Radiation rebound and Quantum Splash in Electron Laser Collision

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The radiation reaction is expected to play a critical role in light-matter interactions at extreme intensity. Knowing key characteristic of this effect will enable its experimental detection and, most importantly, allow to investigate the causality of the electron self interaction with its radiated field. Exploiting the theoretical derivation and 2D PIC simulation, We demonstrate that electron reflection, enabled by the radiation reaction in a head-on collision with an intense laser pulse, can provide a distinct signature to differentiate the classical and quantum features. In classical regime, there is a precipitous threshold of laser intensity to achieve the whole electron bunch reflection. However, this threshold becomes a gradual transition in the quantum regime, where the electron bunch is quasi-isotropically scattered by the laser pulse and this process resembles a water splash.

The interaction of charged particles and ultraintense electric-magnetic field must be accompanied by the emission of radiation and the corresponding recoil[1]. In classical regime, the Landau-Lifshitz (LL) equation[2], avoiding the notorious runaway solution in Lorentz-Abraham-Dirac model, gives a self-consistent determination of electron (mass $m_e$ and charge $e$) dynamics when the continuous radiation reaction (RR) is considered. An analytical solution[3] predicted by the LL equation have elucidated that the electron can be temporarily reflected by the classical RR[4]. In the quantum electrodynamical solution[3] predicted by the LL equation have illustrated radiation reaction (RR) is considered. An analytical model is that the stochastic recoil makes electrons permanently reflected by the coupling effect between transverse expulsion and insufficient longitudinal compensation of the laser ponderomotive force, where the reflection threshold can be determined through theoretical derivation. The second and more important in quantum model is that the stochastic recoil makes electrons have the quasi-isotropic distribution over a wide range of parameters. These phenomena could be experimentally detectable and promising to differentiate the discrete quantum recoil from the classical one.

In a head-on collision process for an electron with initial transverse coordinate $r_0$, three distinct stages are included. i) Slow down, the electron is dissipating energy as it is colliding with the laser pulse. ii) Re-acceleration, the electron is re-accelerated and begins to surf with the laser pulse. Energy losses are minimal in this co-propagating geometry. iii) Transverse expulsion, transverse ponderomotive force expels the electron while it is still moving forward. After it leaves the laser pulse, it retains its forward motion, which is called permanent reflection.

When the electron with a relativistic factor $\gamma_0 \gg 1$, the LL force can be chosen as $\mathbf{f}_{rr} = -(4\pi e / 3\lambda_0) m_e \omega_0 a_s^2 \chi^2 \mathbf{E} \cdot \mathbf{v}$. $\lambda_0$ ($\omega_0$) the laser wavelength (frequency), $a_s = e E_0 / m_e c^2$ the classical electron radius and $\omega_0$ the normalized Schrödinger field. For a linearly polarized Gaussian pulse, the electric and magnetic fields in vacuum can be estimated as $E_y \propto a_0 \exp\{-r^2 / 2a_0^2\} \exp\{-|\phi - \phi_0|^2 / 2\}$ and $B_z = E_y / c$, where the finite laser pulse must contain relatively weak longitudinal fields $E_z$ and $B_z$ to satisfy $\nabla \cdot \mathbf{E} = 0$ and $\nabla \cdot \mathbf{B} = 0$ in propagation. Here, $r = \sqrt{y^2 + z^2}$; $\phi = \omega_0 t + k z$ is the relative phase and $\phi_0$ refers the initial condition. $\phi_0$ is the $1/e$ half pulse width and $\sigma$ denotes the laser spot size. For simplicity, We in-
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is a constant accounting for the effective relativistic fac-
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function of electron’s initial transverse coordinate
\(y\)
introduce the normalization below: \(t \to \omega t, x \to k x, v \to v/c, p \to p/m_c e, E \to eE/m_c \omega_0\) and \(B \to eB/m_c \omega_0\).
Here the small deflection approximation (SDA) is assumed, where the longitudinal momentum \(p_x\) is dominant, i.e. \(p_x^2 \gg p_y^2 = p_{\perp}^2 = p_{\perp 0}^2 + p_{\perp 1}^2\). Under this circumstance, substituting the relations \(p_x \approx \gamma, v_x \approx 1\) and \(\chi_e \approx 2 \gamma a_0/E_S\) into the electron dynamical equation in longitudinal direction, the damped final longitudinal momentum can be derived as

\[
p_{xf} = \frac{\gamma_0}{1 + \sqrt{\frac{\pi}{2 \gamma_0 \eta}} \eta (a_0/a_2)^2 \phi_\perp \exp\left(-2 r_0^2/\sigma_0^2\right)},
\]

(1)

where the coefficient \(\eta = (4 \pi \epsilon / 3 \lambda_0) m_e \omega_0 a_2^2\approx 1983\) for a laser wavelength \(\lambda_0=1 \mu m\). Meanwhile, the final transverse one is estimated as \(p_{yf} \approx \rho \gamma^{1/2} \eta (a_0/a_2)^2 \phi_\perp \exp\left(-2 r_0^2/\sigma_0^2\right)\), where \(\rho\) is a constant accounting for the effective relativistic factor involved in the transverse ponderomotive force. The net contribution of \(p_{yf}\) mainly comes from the longitudinal damping and subsequent transverse ponderomotive expulsion. When the net transverse momentum becomes comparable with damped longitudinal one, the SDA is no longer valid, which can be regarded as the threshold of permanent reflection induced by classical radiation. From \(p_{yf} \gtrsim p_{xf}\), the threshold is expressed as

\[
\xi = \sqrt{\frac{\pi}{2 \rho \eta} \left(\frac{r_0}{\sigma_0}\right)^2} \exp\left(-\frac{r_0^2}{\sigma_0^2}\right) \geq 1.
\]

(2)

Here \(\xi = (p_{x0}/p_{xf})^{1/3}\) and the constant is chosen as \(\rho \approx 0.04\) from simulation. It is worth mentioning that the analogue threshold for two dimension (2D) situation is almost the same except for replacing \(r_0\) by \(y_0\) in Eq. (2).

To testify the threshold defined above, 2D particle-in-cell (PIC) simulations are performed through Epoch [33], where QED module uses the Monte Carlo algorithm consisted of corrected synchrotron photon emission and corresponding recoil [36] [37]. The LL force has been implemented in particle pusher subroutine [38] utilizing the method in Ref. [39]. The simulation box \(100 \mu m \times 100 \mu m\) is uniformly divided by 8000 \(\times\) 8000 grid. A LP laser pulse with the Gaussian envelope in both temporal and spatial space is incident from left boundary, whose parameter is waist radius \(r_0=5 \mu m\), amplitude \(50 \leq a_0 \leq 450\), temporal duration \(\phi_\perp = 6\pi\) and wavelength \(\lambda_0=1 \mu m\). It should be noted that the laser amplitude \(a_0 \gg 1\) accords with the local constant approximation (LCA) [10] [41] and thus impacts issued from the quantum interference [42] can be neglected here. A bunch of electrons, represented by \(10^5\) macro particles with initial energy \(\gamma_0 = 1000\) and uniformly distributed with both longitudinal and transverse length equaling \(5 \mu m\), are injected from left side to counter collide with the laser pulse. Since the electron bunch from wakefield accelerator is underdense, the space charge force of electrons is negligible.

The relations between electron final momentum and its initial transverse position are illustrated in Fig.1(b), where the longitudinal momentum distribution \(p_x=1000/1 + 2.79(a_0/100)^2 \exp(-y_0^2/\sigma_0^2)\) estimated from Eq.(1) is plotted in solid line. The numerical simulation in Fig.1(b) is in good agreement with theoretical derivation for \(a_0=50, 150\) and 250, which confirms the validity of the SDA. The final distribution of \(p_y\) is plotted in Fig.1(d) where the transverse modulation is relatively small for \(a_0 \leq 250\) which guarantees the validity of SDA.

However, SDA is no longer valid for the case \(a_0=350\). The evolution of \(p_x\) and the trajectories of four typical electrons at \(a_0 = 350\) are plotted in Fig.2(a). All of them experience a drastic damping during \(50 T_0 < t < 52 T_0\) as \(R_e \approx 0.4 \approx 4.36\) manifests the entry of RDR [10] [13], where \(a_0 \approx 1/137\) is the fine structure constant. Consequently, they also undergo a temporal reflection [4] and are re-accelerated during \(52 T_0 < t < 65 T_0\). For the electron No.(1) with \(y_0=0.0 \mu m\), it cannot obtain a pronounced transverse expulsion so that the latter down ramp of laser pulse eventually push it to a positive \(p_x=28.7\), which is well agreed with \(p_{xf}|_{a_0=350, y_0=0} = 28.3\) predicted by Eq. (1). From estimated threshold \(\xi \gtrsim 1\) in Eq. (2), the
electrons with initial position of 0.18\(\mu m\) \(< y_0 < 3.53\mu m\) can be permanently reflected. For No.(3) and No.(4), the transverse ponderomotive force makes them slip out of the central axis, and then the temporally reflected electrons have no chance to witness the downramp of laser pulse, whence a permanent rebound occurs. Electron No.(2), the intermediate state between No.(1) and No.(3)/(4), has the inadequate transverse expulsion so that it is only deflected rather than reflected.

In quantum realm, the differential rate of emitting photons (wave vector \(k'\)) with parameter \(\chi_\gamma = |F_{\mu\nu}k'|/E_s\) by an electron with \(\chi_e\) reads

\[
\frac{d^2N}{d\chi_{\gamma}d\theta}(\chi_e, \chi_\gamma) = \frac{\sqrt{3}}{2\pi \tau_c} \frac{\chi_e F(\chi_e, \chi_\gamma)}{\gamma \chi_e}
\]

where \(\tau_c = h/m_e c^2 \approx 1.28 \times 10^{-6}\)fs is the Compton time. Quantum synchrotron emissivity \(F(\chi_e, \chi_\gamma)\) is approximately validated in LCA\[43\]. The stochasticity and uncertainty of the photon emission, determined by Eq.(3), manifests that the discrete quantum recoil is like a virtual inelastic collision resulting in the transition from classical radiation cooling to stochastic heating\[44\]. The QED dynamics are analytically irreducible so that we appeal to the numerical method to investigate the underlying physics. The momentum distributions of the QED electron in Fig.1(c)(e) tend to be more insensitive and stochastic. The averaged insensitivity is because the equivalent quantum RR force \(g(\chi_e)\tau_c\) is modified by a weaken factor \(g(\chi_e) = (3.7\chi_e^3 + 31\chi_e^2 + 12\chi_e + 1)^{-4/9} \approx 1\)\[10, 36\], which makes them more difficult to be deflected. The evolutions of longitudinal momentum and trajectories of four QED electrons, with the identical initial coordinate, are presented in Fig.2(b). There are two random processes intrinsically included in discrete quantum emission, where one is the possibility of photon generation and the other is random value of the emitted energy. The green electron, emitting large amounts of photon energy during 45\(\mu m\) \leq \(x\) \leq 46\(\mu m\), is easily reflected by the counter-propagating pulse. The black electron, with relatively smooth radiation loss, goes through the whole pulse with a small transverse deflection. The electrons marked by red and blue are the intermediate case, where the blue is reflected while the red possesses a positive \(p_x\) eventually.

The definition of electron reflection can be set as \(p_x < 0\), i.e., the polar angle \(\theta = \arctan(p_y/p_x)\) greater than 90°. The angular distributions of electron number ranging from \(a_0 = 250\) to 450 for LL and QED models are shown in Fig.2(c)/(d). For LL case, the averaged polar angle \(\theta = \sum_{i=1}^{N} \theta_i/N\) increases with rising of the laser amplitude \(a_0\) and the full-width at half-maximum (HWFM) bandwidth \(\theta_b\) of the angular distribution is relatively small. On the contrary in Fig.2(d), because of quantum weakening effects, the averaged value \(\theta\) of QED case is a little smaller compared with LL case. Moreover, the bandwidth \(\theta_b\) for QED case in is much wider than LL one due to the QED stochasticity where electrons undergo random emission processes. Therefore the reflection of LL electrons is sensitive to the variation of laser amplitude \(a_0\). However, the QED electrons with identical initial condition experience quite different emission processes. Consequently, the QED case has a broad angular distribution in Fig.2(d) and it even exhibits quasi isotropic
Since it is impossible to track single electron trajectory, measuring of reflection efficiency $\kappa = N_{\text{ref}} / N$, which is the ratio of the number of reflected electron ($N_{\text{ref}}$) to the whole ($N$), is more feasible in experiments. The parameter scans of $\kappa$ with laser amplitude $210 \leq a_0 \leq 450$, spot size $3 \leq \sigma_0 [\mu m] \leq 11$, duration $1.5 \leq \phi_r [2\pi] \leq 4.5$ and initial energy $500 \leq \gamma_0 \leq 2000$, are shown in Fig.3. As most of electrons are distributed nearby the dominant angle in LL case, there is a precipitous boundary in Fig.3(a)(c)(e) between $\kappa = 0$ and 1.0. From Eq.(2), the theoretical reflection efficiency is predicted as $\kappa_{th} = \sum_{i=1}^{N} I_i \{ |p_{xf}| \geq p_{rf} \} / N$, where $I_i \{ |p_{xf}| \geq p_{rf} \}$ equals one (zero) when the condition in bracket is satisfied (unsatisfied). The contour of $\kappa_{th} = 0.9$ derived from Eq.(2) are marked by black dash line in Fig.3(a)(c)(e), which qualitatively show the dependence of radiation rebound threshold on the parameters. In Fig.3(b)(d)(f) the transition has a gradual up-ramp for QED electrons due to the stochastic radiation recoil and uncertain trajectory. From the parameter scans in Fig.3(a)(b), the requirement of electron reflection is increased when the laser transverse spot size $\sigma_0$ varies from 4.3 to 11.0 $\mu m$, due to the weaker transverse ponderomotive force induced by the larger spot size. The abnormal result for $3 \mu m < \sigma_0 < 4.3 \mu m$ is because the spot size is too small to interact with most of electrons and the edge parts are hardly reflected. In Fig.3(c)(d), there is a complementary relationship between the laser temporal duration $\phi_r$ and peak amplitude $a_0$. Fig.3(e) shows the reflection threshold is not sensitive to the initial electron energy for classical case, whereas bigger amplitude $a_0$ is desired to splash electron bunch with larger $\gamma_0$ shown in Fig.3(f).

The electron spatial distribution at time $t = 100T_0$ for $a_0 = 250, 350$ and 450 is shown in Fig.4, where the simulation parameters ($\gamma_0 = 1000, \sigma_0 = 5 \mu m$ and $\phi_r = 6\pi$) are selected from the star marks in Fig.3(a)(b). The (blue) electrons almost completely keep their initial width if no RR is taken into account. In $a_0 = 250$ case, the LL electrons have a smaller transverse spreading compared with QED case. If the field amplitude becomes larger ($a_0 = 350$), the QED electrons behave like the isotropic water splash, but the most of LL electrons are reflected and expelled transversely with the polar angle approximating $130^\circ$. When $a_0$ equals 450, the LL electrons are nearly totally reflected with $\theta > 90^\circ$ and boosted by the ponderomotive force of the co-moving laser pulse; Nevertheless, the QED case still behaves isotropic splashing property nothing but with more electrons being reflected.

In conclusion, we demonstrate that ultra-relativistic electron bunch can be permanently reflected by an ultra intense laser pulse. In the classical RR, there is an obvious threshold which determines whether the electron bunch is rebound or not. Instead, the threshold becomes a smooth transition region in the quantum realm and electron are scattered by the laser pulse like an isotropic quantum splash. With the advent of the multi-PW laser facilities, detecting the rebound and isotropic splash is promising and of great importance to verify the exact model for radiation reaction experimentally, which will pave the avenue to better manipulate the electron dynamics and drive pertinent particle sources.

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