Dynamics of an ensemble of clumps embedded in a magnetized ADAF

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Abstract We investigate the effects of a global magnetic field on the dynamics of an ensemble of clumps within a magnetized advection-dominated accretion flow by ignoring interactions between the clumps and then solving the collisionless Boltzmann equation. In the strong-coupling limit, in which the averaged radial and rotational velocities of the clumps follow dynamics described by an Advection-Dominated Accretion Flow (ADAF), the root mean square radial velocity of the clumps is calculated analytically for different magnetic field configurations. The value of the root mean square radial velocity of the clumps increases by increasing the strength of the radial or vertical components of the magnetic field, but a purely toroidal magnetic field geometry leads to a reduction in the value of the root mean square radial velocity of the clumps in the inner parts by increasing the strength of this component. Moreover, dynamics of the clumps strongly depend on the amount of advected energy so that the value of the root mean square radial velocity of the clumps increases in the presence of a global magnetic field as the flow becomes more advective.

Key words: accretion — accretion disk — black hole physics — magnetohydrodynamics

1 INTRODUCTION

Accretion processes have been extensively studied during recent decades and several types of accretion models have been proposed to explain certain observational features of some astrophysical objects that display this behavior. Most of the models assume that the accretion process can be described as a one-component gaseous fluid. However, there are strong observational and theoretical arguments which imply that at least some of the accreting systems are clumpy so that they consist of cool clumps embedded in a much hotter and more tenuous gaseous fluid. For example, observational evidence shows that the broad-line region (BLR) of active galactic nuclei (AGNs) has a clumpy structure (Rees 1987; Krolik & Begelman 1988; Nenkova et al. 2002; Risaliti et al. 2011; Torricelli-Ciamponi et al. 2014). The broad emission lines in the spectrum of AGNs are attributed to an assembly of clouds which are moving through a hot intercloud medium. The basic properties of clouds are estimated according to photoionization models. These models predict that the typical size of these clouds is $10^{12}\pm1$ cm and their number density is $10^{10}\pm1$ cm$^{-3}$ (e.g., Rees 1987; Krause et al. 2012). Orbital motion of clouds in the BLR is a rich source of information for estimating the mass of the central black hole (e.g., Netzer & Marziani 2010). One can ignore collisions between
the clumps and investigate the orbit of an individual clump in the presence of a central gravitational force and possible radiation field like a two-body classical problem (e.g., Netzer & Marziani 2010; Krause et al. 2011, 2012; Plewa et al. 2013; Khajenabi 2015). Although there are theoretical concerns about the stability of the clumps, it is generally believed that magnetic fields provide a confinement mechanism (e.g., Rees 1987).

Another approach for studying dynamics of the clumps embedded in a hot medium is based on analyzing the collisionless Boltzmann equation as was done by Wang et al. (2012, hereafter WCL). They described the gaseous ambient medium using the classical similarity solutions of Advection-Dominated Accretion Flows (ADAFs) presented by Narayan & Yi (1994) for non-magnetized systems. Although collisions between the clumps have been ignored for simplicity, their interactions with the surrounding gaseous medium were included through a drag force as a function of the relative velocity of the clumps and the gas. In the strong-coupling limit, it was shown that the root mean square radial velocity of the clumps is much larger than radial velocity of the gas flow. Their analysis has been extended to the magnetized case by Khajenabi et al. (2014) where the authors considered a purely toroidal magnetic field geometry for the gaseous component. They found that when magnetic pressure is less than the gas pressure, the averaged radial velocity of clumps decreases at the inner regions of the system whereas it increases at the outer parts, though this enhancement is not very significant unless the system becomes magnetically dominated.

In this work, we extend the analysis by Khajenabi et al. (2014) to include all three components of the magnetic field in the gaseous component. Since properties of the gas flow are significantly modified in the presence of a global magnetic field (Zhang & Dai 2008), the drag force varies depending on the strength of the magnetic field and the assumed magnetic geometry which will eventually lead to a considerable modification in the velocity dispersion of clumps. In the next section, we present our basic assumptions and the equations. In Section 3, a parameter study on the dynamics of the clumps is presented. We conclude with a summary of the results in Section 4.

2 GENERAL FORMULATION

Our analysis for describing an ensemble of clumps is based on the WCL approach which implements the collisionless Boltzmann equation in cylindrical coordinates \((r, \phi, z)\) including the components of the drag force. If we assume the distribution function of clumps is represented by \(F\), then the Boltzmann equation is written as

\[
\frac{\partial F}{\partial t} + \frac{\partial F}{\partial r} v_r + \frac{\partial F}{\partial \phi} v_\phi + \frac{\partial F}{\partial z} v_z + \dot{v}_r \frac{\partial F}{\partial v_r} + \dot{v}_\phi \frac{\partial F}{\partial v_\phi} + \dot{v}_z \frac{\partial F}{\partial v_z} + F\left(\frac{\dot{v}_r}{\partial v_r} + \frac{\dot{v}_\phi}{\partial v_\phi} + \frac{\dot{v}_z}{\partial v_z}\right) = 0, \tag{1}
\]

where \(\dot{r} = v_r, \dot{\phi} = v_\phi/r\) and \(\dot{z} = v_z\). The central object with mass \(M\) is at the origin and the gravitational potential becomes \(\Phi = GM/(r^2 + z^2)^{1/2}\). The components of the drag force are \(F_r = f_r (v_r - V_r)^2\) and \(F_\phi = f_\phi (v_\phi - V_\phi)^2\) where \(f_r\) and \(f_\phi\) are constants of order unity. Here, the radial and rotational velocities of the ADAF that has clumps moving within it are denoted by \(V_r\) and \(V_\phi\) respectively. Thus, dynamical properties of the background medium affect dynamics of the clumps through these components of velocity. Thus, the Boltzmann equation becomes (WCL)

\[
\frac{\partial F}{\partial t} + v_r \frac{\partial F}{\partial r} + v_z \frac{\partial F}{\partial z} + \left(\frac{v_\phi^2}{r} - \frac{\partial \Phi}{\partial r} + F_r\right) \frac{\partial F}{\partial v_r} + \left(F_\phi - \frac{v_r v_\phi}{r}\right) \frac{\partial F}{\partial v_\phi} - \frac{\partial \Phi}{\partial z} \frac{\partial F}{\partial v_z} + 2F \left[F_\phi (v_\phi - V_\phi) + F_r (v_r - V_r)\right] = 0. \tag{2}
\]

It is very difficult to solve this equation analytically in a general case unless we apply further simplifying assumptions. We assume that the clumps are strongly coupled with the background gaseous

\[\text{(2)}\]
medium which implies the mean radial and rotational velocities of the clumps are equal to the radial and the rotational velocity of the ADAF. Under these simplifying assumptions, it is possible to analytically obtain the root mean square radial velocity of the clumps \( \langle v_r^2 \rangle^{1/2} \) (WCL)

\[
\langle v_r^2 \rangle = c^2 \left\{ \frac{1}{2} \left[ \alpha^2 c_1^2 \Gamma_r, \Lambda_q^2 (\Gamma_r, r) + (1 - c_2^2) \Lambda_q^2 (\Gamma_r, r) \right] - \frac{\alpha c_1 c_2}{2 \Gamma_\phi} \Lambda_q^2 (\Gamma_r, r) \right\} + \frac{V_{\text{out}}^2}{c_r^2} e^{-\Gamma_r r_{\text{out}}} \times r \frac{1}{2} \right\} 
\]

where \( \Gamma_r = f_r R_{\text{Sch}} \) and \( \Gamma_\phi = f_\phi R_{\text{Sch}} \) are the coefficients of the drag force. Function \( \Lambda_q \) is introduced by WCL as \( \Lambda_q = \int_{r_{\text{out}}}^{r_\infty} x^{-q} \exp (-\Gamma_{r,x}) \, dx \). The outer boundary condition is at \( r = r_{\text{out}} \), so that \( \langle v_r^2 \rangle = V_{\text{out}}^2 \). Moreover, properties of the ADAF are described using a set of radially self-similar solutions where \( c_1 \) and \( c_2 \) are coefficients of the radial and rotational velocities of the ADAF. In WCL, the standard nonmagnetized ADAF solutions (Narayan & Yi 1994) have been used for their analysis. Then, Khajenabi et al. (2014) extended the analysis by including the purely toroidal component of the magnetic field using similarity solutions of Akizuki & Fukue (2006). However, we extend previous studies by considering all three components of the magnetic field using similarity solutions of Zhang & Dai (2008). Magnetized self-similar solutions of Zhang & Dai (2008) are written as

\[
v_r (r) = -c_1 \alpha \sqrt{\frac{GM}{r}}, \quad v_\phi (r) = c_2 \sqrt{\frac{GM}{r}}, \quad c_s^2 (r) = c_3 \frac{GM}{r}, \quad c_{r,\phi,z}^2 (r) = \frac{B_{r,\phi,z}^2}{4\pi \rho} = 2 \beta_r c_3 \frac{GM}{r},
\]

where the coefficients \( \beta_r, \beta_\phi \) and \( \beta_z \) measure the ratio of the magnetic pressure in three directions with respect to the gas pressure, i.e. \( \beta_{r,\phi,z} = P_{\text{mag},r,\phi,z}/P_{\text{gas}} \). The coefficients \( c_1, c_2 \) and \( c_3 \) are obtained using a set of algebraic equations (Zhang & Dai 2008):

\[
-\frac{1}{2} c_2 \alpha^2 = c_2^2 - 1 - \left[ (s - 1) + \beta_z (s - 1) + \beta_\phi (s + 1) \right] c_3, \quad -\frac{1}{2} c_1 c_2 \alpha = -\frac{3}{2} \alpha (s + 1) c_2 c_3 + c_3 (s + 1) \sqrt{\beta_r \beta_\phi}, \quad c_2^2 = \frac{4}{9 f} \left( \frac{1}{\gamma - 1} + s - 1 \right) c_1.
\]

Here, \( \gamma \) and \( s \) are the adiabatic index of the gas and mass loss parameter, respectively. Also, \( f \) measures the degree to which the flow is advection dominated. Now, we can substitute the above magnetized self-similar solutions into Equation (3) to study dynamics of the clumps in the presence of a global magnetic field.

3 ANALYSIS

We now study the root mean square radial velocity of the clumps \( \langle v_r^2 \rangle^{1/2} \) as a function of the radial distance for different values of the input parameters using the main expression, Equation (3). In all figures, we assume the coefficient of viscosity is \( \alpha = 0.1 \) and the adiabatic index is \( \gamma = 1.4 \).
Fig. 1 The root mean square radial velocity $\langle v_r^2 \rangle$ of clumps versus the radial distance for a central object with one solar mass. Different values for the parameter $\beta_r$ are considered and each curve is labeled by the corresponding value of this parameter. The other input parameters are $\gamma = 1.4$, $s = -0.5$, $f = 0.9$, $\beta_z = 1$ and $\beta_\phi = 1$.

Moreover, the mass of the central object is fixed at one solar mass. The coefficients of the drag force are $\Gamma_r = 5 \times 10^{-2}$ and $\Gamma_\phi = 2.8 \times 10^{-3}$.

Figure 1 shows the root mean square radial velocity of the clumps as a function of the radial distance normalized by $R_{\text{Sch}}$ for different values of $\beta_r$ whereas the other magnetic parameters are fixed at $\beta_z = \beta_\phi = 1$. This figure indicates that the value of $\langle v_r^2 \rangle^{1/2}$ increases with $\beta_r$, although its variation is not very significant.

In Figure 2, we assume that the radial component of the magnetic field does not exist and the toroidal component is fixed, i.e. $\beta_r = 0$ and $\beta_\phi = 1$. We can then vary the parameter $\beta_z$ to study its effect on the radial dynamics of the clumps. Again, we see that the value of $\langle v_r^2 \rangle^{1/2}$ increases as the vertical component of the magnetic field becomes stronger, though its variation with $\beta_z$ is less significant at large values of $\beta_z$. Moreover, in the inner parts of this system, clumps are moving radially faster as the parameter $\beta_z$ increases.

Dependence of the root mean square radial velocity of the clumps on variations in the toroidal component of the magnetic field is more complicated as has already been explored by Khajenabi et al. (2014) for a purely toroidal configuration.

In Figure 3, we assume that $\beta_r = 0$ and $\beta_z = 1$, but different values of $\beta_\phi$ are considered. In comparison to the previous study (Khajenabi et al. 2014), here, the vertical component of the magnetic field is also considered. The value of the root mean square radial velocity of the clumps decreases in the inner parts of the system with $\beta_\phi$ whereas the value of $\langle v_r^2 \rangle^{1/2}$ increases in the outer parts of the system.

Since the radial and rotational velocities of the gas component strongly depend on the amount of advected energy, obviously the clumps experience different values of drag force depending on the advection parameter $f$. We explore the dependence of $\langle v_r^2 \rangle^{1/2}$ on the parameter $f$ for different magnetic field configurations in Figure 4. For purely radial or toroidal magnetic field geometries, the value of $\langle v_r^2 \rangle^{1/2}$ strongly increases with the amount of advected energy. But for a purely vertical magnetic field, this trend changes to a reduction in the root mean square radial velocity of the clumps with increases in the parameter $\beta_z$. However, this reduction is not very significant. Thus, one may
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Fig. 2 Same as Fig. 1, but for different values of $\beta_z$. Here, we have $\beta_r = 0$ and $\beta_\phi = 1$.

Fig. 3 Same as Fig. 1, but for different values of $\beta_\phi$. Here, we have $\beta_r = 0$ and $\beta_z = 1$.

Fig. 4 The root mean square radial velocity $\langle v_r^2 \rangle^{1/2}$ of clumps versus the radial distance for a central object with one solar mass. Here, we explore dependence of the root mean square radial velocity on the amount of advected energy for different magnetic field configurations.

Fig. 5 Same as Fig. 4, but all three components of the magnetic field are considered.

conclude that the value of $\langle v_r^2 \rangle^{1/2}$ generally increases as the flow becomes more advective even in the presence of all three components of the magnetic field (Fig. 5).

In all of the figures, most of the curves cannot extend to the very inner parts. This is actually because of the limitation of similarity solutions. We describe the gaseous component using self-similar solutions and, as is well known, similarity solutions are only valid at regions far from the boundaries. In other words, these similarity solutions for the gas component are not valid at the
very inner parts. But at the intermediate regions, similarity solutions represent dynamics of the gas flow with very good accuracy. Since dynamics of clumps in our model are mainly determined due to interaction of clumps with the gas and similarity solutions for the gas are not valid at the inner boundary, we do not investigate the behavior of clumps in the inner parts based on our solutions.

4 CONCLUSIONS

We studied dynamics of an ensemble of cold clumps embedded in a hot magnetized accretion flow. Although the magnetic field has a vital role in stability and confinement of cold clouds, its role in the orbital motion of these clumps has not been studied. In our work, properties of the gas component are modified in the presence of a global magnetic field and so the drag force on each clump changes accordingly. Compared to the previous study by Khajenabi et al. (2014) who assumed the toroidal component of the magnetic field is dominant, we showed that both the radial and vertical components of the magnetic field also lead to some changes in the root mean square radial velocity of clumps. The value of $(v_{r}^2)^{1/2}$ increases with increasing strength of the radial and vertical components of the magnetic field. Moreover, velocity dispersion of clumps increases as the flow becomes more advective when all components of the magnetic field are considered. Although results of our analysis are not directly applicable to real systems because of limitations of this simplified model, the present study clearly demonstrates the importance of the magnetic field in the dynamics of clumps which cannot be ignored. The results of the paper are obtained within the conditions of strong coupling and simplification of the magnetic field. It is also possible to relax these simplifying assumptions, but then it would be very difficult to obtain analytical solutions, which is the goal of the present study.

As the clumps move toward the central black hole, they will gradually accumulate at the inner parts because of the tidal disruption of the black hole’s gravitational field. In fact, tidal disruption determines the inner edge of the clumpy disk. In the presence of a global magnetic field, we find that on average the clumps are moving radially faster in comparison to a similar configuration that does not have a magnetic field. WCL calculated the capture rate of clumps and found that it is directly proportional to the ratio $(v_{r}^2)^{1/2}/V_r$. Thus, presence of a global magnetic field increases the capture rate of clumps, but the level of enhancement depends on the detailed input parameters that we explored in this study. In other words, the rate that clumps are captured is faster when magnetic fields are considered.

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