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To cite this article: Srinidhi Nagarada Gadde and Richard J.A.M. Stevens 2019 J. Phys.: Conf. Ser. 1256 012026

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Effect of Coriolis force on a wind farm wake

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Abstract. The effect of the Coriolis force on the wake of a wind farm in a conventionally
neutral and a stable boundary layer is investigated using large eddy simulations. We find
that the average flow angle varies with the downstream position in the wind farm. In the
entrance region of the wind farm, the flow blockage and wind veer create a positive gradient
of the vertical flux $\langle u'w' \rangle$ at hub-height, which causes a counter-clockwise flow deflection in
the Northern hemisphere, when observed from above. Further downstream, the vertical entrainment
of air from the upper layers of the boundary layer generates a negative gradient of the vertical
flux $\langle u'w' \rangle$ at hub-height, which results in a clockwise flow deflection in the Northern hemisphere.
The deflection is more pronounced for the stable boundary layer than for the conventionally neutral boundary layer. In addition, we find that the wake deflection has a significant impact
on the power production of turbines further downstream in the wind farm.

1. Introduction
The ever increasing demand for clean energy has led to a growing focus on the development
of wind farms. Large wind farms are generally affected by meso-scale effects such as Earth’s
rotation, and therefore by the Coriolis force. The Coriolis force rotates the wind if there is an
imbalance between pressure gradient and frictional forces. As a consequence of the imbalance,
the wind direction changes with height and follows an Ekman spiral [1]. In the Northern
hemisphere, when observed from above, wind moving from land towards sea is deflected clockwise
by the Coriolis force. This deflection is due to the reduction of the effective roughness length
from land to sea. In contrast, when wind moves from sea to land, the increase in effective ground
roughness length reduces the wind speed and turns the wind counter-clockwise [2]. In addition
to the local variation of roughness length, the wind direction depends on the topography and
entrainment of fluid from the upper layers of the ABL.

Large wind farms induce appreciable entrainment and consequently the effective wind angle
in the wake is different compared to the incoming flow [3]. If an additional wind farm is built in
the wake of the first, the power production of the downstream wind farm gets affected due to
the deflection of the wind by the Coriolis force. In recent years, various studies have considered
the effect of the Coriolis force on wind turbine and large wind farm wakes using large eddy
simulations (LES) or Reynolds averaged Navier Stokes (RANS) simulations [4]. For example,
Abkar and Porté-Agel [5] performed LES of a wind turbine in a stable boundary layer (SBL)
and found that the wind turbine wakes at hub-height are deflected in clockwise direction and
are skewed due to the strong wind veer. Allaerts and Meyers [6] showed using LES that the
wakes deflect counter-clockwise in conventionally neutral boundary layers (CNBL). Similarly, Dörenkämper et al. [7] in their study of the EnBW Baltic 1 wind farm in stably stratified conditions observed a counter-clockwise wake deflection. Recently, van der Laan and Sørensen [8] performed RANS simulations of: (1) a wind farm represented by 25 actuator disks and (2) a wind farm modelled by increased ground roughness. They found that the wind farm, when modelled as roughness, slows down the wind and turns it counter-clockwise. Interestingly, when the wind farm is modelled with actuator disks, the flow turns clockwise. They reasoned that the Coriolis force turns the wind farm wake clockwise in the Northern Hemisphere due to the entrainment of fresh momentum from upper layers of the boundary layer. With the present study, by performing LES of finite-size wind farms in both CNBL and SBLs, we intend to gain a better understanding of the effect of Coriolis force on wind farm wakes.

An introduction to the governing equations, the LES sub-grid scale (SGS) model, and the numerical method is given in section 2. The variation of wind angle in the wind farm, the effect of wind veer on the turbine power production, the effect of vertical entrainment flux, and the wind angle change due to the Coriolis force are discussed in section 3. Conclusions are provided in section 4.

2. Large eddy simulation methodology

In section 2.1, we provide an introduction to the governing equations, LES model, and the wind turbine model used in this study. In section 2.2, the numerical method, boundary conditions, and surface stress modeling are outlined. Finally, details of the computational domain, grid resolution, atmospheric conditions, wind farm layout, and inflow conditions are given in section 2.3.

2.1. Governing equations

In LES, turbulent motions larger than the grid scale are resolved and small scale motions are modeled. We solve the filtered continuity and Navier-Stokes equations with the Boussinesq approximation while solving the filtered advection equation for the temperature, and $f$-plane approximation for the Coriolis forces with the flow driven by the geostrophic wind. The corresponding model equations are:

$$
\partial_t \tilde{u}_i = 0,
$$

$$
\partial_t \tilde{u}_i + \partial_j (\tilde{u}_i \tilde{u}_j) = -\partial_i \tilde{p} - \partial_i \tau_{ij} + g \frac{\tilde{\theta} - \langle \tilde{\theta} \rangle}{\theta_0} \delta_{i3} + f \varepsilon_{ij3} (\tilde{u}_j - G_j) - f_i,
$$

$$
\partial_t \tilde{\theta} + \tilde{u}_j \partial_j \tilde{\theta} = -\partial_j q_j,
$$

where the tilde represents spatial filtering with a spectral cut-off filter, $\tilde{u}_i$ and $\tilde{\theta}$ are the filtered velocity and potential temperature, respectively, $\tau_{ij} = \tilde{u}_i \tilde{u}_j - \langle \tilde{u}_i \tilde{u}_j \rangle$ is the trace-less part of the SGS stress tensor, $g$ is the acceleration due to gravity, $\theta_0$ is the reference temperature, $\delta_{ij}$ is the Kronecker delta, $\langle \rangle$ represents horizontal averaging, $f$ is the Coriolis parameter, $G_j$ is the geostrophic wind which drives the flow, $\epsilon_{ik}$ is the alternating unit tensor, $f_i$ represents the turbine forces, and $q_j = \tilde{u}_j \tilde{\theta} - \tilde{u}_j \tilde{\theta}$ is the SGS heat flux tensor. Viscous terms in the governing equation are neglected as the Reynolds number in ABLs is very high. The SGS stresses and heat fluxes are parameterized as,

$$
\tau_{ij} = \tilde{u}_i \tilde{u}_j - \tilde{u}_i \tilde{u}_j = -2v_T \tilde{S}_{ij} = -2(C_s \Delta)^2 |\tilde{S}| \tilde{S}_{ij}
$$

$$
q_j = \tilde{u}_j \tilde{\theta} - \tilde{u}_j \tilde{\theta} = -v_q \partial_j \tilde{\theta} = -(D_s \Delta)^2 |\tilde{S}| \partial_j \tilde{\theta}
$$

where $v_T$ and $v_q$ are the turbulent and heat flux transfer coefficients, respectively.
where, $\tilde{S}_{ij} = \frac{1}{2} \left( \partial_j \tilde{u}_i + \partial_i \tilde{u}_j \right)$ represents the filtered strain rate tensor, $v_T$ is the eddy viscosity, $C_{s,\Delta}$ is the Smagorinsky coefficient for the flow field at the grid scale $\Delta$, $|S| = \sqrt{2S_{ij}S_{ij}}$ is the strain-rate magnitude, $v_q = v_T Pr_{sgs}^{-1}$ is the eddy diffusivity, $Pr_{sgs}$ is the SGS Prandtl number, and $D_{s,\Delta}$ is the Smagorinsky coefficient for the SGS heat flux. A tuning free, Lagrangian averaged scale-dependent (LASD) model \cite{9}, is used to parameterize the SGS stresses. In the model, the error in the calculation of the SGS coefficients using the Smagorinsky approximation is minimized over the fluid pathlines, which makes the model particularly suited for inhomogeneous flows such as the flow through a wind farm.

The turbine forces are parameterized based on an empirical model for the total thrust force experienced by the wind turbine \cite{10,11},

$$F_t = -\frac{1}{2} \rho C_T U_\infty^2 \pi D^2,$$

where, $C_T$ is the thrust coefficient, and $U_\infty$ is the “upstream” undisturbed reference velocity. Equation \cite{6} is applicable only for LES of isolated turbines \cite{10,11}. For wind farm simulations where the wake of upstream wind turbine interacts with the downstream turbines, $U_\infty$ cannot be readily specified. Consequently, we use the velocity at the rotor disk $U_d$ to model the turbine forces in our wind farm simulations \cite{12,13}. Actuator disk theory is used to relate $U_\infty$ with the rotor disk velocity $U_d$,

$$U_\infty = \frac{U_d}{(1-a)}$$

where $a$ is the induction factor \cite{12}. The velocity is averaged over the rotor disk and the thrust forces are calculated by substituting Eqn. \cite{7}in Eqn. \cite{6} A more detailed derivation can be found in references \cite{12,13}.

To ensure the same geometric pattern of the wind farm layout for both CNBL and SBL cases, we use a PID controller to rotate the inlet flow such that the wind angle at the hub-height is always zero. This allows us to compare the different stability conditions more easily (otherwise the wind direction would not be the same under different stability conditions). In addition, changes in the local wind angle can result in the turbine yaw angle misalignment. To prevent this, we measure the local wind angle averaged over an area equal to the area of the turbine disk at one turbine diameter upstream of each turbine and rotate the turbine such that it is always oriented perpendicular to the average incoming wind direction. Using this control each turbine aims to maximize its own power output given the local incoming wind direction.

2.2. Numerical method

The numerical methods in the present code are similar to the ones used by Albertson and Parlange \cite{14}. We use a pseudo-spectral scheme and periodic boundary conditions in the horizontal directions, and a second-order central difference scheme in the vertical direction. Time integration is performed using a second-order-accurate Adams-Bashforth scheme. The aliasing errors resulting from the non-linear terms are prevented by using the $3/2$ anti-aliasing rule \cite{15}. As we consider very high Reynolds number flows, we neglect the viscous forces in the governing equations as is the common practice in LES of ABL flows. The computational domain is uniformly discretized in the horizontal and wall normal directions. The computational planes are staggered in the vertical direction. The first vertical velocity plane is located at the ground, and the first streamwise and spanwise velocities and potential temperature planes are located at half a grid point away from the ground. The no-slip and free-slip boundary conditions with zero vertical velocity are used at the bottom and top boundaries, respectively. Instantaneous surface shear stress and buoyancy flux at the wall are modelled with the Monin-Obukhov similarity
theory [16] using the resolved grid velocities and temperature at the first grid point,

$$\tau_{i3|w} = -u_*^2 \frac{\widetilde{u}_i}{u_r} = -\left( \ln(z/z_o) - \psi_M \right)^2 \frac{\widetilde{u}_i}{u_r},$$  \hspace{1cm} \text{(8)}$$

and

$$q_{3|w} = \frac{u_* \kappa (\theta_s - \tilde{\theta})}{\ln(z/z_o) - \psi_H}.$$  \hspace{1cm} \text{(9)}$$

where $\tau_{i3|w}$ and $q_{3|w}$ are the instantaneous shear stress and buoyancy flux at the wall, respectively, $u_*$ is the frictional velocity, $z_o$ is the roughness length, $\kappa$ is the von Kármán constant set to 0.4, $\widetilde{u}_r$ is filtered velocity at the first grid level, and $\theta_s$ is the potential temperature at the surface. $\psi_M$ and $\psi_H$ are the stability corrections for momentum and heat [17][18]. For a SBL,

$$\psi_M = -4.8z/L,$$  \hspace{1cm} \text{(10)}$$

$$\psi_H = -7.8z/L,$$  \hspace{1cm} \text{(11)}$$

where, $L = -(u_*^3 \theta_0)/(\kappa g q_{3|w})$ is the Obukhov length. For a CNBL, $\psi_M = 0$ and $\psi_H = 0$, because $q_{3|w} = 0$ and the Obukhov length $L = \infty$.

Our code has been successfully validated and used to simulate flow through a wind farm in neutral boundary layers [4][19][20]. Furthermore, the code has been used for the simulations of both stable and unstable ABLs and due to the scale-dependent nature and the better dissipation characteristics of the LASD model the code gives better results than when a standard Smagorinsky model is used; when compared with observational data. We have analysed important turbulence statistics such as the variance and skewness of horizontal and vertical velocities, buoyancy fluxes, boundary layer height etc. and found that the results agree well with the observations [21][22] as well as reference LES [23] in the literature. This validation study will be published in a future publication.

### 2.3. Computational set-up

In our simulations, wind turbines with a rotor diameter $D$ of 100 m and a hub-height $z_h$ of 100 m are modelled as actuator disks. The wind farm consists of $5 \times 6$ turbines separated by a distance of 7D in the streamwise and 5D in the spanwise direction. Details of the computational domain and wind farm layout are given in Fig. [1]. The inflow conditions are generated by performing a precursor simulation [24] in a 10 km $\times$ 5 km $\times$ 4 km domain. Similar to Stevens et al. [24], the inflow conditions are forced in the wind farm domain by using a fringe layer. Due to the Coriolis force, the wind follows the Ekman spiral. Consequently, for a finite size wind farm, the fringe layer has to be considered both in streamwise and spanwise directions as indicated in Fig. [1]. We consider the SBL studied in the Global Energy and Water Cycle Experiment (GEWEX) ABL study (GABLS−1) [23]. Therefore, the Coriolis parameter is set to $f = 1.39 \times 10^{-4}$ rad/s (corresponding to latitude 73°N), and the geostrophic wind is set to 8 m/s. The surface potential temperature and roughness length are 265 K and 0.1 m, respectively. The boundary layer is capped by an inversion layer of strength 0.01 K/m and is forced by the geostrophic wind with a specified surface cooling rate. For the CNBL the surface cooling rate is set to zero and for the SBL the surface is cooled at the rate of 0.25 K/hour. A Rayleigh damping layer with a damping coefficient of 0.016 s$^{-1}$ is considered at the top 1 km of the computational domain to limit the reflection of gravity waves [25]. The simulations are run for 10 hours and the statistics are collected over the last hour of the simulations.
Figure 1: Details of the computational domains. Black shaded portion of the precursor domain is copied to the wind farm domain at each time step to act as the inflow condition. Turbine positions are represented by orange circles. Results from the simulations are sampled from the area shaded in green. Turbine rows are numbered from 1-6. At hub-height the wind direction is from West to East, which we define as the $0^\circ$ direction.

Figure 2: Left: Horizontally averaged wind velocity magnitude, $V_{mag} = \sqrt{u^2 + v^2}$, where $u$ and $v$ represent the velocities in streamwise and spanwise directions, respectively. Right: Wind angle, $\phi = \tan^{-1}\left( \frac{v}{u} \right)$, variation with height. Solid and dashed lines represent the SBL and CNBL, respectively. The horizontal orange lines indicate the bottom, top, and hub-height of the actuator disks.

We carried out extensive grid independence study of the GABLS–1 SBL in a domain of dimension 400 m x 400 m x 400 m. In addition, we simulated the SBL in the precursor domain and compared the results with the LES results by Beare et al. [23]. In LES, scale-dependent dynamic SGS models provide better predictions of the SBL properties compared to LES using just the Smagorinsky model [26, 27, 5]. Based on grid resolution tests we find that the LASD model, which we use in this study, gives reasonably accurate predictions when a grid with a horizontal resolution of 19.53 m and a vertical resolution of 10.42 m is used. It is worth
mentioning here that this grid resolution we use is similar to the resolution used by Allaerts and Meyers [6] who used a resolution of 30 m and 15 m in the streamwise and spanwise directions, respectively, and Wu and Porté-Agel [28], who used a resolution of 40 m and 16 m in the streamwise and the spanwise directions, respectively, in their LES studies of CNBL. The boundary layers considered in this study are conventionally neutral and moderately stable with reasonably big turbulent eddies unlike a highly SBL. Consequently, the LASD model sustains the turbulence and faithfully predicts all the essential boundary layer features including the surface similarity profiles even at coarse grid resolutions, provided the domain is large enough. Based on the studies, we discretized both the precursor and the wind farm domains with $512 \times 256 \times 385$ grid points, which gives a resolution of 19.53 m in the horizontal directions and 10.42 m in the vertical direction, respectively. Figure 2 gives the velocity profile and wind angle of the ABLs simulated in the precursor domain. We note that the wind veer over the turbine diameter for the CNBL is $6.4^\circ$ and for the SBL it is $15.2^\circ$. It is worth noting here that in a truly neutral boundary layer without a capping inversion the wind veer is less pronounced.

3. Results and discussion

Figure 3 shows contour plots of the wind velocity magnitude, $V_{mag}$, at the hub-height, for the CNBL (Fig. 3: left) and the SBL (Fig. 3: right) simulations. It is clear from the figures that the wind farm wake turns clockwise in the Northern hemisphere. Owing to the changes in local wind angle and veer, the turbines rotate with the wind and deflect the wake. As the wind veer is stronger in the SBL case the wake deflects more in the SBL case than for the CNBL case. A careful observation of the velocity contour plots reveals that the velocity deficit behind the second and subsequent rows is significantly larger than the wake deficit behind the first row. Similar wake variation has been observed by Witha et al. [29] in their study of the EnBW Baltic-1 wind farm for different atmospheric stratifications.

![Figure 3: Velocity magnitude at hub-height. Left: CNBL. Right: SBL](image)

Visual inspection of the contour plot of the SBL shows that the deflection of the wake behind the last turbine in the first row (Fig. 3: right: enclosed in a dashed rectangle) is more pronounced than for the rest of turbines. It is also clear that the length of the wind farm wake increases from row 6 to 1, see the sketch in Fig. 1 for the row numbering. The increase in the length of the wakes is due to the cumulative effect of the veering of the wind by the neighboring turbines rows to their right. As the top row (row 6, Fig. 3) has no neighboring turbine row deflecting the wind clockwise, the effect of wind veer on the wake of this row is weak and consequently the length of the wake is short.

To further assess the impact of the wake turning, we analyze the power produced by the turbines. Figure 4 shows the power production for the individual turbines normalized with the
power produced by the turbines of the sixth row, $P_6$, of their respective columns. In the absence of wind veer, for a streamwise flow of $\phi = 0^\circ$ everywhere, all the entries have to be close to 1, as the effect of the neighboring rows of turbines on the power production is limited. For both the CNBL and the SBL, we find that the wind veer causes a reduction in the power production of the turbines in columns 2 to 5. The effect is largest on the last turbine of the first row $P(1, 5)$ (turbine with the lowest relative power production). The effect is more pronounced for the SBL than for the CNBL, due to the increasing strength of stratification and the resulting wind veer. There is a possibility that the wind farm blockage affect the power production of the outer wind turbine rows (row 1 and 6). When there is a wind farm blockage the result would be symmetric around rows 3 and 4 which is clearly not the case. Although we cannot rule out the wind farm blockage effect, it is clear from Fig. 4 that the main effect of the power degradation is due to the turning of the wakes. For column 5 there is a pronounced downward trend in the power production, this trend would not be influenced by the wind farm blockage effect, which would be an interesting topic of study in itself, but outside the scope of the present study.

![Figure 4](image)

Figure 4: Turbine power production normalized with the power produced by the turbines of sixth row of the respective columns, which is indicated by $P_6$. The production of the downstream turbines in row 1 is lowest due to the cumulative effect of the wakes of the neighboring rows.

Figure 5a shows the variation of spanwise averaged wind angle sampled from the shaded green region (Fig. 1) at hub-height. To check the uncertainty in the calculation of the average wind angle we calculated the average in 10 minute intervals, binned the data into six bins and determined the standard deviation in the wind angle from this. In Fig. 5a this standard deviation is used to indicate the uncertainty of the results. The wake of the wind farm turns clockwise and eventually reaches a constant value. The deflection is more pronounced in the SBL than in the CNBL. At the entrance of the wind farm the flow turns counter-clockwise, reaches a maximum, following this, the change in the wind angle between maximum and minimum wind angle (Fig. 5a) is approximately $3^\circ$ in SBL, and $1.5^\circ$ in CNBL. Interestingly, the variation of the wind angle is strongly related to the variation of turbulence-induced vertical flux $D_{u'w'}$, here the over-bar represents time averaging and $\langle \cdot \rangle$ represents the horizontal averaging over the sampling region, where $u' = u - \overline{u}$ and $w' = w - \overline{w}$ represents the velocity fluctuations in the
streamwise and vertical directions, respectively. The fluxes are calculated by adding the resolved scale flux $\langle \tilde{u}\tilde{w} \rangle$, and the SGS flux $\tilde{r}_{xz}$. Similarly, the vertical flux of spanwise velocity, $\langle \tilde{v}w' \rangle$, is calculated by the summation of the resolved scale flux $\tilde{r}_{0w}$ and the SGS flux $\tilde{r}_{yz}$, where, $v' = v - \bar{v}$, is the fluctuation in the spanwise velocity. When the vertical gradient of $\langle \tilde{u}w' \rangle$ is positive, the flow turns counter-clockwise and negative gradients turn the flow clockwise.

At the entrance and immediate downwind of the first turbine row, the flow is decelerated due to the momentum extraction by the wind turbines, this causes an upward momentum flux causing the growth of an internal boundary layer. The flow blockage due to wind farm results in the upward shift of the low momentum fluid causing a positive gradient of $\langle \tilde{u}w' \rangle$ at the entrance and immediate downwind of the first turbine row the flow turns counter-clockwise. The upward displacement of the streamlines and the consequent variation of the flow has been reported by Wu and Porté-Agel [28] in their studies of wind farms in a CNBL. Figure 5b shows the variation of the momentum flux with height in the entrance region ($x < 1$ km, sampled from green shaded region of Fig. 1 and averaged in the spanwise direction) and the rest of the sampling region. In the entrance region of the wind farm, the vertical gradient of $\langle \tilde{u}w' \rangle$ is positive at the hub-height (Fig. 5b, black circular markers), and this turns the flow counter-clockwise. Further downstream, the vertical gradient of $\langle \tilde{u}w' \rangle$ is negative (Fig. 5b, black square markers), which deflects the flow clockwise. The combined effect of the vertical gradients of $\langle \tilde{u}w' \rangle$ and $\langle \tilde{v}w' \rangle$ is the primary reason behind the turning of the wind farm wake.

Figure 5: (a): Left axis and black curves represent the wind angle in degrees. Variation of wind angle with streamwise distance. Right axis and black curves represent the vertical entrainment flux, $\langle \tilde{u}'w' \rangle$, sampled from the shaded green region depicted in Fig. 1. Shaded region represents the uncertainty in the 1 hour average calculated by the standard deviation of the six 10 minute averages. (b) Spanwise averaged fluxes at the entrance $x < 1$ km (circular markers) and rest of the sampling region $x > 1$ km (square markers) for the SBL case. black and red curves represent $\langle \tilde{u}w' \rangle$ and $\langle \tilde{v}w' \rangle$, respectively.
The mechanism of the wake turning can be better understood by simplifying the Reynolds averaged streamwise and spanwise momentum equations for a neutral Ekman boundary layer,

\[
D_t \overline{u} = (f \overline{v} - f u_g) - \left[ \partial_x (\overline{u'u'}) + \partial_y (\overline{u'v'}) + \partial_z (\overline{u'w'}) \right] - \overline{f_x} \\
D_t \overline{v} = - (f \overline{u} - f u_g) - \left[ \partial_x (\overline{v'v'}) + \partial_y (\overline{v'v'}) + \partial_z (\overline{v'w'}) \right] - \overline{f_y}
\]

(12)

(13)

where \(u_g\) and \(v_g\) are the components of the geostrophic wind acting as the pressure forcing which drives the flow, and \(\overline{f_x}\) and \(\overline{f_y}\) represent the time averaged thrust forces of the turbines.

In the above equations we assume very high Reynolds number and viscous terms are therefore neglected. To further simplify the analysis, assume a statistically steady state such that \(D_t \overline{u}\) and \(D_t \overline{v}\) are negligible. The above equations can be further simplified by performing a horizontal average in the streamwise and spanwise directions. If we also assume horizontal statistical homogeneity, we get, \(\partial_x (\overline{u'w'}) = \partial y (\overline{v'w'}) = 0\). Similar analysis of the vertical entrainment flux associated with turbulence for a pressure driven ABL can be found in Calaf et al. [13]. Due to the aforementioned assumptions, the horizontal gradients of the streamwise and spanwise velocity fluctuations can also be neglected.

Using the this, we obtain the following simplified equations,

\[
\langle \overline{u} \rangle = u_g - \frac{1}{f} \frac{\partial (\overline{v'w'})}{\partial z} - \frac{\overline{f_y}}{f} \\
\langle \overline{v} \rangle = v_g + \frac{1}{f} \frac{\partial (\overline{u'w'})}{\partial z} + \frac{\overline{f_x}}{f}
\]

(14)

(15)

The above equations give an insight into the deflection of the flow. For a boundary layer without a wind farm, \(\overline{f_x} = \overline{f_y} = 0\), the flow direction at any height depends on the vertical gradients of \(\overline{u'w'}\) and \(\overline{v'w'}\). Far away from the ground, pressure forces \(\langle u_g, v_g \rangle\) are balanced by the local velocity, this is the geostrophic equilibrium, known as the Taylor-Proudman theorem [30, 31]. Close to the ground, wind stresses reduce the streamwise velocity and increase the spanwise velocity. Consequently, the flow rotates counter-clockwise as it approaches the ground giving rise to the Ekman spiral. The presence of a wind farm in an Ekman boundary layer introduces additional turbulent fluxes, the gradients of which decide the wind deflection. At the entrance of the wind farm, flow blockage makes \(\frac{\partial (\overline{u'w'})}{\partial z}\) positive, this increases the spanwise velocity (Eqn. 15) and rotates the wind counter-clockwise. Further downstream, \(\frac{\partial (\overline{u'w'})}{\partial z}\) is negative at the hub-height which reduces the spanwise velocity, this reduction deflects the flow clockwise. In addition, the reduced velocity in the wakes induces changes in the local Coriolis forces, this effect turns the flow counter-clockwise. However, van der Laan and Sørensen [8] have shown that this effect is smaller than the effect caused by the wake induced turbulence.

4. Conclusions

We performed LES of a wind farm with 30 turbines, which are represented using an actuator disk model, in CNBL and SBL. We find that the average wind angle varies with downstream position in the wind farm. For the boundary layers in our study, the flow at hub-height deflects counter-clockwise at the entrance of the wind farm and clockwise further downstream. The wind deflection depends on the gradient of the turbulence induced vertical fluxes, \(\overline{u'w'}\). Flow blockage and wind veer create a positive gradient flux at hub-height in the entrance region of the
wind farm, which results in a counter-clockwise flow deflection. Further downstream, vertical entrainment results in a negative gradient flux and a clockwise flow deflection. The variation of turbulent momentum fluxes in the wind farm wake suggests that the imbalance caused by the entrainment of fluid from above the wind farm results in an appreciable clockwise flow deflection at hub-height. An analysis of the power production in the wind farm shows that the wake deflection has a significant influence on the performance of downstream turbines, which is in agreement with the result of RANS simulations by van der Laan et al. \[3\].

Clearly, the present analysis is simplified. For example, the effects of gravity waves induced by the flow blockage are not taken into account. Allaerts and Meyers \[32\] report that the pressure perturbations created by the gravity waves also induce local changes in the wind angle. To further assess the interplay of the effect of the Coriolis force and gravity waves on flow deflection in extended wind farms, more detailed studies are required that consider various CNBL and SBL configurations.

Acknowledgments
This work is part of the Shell-NWO/FOM-initiative Computational sciences for energy research of Shell and Chemical Sciences, Earth and Live Sciences, Physical Sciences, FOM and STW. This work was carried out on the national e-infrastructure of SURFsara, a subsidiary of SURF corporation, the collaborative ICT organization for Dutch education and research. We also acknowledge PRACE for awarding us access to JUWELS based in Germany at Jülich under PRACE project number 2017174146.

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