Fractionalization Waves in Two-dimensional Dirac Fermions:
Quantum Imprint from One Dimension

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Transport of strongly correlated fermions in more than one spatial dimension (1D) remains poorly understood. We consider an exactly solvable case, in which correlations in a system of decoupled 1D chains are imprinted via quantum quench upon two-dimensional Dirac fermions. As a probe, we calculate the density waves emitted from a bump. A nonzero fermion anomalous dimension in the initial state launches relativistic “fractionalization waves” along the chains, while coupling noninteracting chains induces perpendicular dispersion. These could be easily distinguished in an ultracold gas experiment.

A primary focus of modern condensed matter physics is the collective behavior of strongly interacting (“correlated”) matter, and in particular transport in strongly correlated electron systems. Correlations can destroy the notion of well-defined quasiparticles; without the latter, the conventional frameworks that predict transport from the motion of individual particles (e.g., the semi-classical kinetic equation, or the quantum non-linear sigma model \textsuperscript{[1]}) may not apply. In some situations, however, a description in terms of collective dynamics is still tractable: this includes bosonization of one-dimensional (1D) gapless fermion systems \textsuperscript{[2–4]}, and hydrodynamics for high-temperature carriers in higher dimensions \textsuperscript{[5]}.

The quasiparticle picture breaks down when the electron operator acquires an anomalous dimension, due to interactions \textsuperscript{[6, 7]}. This smearsthe one-particle spectral function at arbitrarily low energies, interpreted as the “fractionalization” of the electron into collective excitations. In the absence of a momentum relaxation mechanism, however, this one-particle property typically does not directly affect transport, a two-particle observable. Indeed, although a 1D Luttinger liquid generically exhibits charge fractionalization, zero temperature dc transport through ideal leads is indistinguishable from that through a noninteracting single-channel quantum wire \textsuperscript{[8–10]}.

In this Letter, we consider a case where a nonzero fermion anomalous dimension directly determines density wave dynamics in a two-dimensional (2D) fermion system. Here Luttinger liquid correlations in a system of initially decoupled 1D chains are \textit{imprinted} upon two-dimensional Dirac fermions. This is accomplished via a quantum quench \textsuperscript{[11–20]} that couples together the chains into a 2D pi-flux lattice model (see Fig. 1). To probe the dynamics, we calculate the density waves emitted from an initial density bump \textsuperscript{[21–31]}. We show that a nonzero initial-state-fermion anomalous dimension launches relativistic “fractionalization waves” along the chains, shown in Fig. 2. By contrast, the same quench performed from initially noninteracting chains induces dispersive propagation perpendicular to the chains, see Fig. 3. The key result of this work is that the orthogonal motions of the fractionalized and noninteracting cases should be easily distinguishable in an ultracold fermion experiment.

Similar fractionalization waves dubbed “supersolitons” were previously predicted in 1D quenches, including the continuum sine-Gordon model \textsuperscript{[22]} and the XXZ chain \textsuperscript{[23]}. As in those studies, correlations shape the initial condition, but we ignore interactions in the post-quench evolution. In an ultracold fermion gas it might be possible to turn off the interactions at the time of the quench, but this is not a requirement for us. Interactions are strongly irrelevant (in the sense of the renormalization
group) in the post-quench band structure. Our results should hold over a tunable transient window $0 < t < \tau_{\text{ad}}$, where $1/\tau_{\text{ad}}$ is the particle-particle scattering rate determined by the interactions and the post-quench energy density.

Signatures of Luttinger-liquid physics in ultracold fermion atoms have recently been detected \cite{32, 33}. For our 1D to 2D quench setup, it is crucial that the dynamics are generated by fermions, not bosons. The pi-flux lattice has been realized in various experiments \cite{34–36}. Quenches between 1D chains and 2D planes have been performed in spin-1/2 Hubbard lattice gases \cite{37}. For our 1D to 2D quench setup, it is crucial that the dynamics discussed here.

The system is assumed to initially have $b = 0$, so that the lattice reduces to a set of uncoupled 1D chains. The noninteracting band structure consists of vertical nodal lines. Via instantaneous quantum quench, $b$ is switched to a positive, nonzero value. The pi-flux band structure gaps out the nodal lines except for a pair of Dirac points at \( \{ k_x, k_y \} = \{ \pm \pi/2a, 0 \} \).

We assume that the main effect of the global quench is to excite particle-hole pairs along the nodal lines of the prequench band structure. We therefore retain momentum modes along narrow channels including these, \( \{ k_x \pm \pi/2a \} \leq \Lambda, |k_y| \leq \pi/2a \). Here $\Lambda \ll \pi/a$ is a momentum cutoff. Eq. (1) can then be approximated as

\[
H \simeq v_F \int_{-\Lambda}^{\Lambda} \frac{dk_x}{2\pi} \int_{-\pi}^{\pi} \frac{dk_y}{2\pi} |\psi(\mathbf{k})\rangle \langle \Phi(\mathbf{k}) | + H_I, 
\]

where \( v_F = 2Ja \) is the maximum band velocity. The field \( \langle \Phi(\mathbf{k}) | \rangle = \psi_{\sigma,\tau}(\mathbf{k}) \) is a four-component spinor. The Pauli matrices $\sigma^1, \sigma^2, \sigma^3$ act on the space of right ($\sigma^3 = 1$) and left ($\sigma^3 = -1$) movers in the initial decoupled chains; this is not equivalent to the space of right and left nodal lines.

\[m(k_y) \equiv (b/a) \sin(k_ya) \quad \text{im} \]

in Eq. (2b) plays the role of a $k_y$-dependent “mass,” when the system is viewed as a collection of decoupled 1D chains. Linearizing near $k_y = 0$ with $b = 1$ would give isotropic massless 2D Dirac fermions.
Relative to the homogeneous, noninteracting lattice model in Eq. (1), we have incorporated two additional perturbations on the second line of Eq. (2a). The first is an inhomogeneous external potential $\Phi(x, y)$. We assume a localized potential profile in the prequench state. Via the axial anomaly, this induces an initial density inhomogeneity in the decoupled chains of the form $\rho(x, y) = -\kappa \Phi(x, y)$, where $\kappa$ is the compressibility. After the quench we will set $\Phi = 0$, and we will monitor the evolution of $\rho(t, x, y)$ as a probe of the dynamics.

Although Eq. (1) with $b = 0$ describes decoupled chains for spinless fermions, the field $\psi_{x, \tau}$ consists of four, not two components. The synthetic $\tau$-spin degree of freedom is an artifact of folding into the RBZ, necessary for describing the pi-flux lattice. The term $H_I$ in Eq. (2a) encodes generic short-ranged intrachain fermion-fermion interactions. It is well-known that the low-energy theory for a single-channel, spin-1/2 SU(2)-symmetric quantum wire (described via a four-component field $\psi$) admits four independent local, four-fermion interaction operators $\mathcal{I}_k$. This includes spin current-current and charge unklapp interactions, and these can gap out the spin or charge degrees of freedom. In our case the coupling constants of these operators should be tuned precisely to zero, because these describe interactions between pairs of chains. The admissible interactions [charge-current-current and $U(1)$ stress tensor operators] set the charge velocity and Luttinger parameter $K$ in the prequench Luttinger liquid state of the decoupled chains $[33, 35]$.

At time $t = 0$, we quench on the interchain coupling $b > 0$, and turn off the potential $\Phi$ and interactions in $H_I$. (In fact, we argue later that interactions can remain in place, and will produce a negligible effect on the dynamics up to time $\tau_{\text{at}}$, defined below). Then the time-evolving density profile is determined by the convolution

$$
\rho(t, x, y) = \int dx_1 dx_2 dy_1 \text{Tr} \left[ \hat{G}(t, x_1, y_1) \hat{G}(t, x_2, y_1) \right] \times C_\Phi(x - x_1, x - x_2; y - y_1),
$$

where $\hat{G}(t, x_1, y_1)$ is the causal Green’s function associated to $\hat{h}$, and $C_\Phi(x_1, x_2; y)$ describes the static one-particle fermion correlation function in the initial state at linear order in the external potential $\Phi$, given by $[23, 38]$.

$$
C_\Phi(x_1, x_2; y) = \frac{c_\eta}{2} \left[ \frac{\alpha^2}{(x_{12})^2 + \zeta^2} \right] \frac{\eta^2}{\kappa} \int_{x_{12}} dx K \Phi(x, y),
$$

where $x_{12} = x_1 - x_2$. Here $c_\eta$ and $\alpha$ are positive constants, while $\eta$ is the fermion anomalous dimension. The latter is $\eta = (1/2\beta)(K + K^{-1} - 2)$, where $K$ is the Luttinger parameter $[2, 3, 33]$. The exponent $p = 1 (p = 2)$ for spinless (spin-1/2) fermions within each chain. $K = 1$ gives $\eta = 0$ (noninteracting chains); otherwise $\eta > 0$.

The parameter $\zeta$ is a short-distance regularization that can affect the dynamics at long times $[23]$. Eq. (5) is appropriate for half-filling ($k_F = 0$).

**Results.**—We numerically integrate Eq. (4), using (5) and assuming a Gaussian potential

$$
\kappa \Phi(x, y) = Q (\pi \Delta_x \Delta_y)^{-1} e^{-\left( x/\Delta_x \right)^2 - (y/\Delta_y)^2}.
$$

We set the parameter $\alpha = a = 1$ in Eq. (5) $[38]$. As is clear from Figs. 2 and 3, the qualitative behavior of the post-quench density profile depends crucially on whether or not the initial state is fractionalized. In the initially fractionalized case $[\eta > 0$ in Eq. (5), Fig. 2], the density develops collective excitations that propagate horizontally along the chains at the maximum band velocity $v_F$. These fractionalization waves (“supersolitons” $[22, 23]$) retain their shape as they travel and exhibit power-law growth of amplitude with time. Supersolitons possess positive peaks and negative-density troughs; the total particle number induced by the initial potential on top of the filled Fermi sea is preserved at all times $[38]$. The results in Fig. 2 obtain from Eq. (5) with no short-distance regularization, $\zeta = 0$. Nonzero $\zeta$ can arise due...
to the effects of irrelevant operators [23], but we show in Ref. [28] that qualitatively identical dynamics obtain in this case except at long times, wherein the supersoliton growth is curtailed [23]. By contrast, for the noninteracting initial condition ($\eta = 0$, Fig. [3]), there is no supersoliton and the initial density disperses vertically, perpendicular to the chains.

The density dynamics in Figs. [2] and [3] should be contrasted with one-particle quantum mechanics. The same Green’s function $\hat{G}(t, x, y)$ that enters into Eq. (1) determines the evolution of a Gaussian single-particle wavepacket. Since $\hat{h} \simeq -i\partial^3\partial_x - i\partial^2\partial_y$, the result is a circular wavefront propagating at the “speed of light” [28]. Instead, the fractionalized quench gives $x$-directed supersolitons, while the noninteracting quench gives $y$-dispersing propagation. The difference between the latter and one-particle quantum mechanics is due to Pauli blocking [23].

Further insight into the quench dynamics obtains from the Wigner distribution due to the initial Gaussian bump,

$$\delta n_+(v_x, v_y; R_x, R_y) \propto J(v) \int d^2q \, e^{i\mathbf{q} \cdot \mathbf{R}}$$

$$\times \langle a^\dagger [k(v) - q/2] a [k(v) + q/2] \rangle \phi, \quad (7)$$

where $a(k)$ annihilates a pi-flux conduction band fermion with momentum $k$, and $J(v) \equiv |\partial k_\mu/\partial v_\mu|$ is the Jacobian relating the post-quench band velocities $v_{x,y} \equiv \partial k_{x,y} \sqrt{k_x^2 + m^2(k_y)}$ to the momenta $k_{x,y}$. The expectation value in Eq. (7) is computed at time $t = 0$, i.e. using Eq. (5). The Wigner distribution is plotted for variable $\eta$ in Fig. [4]. We observe a pronounced difference between the fractionalized and non-fractionalized cases. The distribution for the case with $\eta > 0$ has a diverging density of $x$-direction velocities approaching the maximum band velocity. On the other hand, the noninteracting $\eta = 0$ case has a velocity distribution strongly localized to $x$-velocities close to zero. The absence of large $x$-velocities in the latter is due to Pauli blocking: for $k_y = 0$, the momentum $|q_y|$ must exceed $2|k_x|$ in order for the pair of operators in Eq. (7) to create a particle-hole pair in the Fermi sea. Large $|q_x| > 1/\Delta_x$ (and therefore large $v_x$) is suppressed by the initial density profile [Eq. (6)]. Fractionalization ($\eta > 0$) circumnavigates the Pauli blocking restriction on large $x$-velocities [28]. This is because the fermions responsible for the post-quench propagation are not locally related to the effectively noninteracting, but fractionally charged constituents that define the prequench vacuum state [23]. We also note that the kinematic condition

$$v_x^2 + v_y^2/b^2 \leq 1 \quad (8)$$

implies that the accumulation of the Wigner distribution at $v_x = 1$ requires $v_y = 0$, and thus explains why the supersoliton is stable to dispersion in the $y$-direction.

Window for collisionless dynamics.—The correlator in Eq. (3) with $\eta > 0$ arises due to generic short-ranged interactions in the initially decoupled chains. The postquench dynamics captured by Eq. (4) ignore interactions in the subsequent time evolution.

Beyond the density bump dynamics explored here, the main bulk effect of the global quench is to generate a finite density of particle-hole pairs, corresponding to a nonzero average energy per particle. If the interactions are not turned off at the time of the quench, then the system is expected to eventually thermalize to a temperature corresponding to the injected energy density [15].

We can estimate the collision rate $1/\tau_{\text{col}}$ responsible for thermalization in the post-quench evolution. For the low-energy pi-flux Dirac fermions, a short-ranged lattice interaction carries units of energy $\times$ length$^2 \sim U_f a^2$, where $U_f$ is the lattice interaction energy. The subscript “$f$” denotes the interaction strength after the quench, which can differ from the prequench strength $\equiv U_i$. The postquench Fermi’s golden rule collision rate should be of order $1/\tau_{\text{col}} \sim [(U_f a^2)/(b v_F^2)]^2 \epsilon^3$, where $\epsilon$ is the characteristic energy per particle. The two factors of $b^{-1} v_F^{-2}$ arise from the density of states. For $k_F = 0$ (half filling), a crude estimate is $\epsilon \sim b v_F/a \sim b J$, so that

$$b J \tau_{\text{col}} \sim (J/U_f)^2. \quad (9)$$

FIG. 4: Wigner velocity distribution $\delta n_+(v_x, v_y; R_x, R_y)$ imprinted on the pi-flux band fermions at the time of the quench, induced by charge fractionalization and a Gaussian density bump in the initial state of decoupled chains. The distribution is evaluated at the center of the bump $R_x = R_y = 0$. Unlike continuum 2D massless Dirac fermions, the allowed velocities span a disk due to the lattice regularization in the $k_y$-direction, Eqs. (2a) and (8). The parameter $\eta$ is the fermion anomalous dimension [Eq. (3)]. For $\eta > 0$, there is a divergence of the $x$-velocities near the “speed of light” (band velocity $v_F = 1$). At $\eta = 0$ (noninteracting initial condition), large $x$-velocities are suppressed by Pauli blocking (see text).
In the post-quench evolution, time is measured in units of $1/bJ$ ($b = 1$ in Figs. 2 and 3). Eq. 1 implies that the window of time over which collisionless dynamics can occur is set by the dimensionless ratio of $U_f/J$. Small $U_f/J$ will induce a large collisionless window. By contrast, the anomalous dimension $\eta$ responsible for the supersoliton dynamics in Fig. 2 is a function of the ratio $U_i/J$. Taking the latter to be too small will result in $\eta \ll 1$. For spin-1/2 Hubbard chains with repulsive interactions and $U_i/J \sim 1$, it is possible to get $K$ close to 1/2 ($\eta = 1/8$) for particle densities very close but not equal to half-filling [3, 59]. In Ref. [38], we show for example that $\eta = 0.2$ still exhibits the supersoliton over the same time interval as Fig. 2.

A balance should be struck between minimizing collisions over a sufficiently long time window after the quench, and maximizing the correlations in the initial state. At the same time, in an optical lattice setup for ultracold fermions $U_f < U_i$, since lowering the tunneling barriers in the $y$-direction to couple the 1D chains together will necessarily “unsqueeze” the atoms in that direction, decreasing the on-site interaction energy. A further reduction of $U_f$ and (enhancement of $\tau_{el}$) is possible if at the time of the quench, the confinement is simultaneously reduced in the $z$-direction, perpendicular to the plane of the 2D lattice.

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I. Hamiltonian, sublattice operators, and $\tau$-SU(2) pseudospin

The lattice Hamiltonian in Eq. (1) of the main text can be rewritten in terms of operators $c_{A,B}$, which annihilate fermions on the A and B sublattices. These are respectively indicated by squares and circles in Fig. 1 (left panel). The Fourier modes of the sublattice operators carry momenta that span the reduced Brillouin zone (RBZ). Then Eq. (1) can be expressed as

$$H = 2J \int_{\text{RBZ}} \frac{d^2 k}{(2\pi)^2} \Psi^\dagger(k) \left[-\cos(k_x a) \hat{\sigma}^1 + a m(k_y) \hat{\sigma}^2 \right] \Psi(k), \quad \Psi(k) \equiv \begin{bmatrix} c_A(k) \\ c_B(k) \end{bmatrix}. \quad (S1)$$

Here the Pauli matrices $\hat{\sigma}^{1,2,3}$ act on the sublattice space, with $\sigma^3 = \pm 1$ corresponding to sublattices A and B, respectively. The “mass” $m(k_y)$ was defined by Eq. (3). Linearizing Eq. (S1) in the vicinity of the right and left nodal lines of the $b = 0$ prequench Hamiltonian (Fig. 1) gives

$$H \simeq v_F \int_{-\Lambda}^{\Lambda} \frac{dk_x}{2\pi} \int_{-\pi/a}^{\pi/a} \frac{dk_y}{2\pi} \Psi^\dagger(k) \left[k_x \hat{\sigma}^1 \hat{x}^3 + m(k_y) \hat{\sigma}^2 \right] \Psi(k), \quad \Psi(k) \equiv \begin{bmatrix} \Psi(k + \frac{\pi}{2a} \hat{x}) \\ \Psi(k - \frac{\pi}{2a} \hat{x}) \end{bmatrix}, \quad v_F = 2Ja. \quad (S2)$$

$\psi(k) \rightarrow \psi_{\sigma,\tau}(k)$ is now a four-component spinor. The Pauli matrices $\hat{\tau}^{1,2,3}$ act on the right/left nodal line space, with $\tau^3 = \pm 1$ corresponding to the nodal lines $k_x = \pm \pi/2a$.

The final form of the (unperturbed) Hamiltonian on the first and third lines of Eq. (2) obtains from Eq. (S2) after
a basis rotation, wherein

$$\psi(k) \Rightarrow \frac{1}{\sqrt{2}} (1 + i\sigma^2 \tau^3) \psi(k) = \frac{1}{\sqrt{2}} \begin{bmatrix} (c_A + c_B)(k + \frac{\pi}{2a}) \\ -(c_A - c_B)(k + \frac{\pi}{2a}) \\ -(c_A - c_B)(k - \frac{\pi}{2a}) \\ (c_A + c_B)(k - \frac{\pi}{2a}) \end{bmatrix} = \begin{bmatrix} R^{(i)}(k) \\ -L^{(i)}(k) \\ R^{(o)}(k) \\ L^{(o)}(k) \end{bmatrix}. \quad (S3)$$

Here, $R^{(i)}(k)$ and $R^{(o)}(k)$ denote right-movers in the prequench system of decoupled chains. The “$i$” and “$o$” superscripts distinguish modes whose $k_y$-values in the full Brillouin zone $|k_{x,y}| \leq \pi/a$ respectively reside inside or outside of the RBZ shown in Fig. 1. The shift of the outside modes by a sublattice reciprocal lattice vector into the reduced zone effectively sends $c_{A,B} \rightarrow \pm \epsilon_{A,B}$.

In the basis defined by Eqs. (2) and [S3], the $\sigma^{1,2,3}$ Pauli matrices act on the right/left-mover space, while the $\tau^{1,2,3}$ matrices grade the inner/outer mode space.

II. GREEN’S FUNCTIONS AND TIME EVOLUTION

The expression for the density $\rho(t, x, y)$ in Eq. (4) exploits the Heisenberg evolution of the Dirac operator $\psi(t, x, y)$ via the noninteracting pi-flux Hamiltonian $h(k)$ in Eq. (2b). This is given by the convolution

$$\psi(t, x, y) = \int dx_1 dy_1 \hat{G}(t, x - x_1, y - y_1) \psi(x_1, y_1), \quad (S4)$$

where $\psi(x, y)$ is the Schroedinger operator. Setting $v_F = 1$ [Eq. (2)], the causal Green’s function is

$$\hat{G}(t, x, y) = i \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} \int_{-\pi/(2a)}^{\pi/(2a)} \frac{dk_y}{2\pi} e^{-i\omega t + ik_x x + ik_y y} \left[ \frac{\omega + \sigma^3 k_x + \sigma^2 m(k_x)}{(\omega + i\eta)^2 - (k_x)^2 - m^2(k_x)} \right]$$

$$= \int_{-\pi/(2a)}^{\pi/(2a)} \frac{dk_y}{2\pi} e^{ik_y y} \hat{G}^{(1+1)}[t, x; m(k_y)], \quad (S5)$$

where

$$\hat{G}^{(1+1)}(t, x; m) \equiv i \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} e^{-i\omega t + ik_x x} \left[ \frac{\omega + \sigma^3 k_x + \sigma^2 m}{(\omega + i\eta)^2 - k_x^2 - m^2} \right]$$

$$= \theta(t) \left\{ \begin{array}{ccc} \delta(t - x) & 0 & \delta(t + x) \\ 0 & \delta(t - x) & \delta(t + x) \end{array} \right\} + \left[ \begin{array}{ccc} G^{(i)}(t, x; m) & G^{(o)}(t, x; m) \\ -G^{(o)}(t, x; m) & G^{(i)}(t, -x; m) \end{array} \right] \theta(t^2 - x^2), \quad (S6a)$$

$$G^{(i)}(t, x; m) = -\frac{m}{2} \left( \frac{t + x}{\sqrt{t^2 - x^2}} \right) J_1 \left( m \sqrt{t^2 - x^2} \right), \quad G^{(o)}(t, x; m) = -\frac{m}{2} J_0 \left( m \sqrt{t^2 - x^2} \right), \quad (S6b)$$

and $\theta(t)$ denotes the Heaviside step function. In Eq. (S6b), $J_{0,1}(z)$ are Bessel functions of the first kind. The causal Green’s function $\hat{G}^{(1+1)}(t, x; m)$ in Eqs. (S5) and (S6a) describes the propagation of 1+1-D massive Dirac fermions [S1].

III. INITIAL STATE CORRELATION FUNCTION

In the initial state $b = 0$, so that Eq. (2) describes decoupled 1D chains. These are perturbed by the external potential $\Phi(x, y)$ and generic short-ranged intrachain interactions [encoded in $H_I$, Eq. (2a)]. The static one-particle correlation function in the prequench state is given by [S1][S3]

$$\langle \psi_{\sigma_1, \tau_1}(x_1, y_1) \psi_{\sigma_2, \tau_2}(x_2, y_2) \rangle = \delta_{\sigma_1, \sigma_2} \delta_{\tau_1, \tau_2} \delta(y_1 - y_2) (-1)^{1+\sigma_1}$$

$$\times \frac{ie^{\eta\alpha/2} \text{sgn}(x_1 - x_2)}{2\pi |x_1 - x_2|^{1+\eta}} \exp \left\{ (-1)^{1+\sigma_1} i\pi \kappa \int_{x_1}^{x_2} dz \left[ -\Phi(z, y) \right] \right\}. \quad (S7)$$
Here $\sigma_1 = \pm 1$ corresponds to right- and left-moving fermions. The initial Hamiltonian and the correlator in Eq. (S7) are both independent of the “inner/outer” [Eq. (S3)] $\tau$-pseudo-spin space. The positive constant $c_\eta$ is defined as $[S1]$

$$c_\eta \equiv \sqrt{\pi} \Gamma \left(1 + \frac{2}{p}\right) / \Gamma \left(\frac{1 - p}{2}\right).$$

However, the overall normalization of Eq. (S7) is not completely determined for $\eta > 0$ due to the prefactor $\alpha^n$, where $\alpha$ is an ultraviolet length scale that is not defined within the continuum bosonization method $[S3]$ (it could be extracted from numerics $[S1]$, or in favorable cases via the Bethe ansatz). The anomalous dimension $\eta = (1/2p)(K + K^{-1} - 2)$, where $K$ is the Luttinger parameter $[S1,S3]$. The exponent $p = 1$ ($p = 2$) for spinless (spin-1/2) fermions within the chains. The external potential $\Phi(x,y)$ appears in the “gauge string” (phase) as a consequence of the 1+1-D axial anomalous; $\kappa = (\partial n/\partial \mu) = 2^{p-1}K/(\pi u)$ is the compressibility. Here $u$ denotes the interaction-renormalized charge velocity.

Linearizing Eq. (S7) in $\Phi(x,y)$ and discarding the zeroth order term (which does not enter the density dynamics), we obtain the correlator quoted in Eq. (5) of the main text if we set $\zeta = 0$ there. Nonzero $\zeta$ can arise from irrelevant operators; although negligible for long-wavelength, long-time dynamics near equilibrium, irrelevant operators can have a strong effect for far-from-equilibrium (e.g., quantum quench) time evolution $[S1]$. The fractionalization waves shown in Fig. 2 were obtained with $\zeta = 0$. Below we show in Fig. $[S4]$ that retaining $\zeta > 0$ does not qualitatively affect the fractionalized case, except at long times when it regularizes the growth of the “supersolitons.” This is consistent with previous results for an XXZ chain lattice quench in 1+1-D $[S1]$.

For the numerical evaluation of Eq. (4) in the main text, we use a Gaussian potential as in Eq. (6). In fact, since the linearized theory in Eq. (2) keeps momentum modes with $k_y \leq \pi/(2a)$, we retain lattice-scale resolution in the $y$-direction. To be precise, we can resolve pairs of chains in the initial state; the members of a pair are in fact encoded in the $\tau$-pseudo-spin degree of freedom of $\psi_{\sigma,\tau}(x,y)$. Therefore we write

$$\kappa \Phi(x,y) \equiv Q\Phi_x(x)\Phi_y(y = 2aj)$$

$$\Phi_x(x) = \frac{1}{\sqrt{\pi} \Delta_x} \exp \left(-x^2/\Delta_x^2\right), \quad \Phi_y(y = 2aj) = N(a, \Delta_y) \exp \left[-(2aj)^2/\Delta_y^2\right].$$

Here $j \in \mathbb{Z}$ indexes chain pairs. The normalization factor is

$$N(a, \Delta_y) = \left[2a\vartheta_3 \left(0, e^{-4a^2/\Delta_y^2}\right)\right]^{-1} \rightarrow \frac{1}{\sqrt{\pi} \Delta_y} \text{as} \ a \rightarrow 0.$$ 

In this equation $\vartheta_3(u, q)$ is the Jacobi theta function.

**IV. QUENCH DYNAMICS**

The explicit solution to the density $\rho(t, x, y)$ obtains from Eq. (4), using Eqs. (5), (S5), (S6) and (S8). It can be written as follows:

$$\rho(t, x, y) = -\frac{Q}{2} \Phi_x(x - t) \Phi_y(y) + c_\eta Q \left[\mathcal{I}_1(t, x, y) + \mathcal{I}_2(t, x, y)\right] + (x \Rightarrow -x),$$

where

$$\mathcal{I}_1(t, x, y) = 2a \int_{-t}^{t} dx_1 \int_{-\pi}^{\pi} \frac{dk_y}{2\pi} G^{(1)}[t, x; m(k_y)] \frac{\alpha^2}{(t - x_1)^2 + \zeta^2} \frac{\int_{x_1}^{x - x_2} dz \Phi_x(z) \Phi_y(y)}{(x_2 - t)},$$

and

$$\mathcal{I}_2(t, x, y) = a \int_{-t}^{t} dx_1 \int_{-t}^{t} dx_2 \int_{-\pi}^{\pi} \frac{dk_y}{2\pi} \int_{-\pi}^{\pi} \frac{dk_y'}{2\pi} \left\{ G^{(1)}[t, x; m(k_y)] G^{(1)}[t, x; m(k_y')] + G^{(2)}[t, x; m(k_y)] G^{(2)}[t, x; m(k_y')] \right\}$$

$$\times \cos [(k_y' - k_y)y] \frac{\Phi_y(k_y' - k_y)}{(x_2 - x_1)^2 + \zeta^2} \frac{\int_{x_1}^{x - x_2} dz \Phi_x(z)}{(x_2 - x_1)}.$$
The transform of $\Phi_y(y)$ is defined via

$$\tilde{\Phi}_y(p) \equiv \frac{1}{\tilde{\vartheta}_3\left(0, e^{-4a^2/\Delta_y^2}\right)} \sum_j e^{-ip(2aj)} \exp\left[-\frac{4(a_j)^2}{\Delta_y^2}\right] = \tilde{\vartheta}_3\left(0, e^{-4a^2/\Delta_y^2}\right).$$

We evaluate the integrals in Eqs. (S10) and (S11) numerically to compute the density profiles shown in Figs. 2 and 3 of the main text. We provide two independent checks for our numerical integration routine in Figs. S5 and S6 discussed below.

V. ADDITIONAL FRACTIONALIZATION WAVE NUMERICS

In this section we exhibit additional numerical results for the 1D-to-2D quench dynamics described in the main text.

A. Results for $\eta = 0.2$

Fractionalization waves as in Fig. 2 are shown in Fig. S2. All parameters are identical to that in Fig. 2, except that the initial-state-fermion anomalous dimension has been reduced to $\eta = 0.2$ in Fig. S2. The result is shown alongside the noninteracting $\eta = 0$ case for comparison.

B. UV regularization

The results shown in Figs. 2 and S2 were obtained using the initial state correlator in Eq. (5), with the parameter $\zeta = 0$. Nonzero $\zeta$ can arise due to the influence of irrelevant operators [S1]. In Fig. S4 we plot the same evolution.
shown in Fig. 2 of the main text, except that here \( \zeta = 1 \) (in units of the lattice spacing \( a \)). This is plotted alongside the \( \zeta = 0 \) result from Fig. 2 for comparison.

The main effect of nonzero \( \zeta \) is to amputate the power-law growth of the supersolitons at large times [S1], as shown in Fig. S4. A quantitative estimate for \( \zeta \) would require knowledge of the microscopic details for the pi-flux lattice quench implementation. Regardless, the primary effect described in the main paper for the case of a fractionalized initial condition is unchanged, i.e. the transient generation of directed, relativistically propagating waves along the wires.

C. Error control in the numerics

The time evolution of our quench dynamics should conserve particle number. Since the entire weight of the (right-moving piece of the) density bump is encoded in the first term on the right-hand-side of Eq. (S9), it must be that the integrated weight \( \mathcal{N}_1(t) + \mathcal{N}_2(t) = 0 \) at all times. Here

\[
\mathcal{N}_{1,2}(t) \equiv \int dx \, dy \mathcal{I}_{1,2}(t, x, y), \tag{S13}
\]

where \( \mathcal{I}_{1,2}(t, x, y) \) were defined in Eqs. (S10) and (S11).

We define the error ratio

\[
E(t) \equiv \frac{|\mathcal{N}_1 + \mathcal{N}_2|}{|\mathcal{N}_1| + |\mathcal{N}_2|}. \tag{S14}
\]

This quantity is plotted in Fig. S5 for \( \eta \in \{0, 0.2, 0.7\} \) and \( \zeta = 0 \).
FIG. S5: Error ratio $E(t)$ defined by Eq. (S14) for the time-slice profiles given for the various many-body quenches in Figs. 3, S1 ($\eta = 0$), Fig. S2 ($\eta = 0.2$), and Figs. 2, S3 ($\eta = 0.7$).

VI. 1D INITIAL CONDITION: NUMERICS VS. ASYMPTOTIC ANALYSIS

The integrals in Eqs. (S10) and (S11) evaluated at large times can be scrutinized via asymptotic analysis [S1]. For the two-dimensional initial condition in Eq. (6) this is rather complicated. A simpler case takes the initial density inhomogeneity to be uniform in the $y$-direction, $\Phi_y(y) = 1$ [$\hat{\Phi}_y(k_y) = 2\pi \delta(k_y)$]. In this case asymptotic analysis performed on Eqs. (S10) and (S11) along the lines of Appendix A in Ref. [S1] leads to

$$
\rho(t,x) = -\frac{Q^2}{2} \Phi(x-t) - \frac{Q}{\Delta} \left( \frac{\alpha^2 t}{\sqrt{2}\Delta} \right)^{\eta/2} \frac{1}{\sqrt{\pi}} \left( \frac{2}{a} \right)^{\eta} \frac{\Gamma(1-\eta)}{\Gamma(1+\eta/2)} e^{-\left((t-x)^2/2\Delta^2\right)} D_{\eta/2} \left[ \sqrt{2} \left( \frac{t-x}{\Delta} \right)^2 \right] + (x \Rightarrow -x). \tag{S15}
$$

Here $D_{\nu}(x)$ denotes the parabolic cylinder function. In Fig. S6 we compare the result in Eq. (S15) to numerical integration for different values of $\eta$. Beyond the error analysis presented in Fig. S5 for the 2D initial condition, this is a second check on the numerical integration routine used to obtain Figs. 2 and 3 in the main text from Eqs. (S10) and (S11) (since the same routine is employed for the 1D initial condition and the results shown in Fig. S6).

VII. SINGLE-PARTICLE WAVEPACKET DYNAMICS IN THE LATTICE-REGULARIZED DIRAC SEMIMETAL

Eq. (S4) also describes the evolution of a wavepacket in one-particle quantum mechanics, provided that $\psi(t,x,y)$ is re-interpreted as a Dirac (spinor) wave function, with $\psi(x,y)$ the initial condition. We consider a Gaussian initial wavepacket with zero average probability current in the $x$- and $y$-directions:

$$
\psi(x,y) = \frac{1}{\sqrt{\pi \Delta_x \Delta_y}} \exp \left( -\frac{x^2}{2\Delta_x^2} - \frac{y^2}{2\Delta_y^2} \right) |\uparrow_x\rangle, \tag{S16}
$$

where $\hat{\sigma}^{-1} |\uparrow_x\rangle = |\uparrow_x\rangle$. Using the Green’s function in Eq. (S5), we time-evolve from the initial condition in Eq. (S16). The resulting probability density $\psi^\dagger \psi(t,x,y)$ is shown in Fig. S7. To demonstrate that the lattice regularization in the $y$-direction does not significantly affect the dynamics on the time scales we are interested in, we compare the

FIG. S6: Comparison of the numerically-calculated supersoliton profile (red dots) to the analytic asymptotic approximation in Eq. (S15) (blue solid lines). The profile is shown at time $t = 100$, with initial parameters $\Delta_x = 6$, $\eta = 0.4$ (left), and $\eta = 0.6$ (right).
FIG. S7: Time-evolved probability profile for an initial Gaussian wavepacket [Eq. (S16)] for a pure massless Dirac band structure (left) and the lattice-regularized evolution operator in Eq. (S5) (right). The top pair of images corresponds to an isotropic initial condition with \( \Delta_x = \Delta_y = 2.5a \), while the bottom pair of images corresponds to an initial condition with \( \Delta_x = 2a, \Delta_y = 3a \), as in the many-body quench Figs. 2 and 3 in the main text. The images are obtained at time \( t = 9 \), with \( v_F = 1 \).

profiles computed using \( \hat{G}(t, x, y) \) in Eq. (S5) to that obtained from a massless, isotropic 2D Dirac Hamiltonian with purely linear dispersion.

The single-particle quantum evolution shown in Fig. S7 exhibits light-cone propagation of probability, with a circular front for an isotropic initial condition. The key observation is that this bears no resemblance to the density dynamics for the many-particle quench. This is true for the interacting quench with \( \eta > 0 \), which produces supersolitons that propagate relativistically along the chains (Fig. 2), as well as the noninteracting quench with \( \eta = 0 \) (Fig. 3), which produces dispersive transport perpendicular to the chains.

The deviation of the \( \eta = 0 \) quench from one-particle quantum mechanics depicted in Fig. S7 is due to Pauli blocking. As explained in the main text, Pauli blocking suppresses large \( v_x \) in the Wigner velocity distribution for the noninteracting quench, shown in the bottom panel of Fig. 4. The latter plot also demonstrates that the noninteracting quench excites particles with a broad range of \( v_y \). The \( y \)-oriented dispersive propagation shown in Fig. 3 therefore results from the excitation of modes at all \( k_y \) along the “vertical strips” retained in the reduced Brillouin zone [Eq. (2)]. The lattice regularization in the \( y \)-direction produces a much stronger effect for the noninteracting quench, compared to the single-particle quantum mechanics.

VIII. CALCULATION OF THE WIGNER DISTRIBUTION

The post-quench Hamiltonian in Eq. (2) (with vanishing external potential \( \Phi = 0 \), neglecting interactions in \( H_I \)) can be diagonalized in terms of canonically-quantized creation and annihilation operators as

\[
H = v_F \int_{-\Lambda}^{\Lambda} \frac{dk_x}{2\pi} \int_{-\pi/2}^{\pi/2} \frac{dk_y}{2\pi} \varepsilon_k \left[ a^\dagger(k) a(k) + b^\dagger(k) b(k) \right],
\]

where

\[
\varepsilon_k = \sqrt{k_x^2 + m^2(k_y)}.
\]

Here \( a(k) \) \([b(k)]\) annihilates a conduction band particle (valence band hole) with momentum \( k \), we will set \( v_F = 1 \), and \( m(k_y) \) is defined by Eq. (3). Since Eq. (2) is invariant under \( \tau \)-space SU(2) rotations, we suppress \( \tau \)-pseudospin indices throughout this section.

The Dirac spinor \( \psi \rightarrow \psi_\sigma \) is expressed in terms of the creation and annihilation operators via

\[
\begin{bmatrix}
\psi_1(k) \\
\psi_2(k)
\end{bmatrix} = \frac{1}{\sqrt{1 + s^2(k)}} \begin{bmatrix} 1 & 1 \\ is(k) & \frac{1}{\sqrt{1 + s^2(-k)}} \end{bmatrix} \begin{bmatrix} a(k) \\ -is(-k) \end{bmatrix} b^\dagger(-k),
\]

(S19)
where
\[ s(k) = \frac{\epsilon_k - k_x}{m(k_y)} . \] (S20)

The band velocities and momenta are related by
\[ v_x = \frac{k_x}{\epsilon_k}, \quad v_y = \frac{b m(k_y)}{\epsilon_k} \cos(k_y a) , \] (S21)
\[ k_x = \frac{m(v_x, v_y) v_x}{\sqrt{1 - v_x^2}}, \quad k_y = \frac{1}{a} \arcsin \left( \frac{a}{b} m(v_x, v_y) \right) . \] (S22)

Here
\[ m(v_x, v_y) = \frac{\text{sgn}(v_y)}{a} \sqrt{b^2 - \left( \frac{v_y^2}{1 - v_x^2} \right)} \] (S23)
is the mass parameter \( m(k_y) \) [Eq. (3)] rewritten in terms of velocities. The velocities are constrained to the (elliptical) disk in Eq. (8). The Jacobian corresponding to the change of variables \((k_x, k_y) \Rightarrow (v_x, v_y)\) is
\[ J(v_x, v_y) = \left| \frac{\partial(k_x, k_y)}{\partial(v_x, v_y)} \right| = \frac{1}{a^2(1 - v_x^2)} . \] (S24)

As in [S1], we define the ground-state Wigner function for the right-moving fermion \( \psi_1(x) \equiv R(x) \) as
\[ \delta n_R(k_x, k_y; R_x, R_y) \equiv \int dx dx d\eta \, e^{-i k_x x - i k_y y} \left\langle R^\dagger \left( R_x - \frac{x_d}{2} - R_y - \frac{y_d}{2} \right) R \left( R_x + \frac{x_d}{2} + R_y + \frac{y_d}{2} \right) \right\rangle_\Phi , \] (S25)
where the subscript \( \Phi \) denotes the linear response to the external potential \( \Phi(x, y) \) in Eq. (2a). The initial state correlation function for the pre-quench ground state formed from uncoupled chains is
\[ \left\langle R^\dagger \left( R_x - \frac{x_d}{2} - R_y - \frac{y_d}{2} \right) R \left( R_x + \frac{x_d}{2} + R_y + \frac{y_d}{2} \right) \right\rangle_\Phi = \delta(y_d) C_\Phi \left( R_x - \frac{x_d}{2}, R_x + \frac{x_d}{2}; R_y \right) , \] (S26)
where \( C_\Phi(x_1, x_2; y) \) is given by Eq. (5). Ignoring for simplicity the ultraviolet regularization parameter \( \zeta \equiv 0 \), Eq. (S25) evaluates to [S1]
\[ \delta n_R(k_x, k_y; R_x, R_y) = c_\eta^n \eta^\zeta (1 - \eta) \sin \left( \frac{\pi}{2} \Phi_y(R_y) \right) \int \frac{dq}{2\pi} \left[ \frac{Q^\dagger}{\eta} \right] \left[ -Q \Phi_y(q) \right] S(q, k_x, k_y) \times \left\{ \text{sgn} \left[ k_x + \frac{q}{2} \right] \right. \left| k_x + \frac{q}{2} \right| - \text{sgn} \left[ k_x - \frac{q}{2} \right] \left| k_x - \frac{q}{2} \right| \left\} , \right. \] (S27)
where we have assumed the separable potential defined by Eqs. (6) and (S8). For the pure right-mover correlator computed in Eq. (S27), the “structure factor” \( S(q, k_x, k_y) = 1 \); therefore \( \delta n_R \) is independent of \( k_y \). It is simply proportional to the potential profile in the \( y \)-direction \( \Phi_y(R_y) \). Eq. (S27) also holds for the prequench Wigner distribution of the left-mover \( \psi_2 \).

To compute the Wigner function for the conduction band particle creation and annihilation operators in Eq. (S17),
\[ \delta n_+(k_x, k_y; R_x, R_y) \equiv \int dQ_x dQ_y \, e^{i Q_x R_x + i Q_y R_y} \left\langle a^\dagger \left( k_x - \frac{Q_x}{2}, k_y - \frac{Q_y}{2} \right) a \left( k_x + \frac{Q_x}{2}, k_y + \frac{Q_y}{2} \right) \right\rangle_\Phi , \] (S28)
we exploit the decomposition in Eq. (S19). The result for \( \delta n_+(k_x, k_y; R_x, R_y) \) is identical to Eq. (S27), except that now the structure factor \( S(q, k_x, k_y) = B(q, k_x, k_y) \), where
\[ B(q, k_x, k_y) = \beta \left( k_x + \frac{q}{2}, k_y \right) \beta \left( k_x - \frac{q}{2}, k_y \right) + \beta \left( -k_x + \frac{q}{2}, k_y \right) \beta \left( -k_x - \frac{q}{2}, k_y \right) , \quad \beta(k_x, k_y) \equiv \sqrt{\frac{\epsilon_k + k_x}{2\epsilon_k}} . \] (S29)
Although the chiral Wigner function in Eq. (S27) is independent of $k_y$, the conduction-band-particle Wigner function defined by Eq. (S28) depends on it through the structure factor $B(q,k_x,k_y)$. The latter is a function of $k_y$ via the band structure $\varepsilon_{k}$ [Eq. (S18)].

Using Eqs. (S22) and (S24) we can convert to the Wigner velocity distribution defined by Eq. (7). This gives

$$\begin{align*}
\delta n_{+}(v_x,v_y;R_x,R_y) & \propto \frac{\Phi_y(R_y)}{(1 - v_x^2)^2} \int dq \frac{e^{i q R_y}}{2\pi q} \left[ -Q \tilde{\Phi}_x(q) \right] B[q,k_x(v_x,v_y),k_y(v_x,v_y)] \\
& \times \left\{ \sgn \left[ k_x(v_x,v_y) + \frac{q}{2} \right] |k_x(v_x,v_y) + \frac{q}{2}| - \sgn \left[ k_x(v_x,v_y) - \frac{q}{2} \right] |k_x(v_x,v_y) - \frac{q}{2}| \right\} .
\end{align*}$$

(S30)

The plots in Fig. 4 are obtained by numerically integrating Eq. (S30).

We can find asymptotic approximations for Eq. (S30) in the limit of large $v_x \to +1$ (approaching the maximum band velocity) for noninteracting ($\eta = 0$) and interacting ($\eta > 0$) initial conditions. In the noninteracting case, evaluated at $R_x = R_y = 0$ (the center of the pre-quench inhomogeneity), we obtain

$$\begin{align*}
\delta n_{+}(v_x \to 1,v_y;0,0) & \propto \frac{1}{(1 - v_x^2) [m(v_x,v_y) v_x]^2} \exp \left\{ -\frac{v_x^2 [m(v_x,v_y) \Delta_x]^2}{1 - v_x^2} \right\} .
\end{align*}$$

(S31)

The factor $m(v_x,v_y) \Delta_x \propto \Delta_x/a$ exponentially suppresses large $v_x$ velocities. As explained in the main text, this is due to Pauli blocking. The corresponding result for the interacting case with $\eta > 0$ is

$$\begin{align*}
\delta n_{+}(v_x \to 1,v_y;0,0) & \propto \frac{[m(v_x,v_y) v_x]^\eta-1}{(1 - v_x^2)^{3+\eta/2}} \Phi_x(R_x = 0) .
\end{align*}$$

(S32)

In this case, the initial inhomogeneity $\Phi_x(R_x)$ factorizes from the velocity dependence. The non-integrable singularity at $v_x = 1$ is due to the Jacobian, without the exponential suppression that arises in the noninteracting case. The singularity is regularized if we retain the ultraviolet scale $\zeta > 0$ in Eq. (5) [S1].

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