Information flow between stock indices

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Abstract – Using transfer entropy, we observed the strength and direction of information flow between stock indices. We uncovered that the biggest source of information flow is America, while most receivers are in the Asia/Pacific region. According to the minimum spanning tree, the Standard and Poor’s 500 Index (GSPC) is located at the focal point of the information source for world stock markets.

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Introduction. – Economic systems have recently become an active field of research for physicists striving to transfer concepts and methodologies from statistical physics such as phase transition, fractal theory, spin models, complex networks, and information theory to the analysis of economic problems [1–24].

Among the numerous methodologies put forward, time series analysis has proven to be one of the most efficient methods and is widely applied to the examination of characteristics of stock and foreign exchange markets. In order to analyze financial time series, a range of statistical measures have been introduced, including probability distribution [20–26], autocorrelation [23], multi-fractal [27], complexity [18–20], entropy density [19,20], and transfer entropy [11].

Information is a keyword in analyzing financial market data or in estimating the stock price of a given company. It is quantified through a variety of methods such as cross-correlation, autocorrelation, and complexity. However, while they may be appropriate measures for the observation of the internal structure of information flow, they fail to illuminate the directionality of information flow. Schreiber [28] introduced transfer entropy, which measures the dependency in time between two variables and notes the directionality of information flow. This concept of transfer entropy has already been applied to the analysis of financial time series. Marschinski and Kantz [29] calculated information flow between the Dow Jones and DAX stock indexes to better observe interactions between the two huge markets. The present authors [11] measured the direction of information flow between the composite stock index and individual stock prices, while the information flow among individual stocks in a stock market has been estimated by Baek and coauthors [30] in order to measure the internal structure of a stock market.

Through transfer entropy, this paper focuses quantitatively on the direction of information flow between 25 stock markets to determine which market serves as a source of information for global stock indices. Drawn from the economic system, a plethora of empirical data reflecting economic conditions can be obtained. The time series of a composite stock price index provides prime data accurately reflecting economic conditions. Therefore, we analyzed the daily time series of the 25 stock indices listed in table 1 for the period of 2000–2007 using transfer entropy in order to examine the information flow between stock markets and identify the hub.

Transfer entropy. – Transfer entropy, which measures the directionality of a variable with respect to time was recently introduced by Schreiber [28] based on the probability density function (PDF). Let us consider two discrete and stationary process, \( I \) and \( J \). Transfer entropy relates \( k \) previous samples of process \( I \) and \( l \) previous samples of process \( J \) and is defined as follows:

\[
T_{J \rightarrow I} = \sum p(i_{t+1}, i^{(k)}_t, j^{(l)}_t) \log \frac{p(i_{t+1} | i^{(k)}_t, j^{(l)}_t)}{p(i_{t+1} | i^{(k)}_t)},
\]

where \( i_t \) and \( j_t \) represent the discrete states at time \( t \) of \( I \) and \( J \), respectively, \( i^{(k)}_t \) and \( j^{(l)}_t \) denote \( k \) and \( l \) dimensional delay vectors of two time series of \( I \) and \( J \), respectively.

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Table 1: List of 25 markets. We obtain data from the website http://finance.yahoo.com.

| Region   | Code | Country       |
|----------|------|---------------|
| Americas | 1    | MERV          |
|          | 2    | BVSP          |
|          | 3    | GSPTSE        |
|          | 4    | MXX           |
|          | 5    | GSPC          |
|          | 6    | DIA           |
|          | 7    | DJI           |
| Asia/Pacific | 8    | AORD          |
|          | 9    | SSEC          |
|          | 10   | HSI           |
|          | 11   | BSESN         |
|          | 12   | JKSE          |
|          | 13   | KLSE          |
|          | 14   | N225          |
|          | 15   | STI           |
|          | 16   | KS11          |
|          | 17   | TWII          |
| Europe   | 18   | ATX           |
|          | 19   | BFX           |
|          | 20   | FCE.NX        |
|          | 21   | GDAXI         |
|          | 22   | AEX           |
|          | 23   | MIBTEL        |
|          | 24   | SSMI          |
|          | 25   | FTSE          |

The joint PDF $p(i_{t+1}, i_t^{(k)}, j_t^{(l)})$ is the probability that the combination of $i_{t+1}$, $i_t^{(k)}$ and $j_t^{(l)}$ have particular values. The conditional PDF $p(i_{t+1} \mid i_t^{(k)}, j_t^{(l)})$ and $p(i_{t+1} \mid i_t^{(k)})$ are the probability that $i_{t+1}$ has a particular value when the value of previous samples $i_t^{(k)}$ and $j_t^{(l)}$ are known and $i_t^{(k)}$ are known, respectively.

The transfer entropy $T_{J \rightarrow I}$ measures the extent to which the dynamics of process $J$ influences the transition probabilities of another process $I$. Reverse dependency is calculated by exchanging the roles of $i_t$ and $j_t$. The transfer entropy is explicitly asymmetric under the exchange of $i_t$ and $j_t$. It can thus provide information regarding the direction of interaction between two time series.

The transfer entropy is quantified by information flow from $J$ to $I$. The transfer entropy can be calculated by subtracting the information obtained from the last observation of $I$ exclusively from the information about the latest observation $I$ obtained from the final joint observation of $I$ and $J$. This is the basis of transfer entropy. Therefore, transfer entropy can be rephrased as

$$T_{J \rightarrow I} = h_I (k) - h_{IJ}(k,l),$$

where

$$h_I (k) = - \sum p(i_{t+1}, i_t^{(k)}) \log p(i_{t+1} \mid i_t^{(k)}),$$

$$h_{IJ}(k,l) = - \sum p(i_{t+1}, i_t^{(k)}, j_t^{(l)}) \log p(i_{t+1} \mid i_t^{(k)}, j_t^{(l)}).$$

Fig. 1: Plots of (a) the outgoing and (b) the incoming transfer entropy. Plots of (c) the outgoing and (d) the incoming transfer entropy for the shuffled data. Numbers in $x$-axis indicate markets listed in table 1.

Empirical data analysis. – We analyzed the daily data records of 25 stock exchange indices for the period January 2000–December 2007. We adopted the log returns to describe a financial time series $x(n) \equiv \ln(S(n)) - \ln(S(n-1))$, where $S_n$ means the index of $n$-th trading day. We partitioned the real value $x(n)$ into discretized value $A(n)$; $A(n) = 0$ for $x(n) \leq -d$ (decrease), $A(n) = 1$ for $-d < x(n) < d$ (intermediate), $A(n) = 2$ for $x(n) \geq d$ (increase). We set $d = 0.04$ in our analysis. At that value, the probabilities of three states are approximately same for all indices. Additionally, we set $k = l = 1$.

In order to estimate the transfer entropy $T_{J \rightarrow I}$, we use data of the previous day for market $I$ and the latest data for market $J$ when there is no overlap of market opening time between two markets. On the other hand, when there is overlap between two markets, the transfer entropy is obtained from the previous day’s data of both markets.

Figure 1 is the plots of the outgoing and the incoming transfer entropy. Numbers in $x$-axis indicate markets listed in table 1 while the dotted lines in the graphs serve to distinguish the continents. Each stock market interacts with 24 other stock markets, and the transfer entropy for all possible pairs can be calculated. Figure 1(a) represents the transfer entropy from the market indicating $x$-axis to the other 24 markets, that is, the outgoing transfer entropy. Figure 1(b) represents the transfer entropy from the other 24 markets to the market in $x$-axis, that is, the incoming transfer entropy. The markets in the Americas have high values of outgoing transfer entropy, while the value of the transfer entropy for the markets in Asia remains low. Moreover, the incoming transfer entropy for Asia is higher than that of the Americas and Europe. From fig. 1, we can determine that the directional influence for a market index is generally from the Americas to Asia. To clarify the tendencies of information flow, we shuffled
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the data and verified the transfer entropy. In fig. 1(c) and fig. 1(d), the transfer entropy for the shuffled data is generally approaching zero, information flow between markets has nearly disappeared. Moreover, in order to verify whether the value of transfer entropy is obtained by chance, we have compared it with the transfer entropy of shuffled data. We have evaluated the z-score of the transfer entropy for real data using mean and standard deviation of transfer entropies estimated from the 30 ensembles for all pairs of markets. The minimum z-score is 1.32 for the transfer entropy from SSEC to KS11 and the z-score of the transfer entropy from GSPC to AORD shows the maximum value 155.23. In case of the minimum z-score, the probability of the appearance of the value of transfer entropy is less than 0.093. It strongly tells us that the transfer entropy from real data is not obtained by chance.

Figure 2 is the gray-scale map of the transfer entropy between stock markets using the same data as in fig. 1. The direction of information flow is from the x-axis to the y-axis. The darker the lattice, the lower the transfer entropy. The markets in the Americas influence the markets in both Asia and Europe, but influence on the Asian markets is particularly strong. The European markets also influence the Asian markets, but to a lesser degree than the American markets do.

Figure 3 is the gray-scale map of the cross-correlations between stock markets from log-return time series $x(n)$. To evaluate the cross-correlation, we do not reflect time difference between stock markets. The cross-correlation does not represent directionality between markets. Therefore, the graph is symmetrical, centering around the diagonal. In this graph, a light lattice means that the cross-correlation is high. The American and European markets are highly correlated between themselves, while the relatively emerging Asian markets are not. It is not strange that the cross-correlations of inter-continents are a relatively small quantity. Nonetheless, it is a quite astonishing result that the transfer entropy highly exists from America and Europe to the Asia/Pacific continent.

To observe schematically the relation of information flow between the stock markets, the minimum spanning tree is depicted in fig. 4. The colors of nodes indicate the continent housing the markets. The minimum spanning tree is constructed by connecting the highest outgoing (fig. 4(a)) and incoming (fig. 4(b)) transfer entropy. In fig. 4(a), the GSPC is located at the focus of the network, connected with the Asian and European markets. The GSPC plays the role of information source among stock markets. On the other hand, in fig. 4(b) the Australian market, AORD, is located in the center of the minimum spanning tree for incoming transfer entropy.

Conclusions. – We observed the transfer entropy and the cross-correlation of the daily data of 25 stock markets to investigate the intensity and the direction of information flow between stock markets. The value of the outgoing transfer entropy from the American markets is high, as it is from the European markets. The information flow mainly streams to the Asia/Pacific region.

We reveal that the cross-correlations between intra-continental markets are higher than those between inter-continental markets, and the cross-correlations between Americas and Europe are relatively high. We can conclude that the American and European markets are strongly clustered and they are able to be regarded as one economic region, while Asia/Pacific markets are economically separated from American and European market cluster. Therefore, we can infer that American and European stock markets fluctuate in tune with a common deriving mechanism. However, the Asia/Pacific markets fluctuate with another common deriving mechanism. The considerable quantity of the transfer entropy from American and European market cluster to the Asia/Pacific markets is the strong evidence that there is an asymmetry of information flow between the deriving mechanisms.
governing each market clusters. The Asia/Pacific markets are remarkably affected by the American and European market situation. From fig. 4, we can confirm that the GSPC is located in the center of the star network, influencing all the other markets. In this study, we show quantitatively that mature markets drive emerging markets.

Transfer entropy is a proper measure for the investigation of the information flow between global stock markets. Through this measure we can quantify information transportation between underlying mechanisms that govern stock markets even though we cannot concretely identify the mechanism.

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REFERENCES

[1] Eguiluz V. M. and Zimmermann M., Phys. Rev. Lett., 85 (2000) 5659.
[2] Krawiec K., Holyst J. A. and Helbing D., Phys. Rev. Lett., 89 (2002) 158701.
[3] Chowdhury D. and Stauffer D., Eur. Phys. J. B, 8 (1999) 477.
[4] Takahshi T., Int. J. Mod. Phys. C, 16 (2005) 1311.
[5] Kaizoji T., Physica A, 287 (2000) 493.
[6] Kaizoji T. and Kaizoji M., Adv. Complex Syst., 6 (2003) 303.
[7] Pal{ágyi} Z. and Mantegna R. N., Physica A, 269 (1999) 132.
[8] Kaizoji T., Bornholdt S. and Fujiwara Y., Physica A, 316 (2002) 441.
[9] Jung W.-S., Chae S., Yang J.-S. and Moon H.-T., Physica A, 361 (2006) 263.
[10] Jung W.-S., Kwon O., Yang J.-S. and Moon H.-T., J. Korean Phys. Soc., 48 (2006) 1315.
[11] Kwon O. and Yang J.-S., Physica A, 387 (2008) 2851.
[12] Arthur W. B., Durlauf S. N. and Lane D. A., The Economy as an Evolving Complex System II (Perseus Books, Jackson) 1997.
[13] Mantena R. N. and Stanley H. E., An Introduction to Econophysics (Cambridge University Press, Cambridge) 2000.
[14] Bouchaud J.-P. and Potters M., Theory of Financial Risks (Cambridge University Press, Cambridge) 2000.
[15] Mandelbrot B. B., Quant. Finance, 1 (2001) 124.
[16] Kullmann L., Kert{é}sz J. and Mantegna R. N., Physica A, 287 (2000) 412.
[17] Giada L. and Marsili M., Physica A, 315 (2002) 650.
[18] Park J. B., Lee J. W., Yang J.-S., Jo H.-H. and Moon H.-T., Physica A, 379 (2007) 179.
[19] Lee J. W., Park J. B., Jo H.-H., Yang J.-S. and Moon H.-T., physica/0607282 (2006).
[20] Yang J.-S., Kwak W., Kaizoji T. and Kim I.-M., Eur. Phys. J. B, 61 (2008) 389.
[21] Kaizoji T., Econophysics of Stock and other Markets: Proceedings of the Econophys-Kolkata II Series, New Economic Windows (Springer) 2006, p. 3.
[22] Matal K., Pal M., Salunkay H. and Stanley H. E., Europhys. Lett., 66 (2004) 909.
[23] Yang J.-S., Chae S., Jung W.-S. and Moon H.-T., Physica A, 363 (2006) 377.
[24] Silva A. C., Prange R. E. and Yakovenko V. M., Physica A, 344 (2004) 227.
[25] Stanley H. E., Amaral L. A. N., Gabaix X., Gopikrishnan P. and Plerou V., Physica A, 299 (2001) 1.
[26] McCauley J. L. and Gunaratne G. H., Physica A, 329 (2003) 178.
[27] Kim K. and Yoon S.-M., Physica A, 344 (2004) 272.
[28] Schreiber T., Phys. Rev. Lett., 85 (2000) 461.
[29] Marschinski R. and Kantz H., Eur. Phys. J. B, 30 (2002) 275.
[30] Baek S. K., Jung W.-S., Kwon O. and Moon H.-T., physics/0509014 (2005).