General Relativistic Collapse of Neutron Stars to Strange Stars: A Mechanism for Gamma Ray Bursts

Abhas Mitra
Theoretical Physics Division, Bhabha Atomic Research Centre, Mumbai-400085, India
E-mail: amitra@apsara.barc.ernet.in

ABSTRACT
It is known that Neutron Stars may be converted into more compact Strange Stars (SS) on capture/formation of a “seed” of strange matter. It is also known that the binding energy of the nascent hot SS is likely to be radiated as $\nu - \bar{\nu}$ so that an electromagnetic pair fireball (FB) may be created by neutrino annihilation. But we show here that, a General Relativistic treatment of the problem may lead to a FB energy ($Q_{FB} \sim 10^{53}$ erg) which could be higher by as much as four orders than what was estimated previously. Further since the baryonic mass of the thin crust of a strange star is negligible, this FB will generate an extremely relativistic blast wave. Thus this process may be one of the viable routes for the genesis of hitherto unexplained cosmic Gamma Ray Bursts.

Key words: Gamma rays: bursts – Gamma rays: theory Stars: neutron – Relativity – Gravitation

1 INTRODUCTION
It is now clear that a large number of Gamma Ray Bursts (GRBs) involve emission of $\gamma$-rays as large as $Q_\gamma \sim 10^{52} - 10^{54}$ erg under condition of isotropy (Kulkarni et al. 1999). However, many GRB afterglow light curves decline more rapidly than what is expected in simple isotropic fireball models, and it is plausible that such afterglows are actually beamed (Kulkarni et al. 1999, Huang et al. 2000). In such cases, the total (electromagnetic) energy budget could be substantially reduced (often by a factor $\sim 100$). Nonetheless, many afterglows do not fade with such unusual rapidity and the decay of the light curve may be smoothly fitted by a single power on the time scale of months. Such GRBs are likely to be more or less spherical events, and thus, for GRB 971214, we may indeed have $Q_\gamma \approx 3 \times 10^{53}$ erg (Dal Fiume et al 2000). There have been many attempts to explain the energy budget and origin of powerful GRBs with special emphasis on the beamed (Paczynski 1998, Popham, Woosley & Frier 1999, Ruffert et al. 1997). In the following, we put forward an alternative model to explain the origin of GRBs with a value of $Q_\gamma \sim 10^{53}$ erg for which the problem of baryonic contamination is minimal because we are considering the possibility of conversion of an isolated or mass accreting massive neutron star (NS) into a Strange Star (SS). It might appear at first sight that the model is not new at all because starting with Alcock, Farhi & Olinto (1986), several other authors have considered this process as a probable mechanism of GRBs (Ma & Xie 1996, Cheng & Dai 1996). For example, Ma & Xie (1996) studied the phase transition of a neutron star into a “hybrid” star. Cheng & Dai (1996) also pointed out that a NS in a low mass X-ray binary, after accreting sufficient mass may undergo a similar phase transition and power a weak GRB event. While Alcock, Farhi & Olinto attempted to explain an electromagnetic FB of $Q_{FB} \sim 10^{45} - 46$ erg, the other authors, crudely estimated that the value of $Q_{FB} \sim 10^{49}$ erg. Clearly such low values of $Q_{FB}$ are highly insufficient to explain the energetics of presently observed GRBs. Thus while the basic concept involved in this paper is not new (NS $\rightarrow$ SS), this general relativistic model as such is new because it may explain four order higher values of $Q_{FB}$.

Since this paper has similarity in idea with that of Cheng & Dai (1996), we shall briefly recall the main result of the latter paper. Cheng & Dai adopted a different approach and estimated that the new born strange star has a thermal energy $E_{th} \sim 5 \times 10^{52}$ erg. The star cools by emitting this thermal energy in the form of $\nu, \bar{\nu}$ pairs. While the neutrinos traverse out from the core, they impart electromagnetic energy inside the star by processes like $n + \nu_e \rightarrow p + e^-$ and $p + \bar{\nu}_e \rightarrow n + e^+$. Cheng and Dai estimated the optical thickness of the above processes to be $\tau \sim 4.5 \times 10^{-2}$ and concluded that the electromagnetic energy deposited within the body of the star to be $E_2 \sim \tau \times E_{th} \sim 2 \times 10^{52}$ erg, which is obviously incorrect for the given value of $E_{th}$ and $\tau$. Thus, as one can easily verify, Cheng & Dai here overestimated $E_2$ by a factor of 10 and its value works out to be $E_2 \sim 2 \times 10^{51}$ erg. Moreover, Cheng & Dai overlooked here fact that pairs
generated \textit{within the body} of the star cannot come out because the star is exceedingly optically thick (electromagnetic optical thickness is $\sim 2.5 \, \text{g cm}^{-2}$). Thus this electromagnetic energy gets absorbed within the star and reemitted primarily as neutrinos. So the total neutrino flux coming out of the star is $\sim E_{\nu} h \times 5 \times 10^{52} \, \text{erg}$. Now Cheng & Dai estimated that total energy coming out from the star in the form of pairs, due to the $\nu + \bar{\nu} \rightarrow e^+ + e^-$ process, is $E_1 \sim 10^{49} \, \text{erg}$. Thus, the total \textit{external} pair flux energy, in the model of Cheng & Dai should be $E_0 \approx E_1 \sim 10^{49} \, \text{erg}$ and not $E_0 = E_1 + E_2 \sim 5 \times 10^{52} \, \text{erg}$ as erroneously concluded by them.

The general relativistic treatment presented below would show that, actually, the pair fireball (FB) energy can be much more.

2 THE MODEL

We consider an initial scenario where a NS, either isolated or accreting mass in a low mass X-ray binary, is of low compactness. This is possible for a class of low density stiff NS equation of states. For instance, the radius of a $1.4M_\odot$ NS with a (low) fiducial density parameter $\rho_0$ could be (Kalogera & Baym 1996).

$$R = 21.2 \text{ km} \left( \frac{\rho_0}{10^{14} \text{g cm}^{-3}} \right)^{-0.35}$$

Here $\rho_0$ is not to be confused with the central density $\rho_c$. Such low density NSs have a binding energy (BE) insignificant compared to their baryonic mass $E_{\text{B}}^\odot \sim 0.1M_\odot c^2$. Here $M_\odot$ is solar mass and $c$ is the speed of light. Following Olinto (1987) we consider the possibility that the NS is hit by a primordial strange matter clump or seed in the form of a cosmic ray. Once the NS gravitationally captures such a “seed”, the neutrons surrounding the seed will be absorbed by it and be deconfined to be strange matter (Alcock & Olinto 1988, Cheng, Dai & Lu 1998). Thus the strange matter seed can become bigger and bigger until the whole star is converted into a strange star. And this process may be complete extremely fast on a timescale of $10^{-7} \, \text{s}$ (Alcock & Olinto 1988, Farhi & Jaffe 1984) This process may occur for an accreting NS once it starts collapsing after its mass crosses a certain maximum value and the central density increases sufficiently. This might also happen for the rapidly spinning supramassive NS scenario (Vietri & Stella 1998) where the NS starts collapsing after loss of sufficient angular momentum.

The maximum values of masses and radii of strange stars may be represented by (Witten 1984, Cheng, Dai & Lu 1998)

$$M = 2.0 \, M_\odot \left( \frac{B_0}{B} \right)^{1/2}$$

and

$$R = 11.1 \left( \frac{B_0}{B} \right)^{1/2} \text{ km}$$

where the “bag constant” is $B$ and $B_0 = 57 \, \text{MeV fm}^{-3}$ Therefore the surface gravitational redshift of a static spherical strange star is given by (Shapiro & Teukolsky 1983) :

$$z = \left( 1 - \frac{2GM(R)}{Rc^2} \right)^{-1/2} - 1 \approx 0.5$$

Here $r$ is the invariant circumference radius, $c$ is the speed of light, and $M$ is the gravitational mass enclosed within $r = r$

$$M(r) = \int_0^r \rho dV = \int_0^r dM$$

where $\rho$ is the total mass-energy density, $dV = 4\pi r^2 dr$ is coordinate volume element, and the symbol $dM$ is self-explanatory.

The self-gravitational energy of a static relativistic star is given by (Shapiro & Teukolsky 1983)

$$E_\text{g} = \int_0^r \rho dV \left\{ 1 - \left[ 1 - \frac{2M(r)}{r} \right]^{-1/2} \right\}$$

Then recalling the definition of $z$ from Eq. (4), we may write

$$E_\text{g} = - \int z(r) dM \approx \alpha z M \approx -z M$$

where $\alpha \sim 1$ is a model dependent parameter and studying the models of relativistic polytropes, we have numerically verified that for $z < 0.6$, indeed $\alpha > 1$.

The binding energy of a cold star is given by $E_B \approx (1/2) | E_\nu |$. If the NS undergoes a phase transition to become a more compact SS, the total energy to be liberated would be $E_B (SS) - E_B (NS)$. But here we are considering \textit{only those situations where the original NS is not very compact} (Kalogera & Baym 1996) with a canonical value of $z \leq 0.15$, and thus, for the sake of analytical simplicity we neglect the initial B.E. of the NS in comparison to the much larger B.E. of the SS, and to compensate for the initial B.E. we set here $\alpha = 1.0$ although, actually it is marginally higher. Since occurrence of powerful GRBs having a frequency $\sim 10^{-6} - 10^{-7} \, \text{galaxy/yr}$ (under conditions of isotropy) is an extremely rare event compared to the same for supernova events, it may be justified to consider only favourable initial conditions. Further during the SS conversion, an additional energy of $\sim 30 \, \text{MeV/nucleon}$ is liberated due to quark deconfinement (Alcock & Olinto 1988, Fari & Jaffe 1984). For the sake of analytical simplicity, we club all such liberated energy into a single expression $Q_\nu \approx E_B \approx \frac{z M}{2}$. In the case of NS formation, the nascent hot NS radiates most of its BE in the form of $\nu - \bar{\nu}$ on a time scale of $\sim 10 \, \text{s}$ (Shapiro & Teukolsky 1983) and we take this as a fiducial scale for the present problem.

So, given this value of $z = 0.5$ the maximum value of $Q_\nu \approx 1.0 \times 10^{54} M_2 \, \text{erg}$ where of $M = M_2 2M_\odot$. The value of $Q_\nu$, measured near the compact object will be higher by a factor $(1 + z)$: $Q'_\nu = z(1 + z)M/2$. And the locally measured duration of the burst would be $t'_\nu = (1 + z)^{-1} t_\nu$. Therefore, the mean (local) $\nu - \bar{\nu}$ luminosity will be

$$L'_\nu = \frac{Q'_\nu}{t'_\nu} \approx \frac{z(1 + z)^2 M}{2t_\nu} \approx 2 \times 10^{33} z(1 + z)^2 M_2 \, t_{10}^{-1} \text{ erg/s}$$

where $t_\nu = t_{10} \, \text{10s}$. As to the cooling of strange stars, there is an important difference with respect to a NS: while a NS cools via emission of all 3 flavours of $\nu$, $\bar{\nu}$ of approximately equal proportion, a SS cools primarily \textit{via the emission of electron neutrinos} $\nu_e$:

$$u + e^- \rightarrow d + \nu_e, \, u + e^- \rightarrow s + \nu_e, \, d \rightarrow u + e^- + \bar{\nu}_e, \, s \rightarrow u + e^- + \bar{\nu}_e$$

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Thus there will primarily be only two species of neutrino each contributing to a luminosity \( L'_\nu = (1/2)L_\nu \). In contrast normal baryonic matter (like a NS) cools via emission of six species of neutrinos. Consequently, emissivity of electromagnetic pairs is much higher for a SS cooling in comparison to a NS cooling, and we feel that, this point has not been noted earlier. By assuming the radius of the neutrinosphere to be \( R_\nu \approx R \), the value of effective local neutrino temperature \( T' \) obtained from the condition

\[
L'_\nu = \frac{7}{8} \pi R^2 \sigma T'^4
\]

where \( \sigma \) is the Stefan-Boltzmann constant. By equating Eqs. (8) and (10), we find

\[
T' = \left( \frac{4z(1 + z)^2 M_e^2}{7\pi^2 R^2 c_d} \right)^{1/4} \approx 18\text{MeV}
\]

\[
z^{0.25}(1 + z)^{0.5} M_2^{0.25} R_6^{0.5} t_{10}^{-0.25}
\]

where \( R = R_0 10^6 \). For a Fermi-Dirac distribution, under the crude assumption of zero \( \nu \)-chemical potential, the mean (local) energy of the neutrinos is \( E'_\nu \approx 3.15 T' \approx 57\text{MeV} \) (for \( z = .5 \)). The various neutrinos will collide with their respective antiparticles to produce electromagnetic pairs by the \( \nu^+ \nu^- \rightarrow e^+ e^- \) process. The rate of energy generation by pair production per unit volume per unit time, at a distance \( r \) from the center of the star, is given by (Goodman, Dar & Nussinov 1987)

\[
\dot{q}_\varepsilon(r) = \sum_i K_{\nu_i} G_F^2 E_i' L_i^2(r) \frac{\sigma}{12\pi^2 c R_6^2} \varphi(r)
\]

Here, \( L_i^2(r) \sim r^{-2} \) is the \( \nu \)-flux density of a given species above the \( \nu \)-sphere, \( G_F^2 = 5.29 \times 10^{-44} \text{cm}^3 \text{MeV}^{-2} \) is the universal Fermi weak coupling constant squared, \( K_{\nu_i} = 2.34 \) for electron neutrinos and has a value of 0.503 for muon and tau neutrinos. Here the geometrical factor \( \varphi(r) \) is

\[
\varphi(r) = (1 - x)^4(x^2 + 4x + 5); \quad x = [1 - (R_\nu/r)^2]^{1/2}
\]

But since in the present case, most of the neutrinos are of the electronic nature, there is substantial enhancement of the efficiency

\[
\dot{q}_\varepsilon(r) = \frac{K_{\nu_i} G_F^2 E_i' L_i^2(r)}{48\pi^2 c R_6^2} \varphi(r)
\]

Now, a simple numerical integration yields the local value of pair luminosity produced above the neutrinosphere:

\[
L_{\perp} = \int_{0}^{\infty} \dot{q}_\varepsilon 4\pi r^2 dr \approx \frac{K_{\nu_i} G_F^2 E_i' L_i^2}{27\pi^2 c R_6} \approx 1.2 \times 10^{52}
\]

\[
z^{2.25}(1 + z)^{4.5} M_2^{2.25} t_{10}^{-2.25} R_6^{-2} \text{erg/s}
\]

This estimate is obtained by assuming rectilinear propagation of neutrinos near the SS. Actually, in the strong gravitational field near the SS surface the neutrino orbits will be curved with significant higher effective interaction cross-section. Since, most of the interactions take place near the \( \nu \)-sphere, for a modest range of \( z \), we may tentatively try to incorporate this nonlinear effect by inserting a \( (1+z)^2 \) factor in the above expression. On the other hand, the value of this electromagnetic luminosity measured by a distant observer will be smaller by a factor of \( (1+z)^{-2} \), so that eventually, \( L_{\perp} = L_{\perp}^{L_\nu} \) of Eq.(15). And the total energy of the electromagnetic FB at \( \infty \) is

\[
Q_{FB} = t_\nu L_\perp \approx 1.2 \times 10^{53}
\]

\[
z^{2.25}(1 + z)^{4.5} M_2^{2.25} t_{10}^{-2.25} R_6^{-2} \text{erg}
\]

For \( z = 0.5, M_2 \approx 1, R_6 \approx 1, t = 10s \), we obtain a highest value of \( Q_{FB} \approx 1.5 \times 10^{53} \text{erg} \). The efficiency for conversion of \( Q_6 \) into \( Q_{FB} \) in this case is \( \epsilon_6 = Q_{FB}/Q_6 \approx 15\% \). It is known that NSs have an upper limit of mass \( M \approx 3.0M_\odot \) and with the inclusion of rotation the maximum mass could be as high as \( 3.7M_\odot \) (Shapiro & Teukolsky 1983). Similarly for a bag constant \( B = B_0 = 57\text{MeVf/m}^3 \), the maximum mass of a rotating SS is \( 2.4M_\odot \) (Colpi & Miller 1992). At this stage the actual value of \( B_0 \) is uncertain and thus a value of \( M \geq 3M_\odot \) can not be ruled out for a SS. Then Eq. (16) may yield a maximum value of \( Q_{FB} \approx 3.7 \times 10^{53} \text{erg} \), as may be required for GRB 971214.

3 DISCUSSION

In some ways, our model in the General Relativistic extension of the previous work by Cheng & Dai (1996), and it is free from the “baryon pollution problem” in same way. The mass of the baryonic crust of a canonical SS is only \( \Delta M = 2 \times 10^{-5}M_\odot \) (Cheng, Dai & Lu 1998, Alcock & Olinto 1984). For the massive SS, even if the value of \( \Delta M \) is considerably higher, we would always have the degree of baryonic pollution \( \eta = Q_{FB}/\Delta M > 10^2 \), and the genesis of a GRB may be understood (Meszaros & Rees 1992).

For quick comprehension by the readers, let us explain here, why our relativistic model is able to generate a FB energy as high as four orders of magnitude than previous non-relativistic estimates. In a non-relativistic model, one usually considers a NS with mass of \( 1M_\odot \), so that \( M_2 = 0.5 \). The radius of such a NS is taken as 10 Km so that \( R_6 = 1 \). For such a NS \( z \approx 0.15 \). We call this Case I. On the other hand, for the present relativistic model (Case II) involving a heavy NS with mass \( 2M_\odot \), we have \( M_2 = 1 \). The radius of such a NS is 11 Km which is practically the same as in Case I, so that for qualitative understanding, we neglect any variation with respect to \( R \) in the two cases. As mentioned before, in this latter case \( z \approx 0.5 \).

Then for a a burst of duration of 10 s, the essential relativistic fators in Eq. (16) are

\[
Q_{FB} \sim z^{2.25}(1 + z)^{4.5} M_2^{2.25}
\]

For case I, the numerical value of this factor is \( Q_{FB} \sim 0.005 \). And for Case II, the same numerical factor is \( Q_{FB} \sim 1.3 \). Thus, ignoring higher pair emissivity for the moment, the ratio of relativistic factors in the two cases is 260. The previous authors ignored the fact that for SS cooling, pair emissivity is much higher compared to the case of NS cooling. For the latter case (Case I), for a total neutrino luminosity of \( L'_\nu \), the luminosity in each species is \( L'_\nu = (1/6)L_\nu \). In contrast for the former case (Case II), we have \( L'_\nu = (1/2)L_\nu \). The pair emissivity, as per Eq. (12) is \( \propto L'_{\nu}^2 \). The same is also \( \propto E'_\nu \propto T' \propto L'_{\nu}^{2.25} \). Then finally pair emissivity \( \propto L_{\nu}^{2.25} \). Since, for Case II, \( L'_\nu \) is higher of 3, one would obtain a pair emissivity higher by a factor of \( \sim 12 \) here if the value of \( K_{\nu_i} \) were same for all the flavours of neutrinos. But, as mentioned before, while \( K_{\nu_i} = 0.503 \) for \( \mu \) and \( \tau \) neutrinos, its value is 2.34 for electron neutrinos. This fact enhances the
pair emissivity approximately by a factor of 4 for Case II. Considering all the three effects, FB energy becomes higher by a net factor of $260 \times 12 \times 4 \approx 10^4$ in Case II!

It might be possible that upon capturing a sufficiently massive strangelet, not only a NS, but an ordinary star like the Sun can also get converted into a SS (Alcock & Olinto 1988). Here the burning process into strange matter arises via a series of proton capture reactions apart from the usual neutron capture and deconfinement mode (Alcock & Olinto 1988). If the NS $\rightarrow$ SS process is complete within $10^{-7}$ s, it is conceivable that a normal star $\rightarrow$ SS process is completed well within 1 s. And if a massive star gets converted into a massive SS in this way, we would be able to explain origin of GRBs having energy even much higher than $10^{54}$ erg. However, the resultant massive SS must be unstable and is likely to proceed for further collapse.

In the case of a supernova, even in an initial spherical model there could be symmetry breaking Rayleigh-Taylor and Kelvin-Helmoltz instabilities because of the severe density gradients in the presence of extreme and variable gravity (Burrows 2000). There could be additional asymmetry if the SS is magnetized and spins fast, and thus it is plausible that the resultant GRB will be beamed. Finally, the recent discovery of a quark-gluon plasma state in CERN provided as an additional motivation behind this work (Abott 2000).

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