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From noncommutative spacetimes to Lorentz symmetry violation

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Abstract. The current article provides a brief review on noncommutative spacetimes in light of Lorentz invariance violation and especially on how these models are linked to the Lorentz-violating Standard-Model Extension. By embedding a general noncommutative spacetime with a constant background tensor into the Standard-Model Extension, a couple of implications are drawn on Lorentz violation in such models.

1. Introduction
Continuous symmetries and their description by Lie groups play a role in a plethora of topics in physics starting with classical mechanics and reaching as far as to the Standard Model of elementary particle physics. For this reason any physicist has come in touch with this highly interesting and powerful concept. Lie groups themselves are linked to homogeneous spaces, which provide examples in classical, commutative differential geometry.

Besides Lie groups there is another class of groups that are important in particular areas of physics; these are known as quantum groups. In principle quantum groups can occur as generalized symmetry groups in certain lattice models [1]. The analogy to Lie groups being connected to homogenous spaces is that quantum groups are linked to quantum homogeneous spaces. These have established the relatively new field of noncommutative differential geometry. Noncommutative spaces may be characterized by preferred directions in their tangent spaces where the existence of such directions leads to a violation of active rotational invariance. Generalizing these concepts to a noncommutative spacetime shows that this is linked to a violation of particle Lorentz invariance.

Lorentz invariance violation is motivated by the existence of a small-scale structure of spacetime. The latter is expected to occur at the Planck length due to quantum-theoretical energy fluctuations and their influence on spacetime geometry. The concept of such a spacetime foam was introduced by Wheeler in the context of his quantum geometrodynamics [2] and also Hawking worked on it a couple of years later [3, 4]. Successively particular spacetime foam models were considered in [5, 6, 7, 8, 9]. Since any direct way of describing a small-scale spacetime structure seems to be very complicated, an effective approach is to introduce noncommutative spacetime coordinates. Such a noncommutativity must necessarily imply a tensorial object, which gives rise to a fixed background and therefore leads to particle Lorentz violation.

The Standard-Model Extension (SME) is a framework, which contains all Lorentz-violating field operators constructed from Standard-Model fields or the Riemann curvature tensor.
The matter sector is invariant under SU(3)$_c \times$ SU(2)$_L \times$ U(1)$_Y$-transformations as well. Each term is characterized by a field operator and particular controlling coefficients transforming as usual under observer Lorentz transformations and general coordinate transformations, respectively. However they are fixed backgrounds under particle Lorentz transformations and diffeomorphisms, respectively. The minimal SME [11] is restricted to all field operators of mass dimension 4 or lower, whereas the nonminimal SME [13, 14, 15] contains all field operators to arbitrary dimension. This is one of the reasons why the SME is a highly powerful framework for the search of Lorentz violation in nature.

The current paper shall briefly review how a noncommutative spacetime can be mapped to the SME whereby existing experimental bounds on certain controlling coefficients translate to bounds on tensor components, which describe noncommutative spacetimes. The article is organized as follows. In Sec. 2 noncommutative spacetimes will be reviewed where we will elaborate on one particular example. Section 3 will give a brief introduction to the SME and Sec. 4 is dedicated to discussing how to link a particular class of noncommutative spacetimes with the SME. Last but not least, Sec. 5 provides a summary of the most important aspects.

2. Noncommutative spacetimes

A noncommutative spacetime can be considered as an effective approach to modeling a small-scale spacetime structure. Spacetime coordinates are assumed to not commute any more, i.e., $[x^\mu, x^\nu] = i \theta^{\mu\nu}$. The nonvanishing right-hand side of the latter equation must have the same tensor structure as the commutator, which means that an antisymmetric object $\theta^{\mu\nu}$ with two Lorentz indices has to be introduced. Since any particle Lorentz transformation does not transform the coordinates, but physical entities such as particles or fields, $\theta^{\mu\nu}$ is considered as a fixed background under particle Lorentz transformations. This is the reason why any such object manifestly violates particle Lorentz invariance. For extensive, general reviews on noncommutative spacetimes we refer to [16, 17].

To see what implications a noncommutative spacetime may have, a particular model, which was first proposed by Majid and Ruegg, shall be considered [18, 19]:

$$[x^\mu, x^\nu] = i \lambda \theta^{\mu\nu}, \quad \theta^{\mu\nu} = \zeta^\mu \xi^\nu - \xi^\mu \zeta^\nu,$$

(1a)

$$\zeta^\mu = \begin{pmatrix} 0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix}, \quad \xi^\mu = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$  

(1b)

First of all, $\lambda$ is a length scale, which is assumed to lie in the order of magnitude of the Planck length. This is reasonable, since a noncommutative spacetime is supposed to describe Planck-scale effects. Furthermore for this particular setup the tensor $\theta^{\mu\nu}$ is decomposed into two four-vectors $\zeta^\mu$ and $\xi^\mu$. The latter can be interpreted as preferred spacetime directions originating from a background field, which penetrates the vacuum. The background may be a low-energy description of physics at the Planck scale such as a spacetime foam. Note that the mass dimension of the equation is consistent since the vector $\xi^\mu$ only involves dimensionless components.

For this particular model, the tensor $\theta^{\mu\nu}$ also depends on the spacetime point considered due to the four-vector $\zeta^\mu$. The preferred spacetime direction $\xi^\mu$ is the same, whereas $\zeta^\mu$ is different at each point. Additionally this leads to a violation of translation invariance, since the physics supposedly differs at each spacetime point.

Now any noncommutative spacetime gives rise to both a modified calculus and modified Fourier transforms, which imply a modified momentum group. In particular for the Majid-Ruegg model the conventional addition of two four-momenta and the inverse four-momentum
are modified as follows [19] where the corresponding group operation will be denoted by $\oplus$:

$$
\begin{align*}
(E, p) \oplus (E', p') &= \left( E + E', p + \exp(\lambda E)p' \right), \\
(E, p)^{-1} &= \left( -\frac{E}{p}, -\exp(-\lambda E)p \right).
\end{align*}
$$

(2)

Here $E$, $E'$ are energies and $p$, $p'$ spatial momenta. The constant $\lambda$ is the length scale, which appears in the definition of the model in Eq. (1). Besides, both the boost and rotation generators of the Lorentz algebra are altered as well resulting in a modified dispersion law [19] for particles of mass $m$:

$$
\exp(\lambda E) + \exp(-\lambda E) - \frac{2}{\lambda^2} - p^2 \exp(-\lambda E) = m^2.
$$

(3)

For particle energies lying well below the Planck scale, i.e., $\lambda E \ll 1$ the dispersion relation can be expanded giving the more illuminating result

$$
E \approx \sqrt{p^2 + m^2} - \frac{\lambda p^2}{2}.
$$

(4)

Besides the standard relativistic particle energy there is an additional contribution, which grows quadratically with the momentum but is suppressed by the small length scale (or large inverse energy scale) $\lambda$. The latter result is only valid if the energy or the momentum is sufficiently small. If $\lambda$ is assumed to be larger than zero, the particle energy could become negative for a sufficiently high momentum otherwise.

Any noncommutative spacetime model gives rise to a noncommutative field theory by introducing noncommutative fields and a new product for two of these fields called the Moyal product [16, 17]. In [20] unitarity is investigated for such noncommutative field theories. The outcome is that theories with $\theta^{0i} \neq 0$, $i = 1 \ldots 3$ may have issues with perturbative unitarity. Based on this result a model of noncommutative spacetime has no difficulties with perturbative unitarity when the following two conditions hold [21]:

$$
\theta_{\mu \nu} \theta^{\mu \nu} > 0, \quad \theta_{\mu \nu} \tilde{\theta}^{\mu \nu} = 0, \quad \tilde{\theta}^{\mu \nu} = \frac{1}{2} \varepsilon^{\mu \nu \rho \sigma} \theta_{\rho \sigma},
$$

(5)

where $\tilde{\theta}^{\mu \nu}$ is the dual of $\theta^{\mu \nu}$ and $\varepsilon^{\mu \nu \rho \sigma}$ is the four-dimensional Levi-Civita symbol with $\varepsilon^{0123} = 1$. For the Majid-Ruegg model of Eq. (1) one can check that $\theta_{\mu \nu} \tilde{\theta}^{\mu \nu} = 0$, but $\theta_{\mu \nu} \theta^{\mu \nu} = -2x^2 < 0$ with the vector $x = (x^1, x^2, x^3)$ of the spatial coordinates. Therefore the fate of perturbative unitarity for this particular noncommutative spacetime remains unclear at first.

### 3. Standard-Model Extension

It is reasonable to consider models of noncommutative spacetimes as effective descriptions of Planck-scale physics. However there are certain drawbacks with such setups. First, they describe the physics only partially since, e.g., Eq. (1) itself does not provide any modified field equations based on commutative field theory. This makes phenomenological investigations very involved. Second, there may be no observable physical effects even if there is a modified particle dispersion relation. These issues are based on the lack of an underlying effective field theory.

The reasons stated, amongst others, led to the development of the SME, which is an effective field theory comprising all Lorentz-violating operators constructed within the ordinary Standard Model and General Relativity. The great advantages of the SME are that all Lorentz-violating terms can be well organized according to their symmetry properties and the particle sector they belong to. Each contribution leads to modified field equations, Feynman rules, and all the tools available that are needed to consider (quantum) field theory and particle physics processes. For example, only within such a framework it can be understood why certain modifications of
4. Embedding noncommutative spacetimes into the Standard-Model Extension

A very helpful approach to investigating the physical implications of a noncommutative spacetime, i.e., \([x^\mu, x^\nu] = i\delta^{\mu\nu}\) is to embed it into the SME. To get a first idea which sector of the SME a noncommutative spacetime corresponds to, one may be tempted to map modified dispersion relations to each other. First the expanded dispersion relation of the Majid-Ruegg model given by Eq. (4) shall be considered and be identified with a subset of the SME fermion sector coefficients. The dispersion relation is isotropic and since there is a single one, particles and antiparticles do not differ from each other. Therefore only the CPT-even operators \(\hat{m}, \hat{m}_5, \hat{c}^\mu, \hat{d}^\mu, \) and \(\hat{H}^{\mu\nu}\) come into consideration. From these already a large number is excluded for several reasons. The operator \(\hat{m}_5\) can be removed by a chiral transformation in most cases, which is why it will be omitted. Since \(\lambda\) is considered as the Lorentz-violating coefficient in Eq. (4), we must look for coefficients with mass dimension \(-1\). Thus \(\hat{c}^\mu\) and \(\hat{d}^\mu\) are discarded as well, as the corresponding coefficients have even mass dimension. What finally remains are the operators \(\hat{m}\) and \(\hat{H}^{\mu\nu}\) where our interest is based on the isotropic cases. In [15] all isotropic sectors are discussed including operators to dimension 6. On the one hand, the isotropic subspace of the coefficients associated with the dimension-5 operator \(\hat{m}\) is given by:

\[
(m^{(5)\mu\nu}) = \text{diag}(m^{(5)00}, m^{(5)33}, m^{(5)33}, m^{(5)33}),
\]

i.e., there are two distinct nonzero coefficients. The effective coefficient \(c_{\text{eff}}^{(6)0000}\) in Eq. (98) of [15] involves \(m^{(5)00}\) via the second of their Eq. (27). Furthermore, \(c_{\text{eff}}^{(6)00jj}\) and \(c_{\text{eff}}^{(6)jjkk}\) contains the sum \(m^{(5)jj}\) of the coefficients with two spatial indices set equal. Here \(m^{(5)11} = m^{(5)22} \equiv m^{(5)33}\) is considered instead, which is equivalent. With the choice of Eq. (6) the (physical) isotropic fermion dispersion relation reads as follows:

\[
p^0 = \sqrt{\frac{1 - 2m^{(5)00}m_\psi + p^2m^{(5)33}}{2(m^{(5)00})^2}} - \sqrt{\frac{1 - 4m_\psi m^{(5)00} - 4p^2m^{(5)00}(m^{(5)00} + m^{(5)33})}{2(m^{(5)00})^2}}
\]

\[
\approx \sqrt{p^2 + m_\psi^2 \left(1 + m^{(5)00}m_\psi + m^{(5)33} \frac{m_\psi^2 p^2}{p^2 + m_\psi^2}\right)},
\]

where \(m_\psi\) is the fermion mass. Note that \(m^{(5)00}, m^{(5)33}\) have mass dimension \(-1\), which renders the terms in parentheses dimensionless. Furthermore there are spurious dispersion relations, which do not have a limit for vanishing Lorentz-violating coefficients [43]. However they are
considered as unphysical for momenta much smaller than the Planck scale where the SME is an effective framework. On the other hand, the isotropic subspace of the dimension-5 operator $\hat{H}^{\mu\nu}$ is spanned by the nonvanishing coefficients

$$H^{(5)1203} = H^{(5)2301} = H^{(5)3102} = -H^{(5)1302} = -H^{(5)2103} = -H^{(5)3201} \equiv H.$$  

These are the coefficients $\hat{H}^{(5)0ij}$, $j = 1 \ldots 3$ of the dual of $H^{(5)\mu\nu\rho\sigma}$ as indicated in Eq. (97) in [15]. In that case the modified fermion dispersion law is given by:

$$p^0 = \sqrt{\left(p^2 + m^2 - H m_\psi |p| \right)}.$$  

The expansions stated hold as long as $m^{(5)00} m_\psi \ll 1$, $m^{(5)33} |p| \ll 1$, and $H |p| \ll 1$, i.e., only if the Lorentz-violating coefficient and the momentum are sufficiently small, which will be assumed in what follows. All three Lorentz-violating modifications grow linearly with increasing momentum for $p^2 \gg m_\psi^2$. However for $p^2 \ll m_\psi^2$ their momentum dependences differ from each other. The $m^{(5)00}$-term is then independent of the momentum, the $H$-term still grows linearly and the $m^{(5)33}$-term even grows quadratically. Hence in this case the momentum dependences of the $m^{(5)33}$-term and Eq. (4) would coincide. However if the SME-coefficient $m^{(5)33}$ was identified with the Lorentz-violating parameter $\lambda$ of Eq. (4) both dispersion laws would be expected to correspond to each other not only when the particle momentum is much smaller than the particle mass. Therefore it seems that there is no operator in the SME, which produces a dispersion relation similar to Eq. (4) for a reasonably large range of the particle momentum. As a matter of fact it is challenging to identify the SME fermion sector corresponding to the Majid-Ruegg model by such an analysis. This does not mean that are is no possibility of performing this identification. However further studies of this issue are beyond the scope of the paper. That is why we will turn to another class of noncommutative spacetimes: models for which $\theta^{\mu\nu}$ itself is spacetime-independent.

These particular models form the basis of [21]. In the latter article a correspondence between the constant $\theta^{\mu\nu}$ and the SME is established. Since the SME is a commutative field theory, the Seiberg-Witten map [44] has to be used to match $\theta^{\mu\nu}$ to SME coefficients. By doing so, the noncommutative photon field $\hat{A}_\mu$ and the noncommutative spinor field $\hat{\psi}$ are mapped to the commutative photon and spinor field $A_\mu$ and $\psi$, respectively. At next-to-leading order in the commutative fields this map is given by:

$$\hat{A}_\mu = A_\mu - \frac{1}{2} \theta^{\alpha\beta} A_\alpha (\partial_\beta A_\mu + F_{\beta\mu}) + \ldots, \quad \hat{\psi} = \psi - \frac{1}{2} \theta^{\alpha\beta} A_\alpha \partial_\beta \psi + \ldots.$$  

The procedure outlined leads to a modified commutative field theory where two characteristic additional terms will be given as an example:

$$L = \overline{\psi} \left( \frac{i}{2} \gamma^\mu \overleftrightarrow{D_\mu} - m_\psi \right) \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{4}{3} m q \theta^{\alpha\beta} F_{\alpha\beta} \overline{\psi} \psi - \frac{1}{2} q \theta^{\alpha\beta} F_{\alpha\mu} F_{\beta\nu} F^{\mu\nu} + \ldots.$$  

Here $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$ is the electromagnetic field strength tensor, $\overline{\psi} \equiv \psi \gamma^0 \gamma^1$ the Dirac-conjugate spinor, $\gamma^\mu$ are the standard Dirac matrices, $D_\mu$ is the covariant derivative, $m_\psi$ the fermion mass, and $q$ its charge. The first three terms are standard and the remaining two emerge from the noncommutative field theory by applying the Seiberg-Witten map. Note that both of
them involve the noncommutativity tensor $\theta^{\alpha \beta}$. The first describes a photon-fermion interaction and the second is a photon interaction term.

The great advantage of the effective field theory gained from the noncommutative one is that $\theta^{\alpha \beta}$ can be identified with Lorentz-violating coefficients from the SME. Therefore fermions with charge $q$ and photons are considered and the electromagnetic field strength tensor is understood as a sum over a constant background field strength tensor $f_{\mu \nu}$ and a small dynamical fluctuation $F_{\mu \nu}$. Since any realistic noncommutative theory is CPT-even [21], the identification reveals relationships between the noncommutativity tensor and the CPT-even coefficients $c^{\mu \nu}$ of the minimal SME fermion sector and the $(k_F)^{\mu \nu \sigma}$ of the minimal photon sector. For the fermion sector it reads as follows [21]:

$$c^{\mu \nu} = -\frac{q}{2} f^{\mu \lambda} \theta_{\lambda \nu}.$$  \hspace{1cm} (12)

Using this relation and looking for sidereal variations, e.g., in clock-comparison experiments provides conservative bounds on certain coefficients of $\theta^{\mu \nu}$ in a particular frame lying in the order of magnitude of $(10\text{ TeV})^{-2}$ [21]. It is impressive that via low-energy experiments such tight constraints on Planck-scale physics can be derived, which demonstrates the phenomenological power of the SME.

Now the correspondence of Eq. (12) shall be further investigated from a theoretical point of view. The properties of the SME reveal that $c^{\mu \nu}$ is symmetric and traceless: $c^{\mu \nu} = c^{\nu \mu}$ and $c^{\mu \mu} = 0$. The latter property is valid, since the trace of $c^{\mu \nu}$ does not contain any physics, but it can be absorbed by a redefinition of the physical fields. Therefore these conditions shall be applied to the correspondence stated above. If $c^{\mu \nu}$ is claimed to be symmetric, four of the six components of the antisymmetric $\theta^{\mu \nu}$ can already be eliminated. If the electric and magnetic field vectors $\mathbf{E} = (E^1, E^2, E^3)$, $\mathbf{B} = (B^1, B^2, B^3)$ are introduced according to the definition of the contravariant field strength tensor,

$$(f^{\mu \nu}) = \begin{pmatrix}
0 & -E^1 & -E^2 & -E^3 \\
E^1 & 0 & -B^3 & B^2 \\
E^2 & B^3 & 0 & -B^1 \\
E^3 & -B^2 & B^1 & 0
\end{pmatrix}, \hspace{1cm} (13)$$

the components $\theta^{0 i}$ for $i = 1 \ldots 3$ and $\theta^{23}$ can be expressed via $\theta^{12}$, $\theta^{13}$ as follows:

$$\theta^{0 i} = \sum_{j=2,3} (E^j B^i + B^j E^i) \theta^{1 j}, \hspace{1cm} \theta^{23} = \frac{\sum_{i=2,3} (E^1 B^i - E^i B^1) \theta^{1 i}}{(\mathbf{E} \times \mathbf{B})^1}. \hspace{1cm} (14)$$

With the tracelessness condition of $c^{\mu \nu}$ we can eliminate $\theta^{13}$ as well:

$$\theta^{13} = -\frac{E^2 (E^2 - B^2) + 2B^2 \mathbf{E} \cdot \mathbf{B}}{E^3 (E^2 - B^2) + 2B^3 \mathbf{E} \cdot \mathbf{B}} \theta^{12}. \hspace{1cm} (15)$$

By using the relationships of Eqs. (14), (15) the set of coefficients $c^{\mu \nu}$ must read as

$$(c^{\mu \nu}) = q \theta^{12} \frac{\mathbf{E} \cdot \mathbf{B}}{E^3 (E^2 - B^2) + 2B^3 \mathbf{E} \cdot \mathbf{B}} \begin{pmatrix}
\hat{c}^{00} & \hat{c}^{01} & \hat{c}^{02} & \hat{c}^{03} \\
\hat{c}^{01} & \hat{c}^{11} & \hat{c}^{12} & \hat{c}^{13} \\
\hat{c}^{02} & \hat{c}^{12} & \hat{c}^{22} & \hat{c}^{23} \\
\hat{c}^{03} & \hat{c}^{13} & \hat{c}^{23} & \hat{c}^{33}
\end{pmatrix}, \hspace{1cm} (16a)$$

$$\hat{c}^{00} = (E^2 + B^2)/2, \hspace{1cm} (16b)$$

$$\hat{c}^{ij} = \hat{c}^{00} \delta^{ij} - (E^i E^j + B^i B^j), \hspace{1cm} (16c)$$

$$\hat{c}^{0i} = (\mathbf{E} \times \mathbf{B})^i, \hspace{1cm} (16d)$$
where \( i, j \in \{1, 2, 3\} \). There are at least two interesting special cases. First, for orthogonal electric and magnetic fields the coefficients \( c^{\mu\nu} \) vanish as long as the denominator in Eq. (16a) remains nonzero. Hence such a field configuration in a noncommutative spacetime does not imply Lorentz violation in the fermion sector. Second, for the choice \( E_1 = E_2 = B_1 = B_2 = 0 \) there results a fairly simple set of \( c^{\mu\nu} \), which is given by a diagonal matrix:

\[
(c^{\mu\nu}) = c^{00} \text{ diag}(1, 1, 1, -1), \quad c^{00} = \frac{q}{2} \theta^{12} B^3.
\] (17)

The \( z \)-component of the electric field strength vector is completely arbitrary (but nonzero); it does not contribute to \( c^{\mu\nu} \) in this case. The latter choice of Lorentz-violating coefficients gives rise to the following modified fermion dispersion relation:

\[
p^0 = \sqrt{\left(1 - c^{00}\right)^2 \left(p^1\right)^2 + \left(p^2\right)^2 + \left(p^3\right)^2 + \tilde{m}_\psi}, \quad \tilde{m}_\psi \equiv \frac{m_\psi}{1 + e^{\theta 0}}.
\] (18)

Here the fermion mass can be redefined as indicated. The resulting dispersion law is modified in the \( x-y \)-plane and it has a remaining isotropy in this plane. Therefore for a field configuration with parallel electric and magnetic field vectors in a noncommutative spacetime a Lorentz-violating fermion dispersion relation may emerge, which is characterized by a residual isotropy. Attempts to construct a field configuration giving rise to an isotropic Lorentz-violating dispersion relation, did not produce any results. Hence it seems that a noncommutative spacetime with constant tensor components \( \theta^{\mu\nu} \) cannot lead to isotropic Lorentz-violating modifications.

5. Conclusions

To summarize, noncommutative spacetimes provide effective models for a microscopic spacetime structure, which may emerge at the Planck scale. The noncommutativity is characterized by an antisymmetric two-tensor \( \theta^{\mu\nu} \) with six independent components, which may even be spacetime-dependent. As an example a particular model with spacetime-dependent components \( \theta^{\mu\nu} \) was considered with the goal of identifying it with a particular sector of the SME. However no corresponding modified particle dispersion relation was found, which makes a possible identification challenging. In contrast, it has been well-known for over a decade that models with a spacetime-independent \( \theta^{\mu\nu} \) can be embedded into the SME by considering photons and fermions propagating in constant electromagnetic fields.

The identification leads to nonvanishing \( CPT \)-even coefficients \( c^{\mu\nu} \) of the minimal fermion sector and \( (k_F)^{\mu\nu}\rho\sigma \) of the minimal photon sector [21]. In the current paper it was demonstrated that five of the six components of \( \theta^{\mu\nu} \) can be eliminated due to the symmetry properties of \( c^{\mu\nu} \). Therefore any Lorentz-violating dispersion relation in the fermion sector only depends on one of these components and the electric and magnetic background fields. For orthogonal electric and magnetic fields no Lorentz violation in the fermion sector can be observed. For parallel fields there exists a Lorentz-violating dispersion relation having a residual isotropy in a plane.

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