NONTHERMAL SYNCHROTRON RADIATION FROM GAMMA-RAY BURST EXTERNAL SHOCKS AND THE X-RAY FLARES OBSERVED WITH SWIFT

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ABSTRACT

An analysis of the interaction between a spherical relativistic blast-wave shell and a stationary cloud with a spherical cap geometry is performed assuming that the cloud width \( \Delta \rho \ll x \), where \( x \) is the distance of the cloud from the gamma-ray burst (GRB) explosion center. The interaction is divided into three phases: (1) a collision phase with both forward and reverse shocks; (2) a penetration phase when either the reverse shock has crossed the shell while the forward shock continues to cross the cloud, or vice versa; and (3) an expansion phase when both shocks having crossed the cloud and shell, the shocked fluid expands. Temporally evolving spectral energy distributions (SEDs) are calculated for the problem of the interaction of a blast-wave shell with clouds that subtend large and small angles compared with the Doppler (cone) angle \( \theta_0 = 1/\Gamma_0 \), where \( \Gamma_0 \) is the coasting Lorentz factor. The Lorentz factor evolution of the shell/cloud collision is treated in the adiabatic limit. The behavior of the light curves and SEDs on, e.g., \( \Gamma_0 \), shell-width parameter \( \eta \), \( \Delta \rho + \eta V^2/\Gamma_0^2 \) is the blast-wave shell width, and properties and locations of the cloud is examined. Short-timescale variability in GRB light curves, including \( \sim 100 \) keV \( \gamma \)-ray pulses observed with BATSE and delayed \( \sim 1 \) keV X-ray flares found with Swift, can be explained by emissions from an external shock formed by the GRB blast wave colliding with small density inhomogeneities in the “frozen-pulse” approximation (\( \eta \to 0 \)), and perhaps in the thin-shell approximation (\( \eta \to 1/\Gamma_0 \)), but not when \( \eta \approx 1 \). If the frozen-pulse approximation is valid, then external shock processes could make the dominant prompt and afterglow emissions in GRB light curves consistent with short-delay two-step collapse models for GRBs.

Subject headings: gamma rays: bursts — hydrodynamics — radiation mechanisms: nonthermal — relativity

Online material: color figures

1. INTRODUCTION

Important knowledge about the nature of gamma-ray bursts (GRBs) comes from analyses of burst light curves. BATSE, triggering on peak flux over 64, 256, and 1024 ms timescales in the 50–300 keV band, provides the largest database at hard X-ray/soft \( \gamma \)-ray energies, amounting to over 2292 GRBs reported nearly 8 years into the mission (Fishman 1999) and \( \sim 2700 \) BATSE GRBs in total. Examining BATSE GRB light curves gives the impression that the \( \gamma \)-ray activity, so intense during the first \( \sim 1 \)–100 s, begins to weaken with time, all the while displaying pulses with relatively constant widths. If this evidence requires for its explanation a very powerful structured relativistic wind from an active central engine that becomes less vigorous with time, then the implied GRB explosion mechanism is very different than if this data is taken as evidence for a single explosive event where the GRB pulses are made by external shocks from a weakening blast wave that interacts with material in the circumburst medium.

The GRB afterglow revolution, initiated by BeppoSAX and continued with HETE-2 and INTEGRAL, gave for the first time GRB source redshifts and distances and therefore apparent isotropic powers and energies. With the opening angle of the relativistic jet inferred from the achromatic beaming breaks in optical light curves, the absolute energy of a GRB is fixed to the uncertainty in the efficiency of radiation production. The tools of standard relativistic blast-wave physics for the afterglow external shock emissions allow one to test for uniform or wind-formed circumburst media and determine the \( e_e \) and \( e_B \) parameters for assumed jet structures (Panaitescu & Kumar 2002). Fitting the statistical data on the GRB redshift and opening angle distributions can determine the intrinsic jet opening angle and whether a top-hat model for GRB jet structure gives good agreement with the data (Guetta & Piran 2006; Le & Dermer 2007).

A rich new database has been recently opened by Swift (Gehrels et al. 2004). After triggering with BAT in the 15–150 keV range, Swift autonomously slews within \( \approx 100 \) s to a burst so that the XRT, with excellent sensitivity, can get a clear picture of the evolution of the 0.3–10 keV X-ray emission. In large numbers of GRBs, there are steep declines in the X-ray emission at the end of the prompt X-ray luminous phase hundreds of seconds after the start of the GRB (Tagliaferri et al. 2005), an extended plateau phase (Barthelmy et al. 2005), and X-ray flares (Burrows et al. 2005), which in some cases have durations that are a small fraction of the time since the start of the GRB. Extrapolation of the BAT into the XRT band gives an almost continuous X-ray light curve since the GRB trigger (O’Brien et al. 2006b).

The interpretation of this data is complicated not only by the rich detail revealed by Swift but, for statistical studies, by its unusual triggering method, which makes the assignment of a threshold flux problematical (Band 2006). Because the Swift BAT uses a coded mask from which images of the GRB photons can be formed, a lower peak flux than pre-Swift telescopes can be identifiable as a GRB by using both rate and image triggers. A broader dynamic range, from 0.004 to 26 s, is used for the rate trigger. Differences in redshift distributions of Swift GRBs compared with the GRBs detected with BATSE, BeppoSAX, and HETE-2, which had comparable triggering criteria, can, however, mainly be understood by different peak flux sensitivities of Swift and pre-Swift detectors (Le & Dermer 2007).

The X-ray flares in the late prompt and early afterglow (\( t \sim 10^2–10^5 \) s) observed with the Swift satellite have been interpreted as evidence for internal shocks and refreshed energization of the...
relativistic outflow (Zhang et al. 2006; Chincarini et al. 2007). Liang et al. (2006) and Yamazaki et al. (2006) model the steeply declining phase as a superposition of background external shock emission and a curvature pulse produced by shells ejected long after the start of the GRB. This model for energy dissipation in GRBs and production of the GRB light curves is commensurate with the collapsar/failed supernova scenario for long-duration GRBs, where the evolved iron core of a massive star collapses to a black hole. In the collapsar picture, energy dissipation in an accretion torus or through the Blandford-Znajek process drives a narrow collimated jet through the stellar envelope. During the early prompt phase, continued activity drives successive waves of collimated relativistic plasma through the supernova shell. Collisions between the shells make GRB pulses. Later activity and peculiarities in the light curves are attributed to refreshed outflows or internal shocks from late outbursts of the engine. Interpreting rapid variations in the GRB X-ray light curve as activity of the central engine, then the engines in some GRB sources must be on for rest-frame times $\gtrsim 10^4$ s after the start of the event (Burrows et al. 2005, 2006; Falcone et al. 2006).

Early calculations of the spectral and temporal behavior of the GRB pulses in an internal shock model were made by Daigne & Mochkovitch (1998), Kobayashi et al. (1997), and Granot et al. (1999). The efficiency difficulties for the internal shock model (Kumar 1999; Beloborodov 2000; Mimica et al. 2007) are reduced by large Lorentz factor contrasts between different shells (Kobayashi & Sari 2001), which constrains the collision radius and $\nu F_{\nu}$ peak energies under the requirement of Thomson thinness (Guetta et al. 2001). For the internal shock/colliding shell model, as well as for the external shock model, efficiency concerns are ameliorated by the large hadronic energy content in the prompt phase that escapes to form the ultrahigh-energy cosmic rays (Dermer 2007).

A complete description of the collision between two relativistic shells that follows both the hydrodynamics and radiation physics needed to calculate emission spectra and light curves from different pulses in GRBs, necessary to test the internal shell model, has only been recently treated, in a series of papers by Mimica et al. (2004, 2005, 2007). Here we present a complete and systematic analysis of the collision between a cold GRB blast-wave shell and a cloud with a spherical cap geometry in an external shock model. We use these results to check the widespread claims by the Swift team that central engine activity is responsible for the erratic X-ray flares from GRBs (e.g., O’Brien et al. 2006a; Falcone et al. 2006; Romano et al. 2006; Burrows et al. 2007). We dispute these claims and argue that an external shock model can make the prompt $\gamma$-ray pulses and afterglow X-ray flares under the condition of no spreading of the cold blast-wave fluid shell. If this assumption is allowed, then interesting implications for the nature of the collapse process in GRB sources follow. A short-delay two-step collapse process, in fact, provides a competing and compelling alternative to the collapsar/failed supernova model, as discussed below (§ 5).

The emissivities calculated from the interaction of a blast-wave shell and a stationary cloud are integrated over three spatial dimensions to obtain spectral fluxes as a function of observer time. Only the synchrotron component is considered here (for a recent treatment of the synchrotron self-Compton component in external shocks, see Galli & Piro 2007), and other assumptions are made to simplify the analysis, for example, a randomly ordered magnetic field and isotropic electron distribution in the proper frame of the shocked fluid.

1. Long-duration GRBs are referred to henceforth as GRBs; here we do not consider the short hard GRB class or the low-luminosity GRBs, also argued to form a separate class (Liang et al. 2007).

The techniques of the standard blast-wave model are used (e.g., Sari et al. 1998; Piran 1999, 2005; Mészáros 2006), and the results are applied to observations of prompt $\sim 100$ keV emission pulses and $\sim 1$ keV X-ray flares in GRB light curves. The problem is analyzed and broken into three distinct interaction phases, as described in § 2. Calculations of light curves and spectral energy distributions (SEDs) from the emissions of external shocks formed by the interaction of a GRB blast wave with a stationary cloud are presented in § 3. Parameter are chosen to produce short-timescale variability (STV; $\Delta t_{\nu}/\nu \ll 1$, where $\Delta t_{\nu}$ is the variability timescale and $\nu$ is the time since the start of the GRB) in the $\gamma$-ray light curve, which is only possible if the blast-wave shell width can be approximated by a thin shell ($\Delta x < \nu t_{\nu}^3$) or frozen (unspreading) shell. This result is demonstrated analytically in § 4 and illustrated by Monte Carlo simulations of GRB light curves. We conclude that the X-ray flares observed with Swift are compatible with an external shock model if the GRB blast-wave shell interacts with a clumpy surrounding external medium and does not spread transversely. This assumption is discussed in § 5 and is argued to be correct. Implications for the nature of the collapse mechanism and jet formation in long-duration GRBs are also discussed in § 5. The study is summarized in § 6.

The reader who is not interested in the detailed analysis may avoid § 2 and refer directly to the numerical results in § 3. The principal conclusions of the analysis are found in the simplified analytic descriptions in § 4.

2. ANALYSIS OF THE BLAST-WAVE/CLOUD INTERACTION

Consider a GRB that takes place at redshift $z$ and releases energy over a characteristic timescale $\Delta_0/c$ representing the period of activity of the GRB central engine. The corresponding spatial dimension $\Delta_0$ should probably be greater than the Schwarzschild radius of the several solar mass black hole formed in the GRB event; thus, $\Delta_0 \approx 10^6$ cm. Depending on the nature of the central engine of GRBs, $\Delta_0$ could range from a fraction of a second to hundreds of seconds or longer if, as in the collapsar model for the GRB prompt phase, the duration of the highly variable X-ray and $\gamma$-ray flux in a GRB is assumed to reflect the period of activity of the engine. Here we consider a GRB engine where the progenitor neutron star collapses impulsively to a black hole, so that $\Delta_0 \sim 10^4 \Delta_7$ cm, with $\Delta_7 \sim 1$.

The apparent isotropic equivalent $\gamma$-ray energy released by a GRB explosion is written as $E_0$. The apparent $20$ keV–$2$ MeV isotropic rest-frame energies measured from GRBs typically range in values from $\approx 10^{51}$ to $10^{54}$ ergs, with a handful of anomalously low energy GRBs with energies as low as $10^{48}$ ergs (e.g., GRB 980425; Friedman & Bloom 2005; Ghirlanda et al. 2007). A significant number of GRBs have $E_0 \gtrsim 10^{54}$ ergs, so that the total GRB energy must be larger, by a factor of at least several (and possibly much larger if the radiation efficiency is low or energetic hadrons are formed and escape, e.g., as neutrons; Atöyan & Dermer 2003; Dermer 2007). In this study, jet effects are considered as a restriction on the interaction angle, the jet structure is assumed to be “top-hat,” and there is no lateral and very little transverse spreading.

The explosion is assumed to form a fireball with initial Lorentz factor (entropy per baryon) denoted by

$$\Gamma_0 = \frac{E_0}{M_0 c^2} \gg 1,$$

where $M_0$ is the amount of baryonic matter mixed into the initial explosion. In this analysis, we do not consider effects of neutron
decoupling (Derishev et al. 1999; Bahcall & Mészáros 2000; Beloborodov 2003) and, furthermore, assume that the blast-wave energy in the coasting stage is carried primarily by particle rather than field energy (see, e.g., Lyutikov & Blackman 2001 for the latter case).

The blast-wave plasma shell reaches its coasting Lorentz factor $\Gamma_{0}$ at distance $x \approx \Gamma_{0} \Delta_{0}$ from the explosion. At $x \gtrsim x_{\text{spr}} \equiv \Gamma_{0}^{2} \Delta_{0}$, where $x_{\text{spr}}$ is the spreading radius, internal motions within the blast-wave shell are thought to cause it to spread so that $\Delta(x) \approx \eta \Gamma_{0} \Delta_{0}$ (Mészáros et al. 1993; Piran 1999), where $\eta \ll 1$. We consider the case that the shell experiences little spreading, so that $\eta \ll 1$, which has as its asymptotic limit the frozen-pulse approximation ($\eta \to 0$). More discussion about $\eta$ and shell spreading is given in §4.4.

In the stationary frame of the explosion, the shell width $\Delta(x)$ is therefore given by

$$\Delta(x) \approx \Delta_{0} + \eta \frac{x}{\Gamma_{0}}.$$  

(2)

The proper number density of the relativistic shell is given by

$$n(x) = \frac{E_{0}}{4\pi x^{2} \Gamma_{0}^{2} m_{p} c^{2} \Delta(x)}.$$  

(3)

2.1. Geometry of the Blast-Wave/Cloud Interaction

The interaction event is sketched in Figure 1. The blast-wave shell of width $\Delta = \Delta(x_{0})$ collides with a uniform cloud with density $n_{cl}$. The cloud is assumed to have a spherical cap geometry and to subtend a solid angle $\Delta \Omega_{cl} = \pi \theta_{cl}^{2}$ as measured from the explosion center, and it therefore presents a projected area $A_{cl} = \pi \theta_{cl}^{2} x_{0}^{2}$ to the GRB blast wave, where $x_{0}$ is the distance from the origin to the inner edge of the cloud. The angle between the axis through the cloud center and the line of sight to the observer is denoted by $\theta_{i}$. The coordinates $x = (x, \theta, \phi)$ defining the location of the radiating material in the stationary (explosion) frame are measured with respect to the line of sight to the observer, with the angle $\phi$ representing the projection of the azimuth on the plane normal to the direction to the observer; thus, $\phi = 0$ is defined with respect to the projection of the axis through the cloud center on this plane (see Fig. 1a). For simplicity, both the shell and cloud are assumed to be composed of electron-proton plasma.

The $\nu F_{\nu}$ flux, denoted by $f_{\nu}(t)$, measured at observer time $t$ and at dimensionless photon energy $\epsilon = h\nu/m_{e}c^{2}$, is given by

$$f_{\nu}(t) = d_{L}^{-2} \int_{0}^{2\pi} d\phi \int_{-1}^{1} d\mu \int_{0}^{\infty} dx x^{2} f'_{\nu}(\epsilon'; \Omega'; x, t') \delta_{D},$$  

(4)

where $f'_{\nu}$ refers to comoving quantities, $\epsilon' = c\epsilon/\delta_{D} \equiv (1 + z)c/\delta_{D}$, and the integration is over volume in the stationary (explosion) frame (Granot et al. 1999; Kumar & Panaitescu 2000; Dermer 2004). The Doppler factor of the radiating fluid is defined by the expression

$$\delta_{D} = \frac{1}{\Gamma (1 - \beta \mu)},$$  

(5)

where $\mu = \cos \theta$, $\Gamma = \Gamma(x, t', \epsilon)$ is the emitting fluid’s Lorentz factor, and $\beta c = c(1 - \Gamma^{-2})^{1/2}$ is its speed. The differential emissivity

$$f'_{\nu}(\epsilon'; \Omega'; x, t') = \frac{d\mathcal{E}_{\nu}(\Omega')}{dV' dt' d\Omega' d\epsilon'}$$

is defined such that $f'_{\nu}(\epsilon'; \Omega'; x, t') dV' dt' d\Omega' d\epsilon'$ is the differential comoving energy $d\mathcal{E}_{\nu}$ of photons with dimensionless energies between $\epsilon'$ and $\epsilon' + d\epsilon'$ that are radiated during time $dt'$ from comoving volume $dV'$ within the solid angle element $d\Omega'$ in the direction $\Omega'$ defined in the comoving fluid frame. Although the directional properties of the radiation must be considered in the general case of nonthermal synchrotron emission from relativistic electrons in an ordered magnetic field, or for external Compton processes, here we assume that the radiation is emitted isotropically in the comoving frame and assume a nonthermal synchrotron origin for the keV–MeV emission from GRBs. Implicit in this approximation is that the magnetic field is randomly oriented and that the electrons have an isotropic pitch-angle distribution in the fluid frame.

Thus, $f'_{\nu}(\epsilon'; \Omega'; x, t') = f_{\nu}(\epsilon'; x, t')/4\pi$. Because the azimuthal dependence no longer appears in either $f(\epsilon'; x, t')$ or $\delta_{D}$, it is trivial to integrate equation (4) over $\phi$ under the assumption that the emissivity $f'_{\nu}(\epsilon'; x, t') = f_{\nu}(\epsilon'; x, t')$, that is, the emissivity depends only on $x$ and $t$ but not on the angle (except as determined by the physical extent of the cloud). This assumption allows us to treat the hydrodynamics of the blast-wave/cloud interaction in a planar geometry (§2.2). We obtain

$$f_{\nu}(t) = (2\pi d_{L}^{-2})^{-1} \int_{0}^{\theta_{+}} d\theta_{i} \sin \theta_{i} \theta_{i}^{3} \times \int_{0}^{\infty} dx x^{2} f_{\nu}(\epsilon'; x, t') \delta_{D},$$  

(6)

noting that the Doppler factor is, in general, dependent on location. The term $\theta_{+} = \theta_{+}(\theta, \theta_{cl}, \theta_{i})$ represents the maximal azimuthal angle subtended by the cloud for received emission from
radiating plasma located at an angle $\theta$ with respect to the observer’s line of sight (see Fig. 1). From the addition formula for angles in spherical geometry, we have that $\cos \theta_i = \cos \theta \cos \theta_i + \sin \theta \sin \theta_i \cos \phi_i$. Inspection of Figure 1 reveals that when $\theta_i < \theta_cl$, that is, when the line of sight to the observer intersects the cloud volume, then $\phi_i = \pi$ when $\theta \leq \theta_i - \theta_i$. Thus, we have two cases, as follows:

1. $\theta_i < \theta_cl$, 
   \[ \phi_i = \begin{cases} \pi, & \theta_i - \theta_cl \leq \theta \leq \theta_cl - \theta_i, \\ \arccos \Theta, & \theta_cl - \theta_i \leq \theta \leq \theta_i + \theta_cl, \\ 0, & \text{otherwise}; \end{cases} \tag{7} \]

2. $\theta_i > \theta_cl$, 
   \[ \phi_i = \begin{cases} \arccos \Theta, & \theta_i - \theta_cl \leq \theta \leq \theta_cl + \theta_i, \\ 0, & \text{otherwise}, \end{cases} \tag{8} \]

where 
\[ \Theta = \frac{\cos \theta_cl - \cos \theta \cos \theta_i}{\sin \theta \sin \theta_i}. \]

The stationary frame time $t_s$ is related to the observer time $t$ through 
\[ t_s = \frac{t}{(1+z)} = t_s - \frac{x \cos \theta}{c}, \tag{9} \]

where the zero of time in the two frames is defined by the start of the GRB explosion. The differential distance $dx$ traveled by a relativistic blast wave during the differential time elements $dt_s$, $dt'$, and $dt_o$ in the stationary, comoving, and observer frames, respectively, is given by 
\[ dx = \beta c dt_s = \beta \Gamma c dt' = \beta \Gamma c dt_o (1+z). \tag{10} \]

Equations (9) and (10) are needed to determine the emissivity values of $x$ and $t_s$.

### 2.2. Phases of the Interaction Event

The complete interaction is treated in a planar geometry, as illustrated in Figures 1 and 2. The cloud is assumed to have uniform density between radii $x_0$ and $x_2$, so that the cloud width $\Delta x = x_2 - x_0$. A major simplifying assumption of the analysis is that $\Delta x \ll x_0$, so that $n(x) \approx n(x_0)$ throughout the duration of the interaction. Three phases of the interaction event are identified and illustrated in Figure 2.

1. Collision phase (Fig. 2a). Both a forward shock (FS) and reverse shock (RS) are found in this phase; the FS accelerates the cloud material, and the RS decelerates the shell material.

2. Penetration phase (Fig. 2b). This phase bifurcates into two cases, depending on whether the RS crosses the shell before the FS crosses the cloud, or whether the FS crosses the cloud before the RS crosses the shell. In the former case, the shocked fluid produced during the collision phase decelerates as more cloud material is swept up at the FS, during which time a new accelerated particle population is introduced at the decelerating FS, so we denote this process as the deceleration shock (DS). In the latter case, the shocked fluid produced during the collision phase is accelerated by the remaining shell material at the RS, and a new particle population is accelerated at the RS. We only treat the first case of this phase (which is the most important case for GRB studies) in this paper.

3. Expansion phase (Fig. 2c). After the FS crosses the remaining cloud material in the first case of phase 2 or the RS crosses the...
remaining shell material in the second case of phase 2, the shocked fluid, being composed of highly relativistic particles and magnetic fields, expands and adiabatically cools.

2.2.1. Collision Phase

We follow the approach of Sari & Piran (1995) and Kobayashi et al. (1999) to analyze this phase. Let \( \bar{\Gamma} = 1/(1 - \beta^2)^{1/2} \) represent the relative Lorentz factor of unshocked shell material in the rest frame of the shocked fluid. From Figure 2b,

\[
\bar{\Gamma} = \Gamma_0(1 - \beta \beta_0) \approx \frac{1}{2} \left( \Gamma_0 + \frac{\Gamma_0}{\bar{\Gamma}} \right),
\]

where the last relation applies in the regime \( \Gamma_0, \bar{\Gamma} \gg 1 \) considered here. Note that \( \Gamma \) and \( \bar{\Gamma} \) remain constant during the duration of phase 1, as a consequence of the assumption that \( \Delta x \ll x \).

The shocked fluid travels with Lorentz factor \( \Gamma \), and the shock itself moves with Lorentz factor \( \Gamma_x \). When \( \bar{\Gamma} \gg 1 \), then \( \Gamma_x \approx \sqrt{2\bar{\Gamma}} \), implying a compression ratio of \( 4\bar{\Gamma} \) (particle densities will always be referred to the proper frame). Likewise, when \( \bar{\Gamma} \gg 1 \), the Lorentz factor of the RS in the comoving fluid frame is \( \Gamma_{RS} \approx \sqrt{2\bar{\Gamma}} \). When \( \bar{\Gamma} - 1 \ll 1 \), then \( \beta_{RS} \approx (1 - \Gamma_{RS}^{-2})^{1/2} \approx 4\beta/3 \), implying a compression ratio \( \approx 4 \). Thus, \( \bar{\Gamma} \approx \sqrt{2\bar{\Gamma}} \) when \( \bar{\Gamma} > \sqrt{2} \), and \( \beta_{RS} \approx 4\beta/3 \) when \( \bar{\Gamma} \leq \sqrt{2} \). The Lorentz factor of the RS in the stationary frame is \( \Gamma_{RS} = \Gamma_{RS}(1 - \beta \beta_{RS}) \), and \( \beta_{RS} = (1 - \Gamma_{RS}^{-2})^{1/2} \).

When \( \bar{\Gamma} \gg 1 \), the fluid density of the FS material is \( n_{FS} \approx 4\Gamma n_{cl} \). The density of the reverse-shocked fluid is \( n_{FS} \approx (4\bar{\Gamma} + 3)n(\bar{x}) \) for a relativistic RS, and \( n_{FS} \approx (4 + 5\beta^2/4)n(x) \) for a nonrelativistic RS composed of cold shell material; an expression that joins the two regimes is \( n_{FS} \approx 4\bar{\Gamma}n(x) \). The equality of kinetic energy densities at the contact discontinuity implies that

\[
n_{FS}(\bar{\Gamma} - 1) = n_{RS}(\bar{\Gamma} - 1) \approx 4\Gamma n_{cl} \frac{\bar{\Gamma}}{\Gamma} \approx 4\bar{\Gamma}(\bar{\Gamma} - \bar{\Gamma}).
\]

The relativistic shock jump conditions for an isotropic explosion in a uniform circumburst medium therefore give (Sari & Piran 1995; Panaitescu & Mészáros 1999)

\[
n(\bar{x}_0) = F = \frac{E_0}{4\pi^2 H_{cl}^{1/2} n_{cl} \bar{x}} m_p c^3 \Delta(\bar{x}_0) \approx \frac{\bar{x}^{-2}}{\bar{x} - \Gamma} \rightarrow \begin{cases} 2\bar{\Gamma}^2 / \beta_0^2, & \text{Nonrelativistic RS (NRS),} \\ 2\Gamma^2 / \bar{\Gamma}^2, & \text{Relativistic RS (RRS).} \end{cases}
\]

Hence,

\[
\bar{\Gamma} = \frac{1}{\sqrt{1 - \beta_0^2}}, \quad \Gamma = \frac{1}{\Gamma_0(1 + \beta)},
\]

when \( F \gg 4\Gamma_0^2 \) (NRS), and

\[
\bar{\Gamma} \approx 1, \quad \Gamma = \frac{\Gamma_0 F^{1/4}}{2\Gamma_0 - F^{1/2}} \approx \sqrt{\frac{\Gamma_0^2}{2}} F^{1/4},
\]

when \( F \ll 4\Gamma_0^2 \) (RRS). Figure 3 shows the shocked fluid Lorentz factor \( \Gamma \) and the relative Lorentz factor \( \bar{\Gamma} \) as functions of \( F/4\Gamma_0^2 \) for \( \Gamma_0 = 300 \). The accurate numerical results are shown by the solid curves, obtained by solving \( F = \beta(\Gamma^2 - 1)/\beta(\bar{\Gamma}^2 - 1) \). The dotted curves show the approximate analytic expressions, equations (14) and (15), using the expression \( \Gamma = (\Gamma_0/2)^{1 - F^{1/4}} \) in the latter case. The numerical solutions for \( \Gamma \) and \( \bar{\Gamma} \) are used in subsequent calculations.

The FS power is \( dE''/dt_{FS} = A_0 n_0 m_p c^3 \beta(\Gamma^2 - 1) \approx A_0 n_0 m_p c^3 \beta(\bar{\Gamma}^2 - 1) \). The RS power is \( dE''/dt_{RS} = A_0 n_0 m_p c^3 \beta(\bar{\Gamma}^2 - \bar{\Gamma}) \). Thus, \( (dE''/dt_{FS})/(dE''/dt_{FS}) = 1 \), so that equal power is dissipated as internal energy in the FS and the RS. The comoving duration of the FS before penetrating the cloud is \( \Delta t_{FS} = \Delta x_{FS}/\Delta c_{FS} \). The comoving duration of the RS before penetrating the shell is \( \Delta t_{RS} = \Delta x_{RS}/\Delta c_{RS} \). Therefore, case 1, where the RS crosses the shell before the FS crosses the cloud, applies when \( \Delta t_{RS} > \Delta t_{FS} \).

The range of \( x \) occupied by the FS fluid during phase 1 (designated as FS1) at time \( t_0 = t_0 + x \cos \theta / c \) is

\[
x_0 + \beta c(t_0 - t_0) \leq x \leq x_0 + \beta c(t_0 - t_0),
\]

while the range occupied by the RS fluid during phase 1, or RS1, is

\[
x_0 + \beta_{RS} c(t_0 - t_0) \leq x \leq x_0 + \beta_{RS} c(t_0 - t_0),
\]

where \( t_0 \approx x_0 / \beta_0 c \) and \( \beta_0 = (1 - \Gamma_0^{-2})^{1/2} \). The comoving time \( t' = (t_0 / \Gamma_0) + (t_0 - t_0) / \Gamma \), so that the elapsed comoving time since the start of the interaction is \( \Delta t' = (t_0 - t_0) / \Gamma \). In the absence of an interaction, the shell occupies the range \( \beta_0 c t_0 - \Delta \leq x \leq \beta_0 c t_0 \). Thus, the RS leaves the shell when \( \beta_0 c t_0 - \Delta = x_0 + \beta_{RS} c(t_0 - t_0) \), that is, when

\[
t_0 = t_0 + \frac{\Delta}{\beta_0 - \beta_{RS} c}.
\]

This represents the end of stage 1 for the case where the RS crosses the shell before the FS crosses the cloud. Therefore, stage 1 lasts for comoving time

\[
\Delta t''_{RS} = \frac{\Delta}{(\beta_0 - \beta_{RS} c)}. \quad (19)
\]
which may be compared with the approximation derived above. The time measured between the start of the GRB explosion and when the blast wave first encounters the cloud is

$$t_0 = (1 + \epsilon) \frac{\delta \epsilon}{\beta_0 c} \times \begin{cases} 1 - \beta_0, & \theta_1 \leq \theta_3, \\ 1 - \beta_0 \cos(\theta_1 - \theta_3), & \theta_1 > \theta_3. \end{cases} \quad (20)$$

Following the treatment of Sari et al. (1998; for the accuracy of this approach, see Dermer et al. 2000), we let \(n_T(\gamma; x, t') d\gamma\) represent the differential number density of electrons with Lorentz factors between \(\gamma\) and \(\gamma + d\gamma\) in the shocked fluid. Assuming that all swept-up electrons are accelerated by the FS with spectral index \(p_{FS}\), joint normalization of number and power gives a minimum electron Lorentz factor

$$\gamma_{\min,FS} \simeq \epsilon_{FS} \frac{m_p}{m_e} f(p_{FS})(\Gamma - 1), \quad (21)$$

where \(f(p) = (p - 2)/(p - 1)\) and \(\epsilon_{FS}\) represents the fraction of swept-up FS power carried by the injected nonthermal electrons. For electrons accelerated by the RS with spectral index \(p_{RS}\), similar considerations give a minimum Lorentz factor

$$\gamma_{\min,RS} \simeq \epsilon_{RS} \frac{m_p}{m_e} f(p_{RS})(\Gamma - 1). \quad (22)$$

Electrons cool through synchrotron losses during comoving time \(\Delta t'\) since the start of the collision to cooling Lorentz factor

$$\gamma_{c,FS} = \frac{6 \pi m_e c}{\sigma_T B_{FS}^2 \Delta t'}, \quad \gamma_{c,RS} = \frac{6 \pi m_e c}{\sigma_T B_{RS}^2 \Delta t'} \quad (23)$$

for the FS and RS fluids, respectively, assuming dominant synchrotron losses. The magnetic field strength \(B\) in the FS and RS fluids is assigned according to the usual prescription, namely, that the magnetic field energy density is proportional to the downstream energy density of the shocked fluid with proportionality constant \(\epsilon_B \leq 1\). Thus,

$$B_{FS}^2 = 32 \pi n_{e0} m_p c^2 \epsilon_{FS} \beta \Gamma (\Gamma^2 - 1),$$

$$B_{RS}^2 = 32 \pi n_{e0} m_p c^2 \epsilon_{RS} \beta \Gamma (\Gamma^2 - 1). \quad (24)$$

The maximum electron Lorentz factor \(\gamma_{\max}\) is determined by equating the most rapid acceleration rate expected in Fermi processes, \(\dot{\gamma}_{\text{acc}} = \epsilon_2 e B/m_e c\), where the rate factor \(\epsilon_2 \leq 1\), with the synchrotron loss rate \(\sim \gamma_{\text{syn}}\), giving

$$\gamma_{\max,FS} \simeq \left( \frac{6 \pi e_2 FS}{\sigma_T B_{FS}} \right)^{1/2} \simeq 1.2 \times 10^8 e_{2FS} \sqrt{B_{FS}[G]}, \quad (25)$$

and similarly for \(\gamma_{\max,RS}\) at the RS.

We calculate the nonthermal synchrotron radiation in the collision phase spectrum by noting that the emissivity in equation (4) can be expressed through the relation

$$\epsilon' / (\epsilon', \Omega'; x, t') \approx 4 \sigma_T U_B \gamma^3 n(\gamma; x, t'), \quad (26)$$

where \(U_B = B^2/8\pi\) is the magnetic field energy density,

$$\gamma_\times \approx \sqrt{\frac{\epsilon c}{\theta_0 (B/B_\odot)}}, \quad (27)$$

and the critical magnetic field \(B_c = m_e^2 c^3/e h = 4.14 \times 10^{13} \text{ G}\). This follows from the formula for the electron energy-loss rate through synchrotron emission and from the expression for the mean energy of synchrotron photons radiated by an electron with Lorentz factor \(\gamma\). Equation (26) applies to the FS and RS fluids by using values of \(B\) and \(\gamma_\times\) appropriate to each component.

Assuming that the electrons are accelerated by the first-order Fermi shock process and injected into the shocked fluid in the form of a power law with index \(p\), then the density distribution \(n(\gamma; x, t')\) of energized electrons can be approximated by the expression

$$n(\gamma; x, t') \approx \frac{n(x, t')}{s - 1} \left( \frac{\gamma}{\gamma_0} \right)^{1+1-p}/\left( \frac{\gamma_0}{\gamma_{\gamma_0}} \right)^{1-p}, \quad \gamma_0 \leq \gamma \leq \gamma_1, \quad \gamma_1 \leq \gamma \leq \gamma_{\max}. \quad (28)$$

In the slow-cooling regime, \(\gamma_{\min} < \gamma_0 = \gamma_{\min}\), \(\gamma_1 = \gamma_{\min}\), and \(s = p\). In the fast-cooling regime, \(\gamma_{\min} < \gamma_0 < \gamma_1 = \gamma_{\min}\), and \(s = 2\) (Sari et al. 1998). At energies \(\epsilon \leq \epsilon_0 = (B/B_\odot)\gamma^2_\times \beta_\odot (1 + z)\), corresponding to \(\gamma \leq \gamma_0\), the synchrotron spectrum is approximated by the elementary synchrotron emissivity spectrum \(\propto \epsilon_{1/3}\) in a \(\nu F_\nu\) representation. Thus, we can include this branch of the synchrotron spectrum by writing

$$n(\gamma; x, t') \approx \frac{n(x, t')}{s - 1} \left( \frac{\epsilon}{\epsilon_0} \right)^{1/3} \quad (29)$$

for \(\gamma \leq \gamma_0\). Synchrotron self-absorption is neglected here, which is valid for the radiation detected at infrared frequencies and higher considered in this paper.

The proper densities of electrons swept up by the FS and RS are

$$n_{FS}(x, t') = \frac{n_{eFS}}{(\beta_\times - \beta)} \simeq 4 \Gamma n_{dFS}, \quad n_{RS}(x, t') = \frac{\beta_\times \Gamma n_{dRS}}{(\beta_\times - \beta_{RS}) \Gamma}, \quad (29)$$

respectively. In the treatment used here, the total electron density is independent of location within the volume of the FS and RS fluids during phase 1. The collision phase spectrum is calculated by substituting equation (26) into equation (6), making use of equation (27) to relate \(\gamma\) to the received photon energy.

### 2.2.2. Penetration Phase

Only the case where the RS passes through the shell before the FS passes through the cloud is analyzed here. During this phase, the shell is assumed to be hydrodynamically connected so that the entire shocked fluid decelerates with the same Lorentz factor \(\Gamma_2\). As the FS passes through the remaining cloud material, it sweeps up and injects a new nonthermal particle population in addition to the nonthermal electrons left over from phase 1. We denote \(\Gamma = \Gamma_1\), the Lorentz factor of the shocked fluid during phase 1. The equation for blast-wave deceleration of a relativistic blast wave when it sweeps up material from the cloud is given by (Böttcher & Dermer 2000; Dermer & Humi 2001)

$$\Gamma_2(x) = \frac{\Gamma_1 [1 + (2 - \varphi) \epsilon_{2FS}^2 m(x)/E_0]^{1/(2 - \varphi)}}{[1 + (2 - \varphi) \epsilon_{2FS}^2 m(x)/E_0]^{1/(2 - \varphi)}} \quad (30)$$

where \(\varphi\) is the fraction of swept-up energy that is radiated, and \(m(x) = 4\pi n_e \int_{x_{min}}^{x} dx' x'^2 n_{e0}(x') \simeq 4\pi n_e \epsilon_{2FS}^2 (x - x_\Delta)\) is the swept-up mass. Here,

$$x_{\Delta} = x_0 + \frac{\beta_\times \Delta}{\beta_0 - \beta_{RS}} \quad (31)$$
is the distance of the FS from the explosion center at the end of phase 1 (see eqs. [16] and [18]). Defining the deceleration radius $x_d$ through the expression $\Gamma_1 m(x_d) = \frac{E_0 e^{2\Gamma_1 c}}{4\pi m c^2}$ gives the following expression for the deceleration radius for a blast-wave/cloud interaction,

$$x_d = x_e + \frac{E_0}{4\pi m c^2 \Gamma_1^2}$$  \hspace{1cm} (32)

Assuming adiabatic evolution ($\varphi \ll 1$), equation (30) becomes

$$\Gamma_2 = \frac{\Gamma_1}{\sqrt{1 + 2(x - x_\Delta)/(x_d - x_\Delta)}} = \frac{\Gamma_1}{\sqrt{1 + 2\beta_{1s} u - u^2/2\Gamma_1^2}},$$  \hspace{1cm} (33)

where $u = c(t_s - t_0)/(x_d - x_\Delta)$, and $\beta_{1s} = (1 - 1/2\Gamma_1^2)^{1/2}$. The expression on the right-hand side of equation (33) is obtained by integrating $dx = \beta_{1s}(x) dt$, where $\Gamma_{1s}(x) = \sqrt{2\Gamma_1(x)}$.

During phase 2, the particles injected at the RS and FS during phase 1 are no longer subject to further acceleration. We assume that these fluid volumes, denoted RS2 and FS2, respectively, neither expand nor contract during phase 2, and decelerate with Lorentz factor $\Gamma_2(t_s)$. The range occupied by RS2 is therefore

$$\begin{align*}
x_0 + \frac{\beta_{RS}\Delta}{\beta_0 - \beta_{RS}} + x_2(t_s - t_\Delta) & \leq x \\
& \leq x_0 + \frac{\beta_{RS}\Delta}{\beta_0 - \beta_{RS}} + x_2(t_s - t_\Delta).
\end{align*}$$  \hspace{1cm} (34)

For FS2, the range is

$$\begin{align*}
x_0 + \frac{\beta_{1s}\Delta}{\beta_0 - \beta_{RS}} + x_2(t_s - t_\Delta) & \leq x \\
& \leq x_0 + \frac{\beta_{1s}\Delta}{\beta_0 - \beta_{RS}} + x_2(t_s - t_\Delta). \hspace{1cm} (35)
\end{align*}$$

Here,

$$\begin{align*}
x_2(t_s - t_\Delta) &= c \int_{t_\Delta}^{t_s} dt' \beta_{1s}(t') \\
&= \frac{x_d - x_\Delta}{\Gamma_1} \left[ \left( \frac{u^2}{2} - \beta_{1s}\Gamma_1^2 \right) \sqrt{\frac{u^2}{2\Gamma_1^2} - 2\beta_{1s}u' + \Gamma_1^2 - 1} - \frac{\Gamma_1^2}{\sqrt{2}} \ln \sqrt{2} \sqrt{\frac{u^2}{2\Gamma_1^2} - 2\beta_{1s}u' + \Gamma_1^2 - 1} + u' + 2\beta_{1s} - 2\beta_{1s}\Gamma_1^2} \right]_0^u.
\end{align*}$$  \hspace{1cm} (36)

The elapsed comoving time since the start of phase 2, obtained by integrating $dt' = dt/\Gamma_2(t_s)$, is given by

$$\begin{align*}
t_2 &= \frac{x_d - x_\Delta}{\beta_0 - \beta_{RS}} \left\{ \left( \frac{u^2}{2} - \beta_{1s}\Gamma_1^2 \right) \sqrt{1 + \beta_{1s}u - x_\Delta} - \frac{\beta_{1s}\Gamma_1^2}{\sqrt{2}\Gamma_1} \arcsin \left( \frac{\beta_{1s} - u}{2\Gamma_1^2} \right) - \arcsin \beta_{1s} \right\}. \hspace{1cm} (37)
\end{align*}$$

The RS2 and FS2 emissivities are the same as the RS1 and FS1 emissivities, except now $\Delta t' = t' - t_\Delta + (t_s - t_0)/\Gamma_1$ is used to evaluate the cooling Lorentz factor, equation (23). Moreover, because injection and subsequent particle acceleration has ceased in these two phases, the maximum electron Lorentz factor decays by synchrotron losses to a value of

$$\gamma_{2,FS}(t') = \left( \gamma_{\text{max},FS}^{-1} + \frac{\tau T_{BS}^2}{6\pi m^2 c} (t' - t_\Delta) \right)^{-1},$$  \hspace{1cm} (38)

where $\gamma_{\text{max},FS}$ is given by equation (25) for FS2, with a related equation for RS2. Because the FS and RS fluid shells are assumed not to expand during phase 2, the magnetic field remains the same as in phase 1.

The fluid containing the nonthermal electrons and protons injected by the decelerating shock as it sweeps up cloud material in phase 2, denoted DS2, occupies the range

$$\begin{align*}
x_\Delta + x_2(t_s - t_\Delta) & \leq x \leq x_\Delta + x_2(t_s - t_\Delta), \hspace{1cm} (39)
\end{align*}$$

where

$$\begin{align*}
x_2(t_s - t_\Delta) &= c \int_{t_\Delta}^{t_s} dt' \beta_{2s}(t') \\
&= \frac{x_d - x_\Delta}{\Gamma_1^2} \left[ \left( \frac{u^2}{2} - \beta_{2s}\Gamma_1^2 \right) \sqrt{\frac{u^2}{2\Gamma_1^2} - 2\beta_{2s}u' + \Gamma_1^2 - 1} + \beta_{2s}\Gamma_1^2 \sqrt{2\Gamma_1^2 - 1} \right].
\end{align*}$$  \hspace{1cm} (40)

The DS2 emissivity at time $t_s$, determined by the values of $\theta, x,$ and observer time $t_\Delta$ in the evaluation of equation (6), is produced by nonthermal electrons injected at location $x_i$ and time $t_i$. The values of $\theta$ and $t_i$ are determined by the intersection of $x_2(t_s - t_\Delta) = x_\Delta + x_2(t_i - t_\Delta)$, representing the leading edge of the DS that injects particles into the fluid, and $x_\Delta = x - x_2(t_i - t_\Delta)$, representing the worldline connecting the injection coordinates to the received values. The injection coordinates $x_i$ and $u_i = c(t_i - t_\Delta)/(x_i - x_\Delta)$ are therefore obtained by numerically solving $x = x_\Delta - x_2(t_i - t_\Delta)$ using a staggered leapfrog routine.

The received spectrum during phase 2 is given by equation (6), with the $x$-integral limited to the ranges defined above. The emissivity for DS2 is given by equation (26), with values of $\gamma_{\text{min}}, \gamma_c$, and $\gamma_{\text{max}}$ appropriate to the injection value of $\Gamma(t_i)$. The elapsed comoving time since injection used to calculate $\gamma_c$ is obtained from equation (37). The Doppler factor at time $t_s$ uses equation (33) for $\Gamma$. Phase 2 ends when $x = x_2$, that is, at stationary time

$$\begin{align*}
t_2 &= t_\Delta + \frac{\sqrt{2\Gamma_1^2 - 1}}{c \Gamma_2(t_\Delta)} \left[ \sqrt{2\Gamma_1^2 - 1} - 1 - 2 \frac{x_\Delta - x_\Delta}{x_\Delta - x_\Delta} \right]. \hspace{1cm} (41)
\end{align*}$$

2.2.3. Expansion Phase

After the FS passes through the cloud, the shocked fluid has Lorentz factor

$$\Gamma_3 = \Gamma_2(t_\Delta) = \frac{\Gamma_1}{\sqrt{1 + 2(x_\Delta - x_\Delta)/(x_d - x_\Delta)}}.$$  \hspace{1cm} (42)

Because there is no further pressure on the fluid, the shocked fluid is assumed to expand outward along the parallel direction with speed $\beta_{\parallel} c$ and Lorentz factor

$$\Gamma_{\parallel} = 1/\sqrt{1 - \beta_{\parallel}^2}.$$
transverse extent, due to the compression of the quasi-spherical cloud material by a factor $\geq 4 \Gamma_0^2$ in the parallel direction, effects of transverse expansion can be neglected.) No further injection or acceleration takes place in phase 3, and the relativistic electrons cool by synchrotron and expansion losses. The shocked fluid in phase 3, denoted by RS3, FS3, and DS3, consists of fluid from the prior RS, FS, and DS phases. In order that each fluid phase is to have the same fractional volume expansion, the FS fluid is assumed to expand in both transverse directions in the co-moving frame with speed $\beta_e c / 3$, where $\Gamma_3 = 1 / (1 - (\beta_3 c / 3)^2)$.

From these assumptions, we define the four stationary frame Lorentz factors $\Gamma_{RS3} = \Gamma_3 \Gamma_1 (1 - \beta_3 \beta_1)$, $\Gamma_{FS3} = \Gamma_3 \Gamma_{FS3} (1 - \beta_3 \beta_{FS3})$, $\Gamma_{FS3+} = \Gamma_3 \Gamma_{FS3+} (1 + \beta_3 \beta_{FS3})$, and $\Gamma_{DS3} = \Gamma_3 \Gamma_{DS3} (1 + \beta_3 \beta_{DS3})$, along with their associated beta factors $\beta_{RS3}$, $\beta_{FS3}$, $\beta_{FS3+}$, and $\beta_{DS3}$, respectively, that characterize the motions of the radial boundaries of the different fluid layers. The range of RS3 at time $t_s$ is

$$x_0 + \frac{\beta_{RS3} \Delta}{\beta_0 - \beta_{RS3}} + x_2 (t_s - t_0) + c \beta_{RS3} (t_s - t_2) \leq x_0 + \frac{\beta_0 \Delta}{\beta_0 - \beta_{RS3}} + x_2 (t_s - t_0) + c \beta_{FS3} (t_s - t_2). \quad (43)$$

For FS3, the range is

$$x_0 + \frac{\beta_{1} \Delta}{\beta_0 - \beta_{FS3}} + x_2 (t_s - t_0) + c \beta_{FS3} (t_s - t_2) \leq x_0 + \frac{\beta_0 \Delta}{\beta_0 - \beta_{FS3}} + x_2 (t_s - t_0) + c \beta_{FS3+} (t_s - t_2). \quad (44)$$

The range of DS3 is

$$x_0 + x_2 (t_s - t_0) + c \beta_{FS3+} (t_s - t_2) \leq x_0 + x_2 (t_s - t_0) + c \beta_{DS3} (t_s - t_2). \quad (45)$$

Let $R'_i (t') = R_{0i} (t') + 2 \beta_i c (t' - t'_i)$ denote the comoving parallel width of the entire shocked fluid, with additional subscripts $i = RS$, FS, and DS to refer to the widths $R_{0i} = R_{0RS} (t' - t'_i)$, $R_{0FS} = R_{0\beta} (t' - t'_i)$, and $R_{0DS} = R_{0\beta} (t' - t'_i)$. The forward, reverse, and deceleration shocked fluid layers, respectively. The superscript “0” refers to the radial width of the fluid layer at the end of phase 2, and $t'_i$ is the comoving time at the end of phase 2. From equations (43)–(45), $R'_{0RS} = \Gamma_3 (\beta_1 - \beta_{RS}) \Delta t_0$, $R'_{0FS} = \Gamma_3 (\beta_1 - \beta_{FS}) \Delta t_0$, and $R'_{0DS} = \Gamma_3 (x_2 (t_s - t_0) - x_2 (t_s - t_0))$. Conservation of magnetic flux for the transverse magnetic field component $B_{0i}$ implies $B_i (t') = \Gamma_0 R_{0i} (t')$, so that $B_i (t') = \Gamma_0 R_{0i} (t') + \beta_i c (t' - t'_i)^2$. If the electrons are isotropized on timescales that are short compared to the cooling timescale, then the electron energy loss rate due to adiabatic and synchrotron losses is given by

$$- \frac{d\gamma}{dt} = \frac{1}{R_i (t')} \frac{dR_i (t')}{dt} \gamma + \frac{\sigma T_B^2 (t') + B_i^2 (t')}{6 \pi m_e c} \gamma^2, \quad (46)$$

where $B_i$ is the parallel magnetic field component. Assuming that $B_i (t') \gg B_i (t')$, which would occur if the magnetic field is formed by sweeping up and compressing an external field, then equation (46) can be written in the form

$$- \frac{d\gamma}{dt} = \frac{\gamma}{\tau} + \frac{\gamma^2}{\tau^2}, \quad (47)$$

where $\tau \equiv 1 + \frac{2}{3} \beta_1 c (t' - t'_0) / R_{0i}$ and $b \equiv R'_{0i} / \sigma_T B_{0i} (4 \pi \beta_1 m_e c^2)$. This can be solved (Gupta et al. 2006) using the substitution $v = \gamma \tau^{-n}$, with $n = 3$, giving the result

$$\gamma (\tau) = \frac{4 \tau^3}{b (\tau^4 + 1) + 4 \tau^4 / \gamma_i}, \quad (48)$$

where $\gamma_i$ is the initial electron Lorentz factor at $t' = t'_0$ or $\tau = 1$.

The emissivity for RS3 and FS3 is given by equation (26) with

$$n (\gamma; t') = n (\gamma; \tau) \Gamma_3 \left[ \frac{R_i (t')}{R_{0i}} \right]^{-1} \frac{n (\gamma_i; 1)^2}{\gamma_i^2} \frac{1}{\gamma^2} = \frac{n (\gamma_i; 1)^2}{\tau^2 \gamma^2}, \quad (49)$$

$B \rightarrow B_i (t') = B_{0i} / \tau^2$, and $n (\gamma_i; 1)$ is the electron $\gamma$-distribution at the end of phase 2. This expression also applies to the DS3 emissivity, with the emission coordinates $x_s, t_s$ mapped back to the injection coordinates $x_i, t_i$, as was done for DS2. Note the minor inconsistency for phase 3 between the original assumption that the magnetic field is randomly ordered, compared to the dominant perpendicular magnetic field component implied by field compression in this phase. This inconsistency should be relaxed with the use of direction synchrotron emission spectra in more detailed treatments, but in either case, the contribution from the expansion phase is usually much smaller than that from the penetration phase. The Doppler factor during phase 3 is $\delta_D = \Gamma_3 (1 - \beta_3 \mu)^{-1}$.

### 3. RESULTS

Nonthermal synchrotron emission spectra from blast-wave/cloud interactions were computed using the formulae given in §2. Standard parameters are given in Table 1. For the model GRB pulse, we consider a source at $z = 1$, corresponding to the mean redshift of the pre-Swift BATSE/BeppoSAX/HETE-2/INTEGRAL sample. The coating Lorentz factor and apparent isotropic energy release of the blast wave are chosen to be $\Gamma_0 = 300$ and

| Parameter | GRB Pulse | X-Ray Flare |
|-----------|-----------|-------------|
| Redshift $z$ | $1$ | $2$ |
| $d_L$ (10$^{28}$ cm) | $2.02$ | $4.80$ |
| $E_0$ (ergs) | $10^{53}$ | $10^{54}$ |
| $\Gamma_0$ | $300$ | $100$ |
| $\Delta_0$ (cm) | $10^7$ | $10^7$ |
| $\eta$ | $0.1$ | $0.1$ |
| $\epsilon_e$ | $10^{-1}$, $10^{-4}$ | $10^{-3}$, $10^{-4}$ |
| $\tau$ | $0.1$ | $0.1$ |
| $b$ | $2.5$ | $2.5$ |
| $\theta_0$ | $0.01$ | $0.01$ |
| $\theta_i$ | $0.0$ | $0.0$ |
| $x_0$ (cm) | $10^{16}$ | $10^{17}$ |
| $x_2$ (cm) | $1.02 \times 10^{16}$ | $1.02 \times 10^{17}$ |
| $n_0$ (cm$^{-3}$) | $10^5$ | $10^1$ |

Note.—Forward and reverse shock parameters are the same.
The kinematic minimum and maximum times are implied by the idealized cloud location and geometry. From equation (20) for on-axis clouds,

\[ t_{\text{min}} = \frac{(1+z)x_0}{2\Gamma_0^2 c} \approx 3.70 \left( \frac{1+z}{2} \right) \frac{x_{16}}{\Gamma_{100}} \text{ s}, \]

\[ t_{\text{max}} \approx t_{\text{min}} (1 + \Gamma_0^2 \theta_2^2 \Omega_1). \]

where \( \Gamma_{100} = \Gamma_0 / 300 \). Depending on the observed energy, the maximum emission time will also be extended by the timescale for electron cooling. As can be seen from Figure 4, at low densities the flux is weak and the blast wave hardly decelerates. For most purposes, such interactions can be neglected, as they extract only a small amount of energy from that solid angle element of the blast wave. With increasing \( n_{\text{cl}} \), the \( \nu F_{\nu} \) flux increases until the cloud density becomes so thick that the RS becomes relativistic, \( \Gamma \ll \Gamma_0 \), and the emission is received at lower photon energies. At such high densities, the flux received by a distant observer is weak, but may last for a long time, especially at small photon energies.

The model light curves show a sharp rise followed by a more leisurely power-law temporal decay. Some of this rapid rise has to do with the idealized cloud geometry; the use of an actual spherical cloud geometry (which could be mimicked by superposition of interacting annuli) would tend to reduce the sharpness in the rising phase of the GRB light curve. As can be seen from the bottom panel of Figure 4 in a logarithmic representation, the power-law decay of the flux displays a temporal curvature or softening at times well past the peak from cooling breaks in the synchrotron spectrum. This is related to the detection of higher energy photons that were emitted from a higher energy and usually softer part of the spectrum by off-axis emitting sites (Fenimore et al. 1996; Kumar & Panaitescu 2000; Dermer 2004; Liang et al. 2006; Zhang et al. 2007). The pulse shapes with \( n_{\text{cl}} \ll 10^3 \text{ cm}^{-3} \) correspond to kinematic “curvature” pulses (Dermer 2004), where the radiating shell is thin and the emission is radiated promptly on the crossing time defined by the comoving shell thickness, and the spectrum is well described by a single power law in the weak cooling regime. The GRB light curve produced by interactions with such clouds with optimal density \( n_{\text{cl}} \approx 10^3 - 10^5 \text{ cm}^{-3} \) would represent a generic long-duration GRB except that it is rather weak, reaching peak \( \nu F_{\nu} \) fluxes of \( \approx 1 - 2 \times 10^{-8} \text{ erg s}^{-1} \text{ cm}^{-2} \) at \( \approx 500 \text{ keV} \).

The weakness of the flux is largely due to the small value used for the \( \epsilon_B \) parameter. Figure 5 shows the \( \nu F_{\nu} \) SEDs calculated near the peak of the \( \epsilon = 1 \) light curve at observer time \( t = 4.46 \text{ s} \), for three different cases of \( \epsilon_B \) and with \( n_{\text{cl}} = 10^3 \text{ cm}^{-3} \). Other parameters of the event are the same as in Figure 4. The various components making up the spectrum are shown. The DS component during phase 2 makes the dominant contribution to the \( \nu F_{\nu} \) spectra near peak energy release. For the higher magnetic field case \( \epsilon_B = 0.01 \), and even more so for \( \epsilon_B = 0.1 \), the stronger cooling causes the collision phase components to decay away more rapidly.

For the case \( \epsilon_B = 10^{-4} \) in Figure 5 (left), one sees that the peak \( \nu F_{\nu} \) flux, which is important for triggering, is at the level of a few \( 10^{-8} \text{ erg s}^{-1} \text{ cm}^{-2} \). BATSE triggered between \( \epsilon = 0.1 \) and 0.6 at a typical \( \nu F_{\nu} \) flux level of \( \approx 5 \times 10^{-9} \text{ ergs cm}^{-2} \text{ s}^{-1} \), so such a GRB would be hard to detect unless it occurred nearby (\( z \approx 1 \)) and happened to be pointed at us. This cannot be excluded for GRBs without redshift measurements, so our interest for spectral modeling is principally on GRBs that have redshift information. Stronger magnetic fields produce brighter pulses, and when \( \epsilon_B \approx 0.01 \), the model pulse flux is at the level

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3. The total mass of such clouds distributed in all directions around the GRB source is \( \approx 10^{-5} M_\odot \).

4. The \( \nu F_{\nu} \) peak flux measured by a GRB telescope integrating over a finite bandwidth would be a factor of a few larger due to a bolometric spectral correction.
\[ \log_{10}(V_F) \text{[ergs cm}^{-2}\text{s}^{-1}] \]

Fig. 5.—Separate spectral components and the total \( vF_v \) flux observed at 4.46 s after the start of the GRB, near the peak flux of its light curve. This GRB pulse model uses parameters given in Table 1, with \( n_{cl} = 10^{3} \text{cm}^{-3} \) and \( \eta = 1/10 \). The differences between the system are that, from left to right, \( \epsilon_B = 10^{-3}, 10^{-2} \), and 0.1. The total spectrum is given by the thick solid curve with data points, the FS components are given by the solid curves, the RS components are given by the dashed curves, and the DS components are given by the dotted curves. The thick dark gray, gray, and thin gray curves correspond to phases 1, 2, and 3, respectively. [See the electronic edition of the Journal for a color version of this figure.]

\[ \simeq 3 \times 10^{-7} \text{ergs cm}^{-2}\text{s}^{-1} \text{, making it an easily detectable GRB with BATSE at } z \simeq 1, \text{ and even more so when } \epsilon_B \gtrsim 0.1 \text{ and } \epsilon_e \gtrsim 0.1, \text{ although the adiabatic assumption for the DS may then become invalid, a situation that can also arise even when } \epsilon_e \sim 0.1 \text{ if photohadronic energy losses are strong (Dermer 2007).} \]

An interesting result of these calculations is that the spectral index below the \( vF_v \) peak is very hard, in number index harder than \(-0.7\), \(-1\), and \(-4/3\) for \( \epsilon_B = 10^{-4}, 10^{-2}, \) and \( 10^{-1} \), respectively. In no case does a well-defined cooling spectrum with number index \(-3/2\) or softer form. This suggests another argument against the concerns of Ghisellini et al. (2000) that radiative cooling spectra should be observed in GRB spectra. Here, there is not sufficient time until the interaction is over for the electrons to cool, so the spectra remain much harder than a cooling synchrotron spectrum below \( E_{pk} \). What stops the interaction and cooling is simply that, after both shocks have passed through the shell and cloud, the disturbed material rapidly expands, shutting off the subsequent synchrotron emissions as nonradiative adiabatic losses increase and the magnetic field intensity, and therefore the synchrotron losses, decrease.

The spectra formed by the interaction of a relativistic blast wave with a small cloud consisting of both FS and RS emission have the characteristic “Band” shape (Band et al. 1993), except for a lower energy emission component from the RS. The relativistic importance of the RS and FS components increases with increasing \( \epsilon_B \) for systems with nonrelativistic or weakly relativistic RSs. When \( \epsilon_B \gg 10^{-4} \), the RS produces a strong optical flux. Indeed, this is the external shock model explanation for prompt optical emission in GRB emission, first discovered by Akerlof et al. (1999).

Even when \( \epsilon_B = 10^{-2} \), the RS emission produces concavity in the X-ray spectrum that could be detectable in joint spectral fitting of the Swift BAT and XRT data with optical/UV measurements with the UVOT and ground-based robotic optical telescopes. For GRBs as modeled by Figure 5, optical flashes as bright as \( m_V \approx 10-17 \) would be coincident with GRB pulses. The relative amplitude of the RS component is also, however, very sensitive to the value of the shock thickness parameter \( \eta \) and various underlying assumptions for the constancy of the \( \epsilon_e \) and \( \epsilon_B \) parameters with time and equality in the reverse and forward shocks. The rich parameter space could allow detailed modeling of prompt and early afterglow optical light curves from, e.g., RAPTOR, ROTSE, and Super-LOTIS.

Figure 6 shows 511 keV light curves calculated for the standard GRB pulse parameter set with \( \epsilon_B = 10^{-2} \), while varying the cloud density \( n_{cl} \). The same basic behavior as was found in Figure 4 is apparent, except that the pulse is much brighter and decays more rapidly. The optimal cloud density to make the brightest possible pulse is again near \( n_{cl} \approx 10^{3} \text{cm}^{-3} \) for \( \epsilon_B = 10^{-2} \) (see § 4.2). The generic pulse profile found here is typical of a FRED-type GRB. Such a profile could just as easily be produced by a GRB blast wave decelerating in a uniform surrounding medium (Dermer et al. 1999). The external shock model can explain relatively smooth light curves that moreover show various widths and asymmetries. The central question for GRB pulse modeling is, however, whether the rapid variability found in some GRB light curves can be reproduced in an external shock model.

Figure 7 displays the light curve formed for the standard parameter set with \( \epsilon_B = 10^{-3} \) and \( \eta = 1/10 \). Here the cloud has extent \( \theta_{cl} = 0.01 \), and a jet with opening half-angle \( \theta_j \) collides with the cloud. This plot indicates the latitudes from which the pulse flux originates. The high-latitude emission arrives at late times, and most of the fluence is formed by interactions within the Doppler opening angle \( 1/\Gamma_0 \). Although this calculation illustrates the part of the blast-wave/cloud interaction from where the flux originates, the opposite limit, where \( \theta_j/\theta_{cl} \) is of more interest for producing STV in GRB light curves.

Figure 8 shows the emission profiles produced by clouds with density \( n_{cl} = 10^{3} \text{cm}^{-3} \) and typical sizes \( \Delta x_0 = 0.001 \text{au} = 0.3 \gamma_0 \Gamma_0 = 10^{13} \text{cm} \), so that their angular extent is smaller than the Doppler angle. The blast wave is assumed to form a thin shell with \( \eta = 1/\Gamma_0 \). In an idealized circumburst medium where cloud

\[ F \text{ stands for Fast Rise, Exponential Decay, although the decay phase is often better fit by a power law.} \]
properties were roughly the same throughout this region, then for every bright pulse or two, there would be another dimmer by a factor of 10\(^7\) and another 20 pulses dimmer than the brightest pulse by a factor of 10\(^{20}\). Pulse widths of a second or so are formed by the brightest pulses for such clouds, in keeping with the pulse paradigm (Norris et al. 1996). At late times and from larger angles, a lower fluence emission plateau would be formed if the jet and clouds are assumed to maintain roughly uniform properties at these angles.

Because of the unknown distribution of cloud densities, sizes, and locations, as well as effects of GRB jet structure, simulating GRB light curves within an external shock scenario is very model dependent. But simulations of GRB light curves with small clouds randomly distributed in a shell show many of the features of spiky GRB light curves (Dermer & Mitman 1999, 2004; x4.5 below).

The calculations shown here mean that given the thin-shell approximation \(\eta \simeq 1/\Gamma_0\), essentially all BATSE GRBs without redshift information and many BATSE/BeppoSAX/HETE-2 GRBs with redshift information can be modeled within an external shock scenario. The standard jet properties are \(E_0 = 10^{52}\) ergs and \(z = 1\), in accord with pre-Swift observations of GRBs with redshift information (Friedman & Bloom 2005). These pulses have SEDs in agreement with BATSE observations (e.g., Preece et al. 2000), bearing in mind different ways to remedy the “line-of-death” problem (Preece et al. 1998) in a nonthermal synchrotron shock model through the jitter mechanism (Medvedev 2000), radiation reprocessing (Dermer & Böttcher 2000), etc. (see Zhang & Mészáros 2002 for a discussion of GRB spectral break models).

Many X-ray flares observed with Swift can also be modeled within the thin-shell approximation, as illustrated by the calculation shown in Figure 9. Here the standard X-ray flare parameters from Table 1 are used. The principal difference between these parameters, which apply to Swift observations, and those for the BATSE GRB pulse observations is that the source redshift is \(z = 2\) and the apparent isotropic energy release is \(10^{54}\) ergs, as implied by observations of GRBs at such redshifts (Ghirlanda 2007). The flares were detected by the XRT on Swift, so that the mean telescope photon energy is \(\epsilon \lesssim 0.01\). As can be seen, flares at flux levels reaching \(10^{-9}\) ergs cm\(^{-2}\) s\(^{-1}\) can be made by external shocks. Comparison with observations of GRBs with measured redshifts in the Swift catalog (O’Brien et al. 2006b) shows that most flares are well below this level, except for a flare from GRB 050820A at \(z = 2.612\) that reached a \(\nu F_\nu\) flux between
0.3 and 10 keV of $\approx 10^{-8}$ ergs cm$^{-2}$ between 200 and 300 s after the GRB.

The SED for the model GRB X-ray flare peaking hundreds of seconds after the start of the GRB is shown by the inset in Figure 9. In these flares the RS synchrotron component can make an enhanced optical/IR emission component due to the concavity produced by the various RS components. Figure 10 illustrates the light curves of X-ray flares observed at different angles in the frozen pulse approximation.

GRBs which lack redshift information, such as GRB 050502B with its enormous X-ray flare exceeding $10^{-5}$ ergs cm$^{-2}$ (Falcone et al. 2006), pose no difficulty to the external shock model. If the frozen-pulse assumption is allowed, then there is no difficulty in explaining $\gamma$-ray pulses or X-ray flares in GRBs, provided that the surrounding medium of a GRB is clumpy on well-defined density and size scales.

$\dagger$ Because the $\nu F_\nu$ peak flux was above the Swift BAT energy window, the fraction of total GRB fluence in the X-ray flare is unknown.

A straightforward prediction of the external shock model as presented here is that the spectra of X-ray flares tend to become softer with time and will be systematically softer than the pulses in the prompt phase. This is because the single expanding blast wave becomes less dense with radius, so that interactions with surrounding cloud material tend to have weaker FSs. Thus, there will be a systematic reduction of the $\nu F_\nu$ peak energies and a softening of the X-ray flare spectra. Softenings of spectra are observed in flares from individual GRBs like GRB 050502B (Falcone et al. 2006) or GRB 050822 (Godet et al. 2007). This cannot be considered a definitive test of the external shock model, however, because such behavior can also be accounted for by a colliding shell/internal shock model if the relativistic ejecta tend to have lower $\Gamma$ factors or less energy with time. It would be surprising and contrary to the expectations of this model if X-ray flares tend to be harder than the prompt emission.

4. PHYSICS OF PULSE AND FLARE FORMATION IN THE EXTERNAL SHOCK MODEL

We find from this analysis that short pulses or flares can be made if the GRB blast wave undergoes no transverse spreading and
interacts with small dense ($\approx 10^3$ cm$^{-3}$) clouds within $\approx 10^{16}$-$10^{17}$ cm from the GRB explosion. The formation of strong FSs with $\Gamma \approx \Gamma_0$ is necessary to make bright pulses and was the underlying assumption of the GRB light curve simulations in Dermer & Mitman (1999, 2004), where GRB light curves similar to actual GRB light curves were simulated with $\approx 10\%$ efficiency. The implications and plausibility of the assumption of a thin blast-wave shell are now considered.

4.1. Temporal Behavior

The characteristic timescales shown in the preceding figures can be understood from simple considerations (Fenimore et al. 1996; Sari & Piran 1997; Dermer & Mitman 1999; Ioka et al. 2005). Equation (9) gives the time

$$ t = \frac{(1+z)x}{\beta_0 c} (1 - \beta_0 \mu_i) $$

(51)

since the start of the GRB explosion when emission is first received from a cloud at location $x$ and inclination angle $\theta_i = \arccos \mu_i$ that is impacted by a blast wave moving with Lorentz factor $\Gamma_0$. The angular timescale giving the temporal duration over which the emission, emitted at radius $r$ in the angular range $\theta_2 = \theta_i + \theta_{cl} > \theta_1 = \max(0, \theta_i - \theta_{cl})$, is received is simply

$$ \frac{\Delta t_{\text{ang}}}{1+z} = \frac{x}{\beta_0 c} \frac{1}{(1 + \beta_0 \mu_i)} (1 - \beta_0 \mu_i) \theta_1, \theta_2 \ll 1, \Gamma_0 \gg 1 $$

$$ \frac{x}{2c} (\theta_2^2 - \theta_1^2) \approx \left( \frac{x}{2c} \right) \Delta \theta_i \theta_{cl} $$

(52)

when $\theta_i > \theta_{cl}$, and

$$ \frac{\Delta t_{\text{ang}}}{1+z} \approx \frac{x}{2c} (\theta_1^2 - \theta_{cl}^2) $$

(53)

when $\theta_i < \theta_{cl}$.

The radial timescale for emission from two points at different distances $x_1$ and $x_2$ but at the same inclination angle $\theta_i$ is given from equation (20) by

$$ \frac{\Delta t_{\text{rad}}}{1+z} = \frac{\Delta \theta_1}{\beta_0 c} (1 - \beta_0 \mu_i) \approx t_{\text{min}} \left( \frac{\Delta \theta_1}{x} \right) (1 + \theta_1^2 \Gamma_0^2), $$

(54)

where $t_{\text{min}}$ is defined by equation (50), and the inclination and cloud angles are in units of the Doppler angle $\theta_0 = 1/\Gamma_0$ by the relations

$$ v_i = \Gamma_0 \theta_i, \quad v_{cl} = \theta_0 \Gamma_0, $$

respectively. The characteristic duration of a shell/cloud collision is therefore

$$ \Delta t \approx t_{\text{min}} \max \left[ \frac{2v_{cl}}{1} \left( 1 + v_i^2 \right), 4v_{cl} H(v_i - v_{cl}) + (v_i + v_{cl})^2 H(v_{cl} - v_i) \right], $$

(55)

where $H(u) = 1$ if $u \geq 1$ and $H(u) = 0$ otherwise. Note that the angular timescale is independent of cloud location $x$, using $v_{cl} = \Gamma_0 \theta_{cl} = (\Delta \theta_{cl}/x) \Gamma_0$ in equation (55), but depends on cloud size $\Delta \theta_{cl}$ and inclination angle $\theta_i$. The radial timescale is also independent of $x$ when written in terms of cloud size.

The STV condition for the external shock model is $v_{cl} \approx 1$. Thus, we see that the kinematic duration is determined by the angular timescale everywhere except when $v_i \leq 1/2 \Gamma_0$, that is, for nearly on-axis clouds with $\theta_i \leq 1/2 \Gamma_0^2$. The kinematic duration of the pulse $\Delta t \propto 1/\theta_i$ for $\theta_i \geq 1/2 \Gamma_0^2$. The pulse shortening for nearly on-axis events, combined with beaming factors and spectral effects, is shown from analyses and Monte Carlo simulations capable of explaining the range of observed GRB BATSE light curves (Dermer & Mitman 1999, 2004).

4.2. Optimal Cloud Density for Bright Pulses

The optimal density giving the brightest measured flux can be derived from three requirements.

1. A strong FS which, from equation (13), implies a maximum cloud density given by

$$ n_{cl} \leq \frac{E_0}{16 \pi \chi^2 \Gamma_0^2 m_p c^2 \Delta(x)} $$

(56)
2. Significant blast-wave deceleration to provide efficient energy extraction, which requires clouds with thick columns (Dermer & Mitman 1999), that is, with densities

\[ n_{cl} \approx \frac{E_0}{4\pi x^2 \Gamma_0^3 m_p c^2 \Delta_{cl}}. \] (57)

3. The requirement of a strong FS and a thick column therefore translates into the requirement that

\[ \Delta_{cl} \approx 4\Gamma_0^2 \Delta(x) \] (58)

in order to produce STV. The STV condition is

\[ \theta_{cl} = \frac{\Delta_{cl}}{x} = \frac{v_{cl}}{\Gamma_0} \ll \frac{1}{\Gamma_0}. \] (59)

Using equation (2) for the shell width, equation (58) becomes

\[ 4\eta \Gamma_0 < \frac{4\Gamma_0^3 \Delta_0}{x} + 4\eta \Gamma_0 \ll 1, \]

so that the requirement on the width of the radiating shell material in the external shock model for rapid variability is simply that

\[ \eta \approx \frac{1}{4\Gamma_0} \quad \text{when} \quad x \approx 4\Gamma_0^3 \Delta_0 \approx 10^{15} \Gamma_3^{\frac{1}{30}} \Delta_7 \, \text{cm}. \] (60)

It is therefore not possible to have STV if \( \eta \approx 1 \), because then the RS is relativistic, the FS Lorentz factor is low, and Doppler boosting is lost. But it becomes possible in the thin-shell approximation \( \eta \approx 1/\Gamma_0 \). As shown by the light curves and SED calculations, the phenomenology of GRBs can be explained in the frozen pulse \( (\eta = 0) \) and also, possibly, in the thin-shell approximation.

The optimal cloud density for bright variable pulses in the external shock model for rapid variability is simply

\[ n_{cl}[\text{cm}^{-3}] \approx 1.6 \times 10^6 \frac{E_{54}}{x_{16} \Gamma_{30} \Delta_7}. \] (61)

For instance, the calculations shown in Figure 5 with \( \eta = 1/\Gamma_0 \) have \( \Delta_{cl}(x = 10^{16} \, \text{cm}) = 38 \).

4.3. Width of Radiating Shell

Besides the requirement for STV that the unshocked fluid shell be thin or frozen, the width of the shocked fluid shell cannot be too great, for otherwise rapid variability would be washed out by contributions from different parts of the shell.

First, consider a blast wave sweeping up matter from a uniform surrounding circumburst medium with density \( n_0 \) (cm\(^{-3}\)). The swept-up material, compressed by the factor \( 4\Gamma \) at the external shock, has thickness

\[ \Delta_{sh} = \frac{x}{12\Gamma} \quad \text{and} \quad \Delta_{sh}(x) = \frac{x}{12\Gamma^2}. \] (62)

in the proper and explosion frames, respectively. Because the angular extent of the shocked fluid layer in the blast wave covers the full Doppler cone, the variability is already limited to a timescale \( \approx 0.25\Delta_{\text{min}} \) by light travel time limitations. Under these circumstances, rapid variability is not expected, even when considering density jumps (Nakar & Granot 2007).

Now consider the transverse width of the shocked fluid shell colliding with a cloud with Doppler size \( v_{cl} \ll 1 \) that subtends the solid angle \( \Delta_{cl} \) as seen from the center of the explosion. By equating the mass \( m_p \Delta_{cl} \Delta(x) n(x) \Delta_{sh}(x) \) with the cloud mass \( m_p \Delta_{cl} \Delta(x) n_{cl} \Delta_{cl} \), we have, using the relativistic density jump condition \( n(x) = 4\Gamma_0 \),

\[ \Delta_{sh}(x) = \frac{\Delta_{cl}}{4\Gamma} = \frac{v_{cl} x}{4\Gamma^2}, \quad \Delta_{sh}(x) = \frac{v_{cl} x}{4\Gamma^3}, \] (63)

i.e., a shocked fluid layer thinner than required to produce STV.

4.4. Thin Shell Assumption

From the preceding discussion, one can see that the viability of an external shock model for the \( \gamma \)-ray pulses and X-ray flares depends on whether the GRB blast-wave width spreads in the coating phase according to equation (2), with \( \eta \approx 1/\Gamma_0 \). In the gasdynamical study of Mészáros et al. (1993), inhomogeneities in the GRBs model produce a spread in particle velocities of order \( [v - cl]/c \sim \Gamma^{-\frac{3}{2}}, \) so that \( \Delta(x) \sim x/\Gamma_0 \) when \( x \approx 10^8 \Delta_0 \) and \( \eta 

generation and amplification of the magnetic field in the shocked fluid layer (including particle acceleration and diffusion into the cold fluid shell), and magnetic field and particle energy evolution from shell expansion.

4.5. Density Contrast

A further requirement on this model is that the dense clouds are found in a medium sufficiently tenuous that the blast wave has not undergone significant deceleration by sweeping up this material. This constraint can be expressed by the condition that the deceleration radius \( x_d = (3E_0/4\pi m_p c^2 n_0 \Gamma_0^2)^{1/3} \) (Rees & Mészáros 1992; Mészáros & Rees 1993) be much greater than the radius defining the condition for a thick column, equation (57). This translates to the requirement that the density contrast between the surrounding medium density \( n_0 \) and the cloud density \( n_{cl} \) is given by

\[
\frac{n_0}{n_{cl}} \ll \frac{\Gamma_0 \sqrt{36\pi n_{cl} \Delta_{cl}^3}}{E_0/m_pc^2} \sim 10^{-10}\Gamma_300 \left(\frac{n_{cl}/10^3 \text{ cm}^3}{\Delta_{cl}/10^9 \text{ cm}}\right)\frac{E_{54}}{E_54} \tag{64}
\]

This is a severe constraint that can only be realized in an extreme environment such as the one described in more detail below.
4.6. Simulations of GRB Light Curves

Figure 11 shows Monte Carlo simulations, described in more detail in Dermer & Mitman (2004), of GRB light curves under the assumption that the cold blast-wave shell remains thin and the interaction forms a strong FS. The parameters of the simulation are shown in Table 2, and here we use fixed cloud sizes with radius $r_1$ that are “uniformly randomly” distributed in the volume between $R_1$ and $R_2$. In all cases, the surface filling factor $\xi = 10\%$, and a fraction $\zeta = 10\%$ of the blast-wave energy intercepted by the cloud is transformed into radiation received by the detector at energy $m_ec^2c$. The angular extent of the jet is $k_0/\Gamma$, and an occulting factor is included so that the portion of the blast wave that intercepts a cloud at a smaller radius no longer radiates when interacting with clouds at larger radii.

The top four panels of Figure 11 represent the GRB prompt phase detected near 100 keV using parameters appropriate to BATSE, including noise at a level typical of the BATSE detectors. The bottom two panels of Figure 11 make use of parameters that would apply to Swift. The variety of simulated light curves is endless, depending on the seed used in the random number generator and the choices of cloud and blast-wave parameters. Precursor events can be made if some material is found very close to the GRB (Piro et al. 2005, compare Fig. E).

4.7. Criticisms of the External Shock Model

We describe the general criticisms that the external shock model for prompt GRB emissions has endured. Fenimore et al. (1996) demonstrated that an expanding blast wave would, if briefly illuminated over a large portion of its surface covering the Doppler cone, make progressively longer pulses in accordance with the relation $t \simeq (1 + z)x/T_0c$. The tendency for pulse lengthening can be avoided if a small fraction of the blast-wave surface area within the Doppler cone were illuminated at any one time, but this was thought to require an unreasonable number of illuminating regions (“clouds”).

One approach is to estimate a maximum “surface filling factor” of radiating sites consistent with measured variability, using kinematic arguments (Fenimore et al. 1999). Sari & Piran (1997) argued that highly variable light curves cannot be constructed from blast-wave interactions with surrounding density inhomogeneities by superposing pulses made near the outer region of the Doppler cone to show how they overlap due to the large angular timescales; hence, the sources of GRBs must require an active central engine to make discrete pulses. Dermer & Mitman (2004) showed that this argument, made also more recently by Piran (2005) and Ioka et al. (2005), mistakenly uses the angular timescale near the outer edge of the Doppler cone and misses a number of important points. One is that the beaming factors, not considered in these papers, heavily weight the total fluence for nearly on-axis clouds, and even more so the pulse peak flux because of the smaller kinematic variability timescale for nearly on-axis clouds; another is the different (observer) times for the clouds nearly on-axis as compared with those off-axis, so that flux ratios from different parts of the shell are time-dependent; a third is the assumption that clouds are distributed with azimuthal symmetry within the Doppler cone; a fourth is that the beaming factor of the relativistic jet that forms $\gamma$-ray pulses in the prompt phase may be smaller than $\sim \Gamma_0^{-1}$ and the typical angles inferred from optical afterglow breaks, which will reduce the importance of off-axis events. The crucial role of the narrow cold blast-wave fluid shell (not the shocked fluid shell considered by Nakar & Granot 2007) to ensure a strong FS has also been overlooked in past studies.

Ramirez-Ruiz & Fenimore (2000) claim that the lack of pulse width evolution in BATSE GRB light curves favors internal shock models; Kocevski et al. (2007), in the absence of any quantitative consideration of external shocks, argue that XRT pulse spreading in Swift GRBs favors internal shock models. From our simulations, we see that this effect is subtle at best in the unlikely condition when the clouds are “uniformly randomly” situated. Realistic cloud clustering properties or a narrow $\gamma$-ray jet could obscure this effect completely. Lazzati & Perna (2007) derive the timescale ratio $\Delta t_t/\Delta t_0 \sim 0.25$ for a spherical shell and suggest that the observational data indicate a clustering in the value $\Delta t_t/\Delta t_0 \sim 0.25$ that rules out the external shock model, but nowhere consider observational biases for the detection of flare profiles with larger or smaller timescale ratios, or whether this timescale ratio could reflect cloud properties.

As shown in the simulations, STV in the external shock model with moderate ($\sim 10\%$) efficiency is possible if the blast wave interacts with clouds with thick columns and sizes $\Delta_{el} \ll x/T_0$, because then the condition of “local spherical symmetry” (Fenimore et al. 1996) is broken. The shortest pulses carrying a significant fraction of the total fluence are made by interactions of the blast wave with small clouds at angles $\theta \ll 1/T_0$ to the observer line of sight (Dermer & Mitman 2004). Efficiency concerns are reduced if the fraction of energy dissipated as X-rays and $\gamma$-rays is a small fraction of the total (Dermer 2007).

5. DISCUSSION

We have critically examined whether external shock processes can make short-timescale X-ray and $\gamma$-ray variability in the $\gamma$-ray luminous phase and the early afterglow phase, extending to

| Parameter | A | B | C | D | E | F |
|-----------|---|---|---|---|---|---|
| $E_b$ (ergs) | $10^{53}$ | $10^{53}$ | $10^{54}$ | $10^{54}$ | $10^{54}$ | $10^{54}$ |
| Redshift $z$ | 1 | 1 | 1 | 1 | 1 | 1 |
| $k_0$ | 1 | 1 | 0.5 | 0.5 | 0.5 | 0.5 |
| $\Gamma_0$ | 300 | 300 | 300 | 300 | 300 | 100 |
| $\epsilon$ | 0.2 | 0.2 | 0.2 | 0.2 | 0.002 | 0.002 |
| $\epsilon_{\text{PK}}$ | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 | 0.4 |
| $R_1$ (cm) | $10^{16}$ | $10^{16}$ | $10^{16}$ | $10^{16}$ | $10^{16}$ | $10^{16}$ |
| $R_2$ (cm) | $10^{17}$ | $10^{17}$ | $10^{17}$ | $10^{17}$ | $10^{18}$ | $10^{18}$ |
| $r_1$ (cm) | $3 \times 10^{12}$ | $10^{13}$ | $3 \times 10^{12}$ | $10^{13}$ | $10^{13}$ | $3 \times 10^{13}$ |
| $\xi$ | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |
| $\zeta$ | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 | 0.1 |

* Received photon energy and $\epsilon_{\text{PK}}$ in units of $m_ec^2c^2$. 

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5. DISCUSSION

We have critically examined whether external shock processes can make short-timescale X-ray and $\gamma$-ray variability in the $\gamma$-ray luminous phase and the early afterglow phase, extending to
\(\geq 10^4\) s after a GRB trigger. The result is simple: external shocks can only make the erratic variability observed in GRB light curves if the cold fluid shell ejected by the impulsive burst event does not spread significantly from its initial characteristic width \(\Delta_{\text{exp}}\), at the time of the GRB explosion, where \(\Delta_{\text{exp}}\), assumed to be \(\ll 1\) s, is the time of the energy release in the explosion frame. The question of whether the lack of shell spreading is a valid assumption, at least within the Doppler cone primarily sampled by the observer, is an open question but plausibly valid. A crucial point is that an external shock model for the rapid variability is feasible because the GRB blast wave is still in its coasting phase and has not yet reached the adiabatic self-similar phase. Let us follow the implications if this assumption were allowed.

One way to interpret GRB light curve data is to suppose that there is a single impulsive burst of energy, and all subsequent phenomena are a consequence of the collimated ultrarelativistic blast wave interacting with its surroundings (Mészáros & Rees 1993; Dermer & Mitman 1999, 2004). External shocks are the basis for the success that the relativistic blast-wave physics model has had in interpreting GRB afterglows (Mészáros & Rees 1997; Zhang & Mészáros 2004); we also suppose that its success can be translated to the prompt phase. The properties of the surrounding medium, as implied by extensive evidence on GRB host galaxy studies, is determined by the type of GRB progenitor, which is a massive \((\geq 10-20 \, M_\odot)\) star lacking a H envelope whose core collapses to form a Type Ib/c supernova. The supernova remnant and the progenitor stellar wind, including the effects of a possible binary companion (Fryer et al. 2007), form a complicated surrounding that imprints its structure on the GRB light curves (Dermer & Mitman 1999; Wang & Loeb 2000).

The brief impulsive energy releases in GRB explosions, which for neutrino-mediated processes are most efficient for \(\gtrsim\) ms timescales, are compatible with brief collapse events or phase transitions from stellar cores to compact objects, for example, from neutron stars to black holes or neutron stars to quark stars (e.g., Drago et al. 2007; Berezhiani et al. 2003). This collapse event would follow by minutes, hours, days, or more a Type Ib/c supernova that forms a supramassive, rotationally supported neutron star/magnetar. This picture is a two-step collapse model, like the supernova model of Vietri & Stella (1998, 1999), although with a short delay. Such a picture, orthogonal in all respects to the collapsar model except for the primary role of high-mass stars as long-duration GRB progenitors, has a great deal of explanatory and predictive capability. Here we point out some of the attractive features of such a picture.

1. **\(^{56}\)Ni.** The direct collapse of the evolved core of a massive star to a black hole in the collapsar model, avoiding core bounce, requires new routes for the production of \(^{56}\)Ni. This makes the similarity between the peak optical rest-frame \(M_*\) magnitudes (taken as a proxy for \(^{56}\)Ni production) measured in supernovae associated with GRBs and Type Ib/c supernovae not associated with GRBs (Soderberg et al. 2005) a remarkable coincidence. Two-step collapse processes avoid this coincidence by forming \(^{56}\)Ni through neutron star core bounce.

2. **Energy reservoir.** The discovery (Frail et al. 2001; Panaitescu & Kumar 2001; Friedman & Bloom 2005) of a standard energy reservoir, or at least a clustering of beaming-corrected absolute energies toward a value \(\gtrsim 10^{51}\) ergs, modulo an uncertain efficiency to convert explosion energy into \(\gamma\)-rays, fixes the energy budget of the GRB explosion. The collapse from a rotationally supported neutron star to a black hole, or the transition from a degenerate neutron fluid to quark state, could liberate kinetic energies approaching a constant maximum energy that may not defy ability or require excessive assumptions to calculate. If the explosion involves a neutrino or Poynting flux-mediated interaction, values of collimated electromagnetic energy of \(\approx 10^{51}-10^{52}\) ergs could be released in a second collapse to a black hole.

3. **Complex circumburst medium.** A magnetar phase (Usov 1992) follows, even if only by minutes, the first collapse event and drives a strong pulsar wind to disrupt, thread, and render extremely complex the surrounding medium, in particular, the supernova remnant shell produced in the first collapse event. The work of Königl & Granot (2002) shows the profound impact on the circumburst medium that this phase can have for long (month–yr) delays.

4. **Clean environment.** The Usov phase cleans the environment except for dense clumps formed at distances where the strong radiation pressure can be withstood—and these are the clumps that form the clouds in the external shock model. The MHD winds effectively drive out all surrounding material except for very dense clouds, possibly allowing for the existence of density contrasts of magnitude defined by equation (64). The clouds themselves would be expanding, but on a much longer timescale than the period of activity of the system. Close to the explosion trigger, the environment of a rotationally supported neutron star is baryon-clean (Vietri & Stella 1998).

5. **Explanation for chromatic X-ray and optical afterglow beaming breaks.** Panaitescu et al. (2006) compare Swift BAT X-ray light curves with optical light curves taken from a variety of telescopes for six GRBs, finding several instances where the X-ray light curves appear to decay chromatically with respect to the optical light curves. Possible explanations for these observations is evolutions \(c_\gamma\) and \(\delta_\gamma\) parameters and the addition of the (synchrotron self-) Compton for a beamed outflow (illustrated numerically by Dermer et al. 2000), which plausibly explains the light curves of GRB 050922C and the lack of X-ray breaks observed with Swift (Sato et al. 2007). The optical light curves of GRBs 050607, 050802, and 050713A are too poorly sampled to draw definitive conclusions, and the optical and X-ray light curves in GRB 050319 appear to follow each other except for a final high optical datum. This leaves only GRB 050401 to explain. More and better data are clearly needed.

6. **Explanation for the decay and plateau phases observed with Swift.** Within the context of the external shock model for a GRB taking place within a uniform surrounding environment, I recently showed that an intense phothadronic phase, effectively depleting internal GRB blast-wave energy, could cause the X-ray light curves to exhibit rapid flux declines (Dermer 2007). After the discharge, the blast wave regains its nonradiative character to form the plateaus observed with Swift. Because GRBs showing a steep decay phase have significantly decelerated, the blast wave should then have entered the adiabatic self-similar phase when only broad X-ray flares can result from large density contrasts (Nakar & Granot 2007). It is interesting to note that GRBs showing the most extreme decay phases, e.g., GRBs 050315, 050416A, 050713B, 050814, and 050915B, display smooth subsequent light curves (O’Brien et al. 2006b; Chincarini et al. 2007), although GRB 050916 provides a counterexample and a challenge to this interpretation.

7. **High-energy radiation.** Observations of GRBs with the GLAST GBM and LAT will monitor the Compton components in the spectrum of a GRB. Definite correlations between the leptonic synchrotron and synchrotron self-Compton components are expected, which behave in stark contrast to phothadronic \(\gamma\)-ray components that vary independently of the lower energy lepton synchrotron component. GLAST will search for phothadronic
emission components and, in conjunction with detectors such as IceCube, test multiwavelength and multichannel predictions (Razzaque et al. 2004; Dermer et al. 2007).

8. Predictions. With respect to the short-delay supranova model, the confirming prediction is to glimpse the GRB progenitor in its magnetar phase prior to the second step collapse to form the GRB. Analysis of radio, optical, X-ray, or γ-ray emissions to obtain \( P(t) \) and \( \dot{P}(t) \) in the brief interval between the two collapse events is now only possible with broad field-of-view detectors with poor sensitivities. Unfortunately, optical supernova discovery generally happens within a few weeks after the supernova event, by which time a second collapse would have happened. So for the short-delay supranova intervals consistent with the offset between nearby GRBs and their supernova emissions (Zeh et al. 2004), this prediction is not promising except to test the standard supranova model (Vetri & Stella 1998).

A definitive test of the collapsar model is to find a supernova taking place well in advance of the GRB, so that the GRB has no detectable supernova emissions. Development of telescopes with wide field-of-view optical survey capabilities, like Pan-STARRS or SNAP, holds promise to rule out the collapsar model for specific GRBs (C. Dermer & A. Drago 2006, unpublished) and could also provide a database to search for magnetar activity in advance of a GRB.

6. SUMMARY AND CONCLUSIONS

A detailed analysis of the interaction between a relativistic blast wave and a stationary cloud was performed in the limit that the cloud width \( \Delta x \ll x \), so that the shell density remains effectively constant during the interaction. Synchrotron light curves and spectra from such interactions were calculated for a range of cloud densities, sizes, and locations, for blast-wave coasting Lorentz factors \( \Gamma_0 \) = 100 and 300, and for different blast-wave widths.

In order to produce short-timescale variability in the external shock model, the shell width parameter \( \eta \) must be \( \lesssim 1/\Gamma_0 \), as demonstrated both analytically and numerically. If this model assumption is valid, then the external shock model can explain the generic spectral shape of GRB pulses, rapid variability in GRB light curves observed with BATSE, and the delayed X-ray flares observed with Swift. The tendency of the GRB pulses or X-ray flares to diminish in intensity with time is a consequence of the expansion of the blast wave. The durations of GRB pulses reflect the dimensions of the surrounding thick-columned clouds. The typical distances of the clouds that produce γ-ray pulses are \( \approx 10^{15} \text{--} 10^{17} \text{cm} \) from the GRB explosion center, with cloud sizes \( \approx 10^{12} \text{--} 10^{13} \text{cm} \) and densities \( \approx 10^3 \text{ cm}^{-3} \). The clouds that produce X-ray flares observed with Swift are typically found \( \gtrsim 10^{17} \text{ cm} \) from the center of the GRB explosion and have sizes \( \approx 10^{15} \text{ cm} \) and densities \( \approx 10^3 \text{ cm}^{-3} \). This analysis also provides a basis for making accurate calculations of light curves and spectra formed by both forward and reverse shocks in collisions between relativistic shells.

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