Calculation method of CGH for Binocular Eyepiece-Type Electro Holography

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Abstract. We had researched about eyepiece-type electro holography to display 3-D images of larger objects at wider angle. We had enlarged visual field considering depth of object with Fourier optical system using two lenses. In this paper, we extend our system for binocular. In the binocular system, we use two different holograms for each eye. The 3-D image for left eye should be observed like the real object observed using left eye and the same for right eye. So, we propose a method of calculation of computer-generated hologram (CGH) transforming the coordinate system of the model data to make two holograms for binocular eyepiece-type electro holography. The coordinate system of original model data is called the world coordinate system. The left and the right coordinate system are transformed from the world coordinate system. We also propose the method for correcting the installation error that occurs when placing the electronic and optical devices. The installation error is calculated and the model data is corrected using the distance between measured position and setup position of the reconstructed image. Optical reconstruction experiments were carried out to verify the proposed method.

1. Introduction
Holography records light waves from objects, and presents us with beautiful three-dimensional objects\cite{1}. The light waves reflected on the surfaces of objects are called object light waves. And interference pattern of the object light waves and reference light waves, which are used to record the objects and which have its wave length equivalent to the object light waves, is a hologram.

In electro holography, electronic display device is used to display the hologram\cite{2}. The viewing zone and the visual field in electro holography is limited into small area, because resolution of display devices is not sufficient for displaying the hologram, where the viewing zone is the area in which human can observe the image reconstructed from the hologram, and the visual field is the area in which the reconstructed image is placed. Therefore, the size of reconstructed image is limited under the size of display device. In addition, the reconstructed image needs some distance to be observed by the observer because of the characteristic of hologram. As a result, the apparent field of view, which is the angle size of the observed image from the observer, is more limited\cite{3}\cite{4}. To make the apparent field of view wide, the Fourier transform optical system has been used\cite{5}\cite{6}. The distance between the reconstructed image and the observer becomes shorter and the apparent field of view becomes wider in the Fourier transform system. However,
the size of the reconstructed image does not change and the apparent field of view is still limited in the Fourier transform system. In our previous study, we have proposed the method to enlarge the apparent field of view over 45 degree using two Fourier lens, the objective lens and the ocular lens[7]. However, the visual field has limited too narrow to observe with two eyes. In this paper, we combine two set of the previous optical system to make the reconstructed image observable with two eyes at a time. And we propose the calculation method of making the holograms with transforming the coordinate system for each eye. We also propose the method for correcting the installation error which occurs when placing the real objects, displaying devices and optical devices.

2. Theory

2.1. Monocular Eyepiece-type Optical System

The apparent field of view is extended to over 45 degrees with the monocular eyepiece-type optical system that we have proposed[7]. However, because the viewing zone is limited in very narrow region, the represented image is observed by only one eye. The monocular eyepiece-type optical system is consisted of a spatial light modulator(SLM), first lens as objective lens, a light emitting diode(LED) as reference light and second lens as ocular lens (figure 1).

At first, the primary image is reconstructed as a real image by the SLM, objective lens and reference light. And reconstruction of the real image is known as the Fourier transfer hologram. And the real image is enlarged as the virtual image by the ocular lens and the viewing zone is close to the optical system. As a result, the apparent field of view is extended.

The monocular eyepiece-type optical system is based on the conventional theory used in the telescope or microscope. However, while the telescope and microscope consider the real image as a plane image, the real image of the monocular eyepiece-type optical system is three-dimensional image with depth. And the reconstructed image is distorted, because the magnification power is constantly varying with the depth of the reconstructed image. To remove the distortion, we proposed a calculate method reshaping the model data, which is used to simulate the hologram in CGH. When a point of model data \(P(x, y, z)\) indicates the point of the virtual image, the point of the real image \(P'(x', y', z')\) is expressed as

\[
\begin{align*}
    x' &= \frac{f_{\text{ocular}}}{f_{\text{ocular}} + z_{\text{ocular}} - z} x, \\
    y' &= \frac{f_{\text{ocular}}}{f_{\text{ocular}} + z_{\text{ocular}} - z} y, \\
    z' &= z_{\text{ocular}} - \frac{f_{\text{ocular}}}{f_{\text{ocular}} + z_{\text{ocular}} - z} (z_{\text{ocular}} - z),
\end{align*}
\]

where \(f_{\text{ocular}}\) and \(z_{\text{ocular}}\) are focal length and Z-coordinate of the ocular lens. And the expanded

![Figure 1. Monocular eyepiece-type optical system.](image)
apparent field of view $\theta_{\text{apparent}}$ is expressed as

$$\theta_{\text{apparent}} = 2 \arctan \left( \frac{s_{\text{ocular}}}{2f_{\text{ocular}}} + \frac{s_{\text{ocular}} - s_{\text{SLM}}}{2z_{\text{ocular}}} \right),$$

where $z_{\text{ocular}}$ is the Z-position of the ocular lens in the left of the right coordinate system, and $s_{\text{ocular}}$ and $s_{\text{SLM}}$ are the size of the ocular lens and the SLM.

### 2.2. Binocular Eyepiece-type Optical System

We propose a binocular optical system consisted of two monocular eyepiece-type optical system to observe the reconstructed image with two eyes. Two reconstructed image is represented for binocular observing, one image for left eye and the other image for right eye with two set of monocular eyepiece-type optical system. To be consistent the two reconstructed image, we make two coordinate systems for each eye from the coordinate system of model data. We call the original coordinate system the world coordinate system (figure 2). The right optical system slide laterally varying the distance between left and right eye from 60mm to 70mm and the world coordinate system also slide laterally half amount of the movement of the right optical system, because the distance between left and right eye is different with person.

When a point of the world coordinate is $P_W(x_W, y_W, z_W)$, the corresponding points $P_L(x_L, y_L, z_L)$ and $P_R(x_R, y_R, z_R)$ of the left coordinate system and the right coordinate system are calculated by movement and rotation transformation, which are expressed as

$$
\begin{bmatrix}
  x_L \\
y_L \\
z_L \\
1
\end{bmatrix} = \begin{bmatrix}
  \cos \theta_R & 0 & \sin \theta_R & 0 \\
  0 & 1 & 0 & 0 \\
-\sin \theta_R & 0 & \cos \theta_R & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  1 & 0 & 0 & -x_{WO_L} \\
  0 & 1 & 0 & -y_{WO_L} \\
0 & 0 & 1 & -z_{WO_L} \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x_W \\
y_W \\
z_W \\
1
\end{bmatrix},
$$

$$
\begin{bmatrix}
  x_R \\
y_R \\
z_R \\
1
\end{bmatrix} = \begin{bmatrix}
  \cos(-\theta_R) & 0 & \sin(-\theta_R) & 0 \\
  0 & 1 & 0 & 0 \\
-\sin(-\theta_R) & 0 & \cos(-\theta_R) & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  1 & 0 & 0 & -x_{WO_R} \\
  0 & 1 & 0 & -y_{WO_R} \\
0 & 0 & 1 & -z_{WO_R} \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x_W \\
y_W \\
z_W \\
1
\end{bmatrix},
$$

where $\theta_R$ is the angle between the lines of sight and Z-axis of the world coordinate system, and

![Figure 2. Binocular eyepiece-type optical system.](image-url)
2.3. Correcting Installation Error

The installation error of the optical system makes considerable error in reconstructed image, because the real image is enlarged to the virtual image with high-power magnification. However, making the installation error zero is very difficulty and hard work. So, we propose the method of correcting the model data to offset the error with the installation error calculated using measured error of the reconstructed image.

We make three setup parameter $d_1$, $d_2$ and $d_3$ to control the position of the real and the virtual images; $d_1$ is the distance between the SLM and the objective lens; $d_2$ is the distance between the objective lens and the LED and $d_2$ is the same with the focal length of the objective lens; $d_3$ is the distance between the LED and the ocular lens. The installation error $\Delta d$ is consisted of $\Delta d_1$, $\Delta d_2$, $\Delta d_3$, the amount of difference in $d_1$, $d_2$, $d_3$ (figure 3). $\Delta d_1$ and $\Delta d_2$ make error in the position of the real image. When the point of $P_{Rerror}$ is the point influenced by the installation error $\Delta d_1$ and $\Delta d_2$ from setup point $P_R$, the Z-position of $P_{Rerror}$ is expressed as

$$z_{Rerror} = \frac{z_R f_{objective}^2 - z_R (\Delta d_1 + \Delta d_2) f_{objective} + (\Delta d_1 + \Delta d_2)^2 f_{objective}}{f_{objective}^2 + (\Delta d_1 + \Delta d_2)^2 - z_R (\Delta d_1 + \Delta d_2)}, \quad (4)$$

where $z_R$ is the setup Z-position of the real image point $P_R$ and $f_{objective}$ is the focal length of the objective lens. And the Z-position of the virtual image point $P_{Verror}$ influenced by the installation error $\Delta d$ is expressed as

$$z_{error} = \frac{f_{ocular}}{f_{ocular} - (z_{Rerror} + \Delta d_3) z_{Rerror}}, \quad (5)$$

and the X-position and Y-position are calculated as same.

Figure 4 express the relationship of the installation error $\Delta d$ and the Z-positions of the real image and the virtual image. The line of Z-positions moves up with increasing of the install error, which means the virtual image moves far from the observer.

When the amount of error in the reconstructed image $\Delta d_m$ is measured, $\Delta d$ is calculated with $\Delta d_m$ by reverse of equation 5. And the model data is corrected using $\Delta d$ to display the reconstructed image on the setup position. With the correction, the influence of installation error is removed and the reconstructed images becomes to have correct depth and binocular disparity.
2.4. Calculating Hologram
In the proposed method, the CGH is calculated in four steps.

(i) Read the model data and transform the model data to the left and the right data with equation 3.

(ii) Calculate the data for the real image from the data for virtual image which is the left and the right data with equation 1.

(iii) Correct the installation error from the data for the real image with the reverse equation of equation 5. The installation error $z_{error}$ needs to be pre-calculated.

(iv) Calculate the interference pattern $I$ of CGH from the corrected data using reference light $R$.

\[
\begin{align*}
    u_i(x_h, y_h) &= \frac{a_i}{d_i} \exp(-j(kd_i + \phi_i)), \\
    U(x_h, y_h) &= \sum u_i(x_h, y_h), \\
    I(x_h, y_h) &= \text{Re}[U(x_h, y_h)R(x_h, y_h)^*],
\end{align*}
\]

where $u_i(x_h, y_h)$ is the Fresnel-Kirchhoff diffraction on a pixel $(x_h, y_h)$ from i-th point $p_i$ of the corrected data which has its amplitude $a_i$ and phase $\phi_i$; $d_i$ is distance between $p_i$ and $(x_h, y_h, 0)$; $k$ is the wavenumber of light; $U(x_h, y_h)$ is light wave from every point; $R(x_h, y_h)$ is light wave from the reference light;

3. Experiments
We developed an optical system for proposed binocular eyepiece-type electro holography (figure 5-6). To prevent physical interference with the SLMs and the objective lenses, we inserted a prism in the right optical system, which isolated the left and the right optical system. And whole of the right optical system was on a rail to make the right optical system slide left or right. Parameters of experiments are shown in table 1.

To make sure proposed method is correct, we measured $\Delta D_m$ with the optical system and calculated $\Delta D$ using measured $\Delta D_m$. The shape of model is a vertical line, and the distance
of the line from observer is varying from 500mm to 900mm with 100mm of interval. And we made a hologram with proposed correction and reconstructed the image using the hologram and measured the Z-position.

3.1. Correction Installation Error
We made a hologram from model data and reconstructed the image from the hologram. And we measured 6-times, 3-times for left optical system and 3-times for right optical system, using holograms of each depth from the reconstructed image with camera focusing and comparing with real object reflected by half mirror (table 3.1). By assigning the average measured distance to equation 4, \( \Delta d \) was calculated to 39mm.
### Table 1. Parameters of optical system.

| Parameter                                                   | Value          |
|-------------------------------------------------------------|----------------|
| SLM pixel pitch                                            | 8.0 μm         |
| SLM pixels                                                 | 1920 x 1080    |
| Wavelength of reference light                              | 632nm          |
| Focal length of objective lens                             | 300mm          |
| Focal length of ocular lens                                | 100mm          |
| Setup distance between SLM and objective lens \(d_1\)       | 10mm           |
| Setup distance between objective lens and LED \(d_2\)       | 300mm          |
| Setup distance between LED and ocular lens \(d_3\)          | 150mm          |

### Table 2. Measured Z-position of the reconstructed image.

| Setup distance | Average measured distance (left) | Average measured distance (right) |
|----------------|----------------------------------|----------------------------------|
| 500mm          | 351mm                            | 330mm                            |
| 600mm          | 416mm                            | 388mm                            |
| 700mm          | 470mm                            | 460mm                            |
| 800mm          | 522mm                            | 496mm                            |
| 900mm          | 557mm                            | 592mm                            |

![Figure 7. Measured Z-position of the corrected reconstructed image.](image_url)

**3.2. Optical Reconstruction**

To verify the effect of correction, we simulated the holograms and measured the Z-position of the reconstructed image with line shape model corrected using \(\Delta d\). To make the measure more accurately, we set the line position from 450mm to 750mm with 30mm of interval. And we
measured the Z-position of reconstructed image 6-times as like as 3.1. In figure 7, the measured values with the left and the right optical systems seem around the line of theoretical value and the maximum error is 35mm at measured value 665mm of the left when the theoretical value is 665mm.

And we made a hologram which was simulated for reconstructing a triangle and a rectangle placed at 500mm and 1000mm from the observer (figure 8-9). The Z-position of the triangle and the rectangle was close the setup position and the binocular disparity was observed naturally.

4. Conclusion
In this paper, we proposed the calculation method of CGH for binocular eyepiece-type electro holography. Based on our previously monocular eyepiece-type optical system, we combined two optical system for each eye. And we proposed the coordinate transformation from the world coordinate system to the left and the right coordinate systems. We also proposed the correction method to remove the installation error which is too hard to make zero. The correction method is consisted of two phases, measuring the amount of error and correcting the installation error. The optical experiments showed that the correction was functioned well removing the installation error, so the setup model was shown at the setup coordinate. And the experiments also showed that the binocular images had proper binocular disparity and Z-depth, so some observers felt the images natural. However, the proposed correction method needs to be verify its vaidity with more experiments. And if the length of the optical system will be more short, it may be useful for a head-mount-display using electo holography.

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