Magnetic Catalysis of Chiral Symmetry Breaking in QED at Finite Temperature

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Abstract

The catalysis of chiral symmetry breaking by a magnetic field in the massless weak-coupling phase of QED is studied. The dynamical mass of a fermion (energy gap in the fermion spectrum) is shown to depend essentially nonanalytically on the renormalized coupling constant $\alpha$ in a strong magnetic field. The temperature of the symmetry restoration is calculated analytically as $T_c \approx m_{dyn}$, where $m_{dyn}$ is the dynamical mass of a fermion at zero temperature.

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In this talk I will discuss dynamical chiral symmetry breaking in a magnetic field and at finite temperature. The talk is based on the recent papers with V.Miransky and I.Shovkovy [1, 2, 3].

The dynamics of fermions in an external constant magnetic field in QED was considered by Schwinger long ago [4]. In that work, while the interaction with the external field was considered in all orders in the coupling constant, the quantum dynamics was treated perturbatively. There is no dynamical chiral symmetry breaking in QED in this approximation. Also, chiral symmetry breaking is not manifested in the weak coupling phase of QED in the absence of a magnetic field, even if it is treated nonperturbatively. We will show that a constant magnetic field $B$ changes the situation drastically, namely, it leads to dynamical chiral symmetry breaking in QED for any arbitrary weak interaction. The essence of this magnetic catalysis is that electrons are effectively 1+1 dimensional when their energy is much less than the Landau gap $\sqrt{|eB|}$ what was pointed out recently in Refs.[5, 6]. The lowest Landau level (LLL) plays here the role similar to that of the Fermi surface in the BCS theory of superconductivity, leading to dimensional reduction in dynamics of fermion pairing.

The dynamical mass of fermions (energy gap in the fermion spectrum) is:

$$m_{dyn} \simeq C \sqrt{eB} \exp \left[ - \left( \frac{\pi}{\alpha} \right)^{1/2} \right], \quad (1)$$

where the constant $C$ is of order one and $\alpha = e^2/4\pi$ is the renormalized coupling constant.

The effect of magnetic catalysis was studied also in Nambu-Jona-Lasinio (NJL) models in 2+1 [5, 6] and 3+1 dimensions [5], it was extended to the case of external non-abelian chromomagnetic fields and finite temperatures [5, 6], curved spacetime [10], as well as to the supersymmetric NJL model [11], confirming the universality of the phenomenon.

We emphasize that we will work in the conventional, weak coupling, phase of QED. That is, the bare coupling $\alpha^{(0)}$, relating to the scale $\mu = \Lambda$, where $\Lambda$ is an ultraviolet cutoff, is assumed to be small, $\alpha^{(0)} \ll 1$. Then, because of infrared freedom in QED, interactions in the theory are weak at all scales and, as a result, the treatment of the nonperturbative
dynamics is reliable.

An important question of the chiral symmetry restoration in QED at finite temperature was addressed in the recent work of Lee, Leung and Ng [12] who have obtained for the critical temperature $T_c \simeq \frac{\alpha}{\pi} \sqrt{2\pi |eB|}$. However, their $T_c$ can be considered only as a rough upper estimate. We will show that the correct estimate for the critical temperature is $T_c \approx m_{dyn}$ with $m_{dyn}$ given by (1).

The Lagrangian density of massless QED in a magnetic field is:

$$
\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \left[ \bar{\psi}, (i \gamma^\mu D_\mu) \psi \right],
$$

(2)

where the covariant derivative $D_\mu$ is

$$
D_\mu = \partial_\mu - ie (A_\mu^{ext} + A_\mu), \quad A_\mu^{ext} = \left( 0, -\frac{B}{2} x_2, \frac{B}{2} x_1, 0 \right).
$$

(3)

The Lagrangian density (2) is chiral invariant (we will not discuss the dynamics related to the anomalous current $j_5^\mu$, in any case it is not manifested in quenched approximation dealt with in present consideration). When the chiral symmetry is broken there appears a gapless NG boson composed of fermion and antifermion. The dynamical mass for a fermion can be defined by considering the Bethe-Salpeter (BS) equation for NG boson [1, 2] or the Schwinger-Dyson (SD) equation for the dynamical mass function [13, 14]. We consider here the homogeneous BS equation for NG bound state which takes the form:

$$
\chi_{\alpha\beta}(x, y; P) = -i \int d^4x_1 d^4y_1 d^4x_2 d^4y_2 G_{\alpha\alpha_1}(x, x_1) K_{\alpha_1\beta_1; \alpha_2\beta_2}(x_1 y_1, x_2 y_2) \chi_{\alpha_2\beta_2}(x_2, y_2; P) G_{\beta_1\beta}(y_1, y),
$$

(4)

where the BS wave function $\chi_{\alpha\beta}(x, y; P) = \langle 0| T \psi_\alpha(x) \bar{\psi}_\beta(y) |P \rangle$, and the fermion propagator $G_{\alpha\beta}(x, y) = \langle 0| T \psi_\alpha(x) \bar{\psi}_\beta(y) |0 \rangle$. Note that though the external field $A_\mu^{ext}$ (3) breaks the conventional translation invariance, the total momentum $P$ is a good, conserved, quantum number for neutral channels [13], in particular, for this NG boson. Since, as will be shown below, the NG boson is formed in the infrared region, where the QED coupling is weak, one
can use the BS kernel in leading order in $\alpha$:

$$K_{\alpha_1\beta_1;\alpha_2\beta_2}(x_1y_1,x_2y_2) = -4\pi i\alpha\gamma_{\alpha_1\alpha_2}^{\mu} \gamma_{\beta_2\beta_1}^{\nu} D_{\mu\nu}(y_2-x_2)\delta(x_1-x_2)\delta(y_1-y_2),$$

(5)

where the photon propagator

$$D_{\mu\nu}(x) = \frac{-i}{(2\pi)^4} \int d^4k e^{ikx} \left( g_{\mu\nu} - \lambda k_{\mu}k_{\nu} \right) \frac{1}{k^2}$$

(6)

($\lambda$ is a gauge parameter). Then the BS equation takes the form:

$$\chi_{\alpha\beta}(x,y;P) = -4\pi \alpha \int d^4x_1d^4y_1 \left[ S(x,x_1) \gamma_\mu \chi(x_1,y_1;P) \gamma_\nu S(y_1,y) \right]_{\alpha\beta} D_{\mu\nu}(y_1-x_1),$$

(7)

where, since we are working to the lowest order in $\alpha$, the full fermion propagator $G(x,y)$ is replaced by the propagator $S$ of a free fermion (with the mass $m = m_{dyn}$) in a magnetic field $\mathcal{A}$:

$$S(x,y) = \exp \left[ \frac{ie}{2} (x-y)^\mu A_\mu^{ext}(x+y) \right] \tilde{S}(x-y),$$

(8)

where the Fourier transform of $\tilde{S}$ is

$$\tilde{S}(k) = \int ds \exp \left[ is \left( k_0^2 - k_3^2 - k_\perp^2 \tan(eBs) - m_{dyn}^2 \right) \right] \cdot \left( k_0 \gamma_0 - k_3 \gamma_3 + m_{dyn}(1 + \gamma_1 \gamma_2 \tan(eBs)) - k_\perp \gamma_\perp (1 + \tan^2(eBs)) \right).$$

(9)

Here $k_\perp = (k_1, k_2)$, $\gamma_\perp = (\gamma_1, \gamma_2)$. Using the variables, the center-of-mass coordinate, $R = (x+y)/2$, and the relative coordinate, $r = x - y$, equation (7) can be rewritten as

$$\tilde{\chi}_{\alpha\beta}(R,r;P) = -4\pi \alpha \int d^4R_1d^4r_1 \left[ \tilde{S} \left( R - R_1 + \frac{r-r_1}{2} \right) \gamma_\mu \tilde{\chi}(R_1,r_1;P) \gamma_\nu \right. \cdot \left. \tilde{S} \left( \frac{r-r_1}{2} - R + R_1 \right) \right]_{\alpha\beta} D_{\mu\nu}(-r_1) \exp \left[ -ie(r + r_1)^\mu A_\mu^{ext}(R - R_1) \right].$$

(10)

Here the function $\tilde{\chi}_{\alpha\beta}(R,r;P)$ is defined from the equation

$$\chi_{\alpha\beta}(x,y;P) = \langle 0 | T\psi_\alpha(x)\bar{\psi}_\beta(y) | P \rangle = \exp[ie\gamma_\mu A_\mu^{ext}(R)]\tilde{\chi}_{\alpha\beta}(R,r;P).$$

(11)

It is important that the effect of translation symmetry breaking by the external field is factorized in the phase factor in Eq.(11) and equation (10) admits a translation invariant
solution, \( \tilde{\chi}(R, r; P) = \exp(-iPR)\tilde{\chi}(r; P) \). Henceforth we will consider the case \( P = 0 \), corresponding to NG bound state. Then, transforming this equation into momentum space, we get:

\[
\tilde{\chi}_{\alpha\beta}(p) = -4\pi\alpha \int \frac{d^2q_\perp d^2R_\perp d^2k_\perp d^2k_\parallel}{(2\pi)^6} \exp(-iq_\perp R_\perp) \left[ \tilde{S}\left(p_\parallel, p_\perp + eA^{ext}(R_\perp) + \frac{q_\perp}{2}\right) \cdot \gamma^\mu \tilde{\chi}(p_\parallel, p_\perp + eA^{ext}(R_\perp) - \frac{q_\perp}{2}) \right] \alpha\beta \end{equation}

\[
D_{\mu\nu}(k_\parallel - p_\parallel, k_\perp - p_\perp - 2eA^{ext}(R_\perp)), \tag{12}
\]

where \( p_\parallel \equiv (p^0, p^3), p_\perp \equiv (p^1, p^2) \).

The crucial point for the further analysis will be the assumption that \( m_{dyn} \ll \sqrt{|eB|} \) and that the region mostly responsible for generating the mass is the infrared region with \( k \ll \sqrt{|eB|} \). The assumption allows us to replace the propagator \( \tilde{S} \) in Eq.(12) by that projected into LLL. In order to show this, we recall that the energy spectrum of fermions with \( m = m_{dyn} \) in a magnetic field is

\[
E_n(p_3) = \pm \sqrt{m_{dyn}^2 + 2|eB|n + p_3^2}, \quad n = 0, 1, 2, \ldots \tag{13}
\]

(the Landau levels). The propagator \( \tilde{S}(p) \) can be decomposed over the Landau level poles as follows \cite{6,16}:

\[
\tilde{S}(p) = i \exp \left( -\frac{p_\perp^2}{|eB|} \right) \sum_{n=0}^{\infty} (-1)^n \frac{D_n(eB, p)}{p_0^2 - p_3^2 - m_{dyn}^2 - 2|eB|n} \tag{14}
\]

with

\[
D_n(eB, p) = (p^0\gamma^0 - p^3\gamma^3 + m_{dyn}) \left[ (1 - i\gamma^1\gamma^2 \text{sign}(eB))L_n \left( 2\frac{p_\perp^2}{|eB|} \right) \right] - (1 + i\gamma^1\gamma^2 \text{sign}(eB))L_{n-1} \left( 2\frac{p_\perp^2}{|eB|} \right) + 4(p^1\gamma^1 + p^2\gamma^2)L_{n-1}^1 \left( 2\frac{p_\perp^2}{|eB|} \right),
\]

where \( L_n(x) \) are the generalized Laguerre polynomials (\( L_n \equiv L_n^0, L_n^{-1}(x) = 0 \)). Eq.(14) implies that at \( p_\parallel^2, m_{dyn} < \sqrt{|eB|} \), the LLL with \( n = 0 \) dominates and we can take

\[
\tilde{S}(p) \approx i \exp(-\frac{p_\perp^2}{|eB|}) \hat{p}_\parallel + m_{dyn} \frac{1}{p_\perp^2 - m_{dyn}^2} (1 - i\gamma^1\gamma^2 \text{sign}(eB)), \tag{15}
\]
where \( \hat{p}_\parallel = p^0 \gamma^0 - p^3 \gamma^3 \) and \( \hat{p}_\parallel^2 = (p^0)^2 - (p^3)^2 \), and Eq.(12) transforms into the following one (further for concretness we assume \( eB > 0 \)):

\[
\rho(p_\parallel, p_\perp) = 2\alpha l^2 \int d^2 A_\perp d^2 k_\perp d^2 \hat{k}_\parallel e^{-l^2 A_\perp^2} (1 - i\gamma^1 \gamma^2) \gamma^\mu \frac{\hat{k}_\parallel + m_{\text{dyn}}}{k_\parallel^2 - m_{\text{dyn}}^2} \cdot \rho(k_\parallel, k_\perp) \varphi(1 - i\gamma^1 \gamma^2) \mathcal{D}_{\mu\nu}(k_\parallel - p_\parallel, k_\perp - A_\perp),
\]

(16)

where

\[
\rho(p_\parallel, p_\perp) = (\hat{p}_\parallel - m_{\text{dyn}}) \hat{\chi}(p)(\hat{p}_\parallel - m_{\text{dyn}}),
\]

(17)

and \( l = |eB|^{-1/2} \) is the magnetic length. Eq.(16) implies that \( \rho(p_\parallel, p_\perp) = \exp(-l^2 p_\perp^2) \varphi(p_\parallel) \), where \( \varphi(p_\parallel) \) satisfies the equation:

\[
\varphi(p_\parallel) = \frac{\pi\alpha}{(2\pi)^4} \int d^2 k_\parallel (1 - i\gamma^1 \gamma^2) \gamma^\mu \frac{\hat{k}_\parallel + m_{\text{dyn}}}{k_\parallel^2 - m_{\text{dyn}}^2} \varphi(1 - i\gamma^1 \gamma^2) \mathcal{D}_{\mu\nu}(k_\parallel - p_\parallel),
\]

(18)

Thus the BS equation has been reduced to a two–dimensional integral equation. Of course, this fact reflects the two–dimensional character of the dynamics of electrons from LLL, that can be explicitly read from Eq.(15).

We emphasize that the dimensional reduction in a magnetic field does not affect the dynamics of the center of mass of neutral bound states (in particular, this NG boson). Indeed, the reduction \( 3 + 1 \rightarrow 1 + 1 \) in the fermion propagator, in the infrared region, reflects the fact that the motion of charged particles is restricted in directions perpendicular to the magnetic field. Since there is no such a restriction for the motion of the center of mass of neutral particles, their propagator must have a \((3 + 1)\)–dimensional form. This fact was explicitly shown for neutral bound states in NJL model in a magnetic field, in \( 1/N_c \) expansion [3], and for neutral excitations in nonrelativistic systems [17]. This in particular implies that, notwithstanding the dimensional reduction in a magnetic field, the phenomenon of
spontaneous chiral symmetry breaking in QED does not contradict to the Mermin–Wagner–Coleman theorem [18] forbidding the spontaneous breakdown of continuous symmetries at $D = 1 + 1$.

Henceforth we will use Euclidean space with $k_4 = -ik^0$. In order to define the matrix structure of the wave function $\varphi(p_\parallel)$ of the NG boson, note that, in a magnetic field, there is the symmetry $SO(2) \times SO(2) \times P$, where the $SO(2) \times SO(2)$ is connected with rotations in $x_1-x_2$ and $x_3-x_4$ planes and $P$ is the inversion transformation $x_3 \to -x_3$ (under which a fermion field transforms as $\psi \to i\gamma_5\gamma_3\psi$). This symmetry implies that the function $\varphi(p_\parallel)$ takes the form:

$$\varphi(p_\parallel) = \gamma_5(A + i\gamma_1\gamma_2B + \hat{p}_\parallel C + i\gamma_1\gamma_2\hat{p}_\parallel D),$$

(20)

where $\hat{p}_\parallel = p_3\gamma_3 + p_4\gamma_4$ (with $\gamma_\mu$ anti-Hermitian in Euclidean space) and $A, B, C$ and $D$ are functions of $p_\parallel^2$.

In Feynman gauge

$$D_{\mu\nu}(k_\parallel - p_\parallel) = \delta_{\mu\nu} \pi \int_0^\infty dx \exp(-l^2 x/2)$$

and, substituting expansion (20) into equation (18), we find that $B = -A$, $C = D = 0$, i.e.,

$$\varphi(p_\parallel) = A\gamma_5(1 - i\gamma_1\gamma_2),$$

and the function $A$ satisfies the equation

$$A(p) = \frac{\alpha}{2\pi^2} \int \frac{d^2kA(k)}{k^2 + m^2_{\text{dyn}}} \int_0^\infty dx \exp(-xl^2/2)$$

(21)

(henceforth we will omit the symbol $\parallel$ from $p$ and $k$). Eq.(21) coincides with the equation for the dynamical fermion mass function obtained with the help of SD equation for a fermion propagator in [12, 14]. As was shown in [2] (see Appendix C), in the case of weak coupling $\alpha$, the function $A(p)$ remains almost constant in the range of momenta $0 < p^2 \lesssim 1/l^2$ and decays like $1/p^2$ outside that region. To get an estimate for $m_{\text{dyn}}$ at $\alpha << 1$, we set the external momentum to be zero and notice that the main contribution in the integral on the right hand side of Eq.(21) is formed in the infrared region with $k^2 \lesssim 1/l^2$. The latter validates
in its turn the substitution \( A(k) \to A(0) \) in the integrand of (21), and we finally come to the
following gap equation:
\[
A(0) \approx \frac{\alpha}{2\pi^2} A(0) \int \frac{d^2 k}{k^2 + m_{dyn}^2} \int_0^\infty \frac{dx \exp(-l^2 x/2)}{k^2 + x},
\]  
(22)
i.e.,
\[
1 \approx \frac{\alpha}{2\pi} \int_0^\infty dx \frac{\exp(-ax)}{x - 1} \log x, \quad a = \frac{m_{dyn}^2 l^2}{2}.
\]  
(23)
The main contribution in (23) comes from the region \( x \approx 1/a \), thus at \( a << 1 \) we get
\[
1 \approx \frac{\alpha}{4\pi} \log^2 \left( \frac{m_{dyn}^2 l^2}{2} \right),
\]  
(24)
therefrom the expression (1) for \( m_{dyn} \) follows. The exponential factor in \( m_{dyn} \) displays the
nonperturbative nature of this result. It can be shown also that the expression (1) for
the dynamical mass is gauge invariant [1].

More accurate analysis [2], which takes into account the momentum dependence of the
mass function, leads to the result
\[
m_{dyn} \approx C \sqrt{|eB|} \exp \left[ -\frac{\pi}{2\sqrt{2}} \sqrt{\frac{\pi}{2\alpha}} \right].
\]  
(25)
Notice that the ratio of the powers of this exponent and that in Eq.(1) is \( \pi/2 \sqrt{2} \approx 1.1 \), thus
the approximation used above is rather reliable.

To study chiral symmetry breaking in an external field at nonzero temperature we use
the imaginary time formalism. Now the analogue of the equation (21) (with the replacement
\( m_{dyn} \to m^2(T) \) in the denominator) reads
\[
A(\omega_{n'}, p) = \frac{\alpha}{\pi} T \sum_{n = -\infty}^\infty \int_{-\infty}^\infty \frac{dk A(\omega_n, k)}{\omega_n^2 + k^2 + m^2(T)} \int_0^\infty \frac{dx \exp(-l^2 x/2)}{(\omega_n - \omega_{n'})^2 + (k - p)^2 + x},
\]  
(26)
where \( \omega_n = \pi T (2n + 1) \) are Matsubara frequencies.

If we now take \( n' = 0, p = 0 \) in the left hand side of Eq.(26) and put \( A(\omega_n, k) \approx A(\omega_0, 0) = const \) in the integrand, we come to the equation
\[
1 = \frac{\alpha}{\pi} T \sum_{n = -\infty}^\infty \int_{-\infty}^\infty \frac{dk}{\omega_n^2 + k^2 + m^2(T)} \int_0^\infty \frac{dx \exp(-l^2 x/2)}{(\omega_n - \omega_0)^2 + k^2 + x}.
\]  
(27)
After evaluating the sum in (27), we are left with the equation

\[
1 = \frac{\alpha}{\pi} \int_0^\infty \int_0^\infty \frac{dk dx \exp[-\ell^2 x/2]}{[(\pi T)^2 + x - m^2(T)]^2 + (2\pi T)^2(k^2 + m^2(T))} \left\{ \frac{(\pi T)^2 + x - m^2(T)}{\sqrt{k^2 + m^2(T)}} \right\}.
\]

The equation for the critical temperature is obtained from (28) putting \( m(T_c) = 0 \):

\[
1 = \frac{\alpha}{\pi} \int_0^\infty \int_0^\infty \frac{dk dx e^{-2x(\pi T \ell)^2}}{[1/4 + x]^2 + k^2} \left\{ \frac{1/4 + x}{k} \tanh(\pi k) + \frac{1/4 - x}{\sqrt{k^2 + x}} \coth(\pi \sqrt{k^2 + x}) \right\},
\]

(29)

where we also switched to dimensionless variables \( x \rightarrow (2\pi T)^2 x \) and \( k \rightarrow 2\pi T \ell k \).

By assuming smallness of the critical temperature in comparison with the scale put by the magnetic field, \( T_c \ell \ll 1 \), we see that the double logarithmic in field contribution in Eq. (29) comes from the region \( 0 < x \approx 1/2(\pi T \ell)^2, 1/\pi \ll k < \infty \). Simple estimate gives:

\[
1 \approx \frac{\alpha}{\pi} \int_0^\infty dx \int_{1/\pi}^\infty \frac{dk}{1/4 + x} \left\{ \frac{1/4 + x}{k} + \frac{1/4 - x}{\sqrt{k^2 + x}} \right\}
\]

\[
\approx \frac{\alpha}{\pi} \int_0^\infty dx \left\{ \frac{1}{2(1/4 + x)} \log \left[ 1 + (1/4 + x)^2 \pi^2 \right] + \frac{1/4 - x}{(1/4 + x)(1/4 - x)} \right\}
\]

\[
\cdot \log \left[ \frac{(1/4 + x + 1/4 - x) \sqrt{1/\pi^2 + (1/4 + x)^2}}{(1/4 + x) \sqrt{1/\pi^2 + x + 1/4 - x}/\pi} \right] \approx \frac{\alpha}{4\pi} \log^2 \left[ \frac{1}{2(\pi T \ell)^2} \right].
\]

(30)

Thus, for the critical temperature, we obtain the estimate:

\[
T_c \approx |eB| \exp \left[ -\sqrt{\frac{\pi^2}{\alpha^2}} \right] \approx m_{dyn}(T = 0),
\]

(31)

where \( m_{dyn} \) is given by (1). The relationship \( T_c \approx m_{dyn} \) between the critical temperature and the zero temperature fermion mass was obtained also in NJL model in (2+1)- and (3+1)-dimensions \[4, 9\]. The constant \( C \), in the relation \( T_c = C m_{dyn} \), is of order one and can be calculated numerically. We note the photon thermal mass, which is of the order of \( \sqrt{\alpha T} \) [19], cannot change our result for the critical temperature. As is easy to check, the only effect of taking it into account will be the shift in \( x \) for a constant of the order of \( \alpha \) in the
integrand of (30). However, such a shift is absolutely irrelevant for our estimate (31). On the other hand, there are relevant one-loop contributions in the photon propagator due to a magnetic field. Taking them into account (that corresponds to the so-called improved ladder approximation), we get an expression for $m_{dyn}$ of the form (1) with the replacement $\alpha \rightarrow \alpha/2$.

In conclusion, let us discuss possible applications of this effect. One potential application is the interpretation of the results of the GSI heavy–ion scattering experiments [20] in which narrow peaks are seen in the energy spectra of emitted $e^+e^-$ pairs. One proposed explanation [21] is that a very strong electromagnetic field, created by the heavy ions, induces a phase transition in QED to a phase with spontaneous chiral symmetry breaking. The observed peaks are due to the decay of positronium–like states in this phase. The catalysis of chiral symmetry breaking by a magnetic field in QED, discussed in this report, can serve as a toy example of such a phenomenon. In order to get a more realistic model, it would be interesting to extend this analysis to non–constant background fields.

An interesting application of the magnetic catalysis in astrophysics, such as the cooling process of neutron stars due to enhanced Primakoff process in high magnetic field, was mentioned recently in [22].

Yet another application of the effect is connected with the role of chromomagnetic backgrounds imitating gluon condensate in the QCD vacuum [8, 9, 23].

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