Influence of Lattice Defect to Vortex Flow on Type II Superconductor

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Abstract. Adding defect on type II superconductor can influence to vortex dynamic. It declares vortex movement, so speed of vortex is more stable. In this research, we applied lattice defect. In case to study dynamics of the vortex in type II superconductor material, this study was based on numerical solution of the Time-Dependent Ginzburg Landau (TDGL) equations by means of finite difference method with Forward Time Centred Space (FTCS) scheme. Applied external current density $J_e$ and external magnetic field $H_e$ to superconductor material generate a vortex flow from higher magnetic field to lower magnetic field. Furthermore, applied external current density $J_e$ without external magnetic field $H_e$ generates vortex flow between lattice defects.

Keyword: Defect, vortex, superconductor, TDGL, FTCS

1. Introduction

Superconductivity is the properties of which has resistivity approach to zero. Generally, a material which is under critical temperature has superconductivity properties. Superconductor can be categorized into two types are type I and type II. Type I has one critical magnetic field, whereas type II has two critical magnetic fields. In type II, the external magnetic field may penetrate to superconductor material. It enters into superconductor material in the form of vortex [1].

In recent years, research about vortex dynamic is more attention. It can obtain important properties of type II superconductor such as plastic properties of vortex which this property affects on critical current density in superconductor [2]. The vortex dynamic can be studied by time-dependent Ginzburg-Landau equation which is a non-linear differential equation. Thus it can be solved by computational methods [1, 3, 4]. One of the computational methods is UΨ [5–8] This method has an advantage namely converging property for the high magnetic field [8–10].

In previous research about vortex dynamics with a single defect on the material has been successfully studied. From that research, we know that the influence of applied external current on vortex dynamics can be measured by the potential difference on the side of the material. In the presence of a defect, potential difference changes slower than on clean superconductor [11–13]. On the other research, when current density applied with a different value, it can change the vortex flow [14]. From the previous research, vortex dynamics could not show stability. One of the solution to get the stability is watched the dynamics of the vortex in the presence of defects and clean superconductor closely.
2. Methods
In this research, the influence of lattice defect on superconductor type II was studied by numerical methods. The Time-Dependent Ginzburg-Landau (TDGL) equation is used on this research. This equation has been successfully explaining superconductivity phenomenon for any sample which consists of the normal and superconducting part. Based on a modified TDGL equation, vortex dynamic has been successfully studied by [12, 15–17]. It shows that this model has the potential for explaining how the characteristics of vortex when there is a defect on type II superconductor. The normalized TDGL equations with potential gauge can be expressed by [18].

\[ \partial_t \Psi = (\nabla - iA)^2 \Psi + (1-T)(1-\abs{\Psi}^2)\Psi \quad \text{and} \quad \partial_t A = J_s - J \quad \text{for} \quad \Omega_s \] (1)

\[ \partial_t \Psi = (\nabla - iA)^2 \Psi - m_n a_n (0)(1-T)\Psi \quad \text{and} \quad \partial_t A = \frac{J_s \sqrt{m_n}}{\mu_n} - \frac{J}{\mu_n} \quad \text{for} \quad \Omega_n \] (2)

Consider a system of type II superconductor with computational grid size is \( N_x \times N_y \), and the size of typical grid size is \( h_x \times h_y = 0.2\xi_0 \times 0.2\xi_0 \), like on Figure 1. External current density \( J_x \) applied to this material. A computational grid is obtained by divide \( L_x \) with \( N_x \) and every part of the typical grid can be written as \( h_x = L_x/N_x \). And also divided \( L_y \) with \( N_y \) and every part of the typical grid can be written as \( h_y = L_y/N_y \). From this way, we can produce a homogeneity computational grid, shown on Figure 1. Next step is the continue variable \( x \) and \( y \) with range \( 0 \leq x \leq L_x \) and \( 0 \leq y \leq L_y \), change into discrete variable \( x_i = (i-1)h_x \) for \( i = 1, \ldots, N_x + 1 \) and \( y_j = (j - 1)h_y \) for \( j = 1, \ldots, N_y + 1 \).

![Figure 1. (a) Computational grid with area \( S = L_x L_y \), (b) evaluate dots](image)

3. Results and Discussion
In this research, vortex dynamic with adding lattice defect has been simulated. The size of superconductor is \( L_x \times L_y = 50\xi \times 50\xi \), made from niobium = \( \kappa = 1,3 \), \( h_x = h_y = 0.5\xi \), and \( \Delta t = 0.001\xi^2/D \). The superconducting material was on 0 Kelvin condition. When external current density flow on the material, the magnetic induction will appear of \( H_e = J_s L_x / 2\kappa^2 \). Moreover, this material was placed in
external magnetic field $H_{ext}$, so there is resultant between the induction magnetic field and external magnetic field, namely vortex. Vortex enters into superconductor material because of interaction between external current density and potential vector. In the equilibrium state, the induction magnetic field is equal to the external magnetic field, $B_z = H_z$. If the external current density flows in the $x$-direction, then

$$\kappa^2 \frac{\partial B_z}{\partial y} = J_{ext}$$

(3)

It means that the induction magnetic field is variation in interval $B_z^0 + \Delta B_z$ and $B_z^0$. Total current is given by $I = \kappa^2 \Delta B_z$. Ginzburg-Landau free energy can be expressed by

$$\varepsilon = \int f_n - |\Psi|^2 + \frac{1}{2} |\Psi|^4 + \left( |V - iA| \right)^2 + \kappa^2 \left( \nabla \times A - H \right)^2 \, dv$$

(4)

In the last term of equation (4) is a contribution from magnetic field energy density. So that, density energy $B_z$ is $\kappa^2 B_z(y) \frac{\partial B_z(y)}{\partial y}$. The density energy can produce a Lorentz force of

$$F_y = \frac{\partial}{\partial y} \left( \kappa^2 B_z^2(y) \right) = 2\kappa^2 B_z(y) \frac{\partial B_z(y)}{\partial y}$$

(5)

In external current density form, the Lorentz force can be written by

$$F_y(y) = 2B_z(y)J_{ext}$$

(6)

Figure 2 shows the dynamics of vortex when external current density $J_{ext} = 0.016$ applied to the materials. In figure 2(a) is the condition when the vortex does not penetrates to the material yet. In Figure 2(b) show that vortex has penetrating to the material. From 2(b), we can see that the defect is attracting the vortex one by one from it upper side. When the vortex have filling the upper defect, the other vortex will flow between the defect and it will be attracting again with the empty defect and it will happen continuously, like in Figure 2(c) and 2(d) until all of the defect filled. Figure 2(e) show when vortex have filling all the defect in the material. In this condition, vortex couldn’t left or out from the material. It can happen if the external current density is not quite bigger to push vortex to left the material. And on this condition vortex cannot penetrate or left the material.
**Figure 3.** Potential Curve for material with a defect (black line) and without defect (red line) for $J_e = 0.016$

Figure 3 above show potential characteristics curve on dynamics of the vortex in the presence of lattice defect (black line) and a clean superconductor (red line). From Figure 3, all peaks show the condition on the potential difference when vortex penetrate to the material. The value of potential difference can show how the velocity of vortex movement and amount of vortex inside the material, which can be written as $V = h \times V \times L$ [19]. From that equation, the high value of the potential difference is implicating faster movement of the vortex.

Red line a curve in Figure 3 indicate the vortex dynamics in a clean superconductor. Value of potential difference increase indicates that there are many vortexes penetrate to the material and implicate that vortex inside the material move faster. When the value of potential difference decrease, it indicates that penetrating of vortex decrease and also the velocity of the vortex movement is slower than before. When the value of potential difference on equilibrium state, $t = 10000$ to $t = 20000$, it indicates that there are no more penetrate of the vortex, motionless of the vortex, and there is no vortex left the material.

Black line a curve in Figure 3 indicate that vortex dynamics in the material with the existence of lattice defect. From that line, we can see that the curve has more stability than the curve for a clean superconductor. The sharp peak on that line indicates when the defect will pin vortex. The small peaks indicate when vortex penetrate to the material. The value of the potential difference is lower than on clean superconductor, because vortex could not move faster in the presence of a defect. Furthermore, the vortex will get it equilibrium state slower than on clean superconductor.

Comparing to the previous research, Figure 4 below shows the influence of defect on the dynamics of the vortex on type II superconductor. Figure (4) show that the defect influences characteristics $V$-$I$ curve. Figure 4(a) shows the dotted line and black line for the evolution of vortex dynamics for clean superconductor and with a single defect, respectively. For clean superconductor, peak P₁ and P₂ show there is a dissipation of energy when vortex penetrates to the materials. Moreover, peak P₃ and P₄ show when vortex left the material from the other side [11].
Figure 4: (a) $V$-$t$ characteristics (b) $V$-$I$ characteristics

For the material with a single defect on the center of materials, peak $D_1$ show when the first vortex penetrate to the material. We can see from figure 2(a) that peak $D_2$ is sharper than peak $D_1$. This peak is total of raising of first vortex velocity around defect before pinning and second vortex penetration. Peak $D_3$ show the third vortex penetrate to the material. Peak $D_4$ show the raising of second vortex velocity around the defect before pinning. In this case, we can see that there are two vortexes inside the defect. After that, the sharp peaks will not exist anymore on the curve because there is no vortex movement on the materials[11].

The presence of defect on the material will change characteristic curve of $V$-$I$, in Figure 4(b). Figure 4(b) explains that the current $I$ will change in range $0.22 \leq I \leq 0.025$ and not giving an influence of potential difference $V$ changing. That means, in this range, the defect can pin vortex and stopping the movement of the vortex on materials. When current $I$ increase to 0.026, value of potential difference will rise sharply. In this condition happen when vortex released from defect and move faster avoid the defect[11].

4. Conclusion
Based on the discussion above, it can be concluded that the existence of lattice defect in superconductor type II influences on vortex dynamics showing the slower flow of the vortex. It indicate the pinning vortex flow of the system. Furthermore, the defects have role as pinning vortex and increase the superconductivity of material. In another word, the defects also have as essential role to increase the stability of vortex flow. Interestingly, the stability of vortex flow is able to enhance the critical current of the superconductor.

References
[1] D. Lu, “Vortex Dynamics in Type II Superconductors,” arXiv Prepr. arXiv1702.08842, 2017.
[2] F. Anwar, M. Yunianto, R. A. S. Yosi, P. Nurwantoro, B. S. U. Agung, and A. Hermanto, “Simulasi Komputer Pengaruh Efek Proksimitas Pada Vorteks Superkonduktor Berlubang,” Media Fis., (Indonesian) vol. 9, no. 2, pp. 7–16, 2010.
[3] Harsojo, “Dinamika Vorteks Dua Dimensi dan Parameter Kritis pada Superkonduktor Tipe II,” 2009.
[4] R. Rosyida and F. Anwar, “Numerical simulation of dimension effect on critical field of rectangular superconductor Numerical simulation of dimension effect on critical field of rectangular superconductor,” 2017.
[5] C. Bolech, G. C. Buscaglia, and A. López, “Numerical simulation of vortex arrays in thin superconducting films,” Phys. Rev. B, vol. 52, no. 22, pp. 719–722, 1995.
[6] W. D. Gropp, H. G. Kaper, G. K. Leaf, D. M. Levine, M. Palumbo, and V. M. Vinokur, “Numerical simulation of vortex dynamics in type-ii superconductors,” J. Comput. Phys., vol. 123, no. 2, pp. 254–266, 1996.

[7] T. Winiecki and C. S. Adams, “A fast semi-implicit finite-difference method for the TDGL equations,” J. Comput. Phys., vol. 179, no. 1, pp. 127–139, 2002.

[8] G. Buscaglia, C. Bolech, and A. López, “On the Numerical Solution of the Time-Dependent Ginzburg-Landau Equations in Multiply Connected Domains,” pp. 200–214, 2000.

[9] J. Chapman, Q. Du, and M. A. X. D. Gunzburger, “A Ginzburg-Landau Type Model Of Superconducting / Normal Junctions Including Josephson Junctions,” vol. 114, pp. 24–29, 1995.

[10] J. J. Barba, C. C. de Souza Silva, L. R. E. Cabral, and J. A. Aguiar, “Flux trapping and paramagnetic effects in superconducting thin films: The role of de Gennes boundary conditions,” Phys. C Supercond. its Appl., vol. 468, no. 7–10, pp. 718–721, 2008.

[11] M. Machida and H. Kaburaki, “Numerical simulation of flux-pinning dynamics for a defect in a type-II superconductor,” Phys. Rev. B, vol. 50, 1994.

[12] H. Wisodo, “Kajian Model Ginzburg-Landau pada Superkonduktor Mesoskopik dan Potensi Aplikasinya pada SQUID,” Disertasi, 2014.

[13] H. Wisodo, A. Hidayat, P. Nurwantoro, B. S. U. Agung, and E. Latifah, “Peran Vorteks pada Prinsip Kerja SQUID Berdasarkan Ginzburg-Landau Termodifikasi,” pp. 1–8, 2014.

[14] T. Winiecki and C. Adams, “Time-dependent Ginzburg-Landau simulations of the voltage-current characteristic of type-II superconductors with pinning,” Phys. Rev. B, vol. 65, no. 10, pp. 1–5, 2002.

[15] Q. Du, M. D. Gunzburger, and J. S. Peterson, “Computational simulation of type-II superconductivity including pinning phenomena,” Phys. Rev. B, vol. 51, no. 22, p. 16194, 1995.

[16] S. J. Chapman, Q. Du, and M. D. Gunzburger, “A Ginzburg–Landau type model of superconducting/normal junctions including Josephson junctions,” Eur. J. Appl. Math., vol. 6, no. 2, pp. 97–114, 1995.

[17] H. Wisodo, A. Hidayat, P. Nurwantoro, A. Bambang, S. Utomo, and E. Latifah, “Influence of Vortex-Antivortex Annihilation on the Potential Curve for Josephson Junction Based on The Modified Time Dependent Ginzburg-Landau Equations,” vol. 27, no. 1, pp. 52–57, 2014.

[18] Q. Du and J. Remski, “Limiting models for Josephson junctions and superconducting weak links,” J. Math. Anal. Appl., vol. 266, no. 2, pp. 357–382, 2002.

[19] H. Wisodo, P. Nurwantoro, and B. A. S. Utomo, “Pengaruh rapat arus eksternal terhadap gerakan vortex tunggal dalam superkonduktor tipe ii,” Simp. Fis. Nas. 23, 2010.

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