PDE-solved by boundary element method for electrical impedance tomography

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Abstract. In this article a new version of the algorithm for Electrical Impedance Tomography is presented. By describing the problem with differential equations brought to integral equations, the algorithm can be used for many types of tomography. The approach used is particularly useful where it is not possible to formulate boundary conditions at the outer boundary of the region. Influence of the proximity effect on precision of imaging was considered.

1. Introduction
The motivation for writing this work is the need of building relatively universal algorithm which could be able to provide a descent images for several modalities like EIT (Electrical Impedance Tomography) \cite{1-3}, ECT (Electrical Capacitance Tomography) \cite{4}, DOT (Diffuse Optical Tomography) \cite{5}, UTT (Ultra Sound Transmission Tomography) \cite{6-9}, RT (Radio Tomography \cite{10} and the others \cite{11,12}. That will open the way for the hybrid tomograph \cite{10}, combining the possibility of EIT and ECP for example. The main problem in hybridisation is designing compact electrodes system suitable for both tomography types \cite{13}.

The algorithm based on the PDE (Partial Differential Equations) solution (the forward problem) by BEM (Boundary Element Method) is presented in this paper. Immediately arise the question why BEM not the FEM (Finite Element Method)? Firstly, the mathematical similarities of different kinds of nonconventional tomography predispose towards BEM rather than FEM because. In BEM it is enough to change the Green function \cite{14} to switch from, for example EIT to RT. Secondly the boundary conditions are much easier impose, particularly for UTT and RT.

2. Forward problem solution
Electrical properties such as conductivity $\gamma$ [S/m] and electrical permittivity $\varepsilon$ [F/m] define the behaviour of a material under the influence of an external electric field. For example, conductive materials have high conductivity $\gamma$ for both direct (DC) and alternate (AC) current. Dielectric materials have a high permittivity with comparison to conductivity which is going to zero. Let us consider a complex function of conductivity:

\[
\gamma(x, \omega) = \sigma(x) + i \omega \varepsilon(x), \quad i = \sqrt{-1}
\]

where $x$ is a position vector in a 2D space depending on coordinates $x, y$, and $i = \sqrt{-1}$ denotes imaginary unit and $\omega$ is the pulsation (angular frequency). A forward problem solution involves the Laplace equation solution for the EIT:
\[ \nabla \cdot (\gamma \nabla \varphi) = 0 \]  

(2)

with Dirichlet boundary conditions on the voltage electrodes and Neumann boundary conditions on the rest of the boundary nodes. If \( \omega = 0 \) or \( \omega \varepsilon \ll \sigma \), then we are dealing with resistive tomography. But when the imaginary part of complex conductivity is much larger than its real part (resistivity), then the electrical impedance tomography acts as capacitance tomography. In practice three names are used: impedance tomography which refers also to resistance tomography, capacitance tomography and induction, or eddy current tomography, called magnetic tomography by some authors and the optimization methods [15-29].

All three kinds of tomography are included in the group of impedance tomography, just having different characters: resistive, capacitive, or inductive.

In the rest of the paper the algorithm suitable for impedance/resistance tomography will be used, therefore to simplify the text we can introduce a material factor \( k(x,y) \), which in this case, will represent conductivity but in other cases it will represent the permittivity \( \varepsilon(x) \) or permeability \( \mu(x) \).

3. Boundary Element Method

Imaging has been reduced to an inverse problem of optimal dimensioning and optimal location [30]. This problem belongs to the family of the optimal shape design tasks. It could be solved by gradient free optimization method for example [9,10,31].

This approach consists in the assumption that inside the imaging area exists some number of trial objects. Imaging of the region rely on internal objects illustration with respect to their location and their dimensions. As a matter of fact, it is a process of parametrization of the image. Some simplifying assumptions were introduced:

1. inside the region under consideration only single object exists,
2. measurement data are noise free and were computer generated that is why we call them as a synthetic measurement.

For optimal dimensioning and optimal location, the BEM has big advantage over FEM as remeshing is not necessary during the iteration process.

For UTT or High Frequency EIT representing the ECP it is impossible to impose precisely the boundary conditions so the artificial boundary or artificial/approximate boundary conditions should be introduced. Implementation of such a procedure is much easier in BEM than in FEM.

4. Inverse problem in Electric Impedance Tomography

There is one big difference between tomographic and non-tomographic inverse problem. For tomographic problems existence of several projection angles make that input data are much more complicated in comparison to single projection angle (see for example figure 3).

Let us consider a circular cross section of the region and single internal obstacle, for simplification also circular shape as it is shown in figure 1. The environment is heterogeneous, it was assumed that conductivity of the background is equal to 1 S/m, but conductivity of internal object is equal to 1000 S/m (ideal conductor) or 0.001 S/m (ideal insulator).

External boundary was divided by 32 boundary elements with constant state function (interpolation with zero order polynomial). For the sake of attention, the potential value was placed in the middle of the element (see figure 1).

The number of elements is twice as much as the number of electrodes. Now the forward tomography problem could be solved by BEM for subregion with homogeneous materials what is a certain difficulty for BEM (see for example [5]).
The crucial point of an Inverse Problems is the Sensitivity Analysis, which is particularly difficult for BEM. So, to avoid the Sensitivity Analysis which for BEM is complicated and time consuming, the gradient free optimization offered by MATLAB [31] was selected.

5. Definition of the objective function
In order to match the signal calculated in each iteration step to the measured signal, the following objective function has been defined. This objective function will be subject to minimization with a certain constrains:

$$
\Phi = \sum_{j=1}^{p} \Phi_j = \frac{1}{z} \sum_{j=1}^{p} (f_j - v_{0j})^T (f_j - v_{0j}) = \frac{1}{z} (F - V_0)^T (F - V_0) \quad (3)
$$

where:
- $\Phi$ – global objective function calculated for all $p = 16$ positions of the voltage source (so-called projection angles), $j = 1, 2, ..., p$
- $\Phi_j$ – objective function for the $j$-th position of the voltage source,
- $f_j$ – vector of electrodes voltages obtained from calculations in the current iterative step for the assumed distribution of internal objects for the $j$-th position of the voltage source (projection angles),
- $v_{0j}$ – vector of measured voltages for $j$-th position of the voltage source.
- The matrices $F = [f_1, f_2, ..., f_p]^T$ and $V_0 = [v_{01}, v_{02}, ..., v_{0p}]^T$ respectively.

6. Definition of inequality constrains
The imagining problem was turned into the optimization problem means parametrization of an image. The following parameters could sufficiently describe proposed image: radius of internal object, the position vector of the centre of circular internal object. During the optimization process internal object should not cross the external boundary of the region. Mathematically could be express as follows:

$$
\begin{bmatrix}
-1 & 0 \\
0 & -1 \\
1 & 1 
\end{bmatrix} \begin{bmatrix} r_1 \\
r_2 \end{bmatrix} < \begin{bmatrix}
-0.1R_0 \\
0 \\
0.9R_0 
\end{bmatrix} \quad (4)
$$

where: $r_1$ – radius of the internal object, $r_2$ – position vector, $R_0$ – radius of the region to be imaged (see figure 1). The third parameter - an angle remains without of any constrains.

Figure 1. Heterogeneous region under consideration and its discretization by boundary elements.
7. Polar measurement protocol

Let us consider two, among many others, the most popular measurement protocols. For the sake of clarity, the measurement protocol will be presented for the set of sixteen electrodes. The first one named “polar” (polar source connection and polar measurement), for which the source electrodes and measurement electrodes are placed facing each other as it is shown in figure 2.

![Figure 2. Polar protocol of measurements, for simplicity only set of 16 electrodes is shown.](image)

In this case measurement signal is taken for each projection angle and collecting them together the total signal will be constructed as it is shown in figure 3.

(a)
Figure 3. So called “measured” signal and final signal after optimization process.

The build in MATLAB gradient free nonlinear optimization function was applied in the optimization process. Due to this function the sensitivity analysis, was not necessary. Results of optimization are presented in figure 4.

Figure 4. Result of optimization for polar electrodes.

The yellow colour denotes the object in starting point of optimization. During the optimization internal object would be changing position and dimension. It means that the imaging process was transformed to the optimization shape design problem or strictly speaking to optimal dimensioning and optimal location problem.

The green colour means the final position and dimensions of the internal object. Blue dashed line denotes its real position and dimension.

As it is seen, for the polar protocol precision of imaging is less than 10%. Many different experiments for different discretization, different starting positions shows that the influence for the results was rather week. Next experiments were done for the adjacent protocol regarding projection angle and measurements as it is shown in figure 5. It is a common knowledge that adjacent protocol is the better idea when the internal object to be imaged is placed close to the external boundary.
Figure 5. Adjacent protocol of measurements (a) first (b) the last projection angle.

8. Adjacent measurement protocol
As in previous case the supplying voltage source of 10V was applied (see figure 5). The idea of adjacent supplying and measured electrodes is presented in figure 5a for the first projection angle and in figure 5b for the last projection angle. For clarity of this figures only 16 electrodes were considered.

Measured adjacent electrodes signal composed of 15 projection angles is presented in figure 6.

Figure 6. “Measuring” signal for all projection angles in case of adjacent electrodes.

As predicted for objects close to the outer boundary adjoint protocol should provide better results (figure 7). In this case the maximal relative error is equal to 7.4%.
Further improvements in accuracy are expected in increasing the order of the boundary elements. History of gradient free optimization process is presented in figure 8.

It is clear from the above numerical experiment, the change in the measurement protocol has little effect on the results obtained which can be explained by synthetic data. Achieved results are acceptable, however the lack of noise should result in an almost ideal solution.

It remains to be examined the hypothesis that the proximity effect (observed in the Boundary Element Method [5,14]), may have a significant impact on the total error of the method used.

Especially when it comes to obstacles close to the boundary or close to each other. This can be checked experimentally by gradually moving the object away from the outer boundary. On the other hand, it is commonly known that the adjacent measurement protocol is particularly useful if the inner object is close to the boundary.

Using adjacent measurement protocol, the inner object was gradually moved away from the outer boundary, noting the improvement in the accuracy of the solution. The results of this experiment are presented in figure 9.
9. Conclusion

In this paper a new approach to Electrical Impedance Tomography was presented. Based on experiments with synthetic noisy free data the following conclusions can be stated:

1. proposed algorithm based on PDE solved by BEM can be relatively easy adopted to other tomography modalities,
2. converting the imaging problem to the inverse task demands the solution of PDE. Using the BEM remeshing is not demanded in each iteration step as it would be for FEM,
3. proposed method depends on the configuration of the object or objects inside of the region and the starting position and dimensions. Such parameters should be carefully selected depending on the experience of researcher and the case under consideration.

The authors would like to state that all figures were drawn with the aid of MATLAB [31].

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