Abstract

We present results on the mixing parameter \( \Delta m_d \) and tests of the \( CPT \) symmetry in the \( B_d^0 \bar{B}_d^0 \) mixing from the time evolution of dilepton events on the \( \Upsilon(4S) \) resonance. The analysis is based on a 5.9 fb\(^{-1} \) data sample collected by the Belle detector at the KEKB accelerator from January to July of 2001. We obtain \( \Delta m_d = 0.463 \pm 0.008 \) (stat) \( \pm 0.016 \) (syst) ps\(^{-1} \). No evidence for \( CPT \) violation is found.

This is the first determination of \( \Delta m_d \) from time evolution measurements at \( \Upsilon(4S) \), and the first time that an experimental limit on \( |m_{B_d^0} - m_{\bar{B}_d^0}| / m_{B_d^0} \) has been obtained.
1 Introduction

The mixing parameter of the $B_d^0$-$\bar{B}_d^0$ system is equal to the mass difference of the physical (mass) eigenstates of the $B_d^0$ mesons, $\Delta m_d$. $\Delta m_d$ is considered a fundamental parameter of the $B_d^0$ system and a crucial element in the study of $CP$ violation. Its extremely small value (13 orders of magnitude smaller than the $B_d^0$ mass) enhances the sensitivity of the $B_d^0$-$\bar{B}_d^0$ interferometry, making the neutral meson system a powerful probe for physics beyond the Standard Model.

At KEKB, 8.0 GeV electrons and 3.5 GeV positrons collide on the $\Upsilon(4S)$ resonance, producing correlated $B_d^0$-$\bar{B}_d^0$ pairs that live in an antisymmetric state. The time-dependent asymmetry between same-flavor ($B_d^0B_d^0/\bar{B}_d^0\bar{B}_d^0$) and opposite-flavor ($B_d^0\bar{B}_d^0$) decay pairs undergoes an oscillation with frequency equal to the mixing parameter, $\Delta m_d$. The center of mass (CM) of the meson system is moving along the electron beam direction with a Lorentz boost of $\beta\gamma = 0.452$. Since the two $B$ mesons are practically still at the $\Upsilon(4S)$ frame, we can use this boost and their decay vertex separation, $\Delta z$, to calculate the proper time difference between their decay times: $\Delta t = \Delta z/\beta\gamma c$. The typical decay vertex separation is 200 $\mu$m, a distance well within Belle’s vertexing capabilities.

High momentum electrons (or positrons) and muons are used for both the $B$ flavor tagging and the decay vertex determination. The proper time distributions for same- and opposite-flavor $B$ decay pairs can be used for the study of the $B_d^0$-$\bar{B}_d^0$ time evolution and the $\Delta m_d$ measurement. By allowing for a possible non-zero mass or lifetime difference between $B_d^0$ and $\bar{B}_d^0$—encoded in the complex parameter $\cos \theta$ of the system’s “effective” Hamiltonian— we can use the same time distributions to test the conservation of the $CPT$ symmetry in the $B_d^0$-$\bar{B}_d^0$ mixing.

The analysis presented here is based on data samples collected by the Belle detector on the $\Upsilon(4S)$ resonance and 60 MeV below the peak from January to July of 2001, corresponding to integrated luminosities of 5.9 fb$^{-1}$ and 0.6 fb$^{-1}$, respectively.

2 The Belle Detector

The Belle detector is designed to make precise measurements of charged and neutral particles over a wide acceptance. The combination of a silicon vertex detector (SVD) and a central drift chamber (CDC) surrounding the SVD is used for vertexing. For measurements of the charged particle momenta, the SVD and CDC are embedded in a 3.4 m diameter superconducting solenoid. The generated 1.5 T magnetic field runs parallel to the direction of the $z$-axis (defined by the electron beam direction).

For the particle identification (PID), three subsystems are employed: the CDC (measuring the mean energy loss $dE/dx$), the Aerogel Čerenkov Counters or ACC (a
set of 1188 Čerenkov radiation modules located outside the CDC volume) and the Time of Flight system or TOF (128 scintillation counters providing timing information).

A pair of calorimeters, the Electromagnetic Calorimeter or ECL (8736 CsI/Tl crystals) and the Electromagnetic Forward Calorimeter or EFC (320 BGO crystals), is used for the detection and identification of photons and electrons.

Finally, a set of 14 layers of resistive plate counters sandwiched between 4.7 cm thick iron plates and filled up with a gas mixture on high voltage is the \( K_L \) and muon subdetector (KLM). KLM is the only detector system placed outside the coil. The iron also serves as the return path for the magnetic flux.

Ref. [1] gives details on the specifications of the detector components.

3 Analysis

In this analysis we exclusively use leptons for flavor tagging. This mode is referred in the bibliography as “dileptons”: events where both \( B \) mesons are flavor-tagged by leptons. The term “dileptons” implies that both of the \( B \) mesons decay via a semileptonic process \( B^0_d(t), \ B^0_s(t) \rightarrow \ell^\pm X^{\mp} \) (where \( \ell^\pm = e^\pm, \mu^\pm \)) through a \( b \rightarrow c \) quark transition, emitting primary leptons.

3.1 Selection criteria

A set of kinematic and quality criteria is used to separate hadronic events from continuum, QED and beam backgrounds, and to suppress contamination from cascade or fake leptons.

Hadronic events are classified as having at least five tracks, a primary vertex with radial \( (V_r) \) and \( z \) \( (V_z) \) components with respect to the origin satisfying \( V_r < 1.5 \text{ cm} \) and \( |V_z| < 3.5 \text{ cm} \), a total visible energy detected by ECL and CDC greater than 50\% of \( E^{CM}_{\Upsilon(4S)} \) \( (E^{CM}_{\Upsilon(4S)} \) being the energy of \( \Upsilon(4S) \) at CM), a \( z \) component of the total visible energy less than 30\% of \( E^{CM}_{\Upsilon(4S)} \), a total calorimeter energy between 2.5\% and 90\% of \( E^{CM}_{\Upsilon(4S)} \), and a ratio \( R_2 \) of second and zeroth Fox-Wolfram moments [1] less than 0.7.

We select the electrons and the muons from the data sample of hadronic events. For the electron identification, we use the cluster energy, and position and shape of the electromagnetic shower in ECL, the \( dE/dx \) energy loss rate in CDC, the matching between the track in the CDC and the cluster in ECL, and TOF and ACC hit information for the fake lepton suppression. Optimization of the above parameters yields a \( \sim 90\% \) efficiency for electrons and a \( \sim 0.3\% \) misidentification probability for charged tracks with \( p > 1 \text{ GeV/c} \). Electrons consistent with pair production from
photon conversions are removed. For the muon identification, we extrapolate the track in the CDC to the KLM, and we examine the difference between the expected and the actual penetration, as well as the matching quality of the KLM hits with the fitted track. The efficiency is $\sim 85\%$ for muon tracks and the misidentification probability $\sim 2\%$ for particles with $p > 1$ GeV/c.

A "$J/\psi$ veto" is then applied, eliminating events consistent with a $J/\psi$ decay to a dilepton pair. All remaining leptons of the event are ordered according to the magnitude of their momentum at the $\Upsilon(4S)$ rest frame. Dilepton event candidates must have at least two particles identified as leptons. The leptons with the highest and the second highest momenta in the CM are selected as "dileptons". Leptons are required to have a CM momentum and a laboratory polar angle within the ranges $1.1 < p^* < 2.3$ GeV/c and $30^\circ < \theta^{\text{lab}} < 135^\circ$, $r$ and $z$ distances of the decay vertex from the beam interaction point (BIP) satisfying $|\Delta r^{\text{BIP}}| < 0.05$ cm and $|\Delta z^{\text{BIP}}| < 2.00$ cm, and at least two and one hits in the $z$ and $(r, \phi)$ SVD layer strip planes, respectively. This is a combination of cuts aiming at suppressing cascade and fake leptons, as well as selecting events with good vertex resolution for the lepton tracks. To further reduce particle pairs originating from the same $B$, the opening angle between the lepton tracks calculated at CM, $\theta^{\ell \ell}_*$, is required to satisfy $-0.80 < \cos \theta^{\ell \ell}_* < 0.95$. The above values of the cuts have been optimized based on Monte Carlo (MC) studies. Application of all the selection criteria yields 8573 same-sign (SS) and 40981 opposite-sign (OS) dilepton events on the $\Upsilon(4S)$ peak, and 40 SS and 198 OS dilepton events below the resonance.

### 3.2 Vertexing

The proper time difference can be calculated from the $z$ component of the distance between the decay vertices of the $B$ meson pair, $\Delta z$: $\Delta t = \Delta z/\beta\gamma c$. The $z$-vertex position of the leptons is determined by finding the intersection of each of the fitted lepton tracks with an ellipsoid known to contain the $B^0_s$ decay vertices. The dimensions of this volume are calculated by convolving the BIP "profile" ($\sigma_x^{\text{BIP}} \sim 100 - 120$ $\mu$m, $\sigma_y^{\text{BIP}} \sim 5$ $\mu$m, $\sigma_z^{\text{BIP}} \sim 2000 - 3000$ $\mu$m) with the average flight length of the $B$ meson ($\sim 20$ $\mu$m at CM). The center and the dimensions of the BIP profile are determined by the primary vertex position distribution on a run-by-run basis (a few tens of thousand events). The large uncertainty of the $z$ component of the BIP is irrelevant since we are only interested in the difference between the $z$ vertices, $\Delta z$. $\Delta z$ is defined as $z_{\ell^+} - z_{\ell^-}$ for the OS events, whereas for the experimentally indistinguishable leptons of SS events we use the absolute value of $\Delta z$. 

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3.3 General strategy

We classify the contributions to the SS and OS proper time distributions as “signal” (events with two primary leptons from semileptonic $B^\pm$ or $B^0_d$ decays), “background” ($\Upsilon(4S)$ events with at least one cascade or fake lepton) and continuum (all non-$\Upsilon(4S)$ events). The background is further divided into two sub-categories indicating correctly or wrongly tagged events. The $\Delta z$ spectra for SS and OS dileptons can therefore be described as sums of signal ($S$), correctly tagged background ($C$), wrongly tagged background ($W$) and continuum. Each of these contributions is characterized by a (normalized) proper time distribution, and is scaled by the number of source events ($N$) and an overall selection efficiency ($\epsilon$). Parameters that control the populations of source events are the number of $\Upsilon(4S)$ and continuum events in the data sample ($N_{\Upsilon(4S)} = N_{b\bar{b}}$ and $N_{\text{cont}}$), the branching fractions of neutral and charged $B$ pairs in $\Upsilon(4S)$ decays ($f_0$ and $f_\pm = 1 - f_0$), the fraction of mixed events in neutral $B^0_d$ pairs ($\chi_d$), and the semileptonic branching fractions for neutral and charged $B$ mesons ($b_0$ and $b_\pm$). Table 1 summarizes the categories of the events for the SS and OS distributions and the numbers of source events, $N$.

The signal terms for mixed ($B^0_d B^0_d$; “mix”), unmixed ($B^0_d \overline{B}^0_d$: “unm”) and charged ($B^+ B^-$: “chd”) $B$ meson pairs are modeled analytically by smearing theoretical expressions with explicit mixing and CPT-violating dependence:

$$P_{\text{mix}}(\Delta t) = (|\sin \theta|^2/4 \tau_{B^0_d}) e^{-|\Delta t|/\tau_{B^0_d}} [1 - \cos(\Delta m \Delta t)]$$  \hspace{1cm} (1)
\[ P_{\text{unm}}(\Delta t) = \left(1/4\tau_{B_d^0}\right) e^{-|\Delta t|/\tau_{B_d^0}} \left[ 1 + |\cos \theta|^2 + (1 - |\cos \theta|^2) \cos(\Delta m \Delta t) \right] - 2 \text{Im}(\cos \theta) \sin(\Delta m \Delta t) \]  
\[ P_{\text{chd}}(\Delta t) = \left(1/2\tau_{B_{\pm}}\right) e^{-|\Delta t|/\tau_{B_{\pm}}} \]  

A non-zero value for the complex parameter \( \cos \theta \) would indicate \( CPT \) violation \[4\]. A \( \cos \theta \neq 0 \) would also affect the \( B_d^0 \) mixed event fraction, \( \chi_d \):

\[ \chi_d = \frac{|\sin \theta|^2 x_d^2}{\sin \theta^2 x_d^2 + (2 + x_d^2 + x_d^2 |\cos \theta|^2)} \]  

where \( x_d \equiv \tau_{B_d^0} \Delta m_d \); \( 1/\tau_{B_d^0} \equiv (\Gamma_H + \Gamma_L)/2 \) is the mean width of the mass eigenstates, assumed to be known in this analysis \[5\].

If \( CPT \) is a good symmetry, then \( \cos \theta = 0 \), and the above expressions become the standard ones one finds in the bibliography:

\[ P'_\text{mix}(\Delta t) = \left(1/4\tau_{B_d^0}\right) e^{-|\Delta t|/\tau_{B_d^0}} \left[ 1 - \cos(\Delta m \Delta t) \right] \]  
\[ P'_\text{unm}(\Delta t) = \left(1/4\tau_{B_d^0}\right) e^{-|\Delta t|/\tau_{B_d^0}} \left[ 1 + \cos(\Delta m \Delta t) \right] \]  

\[ \chi'_d = \frac{x_d^2}{2(1 + x_d^2)} \]

The detector resolution for the signal terms is determined by the \( \Delta z \) distribution of \( J/\psi \) decays to lepton pairs, which here serves as a “response function”. These leptons are selected with the same kinematic and quality criteria applied on primary lepton pairs from \( B \) decays \[1\] (Sec. \[\Delta \]). The \( \Delta z \) distribution is obtained by finding two separate \( z \) vertices for the two lepton tracks of the \( J/\psi \) decay, and by taking their relative difference. Its width is \( \sigma = 112 \, \mu m \). To properly account for the long tails at large \( |\Delta z| \), the distribution is binned in a lookup table \( g \), which is used to smear the theoretical functions:

\[ \tilde{P}_a(\Delta t_{\text{smea}}) = \frac{\int g(\Delta t_{\text{smea}} - \Delta t) P_a(\Delta t) \, d(\Delta t)}{\int \int g(\Delta t_{\text{smea}} - \Delta t) P_a(\Delta t) \, d(\Delta t) \, d(\Delta t_{\text{smea}})} \]  

with \( a = \text{“mix”, “unm”, “chd”} \).

The modeling of the \( \Upsilon(4S) \) background distributions is done numerically. To this end, we have simulated large MC event samples of generic charged and neutral \( B \) meson \( \Delta z \) distributions. The major component is the combination of a primary and a secondary lepton from a cascade decay (\( e \to s \) or \( \tau \) decay). We separate the contributions from cascade and fake leptons. The number of cascade leptons is scaled to match

\[1\] With the exception of the \( J/\psi \) veto, of course.
the $\mathcal{B}(B \rightarrow D^\pm X)$, $\mathcal{B}(B \rightarrow D^0 X)$ branching ratios measured by CLEO [3]. The number of fake leptons is independently determined with $K_S \rightarrow \pi^+\pi^-$ decays from the same run period. The small discrepancy in the detector’s vertexing performance found between MC and data (~10%) is corrected by applying a convolution with a single Gaussian with $\sigma = 50^{+18}_{-12} \mu m$ to each MC-determined background distribution. This procedure is tested on similarly obtained $\Delta z$ distributions from $J/\psi \rightarrow \ell^+\ell^-$ and $K_S \rightarrow \pi^+\pi^-$ decays and is found to give a satisfactory matching. The strong dependence of the background distributions on the value of $\Delta m_d$ is modeled through a linear interpolation of distributions from two generic $B^0_d$ MC samples, generated with $\Delta m_d = 0.423 \text{ ps}^{-1}$ and $\Delta m_d = 0.464 \text{ ps}^{-1}$. Finally, we have the option of adjusting the steepness of the exponentially falling background distributions. This is useful when we want to perform the fit for a different value of the (fixed) $B^0_d$, $B^\pm$ lifetimes, or calculate the associated systematic error.

The continuum $\Delta z$ distributions are also modeled numerically with MC simulation samples. The relative weight of the continuum contribution is determined by the number of off-resonance data, scaled to account for luminosity and energy differences. The correction to the vertex resolution is applied here as well.

### 3.4 Fitting

The data is fit to a linear combination of analytical and numerical expressions to extract the mixing and the CPT violation parameters. A binned maximum likelihood fit is performed simultaneously to the SS and OS dilepton $\Delta z$ distributions in the region $|\Delta z| < 1.85 \text{ mm}$ [3].

We fix the parameters $\tau_{B^0_d} = 1.548 \text{ ps}$ [3], $\tau_{B^\pm}/\tau_{B^0_d} = 1.06$ [3], $f_\pm/f_0 = 1.05$ [4] and $b_\pm/b_0 = \tau_{B^\pm}/\tau_{B^0_d}$ [4]. Besides $\Delta m_d$ and the (complex) parameter $\cos \theta$, the fit includes two more free parameters, the selection efficiency ratios $\epsilon_S/\epsilon_C$ and $\epsilon_S/\epsilon_W$ for the charged $B$ mesons. The relative selection efficiencies for mixed, unmixed and charged $B$ mesons within each event tag type ($S$, $C$ or $W$) are fixed to their MC values.

The expected total MC distributions for the SS, OS spectra are not normalized separately to the numbers of the events of the two histograms, but only to the total number of data events $N_{\text{tot}} = N_{SS} + N_{OS}$. In other words, the ratio of the populations of the SS and OS total MC distributions is determined by the fit result for the mixing.

Assuming that CPT is a good symmetry, and therefore by fixing $\cos \theta = 0$, we find $\Delta m_d = 0.463 \pm 0.008 \text{ ps}^{-1}$ with $\chi^2/N_{\text{dof}} = 332.5/376$. The efficiency ratios are $\epsilon_S/\epsilon_C = 143.2 \pm 5.1$ and $\epsilon_S/\epsilon_W = 103.6 \pm 15.0$.

When $\cos \theta$ is a free parameter in the fit, we find $\Delta m_d = 0.461 \pm 0.008 \text{ ps}^{-1}$, $Re(\cos \theta) = 0.00 \pm 0.15$ and $Im(\cos \theta) = 0.035 \pm 0.029$, with $\chi^2/N_{\text{dof}} = 331.0/376$. These results are consistent with CPT symmetry. The efficiency ratios in this case
are $\epsilon_S/\epsilon_C = 142.8 \pm 5.1$ and $\epsilon_S/\epsilon_W = 102.8 \pm 14.9$. The fractions of signal events are found to be 32.1% in SS (mixed $B$ pairs) and 77.5% in OS (32.1% unmixed and 45.9% charged $B$ pairs).

Fig. 1 shows the $\Delta z$ distributions for the data and the fitted Monte Carlo distributions, for the SS (top) and OS (bottom) dilepton spectra. The signal in the SS histogram is a small term compared to the background from $B^0_d\bar{B}^0_d$ and $B^+B^-$. A much more evocative way of displaying the $B^0_d - \bar{B}^0_d$ oscillation is by plotting the time dependent asymmetry between SS and OS dileptons:

$$A(\Delta t) = \frac{N^{+-}(\Delta t) - N^{++}(\Delta t)}{N^{+-}(\Delta t) + N^{++}(\Delta t)} \sim \cos(\Delta m \Delta t)$$

To a good approximation, most of the background cancels out, and the time dependent asymmetry reveals a clean oscillation signal, as illustrated in Fig. 2. In the same plot, a convolved cosine term is superimposed on the data points, with the period determined from the fit.

### 3.5 Systematic errors

To estimate the systematic errors of the various assumptions made in the fit, the associated input parameter is varied, typically by $\pm \sigma$, and then the fit is repeated. The shift in the central value is taken to be the systematic error for that category.

The main contributions to the systematic errors are from the uncertainty in the measurements of the $B$ lifetimes and the branching fractions of $\Upsilon(4S)$ to charged and neutral $B$ pairs. We calculate these contributions by adjusting the corresponding parameters by the amount of their uncertainties, one at a time. The lifetime changes affect all $B$-originating terms in the fit (exponentially decaying distributions) and, indirectly, signal or background events with one primary lepton through the propagated change in the semileptonic branching ratios. The change of branching fractions $f_\pm, f_0$ affects the relative populations of charged and neutral $B$ pairs in the distributions (signal and background).

We have made the approximation that the difference between the decay widths of the $B^0_d$ mass eigenstates, $\Delta \Gamma_d$, is zero. The conservative estimation used by most phenomenologists is $\Delta \Gamma_d/\Gamma_d \leq 10^{-2}$ \cite{5}. We report the shift in the fitted values by assuming $\Delta \Gamma_d/\Gamma_d = 1\%$ and by modifying accordingly the proper time distributions \cite{4}. For the systematic errors associated with the response function, we estimate the effect of the $B$ motion in the $\Upsilon(4S)$ frame (which includes the discrepancy of the $J/\psi \Delta z$ distribution from the true dilepton response function) and the error originating from the statistical uncertainties in the determination of the response function. This set of changes only affects the signal terms.
Figure 1: $\Delta z$ distributions for same-sign (top) and opposite-sign (bottom) dileptons for data (points) and the fitted Monte Carlo distributions (histograms). For the Monte Carlo distributions, signal and background from different categories are plotted separately.
Figure 2: Time dependent asymmetry between SS and OS dileptons. The curve is a convoluted cosine with period determined by the fit. The negative $\Delta z$ region for the OS histogram has been folded into the positive region for display purposes.
We change the cascade lepton rates by modifying the branching ratios of $B$ mesons decaying to $D^0$ and $D^\pm$ in the MC by $\pm 4.6\%$ and $\pm 14.3\%$, respectively. We change the levels of fake lepton and continuum contamination in the sample by shifting the fake rates by $\pm 35\%$, and the continuum contribution by $\pm 16$ (SS) and $\pm 7\%$ (OS). The error in the determination of the width of the convolving Gaussian ($\sigma = 50^{+12}_{-18}$$\mu$m) is used as the uncertainty in the detector’s vertexing performance for the background terms ($\sim 30\%$ of the sample). We change the width of the Gaussian and report the shift in the fit central values with the new background distributions. We check the effect of finite MC statistics by varying the number of background events on a bin-by-bin basis by the amount of the statistical errors.

We finally check the effect of binning and several other possible sources of systematic errors, and we find that their contribution is negligible.

Table 2 summarizes the systematic errors for $\Delta m$, $Re(cos \theta)$ and $Im(cos \theta)$. Since the asymmetry between positive and negative errors is small ($\sim 15 - 20\%$), we conservatively take the larger of the values to be the systematic error.

4 Summary

We have measured the mixing parameter $\Delta m_d$ and searched for $CPT$ violation in the $B^0_d$ system from the time evolution of dileptons in $\Upsilon(4S)$ decays. The analysis corresponds to an integrated luminosity of 5.9 fb$^{-1}$ of data collected with the Belle detector. The run period was from January to July of 2000. If we invoke $CPT$ symmetry for $\Delta m_d$ we obtain:

$$\Delta m_d = 0.463 \pm 0.008 \text{ (stat)} \pm 0.016 \text{ (sys)} \text{ ps}^{-1}$$

If we fit simultaneously for $\Delta m_d$ and the $CPT$-violating parameter $cos \theta$ we obtain:

$$\Delta m_d = 0.461 \pm 0.008 \text{ (stat)} \pm 0.016 \text{ (sys)} \text{ ps}^{-1}$$
$$Re(cos \theta) = 0.00 \pm 0.15 \text{ (stat)} \pm 0.06 \text{ (sys)}$$
$$Im(cos \theta) = 0.035 \pm 0.029 \text{ (stat)} \pm 0.051 \text{ (sys)}$$

These results imply $[4]$ for the mass and the lifetime differences between $B^0_d$ and $\bar{B}^0_d$:

$$|m_{B^0_d} - m_{\bar{B}^0_d}|/m_{B^0_d} < 1.6 \times 10^{-14} \text{ and } |\Gamma_{B^0_d} - \Gamma_{\bar{B}^0_d}|/\Gamma_{B^0_d} < 0.161, \text{ at 90\% C.L.}$$

The results are consistent with $CPT$ conservation. This is the first direct measurement and the first published result of $\Delta m_d$ from a time-dependent analysis on the $\Upsilon(4S)$ resonance $[8]$. The limit on $(\Gamma_{B^0_d} - \Gamma_{\bar{B}^0_d})/\Gamma_{\bar{B}^0_d}$ measurement is consistent with previous measurements $[4]$. This is the first measurement on $Re(cos \theta)$ and on the mass difference between $B^0_d$ and $\bar{B}^0_d$. 

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Table 2: Summary of systematic errors (a) with and (b) without invoking the CPT symmetry.

| Input parameter and uncertainty | Fitting for | (a) $\Delta m_d$ | (b) $\Delta m_d$ and $\cos \theta$ |
|---------------------------------|-------------|-------------------|---------------------|
|                                 |             | $\Delta m_d$ (ps$^{-1}$) | $\Delta m_d$ (ps$^{-1}$) | $Re(\cos \theta)$ | $Im(\cos \theta)$ |
| $B$ lifetimes and semileptonic branching ratios |             |                    |                    |                  |
| $\tau_{B^0} = 1.548 \pm 0.032$ ps | -0.004 | $\pm$0.004 | < 0.01 | +0.004 | -0.003 |
| $\tau_{B^\pm}/\tau_{B^0} = 1.06 \pm 0.03$ | +0.006 | +0.006 | < 0.01 | < 0.001 |
| $\Delta \Gamma_d/\Gamma_d < 1\%$ | < 0.001 | < 0.001 | $\pm$0.01 | < 0.001 |
| Fractions of charged and neutral $B$ meson pairs |             |                    |                    |                  |
| $f_{\pm}/f_0 = 1.05 \pm 0.08$ | +0.007 | +0.008 | < 0.01 | $\mp$0.001 |
| $f_{\pm}/f_0 = 1.05 \pm 0.08$ | -0.009 | -0.010 | < 0.01 | $\mp$0.001 |
| $B$ motion in $\Upsilon(4S)$ frame |             | $\pm$0.001 | $\pm$0.001 | $\pm$0.06 | < 0.001 |
| statistics of $J/\psi$ sample |             | $\pm$0.005 | $\pm$0.005 | < 0.01 | $\pm$0.050 |
| $B(B \rightarrow D^0 X)$ ($\pm4.6\%$) | < 0.001 | < 0.001 | < 0.01 | < 0.001 |
| $B(B \rightarrow D^\pm X)$ ($\pm14.3\%$) | < 0.001 | < 0.001 | < 0.01 | $\mp$0.001 |
| Fake rates ($\pm35\%$) |             | +0.003 | $\pm$0.004 | < 0.01 | $\pm$0.001 |
| Continuum |             | +0.001 | +0.002 | < 0.01 | $\mp$0.001 |
| (SS: $\pm16\%$, OS: $\pm7\%$) | -0.002 | -0.001 | < 0.01 | $\mp$0.001 |
| Detector resolution ($\pm_{18}^{12}$ $\mu$m) | $\mp$0.001 | -0.001 | < 0.01 | +0.002 |
| MC statistics |             | +0.004 | +0.005 | < 0.01 | $\pm$0.009 |
| Other background related parameters |             | $\pm$0.001 | $\pm$0.001 | < 0.01 | $\pm$0.002 |
| Other systematic errors |             | +0.012 | +0.014 | $\pm$0.06 | $\pm$0.051 |
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