Learning more about what can be concluded from the observation of neutrinos from a galactic supernova

Solveig Skadhauge

Nordita, AlbaNova University Center,
Roslagstullbacken 23, SE-10691 Stockholm, Sweden

Renata Zukanovich Funchal

Instituto de Física, Universidade de São Paulo,
C. P. 66.318, 05315-970 São Paulo, SP, Brazil

Abstract

We investigate what one can hope to learn about the parameters that describe the neutrino fluxes emitted by the explosion of a galactic supernova using the observations of a megaton-size Water-Cherenkov detector. We calculate the allowed regions that can be obtained by fitting these parameters to a simulated observation of events by such a detector. All four available detection channels (inverse beta decay, charge and neutral current on oxygen and elastic scattering on electrons) are included in the fit and we use a ten dimensional parameters space. Nine parameters describe the initial neutrino fluxes and are referred to as the supernova parameters. Furthermore, we include the dependence on the Chooz mixing angle $\theta_{13}$, which controls the matter effects that the neutrino undergoes in the outer-parts of the supernova. If we do not make any extra assumption on these parameters, we show that one can hope to determine $\theta_{13}$ quite well whereas, except for the parameters describing the $\bar{\nu}_e$ flux, most of the supernova parameters are rather difficult to constrain, even if the four detection channels could be completely separated.

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I. INTRODUCTION

The phenomenon of a core-collapse Supernova (SN) explosion embraces vastly different areas of physics and involves all the known fundamental interactions [1]. The core-collapse itself is driven by gravitational effects; the thermodynamics is controlled by electromagnetic and strong forces and naturally the weak force plays a major role in the energy loss through emission of neutrinos. Evidently, the study of supernovas is a promising playground for testing new physics and to learn about particle properties.

Among the particles one hopes to learn more about are the neutrinos. Indeed the major part of the enormous energy liberated in a core-collapse (type-II) supernova is emitted in the form of neutrinos. Naturally, one might also attempt to use our knowledge of neutrino properties to extract information about the supernova physics from observations of the neutrino fluxes. In particular, as the neutrinos are emitted from the interior of the star, they offer a way to probe the physics of the core-collapse. But independent of whether one attempts to learn about supernova physics or about neutrino properties from a SN observation, ultimately, when comparing data with theory one needs to take into account the uncertainties on the supernova parameters as well as on the neutrino parameters.

Many efforts have been made in order to understand the complex physics involved and to predict the emitted neutrino fluxes [2, 3]. When discussing supernova neutrino there are only three distinguishable flavors, as the $\mu$ and $\tau$ neutrinos, as well as their antineutrinos, have identical properties. We will denote the $\mu$ and $\tau$ neutrinos and antineutrinos by $\nu_x$. Thus $\nu_x$ along with the electron neutrino ($\nu_e$) and the electron antineutrino ($\bar{\nu}_e$) constitute the three supernova neutrino species. The neutrinos from the core-collapse supernova are expected to be almost thermal, since they are in fact trapped inside what is known as the neutrino-sphere. Besides the very early universe a supernova is probably the only place neutrinos thermalize. However, small deviations from the thermal spectrum are expected and how narrow or wide the energy spectra will be is in general described by a pinching parameter. Therefore, it is a good approximation to parametrize each initial neutrino spectrum by three parameters; the average neutrino energy, the total emitted energy (or integrated luminosity) and the pinching parameter.

During the last few years evidences of neutrino masses and mixing have been gathered and today most of the neutrino oscillation parameters have been determined with good precision, from the observation of solar and atmospheric neutrinos as well as other terrestrial neutrino experiments [4, 5, 6, 7, 8]. Some important parameters remain unknown. The absolute neutrino mass scale, $m_0$, is only bounded from above by cosmological data $m_0 \lesssim 0.2 - 0.7$ eV [9]. The same is true for the mixing angle $\theta_{13}$, $\sin^2 \theta_{13} \lesssim 0.04$ [10]. Furthermore, the pattern of neutrino masses is not established yet: we do not know if nature has chosen the
normal \((m_3 > m_2 > m_1)\) or the inverted \((m_2 > m_1 > m_3)\) mass hierarchy, where \(m_1\) \((m_3)\) is the mass of the neutrino state most (least) populated by the \(\nu_e\) component. Finally, we do not know anything about the existence of CP violating phases in the neutrino sector *.

In particular, for supernova neutrinos the neutrino mass hierarchy and the Chooz angle, \(\theta_{13}\), are very important, as the so-called Mikeyev-Smirnov-Wolfenstein (MSW) \(\text{H}\) resonance strongly depends on them [12, 13]. Recently, there has been several investigations of the possible impact of the neutrino-neutrino interactions in the dense neutrino region inside the neutrino-sphere in the supernova [14]. These self interactions may cause collective effects and thus influence the neutrino survival probability as a function of energy. Under realistic supernova density profiles the collective effects are negligible for the normal mass hierarchy. In the case of the inverted hierarchy it seems that a swapping of the \(\nu_e\) and \(\nu_x\) energy spectra above a certain critical energy occurs, and at the same time a swapping of the antineutrino spectra occurs. In the latter case there are indications that the spectra might be smeared out and therefore does not exhibit a sharp interchange of the spectra at a certain energy. In principle, these swapped spectra should then be used as input when calculating the MSW effects in the outer parts of the supernova. However, since the impact of the neutrino-neutrino self interactions is still being debated, we will in this paper only consider the normal mass hierarchy and we can thus neglect the collective neutrino-neutrino effects.

One can imagine two very different sources of supernova neutrino signals. The diffuse supernova neutrino background (DSNB), arising from all past supernova explosions; and the lucky event of a galactic supernova. In the latter case one expect about \(10^4 - 10^5\) neutrino events in a megaton scale detector in a 10 seconds time interval. Therefore, this is practically free of background. On the other hand, background is the major issue for the DSNB detection and one expect only to be able to extract a signal in a rather small energy window and most likely only for the dominant detection channel. Therefore, depending on the detector, there will be sensitivity only to the electron antineutrino flux (Water-Cherenkov and Scintillator) or only to the electron neutrino flux (Liquid Argon). On the contrary, for galactic SN observations one expect to have several detection channels available, each with sensitivity to different neutrino flavor compositions - a feature which is crucial for the pinning down of the neutrino parameters. As is well-known, a galactic supernova explosion is a rare event. About two per century is the best we can hope for. The optical observation is actually likely to be obscured by dust etc., and in this light it seems even more crucial to get a better understanding of the SN neutrino fluxes.

Here we will focus on the investigation of what can be learned from an observation of the

* CP-violation effects are ignored in our analysis. It was explicitly shown in [11] that there is no net CP effect due to standard neutrino oscillations in a SN core-collapse.
neutrino burst from a galactic supernova. We will furthermore restrict ourself to what can be measured by a Water-Cherenkov detector. We aim at figuring out what can be learned about supernova physics as well as what can be learned about the neutrino parameters simultaneously. We thus continue previous studies [15, 16, 17], but we will increase the parameter space, taking into account all important parameters in what optimistically could be referred to as a full parameter space fit. In the previous studies only the dominant detection channel has been considered or some of the parameters have been fixed. In most earlier works the pinching parameters were fixed, but in Ref. [17] it was shown that, when considering only the dominant inverse beta decay detection channel, these constitute important uncertainty factors when attempting to extract information about the supernova parameters.

In fact, the authors of Ref. [17] have discovered a degeneracy, which only appears when including the pinching parameters, between the mean energy $\langle E_x \rangle$, the luminosity $L_x$ and the pinching parameter $\beta_x$ of $\nu_x$ flux. By degeneracy we mean that, for a given allowed point in $(\langle E_x \rangle, L_x, \beta_x)$, i.e. a point where the data are explained (one can always take the input values themself), it is possible to construct a different allowed point which has a larger value of $\langle E_x \rangle$ by increasing $\beta_x$ and decreasing the luminosity $L_x$. As this degeneracy covers almost the complete parameter space, the determination of the correct value of $\langle E_x \rangle$ (as well as $\beta_x$ and $L_x$) becomes very hard. Indeed the allowed region for $\langle E_x \rangle$ was shown to include the whole range from 15 MeV to 30 MeV as expected from supernova simulations [17]. This degeneracy occurs since shifting the parameters in the mentioned fashion, maintains the $\bar{\nu}_e$ energy-spectrum at the Earth almost identical, with only small differences in the low energy spectrum. Due to the threshold (which is about 5 MeV) on the inverse-beta decay channel, the Water-Cherenkov detector is not sensitive to the spectrum at low energies. Clearly the $\bar{\nu}_e$ parameters have also to be adjusted correspondingly, but as the allowed regions are still rather small, we will not speak of a degeneracy in these variables. However, only the inverse beta decay channel has been considered in Ref. [17] and it is not clear to what extend the inclusion of the other detection channels might break down this degeneracy. Indeed in Ref. [15] several degeneracies in the only-inverse-beta-decay case were found to be broken down when considering all four detection channels. This is one of the motivations for the present work.

In our full parameter space fit, we use a total of ten fitting parameters. Nine parameters describe the initial supernova neutrino flux ($\nu_e$, $\bar{\nu}_e$ and $\nu_x$ luminosities, mean energies and pinching parameters), as stated above three parameters are needed for each of the three neutrino species. Furthermore, we also freely vary the most important neutrino parameter, the Chooz angle $\theta_{13}$. However, as explained above we fix another important neutrino parameter, the neutrino mass hierarchy, as normal. The terminology full parameter space should therefore, as usual, be taken with a grain of salt. There might be a number of other pa-
rameters, such as deviation of the supernova density profile from the assumed $\rho^{-3}$ form \cite{19}, effects from sterile neutrinos \cite{20} or from new interactions \cite{11}, deviation from the assumed spectral forms and yet undiscovered effects that may influence the flux of neutrinos from a supernova.

II. THE ANALYSIS PROCEDURE AND THE PARAMETER SPACE COVERAGE

In this section we describe the details of our analysis procedure and the chosen parameter space. As mentioned in the introduction we will investigate how well the supernova and neutrino parameters can be determined by the observation of neutrinos from a galactic SN by a future megaton scale Water-Cherenkov detector. We will take into account all major parameters (with exception of the neutrino mass hierarchy, here assumed to be normal) that can influence the neutrino flux, thus expanding the parameter space as compared to earlier works. We vary a total of 10 parameters to be fitted by data: the $\nu_e$, $\nu_\bar{e}$ and $\nu_x$ luminosities ($L_e$, $L_\bar{e}$ and $L_x$), mean energies ($\langle E_e \rangle$, $\langle E_\bar{e} \rangle$ and $\langle E_x \rangle$), pinching factors ($\beta_e$, $\beta_\bar{e}$ and $\beta_x$), 9 SN parameters, and a single neutrino quantity, $\sin^2 \theta_{13}$.

We will assume that the initial supernova neutrino fluxes emitted at the respective neutrino-spheres can be parametrized with the spectrum as suggested in Ref.\cite{21}. Thus, for each neutrino specie $i = e, \bar{e}, x$, we assume the energy spectrum to be of the form

$$\phi_{i}^{0} = \frac{\beta_{i}^{\beta_{i}+1}}{\Gamma(\beta_{i}+1) \langle E_{i} \rangle^{\beta_{i}+1}} \left(\frac{E_{i}}{\langle E_{i} \rangle}\right)^{\beta_{i}-1} \exp(-\beta_{i} E_{i} / \langle E_{i} \rangle) ,$$

where $\beta_{i}$ is the pinching parameter, $E$ the neutrino energy and $\langle E_{i} \rangle$ the $\nu_{i}$ mean energy. For $\beta_{i} \geq 3$ the spectrum is pinched with suppressed low and high energy tails, whereas for $\beta_{i} \leq 3$ the spectrum is anti-pinched (broader). The $\nu_x$ neutrinos decouple at a smaller radius and will therefore be hotter. Due to different charge-current interaction also the electron antineutrinos will decouple before the electron neutrinos. Correspondingly, a hierarchy of the form $\langle E_{e} \rangle \leq \langle E_{\bar{e}} \rangle \leq \langle E_{x} \rangle$ is expected.

The unoscillated flux at distance $D$ from the SN is given by

$$F_{\nu_{i}}^{0} = \frac{L_{i}}{4\pi D^{2}} \phi_{i}^{0} (E) ,$$

and we will fix the distance to 10 kpc. The neutrinos which are emitted from the interior of the star may undergo various flavor transitions due to MSW matter effects, when passing through the outer layers of the supernova. For the normal hierarchy, the the $\nu_e$ and $\bar{\nu}_e$ survival probabilities, $P_{ee}$ and $P_{\bar{e}e}$, respectively, are approximated by

$$P_{ee} \simeq P_{H}|U_{e2}|^{2} + (1 - P_{H})|U_{e3}|^{2},$$

$$P_{\bar{e}e} \simeq |U_{e1}|^{2},$$

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where $U_{\alpha,i}$, $\alpha = e, \mu, \tau$, $i = 1, 2, 3$ are the Maki-Nakagawa-Sakata neutrino mixing matrix elements and we have used here the standard parameterization. We have fixed the value of the solar mixing angle $\theta_{12} = 0.575$ rad ($\sin^2 \theta_{12} = 0.3$) and the atmospheric mass squared difference $\Delta m_{31}^2$ is set to be $3 \times 10^{-3}$ eV$^2$, as their variation have little impact on the final neutrino fluxes. We will disregard Earth matter effects and therefore the exact value of solar mass squared difference is irrelevant. Also the value of the atmospheric mixing angle is not relevant as the muon and tau neutrinos are indistinguishable. $P_H$ is the hopping probability that can be written as

$$P_H = \exp \left[-\sin^2 \theta_{13} \left(\frac{1.08 \cdot 10^7}{E}\right)^{2/3} \left(\frac{|\Delta m_{31}^2|}{10^{-3}}\right)^{2/3} 4^{1/3}\right].$$

The final fluxes arriving at Earth are simply given by

$$F_{\nu_e} = F^0_{\nu_e} P_{ee} + F^0_{\nu_x} (1 - P_{ee}),$$
$$F_{\bar{\nu}_e} = F^0_{\bar{\nu}_e} P_{\bar{e}\bar{e}} + F^0_{\bar{\nu}_x} (1 - P_{\bar{e}\bar{e}}),$$
$$F_{\nu_\mu} + F_{\nu_\tau} = F^0_{\nu_e} (1 - P_{ee}) + F^0_{\nu_x} (1 + P_{ee}),$$
$$F_{\bar{\nu}_\mu} + F_{\bar{\nu}_\tau} = F^0_{\bar{\nu}_e} (1 - P_{\bar{e}\bar{e}}) + F^0_{\bar{\nu}_x} (1 + P_{\bar{e}\bar{e}}).$$

These neutrino fluxes depend on our 10 dimensional parameter space, which is given by

$$\langle E_i \rangle, \, L_i, \, \beta_i, \, \, \, i = e, \bar{e}, x$$
$$\sin^2 \theta_{13}$$

and we refer to the first 9 parameters as the SN parameters.

There are four known detection channels for a water-Cherenkov detector: the dominant inverse beta decay (IB); the charge current on oxygen (CC-O); the neutral current on oxygen (NC-O) and the elastic scattering on electrons (ELAS). For a detailed description of these four possible channels in a Water-Cherenkov detector, please see [15].

We will analyze three different cases for fitting the supernova and neutrino parameters.

- **Case A:** We assume that only the inverse beta decay channel is available. In this case there is no dependence on the $\nu_e$ parameters. Moreover, since we consider only the normal hierarchy the allowed parameter space will even be independent of the Chooz angle ($\theta_{13}$), since $P_{ee}$ is constant (see eg. Fig.1 of Ref.[15]). Therefore, in this case only 6 parameters are left to be determined by data.

- **Case B:** Here we will assume that there are four available detection channels and the detected neutrino fluxes are sensitive to all 10 parameters.
Case C: In this case we also assume that all four detection channels are available, but we impose the constraint $L_e = L_{e\bar{e}}$, leaving us with 9 free parameters.

The enormous flux of neutrinos which will arrive at the Earth from a galactic supernova, makes case A a highly pessimistic scenario, as it seems very likely that it will be possible to measure and also separate at least some of the other detection channels. Our main motivation for including this case is in order to be able to compare to previous works. Similarly, the condition $L_e = L_{e\bar{e}}$ has been used in previous works, so we include this case for easy comparison. We would like to note that we assume that all the four channels can be completely separated, like it was done in Ref. [15]. This is certainly not a realistic assumption but it allows us to establish what would be the best attainable results for a future Water-Cherenkov detector. We should however point out that with the addition of gadolinium [23] it should be at least possible to do a fairly good separation by using the known directional forms of each event type.

We simulate the signals for a given set of input values of the 10 parameters of Eq. (11) and try to find the allowed regions of these parameters minimizing a $\chi^2$ function. For this purpose we define the simple $\chi^2$ function

$$\chi^2 = \sum_{i=1}^{N_{\text{bin}}} \frac{(N_{i}^{\text{th}} - N_{i}^{\text{obs}})^2}{N_{i}^{\text{obs}}} ,$$

where $N_{i}^{\text{obs}}$ and $N_{i}^{\text{th}}$ are, respectively, the simulated and fitted number of events in the $i$-th energy bin, and we take the number of bins, $N_{\text{bin}}$, to be 40. All bins have a width of 2.5 MeV and we set the threshold at 5 MeV. For case A, the $\chi^2$ function only includes events from the inverse beta decay channel;

$$\chi^2_{\text{Case A}} = \chi^2_{\text{IB}} ,$$

whereas we use all four available channels for cases B and C

$$\chi^2_{\text{Case B, Case C}} = \chi^2_{\text{IB}} + \chi^2_{\text{CC-O}} + \chi^2_{\text{NC-O}} + \chi^2_{\text{ELAS}} ,$$

and we calculate the $3 \sigma$ allowed areas in the projected two dimensional space using 2 degrees of freedom.

Naturally the allowed areas will depend on the assumed true values of the parameters, what we will refer to as the input parameters. We will just look at one illustrative example and for that we have chosen the input values $\langle E_e \rangle = 12$ MeV, $\langle E_{e\bar{e}} \rangle = 15$ MeV, $\langle E_x \rangle = 18$ MeV and furthermore we take all the integrated luminosities to be equal; $L_i = 0.5 \times 10^{32}$ Ergs. The spectral indices are assumed to be $\beta_e = 5$, $\beta_{e\bar{e}} = 5$ and $\beta_x = 4$. Finally, the neutrino parameter $\sin^2 \theta_{13} = 10^{-6}$. These values are all chosen equal to the input values of
Ref. [17] as this allows for an easier comparison. For the detector size we have also taken the same value as in Ref. [17] with a fiducial volume of 720 kton. This value is somewhat high but it should be remembered that the fiducial volume for a galactic supernova detection is larger than that for eg. atmospheric neutrino detection and therefore this value is not unrealistic.

We will use a MINUIT [18] based code for finding the $3\sigma$ allowed regions. A number of constraints on the parameters have also been implemented;

$$5 \leq \langle E_e \rangle / \text{MeV} \leq 17,$$

$$5 \leq \langle E_i \rangle / \text{MeV} \leq 35, \ i = \bar{e}, x$$

$$0 \leq \beta_i \leq 25, \ i = e, \bar{e}, x$$

such as to avoid extremely unrealistic values. Also, for the case A, the ratio between the neutrino fluxes are not allowed to become too large. Especially this means that we take the $\chi^2$ minimum value to be the minimum which is the local minimum closest to the input values. To be specific, we set all parameters to their input values and let MINUIT fall into the local minimum from there.

### III. DISCUSSION

In this section we will present our results, and also compare them to previous works. Furthermore, we investigate ways to break down the degeneracies which exists in the parameter space.

In Fig. we show the $3 \sigma$ allowed areas for the three cases A, B and C in six different planes of two parameter projections. It is seen that the allowed regions are quite different in each case. The allowed regions for case A are the largest and the ones for case C are the smallest, which is clear from the definitions of the three cases. The only exception is for $\beta_{\bar{e}}$ which remains completely undetermined even in case C.

By looking at the contours for case A, we see that we overall agree with the findings of Ref. [17]. Indeed the degeneracy in the $\langle E_x \rangle - \beta_x - L_x$ parameters is also clearly exhibited in Fig. (panels 3 and 5). Although, there are minor differences in the size of the areas, the shapes of the allowed areas are the same as in Ref. [17]. The small differences must be due to slightly different analysis procedures. For instance, the width of the energy bins are different in the two analysis. As already explained there is no sensitivity to the $\nu_e$ parameters as well as to $\sin^2 \theta_{13}$ in case A. The determination of the $\bar{\nu}_e$ parameters are not impressive but still fairly good. The uncertainty on the value of $\langle E_{\bar{e}} \rangle$ is about 5 %, $L_{\bar{e}}$ is determined within 50% and $\beta_{\bar{e}}$ is consistent with values in two distinct island, one including the input value. Clearly the most important and problematic fact is that the $\nu_x$ parameters are left undetermined.
FIG. 1: The 3 σ allowed regions for the three cases A, B and C as explained in section II. The outermost dark (blue) contour assumes that only the inverse beta decay channel is available, and thus only depends on 6 variables (case A). The middle dark (red) contour assumes that all 4 channels are available and in this case there is in total 10 free variables (case B). For the light (cyan) contour we also assume that all 4 detection channels are available, but we implement the condition $L_e = L_{\bar{e}}$, thus leaving only 9 free variables (case C). For the case A, there is no dependence on the electron-neutrino parameters and therefore this case does not constrain any of the variables in frame 1.
The degeneracy extends over the whole realistic range of parameter. Therefore, without further input, it is unfortunately impossible to say something conclusive about the initial $\nu_x$ flux.

In case B we find rather large allowed regions for some of the parameters, but considerably smaller than for case A. Although the degeneracy is partially broken, there is still correlation among the $\nu_x$ parameters so a large elongated area is still seen in panels 3 and 5 of Fig.1. We believe that the main reason for the fact that much of the degenerate area survives is simply that the other detection channels (CC-0, NC-0 and ELAS) have a much smaller number of events. For an idea of the orders of magnitudes of each event type, please see Table II in Ref.15. By comparing with the results of Ref.15 it becomes clear that the inclusion of the pinching parameters have a very large impact on the size of the allowed areas for the average energies, in particular that of the $\nu_x$ neutrinos. This is also illustrated in Fig.2. The fact that, to a large extend the degeneracy is still left even after including all four detection channels, should be viewed as a large obstacle for the determination of the $\nu_x$ parameters. Below we will study some possible ways of overcoming this degeneracy.

For case B it is seen that the determination of the value of $\sin^2 \theta_{13}$ is quite good. It should be remembered that for values of $\sin^2 \theta_{13} \leq 10^{-6}$ there is no dependence on this variable as the H resonance is always non-adiabatic. Furthermore, the values of the $\nu_e$ parameters are somewhat better determined than for the case A. The value of $\langle E_{\bar{e}} \rangle$ is determined within about 3% and $\beta_{\bar{e}}$ is constrained within about 8%. The electron-neutrino parameters are rather difficult to determine as there is not enough statistic in the $\nu_e$ detection channels. Nevertheless, in view of this, the allowed area of $\langle E_{\bar{e}} \rangle$ is rather small, $\langle E_{\bar{e}} \rangle$ is determined within 13% but one can give only a lower limit for the luminosity $L_e > 0.1 \times 10^{32}$ ergs.

Let us next look at the contours for the case C in Fig.1. These contours are in general very small. In particular, the degeneracy in the $\nu_x$ variables is broken down and the averaged $\nu_x$ energy is determined within 5%, which is a big improvement when compared to case B. Also the constraints on $\langle E_\nu \rangle$ is somewhat improved, whereas the determination of $\theta_{13}$ and $\beta_{\bar{e}}$ have not really improved as compared to the case B. In contrast with the other cases, all the luminosities are constrained within about 10% for the case C.

Of course the assumption of identical electron neutrino and electron antineutrino integrated luminosities can be criticized as it is not based on a strictly physical condition. Although the two luminosities are expected to be at least of the same order of magnitude, the ratio of the two is still varying a lot in different SN simulations. In Ref.15 this assumption was made in order to decrease the number of free parameters. Even if this assumption...
seems rather innocent we see that it has a large impact specially on the allowed region for the \( \nu_x \) parameters. In fact, from Fig. 2 it is clear, that either the assumption, \( L_e = L_{\bar{e}} \) or the fixing of the \( \beta_i \)'s, significantly diminish the allowed average neutrino energies, most pronounced for \( \langle E_x \rangle \). One can also observe, by looking at panels 3 and 5 of Fig. 1, that the fixing of \( \beta_x \) alone would allow one to determine quite well all the mean energies and luminosities (as long as the total luminosity is independently constrained).

![Fig. 2: The innermost dark (black) contour shows the 3\( \sigma \) allowed region when fixing the \( \beta_i \)'s at their input values. We assume that all four channels are available. The outermost dark (red) and the light (cyan) contours are identical to the ones in Fig. 1 and shown for easy comparison.](image-url)

However, neither the constraint \( L_e = L_{\bar{e}} \) nor the fixing of the pinching parameters, influence the allowed region for the Chooz angle in a sizable way. In the left panel of Fig. 2, we have explicitly shown this by calculating the contour obtained when fixing the \( \beta_i \)'s at their input values. Therefore, a nice feature that is kept even when including the pinching parameters is the rather fine determination of the Chooz angle as seen in the last panel of Fig. 1. This is particularly important, since for very small \( \theta_{13} \), i.e. \( \sin^2 \theta_{13} \lesssim 10^{-4} \), the observation of a galactic supernova is, at least at present, the only way of determining (or constraining) \( \theta_{13} \).

Another important question that arises is whether the hierarchy can be determined by the observation of a galactic supernova? Here we consider simply the observation of the neutrino fluxes without the measurement of Earth matter or shock wave effects. Let us briefly comment on this point. In this work we have fixed to neutrino mass hierarchy to normal. But, let us in the following discussion assume that the collective neutrino-neutrino effects are negligible even for the inverted neutrinos mass hierarchy. In Ref. [15], a discussion of
the prospects for determining the neutrino mass hierarchy is presented. In this work the $\beta_i$'s are fixed and also the condition $L_e = L_{\bar{e}}$ has been implemented. Despite these assumptions it seems clear to us that the results presented in Ref. [15] about the determination of the neutrino mass hierarchy will be valid even for a 1D dimensional parameter space (i.e. that of Eq. (11) along with the neutrino mass hierarchy). The hierarchy will influence the total number of events in each channel for large values of $\sin^2 \theta_{13}$ (above $10^{-5}$) (see Table II in Ref. [15]). Therefore the hierarchy can be determined by the combined use of all four channels in a Water-Cherenkov detector. The degeneracy that we have observed in the $\nu_x$ parameters on the other hand maintains the total number of events in each channel almost fixed and only changes the spectral form at low energies. Henceforth, we expect that for large values of $\sin^2 \theta_{13}$, the neutrinos mass hierarchy can be determined with very large statistic. Moreover, in the case of the spectral swapping due to collective effects, as described in Ref. [14], it might even be possible to determine the hierarchy for small values of $\sin^2 \theta_{13}$.

One can wonder to what extend our results depend on the particular input we have chosen. Especially if the average energies are further apart, like eg. $\langle E_x \rangle \gg \langle E_{\bar{e}} \rangle$, would the degeneracy in the $\nu_x$ parameters be much milder? Such investigation are left for a future paper.

In the following we will discuss ways to break down the $\langle E_x \rangle$, $\beta_x$ and $L_x$ degeneracy. As the main problem is pinning down the value of $\langle E_x \rangle$, we will focus on the third frame in Fig. 1. One possibility is that some robust features for the supernova parameters will emerge from supernova simulations in the future. These features can then be safely implemented in the fitting procedure and used to constrain the parameter space. This can be thought of as a standard supernova model, in analog to the use of the standard solar model when studying solar neutrinos. If a standard supernova model will be developed one can include a penalty in the $\chi^2$ function when eg. the ratio $L_e/L_{\bar{e}}$ differs from its central value as predicted by this standard model. We do not have such a standard model at our disposal yet, so to demonstrate our point we study some simple cases.

Clearly, if the supernova simulations could tell us rather precisely the value of the pinching parameters, then this degeneracy would be broken. It should be remembered that we fit to the total number of events in the cooling phase. A slightly time varying spectral index, will cause the time integrated spectra to be broader. Therefore, we can expect lower values for the $\beta_i$’s in the time-integrated spectra. By combining the information of the panels 3 and 5 in Fig. 1 one can directly relate constrains on $\beta_x$ to constraints on $\langle E_x \rangle$. Notice that if we take the range value of 3–6, which is presently suggested by the SN simulations, then the constraints on the average $\nu_x$ energy is not improved for the case B.

An appealing possibility is to use ratios of the individual luminosities. Let us define two
FIG. 3: The $3\sigma$ allowed regions for various assumptions about constraints on the supernova luminosity ratios. We assume that all four channels are available in all the contours. The outermost dark (red) and the second smallest (cyan) contours are identical to the respectively case B and case C, in Fig.II and shown for easy comparison. For the second largest (purple) contour we have constrained $0.5 \leq \xi_i \leq 2.0$ and for the third largest (green) contour $2/3 \leq \xi_i \leq 3/2$, where $i = 1, 2$. The smallest (black) contour is calculated assuming $\xi_2 = 1$.

The physical processes that are behind the production of the neutrino flux are related to known physics. Nevertheless, there are a number of processes, such as nucleon-nucleon bremsstrahlung ($NN \rightarrow NN\nu\bar{\nu}$), neutrino-antineutrino annihilation (eg. $\nu_e\bar{\nu}_e \rightarrow \nu\bar{\nu}$) and various scattering reactions between neutrino and antineutrinos of different flavors, which of course complicates the calculation of the neutrino emission. From the present supernova simulations it seems that these ratios can at most be two and should be larger than one-half. It is important to notice that when increasing the value of $\langle E_x \rangle$ within the degeneracy area, the ratios $\xi_1$ as well as $\xi_2$ also increases. In general, for values $\langle E_x \rangle \leq 18$ MeV the two ratios will be below one and for $\langle E_x \rangle \geq 18$ MeV the ratios will be larger than one. In Fig.3 we illustrate how the constrains on these ratios affect the allowed region. The second largest area in Fig.3 corresponds to the suggestions from present supernova simulations and

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a smaller allowed area is obtained. In Fig. 3 we also show the contours when fixing \( \xi_1 = 1 \) (the same as the case C of Fig.1) as well as \( \xi_2 = 1 \). In these cases a very good determination of \( \langle E_x \rangle \) is possible. In conclusion we find that constraining the luminosity ratios can be helpful for pinning down the \( \nu_x \) supernova parameters.

![Graph showing the allowed regions for different cases.](image)

FIG. 4: The dashed dark (black) contour shows the 3\( \sigma \) allowed region when fixing the total luminosity of the supernova to the input value of \( 3 \times 10^{53} \) ergs and leaving all parameters free. We assume that all four channels are available. The outermost dark (red) and the light (cyan) contours are identical to the ones in Fig.1 and shown for easy comparison.

Next, we will look at the possibility that the total energy liberated in the supernova, which we will refer to as \( L_{\text{tot}} \), has been already determined by some other method. In Ref. [22] it is suggested that a measurement of the electron neutrinos from the neutronization burst can be used to determine the distance to the supernova. This along with a optical observation of the SN, might be used to predict the total energy liberated. In Fig.4 we present the results for fixing \( L_{\text{tot}} \) to its input value. This reveals that even in this extreme situation the improvement in the allowed region is minor. Indeed, the neutral currents channels, the NC-O and in parts also the ELAS channel, are already constraining the total luminosity. In case B the total luminosity is determined to about 4\% accuracy (assuming that the distance is known). Clearly constraining \( L_{\text{tot}} \) is not very helpful for determining the supernova parameters. In fact, one might even expect that the fitting procedure (case B) will provide the best determination of \( L_{\text{tot}} \).
IV. CONCLUSION

We have shown that the inclusion of all supernova and neutrinos parameters is important for determining the allowed regions that can be obtained from the observation of the neutrino burst from a galactic supernova. We use a total of 10 parameters ($\langle E_e \rangle$, $\langle E_{\bar{e}} \rangle$, $\langle E_x \rangle$, $L_e$, $L_{\bar{e}}$, $L_x$, $\beta_e$, $\beta_{\bar{e}}$, $\beta_x$ and $\sin^2 \theta_{13}$) and four detection channels (IB, CC-O, NC-O and ELAS) that can be observed by a Water-Cherenkov detector to fit to such an observation. The degeneracy between $\langle E_x \rangle$, $\beta_x$ and $L_x$ when using only the inverse beta decay channel [17], is only broken mildly by the inclusion of the other channels in a Water-Cherenkov detector. This is mainly due to the fact the number of events from these other channels are at least one order of magnitude smaller, and thus, in principle, the degeneracy could be broken by including more channels if statistics was not a limitation. Unfortunately, the supernova parameters are very difficult to determine due to this degeneracy. Especially, the $\nu_x$ parameters cannot be properly identified. We have discussed ways that supernova simulations can help overcoming this problem. A particular good way, seems to be to constrain the ratio of the integrated luminosities of the neutrino flavors.

We have demonstrated that the so-called Chooz angle, $\theta_{13}$, can in principle be determined very well even when freely varying all parameters (including the pinching parameters). In fact, whether or not the pinching parameters are freely varied, does not influence much the allowed $\theta_{13}$ region. This is so because the Chooz angle influence the ratios of the total number of events in each of the four different channels, whereas the degeneracy caused by the pinching parameters maintains the total number of events almost intact. In the same way we expect that the neutrino mass hierarchy can be determined for large value of $\sin^2 \theta_{13}$ by a galactic supernova.

On the other hand, data can only constrain the SN parameters describing the $\bar{\nu}_e$ flux ($\beta_{\bar{e}}$, $\langle E_{\bar{e}} \rangle$ and $L_{\bar{e}}$) and $\langle E_e \rangle$. Without extra assumptions on the luminosities (ratios and/or total) or $\beta_x$ one cannot determine $L_e$, $L_x$ and $\langle E_x \rangle$, even if the four detection channels could be completely separated.

To summarize, the neutrino parameters can be determined quite precisely, whereas it is more difficult to determine the supernova parameters.

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