Analytical calculation of the solid angle defined by a cylindrical detector and a point cosine source with parallel axes

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Abstract

We derive analytical expressions for the solid angle subtended by a right finite circular cylinder at a point source with cosine angular distribution in the case where the source direction is parallel to the cylinder axis. As a subsidiary result, an expression for the solid angle subtended by a disc detector at a spread disc source is also provided, in the case where the two discs have a common symmetry axis which is also coincident with the source direction.

Key words: solid angle, point cosine source, cylindrical detector, cylinder, analytic expressions

1 Introduction

In many situations in radiation physics the value of the solid angle subtended by a circular cylindrical detector at a point source is needed. The case of an isotropic point source has been treated to great extent (Jaffey (1954), Macklin (1957), Masket et al (1955), Masket (1957), Gillespie (1970), Gardner and Verghese (1971), Green et al (1974), Prata (2003b)).

In a recent work (Prata, 2003a) we derived analytical expressions for the solid angle subtended by a cylinder at a point cosine source, under the restriction that the source and cylinder axes are orthogonal to each other. In the present...
work we obtain similar expressions in the case where the source direction is parallel to the cylinder axis. An expression (eq. 33) for the solid angle subtended by a circular disc with symmetry axis parallel to the source direction is also given as an auxiliary result. It should be mentioned that this latter expression appeared previously (Hubbell et al., 1961, eq. 29) in a slightly different context. The authors detailed a quite general procedure to calculate the response of a small detector to an axially symmetric source with arbitrary polar angle distribution, uniformly spread on a circular disc. In that work, $\Omega_{\text{circ}}$ (here eq. 33) is interpreted as the response of a plane detector parallel to a Lambertian uniformly distributed disc source. In the same work the result is also credited to other authors (Herman (1900), Footz (1915)).

With the on-going computer revolution, very complex problems otherwise impossible to tackle, are routinely addressed using numerical methods. Calculations such as the one presented here, amounting to a one-dimensional integration, can be quickly specified and performed in a desk-top computer with minimal effort. Nevertheless, we believe that in the few fortunate occasions where simple expressions exist in closed form, they deserve an interested look and are worth the effort of finding them. There are various reasons for this. No matter how careful the error-checking procedure followed, any numerical result should always be taken with some degree of prudence. In fact, except for the most simple situations, a great deal of effort is normally put into testing the result, rather than into obtaining it. Analytical results can be of some help here by providing reliable tests to computational algorithms. Also, a closed expression can sometimes be given an intuitive interpretation like the one provided for $\Omega_{\text{circ}}$ in section (2.3) and, at the very least, enables the usage of analytical tools to search for extreme values, regions of monotony, reliable approximations, asymptotic expressions, etc. Furthermore, the existence of a closed expression for a point source reduces the complexity of the problem when considering the more realistic case of a planar spread source. The required four-dimensional integral can of course be reduced to a double integral by using the exact expressions deduced here for a point source. In section (2.4) we illustrate this by obtaining an analytical result for the solid angle defined by a cosine source evenly distributed on a circular disc and a disc detector, in the very special situation where the two discs share the same symmetry axis which is also assumed to be coincident with the source direction. Reducing the dimensionality of a numerical integration is of obvious

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2 In fact, some of the results displayed in the present work were checked against a numerical integration.

3 In the case of an uniformly spread source on a disc, a reduction to a double integral could also be achieved with resort to the exact expressions obtained by Hubbell et al. (1961) for the radiation field created by such source at an arbitrary point. The remaining double integral would then be performed over the detector surface rather than over the source surface.
advantage because the calculation is less prone to numerical errors and can be performed faster. In fact, when using quadrature formulae like Simpson’s or Gaussian rules to calculate multidimensional integrals, the total number of points \((N_T)\) scales like \(N_T = N^d\), where \(N\) is the number of quadrature points used in each dimension and \(d\) is the dimensionality of the integral. The number of evaluations of the integrand can then become prohibitively high, regardless of the computing resources available, making necessary the resort to the Monte Carlo method, which, of course, has the drawback of a small convergence rate (only as \(1/\sqrt{N_T}\)). This was discussed in a review by James (1980), who set (in 1980) the feasibility limit to the use of a ten-point Gaussian rule to five dimensions (with limited computer resources available) and to ten dimensions (with ‘unlimited’ computer resources).

To illustrate the behavior of the solid angle sample plots are presented in section (3).

2 Solid Angle Calculation

The solid angle \((\Omega_{surf})\) subtended by a given surface at a point source can be defined as

\[
\Omega_{surf} = \iint_{\text{hitting surface}} f(\Omega) d\Omega ,
\]  

(1)

where \(f(\Omega)d\Omega\) is the source distribution. In the case of a point cosine the distribution is defined with respect to some direction axis specified by the unit vector \(k\) and it is given by \(f(\Omega) = (\Omega \cdot k + |\Omega \cdot k|)/(2\pi)\). The \((2\pi)^{-1}\) factor ensures that \(0 \leq \Omega_{surf} \leq 1\). Setting \(\mu = \Omega \cdot k = \cos(\nu)\), where \(\nu\) is the angle between the two axes, there results that \(f(\Omega) = \{\mu/\pi (\mu \geq 0); 0 (\mu < 0)\}\) so that the source only emits into the hemisphere around \(k\). In the following we shall consider the situation of a right circular cylinder with axis parallel to \(k\). The source is assumed to be at the origin of the coordinate system and the \(z\) axis is chosen both aligned with \(k\) and parallel to the cylinder axis. The solid

\[\text{It should be mentioned that there is always some value of } d \text{ above which the Monte Carlo method is faster than any quadrature rule. This is so because the convergence rate of the quadrature rule is that of the one-dimensional rule. For instance, Simpson’s rule error is proportional } N^{-4} \text{ or to } N_T^{-4/d} \text{ in } d \text{ dimensions, which means Simpson’s rule should, in general, be slower than Monte Carlo for } d > 8.\]
angle is then given by

$$\Omega_{surf} = \pi^{-1} \int_{\phi_{\text{min}}}^{\phi_{\text{max}}} \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \cos(\theta) \sin(\theta) d\theta d\varphi$$

$$= (2\pi)^{-1} \int_{\phi_{\text{min}}}^{\phi_{\text{max}}} (\sin^2(\theta_{\text{max}}) - \sin^2(\theta_{\text{min}})) d\varphi ,$$

(2)

where $\theta$ is the polar angle from the z axis; $\varphi$ is the azimuthal angle in the xy plane and the limiting angles are to be determined from the conditions that $\mu = \cos(\theta) \geq 0$ and that each included $(\theta, \varphi)$ direction hits the surface.

The solid angle $\Omega$ of the whole cylinder can in general be decomposed according to $\Omega = \Omega_{cyl} + \Omega_{circ}$, where $\Omega_{cyl}$ and $\Omega_{circ}$ are the contributions of the cylindrical surface and of one of the end circles. To calculate these quantities we refer to figs. 1 and 2, where, for simplicity, we assume that the cylinder and disc axes lie in the $xz$ plane. To obtain $\Omega_{cyl}$ it is sufficient to consider the situation depicted in fig. 1, where one of the end discs is at the same plane as the source (i.e. $z = 0$). Let $\Omega_{cyl0}(L, r, d)$ denote the solid angle in this case and $\Omega_{circ}(L, r, d)$ the solid angle defined by the disc. In the case of the disc we distinguish the situation shown if fig. 2, where $r > d$, from that where $d > r$. For economy this latter situation is also represented in fig. 1, with reference to the circle at the plane $z = L$.

![Fig. 1. Notation for $\Omega_{cyl0}$ and for $\Omega_{circ} (z = L)$ when $d > r$](image1)

![Fig. 2. Notation for $\Omega_{circ}$ when $d < r$](image2)

Using eq. 2 it follows that
Fig. 3. Definitions of (a) $\rho_\pm$, $\phi_\pm$ and $\phi_o$ ($d > r$) and (b) $\rho_+$, $\phi_+$ ($d < r$)

\[
\Omega_{cyl0}(L, r, d) = \pi^{-1} \int_{\phi_o}^{\phi_o} \left(1 - \rho_2(\varphi)/[L^2 + \rho_2(\varphi)]\right) d\varphi
\]
\[
= \pi^{-1} \int_{0}^{\phi_o} L^2/[L^2 + \rho_2(\varphi)] d\varphi \equiv \pi^{-1} A_-(\phi_o)
\]
\[(3)\]

\[
\Omega_{circ}(L, d > r) = \pi^{-1} \int_{0}^{\phi_o} \left(\rho_2^2(\varphi)/[L^2 + \rho_2^2(\varphi)] - \rho_2^2(\varphi)/[L^2 + \rho_2^2(\varphi)]\right) d\varphi
\]
\[
= \pi^{-1} \int_{0}^{\phi_o} L^2/[L^2 + \rho_2^2(\varphi)] d\varphi - \int_{0}^{\phi_o} L^2/[L^2 + \rho_2^2(\varphi)] d\varphi
\]
\[
\equiv \pi^{-1} [A_-(\phi_o) - A_+(\phi_o)]
\]
\[(4)\]

\[
\Omega_{circ}(L, d < r) = \pi^{-1} \int_{0}^{\pi} \rho_2^2(\varphi)/[L^2 + \rho_2^2(\varphi)] d\varphi
\]
\[
= 1 - \pi^{-1} \int_{0}^{\pi} L^2/[L^2 + \rho_2^2(\varphi)] d\varphi
\]
\[
= 1 - \pi^{-1} [A_+(\pi)]
\]
\[(5)\]

where, from fig.3,

\[
\phi_o \equiv \arcsin(r/d)
\]
\[(6)\]
and

\[ \rho_\pm(\varphi) = d \cos \varphi \pm \sqrt{r^2 - (d \sin \varphi)^2}. \] (7)

The required integrals \( A_\pm \) can be expressed in terms of the integral

\[ I(L, r, d, \phi_+) = \int \frac{L^2}{L^2 + \rho_+^2(\phi_+)} \frac{1}{2} \left( 1 + \frac{r^2 - d^2}{\rho_+^2(\phi_+)} \right) d\phi_+, \] (8)

where

\[ \rho_+(\phi_+) = \sqrt{d^2 + r^2 + 2dr \cos(\phi_+)} \] (9)

To proceed we begin by calculating \( I \).

2.1 Calculation of \( I \)

The integrand in the rhs of eq. 8 can be written as

\[ \frac{1}{2} \left( \frac{L^2 + d^2 - r^2}{L^2 + d^2 + r^2} \frac{1}{1 + m \cos(\phi_+)} + \frac{r^2 - d^2}{d^2 + r^2} \frac{1}{1 - n \cos(\phi_+)} \right), \] (10)

where

\[ m = 2rd/(L^2 + d^2 + r^2) \] (11)

and

\[ n = 2rd/(d^2 + r^2). \] (12)

The integration is straightforward, giving

\[ I = \frac{L^2 + d^2 - r^2}{L^2 + d^2 + r^2} \frac{1}{\sqrt{1 - m^2}} \arctan \left[ \sqrt{\frac{1 - m}{1 + m}} \tan \left( \frac{\phi_+}{2} \right) \right] \]

\[ + \frac{r^2 - d^2}{d^2 + r^2} \frac{1}{\sqrt{1 - n^2}} \arctan \left[ \sqrt{\frac{1 - n}{1 + n}} \tan \left( \frac{\phi_+}{2} \right) \right]. \] (13)
Since
\[
\frac{(r^2 - d^2)}{(d^2 + r^2)} = \begin{cases} \sqrt{1 - n^2} & (r > d); \sqrt{1 - n^2} & (d > r) \end{cases}
\]
and
\[
\frac{(L^2 + d^2 - r^2)}{(L^2 + d^2 + r^2)} = \begin{cases} 1 - m/n(1 + \sqrt{1 - n^2}) & (r > d); 1 - m/n(1 - \sqrt{1 - n^2}) & (d > r) \end{cases}
\]
there results that
\[
I = \frac{1 - m/n(1 + \sqrt{1 - n^2})}{\sqrt{1 - m^2}} \arctan\left[\frac{1 - m}{1 + m} \tan\left(\frac{\phi_+}{2}\right)\right] + \arctan\left[\frac{1 - n}{1 + n} \tan\left(\frac{\phi_+}{2}\right)\right]; (r > d)
\]
(15)
and
\[
I = \frac{1 - m/n(1 - \sqrt{1 - n^2})}{\sqrt{1 - m^2}} \arctan\left[\frac{1 - m}{1 + m} \tan\left(\frac{\phi_+}{2}\right)\right] - \arctan\left[\frac{1 - n}{1 + n} \tan\left(\frac{\phi_+}{2}\right)\right]; (d > r).
\]
(16)

To express \( A_+ \) in the rhs of eqs. 4 and 5 in terms of \( I \), a change of variable to \( \phi_+ \) represented in fig. 3 is made, using \( \phi_+/2 = \arctan\left[\sin(\varphi)\rho_+/\left(r - d + \cos(\varphi)\rho_+\right)\right] \), where \( \rho_+ = \rho_+(\varphi) \) is obtained from eq. 7. It follows that \( A_+ = I \) and, changing the integration limits, that
\[
A_+(\pi) = I\big|_0^\pi
\]
(17)
and
\[
A_+(\varphi_o) = I\big|_{\pi/2+\varphi_o}^{\varphi_o}.
\]
(18)
In the case of $A_-$, the integration variable is first changed to $\phi_-$ shown in fig. 3 and given by $\phi_-/2 = \arctan [\sin(\varphi) \rho_-/(r + d - \cos(\varphi) \rho_-)]$, where, again, $\rho_-$ is defined through eq. 7. Then,

$$A_-(\varphi_o) = -\pi/2 - \varphi_o \int_0^{\pi} \frac{L^2}{L^2 + \rho^2_-(\varphi_-)} \frac{1 + \frac{r^2 - d^2}{\rho^2_-(\varphi_-)}}{2} d\phi_-,$$

where

$$\rho_-(\varphi_-) = \sqrt{d^2 + r^2 - 2dr \cos(\varphi_-)} \quad (19)$$

Making a further change to $\tilde{\phi} = \pi - \phi_-$ yields

$$A_-(\varphi_o) = -\pi/2 + \varphi_o \int_{\pi/2 + \varphi_o}^{\pi} \frac{L^2}{L^2 + \rho^2_+(\varphi)} \frac{1 + \frac{r^2 - d^2}{\rho^2_+(\varphi)}}{2} d\tilde{\phi}, \quad (20)$$

where eqs. 19 and 9 were used to write

$$\rho_-(\varphi_-) = \rho_-(\pi - \tilde{\phi}) = \sqrt{d^2 + r^2 + 2dr \cos(\tilde{\phi})} = \rho_+(\tilde{\phi}).$$

By comparison of eqs. 20 and 8:

$$A_-(\varphi_o) = -I|_{\pi/2 + \varphi_o}^{\pi}.$$

Using eqs. 21, 18 and 17, eqs. 3 to 5 can be written as

$$\Omega_{cyl_0}(L, r, d) = -\pi^{-1}I|_{\pi/2 + \varphi_o}^{\pi} \quad (22)$$

$$\Omega_{circ}(L, d > r) = -\pi^{-1}(I|_{\pi/2 + \varphi_o}^{\pi} + I|_{0}^{\pi/2 + \varphi_o}) = -\pi^{-1}I|_{0}^{\pi} \quad (23)$$

and

$$\Omega_{circ}(L, r > d) = 1 - \pi^{-1}I|_{0}^{\pi} \quad (24)$$

Since $I$ is discontinuous at $d = r$ (see eqs. 15, 16), it should be clear that eqs. 23 and 24 do not mean that $\Omega_{circ}(L, d > r) = \Omega_{circ}(L, r > d) - 1$.

We now turn to eqs. 16 and 15 to evaluate the integrals. Since $\tan(\phi)/\rho = [(d + r)/(d - r)]^{1/2} = [(1 + n)/(1 - n)]^{1/4}$ and making use of $\arctan(z) + \arctan(1/z) = \pi/2, (z > 0)$, one obtains
\[
\pi^{-1}(\arctan \left[ \frac{1}{\sqrt{1-n}} \right] - \frac{1-m/n(1-\sqrt{1-n^2})}{\sqrt{1-m^2}} \arctan \left[ \frac{1}{\sqrt{1-m}} \sqrt{1+n} \right]) \Omega_{cyl0}(L, r, d = 0) = \pi
\]
(25)

For \( \Omega_{\text{circ}} \) there results
\[
\Omega_{\text{circ}}(L, d > r) = 1/2(1 - \frac{1-m/n(1-\sqrt{1-n^2})}{\sqrt{1-m^2}}) \quad (26)
\]
\[
\Omega_{\text{circ}}(L, d < r) = 1/2(1 - \frac{1-m/n(1+\sqrt{1-n^2})}{\sqrt{1-m^2}}) \quad (27)
\]

Defining \( \beta \) and \( \gamma \) by
\[
\tan(\beta/2) = \sqrt{\frac{1}{n}} / (1-n) \quad (28)
\]
and
\[
\cos(\gamma) = m \quad , \quad \text{(29)}
\]

eqs. 25, 26 and 27 can be recast as
\[
\Omega_{\text{circ}}(L, d > r) = 1/2(1 - \frac{1+\cos(\beta)\cos(\gamma)}{\sin(\gamma)}) \quad , \quad \text{(30)}
\]
\[
\Omega_{\text{circ}}(L, r > d) = 1/2(1 - \frac{\cos(\beta) + \cos(\gamma)}{\sin(\gamma) \cos(\beta)}) \quad (31)
\]

and
\[
\pi^{-1}(\arctan \left[ \frac{\beta}{2} - \frac{1+\cos(\beta)\cos(\gamma)}{\sin(\gamma)} \right] \arctan \left[ \cot(\frac{\gamma}{2}) \cot\left(\frac{\beta}{2}\right) \right]) = \Omega_{cyl0}(L, r, d)
\]
(32)

From eq. 28 it is seen that \( \pi/2 \leq \beta \leq \pi \) and that, for \( d > r \), \( \beta = \pi/2 + \varphi_o \).
As functions of $L$, $d$ and $r$, eqs. 26 and 27 can be rewritten to give $\Omega_{\text{circ}}$ in terms of a single expression:

$$\Omega_{\text{circ}}(L, d, r) = \frac{1}{2}(1 - \frac{d^2 + L^2 - r^2}{\sqrt{(r^2 + d^2 + L^2)^2 - 4r^2d^2}}),$$  \hspace{1em} (33)

which is equivalent to Hubbell et al. (1961, eq. 29), except for a factor of $\pi$.

2.2 Special values and continuity

First, the cases $L = 0$ and $d = r$ are discussed. From eqs. 11 and 12, it follows that

$L \to 0 \Leftrightarrow m \to n$

d $\to r \Leftrightarrow \{n \to 1 \land m \to m_1\}$

where $m_1 = 2r^2/(L^2 + 2r^2)$.

Then, using eq. 25,

$\Omega_{\text{cy}l0}(L \to 0, r < d) = \Omega_{\text{cy}l0}(m \to n) = 0$

$\Omega_{\text{cy}l0}(L \neq 0, d \to r^+) = \Omega_{\text{cy}l0}(n \to 1, m \to m_1) = 1/2$

and $\Omega_{\text{cy}l0}$ is thus discontinuous when $L \to 0, d \to r^+$.

Starting from eqs. 26, 27,

$\Omega_{\text{circ}}(L \neq 0, d \to r^\pm) = \Omega_{\text{circ}}(n \to 1, m \to m_1) = 1/2(1 - \sqrt{1-m_1})$

$\Omega_{\text{circ}}(L \to 0) = \{0 \ (d > r), 1/2 \ (d = r), 1 \ (d < r)\}$. One concludes that $\Omega_{\text{circ}}$ is continuous whenever $L \neq 0$.

Finally, for an infinite length cylinder at a skew position from the source (i.e. $d > r$) and with one end at the source plane there is the upper asymptotic value

$$\Omega_{\text{cy}l0}(L \to \infty, r < d) = \Omega_{\text{cy}l0}(m \to 0) = 1/2 - 2/\pi \arctan\left(\sqrt{\frac{1-n}{1+n}}\right)$$  \hspace{1em} (34)

$$= \varphi_o/\pi.$$  \hspace{1em} (35)
The last result can be checked directly using eq. 3 and can also be interpreted as the solid angle defined by an infinite length rectangle.

2.3 Geometrical interpretation of $\Omega_{\text{circ}}$

$\Omega_{\text{circ}}$ can be given a simple geometrical meaning by noticing that

$$\Omega_{\text{circ}} = 1/2(1 - \cos(\delta)),$$

(36)

where $\delta$, shown in fig. 4, is the the aperture of the solid angle cone measured in the plane containing the source axis and the disc center. Eq. 36 is easily proved by writing $\delta = \arctan((d + r)/L) - \arctan((d - r)/L) = \arctan(2rL/(L^2 + d^2 - r^2))$, where in the last step we used $\arctan(z_1) - \arctan(z_2) = \arctan[(z_1 - z_2)/(1 + z_1z_2)]$. There results that $\cos(\delta) = (L^2 + d^2 - r^2)/\sqrt{(L^2 + d^2 - r^2)^2 + (2Lr)^2}$. Since $(L^2 + d^2 - r^2)^2 + (2Lr)^2 = (L^2 + d^2 + r^2)^2 - (2dr)^2$, it is seen that eq. 36 follows directly from eq. 33.

2.4 The solid angle for a parallel coaxial disc source

The solid angle defined by a spread cosine source evenly distributed on a disc and a coaxial, parallel, disc detector can also be obtained in closed form. Referring to fig. 5, we see that the solid angle is given by

$$\Omega_s = \frac{1}{\pi R_s^2} \int_0^{R_s} \int_0^{2\pi} \rho \Omega_{\text{circ}}(L, R_d, \rho) \, d\theta d\rho,$$

(37)

where $R_s$ and $R_d$ are the source and detector radii; and $\Omega_{\text{circ}}(L, R_d, \rho) \rho d\theta d\rho$ is the contribution of an elemental area which can be obtained by putting $r = R_d$ and $d = \rho$ in eq. 33. Then,

$$\Omega_s = \frac{1}{2R_s^2} \left[ \rho^2 - \sqrt{(L^2 + \rho^2 - R_d^2)^2 + (2LR_d)^2} \right]_{\rho=0}^{\rho=R_s}.$$

(38)
A little algebra yields
\[
\Omega_s(L, R_d, R_s) = \frac{R_d}{R_s} \frac{1}{2m_s} \left[ 1 - \sqrt{1 - m_s^2} \right],
\] (39)

or
\[
\Omega_s(L, R_d, R_s) = \frac{R_d \tan(\gamma_s/2)}{2},
\] (40)

where
\[
\sin(\gamma_s) \equiv m_s \equiv \frac{2R_d R_s}{L^2 + R_d^2 + R_s^2}.
\]

Fig. 5. Notation for the coaxial , parallel, circular source and detector set.

3 Results and discussion

In order to define the general position of the cylinder, let \( L_1, L_2 \) be the z coordinates of the end discs and set \( L_1 = L_2 + \text{length} > L_2 \).

The solid angle \( \Omega(L_1, L_2, r, d) \) can be calculated using eqs. 25, 26 27, 11 and 12. The several situations to be considered are summarized in table 1.

As examples, we consider two detectors of radius 1 and lengths 5 (\( L_1 = L_2 + 5 \)) and 10 (\( L_1 = L_2 + 10 \)).

Plots of \( \Omega \) for each detector as a function of \( L_1 \) are shown in figs. 6 and 7, for different values of distance \( d \). When \( d < r \) the curves for the two detectors are essentially the same except for a displacement equal to the difference of the detectors lengths. This is so because for \( L_1 \) smaller than the length of the detector (\( L_2 < 0 \)), the source is inside the detector and \( \Omega = 1 \); for \( L_2 > 0 \) the solid angle is that of the circle (\( \Omega = \Omega_{\text{circ}} \)) and it has the same value for both equal-radius detectors.

When \( d > r \), \( \Omega \) increases with \( L_1 \) as the detector is drawn from 'behind' the source. For the smaller distances (e.g. \( d = 1.5 \)) \( \Omega \) quickly rises to the
Table 1
Expressions for the solid angle of the whole detector. $L_1, L_2$ ($L_1 > L_2$) are the $z$ coordinates of the end discs.

| $L_1, L_2$ | $d, r$ | $\Omega$ |
|-----------|--------|---------|
| $L_1 < 0$ | —      | $0^{(a)}$ |
| $L_1 > 0, L_2 < 0$ | $d < r$ | $1^{(b)}$ |
|           | $d > r$ | $\Omega_{cyl0}(L_1, r, d)$ |
| $L_1 > 0, L_2 > 0$ | $d < r$ | $\Omega_{cyl}(L_2, r, d)$ |
|           | $d > r$ | $\Omega_{cyl0}(L_1, r, d) - \Omega_{cyl0}(L_2, r, d) + \Omega_{cyl}(L_2, r, d)$ |

(a) the source only emits into $z \geq 0$

(b) source inside the detector

asymptotic value (see eq. 34) and remains almost constant until $L_1$ is increased to values bigger than the length of the detector ($L_2 > 0$). Then $\Omega$ falls off as the detector is moved away from the source. It should be clear that $\Omega(d > r)$ is the same for both detectors if $L_1$ is smaller than the length of the shorter detector (i.e. $L_1 \leq 5$).

\[ \text{Fig. 6. Solid angle defined by a cylinder of radius 1 and length 5.} \]

\[ \text{Fig. 7. Solid angle defined by a cylinder of radius 1 and length 10.} \]
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