STOCHASTIC BACKGROUNDS OF GRAVITATIONAL WAVES FROM COSMOLOGICAL POPULATIONS OF ASTROPHYSICAL SOURCES

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Abstract. Astrophysical sources of gravitational radiation are likely to have been formed since the beginning of star formation. Realistic source rates of formation throughout the Universe have been estimated from an observation-based determination of the star formation rate density evolution. Both the radiation emitted during the collapse to black holes and the spin-down radiation, induced by the r-mode instability, emitted by hot, young rapidly rotating neutron stars have been considered. We have investigated the overall signal produced by the ensemble of sources exploring the parameter space and discussing its possible detectability.

1. Introduction

Stochastic backgrounds of gravitational waves are interesting sources for the interferometric detectors that will soon start to operate. Their production is a robust prediction of any model which attempts to describe the evolution of the Universe at primordial epochs. However, bursts of gravitational radiation emitted by a large number of unresolved and uncorrelated astrophysical sources generate a stochastic background at more recent epochs, immediately following the onset of galaxy formation. Thus, astrophysical backgrounds might overwhelm the primordial ones and their investigation provides important constraints on the detectability of signals coming from the very early Universe. The main characteristics of the gravitational backgrounds produced by cosmological populations of astrophysical sources depend both on the emission properties of each single source and on the source rate evolution with redshift. Extra-galactic backgrounds are proved to be mainly contributed by sources at redshifts $z \leq 1 - 2$ and their formation rate can not be simply extrapolated from its local value but must account for the evolution of the overall galactic population [1], [2], [3]. The
model we have adopted for the redshift evolution of the source rate of formation is described in Section 1 and it is based on the star formation history derived by UV-optical observations of star forming galaxies out to redshifts of $\sim 4 - 5$ (see e.g. [4], [5]).

The gravitational wave sources for which the extra-galactic background contributions have been investigated so far are white dwarfs binary systems during the early in-spiral phase [3] and core-collapse SNae. In particular, we have considered the gravitational waves emitted during the core-collapse to a black hole [1] and the gravitational radiation emitted by newly formed, rapidly rotating, hot neutron stars with an instability in their r-modes [2]. The first choice was motivated by the results of numerical simulations of core-collapses: unlike the case of a core-collapse to a neutron star, the gravitational wave emission spectrum produced during a core-collapse to a black hole is rather generic, in the sense that it is sufficiently independent of the initial conditions and of the equation of state of the collapsing star (see, for a recent review, [6]). The second kind of sources were considered because of their high efficiency in producing gravitational signals: though preliminary, the investigations of the r-modes instabilities in highly rotating young neutron stars have proved that a considerable fraction of the star initial rotational energy is converted in gravitational waves, making the process very interesting for gravitational wave detection [7], [8], [9], [10], [11].

A brief description of the characteristics of the source emission spectra is given in Section 2. Finally, in Section 3 we derive the spectra of the corresponding backgrounds, explore the parameter space and discuss their detectability.

2. The source formation rate

In the last few years, the extraordinary advances attained in observational cosmology have led to the possibility of identifying actively star forming galaxies at increasing cosmological look-back times (see e.g. [12]). Using the rest-frame UV-optical luminosity as an indicator of the star formation rate and integrating on the overall galaxy population, the data obtained with the Hubble Space Telescope (HST [13], [14]) Keck and other large telescopes [15], [5] together with the completion of several large redshift surveys [16], [17], [18] have enabled, for the first time, to derive coherent models for the star formation rate evolution throughout the Universe.

A collection of some of the data obtained at different redshifts together with a proposed fit is shown in Figure 1. Because dust extinction can lead to an underestimate of the real UV-optical emission and, ultimately, of the real star formation activity, the data shown in Fig. 1 have been corrected upwards according to the factors implied by the Calzetti dust extinction law (see [19], [5]). Although the strong luminosity evolution observed between redshift 0 and 1-2 is believed to be quite firmly established, the amount of dust correction to be applied at intermediate redshift (thus the amplitude of the curve at $z \sim 1 - 2$) as well as the behaviour of the star formation rate at high redshift is still relatively uncertain. In particular, the decline of the star formation rate density implied by the $< z > \sim 4$ point of the Hubble Deep Field (HDF, see Fig. 1) is now contradicted by the star formation rate density derived from a new sample of Lyman break galaxies with $< z > = 4.13$ [5] which, instead, seems to indicate that the star formation rate density remains substantially constant at $z > 1 - 2$. It has been suggested that this discrepancy might be caused by problems of sample variance in the HDF point at $< z > = 4$ [5]. Thus, we have up-dated the star formation rate model that we have previously
Figure 1. The Log of the star formation rate density in units of $M_{\odot}yr^{-1}Mpc^{-3}$ as a function of redshift for a cosmological background model with $\Omega_M = 1$, $\Omega_{\Lambda} = 0$, $H_0 = 50kms^{-1}Mpc^{-1}$ and a Salpeter IMF (see text). The data points correspond to [16] (cross), [17] (filled pentagon), [18] (filled circles), [14] (filled squares), HDF [13] (filled triangles), [15], [5] (asterisks).

considered in the analysis even though the gravitational wave backgrounds are almost insensitive to the behaviour of the star formation rate at $z > 1 - 2$ because the contributions of very distant sources is very weak [1], [2]. Conversely, if a larger dust correction factor should be applied at intermediate redshifts, this would result in a similar amplification of the gravitational background spectra.

From the star formation history plotted in Figure 1, it is possible to infer the formation rate (number of objects formed per unit time) of a particular population of gravitational wave sources (remnants) by integrating the star formation rate density over the comoving volume element out to redshift $z$ and considering only those progenitors with masses falling in the correct dynamical range for the remnant to form, i.e.,

$$R(z) = \int_0^z dz' \frac{\dot{\rho}_* (z')}{1 + z'} dV \int_{\Delta M} dM' \Phi(M'),$$

where the factor $(1 + z)^{-1}$ takes into account the dilution due to cosmic expansion and $\Phi(M)$ is the initial mass function (IMF) chosen to be of Salpeter type, $\Phi(M) \propto M^{-(1 + x)}$ with $x = 1.7$.

Stellar evolution models have shown that single stars with masses $\geq 8M_{\odot}$ pass through all phases of nuclear burning and end up as core-collapse supernovae leading to a neutron star or a black hole remnant. While there seems to be a general agreement that progenitors with masses in the range $8M_{\odot} \leq M < 20M_{\odot}$ leave neutron star remnants, the value of the minimum progenitor mass which leads to a black hole remnant is still uncertain, mainly because of the unknown amount of fallback of material during the supernova explosion [20], [21]. In our analysis, a reference interval of $25M_{\odot} \leq M \leq 125M_{\odot}$ was considered but we have also investigated the effects of choosing a lower limit of $20M_{\odot}$ and $30M_{\odot}$ as well as an upper limit of $60M_{\odot}$. The rate of core-collapse SNe predicted for three cosmological background models is shown in Figure 2 as a function of redshift. The main difference between the three cosmologies is introduced by the geometrical effect of the comoving volume and is
The rate of core-collapse supernovae (progenitor masses in the range $8M_\odot \lesssim M \lesssim 125M_\odot$) for three different cosmological background models. When varying the cosmological parameters, both the comoving volume element and the star formation rate density are properly modified (see [1]).

significant at $z > 1 - 2$. This implies that the gravitational backgrounds, which are mainly contributed by sources at $z > 1 - 2$, are almost insensitive to the cosmological parameters.

The total black hole formation rate $R_{BH}$ and neutron star formation rate $R_{NS}$ predicted by our model are,

$$R_{BH} = 3.3 - 4.7 s^{-1} \quad R_{NS} = 13.6 - 19.3 s^{-1}$$

depending on the cosmological background model considered. The value predicted by our model for the local core-collapse SNe rate is in good agreement with the available observations [22].

3. The single source emission spectra

The emission spectrum that we have adopted as our model for the gravitational wave radiated from a core which is collapsing to a black hole was that obtained from a fully non-linear numerical simulation of Einstein+hydrodynamic equations of an axisymmetric core collapse [23], [24].

The main properties of the spectrum are shown in Fig. 3 for the collapse of a $1.5M_\odot$ naked core to a black hole at a distance of 15 Mpc and for three assigned values of the angular momentum.

The relevant quantity is the rotational parameter $a = J/(GM^2_{core}/c)$. In fact, there is a maximum in the emission at a frequency which depends on the value of $a$ and whose amplitude, for values of $a$ in the range $0.2 < a < 0.8$, scales as $a^4$. This peak is located at a frequency which is very close to the frequency of the lowest $m = 0$ quasi-normal mode. This means that a substantial fraction of the energy will be emitted after the black hole has formed: it will oscillate in its quasi-normal modes until its residual mechanical energy is radiated away in gravitational waves.

For high values of the rotational parameter, the geometry of the collapse is different
Figure 3. The average energy flux emitted during the axisymmetric collapse of a rotating, polytropic star to a black hole of $M_{\text{core}} = 1.5M_\odot$ at a distance of 15 Mpc. The three curves correspond to assigned values of the rotational parameter $a$.

As the star becomes flattened into the equatorial plane and then bounces vertically, but still continues to collapse inward until the black hole is formed. In this case, a low frequency component appears, with an amplitude which may become comparable to that of the peak corresponding to the quasi-normal modes.

In general, the efficiency of this axisymmetric core-collapse to a black hole is $\Delta E_{\text{GW}}/M_{\text{core}}c^2 \leq 7 \times 10^{-4}$. It should be remembered that less symmetric configurations may result in a more efficient production of gravitational waves.

A number of investigations of relativistic rotating stars has recently led to the discovery of a new class of instability modes, called the r-modes [7], [8], [9], [10], [11]. These modes are characterized by having the Coriolis force as the restoring force and thus they are relevant only for rotating stars. Even though the analyses carried out so far are still preliminary and are based on several approximations, these modes, whose instability is driven by gravitational radiation, appear to efficiently radiate in gravitational waves a large part of the initial rotational energy in a relatively small time interval.

A preliminary estimate of the corresponding emission spectrum was recently obtained in [10] for a polytropic neutron star model with a 1.4$M_\odot$ mass and a radius of 12.53 km. We have adopted their proposed spectrum as our model for the single source emission in order to estimate the gravitational background produced by young, hot, rapidly rotating neutron stars through the r-mode instability [2].

The evolution of the angular momentum of the star is determined by the emission of gravitational waves, which couple to the r-modes through the current multipoles, primarily that with $l = m = 2$. For this mode, the frequency of the emitted gravitational radiation is $\nu = (2/3\pi)\Omega$. The star is assumed to be initially rotating at its maximum spin rate, i.e., at its Keplerian value $\Omega_K$, which corresponds to a gravitational wave frequency of $\sim 1400$ km, for the star model considered [10]. The evolution of $\Omega$ during the phase in which the amplitude of the mode is small can be determined from the standard multipole expression for angular momentum loss, and from the energy loss due to the gravitational emission and to the dissipative effects.
induced by the bulk and shear viscosity. In this phase, $\Omega$ is nearly constant and the instability grows exponentially. After a short time, the amplitude of the mode becomes close to unity and non-linear effects saturate and halt further growth of the mode. This phase lasts for approximately 1 yr, during which the star loses angular momentum radiating approximately $2/3$ of its initial rotational energy in gravitational waves, up to a point where the angular velocity reaches a critical value, $\Omega_c$. This value can be determined by solving the equation $1/\tau(\Omega_c) = 0$, where $\tau$ is the total dissipation time-scale which can be decomposed as a sum of the damping times associated to the gravitational emission, to the shear and to the bulk viscosity. $\tau(\Omega_c)$ is clearly a function of the temperature of the star, and it has been shown that the r-mode instability operates only in hot neutron stars ($10^{10} > T > 10^9$ K) \cite{10}. Above $10^{19}$ bulk viscosity kills the r-mode instability whereas below $10^9$ K superfluidity and other non-perfect fluid effects become important and the damping due to viscosity dominates with respect to the destabilizing effect of the gravitational radiation. For the star model considered, $\Omega_c \approx 566$ Hz, which corresponds to a final spin period of $\approx 11$ ms and to $\nu_{\text{min}} \approx 120$ Hz. Below this critical value, viscous forces and gravitational radiation damp out the energy left in the mode, and the star slowly reaches its final equilibrium configuration.

The qualitative picture that arises from this simple model is believed to be sufficiently reliable, even though various uncertainties and approximations might affect the quantitative results for the initial rotation of the star after collapse, for the spin-down time-scales as well as for the final rotation period \cite{11}. However, in this framework the expression of the energy spectrum can be approximated as follows,

$$\frac{dE_{GW}}{d\nu} \approx \frac{4}{3} E_K \frac{\nu}{\nu_{\text{max}}^2} \quad \text{for} \quad \nu_{\text{min}} \leq \nu \leq \nu_{\text{max}}$$

where $E_K$ indicates the initial rotational energy \cite{10}.

Thus, the mean flux emitted by this source can be written as,

$$f(\nu) = \frac{1}{4\pi d^2} \left( \frac{dE_{GW}}{d\nu} \right) .$$

4. The stochastic backgrounds

In order to evaluate the spectral energy density, $dE/dt dS d\nu$, of the stochastic backgrounds produced by the radiation emitted during an axisymmetric black hole collapse and by the spin-down radiation from newly born neutron stars, we need to convolve the differential rate of sources, $dR(z)$, with the flux emitted by a single source at redshift $z$ as it would be observed today (see \cite{1}, \cite{2}). This means that we account for the luminosity distance damping on the flux emitted by a single source and we redshift the emission frequencies.

The corresponding values of the closure energy densities of gravitational waves can be obtained as follows,

$$\Omega_{GW}(\nu_{\text{obs}}) = \frac{\nu_{\text{obs}}}{c^2 \rho_{\text{cr}}} \frac{dE}{dtdS d\nu} ,$$

where $\rho_{\text{cr}} = 3H_0^2 / 8\pi G$ and are shown in Fig. \ref{fig:energy}. These Figures have been obtained for a flat cosmological background model with zero cosmological constant and with a Hubble constant of $H_0 = h 100 = 50$ km/s/Mpc.
As previously mentioned, the effect of a varying cosmological background is negligible on the final properties of the stochastic backgrounds. In fact, the amplification of the rate at high redshifts shown in Fig. 2 for an open model and a model with a cosmological constant is mostly suppressed by the inverse squared luminosity distance dependence of the single source spectrum for the same models.

The closure density of the black hole collapse background is shown in the left panel of Fig. 4 for three values of the rotational parameter. Since we do not know the distribution of angular momenta, for each curve all the sources of the ensemble were assumed to have the same value of $a$. Depending on this value, the closure density has a maximum amplitude in the range $\sim 10^{-9} - 10^{-10}$ at frequencies between $\sim 2 - 3$ kHz. Even though the final properties of the background depend on the model that we have assumed as being representative of the process of gravitational collapse to a black hole, the relevant features of the energy spectrum we use to model each single event are likely to reasonably represent a generic situation, (see the discussion in [1]). As for the dependence on the formation rate of black holes, the uncertainties which affect the evolution of the star formation rate at high redshifts are completely irrelevant whereas variations induced by different lower and upper mass cut-offs of the progenitor mass range are limited to a factor $< 2$ [1].

As shown in the right panel of Fig. 4, the closure density for the neutron star background has a larger amplitude than the previous case and the main part of the signal is concentrated at lower frequencies. In fact, it is characterized by a wide maximum, ranging from $\sim 0.7 - 1$ kHz, with an amplitude of a few $10^{-8}$. Allowing for variations in $\nu_{\text{min}}$ and $\nu_{\text{max}}$ does not substantially alter the main features of the background although some quantitative differences appear both in the small and large frequency part of the signal (see Fig.s 6 and 8 in [2]).

The neutron star background allows a clear inspection of the impact of the star formation rate evolution on its final properties. In fact, in this case all the sources have been assumed to have the same mass and thus, elements of the ensemble at the same redshift have exactly the same emission properties. Therefore, it is easier to distinguish the effects of the source rate evolution from that of the spectrum of each single event. The right panel of Fig. 5 shows the spectrum of the neutron star
background. The maximum amplitude occurs around $\sim 700$ Hz. This means that the most significant contribution to the background signal comes from neutron stars at their maximum spin rate ($\sim 1400$ Hz, for our model) which are formed at redshifts $z \sim 1 - 2$ where the star formation rate reaches its maximum value before entering its high redshift plateau. Similarly, if one takes into account that the mean value of the core mass which collapses is around $\approx 4 - 5M_\odot$, the corresponding maximum in the contribution of a mean single source occurs at rest-frame frequencies in the range $2 - 3$ kHz. From the left panel of Fig. 5 it is possible to see that the maximum amplitude in the black hole background spectra corresponds to frequencies $\approx 1 - 2$ kHz, depending on the value of the rotational parameter. Thus, the relevant contribution to the final black hole background signal comes from those sources which are formed around $z \sim 1 - 2$.

Moreover, it is important to note that for sources, such as the one we have described, which emit gravitational waves at rest-frame frequencies $\nu > \sim 100$ Hz, at frequencies $1 - 100$ Hz, where cross-correlation between terrestrial interferometers can be accomplished, the stochastic background signal is entirely produced at $0 < z < 1 - 2$. We can conclude that a reliable estimate of astrophysical backgrounds can not set the important effect of the star formation rate evolution aside.

Finally, it is possible to show that the first generation of interferometers will not reach the sensitivities required to observe these backgrounds. In fact, the relevant part of the signal is at relatively high frequencies where, at their actual sites, the interferometers that will soon start to operate can not be cross-correlated.

For the first generation of interferometers, the best signal-to-noise ratio is obtained by cross-correlating VIRGO and GEO600 optimally oriented. Assuming one year of integration, $S/N \sim 2 \times 10^{-3}$. For the same integration time, two LIGO interferometers with advanced sensitivities give $S/N \sim 1.23$ at their actual sites and $S/N \sim 15$ if they were at a distance of $\sim 300$ km.

Though signal-to-noise ratios calculated for interferometer-bar pairs, such as VIRGO-NAUTILUS or GEO600-NAUTILUS are still very low, two hollow spheres with $\sqrt{S_n(200 \text{ Hz})} \sim 10^{-24}$ placed at the same site would reach, in one year of integration, a signal-to-noise ratio $S/N \sim 1 \times 10^4$. So far, the stochastic backgrounds we have

**Figure 5.** The spectra vs the observational frequency corresponding to the background produced by the radiation emitted by an ensemble of axisymmetric black hole collapses (left panel) and by the spin-down radiation emitted by an ensemble of rapidly rotating neutron star (right panel). The three curves on the left panel correspond to assigned values of the rotational parameter. A flat cosmological background model with zero cosmological constant and $h = H_0/100 = 0.5$ is assumed (see text).
described were considered to be continuous. This is always the case for the background produced by the spin-down radiation emitted by rapidly rotating neutron stars, as the signal from each single source is emitted in a relatively long time interval, of the order of 1 yr (see Section 2). Thus, these signals can superimpose and do form a continuous background [1]. Conversely, the background produced by core collapses to black holes has a shot noise character. In fact, the typical duration of the gravitational signal emitted by each source is much shorter than the previous case, of the order of a ms. Thus, the contributions from the elements of the ensemble do not superimpose but rather generate a shot-noise background, characterized by a succession of isolated bursts with a mean separation of the order of 0.1 seconds, much longer than the typical duration of each burst [1]. The peculiar statistical character of this background might be exploited in order to design a specific algorithm which may help its detection.

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