EXCITED HEAVY MESONS DECAY FORMFACTORS IN LIGHT CONE QCD

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August 21, 2018

Abstract

The formfactors of $B_1 \to \pi$ and $D_1 \to \pi$ transitions, where $B_1(D_1)$ is the $1^+$ P-wave $\bar{Q}_3 \gamma_5 q$ meson state, are calculated in the framework of the light cone QCD. Furthermore these formfactors are compared with the pole dominance model prediction using the values of the strong coupling constants $g_{B_1 B^* \pi}$ and $g_{D_1 D^* \pi}$, and a good agreement between these two different descriptions is observed.

PACS numbers: 13.20.He, 13.25.Hw, 13.30.Eg

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1 Introduction

Understanding the formfactors of the hadronic currents is one of the main problems in particle physics. As is well known, the knowledge of the formfactors allows one the determination of the quark mixing parameters, in a unique way [1]. Moreover, it gives us a direct information about the dynamics of the hadronic processes at large distance, where perturbation theory of QCD is invalid. At present there exists no theoretical method for calculating the formfactors, starting from a fundamental QCD Lagrangian. Therefore, in estimating the formfactors, usually phenomenological or semiphenomenological methods are used.

Along these lines, there exists two well known methods for calculating the hadronic formfactors. First of these is the pole model description based on the vector dominance idea, that suggests a momentum dependence dominated by the nearest pole, and the other one is the QCD sum rules method. The pole model approach does not have its roots on the basic principles of the theory and is purely phenomenological. Generally, it is assumed that the vector dominance approximation is valid at zero recoil, that is at $p^2 \rightarrow m^2$, where $p^2$ is the square of the momentum transfer, that depends on the formfactor, and $m$ is the mass of the nearest vector meson. However, there are no reliable arguments in favour of the pole model, for it to be also valid at small $p^2$, that are interesting for practical applications. The effective heavy quark theory predicts somewhat larger region of validity, characterized by $(m^2 - p^2)/m_Q \sim (1 \text{ GeV})$, where $m_Q$ is the heavy quark mass (for more detail see [2, 3] and the references cited therein). It is shown in [4, 5] that the pole behaviour of the formfactors is consistent with the $p^2$ dependence at $p^2 \rightarrow 0$ predicted by the sum rule. This result has been confirmed independently by the calculations carried within the framework of the light cone sum rules [2, 3, 7].

In contrary to the pole model, the QCD sum rules method is based on the first principles and on the fundamental QCD Lagrangian. In this work we employ an alternative version of the QCD sum rules, namely light cone sum rules method for the calculation of the formfactors. We note that using this approach, the formfactors of the $\pi A^0 \gamma^*$ transition [8], the pion formfactors at the intermediate momentum transfer [9], semileptonic B and D decays [10], radiative $B \rightarrow K^* \gamma$ decay formfactor [11] and $B^* \rightarrow B \pi$, $D^* \rightarrow D \pi$ transition formfactors [3] are calculated. The results of all works that have been devoted to the calculation of the formfactors confirm that, the predictions of the pole model are compatible
with that of the QCD sum rules method. Note that, in all these works the transition formfactors between the ground state mesons are considered.

In connection with these observations, there follows an immediate question to whether the same agreement holds for the prediction of the formfactors resulting from the decays of the excited meson states. This article is devoted to find an answer to the question cited above, more precisely, we calculate the formfactors for the excited $1^+$ $P$ wave decays $P_1 \to P^* \pi$, where $P_1(B_1, D_1)$ is the excited $1^+$ $P$ wave meson state, and $P^*$ is the $1^-$ state. This article is organized as follows: In sect.2, we derive the sum rules for the $P_1 \to P^* \pi$ transition formfactors. Sect.3 is devoted to the numerical analysis and discussions.

## 2 Sum Rules for the $P_1 \to P^* \pi$ transition formfactors

For calculation of the sum rules formfactors for the $P_1 \to P^* \pi$ transition, we consider the following correlator,

$$
\Pi_{\mu\nu}(q_2, q) = i \int d^4 x \ e^{i q_2 x} \langle \pi^- (q) | \bar{d} (x) \gamma_\mu Q(x) \bar{Q} (0) \gamma_\nu \gamma_5 u (0) | 0 \rangle ,
$$

where, $\bar{d} \gamma_\mu Q$ and $\bar{Q} \gamma_\nu \gamma_5 u$ are the interpolating currents of $1^-$ and $1^+$, respectively, $Q$ is the heavy quark, $q_2$ is the momentum of the $1^-$ meson and $q$ is the pion’s momentum. When the pion is on the mass shell, i.e., $q^2 = m_{\pi}^2$, the correlator depends on two invariant variables $q_2^2$ and $(q_2 + q)^2$. In further calculations, we will set $m_\pi = 0$.

We start by considering the physical part of eq.(1). Before calculating the physical part of the eq.(1), few words are in order. According to the value of angular momentum of the light degrees of freedom ($s_P^l = \frac{1}{2}, \frac{3}{2}$), the heavy quark effective theory [12, 13], predicts the existence of two multiplets, each including the $1^+$ meson: the first one contains $(0^+, 1^+_1)$ and the second $(2^+, 1^+_2)$ mesons. In terms of the conventional $(2s+1)P_J$ states, the $1^+$ states defined above (physical states), are given by the following linear combinations [12, 14]:

$$
|1^+_\frac{1}{2}\rangle &= \sqrt{\frac{2}{3}} |1_P^1 \rangle + \sqrt{\frac{1}{3}} |3_P^1 \rangle \\
|1^+_\frac{3}{2}\rangle &= \sqrt{\frac{1}{3}} |1_P^1 \rangle - \sqrt{\frac{2}{3}} |3_P^1 \rangle .
$$

Therefore, the physical part of the eq.(1) must contain the contributions from both of the $1^+$ mesons, and it can be written as

$$
\Pi_{\mu\nu} = \sum_{l=\frac{1}{2}, \frac{3}{2}} \frac{\langle \pi^- (q) | \bar{d} \gamma_\mu Q | 1^+_l \rangle \langle 1^+_l | Q \gamma_\nu \gamma_5 u | 0 \rangle}{(q_2 + q)^2 - m_{1_l}^2} .
$$
In \cite{15, 16}, it was shown that in the heavy quark limit, \(m_Q \gg m_q\),
\[
\langle 1^+_1 | \bar{Q} \gamma^\nu \gamma_5 u | 0 \rangle = 0 .
\]
The full theory can only bring small corrections to this result. For this reason the contribution of the \(1^+_2\) meson, in eq.(2), can safely be neglected. Therefore, in future analysis we will consider the contribution of the \(1^+_2\) meson only, and for simplicity we denote it as \(P_1\).

The matrix elements entering in eq.(2) are defined in the standard manner:
\[
\langle P_1 | \bar{Q} \gamma^\nu \gamma_5 u | 0 \rangle = -i f_{P_1} m_{P_1} \epsilon_\nu(p) ,
\]
\[
\langle \pi | \bar{d} \gamma^\nu Q | P_1 \rangle = F_0 \epsilon_\nu(p) + F_1(q\epsilon)(p + q)_\nu + F_2(q\epsilon)(p - q)_\nu ,
\]
where \(f_{P_1}\), \(\epsilon_\nu\) and \(m_{P_1}\) are the leptonic decay constant, vector polarization and mass of the excited \(1^+\) meson, respectively. \(F_0\), \(F_1\) and \(F_2\) are the formfactors that describe the \(P_1 \to \pi\) transition. The contributions of the coefficients of \(F_1\) and \(F_2\) to the decay rate are proportional to
\[
m^2_\pi + (m_{P_1} - m_{P^*})^2 .
\]
From this expression it is obvious that, their contributions are small in comparison to the \(F_0\)'s contribution. Our numerical analysis shows that the contributions of \(F_1\) and \(F_2\) to the decay width constitute about 20\% of \(F_0\), and hence, we shall neglect such terms. Thus,
\[
\Pi_{\mu\nu} = -i \frac{f_{P_1} m_{P_1}}{(q_2 + q)^2 - m_{P_1}^2} F_0 \left( -g_{\mu\nu} + \frac{p_\mu p_\nu}{m_{P_1}^2} \right) ,
\]
and in further analysis we will pay our attention to the structure \(g_{\mu\nu}\) and denote its coefficient by \(\Pi\). The physical part of the above relation takes the following form:
\[
\Pi \left( q_2^2, (q_2 + q)^2 \right) = -i \frac{f_{P_1} m_{P_1}}{m_{P_1}^2} F_0(q_2^2) \frac{\rho(h(q_2^2, s)ds}{s - (q_2 + q)^2} .
\]
The first term on the right hand side is the pole term due to the ground state in the heavy channel, while the continuum and the higher order states are taken into account by the dispersion integral above the threshold \(s_0\).

Let us now consider the theoretical part of the correlator \(\Pi\). This correlator function can be calculated in QCD at large Euclidean momenta \((q_2 + q)^2\). In this case the virtuality of the heavy quark is of the order \(m_Q^2 - (q_2 + q)^2\), and this is a large quantity. Therefore, we can expand the heavy quark propagator in powers of the slowly varying fields residing in
the pion, that acts as the external field on the propagating heavy quark. The expansion in
powers of the external fields is also the expansion of the propagator in powers of a deviation
from the light cone at $x^2 = 0$. The leading contribution is obtained by using the free heavy
quark propagator in eq.(1)

$$S_Q^0(x) = i \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{k + m_Q}{k^2 - m_Q^2}.$$ 

Hence,

$$\Pi_{\mu\nu}(q_2, q) = \int \frac{d^4x d^4k}{(2\pi)^4} \frac{e^{i(q_2 - k)x}}{(m_Q^2 - k^2)} \langle \pi(q)| \bar{q}(x) \gamma_{\mu} (k + m_Q) \gamma_{\nu} \gamma_5 u(0)|0 \rangle.$$ 

We point out that, in eq.(5) and throughout the course of the present analysis, the path-
ordered factor $P \exp \left(i g_s \int_0^1 x_\mu A^\mu (ux) \, du \right)$ has been omitted, since in the Fock-Schwinger
gauge $x_\mu A^\mu = 0$, this factor is trivial. It follows from eq.(5) that, the answer is expressed
via the pion matrix element of the gauge invariant, nonlocal operator with a light cone
separation $x^2 \simeq 0$. Following [17], the two particle pion wave functions are defined as:

$$\langle \pi(q)| \bar{d}(x) \gamma_{\mu} \gamma_5 u(0)|0 \rangle = -i q_\mu f_\pi \int_0^1 du \ e^{iqux} \left[ \varphi_\pi(u) + x^2 g_1(u) \right] +$$

$$+ q_\mu f_\pi \left( x_\mu - x^2 q_\mu \right) \int_0^1 du \ e^{iqux} g_2(u),$$

$$\langle \pi(q)| \bar{d}(x) \gamma_5 u(0)|0 \rangle = \frac{f_\pi m_\pi^2}{m_u + m_d} \int_0^1 du \ e^{iqux} \varphi_P(u),$$

where $\varphi_\pi(u)$ is the leading twist $\tau = 2$, $\varphi_P(u)$ is $\tau = 3$ and $g_1(u)$, $g_2(u)$ are the $\tau = 4$ two
particle pion wave function. Using eq.(5) and eq.(6) after integrating over $x$ and $k$, we get
for the structure $g_{\mu\nu}$:

$$\Pi(q_2, q) = -i \left\{ \frac{m_\pi^2}{m_u + m_d} m_Q \int_0^1 du \ \frac{\varphi_P(u)}{\Delta} + 2m_Q \int_0^1 du \ \frac{g_2(u)}{\Delta^2} -$$

$$- q_2 q \left[ 4 \int_0^1 du \ \frac{(g_1(u) + G_2(u))}{\Delta^2} \left( 1 + \frac{2m_Q^2}{\Delta^2} \right) - \int_0^1 du \ \frac{\varphi_\pi(u)}{\Delta} \right] \right\},$$

where,

$$\Delta = m_Q^2 - (q_2 + qu)^2,$$

$$G_2(u) = - \int_0^u g_2(v) dv.$$ 

Since the higher order twist contributions are taken into account in eq.(8), the terms
up to $\tau = 4$, which comes from the propagator expansion, must also be considered. The
complete expansion of the propagator is given in [18], and it contains the contributions from the nonlocal operators $\bar{q}Gq$, $\bar{q}GGq$, and $\bar{q}qqq$. The only operator we consider here is $\bar{q}Gq$, because the contributions from the other two are very small, and hence they are neglected (for more detail see [17], [18]). Under these approximations, the heavy quark propagator is given by the following expressions [3, 17]:

$$S_Q(x) = S_Q^0(x) - ig_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \int_0^1 du \left[ \frac{1}{2} \frac{k + m_Q}{(m_Q^2 - k^2)^2} G^{\mu\nu}(ux)\gamma_\mu + \frac{1}{m_Q^2 - k^2} ux_\mu G^{\mu\nu}(ux)\gamma_\nu \right]. \tag{9}$$

Substituting second term in eq.(9) into eq.(1), we get,

$$\Pi_{\mu\nu}(q_2, q) = \int \frac{d^4x}{(2\pi)^4} \int \frac{d^4k}{(2\pi)^4} e^{i(q_2-k)x} \langle \pi(q) | \bar{d}(x) \gamma_\mu [ux_\rho G^{\rho\lambda}(ux) \gamma_\lambda + \frac{k + m_Q}{(m_Q^2 - k^2)^2} G^{\rho\lambda}(ux)\sigma_{\rho\lambda}] \gamma_\nu \gamma_5 u(0)|0 \rangle. \tag{10}$$

Using the following identities,

$$\gamma_\mu \gamma_\lambda \gamma_\nu = g_{\mu\lambda} \gamma_\nu + g_{\lambda\nu} \gamma_\mu - g_{\mu\nu} \gamma_\lambda - ie_{\mu\lambda\nu\tau} \gamma^\tau \gamma_5;$$

$$\gamma_\mu \sigma_{\rho\lambda} = i (g_{\mu\rho} \gamma_\lambda - g_{\rho\lambda} \gamma_\mu) + e_{\mu\rho\lambda\tau} \gamma^\tau \gamma_5;$$

and the definition of the three particle pion wave functions [17],

$$\langle \pi(q) | \bar{d}(x) \gamma_\mu g_s G_{\alpha\beta}(ux) u(0)|0 \rangle =$$

$$f_\pi \left\{ q_3 \left(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{qx} - q_\alpha \left(g_{\beta\mu} \frac{x_\beta q_\mu}{qx} \right) \right) \right\} \int D\alpha_1 D\phi_p(\alpha_1) e^{iqx(\alpha_1 + u\alpha_3)} +$$

$$+ \frac{q_\mu}{qx} \left( q_\alpha x_\beta - q_\beta x_\alpha \right) \int D\alpha_1 D\phi_p(\alpha_1) e^{iqx(\alpha_1 + u\alpha_3)} \right\}, \tag{11}$$

$$\langle \pi(q) | \bar{d}(x) \gamma_\mu g_s \tilde{G}_{\alpha\beta}(ux) u(0)|0 \rangle =$$

$$if_\pi \left\{ q_3 \left(g_{\alpha\mu} - \frac{x_\alpha q_\mu}{qx} - q_\alpha \left(g_{\beta\mu} \frac{x_\beta q_\mu}{qx} \right) \right) \right\} \int D\alpha_1 D\tilde{\phi}_p(\alpha_1) e^{iqx(\alpha_1 + u\alpha_3)} +$$

$$+ \frac{q_\mu}{qx} \left( q_\alpha x_\beta - q_\beta x_\alpha \right) \int D\alpha_1 D\tilde{\phi}_p(\alpha_1) e^{iqx(\alpha_1 + u\alpha_3)} \right\}, \tag{12}$$

we obtain from eq.(13), for the structure $g_{\mu\nu}$

$$\Pi = iq_2 q \int_0^1 du \int D\alpha_1 \frac{(1 - 2u)\phi_p(\alpha_1) + \tilde{\phi}_p(\alpha_1)}{[m_Q^2 - (q_2 + q_1 (\alpha_1 + u\alpha_3))^2]^2}, \tag{13}$$

5
where, $D\alpha_i \equiv d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$, and $G_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\mu\nu} G^{\mu\nu}$, is the dual tensor to $G^{\mu\nu}$, and $\varphi$’s are all twist four functions. Collecting all the terms (eqs.(8) and (9)) for the theoretical part of the correlator function, we get,

$$
\Pi^{\text{theor}} = -i \left\{ \frac{m^2_{\pi}}{m_u + m_d} m_Q \int_0^1 du \frac{\varphi_p(u)}{\Delta} + 2m^2_Q \int_0^1 du \frac{g_2(u)}{\Delta^2} + q_2 q \left[ \int_0^1 du \frac{\varphi_\pi(u)}{\Delta} - 4 \int_0^1 du \frac{(g_1(u) + G_2(u))}{\Delta^2} \left( 1 + \frac{2m^2_Q}{\Delta} \right) \right] - q_2 q \int_0^1 du \int D\alpha_i \left( \frac{1 - 2u}{\Delta^2} \varphi_{||}(\alpha_i) + \bar{\varphi}_{||}(\alpha_i) \right) \right\},
$$

where,

$$
\Delta_1 = m^2_Q - [q_2 + q(\alpha_1 + u\alpha_3)]^2.
$$

The sum rule for the formfactor $F_0(q^2_2)$ is obtained by equating the expressions (4) and (14) for the invariant amplitude $\Pi(q^2_2, (q_2 + q)^2)$, which results as:

$$
\frac{f^2}{f_{\pi} m_{P_1} m_{P_2}} \left\{ \int_0^1 du \frac{1}{u} \exp \left[ \frac{m^2_{P_1}}{M^2} - \frac{m^2_Q - q^2_2 u}{u M^2} \right] \Phi_2(u, M^2, q^2) + \frac{1}{2} \int_0^1 du \int \frac{D\alpha_i}{(\alpha_1 + u\alpha_3)^2} \Theta(\alpha_1 + u\alpha_3 - \delta) \times \Phi_3(u, M^2, q^2) \right\},
$$

where,

$$
\delta = \frac{m^2_Q - q^2_2}{s_0 - q^2_2}, \quad \bar{u} = 1 - u,
$$

and,

$$
\Phi_2 = \frac{m^2_{\pi}}{m_u + m_d} m_Q \varphi_p(u) + \frac{m^2_Q - q^2_2}{u} \varphi_\pi(u) + 2 \frac{g_1(u)}{u} \left[ 1 + \frac{m^2_Q - q^2_2}{u M^2} - \frac{m^2_Q(m^2_Q - q^2_2)}{u^2 M^4} \right] + 2 \frac{G_2(u)}{u} \left[ 1 - \frac{m^2_Q - q^2_2}{u M^2} \right],
$$

$$
\Phi_3 = \left[ (1 - 2u) \varphi_{||}(\alpha_i) + \bar{\varphi}_{||}(\alpha_i) \right] \left( 1 - \frac{m^2_Q - q^2_2}{(\alpha_1 + u\alpha_3) M^2} \right).
3 Numerical analysis and discussion

Now we turn to the numerical calculations. In expression (15), we use the following set of parameters: \( m_B = 5.738 \text{ GeV}, \) \( m_D = 2.4 \text{ GeV}, \) \( s_0B = 35 \text{ GeV}^2, \) \( s_0D = 8 \text{ GeV}^2. \) For leptonic decay constants \( f_{B_1} \) and \( f_{D_1}, \) we use the results given in [19]: \( f_{B_1} \approx 0.2 \pm 0.02 \text{ GeV} \) and \( f_{D_1} \approx 0.3 \pm 0.03 \text{ GeV}. \) The highest value of the Borel Parameter \( M^2 \) is fixed by imposing the condition that the continuum contribution is 30\% of the resonance. With the help of this restriction, we calculate the maximum value of the Borel parameter for \( B(D) \) to be \( M_{\text{max}}^2 \approx 20 \text{ GeV}^2 \approx 8 \text{ GeV}^2. \) The minimal value of the same parameter is usually fixed by the condition that the terms proportional to the higher powers of \( 1/M^2 \) are negligible and it is found to have the value \( M^2_{\text{min}} \approx 8 \text{ GeV}^2 \approx 2 \text{ GeV}^2 \) for \( B(D) \) mesons. Our calculations show that, the variation of \( M^2 \) within the above-mentioned limits, changes the result by less than 10\%, which means that the dependence of the formfactor \( F_0(q_2^2) \) on the Borel paremeter \( M^2 \) is quite weak. Note that, a similar situation exists for the \( B \to \pi \) and \( D \to \pi \) transitions too (see for example [2, 3]). Because of that, in our analysis we take \( M^2 = 15 \text{ GeV}^2 \) for the \( B_1 \to \pi \) and \( M^2 = 4 \text{ GeV}^2 \) for the \( D_1 \to \pi \) transition formfactors.

The maximum momentum transfer \( q_2^2 \) at which the sum rule (15) is applicable, is about \( 15 - 20 \text{ GeV}^2 \) for B meson and \( 1 \text{ GeV}^2 \) for the D meson cases, respectively (for more detail see [4, 5]). The explicit form of the pion wave function, used in eq.(15) can be found in [17]. The momentum transfer \( q_2^2 \) dependence of the formfactors \( F_0^B(q_2^2) \) and \( F_0^D(q_2^2) \) are plotted in Fig.1 and Fig.2. From these figures we observe that,

\[
F_0^B(q_2^2 = 0) = 1.2, \\
F_0^D(q_2^2 = 0) = 1.5. \tag{16}
\]

Let us turn our attention to the pole model prediction for \( F_0^B \) and \( F_0^D \) formfactors. In [3], it is shown that the coupling constants \( g_{B^*B\pi} \) and \( g_{D^*D\pi} \) fix the normalization of the formfactors of the \( B \to \pi \) and \( D \to \pi \) transitions, respectively, within the context of pole model (see also, [4, 5]). Using the pole model description in a similar manner, the formfactor \( F_0(q_2^2) \) can be expressed via the \( g_{P_1P^*\pi} \) coupling constant, that is calculated using the same correlator function ([4]) in [19]. Indeed, the formfactor \( F_0(q_2^2) \) defined by the matrix element,

\[
\langle \pi(q)|\bar{d}\gamma_\mu Q|P_1(p)\rangle = F_0(q_2^2)\epsilon_\mu(p) + F_1(q_2^2)(\epsilon q)(p + q)_\mu + F_2(q_2^2)(\epsilon q)(p - q)_\mu, \tag{17}
\]
is predicted to be (for the structure $g_{\mu\nu}$)

$$F_0(q_2^2) = \frac{g_{P_1P^{**}\pi}f_{P^{**}}}{m_{P^{**}}\left(1 - \frac{q_2^2}{m_{P^{**}}^2}\right)}.$$  \hspace{1cm} (18)

In deriving eq.(18), we used the following definition,

$$\langle \pi P^{**}|P_1\rangle \equiv g_{P_1P^{**}\pi}(\epsilon\epsilon^{*}), \hspace{1cm} (19)$$

where $\epsilon$ and $\epsilon^{*}$ are the 4-polarization vectors of the $P_1$ and $P^{**}$ mesons, respectively. The dependence of the formfactors $F_0^B(q_2^2)$ and $F_0^D(q_2^2)$ on $q_2^2$ in the pole model (eq.(18)) are plotted in Fig.1 and Fig.2, where we use $g_{B_1B^{*}\pi} = 24 \pm 3 \text{ GeV}$ and $g_{D_1D^{*}\pi} = 10 \pm 2 \text{ GeV}$ [19]. From these figures we conclude that, in the overlap region both calculations agree, approximately, with each other. Quantitatively, at $q_2^2 = 0$, it follows from eq.(18) that,

$$F_0^B(q_2^2 = 0) = 0.72,$$

$$F_0^D(q_2^2 = 0) = 1.20.$$  \hspace{1cm} (20)

If we compare eqs.(16) and (20), we see that, in the region for which $m_Q^2 - q_2^2 > (1 \text{ GeV}^2)$, $(Q = b, c)$, the numerical agreement between the two approaches is better than 20% for D, but only within 35% for the B meson case.

Finally we would like to point out that, these two approaches lead to absolutely different asymptotic behaviours of the relevant formfactors. Using the HQFT results, we get

$$f_{P_1}\sqrt{m} = \hat{f}_{P_1}, \quad f_{P^{**}}\sqrt{m} = \hat{f}_{P^{**}}$$

and,

$$g_{P_1P^{**}\pi} \simeq \frac{2m}{g} \quad \text{(see also [3])}.$$  \hspace{1cm} (21)

From eq.(18) it follows that, at $m_b \rightarrow \infty$ we get,

$$F_0^{B(D)} \sim (m_{B(D)})^{-1/2}.$$  \hspace{1cm} (21)

Then from eq.(15) we see that, in this limit

$$F_0^{B(D)} \sim (m_{B(D)})^{-3/2}. \quad \text{(see also [20])}$$  \hspace{1cm} (22)

In conclusion, we calculated the formfactors of the excited state 1+ mesons, namely, $B_1 \rightarrow \pi$ and $D_1 \rightarrow \pi$ transitions, in the framework of the light cone QCD sum rules
and compared our results with the pole dominance model predictions. The comparision elaborates that, the agreement between the two descriptions is rather good. This justification demonstrates that, the pole dominance model works quite good for the p-wave meson transitions, as it does for the s-wave ones.
Figure 1: The dependence of the formfactor $F_0(q_2^2)$, for the $B_1 \rightarrow \pi$ transition, on $q_2^2$. The dotted line corresponds to the light cone QCD sum rules prediction and the solid line describes the pole dominance model prediction.
Figure 2: Same as in Fig.(1), but for the $D_1 \to \pi$ transition.
Figure Captions

1. The dependence of the formfactor $F_0(q_2^2)$, for the $B_1 \to \pi$ transition, on $q_2^2$. The dotted line corresponds to the light cone QCD sum rules prediction and the solid line describes the pole dominance model prediction.

2. Same as in Fig.(1), but for the $D_1 \to \pi$ transition.
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