QUANTITATIVE STELLAR SPECTRAL CLASSIFICATION. III. SPECTRAL RESOLUTION

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1. INTRODUCTION

In a previous work, Stock & Stock (1999, hereafter Paper I) developed a quantitative method to obtain stellar physical parameters such as absolute magnitude, intrinsic color, and a metallicity index. These parameters were calculated by the use of pseudo-equivalent widths of absorption features in stellar spectra by means of polynomials. For this purpose, the spectral library of Jones (1999, available at ftp://ftp.noao.edu/catalogs/coudelib) was used which contains about 700 stellar spectra in the bands 3820-4500 Å and 4780-5450 Å with a resolution FWHM of 1.8 Å. The calibration (i.e. the determination of the coefficients of the polynomials) was made using 487 stellar spectra of A-K types. In such spectra, the definition of the true continuum is imprecise. For this reason, in Paper I, the authors worked with pseudo-equivalent widths for the identified features. This was done by selecting one inner and two outer regions for each absorption line. The two outer regions (one on each side of the line) were used to determine the pseudo-continuum, while the integration of the inner region allows us to know the pseudo-equivalent width. This procedure is similar to that used by Worthey et al. (1994).
One of the main goals of the quantitative classification method is its application to a large number of stellar spectra, obtained by means of objective-prism observations, which are a kind of low resolution spectra. Therefore, it is essential to simulate low-resolution spectra. This may be done by applying a mathematical treatment to smooth the spectra of Jones library. Once this is done, it is necessary to re-calibrate the method using those smoothed spectra in order to compare the results with those obtained in Paper I. It is important to know if the method is as accurate as in Paper I when it is used with low resolution spectra.

On the other hand, the equivalent width of a line does not depend, in principle, on the spectral resolution. Considering the way that the pseudo-equivalent widths were defined in Paper I, these could be affected by variations in the spectrum profile. It is also important to analyze the sensitivity of the method to variations in the spectral resolution (due, for instance, to seeing variations during the observations). To do this, first the polynomials are generated applying the method to the smoothed spectra. Then these are applied to spectra of two other resolutions, and the respective results are compared.

A short explanation of how the low-resolution spectra are simulated and how the method is applied to these is given in Sections 2 and 3. Section 4 is dedicated to a discussion of the results, and the main conclusions are summarized in Section 5.

2. LOW-RESOLUTION SPECTRA SIMULATION

The smoothing method consists of taking each pixel and then distributing its intensity among the neighboring pixels using a mathematical function. The spectra of the Jones library were convolved with a Gaussian function, \( \sigma \) (the full width at half maximum) being the smoothing parameter. The number of neighboring pixels used in this process increases as \( \sigma \) increases with the consequent decrease of the spectral details (lower resolution). All the spectra of Jones library were smoothed using 14 different values of \( \sigma \) from 20 to 700.

As was pointed out in Paper I, objective-prism spectra are very useful in galactic structure studies which is the main motivation for the development of this quantitative classification method. Even though the objective-prism spectral dispersion is not a linear function of \( \lambda \) (wavelength), we simulate the low-resolution spectra using a linear relationship. This might be done since the main goal of this work is to analyze the sensitivity of the method to changes in the spectral resolution. Both the pseudo-equivalent widths and the line depths depend on the spectral resolution. Thus, we defined an index as the ratio of these two quantities in order to compare smoothed spectra with real objective-prism spectra. In this way, we could identify which one of the indices better matches the observations. For early-type stars, the hydrogen \( \text{H}\gamma \) and \( \text{H}\delta \) lines were selected whereas for late-type stars the G-band was used.

For the resolutions corresponding to all \( \sigma \) values, these indices (\( \text{H}\gamma \), \( \text{H}\delta \), and G-band) were calculated on typical A0V and G0 stars from the Jones library. Additionally, the same indices were calculated for typical objective-prism spectra of A0V and G0 spectral types obtained by Stock (1997) with the 60 cm Curtis Schmidt telescope at CTIO (Cerro Tololo Interamerican Observatory), using a 4-degree prism and a B filter. As an example, Figure 1 shows the \( \text{H}\gamma \) index for an early-type spectrum taken from the Jones library as a function of both \( \sigma \) (bottom X-axis) and the final smoothed resolution in angstrom (top X-axis). The open square indicates the value of this index for a typical objective-prism spectrum. In this way, we were able to determine that \( \sigma \simeq 250 \) reproduces the resolution (\( \simeq 5 \text{ Å} \)) that best fits objective-prism observations.

In addition to this value, that we will name the “standard resolution”, we chose two other values of \( \sigma \) (150 and 350) to simulate higher and lower spectral resolutions, respectively. The reason for this is to estimate the sensitivity of the method to variations in the resolution. This was carried out by applying the polynomials obtained with the standard resolution to the other two resolutions and comparing the results. As an example, Figure 2 shows the spectra of both an early-type star (HD31295, type A0V) and a late-type star (HD29645, type G0V), taken from the Jones library. The corresponding simulations for \( \sigma = 250 \) (standard resolution) are also showed where the changes in the shape of the line profiles can be seen. Solid circles in this Figure indicate windows defining a given line (in this case \( \text{H}\beta \)) in the smoothed spectra, and the both-sides continuum regions on which we average the fluxes to define the pseudo-continuum. The solid line shows the resulting pseudo-continuum used to calculate the pseudo-equivalent width for this line.

3. APPLICATION OF THE METHOD

The determination of the equivalent widths and the calculation of absolute magnitudes and color indices \((B - V)_0\) were performed by the use of the same codes developed in Paper I. However, the points that define
the pseudo-continuum for each identified feature were properly re-defined for the spectra smoothed with the standard resolution. This was done because the features of the original spectra are very different from those of the smoothed ones as can be seen in Figure 2. The new points and the identified lines are contained in Table 1. For all these lines, the equivalent widths were determined, and polynomials up to the second degree were defined with three lines as independent variables. The stars were divided into groups as in Paper I, i.e. 1=very early stars, 2=early stars, 3=late stars and 4=very late stars. The separation into groups is made in basis of the number of lines falling within certain predefined ranges of equivalent widths, and it does not correspond to any specific spectral types or colors (see Figure 6 in Paper I), therefore the adjectives “early” or “late” must be taken carefully. The range of spectral types is approximately B9-A2 for stars belonging to the group 1, F0-G5 for group 2, G0-K0 for group 3 and G5-M0 for group 4. Nevertheless, group 1 was excluded from the analysis because the number of available spectra was small since we only took into account stars for which the Hipparcos catalogue gives parallaxes with an error less than 20%.

Afterwards, the magnitudes and $(B-V)_0$ colors for spectra with $\sigma = 150$ and 350 were calculated by using the polynomials obtained for each group with the standard resolution spectra. It is important to emphasize here that the polynomials and the coefficients used to calculate absolute magnitudes and colors are, for all the three resolutions, those derived from the standard resolution. This is necessary in order to analyze the sensitivity of the method to the spectral resolution.
| Line | Cont1(Å) | Line(Å) | Cont2(Å) | λ (Å) | Identification          |
|------|----------|---------|----------|-------|-------------------------|
| 1    | 3859.9   | 3863.0  | 3863.0   | 3910.3| Hβ+HeI                  |
| 2    | 3910.3   | 3915.3  | 3915.3   | 3950.2| CaII K                  |
| 3    | 3915.9   | 3950.2  | 3950.2   | 4003.1| CaII H + He             |
| 4    | 4009.3   | 4018.7  | 4018.7   | 4046.7| CaII H + Hε             |
| 5    | 4043.6   | 4056.0  | 4056.0   | 4143.3| FeI                     |
| 6    | 4183.8   | 4205.6  | 4205.6   | 4246.1| FeI                     |
| 7    | 4274.1   | 4280.3  | 4280.3   | 4323.9| G-band                  |
| 8    | 4317.7   | 4321.4  | 4322.7   | 4328.3| FeI, CH                 |
| 9    | 4280.3   | 4317.7  | 4317.7   | 4365.1| Hγ                      |
| 10   | 4358.2   | 4364.4  | 4364.4   | 4411.2| FeI                     |
| 11   | 4420.5   | 4445.4  | 4445.4   | 4473.5| Blend (CaI, MnI, FeI)   |
| 12   | 4819.9   | 4826.1  | 4826.1   | 4904.0| FeI                     |
| 13   | 4904.0   | 4910.2  | 4910.2   | 4947.6| Hβ                      |
| 14   | 4973.6   | 4994.3  | 4994.3   | 5026.6| Blend (FeI, NiI)        |
| 15   | 5028.6   | 5059.7  | 5059.7   | 5090.9| Blend (FeI, FeI)        |
| 16   | 5090.9   | 5159.4  | 5159.4   | 5196.8| MgI                     |
| 17   | 5234.2   | 5243.5  | 5243.5   | 5287.2| Blend (CaI, FeI)        |
| 18   | 5287.2   | 5312.1  | 5312.1   | 5358.8| FeI                     |
| 19   | 5358.8   | 5383.7  | 5383.7   | 5421.1| FeI                     |
Fig. 2. The spectra of HD31295 (left-side panel) and HD29645 (right-side panel) from the Jones library (dashed lines) and the corresponding simulations for $\sigma = 250$ (continuous lines). The black points indicate the ranges defining an example line (H$\beta$) and the continuum regions (see text).

4. ANALYSIS

For the standard resolution ($\sigma = 250$), the best solutions obtained with combinations of three lines are shown in Table 2 ($N_u$ indicates the number of stars used), both for the absolute magnitude and for the $(B - V)_0$ color. We see that absolute magnitudes can be recovered with an average error of about 0.26 magnitudes, and $(B - V)_0$ colors with 0.020 magnitudes for all the groups. Thus, the results obtained with the standard resolution are in agreement with the results obtained in Paper I, the basic difference being that the smoothed spectra simulate properly objective-prism spectra within the assumptions explained in Section 2.

The intrinsic color-magnitude diagram of the stars used in this work is shown in Figure 3. The lengths of the arrows are the differences between the values of the physical parameters determined in Paper I, using the full-resolution spectra, and those determined in the present work for the same points, but using the spectra smoothed with the standard resolution. As reference, the arrows in the box indicate the mean rms of the method for $M_V$ and $(B - V)_0$. Generally, the lengths of the arrows are small, except for a few late-type stars where the solution is not very robust because of the lack of data in that region.

For each group of stars, the effect of spectral resolution variations was analyzed by comparing the obtained stellar parameters for the spectra smoothed with different values of $\sigma$. As was pointed out before, the same polynomial should be used for all the smoothed spectra and for this reason we chose the polynomial obtained for the standard resolution with the best combination of lines (see Table 2). Figure 4 is a color-magnitude diagram for the stars belonging to group 2. In this case, the origin of each arrow is given by the magnitudes and intrinsic colors obtained with the standard resolution ($\sigma = 250$), whereas the lengths of the arrows are given by the differences between the values of the physical parameters calculated for the spectra smoothed with $\sigma = 350$ and $\sigma = 250$. A trend to decrease $(B - V)_0$ when the spectral resolution is decreased can be noted (i.e. when $\sigma$ is increased), although the changes in the intrinsic color are always smaller than the mean error of the adjustment of the applied polynomial. In fact, the averaged value of the difference of colors (between both resolutions) is -0.008 magnitudes with a rms of 0.008 magnitudes, whereas the mean rms resulting from the application of the method is, as mentioned above, 0.020 magnitudes. The observed tendency in the absolute magnitude is that $M_V$ gets brighter as the resolution decreases. The mean value of the differences in magnitude is -0.15 magnitudes (with a rms of 0.06 magnitudes) which is smaller than the mean rms of the solution (0.26
### Table 2

**Best Combinations for the Determination of the Stellar Parameters with the Standard Resolution**

#### Absolute Magnitude

|       | Group 2       |   | Group 3       |   | Group 4       |   |
|-------|---------------|---|---------------|---|---------------|---|
|       | L1  | L2  | L3  | rms | N_u | L1  | L2  | L3  | rms | N_u | L1  | L2  | L3  | rms | N_u |
| 1     | 3   | 16  |     | 0.273 | 153 | 1   | 8   | 12  | 0.256 | 143 | 1   | 2   | 18  | 0.263 | 115 |
| 2     | 4   | 16  |     | 0.274 | 144 | 1   | 9   | 16  | 0.222 | 148 | 1   | 3   | 14  | 0.248 | 116 |
| 3     | 10  | 16  |     | 0.288 | 153 | 1   | 12  | 16  | 0.286 | 155 | 1   | 9   | 14  | 0.222 | 114 |
| 4     | 13  | 16  |     | 0.274 | 152 | 2   | 6   | 8   | 0.223 | 143 | 2   | 3   | 16  | 0.254 | 123 |
| 5     | 16  | 19  |     | 0.273 | 154 | 2   | 6   | 16  | 0.237 | 159 | 2   | 5   | 16  | 0.245 | 119 |
| 6     | 14  | 16  |     | 0.253 | 155 | 2   | 9   | 16  | 0.277 | 162 | 2   | 6   | 10  | 0.264 | 121 |
| 7     | 14  | 16  |     | 0.273 | 156 | 5   | 7   | 16  | 0.269 | 163 | 2   | 7   | 16  | 0.243 | 121 |
| 8     | 16  | 19  |     | 0.292 | 153 | 5   | 9   | 16  | 0.264 | 157 | 2   | 12  | 16  | 0.246 | 120 |
| 9     | 14  | 16  |     | 0.258 | 155 | 5   | 13  | 16  | 0.256 | 158 | 2   | 13  | 16  | 0.254 | 117 |
| 10    | 18  | 19  |     | 0.286 | 155 | 7   | 9   | 16  | 0.282 | 165 | 3   | 7   | 16  | 0.260 | 124 |

#### Intrinsic Color \((B - V)_0\)

|       | Group 2       |   | Group 3       |   | Group 4       |   |
|-------|---------------|---|---------------|---|---------------|---|
|       | L1  | L2  | L3  | rms | N_u | L1  | L2  | L3  | rms | N_u | L1  | L2  | L3  | rms | N_u |
| 1     | 3   | 14  |     | 0.019 | 143 | 3   | 5   | 7   | 0.018 | 157 | 1   | 2   | 3   | 0.023 | 118 |
| 2     | 4   | 7   |     | 0.018 | 143 | 3   | 5   | 7   | 0.018 | 157 | 2   | 3   | 10  | 0.023 | 118 |
| 3     | 5   | 6   |     | 0.018 | 143 | 4   | 5   | 6   | 0.022 | 161 | 2   | 10  | 19  | 0.024 | 114 |
| 4     | 6   | 10  |     | 0.015 | 136 | 5   | 7   | 13  | 0.021 | 157 | 3   | 4   | 13  | 0.024 | 124 |
| 5     | 8   | 9   |     | 0.018 | 142 | 5   | 7   | 17  | 0.019 | 153 | 3   | 5   | 6   | 0.018 | 120 |
| 6     | 2   | 7   |     | 0.019 | 144 | 5   | 7   | 19  | 0.019 | 152 | 3   | 5   | 7   | 0.022 | 122 |
| 7     | 8   | 9   |     | 0.018 | 144 | 5   | 8   | 12  | 0.019 | 152 | 3   | 5   | 12  | 0.021 | 121 |
| 8     | 11  | 12  |     | 0.018 | 145 | 5   | 8   | 14  | 0.021 | 155 | 3   | 5   | 15  | 0.020 | 120 |
| 9     | 14  | 16  |     | 0.018 | 144 | 5   | 10  | 16  | 0.019 | 153 | 3   | 6   | 10  | 0.022 | 121 |
| 10    | 14  | 17  |     | 0.019 | 145 | 5   | 14  | 19  | 0.022 | 159 | 3   | 10  | 17  | 0.022 | 119 |

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magnitudes). Thus, even when a systematic behavior is observed both in magnitude and color, this effect is not significant.

Figure 5 shows a color-magnitude diagram for the stars of the same group (2), but now the stellar parameters determined for the spectra smoothed with $\sigma = 250$ (standard resolution) and $\sigma = 150$ (higher resolution) are compared. As the resolution increases, $(B - V)_0$ increases, but an additional effect is also observed: generally, the increase in the color is higher for the later-type stars of this group. However, the mean value of the differences in color between both resolutions is 0.028 magnitudes (with a rms of 0.014 magnitudes), which is slightly higher than the rms of the method. The $M_V$ magnitude gets fainter as resolution increases, and in this case the differences in $M_V$ between both resolutions is about 0.07 magnitudes with a rms of 0.06 magnitudes (below the mean error of the method).

Therefore, from Figures 4 and 5 we conclude that for this group of stars, both the $(B - V)_0$ colors and the $M_V$ magnitudes tend to decrease when the resolution of the spectra decreases. This systematic behavior could be taken into account to correct it. Nevertheless, we have seen that these changes are of the order or less than the error with which the stellar parameters can be recovered by the use of this method. Thus, the effects of variations in the spectral resolution can be neglected, at least when the method is applied to stars belonging to the second group.

Regarding stars belonging to the group 3, the color-magnitude diagram comparing the results obtained with $\sigma = 250$ with those obtained using $\sigma = 350$ is shown in Figure 6. The corresponding comparison with
Fig. 4. Color-magnitude diagram obtained from the calibration with $\sigma = 250$ (standard resolution), for the stars of group 2. The length of each arrow is the difference between the stellar parameters calculated with $\sigma = 250$ and the parameters calculated with $\sigma = 350$.

$\sigma = 150$ is shown in Figure 7. The effect of variations in the resolution is more perceivable in the stars of this group because of the nature of this kind of spectra. It can be noted that there is no systematic effect when the resolution is changed, i.e. some stars increase and other decrease $(B-V)_0$ (similarly for $M_V$). The averaged value of the difference between the color determined with a given $\sigma$ value and the color determined for $\sigma = 250$ are -0.026 mag (rms = 0.021 mag) for $\sigma = 350$, and 0.005 mag (rms = 0.035 mag) for $\sigma = 150$. For the absolute magnitudes, the results show that the averages of the differences were -0.09 mag (rms = 0.38 mag) and 0.09 mag (rms = 0.47) for $\sigma = 350$ and 150, respectively. Thus, for this group of stars, we see that variations in the spectral resolution produce changes in color and magnitude which are of the order or even higher than the mean error of the adjustment of the polynomial.

The differences between the colors and magnitudes determined by using the spectra of various resolutions become too high for very late stars (group 4), rising up to 3 magnitudes for the absolute magnitude and 1 magnitude for the $(B-V)_0$ color. Obviously, this is due to the fact that these stars have spectra with a great variety of features and details, and the pseudo-continuum of every absorption line cannot be defined without ambiguity. In this case, the method described in Paper I is not suitable to obtain the stellar parameters with the desired accuracy, and another approach is required.
5. CONCLUSIONS

There are two main conclusions drawn from the present study. First, the application of the quantitative classification method (Paper I) to spectra smoothed with the standard resolution ($\sigma = 250$), which simulates typical objective-prism spectra, yielded results in good agreement with the previous ones: it was possible to derive absolute magnitudes with an average error of 0.26 magnitudes and $(B-V)_0$ colors with an error of 0.020 magnitudes. This method can then be applied to objective-prism observations with high accuracy.

Second, the results indicate that changes in the resolution of the spectra will not affect the determination of the measurable parameters for the stars belonging to group 2 (earlier-type stars). Concerning to very early stars (spectral types O-B), the continuum can be easily determined and the variations in the spectral resolution are not a problem, neither for the calibration nor for the derivation of the fundamental parameters (see Stock et al. 2002). But, for later-type stars (groups 3 and 4), it is necessary to apply a different approach because the shape of these spectra is very sensitive to resolution variations. We propose that this group should be divided into subgroups in order to reduce the differences between the parameters derived with several spectral resolutions.

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Fig. 6. Color-magnitude diagram obtained from the calibration with $\sigma = 250$ (standard resolution), for the stars of group 3. The length of each arrow is the difference between the stellar parameters calculated with $\sigma = 250$ and the parameters calculated with $\sigma = 350$.

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Fig. 7. Color-magnitude diagram obtained from the calibration with $\sigma = 250$ (standard resolution), for the stars of group 3. The length of each arrow is the difference between the stellar parameters calculated with $\sigma = 250$ and the parameters calculated with $\sigma = 150$. 
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