EFFECTS OF LOCAL DISSIPATION PROFILES ON MAGNETIZED ACCRETION DISK SPECTRA

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ABSTRACT

We present spectral calculations of non-LTE accretion disk models appropriate for high-luminosity stellar mass black hole X-ray binary systems. We first use a dissipation profile based on scaling the results of shearing box simulations of Hirose et al. to a range of annuli parameters. We simultaneously scale the effective temperature, orbital frequency, and surface density with luminosity and radius according to the standard $\alpha$-model. This naturally brings increased dissipation to the disk surface layers (around the photospheres) at small radii and high luminosities. We find that the local spectrum transitions directly from a modified blackbody to a saturated Compton scattering spectrum as we increase the effective temperature and orbital frequency while decreasing midplane surface density. Next, we construct annuli models based on the parameters of a $L/L_{\text{edd}} = 0.8$ disk orbiting a 6.62 solar mass black hole using two modified dissipation profiles that explicitly put more dissipation per unit mass near the disk surface. The new dissipation profiles are qualitatively similar to the one found by Hirose et al., but produce strong near power-law spectral tails. Our models also include physically motivated magnetic acceleration support based once again on scaling the Hirose et al. results. We present three full-disk spectra, each based on one of the dissipation prescriptions. Our most aggressive dissipation profile results in a disk spectrum that is in approximate quantitative agreement with certain observations of the steep power-law spectral states from some black hole X-ray binaries.

Key words: accretion, accretion disks – black hole physics – X-rays: binaries

Online-only material: color figures

1. INTRODUCTION

Understanding the internal structure of radiating accretion disks remains a major outstanding problem in astrophysics and is the key for connecting theoretical models to the observed spectra from systems such as black hole X-ray binaries and active galactic nuclei. In a series of papers, Hubeny & Hubeny (1998), Hubeny et al. (2000), and Hubeny et al. (2001) adapted the one-dimensional time-independent stellar atmosphere code TLUSTY for accretion disk applications. These authors constructed non-LTE accretion disk annuli models that self-consistently solved the radiative transfer and vertical structure equations. The results were appropriate for supermassive black holes and eventually took into account relativistic effects, Comptonization, and metal opacities. Despite their complexities, these models still have significant physical limitations. In particular, they employed an ad hoc prescription where dissipation per unit mass as a function of height was assumed to be constant.

Recent vertically stratified shearing box simulations have led to significant progress in understanding the internal structure of accretion flows. These calculations evolve the time dependent three-dimensional radiation magnetohydrodynamic equations in a box embedded inside the disk and account for the vertical tidal gravity due to the central black hole. Such simulations have been completed for a wide range of box-integrated radiation to gas pressure ratios of interest (Turner 2004; Hirose et al. 2006, 2009b; Krolik et al. 2007; Blaes et al. 2007). The resulting spatial dissipation profiles generally peak at slightly less than a pressure scale height away from the disk midplane (Blaes et al. 2011). While the underlying physics is still unclear, these dissipation distributions presumably capture the effects of vertically stratified magnetorotational (Balbus & Hawley 1991, 1998) turbulence. Moreover, these studies showed that magnetic pressure support against gravity may be significant or even dominant (in the outer layers) compared with radiation pressure support. A limitation of the shearing box simulations is that they rely on flux limited diffusion for treating photons and do not solve the full radiative transfer equation.

Studies that self-consistently integrate full multi-frequency radiative transfer into the time-dependent three-dimensional magnetohydrodynamic simulations do not yet exist, though significant progress toward that goal is being made (Jiang et al. 2012; Davis et al. 2012). Many authors have instead attempted to incorporate physics results from the shearing box simulations into TLUSTY calculations. For example, Davis et al. (2005) used the time and horizontal averaged dissipation profile as a function of height from the Turner (2004) simulation. Compared with a model based on the ad hoc constant dissipation profile per unit mass prescription, the resulting annulus vertical structure is very different but the emergent spectrum is not. Blaes et al. (2006) included a horizontal and time averaged magnetic acceleration profile derived from the Hirose et al. (2006) simulation. They found that magnetic pressure support produced a disk atmosphere with a larger vertical density scale height and a harder spectrum. More recently, Davis et al. (2009) utilized both the magnetic acceleration and dissipation profiles from Hirose et al. (2009b) in a TLUSTY calculation. The resulting disk spectrum is once again harder than the case without magnetic support but similar to models with the ad hoc dissipation profile. These authors also performed a separate Monte Carlo radiative transfer calculation that propagated photons through a three-dimensional snap shot of the simulation domain. They discovered that the complicated density inhomogeneities in the photospheric regions actually soften the spectrum and largely negate the hardening that resulted from including the significant magnetic pressure support against gravity. Note that all of the above studies relied on the $\alpha$ model to generate radial disk profiles in order to produce full disk spectra.
The stratified shearing box simulations by Hirose et al. (2009b) found no evidence of the radiation pressure dominated thermal instabilities predicted by the standard $\alpha$-model (Shakura & Sunyaev 1976). However, more recent simulations by Jiang et al. (2013) find evidence of thermal runaways. Moreover, these disks may still be vulnerable to viscous (inflow) instabilities (Lightman & Eardley 1974; Hirose et al. 2009a), so whether such flows are ultimately stable is still open to debate. Nevertheless, if we assume that radiation pressure dominated disks can exist in nature, they provide a viable mechanism for powering the high-luminosity steep power law (SPL, also known as very high) and soft (also known as high) states observed in accreting systems. Hirose et al. (2009a) find that the thermal equilibrium relation between surface density and time-averaged (over many thermal times) vertically integrated stress actually agrees with the one predicted by the standard $\alpha$ prescription. Furthermore, these authors found that the time averaged and vertically integrated total thermal pressure (radiation plus gas) does correlate closely with the vertically integrated stress, once again in agreement with the $\alpha$ model.

These results suggest that the $\alpha$ model predictions for the relationships between basic annuli parameters such as surface density $\Sigma$, accretion rate $\dot{M}$ (which is proportional to luminosity), effective temperature $T_{\text{eff}}$, and distance $r/r_g$ (where $r_g$ is the gravitational radius $= GM/c^2$) from the black hole may actually hold, at least in a time averaged sense. In particular, the surface density of a radiation pressure dominated disk annulus will decrease with increasing accretion rate and decreasing distance from the black hole. Additionally, the disk half thickness, which is proportional to its pressure scale height, will increase with higher accretion rate. Moreover, the shearing box simulations collectively revealed that the locations of the (effective and scattering) photospheres and the spatial distribution of turbulent dissipation are both tied to the scale height. While both the dissipation profile and photospheres move vertically outward at higher accretion rates, the simulations also suggest that the photospheres actually move closer to the dissipation peaks as the surface density drops and effective temperature climbs. This means that the fraction of accretion power dissipated near the spectral forming photospheric region increases as we raise the annulus luminosity and move toward the black hole. At high accretion rates, there may be enough dissipation in the upper layers of those annuli close to the black hole to power a corona that can Compton up-scatter a significant number of photons to very high energies. This might be a plausible mechanism for generating the high-energy SPL spectra. Previously, Davis et al. (2005) tried a dissipation profile that is discontinuous at the scattering photosphere, which resulted in dissipating 50% of the flux above the scattering photosphere as well as a power-law like annulus spectrum that qualitatively resembled the SPL.

In this work, we explore this possibility and extend these previous efforts by comparing the effects of three different dissipation profiles on the disk vertical structure and emergent spectrum. The first dissipation profile is a fit to that produced by the shearing box simulations of Hirose et al. (2009b). The two others explicitly put more dissipation per unit mass in the low column density regions. We also include simulation motivated magnetic acceleration profiles in our calculations. We first study the effects of increased dissipation (of turbulent energy into heating the gas) in the disk upper layers as luminosity increases and annulus radius decreases. Next, we compare the effects of the three dissipation prescriptions on the spectrum of a $L/L_{\text{Edd}} = 0.8$ disk. We organize the paper as follows. In Section 2, we summarize our numerical method for solving the vertical accretion disk structure plus radiative transfer equations. Section 3 presents some theoretical estimates regarding the disk structure to help us better understand our results. We then move on to presenting our calculated spectra and disk structures in Section 4. In Section 5, we outline the key physics underlying our numerical results. Finally, we discuss the astrophysical implications of our findings in Section 6 and conclude in Section 7.

2. METHOD

We use the computer program TLUSTY (Hubeny & Lanz 1995) to compute non-LTE time-independent one-dimensional disk annuli vertical structure models and emergent spectra. We incorporate dissipation profiles based on simulations of the turbulence generated by the magnetorotational instability (Balbus & Hawley 1991) that serves as the physical source of stress. The code self-consistently solves the disk structure equations along with angle-dependent radiative transfer but treats electron (both Thomson and Compton) scattering with an angle-averaged Kompaneets source function (Hubeny et al. 2001). We ignore the effects of potentially important physics such as outflow and irradiation. Since the mean free path for electron–ion collisions is generally very short compared with the disk scale height, we assume that the electrons and protons are thermally locked, so that ionized gas can be locally characterized by a single temperature $T_g$. We assume that energy transport is purely vertical and is only by diffusive radiative flux so that the dissipation $Q$ is related to the frequency-integrated flux $F$ via

$$ Q(z) = \frac{dF}{dz}, $$

where $z$ is the height measured vertically upward so that $z = 0$ is the disk midplane. Blaes et al. (2011) discovered that this is unlikely to be the case in nature as the latest shearing box simulations revealed that radiation pressure work and magnetic buoyancy driven radiation advection also contribute significantly to vertical energy transport in radiation pressure dominated disks. In particular, radiation advection can be as important as the radiative diffusion flux near the midplane. However, in the Blaes et al. (2011) simulations, both radiation pressure work and radiation advection flux become small compared with radiative diffusion flux at higher altitudes, so that in the end it is radiative diffusion that carries the dissipated energy into the photospheric region where the spectrum forms, regardless of how the energy transport occurs deep inside the disk. We therefore suspect that ignoring radiation advection will not significantly change our spectral results.

To generate input for TLUSTY, we express the dissipation profile as a function of fractional surface density. In the first case, we fit the horizontal and time averaged dissipation profile from Hirose et al. (2009b) with a broken power law of the form

$$ \frac{Q\Sigma}{\rho} = -\Sigma \frac{dF}{d\Sigma} = F_0 \begin{cases} A \left( \frac{\Sigma}{\Sigma_0} \right)^{0.5}, & \Sigma/\Sigma_0 < 0.11 \\ B \left( \frac{\Sigma}{\Sigma_0} \right)^{0.2}, & \Sigma/\Sigma_0 > 0.11 \end{cases} $$

as shown in Figure 1. Here $F_0$ and $\Sigma_0$ denote the total outgoing flux and midplane surface density, respectively. Here $A \approx 0.65$ and $B \approx 0.33$ are normalization constants computed automatically in the code to ensure that the profile is continuous.
at $\Sigma/\Sigma_0 = 0.11$ and that the total flux is $F_0$ so that
\[
F_0 = \int_0^{\Sigma_0} \frac{dF}{d\Sigma} d\Sigma.
\]
(3)

The surface density $\Sigma$ at height $z$ is given by
\[
\Sigma(z) = \int_{\Sigma}^{\infty} \rho(z') dz',
\]
(4)

where $\rho$ is the depth dependent density. Note that $\rho$ and $\Sigma$ both go to zero at infinity since we defined $z = 0$ to be the disk midplane. Davis et al. (2009) used a similar profile with a power-law index of 0.5 throughout the disk. Here we chose to fit the simulation in slightly more detail, resulting in the break at $\Sigma/\Sigma_0 = 0.11$.

Our other dissipation profiles are given by
\[
\frac{Q\Sigma}{\rho} = -\Sigma \frac{dF}{d\Sigma} = F_0 \zeta \left( \frac{\Sigma}{\Sigma_0} \right)^\zeta,
\]
(5)

where the power-law index $\zeta$ is equal to 0.1 or 0.03. From here on, we shall refer to the 0.1 case as the moderate profile and the 0.03 case as the aggressive profile because the latter dissipates an even higher fraction of the accretion power in the disk upper layers around the photospheres. Note that in both cases, we have artificially increased the dissipation per unit mass $Q/\rho = dF/d\Sigma$ at low surface densities compared with Equation (2). These dissipation profiles are all shown in Figure 2. As it turns out, our calculation shows that these simpler but somewhat artificial prescriptions still result in dissipation profiles $Q(z)$ that qualitatively agree with Hirose et al. (2009b) when viewed as a function of height, as shown in Figure 8. In practice, we manually imposed a cut-off for $\Sigma < 10^{-6}\Sigma_0$ to these single power-law dissipation profiles in order to ensure energy conservation (in other words, to ensure that the total flux produced by the models is equal to $\sigma T_{\text{eff}}^4$ within the calculation domain.

Figure 2 also illustrates the dissipation profile that would result from a vertically local alpha model, in which dissipation locally tracked thermal pressure as a function of height. This is inconsistent with simulations, and puts far less dissipation above the photosphere than our other profiles.

Our key assumption is that the dissipation per unit mass always has the same functional dependence on fractional surface density $\Sigma/\Sigma_0$ regardless of annuli parameters. Given $\Sigma_0$, $\Omega$, and $T_{\text{eff}}$, the total flux $F_0$ ensures the correct normalization for the dissipation profile while the power-law dependence on fractional surface density encodes the depth dependence. We will elaborate further on the implications of this assumption in the next section.

To incorporate magnetic support against gravity, we assume that the magnetic acceleration profile of an annulus is tied to the pressure scale height $H$ and simply expands vertically as we increase $H$ by increasing $T_{\text{eff}}$ and decreasing $\Sigma_0$. In other words, the magnetic acceleration profile has the form
\[
a_{\text{mag}}(x) = a_0 f(x),
\]
(6)

where $x = z/H$ is the vertical position in an annulus normalized to the scale height, and $a_0$ is the midplane acceleration. To get $f(x)$, which represents the height dependence of $a_{\text{mag}}$, we smoothed the time and horizontally averaged magnetic acceleration data from simulation 1112a of Hirose et al. (2009a). We assumed that only the midplane magnetic acceleration $a_0$ changes with annuli parameters in our models. This assumption is consistent with magnetic acceleration profiles from other recent simulations (Hirose et al. 2009a, 2009b). To obtain $a_0$, we assume that the magnetic acceleration simply scales with the radiative acceleration so that $a_0$ is proportional to $k F_0/c$, where $F_0 = \sigma T_{\text{eff}}^4$ is the total (frequency integrated) emergent flux.
from the annulus surface and $\kappa$ is the total (frequency integrated) opacity. In other words, the relative importance of magnetic and radiative accelerations remain unchanged in our disk models as we sweep through the parameter space. We should note that the shearing box accretion disks simulations mentioned earlier have radiation support against gravity becoming more important compared with the magnetic contributions as the radiation to gas pressure ratio of a disk segment is increased. This means that our prescription will likely produce harder spectra (Blaes et al. 2006) than what the highest radiation to gas pressure ratio simulation would predict. However, Davis et al. (2009) found that this hardening is largely compensated by softening due to density inhomogeneities.

We first study the evolution of annuli spectra with midplane surface density $\Sigma_0$, orbital frequency $\Omega$, and effective temperature $T_{\text{eff}}$. Our focus is on low surface density (down to 80 solar mass) and high effective temperature (up to $1.8 \times 10^7$ K) annuli that might represent conditions within the inner parts of high-luminosity disks around stellar mass black holes.

Next, we construct the full spectrum for a disk around a 6.62 solar mass Kerr black hole with spin $a/M = 0.5$ using both of the above dissipation profiles. We divide the disk into 52 individual annuli extending down to $5R_g$ and out to $60R_g$, where $R_g = GM/c^2$ is the gravitational radius. Following previous authors, we compute $\alpha$ model-based $\Sigma_0$ and $T_{\text{eff}}$ radial profiles using the fully relativistic one-zone disk radial structure equations by Novikov & Thorne (1973), including the Riffert & Herold (1995) corrections. We have also employed the standard no-torque inner boundary conditions at the inner most stable circular orbit (ISCO) of the disk (but see, e.g., Krolik et al. 2005). We then use a relativistic transfer function code (Agol 1997) to generate the full disk spectrum as seen by observers at infinity. Finally, we ignore the potentially important effects of radial advection and assume that energy transport is purely vertical. A proper study of these effects requires two-dimensional slim disk models that couple the equations of radial and vertical structure (Sadowski et al. 2011) instead of simply computing the radial profile from height-averaged quantities. For our work, we checked that vertical radiative diffusion time is significantly less than the infall time for all annuli that we modeled.

3. THEORETICAL EXPECTATIONS

In order to better understand the physics behind our numerical results, we first explore the likely consequences of changing $\Sigma_0$, $\Omega$, and $T_{\text{eff}}$ by estimating the variation of photosphere locations, both scattering and effective, with these parameters. We assume a geometrically thin disk in the radiation pressure dominated limit to obtain the basic annuli parameters. To make the calculation easier, we re-parameterize the problem in terms of variations in annulus radial distance $r$ from the black hole and in disk luminosity $L$. For simplicity, we ignore general relativistic corrections for our simple estimates, although all numerical calculations elsewhere in this paper include them. Our annuli parameters are related to $L$ and $r$ via

$$T_{\text{eff}} = \left( \frac{3GM_{\odot}L_{\odot}}{8\pi\eta \kappa c^2} \right)^{1/4} \left( L \right)^{1/4} \left( \frac{1}{r} \right)^{3/4}$$

(7)

and

$$\Omega = \left( \frac{GM}{r^3} \right)^{1/2}.$$

(8)

Here $\eta$, $r_g$, $L_{\odot}$, and $\kappa_T$ denote the accretion efficiency, gravitational radius, Eddington luminosity, and Thomson opacity, respectively. Equation (7) arises out of the fact that $F \propto L/r^3$ if the annulus is far away from the black hole and is thus in the non-relativistic regime. Finally, for a radiation pressure dominated annulus, the pressure scale height is on the order of (and is proportional to) the disk half-thickness, and is therefore given by

$$H = \frac{3k}{2\kappa_T \eta L_{\odot}} \frac{L}{r} r_s = \frac{L}{r^3} \eta L_{\odot}$$

(9)

The equations above do not make any assumptions about the local turbulent stresses. However, to obtain scalings for the surface mass density, we must assume a stress prescription, and we adopt the $\alpha$-model (Shakura & Sunyaev 1973), which in turn implies

$$\Sigma_0 = \frac{8}{3\eta} \frac{\eta}{L/E} \left( \frac{\kappa}{\kappa_T} \right)^{-2} \left( \frac{r}{r_s \sigma_T} \right)^{1/4}$$

(10)

To proceed, consider a semi-infinite plane parallel atmosphere with only free-free and electron scattering opacities. Our numerical models elsewhere in the paper include bound–free opacities and Compton scattering. We define the lower boundary to be at $z = 0$. The effective and scattering photospheres occur at depths $z_e$ and $z_s$ such that

$$\tau_e(z_e) \approx \int_{z_e}^{\infty} \rho(z) \sqrt{\kappa_T} dz = 1$$

(11)

and

$$\tau_s(z_s) = \int_{z_s}^{\infty} \rho(z) \kappa_T dz = 1,$$

(12)

where

$$\kappa_T(z) = \frac{A \rho(z)}{T_g(z)^{7/2}}$$

(13)

is the free–free opacity with $A$ being a constant. Here $\rho$ and $T_g$ are the gas density and temperature, respectively. Also, $\tau_e$ and $\tau_s$ are the frequency averaged effective and scattering optical depths of the media, respectively.

Next, for a disk annulus with pressure scale height $H$, we assume that the density and gas temperature, like magnetic acceleration, are also only functions of $x = z/H$, so that

$$\rho(x) = \rho_0 f(x)$$

(14)

and

$$T_g(x) = T_0 f(x)$$

(15)

where $\rho_0$ and $T_0$ denote the density and temperature at the disk midplane, respectively. The functions $f_0(x)$ and $f_1(x)$ are thus invariant under annuli parameter changes and encode the height dependence of density and temperature.

Recall our prescription that the dissipation per unit mass has the same functional dependence on fractional surface density regardless of annulus parameters. This assumption together with Equation (14) means that we can also write the local dissipation as a function of height in the form of

$$Q(z) = Q_0 q(z),$$

(16)

where the function $q(x)$ is once again independent of annuli parameters. The normalization $Q_0$ depends on annuli parameters through the fact that $Q = dF/dz$ and $F \propto L/r^3$. In particular, this means that $Q_0$ only changes with $r$ but not $L$, at least far away from the black hole where we can ignore relativistic effects.
The more important implication of Equation (16) is that the position of the dissipation profile peaks observed in the Hirose et al. (2009b) simulations remains at the same $x_p = z_p / H$ regardless of annuli parameters. In other words, we have assumed that the dissipation profile is tied to the disk scale height $H$ and simply stretches out as we increase $H$, which is in reasonable agreement with the simulation results in Blaes et al. (2011).

We begin with a disk annulus with luminosity $L_1$ at a distance $r_1$ away from the black hole and scale to a disk at some new $L$ and $r$. Equations (8)–(10) imply

$$\rho_0 = \rho_1 \left( \frac{L_1}{L} \right)^{2} \left( \frac{r}{r_1} \right)^{3/2},$$  \hspace{1cm}  (17)

$$T_0 = T_1 \left( \frac{r}{r_1} \right)^{-3/8},$$  \hspace{1cm}  (18)

and

$$H = H_1 \frac{L}{L_1},$$  \hspace{1cm}  (19)

where $\rho_1$, $T_1$, and $H_1$ are the midplane density, temperature, and scale height of the original disk annulus, respectively. The temperature scaling arises from the total radiative flux $F(r) \propto L / r^3$ and the assumption that radiative diffusion is the dominant mechanism of vertical heat transport. Inserting these scaling relations into Equations (11) and (12), and changing variable from $z$ to $x$, yields

$$\tau_e(x_e) \approx \frac{H_1 \rho_1^{3/2} \sqrt{\kappa T_A}}{T_1^{7/4}} \left( \frac{L_1}{L} \right)^{2} \left( \frac{r}{r_1} \right)^{51/32} \int_{x_e}^{\infty} \frac{f_d(x)}{f_i(x)^{7/2}} dx = 1,$$  \hspace{1cm}  (20)

and

$$\tau_s(x_s) = H_1 \kappa T \frac{L_1}{L} \left( \frac{r}{r_1} \right)^{3/2} \int_{x_s}^{\infty} f_d(x) dx = 1.$$  \hspace{1cm}  (21)

In deriving the above we assumed once again that $f_d(x)$ and $f_i(x)$ are invariant under changing $L$ and $r$. Equations (20) and (21) imply that both $x_e$ and $x_s$ must decrease as we raise the disk luminosity and move toward to the black hole. In contrast, the dissipation profile peak stays at the same $x = x_p$ independent of $L$ and $r$ as discussed above. Hence the spectral forming photospheric region will move inward, at least in scale height units, toward segments of higher dissipation with increasing $L$ and decreasing $r$ (or equivalently, increasing $\Omega$, $T_{\text{eff}}$ and decreasing $\Sigma_0$).

This result suggests that the fraction of accretion power dissipated in the spectral forming region (around the photospheres) will increase as we increase $\Omega$ and $T_{\text{eff}}$ and decrease $\Sigma_0$. The higher dissipation rate may drive the electron temperature to rise sharply over the photon temperature immediately beyond the point where the gas and radiation thermally decouple. Such a vertical temperature structure may result in a hot corona that Compton up-scatters photons from the colder, optically thick disk underneath. This disk–corona geometry has been invoked to explain the SPL (also known as very high) state spectra commonly observed in black hole X-ray binary systems (Kubota & Done 2004). As we shall see in Section 5 below, for this scenario to work, the electron temperature gradient at the disk–corona interface needs to be steep enough to produce a downward sloping power-law spectrum.

![Figure 3](image_url)

**Figure 3.** Gas temperature (bottom panel) and emergent spectrum (top panel) from annuli models selected to illustrate the evolution of spectra with disk parameters. All models here use the broken power-law dissipation profile from Equation (2). The blue curves correspond to parameters from the Hirose et al. (2009b) simulation with $T_{\text{eff}} = 3.46 \times 10^7$ K, $\Omega^2 = 3.61 \times 10^4$ s$^{-2}$ and $\Sigma_0 = 5.31 \times 10^4$ g cm$^{-2}$. The black curve has $T_{\text{eff}} = 1.23 \times 10^7$ K, $\Omega^2 = 9.75 \times 10^5$ s$^{-2}$ and $\Sigma_0 = 1722$ g cm$^{-2}$. Finally, the red curve has $T_{\text{eff}} = 1.81 \times 10^7$ K, $\Omega^2 = 4.51 \times 10^6$ s$^{-2}$ and $\Sigma_0 = 801$ g cm$^{-2}$. The vertical dashed lines on the lower panel indicate the locations of the scattering photospheres for each model.

(A color version of this figure is available in the online journal.)

### 4. RESULTS

#### 4.1. Spectra and Vertical Structure

Figure 3 shows the vertical structures and resulting spectra for a series of three annuli models using the broken power-law dissipation profile from Equation (2). Recall that this dissipation prescription is based on the results from the simulations of Hirose et al. (2009b). To better understand these models, we calculated a frequency-dependent Compton $\gamma$-parameter by integrating outward from the effective photosphere at each frequency. For models that show prominent saturated Compton scattering ($T_{\text{eff}} = 1.81 \times 10^7$ K and $T_{\text{eff}} = 1.23 \times 10^7$ K), the frequency-dependent Compton $\gamma$-parameter is much less than 1 at low frequencies but rises sharply at higher frequencies as scattering begins to dominate over absorption opacity. The model with the original Hirose et al. (2009b) simulation parameters, however, does not display significant Comptonization and its $\gamma$-parameter is much less than unity at all frequencies. The maximum $\gamma$-parameter increases (from about 0.01 to 10 and then 20) as we increase $T_{\text{eff}}$ and $\Omega$ and decrease $\Sigma_0$, in agreement with the evolution of the spectral shapes in this plot. Note that the
lowest surface density (801 g cm\(^{-2}\)) annulus is actually marginally effectively thin, while all other models presented here are effectively thick.

In Figure 4, we show results representative of the series of annuli models that we use later to construct a disk integrated spectrum utilizing the same broken power-law dissipation profile. The disk has \(L/L_{\text{Edd}} = 0.8\) and orbits a black hole with spin parameter \(a/M = 0.05\). Note that none of the models in Figures 3 and 4 display a significant high-energy spectral tail despite covering a wide range of \(T_{\text{eff}}, \Omega\) and \(\Sigma_0\), indicating that simply changing these basic disk parameters does not produce spectra consistent with the SPL state.

Replacing the dissipation profile with the single power-law forms given by Equation (5) results in vastly different annuli spectra for the same \(L/L_{\text{Edd}} = 0.8\) disk. In this case, we have by construction put more dissipation per unit mass into the low surface density upper layers. The annuli spectra now display clear high-energy tails, as shown in Figures 5 and 6.

Finally, Figure 7 shows total disk integrated spectra using each of the three dissipation prescriptions while holding the disk parameters the same. The single power-law dissipation profiles (dashed) put more energy per unit mass into the upper layers, resulting in spectra with strong high-energy tails.

### 4.2. Dissipation and Photospheres

The numerical results confirm our theoretical estimate regarding the photospheres. Figure 8 illustrates that the dissipation profile peaks do indeed remain at roughly the same \(z/H\) (although \(H\) is modified from Equation (9) and varies with \(r\) since our code took relativistic effects into account). Moreover, the scattering photospheres move toward regions of higher dissipation as we decrease \(r\), so that the fractional accretion power dissipated around the photosphere (and in the outer layers) increases as we approach the black hole.

Before moving on, we stress that the functional form of the dissipation profile deep inside the disk is sensitive to details of the thermal and magnetic pressures, but does not actually affect the photon output as the spectral shape really only depends on the dissipation per unit mass near the photospheres. In other words, the fact that the dissipation profiles peak slightly above instead of at the midplane is in some sense a detail that is mostly irrelevant to the spectra.

### 5. Physics of Annuli Spectra Evolution

We saw in the previous section that the two classes of dissipation profiles we used resulted in vastly different spectra. The
more artificial unbroken power-law dissipation profiles represented by Equation (5), which by construction put more energy per unit mass into the low surface density regions, gave rise to extended high-energy spectral tails. On the other hand, the broken power-law prescription from Equation (2) failed to produce a high-energy spectral tail even at the remarkably low midplane surface density of 801 g cm$^{-2}$, but instead evolved from a modified blackbody shape directly into spectra characteristic of saturated Compton scattering as we decreased $\Sigma_0$ while increasing $\Omega$ and $T_{\text{eff}}$.

Comparing Figure 4 with either Figure 5 or 6, the temperature structures resulting from the two sets of profiles indicate two important and related differences. First, in the region right below the scattering photosphere where the atmosphere is marginally optically thick to electron scattering, the single power-law dissipation prescriptions result in sharper upward gas temperature gradients compared to the broken power-law version, largely because the former puts more power into the upper layers. Second, the same extra power also drives the maximum gas temperature for models resulting from the unbroken power-law dissipation profiles much higher.

To make our understanding more quantitative, we compute an approximate Compton-$\gamma$ parameter and associated photon power-law index. We do not include relativistic effects in this simple estimate. In an atmosphere with vertical temperature stratification, we define a $\gamma$-parameter in terms of the scattering optical depth $\tau_e$. If the atmosphere is optically thin to scattering, we have

$$y = \int_0^{\tau_e} \frac{4k_B T_g d\tau_e}{m_e c^2} = \int_{z_{\text{max}}}^{z_e} \frac{4k_B T_g \kappa_T \rho dz}{m_e c^2}, \quad (22)$$

where $\tau_e$ is the scattering optical depth where the up-scattering process effectively begins, and $z_e$ is the corresponding physical height. We integrate inward from the disk surface so that $z_{\text{max}}$ is the maximum height of the annulus. For an optically thick medium, $y$ is the sum of Equation (22) and a $\tau_e > 1$ part:

$$y = \int_0^{1} \frac{4k_B T_g d\tau_e}{m_e c^2} + \int_{1}^{\tau_e} \frac{4k_B T_g \kappa_T \rho dz}{m_e c^2}. \quad (23)$$

The above expressions give the integrated or effective $\gamma$-parameter that an observer sees by looking down a depth $\tau_e$ into the disk. Within the Kompaneets framework, Comptonization of cool source photons by a hotter electron gas should result...
in an intensity spectrum of the form

\[ I_\nu \propto \nu^{m+3}, \tag{24} \]

where

\[ m + 3 = \frac{3}{2} \left( 1 \pm \sqrt{1 + \frac{16}{9y}} \right) \tag{25} \]

is a real constant and we have written the power-law index of \( I_\nu \) in the standard notation. For the up-scattering process under consideration, the \( y \)-parameter is usually significantly less than 1 and the minus sign is generally appropriate.

For a given converged annulus model with a prominent high-energy spectral tail, we integrate Equation (23) from the disk surface downward and compute \( m + 3 \) as a function of height. We then identify the height \( z \) at which the power-law index approximately matches the measured slope of the high-energy tail produced by the TLUSTY spectral calculation. This then gives us the integrated (or effective) \( y \)-parameter of the atmospheric segment that produced the power-law like spectrum along with the an idea of the depth at which the up-scattering process that led to the high-energy tail began.

We use the \( r/r_g = 20.2 \) annulus from the \( \zeta = 0.03 \) run to briefly illustrate the above process. For this model, we find that \( m + 3 \) is approximately equal to the estimated high-energy tail slope of \(-2\) at \( \tau_r \approx 3 \) with an integrated \( y \)-parameter of about 0.39. This means that the Compton up-scattering region has to extend down to about \( \tau_r \approx 3 \) in order to produce the model’s high-energy tail, which is a very sensible result since this is approximately the depth where the upward gas temperature gradient is the steepest. Note that the photon spectrum does not display an exponential cut-off within the frequency range of our calculations but instead has a slightly concave down shape due to the vertical temperature stratification. In an isothermal medium, the photons would eventually reach the gas temperature and the up-scattering would stop. However, in our model atmospheres the photons continuously move upward into regions of even higher temperature. The atmosphere becomes increasingly optically thin as photons propagate toward the disk surface and the up-scattering efficiency decreases, resulting in the continuously decreasing (or increasingly negative) spectral slope with photon energy.

In the case with the single power-law dissipation profiles, the spectral forming region around the scattering photosphere is able to significantly Compton up-scatter photons from the peak of the lower energy blackbody spectrum produced deeper within the disk (below the gas temperature minimum where electrons and photons decouple). On the other hand, the broken power-law profile does not heat the photospheric region sufficiently and hence most lower energy photons red-ward of the thermal spectra maximum are up-scattered into the blackbody peak. This then results in the saturated Compton scattering spectrum from the models based on Equation (2) as the height with the lowest gas temperature (where the up-scattering begins) is still very optically thick to scattering.

It is worth noting that the region between the gas temperature minimum and the scattering photosphere is largely optically thick to electron scattering for models based on any of our dissipation prescriptions, which at first glance might lead us to expect saturated Compton scattering spectra in all cases. To understand why this does not occur, we stress that the physical reasoning in the previous paragraph is only applicable to optically thick scattering dominated photosphere having an outward rising electron temperature gradient. Here it is the marginally optically thick region right below the scattering atmosphere that controls the spectral shapes and, given the right conditions, a significant high-energy tail will result even if the entire scattering dominated region (between the electron temperature minimum and the scattering photosphere) is highly optically thick. In contrast, an unstratified isothermal medium would give rise to a powerful high-energy tail only if the entire scattering dominated region is not so optically thick that the photons can thermalize with the electrons before exiting. In this sense, it is really the degree of deviation of the electron temperature structure in the scattering dominated region from that of an isothermal atmosphere that determines whether the spectrum will develop a powerful tail when the medium is overall very optically thick to scattering.

6. DISCUSSION

We have constructed, via calculations that self-consistently incorporate disk vertical structure, radiative transfer, and physically motivated dissipation prescriptions, annuli models that have an inner region between where the gas temperature decreases outward and an outer layer where the temperature increases, potentially sharply, with height. Such a vertical structure can be viewed as a hot corona covering an optically thick cold disk. Variations upon this basic geometry are commonly invoked to explain and fit the high-energy emissions from black hole X-ray binaries. Our results indicate that the corona, if it exists in nature, may not be isothermal and that it is the sharpness of the temperature gradient in this electron scattering dominated region that determines the emergent photon spectrum.

The locations of the scattering photosphere and gas temperature minimum move vertically inward as we decrease \( r \) as predicted by our estimate in Section 3. In particular, this means that our self-consistent calculation may produce a geometry where, due to strong Comptonization, the disk becomes truncated as the radius decreases and is completely replaced by the coronal structure. This has been considered as another potential geometry underlying the observed very high state spectra (Done & Kubota 2006). The difference between this scenario and one where the disk extends down to the last stable orbit (presumably the models shown in Figures 4–6) may represent the distinction between the so-called extreme very state and very high state geometries as suggested by, for example, Kubota & Done (2004).

We used dissipation prescriptions that are at least qualitatively consistent with the Hirose et al. (2009b) simulations to produce powerful high-energy spectral tails. These dissipation profiles put more energy per unit mass into the low surface density upper layers to give rise to sharply rising gas temperature gradients. The more aggressive \( \zeta = 0.03 \) case actually resulted in a full disk spectrum that is a good match for the steep power-law observations. For example, the steep power-law state observed in well-studied black hole X-ray binary systems such as XTE J1748-288 (\( \Gamma \approx 2.92 \); Revnivtsev et al. 2000; McClintock & Remillard 2006) and XTE J1550-564 (\( \Gamma \approx 2.82 \); Done & Kubota 2006; McClintock & Remillard 2006) have spectral index \( \Gamma - 2 \) close to but less than 1, where \( \Gamma \) is defined such that \( \nu F_\nu \propto \nu^{-(\Gamma - 2)} \). This is in approximate quantitative agreement with the dashed blue curve (face on disk using \( \zeta = 0.03 \)) in Figure 7. However, our edge-on full disk spectra for both single power-law dissipation prescriptions peak at an energy too high compared with observations. Nonetheless, our results therefore
indicate that dissipation physics may explain the observed $\Gamma$ in the X-ray bands of some SPL observations. We should point out a numerical caveat in our work. We had to manually impose a cut-off to the single power-law dissipation profile at $\Sigma < 10^{-6}\Sigma_0$ in order to obtain energy conservation. Without the cut-off, our calculations would only capture about 80% to 85% of the expected radiative flux (given by $\sigma T^4_{\text{eff}}$) because the computational domain is limited by convergence issues to a minimum surface density floor of $10^{-3}\text{ g cm}^{-2}$. However, the atmosphere becomes so tenuous at such low surface densities that the material there perhaps should no longer be considered part of the accretion disk. The gas temperature at the surface density minimum (which is also the maximum $T_{\text{eff}}$ reached by the calculation) is so high that the particles beyond that height likely have enough thermal energy to escape the black hole, especially in high Eddington ratio systems. This indicates that the missing 15% to 20% of the expected radiative flux, which corresponds to power dissipated at heights with surface density lower than $10^{-3}\text{ g cm}^{-2}$, may actually go into something other than photons, most likely a thermally driven wind. A calculation with an even lower dissipation power-law index $\zeta$ would capture even less of the expected flux within the computational domain. It is therefore possible that further increasing the dissipation per unit mass in the disk upper layers will simply result in dissipating a larger fraction of the accretion power into a wind instead of a more prominent high-energy spectral component. This scenario may physically limit the ability of models like ours, where the dissipation of turbulent energy simply heats a thermal electron population, to self-consistently produce a spectrum with an X-ray tail that is significantly harder (lower $\Gamma$) than what we have so far. This means that alternative physics input such as a non-thermal electron energy distribution may be necessary to reproduce some SPL observations where $\Gamma$ is closer to 0.5. This case is particularly plausible given that the spectral tails can extend out to several hundred keV or more in the soft, hard (Ling & Wheaton 2005) and very high states.

7. CONCLUSIONS

We have self-consistently computed the vertical structure and emergent spectra of accretion disk annuli models spanning a range of surface densities, effective temperatures and orbital frequencies. We focused on the effects of three different local dissipation prescriptions. We also incorporated simulation-based magnetic acceleration support against gravity. The first dissipation profile is a power law as a function of fractional surface density, with a break at $\Sigma/\Sigma_0 = 0.11$. This form fits the horizontal and time averaged dissipation profile from the local shearing box simulations of Hirose et al. (2009b). The resulting annuli spectrum transitions directly from that of a modified blackbody to one characteristic of saturated Compton scattering as we decrease $\Sigma_0$ while increasing $T_{\text{eff}}$ and $\Omega$. The full spectrum ($L/L_{\text{Edd}} = 0.8$) thus constructed is largely thermal and shows a hint of the saturated Comptonization feature. The second and third dissipation profiles are unbroken power laws with artificially chosen power-law indexes but are still in qualitative agreement with the simulation results when viewed as a function of height. These prescriptions put more power per unit mass into the disk upper layers compared with the broken power-law case and resulted in annuli spectra with significant high-energy tails, which are prominent even in the full disk spectra. In particular, the more aggressive $\zeta = 0.03$ models gave rise to a disk spectrum that is in approximate quantitative agreement with some SPL observations. Our annuli vertical structure results are consistent with theoretical expectations. As a function of height, the positions of the dissipation profile peaks were indeed tied to the disk scale height, remaining at roughly constant $z/H$ as we varied annuli parameters. Moreover, the photospheres moved inward as a function of $z/H$ as we increased $T_{\text{eff}}$ and $\Omega$ while decreasing $\Sigma_0$, bringing more dissipation to the spectral forming region. With both dissipation profiles, we found that the disk thickness decreased as we approached the black hole, raising the possibly of a truncated inner accretion flow. While our results are promising, we also saw evidence that increasing dissipation in the disk upper layers may not necessarily lead to a more powerful high-energy spectral component. Instead, a larger fraction of the accretion power would go into driving a thermal wind. In addition, non-thermal electron distributions, which we did not consider, are probably necessary to truly understand the full range of high-energy spectral shapes observed in black hole X-ray binary systems.

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REFERENCES

Agol, E. 1997, PhD thesis, Univ. California at Santa Barbara
Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214
Balbus, S. A., & Hawley, J. F. 1998, RevMP, 70, 1
Blaes, O., Hirose, S., & Krolik, J. H. 2007, ApJ, 664, 1057
Blaes, O., Hirose, S., Krolik, J. H., & Davis, S. W. 2006, ApJ, 645, 1402
Blaes, O., Krolik., J. H., Hirose, S., & Shabaltas, N. 2011, ApJ, 733, 110
Davis, S. W., Blaes, O., Hirose, S., & Krolik, J. H. 2009, ApJ, 703, 569
Davis, S. W., Blaes, O., Hubeny, I., & Turner, N. J. 2005, ApJ, 621, 372
Davis, S. W., Stone, J. M., & Jiang, Y.-F. 2012, ApJS, 199, 9
Done, C., & Kubota, A. 2006, MNRAS, 371, 1216
Hirose, S., Blaes, O., & Krolik, J. H. 2009a, ApJ, 704, 781
Hirose, S., Krolik, J. H., & Blaes, O. 2009b, ApJ, 691, 16
Hirose, S., Krolik, J. H., & Stone, J. M. 2006, ApJ, 640, 901
Hubeny, I., Agol, E., Blaes, O., & Krolik, J. H. 2000, ApJ, 533, 710
Hubeny, I., Blaes, O., Krolik, J. H., & Agol, E. 2001, ApJ, 559, 680
Hubeny, I., & Hubeny, V. 1998, ApJ, 505, 558
Hubeny, I., & Lanz, T. 1995, ApJ, 439, 875
Jiang, Y.-F., Stone, J. M., & Davis, S. W. 2012, ApJS, 199, 14
Jiang, Y.-F., Stone, J. M., & Davis, S. W. 2013, ApJ, submitted
Krolik, J. H., Hawley, J. F., & Hirose, S. 2005, ApJ, 622, 1008
Krolik, J. H., Hirose, S., & Blaes, O. 2007, ApJ, 664, 1045
Kubota, A., & Done, C. 2004, MNRAS, 353, 980
Lightman, A. P., & Eardley, D. M. 1974, ApJL, 187, L1
Ling, J. C., & Wheaton, W. A. 2005, ApJ, 622, 492
McIntosh, J. E., & Remillard, R. A. 2006, in Compact Stellar X-Ray Sources, ed. W. H. G. Lewin & M. van der Klis (New York: Cambridge Univ. Press), 157
Novikov, I. D., & Thorne, K. S. 1973, in Black Holes, ed. C. De Witt & B. De Witt (New York: Gordon & Breach), 343
Revnivtsev, M. G., Trudolyubov, S. P., & Borozdin, K. N. 2000, MNRAS, 312, 1151
Riffert, H., & Herold, H. 1995, ApJ, 450, 508
Sadowski, A., Abramowicz, M., Bursa, M., et al. 2011, A&A, 527, 17
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
Shakura, N. I., & Sunyaev, R. A. 1976, MNRAS, 175, 613
Turner, N. J. 2004, ApJL, 605, L45