Damping Problem of Numerical Simulation of Structural Reverberation Time

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Abstract. Finite element simulations of sound insulation are very problematic due to problem of damping. Omitting damping issues in dynamic analysis, which in conclusions leads to acoustical performance of system may reflect in non-reliable results. In this paper study using different values of Rayleigh damping coefficients are used for Explicit analysis to calculate structural reverberation time. To investigate behaviour of structure under different damping the structural reverberation time and the total loss factor is calculated. The modelled slab was 14 [cm] thick and geometry of considered plate is 4.19 x 3.61 [m]. Load on the structure was applied in 3 different points with unit pressure on small area (5x5 [cm]) using amplitude of load as Dirac delta approximation. Results of performed simulations of the slab subjected to impact load were compared with laboratory result of structural reverberation time measurements obtained with accelerometers. As a result of this analysis it can be stated that damping is crucial issue to reflect acoustical behaviour of considered structure. Results shown that \( \beta \)-coefficient of Rayleigh damping affects more structural response than \( \alpha \)-coefficient especially in medium and high frequency. Comparing simulation results with laboratory measurements and formulas to calculate \( \alpha \) - and \( \beta \)-coefficient of Rayleigh damping it can be concluded that formulas used in structural dynamics for estimating damping coefficients in simulation of whole building are not valid for simulation of individual building elements. These formulas can be used only for initial estimation of damping coefficients. In considered situation gave effect of over dampening tested slab. Based on this conclusion additional simulations were performed to conclude that \( \beta \)-coefficient is overestimated by order of magnitude.

1. Introduction

This article provides Finite Element Method (FEM) simulations of structural reverberation time in the concrete slab using explicit method. Aim of this paper is to analyse influence of mass (\( \alpha \)-coefficient) and stiffness (\( \beta \)-coefficient) part of Rayleigh damping of explicit model formulation.

In building acoustics structural reverberation time (\( T_r \)) is crucial to properly estimate value of sound insulation of partition or structure-borne sound radiated from partition using different type of simulations. Structural reverberation time is defined similarly as reverberation time in room. In practice structural reverberation time is used only for bending waves in building elements, but it can also be calculated for quasi-longitudinal and transverse shear waves.

Structural reverberation time as a value characteristic to structural system can be measured following [1]. What is more evaluation method based on measurements is similar to room reverberation time (\( T \)) estimation method with evaluation of decay curves given in standard [2]. The method used in most cases uses backwards-integration of measured impulse responses of structural system. Both hammers and shakers can be applied to the system. Hammers able to record response of structure directly. Excitation of structure using shaker consist of applying sweep noise signal and subsequent signal processing.
Following publications consider problem of structural reverberation time measurements:

[3] – where different methods of structural reverberation time are discussed using various hammers and shaker. For hammer measurements four different type of hammers with different masses were used. Hammers used for measurements had a plastic tip to cover measured range of frequencies – 100 [Hz] to 5 [kHz]. In total 12 measured decay curves for each hammer were obtained – with 4 excitation points and 3 receiving points. For shaker measurements two setup configurations with larger and smaller shaker were used. Shakers were coupled with wall using force transducer and a magnet.

[4] – where hollow core concrete floor is subjected to analysis using different excitation sources. A 200 [mm] thick hollow core concrete floor with dimensions of 4.70 x 3.80 [m] was tested. The structural response of system was obtained with two different shakers (smaller and larger one) with exponential sweep of length 40 [s]. Additionally two impact hammers were applied to measuring process. Three excitation points and four receiving points were set. Different hammer tips were used to provide wider frequency range for excitation.

[5] – where structural reverberation time for concrete slab and masonry walls is measured and prediction of energy flow between structure junctions is done by Transient Statistical Energy Analysis (TSEA). Comparison between TSEA and laboratory measurements consists of five different systems – concrete floor slab, masonry wall, two T junctions of masonry walls and X junction of masonry walls. Laboratory measurements were performed similarly as in previous literature based on standard [1].

2. Assumptions and calculation method

In this paper 3D model in Abaqus Explicit simulation is performed. Utilization of Abaqus software for structural reverberation time was dictated by good control of mesh parameters in order to obtain stable and reliable results. Assumptions for analysed model, as stated below were made.

2.1. Geometry and material data

Given geometry of the considered plate is 4.19 x 3.61 [m] and represents around 15 [m²] to compare results of simulation with measurements presented in discussed literature [5]. What is more, dimensions are also consistent with standards [6, 7], to provide at least 10 [m²] of tested specimen. Thicknesses of the plate is 14 [cm] as it was given in considered literature laboratory results for concrete slab [5]. Material properties are given in table 1, which are also consistent with those used in literature [5].

| Table 1. Material and model data assumed for FEM model. |
|---------------------------------------------------------|
| Element                  | Young modulus E [MPa] | Density ρ [kg/m³] | Poisson ratio ν [-] |
| Concrete floor           | 30000                 | 2400              | 0.20                |

To analyse damping influence on structural reverberation time values of α and β coefficients of Rayleigh damping are set for simulation as in Table 2. For each α-coefficient corresponds β-coefficient. There is existing a method to provide estimated values of Rayleigh damping coefficients in structural dynamics [8, 9]. Given in formulas (1) and (2) where \( f_a \) and \( f_b \) are first and second resonant frequency of system and \( \delta_a \) and \( \delta_b \) are logarithmic damping decrements corresponding to \( f_a \) and \( f_b \). Method discussed in these articles is used to estimate α-coefficient and β-coefficient for whole buildings with different type of structures. These coefficients are estimated basing on in-situ test results. For purpose of this article assumptions of formulas (1) and (2) are not properly fulfilled. Thus results calculated using formulas (1) and (2) may not give proper values of α-coefficient and β-coefficient.

\[
\alpha = \frac{2f_a f_b \delta_b f_a - \delta_a f_b}{f_b^2 - f_a^2} \quad (1)
\]

\[
\beta = \frac{1}{2\alpha \sigma_b^2} \frac{\delta_a f_b - \delta_b f_a}{f_b - f_a} \quad (2)
\]
Using formulas (1) and (2) to calculate give estimation of $\alpha = 9.42$ and $\beta = 5.53e-5$, which was good for estimation of chosen values used for performed simulations. Due to know collision with formulas (1) and (2) assumptions it was decided to divide parameters estimation into two stages. After initial simulations two additional values of $\beta$-coefficient for best fitting $\alpha$-coefficient (equal to 15, see table 2).

| $\alpha$-coefficient of Rayleigh damping [-] | $\beta$-coefficient of Rayleigh damping [-] |
|--------------------------------------------|--------------------------------------------|
| 0                                         | 0                                         |
| 5                                         | 3e-5                                      |
| 10                                        | 6e-5                                      |
| 15                                        | 9e-5                                      |
| 3e-6 (after initial simulations, with $\alpha = 15$ only) |                                        |
| 3e-7 (after initial simulations, with $\alpha = 15$ only) |                                        |

2.2. Mesh and simulation parameters

Linear elements with reduced integration were used, to prevent occurring of hourglass effect. Hourglass effect is a spurious deformation mode of a Finite Element Mesh. It results from the excitation of zero-energy degrees of freedom. It typically manifests as a patchwork of zig-zag or hourglass like element shapes, where individual elements are severely deformed, while the overall mesh section is not deformed [10]. Proper control in Abaqus software was provided. Brick elements are applied to provide most stable results [10]. Size of mesh for all model with thickness varying was set to 7 [cm]. Size of mesh seed is limited by three factors. First one is element thickness, at least two FE for thickness, which gives 7 [cm] element size. Second one is to provide at least quarter of wavelength. For concrete wave velocity is around 3400 [m/s] [11, 12, 13], which gives required highest mesh size of 17 [cm]. Third one is to obtain convergence in explicit analysis, even though brick elements were applied. Application of brick elements does not automatically provide convergence of performed simulations, thus additional measures have to be applied, such as reduction of mesh size. Example of the analyzed model is given in Figure 1 and Figure 2.

**Figure 1.** Model visualization of slab with thickness of 14 [cm] with boundary conditions, X,Y,Z movements blocked at each edge.

**Figure 2.** Mesh applied to slab model with thickness of 14 [cm], mesh seed 7 [cm].
The impact source was modelled as unit pressure applied to surface of 5 x 5 [cm], in 3 random spots on surface of slab, with amplitude given in table 3. At least three random spots have to be applied due to fact that localization of point may have effect on final results. The amount of source points of excitation is taken from standard [1]. Time increment of the simulation was equal 0.00008 [s]. Due to initially predicted structural reverberation times to be estimated around 0.5 [s] total time of analysis was chosen to be equal 1 [s]. With given time of analysis and time step, sampling frequency is equal to 12500 [Hz] and thus limits analysis to frequency of 6250 [Hz]. Due to this convenience, analysis in 1/3rd octave bands can be done to band with centre frequency of 5000 [Hz] which is stated in [1], as highest frequency band for measurements. This treatment excludes aliasing problem near the highest analysed frequency band.

| Time [s] | Amplitude [-] |
|---------|---------------|
| 0.00000 | 0             |
| 0.00008 | 1             |
| 0.00016 | 0             |

Table 3. Amplitude for unit pressure applied as impact.

2.3. Method of obtaining results
Obtaining of simulation results consisted of three stages. First one was to perform FE simulations in Abaqus software and receive accelerations normal to surface with combination of 3 emitting points and 4 receiving points consistent with standard [1]. Second step was to export results of normal accelerations to the element surface and calculate decay curves and plot the cumulative spectral decay (CSD) function. Third step was to calculate and compare total loss factors obtained from structural reverberation time.

The calculation of structural reverberation time was set basing on T20, which states the evaluation range between 5 dB and or 25 dB below the maximum level [1]. Due to fact that there were results of simulations, not an in-situ test, background noise was not an issue. Decay curves where treated mathematically according to standard [2] using backward-integration method.

3. Results and discussion
Following eigenfrequencies up to 200 [Hz] of the model without damping are presented in Table 4.

| Mode number [-] | 1 | 2 | 3 | 4 | 5 | 6 |
|-----------------|---|---|---|---|---|---|
| Frequency [Hz]  | 48.29 | 88.79 | 105.98 | 142.58 | 152.68 | 192.85 |

Table 4. Eigenfrequencies of analysed slabs

3.1. Structural reverberation time with varying damping
Below in Figure 3 to 6 are presented calculated structural reverberation times with various Rayleigh damping coefficients. Values of α-coefficient and β-coefficient are consistent with assumed values in Table 2.

As it can be clearly seen in Figure 6 values of damping coefficients, assumed in this paper, α = 15 and β = 0 are giving closest results of structural reverberation time to those measured in laboratory [5]. Nevertheless, comparing with simulation results and laboratory tests [5] it can be stated that formulas used in general to estimate damping in whole buildings [8, 9] should not be used to estimate damping in individual building elements for broadband analyses without extra precaution. In order of magnitude (10^-5) β-coefficient give stable result of structural reverberation time, but still well below values given in literature [5]. In order to meet laboratory results in simulation, there were additional Rayleigh coefficients given as follows α = 15, β = 3e-6 and α = 15, β = 3e-7. The results of additional simulations are given in Figure 7.
Results presented in Figure 7 shows an interesting behavior with increase of $\beta$- coefficient. Even though it can be thought that damping is increased with increase of $\beta$- coefficient, behavior of structure is counter-intuitive. The stiffness part of Rayleigh damping decreases structural reverberation time (increases damping) in higher frequencies– for $\beta = 3e-7$ above 630 [Hz] frequency band and for $\beta = 3e-6$ above 80 [Hz] frequency band.
3.2. Cumulative spectral decay (CSD)
Below in figures from 8 to 13 are presented CSD plots showing decay in each frequency. Dynamic range of plots is 40 [dB] in order to exceed the 25 [dB] drop required by standard [1]. Extra 15 [dB] were given to show late decay of bending wave in structure. Time plotted equals to 1 [s] and is limited by time of simulation. Frequency range is set between 31.5 [Hz] up to 4000 [Hz] with 1/3rd octave smoothing.

Figure 8. CSD plot for 14 [cm] thick slab with damping coefficients $\alpha=0;\beta=0$.

Figure 9. CSD plot for 14 [cm] thick slab with damping coefficients $\alpha=0;\beta=9e^{-5}$.

Figure 10. CSD plot for 14 [cm] thick slab with damping coefficients $\alpha=10;\beta=0$.

Figure 11. CSD plot for 14 [cm] thick slab with damping coefficients $\alpha=10;\beta=9e^{-5}$.

Figure 12. CSD plot for 14 [cm] thick slab with damping coefficients $\alpha=20;\beta=0$.

Figure 13. CSD plot for 14 [cm] thick slab with damping coefficients $\alpha=20;\beta=9e^{-5}$.
Comparing CSD plots with structural reverberation times reaches to interesting conclusions, which are focused on high frequency ranges. As it can be seen in two adjacent figures with and without β-coefficient of Rayleigh damping, there is a huge change in both frequency spectrum and decay time. Interesting fact is that α-coefficient is not changing character of response of slab in high manner. Also worth mentioning is fact that initial frequency response varies between different values of β-coefficient. Increase of value of β-coefficient tends to decrease and smooth frequency response spectrum in higher frequencies even up to 30 [dB] in 4000 [Hz] frequency band.

3.3. Total loss factor (TLF) \( \eta_{\text{total}} \)

In figures 14 to 18 are presented plots with TLF calculated basing on following formula (3) where \( f \) is frequency in [Hz] and \( T_s \) is structural reverberation time in [s]. Values of α-coefficient and β-coefficient are consistent with assumed values in Table 2.

\[
\eta_{\text{total}} = \frac{2 \pi}{fT_s}
\]

(3)

As it can be concluded, transmission loss factor does not show issues with subresonant zone below 1\textsuperscript{st} mode of structure. This means using only transmission loss factor as a factor to evaluate damping in structural element may give wrong conclusions. On the other hand, transmission loss factor is seems to be good tool to evaluate influence of β-coefficient. Increase of β-coefficient is clearly visible in higher frequencies in transmission loss factor graphs. This behaviour is not clearly visible in structural reverberation time.
4. Conclusions

Basing on performed analysis of structural reverberation time, total loss factor time and frequency domain present in this paper following conclusions can be made.

Damping parameters are crucial to estimate structural reverberation time and following total loss factor. Basing on this fact and taking standard [14] into account, using FEM to estimate acoustic condition (airborne and impact sound insulation) of partition have to be preceded by estimation of damping.

The $\alpha$-coefficient of Rayleigh damping influence decreases with increase of frequency and its order of magnitude in use for slabs is $10$ and slightly meet prediction of literature [8, 9]. The $\beta$-coefficient of Rayleigh damping need to be applied with order of magnitude $10^{-6}$ and has high influence on frequencies above $2^{nd}$ natural frequency of slab. Order of magnitude predicted by [8, 9] is higher than it is applicable. This might be the result of fact that formulas (1) and (2) are commonly used only in low frequencies – up to $100$ [Hz].

Subresonant region found to be insensitive to changes of damping parameters considering transmission loss factor. This way of behaviour resembles single degree of freedom (SDOF) mass-spring system, where amplitude (instead of structural reverberation time) of subresonant zone tends to be unchanged with varying damping. What is more changes in amplitude in SDOF mass-spring in spectrum are also similar to changes in structural reverberation time.

The analysis of influence of $\alpha$-coefficient and $\beta$-coefficient on behaviour of structural element should be performed by bot structural reverberation time and structural loss factor. For $\alpha$-coefficient analysis using structural reverberation time shows higher sensitivity than for $\beta$-coefficient. On the other hand, total loss factor analysis is more sensitive for $\beta$-coefficient changes. Total loss factor is directly used for calculation of sound insulation [14], so it can be concluded that this coefficient is crucial for acoustic simulations. Also the $\beta$-coefficient has very high influence on direct frequency response of considered structure. This fact has to be taken into consideration especially modelling structure-borne noise in room.

Further work will consist of laboratory analysis of different sizes of partition in order to provide formulas analogous to formulas (1) and (2). These formulas are not meant to be applied for single structural elements, that is why they are not predicting $\alpha$-coefficient and $\beta$-coefficient properly. Further test will provide values of $\alpha$-coefficient and $\beta$-coefficient as function of partition material and partition size (absolute values or length to width ratio).
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