Integrated Optimal Design of Speed Profile and Fuzzy PID Controller for Train With Multifactor Consideration

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This work was supported in part by the Fundamental Research Funds for the Central Universities under Grant 2017YJS167, and in part by the Research Funds of the Tianjin Jinhang Computing Technology Research Institute under Grant B17L00010.

ABSTRACT Speed profile design and tracking control are two important issues to ensure the performance of the automatic train operation system. Aiming to realize the maximum optimization probability of train running, this paper constructs an integrated optimal method through speed profile design and fuzzy PID controller design. In the recommended speed profile design, three typical running strategies are included in the proposed comprehensive scheme to extend solution space, and the constructed train performance simulation is combined with NSGA-II algorithm to solve such multiobjective problem. As for recommended speed profile tracking, in order to improve the tracking performance of the train, which is a nonlinear system, a fuzzy PID controller is designed to adaptively adjust PID gains. Note that multiple indicators of punctually, energy consumption, stopping accuracy and comfort are all considered in the integrated design. Taking Beijing Subway Line 8 as the numerical example, the results show that the design of speed profile and fuzzy PID controller are both effective, and energy consumption saves about 10.40% with other better indicators compared with the original data.

INDEX TERMS Urban rail transit, eco-driving, train running simulation, speed profile design, fuzzy PID controller.

I. INTRODUCTION

Many urban rail trains are equipped with automatic train operation (ATO) system to ensure the train automatic running, which provides the possibility of better running performance [1], [2]. The ATO systems of many cities, such as Madrid, London, Beijing, etc. [3]–[6], control the train running according to a predefined recommended speed at each running cycle. Thus, train running performance can be improved through recommend speed profile optimization and speed tracking controller design.

Speed profile optimal design has been studied in many researches, and most of them focus on the eco-driving. As early as 80’s of 20 centuries, Milroy [7] utilized maximum principle to study the train control strategy and focused on energy consumption, the deduced optimal train control strategy is “Maximum traction-Coasting-Maximum braking”. Based on the previous studies, Howlett et al. [8] optimized the train operation process model, and concluded that there exist an optimum control strategy through the minimum theory, which is “Acceleration-Cruising-Coasting-Braking” when energy saving is the aim in long running distance. Khmelnitsky [9] studied the train running process with various gradient data and established the optimization model with the energy-saving objective, the model was solved by the minimum principle to get the optimum train control strategy. Domínguez et al. [10] considered the effect of regenerative braking on train running energy consumption, and proposed an optimal control method which was validated through the simulation of practice subway line. Based on the Pontryagin
maximum principle, Liu et al. [11] considered the effect of steep downhill segment and realized a better energy-efficient driving of urban railway train. Pan et al. [12] discussed the influence of many factors on train energy consumption, but relevant factors were not used to provide a more economy driving strategy. Tian et al. [13] optimized energy consumption through optimal parameter design of cruising plus coasting strategy, and validated the method through driver practice training system (DPTS). Xiao et al. [14] used multi-phase dynamic programming (DP) algorithm to optimize the eco-driving running strategy with the consideration of signal constraints, and the method was validated through numerical simulation.

The previously mentioned studies do not fully considered strategy types or objectives such as train running time delay, stopping accuracy, and passenger comfort. To provide ATO system with a larger solution space, three typical running strategies as well as multiple objectives are introduced in the speed profile optimal design in this study. Referred with relevant studies [3], [15]–[17], this problem is treated as a multiobjective problem (MOP) and solved through non-dominated sorting genetic algorithm II (NSGA-II) algorithm combined with train running simulation (TRS).

As for train running control, the classical proportional-integral-derivative (PID) controller is currently applied on most trains based on empirical formulas [18]. The PID controller, firstly introduced by Minorsky, is widely used for its simple structure and universality [19], [20]. Although the simple structure that PID controller has, it performance is not good enough in nonlinear systems [21]. To deal with the unpredictable factors, the adaptive controller is proposed to tune control parameters. There are many adaptive controllers that involve complicated models of train running control [22]–[24]. It should be noted that the exact model of the nonlinear system, which is hard to be constructed, is the precondition of effective adaptive control. Thus, intelligent algorithms can be introduced to improve the controller performance [25], [26].

Fuzzy logic is utilized in tracking controller by many researchers, which shows good performance for nonlinear systems with variable parameters and disturbances [27]. On the one hand, fuzzy model is widely used in model-free controller design with good performance [28]. On the other hand, approximation-based adaptive fuzzy control is also studied by many researchers [29], [30]. And fuzzy tracking control is applied on various nonlinear systems, such as discrete-time nonlinear networks [31], switched stochastic pure-feedback nonlinear systems [32], switched nonstrict-feedback nonlinear systems [33], uncertain switched nonlinear systems [34], etc.

The train running control system is also a kind of nonlinear system and optimized with fuzzy logic by many researchers. Yasunobu et al. [35] firstly added fuzzy logic into the train control system and proposed a controller based on the fuzzy prediction theory, and the control accuracy of Sendai metro was successfully improved. Zhu et al. [36] constructed a predictive fuzzy controller for automatic train protection system and the effectiveness was validated though the simulation based on Labview. Moaveni et al. [37] proposed a fuzzy supervisory predictive controller for train tracking running with the consideration of wheel slip prevention. Sun et al. [26] designed an adaptive neural-fuzzy sliding mode controller (ANFSMC) to overcome the disturbance and parameter perturbations, adaptive-fuzzy approximator and neural-fuzzy switching law were employed in the controller. Yang et al. [38] proposed a controller based on the combination of neural network and fuzzy logic, the control effect of this method was superior to the single controller that validated by the computer simulation. Yang et al. [39] separated different train running commands under different situations and defined them as control modes, and various control algorithms were introduced to different modes to optimize the speed tracking effect.

Even though there are many effective fuzzy controllers mentioned previously, it is not easy to apply such controllers for the different structure with currently used PID controller. Hence, a model-free fuzzy PID controller is constructed in this study for the comprehensive advantage of robustness, adaptiveness, and inheritance.

Our study focuses on the design of recommended speed profile and tracking controller to provide a comprehensive optimization of ATO system. The main contributions are summarized as follows:

1. Three typical running strategies are included in the proposed comprehensive scheme to extend solution space.

2. TRS is constructed based on control cycle and combined with NSGA-II algorithm to solve the problem of speed profile design.

3. An adaptive FPID controller is constructed to make train running follow the speed profile dynamically and efficiently with better performance.

4. Multiple indicators such as running punctuality, train energy consumption, stopping accuracy, safety and passenger comfort are taken into consideration.

5. Integrated optimization results of speed profile design and fuzzy PID controller are shown in this paper, and all indicators are optimized.

The reminder of this paper is organized as follows. In Section II, the MOP of train running is detailed analyzed. The models of train and route are established as well as the performance indicators. The method to obtain the recommended speed profile is illustrated in Section III, and Section IV describes the FPID controller we constructed. In Section V, the effectiveness of the integrated method is validated through the case study of a running section of Beijing Subway Line 8. In the last, Section VI concludes this paper.

II. PROBLEM STATEMENT

A. TRAIN OPERATION ANALYSIS

There are heavy traffic jams in many cities, and railway system is an efficient way to relieve this situation.
Automatic train control (ATC) system plays an important part in the complicated railway system, and ATO system is the core of ATC system. ATO system aims to improve energy efficiency, guarantee passenger safety and reduce driver workload. The main function of ATO system is adjusting the train speed during train running, which decides running time, energy consumption, stopping accuracy and passenger comfort. Coordinating with automatic train protection (ATP) system and automatic train supervision (ATS) system, ATO system can finish relevant functions.

ATO system controls train running according to the recommended speed. The recommended speed is acquired from the recommended speed profile at each control cycle. Thus, an integrated optimization is proposed through two stages, which are recommended speed profile design and tracking controller design. The study scheme is shown as Figure 1.

B. TRAIN MODEL

1) COMPREHENSIVE SCHEME

There are four kinds of typical ATO commands, traction, braking, cruising, and coasting. Traction command is executed through the train traction system, while braking command is executed by the braking system. Cruising command means running at a specific speed, and coasting command implies neither traction force nor braking force is applied on the train.

Different running strategy leads to different performance on energy consumption, running time, stopping accuracy, and passenger comfort. There are three main train running strategies, specifically are cruising strategy, coasting strategy, and cruising plus coasting strategy [40]. To include typical three running strategies, a 3-dimension vector \( CS \) is used as a comprehensive scheme in this study with three decision variables as (1). \( v_{\text{cruise}} \) is cruising speed (holding speed), \( s_{\text{coast}} \) is coasting position, and \( \alpha_{\text{brake}} \) is output percent of maximum braking force. \( CS \) includes the three different strategies mentioned above to avoid the limitation of a specific strategy. Note that the specific strategy of \( CS \) is decided by the value of variables. The typical speed profiles under relevant scenarios can be referred in Figure 2.

\[
CS = \{v_{\text{cruise}}, s_{\text{coast}}, \alpha_{\text{brake}}\} \tag{1}
\]

Though the proposed comprehensive scheme, all three typical running strategies are included in this study, which enlarge the solution space compared with the single strategy.

Different value of variables can lead to different running strategies:

- **Coasting strategy**: If train arrives at coasting position \( s_{\text{coast}} \) before reaching the cruising speed, then train will execute coasting command, and the cycles of coasting-traction will repeat until final braking as Figure 2(a) shown. In this strategy, the train speed at \( s_{\text{coast}} \) is used as coasting speed, and traction speed is a predefined constant value.

- **Cruising plus coasting strategy**: If train arrives at coasting position \( s_{\text{coast}} \) after reaching the cruising speed and before final braking, the train speed profile will show as Figure 2(b). If needed, coasting-traction will repeat after cruising command in this strategy.

- **Cruising strategy**: If train arrives at coasting position \( s_{\text{coast}} \) after final braking, the train will execute cruising command before final braking as Figure 2(c) shown.
2) FORCES ACTED ON THE TRAIN

Train running can be affected by multiple factors, which mainly include route environment, train performance, running timetable and signal equipment. As far as the forces acted on the train are concerned, train running is influenced by the forces from vertical direction and running direction. The forces affect train running are shown in Figure 3. $F_t$ is traction force, $F_w$ is running resistance, $F_b$ is braking force.

The train is dealt as a particle for the length scale compared with running sections in almost all studies about running strategy. The train running is decided by the resultant force $F_{\text{total}}$ shown as (2). The value of forces shown in (2) is positive when the force direction is the same as train running direction, and negative when the force direction is opposite to the train running direction.

$$ F_{\text{total}} = F_t - F_b - F_w \quad (2) $$

Traction force is generated from mechanical devices of the train and can be controlled by ATO system. Train braking force aims to reduce the train speed. The maximum train traction/braking force is related to the train running speed, and the typical correlation is shown in Figure 4.

The maximum traction force is expressed as (3) according to the traction characteristic curve in Figure 4. $F_{\text{max}}$ is the maximum traction force of the corresponding speed. $v$ is the train running speed. The train traction output force is the percent of the maximum traction force, which is calculated through (4). $F_t$ is the train traction output force, and $\alpha_{\text{tract}}$ is the percent of output force and the maximum traction force, $\alpha_{\text{tract}} \in [0,1]$.

$$ F_{\text{max}} = f_t(v) $$

$$ F_t = \alpha_{\text{tract}} F_{\text{max}} = \alpha_{\text{tract}} f_t(v) \quad (4) $$

Similar to traction force, the maximum braking force under different running speed is expressed as (5). $F_{\text{bmax}}$ is the maximum braking force of the corresponding speed. The train braking output force can be calculated by (6). $F_b$ is the train braking output force, and $\alpha_{\text{brake}}$ is the percent of output force and the maximum braking force, $\alpha_{\text{brake}} \in [0,1]$.

$$ F_{\text{bmax}} = f_b(v) \quad (5) $$

$$ F_b = \alpha_{\text{brake}} F_{\text{bmax}} \quad (6) $$

Except traction force and braking force, resistance is also acted on the train. Train running resistance is composed by basic resistance and additional resistance as (7). $F_w$ is the total resistance, $F_{\text{wbas}}$ is the train basic running resistance of unit mass, $F_{\text{wadd}}$ is the train additional running resistance of unit mass, $M$ is the train mass.

$$ F_w = (F_{\text{wbas}} + F_{\text{wadd}}) \times M \quad (7) $$

Train basic resistance is mainly caused by the mechanical friction and air drag. The train mechanical structure and running states can affect basic resistance. For the complexity of influence factors, basic resistance is hard to be modelled, thus, empirical formula (8) is used to calculate it. $A$, $B$, and $C$ are the parameters of basic resistance per unit mass and obtained through practice measurement.

$$ F_{\text{wbas}} = A + Bv + Cv^2 \quad (8) $$

Train additional resistance is caused by running environments and route conditions, and mainly consists of slope resistance, curve resistance, and tunnel resistance. The train additional resistance of unit mass is calculated through (9). $F_{\text{wil}}$ is slope resistance of unit mass, $F_{\text{wcr}}$ is curve resistance of unit mass, and $F_{\text{ws}}$ is tunnel resistance of unit mass.

$$ F_{\text{wadd}} = F_{\text{wil}} + F_{\text{wcr}} + F_{\text{ws}} \quad (9) $$

Slope resistance is the component force of train weight caused by slope. Slope resistance decelerates train on the upslope and accelerates train on the downslope. The gradient of route is small for the design requirements, thus the slope resistance of unit mass equals to the gradient as (10). $F_{\text{wil}}$ is slope resistance of unit mass, $M$ is the train mass, $g$ is gravity, $\theta$ is the slope angle, and $P$ is the equivalent gradient.

$$ F_{\text{wil}} = \frac{Mg \sin \theta}{M} \times 1000 = 1000 \times \sin \theta \approx 1000 \times \tan \theta = P \quad (10) $$

Train may be affected by multiple slope sections at the same time, thus train is dealt as continuous model to get the equivalent gradient $P$ of each running position as (11). $L$ is train length, $p$ is slope gradient, $\text{is}n$ is the number of different
slope gradient under the train, \( p_{is} \) is the gradient of each slope gradient under the train, and \( s_{is} \) is the corresponding length of \( p_{is} \).

\[
P = \int_0^L \frac{pdp}{L} = \sum_{i=1}^{i=n} \frac{p_{is}s_{is}}{L} \tag{11}
\]

Train curve resistance of unit mass \( F_{wr} \) is calculated by (12) [41]. \( \beta \) is an empirical parameter, and \( R \) is the curve radius of the track.

\[
F_{wr} = \frac{\beta}{R} \tag{12}
\]

For the narrow space of tunnel, air can lead to running resistance named as tunnel resistance in this study. Tunnel resistance of unit mass \( F_{ws} \) is also calculated through empirical formula as shown in (13) [42]. \( L_s \) is the tunnel length, \( \gamma \) is the empirical parameter.

\[
F_{ws} = \gamma L_s \tag{13}
\]

3) TRAIN DYNAMIC MODEL

To figure out the train running process, it is important to get the speed and acceleration of the train. According to Newton’s law, the train acceleration is calculated through (14). \( a \) indicates the train acceleration, \( F_{total} \) is the resultant force, and \( M \) is the train mass.

\[
a = \frac{F_{total}}{M} \tag{14}
\]

Train model is simulated by computer in this paper and refreshed in each simulation time step. The time step of this study is uniform and represented by \( \Delta t \). The acceleration is assumed to be fixed in each simulation time step. The speed and running distance of the train are refreshed by (15). \( V_i \) is the initial speed of \( i \)th time step. \( V_{i+1} \) is the initial speed of \( i+1 \)th time step, which is also the final speed of \( i \)th time step. \( S_i \) is the initial position of \( i \)th time step. \( S_{i+1} \) is the initial position of \( i+1 \)th time step, which is also the final position of \( i \)th time step.

\[
\begin{align*}
V_{i+1} &= V_i + a^*\Delta t \\
S_{i+1} &= S_i + \frac{(V_{i+1} + V_i) \Delta t}{2}
\end{align*} \tag{15}
\]

C. RUNNING PERFORMANCE INDICATORS

Energy saving is the main optimal objective of many train operation researches, and because more accurate control can be provided with technique developing, the requirements become more stringent in energy saving, running safety, stopping accuracy, running on time, and ride comfort. More performance indicators should be considered to evaluate train running. Energy consumption, running time, stopping accuracy and passenger comfort are four control indicators of our study. Note that these indicators are used to evaluate the final results of train controller as well as the Pareto front solutions. The detailed calculation of these objectives are illustrated in this section.

1) ENERGY CONSUMPTION

Train total energy consumption is mainly caused by traction force and auxiliary equipment such as lightings and air-conditional system. The power of auxiliary equipment is nearly the same of the whole running, thus, auxiliary equipment power is not considered for the little difference. The formula of train energy consumption is shown as (16). \( E \) is the train energy consumption, \( i \) is the time step index, \( n \) is the total time step, \( F_i \) is the traction force acted on the train, \( v \) is the train speed, \( \Delta t \) is the step time, \( \delta \) is the transfer coefficient from electronic energy to mechanical energy. \( K_E \) is the train energy consumption indicator which has the same calculation of \( E \) as (17) shown. \( E \) is used in the speed profile design, while \( K_E \) is used to evaluate the final energy consumption of the tracking controller.

\[
E = \sum_{i=1}^{n} F_i v_\Delta t \tag{16}
\]

\[
K_E = \frac{E}{\delta} \tag{17}
\]

2) RUNNING TIME

Train running time is also the objective of speed profile design. Total running time \( T \) of TRS is expressed as (18). \( i \) is the step index, \( n \) is the total time step, and \( \Delta t \) is the step time.

\[
T = \sum_{i=1}^{n} \Delta t = n\Delta t \tag{18}
\]

After the running is finished, punctuality is measured through the comparison between expected running time and practice running time as (19) shown. \( K_T \) is the train running punctuality indicator, \( T \) is practice running time in the simulation, \( T_{req} \) is the required running time obtained from ATS.

\[
K_T = \left| T - T_{req} \right| \tag{19}
\]

It worth to notice that, the multiojective optimized solutions are the Pareto front solutions of speed profile design, and the tracking speed profile is selected according to the required running time from them. Hence, the train final running time can be ensured under the good controller performance.

3) STOPPING ACCURACY

Stopping accuracy requirement is used to ensure the passenger pass between train doors and platform screen doors (PSD). Stopping error is used to measure stopping accuracy with the difference value between the stopping position and the required stopping position. Stopping error should less than the required \( S_r \). In the speed profile design, the constraint of stopping accuracy is (20). \( SE \) is the stopping error and calculated through (21). \( S \) is the train running distance, \( S_p \) is the length of running section.

\[
SE \leq S_p \tag{20}
\]

\[
SE = |S - S_p| \tag{21}
\]

The stopping accuracy indicator of the tracking controller is \( K_{SE} \) and calculated in the same way of \( SE \) which expressed...
as (22).

\[ K_{SE} = SE \] (22)

4) PASSENGER COMFORT

Passenger comfort describes passenger ride feeling, which is affected by acceleration/deceleration, frequent changes among commands, and centrifugal force caused by the curve. The change rate of acceleration is used to express the passenger comfort in this paper, which is jerk in physical. Passenger comfort is hardly calculated in speed profile design, thus it is used as performance indicator of the controller. The passenger comfort shown as (23) is obtained through the differential module applied on acceleration in the Simulink simulation. \( K_C \) is the passenger comfort indicator of the tracking controller, \( a \) is the train running acceleration, \( t \) is the running time.

\[ K_C = \max\left\{ \left| \frac{da}{dt} \right| \right\} \] (23)

Passenger comfort has the nonlinear negative correlation with \( K_C \). A specific value of \( K_C \) can be seen as boundary value, because \( K_C \) has little effect on passengers if it is less than that specific value [43]. The international standard ISO2631 gives the comfort evaluating standard as it is less than that specific value [43]. The international standard ISO2631 gives the comfort evaluating standard as it is less than that specific value [43].

### III. SPEED PROFILE DESIGN

ATO controls train running according to a predefined speed profile, and the speed profile has a big effect on eco-driving. This section uses a MOP solving algorithm to deal with this problem, and the gene accords with the proposed comprehensive scheme \( CS \), which includes typical three strategies. Through the solving algorithm of MOP, Pareto front solutions are obtained with the optimization of train energy consumption, running time, and stopping accuracy. The specific solution with the corresponding speed profile is selected according to the timetable, then the train will track it through the controller.

#### A. PROBLEM WITH MULTIOBJECTIVE

For the features of MOP, the solving method takes the consideration of multiple indicators. To deal with objectives of MOP, linear weighted sum method and nondominated solution method are two mainly solving methods. Different from the subjective weights used in linear weighted sum method, nondominated solution method compares results of each solution in the same objective dimensions to obtain Pareto front solutions.

Heuristic algorithms of MOP such as NSGA-II, strength-Pareto evolutionary algorithm (SPEA), Pareto envelope-base selection algorithm (PESA), and Pareto-archived evolution strategy (PAES) are widely used. Among the algorithms we mentioned, NSGA-II is the most recognizable one for the good solution distribution and convergence. The superior performance of NSGA-II was demonstrated by the study of Deb et al. [45], and this algorithm was applied in the train running strategy design successfully [46]. Hence, NSGA-II algorithm is used to get the optimal solutions of recommended speed profiles under the proposed comprehensive scheme.

Objectives of MOP are usually contradictory and restrictive, and Pareto dominance and Pareto front solutions are defined as following. Note that the optimal problem discussed in this paper is the minimum problem, and the maximum problem can be transferred as the minimum problem.

**Definition 1 (Pareto Dominance):** As if there are two solutions \( x_{(a)} \) and \( x_{(b)} \) of a MOP in \( D \) dimensions. The solution \( x_{(a)} \) is said to dominate the solution \( x_{(b)} \), if Statement 1 and Statement 2 are both satisfied.

**Statement 1:** \( f_j(x_{(a)}) \leq f_j(x_{(b)}) \) for all \( j \in \{1, 2, \ldots, D\} \), which means the solution \( x_{(a)} \) is no worse than the solution \( x_{(b)} \) in all objectives.

**Statement 2:** \( f_j(x_{(a)}) < f_j(x_{(b)}) \) for at least one \( j \in \{1, 2, \ldots, D\} \), which indicates that \( x_{(a)} \) is better than \( x_{(b)} \) in at least objective.

**Definition 2 (Pareto Front Solutions):** For the solution set \( PFS \) (where \( PFS \) represent Pareto front solutions), there is no solution of the whole obtained solutions dominates the solution \( f^* \in PFS \).

In order to get the nondominated solutions, NSGA-II algorithm combined with TRS is used in our study. Each solution of Pareto front solutions is corresponding to an optimal speed profile.

#### B. NSGA-II ALGORITHM

NSGA-II algorithm is inspired by the survival of the fittest from nature. Fitness values are the main standard of selection operation, which means the better that fitness values are, the better inherited property of gene is. The fitness function calculates fitness values of each individual through TRS and the procedures of NSGA-II are described as below [45].

The elitist is used in the algorithm to expand the solution selected space. The parent population set \( P_{gen} \) and offspring population \( Q_{gen} \) are combined to form a new population set \( R_{gen} \). The next parent population \( P_{gen+1} \) is selected from population set \( R_{gen} \) based on nondominated sorting results \( SK_{gen} \) and crowding distance results \( CR_{gen} \). The next offspring population \( Q_{gen+1} \) is obtained through selection, crossover and mutation of next parent population \( P_{gen+1} \). The reservation
NSGA-II Algorithm

\[
\begin{align*}
&\text{gen} = 0, \text{ and initialize } P_0 \\
&Q_0 = \text{selection-crossover-mutation}(P_0) \\
&\text{gen} < \text{required-generation} \\
&P_{\text{gen}} = P_{\text{gen}} \cup Q_{\text{gen}} \\
&FR_{\text{gen}} = \text{get-fitness-value} (P_{\text{gen}}) \\
&SR_{\text{gen}} = \text{nondominated-sort} (FR_{\text{gen}}) \\
&CR_{\text{gen}} = \text{get-crowding-distance} (SR_{\text{gen}}) \\
&P_{\text{gen+1}} = \text{get-next-parent} (SR_{\text{gen}}, CR_{\text{gen}}) \\
&Q_{\text{gen+1}} = \text{selection-crossover-mutation} (P_{\text{gen+1}}) \\
&\text{gen} = \text{gen} + 1
\end{align*}
\]

of excellent individuals and better results are realized through the repetition of above operations.

C. TRAIN RUNNING SIMULATION

The key point of the application of NSGA-II algorithm is how to get the fitness values from \(CS\) of each individual. As Figure 2 described, the set of \(CS\), which includes cruising speed \(v_{\text{cruise}}\), coasting position \(s_{\text{coast}}\), and braking force percent \(\alpha_{\text{brake}}\), is corresponding to a speed profile, thus the comprehensive scheme vector \(CS\) as (1) is used to represent the individual gene. Different values of \(CS\) can result in different speed profiles and different running fitness values.

Simulation is an efficient study method which can save costs and avoid potential risks. The simulation structure is shown as Figure 5. In order to simplify the calculation of fitness values, real number encoding is used and the gene of each individual has practice meaning. Each individual can obtain the speed profile through the TRS, and the indicators of running time, energy consumption, and stopping accuracy also can be got.

Simulation is dealt as a function and called by the NSGA-II algorithm in Matlab. The detailed train and route information is read by the simulation firstly, and the information of cruising speed \(v_{\text{cruise}}\), coasting point \(s_{\text{coast}}\), and output percent of maximum braking force \(\alpha_{\text{brake}}\) are obtained through the resolution of the individual. Then the main loop of train running executes. In every loop iteration, the command judgments are made in sequence of whether final braking or not, whether train position larger than coasting point \(s_{\text{coast}}\) or not, and whether train speed arrive at cruising speed \(v_{\text{cruise}}\) or not. The train executes commands and update train state according to the judgment results. Before the next loop iteration, energy and running time are accumulated. In the last, train running time \(T\), energy consumption \(E\), and stopping error \(SE\) of each individual are recorded and return to NSGA-II algorithm. It is worth noting that the data of train and route are saved in the format of computer files and can be read when simulation starts, this implies the universal of this study by substituting data files.

D. RECOMMENDED SPEED PROFILE

The recommended speed profile is corresponding to the solution selected from Pareto front solutions. The typical Pareto front of our research can be referred in Figure 6. Usually, the solution used as recommended speed profile is selected according to the required running time. For the discontinue distribution of Pareto front solutions in each objective axis, the solution with nearest running time is selected as the recommended speed profile which is expressed as (24).

\[
RSP = (P_{\text{si}}|T_{\text{si}} - T_{\text{req}} = \min\{T_{\text{pi}} - T_{\text{req}}\}, P_{\text{si}} \in P) \quad (24)
\]

Note that each solution is corresponding to a variable set \(CS\) as well as running time \(T\), energy consumption \(E\), and stopping error \(SE\). And each solution as Figure 6 shown possesses these information.
IV. TRACKING CONTROLLER DESIGN
In order to control train running according to the designed recommended speed profile, an effective adaptive controller is necessary. The ATO controller controls train running by speed adjusting to make train running satisfy requirements. ATO is a servo control system, the schematic diagram is shown in Figure 7. Before train running starts, the recommended speed profile is settled. The controller controls forces acted on the train through traction/braking system to reduce the output error. $u(t)$ is the control signal of the train.

![Figure 7](image_url)

**FIGURE 7.** The schematic diagram of ATO system.

A. PID CONTROL AND FUZZY CONTROL
The PID controller controls the system through the linear combination of proportional, integral and derivative three parts. The control law of PID controller is (25). $u(t)$ is the control signal, $e(t)$ is the difference value between practice system output and expected system output, $K_p$ is the proportional gain, $K_i$ is the integral gain, $K_d$ is the differential gain.

$$u(t) = K_pe(t) + K_i \int_0^t e(t)dt + K_d \frac{de(t)}{dt} \tag{25}$$

The proportional component of PID can quickly response to the output error $e(t)$, but accumulative error can’t be eliminated and results in the deviation of system output. The integral component of PID is mainly used to eliminate the accumulative error. However, integral component also introduces the lagging which may reduce the system response speed and bring oscillations. The derivative component of PID reflects the change rate of $e(t)$ and helps the system adjust in advance to improve the dynamic performance.

Fuzzy theory aims to transfer the complicated process and natural language to machine language that computer can figure out. The fuzzy control theory is developed based on the fuzzy theory and performs well in the nonlinear system. The thinking mode and practice experience provide bases of fuzzy logic. The fuzzy control system is consisted of fuzzy controller and plant as Figure 8 shown.

![Figure 8](image_url)

**FIGURE 8.** Structure of fuzzy control system.

The four basic parts of fuzzy controller are fuzzification, rule base, fuzzy interference, and defuzzification. Fuzzification maps the value of inputs to the linguistic descriptions. Rule base imitates the human thinking and stores the rules, which described by the conjunctions such as if-then, and, else. Fuzzy interference can provide fuzzy control values according to the input fuzzy values, membership function and rule base. The fuzzy control value is transferred as control value of plant through defuzzification.

B. FPID CONTROLLER
The FPID controller of ATO system controls the train through PID controller with dynamic $\Delta K_p$, $\Delta K_i$, and $\Delta K_d$ generated by fuzzy logic as Figure 9 shown. Fuzzy logic is the key module of the designed FPID controller, and it is important to design range of inputs and outputs, variables membership function, and fuzzy adaptive rules in fuzzy logic. The proper fuzzy logic design can realize a better control effect of ATO.

![Figure 9](image_url)

**FIGURE 9.** Structure of FPID controller.

1) MEMBERSHIP FUNCTION OF VARIABLES
The outputs of fuzzy logic are $\Delta K_p$, $\Delta K_i$, and $\Delta K_d$, the inputs are speed error $e$ and speed error change rate $\dot{e}$. The two dimensions of inputs can avoid overcomplicated fuzzy rules caused by high input dimensions.

The reasonable domain is necessary to realize effective train control. Based on the practice situations and simulation experiment, the domain of $e$ is $[-16, +16]$, $\dot{e}$ with $[-16, +16]$, $\Delta K_p$ with $[-4, +4]$, $\Delta K_i$ and $\Delta K_d$ both are with $[-3, +3]$. The input space and output space in fuzzy interference are described by lingual variables so that can be dealt by fuzzy rules. The number of lingual description in fuzzy set decides the accuracy of fuzzy logic, and the fuzzy sets of inputs and outputs both are {Negative Big-NB, Negative Medium-NM, Negative Small-NS, Zero-ZO, Positive Small-PS, Positive Medium-PM, Positive Big-PB}.

Membership describes the mapping relationship between the variable and the fuzzy set. With the consideration of variable distribution as well as the requirements of sensitivity and robust, trigonometric membership function is used. And the gently mapping is required in the extreme values, the gauss membership function is used in the both ends of fuzzy set. Figure 10 displays the membership functions of fuzzy inputs edited through Matlab, and the membership functions of fuzzy outputs edited through Matlab are shown in Figure 11.

2) FUZZY ADAPTIVE RULES
Fuzzy outputs are decided by fuzzy rules, and fuzzy rules are obtained through the experience of operators. Fuzzy rules describe the manual control method through conditional
The parameters $K_p$, $K_i$, and $K_d$ of PID controller are modified through fuzzy logic as (26). $K'_p$, $K'_i$, and $K'_d$ are the initial value of parameters of PID controller, $\Delta K_p$, $\Delta K_i$, and $\Delta K_d$ are the outputs of fuzzy logic.

$$\begin{align*}
K_p &= K'_p + \Delta K_p \\
K_i &= K'_i + \Delta K_i \\
K_d &= K'_d + \Delta K_d
\end{align*}$$  
(26)

The experience and knowledge from the driver are the bases of fuzzy rule design. The relationships between fuzzy inputs and fuzzy outputs that this paper used are shown as Table 2.

The rules in Table 2 can also be described through conditional statements such as: if ($e$ is NB) and (ec is NB) then ($\Delta K_p$ is PB)($\Delta K_i$ is NB)($\Delta K_d$ is PS). Similar statements are edited through Matlab fuzzy rule editor. The membership functions of fuzzy inputs and outputs as well as fuzzy rules are saved as .fis file and loaded in the Simulink.

The designed FPID controller obtains the PID revised gains from fuzzy logic, so that the PID gains can realize adaptive adjustment to make better performance. Besides, appropriate initial gains are necessary. This paper uses initial values 20, 1.8, and 0.5 of $K'_p$, $K'_i$, and $K'_d$ respectively, which is based on the simulation tests. After finishing the design of parameters and fuzzy logic, the FPID controller can deal with different tracking error and acquire corresponding parameters.

### C. SIMULINK FPID MODEL

To validate FPID controller we designed, Simulink platform is chosen to simulate the train running control system. Fuzzy logic can adjust gains adaptively according to the various conditions, so that the dynamic performance of ATO system controller is improved. The fuzzy logic inputs are speed error $e$ and speed error change rate $ec$, meanwhile, the outputs are $\Delta K_p$, $\Delta K_i$, and $\Delta K_d$. Fuzzy Logic Controller module is used to execute the designed fuzzy logic as Figure 12 shown. The membership functions and fuzzy rules of fuzzy inputs/outputs are saved as .fis file and loaded in the Simulink.
PID unit controls train running according to the speed error \( e \), the control signal is obtained through the linear combination of proportional, integral, and derivative three parts. The inputs of this part are gains of proportional, integral, and derivative as well as speed error. PID module is constructed as Figure 13 shown.

Transfer Fun module is used as the plant to show the relationship between control signal and train running speed, relevant module and parameters are shown as Figure 14. The transfer function of the train obtained from the study of Zhu et al. [42] is shown as (27) and used in our Simulink model.

\[
G(s) = \frac{0.07128}{s^2 + 0.4356s + 0.0324} \quad (27)
\]

The input data of recommended speed profile is recorded in a file and imported in Simulink. The output data will be recorded as files and figures through To File and Scope modules. To observe the tracking effect of recommended speed profile and get train performance indicators, the data of train running speed profile, acceleration, jerk and speed error are all saved in workspace.

After the complement of main parts of simulation, the whole Simulink model as Figure 15 shown is constructed to verify the speed tracking controller. The recommended speed profile is input through the input module and used as the reference signal of the controller. Then the difference between reference speed and system output speed is combined with interferences as the error, the PID gains are revised through the fuzzy logic according to different error and error change rate. The control signal of train is the output of FPID controller, and speed profile tracking under intelligent adaptive control is realized through this controller design. Relevant results can be obtained through output modules.

V. CASE STUDY AND ANALYSIS

Matlab software and Simulink platform are used to verify the integrated optimization method proposed by this study. Matlab mainly executes the calculation of Pareto front solutions, and Simulink is used to construct the FPID controller and provide the final running results.

A. DATA AND PARAMETERS

Practice data is applied in case study to verify the proposed optimal method. Beijing Subway Line 8 has 35 stations with 45.6 kilometers long. The section between Yuzhili (YZL) station and Pingxufu (PXF) station of Beijing Subway Line 8 is selected as the test section. The distance of this section is 1985.71 m with the required running time 135s, the relevant data of slope, curve, and speed limit are shown as Table 3, Table 4, and Table 5 respectively. Note that the position 0 is the start position of the first station in up direction.

The railway train used in Beijing Subway Line 8 is the type B train with 6 cabs, the relevant parameters are shown in Table 6. The train mass data used in this paper is under the standard passenger load, the fitting equations of traction and braking characteristic curves are shown as (28) and (29). Note that the braking force is negative value for the opposite direction of train running, and the unit of traction/braking force is kN.

\[
F_{\text{max}} = \begin{cases} 
284 & 0 \leq v \leq 41 \\
-0.0025v^3 + 0.5766v^2 - 46.86v + 1406.8 & 41 \leq v \leq 80
\end{cases} \quad (28)
\]

\[
B_{\text{max}} = \begin{cases} 
-238 & 0 \leq v \leq 52 \\
-0.1143v^2 + 19.937v - 963.37 & 52 \leq v \leq 80
\end{cases} \quad (29)
\]

B. RESULTS ANALYSIS

The Pareto front solutions are got through NSGA-II algorithm combined with TRS. Figure 16 shows the distribution of final Pareto front solutions. The range of energy consumption and running times can be observed from the figure. And with the running time gets larger, energy consumption tends to decrease. Stopping errors corresponding to Pareto front solutions are described with colors, and all less than 0.3 m, which means the satisfactory of stopping accuracy. The red point in Figure 16 is the practice running data, and the solution with a better value in at least one dimension can always be found in Pareto front solutions. The strategy distribution is further shown in Figure 17. Cruising strategy mainly with small running time of YZL to PXF section, coasting strategy takes a small part while cruising plus coasting strategy constitutes the majority. The percent of cruising strategy, coasting strategy, and cruising plus coasting strategy are 3.0%, 25.5%, and...
71.5% respectively. And the solution with the least running time is under cruising strategy, which accords with relevant studies [17].

The recommended speed profile is selected according to the required running time as (24) described. Specifically, system compares the running time of each solution and selects the most approximate one. Required running time of YZL-PXF section is 135 s, and relevant position-speed curve is obtained as Figure 18(a) with 135.05 s. The selected solution is under cruising plus coasting strategy, the train accelerates until the speed reaches 66.2 km/h, and cruising command executes then, when the train position arrives at coasting position 3904.75 m, coasting commanded is applied on the train until final braking with 70% maximum force. Corresponding time-speed curve is shown as Figure 18(b). The Pareto front offers the solutions with various running time, and the recommended speed profile can be obtained

### TABLE 3. Slope data of YZL-PXF section.

| Segments (m) | Slope(%) |
|--------------|----------|
| 2654-2800    | -2       |
| 2800-3470    | -10      |
| 3470-4330    | 5.8      |
| 4330-4530    | 22       |
| 4530-4639.71 | 2        |

### TABLE 4. Curve data of YZL-PXF section.

| Segments (m) | Curve radius (m) |
|--------------|------------------|
| 2654.00-2746.12 | 0                |
| 2746.12-2849.24 | 1200             |
| 2849.24-2869.47 | 0                |
| 2869.47-2972.59 | 1200             |
| 2972.59-3653.36 | 0                |
| 3653.36-3816.47 | 1500             |
| 3816.47-4076.01 | 0                |
| 4076.01-4296.96 | 600              |
| 4296.96-4347.59 | 0                |
| 4347.59-4545.59 | 500              |
| 4545.59-4639.71 | 0                |

### TABLE 5. Speed limit of YZL-PXF section.

| Segments (m) | Speed limit (km/h) |
|--------------|--------------------|
| 2654.00-2838.12 | 60                 |
| 2838.12-4572.71 | 80                 |
| 4572.71-4639.71 | 55                 |

### TABLE 6. Relevant parameters of the case study.

| Parameters | Data | Parameters | Data |
|------------|------|------------|------|
| $M$        | 284.1 t | $\beta$ | 600 daN/\text{t}\cdot\text{m} |
| $L$        | 118.87 m | $\gamma$ | 0.00013 daN/\text{t}\cdot\text{m} |
| $A$        | 2.7531 daN/\text{t} | $\delta$ | 85 % |
| $B$        | 0.014 daN/\text{t}\cdot\text{h}/\text{km} | $\Delta t$ | 0.2 s |
| $C$        | 0.00075 daN/\text{t}\cdot\text{h}^2/\text{km}^2 | $S_c$ | 0.3 m |
| $\mu_{\text{slip}}$ | 80 % |
with different running time. The required running time may change in practice for timetable adjustment caused by peak and off-peak hour, unexpected failures, etc. Under such circumstance, the controller can quickly pick the recommended speed profile through the proposed speed profile design method instead of recalculation.

The selected solution is used as recommended speed profile of train tracking controller. In order to show the effectiveness of designed FPID controller, current used PID controller is also made with the same parameters. The final running position-speed curves of designed FPID controller and classical PID controller are shown as Figure 19(a) and Figure 19(b) respectively. The requirements of speed limit are both met of PID controller and FPID controller, which is observed from Figure 19(a), (b). Figure 19(a) shows a more tightly tracking results than that of Figure 19(b), which implies the superiority of the proposed FPID controller. The speed error curve in Figure 20 also validates the better tracking effect of FPID controller. And the acceleration change rate figures of designed FPID controller and classical PID controller are shown in Figure 21, and the comfort level of FPID controller is also better than that of PID controller.

It should be noted that the fuzzy logic used in the proposed tracking controller relies on experience, thus, the controller performance may be improved through further adjustment of fuzzy logic based on more trials. Besides, if the constructed
FPID controller is applied on other trains or lines, fuzzy logic also should be adjusted carefully.

### TABLE 7. Comparison results.

| Data                      | $K_T$ (s) | $K_E$ (kWh) | $K_{SE}$ (s) | $K_C$ (m/s³) |
|---------------------------|-----------|-------------|--------------|--------------|
| Requirements (R)          | -         | -           | <0.300       | <1           |
| Practice data (PD)        | 0.20      | 16.25       | 0.200        | 0.87         |
| Recommended speed profile (RSP) | 0.05      | 14.22       | 0.050        | -            |
| PID Tracking results (PIDTR) | 1.24     | 14.76       | 0.037        | 0.75         |
| FPID Tracking results (FPIDTR) | 0.18     | 14.56       | 0.046        | 0.59         |
| RSP-PD                    | -0.15     | -2.03       | -0.15        | -            |
| FPIDTR-PIDTR              | -1.06     | -0.20       | 0.009        | -0.16        |
| FPIDTR-RSP                | 0.13      | 0.34        | -0.004       | -            |
| FPIDTR-PD                 | -0.02     | -1.69       | -0.15        | -0.28        |
| (FPIDTR-PD)/PD            | -10.00%   | -10.40%     | -77.00%      | -32.18%      |

Through the design of recommended speed profile and tracking controller, the performance of train operation has been obviously improved, the relevant results are shown in Table 7. There are four indicators used to measure the train running performance, specifically are $K_T$, $K_E$, $K_{SE}$, and $K_C$ as (19), (17), (22), and (23) described. The indicators of practice data, recommended speed profile, and simulation tracking of FPID controller and PID controller all meet the requirements of punctually, energy consumption, stopping error, and comfort. It can be seen from Table 7 that all the indicators of recommended speed profile are better than them of practice. Tracking results of FPID controller are not as good as recommended speed profile in practice running, which is caused by system uncertainties. Compared with current PID controller, FPID controller performs better except a minor inferiority in stopping error. And compared with practice data, the results of FPID controller tracking are optimized with 0.02 s, 1.69 kWh, 0.15 m, and 0.28 m/s³ of running time, energy consumption, stopping error, and jerk value respectively. Through the integrated optimization method proposed in this study, running punctually, energy consumption, stopping accuracy, and running comfort are improved with 10.00%, 10.40%, 77.00%, and 32.18% respectively. And the optimization effect will be enhanced if the proposed controller is applied on all trains of the running line.

This studied proposed an integrated optimization method of train running through speed profile design and FPID tracking control. Simulation results show that the proposed integrated optimization method of this study is feasible and effective. The speed profile is optimized through the NSGA-II algorithm with the enlarged solution space of multiple running strategies. And the better effect of tracking controller benefits from the fuzzy parameter tuning law. These operations of the proposed method ensure the optimized results.

### VI. CONCLUSION

The main purpose of this paper is developing an integrated method with multifactor consideration to realize a better train running performance through optimal design of speed profile and fuzzy PID controller. Based on the proposed comprehensive scheme, NSGA-II algorithm combined with TRS is used to obtain the Pareto front solutions and then the recommended speed profile is selected under predefined running time. An intelligent adaptive FPID controller is built in this study to realize a better tracking effect compared with current PID controller. As for performance indicators, which are also used as optimization objectives, energy consumption, train running time, stopping accuracy and passenger comfort are considered in the study. In case study, the indicators of recommended speed profile and controller tracking results show the effectiveness of proposed method in punctually, energy consumption, stopping error, and comfort level. Especially, the final results imply about 10% improvement in energy consumption through the integrated optimization method.

For the complicity of railway system, multiple train running is a research hotspot. As is well-known, conflict often exist between running trains for the tight timetable, and train running timetable affects regenerative braking utilization coefficient of the whole line. Thus, we will focus on the combination of the proposed single train control strategy and the timetable in the future work, in order to control the train with conflict situations and improve the regenerative energy utilization.

### REFERENCES

[1] A. González-Gil, R. Palacin, P. Batty, and J. P. Powell, “A systems approach to reduce urban rail energy consumption,” *Energy Convers. Manage.*, vol. 80, pp. 509–524, Apr. 2014.

[2] X. Zhu, R. Zhang, W. Dai, Z. Zhang, and J. Li, “Performance and safety assessment of ATO systems in urban rail transit systems in China,” *J. Transp. Eng.*, vol. 137, pp. 727–728, Jul. 2013.
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