Hawking radiation of Kerr-Newman-de Sitter black hole

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Abstract. We extend the classical Damour-Ruffini method and discuss Hawking radiation in Kerr-Newman-de Sitter (KNdS) black hole. Under the condition that the total energy, angular momentum and charge of spacetime are conserved, taking the reaction of the radiation of the particle to the spacetime and the relation between the black hole event horizon and the cosmological horizon into consideration, we derive the black hole radiation spectrum. The radiation spectrum is no longer a pure thermal one. It is related to the change of the Bekenstein-Hawking entropy corresponding the black hole event horizon and the cosmological horizon. It is consistent with an underlying unitary theory.

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1 Introduction

In the 1970s, Hawking [1] pointed that the black hole can radiate particles. Vacuum fluctuations near the surface of the black hole would produce virtual particle pairs. When the virtual particles with negative energy go into black hole via the tunnel effect, the energy of the black hole will decrease. At the same time, the particle with positive energy may thread out the gravitation region outside the black hole. Equivalently, the black hole radiates a particle. Gibbons and Hawking also demonstrated that the energy spectrum of radiation is exactly thermal [2]. The Hawking radiation demonstrates that the black hole is no longer an ultimate state of a star and will evolve. Since Hawking did not consider the reaction of the radiation to spacetime, the Hawking radiation spectrum is an exact black body spectrum. So we can only obtain a parameter-temperature from black body spectrum. Thus the black hole radiation will not bring any information about matter in the black hole. It means that if the black hole completely evaporates, all black hole information including the unitary property will disappear. This violates the unitary principle in quantum mechanics. This is a serious challenge to the theoretical basic of quantum mechanics.

Before 2004, Hawking firmly believed that information is not conservative during the black hole evolution. The black hole evolution did not satisfy the unitary principle in quantum theory[3]. However, some physicists advocate that information is conserved during the black hole evolution and the unitary principle should be satisfied [4].

However, the aforesaid two view points have not been proved for 30 years. In 2004, during the 17th International Conference on General Relativity and Gravitation, Hawking brought about a tremendous conservation. He proposed that the information should be conservative during the black hole formation and evaporation process [5]. But Hawking did not proved strictly his proposal.

In 2000, Parikh and Wilczek proposed a semiclassical method for calculating the modified spectrum of the black hole Hawking radiation [6]. In this method, the black hole Hawking radiation is understood as a sort of quantum tunneling. The potential barrier is determined by the energy of emission particles. The key of this method is emphasizing energy conservation during the particle emission process and establishing a good coordinate system at horizon. Using this method Parikh and Wilczek have calculated the emission modified spectrum of particles through a Schwarzschild black hole and a Reissner-Nordström one. The result departs from the purely thermal spectrum. It satisfies the unitary principle and support the result of information conservation. Subsequently, the Hawking radiation modified spectrums of axially symmetric black holes have been calculated [7-35]. They all satisfy the unitary theory and support the result of information conservation.

The core idea of Parikh and Wilczek’s work is considering the total energy conservation of spacetime in the process of black hole radiation, and the energy of the black hole can produce fluctuations. When Ref.[21-23] calculated the radiation spectrum of axisymmetric black hole using the tunneling method proposed by Parikh and Wilczek, they only considered that the energy and charge of the black hole produce fluctuations. The change of the black hole angular momentum is determined by the change of the black hole energy. They did not consider the effect of
the rotation of the black hole radiated particles on the black hole angular momentum.

It is well-known that as a thermodynamic system the black hole has temperature and entropy. For a spacetime that does not include a cosmological term, the state parameters of this thermodynamic system are all embodied on the black hole horizon surface. The black hole event horizon is the "window" that transports information to outside world. The radiation spectrum of this type black hole has been discussed by the tunnel method. However, for the spacetime that include a cosmological term, the state parameters of this thermodynamic system embody on not only the black hole event horizon surface but also the cosmological horizon surface. Both the black hole event horizon and the cosmological horizon are the "windows" that transport information to outside world. Because the black hole event horizon and the cosmological horizon have the same state parameters, there is a correlation between the black hole event horizon and the cosmological horizon. At present when the two horizons have relevance, the study on black hole radiation spectrum has not yet reported.

In this paper, we extend the classical Damour-Ruffini method [36] to discuss the radiation spectrum in the Kerr-Newman-de Sitter(KNds) spacetime. Under the condition that the spacetime total energy, total charge and total angular momentum are conserved, we derive the black hole radiation spectrums after considering the reaction of radiation to the spacetime and the relation between the black hole event horizon and the cosmological horizon. The radiation spectrum is no longer a pure thermal spectrum and deviate from the black body spectrum. It is related to the change of the Bekenstein-Hawking entropy corresponding the black hole event horizon and the cosmological horizon.

2 Kerr-Newman-de Sitter black hole

The Kerr-Newman-de Sitter(KNds) spacetime line element [37,38]:

\[
ds^2 = -\frac{1}{\rho^2} \left( \Delta_r - \Delta_\theta a^2 \sin^2 \theta \right) dt^2 + \frac{\rho^2}{\Delta_r} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 + \frac{1}{\rho^2 \Xi} \left[ \Delta_\theta (r^2 + a^2) \sin^2 \theta \sin^2 \theta d\varphi^2 - \frac{2a}{\rho^2 \Xi} \left( \Delta_\theta (r^2 + a^2) - \Delta_r \right) \sin^2 \theta dt d\varphi, \right. (1)
\]

where

\[
\rho^2 = r^2 + a^2 \cos^2 \theta, \quad \Delta_\theta = 1 + \frac{1}{3} \Lambda a^2 \cos^2 \theta, \quad \Delta_r = (r^2 + a^2) \left( 1 - \frac{1}{3} \Lambda r^2 \right) - 2Mr + q^2,
\]

Here the parameters \( M, a \) and \( q \) are the associated with the mass, angular momentum, and charge parameters of the space-time, respectively, and \( \Lambda \) is the positive cosmological constant. Based on the metric (1), we have

\[
g = \det(g_{\mu\nu}) = -\frac{1}{\Xi^2} \rho^4 \sin^2 \theta. \quad (3)
\]

The contravariant variant of \( g_{\mu\nu} \) is

\[
\frac{\partial^2}{\partial s^2} = -\frac{1}{\rho^2 \Delta_r \Delta_\theta} \left[ \Delta_\theta (r^2 + a^2)^2 - \Delta_r a^2 \sin^2 \theta \right]
\]

\[
\frac{\partial^2}{\partial t^2} + \frac{\Delta_\theta \partial^2}{\rho^2 \partial r^2} + \frac{\Delta_\theta \partial^2}{\rho^2 \partial \theta^2} + \frac{\Xi^2}{\Delta_r \Delta_\theta \rho^2 \sin^2 \theta} \left( \Delta_r - \Delta_\theta a^2 \sin^2 \theta \right)
\]

\[
\frac{\partial^2}{\partial \varphi^2} = \frac{2 \Xi a}{\rho^2 \Delta_r \Delta_\theta} \left[ \Delta_\theta (r^2 + a^2) - \Delta_r \right] \frac{\partial^2}{\partial t \partial \varphi}. \quad (4)
\]

The electromagnetic potential is [39]

\[
A_\mu = -\frac{qr}{\rho^2} \left( 1, 0, 0, -\frac{a \sin^2 \theta}{\Xi} \right). \quad (5)
\]

The horizon surface equation of the Kerr-Newman-de Sitter spacetime is

\[
\Delta_r = -\frac{1}{3} \Lambda (r - r_c) (r - r_+) (r - r_-) (r - r_{-}) = 0. \quad (6)
\]

When \( \frac{1}{3} \Lambda \gg M^2 > a^2 + Q^2 \), equation \( \Delta_r = 0 \) has \( r_c, r_+, r_-, r_{-} \) four real solutions, where \( r_c, r_+, r_- \) are positive and \( r_c > r_+, r_- \) is negative. \( r_c, r_- \) correspond to de Sitter cosmological horizon. \( r_+, r_- \) correspond to Kerr-Newman black hole horizon.

The Abbott and Deser (AD) mass of the KNds solution can be expressed in terms of the horizon radius \( r_+ \), \( a \) and charge \( q \) [37-39]:

\[
E = \frac{M}{\Xi} = \frac{(r_+^2 + a^2)(r_+^2 - 3/\Lambda) - q^2 3/\Lambda}{2\Xi r_+^3/\Lambda}. \quad (7)
\]

The Hawking temperature of the black hole horizon is given by

\[
T_+ = \frac{1}{4\pi} \frac{\Delta'_+ (r_+)}{r_+^2 + a^2} = -\frac{3r_+^4 + r_+^2 (a^2 - 3/\Lambda) + (a^2 + q^2)3/\Lambda}{4\pi r_+ (r_+^2 + a^2)3/\Lambda}. \quad (8)
\]

The entropy associated with the black hole horizon can be calculated as

\[
S_+ = \frac{\pi (r_+^2 + a^2)}{\Xi}. \quad (9)
\]

The angular velocity of the black hole horizon is given by

\[
\Omega_+ = \frac{a \Xi}{r_+^2 + a^2}. \quad (10)
\]
The angular momentum \( J \), the electric charge \( Q \), and the electric potential \( \phi_+ \) are given by
\[
J = \frac{Ma}{\Xi}, \quad Q = \frac{q}{\Xi}, \quad \phi_+ = \frac{qr_+}{r_+^2 + a^2},
\]  
(11)
The quantities obtained above of the black hole horizon satisfy the first law of thermodynamics:
\[
dE = T_+dS_+ + \Omega_+dJ + \phi_+dQ.
\]  
(12)
The cosmological horizon has associated thermodynamic quantities
\[
T_c = -\frac{1}{4\pi} \frac{\Delta_\gamma'(r_+)}{r_+^2 + a^2} = \frac{3r_+^4 + r_+^2(a^2 - 3/A) + (a^2 + q^2)3/A}{4\pi r_+(r_+^2 + a^2)^2/A},

S_c = \frac{\pi(r_+^2 + a^2)}{\Xi}, \quad \Omega_c = \frac{-a\Xi}{r_+^2 + a^2},

J = \frac{Ma}{\Xi}, \quad Q = \frac{q}{\Xi}, \quad \phi_c = \frac{-qr_c}{r_+^2 + a^2}.
\]  
(13)
The Balasubramanian, de Boer and Minic (BBM) mass of KNdS is
\[
\tilde{E} = -\frac{M}{\Xi} = \frac{(r_+^2 + a^2)(r_+^2 - 3/A) - q^23/A}{2\Xi r_+3/A}.
\]  
(14)
The quantities obtained above of the cosmological horizon also satisfy the first law of thermodynamics:
\[
d\tilde{E} = T_c dS_c + \Omega_c dJ + \phi_c dQ.
\]  
(15)

3 Klein-Gordon equation

In curved spacetime, the Klein-Gordon equation of a charged particle is
\[
\frac{1}{\sqrt{-g}} \left( \frac{\partial}{\partial x^\mu} - ieA_\mu \right) \sqrt{-g} g^{\mu\nu} \left( \frac{\partial}{\partial x^\nu} - ieA_\nu \right) \Phi - \mu_0^2 \Phi = 0,
\]  
(16)
where \( \mu_0 \) is the mass of the scalar particle, \( e \) is the charge of the particle. Substituting (11) and (13) into (16), we have
\[
\frac{1}{\Delta_\gamma \Delta_\theta} \left[ \Delta_\theta (r_+^2 + a^2)^2 - \Delta_\gamma a^2 \sin^2 \theta \right] \frac{\partial^2 \Phi}{\partial \theta^2} + 2 \frac{eqr_+}{\Delta_r} \left[ a \Delta_\gamma \theta + (r_+^2 + a^2) \frac{\partial}{\partial r} \right] \Phi

- \frac{\Xi^2}{\Delta_r \Delta_\theta \sin^2 \theta} \left( \Delta_r - \Delta_\theta a^2 \sin^2 \theta \right) \frac{\partial^2 \Phi}{\partial \varphi^2}

\frac{2\Xi a}{\Delta_r \Delta_\theta} \left[ \Delta_\theta (r_+^2 + a^2) - \Delta_r \right] \frac{\partial^2 \Phi}{\partial r \partial \varphi} - \frac{\partial}{\partial r} \Delta_r \frac{\partial \Phi}{\partial r}

- \frac{1}{\sin \theta \Delta_\theta} \left( \sin \theta \Delta_\gamma \right) \frac{\partial \Phi}{\partial \theta} \left[ -\mu_0^2 \rho^2 + e^2 q^2 r_+^2 \frac{1}{\Delta_r} \right] \Phi. \]  
(17)
Separating variable \( t \) and \( (r, \theta) \), and let
\[
\Phi(t, r, \theta, \varphi) = e^{-i\omega t} e^{im\varphi} \psi(r, \theta),
\]  
(18)
can be reduced to
\[
-\omega^2 \frac{1}{\Delta_r \Delta_\theta} \left[ \Delta_\theta (r_+^2 + a^2)^2 - \Delta_r a^2 \sin^2 \theta \right] \psi

+ 2 \frac{eqr_+}{\Delta_r} \left[ (r_+^2 + a^2) \omega - a \Xi m \right] \psi

+ m^2 \frac{\Xi^2}{\Delta_r \Delta_\theta \sin^2 \theta} \left( \Delta_r - \Delta_\theta a^2 \sin^2 \theta \right) \psi

+ \frac{2\Xi a}{\Delta_r \Delta_\theta} \left[ \Delta_\theta (r_+^2 + a^2) - \Delta_r \right] \psi - \frac{\partial}{\partial r} \Delta_r \frac{\partial \psi}{\partial r}

- \frac{1}{\sin \theta \Delta_\theta} \left( \sin \theta \Delta_\gamma \right) \frac{\partial \psi}{\partial \theta} \left[ -\mu_0^2 \Sigma + e^2 q^2 r_+^2 \frac{1}{\Delta_r} \right] \psi. \]  
(19)
Let \( \psi = \chi(\theta)R(r) \), we have
\[
\frac{1}{\sin \theta \Delta_\theta} \left( \sin \theta \Delta_\gamma \right) \frac{\partial \chi(\theta)}{\partial \theta} = \left[ \frac{1}{\Delta_\gamma} \left( \omega a \sin \theta - \frac{m\Xi}{\sin \theta} \right)^2 + \mu_0 a^2 \cos^2 \theta - \lambda \right] \chi(\theta),
\]  
(20)
where \( \lambda \) is separation variable constant, \( \omega \) is the energy of the radiation particle, \( m \) is the projection of the angular momentum of the radiation particle on the rotation axis. \( e \) is the charge of the radiation particle. Letting \( K = (r_+^2 + a^2) \omega - am\Xi - eqr_+ \), Eq. (21) can be reduced to
\[
\lambda + \mu_0^2 r_+^2 - \frac{1}{\Delta_r} \left[ \omega (r_+^2 + a^2) - am\Xi - eqr_+ \right] R(r).
\]  
(21)
4 Tortoise coordinate transformation

Now we introduce the tortoise coordinate $r_*$ through the following equations

$$ dr_* = \frac{1}{\Delta_r} (r^2 + a^2) dr, $$

$$ \frac{d}{dr} = \frac{r^2 + a^2}{\Delta_r} \frac{d}{dr_*}, $$

$$ \frac{d^2}{dr^2} = \left(\frac{r^2 + a^2}{\Delta_r}\right)^2 \frac{d^2}{dr_*^2} $$

$$ + \frac{2Q^2 r - Mr^2 + Ma^2 + (r^2 + a^2)^2 \alpha r/3}{\Delta^2} \frac{d}{dr_*}. $$

Substituting (23) into (22), we have

$$ (r^2 + a^2) \frac{d^2 R(r)}{dr_*^2} + 2r \Delta_r \frac{dR(r)}{dr_*} = \left[\Delta_r (\lambda + \mu_\omega r^2) - K^2\right] R(r). $$

Near the black hole horizon $\Delta_r(r_+) \to 0$, so (24) can be reduced to

$$ \frac{d^2 R(r)}{dr_*^2} + (\omega - \omega_0)^2 R(r) = 0, $$

where $\omega_0 = \mu \Omega + e\phi$. The solution of (25) is

$$ R = e^{\pm i(\omega - \omega_0)r_*}. $$

Considering time factor, near the black hole horizon $r_+$, this solution is

$$ \Psi_{out} = e^{-i\omega t \pm i(\omega - \omega_0)r_*}. $$

Letting $\hat{r} = \frac{\omega - \omega_0}{r_*}$, on the black hole horizon surface we derive the ingoing wave solution

$$ \Psi_{in} = e^{-i\omega (t+\hat{r})} = e^{-i\omega v}, $$

and outgoing wave solution

$$ \Psi_{out}(r > r_+) = e^{-i\omega (t-\hat{r})} = e^{-i\omega v} e^{2i\omega \hat{r}} = e^{-i\omega v} e^{2i(\omega - \omega_0)r_*}. $$

where $v = t + \hat{r}$ is Eddington-Finkelstein coordinate. Because of $\frac{dr}{\Delta_r} = \frac{dr_*}{r_*^2 + a^2}$, near the black hole horizon surface $r_+$, we have

$$ \ln(r - r_+) = \frac{1}{r_*^2 + a^2} \frac{d\Delta_r}{dr} \bigg|_{r=r_+} = 2\kappa_+ r_*, $$

where

$$ \kappa_+ = -\frac{3r_+^4 + r_+^2 (a^2 - 3/\Lambda) + (a^2 + q^2)3/\Lambda}{r_+ (r_+^2 + a^2)3/\Lambda}, $$

$\kappa_+$ is gravitational acceleration on the black hole horizon surface $r_+$. From (30), we have

$$ (r - r_+) = \exp(2\kappa_+ r_*), $$

and the outgoing wave is rewritten as

$$ \Psi_{out}(r > r_+) = e^{i\omega v}(r - r_+)^\frac{1}{\kappa_+} (\omega - \omega_0). $$

Obviously, Eq. (33) has singularity on horizon surface $r_+$, and can only describe outgoing particles outside horizon $r_+$. It can not describe the outgoing particles on the horizon.

5 Analytic extension

We are interested in outgoing wave when the black hole radiation is studied. From (33), the outgoing wave is singular at $r = r_+$, we can extend $\Psi_{out}$ from the outside of the black hole into the inside of the black hole. We take the singularity $r = r_+$ as the center of a circle, and take $|r - r_+|$ as radius. By analytical continuation rotating $-\pi$ through the lower-half complex $r$ plane, we have

$$ (r - r_+) \to |r - r_+| e^{-i\pi} = (r_+ - r) e^{-i\pi}. $$

So we obtain the outgoing wave in the horizon surface $r_+$,

$$ \Psi_{out}(r < r_+) = e^{i\omega v} (r - r_+) \frac{1}{\kappa_+} (\omega - \omega_0) \frac{1}{\kappa_+} (\omega - \omega_0) $$

$$ = e^{i\pi (\omega - \omega_0)/\kappa_+} e^{-i\omega v} e^{2i(\omega - \omega_0)r_*}. $$

Eqs. (35) and (29) describe the outgoing wave of outside and inside of black hole, respectively. So, for outgoing wave of particle with energy $\omega$, charged $e$ and angular momentum $m$, the outgoing rate at the horizon surface is given by

$$ \Gamma_+ = \left| \frac{\Psi_{out}(r > r_+)}{\Psi_{out}(r < r_+)} \right|^2 = e^{-2\pi (\omega - \omega_0)/\kappa_+}. $$

Near the the cosmological horizon $\Delta_c(r_c) \to 0$, by the same method, we can solve equation (25) and derive the outgoing wave of particle with energy $\omega$, charged $e$ and angular momentum $m$, the outgoing rate at the cosmological horizon surface is given by

$$ \Gamma_c = \left| \frac{\Psi_{out}(r > r_c)}{\Psi_{out}(r < r_c)} \right|^2 = e^{2\pi (\omega - \omega_c)/\kappa_c}, $$

where $\omega_c = m \Omega_c + e \phi_c$,

$$ \kappa_c = \frac{3r_c^4 + r_c^2 (a^2 - 3/\Lambda) + (a^2 + q^2)3/\Lambda}{r_c (r_c^2 + a^2)3/\Lambda}, $$

$\kappa_c$ is gravitational acceleration on the cosmological horizon surface.
6 Radiation spectrum

According to the above discussion, we obtain that the total energy, angular momentum and charge of the spacetime are respectively \( M/\Xi + \omega, J + m \) and \( Q + e \) \cite{40}. However, between the black hole event horizon and the cosmological horizon the energy, angular momentum and charge of the radiation particles are respectively \( \omega, m \) and \( e \). Before radiation the energy of the black hole is \( M/\Xi + \omega \), the charge is \( Q + e \), and angular momentum is \( J + m \). So we can take the process of the black hole that radiates particles as the process that the Kerr-Newman-de Sitter black hole transfers from the initial state \( (M/\Xi + \omega, \gamma + e) \) to the final state \( (E, \gamma + Q, \omega + J) \).

Now we replace the energy, angular momentum and charge of the radiation particles with the parameters of the Kerr-Newman-de Sitter black hole. This result embodies the reaction of the radiation to spacetime. When parameters of the Kerr-Newman-de Sitter black hole are used, we must guarantee that the total energy, angular momentum and charge of spacetime are all conserved. That is

\[-\omega = \Delta E, \quad \omega = \Delta E, \quad -e = \Delta Q, \quad -m = \Delta J, \]

where \( \Delta E \) or \( \Delta E \), \( \Delta Q \) and \( \Delta J \) is the change of energy, charge and angular momentum of the black hole event horizon and cosmological horizon before and after radiation, respectively. Substituting \( 39 \) and \( 12 \) into \( 36 \), we obtain the outgoing rate of the outgoing wave on the black hole horizon

\[ \Gamma_+ = e^{\Delta S_+}. \]

Substituting \( 39 \) and \( 15 \) into \( 30 \), we derive the outgoing rate of the outgoing wave on the cosmological horizon surface

\[ \Gamma_+ = e^{\Delta S_+}. \]

Taking the black hole as a thermodynamic system, we discuss the radiation rate that this thermodynamic system radiates particles with energy \( \omega \), angular momentum \( m \) and charge \( e \). According to the discussion, when we do not consider the radiation of cosmological horizon, the emission rate that the black hole event horizon radiates particles with energy \( \omega \), angular momentum \( m \) and charge \( e \) is \( 40 \). After the black hole event horizon radiates particles with energy \( \omega \), angular momentum \( m \) and charge \( e \), from \( 15 \), the change of the entropy corresponding to the cosmological horizon is \( \Delta S_+ \). So we can take \( 40 \) as the probability of the change of the entropy corresponding to the cosmological horizon caused by radiating particles from the black hole horizon. By the same method we can take \( 41 \) as the probability of the change of the entropy corresponding to the black hole horizon caused by radiating particles from the cosmological horizon. Thus for the black hole event horizon because of radiating particles with energy \( \omega \), angular momentum \( m \) and charge \( e \) there are two way that cause the change of the entropy \( \Delta S_+ \).

One way is the black hole radiates particles with energy \( \omega \), angular momentum \( m \) and charge \( e \). The probability is given by \( 40 \). The other is the cosmological horizon radiates particles with energy \( \omega \), angular momentum \( m \) and charge \( e \). And the probability is given by \( 41 \). So for the black hole event horizon because of radiating particles with energy \( \omega \), angular momentum \( m \) and charge \( e \), the probability that entropy change is \( \Delta S_+ \)

\[ \Gamma = \Gamma_+ \Gamma_\gamma = e^{\Delta S_+ + \Delta S_\gamma}. \]

It is known that the radiation spectrum of Kerr-Newman-de Sitter black hole is related not only to the change of the entropy of the black hole horizon but also to the one of the cosmological horizon. And the radiation spectrum satisfies the unitary principle.

7 Conclusion and Discussion

For de Sitter, there is radiation not only from the black hole event horizon but also from the cosmological horizon. Refs.\cite{31,41-43} took the black hole event horizon and the cosmological horizon as two independent horizons, when they studied the quantum tunneling of those spacetimes. They discussed the radiation spectrums respectively and did not consider the relation between the black hole event horizon and the cosmological horizon.

We extend the classical Damour-Ruffini method and discuss Hawking radiation spectrum in the Kerr-Newman-de Sitter (KNdS) black hole under the condition that the total energy, angular momentum and charge of spacetime are conserved. We discuss the particle radiation with arbitrary angular momentum. After considering the reaction to spacetime from radiation particles, the radiation spectrum of the Kerr-Newman-de Sitter black hole is related not only to the entropy change of the black hole horizon but also to the entropy change of the cosmological horizon. The black hole radiation spectrum satisfies the unitary principle. Since

\[ \Delta J = \Delta (Ma/\Xi^2) = -m, \]

where \( m \) takes only zero or integer values, the black hole angular momentum \( J \) is quantized. From \( 13 \), the change of \( m \) is determined not only by the change of \( M \) but also the change of the black hole rotating parameter \( a \). The research on axisymmetric black hole in Refs.\cite{44,45} is under the condition that \( a \) is constant.

When only consider the tunneling radiation of the black hole event horizon, we adopt the same method to Ref.\cite{34}. Starting from Damour-Ruffini method, we discuss the outgoing wave of particles with energy \( \omega \), charge \( e \) and angular momentum \( m \). Our result is \( 11 \) \( (13 \) in Ref.\cite{34}). In Ref.\cite{34} the radiation process was taken as a integration process. Summing up the energy of radiation particles, they derived that the radiation spectrum departed from the black body spectrum. Under the condition that the
Because the Kerr-Newman-de Sitter black hole has the event horizon and the cosmological horizon, and the state parameters that describe two horizons are the same, so the radiations of two horizons are correlative. When we discuss the radiation spectrums of those black holes, we must consider the relevance between the black hole event horizon and the cosmological horizon. In this paper, we consider the radiation spectrums of two correlative horizons.

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