Brane-Bulk energy exchange and agegraphic dark energy

Ahmad Sheykhi *

Department of Physics, Shahid Bahonar University, P.O. Box 76175, Kerman, Iran
Research Institute for Astronomy and Astrophysics of Maragha (RIAAM), Maragha, Iran

We consider the agegraphic models of dark energy in a braneworld scenario with brane-bulk energy exchange. We assume that the adiabatic equation for the dark matter is satisfied while it is violated for the agegraphic dark energy due to the energy exchange between the brane and the bulk. Our study shows that with the brane-bulk interaction, the equation of state parameter of agegraphic dark energy on the brane, $w_D$, can have a transition from normal state where $w_D > -1$ to the phantom regime where $w_D < -1$, while the effective equation of state for dark energy always satisfies $w_{eff}^D \geq -1$.

I. INTRODUCTION

The observed acceleration in the universe expansion rate is usually attributed to the presence of an exotic kind of energy, called “dark energy” [1]. A great variety of dark energy models have been proposed, but most of them are not able to explain all features of the universe, or are artificially constructed in the sense that it introduces too many free parameters to be able to fit with the experimental data. For a recent review on dark energy candidates see [2]. Many theoretical attempts toward understanding the dark energy problem are focused to shed light on it in the framework of a fundamental theory such as string theory or quantum gravity. Although a complete theory of quantum gravity has not established until now, we still can make some attempts to investigate the nature of dark energy according to some principles of quantum gravity. An interesting attempt for probing the nature of dark energy within the framework of quantum gravity (and thus compute it from first principles) is the so-called “Agegraphic Dark Energy” (ADE) proposal. This model is based on the uncertainty relation of quantum mechanics together with the gravitational effect in general relativity. Following the line of quantum fluctuations of spacetime, Karolyhazy et al. [3] argued that the distance $t$ in Minkowski spacetime cannot be known to a better accuracy than $\delta t = \beta t^{2/3} r^{1/3}$ where $\beta$ is a dimensionless constant of order unity. Based on Karolyhazy relation, Maziashvili discussed that the energy density of the metric

* sheykhi@mail.uk.ac.ir
fluctuations of Minkowski spacetime is given by

$$\rho_D \sim \frac{1}{t_p^2 t^2} \sim \frac{m_p^2}{t^2},$$  (1)

where $t_p$ is the reduced Planck time. Throughout this paper we use the units $c = \hbar = k_b = 1$. Therefore one has $l_p = t_p = 1/m_p$ with $l_p$ and $m_p$ are the reduced Planck length and mass, respectively. The ADE model assumes that the observed dark energy comes from the spacetime and matter field fluctuations in the universe. The agegraphic models of dark energy have been examined and constrained by various astronomical observations.

Independent of the challenge we deal with the dark energy puzzle, in recent years, theories of large extra dimensions, in which the observed universe is realized as a brane embedded in a higher dimensional spacetime, have received a lot of interest. According to the braneworld scenario the standard model of particle fields are confined to the brane while, in contrast, the gravity is free to propagate in the whole spacetime. In this theory the cosmological evolution on the brane is described by an effective Friedmann equation that incorporates non-trivially with the effects of the bulk into the brane. An interesting consequence of the braneworld scenario is that it allows the presence of five-dimensional matter which can propagate in the bulk space and may interact with the matter content in the braneworld. It has been shown that such interaction can alter the profile of the cosmic expansion and lead to a behavior that would resemble the dark energy. The cosmic evolution of the braneworld models with energy exchange between the brane and bulk has been studied in the different setups. In these models, due to the energy exchange between the bulk and the brane, the usual energy conservation law on the brane is broken down and consequently it was found that the equation of state of the dark energy may experience the transition behavior. In the context of holographic dark energy braneworld model with bulk-brane interaction has also been studied. Other studies on the dark energy models in the context of braneworld scenarios have been carried out in.

The purpose of the present work is to disclose the effect of the energy exchange between the brane and the bulk in RSII braneworld scenario on the evolution of the universe by considering the flow of energy onto or away from the brane. Employing the agegraphic model of dark energy in a non-flat universe, we obtain the equation of state parameter for ADE density. We shall assume that the adiabatic equation for the dark matter is satisfied while it is violated for the ADE due to the energy exchange between the brane and the bulk. We will show that by suitably choosing model parameters, our model can exhibit accelerated expansion of the universe. In addition, we will present a profile of the $w_D$ crossing $-1$ phenomenon which is in good agreement with observations.
This paper is organized as follows. In section II we review the formalism of bulk-brane energy exchange. In section III we study the original ADE in braneworld where the time scale is chosen to be the age of the universe. In section IV we consider the new model of ADE while the time scale is chosen to be the conformal time instead of the age of the universe. The last section is devoted to conclusions and discussions.

II. BRANEWORLD WITH BRANE-BULK INTERACTION

The theory we are considering is five-dimensional and has an action of the form

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} (R - 2\Lambda) + \int d^5x \sqrt{-\tilde{g}} L^m_{\text{bulk}}$$

$$+ \int d^4x \sqrt{-\tilde{g}} (L^m_{\text{brane}} - \sigma),$$

(2)

where $R$ is the 5D scalar curvature and $\Lambda < 0$ is the bulk cosmological constant. $g$ and $\tilde{g}$ are the bulk and the brane metrics, respectively. We have also included arbitrary matter content both in the bulk and on the brane through $L^m_{\text{bulk}}$ and $L^m_{\text{brane}}$, respectively, and $\sigma$ is the positive brane tension. The field equations can be obtained by varying action (2) with respect to the bulk metric $g_{AB}$. The result is

$$G_{AB} + \Lambda g_{AB} = \kappa^2 T_{AB}. \quad (3)$$

For convenience and without loss of generality, we can choose the extra-dimensional coordinate $y$ such that the brane is located at $y = 0$ and bulk has $\mathbb{Z}_2$ symmetry. We are interested in the cosmological solution with a metric

$$ds^2 = -n^2(t, y) dt^2 + a^2(t, y) \gamma_{ij} dx^i dx^j + b^2(t, y) dy^2,$$

(4)

where $\gamma_{ij}$ is a maximally symmetric three-dimensional metric for the surface ($t=$const., $y=$const.), whose spatial curvature is parameterized by $k = -1, 0, 1$ corresponding to open, flat, and closed universes, respectively. A closed universe with a small positive curvature ($\Omega_k \simeq 0.01$) is compatible with observations [23]. The metric coefficients are chosen so that, $n(t, 0) = 1$ and $b(t, 0) = 1$, where $t$ is cosmic time on the brane. The total energy-momentum tensor has bulk and brane components and can be written as

$$T_{AB} = T_{AB} \mid_{\text{brane}} + T_{AB} \mid_\sigma + T_{AB} \mid_{\text{bulk}}. \quad (5)$$
The first and the second terms are the contribution from the energy-momentum tensor of the matter field confined to the brane and the brane tension

\[ T_A^B|_{\text{brane}} = \text{diag}(-\rho, p, p, p, 0) \frac{\delta(y)}{b}, \]  
\[ T_A^B|_{\sigma} = \text{diag}(-\sigma, -\sigma, -\sigma, -\sigma, 0) \frac{\delta(y)}{b}, \]

where \( \rho \) and \( p \), being the energy density and pressure on the brane, respectively. In addition we assume an energy-momentum tensor for the bulk content of the form

\[ T_A^B|_{\text{bulk}} = \begin{pmatrix}
T_0^0 & 0 & T_0^5 \\
0 & T_i^j \delta_{ij} & 0 \\
-\frac{n^2}{b^2} T_5^0 & 0 & T_5^5
\end{pmatrix}. \]

The quantities which are of interest here are \( T_5^5 \) and \( T_0^5 \), as these two enter the cosmological equations of motion. In fact, \( T_0^5 \) is the term responsible for energy exchange between the brane and the bulk. Inserting the ansatz (4) for the metric, the non-vanishing components of the Einstein tensor \( G_{AB} \) are found to be

\[ G_{00} = 3 \left\{ \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right) - \frac{n^2}{b^2} \left( \frac{a''}{a} + \frac{a'}{a} \left( \frac{a'}{a} - \frac{b'}{b} \right) \right) + k \frac{n^2}{b^2} \right\}, \]
\[ G_{ij} = \frac{a^2}{b^2} \gamma_{ij} \left\{ \frac{a'}{a} \left( \frac{a'}{a} + 2 \frac{n'}{n} \right) - \frac{b'}{b} \left( \frac{n'}{n} + 2 \frac{a'}{a} \right) + 2 \frac{a''}{a} + \frac{n''}{n} \right\} + \frac{a^2}{n^2} \gamma_{ij} \left\{ \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} + 2 \frac{\dot{n}}{n} \right) - 2 \frac{\dot{a}}{a} + \frac{\dot{b}}{b} \left( -2 \frac{\dot{a}}{a} + \frac{\dot{n}}{n} \right) - \frac{\dot{b}}{b} \right\} - k \gamma_{ij}, \]
\[ G_{05} = 3 \left\{ \frac{n'}{n} \dot{a} + \frac{\dot{a}' a'}{a b} - \frac{\dot{a}'}{a} \right\}, \]
\[ G_{55} = 3 \left\{ \frac{a'}{a} \left( \frac{a'}{a} + \frac{n'}{n} \right) - \frac{b'^2}{n^2} \left( \frac{\dot{a}}{a} \left( \frac{\dot{a}}{a} - \frac{\dot{n}}{n} \right) + \frac{\ddot{a}}{a} \right) - k \frac{b^2}{a^2} \right\}. \]

In the above expressions, primes and dots stand for derivatives with respect to \( y \) and \( t \), respectively. Integrating Eqs. (9) and (10) across the brane and imposing \( \mathbb{Z}_2 \) symmetry, we obtain the jumps across the brane

\[ \frac{a_+}{a_0} = -\frac{\kappa^2}{6} (\rho + \sigma), \]
\[ \frac{n'_+}{n_0} = \frac{\kappa^2}{6} (2\rho + 3p - \sigma), \]

where \( 2a'_+ = -2a'_- \) and \( 2n'_+ = -2n'_- \) are the discontinuities of the first derivative, and the subscript “ 0” denotes quantities are evaluated at \( y = 0 \). Substituting the junction conditions (13) and (14)
into the (05) and (55) components of the field equations, we obtain the modified Friedmann equation and the semi-conservation law on the brane

\[
\dot{\rho} + 3H(\rho + p) = -2T_5^0, \tag{15}
\]
\[
2H^2 + \dot{H} + \frac{k}{a^2} = -\frac{\kappa^4}{36} \left[ \sigma (3p - \rho) + \rho (\rho + 3p) \right],
\]
\[
+ \frac{\kappa^2}{3} \left( \Lambda + \frac{\kappa^2 \sigma^2}{6} \right) - \frac{\kappa^2 T_5^5}{3}, \tag{16}
\]
where \(a = a_0 = a(t, 0)\) and \(H = \dot{a}/a\) is the Hubble parameter on the brane. We shall assume an equation of state \(p = w\rho\) which represents a relation between the energy density and pressure of the matter on the brane. We also neglect \(T_5^5\) term by assuming that the bulk matter relative to the bulk vacuum energy is much less than the ratio of the brane matter to the brane vacuum energy [15]. Considering this we get

\[
2H^2 + \dot{H} + \frac{k}{a^2} = \gamma \rho (1 - 3w) - \beta \rho^2 (1 + 3w) + \frac{\lambda}{3}, \tag{17}
\]
\[
\dot{\rho} + 3H \rho (1 + w) = -2T_5^0, \tag{18}
\]
where we have used the usual definition \(\beta = \kappa^4/36, \gamma = \beta \sigma\) and \(\lambda = \kappa^2(\Lambda + \kappa^2 \sigma^2/6)\). Assuming the Randall-Sundrum fine-tuning \(\lambda = \kappa^2(\Lambda + \kappa^2 \sigma^2/6) = 0\) holds on the brane, one can easily check that the Friedmann equation (17) is equivalent to the following equations

\[
H^2 + \frac{k}{a^2} = \frac{\beta \rho^2 + 2\gamma (\rho + \chi)}{3}, \tag{19}
\]
\[
\dot{\chi} + 4H \chi = 2T_5^0 \left( \frac{\rho}{\sigma} + 1 \right). \tag{20}
\]
Equation (19) is the modified Friedmann equation describing cosmological evolution on the brane. The auxiliary field \(\chi\) incorporates non-trivial contributions of dark energy which differ from the standard matter fields confined to the brane. The bulk matter contributes to the energy conservation equation (18) through \(T_5^0\), which is responsible for the energy exchange between the brane and bulk. We are interested in the scenarios where the energy density of the brane is much lower than the brane tension, namely \(\rho \ll \sigma\), therefore our system of equations can be simplified in the following form

\[
H^2 + \frac{k}{a^2} = \frac{1}{3m_p^2} (\rho + \chi), \tag{21}
\]
\[
\dot{\chi} + 4H \chi \approx 2T_5^0 = Q, \tag{22}
\]
\[
\dot{\rho} + 3H \rho (1 + w) = -2T_5^0 = -Q. \tag{23}
\]
Here \(m_p^2 = (8\pi G_4)^{-1}\) is the reduced Planck mass, where \(G_4 = 3\gamma/4\pi\) is the 4D Newtonian constant.
We assume that there are two dark components in the universe, dark matter and dark energy, and
thus the total energy density is \( \rho = \rho_m + \rho_D \), where \( \rho_m \) and \( \rho_D \) are the energy density of dark matter and dark energy, respectively. With the energy exchange between the bulk and brane, the usual energy conservation is broken down. Here we assume that the adiabatic equation for the dark matter is satisfied while it is violated for the dark energy due to the energy exchange between the brane and the bulk.

\[
\begin{align*}
\dot{\rho}_m + 3H\rho_m &= 0, \\
\dot{\rho}_D + 3H\rho_D(1 + w_D) &= -2T_0^\phi = -Q.
\end{align*}
\]

Here \( w_D = p_D/\rho_D \) is the equation of state parameter of ADE and \( Q = \Gamma \rho_D \) stands for the interaction term between the bulk and the brane with interaction rate \( \Gamma \). Therefore, until now we have obtained the set of equations describing the dynamics of our universe in braneworld with bulk-brane interaction.

### III. THE ORIGINAL ADE AND BULK-BRANE INTERACTION

The original ADE density has the form (1) where \( t \) is chosen to be the age of the universe

\[
T = \int dt = \int_0^a \frac{da}{Ha}. 
\]

Thus, the energy density of the original ADE is given by [5]

\[
\rho_D = \frac{3n^2m_p^2}{H^2 T^2},
\]

where the numerical factor \( 3n^2 \) is introduced to parameterize some uncertainties, such as the species of quantum fields in the universe, the effect of curved space-time and so on. The Friedmann equation (21) can be reexpressed as

\[
H^2 + \frac{k}{a^2} = \frac{1}{3m_p^2} (\rho_m + \rho_D + \chi).
\]

If we introduce, as usual, the fractional energy densities such as [21]

\[
\Omega_m = \frac{\rho_m}{3m_p^2 H^2}, \quad \Omega_D = \frac{\rho_D}{3m_p^2 H^2}, \quad \Omega_k = \frac{k}{H^2 a^2}, \quad \Omega_\chi = \frac{\chi}{3m_p^2 H^2},
\]

then, the Friedmann equation (28) can be written as

\[
\Omega_m + \Omega_D + \Omega_\chi = 1 + \Omega_k.
\]

Using Eq. (27), we have

\[
\Omega_D = \frac{n^2}{H^2 T^2}.
\]
We choose the following ansatz for the interaction rate \[24\]
\[
\Gamma = 3b^2(1 + r)H, \tag{32}
\]
where \(b^2\) is a coupling constant and \(r = \chi/\rho_D\) is the ratio of two energy densities \[21\],
\[
r = \frac{\Omega \chi}{\Omega_D} = -1 + \frac{1}{\Omega_D} [1 + \Omega_k - \Omega_m]. \tag{33}
\]
Using the continuity equation \[24\], it is easy to show that
\[
\Omega_m = \Omega_{m0}a^{-3} = \Omega_{m0}(1 + z)^3, \tag{34}
\]
where \(\Omega_{m0} = 0.28 \pm 0.02\) is the present value of all part of the matter confined to the brane. Taking the derivative of Eq. \(27\) with respect to the cosmic time and using Eq. \(31\) we reach
\[
\dot{\rho}_D = -2H\rho_D\sqrt{\Omega_D/n}. \tag{35}
\]
Inserting this equation in the conservation law \(25\) and using Eqs. \(32\)–\(34\) we find the equation of state parameter of the original ADE on the brane
\[
w_D = -1 + \frac{2}{3n}\sqrt{\Omega_D} - b^2\Omega_D^{-1} [1 + \Omega_k - \Omega_{m0}(1 + z)^3]. \tag{36}
\]
One can easily check that \(w_D\) can cross the phantom divide if \(3nb^2(1 + \Omega_k - \Omega_m) > 2\Omega_D^{3/2}\). If we take \(\Omega_D \approx 0.72\), \(\Omega_{m0} \approx 0.28\) and \(\Omega_k \approx 0.01\) for the present time, the phantom-like equation of state for \(w_D\) can be achieved provided \(nb^2 > 0.56\). The joint analysis of the astronomical data for the new agegraphic dark energy gives the best-fit value (with 1σ uncertainty) \(n = 2.7\) \[12\]. Thus, the condition \(w_D < -1\) leads to \(b^2 > 0.2\) for the coupling between dark energy and dark matter. For instance, if we take \(b^2 = 0.25\) we get \(w_D = -1.04\). If we define, following \[25, 26\], the effective equation of state parameter as
\[
w^{\text{eff}}_D = w_D + \frac{\Gamma}{3H}, \tag{37}
\]
then, the continuity equation \(25\) for dark energy can be written in the standard form
\[
\dot{\rho}_D + 3H\rho_D(1 + w^{\text{eff}}_D) = 0. \tag{38}
\]
Substituting Eqs. \(32\), \(33\) and \(36\) into Eq. \(37\), we find
\[
w^{\text{eff}}_D = -1 + \frac{2}{3n}\sqrt{\Omega_D}. \tag{39}
\]
From Eq. (39) we see that \( w_{eff}^D \) is always larger than \(-1\) and cannot cross the phantom divide \( w_{eff}^D = -1 \). Let us study the behavior of \( w_{eff}^D \) in two different stages. In the early time (matter-dominated epoch) where \( \Omega_D \to 0 \) we have \( w_{eff}^D = -1 \). Namely, the effective equation of state mimics a cosmological constant in the matter-dominated epoch. In the late time where \( \Omega_D \to 1 \) we have \( w_{eff}^D = -1 + 2/3n \). Thus we have \( w_{eff}^D < -2/3 \) provided \( n > 2 \) which is consistent with recent cosmological data [12]. Next, we obtain the equation of motion of \( \Omega_D \). Differentiating Eq. (31) and using relation \( \dot{\Omega}_D = \Omega'_D H \), we reach

\[
\Omega'_D = \Omega_D \left( -2 \frac{\dot{H}}{H^2} - \frac{2}{n} \sqrt{\Omega_D} \right),
\]

where the dot is the derivative with respect to the cosmic time and the prime denotes the derivative with respect to \( x = \ln a \). Taking the derivative of both side of the Friedmann equation (28) with respect to the cosmic time, and using Eqs. (22), (24), (30), (31) and (35), it is easy to find that

\[
\frac{\dot{H}}{H^2} = -2 + \frac{3b^2}{2} - \frac{\Omega_k}{2}(2 - 3b^2) + \frac{\Omega_m}{2}(1 - 3b^2) + \Omega_D \left( 2 - \frac{\sqrt{\Omega_D}}{n} \right).
\]

Substituting this relation into Eq. (40), we obtain the equation of motion of the original ADE

\[
\Omega'_D = \Omega_D \left\{ 4(1 - \Omega_D) \left( 1 - \frac{\sqrt{\Omega_D}}{2n} \right) + 2\Omega_k - \Omega_m - 3b^2(1 + \Omega_k - \Omega_m) \right\}.
\]

This equation describes the evolution behavior of the original ADE in braneworld cosmology with brane-bulk energy exchange. For completeness, we give the deceleration parameter

\[
q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2},
\]

which combined with the Hubble parameter and the dimensionless density parameters form a set of useful parameters for the description of the astrophysical observations. Substituting Eq. (41) into (43) we get

\[
q = 3 + \Omega_k - \frac{\Omega_m}{2} - \Omega_D \left( 2 - \frac{\sqrt{\Omega_D}}{n} \right) - \frac{3b^2}{2} \left( 1 + \Omega_k - \Omega_m \right).
\]

If we take \( \Omega_D = 0.72 \), \( \Omega_m0 \approx 0.28 \) and \( \Omega_k \approx 0.01 \) for the present time and choosing \( n = 2.4, b^2 = 2 \) we obtain \( q \approx -0.5 \) for the present value of the deceleration parameter which is in good agreement with recent observational results [27].

### IV. THE NEW ADE AND BULK-BRANE INTERACTION

Soon after the original ADE model was introduced by Cai [5], an alternative model dubbed “new agegraphic dark energy” was proposed by Wei and Cai [6], while the time scale is chosen to
be the conformal time $\eta$ instead of the age of the universe, which is defined by $dt = a d\eta$, where $t$ is the cosmic time. It is important to note that the Karolyhazy relation $\delta t = \beta t^{2/3} t^{1/3}$ was derived for Minkowski spacetime $ds^2 = dt^2 - dx^2$. In case of the FRW universe, we have $ds^2 = dt^2 - a^2 dx^2 = a^2(dy^2 - dx^2)$. Thus, it might be more reasonable to choose the time scale in Eq. (27) to be the conformal time $\eta$ since it is the causal time in the Penrose diagram of the FRW universe. The new ADE contains some new features different from the original ADE and overcome some unsatisfactory points. For instance, the original ADE suffers from the difficulty to describe the matter-dominated epoch while the new ADE resolved this issue. The energy density of the new ADE can be written

$$\rho_D = \frac{3n^2m_\nu^2}{\eta^2},$$

(45)

where the conformal time is given by

$$\eta = \int \frac{dt}{a} = \int_0^a \frac{da}{Ha^2}.$$  

(46)

The fractional energy density of the new ADE is given by

$$\Omega_D = \frac{n^2}{H^2\eta^2}.$$  

(47)

Taking the derivative of Eq. (45) with respect to time and using Eq. (47) we reach ($\dot{\eta} = 1/a$)

$$\dot{\rho}_D = -2H\rho_D \sqrt{\Omega_D} \frac{\sqrt{\Omega_D}}{na}.$$  

(48)

Inserting this equation in the conservation law (25) and using Eqs. (32)-(34) we can find the equation of state parameter

$$w_D = -1 + \frac{2}{3na} \sqrt{\Omega_D} - b^2 \Omega_D^{-1} \left\{ 1 + \Omega_k - \Omega_m0(1 + z)^3 \right\}.$$  

(49)

Again we see that $w_D$ can cross the phantom divide provided $3nab^2(1 + \Omega_k - \Omega_m) > 2\Omega_D^{3/2}$. The effective equation of state $w_D^{\text{eff}}$ reads as

$$w_D^{\text{eff}} = -1 + \frac{2}{3na} \sqrt{\Omega_D}.$$  

(50)

In the late time where $a \to \infty$ and $\Omega_D \to 1$, from Eq. (50) we have $w_D^{\text{eff}} = -1$, while from Eq. (49) it is necessary to have $w_D < -1$. Thus the effective equation of state $w_D^{\text{eff}}$ behaves like a cosmological constant in the late time, while $w_D$ crosses the phantom divide $w_D = -1$. We can also find the equation of motion for $\Omega_D$ by differentiating Eq. (47). The result is

$$\Omega_D' = \Omega_D \left( -2\frac{\dot{H}}{H^2} - \frac{2}{na} \sqrt{\Omega_D} \right).$$  

(51)
Taking the derivative of both side of the Friedman equation \(^{(28)}\) with respect to the cosmic time, and using Eqs. \((22), (24), (30), (47)\) and \((48)\), it is easy to find that
\[
\frac{\dot{H}}{H^2} = -2 + \frac{3b^2}{2} - \frac{\Omega_k}{2}(2 - 3b^2) + \frac{\Omega_m}{2}(1 - 3b^2) + \Omega_D \left(2 - \frac{\sqrt{\Omega_D}}{na}\right). \tag{52}
\]
Substituting this relation into Eq. \((51)\), we obtain the equation of motion of the new ADE
\[
\Omega_D' = \Omega_D \left\{4(1 - \Omega_D)\left(1 - \frac{\sqrt{\Omega_D}}{2na}\right) + 2\Omega_k - \Omega_m - 3b^2(1 + \Omega_k - \Omega_m)\right\}. \tag{53}
\]
The deceleration parameter is now given by
\[
q = 3 + \Omega_k - \frac{\Omega_m}{2} - \Omega_D \left(2 - \frac{\sqrt{\Omega_D}}{na}\right) - \frac{3b^2}{2}(1 + \Omega_k - \Omega_m). \tag{54}
\]
Comparing Eqs. \((48)-(54)\) with their respective equations obtained in the previous section, we see that the scale factor \(a\) enters Eqs. \((48)-(54)\) explicitly. Besides, comparing the results obtained in this work with those presented in \([5, 6, 9]\) for ADE models in standard cosmology we find that the energy exchange between the brane and bulk seriously modifies our basic equations.

V. CONCLUSIONS AND DISCUSSIONS

Among different candidates for probing the nature of dark energy, the holographic dark energy model arose a lot of enthusiasm recently \([28–32]\). However, there are some difficulties in holographic dark energy model. Choosing the event horizon of the universe as the length scale, the holographic dark energy gives the observation value of dark energy in the universe and can drive the universe to an accelerated expansion phase. But an obvious drawback concerning causality appears in this proposal. Event horizon is a global concept of spacetime; existence of event horizon of the universe depends on future evolution of the universe; and event horizon exists only for universe with forever accelerated expansion. In addition, more recently, it has been argued that this proposal might be in contradiction to the age of some old high redshift objects, unless a lower Hubble parameter is considered \([33]\).

In this work we have studied the agegraphic dark energy in the framework of RSII braneworld scenario with bulk-brane energy exchange. Considering the effects of the interaction between the brane and the bulk we have obtained the equation of state for the ADE in a non-flat universe on the brane. We found that although the equation of state parameter of ADE on the brane, \(w_D\), can cross the phantom divide, the effective equation of state parameter \(w^{eff}_D = w_D + \frac{\Gamma}{3}\) is always larger than \(-1\) and cannot cross the phantom divide \(w^{eff}_D = -1\), where \(\Gamma\) is the rate of the bulk-brane interaction. For instance, taking \(n = 2.7\) \([12]\) and \(\Omega_D = 0.72\) for the present time, we found
\( w^{\text{eff}}_D = -0.8 \). This indicates that one cannot generate phantom-like effective equation of state from an ADE in a braneworld model with bulk-brane interaction. For new ADE, in the late time where \( a \to \infty \) and \( \Omega_D \to 1 \), we found \( w^{\text{eff}}_D = -1 \) while \( w_D < -1 \). Thus in the new model of ADE the effective equation of state \( w^{\text{eff}}_D \) mimics a cosmological constant in the late time, while \( w_D \) necessary have a transition to the phantom regime in the presence of bulk-brane interaction.

In agegraphic models of dark energy with bulk-brane interaction, the properties of ADE is determined by the parameters \( n \) and \( b \) together. These parameters would be obtained by confronting with cosmic observational data. In this work we just restricted our numerical fitting to limited observational data. Giving the wide range of cosmological data available, in the future we expect to further constrain our model parameter space and test the viability of this model.

Acknowledgments

I thank the anonymous referee for constructive comments. This work has been supported by Research Institute for Astronomy and Astrophysics of Maragha, Iran.

[1] A.G. Riess, et al., Astron. J. 116 (1998) 1009;
   S. Perlmutter, et al., Astrophys. J. 517 (1999) 565;
   S. Perlmutter, et al., Astrophys. J. 598 (2003) 102;
   P. de Bernardis, et al., Nature 404 (2000) 955.
[2] T. Padmanabhan, Phys. Rep. 380, (2003) 235;
   P. J. E. Peebles, B. Ratra, Rev. Mod. Phys. 75, (2003) 559;
   E.J. Copeland, M. Sami, S. Tsujikawa, Int. J. Mod. Phys. D 15 (2006) 1753.
[3] F. Karolyhazy, Nuovo.Cim. A 42 (1966) 390;
   F. Karolyhazy, A. Frenkel and B. Lukacs, in Physics as natural Philosophy edited by A. Shimony and H. Feschbach, MIT Press, Cambridge, MA, (1982);
   F. Karolyhazy, A. Frenkel and B. Lukacs, in Quantum Concepts in Space and Time edited by R. Penrose and C.J. Isham, Clarendon Press, Oxford, (1986).
[4] M. Maziaashvili Int. J. Mod. Phys. D 16 (2007) 1531;
   M. Maziaashvili, Phys. Lett. B 652 (2007) 165.
[5] R. G. Cai, Phys. Lett. B 657 (2007) 228.
[6] H. Wei and R. G. Cai, Phys. Lett. B 660 (2008) 113.
[7] H. Wei and R. G. Cai, Eur. Phys. J. C 59 (2009) 99.
[8] H. Wei and R. G. Cai, Phys. Lett. B 663 (2008) 1;
    J. Cui, et al., arXiv:0902.0710
    Y. W. Kim, et al., Mod. Phys. Lett. A 23 (2008) 3049;
    Y. Zhang, et al., arXiv:0708.1214
    J. P. Wu, D. Z. Ma, Y. Ling, Phys. Lett. B 663, (2008) 152;
    K. Y. Kim, H. W. Lee, Y. S. Myung, Phys.Lett. B 660 (2008) 118;
    X. Wu, et al., arXiv:0708.0349
    J. Zhang, X. Zhang, H. Liu, Eur. Phys. J. C 54 (2008) 303;
    I. P. Neupane, Phys. Lett. B 673 (2009) 111.
[9] A. Sheykhi, Phys. Lett. B 680 (2009) 113.
[10] A. Sheykhi, Phys. Lett. B 682 (2010) 329;
    A. Sheykhi, Int. J. Mod. Phys. D 18, No. 13 (2009) 2023;
    A. Sheykhi, Phys. Rev. D, in press, arxiv 0908.0606;
    A. Sheykhi, arXiv:0909.0302
[11] M. R. Setare, arXiv:0907.4910
    M. R. Setare, arXiv:0908.0196
[12] H. Wei and R. G. Cai, Phys. Lett. B 663 (2008) 1;
    X. Wu, et al., arXiv:0708.0349.
[13] L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 3370;
    L. Randall, R. Sundrum, Phys. Rev. Lett. 83 (1999) 4690.
[14] Binetruy, P., C. Deffayet and D. Langlois, Nucl. Phys. B 565 (2000) 269.
[15] E. Kiritsis, G. Kofinas, N. Tetradsis, T. N. Tomaras and V. Zarikas, JHEP 0302 (2003) 035;
    E. Kiritsis, N. Tetradsis and T. N. Tomaras, JHEP 0203 (2002) 019;
    P. S. Apostolopoulos and N. Tetradsis, Phys. Rev. D 71 043506 (2005);
    P. S. Apostolopoulos and N. Tetradsis, Phys. Lett. B 633 409 (2006);
    E. Kiritsis, JCAP 0510 014 (2005);
    K. I. Umez, K. Ichiki, T. Kajino, G. J. Mathews, R. Nakamura and M. Yahi, Phys. Rev. D 73 063527 (2006).
[16] R.G. Cai, Y. Gong and B. Wang, JCAP 0603, (2006) 006 .
[17] C. Bogdanos and K. Tamvakis, Phys. Lett. B 646 (2007) 39;
    C. Bogdanos, A. Dimitridis, and K. Tamvakis, Phys. Rev. D 75 (2007) 087303.
[18] V. Sahni, Y. Shtanov, JCAP 0311 (2003) 014.
[19] A. Sheykhi, B. Wang and N. Riaz, Phys. Rev. D 75 (2007) 123513.
[20] M. S. Movahed and A. Sheykhi, Mon. Not. R. Astron. Soc. 388, (2008) 197.
[21] M.R. Setare, Phys. Lett. B 642 (2006) 421.
[22] M. R. Setare, E. N. Saridakis JCAP 0903 (2009) 002;
    P. Moyassari, M. R. Setare, Phys. Lett. B 674 (2009) 237;
E. N. Saridakis, Phys. Lett. B 661 (2008) 335;
E. N. Saridakis, JCAP 0804 (2008) 020;
E. N. Saridakis, Phys. Lett. B 660 (2008) 138.

[23] D. N. Spergel, Astrophys. J. Suppl. 148 (2003) 175;
    C. L. Bennett, et al., Astrophys. J. Suppl. 148 (2003) 1;
    U. Seljak, A. Slosar, P. McDonald, JCAP 0610 (2006) 014;
    D. N. Spergel, et al., Astrophys. J. Suppl. 170 (2007) 377.

[24] B. Wang, Y. Gong, E. Abdalla, Phys. Lett. B 624 (2005) 141;
    D. Pavon, W. Zimdahl, Phys. Lett. B 628 (2005) 206;
    A. Sheykhi Phys. Lett. B 681 (2009) 205.

[25] H. Kim, H.W. Lee, Y.S. Myung, Phys. Lett. B 632 (2006) 605.

[26] M. R. Setare, Phys. Lett. B 642 (2006) 1.

[27] R.A. Daly et al., Astrophysics J. 677 (2008) 1.

[28] A. Cohen, D. Kaplan, A. Nelson, Phys. Rev. Lett. 82 (1999) 4971.

[29] M. Li, Phys. Lett. B 603 (2004) 1.

[30] Q. G. Huang, M. Li, JCAP 0408 (2004) 013.

[31] S. D. H. Hsu, Phys. Lett. B 594 (2004) 13.

[32] M. R. Setare, Eur. Phys. J. C 50 (2007) 991;
    M. R. Setare, JCAP 0701 (2007) 023;
    M. R. Setare, Phys. Lett. B 654 (2007) 1;
    M. R. Setare, Phys. Lett. B 644 (2007) 99.

[33] H. Wei and S. N. Zhang, arXiv:0707.2129.