Efficient two-step entanglement concentration for arbitrary W states

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We present two two-step practical entanglement concentration protocols (ECPs) for concentrating an arbitrary three-particle less-entangled W state into a maximally entangled W state assisted with single photons. The first protocol uses the linear optics and the second protocol adopts the cross-Kerr nonlinearity to perform the protocol. In the first protocol, based on the post-selection principle, three parties say Alice, Bob and Charlie in different distant locations can obtain the maximally entangled W state from the arbitrary less-entangled W state with a certain success probability. In the second protocol, it dose not require the parties to posses the sophisticated single-photon detectors and the concentrated photon pair can be retained after performing this protocol successfully. Moreover, the second protocol can be repeated to get a higher success probability. Both protocols may be useful in practical quantum information applications.

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I. INTRODUCTION

Entanglement is the essential role in quantum information theory [1–3]. Most of the applications of entanglement information processing work only with the maximally entangled states [4–16]. They first produce the entanglement locally and distribute them into different distant locations. However, during the transmission, the particles will inevitably contact the environment. The noisy environment will degrade the entangled state and make it become a nonmaximally entangled state. Generally speaking, the nonmaximally entangled states unusually include two different types. The first type is the mixed state and the second type is the pure less-entangled state. Both of which will make the fidelity of quantum teleportation degraded, quantum dense coding failed, and the quantum cryptography protocol be insecure.

The method of distilling a mixed state into a maximally entangled state is called entanglement purification, which has been studied for several decades [17–32]. Bennett et al. proposed an entanglement purification protocol in 1996 [17]. Another way of distilling a pure less-entangled state into a maximally entangled state is called entanglement concentration, which will be detailed later [33–48]. Bennett et al. proposed an entanglement concentration protocol (ECP) in 1996 [33]. It is so called Schmidt projection protocol. Bose et al. proposed an ECP based on entanglement swapping [34]. This protocol needs collective Bell-state measurement. It was developed by Shi et al., subsequently [35]. An ECP based on the quantum statistics was discussed by Paunković et al. Their protocol requires less knowledge of the initial state than most ECPs [36]. In 2001, Yamamoto et al. and Zhao et al. proposed two similar protocols based on linear optics [37, 38]. Their protocols were both realized experimentally in 2008 [39, 40]. The ECPs based on the cross-Kerr nonlinearity were proposed in 2008 [42, 43]. However, most of the ECPs are focused on two-particle system. They are used to concentrate a two-particle entangled state α|00⟩ + β|11⟩ to the Bell state 1/√2(|00⟩ + |11⟩). In a three-particle system, there are two classes of tripartite-entangled states which cannot be converted into each other by stochastic local operations and classical communication [49]. They are Greenberger-Horne-Zeilinger (GHZ) state and W state. The GHZ state can be described as |GHZ⟩ = 1/√2(|000⟩ + |111⟩) and the W state can be written as |W⟩ = 1/√3(|001⟩ + |010⟩ + |100⟩).

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with the help of joint unitary transformation \[^{50}\]. In 2007, Zhang \textit{et al.} proposed an ECP based on the Bell-state measurement \[^{51}\]. Both joint unitary transformation and Bell-state measurement are not easy to realize in current condition. In 2010, Wang \textit{et al.} proposed an ECP for W state with linear optics \[^{52}\]. Their protocol is focused on a special kind of W state, that is \(\alpha|HHH\rangle + \beta|HVV\rangle + \gamma|VHV\rangle\). Recently, Yildiz proposed an optimal distillation of three-qubit asymmetric W states \[^{53}\] of the form

\[
\begin{align*}
\frac{1}{\sqrt{2}}|001\rangle + \frac{1}{2}|010\rangle + \frac{1}{2}|100\rangle, \\
\frac{1}{2}|001\rangle + \frac{1}{2}|010\rangle + \frac{1}{\sqrt{2}}|100\rangle.
\end{align*}
\]

In this paper, we present two ECPs for concentrating an arbitrary three-photon W state \(\alpha|VHH\rangle + \beta|HHV\rangle + \gamma|HVV\rangle\) to a standard maximally entangled W state \(\frac{1}{\sqrt{4}}(|VHH\rangle + |HHV\rangle + |HVV\rangle + |VHV\rangle)\) with only local operation and classical communication. Here \(|H\rangle\) and \(|V\rangle\) represent the horizontal and vertical polarization of the photon, respectively. Our protocols are quite different from others for we are focused on the arbitrary three-particle W states. Moreover, during concentrating, we do not require two copies of less-entangled states, but only need one pair of less-entangled satate and two single photons. In this way, it is more practical and economical. The two protocols are implemented with different optical elements. In the former, we use the polarization beam splitter (PBS) to perform the parity check and to achieve the whole task. In the latter, we adopt the cross-Kerr non-linearity to construct a quantum nondemolition (QND) measurement. With the help of the QNDs, this protocol becomes more powerful. It does not require the sophisticated single-photon detectors and can be repeated to get a higher success probability.

This paper is organized as follows: In Sec. II, we describe the first protocol with linear optics. We denote it the PBS protocol. We show that an arbitrary less-entangled W state can be concentrated with a certain success probability. In Sec. III, we exploit the QNDs to substitute the PBSs and make the ECP more feasible and efficient in current technology. We denote it the QND protocol. In Sec. IV, we present a discussion and summary.

II. W STATE CONCENTRATION WITH LINEAR OPTICS

In Fig. 1, we show the basic principle of our concentration protocol. In order to explain this protocol clearly, we divide the whole ECP into two steps. In a practical experiment, both steps of the operations should be performed simultaneously. Suppose a pair of less-entangled W state \(\Phi_{a1b1c1}\) is sent to Alice, Bob and Charlie. The photon in the spatial mode \(a1\) is sent to Alice. The photon in the spatial mode \(b1\) is sent to Bob, and \(c1\) belongs to Charlie. The photon pair is initially in the following polarization less-entangled state

\[
\begin{align*}
\Phi_{a1b1c1} &= \alpha|V\rangle_{a1}|H\rangle_{b1}|H\rangle_{c1} + \beta|H\rangle_{a1}|V\rangle_{b1}|H\rangle_{c1} \\
&\quad + \gamma|H\rangle_{a1}|H\rangle_{b1}|V\rangle_{c1}.
\end{align*}
\]

We let \(\alpha, \beta\) and \(\gamma\) be real for simple, with \(\alpha^2 + \beta^2 + \gamma^2 = 1\).

![FIG. 1: Schematic drawing of the first ECP with linear optics. S1 is the partial entanglement source and S2 and S3 are the single photon sources. PBSs transmit the horizontal polarization component and reflect the vertical component. HWP\(_{90}\) and HWP\(_{45}\) can rotate the polarization of the state by 90° and 45°, respectively.](image)

In first step, a single photon in the spatial mode \(a2\) emitted from source S2 is sent to Alice. It is described as

\[
\Phi_{a2} = \frac{\alpha}{\sqrt{\alpha^2 + \beta^2}}|H\rangle_{a2} + \frac{\beta}{\sqrt{\alpha^2 + \beta^2}}|V\rangle_{a2}. \tag{3}
\]

So the whole four-photon system can be described as

\[
\Psi = |\Phi_{a1b1c1} \otimes |\Phi_{a2} = (\alpha|V\rangle_{a1}|H\rangle_{b1}|H\rangle_{c1} \\
&+ \beta|H\rangle_{a1}|V\rangle_{b1}|H\rangle_{c1} + \gamma|H\rangle_{a1}|H\rangle_{b1}|V\rangle_{c1}) \\
&\otimes \left(\frac{\alpha}{\sqrt{\alpha^2 + \beta^2}}|H\rangle_{a2} + \frac{\beta}{\sqrt{\alpha^2 + \beta^2}}|V\rangle_{a2}\right)
\]

\[
= \frac{\alpha^2}{\sqrt{\alpha^2 + \beta^2}}|V\rangle_{a1}|H\rangle_{a2}|H\rangle_{b1}|H\rangle_{c1}
\]

\[
+ \frac{\beta^2}{\sqrt{\alpha^2 + \beta^2}}|H\rangle_{a1}|V\rangle_{a2}|V\rangle_{b1}|H\rangle_{c1}
\]

\[
+ \frac{\alpha \gamma}{\sqrt{\alpha^2 + \beta^2}}|H\rangle_{a1}|H\rangle_{a2}|H\rangle_{b1}|V\rangle_{c1}
\]

\[
+ \frac{\beta \gamma}{\sqrt{\alpha^2 + \beta^2}}|H\rangle_{a1}|V\rangle_{a2}|H\rangle_{b1}|H\rangle_{c1}
\]

\[
+ \frac{\alpha \beta}{\sqrt{\alpha^2 + \beta^2}}|V\rangle_{a1}|V\rangle_{a2}|H\rangle_{b1}|H\rangle_{c1}
\]

\[
+ \frac{\alpha \beta}{\sqrt{\alpha^2 + \beta^2}}|H\rangle_{a1}|H\rangle_{a2}|V\rangle_{b1}|H\rangle_{c1}. \tag{4}
\]

From Eq. (4), after the two photons in spatial modes \(a1\) and \(a2\) both passing through the PBS\(_1\), the
three items $|V\rangle_{a1}|H\rangle_{a2}|H\rangle_{b1}|H\rangle_{c1}$, $|H\rangle_{a1}|V\rangle_{a2}|V\rangle_{b1}|H\rangle_{c1}$ and $|H\rangle_{a1}|V\rangle_{a2}|H\rangle_{b1}|V\rangle_{c1}$ will lead to the two photons in spatial modes $a1$ and $a2$ in the same output mode. But the other three items $|H\rangle_{a1}|H\rangle_{a2}|H\rangle_{b1}|V\rangle_{c1}$, $|V\rangle_{a1}|V\rangle_{a2}|H\rangle_{b1}|H\rangle_{c1}$, and $|H\rangle_{a1}|H\rangle_{a2}|V\rangle_{b1}|H\rangle_{c1}$ will lead to the two output modes of PBS1, both containing exactly one and only one photon. Therefore, if they choose the case that the spatial modes $d1$, $d2$, $b1$ and $c1$ all contain exactly one photon, then the initial state collapses to

$$|\Psi\rangle = \frac{\alpha\gamma}{\sqrt{\alpha^2 + \beta^2}}|H\rangle_{d1}|H\rangle_{d2}|H\rangle_{b1}|V\rangle_{c1} + \frac{\alpha\beta}{\sqrt{\alpha^2 + \beta^2}}|V\rangle_{d1}|V\rangle_{d2}|H\rangle_{b1}|H\rangle_{c1} + \frac{\alpha\beta}{\sqrt{\alpha^2 + \beta^2}}|H\rangle_{d1}|H\rangle_{d2}|V\rangle_{b1}|H\rangle_{c1},$$

with a success probability of

$$P_1 = \frac{\alpha^2(\gamma^2 + 2\beta^2)}{\alpha^2 + \beta^2}.$$ 

The superscription “1” means the first concentration step.

Eq. (5) can be rewritten as

$$|\Psi\rangle = \frac{\gamma}{\sqrt{\gamma^2 + 2\beta^2}}|H\rangle_{d1}|H\rangle_{d2}|H\rangle_{b1}|V\rangle_{c1} + \frac{\beta}{\sqrt{\gamma^2 + 2\beta^2}}|V\rangle_{d1}|V\rangle_{d2}|H\rangle_{b1}|H\rangle_{c1} + \frac{\beta}{\sqrt{\gamma^2 + 2\beta^2}}|H\rangle_{d1}|H\rangle_{d2}|V\rangle_{b1}|H\rangle_{c1}. \quad (7)$$

Now Alice uses $\lambda/4$-wave plate HWP$_{45}$ to rotate the photon in spatial mode $d1$. The unitary transformation of $45^\circ$ rotation can be described as

$$|H\rangle_{d2} \to \frac{1}{\sqrt{2}}(|H\rangle_{d2} + |V\rangle_{d2}),$$

$$|V\rangle_{d2} \to \frac{1}{\sqrt{2}}(|H\rangle_{d2} - |V\rangle_{d2}). \quad (8)$$

After the rotation, $|\Psi\rangle$ can be written as

$$|\Psi\rangle'' = \frac{\gamma}{\sqrt{\gamma^2 + 2\beta^2}}|H\rangle_{d1}|H\rangle_{d2}|H\rangle_{b1}|V\rangle_{c1} + \frac{\beta}{\sqrt{\gamma^2 + 2\beta^2}}|V\rangle_{d1}|V\rangle_{d2}|H\rangle_{b1}|H\rangle_{c1} + \frac{\beta}{\sqrt{\gamma^2 + 2\beta^2}}|H\rangle_{d1}|H\rangle_{d2}|V\rangle_{b1}|H\rangle_{c1}$$

$$+ \frac{1}{\sqrt{\gamma^2 + 2\beta^2}}|H\rangle_{d1}|H\rangle_{d2}|H\rangle_{b1}|V\rangle_{c1}$$

$$- \frac{\beta}{\sqrt{\gamma^2 + 2\beta^2}}|V\rangle_{d1}|V\rangle_{d2}|H\rangle_{b1}|H\rangle_{c1}$$

$$+ \frac{\beta}{\sqrt{\gamma^2 + 2\beta^2}}|H\rangle_{d1}|V\rangle_{b1}|H\rangle_{c1}|V\rangle_{d2}. \quad (9)$$

Therefore, if the photon in spatial mode $d2$ is $|H\rangle_{d2}$, and makes detector $D_1$ fire, the original state is left in the state

$$|\Phi_{d1b1c1}\rangle_{d1b1c1} = \frac{\gamma}{\sqrt{\gamma^2 + 2\beta^2}}|H\rangle_{d1}|H\rangle_{b1}|V\rangle_{c1} + \frac{\beta}{\sqrt{\gamma^2 + 2\beta^2}}|V\rangle_{d1}|H\rangle_{b1}|H\rangle_{c1}$$

$$+ \frac{\beta}{\sqrt{\gamma^2 + 2\beta^2}}|H\rangle_{d1}|V\rangle_{b1}|H\rangle_{c1}. \quad (10)$$

Otherwise, if $D_2$ fires, the original state is left in the state

$$|\Phi_{d1b1c1}\rangle_{d1b1c1} = \frac{\gamma}{\sqrt{\gamma^2 + 2\beta^2}}|H\rangle_{d1}|H\rangle_{b1}|V\rangle_{c1} - \frac{\beta}{\sqrt{\gamma^2 + 2\beta^2}}|V\rangle_{d1}|H\rangle_{b1}|H\rangle_{c1}$$

$$+ \frac{\beta}{\sqrt{\gamma^2 + 2\beta^2}}|H\rangle_{d1}|V\rangle_{b1}|H\rangle_{c1}. \quad (11)$$

In order to get $|\Phi_{d1b1c1}\rangle_{d1b1c1}$, one of the parties, says Alice, Bob or Charlie should perform a local operation of phase rotation on her or his photon. It is the first step of the first ECP.

The second step is analogy with the first one. It is performed by Charlie, shown in Fig. 1. After they get the state $|\Phi_{d1b1c1}\rangle_{d1b1c1}$, another single photon state $|\Phi_{d2}\rangle_{d2}$ emitted from source $S_3$ is sent to Charlie. $|\Phi_{d2}\rangle$ can be written as

$$|\Phi_{d2}\rangle = \frac{\beta}{\sqrt{\gamma^2 + 2\beta^2}}|H\rangle_{d2} + \frac{\gamma}{\sqrt{\gamma^2 + 2\beta^2}}|V\rangle_{d2}. \quad (12)$$

Charlie first rotates the photon by $90^\circ$ in the spatial mode $c2$ with HWP$_{90}$. The $|\Phi_{d2}\rangle$ can be written as

$$|\Phi_{d3}\rangle = \frac{\beta}{\sqrt{\gamma^2 + 2\beta^2}}|V\rangle_{d3} + \frac{\gamma}{\sqrt{\gamma^2 + 2\beta^2}}|H\rangle_{d3}. \quad (13)$$

The combination of four-photon state can be written as

$$|\Phi_{d3}\rangle \otimes |\Phi_{d2}\rangle_{d1b1c1}$$

$$= \frac{\beta\gamma}{\sqrt{\gamma^2 + 2\beta^2}\sqrt{\gamma^2 + 2\beta^2}}|H\rangle_{d1}|H\rangle_{b1}|V\rangle_{c1}|V\rangle_{d3}$$

$$+ \frac{\gamma^2}{\sqrt{\gamma^2 + 2\beta^2}}|H\rangle_{d1}|H\rangle_{b1}|V\rangle_{c1}|H\rangle_{d3}$$

$$+ \frac{\beta^2}{\sqrt{\gamma^2 + 2\beta^2}}|V\rangle_{d1}|H\rangle_{b1}|H\rangle_{c1}|V\rangle_{d3}$$

$$+ \frac{\beta\gamma^2}{\sqrt{\gamma^2 + 2\beta^2}}|V\rangle_{d1}|H\rangle_{b1}|H\rangle_{c1}|H\rangle_{d3}$$

$$+ \frac{\beta^2}{\sqrt{\gamma^2 + 2\beta^2}}|V\rangle_{d1}|V\rangle_{b1}|H\rangle_{c1}|V\rangle_{d3}$$

$$+ \frac{\beta\gamma^2}{\sqrt{\gamma^2 + 2\beta^2}}|V\rangle_{d1}|V\rangle_{b1}|H\rangle_{c1}|H\rangle_{d3}. \quad (14)$$
It is easy to find that the three items \(|H\rangle_{d1}|H\rangle_{b1}|V\rangle_{c1}|H\rangle_{c3}, |V\rangle_{d1}|H\rangle_{b1}|H\rangle_{c1}|V\rangle_{c3}\) and \(|H\rangle_{d1}|V\rangle_{b1}|H\rangle_{c1}|V\rangle_{c3}\) will lead to the two photons in Charlie’s location in the same output mode after passing through the PBS2. But the other three items \(|H\rangle_{d1}|H\rangle_{b1}|V\rangle_{c1}|V\rangle_{c3}, |V\rangle_{d1}|H\rangle_{b1}|H\rangle_{c1}|H\rangle_{c3}\) and \(|H\rangle_{d1}|V\rangle_{b1}|H\rangle_{c1}|H\rangle_{c3}\) will lead to the two photons in different output modes. Therefore, similar to the first step, Charlie chooses the case that two output modes of PBS2 both contain one photon. So Eq. (14) becomes

\[
\Psi'' = \frac{1}{\sqrt{3}}(|H\rangle_{d1}|H\rangle_{b1}|V\rangle_{c1}|V\rangle_{c2} + |V\rangle_{d1}|H\rangle_{b1}|H\rangle_{c1}|H\rangle_{c2}).
\]

with a success probability of

\[
P^2 = \frac{3\beta^2\gamma^2}{(\gamma^2 + \beta^2)(\gamma^2 + 2\beta^2)}.
\]

The superscription ”2” means the second concentration step.

Finally, Charlie rotates his photon in the mode e2 by 45° with HWP_{45} and makes

\[
|H\rangle_{e2} \rightarrow \frac{1}{\sqrt{2}}(|H\rangle_{e2} + |V\rangle_{e2}),
\]

\[
|V\rangle_{e2} \rightarrow \frac{1}{\sqrt{2}}(|H\rangle_{e2} - |V\rangle_{e2}).
\]

If D_3 fires, they will get

\[
|\Phi_1\rangle_{d1b1c1} = \frac{1}{\sqrt{3}}(|H\rangle_{d1}|H\rangle_{b1}|V\rangle_{c1} + |V\rangle_{d1}|H\rangle_{b1}|H\rangle_{c1} + |H\rangle_{d1}|V\rangle_{b1}|H\rangle_{c1}).
\]

If D_4 fires, they will get

\[
|\Phi_2\rangle_{d1b1c1} = \frac{1}{\sqrt{3}}(|H\rangle_{d1}|H\rangle_{b1}|V\rangle_{c1} + |V\rangle_{d1}|H\rangle_{b1}|H\rangle_{c1} + |H\rangle_{d1}|V\rangle_{b1}|H\rangle_{c1}).
\]

Both Eqs. (18) and (19) are the maximally entangled W states. In order to get \(|\Phi_1\rangle_{d1b1c1}, one of these three states, say Alice, Bob or Charlie should perform a local operation of phase rotation on her or his photon.

Thus far, we have fully explained our first ECP. The total success probability \(P_s\) for obtaining a maximally entangled W state is

\[
P_s = P_1P_2 = \frac{\alpha^2(\gamma^2 + 2\beta^2)}{\alpha^2 + \beta^2} \frac{3\beta^2\gamma^2}{(\gamma^2 + \beta^2)(\gamma^2 + 2\beta^2)} = \frac{3\alpha^2\beta^2\gamma^2}{(\alpha^2 + \beta^2)(\gamma^2 + \beta^2)}.
\]

Actually, during the whole procedure, Alice and Charlie use essentially the same principle to perform the concentration protocol. They both pick up the even parity states \(|H\rangle|H\rangle\) and \(|V\rangle|V\rangle\) after the photons passing through the PBSs and discard the odd parity states \(|H\rangle|V\rangle\) and \(|V\rangle|H\rangle\) because the two photons are in the same spatial modes. In fact, the discarded items can also be reused to obtain the maximally entangled W states. So this kind of protocol is a not optimal one.

In the above description, we explain the total ECP by dividing it into two steps. The first protocol is essentially to obtain the state in Eq. (10) from Eq. (2), and the second step is to obtain the genuine maximally entangled W state from Eq. (18). Actually, we should point out that in a practical experiment, we cannot perform this protocol like that. The main reason is that this kind of ECP is based on the post-selection principle. That is to say, they should resort the sophisticated single-photon detectors to check the photon number in the output modes of PBSs. For instance, in the first step, the successful case will make both of spatial modes d1 and d2 contain one photon. However, once the photons are successfully detected, the whole photon-state is destroyed. It is impossible to perform the further step. Thus, the feasible way is to perform the two steps simultaneously, and choose the cases that the spatial modes d1, d2, e1, e2 and b1 contain exactly one and only one photon with the success probability of \(P_s\).

III. W STATE CONCENTRATION WITH CROSS-KERR NONLINEARITY

In Sec. II, we have fully explained our first ECP with linear optics. The whole protocol should resort sophisticated single-photon detectors to check the photon number. Moreover, the whole protocol is based on the post-selection principle and requires Alice and Charlie to perform the two steps simultaneously. These disadvantages may limit its practical application in current quantum information processing.

In this section, we adopt the cross-Kerr nonlinearity to implement a QND, which can play the roles of both parity check and single-photon detector. Before we start this protocol, let us briefly explain the basic principle of the cross-Kerr nonlinearity. The cross-Kerr nonlinearity has been widely studied in constructing of CNOT gates [54], performing entanglement purification protocol [22], ECPs [42, 45], complete Bell-state analysis [55], and other quantum communication and computation processing [56–64]. The Hamiltonian of a cross-Kerr nonlinearity can be described as 

\[
H = h\chi\hat{a}_h\hat{a}_b\hat{a}_b\hat{a}_h,
\]

where the \(h\chi\) is the coupling strength of the nonlinearity. It is decided by the cross-Kerr material. The \(\hat{n}_a(\hat{n}_b)\) is the number operator for mode a(b) [54]. In Fig. 2, two polarized photons are initially prepared in the states \(|\varphi\rangle_{a_1} = c_0|H\rangle_{a_1} + c_1|V\rangle_{b_1}\) and \(|\varphi\rangle_{a_2} = d_0|H\rangle_{a_2} + d_1|V\rangle_{a_2}\). They combine with a coherent beam \(|\alpha\rangle_p\) and inter-
act with the cross-Kerr nonlinearities. So the state of the composite quantum system from the original one
\(|\Psi\rangle_O = |\varphi\rangle_{a_1} \otimes |\varphi\rangle_{a_2} \otimes |\alpha\rangle_p\) evolves to
\[
|\Psi\rangle_T = [c_0d_0|HH\rangle + c_1d_1|VV\rangle]|\alpha e^{i\theta}\rangle_p
+ c_0d_1|HV\rangle|\alpha e^{2i\theta}\rangle_p + c_1d_0|VH\rangle|\alpha\rangle_p.
\] (21)

From Eq. (21), the items \(|HH\rangle\) and \(|VV\rangle\) make the coherent beam \(|\alpha\rangle_p\) pick up a \(\theta\) phase shift. The item \(|HV\rangle\) picks up a 2\(\theta\) phase shift, and the item \(|VH\rangle\) picks up no phase shift. With a general homodyne-heterodyne measurement (X homodyne measurement), one can distinguish \(|HH\rangle\) and \(|VV\rangle\) from \(|HV\rangle\) and \(|VH\rangle\) according to their different phase shift [54]. It plays essentially the same role of parity check.

Now we reconsider the first step of ECP in Sec. II. In Fig. 3, the QND1 and QND2 are described in Fig. 2. The \(|\Phi\rangle_{a_1b_1c_1}\) and \(|\Phi\rangle_{a_2}\) coupled with the coherent state evolves as

\[
|\alpha\rangle \rightarrow |\alpha\rangle
+ \theta |X\rangle \langle X|
\]

\[
= |\Phi\rangle_{a_1b_1c_1} \otimes |\Phi\rangle_{a_2} = (|\alpha\rangle|V\rangle_{a_1}|H\rangle_{b_1}|H\rangle_{c_1}
+ \beta|H\rangle_{a_1}|V\rangle_{b_1}|H\rangle_{c_1}
+ \gamma|H\rangle_{a_1}|H\rangle_{b_1}|V\rangle_{c_1})
\otimes (|\alpha|\sqrt{\alpha^2 + \beta^2}|H\rangle_{a_2} + \beta|\alpha e^{i\theta}|V\rangle_{a_2}|\alpha\rangle)
\rightarrow \frac{\alpha^2}{\sqrt{\alpha^2 + \beta^2}}|H\rangle_{a_1}|H\rangle_{a_2}|H\rangle_{b_1}|H\rangle_{c_1}|\alpha\rangle
+ \frac{\beta^2}{\sqrt{\alpha^2 + \beta^2}}|H\rangle_{a_1}|V\rangle_{a_2}|V\rangle_{b_1}|H\rangle_{c_1}|\alpha e^{i\theta}\rangle
+ \frac{\alpha\gamma}{\sqrt{\alpha^2 + \beta^2}}|H\rangle_{a_1}|H\rangle_{a_2}|H\rangle_{b_1}|V\rangle_{c_1}|\alpha e^{2i\theta}\rangle
+ \frac{\beta\gamma}{\sqrt{\alpha^2 + \beta^2}}|H\rangle_{a_1}|V\rangle_{a_2}|H\rangle_{b_1}|V\rangle_{c_1}|\alpha e^{i\theta}\rangle
+ \frac{\alpha\beta}{\sqrt{\alpha^2 + \beta^2}}|H\rangle_{a_1}|H\rangle_{a_2}|V\rangle_{b_1}|H\rangle_{c_1}|\alpha e^{2i\theta}\rangle
+ \frac{\beta\beta}{\sqrt{\alpha^2 + \beta^2}}|H\rangle_{a_1}|V\rangle_{a_2}|V\rangle_{b_1}|H\rangle_{c_1}|\alpha e^{i\theta}\rangle.
\] (22)

From Eq. (22), after the two photons in spatial modes \(a_1\) and \(a_2\) passing through the QND1, if Alice chooses the \(\theta\) phase shift, the remaining state is the same as Eq. (20), with the same probability \(P_1\). Then following the same step described in Sec. II, if \(D_1\) fires, they will get \(|\Phi\rangle_{d_1b_1c_1}\); and if \(D_2\) fires, they will get \(|\Phi\rangle_{d_2b_1c_1}\).

The second step is analogy with the first one. They choose another single photon \(|\Phi\rangle_{c_2}\) and then rotate it by 90° with HWP\(_{90}\). The \(|\Phi\rangle_{d_1b_1c_1}\) and \(|\Phi\rangle_{c_3}\) combined with the coherent state evolves as

\[
|\Phi\rangle_{c_3} \otimes |\Phi\rangle_{d_2b_1c_1}\rightarrow \frac{\beta\gamma}{\sqrt{\gamma^2 + 2\beta^2}}|H\rangle_{d_1}|H\rangle_{b_1}|V\rangle_{c_1}|V\rangle_{c_2}|\alpha e^{i\theta}\rangle
+ \frac{\sqrt{\gamma^2 + 2\beta^2\gamma^2}}{|\beta\rangle_{d_1}|H\rangle_{b_1}|V\rangle_{c_1}|V\rangle_{c_2}|\alpha\rangle
+ \frac{\beta^2}{\sqrt{\gamma^2 + 2\beta^2}}|V\rangle_{d_1}|H\rangle_{b_1}|H\rangle_{c_1}|V\rangle_{c_2}|\alpha \langle\alpha\rangle
+ \frac{\beta^2}{\sqrt{\gamma^2 + 2\beta^2}}|V\rangle_{d_1}|V\rangle_{b_1}|H\rangle_{c_1}|V\rangle_{c_2}|\alpha e^{2i\theta}\rangle
+ \frac{\beta}{\sqrt{\gamma^2 + 2\beta^2}}|H\rangle_{d_1}|V\rangle_{b_1}|H\rangle_{c_1}|H\rangle_{c_2}|\alpha e^{i\theta}\rangle.
\] (23)

From Eq. (23), if Charlie picks up the \(\theta\) phase shift, the remaining state is essentially the four-photon maximally entangled W state. Thus, following the same way, Charlie measures his photon in mode \(c_2\) after rotating it by 45° with HWP\(_{45}\). Finally, if \(D_3\) fires, they will get the same state Eq. (18). Otherwise, if the detector \(D_4\) fires, they will obtain the same state in Eq. (19), with the same probability \(P_2\).
\[
\begin{align*}
\text{Compared with the first protocol, the function of the QND is also the parity check. Certainly, with the help of the QND, we do not need to measure the photon directly, and the concentrated photon pairs can be remained. During the whole process, Alice and Charlie both pick up the cases that the phase shift is } \theta \text{ and discard the other results. Interestingly, If a suitable cross-Kerr nonlinearity can be provided, and the interaction time } t \text{ can be well controlled, which leads to } \theta = \pi. \text{ In this way, phase shift } 2\theta = 2\pi \text{ and } 0 \text{ will not be distinguished. Therefore, the discarded items in each step by Alice and Charlie are also the nonmaximally entangled W state and can be reconcentrated in the next round. For instance, in Eq. (22), if the phase shift is not } \theta \text{ (} \pi \text{), but } 2\pi \text{ (} 0 \text{), that the whole state collapses to}
\end{align*}
\]
\[
\begin{align*}
|\Psi^\prime\rangle_{a1a2b1c1} &= \frac{\alpha^2}{\sqrt{\alpha^2 + \beta^2}} |V\rangle_{a1}|H\rangle_{a2}|H\rangle_{b1}|H\rangle_{c1} \\
&\quad + \frac{\beta^2}{\sqrt{\alpha^2 + \beta^2}} |H\rangle_{a1}|V\rangle_{a2}|V\rangle_{b1}|H\rangle_{c1} \\
&\quad + \frac{\beta\gamma}{\sqrt{\alpha^2 + \beta^2}} |H\rangle_{a1}|V\rangle_{a2}|H\rangle_{b1}|V\rangle_{c1}.
\end{align*}
\]
\[
\text{(24)}
\]
\[
\text{By measuring the photon in the mode } a2 \text{ after rotating it by } 45^\circ, \text{ they will get another lesser-entangled state of the form}
\]
\[
\begin{align*}
|\Psi^\prime\prime\rangle_{a1a2b1c1} &= \alpha'|V\rangle_{a1}|H\rangle_{b1}|H\rangle_{c1} \pm \beta'|H\rangle_{a1}|V\rangle_{b1}|H\rangle_{c1} \\
&\quad \pm \gamma'|H\rangle_{a1}|H\rangle_{b1}|V\rangle_{c1},
\end{align*}
\]
\[
\text{(25)}
\]
\[
\text{with}
\]
\[
\begin{align*}
\alpha' &= \frac{\alpha^4}{\sqrt{\alpha^4 + \beta^4 + \beta^2\gamma^2}}, \\
\beta' &= \frac{\beta^4}{\sqrt{\alpha^4 + \beta^4 + \beta^2\gamma^2}}, \\
\gamma' &= \frac{\beta^2\gamma^2}{\sqrt{\alpha^4 + \beta^4 + \beta^2\gamma^2}}.
\end{align*}
\]
\[
\text{(26)}
\]
\[
\text{‘+’ or ‘−’ depends on the measurement results. If } D_1 \text{ fires, it is ‘+’, otherwise, it is ‘−’.}
\]
\[
\text{In the second step, if the phase shift in Charlie’s location is not } \theta \text{ yet, then the Eq. (24) becomes}
\]
\[
\begin{align*}
|\Psi^\prime\prime\prime\rangle_{d1b1c1c2} &= \frac{\gamma^2}{\sqrt{\gamma^2 + 2\beta^2\sqrt{\gamma^2 + \beta^2}}} |H\rangle_{d1}|H\rangle_{b1}|V\rangle_{c1}|H\rangle_{c2} \\
&\quad + \frac{\beta^2}{\sqrt{\gamma^2 + 2\beta^2\sqrt{\gamma^2 + \beta^2}}} |V\rangle_{d1}|H\rangle_{b1}|H\rangle_{c1}|V\rangle_{c2} \\
&\quad + \frac{\beta\gamma}{\sqrt{\gamma^2 + 2\beta^2\sqrt{\gamma^2 + \beta^2}}} |H\rangle_{d1}|V\rangle_{b1}|H\rangle_{c1}|V\rangle_{c2}.
\end{align*}
\]
\[
\text{(27)}
\]
\[
\text{By measuring the photon in mode } c2 \text{ after rotating it by } 45^\circ, \text{ it becomes}
\]
\[
|\Psi^\prime\prime\prime\prime\rangle_{d1b1c1} = \gamma''|H\rangle_{d1}|H\rangle_{b1}|V\rangle_{c1} \pm \beta''|V\rangle_{d1}|H\rangle_{b1}|H\rangle_{c1} \\
&\quad \pm \beta''|H\rangle_{d1}|V\rangle_{b1}|H\rangle_{c1}.
\]
\[
\text{(28)}
\]
\[
\text{with}
\]
\[
\begin{align*}
\gamma'' &= \frac{\gamma^2}{\sqrt{\gamma^4 + 2\beta^4}}, \\
\beta'' &= \frac{\beta^2}{\sqrt{\gamma^4 + 2\beta^4}}.
\end{align*}
\]
\[
\text{(29)}
\]
\[
\text{‘+’ or ‘−’ also depends on the measurement result. If } D_2 \text{ fires, it is ‘+’, otherwise, it is ‘−’}
\]
\[
\text{Compared with Eqs. (2) and (10), it is obvious to see that Eqs. (25) and (26) have the same form with Eqs. (2) and (10). That is to say, the states of Eqs. (25) and (26) can be reconcentrated to get a maximally entangled W state in the next round. We take } |\Psi^\prime\rangle_{d1b1c1} \text{ as an example. In detail, Charlie chooses another single photon of the form}
\]
\[
|\Phi\rangle_{c2} = \frac{\beta^2}{\sqrt{\gamma^4 + \beta^4}} |H\rangle_{c2} + \frac{\gamma^2}{\sqrt{\gamma^4 + \beta^4}} |V\rangle_{c2}.
\]
\[
\text{(30)}
\]
\[
\text{After rotating this photon by } 90^\circ, \text{ it becomes}
\]
\[
|\Phi\rangle_{c3} = \frac{\beta^2}{\sqrt{\gamma^4 + \beta^4}} |V\rangle_{c3} + \frac{\gamma^2}{\sqrt{\gamma^4 + \beta^4}} |H\rangle_{c3}.
\]
\[
\text{(31)}
\]
\[
\text{Therefore, states } |\Psi^\prime\rangle_{d1b1c1} \text{ and } |\Phi\rangle_{c3} \text{ combined with the coherent state } |\alpha\rangle \text{ evolves as}
\]
\[
\begin{align*}
|\Phi\rangle_{c3} \otimes |\Psi^\prime\rangle_{d1b1c1}|\alpha\rangle &= \frac{\beta^2\gamma^2}{\sqrt{\gamma^4 + 2\beta^4\sqrt{\gamma^4 + \beta^4}}} |H\rangle_{d1}|H\rangle_{b1}|V\rangle_{c1}|V\rangle_{c2}|\alpha e^{i\theta}\rangle \\
&\quad + \frac{\gamma^4}{\sqrt{\gamma^4 + 2\beta^4\sqrt{\gamma^4 + \beta^4}}} |H\rangle_{d1}|H\rangle_{b1}|V\rangle_{c1}|H\rangle_{c2}|\alpha\rangle \\
&\quad + \frac{\beta^4}{\sqrt{\gamma^4 + 2\beta^4\sqrt{\gamma^4 + \beta^4}}} |V\rangle_{d1}|H\rangle_{b1}|H\rangle_{c1}|V\rangle_{c2}|\alpha e^{i\theta}\rangle \\
&\quad + \frac{\beta^2\gamma^2}{\sqrt{\gamma^4 + 2\beta^4\sqrt{\gamma^4 + \beta^4}}} |V\rangle_{d1}|V\rangle_{b1}|H\rangle_{c1}|H\rangle_{c2}|\alpha e^{i2\theta}\rangle \\
&\quad + \frac{\beta^4}{\sqrt{\gamma^4 + 2\beta^4\sqrt{\gamma^4 + \beta^4}}} |H\rangle_{d1}|V\rangle_{b1}|H\rangle_{c1}|V\rangle_{c2}|\alpha e^{i2\theta}\rangle \\
&\quad + \frac{\beta^2\gamma^2}{\sqrt{\gamma^4 + 2\beta^4\sqrt{\gamma^4 + \beta^4}}} |H\rangle_{d1}|V\rangle_{b1}|V\rangle_{c1}|H\rangle_{c2}|\alpha e^{i\theta}\rangle.
\end{align*}
\]
\[
\text{(32)}
\]
\[
\text{After the photons in the spatial modes } c1 \text{ and } c3 \text{ passing through the QND2, if the homodyne measurement of the coherent state is } \theta, \text{ Eq. (32) will also collapse to the maximally entangled W state with the same form of Eq. (18) or (19), after measuring the photon in the mode } c2. \text{ The success probability } P_3^2 \text{ of obtaining the states (18) and (19) includes two terms. Here the subscription "n" means the second concentration round. The first term is the probability of the first round to get the } 0 (2\pi) \text{ phase shift. From Eq. (27), it equals to}
\]
\[
\frac{\gamma^4 + 2\beta^4}{(\gamma^2 + \beta^4)(\gamma^4 + 2\beta^4)}.\]
\[
\text{The second term is the success probability to get the } \theta \text{ phase shift}.
shift in the second round. From Eq. (32), it equals to \(\frac{3\beta^4}{(\gamma^2 + 2\beta^2)(\gamma^2 + \beta^2)}\). Therefore, the whole success probability is

\[
P_2^2 = \frac{1}{(\gamma^2 + 2\beta^2)(\gamma^2 + \beta^2)} \left(\frac{3\beta^4}{\gamma^4 + 2\beta^4}\right)
\]

\[
= \frac{3\beta^4}{(\gamma^2 + 2\beta^2)(\gamma^2 + \beta^2)}. \tag{33}
\]

On the other hand, in the second concentration round, if the phase shift is not \(\theta\), but \(0 (2\pi)\), after measuring the photon in the mode \(e2\) by rotating \(45^\circ\), the remaining state is

\[
\sqrt{\frac{\gamma^4}{\gamma^4 + 2\beta^4}}|H\rangle_{d1}|H\rangle_{b1}|V\rangle_{c1}|H\rangle_{e2}
\]

\[
\pm \frac{\beta^4}{\sqrt{\gamma^4 + 2\beta^4}}|H\rangle_{d1}|V\rangle_{b1}|H\rangle_{c1}|V\rangle_{e2}
\]

\[
\pm \frac{\beta^4}{\sqrt{\gamma^4 + 2\beta^4}}|H\rangle_{d1}|V\rangle_{b1}|H\rangle_{c1}|V\rangle_{e2}. \tag{34}
\]

Compared with Eq. (28), state of Eq. (34) can also be recombined in a third round. In this way, this protocol can be repeated for \(N\) \((N \rightarrow \infty)\) times in principle, if each round can not get the \(\theta\) phase shift.

Through the above description, if they choose the \(\theta = \pi\) phase shift, the discarded items in Sec. II can be reused to get a higher success probability. However, the natural cross-Kerr nonlinearity is extremely small \(65, 66\). It is hard to reach \(\theta = \pi\). Moreover, using longer interaction time will induce decoherence from losses. It will make the output state become a mixed state. A practical alternative way is to use the coherent state rotation. We take Eq. (22) as an example. If they obtain the state Eq. (22), Alice rotates the coherent state by \(\theta\), then Eq. (22) becomes

\[
\rightarrow \frac{\alpha^2}{\sqrt{\alpha^4 + 2\beta^4}}|V\rangle_{a1}|H\rangle_{a2}|H\rangle_{b1}|H\rangle_{c1}|e^{-i\theta}\rangle
\]

\[
+ \frac{\beta^2}{\sqrt{\alpha^4 + 2\beta^4}}|H\rangle_{a1}|V\rangle_{a2}|V\rangle_{b1}|H\rangle_{c1}|e^{i\theta}\rangle
\]

\[
+ \frac{\alpha\gamma}{\sqrt{\alpha^4 + 2\beta^4}}|H\rangle_{a1}|H\rangle_{a2}|H\rangle_{b1}|V\rangle_{c1}|\alpha\rangle
\]

\[
+ \frac{\beta\gamma}{\sqrt{\alpha^4 + 2\beta^4}}|H\rangle_{a1}|V\rangle_{a2}|H\rangle_{b1}|V\rangle_{c1}|\alpha\rangle
\]

\[
+ \frac{\alpha\beta}{\sqrt{\alpha^4 + 2\beta^4}}|V\rangle_{a1}|V\rangle_{a2}|H\rangle_{b1}|H\rangle_{c1}|\alpha\rangle
\]

\[
+ \frac{\alpha\beta}{\sqrt{\alpha^4 + 2\beta^4}}|H\rangle_{a1}|H\rangle_{a2}|V\rangle_{b1}|H\rangle_{c1}|\alpha\rangle. \tag{35}
\]

From Eq. (35), if there is no phase shift, the remaining state is the same as Eq. (5), with the same probability of \(P_1\). Then following the same step described in Sec.

II, if \(D_1\) fires, they will get \(|\Phi_1\rangle_{d1b1c1}\), and if \(D_2\) fires, they will get \(|\Phi_2\rangle_{d1b1c1}\). Otherwise, they use the \(|X\rangle\langle X|\) homodyne detection \(54, 55\) on the coherent state \(56, 59\), which can make the \(|ae^{i\theta}\rangle\) and \(|ae^{-i\theta}\rangle\) can not be distinguished. Then the remaining state is the same as Eq. (21). In this way, following the same step, it can be recombined to get a higher success probability. The way of coherent state rotation can also be suitable for the second step in our the second protocol.

IV. DISCUSSION AND SUMMARY

By far, we have fully explained our ECPs both with linear optics and cross-Kerr nonlinearity. In order to explain ECPs clearly, for each ECP we divide it into two steps. The first step is operated by Alice, and the second one is operated by Charlie. Bob needs only to retain or discard his photons according to the measurement results from Alice and Charlie by classical communication. In the PBS protocol, the two steps should be performed simultaneously because of the post-selection principle. In the QND protocol, the QND provides us a powerful tool to make quantum nondemolition measurement which does not destroy the photons. This advantage makes each step can be operated independently. Moreover, with QNDs, if they choose the \(\theta = \pi\), or use the coherent state rotation, both steps can be iterated to get a higher success probability. Now let us calculate the success probability in each iteration rounds.

In the first step, we calculate the success probability in each iterated round as

\[
P_1^1 = \frac{\alpha^2(\gamma^2 + 2\beta^2)}{\alpha^2 + \beta^2},
\]

\[
P_2^1 = \frac{\alpha^4(\beta^2\gamma^2 + 2\beta^4)}{(\alpha^4 + \beta^4)(\alpha^2 + \beta^2)},
\]

\[
P_3^1 = \frac{\alpha^8(\beta^6\gamma^2 + 2\beta^8)}{(\alpha^8 + \beta^8)(\alpha^4 + \beta^4)(\alpha^2 + \beta^2)}.
\]

\[
\cdots
\]

\[
P_N^1 = \frac{\alpha^{2N}(\beta^{2N-2}\gamma^2 + 2\beta^{2N})}{(\alpha^{2N} + \beta^{2N})(\alpha^{2N-1} + \beta^{2N-1}) \cdots (\alpha^2 + \beta^2)}. \tag{36}
\]

Here the superscription "1" means the first step. The subscript "1", "2", "3", ..., "N" is the iteration number.

Following the same principle, in the second step, the success probability in each iterated round is

\[
P_1^2 = \frac{3\beta^2\gamma^2}{\gamma^2 + \beta^2},
\]

\[
P_2^2 = \frac{3\beta^4\gamma^4}{(\gamma^2 + 2\beta^2)(\gamma^4 + \beta^4)},
\]

\[
P_3^2 = \frac{3\beta^8\gamma^8}{(\gamma^2 + 2\beta^2)(\gamma^4 + \beta^4)(\gamma^2 + \beta^2)}.
\]
is successful in the
a failure, then they repeat it again until it is successful.

Here, we choose \( \beta = \frac{1}{\sqrt{2}} \), \( \alpha \in (0, \sqrt{2}) \). Curve A is the QND protocol, and Curve B is the PBS protocol. For numerical simulation, we choose \( N = M = 3 \) for approximation.

\[
P^2_M = \frac{3 \beta^M \gamma^2 M \cdots (\gamma^2 + \beta^2)}{\gamma^2 + 2 \beta^2}
\]

Here the superscript "2" means the second step. The subscript "1", "2", "3", \( \cdots ")Mn" is also the iteration number.

With the QNDs, if the initial state is Eq. 2, the whole concentration procedure can be described as follows: They first perform the first step first time. If it is a failure, then they repeat it again until it is successful. Then they go to the second step with the same iteration principle. Interestingly, in the first step, suppose it is successful in the \( K \) \((K = 1, 2, \cdots N)\) iteration, they always obtain Eq. 11, that is the initial state of the second step. Therefore, by repeating both steps, the total success probability is

\[
P_{\text{total}} = P_1^1 (P_1^2 + P_2^2 + \cdots + P_M^2) + P_1^2 (P_1^2 + P_2^2 + \cdots + P_M^2) + \cdots + P_N^1 (P_1^2 + P_2^2 + \cdots + P_M^2)
\]

\[
= \sum_{N=1}^{\infty} P_N^1 \sum_{M=1}^{\infty} P_M^2.
\]

From Eq. 15, the success probability of the PBS protocol is essentially the first term of Eq. 18, that is the case of \( N = M = 1 \). Interestingly, if \( \alpha = \beta = \gamma = \frac{1}{\sqrt{2}} \), the success probability of the PBS protocol is \( P = P_1^1 P_1^2 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \). In this way for the QND protocol,

\[
P_{\text{total}} = \sum_{N=1}^{\infty} P_N^1 \sum_{M=1}^{\infty} P_M^2
\]

We calculate the total success probability of both the PBS and the QND protocol, shown in Fig. 4. We choose \( \beta = \frac{1}{\sqrt{2}} \), and change \( \alpha \in (0, \sqrt{2}) \). For the QND protocol, we choose \( N = M = 3 \) for a good numerical simulation. In Fig. 4, it is shown that both success probability monotonic increase with \( \alpha \), when \( \alpha \in (0, \sqrt{2}) \). They both have a maximally value when \( \alpha = \frac{1}{\sqrt{2}} \).

From Fig. 4, with QNDs, one can get a higher success probability. In this way, they should exploit the cross-Kerr nonlinearity to generate \( \pi \) phase shift on the coherent state, or use the coherent state rotation. However, the largest natural cross-Kerr nonlinearities are extremely weak \( (\chi^{(3)} \approx 10^{-22} m^2 V^{-2}) \). The Kerr phase shift when operating in the optical single-photon regime is about \( \tau \approx 10^{-18} \). With electromagnetically induced transparent materials, it is much larger and can reach \( \tau \approx 10^{-9} \). On the other hand, using cross-Kerr nonlinearity to implement the quantum information processing is still a controversial topic \( [57, 67] \). In 2003, Hofmann et al. pointed out that a \( \pi \) phase shift can be reached with a single two-level atom in a one-side cavity \( [63] \). In Ref. 69, Shapiro argues that the single-photon Kerr nonlinearities do not help quantum computation. Recently, He et al. discussed the cross-Kerr nonlinearity between continuous-mode coherent state and single photons. They believed that their work constitutes significant progress in making the treatment of coherent state and single photon interactions more realistic \( [57] \). Feizpour et al. also showed that it is possible to amplify a cross-Kerr phase shift to an observable value, which is much larger than the intrinsic magnitude of the single-photon-level nonlinearity, with the help of weak measurement \( [71] \). Giant cross-Kerr nonlinearities was also obtained with nearly vanishing optical absorption, investigating the linear and nonlinear interactions of probe and signal pulses which is coupled in a double quantum-well structure with a four-level, double A-type configuration by Zhu and Huang \( [72] \).

In summary, we have presented two ECPs for concentrating arbitrary W states. We exploit both the linear optic element PBS and the nonlinear optics cross-Kerr nonlinearity to achieve the whole task. Compared with other concentration protocols, these protocols do not require the collective measurement. Moreover, they do not need two same copies of less entangled pairs to perform the protocol, which make them be more economical. In the PBS protocol, based on the post-selection principle, one can obtain the maximally entangled W state with certain probability. In the QND protocol, we adopt the QNDs to substitute the PBSs and make the protocol become more powerful. First, the parties can operate the protocol independently. Second, it does not require the sophisticated single-photon detectors. Third, by iterat-
ing this protocol, one can reach a higher success probability. All these features maybe make these two protocols more useful in practical applications.

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