Neutrino Helioseismology

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Abstract

The observed deficit of $^8$B solar neutrinos may call for an improved standard model of the sun or an expanded standard model of particle physics (e.g., with neutrino masses and mixing). In the former case, contemporary fluid motions and thermal fluctuations in the sun’s core may modify nuclear reaction rates and restore agreement. To test this notion, we propose a search for short–term variations of the solar neutrino flux.
The observed deficit of $^8$B solar neutrinos \[1\]–\[4\] may call for an improved standard model of the sun or an expanded standard model of particle physics (\textit{e.g.}, with neutrino masses and mixing). In the former case, contemporary fluid motions and thermal fluctuations in the sun’s core may modify nuclear reaction rates and restore agreement \[5\] \[6\]. To test this notion, we propose a search for short–term variations of the solar neutrino flux.

Models of the sun fit its radius $R_\odot$ and luminosity $L_\odot$ to an assumed initial $^4$He abundance and a convective mixing length \[7\]–\[10\]. While challenged by solar–neutrino observations, they are supported by solar-surface measurements \[11\] of the frequencies of thousands of $p$–waves (pressure waves). These are inverted to yield the sound velocity at depth \[12\] \[13\]. Whilst the result agrees with solar models to better than 1\%, helioseismology provides scant information about the solar core, where $p$ waves are damped \[14\].

Solar $g$ waves (for which gravity is the restoring force) are suppressed toward the surface and difficult to see, but they may well be present. As the sun evolves, the $^3$He abundance in its core develops a positive outward gradient. This leads \[15\] \[16\] to a hydrostatic instability (often ignored in standard solar models) and to the secular growth of radially asymmetric standing $g$ waves of low order $n$ (number of radial nodes) and degree $l$ (multipole moment). Their periods $2\pi/\omega$ are of order one hour \[14\] \[17\]. Since energy–transport times are much larger, $g$ waves correspond to quasi–adiabatic temperature fluctuations about a radial mean:

$$T(r, t, \theta, \phi) = \overline{T}(r) \left[ 1 + A g(r) Y_{lm}(\theta, \phi) \sqrt{2} \cos(\omega t) \right],$$

where $A$ is the amplitude of an oscillation whose angular dependence is that of a spherical harmonic with $\int |Y|^2 d\Omega = 1$ and whose radial eigenfunction $g(r)$ has a maximum of one.

Any $g$ wave present in the sun affects its neutrino–producing processes:

$$p + p \longrightarrow D + e^+ + \nu,$$

$$^7\text{Be} + e^- \longrightarrow ^7\text{Li} + \nu,$$

$$^7\text{Be} + p \longrightarrow \gamma + ^8\text{B} \quad ^8\text{B} \longrightarrow ^8\text{Be} + e^+ + \nu,$$

which we label $a = 1, 7, 8$. Their angularly–averaged rates $\hat{\epsilon}_a(r, t)$ are:

$$\hat{\epsilon}_a(r, t) = \epsilon_a(r) \langle (T/\overline{T})^{N_a} \rangle_\Omega,$$
where the $\epsilon_a(r)$ depend on the local density, nuclear abundances and $T(r)$. The exponents in (3), $N_1 = 4$, $N_7 \simeq -0.5$ and $N_8 \simeq 13$, reflect the $T$ dependences of the reaction rates at fixed abundances [6]. Expanding (3) in powers of $A^2$, we exhibit the time dependence of the rates:

$$\hat{\epsilon}_a(r, t) = \epsilon_a(r) \left(1 + \frac{1}{2} A^2 N_a(N_a - 1) g^2(r) [1 + \cos(2\omega t)] + O(A^4)\right).$$

(4)

The constant in square brackets affects the time–averaged neutrino fluxes; the cosine generates oscillations with twice the $g$ wave frequency.

We integrate (4) over $r$ using an $n = 1$ mode with $g(r) = x \exp(1-x)$, $x = r/(0.15 R_\odot)$ [7]. In solar models [8]–[10], $\epsilon_a(r)$ are roughly of the form $\epsilon_a(r) = y^2 \exp(-y^2)$ with $y = r/(f_a R_\odot)$ and $f_a \simeq 25, 17, 10$ for $a = 1, 7, 8$. For the oscillations of the various components of the solar neutrino flux, we predict:

$$F_a \simeq \overline{F}_a [1 + \lambda_a \cos(2\omega t)], \quad \lambda_{1,7,8} \simeq (4.8, 0.21, 28.5) A^2,$$

(5)

where $\overline{F}_a$ are time–averaged fluxes. Notice that $\lambda_8 > \lambda_1 >> \lambda_7$.

According to (3), the reaction rates $\hat{\epsilon}_a$ exceed those in a steady sun with temperature profile $T(r)$. To keep $L_\odot$ fixed, the solar model must be modified to lower $T$. Gough [6] estimates how the time–averaged neutrino fluxes depart, in the presence of an $n = 1$ $g$-wave, from those of the standard model:

$$\overline{F}_1 \simeq F_1^{SSM},$$
$$\overline{F}_7 \simeq F_7^{SSM} [(1 - 33 A^2 + 267 A^4],$$
$$\overline{F}_8 \simeq F_8^{SSM} [1 - 57 A^2 + 1067 A^4].$$

(6)

The effects of the $g$ wave on the time–averaged flux (5) and its fluctuations (4) are greatest for $^8$B neutrinos. With $A = 0.1$, the $^8$B flux is reduced by 0.54 and that of $^7$Be by 0.70, removing the discrepancy between experiment and theory. We choose this value of $A$ to set the scale for anticipated neutrino flux oscillations.

Future experiments will measure arrival times $t_i$ of thousands of neutrinos. Assume that a $g$ mode of frequency $\omega$ modulates the neutrino fluxes, as in (5). The precise frequency

1 The burning of H to $^4$He is the main source of solar energy, so that $F_1$ is hardly affected. $F_7$ and $F_8$ are suppressed [8], since the effects of reducing $\overline{T}$ win over the enhancement obtained from the time average of (4).
of the $g$ wave is unknown, but its effect can be found by Fourier transforming the data over a frequency range $f_{\text{min}} < f < f_{\text{max}}$. Suppose that $n$ neutrinos are detected in a run of duration $\tau$. Let:

$$ P(f) \equiv \left| \sum_{j=1}^{n} e^{ift_j} \right|. \quad (7) $$

The signature of a $g$ wave is a peak in $P(f)$ at $f = 2\omega$, emerging even if $2\omega$ exceeds $\tau/n$, the mean counting rate. The peak’s expected magnitude is $P_s = \lambda n/2$. Its half–width at half maximum, $\Delta\omega = \sqrt{6}/\tau$, sets the required Fourier resolution. Away from the peak, $P(f)$ fluctuates about $\sqrt{n}$, exceeding $P_s$ with probability $\exp(-P_s^2/2n)$.

The minimum significant signal (with confidence level C.L.) corresponds to a $g$ wave of amplitude:

$$ \lambda_{\text{min}} = \left[ \frac{8}{n} \ln \left( \frac{f_{\text{max}} - f_{\text{min}}}{\tau} \frac{1}{1 - \text{C.L.}} \right) \right]^{1/2}. \quad (8) $$

With $\tau \sim 1$ year and $f_{\text{max}} - f_{\text{min}} \sim$ inverse minutes, the logarithm’s argument is large and its precise value immaterial.

We see from (5) and (8) that for $A \simeq 0.1$, an experiment sensitive to the $pp$ flux must observe $\sim 6 \times 10^4$ events to find a 99%-confidence effect. A real–time $pp$ neutrino detector with this capability has been discussed (Ypsilantis, T. & Seguinot, J., priv. com.). The proposed BOREX experiment (Raghavan, R. et al., Bell Laboratory Report No. ATT-BX-88-01 (1988)) could detect a million $^7$Be neutrinos, but falls short of the $\sim 3 \times 10^7$ events needed to detect the tiny oscillations expected in this case.

Fewer events suffice to detect oscillations of the $^8$B neutrino flux. The Sudbury Neutrino Observatory [18], Super–Kamiokande [19] and Icarus [20] each will time thousands of these neutrinos. We deduce from (5) and (8) that an experiment gathering 3000 (30,000) events can find an $A = 0.08 (0.05)$ signal with 99% confidence, decisively testing whether the suppression of the $^8$B neutrino flux is due to a single $g$–mode.\footnote{If the $^8$B neutrino deficit results from several $g$ modes rather than a dominant one, their Fourier powers are smaller and a model–independent search becomes more difficult.}

If neutrino experiments were to detect the sun’s heartbeat, it is the sun that oscillates, not the neutrino.

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References

[1] Davis Jr., R. et al. in Proceedings of the 21st International Cosmic Ray Conference (ed Protheroe, R.J.) 155–158 (University of Adelaide Press, Adelaide 1990).

[2] Hirata, K.S. et al., Phys. Rev. D44, 2241–2260 (1991).

[3] GALLEX Collaboration (Anselman, P. et al.), Phys. Lett. B285, 376–389 (1992).

[4] Abazov, A.I. et al., Phys. Rev. Lett. 67, 3332–3335 (1991).

[5] Roxburgh, I.W. in The Internal Solar Angular Velocity (ed Durney, B.R. & Sofia, S.) 1–5 (Reidel, Dordrecht 1987).

[6] Gough, D.O., Ann. N.Y. Acad. Sci. (in press).

[7] Schwarzchild, M., Howard, R. & Härm, R., Astroph. J. 125, 233–241 (1957).

[8] Bahcall, J.N. & Ulrich, R.K., Rev. Mod. Phys. 60, 297-372 (1988).

[9] Turk–Chièze, S. in Inside the Sun (eds Berthomieu, G. & Cribier, M.) 125–132 (Kluwer Acad. Pub., Dordrecht 1990).

[10] Christensen–Dalsgaard, J., Gough, D.O. & Thompson, M.J., Astroph. J. 378, 413–437 (1991).

[11] Duvall Jr., T. L. in Inside the Sun, op. cit. 253-264.

[12] Gough, D.O., Solar Physics 100, 65–99 (1985).

[13] Gough, D.O. & Thompson, M.J. in Solar Interior and Atmosphere, (ed Cox, A.N., Livingstone, W.C. & Mathews, M.) 401–478 (Space Science Series, University of Arizona Press, Tuscon 1991).

[14] Christensen–Dalsgaard, J. & Berthomieu, G. in Solar Interior and Atmosphere, ibid. 519-561.

[15] Dilke, F.W.W. & Gough, D.O., Nature 240, 262–264 cont’d 293–294 (1972).

[16] Merryfield, W.J., Toomre J. & Gough, D.O., Astroph. J. 367, 658–665 (1991).

[17] Hill, H. et al. in Solar Interior and Atmosphere, op. cit. 562–617.

[18] Beier, E. in Proc. of the International Symposium on Underground Physics Experiments (ed Nakamura, K.) 165–170 (Institute for Cosmic Ray Research, University of Tokyo 1990).

[19] Totsuka, Y., ibid. 129–164.

[20] Baldo–Ceolin, M. in Massive Neutrinos in Particle Physics and Astrophysics (ed Fackerler, O.W. & Tran Thanh Van, J.) 159–164 (Editions Frontières, Gif sur Yvette 1986).