An Approximation to the Cross Sections of $Z_l$ Boson Production at CLIC by Using Neural Networks

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Abstract

In this work, compatible with our previous study, mass ($M_{Z_l}$) and interaction constant ($g_l$) of massive leptonic (leptophilic) boson ($Z_l$) at CLIC were investigated by using artificial neural networks (ANNs). Furthermore, it was seen that invariant mass distributions for final muons at CLIC after $e^+e^-\rightarrow \gamma,Z,Z_l \rightarrow \mu^+\mu^-$ signal $e^+e^-\rightarrow \gamma,Z \rightarrow \mu^+\mu^-$ background processes were consistently predicted by using ANN. Lastly, for these highly nonlinear data, we have constructed consistent empirical physical formulas (EPFs) by appropriate feed-forward ANN. These ANN-EPFs can be used to derive further physical functions which could be relevant to studying for leptophilic $Z_l$ vector boson.

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I. INTRODUCTION

The gauging of the baryon and lepton numbers has a long history. In 1955 Lee and Yang proposed massless baryonic “photon” [1], later in 1969 Okun considered massless leptonic “photon” [2] in analogy with the baryonic photon. On the other hand gauging of $B - L$ [3, 4] is natural in the framework of Grand Unification Theories. Manifestations of the $Z'$ boson of the minimal $B - L$ model at future linear colliders and LHC have been considered in recent paper [5]. In [6] we have considered phenomenology of massive U(1) boson coupled to lepton charge. In this paper, by using data from our previous study we have obtained some limit values for $M_{Z_l}$ and $g_l$ via artificial neural networks (ANNs).

The physical phenomena involved in massive leptonic (leptophilic) boson ($Z_l$) are characteristically highly nonlinear. Therefore, it may be difficult to construct explicit form of empirical physical formulas (EPFs) relevant to $Z_l$. Then, by various appropriate operations of mathematical analysis, derivation of potentially useful highly nonlinear physical functions for $Z_l$ is of utmost interest. Compatibly a previous theoretical treatment [7], appropriate EPFs relevant to $Z_l$ can be built by using a feed-forward artificial neural network (ANN). As we give more details in Section II, the ANN is a universal nonlinear function approximator [8].

Recently, ANNs have emerged with successful applications in many fields, including Higgs boson search [9,13]. In this study, compatible with our previous study [6], mass ($M_{Z_l}$) and interaction constant ($g_l$) of $Z_l$ at CLIC were investigated by using ANNs. Besides, the invariant mass distributions for final muons ($M_{\mu^+\mu^-}$) at CLIC with $\sqrt{s} = 3 TeV$ for signal and background were consistently obtained by using ANN. In all calculations we have performed, signal and background processes were $e^+e^- \rightarrow \gamma, Z, Z_l \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \gamma, Z \rightarrow \mu^+\mu^-$ respectively. Also, we particularly aim to construct explicit mathematical functional form of ANN-EPFs for nonlinear data relevant to $Z_l$. While the calculated data were intrinsically highly nonlinear, even so train set ANN-EPFs successfully fitted these data. Furthermore, test set ANN-EPFs consistently predicted the data. That is, the physical laws embedded in the data were extracted by the ANN-EPFs.
Figure 1: Fully connected one input-one hidden-one output layer ANN. $x_i$ and $y_i$ are input and output vector components, respectively. Circles are neurons and arrows indicate adaptable synaptic weights. $w_{jk}^i$: weight vector component, where $i$ is a layer index, $jk$ weight component from the $j$th neuron of $i$th layer and to $k$th neuron of $(i+1)$th layer.

II. ANN AND ANN-EPF

The fundamental task of the artificial neural networks (ANNs) is to give outputs in consequence of the computation of the inputs. ANNs are mathematical models that mimic the human brain. They consist of several processing units called neurons which have adaptive synaptic weights [14]. ANNs are also efficient tools for pattern recognition. The ANN consists of three layers named as input, hidden and output (Fig.1). The number of hidden layers can differ, but a single hidden layer is enough for efficient nonlinear function approximation [8]. In this study, one input layer with one neuron, one hidden layer with many ($h$) neuron and one output layer with one neuron ($1 - h - 1$) ANN topology was used for investigation of massive leptonic (leptophilic) boson ($Z_l$) at CLIC with $\sqrt{s} = 3$TeV. Analyses were performed for most convenient hidden neuron numbers in each. The total numbers of adjustable weights are calculated by using formula given in (1),

$$ (p \times h + h \times r = h \times (p + r) = 2h ) \quad (1) $$

where $p$ and $r$ are the input and output neuron numbers, respectively.

The neuron in the input layer collects the data from outside and transmits via weighted connections to the neurons of hidden layer which is needed to approximate any nonlinear function. The hidden neuron activation function can be any well-behaved nonlinear function. In this study, the type of activation functions for hidden layers were chosen as hyperbolic
tangent (2). Note that instead of this function, any other suitable sigmoidal function could also be used. Finally, the output layer neurons return the signal after the analysis.

\[
tanh = \frac{e^x - e^{-x}}{e^x + e^{-x}}
\]  

An ANN software NeuroSolutions v6.02 was used for separate applications. In these applications, ANN inputs were \(M_{Zl}, g_t \) and \(M_{\mu^+\mu^-} \) and corresponding desired outputs were cross sections for each input. For all ANN processing case, the data were divided into two equal separate sets. One of these (50%) belongs to the training stage and the rest (50%) belongs to the test stage. In the training stage, a back-propagation algorithm with Levenberg-Marquardt for the training of the ANN was used. The maximum epoch number (one complete presentation of the all input-output data to the network being trained) was 1000. ANN modifies its weights by appropriate modifications until an acceptable error level between predicted and desired outputs is reached. The error function which measures the difference between outputs was mean square error (MSE) as given in (3),

\[
MSE = \frac{[\sum_{k=1}^{r} \sum_{i=1}^{N} (y_{ki} - f_{ki})^2]}{N}
\]  

where \(N\) is the number of training or test samples, \(y_{ki}\) and \(f_{ki}\) are the desired output and network output, respectively. Then by using ANN with final weights, the performance of the network is tested over test data which are never seen before by network. If the predictions of the test data are well enough, the ANN is considered to have consistently learned the functional relationship between input and output. In this work, the MSE values were between \(8 \times 10^{-38}\) and \(5 \times 10^{-3}\) for the training stage and \(6 \times 10^{-11}\) and \(4 \times 10^{-2}\) for the test stage. In Fig.2, the training MSE values for invariant mass distribution of final muons at CLIC with \(M_{Zl} = 1TeV\) were given as an illustration.

Owing to the fact that a single hidden layer feed-forward ANN is enough for nonlinear function approximation [8], in this paper we used single hidden layer ANNs as previously stated. Here, we only explain the single hidden layer feed-forward ANN functionality. Borrowing from [8], the desired output vector \(\vec{y}\) is approximated by a network multi-output vector \(\vec{f}\) which is defined by (4).
Figure 2: For invariant mass distribution of final muons at CLIC, the training MSE values versus epoch number

\[ \tilde{f} : R^p \rightarrow R^r : \tilde{f}_k(\vec{x}) = \sum_{j=1}^{N} \beta_j G(A_j(\vec{x})); \vec{x} \in R^p, \beta_j \in R, A_j \in A^p, k = 1, \ldots, r \]  

(4)

where \( \vec{x} \) is the ANN input vector, \( A^p \) is the set of all functions of \( R^p \rightarrow R \) defined by \( A(\vec{x}) = \vec{w} \cdot \vec{x} + b \), \( \vec{w} \) is input to hidden layer weight vector, \( b \) is the bias weight. In Fig. 1, the columns of the weight matrices \( w^1 \) and \( w^2 \) correspond to weight vectors defined in \( A(\vec{x}) \) and \( \vec{\beta} \) in (4). However, as can be seen in Fig. 1 and (4), the correspondences \( w^1 \rightarrow A(\vec{x}) \) and \( w^2 \rightarrow \vec{\beta} \) are valid only for single hidden layer feed-forward ANN.

Since a deterministic or random EPF is usually a mathematical vector function \( \vec{y} : R^p \rightarrow R^r \) between the physical variables under investigation, particularly ANN is relevant to EPF construction. Therefore, being a general input-output function estimator, the ANN defined by (4) is particularly relevant in this context. But, although there can be several independent variables \( (p > 1 \text{ in (4)}) \), the number of the dependent variables is usually one. Train data for both independent and dependent physical variables are presented to the input and output layers respectively. Then after an appropriate weight adaptation process, the LFNN estimates the unknowable generally nonlinear EPF.
III. THE CONCRETE ALGORITHM FOR ANN-EPF CONSTRUCTION

To construct appropriate EPF for highly nonlinear cross sections, we used one neuron output ANN vector function $\vec{f}$ in (4). However, due to the fact that it gives only the rough structure of the ANN without generating the final EPF parameters/ final ANN optimal weights, this equation is not sufficient for the complete construction of the desired nonlinear EPF. In order to obtain the final weight vector $\vec{w}_f$ and the corresponding ANN output vector function $\vec{f}_{min} = \vec{f}(\vec{w}_f)$ of (4), we simultaneously used the (3) and (4). More clearly, given the desired input-output experimental data, $\vec{f}_{min}$ is the network output vector function giving the minimum MSE by a convenient ANN weight adaptation. Note that, $\vec{f}_{min}$ is the best nonlinear estimation vector of the theoretically unknown desired output function $\vec{y}: R^p \rightarrow R^r$ (see Fig. 1). In other saying, the unknown vector function $\vec{y}$ is estimated by $\vec{f}_{min}$ which is actually desired nonlinear EPF that we aim to eventually obtain. $\vec{f}_{min}$ totally depends on the structure of the network output vector function $\vec{f}$ and the final weight vector $\vec{w}_f$. In (4), components of the weight are embedded in $A(\vec{x})$ and $\vec{\beta}$. In (4), $\vec{f}$ depends on the apparent forms of $A$ and hidden layer activation. In this paper, setting $\vec{\beta} = w^2$ of Fig. 1, hidden layer activation function is nonlinear tangent hyperbolic and $A$ is the dot product of $w^1$ and $\vec{x}$ of Fig. 1. So, we can construct explicit form of $\vec{f}$. Afterwards, by minimization of (4), we finally obtain $\vec{f}_{min} = \vec{f}(\vec{w}_f)$. Now, the concrete ANN-EPF construction algorithm for nonlinear cross sections is completed. The actual ANN-EPFs results are given in Section IV.

IV. RESULTS AND DISCUSSION

A. ANN-EPFs for train set fittings

During all the training stages, the number of data points was 50% of all data. For a single hidden layer ANN, the train set nno (neural network output) fittings for cross sections versus $M_{Z_l}$ for different $g_l$ values (0.10, 0.20, and 0.30) were given in Fig.3. Here the best fitting was obtained for $h = 3$ (h: hidden layer neuron number). In Fig.4 and 5, the train set nno fittings for cross sections versus $g_l$ and $M_{\mu^+\mu^-}$ were given, respectively. For $M_{\mu^+\mu^-}$, two different $M_{Z_l}$ values were used and hidden layer neuron number which gives the best fitting is 7. In order to show effect of varying $h$, not only best one but also different ones of $h$ were
Figure 3: Calculated and nno train set fittings of cross section versus $M_{Z_l}$ for different $g_l$ values at CLIC with $\sqrt{s} = 3\,TeV$

Figure 4: Calculated and nno train set fitting of cross section versus $g_l$ at CLIC with $\sqrt{s} = 3\,TeV$ for different hidden layer neuron number.

given for $g_l$ in Fig.5. It can be clearly seen in the figures, the nno fittings agree exceptionally well with highly nonlinear calculated data. Additionally, it is clear in Fig.5 that signal is well above the background.
Figure 5: Calculated and nno train set fittings of differential cross section versus $M_{\mu^+\mu^-}$ for SM background and signal at CLIC with $\sqrt{s} = 3$ TeV.

B. Consistency of the constructed ANN-EPFs: Test set predictions

Unless the train set ANN-EPFs are tested over cross section data, these fitted EPFs cannot be used consistently over a desired range of cross section values. If the predictions are consistent with the test data values, then the ANNs can be taken as appropriate ANN-EPFs. The corresponding test set nno predictions of Figs.3-5 were given in Figs.6-8. The number of data points was 50% of all data. As can be seen in Figs.6-8, the nno predictions agree exceptionally well with highly nonlinear experimental values. This obviously indicate that the test set ANNs of cross sections versus $M_{Zl}$, $g_l$ and $M_{\mu^+\mu^-}$ have consistently generalized the train ANN fittings. So that, obtained ANNs can safely be used as ANN-EPFs since the physical law embedded in cross sections versus $M_{Zl}$, $g_l$ and $M_{\mu^+\mu^-}$ data has been successfully extracted by the constructed ANN.

V. CONCLUSION

Future linear colliders, like CLIC, will give a chance for investigation leptophilic vector boson with masses up to the center of mass energy if $g_l \geq 10^{-3}$. It was clearly seen that, ANN method is consistent with simulations. For highly nonlinear cross sections for $M_{Zl}$, $g_l$ and $M_{\mu^+\mu^-}$, we have constructed consistent empirical physical formula (EPFs) by
Figure 6: Calculated and nno test set predictions of cross section versus $M_{Z_l}$ for different $g_l$ values at CLIC with $\sqrt{s} = 3$TeV

Figure 7: Calculated and nno test set prediction of cross section versus $g_l$ at CLIC with $\sqrt{s} = 3$TeV for different hidden layer neuron number.

appropriate ANNs. The test set ANNs of cross sections versus $M_{Z_l}$, $g_l$ and $M_{\mu^+\mu^-}$ have generalized the train ANN fittings. Therefore, the test set ANNs can be surely used as ANN-EPFs since the physical laws embedded in cross sections versus $M_{Z_l}$, $g_l$ and $M_{\mu^+\mu^-}$
Figure 8: Calculated and nno test set predictions of differential cross section versus $M_{\mu^+\mu^-}$ for SM background and signal at CLIC with $\sqrt{s} = 3 TeV$.

data have been successfully extracted by the ANN.

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