Abstract

Wavelet transform (WT) is a powerful statistical tool used in applied mathematics for signal and image processing. The different mother wavelet basis function has been compared to select the optimal wavelet function that represents the Electromyogram signal characteristics of upper limb amputees. Four different EMG electrode has placed on different location of shoulder muscles. Twenty one wavelet functions from different wavelet families were investigated. These functions included Daubechies (db1–db10), Symlets (sym1–sym5), Coiflets (coif1–coif5) and Discrete Meyer. Using mean square error value, the significance of the mother wavelet functions has been determined for Teres, Pectorials and Infraspinatus around shoulder muscles. The most compatible wavelet families Daubechies families were selected to achieve the classification of the shoulder movement.

Keywords

Wavelet Transform, Upper Limb Amputation, Shoulder Muscles, Symlets, Coiflets, Daubechies
1. Introduction

Wavelet analysis is a new development in the area of biomedical signal processing. As a part of wavelet analysis, selection of the mother wavelet is an important step to determine the effect of wavelet transform in decomposition, denoising and reconstruction of different coefficients. The selection of mother wavelet may be either empirical or by visual inspection of the signal or by previous experience and knowledge of the person. The selection of the decomposition level before the mother wavelet is the matter taken up in this section. The different decomposition levels and mother wavelet in the analysis of the signal were used (Hosseini and Gholam 2004), (Moshou, D.; Hostens, I.; Papaioannou, G. & Ramon 2000). Zhang and Luo in 2006 proposed the sym8 wavelet function with 4 decomposition levels with weighted averages rescaling method including soft and hard threshold functions for upper limb prosthesis control. (Angkoon Phinyomark, Phukpattaranont, and Limsakul 2012)(Hussain et al., in 2009) proposed db2, db6, db8, dmev, sym8, sym4 and sym 5 wavelet functions with decomposition level 4 and universal scaling with hard threshold function to determine the muscles contraction. For the feature selection from different hand functions, Phinyomark et al., (Angkoon Phinyomark, Phukpattaranont, and Limsakul 2012)(Phinyomark, A.; Limsakul, C. & Phukpattaranont 2008; Angkoon Phinyomark, Phukpattaranont, and Limsakul 2012) summarized that the best performance can be obtained from db2 with level 2 and db7 with the decomposition level 4. But in (Megahed et al. 2008; Reaz, M. B. I.; Hussain, M. S. & Mohd-Yasin 2006) the researchers recommended for the wavelet functions as db2, db7, sym2, sym5, coif 4, bior5.5 and bior 2.2 with the decomposition level 4 for de-noising. It is clear that for the selection of the mother wavelet, characteristics of the signal and properties of wavelet transform should be carefully matched.

The Fourier transform does not represent abrupt changes efficiently therefor data are not localized in time or in space. Therefore, there is a need of new class function that can accurately analysed the signal and images with abrupt changes localized in time and frequency. To resolve this problem we resorted to wavelet transform techniques to analyse the signal with adaptive resolution property. But because the uses of wavelet techniques there must be optimal selection of wavelet function from various wavelet families. The main objective of this work to introduce a system that can select the optimal mother wavelet from the large number of wavelet families after pre- processing the sEMG signal.(Kaur, Agarwal, and Kumar 2017, 2016)
2. Wavelet Transforms

The wavelet transform is the time scale analysis of the signal. The first wavelet was introduced by Haar in 1909. Then the Gabor function was introduced by the Denis Gabor in 1946. George Zweig discovered the continuous wavelet transform in 1975. In 1982, Grossmann and Morlet tried to observe the signal with the shorter wavelength signal with high frequency instead of equal duration pulses. Then in 1988, complete idea was formulated into the different mathematical tools by Daubechies who introduced the orthogonal wavelet transform. Then Stéphane Mallat with Daubechies jointly gave the filter implementation using discrete wavelet transform. Wavelet analysis allows to isolate and manipulate specific type of pattern hidden in the masses of data. These were designed for the non-stationary data like sEMG signal that were difficult to analyse in time domain. Wavelet has ability to examine the signal simultaneously in both time and frequency. It decomposes a signal into a set of basic functions called wavelets which means a small wave. Wavelet $\psi$ has energy concentrated in time and is a function of zero average. These wavelets are obtained from the mother wavelet $\psi(\cdot)$. The daughter wavelet can be formed by dilations and shifting. It is a two-dimensional array value and is defined as:

$$\psi_{x,b}(t) = \left(\frac{1}{\sqrt{x}}\right) \psi \left(\frac{t-b}{x}\right)$$

where $x$ and $b$ are the scaling factor and translation (shifting) factor respectively. For the function to be wavelet, it should be time limited (Elektrik, Teknikal, and Tunggal 2018). Scaling refers to the stretching or shrinking of the signal in time. There is a centre frequency caused due to the constant of proportionality. The scale and frequency of the signal are reciprocal to the constant of proportionality. The stretched wavelet is corresponding to the lower frequency and the large scale factor whereas the shrunken wavelet is corresponding to the high frequency with the small scale factor. Shifting the wavelet along the length of the signal is called delaying the onset of the wavelet function. In other words, for high resolution in the signal, the mother wavelet contraction captures all the sudden changes appearing in the signal for time domain analysis. The wavelet transform is of two type viz. continuous wavelet transforms (CWT) and discrete wavelet transform (DWT)(Kaur, Kumar, and Agarwal 2017).

2.1 Continuous Wavelet Transforms (CWT)

The continuous wavelet transforms (CWT) can resolve both time and frequency events. It provides better output than the STFT. It is defined as

$$CWT(b, x) = \psi(b, x) = \frac{1}{\sqrt{|x|}} \int x(t) b \left(\frac{t-b}{x}\right) dt$$
\( \psi \) is the analysing function called wavelet function. Scaling is stretching a function denoted by ‘b’ and the shifting or translation is denoted by ‘x’. These scaling and position parameters are continuously varied for getting the cwt coefficients \( C(b, x) \). \( \psi(t) \) is the mother wavelet which implies that it can generate other window functions. In CWT, the most common wavelet functions are Morlet wavelet and the Mexican hat. CWT may be a complex valued variable function or a real valued variable function of the scale and the position. This depends upon the nature of the wavelet function (real or complex).

2.2 Discrete Wavelet Transforms (DWT)

The discrete wavelet transforms has a critical role for processing the different human signals in biomedical engineering. It is obtained by the discretization of the CWT values and used in the time-frequency plane. Its computational time is less than continuous WT. It decomposes the signal into various sub-bands. At the high frequency signal, DWT exhibits good time resolution and at low frequency, it provides a good frequency resolution. Thus, low frequency components are more significant than the high frequency elements. It reduces the computation and provides the adequate and sufficient data of the original signal for analysis and synthesis.

\[
DWT_{i,j}(f) = x_0^{-m/2} \int f(t) \psi(x_0^{-it} - j b_0) dt
\]

The integer value \( i \) and \( j \) can be defined by the solution of a dilation equation or by an analytical expression. The value of \( x_0 \) and \( b_0 \) can be 2 and 1 respectively. The mother wavelet is obtained by dilated, translated and scaled version of the wavelet function and is defined as:

\[
\psi_{x,b}(t) = \left( \frac{1}{\sqrt{x}} \right) \sum_r \psi \left( \frac{r-b}{x} \right) f(n), \quad b>0
\]

DWT applies a series of the low pass filter (l) and high pass filter (h) on data for extracting the high and low frequency components of the signal respectively via a finite impulse response. In this study, DWT is used to extract the features from the signal. It is a multi-resolution technique used in real time, engineering applications employed to set of function called the scaling and wavelet function.

\[
h(l) = (-1)^n l(1 - x)
\]

\[
\phi(p) = \sum_n l(n) \sqrt{2} \phi(2x - n)
\]

\[
\phi(x) = \sum_n h(n) \sqrt{2} \phi(2x - n)
\]

The quadrature mirror filter (QMF) output is

\[
A = \sum_n l(n - 2L)x(n)
\]

\[
D = \sum_n h(n - 2L)x(n)
\]
QMF has been used for splitting the signal in the frequency domain and to provide different sub-bands. The signal \( x(n) \) convolves with \( l(n - 2L) \) and \( h(n - 2L) \) which acts as high pass filter and low pass filter respectively and \( L \) is related to the mother wavelet function. The two components namely approximation component and detailed components are represented by \( cA \) and \( cD \) respectively. When these sub band signals are recombined then the original signal can again be reconstructed. The decomposition level along with the level can be obtained in the DWT technique by multi-level subsets. In this study, the raw data was decomposed by the DWT method. But before the decomposition, the level of decomposition and the optimal wavelet function should be selected according to the application.

3. Methodology used for Optimum Mother Wavelet Selection

The total time for one trial of single movement and then rest position is of approx. 4 second. Sampling rate is 2048 samples/second. The total samples for one trial were \( 4 \times 2048 = 8192 \) samples. The total trials are 8, therefore the total no of samples for a single movement for a subject were 65,536. In this study, the total subjects were six and the four channel data set were used for the analysis purpose. Here, mother wavelet function methodology is described as shown in the Figure 1 and 2 described the denoising of the original signal is introduced while in other extra noise level is added to find out the best mother wavelet which leads to a better filtration performance respectively. The different steps in the methodology are follows. After acquiring the signal form, the multi-channel combination. the next step was to find the optimum mother wavelet for denoising the acquired signal.

![Figure 1: Methodology to find The Optimal Mother Wavelet for Shoulder Muscles Signal](image-url)
Mother wavelet transform, wavelet decomposition, threshold values and reconstruction of the signal are the signs of better performance in wavelet based signal investigation. Mother is chosen on the premise of the similarity with the shoulder sEMG signal. It additionally helps to retain the original signal and improves the frequency spectrum of a de-noised signal. All the wavelet families used in this study are presented in the Table 1.

Table 1: List of used 21 Wavelet Function from Four Different Wavelet Family

| Wavelet Family | Wavelet subtypes                      |
|----------------|--------------------------------------|
| Daubechies (db)| db1, db2, db3, db4, db5, db6, db7, db8, db9, db10 |
| Symlets (sym)  | Sym1, sym2, sym3, sym4, sym5          |
| Coiflet (coif) | Coif1, coif2, coif3, coif4, coif5     |
| Discrete Meyer | dmey                                  |

The setup for the selection of the optimal mother wavelet is given in Figure 1. The main motive behind choosing the optimal mother wavelet from wavelet families was that the reconstructed signal should be free from the artifacts that contaminate the sEMG signal. In this study, different orthogonal families, including (db1-db10), Symlets (sym1–sym5) and Coiflets (coif1–coif5) were selected. The right wavelet family determines the proper analysis and reconstruction of the signal. The error between the reconstructed signal and original signal was calculated as the mean square error (MSE) value defined as

\[
MSE = \frac{\sum_{i=1}^{N}(H_i-R_i)^2}{N}
\]

where \( H_i \) and \( R_i \) represent the sEMG signal and the noised signal respectively. The less value of the MSE indicates the better performance of the wavelet method.

3.1 Wavelet Decomposition

To analyze the sEMG signal, the initial step is the determination of ideal decomposition levels on the premise of the dominant frequency. The output signal is maximal if the input signal looks like the mother wavelet and energy will spread over a large number of coefficients. The variation of the decomposition level is from the first decomposition level to the last decomposition level depending on the factor 1 to \( M = \log_2 N \) where \( N \) is the length of the samples in time domain.

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So, the decomposition levels are dependent on the dominant frequency of the signal. The original signal, denoted by \( s(n) \) was passed through the high pass and low pass filters. The high pass filter coefficients are called detailed coefficients denoted by \( D_{k+1}(n) \) whereas, the low pass filter coefficients are called approximate coefficients denoted by \( A_{k+1}(n) \).

In this study, sampling frequency of the sEMG signal with shoulder muscles was 2048 Hz. Figure 2 shows the first level approximate coefficients (cA1) can obtain with the down sampling of the low pass filter and the first level decomposition coefficient (cD1) can with the same sampling of the high pass filter. In second level decomposition was on the cA part by same down sampling.

**Figure 2:** Wavelet Decomposition Tree with the sEMG Signal (Al-Qazzaz et al. 2015)
method and the second level decomposition coefficients can obtain. The process was repeated up to the desired level of decomposition shown in the Figure 4.17. It shows that the down sampling frequency produced the different decomposition (cD) and approximate coefficients (cA) for five different decomposition levels (1, 2, 3, 4, 5). After thresholding, the signal moves through the reconstruction process as inverse of the decomposition process with the same level. The process generally goes on up to three or maximum four decomposition levels. Here, the sampling frequency of the sEMG signal with shoulder muscles was 2048 Hz. The decomposition levels were selected for different mother wavelet functions.

4. Wavelet Denoising

To get best wavelet denoising method, the additional noises are added and each time the noise decomposition level is varied from low to high noise decomposition level. Figure 4.18, shows the white Gaussian noise with 5dB SNR were added in the signal. This noise level has been removed from the signal by denoising the process before the reconstruction. The small value of MSE proved that the undesired part of the noise was removed and the useful information remained in the signal. To grab this outcome, the DWT procedure involved different steps: threshold selection rule, threshold rescaling and threshold function. In addition to this, the decomposition level and the wavelet function must be evaluated which is described in this section(Angkoon; Phinyomark, Limsakul, and Phukpattaranont 2009; Ngui et al. 2013).

4.1 Thresholding Techniques

The observed coefficients can themselves be considered as the noisy version of the wavelet coefficients and after decomposition, these coefficients can be denoised. So, the thresholding technique is applied to the detailed decomposition level coefficients. After the decomposition, the (Khezri, M. & Jahed 2008) denoising technique is applied to the signal to remove the noise level from the original signal. This procedure removes the level of noise from the original signal. The output will be maximized if the input signal most resembles the mother wavelet. Since, the wavelet transform is linear therefor, it works for additive noise with equal power at all the frequency level as noise affects every single frequency component over the whole signal. Therefore, the thresholding method is used in the wavelet domain.
4.2 Threshold Selection Rule

The main part of the thresholding is to choose the threshold value. Phinyomark et al., (A. Phinyomark, C. Limsakul and Phukpattaranont 2009) and Donald et al., (De Luca et al. 2006) have utilized different universal threshold values and shown that the de-noising capability of this method is better than other thresholding methods like SURE, hybrid and minimax method.

![Diagram showing methodology](image)

**Figure 3**: Methodology to find the Optimal Mother Wavelet Addition of Additive White Gaussian Noise

Universal method was proposed by the Donald and Johnstone which is defined by

$$\text{THR} = \sigma \sqrt{2 \log(N)}$$

where \( \sigma \) the standard deviation and \( N \) is the length of the samples. SURE is selected using the rule of Stein’s Unbiased Estimate of Risk. Mixture of SURE and the threshold was provided by both. Minimizing the value of Risk gives the threshold. Afterwards, Stein proposed the minimax method and tested its performance by using the mean square error values. The different rescaling methods can be used for the smoothening of the threshold. The wavelet threshold functions are described and categorized as Hard and Soft functions. Any of the functions can be used for investigating. The smoother effects are provided with the soft threshold whereas better edge preservation is obtained in hard threshold.

4.3 Reconstruction Process

To obtain the effective sEMG signal, the de-noised signal is reconstructed by using the inverse wavelet transform of the final decomposition level (cD1, cD2, cD3, cD4 and cA4). In the inverse process, the reconstructed approximate (A4) and detailed (D1, D2, D3 and D4) signal can be obtained by up-sampling the signal. The signal is passed through low pass and from the high pass filter and received signal is added for providing an output.(Al-Qazzaz et al. 2015; A Phinyomark, Limsakul, and Phukpattaranont 2011)
5. Results and Discussion

The results were evaluated for optimal mother wavelet in the wavelet transform based on wavelet decomposition, de-noising the signal and reconstruction of the signal. Table 1 described the used four-wavelet family for three different muscle activations. Thereafter, these three-channel data was decomposed and reconstructed by the wavelet families. The combined sEMG data contained the activation of three muscles with three different movements. The three muscles having a different magnitude level with respect to the movement were individually analysed by the wavelet. Mean square error (MSE) method was used to find the best mother wavelet in decomposition level.

5.1 Optimal Wavelet Selection for Trapezius Muscles

From the three-channel data set, the first data set of trapezius was analysed to find out the optimum mother wavelet by using the mean square error. The MSE was calculated from 24 wavelet functions presented in the graph (Figure 4) for all the decomposition levels of the trapezius muscles data set used in this study. The results from the decomposed trapezius data indicate that levels 1, 2 and more than 6 produced a very large value of MSE so they were neglected.

![Performance of db1-db10, Sym1-sym5 and coif1-coif5 Mother Wavelet Families for Trapezius Muscle](image)

**Figure 4:** Performance of the db1-db10, Sym1-sym5 and coif1-coif5 Mother Wavelet Families for Trapezius Muscle
A particular colour depicts the level of the error at the different decomposition level regarding the wavelet function (Figure 4) sEMG signal was decomposed with different decomposition coefficient levels and afterwards reconstructed by denoising the signal using a universal thresholding rule with the soft thresholding function.

5.2 Optimal Wavelet Selection for Teres Muscles Signal

The second channel data was examined by the wavelet family by choosing an optimal mother wavelet for the teres muscles signal. The wavelet universal thresholding rule with soft thresholding function was used. The 1 to 6 level decomposition was done for the sEMG teres signal for different wavelet families. Others levels led to an increased MSE value and were not considered. The calculated MSE value for the teres muscles around the shoulder with three upper limb movements to find out the optimal decomposition level from decomposition levels DL1 to DL6.

![Selected Wavelet Family](image)

**Figure 5:** Performance of Daubechies Wavelet Family for Teres
5.3 Optimal Wavelet Selection of Pectoralis Muscles Signal

Pectoralis muscles data was applied and decomposed with different number of levels for all the wavelet families. Figure 6 shows the calculated MSE value with the different wavelet family. During recording of the data signal from different muscles, noise or various types of artifacts contaminated the sEMG signal. In engineering and clinical application, noise is the main problem in the data signal. Due to the random nature of the sEMG signal, the conventional filters are not able to effectively remove the noise signal. But to choose the optimal wavelet method with the best decomposition level can help to eliminate the artifacts and that was the main aim of this work. Again, due to the stochastic nature of the sEMG signal, it was a challenge to select the best wavelet for the acquired signal from shoulder muscles.

6. Conclusion

The suggestion of different wavelet functions with the decomposition levels from four defined wavelet family with the universal thresholding level using soft thresholding function may be made as follows:

- Wavelet Functions: db3, db4, sym2, sym5, coif4
- Decomposition Levels: 3, 4 and 5
- Threshold selection rule, rescaling method and function
These recommendations can be used for the sEMG shoulder muscles signal for different applications. For getting the best results from the wavelet, denoising method without adding or after adding the noise signal were also the part of the study. The main aim of this thesis to classify the different shoulder signal. For efficient classification of the signal, the various features were extracted from the reconstructed coefficients of db3 with three level wavelet function to form a feature vector for the classifier described.

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