Applications Modified Adomian Decomposition Method for Solving the Second-Order Ordinary Differential Equations

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Abstract: In this paper, we will display an efficient application of the Modified Adomian Decomposition method (MADM) for solving linear and non-linear singular initial value problem in the second order ordinary differential physical equations such as the Schrödinger’s Equation, Hermit differential Equation and capacitance regulation equation. It is shown that the MADM efficiency, simple, easy to use and accuracy because the series solutions converge to the specific answer for every problem if it exist or numerical solution if not exist.

Keywords: Modified Adomian Decomposition Method, Second-Order Ordinary Differential Equations, Physics equations, Numerical Method.

1. Introduction:

In the 1970s, George Adomian (1923-1996) presented another ground-breaking strategy for understanding linear and nonlinear functional equations. From that point forward, this strategy has been known as the Adomian decomposition method (ADM) [1,2]. The ADM has prompted a few alterations on the technique made by different specialists trying to improve the exactness or extend the use of the first strategy. At first, Adomian and Rach [5] in 2001, Wazwaz and Al-Sayed [9] displayed another (MADM) for linear and nonlinear operator, in the new change Wazwaz supplanted the way toward partitioning \( f \) into two segments by a series of infinite components, Hasan in [6] presented another adjustment of ADM, he proposed another differential administrator which can be utilized for singular and nonsingular ODEs. The new modification introduce a promising applications for linear and nonlinear operator. Therefore, we study in this research the applications (ADM) method and its modification in solving physical applications.

2. Analysis of the method

Consider the ordinary differential equation has the form

\[ Lu + Ru + Nu = g(t) \]  

(1)

Where \( N \) is nonlinear operator, \( L \) is the order derivative which is thought to be invertible and \( R \) is the linear differential operator of order less the \( L \) subject of the formulæ. We get

\[ Lu = g(t) - R(t) - Nu \]  

(2)

When a solution (2) for \( Lu \), we get on \( L \)

\[ L^{-1}[L u] = L^{-1}[g(t)] - L^{-1}[Ru] - L^{-1}[Nu] \]  

(3)

we advantageously for initial value problems define \( L^{-1} \) for \( L = \frac{d^n}{dt^n} \) as the \( n \)-fold definite integration from \( 0 \) to \( t \). If \( L \) is a second-order operator, \( L^{-1} \) is a two fold integral and solving (3) for \( y \), we get

\[ y = K + hx + L^{-1}[g(t)] - L^{-1}[Ru] - L^{-1}[Nu] \]  

(4)

Where \( k \) and \( h \) are constant of integration is resolved form a solution (3). The (ADM) consists of rounding resolution of (1) as alien series.
Let \( u(t) = \sum_{n=0}^{\infty} u_n \) (t) \( (5) \)

And decomposing nonlinear operator\( n \) as
\[
N(u) = \sum_{n=0}^{\infty} A_n, \tag{6}
\]

Where \( A_n \) are (ADM) \( [15,16] \) of \( u_0, u_1, u_2, u_3, \ldots, u_n \) given by
\[
A_0 = f(u_0)
\]
\[
A_1 = f'(u_0)u_1
\]
\[
A_2 = f'(u_0)u_2 + \frac{1}{2!} f''(u_0) u_1^2
\]
\[
A_3 = f'(u_0)u_3 + \frac{2}{2!} f''(u_0)u_1u_2 + \frac{1}{3!} f'''(u_0)u_1^3
\]
\[
A_4 = f'(u_0)u_4 + f''(u_0) \left( \frac{1}{2!} u_1^2 + u_1u_2 \right) + \frac{1}{2!} f'''(u_0)u_1^2u_2 + \frac{1}{4!} f''''(u_0)u_1^4
\]
\[
\vdots
\]

Hence
\[
A_n = \frac{1}{n!} \frac{d^n}{da^n} [N(\sum_{n=0}^{\infty} \alpha^n u_n)]_{\alpha=0}, \quad n \geq 0 \tag{7}
\]

To find the \( A_n \)'s by Adomian general formula, these polynomials will be computed as follows.
\[
A_0 = N(u_0)
\]
\[
A_1 = u_1 N(u_0) = \frac{d}{da} N(u_0 + \alpha u_1) \Big|_{\alpha=0}
\]
\[
A_2 = u_2 N'(u_0) + \frac{1}{2!} u_1^2 N''(u_0) = \frac{1}{2!} \frac{d^2}{da^2} N(u_0 + \alpha u_1 + \alpha^2 u_2) \Big|_{\alpha=0}
\]
\[
A_3 = u_3 N'(u_0) + \frac{2}{2!} u_1u_2 N''(u_0) + \frac{1}{3!} u_1^3 N'''(u_0) = \frac{1}{3!} \frac{d^3}{da^3} N(u_0 + \alpha u_1 + \alpha^2 u_2 + \alpha^3 u_3) \Big|_{\alpha=0}
\]
\[
\vdots
\]

Compensation (5) and (6) into (4) yields
\[
\sum_{n=0}^{\infty} u_n = k + hx + L^{-1}[g(t)] - L^{-1}[R(\sum_{n=0}^{\infty} u_n)] - L^{-1}[\sum_{n=0}^{\infty} A_n] \tag{8}
\]

The recursive relationship is found to be
\[
u_0 = g(t)
\]
\[
u_{n+1} = -L^{-1} [R u_n] - L^{-1} [A_n]
\]

Using a relationship (8) can be built in the form
\[
u = \lim_{n \to \infty} \phi_n(u) \tag{9}
\]

Where
\[
\phi_n(u) = \sum_{i=0}^{n} u_i \tag{10}
\]
3 Modified Adomian decomposition method.

Algorithm 1.

Modifications of The (ADM) Based on the operators(MADM1) [6 ]

The first order ordinary differential equation can be consider as :

\[ u' + p(t)u + F(t, u) = g(t), \]  \hspace{1cm} (11)

With boundary condition \( u(o) = K \),

Where \( K \) is constant , \( p(t) \) and \( g(t) \) given function and \( F(t,u) \) is real function. Characterize another differential operator \( L \) regarding the one derivative contained in the issue. Modify (11) in the form

\[ Lu = g(t) - F(t, u) \]  \hspace{1cm} (12)

Where the differential operator is defined by

\[ L(u) = e^{-\int p(t)dt} \frac{d}{dt} \left( e^{\int p(t)dt} u \right) \]  \hspace{1cm} (13)

The converse operator \( L^{-1} \) is hence think about a one-fold integral operator, as underneath,

\[ L^{-1} (\cdot) = e^{-\int p(t)dt} \frac{d}{dt} \int_0^t e^{\int p(t')dt'} (\cdot)dt' \]  \hspace{1cm} (14)

Applying \( L^{-1} \) of (14) to the first two terms \( u' + p(t) \) of eq. (11) We find

\[ L^{-1} (u' + p(t)u) = e^{-\int p(t)dt} \int_0^t e^{\int p(t')dt'} (u' + p(t)u) \, dt' \]

\[ = u - u(0) \varphi(0)e^{-\int p(t)dt} \]

Where \( \varphi(t) = e^{\int p(t)dt} \)

By operating \( L^{-1} \) on (11), we have

\[ u(t) = u(0)e^{-\int p(t)dt} + L^{-1}g(t) - L^{-1}F(t, u) \]  \hspace{1cm} (15)

The ADM introduces the solving by an infinity series of components by eq (5) And the nonlinear function \( F(t,u) \) by an infinite series of polynomials

\[ F(t, u) = \sum_{n=0}^{\infty} A_n \]  \hspace{1cm} (16)

Where the components \( u_n(t) \) of the arrangement \( u(t) \) will be determinately recurrently and \( A_n \) are ADM polynomial that can be built for different classes of nonlinearity as per explicit calculations set by Wazwaz [18]. For nonlinear \( F(u) \), the initial scarcely any polynomials are given by (7). Substituting (5) and (15) into (15) gives:

\[ \sum_{n=0}^{\infty} u_n(t) = u(0)\varphi(0)e^{-\int p(t)dt} + L^{-1}g(t) - L^{-1} \sum_{n=0}^{\infty} A_n \]

To locate the component \( u_n(t) \), we use ADM that proposes the utilization of the recursive relation

\[ u_0(t) = u(0)\varphi(0)e^{-\int p(t)dt} + L^{-1}g(t) \]

\[ u_{n+1}(t) = -L^{-1}(A_n), \quad n \geq 0 \]

Which gives:

\[ u_0(t) = u(0)\varphi(0)e^{-\int p(t)dt} + L^{-1}g(t) \]
\[
u_1(t) = -L^{-1}(A_0) \\
u_2(t) = -L^{-1}(A_1) \\
u_3(t) = -L^{-1}(A_2) \\
u_4(t) = -L^{-1}(A_3) \\
\vdots
\]

(17)

From (7) and (17), we can decide the parts \( u_n(t) \) and as a result the series solution of \( u(t) \) in (5) can be immediately gotten. For numerical study, the \( n^{th} \) \textit{term approximant}

\[
\varphi(t) = \sum_{n=0}^{n-1} u_n(t)
\]

Can be used to approximate the exact solution.

\textbf{Algorithm 2.}

. Modifications of The (ADM) Based on the operators (MADM2) second order ODEs [7].

Consider the underlying worth issue in the second common differential condition in the from

\[
u'' + p(t)u' + f(t, u) = g(t) \quad , \quad u(0) = K , u'(t) = H
\]

(18)

Where \( K \) and \( H \) constants, and \( P(t) \) and \( g(t) \) are given function, and \( F(t,u) \) is a real function.

Technique for solving

we propose the new differential administrator, as beneath

\[
L(\cdot) = e^{-\int p(t)dt} \frac{d}{dt}\left(e^{\int p(t)dt} \frac{d}{dt}(\cdot)\right)
\]

(19)

So ,the problem (12) can be written as follows .

\[
Lu = g(t) - f(t, u)
\]

(20)

The converse operator \( L^{-1} \) is in this manner considered a two-fold integral operator as underneath:

\[
L^{-1}(\cdot) = \int_{0}^{t} e^{\int_{0}^{t} p(t)dt} \frac{d}{dt} \left(e^{\int_{0}^{t} p(t)dt} \frac{d}{dt}(\cdot)\right) dt
dt
\]

By applying on (18) , we have

\[
u(t) = \varphi(t) + L^{-1}[g(t)] - L^{-1}[F(t, u)]
\]

(21)

To such an extent that

\[
(\varphi(t) = 0)
\]

Review that the ADM present the arrangement \( u(t) \) and the nonlinear \( F(t,u) \) by infinite series string using eq.(5) and eq.(16) , where the components \( u_n(t) \) of the solution \( u(t) \) will be specified as see in the past section through eq.(6) . which can be used to construct ADM polynomials, when \( F(u) \) is a nonlinear function . by substituting eq.(5) and eq. (15) into eq. (21), we get :

\[
\sum_{n=0}^{\infty} u_n(t) = \varphi(t) + L^{-1}[g(t)] - L^{-1}[\sum_{n=0}^{\infty} A_n]
\]

(22)

By using ADM, the components \( u_n(t) \) can be specified as

\[
u_0(t) = \varphi(t) + L^{-1}[g(t)] \quad , \quad u_{n+1}(t) = -L^{-1} \sum_{n=0}^{\infty} A_n \quad , \quad n \geq 0
\]

(23)

Which gives formulas as described in eq.(17) . From (23) and (17), we can locate the components \( u_n(t) \) and as result the series solution of \( u(t) \) in (22) can be instantly gotten.
Algorithm 3.

Modifications of The (ADM) based on the operators(MADM3) second order ODEs [ 8 ]

Consider the solitary beginning with issue in the second request conventional differential condition in the structure

$$u'' + \frac{2}{t} u' + f(t, u) = g(t) \quad u(0) = K, \quad u'(0) = H$$  \hspace{1cm} (24)

Where K and H are constants, P(t) and g(t) are given function, and F(t,u) is a real function.

Technique for arrangement we propose the new differential administrator, as beneath

$$L(\cdot) = e^{-\int p(t) dt} \frac{d^2}{dt^2} (e^{\int p(t) dt} \cdot)$$  \hspace{1cm} (25)

So, the problem (18) can be written as follows. $Lu = g(t) - F(t, u)$ The inverse operator $L^{-1}$ is therefore considered a two-fold integral operator as below:

$$L^{-1}(\cdot) = t^{-1} \int_0^t \int_0^t (\cdot) dt \, dt$$  \hspace{1cm} (26)

Applying $L^{-1}$ of (26) to the initial two terms $u'' + \frac{2}{t} u'$ of equation (24) we find

$$L^{-1} \left( u'' + \frac{2}{t} u' \right) = t^{-1} \int_0^t \int_0^t (t u'' + \frac{2}{t} u) \, dt \, dt = u - u(0)$$

By operating on (24), we have:

$$u(t) = K + L^{-1}[g(t)] - L^{-1}[F(t, u)]$$

Review that the ADM present the arrangement $u(t)$ and the non linear function $F(t,u)$ by infinity series through (5) and (16) when we use (7). which can be utilized to build Adomian polynomials, when $F(u)$ is a nonlinear function So, we get

$$\sum_{n=0}^{\infty} u(t) = K + L^{-1}[g(t)] - L^{-1} \sum_{n=0}^{\infty} A_n$$

Through utilizing ADM the components $u_n(t)$ can be determined as

$$u_0(t) = K + L^{-1}[g(t)]$$

$$u_{n+1}(t) = L^{-1}[A_n] \quad n \geq 0$$

Which gives formulas as described in eq.(17). From (24) and (17), we can locate the components $u_n(t)$ and consequently the series solution of $u(t)$ in (21) can be as soon as possible gotten.

4 Application modified Adomain Decomposition method for physical Equations

4.1 Schrödinger’s equation is not time dependent

The non-linear differential equation describes stable states and used when the Hamiltonian is not based on weight, but is dependent only on location[11]

$$u'' - t^2 u = 0 \quad , \quad u(0) = 1 \quad , \quad u'(0) = 0$$  \hspace{1cm} (27)
Using (MADM2) and from eq(19) we get: \( L(u) = t^2 \ u \), Now by applying \( L^{-1} \) to both sides of (26) we have: 
\[
\sum_{n=0}^{\infty} u_n(t) = 1 + L^{-1} \left[ \sum_{n=0}^{\infty} t^2 u_n \right],
\]
Thus we obtain:
\[
u_0 = 1
\]
\[
u_{n+1} = L^{-1} \left[ t^2 u_n \right], \quad n=0,1,2,...
\]
Therefore we have:
\[
u_1 = L^{-1} \left[ t^2 u_0 \right] = \int_{0}^{t} \int_{0}^{t} t^2 \ dt \ dt = \frac{1}{12} \ t^4
\]
\[
u_2 = L^{-1} \left[ t^2 u_1 \right] = \int_{0}^{t} \int_{0}^{t} \frac{1}{12} \ t^6 \ dt \ dt = \frac{1}{672} \ t^8
\]
\[
u_3 = L^{-1} \left[ t^2 u_2 \right] = \int_{0}^{t} \int_{0}^{t} \frac{1}{672} \ t^{10} \ dt \ dt = \frac{1}{88704} \ t^{12}
\]
\[
u_4 = L^{-1} \left[ t^2 u_3 \right] = \int_{0}^{t} \int_{0}^{t} \frac{1}{88704} \ t^{14} \ dt \ dt = \frac{1}{21288960} \ t^{16}
\]
\[
\vdots
\]
And it implies that
\[
u_n(t) = 1 + \frac{1}{12} \ t^4 + \frac{1}{672} \ t^8 + \frac{1}{88704} \ t^{12} + \frac{1}{21288960} \ t^{16} + \ldots
\]
The following table shows a comparison of the absolute errors between the exact solution

| Approximate solutions by ADM. |
|-----------------------------|
| X     | Numerical Methods | ADM | Abs. error     |
| 0.0   | 1.000008333333333 | 1.000008333348214 | 14.8809851461 × 10^{-12} |
| 0.1   | 1.000133336493071 | 1.000133337142903 | 649.8321444433 × 10^{-12} |
| 0.2   | 1.000675094275032 | 1.000675097639920 | 336 × 10^{-11} |
| 0.3   | 1.001234982900800 | 1.00123498760586 | 192086741 × 10^{-11} |
| 0.4   | 1.005214123767444 | 1.005214148958381 | 2519 × 10^{-11} |
| 0.5   | 1.01082496003561  | 1.010825018838814 | 5184 × 10^{-11} |
| 0.6   | 1.015518496127790 | 1.015518768879434 | 27275 × 10^{-11} |
| 0.7   | 1.020094179093968 | 1.020094257257745 | 9616 × 10^{-11} |
| 0.8   | 1.024383604243894 | 1.024383770314804 | 16607 × 10^{-11} |
| 0.9   | 1.028432316528820 | 1.028432749116518 | 43259 × 10^{-11} |

**Table 4.1**: Computing the absolute for solving Schrödinger’s equations.
Figure (4.1): (a) Graph the solution of Schrödinger's equation by MADM

(b) Graph the solution of Schrödinger's equation by Numerical method

4.2 Hermit differential equation

A homogeneous differential equation with a traditional orthogonal pathway that appears in gaussian numerical analysis as well as used in systems theory related to nonlinear factors in gaussian noise and other sciences [10]

$$u'' - 2t u' + 2m u = 0 \quad , \quad u(0) = 0 \quad , \quad u(0) = 1$$

The eq.(28) can be written as: $$L[u] = 2t u' - 2m u \quad , \quad u'(0) = 1 \quad , \quad$$ where $$L = \frac{d^2}{dt^2}$$ is the differential operator. Working on the two sides with the opposite oprator of L ( namely $$L^{-1}[\cdot] = \int_0^t \int_0^t dt d\tau$$ to get: $$u(t) = t + L^{-1}[2t u' - 2m u]$$, by eq.(22) we have:

$$\sum_{n=0}^{\infty} u_n(t) = t + L^{-1}[\sum_{n=0}^{\infty} 2t u'_n - 2m u_n]$$

Thus we obtain:

$$u_0 = t$$

$$u_{n+1} = L^{-1}[2t u'_n - 2m u_n] \quad , \quad n = 0, 1, 2, 3, ...$$

Therefore we have:
\[ u_1 = L^{-1}[2t u'_0 - 2m u_0] = \int_0^t \int_0^t 2t - 2mt \, dt \, dt = \left( \frac{1}{3} - \frac{m}{2} \right) t^3 \]

\[ u_2 = L^{-1}[2t u'_1 - 2mu_1] = \int_0^t \int_0^t 2t^3 - 2mt^3 - \frac{2m}{3} t^3 + \frac{2m^2}{3} t^3 \, dt \, dt = \left( \frac{1}{10} - \frac{2m}{15} + \frac{m^2}{30} \right) t^5 \]

\[ u_3 = L^{-1}[2t u'_2 - 2mu_2] = \int_0^t \int_0^t t^5 - \frac{4m}{3} t^5 + \frac{m^2}{3} t^5 - \frac{m}{5} t^5 + \frac{4m^2}{15} t^5 \]

= \frac{m^3}{15} t^5 \, dt \, dt = \left( \frac{1}{42} - \frac{23}{630} + \frac{m^2}{70} - \frac{m^3}{630} \right) t^7 \]

And it implies that:

\[ u_n(t) = t + \left( \frac{1}{3} - \frac{m}{2} \right) t^3 + \left( \frac{1}{10} - \frac{2m}{15} + \frac{m^2}{30} \right) t^5 + \left( \frac{1}{42} - \frac{23}{630} + \frac{m^2}{70} - \frac{m^3}{630} \right) t^7 + \ldots \]

The accompanying table shows an examination of the total blunders between the numerical solution and approximate solutions by ADM.

| X     | Numerical methods | ADM          | Abs. error  |
|-------|-------------------|--------------|-------------|
| 0     | 0                 | 0            | 0           |
| 0.1   | 0.100233916666667 | 0.10023396531563 | 4986 \times 10^{-11} |
| 0.2   | 0.201886907657691 | 0.201887237006202 | 32935 \times 10^{-11} |
| 0.3   | 0.306456061318320 | 0.306460128546290 | 40673 \times 10^{-11} |
| 0.4   | 0.415602009246113 | 0.415631471426804 | 2946218 \times 10^{-11} |
| 0.5   | 0.531250527088885 | 0.531389995582218 | 13946849 \times 10^{-11} |
| 0.6   | 0.655721027723658 | 0.656218829277806 | 49780155 \times 10^{-11} |
| 0.7   | 0.7918966790000371 | 0.79355074412944 | 145839541 \times 10^{-11} |
| 0.8   | 0.943457176683684 | 0.947150074706408 | 369289802 \times 10^{-11} |
| 0.9   | 1.15205131863528  | 1.123560557741671 | 835542588 \times 10^{-11} |
| 1.0   | 1.313532636139214 | 1.330806309523809 | 1727367338 \times 10^{-11} |

**Table 4.2:** Computing the absolute for solving Hermite differential equation.
Figure (4.2): (a) Graph the solution of Hermit differential equation by MADM  
(b) Graph the solution of 2 Hermit differential equation by Numerical method

4.3 capacitance regulation equation

It can be found from the nonlinear Schro¨dinger equation (NLSE) solution used to regulate the amplification and pressure in pulses of non-linear fiber optic amplifiers and other uses[12]

\[ u'' - \lambda u + 2\kappa u^3 = 0 , \quad u'(0) = 0 , \quad u(0) = 1 \]  

(29)

Using (MADM3) and from eq(23) we get:

\[ L(u) = \lambda u - 2\kappa u^3 , \]

Now by applying \( L^{-1} \) to both sides of (26) we have:

\[ u(t) = 1 + \lambda L^{-1}[u] - 2\kappa L^{-1}[u^3] \]

Using ,eq.(22) we get:

\[ \sum_{n=0}^{\infty} u_n(t) = 1 + \lambda L^{-1}[\sum_{n=0}^{\infty} u_n] - 2\kappa L^{-1}[\sum_{n=0}^{\infty} A_n] \]

Thus we obtain

\[ u_0 = 1 \]
\[ u_{n+1} = \lambda L^{-1} \left[ \sum_{n=0}^{\infty} u_n \right] - 2\kappa L^{-1} \left[ \sum_{n=0}^{\infty} A_n \right], \quad n \geq 0 \]

We compute \( A_n \)'s Adomian polynomials of nonlinear term \( u^3 \), by eq. (7) we get:

\[
\begin{align*}
    u_1 &= \lambda L^{-1} [u_0] - 2\kappa L^{-1} [A_0] = \lambda \int_0^t \int_0^t \frac{\lambda}{2} \tau^2 - \kappa t^2 \, dt \, dt = \left( \frac{\lambda^2}{24} - \frac{\lambda \kappa}{12} \right) t^2 \\
    u_2 &= \lambda L^{-1} [u_1] - 2\kappa L^{-1} [A_1] = \lambda \int_0^t \int_0^t \frac{\lambda}{2} \tau^2 - \kappa t^2 \, dt \, dt = \left( \frac{\lambda^2}{720} - \frac{\lambda \kappa}{72} \right) t^4 \\
    u_3 &= \lambda L^{-1} [u_2] - 2\kappa L^{-1} [A_2] = \lambda \int_0^t \int_0^t \frac{\lambda^2}{24} t^4 - \frac{\lambda \kappa}{12} t^4 \, dt \, dt = \left( \frac{\lambda^3}{40320} - \frac{\lambda^2 \kappa}{20160} \right) t^6 \\
    u_4 &= \lambda L^{-1} [u_3] - 2\kappa L^{-1} [A_3] = \lambda \int_0^t \int_0^t \frac{\lambda^3}{720} t^6 - \frac{\lambda^2 \kappa}{360} t^6 \, dt \, dt = \left( \frac{\lambda^4}{403200} - \frac{\lambda^3 \kappa}{201600} \right) t^8 \\
    &\vdots
\end{align*}
\]

\[ u_n(t) = 1 + \left( \frac{\lambda}{2} - \kappa \right) t^2 + \left( \frac{\lambda^2}{24} - \frac{\lambda \kappa}{12} \right) t^4 + \left( \frac{\lambda^3}{720} - \frac{\lambda^2 \kappa}{360} \right) t^6 + \left( \frac{\lambda^4}{40320} - \frac{\lambda^3 \kappa}{20160} \right) t^8 + \cdots \]

The accompanying table shows an examination of the total blunders between the numerical solution and approximate solutions by ADM.

| X   | Numerical methods | ADM            | Abs. error      |
|-----|-------------------|----------------|-----------------|
| 0   | 0                 | 0              | 0               |
| 0.1 | 0.100083583333333 | 0.10008558410496 | 19751 \times 10^{-11} |
| 0.2 | 0.2006768384444079 | 0.200680792828068 | 395438 \times 10^{-11} |
| 0.3 | 0.302328618203162 | 0.302334584891578 | 596669 \times 10^{-11} |
| 0.4 | 0.405668287324306 | 0.405676577978184 | 829065 \times 10^{-11} |
| 0.5 | 0.511451749945724 | 0.511464311899716 | 1256195 \times 10^{-11} |
| 0.6 | 0.620615596552155 | 0.620640979265638 | 2538271 \times 10^{-11} |
| 0.7 | 0.734344325997967 | 0.734411718233736 | 6739224 \times 10^{-11} |
| 0.8 | 0.854158145581713 | 0.854350463739244 | 19231816 \times 10^{-11} |
| 0.9 | 0.982033078157173 | 0.982554961111168 | 52188295 \times 10^{-11} |
| 1.0 | 1.120572266012892 | 1.121874900654901 | 130263464 \times 10^{-11} |

**Table 4.3:** Computing the absolute for solving capacitance regulation equation.
4.4 Forced Vibration with Damping

We will presently research the circumstance wherein an intermittent outer power is applied to a spring-mass framework. The conduct of this basic framework models that of numerous oscillatory framework with an outer power due, for example, to an engine appended to the framework. We will initially consider the case wherein damping is available and will take a gander at the glorified extraordinary case wherein there is thought to be no damping [17].

The logarithmic estimations can be genuinely muddled right now issue, so we will start with a moderately basic model.

\[ u'' + u' + 1.25u = 3\cos t \quad , \quad u(0) = 2 \quad , \quad u'(0) = 3 \]  

The Exact solution: \[ u(t) = e^{-\frac{t}{17}} \left[ \frac{22}{17} \cos t + \frac{14}{17} \sin t \right] + \frac{12}{17} \cos t + \frac{48}{17} \sin t \]

Using (MADM2) and from eq(19) we get: \[ L(u) = 3\cos t - 1.25u \] , Now by applying \( L^{-1} \) to both sides of (26) we have: \[ u(t) = 3t + 2 + \frac{3}{2}(e^{-x} - \cos t + \sin t) - L^{-1}[1.25 u] \]

Using ,eq.(22) we get:

\[ \sum_{n=0}^{\infty} u_n(t) = 3t + 2 + \frac{3}{2}(e^{-x} - \cos t + \sin t) - L^{-1}[\sum_{n=0}^{\infty} 1.25 u_n] \]

Thus we obtain:
\[ u_0 = 3t + 2 \]
\[ u_{n+1} = \frac{3}{2} (e^{-x} - \cos t + \sin t) - L^{-1}[\sum_{n=0}^{\infty} 1.25u_n] \]

Therefore we have:

\[ u_1 = \frac{3}{2} (e^{-x} - \cos t + \sin t) - L^{-1}[1.25u_0] = \frac{3}{2} (e^{-x} - \cos t + \sin t) - \int_0^t e^{-t} f(t) \left( \frac{15}{4} t + \frac{5}{2} \right) e^t dt = -\int_0^t e^{-t} \left( \frac{55}{16} + \frac{15}{8} \right) t^3 - \frac{25}{8} t^2 + \frac{25}{4} t - \frac{25}{4} e^t \]
\[ u_2 = -L^{-1}[1.25 u_1] = -\int_0^t e^{-t} \left( \frac{55}{16} + \frac{15}{8} \right) t^3 - \frac{25}{8} t^2 + \frac{25}{4} t - \frac{25}{4} e^t \]
\[ u_3 = -L^{-1}[1.25 u_2] = -\int_0^t e^{-t} \left( \frac{55}{16} + \frac{15}{8} \right) t^3 - \frac{25}{8} t^2 + \frac{25}{4} t - \frac{25}{4} e^t \]

The accompanying table shows an examination of the total blunders between the exact solution and approximate solutions by ADM.

| X  | Exact solution | ADM       | Abs. error          |
|----|----------------|-----------|---------------------|
| 0  | 2.003000249291688 | 2.003000249291688 | 14995 \times 10^{-10} |
| 0.01 | 2.005994998334350 | 2.0060009433760 | 5996 \times 10^{-9} |
| 0.02 | 2.0089887447979115 | 2.00902230877115 | 134865 \times 10^{-10} |
| 0.03 | 2.011979986681534 | 2.012003954673375 | 2396799 \times 10^{-11} |
| 0.04 | 2.014968723994395 | 2.015006161474730 | 3743775 \times 10^{-11} |
| 0.05 | 2.017954955074562 | 2.018008847034054 | 5389199 \times 10^{-11} |
| 0.06 | 2.020938678679600 | 2.021012007104855 | 7332843 \times 10^{-11} |
| 0.07 | 2.023919893568451 | 2.024015637441281 | 9574387 \times 10^{-11} |
| 0.08 | 2.026898598501433 | 2.02701973798170 | 1211353 \times 10^{-11} |
| 0.09 | 2.02987492240240 | 2.03002429130985 | 14949969 \times 10^{-11} |
Table 4.4: Computing the absolute for solving Forced Vibration with Damping

|          |          |
|----------|----------|
|          |          |

Figure (4.4): (b) Graph the solution of Forced Vibration with Damping by MADM.

(a) Graph the solution of Forced Vibration with Damping by exact solution.

4.5 Forced Vibrations without Damping

We presently expect subsequently acquiring the condition of movement of an undamped constrained oscillator the sort of movement, having an intermittent variation of sufficiency, shows what is known as a beat. For example, such a wonder happens in acoustics when two tuning forks of almost equivalent recurrence are energized at the same time. In electronics, the variety of the abundance with time is called amplitude modulation[17]

\[ u'' + u = 0.5 \cos 0.8t \]
\[ u'(x) = u(t) = 0 \] (30)

The Exact solution: \( u(t) = -1.388888888889 \cos t + 1.388888888889 \cos 0.8t \)

Using eq.(19), So, the problem (35) can be written as follows.

\[ Lu = 0.5 \cos 0.8 t - u \] (31)

Now, by applying \( L^{-1} \) to both sides of (36) we have:

\[ U(t) = L^{-1}[0.5 \cos 0.8t] \cdot L^{-1}[u] \] and using eq. (7) we have:

\[ u_0 = \int_0^t \int_0^t 0.5 \cos 0.8t \ dt \ dt = -\frac{25}{32} \cos 0.8t + \frac{25}{32} \]

\[ u_1 = -L^{-1}[u_0] = -\int_0^t \int_0^t -\frac{25}{32} \cos 0.8t + \frac{25}{32} \ dt \ dt \]
\[ u_2 = -L^{-1}[u_1] = - \frac{625}{512} \cos 0.8t - \frac{25}{64} t^2 + \frac{625}{512} dt \]

\[ = -1.907348633 \cos 0.8t + \frac{25}{768} t^4 - \frac{625}{1024} t^2 + -1.907348633 \]

\[ u_3 = -L^{-1}[u_2] = - \int_0^t \int_0^t -1.907348633 \cos 0.8t + \frac{25}{768} t^4 - \frac{625}{1024} t^2 \]

\[ + -1.907348633 dt \]

\[ = -2.980232239 \cos 0.8t - \frac{5}{4608} t^6 + \frac{625}{12288} t^4 - 0.9536743165 t^2 \]

\[ + 2.980232239 \]

The accompanying table shows an examination of the total blunders between the exact solution and approximate solutions by ADM.

| X   | Exact solution | ADM    | Abs. error |
|-----|---------------|--------|------------|
| 0   | 0             | 0      | 0          |
| 0.1 | 0.009945424353314 | 0.009945424854420 | 501.105877475 \times 10^{-12} |
| 0.2 | 0.039131144554942 | 0.039131145059283 | 504.340895818 \times 10^{-12} |
| 0.3 | 0.085637927596682 | 0.085637928106681 | 509.999091826 \times 10^{-12} |
| 0.4 | 0.146373678523799 | 0.146373679043976 | 520.177074293 \times 10^{-12} |
| 0.5 | 0.217228338165331 | 0.217228338702370 | 537.039042892 \times 10^{-12} |
| 0.6 | 0.293280877216389 | 0.293280877736165 | 519.775859823 \times 10^{-12} |
| 0.7 | 0.369049032467351 | 0.369049032538159 | 70.808136128 \times 10^{-12} |
| 0.8 | 0.438770461018430 | 0.438770457869818 | 315 \times 10^{-11} |
| 1.9 | 0.496702504765640 | 0.49670245045720 | 1972 \times 10^{-11} |
| 1.0 | 0.537426825341506 | 0.537426735894675 | 8945 \times 10^{-11} |

**Table 4.5**: Computing the absolute for solving Forced Vibrations without Damping
Figure (4.5): (b) Graph the solution of Forced Vibrations without Damig by MADM.

(a) Graph the solution Forced Vibrations without Damig by exact solution.

4.6 Differential-Equation Description of Dispersive Nonlinear Media

This equation discusses briefly the origin of dispersion and its effect on nonlinear optical processes. For simplicity, anisotropic effects are not included. The dispersion medium is a medium with a memory that does not immediately show the polarization density resulting from an applied electric field [17].

\[ u'' + 2u' + 16u + 9.6u^2 = 3.2 \quad \text{,} \quad u(0) = u'(0) = 0 \tag{32} \]

The eq. (37) can be written as:

\[ L[u] = 3.2 - 2u' - 16u - 9.6u^2 \quad \text{,} \quad u'(0) = u(0) = 0 \]

where \( L = \frac{d^2}{dt^2} \) is the differential operator. Working on the two sides with the backwards operator of \( L \) (namely \( L^{-1}[] = \int_0^t \int_0^s [] dt \) to get:

\[ u(t) = \frac{8}{5} t^2 - L^{-1}[2u' + 16u] - 9.6 L^{-1}[u^2] \]

by eq. (22) we have:

\[ \sum_{n=0}^{\infty} u_n(t) = \frac{8}{5} t^2 - L^{-1}[\sum_{n=0}^{\infty} 2u'_n + 16u_n] - 9.6 L^{-1}[\sum_{n=0}^{\infty} A_n] \]

Thus we obtain:
The accompanying table shows an examination of the total blunders between the numerical solution and approximate solutions by ADM.

| X  | Numerical methods | ADM   | Abs. error |
|----|-------------------|-------|------------|
| 0  | 0                 | 0     | 0          |
| 0.01 | 0.000158917332309 | 0.000158917482125 | 149.816 $\times 10^{-12}$ |
| 0.02 | 0.000631215111982 | 0.00063121540205 | 298.22307702 $\times 10^{-12}$ |
| 0.03 | 0.001409939496073 | 0.00140993983114 | 335.066891567 $\times 10^{-12}$ |
| 0.04 | 0.002487787789944 | 0.002487787950548 | 160.603991864 $\times 10^{-12}$ |
Table 4.6: Computing the absolute for solving Differential-Equation Description of Dispersive Nonlinear Media.

| Value | ADM       | Numerical Method | Error |
|-------|-----------|------------------|-------|
| 0.05  | 0.003857125598555 | 0.003857124468790 | $113 \times 10^{-11}$ |
| 0.06  | 0.005510003707400 | 0.005509998494678 | $521 \times 10^{-11}$ |
| 0.07  | 0.007438174717737 | 0.007438159207002 | $1551 \times 10^{-11}$ |
| 0.08  | 0.009633109461855 | 0.009633071352110 | $3811 \times 10^{-11}$ |
| 1.09  | 0.012086013224894 | 0.012085930217609 | $8301 \times 10^{-11}$ |
| 1.00  | 0.014787841800014 | 0.014787676144277 | $16566 \times 10^{-11}$ |

Figure (4.6): (a) Graph the solution of Differential-Equation Description of Dispersive Nonlinear Media by MADM. (b) Graph the solution of Differential-Equation Description of Dispersive Nonlinear Media by Numerical method.
5. Conclusion

In this paper, a modified Adomian Decomposition method was introduced and its application on a set of linear and non-linear second-order ordinary differential equations. It is observed that the MADM is a powerful method because the approximate solution of a non-linear equation as an infinite series which usually converges to the exact solution. In other hand, the method is able to convert the most difficult problem's into simple and easy solutions to problems.

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