Application of A.M. Obukhov’s theory of correlation of vectors for scientific research and engineering calculations of ice drift in the Arctic Ocean.

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Abstract The theory of vectors correlation of Obukhov was used to describe an ice drift in the Arctic Ocean. The main features are defined by an area. The tracks tortuosity in Beaufort Gyre and Transarctic current varies threefold: 1.5 and 4.5. Year to year and seasonal variations indicate the position of the Beaufort Gyre and the speed of the Transarctic drift. The average speed does not exceed 10 cm/s. The drift increases in summer, and in winter it intensifies in Fram Strait. In Fram Strait the velocity month–over–month increases from June to December–March from 2.5 to 6.5 cm/s. Overall correlation drift and wind is 0.95 and 0.85 for ice fields and icebergs. The non-wind drift reproduces the cyclonic circulation between Frantz Josef Land and Novaya Zemlya. The wind drift specifies the coefficients and angles of drift evasion from the wind. The coefficient for total drift is 0.14–0.18, fore wind drift 0.10–0.15, and the angles are 15°–30° and 10°–15° in the Barents Sea. In the Fram Strait the drift and wind directions are stable, the trend considers the modulus and direction. The drift trend has the direction towards the south. It decreases to 5 % of variance in June and increases to 15% from December to March.

1. Introduction, research methods and materials

The report is devoted to the first major result of A.M. Obukhov — the theory of normal correlation of vectors [1, 2]. It was originally created to describe a system of affine vectors. Its applicability to Euclidean vectors was shown later [3, 4, 5]. This is important for the study of ice drift and its dependence on wind and currents in the Arctic Ocean (AO). The greatest efficiency is achieved when it is used in combination with the vector-algebraic method of random vectors [6] and the correlation method Watanabe [7] and Z.M. Gudkovich [8, 9].

By the definition of N.E. Kochin [10] the velocity model is a Euclidean vector \( \mathbf{V} \) with modulus \( V \), direction \( \varphi \), and projections \( V_x, V_y \). Addition according to the parallelogram rule, transformation of coordinates during their rotation, and three kinds of multiplication — scalar, vector, and tensor — are defined. Characteristic \( \mathbf{V} \) as a random vector — probability distribution by gradations of modulus \( V \) and by rhumb lines \( \varphi \) are graphically represented by "roses" (in figure. 2, 3 below).

Moments of distribution — mean velocity vector \( \mathbf{\bar{m}}_V \) and variance tensor \( D \):

\[
D_V = \text{M}\{(\mathbf{V} - \mathbf{\bar{m}}_V) \otimes (\mathbf{V} - \mathbf{\bar{m}}_V)\} = \begin{pmatrix} D_{Vx} & K_{VxVy} \\ K_{VxVy} & D_{Vy} \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}
\]

(1)

Components (1) — variance and covariance of projections \( \mathbf{V} \). The tensor (1) is brought to diagonal form \( (\lambda_{1,2}) \) by a turn of coordinates to an angle \( \alpha \), showing the direction of maximum variability \( \mathbf{V} \). Invariants \( \lambda_{1,2} \) define the distribution of variance in orthogonal directions. Their combinations are also invariants. Linear invariant \( I_1 = \lambda_1 + \lambda_2 \) shows the total variance (in modulo and direction). Adding to (1) the ratio
of $I_1$ and scalar modulus variance $\gamma_i=1-D_i/I_i$ shows the contribution of direction variability to the total variance. The ratio of $I_1$ to the mean velocity modulus $v_i=I_i^{1/2}/m_i$ is the coefficient of variation.

Standard deviation is an ellipse $\sigma_i$, with axes $\lambda_{1,2}$ oriented in the direction of maximum variability $\alpha_i$, its elongation shows the invariant $\chi=\lambda_2/\lambda_1$. Graphically, the variability $V$ is shown by combining the vector $\bar{m}_i$ and the ellipse $\sigma_i$ (see figure 2, 3, 5 below).

The covariance in the system of random vectors $-\bar{V}$ drift and $\bar{G}$ wind is described by tensor

$$COV_{\bar{G}}=\begin{pmatrix} K_{xX} & K_{yG} \\ K_{xG} & D_{yY} \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} + 0.5A \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(2)

In contrast to (1) tensor (2) is asymmetric. The invariant $I_1=\lambda_1+\lambda_2$ of symmetric part (2) shows the connectivity of changes of collinear components $\bar{V}$ and $\bar{G}$, and the invariant $A$ (rotation indicator) shows the connectivity of changes of orthogonal components.

The proportional drift dependence on wind was established by famous F. Nansen’s Fram expedition [11], H. Sverdrup’s Maud expedition [12] and confirmed by subsequent measurements [4, 5, 9, 13—18]. A linear approximation of the dependence is defined by A.M. Obukhov with equation

$$\bar{V}=A_{\bar{G}} \bar{G} + B_{\bar{G}}$$

(3)

Regression — tensor $A_{\bar{G}}=COV_{\bar{G}}D_{\bar{G}}$, free term is a vector $B_{\bar{G}}=m_{\bar{G}}-A_{\bar{G}}m_{\bar{G}}$. Components $COV_{\bar{G}}$ and $D_{\bar{G}}$ and mean velocities $\bar{m}_{\bar{G}}$, $\bar{m}_{\bar{G}}$ drift and wind are defined from the measurement data.

Correlation is an integral part of regression. In the system of vectors ($\bar{V}, \bar{G}$), it is defined by mutual changes of drift and wind moduli and directions. It is important that the correlation not only measures the "closeness" of dependence, but also allows for interpretation in Euclidean space. In the paper [4] the system of indices of collinear $r_{+1}$, orthogonal correlation $r_{-1}$ and indicator $\mu$ by invariants of tensors $D_{\bar{G}}$ (1) and $COV_{\bar{G}}$ (2) is given by

$$r_{+1} = \frac{I^{1/2}}{I^{1/2} I_{G}^{1/2}}$$

$$r_{-1} = \frac{A^{1/2}}{I^{1/2} I_{G}^{1/2}}$$

$$\mu = \frac{r_{+1}^2 + r_{-1}^2}{2} \times 0.5$$

(4)

If the dependence is defined by changes only to moduli $\bar{V}$ and $\bar{G}$, then $r_{-1}=0$. At joint amplification (weakening) of velocities the collinear correlation $r_{+1}>0$, and at amplification of one vector and weakening of another $r_{+1}<0$. If at a stable modulus the vector $\bar{V}$ is reversed to the right of the vector $\bar{G}$, orthogonal correlation $r_{-1}>0$ and vice versa. The indicator $\mu$ shows total correlation and emphasizes the need for a joint analysis of $r_{+1}$ and $r_{-1}$. For independent vectors $\mu=0$, with deterministic dependence $\mu=1$, and with stochastic dependence $\mu<1$. Estimates (4) were used to clarify drift dependence on wind in the entire AO [15, 17] and in its individual regions [5, 14].

The surface circulation and ice drift in the AO are defined by drift dependence on wind and non-wind currents [9, 13, 16]. Wind coefficients $k=V/G$ and angles of drift deflection from wind $\psi=\phi_{\bar{V}}-\phi_{\bar{G}}$
were used to calculate and predict drift dependence on wind. They are usually defined by the total drift. In the paper [5] the dependence of $k$ and $v$ on the type of object (field or iceberg) was confirmed; replacing the total drift $\vec{V}$ with the wind component $\vec{U}$ using formulas (3, 6) clarifies the estimates of $k$ and $v$. Non-wind currents are formed by adjusting the water density field to the wind [9]. They define the "non-wind" drift $\vec{C}$ (formula 7) and are quasi-constant over intervals from weeks to the season. When averaged over the season, their contribution to total drift in the AB averages 60% and is increased in the Fram Strait and Greenland Sea. The velocity $\vec{C}$ in general for the AB is increased in summer and decreased in winter. In the Fram Strait, a winter maximum was recorded due to intensification of the Icelandic cyclone.

The correlation method of Watanabe – Gudkovich was used to separate wind and non-wind components in total drift. In this method, each drift projection $\vec{V}$ is defined by a multiple regression with both wind projections $\vec{G}$, and $C_x$, $C_y$ are non-wind drift projections $\vec{C}$.

$$V_x = a_x G_x + b_x G_y + C_x,$$
$$V_y = a_y G_x + b_y G_y + C_y \quad (5)$$

In the absence of measurements, geostrophic wind was used according to the drift theory of N.N. Zubov [19]. Its characteristics are specified in the paper [9]. Despite the differences in velocity vector models (Obukhov — modulus and direction, Gudkovich — projections), calculations of drift dependence on wind by formulas (3) and (5) are numerically the same. Formula (3) defines wind drift by $B_{Gv} = 0$ as

$$\vec{U} = A_{Gv} \vec{G}, \quad (6)$$

Then non-wind drift is the difference of mean velocities of total and wind drift

$$\vec{C} = \vec{m} - \vec{m}_{v}. \quad (7)$$

The method effectiveness is demonstrated (in figure 4 below) by the map of cyclonic circulation between Franz Josef Land and Novaya Zemlya [5]. Formula (6) is important for specifying wind coefficients $k$ of angles of drift deflection from the wind in engineering calculations and forecasts.

Figure 1a shows a map of seas and major straits in the AO [20]. Figure 1b in the paper [16, 18] shows ice drift scheme in the AO — the Beaufort Gyre in the Amerasian sector, the Transarctic Current from the Chukchi Sea to the Fram Strait, and cyclonic circulations in the Siberian seas. The center position and size of the Beaufort Gyre, ice flow from the AB to the Greenland Sea via the Fram Strait and its transit to the Atlantic via the Danish Strait, ice export from the Siberian seas to the Transarctic Current area, and ice runoff via the Davis Strait. ESV data in figure 1c, d [17] for December 1982 and January 1983 show marked changes from month to month.

Accordingly figure 1 mean ice drift velocity in the AO does not exceed 10 cm/s. In the entire AB, the transarctic drift is intensified in summer. But in the Fram Strait the drift is intensified in winter due to intensification of the Icelandic cyclone. It is an indicator for the entire AO its variability depends on the pressure gradient between Greenland and Scandinavia.

The drift pattern is stable. Interannual and seasonal differences are evident in the center position and size of the Beaufort Gyre, transarctic drift velocity, and ice runoff intensity through the Fram, Denmark, and Baffin Straits. This is shown by drift maps from satellite (ESV) drift measurements in figure 1c, d [17]. In just one month (December 1982, January 1983), center position of the Beaufort Gyre and its intensity changed, and in the Kara Sea there was a drift reversal of about 90°.
The multiyear variability is described by a trend. The Beaufort Gyre has been shown to intensify in recent decades against the background of warming and ice reduction in the AO, positive trends in the Transarctic drift and ice export from the AB. It was explained by a decrease in the ice extent, ice thickness, and concentration of multiyear ice. The conclusion that wind does not affect the drift trend (for 2000-09 in the Fram Strait drift trend was 24% and 18% of variance in winter and summer, and only 2% and 3% for wind) needs to be clarified, as these papers use only drift $\mathbf{V}$ and wind vector $\mathbf{G}$ speed.

Vector trend $\mathbf{\tilde{V}}(t)$ is defined as

$$\mathbf{\tilde{V}}(t) = \bar{m}_V + \bar{a} + \bar{b} + \bar{\epsilon}(t)$$  

(8)

Here, where $\bar{m}_V$ is the mean velocity, components of trend $\bar{a}$ and free term $\bar{b}$ coefficient vectors are defined by projections of $V_x(t)$, $V_y(t)$; $\bar{\epsilon}(t) \rightarrow$ trend anomalies. Trend contribution to the variance
defines the ratio of invariants $\gamma = I^{(c)}_1 / I^{(p)}_1$ of tensors (1) of anomalies and of series itself $D_c$. It is important to compare the directions. If the trend $\vec{a}$ and mean velocity $\vec{m}$ are colinear — the trend shows modulus amplification (weakening), and if they are orthogonal — the reversal (to the left or to the right by a difference sign $\varphi_1 - \varphi_a$).

In Sections 2.1, 2.2 the drift velocity $\vec{V}$ of the North Pole stations, icebergs and ice fields in the Barents Sea were calculated based on GPS measurements of geographic coordinates. Discreteness is 1 day. The fields of mean monthly drift in the entire AO in Section 2.3 were obtained from satellite measurements. There are direct measurements for the wind $\vec{G}$ at the NP. For buoys and ESV data, the wind is defined by reanalysis. Wind drift $\vec{U}$ and non-wind currents $\vec{C}$ were defined according to Obukhov's theory by formulas (3, 6, 7) for total drift $\vec{V}$ and wind $\vec{G}$.

2. Results and discussion

2.1. Drift of the North Pole stations

The Russian North Pole (NP) stations made an epoch in the study of hydrometeorological processes in the Arctic ocean. They provide the most accurate data — contact measurements of drift, wind and currents. An overview of the drift at NP 1—33 (1937-2004) is given in the paper [16] and NP 35 [15]. It is shown that the main features are defined by the region — drift characteristics are different in the Beaufort Gyre and in the Transarctic Current zone. Seasonal features and interannual variability are observed.

The drift NP 37, 40 in the Beaufort Gyre, NP 36 and Ice Base (IB) in the Transarctic Current area is considered here. The drift $\vec{V}$ and wind $\vec{G}$ discreteness is 1 day. Dated tracks are shown in figure. 2a, surface circulation diagram is reproduced from Atlas [20]. Drift start dates, drift end dates, duration and region (latitude and longitude extremes) are shown in table 1. Track parameters - length $L$, modulus of resultant displacement $R$ and tortuosity coefficient $\gamma_L$=L/R are also given here.

| № NP  | Start and finish observation | Boundary region | Parameter track |
|-------|-----------------------------|----------------|----------------|
| LB    | 23.06.2007 - 14.08.2007     | Day: 53        | $L_{min}$: 81.7 | $R_{10}$: 66   | $\gamma_L$: 1.4 |
| NP-36 | 8.09.2008 - 23.08.2009      | Day: 350       | $L_{min}$: 82.3 | $R_{10}$: 76   | $\gamma_L$: 1.5 |
| NP-37 | 8.09.2009 - 31.05.2010      | Day: 266       | $L_{min}$: 80.1 | $R_{10}$: 71   | $\gamma_L$: 1.6 |
| NP-40 | 2.10.2012 - 6.06.2013       | Day: 248       | $L_{min}$: 81.1 | $R_{10}$: 65   | $\gamma_L$: 4.0 |

Figure 2a and table 1 confirm that drift depend primarily on the region. Four NP drift in different years and seasons, the duration of their drift (from 50 to 350 days), its length and wind conditions are different. But two groups are distinguished by track shape and elongation of tracks — drift in Beaufort Gyre (NP 37, 40) and in Transarctic Current (NP 36, IB). Its underlined by estimates of tortuosity coefficient $\gamma_L$ in table 1. For IB and NP 36 it was 1.5, and for NP 37, 40 3 times more $- 4.0\pm 4.7$.

Drift characteristics, its wind component and wind itself at the IB and NP 36, 37 in table 2 are represented by estimates of the scalar modulus (mean $V_{mean}$, maximum $V_{max}$, and standard deviation $\sigma_V$), mean vector (modulus $m_v$ and direction $\varphi_v$), and variance ellipse ($\lambda_1, \lambda_2$, total RMS $I=\lambda_1+\lambda_2$, elongation $\chi=\lambda_1/\lambda_2$, direction of maximum variability $\alpha$ and direction contribution to variance $\gamma_V$ and coefficient of variation $\nu=I/m_1$). Drift and wind recurrence roses by rumbs and modulus gradation are shown in figure. 2b. Figure 2c combines average velocity and wind dispersion ellipses for total and wind drift.
### Table 2. Variability of total drift velocity ($V$), its wind component ($U$) and wind ($G$) of drifting stations NP 36, 37 and the Ice Base (LB).

| № NP | Drift and wind | Scalar speed | Mean vector | Variance speed vector |
|------|----------------|--------------|-------------|-----------------------|
|      | $V$, cm/s      | $V_{cp}$    | $V_{max}$  | $\sigma_V$           | $m_V$ | $\phi_V$ | $\lambda_1$ | $\lambda_2$ | $\chi$ | $\alpha$ | $\gamma_V$ | $\nu$ |
| LB   |                | 11.1        | 31.8       | 7.4                  | 7.6   | 8        | 11.0        | 9.3          | 5.8    | 0.62    | 26         | 54    | 1.4       |
|      | $U$, cm/s      | 10.1        | 27.7       | 5.7                  | 5.2   | 359      | 10.4        | 8.9          | 5.4    | 0.60    | 28         | 70    | 2.0       |
| G, m/s | 4.1          | 10.6        | 2.2        | 2.1                  | 2.1   | 32       | 4.2         | 3.3          | 2.5    | 0.75    | 358        | 73    | 2.0       |
| NP-36 | $V$, cm/s    | 9.0         | 34.3       | 6.3                  | 3.1   | 64       | 10.5        | 7.8          | 7.0    | 0.90    | 7          | 64    | 3.4       |
|      | $U$, cm/s      | 8.5         | 24.1       | 4.1                  | 2.1   | 73       | 9.2         | 6.8          | 6.2    | 0.90    | 25         | 80    | 4.3       |
| G, m/s | 4.8          | 13.5        | 2.8        | 1.2                  | 1.2   | 52       | 5.2         | 3.7          | 3.6    | 0.98    | 14         | 71    | 4.3       |
| NP-37 | $V$, cm/s    | 8.1         | 26.5       | 5.1                  | 1.8   | 110      | 9.5         | 7.1          | 6.3    | 0.88    | 114        | 72    | 5.3       |
|      | $U$, cm/s      | 7.7         | 18.4       | 3.6                  | 1.0   | 195      | 8.5         | 6.4          | 5.6    | 0.88    | 117        | 82    | 8.5       |
| G, m/s | 4.8          | 11.5        | 2.3        | 0.6                  | 0.6   | 121      | 5.3         | 4.1          | 3.4    | 0.83    | 98         | 81    | 8.8       |

**Figure 2.** The North Pole drifting station (IB and NP 36, 37, 40): a – sea ice drift line, b – frequency pattern wind and ice drift speed (module and direction), c – mean speed $m_V$ and MSE variance ellipse $\sigma_V$ of wind and drift – completely and wind component.

According to table 2 and figure 2 for all NP (except for NP 37 in the Beaufort Gyre) the drift is to the right of the wind according to the theory. The differences between resulting drift velocity vectors $\hat{m}_V$ and wind drift $\hat{m}_U$ calculated according to (6), as well as between dispersion ellipses $\sigma_V$ and $\sigma_U$, emphasize the significant contribution of the "non-wind" component $\hat{C}$ according to formula (7) to the total drift $\hat{V}$. Drift features in the region of the Transarctic Current and the Beaufort Gyre most clearly
show the coefficient of variation \( \nu \) and elongation of dispersion ellipses \( \chi \) (table 2, figure. 2c). The coefficient \( \nu \) was 2.0 for IB, about 4.5 for NP 36, and increased to 8.5 in the Beaufort Gyre. The coefficient \( \chi \) is sharply reduced at the IB (0.6), and at the other NPs the distribution of variance in directions is close to circular \( \geq 0.9 \).

Base LB drift feature in comparison with the other NP station is connected with the increased wind direction stability in June-August 2007. Station NP 36 drifted in the same region in summer 2009 (figure. 1). The measurement data showed (in table 3) that in 2007 the frequency of wind directed to the north-west (NW) was 35\%, and in W–NW–N sectors it was 70\% in total. In 2009, wind distribution by rhumb lines is much more even — from 8\% to 20\%.

### 2.2. Drift of icebergs and ice fields in the north—eastern part of the Barents sea

Contact measurements of drift and wind at the NP stations are the most accurate. But they are insufficient for analysis of climatic variability — there are data only in 1 point at any given time. A good basis for analyzing long-term drift variability in the Arctic basin of the AO is data from American automatic buoys. According to this program, since 1979, GPS sensors have been installed on drifting ice objects in the Amerasian sector of the AB and their coordinates have been used to define drift. The points of buoy installation are random and mobile. The AARI has developed a technology for bringing data to regular grid nodes throughout the AB based on matching drift measurements with the pressure field. It is used to monitor hydrometeorological conditions in the AO.

Here are the drift characteristics of 10 icebergs and 20 ice fields in the northeastern part of the Barents Sea in May-August 2009. They were obtained at the AARI. The expedition was unique in the number of buoys in a limited area (10 icebergs and 20 fields). Its results made it possible to obtain detailed drift estimates and compare the drift of icebergs and ice fields [5].

Parameters of icebergs and fields — \( \text{min-max} \) range and the specific spread \( (\text{max-min})/m \) differ markedly (especially for the mass of icebergs and the area of fields). They were \( 9 \pm 17 \) and 0.5 for iceberg height \( (m) \), \( 46 \pm 82 \) and 0.6 for draft \( (m) \), \( 150 \times 3700 \) and 2.7 for mass \( (kg \times 10^6) \); 25\% to 70\% and 3.2 for field area \( (km^2) \). The differences of drift between iceberg-field groups are significant (due to significant height and draft of icebergs compared to the fields), but no regular dependence of drift on morphometric characteristics of icebergs and fields was revealed within the groups.

In figure. 3 for synchronous drift (June 2009) of the iceberg and the field probability distributions by 8 rumb lines and gradations of the velocity modulus are represented by wind \( \hat{G} \) “roses” and total drift \( \hat{V} \). The interoperability of roses of iceberg and field drift with the wind demonstrates drift dependence on wind. Small differences between the field and the iceberg are due to the drift region. Drift roses are more "concentrated" (by rumb lines) compared to the wind. This shows a noticeable contribution of non-wind currents to the drift.

At the bottom of figure. 3, roses are approximated by combining mean velocity and dispersion ellipses, and additionally vector \( \bar{m}_v \) and ellipse \( \sigma_v \) of the drift wind component are shown. They show the right drift turn from the wind and the influence of non-wind currents on the drift. It is manifested by a greater elongation of the drift ellipses \( \sigma_v \) compared to the wind and the difference in velocities of total \( \bar{m}_v \) and wind \( \bar{m}_w \) drift in modulo and even in direction.
Figure 3. Synchronized the drift ice floe and iceberg from daily measurement in June, 2009: a, d – frequency pattern wind speed module and direction (a, c) and completely drift (b, d); e, f – mean speed \( \bar{V} \) and variance ellipse \( \sigma_{V} \) (e – ice floe, f – iceberg) of wind (1), complete drift (2) and wind component ice drift (3), according to [5].

Table 3 shows for June 2009 the spread (within groups of 10 icebergs and 20 fields) of variability parameters of the scalar velocity \( V \) (mean, maximum and RMS), modulus of the mean velocity vector \( m_{V} \), linear invariant \( I_{1} \), elongation of dispersion ellipse \( \chi \) and contribution of direction to the variance \( \gamma_{V} \) for the total drift and its wind component. It shows that the drift of ice fields differs from the icebergs in its greater velocity and enhanced variability. For the total drift, mean velocity, maximum and mean vector modulus values \( V_{\text{mean}}, V_{\text{max}}, m_{V} \) were 13–19, 27–40 and 4.5–11 cm/s for fields and 10–15, 22–34 and 3.5–8 cm/s for icebergs, and variability (invariant \( I_{1} \)) was 13.5–20 and 12–16 cm/s. The contribution of direction variability \( \gamma_{V} \) to the variance according to Table 3 exceeds 50%, the increase of \( \gamma_{V} \) for fields (65–75 %) compared to icebergs (60–70 %) is due to iceberg draft.

Table 3. Spread (min–max) parameter of scalar and vector speed Iceberg and Ice Floe in Barents sea (June, 2009): a – complete drift, b – wind component.

| Complete Drift (a) and Wind Drift (b) | Scalar speed drift | Vector speed drift |
|--------------------------------------|---------------------|-------------------|
|                                      | \( V_{\text{sp}} \) cm/s | \( V_{\text{max}} \) cm/s | \( \sigma_{V} \) cm/s | \( m_{V} \) cm/s | \( (l_{1})^{0.5} \) cm/s | \( \chi \) | \( \gamma_{V} \) % |
| Iceberg a                            | 9.9\,\pm\,15.1      | 21.9\,\pm\,34.2     | 4.7\,\pm\,9.4       | 3.6\,\pm\,8.1      | 11.9\,\pm\,16.2    | 0.48\,\pm\,0.94   | 60\,\pm\,70   |
| b                                    | 9.3\,\,\pm\,13.6   | 18.2\,\,\pm\,27.0   | 4.0\,\,\pm\,6.3     | 2.9\,\,\pm\,5.6    | 10.2\,\,\pm\,14.1  | 0.42\,\,\pm\,0.94 | —           |
| Ice Floe a                           | 12.8\,\,\pm\,19.2  | 27.1\,\,\pm\,40.2   | 8.0\,\,\pm\,11.8    | 4.6\,\,\pm\,10.8   | 13.5\,\,\pm\,20.1  | 0.49\,\,\pm\,0.92 | 65\,\,\pm\,75 |
| b                                    | 11.0\,\,\pm\,17.8  | 24.7\,\,\pm\,41.1   | 6.2\,\,\pm\,9.3     | 3.5\,\,\pm\,12.6   | 12.2\,\,\pm\,16.8  | 0.45\,\,\pm\,0.84 | —           |

Table 4 shows mean values \( m \), min–max ranges and \( R=mi–max \) range of colinear \( r_{1} \), orthogonal correlation \( r_{2} \) and indicator \( \mu \) (4) dependence of mean daily drift velocities of icebergs and fields on the
wind. For icebergs (draft 50–80 m) it is less than for fields (0.85 and 0.95 on average), and the relative spread \((R/m)\) for individual ice objects is small (10–15%). Colinear correlation is many times higher than orthogonal \(r_{ij} \gg r_{ik}\). The main reason for the correlation is unidirectional changes in wind and drift. In accordance with the theory, positive values \(r_i > 0\) show a rightward drift reversal from the wind, and for icebergs (draft 50–80 m) it is greater than for fields (mean values \(r_i 0.20\) and \(0.08\)).

| Table 4. Correlations speed Drift Iceberg and Ice Floe in Barents sea with Wind, June–August 2009 year. |
| Iceberg (a) and Ice Floe (b) | \(r_{11}\) | min-max | \(R\) | \(r_{12}\) | min-max | \(R\) | \(r_{13}\) | min-max | \(R\) |
|----------|----------|---------|-------|----------|---------|-------|----------|---------|-------|
| a        | 0.82     | 0.77–0.89 | 0.12  | 0.20     | 0.12–0.24 | 0.12  | 0.84     | 0.78–0.91 | 0.13  |
| b        | 0.92     | 0.87–0.95 | 0.12  | 0.08     | 0.02–0.22 | 0.20  | 0.94     | 0.88–0.98 | 0.10  |

The "non-wind" drift velocity \(\tilde{C}\) characterizes the surface circulation. The map \(\tilde{C}\) in figure. 4 in May–June 2009 was obtained for drift of all icebergs and fields and in July-September for icebergs only (fields melt in july). It shows that detailed (by the number of drifting objects) drift measurements in northeastern Barents Sea clearly reproduce the cyclonic circulation highlighted in this region from multiyear data (figure 1b). It is important that the approximate constancy of the direction \(\tilde{C}\) in the same region from late May to September confirms the hypothesis of quasi-continuity of the non-wind drift within a season.

Figure. 4. Non–wind drift iceberg (a) and ice floe (b) in June (1), July (2), august (3) and September (4) in 2009 year between the Franc–Josef Land and Novaya Zemlya, according to [5].

In Arctic hydrocarbon production is actively developing, and icebergs and large ice fields are the most dangerous for economic activities. The Ice Management System makes it possible to identify potentially dangerous objects in advance. To calculate their drift according to wind forecast, accurate estimates of wind coefficients \(k=V/G\) and angles of drift deflection from wind are needed. Usually they are defined by the total drift. Replacing the total drift \(\vec{V}\) with the wind component \(\vec{U}\) using Obukhov's formulas (3, 6) significantly clarifies estimates of \(k\) and \(\psi\). In the Barents Sea in 2009, \(k_U\) coefficients of total drift varied in the range of \(0.014–0.18\) (median \(me=0.015\)), for wind drift \(k_U\) decreased to \(0.010–0.015\) \((me=0.012)\). For angles of deflection, the difference between \(\psi_V\) (14–30°, \(me=18°\)) and \(\psi_U\) (10–15°, \(me=12°\)) is noticeable in the reduction of median and spread.

2.3. Drift fields in the AO according to satellite (ESV) data
Drift and wind data at the NP stations are the most accurate, but there are only 1 point. The buoy data describe processes only in the Arctic basin. To characterize modern climatic variability in the entire AO and the adjacent region of the North Atlantic, remote (ESV) drift velocity fields are the best in terms of
spatial coverage. The archive of these fields began in 1979 and is replenished monthly. The drift maps for December 1982 and January 1983 are shown above in figure 1c,d.

The most important process for the entire AO is the AB ice ruboff into the Greenland Sea through the Fram Strait. Estimates of the long-term (1979 to 2018 year) variability and trend of ice and wind drift were obtained for a point at coordinates 82.2°N and 0.0°d. (figure. 1c,d). In accordance with natural seasons highlighted for the AO [16] in Table 5 (in the form of table 2), they are defined for 4 key months: September (ice min), December (ice development), March (ice max), June (ice degradation). In figure 5, average drift velocities $\bar{m}$, wind $\bar{m}$ and dispersion ellipses $\sigma_{\bar{m}}$, $\sigma_{\bar{g}}$ and vectors of corresponding trends $\bar{a}$ are shown in blue. Drift peculiarity in the Fram Strait is stable drift and wind direction (the southern component predominates) at a significant velocity. According to table 5 and figure. 5, average drift velocity $m_{\bar{V}}$ in December and March was 6–6.5 cm/s, decreased to 5 cm/s in September, and was minimal at 2.5 cm/s in June. Drift velocity maxima in December and March were 17–18 cm/s, decreased to 14 cm/s in September, and 11 cm/s in June. Decrease in elongation for drift relative to wind indirectly demonstrates the contribution of the non-wind current to total drift.

### Table 5. Multi-year variance and trend speed month Drift in Wind in Fram strait (82.2° N latitude and 0° longitude) from September, December, March, June 2009 year.

| Month | Drift and wind | Speed | Mean vector | Variance speed vector | Trend |
|-------|----------------|-------|-------------|-----------------------|-------|
|       | $V_{\bar{m}}$ | $V_{\bar{m}}$ | $\sigma_{\bar{V}}$ | $m_{\bar{V}}$ | $\varphi_{\bar{V}}$ | $V_{\bar{V}}$ | $\lambda_1$ | $\lambda_2$ | $\chi$ | $\alpha$ | $\gamma_{\bar{V}}$ | $\gamma_{\bar{g}}$ | $\alpha$ | $\varphi_a$ |
| IX    | $V$           | 5.9   | 14.2        | 5.0        | 171     | 4.4         | 3.6       | 2.6       | 0.70     | 168     | 55            | 10          | 1.3       | 140       |
|       | $G$           | 2.7   | 6.6         | 1.3        | 1.7     | 138        | 2.5       | 2.0       | 1.4      | 0.71    | 189           | 75          | 2         | 0.3       | 178       |
| XII   | $V$           | 8.0   | 17.7        | 4.3        | 5.9     | 198        | 6.9       | 6.1       | 3.2      | 0.52    | 184           | 60          | 15        | 2.2       | 155       |
|       | $G$           | 4.2   | 7.8         | 1.7        | 3.6     | 178        | 2.8       | 2.3       | 1.6      | 0.71    | 168           | 65          | 5         | 0.5       | 160       |
| III   | $V$           | 7.9   | 16.9        | 4.5        | 6.4     | 176        | 6.4       | 5.9       | 2.5      | 0.42    | 157           | 45          | 17        | 2.2       | 170       |
|       | $G$           | 4.2   | 8.6         | 1.8        | 3.8     | 177        | 2.5       | 2.1       | 1.4      | 0.70    | 162           | 50          | 20        | 0.9       | 160       |
| VI    | $V$           | 3.3   | 11.1        | 2.4        | 2.3     | 155        | 3.4       | 2.8       | 1.9      | 0.45    | 177           | 50          | 5         | 0.5       | 180       |
|       | $G$           | 2.4   | 5.8         | 1.1        | 1.5     | 112        | 2.1       | 1.7       | 1.2      | 0.70    | 174           | 73          | 5         | 0.4       | 160       |

**Figure 5.** Mean speed $\bar{m}$, variance ellipses $\sigma_{\bar{V}}$ (in black) and vector trend $\bar{a}$ (in blue) of ice drift (a) and wind (b) and variance trend drift and wind (%).
Table 5 and figure 5 show multi-year trend estimates using formula (8), trend contribution to the variance $\gamma_a$ and modulus (10-year change) $a$ and direction value $\varphi_a$ of vector trend coefficient from 1979 to 2018. Drift trend vector $\vec{a}_t$ and mean drift $\vec{m}_t$ are oriented predominantly to the south (in March the directions coincide, and in September and December there is a slight deflection $\vec{a}_t$ from $\vec{m}_t$ eastward). This confirms the conclusion that in the last decades, the ice drift in the AO is characterized by a positive velocity trend at a relatively stable direction. According to table 5, drift trend variance $\gamma_a$ is relatively small and depends on the season — 15% in the cold season (December and March) and 10% in September. In September and December, wind trend variance is radically smaller (up to 5%), which indirectly confirms that drift acceleration is caused not by wind, but by a decrease in the total ice extent and a decrease in the concentration of multiyear ice. But in March (ice maximum), the variance of drift and wind trends is increased and comparable (20%), and in June (at the stage of rapid ice degradation), drift and wind trends disappear weakly (5%). Therefore, additional studies are needed to clarify the estimates of ice drift trends in the AO.

3. Conclusion
The above results show that the theory of normal correlation of vectors developed by A.M. Obukhov in 1938–40 is relevant for studies of ice dynamics in the AO even at the turn of the XX–XXI centuries in conditions of rapidly changing climatic conditions.

Analysis of ice drift speed in the Arctic Ocean after the year 2000 by the method of A.M. Obukhov confirmed the conclusions of the paper [21] on the intensification of the transpolar drift and the Beaufort Gyre under the conditions of the Arctic current warming. Estimation of wind coefficients and wind drift deviation angles precise the forecast of icebergs and ice fields drift, hazardous to marine activities [5, 14].

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