Hybrid Quantum Computation in Quantum Optics

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We propose a hybrid quantum computing scheme where qubit degrees of freedom for computation are combined with quantum continuous variables for communication. In particular, universal two-qubit gates can be implemented deterministically through qubit-qubit communication, mediated by a continuous-variable bus mode (“qubus”), without direct interaction between the qubits and without any measurement of the qubus. The key ingredients are controlled rotations of the qubus and unconditional qubus displacements. The controlled rotations are realizable through typical atom-light interactions in quantum optics. For such interactions, our scheme is universal and works in any regime, including the limits of weak and strong nonlinearities.

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There are various proposals for realizing quantum computers [1, 2]. At the few-qubit level, some proposals have been demonstrated already in the laboratory. These proof-of-principle demonstrations include schemes based on, for instance, trapped ions [3], linear optics [4, 5] and nuclear spins in liquid-state molecules [6]. For the long-term prospects of scalability, “solid-state” qubits are also of great interest. For their realization, the toolbox and all the fabrication and manufacturing expertise developed for conventional IT could be exploited. However, at present, such solid-state-based schemes lag behind the other approaches and are at best at the one- or two-qubit demonstration level.

For processing photonic qubits directly in an optical quantum computer, the large Kerr-type nonlinearities needed for a two-qubit gate are hard to obtain with single photons. A possible way to circumvent this obstacle is to apply only linear transformations, supplemented by measurement-induced nonlinearities [4, 5]. The simplest forms of these linear-optical gates have been realized already [5]. There are also proposals that combine the advantages of the solid-state and the optical approaches; the main idea of these schemes is to use single photons as a bus to mediate interactions between non-nearest neighbours of solid-state qubits [7, 8, 9, 10, 11, 12]. In principle, this enables one to add arbitrarily many qubits to a system, in order to achieve universality and scalability. Two-qubit gates can be achieved for any pair and there is no need for the qubits to be so close together such that individual addressing is no longer possible.

Significant difficulties with single-photon-based buses arise due to the demanding requirements on the generation and detection of the photons. In particular, successful near-deterministic gate performance depends on efficient detectors that unambiguously detect a single photon. As a result, with typically low practical detector efficiencies the gates will be highly nondeterministic. However, efficient local gates are essential ingredients in, for example, long-distance quantum communication via quantum repeaters [13]. In such schemes, inefficient gates require more expensive quantum resources. In addition, measurement-based gates are typically slow, limited by the measurement speed. It is therefore desirable to circumvent the need for measurements.

All of the above-mentioned proposals for realizing a quantum computer rely exclusively on discrete variables (DV). The quantum information is encoded into qubits (actual, or effective—a 2D subspace in a larger Hilbert space) and, in some cases, qubits are also used as a bus to mediate interactions. This includes the original ion-trap proposal [3] where the two lowest states of a vibrational mode mediate a gate between two ion-qubits (based on two internal ion states). There are now also efficient and practical approaches to quantum communication based on continuous variables (CV) [14]. Inspired by these results, and in order to avoid both direct qubit-qubit interactions and the use of single photons, here we propose the following “hybrid quantum computer”: universal two-qubit gates shall be achieved indirectly through the interaction between the qubits and the quadrature phase amplitude of a common bosonic mode. The CV mode plays the role of a communication bus which we call a “qubus”. This approach brings together the best of both worlds, utilizing DV for processing and CV for communication.

The idea of the CV qubus computer has been applied to ion traps [15, 16, 17, 18] and other systems [19], but here we focus on a quantum optical realization. In this approach, the qubits are either atomic or photonic, and the qubus is an electromagnetic field mode; the CV are the phase-space variables of this field mode. Although efficient homodyne detection of
certain phase-space variables (quadratures) is possible, no measurement will be needed in our scheme. By design, under ideal conditions, the bus mode disentangles automatically from the qubits after a sequence of interactions. Measurement-induced errors are thus avoided and the gates become deterministic, requiring neither measurement-result-dependent post-selection nor any feed-forward operations on the qubits. Moreover, we make no assumptions about the strength of the qubit-qubits interactions; our scheme works in any regime, including the limits of weak and strong nonlinearities. The proposal here relies on two new important concepts: the exact simulation of controlled phase-space displacements via controlled rotations and uncontrolled displacements; and an efficient all-cavity implementation of this simulation.

In contrast to existing CV-mediated proposals for measurement-free ion-trap gates based upon conditional displacements [15, 16, 17, 18], our proposed two-qubit gate is based on conditional rotations. These are obtainable from the fundamental Jaynes-Cummings interaction \( hg(\sigma^- a^\dagger + \sigma^+ a) \) in the dispersive limit [20], which gives

\[
H_{\text{int}} = \hbar \chi \sigma_z a^\dagger a .
\]

Here, \( a (a^\dagger) \) refers to the annihilation (creation) operator of an electromagnetic field mode in a cavity and \( \sigma_z = |0\rangle \langle 0| - |1\rangle \langle 1| \) is the corresponding qubit operator from the set of Pauli operators \( \{\sigma_x, \sigma_y, \sigma_z\} \) for a two-level atom in the cavity (with ground state \( |0\rangle \) and excited state \( |1\rangle \)) [21]. The atom-light coupling strength is determined via the parameter \( \chi = g^2 / \Delta \), where \( 2g \) is the vacuum Rabi splitting for the dipole transition and \( \Delta \) is the detuning between the dipole transition and the cavity field. The Hamiltonian in Eq. (1) generates a conditional phase-rotation of the field mode, dependent upon the state of the atomic qubit. Note that the dispersive interaction for a high-fidelity conditional rotation does not require strong coupling; the only requirement is a sufficiently large cooperativity parameter [22].

It has been pointed out [23] that a suitable set of Hamiltonian terms, including conditional rotations and unconditional displacements \( \{\sigma_x a^\dagger a, \sigma_z a^\dagger a, x\} \), is, in principle, sufficient for universal quantum computation. Here our main concern is how to efficiently utilize these universal resources. Throughout, we use the definition for quadrature operators \( X(\phi) = (a^\dagger e^{i\phi} + ae^{-i\phi}) \) such that \( X(0) = x \) and \( X(\pi/2) = p \) play the roles of “position” and “momentum”, respectively, with \( [x, p] = 2i \) for \( [a, a^\dagger] = 1 \). We now demonstrate how a universal two-qubit gate can be implemented via the Jaynes-Cummings-type interaction from Eq. (1) and additional unconditional displacements.

Our two-qubit gate relies upon the basic principle that a CV mode acquires a phase shift whenever it goes along a closed loop in phase space. This phase shift only depends on the area of the loop and not on its form [23] and it originates from the fact that for any sequence of two displacements, the total displacement operator contains an extra phase factor,

\[
D(\beta_1)D(\beta_2) = \exp \left[ i \text{Im} (\beta_1 \beta_2^*) \right] D(\beta_1 + \beta_2) .
\]

Here \( D(\beta) = \exp(\beta a^\dagger - \beta^* a) \) is the usual quantum optical displacement operator. In this sense such a two-qubit gate can be regarded as a geometric phase gate [23]. In Ref. [19], it was shown how a conditional phase gate on qubits can be realized by creating almost closed loops in phase space through controlled rotations and uncontrolled displacements. However, this gate is imperfect, as even under ideal conditions the CV qubits does not disentangle completely from the qubits, leading to an intrinsic dephasing error. Here, instead of directly applying the interaction in Eq. (1) to create a closed path, we instead simulate controlled displacements via the controlled rotations in Eq. (1). With controlled displacements available it is straightforward to implement a conditional phase gate, as we now describe.

Let us assume that an arbitrary two-qubit state enters the gate such that the total initial state (of the two-qubit-qubits system) may be written as

\[
(c_1|00\rangle + c_2|01\rangle + c_3|10\rangle + c_4|11\rangle)\langle \text{qubits}) ,
\]

with a qubits-probe mode initially in an arbitrary state \( |\text{qubits}\rangle \). The two-qubit gate follows from four conditional

![FIG. 1: a) Circuit diagram of a universal two-qubit gate based on controlled displacements between the qubits and the probe bus. b) Schematic phase space evolution of a coherent qubits on controlled displacements between the qubits and the probe bus.](image)
displacements. The sequence of operations is shown in Fig. 1a). This defines the total unitary operator

\[ U_{\text{tot}} \equiv D(i\beta_2 \sigma_2) D(\beta_1 \sigma_1) D(-i\beta_2 \sigma_2) D(-\beta_1 \sigma_1). \]  

(4)

Using Eq. (2), it is straightforward to show that

\[ U_{\text{tot}} = \exp[2i \operatorname{Re}(\beta_1^* \beta_2) \sigma_1 \sigma_2]. \]  

(5)

Apparently, when this operator acts on the two-qubit-qubus system, the only effect is the generation of phase factors conditional on the two-qubit state. Although it is entangled with the qubits during the gate, the qubus mode finishes in its initial state, disentangled from the qubits. The evolution does not depend on this qubus state—a convenient choice would be a coherent state [24].

For the case of real \( \beta_1 \) and \( \beta_2 \), the effect of the total operation on a bus coherent state, conditional on the state of the qubits, is illustrated in Fig. 1a). By choosing \( \beta_1 \beta_2 = \pi/8 \), a total initial state as in Eq. (3) gives a final pure two-qubit state of

\[ e^{-\frac{i\pi}{8}} U \otimes U (c_1|00\rangle + c_2|01\rangle + c_3|10\rangle - c_4|11\rangle), \]  

(6)

where \( U \equiv e^{i\frac{\pi}{8} \sigma_z} \). Thus, up to a global phase and local unitaries, we obtain a controlled-phase gate.

So far we have assumed that we can perform conditional displacements in order to construct the operation \( U_{\text{tot}} \) of Eq. (4). In quantum optics, it is hard to generate such conditional displacements directly through photon-atom or photon-photon interactions. However, the Jaynes-Cummings-type interaction of Eq. (1) is readily available. We now show that this interaction is sufficient to generate the required conditional displacements. More specifically, we will use a series of pulses and interactions of the type of Eq. (1) in order to effectively simulate a controlled displacement. No approximations will be needed for this purpose, so our method is applicable to any regime of the interaction in Eq. (1), including the weak and the strong coupling limits.

We define conditional rotations as generated by Eq. (1), with an effective interaction time \( \chi t \equiv \theta \). Consider the following operator

\[ U \equiv D(\alpha \cos \theta) e^{-i\theta \sigma_z a_1^a} D(-2\alpha) e^{i\theta \sigma_z a_1^a} D(\alpha \cos \theta), \]  

(7)

consisting of unconditional displacements and conditional rotations. Using \( e^{-i\theta a_1^a a} e^{i\theta a_1^a a} = a e^{i\theta} \), hence \( e^{-i\theta a_1^a D(\alpha) e^{i\theta a_1^a a} = D(a e^{i\theta}) \), and the rule in Eq. (2), we find that the sequence in Eq. (7) exactly realizes a conditional displacement such that

\[ U = D(2i\alpha \sin \theta \sigma_z). \]  

(8)

Figure 2 illustrates the sequence of uncontrolled displacements and controlled rotations to simulate a controlled displacement.

The resultant operation in Eq. (8) corresponds to a conditional displacement by \( 2i\alpha \sin \theta \). The entire sequence of Eq. (4) can now be achieved through uncontrolled displacements and controlled rotations of the probe via the Jaynes-Cummings-type interaction from Eq. (1). This provides an exact mechanism to create the controlled phase gate. Assuming \( \beta_1 = \beta_2 = \sqrt{\pi/8} \), the strength of the conditional rotations for simulating the conditional displacements are determined by the parameter \( d \equiv 2|\alpha| \sin \theta = \sqrt{\pi/8} \approx 0.6 \). For example, with a Jaynes-Cummings coupling and interaction time corresponding to \( \theta \sim 10^{-2} \), unconditional displacements of about \( |\alpha|^2 \approx 10^4 \) photons are needed. However, we may also satisfy \( d \approx 0.6 \) using strong nonlinearities, \( \theta \sim \pi/2 \) with weak qubus displacements of the order \( |\alpha| \sim 1 \).

We shall now compare the controlled phase gate proposed here to the one described in Ref. [19]. Two crucial differences exist, both of which highlight the advantages of the new gate.

- First and foremost the gate of Ref. [19] is only approximate. It has an intrinsic error since the qubus probe does not completely disentangle from the qubits, causing a dephasing effect on the qubits. To keep this error small requires \( |\alpha| \theta^2 \ll 1 \), so the gate only works when \( \theta \ll 1 \). The gate presented here does not have this limitation. In this sense, our scheme here is universal and can be applied to various physical systems, in any coupling regime.

- The second difference is important from a practical point of view and relates to the local single qubit rotations needed to realize the gate in (6). The
gate in Ref. [19] requires single qubit rotations of the form $e^{i|a|^2\sigma_z}$. This places considerable sensitivity on $\alpha$ and $\theta$, requiring them to be known accurately enough to perform single qubit operations that scale as $|\alpha|^2$. In the gate presented here we only require a unitary of the form $e^{i\hat{z}\sigma_z}$, which is independent of both $\alpha$ and $\theta$ and thus much less demanding.

In order to accomplish the sequence $U_{\text{tot}}$ in Eq. (11) via the operation $\mathcal{U}$ from Eq. (7), it appears to be necessary to couple the qubus mode out of the cavity and back into it whenever an unconditional displacement must be applied via an external local oscillator field. However, this rather inefficient feature can be avoided in an all-cavity-based implementation of $\mathcal{U}$. A very natural way to generate the unconditional displacements is to drive the qubus mode directly with a classical pump. Such driving can be represented by the Hamiltonian $H_d = \hbar c X(\phi)$, with $c$ real, effectively resembling a phase-space displacement. For instance, with $\phi = 0$, our system Hamiltonian is of the form $H(\epsilon, \chi\sigma_z) = \hbar c (a^\dagger + a) + \hbar \chi a \sigma_z$. Now applying this operation $U(\epsilon, \chi\sigma_z) = \exp[-i\frac{\pi}{\hbar}H(\epsilon, \chi\sigma_z)t]$ for a time $t$ followed by $U(\epsilon, -\chi\sigma_z)$ [25] for the same time $t$ implements an effective controlled displacement of the form $D_{\sigma_z} = |1 - e^{i\chi t}\sigma_z| \rangle \langle 1 - e^{-i\chi t}\sigma_z|$ which for $\chi t \ll 1$ has the more usual form $D[2i\epsilon \sigma_z]$. This conditional operation $D_{\sigma_z}$ also contains unconditional operations, but this does not effect the operation of the gate [21]. In fact, these unconditional displacements are undone by further conditional operations and so our controlled displacement can be reduced to just two operations. The entire two-qubit gate then requires only eight operations.

A further issue we need to consider is the robustness of our two-qubit gate against noise and errors, for example caused by photon losses in the qubus mode [27]. This is particularly important, as we suggest to place the two atomic qubits in two different cavities in order to avoid the complication of individually addressing more atoms in one cavity. A simple loss model reflects part of the qubus mode from a beam splitter into a second mode that represents the environment. In this case, the controlled displacements $D(\beta\sigma_z)$ can be described as acting upon both the qubus mode, $D_1(\sqrt{1-\eta^2}\beta\sigma_z)$, and the loss mode, $D_2(\eta\beta\sigma_z)$, where $\eta$ is the reflectivity parameter. The first observation is that the controlled displacements on the qubus mode are no longer exactly those required, leading to a smaller phase shift and an error in the gate. It is also possible that the qubus mode will not disentangle exactly from the qubits, if the phase space loops the qubus traverses do not quite close. As long as the degree of loss is known, these two effects can be eliminated by increasing the amplitude $\beta'$ of the controlled displacement such that $\beta = \sqrt{1-\eta^2}\beta'$. The most important effect to consider is therefore the controlled displacements acting on the loss mode, which cause a de-phasing effect on the two-qubit state. This effect scales as $\eta^2 \alpha^2 \sin^2 \theta$ and for $\eta$ small ($\eta \ll 1$) this dephasing effect is minimal [recall $\beta \approx O(1)$].

In conclusion, we have demonstrated how to implement universal two-qubit gates using fundamental atom-light interactions in quantum optics, through qubus-mediated qubit-qubit communication and without direct interaction between the qubits. In this hybrid scheme, the only required interactions lead to controlled rotations of a continuous-variable qubus mode, conditioned on the state of the qubits. Our scheme is universal in the sense that any regime is allowed for the controlled rotations, including interactions in the limit of weak or strong nonlinearities. The resulting phase gate is deterministic and measurement-free, and thus represents a promising approach to implementing quantum logic.

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The atomic system could be a two-level atom or an effective two-level system with an auxillary level (Λ-system).

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The qubus mode can be prepared in an arbitrary state, which could even be mixed. However, the qubus should not evolve independently on timescales comparable with that of the gate, for example, by decoherence.

The sign change here can be realised by a simple bit flip on the matter qubit.

The sequence of operations $U(\epsilon, \chi \sigma_z), U(\epsilon, -\chi \sigma_z), U(-\epsilon, -\chi \sigma_z), U(-\epsilon, \chi \sigma_z)$ would achieve the controlled displacement alone.

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