Cosmology of an asymptotically free scalar field with spontaneous symmetry breaking

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Abstract

We solve Einstein’s equation with Robertson-Walker metric as an initial-value problem, using as the source of gravity a Halpern-Huang real scalar field, which was derived from renormalization-group analysis, with a potential that exhibits asymptotic freedom and spontaneous symmetry breaking. Both properties are crucial to the formulation of the problem. Numerical solutions show that the universe expands at an accelerated rate, with the radius increasing like the exponential of a power of the time. This is relevant to "dark energy" and "cosmic inflation". Extension to the complex scalar field will make the universe a superfluid. The vortex dynamics that emerges offers explanations for other cosmological problems, namely, matter creation, galactic voids, and the “dark mass”.
Scalar fields have been invoked in cosmological models of "dark energy" [1], and "cosmic inflation" [2]. For dark energy, i.e., accelerating expansion of the universe, a scalar field is used as an alternative to Einstein’s cosmological constant. In cosmic inflation, one introduces a scalar field with spontaneous symmetry breaking, i.e., having a potential with a minimum located at a nonzero value of the field. Initially the universe was placed at the "false vacuum" of zero field, and it "inflates" while it "rolls down" the potential towards the true vacuum. It seems appropriate, therefore, to try to solve Einstein’s equation as an initial-value problem, using a scalar field as the source of gravity. Mathematical consistency requires that the scalar potential be asymptotically free. That is, it should be zero at the big bang and grow as the universe expands, instead of having the opposite behavior, as with the familiar $\phi^4$ theory. Spontaneous symmetry breaking is desirable for physical consistency, because the existence of a nonzero vacuum field enables us to neglect quantum fluctuations and treat the entire problem classically. Both requirements are fulfilled in the scalar field found by Halpern and Huang (HH) [3,4] on the basis of renormalization-group (RG) analysis. They show that asymptotic freedom uniquely determines the potential to be a Kummer function, which has exponential behavior at large fields.

We consider an $N$-component $O(N)$-symmetric real scalar quantum field $\phi_n(x)$, with flat-space Lagrangian density $L(x) = \frac{1}{2} \sum_n (\partial \phi_n)^2 - V(\phi^2)$, where $\phi^2 = \sum_n \phi_n^2$, and $\hbar = c = 1$. In the quantum field theory, a high-momentum cutoff $\Lambda$ is the only scale in the system, and all coupling parameters are scaled by appropriate powers of $\Lambda$. As $\Lambda$ changes, the coupling parameters "renormalize" to compensate for the change, such that the theory is preserved, i.e., the partition function remains invariant. They trace out an RG trajectory in the parameter space, the space of all possible scalar theories. The limit $\Lambda \to \infty$ corresponds to a fixed point, at which the system is scale-invariant. One such point is the Gaussian fixed point corresponding to the massless free field, and this is where we would place the system just before the big bang. At the big bang the length scale begins to increase, and asymptotic freedom means that the scalar field leaves this fixed point along a direction tangent to some RG trajectory. The initial direction determines the precise form of the potential.

The HH potential has the form $V = \Lambda^4 U_b$, where the dimensionless potential $U_b$ is given by

$$U_b(z) = c\Lambda^{-b} [M(-2 + b/2, N/2, z) - 1]$$

where $z = 8\pi^2\phi^2/\Lambda^2$, and $c$ is an arbitrary constant. The function $M$ is a Kummer function,
with power series $M(p, q, z) = 1 + zp/q + (z^2/2!)p(p + 1)/q(q + 1) + \cdots$, and asymptotic behavior $z^{b-q}\exp z$. The potential satisfies $\Lambda \partial U_b/\partial \Lambda = -bU_b$ in the neighborhood of the fixed point ($\Lambda = \infty$); it is an eigenfunction of scale transformation with eigenvalue $b$. The case $b = 2$ corresponds to the free field, and $b = 0$ corresponds to the $\phi^4$ theory. Asymptotic freedom corresponds to $b > 0$, and spontaneous symmetry breaking occurs when $b < 2$; hence we choose $0 < b < 2$. This potential is a lowest-order approximation in $\Lambda^{-1}$, derived in flat space-time. Higher-order and curvature corrections have not been calculated, but this should be valid in a neighborhood of the big bang.

In the cosmological problem, we use the Robertson-Walker (RW) metric for a homogeneous space, with scale parameter $a(t)$, where $t$ is the time. Since this is the only scale in the system, we put $\Lambda = 1/a$. This signifies that spacetime curvature supplies the cutoff for the quantum scalar field. Einstein’s equation, together with the scalar-field equation of motion in the RW metric, leads to the following initial-value problem:

$$\dot{H} = ka^{-2} - 4\pi G \left[ \sum_n \phi_n^2 - a (\partial V/\partial a) / 3 \right]$$

$$\ddot{\phi}_n = -3H \dot{\phi}_n - \partial V/\partial \phi_n$$

$$X = 0$$

(2)

where $H = \dot{a}/a$, with a dot denoting time derivative, is the Hubble parameter, $k = 0, \pm 1$ is the curvature parameter, $G$ is the gravitational constant, and $X = H^2 + ka^{-2} - (8\pi G/3) \left( \sum_n \dot{\phi}_n^2/2 + V \right)$. The equation $X = 0$ is Friedman’s equation (00 component of Einstein’s equation), which acts as a constraint on initial conditions. The existence of the the Cauchy problem [5] in general relativity means that $X = 0$. To preserve this when $V$ depends on the cutoff, we have added the $\partial V/\partial a$ term in the first equation. This is equivalent to adding a correction term $-a (\partial V/\partial a) / 3$ to the pressure of the scalar field, making the energy density and pressure $ho = \sum_n \dot{\phi}_n^2/2 + V$, $p = \sum_n \dot{\phi}_n^2/2 - V - a (\partial V/\partial a) / 3$, respectively. A more basic derivation would necessitate a reformulation of the action principle, which we leave for the future.

If the constraint $X = 0$ were ignored in choosing initial conditions, one would generally obtain $H(t) \sim H_\infty$ asymptotically, leading to $a(t) \sim \exp (H_\infty t)$. This would have posed a “fine-tuning” problem, for $H_\infty$ would be naturally on the Planck scale, some 60 orders of magnitude larger than present value. With the constraint $X = 0$, one can show that
the time-averaged behavior is $\langle H(t) \rangle \sim t^{-p}$, corresponding to $\langle a(t) \rangle \sim \exp t^{1-p}$, where $p$ depends on the parameters of the potential, and initial conditions. For $0 < p < 1$, the universe is in accelerated expansion. Thus, the vacuum field supplies dark energy, but its relation to the expansion is neither intuitive nor simple. The power-law decay of $H$ somewhat eases the fine-tuning problem. We shall discuss comparison with observations, including the deceleration era [6], in a separate paper.

Fig.1 shows numerical results for a real scalar field with the HH potential ($N = 1$). They verify the power law as a time-averaged behavior, and give $p = 0.81$. Fig.2 shows results for a superposition of two HH potentials, which give $p = 0.075$. Numerical computations fail after a time, for they tend to violate the constraint $X = 0$, and need to be supplemented by correction procedures.

The problem of cosmic inflation is inseparable from matter creation, which has not been taken into account. We have looked into this problem with the real scalar field, and conclude that efficient matter production calls for extension to a complex scalar field. This work is in progress, and we report on some qualitative expectations.

A complex scalar field corresponds to $N = 2$, and is not qualitatively different from the case $N = 1$ if the components $\phi_1, \phi_2$ are uniform in space. However, new physics emerges when we use the complex representation $\phi, \phi^*$, with $\phi = F \exp (i\sigma)$, and allow the phase $\sigma$ to vary in space. The scalar field then describes a superfluid with superfluid velocity $\mathbf{v} = \nabla \sigma$. Whenever a fluid flows there will be vortices, and in the superfluid case we have quantized vortices. Some immediate consequences of the vortex dynamics are the following.

1. Whenever two quantized vortex lines cross, they could reconnect, as pictured by Feynman [7]. Friction on the superfluid, from quantum and thermal fluctuations, can generate large growing vortex rings, which will emulsify under reconnections and form a vortex tangle [8], with fractal dimension 1.6 [9]. On the vortex lines, reconnection instantaneously creates two cusps, which spring away from each other with theoretically infinite speed. The accompanying burst of kinetic energy could create matter in the form of two opposing jets. The output rate would be Planck energy in Planck time, or $10^{18}$GeV in $10^{-43}$s. As the universe expands, the vortex tangle will eventually decay, but could have created enough matter for nucleogenesis. The demise of the vortex tangle would mark the end of the inflation era.

2. There will be remnant vortex lines, at much reduced density. Their vortex core size must be proportional to the scale $a(t)$ in the RW metric. Inside the core there is no scalar
field, and presumably no matter. Thus, with the expansion of the universe, vortex cores will
grow to become the presently observed voids in galactic space.

3. Reconnection of fat vortex lines in a later universe could create the observed cosmic
jets and gamma-ray bursts.

4. A random potential can pin a superfluid [10]. A rotating galaxy might drag portions
of the superfluid with it, thus acquiring ”dark mass”.

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Figure captions

Fig.1. Halpern-Huang potential $U_b$, for a real field ($N = 1$), with $b = 1, c = 0.1$. Its
argument is $z = 8 (\pi a \phi)^2$. Right panels show $\phi (t)$, and the Hubble parameter $H (t)$ in
a log-log plot. The curvature parameter is $k = -1$. The initial conditions are given by
$\{a_0, \phi_0, \dot{\phi}_0\} = \{1, .01, .1\}$. We obtain $p = 0.81$. All quantities are measured in Planck units,
in which $4\pi G = 1$.

Fig.2. The potential is the superposition $U = 0.1 U_{b=1} - 0.2 U_{b=2}$, with $c = 1$. This makes
$U > 0$ near $\phi = 0$, and gives a ”push” to the initial acceleration of $a (t)$. We set $k = 0$, and
the initial conditions are $\{a_0, \phi_0, \dot{\phi}_0\} = \{1, 0, .1\}$. We obtain $p = 0.075$. 
slope = 0.8
