Finite-size effects on the hadron-quark mixed phase

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Abstract. We show that the hadron-quark mixed phase is restricted to narrow range of baryon chemical potential by the charge screening effect. Accordingly the mixed phase expected in hadron-quark hybrid stars should be narrow. Although the screening would not have large effect in bulk properties of the star such as mass or radius, it change the internal structure of the star very much, which may be tested by the cooling curve, glitch phenomena or gravitational waves.

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INTRODUCTION

Recently it seems to be widely believed that the structured mixed phase (SMP) would appear in the wide density range during the first order phase transitions with many (≥ 2) chemical potentials [1, 2, 3]. Applying the Gibbs conditions to get the equation of state (EOS), one may see non-uniform structures of not only baryon density but also charge density distribution for the mixed phase, and no constant pressure region in EOS. If this is the case, the Maxwell construction (MC) should become meaningless, where the phase equilibrium between two bulk matters with local charge neutrality is assumed. The appearance of such SMP in the hadron-quark deconfinement transition has been expected inside neutron stars and its implications have been discussed [3].

The importance of the finite-size effects in the mixed phase, which has not been fully included in the previous calculations, has been emphasized in recent papers: especially, the charge rearrangement effect induced by the Coulomb interaction should be carefully taken into account. Actually it has been demonstrated [4, 5, 6] that the Debye screening effect should greatly modify the description of the SMP.

To study non-uniform structure, we solve the coupled equations of motion based on the density functional theory [7], using the Wigner-Seitz approximation. In our calculation we can derive the density profiles of all the particle species and determine the configuration of the Coulomb potential exactly without recourse to any approximations included in the recent study.
NUMERICAL RESULTS

We present the thermodynamic potential density $\omega \equiv \Omega/V$ of each uniform matter in Fig. 1(a) as a function of the baryon number chemical potential $\mu_B$: the hadron phase (H) is thermodynamically favorable for $\mu_B$ below 1225 MeV and the quark phase (Q) above it. We also depict the one denoted by “bulk Gibbs” for comparison, where the Gibbs conditions are applied but the finite-size effects are completely discarded. MC can be represented as a point in this figure. We can immediately see that “bulk Gibbs” smoothly connects H and Q, and the mixed phase appears in this wide interval. We plot $\delta \omega$, the difference of the thermodynamic potential density between the mixed phase and each uniform matter, in Fig. 1(b): the curve denoted by “screening” is the result of self-consistent calculation, while the one denoted by “bulk Gibbs” corresponds to that in Fig. 1(a). We also depict another curve denoted by “no screening” to elucidate the charge screening effect, which is given by a perturbative treatment of the Coulomb interaction. Then we can see that the large reduction of the thermodynamical potential from “bulk Gibbs” is mainly given the effect of the surface tension, while the screening effect further reduces it. $\delta \omega$ given by MC appears as a point denoted by a circle in Fig. 1.

FIGURE 1. Thermodynamic potential density. (a) shows the results of each uniform matter and “bulk Gibbs”. (b) shows the difference between the mixed phases and the uniform matter.

(b) where two conditions, $P^Q = P^H$ and $\mu_B^Q = \mu_B^H$, are satisfied. On the other hand the mixed phase derived from “bulk Gibbs” appears in a wide region of $\mu_B$. Therefore, if the region of the mixed phase becomes narrower, it signals that the properties of the mixed phase become close to those given by MC. One may clearly see that the thermodynamic potential becomes close to that given by MC due to the finite-size effects.

Figures 2(a) and (b) show EOS in the cases with and without the screening effect. The pressure becomes more similar to that given by MC by the finite-size effects. Moreover, in Fig. 2(b), it becomes more close to MC by the charge screening effect, which shows a larger pressure in the beginning and weaker one in the end of the phase transition.
FIGURE 2. Pressure as a function of baryon-number density. (a) is the result of “no screening” and (b) “screening”. The results given by “bulk Gibbs” and MC are also presented for comparison.

SUMMARY AND CONCLUDING REMARKS

We have seen that the finite-size effects changes the properties of the hadron-quark mixed phase which is expected in hybrid stars. In particular, the region in the baryon-number chemical potential is restricted by the charge screening effect. We have seen that EOS becomes close to that with MC by the finite-size effects; EOS becomes more similar to that with MC by the charge screening effect.

Let us briefly consider some implication of these our results for compact star phenomena. Glendenning and Pei [3] suggested many SMPs appear in the core region by using “bulk Gibbs”: the mixed phase should appear for several kilometers. However we can say that the region of SMP should be narrow in the $\mu_B$ space and EOS is more similar to that of MC due to the finite-size effects. These results seem to be consistent with those given by other studies. Bejger et al. [8] have examined the relation between the mixed phase and glitch phenomena, and shown that the mixed phase should be narrow if the glitch is generated by the mixed phase in the inner core. On the other hand the gravitational wave asks for density discontinuity in the core region [9]. It is very interesting to study the relation between these phenomena and our results in more detail.

REFERENCES

1. N. K. Glendenning, Phys. Rev. D46 (1992) 1274.
2. H. Heiselberg, C. J. Pethick and E. F. Staubo, Phys. Rev. Lett. 70 (1993) 1355.
3. N. K. Glendenning and S. Pei, Phys. Rev. C52 (1995) 2250.
4. D. N. Voskresensky, M. Yasuhira and T. Tatsumi, Nucl. Phys. A723 (2003) 291.
5. T. Endo, Toshiki Maruyama, S. Chiba and T. Tatsumi, Nucl. Phys. A749 (2005) 333c.
6. T. Endo, Toshiki Maruyama, S. Chiba and T. Tatsumi, hep-ph/0502216.
7. T. Endo, Toshiki Maruyama, S. Chiba and T. Tatsumi, Prog. Theor. Phys. in press; hep-ph/0510279.
8. M. Bejger, P. Haensel and J. L. Zdunik, Mon. Not. Roy. Astron. Soc. 359 (2005) 699.
9. G. Miniutti, J. A. Pons, E. Berti, L. Gualtieri, V. Ferrari, Mon. Not. Roy. Astron. Soc. 338 (2003) 389.