On Universality in Black Hole Thermodynamics\textsuperscript{a}

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The low energy decay rates of four- and five dimensional dyonic black holes in string theory are equivalently described in terms of an effective near horizon \(\text{AdS}_3\) (BTZ) black hole. It is then argued that \(\text{AdS}_3\) gravity provides an universal microscopic description of the low energy dynamics these black holes.

1 Introduction

The last three years have seen dramatic progress in the understanding of the microscopic description of black holes initiated by the identification of certain dyonic black holes and Bogomol’nyi saturated states in string theory\textsuperscript{1,2}. Such configuration arise as vacua in compactified superstring theory. The microscopic description then requires some understanding of the corresponding world sheet conformal field theory\textsuperscript{2}. This was first achieved in\textsuperscript{3} for the 5-dimensional dyonic black hole.

Non extremal as well as four dimensional black holes where considered in\textsuperscript{4} and\textsuperscript{5} respectively. For non-extremal black holes they decay rates also agree with an effective string theory prediction for a large range of parameters\textsuperscript{6}. There the microscopic derivations are less rigorous. However, the effective string theory predictions work much better than they should, i.e. produce the correct result for parameters of the black hole where higher order effects on string theory side are expected\textsuperscript{7}. This hints to the existence of a simpler microscopic description of these black holes that does not involve all degrees of freedom of superstring theory.

Here we elaborate on these ideas. It has been known for some time\textsuperscript{8} that the near horizon geometry of the five- and six dimensional black string is a \(\text{AdS}_3\) (BTZ)\textsuperscript{9} black hole \(\times S^{2(3)}\) respectively. Furthermore, the s-wave decay rates of the four- and five dimensional black holes are identical with the BTZ decay rates\textsuperscript{10}. On the other hand \(\text{AdS}_3\) gravity has an equivalent description in terms of a conformal field theory at the boundary\textsuperscript{11,12}. The scalar field induces an interaction between the left- and right moving sectors of this CFT and the corresponding normalised transition amplitudes indeed reproduce the correct semiclassical s-wave decay rates.

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2 Effective 2+1 dimensional description

The (generalised) five-dimensional Reissner Nordström black hole is obtained by compactification of the 6-dimensional black string with metric given by

\[ ds^2 = \frac{1}{\sqrt{f_1 f_5}} \left( -hf_p^{-1}dt^2 + f_p[dz + \frac{r_p^2}{r^2} \cosh \sigma_p \sinh \sigma_p \frac{dt}{f_p}]^2 \right) \\
+ \sqrt{f_1 f_5} \left( \frac{1}{h}dr^2 + r^2d\Omega_3^2 \right) \]

where

\[ f_1 = 1 + \frac{r^2}{r_1^2}, \quad f_5 = 1 + \frac{r^2}{r_5^2}, \quad h = 1 - \frac{r_0^2}{r^2}, \quad f_p = 1 + \frac{r_p^2}{r^2}, \quad r_p^2 = r_0^2 \sinh^2 \sigma_p. \]

In what follows we assume \( r_p << r_1, r_5 \). Then, with the identifications

\[ l^2 = r_1 r_5, \quad \rho^2 = \frac{R^2}{\ell^2} (r^2 + r_0^2 \sinh^2 \sigma_p), \quad \tau = \frac{l}{R} t \quad \text{and} \quad \phi = z R, \]

the metric (1) approaches near the horizon \( (r << r_1, r_5) \)

\[ ds^2 = -f^2d\tau^2 + f^{-2}d\rho^2 + \rho^2(d\phi - \frac{J}{2\rho^2 dt})^2 + l^2d\Omega_3^2 \]

with

\[ f^2 = \frac{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}{l^2\rho^2} \quad \text{and} \quad J = 2\rho_+^{(0+)} \]

which is recognised as that of the BTZ black hole with mass \( 8G\ell^2 M = \rho_+^2 + \rho_-^2 \) and angular momentum \( 8G\ell J = 2\rho_+^{(0+)} \). Here \( \ell \) is the inverse of the cosmological constant and \( \rho_+(\rho_-) = \frac{R}{\ell} r_0 \cosh \sigma_p (\sinh \sigma_p) \).

2.1 Decay Rate

The greybody factors for scalar fields in the BTZ black hole have been obtained in \([10]\). The resulting absorption cross section is given by

\[ \sigma_{abs} = A_H \frac{\Gamma(a_+ + 1)\Gamma(a_- + 1)}{\Gamma(a_+ + a_- + 1)^2} \quad \text{where} \quad a_\pm = \frac{i\ell^2}{2(\rho_+^{(0+)} \pm \rho_-)} \left( \omega \mp \frac{m}{\ell} \right), \]

or, when expressed in terms of the 5-dimensional parameters,

\[ a_\pm = \frac{i(\omega_5 \mp \frac{m}{\ell})\ell^2}{2r_0} e^{\pm \sigma_5}, \]

hence reproducing the 5-dimensional expression \([11]\) for both, neutral- and charged emission. The result \([11]\) holds provided

\[ \omega_5^2 R^2 - m^2 << 1 \quad \text{and} \quad [\omega_5 \frac{R}{\ell} \cosh \sigma_p - \frac{m}{\ell} \sinh \sigma_p] r_0 \frac{R}{\ell} << 1. \]
It is interesting to compare (6) to the 5-dimensional string-theory condition

\[ \omega_5^2 - \left( \frac{m}{R} \right)^2 \ell^2 \ll 1. \]  

(7)

For 'fat' black holes where \( l > R \) the five dimensional condition is stronger whereas for \( l < R \) the BTZ condition is stronger. Hence, for fat black holes the near horizon region still has a conformal field theory description for frequencies \( 1/\ell < \omega_5 < 1/R \) whereas the 'outside' geometry adds 'non-conformal' effects to the black hole. In the latter case the near horizon region is not big enough to capture all effects of the underlying conformal field theory. One has to include 'outside' scattering to recover the conformal field theory description. Finally, we note that the equivalence continues to hold for charged emission where \( \sigma \) has to be replaced by \( \sigma' \).

The above discussion can be repeated for the 4-dimensional charged black hole and it is not hard to show that the 4-dimensional greybody factors are equally well encoded in the near horizon BTZ geometry. Recently it has been argued\(^{13}\) that the \( \text{AdS}_3 \)-structure holds for extremal black holes and \( p \)-brane geometries in various dimensions. We do not develop this issue further here but note that this is likely to extend the validity of the BTZ description.

\section*{3 Microscopic Theory}

The quantisation of the BTZ-horizon was first proposed by Carlip\(^12\) mapping the horizon degrees of freedom into a WZW-theory. This has lead to a microscopic computation of the entropy in terms of the horizon WZW-theory. An alternative approach\(^14\) concentrates on the asymptotic degrees of freedom. The two description should be equivalent as 3-dimensional gravity is not dynamical. In the asymptotic description one uses the fact that the asymptotic isometries of the black hole metric form a Virasoro algebra with central charge \( c = \frac{3}{2} \). Assuming that Cardy’s formula\(^15\) applies\(^b\) the statistical entropy for large black holes\(^c\) is identical with the geometrical entropy.

In order to describe the decay rate we need to know how the scalar field propagating in the gravitational background affects the boundary conformal field theory. It turns out that its effect is to introduce an interaction of the form

\[ S_{\text{int}} \propto \int dx_+ dx_- I_+ I_- e^{i(\omega + m/\ell)x_+ + i(\omega - m/\ell)x_-}, \]  

(8)

\(^b\)The applicability of Cardy’s formula in its simplest form to the present situation has been questioned recently so that this result ought to be taken with a grain of salt.

\(^c\)The derivation of this result we be presented elsewhere.
where $I_\pm$ are the left- and right moving Kac-Moody currents respectively. The transition amplitudes between various black hole states in the presence of a scalar field is therefore encoded in the set of correlation functions of Kac-Moody currents. To compute the decay rate of a highly excited black hole we sum over final states and thermally average over the initial states. The finite temperature two point function of the Kac-Moody currents is given by

$$\langle I_+(0)I_+(x_+)\rangle_T = 2 \left[ \frac{\pi T}{\sinh(\pi T x_+)} \right]^2$$

(9)
and similarly for the right moving part. The resulting emission rate precisely reproduces the semiclassical result.[4]

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