A TWO-PARAMETER CRITERION FOR CLASSIFYING THE EXPLODABILITY OF MASSIVE STARS BY THE NEUTRINO-DRIVEN MECHANISM

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ABSTRACT

Thus far, judging the fate of a massive star (either a neutron star [NS] or a black hole) solely by its structure prior to core collapse has been ambiguous. Our work and previous attempts find a nonmonotonic variation of successful and failed supernovae with zero-age main-sequence mass, for which no single structural parameter can serve as a good predictive measure. However, we identify two parameters computed from the pre-collapse structure of the progenitor, which in combination allow for a clear separation of exploding and nonexploding cases with only a few exceptions (~1%–2.5%) in our set of 621 investigated stellar models. One parameter is \( M_4 \), defining the normalized enclosed mass for a dimensionless entropy per nucleon of \( s = 4 \), and the other is \( \mu_4 \equiv (dm/ds)/(dr/1000 \text{ km}) \), being the normalized mass derivative at this location. The two parameters \( \mu_4 \) and \( M_4/\mu_4 \) can be directly linked to the mass-infall rate, \( \dot{M} \), of the collapsing star and the electron-type neutrino luminosity of the accreting proto-NS, \( L_\nu \propto M_4 \dot{M} \), which play a crucial role in the “critical luminosity” concept for the theoretical description of neutrino-driven explosions as runaway phenomena of the stalled accretion shock. All models were evolved employing the approach of Ugliano et al. for simulating neutrino-driven explosions in spherical symmetry. The neutrino emission of the accretion layer is approximated by a gray transport solver, while the uncertain neutrino emission of the 1.1 \( M_\odot \) proto-NS core is parameterized by an analytic model. The free parameters connected to the core-boundary prescription are calibrated to reproduce the observables of SN 1987A for five different progenitor models.

Key words: hydrodynamics – neutrinos – stars: massive – supernovae: general

1. INTRODUCTION

Presupernova (pre-SN) stars in the mass range above \( \sim 9 M_\odot \) exhibit large variations of their structure with respect to, e.g., their Fe-core and O-core masses, their binding energies, and their density or entropy profiles above the Fe core (Woosley et al. 2002). These properties vary nonmonotonically with the zero-age main-sequence (ZAMS) mass and can differ considerably even between progenitors with only a small difference of their ZAMS masses (Sukhbold & Woosley 2014). Correspondingly, Ugliano et al. (2012) found that the properties of neutrino-driven SNe like explosion energy, nickel mass, and remnant mass change nonmonotonically with the ZAMS mass. In particular, for the investigated grid of 101 solar-metallicity progenitors binned in 0.2 \( M_\odot \) steps (Woosley et al. 2002), they found islands of nonexploding, black hole (BH) forming cases down to 15 \( M_\odot \), alternating with mass intervals of exploding progenitors. In a few cases individual neighboring progenitors showed the opposite behavior.

Ugliano et al. (2012) used a simple, parametric model for the contracting proto-neutron star (PNS) as a neutrino source to trigger neutrino-driven explosions in spherically symmetric (1D) hydrodynamic simulations, but their basic findings were confirmed by other groups working with semianalytic descriptions in 1D (Pejcha & Thompson 2015) and approximate neutrino transport in two-dimensional (2D) and three-dimensional (3D) hydrodynamic models (Horiuchi et al. 2014; Nakamura et al. 2015). While O’Connor & Ott (2011) suggested that BH formation requires a compactness (normalized enclosed mass–radius ratio) of \( \xi_{2.5} > 0.45 \) with

\[
\xi_M \equiv \frac{M}{R(M)/1000 \text{ km}},
\]

Ugliano et al. (2012) obtained only explosions for \( \xi_{2.5} < 0.15 \), explosions or BH formation for \( 0.15 \leq \xi_{2.5} \leq 0.35 \), and only BH formation for \( \xi_{2.5} > 0.35 \), which implies a larger fraction of BH-formation cases for solar-metallicity stars. Horiuchi et al. (2014) pointed out that a critical compactness of \( \xi_{2.5} \geq 0.2 \) for failed explosions is compatible with a lack of red supergiant SN IIP progenitors above \( \sim 16 M_\odot \) (Smartt et al. 2009) and with a significant excess of the star formation rate compared to the observed SN rate (Horiuchi et al. 2011). Pejcha & Thompson (2015) showed that their parameterization “case (a),” which yields results similar to those of Ugliano et al. (2012), is close to being optimally compatible with a combination of several observational constraints.

How can the nonmonotonocities of the explodability be understood in terms of the pre-SN properties and in the context of the physics of the neutrino-driven mechanism? Are there characteristic parameters of the pre-SN star that decide better about success or failure of the explosion than a single value of the compactness or other, similarly useful parameters like the iron-core mass or the binding energy outside of the iron core? While all these measures reflect trends like an enhanced tendency of BH formation for high compactness, large iron-core mass, or high exterior binding energy, there are still many outliers that do not obey the correlations. For example, a
suitably chosen mass $M$ of the compactness $\xi_M$ allows us to correctly predict explosions in $\lesssim$90% of the cases (Pejcha & Thompson 2015), but the best choice of $M$ is merely empirical and the physical justification of $\xi_M$ as a good diagnostic is unclear.

Here we propose a two-parameter criterion that separates successful explosions from failures with very high reliability. While two compactness values, e.g., $\xi_{1.5}$ and $\xi_{2.5}$, or the iron-core mass and the mean entropy in some suitable mass range begin to show such a disentanglement, we demonstrate that the normalized mass inside a dimensionless entropy per nucleon of $s = 4$,

$$M_s = m(s = 4)/M_\odot, \quad (2)$$

and the mass derivative at this location,

$$\mu_s = \left. \frac{dm/M_\odot}{dr/1000 \text{ km}} \right|_{s=4}, \quad (3)$$

both determined from the pre-SN profiles, allow us to predict the explosion behavior successfully in $\gtrsim$97% of all cases and have a direct connection with the theoretical basis of the neutrino-driven mechanism.

We briefly describe our numerical approach in Section 2, including a detailed discussion of our modeling methodology in comparison to other approaches in the recent literature, present our results in Section 3, and conclude in Section 4.

2. NUMERICAL SETUP AND PROGENITOR MODELS

2.1. Modeling Approach

Our basic modeling approach follows Ugliano et al. (2012) with a number of improvements. To trigger neutrino-driven explosions in spherically symmetric (1D) hydrodynamic simulations, we use a schematic model of the high-density core of the PNS as the neutrino source (for details, see Ugliano et al. 2012). This analytic description is applied to the innermost $1.1 M_\odot$, which are excised from the computational domain, and it yields time-dependent neutrino luminosities that are imposed as boundary values at the contracting, Lagrangian inner grid boundary. On the numerical grid, where neutrino optical depths increase from initially $\sim$10 to finally several thousand, neutrino transport is approximated by the gray treatment described in Scheck et al. (2006) and Arcones et al. (2007). This allows us to account for the progenitor-dependent variations of the accretion luminosity.

Our approach replaces still uncertain physics connected to the equation of state (EOS) and neutrino opacities at high densities by a simple, computationally efficient PNS core model. The associated free parameters are calibrated by reproducing observational properties of SN 1987A. We emphasize that the neutrino emission is sensitive to the time- and progenitor-dependent mass accretion rate. Not only the accretion luminosity increases for progenitors with higher mass accretion rate of the PNS, but also the neutrino loss of the inner core rises with the accreted mass because of compressional work of the accretion layer on the core. Such dependences are accounted for in our modeling of neutron star (NS) core and accretion.

Our numerical realization improves the treatment by Ugliano et al. (2012) in several aspects. We use the high-density EOS of Lattimer & Swesty (1991) with a compressibility of $K = 220 \text{ MeV}$ and below $\rho = 10^{11} \text{ g cm}^{-3}$ apply an $e^\pm$, photon, and baryon EOS (Timmes & Swesty 2000) for nuclear statistical equilibrium (NSE; K. Kifonidis 2004, private communication) with 16 nuclei for $T > 7 \times 10^9 \text{ K}$ and a 14-species alpha network (including an additional neutron-rich tracer nucleus of iron-group material) at lower temperatures (Müller 1986). The tracer nucleus is assumed to be formed in ejecta with $Y_e < 0.49$ and thus tracks the ejection of matter with neutron excess, when detailed nucleosynthesis calculations predict little production of $^{56}\text{Ni}$ (Thielemann et al. 1996).

The network is consistently coupled to the hydrodynamic modeling and allows us to include the contribution from explosive nuclear burning to the energetics of the SN explosions. The collapse phase until core bounce is modeled with the deleptonization scheme proposed by Liebendörfer (2005), using the $Y_e(\rho)$ trajectory of Figure 1 for the evolution of the electron fraction $Y_e$ as a function of density $\rho$ (B. Müller 2013, private communication). This yields good overall agreement with full neutrino transport results and allows for a very efficient computation of large sets of post-bounce models.

2.2. Progenitor Models

We perform collapse and explosion simulations for large progenitor sets of different metallicities, namely: the zero-metallicity z2002 set (30 models with ZAMS masses of 11.0–40.0 $M_\odot$), the low-metallicity ($10^{-4}$ solar) u2002 series (247 models, 11.0–75.0 $M_\odot$), and the solar-metallicity s2002 series (101 models, 10.8–75.0 $M_\odot$) of Woosley et al. (2002) plus a 10.0 $M_\odot$ progenitor (S. E. Woosley 2007, private communication) and a 10.2 $M_\odot$ progenitor (A. Heger 2003, private communication); the solar-metallicity s2014 (151 models, 15.0–30.0 $M_\odot$) and sh2014 series (15 models, 30.0–60.0 $M_\odot$, no mass loss) of Sukhbold & Woosley (2014), supplemented by an additional 36 models with 9.0–14.9 $M_\odot$; the solar-metallicity s2007 series (32 models, 12.0–120.0 $M_\odot$) of S. E. Woosley et al. (2007, private communication); and the n2006 series (8 models, 13.0–50.0 $M_\odot$; Nomoto et al. 2006).

For the core-model parameter calibration we choose five different progenitors, namely, the (red supergiant) model s19.8 of the s2002 series as in Ugliano et al. (2012), and four blue supergiant pre-SN models of SN 1987A: w15.0 (ZAMS mass
of $15 \, M_\odot$; Woosley et al. 1988), w18.0 (18 $M_\odot$, evolved with rotation; S. E. Woosley et al. 2007, private communication), w20.0 (20 $M_\odot$; Woosley et al. 1997), and n20.0 (20 $M_\odot$; Shigeyama & Nomoto 1990). Compactness values and explosion and remnant parameters of these models are listed in Table 1.

The calibration aims at producing the explosion energy and ejected $^{56}$Ni mass of SN 1987A compatible with observations, for which the best values are $E_{\text{exp}} = (1.50 \pm 0.12) \times 10^{51}$ erg (Utrobin 2005), $E_{\text{exp}} \sim 1.3 \times 10^{51}$ erg (Utrobin & Chugai 2011), and $M_{\text{Ni}} = 0.0723 \pm 0.0772$ $M_\odot$ (Utrobin et al. 2014), but numbers reported by other authors cover a considerable range (see Handy et al. 2014 for a compilation). The explosion energy that we accept for an SN 1987A model in the calibration process is guided by the ejected $^{56}$Ni mass (which fully accounts for short-time and long-time fallback) and a ratio of $E_{\text{exp}}$ to ejecta mass in the ballpark of estimates based on light-curve analyses (see Table 1 for our values).

Because of the “gentle” acceleration of the SN shock by the neutrino-driven mechanism (also in 3D simulations; see Utrobin et al. 2014), it is difficult to produce this amount of ejected $^{56}$Ni just by shock-induced explosive burning. $M_{\text{Ni}}$ in Table 1 mainly measures this component but also contains $^{56}$Ni from proton-rich neutrino-processed ejecta. However, also neutrino-processed ejecta and the neutrino-driven wind with a slight neutron excess could contribute significantly to the $^{56}$Ni production. The electron fraction $Y_e$ of these ejecta is set by $\nu_e$ and $\bar{\nu}_e$ interactions and depends extremely sensitively on the properties (luminosities and spectra) of the emitted neutrinos, which our transport approximation cannot reliably predict and which also depend on subtle effects connected to multidimensional physics and neutrino opacities. For these reasons we consider the $^{56}$Ni as uncertain within the limits set by the true $^{56}$Ni yield from our network on the low side and, in the maximal case, all tracer material added to that. We therefore provide as possible $^{56}$Ni production of our models the range $M_{\text{Ni}} \leq M_{\text{Ni,cal}} \leq M_{\text{Ni}} + M_{\text{tracer}}$.

Different from Ugliano et al. (2012), we reduce the compression parameter of the NS core model by a linear relation $\zeta' \propto \xi_{1.75,b}$ (the value of $\xi_{1.75,b}$ is measured at core bounce) for progenitors with $\leq 13.5 \, M_\odot$, i.e., we use the function

$$\zeta' = \zeta \left( \frac{\xi_{1.75,b}}{0.5} \right) \text{ for } M \leq 13.5 \, M_\odot,$$

with $\zeta$ being the value determined from the SN 1987A calibration for a considered progenitor model of that SN. We note that the values of $\xi_{1.75,b}$ are less than 0.5 for all progenitors below $13.5 \, M_\odot$ and close to 0.5 for $M \sim 13.5 \, M_\odot$, for which reason Equation (4) connects smoothly to the $\zeta$ value applied for stars above $13.5 \, M_\odot$ according to the SN 1987A calibration.

The modification of Equation (4) accounts for the reduced burden of the small mass of the accretion layer of these stars with their extremely low compactnesses. Such a modification allows us to reproduce the trend to weak explosions obtained in sophisticated 2D and 3D simulations for low-mass iron-core progenitors (Janka et al. 2012; Melson et al. 2015b; Müller 2015). We point out that the Crab supernova SN 1054 is considered to be connected to the explosion of a $\sim 10 \, M_\odot$ star (e.g., Nomoto et al. 1982; Smith 2013), and its explosion energy is estimated to be up to only $\sim 10^{50}$ erg (e.g., Yang & Chevalier 2015). This fact lends support to the results of recent, self-consistent 1D and multidimensional SN models of $\lesssim 10 \, M_\odot$ stars (e.g., Kitaura et al. 2006a; Fischer et al. 2010; Melson et al. 2015b), whose low explosion energies and low nickel production agree with the Crab observations.

Although as a consequence of our $\zeta$ reduction the explosion times, $t_{\text{exp}}$, tend to be late for stars in the 10.5–12.5 $M_\odot$ range (Figure 3), this behavior also seems to be compatible with self-consistent, multidimensional simulations of stellar explosions in the 11–12 $M_\odot$ range by Müller et al. (2013), Müller (2015), and Janka et al. (2012), where these stars were found to have a long-lasting phase of accretion and simultaneous mass outflow after a quite inert onset of the explosion. A more detailed discussion and justification of our modified treatment of low-ZAMS mass cases will be provided in Section 2.3.4 and can also be found in a follow-up paper by Sukhbold et al. (2015),
where the values of all PNS core-model parameters are tabulated for all calibrations. It must be emphasized that in the context of the present work the detailed treatment of the low-mass stars is not overly important. These stars usually explode fairly easily, independent of the treatment of the PNS core model with the original or with our revised calibration. Therefore, these stars lie far away from the boundary curve that separates exploding from nonexplosions and whose determination will be our main goal in Section 3. For this reason exactly the same separation line is obtained when the core-model parameter values from the SN 1987A calibration are applied to all stars.

In Table 1 and the rest of our paper, time-dependent structural parameters of the stars (like compactness values, \(\mu_d\)) and the iron-core mass \(M_{Fe}\) are measured when the stars possess a central density of \(5 \times 10^{10} \text{g cm}^{-3}\), unless otherwise stated. This choice of reference density defines a clear standard for the comparison of stellar profiles of different progenitors (see Appendix A of Buras et al. 2006). Different from the moment of core bounce, which was used in other works, our reference density has the advantage of being still close to the initial state of the pre-collapse models provided by stellar evolution modeling and therefore to yield values of the structural parameters that are more similar to those of the pre-collapse progenitor data. Our calibration model w15.0, however, must be treated as an exception. Because pre-collapse profiles of this model are not available any more, all structural quantities for this case are given (roughly) at core bounce.

2.3. Methodology and Theory of Neutrino-driven Explosions

2.3.1. Status of “Ab Initio” SN Modeling

“Ab initio,” fully self-consistent simulations of stellar core collapse with state-of-the-art treatment of microphysics and neutrino transport do not lead to explosions in spherical symmetry except for stars with O–Ne–Mg and Fe cores near the low-mass end of SN progenitors (Kitaura et al. 2006; Janka et al. 2008, 2012; Fischer et al. 2010; Melson et al. 2015b). Two-dimensional simulations in the recent past have produced successful explosions and underline the fundamental importance of multidimensional effects, but the true meaning of these results with respect to the neutrino-driven mechanism is not finally clear, and the current situation is diffuse and contradictory.

On the one hand, some of the 2D explosions set in relatively late and might remain on the weak side (e.g., Marek & Janka 2009; Müller et al. 2012a, 2012b, 2013; Suwa 2012; Hanke 2014; Müller & Janka 2014; Suwa et al. 2014; Müller 2015), although such apprehension is speculative because not all simulations could be continued until the explosion energy had saturated (Müller 2015). On the other hand, the Oak Ridge group obtained explosions much earlier after bounce, with shock evolutions being astonishingly similar for 12, 15, 20, and 25 \(M_\odot\) stars and explosion energies fairly compatible with observations (Bruenn et al. 2013, 2014). In contrast, Dolence et al. (2015) did not find any successes in 2D simulations of the same progenitors, but they used a different treatment of gravity, hydrodynamics, EOS, neutrino transport, and neutrino opacities. The exact reasons for the different findings will have to be clarified by detailed tests and comparisons. The situation is even more diffuse because current 3D simulations agree in showing slower explosions compared to 2D calculations or even no explosions (e.g., Hanke et al. 2012, 2013; Couch 2013; Couch & O’Connor 2014; Takiwaki et al. 2014; Tamborra et al. 2014; Mezzacappa et al. 2015), although some studies have proclaimed the opposite behavior (Nordhaus et al. 2010; Burrows 2013; Dolence et al. 2013). So far only a few recent 3D calculations with highly refined neutrino treatment have obtained successful shock revival by the neutrino-driven mechanism (Lentz et al. 2015; Melson et al. 2015a, 2015b). Interestingly, the 3D simulation of a low-mass \((9.6 M_\odot)\) progenitor with detailed neutrino physics, whose explosion energy approached its saturation level, was found to explode more energetically in 3D than in 2D (Melson et al. 2015b). This result is in line with a 2D–3D comparison in the 11–12 \(M_\odot\) range conducted by Müller (2015). In both studies accretion downflows and the re-ejection of neutrino-heated matter were observed to be different in 2D and 3D because of geometry-dependent differences of the Kelvin–Helmholtz instability and flow fragmentation. The 3D models therefore suggest that explosions in 2D are massively affected by the assumption of rotational symmetry around the polar grid axis and by an inverse turbulent energy cascade, which tends to amplify energy on the largest possible scales (Hanke et al. 2012) and also produces numerical artifacts in the post-explosion accretion phase of the neutron star (Müller 2015). It must therefore be suspected that the early onset of explosions and the extremely unipolar or bipolar deformations along the symmetry axis obtained in many 2D models could be artifacts of the imposed symmetry constraints.

2.3.2. Modeling Recipes in Recent Literature

Before 3D modeling will have become a routine task and results will have converged, neutrino-driven explosions of large sets of progenitor stars can be explored for their observational implications only by referring to simplified modeling approaches. Several different recipes have been introduced for this recently. Ugliano et al. (2012) used an analytic PNS core-cooling model in connection with a neutrino transport approximation in 1D hydrodynamic explosion simulations (as briefly summarized in Section 2), thus improving the simpler, time-dependent boundary neutrino luminosity prescribed by previous users of the simulation code (Scheck et al. 2004, 2006, 2008; Kifonidis et al. 2006; Arcones et al. 2007; Arcones & Janka 2011) and the even simpler neutrino lightbulb treatment (without any transport approximation) applied by Janka & Müller (1996) and Kifonidis et al. (2003). O’Connor & Ott (2011) resorted to a scaling parameter \(f_{\text{heat}}\) to artificially enhance the neutrino heating by charged-current processes behind the stalled shock in 1D hydrodynamic models with approximate neutrino treatment. Nakamura et al. (2015) performed an extensive set of 2D simulations with simplified neutrino transport despite the grains of salt mentioned in Section 2.3.1 (see also Horiiuchi et al. 2014, for cautioning against the 2D results). Pejcha & Thompson (2015) applied a semianalytic model to determine the onset times of the explosions, using neutrino luminosities from 1D calculations of accreting PNSs, and estimated explosion properties by analytic arguments. Suwa et al. (2014) also performed 2D simulations and suggested analytic approximations for describing diffusion and accretion components of the neutrino luminosities from PNSs and a free-fall treatment for the collapse of the overlying stellar layers. Perego et al. (2015) invented a method they named “PUSH,” which they applied to
trigger explosions artificially in their general relativistic, 1D hydrodynamic core-collapse and PNS formation modeling with sophisticated neutrino transport. PUSH gradually switches on and off additional neutrino heating of chosen strength during a chosen period of time. This procedure is assumed to mimic the effects of multidimensional hydrodynamics in the postshock region. The extra heating is coupled to the heavy-lepton neutrino emission from the PNS.

All of these recipes contain larger sets of parameters and degrees of freedom, which are either varied in exploring different cases (e.g., Pejcha & Thompson 2015) or adjusted by comparison to more complete models (e.g., Suwa et al. 2014) or by reproducing observational benchmarks like those set by SN 1987A (Ugliano et al. 2012; Perego et al. 2015). The models of Ugliano et al. (2012) and those in the present paper assume that the explosion trigger is tightly coupled to the physics that reflects the main differences between different progenitor stars, namely, to the post-bounce accretion history of the collapsing stellar core and the corresponding accretion luminosities of $\nu_e$ and $\bar{\nu}_e$. It will have to be seen whether this important aspect of the models remains being supported by future developments toward a more complete understanding of the physics of the central engine that powers the explosion in the context of the neutrino-driven mechanism.

2.3.3. Motivation of Modeling Assumptions of This Work

The analytic NS-core model introduced by Ugliano et al. (2012) in combination with their approximate transport solver for treating the accretion component of the neutrino luminosity, as well as neutrino cooling and heating between the PNS and shock, is an attempt to realize the tight coupling of accretion behavior and explodability in close similarity to what is found in current 2D simulations (e.g., those of Marek & Janka 2009; Müller et al. 2012a, 2012b, 2013; Müller & Janka 2014). Since 1D models with elaborate neutrino physics and a fully self-consistent calculation of PNS cooling miss the critical condition for explosions by far, it is not the goal of Ugliano et al. (2012) to closely reproduce the neutrino emission properties of such more sophisticated calculations. Rather, it is the goal to approximate the combined effects of neutrino heating and multidimensional postshock hydrodynamics by a simple and computationally efficient neutrino-source model, which allows for the fast processing of large progenitor sets including the long-time evolution of the SN explosion to determine also the shock breakout and fallback evolution.

Free parameters in the NS core model and the prescribed contraction behavior of the inner grid boundary are calibrated by matching basic observational features (explosion energy, $^{56}$Ni yield, total release of neutrino energy) of SN 1987A. This is intended to ensure that the overall properties of the neutrino-source model are anchored on empirical ground. Of course, the setting of the parameter values cannot be unambiguous when only a few elements of a single observed SN are used for deriving constraints. However, the approximate nature of the neutrino source treatment as a whole does not require the perfectly accurate description of each individual model component in order to still contain the essence of the physics of the system like important feedback effects between accretion and outflows and neutrinos, which govern the progenitor-dependent variations of explodability and SN properties. A reasonable interplay of the different components is more relevant than a most sophisticated representation of any single aspect of the neutrino-source model.

In detail, the basic features of our 1D realization of the neutrino-driven mechanism along the lines of Ugliano et al. (2012) are the following.

1. The possibility of an explosion is coupled closely to the progenitor-dependent strength and evolution of the post-bounce accretion. This is achieved not only by taking into account the accretion luminosity through the approximate neutrino transport scheme but also through the response of the PNS core to the presence of a hot accretion mantle. The evolution of the latter is explicitly followed in our hydrodynamic simulations, which track the accumulation of the collapsing stellar matter around the inner PNS core. The existence of the mantle layer enters the analytic core model in terms of the parameter $m_{\text{acc}}$ for the mass of this layer and the corresponding accretion rate $\dot{m}_{\text{acc}}$.

2. The inner 1.1 $M_\odot$ core of the PNS is cut out and replaced by contracting inner grid boundary and a corresponding boundary condition in our model. This inner core is considered to be the supranuclear high-density region of the nascent NS, whose detailed physics is still subject to considerable uncertainties. This region is replaced by an analytic description, whose parameters $\Gamma$, $R_c(t)$, and $n$ (see Ugliano et al. 2012) are set to the same values for all stars. This makes sense because the supranuclear phase is highly incompressible, for which reason it can be expected that the volume of the core is not largely different during the explosion phase for different PNS masses. Moreover, the neutrino diffusion timescale out of this core is seconds, which implies that its neutrino emission is of secondary importance during the shorter post-bounce phase when the explosion develops. Despite its simplicity, our core treatment still includes progenitor- and accretion-dependent variations through the mass $m_{\text{acc}}$ of the hot accretion mantle of the PNS and the mass accretion rate $\dot{m}_{\text{acc}}$, whose influence on the inner core is accounted for in Equations (1)–(4) of Ugliano et al. (2012) for describing the energy evolution of the core model.

3. The onset of the explosion is considerably delayed (typically between several hundred milliseconds and about a second) with a slow (instead of abrupt) rise of the explosion energy during the subsequent shock acceleration phase, when an intense neutrino-driven wind ejects matter and delivers power to the explosion. Neutrino heating cannot deposit the explosion energy impulsively, because the ejected matter needs to absorb enough energy from neutrinos to be accelerated outward. The rate of energy input to the explosion is therefore limited by the rate at which matter can be channeled through the heating region. A long-lasting period (hundreds of milliseconds to more than a second) of increasing energy is characteristic of neutrino-driven explosions (Figure 2) and is observed as gradual growth of the explosion energy also in 2D explosion models, e.g., by Scheck et al. (2006) and Bruenn et al. (2014). To achieve this behavior in our 1D models, the core-neutrino source needs to keep up high neutrino luminosities for a more extended period of time than found in fully self-consistent SN simulations, where the
The rapid decline of the mass accretion rate at the surface of the iron core and at the interface of silicon and silicon-enriched oxygen layers leads to a strong decrease of the accretion luminosity. The longer period of high neutrino emission is compensated by a somewhat underestimated early post-bounce neutrino luminosity (Figure 2) in order to satisfy the energy constraints set by the total gravitational binding energy of the forming NS.

Figure 2. Models s14.0 (left), s21.0 (middle), and s27.0 (right) of the s2014 progenitor series as exemplary cases of successful explosions with the w18.0 calibration. The top panels display as functions of post-bounce time the radius of the outgoing shock (black line), the mass accretion rate measured at 500 km (blue line; scale on the right side), and the radii of iron core (orange), \( M_4 = m(s = 4) \) (red), \( M_4 + 0.3 \) (red dashed), and trajectory of the final mass cut (after completion of fallback; purple). The second panels from top show the time evolution of the luminosities of \( \nu_e \), \( \nu_x \), and a single species of heavy-lepton neutrinos \( \nu_x \) as labeled in the plot, measured at 500 km (solid lines) and at the inner grid boundary (dashed lines). The third panels from top show the mean energies of all neutrino kinds as radiated at 500 km. The vertical dotted lines indicate the onset time of the explosion as the moment when the outgoing shock passes the radius of 500 km. The bottom panels provide the time evolution of the diagnostic energy of the explosion (integrated energy of all postshock zones with positive total energy; blue line). Also shown are the kinetic energy (red), gravitational energy (black), and internal energy (orange) as integrals over the whole, final SN ejecta between the final mass cut (after fallback) on the one side and the stellar surface on the other. The total (binding) energy (purple) as the sum of these energies ultimately converges to the diagnostic energy, and both of these energies asymptote to the final explosion energy. While this convergence is essentially reached after \( \sim 4 \) s in the case of s14.0, the expansion of shocked matter in the s21.0 model and thus the energy evolution is slowed down at \( \sim 2.7 \) s by the high densities in the stellar core. The s27.0 model becomes gravitationally unbound (i.e., the total binding energy becomes positive) even more slowly because of the very massive stellar core. The convergence of total energy and diagnostic energy takes tens of seconds in this case.
4. The timescale and duration of the growth of the explosion energy in multidimensional models of neutrino-driven SNe are connected to an extended period of continued accretion and simultaneous shock expansion that follows after the revival of the stalled shock (see Marek & Janka 2009). Persistent accretion thereby ensures the maintenance of a significant accretion luminosity, while partial re-ejection of accreted and neutrino-heated matter boosts the explosion energy. In our 1D simulations the physics of such a two-component flow cannot be accurately accounted for. In order to approximate the consequences of this truly multidimensional phase, our 1D models are constructed with two important properties: On the one hand, they refer to a high level of the PNS-core luminosity for about 1 s. On the other hand, they are set up to possess a more extended accretion phase that precedes the onset of the delayed explosion before the intense neutrino-driven wind pumps energy into the explosion. The power and mass loss in this wind are overestimated compared to sophisticated neutrino-cooling simulations of PNSs. However, this overestimation of the wind strength has its justification: The early wind is supposed to mimic the mass ejection that is fed in the multidimensional case by the inflow and partial re-ejection of matter falling toward the gain radius during the episode of simultaneous accretion and shock expansion. The enhanced wind mass counterbalances the extra mass accretion by the PNS during the long phase before the shock acceleration is launched, and this enhanced wind mass is of crucial importance to carry the energy of the neutrino-powered blast.

Figure 2 shows the post-bounce evolution of the stalled SN shock, the onset of the explosion, energy evolution, and the time evolution of the neutrino emission properties for three representative progenitors, namely, s14, s21, and s27 of the s2014 series, which explode successfully with the w18.0 calibration. The shock stagnation at a radius of approximately 200 km lasts between ~700 and 900 ms and can exhibit the well-known oscillatory expansion and contraction phases, which signal proximity to the explosion (see, e.g., Buras et al. 2006; Murphy & Burrows 2008; Fernández 2012). The explosion sets in shortly after $M_d$ (Equation (2)) has fallen through the shock and well before the mass shell corresponding to $M_d + 0.3$ has collapsed. High neutrino luminosities are maintained by high mass accretion rates and, after the onset of the explosion, by a contribution from the core emission (dashed lines in the luminosity panel of Figure 2) that grows until roughly 1 s. The current models underestimate the surface luminosity of heavy-lepton neutrinos compared to more sophisticated simulations because of the chosen modest contraction of the inner boundary of the computational grid (which leads to underestimated temperatures in the accretion layer of the PNS) and because neutrino-pair production by nucleon-nucleon bremsstrahlung is not taken into account. We did not upgrade our treatment in this respect because $\nu_e$ and $\bar{\nu}_e$ are not of immediate relevance for our study since the explosion hinges exclusively on the heating by $\nu_{\mu}$ and $\nu_{\tau}$.

Replacing the inner core of the PNS by a contracting inner boundary of the computational mesh introduces a number of free parameters, whose settings allow one to achieve the desired accretion and neutrino-emission behavior as detailed above. On the one hand, our model contains parameters for the prescription of the contraction of the grid boundary; on the other hand, there are parameters for the simple high-density core model (see Ugliano et al. 2012 and references therein). While the core-model parameters ($\Gamma$, $\zeta$, $R_f(t)$, $n$) regulate the neutrino-emission evolution of the excised, high-density core of the PNS, the prescribed grid-boundary radius, $R_b(t)$, governs the settling of the hot accretion mantle of the PNS. Because of partially compensating influences and dependencies, not all of these parameters have a sensitive impact on the outcome of our study. Again, more relevant than a highly accurate description of individual components of the modeling is a reasonable reproduction of the overall properties of the accretion and neutrino emission history of the stalled SN shock and mass-accumulating PNS. For example, the moderate increase of the mean neutrino energies with time and their regular hierarchy $\langle e_\nu \rangle \leq \langle e_\bar{\nu} \rangle \leq \langle e_\nu \rangle$; Figure 2) are not compatible with the most sophisticated current models (see, e.g., Marek & Janka 2009; Müller & Janka 2014). They reflect our choice of a less extreme contraction of the $1.1 M_\odot$ shell than found in simulations with soft nuclear EOSs for the core matter, where the PNS contracts more strongly and its accretion mantle heats up to higher temperatures at later post-bounce times (compare Scheck et al. 2006 and see the discussion by Pejcha & Thompson 2015). Our choice is motivated solely by numerical reasons (because of less stringent time-step constraints), but it has no immediate drawbacks for our systematic exploration of explosion conditions in large progenitor sets. Since neutrino-energy deposition depends on $L_\nu/\langle e_\nu^2 \rangle$, the underestimated mean neutrino energies at late times can be compensated by higher neutrino luminosities $L_\nu$ of the PNS core.

The neutrino emission from the PNS-core region is parameterized in accordance with basic physics constraints. This means that the total loss of electron-lepton number is compatible with the typical neutronization of the inner $1.1 M_\odot$ core, whose release of gravitational binding energy satisfies energy conservation and the virial theorem (see Ugliano et al. 2012). Correspondingly chosen boundary luminosities therefore ensure a basically realistic deleptonization and cooling evolution of the PNS as a whole and of the accretion mantle in particular, where much of the inner-boundary fluxes are absorbed and reprocessed. Again, a proper representation of progenitor-dependent variations requires a reasonable description of the overall system behavior but does not need a very high sophistication of all individual components of the system.

2.3.4. Calibration for Low-ZAMS Mass Range

Agreement with the constraints from SN 1987A employed in our work (i.e., the observed explosion energy, $^{56}$Ni mass, total neutrino energy loss, and the duration of the neutrino signal) can be achieved with different sets of values of the PNS core-model parameters. Using only one observed SN case, the parameter set is underconstrained and the choice of suitable values is ambiguous. It is therefore not guaranteed that the calibration works equally well in the whole mass range of investigated stellar models.

In particular, stars in the low-ZAMS mass regime ($M_{\text{ZAMS}} \lesssim 12\ldots13 M_\odot$) possess properties that are distinctly different from those of the adopted SN 1987A progenitors and in the mass neighborhood of these progenitors. Stars with $M_{\text{ZAMS}} \lesssim 12\ldots13 M_\odot$ are characterized by very small values of compactness (Equation (1)), binding energy outside of the iron
core and outside of \( M_4 \) (Equation (2)), and mass derivative \( \mu_4 \) (Equation (3)). The progenitor of SN 1054 giving birth to the Crab remnant is considered to belong to this mass range, more specifically to have been a star with mass around 10\( M_\odot \) (Nomoto et al. 1982; Smith 2013; Tominaga et al. 2013). Because of their structural similarities and distinctive differences compared to more massive stars, Sukhbold et al. (2015) call progenitors below roughly 12\( -13 M_\odot \) “Crab-like,” in contrast to stars above this mass limit, which they term “SN 1987-like.”

Stars below \(~10 M_\odot\) were found to explode easily in self-consistent, sophisticated 1D, 2D, and 3D simulations (Kitaura et al. 2006; Janka et al. 2008; Fischer et al. 2010; Wanao et al. 2011; Janka et al. 2012; Melson et al. 2015b) with low energies (less than or around 10\(^5\) erg = 0.1 B) and little nickel production (<0.01\( M_\odot \)), in agreement with observational properties concluded from detailed analyses of the Crab remnant (e.g., Yang & Chevalier 2015). We therefore consider the results of these state-of-the-art SN models together with the empirical constraints for the Crab SN as important benchmarks that should be reproduced by our approximate 1D modeling of neutrino-powered explosions.

The results of Ugliano et al. (2012) revealed a problem in this respect, because they showed far more energetic explosions of stars in the low-mass domain than expected on grounds of the sophisticated simulations and from observations of Crab. Obviously, the neutrino-source calibration used by Ugliano et al. (2012) is not appropriate to reproduce “realistic” explosion conditions in stars with very dilute shells around the iron core. Instead, it leads to an overestimated power of the neutrino-driven wind and therefore overestimated explosion energies. In particular, the strong and energetic wind is in conflict with the short period of simultaneous postshock accretion and mass ejection after the onset of the explosion in \(<10 M_\odot\) stars. Since the mass accretion rate is low and the duration of the accretion phase is limited by the fast shock expansion, the energetic importance of this phase is diminished by the small mass that is channeled through the neutrino-heating layer in convective flows (Kitaura et al. 2006; Janka et al. 2008; Wanao et al. 2011; Janka et al. 2012; Melson et al. 2015b). In order to account for these features found in the most refined simulations of low-mass stellar explosions, the neutrino-driven wind power of our parametric models has to be reduced.

We realize such a reduction of the wind power by decreasing the parameter \( \zeta \), which scales the compression work exerted on the inner (excised) core of the PNS by the overlying accretion mantle (see Equations (1)–(4) in Ugliano et al. 2012), in proportionality to the compactness parameter \( \xi_{1.75, P} \), which drops strongly for low-mass progenitors (see Equation (4)). This procedure can be justified by the much lighter accretion layers of such stars, which implies less compression of the PNS core by the outer weight. Such a modification reduces the neutrino emission of the high-density core and therefore the mass outflow in the early neutrino-driven wind. As a consequence, the explosion energy falls off toward the low-mass end of the investigated progenitor sets. This can be seen in the top panel of Figure 3, which should be compared to the top left panel of Figure 5 in Ugliano et al. (2012).

The \( \zeta \) scaling of Equation (4) is introduced as a quick fix in the course of this work and is a fairly ad hoc measure to cure the problem of overestimated explosion energies for low-mass SN progenitors. In Sukhbold et al. (2015) a different approach is taken, in which the final value of the core-radius parameter of the one-zone model describing the supranuclear PNS interior as the neutrino source (see Ugliano et al. 2012) is modified. This procedure can directly be motivated by the contraction of the PNS found in self-consistent cooling simulations with microphysical high-density EOSs and detailed neutrino transport. Mathematically, the modification of the core radius has a similar effect on the core-neutrino emission to the \( \zeta \) scaling employed here. While we refer the reader to Sukhbold et al. (2015) for details, we emphasize that the consequences for the overall explosion behavior of the low-mass progenitors are very similar for both the \( \zeta \) reduction and the core-radius adjustment applied by Sukhbold et al. (2015). They lead to considerably lower explosion energies for \( M_{ZAMS} \lesssim 12 M_\odot \) stars and a further drop of the explosion energies below \(~9.5 M_\odot\).

As a drawback of this modification, the explosions of some of the low-mass progenitors between \(~10.5 \) and \(~12.5 M_\odot\) set in rather late (>1 s post-bounce; see Figure 3).\(^5\) This, however, is basically compatible with the tendency of relatively slow shock expansion and late explosions that are also found in sophisticated multidimensional simulations of such stars, which, in addition, reveal long-lasting phases of simultaneous accretion and mass ejection after the onset of the explosion (Müller et al. 2013; Müller 2015). This extended accretion phase has only moderate consequences for the estimated remnant masses because the mass accretion rate of these progenitors reaches a low level of \(<\lesssim 0.05 - 0.1 M_\odot s^{-1}\) after a few hundred milliseconds post-bounce, and some or even most of the accreted mass is re-ejected in the neutrino-driven wind.

Besides providing information on the explosion energies and the onset times of the explosion (defined by the time the outgoing shock reaches 500 km), Figure 3 also displays for exploding models the ejected masses of \(^{56}\)Ni and the iron-group tracer element, the baryonic and gravitational remnant masses, the fallback masses, and the total energies radiated by neutrinos. Overall, these results exhibit features very similar to those discussed in detail by Ugliano et al. (2012) for a different progenitor series and a different calibration model. We point out that the fallback masses in the low-mass range of progenitors were overestimated by Ugliano et al. (2012) owing to an error in the analysis (more discussion will follow in Section 3.5; an erratum on this aspect is in preparation).

3. RESULTS

3.1. One- and Two-parameter Classifications

Figure 4 shows \( \xi_{2.5} \) versus ZAMS mass with BH-formation cases indicated by gray and explosions by red bars for the s2002 and s2014 series and all calibrations. The irregular pattern found by Ugliano et al. (2012) for the s2002 progenitors is reproduced and appears similarly in the s2014 set. High compactness \( \xi_{2.5} \) exhibits a tendency to correlate with BHs. But also other parameters reflect this trend, for example, \( \xi_{1.5} \), the iron-core mass \( M_{Fe} \) (defined as the core where \( \sum_{A \geq 46} X_A > 0.5 \) for nuclei with mass numbers \( A \) and mass fractions \( X_A \)), and the enclosed mass at the bottom of the O-burning shell. All three of them are tightly correlated; see Figure 5 as well as Figure 4 of Ugliano et al. (2012). Also, high

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\(^5\) In extremely rare cases one may even obtain failed explosions.
values of the binding energy $E_b(m > M_{Fe})$ outside of $M_{Fe}$ signal a tendency for BH formation, because this energy correlates with $\zeta_{2.5}$ (see Figure 4 in Ugliano et al. 2012). However, for none of these single parameters does a sharp
A boundary value exists that discriminates between explosions and nonexplosions. For all such choices of a parameter, the BH formation limit tends to vary (nonmonotonically) with $M_{\text{ZAMS}}$, and in a broad interval of values either explosion or BH formation can happen. Pejcha & Thompson (2015) tried to optimize the choice of $M$ for $\xi_{1.5}$, but even their best case achieved only 88% correct predictions. Since in the cases of $\xi_{2.5}$ and $\xi_{2.5}$ or $E_{\text{He}}$ (or $E_{\text{He}}$), for example, the threshold value for BH formation tends to grow with higher ZAMS mass, one may hypothesize that a second parameter could improve the predictions.

Placing the progenitors in a two-parameter space spanned by $\xi_{1.5}$ and $\xi_{2.5}$ or, equally good, $M_{\text{Fe}}$ and $\xi_{2.5}$, begins to show a cleaner separation of successful and failed explosions: SNe are obtained for small values of $\xi_{2.5}$, whereas BHs are formed for high values of $\xi_{2.5}$, but the value of this threshold increases with $\xi_{1.5}$ and $M_{\text{Fe}}$. For given $\xi_{1.5}$ (or $M_{\text{Fe}}$) there is a value of $\xi_{2.5}$ above which only BHs are formed. However, there is still a broad overlap region of mixed cases.

This beginning separation can be understood in view of the theoretical background of the neutrino-driven mechanism, where the expansion of the SN shock is obstructed by the...
ram pressure of infalling stellar-core matter and shock expansion is pushed by neutrino-energy deposition behind the shock. For neutrino luminosities above a critical threshold $L_{\nu}$, which depends on the mass accretion rate $M$ of the shock, shock runaway and explosion are triggered by neutrino heating (see Figure 6, left panel, and, e.g., Burrows & Goshy 1993; Janka 2001, 2012; Murphy & Burrows 2008; Nordhaus et al. 2010; Fernández 2012; Hanke et al. 2012; Pejcha & Thompson 2012; Müller & Janka 2015). $M_{\text{Fe}}$ (or $\xi_{1.5}$) can be considered as a measure of the mass $M_{\text{ns}}$ of the PNS as an accretor, which determines the strength of the gravitational potential and the size of the neutrino luminosities. Such a dependence can be concluded from the proportionality $L_{\nu} \propto R_{\nu}^2 T_{\nu}^4$, where $R_{\nu}$ is the largely progenitor-independent neutrinosphere radius and the neutrinospheric temperature $T_{\nu}$ increases roughly linearly with $M_{\text{ns}}$ (see Müller & Janka 2014). On the other hand, the long-time mass accretion rate of the PNS grows with $\xi_{2.5}$, which is higher for denser stellar cores. For each PNS mass explosions become impossible above a certain value of $M$ or $\xi_{2.5}$.

### 3.2. Two-parameter Classification Based on the Theoretical Concept of the Neutrino-driven Mechanism

If the initial mass cut at the onset of the explosion develops at an enclosed mass $M = m(r)$ of the progenitor, we can choose $M = m(r)$ as a suitable proxy of the initial PNS mass, $M_{\text{ns}}$. A rough measure of the mass accretion rate $M$ by the stalled shock around the onset of the explosion is then given by the mass gradient $m'(r) \equiv \frac{dm(r)}{dr} = 4\pi r^2 \rho(r)$ at the corresponding radius $r$. This is the case because $m'(r)$ can be directly linked to the free-fall accretion rate of matter collapsing into the shock from initial radius $r$ according to

$$M = \frac{dm}{dt_{\text{ff}}} = \frac{dm(r)}{dr} \left( \frac{dt_{\text{ff}}(r)}{dr} \right)^{-1} = \frac{2m'(r)}{t_{\text{ff}}[(3/r) - m'(r)/m(r)]} \approx \frac{2}{3} \frac{r}{t_{\text{ff}}} m'(r),$$

(5)

where $t_{\text{ff}} = (r^3/[Gm(r)])$ is the free-fall timescale (Suwa et al. 2014) and the last, approximate equality is justified by the fact that $(m'/m)^{-1} = dr/d\ln(m) \gg r$ outside of the dense stellar core.

Following the critical-luminosity concept now points the way to further improvements toward a classification scheme of explosion conditions: the $L_{\nu} - M$ dependence of the neutrino-driven mechanism suggests that the explosibility of the progenitors may be classified by the parameters $M = m(r)$ and $M \propto m'(r)$ because the accretion luminosity $L_{\nu}^\text{acc} \propto GM_{\text{ns}}/R_{\text{ns}} \propto Mm'(r)$ accounts for a major fraction of the neutrino luminosity of the PNS at the time of shock revival (Müller & Janka 2014, 2015), and, in particular, it is the part of the neutrino emission that reflects the main progenitor dependence. It is important to note that the time-evolving NS and neutrinospheric radii, $R_{\text{ns}} \sim R_{\nu}$, are nearly the same for different progenitors and only weakly time dependent when the explosions take place rather late after bounce. This is true for our simulations (where $t_{\text{exp}} \gtrsim 0.5$ s with few exceptions; Figure 3), as well as for self-consistent, sophisticated models (see, e.g., Figure 3 in Müller & Janka 2014). In both cases the spread of the NS radii and their evolution between $\sim 0.4$ and 1 s after bounce accounts for less than 25% variation around an average value of all investigated models. In Section 3.6 we will come back to this argument and give reasons why the NS radius has little influence on the results discussed in this work. Moreover, the neutrino loss from the low-entropy, degenerate PNS core, whose properties are determined by the incompressibility of supranuclear matter, should exhibit a progenitor dependence mostly through the different weight of the surrounding accretion mantle, whose growth depends on $M$. Such a connection is expressed by the terms depending on $m_{\text{acc}}$ and $m_{\text{ns}}$ in neutrino luminosity of the high-density PNS core in Equation (4) of Ugliano et al. (2012). We therefore hypothesize, and demonstrate below, a correspondence of the $L_{\nu} - M$ space and the $M_{\text{ns}} - m'$ parameter plane and expect that the critical luminosity curve $L_{\nu, \text{crit}}(M)$ maps to a curve separating BH formation and successful explosions in the $M_{\text{ns}} - m'$ plane.\(^6\)

In our simulations neutrino-driven explosions set in around the time or shortly after the moment when matter arriving at the shock possesses an entropy $s \sim 4$. We therefore choose $M_4$ (Equation (2)) as our proxy of the PNS mass, $M_4 \propto M_{\text{ns}}$, and $\mu_4 \equiv m'(r)[M_\odot/(1000 \text{ km})]^{-1} \sim 4$ (Equation (3)) as a corresponding measure of the mass accretion rate at this time, $\mu_4 \propto M$. The product $M_{\text{ns}} m'$ is therefore represented by $M_4 \mu_4$. Tests showed that replacing $M_4$ by the iron-core mass, $M_{\text{Fe}}$, is similarly good and yields results of

\(^6\) Since the shock revival is determined by neutrino heating, which depends on $L_{\nu}(\ell_\nu)$, and since the average squared neutrino energy $\langle \nu^2 \rangle \propto T_\nu^2 \propto M_{\text{ns}}^2$, $M_{\text{ns}}$ increases with time and PNS mass (Müller & Janka 2014), Müller & Janka (2015) discuss the critical condition for shock revival in terms of $L_{\nu}(\ell_\nu)$ as a function of $M_4 M$. This suggests that an alternative choice of parameters could be $M_{\text{ns}} m'$ and $M_{\text{ns}}$ instead of $M_{\text{ns}}$ and $m'$, respectively. Our results demonstrate that the basic physics is already captured by the $M_{\text{ns}} - m'$ dependence.
nearly the same quality in the analysis following below (which points to an underlying correlation between $M_4$ and $M_{\text{p0}}$). In practice, we evaluate Equation (3) for $\mu_4$ by the average mass gradient of the progenitor just outside of $s = 4$ according to

$$\mu_4 \equiv \frac{\Delta m/M_4}{\Delta r/1000 \text{ km}} \bigg|_{s=4} = \frac{(M_4 + \Delta m/M_{\text{p0}}) - M_4}{[r(M_4 + \Delta m/M_{\text{p0}}) - r(s=4)]/1000 \text{ km}},$$

with $\Delta m = 0.3 M_4$ yielding optimal results according to tests with varied mass intervals $\Delta m$. With the parameters $M_4$ and $\mu_4$ picked, our imagined mapping between critical conditions in the $L_{\nu,\text{post}}$ and $Mm' \rightarrow m'$ spaces transforms into such a mapping relation between the $L_{\nu,\text{post}}$ and $M_4\mu_4 = \mu_4$ planes as illustrated by Figure 6.

Figure 7 demonstrates the strong correlation of the mass accretion rate $\dot{M} = dm/dt$ with the parameter $\mu_4$ as given by Equation (6) (panel (c)), as well as the tight correlations between $M_4\mu_4$ and the sum of $\nu_e$ and $\bar{\nu}_e$ luminosities (panel (a)) and the summed product of the luminosities and mean squared energies of $\nu_e$ and $\bar{\nu}_e$ (panel (b)). It is important to note that the nonstationarity of the conditions requires us to average the quantities plotted on the abscissa over time from the moment when the $s = 4$ interface passes through the shock until either the models explode (i.e., the shock radius expands beyond 500 km; open circles) or the mass shell $(M_4 + 0.3 M_4)$ has fallen through the shock, which sets an end point to the time interval within which explosions are obtained (nonexplosively cases marked by filled circles). The time-averaging is needed not only because of evolutionary changes of the preshock mass accretion rate (as determined by the progenitor structure) and corresponding evolutionary trends of the emitted neutrino luminosities and mean energies. The averaging is necessary, in particular, because the majority of our models develop large-amplitude shock oscillations after the accretion of the $s = 4$ interface, which leads to quasi-periodic variations of the neutrino emission properties with more or less pronounced, growing amplitudes (see the examples in Figure 2).

Panel (d) demonstrates that exploding models (open circles) exceed a value of unity for the ratio of advection timescale, $t_{\text{adv}}$, to heating timescale, $t_{\text{heat}}$, in the gain layer, which was considered as a useful critical threshold for diagnosing explosions in many previous works (e.g., Janka & Keil 1998; Janka et al. 2001; Thompson 2001; Thompson et al. 2005; Buras et al. 2006; Marek & Janka 2009; Fernández 2012; Müller et al. 2012b; Müller & Janka 2015). The exploding models also populate the region toward low mass accretion rates (as visible in panels (a)–(c), too), which confirms our observation reported in Section 3.1. In contrast, nonexploding models cluster, clearly separated, in the left, upper area of panel (d), where $t_{\text{adv}}/t_{\text{heat}} < 1$ and the mass accretion rate tends to be higher. For the calculation of the timescales we follow the definitions previously used by, e.g., Buras et al. (2006), Marek & Janka (2009), Müller et al. (2012b), and Müller & Janka (2015):

$$t_{\text{heat}} = \left( \int_{R_g}^{R_0} (e + \Phi) \rho \, dV \right) \left( \int_{R_g}^{R_0} q_\nu \rho \, dV \right)^{-1},$$

$$t_{\text{adv}} = \int_{R_g}^{R_0} \frac{1}{|\dot{v}_r|} \, dr.$$

Here the volume and radius integrals are performed over the gain layer between gain radius $R_g$ and shock radius $R_0$, $e$ is the sum of the specific kinetic and internal energies, $\Phi$ the (Newtonian) gravitational potential, $\rho$ the density, $q_\nu$ the net heating rate per unit of mass, and $v_r$ the velocity of the flow. Again, because of the variations of the diagnostic quantities associated with the time evolution of the collapsing star and the oscillations of the gain layer, the mass accretion rate and timescale ratio are time-averaged from the moment when the $s = 4$ interface passes the shock until either 300 ms later or until the model explodes (shock radius exceeding 500 km).\(^\text{7}\)

\(^\text{7}\) We tested intervals ranging from 100 to 600 ms and observed the same trends for all choices.
Panels (e) and (f) lend support to the concept of a critical threshold luminosity in the $L_{\nu} - \dot{M}$ (and $L_{\nu} - \langle \epsilon_{\nu}^2 \rangle - \dot{M}$) space mentioned above. The two plots show a separation of exploding (open circles) and nonexploding (filled circles) models in a plane spanned by the time-averaged values of the preshock mass accretion rate, $\dot{M}$, and the characteristic neutrino-emission properties, respectively, as obtained in our simulations and measured at 500 km. The abscissas of panels (a) and (b) show the summed luminosities of $\nu_e$ and $\bar{\nu}_e$, $L_{\nu_e} + L_{\bar{\nu}_e}$, and the summed products of luminosities and mean squared energies, $L_{\nu_e} \langle \epsilon_{\nu_e}^2 \rangle + L_{\bar{\nu}_e} \langle \epsilon_{\bar{\nu}_e}^2 \rangle$, of both neutrino species, respectively. For the exploding models the time-averaging is performed from the arrival of the $s=4$ interface at the shock until the explosion sets in (defined by the shock radius reaching 500 km), whereas the averages for nonexploding models cover the time from the $s=4$ interface passing the shock until 0.3 $M_e$ of overlying material have been accreted by the shock. Panel (d) displays the separation of exploding and nonexploding models in the plane spanned by the mass accretion rate and the ratio of advection to heating timescale. Panels (e) and (f) demonstrate this separation in the planes spanned by $\dot{M}$ and $L_{\nu_e} + L_{\bar{\nu}_e}$ or $L_{\nu_e} \langle \epsilon_{\nu_e}^2 \rangle + L_{\bar{\nu}_e} \langle \epsilon_{\bar{\nu}_e}^2 \rangle$, respectively. The time averages of the quantities in panels (d)–(f) are computed from the passage of the $s=4$ interface through the shock until 300 ms later for nonexploding models or until the onset of the explosion otherwise. Gray shading in panels (d)–(f) indicates the regions where explosions fail. (No exact boundary curves are determined for the cases of panels (e) and (f).)
the shock, the time-averaged conditions of the exploding models reach the lower halves of these panels, whereas the time-averaged properties of the nonexploding models define the positions of these unsuccessful explosions in the upper halves. A separation appears that can be imagined to resemble the critical luminosity curve \( L_{\nu,\text{crit}}(M) \) sketched in the left panel of Figure 6.

Because of the strong time dependence of the postshock conditions and of the neutrino emission during the phase of dynamical shock expansion and contraction after the accretion of the \( s = 4 \) interface, it is very difficult to exactly determine the critical luminosity curve that captures the physics of our exploding 1D simulations. Although we do not consider such an effort as hopeless if the governing parameters are carefully taken into account (see, e.g., the discussions in Müller & Janka 2015; Pejcha & Thompson 2015) and their time variations are suitably averaged, we think that the clear separation of successful and failed explosions visible in panels (e) and (f) of Figure 7 provides proper and sufficient support for the notion of such a critical curve (or, more general: condition) in the \( L_{\nu,\text{crit}}-M \) space. Figure 6 illustrates our imagined relation between the evolution tracks of collapsing stellar cores that explode or fail to explode in this \( L_{\nu} \)-space on the left side and the locations of SN-producing and BH-forming progenitors in the \( M_{\Delta}\mu_{4}-\mu_{4} \) plane on the right side. The sketched evolution paths are guided by our results in panels (e) and (f) of Figure 7. In the following section we will demonstrate that exploding and nonexploding simulations indeed separate in the \( M_{\Delta}\mu_{4}-\mu_{4} \) parameter space.

### 3.3. Separation Line of Exploding and Nonexploding Progenitors

The existence of a separation line between BH-forming and SN-producing progenitors in the \( M_{\Delta}\mu_{4}-\mu_{4} \) plane is demonstrated by Figure 8, which shows the positions of the progenitors for all investigated model series in this two-dimensional parameter space. For all five calibrations successful explosions are marked by colored symbols, whereas BH formation is indicated by black symbols. The regions of failed explosions are underlaid in gray. They are bounded by straight lines with fit functions as indicated in the panels of Figure 8,

\[
y_{\text{sep}}(x) = k_1 \cdot x + k_2,
\]

where \( x \equiv M_{\Delta}\mu_{4} \) and \( y \equiv \mu_{4} \) are dimensionless variables with \( M_{\Delta} \) in solar masses and \( \mu_{4} \) computed by Equation (6). The values of the dimensionless coefficients \( k_1 \) and \( k_2 \) as listed in Table 2 are determined by minimizing the numbers of outliers.

The stellar models of all progenitor sets populate a narrow strip in the \( x-y \) plane of Figure 8, left panels. BH-formation cases are located in the upper left part of the \( x-y \) plane. The inclination of the separation line implies that the explosion limit in terms of \( \mu_{4} \) depends on the value of the \( M_{\Delta}\mu_{4} \) and therefore a single parameter would fail to predict the right behavior in a large number of cases. Denser cores outside of \( M_{\Delta} \) with high mass accretion rates (larger \( \mu_{4} \)) prevent explosions above some limiting value. This limit grows for more massive cores and thus higher \( M_{\Delta} \) because not only do larger mass accretion rates hamper shock expansion by higher ram pressure but larger core masses and bigger accretion rates also correlate with an increase of the neutrino luminosity of the PNS as expressed by our parameter \( M_{\Delta}\mu_{4} \). The evolution tracks of successful explosion cases in the left panel of Figure 6 indicate that for higher \( L_{\nu} \propto M_{\Delta} \) the explosion threshold, \( L_{\nu,\text{crit}}(M) \), can be reached for larger values of \( M_{\Delta} \).

Explosions are supported by the combination of a massive PNS, which is associated with a high neutrino luminosity from the cooling of the accretion mantle, on the one hand, and a rapid decline of the accretion rate, which leads to decreasing ram pressure, on the other hand. A high value of \( M_{\Delta} \) combined with a low value of \( \mu_{4} \) is therefore favorable for an explosion because a high accretion luminosity (due to a high accretor mass \( M_{\Delta} \approx M_{\text{eq}} \)) comes together with a low mass accretion rate (and thus low ram pressure and low binding energy) exterior to the \( s = 4 \) interface (see Figure 9). Such conditions are met, and explosions occur readily, when the entropy step at the \( s = 4 \) location is big, because a high entropy value outside of \( M_{\Delta} \) correlates with low densities and a low accretion rate. \( M_{\Delta} \) is usually the base of the oxygen shell and a place where the entropy changes discontinuously, causing (or resulting from) a sudden decrease in density due to burning there. This translates into an abrupt decrease in \( M_{\Delta} \) when the mass \( M_{\Delta} \) accretes. Figure 14 of Sukhbold & Woosley (2014) shows a strong correlation between compactness \( \xi_{2.5} \) and location of the oxygen shell. The decrease of the mass accretion rate is abrupt only if the entropy change is steep with mass, for which \( \mu_{4} \) at \( M_{\Delta} \) is a relevant measure.

Progenitors with \( M_{\text{ZAMS}} \lesssim 22 M_{\odot} \) that are harder to explode often have relatively small values of \( M_{\Delta} \) and an entropy ledge above \( s = 4 \) on a lower level than the entropy reached in more easily exploding stars. The lower neutrino luminosity associated with the smaller accretor mass in combination with the higher ram pressure can prohibit shock expansion in many of these cases. Corresponding to the relatively small values of \( M_{\Delta} \) and relatively higher densities outside of this mass, these cases stick out from their neighboring stars with respect to the binding energy of overlying material, namely, nonexploding models in almost all cases are characterized by \( \text{local} \) maxima of \( E_{\text{sh}}(s=4) = E_{\nu}(m/s_{\text{sh}}>M_{\Delta}) \) (see Figure 9).

In view of this insight, it is not astonishing that exploding and nonexploding progenitors can be seen to start separating from each other in the two-parameter space spanned by \( M_{\Delta} \) and the average entropy value \( \langle s \rangle_{4} \) just outside of \( M_{\Delta} \) (Figure 10). Averaging \( s \) over the mass interval \([M_{\Delta}, M_{\Delta} + 0.5]\) turns out to yield the best results. Exploding models cluster toward the side of high \( \langle s \rangle_{4} \) and low \( M_{\Delta} \), while failures are found preferentially for low values of \( \langle s \rangle_{4} \). The threshold for success tends to grow with \( M_{\Delta} \). However, there is still a broad band where both types of outcomes overlap. The disentanglement of SNe and BH-formation events is clearly better achieved by the parameter set of \( M_{\Delta}\mu_{4} \) and \( \mu_{4} \), which, in addition, applies correctly not only for stars with \( M_{\text{ZAMS}} \geq 15 M_{\odot} \) but also for progenitors with lower masses.

### 3.4. Stellar Outliers

Out of 621 simulated stellar models for the s19.8, w15.0, w18.0, w20.0, and n20.0 calibrations, only 9, 14, 16, 11, and 9 models, respectively, do not follow the behavior predicted by their locations on the one or the other side of the separation line in the \( M_{\Delta}\mu_{4}-\mu_{4} \) plane (see the zoom-ins in the right column of Figure 8). But most of these cases lie very close to the boundary curve, and their explosion or nonexplosion can be affected by fine details and will certainly depend on multidimensional effects. A small sample of outliers is farther away...
from the boundary line. The w20.0 calibration is the weakest driver of neutrino-powered explosions in our set and tends to yield the largest number of such more extreme outliers.

These cases possess unusual structural features that influence their readiness to explode. On the nonexploding side of the separation line, model s20.8 of the s2014 series with \((M_4, \mu_4) \approx (0.142, 0.0981)\) is one example of a progenitor that blows up with all calibrations except w20.0, although it is predicted to fail (see Figure 8). In contrast, its mass neighbor, s20.9 with \((M_4, \mu_4) \approx (0.123, 0.085)\), and its close neighbor in the \(M_4, \mu_4\) space, s15.8 of the s2002 series with \((M_4, \mu_4) \approx (0.140, 0.096)\), both form BHs as expected. The structure of these pre-SN models in the \(s = 4\) region is very similar to \(M_4 = 1.45, 1.45, \text{ and } 1.46\) for s20.8, s20.9, and s15.8, respectively. Although s20.8 reaches a lower entropy level outside of \(s = 4\) than the other two cases and therefore is also predicted to fail, its explosion becomes possible when the next entropy step at an enclosed mass of 1.77 \(M_e\) reaches the shock. This step is slightly farther out (at 1.78 \(M_e\)) in the s20.9 case and comes much later (at \(\sim 1.9 \ M_e\)) in the s15.8 model. Both the earlier entropy jump and the lower preceding entropy level enable the explosion of s20.8, because the associated higher

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**Table 2**

| Calibration Model | \(k_1^a\) | \(k_2^a\) | \(M_{\ i1}^b\) | \(\mu_{\ i1}^b\) | \(M_{\ i4}^b\) |
|-------------------|-----------|-----------|----------------|----------------|-------------|
| s19.8 (2002)      | 0.274     | 0.0470    | 1.529          | 0.0662         | 0.101       |
| w15.0             | 0.225     | 0.0495    | 1.318          | 0.0176         | 0.023       |
| w18.0             | 0.283     | 0.0430    | 1.472          | 0.0530         | 0.078       |
| w20.0             | 0.284     | 0.0393    | 1.616          | 0.0469         | 0.076       |
| n20.0             | 0.194     | 0.0580    | 1.679          | 0.0441         | 0.074       |

**Notes.**

\(a\) Fit parameters of separation curve (Equation (9)) when \(x\) and \(y\) are measured for a central stellar density of \(5 \times 10^{10} \text{ g cm}^{-3}\).

\(b\) Measured for a central stellar density of \(5 \times 10^{10} \text{ g cm}^{-3}\).

\(c\) \(M_4\) and \(\mu_4\) measured roughly at core bounce, because pre-collapse data are not available.

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**Figure 8.** Separation curves between BH formation (gray region, black symbols) and SN explosions (white region, colored symbols) for all calibrations in the plane of parameters \(x = M_4 \mu_4\) and \(y = \mu_4\) (zoom-ins in right panels). Note that the left panels do not show roughly two dozen BH-forming models of the u2002 series, which populate the \(x\)-range between 0.5 and 0.62 and are off the displayed scale. Different symbols and colors correspond to the different progenitor sets. The locations of the calibration models are also indicated in the left panels by crossing blue lines.
Figure 9. Iron-core masses $M_{\text{Fe}}$ (top panel), exterior binding energies ($E_{\text{b,Fe}} = E_{\text{b}} (m > M_{\text{Fe}})$; second panel), normalized masses $M_4$ (third panel), exterior binding energies ($E_{\text{b,4}} = E_{\text{b}} (m > M_4)$; fourth panel), and $\mu_4$ (fifth panel), for models of the s2014 series, the supplementary low-mass progenitors with $M_{\text{ZAMS}} < 15 M_\odot$, and models with $M_{\text{ZAMS}} > 30 M_\odot$ from the s2007 series. A black vertical line marks the boundary between the two progenitor sets. Red bars indicate exploding cases and gray bars nonexploding ones. All quantities are measured when the central density of the collapsing stellar iron core is $5 \times 10^{10} \text{ g cm}^{-3}$. The upper five panels correspond to the w18.0 calibration, the lower five panels to the n20.0 calibration. Mass and parameter values of the calibration models are indicated by vertical and horizontal blue lines, respectively. In the region of $M_{\text{ZAMS}} \lesssim 22 M_\odot$ nonexploding cases, with very few exceptions, correlate with local minima of $M_4$ and pronounced local maxima of $\mu_4$ and $E_{\text{b,4}}$. A high value of $M_4$ combined with a low value of $\mu_4$ is typically supportive for an explosion because a high accretion luminosity (due to a high accretor mass $M_{\text{ns}} \approx M_4$) comes together with a low mass accretion rate (and thus low ram pressure and low binding energy) exterior to the $s = 4$ interface. The iron-core masses and their exterior binding energies show a similar tendency, but significantly less pronounced.
density maintains a higher mass accretion rate and therefore higher neutrino luminosity until the entropy jump at 1.77 \( M_\odot \) falls into the shock. The abnormal structure of the progenitor therefore prevents that the explosion behavior is correctly captured by our two-parameter criterion for the explodability.

On the exploding side, model s15.3 of the s2014 series with \(( M_{4M, t4}, \mu_4 ) \approx (0.146, 0.0797)\) is expected to blow up according to the two-parameter criterion, but does not do so for all calibrations (Figure 8). Similarly, s15.0 of the s2007 series with \(( M_{4M, t4}, \mu_4 ) \approx (0.137, 0.0749)\) fails with the w15.0 and w20.0 calibrations, although success is predicted. We compare their structure with the nearby successful cases of s25.4 (s2014 series, \(( M_{4M, t4}, \mu_4 ) \approx (0.150, 0.0820)\)), s25.2, s25.5 (both from the s2014 series), and s25.8 (s2002 series), all of which group around \(( M_{4M, t4}, \mu_4 ) \approx (0.143, 0.0783)\). The successfully exploding models all have similar density and structure, namely, fairly low entropies \((s \lesssim 3)\) and therefore high densities up to 1.81–1.82 \( M_\odot \), where the entropy jumps to \(s \gtrsim 6\). The high mass accretion rate leads to an early arrival of the \(s = 4\) interface at the shock \((\sim 300\) ms after bounce\)) and high accretion luminosity. Together with the strong decline of the accretion rate afterward, this fosters the explosion. In contrast, the two models that blow up less easily have higher entropies and lower densities so that the \(s = 4\) mass shells (at \(\sim 1.8 M_\odot\)) in s15.0 and at \(\sim 1.82 M_\odot\) in the s15.3) arrive at the shock much later (at \(\sim 680\) and \(\sim 830\) ms post-bounce, respectively), at which time accretion contributes less neutrino luminosity. Moreover, both models have a pronounced entropy ledge with a width of \(\sim 0.05 M_\odot\) (s15.0) and \(\sim 0.08 M_\odot\) (s15.3) before the entropy rises above \(s \sim 5\). This ledge is much narrower than in the majority of nonexploding models, where it stretches across typically \(0.3 M_\odot\) or more. The continued, relatively high accretion rate prohibits shock expansion and explosion. This is obvious from the fact that model s15.0 with the less extended entropy ledge exhibits a stronger tendency to explode and for some calibrations indeed does, whereas s15.3 with the wider ledge fails for all calibrations. Our diagnostic parameter \(\mu_4\) to measure the mass derivative in an interval of \(\Delta M = 0.3 M_\odot\), however, is dominated by the high-entropy level (low-density region) above the ledge and therefore underestimates the mass accretion rate in the ledge domain, which is relevant for describing the explosion conditions. Again the abnormal structure of the s15.0 and s15.3 progenitors prevents our two-parameter classification from correctly describing the explosion behavior of these models.

3.5. Systematics of Progenitor and Explosion Properties in the Two-parameter Plane

In Figure 11 colored symbols show the positions of the progenitors of the s2014 series and those of the supplementary low-mass models with \(M_{\text{ZAMS}} < 15 M_\odot\) in the \(x-y\) plane relative to the separation line \(\text{E}_{\text{Fe}}(x)\) of exploding and nonexploding cases. In the top panel the color-coding corresponds to \(M_{\text{ZAMS}}\), in the middle panel to the iron-core mass, \(M_{\text{Fe}}\), and in the bottom panel to the binding energy of matter outside of the iron core. \(M_{\text{Fe}}\) is taken to be the value provided by the stellar progenitor model at the start of the collapse simulation in order to avoid misestimation associated with our simplified nuclear burning network and with inaccuracies from the initial mapping of the progenitor data. Since we use the pressure profile of the progenitor model instead of the temperature profile, slight differences of the derived temperatures can affect the temperature-sensitive shell burning and thus the growth of the iron-core mass.

While low-mass progenitors with small iron cores and low binding energies populate the region towards the lower left corner with significant distance to the separation curve, stars above 20 \( M_\odot\) with bigger iron cores and high binding energies...
can be mostly found well above the separation curve. However, there are quite a number of intermediate-mass progenitors above the line and higher-mass cases below. In particular, a lot of stars with masses between $\sim 25$ and $30 M_\odot$ cluster around $y_{sep}(x)$ in the $x \sim 0.13$–0.15 region. These stars are characterized by $M_{Fe} \sim 1.4$–1.5 $M_\odot$ and high exterior binding energies. Some of them explode, but most fail (see Figure 9). The ones that group on the unsuccessful side are mostly cases with smaller iron cores, whose neutrino luminosity is insufficient to create enough power of neutrino heating to overcome the ram pressure of the massive infall.

Figure 12 displays the BH-formation cases of the s2014 series without associated SNe by black crosses in the $x$–$y$-plane. Successful SN explosions of this series plus additional MM15ZAMS < $15 M_\odot$ for calibration models w18.0 (left) and n20.0 (right) in the $x$–$y$ parameter plane. Black crosses correspond to BH-formation cases, colored crosses to successful explosions. In the middle and bottom panels, the blue (partly overlapping) symbols correspond to fallback SNe with estimated BH masses (baryonic masses in parentheses) and fallback masses as listed in the legends. The horizontal and vertical lines mark the locations of the calibration models with the colors corresponding to the values of the displayed quantities.

Figure 12. Explosion energies ($1 \text{ B} = 1 \text{ bethe} = 10^{51} \text{ erg}$), postbounce explosion times, gravitational neutron star masses ($M_{ns,g} = M_{ns,b} - E_{\nu,\text{tot}}/c^2$), ejected iron-group material (i.e., $^{56}$Ni plus tracer masses), and fallback masses (from top to bottom) of the s2014 series and the supplementary low-mass progenitors with $M_{ZAMS} < 15 M_\odot$ for calibration models w18.0 (left) and n20.0 (right) in the $x$–$y$ parameter plane. Black crosses correspond to BH-formation cases, colored crosses to successful explosions. In the middle and bottom panels, the blue (partly overlapping) symbols correspond to fallback SNe with estimated BH masses (baryonic masses in parentheses) and fallback masses as listed in the legends. The horizontal and vertical lines mark the locations of the calibration models with the colors corresponding to the values of the displayed quantities.

Our estimates of NS binding energies, $E_{ns,b} = E_{\nu,\text{tot}}$, are roughly compatible with the Lattimer & Yahil (1989) fit of $E_{ns,b} = 1.5 \times 10^{53} (M_{ns,g}/M_\odot)^2 \text{ erg} = 0.084 M_\odot c^2 \text{ erg}$. 

$M_{ns,g} = M_{ns,b} - \frac{1}{c^2} E_{\nu,\text{tot}}$. 

$E_{\nu,\text{tot}}$ is the total neutrino energy carried away by our simulations.
Blue symbols in the middle and bottom panels mark fallback BH-formation cases, for which gravitational and baryonic masses (the latter in parentheses) are listed in the panels. We ignore neutrino-energy losses during fallback accretion for both NS- and BH-forming remnants.

Progenitors in Figure 12 that lie very close to the separation line tend to produce weaker explosions that set in later than those of progenitors with a somewhat greater distance from the line. Moreover, there is a tendency of more massive NSs to be produced higher up along the $x$--$y$ band where the progenitors cluster, i.e., bigger NS masses are made at higher values of $x = M_{\rm df}\mu_4$. Also, the largest ejecta masses of $^{56}\text{Ni}$ and $^{56}\text{Ni}$ plus the tracer are found toward the right side of the displayed progenitor band just below the boundary of the BH-formation region.

Fallback masses tend to decrease toward the lower left corner of the $x$--$y$-plane in Figure 12, far away from the separation curve, where predominantly low-mass progenitors are located, besides five progenitors around $20 M_\odot$, which lie in this region because they have extremely low values of $E_\text{b}(m > M_4)$ (see Figure 9) and exceptionally small values of $\mu_4$ (Figure 11) and develop fast and strong explosions with small NS masses, large masses of ejected $^{56}\text{Ni}$ plus the tracer, and very little fallback. Closer to the separation line the fallback masses are higher, but for successfully exploding models they exceed $0.05 M_\odot$ only in a few special cases of fallback SNe (see Figure 3), where the fallback mass can amount to up to several solar masses. We point out that the fallback masses in particular of stars below $\sim 20 M_\odot$ were massively overestimated by Ugliano et al. (2012). The reason was an erroneous interpretation of the outward refection of reverse-shock-accelerated matter as a numerical artifact connected to the use of the condition at the inner grid boundary. The reverse shock, which forms when the SN shock passes the He/H interface, travels backward through the ejecta and decelerates the outward-moving matter to initially negative velocities. This inward flow of stellar material, however, is slowed down and reflected back outward by the large negative pressure gradient that builds up in the reverse-shock-heated inner region. With this outward reflection, which is a true physical phenomenon and not a boundary artifact, the matter that ultimately can be accreted by the NS is diminished to typically between some $10^{-4} M_\odot$ and some $10^{-2} M_\odot$ of early fallback (see Figure 3).

In a handful of high-mass s2014 progenitors (s27.2 and s27.3 for the w18.0 calibration and s27.4, s29.0, s29.1, s29.2, and s29.6 for the n20.0 set) the SN explosion is unable to unbind a large fraction of the star, so that fallback of more than a solar mass of stellar matter is likely to push the NS beyond the BH-formation limit. Such fallback SN cases cluster in the vicinity of $x \sim 0.13$--0.14 and $y \sim 0.080$--0.081 on the explosion side of the separation line. In the ZAMS mass sequence they lie at interfaces between mass intervals of successfully exploding and nonexploding models, or they appear isolated in BH-formation regions of the ZAMS mass space (see Figures 4 and 9). Their fallback masses and estimated BH masses are listed in the corresponding panels of Figure 12. Naturally, they stick out also by their extremely low ejecta masses of $^{56}\text{Ni}$ and tracer elements, late explosion times (around 1 s post-bounce or later), and relatively low explosion energies ($\sim 0.3$--0.5 B).

### 3.6. Influence of the NS Radius

Basically, the accretion luminosity, which is given by $L_\text{acc} \propto GM_{\text{ns}} M / R_{\text{ns}}$, depends not only on the PNS mass, $M_{\text{ns}}$, and the mass accretion rate, $\dot{M}$, but also on the PNS radius, $R_{\text{ns}}$. One may wonder whether our two-parameter criterion is able to capture the essential physics, although we disregard the radius dependence when using $x = M_{\rm df}\mu_4$ as a proxy of the accretion luminosity.

For this reason, we tested a redefinition of $x = M_{\rm df}\mu_4$ by including a factor $(R_{\text{ns}}/300 \text{ ms})$, i.e., using $\tilde{x} \equiv M_{\rm df}\mu_4 (R_{\text{ns}}/300 \text{ ms})$ instead. Doing so, we found essentially no relevant effects on the location of the boundary curve. In fact, the separation of exploding and nonexploding models in the $\tilde{x}$--$y$-plane is even slightly improved compared to the $x$--$y$ plane, because $(R_{\text{ns}}/300 \text{ ms})$ for most nonexploding models is larger than for the far majority of exploding ones. As a consequence, the nonexploding cases tend to be shifted to the left away from the boundary line, whereas most of the exploding cases are shifted to the right, also increasing their distances to the boundary line. This trend leads to a marginally clearer disentanglement of both model groups near the border between explosion and nonexplosion regions. A subset of the (anyway few and marginal) outliers can thus move to the correct side, while very few cases can become new, marginal outliers. It might therefore even be possible to improve the success rate for the classification of explodability by a corresponding (minor) relocation of the boundary curve. The improvement, however, is not significant enough to justify the introduction of an additional parameter into our two-parameter criterion in the form of $(R_{\text{ns}}/300 \text{ ms})$, which has the disadvantage of not being based on progenitor properties and whose exact, case-dependent value cannot be predicted by simple arguments.

Because the line separating exploding and nonexploding models did not change in our test, the criterion advocated in this paper captures the basic physics, within the limitations of the modeling. While we report here this marginal sensitivity of our two-parameter criterion to the NS radius as a result of the present study, a finally conclusive assessment of this question would require repeating our set of model calculations for different prescriptions of the time-dependent contraction of the inner boundary of our computational grid. The chosen functional behavior of this boundary movement with time determines how the PNS contraction proceeds during the crucial phase of shock revival. In order to avoid overly severe numerical time-step constraints, which can become a serious handicap for our long-time simulations with explicit neutrino transport over typically 20 s, we follow Ugliano et al. (2012) in using a relatively slow contraction of the inner grid boundary. It would be highly desirable to perform model calculations also for faster boundary contractions, which is our plan for future work. In view of this caveat, the arguments and test results discussed in this section should still be taken with a grain of salt.

On grounds of the discussion of our results in the $x$--$y$ plane, one can actually easily understand why the definition of the

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8 The radius $R_{\text{ns}}$ of the PNS is defined by the radial position where the density is $10^{10} \text{ g cm}^{-3}$. As in panels (d)–(f) of Figure 7, the time-averaging for $(R_{\text{ns}}/300 \text{ ms})$ is performed from the passage of the $s = 4$ interface through the shock until the onset of the explosion for successful models and from the infall of the $s = 4$ interface until 300 ms later for nonexploding models. We also employ a normalization factor of 70 km to recover (roughly) the same range of values for $\tilde{x}$ as in the case of $x$. 
separation curve of exploding and nonexploding models in the present paper did not require us to take into account a possible dependence of the accretion luminosity on the NS radius. Instead, we could safely ignore such a dependence when we coined our ansatz that $L_{\nu} \propto \text{Min}(r) \propto M_{\text{ns}} \mu_{14}$. There are two reasons for this. On the one hand, the separation line in the $M_{\text{ns}} \mu_{14} - \mu_{14}$ plane is fairly flat. A variation of the NS radius corresponds to a horizontal shift of the location of data points in the $x$--$y$ plane. However, with the contraction behavior of the NSs obtained in our simulations, only for (relatively few) points in the very close vicinity of the separation curve is such a horizontal shift sufficiently big to potentially have an influence on whether the models are classified as exploding or exploding. On the other hand, the models near the separation line typically blow up fairly late ($t_{\text{exp}} \gtrsim 0.6$ s), and the NS masses lie in a rather narrow range between roughly 1.4 and 1.6 $M_{\odot}$ for the gravitational mass (see Figure 12). For such conditions the variation of the NS radius is of secondary importance (see Figure 3 in Müller & Janka 2014). Low-mass progenitors with less massive NSs, whose radius (at the same time) can be somewhat larger (Figure 3 in Müller & Janka 2014), however, are located toward the lower left corner of the $x$--$y$ plane and therefore far away from the separation curve (compare Figures 11 and 12). An incorrect horizontal placement of these cases (due to the omission of the dependence of the neutrino luminosity on the NS radius) does not have any relevance for the classification of these models.

3.7. Brief Comparison to Previous Works

Our result of a complex pattern of NS- and BH-forming cases as a function of progenitor mass was previously found by Ugliano et al. (2012), too, and was confirmed by Pejcha & Thompson (2015). Aside from differences in details depending on the use of different progenitor sets and different SN 1987A calibration models, the main differences of the results presented here compared to those of Ugliano et al. (2012) are lower explosion energies for progenitors with $M \lesssim 13 M_{\odot}$ (see discussion in Section 2.3.4) and lower fallback masses as mentioned in Section 3.5. Based on simple arguments (which, however, cannot account for the complex dynamics of fallback), Pejcha & Thompson (2015) already expected (in particular for their parameterization (a)) that cases with significant fallback—in the sense that the remnant masses are significantly affected—should be rare for solar-metallicity progenitors. Our results confirm this expectation, although the ZAMS masses with significant fallback are different and less numerous than in the work by Pejcha & Thompson (2015). As pointed out by these authors, fallback has potentially important consequences for the remnant mass distribution, and the observed NS and BH masses seem to favor little fallback for the majority of SNe.

As discussed in detail by Ugliano et al. (2012), our explosion models (as well as parameterization (a) of Pejcha & Thompson 2015) predict many more BH-formation cases and more mass intervals of nonexploding stars than O’Connor & Ott (2011), who made the assumption that stars with compactness $\xi > 0.45$ do not explode. Roughly consistent with our results and those of Ugliano et al. (2012), Horiuchi et al. (2014) concluded on the basis of observational arguments (comparisons of the SN rate with the star formation rate; the red supergiant problem as a lack of SNe IIP from progenitors above a mass of $\sim 16 M_{\odot}$) that stars with an “average” compactness of $\xi_{2.5} \sim 0.2$ should fail to produce canonical SNe (see Figure 15 in Sukhbold et al. 2015).

A correlation of explosion energy and $^{56}\text{Ni}$ mass as found by Pejcha & Thompson (2015) and Nakamura et al. (2015) (both, however, without rigorous determination of observable explosion energies at infinity) and as suggested by observations (see Pejcha & Thompson 2015 for details) is also predicted by our present results (but not by Ugliano et al. 2012 with their erroneous estimate of the fallback masses and the overestimated explosion energies of low-mass progenitors; see Section 2.3.4). Our models yield low explosion energies and low nickel production toward the low-mass side of the SN progenitors (see also Sukhbold et al. 2015). In contrast, the modeling approach by Pejcha & Thompson (2015) predicts a tendency of lower explosion energies and lower $^{56}\text{Ni}$ masses toward the high-mass side of the progenitor distribution (because of the larger binding energies of more massive stars), although there is a large mass-to-mass scatter in all results. This difference could be interesting for observational diagnostics. The modeling approach by Pejcha & Thompson (2015) seems to yield neutrino-driven winds that are considerably stronger, especially in cases of low-mass SN progenitors, than those obtained in the simulations of Ugliano et al. (2012) and, in particular, than those found in current, fully self-consistent SN and PNS cooling models, whose behavior we attempt to reproduce better by the Crab-motivated recalibration of the low-mass explosions used in the present work. Moreover, for the $^{56}\text{Ni}$--$E_{\text{exp}}$ correlation reported by Pejcha & Thompson (2015), a mass interval between $\sim 14$ and $\sim 15 M_{\odot}$ with very low explosion energies and very low Ni production, which does not exist in our models, also plays an important role. Nakamura et al. (2015) found a positive correlation of the explosion energy and the $^{56}\text{Ni}$ mass with compactness $\xi_{2.5}$. We can confirm this result for the nickel production but not for the explosion energy (see Sukhbold et al. 2015, Figure 15 there). A possible reason for this discrepancy could be the consideration of “diagnostic” energies by Nakamura et al. (2015) at model-dependent final times of their simulations instead of asymptotic explosion energies at infinity, whose determination requires seconds of calculation and the inclusion of the binding energy of the outer progenitor shells (see Figure 3 and Figure 6 in Sukhbold et al. 2015, for the evolution of the energies in some exemplary simulations).

Since a comprehensive discussion of explosion energies, nickel production, and remnant masses is not in the focus of the present work, we refer the reader interested in these aspects to the follow-up paper by Sukhbold et al. (2015). For the same reason we also refrain here from more extended comparisons to the progenitor-dependent explosion and remnant predictions of other studies, in particular those of O’Connor & Ott (2011), Nakamura et al. (2015), and Pejcha & Thompson (2015). A detailed assessment of the different modeling methodologies and their underlying assumptions would be needed to understand and judge the reasons for differences of the results and to value their meaning in the context of SN predictions based on the neutrino-driven mechanism. Such a goal reaches far beyond the scope of our paper.

4. CONCLUSIONS

We performed 1D simulations of SNe for a large set of 621 progenitors of different masses and metallicities, including the solar-metallicity s2002 series (Woosley et al. 2002) previously
investigated by Ugliano et al. (2012) and the new s2014 models of Sukhbold & Woosley (2014) with their fine gridding of 0.1 $M_\odot$ in the ZAMS mass.

In order to obtain SN explosions in spherical symmetry, we adopt the methodology of Ugliano et al. (2012), using 1D hydrodynamics and approximate neutrino transport and a PNS-core neutrino source, but with improvements in a number of modeling aspects (e.g., a nuclear high-density EOS and a fully self-consistent implementation of nuclear burning through a small network; see Section 2.1). Explosions are triggered by a neutrino luminosity that is sufficiently large to overcome the critical threshold condition for shock runaway. This luminosity is fed by a progenitor-dependent accretion component during the post-bounce shock-stagnation phase, as well as a component from the high-density core of the nascent NS. The core emission is also progenitor dependent because it scales with the mass of the hot accretion mantle that assembles around the cooler high-density core of the PNS during the pre-explosion evolution. The conditions for an explosion are thus tightly coupled to the progenitor structure, which determines the post-bounce accretion history.

Our approach contains a number of free parameters, whose values are calibrated by reproducing observational properties (explosion energy, $^{56}$Ni mass, total neutrino energy, and signal duration) of SN 1987A with suitable progenitor models of this SN. We consider five different such progenitors for our study, namely, besides the s19.8 star of the s2002 model series of Woosley et al. (2002), which was used by Ugliano et al. (2012) as the calibration model, also 15, 18, and 20 $M_\odot$ models of Woosley and collaborators, as well as a 20 $M_\odot$ model from Shigeyama & Nomoto (1990) (see Section 2.2).

Because 1D simulations cannot properly reproduce the period of continued accretion and simultaneous outflow that characterizes the early expansion of the revived shock in multidimensional simulations and delivers the explosion energy, our 1D models exhibit an extended episode of accretion that is followed by a strong early wind phase. The overestimated mass loss during the latter phase compensates for the enhanced preceding accretion, and the associated recombination energy yields the dominant contribution to the power supply of the beginning explosion. A detailed discussion of our methodology can be found in Section 2.3.

Overall, our results confirm the ZAMS-mass-dependent explosion behavior that was found by Ugliano et al. (2012) for the s2002 model series. For the same explosion calibration this progenitor set and the newer s2014 models have basic features in common. Moreover, for all calibration cases we observe similar irregular patterns of successful explosions alternating with BH-formation events above ~15 $M_\odot$. The largest fraction of BH-formation cases is obtained with the s20.0 calibration model, a 20 $M_\odot$ SN 1987A progenitor of Woosley et al. (1997), which explodes relatively easily and reproduces the SN 1987A $^{56}$Ni yield with a fairly low ratio of explosion energy to ejecta mass of $E_{\exp}/M_{\text{ej}} \sim 0.7$ only. The core neutrino source is correspondingly weak and enables successful SNe in a smaller subset of progenitors. On the other hand, we obtain the closest similarity of the explodability of the investigated progenitor sets when we use the s19.8 (Ugliano et al.’s) calibration model and the Shigeyama & Nomoto (1990) n20.0 SN 1987A progenitor, which possess very similar compactness values $\xi_{2.5}$.

Nonexploding cases tend to correlate with local maxima of the compactness $\xi_{2.5}$ of the total binding energy outside of the iron core, $E_b (m > M_b)$, and, most significantly, local maxima of the total binding energy $E_b (m/M_{\odot} > M_4)$ outside of the mass coordinate $M_4 (s = 4)/M_{\odot}$, where the dimensionless entropy per nucleon reaches a value of $4$. Many (but not all) nonexploding progenitors below ~22 $M_\odot$ also coincide with local minima of $M_{\text{ke}}$ and, in particular, with minima of $M_4$. However, there are no fixed threshold values of any of these characteristic parameters of the pre-collapse progenitor structure that could be used to discriminate favorable from nonfavorable conditions for an explosion.

Guided by such insights, we propose a two-parameter criterion to classify the explodability of progenitor stars by the neutrino-heating mechanism in dependence of the pre-collapse properties of these stars. The two structural parameters that turn out to yield the best separation of successful and unsuccessful cases are $M_4$ and the mass derivative $\mu_4 = dm/dm'$ at $s = 4$ just outside of the $s = 4$ location, which we combine to a parameter $x = M_4 \mu_4$. The parameters $x$ and $y \equiv \mu_4$ are tightly connected to the two crucial quantities that govern the physics of the neutrino-driven mechanism, namely, the mass accretion rate of the stalled shock, $\dot{M}$, and the neutrino luminosity $L_\nu$. The former determines the ram pressure that damps shock expansion and can be mathematically linked to the mass derivative $m'$ (see Equation (5)). The latter is a crucial ingredient for the neutrino heating that is responsible for shock revival and is dominated by the accretion luminosity and the PNS-mantle cooling emission during the crucial phase of shock revival. Both of these scale with $M$ and/or the accretor mass (i.e., the PNS mass) so that $L_\nu \propto M_{\text{cc}} M_4$ expresses the leading dependence. Since the neutrino-driven explosions in our simulations set in shortly after the entropy interface and density jump around $s = 4$ have fallen through the shock (Figure 2), $M_4$ can be taken as a good proxy of $M_{\text{cc}}$ as the accretor mass, and $\mu_4$ can serve as a measure for the mass accretion rate $\dot{M}$ of the PNS.

Higher values of $M_4$ tend to be favorable for an explosion, as shown by Figure 9, where many nonexploding cases (dark gray) correlate with local minima of $M_4$. The reason is that the neutrino luminosity scales (roughly) with $x = M_4 \mu_4$ (the actual sensitivity of the neutrino-energy deposition to $M_4$ is even steeper). Therefore, higher $M_4$ imply greater neutrino luminosities and stronger neutrino heating. In contrast, the influence of $y = \mu_4$ is ambivalent. On the one hand, a high value of $\mu_4$ increases the neutrino luminosity; on the other hand, it also causes a large ram pressure that has to be overcome by neutrino heating. The effect of these competing influences is that a higher value of $M_4$ in association with a lower value of $\mu_4$ favors explosions. Conversely, nonexploding cases in Figure 9 are correlated with local minima of $M_4$ and maxima of $\mu_4$. Visually, this means that explosion cases are preferentially located toward the lower right of the $M_4 \mu_4$ parameter space (see Figures 6, 8, 12).

The parameters $x$ and $y$ therefore span a plane in which successful explosions and failures with BH formation are clearly separated. The progenitor stars populate an astonishingly narrow band that stretches from the lower left corner with the lightest stars to the upper right direction of the $x$–$y$ plane, where the massive progenitors with the biggest iron cores and highest binding energies of overlying material are located (see Figure 11). While SNe can be found in the region of low values of $y$, i.e., for low mass accretion rate, the nonexploding cases
inhabit the domain of high \( y \), but the limiting value of the mass accretion rate that prevents the success of the neutrino-driven mechanism grows with the value of \( x \). Both sectors in the \( x-y \) plane are separated by a boundary line that can be well represented by a linear function \( \gamma_{\text{sep}}(x) \) (Equation (9)) with increasing slope. (The values of the dimensionless coefficients of this linear relation are listed for all calibration models in Table 2.) Because of the physical meaning of the parameters \( x \) and \( y \), there is a close correspondence between the separation line \( \gamma_{\text{sep}}(x) \) and the critical threshold luminosity \( L_{\nu,\text{crit}}(M) \) that has to be exceeded to trigger runaway expansion of the accretion shock by neutrino heating. The rising slope of the separation curve in this context means that for each value of the neutrino luminosity, respectively \( x \), there is an upper limit of the mass accretion rate, respectively \( y \), up to which neutrino-driven explosions are possible, and that this BH-formation threshold value of \( y \) grows with \( x \). The parameters \( x \) and \( y \), computed from the progenitor profiles before collapse, allow one to judge whether a considered star is able to overcome the threshold neutrino luminosity for an explosion, or, in other words, whether its mass accretion rate stays below the critical limit above which the onset of the explosion is prevented.

Only \( \sim 1\%–3\% \) of all investigated progenitors do not follow this discrimination scheme in their final fate but lie on the wrong side of the separation curve. These outliers are characterized by pathologies of their entropy and density profiles that describe the composition-shell structure in the Si-O layers. Such special conditions lead to unusual combinations of mass accretion rate and PNS masses. Our two-parameter criterion expressed by the separation line \( \gamma_{\text{sep}}(x) \) therefore enables one, with a very high significance, to predict the explodability of progenitor stars via the neutrino-driven mechanism by referring to the properties of these stars as captured by the pair of parameters \( x \) and \( y \).

Clausen et al. (2015) explore the interesting possibility that the death of massive stars in NS versus BH formation may be better captured by a probabilistic description. The nonmonotonic variations of explosion versus nonexplosion with ZAMS mass or compactness might be interpreted as stochasticity in the explosion behavior. However, by considering the problem in a more appropriate two-parameter space, our two-parameter criterion unMASKS these putatively random variations as actually deterministic phenomena. Clausen et al. (2015), in contrast, suggest a variety of factors besides ZAMS mass and metallicity, e.g., rotation, binarity, the strength of magnetic fields, and stochastic differences in the pre-collapse structure or even in the explosion mechanism, that might introduce a randomness such that a star of given mass might not form either an NS or a BH but both with a certain probability. If such a diversity in the stellar destiny depends on a causal process, for example, the presence of different amounts of spin, a deterministic description could still apply but would require an extension to a parameter space of more than the two dimensions combined by our current criterion, e.g., by adding extra dimension(s) that capture the role of rotation in the explosion mechanism. If, in contrast, truly stochastic effects like turbulent processes or chaotic fluctuations in the progenitor decide about the stellar fate, a deterministic criterion for explodability would be ruled out and a probabilistic description would be indispensable.

Our study has a number of additional caveats that need to be addressed. The understanding of the explosion mechanism(s) of massive stars is still incomplete, although considerable progress has been achieved in recent years owing to the progress in 2D and 3D modeling and in particular also through improvements in the treatment of the crucial neutrino physics and transport in collapsing stellar cores (see, e.g., Janka et al. 2012; Janka et al. 2012; Burrows 2013; Lentz et al. 2015; Melson et al. 2015a, 2015b; Mezzacappa et al. 2015, and references therein). Without self-consistent 3D explosion simulations for large sets of progenitor stars being possible yet, our study refers to a 1D modeling approach, in which not only the neutrino description is approximated in many aspects, but also the explosions have to be triggered artificially. We employ a boundary condition that replaces the high-density, low-entropy core of the nascent NS as a neutrino source, and we describe the time-dependent behavior of this core and of the coupling between the core and its hot accretion mantle by a simple, analytical model. Calibrating the involved free parameters by observational constraints from SN 1987A for the more massive stars and by comparison to results of sophisticated SN models for low-mass progenitors is intended to anchor our simplified description on empirical ground. Although the elements of this approximate approach appear qualified to capture the essence of the neutrino-heating mechanism in dependence of the progenitor-specific post-bounce accretion evolution (see Section 2.3 for details), confirmation by fully self-consistent, multidimensional SN calculations is ultimately indispensable. It is also evident that our study, which is only concerned with neutrino-driven explosions, cannot yield any information about the possibility and implications of other mechanisms to blow up stars, for example, magnetorotational explosions of rapidly rotating stellar cores, which might be a consequence of magnetar or BH formation and could be associated with hypernovae and gamma-ray burst SNe (see, e.g., Mazzali et al. 2014), as well as ultraluminous SNe (Kasen & Bildsten 2010; Woosley 2010; Sukhbold et al. 2015).

Our study employs pre-collapse models that emerge from 1D stellar evolution calculations of single, nonrotating SN progenitors with prescribed mass-loss histories. The results of our study naturally depend on the post-bounce accretion properties of the collapsing stars. The (iron and low-entropy) core masses and the entropy and density jumps at the composition-shell interfaces play an important role in setting the conditions for the neutrino-heating mechanism, which is obvious from the definition of our parameters \( x \) and \( y \). It is conceivable that multidimensional hydrodynamics could lead to considerable changes of the stellar properties as functions of the progenitor mass (e.g., Arnett et al. 2015), and that asymmetries and perturbations in the shell-burning layers of the pre-collapse core might have important consequences for the development of SN explosions by the neutrino-driven mechanism (e.g., Arnett & Meakin 2011; Couch & Ott 2013, 2015; Couch et al. 2015; Müller & Janka 2015). We are hopeful that the basic insights of our study, in particular the existence of a two-parameter criterion for the explodability—expressed by an SN–BH separation line \( \gamma_{\text{sep}}(x) \) in the \( x-y \) space and the tight connection between this curve and the critical luminosity condition \( L_{\nu,\text{crit}}(M) \) of the neutrino-driven mechanism—possess more general validity. The explosion properties of the progenitor stars as functions of the ZAMS mass, however, not only depend (moderately) on the considered SN 1987A progenitor models but will probably also change
once multidimensional stellar evolution effects are accounted for in the pre-SN conditions.

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