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Weighted multiplier ideals of reduced divisors. (English) Zbl 07605964
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Summary: We use methods from birational geometry to study the Hodge and weight filtrations on the localization along a hypersurface. We focus on the lowest piece of the Hodge filtration of the submodules arising from the weight filtration. This leads to a sequence of ideal sheaves called weighted multiplier ideals. The last ideal of this sequence is a multiplier ideal (and a Hodge ideal), and we prove that the first is the adjoint ideal. We also study the local and global properties of weighted multiplier ideals and their applications to singularities of hypersurfaces of smooth varieties.

MSC:
14F18 Multiplier ideals

Full Text: DOI arXiv

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