BRST quantization of a sixth-order derivative scalar field theory

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Abstract

We study a sixth order derivative scalar field model in Minkowski spacetime as a toy model of higher-derivative critical gravity theories. This model is consistently quantized when using the Becchi-Rouet-Stora-Tyutin (BRST) quantization scheme even though it does not show gauge symmetry manifestly. Imposing a BRST quartet generated by two scalars and ghosts, there remains a non-trivial subspace with positive norm. This might be interpreted as a Minkowskian dual version of the unitary truncation in the logarithmic conformal field theory.

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I. INTRODUCTION

Stelle \[1\] has first introduced the quadratic curvature gravity of \(\alpha \left( R_{\mu\nu}^2 - \frac{R^2}{3} \right) + \beta R^2 \) to improve the perturbative properties of Einstein gravity. In case of \(\alpha \beta \neq 0\), the renormalizability was achieved but the unitarity was violated for \(\alpha \neq 0\), showing that the renormalizability and unitarity exclude to each other. Although the \(\alpha\)-term of providing massive graviton improves the ultraviolet divergence, it induces ghost excitations which spoil the unitarity. The price one has to pay for making the theory renormalizable is the loss of unitarity.

After this work, a first requirement for the quantum gravity is to gain the unitarity, which means that its linearized theory has no tachyon and ghost in the particle content \[2\]. To that end, critical gravities have recently received much attention because they were considered as toy models for quantum gravity \[3-10\]. At the critical point, a degeneracy took place in the AdS spacetime and thus, ghost-like massive gravitons become massless gravitons. Instead of massive gravitons, an equal amount of logarithmic modes were introduced for the critical gravity. However, one has to resolve the non-unitarity problem of the critical gravity theories because these contain higher-derivative interactions \[4\]. It was shown that a rank-2 logarithmic conformal field theory (LCFT) is dual to a critical gravity \[11-13\]. Thus, the non-unitarity of critical gravity is closely related to that of the rank-2 LCFT where the Hamiltonian cannot be diagonalized on the fields due to the Jordan structure \[14, 15\].

In order to address the non-unitarity issue of critical gravity, it has been proposed to truncate log-modes out by imposing the AdS boundary conditions \[16\]. A rank of the LCFT refers to the dimensionality of the Jordan cell. The rank-2 LCFT dual to a critical gravity has a rank-2 Jordan cell and thus, an operator has a logarithmic partner. However, there remains nothing for the rank-2 LCFT after truncation. Dipole-ghost fields \((A, B)\) on AdS\(_3\) space are also dual to the rank-2 LCFT \[17, 18\], whereas they form a BRST quartet to give zero norm state \[19, 21\] after introducing ghosts in Minkowski spacetime \[22\]. Instead, a polycritical gravity was introduced to provide a polycritical point \[23, 24\] whose dual is supposed to be a higher rank LCFT. The LCFT dual to a tricritical gravity has rank-3 Jordan cell \[25\] and an operator has two logarithmic partners of log and log\(^2\). After truncation, there remains a unitary subspace with non-negative inner product. Its dual scalar model was investigated on the BTZ black hole spacetime explicitly \[26\].

In this direction, it is very important to understand the truncation mechanism which
leads naturally to the unitary subspace. However, the fact that the bulk spacetime is AdS and theory is a polycritical gravity prevents us from understanding this truncation scheme well. Hence, we consider a sixth-order derivative scalar field theory in Minkowski spacetime. To avoid a difficulty in dealing with a single sixth-order derivative theory directly [27], we introduce an equivalent three coupled scalar fields with degenerate masses. This model will be quantized by employing the BRST quantization scheme even though it does not show gauge symmetry manifestly. Imposing a BRST quartet generated by two scalars $\phi_1, \phi_3$ and ghosts $c, d$, there remains a non-trivial subspace with positive norm for $\phi_2$. This could be interpreted to be a Minkowskian dual version of the unitary truncation in the LCFT.

Our action consists of three scalar fields $\phi_1, \phi_2, \phi_3$, and ghost fields $c, d$ with degenerate masses in four dimensional spacetime

$$S = -\int d^4x \left[ \partial_\mu \phi_1 \partial^\mu \phi_3 + \frac{1}{2} \partial_\mu \phi_2 \partial^\mu \phi_2 + \phi_1 \phi_2 + m^2 \phi_1 \phi_3 + \frac{1}{2} m^2 \phi_2^2 + \partial_\mu c \partial^\mu d + m^2 cd \right].$$  \(1\)

Without ghosts, the action appeared in [16, 26]. Here, we adopt the conventions of $\eta_{\mu\nu} = \text{diag.}(-+++)$ and $x^\mu = (t, \vec{x})$. We note to stress that the action \(1\) is invariant under BRST transformations

$$\delta \phi_1 = 0, \quad \delta \phi_2 = 0, \quad \delta \phi_3 = c, \quad \delta c = 0, \quad \delta d = \phi_1. \quad (2)$$

Here the ghost numbers are assigned to be $[\phi_i] = 0 \ (i = 1, 2, 3)$, $[c] = -1$, and $[d] = 1$. In the non-degenerate masses, there is no the BRST invariance like \(2\) because this symmetry is not nilpotent.

**II. SIXTH-ORDER TIME DERIVATIVE PARTICLE THEORY**

In order to understand a sixth-order derivative scalar theory, we consider first its sixth-order time derivative version of the action \(1\)

$$S = \int dt \left[ \dot{\phi}_1 \dot{\phi}_3 + \frac{1}{2} \dot{\phi}_2^2 - \dot{\phi}_1 \phi_2 - m^2 \phi_1 \phi_3 - \frac{1}{2} m^2 \phi_2^2 + \dot{c} \dot{d} - m^2 cd \right], \quad (3)$$

where the dot denotes differentiation with respect to time. This action could describe a sixth order derivative harmonic oscillator for $\phi_3$ when coupling to $\phi_1$ and $\phi_2$. Equations of motion can be obtained as

$$\ddot{\phi}_1 + m^2 \phi_1 = 0, \quad (4)$$

$$\ddot{\phi}_2 + m^2 \phi_2 = 0, \quad (5)$$

$$\ddot{\phi}_3 + \phi_1 \phi_2 - m^2 \phi_1 \phi_3 - \frac{1}{2} m^2 \phi_2^2 + \dot{c} \dot{d} - m^2 cd = 0. \quad (6)$$
\[\ddot{\phi}_2 + m^2 \phi_2 = -\phi_1, \quad (5)\]
\[\ddot{\phi}_3 + m^2 \phi_3 = -\phi_2 \quad (6)\]

by varying \(\phi_3, \phi_2,\) and \(\phi_1,\) respectively. Eliminating \(\phi_1\) and \(\phi_2\) leads to the sixth-order time derivative equation for \(\phi_3\)
as
\[\left(\frac{d^2}{dt^2} + m^2\right)^3 \phi_3 = 0. \quad (7)\]

On the other hand, eliminating \(\phi_1\) leads to the fourth order equation for \(\phi_2\)
\[\left(\frac{d^2}{dt^2} + m^2\right)^2 \phi_2 = 0 \quad (8)\]
which is recognized to be the degenerate Pais-Uhlenbeck oscillator with \(m_i = \omega_i^2\) [28]. Its field theory realization was reported in [29].

The solutions to Eqs. (4)-(6) are given by
\[
\phi_1(t) = iN_1 \left(a_1 e^{-imt} - a_1^\dagger e^{imt}\right), \quad (9)
\]
\[
\phi_2(t) = \frac{N_1}{2m^2} \left[ (a_2 + a_1 mt) e^{-imt} + \left(a_2^\dagger + a_1^\dagger mt\right) e^{imt} \right], \quad (10)
\]
\[
\phi_3(t) = -iN_1 \frac{1}{4m^4} \left[ \left(a_3 + \left(a_2 - i\frac{1}{2}a_1\right) mt + \frac{1}{2}a_1^\dagger m^2 t^2\right) e^{-imt} \right.
\]
\[
\left. - \left(a_3^\dagger + \left(a_2^\dagger + i\frac{1}{2}a_1^\dagger\right) mt + \frac{1}{2}a_1^\dagger m^2 t^2\right) e^{imt} \right], \quad (11)
\]
respectively. Note that they are all hermitians. It is also easy to check that the solution [(11) [(10)] solves the higher-order equation [(7) [(8)] directly. Fig. 1 shows the temporal behaviors of the solutions. \(\phi_1\) shows a pure oscillation in time. Even though \(\phi_2\) and \(\phi_3\) are growing linearly and quadratically in time, their growths are milder than an exponentially growing mode. Although these linear and quadratic growths are independent of the presence of the Ostrogradski instability, they show that \(\phi_3\) and \(\phi_2\) are solutions to higher-order time derivative equations [(7) and [(8)]. Also, there is no imaginary propagation speed that would signify a classical instability in the form of a tachyonic mode appeared when replacing \(m^2\) by \(-m^2\) in Eqs. (4)-(6).

On the other hand, equations of motion for the ghosts
\[\ddot{c} + m^2 c = 0, \quad (12)\]
\[\ddot{d} + m^2 d = 0 \quad (13)\]
lead to solutions as

\[ c(t) = -\frac{iN}{4m^4} \left( c_1 e^{-imt} - c_1^\dagger e^{imt} \right), \tag{14} \]
\[ d(t) = iN_1 \left( d_1 e^{-imt} - d_1^\dagger e^{imt} \right), \tag{15} \]

respectively. Fig. 2 shows the temporal behaviors of the ghost solutions as \( \phi_1 \) does show.

Now we are in a position to carry out canonical quantization of (3). This may provide a hint to quantization of its full action (1). Canonical quantization can be started with finding canonical momenta

\[ \pi_1 = \dot{\phi}_3, \quad \pi_2 = \dot{\phi}_2, \quad \pi_3 = \dot{\phi}_1. \tag{16} \]

Canonical Hamiltonian is given by

\[ H_c = \pi_1 \pi_3 + \frac{1}{2} \pi_2^2 + \phi_1 \dot{\phi}_2 + m^2 \phi_1 \phi_3 + \frac{1}{2} m^2 \dot{\phi}_2^2. \tag{17} \]

Expressing (17) in terms of modes \( a_1, a_2, \) and \( a_3 \) in Eqs. (9)-(11), the canonical Hamiltonian becomes

\[ H_c = \frac{N_1^2}{2m^2} \left[ a_2^\dagger a_2 + 2i \left( a_2^\dagger a_1 - a_1^\dagger a_2 \right) - a_3^\dagger a_1 - a_1^\dagger a_3 \right]. \tag{18} \]
From now on, we would take $N_1^2 = 2m^3$ for convenience, if not mentioned otherwise. The equal-time commutation relations between operators are obtained as

$$[a_1, a_3^\dagger] = -1, \quad [a_2, a_2^\dagger] = 1, \quad [a_2, a_3^\dagger] = i, \quad [a_3, a_3^\dagger] = \frac{3}{2}. \quad (19)$$

These can be cast into the following matrix form:

$$[a_i, a_j^\dagger] = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & -i \\ -1 & -i & \frac{3}{2} \end{pmatrix}. \quad (20)$$

Although its Hamiltonian is not diagonal and positive definite, their commutation relations reveal useful information between operators. Clearly, $[a_2, a_2^\dagger] = 1$ represents a standard commutation relation, while others do show non-standard commutations. In order to make the Hamiltonian diagonal, one has to introduce new operators $b_i$ by using transformations

$$a_1 = \frac{3\sqrt{2}i}{8}b_1 - \frac{\sqrt{2}i}{8}b_2 + \frac{i}{2}b_3, \quad a_2 = \sqrt{2}b_1 - \sqrt{2}b_2 + b_3, \quad a_3 = -3b_1 + b_2 - 2(\sqrt{2} + i)b_3. \quad (21)$$

Then, the canonical Hamiltonian can be reduced to the diagonal form

$$\mathcal{H}_c = m \left(-b_1^\dagger b_1 + b_2^\dagger b_2 + b_3^\dagger b_3\right). \quad (22)$$

However, we are afraid to have unusual commutation relations between $b_i$ and $b_j^\dagger$

$$[b_1, b_1^\dagger] = -\frac{133}{64}, \quad [b_1, b_2^\dagger] = -\frac{55}{64}, \quad [b_1, b_3^\dagger] = \frac{19\sqrt{2}}{32}, \quad [b_2, b_1^\dagger] = \frac{55}{64}, \quad [b_2, b_2^\dagger] = \frac{17\sqrt{2}}{32}, \quad [b_3, b_3^\dagger] = \frac{3}{8}, \quad (23)$$

where all commutation relations survive. We call these unusual commutators because all constant factors are nonstandard and signs are negative in the first two expressions.

On the other hand, the canonical momenta for the ghost parts are given by

$$\pi_c = \dot{d}, \quad \pi_d = -\dot{c}, \quad (24)$$

and the canonical Hamiltonian is constructed by using modes

$$\mathcal{H}_{c}^{\text{gh}} = m \left(d_1^\dagger c_1 - c_1^\dagger d_1\right). \quad (25)$$
Their anti-commutation relations are defined to be

\[ \{c, d^\dagger\} = -1, \quad \{d, c^\dagger\} = 1. \quad (26) \]

Finally, the BRST charge can be obtained as

\[ Q_B = a_1^\dagger c_1 - c_1^\dagger a_1, \quad Q_B^2 = 0, \quad (27) \]

and, the total canonical Hamiltonian of \( \mathcal{H}_c + \mathcal{H}_c^{\text{gh}} \) is invariant under the BRST transformation of

\[ \delta a_1 = [Q_B, a_1] = 0, \quad \delta a_2 = [Q_B, a_2] = 0, \quad \delta a_3 = [Q_B, a_3] = c_1, \]
\[ \delta c_1 = [Q_B, c_1] = 0, \quad \delta d_1 = [Q_B, d_1] = a_1, \quad (28) \]

which imply that \( \{a_2\} \) represents unitary Fock space, while \( \{a_1, a_3, c_1, d_1\} \) form a quartet representation of BRST algebra. Here we wish to point out that for dipole ghost fields (fourth-order time derivative theory), the BRST invariant states appears only zero norm combination through the quartet mechanism. This indicates that physical state is the vacuum \[22\]. On the other hand, for the sixth-order time derivative scalar theory, the physical state is not the vacuum but \( \phi_2 \). This shows clearly a way of how a higher derivative harmonic oscillator is free from negative norm states when quantizing it.

III. QUANTIZATION OF SIXTH ORDER DERIVATIVE SCALAR THEORY

In this section, we consider the full action \([1]\) in the field theoretic point of view. Inspired by the canonical quantization in the previous section, we wish to carry out the BRST quantization of the action \([1]\). Usually, the BRST symmetry was found in gauge theories as a symmetry of the gauge-fixed action. Its purpose is definitely to remove the negative norm states associated with the gauge invariance. Physical states are defined as those which have zero ghost number and are invariant under the BRST transformations. Here the BRST transformation \([2]\) is not due to gauge invariance. Surely, it takes into account a feature of giving the sixth-order derivative structure starting from the second-order action \([1]\) via coupling. Since we have a BRST symmetry, there is no doubt to require that the physical states are those which have zero ghost number and are left invariant under the BRST transformation.
Varying the fields $\phi_3$, $\phi_2$, and $\phi_1$ in the action (1), we have equations of motion

\[
\begin{align*}
(\nabla^2 - m^2) \phi_1 &= 0, \\
(\nabla^2 - m^2) \phi_2 &= \phi_1, \\
(\nabla^2 - m^2) \phi_3 &= \phi_2,
\end{align*}
\]

respectively. Making use of an ansatz

\[
\phi_1(x) = \int \frac{d^3k}{(2\pi)^{3/2} \sqrt{2\omega}} \phi_{1k}(\vec{k}\!\cdot\!\vec{x}) e^{i\vec{k}\!\cdot\!\vec{x}}
\]

the equation of motion (29) leads to one dimensional equation for $\phi_{1k}(\vec{k}\!\cdot\!\vec{x})$ as

\[
\left( \frac{d^2}{dt^2} + \omega^2 \right) \phi_{1k}(\vec{k}\!\cdot\!\vec{x}) = 0,
\]

where $\omega^2 = \vec{k}^2 + m^2$. As like in the quantum mechanical model in the previous section, this gives a solution expanded in Fourier modes

\[
\phi_1(x) = \int \frac{d^3k}{(2\pi)^{3/2} \sqrt{2\omega}} iN_1 \left( a_1(\vec{k}) e^{-i\omega t + i\vec{k}\!\cdot\!\vec{x}} - a_1^\dagger(\vec{k}) e^{i\omega t - i\vec{k}\!\cdot\!\vec{x}} \right).
\]

Similarly, we can find the solutions to Eqs. (30) and (31) by using (34) as

\[
\begin{align*}
\phi_2(x) &= \int \frac{d^3k}{(2\pi)^{3/2} \sqrt{2\omega}} \left( -\frac{iN_1}{4\omega^4} \left[ \left( a_3(\vec{k}) + \frac{i}{2}a_1(\vec{k}) \omega t \right) e^{-i\omega t + i\vec{k}\!\cdot\!\vec{x}} + \left( a_3^\dagger(\vec{k}) + \frac{i}{2}a_1^\dagger(\vec{k}) \omega t \right) e^{i\omega t - i\vec{k}\!\cdot\!\vec{x}} \right] \right), \\
\phi_3(x) &= \int \frac{d^3k}{(2\pi)^{3/2} \sqrt{2\omega}} \left( -\frac{iN_1}{4\omega^4} \left[ \left( a_3(\vec{k}) - \frac{i}{2}a_1(\vec{k}) \omega t \right) e^{-i\omega t + i\vec{k}\!\cdot\!\vec{x}} - \left( a_3^\dagger(\vec{k}) - \frac{i}{2}a_1^\dagger(\vec{k}) \omega t \right) e^{i\omega t - i\vec{k}\!\cdot\!\vec{x}} \right] \right).
\end{align*}
\]

On the other hand, for the ghost fields, their equations of motion are given by

\[
\begin{align*}
(\nabla^2 - m^2) c &= 0, \\
(\nabla^2 - m^2) d &= 0,
\end{align*}
\]

whose solutions are found to be

\[
\begin{align*}
c(x) &= \int \frac{d^3k}{(2\pi)^{3/2} \sqrt{2\omega}} \left( -\frac{iN_1}{4\omega^4} \left[ c_1(\vec{k}) e^{-i\omega t + i\vec{k}\!\cdot\!\vec{x}} - c_1^\dagger(\vec{k}) e^{i\omega t - i\vec{k}\!\cdot\!\vec{x}} \right] \right), \\
d(x) &= \int \frac{d^3k}{(2\pi)^{3/2} \sqrt{2\omega}} \left( iN_1 \left[ d_1(\vec{k}) e^{-i\omega t + i\vec{k}\!\cdot\!\vec{x}} - d_1^\dagger(\vec{k}) e^{i\omega t - i\vec{k}\!\cdot\!\vec{x}} \right] \right).
\]
Now, canonical momenta is given by those as in Eq. (16), and canonical Hamiltonian is obtained to be
\[ H_c = \int d^3x \left[ \pi_1 \pi_3 + \frac{1}{2} \pi_2^2 + \phi_1 \phi_2 + m^2 \phi_1 \phi_3 + \frac{1}{2} m^2 \phi_2^2 + \vec{\nabla} \phi_1 \cdot \vec{\nabla} \phi_3 + \frac{1}{2} (\vec{\nabla} \phi_2)^2 \right]. \] (41)

It is tedious but straightforward to express the canonical Hamiltonian in terms of modes \( a_1(\vec{k}), a_2(\vec{k}), \) and \( a_3(\vec{k}) \) as
\[ H_c = \int \frac{d^3k}{2\omega} \left( \frac{N_1^2}{2\omega^2} \right) \left[ a_2^\dagger(\vec{k})a_2(\vec{k}) + 2i \left( a_2^\dagger(\vec{k})a_1(\vec{k}) - a_1^\dagger(\vec{k})a_2(\vec{k}) \right) - a_3^\dagger(\vec{k})a_1(\vec{k}) - a_1^\dagger(\vec{k})a_3(\vec{k}) \right], \] (42)

which corresponds to the field theoretical representation of the canonical Hamiltonian \([18]\).

Importantly, the commutation relations are given by
\[
\begin{align*}
[a_1(\vec{k}), a_2^\dagger(\vec{k}')] &= -2\omega \delta^3(\vec{k} - \vec{k}'), \\
[a_2(\vec{k}), a_2^\dagger(\vec{k}')] &= 2\omega \delta^3(\vec{k} - \vec{k}'), \\
[a_2(\vec{k}), a_3^\dagger(\vec{k}')] &= 2i \omega \delta^3(\vec{k} - \vec{k}'), \\
[a_3(\vec{k}), a_3^\dagger(\vec{k}')] &= 3\omega \delta^3(\vec{k} - \vec{k}'),
\end{align*}
\] (43)

where we have also taken \( N_1^2 = 2\omega^3 \). These can be also cast into the matrix form
\[
[a_i(\vec{k}), a_j^\dagger(\vec{k}')] = 2\omega \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & -i \\ -1 & -i & \frac{3}{2} \end{pmatrix} \delta^3(\vec{k} - \vec{k}'),
\] (44)

which is similar to the two-point correlation functions in rank-3 LCFT \([16]\)
\[ < O^i O^j > \sim \begin{pmatrix} 0 & 0 & CFT \\ 0 & CFT & L \\ -CFT & L & L^2 \end{pmatrix}, \] (45)

where \( i, j = \text{KG, log, log}^2 \). Here \( CFT \) is the CFT correlation function, \( L \) represents log-correlation function, \( L^2 \) denotes \( \log^2 \)-correlation function. A truncation to have a unitary subspace is carried by throwing all modes which generate the third column and row of this matrix containing \( L^2 \). Hence, the only non-zero correlation function is proportional to the ordinary CFT correlation. Using the AdS/LCFT correspondence, the remaining bulk modes have a non-negative scalar product and the truncated theory is unitary. This method can be generalized to arbitrary odd rank but it fails for even rank LCFTs.
By making use of the transformations of
\begin{align*}
a_1(\vec{k}) &= \frac{3\sqrt{2}i}{8}b_1(\vec{k}) - \frac{\sqrt{2}i}{8}b_2(\vec{k}) + \frac{i}{2}b_3(\vec{k}), \\
a_2(\vec{k}) &= \sqrt{2}b_1(\vec{k}) - \sqrt{2}b_2(\vec{k}) + b_3(\vec{k}), \\
a_3(\vec{k}) &= -3b_1(\vec{k}) + b_2(\vec{k}) - 2(\sqrt{2} + i)b_3(\vec{k}),
\end{align*}
the canonical Hamiltonian can be successfully reduced to a diagonal form
\begin{align*}
H_c &= \int \frac{d^3k}{2\omega} \left[ -b_1^\dagger(\vec{k})b_1(\vec{k}) + b_2^\dagger(\vec{k})b_2(\vec{k}) + b_3^\dagger(\vec{k})b_3(\vec{k}) \right].
\end{align*}
Canonical momenta for the ghosts are the same with Eq. (24), and canonical Hamiltonian is given by
\begin{align*}
H^{gh}_c &= \int \frac{d^3k}{2\omega} \omega \left( d_1^\dagger(\vec{k})c_1(\vec{k}) - c_1^\dagger(\vec{k})d_1(\vec{k}) \right).
\end{align*}
Their commutation relations are
\begin{align*}
\{c_1(\vec{k}), d_1^\dagger(\vec{k}')\} &= -2\omega \delta^3(\vec{k} - \vec{k}'), \\
\{d_1(\vec{k}), c_1^\dagger(\vec{k}')\} &= 2\omega \delta^3(\vec{k} - \vec{k}').
\end{align*}
Finally, the BRST charge can be obtained as the Noether charge
\begin{align*}
Q_B = \int d^3x \left( a_1^\dagger(\vec{k})c_1(\vec{k}) - c_1^\dagger(\vec{k})a_1(\vec{k}) \right), \\
Q_B^2 &= 0,
\end{align*}
and the total canonical Hamiltonian of $H_c + H^{gh}_c$ is invariant under the BRST transformations
\begin{align*}
\delta a_1(\vec{k}) &= [Q_B, a_1] = 0, \\
\delta a_2(\vec{k}) &= [Q_B, a_2] = 0, \\
\delta a_3(\vec{k}) &= [Q_B, a_3] = c_1(\vec{k}), \\
\delta c_1(\vec{k}) &= [Q_B, c_1] = 0, \\
\delta d_1(\vec{k}) &= [Q_B, d_1] = a_1(\vec{k}).
\end{align*}
Here we observe that $\{a_2\}$ is invariant under the BRST transformations, while $\{a_1, a_3, c_1, d_1\}$ form a quartet representation of BRST algebra to give zero norm state. For the sixth-order derivative scalar theory, the physical state is not vacuum but $\phi_2$. This shows clearly a way of how a higher derivative scalar field theory is free from negative norm states.

**IV. DISCUSSIONS**

We have clarified the truncation scheme to provide the unitary subspace in the rank-3 LCFT. This was just an obtaining procedure of a unitary subspace after forming a BRST
quartet even for a sixth-order derivative scalar field theory. The correspondence between the rank-3 LCFT and sixth order derivative scalar field theory are given by observing (44) and (45). This is clear because (45) was obtained from the same action (1) on the boundary of the AdS$_3$ spacetime without ghosts. The difference is that we consider (1) in Minkowski spacetime.

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[1] K. S. Stelle, Phys. Rev. D 16, 953 (1977).
[2] N. H. Barth and S. M. Christensen, Phys. Rev. D 28, 1876 (1983).
[3] W. Li, W. Song and A. Strominger, JHEP 0804, 082 (2008) [arXiv:0801.4566 [hep-th]].
[4] H. Lu and C. N. Pope, Phys. Rev. Lett. 106, 181302 (2011) [arXiv:1101.1971 [hep-th]].
[5] S. Deser, H. Liu, H. Lu, C. N. Pope, T. C. Sisman and B. Tekin, Phys. Rev. D 83, 061502 (2011) [arXiv:1101.4009 [hep-th]].
[6] M. Alishahiha and R. Fareghbal, Phys. Rev. D 83, 084052 (2011) [arXiv:1101.5891 [hep-th]].
[7] E. A. Bergshoeff, O. Hohm, J. Rosseel and P. K. Townsend, Phys. Rev. D 83, 104038 (2011) [arXiv:1102.4091 [hep-th]].
[8] M. Porrati and M. M. Roberts, Phys. Rev. D 84, 024013 (2011) [arXiv:1104.0674 [hep-th]].
[9] Y. S. Myung, Y. -W. Kim, T. Moon and Y. -J. Park, Phys. Rev. D 84, 024044 (2011) [arXiv:1105.4205 [hep-th]].
[10] H. Lu, C. N. Pope, E. Sezgin and L. Wulff, JHEP 1110, 131 (2011) [arXiv:1107.2480 [hep-th]].
[11] D. Grumiller and N. Johansson, JHEP 0807, 134 (2008) [arXiv:0805.2610 [hep-th]].
[12] Y. S. Myung, Phys. Lett. B 670, 220 (2008) [arXiv:0808.1942 [hep-th]].
[13] A. Maloney, W. Song and A. Strominger, Phys. Rev. D 81, 064007 (2010) [arXiv:0903.4573]
[hep-th]].

[14] V. Gurarie, Nucl. Phys. B 410, 535 (1993) [hep-th/9303160].

[15] M. Flohr, Int. J. Mod. Phys. A 18, 4497 (2003) [hep-th/0111228].

[16] E. A. Bergshoeff, S. de Haan, W. Merbis, M. Porrati and J. Rosseel, JHEP 1204, 134 (2012) arXiv:1201.0449 [hep-th].

[17] I. I. Kogan, Phys. Lett. B 458, 66 (1999) hep-th/9903162.

[18] Y. S. Myung and H. W. Lee, JHEP 9910, 009 (1999) hep-th/9904056.

[19] C. Becchi, A. Rouet and R. Stora, Annals Phys. 98, 287 (1976).

[20] I. V. Tyutin, LEBEDEV-75-39, arXiv:0812.0580 [hep-th].

[21] T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. 66, 1 (1979).

[22] V. O. Rivelles, Phys. Lett. B 577, 137 (2003) hep-th/0304073.

[23] T. Nutma, Phys. Rev. D 85, 124040 (2012) arXiv:1203.5338 [hep-th].

[24] A. Kleinschmidt, T. Nutma and A. Virmani, Gen. Rel. Grav. 45, 727 (2013) arXiv:1206.7095 [hep-th].

[25] E. A. Bergshoeff, S. de Haan, W. Merbis, J. Rosseel and T. Zojer, Phys. Rev. D 86, 064037 (2012) arXiv:1206.3089 [hep-th].

[26] T. Moon and Y. S. Myung, Phys. Rev. D 86, 084058 (2012) arXiv:1208.5082 [hep-th].

[27] F. J. de Urries and J. Julve, J. Phys. A 31, 6949 (1998) hep-th/9802115.

[28] A. Pais and G. E. Uhlenbeck, Phys. Rev. 79, 145 (1950).

[29] J. B. Jimenez, E. Dio and R. Durrer, JHEP 1304, 030 (2013) arXiv:1211.0441 [hep-th].