Abstract

This paper introduces the notion of cache-tapping into the information theoretic models of coded caching. In particular, the wiretap II channel model in the presence of multiple receivers equipped with fixed-size cache memories, and an adversary who is able to choose symbols to tap into from cache placement, in addition to or in lieu of, delivery transmission, is introduced. The model is hence termed the caching broadcast channel with a wire and cache tapping adversary of type II. The legitimate parties know neither whether cache placement, delivery, or both phases are tapped, nor the positions in which they are tapped. Only the size of the overall tapped set is known. For the instance of two receivers and two library files, the strong secrecy capacity of the model, i.e., the maximum achievable file rate while keeping the overall library strongly secure, is identified. Lower and upper bounds on the achievable strong secrecy file rate are derived when the library has more than two files. Achievability schemes in this paper rely on a code design which combines wiretap coding, security embedding codes, one-time pad keys, and coded caching. A genie-aided upper bound, in which a genie provides the transmitter with user demands before cache placement, establishes the converse for the two files instance. For the library of more than two files, the upper bound is constructed by three successive channel transformations. Our results establish that strong information theoretic security is possible against a powerful adversary who optimizes its attack over both phases of communication in a cache-aided system.
I. Introduction

Caching is proposed to efficiently reduce network traffic congestion by storing partial contents at the cache memories of end users earlier during off-peak times, providing local caching gain \([1]–[3]\). More recently, reference \([4]\) has shown that a careful design of cache contents at the end users in a multi-receiver setting allows the transmitter to send delivery transmissions that are simultaneously useful for many users, providing a global caching gain. This gain depends on the aggregate cache memory of the network and demonstrates the ability of coding over delivery transmission and/or cache contents to offset lack of cooperation between the receivers.

In numerous works to date, coded caching has been studied under various modeling assumptions and for various network configurations, including online, decentralized, hierarchical caching \([5]–[8]\), non-uniform demands \([9]\), more users than files \([10], [11]\), heterogeneous cache sizes \([12], [13]\), improved bounds \([14]–[16]\), interference networks \([17]–[19]\), combination networks \([20], [21]\), device-to-device communication \([22], [23]\), and broadcast channels \([24]–[27]\).

Coded caching with confidentiality requirements has recently been studied in references \([21], [28]–[33]\). These references assume secure cache placement, i.e., the adversary cannot tap into the cache contents or into the communication which performs the cache placement. At the other extreme, if cache placement were to be public, i.e., the adversary were to have perfect access to the cache contents, it follows in a straightforward fashion from \([34], [35]\) that the presence of cache memories cannot increase the secrecy capacity. Given the results of these two extreme settings, this paper considers an intermediate scenario in which the adversary may have partial access to cache placement.

The wiretap channel II, introduced in \([36]\), provides a model for an adversary who has partial access to the legitimate communication, in the form of a threshold on the time fraction during which the adversary is capable of tapping into the communication. In particular, the model considers a noiseless legitimate channel and an adversary who chooses a fixed-size subset of the transmitted symbols to noiselessly observe. Reference \([36]\) showed that, despite this ability to choose the wiretapped symbols, with proper coding, this adversary can be made no more powerful than nature, i.e., the secrecy capacity of the wiretap II model is identical to that of a binary erasure wiretapper channel with the same fraction of erasures. Reference \([37]\) has generalized the wiretap II model to one with a discrete memoryless, i.e., noisy, legitimate channel, and...
derived inner and outer bounds for its capacity-equivocation region. The secrecy capacity for this model was identified in [38]. Reference [39] has introduced a generalized wiretap model which subsumes both the classical wiretap [40] and wiretap II [37] models as special cases. This generalized model was extended to multi-transmitter and multi-receiver networks in [41]–[43]. In all these models, the common theme is the robustness of stochastic wiretap encoding [40] against a type II adversary who is allowed to choose where to tap.

In this paper, we consider an adversary model of type II as in [36]–[39], [41]–[43], but in a cache-aided communication setting. In particular, the adversary noiselessly observes a partial subset of its choosing of the transmitted symbols over the cache placement and/or delivery phases. Thus we term this model the caching broadcast channel with a wire and cache tapping adversary of type II (CBC-WCT II). The legitimate parties do not know whether the cache placement, delivery, or both transmissions are tapped, the relative fractions of tapped symbols in each, or their positions. Only the knowledge of the overall size of the tapped set by the adversary is available to the legitimate terminals.

The challenge in caching stems from the fact that the transmitter, who has access to a library of files, has no knowledge about the future demands of end users when designing their cache contents. This remains to be the case when security against an external adversary is concerned. Additionally, for the adversary model in consideration, the adversary might tap into cache placement, delivery, or both, and where the tapping occurs is unknown to the legitimate parties. We show that even under these unfavorable conditions, strong secrecy guarantees can be provided that are invariant to the positions of the tapped symbols varying between cache placement, delivery, or both.

In coded caching literature up to date, the physical communication which populates the cache memories at end users does not need to be considered in the problem formulation, due to the assumption of secure cache placement. By contrast, in order to model cache placement that is tapped by an adversary, we consider a length-\(n\) communication block over a two-user broadcast wiretap II channel [42]. The sizes of cache memories at the receivers are fixed in this model. We note that introducing variable memory sizes for which a rate-memory tradeoff can be characterized, as in the usual setup for caching, requires considering additional communication blocks for cache placement. Being of future interest, we comment on this extension to multiple
communication blocks for cache placement in the Discussion section, Section VII. We as well provide reasoning for our choice of the broadcast setting for cache placement.

In summary, the contributions of this work are summarized as follows:

1) We introduce the notion of cache-tapping into the information theoretic models of coded caching, in which an adversary of type II is able to tap into a fixed-size subset of symbols of its choosing either from cache placement, delivery, or both transmissions.

2) We characterize the strong secrecy capacity of the model, i.e., the maximum achievable file rate which keeps the overall library strongly secure, for the instance of a transmitter’s library with two files:
   - We devise an achievability scheme which integrates wiretap coding [36], security embedding codes [44], [45], one-time pad keys [34], coded cache placement and uncoded delivery [4].
   - We utilize a genie-aided upper bound, in which a genie provides the transmitter with user demands before cache placement, rendering the model to a two-user broadcast wiretap II channel [42], in order to establish the converse for this case.

3) We derive lower and upper bounds on the strong secrecy file rate when the transmitter’s library has more than two files:
   - We utilize a coding scheme that is similar to the scheme we used for two files. However, the cache placement and delivery schemes we employ to achieve the rates differ from those utilized for two files. In particular, we utilize here uncoded cache placement and a partially coded delivery.
   - We derive the upper bound in three steps. First, we consider a transformed channel with an adversary who can tap an equal fraction of symbols to our model, but is only allowed to tap into the delivery phase. Since this adversary has a more restricted strategy space than the original one, the corresponding secrecy capacity is at least as large as our original model. Next, we utilize Sanov’s theorem in method of types [46, Theorem 11.4.1] to further upper bound the secrecy capacity for the restricted adversary model by the secrecy capacity when the adversary encounters a discrete memoryless binary erasure channel. Finally, the secrecy capacity of the discrete memoryless model is upper bounded by the secrecy capacity of a single receiver setting in which the
receiver requests two files from the library.

The remainder of the paper is organized as follows. Section III describes the communication system proposed in this paper. Section III presents the main results. The proofs for these results are provided in Sections IV, V, and VI. Section VII provides a discussion about the communication model in question and the presented results. Section VIII concludes the paper.

II. SYSTEM MODEL

We first remark the notation we utilize throughout the paper. \( \mathbb{N} \), \( \mathbb{Z} \), and \( \mathbb{R} \) denote the sets of natural, integer, and real numbers, respectively. For \( a, b \in \mathbb{R} \), \([a : b]\) denotes the set of integers \( \{i \in \mathbb{N} : a \leq i \leq b\} \). We use \( A_{[1:n]} \) to denote the sequence of variables \( \{A_1, A_2, \ldots, A_n\} \). For two sets \( A_1 \) and \( A_2 \), \( A_1 \times A_2 \) denotes their Cartesian product. \( A^T \) denotes the \( T \)-fold Cartesian product of the set \( A \). For \( W_1, W_2 \in [1 : M] \), \( W_1 \oplus W_2 \) denotes the bit-wise XOR on the binary bit strings that correspond to \( W_1, W_2 \). We use \( 1_A \) to denote the indicator function for the event \( A \). \( \mathbb{D}(p_x || q_x) \) denotes the Kullback-Leibler divergence between the probability distributions \( p_x \), \( q_x \), defined on the same probability space. \( \{\epsilon_n\}_{n \geq 1} \) denotes a sequence of positive real numbers such that \( \epsilon_n \to 0 \) as \( n \to \infty \).
Consider the communication system depicted in Fig. 1 in which the adversary has the ability to tap into both the cache placement and delivery transmissions. The transmitter observes $D \geq 2$ independent messages (files), $W_1, W_2, \cdots, W_D$, each of which is uniformly distributed over $[1 : 2^{nR_s}]$. Each receiver has a cache memory of size $n/2$ bits. The communication occurs over two phases: placement and delivery. The broadcast channel is noiseless during both phases. The communication model is described as follows:

**Cache placement phase:** During this phase, the transmitter broadcasts a length-$n$ binary signal, $X^n_c \in \{0, 1\}^n$, to both receivers. The codeword $X^n_c$ is a function of the library files, i.e., $X^n_c \equiv f_c(W_{[1:D]})$. The transmitter does not know the receiver demands during cache placement [4]. Each receiver has a cache memory of size $n/2$ bits in which they store a function of $X^n_c$, $M_{c,j} \equiv f_{c,j}(X^n_c)$, where $f_{c,j} : \{0, 1\}^n \mapsto [1 : 2^{n/2}]$ and $j = 1, 2$.

**Delivery phase:** At the beginning of the delivery phase, the two receivers announce their demands $d \equiv (d_1, d_2) \in [1 : D]^2$ to the transmitter. The transmitter, in order to satisfy the receiver demands, encodes $W_{[1:D]}$ and $d$ into the binary codeword $X^n_d \in \{0, 1\}^n$. In particular, for each $d$, the transmitter uses the encoder $f_d : [1 : 2^{nR_s}]^D \mapsto \{0, 1\}^n$ and sends $X^n_d \equiv f_d(W_{[1:D]})$.

**Decoding:** Receiver $j$ utilizes the decoder $g_{d,j} : [1 : 2^{n/2}] \times \{0, 1\}^n \mapsto [1 : 2^{nR_s}]$, in order to output the estimate $\hat{W}_{d,j} \equiv g_{d,j}(f_{c,j}(X^n_c), X^n_d)$ of its desired message $W_{d,j}$, where $j = 1, 2$.

**Adversary model:** The adversary chooses two subsets $S_1, S_2 \subseteq [1 : n]$. The size of the sum of cardinalities of $S_1$ and $S_2$ is fixed. That is, for $|S_1| = \mu_1$, $|S_2| = \mu_2$, $\mu_1, \mu_2 \leq n$, we have $\mu_1 + \mu_2 = \mu$. The subsets $S_1$ and $S_2$ indicate the positions tapped by the adversary during the cache placement and delivery phases, respectively. Over the two phases, the adversary observes the length-2n sequence $Z^n_S = [Z^n_{S_1}, Z^n_{S_2}] \in \mathcal{Z}^{2n}$, $Z^n_{S_j} \equiv [Z_{S_j,1}, Z_{S_j,2}, \cdots, Z_{S_j,n}] \in \mathcal{Z}^n$, $j = 1, 2$,

$$Z_{S_1,i} = \begin{cases} X_{c,i}, & i \in S_1 \\ \?, & i \notin S_1 \end{cases}, \quad \text{and} \quad Z_{S_2,i} = \begin{cases} X_{d,i}, & i \in S_2 \\ \?, & i \notin S_2 \end{cases} \quad (1)$$

The alphabet is $\mathcal{Z} = \{0, 1, \?\}$, where "?" denotes an erasure.

The legitimate terminals know neither the realizations of $S_1$ and $S_2$, nor the values of $\mu_1$ and $\mu_2$. Only $\mu$ is known. Let us define $\alpha_1 = \frac{\mu_1}{n}$ and $\alpha_2 = \frac{\mu_2}{n}$ as the fractions of the tapped symbols in the cache placement and delivery phases, and let $\alpha = \alpha_1 + \alpha_2$ be the overall tapped ratio.
Notice that $\alpha_1, \alpha_2 \in [0, 1]$ and $\alpha \in (0, 2]$.

**Remark 1** We consider that $\alpha$ is strictly greater than zero, i.e., the adversary is present. For $\alpha = 0$, i.e., no adversary, the problem considered in this paper has been extensively studied in the literature, see for example [4], [47]–[49].

A channel code $C_{2n}$ for this model consists of
- $D$ message sets; $W_l \triangleq [1 : 2^{nR_s}]$, $l = 1, 2, \ldots, D$,
- Cache encoder; $f_c : [1 : 2^{nR_s}]^D \mapsto \{0, 1\}^n$,
- Cache decoders; $f_{c,j} : \{0, 1\}^n \mapsto [1 : 2^{\frac{n}{2^j}}]$, $j = 1, 2$,
- Delivery encoders; $\{ f_d : \mathbf{d} \in [1 : D]^2 \}, f_d : [1 : 2^{nR_s}]^D \mapsto \{0, 1\}^n$,
- Decoders; $\{ g_{d,j} : j = 1, 2, \mathbf{d} \in [1 : D]^2 \}, g_{d,j} : [1 : 2^{\frac{n}{2^j}}] \times \{0, 1\}^n \mapsto [1 : 2^{nR_s}]$.

The file rate $R_s$ is said to be achievable with strong secrecy if there exists a sequence of channel codes $\{ C_{2n} \}_{n \geq 1}$ satisfying
\[
\lim_{n \to \infty} \max_{\mathbf{d} \in [1 : D]^2} \mathbb{P} \left( \bigcup_{j=1,2} (\hat{W}_{dj} \neq W_{dj}) \right) = 0, \tag{2}
\]
\[
\text{and} \quad \lim_{n \to \infty} \max_{S_1, S_2 \subseteq [1 : n] : |S_1| + |S_2| \leq \mu} I(W_{[1 : D]}; Z_{S_1}^{W_{S_1}}, Z_{S_2}^{W_{S_2}}) = 0. \tag{3}
\]

That is, $R_s$ is the symmetric secure file rate, under any demand vector and adversarial strategy. The strong secrecy capacity $C_s$ is the the supremum of all achievable $R_s$.

**Remark 2** While we consider the file rate $R_s$ which guarantees reliability for the worst-case demand vector, the average rate for which there exists a prior distribution on the demands has been studied in coded caching literature as well; see for example [9], [14], [50].

**Remark 3** The condition in (3) guarantees strong secrecy against all possible strategies for the adversary, i.e., choices of the subsets $S_1$ and $S_2$ which satisfy the condition $|S_1| + |S_2| \leq \mu$. 

### III. MAIN RESULTS

For clarity of exposition, we first study the model described in Section II when the transmitter has two files in its library, i.e., $D = 2$. We then extend the ideas and the analysis to the case of a library with more than two files, i.e., $D > 2$. For $D > 2$, we utilize a channel coding scheme
that is similar to the scheme we construct for $D = 2$, but the cache placement and delivery schemes that achieve the best rates are different from those used for $D = 2$.

The following theorem presents the strong secrecy capacity for $D = 2$.

**Theorem 1** For $0 < \alpha \leq 2$ and $D = 2$, the strong secrecy capacity for the caching broadcast channel with a wire and cache tapping adversary of type II (CBC-WCT II), described in Section II is given by

$$C_s(\alpha) = 1 - \frac{\alpha}{2}. \quad (4)$$

**Proof:** The proof is provided in Section IV. ■

Theorem 2 below presents an achievable strong secrecy file rate for $D > 2$.

**Theorem 2** For $0 < \alpha \leq 2$ and $D > 2$, the achievable strong secrecy file rate for the CBC-WCT II is

$$R_s(\alpha) \geq \begin{cases} \frac{1}{2} + \frac{3(1-\alpha)}{4D}, & 0 < \alpha < 1 \\ 1 - \frac{\alpha}{2}, & 1 \leq \alpha \leq 2. \end{cases} \quad (5)$$

**Proof:** The proof is provided in Section V. ■

The following theorem upper bounds the secure file rate when $D > 2$.

**Theorem 3** For $0 < \alpha \leq 2$ and $D > 2$, the achievable strong secrecy file rate for the CBC-WCT II is upper bounded as

$$R_s(\alpha) \leq \begin{cases} \frac{1}{2} + \frac{2D-1}{2D(D-1)}(1 - \alpha), & 0 < \alpha < 1 \\ 1 - \frac{\alpha}{2}, & 1 \leq \alpha \leq 2. \end{cases} \quad (6)$$

**Proof:** The proof is provided in Section VI. ■

The following corollary is immediate from Theorems 1, 2, and 3.

**Corollary 1** For $1 \leq \alpha \leq 2$, that is when the adversary can tap longer than one phase of communication, the strong secrecy capacity for the CBC-WCT II is

$$C_s(\alpha) = 1 - \frac{\alpha}{2}. \quad (7)$$
Remark 4 When $\alpha \in [1, 2]$, i.e., $n \leq \mu \leq 2n$, two possible strategies for the adversary are $\{S_1 = [1 : n], S_2 \subset [1 : n]\}$ or $\{S_1 \subset [1 : n], S_2 = [1 : n]\}$. That is, the adversary can tap into all symbols in one phase and a subset of symbols in the other phase. Interestingly, a positive strong secrecy capacity is achievable against such an adversary. We elaborate more on the intuition behind this result in the Discussion section. ■

Unlike for $1 \leq \alpha \leq 2$, for $0 < \alpha < 1$, the lower and upper bounds in (5) and (6) have a gap. For illustration purposes, these bounds are plotted for $\alpha = 0.4$ in Fig. 2.

Remark 5 When $\alpha = 0$, i.e., no adversary, our achievability scheme for $D > 2$ described in Section V reduces to the achievability scheme in [4], which is shown to achieve the optimal rate-memory tradeoff for the case of two users and a library size of three or larger [47], [49]. The upper bound for $D > 2$ derived in this work is to address the intricacies of the adversarial model and is useful only when the adversary is present ($\alpha > 0$), i.e., (6) is loose when $\alpha = 0$. ■
IV. PROOF OF THEOREM 1

In this section, we prove Theorem 1, which identifies the strong secrecy capacity for the model in Section II when $D = 2$. Recall that the demand vector is denoted by $d = (d_1, d_2)$, where $d_1, d_2 \in \{1, 2\}$.

A. Converse

For the model in Theorem 1 when the demand vector $d$ is known to the transmitter during cache placement, the model reduces to a broadcast wiretap channel II, over a length-$2n$ communication block. The strong sum secrecy rate for that model, $2R_s$, is upper bounded by

$$2R_s \leq 2 - \alpha,$$

which follows from our recent work [42, Theorem 1]. Notice that (8) holds for any $d = (d_1, d_2)$ such that $d_1 \neq d_2$, which represents the worst-case demands. Since the demand vector is unknown for the model in consideration, $1 - \frac{\alpha}{2}$ is an upper bound for its strong secrecy capacity.

B. Restricted Adversary Models as Building Blocks

Before proceeding with the achievability proof, it is relevant to take a step back and investigate the secrecy capacity when a known fraction of cache placement, a known fraction of delivery, or both, is tapped. In particular, we consider that the adversary taps into (i) cache placement only, (ii) delivery only, or (iii) both and the relative fractions of tapped symbols in each are known. For these three models, we show that the strong secrecy file rate in (4), i.e., $1 - \frac{\alpha}{2}$, is achievable, and hence determines their strong secrecy capacities. We then use these models as building blocks for when the relative fractions are unknown, and provide the achievability proof in Sections IV-C and IV-D.

1) Setting 1: The adversary taps into cache placement only:

Let $\alpha_1 = \alpha$ ($\alpha_2 = 0$), and $|S_1| = \mu$ ($S_2 = \emptyset$). That is, the adversary taps into cache placement only. Consider that the transmitter and the receivers know that $\alpha_1 = \alpha$. We show that $1 - \frac{\alpha}{2}$ is an achievable strong secrecy file rate for this setting.
The transmitter divides the message $W_l$, $l = 1, 2$, into three independent messages, $W_l^{(1)}$, $W_l^{(2)}$, and $W_{l,s}$, where $W_l^{(1)}$, $W_l^{(2)}$, are uniform over $[1 : 2^{n\frac{1-\alpha-n}{2}}]$, and $W_{l,s}$ is uniform over $[1 : 2^{n\frac{\alpha+n}{2}}]$. Define

$$M_c = \{M_{c,1}, M_{c,2}\}; \quad M_{c,1} = W_1^{(1)} \oplus W_2^{(1)}, \quad M_{c,2} = W_1^{(2)} \oplus W_2^{(2)},$$

(9)

$$M_d = \{W_{d_1}^{(2)}, W_{d_2}^{(1)}, W_{d_1,s}, W_{d_2,s}\}.$$  

(10)

During cache placement, the transmitter maps $M_c$ into $X_n^c$ using stochastic encoding, i.e., wiretap coding [40]. Since the rate of $M_c$ is less than $1 - \alpha$, $M_c$ is strongly secure from the adversary who observes $n\alpha$ symbols of $X_n^c$ [38], [39]. During the delivery phase, the transmitter sends $X_n^d$ as the binary representation of $M_d$ which is of length $n$ bits, since the delivery phase is noiseless and secure.

Using $X_n^c$, noiselessly received during cache placement, receiver $j$, $j = 1, 2$, recovers $M_{c,j}$ and stores it in its cache memory. Notice that the size of $M_{c,j}$, for $j = 1, 2$, is smaller than $\frac{n}{2}$ bits, i.e., the cache size at each receiver. Using $X_n^d$, received noiselessly during delivery, both receivers perfectly recover $M_d$. Using $M_d$ along with its cache contents, i.e., $M_{c,j}$, and for $n$ sufficiently large, receiver $j$ correctly recovers its desired message $W_{d,j}$, $j = 1, 2$.

For secrecy, we show in Appendix A that (3), which reduces to

$$\lim_{n \to \infty} \max_{S_1 \subseteq [1:n]} I(W_1, W_2; Z_{S_1}^n) = 0,$$

(11)

is satisfied. Since $\epsilon_n \to 0$ as $n \to \infty$, the achievable strong secrecy file rate is given by

$$R_s(\alpha) = 2 \times \frac{1 - \alpha}{2} + \frac{\alpha}{2} = 1 - \frac{\alpha}{2}.$$  

(12)

2) Setting 2: The adversary taps into the delivery only:

This setting corresponds to $\alpha_1 = 0$ and $\alpha_2 = \alpha$, and the transmitter and receivers possess this knowledge. Once again, we show that $1 - \frac{\alpha}{2}$ is an achievable strong secrecy file rate.

The transmitter performs the same division of $W_l$, $l = 1, 2$, as in Setting 1. In addition, the transmitter generates the keys $K_1$, $K_2$, each is uniform over $[1 : 2^{n\frac{\alpha+n}{2}}]$, independent from one

\[^1\]Large block-length $n$ is needed in order to ensure a valid subpacketization of the file $W_l$ into the sub-files $\{W_l^{(1)}, W_l^{(2)}, W_{l,s}\}$, for $l = 1, 2$. That is, a bijective map between the file and its sub-files is preserved.
another and from $W_1, W_2$. In this case, we define $M_c, M_d,$ and $\tilde{M}_d$, as follows.

$$M_c = \{M_{c,1}, M_{c,2}\}; \quad M_{c,1} = \{W_1^{(1)} \oplus W_2^{(1)}, K_1\}, \quad M_{c,2} = \{W_1^{(2)} \oplus W_2^{(2)}, K_2\},$$

(13)

$$M_d = \{W_{d_1}^{(2)}, W_{d_2}^{(1)}\},$$

(14)

$$\tilde{M}_d = \{W_{d_1,s} \oplus K_1, W_{d_2,s} \oplus K_2\}.$$

(15)

During cache placement, the transmitter sends $X_c^n$ as the binary representation of $M_c$, and receiver $j$, $j = 1, 2$, stores $M_{c,j}$ in its cache memory. During delivery, the transmitter encodes $M_d$ into $X_d^n$ using wiretap coding, while using $\tilde{M}_d$ as the randomization message. Receiver $j$ recovers $M_d$ and $\tilde{M}_d$, using which, along with $M_{c,j}$, it correctly decodes $W_{d,j}$, for sufficiently large $n$. By contrast, the adversary can only retrieve $\tilde{M}_d$ using which it can gain no information about $W_1$ and $W_2$. We show in Appendix B that (3), i.e.,

$$\lim_{n \to \infty} \max_{S_2 \subseteq [1:n]: |S_2| = \mu} I(W_1, W_2; Z^n_{S_2}) = 0,$$

(16)

is satisfied. The achievable strong secrecy file rate is again $1 - \frac{\alpha}{2}$.

3) Setting 3: The legitimate terminals know the values of $\alpha_1$ and $\alpha_2$:

For this setting, neither $\alpha_1 = 0$ nor $\alpha_2 = 0$. However, the transmitter and receivers know the values of $\alpha_1$ and $\alpha_2$. Under these assumptions, the scheme which achieves the strong secrecy rate of $1 - \frac{\alpha}{2}$ depends on whether $\alpha_1 \geq \alpha_2$. For $\alpha_1 \geq \alpha_2$, we utilize an achievability scheme similar to Setting 1; for $\alpha_1 < \alpha_2$, we utilize an achievability scheme similar to Setting 2.

Case 1: $\alpha_1 \geq \alpha_2$: The transmitter divides $W_l$, $l = 1, 2$, into the independent messages \(\{W_{l}^{(1)}, W_{l}^{(2)}, W_{l,s}\}\); $W_{l}^{(1)}, W_{l}^{(2)}$ are uniform over $[1:2^{n\frac{1-\alpha_1}{2}}]$ and $W_{l,s}$ is uniform over $[1:2^{n\frac{\alpha_1-\alpha_2}{2}}]$. The transmitter forms $M_c$ and $M_d$ as in (9) and (10), and uses wiretap coding to map them into $X_c^n$ and $X_d^n$, respectively. As in setting 1, receiver $j$ correctly decodes $W_{d,j}$.

For the secrecy constraint, notice that $M_c$ and $M_d$ are independent, and their rates are $1 - \alpha_1 - \epsilon_n$ and $1 - \alpha_2 - \epsilon_n$, respectively. Thus, wiretap coding strongly secures both $M_c$ and $M_d$ against the adversary. We show in Appendix C that (3) is satisfied. The achievable strong secrecy file rate is

$$R_s(\alpha) = 2 \times \frac{1 - \alpha_1}{2} + \frac{\alpha_1 - \alpha_2}{2} + \frac{2 - \alpha_1 - \alpha_2}{2} = 1 - \frac{\alpha}{2},$$

(17)
Case 2: $\alpha_1 < \alpha_2$: The transmitter (i) divides $W_l$, $l = 1, 2$, into $\left\{ W_l^{(1)}, W_l^{(2)}, W_l^{(1)}, W_l^{(2)} \right\}$; $W_l^{(1)}$, $W_l^{(2)}$ are uniform over $\left[ 1 : 2^n \frac{1 - \alpha_2 - \alpha_1}{2} \right]$ and $W_l^{(1)}$, $W_l^{(1)}, W_l^{(2)}$ is uniform over $\left[ 1 : 2^n \frac{\alpha_2 - \alpha_1}{2} \right]$; (ii) generates the keys $K_1$, $K_2$, uniform over $\left[ 1 : 2^n \frac{\alpha_2 - \alpha_1}{2} \right]$, independently from $W_1$, $W_2$, (iii) forms $M_c$ as in (13) and encodes it into $X^n_c$ using wiretap coding, (iv) forms $M_d$ as in (14) and forms $\tilde{M}_d$ as

$$\tilde{M}_d = \left\{ W_{d_1,s} \oplus K_1, W_{d_2,s} \oplus K_2, \tilde{W} \right\},$$

where $\tilde{W}$ is independent from all other variables and uniform over $\left[ 1 : 2^n \frac{\alpha_1 + \epsilon_n}{2} \right]$, and (v) encodes $M_d$ into $X^n_d$ using wiretap coding, while using $\tilde{M}_d$ as the randomization message.

As in Setting 2, for $n$ sufficiently large, receiver $j$, $j = 1, 2$, correctly decodes $W_{d_j}$, and the adversary can only retrieve $\tilde{M}_d$ using which it can gain no information about $W_1$, $W_2$. In Appendix D we show that (3) is satisfied. The achievable secrecy rate is

$$R_s(\alpha) = 2 \times \frac{1 - \alpha_2}{2} + \frac{\alpha_2 - \alpha_1}{2} = 1 - \frac{\alpha}{2}. \tag{19}$$

With the aforementioned settings, we showed that the same secrecy rate, i.e., $1 - \frac{\alpha}{2}$, is achievable irrespective of where the adversary taps as long as $\alpha_1$ and $\alpha_2$ are known. The question then arises whether the lack of knowledge about relative fractions of tapped symbols would decrease the secrecy capacity. The following setting we propose provides a hint on the answer.

4) Setting 4: Either $\alpha_1 = 0$ or $\alpha_2 = 0$, the legitimate terminals do not know which is zero:

The adversary taps into either cache placement or delivery, but not both. The legitimate terminals do not know which phase is tapped. We show that the strong secrecy rate $1 - \frac{\alpha}{2}$ is again achievable.

The transmitter performs the same division of $W_1$, $W_2$ as in Settings 1, 2, and generates independent keys $K_1$, $K_2$ as in Setting 2. Let us define

$$M_c = \{ M_{c,1}, M_{c,2} \}; \quad M_{c,1} = W_1^{(1)} \oplus W_2^{(1)}, \quad M_{c,2} = W_1^{(2)} \oplus W_2^{(2)}; \tag{20}$$

$$\tilde{M}_c = \{ K_1, K_2 \}; \tag{21}$$

$$M_d = \left\{ W_{d_1}^{(2)}, W_{d_2}^{(1)} \right\}; \tag{22}$$

$$\tilde{M}_d = \{ W_{d_1,s} \oplus K_1, W_{d_2,s} \oplus K_2 \}. \tag{23}$$

During cache placement, the transmitter encodes $M_c$ into $X^n_c$ using wiretap coding, while using $\tilde{M}_c$ as the randomization message. Receiver $j$, $j = 1, 2$, stores $M_{c,j}$, $\tilde{M}_{c,j}$, in its cache
memory. During delivery, the transmitter uses wiretap coding to encode $M_d$ into $X_n^d$, while using $\bar{M}_d$ as the randomization message. Using its cache contents, along with $M_d$ and $\bar{M}_d$, receiver $j$ correctly decodes $W_{d_j}$. By contrast, the adversary can only retrieve either $\{K_1, K_2\}$ or $\{W_{d_1,s} \oplus K_1, W_{d_2,s} \oplus K_2\}$, but not both, using which it can obtain no information about $W_1$ and $W_2$. We show in Appendix E that (3) is satisfied. The achievable strong secrecy rate is $1 - \frac{\alpha}{2}$.

The lack of knowledge about which phase is tapped is countered by encrypting pieces of information, $\{W_{d_1,s}, W_{d_2,s}\}$, with one-time pad keys $K_1$ and $K_2$, while ensuring that the adversary only retrieves either the keys or the encrypted bits but not both; using which it can gain no information about the messages $W_1$ and $W_2$.

In the following subsection, we generalize this idea to tackle the case when the adversary gets to tap into both phases, with no knowledge about the relative fractions of tapped symbols in each, i.e., the model in Fig. 1. In particular, similar to [45], in which the uncertainty about the wiretapper’s channel is treated by using a security embedding code [44], here, in each phase, we construct an embedding code in which $n\alpha$ single-bit layers are embedded into one another. Doing so, we ensure that, no matter what the values for $\alpha_1$ and $\alpha_2$ are, the adversary can retrieve no more than $n\alpha_1$ bits from cache placement, and $n\alpha_2$ bits from delivery. By designing what the adversary retrieves to be either a set of key bits and/or information bits encrypted with a distinct set of key bits, we guarantee no information on the messages is asymptotically leaked to the adversary. We thus prove that the lack of knowledge about relative fractions of tapped symbols does not decrease the secrecy capacity.

C. Achievability for $\alpha \in (0, 1)$:

We are now ready to present the achievability for the general model considered in this paper. Consider first $\alpha \in (0, 1)$. For simplicity, assume that $n\frac{\alpha_1}{2} = \frac{\mu_1}{2}$ and $n\frac{\alpha_2}{2} = \frac{\mu_2}{2}$ are integers. A minor modification to the analysis can be adopted otherwise.

The transmitter divides $W_l$, $l = 1, 2$, into the independent messages $W_l^{(1)}$, $W_l^{(2)}$, $W_{l,s}$; $W_l^{(1)}$, $W_l^{(2)}$ are uniform over $[1 : 2^{n^{\frac{1-\alpha}{2}}}]$, and $W_{l,s}$ is uniform over $[1 : 2^{n^{\frac{\alpha}{2}}}]$. The transmitter generates the independent keys $K_1, K_2$, uniform over $[1 : 2^{n^{\frac{\mu_1}{2}}}]$ and independent from $W_1, W_2$. For simplicity of exposition, we have ignored the small rate reduction $\epsilon_n$ at this stage, as we will introduce this later into the security analysis. The main ideas of the achievability proof are:
1) The transmitter uses wiretap coding with a randomization message of size $n(\alpha_1 + \alpha_2) = n\alpha$ bits in both the cache placement and delivery phases. As the adversary does not tap into more than $n\alpha$ bits in each phase, a secure transmission rate of $1 - \alpha$ is achievable in each phase, as long as the randomization messages in the two phases are independent. Using coded placement for $W_{r(1)}^1, W_{r(2)}^1, W_{r(1)}^2, W_{r(2)}^2$, a secure file rate of $1 - \alpha$ can be achieved.

2) The randomization messages over the two phases can deliver additional secure information, of rate $\frac{\alpha_2}{2}$ per file, via encryption. The overall achievable file rate is thus $R_s = 1 - \frac{\alpha_2}{2}$. In particular, we utilize the keys $K_1, K_2$, as the randomization message for cache placement. Along with wiretap coding, we employ a security embedding code [44], by using bits of $K_1, K_2$, in a manner that allows the adversary to be able to retrieve only the last $n\alpha_1\frac{n_2}{2}$ bits from each. In the delivery phase, we encrypt additional pieces of information, $W_{d_1,s}$ and $W_{d_2,s}$, with the keys $K_1$ and $K_2$, and utilize this encrypted information as the randomization message. We employ again a security embedding code, in the reverse order, such that the adversary can only retrieve the first $n\alpha_2\frac{n_2}{2}$ bits from each of $W_{d_1,s} \oplus K_1$ and $W_{d_2,s} \oplus K_2$.

3) With the aforementioned construction, the adversary, for any values of $\alpha_1$ and $\alpha_2$ it chooses, can only retrieve a set of key bits and/or a set of information bits encrypted with other key bits. In particular, due to the reversed embedding order, the adversary does not obtain, in the delivery phase, any message bits encrypted with key bits it has seen during cache placement. In addition, since $\{K_1, K_2\}$ is independent from $\{W_{d_1,s} \oplus K_1, W_{d_2,s} \oplus K_2\}$, and is an independent sequence, the adversary can not use the revealed key bits in the cache placement to obtain any information about the bits of $W_{d_1,s} \oplus K_1$ and $W_{d_2,s} \oplus K_2$ that need to be securely transmitted in the delivery phase.

We now explain the achievability scheme in more detail. Let us define $M_c$ and $\tilde{M}_c$ as in (20) and (21). In particular, let

\begin{align}
M_c &= \{M_{c,1}, M_{c,2}\}; \quad M_{c,1} = W_{r(1)}^1 \oplus W_{r(2)}^1, \quad M_{c,2} = W_{r(2)}^1 \oplus W_{r(2)}^2, \tag{24} \\
\tilde{M}_c &= \{\tilde{M}_{c,1}, \tilde{M}_{c,2}\}; \quad \tilde{M}_{c,1} = K_1, \quad \tilde{M}_{c,2} = K_2. \tag{25}
\end{align}

$M_c$ in (24) represents the message to be securely transmitted during cache placement, regardless of the adversary’s choice of $\alpha_1$. $\tilde{M}_c$ in (25) represents the randomization message utilized for wiretap coding in the cache placement.
The transmitter further divides $\tilde{M}_{c,1}$ and $\tilde{M}_{c,2}$ into sequences of independent binary bits, 
\[
\{ \tilde{M}_{c,1}^{(1)}, \tilde{M}_{c,1}^{(2)}, \ldots, \tilde{M}_{c,1}^{(\frac{n}{2})} \} \quad \text{and} \quad \{ \tilde{M}_{c,2}^{(1)}, \tilde{M}_{c,2}^{(2)}, \ldots, \tilde{M}_{c,2}^{(\frac{n}{2})} \},
\]
and generates $X^n_c$ as follows:

**Cache Placement Codebook Generation:** Let 
\[
m_c, \tilde{m}_{c,1} = \left\{ \tilde{m}_{c,1}^{(1)}, \tilde{m}_{c,1}^{(2)}, \ldots, \tilde{m}_{c,1}^{(\frac{n}{2})} \right\},
\]
and 
\[
\tilde{m}_{c,2} = \left\{ \tilde{m}_{c,2}^{(1)}, \tilde{m}_{c,2}^{(2)}, \ldots, \tilde{m}_{c,2}^{(\frac{n}{2})} \right\}
\]
be the realizations of $M_c$, $\tilde{M}_{c,1}$, and $\tilde{M}_{c,2}$ in (24) and (25).

We construct the cache placement codebook $C_{c,n}$, from which $X^n_c$ is drawn, as follows. We randomly and independently distribute all the possible $2^n$ length-$n$ binary sequences into $2^{n(1-\alpha)}$ bins, indexed by $m_c \in \left[ 1 : 2^{n \frac{1-\alpha}{2}} \right]^2$. Each bin $m_c$ contains $2^{n\alpha}$ binary sequences (codewords). Further, we randomly and independently divide each bin $m_c$ into two sub-bins, indexed by $\tilde{m}_{c,1}^{(1)}$,
and each contains \(2^{n\alpha-1}\) codewords. The two sub-bins \(\tilde{m}_{c,1}^{(1)}\) are further divided into smaller bins, indexed by \(\tilde{m}_{c,2}^{(1)}\), and each contains \(2^{n\alpha-2}\) codewords. The process continues, going over \(\tilde{m}_{c,1}^{(2)}, \tilde{m}_{c,2}^{(2)}, \ldots, \tilde{m}_{c,1}^{(\lfloor \frac{n}{2} \rfloor - 1)}, \tilde{m}_{c,2}^{(\lfloor \frac{n}{2} \rfloor - 1)}, \tilde{m}_{c,1}^{(\lfloor \frac{n}{2} \rfloor)}, \tilde{m}_{c,2}^{(\lfloor \frac{n}{2} \rfloor)}\), until the remaining two codewords, after each sequence of divisions, are indexed by \(\tilde{m}_{c,2}^{(\lfloor \frac{n}{2} \rfloor)}\). The codebook \(C_{c,n}\) is described in Fig. 3.

**Remark 6** An alternative representation of the binning procedure described above is that, each of the \(2^{n\alpha}\) binary codewords in the bin \(m_c\), where \(m_c \in \left[ 1 : 2^{n\frac{1-\alpha}{2}} \right]^2\), is randomly assigned to an index \(\left\{ \tilde{m}_{c,1}^{(1)}, \tilde{m}_{c,2}^{(1)}, \tilde{m}_{c,1}^{(2)}, \tilde{m}_{c,2}^{(2)}, \ldots, \tilde{m}_{c,1}^{(\lfloor \frac{n}{2} \rfloor)}, \tilde{m}_{c,2}^{(\lfloor \frac{n}{2} \rfloor)} \right\}\). We chose however to present the former description in order to provide a more detailed explanation of the embedding structure; in particular, the order of embedding, which is a critical component in the achievability scheme.

**Cache Encoder:** Given the messages \(w_1, w_2\), i.e., \(\left\{ w_1^{(1)}, w_1^{(2)}, w_{1,s} \right\}, \left\{ w_2^{(1)}, w_2^{(2)}, w_{2,s} \right\}\), the transmitter generates \(m_c, \tilde{m}_c = \{\tilde{m}_{c,1}, \tilde{m}_{c,2}\}\) as in (24), (25). Using the codebook \(C_{c,n}\), the transmitter sends \(x_c^n\) which corresponds to \(m_c, \tilde{m}_{c,1}, \tilde{m}_{c,2}\), i.e., \(x_c^n \left( m_c, \tilde{m}_{c,1}^{(1)}, \tilde{m}_{c,2}^{(1)}, \ldots, \tilde{m}_{c,1}^{(\lfloor \frac{n}{2} \rfloor)}, \tilde{m}_{c,2}^{(\lfloor \frac{n}{2} \rfloor)} \right)\).

For the delivery phase, as in (22) and (23), define

\[
M_d = \left\{ W_{d_1}^{(2)}, W_{d_2}^{(1)} \right\},
\]
\[
\tilde{M}_d = \left\{ \tilde{M}_{d,1}, \tilde{M}_{d,2} \right\}; \quad \tilde{M}_{d,1} = W_{d_1,s} \oplus K_1, \quad \tilde{M}_{d,2} = W_{d_2,s} \oplus K_2.
\]  

(26) \hspace{1cm} (27)

\(M_d\) in (26) represents the message to be securely transmitted during the delivery phase no matter what the adversary’s choice of \(\alpha_2\) is. \(\tilde{M}_d\) in (27) represents the randomization message utilized for the wiretap coding in the delivery phase.

Similar to cache placement, the transmitter further divides \(\tilde{M}_{d,1}, \tilde{M}_{d,2}\) into sequences of independent binary bits, \(\left\{ \tilde{M}_{d,1}^{(1)}, \ldots, \tilde{M}_{d,1}^{(\lfloor \frac{n}{2} \rfloor)} \right\}, \left\{ \tilde{M}_{d,2}^{(1)}, \ldots, \tilde{M}_{d,2}^{(\lfloor \frac{n}{2} \rfloor)} \right\}\), and generates \(X_d^n\) as follows.

**Delivery Codebook Generation:** Let \(m_d, \tilde{m}_{d,1} = \{\tilde{m}_{d,1}^{(1)}, \ldots, \tilde{m}_{d,1}^{(\lfloor \frac{n}{2} \rfloor)}\}, \tilde{m}_{d,2} = \{\tilde{m}_{d,2}^{(1)}, \ldots, \tilde{m}_{d,2}^{(\lfloor \frac{n}{2} \rfloor)}\}\) be the realizations of \(M_d, \tilde{M}_{d,1}, \tilde{M}_{d,2}\) in (26), (27). We construct the delivery codebook \(C_{d,n}\), from which \(X_d^n\) is drawn, in a similar fashion as the codebook \(C_{c,n}\), but with a reversed indexing of the sub-bins. In particular, we randomly and independently divide all the \(2^n\) binary sequences into \(2^{n(1-\alpha)}\) bins, indexed by \(m_d \in \left[ 1 : 2^{n\frac{1-\alpha}{2}} \right]^2\), and each contains \(2^{n\alpha}\) codewords. We further randomly and independently divide each bin \(m_d\) into two sub-bins, indexed by \(\tilde{m}_{d,1}^{(\lfloor \frac{n}{2} \rfloor)}, \tilde{m}_{d,2}^{(\lfloor \frac{n}{2} \rfloor)}\), and each contains \(2^{n\alpha-1}\) codewords. The process continues, going in reverse order over \(\tilde{m}_{d,2}^{(\lfloor \frac{n}{2} \rfloor)}, \tilde{m}_{d,1}^{(\lfloor \frac{n}{2} \rfloor)}\),
Fig. 4: Codebook construction for the delivery phase, C_{d,n}.

\[ \tilde{m}_{d,2}^{(n-2)}, \ldots, \tilde{m}_{d,1}^{(1)} \] until the remaining two codewords, after each sequence of divisions, are indexed by \( \tilde{m}_{d,2}^{(1)} \). The codebook \( C_{d,n} \) is described in Fig. 4.

**Delivery Encoder:** Given \( w_1, w_2, \) i.e., \( \{w_1^{(1)}, w_1^{(2)}, w_{1,s}\}, \{w_2^{(1)}, w_2^{(2)}, w_{2,s}\} \), and \( d = (d_1, d_2) \), the transmitter generates \( m_d, \tilde{m}_d = \{\tilde{m}_{d,1}, \tilde{m}_{d,2}\} \) as in (26), (27). The transmitter sends \( x_n^d \), from \( C_{d,n} \), which corresponds to \( m_d, \tilde{m}_{d,1}, \) and \( \tilde{m}_{d,2} \), i.e., \( x_n^d( m_d, \tilde{m}_{d,1}^{(n-2)}, \tilde{m}_{d,2}^{(n-2)}, \ldots, \tilde{m}_{d,1}^{(1)}, \tilde{m}_{d,2}^{(1)} ) \).

**Decoding:** Using \( X_n^c \), receiver \( j, j = 1, 2 \), recovers \( M_{c,j}, \tilde{M}_{c,j} \), and stores them in its cache memory. For \( j = 1, 2 \), the combined size of \( M_{c,j} \) and \( \tilde{M}_{c,j} \) does not exceed \( n/2 \) bits. Using \( X_n^d \), both receivers recover \( M_d, \tilde{M}_d \). Using \( M_d, \tilde{M}_d, M_{c,j}, \tilde{M}_{c,j} \), and for \( n \) sufficiently large, receiver \( j \) correctly decodes \( W_{d,j} \).
\textit{Security Analysis:} Let us first slightly modify the construction above as follows. Recall that 
\{\epsilon_n\}_{n \geq 1} is a sequence of positive real numbers such that \(\epsilon_n \to 0\) as \(n \to \infty\). Define

\[
\alpha_\epsilon = \alpha + 2\epsilon_n, \quad \alpha_{1,\epsilon} = \alpha_1 + \epsilon_n, \quad \alpha_{2,\epsilon} = \alpha_2 + \epsilon_n.
\]  
(28)

That is, \(\alpha_{1,\epsilon} + \alpha_{2,\epsilon} = \alpha_\epsilon\). We increase the sizes of \(K_1\) and \(K_2\) into \(\frac{n\alpha_\epsilon}{2}\) bits, from \(n\frac{\alpha}{2}\), and zero-pad the bit strings of \(W_{d_1,s}\) and \(W_{d_2,s}\) accordingly. Additionally, we decrease the sizes of \(W_l^{(1)}, W_l^{(2)}\), \(l = 1, 2\), to \(n\frac{1-\alpha_\epsilon}{2}\) bits, instead of \(n\frac{1-\alpha}{2}\). Once again, we assume that \(\frac{n\alpha_\epsilon}{2}\) and \(\frac{n\alpha_{1,\epsilon}}{2}\) are integers; as minor modifications can be adopted otherwise.

Let us fix the subsets \(S_1, S_2 \subseteq [1 : n]\). For the corresponding (fixed) values of \(\alpha_1\) and \(\alpha_2\), the cache placement codebook \(C_{c,n}\) can be viewed as a wiretap code with \(2^{n(1-\alpha_1,\epsilon)}\) bins. Each bin is indexed by the message

\[
w_c = \left( m_c, \tilde{m}_{c,1}^{(1)}, \tilde{m}_{c,2}^{(1)}, \tilde{m}_{c,1}^{(2)}, \tilde{m}_{c,2}^{(2)}, \ldots, \tilde{m}_{c,1}^{(\epsilon_2,\epsilon)}, \tilde{m}_{c,2}^{(\epsilon_2,\epsilon)} \right). 
\]  
(29)

Each bin \(w_c\) contains \(2^{n\alpha_{1,\epsilon}}\) binary codewords which are indexed by the randomization message

\[
\tilde{w}_c = \left( \tilde{m}_{c,1}^{(\epsilon_2,\epsilon)+1}, \tilde{m}_{c,2}^{(\epsilon_2,\epsilon)+1}, \tilde{m}_{c,1}^{(\epsilon_2,\epsilon)+2}, \tilde{m}_{c,2}^{(\epsilon_2,\epsilon)+2}, \ldots, \tilde{m}_{c,1}^{(\epsilon_1)}, \tilde{m}_{c,2}^{(\epsilon_1)} \right). 
\]  
(30)

Similarly, the delivery codebook \(C_{d,n}\) can be seen as a wiretap code with \(2^{n(1-\alpha_{2,\epsilon})}\) bins, each of which is indexed by the message

\[
w_d = \left( m_d, \tilde{m}_{d,1}^{(\epsilon_1)}, \tilde{m}_{d,2}^{(\epsilon_1)}, \tilde{m}_{d,1}^{(\epsilon_2)}, \tilde{m}_{d,2}^{(\epsilon_2)}, \ldots, \tilde{m}_{d,1}^{(\epsilon_2,\epsilon)+1}, \tilde{m}_{d,2}^{(\epsilon_2,\epsilon)+1} \right). 
\]  
(31)

Each bin \(w_d\) contains \(2^{n\alpha_{2,\epsilon}}\) codewords, indexed by the randomization message

\[
\tilde{w}_d = \left( \tilde{m}_{d,1}^{(\epsilon_2,\epsilon)}, \tilde{m}_{d,2}^{(\epsilon_2,\epsilon)}, \tilde{m}_{d,1}^{(\epsilon_2,\epsilon)+1}, \tilde{m}_{d,2}^{(\epsilon_2,\epsilon)+1}, \ldots, \tilde{m}_{d,1}^{(1)}, \tilde{m}_{d,2}^{(1)} \right). 
\]  
(32)

Let \(\{\mathcal{B}_{w_c} : w_c = 1, 2, \ldots, 2^{n(1-\alpha_{1,\epsilon})}\}\) and \(\{\mathcal{B}_{w_d} : w_d = 1, 2, \ldots, 2^{n(1-\alpha_{2,\epsilon})}\}\) denote the partition, i.e., bins, of the codebooks \(C_{c,n}\) and \(C_{d,n}\), which correspond to the messages \(w_c\) and \(w_d\) in (29) and (31), respectively. Let \(x^{2n} \triangleq (x^n_c, x^n_d)\) denote the concatenation of the two length-\(n\) binary codewords \(x^n_c, x^n_d\). Define the Cartesian product of the bins \(\mathcal{B}_{w_c}\) and \(\mathcal{B}_{w_d}\), as

\[
\mathcal{B}_{w_c \times w_d} \triangleq \{ x^{2n} = (x^n_c, x^n_d) : x^n_c \in \mathcal{B}_{w_c}, x^n_d \in \mathcal{B}_{w_d} \}. 
\]  
(33)
Since the partitioning of the codebooks $C_{c,n}$ and $C_{d,n}$ is random, for every $w_c$ and $w_d$, $B_{w_c,w_d}$ is a random codebook which results from the Cartesian product of the random bins $B_{w_c}$, $B_{w_d}$. Recall that $B_{w_c}$ contains $2^{n\alpha_1,\epsilon}$ and $B_{w_d}$ contains $2^{n\alpha_2,\epsilon}$ length-$n$ binary codewords. Thus, the product $B_{w_c,w_d}$ contains $2^{n\alpha_1 + \alpha_2,\epsilon}$ length-$2n$ binary codewords.

Let $\left\{W^{(1)}_{d_1,s}, W^{(2)}_{d_1,s}, \ldots, W^{(n\alpha_2,\epsilon)}_{d_1,s}\right\}$ and $\left\{K^{(1)}_l, K^{(2)}_l, \ldots, K^{(n\alpha_2,\epsilon)}_l\right\}$ denote the binary bit strings of $W_{d_1,s}$ and $K_l$, $l = 1, 2$. In addition, for notational simplicity, define

$$W^{(1)}_s = \left\{W^{(1)}_{d_1,s}, W^{(1)}_{d_2,s}, \ldots, W^{(n\alpha_2,\epsilon)}_{d_1,s}, W^{(n\alpha_2,\epsilon)}_{d_2,s}\right\},$$

$$W^{(2)}_s = \left\{W^{(n\alpha_2,\epsilon)+1}_{d_1,s}, W^{(n\alpha_2,\epsilon)+1}_{d_2,s}, \ldots, W^{(n\alpha_2,\epsilon)}_{d_1,s}, W^{(n\alpha_2,\epsilon)}_{d_2,s}\right\},$$

$$K^{(1)} = \left\{K^{(1)}_1, K^{(1)}_2, \ldots, K^{(n\alpha_2,\epsilon)}_1, K^{(n\alpha_2,\epsilon)}_2\right\},$$

$$K^{(2)} = \left\{K^{(n\alpha_2,\epsilon)+1}_1, K^{(n\alpha_2,\epsilon)+1}_2, \ldots, K^{(n\alpha_2,\epsilon)}_1, K^{(n\alpha_2,\epsilon)}_2\right\},$$

$$W^{(1)}_{\oplus K} = \left\{W^{(i)}_{d_1,s} \oplus K^{(i)}_1, W^{(i)}_{d_2,s} \oplus K^{(i)}_2 : i = 1, 2, \ldots, n\alpha_2\right\},$$

$$W^{(2)}_{\oplus K} = \left\{W^{(i)}_{d_1,s} \oplus K^{(i)}_1, W^{(i)}_{d_2,s} \oplus K^{(i)}_2 : i = n\alpha_2 + 1, n\alpha_2 + 2, \ldots, n\alpha_2 + 1\right\}.$$

Let $W_c$, $\tilde{W}_c$, $W_d$, and $\tilde{W}_d$ denote the random variables that correspond to the realizations defined in (29)–(32). Using (24)–(27), (29)–(32), and (36)–(39), we have

$$W_c = \left\{M_c, K^{(1)}\right\} = \left\{W^{(1)}_1 \oplus W^{(1)}_2, W^{(2)}_1 \oplus W^{(2)}_2, K^{(1)}_1\right\}, \quad \tilde{W}_c = K^{(2)}.$$

$$W_d = \left\{M_d, W^{(2)}_{\oplus K}\right\} = \left\{W^{(2)}_{d_1}, W^{(1)}_{d_2}, W^{(2)}_{\oplus K}\right\}, \quad \tilde{W}_d = W^{(1)}_{\oplus K}.$$

Notice that $\tilde{W}_c$ and $\tilde{W}_d$ are independent, and each is uniformly distributed. $\{\tilde{W}_c, \tilde{W}_d\}$ is thus jointly uniform. In addition, $\{\tilde{W}_c, \tilde{W}_d\}$ is independent from $\{W_c, W_d\}$. Thus, we can apply the analysis in [38 (94)-(103)] to show that, for every $S_1$, $S_2$, $w_c$, and $w_d$, and every $\epsilon > 0$, there exists $\gamma(\epsilon) > 0$ such that

$$P_{B_{w_c,w_d}} \left( \mathcal{D} \left( P_{Z_{S_1}^n Z_{S_2}^n | W_c = w_c, W_d = w_d} \right) \left| P_{Z_{S_1}^n Z_{S_2}^n} \right) > \epsilon \right) \leq \exp \left( -e^{\gamma(\epsilon)} \right).$$

$P_{Z_{S_1}^n Z_{S_2}^n | W_c = w_c, W_d = w_d}$ is the induced distribution at the adversary when $\mathcal{X}_c^n(w_c, \tilde{w}_c)$ and $\mathcal{X}_d^n(w_d, \tilde{w}_d)$ are the transmitted codewords over cache placement and delivery phases. $P_{Z_{S_1}^n Z_{S_2}^n}$ is the output distribution at the adversary.

The number of the messages $\{w_c, w_d\}$ is $2^{n(2-\alpha_1)}$. Additionally, the number of possible choices
for the subsets $S_1$ and $S_2$ is $\binom{2n}{n} < 2^{2n}$. Thus, the combined number of the messages and the subsets is at most exponential in $n$. Using (42) and the union bound, as in [38, 39], we have

$$\lim_{n \to \infty} \max_{S_1, S_2} I (W_c, W_d; Z^n_{S_1}, Z^n_{S_2}) = 0.$$  \hspace{1cm} (43)

For the sake of completeness, we provide the full proofs for (42) and (43) in Appendix E.

We also have, for any $d = (d_1, d_2), d_1, d_2 \in \{1, 2\}$,

$$I (W_1, W_2; Z^n_{S_1}, Z^n_{S_2}) = I \left( W_1^{(1)}, W_2^{(1)}, W_1^{(2)}, W_2^{(2)}, W_{1,s}, W_{2,s}; Z^n_{S_1}, Z^n_{S_2} \right)$$

$$= I \left( W_1^{(1)}, W_1^{(2)}, W_2^{(1)}, W_2^{(2)}, W^{(2)}, W_s^{(1)}, W_s^{(2)}; Z^n_{S_1}, Z^n_{S_2} \right)$$

$$= I \left( W_1^{(1)} \oplus W_2^{(1)}, W_1^{(2)} \oplus W_2^{(2)}, W_{d_1}, W_{d_2}, W^{(2)}, W_s^{(1)}, W_s^{(2)}; Z^n_{S_1}, Z^n_{S_2} \right)$$

$$= I \left( M_c, M_d, W_s^{(1)}, W_s^{(2)}; Z^n_{S_1}, Z^n_{S_2} \right)$$

$$\leq I \left( M_c, M_d, W_s^{(1)}; W_s^{(2)} @ K; Z^n_{S_1}, Z^n_{S_2} \right)$$

$$= I \left( M_c, W_s^{(1)}, W_d; Z^n_{S_1}, Z^n_{S_2} \right)$$

$$= H (Z^n_{S_1}, Z^n_{S_2}) - H \left( Z^n_{S_1}, Z^n_{S_2} \mid M_c, W_s^{(1)}, W_d \right).$$  \hspace{1cm} (50)

Equation (45) follows since, for $d = (d_1, d_2), Z^n_{S_1}$ and $Z^n_{S_2}$ depend only on $W_1^{(1)}, W_1^{(2)}, W_2^{(1)}, W_2^{(2)}, W_{d_1,s}$, and $W_{d_2,s}$, and by using (34) and (35). Equation (46) follows because there exists a bijection between $\{W_1^{(1)}, W_1^{(2)}, W_2^{(1)}, W_2^{(2)}\}$ and $\{W_1^{(1)} \oplus W_2^{(1)}, W_1^{(2)} \oplus W_2^{(2)}, W_{d_1}^{(2)}, W_{d_2}^{(2)}\}$. Equation (47) follows from (24) and (26). The inequality in (48) follows due to the Markov chain $W_s^{(2)} - \{M_c, M_d, W_s^{(1)}; W_s^{(2)} \} - \{Z^n_{S_1}, Z^n_{S_2}\}$, and the data processing inequality. This Markov chain holds because $\{M_c, M_d, W_s^{(1)}\}$ are independent from $\{W_s^{(2)}; K^{(2)}\}$, and only the encrypted information $W_s^{(2)}$ is transmitted. Equation (49) follows from (41).

The second term on the right hand side of (50) can be lower bounded as

$$H \left( Z^n_{S_1}, Z^n_{S_2} \mid M_c, W_s^{(1)}, W_d \right) = H \left( Z^n_{S_1}, Z^n_{S_2}, W_s^{(1)} \mid M_c, W_d \right) - H \left( W_s^{(1)} \mid M_c, W_d \right)$$

$$= H \left( Z^n_{S_1}, Z^n_{S_2}, W_s^{(1)}; W_s^{(1)} @ K \mid M_c, W_d \right)$$

$$- H \left( W_s^{(1)} \mid M_c, W_d, W_s^{(1)}, Z^n_{S_1}, Z^n_{S_2} \right) - H \left( W_s^{(1)} \right)$$

$$= H \left( Z^n_{S_1}, Z^n_{S_2}, K^{(1)}; W_s^{(1)} @ K \mid M_c, W_d \right)$$
Remark 7 Although the cache placement and delivery codebooks, $\mathcal{C}_{c,n}$ and $\mathcal{C}_{d,n}$, are designed and generated disjointly, in the security analysis, we have considered the Cartesian products of the individual bins of the two codebooks. We were able to do so since the input distributions for generating the two codebooks are identical, i.e., uniform binary. ■
For $\alpha \in [1, 2]$, we adapt the achievability scheme described in Section IV-C as follows. The messages $W_1$ and $W_2$ are uniform over $[1 : 2^{n^{2-\alpha}}]$; $\alpha_\epsilon$ is defined in (28). The transmitter generates the independent keys $K_1$, $K_2$, uniform over $[1 : 2^{n^{2-\alpha}}]$, and independent from $W_1$, $W_2$. In addition, the transmitter, independently from $W_1$, $W_2$, $K_1$, $K_2$, generates the independent randomization messages $\tilde{W}$ and $\tilde{W}_K$, uniformly over $[1 : 2^{n^{(\alpha-1)}}]$.

The messages for cache placement at receivers 1 and 2 are

$$M_{c,1} = K_1, \quad M_{c,2} = K_2. \quad (60)$$

That is, receiver $j$, $j = 1, 2$, stores the key $K_j$ in its cache memory. The message to be securely transmitted during delivery is

$$M_d = \{M_{d,1}, M_{d,2}\}; \quad M_{d,1} = W_{d,1} \oplus K_1, \quad M_{d,2} = W_{d,2} \oplus K_2. \quad (61)$$

Let $\{W_{d,1}^{(1)}, \cdots, W_{d,1}^{(n^{2-\alpha})}\}$, $\{K_1^{(1)}, \cdots, K_1^{(n^{2-\alpha})}\}$, and $\{M_{d,1}^{(1)}, \cdots, M_{d,1}^{(n^{2-\alpha})}\}$ denote the bit strings of $W_{d,1}$, $K_1$, and $M_{d,1}$, $l = 1, 2$.

Notice that, for $\alpha \in [1, 2]$, the adversary can see all symbols in at least one of the phases. Hence, unlike Section IV-C, we cannot utilize randomization messages, $\tilde{W}$ and $\tilde{W}_K$, to carry any information; only keys are stored in the cache memories of the receivers. Additionally, the cache placement and delivery codebooks for this case have a different embedding structure than for $\alpha \in (0, 1)$ in Section IV-C. In particular, the embedding here is performed on the bits of the messages $M_c$ and $M_d$, while the embedding in Section IV-C is performed on the bits of the randomization messages $\tilde{M}_c$ and $\tilde{M}_d$.

**Cache Placement Codebook Generation:** During cache placement, the transmitter generates $C_{c,n}$ as follows. The transmitter randomly and independently divides all the $2^n$ length-$n$ binary sequences into 2 bins, indexed by $K_1^{(1)}$, and each contains $2^{n-1}$ codewords. These two bins are further randomly and independently divided into two sub-bins, indexed by $K_2^{(1)}$, and each contains $2^{n-2}$ codewords. The process continues, going over $K_1^{(2)}$, $K_2^{(2)}$, $\cdots$, $K_1^{(n^{2-\alpha})}$, $K_2^{(n^{2-\alpha})}$, until the remaining $2^{n^{(\alpha-1)}}$ codewords, after each sequence of divisions, are indexed by $\tilde{W}_K$.

**Cache Encoder:** The transmitter sends $X^n_c$ which corresponds to the keys $K_1$, $K_2$, and the
randomization message $\bar{W}_K$, i.e., $X^n_c \left( K_1^{(1)}, K_2^{(1)}, \ldots, K_1^{(n^-\alpha_x)}, K_2^{(n^-\alpha_x)}, \bar{W}_K \right)$.

**Delivery Codebook Generation:** In the delivery phase, the transmitter generates $C_{d,n}$ as follows. The transmitter randomly and independently divides all the $2^n$ length-$n$ binary sequences into two bins, indexed by $M_{d,1}^{(2^n-\alpha_x)}$, and each contains $2^{n-1}$ codewords. These two bins are further randomly and independently divided into two sub-bins, indexed by $M_{d,2}^{(n^2-\alpha_x)}$, and each contains $2^{n-2}$ codewords. The process continues, going in reverse order over $M_{d,1}^{(2^n-\alpha_x)}$, $M_{d,2}^{(n^2-\alpha_x)}$, $\ldots$, $M_{d,1}^{(1)}$, $M_{d,2}^{(1)}$, until the remaining $2^{n(\alpha_x-1)}$ codewords, after each sequence of divisions, are indexed by the randomization message $\bar{W}$.

**Delivery Encoder:** Given $W_1$, $W_2$, $K_1$, $K_2$, $\bar{W}$, and $d = (d_1, d_2)$, the transmitter forms $M_{d,1}$ and $M_{d,2}$ as in (61) and sends $X^n_d$ which corresponds to $M_{d,1}$, $M_{d,2}$, and $\bar{W}$, i.e.,

$X^n_d \left( M_{d,1}^{(n^2-\alpha_x)}, M_{d,2}^{(n^2-\alpha_x)}, \ldots, M_{d,1}^{(1)}, M_{d,2}^{(1)}, \bar{W}_K \right)$.

**Decoding:** Using $X^n_d$, receiver $j$, $j = 1, 2$, recovers $M_{c,j} = K_j$ and stores it in its cache memory. Using $X^n_d$, both receivers recover $M_d = \{M_{d,1}, M_{d,2}\}$. Using $M_{d,j}$, $K_j$, and for $n$ sufficiently large, receiver $j$ correctly decodes $W_{d,j}$.

**Security Analysis:** Fix the subsets $S_1$, $S_2$. Recall that $\alpha_1, \alpha_2 \leq 1$. Since $\alpha \geq 1$, $\alpha_1, \alpha_2 \geq \alpha - 1$. If $\alpha_1 = 1$, then $\alpha_2 = \alpha - 1$, and vice versa. In addition, notice that $1 - \alpha_1, 1 - \alpha_2 \leq 2 - \alpha$.

As in Section IV-C for a fixed value of $\alpha_1$, the codebook $C_{c,n}$ is a wiretap code with $2^{n(1 - \alpha_1, \epsilon)}$ bins, indexed by

$W_c = \left( K_1^{(1)}, K_2^{(1)}, \ldots, K_1^{(n^-\alpha_1, \epsilon)}, K_2^{(n^-\alpha_1, \epsilon)} \right)$. \hspace{1cm} (62)

Each bin $W_c$ contains $2^{n\alpha_1, \epsilon}$ binary codewords, indexed by

$\tilde{W}_c = \left( K_1^{(n^-\alpha_1, \epsilon + 1)}, K_2^{(n^-\alpha_1, \epsilon + 1)}, \ldots, K_1^{(2^-\alpha)}, K_2^{(2^-\alpha)}, \bar{W}_K \right)$. \hspace{1cm} (63)

Similarly, for a fixed value of $\alpha_2$, the codebook $C_{d,n}$ is a wiretap code with $2^{n(1 - \alpha_2, \epsilon)}$ bins, each is indexed by

$W_d = \left( \tilde{M}_{d,1}^{(n^-\alpha_2)}, \tilde{M}_{d,2}^{(n^-\alpha_2)}, \ldots, \tilde{M}_{d,1}^{(n^-\alpha_2 + 1)}, \tilde{M}_{d,2}^{(n^-\alpha_2 + 1)} \right)$. \hspace{1cm} (64)

Each bin $W_d$ contains $2^{n\alpha_2, \epsilon}$ codewords, indexed by

$\tilde{W}_d = \left( \tilde{M}_{d,1}^{(n^-\alpha_2)}, \tilde{M}_{d,2}^{(n^-\alpha_2)}, \ldots, \tilde{M}_{d,1}^{(1)}, \tilde{M}_{d,2}^{(1)}, \bar{W} \right)$. \hspace{1cm} (65)
Let us re-define

$$K^{(1)} = \left\{ K_1^{(i)}, K_2^{(i)} : i = 1, \ldots, n, \frac{1 - \alpha_1}{2} \right\},$$

$$K^{(2)} = \left\{ K_1^{(i)}, K_2^{(i)} : i = n - \frac{1 - \alpha_1}{2} + 1, \ldots, n \frac{2 - \alpha_1}{2} \right\},$$

$$W^{(1)}_{@K} = \left\{ W_{d_1}^{(i)} \oplus K_1^{(i)}, W_{d_2}^{(i)} \oplus K_2^{(i)} : i = 1, \ldots, n, \frac{1 - \alpha_1}{2} \right\},$$

$$W^{(2)}_{@K} = \left\{ W_{d_1}^{(i)} \oplus K_1^{(i)}, W_{d_2}^{(i)} \oplus K_2^{(i)} : i = n - \frac{1 - \alpha_1}{2} + 1, \ldots, n \frac{2 - \alpha_1}{2} \right\},$$

and define

$$W^{(1)} = \left\{ W_{d_1}^{(i)}, W_{d_2}^{(i)} : i = 1, \ldots, n, \frac{1 - \alpha_1}{2} \right\},$$

$$W^{(2)} = \left\{ W_{d_1}^{(i)}, W_{d_2}^{(i)} : i = n - \frac{1 - \alpha_1}{2} + 1, \ldots, n \frac{2 - \alpha_1}{2} \right\},$$

From (62)-(69), we have

$$W_c = K^{(1)}, \quad \tilde{W}_c = \left\{ K^{(2)}, \tilde{W}_K \right\}, \quad W_d = W^{(2)}_{@K}, \quad \tilde{W}_d = \left\{ W^{(1)}_{@K}, \tilde{W} \right\}. \quad (72)$$

Similar to Section IV-C, $W_c, \tilde{W}_d$, are independent and uniformly distributed, and hence $\{W_c, \tilde{W}_d\}$ is jointly uniform. Additionally, $\{W_c, \tilde{W}_d\}$ is independent from $\{W_c, W_d\}$. Thus, (43) is satisfied.

We also have, for any $d = (d_1, d_2)$,

$$I(W_1, W_2; Z^n_{S_1}, Z^n_{S_2}) = I(W^{(1)}, W^{(2)}; Z^n_{S_1}, Z^n_{S_2}) \quad (73)$$

$$\leq I(W^{(1)}, W^{(2)}; Z^n_{S_1}, Z^n_{S_2}) \quad (74)$$

$$= I(W^{(1)}, W_d; Z^n_{S_1}, Z^n_{S_2}) \quad (75)$$

$$= H(Z^n_{S_1}, Z^n_{S_2}) - H(Z^n_{S_1}, Z^n_{S_2} | W^{(1)}, W_d) \quad (76)$$

$$\leq H(Z^n_{S_1}, Z^n_{S_2}) - H(Z^n_{S_1}, Z^n_{S_2} | K^{(1)}, W_d) + \epsilon' \quad (77)$$

$$= I(W_c, W_d; Z^n_{S_1}, Z^n_{S_2}) + \epsilon'. \quad (78)$$

The inequality in (74) follows due to the Markov chain $W^{(2)} = \left\{ W^{(1)}, W^{(2)}_{@K} \right\} - \left\{ Z^n_{S_1}, Z^n_{S_2} \right\}$. Equations (75) and (78) follow from (72). The inequality in (77) follows by using similar steps as in (51)-(57). Using (43) and (78), the secrecy constraint in (3) is satisfied. Since $\epsilon_n \to 0$ as
As $n \to \infty$, the achievable strong secrecy file rate is

$$R_s(\alpha) = \frac{2 - \alpha}{2} = 1 - \frac{\alpha}{2}. \tag{79}$$

This completes the proof for Theorem 1.

V. PROOF OF THEOREM 2

In this section, we extend the achievability scheme presented in Section IV and provide a lower bound on the achievable strong secrecy file rate when $D > 2$. The demand vector is again denoted by $d = (d_1, d_2)$, where $d_1, d_2 \in [1 : D]$. As in Section IV we divide the proof into two cases for the ranges $\alpha \in (0, 1)$ and $\alpha \in [1, 2]$.

A. $\alpha \in [1, 2]$

For $\alpha \in [1, 2]$, we utilize the same achievability scheme in Section IV-D. The reason behind this is, for this range of $\alpha$, only the keys $K_1, K_2$, are transmitted in the cache placement, and stored in receivers 1 and 2 cache memories, respectively. That is, no information messages are stored in the caches, and the user demands are known during the delivery phase. The achievable strong secrecy file rate is $1 - \frac{\alpha}{2}$.

B. $\alpha \in (0, 1)$

The achievability scheme for this case has the same channel coding structure as in the scheme described in Section IV-C. The difference however lies in generating the messages to be securely communicated over cache placement and delivery phases, i.e., $M_c$ and $M_d$. In particular, we utilize here uncoded placement for designing the cache contents, and a partially coded delivery transmission that is simultaneously useful for both receivers.

The transmitter divides $W_i, i \in [1 : D]$, into the independent messages $\{W_{i}^{(1)}, W_{i}^{(2)}, W_{i,t}, W_{i,s}\}$. $W_{i}^{(1)}$, $W_{i}^{(2)}$, are uniform over $[1 : 2^{n\frac{1-\alpha_k}{2D}}]$; $\alpha_k$ is defined in (28). $W_{i,t}$ is uniform over $[1 : 2^{n\frac{(2D-1)(1-\alpha_k)}{4D}}]$, and $W_{i,s}$ is uniform over $[1 : 2^{n\frac{\alpha_k}{2}}]$. The transmitter, independently from $W_{[1:D]}$, generates the independent keys $K_1, K_2$, uniformly distributed over $[1 : 2^{n\frac{\alpha_k}{2}}]$.
Let $M_c = \{M_{c,1}, M_{c,2}\}$. Unlike (24), we utilize here \textit{uncoded placement} for designing $M_{c,1}$ and $M_{c,2}$. We have,

$$M_{c,1} = \left\{ W_{1}^{(1)}, W_{2}^{(1)}, \cdots, W_{D}^{(1)} \right\},$$  
(80)

$$M_{c,2} = \left\{ W_{1}^{(2)}, W_{2}^{(2)}, \cdots, W_{D}^{(2)} \right\}.$$  
(81)

The randomization message for cache placement, $\tilde{M}_c = \{\tilde{M}_{c,1}, \tilde{M}_{c,2}\}$, is identical to (25). That is, $\tilde{M}_{c,1} = K_1$ and $\tilde{M}_{c,2} = K_2$. Receiver $j$ stores $M_{c,j}$ and $\tilde{M}_{c,j}$ in its cache memory.

Unlike (26), we utilize here \textit{partially coded} delivery. The message to be securely transmitted during the delivery phase is

$$M_d = \left\{ W_{d_2}^{(1)} \oplus W_{d_1}^{(2)}, W_{d_1,t}, W_{d_2,t} \right\}.$$  
(82)

The randomization message for delivery, $\tilde{M}_d$, is identical to (27).

\textbf{Remark 8} Notice that the sizes of $M_c$, $M_d$, $\tilde{M}_c$, and $\tilde{M}_d$, are the same as in Section IV-C. In particular, the sizes of $\tilde{M}_c$ and $\tilde{M}_d$ are both $n\alpha$ bits. The size of $M_c$ is given by

$$2 \times D \times n \frac{1 - \alpha}{2D} = n(1 - \alpha) \text{ bits},$$  
(83)

and the size of $M_d$ is given by

$$n \frac{1 - \alpha}{2D} + 2 \times n \frac{(2D - 1)(1 - \alpha)}{4D} = n(1 - \alpha) \text{ bits}.$$  
(84)

\textbf{Codebooks Generation and Encoders:} For the messages $M_c$, $M_d$, $\tilde{M}_c$, and $\tilde{M}_d$ defined above, the cache placement and delivery codebooks, $C_{c,n}$ and $C_{d,n}$, and the cache and delivery encoders, are designed in the same exact manner as in Section IV-C, see Figures 3 and 4.

\textbf{Decoding:} As in Section IV-C using $M_d$, $\tilde{M}_d$, $M_{c,j}$, $\tilde{M}_{c,j}$, and for $n$ sufficiently large, receiver $j$ correctly decodes $W_{d_j}$, $j = 1, 2$.

\textbf{Security analysis:} Let $W_c$, $\tilde{W}_c$, $W_d$, and $\tilde{W}_d$, be defined as in (29)-(32), (40), and (41). Once again, $\tilde{W}_c$ and $\tilde{W}_d$ are independent and uniform, and hence $\{\tilde{W}_c, \tilde{W}_d\}$ is jointly uniform. In addition, $\{W_c, W_d\}$ are independent from $\{\tilde{W}_c, \tilde{W}_d\}$. Thus, (43) holds for this case.
For any \( d = (d_1, d_2) \), we have
\[
I (W_{[1:D]}; Z_{S_1}^n, Z_{S_2}^n) = I \left( \left\{ W_l^{(1)}, W_l^{(2)}; W_{l,t}, W_{l,s} \right\}_{l=1}^D ; Z_{S_1}^n, Z_{S_2}^n \right) \tag{85}
\]
\[
\leq I \left( M_c, \left\{ W_l^{(1)}, W_l^{(2)}; W_{l,t}, W_{l,s} \right\}_{l=1}^D ; Z_{S_1}^n, Z_{S_2}^n \right) \tag{86}
\]
\[
\leq I \left( M_c, W_{d_2}^{(1)} \oplus W_{d_1}^{(2)}; W_{d_1,t}, W_{d_2,t}, W_{d_1,s}, W_{d_2,s}; Z_{S_1}^n, Z_{S_2}^n \right) \tag{87}
\]
\[
= I \left( M_c, M_d, W_{d_1,s}, W_{d_2,s}; Z_{S_1}^n, Z_{S_2}^n \right) \tag{88}
\]
\[
\leq I \left( W_c, W_d; Z_{S_1}^n, Z_{S_2}^n \right) + \epsilon_n', \tag{89}
\]
where (87) follows form the Markov chain \( W_{[1:D]} - \left\{ M_c, W_{d_2}^{(1)} \oplus W_{d_1}^{(2)}; W_{d_1,t}, W_{d_2,t}, W_{d_1,s}, W_{d_2,s} \right\} - \left\{ Z_{S_1}^n, Z_{S_2}^n \right\} \); (88) follows from (82), and (89) follows using similar steps as in (46)-(57). Using (83) and (89), the secrecy constraint in (5) is satisfied.

With \( \epsilon_n \to 0 \) as \( n \to \infty \), the achievable strong secrecy file rate is
\[
R_s(\alpha) = 2 \times \frac{1 - \alpha}{2D} + \frac{(2D - 1)(1 - \alpha)}{4D} + \frac{\alpha}{2} \tag{90}
\]
\[
= \frac{1}{2} + \frac{3(1 - \alpha)}{4D}. \tag{91}
\]
This completes the proof for Theorem 2.

Remark 9 For \( D = 2 \), the achievable secrecy rate in (91) is strictly smaller than the secrecy rate obtained by coded placement and uncoded delivery in Section IV-C i.e., \( 1 - \frac{\alpha}{2} \). ■

Remark 10 An achievability scheme which utilizes coded placement and uncoded delivery, as in Section IV-C achieves the same secure file rate as (91) for \( D = 3 \). However, this scheme achieves a strictly smaller secure file rate for \( D \geq 4 \). In this scheme, \( W_l^{(1)} \) and \( W_l^{(2)} \) are uniform over \( \left[ 1 : 2^n \frac{\log_2 e}{D(D-1)} \right] \). \( W_{l,t} \) is uniform over \( \left[ 1 : 2^n \frac{\log_2 e}{2(D-1)-1} \right] \). \( W_{l,s}, K_1 \) and \( K_2 \), are uniform over \( \left[ 1 : 2^n \frac{\log_2 e}{2(D-1)-1} \right] \). \( M_c = \{ M_{c,1}, M_{c,2} \} \) and \( M_d \) are given by
\[
M_{c,1} = \left\{ W_1^{(1)} \oplus W_2^{(1)}, W_2^{(1)} \oplus W_3^{(1)}, \ldots, W_{D-1}^{(1)} \oplus W_D^{(1)} \right\}, \tag{92}
\]
\[
M_{c,2} = \left\{ W_1^{(2)} \oplus W_2^{(2)}, W_2^{(2)} \oplus W_3^{(2)}, \ldots, W_{D-1}^{(2)} \oplus W_D^{(2)} \right\}, \tag{93}
\]
\[
M_d = \left\{ W_{d_2}^{(1)}, W_{d_2}^{(2)}, W_{d_1,t}, W_{d_2,t} \right\}. \tag{94}
\]
Without loss of generality, let \( d_1 < d_2 \). For any \( d = (d_1, d_2) \), using \( M_{c,j} \) in (92) and (93), receiver
can restore $W_{d_1}^{(j)} \oplus W_{d_2}^{(j)}$ by xor-ing $\{W_{d_1}^{(j)} \oplus W_{d_1+1}^{(j)}\}$, $\{W_{d_1+1}^{(j)} \oplus W_{d_1+2}^{(j)}\}$, \ldots, $\{W_{d_2-1}^{(j)} \oplus W_{d_2}^{(j)}\}$.

The achievable strong secrecy file rate using this scheme is $R_s(\alpha) = \frac{1}{2} + \frac{1-\alpha}{2(D-1)}$.

VI. PROOF OF THEOREM 3

When $\alpha \in [1, 2]$, the upper bound on $R_s$, stated in Theorem 3 for $D > 2$, follows as in Section IV-A. Thus, it remains to prove the upper bound for $\alpha \in (0, 1)$. The proof is divided into the three following steps.

Step 1: We first upper bound $R_s$ by the secrecy capacity when the adversary is restricted to tap into the delivery transmission only, denoted as $C^{\text{Res}}_s$. That is, $C^{\text{Res}}_s$ is the maximum achievable file rate when $\alpha_1 = 0$ and $\alpha_2 = \alpha$. Restricting the adversary to only tap into the delivery phase cannot decrease the secrecy capacity, i.e., $R_s \leq C^{\text{Res}}_s$, since this setting is included in the feasible strategy space for the adversary. The cache placement transmission is secure, and each receiver has a secure cache memory of size $\frac{n}{2}$ bits.

Step 2: We upper bound $C^{\text{Res}}_s$ by the secrecy capacity, i.e., the maximum achievable file rate, when the delivery channel to the adversary is replaced by a discrete memoryless binary erasure channel, with erasure probability $1 - \alpha$, denoted as $C^{\text{DM}}_s$. The proof for this step follows the same lines as in [39, Section V]. The idea is when the binary erasure channel produces a number of erasures greater than or equal to $(1 - \alpha)n$, the adversary’s channel in this discrete memoryless setup is worse than its channel in the former model, i.e., when it encounters exactly $(1 - \alpha)n$ erasures and is able to select their positions. Hence, $C^{\text{Res}}_s \leq C^{\text{DM}}_s$ for this case. The result follows by utilizing Sanov’s theorem in method of types [46, Theorem 11.4.1] to show that the probability of the binary erasure channel causing a number of erasures less than $(1 - \alpha)n$ goes to zero as $n \to \infty$.

Step 3: From Step 1, each receiver has a secure cache of size $\frac{n}{2}$ bits. Since increasing the cache sizes cannot decrease the achievable file rate, we further upper bound $C^{\text{DM}}_s$ with the maximum achievable file rate when each receiver has a cache memory of size $n$ bits, in which it stores $X_c^n$. Receiver $j$, $j = 1, 2$, utilizes both $X_c^n$ and $X_d^n$ in order to decode its desired message $W_{d_j}$, i.e., $W_{d_j} = g_{d,j}(X_c^n, X_d^n)$, $\mathbf{d} = (d_1, d_2)$. This setup is thus equivalent to a single receiver, with a cache of size $n$ bits, who demands two files $W_{d_1}, W_{d_2}$, and utilizes the decoder $g_d \triangleq \{g_{d,1}, g_{d,2}\}$.
Let us denote the maximum achievable file rate for this single receiver model as \( C^\text{SR}_s \). We have \( C^\text{DM}_s \leq C^\text{SR}_s \). In the following, we upper bound \( C^\text{SR}_s \).

Let \( M_D \) denote the fraction of the size-\( n \) bits cache memory dedicated to store (coded or uncoded) information bits, and let \( M_K \) denote the fraction dedicated to store key bits. That is, \( M_D + M_K = 1 \). Let \( S_D \) denote the information bits stored in this memory, i.e., \( S_D = f_D(W_{[1:D]}^1) \) and \( H(S_D) = nM_D \). We utilize the following lemma in order to upper bound \( C^\text{SR}_s \).

**Lemma 1** [33, Lemma 1] For a fixed allocation of \( M_D \) and \( M_K \), and a receiver who demands the files \( W_{d_1} \) and \( W_{d_2} \), the secrecy rate for the single receiver model is upper bounded as

\[
2R^\text{SR}_s \leq \min \{ 1, 1 - \alpha + M_K \} + \frac{1}{n} I(W_{d_1}, W_{d_2}; S_D). \tag{95}
\]

Notice that (95) holds for any demand pair \( d = (d_1, d_2) \) such that \( d_1 \neq d_2 \), i.e., the worst-case demands. Summing over all such demands, we have

\[
2R^\text{SR}_s \leq \min \{ 1, 1 - \alpha + M_K \} + \frac{1}{nD(D - 1)} \sum_{d_1, d_2 \in [1 : D], d_1 \neq d_2} I(W_{d_1}, W_{d_2}; S_D). \tag{96}
\]

The second term on the right hand side of (96) can be written as

\[
\frac{1}{nD(D - 1)} \sum_{d_1, d_2 \in [1 : D], d_1 \neq d_2} I(W_{d_1}, W_{d_2}; S_D) = \frac{1}{nD} \sum_{d_1 \in [1 : D]} I(W_{d_1}; S_D) + \frac{1}{nD(D - 1)} \sum_{d_1, d_2 \in [1 : D], d_1 \neq d_2} I(W_{d_2}; S_D | W_{d_1}) \tag{97}
\]

\[
\leq \frac{1}{nD} \sum_{d_1 \in [1 : D]} I(W_{d_1}; S_D) + \frac{1}{nD(D - 1)} \sum_{d_1 \in [1 : D]} \left( \sum_{d_2 \in [1 : D]} I(W_{d_2}; S_D | W_{d_1}) \right). \tag{98}
\]

For any \( d_1 \in [1 : D] \), we have

\[
\sum_{d_2 \in [1 : D]} I(W_{d_2}; S_D | W_{d_1}) = \sum_{d_2 = 1}^{D} [H(W_{d_2} | W_{d_1}) - H(W_{d_2} | W_{d_1}, S_D)] \tag{99}
\]

\[
\leq \sum_{d_2 = 1}^{D} [H(W_{d_2} | W_1, W_2, \ldots, W_{d_2-1}, W_{d_1}) - H(W_{d_2} | W_1, W_2, \ldots, W_{d_2-1}, W_{d_1}, S_D)] \tag{100}
\]

\[
= I(W_1, W_2, \ldots, W_D; S_D | W_{d_1}) \tag{101}
\]

\[
\leq H(S_D) = nM_D, \tag{102}
\]
where (100) follows because when \( d_2 = d_1 \), \( H(W_{d_2}|W_{d_1}) = H(W_{d_2}|W_1, W_2, \ldots, W_{d_2-1}, W_{d_1}) = 0 \), and when \( d_2 \neq d_1 \), \( H(W_{d_2}|W_{d_1}) = H(W_{d_2}|W_1, W_2, \ldots, W_{d_2-1}, W_{d_1}) = H(W_{d_2}) \).

Similarly, we have

\[
\sum_{d_1 \in [1:D]} I(W_{d_1}; S_D) \leq H(S_D) = nM_D. \tag{103}
\]

Substituting (102) and (103) in (98) gives

\[
\frac{1}{nD(D-1)} \sum_{d_1, d_2 \in [1:D], d_1 \neq d_2} I(W_{d_1}, W_{d_2}; S_D) \leq \frac{2D-1}{D(D-1)} M_D. \tag{104}
\]

Thus, using (96) and (104), \( R_{s}^{SR} \) is further upper bounded as

\[
R_{s}^{SR} \leq \frac{1}{2} \left[ \min \{1, 1 - \alpha + M_K\} + \frac{2D - 1}{D(D-1)} M_D \right]. \tag{105}
\]

Finally, by maximizing over all possible allocations for \( M_D \) and \( M_K \) such that \( M_D + M_K = 1 \), we obtain

\[
C_{s}^{SR} \leq \frac{1}{2} \max_{M_D, M_K: M_D + M_K = 1} \left\{ \min \{1, 1 - \alpha + M_K\} + \frac{2D - 1}{D(D-1)} M_D \right\} \tag{106}
\]

\[
= \frac{1}{2} \left[ 1 + \frac{2D - 1}{D(D-1)} (1 - \alpha) \right]. \tag{107}
\]

Equation (107) follows because, for \( D \geq 3 \), the maximum occurs at \( M_K = \alpha \) and \( M_D = 1 - \alpha \). This completes the proof for Theorem 3.

**Remark 11** An upper bound considering uncoded placement only can be derived as follows. The same analysis as in (95)-(107) carries through with \( I(W_{d_2}; S_D|W_{d_1}) \) in (97) is equal to \( I(W_{d_2}; S_D) \). Hence the right hand side of (104) is replaced by \( \frac{2M_K}{D} \). The resulting bound \( R_{s} \leq \frac{1}{2} + \frac{(1-\alpha)}{D} \) is tighter than (107). ■

**VII. DISCUSSION**

While the fixed-size cache memory setup considered in this paper can be seen as a clean basic model for the intricate problem in consideration, it also allows us to obtain results and insights that are generalizable to more involved cache memory models. In particular, the extension to variable memory sizes can be done by considering multiple communication blocks for cache
placement. Our results and coding scheme readily apply to an adversary model whose tapping capability during the delivery is normalized with respect to tapping during cache placement, i.e., $\mu_1 + B\mu_2 \leq \mu$; $B$ is the number of communication blocks for cache placement. This is a reasonable assumption given that cache placement generally takes place in a longer period than delivery. The problem turns to be more challenging when the adversary optimizes its tapping uniformly over the multiple blocks for cache placement as well as the delivery phase. This is left for future investigation.

It is typical to model the cache placement as a noiseless channel since placement is assumed to occur when networks are not congested and their rates are assumed to be large enough. Here however we model the cache placement as a broadcast channel communication. The broadcast model avails a clean and tractable solution without compromising its generalizability. A time division multiple access (TDMA) model for cache placement is a special case by imposing an additional constraint in which each receiver has to decode its desired file using only one half of the transmitted codeword. Additionally, the broadcast model is in line with the network information theory literature and it does not limit the cache placement to occur over low rate traffic. With the ever-growing user demands, placement and delivery occurring in less asymmetric network loads is likely to be expected in the near future.

Corollary 1 demonstrates that, for the model considered in this paper, when $\alpha \in [1, 2]$, the strong secrecy capacity is equal to $1 - \frac{\alpha}{2}$ for any library size. For $\alpha \in [1, 2]$, $\{S_1 = [1 : n], S_2 \subset [1 : n]\}$ and $\{S_1 \subset [1 : n], S_2 = [1 : n]\}$ are two possible strategies for the adversary. In other words, the adversary can tap into either all transmitted symbols in cache placement and a subset of symbols in the delivery, or all transmitted symbols in the delivery and a subset of symbols in cache placement. Such an adversary limits the communication for cache placement, i.e., the use of cache memories, to exchanging additional randomness (key bits) that allows for communicating a positive secure rate over the two phases. In other words, the cache memories are not utilized to store any data bits, and hence the lack of knowledge of user demands during cache placement is immaterial.

For a library with two files, if the receivers were to have cache memories of size $n$ bits in which they store the transmitted signal during cache placement, the strong secrecy file rate in Theorem 1 is achievable using a simple wiretap code. In particular, the transmitter encodes
\( W = (W_1, W_2) \in [1:2^{n2R}] \) into a length-\( 2n \) binary codeword using a wiretap code, and sends the first \( n \) bits of this codeword during cache placement and the last \( n \) bits during delivery. Each receiver can thus decode both files, and the secrecy of \( W_1 \) and \( W_2 \) against the adversary follows by the results in \([38], [39]\). In caching problems, the relevant setup however is when the receivers have cache memories of limited size with respect to the overall transmission during cache placement. This calls for the limited size cache memory model considered in this paper, which in turn necessitates the use of the more elaborate coding scheme in Section IV.

VIII. CONCLUSION

We have introduced the caching broadcast channel with a \textit{a wire and cache} tapping adversary of type II. In this broadcast model, each receiver is equipped with a fixed-size cache memory, and the adversary is able to tap into a subset of its choosing of the transmitted symbols during cache placement, or delivery, or both. The legitimate terminals have no knowledge about the fractions of the tapped symbols in each phase, or their positions. Only the size of the overall tapped set is known. We have identified the strong secrecy capacity of this model, i.e., the maximum achievable file rate while keeping the overall library secure, when the transmitter’s library has two files. We have derived lower and upper bounds for the strong secrecy file rate when the transmitter has more than two files in its library. We have devised an achievability scheme which combines wiretap coding, security embedding codes, one-time pad keys, and coded caching techniques. The results presented in this paper highlight the robustness of (stochastic) coding against a smart adversary who performs a chosen attack, jointly optimized over both cache placement and delivery phases. Future directions include investigating a tighter upper bound for a library with more than two files, and exploring the extensions of this work to variable cache memory sizes, more than two users, and a noisy legitimate channel.

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APPENDIX A

SECRET CONSTRAINT FOR SETTING 1: PROOF OF (11)

For every $S_1 \subseteq [1 : n]$ satisfying $|S_1| = \mu$, we have

$$\lim_{n \to \infty} I(W_1, W_2; Z^n_{S_1}) = \lim_{n \to \infty} I\left(W^{(1)}_1, W^{(2)}_1, W^{(1)}_2, W^{(2)}_2, W_{1,s}, W_{2,s}; Z^n_{S_1}\right)$$

$$= \lim_{n \to \infty} I\left(W^{(1)}_1, W^{(2)}_1, W^{(1)}_2, W^{(2)}_2; Z^n_{S_1}\right)$$

$$\leq \lim_{n \to \infty} I\left(W^{(1)}_1 \oplus W^{(1)}_2, W^{(2)}_1 \oplus W^{(2)}_2; Z^n_{S_1}\right)$$

$$= \lim_{n \to \infty} I(M_c; Z^n_{S_1}) = 0. \tag{111}$$

Recall that the adversary’s observation over cache placement, $Z^n_{S_1}$, results from sending $M_c = \{M_{c,1}, M_{c,2}\}$, where $M_{c,1} = W^{(1)}_1 \oplus W^{(1)}_2$ and $M_{c,2} = W^{(2)}_1 \oplus W^{(2)}_2$. Thus, (109) follows because $Z^n_{S_1}$ does not depend on $\{W_{1,s}, W_{2,s}\}$ and (110) follows due to the Markov chain $\{W^{(1)}_1, W^{(2)}_1, W^{(1)}_2, W^{(2)}_2\} - \{W^{(1)}_1 \oplus W^{(1)}_2, W^{(2)}_1 \oplus W^{(2)}_2\} - Z^n_{S_1}$. The second equality in (111) follows from [38, Theorem 2], and the fact that the rate of $M_c$ is less than $1 - \alpha$.

APPENDIX B

SECRET CONSTRAINT FOR SETTING 2: PROOF OF (16)

For every $S_2 \subseteq [1 : n]$ satisfying $|S_2| = \mu$, and any $d = (d_1, d_2), d_1, d_2 \in \{1, 2\}$, we have

$$I(W_1, W_2; Z^n_{S_2}) = I\left(W^{(2)}_1, W^{(1)}_2, W_{d_1,s}, W_{d_2,s}; Z^n_{S_2}\right)$$

$$= I\left(W^{(1)}_{d_1}, W^{(2)}_{d_2}; Z^n_{S_2}\right) + I\left(W_{d_1,s}, W_{d_2,s}; Z^n_{S_2}\right) \left| W^{(2)}_{d_1}, W^{(1)}_{d_2}\right.$$ (112)

$$\leq I\left(W^{(1)}_{d_1}, W^{(2)}_{d_2}; Z^n_{S_2}\right) + I\left(W_{d_1,s}, W_{d_2,s}; W_{d_1,s} \oplus K_1, W_{d_2,s} \oplus K_2\right| W^{(2)}_{d_1}, W^{(1)}_{d_2}\right.$$ (113)

$$= I\left(W^{(1)}_{d_1}, W^{(2)}_{d_2}; Z^n_{S_2}\right) + I(W_{d_1,s}, W_{d_2,s}; W_{d_1,s} \oplus K_1, W_{d_2,s} \oplus K_2)$$ (114)

$$= I\left(W^{(1)}_{d_1}, W^{(2)}_{d_2}; Z^n_{S_2}\right)$$ (115)

$$= I(M_d; Z^n_{S_2}). \tag{116}$$

The adversary’s observation over the delivery phase, $Z^n_{S_2}$, results from sending $M_d = \{W^{(2)}_{d_1}, W^{(1)}_{d_2}\}$ and the randomization message $\bar{M}_d = \{W_{d_1,s} \oplus K_1, W_{d_2,s} \oplus K_2\}$. Equation (112) thus follows because $Z^n_{S_2}$ depends only on $W^{(1)}_{d_1}, W^{(2)}_{d_2}, W_{d_1,s},$ and $W_{d_2,s}$. The inequality in (114) follows
from the Markov chain \( \{W_{d_1,s}, W_{d_2,s}\} \rightarrow \left\{ W_{d_1}^{(2)}, W_{d_2}^{(1)}, W_{d_1,s} \oplus K_1, W_{d_2,s} \oplus K_2 \right\} - Z_{S_2}^n \). Equation (115) follows because \( \left\{ W_{d_1}^{(2)}, W_{d_2}^{(1)} \right\} \) are independent from \( \{W_{d_1,s}, W_{d_2,s}, K_1, K_2\} \).

The randomization message of the wiretap code in the delivery phase, \( M_d \), is independent from the message \( M_c \). Thus, using (117) and [58, Theorem 2], we have

\[
\lim_{n \to \infty} \max_{S_2 \subseteq [1:n]: |S_2|=\mu} I(W_1, W_2; Z_{S_2}^n) \leq \lim_{n \to \infty} \max_{S_2 \subseteq [1:n]: |S_2|=\mu} I(M_d; Z_{S_2}^n) = 0. \tag{118}
\]

\section*{Appendix C}

\textbf{Secrecy Constraint for Setting 3 when } \( \alpha_1 \geq \alpha_2 \)

Recall that \( M_c \) and \( M_d \) are defined as in (9) and (10), respectively. For a fixed choice of the subsets \( S_1, S_2 \subseteq [1:n] \) such that \( |S_1| + |S_2| = \mu \), and any \( d = (d_1, d_2) \), \( d_1, d_2 \in \{1, 2\} \), we have

\[
I(W_1, W_2; Z_{S_1}^n, Z_{S_2}^n) = I \left( W_1^{(1)}, W_1^{(2)}, W_2^{(1)}, W_2^{(2)}, W_{1,s}, W_{2,s}; Z_{S_1}^n, Z_{S_2}^n \right) \tag{119}
\]

\[
= I \left( W_1^{(1)} \oplus W_2^{(1)}, W_1^{(2)} \oplus W_2^{(2)}, W_{d_1}, W_{d_2}, W_{d_1,s}, W_{d_2,s}; Z_{S_1}^n, Z_{S_2}^n \right) \tag{120}
\]

\[
= I \left( M_c, M_d; Z_{S_1}^n, Z_{S_2}^n \right) \tag{121}
\]

\[
= I \left( M_c; Z_{S_1}^n \right) + I \left( M_d; Z_{S_1}^n, Z_{S_2}^n | M_c \right) \tag{122}
\]

\[
= I \left( M_c; Z_{S_1}^n \right) + I \left( M_c; Z_{S_2}^n | M_c \right) + I \left( M_d; Z_{S_2}^n | M_c, Z_{S_2}^n \right), \tag{123}
\]

where (120) follows because, for any \( d_1, d_2 \in \{1, 2\} \), there exists a bijective map between

\( \left\{ W_1^{(1)}, W_1^{(2)}, W_2^{(1)}, W_2^{(2)} \right\} \) and \( \left\{ W_1^{(1)} \oplus W_2^{(1)}, W_1^{(2)} \oplus W_2^{(2)}, W_{d_1}, W_{d_2} \right\} \).

From (9) and (10), \( M_c \) and \( M_d \) are independent. The adversary’s observation in cache placement, \( Z_{S_1}^n \), results from sending \( M_c \), while its observation in the delivery phase, \( Z_{S_2}^n \), results from sending \( M_d \). Thus, for a fixed choice of the subsets \( S_1, S_2 \), \( \{M_c, Z_{S_1}^n\} \) are independent from \( Z_{S_2}^n \). We thus have

\[
I \left( M_c; Z_{S_2}^n | Z_{S_1}^n \right) = 0. \tag{124}
\]

In addition, \( \{M_d, Z_{S_2}^n\} \) are independent from \( M_c \). Thus,

\[
I \left( M_d; Z_{S_2}^n | M_c \right) = H \left( Z_{S_2}^n | M_c \right) - H \left( Z_{S_2}^n | M_c, M_d \right) \tag{125}
\]

\[
= H \left( Z_{S_2}^n | M_c \right) - H \left( Z_{S_2}^n | M_d \right) \tag{126}
\]
Theorem 2] to (130), we have defined in (15). For notational simplicity, let us define $d_1, d_2$, and $\tilde{d}$.

Finally, using the Markov chain $\{M_d, Z^n_{S_2}\} - M_c - Z^n_{S_1}$, we have

$$I(M_d; Z^n_{S_1} | M_c, Z^n_{S_2}) = H(Z^n_{S_1} | M_c, Z^n_{S_2}) - H(Z^n_{S_1} | M_c, Z^n_{S_2}, M_d)$$

(128)

$$\leq H(Z^n_{S_1}) - H(Z^n_{S_1} | M_c) = I(M_c; Z^n_{S_1}).$$

(129)

Substituting (124), (127), and (129) in (123),

$$I(W_1, W_2; Z^n_{S_1}, Z^n_{S_2}) \leq 2I(M_c; Z^n_{S_1}) + I(M_d; Z^n_{S_2}).$$

(130)

The rates of $M_c$ and $M_d$ are $1 - \alpha_1 - \epsilon_n$ and $1 - \alpha_2 - \epsilon_n$, respectively. By applying Theorem 2] to (130), we have

$$\lim_{n \to \infty} \max_{S_1, S_2 \subseteq [1:n]: |S_1| + |S_2| = \mu} I(W_1, W_2; Z^n_{S_1}, Z^n_{S_2})$$

$$\leq 2 \lim_{n \to \infty} \max_{S_1 \subseteq [1:n]: |S_1| = \mu_1} I(M_c; Z^n_{S_1}) + \lim_{n \to \infty} \max_{S_2 \subseteq [1:n]: |S_2| = \mu_2} I(M_d; Z^n_{S_2})$$

(131)

$$= 0.$$  

(132)

APPENDIX D

SECRET SECRECY CONSTRAINT FOR SETTING 3 WHEN $\alpha_1 < \alpha_2$

For this case, $M_c$ and $M_d$ are defined in (13) and (14) and the randomization message $\tilde{d}$ is defined in (15). For notational simplicity, let us define

$$M_{c,1 \setminus K_1} = W_1^{(1)} \oplus W_2^{(1)}, \quad M_{c,2 \setminus K_2} = W_1^{(2)} \oplus W_2^{(2)}$$

(133)

$$M_{c \setminus K} = \{M_{c,1 \setminus K_1}, M_{c,2 \setminus K_2}\}.$$  

(134)

For a fixed choice of $S_1, S_2 \subseteq [1 : n]$ such that $|S_1| + |S_2| = \mu$, and any $d = (d_1, d_2)$, $d_1, d_2 \in \{1, 2\}$, we have

$$I(W_1, W_2; Z^n_{S_1}, Z^n_{S_2}) = I(W_1^{(1)} \oplus W_2^{(1)}), W_1^{(2)} \oplus W_2^{(2)}, W_1^{(2)}, W_2^{(2)}, W_{d_1}, W_{d_2}, W_{d_1, s}, W_{d_2, s}; Z^n_{S_1}, Z^n_{S_2})$$

(135)

$$= I(M_{c \setminus K}, M_d, W_{d_1, s}, W_{d_2, s}; Z^n_{S_1}, Z^n_{S_2})$$

(136)
= I \left( M_c | M_d, Z^n_{S_1}, Z^n_{S_2} \right) + I \left( M_d, Z^n_{S_1}, Z^n_{S_2} \big| M_c \right) + I \left( W_{d_1,s}, W_{d_2,s}; Z^n_{S_1}, Z^n_{S_2} | M_d, M_c \right). \tag{137}

From (13), (14), and (15), $M_c$ is independent from $\{M_d, \tilde{M}_d\}$. The adversary’s observation in cache placement, $Z^n_{S_1}$, results from sending $M_c = \{M_c | K_1, K_2\}$, and its observation in the delivery results from sending $M_d = \{W_{d_1}, W_{d_2}\}$ and the randomization message $\tilde{M}_d = \{W_{d_1,s} + K_1, W_{d_2,s} + K_2\}$. We now upper bound each term on the right hand side of (137). For the third term, we have

$$I \left( W_{d_1,s}, W_{d_2,s}; Z^n_{S_1}, Z^n_{S_2} | M_d, M_c \right) \leq I \left( W_{d_1,s}, W_{d_2,s}; \tilde{M}_d | M_d, M_c \right) \tag{138}$$

$$= I \left( W_{d_1,s}, W_{d_2,s}; W_{d_1,s} + K_1, W_{d_2,s} + K_2 \right) = 0, \tag{139}$$

where (138) follows due to the Markov chain $\{W_{d_1,s}, W_{d_2,s}\} - \{M_c | M_d, \tilde{M}_d\} - \{Z^n_{S_1}, Z^n_{S_2}\}$, and (139) follows because $\tilde{M}_d$ is independent from $\{W_{d_1,s}, W_{d_2,s}, M_d, M_c \}$.

For a fixed choice of the subsets $S_1$ and $S_2$, $Z^n_{S_2}$ is independent from $\{M_c, Z^n_{S_1}\}$. Thus, the first term on the right hand side of (137) is bounded as

$$I \left( M_c | Z^n_{S_1}, Z^n_{S_2} \right) \leq I \left( M_c | Z^n_{S_1}, Z^n_{S_2} \right) \tag{140}$$

$$= I \left( M_c | Z^n_{S_1} \right) + I \left( M_c | Z^n_{S_2} \big| Z^n_{S_1} \right) = I \left( M_c | Z^n_{S_1} \right). \tag{141}$$

For the second term on the right hand side of (137), we have

$$I \left( M_d | Z^n_{S_1}, Z^n_{S_2} \big| M_c \right) = I \left( M_d | Z^n_{S_2} \big| M_c \right) + I \left( M_d | Z^n_{S_1} \big| M_c, Z^n_{S_2} \right). \tag{142}$$

Notice that $M_c | K_1$ and $Z^n_{S_2}$ are conditionally independent given $M_d$. Thus,

$$I \left( M_d | Z^n_{S_2} \big| M_c \right) = H \left( Z^n_{S_2} \big| M_c \right) - H \left( Z^n_{S_2} \big| M_d \right) \leq I \left( M_d | Z^n_{S_2} \right). \tag{143}$$

In addition, using the independence between $\{M_d, Z^n_{S_2}\}$ and $\{M_c, Z^n_{S_1}\}$, we have

$$I \left( M_d | M_c, Z^n_{S_1}, Z^n_{S_2} \right) = H \left( Z^n_{S_2} \big| M_c, M_d, Z^n_{S_1} \right) \tag{144}$$

$$\leq H \left( Z^n_{S_1} \right) - H \left( Z^n_{S_1} \big| M_c, K_1, K_2, M_d, Z^n_{S_2} \right) \tag{145}$$

$$= H \left( Z^n_{S_1} \right) - H \left( Z^n_{S_1} \big| M_c \right) = I \left( M_c | Z^n_{S_1} \right). \tag{146}$$
Substituting (143) and (146) in (142) gives

\[ I \left( M_d; Z^n_{S_1}, Z^n_{S_2} \mid M_{c\backslash K} \right) \leq I \left( M_d; Z^n_{S_2} \right) + I \left( M_c; Z^n_{S_1} \right). \]  

(147)

Finally, substituting (139), (141), (147) in (137), and applying [38, Theorem 2], we have

\[ \lim_{n \to \infty} \max_{S_1, S_2 \subset \{1:n\}; |S_1| + |S_2| = \mu} I \left( W_1, W_2; Z^n_{S_1}, Z^n_{S_2} \right) = 0, \]  

(148)

since the rates of \( M_c \) and \( M_d \) are \( 1 - \alpha_1 - \epsilon_n \) and \( 1 - \alpha_2 - \epsilon_n \), respectively.

**APPENDIX E**

**SECRECY CONSTRAINT FOR SETTING 4**

For this setting, \( M_c \) and \( M_d \) are defined in (20) and (22), and the randomization messages \( \tilde{M}_c \) and \( \tilde{M}_d \) are defined in (21) and (23). Notice that, \( M_c \) is independent from \( \tilde{M}_c \); \( M_d \) is independent from \( \tilde{M}_d \), and \( \{M_c, \tilde{M}_c\} \) are independent from \( \{M_d, \tilde{M}_d\} \).

Conditioned on a fixed choice of the subsets \( S_1 \) and \( S_2 \), which satisfies the conditions for this setting, i.e., either \( \{|S_1| = \mu, |S_2| = 0\} \) or \( \{|S_1| = 0, |S_2| = \mu\} \), define the random variable

\[ Z^n_S \triangleq Z^n_{S_1} 1_{\{|S_2|=0\}} + Z^n_{S_2} 1_{\{|S_1|=0\}}. \]  

(149)

Notice that the random variable \( Z^n_S \) only have a well-defined probability distribution when conditioned on the event \( \{S_1, S_2\} \), since a prior distribution on these subsets is not defined. For this fixed choice of the subsets, and any \( d = (d_1, d_2), d_1, d_2 \in \{1, 2\} \), we have

\[ I \left( W_1, W_2; Z^n_{S_1}, Z^n_{S_2} \right) = I \left( W^{(1)}_1 \oplus W^{(1)}_2, W^{(2)}_1 \oplus W^{(2)}_2, W^{(2)}_{d_1}, W^{(2)}_{d_2}, W_{d_1,s}, W_{d_2,s}; Z^n_{S_1}, Z^n_{S_2} \right) \]  

(150)

\[ = I \left( M_c, M_d, W_{d_1,s}, W_{d_2,s}; Z^n_S \right) \]  

(151)

\[ = 1_{\{|S_2|=0\}} I \left( M_c, M_d, W_{d_1,s}, W_{d_2,s}; Z^n_S \mid \{|S_2| = 0\} \right) \]

\[ + 1_{\{|S_1|=0\}} I \left( M_c, M_d, W_{d_1,s}, W_{d_2,s}; Z^n_S \mid \{|S_1| = 0\} \right) \]  

(152)

\[ = 1_{\{|S_2|=0\}} I \left( M_c, M_d, W_{d_1,s}, W_{d_2,s}; Z^n_{S_1} \right) + 1_{\{|S_1|=0\}} I \left( M_c, M_d, W_{d_1,s}, W_{d_2,s}; Z^n_{S_2} \right) \]  

(153)

\[ = 1_{\{|S_2|=0\}} I \left( M_c; Z^n_{S_1} \right) + 1_{\{|S_1|=0\}} I \left( M_d, W_{d_1,s}, W_{d_2,s}; Z^n_{S_2} \right) \]  

(154)
due to the Markov chain $M$ and $\tilde{U}$ where (157) follows from (155), and (158) follows because both limits exist and equal to zero; of the random bins $B$ cache placement and delivery phases are $x$ denotes the induced distribution at the adversary’s output when the transmitted codewords over $C$.

Finally, since $\tilde{M}_c$ is independent from $M_c$, $\tilde{M}_d$ is independent from $M_d$, and the rates of $M_c$ and $M_d$ are both equal to $1 - \alpha - \epsilon_n$, we have

$$\lim_{n \to \infty} \max_{S_1, S_2 \subseteq \{1:n\}: \left| S_1 \right| + \left| S_2 \right| = \mu} I(W_1, W_2; Z^n_{S_1}, Z^n_{S_2}) = \lim_{n \to \infty} \max_{S_1, S_2 \subseteq \{1:n\}: \left| S_1 \right| = \mu} I(W_1, W_2; Z^n_{S_1}, Z^n_{S_2}) = 0,$$

where (157) follows from (155), and (158) follows because both limits exist and equal to zero; by using [38, Theorem 2].

APPENDIX F

PROOFS FOR (42) AND (43)

Let us fix the subsets $S_1$ and $S_2$, and the messages $w_c$ and $w_d$. Consider the Cartesian product of the random bins $B_{w_c}$ and $B_{w_d}$, i.e., $B_{w_c, w_d}$, defined in [35]. Recall that $P_{Z^n_{S_1} Z^n_{S_2}}|_{w_c = w_c, w_d = w_d}$ denotes the induced distribution at the adversary’s output when the transmitted codewords over cache placement and delivery phases are $x^n_c (w_c, \tilde{w}_c)$ and $x^n_d (w_d, \tilde{w}_d)$, i.e., when $(x^n_c, x^n_d)$ belongs to $B_{w_c, w_d}$. In addition, $P_{Z^n_{S_1} Z^n_{S_2}}$ denotes the output distribution at the adversary, induced by the cache placement and delivery codebooks, $C_{c,n}$ and $C_{d,n}$, defined in Figures 3 and 4.

Let $z^n_1, z^n_2 \in \mathcal{Z}^n$, where $\mathcal{Z} \triangleq \{0, 1\} \cup \{?\}$. Define the distribution $Q_{Z^n_{S_1} Z^n_{S_2}}$ as follows:

$$Q_{Z^n_{S_1} Z^n_{S_2}}(z^n_1, z^n_2) = \prod_{i \notin S_1, j \notin S_2} 1_{\{z_{1,i} = ?, z_{2,j} = ?\}} \prod_{i \in S_1, j \in S_2} U_X(z_{1,i}) U_X(z_{2,i}),$$

where $U_X(z)$ is a uniform binary distribution when $z = 0, 1$, and $U_X(z) = 0$ when $z = ?$. 
We thus have

\[
I(W_c, W_d; Z^n_{S_1}, Z^n_{S_2}) = \mathbb{D} \left( P_{w_c w_d z^n_{S_1}, z^n_{S_2}} \mid P_{w_c w_d z^n_{S_1}, z^n_{S_2}} \right) 
\]

\[
= \sum_{w_c, w_d} P_{w_c w_d}(w_c, w_d) \sum_{z^n_{S_1}, z^n_{S_2}} P_{z^n_{S_1}, z^n_{S_2}}(w_c, w_d) \log \left( \frac{P_{w_c w_d} z^n_{S_1}, z^n_{S_2} (w_c, w_d, z^n_{S_1}, z^n_{S_2})}{P_{z^n_{S_1}, z^n_{S_2}} (z^n_{S_1}, z^n_{S_2}) P_{w_c w_d}(w_c, w_d)} \right) 
\]

\[
= \sum_{w_c, w_d} P_{w_c w_d}(w_c, w_d) \sum_{z^n_{S_1}, z^n_{S_2}} P_{z^n_{S_1}, z^n_{S_2}}(w_c, w_d) \log \left( \frac{P_{w_c w_d} z^n_{S_1}, z^n_{S_2} (w_c, w_d, z^n_{S_1}, z^n_{S_2})}{Q_{z^n_{S_1}, z^n_{S_2}} (z^n_{S_1}, z^n_{S_2})} \times \frac{Q_{z^n_{S_1}, z^n_{S_2}} (z^n_{S_1}, z^n_{S_2})}{P_{z^n_{S_1}, z^n_{S_2}} (z^n_{S_1}, z^n_{S_2})} \right) 
\]

\[
= \sum_{w_c, w_d} P_{w_c w_d}(w_c, w_d) \mathbb{D} \left( P_{z^n_{S_1}, z^n_{S_2}}(w_c=w_c, w_d=w_d) \mid Q_{z^n_{S_1}, z^n_{S_2}} \right) - \mathbb{D} \left( P_{z^n_{S_1}, z^n_{S_2}} \mid Q_{z^n_{S_1}, z^n_{S_2}} \right) 
\]

\[
\leq \sum_{w_c, w_d} P_{w_c w_d}(w_c, w_d) \mathbb{D} \left( P_{z^n_{S_1}, z^n_{S_2}}(w_c=w_c, w_d=w_d) \mid Q_{z^n_{S_1}, z^n_{S_2}} \right) 
\]

Define \( Z^{S_1} \triangleq \{ Z_{S_1, i} : i \in S_1 \} \), \( Z^{S_2} \triangleq \{ Z_{S_2, i} : i \in S_2 \} \), \( Z^{S_{i}} \triangleq \{ Z_{S_{i}, i} : i \notin S_1 \} \), \( Z^{S_{i}} \triangleq \{ Z_{S_{i}, i} : i \notin S_2 \} \), and let \( z^{S_1}, z^{S_2}, z^{S_{i}}, z^{S_{i}} \) be the corresponding realizations. Note that \( z^n_{S_1} = \{ z^{S_{i}}, z^{S_{i}} \} \) and \( z^n_{S_2} = \{ z^{S_{i}}, z^{S_{i}} \} \). For each \( S_1, S_2, w_c, \) and \( w_d, \) we have

\[
\mathbb{D} \left( P_{z^n_{S_1}, z^n_{S_2}}(w_c=w_c, w_d=w_d) \mid Q_{z^n_{S_1}, z^n_{S_2}} \right) = \mathbb{D} \left( P_{z^{S_1}, z^{S_2} z^{S_{i}}, z^{S_{i}}}(w_c=w_c, w_d=w_d) \mid Q_{z^{S_1}, z^{S_{i}}, z^{S_{i}}} \right) 
\]

\[
= \sum_{z^{S_1}, z^{S_{i}}, z^{S_2}, z^{S_{i}}} P_{z^{S_1}, z^{S_{i}}, z^{S_2}, z^{S_{i}}}(w_c, w_d) \log \left( \frac{P_{z^{S_1}, z^{S_{i}}, z^{S_2}, z^{S_{i}}}(w_c, w_d, z^{S_1}, z^{S_{i}}, z^{S_2}, z^{S_{i}})}{Q_{z^{S_1}, z^{S_{i}}, z^{S_2}, z^{S_{i}}}(z^{S_1}, z^{S_{i}}, z^{S_2}, z^{S_{i}})} \right) 
\]

\[
= \sum_{z^{S_1}, z^{S_{i}}, z^{S_2}, z^{S_{i}}} P_{z^{S_1}, z^{S_{i}}, z^{S_2}, z^{S_{i}}}(w_c, w_d) \log \left( \frac{P_{z^{S_1}, z^{S_{i}}, z^{S_2}, z^{S_{i}}}(w_c, w_d, z^{S_1}, z^{S_{i}}, z^{S_2}, z^{S_{i}})}{Q_{z^{S_1}, z^{S_{i}}, z^{S_2}, z^{S_{i}}}(z^{S_1}, z^{S_{i}}, z^{S_2}, z^{S_{i}}) Q_{z^{S_1}, z^{S_2}}(z^{S_1}, z^{S_2}) Q_{z^{S_{i}}, z^{S_{i}}}(z^{S_{i}}, z^{S_{i}})} \right) 
\]

\[
= \mathbb{D} \left( P_{z^{S_1}, z^{S_2}}(w_c=w_c, w_d=w_d) \mid Q_{z^{S_1}, z^{S_2}} \right) + \sum_{z^{S_1}, z^{S_{i}}=z^{S_2}} P_{z^{S_1}, z^{S_{i}}, z^{S_2}}(w_c=w_c, w_d=w_d) \left( z^{S_1}, z^{S_2} \right) 
\]
where (170) follows because

\[ P_{Z_1^{S_1} Z_2^{S_2}}|_{W_c=w_c, W_d=w_d, Z_1^{S_1} = z_1^{S_1}, Z_2^{S_2} = z_2^{S_2}} = Q_{Z_1^{S_1} Z_2^{S_2}}|_{Z_1^{S_1} = z_1^{S_1}, Z_2^{S_2} = z_2^{S_2}} \]

(171)

By applying the stronger version of Wyner’s soft covering lemma in [38, Lemma 1] to (170), for every \( \epsilon > 0 \), there exists a \( \gamma(\epsilon) > 0 \) such that

\[ P_{B_{w_c, w_d}} \left( \mathbb{D} \left( P_{Z_1^{S_1} Z_2^{S_2}}|_{W_c=w_c, W_d=w_d} \parallel Q_{Z_1^{S_1} Z_2^{S_2}} > \epsilon \right) \right) = P_{B_{w_c, w_d}} \left( \mathbb{D} \left( P_{Z_1^{S_1} Z_2^{S_2}}|_{W_c=w_c, W_d=w_d} \parallel Q_{Z_1^{S_1} Z_2^{S_2}} > \epsilon \right) \right) \leq \exp \left( -\epsilon^{\alpha(\epsilon)} \right), \]

(172)

since the rate of \( B_{w_c, w_d} \) is slightly greater than \( \alpha \), i.e., \( B_{w_c, w_d} \) contains \( 2^{n_{\max}} \) codewords.

Using (165) and (170), we have

\[ I \left( W_c, W_d; Z_1^{n_{S_1}}, Z_2^{n_{S_2}} \right) \leq \sum_{w_c, w_d} P_{W_c, W_d}(w_c, w_d) \mathbb{D} \left( P_{Z_1^{S_1} Z_2^{S_2}}|_{W_c=w_c, W_d=w_d} \parallel Q_{Z_1^{S_1} Z_2^{S_2}} \right). \]

(173)

Thus,

\[ P_{B_{w_c, w_d}} \left( \max_{S_1, S_2 \subseteq [1:m]; |S_1|+|S_2|=\mu} I \left( W_c, W_d; Z_1^{n_{S_1}}, Z_2^{n_{S_2}} \right) \geq \epsilon \right) \]

(174)

\[ \leq P_{B_{w_c, w_d}} \left( \max_{S_1, S_2} \sum_{w_c, w_d} P_{W_c, W_d}(w_c, w_d) \mathbb{D} \left( P_{Z_1^{S_1} Z_2^{S_2}}|_{W_c=w_c, W_d=w_d} \parallel Q_{Z_1^{S_1} Z_2^{S_2}} \right) > \epsilon \right) \]

(175)

\[ \leq P_{B_{w_c, w_d}} \left( \max_{w_c, w_d} \mathbb{D} \left( P_{Z_1^{S_1} Z_2^{S_2}}|_{W_c=w_c, W_d=w_d} \parallel Q_{Z_1^{S_1} Z_2^{S_2}} \right) > \epsilon \right) \]

(176)

\[ \leq \sum_{w_c, w_d} P_{B_{w_c, w_d}} \left( \mathbb{D} \left( P_{Z_1^{S_1} Z_2^{S_2}}|_{W_c=w_c, W_d=w_d} \parallel Q_{Z_1^{S_1} Z_2^{S_2}} \right) > \epsilon \right) \]

(177)

where (174) follows from (173), and (177) follows from the union bound. Since the combined
number of the messages $w_c, w_d$, and the subsets $S_1, S_2$ is at most exponential in $n$, using the super-exponential decay rate in (172), the probability term on the right hand side of (174) goes to zero as $n$ goes to infinity. Thus, $\max_{S_1, S_2} I \left( W_c, W_d; Z^n_{S_1}, Z^n_{S_2} \right)$ converges to zero almost surely. This completes the proof of (43).

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