Regular and black hole solutions in the Einstein–Skyrme theory with negative cosmological constant

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Abstract
We study spherically symmetric regular and black hole solutions in the Einstein–Skyrme theory with a negative cosmological constant. The Skyrme field configuration depends on the value of the cosmological constant in a similar manner to effectively varying the gravitational constant. We find the maximum value of the cosmological constant above which there exists no solution. The properties of the solutions are discussed in comparison with the asymptotically flat solutions. The stability is investigated in detail by solving the linearly perturbed equation numerically. We show that there exists a critical value of the cosmological constant above which the solution in the branch representing unstable configurations in the asymptotically flat spacetime turns to be linearly stable.

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1. Introduction

The Skyrme model is a unified theory of hadrons proposed by Skyrme [1]. The model consists of meson fields alone which are represented in terms of angular variables to be multi-valued. Associated with this nonlinearity, a topological soliton solution called a skyrmion arises and its topologically conserved charge is interpreted as the number of particle sources, baryon number $B$. Skyrme discussed the particle nature of a $B = 1$ spherically symmetric skyrmion by imposing the hedgehog ansatz on the pion fields. Witten showed that QCD in the large-$N_c$ limit reduces to an effective theory of mesons, and baryons emerge as solitons in this weakly coupled meson theory [2]. A detailed analysis for the property of the $B = 1$ skyrmion as a nucleon was performed in [3] upon quantization of the collective coordinate. Axially symmetric $B = 2$ skyrmions were found and quantized in [4, 5]. The ground state of the solution was shown to have the correct quantum numbers of the deuteron. Remarkably,
multi-skyrmions with $B > 2$ possess various discrete symmetries analogously to multi-BPS monopoles [6, 7].

It has been known that the Einstein–Skyrme (ES) system possesses regular and black hole solutions. The spherically symmetric black hole solution with Skyrme hair [8–11] and self-gravitating skyrmion [11] were investigated. It was shown that there exist two fundamental branches of the solutions and interestingly one of the branches represents stable configuration under linear perturbations [11, 12]. $B = 2$ Skyrme black hole and regular solutions with axisymmetry were constructed in [13]. The review of the black hole solutions with Skyrme hair is given in [14]. All of these solutions are, however, constructed in the asymptotically flat spacetime. Recently, in [15] we considered the Einstein–Skyrme system with a negative cosmological constant and found $B = 1$ asymptotically anti-de Sitter (AdS) black hole solutions. In this paper we develop the previous work of [15] and study $B = 1$ spherically symmetric regular and black hole solutions in the asymptotically AdS spacetime.

In the context of the Einstein–Yang–Mills (EYM) theory, it was shown that regular and black hole solutions are unstable for $\Lambda > 0$ [23–25], but there exist stable solutions for $\Lambda < 0$ [26–28]. We investigate the linear stability of our solutions and discuss in detail to see if the presence of the cosmological constant changes the stability properties of the solutions as in the EYM theory.

There has been an increasing interest in the AdS spacetime. Especially the AdS black hole is an interesting object from the holographic point of view in the form of AdS/CFT correspondence [16, 17]. Braneworld cosmology also indicates that there was a period when spacetime was AdS with a negative cosmological constant in the early universe (for example, see [19–22]). The solutions we obtain in this paper provide a semiclassical framework to study the interaction of a baryon and gravity or a primordial black hole with a negative cosmological constant.

2. The Einstein–Skyrme model

The Einstein–Skyrme system with a cosmological constant $\Lambda$ is defined by the action

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{16\pi G} (R - 2\Lambda) + \frac{F^2}{16} g^{\mu\nu} \text{tr}(L_\mu L_\nu) + \frac{1}{32e^2} g^{\mu\nu} g^{\rho\sigma} \text{tr}([L_\mu, L_\rho][L_\nu, L_\sigma]) \right\}$$

(1)

where $L_\mu = U^\dagger \partial_\mu U$ and $U$ is an $SU(2)$ chiral field. $F_\pi$ is the pion decay constant and $e$ is a dimensionless parameter.

We require that the spacetime recovers the AdS solution at infinity and thus parametrize the metric as

$$ds^2 = -e^{2\Phi(r)} C(r) dt^2 + C(r)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

(2)

where

$$C(r) = 1 - \frac{2Gm(r)}{r} - \frac{\Lambda r^2}{3}.$$  

(3)

The topology of the AdS spacetime is $S^1 \times R^3$ and hence the timelike curves are closed. This can be, however, unwound if we consider the covering spacetime with topology $R^4$.

The $B = 1$ skyrmion can be obtained by imposing the hedgehog ansatz on the chiral field

$$U(\vec{r}) = \cos f(r) + i\vec{n} \cdot \vec{r} \sin f(r).$$

(4)

Introducing the dimensionless variables

$$x = eF_\pi r, \quad \mu(x) = eF_\pi Gm(r), \quad \tilde{\Lambda} = \Lambda/e^2 F_\pi^2$$

(5)
with
\[ C(x) = 1 - \frac{2\mu(x)}{x} - \frac{\Lambda x^2}{3}, \] (6)

one obtains the skyrmion energy as
\[ E_S = 4\pi \frac{F_\pi}{e} \int \sqrt{1 - \left(\frac{C f'}{x^2} + \frac{2\sin^2 f}{x^2}\right)\sin^2 f + \left(2C f'^2 + \sin^2 f\right)\frac{1}{x^2}} e^2 x^2 \mathrm{d}x \] (7)
\[ = 4\pi \frac{F_\pi}{e} \int \left(1 - \frac{Cu f'^2 + \frac{1}{4x^2}v}{x^2}\right) e^2 \mathrm{d}x \] (8)
where we have defined \( u = x^2 + 8\sin^2 f \) and \( v = \sin^2 f(x^2 + 2\sin^2 f) \). The prime denotes the derivative with respect to \( x \). The covariant topological current is defined by
\[ B^\mu = -\frac{e^{\nu\rho\sigma}}{24\pi^2} \frac{1}{\sqrt{-g}} \text{tr}(U^{-1}\partial_\nu U U^{-1}\partial_\rho U U^{-1}\partial_\sigma U) \] (9)
whose zeroth component corresponds to the baryon number density
\[ B^0 = -\frac{1}{2\pi^2} e^{-\delta} \frac{f^2 \sin^2 f}{r^2}. \] (10)

We impose the boundary condition on the profile function as
\[ f(x) \to 0 \quad \text{as} \quad r \to \infty \] (11)
which ensures the total energy (8) is finite. Then the baryon number becomes
\[ B = \int \sqrt{-g} B^0 \mathrm{d}^3x = -\frac{2}{\pi} \int_{f_0}^0 \sin^2 f \mathrm{d}f = \frac{1}{2\pi}(2f_0 - \sin 2f_0). \] (12)

For regular solutions, \( f_0 = f(0) \) should take \( \pi \) in order for the baryon number to be one. For black hole solutions, \( f_0 = f(x_h) \) with event horizon \( x_h \) takes a value less than \( \pi \), which means that the solution possesses fractional baryonic charge. In this case \( f(x_h) \) is a shooting parameter determined numerically so as to satisfy the desired asymptotical behaviour in equation (11).

The Einstein equations with a cosmological constant \( \Lambda \) take the form
\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi GT_{\mu\nu} \] (13)
which reads
\[ G_{00} = 8\pi GT_{00} - \Lambda g_{00} \to 1 - C - C'x = \frac{\alpha}{4} \left(Cuf'^2 + \frac{2v}{x^2}\right) + \Lambda x^2 \] (14)
\[ G_{11} = 8\pi GT_{11} - \Lambda g_{11} \to -1 + C + \frac{(Cuf')'}{e^{2\delta}} + \sin^2 f = \frac{\alpha}{4} \left(Cuf'^2 - \frac{2v}{x^2}\right) - \Lambda x^2 \] (15)
where we have defined the coupling constant \( \alpha = 4\pi GF_\pi^2 \). Consequently, the following two equations are obtained for the gravitational fields
\[ \delta' = \frac{\alpha}{4x} u f'^2, \quad -(C uf')' + 1 = \frac{\alpha}{4} \left(Cuf'^2 + \frac{2v}{x^2}\right) + \Lambda x^2. \] (16)
Taking the variation of the static energy (8) with respect to the profile \( f(x) \), one can get the field equation for matter
\[ f'' = \frac{1}{e^2 Cu} \left[ -(e^\delta Cu)' f' + \left(4C f'^2 + 1 + \frac{4\sin^2 f}{x^2}\right) e^\delta \sin 2f \right]. \] (17)
Thus the coupled field equations to be solved are given by

\[ \delta' = \frac{\alpha}{4x} u f'^2 \]  

(18)

\[ \mu' = \frac{\alpha}{8} \left( C u f'^2 + \frac{2u}{x^2} \right) \]  

(19)

\[ f'' = \frac{1}{e^\delta Cu} \left[ -(e^\delta Cu)' f' + \left( 4C f'^2 + 1 + \frac{4\sin^2 f}{x^2} \right) e^\delta \sin 2f \right]. \]  

(20)

3. Boundary conditions

The boundary conditions for regular solutions are determined by expanding the functions around the origin \( x = 0 \) and comparing the coefficients in each order of \( x \) in the field equations (18)–(20). As a result, we find

\[ f = \pi + f_1 x + O(x^3) \]  

(21)

\[ \delta = \delta_0 + \frac{\alpha}{8} f_1^2 \left( 1 + 8f_1^2 \right) x^2 + O(x^3) \]  

(22)

\[ \mu = \frac{\alpha}{8} f_1^2 \left( 1 + 4f_1^2 \right) x^3 + O(x^4) \]  

(23)

where \( f_1 \) and \( \delta_0 \) are shooting parameters. \( f_1 \) is chosen so as to satisfy equation (11) and \( \delta_0 \) is chosen so as to recover the AdS spacetime asymptotically, that is, \( \delta(x) \to 0 \) as \( x \to \infty \).

Similarly, in order to determine the boundary conditions on the regular event horizon, let us expand the fields around the horizon \( x = x_h \):

\[ f = f_h + f_1 (x - x_h) + O((x - x_h)^2) \]  

(24)

\[ \delta = \delta_h + \delta_1 (x - x_h) + O((x - x_h)^2) \]  

(25)

\[ \mu = \frac{x_h}{2} - \frac{\bar{\Lambda} x_h^2}{6} + \mu_1 (x - x_h) + O((x - x_h)^2). \]  

(26)

Inserting them into the field equations (18)–(20) and comparing the coefficients in each order of \( (x - x_h) \), one obtains

\[ f_1 = \frac{x_h^2 + 4 \sin^2 f_h}{x_h(x_h^2 + 8 \sin^2 f_h)(1 - 2\mu_1 - \bar{\Lambda} x_h^2)} \sin 2f_h \]  

(27)

\[ \delta_1 = \frac{\alpha}{4x_h} \left( x_h^2 + 8 \sin^2 f_h \right) f_1^2 \]  

(28)

\[ \mu_1 = \frac{\alpha}{4} \left( 1 + \frac{2 \sin^2 f_h}{x_h^2} \right) \sin^2 f_h \]  

(29)

where \( f_h \) and \( \delta_h \) are shooting parameters with the desired asymptotic behaviour \( f(x), \delta(x) \to 0 \) as \( x \to \infty \).
4. Numerical results

The profile functions of regular solutions are shown in figure 1 for several values of $|\tilde{\Lambda}|$ with $\alpha = 0.1$ fixed. There are two branches of the solutions for each value of the cosmological constant as well as the coupling constant. We define the solution with the larger value of $f_1$ as an upper branch and the smaller value of $f_1$ as a lower branch. The skyrmion shrinks as $|\tilde{\Lambda}|$ becomes larger in the upper branch and expands slightly in the lower branch. Similar behaviour is observed for the solutions in the asymptotically flat spacetime when $\alpha$ is increased [14]. Thus, the variation of cosmological constant gives a similar effect on the skyrmion as the variation of the coupling constant does. We found the maximum value of the cosmological constant with $\alpha > 0$ above which there exists no solution. When $\alpha = 0$, regular solutions exist for all values of $|\tilde{\Lambda}| \leq 0$. The maximum value of $|\tilde{\Lambda}|$ is plotted as a function of $\alpha$ in figure 2. $|\tilde{\Lambda}|$ decreases monotonically as $\alpha$ increases and at $\alpha \approx 0.162$ only the asymptotically flat spacetime solution exists. For $\alpha > 0.162$, we found no solution with $\Lambda \leq 0$. Figure 3 shows the dependence of the ADM mass $M = \mu(\infty)$ on $\alpha$ and $|\tilde{\Lambda}|$. The ADM mass gives the total energy available in the spacetime and therefore, for regular solutions, it is equivalent to the skyrmion energy $\tilde{E}_S = \mu(\infty)$ where $\tilde{E}_S = e E_S / F_\pi$ in equation (8). As is to be expected, the mass increases as $\alpha$ and/or $|\tilde{\Lambda}|$ increase. It is observed that the presence of the cosmological constant affects the upper branch significantly more than the lower branch.

For black hole solutions, the profile function numerically computed for several values of $|\tilde{\Lambda}|$ is shown in figure 4. The horizon radius and the coupling constant are fixed with $x_h = 0.1$ and $\alpha = 0.02$. There are two branches of solutions for each value of the cosmological constant as was seen in the regular case. The dependence of profiles on the cosmological constant is also similar to the regular case. In the lower branch, however, the change in size is much smaller and is almost unrecognizable.

The black hole skyrmion mass–horizon radius relation is shown in figure 5. The skyrmion energy $\tilde{E}_S$ is related to the black hole skyrmion mass $M_{bh} = \mu(\infty)$ by

$$\tilde{E}_S = M_{bh} - \mu(x_h) = \mu(\infty) - \frac{x_h}{2},$$

(30)
Figure 2. The solid line shows the $\alpha$ dependence of the maximum value of $|\bar{\Lambda}|$ above which there exists no soliton solution.

Figure 3. The dependence of the ADM mass $M$ on the cosmological constant $|\bar{\Lambda}|$ for $\alpha = 0.06, 0.08$ and 0.1.

Since the entropy of the black hole is written by

$$S = \frac{\pi \bar{r}_h}{4\hbar G} = \frac{\pi^2}{\hbar c^2} \left( \frac{x_h^2}{\alpha} \right),$$

one can see that the upper and lower branches correspond to the high- and low-entropy branches respectively. The cosmological constant reduces the entropy of the black hole. The reduction of the entropy is also seen when the coupling constant increases as is inferred from equation (31).

Figure 6 shows the parameter $f_h$ as a function of $|\bar{\Lambda}|$ for $\alpha = 0.0, 0.02, 0.04$ with $x_h = 0.1$ fixed. The value of $f_h$ is directly related to the baryon number as can be seen from equation (12). Thus, in the upper branch, the baryon number becomes smaller as $|\bar{\Lambda}|$ becomes
larger, which represents the baryon more absorbed by the black hole. On the other hand, in the lower branch, the baryon number slightly increases as $|\tilde{\Lambda}|$ becomes larger. This result also shows that the cosmological constant gives a similar effect on the skyrmion as the coupling constant.

We found the maximum value of $|\tilde{\Lambda}|$ above which there exists no black hole solution for each value of the coupling constant. In figure 7, the maximum value of $|\tilde{\Lambda}|$ is shown as a function of $\alpha$. The maximum value decreases monotonically as $\alpha$ increases, and at $\alpha = 0.126$ it becomes zero. Thus at $\alpha = 0.126$, the asymptotically AdS solution does not exist and only the asymptotically flat solution exists. For $\alpha > 0.126$, we found no solution with $\tilde{\Lambda} \leq 0$. 

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**Figure 4.** The profile function $f$ as a function of the radial coordinate $x$ for $|\tilde{\Lambda}| = 0.0, 0.5, 1.0$ with $x_h = 0.1$ and $\alpha = 0.02$ fixed.

**Figure 5.** The horizon radius $x_h$ as a function of the black hole skyrmion mass $M_{bh}$ for $|\tilde{\Lambda}| = 0.0, 1.0, 2.0$ with $\alpha = 0.02$ fixed.
Let us denote that for both the regular and black hole cases, the lower- and upper-branch solutions coalesce at the maximum value of $|\tilde{\Lambda}|$.

5. Linear stability analysis

In this section we shall examine the linear stability of the solutions described in the previous section. Let us consider the time-dependent small fluctuation around the static classical solutions $f_0$, $\delta_0$, and $\mu_0$:

$$f(r, t) = f_0(r) + f_1(r, t)$$

$$\delta(r, t) = \delta_0(r) + \delta_1(r, t)$$
\[ \mu(r, t) = \mu_0(r) + \mu_1(r, t). \quad (34) \]

From the time-dependent Einstein–Skyrme action
\[ S = -\frac{\pi e^2 F^4}{2} \int \left[ \left( -\frac{1}{e^C} f^2 + C f'^2 \right) u + v \right] e^\delta \, dx, \quad (35) \]
one obtains the time-dependent field equation as
\[ (e^\delta Cu f')' + \frac{1}{2} \left( \frac{1}{e^C} f^2 - e^\delta C f'^2 \right) u_f - \frac{e^\delta v_f}{x^2} = \frac{1}{e^C} u_f \quad (36) \]
where we have defined \( u_f = \delta u / \delta f \) and \( v_f = \delta v / \delta f \). The dot denotes the time derivative.

The time-dependent Einstein equations are then given by
\[ G_{00} = 8\pi G T_{00} \to 1 - C - C' x = \frac{\alpha}{4} \left( \frac{1}{e^{2\delta C}} f^2 + C f'^2 \right) u + \frac{2v}{x^2} \quad (37) \]
\[ G_{11} = 8\pi G T_{11} \to -1 + C + \frac{(e^{2\delta C})'}{e^{2\delta C}} x = \frac{\alpha}{4} \left( \frac{1}{e^{2\delta C}} f^2 + C f'^2 \right) u - \frac{2v}{x^2} \quad (38) \]
which gives the following two equations
\[ \delta' = \frac{\alpha}{4x} \left( \frac{1}{e^{2\delta C}} f^2 + f'^2 \right) u \quad (39) \]
\[ -(C_x)' + 1 = \frac{\alpha}{2x^2} v + C \delta' x. \quad (40) \]

Substituting equations (32)–(34) into equations (39) and (40) gives the linearized equations
\[ \delta_i = \frac{\alpha}{2x} \left( 2u_f 0 f_1 + u_f 0 f_0 f_1 \right) \quad (41) \]
\[ -(e^{\delta_0} C_1 x)' = \frac{\alpha}{2x^2} e^{\delta_0} v_f 0 f_1 + e^{\delta_0} C_0 \delta_1 x. \quad (42) \]

Equation (41) and the classical field equation (36) which can be rewritten as
\[ \frac{e^{\delta_0} v_f 0 f_1}{x^2} = \frac{1}{2} \left( e^{\delta_0} C_0 u_f 0 f_0^2 \right)' - \frac{1}{2} \frac{e^{\delta_0} C_0 u_f 0 f_0^2}{x^2} \quad (43) \]
are inserted into equation (42) and as a result one gets
\[ -(e^{\delta_0} C_1 x)' = \frac{\alpha}{2} \left( e^{\delta_0} C_0 u_f 0 f_0 f_1 \right)' \quad (44) \]
This equation can be integrated immediately to obtain
\[ C_1 = -\frac{\alpha}{2x} C_0 u_f 0 f_0 f_1. \quad (45) \]

Similarly, let us linearize the field equation (36). Using equations (41), (43) and (45), one arrives at
\[ (e^{\delta_0} C_0 u_f 0 f_1)' - U_0 f_1 = \frac{1}{e^{\delta_0} C_0} u_f 0 f_1 \quad (46) \]
where
\[ U_0 = -(e^{\delta_0} C_0 u_f 0 f_0)' + \left( \frac{\alpha}{2x} e^{\delta_0} C_0 u_f 0 f_0^2 \right)' - \frac{\alpha}{2x^2} e^{\delta_0} C_0 u_f 0 f_0 f_0^2 + \frac{1}{2} \frac{e^{\delta_0} C_0 u_f 0 f_0^2}{x^2} + \frac{e^{\delta_0} v_f 0 f_1}{x^2}. \quad (47) \]
Figure 8. Regular: the dependence of the ground state energy $\omega_0^2$ on the cosmological constant $|\Lambda|$ and the coupling constant $\alpha$.

Setting $f_1 = \xi(x) e^{i\omega t}/\sqrt{u_0}$, we derive from equation (46)

$$-(e^{i\theta} C_0 \xi')' + \left[ \frac{1}{2\sqrt{u_0}} \left( e^{i\theta} C_0 \frac{u_0'}{\sqrt{u_0}} \right)' + \frac{1}{u_0} U_0 \right] \xi = \omega^2 \frac{1}{e^{i\theta} C_0} \xi. \quad (48)$$

Let us introduce the tortoise coordinate $x^*$ such that

$$\frac{dx^*}{dx} = \frac{1}{e^{i\theta} C_0} \quad (49)$$

with $-\infty < x^* < +\infty$. Equation (48) is then reduced to the Sturm–Liouville equation

$$-\frac{d^2 \xi}{dx^*^2} + \hat{U}_0 \xi = \omega^2 \xi \quad (50)$$

where

$$\hat{U}_0 = e^{i\theta} C_0 \left[ \frac{1}{2\sqrt{u_0}} \left( e^{i\theta} C_0 \frac{u_0'}{\sqrt{u_0}} \right)' + \frac{1}{u_0} U_0 \right]. \quad (51)$$

The classical solution is linearly stable if there exists no negative eigenvalue since imaginary $\omega$ represents exponentially growing modes. Unfortunately, the potential $\hat{U}_0$ has a complicated form and we are unable to discuss the stability analytically. Thus, we solve the wave equation (48) numerically under the boundary conditions that $\xi$ vanishes at the boundaries, which ensure the norm of the wavefunction to be finite. The ground state corresponds to the wavefunction with no node. The $n$th excited state corresponds to the wavefunction with $n$ nodes.

We show the eigenvalue of the ground state as a function of $|\Lambda|$ in figure 8 for regular solutions and in figure 9 for black hole solutions. For all values of $|\Lambda|$, upper-branch solutions have positive eigenvalues, and hence they are stable. Remarkably, the eigenvalues of lower-branch solutions increase as $|\Lambda|$ increases, and at some value, they cross zero to become positive. These figures show clearly how the cosmological constant tilts the eigenvalues to the positive direction. Thus, lower-branch solutions change their stability at the critical value of the cosmological constant. The stability of black hole solutions more strongly depends on the cosmological constant than that of regular solutions.
Regular and black hole solutions in the Einstein–Skyrme theory

The results we have obtained indicate that the presence of a negative cosmological constant stabilizes both regular and black hole solutions. This is consistent with the study of the EYM system where it was shown that the EYM solution is unstable for $\Lambda \geq 0$ [23–25], but the stable EYM solution exists for $\Lambda < 0$ [26, 28].

Another important feature for the ES solutions with $\Lambda < 0$ is that only discrete modes exist in both branches. This is because the potential behaves asymptotically as $x^2$, meaning all the eigenvalues are discretized analogous to the harmonic oscillator eigenvalues.

6. Conclusions

We have studied regular and black hole solutions in the Einstein–Skyrme system with a negative cosmological constant. There exist two fundamental branches of the solutions. For black holes, these correspond to the high- and low-entropy branches respectively. The increase in the absolute value of the cosmological constant $|\Lambda|$ gives similar effects on the skyrmion as effectively increasing the value of the gravitational constant. The skyrmion shrinks in the upper branch and expands in the lower branch as $|\Lambda|$ increases. Particularly, in the black hole case the baryon number is more absorbed by the black hole corresponding to the increase in $|\Lambda|$. There is a maximum value of $|\Lambda|$ above which no solution exists for either value of the coupling constant. We have observed that the critical values of $\alpha$ for black hole solutions to exist are 0.126 and 0.162 for the regular case.

The linear stability was examined in detail by solving the linear perturbed wave equation numerically. In the asymptotically flat case, it was shown that the upper branch is stable and the lower branch is unstable [12, 11]. In the AdS case, however, there exist stable solutions even in the lower branch depending on the value of $|\Lambda|$. We have shown by numerically solving the linearly perturbed equation that the presence of a negative cosmological constant lifts the eigenvalues from negative to positive values. The observation that the negative cosmological constant stabilizes the ES solutions is consistent with the results in the EYM theory where stable regular and black hole solutions exist only when $\Lambda < 0$ [26–28]. Let us give a brief comment on the catastrophe theory applied to the stability analysis of non-Abelian black holes in [29]. It seems that for asymptotically non-flat spacetimes, catastrophe
theory is not applicable to the stability analysis of solutions. Although concrete analysis should be performed for any statement on this matter to be confirmed, we suspect that the cosmological constant needs to be taken into account as an additional control parameter in the parameter space, which makes the dimension of the Whitney surface 3 with $S = S(M, B_{H}, \Lambda)$ in the notation of [29]. This extended catastrophe theory may be applicable for spacetimes with $\Lambda$.

The solutions exhibited in this paper provide a semiclassical framework to study the interaction of a baryon and gravity or a primordial black hole in the presence of a negative cosmological constant. If the universe had gone through the AdS phase in the early epoch as indicated in [19–22], those solutions may have been produced after the hadronization. Since our model predicts the baryon decay by primordial black holes, the Einstein–Skyrme theory should be useful as a simple framework to study such decay process.

The Skyrme model corresponds to QCD in the large $N$ limit and therefore it may be worth understanding the Skyrme model in the context of string and brane theories. Also, interesting applications are to find solutions with nonspherical event horizon [30] or with $B = 2$ axial symmetry [31] which were already discovered in the EYM theory.

Finally, the numerical method employed to integrate the differential equations is based on the fourth-order Runge–Kutta method with a grid size 0.05.

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