Tripling the 99 % mode conversion bandwidth of gratings using a single phase shift

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Achieving large bandwidth at strong mode conversion strengths is a central problem in few-mode optical waveguides. The most uncomplicated modification of a standard mode-converting grating is to incorporate a single extra gap. Such simple design has not been applied yet for broadening conversion spectrum between core modes of any few-mode waveguide e.g. few-mode fiber, possibly due to lacking knowledge of critical parameter control: (a) phase shift value at resonance wavelength, and (b) deterministic method for placement of the phase shift(s) accommodating the change in waveguide properties during grating writing. From semi-analytical considerations, we show in full generality that our design of a single-phase-shifted grating increases the bandwidth at 99 % conversion strength by more than 2.9 times, together with elucidating crucial parameter tolerances. The phase-shift placement is determinable on-the-go from the conversion spectra, thereby obviating any estimation of modified waveguide properties during grating-writing. This offers the longed on-demand in-fiber solution for dispersion control in fiber laser cavities and communication systems.

Two-dimensional waveguides support finite number of diffraction-less propagating modes, thereby simplifying the design of photonic devices. Ignoring polarization, the electric field of an LP mode \([n, m]\) is given by

\[
\begin{align*}
E_{\beta_k}(x, y, \lambda)e^{i(\frac{2\pi}{\lambda}t - \beta_k(\lambda)z)} & \quad \text{propagating forward} \\
E_{\beta_k}(x, y, \lambda)e^{i(\frac{2\pi}{\lambda}t + \beta_k(\lambda)z)} & \quad \text{propagating backward}
\end{align*}
\]

where each mode is identified by its unique propagation constant \(\beta_k(\lambda) > 0\), with transverse electric field \(E_{\beta_k}(x, y, \lambda) \in \mathbb{R}\) and waveguide axis \(z\). The number of modes can be controlled by designing the transverse profile of the waveguide. Operating at single-mode regimes reduce complexity. The enhanced design flexibility of few-mode waveguides come at the expense of increased complexity of differently paced co-propagating modes. Having majority of the power carried by individual modes at different parts the waveguide along with high-purity mode conversion at the intersections circumvents such complexity. A powerful application is dispersion compensation utilizing opposite dispersion signs of different modes \([2]\). Near-zero net cavity dispersion is sought for reducing noise in ultrashort-pulsed fiber laser \([3]\).

Grating mode converters, which are periodic perturbations along the waveguide axis, allow high-purity conversion between co-propagating modes in an inherently mode-selective manner. Power loss from a grating can be made small by keeping the perturbations small and smooth along \(z\), at the expense of having longer length.

Standard gratings are limited in bandwidth at strong conversion strengths (Equation \([5]\)), often falling short of wavelength ranges of interest e.g. bandwidths of EDFA amplifier for fiber communications and gain of ultrafast fiber lasers. Increasing grating bandwidth has been extensively researched, for example: (a) The seminal solution of achieving mode conversion at flat parts of difference in propagation constants near a turning point wavelength \([4]\), limited to the few critical wavelengths, (b) increasing the coupling constant \([5]\) at the expense of increased loss, especially for gratings asymmetric to the transverse waveguide profile, (c) sophisticated in-fiber phase plates fabricated with laser irradiation \([6]\) and offset splicing \([7]\), and (d) the method of inter-playing between the phases of consequent gratings and precise gaps interleaving those \([8]\). The ease of fabrication, low-loss and wide wavelength tunability of phase-shifted gratings have attracted wide attention including achievement of broadband > 99 % mode conversion in bottom-up fabricated polymer waveguide and gratings using an excess of two phase-shifts \([9]\). Recently, the minimal case of a single phase-shift has been explored in two-mode optical fibers with dissipative gratings on the two sides of the phase shift \([10]\), which suffer from sub-optimal interplay of phases between the segments. The use of single-phase shifted gratings to broaden 99 % core-cladding conversion in a single-mode fiber was numerically simulated and demonstrated with arc-writing \([11]\), showing 3-fold enhancement. Application of such compact design for converting between core modes of waveguides has been surprisingly overlooked.

Our generalized fabrication recipe for any mode pair, which is also independent of wavelength-dependent parameter changes during grating writing, increases 99 % conversion bandwidth by > 2.9 times compared to a standard grating. Semi-analytical proof is presented, along with numerical evidence of the criticality of the \(\pi\)-phase shift at the resonance wavelength for a particular example.

For a standard grating of pitch \(\Lambda_{MC}\) which can achieve >99% mode conversion at resonance wavelength \(\lambda_{MC}\), let us designate the number of periods corresponding to maximum mode conversion as \(N_0\), which gives the value of coupling per period \(K(\lambda_{MC}) \approx \frac{\pi}{2N_0}\). Compared to the standard grating, greater than 2.9 times increase the 99 % mode conversion bandwidth can be achieved by the following design (figure \([1]\): (a) first writing \(N_1\) periods such that the mode conversion at resonance wave-
length $\lambda_{MC}$ just reaches $50\%$ or above, (b) followed by an extra gap adding a phase of $\pi$ at $\lambda_{MC}$, (c) followed by writing the minimum $N_2$ number of marks such that $\cos^2(K(N_2 - N_1)) \leq 10^{-2}$. Additionally, harnessing the simplicity of the linear algebraic model of a single-phase-shifted grating, we identify the bimodal spectral shape of the $>99\%$ mode conversion range.

![Schematic of single-phase-shifted grating of pitch $\Lambda_{MC}$, with $N_1$ and $N_2$ full periods in the two segments with an extra gap of $\Lambda_s$ in between.](image)

**FIG. 1.** Schematic of single-phase-shifted grating of pitch $\Lambda_{MC}$, with $N_1$ and $N_2$ full periods in the two segments with an extra gap of $\Lambda_s$ in between.

Perturbations locally change $\beta_k(\lambda) \rightarrow \beta_k(\lambda, z)$. The resonance wavelength $\lambda_{MC}$ is determined by phase-matching over any period length $\Lambda_{MC}$ of the grating $\Phi(\lambda) = \int_{z}^{z+\Lambda_{MC}} \beta_1(\lambda, z) - \beta_2(\lambda, z) - 2\pi dz$. Direct method to quantify the additional phase $\Delta \phi(\lambda)$ added per perturbation has recently become available [12, 13].

$$\Phi(\lambda) = \Lambda_{MC} \left( \beta_1(\lambda_{MC}) - \beta_2(\lambda_{MC}) \right) + \Delta \phi(\lambda)$$

(1)

Using $\frac{\Lambda_{MC}}{2\sqrt{\frac{\phi(\lambda)}{\Lambda_s}}}$ length of the pristine waveguide to induce $\pi$ phase shift at $\lambda_{MC}$ is erroneous and leads to subpar performance, which may have bottlenecked experimental efforts so far. Equation (1) can additionally be used to estimate both $\lambda_{MC}$ and $(\beta_1(\lambda_{MC}) - \beta_2(\lambda_{MC}))$ in any new waveguide and writing setup using a single experiment, although that is not a requisite for this method.

We will prove our method only for low-loss gratings. In case of non-negligible loss with separate "Discretized-medium" approximation [14], we always saw broadband performance, which may have bottlenecked experimental efforts so far. Equation (1) can additionally be used to estimate both $\lambda_{MC}$ and $(\beta_1(\lambda_{MC}) - \beta_2(\lambda_{MC}))$ in any new waveguide and writing setup using a single experiment, although that is not a requisite for this method.

An extra gap of $\Lambda_s$ after any segment of full periods adds a phase of $\xi(\lambda) = \beta_1(\lambda) - \beta_2(\lambda) \Lambda_s$ without mode conversion, corresponding to $K(\lambda) = 0$, $N = 1$, $\Phi(\lambda) = \xi(\lambda)$ in equation (2). For $K$ segments with $N_1 \leq k < N_s$ periods and gaps of $\Lambda_s$ between segments, the complex transmission at $z_{(N_k)} = z + \sum_{k=1}^{N_s} N_k \Lambda_{MC} + K \Lambda_s$ is given by

$$\begin{bmatrix} E_1(\lambda, z(N_k)) \\ E_2(\lambda, z(N_k)) \end{bmatrix} = T(\lambda, \{N_k\}) \begin{bmatrix} E_1(\lambda, z) \\ E_2(\lambda, z) \end{bmatrix}$$

(3)

For $K = 2$ with an extra gap introducing a phase shift of $\xi(\lambda)$, the entries of the complex transmission matrix $T_{ij}(\lambda, N_2, N_1)$, $i, j = 1, 2$ in equation (3) are especially simple to write. Since for low-loss gratings $|T_{1,1}|^2 + |T_{2,2}|^2 \approx |T_{1,1}|^2 + |T_{2,2}|^2 \approx 1$ and $T_{2,1} = T_{1,2}^*$, only studying the spectral response of $|T_{1,1}|^2$ suffices.

For conciseness, let us use the notations $\alpha_1 = (N_2 - N_1)/N_0$, $\alpha_2 = (N_2 + N_1)/N_0$, $K = K(\lambda)$, $\phi = \phi(\lambda)$, $\xi = \xi(\lambda)$ and $C(\lambda) = \Gamma(\lambda)N_0$. Using the normalized parameter $x(\lambda) = K(\lambda)/\Gamma(\lambda) = (1 + \frac{\phi(\lambda)}{\phi(\lambda)})^2 \geq 1$ we get

$$e^{-i\xi(\lambda) T_{1,1}(\lambda, \{N_1, N_2\})} =$$

$$\frac{K^2}{T^2} \cos(2\xi) \cos((N_2 - N_1)\Gamma) + \frac{\phi^2}{T^2} \cos((N_2 + N_1)\Gamma)$$

$$+ i \left( \phi \sin((N_2 - N_1)\Gamma) + \frac{K^2}{T^2} \sin(2\xi) \sin(N_2\Gamma) \sin(N_1\Gamma) \right)$$

$$= \frac{1}{x^2} \cos(2\xi) \cos(\alpha_1 C x) + (1 - \frac{1}{x^2}) \cos(\alpha_2 C x) +$$

$$i \left( \sqrt{1 - \frac{1}{x^2}} \sin(\alpha_2 C x) + \frac{1}{x^2} \sin(2\xi) \sin(N_2 N_0 C x) \sin(N_1 N_0 C x) \right)$$

(4)

For a standard grating with maximum mode conversion ($N_1 = N_0$, $N_2 = 0$), $|T_{1,1}|^2 \leq 10^2$ is satisfied for

$$1 \leq x(\lambda) \leq 1.005$$

(5)

With $\xi(\lambda) = 1$, we do some simplifications:

- Assume $C(\lambda) = K(\lambda)N_0 \approx K(\lambda) N_0 = \frac{\pi}{2}$ to be constant based on $\frac{\partial K(\lambda)}{\partial \lambda} \ll \frac{\phi(\lambda)}{\phi(\lambda)}$ [9].

- Since $\xi(\lambda)$ is unknown without further experimentation [13], use bounded variation $g(x, -s) \leq 2\xi(x) \leq g(x, s) = 2\pi(1 + sx - s)$ to normalize $\sin(2\xi(\lambda))$. For most waveguides, $s = 0.02$ suffices.

- Replace $\cos(2\xi(\lambda))$ by 1, since $\cos(2\xi(\lambda)) \approx 1.0000$ for any $1 \leq x(\lambda) \leq 1.045$ and $0 \leq s \leq 0.02$. 

With these simplifications, finding the range \(1 \leq \lambda \leq x_{0,01}(N_0, s)\) satisfying \(|T_{1,1}|^2 \leq 0.01\) is equivalent to finding \(x_{0,01}(N_0, s) = \min_{-s \leq s' \leq s} x_{0,01}(N_0, s')\) such that \(u^2(x_{0,01}(N_0, s'), N_0, s') + v^2(x_{0,01}(N_0, s'), N_0, s') = 0.01\). The material independent \(u(x, N_0, s')\) and \(u(x, N_0, s')\) are given by equations (6) and (7).

\[
v(x, N_0, s') = \sqrt{1 - \frac{1}{x^2}} \sin(\alpha_2 \pi x) + \left[\frac{1}{x^2} \sin\left(g(x, s')\sin(\frac{\alpha_1 + \alpha_2}{2} \pi x)\sin(\frac{\alpha_1 - \alpha_2}{2} \pi x)\right)\right]
\]

\[
u(x, N_0, s') \approx \frac{1}{x^2} \cos(\frac{\alpha_1}{2} \pi x) + (1 - \frac{1}{x^2}) \cos(\frac{\alpha_2}{2} \pi x)
\] (6)

(7)

For any given \(N_0\) and \(s'\), \(x_{0,01}(N_0, s')\) can be numerically calculated. Example of calculated \(u^2(x, N_0, s') + v^2(x, N_0, s')\) for \(N_0 = 30\) and \(s' = 0.02\) is shown in figure 2. Since \(\phi(\lambda)/K \approx \sqrt{x^2(\lambda) - 1}\), the relative enhancement of 99% conversion bandwidth compared to a standard grating with identical parameters can be estimated by the ratio \(\Omega(N_0, s) = \frac{x_{0,01}(N_0, s)^2 - 1}{\sqrt{1.005 - 1}}\). Computed \(\Omega(N_0, s)\) for \(16 \leq N_0 \leq 1000\) and \(s = 0.02\) are presented in figure 3 showing at least 2.9 times relative bandwidth enhancement. Although smaller \(N\) corresponds to larger absolute bandwidth, it is harder to control due to large jumps between spectra during grating writing.

In order to predict the spectral shape above 99% mode conversion, we notice that for any \(16 \leq N_0 \leq 1000\), \(-0.02 \leq s' \leq 0.02\) and \(1 \leq x \leq 1.045\), we have \(v^2(x, N_0, s') < 10^{-3}\). Thus the spectral shape of \(|T_{1,1}|^2\) is dominated by \(u^2(x, N_0, s')\). To prove that \(u(x, N_0, s')\) attains zero only once in \(1 \leq x \leq 1.045\), let us check its sign at \(x = 1\) and \(x = \frac{1}{\alpha_1}\).

![FIG. 2. Illustration of \(u^2(x, N_0, s')\) (Dotted black curve), \(v^2(x, N_0, s')\) (Dashed black curve) and \(u^2(x, N_0, s') + v^2(x, N_0, s')\) (Solid gray curve) with \(N_0 = 30\) and \(s' = 0.02\). The dashed gray horizontal lines represent \(y = 1 \times 10^{-2}\) (on top) and \(y = 10^{-3}\) (on bottom).](image)

The 3-fold enhancement of 99% conversion bandwidth is a guaranteed estimate for any waveguide satisfying \(s \leq 0.02\). The actual enhancement can be larger, as we can see from simple simulation of LP01-LP02 mode conversion in a step-index fiber of radius 5.0 \(\mu\)m and 14% GeO2 doping in the core. \(\beta_k(\lambda)\) were calculated semi-analytically using V-b diagram [15] and Sellmeier coefficients [16]. Bandwidth enhancement calculated directly from spectra for \(N_0 = 50\) is illustrated in figure 3 for different values of \(2\xi(\lambda_{MC})/\Phi(\lambda_{MC})\), which shows 3-fold enhancement in the range \(0.97 > 2\xi(\lambda_{MC})/\Phi(\lambda_{MC}) > 1.03\), peaking at \(\xi(\lambda_{MC}) = \pi\). For \(2\xi(\lambda_{MC})/\Phi(\lambda_{MC}) < 0.93\) and \(2\xi(\lambda_{MC})/\Phi(\lambda_{MC}) > 1.08\) the normalized transmission intensity never goes below 0.01, thereby barring any enhancement. Degradation of bandwidth enhancement for inexact \(\xi(\lambda_{MC})\) is directly illustrated by calculated spectra over \(N_2\), as illustrated in 3 for assumed \(\Phi(\lambda) = 1.04\lambda_{MC} (\beta_3(\lambda) - \beta_2(\lambda))\) and inexact \(\xi(\lambda) = \pi/1.04\). Below the critical wavelength \(\lambda_{MC}\) (\(\frac{\beta_3(\lambda)}{\beta_2(\lambda)} > 0\)), amongst the coalescing spectra dips on the two sides of \(\lambda_{MC}\), the spectral dip with smaller \(\lambda\) reaches minimum first with increasing \(N_2\). The behavior switches above the critical wavelength. Whenever \(\xi(\lambda) \neq \pi\), the minima of the spectral dips of \(T_{1,1}(\lambda)\) on two sides of \(\lambda_{MC}\) evolve at different rates with \(N_2\), hampering flattening of the conversion spectrum.

Equation (5) does not capture the effect of duty cycle, which can be addressed by simulating gratings us-

![FIG. 3. Calculated bandwidth enhancement of a \(\pi\)-phase-shifted grating compared to a standard grating, given by \(\Omega(N_0, s) = \sqrt{x(N_0, s)^2 - 1}/\sqrt{1.005 - 1}\). In particular, we find \(\Omega(N_0, s) > 3\) for all \(N_0 \geq 32\).](image)
FIG. 4. (a) Calculated LP\textsubscript{01} transmission spectra after LP\textsubscript{01}-LP\textsubscript{02} mode conversion by a single-phase-shifted grating with inexact phase shift of $\xi(\lambda_{MC}) = \pi/1.04$, plotted for different $N_2$. $N_1 = 15$ corresponding to maximum transmission for $N_0 = 30$ periods of a uniform grating with identical parameters. For the spectral dips at shorter wavelengths than the critical wavelength $\lambda_T$ given by $\partial \Phi(\lambda_T) / \partial \lambda = 0$, the dip with smaller wavelength grows and reverts earlier. For wavelengths larger than $\lambda_T$, the spectral dip with larger wavelength grows and reverts earlier. (b) For different values of $\frac{2\xi(\lambda_{MC})}{\Phi(\lambda_{MC})}$ and $N_1 = 15$, the best bandwidth enhancement calculated over different $N_2$.

In conclusion, we prove at least 2.9 times bandwidth enhancement at above 99 % mode conversion by a $\pi$-phase-shifted grating which has minimum number of periods $N_1$ and $N_2$ in first and second segment such that the transmission intensity of the original mode at the resonance wavelength is $\leq 50\%$ after writing only $N_1$ periods and $< 10^{-2}$ after writing both segments. This deterministic solution makes broadband mode converter gratings independent of material parameters, mode-pair and even grating-writing method, thereby relaxing major experimental bottlenecks.

I. DISCLOSURES

The author declares no conflicts of interest.

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