Shear strength evaluation in the existence of axial compressive loads for reinforced concrete beams

Mohamed S. Issa\textsuperscript{a} and Ahmed A. El-Abbasy\textsuperscript{b}

\textsuperscript{a}Reinforced Concrete Institute, Housing and Building National Research Center, Giza, Egypt; \textsuperscript{b}Civil Engineering Department, Jazan University, Jazan, Saudi Arabia

**ABSTRACT**

Response-2000 is a program developed to calculate the shear capacities of beams and columns under any combinations of shear, moment, and axial loads. It is based on the modified compression field theory. The capabilities of the program Response-2000 to obtain shear strength capacities in the existence of axial compressive loads are demonstrated. The comparison with the experimental tests proved that the program gives good estimate for the failure shear in this case. The shear provisions of the Egyptian reinforced concrete code (ECP203-2017) are also evaluated against the experimental results. It is found that the accuracy of the obtained results using Rresponse-2000 is better than those obtained using the shear equations of the Egyptian reinforced concrete code (ECP203-2017). The program Response-2000 is then used to generate 336 results for 7 different cross-sections considering different concrete compressive strengths, transverse reinforcement, and values of axial loads. These generated results along with 33 experimental results for other 7 cross-sections are used to develop a new formula for the shear strength capacity in the existence of axial compressive loads. This formula is found to give better results when verified against experimental data.

**ARTICLE HISTORY** Received 30 December 2020; Revised 18 May 2021; Accepted 11 June 2021

**KEYWORDS** Shear strength; axial compressive load; Egyptian code 2017; Response-2000 program; reinforced concrete beams

**Introduction**

Beams subjected to combined bending, shear, axial force, and torsion are encountered for example in the cases of buildings and frames which resist wind or seismic forces. The analysis of these beams has attracted lots of research. Halim et al. (2011) \cite{1} used experimental data and finite elements package to evaluate the ACI318 and AASHTO shear and combined shear and torsion provisions for shear-critical beams. They indicated the possibility of deriving shear–torsion interaction equations and
reported that they plotted the interaction envelope points. A MathCAD design program employing the modified compression field theory and truss analogy was developed for sections under combined shear and torsion or shear only. Metwally (2012) [2] evaluated the program Response-2000 [3] in terms of its capability for predicting the shear strength of reinforced and prestressed concrete members under moment and shear. He found that it can predict the experimental tests accurately. Esfandiari and Adebar (2009) [4] presented a procedure for evaluating the shear capacity of reinforced concrete girders which is similar to the 2008 AASHTO LRFD method. They compared their method with the modified compression field procedure for the case of an element under uniform shear. They also compared their method with the program Response-2000 for beams under shear and bending moment. A comparison with test results for reinforced and prestressed beams under shear and moment and a comparison with the predictions of the AASHTO LRFD and ACI318 codes were made. Esfandiari and Adebar (2009) [4] found that their method gives good shear strength predictions. Their equations start by defining the nominal shear resistance as follows:

Nominal shear resistance \( (V_n) = \) part from concrete \((V_c) + \) part from stirrups \((V_s)\)

\[
V_n = V_c + V_s = \beta \sqrt{f'_c b_w d_v} + \frac{A_v f_y d_v}{s} \cot \theta
\]

where

- \( \beta \) = factor measures contribution of concrete
- \( f'_c \) = concrete compressive strength
- \( b_w \) = width of web
- \( d_v \) = shear depth of beam
- \( = d \)
- \( A_v \) = stirrups area within \( s \)
- \( f_y \) = stirrups yield strength (stirrups are assumed reached yielding)
- \( \theta \) = angle of the concrete major compressive stress on the longitudinal axis
- \( = 45^\circ \)

Dividing the equation of nominal shear by \( b_w d_v \) (shear area) we obtain

\[
\frac{V_n}{b_w d_v} = \frac{\beta \sqrt{f'_c}}{b_w d_v} + \frac{A_v f_y}{b_w s} \cot \theta
\]

where

\[
\rho_s = \frac{A_v}{b_w s}
\]

= stirrups ratio
The modified compression field theory (MCFT) is used to define $\theta$ and $\beta$ assuming no normal stress in z-direction. This way the relationship between shear stress and strain can be obtained. The work of Esfandiari and Adebar (2009) [4] calculates the axial strain $\varepsilon_x$ from the amount of tensile reinforcement and applied forces. There are three shear failure modes. These are: 1-longitudinal reinforcement yielding; two-transverse reinforcement yielding; and 3-concrete crushing after transverse reinforcement yielding. The $\theta$ is related to the major compressive strain, which in turn is a function of the shear stress $\nu/f'_c$. As the shear stress ratio is not determined during evaluation, $\rho z f_y/f'_c$ is used in the following relation

$$\theta = \theta_0 + \Delta \theta \varepsilon_x$$

For the case when the stirrups are yielding

$$\theta_0 = \left(85 \frac{\rho z f_y}{f'_c} + 19.3\right) \left(-50 \varepsilon_y + 1.1\right)$$

$$\Delta \theta = 1000 \left[37.5 \left(-200 \varepsilon_y + 1.4\right) - \theta_0\right]$$

and at crushing of concrete

$$\theta_0 = 119 \frac{\rho z f_y}{f'_c} + 15.6$$

$$\Delta \theta = 15,000 \frac{\rho z f_y}{f'_c} + 2000$$

The authors provide figures, which can be reviewed in their work, for $\beta$ in terms of axial strain $\varepsilon_x$. For the case of concrete crushing, they provide the following equation

$$\beta = 0.65 \frac{\rho z f_y}{f'_c} \text{MPa}$$

A safe approximation for shear which results in yielding of both stirrups and longitudinal steel is obtained by the following proposed expression

$$\nu = \sqrt{\rho z f_y n_{vc}}$$

where

$$n_{vc} = 2 \nu_c \cot \theta + \rho z f_y \cot^2 \theta$$

Hawkins et al. (2005) [5] compared the results of Response-2000 [3] to 149 experimental tests for beams with minimum stirrups. They found that the average measured-to-predicted shear capacities are 1.02 for
reinforced beams and 1.11 for prestressed beams. Collins et al. (1996) [6] developed a unified method to design reinforced concrete beams under shear and axial tension or compression. Their method adopts the concept of modified compression field theory. They indicated that the maximum longitudinal strain $\varepsilon_x$ in the beam web can be taken as the reinforcement tension strain as follows

$$
\varepsilon_x = \frac{M_u}{d_v} + 0.5N_u + 0.5V_ucot\theta
$$

where

- $M_u$ = ultimate moment
- $V_u$ = ultimate shear
- $N_u$ = ultimate axial load (positive for tension, negative for compression)
- $d_v$ = effective shear depth
  = flexural lever arm $\geq 0.9d$
- $E_s$ = elastic modulus of reinforcement
- $A_s$ = area of tension longitudinal reinforcement
- $\theta$ = inclination angle on the longitudinal axis of principal compressive stress of cracked concrete

The major tensile strain $\varepsilon_1$ is related to the axial strain $\varepsilon_x$, and the major compressive strain $\varepsilon_2$ by the following relation

$$
\varepsilon_1 = \varepsilon_x + (\varepsilon_x - \varepsilon_2)cot^2\theta
$$

The nominal shear strength $V_n$ is given by

$$
V_n = V_c + V_s
$$

$$
V_n = \beta \sqrt{f'_c b_v d_v} + \frac{A_v f_y}{s} d_v cot \theta
$$

where

- $b_v$ = web width
  = minimum web width within $d_v$
- $A_v$ = shear reinforcement area within $s$
- $s$ = shear reinforcement spacing
- $f_y$ = stirrups yield strength
- $f'_c$ = concrete compressive strength
- $\beta$ = factor related to the capacity of cracked concrete to transfer shear and equals

$$
\beta = \frac{0.33 cot \theta}{1 + \sqrt{500\varepsilon_1}} \leq \frac{0.18}{0.3 + \frac{24w}{\pi + 16}} MPa
$$

where

- $w$ = width of crack
The largest aggregate size

The largest compressive stress \( f_2 \) can be calculated as

\[
f_2 = \frac{V_n}{b_v d_v} (\tan \theta + \cot \theta)
\]

The largest tensile strain \( \varepsilon_1 \) may be found from

\[
\varepsilon_1 = \varepsilon_x + \left[ \varepsilon_x + 0.002 \left( 1 - \sqrt{1 - \frac{V_n}{b_v d_v f'_c} (\tan \theta + \cot \theta)(0.8 + 170 \varepsilon_1)} \right) \right] \cot^2 \theta
\]

To use the equation of \( V_n \) to calculate the needed shear reinforcement, the engineer shall define values for \( \beta \) and \( \theta \). The authors give table for the values of \( \beta \) and \( \theta \) in terms of the axial strain \( \varepsilon_x \) and the level of shear stress \( \frac{V_n}{b_v d_v f'_c} \). For any section of the beam under \( N_u, V_u, \) and \( M_u \), the shear strength may be calculated as

\[
V_u \leq \varphi V_n
\]

where

\( \varphi = \) strength reduction factor \( = 0.85 \)

The amount of shear reinforcement for the section can be found from

\[
V_s \geq \frac{V_u}{\varphi} - V_c
\]

Ford et al. (1981) [7] tested nine columns to trace their deformation capacity. They provided analytical study for the axial load versus moment curvature and obtained the descending branch of the relationship. Osei-Antwi et al. (2014) [8] proposed an analytical method to estimate the shear and axial stresses in sandwich beams. The method proposes formulation for defining the shear and bending stiffness, which was verified by finite elements and experimental tests. The new method proved to be accurate. They define the bending stiffness, \( D \), as the summation of rigidities of the core and face sheets. The model has the following form for the case of two cores

\[
D_{ml} = E_{f_1} I_{f_1} + E_{c_1} I_{c_1} + E_{c_2} I_{c_2} + E_{f_2} I_{f_2}
\]

where

- Subscript \( ml \) refers to multilayer sandwich beam.
- Subscript \( f \) refers to face sheet.
- Subscript \( c \) refers to core.

\( E_{f_1}, I_{f_1} = \) Elastic modulus and moment of inertia of the top face sheet about the neutral axis of the entire cross section

\( E_{f_2}, I_{f_2} = \) Elastic modulus and moment of inertia of the bottom face sheet about the neutral axis of the entire cross section

\[f_1, f_2\]
\( E_{c1}, I_{c1} = \) Elastic modulus and moment of inertia of the core layer 1 about the neutral axis of the entire cross section

\( E_{c2}, I_{c2} = \) Elastic modulus and moment of inertia of the core layer 2 about the neutral axis of the entire cross section

The shear stiffness, \( S \), for the case of single core is the shear modulus of the core, \( G_c \), times the cross-section of the core, \( A_c \). For the case of two cores, the authors use the energy method to expand the model to this case and the derived shear stiffness is as follows

\[
S_{ml} = bd^2 \left[ \frac{G_{c1}G_{c2}}{G_{c1}h_{c2} + G_{c2}h_{c1}} \right]
\]

where

- \( d \) = distance between the axis of the face sheets
- \( b \) = width of beam
- \( h_{c2}, h_{c1} \) = second and first core thicknesses, respectively
- \( G_{c1}, G_{c2} \) = shear moduli of cores 1 and 2, respectively

The axial stresses in the core and face sheets of sandwich beams, assuming that the distribution of strain is plane, are given by

\[
\sigma_{cx} = \frac{M_z E_c y}{D}
\]

\[
\sigma_{fx} = \frac{M_z E_f y}{D}
\]

where

- \( M_z \) = bending moment at the considered section of the beam
- \( y \) = depth from the neutral axis

The shear stresses in the core and face sheets are given by

\[
\tau_c = \frac{V_y \sum'(S_c E_c)}{D}
\]

\[
\tau_f = \frac{V_y S_f E_f}{D}
\]

where

- \( S_c, S_f \) = first moment of area of the core and the face sheet
- \( V_y \) = shear force at the considered section of the beam

Ou and Nguyen (2016) [9] proposed flexure-shear-axial interaction approach which accounts for corrosion of reinforcement. The shear effect on confinement of concrete and buckling of reinforcement, and the softening effect of transverse reinforcement on concrete flexural compression zone are considered. The proposed approach uses corroded models for bond, steel, and concrete. The method was generally able
to capture the behavior of uncorroded and corroded beams. Rajapakse et al. (2019) [10] developed a finite element approach based on fiber force formulation to evaluate the response of frames or walls considering shear force-axial force-bending moment interaction. Their approach calculates the stresses corresponding to certain strain using the modified compression field theory. The proposed element proved its capability when compared to experimental results. Kirkland et al. (2015) [11] studied the axial force-shear-moment interaction for composite beams and developed a finite element formulation which gave good predictions compared with experimental tests. They developed a design approach for shear and flexure interaction of axially loaded beams. Lai et al. (2019) [12] assessed axial force-moment interaction of concrete encased steel columns made of a range of concrete and steel grades. They collected from the literature experimental tests covering steel yield strength ranging from 280 MPa to 913 MPa and concrete strength ranging from 20 MPa to 104 MPa. They used these data to evaluate the approach of EN 1994-1-1 for calculating the ultimate strength of these columns. Then, they performed an analytical study to obtain the interaction curves for axial-moment capacities of cross-sections of concrete encased steel columns. They reported that the EC4 method yields un-conservative values. Lai et al. (2019) [12] also presented a simple method to calculate the strength of composite columns under axial compression and bending. This general method is given here for comparison purpose. To plot the interaction curve, four points are calculated. These points are the pure compression, point close to the pure bending, and points for some strain states. The pure compression point is obtained using the following relation

\[ N_{pl} = a_c f_c A_c + f'_{ys} A_s + f'_{yr} A_l \]

where
- \( N_{pl} \) = cross-sectional plastic strength
- \( A_c, A_s, \) and \( A_l \) = concrete area, steel section area, and longitudinal reinforcement area, respectively.
- \( f_c \) = compressive strength of concrete
- \( f'_{ys} \) and \( f'_{yr} \) = steel section effective yield strength, and longitudinal reinforcement effective yield strength, respectively (\( f'_{y} = \min(E, \varepsilon_{co} f_y) \))
- \( a_c \) = factor which accounts for difference between concrete in cylinder and concrete in column, size effect, and the change in the strength of concrete over the length of the column
- \( = 0.85 \)
- \( E \) = elastic modulus of steel
- \( \varepsilon_{co} \) = peak strain
The bending and compression points are obtained for three strain cases. The ultimate strain of concrete, which is 0.003, is assumed for the three points. One of the points is when the geometric centroid of the composite cross-section lies on the neutral axis. In this case, they assume that the steel section is in the elastic range and the stress in concrete is nonlinear and is represented by the rectangular stress block. The effective height coefficient $\beta$ and the effective strength coefficient $\alpha$ are calculated using the following equations.

\[
\beta = 0.85 - \frac{1}{500} (f_c - 20) \quad \text{where} \quad 0.85 \geq \beta \geq 0.67
\]

\[
\alpha = 0.85 - \frac{1}{1000} (f_c - 20) \quad \text{where} \quad 0.85 \geq \alpha \geq 0.75
\]

The strain at the centroid of the flange of steel section, at the tip of web of steel section, and at reinforcing bar is calculated as follows:

\[
\varepsilon_f = \frac{d_f \times \varepsilon_{cu}}{x}
\]

\[
\varepsilon_w = \frac{d_w \times \varepsilon_{cu}}{x}
\]

\[
\varepsilon_r = \frac{d_r \times \varepsilon_{cu}}{x}
\]

where

$d_f$, $d_w$, and $d_r =$ distance from the centroid of flange of steel section, tip of web of steel section, and centroid of reinforcing bars, respectively, to concrete surface

$x =$ neutral axis depth

$\varepsilon_{cu} =$ ultimate strain of concrete

$= 0.003$

From the longitudinal strains calculated above, the stresses can be found assuming elastic-perfectly plastic behavior. Thus, the bending moment and axial force may be obtained for the cross-section.

Xu and Zhang (2012) [13] suggested a hysteretic model for the interaction of flexural-shear-axial strengths of reinforced concrete columns. The model was used in a finite element program which was verified with experimental tests for columns under different axial loads. They also obtained the seismic response of a prototype bridge. Kocer et al. (2019) [14] calculated the shear and moment strengths of reinforced concrete columns and their displacements based on artificial neural network. They reported that the success of this method for calculating the shear and moment strengths was good while for displacements it was not sufficient.
Popovic (2012) [15] reported that the shear design equations of the FIB model code (2010) [16] are simplifications of the modified compression field theory. He presented a method for the bending moment-shear force-axial force interaction curves to be used in the design of reinforced concrete beams. He also studied the effect of the axial forces on the shapes of the interaction diagrams. Popovic (2012) [15] reported that the longitudinal strain can be obtained by the following relation after some approximations:

$$\varepsilon_x = \frac{M_{Ed}}{d_v} + V_{Ed} + 0.5N_{Ed}$$

where
- $M_{Ed}$ = applied design moment
- $V_{Ed}$ = applied design shear force on the section shear area
- $N_{Ed}$ = applied design axial force (positive for tension and negative for compression)
- $d_v$ = effective shear depth (flexural lever arm)
- $A_s$ = area of tensile flexural reinforcement
- $E_s$ = elastic modulus of tensile flexural reinforcement

The shear capacity of the beam is calculated from

$$V_{Rd} = V_{Rd,c} + V_{Rd,s} \geq V_{Ed}$$

where
- $V_{Rd}$ = design shear capacity
- $V_{Rd,c}$ = concrete design shear capacity
- $V_{Rd,s}$ = design shear capacity of shear reinforcement

The design shear capacity of concrete for the case where minimum shear reinforcement exists is given by

$$V_{Rd,c} = k_v \sqrt{f_{ck}} b_w d_v$$

where
- $f_{ck}$ = concrete cylinder characteristic compressive strength, MPa ($\sqrt{f_{ck}} \leq 8$MPa)
- $\gamma_c$ = concrete partial safety factor
- $b_w$ = width of beam
- $k_v$ = coefficient which considers aggregate interlock and shear reinforcement greater than the minimum

$$k_v = \frac{0.4}{1 + 1500\varepsilon_x}$$

The design shear capacity of vertical stirrups is given by
V_{Rd,s} = \frac{A_{sw}}{s_w} d_f y_{wd}(\cot \theta)

where

\begin{align*}
A_{sw} &= \text{stirrup area} \\
s_w &= \text{stirrup spacing} \\
f_{ywd} &= \text{design yield strength of stirrups} \\
\theta &= 29^\circ + 7000 \varepsilon_x
\end{align*}

Lodhi and Sezen (2012) [17] reported that reinforced concrete columns with transverse reinforcement not designed for earthquake codes may suffer shear failure in the event of strong seismic load. The authors applied two models for displacement-based study of reinforced concrete columns. Their proposed method gave results consistent with experimental data. Saritas and Filippou (2009) [18] presented a beam element which considers shear-flexure-axial coupling. The coupling is attained using numerical integration of the axial model for the material over the depth. The examples proved the quality of their beam element. Mullapudi and Ayoub (2013) [19] presented a three dimensional concrete model for fiber analysis of reinforced concrete. The model considers the interaction between shear, bending, axial force, and torsion. The shear behavior is considered using Timoshenko approach. The capability of the model has been evaluated. Zhang et al. (2017) [20] suggested a mechanics approach to model reinforced concrete beams under shear. They also extended their approach to consider axial load and developed a closed form solution. They validated their method using experimental results. Their closed-form solution has the following form:

\[
V_{sl} = \frac{b d_{NA} A + \sum_{i=1}^{n} P_{stpi} + \frac{C}{Jd} \left[ \sum_{i=1}^{n} P_{stpi} d_{stpi} + P_a (d - \frac{h^2}{2}) \right]}{1 - C \frac{\frac{h^2}{2} - d \cot \beta_{CDC}}{Jd}}
\]

where

\[
\begin{align*}
b &= \text{width of the reinforced concrete beam} \\
d_{NA} &= \text{neutral axis depth for the case of reinforced concrete beam with axial load} \\
A &= \text{shear friction material property before sliding} \\
P_{stpi} &= \text{tensile force by the legs of stirrup number i} \\
n &= \text{number of stirrups crossed by diagonal crack} \\
C &= \text{coefficient which is a function in } \beta_{CDC} \text{ and } B \\
\beta_{CDC} &= \text{critical diagonal plane angle} \\
B &= \text{shear friction material property before sliding} \\
Jd &= \text{lever arm between } P_{rt} \text{ and } P_{conc}
\end{align*}
\]
\[ P_{rt} = \text{tensile reinforcement force} \]
\[ P_{conc} = \text{concrete compression force} \]
\[ d_{stpi} = \text{the distance from the crack at tensile reinforcement to the i-th stirrup} \]
\[ P_a = \text{axial force on the cross-section} \]
\[ d = \text{effective depth of beam} \]
\[ h = \text{overall depth of beam} \]
\[ M_a = \text{bending moment at the considered section} \]
\[ V_a = \text{shear force at the considered section} \]
\[ M_a = V_a * a \]
\[ a = \text{the effective shear span} \]

Rasheed and Abouelleil (2015) [21] addressed the analysis of columns subjected to axial-moment-shear interaction. They compared different procedures for calculating the shear strength and paid more attention to the AASHTO LRFD 2014 procedure. They validated with experimental tests and improved the accuracy of AASHTO LRFD method for calculating the shear strength. The authors used the AASHTO LRFD equations with the following two modifications. The first modification is an equation for calculating the inclination angle (\( \alpha \)) of spiral reinforcement for circular columns in relation to the longitudinal axis. The equation has the following form:

\[
\alpha = \cos^{-1}\left(\frac{s}{\sqrt{\left(D_r^2\right) + \left(s/2\pi\right)^2}}\right)
\]

where

- \( s \) = spiral spacing (pitch)
- \( D_r \) = diameter of circular helix

The second modification is the equation for shear strength of transverse steel (\( V_s \)) which has the following form for the case of hoops perpendicular on the center line of circular column:

\[
V_s = \frac{\pi}{2} A_{sh} f_y
\]

where

- \( A_{sh} \) = area of single hoop
- \( f_y \) = yield strength of hoop

Response-2000 [3] is a two-dimensional nonlinear program for sectional analysis according to the modified compression field theory. It considers beams and columns subjected mainly to shear. The program can integrate the behavior of the section to beam segments. Response-2000 assumes that there is no clamping stress over the beam depth and that plane sections remain plane after deformation. Response-2000 gives
diagrams over the depth of the cross-section. It gives the shear-strain versus shear and the curvature versus moments which allow noting the flexural failures versus shear failures. Also, the program gives information about cracking of concrete and yielding of reinforcement. Longitudinal strains and transverse strains are also supplied. Among other produced information is the load-deformation curve.

The aim of this paper is to evaluate the Response-2000 analysis results and the results from the Egyptian reinforced concrete code (ECP203-2017) [22] model for the case of reinforced concrete beams under shear and axial compressive load. The evaluation is to be made comparing with experimental results. Further, the work uses the program to develop extra data which are to be used to obtain new formula for the shear strength in the existence of axial compressive load. This formula is then tested.

**Shear model of the Egyptian code for reinforced concrete (ECP203-2017) [22]**

The Egyptian code for reinforced concrete (ECP203-2017) [22] defines the shear strength of cracked concrete when the shear is resisted by concrete and steel as

$$V_{cu} = 0.12 \sqrt{f_{cu} \gamma_c} \text{ MPa}$$

where

- $f_{cu} =$ standard cube 28-day compressive strength, in MPa
- $\gamma_c =$ concrete strength reduction factor = 1.5

Beams with web reinforcement develop the flexural capacity after shear-inclined cracking. The shear is carried by the uncracked concrete before flexural cracking. Between flexural and inclined cracking this shear is taken by concrete and steel. At last, the stirrups crossing the crack yield and the inclined crack open rapidly and a splitting dowel failure occurs, the compression zone crushes or the web crushes. The last two components present a brittle failure and are lumped by $V_{cu}$ which is taken equal to the inclined cracking shear. Axial compressive forces increase the inclined cracking load. This is because they delay the flexural cracking and reduce its penetration into the beam (Wight and MacGregor, 2012 [23]).

In the case where there is compressive axial load, the above equation is to be multiplied by the following factor

$$\delta_c = 1 + 0.07 \frac{P_u}{A_c} \leq 1.5$$

where

- $P_u =$ the ultimate axial compressive load substituted for as positive, in N
\[ A_c = \text{the area of cross-section, in mm}^2 \]

**Experimental data used for evaluating the program and the ECP203-2017 [22]**

Mattock and Wang (1984) [24] tested simply supported beams under different values of axial compressive loads and subjected to either two-point loads at equal distances (specimens C205 and C210) or one-point load at the middle (specimens C305 and C310). The compression reinforcement was very small or equal to the tension reinforcement. The area of tension reinforcement is 982 mm\(^2\) or 1232 mm\(^2\) and the yield strength ranges from 366.9 MPa to 394.6 MPa. The shear reinforcement consists of closed stirrups made of 6 mm diameter bars of yield strength equal to 361.1 MPa and placed at spacing from 80 mm to 160 mm. The cube compressive strength of concrete ranges from 29.6 MPa to 34.8 MPa. The ultimate shear strengths of these beams are evaluated using the program Response-2000 [3] and the shear equations of the Egyptian code for reinforced concrete (ECP203-2017) [22]. When using the ECP303-2017 equations, the concrete strength reduction factor is taken equal to 1.0. The data for the tested beams and the analysis results are shown in Tables 1A and 2. The average error obtained when using the software Response-2000 is 34.1\% while the average error obtained with the Egyptian reinforced concrete code (ECP203-2017) is 38.7\%. Both are on the conservative side with the Egyptian code being less conservative for beams under one point load (see Figures 1 and 2). Also, Table 1B gives the experimental data of Haddadin et al. (1971) [25] which are used for the verification of the proposed equation. The stirrups for their beams where of two branches of area from 64.52 mm\(^2\) to 141.94 mm\(^2\), of spacing ranges from 63.5 mm to 190.5 mm, and of yield strength ranges from 344.75 MPa to 456.45 MPa. The cube compressive strength of concrete ranges from 19.31 MPa to 50.68 MPa. The area of tension reinforcement ranges from 1290.32 MPa to 3870.96 MPa and of yield strength 517.13 MPa.

**Added results using Response-2000 and the developed shear strength equation**

The program Response-2000 [3] is used to generate 336 results for the specimens shown in Table 3. The parameters are the concrete compressive strength, the cross-section, the transverse reinforcement, and the value of axial compressive load. The concrete strengths \(f'c\) are 30 and 70 MPa. The cross-section widths range from 350 to 450 mm and the cross-section heights range from 800 to 1900 mm. The spacings of
transverse reinforcements are 100 and 150 mm. The value of the axial load ranged between 0 and 0.20Acf,fcu. The added cross-sections are assumed to be subjected to either shear only or shear and moment equals to 1.333*shear value. The last column of Table 3 presents different values of the axial load for the same cross-section. The obtained values of concrete ultimate shear strengths (Vcu) are shown in Table 4 for the cross-sections of no moment and Table 5 for the cross-sections with shear and moment. The last column of both tables gives the ultimate concrete shear strength corresponding to each axial load in Table 3.

All the test results and the generated results are used to develop an empirical relationship for the ultimate shear strength of concrete (Vcu) in the existence of axial compressive load. The procedure adopted for developing this equation consists of plotting relation between the axial compressive

| Specimen Number | Axial Load (Pc), kN | fc, MPa | A, mm² | f', MPa | Stirrup Spacing, mm |
|-----------------|---------------------|---------|--------|--------|-------------------|
| C205-D10       | 0.0                 | 29.6    | 1232   | 368.5  | 157               |
| C205-D11       | 130.0               | 29.6    | 1232   | 368.5  | 157               |
| C205-D13       | 486.0               | 29.6    | 1232   | 368.5  | 157               |
| C205-D15       | 650.0               | 29.6    | 1232   | 368.5  | 157               |
| C205-D16       | 873.0               | 29.6    | 1232   | 368.5  | 157               |
| C205-D20       | 0.0                 | 31.0    | 982    | 394.6  | 157               |
| C205-D21       | 124.0               | 31.0    | 982    | 394.6  | 157               |
| C205-D22       | 248.0               | 31.0    | 982    | 394.6  | 157               |
| C205-D24       | 496.0               | 31.0    | 982    | 394.6  | 157               |
| C205-D26       | 744.0               | 31.0    | 982    | 394.6  | 157               |
| C210-D0A       | 0.0                 | 34.8    | 1232   | 368.5  | 157               |
| C210-D2        | 226.0               | 34.8    | 1232   | 368.5  | 157               |
| C210-D4        | 452.0               | 34.8    | 1232   | 368.5  | 157               |
| C210-D6        | 678.0               | 34.8    | 1232   | 368.5  | 157               |
| C210-S0        | 0.0                 | 29.3    | 1232   | 368.5  | 1232              |
| C210-S1        | 97.0                | 29.3    | 1232   | 368.5  | 1232              |
| C210-S2        | 544.0               | 29.3    | 1232   | 368.5  | 1232              |
| C210-S4        | 680.0               | 29.3    | 1232   | 368.5  | 1232              |
| C210-S6        | 800.0               | 29.3    | 1232   | 368.5  | 1232              |
| C305-D0        | 0.0                 | 33.1    | 1232   | 368.5  | 157               |
| C305-D1        | 109.0               | 33.1    | 1232   | 368.5  | 157               |
| C305-D2        | 218.0               | 33.1    | 1232   | 368.5  | 157               |
| C305-D4        | 436.0               | 33.1    | 1232   | 368.5  | 157               |
| C305-D6        | 654.0               | 33.1    | 1232   | 368.5  | 157               |
| C310-D10       | 0.0                 | 30.9    | 1232   | 366.9  | 157               |
| C310-D11       | 111.0               | 30.9    | 1232   | 366.9  | 157               |
| C310-D13       | 333.0               | 30.9    | 1232   | 366.9  | 157               |
| C310-D16       | 666.0               | 30.9    | 1232   | 366.9  | 157               |
| C310-D20       | 0.0                 | 31.2    | 1232   | 366.9  | 157               |
| C310-D21       | 114.0               | 31.2    | 1232   | 366.9  | 157               |
| C310-D23       | 340.0               | 31.2    | 1232   | 366.9  | 157               |
| C310-D25       | 568.0               | 31.2    | 1232   | 366.9  | 157               |
| C310-D27       | 722.0               | 31.2    | 1232   | 366.9  | 157               |
Figure 1. Comparison between test results, Response-2000, and current code for ultimate shear (variation with axial compressive load, shown lines are trend lines.).

Figure 2. Comparison between test results, Response-2000, and current code for ultimate shear (variation with concrete compressive strength, shown lines are trend lines.).
Table 1B. Properties of the test specimens of Haddadin et al. (1971) [25].

| Specimen Number | Axial Load ($P_{au}$), kN | $f_{cu}$, MPa | $A_{cu}$, mm$^2$ | $f_{p}$, MPa | Stirrup Spacing, mm | Area of Stirrups, mm$^2$ | Yield Strength of Stirrups, MPa |
|-----------------|-------------------------|---------------|-----------------|-------------|------------------|-------------------------|-------------------------------|
| A3C             | 351.39                  | 42.49         | 2580.64         | 517.13      | 190.5            | 141.94                 | 344.75                        |
| A4C             | 351.39                  | 35.47         | 2580.64         | 517.13      | 101.6            | 141.94                 | 344.75                        |
| A5C             | 351.39                  | 33.83         | 2580.64         | 517.13      | 63.5             | 141.94                 | 344.75                        |
| B3C             | 351.39                  | 34.65         | 2580.64         | 517.13      | 190.5            | 141.94                 | 344.75                        |
| C2C             | 351.39                  | 34.91         | 2580.64         | 517.13      | 190.5            | 64.52                  | 358.54                        |
| C3C             | 351.39                  | 33.35         | 2580.64         | 517.13      | 190.5            | 141.94                 | 344.75                        |
| C3C'            | 175.7                   | 34.13         | 2580.64         | 517.13      | 190.5            | 141.94                 | 344.75                        |
| C4C             | 351.39                  | 32.88         | 2580.64         | 517.13      | 101.6            | 141.94                 | 344.75                        |
| D3C             | 351.39                  | 37.36         | 2580.64         | 517.13      | 190.5            | 141.94                 | 344.75                        |
| E3C             | 351.39                  | 19.31         | 2580.64         | 517.13      | 190.5            | 141.94                 | 344.75                        |
| F3C             | 351.39                  | 50.68         | 2580.64         | 517.13      | 190.5            | 141.94                 | 456.45                        |
| G3C             | 351.39                  | 39.69         | 2580.64         | 517.13      | 127.0            | 141.94                 | 456.45                        |
| G4C             | 351.39                  | 31.50         | 2580.64         | 517.13      | 76.2             | 141.94                 | 456.45                        |
| H1C             | 351.39                  | 38.09         | 1290.32         | 517.13      | 190.5            | 141.94                 | 344.75                        |
| H2C             | 351.39                  | 37.58         | 3870.96         | 517.13      | 190.5            | 141.94                 | 344.75                        |
| J3C             | 351.39                  | 38.61         | 2580.64         | 517.13      | 190.5            | 141.94                 | 344.75                        |

Table 2. Experimental and analytical ultimate shear of the test specimens of Mattock and Wang (1984) [24].

| Specimen Number | $V_u$, kN from test | $V_u$, kN from ECP2000 | $V_u$, kN from ECP2000 | Error for Response-2000 | Error for ECP2000 |
|-----------------|---------------------|------------------------|------------------------|-------------------------|-------------------|
| C205-D10        | 140                 | 106.5                  | 71.05                  | 0.24                    | 0.49              |
| C205-D11        | 187.5               | 106.5                  | 76.40                  | 0.43                    | 0.59              |
| C205-D13        | 195.0               | 104.8                  | 86.47                  | 0.46                    | 0.56              |
| C205-D15        | 165.0               | 102                    | 86.47                  | 0.38                    | 0.48              |
| C205-D16        | 140.0               | 79.9                   | 86.47                  | 0.43                    | 0.38              |
| C205-D20        | 125.0               | 107.2                  | 71.77                  | 0.14                    | 0.43              |
| C205-D21        | 172.5               | 107.4                  | 76.99                  | 0.38                    | 0.55              |
| C205-D22        | 210.0               | 107.7                  | 82.21                  | 0.49                    | 0.61              |
| C205-D24        | 195.0               | 107.7                  | 87.56                  | 0.45                    | 0.55              |
| C205-D26        | 207.5               | 102.3                  | 87.56                  | 0.51                    | 0.58              |
| C210-D10 D0A    | 180.0               | 142                    | 113.85                 | 0.21                    | 0.37              |
| C210-D2         | 210.0               | 158                    | 123.93                 | 0.25                    | 0.41              |
| C210-D4         | 200.0               | 151.7                  | 130.58                 | 0.24                    | 0.35              |
| C210-D6         | 200.0               | 132.2                  | 130.58                 | 0.34                    | 0.35              |
| C210-S0         | 200.0               | 144.9                  | 111.10                 | 0.28                    | 0.44              |
| C210-S1         | 235.0               | 157                    | 115.05                 | 0.33                    | 0.51              |
| C210-S2         | 290.0               | 174                    | 126.44                 | 0.40                    | 0.56              |
| C210-S4         | 300.0               | 178.6                  | 126.44                 | 0.40                    | 0.40              |
| C210-S6         | 285.0               | 183.5                  | 126.44                 | 0.36                    | 0.56              |
| C305-D0         | 110.0               | 97.8                   | 72.82                  | 0.11                    | 0.34              |
| C305-D1         | 125.0               | 103.3                  | 77.56                  | 0.17                    | 0.38              |
| C305-D2         | 150.0               | 100.6                  | 82.31                  | 0.33                    | 0.45              |
| C305-D4         | 150.0               | 96.4                   | 89.13                  | 0.36                    | 0.41              |
| C305-D6         | 150.0               | 84.1                   | 89.13                  | 0.44                    | 0.41              |
| C310-D10        | 135.0               | 105.9                  | 111.92                 | 0.22                    | 0.17              |

(Continued)
Table 2. (Continued).

| Specimen Number | $V_u$, kN from test | $V_u$, kN Response-2000 | $V_u$, kN from ECP203-2017 | Error for Response-2000 | Error for ECP203-2017 |
|-----------------|---------------------|-------------------------|--------------------------|-------------------------|------------------------|
| C310-D11        | 180.0               | 113.2                   | 116.59                   | 0.37                    | 0.35                   |
| C310-D13        | 175.0               | 112.1                   | 125.92                   | 0.36                    | 0.28                   |
| C310-D16        | 160.0               | 80.1                    | 127.68                   | 0.50                    | 0.20                   |
| C310-D20        | 142.5               | 105.9                   | 112.08                   | 0.26                    | 0.21                   |
| C310-D21        | 170.0               | 113.5                   | 116.89                   | 0.33                    | 0.31                   |
| C310-D23        | 187.5               | 112.7                   | 126.43                   | 0.40                    | 0.33                   |
| C310-D25        | 165.0               | 92                      | 127.91                   | 0.44                    | 0.22                   |
| C310-D27        | 150.0               | 75.7                    | 127.91                   | 0.50                    | 0.15                   |

Table 3. Data of the added specimens for the two cases Moment ($M$) = ‘0’ and Moment ($M$) = ‘Shear*1.333’ kN.m.

| Specimen Name      | Cross-Section, mm*mm | $f_{cu}$, MPa | Stirrup Spacing, mm | Axial Compressive Load ($P_u$), kN |
|--------------------|----------------------|---------------|---------------------|----------------------------------|
| B1-30-100          | 350*800              | 30            | 100                 | 0.252, 420, 1176, 1512 & 1680    |
| B1-30-150          | 350*800              | 30            | 150                 | 0.252, 420, 1176, 1512 & 1680    |
| B1-70-100          | 350*800              | 70            | 100                 | 0.588, 980, 2744, 3528 & 3920    |
| B1-70-150          | 350*800              | 70            | 150                 | 0.588, 980, 2744, 3528 & 3920    |
| B2-30-100          | 350*1100             | 30            | 100                 | 0.346.5, 577.5, 1617, 2079 & 2310|
| B2-30-150          | 350*1100             | 30            | 150                 | 0.346.5, 577.5, 1617, 2079 & 2310|
| B2-70-100          | 350*1100             | 70            | 100                 | 0.806.9, 1347.5, 3773, 4851 & 5390|
| B2-70-150          | 350*1100             | 70            | 150                 | 0.806.9, 1347.5, 3773, 4851 & 5390|
| B3-30-100          | 350*1300             | 30            | 100                 | 0.409.5, 782.5, 1911, 2347 & 2730|
| B3-30-150          | 350*1300             | 30            | 150                 | 0.409.5, 782.5, 1911, 2347 & 2730|
| B3-70-100          | 350*1300             | 70            | 100                 | 0.955.5, 1592.5, 4459, 5733 & 6370|
| B3-70-150          | 350*1300             | 70            | 150                 | 0.955.5, 1592.5, 4459, 5733 & 6370|
| B4-30-100          | 400*1400             | 30            | 100                 | 0.504, 840, 2352, 3024 & 3360    |
| B4-30-150          | 400*1400             | 30            | 150                 | 0.504, 840, 2352, 3024 & 3360    |
| B4-70-100          | 400*1400             | 70            | 100                 | 0.1176, 1960, 5488, 7056 & 7840  |
| B4-70-150          | 400*1400             | 70            | 150                 | 0.1176, 1960, 5488, 7056 & 7840  |
| B5-30-100          | 400*1700             | 30            | 100                 | 0.612, 1020, 2856, 3672 & 4080  |
| B5-30-150          | 400*1700             | 30            | 150                 | 0.612, 1020, 2856, 3672 & 4080  |
| B5-70-100          | 400*1700             | 70            | 100                 | 0.1428, 2380, 6664, 8568 & 9520  |
| B5-70-150          | 400*1700             | 70            | 150                 | 0.1428, 2380, 6664, 8568 & 9520  |
| B6-30-100          | 400*1800             | 30            | 100                 | 0.648, 1080, 3024, 3888 & 4320  |
| B6-30-150          | 400*1800             | 30            | 150                 | 0.648, 1080, 3024, 3888 & 4320  |
| B6-70-100          | 400*1800             | 70            | 100                 | 0.1512, 2520, 7056, 9072 & 10080 |
| B6-70-150          | 400*1800             | 70            | 150                 | 0.1512, 2520, 7056, 9072 & 10080 |
| B7-30-100          | 450*1900             | 30            | 100                 | 0.769.5, 1282.5, 3591, 4617 & 5130|
| B7-30-150          | 450*1900             | 30            | 150                 | 0.769.5, 1282.5, 3591, 4617 & 5130|
| B7-70-100          | 450*1900             | 70            | 100                 | 0.1795.5, 2992.5, 8379, 10773 & 11970|
| B7-70-150          | 450*1900             | 70            | 150                 | 0.1795.5, 2992.5, 8379, 10773 & 11970|

load and concrete shear strength for each combination of cross-section, stirrup, concrete compressive strength, and moment. For each plotted group of points a curve fitting is performed. The coefficients in the proposed equation are thus obtained and averaged. In proposing the new equation the
Table 4. Concrete ultimate shear strengths ($V_{cu}$) from Response-2000 [Moment (M) = ‘0’].

| Specimen Name | Concrete Shear Strength ($V_{cu}$) from Response-2000, kN |
|---------------|----------------------------------------------------------|
| B1-30-100     | 333.2, 349.3, 350.0, 407.8, 445.4 & 471.6             |
| B1-30-150     | 348.8, 361.8, 375.6, 472.7, 510.8 & 539.3             |
| B1-70-100     | 374.9, 483.5, 546.3, 828.1, 1007.5 & 1112.3            |
| B1-70-150     | 427.6, 598.7, 543.7, 808.7, 923.7 & 930.9             |
| B2-30-100     | 236.3, 266.8, 286.6, 390.7, 451.8 & 638.9             |
| B2-30-150     | 381.4, 453.9, 386.5, 492.3, 540.7 & 601.8             |
| B2-70-100     | 554.5, 676.0, 816.8, 1045.8, 1291.9 & 1456.0          |
| B2-70-150     | 490.9, 777.4, 847.8, 1098.5, 1351.2 & 1413.2          |
| B3-30-100     | 644.7, 674.6, 691.0, 750.3, 793.3 & 820.8             |
| B3-30-150     | 709.3, 727.0, 737.4, 793.3, 827.6 & 932.4             |
| B3-70-100     | 1384.0, 1404.5, 1415.4, 1531.5, 1802.9 & 1843.4      |
| B3-70-150     | 1290.3, 1305.3, 1313.8, 1538.4, 1628.6 & 1641.6      |
| B4-30-100     | 422.2, 477.3, 497.6, 628.4, 708.3 & 758.2             |
| B4-30-150     | 446.7, 610.2, 697.1, 693.4, 861.9 & 930.8             |
| B4-70-100     | 692.8, 974.6, 1278.4, 1544.2, 1942.9 & 2150.4        |
| B4-70-150     | 592.7, 1213.4, 1253.8, 1587.4, 1887.9 & 1890.3      |
| B5-30-100     | 939.4, 967.4, 975.2, 1111.7, 1154.5 & 1188.5          |
| B5-30-150     | 1002.1, 1023.2, 1034.3, 1118.9, 1260.4 & 1339.6      |
| B5-70-100     | 1926.8, 1975.7, 1989.8, 2190.4, 2574.6 & 2622.8      |
| B5-70-150     | 1780.4, 1799.9, 1820.1, 2208.7, 2284.9 & 2266.4      |
| B6-30-100     | 1051.3, 1082.8, 1106.3, 1225.5, 1289.5 & 1329.3      |
| B6-30-150     | 1122.9, 1160.3, 1171.9, 1309.5, 1306.0 & 1390.4      |
| B6-70-100     | 2144.2, 2186.6, 2276.6, 2480.5, 2724.9 & 2908.6      |
| B6-70-150     | 2071.9, 2051.7, 2085.4, 2187.5, 2578.1 & 2592.1      |
| B7-30-100     | 1055.4, 1084.6, 1132.2, 1297.3, 1419.4 & 1396.3      |
| B7-30-150     | 1239.9, 1256.4, 1286.9, 1388.3, 1457.6 & 1593.6      |
| B7-70-100     | 2429.8, 2521.2, 2566.6, 2814.1, 3304.1 & 3383.7      |
| B7-70-150     | 2343.8, 2382.8, 2423.2, 2799.7, 3067.9 & 3094.8      |

Table 5. Concrete ultimate shear strengths ($V_{cu}$) from Response-2000 [Moment (M) = ‘shear*1.333’ kN.m].

| Specimen Name | Concrete Shear Strength ($V_{cu}$) from Response-2000, kN |
|---------------|----------------------------------------------------------|
| B1-30-100     | 55.90, 71.57, 81.93, 0.0, 9.69, and 32.69               |
| B1-30-150     | 92.80, 110.03, 125.14, 199.95, 218.98, and 234.99      |
| B1-70-100     | 102.45, 164.11, 201.61, 411.05, 374.49, and 437.39    |
| B1-70-150     | 140.63, 196.73, 244.01, 453.59, 532.34, and 532.69    |
| B2-30-100     | 0.0, 0.0, 0.0, 0.0, 0.0, and 0.0                        |
| B2-30-150     | 0.0, 0.0, 0.0, 0.0, 0.0, and 0.0                        |
| B2-70-100     | 0.0, 0.0, 0.0, 0.0, 0.0, and 0.0                        |
| B2-70-150     | 0.0, 0.0, 0.0, 0.0, 0.0, and 0.0                        |
| B3-30-100     | 736.37, 752.17, 777.27, 778.77, 797.97, and 811.27    |
| B3-30-150     | 764.85, 780.05, 791.75, 822.65, 865.15, and 886.25    |
| B3-70-100     | 1102.37, 1287.77, 1205.07, 1579.27, 1711.37, and 1665.37 |
| B3-70-150     | 1259.35, 1384.45, 1381.15, 1469.35, 1535.35, and 1506.05 |
| B4-30-100     | 0.0, 0.0, 0.0, 0.0, 0.0, and 0.0                        |
| B4-30-150     | 71.19, 102.91, 330.79, 485.29, 622.21, and 640.31     |
| B4-70-100     | 0.0, 0.0, 123.17, 980.67, 1323.47, and 1433.97        |
| B4-70-150     | 7.81, 477.23, 586.04, 1119.91, 1434.31, and 1600.01   |
| B5-30-100     | 1074.67, 1093.17, 1069.37, 1198.77, 1182.57, and 1212.67 |
| B5-30-150     | 1098.71, 1116.91, 1125.91, 1203.71, 1220.61, and 1263.31 |
| B5-70-100     | 1873.27, 2130.17, 2150.47, 2271.47, 2335.77, and 2356.77 |

(Continued)
Table 5. (Continued).

| Specimen Name | Concrete Shear Strength ($V_{cu}$) from Response-2000, kN |
|---------------|---------------------------------------------------------|
| B5-70-150     | 1933.41, 1800.28, 1836.68, 1938.02, 1972.32, and 1990.22 |
| B6-30-100     | 839.57, 997.37, 1087.17, 1261.27, 1377.37, and 1341.87 |
| B6-30-150     | 1214.37, 1191.98, 1225.58, 1304.28, 1339.58, and 1390.88 |
| B6-70-100     | 1403.57, 1595.87, 1786.67, 2452.87, 2527.97, and 2589.17 |
| B6-70-150     | 1455.62, 1753.98, 1933.88, 2185.89, 2224.99, and 2215.39 |
| B7-30-100     | 1901.17, 1928.27, 1987.27, 2117.37, 2112.17, and 2146.27 |
| B7-30-150     | 1884.05, 1914.85, 1969.75, 1960.85, 1992.35, and 2019.95 |
| B7-70-100     | 3132.27, 3421.97, 3529.77, 3644.77, 3790.57, and 3775.67 |
| B7-70-150     | 3032.22, 2996.12, 3031.02, 3158.32, 3142.32, and 3193.32 |

Table 6. Concrete ultimate shear strengths ($V_{cu}$) from the proposed equation [Moment (M) = ‘0’].

| Specimen Name | ($V_{cu}$) Response-2000/$V_{cu}$ Proposed Equation |
|---------------|--------------------------------------------------|
| B1-30-100     | 1.04, 1.00, 0.95, 0.90, 0.93, & 0.99             |
| B1-30-150     | 1.09, 1.04, 1.02, 1.04, 1.07, & 1.13             |
| B1-70-100     | 0.77, 0.82, 0.83, 1.13, 1.38, & 1.52             |
| B1-70-150     | 0.88, 1.02, 0.83, 0.84, 0.84, & 0.80             |
| B2-30-100     | 0.53, 0.55, 0.56, 0.61, 0.67, & 0.95             |
| B2-30-150     | 0.85, 0.93, 0.75, 0.77, 0.81, & 0.90             |
| B2-70-100     | 0.81, 0.82, 0.89, 1.02, 1.26, & 1.42             |
| B2-70-150     | 0.72, 0.94, 0.92, 1.07, 1.32, & 1.38             |
| B3-30-100     | 1.21, 1.16, 1.13, 0.99, 0.99, & 1.03             |
| B3-30-150     | 1.33, 1.25, 1.20, 1.05, 1.03, & 1.17             |
| B3-70-100     | 1.70, 1.42, 1.29, 1.25, 1.48, & 1.51             |
| B3-70-150     | 1.58, 1.32, 1.19, 1.26, 1.33, & 1.34             |
| B4-30-100     | 1.07, 1.11, 1.10, 1.12, 1.16, & 1.20             |
| B4-30-150     | 0.68, 0.85, 0.92, 0.74, 0.87, & 0.94             |
| B4-70-100     | 0.69, 0.80, 0.94, 1.02, 1.29, & 1.43             |
| B4-70-150     | 0.59, 1.00, 0.92, 1.05, 1.25, & 1.25             |
| B5-30-100     | 1.17, 1.10, 1.05, 0.97, 0.96, & 0.98             |
| B5-30-150     | 1.24, 1.17, 1.12, 0.98, 1.04, & 1.11             |
| B5-70-100     | 1.57, 1.33, 1.20, 1.19, 1.40, & 1.42             |
| B5-70-150     | 1.45, 1.21, 1.10, 1.20, 1.24, & 1.24             |
| B6-30-100     | 1.23, 1.16, 1.13, 1.01, 1.01, & 1.04             |
| B6-30-150     | 1.31, 1.25, 1.19, 1.08, 1.02, & 1.08             |
| B6-70-100     | 1.64, 1.38, 1.29, 1.27, 1.39, & 1.49             |
| B6-70-150     | 1.55, 1.30, 1.18, 1.12, 1.32, & 1.32             |
| B7-30-100     | 1.75, 1.63, 1.59, 1.40, 1.41, & 1.39             |
| B7-30-150     | 1.70, 1.57, 1.51, 1.30, 1.27, & 1.36             |
| B7-70-100     | 2.03, 1.73, 1.57, 1.52, 1.73, & 1.76             |
| B7-70-150     | 1.82, 1.53, 1.39, 1.41, 1.52, & 1.54             |

The form of the equation of the Egyptian code is adopted. The change is in using different coefficients as follows:
Table 7. Concrete ultimate shear strengths ($V_{cu}$) from the proposed equation [Moment (M) = ‘shear’$^*1.333$’ kN.m].

| Specimen Name | ($V_{cu}$) Response-2000/$V_{cu}$ Proposed Equation |
|---------------|-----------------------------------------------------|
| B1-30-100     | 0.18, 0.21, 0.22, 0.0, 0.02, & 0.07                |
| B1-30-150     | 0.29, 0.32, 0.34, 0.44, 0.46, & 0.49                |
| B1-70-100     | 0.21, 0.28, 0.31, 0.56, 0.51, & 0.6                  |
| B1-70-150     | 0.29, 0.33, 0.37, 0.62, 0.73, & 0.73                |
| B2-30-100     | 0.0, 0.0, 0.0, 0.0, 0.0, & 0.0                      |
| B2-30-150     | 0.0, 0.0, 0.0, 0.20, 0.33, & 0.37                   |
| B2-70-100     | 0.0, 0.0, 0.0, 0.31, 0.54, & 0.58                   |
| B2-70-150     | 0.0, 0.30, 0.37, 0.58, 0.75, & 0.84                 |
| B3-30-100     | 1.38, 1.29, 1.27, 1.03, 1.0, & 1.01                 |
| B3-30-150     | 1.43, 1.34, 1.29, 1.09, 1.08, & 1.11                |
| B3-70-100     | 1.35, 1.31, 1.10, 1.29, 1.40, & 1.36                |
| B3-70-150     | 1.55, 1.40, 1.26, 1.20, 1.26, & 1.23                |
| B4-30-100     | 0.0, 0.0, 0.0, 0.12, 0.27, & 0.37                   |
| B4-30-150     | 0.11, 0.14, 0.44, 0.52, 0.63, & 0.65                |
| B4-70-100     | 0.0, 0.0, 0.09, 0.65, 0.88, & 0.95                  |
| B4-70-150     | 0.01, 0.39, 0.43, 0.74, 0.95, & 1.06                |
| B5-30-100     | 1.33, 1.25, 1.15, 1.05, 0.98, & 1.00                 |
| B5-30-150     | 1.36, 1.27, 1.22, 1.05, 1.01, & 1.05                |
| B5-70-100     | 1.52, 1.43, 1.29, 1.23, 1.27, & 1.28                |
| B5-70-150     | 1.57, 1.21, 1.11, 1.05, 1.07, & 1.08                |
| B6-30-100     | 0.98, 1.07, 1.11, 1.04, 1.07, & 1.05                |
| B6-30-150     | 1.42, 1.28, 1.25, 1.08, 1.05, & 1.09                |
| B6-70-100     | 1.08, 1.01, 1.01, 1.25, 1.29, & 1.32                |
| B6-70-150     | 1.12, 1.11, 1.10, 1.12, 1.14, & 1.13                |
| B7-30-100     | 1.87, 1.74, 1.70, 1.47, 1.39, & 1.41                |
| B7-30-150     | 1.85, 1.73, 1.69, 1.36, 1.31, & 1.33                |
| B7-70-100     | 2.02, 1.82, 1.68, 1.57, 1.63, & 1.62                |
| B7-70-150     | 1.95, 1.60, 1.45, 1.36, 1.35, & 1.37                |

\[
V_c = \left( 1 + 0.10 \frac{P_u}{A_c} \right) \times 0.20 \sqrt{\frac{f_{cu}}{Y_c}}
\]

where

\[
\left( 1 + 0.10 \frac{P_u}{A_c} \right) \leq 1.5
\]

Tables 6 and 7 show the ratio between ultimate concrete shear strength from Response-2000 to the proposed equation for the 336 generated results. Also, this equation is used to calculate the ultimate shear capacity of the experimental beams, where the concrete strength reduction factor is taken equal to 1.0. The obtained average error is 24.7\% for the beams of Mattock and Wang (1984) [24] and is 22.0\% for the beams of Haddadin et al. (1971) [25] as shown in Tables 8 and 9, respectively. It is clear that the degree of conservatism for this equation is less than that of the Egyptian code. Figures 3 and 4 show the variation of concrete shear strength calculated from the Response-2000 [3] and the proposed
Table 8. Ultimate shear of the test specimens of Mattock and Wang (1984) [24] over the ultimate shear calculated from the proposed equation.

| Specimen Name | \((V_u)_{\text{test}}/(V_u)_{\text{Proposed Equation}}\) |
|---------------|------------------------------------------------------|
| C205-D10      | 1.35                                                 |
| C205-D11      | 1.64                                                 |
| C205-D13      | 1.78                                                 |
| C205-D15      | 1.64                                                 |
| C205-D16      | 1.74                                                 |
| C205-D20      | 1.53                                                 |
| C205-D21      | 1.80                                                 |
| C205-D22      | 1.66                                                 |
| C205-D24      | 1.41                                                 |
| C205-D26      | 1.19                                                 |
| C210-D0A      | 1.32                                                 |
| C210-D2       | 1.31                                                 |
| C210-D4       | 1.22                                                 |
| C210-D6       | 1.22                                                 |
| C210-S0       | 1.52                                                 |
| C210-S1       | 1.67                                                 |
| C210-S2       | 1.85                                                 |
| C210-S4       | 1.91                                                 |
| C210-S6       | 1.81                                                 |
| C305-D0       | 1.16                                                 |
| C305-D1       | 1.18                                                 |
| C305-D2       | 1.28                                                 |
| C305-D4       | 1.23                                                 |
| C305-D6       | 1.23                                                 |
| C310-D10      | 1.02                                                 |
| C310-D11      | 1.25                                                 |
| C310-D13      | 1.10                                                 |
| C310-D16      | 1.01                                                 |
| C310-D20      | 1.07                                                 |
| C310-D21      | 1.18                                                 |
| C310-D23      | 1.17                                                 |
| C310-D25      | 1.03                                                 |
| C310-D27      | 0.94                                                 |

Table 9. Ultimate shear of the test specimens of Haddadin et al. (1971) [25] over the ultimate shear calculated from the proposed equation.

| Specimen Name | \((V_u)_{\text{test}}/(V_u)_{\text{Proposed Equation}}\) |
|---------------|------------------------------------------------------|
| A3C           | 1.19                                                 |
| A4C           | 1.30                                                 |
| A5C           | 1.15                                                 |
| B3C           | 1.39                                                 |
| C2C           | 1.27                                                 |
| C3C           | 1.31                                                 |
| C3C'          | 1.35                                                 |
| C4C           | 1.05                                                 |
| D3C           | 1.05                                                 |
| E3C           | 1.28                                                 |
| F3C           | 1.50                                                 |
| G3C           | 1.64                                                 |
| G4C           | 1.25                                                 |
| G5C           | 1.06                                                 |
| H1C           | 1.50                                                 |
| H2C           | 1.50                                                 |
| J3C           | 1.37                                                 |
Figure 3. Comparison between Response-2000, and proposed equation for concrete shear strength (variation with axial compressive load, shown lines are trend lines.).

Figure 4. Comparison between Response-2000, and proposed equation for concrete shear strength (variation with concrete compressive strength, shown lines are trend lines.).
equation with the axial load and the concrete compressive strength, respectively. It is very clear that the increase in either the axial normal force or the concrete compressive strength results in increase in the ultimate concrete shear capacity. The increase in shear capacity with the increase in the axial compression is due to the reduction in the major tensile stress in concrete.

Conclusions

1-The program Response-2000 gives good estimates for the shear strength in the case of existing axial compressive force. The accuracy of the program is better than that of the equations of the current Egyptian code. Both the program and the code are on the conservative side.

2-A new equation for the shear strength of beams in the case of existing axial compressive load is developed. This equation has less error, and still on the conservative side, when compared to the current code equation.

Disclosure of potential conflicts of interest

No potential conflict of interest was reported by the author(s).

References

1. Halim A, Rasheed HA, Esmaeily A Software for AASHTO LRFD combined shear and torsion computations using modified compression field theory and 3D truss analogy. Report No. k-TRAN:KSU-08-5 October 2011; Kansas Department of Transportation, USA, 93 pp.
2. Metwally IM. Evaluate the capability and accuracy of response-2000 program in prediction of the shear capacities of reinforced and prestressed concrete members. HBRC J. 2012;8(2):99–106.
3. Bentz EC, Collins MP Response-2000. http://www.ecf.utoronto.ca 2019.
4. Esfandiari A, Adebar P. Shear strength evaluation of concrete bridge girders. ACI Struct J. 2009 July-August;106(4):416–426.
5. Hawkins NM, Kuchma DA, Mast RF, et al. Simplified shear design of structural concrete members. NCHRP Report 549 2005; American Association of State Highway and Transportation Officials, Washington, D.C.
6. Collins MP, Mitchell D, Adebar P, et al. A general shear design method. ACI Struct J. 1996;93(1):36–45.
7. Ford JS, Chang DC, Breen JE. Behavior of concrete columns under controlled lateral deformation. ACI J Proc. 1981 January;78(1).
8. Osei-Antwi M, De Castro J, Vassilopoulos AP, et al. Modeling of axial and shear stresses in multilayer sandwich beams with stiff core layers. Compos Struct. 2014;116:453–460.
9. Ou Y, Nguyen ND. Modified axial-shear-flexure interaction approaches for uncorroded and corroded reinforced concrete beams. Eng Struct. 2016;128:44–54.
10. Rajapakse RMCM, Wijesundara KK, Nascimbene R, et al. Accounting axial-moment-shear interaction for force-based fiber modeling of RC frames. Eng Struct. 2019;184:15–36.
11. Kirkland B, Kim P, Uy B, et al. Moment-shear-axial force interaction in composite beams. J Const Steel Res. 2015;114:66–76.
12. Lai B, Liew JYR, Hoang AL, et al. Approach to evaluate axial force-moment interaction curves of concrete encased steel composite column. Eng Struct. 2019;201:1–15.
13. Xu S-Y, Zhang J. Axial-Shear-Flexure interaction hysteretic model for RC columns under combined actions. Eng Struct. 2012;34:548–563.
14. Kocer M, Ozturk M, Arslan MH. Determination of moment, shear and ductility capacities of spiral columns using artificial neural network. J Build Eng. 2019;26:1–15.
15. Popovic B. The Influence of the reinforced concrete beams axial force on the shape of the shear force-bending moment interaction diagram. 2nd International Scientific Meeting (GTZ 2012) 7-9 June 2012; Tuzla: 235–242.
16. International Federation for Structural Concrete (FIB). FIB Model Code 2010-Volume 2. FIB Bulletin 56 April 2010; 311
17. Lodhi MS, Sezen H. Estimation of monotonic behavior of reinforced concrete columns considering Shear-Flexure-Axial load interaction. Earthq Eng Struct Dyn. 2012;41:2159–2175.
18. Saritas A, Filippou FC. Inelastic Axial-Flexure-Shear coupling in a mixed formulation beam finite element. Int J Non Linear Mech. 2009;44(8):913–922.
19. Mullapudi T, Ayoub A. Analysis of reinforced concrete columns subjected to combined axial, flexure, shear, and torsional loads. ASCE J Struct Eng. 2013;139 (4):561–573.
20. Zhang T, Visintin P, Oehlers DJ. Shear strength of RC beams subjected to axial load. Aust J Civ Eng. 2017;15(1):32–48.
21. Rasheed H, Abouelleil A KDOT column expert: ultimate shear capacity of circular columns using the simplified modified compression field theory. Report No. 25-1121-0003-262 August 2015; Mid-America Transportation Center, USA, 128.
22. Committee for Egyptian Concrete Code. Egyptian code for design and construction of reinforced concrete structures (ECP203-2017). Housing and Building National Research Center 2017; Giza, Egypt, 9 chapters.
23. Wight JK, MacGregor JG. Reinforced concrete mechanics and design. USA: Pearson Publishing; 2012. p. 1157.
24. Mattock AH, Wang Z. Shear strength of reinforced concrete members subject to high axial compressive stress. ACI J. 1984;81:28:287–298.
25. Haddadin MJ, Hong S-T, Mattock AH Stirrup effectiveness in reinforced concrete beams with axial force. Journal of the Structural Division, Proceedings of the American Society of Civil Engineers 1971; 97(ST9): 2277–2297.