QUARK LEPTON MASS HIERARCHIES
AND THE BARYON ASYMMETRY

W. Buchmüller\textsuperscript{a}, T. Yanagida\textsuperscript{a,b}
\textsuperscript{a} Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany
\textsuperscript{b} Department of Physics, University of Tokyo, Tokyo, Japan

Abstract

The mass hierarchies of quarks and charged leptons as well as a large $\nu_\mu$-$\nu_\tau$ mixing angle are naturally explained by the Frogatt-Nielsen mechanism with a nonparallel family structure of chiral charges. We extend this mechanism to right-handed neutrinos. Their out-of-equilibrium decay generates a cosmological baryon asymmetry whose size is quantized in powers of the hierarchy parameter $\epsilon^2$. For the simplest hierarchy pattern the neutrino mass $\bar{m}_\nu = (m_{\nu_\mu} m_{\nu_\tau})^{1/2} \sim 10^{-2}$ eV, which is inferred from present indications for neutrino oscillations, implies a baryon asymmetry $n_B/s \sim 10^{-10}$. The corresponding baryogenesis temperature is $T_B \sim 10^{10}$ GeV.
In the standard model left- and right-handed quarks and leptons can be grouped into the SU(5) multiplets \(10 = (q_L, u_R^c, l_L^c, e_R^c)\), \(5^* = (d_R^c, l_L)\) and \(1 = \nu_R^c\), such that the Yukawa interactions with the Higgs fields take the form,

\[
\mathcal{L} = h_{ij}^{(u)} 10_i 10_j H_1(5) + h_{ij}^{(d)} 5^*_i 10_j H_2(5^*) + h_{ij}^{(\nu)} 5^*_i 1_j H_1(5) + h_{ij}^{(e)} 1_i 1_j S(1) .
\] (1)

Here \(i, j = 1 \ldots 3\) are generation indices. The expectation values of the Higgs multiplets \(H_1\) and \(H_2\) generate ordinary Dirac masses of quarks and leptons, whereas the expectation value of the singlet Higgs field \(S\) generates the Majorana mass matrix of the right handed neutrinos.

The masses of up-quarks, down-quarks and charged leptons approximately satisfy the following mass relations,

\[
m_t : m_c : m_u \simeq 1 : \epsilon^2 : \epsilon^4 ,
\] (2)

\[
m_b : m_s : m_d \simeq m_\tau : m_\mu : m_e \simeq 1 : \epsilon : \epsilon^3 ,
\] (3)

where, for masses defined at the unification scale \(\Lambda_{GUT}\), \(\epsilon^2 \sim 1/300\). According to the Frogatt-Nielsen mechanism [1] these mass hierarchies are related to a spontaneously broken symmetry. Each SU(5) multiplet of fermions \(\psi_i\) carries a corresponding charge \(Q_i\), and the Yukawa couplings then scale like

\[
h_{ij} \propto \epsilon^{Q_i + Q_j} .
\] (4)

It is usually assumed that the mass hierarchy is generated by the expectation value of a singlet field \(\Phi\) with charge \(Q_\Phi = -1\) via a nonrenormalizable interaction with a scale \(\Lambda > \Lambda_{GUT}\), i.e. \(\epsilon = \langle \Phi \rangle / \Lambda\).

The observed mass hierarchy of up-quarks and the large t-quark mass yield for the 10-plets uniquely the charges listed in table 1. Here we have assumed that Yukawa couplings are not larger than \(\mathcal{O}(1)\). From the mass hierarchy of the charged leptons one infers that the second and third generation \(5^*\)-plet have the same charge \((a)\), which differs by one unit from the charge of the first generation \(5^*\)-plet. The value of the b-quark mass allows

| \(\psi_i\) | 10_3 | 10_2 | 10_1 | 5^*_3 | 5^*_2 | 5^*_1 | 1_3 | 1_2 | 1_1 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \(Q_i\) | 0 | 1 | 2 | a | a | a+1 | b | c | d |

Table 1: Chiral charges for the fermions of the standard model; \(a=0\) or \(1\), and \(0 \leq b \leq c \leq d\).
Such a nonparallel family structure has previously been identified in a detailed study of U(1) generation symmetries. Note, that the U(1) may be replaced by an anomaly-free discrete $\mathbb{Z}_5$ symmetry with the charge assignment in table 1.

The difference between the observed down-quark mass hierarchy and the charged lepton mass hierarchy can be accounted for by introducing an additional $45$-plet of Higgs fields. The following discussion does not depend on this and could also be carried out directly for the quark and lepton multiplets of the standard model gauge group. The chiral charges of the lepton doublets would then correspond to the charges of the $5^*$-plets given in table 1.

Masses for the standard model neutrinos are generated by the unique dimension-5 operator

$$\mathcal{L}_{\Delta L=2} = f_{ij} \ 5^*, 5^* j H_1(5) H_1(5) ,$$

which is induced by the exchange of heavy particles in generic seesaw models. The chiral charges of the $5^*$-plets lead to the neutrino mass matrix

$$m_{\nu_{ij}} = f_{ij} v^2 \propto \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} ,$$

where we have only listed the order in $\epsilon$ for the different matrix elements. Diagonalization clearly yields a large $\nu_\mu$-$\nu_\tau$ mixing angle, which is needed to explain the atmospheric neutrino deficit by $\nu_\mu$-$\nu_\tau$ oscillations. As described above, this is a direct consequence of the mass hierarchy of the charged leptons which leads to the charge assignment given in table 1. The masses of the two eigenstates $\nu_\mu$ and $\nu_\tau$ depend on factors of order one, which are omitted in (5), and may easily differ by an order of magnitude. They can therefore be consistent with the mass differences $\Delta m^2_{\nu_\mu\nu_\mu} \simeq 4 \cdot 10^{-6} - 1 \cdot 10^{-5}$ eV$^2$ inferred from the MSW solution of the solar neutrino problem and $\Delta m^2_{\nu_\mu\nu_\tau} \simeq (5 \cdot 10^{-4} - 6 \cdot 10^{-3})$ eV$^2$ associated with the atmospheric neutrino deficit. In the following we shall use for numerical estimates the average of the neutrino masses of the second and third family, $\bar{m}_\nu = (m_{\nu_\mu}, m_{\nu_\tau})^{1/2} \sim 10^{-2}$ eV.

Consider now the simplest class of seesaw models where three right-handed neutrinos $\nu_{Rj}$ are added to the standard model. According to (5) and the charge assignment in table 1, the Dirac neutrino mass matrix has the form

$$m_D = h^{(\nu)} v = \epsilon^a \begin{pmatrix} A \epsilon^{d+1} & B \epsilon^{c+1} & C \epsilon^{b+1} \\ D \epsilon^d & E \epsilon^c & F \epsilon^b \\ G \epsilon^d & H \epsilon^c & K \epsilon^b \end{pmatrix} v ,$$
where \( A, \ldots, K \) are constants of order one. Correspondingly, the Majorana mass matrix of the right-handed neutrinos reads,

\[
M = h^{(s)} \langle S \rangle = \begin{pmatrix} \alpha \epsilon^2 \alpha & 0 & 0 \\ 0 & \beta \epsilon^2 \beta & 0 \\ 0 & 0 & \gamma \epsilon^2 \gamma \end{pmatrix} \langle S \rangle .
\] (8)

Here \( \alpha, \ldots, \gamma \) are constants of order one and, without loss of generality, we have chosen a basis where \( M \) is diagonal and real. Note, that this diagonalization does not change the hierarchy structure of the Dirac mass matrix (7). The corresponding mass eigenstates are the Majorana neutrinos \( N_i \simeq \nu_{R_i} + \nu_{R_i}^c \). In terms of the Dirac mass matrix (7) and the Majorana mass matrix (8) the Majorana mass matrix of the light neutrinos is given by

\[
m_\nu = m_D \frac{1}{M} m_D^T .
\] (9)

In this expression for the light neutrino masses the dependence on the chiral charges of the heavy neutrinos drops out and one obtains the hierarchy structure (3).

Heavy Majorana neutrinos are likely to play a crucial role for the cosmological baryon asymmetry. At high temperatures, above the critical temperature of the electroweak phase transition, baryon and lepton number violating processes are in thermal equilibrium [10]. As a consequence, a primordial lepton asymmetry generated by the out-of-equilibrium decay of heavy Majorana neutrinos is partially transformed into a baryon asymmetry [11]. The dominant contribution to the lepton asymmetry is produced in the decays of \( N_1 \), the lightest of the heavy Majorana neutrinos. The corresponding baryon asymmetry is given by,

\[
Y_B = \frac{n_B}{s} = \kappa \frac{\varepsilon_1}{g_*} .
\] (10)

Here \( \varepsilon_1 \) is the CP-asymmetry in the decay of \( N_1 \), \( C \) is the ratio of lepton and baryon asymmetry, which equals -8/15 in the (supersymmetric) standard model with 2 Higgs doublets, \( g_* \sim 100 \) is the number of effectively massless degrees of freedom and \( \kappa \) is a dilution factor. Its value reflects the effect of the various lepton number conserving and violating processes in the plasma. In order to determine \( \kappa \) reliably one has to solve the full Boltzmann equations. To obtain a large asymmetry, the number density of the heavy neutrinos at high temperatures has to be large enough, they have to fall out of equilibrium at \( T \sim M_1 \), and the \( \Delta L = 1 \) and \( \Delta L = 2 \) washout processes have to be sufficiently suppressed. Typical values of the dilution factor are then \( \kappa \sim 10^{-1} - 10^{-2} \), which corresponds to a baryon asymmetry

\[
Y_B \sim (10^{-3} - 10^{-4}) \varepsilon_1 .
\] (11)

4
The CP asymmetry $\varepsilon_1$ can be computed in terms of the neutrino mass matrices. For degenerate heavy neutrinos, i.e. $M \propto 1$, $\varepsilon_1$ vanishes. We therefore assume hierarchical heavy neutrino masses, with $d \geq 1$. In this case $M_1 \ll M_{2,3}$, and one obtains \cite{12,13},

$$
\varepsilon_1 = \frac{3}{16\pi v^2} \frac{1}{(m_D^\dagger m_D)_{11}} \sum_{i=2,3} \text{Im} \left[ (m_D^\dagger m_D)_{1i}^2 \right] \frac{M_1}{M_i}.
$$

(12)

Note, that the CP asymmetry depends on $m_D$ only through $m_D^\dagger m_D$. Hence, $\varepsilon_1$ is not directly related to the $\nu_\mu - \nu_\tau$ mixing angle in the leptonic charged current.

From eqs. (7), (8), (9) and (12) one easily obtains the dependence of the CP asymmetry and the light neutrino masses on the hierarchy parameter $\varepsilon$,

$$
\varepsilon_1 \sim 10^{-1} \varepsilon^{2(a+d)},
$$

(13)

$$
\bar{m}_\nu \sim \varepsilon^{2(a+d)} \frac{v^2}{M_1}.
$$

(14)

Using $\bar{m}_\nu \sim 10^{-2}$ eV, the corresponding mass of $N_1$, the lightest of the heavy neutrinos, is given by

$$
M_1 \sim \varepsilon^{2(a+d)} 10^{15}\text{GeV}.
$$

(15)

Note, that the values of $\varepsilon_1$ and $M_1$ do not depend on the charges $b$ and $c$ of the heavy neutrinos $N_3$ and $N_2$, respectively.

From eqs. (11) and (13) one obtains for the baryon asymmetry,

$$
Y_B \sim (10^{-4} - 10^{-5}) \varepsilon^{2(a+d)}.
$$

(16)

Hence, the baryon asymmetry is quantized in powers of the hierarchy parameter $\varepsilon^2$. Its magnitude is fixed by the neutrino charges.

Consider first the simplest case of hierarchical neutrino masses with Yukawa couplings of the third family $\mathcal{O}(1)$, i.e. $a = b = 0$, $c = 1$, $d = 2$. One then has

$$
M_1 \simeq 10^{10}\text{GeV}, \quad M_3 \simeq 10^{15}\text{GeV}; \quad Y_B \sim 10^{-9} - 10^{-10}.
$$

(17)

These are precisely the parameters chosen in \cite{14}. Solving the Boltzmann equations indeed yields an asymmetry $Y_B \sim 10^{-10}$, the baryogenesis temperature is $T_B \sim M_1 \sim 10^{10}\text{GeV}$ and $B - L$ is broken at the GUT scale.

The same results are obtained for $c = 0$. Alternatively, one may have smaller Yukawa couplings together with a smaller mass hierarchy. This corresponds to $a = 0$, $b = c = 1$, $d = 2$ or $a = 1$, $b = c = 0$, $d = 1$. In both cases baryogenesis temperature and baryon asymmetry remain the same, but $M_2 \sim M_3 \sim 10^{12}\text{GeV}$. Hence, the scale of $B - L$ breaking may be below the GUT scale.
In all cases considered so far the CP asymmetry $\varepsilon_1 \sim 10^{-6}$. One may think that the charge assignement $a = b = c = 0, d = 1$, which yields a CP asymmetry $\varepsilon_1 \sim 10^{-4}$, may lead to a larger baryon asymmetry. However, one then has $M_1 \sim 10^{12}$ GeV. The dependence of the baryon asymmetry on $\tilde{m}_1 = (m_D^\dagger m_D)_{11}/M_1$ and $M_1$ has been studied in the non-supersymmetric and in the supersymmetric case by solving the full Boltzmann equations [15]. It turns out that the dependence on $M_1$ is model dependent. For the above choice of parameters, with $\tilde{m}_1 \sim \tilde{m}_\nu \sim 10^{-2}$ eV, the washout processes in the supersymmetric case are too strong, and the generated asymmetry lies far below the observed value $Y_B \sim 10^{-10}$. Hence, the observed baryon asymmetry corresponds to the maximal asymmetry.

The baryogenesis temperature $T_B \sim 10^{10}$ GeV is naturally obtained in hybrid models of inflation. For supersymmetric theories such a large reheating temperature imposes stringent constraints on the mass spectrum of superparticles, in particular on the gravitino mass. Consistency with primordial nucleosynthesis requires either a very light gravitino, $m_G < 1$ keV [16], a heavy gravitino, $m_G > 2$ TeV [17], or a gravitino in the mass range $m_G = (10 - 100)$ GeV as LSP [18], with a higgsino type neutralino as next-to-lightest superparticle. In the last case gravitinos could be the dominant component of dark matter.

There are other mechanisms of baryogenesis. In particular in supersymmetric theories a very large baryon asymmetry may be produced by the coherent oscillation of scalar fields carrying baryon and lepton number [19]. The generated asymmetry then depends on the initial configuration of scalar fields. This is in contrast to leptogenesis, as described above. Here, the baryon asymmetry is entirely determined by neutrino masses and mixings which, at least in principle, can be measured in precision experiments probing the leptonic charged current.

Crucial tests of leptogenesis are the Majorana nature of neutrinos and CP violation in the leptonic charged current. Note, however, that the expected electron neutrino mass $m_{\nu_e} \sim 10^{-5}$ eV is rather small. Its detection in neutrino-less $\beta\beta$-decay is an experimental challenge.

Leptogenesis requires that the neutrino mass $\tilde{m}_1 \sim \tilde{m}_\nu = (m_{\nu_\mu} m_{\nu_\tau})^{1/2}$ is very small in order to satisfy the out-of-equilibrium condition for the decaying heavy neutrino. According to present indications for neutrino oscillations, which yield $\tilde{m}_\nu \sim 10^{-2}$ eV, this is indeed the case. For the simplest hierarchy pattern one obtains a baryon asymmetry $Y_B \sim 10^{-10}$, in agreement with observation.
References

[1] C. D. Frogatt, H. B. Nielsen, Nucl. Phys. B 147 (1979) 277

[2] J. Sato, T. Yanagida, Talk at Neutrino’98, hep-ph/9809307

[3] J. Bijnens, C. Wetterich, Nucl. Phys. B 292 (1987) 443

[4] H. Georgi and C. Jarlskog, Phys. Lett. B 86 (1979) 297

[5] T. Yanagida, in: Workshop on unified Theories, KEK report 79-18 (1979) p. 95; M. Gell-Mann et al., in: Supergravity, Proc. of the Stony Brook Workshop 1979, eds. P. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979), p. 315

[6] Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 81 (1998) 62

[7] N. Irges, S. Lavignac, P. Ramond, Phys. Rev. D 58 (1998) 035003

[8] N. Hata, P. Langacker, Phys. Rev. D 56 (1997) 6107

[9] S. P. Mikheyev, A. Y. Smirnov, Nuovo Cim. 9C (1986) 17; L. Wolfenstein, Phys. Rev. D 17 (1978) 2369

[10] V. A. Kuzmin, V. A. Rubakov, M. E. Shaposhnikov, Phys. Lett. B 155 (1985) 36

[11] M. Fukugita, T. Yanagida, Phys. Lett. B 174 (1986) 45

[12] L. Covi, E. Roulet, F. Vissani, Phys. Lett. B 384 (1996) 169; M. Flanz, E. A. Paschos, U. Sarkar, Phys. Lett. B 345 (1995) 248; Phys. Lett. B 384 (1996) 487 (E)

[13] W. Buchmüller, M. Plüümacher, Phys. Lett. B 431 (1998) 354

[14] W. Buchmüller, M. Plüümacher, Phys. Lett. B 389 (1996) 73

[15] M. Plüümacher, Z. Phys. C 74 (1997) 549; Nucl. Phys. B 530 (1998) 207

[16] H. Pagels, J. R. Primack, Phys. Rev. Lett. 48 (1982) 223

[17] M. Kawasaki, T. Moroi, Progr. Theor. Phys. 93 (1995) 879

[18] M. Bolz, W. Buchmüller, M. Plüümacher, hep-ph/9809381

[19] I. Affleck, M. Dine, Nucl. Phys. B 249 (1985) 361