Δ resonances in Ca+Ca, Ni+Ni and Au+Au reactions from 1 AGeV to 2 AGeV: Consistency between yields, mass shifts and decoupling temperatures

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The Ultra-relativistic Quantitative Molecular Dynamics (UrQMD) transport approach is used to calculate Δ(1232) yields in Ca+Ca, Ni+Ni and Au+Au collisions between 1 AGeV and 2 AGeV. We compare and validate two different methods to extract the yields of Δ(1232) resonances in such low energy nuclear collisions: Firstly, the π− spectra at low $p_T$ are used to infer the Δ(1232) yield in A+A collisions, a method employed by the GSI/FOPI experiment. Secondly, we employ the invariant mass method used by the HADES collaboration, which has recently reported data in the $Δ^{++} → π^+ + p$ channel. We show that both methods are compatible with each other and with the theoretical calculations, indicating that the new HADES results are compatible with the previous FOPI measurements. Then we use the $Δ/\text{nucleon}$ ratio to extract the kinetic decoupling temperatures of the Δ(1232) resonances. We find that the extracted temperatures are consistent with the predicted mass shift of the Δ resonance and the freeze-out parameters estimated from complementary studies (blast wave fits, coarse graining).

I. INTRODUCTION

Heavy ion collisions, carried out in today’s largest particle accelerators, provide excellent opportunities to study nuclear and sub-nuclear matter at extreme conditions. With increasing energy, the possibility to produce novel and exotic states of matter becomes accessible. In the GSI/NICA/FAIR energy regime covering the collision energy range from 1-20 AGeV the exploration of novel and exotic states of matter is studied. Conclusions about the properties of subatomic matter are drawn from particle distributions, e.g. by PHENIX [1], STAR [2], ALICE [3] and CMS [4]. Due to the explosive nature of heavy ion reactions, the time scales of the reactions do not allow for a direct observation of the reaction zone, but one needs to infer the properties of the created QCD-matter at the different stages of the reaction indirectly, e.g. via flow measurements or penetrating probes. Nevertheless it remains difficult and often ambiguous to pin down specific values for the parameters (temperature, density, expansion velocity, transport properties) of the emission source. In this paper we want to answer one of these questions, namely the value of the decoupling temperature, with the help of the Delta resonance.

Historically the Δ(1232) has long been of major interest as its discovery lead to the concept of color charge to maintain the Pauli principle in the $Δ^{++}$ and the $Δ^{-}$ states. For our current exploration, the Δ(1232) is a prime candidate because due to its relatively low mass it is the most abundantly created baryonic resonance. Apart from leptonic decays with very small branching ratios, the Δ(1232) decays always into a π and a N and is therefore rather easy to produce and to measure. As was shown in [10, 11] its spectral function is linked to the temperature of the system at the decoupling surface. Interestingly, a complementary method based on the chemical yields of short-lived resonances to extract the decoupling properties was developed shortly after this finding [12] and will also be used here to analyze and compare the model results.

The experimental measurement of a decaying Δ(1232) resonance can be accomplished in several ways. Today, the most common way is to detect it in the invariant mass distribution of charged π+N pairs. Measurements using this method have recently been reported by the HADES collaboration at GSI [13]. Nowadays, the 2-particle (or even n-particle) invariant mass reconstruction is the method of choice, if sufficient statistics and detector resolution permit its use. Sometimes, however, also simpler methods (using only 1-particle information) can be employed, e.g. high $p_T$ single electrons can be seen as proxies for $D$-mesons [14]. In the case of the Δ(1232) a simplified experimental analysis can also be performed for nucleus-nucleus reactions at low energies [15]. Here, one uses the tight correlation between the pion yield and the Δ(1232) yield: The Delta, having a lifetime of $\sim 1$ fm and being the most dominant source of pions at low energies allows to detect Δ resonances via the π spectra. The observed transverse momentum distribution of pions has two contributions, direct pions and additional pions emerging from decaying Δ(1232). These pions originating from Delta decays populate low transverse momenta and dominate the spectrum below $p_T ≈ 400$ MeV. Thus, low $p_T$ enhancement, together with theoretical calculations, can be used to investigate the Δ(1232) resonance without invariant mass reconstruction. Pioneering
results employing this method have been published for Si+Au reactions at 14.6 AGeV at the AGS \cite{17}, by the E814 collaboration in Si+Al, Si+Al and Si+Pb reactions at 14.6 AGeV at the AGS \cite{18}, and by the FOPI collaboration in Ni+Ni collisions between 1 AGeV and 2 AGeV at GSI \cite{16}. The obtained $\Delta$/nucleon ratios \cite{16} can then be used to estimate the freeze-out temperature at the decoupling hyper-surface of the $\Delta$ using the hadrochemical equilibrium model \cite{19}.

In this paper we will be a) providing a consistency check between different methods to estimate the abundance of $\Delta$ resonances to connect the FOPI data \cite{10} to the recently measured HADES data \cite{13} and b) extracting the decoupling temperature of the $\Delta$ from a chemical analysis of the $\Delta$/nucleon yield ratio to compare to kinematic freeze-out studies and apparent mass shifts. To this aim, we will employ the UrQMD model (v3.4) \cite{21,22} which has been used to explore a wide range of reactions in this energy regime, e.g. flow \cite{6,8}, strangeness \cite{23}, clusters \cite{24}, and di-leptons \cite{25}. The different techniques to extract $\Delta$ yields discussed above are then applied to Ca+Ca (Ar+KCl) collisions at 1.76 AGeV and Au+Au collisions at 1.23 AGeV to predict the relative $\Delta$(1232) abundance at the HADES energies. For alternative studies we refer to \cite{15,26,28}.

II. THE URQMD MODEL

We employ the Ultra relativistic Quantum Molecular Dynamics (UrQMD) transport model in its most recent version (v3.4) \cite{21,22} in cascade mode. It is based on the covariant propagation of hadrons and their interactions by elastic and/or inelastic collisions. Relevant cross sections are taken, if available, from experimental data or derived from effective models. UrQMD includes mesonic and baryonic resonances up to masses of 2 GeV and has a longstanding history to reproduce hadron yields and spectra and forecast new phenomena. In case of the GSI energy regime investigated in the present study, UrQMD has already been proven to describe various observables.

For recent results, we refer the reader to explorations of Au+Au collisions at 1.23 AGeV \cite{9,8,23,25}, and Ca+Ca collisions at 1.76 AGeV \cite{29} or in Ar+KCl collisions at 1.76 AGeV \cite{23,25} which are described very well.

A major advantage of a transport simulation is the opportunity to analyze the full time evolution of each event. This means, in contrast to the two analysis methods available to experimentalists, we can track down each decaying $\Delta$(1232) resonance and follow its individual daughter particles until their next interaction. This allows to define microscopically, if a given $\Delta$ might be observable or not. E.g., if both daughters re-scatter only elastically (probably many times) and do not re-scatter inelastically then the decaying $\Delta$ is considered as reconstructable. This method has already proven to be reliable and is described in detail in \cite{9,11}.

III. PARTICLE SPECTRA

To benchmark the model and the reconstruction methods we calculate Ca+Ca, Ni+Ni, and Au+Au reactions with the UrQMD model. The uppermost 4% of the total cross section are selected via the geometrical interpretation of the cross section using a sharp sphere approximation. This translates to impact parameters of $b_{\text{max}} = 1.6 \text{ fm (Ca+Ca)}$, $b_{\text{max}} = 1.9 \text{ fm (Ni+Ni)}$ and $b_{\text{max}} = 2.8 \text{ fm (Au+Au)}$ respectively.

Let us start with the transverse mass spectra in Fig. 1 for the Ni+Ni systems at three different beam energies (1.06 AGeV, 1.45 AGeV, 1.93 AGeV). (The transverse mass spectra for Ca+Ca and Au+Au are shown in the right panel of Fig. 3 and will be discussed in more detail later.) For each energy, we split the total $\pi^{-}$ yield (circles) into a contribution of $\pi^{-}$ originating directly from $\Delta$ decays (triangles) and $\pi^{-}$ originating from the decay of other resonances (squares). The influence of the $\Delta$ decay is clearly visible for all three shown energies and dominates the pion yield up to transverse masses of $\approx 0.4$ GeV. Assuming $\Delta$ resonances with nominal mass and at rest, it is clear that the transverse mass of an emitted pion in the local rest frame of the $\Delta$ amounts to a maximal value of $\approx 0.3$ GeV matching the observed upper $p_T$ limit in the Lorentz-boosted frame. Thus the $\Delta$ is the major contributor to the pion spectrum at low $p_T$. Pions at higher $p_T$ are created via the decay of mesonic resonances or heavier baryonic resonances and populate the (strongly suppressed) high $p_T$ tail of the distribution.

After setting the stage, we can now investigate the rapidity distribution of pions in Ni+Ni collisions at the same three energies as above as shown in Fig. 2 (upper panel). The beam energy increases from left to right from 1.06 AGeV (left panel), over 1.45 AGeV (middle panel) to 1.93 AGeV (right panel). Note that the distributions are scaled to normalized rapidity $y_0 = \gamma / y_{c.m.s} - 1$ to take care of the different collision energies. For each energy, we show the rapidity densities of all $\pi^{-}$ (red solid line), $\pi^{-}$ with $p_T \leq 0.3$ GeV (dubbed $\pi_{\text{low}}$, red dashed line) and of $\pi^{-}$ originating from $\Delta$(1232) decays (dubbed $\pi_{\Delta}$, red dotted line). The calculations are compared to the FOPI data taken from \cite{16} showing all $\pi^{-}$ (full black circles) and low $p_T$ $\pi^{-}$ (empty black circles).

Generally, we observe a good agreement between the UrQMD calculation and the FOPI measurement for both explored quantities, the rapidity distribution of all $\pi^{-}$ and the rapidity distribution of the low $p_T$ pions. As speculated above, the model calculation also shows clearly that the low $p_T$ pions provide an excellent approximation (within 10%) for the pions coming from the decay of a $\Delta$(1232) as seen from the comparison of the dotted and dashed lines.

The lower panel of Fig. 2 shows the ratio of the low $p_T$ $\pi^{-}$ to all $\pi^{-}$ (UrQMD: red dashed line, FOPI: black line with circles) as a function of normalized rapidity. Generally this ratio shows a mild rapidity dependence, both in the simulation and the data. Generally, the rela-
tive abundance of low \( p_T \) pions decreases with increasing energy due to additional production channels and radial flow. To simplify the extraction of the Delta yield (and to follow the FOPI analysis), we approximate the rapidity dependent ratio, by its average \( \langle \pi^-/\pi_{\text{all}} \rangle \) (cyan dashed line) calculated for \( |y(0)| \leq 1.5 \) as done in \cite{10} which is a good approximation in the 10% level.

Let us now use this method to analyze the recent HADES data obtained in different systems and at different energies using the same method. The right part of Fig. 3 analyzes the transverse mass distributions of pions for the same systems. The upper left panel of Fig. 3 we show the rapidity distributions of pions for the Ca+Ca system at 1.76 AGeV (red) as obtained from the UrQMD simulation. Again, we split the total \( \pi^- \) yield (circles) into a contribution of \( \pi^- \) from \( \Delta \) decays (triangles) and \( \pi^- \) originating from the decay of other resonances (squares). The contribution of the \( \Delta \) decay is clearly visible for both systems and dominates the pion yield up to transverse masses of \( \approx 0.4 \) GeV. In the upper left panel of Fig. 3 we show the rapidity distributions of pions for the same systems. The \( p_T \)-integrated \( \pi^- \) distributions are shown as solid lines, the low transverse momentum \( \pi^- \), i.e. \( p_T \leq 0.3 \) GeV are depicted by dashed lines and the \( \pi^- \) originating from \( \Delta(1232) \) decays are indicated by dotted lines. As before, we observe that the low \( p_T \) pions provide a good proxy for the pions stemming from the decay of Delta resonances (within 20% for the Au+Au case). The ratio of the low \( p_T \) \( \pi^- \) to all \( \pi^- \) as a function of rapidity is shown in the lower left panels of Fig. 3 see legend. As discussed above, we observe again that the importance of low transverse momenta decreases with increasing energy.

IV. COMPARISON OF RESONANCE ABUNDANCES

We can now estimate the Delta yields and the fraction of Delta resonances from all participating baryons. We do this in two different ways: Firstly, we follow the FOPI approach to estimate the \( \Delta(1232) \) abundance, i.e., we multiply the number of pions with the average \( \langle \pi^-_{\text{low}}/\pi_{\text{all}} \rangle \) ratio at freeze-out and employ a scaling factor, taken from the isobar model \cite{31} to account for the isospin asymmetry, as depicted in Eq. 1:

\[
n(\Delta) \approx n(\pi^-) \times f_{\text{isobar}} \times \left\langle \frac{\pi^-_{\text{low}}}{\pi_{\text{all}}} \right\rangle.
\]  

The \( f_{\text{isobar}} \) factor has the numerical values 2.84 \((^{58}\text{Ni}+^{58}\text{Ni})\), 2.24 \((^{197}\text{Au}+^{197}\text{Au})\) and 3.0 \((^{40}\text{Ca}+^{40}\text{Ca})\). The obtained abundance of Delta resonances serves as input to calculate the ratio of excited nucleons to nucleons as shown by Eq. 2:

\[
R = \frac{n(\Delta)}{A_{\text{participant}}}.
\]
where $A_{\text{participant}}$ includes all ground state nucleons and all decay nucleons from resonances, i.e. $A_{\text{participant}} \approx n(\Delta) + n(\text{nucleon})$ at the Delta freeze-out surface.

We are now in the position to compare the different methods for different systems and collision energies. Fig. 3 shows the fraction of $\Delta(1232)$ resonances to all participating nucleons in dependence of the beam energy. Two model calculations are compared to the data by the FOPI collaboration: The red circles show the results using the microscopically reconstructed true Delta resonances from UrQMD, the blue squares show the Delta yield extracted via the low $p_T$ pion yield. The experimental data points of FOPI are shown as black triangles [16].

V. TEMPERATURE EXTRACTION

Finally, we can use the obtained $\Delta/(\Delta+\text{nucleons})$ ratios to calculate the temperature of the decoupling hypersurface of the observable $\Delta$ resonances. To this aim we use the thermal model in Boltzmann approximation:

$$n_i = (2s_i + 1)(2l_i + 1)V\frac{T^3e^{\mu/T}}{2\pi^2}m_i^2\frac{e^{m_i/T}}{T^2}K_2\left(\frac{m_i}{T}\right).$$

(3)

Here, $s_i$ is the spin, $l_i$ the iso-spin of the hadron, $m_i$ is its mass, while $T$ is the temperature, $V$ the volume and $\mu$ is the chemical potential, $K_2$ denotes the Bessel function. Note that $n_{\text{nucleon}}$ includes the decay contributions from the Deltas. Table I shows the estimated temperatures calculated from the Delta yields with the two different methods. The simulation results are compared to the FOPI results [16]. The 1st column shows the energy and the system while the estimated temperatures from the corresponding methods are shown from column 2 to column 5. The 2nd column shows the temperature results obtained by using the microscopically reconstructed $\Delta(1232)$ yield from UrQMD, the 3rd column shows the temperature calculated with the low $p_T$
method from UrQMD and both are compared to the FOPI results [16] using the low pr pion method in the 4th column and using a radial flow analysis in column 5.

| E_{lab} [AGeV] | T [MeV] | T_{UrQMD} | T_{FOPI} | T_{Flow} |
|---------------|-------|-----------|----------|---------|
| (System)      |       |           |          |         |
| 1.06 (Ni+Ni)  | 74    | 72        | 75±5     | 79±10   |
| 1.23 (Au+Au)  | 76    | 72        | –        | –       |
| 1.45 (Ni+Ni)  | 85    | 82        | 80±7     | 84±10   |
| 1.76 (Ca+Ca)  | 92    | 91        | –        | –       |
| 1.93 (Ni+Ni)  | 95    | 93        | 89±9     | 92±12   |

Table I. Temperatures obtained from the ∆/(A_{part}) ratio of UrQMD using the thermal model. The FOPI results are taken from [16].

As can be seen from Tab. I, the estimated temperatures of the different methods used are in good agreement with each other and the measurements by the FOPI collaboration. With increasing collision energy the temperature rises from ~73 MeV to ~94 MeV which is in line with [20]. The temperature extracted at 1.23 AGeV in the Au+Au system re-confirms the value obtained by the analysis of the predicted and measured mass shift of the ∆(1232) which suggested a freeze-out temperature of 81 MeV and a mass shift of ~47 MeV [10] [11]. Based on the extracted temperatures, we predict kinetic mass shifts of ~56 MeV (Ni+Ni at 1.06 AGeV), ~55 MeV (Au+Au at 1.23 AGeV), ~49 MeV (Ni+Ni at 1.45 AGeV), ~45 MeV (Ca+Ca at 1.76 AGeV) and ~43 MeV (Ni+Ni at 1.93 AGeV) using the BW×PS formula described in [10] [11] for the different GSI energies. These mass shifts seem to be confirmed by the estimates provided in [32].

VI. SUMMARY

In this article we have re-analyzed FOPI data tackling the reconstruction of the ∆(1232) yield from the measured pion yield in Ni+Ni collisions between 1 AGeV and 2 AGeV. The method uses the fact that pions at low transverse momenta originate dominantly from the decay of ∆(1232) resonances. Thus, the pion yield can be scaled by the ratio of low pr pions to all pions and the isospin asymmetry factor of the system to obtain the ∆(1232) yield. We compared this method to the reconstruction of the detectable ∆(1232) by following the daughter particles of each decaying Delta resonance in the simulation. We furthermore compared both methods to predict the relative ∆(1232) abundance for the HADES experiment in Ca+Ca and Au+Au collisions. The results are in very good agreement confirming that both methods can be used to obtain Delta yields in the investigated energy regime. Finally we used the thermal model to estimate the decoupling temperature of the ∆(1232)’s. The mean values for the kinetic freeze-out temperature increase from 73 MeV to 94 MeV. The extracted temperature value for Au+Au and Ni+Ni re-confirms our previous estimates for the freeze-out temperature of the Delta resonances and supports our previous finding of a kinematic mass shift related to the freeze-out temperature.

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