Di-Jet Conical Correlations Associated with Heavy Quark Jets in anti-de Sitter Space/Conformal Field Theory Correspondence

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We show that far zone Mach and diffusion wake “holograms” produced by supersonic strings in anti-de Sitter space/conformal field theory (AdS/CFT) correspondence do not lead to observable conical angular correlations in the strict $N_c \to \infty$ supergravity limit if Cooper-Frye hadronization is assumed. However, a special nonequilibrium “neck” zone near the jet is shown to produce an apparent sonic boom azimuthal angle distribution that is roughly independent of the heavy quark’s velocity. Our results indicate that a measurement of the dependence of the away-side correlations on the velocity of associated identified heavy quark jets at the BNL Relativistic Heavy Ion Collider and CERN LHC will provide a direct test of the nonperturbative dynamics involved in the coupling between jets and the strongly-coupled Quark-Gluon Plasma (sQGP) implied by AdS/CFT correspondence.

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The reported observation of Mach cone-like correlations between tagged (near side) jets fragments and away-side associated moderate $p_T < 4$ GeV/c hadrons [1] has generated interest because it may provide further evidence [2, 3] for fast relaxation times and the near perfect fluid property of the sQGP produced in Au+Au reactions at RHIC [4]. While perturbative quantum chromodynamics (pQCD) correctly predicted the differential suppression of light quark and gluon jets [5], recent data on non-photonic single electrons (from heavy quark jet fragment decay) [6] have challenged pQCD interpretations of jet quenching phenomena [7]. The combined observations of high suppression of heavy quark jets, large elliptic anisotropy of both light and heavy quark jet fragments, and the Mach-like away-side correlations have motivated novel approaches to try to explain these phenomena in terms of the Anti-de Sitter/Conformal Field Theory (AdS/CFT) correspondence [8]. We shall show below that AdS/CFT models do predict novel phenomena especially in the heavy quark jet sector where future decisive experiments can be used to verify this new geometric modeling of strongly-coupled gauge theory physics.

The classical Nambu-Goto string drag solutions in an $AdS_5$ black brane background were proposed (see [9]-[12], [10]-[12], and [19] and refs. therein) as a detailed dynamical holographic description of the possible response of a strongly coupled 3+1D Supersymmetric Yang Mills (SYM) plasma (an analog of sQGP) to the energy loss of a heavy quark jet modeled by the endpoint of a supersonic string. Even though supergravity ($N_c \to \infty$ and $\lambda = g_{YM}^2 N_c \to \infty$) descriptions do not lead to true pQCD-like jet structures [20], the $AdS_5$ black brane string calculus provides a rare solvable example of how at least one strongly coupled gauge system could react to both subsonic and supersonic disturbances. In particular, these solutions feature both the familiar and universal far zone near equilibrium Mach sonic boom and diffusion wave collective flow phenomena shown in Fig. 1 as well as particular model dependent “Neck” and “Head” zones corresponding to the non-equilibrium dynamics involving coupled non-Abelian fields and hydrodynamic effects. Associated Mach-like conical angular correlations in nuclear collisions have been mainly studied in the past assuming near equilibrium hydrodynamic models [2, 3] and also pQCD jet energy loss models [21]. In this letter...
we study the different sources of associated correlations taking into account the Mach wake and diffusion zones shown in the insert of Fig. 1. General arguments based on the large $N_c$ limit are used to demonstrate that the yield from the hydrodynamic zone does not produce a conical pattern. The roughly velocity-independent, strong conical flow from the neck region is a true prediction of the AdS/CFT string drag model that can be verified in the near future at RHIC and LHC.

In the AdS/CFT trailing string scenario, the string disturbs the bulk 5-dimensional metric from which $T^\mu\nu(x)$ can be computed on the holographic Minkowski boundary of $AdS_5$ [10]. Because the heavy quark string is assumed to move at a constant speed, $v$, through the plasma there is an external source of stress, $\partial_\nu T^\mu\nu = J^\nu$ with $J^\mu(x) = J \delta(z - vt) \delta^3(\vec{x})(v,1,0,1)$ and $J = -dP/dt = \frac{\Gamma}{3} \sqrt{\lambda T_0^2} v^2/(1 - v^2)$ [10], where $T_0$ is the temperature of the undisturbed background plasma. Beyond a certain “speed limit” $\left| p_T(max) \right| \sim 4M_Q^2/(\lambda T_0^2)$, where $M_Q$ is the heavy quark mass, the classical string drag picture breaks down due to the appearance of a hor- zon but at LHC there could be a sizable window $p_T \approx 30$ GeV where it could be applied [22].

In the supergravity $N_c \to \infty$ limit, the disturbances in the energy-momentum tensor $\delta T^\mu\nu$ caused by the moving string are of order $\sqrt{\lambda/N_c^2}$ with respect to the background plasma energy density, $\varepsilon_{SYM} = K T_0^4$, with $K \propto N_c^2$ [23]. Therefore, the disturbances are arbitrarily small in this limit because $\sqrt{\lambda/N_c^2} \to 0$. An immediate consequence of this is that linearized first-order Navier-Stokes hydrodynamics is predicted to be a very good description of the quark’s holographic wake down to short distance scales away from the heavy quark [12, 18, 24]. In general, the flow velocity $U^\mu(X) = \left(\sqrt{1 + U^2}, \vec{U}\right)$ can be obtained from $T^\mu\nu(X)$ by boosting the system to its local Landau rest frame. We emphasize that the Coulomb stress contribution has been subtracted as in [12]. When $N_c \to \infty$ the flow velocity reduces to $U^\mu = T^\mu_0/(4P_0)$, where $P_0 = \varepsilon_{SYM}/3 \propto N_c^2/\lambda$ is the background $N_c \sim 4$ SYM plasma pressure [18, 24]. Since $T^\mu_0$ is only proportional to $\sqrt{\lambda}$, we see that $U \propto O(\sqrt{\lambda}/N_c^2)$ is also tiny in the strict supergravity limit. Within this approximation, the local temperature is $T(X) = [8 T_0^{00}(X)/3\pi^2(N_c^2 - 1)]^{1/4}$. Thus, the local temperature fluctuates as $T(X) = T_0 + \Delta T(X)$ with $\Delta T(X)/T_0 = O(\sqrt{\lambda}/N_c^2)$. On the other hand, in the Head and Neck zones the stress grows rapidly and the stress perturbation becomes comparable to the background. Since the Head zone is defined by a large amplitude Lorentz contracted Coulomb field [10], the near perfect fluid hydrodynamic approximation must break down close to the quark. In Ref. [24] the analytic near field Yarom solution, $T_1^{\mu\nu}(X)$ [11], was used to compute a “Knudsen” boundary defined by the closed surface inside which the effective Knudsen number $Kn_1 \equiv \Gamma_s |\vec{\nabla} \cdot \vec{S}_\nu|/|\vec{S}_\nu| = 1$. Here $\Gamma_s = 1/(3\pi T_0)$ is the sound attenuation length and $S_\nu = -T_{\nu i}^Y$ is the Yarom momentum flux. Inside this boundary local equilibrium cannot be maintained even by uncertainty principle limited thermalization rates [25, 26].

For $v = 0.99$, we found $Kn \geq 1$ in a region $|x_1| \equiv x - vt < 2/\pi T_0$ and $x_1$ extending into the far zone studies in [18]. Parametrically, the Knudsen zone is defined by

\[ T_{Kn}^{\mu\nu} \equiv \theta(Kn(x) - 1) T_1^{\mu\nu}(x) \sim \sqrt{\lambda T_0^2} \zeta^{\mu\nu} \frac{\gamma^2}{x_1^2 + \gamma^2 x_1^2} \]  

where $\gamma^{-2} = 1 - v^2$ and $\zeta^{\mu\nu}(x)$ is a dimensionless angular function inside the boundary. Even Navier-Stokes viscous hydrodynamics is inapplicable in this zone. Because $T_{Kn}^{\mu\nu} \sim \sqrt{\lambda T_0^2}/|x|^2$ we see that this zone can be considered as a dynamic interference “Neck” zone between the near equilibrium $\sim T_0^4$ plasma stress dominated far zone and a near Head zone dominated by the stress of the Lorentz boosted $\sim \sqrt{\lambda}/|x|^4$ Coulomb field of the heavy quark. The AdS/CFT analogs of electro-magneto-hydrodynamic effects play the dominant role in this Neck zone. Within the Neck zone, there is also an inner core Head zone where the stress becomes dominated by the external Coulomb field of the quark. The Head zone can therefore be defined as in Ref. [21] by equating the analytic Coulomb energy density $\varepsilon_C(x_1, x_\perp)$, to the analytic near zone Yarom energy density $\varepsilon_{YM}(x_1, x_\perp)$, $\sim \sqrt{\lambda}/N_c^2$. This Coulomb head boundary is approximately given by

\[ x_1^2 + \gamma^2 x_1^2 = \frac{1}{(\pi T_0)^2} \frac{1}{\gamma^2 x_1^2 + 2 + \gamma^2 x_1^2}. \]  

The Head zone is thus a Lorentz contracted surface with a maximal longitudinal thickness near $\Delta x_\perp \pi T_0 \sim 1/\gamma^{3/2}$ and $\Delta x_1, C \pi T_0 \sim 1/\gamma^{3/2}$ in agreement with Ref. [27]. The numerical results found in [24] indicated that for $v = 0.99$ jets, the nonequilibrium dynamic screening Knudsen neck zone is much thicker than the contracted Head zone, in agreement with the parametric dependence $\Delta x_1 \Gamma_s \sim 2/(\pi T_0) \sim 6\Gamma_s$, expected from uncertainty principle bounded dissipation rates [25, 26].

The weakest link between any strongly coupled description and experimental data is the freeze-out: the need to convert the strongly interacting fluid into weakly interacting (and eventually free) particles. While the dynamics of how this happens in nature is very poorly understood from first principles, it should be noted that, as long as we use momentum observables and the hadronic rescattering is reasonably short, momentum conservation at freeze-out should limit the effect it can have on early time observables. The usual ansatz used is the Cooper-Frye (CF) formula, where the conversion of the fluid into free particles is achieved instantaneously at a critical surface $\Sigma_x$. Assuming such a freeze-out scheme [3, 29],
we can obtain particle distributions and correlations from the flow velocity field $U^\mu(X)$ and temperature $T(X)$.

For associated (massless) particles with $P^\mu = (p_T, p_T \cos(\pi - \phi), p_T \sin(\pi - \phi), 0)$ the momentum distribution at mid rapidity $y = 0$ is

$$\left. \frac{dN}{dp_T dp_T dy d\phi} \right|_{y=0} = \int_{\Sigma_T} d\Sigma_\mu P^\mu \left[ f(U^\mu, P^\mu, T) - f_{eq} \right]$$

(3)

where $p_T$ is the transverse momentum, $\Sigma(X)$ is the freeze-out hypersurface, and $f_0 = \exp(-U^\mu P^\mu/T(X))$ is a local Boltzmann equilibrium distribution that is approximately valid in the near equilibrium “far” zone $\Sigma_T = V\theta(1-Kn)$. We subtract the isotropic SYM background yield via $f_{eq} \equiv f|_{U^\mu=0,T=T_0}$. Viscous corrections to the Boltzmann distribution function \[30\] produce subleading contributions in $1/N_c$ that are negligible in the supergravity limit. Since the medium is static and infinite we use the isochronous ansatz $d\Sigma^\mu = x_\perp dx_\perp dx_\phi (1, 0, 0, 0)$. More realistic prescriptions such as the isothermal freeze-out are more adequate and applicable when considering expanding media.

In the large $N_c \to \infty$ limit, $\Delta T \ll T_0$ and $\vec{p} \cdot \vec{U} \ll T_0$ as noted before and the Boltzmann exponent can be expanded up to corrections $O(\lambda/N_c^4)$. There is axial symmetry with respect to the trigger jet axis defined here as $n^\mu \equiv (0, 1, 0, 0)$, where the “$x_1 = z - vt$” axis of the away-side jet corresponds to $-n^\mu$ and the nuclear beam axis corresponds to the “$y$” axis. In these coordinates, $U^\mu(x_1, x_\perp) = (U^0, U_1, U_\perp \sin \varphi, U_\perp \cos \varphi)$. The associated away-side azimuthal distribution at mid-rapidity $f(\phi) = dN/p_T dp_T dy d\phi|_{y=0}$ with respect to the beam axis is then given after integrating over $\varphi$ by

$$f(\phi) = 2\pi p_T \int_{\Sigma_T} dx_1 dx_\perp x_\perp \times$$

$$\left( \exp \left\{ \frac{-p_T}{T} [U_0 - U_1 \cos(\pi - \phi)] \right\} I_0(a_\perp) - e^{-p_T/T_0} \right)$$

where $a_\perp = p_\perp U_\perp \sin(\pi - \phi)/T$ and $I_0$ is the modified Bessel function. In the supergravity approximation $a_\perp \sim O(\sqrt{x_T}) \ll 1$ and, thus, we can expand the Bessel function $\lim_{x \to 0} I_0(x) = 1 + \frac{x^2}{4} + O(x^4)$ to get the approximate equation for the distribution

$$f(\phi) \approx e^{-p_T/T_0} \frac{2\pi p_T^2}{T_0} \left[ \langle \Delta T \rangle/T_0 + \langle U_1 \rangle \cos(\pi - \phi) \right]$$

(5)

where deviations from isotropy are then controlled by the following global moments $\langle \Delta T \rangle = \int_{\Sigma_T} dx_1 dx_\perp x_\perp \Delta T$ and $\langle U_1 \rangle = \int_{\Sigma_T} dx_1 dx_\perp x_\perp U_1$.

It is clear that in the strict $N_c \gg 1$ limit the azimuthal distribution only has a trivial broad peak at $\phi = \pi$. A double-peaked structure in the away-side of the jet correlation function can only arise from the Mach zone when $N_c$ is reduced so that $I_0$ in Eq. (5) deviates from unity (see Ref. [3] where it was shown that the angular correlations associated with very soft particles do not have a conical structure within linearized hydrodynamics).

When $N_c = 3$ the simplifications due to the supergravity approximation are not strictly valid but it is of interest to use the numerical solutions for $T^{0\perp}$ computed by Gubser, Pufu, and Yarom (GPY) \[13\] to check how large the induced stress can be for “realistic” parameters. As seen in Fig. 1, even for $\lambda = 5.5$, $N_c = 3$ the deviations remain less than 10% outside the Neck region. We chose our CF volume to be defined by the forward light-cone that begins at $-14/(\pi T_0) < x_1 < 2/(\pi T_0)$ and $x_\perp < 14/(\pi T_0)$, which corresponds to a cylinder of roughly $L \sim R \sim 5$ fm at $T_0 = 0.2$ GeV. Note that $T(X)$ is not well defined in a small region within the Head zone. In practice, to avoid this issue we took $x_1 \pi T_0 > 0.2$ when computing the hadronic yield. Note, however, that we do not consider the Head yield in the following. We take $p_T = 4 - 7 \pi T_0 \sim 2.5 - 4.4$ GeV for the associated hadron as a typical momentum of interest at RHIC. The Neck zone was defined by the condition $\Delta T/\varepsilon_{SYM} > 0.3$ corresponding to roughly the small square in Fig. 1 [31]. The far zone minus the Neck zone gave rise to the blue azimuthal distributions in Fig. 2. There, three jet velocities, $v = 0.9, 0.75, 0.58$ are shown where $p_T = 4-5, 5-6, 6-7 \pi T_0$, respectively. In all cases the far zone distribution peaks at $\phi = \pi$ even for these more “realistic” parameters. We have also checked that no Mach dip appears in that region even for much higher $p_T \sim 20 \pi T_0$. An

![FIG. 2: (Color online) Mid-rapidity azimuthal away-side associated angular distribution from the Cooper-Frye freeze-out of the $(T(x), U(x))$ fields extracted from $\vec{U}$, based on the AdS/CFT string drag model. Three cases for various heavy quark jet velocity and associated hadron transverse momentum ranges, 1 : $v/c = 0.9, p_T/\pi T_0 = 4-5$, 2 : $v/c = 0.75, p_T/\pi T_0 = 5-6$, and 3 : $v/c = 0.58, p_T/\pi T_0 = 6-7$, are compared. Note the scale factors in the plots. The short arrows show the expected Mach angles. The yields from the Neck region (solid red), Mach and diffusion zones (dotted blue), and the sum from all contributions (dashed black) are shown in this plot.](image-url)
plication of CF freeze-out without linearization (i.e. Eq. (4)) to the Neck zone leads to red curves that exhibit a clear double shoulder structure remarkably close to the expected Mach angles for $v = 0.9$. The total dashed black distribution could be easily misinterpreted in this case as a signature for Mach cone emission. However, this angular structure is shown to be almost independent on the jet velocity. We have verified that curves from the Neck region also follow if the numerical GPY solution is replaced by the analytic Yarom solution in the Neck zone. While CF may not be a reliable hadronization scheme for the Neck zone, it correctly shows the presence of the strong transverse Poynting vector flow field seen in the insert of Fig. 1. The conservation of energy flux implies that this correlation may be robust to more general hadronization schemes as we will show elsewhere.

In general, the underlying flow of the medium is expected to modify the effective Mach cone angle $\alpha_{\text{eff}}$ [29, 32]. The preliminary experimental finding [33] that the location of the experimental peaks does not change with either centrality or the orientation with respect to the reaction plane casts doubt that the origin of the measured associated conical correlations are due to ordinary Mach cones. A confirmation of these results may give support to the new source of conical correlations found in this paper for associated heavy quark jets.

In conclusion, while AdS/CFT string solutions feature Mach wakes in coordinate space, their signal is too weak in supergravity to produce observable double shoulder correlations. However, the nonequilibrium transverse flux from the Neck zone could imitate Mach cone-like correlations, without, however, the dependence of the angle on the jet velocity expected from Mach’s law. The new results for azimuthal correlations from the near and far zones presented above are generic to AdS/CFT based modeling of heavy quark jets. We propose that a measurement of the jet velocity dependence of the away-side distributions associated with future tagged heavy quark jets at RHIC and LHC can be used to look for the novel source of conical correlations discussed in this letter.

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