1. INTRODUCTION

The ultrahigh-energy (UHE) range of the cosmic-ray (CR) spectrum is generally broken into three parts: (1) the steeper knee-to-ankle segment ($\sim 10^{15.5} - 10^{18.6}$ eV), (2) the flatter "trans-ankle" CRs below the Greisen–Zatsepin–Kuzmin (GZK) cutoff ($10^{19.6}$ eV), and (3) trans-GZK CRs. Trans-ankle CRs are probably extragalactic, showing little anisotropy at $E \lesssim 10^{19.6}$ eV, and some anisotropy at $E \gtrsim 10^{19.6}$ eV toward the local supercluster.

In this Letter, we consider the hypothesis of sub-ankle UHECR origin from long gamma-ray bursts (GRBs) (Levinson & Eichler 1993; Wick et al. 2004; Calvez et al. 2010). We show that (1) Galactic GRBs are sufficiently good accelerators and sufficiently powerful to account for sub-ankle UHECRs, but that (2) the UHECR near-isotropy limits the current Galactic UHECR output per unit star-forming mass to a value far less than what is energetically required to account for trans-ankle extragalactic UHECRs by extragalactic GRBs. Conclusion (2) can be generalized to any hypothetical UHECR source whose rate density, like that of long GRBs, is in proportion to star formation. This challenges any model in which such sources account for all UHECRs.

Many authors have proposed that GRBs make the CRs above $10^{19}$ eV, but for this energy range there remains the alternative hypothesis that they come from active galaxies (The Pierre AUGER Collaboration et al. 2010). Doubts remain that GRBs could supply the highest-energy CRs. The problems include disparity in the total energetics of each (Eichler et al. 2010, hereafter Paper I, and references therein), adiabatic losses, which lower the maximum energy such that the acceleration be in a compact region, and the isotropy problem. The isotropy problem, discussed here and in an accompanying paper (Pohl & Eichler 2011, hereafter Paper III), is basically that stars in the Milky Way are distributed anisotropically relative to the Earth. If they—or sources similarly distributed—were responsible for UHECRs, the UHECRs should also show anisotropy, the Galactic magnetic field notwithstanding. Paper III studies in detail the propagation of CRs from Galactic GRBs and specifically compares with data the expected anisotropy, composition, and intermittency behavior.

2. PARTICLE ACCELERATION IN OUTFLOWS: A GENERAL DISCUSSION

To be efficiently shock accelerated, a CR that has crossed the shock toward the upstream must be overtaken again by the shock about $n \equiv \beta / \beta_S$ times, where $\beta$, the velocity of the particle in units of $c$, exceeds $\beta_S$, the velocity of the shock. If the shock moves at Lorentz factor $\Gamma_S$, then a CR that is overtaken by a spherical blast wave at radius $R_S$ must have been deflected (by gyration or scattering) through an angle $\Delta \theta \gtrsim 1 / \Gamma_S$ while residing upstream within a time $\Delta t \sim R_S / \beta cn = \beta_S R_S / \beta^2 c$.\footnote{Here it is assumed that, whether the propagation is stochastic or scatter-free, each reorientation or reversal of direction must happen within a CR path length of order $\beta_S R_S / \beta$, as the cumulative time a CR spends within a gyroradius $r_S$ of the shock is only $\approx R_S / \beta c$. This assumption may be controversial for scatter-free propagation in a perpendicular magnetic field, because there is no rigorous proof to our knowledge of the impossibility of more prolonged trapping, but neither are we aware of any counterexample in view of systematic drift (see below).}

In other words, a necessary condition for efficient shock acceleration is that $\Delta \theta / \Delta t = Ze B / \gamma mc \gtrsim \beta^2 c / (\beta_S R_S \Gamma_S)$. Defining the maximum kinetic energy $E_{\text{max}}$ for convenience to be $\sim \beta_{\text{max}}^2 c = \gamma_{\text{max}} c^2 m_e$, we may write

$$E_{\text{max}} \lesssim Ze B \beta_S^2 R_S \Gamma_S.$$ \hspace{1cm} (1)

which generalizes previous results for diffusive shock acceleration (DSA) (Eichler 1981; Forman & Drury 1983; and references therein), and for scatter-free shock drift, when $\Gamma_S = 1$.\footnote{Note that the expression $E_{\text{max}} = Ze B R_S$, often taken from Figure 1 of Hillas (1984), is consistent with Equation (1) only if $\beta_S$ and $\Gamma_S$ are both of order unity.}

Note that scatter-free gyration even in perpendicular shocks, though it gives a much thinner precursor than stochastic propagation, cannot in general confine a CR particle to a subrelativistic blast wave (e.g. a supernova remnant) in all three dimensions, for the particle would generally drift off to the side within a
time $\sim R_S/\beta c$ after gaining the potential difference $\sim Z e B \beta S R_S$ in energy. Also note that we have neglected adiabatic losses.

Random scattering, for a given magnetic field amplitude, changes the CR’s direction more slowly than undisturbed gyration and, for relativistic shocks, usually makes it harder for the shock to catch up with a particle. Therefore if turbulent magnetic field amplification (MFA) increases the field strength to a value $B_{\text{rms}}$, Equation (1) should not be used with $B = B_{\text{rms}}$ if the coherent scattering angle is less than $1/\Gamma S$, i.e. if the coherence length of the field $l$ is less than $r_S/\Gamma S$.

For random small deflections of $\delta \theta$, the mean free path $\lambda$ is about $\beta c/\delta \theta$, where $\delta \theta$ is the angular diffusion coefficient, $D_{\theta \theta}$, is given by

$$D_{\theta \theta} = \langle \delta \theta \rangle^2 / \delta t \sim [Z e B_{\text{rms}} / \beta \gamma m c^2]^2 / (\eta c) = \frac{r_S^2}{\lambda} \beta c,$$

where $\delta t \sim 1/\beta c$ is the scattering coherence time over which the particle scatters by an angle $\delta \theta$, $l$ is the coherence length of the magnetic field, and $r_S$, in a turbulenty enhanced magnetic field, is defined as $r_S \equiv \beta m c^2 / Z e B_{\text{rms}}$. The condition for efficient acceleration is now $\delta \theta^2 = D_{\theta \theta} \delta t \geq 1/\Gamma S$, or $r_S^2/\lambda \geq \beta S \Gamma S / \beta$. Finally, we have

$$E_{\text{max}} \lesssim Z e B_{\text{rms}} [\beta S \beta l R_S] \lesssim \Gamma S,$$

which, for a given field strength is less than the previous expression for $E_{\text{max}}$ when $l \leq r_S/\Gamma S$.

This expression for $E_{\text{max}}$ implies that, if $l \leq r_S/\Gamma S$, MFA raises $E_{\text{max}}$ only if it raises the value of $B_{\text{rms}}$. Simply tangling the field on a small scale so that its strength varies as $1/l^\eta$, $\eta \leq 1/2$, does not raise $E_{\text{max}}$.

For a self-similar, energy-conserving relativistic blast wave in the interstellar medium (Blandford & McKee 1976)

$$R_S \approx (17E / 16\pi \rho c^2)^{1/3} \Gamma S^{-2/3} \approx (6 \times 10^{18} \text{ cm}) \left( \frac{E_{54}}{n_0} \right)^{1/3} \Gamma S^{-2/3} \approx \left( \frac{E_{54}}{n_0} \right)^{1/3} \Gamma S^{-2/3},$$

where the total energy of the blast is $10^{54} E_{54}$ erg and the ambient nucleon density is $n_0$ cm$^{-3}$. This suggests, again taking the limit for $\delta \theta/\delta t$ as arising from coherent gyration, that the gyroradius of maximally energetic escaped particle is

$$r_{g,\text{max}} \approx \Gamma S R_S \approx (6 \times 10^{18} \text{ cm}) \left( \frac{E_{54}}{n_0} \right)^{1/3} \Gamma S^{-4/3}.$$  

In the early stages of a powerful GRB blast wave, $0.1 \lesssim E_{54} \lesssim 10$, $\Gamma S \sim 10^3$, while $10^{-2} \lesssim n_0 \lesssim 1$. Escaping particles, therefore, should obey $10^{18.5} \lesssim r_g \lesssim 10^{20} \text{ cm}$, and contribute to the Galactic CR component in the corresponding energy range. They should be represented in the flux we observe and in the quantity $w_G$, which is defined below to be the CR power per unit baryon mass within the Galactic solar circle.

The range $10^{18.5} \lesssim r_g \lesssim 10^{20} \text{ cm}$ in the Galaxy corresponds to the energy range $10^{16} Z [B/10 \mu G] eV \lesssim E \lesssim (10^{18.5}) Z [B/10 \mu G] eV$, precisely the range, to within the uncertainties, of the “knee-to-ankle” portion of the CR spectrum, which is said to evade the capabilities of supernovae. Relativistic blast waves in the Galaxy fill in this range nicely.

Ultrarelativistic shocks are likely to be quasi-perpendicular in the frame of the shock, and, on these grounds, their ability to accelerate particles efficiently has been questioned. We argue that it is a priori a fair concern, but note that nonthermal spectra in GRB afterglows seem to indicate that shock acceleration works there just fine. The many ways to evade the arguments against shock acceleration in quasi-perpendicular geometries are not the subject of this Letter.

3. THE CR POWER PER UNIT BARYON MASS

If we were to suppose that the mechanism supplying the sub-anke CRs somehow extends well beyond the ankle, we would encounter the problem that the Galactic component of these CRs would be highly anisotropic, assuming their source distribution would be concentrated inside the solar circle in the Galaxy, because their transport is no longer fully diffusive and includes many Lévy flights. This, in addition to the sharp change in spectral index at the ankle, is reason to suppose that Galactic GRBs limit their output to CRs below the ankle. Even in the sub-anke range, the observational limits on anisotropy pose strong constraints on the models. Below and in Paper III, this is further quantified.

Let $f_{sc,b} M_{sc} = (2 \times 10^{44} \text{ g}) f_{sc,b}$ be the total baryonic mass within the solar circle, where $M_{sc} \sim 2 \times 10^{44} \text{ g}$. Using all the sky UHECR integral flux $F[E_1, E_2]$ in energy interval $[E_1, E_2]$ (in units of EeV) implied by Auger, and assuming that the fluxes above the ankle are extragalactic and uniform in the cosmos, we find that the UHECR source power per unit baryon mass in the $[4, 40]$ range, $w_{[4, 40]}$, is

$$w_{[4, 40]} \approx \frac{F[4, 40]}{\lambda_{[4, 40]} \Omega_{B} \rho_c} \approx \left( 40 \text{ erg g}^{-1} \text{ yr}^{-1} \right) \left( \frac{\lambda_{[4, 40]} \Omega_{B}}{\text{Gpc}} \right)^{-1},$$

where $F_{[4, 40]} = 0.017 \text{ erg cm}^{-2} \text{ yr}^{-1}$ (Eichler et al. 2010; Abraham et al. 2010), and the implied luminosity from within the Galaxy’s solar sphere, $E_{[4, 40]}$, is

$$E_{[4, 40]} = w_{[4, 40]} \bar{c} M_{sc} f_{sc,b} \approx (2.5 \times 10^{38} \text{ erg s}^{-1}) \left( \frac{\lambda_{[4, 40]} \Omega_{B}}{\text{Gpc}} \right)^{-1} f_{sc,b}^{-1}.$$  

Here, $\lambda_{[4, 40]} (E_1, E_2)$ is the “horizon” range of UHECRs in the $[E_1, E_2]$ range ($\lambda_{[4, 40]} \sim 1 \text{ Gpc}$), $\Omega_{B} \rho_c \approx 1.4 \times 10^{-31} \text{ g cm}^{-3}$ is the cosmic density in baryons, and $f_{sc} = w_{[4, 40]} / w_{[4, 40],G}$ is the ratio of the average UHECR source power per unit baryon mass that to our Galaxy. To be precise, the subscript “G” denotes the Galactic value within the solar sphere. Because spiral galaxies like our own comprise about half the cosmic mass, with the other half in galaxies with less star formation and hence probably lower UHECR source power, one may estimate the value of $f_{sc}$ to be about 1/2 if UHECR sources are distributed in proportion to star formation.

On the other hand, if the solar system fairly samples the outward flux of CRs within the energy range $[E_1, E_2]$ through the solar sphere, i.e., if the local flux equals the average over the solar sphere, then the inferred power is given by

$$\dot{E}_{[E_1, E_2]} = 4\pi F_{[E_1, E_2]} R_{sc}^2 \bar{E}_{[E_1, E_2]} \approx 4\pi R_{sc}^2 \int_{E_1}^{E_2} E \bar{E}(E) f(E) dE,$$

where $R_{sc} = 8 \text{ kpc}$ is the radius of the solar circle and $\bar{E}_{[E_1, E_2]}$ is the average ratio of enthalpy flux (in the anticenter direction, defined to be $\mu = \cos \theta = 1$), $\int_{E_1}^{E_2} dE \int d\mu [E \mu f(E, \mu)]$ (where the integral runs from $E_1$ to $E_2$), to energy density $4\pi \int_{E_1}^{E_2} dE [E f(E)/c] \approx 2\pi \int_{E_1}^{E_2} dE \int d\mu [E f(E, \mu)/c]$. It is

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6 The horizon range is shorter than the instantaneous range because the expansion of the universe enhances the losses both by adiabatic deceleration of the particles and a raising of the background photon energy density and the losses it causes in the past relative to the present.
measured directly for each energy bin with CR anisotropy measurements. The current experimental limits on $\beta$ set by the Auger Observatory are, to 99% confidence, $\beta \leq 0.004$ in the [0.4,4] EeV range and $\beta \leq 0.025$ in the [4,40] EeV range (Abreu et al. 2011). Here we have used the facts that most of the energy flux is toward the low end of these ranges, where the limits on anisotropy are the strongest, and that $\beta$ is 1/3 of the first harmonic amplitude given by Abreu et al. (2011). Under this assumption, we obtain

$$E_{[4,40]} - F_{[4,40]} 4\pi R_{sc}^2 \beta_{[4,40]} \lesssim 1 \times 10^{35} \text{ erg s}^{-1}, \quad (8)$$

and correspondingly

$$E_{[0.4,4]} - F_{[0.4,4]} 4\pi R_{sc}^2 \beta_{[0.4,4]} \lesssim 2 \times 10^{35} \text{ erg s}^{-1}. \quad (9)$$

Equations (6) and (8) together with the constraints on $\beta$ imply that

$$f_b = \frac{w_{[4,40]}}{w_{[4,40],G}} \gtrsim 2500 f_{sc,b} \left( \frac{\lambda_{[4,40]}}{\text{Gpc}} \right)^{-1}. \quad (10)$$

Note that all UHECR sources that have a power scaling with star-forming mass, e.g., the hypernova scenario for long GRBs, should have a high likelihood of being present in the Galaxy, i.e., $f_b < 1$. We conclude that if (1) the sources of UHECRs are fairly represented in our own Galaxy and (2) the solar system location fairly samples these CRs at present, then the hypothesis that such sources in other galaxies maintain an extragalactic flux at the observed level would be inconsistent with the observed CR flux. There would be more CR production within the solar circle than allowed by observation. This is a challenge to any theory of UHECR origin from long GRBs.

### 4. DISCUSSION

The limit on inferred source power per unit baryon mass required to sustain Galactic UHECRs in the [4–40] EeV range that is imposed by the observed anisotropy limits is smaller, by more than three orders of magnitude, than what is required for an extragalactic origin, as calculated in Eichler et al. (2010), and it corresponds far better to the power per unit mass of gamma rays from GRBs. This numerical coincidence fits the hypothesis of a GRB origin for the Galactic component of UHECRs, without invoking a much larger unseen energy reservoir for GRBs. In fact, it would allow a Galactic origin for UHECRs above the ankle were it somehow possible to trap these CRs within the Galaxy effectively enough to obey the isotropy constraint. It remains to be shown that applying the hypothesis of UHECRs from Galactic GRBs to sub-ankle Galactic CRs obeys the isotropy constraint, and this analysis is done in Paper III.

Although the discussion, for historical reasons, has used GRBs as a standard for power production, it is independent of GRBs. The highest-energy CRs, whatever their source, are surely extragalactic, and apparently produced with a higher power per unit (star-forming) mass than that contributed by the matter within the Galactic solar sphere, given the observed limits of UHECR outflow from this sphere. This challenges any theory of their origin from matter and phenomena of the sort to be found within 10 kpc or so of the Galactic center.

We have considered several alternative possibilities. Active galactic nuclei are an obvious possibility, as they are not represented by our Galaxy, i.e., $w/w_G \gg 1$.

Another possibility is that the sources are white dwarfs or neutron-star mergers from binaries in a very extended halo, and that they spend very little of their time within the solar sphere. Conceivably this could include short GRBs, although their total energy output in the cosmos is probably an order of magnitude less than even that of long GRBs, so the question of total energetics would still loom large. On the other hand, short GRBs, not being tied to the stellar formation rate, need not suffer the recent decline in rate relative to earlier epochs, and so could be an order of magnitude more common, relative to long bursts, at present than in earlier epochs. In any case, one would still have to check that the implied flux is below observed levels and of a suitable angular distribution. Why, for example, would there be correlation above the GZK cutoff with the local supercluster? If one is willing to attribute sub-GZK CRs above the ankle to a different class of sources from those above the GZK cutoff, then short GRBs in the Galactic halo may account for the former, provided they are distant enough to respect the strong limits on anisotropy.

In an effort to accommodate the hypothesis of a GRB origin for all UHECRs, we have also considered the possibility that our present location does not fairly sample the UHECRs exiting the solar sphere, and that the large UHECR output that would be necessary to supply all of the UHECRs at energies where their flux would be extragalactic could then mostly evade the solar system. They conceivably could, for example, be blown out in jets that have avoided our location and/or with an intermittency that excluded the present epoch receiving a fair representation of the time average. If GRB blasts were to escort all their CRs safely out of the Galaxy in narrow jets that avoid our location, there would be less of an anisotropy problem associated with a GRB origin for extragalactic UHECRs. But this scenario would differ from the common view that GRB blasts slow down to subrelativistic Lorentz factors, spreading in angle, within the Galaxy. If the UHECRs escape the jet, then according to Equation (4), they probably get significantly deflected before escaping the Galaxy at large, and it is not obvious that they could then remain sufficiently collimated to conform to an avoidance scenario. A single jet, if it leaks UHECRs, contaminates the sky with a strongly anisotropic component at the energy range in which CRs are strongly scattered by the Galactic magnetic field, but not contained by the jet. If, on the other hand, the jet contains all the UHECRs, then the latter suffer enormous adiabatic losses. The question is whether all but $\sim 10^{-3}$ of the UHECRs can avoid leaking or escaping into the Galaxy at large and mixing with its CR population. This would appear to require a scenario in which CRs at $\sim 10^{19}$ eV would be extremely well confined to the shock (strong scattering) without suffering adiabatic losses and without being scattered out of the shock’s path into the interstellar medium.

Intermittency may explain a low flux from Galactic long GRBs, if the time between GRBs per Milky Way type galaxy were more than $R_{sc}/(3\beta) \sim 10^4 \beta$ yr, the escape time of CRs from the Galaxy. The collimation of GRB jets to within several degrees however, which is now believed to be the case, suggests that GRBs are as frequent as $10^{-7} f_b$ per yr per $M_{sc}$, where $f_b$ is the beaming factor, believed to be of order $10^{-1.5}$ to $10^{-3}$. Specifically, the local rate of GRBs that we detect is $R = R_1 \text{ Gpc}^{-3} \text{yr}^{-1}$, with $R_1 \sim 1$. The expected rate $R_G$ within the Galaxy should then be

$$R_G \simeq (1.6 \times 10^{-9} \text{ yr}^{-1}) f_{sc,b} R_1 f_b f_\Omega \Omega_B \simeq (2.5 \times 10^{-5} \text{ yr}^{-1}) R_1 f_{sc,b} \times \left( \frac{f_b}{10^{-2.5}} \right)^{-1} \left( \frac{f_\Omega \Omega_B}{0.02} \right)^{-1}. \quad (11)$$
Given the near isotropy of UHECRs, if a good fraction of them were produced within the solar circle, their escape time from the Galaxy, $R_{sc}/\bar{\beta}c$ would be larger than $1/R_G$, and there would necessarily be many Galactic GRBs per escape time. Moreover, if the escape is exponential, more than 10 escape times would be necessary to clear all but $10^{-3}$ of the CRs released by the GRBs. These considerations suggest that supply intermittency from GRBs is most plausible when $\bar{\beta} \sim 1$, but it is doubtful that the assumption of $\bar{\beta} \sim 1$ is consistent with the strong limits on anisotropy.

On the other hand, the original scenario of Levinson & Eichler (1993), in which Galactic GRBs supply the Galactic CR component below the ankle, need not make significant energy demands on the GRBs, because there is no constraint imposed on the required output per unit mass other than what the local Galactic flux dictates. Given the low anisotropy of UHECRs, it is likely that those below the ankle are confined to the Galaxy for nearly $10^9$ yr, in which case the rate of GRBs and their energy output are consistent with a CR production per GRB that is less than that of gamma radiation per GRB. This however, would be incompatible with the hypothesis that GRBs produce UHECRs above the ankle as well (Eichler et al. 2010).

While the low observed anisotropy eases the energy demands on sources of Galactic UHECRs, it imposes a strong constraint of its own. It remains to be shown that sources of UHECRs, if distributed as luminous stars in our Galaxy, indeed satisfy the constraint of low anisotropy. This question depends on the question of the mean free paths of UHECRs, hence on their composition, and is taken up in a companion paper (Paper III).

Here we have concluded that GRBs are energetically sufficient to provide the sub-ankle Galactic CRs, given what is known about shock acceleration, relativistic blast waves, and GRB parameters. If the hypothesis of GRB origin for sub-ankle UHECRs is true, it may have implications for the Galactic magnetic field and/or the distribution of those GRBs.

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