Spectroscopy of Family Gauge Bosons

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Abstract

Spectroscopy of family gauge bosons is investigated based on a U(3) family gauge boson model proposed by Sumino. In his model, the family gauge bosons are in mass eigenstates in a diagonal basis of the charged lepton mass matrix. Therefore, the family numbers are defined by \((e_1, e_2, e_3) = (e, \mu, \tau)\), while the assignment for quark sector are free. For possible family-number assignments \((q_1, q_2, q_3)\), under a constraint from \(K^0-\bar{K}^0\) mixing, we investigate possibilities of new physics, e.g. production of the lightest family gauge boson at the LHC, \(\mu^-N \rightarrow e^-N\), rare \(K\) and \(B\) decays, and so on.

PCAC numbers: 11.30.Hv, 12.60.-i, 14.70.Pw,

1 Introduction

The most exciting subject in particle physics is to understand the origin of “flavor”. It seems to be very attractive to understand “families” (“generations”) in quarks and leptons from concept of a symmetry [1]. Since the observed masses of quarks and leptons are in range of \(10^{-3} - 10^2\) GeV, we may suppose a possibility that the lightest family gauge boson can be observed by terrestrial experiments, e.g. at the LHC.

However, when we try to consider such a visible family gauge boson model, we always meet with constraints from the observed pseudo-scalar-anti-pseudo-scalar meson mixings \(P^0-\bar{P}^0\) \((P = K, D, B, B_s)\). The constraints are too tight to allow family gauge bosons with lower masses. It is usually taken that a scale of the symmetry braking is considerably high (e.g. an order of, at least, \(10^4\) TeV). However, there is a family gauge boson model [2] in which such severe constraints from the \(P^0-\bar{P}^0\) mixings can be considerably loosen. In the model, the family gauge symmetry is U(3), so that a number of the family gauge bosons are nine (not eight), and quarks and leptons interacts with the family gauge bosons \(A_{ij}\) is given by

\[
H_{\text{fam}} = \frac{g_F}{\sqrt{2}} \left[ (\bar{e}_i \gamma_{\mu} e_j) + (\bar{\nu}_i \gamma_{\mu} \nu_j) + U^u_{ik} U^u_{jl} (\bar{u}_k \gamma_{\mu} u_l) + U^d_{ik} U^d_{jl} (\bar{d}_k \gamma_{\mu} d_l) \right] (A_{ij})^\mu, \tag{1.1}
\]

where \((u^0_i, d^0_i)\) are eigenstates of the family symmetry U(3) and those are define by \((u^0_i, d^0_i) = (U^u_{ij} u_j, U^d_{ij} d_j)\). (The expression (1.1) is based on an extended version [2] of the Sumino model [3]. See in the next section.) Note that in the limit of no quark mixing, the family number is exactly conserved, so that the whole \(P^0-\bar{P}^0\) mixings are forbidden. (A brief review is given in the next section.)
Another remarkable point in the Sumino model is that the family gauge coupling constant $g_F$ and ratios among the family gauge boson masses $M_{ij}$ are not free, and when once a model is settled, $g_F$ and $M_{ij}/M_{kl}$ are fixed. Therefore, the model can give a clear answer to observations.

The family number in the Sumino model [3] is defined by the charged lepton sector $e_i = (e, \mu, \tau)$ and the gauge boson masses are given proportionally to the charged lepton masses. On the hand, family number in the quark sector may be $d_0^i = (d^0, s^0, b^0)$, but it may be an inverted assignment $d_0^i = (b^0, s^0, d^0)$, and also a twisted assignment $d_0^i = (b^0, d^0, s^0)$. (Of course, we consider the same assignments for $u_0^i$ because of $SU(2)_L$ symmetry.) There are six possible assignments of $(u_0^i, d_0^i)$ correspondingly to $e_i = (e, \mu, \tau)$. (Hereafter, for convenient, we will denote $q_0^i$ as $q_i$ simply.)

In the present paper, based on the Sumino model [3] (and also an extended Sumino model [2]), we investigate visible effects of the family gauge bosons, i.e. the deviations from the $e$-$\mu$-$\tau$ universality, rare $K$ and $B$ decays, $\mu$-$e$ conversion, direct production of the lightest family gauge boson, and so on. We will conclude that the case with a twisted assignment $d_0^i = (b^0, d^0, s^0)$ can give rich phenomenology to us.

2 Sumino mechanism

Priori to our investigation, let us give a brief review of the Sumino model and its extended version.

The necessity of the family gauge bosons was first pointed out by Sumino [3]. Sumino has paid why the charged lepton mass relation [4]

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}, \quad (2.1)$$

is well satisfied by the pole masses (not by the running masses). The running masses $m_{ei}(\mu)$ are given by [5]

$$m_{ei}(\mu) = m_{ei} \left[ 1 - \frac{\alpha_{em}(\mu)}{\pi} \left( 1 + \frac{3}{4} \log \frac{\mu^2}{m^2_{ei}(\mu)} \right) \right]. \quad (2.2)$$

If the factor $\log(m_{ei}^2/\mu^2)$ in Eq.(2.2) is absent, then the running masses $m_{ei}(\mu)$ are also satisfy the formula (2.1). Sumino has required that contribution of family gauge bosons to the charged lepton mass $m_{ei}(\mu)$ cancels the factor $\log(m_{ei}^2/\mu^2)$ due to photon. Therefore, masses of the gauge bosons have to be given by

$$M_i^2(A_j^i) \equiv M_{ij}^2 = k(m_{ei}^n + m_{ej}^n), \quad (2.3)$$

2
\( n = 1 \) in the original Sumino model \([3]\), with the cancellation condition

\[
\left( \frac{g_F}{\sqrt{2}} \right)^2 = \frac{2}{n} e^2 = \frac{4}{n} \left( \frac{g_w}{\sqrt{2}} \right)^2 \sin^2 \theta_w.
\] (2.4)

Here, the family gauge boson coupling constant \( g_F \) is defined by

\[
\mathcal{H}^\text{Sumino}_{\text{fam}} = \frac{g_F}{\sqrt{2}} \sum_{f=u,d,\nu,e} (\bar{f}_i^L \gamma_\mu f_L j - \bar{f}_R j^\mu f_R^i) (A_i^j)\mu.
\] (2.5)

The unfamiliar current form in Eq.(2.5) is due to an assignment \((f_L, f_R) = (3, 3^*)\) of \(\text{U}(3)\) family symmetry \((f = u, d, \nu, e)\). The assignment is inevitably required in order to obtain the minus sign for the cancellation between \(\log m_{e_i}^2\) and \(\log M_{ii}^2\).

However, the assignment \((f_L, f_R) = (3, 3^*)\) in the original Sumino model causes the following problems: (i) The model cannot be anomaly free. (ii) Effective current-current interactions with \(\Delta N_{\text{fam}} = 2\) \((N_{\text{fam}}\text{ is a family number})\) appear inevitably.

In order to evade these problems, an extended version of the Sumino model (K-Y model) \([2]\) has been proposed by Yamashita and the author: (i) \(\text{U}(3)\) assignment is \((f_L, f_R) = (3, 3)\), so that the model is anomaly free. (ii) In order to obtain the minus sign of cancellation, the family gauge boson masses are given by an inverted mass hierarchy

\[
M_{ij}^2 = k \left( \frac{1}{m_{e_i}^2} + \frac{1}{m_{e_j}^2} \right),
\] (2.6)

\((n = 1 \text{ in the K-Y model})\). The cancellation condition is given by

\[
\left( \frac{g_F}{\sqrt{2}} \right)^2 \simeq \frac{1}{n} \frac{3}{2} \zeta e^2.
\] (2.7)

Here, the coupling constant \(g_F\) is defined by

\[
\mathcal{H}^{K-Y}_{\text{fam}} = \frac{g_F}{\sqrt{2}} \sum_{f=u,d,\nu,e} (\bar{f}_i^L \gamma_\mu f_L j - \bar{f}_R j^\mu f_R^i) (A_i^j)\mu.
\] (2.8)

(For convenience of comparison with the Sumino model, the coupling constant \(g_F\) in the original K-Y model \([2]\) has been changed into \(g_F/\sqrt{2}\).) Note that, differently from the original Sumino model, the cancellation in the K-Y model is satisfied only approximately. The factor \(\zeta\) in Eq.(2.7) is a fine tuning factor which gives \(K(\mu) \simeq 2/3\) almost independently of \(\mu\), and it is numerically given by \(\zeta = 1.752\) in the case of \(n = 1\).
In the present investigation, it is essential that the family gauge boson interactions are given by Eq.(1.1). The interaction (1.1) has been derived from the following scenario: The family symmetry breaking is not caused by scalars $3$ and/or $6$ of $U(3)$, but it is caused by a scalar $(3,3^*)$ of $U(3) \times U(3)'$, which are broken at $\Lambda$ and $\Lambda'$ ($\Lambda \ll \Lambda'$), respectively. (In the original Sumino model, the scalar was $(3,3)$ of $U(3) \times O(3)$. In the present investigation, the difference is not essential.) Therefore, a direct gauge boson mixing $A^j_i \leftrightarrow A^i_j$ ($i = 1, 2, 3$) does not appear in this model. The $U(3) \times U(3)'$ is broken by a scalar $\Psi^i$ which is $(3,3^*)$ of $U(3) \times U(3)'$, i.e. by a vacuum expectation value (VEV) $\langle \Psi^i \rangle = v_i \delta_i^a$. In the limit of $\Lambda' \gg \Lambda$, we obtain the $(3,3)$ family current interaction (1.1). In the quark sector, since quark mass matrices $M_u$ and $M_d$ are, in general, not always diagonal on the diagonal basis of $M_e$, so that family number violations at tree level are caused only through the mixing matrices among up- and down-quarks, $U^u \neq 1$ and $U^d \neq 1$.

The gauge boson masses $M_{ij}$ are dominantly generated by VEV of scalar $\Psi^i$ which is $(3,3^*)$ of $U(3) \times U(3)'$, and whose VEV is given by $\langle \Psi^i \rangle = \delta_i^a v_i$. Then, we obtain family gauge boson masses
\[
M_{ij}^2 = \frac{1}{2} g_3^2 (|v_i|^2 + |v_j|^2) + \cdots , \quad (2.9)
\]
where “$+ \cdots$” denotes contributions from other scalars which are negligibly small, so that the family gauge boson masses $M_{ij} \equiv M(A^i_j)$ satisfy relations
\[
2M_{ij}^2 = M_{ii}^2 + M_{jj}^2 . \quad (2.10)
\]
(Note that the, in the Sumino model, family gauge bosons acquire their masses dominantly by a scalar $\Phi = (3,3^*)$ of $U(3) \times U(3)'$ which also gives masses of the charged leptons, while, in the K-Y model, the gauge bosons dominantly acquire their masses by the scalar $\Psi$ which is different scalar from $\Phi$, and where a relation $|\langle \Psi \rangle|^2 \gg |\langle \Phi \rangle|^2$ has been assumed. Exactly speaking, the relation (2.10) is satisfied only approximately in both models.)

In the present paper, we investigate the following two Cases A and B which satisfy the Sumino cancellation mechanism:

**Case A**: The inverted gauge boson mass hierarchy (K-Y model like)

The mass ratios are given by Eq.(2.6):
\[
M_{33} : M_{32} : M_{22} : M_{31} : M_{21} : M_{11} = 1 : \sqrt{\frac{a^2 + 1}{2}} : a : \sqrt{\frac{b^2 + 1}{2}} : \sqrt{\frac{b^2 + a^2}{2}} : b \quad (2.11)
\]
where
\[
a = \frac{M_{22}}{M_{33}} = \left(\frac{m_\tau}{m_\mu}\right)^{n/2} , \quad b = \frac{M_{11}}{M_{33}} = \left(\frac{m_\tau}{m_e}\right)^{n/2} . \quad (2.12)
\]

**Case B**: The normal gauge boson mass hierarchy (the original Sumino model type)
The mass ratios are given by Eq.(2.3):

\[ M_{11} : M_{12} : M_{13} : M_{23} : M_{33} = 1 : \sqrt{a^2 + 1} : a : \sqrt{b^2 + 1} : \frac{b^2 + a^2}{2} : b, \] (2.13)

where

\[ a = \frac{M_{22}}{M_{11}} = \left(\frac{m_\mu}{m_e}\right)^{n/2}, \quad b = \frac{M_{33}}{M_{11}} = \left(\frac{m_\tau}{m_e}\right)^{n/2}. \] (2.14)

In the original Sumino model, the currents with an unwelcome form as shown in Eq.(2.5) appear inevitably. We want less contribution of the family gauge bosons to the $P^0-\bar{P}^0$ mixing. Therefore, in the present investigation in Case B, we slightly change the original Sumino model into a modified model where leptons $\ell_i = (\nu_i, e_i)$ are still assigned to $(\ell_L, \ell_R) = (3, 3^*)$, while quarks $q_i = (u_i, d_i)$ are assigned to $(q_L, q_R) = (3, 3)$, so that the quark sector is anomaly free.

In Case B, the gauge boson interactions are given by

\[ \mathcal{H}^{(B)}_{\text{fam}} = \frac{g_F}{\sqrt{2}} \left[ \sum_{i=e,\mu} (\bar{f}_i \gamma_{\mu} f_L - \bar{f}_R \gamma_{\mu} f_R) + \sum_{f=u,d} (\bar{f}_i \gamma_{\mu} f_j) (A^j_{\mu}) \right], \] (2.15)

instead of Eq.(2.5). However, the lepton currents with the unwelcome form still appear. (We will provide additional heavy leptons in order to remove anomaly in the lepton sector.)

### 3 Quark family arrangements and $P^0-\bar{P}^0$ mixing

Effective quark current-current interactions with $\Delta N_{\text{fam}} = 2$ are given by

\[ H^{\text{eff}} = \frac{1}{2} g_F^2 \left[ \sum_i (\lambda_i)^2 \frac{M_{ii}^2}{M_{ii}^2} + 2 \sum_{i<j} \lambda_i \lambda_j \frac{M_{ij}}{M_{ij}^2} \right] (\bar{q}_k \gamma_{\mu} q_l) (\bar{q}_k \gamma_{\mu} q_l) \] (3.1)

where

\[ \lambda_1 = U_{1k}^{q*} U_{1l}^q, \quad \lambda_2 = U_{2k}^{q*} U_{2l}^q, \quad \lambda_3 = U_{3k}^{q*} U_{3l}^q. \] (3.2)

For example, in a case of $K^0-\bar{K}^0$ mixing, $\lambda_i$ are given by

\[ \lambda_1 = U_{11}^{d*} U_{12}^d, \quad \lambda_2 = U_{21}^{d*} U_{22}^d, \quad \lambda_3 = U_{31}^{d*} U_{32}^d. \] (3.3)

These $\lambda_i$ with $k \neq l$ satisfy a unitary triangle condition

\[ \lambda_1 + \lambda_2 + \lambda_3 = 0. \] (3.4)
We define the effective coupling constant $G^{eff}$ in the current-current interaction as

$$G^{eff} = \frac{1}{2} g_F^2 \left[ \frac{\lambda_1^2}{M_{11}^2} + \frac{\lambda_2^2}{M_{22}^2} + \frac{\lambda_3^2}{M_{33}^2} + 2 \left( \frac{\lambda_1 \lambda_2}{M_{12}^2} + \frac{\lambda_2 \lambda_3}{M_{23}^2} + \frac{\lambda_3 \lambda_1}{M_{31}^2} \right) \right].$$

(3.5)

Note that all family gauge bosons contribute to the $P^{0} - \bar{P}^{0}$ mixing as seen in Eq.(3.1).

In order to demonstrate numerical results, we tentatively assume $U^u \simeq 1$ and $U^d \simeq V_{CKM}$ ($V_{CKM}$ is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [7]). Alternative case with $U^u \simeq V_{CKM}^\dagger$ and $U^d \simeq 1$ can give no contributions to $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$ and $B^0_s - \bar{B}^0_s$ mixings, so that it is good news for the present purpose. However, the case brings a more severe constraint on the gauge boson masses from the observed value of $D^0 - \bar{D}^0$ mixing.

The assumption $U^d \simeq V_{CKM}$ leads to values of $\lambda_i$,

$$\lambda_1 \approx 0.220, \quad \lambda_2 \approx -0.219, \quad \lambda_3 \approx -0.00035.$$ 

(3.6)

Therefore, in the limit of $\lambda_3 \approx 0$ and $\lambda_1 \approx -\lambda_2$, we obtain approximate relation

$$G^{eff}_K \approx \frac{g_F^2}{2} \frac{\lambda_1^2}{M_{11}^2} \left( or \ G^{eff}_K \approx \frac{g_F^2}{2} \frac{\lambda_2^2}{M_{22}^2} \right).$$

(3.7)

Thus, the $K^0 - \bar{K}^0$ mixing put a severe constraint on the lower bound of the family gauge boson mass $M_{11}$ for $M_{11} < M_{22}$ (or $M_{22} < M_{11}$). When we use the observed value [6] $\Delta m_{CP}^{obs} = (3.484 \pm 0.006) \times 10^{-18}$ TeV and a tentative standard model (SM) value [8] $\Delta m_{SM}^{fam} \sim 2 \times 10^{-18}$ TeV, we obtain a lower limit of the value $M_{11}/(g_F/\sqrt{2})$ [or $M_{22}/(g_F/\sqrt{2})$] $\sim 340$ TeV, where we have used a vacuum-insertion approximation (with no QCD correction)

$$\Delta m_{fam}^{fam} = \frac{1}{6} G^{eff}_K f_K f_K (1 + 2S_K),$$

(3.8)

and $S_K = m_K^2/(m_s + m_d)^2$. If we give the parameters $a$ and $b$ in Eq.(2.12) [or (2.14)], we can estimate $G^{eff}$ without approximation (3.7). In the next section, we will calculate constraints for $M_{ij}/(g_F/\sqrt{2})$ directly from Eq.(3.5) and by using $V_{CKM}$ with $CP$ violation phase.

Here, note that the CKM matrix $V_{CKM}$ is defined in the generation basis $u_i = (u, c, t)$ and $d_i = (d, s, b)$. Therefore, the notations $M_{ij}$ in Eqs.(3.1) are different from those defined by the diagonal bases of the charged lepton mass matrix $M_e$. In this paper, we investigate various assignments of $q_i = (q_1, q_2, q_3)$. As far as quark sector is concerned, the use of generation basis $d_i = (d, s, b)$ is convenient. Therefore, hereafter, for example, for Case B1 with family number $d_i = (b, s, d)$ (the case is defined in the next section), we denote $M_{11}$, $M_{12}$, $M_{22}$, · · · as $M_{bb}$, $M_{bs}$, $M_{ss}$, · · ·, respectively, in order to distinguish those from $M_{ij}$ defined in the family numbers. (For
convenience, we use down-quark names as the quark family numbers.) The physics is highly
dependent on the quark family assignments. The details are discussed in the next section.

4 Which quark-family assignment is favorable?

We find that $K^0$-$\bar{K}^0$ mixing puts the most severe constraints on the family gauge boson
masses $M_i$ compared with other $P^0$-$\bar{P}^0$ mixings. As seen in Eqs.(3.6) and (3.7), because of
$|\lambda_b|^2 \ll |\lambda_s|^2 \simeq |\lambda_d|^2$, the observed $K^0$-$\bar{K}^0$ mixing put a constraint on $M_{dd}$ or $M_{ss}$, but it does
not put a constraint on $M_{bb}$. Therefore, for our purpose of visible family gauge bosons, we should
regard the third generation quark $(t, b)$ as $(t, b) = (u_3, d_3)$ in Case A with the inverted gauge
boson mass hierarchy, and $(t, b)$ as $(t, b) = (u_1, d_1)$ in Case B with the normal gauge boson mass
hierarchy. As a result, we have the following four candidates of the quark family assignments:

Case A\(1\): $(d_1, d_2, d_3) = (d, s, b)$, Case A\(2\): $(d_1, d_2, d_3) = (s, d, b)$, Case B\(1\): $(d_1, d_2, d_3) = (b, s, d)$
and Case B\(2\): $(d_1, d_2, d_3) = (b, d, s)$. Cases A\(1\) with $n = 1$ and B\(1\) with $n = 1$ correspond to the
K-Y model and the Sumino model, respectively.

In Table 1, we list gauge boson masses $M_{ij}$ estimated from $\Delta m_{K}^{\text{fam}} \sim 1.4 \times 10^{-8}$ TeV for
these four cases with typical values of $n$, where $n$ is defined by Eq.(2.12) \[or Eq.(2.14)\]. \[Exactly
speaking, since the value $\Delta m_{K}^{\text{fam}} \sim 1.4 \times 10^{-8}$ TeV means those which we can take as large as
possible, the values of $\tilde{M}_{ij}$ in Table 1 (\(\tilde{M}_{ij}\) are defined by Eq.(4.1) below) denote lower limits
of $\tilde{M}_{ij}$\]. In the evaluations of $\lambda_i$, we have taken not only of the magnitudes of $V_{CKM}$ elements,
but also the $CP$ phase \[6\] into consideration. In Table 1, for convenience, numerical values of
masses are given by

$$\tilde{M}_{ij}^2 \equiv \frac{M_{ij}^2}{g_F^2/2}. \quad (4.1)$$

As far as we treat four-Fermi current-current interactions, the value $\tilde{M}_{ij}$ are practically useful
rather than $M_{ij}$. Real mass values $M_{ij}$ are needed only when we discuss a direct observation of
$A^1_i$. In the numerical estimates of $\tilde{M}_{ij}$, note that the expression $M_{ij}$ given by Eq.(2.11) \[and also
Eq.(2.13)\] have been described in the family numbers which defined by $(e_1, e_2, e_3) = (e^-, \mu^-, \tau^-)$,
while the formula (3.5) with Eq.(3.2) have been described by using the quark generation-number
$(d_1, d_2, d_3) = (d, s, b)$.

As seen in Table 1, Case A\(1\) and Case A\(2\) lead to large values of $\tilde{M}_{ij}$, so that these cases
are not interesting to us. Case A with $n \geq 3$ can have $\tilde{M}_{33}$ smaller than a few TeV, but the
case gives $\tilde{M}_{11} \sim 10^6$ TeV. Phenomenology in Case A\(1\) with $n = 1$ has already been investigated
in Refs.[2, 9]. Phenomenology for Case B with $d_i = (d, s, b)$ has investigated in Ref.[10]. The
results for visible effects of the family gauge bosons was negative.

We consider that Case B with $n = 2$ is phenomenologically most attractive, because the
lightest family gauge boson $A^1_1$ has mass of an order of a few TeV which is visible at the LHC
(remember $M_{11} = (g_F/\sqrt{2})\tilde{M}_{ii}$). Besides, even the heaviest gauge boson has, at most, a mass
Table 1: Family gauge boson masses estimated from $\Delta m_K^{fam} \sim 1.4 \times 10^{-8}$ TeV. Here, we have used parameter values $a = (m_{\tau}/m_{\mu})^{n/2} = (16.8167)^{n/2}$ and $b = (m_{\tau}/m_{e})^{n/2} = (3477.15)^{n/2}$ for Case A, and $a = (m_{\mu}/m_{e})^{n/2} = (206.768)^{n/2}$ and $b = (m_{\tau}/m_{e})^{n/2} = (3477.15)^{n/2}$ for Case B. In this table, for convenience, numerical values of masses are given by $\tilde{M}_{ij} \equiv M_{ij}/(g_F/\sqrt{2})$ in a unit of TeV.

| Case | Family gauge boson masses |
|------|---------------------------|
| (A)  | $M_{11} > M_{12} > M_{13} > M_{22} > M_{23} > M_{33}$ |
|      | $\tilde{M}_{\alpha}$<br>$b > \sqrt{\frac{a^2+b^2}{2}} > \sqrt{\frac{b^2+1}{2}} > \sqrt{\frac{a^2+1}{2}} > a > 1$ |
| (A1) | $M_{dd}$ > $M_{ds}$ > $M_{db}$ > $M_{ss}$ > $M_{sb}$ > $M_{bb}$ |
| $n = 1/2$ | 1209 | 884.5 | 862.5 | 319.0 | 251.5 | 157.5 |
| $n = 1$ | 5062 | 3588 | 3580 | 352.0 | 256.2 | 85.8 |
| $n = 2$ | 73342 | 51861 | 51860 | 354.7 | 251.3 | 21.1 |
| $n = 3$ | $1.1 \times 10^6$ | $7.4 \times 10^5$ | $7.4 \times 10^5$ | 356.0 | 251.8 | 5.16 |
| (A2) | $M_{ss}$ > $M_{sd}$ > $M_{sb}$ > $M_{dd}$ > $M_{db}$ > $M_{bb}$ |
| $n = 1/2$ | 1205 | 881.4 | 859.5 | 317.8 | 250.7 | 156.0 |
| $n = 1$ | 5042 | 3574 | 3566 | 350.7 | 255.2 | 85.5 |
| $n = 2$ | 73035 | 51644 | 51644 | 353.2 | 250.2 | 21.0 |
| $n = 3$ | $1.2 \times 10^6$ | $7.5 \times 10^5$ | $7.5 \times 10^5$ | 354.5 | 250.7 | 5.14 |
| (B)  | $M_{11} < M_{12} < M_{22} < M_{13} < M_{23} < M_{33}$ |
|      | $\tilde{M}_{\alpha}$<br>$b > \sqrt{\frac{a^2+b^2}{2}} > \sqrt{\frac{b^2+1}{2}} > \sqrt{\frac{a^2+1}{2}} > a > 1$ |
| (B1) | $M_{bb}$ < $M_{bs}$ < $M_{ss}$ < $M_{bd}$ < $M_{sd}$ < $M_{dd}$ |
| $n = 1/2$ | 63.5 | 176.0 | 240.7 | 347.5 | 384.4 | 487.4 |
| $n = 1$ | 22.5 | 229.8 | 324.2 | 940.2 | 967.6 | 1329 |
| $n = 2$ | 1.77 | 258.3 | 365.3 | 4344 | 4352 | 6144 |
| (B2) | $M_{bb}$ < $M_{bd}$ < $M_{dd}$ < $M_{bs}$ < $M_{ds}$ < $M_{ss}$ |
| $n = 1/2$ | 63.1 | 174.9 | 239.2 | 345.4 | 382.0 | 484.3 |
| $n = 1$ | 22.4 | 228.7 | 322.7 | 935.8 | 963.1 | 1323 |
| $n = 2$ | 1.76 | 257.3 | 363.9 | 4327 | 4334 | 6119 |
of an order of $10^4$ TeV.

5 Phenomenology of the family gauge bosons in Cases B$_1$ and B$_2$

In this section, let us investigate phenomenology of the family gauge bosons in Cases B$_1$ and B$_2$ with $n = 2$. From a point of view of model-building, too, the case $n = 2$ is not so unlikely, because we can consider a VEV relation $\langle \Psi \rangle_i^\alpha = \langle \Phi \rangle_i^\beta \langle \bar{E} \rangle_j^\beta \langle \Phi \rangle_j^\alpha$, where $\langle \bar{E} \rangle = 1$. In this case, from Eq.(2.4), the gauge coupling constant $g_F/\sqrt{2}$ is given by

$$\frac{g_F}{\sqrt{2}}|_{n=2} = e = 0.30684,$$

where, for convenience, we have used [6] $\alpha(m_r) = 1/133.471$.

5.1 Direct production of the lightest gauge boson $A_1^1$

From the value given in Table 1 and the value (5.1), the mass of gauge boson $A_1^1$ is

$$M_{11} \simeq 0.543 \text{ TeV} \ (0.540 \text{ TeV}) \text{ for Case B}_1 \ [\text{Case B}_2],$$

It should be noted that the gauge boson $A_1^1$ can interact only with the third generation quarks ($t, b$), although it does with the first generation leptons ($\nu_e, e$) for leptons. Therefore, the gauge boson $A_1^1$ will be produced by gluon fusion (Fig.1) as

$$p + p \rightarrow A_1^1 + b + \bar{b} + X \rightarrow e^+e^- + X,$$

at the LHC. (In future, we will also observe $A_1^1$ production in the ILC as $e^+ + e^- \rightarrow A_1^1$.)

![Figure 1: $A_1^1$ production at the LHC.](image)

We have decay modes of $A_1^1$ into $t + \bar{t}$, $b + \bar{b}$, $e^- + e^+$ and $\nu_e + \bar{\nu}_e$ with branching fractions as follows:

$$Br(A_1^1 \rightarrow t\bar{t}) = Br(A_1^1 \rightarrow b\bar{b}) = \frac{6}{15} = 40\%,$$

$$Br(A_1^1 \rightarrow e^-e^+) = \frac{2}{15} = 13.3\%, \ Br(A_1^1 \rightarrow \nu_e\bar{\nu}_e) = \frac{1}{15} = 6.7\%.$$
Note that the branching ratio $Br(A_1^1 \to \nu_e \bar{\nu}_e) = 1/15 = 6.7\%$ is one in the case of Majorana neutrinos. If neutrinos are Dirac neutrinos, the branching ratios is given $Br(A_1^1 \to \nu_e \bar{\nu}_e) = 2/16 = 12.5\%$. The large difference between both is due to the large leptonic branching ratio in the family gauge boson decays. Therefore, in future, when data of the direct production of $A_1^1$ are accumulated, we will be able to conclude whether neutrinos are Dirac or Majorana by observing whether $Br(A_1^1 \to \nu_e \bar{\nu}_e)$ is 6.7\% or 12.5\%.

The search for $A_1^1$ production at the LHC is done by a similar way of the $Z'$ search (for a review, see, for example, [11]). Although there has been an experimental report on $Z'$ search [12], the result cannot be applicable for $A_1^1$ search, because $A_1^1$ cannot interact with the first generation quarks, so that the cross section is considerably small compared with $Z'$ production.

The cross section of $A_1^1$ in the original Sumino model has been discussed in Ref.[10], but the case was a different family gauge boson $A_1^1$ which can interact with the first generation quarks.

Since the purpose of the present paper is to give an overview of the family gauge bosons with visible energy scale, estimate of the production rate $\sigma(pp \to A_1^1)$ will be given elsewhere.

If the real mass $M_{11}$ is smaller than 500 GeV, we may expect an observation at the ILC in future, too.

5.2 Contribution of family gauge bosons to the rare decay $K^+ \to \pi^+ \nu \bar{\nu}$

Let us estimate contributions of family gauge bosons to the rare decay $K^+ \to \pi^+ \nu \bar{\nu}$, because only a finite value of the branching ratio has been reported [6] at present:

$$Br(K^+ \to \pi^+ \nu \bar{\nu})_{obs} = (1.7 \pm 1.1) \times 10^{-10}. \quad (5.5)$$

It is usually taken that this value is consistent with the standard model prediction [13]

$$Br(K^+ \to \pi^+ \nu \bar{\nu})_{SM} = (0.80 \pm 0.11) \times 10^{-10}. \quad (5.6)$$

We are interested in whether Case B is consistent or not with the present experimental result (5.5).

In the present model, all family gauge bosons can, in principle, contribute to each rare decay mode. For example, in Cases B1 and B2, a transition $K \to \pi$ is mediated by the gauge bosons $A_s^d \equiv A_2^3$ and $A_s^d \equiv A_3^2$, respectively. However, as seen in Table 1, the mass of $M_{23}$ is of the order of $10^3$ TeV, so that the effect is invisible. Remember that family-number violating transitions are possible in the quark sector. Since the effective mass value of $\tilde{M}_{11} \equiv \tilde{M}_{bb}$ is too small, the contribution of $A_1^1$ is dominated compared with other gauge boson exchanges even considering the existence of the suppression factor $|U_{bd}^d U_{bs}^d|$ (the value is 0.0155 in the approximation $U^d \simeq V_{CKM}$). Then, the branching ratio due to the family gauge boson exchange $A_1^1$ are estimated as follows: $K^+ \to \pi^+ e^- \mu^+$ as follows:

$$\frac{Br(K^+ \to \pi^+ \nu_e \bar{\nu}_e)_{fam}}{Br(K^+ \to \pi^0 \mu^+ \nu_\mu)} = \frac{1}{2} \frac{f(m_{\pi^+}/m_K)}{f(m_{\pi^0}/m_K)} \frac{e^2}{(r_{11})^4} \eta_B \xi^2 \quad (5.7)$$
where

\[(r_{ij})^2 = \frac{(g_F^2/2)/M_{ij}^2}{(g_w^2/8)/M_W^2} = \frac{2v_H^2}{M_W^2},\]  

(5.8)

\(v_H = 246\ \text{GeV},\) and \(f(x)\) is a phase space function \(f(x) = 1 - 8x^2 + 8x^6 - x^8 - 12x^4 \log x^2.\) (We have neglected the lepton masses.) Here, the factor \(\xi\) denotes mixing effects in quarks, and in this case, \(\xi\) is given by

\[\xi = \frac{|V_{ud}V_{ts}|}{|V_{us}|},\]  

(5.9)

where we have used the approximation \(U^d \simeq V_{CKM}.\) The factor \(\frac{1}{2}\) in the denominator of Eq.(5.7) is due to \(\pi^0 = (u\bar{u} - d\bar{d})/\sqrt{2}.\) The factor \(\eta\) denotes difference of effective current-current interactions: When we denote the currents for weak interactions \((\bar{\nu}\gamma_\mu(1-\gamma_5)e)\) as \((V - A)\) symbolically, the factor \(\eta_B\) for a final state of \(\nu\bar{\nu}\) is given by \(\eta_B^B = [(|V|^2 + |A|^2)/(|V|^2 + |A|^2) = 1/4\) because only the left-handed neutrino \(\nu_L\) can contribute as seen in Eq.(2.15). [In contrast to the case \(\nu\bar{\nu},\) for a final state of \(e^+e^-,\) it is given by \(\eta_{ee}^B = 1/2.]\)

We obtain numerical results

\[Br(K^+ \to \pi^+\nu_e\bar{\nu}_e)_{fam} = 1.1 \times 10^{-10} (0.91 \times 10^{-10}) \text{ for } \tilde{M}_{11} = 1.8 (1.9) \text{ TeV,} \]  

(5.10)

form Eq.(5.7). This value is just favorable to the difference between the observed one (5.5) and the SM one (5.6), i.e. \((1.7 - 0.8) \times 10^{-10}.\) (However, it should be noted that result (5.10) is only an approximate one, because we have neglected interference with the final state mode from the standard model. The numerical result should be taken rigidly.)

For rare \(B\) and \(K\) decays, we can estimate their branching ratios by a similar way to Eq.(5.7). We investigate only the decay modes via the family gauge boson \(A_2^1,\) because other gauge bosons are considerably heavy, so that such gauge boson effects are obviously invisible. Note that since the family number in the quark sector is assigned unconventionally, for example, the gauge boson \(A_2^1\) causes the decay \(B \to K + e^+ + \mu^-\) with mixing factor \(\xi = |U_{bb}U_{ss}|/|V_{us}|\) for Case B1 with \(A_{11}^2 \equiv A_s^8,\) and \(B \to \pi + e^+ + \mu^-\) with mixing factor \(\xi = |U_{bb}U_{dd}|/|V_{us}|\) for Case B2 with \(A_{11}^2 \equiv A_b^d.\) Differently from the decay \(K \to \pi\nu_e\bar{\nu}_e,\) the lightest gauge boson \(A_{11}^1\) cannot contribute. Since rare \(B\) and \(K\) decays via the lightest family gauge boson \(A_{11}^1\) yields final states \(e^+e^-\) and \(\nu_e\bar{\nu}_e,\) such decay modes are confused with decay modes via photon and \(Z\) boson. The lightest gauge boson \(A_{1j}^j\) with \(i \neq j\) is \(A_{21}^1.\) The branching ratios of decay modes via \(A_{21}^1\) are, for example, as follows:

\[\text{Case B1: } Br(B^+ \to K^+\mu^-e^+) \simeq 2.1 \times 10^{-11}, \quad Br(B^0 \to K^0\mu^-e^+) \simeq 2.1 \times 10^{-11},\]

\[\text{Case B2: } Br(B^+ \to \pi^+\mu^-e^+) \simeq 2.1 \times 10^{-11}, \quad Br(B^0 \to \pi^0\mu^-e^+) \simeq 1.0 \times 10^{-11},\]  

(5.11)

11
where we have assumed $U^d \simeq V_{CKM}$. These results are invisible for a time, because the present experimental lower limits [6] are $Br(B^+ \to K^+\mu^−e^+) < 9.1 \times 10^{-8}$ and $Br(B^+ \to \pi^+\mu^−e^+) < 1.7 \times 10^{-7}$. The family gauge boson $A^1_2$ can also contribute to rare $K$ decays. However, the predicted branching ratios are of orders of $10^{-15} - 10^{-17}$ because small values of quark mixing factors, so that the effects invisible.

\[ R(Au) \equiv \frac{\sigma(\mu^+ + Au \to e^- + Au)}{\sigma(\mu \text{ capture})} < 7 \times 10^{-13}. \]  
\[ (5.12) \]

The reaction $\mu^−N(A, Z) \to e^-N(A, Z)$ is caused by an exchange of the family gauge boson $A^1_2$. It means the exchange of $A^1_2 = A^b \ll A^d$ in Case B [Case B]. At present, we do not know values of $|U_{ij}^q|$ ($q = u, d$). Therefore, it is not practical, at this stage, to estimate a $\mu-e$ conversion rate strictly. Instead, we roughly estimate a $\mu-e$ conversion rate in the quark level as follows:

\[ R_q \equiv \frac{\sigma(\mu^− + q \to e^- + q)}{\sigma(\mu^− + u \to \nu_\mu + d)} \simeq \left(\frac{\xi g^2 / 2 M_W^2}{M_W^2 + s^2_W / 8} \right)^2 = (\xi (r_{12})^2)^2, \]  
\[ (5.13) \]

where $q = u, d$, and $(r_{12}^q)$ is defined by Eq.(5.8). (Although the estimated value $R_q$ has different physical meaning from the value $R(Au)$, we consider that the order of the value $R_q$ can provide one with useful information.) In Eq.(5.13), $\xi$ is a quark mixing factor similar to Eq.(5.9), and the value of $\xi$ is given by $\xi = [U_{ud}^d U_{ub}^d]/|V_{ud}| = 2.00 \times 10^{-3}$ [$\xi = [U_{dd}^d U_{bd}^d]/|V_{ud}| = 0.867 \times 10^{-2}$] in Case B [in Case B] under the approximation $U^d \simeq V_{CKM}$. In this approximation, we may regard the ratios $R_q$ as $R_u \ll R_d$, so that we can neglect contribution to nucleon from $R_u$ compared with that from $R_d$. Then, we can roughly estimate values of $R_q$

\[ R_q \simeq R_d \sim 1.32 \times 10^{-17} (2.52 \times 10^{-16}) \text{ for Case B1 (Case B2)}, \]  
\[ (5.14) \]

where we have used $\bar{M}_{12} = 260$ TeV from Table 1.

In the near future, the COMET experiment [16] will reach a single-event sensitivity of $2.6 \times 10^{-17}$. Therefore, the value $R_q \sim 10^{-16}$ in Case B2 become within reach of our observation, but the value $R_q \sim 1.32 \times 10^{-17}$ in Case B1 is critical for its observation.
Since the decay $\mu^- \to e^- + \gamma$ is highly suppressed in the present scenario, if we observe $\mu^{-}N \to e^{-}N$ without observation of $\mu^- \to e^- + \gamma$, then it will strongly support our family gauge boson scenario. (The decay $\mu^- \to e^- + \gamma$ can occur through a quark-loop diagram. However, such a diagram is highly suppressed.)

### 5.4 Deviations from the $e$-$\mu$-$\tau$ universality

Previously, we pointed out [9] a possibility of a deviation from the $e$-$\mu$ universality in tau decays $\tau \to \mu \nu \bar{\nu}/e\nu$ by assuming $\tilde{M}_{23} \ll \tilde{M}_{31}$. However, in the present model, we cannot observe such a deviation because the mass spectrum in the present model gives $\tilde{M}_{23} \simeq \tilde{M}_{31}$, and besides, we have a large value $\tilde{M}_{23} \sim 10^3 \text{ TeV}$ in Case B.

On the other hand, we have a possibility of sizable deviations from the $e$-$\mu$-$\tau$ universality in the $\Upsilon$ decays $\Upsilon \to \tau^+\tau^-/\mu^+\mu^-/e^+e^-$, because the value of $M_{11} \equiv M_{bb}$ is considerably small in Case B. We have matrix elements for the decays $\Upsilon \to \tau^+\tau^-/\mu^+\mu^-/e^+e^-$, as follows:

$$M = M_{SM} \text{ and } M_{ee} = M_{SM} + M_{fam} = M_{SM}(1 - \varepsilon),$$

where

$$\varepsilon \simeq \frac{g_{F}'^2/2}{(e/3)^2} \frac{M_2^2}{M_1^2 - M_1^2} \simeq \frac{9}{e^2} \frac{M_2^2}{M_1^2} = 2.64 \times 10^{-3}. \quad (5.15)$$

Therefore, we can expect a deviation

$$1 - \frac{Br(\Upsilon \to e^+e^-)}{Br(\Upsilon \to \mu^+\mu^-)} \simeq 2\varepsilon = 0.0053. \quad (5.16)$$

At present, we have not observed such a deviation [6]. However, the value (5.16) will become visible in future experiments.

### 6 Concluding remarks

We have investigated possibility of visible family gauge boson effects for six family assignments in the quark sector $(d_1, d_2, d_3) = (d, s, b)$, $(d_1, d_2, d_3) = (b, d, b)$, and so on, under the Sumino cancellation condition. In the Sumino model, the family number is defined by the diagonal basis of the charged lepton mass matrix $M_e = \text{diag}(m_e, m_\mu, m_\tau)$. The $P^u, \bar{P}^d$ mixings ($P = K, D, B, B_s$) are caused only through quark mixings $U^u \neq 1$ and $U^d \neq 1$. We have found that the most interesting case is Case B$_2$, $(d_1, d_2, d_3) = (b, d, b)$. In Case B$_2$, a direct production of $A_1$ at the LHC, $\mu$-$e$ conversion $\mu^-N \to e^-N$, and a deviation from $e$-$\mu$-$\tau$ universality in the $\Upsilon$ decay will be observed in future experiments. Also, Case B$_1$, $(d_1, d_2, d_3) = (b, s, d)$, is attractive, although the case is somewhat hard to observe in $\mu^-N \to e^-N$ compared with Case B$_2$.

In Case B, the leptons take a Sumino-like structure (so that Sumino’s cancellation mechanism is satisfied), while quarks take a twisted family-number assignment. At present, there is
no theoretical ground for such family-number assignments. In order to make the twisted family-number assignment \((d_1, d_2, d_3) = (b, d, s)\) more reliable, we, at least, have to build a unified mass matrix model of quarks and leptons under such the twisted family-number assignment. It is a task in future.

We hope that many physicists turn their attention to a possibility of visible family gauge bosons and of a twisted family-number assignment versus generation-numbers.

Acknowledgments

The author thanks T. Yamashita for valuable and helpful conversations. He also thanks H. Yokoya for helpful comments on the direct production of \(A_i^j\) at the LHC, Y. Kuno and H. Sakamoto for useful comments on experimental status of \(\mu-e\) conversion, and M. Tanimoto for valuable information on the recent estimates of \(K^0-\bar{K}^0\) mixing.

References

[1] T. Maehara and T. Yanagida, Prog. Theor. Phys. 60 (1978) 822. For a recent work, for instance, see A. J. Buras, M. V. Carlucci, L. Merlo and E. Stamou, JHEP 1203 (2012) 088. For a work of the family gauge bosons in astrophysics, e.g. see Z.G.Berezhiani and M.Yu.Khlopov, Yadernaya Fizika 51 (1990) 1479 [Sov.J.Nucl.Phys. 51 (1990) 935].

[2] Y. Koide and T. Yamashita, Phys. Lett. B 711 (2012) 384.

[3] Y. Sumino, Phys. Lett. B 671 (2009) 477.

[4] Y. Koide, Lett. Nuovo Cim. 34 (1982) 201; Phys. Lett. B 120 (1983) 161; Phys. Rev. D 28 (1983) 252.

[5] H. Arason, et al., Phys. Rev. D 46 (1992) 3945.

[6] J. Beringer et al., Particle Data Group, Phys. Rev. D 86 (2012) 010001.

[7] N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531; M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49 (1973) 652.

[8] I.I. Bigi and A.I. Sanda, in CP Violation, Cambridge Univ. Press, p.174.

[9] Y. Koide, Phys. Rev. D 87 (2013) 016016.

[10] Y. Koide, Y. Sumino and M. Yamanaka, Phys. Lett. B 695 (2011) 279.

[11] J. Hewett and T. Rizoo, Phys. Rep. 183 (1989) 193; A. Leike, Phys. Rep. 317 (1999) 43.

[12] A. Duperrin, CDF collaboration, arXiv:1112.4443 [hep-ex]; D. Acosta, et al., CDF collaboration, Phys. Rev. Lett. 95 (2005) 131801.
[13] G. Isidori, F. Mescia and C. Smith, Nucl. Phys. B 718 (2005) 319. Also, see A. Buras, F. Schwab and Selma, Rev. Mod. Phys. 80 (2008) 965.

[14] Y. Kuno and Y. Okada, Rev. Mod. Phys. 73 (2001) 151.

[15] R. Kitano, M. Koike and Y. Okada, Phys. Rev. D 66 (2002) 096002.

[16] Y. Kuno, Prog. Theor. Exp. Phys. (2013) 022C01.

[17] W. H. Bertl et al., SINDRUM II collaboration, Eur. Phys. J. C 47 (2006) 337.