(\Lambda_F)}{\alpha_{\text{QCD}}(\Lambda)}\). It would leave the system in the chirally symmetric phase and prevent the whole symmetry breaking mechanism.

We choose the discriminating value of \(\alpha_F^{IR}\) being 0.5 quite arbitrarily but motivated by QCD running coupling constant which is measured (still being in a perturbative regime) at the scale \(1.7\) GeV \(\gtrsim 3\) having the value \(\alpha_s(1.7\text{ GeV}) \approx 0.35\) [27].

A zero of the two-loop \(\beta\)-function (B1) gives an estimate of the perturbative infrared fixed point
\[
\alpha_F^{*, IR} = -4\pi \frac{-18 + N_{3}^{\nu R} + 5N_{6}^{\nu R} + 6N_{8}^{\nu R}}{-21 + 19N_{3}^{\nu R} + 125N_{6}^{\nu R} + 144N_{8}^{\nu R}}.
\]

4. Chirality and non-vector-like nature

Putting all together we get only few possible right-handed neutrino flavor settings defining still viable models. We list them in Tab. II. The models fall into various classes according to two criteria, their chirality and their approximate vector-like nature.

The models containing right-handed neutrinos in both \(3\), \(\overline{3}\), or in \(8\), allow the gauge invariant hard Majorana mass term. Therefore they are non-chiral possessing a hard Majorana mass parameter. The origin of such mass parameter is not explained by the model and it would have been assumed to follow from yet another dynamics operating at higher energy scale. In this sense the chiral models appear to be more complete and more fundamental.

From the high energy (around \(\Lambda_F\)) perspective, the versions of the model that contain only \(3\), or \(\overline{3}\), are approximately vector-like with small non-vector-like perturbation given by the Standard Model gauge dynamics. In that case the dynamics resembles the dynamics of QCD and presumably prefers pairing in the \(3 \times \overline{3}\) that
TABLE II: All viable versions of the flavor gauge model.

|         | ν_R representation setting | chiral | vector-like | approx. |
|---------|-----------------------------|--------|-------------|---------|
| case I  | 3 × 3                       | yes    | yes         |         |
|         | 3 × 3, 1 × (3, 3)           | no     | yes         |         |
|         | 3 × 3, 2 × (3, 3)           | no     | yes         |         |
|         | 3 × 3, 3 × (3, 3)           | no     | yes         |         |
|         | 1 × 6, 4 × (3, 3)           | yes    | no          |         |
|         | 1 × 6, 8 × 3               | no     | no          |         |

| case II | 5 × 3                       | yes    | yes         |         |
|         | 5 × 3, 1 × (3, 3)           | no     | yes         |         |
|         | 5 × 3, 2 × (3, 3)           | no     | yes         |         |
|         | 1 × 6, 2 × (3, 3)           | yes    | no          |         |
|         | 1 × 6, 2 × 3, 1 × (3, 3)    | no     | no          |         |

In this paper we will pursue the triplets denoted by (333) was analyzed in the paper [3]. The only case I version which is both non-vector-like and that certainly break the flavor symmetry.

On the other hand, the versions of the model that be energetically more favorable than the flavor preserving breaking fermion self-energies are then only believed to around Λ due to the flavor anomaly (for detailed analysis of the global symmetries see [3]).

Additionally to the global symmetries of the electroweakly charged fermion sector of the model

\[
U(1)_B × U(1)_{LEW} × U(1)_B × U(1)_L ,
\]

(6)

(for detailed analysis of the global symmetries see [3]) the sterile sector provides another global symmetry of the classical Lagrangian (5): a large sterility symmetry \(G_S\), with both Abelian and non-Abelian components. It is not ordered by anyone and comes out accidentally.

The electroweak lepton number, \(L_{EW}\), is defined by its current

\[
J^\mu_{LEW} = \bar{\nu}_L \gamma^\mu \nu_L + \bar{e}_L \gamma^\mu e_L + \bar{\nu}_R \gamma^\mu \nu_R .
\]

(7)

As well as an Abelian part of the sterility symmetry (see below), it is broken heavily by the flavor instanton effects due to the flavor anomaly

\[
\partial_\mu J^\mu_{LEW} = \frac{\alpha^2}{32\pi^2} F_{\alpha\beta} F^{\alpha\beta} .
\]

(8)

(We neglect the electroweak anomaly.) Nevertheless one can always find some linear combinations of the electroweak lepton number and the sterility symmetry which are flavor anomaly free. One of them plays a role of the conserved lepton number \(L\). The setting of the right-handed neutrinos defines manifestly chiral model.

In this section we define the non-minimal and preferred versions of the flavor gauge model with the triplet right-handed electron and with four right-handed neutrino anti-triplet and one right-handed neutrino sextet by writing the Lagrangian of their neutrino sector. Next we identify its sterility symmetry \(G_S\).

III. NEUTRINO LAGRANGIAN AND ITS SYMMETRIES

In this section we define the non-minimal and preferred versions of the flavor gauge model with the triplet right-handed electron and with four right-handed neutrino anti-triplet and one right-handed neutrino sextet by writing the Lagrangian of their neutrino sector. Next we identify its sterility symmetry \(G_S\).

A. Lagrangian of neutrino sector

The Lagrangian describing the neutrino flavor gauge dynamics is given by

\[
\mathcal{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_\nu ,
\]

(4)

\[
\mathcal{L}_\nu = & \text{ Tr } \gamma^\mu \left( i \partial_\mu - h C_{\mu}^a T^a \right) \nu_L \\
& + \sum_r T_{\nu R}^a \gamma^\mu \left( i \partial_\mu + h C_{\mu}^a T^a + \right) \nu_{R} ,
\]

(5)

where the field strength tensor of flavor gauge bosons \(C^a\) is given by \(F_{\mu\nu} = \partial_\mu C^a_\nu - \partial_\nu C^a_\mu + h f^{abc} C^b_\mu C_c^\nu\). \(T^a_\nu\) are SU(3)_F generators for a representation \(r\) of the right-handed neutrino multiplet.\(^3\) The sum runs over one sextet with \(T^a_6\) and four anti-triplets with \(T^a_3 = -[T^a_3]^* = -\frac{1}{2} \lambda_3^a\).

This non-minimal version is chiral, i.e., it does not allow the Majorana mass term, \(-\frac{1}{2} M_{\nu R}^2 \nu R\), relevant for the non-chiral models.

B. Global symmetries of neutrino sector

Additionally to the global symmetries of the weakly charged fermion sector of the model

\[
U(1)_B × U(1)_{LEW} × U(1)_B × U(1)_L ,
\]

(6)

(We neglect the electroweak anomaly.) Nevertheless one can always find some linear combinations of the electroweak lepton number and the sterility symmetry which are flavor anomaly free. One of them plays a role of the conserved lepton number \(L\).

The setting of the right-handed neutrinos defines manifestly chiral model. The chirality provides quite large accidental sterility symmetry \(G_S\) of the right-handed neutrino sector. The sterility symmetries are

\[
G_S = U(1)_S \times U(1)_S × SU(4)_S .
\]

(9)

The corresponding Noether currents are

\[
J^\mu_{S_0} = \text{ Tr } \xi_R^\mu \nu_{R} \xi_R^* ;
\]

(10a)

\[
J^\mu_{S_3} = \frac{1}{4} C_R^\mu \xi_R^* ;
\]

(10b)

\[
J^\mu_{S_4} = S_R^\mu \left[ S_L^\nu \gamma^\nu \gamma^\mu \xi_R^* \right] ;
\]

(10c)

\(^3\) If the index \(r\) is not used we mean the generators for the fundamental triplet representation, given by the Gell-Mann matrices \(T^a = \frac{1}{2} \lambda^a\).

\(^4\) \(L_{EW}\) denotes the lepton number counting the electroweakly charged leptons, \(e, \nu_L\), and not the right-handed neutrinos \(\nu_R\).
neutrino multipin n in the Nambu–Gorkov formalism

\[
\begin{pmatrix}
\nu_L + (\nu_L)^c \\
\nu_R^3 + (\nu_R^3)^c \\
\nu_R^4 + (\nu_R^4)^c \\
\nu_R^5 + (\nu_R^5)^c \\
\nu_R^6 + (\nu_R^6)^c
\end{pmatrix}
\]

(14)

where the flavor indices are suppressed.

The Lagrangian (5) is then rewritten as

\[
\mathcal{L}_\nu = \frac{1}{2} \bar{\nu}_a \gamma^\mu (i \partial_\mu + \hbar C_a^\mu \tau^a) \nu_a,
\]

(15)

where the flavor generators \( \tau^a \) in multi-component space are given in (A3).

The chiral invariance underlying the gauge dynamics forbids to write the neutrino mass term directly into the Lagrangian. The neutrino masses arise as poles of the full propagator \( S(p) \equiv [\hat{p} - \Sigma(p^2)]^{-1} \), thus as solutions of the equation

\[
\det (p^2 - \Sigma(p^2)\Sigma'^\dagger(p^2)) = 0.
\]

(16)

The neutrino self-energy \( \Sigma(p^2) \) is given as

\[
\Sigma(p^2) = \Sigma(p^2)P_L + \Sigma^\dagger(p^2)P_R,
\]

(17)

where the symmetric \( 21 \times 21 \) matrix \( \Sigma(p^2) \) can be written block-wise as

\[
\Sigma = \begin{pmatrix}
\Sigma_L & \Sigma_D \\
\Sigma_D^\dagger & \Sigma_R
\end{pmatrix}
\]

(18)

or in more detail

\[
\Sigma = \begin{pmatrix}
L^{3\times 3} & P_n^{3\times 3} & P_n^{6\times 3} \\
P_n^{3\times 3} & P_n^{3\times 3} & P_n^{6\times 3} \\
P_n^{6\times 3} & P_n^{6\times 3} & R^{6\times 6}
\end{pmatrix}
\]

(19)

By definition the self-energy matrix is symmetrical: the diagonal blocks, \( L^{3\times 3}, R^{3\times 3} \) and \( R^{6\times 6} \) are symmetrical matrices, and \( D_n^{3\times 3} = [D_n^{3\times 3}]^\dagger \), \( D_3^{3\times 6} = [D_3^{6\times 3}]^\dagger \) and \( R_6^{6\times 6} = [R_6^{6\times 6}]^\dagger \).

In the approximation of the truncated Schwinger–Dyson equation with the wave function renormalization omitted the self-energy is subject of the equation\(^7\)

\[
\Sigma(p^2) = i \int_k \bar{h}_\nu^a(\mathbf{k} + \mathbf{p}) \nu^a \Sigma(k^2) \left[ k^2 - \Sigma(k^2) \Sigma(k^2) \right]^{-1} \nu^b,
\]

(20)

---

5 We ignore here the flavor anomalies of the charged fermion currents corresponding to (6). Their flavor anomalies would otherwise provide some charged fermion component of the heavy sterile majoron, see later. Ignoring their flavor anomalies allows us to treat the heavy sterile majoron as a neutrino and flavor gauge boson composite only.

6 Here, we neglect the wave function renormalization.

7 We use the short-hand notation for integration \( \int_k \equiv \int \frac{d^4 k}{(2\pi)^4} \).
where for the flavor effective coupling we accept the heuristic Ansatz
\[
\bar{h}_{ab}(q) \gtrsim \frac{h^2}{q^2} \Pi_{ac}(q) [1 + \Pi(q)]_{cb}^{-1} \gtrsim -\frac{h^2 M_{ac}^2}{q^2} [q^2 - M_{1b}^2]_{cb}^{-1},
\]
where \( h \) is a non-perturbative infrared fixed point of the flavor gauge dynamics, \( \Pi_{ab}(q) \) is the flavor gauge boson self-energy, and \( M_{ab}^2 \) is the flavor gauge boson mass matrix. (The rationale of the Ansatz is given in [3].)

\section*{B. Flavor symmetry breaking}

The flavor symmetry breaking and the fermion mass generation via formation of the chirality changing self-energies are induced by the strong flavor dynamics. Therefore it is essentially non-perturbative phenomenon, hard to control. This fact is condensed in the Schwinger–Dyson equation (20) and in our impotence to solve it.

At least some qualitative understanding can be gained if we treat the self-energies \( \Sigma \), the flavor symmetry breaking order parameters, as condensates formed by the pairing of the flavored fermion chiral components.

In a regime of very high energies (> \( \Lambda_F \)) the system is fully symmetric, the flavor gauge bosons are massless and the power of attraction, mediated by the massless flavor gauge bosons, can be estimated by the Most Attractive Channel (MAC) method [28].

The attractiveness of different pairing channels
\[
r_1 \times r_2 \rightarrow r_{\text{pair}}
\]
is roughly measured by the quantity
\[
\Delta C_2 = C_2(r_1) + C_2(r_2) - C_2(r_{\text{pair}}),
\]
where \( C_2(r) \) is the quadratic Casimir invariant for the representation \( r \), see Tab. III in the appendix B.

Decreasing the energy scale, the attractiveness of different pairing channels increases differently. Once the most attractive channel produces the flavor symmetry breaking at the energy scale \( \Lambda_F \), the MAC method loses its plausibility for the remaining pairing channels since the flavor gauge bosons become massive.

\subsection*{1. drawbacks of the minimal version}

The minimal version analyzed in [3], where all fields are in triplets or anti-triplets, is approximately vector-like above the huge scale \( \Lambda_F \) because there we can neglect QCD and electroweak effects. The most attractive channel is \( 3 \times \overline{3} \rightarrow 1 \) with \( \Delta C_2 = 8/3 \). It causes several shortcomings of the minimal version:

1) The most attractive channel is a flavor singlet, i.e., it does not break the flavor symmetry. It suggests that the flavor gauge dynamics should rather confine below \( \Lambda_F \).

2) Even if we assume that the QCD and electroweak dynamics are sufficiently relevant at \( \Lambda_F \) to cure previous shortcoming by inducing the necessary non-vector-like nature, it still remains difficult to justify tininess of neutrino masses, simply, because there is no natural reason for the see-saw pattern of neutrino mass matrix.

3) If at all, the breaking of the electroweak and the flavor symmetry happens at once. The separation of the flavor scale \( \Lambda_F \) and the electroweak symmetry breaking scale \( \Lambda_{\text{EW}} \) is not obvious. Necessary relation \( \Lambda_F \gg |\Sigma_n| \) has to be achieved by critical scaling [29, 30].

\subsection*{2. Advantages of the non-minimal version}

The non-minimal version (63333) naturally and straightforwardly leads to the complete flavor symmetry breaking and cures the first two weak points immediately. On top of that it provides the separation of flavor and electroweak symmetry breaking. Requirement of the critical scaling, however, remains unavoidable.

The attractive channels (A.C.), governing different parts of the neutrino self-energy written in the Nambu–Gorkov formalism, are (compare with (19))
\[
(A.C.) = \left( \begin{array}{ccc}
3 \times 3 & 3 \times \overline{3} & 6 \times 3 \\
3 \times \overline{3} & 3 \times \overline{3} & 6 \times \overline{3} \\
3 \times 6 & 3 \times 6 & 6 \times \overline{6}
\end{array} \right).
\]

(24)

The measure (23) of the attractiveness of the channels is
\[
(\Delta C_2) = \left( \begin{array}{ccc}
4/3 & 8/3 & 5/3 \\
8/3 & 4/3 & 10/3 \\
5/3 & 10/3 & 10/3
\end{array} \right).
\]

(25)

It naturally follows that, decreasing the energy scale, the right-handed neutrino pairing of Majorana type with \( \Delta C_2 = 10/3 \) happens first. This fact brings nice features:

1) It breaks the flavor symmetry providing no confinement.

2) It suggests the see-saw pattern of neutrino mass matrix.

3) It does not break the electroweak symmetry what is postponed to lower energies.

\subsection*{3. Effective description of the flavor symmetry breaking}

We can quantify the anti-sextet and the four triplet pairings by, so called, sterility condensates
\[
\langle 0 | \frac{1}{4} \epsilon_{ACE} \epsilon_{BDF} (\frac{\partial^2 \Pi}{\partial q^2})^{c} \delta_{cR}^{EF} | 0 \rangle \propto \Lambda_F^2 \langle 0 | \Phi_{6}^{AB} | 0 \rangle, \quad (26a)
\]
\[
\langle 0 | (\frac{\partial \Pi}{\partial q^2})^{c} \delta_{cR}^{n} | 0 \rangle \propto \Lambda_F^2 \langle 0 | \Phi_{3}^{n-1} | 0 \rangle. \quad (26b)
\]
where we have introduced auxiliary scalar fields $\Phi_6$ and $\Phi_7^n$ of mass dimension one. The index $n = 1, \ldots, 4$ is the SU(4)$_S$ sterility index. The indices, $A, B, C, \ldots = 1, \ldots, 3$, are the indices of the fundamental flavor representation, and $\epsilon^{ABC}$ is the totally anti-symmetric tensor. The auxiliary fields transform as an anti-sextet and a triplet, respectively, under the flavor rotations $U = e^{i\alpha^a T^a}$

$$
\begin{align*}
\Phi_6' &= U^{(T)} \Phi_6 U^\dagger, \\
\Phi_3^n' &= U^{(n)} \Phi_3 .
\end{align*}
$$

(27a, 27b)

These flavor transformation properties follow from the flavor transformation properties of the elementary right-handed neutrino fields (for their definitions see (11))

$$
\begin{align*}
\xi_0' &= U \xi_R U^{(T)}, \\
\eta_R^n' &= U^{(n R)} \eta_R .
\end{align*}
$$

(28a, 28b)

and the fact that the totally anti-symmetric tensor $\epsilon^{ABC}$ is flavor invariant

$$
U^{(A D)} U^{(B E)} U^{(C F)} e^{(D E F)} = e^{A B C} .
$$

(29)

The quantum numbers $(L, S_3 - S_6, S_3 + S_6, SU(4)_S)$ of the scalar fields are

$$
\begin{align*}
\Phi_6 &: = (2 - 2a, -2a, +2a, 1) , \\
\Phi_3^n &: = \left( \begin{array}{c}
1 - 3a \\
\frac{3}{2a} - \frac{3}{4a} + \frac{5}{4a} \\
\end{array} \right) .
\end{align*}
$$

(30a, 30b)

$\Phi_6$ and $\Phi_3^n$ are 18 complex scalar fields. They can be expressed in terms of twice as many real scalar fields from which several are the Nambu–Goldstone fields of broken flavor and sterility symmetries.

$$
\begin{align*}
\Phi_6(x) &= e^{-2i\alpha(x)} e^{2i\beta(x)} x \\
&\times e^{-i\phi(x) T^a} \begin{pmatrix}
\Delta_1(x) & 0 & 0 \\
0 & \Delta_2(x) & 0 \\
0 & 0 & \Delta_3(x)
\end{pmatrix} e^{i\phi(x) T^a} , \\
\Phi_3^n(x) &= e^{-2i\alpha(x)} e^{i\beta(x)} e^{i\gamma(x) s^i} \times
\begin{pmatrix}
\Phi_3^1(x)^T \\
\Phi_3^2(x)^T \\
\Phi_3^3(x)^T
\end{pmatrix}
\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & \delta_4(x) \\
0 & \delta_3(x) & \delta_1(x)
\end{pmatrix} \\
&\times e^{i\phi(x) T^a} .
\end{align*}
$$

(31, 32)

The 25 Nambu–Goldstone bosons are (for majorons see section V):

- 15 of $\gamma^i(x)$ corresponding to broken SU(4)$_S$: non-Abelian light majorons $\eta^i$
- 1 of $\alpha(x)$ corresponding to broken $S_3 - S_6$: Abelian light majoron $\eta^0$
- 1 of $\beta(x)$ corresponding to broken $S_3 + S_6$: super-heavy majoron $H$

Further there are 7 real and 2 complex scalars and in general they all can develop 9 CP-preservation vacuum expectation values $\phi_A$ and $\varphi_k$ and 2 CP-violating phases $\zeta_k$.

- 3 real components of sextet field $\Delta_A(x) \rightarrow \Delta_A(x) + \phi_A, A = 1, 2, 3$
- 4 real anti-triplet fields $\delta_k(x) \rightarrow \delta_k(x) + \varphi_k, k = 1, 2, 3, 4$
- 2 complex anti-triplet fields $\varepsilon_k(x) \rightarrow \varepsilon_k(x) + \varphi_k e^{i\zeta_k}, k = 5, 6$

A general form of the condensate $\langle 0 | \Phi_6 | 0 \rangle$ is

$$
\langle 0 | \Phi_6 | 0 \rangle = \begin{pmatrix}
\phi_1 & 0 & 0 \\
0 & \phi_2 & 0 \\
0 & 0 & \phi_3
\end{pmatrix}
$$

(33)

as follows from (31). A general form of the condensates $\langle 0 | \Phi_3^n | 0 \rangle$ follows from (32). In general they are complex and have nontrivial mutual angle and also non-trivial angle with respect to the $\langle 0 | \Phi_6 | 0 \rangle$.

Not only for the sake of concreteness we choose here a special form of the triplet condensates

$$
\begin{align*}
\langle 0 | \Phi_3^{1=1,2,3} | 0 \rangle &= 0 , \\
\langle 0 | \Phi_3^{2=4} | 0 \rangle &= \begin{pmatrix}
\varphi_4 & \varphi_5 & \varphi_6
\end{pmatrix} .
\end{align*}
$$

(34a, 34b)

The main reason for this choice is that it leaves the SU(3)$_S$ sterility subgroup unbroken what is necessary to protect the seesaw mechanism (see Sect. IV C). Without the special form the general condensates would break the sterility symmetry completely.

The condensates break the flavor symmetry completely while the electroweak symmetry breaking is postponed to the lower energies where the pairing of the electroweakly charged fermions occurs. The sextet sterility condensation is very similar to the sextet color superconductivity [31].

4. Masses from the sterility condensation

The sterility condensation produces masses of all flavor gauge bosons. The masses can be estimated from the lowest order gauge invariant kinetic terms of the effective Lagrangian for the effective scalar fields

$$
\mathcal{L}_{\text{M}_\text{gauge}} = (D^\mu \Phi_3^\dagger) D_\mu \Phi_3 + \text{Tr}(D^\mu \Phi_6) D_\mu \Phi_6 ,
$$

(35)
where
\[
D_\mu \Phi_6 = \partial_\mu \Phi_6 + i h C^\alpha_\mu (T^a \Phi_6 + \Phi_6 T^a), \quad (36a)
\]
\[
D_\mu \Phi^a_3 = (\partial_\mu - i h C^\mu_\alpha T^a) \Phi^a_3. \quad (36b)
\]

In the effective Lagrangian $\mathcal{L}_{M_{\text{gauge}}}$ (35) we substitute the effective scalar fields for their vacuum expectation value $\Phi \rightarrow \langle 0 | \Phi | 0 \rangle$, and we get the mass matrix for the gauge bosons
\[
M^2_{\text{gauge}} = M^2_6 + M^2_3, \quad (37)
\]
where the mass matrices $M^2_6$ and $M^2_3$ with the specific form of the condensates (33) and (34) are in the Appendix (C1).

The sterility condensation produces also Majorana masses for right-handed neutrinos. The masses can be estimated from Yukawa terms of the effective Lagrangian for the effective scalar fields
\[
\mathcal{L}_{M_R} = y_{36} \langle \Phi_6^\dagger \rangle_R \xi_R \Phi_3^R + y_6 e^{ACE} e^{BD\ell} (c_R^{AB})^2 c_R^{CD} (\Phi_6^C)^\dagger + \text{h.c.}, \quad (38)
\]
where the effective Yukawa coupling constants
\[
y_{36} = \frac{4}{9} h^2, \quad y_6 = \frac{4}{9} h^2
\]
are obtained from the effective four-neutrino interaction $\sim (\bar{\nu}_i \gamma^\mu \nu_i)(\bar{\nu}_j \gamma^\mu \nu_j)$.

In the effective Lagrangian $\mathcal{L}_{M_R}$ (38) we can substitute the scalars for the condensates and get the Majorana mass matrix for the sterile neutrinos\(^8\)
\[
M_R = \frac{4}{9} h^2 \begin{pmatrix} 0 & \langle \Phi_3^R \rangle_R^\dagger \\ \langle \Phi_3^R \rangle_R & \langle \Phi_6 \rangle \end{pmatrix}. \quad (39)
\]
The mass matrix $M_R$ has generically at least six zero eigenvalues. With the special choice of sterility condensates (33) and (34), there are nine zero eigenvalues.

C. Neutrino phenomenology

The neutrino masses are given as roots of the equation (16) where the momentum dependence of $\Sigma(p^2)$ makes the calculation difficult. For qualitative purpose it is sufficient to substitute the self-energy by a constant $N \times N$ symmetric mass matrix $M$, where $N = 21$ for our case.

The mass spectrum can be found as eigenvalues $m_1, \ldots, m_N$ of $M^9$
\[
\bar{\nu} \nu = \bar{\nu} U^\dagger \begin{pmatrix} m_1 & \cdots & m_N \end{pmatrix} U \nu \quad (40a)
\]
\[
e \equiv (\nu_1')^\dagger \cdots (\nu_N')^\dagger \begin{pmatrix} 0 & m & \cdots \\ m & \ddots & \cdots \\ \vdots & \ddots & \ddots \end{pmatrix} \begin{pmatrix} \nu_1' \\ \vdots \\ \nu_N' \end{pmatrix}, \quad (40b)
\]
where $U$ is the diagonalizing unitary transformation matrix.

Three types of mass eigenstates can arise: (i) In the most general scenario, when no selection rule is in work, all eigenvalues come out nonzero and different. In our case, they correspond to 21 massive Majorana neutrinos. (ii) The zero eigenvalues correspond to massless Weyl neutrinos. (iii) It can happen that pairs of degenerate eigenvalues appear (see e.g. in (40b)). Each pair then corresponds to a massive Dirac neutrino with its chiral components given as, e.g.,
\[
\nu_L = \nu_2 + i \nu_3', \quad (41a)
\]
\[
\nu_R = (\nu_2')^c - i (\nu_3')^c. \quad (41b)
\]
The presence of the pair degeneracy signals the $O(2) \sim U(1)$ symmetry of the mass matrix, a subgroup of either flavor, or sterility, or both. The symmetry corresponds to a quantum number carried by the Dirac neutrino.

In the minimal version (333), the left- and right-handed Majorana, and Dirac entries of the neutrino mass matrix arise from the pairing channels of the same flavor structure $3 \times 3$ or $\overline{3} \times \overline{3}$, thus of the same strength of attraction. That does not indicate the see-saw pattern of the neutrino mass matrix at all. The reason for the tiny masses of electroweak neutrinos has to be fully left on a huge amplification effects [3]. It is then natural to expect that the remaining nine sterile neutrinos turn out to be of small mass as well. In the same time, the dynamics should be also responsible for sufficient suppression of the right-handed admixtures within the electroweak neutrinos, for what it is difficult to find some natural reason. On contrary, the non-minimal version (63333) naturally leads to the dynamically generated see-saw pattern of the neutrino mass matrix. The see-saw pattern is useful not only for explanation of tiny masses of the electroweak neutrinos, but also for suppression of their oscillations into the sterile neutrinos.

The key point is the presence of the sextet right-handed neutrinos. Their privileged role makes the situation clearer, separates the study of the right-handed

\(^8\) Here the condensates should be rewritten in the $\nu_R$-formalism (14), not in the matrix $\xi_R$-formalism.

\(^9\) If the mass matrix $M$ is complex then we have to find eigenvalues of $M^\dagger M$ to determine the mass spectrum.
neutrinos from other fermions, and allows us to switch into the approximative description by condensates.

Within the (63333) version we have demonstrated so far the massiveness only of the right-handed neutrinos. But of course we expect that ultimately at lower energy scale all elements of the full neutrino mass matrix given by (19) become non-vanishing and all mass eigenstates become massive. The sole fact that there is an odd number of neutrino degrees of freedom indicates that at least one neutrino must be of Majorana type.

By the construction above we want to show that the right-handed Majorana elements dominate the whole neutrino mass matrix. This is exactly what is needed for the see-saw mechanism to work. Due to the strength of the sextet neutrino condensation the see-saw pattern occurs dynamically and naturally.

Nevertheless, the system with the general sterility condensation scheme is not directly able to accommodate all three light electroweak neutrinos. The see-saw mechanism is triggered by switching on the Dirac elements occuring dynamically and naturally.

Of the sextet neutrino condensation the see-saw pattern for the see-saw mechanism to work. Due to the strength of the sextet neutrino condensation the see-saw pattern occurs dynamically and naturally.

The necessary residual sterility symmetry can be achieved by imposing the special form (34) of the triplet condensation which is equivalent to dynamically natural relations that $\langle 0|\Phi_i^n|0\rangle = 0$ or generally $R^{\mathbf{3}\times6}_n = R^{\mathbf{3}\times6}_n$, and $D^{\mathbf{3}\times3}_n = D^{\mathbf{3}\times3}_n$, see (19). The seesaw-mechanism-protecting residual sterility symmetry is then $SU(3)_S \subset SU(4)_S$ generated by $S_i$, with $i = 1, \ldots, 8$.

V. MAJORONS

We leave the line of the previous section where, by phenomenological preferences, we have been lead to the particular pattern of the neutrino self-energy. We assume general pattern providing maximal chiral symmetry breaking occurring at one energy scale $\Lambda_F$. The sterility symmetry together with the lepton number is broken spontaneously breaking the lepton number and the sterility symmetry $G_S$, see section III B.

The majorons corresponding to the anomaly free part of the sterility symmetry are called the light sterile majoron $H$. The majorons corresponding to the anomaly free part of the sterility symmetry are called the light sterile majorons $\eta_i$ in QCD. This majoron is called the heavy sterile majoron $H$.

The spontaneously broken anomaly free lepton number $L$ gives rise to the standard majoron $J$ [20, 21]. It is always present in all versions of the model.

The majorons $\eta$ and $J$ are the true Nambu–Goldstone bosons. They nevertheless do not present a phenomenological problem in the form of new long range force. The argument is simple: The Nambu–Goldstone bosons mediate spin-dependent tensor force among fermions which vanishes with cube of distance [22].

What more, the majorons can eventually acquire mass by gravitational effects of the order of, say, few keV [33–35]. That would of course drastically shorten the force range. In the formulation of the issue we omit these effects and treat the majorons $\eta$ and $J$ as massless. During the phenomenological analysis, nevertheless, we keep this possibility open and call them collectively as light majorons.
A. Light majorons

All versions of the model predict the existence of the standard majoron $J$ from the spontaneously broken lepton number $L$ (13). The standard majoron couples to the lepton number current

$$ (0, \mathcal{J}_L^\mu(0)|J(q)) = i q^\mu F_J $$

(43)

with the strength of the standard majoron decay constant $F_J$. The anomaly free lepton number is broken by the formation of all Dirac, $\Sigma_D$, left-handed Majorana, $\Sigma_L$, and right-handed Majorana, $\Sigma_R$, components of the neutrino self-energy. Therefore the standard majoron is created from vacuum by a linear combination of interpolating operators

$$ J \sim \langle \overline{\nu_R} \nu_L + \overline{\nu_L} \nu_R \rangle . $$

(44)

In the version (63333) sixteen light sterile majorons $\eta_0$ and $\eta_i$, $i = 1, \ldots, 15$, as the Nambu–Goldstone bosons couple to the sterility currents

$$ (0, \mathcal{J}_{\eta_0}^\mu(0)|\eta_0(q)) = i q^\mu F_\eta ; $$

$$ (0, \mathcal{J}_{\eta_i}^\mu(0)|\eta_i(q)) = i q^\mu F_\eta \delta_{ij} $$

(45a)

(45b)

with the strength of the sterile majoron decay constant $F_\eta$. The sterility symmetry is spontaneously broken by the formation of both Dirac, $\Sigma_D$, and right Majorana, $\Sigma_R$, components of the neutrino self-energy. Therefore the sterile majorons are created from vacuum by a linear combination of interpolating operators

$$ \eta_i \sim \langle \overline{\nu_R} S_i \nu_L + \overline{\nu_L} S_i \nu_R \rangle , \quad \overline{\nu_R} S_i \nu_R ; $$

$$ \eta_0 \sim \langle \overline{\nu_R} \nu_L + \overline{\nu_L} \nu_R \rangle . $$

(46a)

(46b)

In the following we will use the common notation for the generators relevant for the light majorons, $\eta_0, \alpha = 0, 1, \ldots, 16$. It denotes the vector of the lepton number and sterility generators

$$ S_0 - S_6 : \quad X_0 = s_0 ; $$

$$ \text{SU}(4)_S : \quad X_i = s_i, \text{ where } i = 1, \ldots, 15 ; $$

$$ L : \quad X_{16} = l . $$

(47a)

(47b)

(47c)

where the lepton number generator in the Nambu–Gorkov formalism $l$ is introduced in (A4), and the sterility symmetry generators in the Nambu–Gorkov formalism $s_0$ are introduced in (A7).

Not counting their mutual interactions, the majorons interact mainly with neutrinos. Such interactions can be described generally by effective Yukawa majoron-neutrino term

$$ L_{\text{eff},\nu \nu} = y_{\nu \nu} \langle \bar{\nu} X_0 \eta_0 \rangle + y'_{\nu \nu} \langle \bar{\nu} X_i \eta_i \rangle _\alpha ; $$

$$ L_{\text{eff},J \nu} = y_{J \nu} \langle \bar{\nu} X_{16} \eta_0 \rangle J . $$

(48a)

(48b)

At that level the effective Yukawa coupling constants $y_{\nu \nu}$, $y'_{\nu \nu}$ and $y_{J \nu}$ are mere parameters. Nevertheless the majoron-neutrino coupling strength can be related to more fundamental quantities of the model, like to the flavor symmetry breaking neutrino self-energy $\Sigma(p)$.

For that purpose we follow standard procedure [36, 37] to insist on the fulfillment of the Ward–Takahashi identity for a proper vertex $\Gamma^\mu_\alpha(p + q, p)$ corresponding to the Green functions $G^\mu_\alpha(x, y, z) \equiv \langle 0| T \mathcal{J}^\mu_\alpha(x) \eta(y) \eta(z) |0 \rangle$, $G^\mu_\alpha(x, y, z) \equiv \langle 0| T \mathcal{J}^\mu_\alpha(x) \eta(y) \eta(z) |0 \rangle$ and $G^\mu_{\eta \eta}(x, y, z) \equiv \langle 0| T \mathcal{J}^\mu_\eta(x) \eta(y) \eta(z) |0 \rangle$ of the lepton number and sterility currents coupled to the neutrino fields. The Ward–Takahashi identity reads

$$ q_0 \Gamma^\mu_\alpha(p + q, p) = S^{-1}(p + q) X_\alpha - \gamma_0 X_\alpha \gamma_0 S^{-1}(p) . $$

(49)

When the dynamics develops symmetry breaking neutrino self-energy, the Ward–Takahashi identity (49) does not vanish for $q \to 0$. It determines uniquely only the leading $\mathcal{O}(q^{-1})$ part $\Gamma^\mu_\alpha_{\alpha,\text{lead}}$ of the proper vertex $\Gamma^\mu_\alpha$

$$ \Gamma^\mu_\alpha(p + q, p) = \Gamma^\mu_{\alpha,\text{lead}}(p + q, p) + \mathcal{O}(q^0) , $$

(50)

where

$$ \Gamma^\mu_{\alpha,\text{lead}}(p + q, p) = -\frac{q^\mu}{q^2} (\Sigma(p) X_\alpha - \gamma_0 X_\alpha \gamma_0 \Sigma(p)) . $$

(51)

Physically, we interpret the pole in terms of the exchange of the massless Nambu–Goldstone boson, i.e., majoron. Following this interpretation we pick up the Nambu–Goldstone part of the proper vertex

$$ \Gamma^\mu_{\alpha,\text{NG}}(p + q, p) \approx \frac{q^\mu}{q^2} I_{\alpha,\beta}(q^2) P_{\gamma}(p + q, p) , $$

(52)

and approximate it by a ‘one-loop’ expression

$$ \Gamma^\mu_{\alpha,\text{NG}}(p + q, p) \approx \frac{q^\mu}{q^2} I_{\alpha,\beta}(q^2) P_{\gamma}(p + q, p) , $$

(53)

where the massless majoron propagator $\delta_{\alpha,\beta}$ connects the neutrino loop $q^\mu I_{\alpha,\beta}(q^2)$ and the majoron-neutrino vertex $P_{\gamma}(p + q, p)$ both regular for $q = 0$.

Comparing the two expressions (50) and (52) for $q \to 0$ we get the expression for the majoron-neutrino vertex for $q = 0$

$$ P_{\alpha}(p, p) = I^{-1}_{\alpha,\beta}(0) \left[ \Sigma(p) X_\beta - \gamma_0 X_\beta \gamma_0 \Sigma(p) \right] . $$

(54)

The loop function $I_{\alpha,\beta}(0)$ from the diagram (53) is

$$ I_{\alpha,\beta}(0) = i \lim_{q \to 0} \int_k \text{Tr} \left( \frac{q}{q^2} X_\alpha S(k - q) P_{\beta}(k - q, k) S(k) \right) . $$

(55)

For the sake of simplicity we write explicit formula for the loop function $I_{\alpha,\beta}(0)$ only within the approximation of
constant self-energies, $\Sigma(p) \to M = \Sigma(0)$. The approximated $I_{\alpha\beta}(0)$ then if plugged into (54) gives us an upper estimate of magnitude of the majoron-neutrino vertex $\tilde{P}_\alpha(p, p)$.

Due to the limit in (55), we need to expand the $q$-dependent quantities up to $O(q^2)$ order. The expansion of the neutrino propagator is

$$S(k - q) = \tilde{S}(k) + \tilde{S}(k)q \tilde{S}(k) + O(q^2)$$  \hspace{1cm} (56)

and we assume that the expansion for the majoron-neutrino vertex is

$$\tilde{P}_\beta(k - q, k) = \tilde{P}_\beta(k, k) + O(q^2),$$  \hspace{1cm} (57)

where the tilde means the constant self-energy approximation of the quantity.

The loop function $I_{\alpha\beta}(0)$ necessary for the majoron-neutrino vertex (54) is given by relation

$$[I(0)I^T(0)]_{\alpha\beta} = \lim_{q \to 0} \int \Tr(q^2X_\alpha\tilde{S}(k)q\tilde{S}(k)) \times$$  \hspace{1cm} (58)

$$\times [MX_\beta - \gamma_0X_\beta\gamma_0M] \tilde{S}(k) .$$

B. Heavy sterile majoron

The sterile majoron $H$ couples to the anomalous current of Abelian sterility symmetry $U(1)_{S_3 + S_a}$

$$\langle 0|J^\mu_{S_3+S_a}(0)|H(q)\rangle = iq^\mu F_S .$$  \hspace{1cm} (59)

The heavy sterile majoron is created from vacuum by neutrino interpolating fields (46b), and additionally, by flavor gauge boson component which is a topologically nontrivial field configuration

$$H \sim \tilde{F}^{\mu\nu}_F .$$  \hspace{1cm} (60)

The majoron $H$ acquires huge mass due to the strong flavor axial anomaly (12). The value of its mass can be estimated according to the $\eta'$ mass analysis in QCD [38–40] as

$$m_H^2 = \frac{\chi(0)}{F^2} \sim \Lambda_F^2 ,$$  \hspace{1cm} (61)

where the flavor topological susceptibility is estimated as $\chi(0) \sim \Lambda_F^4$, and the decay constant as $F_S \sim \Lambda_F$.

The anomalous coupling of $H$ to the flavor gauge bosons is given as

$$\mathcal{L}_{HCC} = \frac{h^2}{32\pi^2} \frac{H}{F_S} \tilde{F}^{\mu\nu}_F .$$  \hspace{1cm} (62)

The effective coupling of the heavy majoron with the neutrinos is

$$\mathcal{L}_{H\nu \nu} \sim \frac{m_n}{F_S} H\bar{\nu}\gamma_5\nu .$$  \hspace{1cm} (63)

Because the interaction is proportional to the neutrino mass, the only significant interaction is with the heavy sterile neutrinos, the heavy sterile majoron is fairly invisible.

C. Majoron phenomenology

1. Light majorons:

Suppose that light majorons have mass of the order of several keV due to the gravitational effects. Then they are suitable candidates for warm dark matter [41]. Important characteristic is their decay width. They can decay only to $N_{\text{light}}$ sufficiently light neutrinos with mass $m_{\text{light}} < M_f/2$, i.e., at least to the three electroweak neutrinos.

In the following we omit the differences among the light majorons and estimate the decay width only for standard majoron $J$ of mass $M_f$. The matrix element $\mathcal{M}$ for the decay is simply governed by the effective majoron-neutrino interaction (48):

$$i\mathcal{M} = \int y_{J\nu\nu} \, .$$  \hspace{1cm} (64)

The decay width $\Gamma$ is given by

$$\Gamma(J \to nn) = \frac{N_{\text{light}}}{8\pi} y^2_{J\nu\nu} M_J \left(1 - \frac{4m^2_{\text{light}}}{M_J^2}\right)^{3/2} ,$$  \hspace{1cm} (65)

where the effective Yukawa coupling is given by the majoron-neutrino vertex $P(p, p)$

$$y_{J\nu\nu} \sim P(p, p) \approx \frac{m_{\text{light}}}{\sum \nu m_{\nu}} .$$  \hspace{1cm} (66)

We have estimated the neutrino loop $I(0)$ (55) by a sum of all neutrino mass eigenvalues $m_\nu$, $I(0) \approx \sum \nu m_\nu$.

Now, in the version (333), we expected that masses of all neutrino eigenstates turn out to be of the same order, thus of the order of the electroweak neutrino mass. That is why we can estimate the effective Yukawa coupling as $y_{J\nu\nu}^{(333)} \approx 10^{-1}$ and neglect ratio $m_{\text{light}} / M_J$. For the decay width we get an estimate

$$\Gamma^{(333)}(J \to nn) \approx 10^{-3} M_f .$$  \hspace{1cm} (67)

On the other hand, in the version (63333) where the see-saw mechanism is in work, we could expect that only the $N_{\text{light}} = 3$ electroweak neutrinos are very light, $m_{\text{light}} < M_f$. Then the decay width (65) becomes

$$\Gamma^{(63333)}(J \to nn) \sim \frac{N_{\text{light}} m^2_{\text{light}} M_f}{N^2_{\text{heavy}} \Lambda_F^2} \approx 10^{-50} M_f ,$$  \hspace{1cm} (68)

where the sum of neutrino mass eigenvalues is dominated by $N_{\text{heavy}}$ super-heavy neutrinos of mass $\sim \Lambda_F \approx 10^{14}$ GeV.

That makes a qualitative difference between the two versions of the model. While in the version (333) the light majorons are short-lived, in the version (63333) the
light majorons are practically stable. From this point of view the version (333) resembles more the triplet majoron models [42], while the version (63333) resembles the singlet majoron models [20].

2. Heavy sterile majoron:

The coupling of the heavy sterile majoron to the flavor anomaly has important consequences for the CP properties of the flavor model. There is no reason why there should not be the $\theta$-term of flavor gauge dynamics in the effective Lagrangian. The $\theta$-parameter shifted by phase that makes the neutrino masses real is eliminated by the Peccei–Quinn mechanism [43, 44], where the heavy sterile majoron plays a role of the composite axion.

The heavy sterile majoron could decay to the heavy flavor gluons due to the direct interaction (62) induced by the flavor anomaly. The decay would be kinematically allowed if the heavy sterile majoron is heavier than twice the mass of $N_C \leq 8$ lighter flavor bosons, $M_H < 2M_C$. For the sake of rough estimate of the decay width, we omit the non-Abelian character of the flavor gauge bosons and also the differences of their masses using a common mass $M_C$. The matrix element $M$ is given by the vertex (62)

\[
iM = \frac{\hbar^2}{32\pi^2F_S} \varepsilon_\mu(p)\varepsilon_\nu(k)\epsilon^{\mu\nu\alpha\beta}p_\alpha k_\beta ,
\]

where $\varepsilon_{\mu}$ is the polarization vector of the flavor gauge boson $C_\mu$. The decay width then follows

\[
\Gamma(H \to CC) = \frac{N_C\hbar^4}{64\pi(32\pi^2)^2} \frac{M_H^2}{F_S^2} \left(1 - \frac{4M_C^2}{M_H^2}\right)^{3/2} .
\]

After some order assumptions $N_C\hbar^4 \approx 100$, and $M_H \sim M_C \sim F_S \sim \Lambda_F$ we get an estimate

\[
\Gamma(H \to CC) \approx 10^{-4} \Lambda_F
\]

leading to enormously fast decay.

In the version (63333), if it is kinematically allowed, a decay to $N_{\text{heavy}}$ super-heavy neutrinos of mass $m_{\text{heavy}} \sim \Lambda_F$ (that are absent in the version (333)), gives a contribution to the heavy sterile majoron decay width of comparable size with (70). From the effective vertex (63) the decay width follows

\[
\Gamma(H \to nn) = \frac{N_{\text{heavy}} m_{\text{heavy}}^2}{8\pi} \frac{M_H^2}{F_S^2} \left(1 - \frac{4m_{\text{heavy}}^2}{M_H^2}\right)^{3/2} .
\]

and a rough estimate is

\[
\Gamma(H \to nn) \approx 10^{-4} \Lambda_F .
\]

VI. CONCLUSIONS

The intention of this paper was to investigate the sterile particle sector of the flavor gauge model [1–3] of the electroweak symmetry breaking.

The model possesses a nice feature that its consistency requires the existence of a definite number of right-handed neutrino fields. Together with the left-handed neutrinos they form Majorana mass eigenstates, what is triggered by the formation of their self-energies dynamically.

The neutrino self-energies break the global symmetries giving rise to majorons. We cannot compute any fermion mass spectrum. But if neutrinos acquire Majorana masses dynamically, majorons must exist. The existence of the standard majoron as a consequence of the spontaneous lepton number breaking is an inevitable outcome of all versions of the model. The existence of the set of light sterile majorons, and one super-heavy sterile majoron depends on whether the sterile symmetry is broken, and their particular spectrum depends on how it is broken and differs from version to version. Majorons are both left- and right-handed neutrino composites. If the standard and the light sterile majorons acquire mass from the gravitational effects, they are excellent candidates for a warm dark matter [32, 41]. The heavy sterile majoron is too unstable to account for any amount of matter of the Universe.

In any case, the heavy sterile majoron provides the Peccei–Quinn mechanism that eliminates the flavor $\theta$-term from the effective Lagrangian. The heavy sterile majoron is the composite invisible flavor axion. In this paper we have ignored anomalies of charged fermion Abelian currents in order to concentrate only on the neutrino sector as much as possible. The model in its completeness is analyzed in [8]. In the simplified case, due to the presence of flavor axion, the heavy sterile majoron, the model does not suffer from a $CP$ violation originating from the non-trivial topology of the flavor gauge dynamics. The only sources of the $CP$ violation remain to be un-transformable phases of the neutrino mixing matrices in the flavor gauge interactions. The $CP$ violating phases originate from the non-trivial neutrino self-energies $\Sigma$.

The sterile particle spectrum is the first main result of the paper. It is qualitatively common to all chiral versions of the model. The analysis is, nevertheless, based on the crucial assumption that the flavor symmetry scenario actually happens.

As the second main result of the paper we brought several heuristic but meaningful arguments why we see the non-minimal chiral version of the model with sextet right-handed neutrinos favored. To our surprise, such version has appeared to be also phenomenologically most suitable.
First of all, better understanding of the flavor symmetry self-breaking has been reached within the (33333) version. Just in the analogy with the color superconductivity, at the extremely high energy scale $\Lambda_F$ the right-handed neutrino fields form the Majorana condensates that break flavor but not electroweak symmetry. The right-handed neutrinos and flavor gauge bosons acquire extremely high masses. The presence of the sextet right-handed neutrino fields is crucial: Their pairing channels are the most attractive, therefore their condensation happens at the highest energy scale which is naturally separated from the energy scale where the rest of fermion self-energies are formed and the electroweak symmetry is broken. This lower scale is, nevertheless, connected to the scale where the QCD axion is formed, i.e., $10^9 - 10^{12}\text{GeV}$ [3]. So there is no advantage against the version (333) in explaining the smallness of the charged fermion masses. We still need the huge amplification of scales. It turns out that the right-handed neutrino Majorana self-energies must be generated at much higher scale $\Lambda_F$ than $10^{12}\text{GeV}$.

Second, the strongly coupled right-handed neutrino condensate formed at this very high energy scale is phenomenologically welcome. (i) It can generate the baryogenesis and drive the inflation of the Universe [45, 46]. (ii) It naturally provides the see-saw pattern of the neutrino mass matrix and suggests $\Lambda_F \gtrsim 10^{14}\text{GeV}$.

Third, in order to reach the presence of the three light electroweak neutrinos in the particle spectrum, we were forced to assume special (but not unnatural) form of the neutrino mass matrix. The form preserves the residual sterility symmetry that protects the see-saw mechanism. It also protects smallness of masses of a number of decoupled sterile neutrinos that can possibly account for fermionic warm dark matter [17–19].

Acknowledgments

The author gratefully acknowledges discussions with J. Hošek, G. Barenboim, J. Novotný, and P. Beneš. The work was supported by the Grant LA08015 of the Ministry of Education of the Czech Republic.

Appendix A: Nambu–Gorkov formalism

The neutrino fields are accommodated within the Nambu–Gorkov multispinor $n$ defined in (14). Its canonical anti-commutation relations then follow

$$\{n_{\alpha i}(x), n_{\beta j}^\dagger(y)\}_{\text{E.T.}} = \delta_{ij} \delta_{\alpha \beta} \delta^{(3)}(x-y), \quad (A1a)$$

$$\{n_{\alpha i}(x), n_{\beta j}(y)\}_{\text{E.T.}} = \delta_{ij} [C\gamma_0]_{\alpha \beta} \delta^{(3)}(x-y), \quad (A1b)$$

where $C$ is charge conjugation matrix.

The flavor transformations

$$n' = e^{i\theta^a s^a} n, \quad (A2)$$

are generated by the flavor generators

$$t^a = \left( T^a_R P_R - [T^a_R]^T P_L \right) \frac{1}{2} \mathbb{1}_{4 \times 4} \left( T^a_R P_L - [T^a_R]^T P_R \right) \frac{1}{2} \mathbb{1}_{4 \times 4} \left( T^a_R P_R - [T^a_R]^T P_L \right) .$$

The lepton number transformation of the neutrino fields is

$$n' = e^{i\theta^l l} n, \quad (A3)$$

where $l$ denotes the corresponding generator

$$l = \left( -L_{\text{EW}} \gamma_5 \frac{1}{2} \mathbb{1}_{4 \times 4} \gamma_5 (1 - a) \gamma_5 \right) . \quad (A4)$$

The sterility transformations of the neutrino fields are

$$n' = e^{i\theta_s s^a} n \quad (A5)$$

and the corresponding currents of the sterility symmetry are compactly rewritten as

$$j_{S,a} = \frac{1}{2} \bar{n} \gamma^\mu s^a n . \quad (A6)$$

where $s_a$ schematically denotes generators of all the sterility symmetries

$$S_3 - S_6 : \quad s_0 = \left( \begin{array}{cc} 0 & \frac{1}{2} \mathbb{1}_{4 \times 4} \gamma_5 \\ \gamma_5 & -\gamma_5 \end{array} \right) ;$$

$$\text{SU(4)}_S : \quad s_i = \left( \begin{array}{cc} 0 & S_i P_R - S_i^T P_L \\ S_i P_L - S_i^T P_R & 0 \end{array} \right) , \quad i = 1, \ldots, 15 ;$$

$$S_3 + S_6 : \quad s_{16} = \left( \begin{array}{cc} 0 & \frac{1}{2} \mathbb{1}_{4 \times 4} \gamma_5 \\ \gamma_5 & 0 \end{array} \right) . \quad (A7a)$$

Appendix B: Two-loop $\beta$-function

Two-loop $\beta$-function is given by [47]

$$\beta(h) =$$

$$-\frac{h^3}{(4\pi)^2} \left[ \frac{11}{3} C(8) - \frac{2}{3} N^{\text{EW}} C(3) - \frac{2}{3} \sum_r N_r \gamma_r C(r) \right]$$

$$-\frac{h^3}{(4\pi)^3} \left[ \frac{34}{3} C(8)^2 - N^{\text{EW}} \left( 2C_2(3) + \frac{10}{3} C(8) \right) C(3) \right]$$

$$- \sum_r N_r \gamma_r \left( 2C_2(r) + \frac{10}{3} C(8) \right) C(r) \right] , \quad (B1)$$

where the coefficient $C(r)$ reflects the flavor symmetry representation of the right-handed neutrino field, and is
related to the quadratic Casimir invariant $C_2(r)$. Their definitions and their relation are

\[ \delta^{ab} C(r) = \text{Tr} T^a_r T^b_r, \]  
(\text{B2a})

\[ d(r) C_2(r) = \text{Tr} T^a_r T^a_r, \]  
(\text{B2b})

\[ d(r) C_2(r) = d(G) C(r). \]  
(\text{B2c})

For completeness we mention also the anomaly coefficient $A(r)$ important for the anomaly analysis. It is related to the cubic Casimir invariant $C_3(r)$. The relevant formulas are

\[ \frac{1}{2} \epsilon^{abc} A(r) = \text{Tr} T^a_r \{ T^b_r, T^c_r \}, \]  
(\text{B3a})

\[ d(r) C_3(r) = \epsilon^{abc} \text{Tr} T^a_r T^b_r T^c_r, \]  
(\text{B3b})

\[ 2d(r) C_3(r) = \frac{5}{6} d(G) A(r). \]  
(\text{B3c})

The values for some of the lowest representations are listed in Tab. III.

**Appendix C: Flavor gauge boson mass matrices**

\[ M_6^2 = \frac{\alpha^2}{h^2} \left( \begin{array}{cccccccc}
(\phi_1 + \phi_2)^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & (\phi_1 - \phi_2)^2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2(\phi_1^2 + \phi_2^2) & 0 & 0 & 0 & 0 & \frac{2}{\sqrt{3}} (\phi_1^2 - \phi_2^2) \\
0 & 0 & 0 & (\phi_1 + \phi_3)^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & (\phi_1 - \phi_3)^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array} \right). \]  
(\text{C1a})

\[ M_3^2 = \frac{\alpha^2}{4} \left( \begin{array}{cccccccc}
(\phi_4^2 + \phi_5^2) & 0 & 0 & \varphi_5 \varphi_6 & 0 & \varphi_4 \varphi_6 & 0 & \frac{2}{\sqrt{3}} \varphi_4 \varphi_5 \\
0 & (\phi_4^2 + \phi_5^2) & 0 & 0 & \varphi_5 \varphi_6 & 0 & -\varphi_4 \varphi_6 & 0 \\
0 & 0 & \varphi_5 \varphi_6 & 0 & -\varphi_5 \varphi_6 & 0 & 0 & \frac{1}{\sqrt{3}} (\varphi_4^2 - \varphi_5^2) \\
\varphi_5 \varphi_6 & 0 & \varphi_4 \varphi_6 & 0 & \varphi_4 \varphi_5 & 0 & 0 & 0 \\
0 & \varphi_5 \varphi_6 & 0 & 0 & \varphi_4 \varphi_6 & 0 & \varphi_4 \varphi_5 & 0 \\
\varphi_4 \varphi_6 & 0 & -\varphi_5 \varphi_6 & \varphi_4 \varphi_5 & 0 & \varphi_4 \varphi_5 & 0 & 0 \\
0 & -\varphi_4 \varphi_6 & 0 & 0 & \varphi_4 \varphi_5 & 0 & \varphi_4 \varphi_5 & 0 \\
\frac{2}{\sqrt{3}} \varphi_4 \varphi_5 & 0 & \frac{1}{\sqrt{3}} (\varphi_4^2 - \varphi_5^2) & -\frac{1}{\sqrt{3}} \varphi_4 \varphi_6 & 0 & \frac{1}{\sqrt{3}} \varphi_4 \varphi_5 & 0 & 0 \\
\end{array} \right). \]  
(\text{C1b})

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