Multiple D$p$-branes in Weak Background Fields

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Abstract

We find the terms in the nonabelian world-volume action of a system of many D$p$-branes which describe the leading coupling to all type II supergravity background fields. These results are found by T-dualizing earlier results for D0-branes, which in turn were determined from calculations of the M-theory description of the supercurrent of 11D supergravity. Our results are compatible with earlier results on the supersymmetric Born-Infeld action for a single D-brane in a general background and with Tseytlin’s symmetrized trace proposal for extending the abelian Born-Infeld action to a nonabelian theory. In the case $p = 3$, the operators we find on the D-brane world-volume are closely related to those which couple to supergravity fields in the $\text{AdS}_5 \times S^5$ IIB supergravity background. This gives an explicit construction, including normalization, of some of the operators used in the celebrated AdS/CFT correspondence for 3-branes. We also discuss the S-duality of the action in the case $p = 3$, finding that the S-duality of the action determines how certain operators in the $\mathcal{N} = 4$ 4D SYM theory transform under S-duality. These S-duality results give some new insight into the puzzle of the transverse 5-brane in M-theory.
1 Introduction

Since Polchinski’s realization in 1995 [1] that the Dirichlet $p$-branes of string theory carry Ramond-Ramond (R-R) charge and should be identified with R-R charged black brane solutions of supergravity, D-branes have become one of the most important tools in analyzing string theory and M-theory. D-branes have been used to construct and study black holes and supersymmetric field theories with many interesting properties. It is even believed that D-branes can be used to describe all of string theory and M-theory in certain regimes and in certain backgrounds, through the M(atrix) theory [2] and AdS/CFT proposals [3]. (For reviews and further references see [4, 5] for general background on D-branes, [6] for black holes from D-branes, [7] for field theories from D-branes, [8, 9, 10, 5] for M(atrix) theory and [11] for the AdS/CFT correspondence.)

To understand the behavior of D-branes in different contexts, it is of fundamental importance to have a description of their dynamics in a general string theory background. To date, our understanding of this description is somewhat incomplete. For a single D-brane it is known that the classical dynamics in a general string background should have a bosonic part described by the Born-Infeld action [12] combined with Wess-Zumino terms describing the coupling to R-R background fields [13] and to the curvature of the background metric [14, 15, 16]. The fermionic part of the theory can be included into a $\kappa$-symmetric action both in flat space [17, 18] and in a general type II supergravity background [19, 20, 21]. For a single Dirichlet $p$-brane this story is essentially complete: the degrees of freedom in the world-volume theory are the $9-p$ transverse scalar fields $X^i$, a world-volume $U(1)$ gauge field $A_\alpha$ and fermionic fields which complete a supersymmetry multiplet with 16 supercharges. In the low-energy limit the supersymmetric Born-Infeld action for a D$p$-brane reduces to the maximally supersymmetric $U(1)$ Yang-Mills action in $p+1$ dimensions.

For a system of multiple D$p$-branes, the world-volume action is much less well understood than for a single brane. In the low-energy limit, the action for a system of $N$ parallel D$p$-branes in a flat supergravity background reduces to the maximally supersymmetric $U(N)$ Yang-Mills theory in $p+1$ dimensions [22]. The extension of this action to a full supersymmetric or $\kappa$-symmetric nonabelian Born-Infeld action is not known. It was suggested by Tseytlin [23] that at least for the bosonic terms, such an extension can be found by using a symmetrized trace to resolve ordering ambiguities inherent in the higher order terms of the Born-Infeld action. Although this proposal has not yet been derived from any more fundamental principles, the symmetrized trace gives results compatible with other approaches such as direct string computation [24, 25] and M(atrix) theory calculations [26, 27], at least at low order in the field strength $F$ (see, however, [28, 29] for a related puzzle). A recent review of the state of knowledge regarding both the abelian and nonabelian Born-Infeld action is given in [30].

In this paper we find a new set of terms in the action describing a system of multiple D$p$-branes in a general type II supergravity background. The new terms which we identify are terms in the action which couple the world-volume fields linearly to the background
supergravity fields and all their higher derivatives. For a given derivative of a background field, we determine the lowest dimension operator in the world-volume theory which couples to the background field derivative at linear order. In a previous paper \cite{31} we described this set of couplings for a system of multiple D0-branes. These results were achieved by transforming earlier results for matrix theory in a weak background \cite{32} using the limiting procedure suggested by Seiberg and Sen \cite{33, 34} for relating the matrix description of M-theory to the N D0-brane action in IIA string theory. In this paper we T-dualize the results of \cite{31} to find the leading linear part of the nonabelian Born-Infeld action for a Dp-brane of arbitrary dimension. In addition, we show how T-duality may also be used starting with the known abelian D9-brane action to derive non-abelian terms in all lower dimensional Dp-brane actions, providing an alternate derivation of some of our results and giving a large set of additional terms.

The paper is organized as follows. In Section 2 we review our results for D0-branes and perform the T-duality transformation to find terms in the action for a D-brane of arbitrary dimension. In Section 3 we compare our results in the abelian case to the known \(\kappa\)-symmetric action for a single Dp-brane. We find that the two methods are consistent, though there is a subtlety in the comparison of fermion terms. We further show how the known abelian D9-brane action may be used to derive a large set of additional terms in the non-abelian actions for lower dimensional Dp-branes using T-duality. In Section 4 we discuss the S-duality of our action in the case \(p = 3\) and comment on a connection with the transverse 5-brane of matrix theory. Section 5 contains some comments on the connection of our results with the AdS/CFT correspondence, including a discussion of how the actions we derive may be used in the study of D-brane black holes. We conclude in section 6 with a brief discussion of the expected corrections to the actions we have derived as well as possible further developments.

While we were completing this paper we received a preprint by Myers \cite{35} which contains some closely related results. In particular, the treatment in \cite{35} is closely related to our discussion in section 3.2.

2 Linear brane-background couplings

In this section we determine the lowest dimension operators on a Dp-brane of arbitrary dimension which couple linearly to the derivatives of the type IIA background supergravity fields. We assume that the background fields represent small variations around a flat space background.

2.1 Review of D0-brane results

We begin by reviewing the method and results of our earlier paper \cite{31} describing D0-branes in weak background fields.

The main idea was to learn about D0-branes using matrix theory results by exploiting the relationship between the action for D0-branes in type IIA string theory and the matrix
theory action which describes the DLCQ of M-theory. Given the action for D0-branes in a specified background, Seiberg and Sen have given a very explicit prescription for how to derive the matrix model of M-theory in the related background. For the case of flat space, the matrix theory action arises as the lowest dimension part of the string theory action describing the dynamics of D0-branes in flat space. Similarly, the action for matrix theory in a general 11-dimensional supergravity background should arise from leading terms in the action for D0-branes in a general type IIA supergravity background. In previous work, we derived explicitly the action for matrix theory in the presence of arbitrary weak 11-dimensional supergravity fields, so our strategy in was to use these results and reverse the Seiberg-Sen prescription to deduce the leading terms in the D0-brane action in the presence of arbitrary type IIA supergravity fields.

In the general background matrix theory action, the 11-dimensional supergravity background fields and their derivatives couple to matrix theory expressions for the multipole moments of the various conserved 11-dimensional supergravity currents. Naturally, the background metric couples to the matrix theory expressions for the multipole moments of the membrane current, \( J^{JJK(k_1 \cdots k_n)} \), and magnetically to moments of the fivebrane current \( M^{IJKLMN(k_1 \cdots k_n)} \). The complete expressions for \( T, J, \) and \( M \) were derived in by comparing a matrix theory calculation of the long-distance interaction potential between two completely general systems with the analogous potential in supergravity. These expressions are reproduced for convenience in the appendix.

Turning to the D0-brane action, the most general form for the terms linear in bosonic background fields can be written as

\[
S_{\text{linear}} = S_{\text{flat}} + \int dt \sum_{n=0}^{\infty} \frac{1}{n!} \left[ \frac{1}{2} \left( \partial_{k_1} \cdots \partial_{k_n} h^{IJA}_\mu \right) I_h^{\mu(k_1 \cdots k_n)} + \left( \partial_{k_1} \cdots \partial_{k_n} \phi \right) I_\phi^{(k_1 \cdots k_n)} 
+ \left( \partial_{k_1} \cdots \partial_{k_n} C_\mu \right) I_0^{\mu(k_1 \cdots k_n)} + \left( \partial_{k_1} \cdots \partial_{k_n} B_\mu \right) I_s^{\mu(k_1 \cdots k_n)} + \left( \partial_{k_1} \cdots \partial_{k_n} \tilde{B}_\mu \right) I_\tilde{B}^{\mu(k_1 \cdots k_n)} + \left( \partial_{k_1} \cdots \partial_{k_n} C^{(3)}_{\mu \nu \lambda} \right) I_2^{\mu \nu \lambda(k_1 \cdots k_n)} + \left( \partial_{k_1} \cdots \partial_{k_n} \tilde{C}^{(3)}_{\mu \nu \lambda} \right) I_4^{\mu \nu \lambda \rho \sigma (k_1 \cdots k_n)} \right]
\]

Here, all fields and their derivatives are evaluated at the origin of 9-dimensional space, around which point we have Taylor expanded the action. \( S_{\text{flat}} \) is the flat space action for \( N \) D0-branes, whose leading terms are the dimensional reduction of D=10 SYM theory to 0+1 dimensions. The complete form of the higher order terms in the flat space action is not known, but the subleading terms vanish in the matrix theory limit. We assume that the background satisfies the source-free IIA supergravity equations of motion so that the dual fields \( \tilde{C}, \tilde{B}, \tilde{C}^{(3)} \) are well-defined 7-, 6- and 5-form fields given (at linear order) by

\[
d\tilde{C} = *dC, \quad d\tilde{B} = *dB, \quad d\tilde{C}^{(3)} = *dC^{(3)}.
\]

The sources \( I_{2n} \) are associated with (multipole moments of) Dirichlet 2\( n \)-brane currents, while the sources \( I_s \) and \( I_5 \) are associated with fundamental string and NS5-brane currents respectively.
Starting from the general form (1), we applied the Seiberg-Sen prescription to arrive at an expression for the general background matrix theory action in terms of the unknown currents $I$. Demanding equivalence with our previously derived explicit results for the matrix theory action, we were led to a set of constraints on the currents $I$ which could then be solved to determine the leading (lowest dimension) term in each current (as well as some subleading terms) in terms of the matrix theory expressions $T$, $J$, and $M$ listed in the appendix. The results for the lowest dimension operators appearing in the monopole (integrated) D0-brane currents are (with NS-NS currents in the left column and R-R currents in the right column)

\[
\begin{align*}
I^0_{h} &= T^{++} + T^{+-} + (I^0_{h})_8 + \mathcal{O}(X^{12}) \\
I^{0i} &= T^{+i} + T^{-i} + \mathcal{O}(X^{10}) \\
I^{ij}_{h} &= T^{ij} + (I^{ij}_{h})_8 + \mathcal{O}(X^{12}) \\
I_{\phi} &= T^{++} - \left(\frac{1}{3}T^{+-} + \frac{1}{3}T^{ij}\right) + (I_{\phi})_8 + \mathcal{O}(X^{12}) \\
I^0_{s} &= 3J^{+-} + \mathcal{O}(X^8) \\
I^{ij}_{s} &= 3J^{+ij} - 3J^{-ij} + \mathcal{O}(X^{10}) \\
I^{ijklmn}_{5} &= \mathcal{O}(X^8) \\
I^{0ijklm}_{5} &= \mathcal{O}(X^{10})
\end{align*}
\]

while the higher moments $I^{(k_1 \cdots k_n)}_i$ of terms on the LHS of each equation are given by the higher moments of terms on the RHS. The subleading terms mentioned explicitly in these operators, with dimensions denoted by a subscript, also satisfy the following relations, which may be derived from the agreement with matrix theory and the constraint of current conservation

\[
\begin{align*}
\frac{1}{2}I^0_{h} + \frac{1}{2}I^{ii}_{h} + \frac{3}{2}I_{\phi} &= T^{--} \\
(I^{ij}_{h})_8 &= (\partial_\phi T^{-i(j)} + \partial_i T^{+i(j)})_8
\end{align*}
\]

It is believed that the operators $I^{\mu\nu\lambda\sigma\rho}_5$ representing NS5-brane current vanish identically; this issue is discussed further in section 4. There are additional terms analogous to those in (1) describing the coupling to the fermionic background fields of the IIA theory; these can be determined from the results of [32] by applying the Seiberg-Sen limiting procedure.

It may seem surprising that a system of D0-branes will couple to all the supergravity fields $B_{\mu\nu}, C_{\mu\nu\lambda}^{(3)}, \ldots$ for which the usual sources are the fundamental string, the D2-brane, etc. In fact, however, this result follows from T-duality from the well-known results that fundamental strings, momentum and lower-dimensional D($p - 2k$)-branes can be described in terms of the field strength of the $U(N)$ gauge field in the world-volume of a Dp-brane. For example, the fact that in the system of D0-branes the operator $J^{+ij} \sim [X^i, X^j]$ carries D2-brane charge is T-dual to the statement that $F^{ij}$ flux on a Dp-brane describes a D($p-2$)-brane extended in the directions perpendicular to the $i, j$ directions [32]. In matrix theory this is familiar as the mechanism by which a regularized membrane is composed of $N$ pointlike...
partons \[30.\] Similarly, the statement that the operator \(M^{+ijkl} \sim X^i X^j X^k X^l\) carries D4-brane charge \[31.\] is T-dual to the statement that the instanton density \(F \wedge F\) on a Dp-brane corresponds to D\((p - 4)\)-brane density \[32.\]. The results of \[33.\] can be interpreted as giving explicit expressions for how the moments of the currents associated with all higher-dimensional branes can be expressed in terms of the matrix degrees of freedom of a system of D0-branes. The subset of these moments associated with conserved brane charges were constructed using the supersymmetry algebra in \[34.\].

We would like to emphasize that all the D0-brane operators determined explicitly by the comparison to matrix theory are defined in terms of a symmetrized trace over the non-abelian fields. However, it is not clear whether this prescription remains true for the higher dimension operators that we are unable to determine.

### 2.2 T-duality of supergravity backgrounds

We would now like to T-dualize our results for the D0-brane case to find terms in the non-abelian Born-Infeld and Wess-Zumino actions for arbitrary D-branes which are linear in the background fields. To do this, we must understand the action of T-duality both on the supergravity background fields and the fields living on the D-brane. We discuss T-duality on the background fields in this subsection and T-duality of the D-brane field theory in the following subsection.

The usual T-duality of string theory on a compact circle can be generalized to arbitrary backgrounds which are independent of the compactification direction \[35.\]. We are only interested at this point in the linear parts of the T-duality transformation. We begin, however by recalling the complete transformation rules for a set of background fields which are independent of a set of \(n\) toroidally compactified coordinates \(x^\alpha\) on which we perform the T-duality transform. We will use the nonlinear form of the T-duality transformation rules in Section 3 when we discuss the nonabelian Born-Infeld action. Throughout this section we use barred indices \(\bar{\alpha}, \bar{\beta}, \ldots\) for the compact spatial directions in which a T-duality is performed and indices \(\mu, \nu, \ldots\) for the remaining \(10 - p\) space-time dimensions (including 0).

It turns out that the complete T-duality rules are greatly simplified if we combine the background fields into the combinations in which they appear in the Born-Infeld and Wess-Zumino actions. Thus, we define fields

\[
G_{\mu\nu} = g_{\mu\nu} - B_{\mu\nu}
\]

\[
C^{(n)} = C^{(n)} - C^{(n-2)} \wedge B + \frac{1}{2!} C^{(n-4)} \wedge B \wedge B + \ldots
\]

We now suppose that the fields are independent of \(n\) coordinates \(x^\bar{\alpha}\) and apply the rules in \[36.\] in order to T-dualize in each of the directions. For the fields we have just defined, the
complete non-linear transformation rules are simply\[1\]

\[
\begin{align*}
G_{\mu\nu} & \rightarrow G_{\mu\nu} - G_{\mu i} G^i_{\bar{\alpha} \bar{\beta}} G_{\bar{\beta} \nu} \\
G_{\mu \bar{\alpha}} & \rightarrow G^{\bar{\alpha} \bar{\beta}} G_{\bar{\mu} \bar{\beta}} \\
G_{\bar{\alpha} \mu} & \rightarrow -G_{\beta \mu} G^{\bar{\beta} \bar{\alpha}} \\
G_{\bar{\alpha} \bar{\beta}} & \rightarrow G^{\bar{\alpha} \bar{\beta}} \\
\phi & \rightarrow \phi - \frac{1}{2} \ln(\det(G_{\bar{\alpha} \bar{\beta}})) \\
C^{(q)}_{\mu_1 ... \mu_{q-k} \bar{\alpha}_1 ... \bar{\alpha}_k} & \rightarrow \frac{1}{(n-k)!} \epsilon^{\bar{\alpha}_1 ... \bar{\alpha}_n} C^{(q-2k+p)}_{\mu_1 ... \mu_{q-k} \bar{\alpha}_{k+1} ... \bar{\alpha}_n}
\end{align*}
\]

Here, $G^{\bar{\alpha} \bar{\beta}}$ is the inverse of $G_{\bar{\alpha} \bar{\beta}}$, so we see that under T-duality, the matrix $G^{\bar{\alpha} \bar{\beta}}$ which we have assumed to be constant transforms into its inverse.

In this section we will only use the linearized forms of these T-duality relations. Writing $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, these read

\[
\begin{align*}
h_{\mu\nu} & \rightarrow h_{\mu\nu} \\
b_{\mu\nu} & \rightarrow b_{\mu\nu} \\
h_{\mu \bar{\alpha}} & \leftrightarrow -b_{\mu \bar{\alpha}} \\
h_{\bar{\alpha} \mu} & \leftrightarrow b_{\bar{\alpha} \mu} \\
h_{\bar{\alpha} \bar{\beta}} & \rightarrow -h_{\bar{\alpha} \bar{\beta}} \\
b_{\bar{\alpha} \bar{\beta}} & \rightarrow -b_{\bar{\alpha} \bar{\beta}} \\
\phi & \rightarrow \phi - \frac{1}{2} \sum_{\bar{\alpha}} h_{\bar{\alpha} \bar{\alpha}} \\
C^{(q)}_{\mu_1 ... \mu_{q-k} \bar{\alpha}_1 ... \bar{\alpha}_k} & \rightarrow \frac{1}{(n-k)!} \epsilon^{\bar{\alpha}_1 ... \bar{\alpha}_n} C^{(q-2k+p)}_{\mu_1 ... \mu_{q-k} \bar{\alpha}_{k+1} ... \bar{\alpha}_n}
\end{align*}
\]

### 2.3 T-duality of D-brane fields

We now turn to the action of T-duality for the fields living on the brane. The low-energy action for a Dp-brane living in flat space is simply the dimensional reduction of $D = 10$ SYM theory to $p + 1$ dimensions. If we retain the $D = 10$ notation for all of the Dp-brane theories and simply reinterpret the expressions appropriately, (i.e. $F_{ai} = D_a X^i$, $F_{ij} = i[X^i, X^j]$) then the expressions for the low-energy actions are identical in each case. In other words, the action of T-duality on expressions written in terms of $D = 10$ quantities is merely an appropriate reinterpretation of the notation. The only subtlety in this interpretation arises when we consider the transverse fields $X^i$ associated with a compact direction. In this situation, these fields are formally described by infinite matrices containing $N \times N$ blocks indexed by an integer associated with string winding number. The components of these \[1\]The transformation laws for the RR fields have additional terms in the case of massive type IIA supergravity.
matrices are T-dual to the momentum modes of the corresponding gauge field component \( A_i \) on the T-dual brane. The details of this correspondence are worked out in [1].

The matrix theory expressions for the moments of the \( D = 11 \) supergravity currents may all be rewritten naturally in \( D = 10 \) language. Throughout the remainder of the paper we use these expressions with the understanding that they should be interpreted in terms of the appropriate \((p + 1)\)-dimensional SYM variables in the \( p \)-brane action.

### 2.4 T-duality results: The Dp-brane action

Rewriting the known terms in the D0-brane action using the \( D = 10 \) expressions for the matrix theory supercurrent components and transforming using the linearized T-duality rules (3), it is a fairly straightforward task to find the terms of interest in all of the higher Dp-brane actions.

Starting from the D0-brane action (1) and applying the rules above, we find that the Dp-brane action may be written\(^2\)

\[
S_{\text{NS-NS}}^{\text{Dp}} = (\phi - \frac{1}{2} h_{\hat{a}\hat{b}}) \mathcal{I}_\phi + \frac{1}{2} h_{00} I_{h}^{00} + \frac{1}{2} h_{ij} I_{h}^{ij} - \frac{1}{2} h_{\hat{a}\hat{b}} I_{h}^{\hat{a}\hat{b}} + h_{0i} I_{h}^{0i} + 2h_{\hat{a}i} I_{h}^{\hat{a}i} - 2h_{0\hat{a}} I_{h}^{0\hat{a}}
\]

\[+ B_{ij} I_{s}^{ij} - B_{\hat{a}\hat{b}} I_{s}^{\hat{a}\hat{b}} + 2B_{0i} I_{s}^{0i} + B_{\hat{a}i} I_{s}^{\hat{a}i} + B_{\hat{a}\hat{b}} I_{s}^{\hat{a}\hat{b}} + \{\text{higher moment terms}\} + \{\text{nonlinear terms}\}\]

(7)

and

\[
S_{\text{RR}}^{\text{Dp}} = \sum \frac{(2n + p + q)!}{(n + p - q)! (q - n)! n!} \epsilon^{\hat{a}_1 \cdots \hat{a}_p} \mathcal{C}(q)_{i_1 \cdots i_n \hat{a}_{n+p-q+1} \cdots \hat{a}_p} I_{2n+p-q-1}^{i_1 \cdots i_n \hat{a}_1 \cdots \hat{a}_{n+p-q}}
\]

\[+ \sum \frac{(2n + p - q + 2)!}{(n + p - q + 1)! (q - n - 1)! n!} \epsilon^{\hat{a}_1 \cdots \hat{a}_p} \mathcal{C}(q)_{i_1 \cdots i_n \hat{a}_{n+p-q+2} \cdots \hat{a}_p} I_{2n+p-q-1}^{i_1 \cdots i_n \hat{a}_1 \cdots \hat{a}_{n+p-q+1}} + \{\text{higher moment terms}\} + \{\text{nonlinear terms}\}\]

(8)

where \( q, n \) are even/odd in the IIA/IIB theory.

The currents in these expressions are written in terms of the 0-brane currents \( I_x \) given in (3) which in turn are given in terms of the \( T \)'s, \( J \)'s and \( M \)'s listed in the appendix, and are to be interpreted as \( D = 10 \) expressions dimensionally reduced to \( p + 1 \) dimensions. The higher moment terms will be of exactly the same form as the terms listed above, with arbitrary derivatives of each background field coupling to the appropriate higher moment of the corresponding current. For example, the lowest dimension terms coupling to \( h_{ij} \) will be

\[
\frac{1}{2n!} \partial_{k_1} \cdots \partial_{k_n} h_{ij} T^{ij(k_1 \cdots k_n)}
\]

\[= \frac{1}{2n!} \partial_{k_1} \cdots \partial_{k_n} h_{ij} \text{STr} \left( (-D_a X^i D_a X^j - [X^i, X^k][X^k, X^j]) X^{k_1} \cdots X^{k_n} \right) + \{\text{fermions}\}\]

\(^2\)To write the action in the Einstein frame, we make the substitution \( h_{\mu\nu} \rightarrow h_{\mu\nu} + \frac{1}{2} \epsilon_{\mu\nu\phi} \)

\(^3\)Here, we use hatted indices for worldvolume spatial indices (i.e. excluding 0), while indices \( i, j, \ldots \) denote transverse spatial indices.
As a consequence of our derivation from the D0-brane action, the index 0 appears to be on a different footing than the remaining worldvolume indices \( \hat{a}, \hat{b}, \ldots \) in the expressions (7) and (8). For example, \( B_{\hat{a}}^{\hat{b} \hat{c}} \) couples to
\[
-3J^{+\hat{a} \hat{b}} + 3J^{-\hat{a} \hat{b}} + \ldots
\]
while \( B_{0\hat{a}} \) couples to
\[
-I^{0\hat{a}} - T^{+\hat{a}} - T^{-\hat{a}} + \ldots
\]
On the other hand, we would expect the D\( p \)-brane action to exhibit \( p+1 \) dimensional Lorentz symmetry in the worldvolume directions, therefore, \( B_{\hat{a}}^{\hat{b}} \) (including 0 indices) should couple to a Lorentz tensor \( I_{\hat{a}}^{\hat{b}} \). If so, it must be the case that
\[
-3J^{+\hat{a} \hat{b}} + 3J^{-\hat{a} \hat{b}} + \ldots = I^{\hat{a}}_{\hat{b}}
\]
\[
-T^{+\hat{a}} - T^{-\hat{a}} + \ldots = 2I^{0\hat{a}}
\]
for some tensor \( I_{\hat{a}}^{\hat{b}} \). Examining the expressions for \( T \) and \( J \) in the appendix, we see that the \( D = 10 \) notation makes it clear that \( T^{+\hat{a}} + T^{-\hat{a}} \) and \( J^{+\hat{a} \hat{b}} - J^{-\hat{a} \hat{b}} \) are indeed \( (0\hat{a}) \) and \( (\hat{a}\hat{b}) \) components of a Lorentz tensor. In a similar way, we find that \( T^{ij} \) is related to \( J^{+i-i} \), and \( J^{ijk} \) is related to \( M^{+-ijkl} \). This was not obvious from the expressions for \( T, J, \) and \( M \) as they appeared in previous papers. As a result of these relationships, it turns out that both expressions (7) and (8) are in fact Lorentz invariant. This provides a first check of our results, and fixes several signs in the currents which were undetermined in [32].

Another obvious test of the results we have derived is to compare with the case \( N = 1 \), where our results should match with the known abelian \( Dp \)-brane actions. In particular, we note that for the case \( p = 9 \), the world volume theory contains no scalar fields, and all of the terms we have written are symmetrized traces of expressions involving only the world-volume field strength, the fermions, and covariant derivatives on the fermions. As a result, if we specialize to the abelian \( U(1) \) case for \( p = 9 \), the action will look the same as in the nonabelian theory, apart from covariant derivatives becoming usual derivatives and the ordering in products of field strengths \( F_{\mu\nu} \) which must be taken into account in the nonabelian theory. This similarity between the terms in the \( U(1) \) and \( U(N) \) actions for the 9-brane is in contrast to all of the lower-dimensional brane actions which have terms containing \( F_{ij} = i[X^i, X^j] \) and \( D_i\Theta = i[X^i, \Theta] \) which vanish in the abelian theory. However, all of these terms T-dualize to terms in the \( p = 9 \) action which do not vanish in the abelian case. Therefore a comparison of our \( p = 9 \) results with the \( p = 9 \) abelian Born-Infeld and Wess-Zumino actions will provide a check not only of the complete set of terms in our \( p = 9 \) action, but also of strictly non-abelian terms in \( p < 9 \) actions \[^4\]. We perform this check in section 3.1, and in section 3.2, we will reverse this logic and use the full \( p = 9 \) abelian

\[^4\]Since we must assume that the background fields are independent of directions transverse to the brane in order to T-dualize to \( p = 9 \), we will not be able to check terms coupling to transverse derivatives of the background fields in this way.
Born-Infeld and Wess-Zumino actions to deduce a large set of additional terms in the non-abelian $p$-brane actions for $p < 9$. In preparation for these comparisons, and as an example of the results from this section, we now write explicitly the terms we have derived for the case $p = 9$.

### 2.5 Example: D9-brane

In the case $p = 9$, there are no transverse indices $i$ so the story simplifies considerably. The terms of lowest dimension in the nonabelian theory coupling linearly to the NS-NS and R-R background fields are given by

\[
S_{\text{D9-NS}} = \text{STr} \left\{ \phi - \frac{1}{2} h_{cc} \left( 1 + \frac{1}{4} F_{ab} F^{ab} - \frac{i}{2} \bar{\theta} \Gamma^a \theta + (I_{\phi})_8 + \cdots \right) \right. \\
- \frac{1}{2} h_{ab} \left( F_{bc} F^c - \frac{i}{2} \bar{\theta} \Gamma_a D^b \theta + (I^a_{bc})_8 + \cdots \right) \\
+ \frac{1}{2} B_{ab} \left( F^{ab} + F^{ac} F_{cd} F^{db} + \frac{1}{4} F_{cd} F^{cd} \right) \\
- \frac{i}{4} F_{cd} \bar{\theta} \Gamma^{cd} D^a \theta - \frac{i}{4} F_{cd} \bar{\theta} \Gamma^{cd} \Gamma^ba D^c \theta + \frac{i}{8} D^d F_{de} \bar{\theta} \Gamma^{deab} \theta + \cdots \right\} 
\]

and

\[
S_{\text{D9-R-R}} = \int d^{10} \sigma \epsilon^{abcdefghij} \text{STr} \left[ c_{10} C_{abcdefghij}^{(10)} + c_8 C_{abcdefghij}^{(8)} \right. \\
+ c_6 C_{abcdefghij}^{(6)} \left( F_{gh} F_{ij} + i \bar{\theta} \Gamma^{ghi} D_j \theta \right) \\
+ c_4 C_{abcdefghij}^{(4)} \left( F_{ef} F_{gh} F_{ij} + i F_{ef} \bar{\theta} \Gamma^{ghi} D_j \theta \right) \\
+ c_2 C_{abcdefghij}^{(2)} \left( F_{cd} F_{ef} F_{gh} F_{ij} + F_{cd} F_{ef} i \bar{\theta} \Gamma^{ghi} D_j \theta + O(\theta^4) \right) \\
\left. + c_0 C_{abcdefghij}^{(0)} \left( F_{ab} F_{cd} F_{ef} F_{gh} F_{ij} + \{O(\theta^2)\} \right) \right]\]

where

\[
c_{2m} \equiv \frac{1}{(2m)!((5-m)!2^{5-m})}
\]

Note that in order to T-dualize in all 9 spatial directions we have assumed that the background fields are independent of all spatial coordinates, so there are no higher moment contributions to this action. Of course, we expect that spatial variations in the background fields can be incorporated into this action by simply assuming that all world-volume fields as well as background fields in the action have spatial dependence. We will discuss this issue in some further detail in section 3.2.

---

5To derive the terms below by T-duality, we have assumed the background fields are constant, in which case the fermion terms are all total derivatives. These fermion terms do not seem to appear in the $\kappa$-symmetric actions, so it seems that we should integrate by parts before restoring spacetime dependence of the background.

6The definition of $c_{2m}$ here reflects a change from \[8\] to more standard normalization conventions for the R-R fields.
3 Comparison with the abelian theory

In this section, we compare our results to the known abelian D-brane theories. In section (3.1), we show that the actions we have derived agree with a gauge fixed version of the abelian $\kappa$-symmetric D-brane actions. In section (3.2), we show that the abelian D9-brane action may be T-dualized to determine non-abelian terms in the lower D-brane actions. This provides an alternate derivation of some of our results plus a large set of additional higher order terms for both the Born-Infeld and Wess-Zumino actions.

3.1 Abelian $\kappa$-symmetric D-brane theory

We now compare our results to the known abelian $\kappa$-symmetric action for a D$p$-brane. For a detailed discussion of these actions, see the original references [17, 18, 19, 20, 21].

The $\kappa$-symmetric D$p$-brane actions are most naturally understood in terms of the super-space formalism for type IIA/IIB supergravity. In addition to the usual bosonic coordinates, type IIA/IIB superspace contains fermionic coordinates which form a pair of Majorana-Weyl spinors of opposite/the same chirality.

The complete action may be written as the sum of a Born-Infeld type action and a Wess-Zumino action which take their usual form,

\[
S_{\text{DBI}} = -\int d^{p+1}\sigma e^{-\phi} \sqrt{-\det(g_{ij} + F_{ij})}
\]

\[
S_{\text{WZ}} = \int e^{\mathcal{F}} \sum_q C^{(q)}
\]

except that now the background fields are replaced by fields and differential forms on super-space. Here, we define

\[F_{ij} = F_{ij} - B_{ij}^{uv}\]

where $F_{ij}$ is the usual field strength of the world volume gauge field but now $B_{ij}^{uv}$ is the pullback of the superspace two-form $B_{MN}$ to the worldvolume. In the Wess-Zumino term, the sum runs over even RR-forms in the IIB case and odd RR-forms in the IIA case, which are also defined as pull-backs of superspace forms. There are additional curvature terms which must appear in (11) to cancel anomalies in the world-volume theory [14, 15, 16]. These terms are linear in the background R-R fields and at least quadratic in curvature so they will not affect our discussion of the linearized couplings and we will ignore them here.

The superspace geometry is described in terms of a super-vielbein one form

\[E^A = dZ^M E^A_M\]

and connection one form \[\omega^A_B.\] In terms of these, the pullback of the metric is given by

\[g_{ij} = \partial_i Z^M E^a_M \partial_j Z^N E^b_N \eta_{ab}\]

\(^7\)We use $M, N, \ldots$ superspace coordinate indices, and $A, B, \ldots$ for local frame indices. We will use lower case Latin indices $a, b, m, n, \ldots$ refer to bosonic coordinates and lower case Greek indices to refer to fermionic coordinates.
The fermionic components of the various forms are determined by constraints on their field strengths, which may be found in [20]. These constraints are simplest in the basis defined by the one-forms $E^A$, however it is important to write the components with all bosonic indices in the coordinate basis rather than the frame basis so that the supergravity fields appearing in the final action are the usual ones. For example, we have in the type IIB case,

$$B^wv_{ij} = \partial_j Z^N E^b_N \partial_i Z^M E^A_M B_{AB}$$

$$= (\partial_j Z^n E^b_n + \partial_j Z^v E^b_v)(\partial_i Z^m E^a_m + \partial_i Z^\mu E^a_\mu)B_{ab}$$

$$+2(\partial_j Z^n E^a_n + \partial_j Z^v E^a_v)(\partial_i Z^m E^a_m + \partial_i Z^\mu E^a_\mu)B_{a\beta}$$

$$+ (\partial_j Z^n E^a_n + \partial_j Z^v E^a_v)(\partial_i Z^m E^a_m + \partial_i Z^\mu E^a_\mu)B_{a\beta}$$

$$= (\partial_j X^m - i e^m_b \bar{\Theta} \Gamma^b \partial_j \Theta)(\partial_i X^m - i e^m_a \bar{\Theta} \Gamma^a \partial_i \Theta)B_{mn}$$

$$+ i(\bar{\Theta} \sigma_3 \Gamma_a \partial_j \Theta)\partial_i X^m e^a_m - i(\bar{\Theta} \sigma_3 \Gamma_a \partial_i \Theta)\partial_j X^m e^a_m$$

$$+ \frac{1}{2}(\bar{\Theta} \sigma_3 \partial_j \Theta)(\bar{\Theta} \Gamma_a \sigma_3 \partial_i \Theta) - \frac{1}{2}(\bar{\Theta} \Gamma_a \sigma_3 \partial_j \Theta)(\bar{\Theta} \Gamma_a \sigma_3 \partial_i \Theta)$$

Here we have used the fact that the superspace constraints on the torsion may be solved by

$$E^a_m = e^a_m, \quad E^a_m = 0, \quad E^a_\mu = i \delta^a_\mu (\Gamma^a \Theta)_\alpha, \quad E^a_\mu = \delta^a_\mu.$$

while the constraints on the field strength of $B$ may be solved by

$$B_{a\beta} = -i(\Gamma_a \sigma_3 \Theta)_\beta, \quad B_{a\beta} = -(\Gamma^a \Theta)_\alpha(\Gamma_a \sigma_3 \Theta)_\beta$$

The matrix $\sigma_3$ is a 2 \times 2 Pauli matrix which acts on the index labeling the two fermionic superspace coordinates.

The Born-Infeld and Wess-Zumino actions above are separately invariant under 32 global supersymmetries which are guaranteed by their superspace covariant form. In addition, the combination is invariant under a 16-dimensional local fermionic $\kappa$-symmetry.

To relate these $\kappa$-symmetric actions, which have two spacetime Majorana-Weyl spinors and ten spacetime coordinates, to our action which is written in terms of a single worldsheet Majorana-Weyl spinor and $9-p$ scalars, we must use $\kappa$-symmetry to set one linear combination of the fermionic coordinates to zero and use reparametrization invariance to identify the first $p+1$ spacetime coordinates with the $p+1$ worldsheet coordinates (the static gauge). Since the local $\kappa$-symmetry is 16-dimensional, it is completely used up in eliminating half of the fermion fields.

The fermion terms in the resulting action will depend on which combination of the fermions we choose to set to zero. To reproduce the results we have derived in the previous section, we must therefore make a specific choice for the fixing of $\kappa$-symmetry, however it is not obvious what this choice should be. Let us focus on the case $p = 9$. In this case, it seems that any Lorentz invariant choice would amount to setting

$$\Theta_1 = a \Theta, \quad \Theta_2 = b \Theta.$$
One natural choice, taking \((a, b) = (0, 1)\) or \((a, b) = (1, 0)\) was demonstrated for the flat space action in \([18]\). With this choice, all terms in the Wess-Zumino action independent of the background fields vanish, leaving only the Born-Infeld action. It may be checked that this choice gives an action whose fermion terms do not agree with our results. In particular, it gives a dimension 4 fermion operator coupling to \(B\), while in our results, \(B\) couples only to operators of dimension 2 and 6. As it turns out, the abelian terms in the actions we have derived seem consistent with another natural choice\(^8\), \(a = b = 1/\sqrt{2}\), that is, we set

\[
\Theta_1 = \Theta_2 = \frac{1}{\sqrt{2}}\Theta.
\]

This is the \(p = 9\) version of “Killing Gauge” \([12]\).

To implement this gauge choice, we simply replace

\[
1, \sigma_1 \rightarrow 1, \quad \sigma_2, \sigma_3 \rightarrow 0
\]

in all fermion bilinears.

With our choice of gauge the expressions for \(g_{ij}\) and \(F_{ij}\) simplify to

\[
\begin{align*}
g_{ij} &= (e_i^a - i \bar{\Theta} \Gamma^a \partial_i \Theta) \, (e_j^b - i \bar{\Theta} \Gamma^b \partial_j \Theta) \, \eta_{ab} \\
F_{ij} &= F_{ij} - B_{ij}^{uv} \\
&= F_{ij} - (\delta_i^a - ie_i^b \bar{\Theta} \Gamma^b \partial_j \Theta)(\delta_j^m - ie_j^m \bar{\Theta} \Gamma^a \partial_i \Theta) B_{mn}
\end{align*}
\]

(12)

We are now in a position to explicitly write down the gauge fixed forms of the actions \([11]\) and compare with the abelian version of our results. For the bosonic terms, we find complete agreement between the two methods for all of the operators we have derived. Comparing the fermion terms, we find something unexpected. The gauge fixed \(\kappa\)-symmetric action matches our results precisely if we assume that the \(p\)-form fields appearing in the matrix theory derived expressions \((9,10)\) are to be identified with world-volume fields rather than spacetime fields. We do not understand why this should be the case, however we note that since the D9-brane fills spacetime, the distinction between worldvolume indices and spacetime indices is somewhat subtle here.

As an example of this, we compute the operator coupling to \(B_{mn}\) in the gauge fixed action. From \((11)\), we see that \(B\) appears in the action only in the combination \(\mathcal{F}\). Thus, the complete dependence on \(B\) may be easily obtained from the flat space action by making the substitution \(F_{ij} \rightarrow \mathcal{F}_{ij}\). Up to dimension 8, the background independent part of the action, including contributions from both the Born-Infeld and Wess-Zumino terms is

\[
S_{D9} = -\frac{1}{4} F_{ab} F_{ab} + \frac{i}{2} (\bar{\Theta} \partial \Theta)
\]

\(8\)The factor \(1/\sqrt{2}\) is chosen to match the normalization conventions used above.

\(9\)In computing the Wess-Zumino terms, the relevant fermion terms in the superspace components of the R-R field strength are

\[
\begin{align*}
C_{a_1 \cdots a_{2n-1} \alpha}^{(2n)} &= e^{-\phi} (\Gamma_{a_1 \cdots a_{2n-1}}(-\sigma_3)^n \sigma_2 \Theta)_{\alpha} \\
C_{a_1 \cdots a_{2n-2} \alpha \beta}^{(2n)} &= -ie^{-\phi} (\Gamma^{a_{2n-1}} \Theta)_{\alpha} (\Gamma_{a_1 \cdots a_{2n-1}}(-\sigma_3)^n \sigma_2 \Theta)_{\beta} + \ldots
\end{align*}
\]
The omitted terms give no contribution with our choice of gauge, while the remaining components only contribute terms of dimension greater than 8. Only the components of $C^{(4k+2)}$ give a non-vanishing contribution in our gauge.  

\[ \frac{1}{2} B_{wv} \to (\delta_j^n - i\epsilon_j^n \bar{\Theta}^b \partial_j \Theta)(\delta_i^m - i\epsilon_i^m \bar{\Theta}^a \partial_i \Theta)B_{mn} \]

however, this has the effect of eliminating the term $\frac{i}{2}(\bar{\Theta} \Gamma^b \partial_c \Theta)F_{ac}$, so there is a single term disagreement if we try to identify the $B$ in (8) with the $B$ having spacetime indices. The same effect appears when comparing the operators coupling to the Ramond-Ramond fields.

It is possible that a different method of fixing $\kappa$-symmetry and reparametrization symmetry would give a gauge fixed action in which the spacetime $B$ appeared as in expression (8), however, it is difficult to see how a change in the gauge fixing could affect only the single term without changing the others that already agree. We leave it as a puzzle to understand why agreement is obtained by comparison with the pulled-back world-volume $p$-forms rather than spacetime $p$-form fields in the D9-brane action.

One could repeat this analysis for the cases $p < 9$ by gauge fixing the other abelian $\kappa$-symmetric $Dp$-brane actions and comparing with our results, however, as discussed above, the comparison for $p = 9$ already provides a check for all terms in the lower $Dp$-brane actions which do not contain derivatives of the background fields in directions transverse to the brane. In fact, since many terms for $p < 9$ vanish in the abelian case but T-dualize to $p = 9$ terms which do not vanish, a comparison with the lower dimensional $\kappa$-symmetric actions would actually be a less stringent check than the one we have already performed. Rather, we will now exploit the connection between terms in the abelian 9-brane action and non-abelian terms in the lower brane actions to deduce a large set of additional terms for these actions.

The omitted terms give no contribution with our choice of gauge, while the remaining components only contribute terms of dimension greater than 8. Only the components of $C^{(4k+2)}$ give a non-vanishing contribution in our gauge.  

\[ \frac{1}{8}(F_{ab}F_{bc}F_{cd}F_{da} - \frac{1}{4}F_{ab}F_{cd}F_{cd}) + \frac{i}{16}(\bar{\Theta} \partial (\Theta) F_{ab}F_{ab} + \frac{i}{4}(\bar{\Theta} \Gamma^a \partial_a \Theta)F_{bc}F_{cd} + \frac{i}{32}\bar{\Omega}_{abcd} \partial_a \Theta F_{bc}F_{cd} + \frac{1}{16}(\bar{\Theta} \partial (\Theta) (\bar{\Theta} \partial (\Theta)) - \frac{1}{16}(\bar{\Theta} \Gamma^a \partial_a \Theta)(\bar{\Theta} \Gamma^{,b} \partial_b \Theta) + \ldots \]

To match conventions with the results in section 2, we have made a field redefinition $\Theta \to 2\Theta$.

Making the substitution $F_{ij} \to F_{ij} - B_{ij}^w$ and keeping terms linear in $B_{ij}^w$, we find that up to dimension 6 operators, the terms coupling to $B_{ij}^w$ are \[ \frac{1}{2} F_{ij} + F_{ac}F_{cd}F_{db} + \frac{1}{4} F_{ab}F_{cd}F_{cd} \]

\[ + \frac{i}{2}(\bar{\Theta} \Gamma^c \partial_b \Theta)F_{ac} + \frac{i}{2}(\bar{\Theta} \Gamma^b \partial_c \Theta)F_{ac} + \frac{i}{4}(\bar{\Theta} \Gamma^{abc} \partial_d \Theta)F_{cd} + \frac{i}{4}(\bar{\Theta} \Gamma^{acd} \partial_b \Theta)F_{cd} + \ldots \]

This expression agrees precisely with the abelian version of our results (8) if we identify the $B$ appearing in that equation with $B_{ij}^w$. We would have expected that the two expressions should have been compared after making the further substitution

\[ B_{ij}^w \to (\delta_j^n - i\epsilon_j^n \bar{\Theta}^b \partial_j \Theta)(\delta_i^m - i\epsilon_i^m \bar{\Theta}^a \partial_i \Theta)B_{mn} \]
3.2 Non-abelian terms from the abelian \( p = 9 \) action

In this subsection we compare our results derived from matrix theory and the D0-brane action by T-dualizing in \( p \) directions to the action found by T-dualizing the Born-Infeld action of a D9-brane in \( 9 - p \) directions. A related discussion is given in the recent paper of Myers \(^{[33]}\).

3.2.1 Bosonic Born-Infeld terms

Let us first consider the bosonic terms in the Born-Infeld action for the case \( p = 9 \). If, following the proposal of Tseytlin for the flat space case, we simply define the non-abelian Born-Infeld action for D9-branes to be that set of terms not containing any commutators of \( F \)'s \(^{[11]}\) then the remaining action will be (by definition) a symmetrized trace over products of \( F \)'s with indices contracted in various ways. As argued above, the restriction to the abelian case will clearly give exactly the same set of terms, the only difference being that the symmetrized trace may be dropped, since all the \( F \)'s commute already. Thus, we conclude that the complete set of terms in the non-abelian action not containing commutators of \( F \)'s should be obtained from the abelian case by imposing a symmetrized trace, that is

\[
S = \int d^{10}x \text{Str} \left( e^{-\phi} \sqrt{-\det (g_{ab} - B_{ab} + F_{ab})} \right)
\]

(13)

This is a simple generalization of the proposal by Tseytlin for the flat space case. Starting from this action, we may now apply the T-duality rules above to find a large set of terms in the lower D\( p \)-brane actions. The resulting actions include terms with explicit commutators \([X^i, X^j]\) and are therefore are a much less obvious generalization of the corresponding abelian actions than (13).

In our case, dualizing to determine the other D\( p \)-brane actions is more complicated than for flat space, due to the presence of background supergravity fields, however, the simple form of T-duality rules derived above will allow us to proceed without much difficulty.

We choose \( n \) directions labeled by \( x^i \) over which the T-duality is to be performed and assume for now that the background fields are independent of these directions. Using our results for the D0-brane action, we will later be able to reinstate dependence on these directions (which will be transverse to the brane). To perform the T-duality, we first rewrite the D9-brane action in terms of the field \( G = g - B \) defined above, distinguishing between indices \( i, j, \ldots \) in the directions to be dualized and indices \( a, b, \ldots \) in the non-dualized directions that will become transverse and world-volume indices respectively. We have

\[
S = \int d^{10}x \text{Str} \left( e^{-\phi} \sqrt{-\det (G_{ab} + F_{ab})} \right)

= \int d^{10}x \text{Str} \left( e^{-\phi} \sqrt{-\det \left( \frac{G_{ab} + F_{ab}}{G_{ib} + F_{ib}} \frac{G_{aj} + F_{aj}}{G_{ij} + F_{ij}} \right)} \right)
\]

\(^{11}\)this is analogous to the abelian restriction to terms not containing derivatives of \( F \)'s since we may rewrite \([F_{ab}, F_{cd}] = D_{[a}D_{b]}F_{cd}\).
\[ = \int d^{10}x \text{Str} \left( e^{-\phi} \sqrt{-\det(G_{ij} + F_{ij}) \det(G_{ab} + F_{ab} - (G_{ai} + F_{ai})(G_{ij} + F_{ij})^{-1}(G_{jb} + F_{jb}))} \right) \]

where in the last line, we have used the identity
\[
\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det \left\{ \begin{pmatrix} 1 & 0 \\ 0 & D \end{pmatrix} \begin{pmatrix} A & B \\ D^{-1}C & 1 \end{pmatrix} \right\} = \det(D) \det(A - BD^{-1}C)
\]

We now dimensionally reduce the world volume fields to \(p+1\) dimensions and replace the background fields by their duals. The resulting expression is
\[
S^{Dp}_{BI} = T_p \int d^{p+1}\sigma \text{Str} \left\{ e^{-\phi} \left( -\det[\delta_{ij} + G_{ik}F_{kj}] \right) \right\}^{1/2}
\]

Here, \(G^{ij}\) is the inverse of \(G_{ij} = g_{ij} - B_{ij}\) and the term \((G + GF G)^{ij}\) is the inverse of \((G_{ij} + G_{ik}F_{kj}G_{lj})\). We also recall that \(F_{ai} = D_a X^i\), \(F_{ij} = i[X^i, X^j]\), and \(F_{ab}\) is the usual world volume field strength.

In the absence of background fields, it may be checked that this action reduces to Tseytlin’s non-abelian flat space action. In the abelian case with background fields, the action becomes
\[
S^{Dp}_{U(1)} = T_p \int d^{p+1}\sigma e^{-\phi} \sqrt{-\det(G_{ab} + G_{ai}F_{ai} + G_{ib}F_{ai} + G_{ij}F_{ai}F_{bj} + F_{ab})}
\]

which is equivalent to the usual covariant action
\[
S^{Dp} = T_p \int d^{p+1}\sigma e^{-\phi} \sqrt{-\det(G_{\mu\nu}\partial_a X^\mu \partial_b X^\nu + F_{ab})}
\]
in the static gauge where we identify \(X^a = \sigma^a\).

By expanding (14) and keeping only terms linear in the background supergravity fields, one may verify that the leading terms are exactly the bosonic terms derived in section 2.

It is interesting to compare how the matrix theory and DBI approaches describe coupling to background fields with spatial dependence. First consider those directions perpendicular to the \(p\)-brane. From the matrix theory approach, we may have dependence of the background supergravity fields on these transverse directions, which are described by the higher moment terms in (\(\mathbf{1}\)). We do not know how to explicitly T-dualize background fields depending on the compact coordinate, so we cannot determine these terms in the \(p\)-brane DBI action by T-dualizing from the 9-brane. For the bosonic terms, however, it is easy to see that we may reproduce the dependence found from the matrix theory approach in the action simply by taking the background fields to be functions of the world-volume coordinates and...
the transverse scalar fields and then formally expanding in the transverse scalars inside the symmetrized trace. For example

$$\phi = \phi(\sigma_o, \ldots, \sigma_p, X^{p+1}, \ldots, X^9)$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} \{\partial_{k_1} \cdots \partial_{k_n} \phi\}(\sigma_o, \ldots, \sigma_p, 0, \ldots, 0) X^{k_1} \cdots X^{k_n}$$

It seems natural to assume that this prescription also applies to the full non-linear action (14), so we interpret all background fields in this expression to be expansions of the form (15). Such a prescription has also been verified in certain cases by explicit string theory scattering calculations [43].

In contrast to the situation for the transverse coordinates, the Born-Infeld approach gives an action in which the background fields have a natural dependence on the world-volume coordinates $\xi^a$, but such a dependence cannot be included when T-dualizing from the 0-brane action. It is quite straightforward to extend the results of Section 2 by making all D-brane fields and background fields explicitly dependent on the world-volume coordinates.

It would be interesting to invert this chain of argument and ask what we can learn from this discussion about the action of T-duality in the presence of a background field which depends on the compact coordinate. In the language of the world-volume theory of the $D_p$-brane wrapped around a torus $T^p$, dependence of the background fields on the compact degrees of freedom is naturally incorporated in the spatial dependence of the background fields and the world-volume fields. Formally inverting the T-duality transformation [41] to get a theory of (formally infinite) D0-brane matrix degrees of freedom, we find a nontrivial coupling between block components of the D0-brane matrices and the Fourier modes of the original background fields. Whether such a background can be interpreted as simply a nontrivial classical supergravity background for the D0-brane system is not clear; this may be a more general class of backgrounds with no classical description, if indeed these backgrounds are consistent. We leave a further investigation of this question to further work.

3.2.2 Bosonic Wess-Zumino terms

We now adopt a similar method to derive terms in the non-abelian $D_p$-brane Wess-Zumino actions.

We again start from the Abelian 9-brane action, which in terms of the field $C \equiv C e^{-B}$ defined above may be written

$$S_{WZ}^{D_9} = \int \sum_{m=0}^{5} \frac{1}{(5-m)!} C^{(2m)} \{\Lambda F\}^{5-m}$$

$$= \int d^{10} \sigma \epsilon^{a_0 \cdots a_9} \sum_{m=0}^{5} \frac{1}{(2m)! (5-m)! 2^{5-m} m!} C^{(2m)}_{a_1 \cdots a_{2m}} F_{a_{2m+1} a_{2m+2} \cdots a_{9} a_{10}}$$

Here, the $F$’s are already symmetrized, so there is no ordering ambiguity in passing to the non-abelian version. Using the T-duality rules for the background fields (14), we then find
that the Dp-brane WZ action is

\[ S^{Dp}_{WZ} = \int d^{p+1} \sigma e^{a_0 \cdots a_p} \sum_{q} \sum_{n=\max(0,q-p-1)}^{\min(q,9-p)} e_{q,n}^p \text{STr} \left\{ C^{(q)}_{a_0 \cdots a_q \cdots a_{n-1} \cdots i_n} F^{n+(p-q+1)/2}_{(a_q \cdots a_p i_1 \cdots i_n)} \right\} \]

where

\[ e_{q,n}^p \equiv \frac{(-1)^{n(n-1)/2}(p+2n-q)!!}{n! (q-n)! (n+p-q+1)!}. \]

In (16), the indices in brackets are to be assigned pairwise to the product of \( F \)'s and then symmetrized over all possible orderings. As for the Born-Infeld action, the background fields should be taken as functions of the world volume coordinates with scalar matrices inserted in place of the transverse coordinates, in order to reproduce the bosonic higher moment terms derived from matrix theory. Again, we may compare terms linear in the background fields from this action to the terms we have derived from the D0-brane results, and we find that both methods give a consistent answer \(^{12}\).

For \( p < 9 \), this action contains many terms involving \( F_{ij} \) which do not appear in the abelian case. Specifically, while the abelian action has couplings involving only R-R q-forms with \( q < p + 1 \) the non-abelian action contains terms involving all allowed R-R fields (i.e. q odd/even for p even/odd). Physically, just as a given \( p \)-brane can carry \( p - 2n \) brane charge measured locally by \( \left( F_{a_1 a_2 \cdots a_{2n-1} a_{2n}} \right) \), a collection of \( p \)-branes can carry (moments of) \( p + 2n \) brane charge measured by \( \text{Tr} \left( F_{i_1 i_2 \cdots i_{2n-1} i_{2n}} \right) \) (with insertions of \( X \)'s). As mentioned in Section 2.1, this is perhaps most familiar from the matrix theory point of view, where membrane (D2-brane) states and longitudinal 5-brane (D4-brane) states may be constructed out of zero-branes.

### 3.2.3 Fermion terms from \( p = 9 \)

As for the bosonic terms, one could use the \( p = 9 \) abelian action to deduce the symmetrized trace terms with background fields independent of the transverse directions for the fermionic terms in all the non-abelian \( p \)-brane actions by imposing a symmetrized trace on the \( p = 9 \) action and dimensionally reducing. However, while for the bosonic terms we were able to restore dependence of the background fields on the transverse directions simply by making them functions of the matrices \( X^i \), such a prescription does not give all of the fermion terms coupled to transverse derivatives of the background fields, as may be seen in specific cases. Some of these terms would appear in the abelian \( p \)-brane actions and should be derivable by considering the \( \kappa \)-symmetric \( p \)-brane action, but there is no clear way to find terms with commutators apart from our methods in the section 2. Thus, it is not possible to reproduce all of our results simply by considering the abelian actions.

\(^{12}\)The normalization conventions in the R-R fields in (8) are slightly different from the usual ones used here but the relation between the two conventions may be easily read off by comparing (8) and (14).
D3-branes and S-duality

The SL(2, \mathbb{Z}) S-duality symmetry of type IIB string theory maps a D3-brane into another D3-brane \[^4\]. We would like to see how this S-duality invariance is manifested in the multiple D\(p\)-brane action we have derived in the case \(p = 3\). For the case of a single D3-brane, the combination of Born-Infeld and Wess-Zumino actions have been shown explicitly to lead to equations of motion which possess an S-duality invariance involving simultaneous transformations of the background fields and the world-volume fields on the D3-brane \[^45, 46, 47, 48\]. Since we do not have the full non-abelian D3-brane actions it will not be possible to demonstrate the complete S-duality of the theory with many D3-branes in a type IIB supergravity background. However, using the linear couplings we have derived and the known transformation properties of the supergravity fields, we will show that the requirement of S-duality gives information about the transformation properties of various operators on the world-volume. We find a new prediction for the S-dual of certain operators constructed from the transverse scalar fields in the SYM theory, and we show that this duality transformation helps resolve an outstanding puzzle regarding the transverse 5-brane of matrix theory.

4.1 S-duality and the linearized action

We focus on the \(Z_2\) subgroup of the S-duality group generated by the transformation \(\tau\) which exchanges the NS-NS and R-R two form fields. Under this transformation, background fields of IIB string theory transform at linear order as\[^13\]:

\[
\begin{align*}
\phi & \rightarrow -\phi \\
C^{(0)} & \rightarrow -C^{(0)} \\
B_{\mu\nu} & \rightarrow -C^{(2)}_{\mu\nu} \\
C_{\mu\nu} & \rightarrow B^{(2)}_{\mu\nu} \\
g_{ij} & \rightarrow g_{ij} \\
C^{(4)} & \rightarrow C^{(4)}
\end{align*}
\]

In order to check that our action satisfies S-duality we write explicitly some of the terms in the D3-brane action, beginning with the coupling of the world-volume bosonic fields to the NS-NS and R-R scalar fields. The action in the absence of background fields is

\[
S_0 = \text{STr} \left( -\frac{1}{4} F^2 - \frac{1}{8} (F^4 - \frac{1}{4} (F^2)^2) + \cdots \right)
\]  

\(^{13}\)Note that these transformation rules are written in Einstein frame. Although we have for the rest of this paper been working in string frame, the terms in the action which we consider in this section are the same in both frames so it is consistent to use the Einstein frame transformation rules in combination with the terms in the action produced by specializing \(^{18}\) to the case \(p = 3\).
The couplings to the background scalars are
\[ \phi \cdot \left( \frac{1}{4} F_{ab} F^{ab} + \cdots \right) \] (18)
and
\[ C^{(0)} \cdot \left( \frac{1}{2} F \wedge F + \cdots \right) \] (19)
The couplings to the NS-NS and R-R 2-form fields are
\[ \frac{1}{2} B_{ab} \cdot \text{STr} \left( F^{ab} + \frac{1}{4} F^{ab} F^2 + F^{a\mu} F_{\mu} F^{\nu b} + \cdots \right) \]
\[ - B_{ai} \cdot \text{STr} \left( F^{a\mu} F_{\mu i} + \cdots \right) \]
\[ - \frac{1}{2} B_{ij} \cdot \text{STr} \left( F_{ij} + \cdots \right) \] (20)
and
\[ - \frac{1}{4} \epsilon^{abcd} C^{(2)}_{ab} \cdot \text{STr} \left( F_{cd} + \cdots \right) \]
\[ \frac{1}{2} \epsilon^{abcd} C^{(2)}_{ai} \cdot \text{STr} \left( F_{bc} F_{di} + \cdots \right) \]
\[ \frac{1}{4} \epsilon^{abcd} C^{(2)}_{ij} \cdot \text{STr} \left( F_{ab} F_{ci} F_{dj} - \frac{1}{4} F_{ab} F_{cd} F_{ij} + \cdots \right) \] (21)

In the abelian case, the field strength of the gauge field on the world-volume of the D3-brane transforms at lowest order by
\[ F_{ab} \rightarrow \frac{1}{2} \epsilon_{abcd} F^{cd}. \] (22)
There are higher order corrections to this transformation law from the Born-Infeld action [45, 46, 47] which we will not need to consider here. From (22) it is straightforward to compute the dual of the higher order terms in the field strength
\[ F_{ab} F^{ab} \rightarrow - F_{ab} F^{ab} + \cdots \]
\[ F \wedge F \rightarrow - F \wedge F + \cdots \] (23)

We have no systematic knowledge of the S-duality transformation properties of the world-volume operators in the nonabelian \( \mathcal{N} = 4 \) \( U(N) \) gauge theory. It is straightforward, however, to generalize the relations from the abelian theory to analogous relations in the non-abelian theory by simply performing a symmetrized trace on both sides of the relations (22, 23). This gives
\[ \text{STr} F_{ab} \rightarrow \text{STr} \left( \frac{1}{2} \epsilon_{abcd} F^{cd} + \cdots \right) \]
\[ \text{STr} \left( F_{ab} F^{ab} \right) \rightarrow \text{STr} \left( - F_{ab} F^{ab} + \cdots \right) \]
\[ \text{STr} (F \wedge F) \rightarrow \text{STr} \left( - F \wedge F + \cdots \right) \] (24)

19
Note that in the nonabelian theory the duality transformation of the higher order terms in $F$ are not automatic consequences of the duality transformation of the linear term.

Although the full abelian Born-Infeld action is not invariant under S-duality, the equations of motion of this theory are invariant \([44, 45, 46]\). We also expect that the terms describing the leading operators coupling linearly to the background fields should transform among themselves under S-duality. The simplest argument for this is that we would expect a system of D-branes which couple to a given background supergravity field such as $B$ through an operator $\mathcal{O}_B$ to couple to the S-dual supergravity field $\tilde{B}$ (in this case $-C^{(2)}$) through the S-dual operator $\tilde{\mathcal{O}}_B = \mathcal{O}_{\tilde{B}}$. In the AdS/CFT context a similar argument was given in \([49]\).

From the proposed nonabelian S-duality relations (24), it is straightforward to check that the leading terms in the action (18,19) which are linear in the background dilaton and axion fields transform into themselves under S-duality. Note that we have so far avoided discussing the terms depending on the world-volume scalars $X_i$. The duality transformation properties of the operators containing these fields are not known, although we will now proceed to show that something can be learned about how these operators transform from other terms in our action.

We now turn to an analysis of the S-duality properties of the terms coupling to the background NS-NS and R-R 2-form fields. The lowest dimension operators coupling to $B_{ab}$ and $C^{(2)}_{ab}$ are indeed related by nonabelian S-duality according to the prescription above. While the terms coupling to $B_{ai}$ and $C^{(2)}_{ai}$ do not seem to be exactly S-dual, the leading operators at least have the same dimension; we will return to these operators momentarily.

When we consider the operators coupling to the 2-form fields $B_{ij}$ and $C^{(2)}_{ij}$ with purely transverse polarization indices, the S-duality of our action seems at first glance to break down completely. There is a dimension 2 operator coupling to the NS-NS 2-form field, while the first operator coupling to the R-R field has dimension 6. In order to resolve this apparent contradiction, we need to recall that each of these fields is related to a dual 6-form field to which the NS-NS and R-R 5-branes are electrically coupled. In particular, consider the R-R 2-form field. This field is related to the R-R 6-form field through

$$\partial_a C^{(2)}_{bcdefg} = \frac{1}{3!} \epsilon_{abcdfgklm} \partial^k (C^{(2)})^{lm}$$

Taking into account operators coupling to the R-R 6-form, which may be read off from (8), we find that the complete coupling of a derivative of the transversely polarized R-R 2-form field is

$$\partial_k C^{(2)}_{ij} \left( -\frac{1}{12} \epsilon_{ijklmn} \text{Tr} \left( X^i [X^m, X^n] \right) + \frac{1}{4} \text{Tr} \left( X^k \epsilon^{abcd} (F_{ab} F_{ci} F_{dj} - \frac{1}{4} F_{ab} F_{cd} F_{ij}) \right) \right) + \cdots$$

Let us compare this to the terms coupling to the transversely polarized NS-NS 2-form field. Since $\text{Tr} [A, B]$ vanishes for any $U(N)$ matrices $A, B$, the first nonvanishing term appears in the first moment and is given by

$$-\frac{1}{2} \partial_k B_{ij} \left( \text{Tr} \left( X^k [X^i, X^j] \right) + \cdots \right)$$
If we assume that S-duality acts on the combination of transverse fields $Tr\left([X^i, X^j]X^k\right)$ through
\[ S : Tr\left([X^i, X^j]X^k\right) \rightarrow -\frac{1}{6}\epsilon^{ijklmn}Tr\left([X^l, X^m]X^n\right), \tag{28} \]
then the term (27) is precisely the S-dual of the leading operator in (26). Thus, we see that the condition that our action is S-duality invariant can be satisfied and indeed may give new information about how operators formed from the transverse fields transform under S-duality.

The mechanism we have just discovered can be seen to solve other apparent problems with the S-duality of the couplings (20) and (21). As noted above, the dimension 4 operators coupling to the fields $B_{ai}$ and $C_{ai}^{(2)}$ are not completely equivalent under S-duality. In particular, the operator coupling to the NS-NS field contains terms of the form $F_{ai}F_{ij}$ while the operator coupling to the R-R field contains no such terms. However, taking into account contributions from the 6-form using (25) we see that the complete coupling of derivative of this background field is
\[ \partial_j C_{ai}^{(2)} \cdot STr\left(\frac{1}{2}\epsilon^{abcd}F_{bc}F_{di}X^j + \frac{1}{6}\epsilon^{ijklmn}F_{ai}F_{mn}X^k + \cdots\right) \tag{29} \]
which has the correct S-duality relation with the term coupling to $\partial_j B_{ai}$ as long as (28) continues to hold with an insertion of $F_{ai}$.

We have thus seen that the S-duality invariance of our linearly coupled action can be explained by S-duality relations on the operators appearing in the maximally symmetric super Yang-Mills theory such as (28). It is interesting to compare this result with the conjecture of Intriligator [49] regarding the S-duality transformation properties of short operators in the SYM theory in the context of the AdS/CFT correspondence. The operators in the SYM theory are characterized by their transformation properties under the superconformal symmetry group $PSU(2,2|4)$. The operators which linearly couple to the background supergravity fields live in short representations of the superconformal algebra, both in the AdS/CFT correspondence and also in the action we are discussing here. (We will discuss the AdS/CFT correspondence and its connection to our action in more detail in the next section.) The operators in short representations of $PSU(2,2|4)$ were classified in [50], and an explicit table listing which operators in the SYM theory correspond to which representations is given in [49]. In general, the short operators are uniquely determined by their $SO(4)$ and $SU(4)$ transformation properties, and must therefore transform into themselves under S-duality up to a sign. In [49], a conjecture was made for the modular weights of each of the short operators in the theory. The operators considered in (28) can be combined into self-dual and anti-self-dual combinations under the 6-dimensional duality transformation arising from contraction with $\epsilon^{ijklmn}$. Each of these combinations lives in an irreducible representation of $SO(6) \approx SU(4)$. Our prediction that
\[ S : Tr\left([X^i, X^j]X^k\right) \rightarrow -\frac{1}{6}\epsilon^{ijklmn}Tr\left([X^l, X^m]X^n\right) \tag{30} \]
\[ \text{We do not need to worry about the constant pieces since constant } B_{ai} \text{ and } C_{ai} \text{ can be gauged away.} \]
corresponds to the statement that the self-dual and anti-self-dual combinations of these operators under the 6-dimensional epsilon symbol also transform into themselves under S-duality so that one combination is self-S-dual and the other is anti-self-S-dual.

### 4.2 S-Duality and transverse 5-branes in matrix theory

We conclude this section with a brief description of how this discussion of S-duality relates to the problem of the transverse 5-brane in matrix theory\(^\text{15}\). Consider a configuration of the D3-brane theory on a 3-torus with transverse fields \(X^4, X^5, X^6\) given by

\[
X^{3+i} = r J^i
\]

where \(J^i\) are the generators of the N-dimensional representation of SU(2). This configuration contains a D5-brane with the geometry \(T^3 \times S^2\) where \(S^2\) is a sphere of radius \(r\) in the space spanned by \(X^4 - X^6\). Under T-duality on the \(T^3\) this becomes the standard membrane sphere of matrix theory \(^\text{11}\). If the D5-brane configuration is acted on by S-duality, the resulting configuration contains a NS5-brane of geometry \(T^3 \times S^2\). The T-dual of this is an NS5-brane constructed from D0-branes, which should carry the first moment of the NS5-brane charge \(I_5^{012345(6)}\), and which should couple (magnetically) to the first derivative of the NS-NS 2-form field of the IIA theory. But we do not know any expression for the NS5-brane charge of a system of 0-branes. Such an expression would correspond to a charge for the transverse 5-brane of matrix theory, which is believed to vanish identically.

This apparent puzzle can be resolved by observing that even if the moment \(I_5^{012345(6)}\) of the NS5-brane charge vanishes, the NS5-brane configuration we have just described couples correctly to the 2-form field of the IIA theory. The initial configuration carried a nonzero charge \(\text{Tr} (X^4 X^5 X^6)\). The S-dual IIB NS5-brane configuration thus carries a charge \(\text{Tr} (X^7 X^8 X^9)\). The same charge is carried by the T-dual IIA configuration which should describe \(N\) 0-branes as well as an NS5-brane with geometry \(T^3 \times S^2\). In the IIA background-dependent D0-brane action \(^\text{11}\) this charge couples to the field \(\partial_7 B_{89}\). But this field is dual to the component \(\partial_{[6} B_{012345]}\) which we expect to describe the appropriate dipole moment of the NS5-brane configuration. Thus, we see that although there is no operator describing transverse NS5-branes in matrix theory, and no corresponding NS5-brane operator \(I_5^{0ijklm}\) in the IIA theory, it is still possible to construct 0-brane configurations which describe finite volume NS5-branes with higher multipole moments, which couple correctly to the background gravitational fields. In matrix theory, such compact transverse M5-brane configurations will couple correctly to the supergravity 3-form field even though the operator \(M^{+ijklm(n)}\) is identically zero.

\(^{15}\)Thanks to Ofer Aharony for discussions related to this issue
5 Relation to the AdS/CFT correspondence

In this section we discuss applications of our results to the study of D-brane black holes and a relation to the AdS/CFT correspondence.

5.1 Absorption by black holes

The most obvious application of the actions we have derived is to the study of D-branes interacting with weak bulk supergravity fields. For example, the probability for absorption of a particle $\phi$ by a stack of coincident branes is exactly determined by the operators coupling linearly to $\phi$ in the nonabelian D$p$-brane action. If $\phi$ couples to an operator $O_\phi$, then the absorption cross-section for $\phi$ may be read off from the two point function (see eg. [52])

$$\langle O_\phi(x)O_\phi(0) \rangle$$

At low energies, only the lowest dimension part of the operator will contribute, and these lowest dimension parts have been completely determined by our results for all supergravity fields. A specific example which has already appeared [53], is the study of dilaton absorption by D3-branes. In that paper, we used our results to determine the lowest dimension operators coupling to all derivatives of the dilaton field in the nonabelian D3-brane action. Using these operators, we were able to compute the low-energy absorption cross-section for all partial waves of the dilaton. We found that the results precisely agreed with the classical supergravity calculation for dilaton partial wave absorption in the D3-brane geometry, giving evidence for the equivalence between the D3-brane world-volume theory and physics of supergravity near the branes.

The exact numerical agreement found provided a stringent check of our results here, including normalizations. In particular, we note that the symmetrized trace ordering prescription was crucial for agreement. An important point to note about the comparison is that the supergravity calculation is reliable only in a limit corresponding to strong t’Hooft coupling in the world-volume theory, which ensures weak curvatures in supergravity. The agreement with our gauge theory calculation, carried out at weak coupling, implies that the calculated two point functions are not renormalized. Using the operators we have derived, it should be possible to perform an explicit gauge theory calculation showing that the first $g^2N$ correction vanishes. This would be an excellent test of our results, since the “purely nonabelian” terms in our operators, involving commutators of nonabelian fields, contribute to the two point function only at this subleading order.

5.2 Particle-operator correspondence

Particle absorption calculations of the type just discussed provided early motivation for the AdS/CFT conjecture [3], which in the most well-known example states that the physics of supergravity in the near-horizon geometry of D3-branes ($AdS^5 \times S^5$) is exactly described by
the low energy physics of the D3-brane world-volume fields, namely $N = 4$, $D = 4$ Super-Yang-Mills theory. In this subsection, we describe how the actions we have derived may be used to obtain useful information about the AdS/CFT correspondence, and also how the AdS/CFT correspondence may be used to obtain useful information about matrix theory.

Consider again the process of particle absorption by D3-branes. At very low energies, the near horizon region and the asymptotically flat part of the D3-brane geometry almost decouple. In this limit, particle absorption may be understood as a particle $P_0$ in the asymptotic region producing a very weak excitation in the transition region between Minkowski space and $AdS^5 \times S^5$ which in turn excites one of the particles $P$ of $AdS^5 \times S^5$ (see for example [54]). In the D-brane world-volume picture, the particle $P_0$ turns on an operator $O_P$ in the gauge theory which governs the decay of $P_0$ into particles on the world-volume. The exact correspondence between particles on $AdS^5 \times S^5$ and operators in the $N = 4$ Super-Yang-Mills world-volume theory is precisely the correspondence between the particles $P$ and the operators $O_P$ which are excited by a given asymptotic particle $P_0$.

This logic suggests a precise way of using our actions to determine the operator $O_P$ corresponding to a given supergravity particle $P$ in $AdS^5 \times S^5$, including normalization. We consider the full supergravity solution corresponding to a stack of a large number of D3-branes in the limit where the near horizon geometry is almost decoupled. As for $AdS^5 \times S^5$, this space will have a set of independently propagating excitations (normal modes). Very near the brane, the equations of motion reduce to those for $AdS^5 \times S^5$, so one of the particles (normal modes) $P'$ of the full supergravity solution must correspond to our particle $P$. In the asymptotically flat region, $P'$ will look like some freely propagating particle $P_0$. This is precisely the particle, discussed above, that when sent in from infinity will excite the particle $P$ in AdS. Thus, to determine the gauge theory operator corresponding to $P$, we simply read off the leading operator coupling to $P_0$ in the nonabelian D3-brane action expanded about flat space.

Though this prescription is well defined in principle, it is usually a nontrivial task to determine the particle $P_0$ in the asymptotic region which corresponds to the AdS particle $P$. It requires diagonalizing the supergravity equations of motion in the two regions (and sometimes in an intermediate region) and matching the solutions on the overlaps. In particular, the combination of supergravity fields describing $P_0$ may in general be different than that describing the particle $P$ in the $AdS$ region.

In [55], Das and Trivedi gave a slightly different prescription, also using the D3-brane action, for determining the relevant operators. Rather than choosing the operator coupling to $P_0$ in the expansion of the flat space D3-brane action, they suggested taking the operator coupling to the particle $P$ in the D3-brane action expanded about AdS space. It may be that both prescriptions give the same result. Certainly in the case of a minimally coupled scalar, the supergravity fields defining $P$ and $P_0$ are the same, so the two methods will give the same operator at least up to normalization. For fields with more complicated propagators, further work is needed to make the correspondence between operators on the D-brane and
asymptotic fields in the extremal geometry precise.

5.3 AdS/CFT and matrix theory

Our results also suggest a precise link between matrix theory and the AdS/CFT conjecture (for previous discussions of connections between these conjectures, see [56, 57, 58, 59, 60]). In short, the $D = 10$ Super-Yang-Mills theory operators whose dimensional reduction to $D = 3 + 1$ correspond to particles in $AdS^5 \times S^5$ are linear combinations of the operators whose dimensional reductions to $D = 0+1$ describe the 11-dimensional supergravity currents corresponding to a given matrix theory configuration. On the AdS side, we have a series of operators corresponding to an infinite tower of Kaluza-Klein modes for each particle, whereas in matrix theory, the series of operators corresponds to the infinite set of multipole moments of a given current

16These matrix theory operators have also been related to the AdS/CFT correspondence by relating them to a tower of Kaluza-Klein states in the near-horizon geometry of $N$ D0-branes [51].

An explicit example of this was given in [53], where the Yang-Mills operator corresponding to the $l$-th Kaluza-Klein mode of the dilaton was found to be a combination of the $l$th multipole moment of various components of the Matrix Theory stress-energy tensor

17Here, the indices $i, k$ are transverse to the brane, while the index $\hat{a}$ runs over the spatial brane directions.

\[
O^{k_1 \cdots k_l}_\phi = \frac{1}{6} T^{ij(k_1 \cdots k_l)} - \frac{1}{3} T^{\hat{a}\hat{a}(k_1 \cdots k_l)} - \frac{1}{3} T^{++(k_1 \cdots k_l)}.
\]

(32)

One particularly interesting aspect of this equivalence is that whereas the matrix theory expressions for the supergravity currents were determined by a general a one-loop gauge theory calculation, the operators corresponding to particles in $AdS$ may be determined (except for normalization) simply by acting with tree level supersymmetry generators on the chiral primary operators

\[
\text{STr} \left( X^{i_1} \cdots X^{i_l} \right) - \{\text{traces}\}
\]

(33)

By the correspondence, it should therefore be possible to obtain the results of one-loop matrix theory calculations simply by computing the action of tree level supersymmetry generators on chiral primary fields. This observation was put into practice in [53] where the four fermion terms in the operators corresponding to all partial waves of the dilaton field (and therefore in the matrix theory currents on the right side of (32)) were computed by acting with four supercharges on the chiral primary operators (33). The matrix theory operator corresponding to the integrated component $T^{--}$ of the stress energy tensor is essentially the complete one-loop matrix theory effective action, which has the form

\[
\text{STr} \left( F^4 - \frac{1}{4} (F^2)^2 + \text{fermions} \right).
\]

On the AdS side, we recognize this as the highest dimension operator $O_8$ corresponding to the Weyl mode of the metric. So a complete calculation of the one-loop Super-Yang-Mills theory effective action appears to be equivalent to simply applying eight supercharges...
to the chiral primary operator $\text{STr} (X^i X^j X^k X^l)$. We expect that the agreement between these two approaches to calculating the dimension 8 operator is a result of the high degree of supersymmetry in the system—indeed, another way to determine the structure of this operator would be to generalize the methods of \cite{62} to show that in the $SU(N)$ theory this operator is uniquely determined by supersymmetry.

Finally, we note that since all $N = 4$ SYM operators corresponding to particles in $AdS^5 \times S^5$ are constrained by supersymmetry to be symmetrized (the short representations to which these operators belong are obtained by acting with supercharges on the symmetrized chiral primary operator), an equivalence between these operators and the matrix theory current operators would provide an explanation for the symmetrized trace both in the matrix theory operators and in leading terms in the D-brane actions which we have derived. In this case, the terms in the Dp-brane actions which vanish in the matrix theory or AdS/CFT limits\(^{18}\) may not be subject to the same constraints from supersymmetry and therefore might be expected to contain non-symmetrized terms. We discuss this point further below.

6 Discussion

In this paper, we have derived the leading operators in all Dp-brane actions coupling linearly to each supergravity field, \((\mathbf{7,8})\). Our results have been obtained by applying the rules of T-duality to previous results for the D0-brane action. Using the same T-duality rules we have shown that the abelian D9-brane action may be used to deduce a large set of additional terms in the symmetrized part of the nonabelian actions for all lower Dp-branes. These expressions, which include terms with arbitrary numbers of background and world-volume fields appear as equations (14) and (16) respectively. These expressions do not represent the complete Dp-brane actions, and we now summarize the types of corrections that we expect to occur.

6.1 Higher order corrections

All of the leading operators that we have derived from matrix theory results are written in terms of a symmetrized trace over the nonabelian fields. We have argued above that this symmetrized trace prescription may be viewed as a constraint of supersymmetry on the terms in the D-brane actions which survive in the matrix theory or AdS/CFT limits. For higher order terms that vanish in these limits, the constraints no longer apply, so we do not have any reason to believe that the symmetrized trace prescription continues to hold. In particular, the higher order expressions (14) and (16) derived from the D9-brane action have been written in terms of a symmetrized trace simply because we have ignored possible terms with commutators of $F$’s. Indeed, it has been argued that such terms are necessary at order $F^6$ in the flat space nonabelian Born-Infeld action \cite{23}. These commutator terms may be viewed as

\(^{18}\)Note that it appears to be the same set of terms which are preserved in both limits.
higher derivative terms which vanish in the abelian theory, since $[F_{ab}, F_{cd}] = D_{[a} D_{b]} F_{cd}$. The Dp-brane actions also contain explicit higher derivative terms which do not vanish for the Abelian case. These are excluded in the definition of the Born-Infeld action; for a discussion see [30] and references therein.

A further type of correction to the actions we have derived involves terms of higher order in the background fields. The expressions (14) and (16) derived from the D9-brane actions contain terms at all orders in the background fields, however it may be shown that these expressions are not complete since they do not satisfy the "geodesic length criterion" proposed by Douglas [63]. This criterion is relevant when we consider two parallel separated branes, described in the world-volume theory by giving an expectation value to the diagonal scalar fields which correspond to the positions of the two branes. For non-zero separation, the off-diagonal scalar fields which arise from strings stretching between the branes should acquire a mass equal to the geodesic distance between the branes. At linear order in the background metric, it was shown in [31] that this geodesic distance criterion is exactly satisfied for the D0-brane action we have found. However, for agreement at second order in the metric, it is possible to show explicitly that additional terms involving $(\partial h)^2$ are required beyond those appearing in the action (14). Some of these terms have been worked out in the case of D-branes on Kahler manifolds in [64, 65], but for the most part these terms are unknown. Another set of terms which appear at higher order in the background fields arise from the anomalous couplings discussed in [14, 15, 16]. These terms presumably all have nonabelian counterparts which remain to be investigated.

Finally, we note that there may be terms in the Dp-brane action involving more than a single trace over world-volume fields. These would seem necessary in order to reproduce string theory amplitudes whose worldsheet involved more than a single boundary, for example an annulus diagram with various world-volume fields inserted on each boundary. It is possible that some of these terms could be derived using our methods by performing a matrix theory calculation at higher than one loop.

We have listed all possible additional terms which may appear in a nonabelian Born-Infeld type action in a general background, based on our understanding of the fields and symmetries of the theory as well as the idea that such an action is defined as a sum over string backgrounds. It is not completely clear that there is really a unique well-defined supersymmetric nonabelian Born-Infeld action which will satisfy all the criteria which the abelian Born-Infeld action satisfies and which additionally satisfies Douglas' geodesic length condition and the condition discussed in [28] that the string spectrum describing small fluctuations around a fixed D-brane background should be reproduced by the NBI action. The terms we have found here are good evidence that it is possible to extend the nonabelian Super-Yang-Mills theory describing multiple Dp-branes to include higher order terms. One might argue, however, that these linear couplings between background fields and the multiple Dp-brane world-volume theory are protected by supersymmetry and are therefore uniquely determined while the definition of the higher order terms in the nonabelian Born-Infeld ac-
tion might be more ambiguous due to the lack of supersymmetric protection. It will be very interesting to see whether further work will reveal a well-defined NBI action at all orders.

6.2 Further directions

The most obvious direction in which this work could be pursued further is to try to use higher-order matrix theory calculations to extend the Born-Infeld action to include terms coupling to higher powers of the background fields. The next concrete step would be to generalize the 3-graviton calculation of Okawa and Yoneya \[66\] to a general 3-body calculation, which would be a general 2-loop calculation in an $SU(N)$ gauge theory with a block-diagonal background containing 3 blocks of arbitrary size. As argued in \[32\], this general result would indicate the structure of the quadratic coupling of a matrix theory object to background fields, and by using the Seiberg-Sen limit this result should translate into a well-defined result for the quadratic couplings of a system of $N$ D0-branes to the background fields. By using T-duality as in this paper, this would lead to the quadratic couplings of a system of $D_p$-branes of arbitrary dimension to the background fields. The correspondence between the matrix theory calculation just described and supergravity interactions at the first nonlinear order depends upon the nonrenormalization of the general 2-loop $SU(N)$ calculation. The agreement of the 2-loop calculation in the case of $SU(3)$ with supergravity gives hope that such a nonrenormalization theorem may hold for general $N$. Note, however, that the methods of \[62\] do not apply to the 2-loop $SU(N)$ calculation when $N > 3$ so the terms we need may not be renormalized \[67\]. If this is the case this approach may not help in understanding the nonlinear terms in the NBI action until a better understanding is found of how matrix theory behaves in the large $N$ limit.

There are many other problems to which the work described here can be applied. It would be nice to see how far the results of \[53\] can be generalized to describe absorption of particles by $D_p$-branes at leading and higher orders. The relationship between T-duality and spatial dependence of background supergravity fields is also an interesting question for further investigation. It would also be interesting to carry out a more systematic analysis of the S-duality of the linear terms we have found in the NBI action; this should give a complete set of results for the S-duality transformation properties of all the short operators in maximally supersymmetric 4D SYM theory.

Acknowledgments

We would like to thank Ofer Aharony, Lorenzo Cornalba, Ken Intriligator, Karl Millar, Samir Mathur, Rob Myers, Ricardo Schiappa and John Schwarz for helpful conversations. We would like to thank Rob Myers for sharing an early draft of his preprint \[35\] with us. The work of MVR is supported in part by the Natural Sciences and Engineering Research Council of Canada (NSERC). The work of WT is supported in part by the A. P. Sloan Foundation and in part by the DOE through contract #DE-FC02-94ER40818.
A Supercurrents from matrix theory

We reproduce here for convenience the matrix theory forms of the multipole moments of the 11D supercurrent found in [27, 32]. Dropping a factor of $1/R$ from each expression, the stress tensor $T^{ij}$, membrane current $J_{ijk}$ and 5-brane current $M_{ijklmn}$ have integrated (monopole) components

\[ T^{++} = \text{STr} (1) = N \]
\[ T^{+i} = -\text{STr} (F^{0i}) \]
\[ T^{+i} = \text{STr} (F_{0}^{\mu} F_{\mu}^{\mu} + \frac{1}{4} F_{\mu\nu}^{ij} F_{\mu\nu}^{ij} + \frac{i}{2} \bar{\Theta} \Gamma^{0} D_{0} \Theta) \]
\[ T^{ij} = \text{STr} (F_{\mu}^{i} F_{\mu}^{j} + \frac{i}{4} \bar{\Theta} \Gamma^{i} D_{j} \Theta + \frac{i}{4} \bar{\Theta} \Gamma^{j} D_{i} \Theta) \]
\[ T^{-i} = -\text{STr} (F_{0}^{\mu} F_{\mu}^{\nu} F_{\nu}^{i} + \frac{1}{4} F_{\mu}^{0i} F_{\mu}^{\mu} F_{\nu}^{0i} F_{\nu}^{\mu} + \frac{i}{8} F_{\mu}^{0i} \bar{\Theta} \Gamma^{0} \Gamma^{i} D_{0} \Theta - \frac{i}{4} F_{\mu}^{0i} \bar{\Theta} \Gamma^{\mu} \Gamma^{0i} D_{\mu} \Theta \]
\[ T^{--} = \frac{1}{4} \text{STr} (F_{\mu}^{i} F_{\mu}^{\nu} F_{\lambda}^{\delta} F_{\delta}^{i} - \frac{1}{4} F_{\mu}^{i} F_{\mu}^{\mu} F_{\gamma}^{i} F_{\gamma}^{\delta} \]
\[ + i F_{\mu}^{i} F_{\gamma}^{i} \bar{\Theta} \Gamma^{\mu} \Gamma^{\gamma} D_{\mu} \Theta + \{\text{four fermion terms}\}) \]
\[ J^{+ij} = -\frac{1}{6} \text{STr} (F^{ij}) \]
\[ J^{+i} = \frac{1}{6} \text{STr} (F_{0}^{i} F_{\mu}^{\nu} F_{\nu}^{\mu} - \frac{1}{4} F_{\mu}^{i} F_{\mu}^{\mu} F_{\gamma}^{i} F_{\gamma}^{\delta} - \frac{i}{4} \bar{\Theta} \Gamma^{i} D_{0} \Theta) \]
\[ J^{ij} = + \frac{1}{6} \text{STr} (F_{0}^{i} F_{jk}^{k} + F_{0j}^{ij} F_{ki}^{k} + F_{0k}^{ij} F_{ik}^{k} - \frac{3i}{4} \bar{\Theta} \Gamma_{0}^{ij} D_{k} \Theta + \frac{i}{4} \bar{\Theta} \Gamma^{ij} D_{k} \Theta) \]
\[ J^{-ij} = \frac{1}{6} \text{STr} (F^{ij} F_{\mu}^{\nu} F_{\nu}^{ij} + \frac{1}{4} F_{\mu}^{ij} F_{\nu}^{ij} F_{\mu}^{\nu} \]
\[ - \frac{i}{8} F_{\mu}^{ij} \bar{\Theta} \Gamma^{ij} D_{i} \Theta + \frac{i}{8} F_{\mu}^{ij} \bar{\Theta} \Gamma^{i} D_{j} \Theta - \frac{i}{4} F_{\mu}^{ij} \bar{\Theta} \Gamma^{ij} D_{\mu} \Theta \]
\[ + \frac{1}{8} \bar{\Theta} \Gamma^{ij} D_{i} \Theta \]
\[ M^{+ijkl} = \frac{1}{12} \text{STr} (F_{ij}^{ijkl} + F_{ik}^{ij} F_{lj}^{kl} + F_{lj}^{ij} F_{ik}^{kl} - i \bar{\Theta} \Gamma^{ijkl} D_{ij} \Theta) \]
\[ M^{-ijklm} = - \frac{5}{4} \text{STr} (F_{0}^{ijkl} F_{m}^{ijkl} + \frac{i}{2} F_{ijkl}^{ijkl} \bar{\Theta} \Gamma^{ijkl} D_{m} \Theta) \]

Here, STr denotes a symmetrized trace in which we average over all possible orderings of the matrices the trace, with commutators being treated as a unit. Time derivatives are taken with respect to Minkowski time $t$. Indices $i, j, \ldots$ run from 1 through 9, while indices $a, b, \ldots$ run from 0 through 9. In these expressions we have used the definitions $F_{0i} = \dot{X}^{i}, F_{ij} = i[X^{i}, X^{j}]$. We do not know of a matrix form for the transverse 5-brane
current components $M^{+ijklm}$, $M^{ijklmn}$, and in fact comparison with supergravity suggests that these should be 0 for any matrix theory configuration.

The higher multipole moments of these currents contain one set of terms which are found by including the matrices $X^k_1, \ldots, X^k_n$ into the symmetrized trace as well as more complicated spin contributions. We may write these as

$$
T^{IJ(i_1 \cdots i_k)} = \text{Sym} (T^{IJ}; X^{i_1}, \ldots, X^{i_k}) + T^I_{\text{fermion}}^{J(i_1 \cdots i_k)}
$$

$$
J^{JK(i_1 \cdots i_k)} = \text{Sym} (J^{JK}; X^{i_1}, \ldots, X^{i_k}) + J^J_{\text{fermion}}^{K(i_1 \cdots i_k)}
$$

$$
M^{JKLMN(i_1 \cdots i_k)} = \text{Sym} (M^{JKLMN}; X^{i_1}, \ldots, X^{i_k}) + M^J_{\text{fermion}}^{KLMN(i_1 \cdots i_k)}
$$

(34)

where some simple examples of the two-fermion contribution to the first moment terms are

$$
T^{+i(j)}_{\text{fermion}} = -\frac{1}{8R} \text{Tr} (\bar{\Omega} \Gamma^{[0ij]} \Theta)
$$

$$
T^{+(i)}_{\text{fermion}} = -\frac{1}{16R} \text{Tr} (F_{\mu
u} \Theta \gamma^{[\mu\nu]} \Theta - 4 \bar{\Theta} F_{0\mu} \gamma^{[0\mu]} \Theta)
$$

$$
T^{ij(l)}_{\text{fermion}} = \frac{1}{8R} \text{Tr} (F_{j\mu} \bar{\Theta} \gamma^{[\mu\nu]} \Theta + \bar{\Theta} F_{\mu} \gamma^{[\mu\nu]} \Theta)
$$

$$
J^{+ij(k)}_{\text{fermion}} = \frac{i}{48R} \text{Tr} (\bar{\Theta} \Gamma^{[ijk]} \Theta)
$$

$$
J^{+(i)}_{\text{fermion}} = \frac{1}{48R} \text{Tr} (F_{0\mu} \Theta \gamma^{[\mu\nu]} \Theta + \bar{\Theta} F_{\mu} \gamma^{[\mu\nu]} \Theta)
$$

$$
M^{+ijklm}_{\text{fermion}} = -\frac{i}{16R} \text{Str} (\Theta F^{[jk]} \Gamma^{[ilm]} \Theta)
$$

The remaining two-fermion contributions to the first moments and some four-fermion terms are also determined by the results in [32].

There are also fermionic components of the supercurrent which couple to background fermion fields in the supergravity theory. We have not discussed these couplings in this paper, but the matrix theory form of the currents is determined in [32].

There is also a 6-brane current appearing in matrix theory related to nontrivial 11D background metrics. The components of this current as well as its first moments are

$$
S^{ijklmn} = \frac{1}{R} \text{Str} (F_{[ij} F_{kl} F_{mn]})
$$

$$
S^{ijklmn(p)} = \frac{1}{R} \text{Str} (F_{[ij} F_{kl} F_{mn]} X_p - \theta F_{[kl} F_{mn] \gamma_{pqr]} \theta)
$$

$$
S^{ijklmp} = \frac{7}{R} \text{Str} (F_{[ij} F_{kl} F_{mn} \bar{X}_p) + (\theta^2, \theta^4 \text{ terms})
$$

$$
S^{ijklmp(q)} = \frac{7}{R} \text{Str} (F_{[ij} F_{kl} F_{mn} \bar{X}_p X_q - \theta \bar{X}_{[ij} F_{kl} F_{mn] \gamma_{pqr]} \theta + \frac{i}{2} \theta F_{[jk} F_{lm} F_{np} \gamma_{qr]} \theta)
$$

(35)

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