Disentangling the Black Hole Vacuum

Sabine Hossenfelder∗

Nordita
KTH Royal Institute of Technology and Stockholm University
Roslagstullsbacken 23, SE-106 91 Stockholm, Sweden

Abstract

We study the question whether disentanglement of Hawking radiation can be achieved with any local operation. We assume that the operation we look for is unitary and can be described by a Bogoliubov transformation. This allows to formulate requirements on the operation of disentanglement. We then show that these requirements can be fulfilled by a timelike boundary condition in the near-horizon area and that the local observer does not notice the presence of the boundary and does not encounter a firewall.

1 Introduction

The black hole information loss problem, ignited by Hawking’s seminal work [1][2], has kept theoretical physicists busy for more than three decades now. A new spin was put on this problem in a recent paper [3] (hereafter referred to as AMPS) that drew attention to an apparent inconsistency in one of the most popular solution attempts to the black hole information loss problem, black hole complementarity [4]. Black hole complementarity attempts to recover information that fell into the black hole by allowing it to both fall in and remain available outside. AMPS argued, using very few assumptions, that a local observer at the horizon must notice if the radiation outgoing to infinity is not a thermal mixed state. If black hole complementarity was correct, so the argument, the local observer would encounter a ‘firewall’, implying an inconsistency with the equivalence principle. This inconsistency challenges our present understanding of black hole physics and consequently has drawn a lot of attention. For a summary and discussion of many follow-up works please refer to [5].

As previously pointed out in [6], the central assumption used by AMPS is that the entanglement necessary for the purity of the outgoing radiation is of a specific form and in particular contains correlations over long times. This assumption is reasonable in that

∗hossi@nordita.org
this is a state one ‘typically’ expects, but raises the question of whether it is the correct way to purify the mixed state.

In [7] it was suggested to resolve the black hole firewall puzzle by disentangling the Hawking modes inside and outside the horizon which realizes a specific purification procedure. The disentanglement was proposed to be achieved by the operation of a CNOT swap that mixes annihilation and creation operators in a suitable way. This swap is assumed to affect only the local observer. The result is that the state at $I^+$ can be pure even though the infalling observer does not notice any deviation from the normally mixed radiation.

However, besides the question of how, physically, such a swap would happen for the local observer only and how to interpret it, it also remains to be seen whether the CNOT swap itself is compatible with the postulates of black hole complementarity that were by AMPS claimed to be internally incompatible. In particular one may worry whether the swap can be achieved by any local operation or if at least the necessary non-locality can be confined to a region sufficiently close by the event horizon.

This paper addresses the question what has to happen in the near horizon region to remove the firewall and to have the outgoing radiation be pure. This question can be formulated as a requirement on the Bogoliubov transformations, which can be read as a boundary condition that enables an interaction. We will see that the swap that is necessary to remove the firewall can indeed be created locally and in the near horizon region, and this boundary and its effect can be interpret as the stretched horizon.

Alas, it also will become clear that the same operation that removes the firewall when the state at $I^-$ is the vacuum state, cannot also be used to transfer infalling information into the outgoing radiation. It thus remains to be seen whether information can be preserved for arbitrary initial states, including the information of the infalling observer themselves. The aim of this paper is thus not to solve the black hole information loss problem but to identify conditions for the absence of the firewall.

We use units in which $m_{\text{pl}} = \hbar = c = 1$.

2 Nomenclature

As usual we use advanced and retarded time coordinates. We denote with $(U, V)$ the coordinates inside the collapsing matter distribution, and with $(u, v)$ the coordinates outside, where

$$v = r^* + t, \quad u = r^* - t, \quad r^* = r + 2M + 2M \ln\left((r - 2M)/2M\right),$$

and $r, t$ are the standard Schwarzschild-coordinates. The surface of the collapsing matter is located at $R(u, v) = 0$ (see Figure 1). A complete set of in-modes that solve the wave-equation of a massless scalar field $\Phi$ in the black hole background can be composed using
up-modes and dn-modes (following the notation of [8])

\[ \begin{align*}
\phi_{\text{dn}}(v) & \sim \Theta(v) e^{-i\omega \ln(\kappa v)/\kappa}, \\
\phi_{\text{up}}(v) & \sim \Theta(-v) e^{i\omega \ln(-\kappa v)/\kappa},
\end{align*} \]

(2) (3)

where \( \Theta \) is the Heaviside step function, \( 4M = 1/\kappa \), and normalizations are omitted because we will not need them (we only care about the difference to the standard case). The up-modes cover only the region before the last escaping ray at \( v = 0 \), and the dn-modes only cover the space after that ray.

Here and in the following we will suppress an index \( \vec{k} \) referring to the momentum of the mode, or \( \omega, l, m \) referring to its frequency and spherical harmonics composition respectively. One thus has to keep in mind that the following vectors and matrices are a compressed notation that represents infinitely many entries. We will neglect spherical harmonics and backscattering on the potential wall.

It is common to use the linear combinations

\[ \begin{align*}
\phi_{\text{d}}(v) & = c (\phi_{\text{dn}} + w \bar{\phi}_{\text{up}}) , \\
\phi_{\text{p}}(v) & = c (\phi_{\text{up}} + w \bar{\phi}_{\text{dn}}) \\
w & = e^{-\pi \omega/\kappa} , \\
c & = (1 - w^2)^{-1/2},
\end{align*} \]

(4) (5)

(a bar denotes complex conjugation) for the in-modes, which is sometimes referred to as Wald’s base [11]. The usefulness of these (orthogonal) modes comes from them being of positive frequency with respect to \( v \), a consequence of the equalities

\[ \int_{-\infty}^{\infty} dv e^{-i\omega v} \phi_{\text{d}}(v) = \int_{-\infty}^{\infty} dv e^{-i\omega v} \phi_{\text{p}}(v) = 0 . \]

(6)

In these integrals, the prefactor \( w \) eats up the branch-cut contribution of the logarithm, resulting in a cancellation of the two parts of the integrals over \( v > 0 \) and \( v < 0 \). For details, see [8], Appendix H.4. Since \( \phi_{\text{d}} \) and \( \phi_{\text{p}} \) are of positive frequency, one can use them to define the in-vacuum while at the same time simplifying the Bogoliubov-transformations. The field can be expressed in this basis as

\[ \Phi_{\text{in}}^T = (\phi_{\text{d}}, \phi_{\text{p}}) , \]

(7)

where T denotes transposition and is just used for better display of the vector. The corresponding annihilation and creation operators will be denoted as

\[ \begin{align*}
a_{\text{in}} & = (a_{\text{d}}, a_{\text{p}}) , \\
a_{\text{in}}^\dagger & = (a_{\text{d}}^\dagger, a_{\text{p}}^\dagger)
\end{align*} \]

(8)

and are by default row-vectors. For the out-base one can use

\[ \Phi_{\text{out}}^T = (\phi_{\text{out}}, \phi_{\text{dn}}) , \]

(9)

where \( \phi_{\text{out}} \sim e^{-i\omega u} \). The in-base and out-base are related by the Bogoliubov transformation

\[ \Phi_{\text{in}} = \bar{A}^T \Phi_{\text{out}} - \bar{B}^T \Phi_{\text{out}} . \]

(10)
Without backscattering, these matrices can be calculated from the scalar products of the in-modes and out-modes and are \[8\]

\[
\mathbf{A} = c \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{B} = -wc \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.
\] (11)

The important physical combinations of the Bogoliubov matrices are firstly

\[
\mathbf{B}^\dagger \mathbf{B}^T = c^2 w^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.
\] (12)

The out-out component of \(\mathbf{B}^\dagger \mathbf{B}^T\) gives the particle-spectrum of the in-vacuum for the out-observer

\[
\langle 0_{\text{in}} | a_{\text{out}}^\dagger a_{\text{out}} | 0_{\text{in}} \rangle = (\mathbf{B}^\dagger \mathbf{B}^T)_{\text{out, out}} = c^2 w^2 = \frac{1}{\exp(2\pi \omega/\kappa) - 1}.
\] (13)

Here \( |0_{\text{in}}\rangle\) denotes the vacuum annihilated by \(a_{\text{in}}^\dagger\). One finds the expected thermal spectrum with temperature \(T = \kappa/(2\pi)\).

And secondly, the \(S\)-matrix can be written as \[8\]

\[
|0_{\text{in}}\rangle = S |0_{\text{out}}\rangle = e^{iW} e^F \exp \left( \frac{1}{2} a_{\text{out}}^\dagger \mathbf{V} (a_{\text{out}}^\dagger)^T \right) |0_{\text{out}}\rangle, \quad \mathbf{V} = -\mathbf{B} (\mathbf{A}^{-1}).
\] (14)

Here \(W\) is a pure phase and not relevant in the following, and

\[
\mathbf{F} = \frac{1}{2} a_{\text{out}}^\dagger \mathbf{A}^{-1} \mathbf{B} a_{\text{out}}^T + a_{\text{out}}^\dagger \left( (\mathbf{A}^{-1})^T - 1 \right) a_{\text{out}}^T
\] (15)

will not contribute to the transition element after normal ordering. With the above it is

\[
\mathbf{A}^{-1} = \frac{1}{c} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},
\] (16)

and thus

\[
\mathbf{V} = w \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.
\] (17)

The relevant property that is eventually responsible for the outgoing state being mixed is that \(\mathbf{V}\) is non-diagonal. Its non-vanishing entries are in the out-dn and dn-out components. This means that the out-vacuum contains entangled pairs of particles in the out-modes and dn-modes. If the dn-modes behind the horizon are lost to the outside observer, the state at \(I^+\) is mixed and lacks information.
3 Requirements

Whatever happens in the near horizon region, if unitarity is to be maintained, it must induce a suitable Bogoliubov transformation from the usual out-state into a new out-state

\[ \tilde{\Phi}_{\text{out}} = C\Phi_{\text{out}} + D\bar{\Phi}_{\text{out}} \]  

(18)

Our aim is now to identify exactly what properties this transformation must have.

1. The operation we are looking for should change the correlations in the radiation received at \( I^+ \), but not the spectrum. If it would alter the spectrum, one would run into the same problem as pointed out by AMPS, namely that, when tracing the modes from \( I^+ \) to the vicinity of the horizon, they will significantly deviate from ‘empty’ space (on scales below the curvature radius) and the local observer will notice.

The reason for this problem goes back to the divergence of the stress-energy, which is the relevant observable. As has been shown in [12, 13], the renormalized vacuum expectation value of the stress-energy tensor in the near horizon region is finite for the local observer if and only if the spectrum at \( I^+ \) is the normal thermal spectrum with temperature determined by the black hole’s mass. Any deviations from this thermal spectrum, regardless how small, will be blueshifted and generate drama for an observer who has to pass through. The stress-energy is determined by the spectral distribution of the modes and the absence of drama requires that this distribution should not be altered. This means that the operation we are looking for should not create additional particles, i.e., it should not mix positive and negative frequencies of the outgoing radiation, and thus \( D = 0 \), and \( C^{-1} = C^T \). The transformation (18) will then necessarily mix positive and negative frequencies of the ingoing state.

2. The transformation (18), when combined with the normal transformation (10), should no longer result in a mixing of dn-modes and out-modes. With \( \tilde{A} := CA \) and \( \tilde{B} := CB \) we denote the new transformation from the in-states to the out-states as

\[ \Phi_{\text{in}} = \tilde{A}^T \tilde{\Phi}_{\text{out}} - \tilde{B}^T \tilde{\bar{\Phi}}_{\text{out}} \]  

(19)

The second requirement then takes the form that

\[ \tilde{V} := -\tilde{B}\tilde{A}^{-1} = \text{diag} \]  

(20)

should not have off-diagonal entries. Since we do not produce additional particles and the spectrum outside should not change, this means

\[ \tilde{V} := V \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]  

(21)
3. Momentum conservation must be maintained, i.e., the matrix $C$ should be proportional to $\delta(p - \tilde{p})$, where $p$ is the total momentum ingoing to the region close by the horizon, and $\tilde{p}$ is the total momentum outgoing from that region. Since we are using spherical coordinates and spherical harmonics, this $\delta$-function should be read as being factorized into spherical coordinates as well so it matches the modes.

4 Boundary condition

We will now see if there is a local boundary condition that can induce the desired transformation. For this, it is helpful to first depict what the transformation must do, see Figure 1. For the observer at $I^+$ to obtain all the information from $I^-$, an out-mode must contain information both from the region $v < 0$ as well as from the region $v > 0$. This means that the stretched horizon must act similar to a partially reflecting mirror, which is most easily seen when traced backwards, see Figure 1 right.

To obtain a boundary condition, we now put a surface at $S(u, v) = 0$ and denote by $u_0, v_0$ its crossing point with the surface of the collapsing matter

$$ (u_0, v_0) : S(u_0, v_0) = 0 \land R(u_0, v_0) = 0 \ . $$

Outside of the collapsing matter, we locate the surface at

$$ S(u, v) = u - \kappa \ln(v/\kappa) + y \ , $$

where $y$ is a (real positive) constant that will only contribute a phase. Inside, for $v_0 \geq v > 0$, we situate $S(U, V) = 0$ at

$$ S(U, V) = UV - \kappa^2 e^y \ . $$

At that surface $S = 0$ we put a partial reflecting condition with the following property expressed in the out-basis as

$$ \Phi_{\text{out}}\big|_{\text{to}} - C\Phi_{\text{out}}\big|_{\text{away}} = 0 \ , $$

where ‘to’ means incoming to $S(u, v) = 0$ (i.e., coming from $I^-$) and ‘away’ means outgoing from $S(u, v) = 0$ (i.e., towards $I^+$ and the singularity) in Figure 1. To connect this to our previous notation, the state incoming to the boundary is $\tilde{\Phi}_{\text{out}}$.

We take $C$ to be

$$ C = \begin{pmatrix} i/\sqrt{2} & -i/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} Y \delta^4(p - \tilde{p}) \text{Vol} \ , $$

$$ Y = \frac{1}{2} \begin{pmatrix} 1 + e^{iy} & i - ie^{-iy} \\ i - ie^{iy} & 1 + ie^{-iy} \end{pmatrix} \ . $$
Figure 1: Causal Diagram of black hole formation. The wavy line demarks the singularity, the fat black line the horizon, and the thin black line marked with $S$ is the stretched horizon. The grey shaded region is the inside of the collapsing matter; its surface is at $R(u, v) = 0$. The (red and green) diagonal lines depict modes. The solid parts of these modes depict the usual ray-tracing, the dashed parts depict the new contribution from the boundary condition at $S$. Left: The outgoing radiation contains information both from the region $v \leq 0$ and $v \geq 0$. Right: The same setting but traced backwards.

The matrix $Y$ is only there to adjust the phase factor. $p$ is the total momentum of the $\Phi$-modes incoming to the boundary and $\tilde{p}$ is the total momentum of the $\tilde{\Phi}$-modes outgoing from the boundary in spherical coordinates.

We will now go on to demonstrate that this boundary condition fulfills our requirements. Recall that we are suppressing an index $\omega$ labelling the modes. The above ansatz for $C$ means that we take the boundary condition to not depend on the frequency.

To begin with, it is $C\tilde{C}^T = 1$, so we can interpret this as the result of a scattering process that comes about by some unknown physics located at $S = 0$, and will not ruin unitarity. The volume factor is there to take into account a suitable normalization procedure for plane waves as usual. In the following we will look at a state (the vacuum state) that preserves the four momentum at the boundary and we will take this factor to
be one, though we will come back to the requirement of momentum conservation in the discussion. Before we come to the discussion though, note that here and for the rest of this and the next section we are not talking about what happens to infalling information, we are only considering the case in which the field is in the vacuum state at $\mathcal{I}^-$. If we make a plane-wave ansatz for $\Phi$, the reflecting condition \((23)\) will convert $\Phi^T_{\text{away}} = (\exp(-i\omega u), 0)$ coming from $\mathcal{I}^+$ to

$$\phi^T = \left(ie^{-i\omega u}/\sqrt{2}, -ie^{-i\omega \ln(v/\kappa)}/\sqrt{2}\right)$$

(28)

moving towards $S = 0$. As anticipated, the phase shift from $y$ is cancelled by $Y$. This means the boundary condition converts both out-modes and dn-modes into combinations of out- and dn-modes at the same frequency. Note that the location of $S$ is essential for this to work. Needless to say, this particular scattering process would not preserve momentum because part of the wave is reflected. Alas, in the vacuum state there will be another contribution from $\phi_{\text{dn}}$ that provides the missing momentum. The requirement of momentum conservation in the chosen base means that the mixture of modes dependent on $u$ and $v$ has to stay the same at the boundary. Since $C$ mixes them in equal parts, this means that the state ingoing to the boundary must also have had contributions in equal parts, ie $\tilde{\phi}^T \sim (\phi_{\text{out}}, \phi_{\text{dn}})$, the relevant point being that the prefactors in both components are identical.

For the annihilation and creation operators, the boundary condition performs the following mixing

$$\tilde{a}_{\text{out}} = a_{\text{out}} C^T, \quad \tilde{a}^\dagger_{\text{out}} = a^\dagger_{\text{out}} C^T.$$  

(29)

The physical picture is the following. We take the in-modes and move them forward to $\mathcal{I}^+$ or to the inside of the horizon respectively. On the way, we have to pass $S = 0$ and have to take into account the boundary condition. By this, the solutions of the wave-equation change. If we start with defining the in-vacuum as the physically real vacuum, then the out-state that corresponds to the in-vacuum is now a different state. It differs from the ‘normal’ out-vacuum that contains thermal radiation (without the boundary condition) by the operation induced by $C$. The early ingoing modes do not encounter a boundary condition. This seems natural, because in the early phase the matter distribution is nothing like a black hole. The dominant contribution for the Hawking-radiation comes from the near-horizon region which is why the approximation around these rapidly oscillating modes is excellent. Therefore it does not really matter what happens with the early ingoing modes and we will not further specify this region.

In the new basis we have

$$\tilde{B} \tilde{B}^T = CB\tilde{B}^T C^T = e^2 w^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

(30)

ie the spectrum remains unchanged as expected. And further

$$|0_{\text{in}}\rangle = e^{iW} \exp\left(\frac{1}{2} a_{\text{out}}^\dagger \tilde{V} (a_{\text{out}}^\dagger)^T\right) |0_{\text{out}}\rangle,$$

(31)
(since the boundary condition does not mix positive with negative frequencies, the $F$ matrix does not come in) where now

$$
\tilde{V} = \tilde{C}VC^T = w \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} .
$$

(32)

Thus, with the additional boundary condition, we get pairs outside and pairs inside while the spectrum remains the same. Inserted into the exponent of the scattering matrix (14), the state factorizes into a part inside and a part outside the horizon.

Inside the matter distribution, the order of operations is changed, in that a mode coming from $I^-$ first encounters the stretched horizon and then the surface of the matter where the usual Bogoliubov transformation is caused. The location of the boundary is easily adjusted (again taking into account the phase shift) so as to have the same effect as outside the matter. Again, note that the location of the boundary is relevant for the mixing to happen between the out-modes and dn-modes. The boundary condition cannot be assigned to an arbitrary timelike surface.

To understand better what is going on, we can ask what in-state would be needed to get the disentangled out-state directly, without the boundary condition. We will call this vacuum $|\tilde{0}_{\text{in}}\rangle$. There are two ways to look at this. First, we have

$$
|\tilde{0}_{\text{in}}\rangle = e^{iW} \exp \left( \frac{1}{2} a^\dagger_{\text{out}} \tilde{V} (a^\dagger_{\text{out}})^T \right) |0_{\text{out}}\rangle ,
$$

(33)

which, combined with the relation for the new out-vacuum (31) gives

$$
|\tilde{0}_{\text{in}}\rangle = e^{iW} \exp \left( \frac{1}{2} a^\dagger_{\text{out}} \left( V - \tilde{V} \right) (a^\dagger_{\text{out}})^T \right) |0_{\text{in}}\rangle ,
$$

(34)

where

$$
a^\dagger_{\text{out}} \left( V - \tilde{V} \right) (a^\dagger_{\text{out}})^T = w \left( a^\dagger_{\text{out}} a^\dagger_{\text{dn}} + a^\dagger_{\text{dn}} a^\dagger_{\text{out}} - a^\dagger_{\text{out}} a^\dagger_{\text{out}} - a^\dagger_{\text{dn}} a^\dagger_{\text{dn}} \right) .
$$

(35)

This just means that we are adding the entangled Hawking pairs and then subtracting the disentangled ones. This expression is however hard to interpret because it contains the out-operators.

Another way to look at this is to first define the new in-state that is the normal tracing back of the new out state

$$
\tilde{\Phi}_{\text{in}} = \tilde{A}^T \tilde{\Phi}_{\text{out}} - B^T \tilde{\Phi}_{\text{out}} .
$$

(36)

Then we insert $\tilde{\Phi}_{\text{out}} = C\Phi_{\text{out}}$ and transform back into the normal in-basis with the inverse transformation

$$
\Phi_{\text{out}} = A\Phi_{\text{in}} + B\tilde{\Phi}_{\text{in}} .
$$

(37)

\footnote{At least not as long as $C$ is assumed to be independent of position and frequency.}
This gives
\[ \tilde{\Phi}_{\text{in}} = \tilde{C}^T \Phi_{\text{in}} - \tilde{D}^T \tilde{\Phi}_{\text{in}}, \] (38)
with
\[ \tilde{C}^T = \tilde{A}^T \tilde{C} \tilde{A} - \tilde{B}^T \tilde{C} \tilde{B}, \quad \tilde{D}^T = \tilde{A}^T \tilde{C} \tilde{B} - \tilde{B}^T \tilde{C} \tilde{A}. \] (39)

Working out the products, one finds
\[ \tilde{C} = \frac{c^2}{\sqrt{2}} \left( \begin{array}{cc} 1 - iw^2 & i - w^2 \\ 1 + iw^2 & -i - w^2 \end{array} \right), \] (40)
\[ \tilde{D} = \frac{c^2 w}{\sqrt{2}} \left( \begin{array}{cc} 1 - i & -1 + i \\ 1 + i & -1 - i \end{array} \right), \] (41)
and
\[ -\tilde{D} \tilde{C}^{-1} = \frac{w}{1 + w^2} \left( \begin{array}{cc} -i & 1 \\ -1 & -i \end{array} \right), \] (42)
\[ -\tilde{D} \tilde{C}^{-1} = \frac{w}{1 + w^2} \left( \begin{array}{cc} -i & 1 \\ -1 & -i \end{array} \right), \] (43)

There is no good physical interpretation for comparing these two in-vacua, but one can extract from this expression what one could have expected already from (35), namely that (when expressed in terms of up-modes and dn-modes), the necessary in-vacuum to create a disentangled out-vacuum without the additional boundary condition must have had correlations between the regions \( v > 0 \) and \( v < 0 \). It is thus not an initial state that we would plausibly have for a collapsing matter distribution but instead a delicately finetuned one.

Since \( S \) contains a \( \ln(v) \) it will not be very close to the horizon in any general way at late times. But one can use the constant \( y \) to get it to stay as closely as desired within any finite amount of time. Since the black hole eventually evaporates, a finite time is sufficient.

### 4.1 A simplified qubit model

We can extract the main idea in a simplified qubit version. Instead of considering the Hawking spectrum with a continuum of modes, we will just look at two different states, denoted + and −. Each of them can either leave the black hole horizon towards future infinity (corresponding to the out-mode) or fall in (corresponding to the dn-mode). We will denote these four states as \( |O+\rangle, |O-\rangle \) and \( |D+\rangle, |D-\rangle \).
The usual Hawking state with entanglement between the inside and outside region then corresponds to
\[
\frac{1}{\sqrt{2}} \left( \frac{1}{\sqrt{2}} (|O+|D+ + |D+|O+) + \frac{1}{\sqrt{2}} (|O-|D- + |D-|O-) \right) .
\] (45)

The scattering at $S$ (compare to Eq. (26)) performs the following operation
\[
|O+\rangle \rightarrow \frac{1}{\sqrt{2}} (|O+\rangle + i|D+\rangle) ,
|O-\rangle \rightarrow \frac{1}{\sqrt{2}} (|O-\rangle + i|D-\rangle) ,
|D+\rangle \rightarrow \frac{1}{\sqrt{2}} (|O+\rangle - i|D+\rangle) ,
|D-\rangle \rightarrow \frac{1}{\sqrt{2}} (|O-\rangle - i|D-\rangle) .
\] (46)
(47)

That the same operation takes place for both the $+$ and $-$ states corresponds to the frequency-independence of the boundary condition. With this operation, the qubit version of the Hawking state goes to
\[
\frac{1}{2} (|O+\rangle|O+\rangle + |D+\rangle|D+\rangle + |O-\rangle|O-\rangle + |D-\rangle|D-\rangle) ,
\] (48)
so the entanglement between the inside and outside modes is gone. Note that the probability to measure any term in this expression is reduced by a factor $1/2$ over the usual case. But since the outside particles now always come in pairs, the spectrum (number density as well as spectral distribution) remain the same.

5 Revisiting the firewall argument

The firewall argument [3] in brief goes like this. If the state at $I^+$ is a pure state with sufficient correlations between the early and the late modes, then one can construct an excitation in the late state and trace it back to the horizon, where it becomes a dramatically large deviation from the local observer's vacuum. The infalling observer must notice, thus violating the equivalence principle. The essence of this firewall problem had been identified already in [14] and was discussed previously in [15]. The important contribution from [3] is that the explicit construction of the traced-back excitation allows to identify exactly what property of the state at $I^+$ is problematic and, seeing what the problem is, to circumvent it.

In the case with the additional boundary condition proposed here, the state at $I^+$ does have the energy spectrum of the Hawking radiation, and there are no correlations between early and late modes, instead there are correlations between energy modes. This remains so also when constructing localized wave-packets; these then always have to come in pairs at the same time, up to the uncertainty that comes from the width of the now localized wave-packages. One can thus turn the AMPS argument around and say that if we trace back a late state of this disentangled vacuum, then the state close by the horizon must also have the same energy spectrum as normally, otherwise the state at $I^+$
must have been different. The infalling observer cannot notice anything unusual because the boundary condition does not mix positive with negative frequency modes and thus he does not notice any change to the vacuum. The renormalized stress-energy delivers the usual result because there are no excitations over the thermal spectrum.

The observer himself does not notice the crossing of the boundary because any partial reflection of a state that is not balanced from the other side of the boundary would violate momentum conservation. Thus, the observer, or the modes he is composed of respectively, cannot scatter on the boundary. There is no drama, because drama would require energy or momentum, neither of which is available. Instead of speaking of a boundary condition which suggests it acts on all modes, it might be more useful to think of the requirement on the stretched horizon as facilitating an interaction that necessarily maintains momentum conservation.

Another way to look at this is that a local observer who hovers at a constant radius $r_0$ outside the black hole horizon uses his local modes which are blue-shifted relative to the modes at asymptotic infinity. These modes are not complete, but they can be completed, resulting in the same set of modes and transformations as previously. The hovering observer thus sees the same thermal spectrum as the observer at $I^+$, but with temperature $T = \gamma \kappa / (2\pi)$, where $\gamma = (1 - 2M/r_0)^{-1/2}$. This temperature diverges as the observer gets close to the horizon, because hovering there would require an infinite acceleration. In the normal case (without the boundary) he cannot locally distinguish between hovering nearby the black hole horizon and acceleration in flat space. He sees the same as the Rindler observer in Minkowski space, and the equivalence principle holds.

The boundary condition does the same to the local observer’s modes as it does to the Hawking modes: Only suitable combinations of out-modes and dn-modes that can scatter on the boundary, and this scattering serves to disentangle them. The observer now finds himself in a thermal bath, but one that has pairwise correlations. Can he measure the difference?

First we note that the equivalence principle in its usual form is ambiguous on exactly what the local observer should see. He should see the same as in flat space, but flat space with a boundary condition (think of the moving mirror or the Casimir effect) is still flat space, yet it makes a difference to the observables. If the local observer would come to conclude that he is in Rindler space with a (semi-transparent) boundary condition, would or wouldn’t this amount to a violation of the equivalence principle?

Having said that, let us ask though what the local observer has to do to notice a difference to empty Minkowski-space, ie absent any boundary conditions.

Let us assume the observer has measured a typical particle with energy $E \sim T$ to precision $\Delta E$. Absent the boundary condition, the next particle is entirely uncorrelated and arrives with a probability distributed according to the Boltzmann statistic, emitted according to the Stefan-Boltzmann law. On the average, it takes a time of $\Delta t \sim 1/(M^2T^3) \sim 16M$ until the black hole emits a particle of energy $T$, where $t$
is the coordinate time, not the observer’s proper time, which is $\tau = t\gamma$. The observer at a radius $r_0$ obtains the emitted particles at a ratio reduced by a factor $(2M)^2/r_0^2$ relative to the flux nearby the horizon. The farther away he is from the horizon, the longer he has to wait for the next particle since the apparent luminosity decreases.

The next particle thus comes on the average within a time window $\Delta t \sim 16M(r_0^3/(2M)^2)$. In the presence of the boundary condition, it has (with probability 1/2) the same energy as the previous one, up to the uncertainty $\Delta E$. If the uncertainty $\Delta E$ is comparable to $T$, the observer cannot tell the difference to the uncorrelated case (or the statistical significance of his conclusion is very low, to be more precise). The observer can thus not make the detector arbitrarily small. It has to have at least have a size of $\sim 16M$, where the size is measured in the asymptotically flat coordinates. Taken together this means the local observer’s measurement needs at least extensions $\Delta r \sim 16M$ and $\Delta t \sim 16M(r_0^3/(2M)^2)$.

We now have to compare this to the typical size of the locally flat region, the region within which curvature effects are negligible. To this end, we can estimate the tital forces acting on the local observer’s apparatus, caused by the difference in acceleration between the two radial positions at distance $\Delta r$. If the observer was accelerating in flat space, the same acceleration for both radial positions $r_0$ and $r_0 + \Delta r$ would not create any relative motion. In the Schwarzschild background, the same acceleration between both radial positions will slowly stretch the apparatus. This is essentially geodesic deviation, except that the apparatus is not moving on a geodesis.

In the limit of small velocities, the difference in acceleration is approximately given by

$$\frac{d^2(\Delta r)}{dt^2} \sim \frac{M}{r_0^2} - \frac{M}{(r_0 + \Delta r)^2},$$

and thus over a time of $\Delta t$, the stretch that the detector acquires is

$$\Delta r(\Delta t) \sim \left(\frac{M}{r_0^2} - \frac{M}{(r_0 + \Delta r)^2}\right) \Delta t^2.$$ 

(50)

Since $r_0 > 2M$, one finds that $\Delta r(\Delta t)/\Delta r > 1$. This means that the observer can measure the effects of curvature on his detector in the time he has to wait for sufficiently many particles.

So the local observers still see a thermal spectrum with the redshifted temperature $T = \gamma\kappa/(2\pi)$. They can distinguish the entangled from the disentangled thermal spectrum, but cannot do it locally.

6 Discussion

So we have been able to construct a local boundary condition at a time-like surface close by the black hole horizon that reconciles the purity of the outgoing radiation with that of the ingoing vacuum state, thus removing the firewall problem.
This became possible by noting that a well-behaved stress-energy requires a thermal energy spectrum [12][13], but not necessarily a thermal state. While a thermal spectrum is normally composed of uncorrelated particles, there are correlated states that have the same energy spectrum. The spectral energy distribution just does not completely specify the state. To see how this can be consider a sequence of uncorrelated (fair) coin tosses. Now consider a sequence of tosses with always twice the same result in a row. The probability distribution (the spectrum) in both cases is the same. The difference is in the correlation between subsequent tosses (the entanglement). To remove the firewall we need an out-state that has correlations but still a thermal energy spectrum. The state considered by AMPS is not of this type because it necessarily has excitations over thermality at late times.

The construction suggested here represents an explicit example for the general argument in [6] that the problematic assumption that leads to the existence of a firewall is not to be found in the axioms of black hole complementarity but in the specific way that the state at $I^-$ was assumed to be entangled.

However, a series of pure states with a thermal spectral distribution might be pure but it still does not contain information (other than the temperature). That is to say, disentangling the vacuum by a suitable boundary condition on the stretched horizon may remove the firewall but in and by itself that does not address the black hole information problem. For this, one would have to find a way to actually encode information from infalling matter (observers) into the outgoing state while still keeping the radiation pure and the spectrum thermal.

This brings us back to the question what happens to in-states other than the vacuum. As noted previously, due to momentum conservation, nothing will happen to the state itself at the stretched horizon. There cannot be anything happening, because there is no energy there that could make anything happen. However, recall also that we assumed the $C$ matrix does not depend on the frequency. This does not necessarily have to be so. One could for example add a relative phase between different modes. This would not affect the spectrum but still constitute an observable change.

If we decide to either shift or not shift a certain mode by, say, a phase of $i\pi$, then we have one bit of information per mode that can be distinguished and measured (each mode is either shifted or not relative to one reference mode). The black hole of mass $M$ typically emits $M^2$ particles with approximate energy of $1/M$, which gives about $2^{M^2}$ states. This estimate shouldn’t be taken too seriously though because we have neither taken into account exactly how good a phase shift can be measured given the uncertainty of wavepackets that can be constructed, nor have we accounted for the spherical harmonics. This is just to illustrate that it seems possible to use the freedom in $C$ to construct sufficiently many different states. In the end though, the boundary condition must come about by some quantum property of the background geometry, and the number of states must be determined by the possible microstates of the background.

Of course this only moves the problem elsewhere, because then we have to wonder...
how the information got from the in-state into the boundary condition. It is beyond the scope of the paper to devise a mechanism for how the stretched horizon may read and release information of matter passing through, but we can take away that, to resolve the black hole information loss problem, the boundary condition must depend on the ingoing state.

An entirely different question is whether there is an equation of motion or an interaction term that generates the boundary condition (5)? There are some examples of different reflecting conditions in [9, 10] but these cannot be directly applied here. One can come up with an equation by noting that when we reverse the boundary condition, we must get a solution of the normal wave-equation. Thus

$$\partial_u \partial_v \Phi_{\text{in}} \left( 1 \Theta(S) + C^{-1} \Theta(-S) \right) = 0 .$$

(51)

One can then take the derivatives, but this does not shed much light on the microscopic origin of the \(C\)-operation.

The here proposed boundary condition also circumvents another difficulty with the setting proposed in [7], that is the doubling of the initial state. The problem with this is that in a space-time with several black holes each of them would swallow a copy of the initial state and it is not clear how to generalize the construction of the Hilbert-space then. Here, we instead split the incoming states and recombine them differently so as to disentangle the Hawking-radiation, which makes the doubling unnecessary.

Finally, it would be interesting to see whether the boundary condition proposed here can be found by means of the gauge-gravity duality or is related to the recent development reported in [16, 17].

### 7 Conclusion

The requirement that the state at \(\mathcal{I}^+\) which corresponds to the in-vacuum is a disentangled, pure, version of the usual mixed Hawking state with the same spectral distribution allows to formulate a boundary condition in the region close by the horizon. This boundary enables an interaction and prevents a mismatch between the traced-back pure radiation at \(\mathcal{I}^+\) and the moved-forward vacuum from \(\mathcal{I}^-\) that gives rise to the firewall paradox; it can be interpreted as the stretched horizon. While this in an by itself does not address the black hole information loss problem, the necessary boundary condition is not unique, and this non-uniqueness might potentially be used to encode information in the outgoing radiation.

### Acknowledgements

I thank Raffael Bousso, Steve Giddings, Steve Hsu, Ted Jacobson and Don Marolf for feedback, and Joe Polchinski for spotting several mistakes in an early draft. I thank Stefan Scherer and Lárus Thorlacius for helpful discussions.
References

[1] S. W. Hawking, “Particle Creation by Black Holes,” Commun. Math. Phys. 43, 199 (1975) [Erratum-ibid. 46, 206 (1976)].

[2] S. W. Hawking, “Breakdown of Predictability in Gravitational Collapse,” Phys. Rev. D 14, 2460 (1976).

[3] A. Almheiri, D. Marolf, J. Polchinski and J. Sully, “Black Holes: Complementarity or Firewalls?,” JHEP 1302, 062 (2013) [arXiv:1207.3123 [hep-th]].

[4] L. Susskind, L. Thorlacius and J. Uglum, “The Stretched horizon and black hole complementarity,” Phys. Rev. D 48, 3743 (1993) [hep-th/9306069].

[5] A. Almheiri, D. Marolf, J. Polchinski, D. Stanford and J. Sully, “An Apologia for Firewalls,” JHEP 1309, 018 (2013) [arXiv:1304.6483 [hep-th]].

[6] S. Hossenfelder, “Comment on the black hole firewall,” [arXiv:1210.5317 [gr-qc]].

[7] E. Verlinde and H. Verlinde, “Passing through the Firewall,” [arXiv:1306.0515 [hep-th]].

[8] V. Frolov and I. Novikov, “Black Hole Physics: Basic Concepts and New Developments”, Kluwer Academic Publishers (1998).

[9] J. Haro and E. Elizalde, “Physically sound Hamiltonian formulation of the dynamical Casimir effect,” Phys. Rev. D 76, 065001 (2007) [arXiv:0705.0597 [hep-th]].

[10] J. Haro and E. Elizalde, “Black hole collapse simulated by vacuum fluctuations with a moving semitransparent mirror,” Phys. Rev. D 77, 045011 (2008) [arXiv:0712.4141 [quant-ph]].

[11] R. M. Wald, “On Particle Creation by Black Holes,” Commun. Math. Phys. 45, 9 (1975).

[12] P. Candelas, “Vacuum Polarization in Schwarzschild Space-Time,” Phys. Rev. D 21, 2185 (1980).

[13] D. A. Lowe and L. Thorlacius, “Pure states and black hole complementarity,” Phys. Rev. D 88, 044012 (2013) [arXiv:1305.7459 [hep-th]].

[14] S. B. Giddings, “Quantum mechanics of black holes,” [hep-th/9412138] [hep-th/9412138]

[15] S. D. Mathur, “The Information paradox: A Pedagogical introduction,” Class. Quant. Grav. 26, 224001 (2009) [arXiv:0909.1038 [hep-th]].

[16] K. Papadodimas and S. Raju, “The Black Hole Interior in AdS/CFT and the Information Paradox,” [arXiv:1310.6344 [hep-th]].

[17] K. Papadodimas and S. Raju, “State-Dependent Bulk-Boundary Maps and Black Hole Complementarity,” [arXiv:1310.6335 [hep-th]].