Heat Transfer Simulation Study of Hot Oil Tubes Based on COMSOL

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Abstract: Heat insulation board is added on the adjacent side of hot oil pipeline and other pipelines to increase thermal resistance and reduce the influence of hot oil pipeline on the temperature of the pipeline. Because the heat conduction problem of the soil around the hot oil pipeline after the addition of heat insulation board becomes very complex, this paper studies and analyzes the heat transfer process of the hot oil pipeline through constructing an equivalent simplified mathematical calculation model and heat transfer simulation based on COMSOL.

1. Introduction
At the beginning of the design of the directly buried heating oil pipeline, the space of the underground pipe is often insufficient, and the space of the heating pipeline is seldom reserved at the beginning of the design[1]. As a result, it is difficult to find the pipe position or the distance from the adjacent pipeline can not meet the minimum net distance requirement of the code in the subsequent design of the heating pipeline. In the actual design and construction, when the minimum net distance cannot be satisfied, heat insulation board is added on the side adjacent to the heating pipeline and other pipelines to increase the thermal resistance and reduce the influence of the heating pipeline on the temperature of the pipeline[2]. Because the heat conduction problem of the soil around the heating pipeline after the addition of heat insulation board becomes very complex, there is no corresponding theoretical calculation method at present. If the numerical simulation method is adopted, it will not only have a long time period, but also be difficult for the general engineering designers to operate the corresponding simulation software[3].

In this paper, the change of soil temperature field after heating of hot tubing is studied and analyzed by constructing an equivalent simplified mathematical calculation model and heat transfer simulation based on COMSOL.

2. Model construction

2.1. Assumptions of the model
(1) It is assumed that the contact between each material layer is good, and the temperature on the contact surface is approximately equal.
(2) Each material layer can be considered as an isotropic continuum.
(3) The thermal conductivity, specific heat capacity and density of each material layer are constant and do not change with temperature.
(4) Each material layer has no internal heat source and will not produce excess heat in addition to heat transfer.
(5) The thermal contact resistance between the contact surfaces of each material layer is ignored.
(6) There is only heat transfer between the air and the material layer and between each material layer, without considering the thermal radiation.
(7) The function fitted in the process conforms to statistical principles.
(8) The experimental process is a random process, and the experimental results are random variables.

2.2. Fourier Law
Fourier discovered and summarized the relationship between heat flux and temperature gradient through a large number of studies on the heat conduction process of substances, which is called Fourier's law[4].

\[ q = -\lambda \frac{\partial t}{\partial x} \]  

Where \( \lambda \) is the thermal conductivity, which is related to the nature of the material itself, the larger the particle, the stronger the thermal conductivity. \( q \) is heat flux, that is, heat transfer per unit area per unit time.

2.3. Thermal conductivity differential equation
In the process of heat conduction, microelements are intercepted for analysis, as shown in the figure below:

Fig. 1 Thermal conductivity analysis of microelement

Assuming that there is no heat source inside, according to the law of conservation of energy, there is:

\[ I = -(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z})dxdydzd\tau \]  

In other words, the net heat flowing in and out of the microelement along the directions of \( x \), \( y \), and the main axis is the same in DT time. Combined with the above equation, the thermal conductivity differential equation can be obtained:

\[ \frac{\partial t}{\partial \tau} = \frac{A}{\rho c} \left( \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right) + \phi \frac{\partial \phi}{\partial \tau} \]  

In the formula remember \( A = \lambda / \rho c \) is called the diffusion rate, reflecting the ability of the temperature of the parts of the object to tend to the same in the heat conduction, \( \phi \) is the heat flow. The thermal conductivity differential equation is based on the Fourier law and the law of conservation of energy, which reflects the relationship between temperature changes with time and space[6].

2.4. Unsteady heat conduction model of one-dimensional system
The unsteady heat conduction process refers to the heat conduction process in which the temperature of the object changes with time. The characteristics of the unsteady process are (1) the heat balance is not reached in the heat transfer process; (2) the heating process of the object is not uniform; (3) The temperature variation of the material is related to the initial temperature distribution; (4) after a period of unsteady heat conduction, the temperature of the object everywhere reached a new steady state.

2.4.1. Single layer plate heat transfer under unsteady condition Taking the material layer I as an example, suppose there is an infinite plate of thickness \( \delta \), the material thermal conductivity of the plate is \( \lambda \), the density is \( \rho \), and the specific heat capacity of the plate is \( C_p \). In the initial state, the temperature of the plate and the surrounding medium is uniform and \( T_0 \). Then , suddenly place one side of the plate under a constant
temperature of TW, and keep T0 constant on the other side. A coordinate system with T0 as the far point is established. At this time, one-dimensional transient unsteady heat transfer under constant temperature boundary condition is generated. The thermal differential equation established is as follows:

\[
\frac{\partial t}{\partial \tau} = \frac{\lambda}{\rho c} \frac{\partial^2 t}{\partial x^2} \quad \tau > 0, 0 < x < \delta
\]  

(4)

The initial conditions are:

\[
\tau = 0, t = t_0, 0 < t < t_w
\]  

(5)

The corresponding boundary conditions are as follows:

\[
\frac{\partial t}{\partial x} \bigg|_{x=0} = 0, \tau > 0
\]  

(6)

\[
x = \delta, t = t_w
\]  

(7)

The analytical solution of the formula obtained by the separation of variables method is as follows:

\[
\frac{\theta x - \tau}{\theta_0} = \frac{\lambda}{\rho c} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi}{2}\right) \cos\left(\frac{n\pi}{2\delta} x\right) e^{\left(\frac{-n\pi}{2\delta} \frac{a}{\tau}\right)}
\]  

(8)

\(\theta x - \tau\) is the excess temperature, \(\beta = t - t_w, a = \frac{\lambda}{\rho c}\).

2.4.2. Heat transfer temperature distribution of semi-infinite plate under unsteady condition

According to the theoretical knowledge of heat transfer, it is introduced and solved:

\[
\frac{\beta}{\pi} = \cot \beta
\]  

(9)

Where \(\beta\) is an infinite number of solutions of \(\beta\), which is introduced in the process of solving the differential equation. \(\beta_1, \beta_2, \beta_3, ... \beta_n\) is the eigenvalue. Let \(B_i = \frac{\beta_i}{\pi}\), and \(B_i\) is called Beworth number.

According to relevant thermodynamic knowledge, the constant temperature boundary condition in this system is the corresponding convection boundary condition when Bewon number \(B_i \rightarrow \infty\). Therefore, the above equation can be rewritten as:

\[
\frac{\theta x - \tau}{\theta_0} = \sum_{n=1}^{\infty} \frac{2 \sin \beta_n}{\beta_0 + \sin \beta_n \cos \beta_n} \cos\left(\frac{\beta_n}{\delta} x\right) e^{\left(\frac{-\beta_n}{\delta} \frac{2 \pi a}{\tau}\right)}
\]  

(10)

In other words, the analytical solution of the heat transfer process of an infinite plate is obtained. The analytical solution can theoretically calculate the temperature value at any time distance from the given point at any distance, that is, the temperature distribution.

3. Solution of the model

3.1. Temperature analysis of material layer under constant temperature on both sides

According to the temperature analytical formula obtained above, the temperature of T_0 at any time under the constant temperature condition at both ends of the first layer material and the temperature at the distance from the high temperature point X can be analyzed. In the process of solving the equation, according to the relevant literature, there are infinite number of formulas, but within the error allowable range, only the first few items can be reserved for solving the simplified model according to the specific situation. When \(n=6\) is taken as the following and prior literature is consulted, it can be obtained:

| i  | 1   | 2   | 3   | 4   | 5   | 6   |
|----|-----|-----|-----|-----|-----|-----|
| \(\beta_i\) | 1.1925 | 3.8088 | 6.704 | 9.724 | 12.7966 | 15.8945 |
| \(\frac{2 \sin \beta_i}{\beta_0 + \sin \beta_i \cos \beta_i}\) | 1.210237 | -0.28815 | -0.115444 | -0.05892 | 0.035057 | -0.02307 |
Therefore, the analytical solution of the temperature of the single-layer material layer under constant temperature on both sides can be obtained. The analytical solution can be calculated by using Matlab to obtain the temperature analytic images with different corresponding values:

![Fig. 2 Images of each analytical solution when n=1 to n=6](image)

![Fig. 3 Temperature variation over time on contact surface I](image)

By comparing the measured temperature outside the tubing with the time variation image, it can be seen that the variation trend of the two is the same, which indirectly verifies the rationality of the analytical solution within the allowable error range.

### 3.2. Temperature analysis of the material layer under the condition of uneven heating on one side and constant temperature on the other side

Different from the first layer, the contact surfaces at the second layer and subsequent layers have several characteristics: (1) The side is the material layer of the previous layer that transfers heat, which is not the normal temperature state of type 1. The other side can be regarded as the original temperature state because the material has certain inertia, that is, it needs to overcome its own thermal resistance to do work when heating up. (2) Due to the thermal resistance of the material when heat is transferred, the heat transferred per unit time is not uniform in the non-equilibrium state.

Because of the above problems, it is necessary to carry on the corresponding special processing to the data. According to 3.2.1, the data table of contact surface 1 changing with time can be obtained. The data is now divided into several groups, and the cut-off points are listed as follows:
Table 2 Cutting point temperatures of different groups

| Group number | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|--------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| The cut-off temperature | 63.59 | 64.92 | 66.6 | 67.73 | 68.67 | 69.55 | 70.61 | 71.48 | 72.56 | 73.41 |

Heat transfer in a certain stage can be regarded as uniform heat transfer within a certain period of time, and the calculated average value of each time stage can be taken as the initial temperature of contact surface II, which can be converted into the temperature analysis of material layer under constant temperature on both sides of type I. The temperature distribution of the corresponding contact surfaces with time can be obtained from the above equation.

4. Simulation solution

The simulation model was established with COMSOL simulation software to simulate the one-dimensional heat transfer model, and the obtained thermal diagram was as follows:

![Thermal simulation image of COMSOL](image)

In the simulation results, data with a cross section of $x=0.6\text{mm}$ was selected for analysis, and 100 groups of data with a step length of 10 were selected to get the numerical value solved by the simulation software.

Table 3 Comparison value table

| Time (seconds) | Temperature (degrees) | Time (seconds) | Temperature (degrees) | Time (seconds) | Temperature (degrees) |
|----------------|-----------------------|----------------|-----------------------|----------------|-----------------------|
| 10.00          | 37.16                 | 340.00         | 71.39                 | 670.00         | 73.51                 |
| 20.00          | 51.06                 | 350.00         | 71.53                 | 680.00         | 73.54                 |
| 40.00          | 56.97                 | 370.00         | 71.81                 | 700.00         | 73.59                 |
| 50.00          | 58.44                 | 380.00         | 71.95                 | 710.00         | 73.60                 |

The temperature between the contact surfaces of each material layer can be calculated by the formula as follows:

Table 4 Temperature between contact surfaces of each material layer

| The layer number | I          | II         | III        |
|-----------------|------------|------------|------------|
| Theoretical steady-state temperature /°C | 72.74765 | 65.09093 | 48.0004 |
| The obtained equilibrium temperature /°C | 74.31625 | 62.20238 | 45.19426 |

According to the unsteady heat transfer differential equation, the temperature of the contact surface of layer I in the equilibrium state can be obtained. It can be seen that there is still a certain gap between the two. Considering the influence of some constant parameters in the unsteady state equation, it also reflects the
correctness of the unsteady state temperature distribution function from one level.

By using R-series expansion method, the derivative of the governing equation is replaced and discretized by the difference quotient of the function values at the nodes of the network, and the algebraic equation with the values at the nodes of the grid as unknown is established.

5. Test of the model
In view of the systematic error, random error and fault error existing in the model, now aiming at the temperature data solved by the established model in the specific problem, combined with the data from the Comsol simulation software operation, the least square method is used to carry out function fitting, and it is found that the fitting degree is very high.

Among them, the fitting function is \( F(x) = 0.9684\times + 1.914 \), the fitting coefficient is 0.9684, \( R=0.9988 \), and the degree of fitting is extremely high. Therefore, this model is considered to be highly reasonable.

6. Conclusions
In this paper, the heat transfer process under unsteady state is analyzed firstly. Combined with the relevant heat transfer theories, the heat transfer model of single-layer plate under unsteady state is established, and the one-dimensional partial differential heat transfer equation under unsteady state is derived. The temperature distribution data of material internal contact surface I is obtained by using the analytical solution obtained by the separation of variables method. Finally, the simulation results of COMSOL software are used to verify the rationality of the model.

References
[1] Wang Qing, Yu Sheng, Ma Xinyi, Zhang Qiang, Dai Min, (2021). Simulation and design of thermal insulation system based on COMSOL. Cement (03),44-47.
[2] Wei Ming, Wang Bo, Liu Shunbo, (2020). Simulation of heat and moisture coupling transfer of deep underground engineering envelope based on COMSOL software. Science Technology and Engineering (14),5729-5736.
[3] Lan Ling Ling. (2020). Numerical simulation of heat transfer characteristics of corrugated fin based on COMSOL. Journal of jiujiang university (natural science edition) (01), 32-38.
[4] Deng Changzheng, Yang Nan, Deng Haifeng, Zhang Nan & Feng Zhen,(2020). Simulation analysis of temperature field of 10 kV high voltage switchgear based on COMSOL. Electrical Materials (01),32-36.
[5] Xie Siyuan, Liu Weijun & Song Kaiyun.(2020). Reliability evaluation of thermal conductivity of multilayer composite panels by COMSOL simulation. Industry & Technology Forum (04),47-49.