Dynamics in mesoscopic superconducting rings: relaxation process and vortex-antivortex pairs

Mathieu Lu-Dac, Viktor V. Kabanov
Jožef Stefan Institute, Jamova 39, SI 1000 Ljubljana, Slovenia
E-mail: mathieu.lu-dac@ijs.si, viktor.kabanov@ijs.si

Abstract. We investigate the behavior of a mesoscopic 1D ring or 2D torus in an external magnetic field by simulating the Time Dependant Ginzburg Landau (TDGL) equations with periodic boundary conditions. In 1D, we analyze the stability and the different possible evolutions for the phase-slip phenomena starting from a metastable state. We used those results to observe the dynamics of a vortex-antivortex pair dissociation in 2D.

1. Introduction

Nonequilibrium phenomena in superconductors are a challenging area for both experimental and theoretical research. They are also crucial for the development of applications as they are the key to the appearance of resistive states in superconducting samples and to the possible use of vortex dynamics. In 1D, the resistive phase-slip process foreseen by W. A. Little [1] has been described quantitatively by the LAMH theory developed by J. S. Langer and V. Ambegaokar [2] and extended by D. E. McCumber and N. B. Halperin [3, 4]. The LAMH theory describes thermally activated phase-slips (TPS). It evaluates the resistance of a 1D superconductor when driven out of thermodynamical equilibrium by a voltage or current source. Since then, this theory has been accepted to a large extent in rather good agreements with experiments (for a more complete review of theoretical and experimental works, see [5]).

More recently, simulations were carried out on different versions of the time dependant Ginzburg Landau (TDGL) equations in order to investigate the dynamics of the process. In particular, the case of the 1D ring has raised interest since it exhibits multiple metastable states which can be reached by phase-slip processes. In [6], the ring is submitted to an electromotive force which constantly accelerates the superconducting electrons. Another approach has been made in [7], where the magnetic field is gradually increased.

In the present work, we consider a superconducting ring of thickness $d$, radius $R$ and length $L$ as represented on figure 1. For simplicity, we consider $d << \xi << \lambda_{eff}$, $R \approx \xi$ and $R << \lambda_{eff}$, where $\xi$ is the coherence length and $\lambda_{eff}$ is the Pearl [8] penetration depth. The first two conditions allow us to treat the ring as one dimensional and the last two conditions account for the mesoscopic size of the ring. We submit this 1D ring to a perpendicular constant magnetic field $H$, which determines the properties (superfluid density, current etc.) of the stable state. Our approach is different because we observe a relaxation process from the initial state to a more stable one, without any external parameter influencing the dynamics.

We also simulated vortex-antivortex dynamics in 2D, starting from a phase-slip line situation. Indeed, the vortex-antivortex pair dissociation is analog to the phase-slip in 1D.
2. The time dependant equations

We use the simplest version of the TDGL equations that takes into account the presence of magnetic field and the possibility of charge imbalance by introducing the vector potential $\mathbf{A}$ and the electrostatic potential $\Phi$ as derived by Gor’kov and Kopnin[9] in CGS units.

We used the following dimensionless variables: vector potential $a = \frac{2\pi \xi}{\Phi_0}$, time $t = \frac{c}{4\pi \lambda_{eff} \sigma_n}$, spatial coordinates $r = \frac{\tilde{r}}{\xi}$, electrostatic potential $\Phi = \tilde{\Phi} \frac{8\pi^2 \lambda_{eff}^2}{c \Phi_0}$ and order parameter $\psi = \Psi e^{i \theta}$ where $\gamma$ is a positive constant accounting for the slow relaxation of the order parameter. $\Psi(\tilde{r}, t) = \rho e^{i \tilde{\theta}}$ is the Ginzburg Landau order parameter which depends on the coordinate $\tilde{r}$ and the time $t$. $m$ and $e$ are the electron mass and charge, $\sigma_n$ is the normal state resistivity of the material, $\hbar$ the reduced Planck constant and $c$ the speed of light. $\alpha$ and $\beta$ are the coefficient appearing when deriving the free energy (see [10] for example). The spacial derivatives have also been modified by using $r = \frac{\tilde{r}}{\xi}$.

The Pearl penetration depth $\lambda_{eff} = \lambda^2 / d$ has been introduced instead of the London penetration depth $\lambda = \sqrt{\frac{m c^2}{8\pi^2 e^2 |\alpha|}}$ since the thickness $d$ is small. The coherence length is defined by $\xi = \sqrt{\frac{\hbar^2}{4m|\alpha|}}$.

There are two characteristic time scales: $\tau_\rho = \frac{\gamma \hbar}{|\alpha|}$ corresponds to the characteristic time of the evolution of the amplitude of the order parameter whereas $\tau_\theta$ accounts for the dynamics of the phase. The ratio between those two characteristic times is the only (dimensionless) parameter left when using the dimensionless variables: $u = \frac{\tau_\rho}{\tau_\theta}$.

We neglect the corrections to the vector potential due to the small dimensions of the ring and large Ginzburg-Landau parameter $\kappa = \frac{\lambda_{eff}}{\xi}$. Moreover, using the electroneutrality relation $div \mathbf{j} = 0$ as discussed in [9] we obtain:

$$u \left( \frac{\partial \psi}{\partial t} + i \Phi \psi \right) = \psi - \psi |\psi|^2 - (i \nabla + a)^2 \psi, \quad (1)$$

$$\nabla^2 \Phi = -\nabla \left( \frac{i}{2} (\psi^* \nabla \psi - \psi \nabla \psi^*) + a |\psi|^2 \right). \quad (2)$$

With a similar derivation of the stationary solutions as in [11], we obtain a family of solutions in the well known twisted plane wave form:

$$\psi_k = \sqrt{1 - (a - k)^2} e^{ikx + \theta_0}, \quad (3)$$
Figure 3. Distribution of the order parameter amplitude $\rho$ (a), phase $\theta$ (b) and electrostatic potential $\Phi$ (c) at different times during a single phase-slip event. We remind that all variables are dimensionless.

where $k$ and $\theta_0$ are real numbers and $x$ the longitudinal spatial coordinate.

We then analyze the stability of a solution in the form (3) by linearizing the equations disturbed by a perturbation $y_r(x,t) + iy_i(x,t)$, and passing to Fourier transform $\hat{y}_r(q,t) + \hat{y}_i(q,t)$.

After putting dimensions back, we find a stability condition relating the winding number or vorticity $n = \frac{1}{2\pi} \oint d\theta x$ with the number of flux quanta penetrating the ring $\frac{\Phi}{\Phi_0}$:

$$|n - \frac{\phi}{\phi_0}| \leq \frac{R}{\xi \sqrt{3}}.$$  

(4)

This condition is consistent with the ground state found by minimizing the free energy with a solution in the form (3). As seen on figure 2, $|n - \frac{\phi}{\phi_0}|$ must be minimal.

3. Simulations

3.1. Phase-slip in 1D

We simulate the evolution after an infinitesimal perturbation from the solution:

$$\psi_0 = \sqrt{1 - a^2}$$  

(5)

to the more stable solution after $n_1$ phase-slips:

$$\psi_1 = \sqrt{1 - (a - \frac{n_1 2\pi}{L})^2} e^{i(\frac{2\pi}{L} x + \theta_0)},$$

(6)

which can be the ground state or a new metastable state closer to the ground state.

As described in the LAMH theory, the amplitude of the order parameter vanishes in a very narrow region, and for a very short time (see figure 3a). It then relaxes to the new (meta)stable state defined by equation (6). The phase of the order parameter takes a sharper sinusoidal form until the minimum and maximum disconnect as seen on figure 3b, at the very moment when the amplitude vanishes and in the same region. Afterwards, it relaxes to a sawtooth pattern corresponding to the new state. The superconducting current behaves in a similar way as the amplitude of the order parameter and vanishes in the region and time of the phase-slip, which becomes a normal resistive region. Unlike for the amplitude, the process is done with a decrease of the supercurrent, which is a physical evidence that the new state is more stable. Accordingly to the Josephson equation $\frac{d\Delta \theta}{dt} = \frac{2eV}{\hbar}$, the electric scalar potential (on figure 3c) and the phase are directly connected.

Giving characteristic values for the process can be deceiving as the values strongly depend on the material and the thickness of the ring. Nevertheless, we give the example of NbN for which we find the characteristic time $\tau_0 \approx 1\text{ps}$, which makes the total time for a single phase-slip to occur of the order of $100\text{ps}$, and the voltage appearing of around $10\mu\text{V}$.
3.2. Vortex-Antivortex in 2D

We used the same set of equations with periodic boundary conditions in 2D to simulate an empty torus of length $L$. We confirmed the topological analogy between the 1D phase-slip and the 2D vortex-antivortex pair dissociation. Indeed, we simulated the relaxation of the superconducting state from a state where a phase-slip line has been introduced (we use $x$ and $y$ as the spatial coordinates):

$$
\begin{align*}
    \psi(x, y, t = 0) &= \sqrt{1 - a^2}, \quad \text{for } 0 < x < l_1 \text{ or } l_2 < x < L \\
    \psi(x, y, t = 0) &= \sqrt{1 - (a - \frac{2\pi}{L} x)^2} e^{i \frac{2\pi x}{L}}, \quad \text{for } l_1 < x < l_2
\end{align*}
$$

From this initial state, we observe a vortex-antivortex pair dissociation (the current is flowing in opposite direction around the two normal cores). The amplitude is plotted on figure 4. Introducing more phase-slip lines creates as many pairs as phase-slip lines introduced.

4. Conclusion

We successfully derived a stability condition for the TDGL in the case of a 1D ring and static magnetic field. This intuitive condition is consistent with the shape of the free energy. The simulations confirm our predictions and enable us to give characteristic values for the phase-slip process. In 2D we confirmed the topological analogy with the phase-slip.

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