Transverse momentum, factorization
and the HERMES experiment

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Abstract
I present some results about transverse momentum dependent distribution and
fragmentation functions. Firstly I illustrate a simple model, with predictive power
about the energy behavior, for T-odd, chiral odd functions. Moreover I propose a
slight modification in extracting transversity from HERMES data, so as to apply
correctly factorization. Lastly I suggest a method for determining the quark
transverse polarization in an unpolarized or spinless hadron.

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1 Introduction

It is well-known that the transversity function[1-4] is quite difficult to determine experimentally[5-9], as well as all chiral odd functions. The most promising methods are based on the azimuthal asymmetries, which are sensitive to transverse momentum dependent (t.m.d.) distribution and fragmentation functions. A big contribution in this sense has been given by the HERMES experiment[10], whose data analysis is strongly based on the T-odd functions[11] and on the factorization assumption[11, 12, 13], essential for extracting transversity. In this talk I revise that analysis, proposing improvements which allow to apply correctly factorization. I also present a simple model for T-odd functions, predicting their behaviors as functions of the energy scale $Q$.

After a short review of the azimuthal asymmetries (sect. 2), I consider (sect. 3) the single spin asymmetry: I define the cross sections and asymmetries measured in the HERMES experiments, moreover I introduce the correlation matrix, dedicating a particular attention to the T-odd functions, for which I have elaborated a simple model. In sect. 4 I revise the HERMES data analysis, proposing a modified weight function for extracting transversity and the distribution function $h_1^+$, which is related to the quark transverse polarization in an unpolarized proton. In sect. 5 I suggest an alternative method for extracting the T-odd, chiral odd fragmentation function for the pion. Lastly I give a short summary in sect. 6.

2 Azimuthal asymmetries

The t.m.d. functions were introduced at first by Ralston and Soper[1]. The importance of such functions has been pointed out in the last years[13, 14, 15], since when people started to plan measurements of azimuthal asymmetries[16-19]. Indeed, such functions contain nontrivial information on the internal structure of the nucleon[14]. In particular, T-odd functions allow, in principle, to measure transversity in a semi-inclusive deep inelastic scattering (SIDIS) experiment[11, 12, 13]. In order to extract t.m.d. functions, one usually considers the weighted asymmetries[17-19], which will
be defined in the following sections. Here I shall be concerned with asymmetries relative to two types of experiments:

a) SIDIS\cite{10, 20, 21}:

\[ \ell \bar{p}(p^\uparrow) \to \ell' \pi X, \tag{1} \]

\( \ell \) denoting a charged lepton;

b) Electron-positron annihilation into two jets, observing a final pion\cite{4, 16}:

\[ e^+e^- \to \pi X; \tag{2} \]

In particular reaction (2) allows to extract a T-odd fragmentation function\cite{11}, to be used in the HERMES data analysis of reaction (1), in order to get transversity\cite{4, 16}. This procedure is based on factorization\cite{11-13}, which has not been proven for t.m.d. functions. However this property is generally assumed, provided the energy scale \( Q \) is not too large, otherwise the Sudakov damping\cite{22} highly suppresses the asymmetry and deeply modifies its dependence of on t.m.d. functions.

\section{Spin asymmetries at HERMES}

\subsection{General formulae}

The HERMES experiments concern reaction (1). In particular, the experiment which has been realized\cite{10} employes a longitudinally polarized proton target, while for the next future the use of a transversely polarized target has been planned\cite{21}. In one-photon approximation, the essential part of the reaction consists of

\[ \gamma^* p(p^\uparrow) \to \pi X. \tag{3} \]

But the proton polarization - be it longitudinal or transverse with respect to the incident lepton beam - has always a longitudinal and a tranverse component with respect to the virtual photon momentum, which is the most relevant direction in this approximation. Since the Hamiltonian of a quantum system is linear with respect to spin, the effects on these two components of the polarization may be studied
independently of each other. This is why transversity can be inferred also from a longitudinally polarized target.

I consider reaction (3) with transverse polarization, for which I define the single spin azimuthal asymmetry

\[ A = \frac{d\sigma_\uparrow - d\sigma_\downarrow}{d\sigma_\uparrow + d\sigma_\downarrow}. \]  

(4)

Here \( d\sigma_\uparrow \) refers to cross sections with opposite polarizations. Moreover

\[ d\sigma_\uparrow - d\sigma_\downarrow \propto d\Gamma L_{\mu\nu}^a H_{\mu\nu}^a, \]  

(5)

\[ L_{\mu\nu} = k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu} k \cdot k', \]  

(6)

\[ H_{\mu\nu}^a = \int d^2 p_\perp \text{Tr} \left[ \gamma_\mu \Phi_{\chi.o.}(x, p_\perp) \gamma_\nu \hat{\Phi}_{\chi.o.}(z, P_\perp) \right]. \]  

(7)

d\( \Gamma \) is the phase space element. \( k \) and \( k' \) are the initial and final four-momentum of the lepton. \( \Phi_{\chi.o.}, x \) and \( p_\perp \) are, respectively, the chiral odd component of the correlation matrix\[14\], the longitudinal fractional momentum and the transverse momentum of a quark inside the proton. \( \hat{\Phi}_{\chi.o.}(z, P_\perp) \) is the chiral odd component of the correlation matrix of a quark fragmenting into a pion, whose longitudinal fractional momentum and transverse momentum are respectively \( z \) and \( P_\perp \). This last is such that the transverse momentum of the pion with respect to the photon, \( i.e. \),

\[ \Pi_\perp = P_\perp + z p_\perp, \]  

(8)

is kept fixed.

### 3.2 Parametrization of the T-odd correlation matrix

I consider a frame where the proton is transversely polarized and has a large momentum, say \( \mathcal{P} \). In this frame the chiral odd component of the correlation matrix can be parametrized as

\[ \Phi_{\chi.o.} = \frac{1}{4} x \mathcal{P} \gamma_5 \left\{ [\mathcal{S}, \hat{n}_+ \left( h_{1T} + h'_1 \frac{p_1}{\mu} \right) + [\hat{n}_1, \hat{n}_+] h''_1 \frac{p_2}{\mu} \right\}. \]  

(9)

Here \( h_{1T} \) is the t.m.d. tranversity function, while \( h'_1 \) and \( h''_1 \) are two new functions, to be illustrated below. Moreover

\[ p_1 = p_\perp \cdot S \times n, \quad p_2 = p \cdot S, \]  

(10)

\[ \sqrt{2} n_+ \equiv (1, n), \quad n_1 \equiv (0, S \times n). \]  

(11)
S and \(n\) are unit vectors respectively in the direction of the proton polarization and of the proton momentum. Lastly \(\mu\) is an undetermined mass scale, which was set equal to the the proton rest mass by various authors\[1, 14, 23\]; as I shall show in the next subsection, this is not the most suitable choice. Furthermore, in reaction (3), it is convenient to identify \(P\) with \(Q/2x\), where \(Q\) is the virtual photon momentum; this amounts to considering a Breit frame where the active quark has an initial longitudinal momentum equal to half photon momentum, but in the opposite direction\[24\].

The second and third term of parametrization (4) are T-odd and give a nonvanishing contribution also in the case of an unpolarized proton, since they are even under the exchange \(S \rightarrow -S\). I show that the sum of these two terms must be independent of \(S\). To this end, I consider the probability density \(q_\uparrow(S; x, p_{\perp})\) for a quark to have a positive spin component along a given unit vector \(s\) not parallel to \(n\). I distinguish a T-even and a T-odd component of this density:

\[
q_\uparrow(S; x, p_{\perp}) = q_\uparrow^{(e)}(S; x, p_{\perp}) + q_\uparrow^{(o)}(S; x, p_{\perp}).
\] (12)

The combined action of parity inversion and time reversal yields

\[
q_\downarrow(-S; x, p_{\perp}) = q_\uparrow^{(e)}(S; x, p_{\perp}) - q_\uparrow^{(o)}(S; x, p_{\perp}),
\] (13)

where \(q_\downarrow\) is the probability density for a quark to have a negative spin component along \(s\). Analogous relations can be written for \(q_\downarrow(S; x, p_{\perp})\). Then, in an unpolarized proton, the difference \(q_\uparrow - q_\downarrow\) results in

\[
\delta q_{\perp} = \frac{1}{2} \left\{ q_\uparrow(S; x, p_{\perp}) - q_\downarrow(S; x, p_{\perp}) - [q_\uparrow(-S; x, p_{\perp}) - q_\downarrow(-S; x, p_{\perp})] \right\}
= q_\uparrow^{(o)}(S; x, p_{\perp}) - q_\downarrow^{(o)}(S; x, p_{\perp}) = \delta q_{\perp}^{(o)}.
\] (14)

But the quark density matrix of an unpolarized proton is \(S\)-independent, and the same must be of the quark polarization \(\delta q_{\perp} = \delta q_{\perp}^{(o)}\). To impose this condition in eq. (9), I set

\[
h'_1 = -h''_1 = h'^{\perp}_1.
\] (15)

Therefore the sum of the last two terms of eq. (9) can be written as

\[
\Phi_{\chi, \alpha}^T = \frac{xP}{4\mu} \gamma_5 [\not{p}_{\perp}, \not{h}_+^{\perp}] h'^{\perp}_1,
\] (16)
where

\[ r_\perp = p_1 S - p_2 n_1 \equiv (0, -p_2, p_1, 0). \]  

(17)

Eq. (16) implies

\[ \delta q_\perp = -\frac{r_\perp \cdot s_0}{\mu} h^1_1, \]  

(18)

where \( s_0 \equiv (0, s) \). Eq. (18) exhibits the meaning of the function \( h^1_1 \), already introduced by Boer and Mulders [25].

In the case of quark fragmenting into a pion, a T-odd, chiral odd fragmentation function can be defined in quite a similar way, substituting \( \mathcal{P} \) by \( \varepsilon \mathcal{P} \) and \( p_\perp \) by \( P_\perp \).

Such a function - to be denoted as \( \hat{h}^1_1 \) in the following - describes the Collins effect [11].

3.3 A model for T-odd functions

A proton may be viewed as a bound state of the active quark with a set \( X \) of spectator partons. In order to take into account coherence effects, I project the bound state onto scattering states with a fixed third component of the total angular momentum with respect to the proton momentum, \( J_z \), and with a spin component \( s = \pm 1/2 \) of the quark along the unit vector \( s \) introduced in subsect. 3.2. For the sake of simplicity, I assume that \( X \) has spin zero, moreover I choose a state with \( J_z = 1/2 \). Then

\[ |J_z = 1/2; s; X\rangle = \alpha |\to, L_z = 0; s; X\rangle + \beta |\to, L_z = 1; s; X\rangle. \]  

(19)

Here \( \to (\leftarrow) \) and \( L_z \) denote the components along \( n \), respectively, of the quark spin and orbital angular momentum, while \( \alpha \) and \( \beta \) are Clebsch-Gordan coefficients. Then the probability of finding a quark with \( J_z = 1/2 \) and spin component \( s \) along \( s \), in a longitudinally polarized proton with a positive helicity, is

\[ |\langle P, \Lambda = 1/2 | J_z = 1/2; s; X\rangle|^2 = \alpha^2 |\langle P, \Lambda = 1/2 | \to, L_z = 0; s; X\rangle|^2 \]

\[ + \beta^2 |\langle P, \Lambda = 1/2 | \to, L_z = 1; s; X\rangle|^2 + I, \]  

(20)

\[ I = 2\alpha\beta Re \left[ \langle P, \Lambda = 1/2 | \to, L_z = 0; s; X\rangle \right]\langle (\leftarrow, L_z = 1; s; X) | P, \Lambda = 1/2 \rangle]. \]  

(21)

Expanding the amplitudes in partial waves yields

\[ I = 2 \sum_{l,l'=0}^{\infty} Re \left[ ie^{-i\phi} A_l B_{l'} \right] P_l(\cos \theta)P_{l'}(\cos \theta). \]  

(22)
Here $A_l$ and $B_l$ are related to partial wave amplitudes; moreover $\theta$ and $\phi$ are respectively the polar and the azimuthal angle of the quark momentum, assuming $n$ as the polar axis and, as the azimuthal plane, the one through $n$ and $s$. In the Breit frame one has

$$P_l(\cos \theta) \sim 1, \quad P_l^1(\cos \theta) \sim \frac{|P_\perp|}{x P}.$$  \hfill (23)

Then eq. (22) yields

$$I \sim \frac{|P_\perp|}{x P} (A \cos \phi + B \sin \phi),$$  \hfill (24)

where $A$ and $B$ are real functions made up with $A_l$ and $B_l$. Since $s$ is an axial vector, parity conservation implies $A = 0$. Therefore eqs. (20) and (24) imply that the interference term $I$ is T-odd and that the final quark is polarized perpendicularly to the proton momentum and to the quark momentum, independent of the proton polarization. Comparing eq. (24) with eq. (18) yields

$$\mu = x P.$$  \hfill (25)

A similar reasoning can be applied to the Collins effect, for which one has

$$\mu = x z P.$$  \hfill (26)

Eqs. (24) and (20) predict that both the Collins effect and the quark transverse polarization in an unpolarized (or spinless) hadron decrease as $P^{-1}$, where, as illustrated in subsect. 3.2, $P$ is related to the energy scale $Q$.

## 4 Data Analysis in SIDIS

### 4.1 Extracting the transversity function from HERMES data

Eqs. (5), (7) and (9) imply that the numerator of the SIDIS asymmetry (4) is of the form

$$d\sigma_\uparrow - d\sigma_\downarrow \propto \int d^2 p_\perp \left[ \frac{P_\perp}{x z P} h_{1T} + \frac{P_\perp \cdot P_\perp}{z x^2 P^2} h_{1T} \right] \hat{h}_{1T},$$  \hfill (27)
assuming the constraint (8). Here

$$P_1 = P_\perp \cdot S \times n.$$  

(28)

In order to extract the transversity function, i.e., \( h_1 = \int d^2 p_\perp h_{1T} \), I define the following weighted asymmetry:

$$\langle A_1 \rangle = \frac{\sum_n d\sigma^{(n)} \Pi_1^{(n)}}{M_P \sum_n d\sigma^{(n)}}, \quad \Pi_1 = P_1 + zp_1.$$  

(29)

Here \( M_P \) is the proton rest mass and \( d\sigma^{(n)} \) the differential cross section at a fixed transverse momentum, the sum running over the data. Eq. (27) implies

$$\sum_n d\sigma^{(n)} \Pi_1^{(n)} \propto h_1(x) \hat{h}_1 \cdot (z), \quad \hat{h}_1 = \int d^2 P_\perp P_\perp^2 \hat{h}_1.$$  

(30)

This allows to extract \( h_1(x) \), provided the function \( \hat{h}_1 \) is known. This function - which does not coincide with the Collins function \([11, 10]\) and from now on will be named the modified Collins function - has to be inferred from an independent experiment, for example from reaction (3), as I shall exhibit in the next section.

### 4.2 How to single out the function \( h_1^\perp \)

\( h_1^\perp \) can be determined by using the weighted asymmetry

$$\langle A'_1 \rangle = \frac{\sum_n(d\sigma^{(n)}_+ - d\sigma^{(n)}_-)(\Pi_1^{(n)})^2}{M_P^2 \sum_n d\sigma^{(n)}}.$$  

(31)

Indeed, formula (27) yields

$$\langle A'_1 \rangle \propto h_1^\perp(x) \hat{h}_1^\perp(z),$$  

(32)

where \( h_1^\perp(x) \) is defined analogously to the second formula (30), with \( p_\perp \) instead of \( P_\perp \). Also this function can be inferred only if the modified Collins function \( \hat{h}_1^\perp(z) \) is known. Formula (27) implies that the weight functions \( \Pi_1^2 \) and \( (\Pi_\perp \cdot S)^2 \) could be used instead of \( \Pi_1^2 \).
5 Determining the modified Collins function

The modified Collins function can be extracted from reaction (3). The helicity of the virtual, timelike photon is directed along the direction of the two initial lepton beams in the laboratory frame. Therefore the final pion distribution presents an azimuthal asymmetry around the jet direction, with respect to the plane $\Omega$ containing the jet and beam directions. In this case the asymmetry is proportional to the difference

$$d\sigma'_\uparrow - d\sigma'_\downarrow \propto d\Gamma' L'_{\mu\nu} Tr \{\gamma^\mu \Phi_{\chi.o.} \gamma^\nu \rho_{\chi.o.}\} \propto \Pi'_1 \hat{h}_1. \quad (33)$$

Here $\Pi'_1 = \Pi'_\perp \cdot n_\perp$, where $\Pi'_\perp$ is the transverse momentum of the pion and $n_\perp$ the unit vector perpendicular to $\Omega$. Moreover $d\Gamma'$ is the phase space element and $L'_{\mu\nu}$ the symmetric leptonic tensor (3), made up with the four-momenta of the two initial leptons. Lastly $\rho_{\chi.o.}$ is the chiral odd component of the density matrix of the parton whose fragmentation is not observed.

I adopt the weighted asymmetry (29), substituting $\Pi_1$ with $\Pi'_1$. Owing to eq. (33), this asymmetry, which I call $\langle A_c^1 \rangle$, results in

$$\langle A_c^1 \rangle \propto \hat{h}_{1(1)}(z). \quad (34)$$

The weight function I have suggested for SIDIS and for $e^+e^-$ annihilation differs from the one used in the data analysis of the HERMES experiment, i.e.,

$$\sin \phi = \frac{\Pi_1}{|\Pi_\perp|}, \quad (35)$$

where $\phi$ is known as the Collins angle. This weight function, unlike those proposed in this talk, does not allow to factorize the integral in the first eq. (30) into a function of the sole $x$ times a function of the sole $z$. This is due to the factor $|\Pi_\perp|^{-1}$. For the same reason, if (33) is used, the T-odd fragmentation function extracted from $e^+e^-$ annihilation data is different from the one which appears in the formula of SIDIS asymmetry. Therefore it is quite difficult to extract $h_1$ by means of this method. The modification proposed (see the second eq. (29)) is essential for factorization.

I have neglected quark flavor throughout my talk. Including this quantum number would involve problems similar to those pointed out by E. Leader in this conference.
6 Summary

Here I recall the main results illustrated in the present talk.

- I have elaborated a simple, and yet sufficiently general, model for T-odd functions, which allows to predict the $Q$-dependence of such functions.

- I have defined some weight functions, different from those used previously, and in particular a modification to the Collins function, which allows to apply correctly factorization in extracting chiral odd functions - and above all transversity - from data.

- In particular I have proposed the weighted asymmetry $\langle A_1^{(c)} \rangle$ (eq. (34)) for extracting the modified Collins function from reaction (2). Moreover I have suggested the weighted asymmetries $\langle A_1 \rangle$ (first eq. (29)) and $\langle A'_1 \rangle$ (eq. (32)) for determining, respectively, the transversity and the function $h_1^{\perp}$ from reaction (1).

- According to my model, $\langle A_1 \rangle$ and $\langle A_1^{(c)} \rangle$ decrease as $Q^{-1}$, while $\langle A'_1 \rangle$ decreases as $Q^{-2}$.

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