Satellite luminosities in galaxy groups

Ramin A. Skibba\textsuperscript{1,2}*, Ravi K. Sheth\textsuperscript{3} & Matthew C. Martino\textsuperscript{3}*

\textsuperscript{1}Department of Physics & Astronomy, University of Pittsburgh, Pittsburgh, PA 15260, USA
\textsuperscript{2}Max Planck Institute for Astronomy, Königstuhl 17, D-69117 Heidelberg, Germany
\textsuperscript{3}Department of Physics & Astronomy, University of Pennsylvania, Philadelphia, PA 19104, USA

ABSTRACT

Halo model interpretations of the luminosity dependence of galaxy clustering assume that there is a central galaxy in every sufficiently massive halo, and that this central galaxy is very different from all the others in the halo. The halo model decomposition makes the remarkable prediction that the mean luminosity of the non-central galaxies in a halo should be almost independent of halo mass—the predicted increase is about twenty percent while the halo mass increases by a factor of more than twenty. In contrast, the luminosity of the central object is predicted to increase approximately linearly with halo mass at low to intermediate masses, and logarithmically at high masses. We show that this weak, almost non-existent mass-dependence of the satellites is in excellent agreement with the satellite population in group catalogs constructed by two different collaborations. This is remarkable, because the halo model prediction was made without ever identifying groups and clusters. The halo model also predicts that the number of satellites in a halo is drawn from a Poisson distribution with mean which depends on halo mass. This, combined with the weak dependence of satellite luminosity on halo mass, suggests that the Scott effect, such that the luminosities of very bright galaxies are merely the statistically extreme values of a general luminosity distribution, may better apply to the most luminous satellite galaxy in a halo than to BCGs. If galaxies are identified with dark halo substructure at the present time, then central galaxies should be about 4 times more massive than satellite galaxies of the same luminosity, whereas the differences between the stellar mass-to-light ratios should be smaller. Therefore, a comparison of the weak lensing signal from central and satellite galaxies of the same luminosity should provide useful constraints on these models. We also show how the halo model may be used to constrain the stellar mass associated with intracluster light: the mass fraction in the ICL is expected to increase with increasing halo mass.

Key words: methods: analytical - galaxies: formation - galaxies: haloes - dark matter - large scale structure of the universe

1 INTRODUCTION

The halo model (see Cooray & Sheth 2002 for a review) has become the preferred language in which to interpret measurements of galaxy clustering. Recently, Zehavi et al. (2005) have expressed the luminosity dependence of clustering in the Sloan Digital Sky Survey (SDSS, York et al. 2000) Second Data Release (DR2, Abazajian et al. 2004) in terms of the halo model. Skibba et al. (2006) show that, if Zehavi et al.’s halo model decomposition is correct, then the luminosity of the central galaxy in a halo depends strongly on halo mass, whereas the luminosities of satellite galaxies depend only weakly on the masses of their host haloes. The main goal of this paper is to test this prediction. We do this in Section 2 by studying the satellite population in the group catalog provided by Berlind et al. (2006). The abundance of groups decreases and the clustering strength increases with increasing richness, as expected (Berlind et al. 2007). This suggests that the test we perform is unlikely to have been biased by incompleteness effects in the catalog. As an additional check, we show that the satellite population in the group catalogs of Yang et al. (2005a) are similar to those from Berlind et al.

Dark matter haloes have substructure (e.g. Tormen 1997; Tormen, Diaferio & Syer 1998; Gao et al. 2004a). If we identify subhaloes with satellite galaxies (e.g. Kravtsov et al. 2004; Conroy et al. 2007), then the halo model makes
specific predictions about how center and satellite galaxies of the same luminosity differ; this difference is the subject of Section 3. These predictions can also be tested by studying how stellar and total mass-to-light ratios depend on environment; how the luminosity function of clusters (after removing the BCG) depends on cluster richness; and how the amount of intracluster light depends on cluster richness. The connections between these tests and the halo model are discussed in a final section which summarizes our findings. Throughout, we assume a spatially flat cosmology with \( \Omega_0 = 0.3, \Lambda_0 = 1 - \Omega_0 \) and \( \sigma_8 = 0.9 \), and we write the Hubble constant as \( H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1} \).

An Appendix presents a few inconsistencies between the halo model of Zehavi et al. (2005) and the group catalog of Berlind et al. (2006). It argues that while these may be due to Zehavi et al.’s assumption that \( \sigma_8 = 0.9 \), they are unlikely to invalidate our findings.

2 THE HALO MODEL AND GROUP GALAXIES

The halo model decomposition provides a prescription for how the galaxy population in a halo depends on halo mass. In practice, halo mass is not an observable, so comparison of this prediction with the objects in a group catalog is not straightforward. However, the halo model decomposition can be re-written so that observable quantities are predicted: these include the number density of groups containing \( N \) galaxies more luminous than some threshold luminosity, as well as the average luminosities of the central and satellite galaxies as a function of \( N \). Specifically,

\[
\frac{\text{d}n(M)}{\text{d}M} = \frac{n_{\text{grp}}(N)}{\text{d}N},
\]

and

\[
\langle L_{\text{cen}} \rangle = \int_{M_{\text{min}}}^{\infty} \text{d}M \frac{\text{d}n(M)}{\text{d}M} p(N|M) L_{\text{cen}}(M),
\]

where \( n_{\text{grp}}(N) \) is the halo mass function (we use the parametrization given by Sheth & Tormen 1999), and the distribution \( p(N|M) \) has mean

\[
\langle N|M, \geq L \rangle = 1 + \langle N_{\text{sat}}|M, \geq L \rangle = 1 + \left[ \frac{M}{M_1(L)} \right]^{\alpha(L)},
\]

with \( N_{\text{sat}} \) drawn from a Poisson distribution (Zehavi et al. 2005). Here \( M_1(L) \approx 23 M_{\text{min}}(L) \), where \( M_{\text{min}} \) denotes the minimum mass required to host a galaxy of luminosity \( L \) or greater, and \( \alpha \approx 1 \).

The minimum mass scales with \( r \)-band \( L \) (our \( r \)-band is actually the SDSS \( r \) filter shifted to \( z = 0.1 \), sometimes denoted \( 0.1r \)) as

\[
\left( \frac{M_{\text{min}}}{10^{12} \text{M}_\odot} \right) \approx \exp \left( \frac{1.1 \times 10^{10} \text{h}^{-2} \text{L}_\odot}{L} \right) - 1,
\]

so

\[
\frac{L_{\text{cen}}}{1.1 \times 10^{10} \text{h}^{-2} \text{L}_\odot} \approx \ln \left( 1 + \frac{M}{10^{12} \text{h}^{-1} \text{M}_\odot} \right).
\]

Figure 1 compares equation (2) with the mean satellite luminosity in groups containing \( N_{\text{gal}} \) members each more luminous than \( M_r \leq -19.9 \) (lower set of symbols with error bars). Triangles and squares show results from the group catalogs of Berlind et al. (2006) and Yang et al. (2005a), respectively, and the points have been slightly offset in \( \log(N_{\text{gal}}) \), for clarity. Lower solid line shows the halo model prediction of this quantity (equation 2). Upper set of symbols with error bars show that the mean central luminosity \( L_{\text{cen}} \) is a stronger function of \( N \) than is \( L_{\text{sat}} \). Solid line shows that the halo model (equation 2) correctly predicts the stronger \( N \) dependence, but overpredicts the actual luminosities.

Figure 2 of Skibba et al. (2006) shows that this is a much weaker function of \( M \) than is \( L_{\text{cen}} \). To see why, notice that if \( \alpha(L) \) were independent of \( L \), then the mean satellite luminosity would be independent of \( M \). This suggests that the \( L \)-dependence of \( \alpha \) reflects the mass dependence of satellite luminosities: the halo model prediction that satellite luminosities depend only weakly on halo mass is a consequence of the fact that \( \alpha \) is only weakly dependent on \( L \).

Figure 1 compares equation (3) with the mean satellite luminosity in the \( M_r \leq -19.9 \) group catalog of Berlind et al. (2006). This catalog is drawn from the SDSS Fourth Data Release (DR4, Adelman-McCarthy et al. 2006); it is volume-limited over the redshift range \( 0.015 < z < 0.100 \), and consists of 21301 galaxies in 4119 groups having three or more members. The groups were identified using a redshift-space friends-of-friends algorithm, which was tuned to iden-
identify galaxy systems that occupy the same host dark matter halo, based on halo occupation distribution models, the group multiplicity function, and distributions of group sizes and velocity dispersions. See Berlind et al. for details of the algorithm and resulting catalogs.

We also compare to a group catalog of Yang et al. (2005a) and Weinmann et al. (2006), which is drawn from the SDSS DR2 (Abazajian et al. 2004). When restricted to the same volume limited cuts, it consists of 10475 galaxies in 3260 groups having two or more members. The groups were identified using a different redshift-space friends-of-friends algorithm. See Yang et al. for details of the algorithm, and Weinmann et al. for their application to the SDSS. The two sets of symbols in Figure 1 show that, whereas the Yang et al. catalog has slightly brighter central galaxies at fixed $N$, the two group catalogs predict almost the same scaling of $(L_{\text{sat}})$ with $N$.

The halo model parameters associated with this luminosity threshold are $M_{\text{min}} \approx 10^{12}h^{-1}M_\odot$ and $\alpha = 1.13$, according to the halo occupation distribution analysis of Zehavi et al. (2005). The symbols in Figure 1 show that $L_{\text{sat}}$ increases only very weakly with $N$ (essentially because it increases only weakly with $M$), whereas $L_{\text{cen}}$ is a stronger function of $N$. The solid curves show that, although equation (2) overpredicts the luminosities of the central galaxies, equation (3) reproduces the scaling of $L_{\text{sat}}$ with $N$ quite well. Although the agreement is not perfect, it is nevertheless remarkable, because the halo model decomposition into central and satellite objects is done without ever identifying groups and clusters in the SDSS dataset. Therefore, this agreement with the satellite population in an actual group catalog represents a nontrivial success of the approach. (The discrepancy for the central galaxies is not a central issue of this paper—it is discussed further in the Appendix.)

Equations (2) and (3) imply that the satellite galaxy luminosity function is given by

$$\phi_{\text{sat}}(\geq L|M) = \left( \frac{M_{12}}{23} \right)^{\alpha(L)} \left( \exp(L_{10}) - 1 \right)^{-\alpha(L)}$$

(8)

where $M_{12} = M/10^{12}h^{-1}M_\odot$ and $L_{10} = (L/1.1)/10^{10}h^{-2}L_\odot$. If $\alpha(L)$ is independent of $L$, then $M$ determines the normalization of the satellite galaxy luminosity function but not its shape. (This is not quite true, because a given $M$ would not host satellites more luminous than $L_{\text{min}}$, so, strictly speaking, it is the faint end shape of the satellite galaxy luminosity function which would be independent of halo mass $M$.) If $\alpha$ is independent of $L$, then the cumulative function is $\phi_{\text{sat}}(\geq L|M) \propto \exp(-\alpha L_{10})$ at $L_{10} \gg 1$, so the luminosity function itself also falls as $\exp(-\alpha L_{10})$ at $L_{10} \gg 1$. In the other limit, $L_{10} \ll 1$, the cumulative function is $\phi_{\text{sat}}(\geq L|M) \propto L_{10}^{-\alpha}$ so $\phi_{\text{sat}}(L|M) \propto L_{10}^{\alpha-1}$. This shows explicitly that the satellite galaxy luminosity function should be reasonably well fit by a Schechter-like form, even though this form played no explicit role in the halo-model parameterization. Note that $L_{10} = 1$ is indeed close to the value of $L_L$ associated with a Schechter function fit to the SDSS luminosity function (Blanton et al. 2003).

### Satellite luminosities in galaxy groups

For completeness the luminosity function is

$$L\phi_{\text{sat}}(L|M) = \alpha L_{10} \left( \frac{M_{12}/23}{\exp(L_{10}) - 1} \right)^{\alpha(L)} \times \left\{ \frac{\exp(L_{10})}{\exp(L_{10}) - 1} - \frac{\ln(M/M_1)}{L_{10}} \frac{d\ln \alpha}{d \ln L} \right\}$$

(9)

If $\alpha$ does not depend on $L$, then this simplifies to

$$L\phi_{\text{sat}}(L|M) = \left( \frac{M_{12}}{23} \right)^{\alpha} \frac{\alpha L_{10} e^{-\alpha L_{10}}}{\left[ 1 - \exp(-L_{10}) \right]^{1+\alpha}}.$$  

(10)

### 3 Galaxies as Halo Substructure

Dark matter haloes are expected to have substructure. Recent work suggests that numerical simulations now give reliable estimates of the subhalo mass function:

$$\frac{dN(m|M)}{dm} = 0.01 \left( \frac{M}{10^{12}h^{-1}M_\odot} \right)^{0.1} \left( \frac{M}{m} \right)^{0.9} \frac{dm}{m}$$

(11)

(Gao et al. 2004b). Hence, the number of subhaloes more massive than $m$ is

$$N(\geq m|M) = \frac{0.01}{0.9} \left( \frac{M}{10^{12}h^{-1}M_\odot} \right)^{0.1} \left[ \left( \frac{M}{m} \right)^{0.9} - 1 \right].$$

(12)

If we use $M_1$ to denote the value of $M$ at which the number of subhaloes is unity, then the expression above implies that

$$\left( \frac{M_1}{m} \right) \approx 90 \left( \frac{10^{12}h^{-1}M_\odot}{m} \right)^{0.1} \approx \frac{90}{m_{12}}^{0.1},$$

(13)

where $m_{12}$ is the subhalo mass in units of $10^{12}h^{-1}M_\odot$. This shows that the mass required to host at least one subhalo of mass $m$ is about 90 times greater than $m$. This is substantially larger than the value of $2m$ that one might naively have guessed from mass conservation.

Suppose we identify satellite galaxies with subhaloes, and require that the relation between satellite galaxy and subhalo mass is monotonic and deterministic (e.g. Kravtsov et al. 2004 and Conroy et al. 2006, although these authors argue that it is more reasonable to use subhalo circular velocity rather than mass). Then requiring that

$$N(\geq m|M) = \left[ \frac{M}{M_1} \right]^{\alpha(L)}$$

(14)

provides a constraint on the mass-to-light ratio of subhaloes. We are particularly interested in quantifying how different this ratio is for satellite galaxies compared to centrals. We expect a difference simply because the mass required to host two galaxies above some luminosity is of order twenty times larger that the mass required to host one, and 20 is much smaller than 90.

The expression above requires

$$\frac{M_{12}/90}{m_{12}^{0.9}} = \left[ \frac{M_{12}/23}{\exp(L_{10}) - 1} \right]^{\alpha(L)},$$

(15)

(recall $L_{10}$ is the luminosity in units of $1.1 \times 10^{10}h^{-2}L_\odot$), which can be rearranged to

$$m_{12}^{0.9} = \frac{M_{12}/90}{(M_{12}/23)^{\alpha(L)}} \left[ \exp(L_{10}) - 1 \right]^{-\alpha(L)}. $$

(16)
In contrast, the relation for centrals is \( M_{12} = \exp(L_{10}) - 1 \) (cf. equation \[15\]). To see what this difference implies, it is instructive to consider the case when \( \alpha = 1 \) independent of \( L \). Then

\[
m_{12} = 10^9 \left[ \frac{\exp(L_{10}) - 1}{\exp(13.643) - 1} \right]^{1/9} \left[ \exp(L_{10}) - 1 \right].
\]

The first line shows that satellites with \( L_{10} = 1 \) (recall this is approximately \( L_\ast \)) are 0.233 times less massive than centrals of the same luminosity. The second line shows that satellites are less massive than centrals provided \( L_{10} < 13.643 \). Notice that these relations do not depend on the mass \( M_{12} \) of the parent halo. If the smaller mass for the satellites is due to tidal stripping (this assumes no change in the luminosity of the satellite as it falls onto its parent halo), then \( \alpha = 1 \) suggests that, on average, a galaxy loses the same fraction of its original dark matter halo whether it falls into a small group or a massive cluster. If \( \alpha \neq 1 \), then the mass fraction which survives scales with parent halo mass as \( M_{12}^{(1-\alpha)/9} \), although the overall scaling depends on \( L_{10} \). However, if \( \alpha = 0.9 \), then the mass fraction which survives increases with parent halo mass as \( M_{12}^{1/9} \), independent of luminosity. If \( \alpha > 1 \), then the mass fraction which survives decreases with increasing \( M_{12} \). Our neglect of even passive evolution means that we are slightly overestimating the mass of the satellite prior to stripping, so we are slightly overestimating the mass fraction which is stripped.

The analysis above assumes that halo substructure at a given time is related to the galaxy population at the same time. It is not obvious that this is reasonable: equation \[11\] for the subhalo population is calibrated from simulations of dark matter clustering. Some of the subhaloes which disrupt in these simulations are expected to survive if the effects of baryons are included. This is because baryons cool into the center of their host halo, thus inhibiting disruption (Gao et al. 2004a; Diemand et al. 2004; Macciò et al. 2006). This particularly affects the galaxy population close to the halo center: subhaloes near the host halo center tend to be more tidally stripped, making the mass-to-light ratio of satellites smallest close to the center, and larger at larger radii (Tormen 1997; Hayashi et al. 2003; Gill et al. 2005; Gao et al. 2004a; Nagai & Kravtsov 2005). As a consequence of this effect, models which seek to identify \( z = 0 \) subhaloes in dark matter only simulations with \( z = 0 \) galaxies will have trouble accounting for the ‘orphan’ satellite galaxies which should remain after their subhaloes have been disrupted; these orphans are expected to contribute to the abundance and small-scale clustering of faint galaxies (e.g. Wang et al. 2006). Our analysis does not account for this effect, because it will only increase the difference between the mass-to-light ratios of centrals and satellites of the same luminosity.

4 DISCUSSION

Halo model interpretations of the observed luminosity dependence of galaxy clustering suggest that central galaxies in haloes are different from all the others—the satellites. Whereas the luminosity of the central object is predicted to be a relatively strong function of halo mass (equation \[9\]), the mean luminosity of satellite galaxies (i.e., those which are not central galaxies) should depend only weakly on halo mass (equation \[10\]). Since the number of galaxies in a halo is also predicted to increase steeply with increasing halo mass, the luminosity of the central galaxy is expected to depend strongly on the number of galaxies in a group, whereas the average luminosity of the satellites is expected to depend little if at all on the group ‘richness’. Figure \[1\] shows that this prediction is in good quantitative agreement with a direct measurement of this trend in a catalog of galaxy groups.

In fact, not just the mean, but the entire shape of the satellite galaxy luminosity function, is predicted to be approximately independent of halo mass, having a steep power-law form at \( L < 10^{10} h^{-2} L_\odot \) and an exponential cutoff at \( L > 10^{10} h^{-2} L_\odot \) (equations \[8\] and \[10\]). Thus, the satellite luminosity function is predicted to have a form like that proposed by Schechter (1976), even though this functional form played no role in the halo model parametrization.

The approximate mass independence of \( \phi_{\text{sat}}(L) \) suggests that approaches which use the halo model to infer how the number of galaxies above some \( L \) depends on halo mass (e.g. Zehavi et al. 2005; sometimes called the HOD approach) may help simplify halo model decompositions which are based on the conditional luminosity function (e.g. Yang et al. 2003; Cooray 2006; van den Bosch et al. 2007). This is because the CLF approach must separately parametrize how the luminosity function of central and satellite galaxies depends on halo mass: the HOD-based analysis here suggests that ignoring the mass dependence of the satellite galaxy LF (thus reducing the numbers of free parameters to be fitted) should be a reasonable first approximation.

The quantitative parts of our discussion were based on the halo model parameters derived by Zehavi et al. (2005) from a consideration of clustering as a function of luminosity in the SDSS main galaxy sample, and the group catalogs are from Berlind et al. (2006) and Yang et al. (2005a). Appendix A shows that, in some respects, the Zehavi et al. numbers are inconsistent with the Berlind et al. catalog. While this may be due to inaccuracies in the group catalog, it may also be due to Zehavi et al.’s assumption that \( \sigma_8 = 0.9 \) (Berlind et al. and Yang et al. assumed this same value when calibrating their group-finders). While changing \( \sigma_8 \) within reasonable bounds is unlikely to invalidate our findings, we hope that the discussion which follows, showing the various implications such measurements can have, will motivate a reanalysis of the SDSS main galaxy sample, but with a lower value of \( \sigma_8 \).

The mass-independence of the satellite luminosity function has implications for the stellar mass-to-light ratios of satellite galaxies. Since \( M_*/L \) is strongly correlated with galaxy color (e.g., Bell et al. 2003), the approximate independence of \( L_{\text{sat}} \) on halo mass means that we should expect that satellite color is an excellent indicator of satellite stellar mass, regardless of the mass of the host halo.

The approximate independence of satellite luminosity functions on group size also suggests that the extreme value statistics of the sort pioneered by Scott (1957) should provide a good description of the luminosity function of the most luminous satellites. Note that most previous work has used extreme value statistics to model the luminosity function of the central rather than satellite galaxies (e.g. Bhavsar & Barrow 1985). This is the subject of work in progress (also
see Vale & Ostriker 2007). More recent work has phrased this discussion in terms of the luminosity gap between the first and second, or second and third ranked galaxies in clusters. For example, Milosavljevic et al. (2006) suggest that the gap between first and second ranked galaxies is correlated with the dynamical age of the system: “fossil” groups, poor clusters, and rich clusters are distinguished by the time since their last major merger, so their luminosity gaps differ. In addition, van den Bosch et al. (2007) find that the average luminosity gap and the fossil group fraction both increase with decreasing host halo mass.

This last finding is particularly easy to understand in the context of our results. At small halo masses, the luminosity of the central galaxy grows linearly with halo mass, but the growth is only logarithmic at large masses (equation [2]). On the other hand the number of satellites grows slightly more strongly than linearly (equation [1]). If the satellite luminosity function is independent of halo mass, then massive haloes are allowed more draws from the universal satellite luminosity function. If this function has an exponential tail, and equations [2] and [10] suggest that it does, then the most luminous of these draws grows logarithmically with the number of draws, so it grows logarithmically with halo mass. Thus, the luminosity gap is larger at small masses, and decreases at larger masses. This effect is further helped by the fact that (i) the satellite luminosity function is not quite independent of halo mass—mean satellite luminosity increases slightly with halo mass; (ii) in equation [1], \( M_1(L) \approx 23 \, M_{\text{min}}(L) \), but the factor of 23 is replaced by a smaller factor at large \( L \). In effect, this allows for even more luminous satellites in massive haloes.

If satellite galaxies are associated with the subhaloes of dark matter haloes, then the halo model predicts that central and satellite galaxies of the same luminosity should differ in mass by factors of about 90/23 \( \approx 4 \) (the centrals being more massive), whereas the stellar masses at fixed luminosity are unlikely to be very different (equation [11]). Weak-lensing analyses should soon be able to test this prediction (e.g. Yang et al. 2006), as should analyses of satellite dynamics (e.g. McKay et al. 2002). In practice, there is likely to be more scatter between subhalo mass and luminosity than there is between parent halo mass and luminosity; we expect this to alter our conclusions qualitatively but not qualitatively. This too can be checked by lensing analyses. At the time or writing, Limousin et al. (2007) have concluded a weak-lensing study of five clusters. They report a difference between central and satellite masses (at fixed luminosity) of about a factor of 5. Dynamical mass is proportional to \( R \sigma^2 \), so since tidal stripping usually does not significantly affect velocity dispersion, the smaller sizes of cluster galaxies compared to field galaxies at the same luminosity implies that the satellite galaxies are less massive by a similar factor. The measurements are still quite uncertain, but constraints from lensing are improving. We look forward to more such data, since our analysis has shown that such studies are rather closely related to studies of the luminosity dependence of galaxy clustering.

If the difference between the factors of 23 (in the halo model description of the mass required to host one satellite) and 90 (in the subhalo mass function) is associated with mass lost to stripping processes as satellites become incorporated into parent haloes, then the mass of a satellite prior to stripping is about 90/23 \( \approx 4 \) times larger than its current mass: about 80\% of its mass is stripped. This is slightly larger than the \( \sim 60\% \) mass-loss factors seen in simulations which only include the dark matter component (e.g. Nagai & Kravtsov 2005). Given that the halo-model argument is based on relating the subhalo population in simulations to observations, it is remarkable that the two estimates are similar.

Our analysis of the connection between subhaloes, galaxies and the halo model has another interesting consequence. The mass fraction in subhaloes is

\[
\int_0^M \frac{m}{M} \frac{dN(m|M)}{dm} \, dm = 0.1 \left( \frac{M}{10^{14} h^{-1} M_\odot} \right)^{0.1},
\]

where we have used equation (11) for the subhalo mass function, \( dN/dm \). If stars only form in sufficiently massive objects, the lower limit to this integral may be greater than zero: this will change the quantitative estimates which follow, but not the qualitative conclusions.

Our estimate of the mass lost to stripping processes, when combined with equation (13) for the mass fraction in subhaloes, leaves about half the mass of a \( 10^{12} h^{-1} M_\odot \) parent halo unaccounted for. For a \( 10^{15} h^{-1} M_\odot \) mass halo this fraction is about twenty percent. For comparison, using the model of subhalo mass-loss due to stripping in Vale & Ostriker (2005; see their Figure 1) with equation (15), these corresponding fractions are about thirty percent and twenty percent, when subhaloes with \( m > 10^{13} h^{-1} M_\odot \) are considered. (These mass fraction estimates are highly sensitive to their model of the amount of mass stripped from the numerous very low mass haloes, however.) Presumably this mass is associated with the central galaxy itself, and/or with subhaloes that were completely disrupted by the parent halo. If these objects hosted stars, then these stars may have been incorporated into the central object, or they may now contribute to intracluster light. Indeed, results from recent studies of intracluster light support the idea that much of the light in the ICL comes from the stripping, disruption, and merging of satellite galaxies (Gonzalez et al. 2005; Zibetti et al. 2005).

Consider a \( 10^{15} h^{-1} M_\odot \) cluster, for which the halo model predicts about eighty percent of the stellar mass is associated with satellite galaxies; the rest is in the BCG or in the intracluster medium. If the luminosity and color of the BCG are observed (as is the case for the SDSS), reasonable assumptions about its stellar mass allow one to predict the stellar mass associated with the ICL. For example, the halo model says the BCG is about 4.5 times more luminous than the satellite galaxies brighter than \( M_r < -19.9 \), and that there should be about 70 such satellites. If the stellar mass to light ratio is independent of \( L \) for the red galaxies in a cluster this should be a reasonable assumption, then the stellar mass of the BCG counts like an additional 4.5 satellites. This makes the stellar mass accounted for (74.5/70) times 80\%, leaving about 15\% of the stellar mass for the ICL or elsewhere.

At \( 10^{14} h^{-1} M_\odot \) the halo model has about 15\% of the total mass in subhaloes, whose associated satellite galaxies contain about 60\% of the total stellar mass. There are about 5 satellite galaxies brighter than \( M_r < -19.9 \), and the BCG is a little more than 3 times the average luminosity of these satellites. This leaves \( 1 - 0.6(8/5) \) or about 5\% of the...
stellar mass for the ICL. Thus, the halo model predicts the ICL fraction to increase with host halo mass, in agreement with other recent work (Purcell et al. 2007; Murante et al. 2007). Despite the crudeness of these estimates, we think our discussion illustrates how the halo model can be related to recent studies of intracluster light. This may be particularly useful in view of current uncertainties about the fate of ‘orphan’ satellites in simulations (Gao et al. 2004a; Conroy et al. 2007).

ACKNOWLEDGEMENTS

We thank Andreas Berlind for helpful discussions about his group catalog, and Frank van den Bosch for providing the group catalog of Yang et al., and for discussions about our conclusions. RAS thanks Sebastian Jester for help with using SDSS CasJobs to obtain photometric information for groups with fiber collided galaxies. This work was supported in part by NSF grant 0520647.

Funding for the SDSS and SDSS-II has been provided by the Alfred P. Sloan Foundation, the Participating Institutions, the National Science Foundation, the U. S. Department of Energy, the National Aeronautics and Space Administration, the Japanese Mombukagakusho, the Max Planck Society, and the Higher Education Funding Council for England. The SDSS Web Site is http://www.sdss.org/

The SDSS is managed by the Astrophysical Research Consortium for the Participating Institutions. The Participating Institutions are the American Museum of Natural History, Astrophysical Institute Potsdam, University of Basel, University of Cambridge, Case Western Reserve University, University of Chicago, Drexel University, Fermilab, the Institute for Advanced Study, the Japan Participation Group, Johns Hopkins University, the Joint Institute for Nuclear Astrophysics, the Kavli Institute for Particle Astrophysics and Cosmology, the Korean Scientist Group, the Chinese Academy of Sciences (LAMOST), Los Alamos National Laboratory, the Max-Planck-Institute for Astronomy (MPIA), the Max-Planck-Institute for Astrophysics (MPA), New Mexico State University, Ohio State University, University of Pittsburgh, University of Portsmouth, Princeton University, the United States Naval Observatory, and the University of Washington.

REFERENCES

Abazajian K. et al., 2004, AJ, 128, 502
Adelman-McCarthy J. K. et al., 2006, ApJS, 162, 38
Bell E. F., McIntosh D. H., Katz N., Weinberg M. D., 2003, ApJS, 149, 289
Berlind A. A., Blanton M. R., Hogg D. W., Weinberg D. H., Davé R., Eisenstein D. J., Katz N., 2005, ApJ, 629, 625
Berlind A. A. et al., 2006, ApJS, 167, 1
Berlind A. A., Kazin E., Blanton M. R., Pueblas S., Scoccolinaro R., Hogg D. W., 2007, ApJ, submitted
Bhavsar S. P., Barrow J. D., 1985, MNRAS, 213, 857
Blanton M. R. et al., 2003, ApJ, 592, 819
Cooray A., 2002, Phys. Rep., 372, 1
Cooray C., Wechsler R. H., Kravtsov A. V., 2006, ApJ, 647, 201
Cooray C., Wechsler R. H., Kravtsov A. V., 2007, astro-ph/0703374

APPENDIX A: INCONSISTENCIES BETWEEN THE HALO MODEL OF ZEHAVI ET AL. AND THE GROUP CATALOG OF BERLIND ET AL.

The main text used the halo model parameters of Zehavi et al. (2005). Figure [A1] shows that this halo model underpredicts the abundance of groups in Berlind et al.’s $M_r < -19.9$ group catalog, and it overpredicts the central galaxy luminosities in these groups. That is, a model that correctly describes the abundance and clustering of SDSS galaxies as a function of luminosity is not consistent with the multiplicity function and central luminosities of this catalog of SDSS
Figure A1. The halo model of Zehavi et al. (2005) underestimates the abundances of Berlind et al.’s galaxy groups. We have performed the same analysis with the Berlind et al. group catalog used in Figure 1. We k-corrected, evolution-corrected, and extinction-corrected them the same way as was done with the galaxies with spectra. We repeated our analysis of $⟨L_{cen}|N_{gal}⟩$, and found it was indeed biased low, but only by 7% on average. In addition, given knowledge of the frequency of fiber collisions as a function of $N_{gal}$, we estimated the effect by modifying equation (2), and we similarly found a bias of about 7%. The fiber collision-corrected measurement and halo model prediction of $⟨L_{cen}|N_{gal}⟩$ are shown in the lower panel of Figure A1 and it is clear that the effect of fiber collisions is smaller than the uncertainties in the measurements and less significant than the discrepancy with the model.

We believe the discrepancies in the multiplicity function and central galaxy luminosities in the figure may be a consequence of Zehavi et al.’s assumption that $σ_8 = 0.9$, which is somewhat larger than the value $σ_8 ≈ 0.8$ suggested by more recent analyses of other data sets (e.g. van den Bosch et al. 2007). We argue below that $σ_8 = 0.8$ is likely to reduce the discrepancies shown in Figure A1 without changing the main point of our paper: that satellite luminosities depend only weakly on halo mass, and that the dark haloes which surround central galaxies are more massive than those which surround satellites of the same luminosity.

If $σ_8$ is lower, then the number of massive haloes is lower as is the clustering of the dark matter. To match the observed abundances and clustering in the main galaxy catalog, the number of galaxies in more massive haloes must be increased relative to the numbers from Zehavi et al. This will have the effect of increasing the number of groups with large $N_{gal}$, thus reducing the discrepancy seen in the top panel of Figure A1. In addition, if $σ_8$ is smaller, then a given value of $N_{gal}$ would be associated with smaller mass haloes, making the mass to light ratio of haloes smaller. Such a change would help alleviate a source of tension between halo models and observations: the mass-to-light ratio of $10^{14}h^{-1}M_\odot$ haloes in our model with the Zehavi et al. halo occupation distribution is about 900±100 for $M_{r} \leq -20$, and although this is consistent with similar models for the same cosmology (Tinker et al. 2005), recent observations have lower mass-to-light ratios that require lower values of $σ_8$ and $Ω_M$ to model them consistently with the abundance and clustering of galaxies (van den Bosch et al. 2007).

Also, Berlind et al. (2007) have argued that the clustering of these groups are indeed better fit by a cosmological model in which $σ_8 = 0.8$, though they do not show if the associated group abundances are consistent with this value (Sheth & Tormen 1999 show that, in hierarchical models, abundance and clustering are very closely related, so it is likely that the abundances are also better fit by the lower $σ_8$ value).

For all these reasons, it is likely that repeating Zehavi et al.’s (2005) analysis of the SDSS main galaxy sample, but with a lower value of $σ_8$, would be very interesting. While such a reanalysis is beyond the scope of this work, we note that our main results are unlikely to be altered by such a change. This is because changing $σ_8$ by $\sim 10\%$ is unlikely to change the fact that $α$ in equation (4) is only a weak function of $L$, and it is this weak dependence which makes satellite luminosities only weakly dependent on halo mass (c.f. discussion following equation (7)).

Finally, it is important to note that the friends-of-
Figure A2. Same as Figure A1 but with $N_{\text{gal}} \rightarrow (N_{\text{gal}} - 1)$ for the group catalog measurement. Now there is good agreement with the halo model (dashed curve). For comparison, the halo model prediction for $\sigma_8 = 0.8$ and $\Omega_M = 0.25$, constrained by the correlation function and luminosity function at $\Omega_M = 0.3$, is also shown (solid curve).

The friends (FoF) algorithm adopted by Berlind et al. (2006) is not an unbiased group-finder, especially at low $N_{\text{gal}}$. As they show in their Figures 12 (in real space) and 14 (in redshift space) in their appendix, for a particular choice of linking length, the FoF algorithm correctly recovers the “true” multiplicity function in mock catalogs only for $N_{\text{gal}} > 10$. At $N_{\text{gal}} \leq 10$, the algorithm is increasingly biased with decreasing $N_{\text{gal}}$: it finds too many groups, merging too many lower $N_{\text{gal}}$ groups and isolated galaxies together. The problem is worse in redshift space, where $N_{\text{gal}} < 8$ groups are over-counted by 50% or more, which clearly shows that the small error bars at low $N_{\text{gal}}$ in Figure A1 are misleading.

In Figure A2 we show that if, on average, the FoF group-finder tends to link one galaxy too many to each group, then the resultant multiplicity function agrees very well with the halo model prediction (solid curve). For comparison, we also show the halo model prediction for $\sigma_8 = 0.8$ and $\Omega_M = 0.25$ (dashed curve), with the halo occupation distribution constrained by the correlation function and luminosity function at $\Omega_M = 0.3$. In particular, for this calculation we are assuming that the change to $\Omega_M$ is sufficiently small that the $\phi(L)$ (Blanton et al. 2003) and $w_p(r_p|L)$ (Zehavi et al. 2005) to which the halo model is fit are unchanged. This is not an unreasonable assumption, since the luminosity functions and correlation functions of volume-limited SDSS catalogs have been found to be not significantly affected by such a small change in cosmology (F. C. van den Bosch, private communication). We then changed $\sigma_8$ in the halo model, thus changing the linear power spectrum and the halo mass function, and found what new halo occupation distribution is required that still correctly describes $\phi(L)$ and $w_p(r_p|L)$. The resulting best-fitting halo occupation distributions for $M_r < -20$ have somewhat similar $M_{\text{min}}(L)$, but with significantly lower $M_f(L)$ and a steeper $\langle N_{\text{sat}}|M \rangle$ slope (see equation 4). However, the resulting multiplicity function predicted by the halo model is similar for both $\sigma_8 = 0.8$ and 0.9, which suggests that the discrepancy in Figure A1 may be due as much to the group-finder itself as to the assumed cosmology.