Quenching Effects in the Hadron Spectrum

C. Allton
Department of Physics, University of Wales Swansea, Swansea SA2 8PP, U.K.

Abstract. Lattice QCD has generated a wealth of data in hadronic physics over the last two decades. Until relatively recently, most of this information has been within the “quenched approximation” where virtual quark–anti-quark pairs are neglected. This review presents a descriptive discussion of the effects of removing this approximation in the calculation of hadronic masses.

1 The Quenched Approximation

In a quantum field theory involving gauge and fermion degrees of freedom, such as QCD, we have the following path integral formalism for the expectation value of a quantity \( \Omega \):

\[
\langle \Omega \rangle = \frac{1}{Z} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}A \ \Omega(\psi, \bar{\psi}, A) \ e^{-S_E(\psi, \bar{\psi}, A)}
\]

(1)

where \( Z \) is the usual path integral and the Euclidean action \( S_E \) is defined in terms of the usual field strength tensor \( F_{\mu\nu} \),

\[
S_E = \int d^4x \left\{ \bar{\psi}(x) \left( \slashed{D} + m \right) \psi(x) + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right\}.
\]

(2)

The gauge degrees of freedom, \( A \), are bosonic, but the fermionic degrees of freedom, \( \psi \) are fermionic and hence anti-commute. These are difficult to deal with in a computer simulation, but fortunately, since they occur as they can be integrated \( \psi \) analytically resulting in the usual determinant factor:

\[
\langle \Omega \rangle = \frac{1}{Z} \int \mathcal{D}A \ \Omega(\psi, \bar{\psi}, A) \det(\slashed{D} + m) \ e^{-S_E(\psi, \bar{\psi}, A)}.
\]

(3)

Simulations of this quantum field theory are performed on a space–time lattice by simply replacing all the continuous derivatives and integrals with finite differences and sums over gauge configurations. Hence, we have on the lattice:

\[
\langle \Omega \rangle = \frac{1}{Z} \sum_{\{U\}} \Omega(\psi, \bar{\psi}, A) \ e^{-S_g det(\slashed{D} + m)}
\]

(4)

The (naive) lattice Euclidean action is

\[
S = S_F + S_g = \sum_x \left\{ \bar{\psi}(\slashed{D} + m) \psi(x) \right\} - \frac{1}{g_0^2} \sum_p Tr(U_p + U_p^\dagger)
\]

(5)
where \( a \) is the lattice spacing, the link variable \( U_\mu(x) \) now carries the gauge degrees of freedom, and \( U_p \) is the trace of the product of link variables around a plaquette,

\[
U_p = U_\mu(x) U_\nu(x + \hat{\mu} a) U_\mu^\dagger(x + \hat{\nu} a) U_\nu^\dagger(x).
\]

(6)

This formalism maintains gauge invariance even on a lattice [1].

Variations of the naive lattice action can be made to improve its convergence to the continuum action in two areas:

- the naive action suffers from fermion doubling – each lattice quark flavour corresponds to \( 2^d \) continuum flavours, where \( d \) is the space–time dimension;
- lattice actions in general suffer from discretisation errors which enter when the continuum derivatives in (2) is replaced by the finite difference in (5).

Two methods are generally used to overcome the first difficulty – the Wilson/clover family of actions, and the staggered action. Both of these actions can be tweaked so that their lattice systematic error (the second difficulty above) are reduced, and then they are termed “improved”.

Simulations using the lattice formalism can be performed by replacing the naive sum in (4) with a Monte Carlo estimate. This introduces a statistical error \( \mathcal{O}(1/\sqrt{N_{cfg}}) \) in the estimate of \( \langle \Omega \rangle \) where \( N_{cfg} \) is the number of configurations in the Monte Carlo sum.

The lattice prescription of formulating a Quantum Field Theory has á priori no model assumptions – it is derived exactly from the full continuum formalism with no approximations. However the parameter values in real computer simulations of lattice QCD are far from their experimental values. This is due to limitations in current computer power! Table 1 lists the values of the parameters in typical lattice simulations along with their experimental values. Thus typical lattice simulations must inevitably rely on some extrapolation of lattice data. Note that the bare lattice gauge coupling, \( g_0 \), is not listed in table 1. This because the information about \( g_0 \) is contained within the \( a \) value, through dimensional transmutation. Our usual intuition about high momentum transfers (short–distance physics) corresponding to the weakly coupled regime (small values of \( g_0 \)) in asymptotically free theories such as QCD, is directly applicable to lattice simulations. So we have \( g_0 \to 0 \) as \( a \to 0 \).

Equations (4 & 5) correctly define the full continuum theory in the limit as the lattice spacing \( a \to 0 \). However, it is extremely expensive to simulate with (4 & 5). Figure 1 shows the estimated cost of lattice calculations as a function of quark mass using the formula in [2] for the “clover” action. (Actually, the horizontal axis of this plot is \( M_{PS} \), but, from the PCAC relation, we have \( M_{PS} \propto \sqrt{m_q} \).) In Fig. 1 we have assumed: (i) a lattice spacing of \( a = 0.1 \text{ fm} \); (ii) a lattice volume of \( L^4 = (3 \text{ fm})^4 \); and (iii) that there are \( N_{cfg} = 200 \) configurations in the ensemble sum in (4). (These are very conservative assumptions!) As can be seen, for even modest values of \( M_{PS} \approx \frac{1}{2}M_K \sim 250 \text{ MeV} \), full simulations require Tera-scale computing.

Recent advances in lattice actions, e.g. using an improved staggered action, have meant that CPU requirements are not quite so pessimistic [3].
Table 1. Typical parameter values in current lattice simulations

| parameter | typical value | experimental value |
|-----------|---------------|--------------------|
| $m_q$     | $\gtrsim m_s/2 \approx 50 \text{ MeV}$ | $m_u, m_d \approx 5 \text{ MeV}$ |
| $a$       | $0.05 - 0.20 \text{ fm}$ | 0 |
| $L$       | $2 - 4 \text{ fm}$ | $\infty$ |
| $N_{cfg}$ | $\mathcal{O}(100)$ | $\infty$ |
| $N_f$     | 0, 2 or 2 + 1 | “2+1” |

For this reason, [4] introduced the “quenched” approximation where the true QCD vacuum is replaced with one with no quarks present (i.e. $N_f = 0$ in Table 1). Specifically the quenched approximation is defined as follows:

- $\det(\Delta + m)$ is replaced by unity, thereby removing the quark–anti-quark loops from the vacuum configurations;
- the coupling $\beta = 6/g_0^2$ is shifted to try to counteract (as much as is possible) the removal of these $q - \bar{q}$ pairs. Typically this shift is of the following order $\beta^Q \approx \beta^{full} + 0.6$ where $\beta^{Q,full}$ refer to the coupling in the quenched and full theories.

In this way the quenched approximation can be viewed as an effective field theory, i.e. it contains a subset of all the interactions, and the couplings of the quenched theory have to be tuned to take care of these missing diagrams. Figure 2 shows the diagrams which are present in both the quenched and full theory, and those which are present only in the full theory.

Quenched simulations are several orders of magnitude faster than full (unquenched) simulations, and full simulations have only been performed in earnest in the last 5 years or so. Typical statistical and systematic errors of state-of-the-art full simulations are of the same order now as quenched simulations’ errors were a decade ago. Particle physics is primarily concerned with the comparison of theory with experiment, and when theoretical calculations have inherent errors, it is crucial to understand and quantify their scale.

The main aim of this chapter is to determine the systematic effect introduced in the hadron spectrum by the quenched approximation. We will find that uncovering quenching effects is more difficult than one would first imagine for two reasons:

- the quenched approximation proves to be surprisingly successful for many hadronic quantities, i.e. it reproduces much of the hadron spectrum at the 5–10% level. Assuming that QCD is the theory of the strong interaction implies that removing this approximation makes a relatively small effect!
- current full simulations have statistical errors of a few percent (since they are highly cpu-intensive) and so discerning the quenching effects with this relatively noisy data can prove difficult;
While the quenched approximation proves to be unintuitively accurate for many hadronic quantities, there are some quantities where it either fails drastically, or has pathologies when the valence quark masses in the hadrons becomes vanishing. Examples of this include:

- The $\eta$ and $\eta'$ mesons are degenerate in the quenched theory, whereas they not degenerate in full QCD. This is because the quenched theory excludes diagrams involving disconnected $q - \bar{q}$ loops. (See [5] for a description of lattice simulations of $\eta$ and $\eta'$.)
- The chiral limit of quenched QCD suffers from “chiral logs” $\sim \log(m_q/\Lambda_\chi)$ where $m_q$ is the quark mass and $\Lambda_\chi$ is a mass parameter proportional to pion decay constant. These logarithms enter the chiral perturbation theory expansion of various hadronic masses in the quenched theory, spoiling their chiral limit.
- The hyperfine mass splittings in heavy-mesons in the quenched theory is wrong by up to 10% or more, with the sign of the discrepancy depending on the states considered. This systematic error is greatly reduced when the full theory is considered.

We will discuss some of these issues in later sections.

The next section briefly reviews the best current results obtained from the quenched approximation. It outlines the accuracy of this approximation for the hadronic spectrum. Section 3 presents recent results from full (i.e. unquenched) simulations and we attempt to uncover estimates of quenching effects in Sect. 4.

2 Results from the Quenched Approximation

While there have been many papers published using lattice simulations in the quenched approximation we will concentrate on the work of the CP-PACS collaboration who have produced one of the most accurate quenched study of the light hadron spectrum in [7]. Their calculations used an improved clover action simulated at volumes of around $2.5^3 \times 1.12$ with several quark masses and lattice spacings. They are thus able to perform continuum ($a \to 0$) and chiral ($m_q \to m_{u,d}$) extrapolations (see Table 1).

An impressive summary of their quenched spectrum for the light hadrons is shown in Fig. 3 as the open symbols. Their lattice data are shown after the appropriate continuum and chiral extrapolations and is taken from table XV of [7]. As can be seen from the middle panel of Fig. 3 discrepancies between the quenched lattice results and the experimental values are $\mathcal{O}(5 − 10\%)$. It is important to note that this relatively small difference is only discernible due

2 A search on the SPIRES database for “quenched” returns more than 500 papers, and this does not include papers which use quenched simulations but where the authors have not included this word in the paper’s title!

3 Another paper from the CP-PACS Collaboration studies even larger lattice of up to $64^3 \times 112$ but the lattice action employed in this work is the pure Wilson action which has $\mathcal{O}(a)$ errors.
to the tiny errors in the lattice data of $O(1 − 2\%)$. Quenched calculations of an earlier generation \cite{10}, with correspondingly larger errors, were not able to uncover deviations from experimental values.

Figure 3 contains two sets of lattice data points: those obtained from the $K$ and $\phi$ inputs. These refer to the hadron whose mass was used to set the strange quark mass in the lattice calculation. (The $\rho$ mass was used to define the lattice spacing, $a$. ) The fact that there are differences between these two sets of data is itself an indication of the failure of the quenched approximation, i.e. an exact calculation’s result wouldn’t depend on how the scale was set.

The CP-PACS collaboration also find the mass splittings, such as the hyperfine splitting in the meson sector and the splittings in the decuplet (baryon) sector are smaller than experimental values.

A further indication of the failure of the quenched approximation is in the determination of the strange quark mass $m_s$. As mentioned above, this quantity can again be calculated using either the $K−$ or $\phi−$meson as input, but the deviation between the two results is at the $3−4\sigma$ level.

Moving to the heavy–hadron mass spectrum, a recent publication, using an improved staggered action, studied the splittings in the heavy–meson sector \cite{3}. The left–hand plot in Fig. 7 (taken from \cite{3}) shows the quenched predictions of various splittings from \cite{3}. This clearly shows a discrepancy between the quenched results and experimental values. As we will see in Sect. 3.4, this discrepancy disappears when we remove the quenched approximation.

3 Results from Full (unquenched) Simulations

This section will give a flavour of current full QCD lattice simulations by concentrating on the CP-PACS \cite{7} and UKQCD \cite{11} collaborations’ results for the light hadron spectrum, and the work of \cite{3} for the heavy–meson spectrum. Both the CP-PACS and UKQCD collaborations used two flavours of improved clover fermions whereas the collaboration in \cite{3} used an improved staggered action (which has a cpu advantage over the Wilson action, see Sect. 1 and \cite{12}).

Table 2 displays the parameters used in these collaborations’ work. Note that we have differentiated the sea and valence quark masses in this table (c.f. table 1). The sea quarks are those which always appears in quark loops and are not connected to the source/sink operators (e.g. the quark loop in Fig. 2), and the valence quarks are those which enter the source/sink interpolating operators.

As can be seen from table 2 the CP-PACS collaboration have performed QCD simulations at parameter values closer to the experimental values and has larger statistics than the UKQCD collaboration (see also Table 1). However the UKQCD collaboration chose a subset of its parameter values so that the lattice spacing remained fixed as the sea quark mass, $m_{sea}^q$, varied. This meant that $O(a)$ effects could more readily be disentangled from dynamical quark effects. Furthermore, the UKQCD lattice action has the technical advantage that it has no $O(a)$ lattice systematic errors.
In the simulations of \cite{3} extremely light quarks were able to be studied due to the use of the improved staggered action. (This seems to have become the action of choice for most dynamical simulations.) Also \cite{3} simulates with the more physical value of 2+1 quark flavours (see table 1).

### Table 2

| Parameter         | CP-PACS \cite{7} | UKQCD \cite{11} | Davies et al. \cite{3} |
|-------------------|------------------|-----------------|------------------------|
| $m^\text{sea}_q$  | 0.5$m_s - 1.8m_s$ | 0.6$m_s - 2.0m_s$ | 0.17$m_s - 0.5m_s$ |
| i.e. $\mathcal{O}(50 - 180)$ MeV | i.e. $\mathcal{O}(60 - 200)$ MeV | i.e. $\mathcal{O}(17 - 50)$ MeV |
| $m^{\text{val}}_q$ | 0.25$m_s - 2.1m_s$ | 0.6$m_s - 2.4m_s$ | 0.12$m_s - m_s$ |
| i.e. $\mathcal{O}(25 - 210)$ MeV | i.e. $\mathcal{O}(60 - 240)$ MeV | i.e. $\mathcal{O}(12 - 100)$ MeV |
| $a$               | 0.09 – 0.25 fm   | $\sim 0.11$ fm  | 0.09 fm & 0.12 fm |
| $L$               | $\sim 2.5$ fm    | $\sim 1.7$ fm   | $\sim 2.5$ fm         |
| $N_{cf}$          | $\mathcal{O}(1000)$ | $\mathcal{O}(200)$ |
| $N_f$             | 2               | 2               | 2 + 1                 |

Rather than give the full details of the results from these collaboration’s work, a summary is presented in the following. In the next section we attempt to understand the discrepancies between this section’s full QCD results and quenched simulations from Sect. \ref{sect:2}.

#### 3.1 Meson spectrum

In Fig. \ref{fig:4} we plot the vector and pseudoscalar meson mass taken from \cite{7} together with the experimental points. In this figure, the lattice spacing, $a$, was determined from the $K$ and $K^*$ meson masses using the method described in \cite{13}. The huge number of data points corresponds to the fact that there are 16 different ($\beta, m^\text{sea}_q$) combinations in \cite{7} and that there are 9 different valence quark combinations for each of these ($\beta, m^\text{sea}_q$) ensembles. Also plotted are the experimental data points corresponding to the ($\pi, \rho$), ($K, K^*$), and ($\eta_s, \phi$).\footnote{Note that $\eta_s$ is not a physical particle, since there is no pure $s - \bar{s}$ pseudoscalar meson due to mixing with the $u,d$ quarks. The $\eta_s$ mass shown here is defined as $M_{\eta_s}^2 = 2M_K^2 - M_\pi^2$ which follows from the PCAC relationship $M_{P_S}^2 \propto m_q$.}

One of the main points to be taken from this plot is that the systematics involved in lattice simulations are clearly under control. Variations amongst this data in Fig. \ref{fig:4} is $\lesssim 1\%$, while the lattice spacing and sea quark mass vary by around a factor of three: $a \sim 0.09 - 0.25$ fm, and $m^\text{sea}_q \sim 0.5 - 1.8m_s$. A close
analysis of the data has been used to extrapolate away these residual systematic effects in $m_{\text{sea}}$ and $a$ [7] (see also [14]).

Figure 3 also shows the hadronic spectrum, including the three mesons, $K, K^*$ and $\phi$ taken from [7]. These have been obtained by chirally extrapolating data analogous to that in Fig. 4 to the physical quark masses. As can be seen (particularly in the lower panel of Fig. 3) the full QCD simulated results are in very good agreement with experiment.

As mentioned above, the $K, K^*$ mass is used to set the lattice spacing, $a$, in Fig. 4 [13]. This means that the lattice data and the experimental $K, K^*$ point agree by construction. However, the slope of the lattice data is a real lattice prediction. In the next subsection we study this gradient.

### 3.2 $J$–parameter

In this section we analyse the gradient $dM_V/dM_{PS}^2$ of the lattice data. The dimensionless quantity used to study this is defined [15]

$$J = M_V \left. \frac{dM_V}{dM_{PS}^2} \right|_{K, K^*}. \quad (7)$$

Note that we define the experimental value of the $J$–parameter by approximating the derivative in (7) by a finite difference:

$$J_{\text{discrete}} = M_{K^*} \frac{M_{K^*} - M_\rho}{M_K^2 - M_\pi^2}. \quad (8)$$

Therefore the lattice estimate of $J$ is obtained by taking the derivative in (7) w.r.t. variations in $M_{PS}^2(m_{\text{sea}}^0, m_{\text{val}}^0)$ at fixed $m_{\text{sea}}^0$ but varying valence quark mass, $m_{\text{val}}^0$ (i.e. the experimental/physical sea quark mass is clearly fixed!).

The $J$–parameter has been studied in both [7] and [11], but we concentrate here on the analysis in [11]. Figure 5 plots the $J$–parameter from [11]. The $J$–parameter is calculated at each of three $m_{\text{sea}}^0$ values separately. These $J(m_{\text{sea}}^0)$ values are then extrapolated in $m_{\text{sea}}^0 \propto M_{PS}^2$ to the physical point $m_{\text{sea}}^0 \approx 0$. This extrapolated $J$ value is shown as a banded region in Fig. 5.

As can be seen from Fig. 5 the individual $J(m_{\text{sea}}^0)$ values are significantly smaller than the experimental value. However there is a clear trend in $m_{\text{sea}}^0$ which tends towards the experimental value.

### 3.3 Baryons

The vector and pseudoscalar light–meson sector at various quark masses is defined by the plot in Fig. 4. Traditionally, the corresponding plot containing information about the light–baryon sector (specifically the nucleon) is the “Edinburgh” plot where the nucleon mass is plotted against the pion mass (with both masses normalised by the vector meson mass). Figure 6 shows this plot for the CP-PACS [7] and UKQCD collaborations for their unitary data, i.e. where
As can be seen, there is a relatively large spread in the data, but there is a tendency for the data to approach the experimental point as the quark masses decrease.

After this chiral extrapolation is performed, the CP-PACS collaboration [7] obtained the baryonic spectrum seen in Fig. 3. This is an impressive array of data spanning octet and decuplet sectors. As can be seen Fig. 3, the nucleon and $\Delta$ differ from experiment by around 10%, whereas the $\Xi, \Xi^*$ (with quark content $lss$) and particularly the $\Omega$ (with quark content $sss$) are in perfect agreement with experiment. This implies that lattice simulations become more accurate as the strange quark content increases [7].

Since lattice simulations normally have valence quarks which span the mass of the strange (see table 2), the spectrum calculation of baryons containing purely strange valence quarks requires no valence quark chiral extrapolation. However, the level of chiral extrapolation required obviously becomes more and more significant as the light quark (i.e. $u$ and $d$) content of the baryon increases. This suggests that chiral extrapolation procedures need more careful consideration in order to resolve the discrepancy above (see [10]). Note that the authors of [7] themselves argue that this discrepancy could be due to finite volume effects which impinge upon baryons composed of light quarks more than those composed of strange quarks. This could presumably also be a factor, particularly since finite volume effects are likely to increase the mass (which is in the direction of the observed discrepancy in Fig. 3) and be most relevant for the lightest baryons.

### 3.4 Heavy–Meson Mass Splittings

There has been a recent study of the heavy–meson spectrum in [3] which uses 2+1 flavours of quarks, i.e. 2 light degenerate flavours which play the role of the $u$ and $d$ quarks, and one heavier (but still dynamical) quark which plays the role of the $s$ quark. This is obviously closer to the real world than the simulations of the CP-PACS and UKQCD collaborations (see tables 1 & 2).

We reproduce, in Fig. 7 (taken from [3]) a graph showing the ratio of lattice prediction to experiment for some heavy–meson mass splittings. As can be seen, the lattice results in the full theory (right–hand plot) are within 1σ of their experimental values.

### 4 Quantifying Quenching effects

#### 4.1 Hadron Spectrum

In Sects. [2] and [3] we have outlined some results for the hadronic spectrum for both the quenched approximation and full QCD. Comparing these results we note firstly that quenched results are generally within 10% of their experimental value for a wide variety of quantities. This is an unexpectedly good level of agreement which will be discussed later in this section.
Studying the light–meson sector, we note that the full theory is able to accurately reproduce the $K, K^*$ and $\phi$ masses to within around 1%, far better than the quenched theory (see lower panel of Fig. 3). We note, however, that the “chiral” slope, defined via the $J-$parameter, (7), is still several $\sigma$ away from its experimental value at the simulated values of $m_{\text{sea}}$ (see Fig. 5), and that the chiral extrapolation, $m_{\text{sea}} \rightarrow m_{u,d}$ is required to make contact with experiment.

This situation is mirrored in the baryonic spectrum. Figure 3 shows the remarkable prediction from the CP-PACS collaboration of eight baryonic masses. In general terms, the agreement between theory and experiment is enhanced when the quenched approximation is removed. Note also that there is no discrepancy in the full theory between predictions using the $K$ and $\phi$ mesons to set the strange quark mass. The same is not true in the quenched data (see Fig. 3). The level of agreement between the full theory prediction and experiment is most profound for baryons containing the largest strange quark content. We argued in Sect. 3.3 that this could imply that the lattice data at the simulated values of $m_\bar{q}$ (roughly around $m_s$) are correct, but that the chiral extrapolation procedure $m_q \rightarrow m_{u,d}$ is going astray. As can be seen in Fig. 6, which is roughly the baryonic equivalent of Fig. 4, the chiral extrapolation required to reach the $u,d$ quarks is substantial.

Moving to the heavy–meson sector, we summarised in Sect. 3.4 results from [3]. These show excellent agreement between full simulation results and experiment for a variety of quantities, especially splittings in the $\upsilon$ spectrum. A corresponding quenched analysis shows discrepancies of $\sim 10\%$.

4.2 Why is the Quenched Approximation so good?

While there are obvious failures in the quenched approximation’s ability to reproduce the real world, it does much better than naive expectations: one would imagine that removing all $q - \bar{q}$ diagrams from the theory would have a drastic effect on the hadron spectrum. Figure 3 shows that this is not the case. Why then does the quenched approximation perform so well?

One can obtain a handle on this issue by studying the static quark potential (which is the quantum mechanical potential between two infinitely heavy quarks). Figure 5 shows UKQCD results for this quantity for both the quenched and full theory [11]. The curve shown in the graph is the “string model”, $V(r) = e/r + \sigma/r + \text{const}$ [17]. Note that the data is defined to agree in value and slope exactly at $r = r_0$ (the hadronic scale defined in [18]) [11]. A close up of the difference between the lattice potential and the string model at short distances is shown in Fig. 6. As can be seen from Figs. 8 & 9 the discrepancy between the quenched and full theories is negligible across the whole range of $r$ except at very small distances where the deviation is discernible, but small. This implies that only physical quantities particularly sensitive to this short–distance scale will be affected by the quenched approximation. Hadronic states are most sensitive to “medium” distance scales $r \approx r_0 \approx 0.5 \, fm$ where (from Fig. 8) the two theories’ data overlay each other.
Thus the quenched and full theories should agree at the same level as quarks in QCD can be approximated as moving in a static quark potential. This observation presumably has relevance to the age–old question: Why does the (non-relativistic) quark model perform so well?

It is worth noting that, from Sect.\textsuperscript{4} the quenched approximation is defined not just by replacing the quark determinant by unity, but also by renormalising the coupling $g_0$. In fact, if you attempt to perform quenched and full simulations at the same value of $g_0$, then the lattice spacing, $a$ (or equivalently the cut-off $\sim 1/a$) will differ by a factor of around four. This is telling us that the virtual quark loops really are affecting the dynamics of the simulation. The apparent contradiction between this fact, and what we have seen above, i.e. that the quenched approximation reproduces the full theory (at the 5\%–10\% level) is resolved as follows. The lattice only actually predicts dimensionless quantities, normally expressed as, e.g. $M \times a$, where $M$ is some mass. In this way the lattice is able to predict dimensionless ratios of physical quantities only, e.g. $M_1/M_2$. Although switching the quark determinant on and off does directly affect the lattice spacing, and therefore $Ma$, it seems to have little effect on the ratio $M_1/M_2$. In other words the physical prediction from the lattice of $M_1$, which can be obtained from $M_1/M_2 \times M_2^{\text{exp}}$, doesn’t seem to be greatly affected by quenching. This is telling us something remarkable: for a wide variety of hadronic masses (and the static quark potential), the removal of virtual quark loops from the theory can be counter-balanced simply by an adjustment in the coupling, $g_0$.

There is one final reason why quenching has only a modest affect on the hadronic spectrum, compared to other physical quantities. In order to extract a hadron mass from a lattice simulation, the quantity $\Omega = C(t)$ is calculated (see \textsuperscript{4}) where $C(t)$ is a two–point correlation function between hadronic currents. In Euclidean space–time, we have

$$C(t) \rightarrow \mathcal{M}^2 e^{-Mt} \quad \text{as} \quad t \rightarrow \infty$$

where $\mathcal{M}$ is a matrix element between the vacuum and the hadronic ground state. Lets assume that we are performing a quenched calculation of $C(t)$ and that it has a relative error of $\varepsilon$ due to this quenched approximation, i.e.

$$C(t)^Q = C(t)^{\text{full}}(1 + \varepsilon),$$

where $C(t)^Q, C(t)^{\text{full}}$ are the quenched and full correlation functions respectively. Because the mass, $M$, appears in the argument of the exponential, a relatively small adjustment in $M$ can mop up the quenching error $\varepsilon$, whereas a larger relative change would required of the matrix element, $\mathcal{M}$.

Obviously this analysis is a little simplistic but it does illustrate that we can expect quenching errors in matrix elements (such as decay constants) to be larger than in masses.
5 Conclusions

Lattice QCD is an approach to solving field theories, such as QCD, which involves no model assumptions. Given a fast enough computer, the lattice can be used to solve QCD on any finite volume and with any non-zero quark mass resulting in an absolute theoretical prediction of QCD. However, in order to make the problem tractable on current computers, certain parameters of QCD need to take non-physical values (see Table I). The parameter under study in this chapter is the number of quark flavours in the vacuum, $N_f$. Setting $N_f = 0$ is called the quenched approximation and corresponds to ignoring virtual $q - ar{q}$ pairs in the vacuum. The approximation $N_f = 0$ is seemingly a particularly brutal approximation and, furthermore, there is little theoretical guidance as to its effect. Thus we are usually forced to “measure” its effect a posteriori by analysing data from lattice simulations.

In this chapter we have studied the hadronic spectrum with and without the quenched approximation, in particular light mesons and baryons, and heavy–mesons. By comparing quenched data with experimental masses, we have shown that quenching effects in the light–hadron spectrum are relatively small ($5 - 10\%$), with a slightly larger discrepancy in the heavy–meson spectrum. It is only recently that full QCD simulations have been able to produce data with statistical and systematic errors beneath this level. With this new generation of data, we are now able to state that full QCD lattice results have better agreement with experiment than quenched results.

We have outlined some reasons why the quenched approximation is so relatively successful, and we have found evidence that the chiral extrapolation techniques currently being used in full QCD simulations require further consideration.

In the future, more precise calculations with, and without the quenched approximation will surely enhance our understanding of the underlying physics of QCD.

Acknowledgements

The author would like to thank the CSSM in Adelaide for their kind hospitality.

References

1. K.G. Wilson, Phys. Ref. D10 (1974) 2445.
2. H. Wittig, Nucl.Phys.Proc.Suppl. 106 (2002) 197-198, hep-lat/0203021
3. C. T. H. Davies et al., hep-lat/0304004
4. H. Hamber and G. Parisi, Phys. Rev. Lett 47 (1981) 1792, E. Marinari, G. Parisi and C. Rebbi, Phys. Rev. Lett 47 (1981) 1795, D. Weingarten Phys. Lett 109B (1982) 57.
5. C. Michael, Phys. Scripta T99 (2002) 7, hep-lat/0111056
6. S. Sharpe, Phys. Rev. D46 (1992) 3146, C. Bernard and M. Golterman, Phys. Rev. D46 (1992) 853.
7. CP-PACS Collaboration, A.A. Khan et al., Phys.Rev. D65 (2002) 054505, hep-lat/0105015
8. CP-PACS Collaboration, S. Aoki et al., Phys.Rev. D67 (2003) 034503, hep-lat/0206009
9. B. Sheikholeslami and R. Wohlert, Nucl. Phys. B259 (1985) 572.
10. F. Butler, H. Chen, J. Sexton, A. Vaccarino, D. Weingarten Phys.Rev.Lett. 70 (1993) 2849, hep-lat/9212031
11. UKQCD Collaboration, C.R. Allton et al., Phys. Rev. D65 (2002) 054502, hep-lat/0107021
12. C.R. Allton, V. Gimenez, L. Giusti, F. Rapuano, Nucl.Phys. B489 (1997) 427-452, hep-lat/9611021
13. W. Armour, C.R. Allton, D. Leinweber, A.W. Thomas, R. Young in preparation.
14. UKQCD Collaboration, P. Lacock and C. Michael, Phys. Rev. D52 (1995) 5213, hep-lat/9506009
15. A. Thomas et al., ibid.
16. M. Luscher, Nucl. Phys. B180 (1981) 317.
17. R. Sommer, Nucl. Phys. B411 (1994) 839. hep-lat/9310022
Fig. 1. The computer time in Teraflop–years required for a full lattice QCD simulation as a function of pseudoscalar meson mass using the formula for clover actions in [2]. We have assumed (i) a lattice spacing of $a = 0.1 \, fm$; (ii) a lattice volume of $(3 \, fm)^4$; and (iii) that 200 configurations in the ensemble sum in (4) are required. The physical points corresponding to the $\pi$– and K-mesons are shown by vertical lines.
Fig. 2. Diagrams which are present in (a) both quenched QCD and full QCD, and (b) present only in full QCD. The full lines are quarks and the spirals are gluons.
Fig. 3. The light hadron spectrum from CP-PACS [7], tables XV and XII. Both the quenched and full QCD results are shown, together with the experimental value. The two hadrons on the left do not contain strange quarks, whereas the other hadrons do. The lattice spacing was set from the $\rho$-mass. Two methods were used to set the strange quark mass: from (i) the $K$ and (ii) the $\phi$-mass. The results of both definitions are shown. In the middle and lower plots, the relative deviation ($= M_{\text{lattice}}/M_{\text{exp}}$) is shown. The lower plot is a close-up of the middle plot showing the relative deviation for the strange mesons.
Fig. 4. The light meson masses from CP-PACS \[7\] (see also \[14\]). In this figure, we have set the lattice spacing from the method described in \[13\]. Also shown are the experimental points.
Fig. 5. $J$ versus $(M_{PS}^{unitary})^2$ using the approaches described in the text from [11]. $M_{PS}^{unitary}$ is the pseudoscalar meson mass comprising of degenerate valence quarks which are themselves degenerate with the sea quarks. (Note, from PCAC, $(M_{PS}^{unitary})^2 \propto m_{sea}^2$.) The quenched data points have been plotted at $(M_{PS}^{unitary})^2 = 1.3 \text{GeV}^2$ for convenience. The banded region at the left of the graph is the result of the extrapolation $m_{sea}^2 \to 0$ for the full QCD data. The experimental value of $J = 0.48(2)$ is also shown.
**Fig. 6.** The “Edinburgh plot” for selected full QCD data from [7,11]. The lattice data points shown are the unitary points (i.e. $m_q^{\text{valence}} = m_q^{\text{sea}}$). The experimental point is shown, along with the static limit ($m_q \to \infty$).

**Fig. 7.** Heavy–meson mass splitting (together with some light hadron quantities) taken from [3].
Fig. 8. The static quark potential from the UKQCD Collaboration [11]. The parameters $c$ and $k$ refer to a coefficient of an improvement term in the action and the sea quark mass parameter respectively.
Fig. 9. The deviation of the static quark potential in Fig. 8 from the string model.