Weak Decays of Heavy Baryons in Light-Front Approach

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In this work, we perform a analysis of semi-leptonic and nonleptonic weak decays of heavy baryons: Λ_b, Ξ_b, Ω_b and Λ_c, Ξ_c, Ω_c. For nonleptonic decay modes, we study only the factorizable channels induced by the external W-emission. The two spectator quarks in baryonic transitions are treated as a diquark and form factors are calculated in the light-front approach. Using the results for form factors, we also calculate some corresponding semi-leptonic and nonleptonic decay widths. We find that our results are comparable with the available experimental data and other theoretical predictions. Decay branching fractions for many channels are found to reach the level $10^{-3} \sim 10^{-2}$, which are promising to be discovered in the future measurements at BESIII, LHCb and BelleII. The SU(3) symmetry in semi-leptonic decays is examined and sources of symmetry breaking are discussed.

I. INTRODUCTION

Quite recently, the LHCb collaboration announced the discovery of the doubly charmed baryon $\Xi^{++}_{cc}$ [1]. Undoubtedly this discovery will open a new door to study strong interactions in the presence of a pair of heavy quarks. Accordingly it has triggered great theoretical interests in studying doubly heavy baryons from different aspects [2–21].

Inspired by this discovery, we also expect a renaissance in the study of singly bottom or charm baryons. Particularly there are rapid progresses in the study of $\Lambda_c$ decays at BESIII [22–27] and some recent studies on $\Lambda_b$ and $\Lambda_c$ decays by LHCb can be found in Refs. [28–33]. It is anticipated that many more decay modes will be established in future. Thus an up-to-date theoretical analysis is highly demanded, and this work aims to do so.

Quark model is a very successful tool in classifying mesons and baryons. A heavy baryon is composed of one heavy quark $c/b$ and two light quarks. Light flavor SU(3) symmetry arranges the singly heavy baryons into the presentations $\mathbf{3} \otimes \mathbf{3} = \mathbf{6} \oplus \mathbf{3}$, as can be seen from Fig. 1. For charmed baryons, the irreducible representation $\bar{\mathbf{3}}$ is composed of $\Lambda_c^+$ and $\Xi_c^{++}$ while the sextet is composed of $\Sigma_c^{++,+,0}$, $\Xi_c^{++,0}$ and $\Omega_c^0$. They all have spin 1/2, but only 4 of them weakly decay predominantly: $\Lambda_c^+$ and $\Xi_c^{++,0}$ in the representation $\mathbf{3}$ and $\Omega_c^0$ in the representation $\mathbf{6}$. Others can decays into the lowest-lying states via strong or electromagnetic interactions. This is similar for bottomed baryons. In this work, we will focus on weak decays of singly heavy baryons and more explicitly we will consider only the following channels:
• charm sector:

\[
\begin{align*}
\Lambda_c^+(cud) & \rightarrow n(dud)/\Lambda(sud), \\
\Xi_c^+(cus) & \rightarrow \Sigma^0(dus)/\Lambda(dus)/\Xi_c^0(sus), \\
\Xi_c^0(cds) & \rightarrow \Sigma^-(dds)/\Xi^-(sds), \\
\Omega_c^0(css) & \rightarrow \Xi^-(dss);
\end{align*}
\]

• bottom sector:

\[
\begin{align*}
\Lambda_b^0(bud) & \rightarrow p(uud)/\Lambda_c^+(cud), \\
\Xi_b^0(bus) & \rightarrow \Sigma^+(uus)/\Xi_c^+(cus), \\
\Xi_b^-(bds) & \rightarrow \Sigma^0(uds)/\Lambda(uds)/\Xi_c^0(cds), \\
\Omega_b^-(bss) & \rightarrow \Xi^0(uss)/\Omega_c^0(css).
\end{align*}
\]

In the above, we have listed the quark contents of the baryons in the brackets and placed the quarks that participate in weak decay in the first place.

The light baryons in the final state are composed of 3 light quarks and belong to the baryon octet. Their wave functions, including the flavor and spin spaces, have the form [34]

\[
B_8 = \sqrt{\frac{1}{2}} (p_S\chi(M_S) + p_A\chi(M_A)).
\] (1)

Here \(p_{S(A)}\) stands for the mixed symmetric (antisymmetric) \(8\) in the SU(3) representation decomposition \(3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1\) in the flavor space, while \(\chi(M_{S(A)})\) stands for the mixed symmetric (antisymmetric) \(2\) in the SU(2) representation decomposition \(2 \otimes 2 \otimes 2 = 4 \oplus 2 \oplus 2\) in the spin space. Here the “mixed symmetric (antisymmetric)” means the state is symmetric (antisymmetric) under interchange of the first two quarks. The wave functions for baryons in the initial and final states are collected in the Appendix [A].

On the theoretical side, the singly heavy baryon decays have been investigated by various theoretical methods, and some of them can be found in Refs. [35–56]. In this work, we will adopt the light-front approach. This method has been widely used to study the properties of mesons [57–74]. Its application to baryons can be found in Refs. [75–78]. In the transition form factors, the two spectator quarks do not change and can be viewed as a diquark. In this diquark scheme, the two quarks are treated as a whole system, and thus its role is similar to that of the antiquark in the meson case, see Fig. 2. In the process like \(\Lambda_b \rightarrow \Lambda_c\), where the light quarks \(u\) and \(d\) are considered to form a scalar diquark, which is denoted by \([ud]\), while in the process like \(\Omega_b \rightarrow \Omega_c\), the light \(s\) quarks are believed to form an axial-vector diquark, which is denoted by \({ss}\).

Some recent works have been devoted to investigate the singly heavy baryon decays with the help of flavor SU(3) symmetry [79–82]. Based on the available data, the SU(3) analysis can give predictions on a great number of decay modes ranging from semi-leptonic decays to multi-body...
nonleptonic decays. However, as we know, in the case of c quark decay, SU(3) symmetry breaking effects are sizable and cannot be omitted. A quantitative study of SU(3) symmetry breaking effects will be conducted within the light-front approach.

The rest of the paper is arranged as follows. In Sec. II, we will present briefly the framework of light-front approach under the diquark picture, and the wave function overlapping factors are also given. Our results are shown in Sec. III, including the results for form factors, predictions on semi-leptonic and nonleptonic decay widths, and detailed discussions on the SU(3) symmetry and sources of symmetry breaking. A brief summary will be given in the last section.

II. THEORETICAL FRAMEWORK

In this section, we will briefly overview the theoretical framework for form factors: the light-front approach. More details can be found in Refs. [75] and [4]. It is necessary to point out that the physical form factor should be multiplied by a factor due to the overlap of wave functions in the initial and final states.
A. Form factors

The transition matrix elements are parameterized as

\[
\langle B'(P', S'_z)|V_\mu|B(P, S_z)\rangle = \tilde{u}(P', S'_z) \left[ \gamma_\mu f_1(q^2) + i\sigma_\mu \nu \frac{q_\nu}{M} f_2(q^2) + \frac{\eta_\mu}{M} f_3(q^2) \right] u(P, S_z),
\]

\[
\langle B'(P', S'_z)|A_\mu|B(P, S_z)\rangle = \tilde{u}(P', S'_z) \left[ \gamma_\mu g_1(q^2) + i\sigma_\mu \nu \frac{q_\nu}{M} g_2(q^2) + \frac{\eta_\mu}{M} g_3(q^2) \right] \gamma_5 u(P, S_z),
\]

where \( q = P - P' \), \( M \) denotes the mass of the parent baryon \( B \), and \( f_j, g_i \) are form factors.

In the light-front approach, hadron states are expanded in terms of quark states superposed with a wave function, where the momentum and other quantum numbers are considered simultaneously. Then the weak transition matrix element can be obtained as

\[
\langle B'(P', S'_z)|(V - A)\mu|B(P, S_z)\rangle = \int \{d^3p_2\} \frac{\phi'(x', k'_1)\phi(x, k_1)}{2\sqrt{p_1^+ p_1^{+*} (p_1 \cdot P + m_1 M_0) (p'_1 \cdot P' + m'_1 M'_0)}} \times \tilde{u}(P', S'_z) \Gamma'(p_1' + m_1') \gamma_\mu (1 - \gamma_5) (\bar{\phi}_1 + m_1) \Gamma u(P, S_z).
\]

From Eqs. (2) and (3), we can extract the explicit expressions of form factors:

\[
f_1(q^2) = \frac{1}{8P^+ P^{'+*}} \int \frac{d^3x d^3k_1}{2(2\pi)^3} \frac{\phi'(x', k'_1)\phi(x, k_1) [k_1 \cdot k_1' + (x_1 M_0 + m_1)(x_1' M_0' + m_1')]}{\sqrt{((m_1 + x_1 M_0)^2 + k_1^2) [(m'_1 + x_1' M_0')^2 + k'_1^2]}},
\]

\[
g_1(q^2) = \frac{1}{8P^+ P^{'+*}} \int \frac{d^3x d^3k_1}{2(2\pi)^3} \frac{\phi'(x', k'_1)\phi(x, k_1) [-k_1 \cdot k_1' + (x_1 M_0 + m_1)(x_1' M_0' + m_1')]}{\sqrt{((m_1 + x_1 M_0)^2 + k_1^2) [(m'_1 + x_1' M_0')^2 + k'_1^2]}},
\]

\[
f_2(q^2) = \frac{1}{8P^+ P^{'+*}} \int \frac{d^3x d^3k_1}{2(2\pi)^3} \frac{\phi'(x', k'_1)\phi(x, k_1) [(m_1 + x_1 M_0) k_1' \cdot q_1 + (m'_1 + x_1' M_0') k_1 \cdot q_1]}{\sqrt{((m_1 + x_1 M_0)^2 + k_1^2) [(m'_1 + x_1' M_0')^2 + k'_1^2]}}
\]

\[
g_2(q^2) = \frac{1}{8P^+ P^{'+*}} \int \frac{d^3x d^3k_1}{2(2\pi)^3} \frac{\phi'(x', k'_1)\phi(x, k_1) [-(m_1 + x_1 M_0) k_1' \cdot q_1 - (m'_1 + x_1' M_0') k_1 \cdot q_1]}{\sqrt{((m_1 + x_1 M_0)^2 + k_1^2) [(m'_1 + x_1' M_0')^2 + k'_1^2]}}
\]

\[
(4)
\]

for a scalar diquark involved in the initial and final baryons, or

\[
f_1(q^2) = \frac{1}{8P^+ P^{'+*}} \int \frac{d^3x d^3k_1}{2(2\pi)^3} \frac{\phi'(x', k'_1)\phi(x, k_1)}{6\sqrt{x_1 x'_1 (p_1 \cdot P + m_1 M_0) (p'_1 \cdot P' + m'_1 M'_0)}} \times \text{Tr}[(\bar{\rho} + M_0) \gamma^+(\bar{\rho}' + M'_0) \gamma_5 \gamma_\alpha (\bar{\rho}_1' + m_1') \gamma^+(\bar{\rho}_1 + m_1) \gamma_\beta \gamma_5 \gamma_\beta] \left( \frac{p_1^\alpha p_2^\beta}{m_2^2} - g^{\alpha \beta} \right),
\]

\[
g_1(q^2) = \frac{1}{8P^+ P^{'+*}} \int \frac{d^3x d^3k_1}{2(2\pi)^3} \frac{\phi'(x', k'_1)\phi(x, k_1)}{2\sqrt{x_1 x'_1 (p_1 \cdot P + m_1 M_0) (p'_1 \cdot P' + m'_1 M'_0)}} \times \text{Tr}[(\bar{\rho} + M_0) \gamma_5 \gamma_\alpha (\bar{\rho}' + M'_0) \gamma_5 \gamma_\alpha (\bar{\rho}_1' + m_1') \gamma^+ \gamma_5 (\bar{\rho}_1 + m_1) \gamma_5 \gamma_\beta] \left( \frac{p_1^\alpha p_2^\beta}{m_2^2} - g^{\alpha \beta} \right),
\]

\[
f_2(q^2) = \frac{1}{8P^+ P^{'+*} q_1^i} \frac{1}{2(2\pi)^3} \int \frac{d^3x d^3k_1}{2(2\pi)^3} \frac{\phi'(x', k'_1)\phi(x, k_1)}{2\sqrt{x_1 x'_1 (p_1 \cdot P + m_1 M_0) (p'_1 \cdot P' + m'_1 M'_0)}} \times \text{Tr}[(\bar{\rho} + M_0) \sigma^{i+} (\bar{\rho}' + M'_0) \gamma_5 \gamma_\alpha (\bar{\rho}_1' + m_1') \gamma^+ (\bar{\rho}_1 + m_1) \gamma_5 \gamma_\beta] \left( \frac{p_1^\alpha p_2^\beta}{m_2^2} - g^{\alpha \beta} \right),
\]

\[
(4)
\]
$$\frac{g_2(q^2)}{M} = \frac{1}{8P + P'} \frac{i q^2}{q^2} \int \frac{dx_2 d^2k_\perp}{2(2\pi)^3} \frac{\varphi'(x', k'_\perp)\varphi(x, k_\perp)}{2\sqrt{x_1 x'_1} (p_1 \cdot P + m_1 M_0)(p'_1 \cdot P' + m'_1 M'_0)}$$
\[\times \text{Tr}[(\vec{P} + M_0)\sigma^i \gamma_5 (\vec{P}' + M'_0)\gamma_5 \gamma_5 (p'_1 + m'_1)\gamma^i \gamma_5 (p_1 + m_1)\gamma_5 \gamma_5 g_{\alpha\beta}](\frac{p_1^2 p'_1^2}{m_2^2} - g_{\alpha\beta}) \] (5)

if an axial-vector diquark is involved.

**B. Spin and flavor wave functions**

In the last subsection, we have presented the explicit expressions of form factors. It should be noted that, the physical transition form factor should be multiplied by the corresponding overlapping factor:

$$f_1^{\text{physical}}(q^2) = \text{the overlapping factor} \times f_1^{\text{in Sec. II A}}(q^2).$$ (6)

From the discussions in the Appendix we can obtain these factors and the corresponding results are collected in Tab. I.

| TABLE I: The overlapping factors in the transitions | overlapping factors |
|-----------------------------------------------------|---------------------|
| $\Lambda_c^+ (cud) \rightarrow n(dud)/\Lambda(sud)$ | $-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}$ |
| $\Xi_c^+ (cus) \rightarrow \Sigma^0(dus)/\Lambda(dus)/\Xi^0(sus)$ | $\frac{1}{2}, \frac{1}{2\sqrt{3}}, -\frac{1}{\sqrt{2}}$ |
| $\Xi_b^0(cds) \rightarrow \Sigma^- (dds)/\Xi^- (sds)$ | $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}$ |
| $\Omega_b^0(css) \rightarrow \Xi^- (dss)$ | $-\frac{1}{\sqrt{3}}$ |
| $\Lambda_b^0 (bud) \rightarrow p(uud)/\Lambda_0^+ (cud)$ | $\frac{1}{\sqrt{2}}, 1$ |
| $\Xi_b^0 (bus) \rightarrow \Sigma^+ (usd)/\Xi_0^+ (cus)$ | $\frac{1}{\sqrt{2}}, 1$ |
| $\Xi_b^- (bds) \rightarrow \Sigma^0 (uds)/\Lambda (uds)/\Xi_b^0 (cds)$ | $\frac{1}{2}, -\frac{1}{2\sqrt{3}}, 1$ |
| $\Omega_b^- (bss) \rightarrow \Xi^0 (uss)/\Omega_b^0 (css)$ | $-\frac{1}{\sqrt{3}}, 1$ |

**III. NUMERICAL RESULTS AND DISCUSSIONS**

All inputs to calculate the form factors will be collected in the first subsection. What follows is the numerical results for form factors, semi-leptonic and nonleptonic processes. Some remarks will also be given. The last subsection is devoted to an SU(3) analysis for semi-leptonic processes.

**A. Inputs**

The quark masses used in the model are given as

$$m_u = m_d = 0.25\text{GeV}, \quad m_s = 0.37\text{GeV}, \quad m_c = 1.4\text{GeV}, \quad m_b = 4.8\text{GeV}.$$
These values are widely adopted in Refs. [66–74]. The diquark masses are chosen as
\[ m_{[ud]} = 0.50\text{GeV}, \quad m_{[us]} = m_{[ds]} = 0.60\text{GeV}, \]
\[ m_{[uu]} = m_{[ud]} = m_{[dd]} = 0.77\text{GeV}, \quad m_{[us]} = m_{[ds]} = 0.87\text{GeV}, \quad m_{[ss]} = 0.97\text{GeV}. \]
Here the square brackets (curly braces) denote a scalar (an axial-vector) diquark.

The shape parameters are given as
\[
\beta_{b[ud]} = 0.66\text{GeV}, \quad \beta_{b[us]} = \beta_{b[ds]} = 0.68\text{GeV}, \quad \beta_{b[ss]} = 0.78\text{GeV},
\]
\[
\beta_{c[ud]} = 0.56\text{GeV}, \quad \beta_{c[us]} = \beta_{c[ds]} = 0.58\text{GeV}, \quad \beta_{c[ss]} = 0.66\text{GeV},
\]
\[
\beta_{s[ud]} = 0.45\text{GeV}, \quad \beta_{s[us]} = \beta_{s[ds]} = 0.46\text{GeV},
\]
\[
\beta_{d[ud]} = 0.40\text{GeV}, \quad \beta_{d[us]} = \beta_{d[ds]} = 0.41\text{GeV}, \quad \beta_{d[ss]} = 0.44\text{GeV},
\]
\[
\beta_{u[ud]} = 0.40\text{GeV}, \quad \beta_{u[us]} = \beta_{u[ds]} = 0.41\text{GeV}, \quad \beta_{u[ss]} = 0.44\text{GeV}.
\]

Some remarks on the above parameters are given in order.

- The masses and lifetimes of the parent baryons are collected in Tab. II and the masses of the daughter baryons are shown in Tab. III [83].

TABLE II: Masses and lifetimes of parent baryons. \( \Lambda \) and \( \Xi \) are in the representation \( \overline{3} \) while \( \Omega \) are in the representation \( 6 \).

| baryon | \( \Lambda^+ \) | \( \Xi^+ \) | \( \Xi^0 \) | \( \Xi^0_c \) | \( \Lambda_b^0 \) | \( \Xi_b^0 \) | \( \Xi_b^0 \) | \( \Omega_b^- \) |
|--------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| mass/GeV | 2.286 | 2.468 | 2.471 | 2.695 | 5.620 | 5.792 | 5.795 | 6.046 |
| lifetime/fs | 200 | 442 | 112 | 69 | 1466 | 1464 | 1560 | 1570 |

TABLE III: Masses of daughter baryons. They form the baryon octet.

| baryon | \( p \) | \( n \) | \( \Lambda \) | \( \Sigma^+ \) | \( \Sigma^0 \) | \( \Sigma^- \) | \( \Xi^0 \) | \( \Xi^- \) |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|
| mass/GeV | 0.938 | 0.940 | 1.116 | 1.189 | 1.193 | 1.197 | 1.315 | 1.322 |

Fermi constant and CKM matrix elements are also taken from PDG [83]:
\[
G_F = 1.166 \times 10^{-5}\text{GeV}^{-2},
\]
\[
|V_{ud}| = 0.974, \quad |V_{us}| = 0.225, \quad |V_{ub}| = 0.00357,
\]
\[
|V_{cd}| = 0.225, \quad |V_{cs}| = 0.974, \quad |V_{cb}| = 0.0411. \quad (7)
\]
B. Form factors

Results for form factors are collected in Tab. [V] for charmed baryons and Tab. [VI] for bottomed baryons. The following expressions have been used to access the $q^2$ distribution:

$$F(q^2) = \frac{F(0)}{1 \mp \frac{q^2}{m_{\text{fit}}^2} + \delta \left(\frac{q^2}{m_{\text{fit}}^2}\right)^2},$$

where the $F(0)$ is the form factor at $q^2 = 0$. The $m_{\text{fit}}$ and $\delta$ are two parameters to be fitted from numerical results. For the form factor $g_2$, a plus sign is adopted in Eq. (8) otherwise the fitted parameter $m_{\text{fit}}$ becomes purely imaginary. The minus sign is adopted for all the other situations.

Some comments are given in order.

- Only the scalar diquark contributes to the $\Lambda_Q$ and $\Xi_Q$ decays and only the axial-vector diquark contributes to the $\Omega_Q$ decays, where $Q = c/b$.

- It should be noted that, in Tabs. [V] and [VI] the overlapping factors are not taken into account. The physical transition form factor should be multiplied by the corresponding overlapping factor, see Eq. (6).

- An advantage of the results in Tab. [V] is that, they can be directly be used to explore SU(3) symmetry and its breaking effects. In fact, if we take the approximations

$$m_d = m_s,$$

$$m_{ud} = m_{us} = m_{ds} = m_{ss},$$

$$\beta_{c[ud]} = \beta_{c[us]} = \beta_{c[ds]} = \beta_{c:ss},$$

$$\beta_{d[ud]} = \beta_{d[us]} = \beta_{d[ds]} = \beta_{d:ss} = \beta_{s[us]} = \beta_{s[ds]} = \beta_{s:ss}$$

and

$$m_{\Lambda^+_c} = m_{\Xi^+_c} = m_{\Xi^0_c} = m_{\Omega^0_c},$$

all the form factors will be the same. From the results in Tab. [V] we can see that the SU(3) symmetry is not severely broken.

In Tab. [IV] we compare our results with other theoretical predictions in Refs. [39, 84, 85].

Some comments are given as follows.

- In Tab. [IV] the physical form factors are shown, see Eq. (6).

- It can be seen that, our results are comparable to other predictions. However, there still exists an uncertainty about the sign of $g_2(0)$. The sign of $g_2(0)$ in this work is same as that derived by LCSR method in Ref. [85] but different from those obtained by other quark models. More careful analysis should be devoted to fixing this problem.
TABLE IV: A comparison with other results for the form factors at the maximum recoil $q^2 = 0$. The physical form factors are shown in “this work” with the help of Eq. (3).

|                  | $f_1(0)$ | $f_2(0)$ | $f_3(0)$ | $g_1(0)$ | $g_2(0)$ | $g_3(0)$ |
|------------------|----------|----------|----------|----------|----------|----------|
| $\Lambda_c \rightarrow n$ |          |          |          |          |          |          |
| this work        | 0.513    | -0.266   | -        | 0.443    | -0.034   | -        |
| Quark model [39] | 0.627    | -0.259   | 0.179    | 0.433    | 0.118    | -0.744   |
| Quark model [84] | 0.470    | -0.246   | 0.039    | 0.414    | 0.073    | -0.328   |
| $\Lambda_c \rightarrow \Lambda$ |          |          |          |          |          |          |
| this work        | 0.468    | -0.222   | -        | 0.407    | -0.035   | -        |
| Quark model [39] | 0.700    | -0.295   | 0.222    | 0.448    | 0.135    | -0.832   |
| Quark model [84] | 0.511    | -0.289   | -0.014   | 0.466    | 0.025    | -0.400   |
| LCSR [85]        | 0.517    | -0.123   | -        | 0.517    | -0.123   | -        |

- The form factors $f_3$ and $g_3$ are not obtained in this work because we have taken the $q^+ = 0$ frame, while another method adopted in Refs. [86, 87] may be applied to extract these form factors.

- Also note that, in Refs. [39, 84, 85], only few channels are investigated but this work aims to give a comprehensive investigation to the heavy baryon decays. Only in this way, SU(3) symmetry and sources of SU(3) symmetry breaking can be seen clearly.

C. Semi-leptonic results

The differential decay width for semi-leptonic process reads

$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_T}{dq^2},$$

with the polarized decay widths are given as

$$\frac{d\Gamma_L}{dq^2} = \frac{G_F^2 |V_{CKM}|^2}{(2\pi)^3} \frac{q^2 p}{24M^2} \left( |H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 \right),$$

$$\frac{d\Gamma_T}{dq^2} = \frac{G_F^2 |V_{CKM}|^2}{(2\pi)^3} \frac{q^2 p}{24M^2} \left( |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \right).$$

Here the $q^2$ is the lepton pair invariant mass, $p = \sqrt{Q_+ Q_-}/2M$, $Q_\pm = (M \pm M')^2 - q^2$, and $M$ ($M'$) is the mass of the parent (daughter) baryon.

The helicity amplitudes are related to the form factors through the following expressions:

$$H_{\frac{1}{2},0}^V = -i \sqrt{Q_-} \sqrt{q^2} \left( (M + M') f_1 - \frac{q^2}{M} f_2 \right),$$

$$H_{\frac{1}{2},1}^V = i \sqrt{2Q_-} \left( -f_1 + \frac{M + M'}{M} f_2 \right),$$
The negative helicity amplitudes are given as

\[ \epsilon_{1} \to n \to -1.71 \pm 0.21 \]
\[ \epsilon_{2} \to \Lambda \to -1.81 \pm 0.20 \]
\[ \epsilon_{3} \to \Sigma_{0} \to -1.58 \pm 0.25 \]
\[ \epsilon_{4} \to \Sigma_{+} \to -0.054 \pm 0.32 \]
\[ \epsilon_{5} \to \Sigma^{-} \to -0.054 \pm 0.32 \]
\[ \epsilon_{6} \to \Xi_{0} \to -0.065 \pm 0.28 \]
\[ \epsilon_{7} \to \Xi_{-} \to -0.065 \pm 0.32 \]
\[ \epsilon_{8} \to \Omega_{-} \to -0.065 \pm 0.28 \]
\[ \epsilon_{9} \to \Xi_{0} \to 0.002 \pm 0.40 \]
\[ \epsilon_{10} \to \Xi_{-} \to -0.182 \pm 0.14 \]

The negative helicity amplitudes are given as

\[ H_{A,0}^A = -i \sqrt{Q_+} \frac{(M - M')g_1 + q^2}{Mg_2}, \]
\[ H_{A,1}^A = i \sqrt{2Q_+} \left( -g_1 - \frac{M - M'}{Mg_2} \right). \]  

The negative helicity amplitudes are given as

\[ H_{A,0}^V = H_{A,0}^V \quad \text{and} \quad H_{A,1}^V = -H_{A,1}^V. \]

The helicity amplitudes for the left-handed current are obtained as

\[ H_{V,0}^V = H_{V,0}^V - H_{V,1}^V. \]

Numerical results are given in Tabs. VII and IX. Comparisons with some recent works and the experimental results can be found in Tabs. VIII and X.

D. Non-leptonic results

For nonleptonic decays, we are constrained to consider only the processes of a W boson emitting outward. The naive factorization assumption is employed. The decay width for the \( B \to B'P \) (\( P \) denotes a pseudoscalar meson) is given as

\[ \Gamma = \frac{p}{8\pi} \left( \left( M + M' \right)^2 - m^2 \right) |A|^2 + \frac{(M - M')^2 - m^2}{M^2} |B|^2, \]
TABLE VI: Same as Tab. [V] but for the bottomed baryon case.

| \( F \) | \( F(0) \) | \( m_{\text{fit}} \) | \( \delta \) | \( F' \) | \( F'(0) \) | \( m_{\text{fit}} \) | \( \delta \) |
|---|---|---|---|---|---|---|---|
| \( f_1^{1/2} \rightarrow p \) | 0.282 | 4.66 0.30 | | | \( f_2^{1/2} \rightarrow p \) | -0.084 | 3.94 0.37 |
| \( g_1^{1/2} \rightarrow p \) | 0.273 | 4.81 0.32 | | | \( g_2^{1/2} \rightarrow p \) | -0.012 | 3.67 0.37 |
| \( f_1^{3/2} \rightarrow \Lambda_c^+ \) | 0.670 | 5.62 0.23 | | | \( f_2^{3/2} \rightarrow \Lambda_c^+ \) | -0.132 | 4.67 0.32 |
| \( g_1^{3/2} \rightarrow \Lambda_c^+ \) | 0.656 | 5.73 0.24 | | | \( g_2^{3/2} \rightarrow \Lambda_c^+ \) | -0.012 | 3.43 0.28 |
| \( f_1^{1/2} \rightarrow \Sigma \) | 0.260 | 4.46 0.34 | | | \( f_2^{1/2} \rightarrow \Sigma \) | -0.086 | 3.84 0.40 |
| \( g_1^{1/2} \rightarrow \Sigma \) | 0.251 | 4.60 0.36 | | | \( g_2^{1/2} \rightarrow \Sigma \) | -0.012 | 3.56 0.41 |
| \( f_1^{1/2} \rightarrow \Xi \) | 0.654 | 5.42 0.27 | | | \( f_2^{1/2} \rightarrow \Xi \) | -0.143 | 4.59 0.34 |
| \( g_1^{1/2} \rightarrow \Xi \) | 0.640 | 5.53 0.28 | | | \( g_2^{1/2} \rightarrow \Xi \) | -0.012 | 4.33 0.30 |
| \( f_1^{1/2} \rightarrow \Sigma \) | 0.260 | 4.46 0.34 | | | \( f_2^{1/2} \rightarrow \Sigma \) | -0.086 | 3.84 0.40 |
| \( g_1^{1/2} \rightarrow \Sigma \) | 0.251 | 4.60 0.36 | | | \( g_2^{1/2} \rightarrow \Sigma \) | -0.012 | 3.56 0.41 |
| \( f_1^{1/2} \rightarrow \Xi \) | 0.260 | 4.46 0.34 | | | \( f_2^{1/2} \rightarrow \Xi \) | -0.086 | 3.84 0.40 |
| \( g_1^{1/2} \rightarrow \Xi \) | 0.251 | 4.60 0.36 | | | \( g_2^{1/2} \rightarrow \Xi \) | -0.012 | 3.56 0.41 |
| \( f_1^{1/2} \rightarrow \Sigma \) | 0.654 | 5.42 0.27 | | | \( f_2^{1/2} \rightarrow \Sigma \) | -0.143 | 4.59 0.34 |
| \( g_1^{1/2} \rightarrow \Sigma \) | 0.640 | 5.53 0.28 | | | \( g_2^{1/2} \rightarrow \Sigma \) | -0.015 | 4.33 0.30 |
| \( f_1^{1/2} \rightarrow \Xi \) | 0.169 | 3.30 0.64 | | | \( f_2^{1/2} \rightarrow \Xi \) | 0.193 | 3.45 0.49 |
| \( g_1^{1/2} \rightarrow \Xi \) | -0.033 | 4.38 0.20 | | | \( g_2^{1/2} \rightarrow \Xi \) | -0.041 | 4.32 0.65 |
| \( f_1^{1/2} \rightarrow \Xi \) | 0.566 | 3.92 0.49 | | | \( f_2^{1/2} \rightarrow \Xi \) | 0.531 | 4.08 0.41 |
| \( g_1^{1/2} \rightarrow \Xi \) | -0.170 | 4.80 0.23 | | | \( g_2^{1/2} \rightarrow \Xi \) | -0.031 | 9.02 5.05 |

TABLE VII: Semi-leptonic decays for charmed baryons.

| channels | \( \Gamma / \text{GeV} \) | \( B \) | \( \Gamma_L / \Gamma_T \) |
|---|---|---|---|
| \( \Lambda_c^+ \rightarrow ne^+\nu_e \) | 6.62 \times 10^{-15} | 2.01 \times 10^{-3} | 1.78 |
| \( \Lambda_c^+ \rightarrow \Lambda e^+\nu_e \) | 5.36 \times 10^{-14} | 1.63 \times 10^{-2} | 1.96 |
| \( \Xi_c^+ \rightarrow \Sigma^0 e^+\nu_e \) | 2.79 \times 10^{-15} | 1.87 \times 10^{-3} | 1.85 |
| \( \Xi_c^+ \rightarrow \Lambda e^+\nu_e \) | 1.22 \times 10^{-15} | 8.22 \times 10^{-4} | 1.79 |
| \( \Xi_c^+ \rightarrow \Xi^0 e^+\nu_e \) | 8.03 \times 10^{-14} | 5.39 \times 10^{-2} | 1.98 |
| \( \Xi^0_c \rightarrow \Sigma^- e^+\nu_e \) | 5.57 \times 10^{-15} | 9.47 \times 10^{-4} | 1.86 |
| \( \Xi^0_c \rightarrow \Xi^- e^+\nu_e \) | 7.91 \times 10^{-14} | 1.35 \times 10^{-2} | 1.98 |
| \( \Omega^0_c \rightarrow \Xi^- e^+\nu_e \) | 2.08 \times 10^{-15} | 2.18 \times 10^{-4} | 7.94 |

where \( p \) is the magnitude of the three-momentum of the daughter baryon \( B' \) in the rest frame of the parent baryon \( B \). \( M \) (\( M' \)) is the mass of the parent (daughter) baryon. For \( B \rightarrow B'V(A) \) (\( V \) denotes a vector meson while \( A \) denotes an axial-vector meson) decay, the decay width is

\[
\Gamma = \frac{p(E' + M')}{4\pi M} \left( 2(|S|^2 + |P_2|^2) + \frac{E^2}{m^2}(|S + D|^2 + |P_1|^2) \right),
\]

where \( E \) (\( E' \)) is the energy of the meson (daughter baryon) in the final state, and

\[ S = -A_1, \]
TABLE VIII: A comparison with some recent works for semi-leptonic charmed decays.

| channel                  | this work       | other theoretical predictions                                      | experiment |
|--------------------------|-----------------|--------------------------------------------------------------------|------------|
| $\Lambda_c^+ \rightarrow ne^+\nu_e$ | $2.01 \times 10^{-3}$ | $(2.7 \pm 0.3) \times 10^{-3}$ [80], $2.07 \times 10^{-3}$ [84], $(4.10 \pm 0.26) \times 10^{-3}$ [53] | -          |
| $\Lambda_c^+ \rightarrow \Lambda e^+\nu_e$ | $1.63 \times 10^{-2}$ | $2.72 \times 10^{-2}$ [88], $(3.80 \pm 0.22) \times 10^{-2}$ [52] | $(3.6 \pm 0.4) \times 10^{-2}$ |
| $\Xi_c^+ \rightarrow \Sigma^0 e^+\nu_e$ | $1.87 \times 10^{-3}$ | $(0.8 \pm 0.1) \times 10^{-3}$ [80] | -          |
| $\Xi_c^+ \rightarrow \Lambda e^+\nu_e$ | $8.22 \times 10^{-4}$ | $(2.5 \pm 0.4) \times 10^{-4}$ [80] | -          |
| $\Xi_c^0 \rightarrow \Xi^- e^+\nu_e$ | $5.39 \times 10^{-2}$ | $(3.38 \pm 0.19) \times 10^{-2}$ [81], $(3.0 \pm 0.5) \times 10^{-2}$ [80] | -          |
| $\Xi_c^0 \rightarrow \Lambda e^+\nu_e$ | $9.47 \times 10^{-4}$ | $(60 \pm 8) \times 10^{-4}$ [80] | -          |
| $\Xi_c^0 \rightarrow \Xi^- e^+\nu_e$ | $1.35 \times 10^{-2}$ | $(4.87 \pm 1.74) \times 10^{-2}$ [81], $(11.9 \pm 1.6) \times 10^{-2}$ [80] | -          |

TABLE IX: Semi-leptonic decays for bottomed baryons.

| channels     | $\frac{\Gamma}{GeV}$ | $B$ | $\frac{\Gamma}{\Gamma_T}$ |
|--------------|-----------------------|-----|---------------------------|
| $\Lambda_b^0 \rightarrow p e^- \bar{\nu}_e$ | $1.41 \times 10^{-16}$ | $3.14 \times 10^{-4}$ | 1.25 |
| $\Lambda_b^0 \rightarrow \Lambda^+ e^- \bar{\nu}_e$ | $3.96 \times 10^{-14}$ | $8.83 \times 10^{-2}$ | 1.71 |
| $\Xi_b^0 \rightarrow \Sigma^+ e^- \bar{\nu}_e$ | $1.27 \times 10^{-16}$ | $2.83 \times 10^{-4}$ | 1.27 |
| $\Xi_b^0 \rightarrow \Xi^+ e^- \bar{\nu}_e$ | $3.97 \times 10^{-14}$ | $8.83 \times 10^{-2}$ | 1.70 |
| $\Xi_b^- \rightarrow \Sigma^0 e^- \bar{\nu}_e$ | $6.37 \times 10^{-17}$ | $1.51 \times 10^{-4}$ | 1.27 |
| $\Xi_b^- \rightarrow \Xi^0 e^- \bar{\nu}_e$ | $2.29 \times 10^{-17}$ | $5.42 \times 10^{-5}$ | 1.25 |
| $\Omega_b^- \rightarrow \Xi^- e^- \bar{\nu}_e$ | $3.97 \times 10^{-14}$ | $9.42 \times 10^{-2}$ | 1.70 |
| $\Omega_b^- \rightarrow \Omega^0 e^- \bar{\nu}_e$ | $1.18 \times 10^{-17}$ | $2.82 \times 10^{-5}$ | 1.72 |

\[
P_1 = -\frac{p}{E} \left( \frac{M + M'}{E' + M'} B_1 + B_2 \right),
\]
\[
P_2 = \frac{p}{E' + M'} B_1,
\]
\[
D = -\frac{p^2}{E(E' + M')}(A_1 - A_2).
\]

$A$, $B$, $A_{1,2}$ and $B_{1,2}$ are given as

\[
A = -\lambda f_P(M - M') f_1(m^2),
\]
\[
B = -\lambda f_P(M + M') g_1(m^2),
\]
\[
A_1 = -\lambda f_V m \left[ g_1(m^2) + g_2(m^2) \frac{M - M'}{M} \right],
\]
\[
A_2 = -2\lambda f_V m g_2(m^2),
\]

TABLE X: A comparison with some recent works for semi-leptonic bottomed decays.

| channel     | this work       | other theoretical predictions                                      | experiment |
|-------------|-----------------|--------------------------------------------------------------------|------------|
| $\Lambda_b^0 \rightarrow p e^- \bar{\nu}_e$ | $3.14 \times 10^{-4}$ | $2.9 \times 10^{-4}$ [84], $(4.80 \pm 0.99) \times 10^{-4}$ [51] | $(4.1 \pm 1.0) \times 10^{-4}$ |
| $\Lambda_b^0 \rightarrow \Lambda^+ e^- \bar{\nu}_e$ | $8.83 \times 10^{-2}$ | $6.9 \times 10^{-2}$ [88], $(5.32 \pm 0.35) \times 10^{-2}$ [51] | $(6.2^{+1.4}_{-1.3}) \times 10^{-2}$ |
\[ B_1 = \lambda f_V m \left[ f_1(m^2) - f_2(m^2) \frac{M + M'}{M} \right], \]
\[ B_2 = 2\lambda f_V m f_2(m^2). \] (17)

Here \( \lambda = \frac{G_F}{\sqrt{2}} V_{CKM} \), the first CKM matrix element corresponds to the process of \( B \to B' \) and the second comes from the production of the meson. \( M(M') \) is the mass of the parent (daughter) baryon and \( m \) is the mass of the emitted meson.

For the decay mode with an axial-vector meson involved, \( f_V \) should be replaced by \( -f_A \) in the expressions of \( A_{1,2} \) and \( B_{1,2} \) in Eqs. (17).

Note that the P-wave meson \( a_1 \) emission case is included. The naive factorization can still work for these processes [92].

The masses of the mesons in the final states can be taken from Ref. [83]. The decay constants are adopted as [61, 74, 93]

\[ f_\pi = 130.4 \text{MeV}, \quad f_\rho = 216 \text{MeV}, \quad f_{a_1} = 238 \text{MeV}, \quad f_K = 160 \text{MeV}, \quad f_{K^*} = 210 \text{MeV}, \]
\[ f_D = 207.4 \text{MeV}, \quad f_{D^*} = 220 \text{MeV}, \quad f_{D_s} = 247.2 \text{MeV}, \quad f_{D_s^*} = 247.2 \text{MeV}. \] (18)

The numerical results are given in Tabs. XI, XIII and XIV. Comparisons with some recent works [54, 81] and the experimental results [83] can be found in Tab. XII and Tab. XV.

E. SU(3) analysis for semi-leptonic decays

From the overlapping factors above, we would expect the following relations

\[ \frac{2\Gamma(\Lambda_0^+ \to n e^+ \nu_e)}{|V_{cd}|^2} = \frac{3\Gamma(\Lambda_0^+ \to \Lambda e^+ \nu_e)}{|V_{cs}|^2} \]
\[ = \frac{4\Gamma(\Xi_c^0 \to \Sigma^0 e^+ \nu_e)}{|V_{cd}|^2} = \frac{12\Gamma(\Xi_c^+ \to \Lambda e^+ \nu_e)}{|V_{cd}|^2} = \frac{2\Gamma(\Xi_c^+ \to \Xi^0 e^+ \nu_e)}{|V_{cs}|^2} \]
\[ = \frac{2\Gamma(\Xi_c^0 \to \Sigma^- e^+ \nu_e)}{|V_{cd}|^2} = \frac{2\Gamma(\Xi_c^0 \to \Xi^- e^+ \nu_e)}{|V_{cs}|^2} \] (19)

for c-baryon sector and

\[ \Gamma(\Lambda_b^0 \to p e^- \bar{\nu}_e) = \Gamma(\Xi_b^0 \to \Sigma^+ e^- \bar{\nu}_e) = 2\Gamma(\Xi_b^- \to \Sigma^0 e^- \bar{\nu}_e) = 6\Gamma(\Xi_b^- \to \Lambda e^- \bar{\nu}_e), \]
\[ \Gamma(\Lambda_b^0 \to \Lambda_c^+ e^- \bar{\nu}_e) = \Gamma(\Xi_b^0 \to \Xi_c^+ e^- \bar{\nu}_e) = \Gamma(\Xi_b^- \to \Xi_c^0 e^- \bar{\nu}_e) \] (20)

for b-baryon sector, if the flavor SU(3) symmetry is respected. These relations for the charmed baryons are consistent with those in Refs. [79, 80], while the ones for the bottomed baryons, as far as we know, are first derived by this work.

In the following, we will investigate the SU(3) symmetry breaking effects. The corresponding results are collected in Tabs. XVI and XVII. Take \( \Lambda_c^+ \to n e^+ \nu_e \) and \( \Lambda_c^+ \to \Lambda e^+ \nu_e \) as examples. We can see that:
If we consider only the difference of daughter baryon mass but take all the other parameters as the same, we get the precise SU(3) symmetry prediction $\Gamma(\Lambda_c^+ \to n\nu_e)/(\frac{1}{2}|V_{cd}|^2) = \Gamma(\Lambda_c^+ \to \Lambda e^+\nu_e)/(\frac{1}{3}|V_{cs}|^2)$. This prediction is also obtained in Refs. [79,80].

If we consider only the difference of daughter baryon mass but take all the other parameters as the same, we get a ratio 0.538. It means that SU(3) symmetry is broken by about 50% between these two modes. The more accurate number is 35% (see Tab. XVI), when all the other relevant impacts are taken into account.
| channels         | $\Gamma$/GeV | $B$   | channels         | $\Gamma$/GeV | $B$   |
|------------------|--------------|-------|------------------|--------------|-------|
| $\Lambda_b^{-} \to p\pi^-$ | $3.99 \times 10^{-18}$ | $8.90 \times 10^{-6}$ | $\Lambda_b^{-} \to pp^{-}$ | $1.17 \times 10^{-17}$ | $1.17 \times 10^{-17}$ |
| $\Lambda_b^{-} \to p\bar{a}_1$ | $1.56 \times 10^{-17}$ | $3.48 \times 10^{-5}$ | $\Lambda_b^{-} \to pK^-$ | $3.22 \times 10^{-19}$ | $7.18 \times 10^{-7}$ |
| $\Lambda_b^{-} \to pK^{*-}$ | $6.02 \times 10^{-19}$ | $1.34 \times 10^{-6}$ | $\Lambda_b^{-} \to pD^-$ | $5.76 \times 10^{-19}$ | $1.28 \times 10^{-6}$ |
| $\Lambda_b^{-} \to pD^{*-}$ | $8.95 \times 10^{-19}$ | $1.99 \times 10^{-6}$ | $\Lambda_b^{-} \to pD_s^-$ | $1.54 \times 10^{-17}$ | $3.44 \times 10^{-5}$ |
| $\Lambda_b^{-} \to pD_s^{*-}$ | $2.19 \times 10^{-17}$ | $4.88 \times 10^{-5}$ | $\Lambda_b^0 \to \Lambda_c^+ \pi^-$ | $3.83 \times 10^{-15}$ | $8.53 \times 10^{-3}$ |
| $\Lambda_b^0 \to \Lambda_c^+ \pi^-$ | $3.83 \times 10^{-15}$ | $8.53 \times 10^{-3}$ | $\Lambda_b^0 \to \Lambda_c^+ \rho^-$ | $1.09 \times 10^{-14}$ | $2.44 \times 10^{-2}$ |
| $\Lambda_b^0 \to \Lambda_c^+ a_1^-$ | $1.40 \times 10^{-14}$ | $3.12 \times 10^{-2}$ | $\Lambda_b^0 \to \Lambda_c^+ K^-$ | $3.04 \times 10^{-16}$ | $6.78 \times 10^{-4}$ |
| $\Lambda_b^0 \to \Lambda_c^+ K^{*-}$ | $5.59 \times 10^{-16}$ | $1.24 \times 10^{-3}$ | $\Lambda_b^0 \to \Lambda_c^+ D^-$ | $4.26 \times 10^{-16}$ | $9.49 \times 10^{-4}$ |
| $\Lambda_b^0 \to \Lambda_c^+ D^{*-}$ | $6.90 \times 10^{-16}$ | $1.54 \times 10^{-3}$ | $\Lambda_b^0 \to \Lambda_c^+ D_s^-$ | $1.10 \times 10^{-14}$ | $2.46 \times 10^{-2}$ |
| $\Lambda_b^0 \to \Lambda_c^+ D_s^{*-}$ | $1.64 \times 10^{-14}$ | $3.65 \times 10^{-2}$ | $\Xi_b^- \to \Sigma^+ \pi^-$ | $3.55 \times 10^{-18}$ | $7.91 \times 10^{-6}$ |
| $\Xi_b^- \to \Sigma^+ a_1^-$ | $1.41 \times 10^{-17}$ | $3.13 \times 10^{-5}$ | $\Xi_b^- \to \Sigma^+ K^-$ | $2.87 \times 10^{-17}$ | $6.40 \times 10^{-7}$ |
| $\Xi_b^- \to \Sigma^+ K^{*-}$ | $5.39 \times 10^{-19}$ | $1.20 \times 10^{-6}$ | $\Xi_b^- \to \Sigma^+ D^-$ | $5.30 \times 10^{-19}$ | $1.18 \times 10^{-6}$ |
| $\Xi_b^- \to \Sigma^+ D^{*-}$ | $8.23 \times 10^{-19}$ | $1.83 \times 10^{-6}$ | $\Xi_b^- \to \Sigma^+ D_s^-$ | $1.42 \times 10^{-17}$ | $3.17 \times 10^{-5}$ |
| $\Xi_b^- \to \Sigma^+ D_s^{*-}$ | $2.02 \times 10^{-17}$ | $4.50 \times 10^{-5}$ | $\Xi_b^- \to \Xi^\circ \pi^-$ | $3.76 \times 10^{-15}$ | $8.37 \times 10^{-3}$ |
| $\Xi_b^- \to \Xi^\circ a_1^-$ | $1.38 \times 10^{-14}$ | $3.08 \times 10^{-2}$ | $\Xi_b^- \to \Xi^\circ K^-$ | $3.00 \times 10^{-16}$ | $6.67 \times 10^{-4}$ |
| $\Xi_b^- \to \Xi^\circ K^{*-}$ | $5.51 \times 10^{-16}$ | $1.23 \times 10^{-3}$ | $\Xi_b^- \to \Xi^\circ D^-$ | $4.26 \times 10^{-16}$ | $9.49 \times 10^{-4}$ |
| $\Xi_b^- \to \Xi^\circ D^{*-}$ | $6.90 \times 10^{-16}$ | $1.54 \times 10^{-3}$ | $\Xi_b^- \to \Xi^\circ D_s^-$ | $1.11 \times 10^{-14}$ | $2.46 \times 10^{-2}$ |
| $\Xi_b^- \to \Xi^\circ D_s^{*-}$ | $1.64 \times 10^{-14}$ | $3.65 \times 10^{-2}$ | $\Xi_b^- \to \Sigma^0 \pi^-$ | $1.78 \times 10^{-18}$ | $4.22 \times 10^{-6}$ |
| $\Xi_b^- \to \Sigma^0 a_1^-$ | $7.03 \times 10^{-18}$ | $1.67 \times 10^{-5}$ | $\Xi_b^- \to \Sigma^0 K^-$ | $1.44 \times 10^{-19}$ | $3.41 \times 10^{-7}$ |
| $\Xi_b^- \to \Sigma^0 K^{*-}$ | $2.70 \times 10^{-19}$ | $6.40 \times 10^{-7}$ | $\Xi_b^- \to \Sigma^0 D^-$ | $2.65 \times 10^{-19}$ | $6.29 \times 10^{-7}$ |
| $\Xi_b^- \to \Sigma^0 D^{*-}$ | $4.12 \times 10^{-19}$ | $9.76 \times 10^{-7}$ | $\Xi_b^- \to \Sigma^0 D_s^-$ | $7.13 \times 10^{-18}$ | $1.69 \times 10^{-5}$ |
| $\Xi_b^- \to \Sigma^0 D_s^{*-}$ | $1.01 \times 10^{-17}$ | $2.40 \times 10^{-5}$ | $\Xi_b^- \to \Lambda \pi^-$ | $6.03 \times 10^{-19}$ | $1.43 \times 10^{-6}$ |
| $\Xi_b^- \to \Lambda a_1^-$ | $2.39 \times 10^{-18}$ | $5.66 \times 10^{-6}$ | $\Xi_b^- \to \Lambda K^-$ | $4.88 \times 10^{-20}$ | $1.16 \times 10^{-7}$ |
| $\Xi_b^- \to \Lambda K^{*-}$ | $9.15 \times 10^{-20}$ | $2.17 \times 10^{-7}$ | $\Xi_b^- \to \Lambda D^-$ | $9.02 \times 10^{-20}$ | $2.14 \times 10^{-7}$ |
| $\Xi_b^- \to \Lambda D^{*-}$ | $1.40 \times 10^{-19}$ | $3.32 \times 10^{-7}$ | $\Xi_b^- \to \Lambda D_s^-$ | $2.43 \times 10^{-18}$ | $5.75 \times 10^{-6}$ |
| $\Xi_b^- \to \Lambda D_s^{*-}$ | $3.44 \times 10^{-18}$ | $8.16 \times 10^{-6}$ | $\Xi_b^- \to \Xi^c_0 \pi^-$ | $3.76 \times 10^{-15}$ | $8.93 \times 10^{-3}$ |
| $\Xi_b^- \to \Xi^c_0 a_1^-$ | $1.38 \times 10^{-14}$ | $3.28 \times 10^{-2}$ | $\Xi_b^- \to \Xi^c_0 K^-$ | $3.00 \times 10^{-16}$ | $7.11 \times 10^{-4}$ |
| $\Xi_b^- \to \Xi^c_0 K^{*-}$ | $5.51 \times 10^{-16}$ | $1.31 \times 10^{-3}$ | $\Xi_b^- \to \Xi^c_0 D^-$ | $4.27 \times 10^{-16}$ | $1.01 \times 10^{-3}$ |
| $\Xi_b^- \to \Xi^c_0 D^{*-}$ | $6.91 \times 10^{-16}$ | $1.64 \times 10^{-3}$ | $\Xi_b^- \to \Xi^c_0 D_s^-$ | $1.11 \times 10^{-14}$ | $2.62 \times 10^{-2}$ |
| $\Xi_b^- \to \Xi^c_0 D_s^{*-}$ | $1.64 \times 10^{-14}$ | $3.90 \times 10^{-2}$ |

**TABLE XIII:** Nonleptonic decays for $\Lambda_b$ and $\Xi_b$. 
We can see from Tabs. XIV and XV:

- The SU(3) symmetry breaking is sizable for c-baryon decays while it is small for the b-baryon decays. This can be understood due to a much smaller phase space in c-baryon decays, and thus the decay width significantly depends on the mass differences of the baryons in the initial and final states.

- SU(3) symmetry is broken more severely in the $c \to s$ processes than in the $c \to d$ processes because of the larger mass of $s$ quark than $u$ and $d$ quarks. The typical value of SU(3) symmetry breaking for $c \to s$ processes is 35% while that for $c \to d$ processes is 15%.

## F. Uncertainties

In this subsection, we will look more carefully at the dependence of our results on the model parameters. Taking $\Lambda_c^+ \to \Lambda$ transition as an example. Varying the model parameters $m_{ds} = m_{[ud]}$,
Some comments are given in order.

- All the form factors are not very sensitive to the diquark mass $m_{di}$.
- $g_2$ is one order of magnitude smaller than the other form factors, and it is sensitive to $\beta_i$ and $\beta_f$, while $f_1$, $f_2$ and $g_1$ are still not very sensitive to these shape parameters.
• It can be seen from Eq. [22] that, these decay widths are not sensitive to the model parameters. There exists at most about 10% deviation in these decay widths.

IV. CONCLUSION

In this work, we have calculated the transition form factors of the singly heavy baryons using the light-front approach under the diquark picture. These form factors are then used to predict semi-leptonic and nonleptonic decays of singly heavy baryons. Most of our results are comparable to the available experimental data and other theoretical results. We have also derived the overlapping factors that can be used to reproduce the SU(3) predictions on semi-leptonic decays. Using the calculated form factors, we pointed out that the SU(3) symmetry breaking is sizable in the charmed baryon decays while in the bottomed case, the SU(3) symmetry breaking is small. Most of the results in this work can be examined at experimental facilities at BEPCII, LHC or BELLEII.

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Appendix A: Wave functions in initial and final states

1. Wave functions in the standard flavor-spin basis

The wave functions in the flavor space can also be found in Ref. [94]. The wave functions of the singly heavy baryons in the initial states in the standard flavor-spin basis are given as follows.

For $B^6_{cqq}$ ($\Sigma_c^{++}, \Omega_c^0$), we have

$$|B^6_{cqq}, \uparrow \rangle = (qqc) \left( \frac{1}{\sqrt{6}}(\uparrow \downarrow \uparrow + \downarrow \uparrow \uparrow - 2 \uparrow \uparrow \downarrow) \right) ,$$  \hspace{1cm} (A1)

where $q = u, d, s$ for $\Sigma_c^{++}, \Omega_c^0$, respectively.

For $B^6_{cq_1q_2}$ ($\Sigma_c^+, \Xi_c^{'+,0}$), we have

$$|B^6_{cq_1q_2}, \uparrow \rangle = \left( \frac{1}{\sqrt{2}}(q_1q_2 + q_2q_1)c \right) \left( \frac{1}{\sqrt{6}}(\uparrow \downarrow \uparrow + \downarrow \uparrow \uparrow - 2 \uparrow \uparrow \downarrow) \right) ,$$  \hspace{1cm} (A2)

where $(q_1, q_2) = (u, d), (u, s), (d, s)$ for $\Sigma_c^+$ and $\Xi_c^{'+,0}$, respectively.

For $B^3_{cqq}$ ($\Lambda_c^+, \Xi_c^{'+,0}$), we have

$$|B^3_{cqq}, \uparrow \rangle = \left( \frac{1}{\sqrt{2}}(q_1q_2 - q_2q_1)c \right) \left( \frac{1}{\sqrt{2}}(\uparrow \downarrow \uparrow - \downarrow \uparrow \uparrow) \right) ,$$  \hspace{1cm} (A3)
where \((q_1, q_2) = (u, d), (u, s), (d, s)\) for \(\Lambda_c^+\) and \(\Xi_c^{+,0}\), respectively.

The wave functions of the baryon octet in the final states in the standard flavor-spin basis are given as follows.

For \(B_{q_1q_2q_3}(p, n, \Sigma^{+-}, \Xi^{0-})\), we have

\[
|B_{q_1q_2q_3}, \uparrow\rangle = \frac{1}{\sqrt{2}} \left\{ \left( \frac{1}{\sqrt{6}}(q_1q_2q_3 + q_2q_1q_3 - 2q_1q_2q_3) \right) \left( \frac{1}{\sqrt{6}}(\uparrow\downarrow\uparrow + \downarrow\uparrow\downarrow - 2\uparrow\downarrow\downarrow) \right) \\
+ \left( \frac{1}{\sqrt{2}}(q_1q_2q_3 - q_2q_1q_3) \right) \left( \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\downarrow) \right) \right\},
\]

(A4)

where \((q_1, q_2) = (u, d), (d, u), (u, s), (d, s), (s, u), (s, d)\) for \(p, n, \Sigma^{+-}, \Xi^{0-}, \Xi^{3-}\), respectively.

For \(\Sigma^0\) and \(\Lambda\), we have

\[
|\Sigma^0, \uparrow\rangle = \frac{1}{\sqrt{2}} \left\{ \left( \frac{1}{\sqrt{12}}(sdu + dsu + sud - 2uds - 2uds) \right) \left( \frac{1}{\sqrt{6}}(\uparrow\downarrow\uparrow + \downarrow\uparrow\downarrow - 2\uparrow\downarrow\downarrow) \right) \\
+ \left( \frac{1}{\sqrt{2}}(sdu - dsu - sud + usd) \right) \left( \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\downarrow) \right) \right\},
\]

(A5)

\[
|\Lambda, \uparrow\rangle = \frac{1}{\sqrt{2}} \left\{ \left( \frac{1}{\sqrt{12}}(sdu + dsu - sud - usd) \right) \left( \frac{1}{\sqrt{6}}(\uparrow\downarrow\uparrow + \downarrow\uparrow\downarrow - 2\uparrow\downarrow\downarrow) \right) \\
+ \left( \frac{1}{\sqrt{12}}(sdu - dsu - sud + usd - 2uds + 2uds) \right) \left( \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\downarrow) \right) \right\}.
\]

(A6)

2. Wave functions in the diquark basis

From the coupling of two angular momenta \(j_1 = 1\) and \(j_2 = \frac{1}{2}\), we know that

\[
|J = \frac{1}{2}, M = \frac{1}{2}\rangle = \sqrt{\frac{3}{5}}|m_1 = 1, m_2 = \frac{1}{2}\rangle - \sqrt{\frac{1}{5}}|m_1 = 0, m_2 = \frac{1}{2}\rangle.
\]

So, it is natural to define the baryon state with an axial-vector diquark as follows.

\[
|q_1(q_2q_3)_{A}, \uparrow\rangle \equiv \frac{\sqrt{2}}{3} q_1 \downarrow (q_2q_3)_{11} - \frac{\sqrt{1}}{3} q_1 \uparrow (q_2q_3)_{10},
\]

(A7)

where \((q_2q_3)_{11} = (q_2q_3)(\uparrow\uparrow)\) and \((q_2q_3)_{10} = (q_2q_3)\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)\right)\). Meanwhile the baryon state with a scalar diquark can be defined as

\[
|q_1(q_2q_3)_{S}, \uparrow\rangle \equiv q_1 \uparrow (q_2q_3)_{S},
\]

(A8)

where \((q_2q_3)_{S} = (q_2q_3)_{00} = (q_2q_3)\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\right)\).

One can prove the following equations

\[
q_1q_2q_3 \left( \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow\downarrow) \right) = -\frac{1}{2} |q_1(q_2q_3)_{S}, \uparrow\rangle - \frac{\sqrt{3}}{2} q_1(q_2q_3)_{A}, \uparrow\rangle,
\]

(A9)
where (the diquark basis can be derived as follows.

\[ q_1 q_2 q_3 \left( \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\downarrow - 2 \uparrow\uparrow\downarrow) \right) = -\frac{\sqrt{3}}{2} q_1 (q_2 q_3)_s \uparrow + \frac{1}{2} q_1 (q_2 q_3)_A \uparrow. \]  

(A10)

Equipped with above expressions, the baryon wave functions in the initial and final states in the diquark basis can be derived as follows.

For \( B_{cqq}^6 (\Sigma_c^{++},0 \text{ and } \Omega_c^0) \), we have

\[ B_{cqq}^6 = -c(qq)_A, \]  

(A11)

where \( q = u, d, s \) for \( \Sigma_c^{++} \) and \( \Omega_c^0 \), respectively.

For \( B_{cqq_1 q_2}^6 (\Sigma_c^+ \text{ and } \Xi_c^{'+},0 \text{ and } \Xi_c^{'+},0) \), we have

\[ B_{cqq_1 q_2}^6 = \frac{1}{\sqrt{2}} (c(q_1 q_2)_A - c(q_2 q_1)_A), \]  

(A12)

where \( (q_1, q_2) = (u, d), (u, s), (d, s) \) for \( \Sigma_c^+ \) and \( \Xi_c^{'+},0 \), respectively.

For \( B_{cqq_1 q_2}^3 (\Lambda_c^+ \text{ and } \Xi_c^+,0) \), we have

\[ B_{cqq_1 q_2}^3 = \frac{1}{\sqrt{2}} (c(q_1 q_2)_S - c(q_2 q_1)_S), \]  

(A13)

where \( (q_1, q_2) = (u, d), (u, s), (d, s) \) for \( \Lambda_c^+ \) and \( \Xi_c^+,0 \), respectively.

For \( B_{qq_1 q_2} (p, n, \Sigma_c^{+,0} \text{ and } \Xi_c^{0,-}) \), we have

\[ B_{qq_1 q_2} = \frac{1}{2\sqrt{3}} \left( -q_1 (q_1 q_2)_A - q_2 (q_1 q_2)_A + 2q_2 (q_1 q_1)_A + \sqrt{3} q_1 (q_1 q_2)_S - \sqrt{3} q_1 (q_2 q_1)_S \right), \]  

(A14)

where \( (q_1, q_2) = (u, d), (u, s), (d, s), (s, u), (s, d) \) for \( p, n, \Sigma_c^{+,0}, \Xi_c^{0,-} \), respectively.

For \( \Sigma_c^0 \) and \( \Lambda \), we have

\[ \Sigma_c^0 = \frac{1}{2\sqrt{6}} (2s(ud)_A + 2s(du)_A - d(us)_A - d(su)_A - u(ds)_A - u(sd)_A \]  

\[ + \sqrt{3} u(ds)_S - \sqrt{3} u(sd)_S + \sqrt{3} d(us)_S - \sqrt{3} d(su)_S), \]  

(A15)

\[ \Lambda = \frac{1}{2\sqrt{6}} (\sqrt{3} d(us)_A + \sqrt{3} d(su)_A - \sqrt{3} u(ds)_A - \sqrt{3} u(sd)_A \]  

\[ + 2s(ud)_S - 2s(du)_S + d(us)_S - d(su)_S - u(ds)_S + u(sd)_S). \]  

(A16)

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