Binary Black Holes in Stationary Orbits

Sandip K. Chakrabarti
Tata Institute of Fundamental Research, Homi Bhabha Road, Colaba, Bombay, 400005, INDIA

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Abstract

We show that under certain astrophysical conditions a binary system consisting of two compact objects can be stabilized against indefinite shrinking of orbits due to the emission of gravitational radiation. In this case, the lighter binary companion settles down to a stable orbit when the loss of the angular momentum due to gravitational radiation becomes equal to its gain from the accreting matter from the disk around the more massive primary. We claim that such systems can be stable against small perturbations and can be regarded as steady emitters of gravitational waves of constant frequency and amplitude. Furthermore, X-rays emitted by the secondary can also produce astrophysically interesting situations when coupled with gravitational lensing and Doppler effects.

It is well known that a binary system is unstable against gravitational radiation. The radiation carries away both energy as well as angular momentum and the orbits shrink indefinitely. In the case of active galaxies and Quasars, it is possible to envisage a situation, such that there is a large accretion disk ($\sim 10^6$ Schwarzschild radius) around the central black hole (of mass $M_1 \sim 10^{6-9}M_\odot$) and a large number of smaller objects (of mass $M_2 \sim 0.1 - 100M_\odot$) such as ordinary stars, white dwarfs, neutron stars and black holes orbiting around it at various orbits.
Let us concentrate on one such system, binary in nature, where the secondary companion is also compact—presumably, a neutron star or a black hole. Since a companion in an eccentric orbit will lose angular momentum rather rapidly, it is safe to assume that the orbit is circular. We also assume that the secondary is in the same plane as the accretion disk itself. This scenario is general enough, since the secondary, in any other plane would be brought to the equatorial plane due to the Lens-Thirring effect. The shrinking orbit of the binary will eventually bring it close to the central massive body, inside the denser part of the accretion disk.

In the case of AGNs and Quasars, the angular momentum of the surrounding accretion disk close to the hole could be almost constant \((l \sim 4GM_1/c)\) giving rise to thick accretion disks. In between the inner edge of such a disk (around \(2r_s\)) and the center of the disk (which could be anywhere between the marginally stable orbit at \(3r_s\) and, say, \(10 - 50r_s\) depending upon the angular momentum distribution) such a disk is super-Keplerian. (see, e.g., Paczynski and Wiita, 1980; here \(r_s = 2GM_1/c^2\) is the Schwarzschild black hole radius, \(G\) and \(c\) are the gravitational constant and the velocity of light respectively). That radiation pressure could cause such a change in the angular momentum distribution has been well demonstrated by Maraschi, Reina and Treves (1976). In the absence of a rigorous understanding of the viscous processes which determine the exact distribution of angular momentum inside the disk, it is customary to use power law distribution of angular momentum (e.g. Paczynski and Wiita, 1980; Chakrabarti 1985). Fig. 1 shows the comparison of a few typical angular momentum distributions as functions of the Schwarzschild radius of the primary. Here, \(l_k\) denotes the Keplerian distribution, with a minimum at the marginally stable orbit \(r_{ms} = 3r_s\); \(l_p\) is a power law distribution \(l_p = 2.8(r/r_s)^{0.25}\) and \(l_{mb}\) is the marginally bound angular momentum \(=4GM_1/c\). The power law distribution intersects the Keplerian curve at two points: at \(r_i\), the inner edge \((i)\) of the disk, and at \(r_c\), the center \((c)\) of the disk. Therefore, once the orbiting black hole or a neutron star (which is on a Keplerian orbit) is inside the region between \(i\) and \(c\), it will accrete mass as well as angular momentum, from the disk matter. We show below that it is possible to have a situation where the
orbital angular momentum gained by the binary companion balances exactly the loss due to gravitational waves and the binary attains a stable orbit, emitting steadily both the gravitational radiation as well as X-rays and gamma rays with possible modulations due to redshifts and Doppler shifts due to its orbital motion, as well as gravitational lensing due to the presence of the massive primary.

We like to mention here that a similar consideration of replenishment of angular momentum through accretion onto a rapidly rotating neutron star radiating angular momentum through gravitational waves was considered by Wagoner (1984). He finds that it is possible to stabilize the neutron star from any further loss of angular momentum and energy.

The rate of loss of energy \( \frac{dE}{dt} \) in a binary system consisting of two point masses \( M_1 \) and \( M_2 \) in circular orbits, having orbital period \( P \) (in hours) is given by (see, Peters and Matthews, 1963; Lang, 1980),

\[
\frac{dE}{dt} = 3 \times 10^{33} \left( \frac{\mu}{M_\odot} \right)^2 \left( \frac{M}{M_\odot} \right)^{4/3} \left( \frac{P}{1 \text{hr}} \right)^{-10/3} \text{ergs sec}^{-1},
\]

(1)

where,

\[
\mu = \frac{M_1 M_2}{M_1 + M_2}
\]

and

\[
M = M_1 + M_2.
\]

The orbital angular momentum loss rate would be,

\[
\frac{dL_\perp}{dt} = \frac{1}{\Omega} \frac{dE}{dt}
\]

(2)

where \( \Omega = \sqrt{GM_1/r^3} \) is the Keplerian angular velocity of the secondary black hole orbiting at a mean radius of \( r \). The negative sign signifies a loss.

For the problem in hand, we assume that \( M_1 > M_2 \). In fact, we shall be typically interested in the cases, where, the primary has \( M_1 \sim 10^{6-8} M_\odot \) and the companion has \( M_2 \sim 10^{0-2} M_\odot \). We assume further that the primary is accreting at the rate at least
close to its critical rate, $\frac{\dot{M}_{1,Ed}}{\eta}$, where $\dot{M}_{1,Ed}$ is the Eddington rate of the primary and $\eta$ is the release of the binding energy loss by the accreting matter, typically 6 per cent, in case the primary is a Schwarzschild black hole. Thus, disk can supply matter, several times the Eddington rate to the secondary as it orbits around the central hole.

Let us assume that the secondary is accreting $\dot{M}_2$ amount of matter from the disk per second. If it accretes specific angular momentum at a fraction $\alpha < 1.0$ of the marginally bound value $4GM_1/c$, the rate of gain of angular momentum by the companion is given by,

$$\frac{dL_+}{dt} = \frac{4GM_1}{c}\alpha\dot{M}.$$  \hspace{1cm} (3)

Here $\alpha$ is assumed to be a constant for simplicity, even though its value is necessarily becomes zero at the inner edge $r_i$ as well as at the center $r_c$ of the disk. It is negative for $r > r_c$.

Equating (2) and (3) we obtain the equilibrium radius as (Chakrabarti 1992),

$$r_{eq} = 15.63\frac{M_1^{1/21}}{[\alpha M_2]^{2/7}}\left(\frac{M}{M_\odot}\right)^{4/7}\left(\frac{M}{M_\odot}\right)^{8/21}\left(\frac{10^8M_\odot}{M_1}\right)$$  \hspace{1cm} (4)

in units of the Schwarzschild radius of the primary. For an illustration, we choose $M_1 = 10^8M_\odot$, $\alpha = 0.05$ and $\dot{M}_2$ as the critical rate onto the secondary. If we choose, $M_2 = 1M_\odot$, we obtain, $r_{eq} = 23r_s$.

It is to be noted that in want of a rigorous distribution of angular momentum in the disk closer to the hole, we have chosen the rate of accretion of angular momentum to be constant (eqn. 3), though globally it should vary significantly. As is obvious from Fig. 1, the difference of angular momentum which could be transported to the orbiting black hole, $l_p - l_K$ decreases sharply as one goes from from $r_{ms}$ to the center ($c$) of the disk at $r_c$. Thus, $\alpha$ sharply falls to zero at $r_c$ and the transport can only take place upto the center of the disk. The $\dot{M}_2$ itself may also fall very rapidly with radial distance due to the sudden increase in thickness of the disk close to the center, causing lesser matter to be accreted onto the secondary. For $r > r_c$, the transport acts in the opposite
direction causing the secondary object to shrink faster than the rate prescribed by the
loss of the gravitational waves. This effect need not be very strong, since the density of
matter falls off sharply with distance and therefore the actual accretion of matter onto
the secondary with sub-Keplerian angular momentum will be smaller.

It is easy to see that the equilibrium orbit at \( r_{eq} \) is stable as long as \( r_{mb} < r_{eq} < r_c \)
and the product \( \alpha \dot{M}_2 \) locally falls off faster than \( r^{-7/2} \). That the second condition is
attainable could be seen in the following way. For simplicity, let us consider an wedge
shaped disk, where the density \( \rho(r) \propto r^{-3/2} \). Let the accreting angular momentum
onto the black hole be of the form \( \alpha(r) \propto r^{-\beta} \), where \( \beta \) is a constant. The rate of
accretion onto the secondary \( \dot{M}_2(r) \propto \rho(r)v(r) \propto r^{-5/2+\beta} \), where \( v(r) = \alpha(r)/r \) is the
relative velocity between the disk matter and the black hole. Therefore, the product
\( \alpha \dot{M}_2 \propto r^{-5/2+2\beta} \) could be faster than \( r^{-7/2} \) when \( \beta > 1/2 \). In this case, as the secondary
black hole is perturbed to a higher radius, it accretes lesser angular momentum from
the disk and it comes back to \( r_{eq} \) through gravitational radius. When it is pushed closer
to the black hole it gains more angular momentum than it losses, and therefore returns
back to \( r_{eq} \). It is possible that, in reality, \( r_{eq} \) would settle down closer to the center
of the disk. If, \( r_{eq} < r_{ms} = 3r_s \), the accretion of angular momentum increases with
radial distance. This makes the equilibrium orbits in this region unstable under small
perturbations.

Being on a stationary orbit, such a source will be emitting steady gravitational
waves with frequency,
\[
f = \frac{1}{\pi} \left( \frac{GM}{r_{eq}^3} \right)^{1/2}
\]
and the constant amplitude of metric perturbation on earth caused by such a source
would be (e.g., Sathyaprakash and Dhurandhar, 1991)
\[
|A| = 5.765 \times 10^{-19} \frac{\mu}{1M_\odot} \left( \frac{M}{10^6 M_\odot} \right)^{2/3} f^{2/3} \left( \frac{D}{1Mpc} \right)^{-1}
\]
\( D \) being the distance of the source from earth. Though the frequency is much lower
\((10^{-6} \text{ to } 0.1\text{Hz})\) than the present observable limits \((10 - 100\text{Hz})\), the steady source can
cause periodic variation of the distance between a pulsar and the earth which is reflected on the post-fit residual. Such variation is observed in the case of PSR1937+21 (J. Taylor, private communication).

We have demonstrated that under certain astrophysical conditions, when the accretion disk around a massive black hole is super-Keplerian, the orbit of a lighter compact object, such as a black hole, neutron star or a white dwarf can be stabilized against the loss of angular momentum due to gravitational wave. Such a system can be a steady source of gravitational waves. The accreting secondary would also emit X-rays and gamma rays which may be modulated due to red-shifts, Doppler effects and gravitational bending of light. Indeed, NGC6814, a low luminosity syfert galaxy, which shows steady periodicity of X-ray flaring over at least five years, could be easily explained by assuming a white dwarf in a stationary orbit at $6r_s$ around a black hole (Chakrabarti and Bao, 1992). Though we concentrated our discussion in the context of thick accretion disks, it is easy to see that even for geometrically thin disks stabilization of orbits may be possible, provided the disk remains super-Keplerian and the stability conditions are satisfied. In cases where stabilization is not possible, particularly when the disk is not sufficiently super-Keplerian, or, when the component masses are comparable so that not enough mass accretion takes place onto the secondary, the behavior of the gravitational wave emitted should still be greatly influenced by the presence of the disk. Work along this line is in progress and will be reported elsewhere.
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Fig. 1: The angular momentum distribution \((l_p)\) inside a typical radiation pressure dominated disk is compared with the Keplerian distribution \(l_k\) and with the marginally bound value \(l_{mb}\). The inner edge of the disk is located at \(i\) and the center is located at \(c\). An orbiting black hole between \(i\) and \(c\) accretes matter as well as angular momentum from the disk.