Non-Hermitian electronics multipods of electromagnetically induced transparency (EIT) and absorption (EIA)

Senghor Tagouegni1,2 · Fernande Fotsa-Ngaffo2,3 · Aurélien Kenfack-Jiotsa2

Received: 7 March 2020 / Accepted: 5 February 2022 / Published online: 10 March 2022
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Abstract
We study a non-Hermitian electronic dimers system based on an imaginary resistor (Z) in a (N + 2) level atomic multipod configuration. Non-Hermitian systems depend on a gain/loss parameter and are specifically marked by a degeneracy exhibited at an exceptional point separating different phases of complex modes dynamics. Interestingly, the structural characterization and the dispersive properties reveal a broad range of strong coupling where the interplay between the control and the probe field induce a simultaneous EIT, EIA and ATS. Here, by identifying the underlying physical mechanisms, we show that multiple windows of transparency can be strongly enhanced by the incorporation of several dimers in the multipod network. On the other hand, if the pumping field is resonant in the weak regime, multiple EIT and EIA windows result in the number of dimers. Remarkably, the proposed system embedded a multiple coupling mechanism whose modulation induces a coupling-less point (CPLP) whereby the energy cross. At this point EIT and related phenomena vanish.

Keywords Non-Hermitian · Couplingless · Multiple EIT · Multiple EIA · ATS

1 Introduction
Electromagnetically Induced Transparency (EIT) and related phenomena are prominent examples of coherent interactions between optical fields and multilevel atoms which have spurred the development of new materials with optimized optical properties (Harris et al.)
The coherent preparation produces remarkable changes in the dispersive properties of a medium which leads to quantum interference, namely the fano interferences, between the excitation pathways that control the optical response. The EIT has also been demonstrated to occur via the splitting of energy levels into dressed states by strong coupling fields. In general the transparency of the absorbing medium is increased at large control intensity of the pumping field, leading to the appearance of two dressed states which correspond to the Autler–Townes splitting (ATS) phenomenon (Autler and Townes 1955; Cohen-Tannoudji 1996). Such phenomena have given rise to a range of properties including lasing without inversion (Mompart and Corbalán 2000), ultra-slow light (Totsuka et al. 2007), stopping of light pulses (Liu et al. 2001; Phillips et al. 2001) quantum memory (Hétet et al. 2008), enhanced nonlinear optical processes (Wang et al. 2001), plasmonic sensing with narrow linewidths (Liu et al. 2010), optical switches (Bermel et al. 2006), just to name but a few. These possibilities opened new avenues for optical information storage and quantum information processing. While EIT renders a narrow spectrum of perfect transmission, its complementary effect known as electromagnetically induced absorption (EIA) defines a band of frequency in which enhanced absorption due to the wave coherence is observed. Traditionally, EIA occurs in a degenerated two-level system (Lezama et al. 1999). The EIA effect supports some exciting wave propagation concept like the anomalous dispersion, detection of foliage moisture and other types of sensing (Ramzan et al. 2016, 2018; Siddiqui et al. 2016).

The realization of EIT in atomic systems has been a difficult task imposed by some restrictions (Wang et al. 2016). Interestingly, The EIT effect was demonstrated in classical optical coupled resonators due to its flexible design and easy implementation (Smith et al. 2004). However, most of these can be tuned by mechanical alignment making it difficult to realize the dynamic control of the transparency window, which limits the practical application of EIT-like effect. In addition to a three-level type configuration (Dastidar and Dutta 2008), EIT has also been realized in various multilevel configurations (Petrosyan and Malakyan 2004; Huy et al. 2019; Bharti and Wasan 2014; Paspalakis and Knight 2002), and quite recently, it was extended to the N and (N + 1)-level systems (Paspalakis and Knight 2002; McGlown et al. 2001). These systems exhibited multiple EIT with highly desired applications in the bifurcation of quantum information in multiple channels.

Experimental realization were carried on by Garrido Alzar et al. (Garrido Alzar et al. 2002) who reproduced the EIT behavior using two linearly coupled LRC circuits. By changing the strength of the coupling parameter, the authors have successfully controlled the response of the system going from the analog of EIT to that of the ATS effect. Joshua Harden et al. (Harden et al. 2011), Zhengyang Bai et al. (Bai et al. 2013), reproduced double EIT (DEIT) through coupled LRC circuits, in a four-level atomic system, in the Y-inverted and tripod configurations, respectively.

In practice, manipulating the LRC circuits in higher frequency regimes may in general lead to miniaturization of devices, higher storage capacities, and larger data transfer rates, but introduces a frequency dependent phase shift and delay effects in two different points of the circuit (Tagouegni et al. 2020; Tabeu et al. 2019) which can hinder the judicious control of EIT.

In this paper, we present an alternative approach of designing electrical analogs of multiple Electromagnetically Induced Transparency (EIT) and/or Absorption (EIA) and related phenomena. Our models are based on (N + 1) coupled ZRC circuits in which the inductor \(L\) is merely replaced by the imaginary resistor of an impedance \(Z\) (\(Z = jr\), with \(j^2 = -1\)). This later presents a major asset with the ability to generate a frequency independent phase
shift, unlike the inductor/capacitor reactive elements (Tagouegni et al. 2020; Tabeu et al. 2019). The experimental realizations of the imaginary resistor have been suggested in the literature using a gyrator (Shouno and Ishibashi 2008; Fujii and Shouno 2019; Nakagawa et al. 2013; Elwakil and Maundy 2015). The imaginary resistor allows an efficient manipulation between the lower and higher frequencies. In addition, the ZRC oscillator has a natural frequency $\omega_0 = 1/rC$, which can be positive or negative according to the real value of the resistance $r$ associated to the imaginary resistor and the capacitance $C$ of the circuit. Within this concept, the ZRC multi dimer can exhibits quantum interference phenomena such as EIT, EIA and ATS with an optimal modulation of the couplings and the gain/loss parameter.

Our paper is organized as follows: in Sect. 2, we present the model equations and the eigenmodes analysis. Here, a structural characterization allows to identify for the system a weak and a strong coupling regimes, and then to emphasize the couplingless point (CPLP). In Sect. 3, the steady state dynamic and dispersive properties of the systems are investigated.

### 2 Equations of model and eigenmodes analysis

We consider a ZRC multi dimer obtained by coupling $(N+1)$ oscillators ZRC as shown in Fig. 1a. Each $Z_i R_i C_i$ circuit consists of a linear capacitor $C_i$, a real resistor $R_i$ and an imaginary resistor of impedance $Z_i$, where $Z_i = jr_i (j^2 = -1)$, all arranged in series. The sub-index $i = (0...N)$ indicates the oscillator’s number, and $N$ ($N \geq 1$) is the total number of cells coupled to the main loop oscillator $Z_0 R_0 C_0$ for which a harmonic voltage source $u(t)$ has been added in series with the resistor $R_0$. The coupling between loops is realized thanks to the capacitor $C_C$ and another imaginary resistor $Z_C (Z_C = jr_C)$, also arranged in series and belonging to the set of circuit meshes.

![Fig. 1a ZRC Multi dimer circuit used for a single and multiple EIT, ATS and EIA windows.](image)

![Fig. 1b Energy diagram of a $(N+2)$ level atomic system in a multi-pod type configuration consisting of $(N+1)$ lower levels $|i\rangle (i = 0...N)$ of which $N$ pumping lasers couple near resonantly each state to the upper state $|e\rangle$, while a probe laser couples level $|0\rangle$ to $|e\rangle$.](image)
In this model, the inductor has been replaced by an imaginary resistor as compared to the analog models using LRC circuits (Garrido Alzar et al. 2002; Harden et al. 2011; Bai et al. 2013).

Figure 1b illustrates a (N + 2) level atomic system in a multipod type configuration, where the (N + 1) lower states |i⟩ which can be degenerated or not, are coupled to a single excited upper level state |e⟩. The atomic system is initially prepared in a particular state |0⟩ and interacts with a probe laser that couples the lower state |0⟩ to an excited state |e⟩. Adding a total number of N light sources (each being in a state |i⟩) induces an atomic transition of frequency ωi from |i⟩ to |e⟩. The absorption spectrum of the probe laser is modified once the N control fields simultaneously interact with the medium, which results in the interferences between the different transitions pathways of the atom.

In what follows, our objective is to demonstrate that the ZRC multi dimer circuit (Fig. 1a) is analog to the (N + 2) level atomic system in a multipod type configuration (Fig. 1b). In this analogy, the loop Z0R0C0 of frequency ω0 models the atom. This represents the central loop where the other loops are connected to. The harmonic voltage u(t) is the probe laser field. Once the switch SWi is closed, each loop of frequency ωi acts as a control field to the central loop.

If the switches SW i, i = (0...N) are closed, the Kirchhoff’s laws leads to the following coupled equations:

$$\left\{ \begin{array}{l} (\beta_0 + j) \frac{dQ_0}{dt} - j\beta_{c0} \sum_{n=1}^{N} \frac{dQ_n}{dt} + \omega_0 Q_0 - \Omega_0 \sum_{n=1}^{N} Q_n = u/r_{ei}, \quad \text{for } i = 0 \\
(\beta_i + j) \frac{dQ_i}{dt} - j\beta_{ci} \left( \frac{dQ_0}{dt} - \sum_{n=1, n\neq i}^{N} \frac{dQ_n}{dt} \right) + \omega_i Q_i - \Omega_i \left( Q_0 - \sum_{n=1, n\neq i}^{N} Q_n \right) = 0, \quad \text{for } i \neq 0 \end{array} \right.$$

where $j = \sqrt{-1}$, $i = 0...N$, $\beta_i = R_i/r_{ei}$, $\beta_{ci} = r_c/r_{ei}$, $\omega_i = 1/r_{ei}C_{ei}$, $\Omega_i = 1/r_{ei}C_{ei}$, $r_{ei} = r_i + r_c$ and $C_{ei} = C_CC/(C_i + C_C)$.

$Q_i$ and $dQ_i/dt$ represent the charge and the current in each loop, respectively. The so-called ZRC multi dimer system leads to first order coupled ordinary differential equations, which can be easily analyzed. Moreover, the system can support negative frequencies, without altering it operating mode, therefore it opens avenue to investigate various quantum phenomena through positive and negative frequencies. Table 1 summarizes the different possible combinations of the circuit elements to have positive or negative charges in the loop. The experimental realization of the negative resistor can be found in ref. (Schindler et al. 2012). The setup is the same as in the analog model, but one of the parameters is negative, that is, $\beta_i < 0$.

One of the key parameters of the system is the gain/loss parameter $\beta_i = R_i/r_{ei}$. This parameter defines the energy state of the system which can be Hermitian or non-Hermitian depending of the nature of $\beta_i$. The system is Hermitian when $\beta_i = 0$. As soon as $\beta_i \neq 0$, the system becomes non-Hermitian. $\beta_i$ is thus called the non-Hermitian parameter responsible

| $r_c$ | $C_C$ | $C_{ei}$ | $R_i$ |
|-------|-------|---------|-------|
| $r_c > 0$ | $C_C > 0$ | $C_{ei} > 0$ | $R_i > 0$ |
| $r_c < 0$ | $C_C < 0$ | $C_{ei} < 0$ | $R_i < 0$ |

**Table 1** Combination of circuit elements for either positive or negative charges

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to the amplification or the damping into the loop. When \( r_{ei} \) and \( R_j \) have the same signs (i.e. \( \beta_j > 0 \)), the loop is said ‘‘loss’’, otherwise, it is ‘‘gain’’. When \( \beta_j \) exceeds a threshold value, the system is marked by a degeneracy and the threshold is called the exceptional point (EP). Non-Hermitian electronic systems have been intensively investigated in the literature (Tagouegni et al. 2020; Tabeu et al. 2019; Schindler et al. 2012; Fotsa-Ngaffo et al. 2017). These systems have exhibited unconventional properties such as unidirectional invisibility, thresholdless transition (Fotsa-Ngaffo et al. 2017), simultaneously coherent perfect absorber lasing (Bai et al. 2016) and many more (Christodoulides and Yang 2018), with desired applications in the generation of new multifunctional optoelectronic devices.

The aim of the next section is to give details on the structural characterization of the different non-Hermitian systems. The circuits are investigated in the absence of an external source. In particular, we intend to reveal the ranges of frequencies that could exhibit EIT, EIA and ATS quantum phenomena.

### 2.1 The ZRC dimer

The ZRC dimer is obtained by coupling only two oscillators \( ZRC_i (i = 0, 1) \) as schematically described by the Fig. 1a when \( N=1 \). We assume that both loops have the same natural frequency \( \omega_0 = 1/rC \), where \( r = r_{ei} \) and \( C = C_{ei} \), \( \omega_0 \) being positive or negative (see Table 1 for the conditions). As in Fotsa-Ngaffo et al. (2017), the active controlling coupling can be used to refer the ZRC dimer. For convenience, for the dimer \( Z \), only the imaginary resistive coupling (\( ZC \)) is active while in the case of the dimer \( ZC \), the series combination \( ZC-C_C \) is active. Rewriting Eq. (1) for \( i = 0, 1 \) and assuming the harmonic solutions \( Q_i(t) = A_0 e^{i\omega t} + c.c \) (where c.c is a complex conjugate) for the charges leads to the following matrix representation:

\[
\begin{pmatrix}
\delta + j\omega \beta_0 & \omega \beta_C - \Omega \\
\omega \beta_C - \Omega & \delta + j\omega \beta_1
\end{pmatrix}
\begin{pmatrix}
A_0 \\
A_1
\end{pmatrix} =
\begin{pmatrix}
0 \\
0
\end{pmatrix}
\]  

(2)

where \( \beta_C = r_C/r \), \( \Omega = 1/rC_C = \kappa \omega_0 \), \( r = r_{0,1} + r_C \), \( \kappa = C/C_C \), \( \omega_0 = 1/rC \), \( C = \frac{C_{0,1} C}{C_{0,1} + C_C} \) and \( \delta = \omega_0 - \omega \) is the frequency detuning.

We are interested in the behavior of the system near the resonance. For that, we use the approximation \( |\omega - \omega_0| \ll \omega, \beta \omega \approx \beta \omega_0 \) (Rodriguez 2016). Therefore, Eq. (2) can be rewritten as:

\[
\begin{pmatrix}
\omega_0 \\
\omega_0
\end{pmatrix}
\begin{pmatrix}
1 + j\beta_0 & c \\
c & 1 + j\beta_1
\end{pmatrix}
- \omega
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
A_0 \\
A_1
\end{pmatrix} =
\begin{pmatrix}
0 \\
0
\end{pmatrix}
\]  

(3)

where \( c = \beta_C - \kappa \) is the effective coupling of the system. Equation (3) has non-trivial solutions if the determinant of the left-most term vanishes. Then, we need to solve the characteristic equation \( |H - \omega I| = 0 \), where \( I \) is the \( 2 \times 2 \) identity matrix and \( H \) is a non-Hermitian Hamiltonian (Rodriguez 2016) written as:

\[
H = \omega_0
\begin{pmatrix}
1 + j\beta_0 & c \\
c & 1 + j\beta_1
\end{pmatrix}
\]  

(4)

The \( 2 \times 2 \) matrix \( H \) has a form of a two state non-Hermitian Hamiltonian describing open quantum systems. One of its particularity is to admit a pair of two complex conjugate mode frequencies instead of four, as with the LRC coupled systems. For this later case, the...
eigenfrequencies are those of an effective Hamiltonian $H_{\text{eff}}$, which results from the similarity transformation of the $4 \times 4$ matrix $L$ describing the system, i.e. $H_{\text{eff}} = jL$ (Schindler et al. 2012).

Through diagonalization of $H$, the eigenfrequencies $\omega_\pm$ of the ZRC dimer can be expressed as follows:

$$\omega_\pm = \omega_0 \left(1 + jB \pm \sqrt{c^2 - \beta_0^2} \right)$$

with \[ B = (\beta_1 + \beta_0) / 2 = \beta_0 (\varepsilon + 1) / 2, \quad \beta = (\beta_1 - \beta_0) / 2 = \beta_0 (\varepsilon - 1) / 2, \quad c = \beta_c - \kappa, \]

\[ \beta_c = r_c / r \text{ and } \kappa = \Omega / \omega_0. \] Here $\varepsilon = \beta_1 / \beta_0$ is the gain/loss ratio.

Before any analysis, it is instructive to note that the structural characterization done here consists in studying the behavior of the eigenfrequencies $\omega_\pm$. This allows us to analyze the real and the complex energy band frequencies which definitively have impact on the quantum interference dynamic.

It is obvious that when $\beta_0 = 0$, $\forall \varepsilon$ (no gain or loss in the loops), $\omega_\pm = \omega_0 (1 \pm c)$, all frequencies are real. The Hamiltonian $H$ is Hermitian.

In the non-Hermitian case, $\beta_0 \neq 0$, $\forall \varepsilon$. The eigenfrequencies in Eq. (5) can be real or complex in general, depending on the $\varepsilon$ parameter. In particular, three cases can be considered:

1. If $\varepsilon = -1$, i.e. $\beta_1 = -\beta_0 \neq 0$, the loss and the gain are exactly compensated. This gives rise to a particular class of non-Hermitian systems called the parity time symmetric (PTS) (Schindler et al. 2012; Fotsa-Ngaffo et al. 2017; Bai et al. 2016; Christodoulides and Yang 2018) of which the Hamiltonian $H$ commutes with the join Parity-Time symmetry operator $PT = \sigma_z K$ (where $\sigma_z$ is the first Pauli matrix and $K$ is the complex conjugation operation). Under the PTS conditions, the eigenfrequencies in Eq. (5) are reduced to $\omega_\pm = \omega_0 \left(1 \pm \sqrt{c^2 - \beta_0^2} \right)$. Then, the eigenspectrum displays a spontaneous symmetry breaking at the coupling threshold $c = c_{\text{th}}$, where $c_{\text{th}} = |\beta_0|$ known as the EP, separates two phases: The PT unbroken phase ($\beta_0 < |c|$) where the frequencies are totally real, and the PT broken phase ($\beta_0 > |c|$) for which the frequencies are complex. It is important to mention that PTS systems have demonstrated many intriguing features in several areas of physics (Christodoulides and Yang 2018).

2. If $\varepsilon = 1$, i.e. $\beta_1 = \beta_0 \neq 0$, the system is either amplifying or dissipative. The eigenfrequencies are complex and can be expressed as $\omega_\pm = \omega_0 (1 \pm c + j\beta_0)$. The imaginary part of the frequencies $\omega_\pm$ are constant and the real parts coalesce at the particular point where the effective coupling vanishes.

$$c = 0 \iff \beta_c = \kappa \neq 0$$

This point is referred to as the couplingless point (CPLP) in the case of the dimer ZC.

3. The third case $\varepsilon \neq \pm 1, \forall \beta_0 \neq 0$, the gain and the loss are not compensated in the non-Hermitian system. As a consequence, the eigenfrequencies $\omega_\pm$ remains complex and display a phase transition at two EPs which separate complex phases. At the transition points, the effective coupling strength satisfies to $c = c_{\text{th}}$, where $c_{\text{th}} = |\beta| = |\beta_0|\varepsilon - 1|/2$. 
In what follows we will illustrate the analysis made above considering two coupled systems, one with the single coupling, and another with a double coupling. The aim is to demonstrate that a double coupling gives rise to a new phenomenon that we cannot observe in the simple coupling. In particular, we want to emphasize the couplingless point (CPLP) that we have illustrated in Eq. (6). For a double coupled system, the coupling parameters are non-zero, i.e. $\beta_C \neq 0$, $\kappa \neq 0$ (dimer ZC), for example, we set $\kappa = 0.75$ which remains constant through the paper; whereas for a simple coupled system, $\kappa = 0$ and $\beta_C \neq 0$ (dimer Z). In both cases, we choose $\beta_0 = 0.5 \neq 0$ and it can be easily verified from Eq. (5) that these systems are non-Hermitian.

In Fig. 2, the real (first column) and the imaginary (second column) parts of the normalized eigenfrequencies $\omega_{\pm}/|\omega_0|$ are shown as a function of the controlling coupling parameter $\beta_C$ and the gain/loss ratio $\epsilon$. The spectra are obtained with the negative frequency $\omega_0 < 0$, since the eigenfrequencies are the odd function of $\omega_0$. The exceptional points are indicated as a coupling threshold $\beta_C^{th}$ with a dashed magenta curves in 3D (top view) plots [(a), (b) for the dimer Z, and (d), (e) for the dimer ZC]. As it can be expected, the frequencies $\omega_{\pm}$ come in complex conjugate pairs and two coupling regimes are exhibited: the weakly driving regime ($|c| < |\beta|$), characterized by the coalescence of the real parts of the eigenfrequencies and the splitting of their imaginary parts; and the strongly coupling regime ($|c| > |\beta|$), where the imaginary parts of the eigenfrequencies cross and the real parts split. The splitting analysis will be studied in detail in the following. In the case of the dimer ZC, the EPs are symmetric with respect to the CPLP. To further complete the understanding of the eigenanalysis, the cross section of both real (solid) and imaginary (dashed) parts is made in the third column (see Fig. 2c for the dimer Z and Fig. 2f for the dimer ZC). The plots are obtained for different values of $\epsilon$ parameter and agree well the observations made from the 3D plots. Indeed, in the PTS case $\epsilon = -1$ (red color), we note that, whether

![Fig. 2](image-url) 3D top view of the real (first column) and imaginary (second column) parts of the normalized eigenfrequencies $\omega_{\pm}/|\omega_0|$ of the ZRC dimer as function of the controlling coupling parameter and the gain/loss ratio $\epsilon$, in the case of dimer Z of which $\epsilon = \beta_c$ [plots (a), (b)] and of the dimer ZC when fixed $\kappa = 0.75$ [plots (d), (e)]. The coupling threshold $\beta_C^{th}$ is indicated with the dashed magenta color. Third column: cross section of the real (solid) and the imaginary (dashed) parts of $\omega_{\pm}$ with different values of $\epsilon$: c: dimer Z and f: dimer ZC (when $\kappa = 0.75$). The values of $\epsilon$ are indicated on the plots. For all plots $\beta_0 = 0.5$.
for the dimer Z or for the dimer ZC, the EPs separate exactly the real modes from the complex ones. When $\varepsilon \neq -1$ these modes are complex [see $\varepsilon = 0$ (green color) or $\varepsilon = 1.5$ (blue color)]. The case $\varepsilon = 1$ (black color) supports the discussions made above on the CPLP, as a point of zero effective coupling of the dimer ZC. This CPLP effectively occurs when $\beta_C = \kappa = 0.75$.

To emphasize on the structural characterization, we can as well predict the coupling range of the system for which quantum phenomena could occur using the splitting analysis. To do so, let’s consider again the ZRC dimer. We assume that the coupled loops oscillate with different natural frequencies; for example $\omega_0$ for the first oscillator and $\omega_1 = \omega_0(1 + \Delta)$ for the second one, where $\Delta$ is a small variation. After a little bit of algebra, the eigenfrequencies of the non-Hermitian system ($\beta_0 \neq 0$) under these considerations are found to be:

$$\omega_{\pm} = \omega_0 \left( 1 + \frac{\Delta}{2} + jB \pm \frac{1}{2} \sqrt{\Delta^2 - 4} \beta^2 + 4c^2 + j4\beta \Delta \right)$$

(7)

where the parameters are defined as in Eq. (5).

In Fig. 3, the real parts of the frequencies obtained in Eq. (7) and the splitting $\omega_+ - \omega_-$ are shown as a function of the variation $\Delta$ when $\varepsilon = 0$.

In absence of the coupling i.e., $\beta_C = \kappa = 0$ (see Fig. 3a), the frequency $\omega_0$ (solid green) remains constant, while both $\omega_1$ (solid red) and the splitting (dashed black) decrease as $\Delta$ increases. The splitting cancels when $\Delta = 0$, where the two frequencies intersect.

Once the weak coupling is established between loops, for example in the case of the dimer Z when $\beta_C = 0.02$ (see Fig. 3a) and for the dimer ZC when $\beta_C = 0.71$ (Fig. 3d), one of the eigenfrequencies decreases while another remains constant of $\omega_0$ as $\Delta$ is increasing. The two frequencies cross at $\Delta = 0$. There, the splitting is also equal to zero. We will show in the following that the crossing of frequencies induces destructive or constructive interferences in the system, which favor to mimic EIT or EIA quantum phenomena, respectively.

---

**Fig. 3** Eigenfrequencies of the dimer as function of the variation $\Delta$. a Uncoupled loops ($c = 0$, with $\beta_C = \kappa = 0$). Dimer Z in weak b and strong c coupling regime. Dimer ZC in weak d and strong e coupling regime. f Dimer ZC at the couplingless point ($c = 0$, with $\beta_C = \kappa = 0.75$). In all plots, $\beta_0 = 0.5$, $\omega_0 < 0$ and $\varepsilon = 0$
On the contrary, when the coupling becomes strong, for example in the case of the dimer Z when \( \beta_C = 0.02 \) (Fig. 3a) and for the dimer ZC when \( \beta_C = 0.71 \) (Fig. 3d), both eigenfrequencies \( \omega_\pm \) decrease as \( \Delta \) is increasing. Remarkably, it can be seen that the two frequencies do not intersect at \( \Delta = 0 \) as in the previous case, however a splitting is observed between them. This reveals the well-known anticrossing behavior, a fingerprint of strong coupling (Frimmer and Novotny 2014). We will also show later that the modes separation is responsible to the appearance of the doublets of Autler Townes splitting phenomenon.

For the double coupled system at the CPLP, that is the dimer ZC when \( \beta_C = \kappa = 0.75 \) (Fig. 3f), one of the eigenvalues remains equal to \( \omega_0 \) while another decreases as \( \Delta \) is increasing from \(-2\) to 0. The two curves intersect each other at \( \Delta = 0 \), exactly as in the absence of the coupling, even if the imaginary part of the splitting does not vanishes as shown in inset. At the CPLP, system then seems as an uncoupled one. Later on, we will see that the CPLP favors the disappearance of quantum interferences phenomena.

In the next section, the structural analysis will be discussed in the case of the ZRC multi dimer system.

### 2.2 The ZRC multi dimer

Let us now focus on the ZRC multi dimer circuit, as depicted in Fig. 1a when \( N > 1 \). The ZRC multi dimers are modeled so that the multipod loops are connected to the central one \((Z_0 R_0 C_0)\). For example, in the cases of the bi and the tri dimer, \( N = 2 \) and \( N = 3 \) loops are respectively connected to the central loop. Considering the harmonic solutions \( Q_i(t) \) for the loop \( i(i = 0...N) \), the Eq. (1) can be rewritten in the following matrix, in the absence of a voltage source:

\[
\begin{pmatrix}
  x_0 & c_0 & 0 & \cdots & c_0 \\
  c_1 & -c_1 & -1 & \cdots & -c_1 \\
  c_2 & -c_2 & \ddots & \ddots & \ddots \\
  \vdots & \vdots & \ddots & \ddots & \ddots \\
  c_N & -c_N & -c_N & \cdots & -c_N \\
\end{pmatrix}
\begin{pmatrix}
  A_0 \\
  A_1 \\
  A_2 \\
  \vdots \\
  A_N \\
\end{pmatrix} = 
\begin{pmatrix}
  0 \\
  0 \\
  0 \\
  \vdots \\
  0 \\
\end{pmatrix}
\]

\[(8)\]

where \( i = 0...N \), \( c_i = \beta_{C_i} - \kappa \beta_{C_i} = r_{C_i}/r_{ei}, \quad \kappa_i = \Omega_i/\omega_0 x_i = \delta_i/\omega_0 + j\beta_i, \quad \delta_i = \omega_i - \omega \).

The eigenfrequencies of the system can therefore be investigated assuming identical values of the imaginary resistances in the circuit \( r_{il(i=0...N)} = r \) while the capacitances are distinct. Thus \( \beta_{C_i} = \beta_{C}, \quad \kappa_i = \kappa \) and \( c_i = c = \beta_{C_i} - \kappa \). Using the approximation \( |\omega - \sigma| \ll \omega \), \( \beta_i \omega \approx \beta \sigma = \beta_0 \omega_0 \) (where \( \sigma = \omega_0 \) is the mean frequency), after a few steps of algebra, the determinant of the left most term in Eq. (8) is found to be:

\[
D(\omega) = \prod_{i=0}^{N} x_i - N c^{N+1} - \sum_{p=1}^{N} \sum_{i=0}^{N-p+2} \sum_{k=i+1}^{N-p+2} \cdots \sum_{s=i+q}^{N-p+q} (N - p)c^{N-p+1} x_i x_k \cdots x_s
\]

\[(9)\]

where \( 1 \leq p \leq N, \quad 2 \leq n \leq N, \quad N \geq 1, \quad q \geq 2, \quad u \geq 3, \) and the other parameters are given as in Eq. (8). It is obvious that the system is non-Hermitian when \( \beta_0 \neq 0, \forall \epsilon \) such that \( \beta_i = \epsilon \beta_0 \). As the number \( N \) of the coupled loops increases from \( N = 2 \) to \( N > 2 \), the complex roots of \( D(\omega) \) can be numerically calculated. The real parts Re\((\omega/|\omega_0|)\) of the roots determine the positions of the normal modes of the system; the imaginary parts Im\((\omega/|\omega_0|)\), describe their widths. We investigate the eigenmodes of the multi dimer ZC (\( \kappa = 0.75, \quad \beta_0 = 0.5 \) and \( \epsilon = 0 \)). The other system parameters are setting as follows [see Eq. (10)]:
It can be seen from Eq. (10) that, with this choice of parameters the natural frequencies of the multipod loops are distinct, and the system is non-Hermitian.

Figure 4 shows the real [(a), (b) and (c)], and the imaginary [(d), (e) and (f)] parts of the roots of $D(\omega)$ as a function of $\beta_C$ in the case of the multi dimer ZC.

As illustrated, the spectrum of the roots of $D(\omega)$ can be analyzed as a function of $\beta_C$, in three domains as $\beta_C$ is increasing from $-1$ to $2$:

1. Before the CPLP, the real parts of the normal modes $\text{Re}(\omega_{\text{ii}(\omega_0)})$ related to the multipod loops evolve almost linearly compared to the central mode $\text{Re}(\omega_0)$. They undergo an abrupt transition after which, all the modes of the coupled system converge through the CPLP.

2. At the CPLP ($\beta_C = \kappa = 0.75$), the modes match with the natural frequencies of the uncoupled system.

3. After the CPLP, the frequencies undergo a second transition less abrupt than the first, and return to their behavior before the first transition. This leads to the splitting of the central mode with respect to those of the multipod. These features are also well observed in the imaginary parts of the roots of $D(\omega)$. Remarkably, the eigenfrequencies of the multi dimer are not symmetric with respect to the CPLP. The transition points become

\[ \begin{align*}
\text{bi dimer (N = 2)} & : \omega_1 = \omega_0 - 0.1\omega_0, \ \omega_2 = \omega_0 + 0.1\omega_0; \\
\text{tri dimer (N = 3)} & : \omega_1 = \omega_0 - 0.1\omega_0, \ \omega_2 = \omega_0 + 0.02\omega_0, \ \omega_3 = \omega_0 + 0.1\omega_0; \\
\text{tetra dimer (N = 4)} & : \omega_1 = \omega_0 - 0.1\omega_0, \ \omega_2 = \omega_0 + 0.1\omega_0, \ \omega_3 = \omega_0 - 0.2\omega_0, \ \omega_4 = \omega_0 + 0.2\omega_0.
\end{align*} \tag{10} \]

Fig. 4 The normalized roots of $D(\omega)$ as function of the imaginary resistive coupling parameter $\beta_C$ for the multi dimer ZC (when $\kappa = 0.75$). First column: The bi dimer ZC (N = 2). Second column: The tri dimer ZC (N = 3) and in last column: The tetra dimer ZC (N = 4). The upper row shows the real parts the roots $[\text{Re}(\omega/\omega_0)]$ and the down row shows their imaginary parts $[\text{Im}(\omega/\omega_0)]$. The other plot parameters are: $\varepsilon = 0, \ \beta_0 = 0.5$ and $\omega_0 < 0$. The inset in (a) corresponding to the case of the bi dimer Z ($\kappa = 0$) allows to show the robustness of the multiple coupling.
closer as the N number of the multipods increases. The frequencies’ phase transitions are of great interest in quantum interferences phenomena in physics. For example, we demonstrate in the following that, the reduction of the gap observed between them is consequent to the increasing of the number of interferences, giving rise to multiple EIT/ EIA phenomena. These observations for the multi dimer with double coupling (multi dimer ZC) around the CPLP, is exhibited in the uncoupled multi dimer system, such as illustrated in the inset Fig. 4a, the case of bi dimer Z. Note that this CPLP was already analyzed in the ZRC single dimer. This demonstrates the efficiency in the cancellation of the coupling modulation of the capacitive coupling ($\kappa$) by the imaginary resistor ($\beta_C$) coupling. In other words, the cancellation is robust throughout the N multipod coupling system.

To better understand the interference phenomena, we will focus in the next section on the steady state dynamics of the systems.

3 Steady state dynamic and dispersive properties

In this section, we analyze the dynamic and dispersive properties of the system in presence of the harmonic voltage source $u(t)$. In this way, we first study the dynamic of the free single ZRC oscillator. Then, we discuss the ZRC coupled systems.

3.1 Single ZRC oscillator

Let us consider the $Z_0 R_0 C_0$ central loop.

In the absence of any coupling (SW off), the first row of Eq. (1) can be rewritten as:

$$\left(\beta_0 + j\right) \frac{dQ_0}{dt} + \omega_0 Q_0 = u/r_0$$

(11)

where $u(t) = V_0 e^{j\omega t} + c.c$. $c.c$ is the complex conjugate, $\omega$ is the voltage frequency, $V_0$ its amplitude and $\beta_0^{-1} = (r_0/r_0)^{-1}$ is the circuit’s quality factor. It has been shown in ref. (Tabeu et al. 2019) that the quality factor displays an important role to determine the nature of the oscillations in a ZRC oscillator. If the quality factor becomes large ($\beta_0^{-1} \to \infty$), the circuit oscillates with constant amplitude.

The general solution of Eq. (10) can be written in the form:

$$Q_0(t) = Ae^{-\omega_0 t/(\beta_0+j)} + (A e^{j\omega t} + c.c)$$

(12)

where $A = V_0/[r_0(\delta + j\beta_0\omega)]$ is a complex amplitude, $c.c$ is the complex conjugate, $\delta = \omega_0 - \omega$ is the frequency detuning and $\alpha$ is a constant amplitude. In the steady state regime, the first term of $Q_0(t)$ can be neglected and we can only consider that $Q_0(t) \approx Ae^{j\omega t} + c.c$. Hence, we can evaluate the dissipated power through the loop as $P_0(t) = (V_0 e^{j\omega t} + c.c) \dot{Q}_0(t)$. The average of a dissipated power during one period $T = 2\pi/|\omega|$ of oscillation is calculated using the expression:

$$P_0(\omega) = \frac{1}{2\pi/\omega} \int_0^{2\pi/\omega} [P_0(t)] dt = \frac{1}{2\pi/\omega} \int_0^{2\pi/\omega} [(V_0 e^{j\omega t} + c.c) \dot{Q}_0(t)] dt$$

(13)
Since we are interested in the response of the system near (or at the resonance), we assume \( |\omega - \omega_0| \ll \omega \) and \( \beta \omega \approx \beta_0 \omega_0 \). Under this approximation and using a few steps of algebra, the electric power in Eq. (13) can be obtained as:

\[
P_0(\omega) \approx \frac{j p_0}{\chi_0}
\]

where \( j = \sqrt{-1} \), \( p_0 = |V_0|^2/r_\omega \), \( \chi_0 = \delta_0/\omega_0 + j \beta_0 \), \( \delta_0 = \omega_0 - \omega \).

In Fig. 5, the real (solid) and the imaginary (dash/dot) parts of the steady state normalized electric power \( P_0/p_0 \) dissipated by the \( Z_0R_0C_0 \) loop are depicted as a function of the normalized voltage frequency \( \omega/\omega_0 \) for given values of the non-Hermitian parameter \( \beta_0 \). The green and the blue curves are used for the loss loop \( (\beta_0 = 0.5 > 0) \) while the purple and the red colors refer to the gain loop \( (\beta_0 = -0.5 < 0) \). As it can be noticed, the absorption profile presents a lorentzian form with the maximum of the absorption centered at \( \omega = \omega_0 \). Remarkably, for a loss loop the lorentzian is positive, corresponding to a typical absorption in a real atomic system. On the contrary, it is negative in the case of the gain loop, which can be interpreted as enhancement of the power. Indeed, the power is amplified instead of being absorbed (Hai-Chao and Guo-Qin 2014). The imaginary part of the power represents the dispersion in an atomic systems. In the \( Z_0R_0C_0 \) oscillator, the slope of a dispersion around the absorption’s peak is positive or negative depending on the sign of \( \omega_0 \). It is positive when \( \omega_0 < 0 \), indicating that, the maximum of absorption (or amplification of a power) is obtained at the resonance, where the voltage frequency is equal to the natural loop’s frequency. The negative frequencies have been suggested to many past researches (Rubino et al. 2012; Conforti et al. 2013; Lourés et al. 2015; Pendharker et al. 2017; Nezlin 1976). However, their use in the context of quantum analog system is often overlook.

3.2 The dynamic of the ZRC dimer

From the eigenfrequencies of the ZRC dimer illustrated in Fig. 2, two main coupling regimes have emerged and border by the EPs of the non-Hermitian system. We now
attempt to understand the physical significance of the coupling regimes on the dynamic of two coupling ZRC oscillators.

Firstly the dynamic of the system can be investigated in the absence of the external voltage. According to Eq. (3), the eigenvectors $A_{i+}$, $A_{i-}(i = 0, 1)$ describing the normal modes of the coupled system satisfy to Dolfo and Vigué (2018):

$$A_{1\pm} = \alpha_{\pm}A_{0\pm}$$

(15)

where $\alpha_{\pm} = \frac{\alpha_0 + (\beta_0 - 1)\omega_0}{\Omega - \omega_0 \beta_C}$, $\beta_C = r_C/r$, $\Omega = 1/rC_C$, $r = r_{0,1} + r_C$ and $\omega_0 = 1/rC$. The eigenfrequencies $\omega_{\pm}$ are given in Eq. (5).

If $c \neq c_{ib} = \beta_{0}(\epsilon - 1)/2$, the general solution $Q_{0,1}(t)$ for the ZRC dimer can be written as:

$$Q_i(t) = A_{i+}e^{\imath \omega_i t} + A_{i-}e^{-\imath \omega_i t}$$

(16)

where $i = (0, 1)$. The amplitudes $A_{i\pm}$ are found from the initial conditions of the oscillators: $A_{0\pm} = [\alpha_0 Q_0(0) - Q_1(0)]/(\alpha_+ - \alpha_-)$ and $A_{1\pm} = [Q_1(0) - \alpha_+ Q_0(0)]/(\alpha_- - \alpha_+)$. $Q_0(0)$ being the initial charges of the capacitors. We have calculated the amplitude squared $|Q_i|^2$ associated to each loop of the dimer and the response is significantly describing the transfer of energy from one oscillator to another. Let’s discuss the dynamic of the dimer by considering the following parameter for the non-Hermitian system $\beta_0 = 0.1$ and $\epsilon = 0$. With these considerations, the EPs are found to be $\beta_{C1} = -0.05$ and $\beta_{C2} = 0.05$ in the case of the dimer Z. In the case of the dimer ZC ($\kappa = 0.75$), they are obtained as $\beta_{C0,1} = 0.7$ and $\beta_{C0,2} = 0.8$. Then, the weakly coupled regimes are in the ranges $-0.05 < \beta_C < 0.05$ for the dimer Z and $0.7 < \beta_C < 0.8$ for the dimer ZC, respectively. In both cases, the strongly coupled regime is found everywhere else. In Fig. 6, the quantities $|Q_0|^2$ and $|Q_1|^2$ are plotted as a function of the time for different values of the active coupling $\beta_C$ in each coupling regime. The initial conditions of the dimer were chosen such as to introduce the signal into the coupling $Z_1R_1C_1$ loop (oscillator 1) $Q_0(0) = 0$, $Q_1(0) = 1$. As illustrated, in presence of a strong coupling, once a signal is put into oscillator 1, it is noticed a rapid periodic exchange of energy between both oscillators loops. This exchange is made in an oscillatory process so that, when the energy of one oscillator reaches its maximum value, the energy of the other oscillator is minimum. Also, note that these oscillations are very fast in the dimer ZC. On the contrary in presence of the weak coupling, it is noticed when $\omega_0$ is positive that, the energy is irreversibly transferred from oscillator 1 to 0 of which the energy remains almost closed to zero. However, if $\omega_0$ is negative, the energy of oscillator 1 takes a relatively long time to reach oscillator 0, of which the energy increases exponentially. At the CPLP, a particular point of a weak coupling regime, a different behavior is remarkable. Indeed, once the signal is introduced in oscillator 1, it remains there. There is no exchange of the energy with the oscillator 0 whose the amplitude remains zero. This confirm that at this point, the system seems effectively as it is uncoupled.

Secondly, let’s investigate the system taking into account the external voltage source. In this case, the exact solutions $Q_i(t)$ of Eq. (1) can be written as a sum of two terms $Q_i(t) = Q_i^0(t) + Q_i^C(t)$. $Q_i^0(t)$ is the complementary solution which linearly combines the two normal modes frequencies of the dimer i.e. $Q_i^0(t) = A_{0,\pm}e^{\imath \omega_0 t} + A_{1,\pm}e^{\imath \omega_1 t}$, where the eigenfrequencies $\omega_{\pm}$ were obtained in Eq. (5). $A_{0,\pm}$ are the complex amplitudes. $Q_i^C(t)$ is a particular solution. It can be taken in the form of the voltage as $Q_i^C(t) = \rho_0 e^{\imath \Omega t} + c.c$, where $\rho_0$ is the amplitude. In the steady state in general, only a particular solution remains. It is interesting to analyze the power absorbed or dissipated by the oscillators from the external
voltage. Hence, substituting \( Q_0(t) \) into Eq. (13), the steady state power dissipated by the loop \( Z_0R_0C_0 \), \( P_0(\omega) \) is found to be:

\[
P_0(\omega) = \frac{|V_0|^2}{r_c \omega} \frac{j\omega (\delta + j\beta_1 \omega)}{(\delta + j\beta_0 \omega)(\delta + j\beta_1 \omega) - (\Omega - \beta_c \omega)^2}
\]  

(17)

where \( \delta = \omega_0 - \omega \) and \( \Omega = \kappa \omega_0 \).

Using the approximation \( |\omega - \omega_0| \ll \omega, \beta_0 \omega \approx \beta \omega_0 \), Eq. (17) can be rewritten as:

\[
P_0(\omega) \approx j \frac{p_0 \chi_1}{\chi_0 \chi_1 - c^2}
\]  

(18)

where \( j = \sqrt{-1}, p_0 = V_0^2/r, c = \beta_c - \kappa, \chi_{0,1} = \delta/\omega_0 + j\beta_{0,1} \) and \( \delta = \omega_0 - \omega \) is the frequency detuning between the loops and the voltage frequencies.

In the context of electronics, it is well known that the real part of the dissipated power (i.e. the active power) determines the absorption. Its imaginary part (i.e. the reactive power) simulates the dispersion properties of the atomic medium. Considering the central loop as a loss loop \( (\beta_0 = 0.1 > 0) \), and modulating the non-Hermitian parameter ratio \( \epsilon \) from the positive to negative values, the dissipated power can be investigated according to the coupling strength of dimers formed by a double and single coupling.
In Fig. 7, the real (solid blue) and the imaginary (dashed red) parts of the normalized dissipated power \( \frac{P}{P_0} \) are reported as a function of the normalized voltage frequency \( \omega/\omega_0 \).

As it can be seen, if a weak coupling is established in the dimer ZC (\( \kappa = 0.75, \beta_C = 0.71, \omega_0 > 0 \)), a narrow dip appears in the absorption spectrum of the \( Z_0 R_0 C_0 \) loop when \( \varepsilon = 0.2 \), leading two absorption peaks (Fig. 7a). The dispersion curve is also modified around the resonance (zero detuning). When \( \varepsilon = 0 \), a zero absorption, so a transparency domain occurs in the spectrum at a zero frequency detuning \( \delta = 0 \). Then, the dispersion curve becomes much steeper near the central frequency. This behavior is similar to the EIT effect. In atomic context, it has been interpreted as a direct manifestation of destructive interferences between the normal modes of oscillations of the system. This description matches perfectly with the experimental demonstrations made previously with classical systems (Garrido Alzar et al. 2002; Harden et al. 2011; Bai et al. 2013). When a small negative dissipation (\( \varepsilon = -0.1 \)) is

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Fig. 7 Absorbed power (dissipated) by the main loop of the dimer ZC [from (a) to (c)] when the natural frequencies of the loops are positive \( \omega_0 > 0 \) and the one of the dimer Z (d) when natural frequencies are negative \( \omega_0 < 0 \). The solid blue and dashed red correspond to the real and imaginary parts of the absorbed power when the main loop is loss \( \beta_0 = 0.1 > 0 \). The insets in green show the real part of the power corresponding to the main loop gain \( \beta_0 = -0.1 < 0 \). The other plot parameters are identical to those used in Fig. 5.
introduced into the pumping $Z_1R_1C_1$ loop, a negative absorption occurs in the transparency zone. The negative absorption refers to the amplification (Hai-Chao and Guo-Qin 2014). The later behavior is referred as the Electromagnetically Induced Amplified Transparency (EIAT). For a further decrease of $\varepsilon$ until the value $\varepsilon = -0.18$, an enhanced absorption peak occurs in the profile where the dip was expected to be. This mimics the absorption spectrum of atomic systems with three energy levels in lambda ($\Lambda$) configuration, in the regime of Electromagnetically Induced Absorption (EIA) (Lezama et al. 1999). EIA has been interpreted for atomic systems, as a manifestation of constructive interferences between the normal modes of oscillations.

Figure 7b shows the absorption profile of the $Z_0R_0C_0$ loop when a strong coupling ($\kappa = 0.75, \beta_C = 0.2, \omega_0 > 0$) is established in the dimer system. If much losses are introduced into the pumping loop ($\varepsilon = 3$), it is observed in the spectrum, two absorption peaks separated by a large transparency window. These peaks are clearly identified when the losses decrease to zero (i.e. $\varepsilon = 0$). The phenomenon is similar to the well-known Autler–Townes Splitting (ATS) effect which is generally interpreted as a consequence of a gap between two atomic resonances peaks. The peaks are located on both sides of the central frequency $\omega_0$. For a further decreasing of the damping, such as to introduce a negative dissipation into a pumping oscillator ($\varepsilon = -0.8$), a negative absorption occurs in the spectrum. This leads the enhancement of the absorption peaks. Such a behavior can be generalized to the values of $\varepsilon$ in the domain between 0 and $-1$, where $\varepsilon = -1$ corresponds to the Parity Time Symmetric (PTS) case. However, when $\varepsilon$ is less than $-1$, the peaks become inverted so that, only a negative absorption (amplification) is observed in the profile. The behavior can be called Autler–Townes splitting with amplification (ATSA).

The question we can ask now is what happens at the CPLP where the effective coupling cancels? To answer to this question, we substitute the couplingless condition given in Eq. (6) into the Eq. (18) and the expression of the absorbed power by the main loop is reduced to the power of a free $Z_0R_0C_0$ loop as obtained in Eq. (14). As we could predict, the cancellation of the effective coupling has the effect to decouple parts of the system such that, no exchange of energy can be observed between them. Therefore, no interference could be obtained. The absorption spectrum is then reduced to that of a free oscillator, a lorentzian curve illustrated in Fig. 7c. Accordingly, all the other replicated atomic phenomena disappear. These results indicate that operating at the CPLP could be an efficient way to control applications of EIT with real atomic systems. Except for the CPLP, all the results discussed here with the ZC dimer when $\omega_0 > 0$, can also be obtained with a single coupled dimer. For example, Fig. 7d illustrates the absorption spectrum in the case of the dimer $Z$ ($\beta_C = 0.02$ and $\beta_C = 0.5$) when $\omega_0 < 0$. The transparent window observed agrees well the observation made with the dimer ZC when $\omega_0 > 0$. This demonstrates that EIT and related phenomena can be observed with negative frequencies of the system, and show a wide range of frequency for investigations with ZRC circuits. Also mention that the green curves illustrated in the insets show the active power obtained in the same conditions, however with the main loop gain ($\beta_0 = -0.1 < 0$).

Knowing now the dynamics of the dimer system, in the next paragraph, we will focus on the dynamics of the multidimer systems.

### 3.3 Dynamic of the ZRC multi dimer

Let us consider the multi dimer circuit. In the previous section, we have seen from the absorption spectrum of the ZRC dimer ($N=1$) that, the circuit allows to reproduce EIT,
EIAT, EIA or ATS. What happens in the spectrum when \( N > 1 \)? Is the multi dimer circuit could be used to replicate a multiple EIT, EIA or ATS windows? Indeed it has been demonstrated (Harden et al. 2011) that, to obtain multiple transparency windows, each part of the circuit must oscillates with its own natural frequency \( \omega_i (i = 0 \ldots N) \), which differs from one loop to another. Considering harmonic solutions for the charges, the dynamic of the multidimer can be described by the Eq. (8) when adding the voltage term \( u(t) = V_0 e^{i \omega_0 t} + \text{c.c} \) in its right term. Solving such equation, the amplitudes \( A_j (i = 0 \ldots N) \) can be obtained and accordingly, the charge solution \( Q_0 (t) \) through the central loop. Thanks to Eq. (13), we have calculated the expression of the steady state frequency-dependence power \( P_0 (\omega) \), which is dissipated by the main loop as a function of the \( N \) number (\( N \geq 1 \)) of multipod loops. Under the approximation \( |\omega - \sigma| \ll \omega, \beta_\omega \approx \beta_\sigma, \) where \( \sigma = \omega_0 \) is the mean frequency of the system and, using steps of algebra, the steady state power \( P_0 (\omega) \) is found to be:

\[
P_0 (\omega) = \frac{j}{D(\omega)} \left( \prod_{i=1}^{N} \chi_i - (N-1)c^N - \sum_{n=2}^{N-n} \sum_{m=1}^{N-n+1} \sum_{h=m+1}^{N-n+u} \ldots \sum_{v=m+u}^{N-n+1} (N-n)c^{N-n+1} \chi_m \chi_n \chi_h \ldots \chi_v \right),
\]

where \( i = 0 \ldots N, \ c_i = \beta_c - k_i, \ \beta_c = r_C/r_{ei}, \ k_i = \Omega_i/\omega_0, \ \chi_i = \delta_i/\omega_0 + j\beta_i, \ \delta_i = \omega_i - \omega, \ \beta_i = R_i/r_{ei}, \ \omega_i = 1/r_{ei} C_{ei}, \ r_{ei} = r_i + r_e \) and \( C_{ei} = C_i C_C/(C_i + C_C) \). The denominator \( D(\omega) \) was expressed in Eq. (8):

\[
D(\omega) = \prod_{i=0}^{N} \chi_i - Nc^{N+1} - \sum_{p=1}^{N} \sum_{i=0}^{N-p+1} \sum_{k=i+1}^{N-p+2} \ldots \sum_{s=i+q}^{N-p+q} (N-p)c^{N-p+1} \chi_i \chi_k \ldots \chi_s
\]

\[
1 \leq p \leq N, \ 2 \leq n \leq N, \ N \geq 1, \ q \geq 2, \ u \geq 3
\]

In the following, we will focus the analysis on three cases of the multi dimer ZC circuits: the bi dimer, the tri dimer and the tetra dimer. The aim is to show that the interferences in the system can allow to reproduce the multiple EIT, EIA and ATS, with the number of windows consistent with the \( N \) number of the multipods. Assuming Eq. (10) for the multipod loops frequencies’ and the central loop as a loss one \( (\beta_0 = 0.5 > 0) \), the steady state power \( P_0 (\omega) \) can be investigated when controlling the coupling parameter \( \beta_C \) and the non Hermitian parameter ratio \( \epsilon \).

In Fig. 8, the real (solid blue) and the imaginary (dashed red) parts of the normalized dissipated power \( P_0 (\omega)/p_0 \) are reported as a function of the normalized frequency \( \omega/|\omega_0| \) for different values of the \( \epsilon \) parameter.

### 3.3.1 Case 1: the bi dimer ZC (\( N = 2 \))

As illustrated in Fig. 8a, in presence of the weak coupling \( (\ k = 0.75, \ \beta_C = 0.85, \) it is noticed in the absorption spectrum when \( \epsilon = 0 \), the occurrence of two symmetric dips centered at \( \omega = 0.9\omega_0 \) and \( \omega = 1.1\omega_0 \) on both sides of the central peak. This gives rise to two EIT windows. The behavior is similar to the quantum phenomenon of a double Electromagnetically Induced Transparency (DEIT). Accordingly, two steep slopes are observed in the dispersion curve near the transparency domain. It is interesting to note that these results are consistent with DEIT demonstrated in ref. (Harden et al. 2011) where the LRC circuits have been used. However, notice that one difference between our results and the previous...
ones is that for the previous ones, the frequencies of the system were positive, while for our system, the frequencies can be as well positive or negative, as described above.

By decreasing $\varepsilon$ to the value $\varepsilon = -0.008$, a phenomenon similar to the EIAT appears in the absorption spectrum. However it is centered at two frequencies, where a negative power is observed. Accordingly, this new behavior is called double EIAT (DEIAT). For a further decreasing of $\varepsilon$ to $-0.002$, and then to the value $-0.1$, it is remarked in the spectrum the occurrence of one EIA peak where the first EIT dip was observed and then another one, where the second dip was expected to be. As a consequence, it results two enhanced absorption peaks. Then, the bi dimer ZC circuit allows to replicate double EIA (DEIA). So, the bi dimer allows the replication of two EIT, EIAT or EIA windows in the weak coupling regime.

Figure 8b shows in presence of the strong coupling ($\kappa = 0.75, \beta_C = 0.2$) that, when a dissipation is introduced into the coupling fields ($\varepsilon = 0.1$), it is observed two dips inside the absorption profile. One of the dips is more pronounced and centered at the frequency $\omega_0$ while the other one is very narrow. Both dips are separated by a large transparency domain, similar

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**Fig. 8** The real and imaginary parts of the steady state normalized dissipated power $P_0/p_0$ as a function of the normalized frequency $\omega/[\omega_0]$ for different cases of the multi dimer ZC according to the coupling regime: weak coupling (first column), strong coupling (second column). In all plots, we set $\omega_0 < 0, \beta_0 = 0.5$ and $\kappa = 0.75$. (a) and (b): bi dimer ZC ($N = 2$) (c) and (d): tri dimer ZC ($N = 3$) (e) and (f): tetra dimer ZC ($N = 4$). The other plot parameters are inserted in text.
to the ATS effect described above. The location of the peaks is in perfect accordance with the eigenmodes behavior presented in Fig. 4, where the multipod modes frequencies are closed together while the central mode splits. A further decrease of $\epsilon$ to the value $\epsilon = 0$ reduces the dips to zero, leading to the phenomenology of EIT. The system then allows to reproduce simultaneously EIT and ATS windows instead of two ATS windows as we could expected in the strong coupling regime. As one would imagine, by lowering $\epsilon$ to negative values for example $\epsilon = -0.02$, a negative power, typical to the EIAT effect observed in the weak coupling for $\epsilon = 0$, occurs in the region of transparency between the two EIT peaks. For $\epsilon = -0.03$, it is not surprising to note that the spectrum just pass from EIAT to the EIA behavior exhibited in the case of the dimer ZC. In all cases, the dispersion curve is modified according to the phenomenon reproduced. The bi dimer can then allow to replicate simultaneously ATS and EIT, EIAT or EIA windows in the strong coupling regime.

### 3.3.2 Case 2: the tri dimer ZC (N = 3)

In the presence of the weak coupling ($\kappa = 0.75, \beta_C = 0.803$) as illustrated in Fig. 8c, if $\epsilon = 0$, the absorption spectrum shows three narrow dips centered at the naturel loops frequencies $0.9\omega_0, 1.02\omega_0$ and $1.1\omega_0$ [see Eq. (10)], thus exhibiting three EIT windows. This behavior is similar to the Triple EIT (TEIT) effect observed in atomic context (Wang et al. 2019). TEIT was also studied in a superconducting quantum circuit with a four-level V-type energy spectrum (Tiaz et al. 2019). A further decreasing of $\epsilon$ to $-0.003$, then to $-0.006$ and thereafter to $-0.04$ allows to the spectrum to switch from the triple EIAT (TEIAT) to a double EIA (DEIA) and then to triple EIA (TEIA), respectively. So, a tri dimer can provides up to three EIT, EIAT or EIA windows in presence of the weak coupling.

In presence of a strong coupling ($\kappa = 0.75, \beta_C = 0.58$) as shown in Fig. 8d, when $\epsilon$ takes respectively the values $\epsilon = 0, \epsilon = -0.002$, and $\epsilon = -0.01$, the spectrum presents similar behaviors as those obtained in the strong coupling regime with the bi dimer ZC. However, the number of EIT, EIAT dips or EIA peaks is increased to two, whereas the number of ATS window remains constant to one.

### 3.3.3 Case 3: the tetra dimer ZC (N = 4)

As illustrated in Fig. 8e, in the presence of the weak coupling ($\kappa = 0.75, \beta_C = 0.78$), it is observed in the profile when $\epsilon = 0$, four EIT windows, called a quadruple EIT (QEIT). By playing on the value of $\epsilon$, the tetra dimer allowed also to reproduce quadruple EIAT (QEIAT) windows when $\epsilon = -0.0007$, then a TEIA when $\epsilon = -0.004$ and finally quadruple EIA (QEIA) if $\epsilon = -0.01$.

In Fig. 8f where the tetra dimer is in presence of the strong coupling ($\kappa = 0.75, \beta_C = 0.2$), the system allows again to mimic TEIT ($\epsilon = 0$), TEIAT ($\epsilon = -0.001$), DEIA ($\epsilon = -0.006$) and TEIA ($\epsilon = -0.0025$) simultaneously accompanied in each case by just an ATS dynamic.

### 4 Conclusion

We have proposed a non-Hermitian electronic dimers, based on the imaginary resistor, which is analog to a $(N+2)$ level atomic system in the multipod type configuration. The so-called ZRC multi dimer system is described by the equations of the first order. Under the resonance approximations, we have successfully investigated on the structural
characterization. Two main coupling regimes have emerged revealing a couplingless point (CPLP) associated to multiple couplings. In the weakly coupling regime, the eigenfrequencies cross. This demonstrates a non-reversible energy transfer between the coupled oscillators. On the other hand, in the strong coupling regime, the eigenfrequencies exhibit a splitting behavior. This indicates exchange between energy in the coupling oscillators in a periodical way. The proposed model is highly desired to develop quantum interference dynamic in dispersive media. When the central loop is coupled to N multipod loops, its absorption profile replicates the N-EIT, N-EIAT or N-EIA dynamic in the weakly coupling. When the strong coupling is established, the (N-1) EIT, (N-1) EIAT or (N-1) EIA with the ATS dynamic is simultaneously observed. It is remarkable that, at the CPLP all the quantum phenomena disappears. This reveals the efficiency of the imaginary resistive coupling in the cancellation of the capacitive coupling effect.

Acknowledgements Authors would like here to acknowledge TABEU Stéphane Boris of HITASTEC (High-tech and Slow Technology) for helpful discussions.

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