Numerical Verification of the Elastic-Plastic Menétrey-William Model (M-W-3) for Masonry Shear Walls Made of Calcium Silicate Masonry Units

Radosław Jasiński

1 Department of Building Structures, Silesian University of Technology, Akademicka 5, 44-100 Gliwice, Poland

Radoslaw.Jasinski@polsl.pl

Abstract. This paper is a continuation of the author's papers on numerical verification of masonry structures. It describes the proposed method of validating the elastic and plastic Menétrey-William model (M-W-3) implemented in the ATENA software. Material parameters of the model were determined from triaxial tests on masonry units. This paper presents a practical use of the calibrated numerical model for stiffening walls. Flat, four-node finite elements (2D model) were used with two degrees of freedom for each node. Simulation covered stiffening walls of various length (l = 4.5 m and 2.25 m), height of 2.45 m, and thickness of 0.18 m. Sequential loading in two stages was performed on walls. Precompression stresses σc of 0.1; 0.75 and 1.5 N/mm² were applied in the first stage, and walls were exposed to monotonic horizontal shearing in the second stage. Calculations were performed at the plane stress (σ1≠0, σ2≠0, σ3=0). The obtained calculations were compared with results for unreinforced walls made of calcium silicate masonry units tested under the identical initial-boundary conditions. The relationship between shear stress and non-dilatational strain angle values of cracking and failure stresses were compared. Calculations showed that shear stresses at the time of cracking demonstrated the closest similarity. The biggest differences occurred for deformation at the moment of destruction.

1. Introduction

The numerical implementation of advanced models of materials can be used to analyse phenomena observed in structures. Depending on the adopted strategy of simulation, the linear-elastic, non-linear elastic and post-elastic states of the structure can be followed. A type of the structure (reinforced concrete, steel, wood) determined the type of used material model. Non-linear models of the material implemented with the finite element method are usually used for simulating masonry structures. Some models are based on experimental observations, and differ in constitutive relations and strength criteria. Considering connections between masonry units, mortar and their contact with masonry units [1,2,3,4], the following simulation techniques can be applied: macro-models, mezzo-models and micro-models. The macro-model treats the masonry structure as material having the identical mechanical parameters. Such a model is applied in practical computations for large masonry structures (finite elements include even more than one masonry unit). Another commonly used method is the mezzo-model [5]. And the last commonly used model - the micro-model identifies the masonry structure as a heterogeneous model. The paper presents results from numerical calculations for stiffening walls made of calcium silicate masonry units. The plastic and elastic Menetrey-William
model was used, whose parameters and calibration have been described in the papers [6,7,8]. Numerical calculations for shear walls that are the subject of studies conducted by the author [9] were preceded by the two-stage calibration process (empirical homogenization) performed on smaller models and described in the paper [8]. In stage 1, numerical deep beam micro-models of walls compressed perpendicularly to the plane of bed joints were prepared. Mortar in bed joints of models were replaced with interface elements. Two material models used for brittle materials (rock, concrete) were used for masonry units. The material model applied in calculations for walls in diagonal compression was chosen on the basis of calculations. In stage 2, numerical deep beam micro-models of diagonally sheared walls, in which masonry units were modelled using the material model from stage 1. By performing numerical calculations, selected material parameters of contact elements were verified.

2. Applied material models

2.1. The Menétrey-Willam model of masonry units

The yield surface of Menétrey–Willam [10], used in calculations, is a modified version of the empirical model developed by Hoek and Brown [11] (used for rock description) changed by Weihe [12] who introduced the elliptic function of eccentricity e depending on Lode angle Θ. The used boundary surface was appropriate for adjusting the shape at octahedral section to tests performed on particular materials. The final form of a criterion used in that model was elaborated by Menétrey–Willam [10] who expressed a three-parameter yield surface M-W-3 as follows:

\[ f^W(\hat{e}, \rho, \Theta) = \left( \sqrt{\frac{5}{2}} \frac{\rho}{k(\kappa) f_c} \right)^2 + m \left( \frac{\rho}{\sqrt{6} k(\kappa) f_c} r(\Theta, e) + \frac{\xi}{\sqrt{3} k(\kappa) f_c} \right)^2 - c(\kappa) = 0 \]  

where:

\[ m = \frac{3 (k(\kappa) f_c)^2 - (\lambda_t f_t)^2}{k(\kappa) f_c \lambda_t f_t} \frac{e}{e + 1} \]  

– a parameter equivalent to cohesion,

\[ r(\Theta, e) = \frac{4 (1 - e^2 \cos^2 \Theta + (2e - 1)^2)}{2(1 - e^2 \cos \Theta + (2e - 1) \sqrt{4(1 - e^2 \cos^2 \Theta + 5e^2 - 4e})} \] – elliptical function – Fig. 2,

\[ e = \text{eccentricity of the elliptical function assuming values from the range } e \in (0,5;1,0), \]

\[ f_c, f_t = \text{uniaxial compressive and tensile strength}, \]

\[ \lambda_t \geq 1 \] – scaling parameter for M-W-3 surface.

The behaviour of the material under tension was described using Rankine criterion, the model of rotating smeared cracks or cracks developing in uniform directions, and the exponential function of fatigue. The boundary surface M-W-3 in deviatoric cross-section is composed of three tangential curves along compressive meridians. The Rankine surface is pyramidal due to intersecting planes expressing the condition \( \sigma_k \leq f_1 \). Fig. 1 illustrates symmetrical boundary surfaces. The M-W-3 surface as an elliptical function is shown in Fig. 2. When eccentricity \( e \) is 0.5, the deviatoric cross-section of failure surface is in the shape of an equilateral triangle. For \( e = 1.0 \), curves forming the deviator cross-section take on a shape of circle. A curve, whose shape is similar to ellipse in the zone of biaxial compression values \( \sigma_1 - \sigma_2, \sigma_3 = 0 \), is a track of boundary surface in the plane of principal stresses. In the hydrostatic cross-section, the surface is formed by parabolic meridians intersecting at the tension point corresponding to triaxial tension. The ellipse extreme corresponds to material strength to biaxial compression \( f_{bc} \). Concrete strength to biaxial stress was empirically determined as \( f_{bc} = 1.14 f_c \), and the corresponding eccentricity of elliptical function was \( e = 0.52 \). For masonry units, the majority of tests covered solid brick [13]. The obtained values of solid brick strength to biaxial compression \( f_{bc} \) were within the range 1.02–1.14\( f_c \), and the corresponding eccentricity values were \( e = 0.501–0.511 \).
Figure 1. Relative position of Rankine and M-W-3 surfaces acc. to parameter value \( \lambda_t \): a) view of surfaces in principal stress space, b) axiatoric sections, c) deviatoric sections; 1 – Rankine surface, 2 – M-W-3 surface at \( k = 1 \) (yield strength), 3 – M-W-3 surface at \( k = k_o \) (end of elastic stage).

A temporary shape and location of the M-W-3 surface at the reinforcement phase was defined by the reinforcement function \((\kappa)\), which depended on the reinforcement/weakening parameters. The failure surface M-W-3 expressed that function as a scaling factor of material compressive strength \((f_c)\). The function of reinforcement had the following elliptic form:

\[
k(\kappa) = k_\kappa \left( \varepsilon_{v,t}^p \right) = k_o \left( 1 - k_o \right) \left[ 1 - \left( \frac{\varepsilon_{v,t}^p - \varepsilon_{v,t}^0}{\varepsilon_{v,t}^0} \right)^2 \right],
\]

where \( \varepsilon_{v,t}^p \) is the plastic volumetric strain from the uniaxial compression (the onset of softening), and \( k_o \) is the value defining the initial surface of plasticity that reduces the elastic state (the onset of plasticity). At the end of the process, the function of reinforcement maintained the constant value, and the material came to the weakening phase controlled by the weakening function \((c)\) which simulated decohesion by shifting the yield surface towards the negative part of the hydrostatic axis. For the uniaxial compression, that function had the following form:
where: \( n_1 = \frac{p_t}{p_{tv}} \), \( n_2 = \left( \frac{p_t}{p_{tv}} + 1 \right) k_{tv} \).

Parameter \( t_v \), describing the volumetric strain, controlled the slope of the weakening function. The value of the weakening function \( c \) took the value equal to 1, at the phase of reinforcement, and was equal to 0 at the complete softening phase of the material with decohesion. Parameters adopted for models of walls made of silicate masonry units are presented in Table 1.

![Figure 2. Elliptical curve shape 0.5 ≥ r(\Theta, e) ≥ 1.0](image)

### Table 1. Parameters of the plastic and elastic model used in calculations

| No. | Parameter | Values used in calculations |
|-----|-----------|-----------------------------|
| 1   | Uniaxial compressive strength \( f_b \), N/mm² | 17.7 |
| 2   | Plastic strain under compression \( \varepsilon_{cp} \) | \( 1.72 \times 10^{-3} \) |
| 3   | Uniaxial compressive strength \( f_{bt} \), N/mm² | 1.39 |
| 4   | Initial modulus of elasticity \( E_c \), N/mm² | 8591 |
| 5   | Poisson's ratio \( \nu \) | 0.17 |
| 6   | Fracture energy \( G_f \), MN/m | \( 8.57 \times 10^{-4} \) |
| 7   | Weakening function at tension | -- |
| 8   | Displacement \( w_c \) at tension, m | \( 3.16 \times 10^{-4} \) |
| 9   | Crack spacing \( s_{max} \), mm | 0.5 mm |
| 10  | Coefficient of tensile strength reduction at the weakening phase \( c_{ts} \) | 0 |
| 11  | Model of cracks | -- |
| 12  | Boundary displacement under compression \( w_d \) | 0.05 mm |
| 13  | Reduced compressive strength in the direction parallel to cracks | 0.8 |
| 14  | Stiffness reduction coefficient under shearing \( s_F \) | 20 |
| 15  | Average size of aggregate | 2 mm |
| 16  | Eccentricity of elliptical function \( e \) | \( e = 0.504 \) |
| 17  | Direction of plastic flow \( \beta \) | \( \beta = 0 \) |

2.2. Mohr-Coulomb interface model

Interface elements were used between masonry units to simulate the behaviour at the interface between two materials, in this case masonry units and mortar in bed joints and masonry units in head joints, and the connection between masonry units and the stand elements. Constitutive relations for the flat state regarding horizontal and standard displacements, were expressed in the following way:
\[
\begin{bmatrix}
\tau \\
\sigma
\end{bmatrix} = \begin{bmatrix}
K_u & 0 \\
0 & K_{nn}
\end{bmatrix} \begin{bmatrix}
\Delta u \\
\Delta v
\end{bmatrix},
\]

where: \(\tau\) – shear stress, \(\sigma\) – normal stress.

The failure surface was expressed using the Mohr-Coulomb criterion (Fig. 3) for normal compressive stresses (7) and elliptical cap at the side subjected to tension (8), in the following form:

\[
|\tau| \leq f_{vo} - \tan \alpha \sigma, \text{ when } \sigma \leq 0,
\]

\[
\tau = \tau_0 \sqrt{1 - \left(\frac{\sigma - \sigma_c}{\sigma_c}\right)^2}, \quad \tau_0 = \frac{1}{f_{vo} - 2f_1 \tan \alpha}, \quad \sigma_c = -\frac{f_1^2 \tan \alpha}{f_{vo} - 2f_1 \tan \alpha}
\]

\[
\text{when, } 0 < \sigma \leq f_t.
\]

For normal stress greater than tensile strength, the criterion was expressed as:

\[
\tau = 0, \text{ when } \sigma > f_t.
\]

For shear stresses greater than limit values described by the relationships (7) and (8), the boundary surface was reduced to the residual surface corresponding to the dry friction surface described by the following equations:

\[
|\tau| \leq \tan \alpha \sigma \text{ when } \sigma \leq 0,
\]

\[
\tau = 0 \text{ when } \sigma > f_t.
\]

A cap material was used in the interval of tensile stresses by Mohr-Coulomb criterion replacing straight lines with the ellipse intersecting the axis of normal stresses at the point corresponding to tensile strength \(f_t\). The intersection point between the ellipse and the axis of shear stresses \(\tau\) corresponded to cohesion \(c\), and was tangential at that point to the Mohr-Coulomb line (Fig. 3).

Therefore, parameters of interface elements had to meet the following conditions:

\[
f_t < \frac{f_{vo}}{\tan \alpha}, \quad f_t < f_{vo},
\]

\[
c > 0, \quad f_t > 0, \quad \tan \alpha > 0.
\]

**Figure 3.** Failure criterion used for contact elements: 1 – boundary surface, 2 – residual surface

The relationships shear stress-displacement, and normal stress-displacement until achieving the failure surface by shear stress, was linear defined by the initial shear stiffness \(K_u\) and the initial normal
stiffness $K_{in}$. Table 2 demonstrates parameters of interface elements used in walls made of calcium silicate masonry units and used in calculations.

**Table 2. Parameters of interface elements for calculating walls made of calcium silicate masonry units**

| No. | Parameter                        | Bed joint          | Head joint         |
|-----|----------------------------------|--------------------|--------------------|
| 1   | Normal stiffness $K_{in}$, MN/m  | $3.92 \times 10^6$ | $3.92 \times 10^6$ |
| 2   | Shear stiffness $K_{In}$, MN/m   | $1.67 \times 10^6$ | $1.67 \times 10^6$ |
| 3   | Tensile strength $f_t$, MN/m     | 0.24               | 0                  |
| 4   | Cohesion $f_{co}$                | 0.70               | 0                  |
| 5   | Friction coefficient $\tan \alpha$ | 0.697            | 0.610              |
| 6   | Normal stiffness $K_{in,min}$, MN/m | $3.92 \times 10^4$ | $3.92 \times 10^4$ |
| 7   | Shear stiffness $K_{In,min}$, MN/m | $1.67 \times 10^4$ | $1.67 \times 10^4$ |
| 8   | Fracture energy under shearing $G_{II}$, MN/m | $2.70 \times 10^{-4}$ | - |
| 9   | Displacement $u_{1c}$, m         | $8.45 \times 10^{-4}$ | - |
| 10  | Equivalent displacement $u_{eq}$, m | $5.63 \times 10^{-4}$ | - |

3. Calculated results

Numerical models of whole walls and stand elements were prepared. The model of each masonry units was prepared individually. Also, lintels were modelled in walls with openings. Interface elements were applied at the interface between masonry units. Models were simulated using flat (deep beam), 4-node finite elements of **CCIsoQuad** type, of different mechanical (properly calibrated) and geometrical (thickness) parameters. Figures 4 and 5 compare images of cracks in test elements and numerical models of HOS and HOS-H series without openings [14, 15], made of calcium silicate masonry units, onto which stress states expressed as the parameter of strengthening $k$ or weakening $c$ function (equations (4) and (5)) were overlapped. Values of the areas close to 1 represented the end of the strengthening process and the onset of the weakening process, that is, the point at which the stress path reached the failure surface. Other areas, for which values were lower than 1, represented either the strengthening state or the weakening state.

Regardless of the length, the greatest width of cracks was found in the central area of the numerical model. Cracks related to the destruction of material of masonry units (due to compression) were formed in corner zones as in case of real research models. The direction of cracks with the greatest width overlapped with the diagonal of the model. Table 3 and Fig. 6 show values of cracking and failure stresses and the corresponding values of angles of non-dilatational strain and deformation of all walls.

**Table 3. Results from numerical calculations for unreinforced walls without openings**

| Series $(l \times h \times t)$ | $\sigma_c$, N/mm² | Stresses | Angles of non-dilatational strain |
|-----------------------------|-------------------|----------|---------------------------------|
|                             | $\tau_{cr,cal}$   | $\tau_{cal}$ | $\tau_{u,cal}$ | $\theta_{cr,cal}$ | $\theta_{cal}$ | $\theta_{u,cal}$ |
| HOS (4.5×2.45×0.18m)       | 0                 | 0.059    | 0.86  | 0.107 | 1.01 | 0.172 | 0.98 | 2.05 | 0.96 |
|                             | 0.1               | 0.099    | 0.80  | 0.291 | 0.93 | 0.084 | 0.97 | 2.43 | 0.36 |
|                             | 1.5               | 0.345    | 1.00  | 1.016 | 1.07 | 0.165 | 0.84 | 0.85 | 0.39 |
| HOS-H (2.25×2.45×0.18m)    | 0.1               | 0.233    | 0.97  | 0.779 | 1.85 | 0.360 | 0.97 | 1.852 | 0.57 |
|                             | 0.75              | 0.511    | 0.93  | 0.944 | 1.10 | 0.368 | 1.04 | 1.138 | 0.52 |
|                             | 1.5               | 0.920    | 1.01  | 1.015 | 1.06 | 0.836 | 1.07 | 1.056 | 0.65 |
Figure 4. Comparison of cracking patterns of test elements and numerical models of HOS series walls made of calcium silicate masonry units: a) wall only subjected to shearing actions, b) wall under shearing and compression to 0.1 N/mm², c) wall under maximum shearing and compression to 1.5 N/mm²
Figure 5. Comparison of cracking patterns of test elements and numerical models of HOS-H series walls made of calcium silicate masonry units: a) wall under shearing and compression to 0.10 N/mm$^2$, b) wall under shearing and compression to 0.75 N/mm$^2$, c) wall under shearing and maximum compression to 1.5 N/mm$^2$
Figure 6. Comparison of shear stresses-shear strain angle relationship from tests and numerical calculations: a) walls of HOS series made of calcium silicate masonry units, b) walls of HOS-H series made of calcium silicate masonry units

Until the moment of cracking, relationships between stress and non-dilatation strain determined by numerical calculation by FEM were almost identical to test results. Average ratios of stresses calculated and determined from tests at the time of cracking were $\frac{\tau_{cr,cal}}{\tau_{cr}} = 0.89$, and the corresponding ratios of angles of non-dilatational strain were $\frac{\Theta_{cr,cal}}{\Theta_{cr}} = 0.98$. Clear differences were found at the phase after cracking. Test elements demonstrated considerably greater deformations at the same values of stresses. Average ratios of calculated and determined shear stresses and the corresponding deformation angle were equal to $\frac{\tau_{u,cal}}{\tau_{u}} = 1.18$, $\frac{\Theta_{u,cal}}{\Theta_{u}} = 0.54$. Differences resulting from some kind of deficiency in the numerical model, in which the possibility of tightening head joints without mortar generates serious deformations.

4. Conclusions

Using in numerical models the method of empirical homogenization and interface finite elements replacing bed and head joints, the FEM calculations can lead to following conclusions:

- considering the morphology of cracks, obtained results were very similar to test results. Regardless of the wall shape, cracks were formed approximately along the wall diagonal. The greatest width of cracks was observed in the central part of the wall. Cracks caused by the material failure due to compression were formed in the support zones.
- Calculated average stresses and angles of non-dilatational strains with reference to test results are as follows:
  - cracking: $\frac{\tau_{cr,cal}}{\tau_{cr}} = 0.89$, $\frac{\Theta_{cr,cal}}{\Theta_{cr}} = 1.18$,
  - failure: $\frac{\tau_{u,cal}}{\tau_{u}} = 0.98$, $\frac{\Theta_{u,cal}}{\Theta_{u}} = 0.54$.

References
[1] P.B. Lourenço, Computational Strategies for Masonry Structures. PhD Thesis, Delf University Press, 1996.
[2] P.B. Lourenço, J.G. Rots, On the Use of Homogenization Techniques for the Analysis of Masonry Structures. Masonry International, Vol. 11, No. 1, 1997, pp. 26 - 32.
[3] L. Gambrotta, S. Logomarsino, Damage models for the seismic response of trick masonry shear walls. Part I. The mortar joint model and its applications. Earthquake Engineering and Structural Dynamic, Vol. 26, 1997, pp. 423 – 439.
[4] L. Gambrotta, S. Logomarsino, Damage models for the seismic response of trick masonry shear walls. Part II. The continuum model and its applications. Earthquake Engineering and Structural Dynamic, Vol. 26, 1997, pp. 441 – 462.
[5] J. Lopez, S. Oller, E. Onnte, J. Lubliner, A Homogeneous Constitutive Model for Masonry.
International Journal for Numerical Methods in Engineering, Vol. 46, 1999, pp. 149 – 156.

[6] R. Jasiński, Identification of the parameters of Menetrey-Willam failure surface of calcium silicate units. IOP Publishing. IOP Conf. Series: Materials Science and Engineering. Vol. 245, 2017, 032045, DOI:10.1088/1757-899X/245/3/032045.

[7] R. Jasinski, Proposal of procedure for identification of Menetrey–Willam (M-W-3) plasticity surface of homogeneous and hollow masonry units. Engineering Structures and Technologies. Volume 11 Issue 2, 2019, pp. 40–49. doi.org/10.3846/est.2019.10582.

[8] R. Jasinski, Validation of Elastic-Brittle, and Elastic-Plastic FEM Model of the Wall Made of Calcium Silicate and AAC Masonry Units. IOP Conf. Series: Materials Science and Engineering 603 (2019) 032001. doi:10.1088/1757-899X/603/3/032001.

[9] W.F. Chen, A.F. Saleeb, Constitutive Equations For Engineering Materials, John Willey & Sons, 1982.

[10] P. Menetrey, K.J. Willam, Triaxial failure criterion for concrete and its generalization. ACI Structural Journal, Vol 92, No. 3, 1995, pp. 311 – 318.

[11] E. Hoek, E.T. Brown, Empirical Criterion for Rock Masses. Journal of the Geotechnical Engineering Division, Vol. 106, No. GT9/1980, pp. 1013 – 1035.

[12] S. Weihe, Implicit Integration Schemes for Multi-Surface Yield Criteria Subjected to Hardening/Softening Behavior. MSc thesis. University of Colorado-Bulder, 1989.

[13] Ł. Drobiec, FEM Micro-Model for Masonry Reinforced in Bed Joints. Proceedings of the British Masonry Society, No. 10, October/November 2006, Published by the Society Stoke-on-Trent.

[14] R., Jasiński, Research of Influence of the Shape of Unreinforced Masonry Shear Walls Made of Calcium Silicate Masonry Units. IOP Conf. Series: Materials Science and Engineering. Vol. 471, 2019. doi:10.1088/1757-899X/471/2/022009.

[15] R. Jasiński, Research and modelling of masonry shear walls. PhD DsC Thesis. Silesian University of Technology, Gliwice, Poland 2017. (In Polish).