Waveguide Radiation of the Combined Vibrator-Slot Structures

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Abstract—A problem of electromagnetic wave diffraction by a longitudinal slot cut in a waveguide wide wall is solved by a generalized method of induced electro-magneto-motive forces (EMMF). The slot radiates in a half-space above a perfectly conducting plane where two vertical impedance monopoles are arbitrarily located. To control electrodynamic characteristics of the radiator, a passive impedance monopole is placed in the waveguide. The paper is aimed at the study of the electrodynamic characteristics of waveguide vibrator-slot structures, analogous to the known Clavin element, with two identical impedance monopoles on both sides of the narrow half-wave slot. The influence of the geometric structure parameters on the directional characteristic of Clavin type element: relative level of sidelobes in the $E$-plane and the RP width differences in the main polarization plane at $-3$ dB level was analyzed. It was shown that the directional and energy characteristics of the radiators: radiation and reflection coefficients, antenna directivity, and gain can be varied within wide limits by changing the electrical length and/or distributed surface impedance of the vibrators.

1. INTRODUCTION

In the modern practice, slotted structures are used as stand-alone small-sized antennas, elements of complex antenna arrays, and devices of antenna-feeder tracts [1–3]. To excite slot radiators and control their electrodynamic characteristics, vibrator-type elements are often used [4]. In combined vibrator-slot structures, the vibrator elements can have a different configuration and can be located in any electrodynamic volume coupling through a slot. For example, structures consisting of wire radiators located over an infinite perfectly conducting plane with a hole radiating into half space over a plane were considered in [5–10]. Radiators with vibrators located in various waveguide tracts and resonators were studied in [11–20]. A special place among combined vibrator-slot structures is occupied by Clavin elements, consisting of narrow radiating slot and two identical passive vibrators (monopoles) located on both sides of the slot cut in plane screen [21–28]. The Clavin element is characterized by similar radiation patterns (RPs) in the $E$- and $H$-planes. They are used as stand-alone radiators, primary feed antennas, and also as part of multi-element phased arrays [28]. However, in publications on this subject by other authors [11–16, 21–25], only perfectly conducting vibrators were considered. New possibilities for controlling the characteristics of vibrator-slot radiating structures by using monopoles with distributed surface impedance were proposed in [3, 4, 17–20, 26, 27].

In this paper, the excitation of electromagnetic fields by the combined radiator consisting of a longitudinal slot cut in a broad wall of the rectangular waveguide and vertical impedance monopoles is solved by the generalized method of induced EMMF [1, 3, 4, 29]. The dimensions of the impedance monopoles and their locations in space above the plane are arbitrary. The electrodynamic characteristics of the combined radiator can also be controlled by a passive impedance monopole located in the waveguide. The radiation characteristics of vibrator-slot structures similar to the Clavin element are thoroughly studied.

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2. PROBLEM FORMULATION AND INITIAL EQUATIONS IN THE GENERAL CASE

Consider a microwave device consisting of a rectangular waveguide section with a longitudinal slot cut in its wide wall and two asymmetric impedance vibrators (monopoles) located near the slot outside the waveguide and a monopole located inside the waveguide in the plane \(\{z=0\}\) parallel to its narrow wall (Fig. 1). The \(H_{10}\)-wave propagates in the waveguide with cross-section \(\{a \times b\}\) (the region is marked as \(Wg\)) from the direction \(z = -\infty\). Parameters of the waveguide filling material are \(\varepsilon_1, \mu_1\). The slot radiates into half-space marked as \(Hs\) located above an infinite perfectly conducting plane. The material parameters of the half-space are \(\varepsilon_2, \mu_2\). The lengths and radii of the outside monopoles are \(L_1, L_2\) and \(r_1, r_2\), and outer vibrator is displaced relative to the longitudinal slot axis at \(x_{d1}\) and \(x_{d2}\). The length and radius of the internal monopole are \(L_4\) and \(r_4\).

**Figure 1.** The geometry of the vibrator-slot structure and accepted notations.

Let the slot width and radii of vibrators satisfy the thin radiator approximations: \(\frac{r_m}{2L_m} \ll 1\), \(\frac{\lambda_m}{\lambda_{1,2}} \ll 1\), \(m = 1, 2, 4\), \(\frac{d}{2L_m} \ll 1\), \(\frac{d}{\lambda_{1,2}} \ll 1\) (\(\lambda_{1,2}\) are wavelengths in the corresponding media) and the electric vibrator currents and equivalent magnetic slot current satisfy the boundary conditions \(J_m(\pm L_m) = 0\), \(J_3(\pm L_3) = 0\) (\(L_m\) are the end coordinates of the monopole mirror images in the plane and waveguide bottom). Then the following system of integral equations can be written [29]

\[
\left(\frac{d^2}{ds_1^2} + k_1^2\right) \left\{ \int_{-L_1}^{L_1} J_1(s'_1) G_{s_1}^{HsE}(s_1, s'_1) \, ds'_1 + \int_{-L_2}^{L_2} J_2(s'_2) G_{s_2}^{HsE}(s_1, s'_2) \, ds'_2 \right\} - ik\varepsilon_{s_1} \text{rot} \int_{-L_3}^{L_3} J_3(s'_3) G_{s_3}^{HsM}(s_1, s'_3) \, ds'_3 = i\omega\varepsilon_{s_1} z_{s_1}(s_1) J_1(s_1),
\]

\[
\left(\frac{d^2}{ds_2^2} + k_2^2\right) \left\{ \int_{-L_2}^{L_2} J_2(s'_2) G_{s_2}^{HsE}(s_2, s'_2) \, ds'_2 + \int_{-L_1}^{L_1} J_1(s'_1) G_{s_1}^{HsE}(s_2, s'_1) \, ds'_1 \right\} - ik\varepsilon_{s_2} \text{rot} \int_{-L_3}^{L_3} J_3(s'_3) G_{s_3}^{HsM}(s_2, s'_3) \, ds'_3 = i\omega\varepsilon_{s_2} z_{s_2}(s_2) J_2(s_2),
\]

\[
\frac{1}{\mu_1} \left(\frac{d^2}{ds_3^2} + k_1^2\right) \int_{-L_3}^{L_3} J_3(s'_3) G_{s_3}^{WgM}(s_3, s'_3) \, ds'_3 + \frac{1}{\mu_2} \left(\frac{d^2}{ds_2^2} + k_2^2\right) \int_{-L_3}^{L_3} J_3(s'_3) G_{s_3}^{HsM}(s_3, s'_3) \, ds'_3 - i\omega H_{0s_3}(s_3)
\]

\[
+ ik\varepsilon_{s_3} \text{rot} \left\{ \int_{-L_1}^{L_1} J_1(s'_1) G_{s_1}^{HsE}(s_3, s'_1) \, ds'_1 + \int_{-L_2}^{L_2} J_2(s'_2) G_{s_2}^{HsE}(s_3, s'_2) \, ds'_2 \right. \\
+ \left. \int_{-L_4}^{L_4} J_4(s'_4) G_{s_4}^{WgE}(s_3, s'_4) \, ds'_4 \right\} = 0.
\]
In this expression, predefined current distribution functions, which can be obtained by solving the equations for currents $J(s_4, s'_4)ds_4$,

$$
\left( \frac{d^2}{ds_4^2} + k_1^2 \right) \left\{ \int_{-L_4}^{L_4} J_4(s'_4)G_{s_4}^{WgE}(s_4, s'_4)ds'_4 \right\}
$$

$$
-ik \varepsilon_s \text{rot} \int_{-L_3}^{L_3} J_3(s_3)G_{s_3}^{HsM}(s_4, s'_3)ds'_3 = -i\omega \varepsilon_1 \left[ E_{0s_4}(s_4) - z_{i4}(s_4)J_4(s_4) \right],
$$

where $z_{im}(s_m)$ are the internal linear impedances of the monopoles (Ohm/m); $H_{0s_3}(s_3)$ and $E_{0s_4}(s_4)$ are the projections of the fields of extraneous sources on the slot and monopole axes; $G_{s_1,2}^{HsE}(s_1,3, s'_1,2)$, $G_{s_3}^{WgE}(s_3, s'_3)$, and $G_{s_3}^{HsM,wgM}(s_1,2,3, s'_1,2, s'_3)$ are the components of the electric (E) and magnetic (M) tensor Green functions for the vector potential of corresponding electrodynamic volumes $[1, 3, 4]; k = 2\pi/\lambda$, $\lambda$ is the wavelength in free space; $k_{1,2} = k\sqrt{\varepsilon_{1,2}\mu_{1,2}} = 2\pi/\lambda_{1,2}$, $\varepsilon_{s_m}$, and $\varepsilon_{s_3}$ are unit vectors directed along the vibrator and slot axes; $s_m$ and $s_3$ are the local coordinates associated with axes of vibrators and slot.

If $\varepsilon_{1,2} = \mu_{1,2} = 1$ and $H_{0s_3}(s_3) = H_0 \cos \frac{2\pi a}{\lambda} e^{-ik_3 s_3} = H_0^0(s_3) + H_0^a(s_3)$, $E_{0s_4}(s_4) = H_0 \frac{k}{k_3} \sin \frac{2\pi a}{\lambda} \cos \theta$ (where $H_0$ is the amplitude of the $H_{10}$-wave), the slot current can be presented as $J_3(s_3) = J_3^a(s_3) + J_3^0(s_3)$.

In this expression, $J_3^a(s_3)$ and $J_3^0(s_3)$ are symmetric and antisymmetric components of the slot current relative to the slot center. Hence, Equation (1) can be transformed to:

$$
\left( \frac{d^2}{ds_4^2} + k_1^2 \right) \left\{ \int_{-L_4}^{L_4} J_4(s'_4)G_{s_4}^{HsE}(s_4, s'_4)ds'_4 + \int_{-L_2}^{L_2} J_2(s'_2)G_{s_2}^{HsE}(s_1, s'_2)ds'_2 \right\}
$$

$$
+ik \int_{-L_3}^{L_3} J_3^a(s'_3)\tilde{G}_{s_3}^{HsM}(s_1, s'_3)ds'_3 = i\omega z_{i1}(s_1)J_1(s_1),
$$

$$
\left( \frac{d^2}{ds_2^2} + k_2^2 \right) \left\{ \int_{-L_2}^{L_2} J_2(s'_2)G_{s_2}^{HsE}(s_2, s'_2)ds'_2 + \int_{-L_1}^{L_1} J_1(s'_1)G_{s_1}^{HsE}(s_2, s'_1)ds'_1 \right\}
$$

$$
+ik \int_{-L_3}^{L_3} J_3^a(s'_3)\tilde{G}_{s_3}^{HsM}(s_2, s'_3)ds'_3 = i\omega z_{i2}(s_2)J_2(s_2),
$$

$$
\left( \frac{d^2}{ds_3^2} + k_3^2 \right) \int_{-L_3}^{L_3} J_3^0(s'_3) \left[ G_{s_3}^{WgM}(s_3, s'_3) + G_{s_3}^{HsM}(s_3, s'_3) \right] ds'_3
$$

$$
-ik \left\{ \int_{-L_4}^{L_4} J_4(s'_4)G_{s_4}^{WgE}(s_4, s'_4)ds'_4 + \int_{-L_2}^{L_2} J_2(s'_2)G_{s_2}^{WgE}(s_3, s'_2)ds'_2 \right\}
$$

$$
+ \int_{-L_4}^{L_4} J_4(s'_4)\tilde{G}_{s_4}^{WgE}(s_4, s'_4)ds'_4 = -i\omega H_0^0(s_3),
$$

$$
\left( \frac{d^2}{ds_3^2} + k_3^2 \right) \int_{-L_3}^{L_3} J_3^a(s'_3) \left[ G_{s_3}^{WgM}(s_3, s'_3) + G_{s_3}^{HsM}(s_3, s'_3) \right] ds'_3 = -i\omega H_0^0(s_3),
$$

$$
\left( \frac{d^2}{ds_4^2} + k_4^2 \right) \int_{-L_4}^{L_4} J_4(s'_4)G_{s_4}^{WgE}(s_4, s'_4)ds'_4 -ik \int_{-L_3}^{L_3} J_3^a(s'_3)\tilde{G}_{s_3}^{WgM}(s_4, s'_3)ds'_3
$$

$$
= -i\omega \left[ E_{0s_4}(s_4) - z_{i4}(s_4)J_4(s_4) \right].
$$

The solution to the equation system (2) can be found by the generalized method of induced EMMF, using the functions $J_m(s_m) = J_{0m}f_m(s_m)$ and $J_{3,0}^a(s_3) = J_{03}^aJ_{3,0}^a(s_3)$ as approximating expressions for the currents. In the above expressions, $J_{0m}$ and $J_{03}^a$ are current amplitudes; $f_m(s_m)$ and $f_{3,0}^a(s_3)$ are predefined current distribution functions, which can be obtained by solving the equations for currents in a stand-alone vibrator and slot by the averaging method $[1, 4]$. The following expressions for the distribution function can be obtained:

$$
J_1(s_1) = \cos k_1^3 L_1,
$$

$$
J_2(s_2) = \cos k_2^2 L_2,
$$

$$
J_3^a(s_3) = \cos k_3 s_3 \cos k_3 L_3 - \cos k_3 L_3 \cos k_3 s_3,
$$

$$
J_3^0(s_3) = \sin k_3 s_3 \sin k_3 L_3 - \sin k_3 L_3 \sin k_3 s_3,
$$

$$
J_4(s_4) = \cos k_4^3 L_4,
$$

$$
J_5(s_5) = \cos k_5^2 L_5.
$$
where \( k_g = \frac{2\pi}{\lambda_g} = \sqrt{k^2 - k_c^2}, \) \( k_c = \frac{2\pi}{\lambda_c} = \frac{\pi}{a}, \) \( \lambda_g \) is the wavelength in the waveguide; \( \lambda_c \) is the critical wavelength of \( H_{10} \)-wave; \( \bar{k}_m = k + \frac{2\pi z_{im}}{Z_0}, \) \( z_{im} = \frac{1}{2L_m} \int_{L_m} z_{im}(s_m)ds_m \) are the impedances averaged over the vibrator length, and \( Z_0 = 120\pi \) Ohm.

Let us first multiply each of Equations (2a), (2b), (2c), and (2d) by functions \( f_1(s_1), f_2(s_2), f_3(s_3), \) \( f_4(s_4) \) and \( f_1(s_4) \) respectively, and then integrate the multiplication results over the vibrator and slot length lengths. Thus, the following system of linear algebraic equations (SLAE) can be obtained

\[
\begin{align*}
J_01 (Z_{11} + F_1^2) + J_02 Z_{12} + J_{03}^a Z_{13} &= 0, \\
J_02 (Z_{22} + F_2^2) + J_01 Z_{21} + J_{03}^s Z_{23} &= 0, \\
J_{03}^s (Z_{33}^s W_g + Z_{33}^s H_s) + J_01 Z_{31} + J_02 Z_{32} + J_{04} Z_{34} &= \frac{i\omega}{2k} H_3^s, \\
J_{03}^a (Z_{33}^a W_g + Z_{33}^a H_s) &= -\frac{i\omega}{2k} H_3^a, \\
J_04 (Z_{44} + F_4^2) + J_{03}^a Z_{43} &= -\frac{i\omega}{2k} E_4,
\end{align*}
\]

where \( H_3^s = H_0 \cos \frac{\pi x_{0sl}}{a} \int_{-L_3}^{L_3} \cos k_g s_3 f_3^s(s_3)ds_3, \) \( H_3^a = -i H_0 \cos \frac{\pi x_{0sl}}{a} \int_{-L_3}^{L_3} \sin k_g s_3 f_3^a(s_3)ds_3, \) \( E_4 = H_0 \frac{k_s}{\sqrt{\alpha}} \sin \frac{\pi x_{0lv}}{a} \int_{-L_4}^{L_4} f_4(s_4)ds_4, \) \( Z_{mn} \) \((m, n = 1, 2, 3, 4)\) and \( F_m^2 \) are dimensionless coefficients. When the amplitudes of currents \( J_{0m} \) and \( J_{03}^a \) are found as the solution of system in Eq. (4), the electrodynamic characteristics of this vibrator-slot structure can be easily obtained.

3. GENERAL ANALYSIS OF THE CLAVIN ELEMENT

The following relations for the Clavin element are valid: \( 2L_1 = 2L_2 = 2L_v, r_1 = r_2 = r, \alpha_1 = \alpha_2 = \alpha, \) \( 2L_4 = 2L_v, r_4 = r_v, \alpha_4 = \alpha_V, \) \( Z_{51}(s_1) = Z_{52}(s_2) = Z_{53}(s_3) = Z_{54}(s_4) = Z_{SV}(s_V) = 2\pi r_v z_{iv}(s_v)/Z_0, \) \( Z_{51}(s_1) = Z_{SV}(s_V) = 2\pi r_v z_{iv}(s_v)/Z_0, \) \( \hat{k}_1 = \hat{k}_2 = \hat{k} = k + i(\alpha/r)V \) \( \hat{k}_4 = \hat{k}_V = k + i(\alpha_V/r_v)Z_{SV}^0, \) \( x_{d1} = x_{d2} = x_d, \) \( F_1^2 = F_2^2 = F_3^2 = F_4^2 = F_V^2, \) \( H_{33}^a = H_{33}^s = \left( Z_{33}^{s,a W_g} + Z_{33}^{s,a H_s} \right) = Z_{33}^{s,a \Sigma}, \) \( f_1(s_1) = f_2(s_2) = f_4(s_4), \) \( f_{33}^{s,a}(s_3) = f_{33}^{s,a}(s_4), \) \( Z_{11} + F_1^2 = Z_{22} + F_2^2 = Z_v + F_3^2 = Z_{33}^{s,a \Sigma}, \) \( Z_{12} = Z_{21} = Z_{44}, Z_{13} = Z_{23} = -2Z_{31} = -2Z_{32} = Z_c, Z_{34} = -Z_{43} = Z_{Wg}. \)

Therefore, the SLAE (4) can be transformed as follows:

\[
\begin{align*}
J_{0v} Z_{Vv}^c + J_{0w} Z_{Vv} + J_{03}^s Z_{Vc} &= 0, \\
J_{03}^s Z_{Vc} - J_{0v} Z_c + J_{0v} Z_{Wg} &= -\frac{i\omega}{2k} H_{3l}^s, \\
J_{03}^a Z_{Vc} - \frac{i\omega}{2k} H_{3l}^a, \\
J_{0v} Z_{Vl}^c - J_{03}^a Z_{Wg} &= -\frac{i\omega}{2k} E_V.
\end{align*}
\]

The solution of equations system (5) can be written as:

\[
\begin{align*}
J_{0v} &= \frac{i\omega}{2k} H_V \frac{Z_c}{Z_{sl} Z_v + Z_c^2}, \quad J_{0v}^a = -\frac{i\omega}{2k} \left[ E_V \frac{1}{Z_{sl}^c} + H_V \frac{\tilde{Z}_v Z_{Wg}}{(Z_{sl} Z_v + Z_c^2) Z_v^c} \right], \\
J_{03}^s &= -\frac{i\omega}{2k} H_{3l}^s \frac{Z_{Vl}^c}{Z_{sl} Z_v + Z_c^2}, \quad J_{03}^a = -\frac{i\omega}{2k} H_{3l}^a \frac{1}{Z_{sl} Z_v + Z_c^2},
\end{align*}
\]

where \( \tilde{Z}_v = Z_{Vl}^c + Z_{Wg}, \) \( \tilde{Z}_d = Z_{sl}^s + Z_{Wg}^2 Z_{Wg}^c, \) \( H_V = H_{3l}^s - E_V Z_{Wg}/Z_{Wg}^c. \) The components of the tensor Green functions for the Clavin element can be written as:

\[
G_{s_v}^{HsE} (s_v, s_v') = \frac{e^{-i k \sqrt{(s_v - s_v')^2 + r^2 (4x_d^2)}}}{\sqrt{(s_v - s_v')^2 + r^2 (4x_d^2)}}, \quad G_{s_{sl}}^{HsM} (s_{sl}, s_{sl}') = 2 \frac{e^{-i k \sqrt{(s_v - s_v')^2 + (d_e/4)^2}}}{\sqrt{(s_v - s_v')^2 + (d_e/4)^2}}.
\]
\[ G_{s,s'}^{WgM}(s_{sl}, s'_{sl}) = \frac{2\pi}{ab} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\varepsilon_m \varepsilon_n}{k_{zt}} e^{-k_{zt}s_{sl}-s'_{sl}} \cos k_{zt}x_0 \cos k_{zt} \left( x_0 + \frac{d_{e}}{4} \right), \]

\[ \tilde{G}_{s,s'}^{HsE}(s_{sl}, s'_{sl}) = \frac{\partial}{\partial x_{Hs}} G_{s,s'}^{HsE}(x_{Hs}, 0, z_{Hs}(s_{sl}); x_d, y_{Hs}^{'}(s'_{sl}), 0) \text{ at } x_{Hs} = 0, \]

\[ G_{s,s'}^{HsM}(s_v, s_{sl}') = \frac{\partial}{\partial x_{ws}} G_{s,s'}^{HsM}(x_{ws}, y_{Hs}(s_v), 0, 0, z_{Hs}^{'}(s_{sl}')) \text{ at } x_{ws} = x_d, \]

\[ \tilde{G}_{s,s'}^{WgE}(s_{sl}, s'_{sl}') = \frac{\partial}{\partial x_{wg}} G_{s,s'}^{WgE}(x_{wg}, 0, z_{wg}(s_{sl}); x_{0w}, y_{wg}(s'_{sl}'), 0) \text{ at } x_{wg} = x_{0w}, \]

\[ G_{s,s'}^{WgM}(s'_{sl}, s_{sl}') = \frac{\partial}{\partial x_{wg}} G_{s,s'}^{WgM}(x_{wg}, y_{wg}(s_{sl}'), 0; x_{0w}, 0, z_{wg}^{'}(s'_{sl}')) \text{ at } x_{wg} = x_{0w}. \]

where \( \varepsilon_n = \left\{ \begin{array}{ll} 1, & n = 0 \\ 2, & n \neq 0 \end{array} \right\} \), \( k_x = \frac{m\pi}{a} \), \( k_{y} = \frac{n\pi}{b} \), \( k_{z} = \sqrt{k_{x}^{2} + k_{y}^{2} - k_{z}^{2}} \), \( m \) and \( n \) are integers; \( d_{e} \approx d \exp\left(-\frac{\pi h}{2a}\right) \) is the effective slot width \([1]\), which takes into account the thickness \( h \) of the waveguide wall. All quantities in the expressions for the current amplitudes in Eq. (6) are:

\[ Z_{v(vv)} = \frac{1}{2k} \int_{-L_{v}}^{L_{v}} f_{v}(s) \left( \int_{-L_{v}}^{L_{v}} f_{v}(s') \left( \frac{d^{2}}{ds^{2}} + k^{2} \right) G_{s,s'}^{HsE}(s, s') ds' \right) ds, \]

\[ F_{v(vv)}(s) = \int_{-L_{v}}^{L_{v}} f_{v}(s') \frac{e^{-ik\sqrt{(s-s')^{2}+r^{2}(4x_{d}^{2})}}}{\sqrt{(s-s')^{2}+r^{2}(4x_{d}^{2})}} ds', \]

\[ F_{v}^{Z} = -\frac{i}{r} \int_{0}^{L_{v}} f_{v}^{Z}(s) \tilde{Z}_{S}(s) ds, \]

where \( \tilde{Z}_{S}(s) = \tilde{R}_{S} + i\tilde{X}_{S}\phi(s), \tilde{Z}_{SV}(s) = \tilde{R}_{SV} + i\tilde{X}_{SV}\phi_{V}(s), \phi(s) \) and \( \phi_{V}(s) \) are predefined functions.
Then, the final expressions for the currents based on Eq. (3) can be written as:

\[
F_e^s = \frac{k \cos g_L k_L}{k_x^2 + k_y^2} \left[ k_z \cos k_L k_L \left( 1 - e^{-2k_L L} \right) + k \sin k_L k_L \left( 1 + e^{-2k_L L} \right) \right] - \frac{k \cos k_L k_L}{k_x^2 + k_y^2} \left[ k_z \cos k_L k_L \left( 1 - e^{-2k_L L} \right) + k \sin k_L k_L \left( 1 + e^{-2k_L L} \right) \right],
\]

\[
F_k^s = 2 \cos g_L k_L \frac{\sin k_L k_L \cos k_L k_L - (g_k/k) \cos k_L k_L \sin g_L k_L}{1 - (g_k/k)^2} - \cos k_L k_L \frac{2g_L k_L + 2g_L k_L}{2(g_k/k)},
\]

\[
Z_{sl}^{M_W} = \frac{2\pi}{ab} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\varepsilon_m \varepsilon_n}{k^2} \cos k_x x_0 \cos k_x \left( x_0 + \frac{dx}{4} \right) \times \left\{ \begin{array}{l} \left[ -\sin g_L k_L \left( \frac{k}{k_x} \cos k_L k_L + \sin k_L k_L \right) \right] F_e^a \\ + \left[ \frac{\sin k_L k_L}{k_x^2 + k_y^2} \left( k_z^2 + k_y^2 \right) \left( \frac{k_z^2}{k_y} \cos k_L k_L + \sin k_L k_L \right) \right] F_e^a + \frac{k_z^2}{k_y} F_e^a \end{array} \right\},
\]

\[
F_e^a = \frac{k \sin g_L k_L}{k_x^2 + k_y^2} \left[ k_z \sin k_L k_L \left( 1 + e^{-2k_L L} \right) - k \cos k_L k_L \left( 1 - e^{-2k_L L} \right) \right] - \frac{k \sin k_L k_L}{k_x^2 + k_y^2} \left[ k_z \sin g_L k_L \left( 1 + e^{-2k_L L} \right) - g_k \cos g_L k_L \left( 1 - e^{-2k_L L} \right) \right],
\]

\[
F_k^a = 2 \sin g_L k_L \frac{\cos k_L k_L \sin k_L k_L - (g_k/k) \sin k_L k_L \cos g_L k_L}{1 - (g_k/k)^2} - \sin k_L k_L \frac{2g_L k_L + 2g_L k_L}{2(g_k/k)},
\]

\[
Z_V = \frac{4\pi}{ab} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\varepsilon_n}{kk_z \left( k_v^2 - k_y^2 \right)} e^{k_L z_V} \sin^2 k_x x_0 \left[ \sin k_v L_V \cos k_y L_V - \left( k_v/k_y \right) \cos k_v L_V \sin k_y L_V \right]^2,
\]

\[
Z_W = \frac{4\pi}{ab} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\varepsilon_n k_x k_v}{kk_z \left( k_v^2 - k_y^2 \right)} e^{k_L z_V} \sin k_x x_0 \cos k_x x_0 \times F_e^a \left[ \sin k_v L_V \cos k_y L_V - \left( k_v/k_y \right) \cos k_v L_V \sin k_y L_V \right] + F_e^a \left[ \sin k_v L_V - k_v L_V \cos k_v L_V \right],
\]

\[
H_{sl}^{(a)} = \frac{1}{k} \frac{1}{k} H_0 \frac{\pi x_{0s}}{a} F_k^{(a)},
\]

\[
E_V = H_0 \frac{k}{k_y k_v} \frac{\pi x_{0V}}{a} f \left( \tilde{k}_v L_V \right), F_V = \sin \tilde{k}_v L_V - \tilde{k}_v L_V \cos \tilde{k}_v L_V.
\]

Then, the final expressions for the currents based on Eq. (3) can be written as:

\[
J_v(s_v) = -\frac{i\omega}{2k^2} H_0 J_v f_v(s_v), \quad J_V(s_V) = -\frac{i\omega}{2k^2} H_0 J_v f_V(s_V),
\]

\[
J_{sl}(s_{sl}) = -\frac{i\omega}{2k^2} H_0 \left[ J_{sl}^a f_{sl}^a(s_{sl}) + iJ_{sl}^a f_{sl}^a(s_{sl}) \right],
\]

where

\[
J_v = -\frac{Z_{sl} F_{sl,V}}{Z_{sl} Z_v + Z_c^2}, \quad J_V = \frac{k^2 \sin(\pi x_{0V}/a) F_V}{k_k k_v V^2} + \frac{Z_{sl} Z_{sl} F_{sl,V}}{(Z_{sl} Z_v + Z_c^2) Z_c^2},
\]

\[
J_{sl}^a = \frac{Z_{sl} F_{sl,V}}{Z_{sl} Z_v + Z_c^2}, \quad J_{sl}^a = -\frac{\pi x_{0sl}}{a} \frac{F_k^a}{Z_{sl}^2}, \quad F_{sl,V} = \frac{\pi x_{0sl}}{a} F_k^a - \frac{k^2}{k_k k_v} \sin \frac{\pi x_{0V}}{a} F_V.
\]
Then, in accordance with formulas (3), we obtain:

\[ S_{11} = \frac{2\pi}{iabk_gk} \left[ k_2^2 \cos \frac{\pi x_{0sl}}{a} (J_{sl}^sF_k^s + J_{sl}^aF_k^a) - \frac{2k_3}{k_V} \sin \frac{\pi x_0V}{a} J_VF_V \right] e^{2ik_gz}, \] (9)

\[ S_{12} = 1 - \frac{2\pi}{iabk_gk} \left[ k_2^2 \cos \frac{\pi x_{0sl}}{a} (J_{sl}^sF_k^s - J_{sl}^aF_k^a) + \frac{2k_3}{k_V} \sin \frac{\pi x_0V}{a} J_VF_V \right], \] (10)

\[ |S_{12}|^2 = 1 - |S_{11}|^2 - |S_{12}|^2. \] (11)

Formulas (8)–(11) are obtained by assuming that \( H_0 = 1 \), and the vibrators are losses. In the spherical coordinate system shown in Fig. 2, the far-zone electric field of the Clavin element is defined by the expression

\[ \vec{E}(R, \theta, \varphi) = \frac{i k^2}{\omega} e^{-ikR} \left[ \vec{\vartheta}^0 \sin \theta \left( \vec{E}_1 e^{-ikx_1x_1 \sin \theta \sin \varphi} + \vec{E}_2 e^{ikx_2x_2 \sin \theta \sin \varphi} \right) + \left( \varphi^0 \cos \theta \cos \varphi + \vec{\vartheta}^0 \sin \varphi \right) 2\vec{E}_3 \right], \] (12)

where \( \vec{\vartheta}^0 \) and \( \varphi^0 \) are unit vectors, \( \vec{E}_1 = J_{01}f_{C1}, \vec{E}_2 = J_{02}f_{C2}, \vec{E}_3 = J_{03}f_{C3} + J_{03}f_{C3}^a \);

\[ f_{C1} = \int_{-L_1}^{L_1} f_1(z)e^{ikz \cos \theta} dz, \quad f_{C2} = \int_{-L_2}^{L_2} f_2(z)e^{ikz \cos \theta} dz, \quad f_{C3}^{(a)} = \int_{-L_3}^{L_3} f_3^{(a)}(x)e^{ikx \sin \theta \cos \varphi} dx. \]

Then, in accordance with formulas (3), we obtain:

\[ f_{Cm} = \frac{2}{k_m^2 - (k \cos \theta)^2} \left[ k_m \cos (kL_m \cos \theta) \sin (k_mL_m) - k \sin (kL_m \cos \theta) \cos (k_mL_m) \cos \theta \right] \]

\[ -2L_m \cos (k_mL_m) \sin (kL_m \cos \theta) \cos (k_mL_m) \cos \theta, \quad m = 1, 2; \]

\[ f_{C3}^s = \frac{2 \cos (k_3L_3)}{k - k(sin \theta \cos \varphi)^2} \left[ \cos (kL_3 \sin \theta \cos \varphi) \sin (kL_3) - \sin (kL_3 \sin \theta \cos \varphi) \cos (kL_3) \sin \theta \cos \varphi \right] \]

\[ - \frac{2 \cos (kL_3)}{k_L^2 - (k \sin \theta \cos \varphi)^2} \left[ k_g \cos (kL_3 \sin \theta \cos \varphi) \sin (k_gL_3) \right. \]

\[ - k \sin (kL_3 \sin \theta \cos \varphi) \cos (k_gL_3) \sin \theta \cos \varphi \right], \]

\[ f_{C3}^a = \frac{2i \sin (k_3L_3)}{k - k(sin \theta \cos \varphi)^2} \left[ - \sin (kL_3 \sin \theta \cos \varphi) \cos (kL_3) + \cos (kL_3 \sin \theta \cos \varphi) \sin (kL_3) \sin \theta \cos \varphi \right] \]

\[ - \frac{2i \sin (kL_3)}{k_L^2 - (k \sin \theta \cos \varphi)^2} \left[ - k \sin (kL_3 \sin \theta \cos \varphi) \cos (kL_3) \right. \]

\[ + k \cos (kL_3 \sin \theta \cos \varphi) \sin (kL_3) \sin \theta \cos \varphi]. \]
After substitution the amplitude where

\[ Z = \text{...} \]

If the combined radiator does not include the internal vibrator (Fig. 3), the SLAE (4) can be simplified to

4. VIBRATOR-SLOT RADIATOR BASED ON A HOLLOW RECTANGULAR WAVEGUIDE

If the combined radiator does not include the internal vibrator (Fig. 3), the SLAE (4) can be simplified to

\[
\begin{align*}
J_{01}Z_{11}^2 + J_{02}Z_{12} + J_{03}^*Z_{13} &= 0, \\
J_{02}Z_{11}^2 + J_{01}Z_{21} + J_{03}^*Z_{23} &= 0, \\
J_{03}Z_{13}^* + J_{01}Z_{31} + J_{02}Z_{32} &= -\frac{i\omega}{2k}H_3^s, \\
J_{03}Z_{13}^* &= -\frac{i\omega}{2k}H_3^s,
\end{align*}
\]

where \( Z_{11}^2 = Z_{11} + F_1^2 \), \( Z_{22}^* = Z_{22} + F_2^2 \), \( Z_{33}^{s(\Sigma)} = Z_{33}^{s(a)Wg} + Z_{33}^{s(a)Hs} \).

Solution of equations system (13) can be written as

\[
\begin{align*}
J_{01} = \frac{i\omega}{2k}H_3^s \frac{Z_{11}^2 Z_{13} - Z_{13} Z_{12}}{Z_{12} + Z_{32} Z_{11}}, \\
J_{02} = \frac{i\omega}{2k}H_3^s \frac{Z_{11}^2 Z_{23} - Z_{13} Z_{21}}{Z_{33}^\Sigma Z_{12} + Z_{31} Z_{22} + Z_{32} Z_{11}}, \\
J_{03}^* = -\frac{i\omega}{2k}H_3^s \frac{Z_{12}^*}{Z_{33}^\Sigma Z_{12} + Z_{31} Z_{22} + Z_{32} Z_{11}}, \\
J_{03}^* = -\frac{i\omega}{2k}H_3^s \frac{1}{Z_{33}^\Sigma}.
\end{align*}
\]

Since \( H_3^{s(a)} = \frac{1}{1-k^2} \frac{\pi x_0}{a} f_k^{s(a)} \), the final expressions for the currents can be presented as:

\[
\begin{align*}
J_1(s_1) &= \frac{i\omega}{2k^2}H_0 f_1(s_1), \\
J_2(s_2) &= \frac{i\omega}{2k^2}H_0 f_2(s_2), \\
J_3(s_3) &= -\frac{i\omega}{2k^2}H_0 \left[ f_3^s(s_3) + i f_3^a(s_3) \right],
\end{align*}
\]

where

\[
\begin{align*}
J_1 &= \cos \frac{\pi x_0}{a} f_k^s \frac{Z_{11}^\Sigma Z_{13} - Z_{13} Z_{12}}{Z_{33}^\Sigma Z_{12} + Z_{31} Z_{22} + Z_{32} Z_{11}}, \\
J_2 &= \cos \frac{\pi x_0}{a} f_k^s \frac{Z_{11}^\Sigma Z_{23} - Z_{13} Z_{21}}{Z_{33}^\Sigma Z_{12} + Z_{31} Z_{22} + Z_{32} Z_{11}}, \\
J_3 &= \cos \frac{\pi x_0}{a} f_k^s \frac{Z_{11}^\Sigma Z_{12}^*}{Z_{33}^\Sigma Z_{12} + Z_{31} Z_{22} + Z_{32} Z_{11}}, \\
J_3 &= -\cos \frac{\pi x_0}{a} f_k^s \frac{1}{Z_{33}^\Sigma}, \\
Z_{12} &= Z_{11}^\Sigma Z_{22}^* - Z_{12} Z_{21}, \\
Z_{11} &= Z_{13} Z_{21} - Z_{11}^\Sigma Z_{23}, \\
Z_{22} &= Z_{23} Z_{12} - Z_{22}^\Sigma Z_{13}.
\end{align*}
\]

After substitution the amplitude \( H_0 = 1 \) in formulas (15a) and (15b), the radiation field is determined.
by formula (12). The energy characteristics of this structure can be obtained in the form:

\[
S_{11} = - \frac{2\pi k^2}{iabk} \cos \frac{\pi x_0}{a} \left( J_{31}^a F_k^s + J_{30}^a F_k^0 \right) e^{2iksz},
\]

\[
S_{12} = 1 - \frac{2\pi k^2}{iabk} \cos \frac{\pi x_0}{a} \left( J_{31}^a F_k^s - J_{30}^a F_k^0 \right),
\]

\[
|S_{12}|^2 = 1 - |S_{11}|^2 - |S_{12}|^2,
\]

The components of the Green functions for this case can be written in the form:

\[
G_{s_{1(2)}}^{HsE} \left( s_{1(2)}, s'_{1(2)} \right) = \frac{e^{-ik\sqrt{\left(s_{1(2)} - s'_{1(2)}\right)^2 + r_{1(2)}^2}}}{\sqrt{\left(s_{1(2)} - s'_{1(2)}\right)^2 + r_{1(2)}^2}} \sqrt{\left(s_{1(2)} - s'_{2(1)}\right)^2 + (x_d + x_d')^2}.
\]

\[
G_{s_{2(1)}}^{HsE} \left( s_{1(2)}, s'_{2(1)} \right) = \frac{e^{-ik\sqrt{\left(s_{1(2)} - s'_{2(1)}\right)^2 + (x_d + x_d')^2}}}{\sqrt{\left(s_{1(2)} - s'_{2(1)}\right)^2 + (x_d + x_d')^2}}.
\]

\[
G_{s_{33}}^{HsM} \left( s_{3}, s'_{3} \right) = 2\frac{e^{-ik\sqrt{(s_3 - s'_3)^2 + (d_e/4)^2}}}{\sqrt{(s_3 - s'_3)^2 + (d_e/4)^2}}.
\]

Then, the corresponding coefficients in formulas (14) and (16) are:

\[
Z_{11(22)} = \frac{1}{2k} \int_{-L_{1(2)}}^{L_{1(2)}} f_1(s_{1(2)}) \int_{-L_{1(2)}}^{L_{1(2)}} f_1(s'_{1(2)}) \left( \frac{d^2}{ds_1^2} + k^2 \right) G_{s_{1(2)}}^{HsE} \left( s_{1(2)}, s'_{1(2)} \right) ds_1 \left( s_{1(2)} \right) ds_{1(2)},
\]

\[
F_1(s_{1(2)}) = \int_{-L_{1(2)}}^{L_{1(2)}} f_1(s'_{1(2)}) \frac{e^{-ik\sqrt{\left(s_{1(2)} - s'_{1(2)}\right)^2 + r_{1(2)}^2}}}{\sqrt{\left(s_{1(2)} - s'_{1(2)}\right)^2 + r_{1(2)}^2}} ds_{1(2)}.
\]

\[
Z_{12} = Z_{21} = \frac{1}{2k} \int_{-L_{1}}^{L_{1}} f_1(s_1) \int_{-L_{2}}^{L_{2}} f_2(s'_2) \left( \frac{d^2}{ds_1^2} + k^2 \right) G_{s_{22}}^{HsE} \left( s_{1}, s'_{2} \right) ds_1,
\]

\[
F_{12}(s_{1(2)}) = \int_{-L_{1}}^{L_{1}} f_2(s'_2) \frac{e^{-ik\sqrt{(s_1 - s'_2)^2 + (x_d + x_d')^2}}}{\sqrt{(s_1 - s'_2)^2 + (x_d + x_d')^2}} ds'_2,
\]

\[
F_{1(2)}^Z = - \frac{i}{r_{1(2)}} \int_{0}^{L_{1(2)}} f_1(s_{1(2)}) \bar{Z} s_{1(2)} \left( s_{1(2)} \right) ds_{1(2)}.
\]
where \( Z_{S1(2)}(s_{1(2)}) = \hat{R}_{S1(2)} + i\hat{X}_{S1(2)}\phi_{1(2)}(s_{1(2)}), \phi_{1(2)}(s_{1(2)}) \) are predefined functions,

\[
Z_{1(2)} = 2i x_{d1(2)} \int_{-L_{1(2)}}^{L_{1(2)}} f_{1(2)}(s_{1(2)}) \left\{ \int_{-L_{3}}^{L_{3}} \left( \begin{array}{c} \frac{e^{-ik\sqrt{s_{1(2)}^2 + s_{3}^2 + x_{d1(2)}^2}}}{\sqrt{s_{1(2)}^2 + s_{3}^2 + x_{d1(2)}^2}} \\ \left( s_{1(2)}^2 + s_{3}^2 + x_{d1(2)}^2 \right)^{3/2} \\ \times \left( s_{1(2)}^2 + s_{3}^2 + x_{d1(2)}^2 \right)^{1/2} \end{array} \right) ds_{3} \right\} ds_{1(2)},
\]

\[
Z_{31(2)} = ix_{d1} \int_{-L_{3}}^{L_{3}} f_{3}(s_{3}) \left\{ \int_{-L_{1(2)}}^{L_{1(2)}} f_{1(2)}(s_{1(2)}) \left( \begin{array}{c} e^{-ik\sqrt{s_{1(2)}^2 + s_{3}^2 + x_{d1(2)}^2}} \\ \left( s_{1(2)}^2 + s_{3}^2 + x_{d1(2)}^2 \right)^{3/2} \\ \times \left( s_{1(2)}^2 + s_{3}^2 + x_{d1(2)}^2 \right)^{1/2} \end{array} \right) ds_{3} \right\} ds_{3},
\]

The expressions \( Z_{33}^{(a)\Sigma} = Z_{33}^{(a)Wg} + Z_{33}^{(a)Hs} \) and \( F_{k}^{(a)} \) analogues to that defined in Section 3 can be used after replacing index \( sl \) by \( 33 \).

Thus, the equation system for the Clavin element on the waveguide can be written as

\[
\begin{align*}
J_{0v1}Z_{v}^\Sigma - J_{0v}Z_{vu} + J_{0sl}Z_{c} &= 0, \\
J_{0sl}Z_{sl}^\Sigma - J_{0v}Z_{c} &= -\frac{i\omega}{2k}H_{sl}^a, \\
J_{0sl}Z_{sl}^\Sigma &= -\frac{i\omega}{2k}H_{sl}^a.
\end{align*}
\]

The complex current amplitudes obtained by solving the equation system (21) are presented as:

\[
J_{0v1} = -\frac{i\omega}{2k}H_{sl}^a \frac{Z_{c}^{\Sigma}}{Z_{sl}^{\Sigma} (Z_{v}^\Sigma - Z_{vu}) - Z_{c}^{\Sigma}}, \quad J_{0v2} = -J_{0v1},
\]

\[
J_{0sl} = -\frac{i\omega}{2k}H_{sl}^a \frac{Z_{v}^\Sigma - Z_{vu}}{Z_{sl}^{\Sigma} (Z_{v}^\Sigma - Z_{vu}) - Z_{c}^{\Sigma}}, \quad J_{0sl}^a = -\frac{i\omega}{2k}H_{sl}^a \frac{1}{Z_{sl}^{\Sigma}}.
\]

The current distribution along the vibrators and slot can be determined by the following expressions:

\[
J_{v}(s) = \frac{i\omega}{2k^2}H_{0}\cos \frac{\pi x_{0}}{a} J_{v}(s), \quad J_{sl}(s) = \frac{i\omega}{2k^2}H_{0}\cos \frac{\pi x_{0}}{a} \left[ J_{sl}^s f_{sl}(s) + iJ_{sl}^a f_{sl}(s) \right],
\]

where \( J_{v} = \frac{Z_{v}F_{k}^a}{Z_{v}^2 + Z_{vu} + Z_{c}^2}, J_{sl}^s = \frac{(Z_{v}^\Sigma + Z_{vu})F_{k}^a}{Z_{v}^\Sigma (Z_{v}^\Sigma + Z_{vu}) + Z_{c}^2}, J_{sl}^a = \frac{F_{k}^a}{Z_{sl}^{\Sigma}} \).

All coefficients defined in Section 3 and the energy characteristics for this structure are defined by expressions (17)–(19) after replacing index \( 33 \) by index \( sl \).

5. NUMERICAL AND EXPERIMENTAL RESULTS

The combined Clavin radiator consists of two identical perfectly conducting monopoles located on the plane screen symmetrically to the slot axis [21]. Clavin et al. have shown experimentally that RPs in the \( H \)-plane (\( \varphi = 0^\circ \)) and \( E \)-planes (\( \varphi = 90^\circ \)) are approximately identical if the monopoles with length \( L_{v} = 0.375\lambda \) are placed symmetrical to the slot axis at distances \( x_{d} = 0.086\lambda \). It is quite clear that according to expression (12), the RP in the \( H \)-plane has only the \( E_{\varphi} \) component, which coincides with the RP of stand-alone slot since the monopole currents with equal amplitudes are in anti-phase relative to each other. The RP in the plane \( \varphi = 0^\circ \) for the configuration with radiators placed over the infinite screen is shown in Fig. 4(a) and marked by a solid curve. According to Eq. (12), the RP in the \( E \)-plane has only the \( E_{\varphi} \) component, and its shape can be made closer to the \( H \)-plane RP shape by varying the vibrator currents. The RP for this case is shown in Fig. 4(a) by the dashed curve 1. Some improvement in the coincidence of the RP of the radiator in two planes in comparison with that observed in Fig. 4(a) in [21] was achieved due to the use of L-shaped vibrators.

The external problem solution by using trigonometric approximations for the slot and vibrator currents [23] have allowed to obtain the radiator parameters \( L_{v} = 0.365\lambda, x_{d} = 0.065\lambda \) which are a little
Since the ratios $\frac{2}{\lambda}$ of side radiation and the distance between the slot and monopole. These plots allow us to obtain $\theta$ and $x$ in Fig. 4(a) are marked in Fig. 4(b) by circles. It is possible to conclude that there exist two options to obtain RPs with almost equal RP widths in the $E$- and $H$-plane $\Delta \theta$ as functions of $L_v/\lambda$ and $x_d/\lambda$.

The influence of parameters $L_v/\lambda$ and $2x_d/\lambda$ on the directivity characteristics of the Clavin type radiators is studied by computing the difference $\Delta \theta$ between the RP widths in $E$- and $H$-planes at $-3$ dB level and the level of lateral radiation $\delta$, i.e., the maximum of $E$-plane RP.

The plots of $\delta$ and $\Delta \theta$ as functions of $x_d/\lambda$ and $L_v/\lambda$ are shown Fig. 4(b) where $\delta$ and $\Delta \theta$ are represented by the color scale level and level curves. The simulation results presented in Fig. 4(b) make it possible to conclude that there exist two options to obtain RPs with almost equal RP widths in the $E$ and $H$ planes. These options presented in Fig. 4(a) by curves 2 and 3 allow us to obtain RPs with a minimum level of side lobes equal to $-20$ dB ($L_v=0.3125\lambda$, $x_d=0.086\lambda$, curve 2) or with a low level of side radiation $\delta = -31$ dB ($L_v=0.3\lambda$, $x_d=0.086\lambda$, curve 3). Parameters $L_v$ and $x_d$ for the curves in Fig. 4(a) are marked in Fig. 4(b) by circles.

The energy characteristics of radiator were obtained with following parameters: $\lambda = 32$ mm, $a \times b = 23 \times 10$ mm$^2$, $h = 1$ mm, $2L_s = 16$ mm ($2L_s = 0.5\lambda$), $d = 1.5$ mm, $x_0 = 2.5$ mm, $r = 0.17$ mm. Since the ratios $2r_v/L_v$ and $[d/(2L_s)]$ do not exceed 0.1, the thin wire and narrow slot approximations were used during simulation. The energy characteristics of the combined Clavin radiators, whose RPs are presented in Fig. 4(a), are summarized in Table 1.

Table 1. The energy characteristics of combine Clavin radiators.

| Geometric parameters | $|S_{11}|$ | $|S_{12}|$ | $|S_{13}|^2$ | $D$ |
|----------------------|----------|----------|-------------|-----|
| $L_v = 0.375\lambda$, $x_d = 0.086\lambda$ | 0.251    | 0.854    | 0.157       | 6.366 |
| $L_v = 0.3125\lambda$, $x_d = 0.086\lambda$ | 0.207    | 0.074    | 7.485       |
| $L_v = 0.3\lambda$, $x_d = 0.086\lambda$ | 0.184    | 0.057    | 7.854       |

The simulation results are presented in Fig. 5, where the radiation and reflection coefficients, directivity factor ($D$), and the gain ($G$) of the combined radiator are shown as the function of monopole electric length and the distance between the slot and monopole. These plots allow us to obtain the
required energy characteristics and directivity factor by selecting the geometric parameters of the radiator. The characteristic pairs of parameters are marked in Fig. 4(b) by circles.

It turned out that the lowest level of lateral radiation ($\delta = -31$ dB) for the radiator with the geometric parameters ($L_v = 0.3 \lambda$, $x_d = 0.086 \lambda$) can be obtained if radiation coefficient is rather low. This can be explained by the phasing conditions for the radiation fields of the slot and vibrators. Really, for full compensation of the radiation field of the slot along the plane in the far zone, a pair of vibrators should induce an equivalent electric field in the geometric center of the slot, equal in amplitude to the field of the slot and anti-phased one. Consequently, the compensation of intrinsic slot field can significantly reduce the radiating capacity of the slot. As from Fig. 5(b), the $|S_\Sigma|^2$ level increases if the distance between the vibrators is increased. This inevitable violates the phase relations and reduces the level of lateral radiation. As an example, consider the RP of the combined radiator with geometric parameters $L_v = 0.3 \lambda$, $x_d = 0.131 \lambda$, presented in Fig. 4(a) (curve 4). Comparing curves 1 and 4 shows that the radiation coefficient increases up to $|S_\Sigma|^2 = 0.184$ while the difference between the RPs’ widths decreases ($\Delta \theta = -5^\circ$). If the distance between the vibrators is further increased under conditions that $\delta = -20$ dB, the radiation and reflection coefficients become relatively large, $|S_\Sigma|^2 = 0.403$, $|S_{11}| = 0.525$. The directivity factor also increases ($D = 8.273$), and $\Delta \theta = -15^\circ$ (curve 5 in Fig. 4(a)).

The analysis of the plots in Fig. 5 shows that the energy characteristics of the combined radiator can

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**Figure 5.** The energy characteristics of the combined radiator as functions of the electric monopole length and distance between the slot and vibrator: (a) $|S_{11}|$, (b) $|S_\Sigma|^2$, (c) $D$, and (d) $G$. 
be controlled by varying the electric length of the vibrators. As known [4], the vibrator electric length can be varied by coating its surface with constant imaginary impedance. In this case, it can be assumed that the RP shape will not be substantially varied, since the RP of monopoles with electric lengths in the range $0 < L_v/\lambda < 0.3$ is similar to that of a perfectly conducting radiator. This assumption for the vibrator with constant impedance distribution can be verified for one a priori chosen impedance value.

The numerical results have shown that monopoles with inductive surface impedance make it possible to obtain specified radiator characteristics with shorter monopoles. For example, the electric length of the monopole with constant impedance $\bar{Z}_S = 0.1i$ can be reduced relative to their physical length by about 30%. The characteristics of the Clavin radiators are presented in Fig. 6 and Fig. 7. As can be seen, the parameter $\Delta \theta$ minimum and predefined level of lateral radiation $\delta$ can be obtained by varying the monopole length and distance between the slot and vibrators. The energy characteristic $|S_{11}|$, $|S_\Sigma|^2$, $D$, and RP parameters $\delta$ and $\Delta \theta$ for the curves in Fig. 6(a) are presented in Table 2.

![Figure 6](image-url)

**Figure 6.** Characteristics of the Clavin radiator with impedance monopoles ($Z_S = 0.1i$): (a) RP; (b) the level of lateral radiation $\delta$ and difference between the RP widths in $E$- and $H$-planes $\Delta \theta$ as functions of $L_v/\lambda$ and $x_d/\lambda$.

**Table 2.** The RP parameters and energy characteristics of the Clavin radiator with impedance monopoles.

| $\delta$, dB | $\Delta \theta$, $^\circ$ | $|S_{11}|$ | $|S_\Sigma|^2$ | $D$ | Curve in Fig. 6(a), No. |
|--------------|------------------|----------|--------------|-----|---------------------|
| -20          | -7               | 0.17     | 0.05         | 7.74| 1                   |
| -32.5        | -11.3            | 0.15     | 0.04         | 8.1 | 2                   |

As can be seen from Fig. 7, when the distance between the vibrators is increased, and the length of the monopoles is decreased; the radiation coefficient $D$ and, hence, the gain $G$ are increased. At the same time, as can be seen from curve 3 in Fig. 6(a), parameter $\Delta \theta$ increases to $\Delta \theta = -23.7^\circ$. Thus, the reflection and radiation coefficients can be varied over a wide range with the low lateral radiation level by fitting the length of the vibrators, distance between them, and/or their surface impedance.

It is quite clear that phase relationships for the fields in the combined waveguide radiator depend not only upon the vibrator geometry, but also upon the slot length, since when it deviates from resonant dimension, the slot intrinsic field becomes asymmetric due to longitudinal slot excitation. The simulation results have shown that the radiation coefficient can be increased by slight increasing the slot length relative to 0.5$\lambda$. The energy characteristics and directivity of the combine radiator with parameters: $\lambda = 32$ mm, $a \times b = 23 \times 10$ mm$^2$, $h = 1$ mm, $d = 1$ mm, $x_0 = a/4$, $r = 0.17$ mm, $x_d = 0.086\lambda$ for perfectly conducting and impedance ($Z_S = 0.1i$) vibrators placed on the waveguide. The RPs and
energy characteristics as function of the electric length of the slot and vibrators plotted by using the above parameters are shown in Figs. 8–11. Thus, the RPs of equal width in the E- and H-planes with the lateral radiation level $\delta = -20$ dB can be obtained if the perfectly conducting monopoles are used with parameters $L_v = 0.3\lambda$, $2L_s = 0.57\lambda$ (curve 1 in Fig. 11(a)). The RP closest in width ($\Delta \theta = -3.4^\circ$) can be obtained if the impedance monopoles with parameters $L_v = 0.22\lambda$, $2L_s = 0.57\lambda$ are used (curve 1 in Fig. 11(b)). The radiation coefficients obtained with these parameters are equal to $|S_{11}|^2 = 0.493$, $|S_{\Sigma}|^2 = 0.497$. The RPs with the lowest levels of lateral radiation, maximal radiation coefficient $|S_{\Sigma}|^2$, and the highest gain are presented in curve 2 and curve 3 of Fig. 11. The parameters $L_v/\lambda$ and $2L_s/\lambda$ used for simulation are marked in Fig. 8 by circles.

All the curves presented above were plotted by using the parameters normalized at free space wavelength. This ensures simple evaluation of the radiator characteristics at the operating wavelength. The reliability of the proposed mathematical model for the combine radiator was confirmed by comparison with experimental data and the results found in literature. For example, the plots of the simulated and experimental reflection coefficients as functions of relative wavelength $\lambda/\lambda_0$ ($\lambda_0$ is wavelength in free space) for the single slot are presented in Fig. 12.
Figure 8. The level of lateral radiation $\delta$ and the RP width difference in orthogonal planes $\Delta \theta$ (dashed curves) as the function of $L_v/\lambda$ and $x_d/\lambda$: (a) $\bar{Z}_S = 0$, (b) $\bar{Z}_S = 0.1i$.

Figure 9. Energy characteristics of the Clavin radiator with perfectly conducting monopoles ($\bar{Z}_S = 0$) as function of $L_v/\lambda$ and $x_d/\lambda$: (a) reflection coefficient $|S_{11}|$, (b) radiation coefficient $|S_{\Sigma}|^2$, (c) directivity $D$, (d) gain $G$. 

Note: The figures depict various parameters and their variations with respect to the specified dimensions, providing insights into the performance characteristics of the Clavin radiator under different conditions.
6. NUMERICAL RESULTS FOR THE COMBINED RADIATOR WITH THE TUNING MONOPOLE IN THE WAVEGUIDE

The energy coefficients of the Clavin radiator placed near the longitudinal slot cut in the broad wall of rectangular waveguide with the vibrator inside the waveguide are plotted as a function of the monopole parameters $x_{0v}, L_V$, presented in Fig. 13. The tuning vibrator is located in the plane $\{x_{0y}\}$ inside the waveguide parallel to its narrow walls as shown in Fig. 1. The parameters of the radiator are as follows: $a \times b = 23 \times 10 \text{ mm}^2$, $h = 1 \text{ mm}$, $\lambda = 32 \text{ mm}$, $2L_s = 16 \text{ mm}$, $d = 1.5 \text{ mm}$, $x_0 = 2.5 \text{ mm}$, $r = 0.17 \text{ mm}$, $L_v = 0.3125\lambda \text{ mm}$, $x_d = 0.086\lambda \text{ mm}$. The radius and surface impedance of the monopole located in the waveguide are $r_V = 0.25 \text{ mm}$ and $\bar{Z}_{SV} = 0$.

As shown in the previous section, the optimal RP with equal width in the $E$- and $H$-planes can be formed by the radiating structure without tuning vibrator at $\lambda = 32 \text{ mm}$. This structure is characterized by the low radiation coefficient, $|S_{\Sigma}|^2 = 0.074$, and sufficiently high reflection coefficient. If the monopole is placed inside the waveguide, this relationship between the energy characteristic can be changed by fitting the monopole length $L_V$ and its displacement $x_{0v}$ relative to the waveguide wall.
The simulation results have shown that the passive monopole weakly influences the slot, since the monopole is placed directly in the region under the slot aperture. This effect can be explained by the general electrodynamical property of thin vibrators, i.e., by the absence of radiation (scattering) in the direction of their longitudinal axes. Furthermore, the longer the monopole is, the smaller its influence is on the slot. As can be seen from Fig. 13, if the monopole length is \( L_v = 8.5 \, \text{mm} \), and the displacement \( x_{0v} \) varies in the range \( 1.75 \, \text{mm} \leq x_{0v} \leq 3.25 \, \text{mm} \), the transmission coefficient \( |S_{12}| \) decreases at about 5%. However, the internal monopole improves the radiator matching with the waveguide by almost 10 times. This effect is important for using combined radiators in multi-element linear antenna arrays with dimensions of the order of hundreds \( \lambda \). Such an array, for example, can be used on spacecrafts.

The spatial separation of the slot and the monopole inside the waveguide can significantly increase the radiation coefficient, while the directional characteristics of the combined radiator are not varied. For example, if the monopole with parameters \( L_v = 7.2 \, \text{mm}, x_{0v} = 17.5 \, \text{mm} \) is used, the radiation coefficient increases up to \( |S_{\Sigma}|^2 = 0.4 \) (Fig. 13). Thus, the additional inhomogeneity in the waveguide...

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**Figure 11.** The RP of the Clavin radiator located on the waveguide in the main polarization planes as function of the angle \( \theta \) for \( \varphi = 0^\circ \) and \( \varphi = 90^\circ \): (a) perfectly conducting vibrator, \( \bar{Z}_S = 0 \), (b) impedance vibrators, \( \bar{Z}_S = 0.1i \).

**Figure 12.** Dependence of the reflection coefficient for the combine radiator on the ratio \( \lambda/\lambda_c \) (\( \lambda_c \) is critical wavelength of the waveguide).
in the form of the monopole increases the reflection coefficient relative to the stand-alone resonant slot to $|S_{11}| = 0.55$.

7. CONCLUSION

In this article, the mathematical model of a combined waveguide structure with the Clavin type radiator is constructed based on the solution of the diffraction problem in a strict formulation by the generalized method of induced EMMF. In contrast to the publications of other authors known in the literature, vibrator elements in the form of impedance monopoles are considered for the first time. The influence of the vibrator lengths and the distance between the vibrators on the directional characteristics of the combined radiator were analyzed to obtain the optimal level of lateral radiation in the $E$-plane and the difference in the RP widths in $E$- and $H$-planes at the level of $-3$ dB. It was shown that the directivity and energy characteristics of the combined radiator: the RP form, radiation and reflection coefficients,
directivity factor and gain can be controlled in a wide range by varying the electric vibrator length, distance between the vibrators, and their surface reactive impedances. It was shown that the combined radiators with optimal characteristics can be realized by using shorter vibrators with inductive surface impedance than perfectly conducting vibrator. It was found that the slot radiation coefficient can be increased by increasing the slot length by 15–20% relative to its resonance length, while the RPs in the $E$ and $H$ planes do not vary. The simulation results have confirmed the possibility to effectively control the energy characteristics of the combined radiator by using the tuning monopole in the waveguide. Since the multi-parameter resonant tuning of the radiator is difficult to achieve experimentally, this is a separate problem requiring further studies. The results obtained can be useful in the design of stand-alone waveguide radiator and multi-element waveguide arrays, with combined radiators based on the Clavin elements, including devices with non-mechanical controlling the electrodynamic characteristics.

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