The age of the universe, the Hubble constant and the accelerated expansion

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Abstract

The idea of an accelerating universe comes almost simultaneously with the determination of Hubble’s constant by one of the Hubble Space Telescope Key Projects. The age of the universe dilemma is probably the link between these two issues.

The age of the universe is calculated by two different ways. Firstly, a lower limit is given by the age of the presumably oldest objects in the Milky Way, e.g., globular clusters. Their ages are calculated with the aid of stellar evolution models which yield 14 Gyr and 10% uncertainty. These are fairly confident figures since the basics of stellar evolution are quite solid. Secondly, a cosmological age based on the Standard Cosmology Model derived from the Theory of General Relativity. The three basic models of relativistic cosmology are given by the Friedmann’s solutions of Einstein’s field equations. The models are characterized by a decelerated expansion from a spatial singularity at cosmic time \( t = 0 \), and whose magnitude is quantified by the density parameter \( \Omega_0 \), the present ratio of the mass density in the universe to the so-called critical mass density. The critical Friedmann model has the critical mass density, and therefore, it has \( \Omega_0 = 1 \), which implies a spatially flat geometry. The present observed density parameter of the universe is approximately \( \Omega_0 = 0.01 \), all made of baryonic matter, the usual matter in stars, planets and human beings. But the total mass density parameter — baryonic plus non baryonic, visible and dark — is \( \Omega_m = 0.3 \), derived from large-scale structure dynamics. Given that the non-critical Friedmann models are highly unstable at time \( t = 0 \), meaning that any minute difference from a critical model would result, at time \( t = t_0 \) (now), an immensely large difference from \( \Omega_0 = 1 \), it is generally accepted that the density parameter is precisely equal to 1. The discrepancy with the observed \( \Omega_0 \) is considered as circumstantial evidence of the incompleteness nature of science. Eventually, one should find the reason for the difference. The fiducial cosmological age of the universe is thus naturally given by the age of the critical model.
It amounts to $t_\circ = (2/3)H_\circ^{-1}$, with $H_\circ$ being the present Hubble constant. Or, $t_\circ = 6.5 \, h^{-1}$ Gyr ($h$ being the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$). Before the 1990s, $h$ was rather uncertain, ranging from 0.5 to 1. The lowest $h$ puts the fiducial cosmological age at acceptable — yet marginal — agreement with the stellar evolution age: $t_\circ = 13$ Gyr.

Then, two important projects in observational astronomy began in the early 1990s. The *Hubble Space Telescope Key Project to Measure the Hubble Constant* and the efforts on using SNe Ia as standard candles to measure the cosmological deceleration. In 1995, Perlmutter et al. publish their result on one supernova at redshift $z=0.458$ and conclude that the universe is indeed decelerating at that cosmic epoch. Some years later, Madore et al. (1998) publish their first result on the HST Key Project: $H_\circ$ is in the range 70 to 73 km s$^{-1}$ Mpc$^{-1}$. The fiducial cosmological age sits now in an uncomfortable narrower band, namely, 8.9–9.3 Gyr, inconsistent with the stellar evolution age. But, in the next year, Perlmutter et al. publish their new results with an extended sample of SNe. They find now that the universe is in an accelerated expansion. The Friedmann critical model, modified to include a cosmological constant that drives the accelerated expansion, implies an age $t_\circ = (2/3)H_\circ^{-1} \Omega_m^{-1/2} \ln[(1-\Omega_m)^{-1/2}+\Omega_m^{1/2}(1-\Omega_m)^{-1/2}] \approx (2/3)H_\circ^{-1}(1-\Omega_m)^{-0.3}$, where $\Omega_m$ is the density parameter associated to the cosmological constant (see Figure 1). With $\Omega_m + \Omega_\Lambda = 1$, and $\Omega_m = 0.3$, one has now $t_\circ = 12.8–13.3$ Gyr, again matching stellar ages.

The age dilemma seems to come to an end. The solution is substantiated with two further amendments. Freedman et al. (2001) published the final results of the HST Key Project on $H_\circ$. They confirm the previous range with the value $H_\circ = 72$ km s$^{-1}$ Mpc$^{-1}$, and 10% uncertainty as initially aimed. Riess et al. (2004), with an enlarged SNe Ia sample, the so-called Gold Sample, confirmed the accelerating universe, adding the new finding that a transition from a decelerated phase occurs at a redshift of $0.46 \pm 0.13$, which matches Perlmutter’s 1995 sole SN with extraordinary precision.

One big problem posed to the Standard Model is solved but others are raised. The existence and identification of the so-called *dark energy* is the most significant. Dark energy — a generic name for what might be a cosmological constant or other candidates — constitutes approximately 70% of the mass-energy content of the universe and is responsible for its accelerated expansion.

Do you believe the accelerated expansion? Many do. Some do not. One amongst the latter is John Archibald Wheeler (1911-2008). And he gives two reasons (Taylor and Wheeler 2000, p. G-11): “(1) Because the speed-up argument relies too trustingly on the supernovas being standard candles. (2) Because such an expansion would, it seems to me, contradict a view of cosmology too simple to be wrong.” Wheeler’s second reason comes from his preferred cosmological model, namely, a closed Friedmann model (see p. G-1). He optimistically goes on saying that “Such clashes between theory and experiment have often triggered

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decisive advances in physics. We can hope that some decisive advance is in the offing.”

References

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Figure 1: Top panel: the age of the universe in an accelerated model as a function of $\Omega_{\Lambda_0}$. Filled circles are the solution of Friedmann’s equation with cosmological constant and the solid curve is an approximation to the exact solution. Bottom panel shows the scale factor for the decelerated Einstein-de Sitter universe (solid curve) and for an universe with an accelerated phase at recent epochs (dashed curve). The age of the universe in both models is shown and corresponds to the scale factor of unity. Notice the changing of concavity of the dashed curve just before the scale factor of 1, meaning that the expansion has changed from a decelerated to an accelerated phase.