Neutrino Masses and Beyond from Supersymmetry

Otto C.W. Kong

Department of Physics, National Central University, Chung-li, TAIWAN 32054
Theory Group, KEK, Tsukuba, Ibaraki, 305-0801, Japan
E-mail: otto@phy.ncu.edu.tw

Abstract

A generic form of the supersymmetric SM naturally gives rise to the lepton number violating neutrino masses and mixings, without the need for extra superfields beyond the minimal spectrum. Hence, SUSY can be considered the origin of beyond SM properties of neutrinos. We have developed a formulation under which one can efficiently analyze the model. Various sources of neutrino masses are discussed in details. Such mass contributions come from lepton number and flavor violating couplings that also give rise to a rich phenomenology of the neutrinos and other leptons, to be discussed.

* Talk presented at NOON 2003 (Feb 10-14), Kanazawa, Japan
— submission for the proceedings.
A generic form of the supersymmetric SM naturally gives rise to the lepton number violating neutrino masses and mixings, without the need for extra superfields beyond the minimal spectrum. Hence, SUSY can be considered the origin of beyond SM properties of neutrinos. We have developed a formulation under which one can efficiently analyze the model. Various sources of neutrino masses are discussed. Such mass contributions come from lepton number and flavor violating couplings that also give rise to a rich phenomenology of the neutrinos and other leptons, also to be discussed.

1. Introduction

From the theoretical point of view, low-energy supersymmetry (SUSY) is by far the most popular candidate theory for physics beyond the Standard Model (SM). On the experimental size, we now do have results confirming beyond SM properties of neutrinos, which at least includes oscillations among different neutrino species. The most natural way to have neutrino oscillations is to have massive neutrinos. Here, we are particularly interested in properties of such massive neutrinos that could actually be considered as arising from SUSY.

Within the SM, neutrino mass terms may be described by VEVs of dimension five operators of the form

\[ L_i \frac{\langle H \rangle \langle H \rangle}{M} L_j , \]

*Work partially supported by grant NSC91-2112-M-008-042 of the National Science Council of Taiwan.
where $M$ denotes some high energy scale. The non-renormalizable dimension five operators should be considered as obtained from integrating out some beyond SM physics underlying, physics of which can be probed only at scale beyond $M$. Neutrino masses are usually classified as Dirac or Majorana. Dirac mass terms involve singlet fermions usually named right-hand neutrinos ($\nu_R$ or $\nu_S$) giving rise to terms of the form

$$\bar{\nu}_{s_k} \langle H \rangle L_j.$$  

Lepton number violating Majorana mass terms at scale $M$ are typically introduced. Integrating out the heavy neutrino degrees of freedom leaves the seesaw induced effective SM neutrino Majorana masses. Direct introduction of such Majorana masses without heavy neutrino degrees of freedom have also been considered. The simplest way to do that is to introduce a Higgs triplet with VEV, giving rise to

$$L_i \langle T \rangle L_j.$$  

An effective triplet VEV $\langle T \rangle = \frac{\langle H \rangle \langle H \rangle}{M}$ is always needed, though it can also come from a loop diagram. Typically, we do need extra scalar bosons in the theory one way or another\(^1\).

So, the old question of whether neutrinos are Dirac or Majorana is not quite the right question to ask. Experimentally speaking, a (physical) neutrino is just a light neutral fermion that experiences weak interactions. As long as low energy phenomenology is concerned, we only need to know how many neutral fermion degrees of freedom are within reach, and what is the generic mass matrix.

2. Supersymmetry and Neutrinos

A supersymmetric extension of the SM has four extra neutral fermions apart from the SM ones. And nonzero masses are generally admissible for the full set of seven neutral fermions including the neutrinos.

From the early history of supersymmetry (SUSY), there had been thinking about its usage in the obviously non-supersymmetric low-energy phenomenology. One of the first idea was the identification of the neutrino as a goldstino, i.e. the Goldstone mode from (global) SUSY breaking\(^2\). Nowadays, the question : “Is the masslessness of the neutrino a result of SUSY (breaking)?” is obvious an uninteresting one. Nevertheless, neutrinos and SUSY just may have everything to do with one another; after all, nonzero masses of neutrinos may be a result of SUSY. The latter is related to the notion of R-parity violation.
The notion of R parity came about also early in the history of SUSY. In those days, baryon and lepton number symmetries might look even better than the standard model (SM) itself. R parity then seemed quite natural. However, global symmetries are since understood to be far less than sacred. The basic theoretical building blocks of the SM are nothing more than the field spectrum and the gauge symmetries, while we have now strong evidence of nonzero neutrino masses that very likely cannot be fit into the pure Dirac mass picture. Why should one stick to R-parity conservation?

If the accidental symmetries of baryon number and lepton number in the SM are to be preserved in the supersymmetric SM, they would have to be added in by hand, i.e. imposed as extra global symmetries on the Lagrangian. R parity, defined in terms of baryon number, lepton number, and spin as $R = (-1)^{3B+L+2S}$ does exactly that. This is, however, at the expense of making particles and superparticles having a categorically different quantum number. R parity is actually not the most effective discrete symmetry to control superparticle mediated proton decay resulted from having both $B$ and $L$ violation, but is most restrictive in terms of what is admitted in the Lagrangian, or the superpotential alone. Most importantly, it separates the four extra neutral fermion states (called neutralinos) from the SM neutrinos, and keeps the latter massless. Giving up the ad hoc notion of R parity, we naturally have massive neutrinos within the supersymmetric SM without the need to add any extra superfields to the minimal spectrum. In this way, one obtain massive neutrinos, from supersymmetry. More interestingly, the theory also gives a rich range of lepton number violating phenomenology from the same set of couplings that are responsible for the neutrino masses.

3. The generic supersymmetric Standard Model

The generic supersymmetric SM is a supersymmetrized SM with no extra symmetry, R parity or otherwise, imposed. The model Lagrangian is simply the most general one constructed using the necessary (minimal) superfield spectrum, the gauge symmetries and renormalizability requirement, as well as the idea that SUSY is softly broken. One does expect some mechanism or symmetry to take care of the proton decay problem, which may also be naively taken as having a large enough suppression among the B violating couplings, from the phenomenological point of view. The lepton number and flavor violating couplings are good for incorporating the beyond SM properties of the neutrinos.

The most general renormalizable superpotential for the generic super-
symmetric SM can be written as

\[
W = \varepsilon_{ab} \left[ \mu_\alpha \hat{L}_a^\alpha \hat{L}_b^\beta \hat{H}_d^\beta \hat{U}_k^c + \chi_{ijk} \hat{L}_a^\alpha \hat{Q}_i^j \hat{D}_k^c \right] + \frac{1}{2} \lambda_{\alpha\beta k} \hat{L}_a^{\alpha} \hat{L}_b^{\beta} \hat{E}_k^c + \frac{1}{2} \lambda'_{ijk} \hat{U}_i^c \hat{D}_j^c \hat{D}_k^c,
\]

where \((a, b)\) are \(SU(2)\) indices, \((i, j, k)\) are the usual family (flavor) indices, and \((\alpha, \beta)\) are extended flavor index going from 0 to 3. In the limit where \(\lambda_{ijk}, \lambda'_{ijk}\) and \(\mu_i\) all vanish, one recovers the expression for the R-parity preserving case, with \(\hat{L}_0\) identified as \(\hat{H}_d\). Without R-parity imposed, the latter is not \(a\ priori\) distinguishable from the \(\hat{L}_i\)'s. Note that \(\lambda\) is antisymmetric in the first two indices, as required by the \(SU(2)\) product rules, as shown explicitly here with \(\varepsilon_{12} = -\varepsilon_{21} = 1\). Similarly, \(\lambda'\) is antisymmetric in the last two indices, from \(SU(3)_C\).

Doing phenomenological studies without specifying a choice of flavor bases is ambiguous. It is like doing SM quark physics with 18 complex Yukawa couplings, instead of the 10 real physical parameters. As far as the SM itself is concerned, the extra 26 real parameters are simply redundant. There is simply no way to learn about the 36 real parameters of Yukawa couplings for the quarks in some generic flavor bases, so far as the SM is concerned. For instance, one can choose to write the SM quark Yukawa couplings such that the down-quark Yukawa couplings are diagonal, while the up-quark Yukawa coupling matrix is a product of (the conjugate of) the CKM and the diagonal quark masses, and the leptonic Yukawa couplings diagonal. Doing that is imposing no constraint or assumption onto the model. On the contrary, not fixing the flavor bases makes the connection between the parameters of the model and the phenomenological observables ambiguous.

In the case of the GSSM, the choice of flavor basis among the 4 \(\hat{L}_\alpha\)'s is a particularly subtle issue, because of the fact that they are superfields the scalar parts of which could bear VEVs. A parameterization called the single-VEV parameterization (SVP) has been advocated since Ref.\(^5\). The central idea is to pick a flavor basis such that only one among the \(\hat{L}_\alpha\)'s, designated as \(\hat{L}_0\), bears a non-zero VEV. There is to say, the direction of the VEV, or the Higgs field \(H_d\), is singled out in the four dimensional vector space spanned by the \(\hat{L}_\alpha\)'s. Explicitly, under the SVP, flavor bases are chosen such that: 1/ \(\langle \hat{L}_0 \rangle \equiv 0\), which implies \(\hat{L}_0 \equiv \hat{H}_d\); 2/ \(\chi_{ijk} \equiv \lambda_{ijk} = -\lambda_{jik}\) \(= \frac{\sqrt{2}}{v_0} \text{diag}\{m_1, m_2, m_3\}\); 3/ \(\chi_{ijk} \equiv \lambda_{ijk}\) \(= \frac{\sqrt{2}}{v_0} \text{diag}\{m_d, m_s, m_b\}\); 4/ \(\chi_{ijk} \equiv \frac{\sqrt{2}}{v_0} V_{CKM}^T \text{diag}\{m_u, m_c, m_t\}\), where \(v_0 \equiv \sqrt{2} \langle \hat{L}_0 \rangle\) and \(v_u \equiv \sqrt{2} \langle \hat{H}_u \rangle\). The parameterization is optimal,
apart from some minor redundancy in complex phases among the couplings. We simply assume all the admissible nonzero couplings within the SVP are generally complex. The big advantage of the SVP is that it gives the complete tree-level mass matrices of all the states (scalars and fermions) the simplest structure.\(^6\)

4. Neutrino Masses in the GSSM

The GSSM has seven neutral fermions corresponding to the three neutrinos and four, heavy, neutralinos. The heavy states are supposed to be mainly gauginos and higgsinos, but there is now admissible mixings among all seven neutral electroweak states. In the case of small \(\mu_i\)'s of interest, it is convenient to use an approximate seesaw block diagonalization to extract the effective neutrino mass matrix. Note that the effective neutrino mass here is actually written in a basis which is approximately the mass eigenstate basis of the charged leptons, i.e., the basis is roughly \((\nu_e, \nu_\mu, \nu_\tau)\). The tree-level result is very well-known.\(^7\) There have also been many papers devoted to the studies of radiatively generated neutrino masses from R-parity violation. Here, we focus only on discussions under, essentially, the current formulation.\(^8,9,10,11,12\)

The neutral fermion mass matrix \(M_\nu\) can be written in the form of block submatrices:

\[
M_\nu = \begin{pmatrix}
M_n & \xi \\
\xi^T & m_\nu
\end{pmatrix},
\]

where \(M_n\) is the upper-left \(4 \times 4\) neutralino mass matrix, \(\xi\) is the \(3 \times 4\) block, and \(m_\nu\) is the lower-right \(3 \times 3\) neutrino block in the \(7 \times 7\) matrix. Starting with the generic formula

\[
M(p^2) = M(Q) + \Pi(p^2) - \frac{1}{2} \left[ M(Q) \Sigma(p^2) + \Sigma(p^2) M(Q) \right]
\]

casted in the electroweak basis, the effective neutrino mass matrix at 1-loop level may be obtained as

\[
(m_\nu)_{1}\approx -\xi M_n^{-1} \xi^T + \Pi_\nu + \xi M_n^{-1} \xi^T + \xi M_n^{-1} \Pi_\xi
\]

\[
+ \frac{1}{2} \Sigma_\nu \xi M_n^{-1} \xi^T + \frac{1}{2} \xi M_n^{-1} \xi^T \Sigma_\nu + \xi M_n^{-1} \Pi_n M_n^{-1} \xi^T,
\]

where the \(\Pi\)'s and \(\Sigma\)'s denote two-point functions to be evaluated at 1-loop order. There are many pieces of contributions involving various lepton number violating couplings. A neutrino mass term involves violation of lepton number by two units, and basically any combination of two lepton number...
violating couplings in the model contributes. Moreover, to the extent that we are quite ignorant about the related phenomena, it is dangerous to make any assumption about the relative strength of such contributions, which is done quite often. We have indeed argued previously that the maximal mixing observed among neutrino flavors is likely to indicate a flavor structure here very different from what we see among the other SM fermions\textsuperscript{7}. Hence, it is rather necessary to check all the possible contributions and have the general result ready. We have given exactly such a listing\textsuperscript{10}.

To keep within the length limit, we have to satisfy with only illustrating some general feature of the detailed results. We are interesting in 1-loop two-point functions with fermions and scalars, both charged or both neutral, running in the loop. An exact evaluation requires using mass eigenstates for the running particles. We have given exact tree-level mass matrices for the five charged fermions, seven neutral fermions, eight charged scalars, and ten neutral scalars\textsuperscript{6}. Perturbational formulae for the elements of the diagonalization matrices are available\textsuperscript{6}. The latter are very useful for an analytical understanding of the lepton flavor violating origin/structure of each of the neutrino mass term. For instance, we have the charged loop contribution

\[
\Pi_{\nu_{ij}}^C = -\frac{\alpha_{em}}{8\pi \sin^2\theta_W} C_{R \, inm}^{\nu_j} C_{L \, jnm}^{\nu_i} M_{\chi_n} B_0(p^2, M_{\chi_n}^2, M_{\tilde{\ell}_m}^2), \tag{5}
\]

where \(C_{R \, inm}^{\nu_j}\) and \(C_{L \, jnm}^{\nu_i}\) are the effective couplings of the \(\nu_i\) and \(\nu_j\) to the \(m\)-th charged scalar \(\tilde{\ell}_m\) and \(R\) and \(L\)-handed parts of the \(n\)-th charged fermion \((\chi_n^-)\), respectively. Taking the \(\lambda\)-coupling term in \(C_{R \, inm}^{\nu_j}\) and the gauge coupling term in \(C_{L \, jnm}^{\nu_i}\), in particular, would resulted in a contribution proportional to

\[
\frac{\lambda_{ijh}}{g_2} V_{(h+2)m}^{\ast} M_{\chi_n} U_{1n} D_{(k+2)m}^j D_{(j+2)m}^{\ast} \approx \frac{\lambda_{ijh}}{g_2} \frac{m_{R \chi_n}^2 M_s}{M_s}. \tag{6}
\]

We note here that the result is actually very sensitive to the \(i \leftrightarrow j\) symmetrization. The dominant result in the expression above is from the case with the \((j+2)\)th charged scalar running in the loop. This is approximately the \(\tilde{\ell}_j^\pm\) slepton. The symmetrization and the fact that \(\lambda_{ijh} = -\lambda_{jih}\) suggest a perfect cancellation of the result in the limit of degenerate sleptons which correspond roughly to the \(\tilde{\ell}^-\) and \(\tilde{\ell}^+\) states\textsuperscript{9,10}.

5. Beyond Neutrino Masses

Our approach easily connect the neutrino mass results with a wide range of other phenomenologies. An interesting class of such we have been studying
is electromagnetic dipole moments of the various fermions. Published results are available for diagonal electric dipole moments of quark contributing to neutron electric dipole moment\(^ {14}\) and an example of transitional moments for the charged lepton giving rise to the \(\mu \rightarrow e \gamma\) decay\(^ {15}\). One common important feature among such dipole moment contributions is an interesting kind of contributions coming from a combination of a bilinear and a trilinear lepton number violating coupling — a \(\mu_i^* \lambda'_{i11}\) for \(d\)-quark dipole moment and a \(\mu_i^* \lambda_{i21}\) for \(\mu \rightarrow e \gamma\). The constraints on the couplings we obtained from studies are actually close to comparable to neutrino mass constraints. Analyzes of similar type of constraints from \(b \rightarrow s \gamma\) and neutrino dipole moment and radiative decays have are in progress.

We summarize results from our numerical study on the \(BR < 1.2 \times 10^{-11}\) experimental constraint on \(\mu \rightarrow e \gamma\)\(^ {15}\) in the following table for your interest.

\[
\begin{array}{|c|c|}
\hline
\text{Constraint} & \text{Value} \\
\hline
|\mu_i^* \lambda'_{i11}|, & < 1.5 \times 10^{-7} \\
|\mu_i^* | & < 0.53 \times 10^{-4} \\
|\lambda_{i21} \lambda'_{i31}|, & < 2.2 \times 10^{-4} \\
|\lambda_{i32} \lambda_{i31}|, & < 1.1 \times 10^{-4} \\
|B_{i1} \lambda_{i21}|, & < 2.0 \times 10^{-3} \\
|B_{i1} | & < 1.1 \times 10^{-5} \\
\hline
\end{array}
\]

The numbers are based inputs as given by

\[
\begin{array}{|c|c|c|c|c|}
\hline
M_1 (GeV) & M_2 (GeV) & \mu_0 (GeV) & \tan\beta \\
\hline
100 & 200 & 100 & 10 \\
\hline
\tilde{m}_e^2 (10^4 \text{ GeV}^2) & \tilde{m}_e^2 (10^4 \text{ GeV}^2) & A_e (\text{GeV}) \\
\hline
\text{diag}\{2,1,1,1\} & \text{diag}\{1,1,1\} & 100 \\
\hline
\end{array}
\]

6. Concluding Remarks

Supersymmetry could be considered a source of neutrino masses and other beyond SM properties of neutrinos. Promoting the field multiplet spectrum of SM to superfields gives naturally lepton number and flavor violating couplings admissible by the gauge interactions. In that sense, the result generic supersymmetric SM is the simplest supersymmetric model incorporating neutrino masses. Other alternatives require extra superfields
beyond the minimal spectrum, and usually also ad hoc global symmetries, in some case with specifically assumed symmetry breaking patterns. Another attractive feature of the generic supersymmetric SM is that the same set of couplings giving the neutrino masses also give rise to a width range of lepton number and flavor violating interactions. There is then correlation between the neutrino masses and other (collider) phenomenologies to be explored. Our formulation, called single-VEV parameterization, has been demonstrated to give a very effective framework to simplify any analytical studies of the model, making the task within easy reach. The whole discussion here is based on a purely phenomenological perspective. We are suggesting studying all the experimental constraints we could obtained on the set of couplings without theoretical bias. The hope to that we could eventually find some pattern among them and learn about the problem of the flavor structure among them. The lesson we learned so far ,from neutrino masses and mixings, is that the usually hierarchical flavor structure established among the Yukawa couplings of the quarks and charged leptons simply does not apply here. However, the lepton number violating couplings revealed through neutrino properties and otherwise may one day help to shed a light on the general flavor problem.

References

1. A very good example is given by the Zee model; A. Zee, Phys. Lett. 93B, 389 (1980).
2. D.V. Volkov and V.P. Akulov, Phys. Lett. 46B, 109 (1973).
3. G. Farrar and P. Fayet, Phys. Lett. 76B, 575 (1978).
4. L.E. Ibáñez and G.G. Ross, Nucl. Phys. B368, 3 (1992).
5. M. Bisset, O.C.W. Kong, C. Macesanu, and L.H. Orr, Phys. Lett. B430, 274 (1998); Phys. Rev. D62, 035001 (2000).
6. O.C.W. Kong, IPAS-HEP-k008, hep-ph/0205205, to be published in Int. J. Mod. Phys. A (2003).
7. See, for example, O.C.W. Kong, Mod. Phys. Lett. A14, 903 (1999).
8. K. Cheung and O.C.W. Kong, Phys. Rev. D61, 113012 (2000).
9. O.C.W. Kong, JHEP 0009, 037 (2000).
10. S.K. Kang and O.C.W. Kong, IPAS-HEP-k009, hep-ph/0206009.
11. Y. Grossman and H.E. Haber, hep-ph/9906310.
12. S. Davidson and M. Losada, JHEP 0005, 021 (2000); Phys. Rev. D65, 075025 (2002).
13. E.J. Chun and S.K. Kang, Phys. Rev. D61, 075012 (2000).
14. Y.-Y. Keum and O.C.W. Kong, Phys. Rev. Lett. 86, 393 (2001); Phys. Rev. D63, 113012 (2001).
15. K. Cheung and O.C.W. Kong, Phys. Rev. D64, 095007 (2001).