MEASUREMENT OF THE W BOSON MASS AT LEP

ARNO STRAESSNER
DPNC, University of Geneva,
24 Quai Ernest-Ansermet, 1211 Geneva 4, Switzerland
e-mail: Arno.Straessner@cern.ch

The mass of the W boson is measured at LEP by fully reconstructing the W boson decays. The measurement techniques and systematic uncertainties are presented. The current measurement of the mass of the W boson at LEP yields 80.412 ± 0.042 GeV.

1 Introduction

At LEP, W bosons are produced in the reaction $e^+e^- \rightarrow W^+W^-$ with the subsequent decay of the W’s into quark pairs, $qq'$, or a lepton and a neutrino, $\ell\nu$. About 40000 W pair events are registered by the four experiments ALEPH, DELPHI, L3 and OPAL, corresponding to a total luminosity of 2.8 fb$^{-1}$.

One of the main goals of the LEP programme is to determine the mass of the W boson, $M_W$, from the reconstructed invariant mass spectra. Involved techniques are used to obtain an optimal statistical precision. However, in the fully hadronic channel, systematic uncertainties are important, like final state interactions (FSI) between the hadronically decaying W bosons. These uncertainties reduce the sensitivity of this channel. The recent activities concentrate on increasing the weight of the hadronic events to obtain a globally more precise result on the LEP W mass.

2 Extraction of the W Mass

In the semi-leptonic and fully hadronic WW decay channels the complete four-fermion final states, shown in Figure 1a, can be reconstructed. The most sensitive observable for the W mass measurement is the invariant mass of the decaying W bosons. To improve the resolution on this quantity, a kinematic fitting procedure is applied. The reconstructed four-momenta of the final state fermions are varied within their resolution and kinematic constraints, like energy-momentum conservation and equal invariant masses in each event, are imposed. In fully hadronic events the resolution is diluted due to the different pairing combinations of the final state jets. In general, the most probable pairings according to the kinematics are chosen. Figure 1b shows an example of the invariant mass spectrum measured in the $qqq$ channel.

In $\ell\nu\ell\nu$ final states the event kinematic can not be reconstructed completely and the leptonic energy spectrum and a pseudo-mass are chosen as optimal W mass estimators.

To obtain W mass and width, three basic extraction methods are used: in the Monte Carlo reweighting procedure, the underlying $M_W$ and $\Gamma_W$ values of the Monte Carlo prediction are varied and compared to the measured spectra; in the convolution method the theoretically


Invariant mass spectrum in the \( qqqq \) final state as measured with the ALEPH detector.

predicted spectra are folded with resolution functions and fitted to data; in the Breit-Wigner method the resonance curve is fitted to data and possible bias is calibrated with Monte Carlo samples. All methods exploit information from several event observables, including the invariant masses obtained in kinematic fits with different constraints, as well as their uncertainties.

With the current techniques an equal statistical precision of 32 MeV and 35 MeV is reached in the \( qq\ell\nu\) and \( qqqq \) channel, respectively.

3 Systematic Uncertainties

A common source of systematic uncertainty in the \( qq\ell\nu\) and \( qqqq \) channels is the theoretical description of photon radiation that is used in the Monte Carlo simulation. In total, the uncertainty on initial state radiation (ISR), final state radiation (FSR) and \( O(\alpha) \) electroweak corrections amounts to 8 MeV in all channels. The current numbers are based on reweighting of Monte Carlo events according to possible differences in the theoretical prediction of the photon spectrum. However, recent comparisons between two different Monte Carlo generators, YFSWW\(^{11}\) and RacoonWW\(^{12}\), show larger differences. The uncertainty is therefore expected to be underestimated and may increase to \( 10 - 15 \) MeV for the final LEP W mass measurement.

Another common systematic error source is the description of the hadronisation of quarks. The uncertainty is mainly derived from the difference between the hadronisation models Pythia, Herwig and Ariadne\(^{12}\). Especially the simulated rate of heavy baryons influences the jet masses and therefore the derived invariant masses. A reweighting to the measured baryon rates is performed by Opal. Delphi also compares mixed Lorentz boosted Z decays, that are arranged to reproduce W-pair decay kinematics, to the hadronisation models. Since the agreement between the different models and data have improved, the current systematic uncertainty of \( 18 - 19 \) MeV may be reduced in future.

A similar reduction of systematic uncertainty, which is not yet included in the LEP combined W mass value, is originating from the LEP beam energy uncertainty. The final LEP energy calibration has improved\(^{11}\) with respect to the current calibration applied. It will result in a much smaller uncertainty of only 10 MeV, with only a small change of the average LEP beam energies. The LEP experiments also performed a cross-check of the LEP energy measurement by reconstructing the Z boson mass in radiative fermion-pair events, as shown in Figure 2. Both Opal and L3 obtain results in good agreement with the LEP energy calibration\(^{15}\).
Z decays at high centre-of-mass energies as well as in Z peak calibration data are analysed to test the detector simulation. Energy and angular measurement of leptons and jets in data are compared to Monte Carlo simulation. Possible differences are corrected and the remaining uncertainty on the difference is taken as systematic uncertainty, which amounts to 14 MeV on the combined W mass result.

4 Final State Interactions

A special complication in reconstructing the invariant masses appears in the fully hadronic channel. Hadronic FSI may introduce cross-talk between the two decaying W bosons of one event. Bose-Einstein Correlations (BEC) lead to an increased production of identical bosons, like pions and kaons, close in phase space. Colour Reconnection (CR) changes the colour flow between the four quarks in the final state. By reconnecting colour strings between the previously colour-neutral di-quark systems from each W decay, momentum is transferred between the W’s and the hadronisation process is altered.

As listed in Table 1, BEC and CR may change the reconstructed W mass by 35 MeV and 90 MeV, respectively. The increase of CR effects with centre-of-mass energy is taken into account.

At LEP, FSI effects are measured also in other observables, which are used to constrain the various BEC and CR models. The BEC measurements in W pairs mainly concentrate on charged pion production. Assuming a spherical and gaussian shaped source emitting the pions, the two-particle correlation function $C$ can be written as:

$$C(Q_{\pi\pi}) = 1 - \lambda \exp(-R^2Q_{\pi\pi}^2),$$

where $Q_{\pi\pi}$ is the square of the four-momentum difference of the two pions, $-(p_{\pi,1} - p_{\pi,2})^2$. The parameter $\lambda$ is the correlation strength and $R$ is the inverse radius of the source. When analysing semi-hadronic W events, it is found that the BEC between pions coming from the same W boson agree very well with the correlations observed in Z decays, if $Z \rightarrow bb$ is suppressed.

Important for the mass measurement are, however, the correlations between pions from different W bosons. If there are no such correlations the two-particle density in fully hadronic
events can be split into three terms:

$$\rho^{WW}(1,2) = \rho^{W+}(1,2) + \rho^{W-}(1,2) + 2\rho^{W+}(1)\rho^{W-}(2).$$

The first two terms on the right-hand side of the equation are the density functions for pions coming from the same W. They can be determined in semi-hadronic events. The last term describes the case when one pion comes from one W boson and the second from the other W boson. This part can be constructed from a sample of mixed semi-hadronic event:

$$\rho^{W+}(1)\rho^{W-}(2) = \rho^{WW}_\text{mix}.$$ 

If the equation holds, i.e. in absence of BE correlations between two W’s, the following ratio and difference of densities

$$D = \frac{\rho^{WW}}{2\rho^W + 2\rho^{WW}_\text{mix}}$$

$$\Delta \rho = \frac{\rho^{WW} - 2\rho^W - 2\rho^{WW}_\text{mix}}{2\rho^W + 2\rho^{WW}_\text{mix}}$$

are equal to 1 and 0, respectively. Figure 3a shows the quantity D as a function of Q as measured by Delphi. The Delphi data are consistent with moderate BEC between pions from different W’s. However, the combination of all LEP experiments prefers the absence of those correlations, as shown in Figure 3b. Monte Carlo studies show that the corresponding W mass shift is proportional to the BEC strength, so that the systematic uncertainty on $M_W$ due to BEC can in future be reduced to $10^{-15}$ MeV using the direct BEC measurement.

A similar approach is made in the reduction of the CR uncertainty. Different models predict CR, for example the Sjöstrand-Khose (SK) models 1 and 2, the Herwig model, or the Ariadne models 1-3. CR mainly manifests in a distortion of the W mass distribution. Effects on particle multiplicities are too small to be detected in data. The only other observable sensitive to CR is the particle flow in the regions between the quark jets. Figure 4a shows the angular distribution of particles in $qq\ell\nu$ events, after rescaling the angle of the particle to the next jet in such a way that each of the four jets is positioned at an integer angular value. A ratio $R$ is then calculated between the particle flow of the regions that connect two jets of one W (A+B) and the region that connects two jets of different W’s (C+D):

$$R = \frac{d\phi/dN(A + B)}{d\phi/dN(C + D)}.$$ 

To quantify the difference in the two regions A+B and C+D, the $R$ distribution is integrated. A variable $r$ is then constructed as the ratio of integrals $R_N$ obtained in data and in a Monte

---

Table 1: Systematic uncertainties in the W mass measurement.

| Source            | Uncertainties on $M_W$ in MeV |
|-------------------|-------------------------------|
|                   | $qq\ell\nu$ | $qqqq$ | $ffff$ |
| ISR/FSR           | 8            | 8      | 8      |
| Hadronisation     | 19           | 18     | 18     |
| LEP Beam Energy   | 17           | 17     | 17     |
| Detector Systematics | 14      | 10     | 14     |
| Colour Reconnection | –        | 90     | 9      |
| BE Correlations   | –            | 35     | 3      |
| Other             | 4            | 5      | 4      |
| **Total Systematic** | **31**  | **101**| **31** |
| **Statistical**   | **32**       | 35     | 29     |
| **Total**         | **44**       | 107    | 43     |
| **Statistical in absence of Systematics** | **32** | 28     | 21     |
Carlo without CR:

\[ r(\text{data}) = \frac{R_N(\text{data})}{R_N(\text{MC without CR})} \]

Figure 4b shows the result of the LEP experiments, using all data. In case all LEP experiments are combined, the SK1 model with 100% reconnection would yield a value of \( r(\text{SK1} - 100\%) = 0.891 \), while a value of \( r(\text{data}) = 0.969 \pm 0.015 \) is obtained in the LEP measurement. Data therefore excludes strong CR effects as predicted by SK1 with \( 5.2 \sigma \). The preferred values of the SK1 model parameter \( k_i \) are in the range \([0.39, 2.13]\). The upper boundary of \( k_i \) is used to estimate the systematic uncertainty on \( M_W \) due to CR. Averaged over all centre-of-mass energies one yields a possible mass shift of 90 MeV. This shift is larger than those observed in other models, like Ariadne 2 or Herwig.

A further constraint on the CR effects can be put by the mass measurement itself. An indication that CR cannot be strong comes from the comparison of the W mass measured in \( qqqq \) and \( qq\ell\nu \) events:

\[ \Delta M_W = M_W(qqqq) - M_W(qq\ell\nu) = +22 \pm 43 \text{ MeV} , \]

where systematic errors due to possible FSI are removed. The observed value is compatible with zero.

In addition, the W mass bias due to CR can also be reduced by modifying the jet algorithms that are used to reconstruct the W decay products. When reducing the cone size of the jets or by restricting the energy range of the objects clustered to a jet, the CR mass bias can be reduced, as shown in Figure 4b. Recent investigations have shown that this is the case for all CR models used. If one now varies the cone size or the energy cut-off, the mass shift observed in data and for the CR models can be compared. This result can also be combined with the particle flow measurement since the correlations between the measurements are small.

The four LEP experiments foresee to perform such an alternative mass analysis with reduced sensitivity to CR effects. This will enhance the weight of the \( qqqq \) channel in the LEP combination, which is currently only 0.10.
5 Results

Figure 5a shows the individual results obtained by the LEP experiments. With the current systematic errors and all correlations properly included, the LEP W mass measurement split into $qq\ell\nu$ and $qqqq$ channel yields:

$$M_W(qq\ell\nu) = 80.411 \pm 0.032\text{(stat.)} \pm 0.030\text{(syst.)} \text{ GeV}$$
$$M_W(qqqq) = 80.420 \pm 0.035\text{(stat.)} \pm 0.101\text{(syst.)} \text{ GeV} \ ,$$

with a correlation coefficient of 0.18. The combined mass value for all channels is

$$M_W(ffff) = 80.412 \pm 0.029\text{(stat.)} \pm 0.031\text{(syst.)} \text{ GeV} \ ,$$

with a good $\chi^2$/d.o.f of 28.2/33. Including the result derived from the cross-section measurement at the W-pair production threshold does not change numerically the above result:

$$M_W^{\text{LEP}} = 80.412 \pm 0.042 \text{ GeV} \ .$$

The method of direct reconstruction is also well suited to measure the width of the W boson, $\Gamma_W$. A combined fit to the LEP data yields:

$$\Gamma_W^{\text{LEP}} = 2.150 \pm 0.068\text{(stat.)} \pm 0.060\text{(syst.)} \text{ GeV} \ ,$$

and agrees well with the Standard Model (SM) prediction of $\Gamma_W = 2.099$ using the LEP W mass cited above.

As shown in Figure 5b, the LEP W mass agrees well with the measurement at $p\bar{p}$ colliders. The combination of these direct $M_W$ measurements is also in good agreement with the indirect determination from the other electroweak data. The result obtained in $\nu N$ scattering by NUTEV, which is derived from the measurement of the electroweak mixing angle, $\sin^2 \theta_W$, deviates from the LEP result by 2.9 $\sigma$. However, there is no systematic effect found that may explain this difference.
The W mass is an important parameter in the Standard Model. The precise measurement of $M_W$ probes the SM at the level of its radiative corrections. A comparison of the direct and indirect W mass determinations is shown in Figure 5a in the plane of W mass and top quark mass, $M_t$, that is measured at the Tevatron. Good agreement between the two sets of measurements is observed. Also shown is the SM prediction for various values of the mass of the Higgs boson, $M_H$. The measurements prefer small values of $M_H$.

In a supersymmetric extension of the theory, the Minimal Supersymmetric Standard Model (MSSM), the preferred $M_W$-$M_t$ area differs slightly from the SM prediction. Additional radiative correction terms, involving supersymmetric particles, shift the $M_W$-$M_t$ band to larger values of $M_W$. The MSSM prediction is shown in Figure 5b. The experimental precision of the electroweak measurements is not accurate enough to decide between the two models. However, an increased precision is expected from measurements at the Tevatron, the future LHC and linear colliders. The electroweak data gives also motivation to continue the search for the Higgs boson in a low mass range, as the last missing piece of the SM, and for supersymmetric particles at the Tevatron and at LHC experiments.

6 Conclusion

The mass of the W boson is measured at LEP to $M_W^{\text{LEP}} = 80.412 \pm 0.042$ GeV and the width to $\Gamma_W^{\text{LEP}} = 2.150 \pm 0.091$ GeV. It is an important contribution to the tests of the Standard Model and its supersymmetric extensions. Exploiting the recent results on systematic uncertainties, the $M_W$ measurement is expected to reach a final accuracy of about 35 MeV, completing the many precision tests of the SM performed at LEP.

Acknowledgements

I would like to thank the experiments ALEPH, DELPHI, L3 and OPAL for making their most recent and preliminary results available. I also like to thank the LEP Electroweak and WW Working Groups as well as Georg Weiglein and collaborators for preparing their results in form of nice graphs and plots.
Figure 6: a) Contour curves for the direct and indirect measurement of $M_W$ and $M_t$ compared to the SM prediction for different values of $M_H$. b) The direct $M_W$ and $M_t$ measurements compared to the MSSM prediction. The exclusion limits from searches for new particles at LEP are taken into account in the calculation. The MSSM and SM areas overlap in the region where the mass of the SM Higgs boson is in the MSSM range, i.e. for $M_H \sim 130$ GeV.

References

1. S. Jadach, W. Placzek, M. Skrzypek, B. F. L. Ward and Z. Was, Comp. Phys. Comm. 140 (2001) 432.
2. A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, Comp. Phys. Comm. 153 (2003) 462.
3. T. Sjöstrand et al., Comp. Phys. Comm. 135 (2001) 238; L. Lönnblad, Comp. Phys. Comm. 71 (1992) 15; G. Corcella et al., hep-ph/0011363; G. Marchesini et al., Comp. Phys. Comm. 67 (1992) 465.
4. The LEP Energy Working Group, Memo "Final LEP 2 centre-of-mass energy values", see http://lepecal.web.cern.ch/LEPECAL/.
5. The ALEPH Collaboration, Internal Note 2003-002; The OPAL Collaboration, Internal Note PN 520; The L3 Collaboration, Phys. Lett. B 585 (2004) 42.
6. The LEP Collaborations and the LEP Electroweak Working Group, Internal Note LEPEWWG/2003-02.
7. T. Sjöstrand and V.A. Khoze, Z. Phys. C 62 (1994) 281; L. Lönnblad, Z. Phys. C 70 (1996) 107; G. Corcella it et al., JHEP 01 (2001) 010.
8. The ALEPH Collaboration, Internal Note 2002-015.
9. The LEP Collaborations and the LEP WW Working Group, Internal Note LEPEWW/FSI/2002-02.
10. The DELPHI Collaboration, Internal Note 2003-021-CONF-641.
11. The LEP Collaborations and the LEP WW Working Group, Internal Note LEPEWWG/MASS/2003-01.
12. The LEP Collaborations and the LEP Electroweak Working Group, Internal Note LEP-EWWG, to be published, see http://lepewwg.web.cern.ch/LEPEWWG/plots/winter2004/.
13. S. Heinemyer and G. Weiglein, "The MSSM in the Light of Precision Data", hep-ph/0307177