Gravitation vs. Rotation in 2+1 Dimensions

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Abstract

We investigate rotation and rotating structures in (2+1)-dimensional Einstein gravity. We show that rotation generally leads to pathological physical situations.

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INTRODUCTION

Rotation in (2+1)-dimensions has recently stimulated much debate following Gott’s [4] claim that cosmic strings could act as time machines by creating closed timelike curves (CTCs) as they pass by each other. Gott’s solution corresponds to point sources in (2+1)-dimensions with relative angular momentum, and can be mapped asymptotically to a spinning point source metric. Such spinning point source solutions contain CTCs and these closed time circuits are at the heart of the time machine proposal. While it has been shown [2] that superluminary velocities are required for Gott’s solution to support timelike lines, the study of rotating, many particle [3] and extended source solutions has only been partially completed in (2+1)-dimensions [4–6].

Of particular interest are rotating structures since our previous work [7,8] proved that all hydrostatic structures cause spatial compactification. Since it makes no sense to talk about such structures rotating, it is clear that any generalisation of the static results to include rotation will be non-trivial. This is in sharp contrast to the situation in (3+1)-dimensions where the solution for a slowly rotating star can be found by giving a static star a small, uniform spin. Changes to the pressure and density profiles are found at linear order in the angular velocity while changes to the structure (oblate flattening) only occur at second order.

We begin by considering the coordinate systems best suited for studying stationary solutions to Einstein’s equations in (2+1)-dimensions. From these we recover the spinning point source solution first given by Deser et al. [9]. We then turn to the problem of rotating structures. In order to simplify the analysis we restrict our attention to the study of solutions with either shear or vorticity, but not both.

We find that the class of solutions with zero vorticity all encounter in finite shear while the class of solutions with zero shear all contain CTCs - unless the structure is supported by tension rather than pressure. The only solution to avoid some serious pathology is the rotating annulus with energy density equal to tension. The rotating closed string solution of Clément [6] is closely related to this rotating annular structure.

I. STATIONARY COORDINATE SYSTEMS

The most general circularly symmetric, stationary metric can be written as

$$ds^2 = (e^{\nu} dt + \lambda d\theta)^2 - e^{2\eta} dr^2 - r^2 d\theta^2,$$  \hspace{1cm} (1.1)

where the metric functions \(\nu\), \(\eta\) and \(\lambda\) are all functions of \(r\) only. Clearly, closed timelike \((ds^2 > 0)\) lines can occur on circles of radius \(r < \lambda\). The vacuum Einstein equations are satisfied if \(\nu\), \(\eta\) and \(\lambda\) are all constants. Choosing \(\nu = 0\) as our scaling of time, the other constants can be related to the mass, \(M\), and spin, \(S\), of the point source, giving the line element [9]

$$ds^2 = dt^2 + 2GS dt d\theta - \frac{1}{(1 - GM)^2} dr^2 - (r^2 - (GS)^2) d\theta^2.$$  \hspace{1cm} (1.2)
We see that for \( r < GS \) the metric admits CTCs and suffers from having an ill-defined proper spatial volume.

While general coordinate invariance tells us that all coordinate systems are on an equal footing, operational experience shows us that some are “more equal” than others. As an example of this, applying the coordinate transformations
\[
\tau = t + (GS) \theta, \quad \rho = (1 - GM)^{-1} r \\
\phi = (1 - GM) \theta
\]
and
\[
(1.2)
\]
gives
\[
ds^2 = d\tau^2 - d\rho^2 - \rho^2 d\phi^2 .
\]
All obvious signs that a point source inhabits this spacetime have been hidden in the coordinates. The time coordinate has become periodic and the angular coordinate no longer ranges from 0 to 2\( \pi \).

Another example is the “fake rotation” metric
\[
ds^2 = \frac{1}{1 + \Omega_0^2 r^2} dt^2 + 2\Omega_0 r^2 dtd\theta - \frac{1}{1 + \Omega_0^2 r^2} dr^2 - \frac{r^2}{1 + \Omega_0^2 r^2} d\theta^2 ,
\]
with \( \Omega_0 = \)constant, which is just a fancy way of writing Minkowski space. This metric arose in the course of studying rotating structures and corresponds to uniformly rotating empty space.

To simplify some calculations we will work with the metric
\[
ds^2 = e^{2\kappa} dt^2 + 2\omega dtd\theta - e^{2\gamma} dr^2 - r^2 d\theta^2 ,
\]
rather than the metric in Eqn. (1.1). In this coordinate system the spinning point source metric takes the form
\[
ds^2 = dt^2 + 2GS dtd\theta - \frac{1}{(1 - GM)^2 (1 + (GS/r)^2)} dr^2 - r^2 d\theta^2 ,
\]
so that the region containing CTCs has been mapped onto the region \(-(GS) < r < 0\). The singular behaviour of the metric at \( r = 0 \) signals the transition into the region containing CTCs.

### II. SOLVING EINSTEIN’S EQUATIONS

Before solving Einstein’s equations, it is instructive to calculate the expansion, shear and vorticity of the fluid. We are principally interested in the scalar invariants of these quantities, \( \Theta, \sigma \) and \( \xi \), respectively, which are defined in terms of the fluids’ three velocity, \( u^\mu \), as follows
\[
\Theta = u^\alpha \gamma^\alpha , \quad \sigma^2 = \frac{1}{2} \sigma^{\alpha\beta} \sigma_{\alpha\beta} , \quad \xi^2 = \frac{1}{2} \xi^{\alpha\beta} \xi_{\alpha\beta} ,
\]
where
\[
\sigma^{\alpha\beta} = \frac{1}{2} (u^\mu P^\alpha_{\mu\beta} + u^\beta_{\mu} P^\mu_{\alpha\beta}) - \frac{1}{2} \Theta P^{\alpha\beta} ,
\]
\[
\xi^{\alpha\beta} = \frac{1}{2} (u^\mu P^\alpha_{\mu\beta} - u^\beta_{\mu} P_{\mu\alpha} ) ,
\]
\[
P^{\alpha\beta} = g^{\alpha\beta} - u^\alpha u^\beta .
\]
In terms of the metric in Eqn. (1.3) we find

\[ \Theta = 0 , \]
\[ \sigma^2 = (\Omega')^2 f(r) , \]  
\[ \xi^2 = (\omega - \Omega r^2)^2 g(r) , \]

(2.2)
(2.3)
(2.4)

where \( \Omega = u^\theta / u^t \) is the rotational velocity and \( f(r) \) and \( g(r) \) are complicated expressions involving the metric functions. The prime denotes differentiation with respect to \( r \). The above decomposition of \( u^\alpha_{\beta} \) allows us to study solutions with either shear or vorticity, or both. Obviously any solution with neither shear nor vorticity is non-rotating.

### A. Vorticity Free Solutions

The vorticity free case is characterised by \( \omega = \Omega r^2 \). With this identification the fluid’s three velocity takes the form

\[ u^t = (e^{2\kappa} + \Omega^2 r^2)^{-\frac{1}{2}} , \quad u^r = 0 , \quad u^\theta = \Omega u^t , \]
\[ u_t = (e^{2\kappa} + \Omega^2 r^2)^{\frac{1}{2}} , \quad u_r = 0 , \quad u_\theta = 0 . \]  

(2.5)

The radial component of the energy-momentum conservation equation \( T^r_{;\alpha} = 0 \) gives the hydrostationary equilibrium condition

\[ p' = -\frac{1}{2}(\rho + p) \left[ \ln(e^{2\kappa} + \Omega^2 r^2) \right]' . \]  

(2.6)

The expression for the shear simplifies to

\[ \sigma^2 = \frac{(\Omega' re^{-\gamma})^2}{2(e^{2\kappa} + \Omega^2 r^2)} . \]  

(2.7)

If we take the limit in which the fluid is essentially a “test fluid” (a fluid of test particles) in the field of a point source, we find

\[ \sigma^2 = \frac{2G^2S^2(1 - GM)^2}{r^4} . \]  

(2.8)

The singular nature of the shear at \( r = 0 \) signals that the region below \( r = 0 \) contains CTCs. We note that the shear vanishes if \( S = 0 \) or \( M = 1/G \). This is a useful consistency check since the spinless case should produce no shear while the limiting case \( M = 1/G \) corresponds to the cylindrical metric

\[ ds^2 = dt^2 + 2GS dt d\theta - dr^2 - a^2 d\theta^2 , \]  

(2.9)

with \( a \) an arbitrary constant. The above spacetime is actually non-rotating since it is spatially compact so again we expect the shear to vanish. The non-rotating nature of this metric can be seen by implementing a suitable coordinate transformation or by realising that a test particle on a rotating cylinder feels no centrifugal force [10].
Turning now to the general case, the Einstein equation $G^t_\theta = 0$ gives the condition

$$\gamma'(e^{2\kappa}r^2 + \omega^2)(2\omega r - \omega' r^2) + (\omega \omega' + \kappa' e^{2\kappa} r^2)(2\omega r - \omega' r^2)$$

$$+ \omega'' r^2 (e^{2\kappa} r^2 + \omega^2) - 2\omega^3 - r^3 \omega' e^{2\kappa} = 0 .$$

(2.10)

This equation is trivially satisfied if $\omega = 0$ or $\omega = \Omega_0 r^2$, where $\Omega_0$ is a constant, i.e. if the shear vanishes. Indeed, all components of Einstein’s equations reduce to their non-rotating expressions for these values of $\omega$. This is because solutions without shear or vorticity are non-rotating. The unique, non-trivial solution to Eqn.(2.10) is given by

$$e^\gamma = \frac{A \Omega' r^3}{(e^{2\kappa} + \Omega^2 r^2)^{1/2}} \quad (\Omega \neq \text{constant}),$$

(2.11)

where $A$ is a constant of integration. We see that the point source solution is recovered if $A = 1/(2GS(1 - GM))$, $\Omega = GS/r^2$ and $\kappa = 0$. The point source is the only example of when the above solution can be used to recover a static limit. This is because the point source solution does not have to have $M = 1/G$ and so does not necessarily cause spatial compactification. In all other cases Eqn.(2.11) is an additional constraint which does not exist in the non-rotating case ($\Omega = \text{constant}$), and which is generally incompatible with the static limit.

Further insight can be gained by inserting Eqn.(2.11) into the expression for the shear scalar (2.7), which yields

$$\sigma^2 = \frac{1}{2A^2 r^4} ,$$

(2.12)

thus all vorticity free solutions encounter infinite shear at $r = 0$. In the coordinate system of Eqn.(1.1) the shear is given by $\sigma^2 = 1/(2A^2[r^2 - \lambda(r)^2])$ so the shear becomes singular at $r = \lambda(r)$, which marks the boundary of the region containing CTCs. Even if CTCs are avoided ($r > \lambda(r)$) there will still be a shear singularity at the origin since $r \to 0$ then demands $[r^2 - \lambda(r)^2] \to 0$. Thus all vorticity free solutions are plagued by infinite shear - with the possible exception of rotating annuli which do not have matter extending to the origin.

B. Shear Free Solutions

The shear-free case is characterised by having $\Omega$ constant. Adopting the metric parameterisation of Eqn.(1.1) and choosing a co-rotating frame with:

$$u^t = e^{-\nu} , \quad u^r = 0 , \quad u^\theta = 0 ,$$

$$u_t = e^{\nu} , \quad u_r = 0 , \quad u_\theta = \lambda ,$$

(2.13)

we find that the vorticity scalar is given by

$$\xi = \frac{\lambda \nu' - \lambda'}{2r} e^{-\nu} .$$

(2.14)
The equation \( G^\theta_\ell = 0 \) then demands
\[
\frac{e^{\nu-\eta}}{r} (2\xi\nu' + \xi') = 0 ,
\] (2.15)
so that \( \xi = \xi_0 e^{-2\nu} \). The remaining independent Einstein equations then read
\[
\frac{\eta'e^{-2\eta}}{r} + 3\xi^2_0 e^{-4\nu} = 2\pi G\rho ,
\] (2.16)
\[
\frac{\nu'e^{-2\eta}}{r} + \xi^2_0 e^{-4\nu} = 2\pi Gp ,
\] (2.17)
\[
(\nu'' - \nu'\eta' + \nu'\nu')e^{-2\eta} + \xi^2_0 e^{-4\nu} = 2\pi Gp .
\] (2.18)
These can be manipulated to give the equilibrium condition \( p' = -(\rho + p)\nu' \). As first shown in [8], Eqns.(2.17) and (2.18) can be combined to yield
\[
\frac{\nu'}{r} = C e^{\eta-\nu} ,
\] (2.19)
which can then be inserted into Eqn.(2.14) to give
\[
\lambda = \frac{\xi_0}{C} \left( e^{-\nu} + Be^\nu \right) .
\] (2.20)
The arbitrary constant \( B \) cannot be determined from Einstein’s equations, i.e. the equations are unaffected by the transformation \( \lambda \to \lambda + A e^\nu \) where \( A \) is an arbitrary constant. Such arbitrary terms can always be removed by a coordinate transformation. The transformation that achieves this is \( t \to t - (B\xi_0/C ) \theta \). Naturally this transformation does not affect the geometry, but it does change the topology of the solution as the time coordinate becomes periodic, just as it did for the metric in Eqn.(1.3).

If we demand that the origin be part of our spacetime \( \nu(0) = \eta(0) = \lambda(0) = 0 \) then \( B = -1 \). For this choice of \( B \) we find for small \( r \) that \( \lambda^2 - r^2 = -r^2(1 - \xi^2_0 r^2 + O(r^4)) \) so there are no CTCs near the origin. While the metric is healthy near the origin, there will generally be CTCs towards the edge of the rotating structures as the following argument demonstrates.

Following the steps used in [8], we can use Eqn.(2.19) to find a general expression for the pressure in terms of the metric functions. The solution is
\[
p = \frac{1}{2\pi G} \left( Ce^{-\eta-\nu} + \xi^2_0 e^{-4\nu} \right) .
\] (2.21)
The above equation shows that all solutions with \( C > 0 \) must be supported by pressure while solutions with \( C < 0 \) could be supported by either tension or pressure. However, Eqn.(2.19) and the equilibrium condition \( p' = -(\rho + p)\nu' \) demand that solutions with \( C < 0 \) are supported by tension rather than pressure. If this were not the case, the pressure would be an increasing function of radius which is unacceptable.

In the non-rotating limit \( (\xi_0 = 0) \) the above expression can be used to prove that \( e^{-\eta} \to 0 \) at the edge of the structure which in turn proves that all hydrostatic structures cause spatial
compactification. For $\xi_0$ non-zero however, this is no longer true since for $C > 0$ the requirement that $p \rightarrow 0$ implies that $e^{-\nu} \rightarrow 0$ so the temporal metric component is singular at the edge of the structure. Returning to our consideration of CTCs, we see that the quantity $\lambda^2 - r^2$ approaches $[(\xi_0/C)^2 e^{2\nu} - r^2]$ at the edge of the structure, and this quantity is positive for structures with finite radius. All such structures will thus encounter CTCs. The only solutions with $C > 0$ to escape this pathology would have pressure profiles which approach zero as $r \rightarrow \infty$, however these solutions would fill the entire space which takes us back to the question of a rotating universe. Since the total angular momentum of a universe must be zero, and since the rotational velocity in this solution is always of the one sign, we conclude that no solutions of this kind can exist.

The only hope for finding solutions free of CTCs comes when $C < 0$ and the structures are supported by tension rather than pressure. The structure may then have $p \rightarrow 0$ without $e^\nu \rightarrow \infty$. It is not surprising that only the tension case can hope to tame the growth of $e^\nu$. Without tension to oppose the centrifugal force, gravity alone must try and hold the structure together which in turn leads to the divergence of $e^\nu$. We shall give explicit examples of such solutions in the following section.

### III. EXACT SOLUTIONS FOR ROTATING STRUCTURES

It is instructive to consider some explicit examples which serve to illustrate some of the general results that we have been discussing. We shall begin by considering some earlier attempts at constructing rotating structures and relate these solutions to our general picture.

The rotating perfect fluid solution of Williams belongs to the class of solutions with zero shear and non-vanishing torsion. From our general arguments we expect this solution to contain CTCs for large $r$ and this is indeed the case. In the language of our paper, the Williams solution has constant angular velocity $\Omega_0$, and density, pressure and vorticity given by

\begin{align}
\rho &= 3p = \frac{3\Omega_0^2}{\pi G (1 + \Omega_0^2 r^2)^2}, \\
\xi &= \frac{\Omega_0}{1 + \Omega_0^2 r^2}.
\end{align}

The metric is given by

\begin{align}
ds^2 &= \left(\frac{1 - \Omega_0^2 r^2}{1 + \Omega_0^2 r^2}\right) dt^2 + \left(\frac{4\Omega_0 r^2}{1 + \Omega_0^2 r^2}\right) dt d\theta - dr^2 - r^2 \left(\frac{1 - \Omega_0^2 r^2}{1 + \Omega_0^2 r^2}\right) d\theta^2.
\end{align}

This metric encounters CTCs for $r > 1/\Omega_0$, and while the “gravitational mass” of the structure is $M_G = 3/G$, a calculation of the proper mass reveals that $M = (1 + i)/G$.

The interior solutions for rotating cosmic strings found by Jensen and Soleng appear to contradict our general results as they are able to construct solutions supported by pressure that are free of CTCs. This apparent conflict stems from the fact that they considered solutions supported by an arbitrarily chosen energy production mechanism. In the limit that this heat source is removed, the rotation vanishes and the solution reduces to empty
space. If we had allowed such an ad hoc support mechanism in our current work, the additional freedom would have allowed any choice of metric functions to be a solution to Einstein’s equations.

As has already been mentioned, the closed rotating string solution of Clément is an interesting example since it belongs to the class of solutions where CTCs and shear singularities can be avoided. Indeed, we can obtain a very similar solution by studying uniformly rotating tension stars.

For the equation of state $\rho = -\alpha p = \alpha T$, ($\alpha \geq 1$) the equation $T' = (\rho - T)\nu' = 0$ yields

$$T = T_0 e^{(\alpha - 1)\nu}.$$  \hfill (3.4)

Using Eqns. (2.21) and (2.19) we find

$$e^{(\alpha + 1)\nu} - \frac{\gamma(\alpha + 1)}{2} e^{-2\nu} = 1 - \frac{\gamma(\alpha + 1)}{2} - (\alpha + 1)(\gamma + 1)^2 \pi G T_0 r^2 ,$$  \hfill (3.5)

where $\gamma = \xi_0^2 / (2 \pi G T_0)$. The edge of the structure is given by $T \to 0$ which implies $e^\nu \to 0$ and $r \to \infty$. More precisely, we find

$$e^{-2\nu} = 2 \pi G T_0 \left( \frac{\gamma + 1}{\gamma} \right) r^2 + 1 - \frac{2}{\gamma(\alpha + 1)} + O(r^{-(\alpha + 1)}) ,$$  \hfill (3.6)

$$\lambda^2 = r^2 - \frac{2 + \gamma(\alpha + 1)}{(\alpha + 1)(\gamma + 1)^2 2 \pi G T_0} + O(r^{-2}) ,$$  \hfill (3.7)

$$\lambda e^{\nu} = \frac{2}{(\gamma + 1)} \left( \frac{\gamma}{2 \pi G T_0} \right)^{1/2} + O(r^{-2}) .$$  \hfill (3.8)

The expression for $\lambda$ shows that the solutions do not encounter CTCs at large $r$. In fact the metric is actually non-rotating at large $r$ since the gauge transformation

$$\theta \to \theta - \left[ \frac{\sqrt{\gamma} 2 \pi G T_0 (\gamma + 1)(\alpha + 1)}{2 + \gamma(\alpha + 1)} \right] t ,$$  \hfill (3.9)

reduces the metric to a static, cylindrical form. However, we know that this class of solutions is shear-free, so since the solution is non-rotating at large $r$ it must be non-rotating everywhere.

There is one exceptional case which avoids this rather disappointing conclusion. When $\alpha = 1$, $T = T_0$ and the edge of the structure is wherever we want it to be. We can even choose to cut the centre out of such structures as there is no tension gradient to support. The general solution is given by

$$e^{2\nu} = \frac{1}{2} \left( a - br^2 + \sqrt{4\gamma + (a - br^2)^2} \right) ,$$  \hfill (3.10)

$$e^{2\eta} = \frac{b e^{2\nu}}{2 \pi G T_0 (4 \gamma + (a - br^2)^2) .}$$  \hfill (3.11)

The constants $a$ and $b$ are determined by matching the metric functions at the inner edge of the annulus, $r = R_i$. If we choose $R_i \neq 0$ we may then match onto the fake rotation metric
and set \( B = 0 \). This was the choice made by Clément for the rotating string case. We can recover Clément’s solution in the limit of vanishing annular width and infinite density (keeping the mass finite as we take the limit).

If we choose instead to match onto “normal” Minkowski space we find \( B = -1, a = 1 - \gamma + 2\pi GT_0(1 + \gamma)^2 R_i^2 \) and \( b = 2\pi GT_0(1 + \gamma)^2 \). A tedious calculation reveals that the metric is free of CTCs and that the Euler number of the spatial manifold can never equal one (spatially compactified) unless \( \gamma = 0 \), in which case the structure is non-rotating [11].

This is most easily demonstrated if we choose \( R_i = 0 \) and \( \gamma \ll 1 \). Then the Euler number is given by

\[
E = 1 + \sqrt{1 - 2\pi GT_0 R_0^2} - \frac{\gamma}{2}(2\pi GT_0 R_0^2)^2 (1 - 2\pi GT_0 R_0^2)^{-3/2} + O(\gamma^2), \tag{3.12}
\]

and we see that \( E > 1 \) for any real value of \( R_0 \) (when \( \gamma \neq 0 \)).

Before leaving the rotating annulus or string solutions it is worth pointing out that the “rotating dust string” solution given by Clément is actually non-rotating. When the tension vanishes, \( \nu' = 0 \), which in turn demands that \( \xi_0 = 0 \) so the string is non-rotating. This can be seen directly from Clément’s solution as the exterior metric is given by Eqn.(2.9) while the interior metric is given by Eqn.(1.4). Both of these metrics actually describe static rather than stationary solutions.

We conclude our collection of illustrative examples with an example of our own from the class of solutions with \( C > 0 \). Choosing the equation of state \( \rho = 3p \) we find

\[
p = p_0 \left( 1 - \left( \frac{r}{R} \right)^2 \right)^2, \tag{3.13}
\]

\[
\xi = \sqrt{2\pi Gp_0 - 1/R^2} \left( 1 - \left( \frac{r}{R} \right)^2 \right)^2, \tag{3.14}
\]

and

\[
ds^2 = \frac{1}{1 - (r/R)^2} dt^2 + \frac{2r^2 \xi}{(1 - (r/R)^2)^3} dt d\theta - \frac{1}{(1 - (r/R)^2)^3} dr^2 - r^2 \left( \frac{1 - 2\pi Gp_0 r^2}{1 - (r/R)^2} \right) d\theta^2. \tag{3.15}
\]

As we expect from our general results, the above metric contains CTCs since reality of \( \xi \) demands \( R^2 \geq 1/(2\pi Gp_0) \) so we see that the angular coordinate becomes timelike for \( (1/2\pi Gp_0) < r^2 < R^2 \). The CTCs only disappear in the non-rotating limit, \( R^2 = 1/(2\pi Gp_0) \). A similar exact solution with \( C > 0 \) can be found for the equation of state \( \rho = 5p \). This solution also admits CTCs as expected.

IV. CONCLUSIONS

We have found that the (2+1)-dimensional analogues of rotating stars are plagued by various pathologies. The vorticity free solutions encounter infinite shear while the shear free solutions generally encounter CTCs. It would be interesting to investigate the behaviour of
solutions with both shear and vorticity to see if the combination can (miraculously) escape the pathologies of the limiting cases. We suspect that the root of the problem lies in the fact that all these solutions are spatially compactified in the static limit and this produces a barrier to rotation.

The only rotating structures to escape these pathologies are tension annuli. This is consistent with the fact that tension annuli do not necessarily cause spatial compactification in the static limit.

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[11] In our earlier treatment of the tension annulus we incorrectly stated that $E = GM = 1$. This does not have to be the case.