TOWARD UNDERSTANDING RICH SUPERCLUSTERS

M. Einasto,1 E. Saar,1 V. J. Martínez,2 E. Einasto,1 L. J. Leivamägi,1 E. Tago,1 J.-L. Starck,3
V. Müller,4 P. Heinämäki,5 P. Nurmi,5 S. Paredes,6 M. Gramann,1 and G. Hütsi1

ABSTRACT

We present a morphological study of the two richest superclusters from the 2dFGRS (SCI 126, the Sloan Great Wall, and SCI 9, the Sculptor supercluster). We use Minkowski functionals, shapefinders, and galaxy group information to study the substructure of these superclusters as formed by different populations of galaxies. We compare the properties of grouped and isolated galaxies in the core region and in the outskirts of superclusters. The fourth Minkowski functional $V_4$ and the morphological signature $K_1-K_2$ show a crossover from low-density morphology (outskirts of supercluster) to high-density morphology (core of supercluster) at mass fraction $m_r \approx 0.7$. The galaxy content and the morphology of the galaxy populations in supercluster cores and outskirts are different. The core regions contain a larger fraction of early-type, red galaxies and richer groups than the outskirts of superclusters. In the core and outskirts regions the fine structure of the two prominent superclusters as delineated by galaxies from different populations also differs. The values of the fourth Minkowski functional $V_4$ show that in the supercluster SCI 126 the population of early-type, red galaxies is more clumpy than that of late-type, blue galaxies, especially in the outskirts of the supercluster. On the contrary, in the supercluster SCI 9, the clumpiness of the spatial distribution of galaxies of different type and color is quite similar in the outskirts of the supercluster, while in the core region the clumpiness of the late-type, blue galaxy population is larger than that of the early-type, red galaxy population. Our results suggest that both local (group/cluster) and global (supercluster) environments are important in forming galaxy morphologies and colors (and determining the star formation activity). The differences between the superclusters indicate that these superclusters have different evolutionary histories.

Subject headings: galaxies: clusters: general — large-scale structure of universe

Online material: color figures

1. INTRODUCTION

Huge superclusters that may contain several tens of rich (Abell) clusters are the largest coherent systems in the universe with characteristic dimensions of up to $100 \, h^{-1} \text{Mpc}$. As they are very large, and dynamical evolution takes place at a slower rate for larger scales, superclusters have retained memory of the initial conditions of their formation and of the early evolution of structure (Kofman et al. 1987).

While we might be able to explain the structure and properties of most (average) superclusters, explaining rich superclusters is still a challenge. To start with, even their existence is not well explained by the main contemporary structure modeling tool, numerical simulations. There are a number of rich superclusters in our close cosmological neighborhood (we list them below), but the comparison of the luminosity functions of observed and simulated (Millennium) superclusters shows that the fraction of very rich superclusters in simulations is much lower than in observed samples (Einasto et al. 2006). The extreme cases of observed objects usually provide the most stringent tests for theories; this motivates the need for a detailed understanding of the richest superclusters.

The richest relatively close superclusters are the Shapley supercluster (Proust et al. 2006 and references therein) and the Horologium-Reticulum supercluster (Rose et al. 2002; Fleenor et al. 2005; Einasto et al. 2003b). The two superclusters studied in this paper, the Sloan Great Wall and the Sculptor supercluster, belong also to this category of very rich superclusters.

The formation of rich superclusters had to begin earlier than smaller structures; they are sites of early star and galaxy formation (e.g., Mobasher et al. 2005) and the first places where systems of galaxies form (e.g., Venemans et al. 2004; Ouchi et al. 2005). Thus, future deep surveys like the ALHAMBRA Deep survey (Moles et al. 2005) are likely to detect (core) regions of rich superclusters at very high redshifts. The supercluster environment affects the properties of groups and clusters located there (Einasto et al. 2003b; Plionis 2004). Rich superclusters contain high-density cores that are absent in poor superclusters (Einasto et al. 2007c, hereafter Paper III). The fraction of X-ray clusters in rich superclusters is larger than in poor superclusters (Einasto et al. 2001, hereafter E01), and the core regions of the richest superclusters may contain merging X-ray clusters (Rose et al. 2002; Bardelli et al. 2000). The richest superclusters are more filamentary, less compact, and more asymmetrical than poor superclusters (Einasto et al. 2007a, hereafter Paper II). Moreover, as we noted above, the fraction of very rich superclusters in observed catalogs is larger than models predict (Einasto et al. 2006).

In the present paper we continue the study of superclusters selected on the basis of the 2dF Galaxy Redshift Survey (2dFGRS). Our paper is devoted to a detailed study of the two richest superclusters in the 2dFGRS. We chose them from the catalog of superclusters of the 2dFGRS by Einasto et al. (2007b, hereafter Paper I).
These are the supercluster SCl 126 in the northern sky and the supercluster SCl 9 (the Sculptor supercluster) in the southern sky (see Fig. 1).

The supercluster SCl 126 is the most prominent supercluster defined by Abell clusters in the northern 2dFGRS (SCl 126 in the catalog of superclusters by E01, N152 in Paper I) in the direction of the Virgo constellation. This supercluster has also been called the Sloan Great Wall (Hoyle et al. 2002; Vogele et al. 2004; Gott et al. 2005; Nichol et al. 2006). The presence of this supercluster affects the measurements of the correlation function (Croton et al. 2004) and of the genus and Minkowski functionals of the SDSS and 2dF redshift surveys (Park et al. 2005; Saar et al. 2007). The “meatball” shift in the measurements of the topology in the SDSS data is partly due to this supercluster (Gott et al. 2008).

The Sculptor supercluster, the most prominent supercluster in the southern strip of the 2dFGRS, is among the three richest superclusters in E01; it contains 25 Abell clusters, 6 of which are also X-ray clusters. Zappacosta et al. (2005) found evidence for the presence of warm-hot diffuse gas there, which is associated with the intercluster galaxy distribution in this supercluster.

In a recent paper (Einasto et al. 2007d, hereafter E07d) we studied the morphology of rich superclusters (their shape and internal structure), using Minkowski functionals and shapefinders. Our calculations in E07d showed that the morphologies of the richest superclusters from the 2dFGRS are different from each other: the supercluster SCl 126 resembles a very rich filament (wall) with a high-density core region, while the supercluster SCl 9 can be described as a collection of spiders (a multi-spider), consisting of a large number of cores connected by relatively thin filaments.

The main aim of the present paper is to understand whether the differences in the overall morphology of these two rich superclusters under study are also reflected by their fine structure as determined by the distribution of galaxies of different luminosity, color, and spectral type in the core region and in the outskirts of superclusters.

The first study to show that in a supercluster early- and late-type galaxies trace the structure of the supercluster in a different manner was performed by Giovanelli et al. (1986), who demonstrated that in the Perseus supercluster, elliptical galaxies are mainly located along the central body of the supercluster, while spiral galaxies are distributed in the outer regions of the supercluster. The presence of a large-scale segregation of galaxies of different type in nearby superclusters was shown also by Einasto & Einasto (1987). In Paper III we showed that rich superclusters have a larger fraction of red, non-star-forming galaxies than poor superclusters. Recently, galaxy populations have been studied in core regions of some very rich superclusters (in the Shapley supercluster, Haines et al. 2006b; in the Pisces-Cetus supercluster, Porter & Raychaudhury 2005). These studies showed that rich clusters in the core regions of superclusters contain a large fraction of passive galaxies, while actively star-forming galaxies are located between the clusters.

Thus, studies of rich superclusters that contain a large variety of environments and possibly a variety of evolutionary phases give us a possibility to study the properties of cosmic structures and the properties of galaxies therein in a consistent...
way, helping us to understand the role of environment in galaxy evolution.

One method to quantify the structure of superclusters is to use the Minkowski functionals. This type of study has been called morphometry by Hikage et al. (2003). Minkowski functionals, genus, and shapefinders (defined using Minkowski functionals) have been used earlier to study the three-dimensional (3D) topology of the large-scale structure from the 2dF survey and SDSS (Saar et al. 2007; Hikage et al. 2003; Park et al. 2005; Gott et al. 2008; James et al. 2007) and to characterize the morphology of superclusters (Saahni et al. 1998; Sheth et al. 2003; Shandarin et al. 2004; Basilakos et al. 2001, 2006; Kolokotronis et al. 2002; Basilakos 2003) from observations and simulations. These studies concern only the “outer” shapes of superclusters and do not treat their substructure. We expanded this approach in E07d by using the Minkowski functionals and shapefinders to analyze the full density distribution in superclusters, at all density levels.

The fourth Minkowski functional $V_3$ (the Euler characteristic) gives us the number of isolated clumps (or voids) in the region (Saar et al. 2007), meaning that we can use it to study the clumpiness of the galaxy distribution inside superclusters, i.e., the fine structure of superclusters. We calculate the fourth Minkowski functional $V_3$ for galaxies of different populations in superclusters for a range of threshold densities, starting with the lowest density used to determine superclusters, up to the peak density in the supercluster core. This analysis shows how the fourth Minkowski functional can be used in studies of the fine structure of superclusters as delineated by different galaxy populations. This study is of an exploratory nature since this is the first time that this method is used for studies of the fine structure of galaxy populations of individual superclusters. Employing Minkowski functionals, we can see in detail how the morphology of superclusters is traced by galaxies of different type.

In our analysis we use also the shapefinders calculated on the basis of the Minkowski functionals. In addition, we compare the galaxy content of groups of different richness in the core regions and in the outskirts of superclusters.

The paper is composed as follows. In § 2 we describe the galaxy data, the supercluster catalog, and the data on the richest superclusters. In § 3 we compare the overall galaxy content of the superclusters. In § 4 we describe the use of the fourth Minkowski functional (the Euler characteristic) to study the fine structure of the superclusters as delineated by different galaxy populations. In § 5 we study the galaxy content in the core regions and in the outskirts of the superclusters and compare galaxy populations in groups of various richness. In §§ 6 and 7 we discuss our results and give the conclusions. In the appendices we give a definition of the Minkowski functionals and shapefinders and describe different kernels used to calculate the density fields of superclusters.

2. DATA

2.1. Rich Supercluster Data

We used the 2dFGRS final release (Colless et al. 2001, 2003) and the catalog of superclusters of galaxies from the 2dF survey (Paper I), applying a redshift limit $z \leq 0.2$. When calculating (comoving) distances, we used a flat cosmological model with the standard parameters, the matter density $\Omega_m = 0.3$ and the dark energy density $\Omega_{\Lambda} = 0.7$ (both in units of the critical cosmological density). Galaxies were included in the 2dFGRS, if their corrected apparent magnitude $b_i$ lied in the interval from $b_1 = 13.5$ to $b_2 = 19.45$. We used weighted luminosities to calculate the luminosity density field on a grid of cell size $1$ h$^{-1}$ Mpc smoothed with an Epanechnikov kernel of radius $8$ h$^{-1}$ Mpc; this density field was used to find superclusters of galaxies. We defined superclusters as connected nonpercolating systems with densities above a certain threshold density; the actual threshold density used was 4.6 in units of the mean luminosity density. A detailed description of the supercluster-finding algorithm can be found in Paper I.

In our analysis we also used the data about groups of galaxies from the 2dFGRS (Tago et al. 2006). Groups in this catalog were determined using the friends-of-friends (FoF) algorithm, in which galaxies are linked together into a system if they have at least one neighbor at a distance less than the linking length. For details about the group-finding algorithm and the analysis of the selection effects see Tago et al. (2006).

For the present analysis we select the richest superclusters from the catalog of the 2dF superclusters. The data on these superclusters are given in Table 1. In this table we give the central coordinates, redshifts, and distances of the superclusters, the number of galaxies, groups, and Abell and X-ray clusters in the superclusters, the mean values of the luminosity density within superclusters, and their total luminosities (from Paper II). In our morphological analysis we use volume-limited samples of galaxies from these superclusters. The luminosity limits for each supercluster sample are also given in Table 1. We plot the sky distribution of galaxies and Abell and X-ray clusters in Figure 2; the location of these superclusters in space can be seen in Figure 1.

The most prominent Abell supercluster in the northern 2dF survey is the supercluster SCI 126 that lies in the direction toward the Virgo constellation at a redshift $z = 0.085$ (Paper I). This supercluster contains seven Abell clusters: A1620, A1650,

---

TABLE 1

| No. | Right Asc. | Decl. | Distance | $z$ | $N_{gal}$ | $M_{lim}$ | $N_{col}$ | $N_{gr}$ | $N_{ACO}$ | $N_X$ | $\delta_m$ | $L_{tot}$ |
|-----|-----------|-------|----------|----|----------|----------|----------|---------|----------|--------|----------|----------|
| SCI 126 | (1N152) | 194.71 | -1.74 | 251.2 | 0.085 | 3591 | -19.25 | 1308 | 18 | 40, 2 | 9 | 4 | 7.7 | 0.378E+14 |
| SCI 9 | (N34) | 9.85 | -28.94 | 326.3 | 0.113 | 3175 | -19.50 | 1176 | 24 | 26 | 9 | 12 (25) | 2 (6) | 8.1 | 0.497E+14 |

**Notes:** The supercluster ID after E01 with the ID of Paper I in parentheses; shown are the equatorial coordinates, the comoving distance $D$ for our cosmology, redshift $z$, the number of galaxies $N_{gal}$ in the supercluster, the magnitude limit $M_{lim}$, and the number of galaxies $N_{col}$ for the volume-limited supercluster: $N_{gal}$ and $N_{gr}$ are the numbers of the density field clusters and groups, respectively, according to Paper I. $N_{ACO}$ gives the number of Abell clusters in this part of the supercluster that is covered by the 2dF survey; the number inside parentheses is the total number of Abell clusters in this supercluster, by the E01 list. $N_X$ is the number of X-ray clusters, $\delta_m$ is the mean value of the luminosity density in the supercluster (in units of the mean survey density), and $L_{tot}$ is the total luminosity of the supercluster (in solar units).
A1651, A1658, A1663, A1692, and A1750. Of these clusters, A1650, A1651, A1663, and A1750 are X-ray clusters. The supercluster SCl 126 is almost completely covered by the 2dF survey volume; only a small part of it lies outside the survey (Einasto et al. 2003b). All Abell and X-ray clusters from this supercluster (nine and six, correspondingly) are located in the region of the survey.

The richest supercluster in the southern sky is the Sculptor supercluster at a redshift \( z = 0.113 \) (Paper I). This supercluster contains also several X-ray clusters and the largest number of Abell clusters in our supercluster sample, 25. However, only 12 of them are located within the region covered by the 2dF supercluster S34. These are A88, A122, A2751, A2759, A2778, A2780, A2794, A2798, A2801, A2844, and two X-ray clusters, A2811 and A2829.

If we assume that the mean mass-to-light ratio is about 400 (in solar units), then we can estimate the masses of the richest superclusters of our sample, using the estimates of the total luminosity of the superclusters (Table 1): \( M_{SCl9} = 2 \times 10^{16} h^{-1} M_\odot \), \( M_{SCl126} = 1.5 \times 10^{16} h^{-1} M_\odot \). These estimates are lower limits only, since the 2dF survey does not fully cover these superclusters. These masses are of the same order as the masses of other known very rich superclusters. For example, Proust et al. (2006) estimate that the total mass of the Shapley supercluster is at least \( M_{tot} = 5 \times 10^{16} h^{-1} M_\odot \). Fleenor et al. (2005) give the same estimate for the total mass of the Horologium-Reticulum supercluster. Porter & Raychaudhury (2005) estimated that the total mass of the Pisces-Cetus supercluster is at least \( M_{tot} = 1.5 \times 10^{16} h^{-1} M_\odot \). Thus, our two superclusters have total masses similar to other rich superclusters.

2.2. Populations of Galaxies Used in the Present Analysis

We characterize galaxies by their luminosity, the spectral parameter \( \eta \), and the color index “col” (Madgwick et al. 2002, 2003a; De Propris et al. 2003; Cole et al. 2005) as follows.

We divided galaxies into the populations of bright (\( B \)) and faint (\( F \)) galaxies, using an absolute magnitude limit \( M_{B} = -20.0 \) (here and in the following we give absolute magnitudes for \( h = 1, H = 100 \text{ km s}^{-1} \text{ Mpc}^{-1} \); for any other value of \( h \), the absolute magnitude \( M = M_{B} + 5 \log h \). We chose this limit close to the characteristic luminosity \( M^* \) of the Schechter luminosity function. The value of \( M^* \) is different for different galaxy populations (Madgwick et al. 2003a; De Propris et al. 2003; Croton et al. 2005), having values from \(-19.0 \) to \(-20.9 \). We used an absolute magnitude limit \( M_{B} = -20.0 \) as a compromise between the different values.

The absolute magnitude limits for the faintest galaxies in our superclusters (volume-limited samples, \( M_{lim} \)) are given in Table 1. Actually, our faint galaxy population is brighter than the faint galaxies analyzed in several other superclusters, e.g., in the Shapley supercluster (Mercurio et al. 2006; Haines et al. 2006b), in the supercluster A2199 (a part of the Hercules supercluster SCl 160 in the E01 list) in Haines et al. (2006a), and in the supercluster A901/902 (Gray et al. 2004).

We used the spectral parameter \( \eta \) to divide galaxies into the populations of early (\( E \)) and late (\( S \)) type. We used for this purpose the spectral parameter limit \( \eta = -1.4 \) (\( \eta \leq -1.4 \) for early-type galaxies, and \( \eta > -1.4 \) for late-type galaxies) and excluded galaxies with uncertain determination of \( \eta \). More detailed morphological types were defined as follows: type 1 (Kennicutt 1992): \( \eta < -1.4 \); type 2: \(-1.4 \leq \eta < 1.1 \); and type 3: \( 1.1 \leq \eta \).

Moreover, the spectral parameter \( \eta \) is correlated with the equivalent width of the H\( \alpha \) emission line, thus being an indicator of the star formation rate in galaxies. Madgwick et al. (2003b) call galaxies with \( \eta > -1.4 \) (late-type galaxies) “generally star-forming.” Thus, the spectral parameter gives us information on the types and star formation activity of galaxies.

We used the rest-frame color index, \( \text{col} = (B - R)_0 \), to divide galaxies into the populations of red galaxies (\( r \)), \( \text{col} > 1.07 \), and blue galaxies (\( b \)), \( \text{col} < 1.07 \) (Cole et al. 2005; Wild et al. 2005). Wild et al. (2005) suggest that red galaxies are mostly passive and blue galaxies are actively star forming. However, since there exist also red galaxies showing signs of star formation (Wolf et al. 2005; Haines et al. 2008), we name our populations as red and blue.

In the next section we analyze the relationship between the spectral parameters and color indices of galaxies in our superclusters in more detail.

In our analysis we use the following ratios: \( E/S \), the ratio of the numbers of early- and late-type galaxies; \( r/b \), the ratio of the numbers of red and blue galaxies; and \( B/F \), the ratio of the numbers of bright (\( M_B \leq -20.0 \)) and faint (\( M_B > -20.0 \)) galaxies.

3. GALAXY CONTENT OF RICH SUPERCLUSTERS

First, we study an overall galaxy content of the rich superclusters (see Table 2). We plot the differential luminosity functions...
and the distributions of the spectral parameter \( \eta \) and the color index \( \text{col} \) for galaxies from superclusters SCl 126 and SCl 19 in Figure 3. For comparison we plot also the distribution of the spectral parameter \( \eta \) and the color index \( \text{col} \) for field galaxies, chosen from the same redshift interval as our two superclusters, and having the same absolute magnitude limit \( (M_{\text{bj}} = -19.50; \text{see below}) \). There are 1975 galaxies in the northern field sample, N(field), and 2927 in the southern field sample, S(field). The number of galaxies in field samples is rather small due to the absolute magnitude limit used here, since, in general, galaxies in the field are fainter than in superclusters (Paper III). The comparison with field galaxies shows whether the possible differences between the distributions of spectral parameters and colors may be due to redshift differences between the two superclusters.

All the distributions (probability densities) shown in this paper have been obtained using the R environment (Ihaka & Gentleman 1996). This package does not provide the customary error limits;

\footnote{The “stats” package available at http://www.r-project.org.}

we discuss these limits in Appendix A and show that they are small (about 4%–5%).

In Table 3 we give the ratios of the numbers of galaxies of different type in the superclusters. In this table we give these ratios for SCl 126 for two magnitude limits, the original limit, which corresponds to the volume-limited sample (Table 1), and the SCl 9 magnitude limit \( M_{\text{bj}} = -19.50 \), which enables us to compare these ratios with those calculated for the supercluster SCl 9.

The Kolmogorov-Smirnov test, Figure 3, and Table 3 show differences in the overall galaxy content of the two richest 2dFGRS superclusters.

The Kolmogorov-Smirnov test shows that the probability that the distributions of luminosities of the two superclusters are drawn from the same parent sample is 0.135, having marginal statistical significance. The most important difference between the luminosities of the galaxies of two superclusters is that the brightest galaxies in SCl 126 are brighter than those in SCl 9. In \( \frac{9}{5} \) we show that there are also other differences in the distribution of the brightest galaxies in the two superclusters under study.

Next, Table 3 shows that the ratio of the numbers of red and blue galaxies \((r/b)\) is slightly larger in the supercluster SCl 126, but the peak value of the color index of galaxies in this supercluster is smaller than in SCl 9. The distributions of the spectral parameter \( \eta \) and the color index \( \text{col} \) both show large differences in the early-type (red) galaxy content of the richest superclusters: in the supercluster SCl 9, galaxies have larger color index values and smaller spectral parameter values than in supercluster SCl 126.

According to the Kolmogorov-Smirnov test, the difference between the distributions of the spectral parameter \( \eta \) and the color index \( \text{col} \) of the two superclusters is highly significant. However, the comparison with the field galaxies of the same distance interval and absolute magnitude limit as the galaxies from superclusters shows that the main reason for the differences of spectral parameters and colors is the difference in their distances (Fig. 3). This effect is difficult to quantify exactly, as the color distributions for supercluster and field galaxies differ in detail but are close on average. We illustrate their similarity by comparing in Table 4 the quartiles for the distributions of the spectral parameter \( \eta \) and the color index \( \text{col} \), for parameter intervals that correspond to early-type and red galaxies \((\eta \leq -1.4 \text{ and } \text{col} \geq 1.07, \text{ correspondingly})\), for the absolute magnitude limit \( M_{\text{bj}} = -19.50 \). These quartiles are very similar for the supercluster and field galaxies.

These differences, however, do not affect our calculations of the Minkowski functionals for individual superclusters. So there

---

**TABLE 2**

Superclusters SCl 126 and SCl 9: The Number of Galaxies in Different Galaxy Populations, for the Whole Supercluster (Volume-limited Samples), for the Core Region (D1), and for the Outskirts of the Superclusters (D2)

| ID (1) | SCl 126 | SCl 9 |
|-------|---------|-------|
|       | All \( M_{\text{bj}} \leq -19.50 \) | D1 | D2 | All \( M_{\text{bj}} \leq -19.50 \) | D1 | D2 |
| \( N_{\text{gal}} \) | 1308 | 932 | 488 | 820 | 1176 | 342 | 834 |
| \( N_{\text{gal}}^{10} \) | 405 | 308 | 227 | 181 | 247 | 137 | 108 |
| \( N_{\text{gal}}^{126} \) | 576 | 410 | 172 | 410 | 576 | 137 | 442 |
| \( N_{\text{gal}}^{126} \) | 327 | 214 | 89 | 229 | 353 | 68 | 284 |
| \( E \) | 809 | 603 | 340 | 469 | 722 | 236 | 536 |
| \( S \) | 490 | 322 | 145 | 345 | 393 | 102 | 291 |
| \( r \) | 937 | 685 | 377 | 560 | 835 | 251 | 584 |
| \( b \) | 371 | 247 | 111 | 260 | 341 | 91 | 250 |
| \( B \) | 400 | 400 | 144 | 256 | 556 | 168 | 388 |
| \( F \) | 908 | 532 | 344 | 564 | 620 | 174 | 446 |

**Notes.—** Col. (1): Population ID \((2,2)\). \( N_{\text{gal}} \) is the number of galaxies in a given density region; \( N_{\text{gal}}^{126} \) the number of galaxies in poor groups with two to nine member galaxies; \( N_{\text{gal}}^{10} \) the number of galaxies in rich groups with at least 10 member galaxies; \( N_{\text{gal}}^{126} \) the number of galaxies that do not belong to groups. Cols. (2)–(4): Number of galaxies in a given population in SCl 126. Cols. (6)–(8): Number of galaxies in a given population in SCl 9.

---

**Fig. 3.**—Distribution of the absolute magnitude (left), the spectral parameter \( \eta \) (middle), and the color index \( \text{col} \) (right) of galaxies in superclusters SCl 126 (solid line) and SCl 9 (dashed line), both for the absolute magnitude limit \( M_{\text{bj}} \leq -19.50 \).
is no need to correct the colors and spectral parameters for this effect.

Next, we present the color-magnitude diagrams for our two superclusters under study (Fig. 4). Interestingly, this figure shows that there is no correlation between the color and magnitude among the red galaxies in our superclusters. In this respect the color-magnitude diagrams for the superclusters SCl 126 and SCl 9 are different from those for some other superclusters, e.g., for the supercluster A901/902 (Gray et al. 2004) and for the core of the Shapley superclusters (Haines et al. 2006b). The reasons for that are probably the use of total magnitudes in our study (see also Cross et al. 2001, in this case the color-magnitude relation is almost flat (Scodeggio 2001), and the use of the data for relatively bright galaxies only. The studies by Gray et al. (2004) and Haines et al. (2006b) include much fainter galaxies than those used in our study; for fainter galaxies the slope of the color-magnitude diagram is larger than for bright galaxies (see also Metcalfe et al. 1994) even when using total magnitudes, as mentioned by Scodeggio (2001). Thus, Figure 4 suggests that we can use galaxy colors for classification without further corrections.

Finally, we take a closer look at the classification of galaxies by their spectral parameters and colors. For that we plot in Figure 5 the joint distribution of the spectral parameter $\eta$ and the color index col for the superclusters SCl 126 and SCl 9, in analogy to Figure 2 of Wild et al. (2005). We see that most galaxies populate an area at the lower right quadrant, where the galaxies have spectra characteristic of early-type galaxies (with no star formation) and have red colors. The upper left quadrant of this figure is populated by late-type, blue galaxies. But there are also galaxies that have spectra characteristic of late-type ("generally star-forming") galaxies and at the same time have red colors. The number of these galaxies is small: 161 in the supercluster SCl 126, and 114 in the supercluster SCl 9. There are also galaxies with blue colors and spectra characteristic of early-type galaxies (41 in SCl 126 and 62 in SCl 9). A detailed analysis of the properties of these galaxies is beyond the scope of the present paper, but below we discuss where these galaxies are located in superclusters and whether the differences between the populations of early-type and red galaxies (and late-type and blue galaxies) influence the calculation of the Minkowski functionals.

### 4. MINKOWSKI FUNCTIONALS

#### 4.1. Method

The supercluster geometry (morphology) is given by their outer (limiting) isodensity surface and its enclosed volume. When increasing the density level over the outer threshold overdensity $\delta = 4.6$ (§ 2.1), the isodensity surfaces move into the central parts of the supercluster. The morphology and topology of the isodensity contours are (in the sense of global geometry) completely characterized by the four Minkowski functionals $V_0 - V_3$.

For a given surface the four Minkowski functionals (from the first to the fourth) are proportional to the enclosed volume $V$, the area of the surface $S$, the integrated mean curvature $C$, and the integrated Gaussian curvature (or Euler characteristic) $\chi$. The Euler characteristic describes the topology of the surface; at high densities it gives the number of isolated clumps (balls) in the region, and at low densities it gives the number of cavities (voids); see, e.g., Saar et al. 2007). This is the functional we use in this paper (see Appendix C for details).

We calculate also the shapefinders $H_1$ (thickness), $H_2$ (width), and $H_3$ (length), which have dimensions of length, and dimensionless shapefinders $K_1$ (planarity) and $K_2$ (filamentarity) (see Appendix C for definitions). In E07d we showed that in the $K_1-K_2$ shapefinder plane the morphology of superclusters is described by a curve that is characteristic of multibranching filaments; we call it a morphological signature.

In order to estimate the Minkowski functionals, we have to ensure that the mean density is constant throughout the volume we study. This is the main reason why we use volume-limited samples of supercluster galaxies. This also means that we have to recalculate the (luminosity) density field; the field used to select superclusters was calculated on the basis of the full sample.

To obtain the density field for estimating the Minkowski functionals, we used a kernel estimator with a $B_3$ box spline as the smoothing kernel, with the total extent of 16 $h^{-1}$ Mpc (for a detailed description see Saar et al. 2007; E07d). This kernel covers exactly the 16 $h^{-1}$ Mpc extent of the Epanechnikov kernel, used to obtain the original density field, but it is smoother and resolves better density field details (its effective width is about 8 $h^{-1}$ Mpc). The density field for both superclusters is shown in Figure 6.
As the argument labeling the isodensity surfaces, we chose the (excluded) mass fraction \( m_f \), the ratio of the mass in regions with density lower than the density at the surface to the total mass of the supercluster. When this ratio runs from 0 to 1, the isosurfaces move from the outer limiting boundary into the center of the supercluster, i.e., the fraction \( m_f = 0 \) corresponds to the whole supercluster and \( m_f = 1 \) to its highest density peak. This is the convention adopted in all papers devoted to the morphology of the large-scale galaxy distribution.

At small mass fractions the isosurface includes the whole supercluster and the value of the fourth Minkowski functional \( V_3 = 1 \). As we move to higher mass fractions, the isosurface includes only the higher density parts of superclusters. Individual high-density regions in a supercluster, which at low mass fractions are joined together into one system, begin to separate from each other, and the value of the fourth Minkowski functional \( V_3 \) increases. At a certain density contrast (mass fraction) \( V_3 \) has a maximum, showing the largest number of isolated clumps in a given supercluster. At still higher density contrasts only the high-density peaks contribute to the supercluster and the value of \( V_3 \) decreases again.

In E07d we showed that, according to Minkowski functionals, the supercluster SCl 126 resembles a multibranching filament with a high-density core at a mass fraction \( m_f = 0.95 \). The maximum value of the fourth Minkowski functional, \( V_3 \), in this supercluster is 9. The main body of this filament is seen in Figure 2 (region D1; see below for definition).

The supercluster SCl 9 can be described as a multispiders rather than a rich filament; i.e., this supercluster consists of a large number of relatively isolated clumps or cores connected by relatively thin filaments, in which the density of galaxies is too low to contribute to the higher density parts of this supercluster. The maximum value of the fourth Minkowski functional, \( V_3 \), in this supercluster is 15; i.e., this supercluster is more clumpy than the supercluster SCl 126. The main cores of this supercluster are seen in Figure 2 as region D1.

Now we find the Minkowski functional \( V_3 \) and morphological signature \( K_1 - K_2 \) for rich superclusters separately for galaxies from different populations, as marked by their luminosity, the spectral parameter \( \eta \), and the color index \( \text{col} \) (Figs. 7 and 8). To understand better the morphological signature, we show the shapefinders \( H_1 \), \( H_2 \), and \( H_3 \) (Fig. 9). To not overcrowd the paper with figures, we show the shapefinders for the supercluster SCl 126 for bright and faint galaxies only.

### 4.2. SCl 126

The top left panel of Figure 7 shows the \( V_3 \) curves for bright and faint galaxies in the supercluster SCl 126. At small values
of the mass fraction $m_f$ the values of $V_3$ are small. At a mass fraction $m_f \approx 0.4$ the values of $V_3$ of both bright and faint galaxies increase rapidly, which shows that at this density level the supercluster is split into several clumps. For bright galaxies the $V_3$ curve has some small plateaus at $m_f \approx 0.6$ and $m_f > 0.8$; this curve reaches maximum value of about 15 at mass fraction $m_f \approx 0.8$.

For faint galaxies the value of $V_3$ increases at mass fraction $m_f \sim 0.4$. In the whole mass fraction interval the values of $V_3$ for faint galaxies are smaller than those for bright galaxies, about 5. This indicates that the overall distribution of bright galaxies is clumpy, while the distribution of faint galaxies is more homogeneous. The peaks of $V_3$ values at very high mass fractions, where the maximum value of $V_3$ for faint galaxies is 10, are due to high-density cores in this supercluster. The $V_3$ curve for the full supercluster is very similar to the $V_3$ curve for faint galaxies, showing that faint galaxies trace the structure of the supercluster rather well.

The bottom left panel of Figure 7 shows the shapefinders $K_1$ (planarity) and $K_2$ (filamentarity) for bright and faint galaxies.

![Figure 7](image-url)

**Fig. 7**—Supercluster SCl 126. Top panels: Minkowski functional $V_3$ (the Euler characteristic) vs. the mass fraction $m_f$ for bright ($B$, $M < -20.0$) and faint ($F$, $M > -20.0$) galaxies (top left), for early- and late-type galaxies (top middle), and for red and blue galaxies (top right). Bottom panels: Morphological signatures $K_1$ (planarity)–$K_2$ (filamentarity) for the same populations. Triangles show the values of $K_1$, $K_2$, where the mass fraction $m_f = 0.0$ (the whole supercluster), and filled circles show the values of $K_1$, $K_2$, which correspond to $m_f = 0.7$. In the left panels we also plot the $V_3$ and $K_1$, $K_2$ curves for the whole supercluster (light gray line).
They are calculated on the basis of shapefinders $H_1 - H_3$ (Fig. 9).

Of these, the shapefinder $H_1$ is the smallest and characterizes the thickness of superclusters. The shapefinder $H_2$ is an analogy of the width of a supercluster, but it contains information about both the area and curvature of an isodensity surface. The shapefinder $H_3$ is the largest and describes the length of the superclusters. This is not the real length of the supercluster, but a measure of the integrated curvature of the surface.

Figure 9 shows that at all mass fractions, the thickness $H_1$ and the width $H_2$ of the supercluster as determined using data for bright galaxies are smaller than those calculated using data for faint galaxies. Thus, at all density levels the distribution of bright galaxies is more compact than the distribution of faint galaxies (which is also less clumpy). However, the shapefinder $H_3$, which is calculated as $H_3 = C/4\pi$ (the integrated mean curvature $C$), reflects the clumpiness of a population, and since the distribution of bright galaxies is much more clumpy (according to $V_3$), the value of $H_3$ for bright galaxies is larger than this value for faint galaxies (Fig. 9, right panel). This makes also the filamentarity $K_2$ for bright galaxies larger than the filamentarity of faint galaxies. As a result, in the morphological signature (Fig. 7, bottom left panel) the values of $K_2$ for bright galaxies are larger. The morphological signature of faint galaxies is similar to that of the whole supercluster, which is another indication that faint galaxies trace the structure of the supercluster well.

In this figure we marked the values of $K_1$ and $K_2$ at the mass fraction $m_f \approx 0.7$. Interestingly, we see that at this mass fraction (density level) the characteristic morphology of the supercluster changes, as seen from the change of the morphological signature.

Fig. 8.—Supercluster SCI 9. Top panels: Minkowski functional $V_3$ (the Euler characteristic) vs. the mass fraction $m_f$ for bright ($B$, $M \leq -20.0$) and faint ($F$, $M > -20.0$) galaxies (top left), for early- and late-type galaxies (top middle), and for red and blue galaxies (top right). Bottom panels: Morphological signatures $K_1$ (planarity)–$K_2$ (filamentarity) for the same populations. Symbols as in Fig. 7.

Fig. 9.—Left to right: Shapefinders $H_1$ (thickness), $H_2$ (width), and $H_3$ (length) (in $h^{-1}$ Mpc) for the supercluster SCI 126, for bright ($B$, $M \leq -20.0$) and faint ($F$, $M > -20.0$) galaxies.
The values of the fourth Minkowski functional, \( V_3 \), for galaxies of different type (Fig. 7, top middle panel; classified by the spectral parameter \( g \)) show that the clumpiness of early-type galaxies starts to increase at low values of the mass fraction, \( m_f \approx 0.2-0.3 \). At the mass fraction value 0.5 the value of \( V_3 \) has a small peak followed by a minimum; at the mass fraction \( m_f \approx 0.7 \) the value of \( V_3 \) increases again, and it has a peak value of about 10 at the mass fraction of about 0.9.

The distribution of late-type galaxies in this supercluster is much less clumpy, as shown by the values of \( V_3 \). The number of isolated clumps of these galaxies grows only at rather high mass fraction values, \( m_f > 0.6 (V_3 = 6) \), and has a peak at \( m_f \approx 0.9 \) (here \( V_3 = 12 \)). The largest differences in the clumpiness of the spatial distribution of galaxies of different type are, according to the fourth Minkowski functional, at mass fractions \( m_f < 0.6 \), which describe the outer (lower density) region of the supercluster.

The bottom middle panel of Figure 7 shows the morphological signature for galaxies of different type. The value of \( K_2 \) (filamentarity) that contains information about both the spatial extent and clumpiness of the data is larger for late-type galaxies. In this figure an important feature is that, again, at the mass fraction (density level) \( m_f \approx 0.7 \) the characteristic morphology of the supercluster changes.

The curves of the fourth Minkowski functional for red galaxies are rather similar to those of early-type galaxies (Fig. 7, top right panel; the color index col). Here we again see an increase of the values of \( V_3 \) at the mass fractions \( m_f \approx 0.4 \) and \( m_f \approx 0.6 \), and a maximum at \( m_f \approx 0.9 \) (\( V_3 = 10 \)).

This panel shows that blue galaxies form a few isolated clumps already at relatively low mass fraction values, but those clumps have low density, and at higher mass fractions some of them do not contribute to the supercluster any more; the value of \( V_3 \) decreases (being smaller than 6) and then increases again at mass fractions \( m_f \approx 0.6-0.7 \). At this mass fraction the clumpiness of the blue galaxy distribution becomes comparable to that of red galaxies. At very high values of the mass fraction, \( m_f > 0.9 \), blue galaxy clumps are not seen, and the value of \( V_3 \) decreases rapidly. The value of \( V_3 \) for red galaxies is larger than for blue galaxies at almost all density levels (mass fraction values).

At the mass fraction \( m_f \approx 0.7 \) (Fig. 7, bottom right panel) the characteristic morphology of the supercluster, as described by red and blue galaxies, changes.

Thus, Figure 7 shows that the differences in clumpiness between galaxies of different type and color (and star formation rate) in the supercluster SCl 126 are the largest, according to the fourth Minkowski functional, at mass fractions \( m_f < 0.7 \), which describe the outer (lower density) regions of the supercluster. In outer parts of this supercluster, the distribution of late-type and blue galaxies is much less clumpy (more homogeneous) than the distribution of early-type, red galaxies.

The fourth Minkowski functional \( V_3 \) for early-type galaxies (Fig. 7, middle panels) has a small peak at mass fractions of about \( 0.4 < m_f < 0.6 \) where the value of \( V_3 = 10 \). This indicates that at this intermediate density level early-type galaxies form some isolated clumps that still higher density level (mass fraction) do not contribute to the supercluster any more, and the value of \( V_3 \) decreases again. At the same mass fractions \( V_3 \) for red galaxies (right panels) has a value \( V_3 = 7 \) with no peak. Therefore, these additional isolated clumps are due to early-type, blue galaxies. The galaxies that are classified as late type but have red colors are located in intermediate-density filaments that connect clumps of early-type galaxies to the main body of the supercluster.

4.3. SCl 9

Next, let us study the values of \( V_3 \) for galaxies of different populations in the supercluster SCl 9 (Fig. 8, top right panel). Here we see that the \( V_3(m_f) \) curve is rather different from that for SCl 126; at small mass fractions, the values of \( V_3 \) for bright and faint galaxies (top left panel) are small, but at a mass fraction value of about 0.4, the values of \( V_3 \) increase. This increase is more rapid for bright galaxies than for faint galaxies. The values of \( V_3 \) for bright galaxies reach a maximum at a mass fraction \( m_f \approx 0.7 \), and the maximum values of \( V_3 \) are larger than in the supercluster SCl 126, about 18. For faint galaxies the \( V_3 \) curve reaches the maximum value, 15, at the mass fraction \( m_f \approx 0.7 \). The clumpiness of both the bright and faint galaxy distribution in the supercluster SCl 9 is larger than that in the supercluster SCl 126. Also in this supercluster the \( V_3 \) curve for the full supercluster is very similar to that of the \( V_3 \) curve for faint galaxies, showing that faint galaxies trace the structure of the supercluster well.

In the top middle panel of Figure 8 (\( V_3 \) for early- and late-type galaxies, defined by the spectral parameter \( g \)) we see a continuous increase of the number of isolated clumps as delineated by galaxies of different type. Interestingly, up to mass fractions of about \( m_f \approx 0.7 \) the \( V_3 \) curves for galaxies of different type almost coincide (\( V_3 = 12 \)), which is opposite to what we saw in SCl 126. Then the number of clumps in the distribution for late-type galaxies increases rapidly and reaches the maximum value, 20, at mass fractions between 0.7 and 0.8. The distribution of late-type galaxies in the core region of the supercluster SCl 9 is even more clumpy than the distribution of early-type galaxies.

The curves for \( V_3 \) for red and blue galaxies (divided using the color index col; Fig. 8, top left panel) are quite similar to those we saw above. Up to the mass fractions \( m_f \approx 0.7 \) (the outer regions of the supercluster) the \( V_3 \) curves for galaxies of different color almost coincide. In the core region of the supercluster the distribution of blue galaxies is very clumpy, the maximum number of clumps is 17, while the maximum number of isolated clumps in the distribution of red galaxies is 12. Larger values of \( V_3 \) for blue galaxies suggest again that in this supercluster, blue galaxies are located in numerous clumps, while the distribution of red galaxies is smoother.

This figure shows that at intermediate mass fractions, \( 0.4 < m_f < 0.6 \), the values of \( V_3 \) for blue galaxies show a small peak not seen in the \( V_3 \) curve for the late-type galaxies. This peak is generated by those galaxies that have blue colors but spectra characteristic of early-type galaxies. This is consistent with the overall large clumpiness of blue galaxies in this supercluster.

In the bottom panels of Figure 8 we plot the morphological signature for galaxies of different populations in SCl 9. These figures show that also in this supercluster the filamentarity of bright galaxies is larger than the filamentarity of faint galaxies. The morphological signature of faint galaxies is similar to that of the whole supercluster. The filamentarity \( K_2 \) for late-type, blue galaxies is larger than that for early-type (and red) galaxies due to their larger clumpiness. However, we see in these figures again that at the mass fraction \( m_f \approx 0.7 \) the morphology of the supercluster changes.

4.4. Summary

The differences in the distribution of galaxies from different populations in the superclusters SCl 126 and SCl 9 are related to different overall morphology of these superclusters. In particular, in the supercluster SCl 126 at mass fractions \( m_f < 0.7 \) the differences in the fine structure formed by galaxies of different...
populations are large: here the maximum values of $V_3$ are, correspondingly, 8 for red galaxies and 5 for blue galaxies, showing that the distribution of blue, late-type galaxies is more homogeneous than the distribution of red, early-type galaxies. At mass fractions $m_f > 0.7$ the clumpiness of the distribution of galaxies of different type and color in the supercluster SCl 126 is similar. In contrast, in the supercluster SCl 9 the clumpiness of galaxy populations of different type and color is quite similar in the outskirts of the supercluster (e.g., the values of $V_3$ are the same, 7–8 for both the early- and late-type galaxies). At mass fractions $m_f > 0.7$ in this supercluster, the clumpiness of the late-type, blue galaxy population is larger (the maximum value of $V_3$ is 17) than the clumpiness of the early-type, red galaxy population (for those galaxies the maximum value is $V_3 = 12$). Overall, the clumpiness of all galaxy populations in the supercluster SCl 126 is smaller than the clumpiness of galaxy populations in SCl 9.

This shows that in superclusters under study there exists a region of mass fractions $m_f \approx 0.7$ where the behavior of the fourth Minkowski functionals for the galaxies of different type and the behavior of the morphological signature show a change in morphology. At this mass fraction we see a crossover from low-density morphology to high-density morphology. Based on that, we choose the mass fraction $m_f = 0.7$ to delimit the supercluster cores. In the following analysis we use this density level as separating the high-density core regions of superclusters (D1) from their outer regions with lower densities (outskirts, D2; see Fig. 2).

The density level $m_f = 0.7$ approximately corresponds to the region where the luminosity density contrast $\delta = 10$ (Papers I and III). In Paper III we showed that high-density cores with $\delta > 10$ are characteristic of rich superclusters. Typical densities in poor superclusters are lower, $\delta < 10.0$, comparable to densities in the outskirts of rich superclusters.

In both superclusters, about $\frac{4}{7}$ of late-type galaxies with red colors, as well as those galaxies showing blue colors and early-type spectra, are located at intermediate densities in the outskirts regions. Figures 7 and 8 show that although due to these galaxies the values of the fourth Minkowski functional $V_3(m_f)$ for galaxies of different type and color differ in some details, the overall shapes of $V_3(m_f)$ curves show no systematic differences.

Figure 2 shows the main body of the supercluster SCl 126 as a rich filament (region D1; E07d). The main core of the supercluster at R.A. $\sim 195^\circ$ contains four Abell clusters (three of them are also X-ray sources). This region has a diameter of about 10 $h^{-1}$ Mpc (Einasto et al. 2003b). The X-ray cluster at R.A. $\sim 203^\circ$ is Abell 1750, a merging binary cluster (Donelly et al. 2001; Belsote et al. 2004).

In the supercluster SCl 9 the regions of highest density form several separate concentrations of galaxies (multispider; Fig. 2 and E07d). One of them, the main center of SCl 9, contains five Abell clusters and one X-ray cluster. There are Abell clusters also in the outskirts of this supercluster. In this supercluster the total number of Abell clusters is larger than in the supercluster SCl 126, but they do not form such a high concentration of Abell and X-ray clusters as is observed in the core region of SCl 126.

In the following section we study the distribution of galaxies from various populations in the core region and in the outskirts of superclusters, using additionally the information about the group membership for galaxies from different populations.

We mention here that the values of $V_3(m_f)$ reflect the distribution of galaxies at scales determined by the width of our smoothing kernel, B3, and this corresponds to larger scales than the typical scale at which the relation between galaxy type and local density applies; i.e., our results do not reflect directly features in galaxy distribution with characteristic scale less than about 1–2 $h^{-1}$ Mpc.

Thus, the difference in clumpiness of galaxies of different type at high mass fractions (in supercluster cores) does not contradict the presence of the luminosity-density relation at small scales. At the same time, the brightest galaxies of both early- and late-type galaxies are located in high-density regions, and this is seen also in $V_3(m_f)$ curves at high mass fraction (Hamilton 1988; Einasto 1991; Park et al. 2007).

5. SUBSTRUCTURE OF SUPERCLUSTERS IN CORE REGION AND IN OUTSKIRTS: RICH AND POOR GROUPS

5.1. Core and Outskirts

Groups of galaxies are additional tracers of substructure in superclusters. The group richness is a local density indicator, if we compare the properties of galaxies in various local environments (Paper III). We divide groups by their richness as follows: rich ($N_{gal} \geq 10$) groups and clusters (we denote this sample as g10) and poor groups ($N_{gal} < 10$, g2). In Table 3 we give the fractions of galaxies in these groups for the whole superclusters.

Let us study the galaxy content of groups in the cores and in the outskirts of our superclusters. We plot the differential luminosity functions and the distributions of the spectral parameter $\eta$ and the color index col for galaxies in groups of different richness in Figures 10 and 11, for the core regions and outskirts of the superclusters SCl 126 and SCl 9, respectively. In Table 5 we present the ratios of the numbers of bright and faint galaxies $B/F$ in superclusters and the ratios of early- and late-type galaxies $E/S$, as classified by the spectral parameter $\eta$. We also calculate the ratio of the numbers of red and blue galaxies, $r/b$, classified by the color index col. We give here the statistical significance of the differences between the galaxy content in core regions and in outskirts in these two superclusters according to the Kolmogorov-Smirnov test.

5.2. SCl 126

The top panels of Figure 10 show the luminosities of galaxies in groups of different richness in the supercluster SCl 126. This figure shows that the brightest galaxies reside in rich groups ($N_{gal} \geq 10$), both in the core region and in the outskirts of the supercluster. Luminosities of galaxies in poor groups are fainter, and galaxies that do not belong to groups are the faintest, although in the core region of the supercluster there are also some bright galaxies among them.

According to the spectral parameter $\eta$ (Fig. 10, middle panels), rich groups in the core region of the supercluster SCl 126 are populated mainly with early-type, quiescent galaxies (type 1 galaxies, $\eta < -1.4$). The fraction of late-type galaxies in them is very small. The fraction of late-type galaxies increases, as we move to poor groups. In the core region of the supercluster the fraction of late-type galaxies is largest among those galaxies that do not belong to groups.

In the outskirts, D2, rich groups are still mainly populated with early-type galaxies, but the fraction of late-type galaxies in them is larger ($E/S = 2.81$) than in rich groups in the core region, where $E/S = 4.35$. The fraction of late-type galaxies in poor groups is larger; the fraction of these galaxies is the largest among those galaxies that do not belong to any group.

In the supercluster outskirts, the fraction of late-type galaxies among those galaxies that do not belong to groups is larger than among this population in the supercluster core.
Table 5 shows that, according to the Kolmogorov-Smirnov test, the differences between the distributions of the spectral parameter $\eta$ for galaxies in the core region and in the outskirts have very high statistical significance. The differences in the galaxy content of groups and of those galaxies that do not belong to groups from the core region D1 and in the outskirts D2 are smaller, and their statistical significance is marginal.

The distribution of the color index $col$ for galaxies in the supercluster SCl 126 (Fig. 10, bottom panels) shows that members of rich groups from the core region D1 and in the outskirts D2 are mainly red galaxies, $col > 1.07$. The fraction of blue galaxies is larger in poor groups, and this fraction is the largest among those galaxies that do not belong to groups. In the outskirts region, even in rich groups there is a larger fraction of galaxies having $col < 1.07$, in comparison with the core region. Mostly, blue galaxies can be found among those galaxies that do not belong to groups. The differences between the galaxy content of poor groups from the core region and the outskirts are smaller.

The statistical significances of these differences are given in Table 5. This table shows that, according to the Kolmogorov-Smirnov test, the differences between the distributions of the color index $col$ for galaxies in the core region and in the outskirts have very high statistical significance. Among other populations, the statistical significance of differences is marginal.
In other words, galaxies of different type in the supercluster SCl 126 are segregated: red galaxies are preferentially located in rich groups, and blue galaxies in poor groups and among those galaxies not in any group. This is in good accordance with the results about the fine structure obtained with the fourth Minkowski functional $V_3$, where we saw that red galaxies in the supercluster SCl 126 are more clumpy while blue galaxies formed less clumpy populations around them.

5.3. SCl 9

The distribution of galaxies from different populations in the supercluster SCl 9 is shown in Figure 11 and Table 5.

We see, in accordance with the data for SCl 126, that galaxies in rich groups are, on average, brighter than galaxies in poor groups; galaxies that do not belong to groups are fainter than group galaxies. The largest difference between the distribution of galaxies by luminosity in the superclusters SCl 126 and SCl 9 is that in the supercluster SCl 9 the luminosities of the brightest galaxies in rich and poor groups are of the same order.

In the supercluster SCl 9 the fraction of early-type, red galaxies is again the largest in rich groups and the smallest among those galaxies that do not belong to groups. However, in this supercluster rich groups contain also a large fraction of late-type, blue galaxies, especially in the outskirts of the supercluster. The fraction...
of blue galaxies in poor groups of the core region is also relatively large. Thus, in contrast to the supercluster SCl 126, in the supercluster SCl 9 early- and late-type and red and blue galaxies reside together in groups of different richness. This is the same structure that was shown by the fourth Minkowski functional: blue galaxies form numerous small clumps.

Table 5 shows that, according to the Kolmogorov-Smirnov test, the differences between the distributions of the spectral parameter $\eta$ for galaxies in the core region and in the outskirts in the supercluster SCl 9 are statistically significant. The differences between the distributions of the color index col for galaxies in the core region and in the outskirts have a lower statistical significance. The differences between the distributions of luminosities for galaxies in the core region and in the outskirts have a marginal statistical significance only.

### 5.4. Summary

Table 5 shows that the overall galaxy content of supercluster cores and outskirts is different. In the core regions, the fraction of galaxies located in rich groups is about 0.45, while the fraction of galaxies in rich groups in the outskirts region is about 0.20. The fraction of those galaxies that do not belong to any group in the core region is smaller than in the outskirts region.

The core regions contain about 1.5 times more of early-type, red galaxies than the outskirts. The statistical significance of these differences is high. What we see here is evidence for a large-scale morphology-density relation in superclusters: high-density cores of superclusters contain relatively more early-type, red galaxies than the lower density outer regions.

Interestingly, the galaxy content of groups in the superclusters SCl 126 and SCl 9 is not similar. In the supercluster SCl 126 galaxies of different type are segregated: red galaxies are preferentially located in rich groups, and blue galaxies in poor groups and among those galaxies not in any group. In the supercluster SCl 9 early- and late-type and red and blue galaxies reside together in groups of different richness.

This is in a good accordance with the results about the fine structure of superclusters delineated by galaxies from different populations obtained using the fourth Minkowski functional $F_3$.

In both superclusters, those galaxies classified as late type but having red colors, as well as blue galaxies with early-type spectra, are mostly located in poor groups or they do not belong to any group; only about 10% of them are located in rich groups.

Another important difference between these superclusters is that in the supercluster SCl 126 the brightest galaxies reside in rich groups while in the supercluster SCl 9 the luminosity of the brightest galaxies in rich and poor groups is the same.

In summary, the high-density cores of the superclusters contain relatively more early-type, quiescent, red galaxies than the lower density regions, where there are more blue, star-forming galaxies. In the core regions the fraction of galaxies in rich groups is larger than this fraction in the outskirts. This shows that both the local (group/cluster) and the global (supercluster) environments...
are important in influencing types, colors, and star formation rates of galaxies.

Earlier studies (Davis & Geller 1976; Dressler 1980; Einasto & Einasto 1987; Phillipps et al. 1998; Norberg et al. 2001, 2002; Zehavi et al. 2002; Goto et al. 2003; Hogg et al. 2003, 2004; Balogh et al. 2004; De Propris et al. 2003, 2004; Madgwick et al. 2003b; Croton et al. 2005; Blanton et al. 2005, 2006, among others) have shown the difference between the galaxy populations in clusters and in the field. Our results show that this difference exists also between the core regions and the outer regions of rich superclusters (see also Paper III).

6. DISCUSSION

6.1. Selection Effects

Both rich superclusters, SCl 126 and SCl 9, are not fully covered by the survey volume of the 2dFGRS. However, in the case of the supercluster SCl 126 only its small, outer part remains outside the survey (see references in §2). All rich clusters in this supercluster are included. Thus, most probably, including the full supercluster may add a relatively small number of mainly blue galaxies to this supercluster; this would not change our results about the galaxy content and fine structure in the supercluster core region and about rich groups in the outskirts of the supercluster.

With the supercluster SCl 9, the situation is more complicated. The part that remains outside contains about half of the Abell clusters in this supercluster (§2). Therefore, including them would increase the maximum values of the fourth Minkowski functional, \( V_3 \), since this characteristic counts the number of isolated clumps in an object under study. Thus, the differences between the values of \( V_3 \) for superclusters SCl 126 and SCl 9 would even increase. Even in the present analysis we saw that the values of \( V_3 \) for blue galaxies for SCl 126 are larger than those values for SCl 126. Thus, even if in the remaining part of the supercluster blue galaxies were distributed homogeneously such as not to increase the present values of \( V_3 \) (not a very probable case!), the differences between the \( V_3 \) values for galaxies of different type would not disappear.

Similarly, the differences in the galaxy content of groups from the core region and the outskirts would not disappear unless the distribution and clumpiness of galaxies in the (excluded) part of the supercluster SCl 9 were completely different from what we see in its 2dFGRS part. Probably we should not expect to see large variations in the galaxy content of different parts of one supercluster. Moreover, in Paper III we showed that the scatter in galaxy populations of superclusters is small.

For a more detailed study we plan to analyze a larger number of rich superclusters. For that, we started to generate supercluster catalogs using the SDSS data. In particular, the region of the supercluster SCl 126 is covered by both surveys, therefore giving us a good chance to compare the properties of this supercluster, enclosed by both data sets.

6.2. Comparison with Other Very Rich Superclusters

The richest relatively nearby supercluster is the Shapley supercluster (Proust et al. 2006 and references therein; Bardelli et al. 2000; Quintana et al. 2000). The main core of this supercluster contains at least two Abell clusters and two X-ray groups. Haines et al. (2006b) studied the galaxy populations in the core region of this supercluster using data for galaxies with fainter absolute magnitude limit than used in our study; thus, we can compare their results with ours qualitatively only. They demonstrated that the colors of galaxies in the core region of the Shapley supercluster depend on their environment, with redder galaxies being located in clusters. They also found a large amount of faint blue galaxies between the clusters. We found the same trends with colors, especially in the core region of the supercluster SCl 126. Haines et al. (2006b) found also that in the core of the Shapley supercluster, where the fraction of blue galaxies is the lowest, the X-ray emission is the strongest. As seen above, we also find that in rich groups, which have X-ray sources, the fraction of blue galaxies is very small (the core region of SCl 126).

Porter & Raychaudhury (2005, 2007) studied recently another rich supercluster partly covered by the 2dF survey: the Pisces-Cetus supercluster (SCI 10 in E01). They used data for Abell clusters, galaxy groups (by Eke et al. 2004), and galaxies in this supercluster. Porter & Raychaudhury (2005, 2007) used the spectral parameter \( \eta \) to determine the star formation rates for galaxies in groups in this supercluster. They concluded that galaxies in rich clusters have lower star formation rates than galaxies in poor groups, in agreement with our results. Porter & Raychaudhury (2005, 2007) also demonstrated that in the filament between the clusters in this supercluster the fraction of star-forming galaxies is higher at larger distance from clusters than close to clusters.

Gray et al. (2004) studied the environment of galaxies of different colors in the supercluster A901/902. They divided galaxies into red (quiescent) and blue (star-forming) populations, using \((U - V)\) colors of galaxies brighter than \(M_v \approx -14\). Gray et al. (2004) obtained strong evidence that the highest density regions in clusters are populated mostly with red, quiescent galaxies, while blue, star-forming galaxies dominate in outer/lower density regions of clusters. Our samples do not contain very faint galaxies; for brighter galaxies, we find qualitatively the same trends in our superclusters.

Wolff et al. (2005) showed that in the supercluster A901/902 there exists a population of dusty red galaxies that show red colors and also signatures of star formation. In this supercluster these galaxies are located at intermediate densities. This agrees well with our finding about galaxies with mixed classification (galaxies with red colors and spectra typical of late-type galaxies): these galaxies populate mostly intermediate-density regions of superclusters, being members of poor groups or not belonging to any group.

Hilton et al. (2005) find that the fraction of early spectral type galaxies is significantly higher in clusters with a high X-ray flux. Several of the X-ray clusters studied by Hilton et al. (2005) belong to the core region of SCl 126 (A1650, A1651, A1663, and A1750), where we also found a high fraction of early-type galaxies. One of the clusters under study by Hilton et al. (2005) is located in the supercluster SCl 9 (A2811).

Balogh et al. (2004) compared the populations of star-forming and quiescent galaxies in groups from 2dFGRS and SDSS using the strength of the \( H\alpha \) emission line as a star formation indicator. The spectral parameter \( \eta \) is correlated with the equivalent width of the \( H\alpha \) emission line: approximately, early-type galaxies correspond to quiescent galaxies and late-type galaxies to star-forming galaxies. Balogh et al. (2004) found that the relative number of star-forming galaxies in groups with high velocity dispersion is lower than in groups with low velocity dispersion. We found that rich groups contain a smaller fraction of late-type galaxies than poor groups, qualitatively in accordance with Balogh et al. (2004).

Plionis (2004) showed that the dynamical status of groups and clusters of galaxies depends on their large-scale environment. Here we show that also the richness of a group and its galaxy content depend on the large-scale environment. The first of these effects was described as an environmental enhancement of group richness in Einasto et al. (2003a, 2005).
Thus, our present data reveal the dependency of the properties of galaxies in superclusters on both the local density, in groups (as shown also by the 2DF team; e.g., Balogh et al. 2004 and references therein), and on the global density (see also Paper III) in the supercluster environment. Moreover, we showed that the fine structure of superclusters, as determined by galaxies from different populations, differs.

The differences in the distribution of galaxies from different populations in individual superclusters are related to a different overall morphology of these superclusters.

6.3. Morphology of Superclusters and Their Formation and Evolution

We showed that there exist several differences between the morphologies of galaxy populations of individual rich superclusters, which cannot be explained by selection effects only.

The supercluster SCI 126 has a very high density core with several Abell and X-ray clusters in a region with dimensions less than \(10 \, h^{-1} \text{Mpc}\). Such very high densities of galaxies have been observed so far only in very few superclusters. Among them are the Shapley supercluster (Bardelli et al. 2000), the Aquarius supercluster (Caretta et al. 2002), and the Corona Borealis supercluster (Small et al. 1998). A very small number of such high-density cores of superclusters is consistent with the results of \(N\)-body simulations, which show that such high-density regions (the cores of superclusters that may have started the collapse very early) are rare (Gramann & Suhhonenko 2002). The fraction of early-type, red galaxies in the core region of the supercluster SCI 126 is very high (higher than in SCI 9), both in groups and among those galaxies that do not belong to groups.

This, together with the presence of a high-density core and overall more homogeneous structure than in the supercluster SCI 9, may be an indication that the supercluster SCI 126 has formed earlier (is more evolved dynamically by the present epoch) than the supercluster SCI 9.

However, in another part of the core region of the supercluster SCI 126 the X-ray cluster A1750 shows signs of merging, according to both X-ray and optical data (Donelly et al. 2001; Belsole et al. 2004).

Donelly et al. (2001) studied this cluster using observational data from ROSAT and ASCA, as well as the velocities of galaxies in this cluster. Their analysis suggests that we observe an on-going merger in this binary cluster.

Additional evidence about complex merger events in this cluster was provided by Belsole et al. (2004), who used XMM-Newton data to study in detail the surface brightness, temperature, and entropy distribution in this cluster. Their data indicate that two components of this binary cluster have just started to interact. In addition, their data suggest that there occurred another merging event in one of the clusters, perhaps in the past 1–2 Gyr. They relate these merging events also to the large-scale environment of this cluster (it is a member of a very rich supercluster, SCI 126). In summary, these studies suggest that the region around the rich cluster A1750 in the supercluster SCI 126 may be dynamically younger than the main core region.

This all shows that the formation and evolution history of superclusters is a complex subject. We can try to model it, but the best way is to follow their real evolution in time, looking for superclusters in deep surveys. An especially promising project is the ALHAMBRA Deep survey by Moles et al. (2005), which will provide us with data for (possible) galaxy systems at very high redshifts that can be searched for and analyzed using morphological methods. Comparing superclusters at different redshifts will clarify many questions about their evolution.

In order to have a larger sample of local supercluster templates, we plan to continue our studies of rich superclusters. We mentioned above that we plan to use for that data from the SDSS to extend the sample of relatively nearby superclusters.

In E07d we showed that the richest superclusters in the Millennium Simulation do not describe the large morphological variety of the observed superclusters. Moreover, Minkowski functionals show that the fine structure as delineated by bright and faint galaxies in Millennium Simulations does not follow the fine structure delineated by galaxies of different luminosities in observed superclusters. This suggests that the model does not yet explain all the features of observed superclusters.

Earlier studies of the Minkowski functional of the whole SDSS region (Gott et al. 2008 and references therein) also conclude that \(N\)-body simulations with very large volume and more power at large scales are needed to model structures like the supercluster SCI 126 more accurately than present simulations. Similar conclusions were reached by Einasto et al. (2006). Also the galaxy formation and its environmental dependence are not yet well understood. Our present results indicate that the properties of galaxies and their evolution history have been affected by both local and global densities in superclusters. The details of these processes have to be modeled in future simulations.

7. CONCLUSIONS

We have presented a morphological study of the two richest superclusters from the 2dfGRS. We studied the internal structure and galaxy populations of these superclusters. Our main conclusions are the following:

1. The values of the fourth Minkowski functional \(V_3\), which contain information about both the local and global morphology, show the fine structure of superclusters as determined by galaxies from different populations. The fourth Minkowski functional \(V_3\) and the morphological signature \(K_1-K_2\) show a crossover from low-density morphology (outskirts of supercluster) to high-density morphology (core of supercluster) at a mass fraction \(m_f \approx 0.7\).

2. In the supercluster SCI 126, the functional \(V_3\) shows that the number of clumps in the distribution of red galaxies is larger than the number of clumps determined by blue galaxies, especially in the outer regions of the supercluster, where the maximum values of \(V_3\) are, correspondingly, 8 for red galaxies and 5 for blue galaxies. Thus, in the outskirts of the supercluster, blue galaxies form a more homogeneous population than red galaxies. In the core region, the clumpiness of galaxy populations is of the same order.

3. In the supercluster SCI 9, the values of \(V_3\) are large for both early-type, red galaxies and late-type, blue galaxies. In the outskirts of the supercluster the differences between the clumpiness of galaxy populations are small (the values of \(V_3\) are the same, 7–8 for both early- and late-type galaxies), while in the core region of this supercluster the clumpiness of the late-type, blue galaxy population is larger (the maximum value of \(V_3\) is 17) than the clumpiness of the early-type, red galaxy population, where the maximum \(V_3\) is 12.

4. Our superclusters contain galaxies with mixed classifications: these galaxies show spectra typical of late-type galaxies and red colors, or they are early-type galaxies with blue colors. These galaxies are mostly located at intermediate densities in the outskirts of our superclusters, being members of poor groups or not belonging to any group. Due to these galaxies, at intermediate mass fractions the curves of the fourth Minkowski
functional $V_j$ for early-type galaxies and for red galaxies, as well as for late-type galaxies and blue galaxies, differ in some
details.

5. Groups in high-density cores of superclusters are richer
than in lower density (outer) regions of superclusters. In high-
density cores, groups contain relatively more early-type, red gal-
axies than the groups in lower density regions in superclusters,
where there are more late-type, blue galaxies. Therefore, both the
richness of a group and its galaxy content depend on the large-
scale environment where it resides.

6. In cores of superclusters, the fractions of early-type, red
galaxies are larger than these fractions in the outskirts. In the
supercluster SCI 126 the morphological segregation of red and
blue galaxies is stronger than in the supercluster SCI 9. In SCI 126,
the most luminous galaxies in rich groups have larger luminosities
than most luminous galaxies in poor groups, while in SCI 9 the
luminosities of the brightest galaxies in rich and poor groups are
comparable.

7. The differences in overall morphology, fine structure, and
galaxy content of the supercluster suggest that there are differ-
ences in their evolutionary history that affect their present-day
properties.

Our study shows the importance of the role of superclusters
as a high-density environment that affects the evolution and the
present-day properties of their member galaxies and the groups/
clusters of galaxies that constitute the supercluster.

The forthcoming Planck satellite observations will determine
the anisotropy of the cosmic background radiation with unpre-
cedented accuracy and angular resolution. As a by-product, Planck
measurements will provide an all-sky survey of massive clusters
via the Sunyaev-Zeldovich (SZ) effect. For the Planck project,
detailed information of supercluster properties is important, help-
ing to correlate the SZ signals with the imprints of local super-
clusters. As a continuation of the present work, we are preparing
supercluster catalogs for the Planck community.

We thank the anonymous referee for careful reading of the
manuscript and for many useful comments that helped to
improve the paper. We are pleased to thank the 2dFGRS Team for
the publicly available data releases. The present study was sup-
pported by the Estonian Science Foundation grants 6104 and
7146 and by the Estonian Ministry for Education and Science
research project TO 0060058S98. This work has also been sup-
pported by the University of Valencia through a visiting professor-
ship for Enn Saar and by the Spanish MEC project AYA2006-14056
(including FEDER). J. E. thanks Astrophysikalisches Institut
Potsdam (using DFG-grant 436 EST 17/4/06) and the Aspen
Center for Physics for hospitality, where part of this study was
performed. P. H. and P. N. were supported by Planck science in
Metsähovi, Academy of Finland. In this paper we have used R, a
language for data analysis and graphics (Ihaka & Gentleman 1996).

APPENDIX A

ESTIMATING PROBABILITY DENSITIES

As said in the main text, all the distributions (probability densities) shown in this paper have been obtained using the R environment
(Ihaka & Gentleman 1996; see footnote 9). We do not show the customary error limits in our figures; here we explain why.

Figure 12 shows the differential luminosity function histogram with Poisson error limits. This figure shows, first, that Poisson errors
are small and, second, that these errors are not very useful since they are defined mainly by the bin widths: the histograms for the
different bin widths clearly do not coincide within the formal error limits and, therefore, cannot represent the true density distribution.
As known for long in statistics (see, e.g., a good pedagogical presentation by Wand & Jones 1995), the most important part in prob-
ability density estimation is the right choice of the bin (kernel) width, and the “density” command in the stats package does that, min-
imizing the MISE (mean integral standard error) of the estimate. Also, it is long known that kernel estimates

$$f(x) = \frac{1}{N} \sum_{i=1}^{N} K(x - x_i; h),$$

where the data are $\{x_i\}, \ i \in [1, N]$ and $K(x; h)$ are suitable kernels of a width $h$, are preferred to binning. Both these estimates depend
on the bin (kernel) widths that can be found by optimizing the MISE, but histograms depend, additionally, on the placing of the bins.
Also, the minimum MISE in case of binning is larger than for kernel estimates.

The MISE is defined as

$$\text{MISE} = E \int \left[ \hat{f}(x) - f(x) \right]^2 \, dx$$

$$= \int \text{Var}(\hat{f}(x)) \, dx + \int \left[ \text{Bias}(\hat{f}(x)) \right]^2 \, dx. \quad (A1)$$

The two terms in the last equality depend in a different way on the kernel width (bandwidth). When we increase this width, the variance decreases, but the bias increases; this is what happens when we take wider bins for Figure 12. The stats package chooses the optimal bandwidth as that which minimizes the MISE. We could also try to minimize the local mean standard error MSE$(x)$, but this asks for adaptive density estimation (using bandwidths that depend on $x$), and the procedure is much more complex.

For a kernel estimate, when binning is replaced by convolution with a suitable kernel, the minimum MISE is of the order of $n^{-4/5}$,
where $n$ is the size of the sample, and the optimal kernel width is of the order of $\hat{\sigma}^{-1/2}$, where $\hat{\sigma}$ is the estimate of the rms deviation of
the data (see, e.g., Silverman 1986). The expression “of the order of” can be mostly read as “equal to,” as the proportionality coefficients
are usually close to unity. This kernel width means that there are about \( n \times n^{-1/5} = n^{4/5} \) points per kernel: we get our accustomed Poissonian error, but only for the optimal kernel width. However, this is only a rough estimate, as the local error, the mean standard error \( \text{MSE}(x) \), depends on the true density, being larger in the regions where this density has minima or maxima, and is difficult to estimate. Thus, the stats package does not provide the MSE, and we do not show error bars in our figures.

You can compare histograms and kernel densities in Figure 13. Here we generated a random sample of 1000 values for a weighted sum of two normal distributions, calculated its histogram, and applied the R procedure to obtain the optimized kernel density. The optimal kernel width is 0.36, 3 times smaller than the bin width; however, the kernel density follows well the smooth true density and recovers the details much better than the histogram does.

As the total number of galaxies in our superclusters is large, the differences between the estimated and true densities are small. These differences can be easily estimated. Let us take SCl 9 as an example (SCl 126 has more galaxies, and the errors are slightly smaller). The total number of galaxies in SCl 9 is 1176, so the MISE for the distributions involving the whole supercluster is \( \approx 0.0035 \), and about 0.006 for separate populations (the corresponding rms errors are 0.06 and 0.08). The local errors (MSE) are, on average, a few times smaller (by the ratio of the total range of a random variable to \( \hat{\sigma} \)). Our use of a constant width kernel does not allow us to better estimate the local errors, but we can always strictly compare densities, using the full data, i.e., the integral distributions (e.g., the Kolmogorov-Smirnov test).

![Figure 12](image12.png)

Fig. 12.—Differential luminosity function histograms \( F = dN/dM \), where \( M \) is the absolute magnitude of a galaxy, for galaxies in the supercluster SCl 126; here the solid line shows the luminosity function and the dashed lines indicate the Poisson errors. Black lines show the histogram for the 0.5 mag bins, and gray lines show the histogram for the 0.25 mag bins.

![Figure 13](image13.png)

Fig. 13.—Comparison of the histogram and kernel density estimates. The true density is shown by a thin line, and the kernel estimate obtained by the R stats package for a sample of size 1000 is shown by a thick line. For comparison, we show a histogram (obtained also by R). It is clearly seen that the histogram is inferior to the kernel estimate.
APPENDIX B

SPATIAL DENSITIES

When studying the morphology of superclusters of galaxies, a necessary step is to convert the spatial positions of galaxies into spatial densities. The standard approach is to use kernel densities (see, e.g., Silverman 1986):

\[ \varrho(x) = \sum_i K(x - x_i; h)m_i, \]

where the sum is over all galaxies, \( x_i \) are the coordinates of the \( i \)th galaxy, and \( m_i \) is its mass (or luminosity, if we are estimating luminosity densities; for number densities we set \( m_i = 1 \)). The function \( K(x; h) \) is the kernel of the width \( h \), and a suitable choice of the kernel determines the quality of the density estimate. If the kernel widths depend either on the coordinate \( x \) or on the position of the \( i \)th galaxy, the densities are adaptive.

Kernels have to be normalized and symmetrical:

\[ \int K(x; h) \, dV = 1, \quad \int xK(x; h) \, dV = 0. \]

For the usual case, when densities are calculated for a spatial grid, good kernels are generated by box splines \( B_f \) (usually used in \( N \)-body mass assignment). Box splines have compact support (they are local), and they are interpolating on a grid:

\[ \sum_i B_f(x - i) = 1, \]

for any \( x \), and a small number of indices that give nonzero values for \( B_f(x) \). In this paper we use the popular \( B_3 \) spline

\[ B_3(x) = \frac{1}{12} \left( |x - 2|^3 - 4|x - 1|^3 + 6|x|^3 - 4|x + 1|^3 + |x + 2|^3 \right) \]

(this function is different from zero only in the interval \( x \in [-2, 2] \)). We define the (one-dimensional) \( B_3 \) box spline kernel of width \( h = N \) as

\[ K^{(1)}_B(x; N) = B_3(x/N)/N. \]

This kernel preserves the interpolation property (mass conservation) for all kernel widths that are integer multiples of the grid step, \( h = N \). The 3D \( K^{(3)}_B \) box spline kernel we use is given by the direct product of three one-dimensional kernels:

\[ K_B(x; N) \equiv K^{(3)}_B(x; N) = K^{(1)}_B(x; N)K^{(1)}_B(y; N)K^{(1)}_B(z; N), \]

where \( x \equiv \{x, y, z\} \). Although it is a direct product, it is isotropic to a good degree (to a few percent in the outer regions).

As seen before, the best (optimal) kernel width is usually determined by minimizing the MISE; in our case this is about \( 6 \pm h \) Mpc\(^{-1}\). We have defined (see E07d) superclusters as density enhancements of a common scale of 8 \( h \) Mpc\(^{-1}\), so we use this value.

APPENDIX C

MINKOWSKI FUNCTIONALS AND SHAPEFINDERS

Consider an excursion set \( F_{\phi_0} \) of a field \( \phi(x) \) [the set of all points where density is larger than a given limit, \( \phi(x) \geq \phi_0 \)]. Then, the first Minkowski functional (the volume functional) is the volume of this region (the excursion set):

\[ V_0(\phi_0) = \int_{F_{\phi_0}} \, d^3x. \quad (C1) \]

The second Minkowski functional is proportional to the surface area of the boundary \( \delta F_{\phi_0} \) of the excursion set:

\[ V_1(\phi_0) = \frac{1}{6} \int_{\delta F_{\phi_0}} \, dS(x) \quad (C2) \]

(but not the area itself; notice the constant). The third Minkowski functional is proportional to the integrated mean curvature of the boundary:

\[ V_2(\phi_0) = \frac{1}{6\pi} \int_{\delta F_{\phi_0}} \left[ \frac{1}{R_1(x)} + \frac{1}{R_2(x)} \right] \, dS(x), \quad (C3) \]
where \( R_1(x) \) and \( R_2(x) \) are the principal radii of curvature of the boundary. The fourth Minkowski functional is proportional to the integrated Gaussian curvature (the Euler characteristic) of the boundary:

\[
V_3(\phi_0) = \frac{1}{4\pi} \int_{\partial F_{\phi_0}} \frac{1}{R_1(x)R_2(x)} \, dS(x). \tag{C4}
\]

At high (low) densities this functional gives us the number of isolated clumps (voids) in the sample (Martínez et al. 2005; Saar et al. 2007).

As the argument labeling the isodensity surfaces, we chose the (excluded) mass fraction \( m_f \), the ratio of the mass in regions with density lower than the density at the surface to the total mass of the supercluster. When this ratio runs from 0 to 1, the isosurfaces move from the outer limiting boundary into the center of the supercluster, i.e., the fraction \( m_f = 0 \) corresponds to the whole supercluster and \( m_f = 1 \) to its highest density peak.

We use directly only the fourth Minkowski functional in this paper; the other functionals are used to calculate the shapefinders (Sahni et al. 1998; Shandarin et al. 2004). The shapefinders are defined as a set of combinations of Minkowski functionals: \( H_1 = 3V/S \) (thickness), \( H_2 = S/C \) (width), and \( H_3 = C/4\pi \) (length). The shapefinders have dimensions of length and are normalized to give \( H_1 = R \) for a sphere of radius \( R \). For a convex surface, the shapefinders \( H_i \) follow the inequalities \( H_1 \leq H_2 \leq H_3 \). Oblate ellipsoids (pancakes) are characterized by \( H_1 \ll H_2 \approx H_3 \), while prolate ellipsoids (filaments) are described by \( H_1 \approx H_2 \ll H_3 \).

Sahni et al. (1998) also defined two dimensionless shapefinders \( K_1 \) (planarity) and \( K_2 \) (filamentarity): \( K_1 = (H_2 - H_1)/(H_2 + H_1) \) and \( K_2 = (H_3 - H_2)/(H_3 + H_2) \).

In the \((K_1, K_2)\)-plane filaments are located near the \( K_2 \)-axis, pancakes near the \( K_1 \)-axis, and ribbons along the diagonal, connecting the spheres at the origin with the ideal ribbon at \((1, 1)\).

**C1. ALGORITHMS**

Different algorithms are used to calculate the Minkowski functionals; here we use a simple grid-based algorithm, based on integral geometry (Crofton’s intersection formula), proposed by Schmalzing & Buchert (1997).

To start with, we find the density thresholds for given filling fractions \( f \) by sorting the grid densities. This is a common step for all grid-based algorithms. Vertices with higher densities than the threshold form the excursion set. This set is characterized by its basic sets of different dimensions: points (vertexes), edges formed by two neighboring points, squares (faces) formed by four edges, and cubes formed by six faces. The algorithm counts the numbers of elements of all basic sets and finds the values of the Minkowski functionals as

\[
\begin{align*}
V_0 &= a^2 N_0, \\
V_1 &= a^2 \left( \frac{2}{3} N_2 - \frac{4}{3} N_3 \right), \\
V_2 &= a \left( \frac{2}{3} N_1 - \frac{4}{9} N_2 + \frac{2}{9} N_3 \right), \\
V_3 &= N_0 - N_1 + N_2 - N_3,
\end{align*}
\]

where \( a \) is the grid step, \( N_0 \) is the number of vertexes, \( N_1 \) is the number of edges, \( N_2 \) is the number of squares (faces), and \( N_3 \) is the number of basic cubes in the excursion set.

**APPENDIX D**

**CLUMPINESS: MORPHOLOGICAL TEMPLATES**

In E07d we generated a series of empirical models that served as morphological templates to understand the behavior of shapefinders. These models showed that the morphological signature of rich superclusters corresponds to a multibranched filament; the simplest model for that was a long filament with short filaments across it. In E07d we did not study the fine structure of superclusters as expressed by their clumpiness. In these models the locations of points that mimic positions of galaxies were generated randomly. Thus, our empirical models in that paper do not recover completely the inner structure of superclusters.

For this appendix, we generated a series of empirical models to understand better the substructure of superclusters. These models will also help us to determine how well our methods distinguish different types of substructure.

In these models, the overall distribution of points (which mimic individual galaxies) resembles a thin filament with a size \( 10 \times 20 \times 100 \) (in grid units), as in E07d. We place inside this filament a series of clusters, located randomly. Their richness, number, and size vary so that the total number of objects is always 1000 or 500 (approximately the number of galaxies in superclusters under study and in individual galaxy populations). We used the richness values 5, 10, and 20; the number of clusters is, correspondingly, 100, 50, and 25. The size of clusters was 1, 5, and 10 in grid units. Small clusters mimic real groups and clusters, while large clusters mimic overdensity regions. Clusters are located randomly and may overlap, forming additional overdensity regions, as, for example, in cores of superclusters, where also galaxy populations are mixed.

We plot the Euler characteristics and morphological signatures for these models in Figure 14.

The models used in this figure are as follows. In the first set the number of clusters is 25, and each cluster has 20 galaxies in it. The sizes of clusters are 1 and 5. In the third model in this set, these clusters are combined together. In the latter model clusters have the same center coordinates. Thus, the model CL125525 (the combined models CL125 and CL525) mimics the distribution where groups are surrounded by lower density galaxies (e.g., ellipticals by spirals).
In the second set the number of clusters is 50, with 10 member galaxies, and in the combined model, cluster centers again coincide (the models CL150, CL550, and CL150550). In the third set (CL125150 and CL525150) we added to the models with 25 clusters (sizes 1 and 5) the model CL150, to have a smaller number of rich groups and a large number of poor groups, but in this case the centers of groups do not coincide. These models mimic the situation where different populations of galaxies are located in clumps of different richness and size, but in separate systems (partly, since clumps may overlap randomly). We plot here also the curves for a simple filament for comparison.

We do not show in this figure models with 100 small clusters and those with cluster size 10; in all these models the values of $V_3$ were smaller than in real superclusters. In the case of large clusters the value of $K_2$ becomes too large.

Figure 14 shows that, first, the morphological signature is recovered correctly by our empirical models. Second, the $V_3(m_f)$ curves for different models are different, showing that they discriminate well different substructure.

These models show that the supercluster SCl 126 is better modeled by a small number of richer/bigger clumps, and the supercluster SCl 9 is better modeled by a large number of smaller clumps. Clusters overlap, thus the maximum values of $V_3$ are smaller than the original numbers of clusters in the models. The case when any population of galaxies is located randomly within the supercluster volume does not correspond to the real distribution of galaxies. Of course, our very simple models in which clusters of one galaxy population are surrounded by low-density “clouds” of galaxies from another population do not recover well the details of clumpiness of different galaxy populations of observed superclusters. In real superclusters, populations of galaxies are more strongly mixed.

REFERENCES

Balogh, M., et al. 2004, MNRAS, 348, 1355
Bardelli, S., Zucca, E., Zamorani, G., Moscardini, L., & Scarlama, R. 2000, MNRAS, 312, 540
Basilakos, S. 2003, MNRAS, 344, 602
Basilakos, S., Plionis, M., & Rowan-Robinson, M. 2001, MNRAS, 323, 47
Basilakos, S., Plionis, M., Yepes, G., Gottlöber, S., & Turchaninov, V. 2006, MNRAS, 365, 539
Belske, E., Pratt, G. W., Sauvageot, J.-L., & Bourdin, H. 2004, A&A, 415, 821
Blanton, M. R., Eisenstein, D., Hogg, D. W., Schlegel, D. J., & Brinkmann, J. 2005, ApJ, 629, 143
Blanton, M. R., Eisenstein, D., Hogg, D. W., & Zehavi, I. 2006, ApJ, 645, 977
Caretta, C. A., Maia, A. G., Kawasaki, W., & Willmer, C. N. A. 2002, AJ, 123, 1200
Cole, S., et al. 2005, MNRAS, 362, 505
Colless, M. M., et al. 2001, MNRAS, 328, 1039
——. 2003, preprint (astro-ph/0306581)
Cross, N. J. G., et al. 2001, MNRAS, 324, 825
Croton, D. J., et al. 2004, MNRAS, 352, 1232
——. 2005, MNRAS, 356, 1155
Davis, M., & Geller, M. J. 1976, ApJ, 208, 13
De Propris, R., et al. 2003, MNRAS, 342, 725
——. 2004, MNRAS, 351, 125
Donelly, R. H., Forman, W., Jones, C., Quinlan, H., Ramirez, A., Churazov, E., & Gilfanov, M. 2001, ApJ, 562, 254
Dressler, A. 1980, ApJ, 236, 351
Einasto, J., Tago, E., Einasto, M., Saar, E., Suhhonenko, I., Hütta, P., & Heinämäki, P. 2005, A&A, 439, 45
Einasto, J., et al. 2006, A&A, 459, L1
——. 2007a, A&A, 462, 397 (Paper II)
——. 2007b, A&A, 462, 811 (Paper I)
Einasto, M. 1991, MNRAS, 252, 261
Einasto, M., & Einasto, J. 1987, MNRAS, 226, 543

Fig. 14.—Euler characteristic (top panels) and the morphological signature $K_1$-$K_2$ (bottom panels) for the empirical models. Left panels: $N_{cl} = 25, N_{gal} = 20$, and the size of the clusters is 1 and 5 (in grid units), and the combined model where the centers of clusters coincide. Solid line: model CL125; dashed line: model CL525; dot-dashed line: model CL125525. Middle panels: $N_{cl} = 50, N_{gal} = 10$, and the size of clusters is 1 and 5 (in grid units), and the combined model where the centers of clusters coincide. Solid line: model CL150; dashed line: model CL550; dot-dashed line: model CL150550. Right panels: Combined models where the centers do not coincide. “Fil” denotes a simple filament with randomly distributed points. Solid line: model CL125150; dashed line: model CL525150; dot-dashed line: model “Fil.”
