Microseismic source location using a 3D velocity model: From the ray tracing method to waveform inversion

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Abstract. Microseismic (MS) source location is important in MS monitoring, providing the basis for determining fracture zones and calculating seismic source parameters (e.g., event magnitude and focal mechanisms). To date, homogenous, 1D, and simple 3D velocity models have been adopted in MS source location. However, a mine is usually characterized by a strong velocity heterogeneity due to engineering geology, 3D geostress, and excavation. In this work, we adopted a travel-time tomography-based high-resolution 3D velocity model and then applied 3D ray tracing based on the shooting method, 3D Gaussian beam-based reverse-time method, and waveform inversion method to MS source location. A semiautomatic waveform-cut method based on the cross-correlation (WCC) technique was developed for a quick, robust, and precise determination of the direct P-phase relative delay times. Additionally, the spectral element method for wavefield modeling, multiscale grid (coarse grid + fine grid) 3D waveform inversion, and L-BFGS iterative method were applied in the waveform inversion. Our results show that the commonly used ray tracing method may be affected by multiray path effects and waveform focusing/defocusing during wavefield propagation, whereas the Gaussian beam method has a frequency-dependent width, and the waveform inversion method has a broader frequency width, which can effectively overcome the issues of ray tracing. The average location errors of eight blasting events obtained using the 3D ray tracing, Gaussian beam, and waveform inversion methods are 26.2, 17.0, and 17.6 m, respectively, which are smaller than those of previous researches obtained using the homogenous-velocity model (average location error > 40 m). In conclusion, the high resolution of the location methods based on 3D velocity models can provide an effective approach to improve the MS source location accuracy and exhibit broad application potential.

Keywords: microseism, source location, 3D velocity, ray tracing, waveform inversion.

1 Introduction

With increasing attention being paid to the safety of engineering constructions, microseismic (MS) monitoring has been widely used in recent years. The key technical issues of MS monitoring include monitoring planning, data processing, and MS event location [1]. MS event location is a fundamental aspect in the computation of the event magnitude and focal mechanisms. MS source location can provide a good basis for hazard assessment. The location accuracy of an MS event is closely related to the data quality of the P-wave arrival times, the solution method of the objective function, and the velocity model. In this study, we focus on the influence of the velocity model on the location accuracy.
Until now, a homogenous-velocity model has usually been adopted for MS event location in mining fields. Moreover, researchers have demonstrated that the accuracy of event location using a 1D velocity model is higher than that obtained using a homogenous-velocity model. Recently, a few studies have introduced a simple 3D velocity model into MS event location and obtained a good location accuracy. In this study, a high-resolution 3D velocity model was combined with the ray tracing, Gaussian beam, and waveform inversion methods for MS source location. The remaining of this article is arranged as follows: The state of the art of MS event location is discussed in Section 2. The fundamentals of the ray tracing, Gaussian beam, and waveform inversion methods are described in Section 3. The three location methods are applied to eight blasting events in Section 4. The MS location accuracy is discussed in Section 5. Finally, a brief conclusion and outlook are provided in Section 6.

2 State of the art of MS event location

In seismology, MS event location is a classical inversion problem. Researchers have used travel-time-based graphical location since the 19th century. Later, trigger time-based objective functions have been proposed to inverse the event location. Among these approaches, Geiger’s method [2] is the most famous. This method transforms the location objective function into a linear problem with a good inversion efficiency. The double-difference location method [3] is another commonly used location method, which uses two close events and the difference of their P-wave arrival time at the same station to reduce the effect of heterogeneity in the ray path. Grid search, simplex algorithm, simulated annealing, particle swarm optimization algorithm, and Bayesian inversion have also been applied to solve the location objective function. In addition, the reverse-time migration technique has been applied to MS event location; this technique determines the event location using the focusing point in the imaging condition of reverse-time migration [4] to eliminate the influence of large-arrival-time picking errors.

The above location method based on ray tracing may be affected by the multipath effect. To handle this issue, the Gaussian beam technique has been proposed for event location. This approach locally solves the wave equation, has a certain ray width, and can avoid the multipath effect of ray tracing. For example, Rentsch et al. [5] verified the applicability of the Gaussian beam technique in event location methods based on 1D and 2D velocity models via synthetic tests. McMechan [6] modeled the wavefield through a finite difference method and stacked the back-propagating wavefield to obtain the event location. The location technique based on the wave equation models the wavefield better than the Gaussian beam method, where the wave equation method has been the most advanced event location technique in seismology. Kim et al. [7] jointly inverted the earthquake moment tensor and location based on waveform inversion, whereas Tong et al. [8] treated the moment tensor as a known parameter and inversed only the event location, which requires much less computation time than the joint inversion. A homogenous-velocity model is usually adopted in MS source location. However, many studies have observed that using a constant value for the velocity will affect the location result. Dong et al. [9] treated the velocity as an unknown parameter in source location and obtained a more accurate location even if their approach was still based on homogenous-velocity models. Models using a 1D velocity in the vertical [10] and horizontal [11] directions were then employed for MS source location. Recently, models using 3D velocities combining tunnels and homogenous velocity were developed for MS source location [12], which largely improved the location accuracy. However, since the mining environment is far more complex than the above velocity models, a higher-resolution 3D velocity model should be considered in MS source location. Fortunately, numerous mining imaging studies offer a solid basis for MS source location based on a 3D velocity model.

3 Theoretical basis

3.1 3D ray tracing

The L1 norm-based trigger-time difference location (TD1) method was selected for the ray tracing location method. Its objective function is defined as [13]:

\[ \text{minimize } TD1(x_o, y_o, z_o) = \sum_{i=1}^{N} \sum_{j=1}^{M} \left| t_i - t_{ij} - (t_{ij} - t_{ij}) \right| \]
where \( x_0, y_0, \) and \( z_0 \) are the coordinates of the MS source location, \( t_i \) and \( t_j \) are the P-wave arrival times of the \( i^{th} \) and \( j^{th} \) sensors, respectively, and \( t_{i0} \) and \( t_{j0} \) are the P-wave travel times from the source to the \( i^{th} \) and \( j^{th} \) sensors, respectively.

Ray tracing was adopted to compute the P-wave travel-time in a high-resolution 3D velocity model. Firstly, the 3D velocity model was discretized into 3D grids, and the velocity in each 3D grid was treated as a constant. Then, the shooting method was used to find the shortest travel-time ray path (Fig. 1 gives an example). The ray path between the grids follows Snell’s law:

\[
\sin \theta_i = \frac{v_i}{v_m} = \frac{\sin \theta_j}{\sin \theta_k} = \cdots = \frac{\sin \theta_n}{\sin \theta_p} = p , \tag{2}
\]

where \( \theta_i \) is the angle measured from the normal to the boundary, \( v_i \) is the seismic velocity in the \( i^{th} \) medium along the ray path between two media, and \( p \) is the ray parameter.

The absolute P-wave arrival time can be easily affected by background noise and propagation attenuation, which may result in a bad P-wave arrival time. In this study, a windowed cross-correlation (WCC) technique is proposed to pick the relative P-wave arrival time, which eliminates the influence of the waveform tail, differently from the full waveform cross-correlation approach. The WCC between the windowed waveforms \( u(x_1,t) \) and \( u(x_2,t) \) is defined as:

\[
WCC(u(x_1,t),u(x_2,t),m) = \sum_{t=-\infty}^{\infty} u(x_1,t)u(x_2,t+m) . \tag{3}
\]

### 3.2 Gaussian beam migration

Unlike infinite-frequency ray tracing, a Gaussian beam has a frequency-dependent width. The coordinate diagram of a 3D Gaussian beam is shown in Fig. 2.
The wavefield solution of a 3D Gaussian beam at point $Q$ in the central ray coordinate system can be calculated according to [14]:
\[
U_{ib}(s, q_1, q_2, \omega) = \frac{V(s) \det [Q(s_0)]}{V(s_0) \det [Q(s)]} \exp \left[ i \omega t + \frac{i \omega}{2} q^T M(s) q \right],
\]
where $V(s_0)$ is the initial velocity of the central ray starting point, and $V(s)$ is the velocity at point $Q$. $q^T = (q_1, q_2)$, where $q_1$ and $q_2$ are the coordinates along the basis vectors $e_1$ and $e_2$, respectively. $\omega$ is the frequency of the Gaussian wavefield. $M(s) = P(s)/Q(s)$, where $Q(s)$ and $P(s)$ are dynamic parameter matrices of the Gaussian beam.

The 3D Green’s function can be calculated through the stacking integral of Gaussian beams with different ray parameters [14]:
\[
G(x, x\prime, \omega) = \frac{i \omega}{2\pi} \int \int \frac{dp_\rho dp_\tau}{p_\rho} U_{ib}(x, x\prime, p, \omega),
\]
where $x = (x, y, z)$ and $x\prime = (x\prime, y\prime, z\prime)$ are the source and receiver locations, respectively, $p_x = \sin \theta \cos \phi / V(s_0)$, $p_y = \sin \theta \sin \phi / V(s_0)$, and $p_z = \cos \theta / V(s_0)$.

For a point source, the synthetic waveform $u_i(x, x\prime, \omega)$ at point $x$ and in the direction $i$ is the convolution between the partial derivative of the Green’s function $G_{ij,k}(x, x\prime, \omega)$ and the source moment tensor $M_{jk}(x\prime)$:
\[
u_i(x, x\prime, \omega) = \sum_{j=1}^{3} \sum_{k=1}^{3} G_{ij,k}(x, x\prime, \omega) M_{jk}(x\prime).
\]

Windowed P-waves are used for the Gaussian beam reverse-time migration (GBRTM) location, and the point with the maximum focusing energy in the migration is treated as the source location.

### 3.3 Waveform inversion

Waveform inversion takes advantage of waveform fitting to invert the source location. The objective function misfit is usually adopted to update the model parameters, i.e., $x_{k+1} = x_k + \alpha_{k+1} p_{k+1}$, where $k$ is the iteration number, $\alpha_k$ is the updating step length, and $p$ is the direction vector. L-BFGS was selected to conduct the iteration, enabling the determination of $p$ after obtaining the gradient vector $\nabla \chi$ according to [15]:
\[
\nabla \chi = \frac{\partial \chi(u, d)}{\partial u} = k \frac{\partial \chi(u, d)}{\partial u},
\]
where $k = \frac{\partial u}{\partial x_{k}}$ is the Frechet derivative (gradient) of the wavefields with respect to the source location $x_k = [x_0, y_0, z_0]$.

Huang et al. [15] expressed the Frechet derivatives based on an acoustic wave equation:
\[
\frac{\partial \omega(x, \omega)}{\partial x} = \int \int \kappa_1(x, \omega) + \int \int \kappa_2(x, \omega) d^2 x
\]
where $\kappa_1(x, \omega) = \varphi^2 \varphi u(x, \omega, \omega) + \frac{\omega^2}{c^2(x)} \varphi u(x, \omega, \omega)$, and $\kappa_2(x, \omega) = -2\omega^2 \varphi(x) u(x, \omega, \omega) G(x, x, \omega) / c^2(x)$, $c(x)$ is the velocity at point $x$.

### 4 Applications

#### 4.1 Dataset

The institute of mine seismology (IMS) monitoring system of the Yongshaba mine (China) was chosen as the study area. This system comprises 28 sensors (Fig. 3), which are located at depths of 930 m (12 sensors), 1080 m (12 sensors), and 1120 m (4 sensors). The sensor coordinates have been listed in Ref. [16]. The 3D velocity model inverted by Wang et al. [17] was adopted for obtaining the source location, and a slice of the velocity model is shown in Fig. 1. The eight blasting events with testing time and coordinates listed in Table 3 of Ref. [9] were taken to evaluate these three different location methods.
Fig. 3 Sensor locations of the IMS in the Yongshaba mine. The background shows the sensor elevation.

4.2 Typical examples

Blasting events 1, 3, and 3 were selected to evaluate the performance in obtaining the source location of the ray tracing, Gaussian beam, and waveform inversion approaches, respectively. The 3D ray tracing location for the first blasting event based on WCC pickings is shown in Fig. 4. A technique combining coarse and fine grids (light blue stars) is proposed to determine the source location. Firstly, a coarse grid with 50-m spacing was generated to obtain an approximate location. Then, a fine grid with 10-m spacing around the approximate location was used to refine the location result. The misfit of the objective function is only ~0.004 s, and the location obtained from the 3D ray tracing method is very close to the premeasured location.

Fig. 4. Location obtained from the 3D ray tracing method for the first blasting event based on the WCC pickings. The red and blue stars correspond to the 3D ray tracing location and the premeasured location, respectively. The map in the inset shows the misfit of Eq. (1).

The GBRTM location result for the third blasting event based on the windowed waveforms is shown in Fig. 5. The noise may be caused by the structural heterogeneity in the back-propagating wavefields for the Gaussian beam modeling. Moreover, the wavefield assumes an increasingly irregular shape as the travel distance increases. A few back-propagating wavefields are slightly far away from the energy focusing point of the image, which may be due to the insufficient accuracy of the velocity model. Though the stacking amplitude image shows some noise, the location where the energy focusing is close to the premeasured location.

Fig. 5. GBRTM location for the third blasting event obtained from the windowed waveforms. The wavefield corresponds to the time at which the focusing energy is maximum. The map in the inset shows the stacking amplitude image near the premeasured source location. The green and black stars correspond to the GBRTM location and premeasured location, respectively.
Fig. 6. Multiscale waveform location for blasting event 3. (a) Waveform inversion location results, (b) enlarged map of the location obtained after iteration, and (c) normalized waveform fitting in the iteration.

The waveform inversion location result for the third blasting event obtained from the windowed waveforms is shown in Fig. 6. Firstly, coarse-grid-based 3D ray tracing was used to obtain an approximate event location (the yellow star in Fig. 6b). Then, low-frequency-band waveform inversion was conducted for the source location iteration (the blue stars in Fig. 6b and c). The iteration converged quickly, whereas the normalized misfit decreased to about 50%. Subsequently, a low-frequency band (0–120 Hz) was used to refine the location, and the normalized misfit decreased only about 7%. The location obtained after iteration is closer to the premeasured location, and the final location error is only 15.4 m.

4.3 Location results

The 3D ray tracing, Gaussian beam, and waveform inversion methods were applied to find the location of these eight blasting events. The location results for these events obtained in previous studies are listed in Table 1. It can be easily seen that the location method based on the 3D velocity model provides much better results than the model based on homogenous velocity, though intensive work has been conducted on the latter. In other words, the velocity model used plays an important role in MS event location. Furthermore, the location results obtained from the 3D Gaussian beam and waveform inversion methods are slightly better than those obtained from the 3D ray tracing method. Detailed reasons for this are provided in the discussion section.

| Event ID | Event location (m) | Location error (m) |
|----------|--------------------|--------------------|
|          | X      | Y  | Z  | Shang et al. [18] | Dong et al. [19] | Li et al. [13] | Peng et al. [16] | 3D ray tracing | Gaussian beam | Waveform inversion |
| E1       | 381683 | 2997760 | 1107 | 84.8 | 61.1 | 31.1 | 45.6 | 32.6 | 23.0 | 26.4 |
| E2       | 381653 | 2997405 | 1099 | / | / | / | 49.0 | 22.4 | 21.1 | 15.4 |
| E3       | 381194 | 2996224 | 1014 | 27.6 | 28.7 | 49.4 | 42.3 | 14.3 | 5.1 | 12.7 |
| E4       | 381684 | 2997777 | 1107 | 3878.2 | / | 30.8 | 35.8 | 29.8 | 24.2 | 23.7 |
| E5       | 381503 | 2997036 | 1028 | / | / | / | 26.4 | 28.5 | 7.2 | 7.5 |
| E6       | 381590 | 2997278 | 1053 | 70.1 | 52.4 | 49.0 | 68.1 | 30.8 | 23.8 | 31.6 |
| E7       | 381526 | 2997584 | 1044 | 55.0 | / | 48.8 | 32.2 | 24.5 | 18.2 | 7.9 |
| E8       | 381442 | 2998029 | 1017 | 143.5 | / | 42.8 | 26.3 | 26.6 | 13.8 | 16.4 |
| Average  |       |     |     | 709.9 | 47.4 | 42.0 | 40.7 | 26.2 | 17.0 | 17.6 |
5 Discussion

MS event location is generally performed using travel-time-based ray tracing location, which assumes that the wavefield propagation has an infinite frequency. However, the recorded waveform has a finite-frequency band, especially for high-frequency energy, which quickly attenuates. Therefore, a simple infinite-frequency approximation for the ray theory fails to account for the travel-time delay or advance caused by medium disturbances [20]. A Gaussian beam with a limited frequency band and waveform modeling, which takes into account the finite-frequency band, can better describe wavefield propagation. Thus, the location methods based on the Gaussian beam and waveform can determine the source location with better accuracy than the ray tracing method.

Table 1 demonstrates that the 3D velocity model can greatly improve the accuracy of the location results. The 3D velocity model adopted in this study is based on ray tracing, which has a resolution of about 50 m. Therefore, a higher-resolution velocity inversed by the finite-frequency tomography or full waveform techniques can be adopted to improve the accuracy of the source location. Moreover, a high-resolution 3D velocity model that considers tunnels, cavities, and back filling can further improve the accuracy of the source location.

6 Conclusions

In this study, a high-resolution 3D velocity model was used for MS source location, and the 3D ray tracing, Gaussian beam, and waveform inversion methods were applied to eight blasting events. The main findings of this study are as follows: (1) The commonly used ray tracing method may be affected by multiray path effects and waveform focusing and defocusing during wavefield propagation; (2) The Gaussian beam method has a frequency-dependent width, and the waveform inversion method has a broader frequency width, which can effectively overcome the problems of the ray tracing method. (3) The average location errors of the eight blasting events obtained from the 3D ray tracing, Gaussian beam, and waveform inversion methods are 26.2, 17.0, and 17.6 m, respectively, which are smaller than those obtained from models based on homogenous velocity (average location error > 40 m). It should be mentioned that only eight blasting events were used to obtain the above conclusions; thus, more tests should be conducted to further verify the proposed methods. Further improvement of the accuracy of source location can be obtained considering a high-resolution 3D velocity model based on finite-frequency tomography techniques and full waveform inversion.

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