Numerical modelling of the reinforced concrete beam shear failure

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Abstract. Shear failure of reinforced concrete members is a complex process, which depends on a huge number of different factors. It is less investigated compared to other types of failure. Modern numerical methods, including finite-element method, allow predicting complex behavior of different structures when loaded. This article deals with the detailed nonlinear analysis of the shear failure of the reinforced concrete beam, using Drucker-Prager yield criterion. The results generated based on the finite-element model, developed in ANSYS software, were compared to the results of a laboratory experiment. In addition, the main advantages and disadvantages of this approach were described.

1. Introduction

From year to year in building field there is a tendency for a constant increase of the volume of construction work along with significant complication of structural decisions. Using complex, innovative and effective solutions that go beyond the standard designing require the high accuracy calculations and obtaining the detailed information about stress-strain state both of the individual structures and the whole building. One of the most effective ways of obtaining the load capacity of the structure is laboratory test. Laboratory tests allow evaluating the load capacity of the reinforced concrete structure with a high accuracy. The main disadvantage of this approach is the high cost and high labour intensity.

As a rule, the failure of the flexural reinforced concrete structures occurs in two ways: failure of the normal section or shear failure [1,2]. Normal section failure occurs due to the significant bending moments, while shear failure is a result of a crosscutting (shear) force. Furthermore, the failure process of the flexural reinforced concrete structures is well described and studied unlike the shear failure, that despite the huge variety of different experiments and investigations [3-5] is not well researched, which is causing ineffective calculation methods in the building codes of practices [6-11].

Another approach, which allow predicting the behavior of construction under the load is using the finite-element method. The emergence of powerful software and the use of the finite element method has made it possible to reproduce the complex nonlinear behavior of reinforced concrete structures.
The aim of this work is demonstrating the effectiveness of modeling reinforced concrete structures, failure due to the shear effort, using Drucker-Prager yield criterion.

2. Laboratory test
As a basis of the finite element model was used the results of the reinforced concrete beam laboratory tests, failure due to the shear force [12]. A summary of the laboratory results for beam is shown in Table 1. Beam deformations were measured with tensoresistors, which were placed on the concrete and bars surfaces.

The beam has cross section of 210x100 mm and the length 1000 mm as shown in Figure 1.

![Figure 1. Geometrical dimensions of beam.](image1)

The beam is reinforced with 12 mm diameter bars as a longitudinal reinforcement and 10 mm diameter bars as a transverse reinforcement as shown in Figure 2.

![Figure 2. Reinforcement drawing of beam.](image2)

Reinforcement steel has the following properties: the grade of reinforcement bars is A500C [2] both for longitudinal and transverse reinforcement, tensile and compressive steel strength equals to 610 MPa. Elastic modulus is $2 \times 105$ MPa and Poisson ratio - 0.3. The cross-section of bars was determined by weighing. The strength properties of concrete were determined by the laboratory tests.
of cubes having size respectively 150x150x150 mm and 150x150x600 mm. The concrete has compressive strength 21.7 MPa and tensile strength 1.64 MPa. Elastic modulus is 37440 MPa.

The beam is simply supported with hinge at one end and roller at other end. Point load is applied in the middle of the span as shown in Figure 3.

![Figure 3. Beam loading scheme.](image)

The destruction of laboratory specimen occurred due to the cutting of the compressive zone on early formed inclined crack.

| Normal crack formation force, kN | 40,00 |
|----------------------------------|-------|
| Inclined crack formation force, kN | 40,00 |
| Failure force, kN | 96,94 |

### 3. Numerical modelling

The finite-element modeling was carried out in software ANSYS. The geometry of numerical model is identical to the laboratory sample as shown in Figure 4.

![Figure 4. Geometry of the numerical model.](image)

The SOLID 186, higher order 3-D 20-node solid element was used for modeling of concrete [13]. Drucker-Prager yield criterion was used for modeling of complex nonlinear behavior of concrete [14]. This yield surface is a combination of compression and tension Drucker-Prager yield surfaces as shown in Figure 5.
Figure 5. Composite Drucker-Prager yield surface.

Yield surface is built in three dimensional principal stress space. The Drucker-Prager yield surface can be described by two equations:

For compressive load:

\[ f_{DPc} = \frac{\sigma_e}{\sqrt{3}} + \beta_c \sigma_m - \sigma_{yc} \Omega_c \]  \hspace{1cm} (1)

where \( \beta_c \) and \( \sigma_{yc} \) can be calculated with \( R_{bc} \), which is determine the angle between Drucker-Prager's compressive yield surface and axis of the principal stress axis; \( R_c \) – uniaxial compressive strength; \( \sigma_e \) – equivalent stresses, \( \sigma_m \) –average stresses; \( \Omega_c \) – hardening function.

\[ \beta_c = \frac{\sqrt{3}(R_b - R_c)}{2R_b - R_c} \]  \hspace{1cm} (2)

\[ \sigma_{yc} = \frac{R_b R_c}{\sqrt{3}(2R_b - R_c)} \]  \hspace{1cm} (3)

For tensile load:

\[ f_{DPt} = \frac{\sigma_e}{\sqrt{3}} + \beta_t \sigma_m - \sigma_{yt} \]  \hspace{1cm} (4)

where \( \beta_t \) and \( \sigma_{yt} \) can be calculated as follows:

\[ \beta_t = \frac{\sqrt{3}(R_t \Omega_t - R_c \Omega_c)}{R_c \Omega_c + R_t \Omega_t} \]  \hspace{1cm} (5)

\[ \sigma_{yt} = \frac{2R_c \Omega_c R_t \Omega_t}{\sqrt{3}(R_c \Omega_c + R_t \Omega_t)} \]  \hspace{1cm} (6)

where \( \Omega_t \) – softening function, \( R_t \) - uniaxial tensile strength.

The linear form of hardening/softening function was used for structure as shown in Figure 6.
Figure 6. Softening in compression (a) and tension (b).

The REINF264 [15] 3-D discrete link was used for modeling of reinforcement. This element is mesh-independent, that allows producing fast solution with high convergence rate and effectively works with complex geometry.

4. Results and discussion
The Figure 7 demonstrates a load-deflection curve of the numerical model.

Figure 7. Deflection-load curve of the numerical model.

Having analyzed the deflection-load curve, three main stages of the construction work can be identified [16-18], namely:

- Elastic stage
- Plastic stage
- Failure stage

The elastic stage is occurred with the loads approximately 10% from failure load. This stage can be characterized this relatively low deformations of the concrete and reinforcement. The plastic stage starts after reaching the concrete of the tensile area the tensile strength. This stage can be characterized by the active involvement of the tensile area's reinforcement in work of the construction, the stresses in which is significant growing.

The inclined and normal cracks are formed in beam under the load approximately equals to 40,00 kN, that is very close to results of the laboratory test. The Figure 8 bellow is show the crack pattern under the load of 40,00 kN.
Under the load approximately equals to 94% of the failure load the reinforcement of the tensile area reach the yield stress [19]. At the same time the last stage of the beam's work starts - failure stage. The crack pattern of the finite element model before the failure is virtually identical to cracks of the laboratory sample, as shown in Figure 9.

The failure of the finite-element model was accompanied by the exponential deformation growth, as in the laboratory specimen [20]. The failure load for numerical model is equal to 98,05 kN, that is very close the failure load of the laboratory test, equals to 96,04 kN. Failure of the finite-element model occurred with the parallel shifting the parts of the beam against each other in the moment before the destruction occurring due to the significant shear force [4,5], that is clearly visible on the Figure 10, imagines the total deflection the beam.

5. Conclusions
The results of the finite-element modeling of the shear failure of the reinforced concrete beam, using Drucker-Prager yield criterion is accurate enough describing the nonlinear behavior of the beam, tested in laboratory conditions.

The main disadvantage of this approach is the need for the using the computer with considerable power. Beam is the relatively small structure, calculations of which is not require much time.
However, using this approach for the calculation of the whole building can take a lot of time and require a great computing power. Currently, there are powerful enough computers, which allow to significantly reducing the calculation time. However, such a computers are very expensive.

The obvious advantage of this approach is the high calculation accuracy, allows to predict the complex nonlinear behavior of the structure. This approach allows obtaining the detailed information about stress-strain state of the construction. In addition, it enable to carry out the volumetric calculations, that impossible in laboratory conditions.

Despite on the high accuracy of the results, this approach cannot fully displace the laboratory tests of constructions. However, using of this method allows to significantly reducing the number of laboratory tests. Thus, laboratory experiments can be used for creation and verification of the finite-element model, which in turn can be used for the further calculations and investigations. This method enable to reduce financial implications and saving time. Such an approach is currently used in many field, such as medicine, aircraft, machinery and so on.

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