Parity-violating two-pion exchange nucleon-nucleon interaction

N. Kaiser

Physik Department T39, Technische Universität München, D-85747 Garching, Germany

Abstract

We calculate in chiral perturbation theory the parity-violating two-pion exchange nucleon-nucleon potentials at leading one-loop order. At a distance of $r = m^{-1} \pi \simeq 1.4$ fm they amount to about $\pm 16\%$ of the parity-violating $1\pi$-exchange potential. We evaluate also the parity-violating effects arising from $2\pi$-exchange with excitation of virtual $\Delta(1232)$-isobars. These come out to be relatively small in comparison to those from diagrams with only nucleon intermediate states. The reason for this opposite behavior to the parity-conserving case is the blocking of the dominant isoscalar central channel by CP-invariance. Furthermore, we calculate the T-matrix related to the iteration of the parity-violating $1\pi$-exchange with the parity-conserving one. The analytical results presented in this work can be easily implemented into calculations of parity-violating nuclear observables.

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Nuclear parity violation is an important tool to study the standard model of strong and electroweak interactions [1, 2]. In the two-nucleon system parity violation has been traditionally represented by parity-violating one-meson exchange where the strong and weak interactions are parametrized through parity-conserving and parity-violating meson-nucleon vertices. Due to CP-invariance, there exists no parity-violating coupling of neutral scalar or pseudoscalar mesons to nucleons (Barton’s theorem). Therefore, standard parametrizations of the parity-violating NN-potential involve the exchange of charged pions ($\pi^\pm$) as well as vector mesons ($\rho^\pm, 0$ and $\omega$). The pertinent parity-violating meson-nucleon coupling constants $h_{\pi, \rho, \omega}(0, 1, 2)$ have been calculated in quark models [3] and soliton models [4]. Due to a variety of uncertainties (e.g. quark wavefunctions, strong interaction enhancements, choice of particular soliton model) only “reasonable” ranges could be derived so far. For the parity-violating $\pi NN$-coupling constant $h_\pi$ an experimental upper bound is known to be $|h_\pi| < 1.43 \cdot 10^{-7}$ [1]. An improved determination of this coupling constant is expected from a measurement of the photon asymmetry $A_\gamma$ in the radiative capture of thermal neutrons $\vec{n}p \rightarrow d\gamma$ [5, 6, 7].

Recently, nuclear parity violation has been reformulated in the framework of effective field theory [8, 9]. As in the case of the parity-conserving NN-interaction one exploits the separation of scales. At low energies only the pions are kept as explicit degrees of freedom while all heavier particles are integrated out and their dynamical effects are subsumed in contact terms. At very low energies $E_{\text{cm}} < 10$ MeV even the pion can be integrated out and then one is working with the pionless version of the effective field theory for nuclear parity violation. It has been shown in ref.[8] that at leading order $O(q)$ there are in total five parity-violating low-energy constants associated with the respective contact operators linear in the nucleon momenta. One therefore requires a minimum of five independent, low-energy observables. For the effective field theory with dynamical pions there appear up to order $O(q)$ three more parameters: the parity-violating $\pi N$-coupling constant $h_\pi$, a next-to-next-to-leading order correction to it, and a new electromagnetic operator [8]. This new electromagnetic operator is a specific feature of
the systematic effective field theory framework and entirely absent in the one-meson exchange phenomenology.

Clearly, a lot of work is still necessary e.g. in order to pin down the parity-violating low-energy constants and thus to reach a level where the effective field theory framework becomes predictive. In that situation it seems worthwhile to investigate separately the hierarchy of long-range pion-induced parity-violating NN forces. This is the purpose of the present short paper. We will compare directly the parity-violating $2\pi$-exchange potentials at leading one-loop order with the parity-violating $1\pi$-exchange. For the parity-conserving NN-potential the $2\pi$-exchange with excitation of the low-lying spin-isospin-$3/2$ $\Delta(1232)$-resonance plays a major role. The corresponding potentials exceed the ones from diagrams with only nucleon intermediate states typically by an order of magnitude [10, 11]. (Recently, it has been found that this feature remains when electromagnetic (one-photon exchange) corrections are included [12].) Therefore it is important to check whether a similar dominance of the $\Delta$-induced processes holds for the parity-violating $2\pi$-exchange interaction. We find that this is not the case. The basic reason for this opposite behavior is CP-invariance which forbids parity-violating effect in the isoscalar central channel. As a further possibly sizeable one-loop contribution we calculate the $T$-matrix related to the iteration of the parity-violating $1\pi$-exchange with the parity-conserving one.

Let us begin with recalling the parity-violating pion-nucleon vertex. It has the form:

$$\mathcal{L}_{\text{pv}} = \frac{h_\pi}{\sqrt{2}} \bar{N}(\vec{\pi} \times \vec{\tau})^3 N, \quad (1)$$

where $N$ denotes a nucleon Dirac-spinor and $h_\pi \sim 10^{-7}$ is the weak $\pi NN$-coupling constant. In the center-of-mass frame the $T$-matrix of parity-violating $1\pi$-exchange is readily computed as:

$$T_{\text{pv}}^{(1\pi)} = -i \frac{g_A h_\pi}{2\sqrt{2} f_\pi} (\vec{\tau}_1 \times \vec{\tau}_2)^3 \left( \frac{\vec{\sigma}_1 + \vec{\sigma}_2}{m_\pi^2 + q^2} + O(M_N^{-2}) \right). \quad (2)$$

Here, $g_A \simeq 1.3$ is the nucleon axial vector coupling constant and $f_\pi = 92.4$ MeV denotes the pion decay constant. $\vec{\sigma}_{1,2}$ and $\vec{\tau}_{1,2}$ are the usual spin- and isospin-operators of the two nucleons and $\vec{q}$ stands for the momentum transfer. Note that relativistic corrections start at order $M_N^{-2}$ (with $M_N = 939$ MeV the nucleon mass) and therefore are negligibly small (typically less than 1%). Fourier transformation of the static term in Eq.(2) leads to a parity-violating NN potential in coordinate space:

$$V_{\text{pv}}(\vec{r}) = (\vec{\tau}_1 \times \vec{\tau}_2)^3 \left( \frac{\vec{\sigma}_1 + \vec{\sigma}_2}{m_\pi^2 + q^2} + O(M_N^{-2}) \right) \cdot \hat{r} U(r), \quad (3)$$

with the radial dependence given by the derivative of a Yukawa function:

$$U(r)^{(1\pi)} = -\frac{g_A h_\pi}{8\sqrt{2\pi} f_\pi} \frac{e^{-m_\pi r}}{r^2} \frac{1 + m_\pi r}{(1 + m_\pi r)}. \quad (4)$$

The second term in Eq.(3) proportional to the cross product $\vec{\sigma}_1 \times \vec{\sigma}_2$ of spin-operators has also been introduced because the associated “vector-type” potential $W(r)$ does receive contributions from parity-violating $2\pi$-exchange.

Next, we come to the parity-violating two-pion exchange. Some representative diagrams with leading order vertices are shown in Fig.1. These are to be supplemented by further diagrams with the parity-violating vertex at a different position and/or both nucleon lines interchanged. We are interested only in the nonpolynomial or finite-range parts of these one-loop diagrams (disregarding the zero-range $\delta^3(\vec{r})$-terms from the polynomial pieces). For that purpose it is sufficient to calculate their spectral functions or imaginary parts using the Cutkosky cutting rule. The pertinent two-body phase space integral is most conveniently performed
in the $\pi\pi$ center-of-mass frame where it becomes proportional to a simple angular integral:

$$\sqrt{\mu^2 - 4m_{\pi}^2/(32\pi\mu)} \int_1^1 dx,$$

with $\mu > 2m_{\pi}$ the $\pi\pi$ invariant mass. Using this technique we get from the leading one-loop diagrams in Fig. 1:

$$\text{Im} T^{(2\pi)}_{pv} = \frac{g_A h_{\pi} \sqrt{\mu^2 - 4m_{\pi}^2}}{64\sqrt{2}\pi f_{\pi}^2 \mu} \left\{ i(\vec{\tau}_1 \times \vec{\tau}_2)^3 (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{q} \left[ g_A^2 \frac{3\mu^2 - 8m_{\pi}^2}{\mu^2 - 4m_{\pi}^2} - 1 \right] 
- 4g_A^2 i(\tau_3^1 + \tau_3^2)(\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \vec{q} \right\}. \quad (5)$$

The notation $\text{Im} T^{2\pi}_{pv}$ is meant here such that one is taking the imaginary part of the loop functions standing to the right of the spin- and isospin factors. For the box diagrams with two nucleon propagators one encounters the nontrivial angular integral:

$$\int_1^1 dx (x - i0)^{-2} = -2.$$

The one-loop $T$-matrix in momentum space can be reconstructed from the spectral function via a once-subtracted dispersion relation:

$$T^{(2\pi)}_{pv} = -\frac{2g^2}{\pi} \int_{2m_{\pi}}^{\infty} d\mu \frac{\text{Im} T^{(2\pi)}_{pv}}{\mu(\mu^2 + q^2)}, \quad (6)$$

and it agrees (up to an irrelevant subtraction constant) with the result of ref.[8] (see Eqs.(120-122) therein). With the help of the mass spectra in Eq.(5) one can also calculate directly the parity-violating $2\pi$-exchange potential in coordinate space. One finds the following radial dependences of the scalar-type potential:

$$U(r)^{(2\pi)} = \frac{g_A h_{\pi} m_{\pi}}{\sqrt{2}(4\pi f_{\pi} r)^3} \left\{ (8g_A^2 - 2)m_{\pi} r K_0(2m_{\pi} r) + (4g_A^2 m_{\pi}^2 r^2 + 9g_A^2 - 3) K_1(2m_{\pi} r) \right\}, \quad (7)$$

and the vector-type potential:

$$W(r)^{(2\pi)} = -\frac{g_A^3 h_{\pi} m_{\pi}}{(2\sqrt{2}\pi f_{\pi} r)^3} \left\{ 2m_{\pi} r K_0(2m_{\pi} r) + 3 K_1(2m_{\pi} r) \right\}, \quad (8)$$

with $K_{0,1}(2m_{\pi} r)$ two modified Bessel functions. Their asymptotic behavior for large distances $r$ is: $U(r)^{(2\pi)} \sim e^{-2m_{\pi} r} r^{-3/2}$ and $W(r)^{(2\pi)} \sim e^{-2m_{\pi} r} r^{-5/2}$. Moreover, in the chiral limit $m_{\pi} = 0$ these potentials follow a simple $r^{-4}$ form, a feature which could also be guessed by counting mass dimensions.

We are now in the position to present numerical results. The numbers in the first and second row of Table I give the ratio between the parity-violating $2\pi$-exchange potentials $U(r)^{(2\pi)}$ and $W(r)^{(2\pi)}$ and the parity-violating $1\pi$-exchange potential $U(r)^{(1\pi)}$ for distances $1.0 \text{ fm} \leq r \leq 3$.
Table I: Ratio of parity-violating $2\pi$-exchange potentials to the parity-violating $1\pi$-exchange potential $U(r)^{(1\pi)} \sim e^{-m_{\pi}r} (1 + m_{\pi}r)/r^2$ as a function of the nucleon distance $r$.

| $r$ [fm] | 1.0  | 1.1  | 1.2  | 1.3  | 1.4  | 1.5  | 1.6  | 1.7  | 1.8  | 1.9  |
|----------|------|------|------|------|------|------|------|------|------|------|
| $U^{(2\pi)}/U^{(1\pi)}$ | -0.332 | -0.271 | -0.224 | -0.188 | -0.159 | -0.136 | -0.117 | -0.102 | -0.089 | -0.078 |
| $W^{(2\pi)}/U^{(1\pi)}$ | 0.420 | 0.331 | 0.265 | 0.215 | 0.177 | 0.146 | 0.122 | 0.103 | 0.087 | 0.074 |
| $U^{(\Delta)}/U^{(1\pi)}$ | -0.190 | -0.134 | -0.097 | -0.072 | -0.055 | -0.042 | -0.033 | -0.026 | -0.021 | -0.017 |
| $W^{(\Delta)}/U^{(1\pi)}$ | 0.322 | 0.229 | 0.167 | 0.125 | 0.095 | 0.073 | 0.057 | 0.045 | 0.036 | 0.029 |

1.9 fm (see also Fig. 2). One sees that in the region around the pion Compton wavelength $r = m_{\pi}^{-1} \simeq 1.4$ fm this ratio varies between 10% and 25% (with opposite sign for the scalar and vector type potential). Such a suppression of the parity-violating $2\pi$-exchange interaction as deduced here from the ratio of coordinate space potentials is quantitatively consistent with the results of ref.[5]. There the combined effect of a cut-off regularized $2\pi$-exchange, short-distance counterterms and the new electromagnetic operator on the photon asymmetry $A_{\gamma}(\vec{n}p \rightarrow d\gamma)$ has been quantified as a (minus) 10 $\sim$ 20% correction to the dominant one-pion exchange contribution.

Fig. 2: Ratios of parity-violating $2\pi$-exchange potentials to the parity-violating $1\pi$-exchange.

Fig. 3: Diagrams related to parity-violating $2\pi$-exchange with single $\Delta(1232)$-isobar excitation.

As mentioned in the introduction the parity-conserving $2\pi$-exchange interaction does not come primarily from leading order diagrams as in Fig. 1. The by far dominant contribution
arises from processes where the low-lying $\Delta(1232)$-resonance is excited in the intermediate state [10]. Equivalently, one can obtain this dominant contribution from $\pi\pi NN$ contact vertices proportional to the large low-energy constants $c_3$ and $c_4$ (which effectively include the important $\Delta$-dynamics). It is therefore important to check whether a similar dominance of the $\Delta$-induced $\pi N\Delta$-coupling since it is of higher order than the parity-violating exchange diagrams with single $\Delta(1232)$-isobar excitation. We do not consider a parity-violating state [10]. Equivalently, one can obtain this dominant contribution from processes where the low-lying $\Delta(1232)$-resonance is excited in the intermediate

$$\frac{g_A^3 h_\pi}{64\sqrt{2}\pi f_{\pi}^2 \mu} \left\{ i(\vec{\tau}_1 \times \vec{\tau}_2)^3 (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{q} \left[ -\sqrt{\mu^2 - 4m_\pi^2} + \frac{1}{\Delta}(\mu^2 + 2\Delta^2 - 2m_\pi^2) \right] ight. $$

$$ \times \arctan\left( \frac{\sqrt{\mu^2 - 4m_\pi^2}}{2\Delta} \right) + i(\tau_1^3 + \tau_2^3) (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \vec{q} \left[ \frac{\pi}{4\Delta}(4m_\pi^2 - \mu^2) \right]$$

$$ + 2\sqrt{\mu^2 - 4m_\pi^2} + \frac{1}{\Delta}(4m_\pi^2 - 4\Delta^2 - \mu^2) \arctan\left( \frac{\mu^2 - 4m_\pi^2}{2\Delta} \right) \right\}, \quad (9)$$

where the term $\arctan(\sqrt{\mu^2 - 4m_\pi^2}/2\Delta)$ is typical for a $\Delta(1232)$-isobar propagating in a (pion) loop [10]. Here, $\Delta = 293$ MeV denotes the delta-nucleon mass splitting. It is treated as a small scale comparable to the pion mass $m_\pi$ and the momentum transfer $q$. In Eq.(9) we have already inserted the empirically well-satisfied relation $g_{\pi N\Delta} = 3g_A M_N/\sqrt{2}f_\pi$ for the $\pi N\Delta$-coupling constant. With the help of the spectral $\text{Im} T_{pv}^{(\Delta)}$ one can calculate the T-matrix in momentum space, using Eq.(6), or the scalar and vector type potentials in coordinate space:

$$ U(r)^{(\Delta)} = \frac{g_A^3 h_\pi}{(4\sqrt{2}\pi f_{\pi}^2)^{3/2}} \int_{2m_\pi}^{\infty} d\mu e^{-\mu r} \left[ -\sqrt{\mu^2 - 4m_\pi^2} ight. $$

$$ \left. + \frac{1}{\Delta}(\mu^2 + 2\Delta^2 - 2m_\pi^2) \arctan\left( \frac{\mu^2 - 4m_\pi^2}{2\Delta} \right) \right], \quad (10)$$

$$ W(r)^{(\Delta)} = \frac{g_A^3 h_\pi}{(4\sqrt{2}\pi f_{\pi}^2)^{3/2}} \int_{2m_\pi}^{\infty} d\mu e^{-\mu r} \left[ \frac{\pi}{4\Delta}(4m_\pi^2 - \mu^2) \right. $$

$$ \left. + 2\sqrt{\mu^2 - 4m_\pi^2} + \frac{1}{\Delta}(4m_\pi^2 - 4\Delta^2 - \mu^2) \arctan\left( \frac{\mu^2 - 4m_\pi^2}{2\Delta} \right) \right], \quad (11)$$

The first term in square brackets of Eq.(11) is the leading one in an expansion in powers of $1/\Delta$ and it makes up about 60% of the total vector type potential $W(r)^{(\Delta)}$. The functional form of this piece reads: $-\mu e^{-2m_\pi r}(1 + m_\pi r^2)/r^5$. It is also interesting to note that if one would work with $\pi\pi NN$ contact vertices proportional to the large low-energy constants $c_{3,4}$ (which represent partly the $\Delta$-dynamics) one would get a nonvanishing contribution only from $c_4$. It corresponds precisely to this leading $1/\Delta$ term just mentioned.

Let us again turn to numerical results. The numbers in the third and fourth row of Table I give the ratio between the parity-violating $2\pi$-exchange potentials $U(r)^{(\Delta)}$ and $W(r)^{(\Delta)}$ and the parity-violating $1\pi$-exchange potential $U(r)^{(1\pi)}$. First, one observes that the $\Delta$-induced potentials are of the same sign as the ones from diagrams with only nucleon intermediate states. Secondly, they are suppressed by a factor 2 to 4 (in the region around the pion Compton
wavelength \( r = m_\pi^{-1} \). This behavior is completely opposite to the parity-conserving case. The primary reason for that is CP-invariance which excludes parity-violating effects in the isoscalar central channel (Barton’s theorem). We note once more that the \( c_3 \) contact vertex which is solely responsible for strong isoscalar central NN-attraction leads to a vanishing contribution to the parity-violating \( 2\pi \)-exchange. We can therefore conclude that the calculation of ref.[5] which aims at an improved determination of the weak \( \pi NN \)-coupling constant \( h_\pi \) from the photon asymmetry \( A_\gamma(\bar{n}p \rightarrow d\gamma) \) is not affected much by the inclusion of \( \Delta \)-induced \( 2\pi \)-exchange corrections. 

The (right) planar box diagram in Fig.1 contains also a two-nucleon reducible piece: the iteration of the parity-violating \( 1\pi \)-exchange with the parity-conserving \( 1\pi \)-exchange. It is distinguished by the feature that its energy denominator is given by the difference of nucleon kinetic energies. This enhances the corresponding T-matrix by a large scale factor, the nucleon mass \( M_N = 939 \text{ MeV} \). For the center-of-mass kinematics \( N_1(\vec{p}) + N_2(-\vec{p}) \rightarrow N_1(\vec{p}^\prime) + N_2(-\vec{p}^\prime) \) we obtain the following expression for the T-matrix:

\[
T_{\text{pv}} = \frac{g_\Delta^3 h_\pi M_N}{\pi \sqrt{2}(4f_\pi)^3} \left\{ (\vec{\tau}_1 \times \vec{\tau}_2)^3 \left[ i(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{q} \left( (2m_\pi^2 + q^2)G_0 - 2\Gamma_0 \right) \right. \right.
\]
\[\left. + (\vec{\sigma}_1 \cdot \vec{q} \vec{\sigma}_2) \cdot \vec{q} \times \vec{p} + \vec{\sigma}_2 \cdot \vec{q} (\vec{\sigma}_1 \cdot \vec{q} \times \vec{p}) (2G_0 + 4G_1) \right] 
\]
\[+ 2(\vec{\tau}_1^3 - \vec{\tau}_2^3) \left[ (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot (\vec{p} + \vec{p}^\prime) \left( 2\Gamma_0 + 2\Gamma_1 - (2m_\pi^2 + q^2)(G_0 + 2G_1) \right) \right. \]
\[\left. + i(\vec{\sigma}_1 \cdot (\vec{p} + \vec{p}^\prime) \vec{\sigma}_2 \cdot \vec{q} \times \vec{p} + \vec{\sigma}_2 \cdot (\vec{p} + \vec{p}^\prime) \vec{\sigma}_1 \cdot \vec{q} \times \vec{p}) (2G_0 + 8G_1 + 8G_3) \right\}, \tag{12}
\]
where \( p = |\vec{p}| = |\vec{p}^\prime| \) is the center-of-mass momentum and \( \vec{q} = \vec{p}^\prime - \vec{p} \) the momentum transfer. The occurring complex-valued loop functions read:

\[
\Gamma_0(p) = \frac{1}{4p} \left[ 2 \arctan \frac{2p}{m_\pi} + i \ln \left( 1 + \frac{4p^2}{m_\pi^2} \right) \right], \tag{13}
\]
\[
\Gamma_1(p) = \frac{1}{2p^2} \left[ m_\pi + i p - (m_\pi^2 + 2p^2)\Gamma_0(p) \right], \tag{14}
\]
\[
G_0(p, q) = \frac{1}{q \sqrt{m_\pi^4 + p^2(4m_\pi^2 + q^2)}} \left[ \arcsin \frac{qm_\pi}{\sqrt{(m_\pi^2 + 4p^2)(4m_\pi^2 + q^2)}} \right. \right.
\]
\[\left. + i \ln \frac{p q + \sqrt{m_\pi^4 + p^2(4m_\pi^2 + q^2)}}{m_\pi \sqrt{m_\pi^2 + 4p^2}} \right], \tag{15}
\]
\[
G_1(p, q) = \frac{\Gamma_0(p) - q^{-1} \arctan \frac{q}{2m_\pi} - (m_\pi^2 + 2p^2)G_0(p, q)}{4p^2 - q^2}, \tag{16}
\]
\[
G_3(p, q) = \frac{1}{2} \frac{\Gamma_1(p) - p^2G_0(p, q) - 2(m_\pi^2 + 2p^2)G_1(p, q)}{4p^2 - q^2}, \tag{17}
\]
where \( \Gamma_{0,1}(p) \) originate from loop integrals with one pion propagator and \( G_{0,1,3}(p, q) \) from loop integrals with two pion propagators. Interestingly, the one-loop T-matrix \( T_{\text{pv}} \) in Eq.(12) related to iterated \( 1\pi \)-exchange is ultraviolet convergent, since the parity-violating \( \pi N \)-vertex is momentum independent. \( T_{\text{pv}} \) represents a complex-valued unitarity correction which cannot be Fourier-transformed into a coordinate space potential. This inhibits a direct comparison with the parity-violating \( 1\pi \)- and \( 2\pi \)-exchange potentials. It should also be noted that \( T_{\text{pv}} \) is
partly accounted for by the use of realistic nucleon wavefunctions in perturbative calculations of parity-violating observables. Nevertheless, its explicit inclusion may be desirable under certain circumstances.

In summary, we have calculated in this work the long-range tail of the parity-violating $2\pi$-exchange interaction. The leading order diagrams with only nucleon intermediate states amount to a $10 - 20\%$ correction of the parity-violating $1\pi$-exchange. The diagrams with excitation of virtual $\Delta$-isobars are significantly suppressed. This completely opposite behavior to the parity-conserving case is a consequence of CP-invariance which forbids parity-violating effects in the isoscalar central channel. The analytical results presented in this work can be easily implemented into calculations of parity-violating nuclear observables.

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