Statistical nature of cluster emission in nuclear liquid-vapour phase coexistence

Y. G. Ma,1,2,3 D. D. Han,4 W. Q. Shen,1,2,5 X. Z. Cai,1 J. G. Chen,1 Z. J. He,1 J. L. Long,1 G. L. Ma,1 K. Wang,1 Y. B. Wei,1 L. P. Yu,1 H. Y. Zhang,1 C. Zhong,1 X. F. Zhou,1 and Z. Y. Zhu1

1Shanghai Institute of Nuclear Research, Chinese Academy of Sciences, P. O. Box 800-204, Shanghai 201800, CHINA
2China Center of Advanced Science and Technology (World Laboratory), P. O. Box 8730, Beijing 100080, CHINA
3Cyclotron Institute, Texas A&M University, College Station, Texas 77843-3366, USA
4Department of Electric Engineering, East China Normal University, Shanghai 200062, CHINA
5Physics Department, Ningbo University, Ningbo 315211, CHINA

(Dated: February 7, 2022)

The emission of nuclear clusters is investigated within the framework of isospin dependent lattice gas model and classical molecular dynamics model. It is found that the emission of individual cluster which is heavier than proton is almost Poissonian except near the transition temperature at which the system is leaving the liquid-vapor phase coexistence and the thermal scaling is observed by the linear Arrhenius plots which is made from the average multiplicity of each cluster versus the inverse of temperature in the liquid vapor phase coexistence. The slopes of the Arrhenius plots, i.e. the "emission barriers", are extracted as a function of the mass or charge number and fitted by the formula embodied with the contributions of the surface energy and Coulomb interaction. The agreement are obtained in comparison with the data for low energy conditional barriers. In addition, the possible influences of the source size, Coulomb interaction and "freeze-out" density and related physical discussions are addressed.

PACS numbers: 25.70.Pq, 05.70.Jk, 24.10.Pa, 02.70.Ns

I. INTRODUCTION

Fragmentation phenomenon is an important subject in many different research fields, such as in nuclear physics and cluster physics etc. It is found that fragmentation phenomenon has a close relation to liquid-to-gas phase transition in heavy ion collision physics and cluster physics. In heavy ion collisions, the fragmentation can occur from a short-lived hot excited nuclei with moderate temperature which can be formed by the collisions between heavy ions at low-intermediate energy. Usually the hot nuclei finally de-excite by the different decay modes, such as the light particle evaporation and the emission of multiple intermediate mass fragment (IMF), i.e. multifragmentation. Even though extensive studies on fragmentation have been carried out experimentally and theoretically, it is still an open question to clarify definitely whether the fragmentation is statistical or dynamical, sequential or simultaneous. Among such efforts, the reducibility and thermal scaling in multiple fragment emission process have been explored and seems to show one a possible interpretation picture to look and understand the multifragmentation. Originally, Moretto et al. observed that the experimental Z-integrated fragment multiplicity distributions $P_n^m$ are binomially distributed, i.e.

$$P_n^m(p) = \frac{m!}{n!(m-n)!} p^n(1-p)^{m-n}, \quad (1)$$

in each transverse energy ($E_t$) window, where $n$ is the number of emitted fragments and $m$ is interpreted as the number of times the system tries to emit a fragment. The probability of emitting $n$ fragments can be reduced to a single-particle emission probability $p$ which gives linear Arrhenius plots (i.e. excitation functions) when $ln(1/p)$ is plotted vs $1/\sqrt{E_t}$. By assuming a linear relationship between $\sqrt{E_t}$ and temperature $T$, Moretto et al. proposed that the linearity of the observed $ln(1/p)$ vs $1/\sqrt{E_t}$ can be interpreted as a thermal scaling of the multifragmentation process. In this case, these linear Arrhenius plots suggest that $p$ has the Boltzman form $p \propto e^{-B/T}$ with a common fragment barrier $B$. However, since the binomial decomposition has been performed on the Z-integrated multiplicities, typically associated with $3 \leq Z \leq 20$, the Arrhenius plot generated with the resulting one fragment probability $p$ is an average over a range of Z values. However, some debates and comments on the above binomial distribution and the thermal scaling were also raised.

Later, instead of analyzing for Z-integrated multiplicities, the behavior of individual fragment species of a given $Z$ for higher resolution experimental data was investigated and found that the $n$-fragment multiplicities $P(n)$ obey a nearly Poissonian distribution,

$$P(n) = \frac{\langle n \rangle^n e^{-\langle n \rangle}}{n!}, \quad (2)$$

where $n$ is the number of fragments of a given $Z$ and the average value $\langle n \rangle$ is a function of the total transverse energy $E_t$, and were thus reducible to a single-fragment probability proportional to $e^{-B/T}$ providing that $T \propto \sqrt{E_t}$, i.e. there exists also a ther-
nal scaling law. More recently, the common features of Poissonian reducibility and thermal scaling can also be revealed in percolation and the Fisher droplet model [17, 18].

In the present work, we would like to make a theoretical re-examination on the Poissonian reducibility and its thermal scaling rather than the binomial reducibility and its thermal scaling. Unlike experiment, we will adopt the true temperature as model input to check the Poissonian reducibility and thermal scaling in the frameworks of the isospin dependent lattice gas model (I-LGM) [19, 20] and followed by the isospin dependent classical molecular dynamics (I-CMD) of Das Gupta and Pan [21]. By investigating the variances and average multiplicities of cluster multiplicity distributions as a function of temperature, we will illustrate that the Poissonian reducibility and its thermal scaling is valid for the fragment emission barrier” sorted by the different mass number, plots and find the thermal scaling is valid only in the liquid vapor coexistence phase. Further we extract the emission barrier” sorted by the different mass number, light isotope, and charge number in I-LGM and I-CMD and use the formula embodied with the contributions of the surface energy and Coulomb interaction to make a systematic fit. The dependence of the model, the source size, Coulomb interaction and "freeze-out" density are presented. Finally, a summary and outlook is given in Sec. IV.

II. DESCRIPTION OF MODELS

Originally, the lattice gas model was developed to describe the liquid-gas phase transition for atomic system by Lee and Yang [22]. The same model has already been applied to nuclear physics for isospin symmetrical systems in the grand canonical ensemble [19, 20] with a sampling of the canonical ensemble [19, 20, 24, 25, 26, 27, 28, 29, 30], and also for isospin asymmetrical nuclear matter in the mean field approximation [31]. In this work, we will adopt the lattice gas model which was developed by Das Gupta et al. [19, 20]. In addition, a classical molecular dynamical model of Das Gupta et al. [21] is also used to compare with the results of lattice gas model. For completeness of the paper, here we make a brief description for the models.

In the lattice gas model, $A = N + Z$ nucleons with an occupation number $s_i$ which is defined $s_i = 1 (-1)$ for a proton (neutron) or $s_i = 0$ for a vacancy, are placed on the $L$ sites of lattice. Nucleons in the nearest neighboring sites have interaction with an energy $\epsilon_{s_i,s_j}$. The Hamiltonian is written as

$$E = \sum_{i=1}^{A} \frac{P_i^2}{2m} - \sum_{i<j} \epsilon_{s_i,s_j}s_is_j,$$

where $P_i$ is the momentum of the nucleon and $m$ is its mass. The interaction constant $\epsilon_{s_i,s_j}$ is chosen to be isospin dependent and be fixed to reproduce the binding energy of the nuclei [21]:

$$\epsilon_{nn} = \epsilon_{pp} = 0.924 MeV, \epsilon_{pn} = -5.33 MeV.$$ (4)

Three-dimension cubic lattice with $L$ sites is used which results in $\rho_f = \frac{A}{L^3}$ of an assumed ”freeze-out” density of disassembling system, in which $\rho_0$ is the normal nuclear density. The disassembly of the system is to be calculated at $\rho_f$, beyond which nucleons are too far apart to interact. Nucleons are put into lattice by Monte Carlo Metropolis sampling. Once the nucleons have been placed we also ascribe to each of them a momentum by Monte Carlo samplings of Maxwell-Boltzmann distribution.

Once this is done the I-LGM immediately gives the cluster distribution using the rule that two nucleons are part of the same cluster if

$$P_r^2/2\mu - \epsilon_{s_i,s_j}s_is_j < 0,$$

where $P_r$ is the relative momentum of two nucleons and $\mu$ is their reduced mass. This prescription is evidenced to be similar to the Coniglio-Klein’s prescription [32] in condensed matter physics and be valid in I-LGM [19, 20, 26, 27]. To calculate clusters using I-CMD we propagate the particles from the initial configuration for a long time under the influence of the chosen force. The form of the force is chosen to be also isospin dependent in order to compare with the results of I-LGM. The potential for unlike nucleons is [21, 24, 33]

$$v_{np}(r)(\frac{r}{r_0} < a) = A \left[ B(\frac{r_0}{r})^p - (\frac{r_0}{r})^q \right] \exp(\frac{1}{\frac{r_0}{a} - a}),$$

$$v_{np}(r)(\frac{r}{r_0} > a) = 0.$$ (6)

In the above, $r_0 = 1.842 fm$ is the distance between the centers of two adjacent cubes. The parameters of the potentials are $p = 2, q = 1, a = 1.3, B = 0.924$, and $A = 1966$ MeV. With these parameters the potential is minimum at $r_0$ with the value $-5.33$ MeV, is zero when the nucleons are more than 1.3$r_0$ apart and becomes stronger repulsive when $r$ is significantly less than $r_0$. The potential for like nucleons is written as

$$v_{pp}(r)(r < r_0) = v_{np}(r) - v_{np}(r_0),$$

$$v_{pp}(r)(r > r_0) = 0.$$ (7)
This means there is a repulsive core which goes to zero at \( r_0 \) and is zero afterwards. It is consistent with the fact that we do not put two like nucleons in the same cube. Essentially the classical molecular dynamics is based on the idea that the equation of state of classical particles interacting through attractive and repulsive Yukawa or Lennard-Jones potentials leads to an equation of state similar to a Van de Waals one. This kind of approach was first proposed in Ref. [34] to treat the nuclear collision with the similar potential as used in this paper. Extensive studies using the similar CMD to explore the liquid gas phase transition and fluctuations were also performed in some works, such as Ref. [35, 36, 37, 38].

The system evolves for a long time from the initial configuration obtained by the lattice gas model under the influence of the above potential. At asymptotic times the clusters are easily recognized. The cluster distribution and the quantities based on it in the two models can now be compared. In the case of proton-proton interactions, the Coulomb interaction can also be added separately and compared with the cases where the Coulomb effects are ignored.

III. RESULTS AND DISCUSSIONS

In this paper we choose the medium size nuclei \( ^{129}\text{Xe} \) as a main example to analyze the behavior of individual fragment emission during nuclear disassembly with the help of I-LGM and I-CMD. In addition, the systems with \( A_{sys} = 80 \) (\( Z_{sys} = 33 \)) and 274 (\( Z_{sys} = 114 \)) are also studied to investigate the possible source size dependence. One part of this work has been reported in Ref. [39] for \( ^{129}\text{Xe} \) in the I-LGM calculation with a fixed "freeze-out" density. In this work, a lot of new calculations are largely included.

In both model calculations, the "freeze-out" density \( \rho_f \) is mostly chosen to be about 0.38 \( \rho_0 \), since the experimental data can be best fitted by \( \rho_f \) between 0.3\( \rho_0 \) and 0.4\( \rho_0 \) in the previous LGM calculations [13, 10], which corresponds to 7\(^3\) cubic lattice is used for Xe, 6\(^3\) for \( A_{sys} = 80 \) and 9\(^3\) for \( A_{sys} = 274 \). Under the condition of the fixed "freeze-out" density, the only input parameter of the models is the temperature \( T \). In the I-LGM case, \( \rho_f \) can be thought as the "freeze-out" density but in the I-CMD case \( \rho_f \) is, strictly speaking, not a "freeze-out" density but merely defines the starting point for time evolution. However since classical evolution of a many particle system is entirely deterministic, the initialization does have in it all the information of the asymptotic cluster distribution, we will continue to call \( \rho_f \) as the "freeze-out" density. 1000 events are simulated for each \( T \) which ensures enough statistics.

A. Poissonian Reducibility

One of the basic characters of the Poisson distribution Eq. (2) is the ratio \( \sigma_{n_i}^2 / \langle n_i \rangle \to 1 \) where \( \sigma_{n_i}^2 \) is the variance of the multiplicity distribution and \( \langle n_i \rangle \) is the mean value of the multiplicity distribution. The first step we are showing is this ratio. We obtain these ratios for clusters classified with different masses (\( A \)), light isotopes (\( ISO \)) and atomic numbers (\( Z \)) for the disassembly of \( ^{129}\text{Xe} \) as a function of temperature in the framework of I-LGM and I-CMD with Coulomb in Fig. 1. Obviously, most of the ratios are close to one, which indicates that basically these cluster production obeys the Poisson distributions, i.e. a cluster is formed independently from one another. Of course, we also notice that the values of protons are almost lower than the unique, i.e. it is narrower than the Poisson distribution. This could be due to protons can be easily separated from some unstable multi-nucleon clusters. In this case, this kind of proton production is obviously related to the parent cluster and then a narrower distribution of mixed protons could reveal. On the contrary, some points are slightly larger than 1 and this behavior becomes obvious in the mediate temperature range, which is happening because at this temperature the system is leaving the liquid vapor phase coexistence [19], where the large fluctuation [41] makes the Poissonian reducibility broke-down.

Besides the above Poisson condition is basically sustained, the excitation function of the average multiplicity of \( n \)-multiple individual cluster emission can be well fitted with the Poisson distribution. For some examples, Fig. 2 shows the quality of the Poissonian fits to the average multiplicity of \( n \)-multiple individual cluster emission in the different temperature for \( ^{129}\text{Xe} \) in the I-LGM case. In each panel of this figure, we first plot the probability of \( n \)-multiple emission species ( \( P(n) \) ) as a function of temperature (as shown by the open symbols), and then we connect the Poisson probabilities in the different temperature as lines in terms of Eq. (2) due to we know \( n \) and its average value \( \langle n \rangle \) over all \( n \)-multiple emission in each temperature (as shown by the lines). Obviously, these Poissonian fits are quite good for almost species with \( Z \geq 2 \) over the entire range of \( T \). The similar good Poissonian fit is overall obtained in the cases of I-CMD.

As a consequence, the Poissonian reducibility is valid in the thermal-equilibrium lattice gas model or molecular dynamics, which illustrates that the cluster production is almost independent each other in the studied models.

B. Thermal Scaling

Naturally, we want to know if there exists a thermal scaling law in the thermal-equilibrium LGM and CMD models. To this end, the temperature dependence of the mean yield (\( \langle n \rangle \)) of individual clusters is investigated. In order to compare a recent well-known thermal Arrhenius-type plot in nuclear multifragmentation phe-
nomenon \[10, 11\], we plot \(\ln\langle n\rangle\) versus \(1/T\). Fig. 3 gives a family of these plots for the disassembly of \(^{129}\text{Xe}\) within the framework of I-LGM (left panels) and I-CMD with Coulomb interaction (right panels). Again, as Fig. 1 the clusters are classified according to their masses (upper panels), the light isotopes (the middle panels) and the charge numbers (the lower panels). For all the panels, the obtained Arrhenius plots are linear for the lower \(T\) side, and their slopes generally increase with increasing \(A\) or \(Z\) value. At these temperatures, one can anticipate one large fragment surrounded by many small clusters. However, contrary tendency reveals in the high \(T\) side where \(\ln\langle n\rangle\) increases with \(1/T\), i.e. decreases with increasing \(T\). In this case, nuclear Arrhenius plots of \(\langle n\rangle\) with \(1/T\) are invalid but the Poissonian reducibility still remains (see Fig. 2). This behavior of \(\langle n\rangle\) at higher \(T\) is related to the branch of the fall of the multiplicity of IMF \(\langle N_{\text{IMF}}\rangle\) with \(T\) where the disassembling system is in vaporization \[12, 13, 14\] and hence only the lightest clusters are dominated and the liquid residue is vanishing with increasing \(T\). Above the temperature at which the Arrhenius-type plot begins to deviate from the linearity, the fragments are no longer in coexistence with the big liquid drop. In this case, the supply of droplets to the vapor has been exhausted and the system is only in a single (gas) phase. In other words, the Arrenhius law looks valid only when the disassembling system is in coexistence phase of liquid and vapor. Recognizing this phenomenon, in the following sections we only focus on the branch of lower temperature (i.e. in phase coexistence) where the thermal scaling exists to discuss the Arrenhius law and their slopes.

C. “Emission Barriers”

1. Model dependence

From Fig. 3 the slope parameter can be directly extracted in the lower \(T\) side as a function of \(Z\) or \(A\). In Ref. \[15\] Moretto et al. has interpreted these slope parameters as “emission barriers” of specific individual fragments. This “emission barrier” is usually associated with pictures of sequent emission from a compound-like nucleus. Fig. 4 gives the “emission barrier” of individual fragments with different \(A\), ISO and \(Z\) in the framework of I-LGM, I-CMD with/without Coulomb interac-
Arrenhius-type plot: the average yield per event of different clusters classified with $A$ (top), ISO (middle) and $Z$ (bottom) as a function of $1/T$. The left panels show the results with I-LGM calculation and the right present the results with I-CMD including Coulomb force. The solid lines are fits to the calculations using a Boltzman factor for $\langle n_1 \rangle$. The symbols are illustrated on the figure.

The error bar in the figure represents the error in the extraction of the slope parameter. The first indication from this figure is that the "emission barrier" in the I-LGM case is the nearly same as the I-CMD case without Coulomb force, which supports that I-LGM is equivalent to I-CMD without Coulomb interaction rather well when the nuclear potential parameter is suitably chosen, but I-LGM is a quick model to analyze the behavior of nuclear dissociations. The inclusion of long-range Coulomb interaction makes the "emission barrier" of individual fragments much lower since the repulsion of Coulomb force reduces the attractive role of potential and hence make clusters escape easily. The second indication is that the "emission barrier" increases with $A$ (or $Z$) values and tend to be saturated at high $A$ (or $Z$) ones. Similar experimental trends have been observed for individual fragments with different $Z$ in Ref.[15] or different $A$ in Ref.[17]. To make a comparison with the experimental data, we plot the data for low energy conditional barriers for $^{82}Kr+^{12}C \rightarrow ^{94}Mo$ from Ref.[18] normalized to $Z=6$ for I-LGM case and I-CMD case, respectively. Overall good agreements between the calculation and data are obtained. We noticed that the middle panel of Fig. 3 shows that insensitivity of "emission barrier" of ISO on $A$ in the fixed atomic number $Z$, which indicates that the $Z$ dependence of barrier is perhaps more intrinsic and the $A$ dependence is basically due to the average effect over the species with the same $A$ but different $Z$.

Source size and Coulomb interaction dependence

On the origin of these barrier, the surface energy and Coulomb energy would play the roles. If the cluster emission is mainly controlled by its surface energy, it would suggest barriers proportional to $Z^{2/3}(A^{2/3})$. In the case of I-LGM and I-CMD without Coulomb, we can try to fit...
the barrier for the particles with different mass number by

\[ B_{\text{Coul,off}} = c_1 \times A_i^{2/3}, \]  

(8)

or for the particles with different charge number by

\[ B_{\text{Coul,off}} = c_1 \times ((A/Z)_{\text{fit}} \times Z_i)^{2/3}, \]  

(9)

where \((A/Z)_{\text{fit}}\) is a fit coefficient of \(A/Z\) for emitted particles, and \(A_i (Z_i)\) is the mass (charge) of particle. \(c_1\) is the fit constant for surface energy term. The solid line in the Fig. 4a is a function of Eq. (8) with \(c_1 = 8.469\) and the solid line in the Fig. 4c is a function of Eq. (9) with \(c_1 = 8.469\) and \((A/Z)_{\text{fit}} = 1.866\). These excellent fits imply that the surface energy play a major role in controlling the cluster emission when the long range Coulomb force is not considered. However for the cluster emission with the Coulomb field, we can assumed that the barrier is mainly constituted by the surface energy term and an additional Coulomb term as

\[ B_{\text{Coul,on}} = c_2 \times A_i^{2/3} - \frac{1.44 \times A_i/(A/Z)_{\text{fit}} \times Z_{\text{res}}}{r_{\text{Coul}}(A_i^{1/3} + ((A/Z)_{\text{fit}} \times Z_{\text{res}})^{1/3})}, \]  

(10)

for the particles classified with different mass number, or

\[ B_{\text{Coul,on}} = c_2 \times ((A/Z)_{\text{fit}} \times Z_i)^{2/3} - \frac{1.44 \times Z_i \times Z_{\text{res}}}{r_{\text{Coul}}((Z_i \times (A/Z)_{\text{fit}})^{1/3} + (Z_{\text{res}} \times (A/Z)_{\text{fit}})^{1/3})}, \]  

(11)

for the particles classified with different charge number, where \(c_2\) is a fit constant for surface term and \(r_{\text{Coul}}\) is chosen to be 1.22 fm. \(Z_{\text{res}}\) is a fitted average charge number of the residue. \((A/Z)_{\text{fit}}\) is chosen to be \(1.866\), as taken from the fits for I-LGM. The overall fits for \(A\) and \(Z\) dependent barrier in the case of I-CMD with Coulomb force give \(c_2 = 12.921\) and \(Z_{\text{res}} \sim 41\) with the dot-dashed line in Fig. 4a and 4c. The excellent fit supports that the Coulomb energy plays another important role in the cluster emission.

In the case of I-LGM and I-CMD without Coulomb, one would expect the barrier for each \(Z (A)\) to be nearly independent of the system studied if only the surface energy is substantial to the "emission barrier". The left panels of the Fig. 5 show the results for \(B_A, B_{\text{ISO}}\) and \(B_Z\) for three different systems in the I-LGM case. The same "freeze-out" density of 0.38\(\rho_0\) and the same \(N/Z\) is chosen for the systems of \(A_{\text{sys}} = 80\) and \(A_{\text{sys}} = 274\). Actually, it appears to have no obvious dependence of "emission barrier" on source size as expected for the role of surface energy. The solid line in the figure is the same as in Fig. 4. However, when the long-range Coulomb interaction is considered, the "emission barrier" reveals a source size dependence. The right panels of Fig. 5 give the "emission barriers" \(B_A, B_{\text{ISO}}\) and \(B_Z\) in the case of I-CMD with Coulomb force. It looks that the barrier increase with the decreasing of charge of system, which can be explained with the Eq. (10) and (11) where the decreasing of the residue \(Z_{\text{res}}\) will result in the decreasing of the Coulomb barrier and hence the increasing of the "emission barrier". The lines represent the fits with the Eq. (10) and (11) for three different mass systems.

3. "Freeze-out" density dependence

In the above calculations, the "freeze-out" density of systems is fixed at \(\sim 0.38\rho_0\). Considering the "freeze-out" density is an important debating variable in the later stage of heavy ion collisions, here we will discuss the possible influence of "freeze-out" density on the "emission barrier" of clusters. The calculations at the "freeze-out" density of 0.177\(\rho_0\) and 0.597\(\rho_0\) for \(^{129}\)Xe, corresponding to 9\(^3\) and 6\(^3\) cubic lattices respectively, are supplemented to compare. Fig. 6 gives the results of \(B_A, B_{\text{ISO}}\) and \(B_Z\) at different density. It looks that there are no obvious "freeze-out" density dependence in the both cases of I-LGM and I-CMD. This is also consistent with that assumption that the surface energy is the dominant role in controlling the cluster emission.

IV. SUMMARY AND OUTLOOK

In conclusion, the Poissonian reducibility and thermal scaling of the emitted clusters is explored in the lattice
gas model and molecular dynamical model of Das Gupta and Pan. It indicates of a statistical nature of such cluster emission. The Poissonian reducibility illustrates that the clusters are produced independently each other and stochastic. But near the transition temperature at which the system leaves the liquid vapor phase coexistence, the large fluctuation breaks down the Poissonian reducibility. The thermal scaling exists while the disassembling system is at coexistence phase. In this case, one can anticipate one large fragment (liquid) surrounded by many small clusters (vapor). The calculations are qualitatively consistent with the recent experimental observation of Poissonian reducibility and thermal scaling by Moretto/Wozniak’s group even though the system studied is different and the temperature was supposed to be proportional to the total transverse energy in their experiments. This also supports that the lattice gas model and classical molecular dynamics is a useful tool to simulate the nuclear disassembly.

Further, based on the Arrhenius law in the liquid phase, we extracted the “emission barrier” for clusters with the different mass, light isotope mass and charge number. The good agreement with the experimental data of the low energy conditional barrier is obtained if the data was normalization to \( Z = 6 \) for the calculation is used. Also, the systematic fits with the formula embodied with the surface energy and Coulomb interaction were performed and the overall good fits were reached. The results suggest that the cluster emission is mainly controlled by both the surface energy and the Coulomb interaction. In the framework of the lattice gas model and molecular dynamics model without the Coulomb interaction, the “emission barrier” relies on the cluster charge with the \( Z^{2/3} (A^{2/3}) \) law and it does not depend on the source size and “freeze-out” density, which indicates that the surface energy play a dominant role to control the cluster emission. However, in the framework of molecular dynamics model with the Coulomb force, the “emission barrier” will decrease strongly according to the Eq. (10) and (11) and it decreases with the increasing of the source size, illustrating that the Coulomb interaction also play another weighty role to control the cluster emission.

Due to the present models are basically statistical, even
for the present molecular dynamics model since it uses I-LGM predictions as the initial conditions for I-CMD. In this case the I-CMD results can not be considered as independent ones. Hence, the dynamical transport models, such as quantum molecular dynamics \[46\], Fermion molecular dynamics \[47\] and anti-symmetrized molecular dynamics \[48\] etc., will be very valuable to explore the statistical and/or dynamical features of cluster emission since these dynamical models include some essential ingredients of nuclei and their collisions, such as Fermi motion, Pauli blocking, mean field effect and collision term etc. They can produce clusters by stochastic motion without the initial thermal equilibrium assumption and "clustering". In fact, some studies have been carried on for investigating the binomial scaling of cluster emission in terms of an elementary binary decay mechanism using a simultaneous statistical multifragmentation model and a molecular dynamics model, such as Ref. \[49\]. A very different feature has been found in that study.

In the same spirit, it is also meaningful to use a molecular dynamics model followed by a statistical decay model \[50\] to explore the Poissonian reducibility and thermal scaling. The work along this line is also in progress.

Finally we would like to emphasize that the present work is based on the canonical model calculation, i.e., the temperature is fixed in the Monte Carlo simulations, which is appropriate for a system in contact with a large heat bath. However, a microcanonical treatment to the multiplicity fluctuation should be also very interesting since the energy conservation will probably play a different role in fragment fluctuation. Recently, Pratt and Das Gupta employed a fragment gas model to discuss the importance of energy conservation (canonical vs microcanonical) in affecting these fluctuation \[51\]. By investigating a correlation coefficient which was defined as the variance of multiplicity distribution relative to its mean in such a way that it is positive or negative for super- or sub-Poissonian distributions, they found that the correlation coefficient developed a singularity at the critical point when plotted against the temperature in canonical calculations; whereas in microcanonical calculations the correlation coefficient reached a gentle maximum for excitation energies in the fragmentation region. The energy conservation (i.e. microcanonical case) results in decreasing of multiplicity fluctuation and the multiplicity distribution always returns to be sub-Poissonian. However, even though the singular behavior of the correlation coefficient was muted in a microcanonical treatment, the behavior of the correlation coefficient as a function of the excitation energy was unique to the process of statistical fragmentation. Of course, this fluctuation phenomenon in different statistical ensembles could be model dependent. For instance, in a recent paper \[52\], Das et al. showed that microcanonical calculation of LGM has no serious departures from canonical results, e.g., the caloric curve and fluctuation of IMF distribution are very similar between canonical and microcanonical treatment. Detailed analysis and comparisons of the reducibility and thermal scaling with the microcanonical LGM are certainly welcome.

Acknowledgments

Authors are grateful to Prof. S. Das Gupta and Dr. J.C. Pan for providing the original LGM and CMD codes kindly. This work was supported in part by the Major State Basic Research Development Program of China under Contract No. G200077400 and the National Science Foundation of China under Grant No. 10135030 and 19725521.

[1] M.L. Gilkes, S. Albergo, F. Bieser, F.P. Brady, Z. Cacci, D.A. Cebra, A.D. Chacon, J.L. Chance, Y. Choi, S. Costa, J.B. Elliott, J.A. Hauger, A.S. Hirsh, E.L. Hjort, A. Insolia, M. Justice, D. Keane, J.C. Kintner, V. Lindenstruth, M.A. Lisa, U. Lynen, H.S. Matis, M. McMahan, C. McParland, W.F.J. Müller, D.L. Olson, M.D. Partian, N.T. Forile, R. Potenza, G. Rai, J. Rasmussen, H.G. Ritter, J. Romanski, J.L. Romero, G.V. Russo, H. Sann, R. Scharenberg, A. Scott, Y. Shao, B.K. Srivastava, T.J.M. Symons, M. Tinekell, C. Tuve, S. Wang, P. Warren, H.H. Wieman, and K. Wolf, Phys. Rev. Lett. 73 (1994) 1590.

[2] J. Pochodzalla, T. Mohlenkamp, T. Rubehn, A. Schuttaufl, A. Worner, E. Zude, M. Begemann-Blaiich, Th. Blaisch, H. Emling, A. Ferrero, C. Gross, G. Iacme, I. Iori, G.J. Kunde, W.D. Kunze, V. Lindenstruth, U. Lynen, A. Moroni, W.F.J. Muller, B. Ocker, G. Raciti, H. Sann, C. Schwarz, W. Seidel, V. Sorfling, J. Stroth, W. Trautmann, A. Trzcinski, A. Tucholski, G. Verde, and B. Zwieglinski, Phys. Rev. Lett. 75, 1040 (1995).

[3] Y.G. Ma, A. Siwek, J. P´eter, F. Gulminelli, R. Dayras, L. Naipas, B. Tamain, E. Vient, G. Auger, Ch.O. Bacri, J. Benlliure, E. Bisquer, B. Borderie, R. Bougault, R. Brou, J.L. Charvet, A. Chibi, J. Colin, D. Cusull, E. De Filippo, A. Demeyer, D. Dore, D. Durand, E. Écomard, Ph. Eudes, E. Gerlic, D. Gourio, D. Guinet, R. Laforest, P. Lautesse, J.L. Laville, L. Lebreton, J.F. Lecolley, A. Le Fèvre, T. Lefort, R. Legrain, O. Lopez, M. Louvel, J. Lukasik, N. Marie, V. Metivier, A. Ouatierza, M. Parkog, E. Plagnol, A. Rahman, T. Reposeur, M.F. Rivet, E. Rosato, F. Saint-Laurent, M. Squalli, J.C. Steckmeyer, M. Stern, L. Tassan-got, C. Volant, and J.P. Wieleczko, Phys. Lett. B 41 (1997).

[4] M. D’Agostino, F. Gulminelli, Ph. Chomaz, M. Bruno, F. Cannata, R. Bougault, F. Gramegna, I. Iorim, N. Le Neidre, G. V.Margaglioni, A. Moroni, and G. Vannini, Phys. Lett. B 390, 41 (1997).

[5] M. Kleine Berkenbusch, W. Bauer, K. Dillman, S. Pratt, L. Beaulieu, K. Kwiatkowski, T. Lefort, W.C. Hsi, V.E. Viola, S.J. Yennello, R.G. Koroteling, and U. Breuer,
S. Aiello, G. Immé, V. Pappalardo, and G. Raciti, R. J. Charity and L. G. Sobotka, I. Iori, A. Moroni, R. Scardoni, and A. Ferrero, W. Seidel, Th. Blaich, L. Stuttge and A. Cosmo, W. A. Friedman, and G. Peilert, Phys. Rev. Lett. \textbf{71}, 1502 (1993).

[44] Y.G. Ma and W.Q. Shen, Phys. Rev. C \textbf{51}, 710 (1995).
[45] K.X. Jing, L.G. Moretto, A.C. Veeck, N. Colonna, I. Lhenry, K. Tso, K. Hanold, W. Skulski, Q. Sui, and G.J. Wozniak, Nucl. Phys. A \textbf{645}, 203 (1999).
[46] J. Aichelin, Phys. Rep. \textbf{202}, 233 (1990).
[47] H. Feldmeier and J. Schnack, Rev. Mod. Phys. \textbf{72}, 655 (2000).

[48] H. Horiuchi, Nucl. Phys. A \textbf{522}, 257c (1991); A. Ono and H. Horiuchi, Phys. Rev. C \textbf{53}, 845 (1996).
[49] R. Donangelo and S.R. Souza, Phys. Rev. C \textbf{56}, 1504 (1997).
[50] Y.G. Ma, R. Wada, K. Hagel, M. Murray, J.S. Wang, L.J. Qin, A. Makeev, P. Smith, J.B. Natowitz, and A. Ono, Phys. Rev. C \textbf{65}, 051602(R) (2002).
[51] S. Pratt and S. Das Gupta, Phys. Rev. C \textbf{62}, 044603 (2000).
[52] C. B. Das, S. Das Gupta and S. K. Samaddar, Phys. Rev. C \textbf{63}, 011602(R) (2000).