Application of the hereditary creep theory in assessing the effect of temperature influence on the strength of carbon fibre plastics

I Gadolina¹*, A Berezin¹, I Maidanov² and S Smelov²

¹ IMASH RAS, Moscow, Russia
² ORPE Technologiya named after A.G. Romashin, Russia

Email: gadolina@mail.ru

Abstract. One of the problems, which engineers face in using of carbon-reinforced composites, is the large scatter of the strength characteristics. To diminish scatter the method of the merger of tests results under varied climatic conditions might be applied. The aim of this merge is diminishing the scatter of the strength test results, and therefore improving the strength estimation accuracy. It is also in connection with the problem of the impact of climatic changes on the material properties. The climate impact is significantly larger in composites than in metals. To deal with those two problems the heredity mechanics developed earlier by Russian academician Ju. N. Rabotnov and his successors has been applied. The developed earlier modification of this theory, related to climate influence, has been applied to the problem of experimental data merger, obtained under varying temperatures, into one generalized set. For consideration of the orthotropy properties which are specific for reinforced composites, it was proposed the experimental investigation of the specimens cut in three different directions. The Rabotnov’s heredity equation should be written in tensor form. The property of viscoelasticity, which is inherent for carbo-plastics materials (especially under the elevated temperatures), is utilized in the decision of this problem. Experimental data and results of the model application are presented.

1. Introduction

The strength properties of fibre-reinforced composites have a high degree of variability. The reasons for the variation of the characteristics are due to many factors, including factors of production of raw materials and prepreg, material processing, manufacturing techniques for the parts, the sequence of layering, environmental conditions and testing technologies. It is also should be considered the existence of changes in the mechanisms of formation and growth of micro defects under various external conditions. At the same time, the full identification of the process requires the use of fine physical research methods, as well as mathematical modelling of the test process. Therefore, mathematical modelling must be complete and reliable, including proofs of the uniqueness and existence of solutions, equations describing processes, and consistency with the physical nature of the simulated process.

In reality, not in all cases, it is possible to find the causes of the origin of defects and damage. At the same time, the cost of composites testing is relatively high - samples are often laid out manually. This fact, combined with the need for additional tests necessary due to anisotropy, i.e. the heterogeneity of the physical (physic-chemical) properties of the nature of composite materials, leads to the need to obtain a larger volume of specific properties in comparison with tests of metallic materials. In this case, there
is a need to use improved statistical methods to assess the reliability of experimentally obtained estimates of the strength characteristics of composites.

2. The relevance of the problem
There is a need to use improved statistical methods to assess the reliability of experimental estimates of strength characteristics of composites. The qualification procedure is the establishment of statistical design tolerances for each mechanical property. The purpose of this analysis is to solve the problem of combining heterogeneous samples into one to diminish the variance of the estimate [1].

In figure 1 four sets distribution function of measured compression strength of the unidirectional carbon-reinforced specimen in the fibre direction under varied climate conditions (sets 1 … 4) are shown. Looking at this figure it is noticeable, that the strength characteristics vary.

![Figure 1](image1.png)

**Figure 1.** Distribution of strength for composite samples in four varied climate conditions.

To apply the procedure of scatter diminishing the method of several sets merger [1] is employed. First, all the data are divided by their own mean values. After that their mean values all become unity, and the variances will differ. In Figure 2 the normalized strength properties are shown in a special plot.

![Figure 2](image2.png)

**Figure 2.** Four normalized sets in a plot graph.

![Figure 3](image3.png)

**Figure 3.** Graphical checking the normality hypothesis of generalized sample distribution [2].
Figure 3 shows the relative closeness of the generalized sample experimental points towards the straight line. This fact could confirm not a contradiction to the hypothesis of normal distribution. Due to this the allowance intervals for generalized sample might be constructed according to [1].

All the sets (Figure 1) has the same object volume, namely, \( n_1=n_2=n_3=n_4=17 \). The mergered set consist of \( N \) elements, \( N=\sum n_i=68 \). Statistical data are shown in Table 1.

**Table 1.** Strength characteristics of composite material under varied climate conditions [1].

| SAMPLE INDEX | 1 | 2 | 3 | 4 |
|--------------|---|---|---|---|
| NOTATION     | Low temperature on dry thermometer | Room temperature by wet thermometer | High temperature on dry thermometer | Increased temperature by wet thermometer |
| NUMBER       | 17 | 17 | 17 | 17 |
| MEAN, KSI    | 107,1744 | 99,07068 | 79,42751 | 61,93163 |
| SD, KSI      | 4,3157 | 3,773153 | 7,370923 | 3,581152 |
| V            | 0,040268 | 0,038085 | 0,092801 | 0,057824 |

Note: The standard deviation of merger sample (normalized) is \( sd_{\text{merge}}=0,06 \).

Since the dimension of the mergered sample \( N=68 \) is greater, than \( n_i=17 \), the tolerant coefficients in estimated basing on greater number of observations are less and the precision grows up.

3. Application of heredity mechanics theory to the problem of the scatter diminishing

Due to the presence of rheological properties in polymer materials, it is possible to apply the theory of hereditary mechanic [3] for processing experiment results at different temperatures. Application of the proposed method to different composites require thoughtfully investigation of their matrix and fiber properties.

In [4] basing on the relations of the algebra of resolvent operators, the Volterra correspondence principle and the Rabotnov non-linear relation, a model describing strain anisotropy of unidirectional composites under time-varying loading was developed. The model described in [4] includes the expression for the elastic modulus recorded taking into account anisotropy. The dynamic part of that expression was selected in such a manner, that the authors could describe the behaviour of the material in time. The advantage of using hereditary equations written in integral form is that they can take into account the influence of loading history, temperature, and other factors.

The nonlinear relationship between stress \( \sigma \) and strain \( \varepsilon \) can be written as an integral equation [3] for an isotropic material and under uniaxial loading:

\[
\varphi(\varepsilon) = \sigma + \int_0^\tau K(t-\tau)\sigma(\tau) d\tau
\]  

(1)

In equation (1): \( \varphi(\varepsilon) \) is a nonlinear function-the instantaneous deformation curve, \( K(t-\tau) \) is the creep core that characterizes the memory of the material; \( \tau \) is the instant time for integration.

Equation (1) is valid for the case of uniaxial loading of the isotropic material.

In the case of anisotropy, equation (1) is not applicable. In this case, the heredity equation should be written in the tensor form [5]:

\[
\varphi(\varepsilon_{ij}) = \sigma_{ij} + \int_0^\tau K_{ijkl}(t-\tau)\sigma_{kl}(\tau) d\tau
\]  

(2)

In equation (2): \( K_{ijkl}(t-\tau) \) is the creep kernel in tensor form, \( \varepsilon_{ij} \) is the strain and \( \sigma_{kl}(\tau) \) is the stress.

To apply the equation (2) in the case of orthotropy (different properties in two directions in a plane problem) and in the conditions of a plane stress state, it is necessary to record the deformation curves for unidirectional carbon-reinforced composite in 3 directions. The specimens should be cut as indicated in Figure 4. In this case, the Poisson's coefficients, elastic modulus, and strength in the specified
directions will be determined. It is also necessary to construct deformation curves in these directions. If this data is available, it is possible to define properties in all other directions (0 ... 90°) [5].

Figure 4. Cutting the specimens for investigation unidirectional carbon-reinforced composites.

Figure 5. Deformation curves for the unidirectional carbon-reinforced composite cut in varied directions.

Due to orthotropy property, the deformation curves will be different in three directions. In Figure 5 those curves for orthotropic material are shown schematically for testing at room temperature.

In [6] the climate factors were introduced into the defining Rabotnov’s ratio (2), namely, the functions of temperature $f_1(T)$ and humidity $f_2(W):$

$$\varphi(\varepsilon) = \sigma + \int_0^t \frac{k}{(t-\tau)\alpha} f_1(T)f_2(W)\sigma(\tau)d\tau$$  \hspace{1cm} (3)

The parameters of the defining equation in form (3) are the parameters $k$ and $\alpha$.

To determine these two parameters, it is necessary to conduct tests with a constant rate of stress change ($\sigma = \text{const}$). For some arbitrary value $\varepsilon^*$, one should fix the values $\sigma_1$ and $\sigma_2$ corresponding to any two different temperatures, for example, room temperature $T_1$ and some other temperature $T_2$. As a result, two relations will be obtained:

$$\varphi(\varepsilon^*) = \sigma_1\left(1 + \frac{k_0}{(1-\alpha)(2-\alpha)} T_1^{\gamma_1} t_1^{1-\alpha}\right)$$  \hspace{1cm} (4)

$$\varphi(\varepsilon^*) = \sigma_2\left(1 + \frac{k_0}{(1-\alpha)(2-\alpha)} T_2^{\gamma_2} t_2^{1-\alpha}\right)$$  \hspace{1cm} (5)

Core parameters $k$ and $\alpha$ can be calculated from ratios (4) and (5) based on experimental data.

In [6], data on testing and calculation of deformation curves of the anti-friction polymer material ROM are presented. The calculated deformation curves obtained by the author are close to the experimental ones (Figure 6). Due to the peculiarities of anti-friction polymer ROM the strength increases with the temperature increase. Under small temperature changes some structures transformation is possible and due to the interaction of fractions and small increment
of strength might be registered with temperatures increasing. A similar approach may be applied to the problem of combining sets to estimate the average strength value in the environment #2 based on the results of tests in the environment #1. In equations (4-5) the temperatures do not affect the curing time, because the curing has been already finished.

Figure 6. Deformation curves of ROM (polyformaldehyde-antifriction material) at different temperatures T: 1-24°C; 2-70°C; 3-90°C; 4- instantaneous deformation curve \( \phi (\varepsilon) \). Solid lines - experiment, dotted lines - calculation [6].

4. Conclusions

Because of the large scatter of the strength test results intrinsic to composite material some mathematical and physical procedures might be needed to consider with this scatter in case of unidirectional carbon-reinforced composite investigation.

The proposed method might serve as a first step while planning the climatic testing of the uniaxial orthotropy carbon-reinforced composite material. Some testing results are shown. Following this approach, the investigators can 1) predict the behavior of the given material under different climatic condition; 2) create the generalized sample out of few sets, obtained under varied condition to diminish the strength estimate variance.

References

[1] Material qualification and equivalence for polymer matrix material systems: an Updated procedure. DOT/FAA / AR-03/19. The service of aviation research Washington, DC. 20591. The U.S. Department of transportation — Federal aviation administration of the United States. September 2003.

[2] Team R Core 2020. R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria.

[3] Rabotnov Yu N 1977 Elements of hereditary mechanics of solid bodies. M.: Nauka, 383 p.

[4] Dumansky A M, Alimov M A and Hao L 2019 Anisotropy of nonlinear behavior of unidirectional CFRP under strain-rate loading. AIP Conf. Proc. 2171, 030003

[5] Berezin A V 1996 Nonlinear behavior of composite materials. Mech. Compos. Mater. St. 2(1) 110-128

[6] Alekseeva S I 2001 Model of nonlinear hereditary environment taking into account temperature and humidity// Reports of the Academy of Sciences 4 471-3.