ON THE ESTIMATE OF MAGNETIC NON-POTENTIALITY OF SUNSPOTS DERIVED USING HINODE SOT/SP OBSERVATIONS: EFFECT OF POLARIMETRIC NOISE

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ABSTRACT

The accuracy of Milne–Eddington (ME) inversions, used to retrieve the magnetic field vector, depends upon the signal-to-noise ratio (S/N) of the spectro-polarimetric observations. The S/N in real observations varies from pixel to pixel; therefore the accuracy of the field vector also varies over the map. The aim of this work is to study the effect of polarimetric noise on the inference of the magnetic field vector and the magnetic non-potentiality of a real sunspot. To this end, we use the Hinode SOT/SP vector magnetogram of a real sunspot NOAA 10933 as an input to generate synthetic Stokes profiles under ME model assumptions. We then add normally distributed polarimetric noise of the level 0.5% of continuum intensity to these synthetic profiles and invert them again using the ME code. This process is repeated 100 times with different realizations of noise. It is found that within most of the sunspot areas (>90% area) the spread in the (1) field strength is less than 8 G, (2) field inclination is less than 1°, and (3) field azimuth is less than 5°. Further, we determine the uncertainty in the magnetic non-potentiality of a sunspot as determined by the force-free parameter αg and spatially averaged signed shear angle (SASSA). It is found that for the sunspot studied here these parameters are αg = −3.5 ± 0.37 (×10⁻⁹ m⁻¹) and SASSA = −1.68 ± 0.014. This suggests that the SASSA is a less dispersed non-potentiality parameter as compared to αg. Further, we examine the effect of increasing noise levels, viz. 0.01%, 0.1%, 0.5%, and 1% of continuum intensity, and find that SASSA is less vulnerable to noise as compared to the αg parameter.

Key words: Sun: flares – Sun: photosphere – Sun: surface magnetism

1. INTRODUCTION

Accurate determination of the vector magnetic field is very important for the study of magnetic non-potentiality in active regions. The evolution of the active region magnetic field toward an increasingly non-potential state leads to the buildup of magnetic-free energy, i.e., the energy above magnetic potential energy. This free magnetic energy is believed to drive the eruptive phenomena like flares and coronal mass ejections (CMEs). Prediction of such phenomena on the Sun is very important for space weather forecasting and requires good knowledge of the non-potentiality of the magnetic field in solar active regions.

The force-free parameter αg has been studied to infer the non-potentiality of sunspots for a long time (Pevtsov et al. 1994, 1995; Abramenko et al. 1996; Bao & Zhang 1998; Hagino & Sakurai 2004, 2005; Nandy 2006). Using the second moment of minimization (Hagino & Sakurai 2004; Tiwari et al. 2009a) the relation between αg and the vector field components is given by the following relation:

\[ \alpha_g = \frac{\sum (\frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial y}) B_z}{\sum B_z^2}. \]  

The global alpha (αg) actually gives twice the degree of twist per unit axial length and not the twist (see Appendix A of Tiwari et al. 2009a). Thus, if the length of the magnetic field structure in a volume is given, a global twist can be obtained from alpha for force-free fields.

Another parameter, spatially averaged signed shear angle (SASSA), henceforth denoted as \( \hat{\Psi} \), was recently proposed by Tiwari et al. (2009b) as a measure of magnetic non-potentiality in sunspots. This parameter is the spatial average of the angle between observed magnetic field and potential field azimuth. It is derived from the following relation:

\[ \hat{\Psi} = \frac{\arctan \left( \frac{B_{yo}B_{zp} - B_{yp}B_{zo}}{B_{zo}B_{xp} + B_{yo}B_{yp}} \right)}{\pi}. \]  

where \( B_{xo}, B_{yo}, B_{xp}, B_{yp} \) are observed and potential transverse components of the magnetic field, respectively. The angled braces represent the spatial average taken over all pixels except those below noise (see Section 2.3). This parameter thus gives average shear angle on the photospheric boundary and is independent of the force-free nature as well as the shape of the sunspots (Venkatakrishnan & Tiwari 2009). The high-quality Hinode data have also allowed these authors to study the contribution of local alpha values of umbral and penumbral structures to the global alpha value of the sunspots. Further, Gosain et al. (2009) evaluated the \( \hat{\Psi} \) of active region NOAA 10930 during the X-class flare of 2006 December 13 using vector magnetograms obtained a few hours before and after the flare by the Spectropolarimeter (SP) instrument (Lites et al. 2007; Ichimoto et al. 2008) with Solar Optical Telescope (SOT) on board the Hinode satellite (Kosugi et al. 2007; Tsuneta et al. 2008), and found that the \( \hat{\Psi} \) decreased after the flare. However, high-cadence vector magnetograms are needed to follow the evolution of non-potentiality, characterized by \( \hat{\Psi} \), during the flare interval. Magnetographs based on tunable filters like the imaging vector magnetograph (IVM; Mickey et al. 1996) and the solar vector magnetograph (SVM; Gosain et al. 2004, 2006) from the ground and the recently launched Helioseismic and Magnetic Imager on board the Solar Dynamics Observatory (SDO) from space (Scherrer & SDO/HMI Team 2002) can provide such high-cadence magnetograms of active regions.

Tiwari et al. (2009a) had earlier evaluated the effect of polarimetric noise on the magnetic twist parameter αg and
magnetic energy using a synthetic bipole (Low 1982). However, for real sunspots the distribution of magnetic field vector is not the same as in the case of synthetic bipole. Real sunspots have umbra and penumbra besides fine structure, as seen by high-resolution ground-based and space-based Hinode observations. In a recent study by Su et al. (2009) using Hinode observations, it was found that the current and $\alpha$ distribution in a sunspot is not smooth but has a fine mesh-like structure with mixed polarity patches of $\alpha$ in the umbra and a radial spine-like pattern with alternating polarity in the penumbra of a sunspot. At the umbra penumbra boundary they found an incomplete annular ring with current and $\alpha$ values of sign opposite to that of global $\alpha$, i.e., $\alpha_p$. Further, Tiwari et al. (2009b) have shown that the distribution of $\alpha$ in the penumbra almost cancels to zero while in the umbra there is a net value which bears the same sign as $\alpha_p$ and has the same magnitude as $\alpha_p$. Further, the new parameter $\Psi$ that has been introduced recently by Tiwari et al. (2009b) needs to be assessed for robustness in the presence of uncertainties in the magnetic field parameters of a real sunspot. This work therefore extends the results of Tiwari et al. (2009a) to the case of a real sunspot observed by Hinode and to the new parameter $\Psi$.

The uncertainties in vector magnetic field obtained by fitting the observed Stokes profiles with model profiles can arise mainly due to the following reasons: (1) invalidity of the model atmosphere used, e.g., the Milne–Eddington (ME) model cannot fit asymmetric Stokes profiles which arise due to variation of physical parameters within the line-forming region, and (2) polarimetric noise, e.g., sensitivity in determining the field parameters depends on the signal-to-noise ratio ($S/N$) of the Stokes profiles. The former reason has been evaluated by Westendorp Plaza et al. (2001) by comparing ME and Stokes Inversion based on Response-Functions (SIR; Ruiz Cobo & del Toro Iniesta 1992) inversions. SIR inversions yield variation of physical parameters within the line-forming region, by exploiting the asymmetric nature of Stokes profiles. They have shown that using ME inverted field parameters is equivalent to using field parameters averaged over the line-forming region by SIR inversions. So, neglecting the effect of Stokes asymmetry, we focused on the errors in the field measurements due to polarimetric noise.

This paper is organized as follows. In Section 2, we describe the Hinode SOT/SP observations and the method of simulating noisy Stokes profiles and their inversion. The results are presented in Section 3. The importance of the non-potentiality parameters and the inaccuracy in their determination as a result of polarimetric noise is discussed in Section 4. Section 5 gives the conclusions based on the present study and the future studies required.

2. DATA AND METHOD

2.1. Observational Data

The high-resolution spectro-polarimetric observations were made by the SP instrument working with SOT on board the Hinode space mission. We choose a regular isolated sunspot in NOAA 10933 during 2007 January 5, when it was located close to the disk center ($\mu = 0.99$) during 07:20 UT. The sunspot was scanned with “fast-map” observing mode by the SP instrument. In this mode the spatial sampling along the slit is $0.32$ per pixel and across the slit is $0.29$ per pixel with an integration time of 1.6 s. The spectral lines used for polarimetric measurement are Fe i 6301.5 and 6302.5 Å line pair. The vector magnetic field for this sunspot was derived using the ME code named MERLIN, provided by the Community Spectro-polarimetry Analysis Center (CSAC; Lites et al. 2007). It is an inversion code based on the least-squares fitting of the observed Stokes profiles using the Levenberg–Marquardt algorithm and is quick due to parallel computing. It assumes a standard ME atmosphere to retrieve the magnetic field vector, line-of-sight velocity, source function, and Doppler broadening as well as the macro-turbulence and the stray light filling factor. For Monte Carlo estimation of error in magnetic parameters we use another ME inversion code named HeLIx (Lagg et al. 2004). We compared the magnetic field parameters retrieved by the two codes after inverting the SOT/SP data set used in the present study. We made difference maps of the retrieved magnetic field parameters, i.e., $\delta M = M_{\text{MERLIN}} - M_{\text{HeLIx}}$, where $M = B$ (field strength), $\gamma$ (inclination), or $\psi$ (azimuth). Within the sunspot, the two codes retrieve identical magnetic field parameters giving zero mean of $\delta M$ and (1σ) standard deviation of 50 G, 0.75 and 2.5 in field strength, inclination, and azimuth angle, respectively. These values are comparable to the standard errors of inversion by MERLIN, reported in Figure 3. Standard errors mean that if the same inversion code is repeatedly applied to a given observed (noisy) profile, it will lead to the same model atmosphere within the error bars given by the standard error estimate, provided the minimum of the merit function is reached (Bellot Rubio et al. 2000). So, the comparison suggests that the two codes are identical so that the differences in the output are of the same magnitude as the difference of a particular code with itself. The reason for choosing HeLIx over MERLIN in our Monte Carlo error estimation is the built-in routines for adding noise to the synthetic Stokes profiles in the HeLIx code which helps us save time as we avoid adding noise offline. The azimuthal ambiguity of the transverse component of the magnetic field vector was resolved using the acute angle method (Harvey 1969).

2.2. The Monte Carlo Approach for the Determination of Uncertainties

The uncertainties in the field parameters derived using ME inversion of observed Stokes profiles, purely as a result of polarimetric noise present in the observed data, are considered here. There are two ways of determining the uncertainties.

1. **Standard errors.** Basically, in the Stokes profile inversions, a merit function is defined as

$$\chi^2 = \frac{1}{\nu} \sum_{k=1}^{4} \sum_{i=1}^{M} \left[ I_{\text{obs}}^{\lambda_i}(\lambda_i) - I_{\text{mod}}^{\gamma_i}(\alpha, \lambda_i) \right]^2 \frac{w_{ki}^2}{\sigma_{ki}^2},$$

where $k = 1$ to 4 represent the four Stokes parameters, $i = 1$ to $M$ represent the number of wavelength samples for observed and synthetic Stokes profiles, $\nu$ represents the number of degrees of freedom (i.e., the number of observables minus the number of free parameters), and the vector $\alpha$ consists of the parameters of the model, $\sigma_{ki}$ represents the noise in the observations and $w_{ki}$ represents the weight given to the data points. This merit function, $\chi^2$, is then minimized by using the nonlinear least-squares method to obtain $\chi_{\text{min}}$. The standard errors in the model parameters basically depend upon the curvature of the $\chi^2$ function near the region of minimum in the parameter space (Press et al. 1986). More details about the estimation of the standard errors in the model parameters derived by Stokes inversion are given in Bellot Rubio et al. (2000).

2. **Monte Carlo errors.** In the Monte Carlo method, we create several artificial realizations of the spectro-polarimetric
data. This is done by generating the synthetic Stokes profiles and adding different realizations of noise to them. These artificial data sets are then inverted to obtain the magnetic parameters, leading to a collection of results. The errors are then determined by finding the spread or standard deviation in the derived parameters. It may be noted that the Monte Carlo errors have no obvious relationship with the standard errors.

Westendorp Plaza et al. (2001) have shown that in the case of SIR inversion of a single profile, the standard errors are slightly larger than the estimates from the Monte Carlo method. However, this depends on the distribution of the field vector, and so the comparison of errors by the two methods may vary from pixel to pixel. Further, the Monte Carlo error estimates depend upon the noise in the data as well as the noise in the process of inversion. Thus, these are useful in establishing the statistical significance of the results.

We use the Monte Carlo approach to determine the statistical spread or uncertainty in the magnetic field parameters as well as the non-potentiality parameters of the active region arising due to polarimetric noise. The approach is as follows.

1. First, we take real observations of a sunspot by Hinode SOT/SP and invert it with an ME code to retrieve the magnetic field vector.
2. Resolve the azimuthal ambiguity in the transverse field using the acute angle method.
3. Consider this vector field as the “true field” \([B_0, \gamma_0, \psi_0]\), and the derived quantities \(\alpha(B_0, \gamma_0, \psi_0)\) and \(\Psi(B_0, \gamma_0, \psi_0)\) as “true non-potentiality parameters” \([\alpha_0, \Psi_0]\), for reference.
4. Using the “true field” value at each pixel we generate synthetic Stokes profiles and add a polarimetric noise of level 0.5% of continuum intensity. These profiles are then inverted again under ME approximation to derive field vectors \([B, \gamma, \psi]\) and \([\alpha_g, \tilde{\Psi}]\). This process is repeated 100 times, each time with a different realization of polarimetric noise. The uncertainty in the field strength, inclination, and azimuth for each pixel and the uncertainty in the parameters \(\alpha_g\) and \(\tilde{\Psi}\) for the vector map are then estimated from the 1\(\sigma\) standard deviation in the 100 values of these parameters. This standard deviation is called the Monte Carlo error.

This simulation gives us an idea about the spread (standard deviation) in the derived field vector and the non-potentiality parameters of a real sunspot, arising due to polarimetric noise. This knowledge of standard deviation in the magnetic non-potentiality parameters \(\alpha_g\) and \(\tilde{\Psi}\) is important in determining whether the observed changes in these parameters, for example in relation to flares, are significant or not. Such studies are yet to be done and the present work will establish the level of uncertainties in the parameter \(\tilde{\Psi}\) or \(\alpha_g\) due to polarimetric noise in modern observations, such as from Hinode.

2.3. Method of Adding the Noise

We first generate synthetic Stokes profiles corresponding to the “true field” using ME-based Stokes profile synthesis and the inversion code named HeLIx (Lagg et al. 2004). To these synthetic profiles we add normally distributed random noise with the 3\(\sigma\) level of 0.5% of continuum intensity (\(I_c\)). The noise, \(N(\lambda)\), is added to the synthetic Stokes profiles, \(S_{\text{syn}}(\lambda)\), as follows.

1. First, a pseudo-random-number sequence is generated which is normally distributed with a zero mean and a 3\(\sigma\) standard deviation of given level, say \(L\). In our case, \(L = 0.5\% \times I_c\),

\[
N(\lambda) = \text{Random\_Number}(\text{Seed}, L).
\]

2. This sequence is then added to the synthetic Stokes profiles to yield the noisy Stokes profile, \(S_{\text{noi}}(\lambda)\),

\[
S_{\text{noi}}(\lambda) = S_{\text{syn}}(\lambda) + N(\lambda).
\]

The noise level is estimated from the observed signals in the continuum of the Stokes spectra. A continuum window between 6302.83 Å and 6303.28 Å is selected for monitoring the noise in the Stokes signal. The left panel of Figure 1 shows the histogram of noise in the observed Stokes profiles for a large number of pixels (1024 pixels) for a typical SOT/SP scan in “fast-map” observing mode. A Gaussian fit to this distribution yields a 3\(\sigma\) value of 0.5% of \(I_c\). This is the noise level that we used in our simulations. The histogram in the right panel of Figure 1 shows the distribution of artificial noise that we add to the synthetic profiles. These profiles are then inverted with the HeLIx code. This process is repeated 100 times and so for each pixel we have 100 values distributed around a mean value.

While modern spectro-polarimetric observations typically have noise levels of the order of 0.5% of \(I_c\), archived observations from the ground-based instruments, which might be used for synoptic studies, may have higher levels of noise. Therefore, we also carried out an exercise to check the variation in the magnetic as well as non-potentiality parameters, \(\tilde{\Psi}\) and \(\alpha_g\), with increasing polarimetric noise levels. We added four different levels of noise, i.e., 0.01%, 0.1%, 0.5%, and 1% of continuum intensity.

2.4. Method of Computing \(\alpha_g\) and \(\tilde{\Psi}\)

The computation of the non-potentiality parameter \(\alpha_g\) and \(\tilde{\Psi}\) is done as proposed by Tiwari et al. (2009a, 2009b). The expression for computation of these parameters is given in Equations (1) and (2) in Section 1. Only those pixels which have field values above a certain noise level are analyzed. This filtering is done in the following way: we select a quiet region on the Sun and evaluate 1\(\sigma\) deviation in the three vector field components \(B_x, B_y,\) and \(B_z\) separately. The box selected for this estimation is shown in the top-left panel of Figure 2. For transverse vector fields, we take the 1\(\sigma\) noise level as the resultant deviations obtained in \(B_x\) and \(B_y\). Only those pixels where the transverse and line-of-sight (LOS) fields both are together greater than twice the above mentioned noise level of 1\(\sigma\) are analyzed. The 1\(\sigma\) value for longitudinal and transverse field inside the box is 380 and 258 G, respectively.

3. RESULTS

3.1. Effect of Noise on Magnetic Field Parameters

Figure 2 shows the effect of noise on magnetic field parameters for the NOAA 10933 sunspot. The top row shows the initial magnetic field parameters \([B_0, \gamma_0, \psi_0]\) derived by ME inversion of Stokes profiles obtained by Hinode SOT/SP scan during 07:20 UT on 2009 January 5. The middle row shows the map of Monte Carlo error, i.e., the 1\(\sigma\) standard deviation in the magnetic field parameters, derived using the Monte Carlo method. The bottom row shows the map of standard error in the field
Figure 1. Left panel shows the histogram of the noise in the Stokes profiles ($S/I_c$) for typical SOT/SP “fast-mode” scan. The right panel shows the histogram of normally distributed pseudo-noise added in the synthetic profiles for Monte Carlo simulations. The solid line shows the fitted Gaussian with $1\sigma$ noise level is about 0.17% of $I_c$.

Figure 2. Top row shows the maps of magnetic field strength, inclination, and azimuth for the sunspot in NOAA 10933 derived using ME inversion. The solid and dashed line contours mark the penumbral and umbral regions, respectively. The white box in the top left panel marks the box used for estimation of noise levels in magnetic field components. The middle row shows the error maps ($1\sigma$ standard deviation) for field strength, inclination, and azimuth derived using the Monte Carlo method. The bottom row shows the maps of the standard error in ME fitting for the field strength, inclination, and azimuth.
parameters from the least-squares fit of the Stokes profiles. The maximum error (taking into account both the standard error and the Monte Carlo error estimate) inside the sunspot is less than 50 G for the field strength, while for inclination and azimuth it is less than a few degrees. The azimuth errors are typically largest in the umbral and plage region where the field is almost vertical and the azimuth is not well defined.

In Figure 2, we isolate the umbral and penumbral regions by using continuum intensity thresholds. We compare the standard errors and the Monte Carlo errors in these two regions in Figure 3. The number of pixels in the umbral and penumbral regions plotted in Figure 3 is 1953 and 13615, respectively. It may be seen that

1. In the umbral and penumbral regions the Monte Carlo errors are typically less than 5 G and 8 G for field strength, less than 1 and 0.5 for field inclination, and less than 5° and 2° for field azimuth, respectively.
2. The standard error is larger than Monte Carlo error estimates in both the umbral and penumbral regions for field strength as was found by Westendorp Plaza et al. (2001) in the case of the inversion of a single profile. For field azimuth and inclination it has an opposite relation, except for the middle panel of the bottom row.

The standard error depends upon the sensitivity of the \( \chi^2 \) to the changes in a particular parameter. It can happen that the Stokes profiles might not have significant sensitivity to the changes in that parameter. This results in a large standard error. On the other hand, the Monte Carlo method examines the changes in the derived parameters resulting from the changes in the Stokes profiles arising due to random fluctuations in the profile as a result of different realizations of noise being added. Since both errors depend upon different origins of the profile fluctuations, they need not produce the same results.

In general, considering all the panels in Figure 3, it may be concluded that the standard errors are larger than the Monte Carlo errors. Here, one should bear in mind that standard errors shown in Figure 3 are for inversion of real Stokes profiles which possess asymmetry, apart from the polarimetric noise, while Monte Carlo errors correspond to repeated inversion of purely synthetic, and therefore symmetric (or antisymmetric for Stokes-V) profiles with polarimetric noise. Therefore, the main source of difference between the two types of errors could be attributed to the presence of Stokes asymmetry in the real Stokes profiles leading to a large value of the standard error. Ideally, one would like to perform a Monte Carlo error estimate by obtaining several simultaneous observational data sets of a real sunspot where the Stokes profiles would have inherent Stokes asymmetry as well as different realization of polarimetric noise. However, in the absence of such a possibility we used the present method to study the effect of polarimetric noise alone, neglecting Stokes asymmetry. The results of the present study are therefore only the lower limits of the errors that can occur practically in real observations as well as in the non-potentiality parameters \( \Psi \) and \( \alpha_g \).

### 3.2. Effect of Noise on Azimuthal Ambiguity Resolution

It is well known that the Zeeman effect diagnostics cannot detect the direction of the transverse field component and so a 180° ambiguity remains in the determination of the field azimuth. Various methods, however, have been developed by researchers to resolve this ambiguity using different arguments (Metcalfe et al. 2006). One of the most common and widely applicable methods is the so-called acute angle method. In this method, the angle between the observed and potential transverse fields, i.e., \( \theta = \arccos (B_{\text{obs}} \cdot B_{\text{pot}}) / (|| B_{\text{obs}} || \cdot || B_{\text{pot}} ||) \), is computed and the solution for which the value of \( \theta \) is an acute angle is considered as the correct solution. However, in the presence of polarimetric noise in the observations, this method can also fail, especially in situations where the observed field is highly sheared. Such highly sheared regions...
are found near the polarity inversion line (PIL) of the active regions.

Fortunately, in a normal round sunspot, like the one used in our simulations, there are no high-shear regions and therefore the azimuthal ambiguity is easily solved with the acute angle method. Nevertheless, we need to check the effect of polarimetric noise on the azimuth ambiguity resolution in our simulations before we examine the uncertainty in the \( \hat{\Psi} \) and \( \alpha_g \) parameters of the sunspot in 100 realizations. After resolving the azimuthal ambiguity using the acute angle method for the 100 realizations of the sunspot vector maps, we made a map of 1σ standard deviation for the azimuth angle, as shown in the middle-right panel of Figure 2. If the ambiguity is not resolved properly, there will be fluctuations of the order of 180°, leading to a large value of standard deviation in azimuth value for a given pixel. However, in the right panel of the middle row in Figure 2 we see the following:

1. The resolution of the azimuth angle is quite stable for the most part within the sunspot, especially in the penumbral region. Outside the sunspot, in the quiet and facular areas, the errors are large. This is mainly due to poor S/N in these areas as a result of weaker and/or vertical fields.

2. As we go toward the umbra the values of standard deviation are large. We examined the values of the field azimuth in these regions and found that the values do not vary to the extent of 180° and therefore the large errors in the umbral region is not due to the ambiguity solver but due to poor S/N in Stokes-Q and U observations. Only in the very central part of the umbra, where the field inclination is close to 90°, the azimuth loses its meaning and so we see a large spread.

Further, we also checked the effect of the ambiguity solver with increasing noise, i.e., four different levels of noise of 0.01%, 0.1%, 0.5%, and 1% of continuum intensity. Here also the azimuth ambiguity resolution is stable for most part of the sunspot.

3.3. Effect of Noise on \( \hat{\Psi} \) and \( \alpha_g \)

Figure 4 shows the histogram of \( \hat{\Psi} \) and \( \alpha_g \) corresponding to 100 Monte Carlo realizations of the vector field. The following can be noticed:

1. The \( \hat{\Psi} \) is not affected much by noise. The distribution of \( \hat{\Psi} \) corresponding to 100 realizations show less scatter (~1%) with \( \hat{\Psi} = -1.68 \pm 0.014 \).

2. The values of \( \alpha_g \) are more affected. The distribution shows large scatter (~10%) in values with \( \alpha_g = -3.5 \pm 0.37 \times 10^{-9} \) m\(^{-1} \).

These results show that the \( \hat{\Psi} \) may be more reliable as compared to \( \alpha_g \) for typical polarimetric noise present in modern spectro-polarimetric observations as it is a lower dispersion parameter.

We also check the effect of increasing noise in the polarimetric measurements on the magnetic as well as non-potentiality parameters of the same sunspot. In order to compare the effect of increasing noise on the field parameters in the sunspot we isolate the umbral and penumbral regions, as was done in Figure 2, and do a scatter plot between the input and output field parameters, for different levels of noise. The scatter plot for umbral and penumbral regions is shown in Figures 5 and 6, respectively. It can be seen that, in the umbra, where the field is mostly vertical, the azimuth determination is more affected with increasing noise than the field strength and inclination. Especially for 0.5% and 1% noise levels the spread is large in the azimuth and inclination values. In comparison, the scatter plot for the penumbral region, where the field is not so vertical, shows that the azimuth is determined with less spread even for 0.5% and 1% noise levels, respectively.

Table 1 lists the non-potentiality parameters, derived from the ME inverted magnetic field vector, after adding noise of different levels in the synthetic Stokes profiles. In Table 1, the spatial fluctuations of the shear angle \( \Psi \) as well as local \( \alpha \) in the map, are represented as \( \sigma_\Psi \) and \( \sigma_\alpha \). Starting with no noise and then adding random noise of 0.01%, 0.1%, 0.5%, and 1% of continuum intensity in the Stokes profiles we get the following results: (1) the sign of the twist in the sunspot magnetic field, as inferred by both \( \alpha_g \) and \( \hat{\Psi} \), is negative and is not affected even when the S/N of Stokes profiles is poor; (2) the absolute value of \( \hat{\Psi} \) tends to decrease systematically with increasing noise in the Stokes profiles; and (3) the fluctuation of 2.5% in the \( \hat{\Psi} = -1.65 \pm 0.04 \) is less than the fluctuation of about 5% in \( \alpha_g = 3.15 \pm 0.17 \times 10^{-9} \) m\(^{-1} \).

4. DISCUSSION

Developing quantitative measures of magnetic non-potentiality in solar active regions is very important for flare research. The line-of-sight magnetic field alone is insufficient for this purpose, therefore people started measuring vector
Figure 5. Scatter plots between the input and derived magnetic field parameters after different levels of noise are added to Stokes profiles. The top of each panel shows the $3\sigma$ level of normally distributed noise that is added to the synthetic profiles. The $x$-axis shows the input magnetic field parameter used in the profile synthesis and the $y$-axis shows the output value of the parameter derived from the noise added synthetic profile.

| Noise (% of $I_c$) | $\hat{\Psi}$ (°) | $\sigma_{\hat{\Psi}}$ (°) | $\sigma_{g}(m^{-1})$ | $\sigma_{\alpha_g}(m^{-1})$ |
|------------------|------------------|-----------------|-----------------|-----------------|
| 0                | -1.692           | 15.724          | -3.334 x 10^{-9} | 3.572 x 10^{-7}  |
| 0.01             | -1.688           | 15.726          | -3.360 x 10^{-9} | 3.573 x 10^{-7}  |
| 0.1              | -1.685           | 15.702          | -3.221 x 10^{-9} | 3.574 x 10^{-7}  |
| 0.5              | -1.648           | 15.636          | -2.976 x 10^{-9} | 3.562 x 10^{-7}  |
| 1                | -1.604           | 15.364          | -3.026 x 10^{-9} | 3.539 x 10^{-7}  |

Table 1: Effect of Increasing Polarimetric Noise in the Estimation of $\hat{\Psi}$ and $\alpha_g$

magnetic fields. The stresses in the magnetic field were quantified in terms of the magnetic shear angle, by comparing the observed field with the potential field (or current-free fields). The distribution of shear angle over active regions showed that the filament bearing PIL regions are characterized by high shear and are potential sites for flares. Shear, as a local measure of non-potentiality, can be easily extended as a whole active region measure, like SASSA $\hat{\Psi}$ (Tiwari et al. 2009b).
Other physical measures of whole active region non-potentiality are the virial estimate of free energy and $\alpha_g$. Both of these measures require the photospheric magnetic field to be force-free. However, the force-free condition may not be justified as the plasma $\beta$ in the photosphere is not much smaller than unity and so the non-magnetic forces are not negligible. The effect of the polarimetric noise on the virial free-energy estimate was determined by Klimchuk et al. (1992) and Tiwari et al. (2009a). However, the results of these studies are valid for analytic force-free field solution of Low (1982). While, in real sunspots, the effect of non-force-freeness of photospheric field on virial energy and $\alpha_g$ may be larger than the effect of polarimetric errors quantified by Klimchuk et al. (1992) and Tiwari et al. (2009a). The whole active region non-potentiality characterized by SASSA $\hat{\Psi}$ is, however, free of force-free assumptions and in the present work we studied the effect of polarimetric noise on this parameter.

We simulated polarimetric profiles with a noise level of 0.5% of $I_c$, which is typical of Hinode SOT/SP observations as shown in Figure 1. It must be noted that the synthesis of these profiles is done under the assumption of the ME model atmosphere and so the present results pertain to symmetric Stokes profiles only.
The asymmetry of the Stokes profiles would lead to a systematic rather than random effect which needs to be quantified in a separate study. In 100 realizations of such noisy symmetric Stokes profiles we found that the parameter $\Psi$ is statistically more stable than $\alpha_g$. The reason for this stability of $\Psi$ as compared to $\alpha_g$ can be understood as follows. In Equation (2), $\Psi$ is derived as a simple summation of angles. So, the random errors would cancel each other in summation, while in Equation (1), we note that $\alpha_g$ depends upon the three components $B_x$, $B_y$, and $B_z$, and is not a simple summation and therefore the random errors in the three parameters would not vanish statistically. Thus, $\alpha_g$ would be more vulnerable to the noise as compared to $\Psi$.

Further, in the presence of different noise levels, i.e., 0.01%, 0.1%, 0.5%, and 1% of $I_0$, it is found that the standard deviation in $\Psi$ is less vulnerable to increasing noise than $\alpha_g$. The absolute value of $\Psi$ tends to reduce systematically with increasing noise.

One should also keep in mind that the accuracy of derived magnetic field parameters also depends upon the number of free parameters (parameters of the model atmosphere) used in the fitting procedure. The present results pertain to a single-component model atmosphere with a stray-light component. The results would be different when one employs a greater number of free parameters, like a two-component model atmosphere (Leka 2001; Lites et al. 2002) or the depth-dependent (stratified) model atmosphere used in SIR inversions. In general, the uncertainty in the best-fit model parameters will deteriorate with the increasing number of model (or free) parameters.

5. CONCLUSION

We generated an ensemble of artificial spectro-polarimetric data sets for a given sunspot. This ensemble of data is then inverted to give an ensemble of vector maps. These maps are then used to estimate uncertainties in the field parameters as well as in the non-potentiality parameters, $\alpha_g$ and $\Psi$, of the sunspot.

The standard errors of ME fitting are given by the inversion codes according to Press et al. (1986) and Bellot Rubio et al. (2000). The Monte Carlo errors give us an independent method to cross-check the standard error estimates. In Figure 3, we show that the Monte Carlo errors for field strength, inclination, and azimuth in the sunspot is determined within $\pm 8$ G, $\pm 1^\circ$, and $\pm 5^\circ$, respectively. A comparison shows that the Monte Carlo errors in field strength are typically smaller than the standard errors, while it is the opposite for the field azimuth and inclination, except for the field inclination in the penumbra where again the Monte Carlo errors are less than the standard errors. In general, the standard errors are more conservative estimates because they include the effects of polarimetric noise as well as those of Stokes asymmetry, while Monte Carlo errors account only for the polarimetric noise.

The effect of polarimetric noise on the parameters characterizing the non-potentiality of a sunspot magnetic field suggests that $\Psi$ is more robust than $\alpha_g$ (as shown by the histogram in Figure 4). Further, $\Psi$ appears to be a stable parameter with increasing noise in the polarimetric data (Table 1).

Thus, $\Psi$ will be useful for studying the evolution of non-potentiality of the active region magnetic fields, which in turn, can help in the prediction of flare occurrence. The parameter $\Psi$ in a large number of active regions as well as the evolution of $\Psi$ in flaring regions will be evaluated in a future work.

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