Some comments on calculations of the scalar radius of the pion and the chiral constant $\bar{l}_4$

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Abstract

The pion scalar radius is given by $\langle r_S^2 \rangle = (6/\pi) \int_{4M^2}^{\infty} dt \frac{\delta_S(t)}{t^2}$, with $\delta_S$ the phase of the scalar form factor. Below $K\bar{K}$ threshold, $\delta_S = \delta_0$, $\delta_0$ being the isoscalar, S-wave $\pi\pi$ phase shift. Between $K\bar{K}$ threshold and $t^{1/2} \sim 1.5$ GeV I argued, in two previous letters, that one can approximate $\delta_S \sim \delta_0$, because inelasticity is small, compared with the errors. This gives $\langle r_S^2 \rangle = 0.75 \pm 0.07$ fm$^2$ and the value $\bar{l}_4 = 5.4 \pm 0.5$ for the one-loop chiral perturbation theory constant, compared with the values given by Leutwyler and collaborators, $\langle r_S^2 \rangle = 0.61 \pm 0.04$ fm$^2$ and $\bar{l}_4 = 4.4 \pm 0.3$. At high energy, $t^{1/2} > 1.5$ GeV, I remarked that the value of $\delta_S$ that follows from perturbative QCD agrees with my interpolation and disagrees with that of Leutwyler and collaborators. In a recent article, Caprini, Colangelo and Leutwyler claim that my estimate of the asymptotic phase $\delta_S$ is incorrect as it neglects higher twist contributions. Here I remark that, when correctly calculated, higher twist contributions are likely negligible. I also show that chiral perturbation theory gives $\bar{l}_4 = 6.60 \pm 0.43$, compatible with my estimate but widely off the value $\bar{l}_4 = 4.4 \pm 0.3$ of Leutwyler and collaborators.
1. Introduction.

I will be concerned here mainly with four papers: they will be denoted by Y1,[1] Y2,[2] EY[3] and CCL.[4] \( F_V(t) \), resp., \( F_S(t) \) will be the vector and scalar form factors of the pion; \( \langle r^2_P \rangle \) and \( \langle r^2_S \rangle \) the respective square radii. \( \delta_V(t) \) will be the phase of \( F_V(t) \) and \( \delta_S(t) \) the phase of \( F_S(t) \). Finally, \( \delta_1(t) \), \( \delta_0(t) \) are the P and S0 phase shifts for \( \pi \pi \) scattering. The pion scalar radius is given by

\[
\langle r^2_S \rangle = \frac{(6/\pi)}{4m^2} \int_{4M^2}^{\infty} dt \delta_S(t)/t^2.
\]

For the vector radius one has a similar formula,

\[
\langle r^2_V \rangle = \frac{(6/\pi)}{4m^2} \int_{4M^2}^{\infty} dt \delta_V(t)/t^2.
\]

Below \( \bar{K}K \) threshold, \( \delta_S(t) = \delta_0(t) \) and \( \delta_V(t) = \delta_1(t) \). In fact, this second equality probably holds as a good approximation up to \( t \sim 2 - 3 \text{ GeV}^2 \), where inelasticity begins to be important.

Between \( \bar{K}K \) threshold and \( t^{1/2} \sim 1.5 \text{ GeV} \) I argued, in Y1 and Y2, that one can approximate \( \delta_S \sim \delta_0 \), because inelasticity is small compared with the errors. Using the experimental value for \( \delta_0 \), this gives \( \langle r^2_S \rangle = 0.75 \pm 0.07 \text{ fm}^2 \) and the value \( \tilde{l}_4 = 5.4 \pm 0.5 \text{ for the one-loop chiral perturbation theory (ch.p.t.) constant \( \tilde{l}_4 \), to be compared with the values given by Leutwyler and collaborators:}^{[5,6]} \langle r^2_S \rangle = 0.61 \pm 0.04 \text{ fm}^2 \) and \( \tilde{l}_4 = 4.4 \pm 0.3 \).

I also argued in Y1 that at high energy, \( t > 2 \text{ GeV}^2 \), the value of \( \delta_S \) that follows from perturbative QCD agrees with my interpolation and disagrees with that of Leutwyler and collaborators. In a recent article, Caprini, Colangelo and Leutwyler (CCL[4]) claim that my estimate of the asymptotic phase \( \delta_S \) is incorrect as it neglects higher twist contributions. Here I remark that, when correctly calculated, higher twist contributions are very likely negligible. I also show that ch.p.t. gives the number \( \tilde{l}_4 = 6.6 \pm 0.4 \), compatible within reasonable two loop corrections with my estimate; but widely off the value \( \tilde{l}_4 = 4.4 \pm 0.3 \) of Leutwyler and collaborators.

2. The asymptotic \( F_V(t) \), \( F_S(t) \)

It is generally assumed that one can approximate, at high energy, \( F_V(t) \) by its asymptotic form at leading twist (l.t.) the expression for which, neglecting quark and pion masses, has been known for a long time:

\[
F_V(t) \underset{t \to \infty}{\sim} F_V^{(l.t.)}(t) = \frac{12\pi C_F f_\pi^2 \alpha_s(t)}{-t} = \frac{12\pi}{33 - 2n_f} \frac{12\pi C_F f_\pi^2}{-t \log(-t/A^2)}. \tag{1}
\]

(For the definition of leading twist and twist three contributions to the pion form factor, see ref. 3).

In 1983, Espriu and I calculated the twist three (t3) correction to this,[3] obtaining an infrared divergent result,

\[
F_V^{(t_3)}(t) \underset{t \to \infty}{\sim} \frac{f_\pi^2 m_\pi^2}{(m_u + m_d)^2} \frac{C}{-t \log(-t/M_0)} \log^{3+2d_m} t.
\]

(2)

the \( m \) are the invariant quark masses, \( m(t) = \frac{1}{2} \tilde{m} \log^{d_m} t/A^2 \), and \( M_0 \) is a cutoff mass (that may be taken equal to \( A \) or \( m_\pi \); it is irrelevant which, as the difference is subleading). We were not able to calculate the numerical constant \( C \); for the other terms, cf. EY, especially the expressions for \( B_0(\nu) \) and the first formula in the second column of p. 189 in ref. 3. So, the full form factor would be

\[
F_V(t) \underset{t \to \infty}{\sim} F_V^{(l.t.)}(t) + F_V^{(t_3)}(t) \underset{t \to \infty}{\sim} \frac{12\pi C_F f_\pi^2 \alpha_s(t)}{-t} + \frac{f_\pi^2 m_\pi^2}{(m_u + m_d)^2} \frac{C}{-t \log(-t/M_0)} \log^{3+2d_m} t \log(-t/M_0)^{3+2d_m}.
\]

(3)

the last because, at large \( t \), the (t3) contribution dominates the (l.t.) one. Note that the (t3) contribution also dominates the (l.t.) one in the chiral limit, for all \( t \).
In this case, (6) would dominate over (5), both at large $t$ as shown in Y2, Eq. (6.4). But, according to CCL, the $(t_3)$ contribution is due to the different pion wave functions involved, axial $\phi$ and in the chiral limit, and thus, in particular at large $t$, we would have

$$F_{t^3}^{(t)}(t) \sim \frac{\text{Const.}}{-t^{3/2}}$$  \hfill (4)

instead of (2), and the (l.t.) piece dominates: one recovers (1), for all of $F_V$.

In EY$^3$ we were not fully convinced by the Brodsky–Lepage arguments [although we quoted them, including the result in Eq. (4) in the present paper], because a fit to data with (1) was much worse than a fit with (2) or (3), as we remarked in EY. Thus, in EY we concluded that “one really cannot calculate the pion form factor in perturbative QCD.”

However, some time later, Isgur and Llewellyn Smith$^7$ considered the partonic wave function of the pion $\phi(\xi, t)$, for finite $t$. They concluded that the asymptotic limit is reached slowly and that, when reasonable $\phi(\xi, t)$ for finite $t$ are used in the (l.t.) expression for $F_V$, the corresponding (l.t.) expression fits data well, even at subasymptotic energies. [An important remark is that the phase of $F_V$ is not affected by the change in the wave function piece, which is real]. Because of this, in ref. 8 I used much milder words: I merely wrote that “There are, unfortunately, a number of snags” with the behavior (1), and proceeded to quote the EY, Brodsky–Lepage and Isgur and Llewellyn Smith$^7$ remarks, without drawing a definite conclusion.

For $F_S(t)$ the situation is very similar. To (l.t.) and leading order in perturbation theory,

$$F^{(l.t.)}_S(t) \sim \frac{576\pi^2}{33 - 2n_f} \frac{f_{V}^2(\hat{m}_u + \hat{m}_d)^2}{-t(\log(-t/A^2)^2d_m)}$$ \hfill (5)

as shown in Y2, Eq. (6.4). But, according to CCL, the $(t_3)$ contribution is$^1$

$$F^{(t_3)}_S(t) \sim \frac{48\pi^2 f_{V}^2M_\pi^2}{33 - 2n_f} \frac{\log(-t/M_0)}{-t}$$ \hfill (6)

In this case, (6) would dominate over (5), both at large $t$ and in the chiral limit, and thus, in particular at large $t$, we would have

$$F_S(t) \sim \frac{576\pi^2}{33 - 2n_f} \frac{f_{V}^2(\hat{m}_u + \hat{m}_d)^2}{-t(\log(-t/A^2)^2d_m)} + \frac{48\pi^2 f_{V}^2M_\pi^2}{33 - 2n_f} \frac{\log(-t/M_0)}{-t}$$ \hfill (7)

However, if one takes into account the Brodsky–Lepage remarks, the $(t_3)$ part should be calculated à la Brodsky–Lepage–Drell, would behave as $t^{-3/2}$, and hence would be negligible: one would recover the behaviour (5) given in Y1, for all of $F_S$. Note that the situation is identical for $F_V$ and $F_S$; the difference between (l.t.) and $(t_3)$ contribution is due to the different pion wave functions involved, axial $\phi$ or pseudoscalar, $\phi_P$, and these are the same for $F_V$ and $F_S$.

3. The asymptotic $\delta_V(t)$, $\delta_S(t)$

The phases of the form factors depend crucially on what happens to the $(t_3)$ contributions. If we believe the naive calculations in EY, or CCL, for the $F^{(t_3)}$, i.e., that the behaviour of the form factors is given by (3)

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$^1$ I am not sure of the correctness of the expression given in (6) for the $(t_3)$ piece; in EY we could not calculate analytically the twist-three wave function of the pion, $\phi_P$, hence we could not get the constant in Eq. (3), either. In fact we did not get constant asymptotic $\phi_P$ but found it growing like a power of $\log t$. I have the impression that CCL have forgotten that one has to diagonalize the anomalous dimension matrix, i.e., that you have to calculate what we call $T_{nm}$ in EY, which enters in the final result: see the detailed discussion in refs. 3, 8. However, since the conclusions are not altered by this, we will forget this problem here.
and (6), then,
\[ \delta_V(t) \approx \pi \left( 1 - \frac{1}{\log t/M_0} \right) [\text{with (3)}], \]
\[ \delta_S(t) \approx \pi \left( 1 - \frac{1}{\log t/M_0} \right) [\text{with (6)}], \]
both phases approach the asymptotic value \( \pi \) from below. However, if we accept the Brodsky–Lepage arguments, one gets instead dominant (l.t.) and thus
\[ \delta_V(t) \approx \pi \left( 1 + \frac{1}{\log t/M_0} \right) [\text{with (1)}], \]
\[ \delta_S(t) \approx \pi \left( 1 + 2d_m \frac{1}{\log t/M_0} \right) [\text{with (5)}], \]
and the value \( \pi \) is now approached from above.

The last results, (9), were what I used in Y1, Y2. In fact, and because (as one can easily imagine) I was aware of the (t3) threat, I started by checking, in Y1, the behaviour (9) for \( \delta_V(t) \): here one has an independent evaluation of \( \langle r^2 \rangle \), from the experimental pion form factor \( \delta \) with which to compare. Looking at pp. 102, 103 in Y1, and correcting for an error there (which alters nothing numerically; see the Erratum in Y1), we find the following result:
\[ \langle r^2 \rangle = \begin{cases} 0.434 \text{ fm}^2 : & \text{with (9)}; \\ 0.303 \text{ fm}^2 : & \text{with (8)}. \end{cases} \]
We have assumed the asymptotic region to begin at \( t = 3 \text{ GeV}^2 \) and taken \( \delta_V(t) \approx \delta_4(t) \) below this. Experimentally, \( \langle r^2 \rangle \) is \( 0.432 \pm 0.006 \) (the error here includes electromagnetic effects). The agreement of what one gets with (9) with experiment is very good; it only depends a little on the values of \( \Lambda \) or the matching point, that I took to be 1.7 GeV, provided we keep them reasonable. However, and this is more important, if using the naive (t3) result, given in Eq. (8), the value obtained is a disaster: below experiment by 30%. It is because of this that I took seriously, in Y1 (and in Y2) the Brodsky–Lepage arguments and hence the (l.t.) evaluation of the phase of the form factors, Eq. (9).

Nevertheless, I concede that the situation is not clear. While there exist a number of indications favouring the results of Y1, Y2 for \( \langle r^2 \rangle \), one cannot definitely exclude the results of the evaluations of the Leutwyler group, as already remarked in Y2. One may therefore summarize the situation as follows: First of all, the experimental data for \( \pi \pi \) scattering in the important region \( 2m_K \leq t^{1/2} \leq 1.5 \text{ GeV} \) are ambiguous. Different analyses produce mutually incompatible results, so the value for \( \langle r^2 \rangle \) depends on which data one uses. If choosing the central value in the fit of Hyams et al.\( ^{[1]} \) one gets \( \langle r^2 \rangle \) \( = 0.61 \pm 0.04 \text{ fm}^2 \) and hence \( \bar{S}_4 = 4.4 \pm 0.3 \) as in ref. 5. However, and as discussed in Y2, if one uses different sets of data (notably, taking into account information from \( \pi \pi \to K \bar{K} \) scattering) one finds a larger value, \( \langle r^2 \rangle = 0.75 \pm 0.07 \text{ fm}^2 \), leading to \( \bar{S}_4 = 5.4 \pm 0.5 \). It could be possible, in principle, to discriminate between both solutions by using the asymptotic value for the phase of \( F_S(t) \) that follows from perturbative QCD; the result, however, is infrared divergent and thus, as happens for the twist three contribution to the electromagnetic form factor of the pion, a cut-off becomes necessary and the result is ambiguous. Therefore, one should probably accept a conservative estimate, covering both evaluations,
\[ \bar{S}_4 = 5 \pm 1. \]

4. \( \bar{S}_4 \) in chiral perturbation theory

But this is not all. As I remarked in Y1, the number of low energy observables in \( \pi \pi \) scattering, to one loop in c.p.t., is such that one can determine the value of \( \bar{S}_4 \) by fitting them, even if leaving \( \bar{S}_4 \) free.

In the present note I elaborate on this. To do so, I will use the values for scattering lengths and effective range parameters given in Table I. In units with the charged pion mass \( M_\pi = 1 \), one has
These $a$s and $b$s can be described in terms of only four chiral coupling constants, $\bar{l}_i$, $i = 1, 2, 3, 4$. In fact we will include in the fits the estimate of Gasser and Leutwyler\cite{11} for $\bar{l}_3$ which gives

$$\bar{l}_3 = 3 \pm 2.5.$$ \hspace{1cm} (12)

This has little influence the other parameters, but imposing (12) avoids unphysical values for $\bar{l}_3$; otherwise the fits tend to give large negative values for this parameter, with very large errors.

We turn to the determination of $\bar{l}_4$ from ch.p.t. We will use the one loop expressions for $a$s and $b$s in terms of $\bar{l}_4$ given in ref. 11,

\[
\begin{align*}
a_0^{(0)} &= \frac{7M_\pi}{32\pi f_\pi^2} \left\{ 1 + \frac{5M_\pi^2}{84\pi^2 f_\pi^2} [\bar{l}_1 + 2\bar{l}_2 - \frac{3}{8}\bar{l}_3 + \frac{21}{10}\bar{l}_4 + \frac{21}{4}] \right\}, \\
a_0^{(2)} &= \frac{M_\pi}{16\pi f_\pi^2} \left\{ 1 - \frac{M_\pi^2}{12\pi^2 f_\pi^2} [\bar{l}_1 + 2\bar{l}_2 + \frac{3}{8}] + \frac{M_\pi^2}{32\pi^2 f_\pi^2} [\bar{l}_3 + 4\bar{l}_4] \right\}, \\
a_1^{(1)} &= \frac{1}{24\pi M_\pi f_\pi^2} \left\{ 1 - \frac{M_\pi^2}{12\pi^2 f_\pi^2} [\bar{l}_1 - \bar{l}_2 + \frac{65}{48}] + \frac{M_\pi^2}{8\pi^2 f_\pi^2} \bar{l}_4 \right\}; \\
b_0^{(0)} &= \frac{1}{4\pi M_\pi f_\pi^2} \left\{ 1 + \frac{M_\pi^2}{12\pi^2 f_\pi^2} [2\bar{l}_1 + 3\bar{l}_2 - \frac{13}{16}] + \frac{M_\pi^2}{8\pi^2 f_\pi^2} \bar{l}_4 \right\}, \\
b_0^{(2)} &= \frac{1}{8\pi M_\pi f_\pi^2} \left\{ 1 - \frac{M_\pi^2}{12\pi^2 f_\pi^2} [\bar{l}_1 + 3\bar{l}_2 - \frac{5}{16}] + \frac{M_\pi^2}{8\pi^2 f_\pi^2} \bar{l}_4 \right\}, \\
b_1^{(1)} &= \frac{1}{288\pi^3 M_\pi f_\pi^4} \left\{ -\bar{l}_1 + \bar{l}_2 - \frac{97}{120} \right\}.
\end{align*}
\]

(for the D waves, see below) and we will consider several fitting strategies:

i) We include in the fit the constraints (11, 12), as well as the experimental numbers (PY) for $a$s and $b$s given in Table I. Then, we get a $\chi^2$/d.o.f. = 8.8/(10 - 4) and the numbers

$$\bar{l}_1 = -1.30 \pm 0.23, \quad \bar{l}_2 = 6.04 \pm 0.07, \quad \bar{l}_3 = 2.2 \pm 2.5, \quad \bar{l}_4 = 6.32 \pm 0.61.$$ \hspace{1cm} (13a)

The clear excess of the $\chi^2$/d.o.f. over unity indicates that, at the level of accuracy of ref. 14, the two loop effects are not negligible.
ii) In fact, the errors in (13a) are purely nominal, as the fit depends on the way we have treated the higher order corrections. For example, if in the one loop expressions for \( a_2^{(l)} \), \( b_1^{(l)} \),

\[
a_2^{(0)} = \frac{1}{1440 \pi^3 M f_\pi} \left\{ \bar{l}_1 + 4 \bar{l}_2 - \frac{53}{3} \right\}, \\
a_2^{(2)} = \frac{1}{1440 \pi^3 M f_\pi} \left\{ \bar{l}_1 + \bar{l}_2 - \frac{103}{30} \right\}, \\
b_1^{(1)} = \frac{1}{288 \pi^3 M f_\pi^4} \left\{ -\bar{l}_1 + \bar{l}_2 - \frac{27}{120} \right\},
\]

we replace the physical \( f_\pi \) by its value in the chiral limit, \( f \),

\[
f_\pi = f \left\{ 1 + \frac{M^2}{16 \pi^2 f_\pi^4} \bar{l}_4 \right\},
\]

which is allowed since the difference is of higher order in ch.p.t., we find a \( \chi^2 / \text{d.o.f.} = 9.7 / (10 - 4) \) and the central values

\[
\bar{l}_1 = -0.33, \quad \bar{l}_2 = 4.56, \quad \bar{l}_3 = 2.1, \quad \bar{l}_4 = 6.93.
\]

\( \bar{l}_1 \) and \( \bar{l}_2 \) are well outside the error bars given in (13a).

iii) We can get safer error estimates, which take into account at least some of the uncertainty due to higher order effects, by averaging (13a) and (13b), weighted with the respective \( \chi^2 / \text{d.o.f.} \), and enlarging the error by including, as an extra error, the difference between the result of this operation and (13a). In this way we find an estimate that covers both (13a) and (13b):

\[
\bar{l}_1 = -0.84 \pm 0.51, \quad \bar{l}_2 = 5.34 \pm 0.70, \quad \bar{l}_3 = 2.2 \pm 2.5, \quad \bar{l}_4 = 6.61 \pm 0.68.
\]

iv) The two loop corrections to \( \pi \pi \) scattering have also been calculated.\(^{[15]}\) It turns out that the scattering amplitude depends on six combinations of one and two loop coupling constants. Including two loop effects it is possible to give a virtually perfect fit to experimental \( \pi \pi \) scattering, up to the F wave; but, unlike for the one loop case where eight scattering lengths and effective range parameters are fitted with only two \( \bar{l} \)'s, we now have six parameters for 14 or 16 observables (see below), so the agreement, although better, is less impressive than before. We find

\[
\bar{l}_1 = -0.80 \pm 0.21, \quad \bar{l}_2 = 5.5 \pm 0.35, \quad \bar{l}_3 = 1.9 \pm 2.4, \quad \bar{l}_4 = 6.60 \pm 0.43.
\]

The numbers in (13d) have been obtained as follows: one fits the two loop expressions for scattering lengths and effective range parameters to the data deduced from experiment in ref. 14, for all waves up to the F-wave. One also includes in the fit the constraint given by Eqs. (13c) for the \( \bar{l}_i \). We then find a \( \chi^2 / \text{d.o.f.} = 3.8 / (14 - 10) \), if not including the F-wave in the fit, and a \( \chi^2 / \text{d.o.f.} = 7.9 / (16 - 10) \) if including \( a_3^{(1)} \) and \( b_3^{(1)} \) in the fit. Averaging them and including their spread as an extra error (that can be attributed to three loop effects) we get (13d).

This result in (13d), \( \bar{l}_4 = 6.60 \pm 0.43 \), which is compatible with all others, is what we consider the more reliable one.

v) The previous determinations include the constraint (11) in the fits. We could also fit leaving \( \bar{l}_4 \) completely free. In this case we find

\[
\bar{l}_1 = -1.27 \pm 0.21, \quad \bar{l}_2 = 6.04 \pm 0.06, \quad \bar{l}_3 = 2.4 \pm 2.5, \quad \bar{l}_4 = 7.1 \pm 0.8,
\]

with only the experimental errors in Table I, which is compatible with (13). We would find results similar to (13b, c, d) if including estimated higher order effects: the errors for \( \bar{l}_4 \) are slightly larger than before, and the central value is also only slightly larger. This confirms the result in (13d) for \( \bar{l}_4 \).
vi) Finally, we may also replace in the fits the “Experimental” PY data with those of ACGL or ABT in Table I. With only the errors there, and leaving $\overline{l}_4$ free, we get

$$\begin{align*}
\overline{l}_1 &= -2.3 \pm 0.1, \\
\overline{l}_2 &= 6.20 \pm 0.05, \\
\overline{l}_3 &= 1.0 \pm 2.0, \\
\overline{l}_4 &= 6.6 \pm 0.3, \quad \text{[CGL]} \\
\overline{l}_1 &= -2.3 \pm 0.6, \\
\overline{l}_2 &= 6.0 \pm 0.3, \\
\overline{l}_3 &= 2.9 \pm 2.5, \\
\overline{l}_4 &= 5.4 \pm 1.4, \quad \text{[ACGL]} \\
\overline{l}_1 &= -1.32 \pm 0.55, \\
\overline{l}_2 &= 6.84 \pm 0.28, \\
\overline{l}_3 &= 1.1 \pm 1.9, \\
\overline{l}_4 &= 6.71 \pm 0.48. \quad \text{[ABT]}
\end{align*}$$

The numbers that follow for $\overline{l}_4$, for all the fits, support the estimate in Y1, Y2, $\overline{l}_4 = 5.4 \pm 0.5$, against that of Donoghue, Gasser and Leutwyler and Ananthanarayan et al., $\overline{l}_4 = 4.4 \pm 0.3$. It is particularly remarkable that the CGL evaluation of low energy parameters, which produces the parameters in the corresponding column in Table I, and which I used for the evaluation CGL in Eq. (15), was made imposing for $\overline{l}_4$ the value of the Leutwyler group, $\overline{l}_4 = 4.4 \pm 0.3$: requiring consistency of the low energy parameters, however, pushes $\overline{l}_4$ to a value compatible with mine in Y1, Y2 and definitely incompatible with the $\overline{l}_4 = 4.4 \pm 0.3$ input. In fact, and apart from the fit using the ACGL low energy parameters, whose large errors for $\overline{l}_4$ preclude drawing any clear conclusion from it, all other fits agree, in central value and error, with (13d). To get results for the value of $\overline{l}_4$ from the scalar radius of refs. 4, 5 consistent with what one gets from $\pi\pi$ scattering, one would require that the two loop corrections to the results of the Leutwyler group for $\langle r_S^2 \rangle$ be of 50% or more.

5. Reggeistics

Some comments are also made in ref. 4 about Reggeistics. I will only note here the surprising fact that, although a wealth of experimental data exist for high energy $\pi\pi$ cross sections[16] which were included in the fits in refs. 17, Caprini, Colangelo and Leutwyler[4] do not use them to derive what they call “our estimate” (see Fig. 2 in ref. 4) and they do not compare what follows from said estimate with the $\pi\pi$ data. The interested reader may find more detailed discussions of the Reggeistics of the Leutwyler group in refs. 14, 17, 18.

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2 The difference, $1.2 \pm 0.7$, i.e., a $20 \pm 16\%$ of the value following from the scalar form factor, is easily attributed to two loop effects in $\langle r_S^2 \rangle$. 
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