Topological changes of two-dimensional magnetic textures

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We investigate the interaction of magnetic vortices and skyrmions with a spin-polarized current. In a square lattice, fixed classical spins and quantum itinerant electrons, evolve according to the coupled Landau-Lifshitz and Schrödinger equations. Changes in the topology occur at microscopic time and length scales, and are shown to be triggered by the nucleation of a nontrivial electron-spin structure at the vortex core.

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Itinerant magnetism is a fascinating state of matter where the interplay of short range coupling (exchange, spin-orbit, crystal anisotropy) and long range dipolar interactions lead to a variety of spatial orders. Experiments show in particular that magnetic structures with nontrivial topology naturally arise in nanosized ferromagnets and chiral metals. Vortices are present in confined geometries with closed magnetic flux lines, such as permalloy nanodots [1–3]; the Dzyaloshinskii-Moriya spin-orbit coupling in magnetic metals with inversion asymmetry like bulk MnSi or Fe atomic films, favors helical ordering in the form of a skyrmion lattice [4–6]. The existence of inhomogeneous metastable states in two-dimensional isotropic ferromagnets, distinct from the usual domains, was theoretically predicted by Belavin and Polyakov [7], who exhibited a solution asymptotically uniform with a central core with reversed magnetization. Lattices of skyrmions were shown to be thermodynamically allowed in chiral magnets, within a range of applied magnetic fields [8]. The topology of these magnetization fields can be characterized by their degree, or topological charge [9]; the skyrmion configuration realizes a map between the plane (the ferromagnetic film) and the sphere (the directions of the magnetization vector); it has therefore an integer topological charge [7, 10]. From a topological point of view, the isolated vortices observed in nanomagnets, are more exotic, since their topological charge is half an integer [11]. Vortices with an out-of-plane core magnetization, can be viewed as half skyrmions, sometimes called merons [12, 13], because only a half sphere is mapped. In disk magnets, their stability is ensured by the constraint of a tangent magnetization at the boundary that minimizes the dipolar magnetic energy [14].

Interestingly, experiments reveal that these topological configurations can be manipulated not only by external magnetic fields, but also using purely electric means, by a spin polarized current, through the spin-transfer torque mechanism [15, 16]. The polarity of a vortex core can be reversed by applying a short pulse of an in-plane magnetic field [17], or by a current [18]. More recently, ultrafast switching of an uniform magnetization, with the temporary formation of a magnetic singularity, was achieved in experiments using laser pulses of circularly polarized light [19, 20], a technique that can in principle also be effective in vortex switching via a topological inverse Faraday effect [21]. In this Letter we investigate the topological changes of magnetic textures induced by a spin-polarized current, using a semiclassical two-dimensional lattice model, in which the itinerant electrons are quantum, and the fixed spins, classical. This approach allows us to take into account nonlocal effects that are fundamental in the mechanisms involving magnetic singularities (in the continuum limit) and spin-transfer torque. As underlined by Miltat and Thiaville [22], the nucleation of Bloch points and vortex cores are at the edge of quantum magnetism.

The dynamics of the magnetization at the microscopic scale is governed by the Landau-Lifshitz equation [23],

$$\hbar \frac{\partial}{\partial t} \mathbf{S} = \mathbf{S} \times \mathbf{f} - \alpha \mathbf{S} \times (\mathbf{S} \times \mathbf{f}),$$

where \( \mathbf{S} \) is the dimensionless spin (\( \mathbf{S} = -m \), the unit magnetization vector) and \( \mathbf{f} \) the effective field derived from the free energy functional (\( \alpha \) is the damping constant, and \( \mathbf{f} \) has the dimensions of energy). This equation takes into account the exact conservation of the magnetization norm to its saturation value. In addition, the special mathematical form of Landau-Lifshitz equation, ensures the conservation of the topological charge [24], defined by,

$$Q = \int_{\mathbb{R}^2} \frac{dx}{4\pi q} q = \mathbf{S} \cdot \partial_x \mathbf{S} \times \partial_y \mathbf{S},$$

in a two-dimensional system, where \( \mathbf{x} = (x,y) \). It is worth noting, that the conservation of the magnetization topology is independent of the effective field specific form, and it holds even in the presence of norm preserving dissipation and time-dependent external fields. However, the topology conservation is violated by stochastic perturbations, related for instance to thermal or quantum fluctuations.

The Landau-Lifshitz micromagnetic approach was extensively used in recent years to investigate the dynamics of magnetic textures involving monopoles and vortices. In spite of the fact that the Landau-Lifshitz equation...
particular, these simulations revealed the importance of magnetic simulations on discrete lattices proved to be useful in describing complex topological changes [25]. In particular, these simulations revealed the importance of the excitation of gyration modes and vortex-antivortex annihilation in vortex core reversal [26–30].

We consider a periodic square lattice of fixed spins \( \mathbf{S} \) (classical, \( |\mathbf{S}| = 1 \)) and a single electron that can jump between neighboring sites \( \mathbf{i} = (x, y) = x\hat{\mathbf{e}}_x + y\hat{\mathbf{e}}_y \), and \( j \) (the hopping energy is \( \epsilon \), the electron charge \( -e \), the lattice parameter \( a \) and size \( L \)). Periodic boundary conditions ensure a well defined topology of the total system. A constant electric field \( E\hat{\mathbf{e}}_y \) is applied to create an electron current; this current is polarized by a fictive magnetic field acting only on the electrons \( \mathbf{B}_p \). Electrons and fixed spins are coupled through an exchange interaction \( J_s \), which in ferromagnets can be much larger than the Curie energy \( J \) [31]. The electron Hamiltonian is,

\[
H_e = -e \sum_{\langle \mathbf{i}, \mathbf{j} \rangle} e^{i\phi_{\mathbf{i}, \mathbf{j}}(t)} c_{\mathbf{i}}^\dagger c_{\mathbf{j}} - J_s \sum_{\mathbf{i}} \mathbf{S}_\mathbf{i} \cdot (c_{\mathbf{i}}^\dagger \mathbf{\sigma} c_{\mathbf{i}}) + H_p
\]  (3)

where \( c_{\mathbf{i}} = (c_{\mathbf{i}}^\uparrow, c_{\mathbf{i}}^\downarrow) \) is the annihilation operator at site \( \mathbf{i} \) and spin up \( \uparrow \) or down \( \downarrow \). To preserve the lattice periodicity we used a gauge transformation of a time-dependent vector potential \( E(t)\hat{\mathbf{e}}_x \). This allows us to take into account the constant electric field through the phase \( \phi_{\mathbf{i}, \mathbf{j}}(t) = (i - j) e\epsilon a E t / \hbar \), which is zero if the neighboring sites \( \mathbf{i}, \mathbf{j} \) are not in the \( x \) direction. The second term accounts for the interaction energy with the fixed spins (\( \mathbf{\sigma} \) are the Pauli matrices). The last term,

\[
H_p = -\mu_e \mathbf{B}_p \cdot \sum_{\mathbf{i}} c_{\mathbf{i}}^\dagger \mathbf{\sigma} c_{\mathbf{i}}
\]

allows the current to polarize in the direction \( \mathbf{B}_p \) (\( \mu_e \) is the electron magnetic moment). The magnetic energy is the sum of exchange \( J > 0 \), anisotropy \( K \) (positive or negative for easy-plane or easy-axis cases, respectively), and Dzyaloshinskii-Moriya \( D \), terms:

\[
H_S = \frac{J}{2} \sum_{\mathbf{i}} (\nabla \mathbf{S}_\mathbf{i})^2 + K \sum_{\mathbf{i}} S_{z\mathbf{i}}^2 - \frac{D}{2} \sum_{\mathbf{i}, \mathbf{j}} \mathbf{S}_\mathbf{i} \cdot (\nabla \times \mathbf{S}_\mathbf{j})
\]  (4)

where \( \nabla \) is here the discrete gradient operator. In the following we use units such that \( a = e = \hbar = e = 1 \). The typical microscopic scales are \( a \approx 0.3 \text{nm} \) and \( e \approx 1 \text{eV} \) for a ferromagnet, or \( e \approx 0.1 \text{eV} \) for MnSi, given a time unit \( t_0 \approx 1 - 10 \text{fs} \); the unit of electric field is about \( 10^8 \text{V m}^{-1} \). These small time and length scales, related to the electron kinetic energy and lattice spacing, are necessary to track the changes in topology.

The system evolution is governed by the Schrödinger equation for the electrons,

\[
i\hbar \frac{\partial c_{\mathbf{i}}(t)}{\partial t} = H_e(t, \mathbf{S}_\mathbf{i}) c_{\mathbf{i}}(t),
\]  (5)

and (1) for the fixed spins, with \( \mathbf{f}_i = -\partial H_S / \partial \mathbf{S}_i \),

\[
\mathbf{f}_i = J \nabla^2 \mathbf{S}_i - K S_{z\mathbf{i}} + D \nabla \mathbf{S}_i + J_s n_e \mathbf{S}_i,
\]  (6)

where \( s_i \) and \( n_e \) are the electron spin and number of electrons per site, respectively. The last term that gives the spin-transfer torque, is calculated using the quantum mean of the spin operator: \( n_e \mathbf{S}_i = \langle c_{\mathbf{i}}^\dagger \mathbf{\sigma} c_{\mathbf{i}} \rangle \). At variance to the usual linear response or quasi-adiabatic approximation, leading to a modified Landau-Lifshitz equation [31–33], we keep the full electron dynamics (5) (see [34] for a related model).

We solved numerically the system (1)-(5) using various initial spin textures that relaxed to a stable state before injecting the electrons. As an illustration of the rich phenomenology we show in Figs. 1 and 2 the topological evolution of skyrmions and vortices in the presence of initial free electrons [35]. The topological charge (2) decreases or increases by integer steps \( \Delta Q = 1 \). Variations with \( \Delta Q > 1 \) result from the superposition of simultaneous and separated in space \( \Delta Q = 1 \) events. In Fig. 1 we also plot \( Q_+(t) \) computed from the integral of \( |q| \), as in (2), which is a measure of the number of vortices present in the system at time \( t \). In these examples we used a strong electric field \( (E \approx 10^{-3}) \) in order to clearly display the current-vortex interactions [36].
There exists a threshold of the electric field and electron density values, below which vortex reversal is absent; it is interesting to note that in such a case the electron current, although present, is localized in channels that avoid the vortex cores. The spatial inhomogeneity and localization of the electrons is a general feature, systematically observed, showing that the systems is relatively far from the quasi-adiabatic regime. The spontaneous nucleation and annihilation of vortices are in fact driven by the strong inhomogeneity of the electron spin distribution.

We may distinguish two ways leading to a topological change, according to the value of $\Delta Q$: The nucleation and annihilation of same polarity vortex-antivortex pairs that do not change the total topological charge, $\Delta Q = 0$; and the reversal of a vortex core, the suppression of a skyrmion, or other vortex interactions involving a change $\Delta Q = 1$. Figure 2 shows the magnetization at selected times, for an initial skyrmion lattice and an array of vortices, displaying a variety of topological change events. The skyrmion lattice may be considered as a superposition of bounded meron-antimeron pairs, double-periodically distributed in the plane, and having a charge $Q = 8 \times (1/2)$ per cell. Under the action of a strong $+x$-spin electron current, they wander around as almost independent $Q = 1/2$ structures (Fig. 2, $t = 2000$), and when equal charge pairs come close together, they annihilate emitting a burst of spin waves (Fig. 2, $t = 4680$, 4760 and $t = 9004$, 9016). Lately, a uniform magnetization state is reached (Fig. 2, $t = 9600$). The evolution of a vortex array, subject to a spin-up current, is representative, in addition to the vortex annihilation event (Fig. 2, $t = 1424$, 1492), of other interesting processes such as the nucleation of vortex-antivortex $Q = 0$ pairs (Fig. 2, $t = 1932$, 2000), and the in-situ core switching (Fig. 2, $t = 2644$, 2764). Each of these events is easily correlated to a sudden change in $Q_+$, Fig. 1c.

In general, the mechanism of a $\Delta Q = 1$ topological change entails the formation of a virtual structure (a singularity in the continuum limit) with a net unit charge, opposite to the initial charge. For instance, in the case of a $Q = 1/2$ vortex, a bump of opposite polarity forms near the core, even when the vortex is almost at rest, as a result of the torque exerted by the electrons. At the time when the two peaks approach close enough, at most a few lattice steps away, a virtual antivortex should appear to provide the necessary charge to annihilate the original vortex, then allowing the growing bump to become a new vortex. This pathway through the $\Delta Q = 1$ change is, with respect to the topology of the magnetization field, similar to the core reversal phenomenology observed in micromagnetic simulations for moving [28, 37] or static structures [38, 39]. However, at variance to these models where the driven mechanism is an external time-dependent magnetic field, here we take into account the self-consistent interaction with the electron current (maintained by an external, constant, electric field). In addition to the precession impressed around the local direction of the spin-polarized current, the action of the moving electrons $s$ on the fixed spins texture is twofold: First they reduce the vortex core size, through a non-local interaction with the surrounding spin currents and waves; and second, they are able, by a local spin-transfer torque, to reverse the orientation of individual spins (strong non-adiabatic effect). The annihilation of a skyrmion core, presented in Fig. 3, is significant of the role played by the itinerant spins in the $\Delta Q = 1$ topological change.

Figure 3 presents the configuration of the Belavin-Polyakov skyrmion in the initial stage of the topological change, corresponding to $t = 1152$ in Fig. 1a. The skyrmion core was previously deformed by the spin-up polarized current, increasing the gradient of $S_z$ in the $x$-direction (Fig. 3a); as in the case of a meron core switching, the core of the skyrmion is ultimately reversed, leav-
ing a $Q = 0$ final state (at time $t = 1168$). One of the main characteristics of the free spins is its quasi-stochastic distribution, as one observes in Fig. 3b. Reminding that the differentiability of the effective field $f$, is a necessary condition for the conservation of the topological charge, the spatio-temporal intermittency of the $s = \langle c^\dagger \sigma c \rangle$ field is a crucial ingredient in the microscopic mechanism of the topological change. The origin of this complex behavior is the multiple quantum scattering of the electron waves on the magnetization inhomogeneities, as can be verified by following the evolution of their wavefunction (compare the fixed and itinerant spin distributions of Fig. 3). We also show in Fig. 3 the effective internal magnetic field created by the fluctuating spin texture of the itinerant electrons,

$$b = n \cdot \partial z n \times \partial_y n, \quad n = s / s.$$  \hspace{1cm} (7)

It naturally arises when imposing the electron spin direction as the natural quantization axis, leading to an effective gauge vector potential $a = (n \times \nabla n) \cdot \sigma$. This is opposite to the usual gauge transformation that takes the magnetization as the reference frame, to locally rotate the quantization axis (used to eliminate the electron degrees of freedom from the action [31, 40]). The remarkable fact about this quantity is that it concentrates at the vortex core, presenting a strong gradient precisely in the region where the core is reversing. The relation between the internal $b$ field and the topological charge density of the electron spin field, let us interpret the rotating opposite polarity peaks at the center of the vortex core, as being the signature of a non-trivial topological structure that will trigger the formation of the $Q = -1$ charge necessary to the transition. Therefore, this process is in some sense the opposite of the quasi-adiabatic mechanism: the magnetization vectors, at a microscopic spatio-temporal scale, follow the dynamics imposed by the itinerant electron spins, which are the source of the topological change.

In summary, we investigated the topological changes in a two-dimensional ferromagnet driven by a self-consistent electron current. The Landau-Lifshitz equation is coupled through the spin-transfer torque term with the Schrödinger equation for the itinerant spins. At variance to the continuous micromagnetic models, the system discreetness and more importantly, the stochastic behavior of the driven term, broke the conservation of the topological charge. We observed that both, local and nonlocal interactions, play a role in the transition between different topological configurations. In particular, the electron current tends to concentrate in channels that avoid the vortex cores —strong gradients of the magnetization act as potential barriers, scattering off the electron waves. The phenomenology of a $\Delta Q = 1$ change of an initial $Q = \pm 1$, $\pm 1/2$ vortex, although rich, reduces to a single topological mechanism, the nucleation of a $Q = \pm 1$ charge that annihilates the old structure, letting the new structure with the opposite charge or $Q = 0$. The interesting point is that this mechanism do not arise spontaneously but is triggered, above a threshold, by the spin-polarized current. This current is strongly fluctuating, up to the lattice and time unit scales, which are the relevant scales for the topological change. At the heart of the topological change is the formation of an electron non-trivial structure that induces the switching mechanism of the magnetization, in a strongly non-adiabatic process.

Our model is limited to a simple geometry and boundary conditions; although it would be interesting to explore more complex situations, the main open question is how to obtain a continuous limit (necessary to model larger systems at longer time scales) that takes into account the effective dissipative coupling with the electrons (see the recent papers [40, 41] where dissipation and dissipationless mechanisms are analyzed).

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