Extracting and Pre-Processing Event Logs

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Abstract. Event data is the basis for all process mining analysis. Most process mining techniques assume their input to be an event log. However, event data is rarely recorded in an event log format, but has to be extracted from raw data. Event log extraction itself is an act of modeling as the analyst has to consciously choose which features of the raw data are used for describing which behavior of which entities. Being aware of these choices and subtle but important differences in concepts such as trace, case, activity, event, table, and log is crucial for mastering advanced process mining analyses.

This text provides fundamental concepts and formalizations and discusses design decisions in event log extraction from a raw event table and for event log pre-processing. It is intended as study material for an advanced lecture in a process mining course.

1 Event Data

Event data is the basis for all process mining analysis. Most process mining techniques assume that their input is in the form of a simple event log such as the following:

\[ L = \begin{bmatrix} \langle A, B, C, D \rangle^{10}, \\ \langle A, C, B, D \rangle^{5}, \\ \langle A, B, A, D \rangle^{3}, \\ \langle A, E, D \rangle^{1} \end{bmatrix}. \]

This simple event log is defined over an alphabet \( \Sigma = \{A, B, C, D, E\} \) which is a set of activity names that have been observed. Each \( a \in \Sigma \) is the name of an activity, i.e., a specific action that can be executed or observed. For now, we consider each activity name as “atomic”—later in this chapter we will see that activities themselves can have some “structure” themselves.

A trace \( \sigma \in \Sigma^* \) is a finite sequence of activities. It describes that this sequence of activities had been observed at some point in the past. Each occurrence of an activity in a trace \( \sigma \) is called an event. For example, the trace \( \langle A, B, A, D \rangle \) describes we first observed \( A \), then \( B \) followed by another occurrence of \( A \), and finally we observed \( D \).

\footnote{Recall that the star “*” after \( \Sigma \) is the Kleene star which we use when constructing the set of all possible finite sequences over the elements of set \( \Sigma \).}
Exercise 1. Which other traces can be built from \( \Sigma = \{A, B, C, D, E\} \) that are not in \( L \)?

A **simple event log** \( L \in B(\Sigma^*) \) is a multiset\(^2\) of traces describing that various traces that have been observed and *how often* each trace has been observed, e.g., \( \langle A, B, A, D \rangle^3 \) was observed 3 times.

Exercise 2. Why do we only study finite sequences of activities when analyzing event logs (recorded historic executions) of processes?

However, event data is *not* recorded in this form in practice. First of all, an event records multiple attributes, not just the name of an activity. Secondly, event data is recorded as it occurs, and thus never grouped into traces or event logs.

A central part of process mining comprises actually obtaining event data from various data sources, and transforming it into an event log. We will see that both steps are non-trivial and allow for many choices. After the event log has been created, it rarely has sufficient quality to be used for any process mining analysis. Consequently, we have to pre-process the event log.

In the following, we first introduce a generic event data model and the notion of an event table in Sect. 2. Then, we explain in Sect. 3 how to extract structured event logs from such an event table and kinds of choices that can be made. We then introduce in Sect. 4 the notion of event classifiers required to turn structured event logs into the simple event logs explained above. We introduce the three central pre-processing operations on structured event logs in Sect. 5.

### 2 Events and Event Table

The most common direct or “raw” logging format for events is an **event table** or **event stream** as shown in Table 1. Each row in this table is one **event record**. Each column is an **attribute** where the column header defines the **attribute name**. The contents of a table cell for event \( e \) in column \( a \) is the **attribute value** event \( e \) has for attribute \( a \).

The event table can be considered as “raw” data as besides providing attributes per event, the data has no further structure. Specifically notice that no traces are recognizable in this event table.

The following definitions formally define events described by attributes and an event table.

- Let \( AN \) be a set of **attribute names**.
- Let \( Val \) a set of **values**.
- Let \( \mathcal{E} \) be the universe of events.

\(^2\) A multi-set is also called a *bag*, which explains the symbol \( B \) we use for constructing the multiset over \( \Sigma^* \). Recall that a multiset can contain the same element \( \sigma \in \Sigma^* \) multiple times.
Table 1. Event Table

| order | time             | action            | life-cycle | user      | customer | item        | delivery | type |
|-------|------------------|-------------------|------------|-----------|----------|-------------|----------|------|
| 23    | 19/12/2018 15:46 | receive payment   | complete   | System    | A7001    | phone       | online   |      |
| 23    | 19/12/2018 16:30 | archive           | complete   | Diana     | A9494    | phone       | online   |      |
| 35    | 20/12/2018 11:02 | receive order     | complete   | System    | A8760    | phone       | online   |      |
| 41    | 20/12/2018 11:03 | receive order     | complete   | Ellen     | A9920    | phone       | online   |      |
| 56    | 20/12/2018 11:04 | receive order     | complete   | System    | A7001    | online      | online   |      |
| 72    | 20/12/2018 11:05 | receive order     | complete   | Charles   | A9494    | online      | online   |      |
| 56    | 20/12/2018 11:12 | pack order        | start      | Bob       | A7001    | online      | online   |      |
| 35    | 20/12/2018 12:09 | pack order        | start      | Alice     | A8760    | phone       | online   |      |
| 35    | 20/12/2018 12:10 | add item          | complete   | Bob       | A8760    | Walkman     | 432      | phone|
| 35    | 20/12/2018 12:10 | add item          | complete   | Bob       | A8760    | Gameboy     | 432      | phone|
| 35    | 20/12/2018 12:15 | ship parcel       | complete   | Charles   | A8760    | 432         | phone    |      |
| 72    | 20/12/2018 14:05 | pack order        | start      | Alice     | A9494    | online      | online   |      |
| 72    | 20/12/2018 14:08 | add item          | complete   | Alice     | A9494    | VHS Player  | 775      | online|
| 72    | 20/12/2018 14:10 | pack order        | suspend    | Diana     | A9494    | online      | online   |      |
| 35    | 20/12/2018 16:00 | pack order        | complete   | Alice     | A8760    | phone       | online   |      |
| 56    | 21/12/2018 09:06 | receive payment   | complete   | System    | A7001    | online      | online   |      |
| 56    | 21/12/2018 09:17 | add item          | complete   | Alice     | A7001    | Walkman     | 623      | online|
| 56    | 21/12/2018 09:23 | pack order        | abort      | Charles   | A7001    | online      | online   |      |
| 56    | 21/12/2018 10:15 | archive           | complete   | Diana     | A9494    | online      | online   |      |
| 72    | 22/12/2018 09:36 | receive payment   | complete   | Diana     | A9494    | online      | online   |      |
| 35    | 22/12/2018 07:23 | receive payment   | complete   | System    | A8760    | phone       | online   |      |
| 35    | 22/12/2018 07:24 | archive           | complete   | Diana     | A9494    | phone       | online   |      |
| 72    | 22/12/2018 08:05 | pack order        | resume     | Alice     | A9494    | online      | online   |      |
| 72    | 22/12/2018 08:07 | add item          | complete   | Alice     | A9494    | VHS Tapes  | 775      | online|
| 72    | 22/12/2018 09:01 | ship parcel       | complete   | Diana     | A9494    | 775         | online   |      |
| 41    | 23/12/2018 23:46 | add item          | complete   | Bob       | A8920    | VHS Player  | 514      | phone|
| 41    | 23/12/2018 23:49 | ship parcel       | complete   | Ellen     | A8920    | 514         | phone    |      |
| 41    | 23/12/2018 23:51 | add item          | complete   | Bob       | A7001    | Gameboy     | 623      | phone|
| 41    | 23/12/2018 23:59 | ship parcel       | complete   | Bob       | A7001    | 623         | phone    |      |
| 41    | 27/12/2018 09:01 | pack order        | complete   | Alice     | A8920    | phone       | phone    |      |
| 41    | 27/12/2018 09:02 | archive           | complete   | Alice     | A9494    | phone       | phone    |      |

Definition 1 (Event). An event $e \in \mathcal{E}$ describes that a specific discrete observation has been made (by a sensor, a system, a human observer, etc.). The observation itself is described by attribute-value pairs through the partial function $\pi : \mathcal{E} \times AN \rightarrow Val$.

For each event $e \in \mathcal{E}$ and each attribute name $a \in AN$, $\pi(e,a) = v$ defines the value $v$ of attribute $a$. We write $\pi(e,a) = v$ if attribute $a$ is undefined for $e$ (has no value). We also write $\pi_a(e) = v$ or $e.a = v$ for $\pi(e,a) = v$.

For each event $e \in \mathcal{E}$, we require that

- the attribute time is defined, i.e., $\pi_{time}(e) \neq \perp$.
- $e$ carries a value $\pi_a(e) \neq \perp$ for some other attribute $a \in AN, a \neq time$.

In other process mining literature, you also find the notation $\#a(e) = v$ instead of $\pi_a(e) = v$ or $e.a = v$ to describe that event $e$ has attribute $a$ with value $v$.

By the two requirements on each event $e \in \mathcal{E}$ in Def. 1 we ensure that each event has a timestamp $\pi_{time}(e)$ and records at least one meaningful observation $\pi_a(e)$ (but it can record more). To be able to analyze processes in a meaningful

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3 A partial function does not have a value for each argument.
way, we need the events we analyze to share some common ground: They should refer to the same kinds of observations, i.e., share some attributes. Therefore, an event table is a sequence of events, that all have the same attribute $a$ defined. We can think of attribute $a$ as the activity name or measurement that was recorded.

**Definition 2 (Event Table).** An event table is a finite sequence $ET = \langle e_1, \ldots, e_n \rangle$ of events $e_1, \ldots, e_n \in E$ of events with $\pi_a(e_i) \neq \perp$ for some attribute $a \in ET$ and all $1 \leq i \leq n$.

We write $e_i \in ET, 1 \leq i \leq n$ when referring to an event in $ET$.

Definitions 1 and 2 define the absolute bare minimum for analyzing events: all events $e$ have a timestamp $\pi_{\text{time}}(e)$ and record the some observation (or value) $\pi_a(e_i)$. In this bare form, an event table could even specify a time-series. However, most events carry many additional attributes which we exploit in process mining.

Table 1 shows an event table according to Def. 1 and Def. 2.

**Exercise 3.** Choose any event from Table 1 and give its formal definition according to Def. 1.

Strictly speaking, Definition 2 does not define an event table in the sense of the data model of relational databases, but rather just a finite stream of events of attribute-value pairs. However, the table format representation is convenient and data in this form is often stored and exchanged using the Comma Separated Value (CSV) format.

**Exercise 4.** What are the differences between Definition 2 and the data model of relational databases?

We can reorder the events/rows in an event table to better understand its contents. Table 2 reorders the events of Table 1 by grouping them by attribute order and the sorting all events per order on attribute time. In Table 2, we added a column assigning each event a unique identifier to be able to refer to them individually, e.g., $e_1$ is the first event in this table.

In this sorted event table, we can start recognizing the traces we discussed in Sect. 1. However, the traces are no objects yet in their own right. We discuss how to obtain traces and structured event logs next.

## 3 Extracting Structured Event Logs from an Event Table

An event table only records for each event its timestamp and some observation such as an activity name. The essential difference between an event table and an event log is the presence of an additional attribute called the case identifier. It allows to group events into cases and traces and compare multiple sequences of events to each other.
3.1 Entities and Case

We use the term Case to refer to an entity or object that we are “tracking” over time in terms of the events in which this entity is involved.

For example, for the first event in Table 2, we can recognize that three types of entities were involved.

- order (for which we find the order id “23” as attribute value)
- user (for which we find the user name “System” as attribute value)
- customer (for which we find an identifier “A7001” as attribute value)

Other events also refer to a fourth entity type delivery.

In contrast, the attribute item does not refer to an entity type because its values describe sets or classes of similar objects but do not identify a unique entity or object. Recognizing which attributes of an event refer to entity types requires domain knowledge or additional context information.

To obtain a structured event log from an event table, we have to recognize from all attribute names the entity types, and then select one of these attributes referring to an entity type as the case identifier attribute. The attribute values for c we find among all events are the cases we find in the data.

Definition 3 (Case identifier, cases). Let ET = ⟨e₁, . . . , eₙ⟩ be an event table.

The set of attribute names in ET is

\[ AN(ET) = \{ a \in AN \mid \exists e_i \in ET, \pi_a(e_i) \neq \perp \} \].

If we select an attribute id ∈ AN(ET) as case identifier, then

Cases(ET, id) = \{ \pi_{id}(e_i) \mid e_i \in ET \}

is the set of cases for this case identifier.

By choosing one entity type as case identifier, we decide to reformat the event data in a way that “tracks” what has happened to all entities of this type.

Note that Definition 3 allows to pick any attribute as case identifier, not just those that refer to entity types. For example, we could even pick attribute name action ∈ AN(ET). The next steps in building a structured event log work with any chosen case identifier. However, the subsequent analysis entirely depends on how sensible this choice of a case identifier was for the particular analysis question. In other words, we have to understand which analysis question we try to answer, and then identify the corresponding attribute name (e.g., of an entity type of interest) that we want to use as case identifier.

This also means that at this point we implicitly require that each event e has three mandatory attributes that are different from each other (i.e., we do not choose a = c or a = c = time):

1. the timestamp \( \pi_{time}(e) \) (see Def. 1)
2. a recorded action or activity \( \pi_a(e) \) (see Def. 2)
3. A case identifier $\pi_c(e)$ (see Def. 3)

However, except for $\pi_{time}(e)$, activity and case identifier are not pre-determined by the event table. They are choices we make.

**Exercise 5.** Can two events happen at the same time? Do they have to be in different cases? Do they have to have different activities?

| # | order | time       | action           | life-cycle | user   | customer | item     | delivery | type   |
|---|-------|------------|------------------|------------|--------|----------|----------|----------|--------|
| 1 | 23    | 19/12/2018 | receive order    | complete   | System | A7001    | online   |          |        |
| 2 | 23    | 19/12/2018 | archive          | complete   | Diana  | A7001    | online   |          |        |
| 3 | 35    | 20/12/2018  | receive order    | complete   | System | A8760    | phone    |          |        |
| 4 | 35    | 20/12/2018  | pack order       | start      | Alice  | A8760    | phone    |          |        |
| 5 | 35    | 20/12/2018  | add item         | complete   | Bob    | A8760    | Gameboy  | 432      | phone  |
| 6 | 35    | 20/12/2018  | ship parcel      | complete   | Charles| A8760    | 432      | phone    |        |
| 7 | 35    | 20/12/2018  | pack order       | complete   | Alice  | A8760    | phone    |          |        |
| 8 | 35    | 22/12/2018  | receive payment  | complete   | System | A8760    | phone    |          |        |
| 9 | 35    | 22/12/2018  | archive          | complete   | Diana  | A8760    | phone    |          |        |
|10 | 35    | 22/12/2018  | receive order    | complete   | Ellen  | A8920    | phone    |          |        |
|11 | 41    | 20/12/2018  | pack order       | start      | Bob    | A8920    | phone    |          |        |
|12 | 41    | 23/12/2018  | receive order    | complete   | Ellen  | A8920    | phone    |          |        |
|13 | 41    | 23/12/2018  | add item         | complete   | Bob    | A8920    | VHS Player| 514      | phone  |
|14 | 41    | 23/12/2018  | ship parcel      | complete   | Ellen  | A8920    | 514      | phone    |        |
|15 | 41    | 23/12/2018  | add item         | complete   | Bob    | A7001    | Gameboy  | 623      | phone  |
|16 | 41    | 23/12/2018  | ship parcel      | complete   | Bob    | A7001    | 623      | phone    |        |
|17 | 41    | 27/12/2018  | pack order       | complete   | Alice  | A8920    | phone    |          |        |
|18 | 41    | 27/12/2018  | archive          | complete   | Alice  | A8920    | phone    |          |        |
|19 | 56    | 20/12/2018  | receive order    | complete   | System | A7001    | online   |          |        |
|20 | 56    | 20/12/2018  | pack order       | start      | Bob    | A7001    | online   |          |        |
|21 | 56    | 21/12/2018  | receive payment  | complete   | System | A7001    | online   |          |        |
|22 | 56    | 21/12/2018  | add item         | complete   | Alice  | A7001    | Walkman  | 623      | online |
|23 | 56    | 21/12/2018  | pack order       | abort      | Charles| A7001    | online   |          |        |
|24 | 56    | 21/12/2018  | archive          | complete   | Diana  | A7001    | online   |          |        |
|25 | 72    | 20/12/2018  | receive order    | complete   | Charles| A9494    | online   |          |        |
|26 | 72    | 20/12/2018  | pack order       | start      | Alice  | A9494    | online   |          |        |
|27 | 72    | 20/12/2018  | add item         | complete   | Alice  | A9494    | VHS Player| 775      | online |
|28 | 72    | 20/12/2018  | pack order       | suspend    | Diana  | A9494    | online   |          |        |
|29 | 72    | 22/12/2018  | receive payment  | complete   | Diana  | A9494    | online   |          |        |
|30 | 72    | 22/12/2018  | pack order       | resume     | Alice  | A9494    | online   |          |        |
|31 | 72    | 22/12/2018  | add item         | complete   | Alice  | A8494    | VHS Tapes | 775      | online |
|32 | 72    | 22/12/2018  | ship parcel      | complete   | Charles| A9494    | 775      | online   |        |

**Table 2. Event Table**

For the example of Table 2, we can see four candidates for case identifiers based on the entity-types we found in the event table: order, delivery, user, and customer.

**Exercise 6.** What are the cases for case identifier delivery in Table 2?

If we have selected an attribute $id$ as case identifier, then we say an event $e$ is correlated to a case $c$ if its $id$-attribute refers to $c$, i.e., $\pi_{id}(e) = c$.

**Definition 4 (Correlation to a case).** Let $ET$ be an event table. Let $c \in \text{Cases}(ET, id)$ be a case for a case identifier $id \in \text{AN}(ET)$. 
Event $e \in ET$ is correlated to $c$ iff $\pi_{id}(e) = c$. The set of all events correlated to $c$ is

$$\text{corr}(ET, id, c) = \{e \in ET \mid \pi_{id}(e) = c\}.$$ 

For example, events $e_1$ and $e_2$ are correlated to $\text{order} = 23$ in Table 2. Note that if an event $e$ does not have attribute $id$ defined, i.e., $\pi_{id}(e) = \bot$, then it is not correlated to any case of this case identifier.

Exercise 7. Which events are correlated to $\text{order} = 35$?

Exercise 8. Which events are correlated to $\text{delivery} = 623$?

Exercise 9. What happens when we choose time to be an activity name or a case identifier?

If all events correlated to a case $c$ carry the same value $v$ for an attribute $x$, then we call $x$ a case attribute of $c$.

Definition 5 (Case attribute). Let $ET$ be an event table. Let $c \in \text{Cases}(ET, id)$ be a case for a case identifier $id \in AN(ET)$.

Attribute $x \in AN(ET)$ is a case attribute of $c$ iff for all $e, e' \in \text{corr}(ET, id, c)$ holds $\pi_x(e) = \pi_x(e') = v \neq \bot$. We then lift the function $\pi(\cdot)$ from events to cases and write $\pi_x(c) = v$.

Attribute $x$ is a global case attribute iff it is a case attribute for every case $c \in \text{Cases}(ET, id)$.

The case identifier is always a global case attribute. A global case attribute has to be defined for each case, but each case can have its own value.

Exercise 10. What are the case attributes of Table 2 for case identifier $\text{order}$?

3.2 Trace

A trace is the sequence of events correlated to a case and ordered by time. For example, the trace of $\text{order} = 23$ in Table 2 is $\langle e_1, e_2 \rangle$.

Definition 6 (Trace of a case). Let $ET = \langle e_1, \ldots, e_n \rangle$ be an event table. Let $id \in AN(ET)$ be the selected case identifier.

A sequence $\langle e_1, \ldots, e_k \rangle$ of events is a trace of case $c \in \text{Cases}(ET, id)$ iff

1. $\{e_1, \ldots, e_k\} = \text{corr}(ET, id, c)$, i.e., it consists of all events of $ET$ correlated to $c$, and
2. for each $i = 1, \ldots, k-1$ holds $\pi_{time}(e_i) \leq \pi_{time}(e_{i+1})$, i.e., events are ordered by time.

Note that there may be more than one way to sequentialize the events $\{e_1, \ldots, e_k\} = \text{corr}(ET, id, c)$ correlated to a case $c$. This happens where two or more events $e_i, e_{i+1}$ have the same time-stamp $\pi_{time}(e_i) = \pi_{time}(e_{i+1})$.

Exercise 11. What is the trace of $\text{order} = 35$?

Exercise 12. What is the trace of $\text{delivery} = 623$?
3.3 Structured Event Log

A structured event log is a set of cases where each case is associated with exactly one trace for this case as a case attribute.

**Definition 7 (Structured Event Log).** Let $ET = \langle e_1, \ldots, e_n \rangle$ be an event table. Let $id \in \text{AN}(ET)$ be the selected case identifier.

The structured event log $L$ is the set $L = \text{Cases}(ET, id)$ of cases for case identifier $id$ so that additionally each case $c \in L$ gets assigned a trace $\langle e_1, \ldots, e_k \rangle$ of $c$ as trace attribute $\pi_{\text{trace}}(c) = \langle e_1, \ldots, e_k \rangle$.

For example, the structured event log of Table 2 has the cases $L = \{23, 35, 41, 56, 72\}$ for order and the following traces:

- $\pi_{\text{trace}}(23) = \langle e_1, e_2 \rangle$ where
  - $e_1$ has
    - $\pi_{\text{order}}(e_1) = 23$
    - $\pi_{\text{time}}(e_1) = 19/12/2018\ 15:46$
    - $\pi_{\text{action}}(e_1) = \text{receive payment}$
    - ...
  - $e_2$ has
    - $\pi_{\text{order}}(e_2) = 23$
    - $\pi_{\text{time}}(e_2) = 19/12/2018\ 16:30$
    - $\pi_{\text{action}}(e_2) = \text{archive}$
    - ...
- $\pi_{\text{trace}}(35) = \langle e_3, e_4, e_5, \ldots, e_{10} \rangle$
- $\pi_{\text{trace}}(41) = \langle e_{11}, e_{12}, e_{13}, \ldots, e_{18} \rangle$
- $\pi_{\text{trace}}(56) = \langle e_{19}, \ldots, e_{24} \rangle$
- $\pi_{\text{trace}}(72) = \langle e_{25}, \ldots, e_{32} \rangle$

A structured event log has a simple hierarchical structure. At the top-level are the cases $L = \{e_1, \ldots, e_k\} = \text{Cases}(ET, id)$. Each case has case attributes as “children”, one of them is the trace $\pi_{\text{trace}}(c)$. Each event $e$ in a trace has event attributes as children, including $\pi_{\text{time}}(e)$ (time-stamp), $\pi_a(e)$ (the observed activity), and $\pi_c(e)$ (the case identifier).

1. A structured event log $L$ consists of a set of cases $L = \{c_1, \ldots, c_n\} \subseteq \text{Val}$, i.e., values for some case identifier.
2. Each case $c \in L$ defines a trace $\pi_{\text{trace}}(c) = \langle e_1, \ldots, e_k \rangle \in \mathcal{E}^*$ as a sequence of events ordered by time, i.e., $\pi_{\text{time}}(e_i) \leq \pi_{\text{time}}(e_{i+1})$ for each $i = 1, \ldots, k-1$.
3. The events in $\pi_{\text{trace}}(c)$ are all correlated to the case, i.e., $\pi_c(e_i) = c$. However, most XES event logs do not store the case identifier as an event attribute again.
4. There is at least one attribute $a$ (e.g., the activity name) defined by each event $\pi_a(e)$ in each trace $e \in \pi_{\text{trace}}(c), c \in L$.
5. Cases do not share events, i.e., there is no event $e \in \mathcal{E}$ with $e \in \pi_{\text{trace}}(c), \pi_{\text{trace}}(e'), c, c' \in L, c \neq c'$. 
This hierarchical structure is formalized in the XES-standard \[2, 6, 7\]. See also other formalizations of event logs \[1\].

**Exercise 13.** Provide the cases and traces of the structured event log of Table 2 for case identifier delivery.

**Exercise 14.** Which other meaningful structured event logs can you extract from Table 2?

## 4 Event Classifiers and Simple Event Logs

The nested hierarchy of a structured event log contains all information about all events. However, analysis techniques operating on events, prefer a flat data structure where

- a structured event \(e \in E\) with its various attributes is represented by a single attribute value \(\pi_a(e)\), e.g., the activity name,
- a structured case \(c\) with its various case attributes is not represented its trace \(\pi_{\text{trace}}(c) = \langle e_1, \ldots, e_k \rangle\) but rather in its simplified form \(\langle \pi_a(e_1), \ldots, \pi_a(e_k) \rangle\).

For example, we can transform \(\langle e_1, e_2 \rangle\) of Table 2 into \(\langle \text{receive payment}, \text{archive} \rangle\). This representation allows easily searching for patterns in the sequences of activity names.

### 4.1 Event Classifiers and Event Classes

However, as for Definition 3 of the case identifier, events do not have a canonical or standard attribute \(a\) by which it must be represented in this simplified way. Rather, we again can pick.

Literature introduces for this purpose the definition of an event classifier.

**Definition 8 (Event Classifier).** An event classifier is a function with signature

\[
\text{class} : E \rightarrow \text{Value}
\]

that maps each event to a value. The value \(\text{class}(e)\) is called the event class. Any two events \(e, e'\) with \(\text{class}(e) = \text{class}(e')\) belong to the same event class, which means they describe the same kind of observation.

Usually, the event classifier is defined over event attributes which can be a single attribute or a combination of attributes. For example,

- The standard event classifier is the activity name classifier \(\text{class}_{\text{act}}(e) = \pi_a(e)\) where \(a \in AN\) is the attribute we identify as the activity name.

For Table 2, the activity name classifier is \(\text{class}_{\text{act}}(e) = \pi_{\text{action}}(e)\). For example, events \(e_4, e_8\) have the same activity name class \(\text{class}_{\text{act}}(e_4) = \text{class}_{\text{act}}(e_8) = \text{pack order}\).
The activity+life-cycle classifier combines the activity name $a$ with the event life-cycle attribute $lc$ (if it exists in the event log), i.e., $\text{class}_{\text{act+life-cycle}}(e) = (\pi_a(e), \pi_{lc}(e))$.

For Table 2, the activity+life-cycle classifier is $\text{class}_{\text{act+life-cycle}}(e) = (\pi_{\text{action}}(e), \pi_{\text{life-cycle}}(e))$.

For example, events $e_4$ belong to different event classes for this classifier: $\text{class}_{\text{act+life-cycle}}(e_4) = \text{pack order}+\text{start}$ and $\text{class}_{\text{act+life-cycle}}(e_8) = \text{pack order}+\text{complete}$.

Event classes over multiple attributes are also represented with a ‘+’, e.g., $\text{class}_{\text{act+life-cycle}}(e_4) = \text{pack order}+\text{start}$.

The resource classifier $\text{class}_{\text{res}}(e) = \pi_r(e)$ where $r \in AN$ is the attribute deferring to the user, machines, or resource that participated in the event.

For Table 2, the resource classifier is $\text{class}_{\text{res}}(e) = \pi_{\text{user}}(e)$. For example, events $e_4$ and $e_8$ belong to the same event resource event class: $\text{class}_{\text{res}}(e_4) = \text{class}_{\text{res}}(e_8) = \text{Alice}$.

We can in principle choose any combination of attributes for the event classifier. This essentially corresponds to feature selection in data mining: we choose the event attributes we think are most relevant for the analysis task at hand. If the event has no value defined for the selected event classifier, e.g., $\text{class}_{\text{item}}(e) = \pi_{\text{item}}(e)$ and $\text{class}_{\text{item}}(e_1) = \perp$, then the event will be omitted from the analysis.

It is also possible to derive new event attributes based on other events in the trace or even the entire event log, and to use these subsequently as event classifiers. This would correspond to feature engineering.

Exercise 15. Identify another meaningful event classifier from Table 2.

Definition 9 (Event Classes of an Event Log). Given a structured event log $L$ (according to Def. 7) and an event classifier class, the set of event classes in $L$ is the set $\Sigma_{\text{class}}(L) = \{ \text{class}(c) | c \in L, e \in \pi_{\text{trace}}(c), \text{class}(c) \neq \perp \}$.

Exercise 16. What are the event classes of the event log in Table 2 for the classifier $\text{class}(e) = \pi_{\text{customer}}(e)$?

4.2 Simple Event Log

If we have fixed an event classifier $\text{class}$, we can represent each trace in a log $L$ by the sequence of event classes, e.g., the sequence of activity names. However, we omit all $\perp$ values.

Definition 10 (Simple Trace). Let $L$ be a structured event log, let $(e_1, \ldots, e_k) = \pi_{\text{trace}}(c), c \in L$ be a trace. Let class be an event classifier.

The simple trace of $c$ is the sequence

$$\text{simple}_{\text{class}}(c) = (\text{class}(e_1), \ldots, \text{class}(e_k))|_{\Sigma_{\text{class}}(L)}$$

where we replace each event $e_i$ by $\text{class}(e_i)$ and then project the resulting sequence onto all valid event classes $\Sigma_{\text{class}}(L)$, i.e., all values that are not $\perp$.

\footnote{We write $\pi_{\Sigma'}$ for the projection of a trace $\sigma \subseteq \Sigma^*$ onto a subset $\Sigma' \subseteq \Sigma$ of some alphabet.}
The simple trace for case 23 and the activity event classifier $\text{class}_{\text{act}}(e) = \pi_{\text{action}}(e)$ is
\[
\langle \text{receive payment, archive} \rangle.
\]

The simple trace for case 23 and the activity event classifier $\text{class}(e) = \pi_{\text{customer}}(e)$ is
\[
\langle \text{A7001} \rangle.
\]

We obtain the simple event log of $L$ by collecting all simple traces of all cases $L$ in a multiset. Recall from Sect. 1 that $\sigma \in \Sigma^*$ is a finite sequence of activity names and $B(\Sigma^*)$ is a multi-set (bag) of finite sequences.

**Definition 11 (Simple Event Log).** Let $L$ be a structured event log. Let $\text{class}$ be an event classifier. Let $\Sigma = \Sigma_{\text{class}}(L)$

The simple event log is the multiset simple $\text{class}(L) = L' \in B(\Sigma^*)$ where $L'(\sigma) = |\{c \in L \mid \text{simple}_{\text{class}}(c) = \sigma\}|$, i.e., there are as many copies of $\sigma$ as there are cases which have the same simple trace $\text{simple}_{\text{class}}(c) = \sigma$.

The simple event log of Table 2 for the $\text{action}$ classifier is (we abbreviate each action name for succinctness):
\[
L' = [\langle \text{RP, AR} \rangle^1, \\
\langle \text{RO, PO, AI, AI, SP, PO, RP, AR} \rangle^1, \\
\langle \text{RO, PO, AI, SP, AI, SP, PO, AR} \rangle^1, \\
\langle \text{RO, PO, RP, AI, PO, AR} \rangle^1, \\
\langle \text{RO, PO, AI, PO, RP, PO, AI, SP} \rangle^1].
\]

The simple traces in a simple event log are also called *trace variants* of the event log as they show the principle ways the object that is tracked by the case identifier “moves” through the data.

**Exercise 17.** What is the simple event log for the $\text{activity+life-cycle}$ classifier?

**Exercise 18.** What is the simple event log for the $\text{item}$ classifier?

Note that each simple trace is a finite sequence $\sigma \in \Sigma^*$ over some alphabet of event classes $\Sigma = \Sigma_{\text{class}}(L)$ and that the simple event log is a multiset of simple traces. We now have a complete procedure for obtaining a simple event log, as outlined in Sect. 1 from event data as it is recorded in practice, i.e., an event table.

1. Find meaningful entity identifiers in the attributes of the event table that correspond to your analysis question.
2. Select one entity identifier as case identifier $id$.
3. Construct the structured event log $L$ for this case identifier by correlating events and ordering them over time.
4. Find meaningful event attributes to summarize or classify the observation that is recorded in the event.
5. Select or define one event classifier \textit{class}.
6. Derive the simple event log from $L$ for this event classifier \textit{class}.

Given an event table $ET$, any simple event log is fully defined by two decisions: the case identifier $id$ and the event classifier \textit{class}. However, these two choices are powerful and allow you to derive many different views.

Exercise 19. What is the simple event log for case identifier \textit{customer} and event classifier \textit{order}?

Almost all process mining software contains a view to visualize the event log in the form of a simple event log; for example the “Explore Event Log” visualizer of ProM\textsuperscript{5} shown in Fig. 1 visualizes event logs as simple event logs and allows identifying patterns through color-coding the event classes.

![Fig. 1. The Explore Event Log visualizer of ProM](http://www.promtools.org/)

5 Pre-Processing Event Logs

Analyzing event data, just like any data analysis, requires pre-processing to remove data points that are not relevant for the specific analysis question at hand.

There are three basic pre-processing operations on event logs that allow us to reduce or “filter” the data in three fundamentally different ways. Most other pre-processing operations are a combination of these three operations. They are defined on the data model of the structured event log (Def. 7).

1. \textit{Selection} of traces reduces the set of cases in $L$ to those that satisfy a specific property. All other cases are removed. The pre-processed log $L'$ contains just
a subset of the cases in \( L \), i.e., \( L' \subseteq L \) and each cases keeps all its properties, especially all events in its trace.

Figure 2 (top) illustrates the selection of \( L \) to all cases whose traces end with an event with activity name \( C \). The resulting log \( L' \) does not contain the cases whose traces end with \( B \) or \( A \).

2. **Projection** removes from each trace in \( L \) all events that do not satisfy a particular property. The resulting event log \( L' \) keeps all its cases, but their traces may contain fewer events or even be empty.

Figure 2 (left) illustrates the projection of \( L \) to all events with activity attribute \( A \) or \( C \). The resulting event log does not contain any event with activity attribute \( B \) anymore.

3. **Aggregation** groups in each trace multiple subsequent events \( e_1, \ldots, e_k \) with the same property into a new event \( e^* \) whose properties are derived from \( e_1, \ldots, e_k \); \( e_1, \ldots, e_k \) are then replaced by \( e^* \). The pre-processed log \( L' \) keeps all its cases, but the traces may have fewer events and may contain a new aggregated event with new properties that were not explicitly visible in \( L \).

Figure 2 (bottom right) illustrates the aggregation of subsequent events with the same activity name. For example, the subsequence \( \langle B, B \rangle \) in the second case was replaced by a single \( B \).

**Fig. 2.** Pre-Processing Operations on Event Logs

**Definition 12 (Selection).** Let \( L \) be a structured event log. Let \( \varphi(c) \) be a predicate over the case attributes and event attributes of \( L \). The selection of \( L \) wrt. \( \varphi \) is the subset

\[
\text{Select}_{\varphi}(L) = \{ c \in L \mid \varphi(c) = \text{true} \}.
\]

Here are several example selection predicates for the event log in Table 2.
Here are several example projection predicates for the event log in Table 2:

\[ \varphi_1(c) \equiv \pi_{\text{type}}(c) = \text{online} \] (only cases of type “online”)
\[ \varphi_2(c) \equiv \pi_{\text{trace}}(c) = \langle e_1, \ldots, e_n \rangle \land \pi_{\text{action}}(e_1) = \text{receive order} \] (only cases starting with “receive order”)
\[ \varphi_3(c) \equiv \pi_{\text{trace}}(c) = \langle e_1, \ldots, e_n \rangle \land \pi_{\text{time}}(e_n) - \pi_{\text{time}}(e_1) < 24h \] (only cases completing within 24 hours)
\[ \varphi_4(c) \equiv \exists \epsilon' \in L \mid \text{simple class}(c) = \text{simple class}(\epsilon') \geq 10 \] (for some event classifier class (only cases whose trace variant, i.e., simple trace, occurs at least 10 times in the event log).

Note that \( \varphi_2(c) \) is not purely local to the case \( c \) but rather “reaches out” into the entire event log \( L \).

**Exercise 20.** Which cases are selected by \( \varphi_1(c)-\varphi_4(c) \)?

**Definition 13 (Projection).** Let \( L \) be a structured event log. Let \( \psi(e) \) be a predicate over the event attributes of \( L \).

Let \( \sigma = \langle e_1, \ldots, e_n \rangle = \pi_{\text{trace}}(c), c \in L \) be a trace. The projection of \( \sigma \) onto \( \psi \) is the projection of \( \sigma \) onto all events \( e_i \) where \( \psi(e_i) = \text{true} \), i.e.,

\[ \text{Proj}_\psi(\sigma) = \langle e_1, \ldots, e_n \rangle | \psi(e_i) = \text{true} \).

We obtain the projection of \( L \) into \( \psi \), written \( \text{Proj}_\psi(L) \), by setting \( \pi_{\text{trace}}(c) := \text{Proj}_\psi(\pi_{\text{trace}}(c)) \).

Here are several example projection predicates for the event log in Table 2:

\[ \psi_1(e) \equiv \pi_{\text{life-cycle}}(e) = \text{complete} \] (only “complete” events)
\[ \psi_2(e) \equiv \pi_{\text{delivery}}(e) \neq \bot \] (only events with a reference to a delivery)
\[ \psi_3(e) \equiv \pi_{\text{type}}(e) = \text{online} \]
\[ \psi_4(e) \equiv \pi_{\text{user}}(e) \in \{\text{Alice, Bob}\} \] (only events where Alice or Bob are involved)
\[ \psi_5(e) \equiv c = \pi_{\text{order}}(e) \land \pi_{\text{trace}}(c) = \langle e_1, \ldots, e_n \rangle \land e = e_i \land \forall j = i + 1, \ldots, n \land \pi_{\text{action}}(e_i) \neq \pi_{\text{action}}(e_j) \] (only the last occurrence of each activity in a trace)
\[ \psi_6(e) \equiv \exists \epsilon' \in L, e' \in \pi_{\text{trace}}(c), \pi_{\text{act}}(e) = \pi_{\text{act}}(e') \geq 5 \] (only events of activities which occur at least 5 times in the event log \( L \).

Note that when we constructed the event log from the event table, each event \( e \) had the chosen case identifier \( id \) as event attribute, i.e., \( \pi_{id}(e) = c \) refers to the case. When constructing the event log, we used the value \( c \) to construct the case itself. This means, we can “reach” the case \( c \) from an event \( e \), and once we have the case \( c \), we can “reach” the entire trace \( \pi_{\text{trace}}(c) \) that also contains \( e \). We use this in \( \psi_6(e) \) to reason about whether \( e \) is not the last event in the trace of the same activity.

Such projection attributes are not possible in all process mining software. It can only be defined if the event \( e \) actually has a reference to the case \( c \) and the data structure in which the event is stored allows to resolve this reference. Similarly, \( \psi_6(e) \) requires that the entire event log (or statistics about the event log) are accessible.
Exercise 21. Apply \( \text{Proj}_{\varphi_2}(L) \) on the event log of Table 2 and derive the simple event log for the activity name event classifier.

Exercise 22. What is the difference between \( \text{Select}_{selectP}(L) \) and \( \text{Proj}_{projP_2}(L) \)?

For aggregation, we do not provide a full formal definition, but outline what has to be defined. Aggregation in a case \( Agg \) has to be defined. Aggregation in a case \( Agg_{g,r}(c) \) requires two functions \( g \) and \( r \):

- A grouping classifier \( g : \mathcal{E} \to \mathcal{V} \) which maps each event to a value, similar to an event classifier.

- With \( g \), we partition the trace \( \pi_{\text{trace}}(c) = \sigma \) into maximal subsequences \( \sigma_i(e_{i_1}, \ldots, e_{i_n}) \) so that \( g(e_j) = g(e_{j+1}) \) for all \( i_1 \leq j < i_n \), e.g., all subsequences with the same activity name. This results in a sequence of \( k \) such sub-sequences of various lengths, i.e., \( g(\sigma) = \langle \sigma_1, \ldots, \sigma_k \rangle \), for instance, \( \langle \langle e_1, e_2 \rangle, \langle e_3 \rangle, \langle e_4, e_5 \rangle \rangle \).

- A replacement function \( r : \mathcal{E}^+ \to \mathcal{E} \) that replaces any non-empty subsequence \( \langle e_{i_1}, \ldots, e_{i+m} \rangle \) by a new event \( r(e_{i_1}, \ldots, e_{i+m}) = e' \) and defines the event attributes for \( e' \) based on the attribute values of \( \langle e_{i_1}, \ldots, e_{i+m} \rangle \); \( r \) specifically has to set the timestamp of \( e' \) to be within \( \pi_{\text{time}}(e_i) \leq \pi_{\text{time}}(e') \leq \pi_{\text{time}}(e_{i+m}) \). For a singleton sub-sequence \( \langle e_i \rangle \), the replacement function should just return the event \( e_i \), i.e., the event remains unchanged.

- \( Agg_{g,r}(c) \) apply \( r \) to each sub-sequence \( \langle e_{i_1}, \ldots, e_{i+m} \rangle \) obtained from \( g \) which results in a new sequence of events, i.e., \( Agg_{g,r}(c) = \langle r(\sigma_1), \ldots, r(\sigma_k) \rangle \) where \( g(\sigma) = \langle \sigma_1, \ldots, \sigma_k \rangle \) and \( \sigma = \pi_{\text{trace}}(c) \). For example, \( \langle r(\langle e_1, e_2 \rangle), r(\langle e_3 \rangle), r(\langle e_4, e_5 \rangle) \rangle = (e_{12}, e_3, e_{45}) \).

- Set \( \pi_{\text{trace}}(c) := Agg_{g,r}(c) \)

For example, we can use \( g(e) = \pi_{\text{action}}(e) \) to find all subsequences where the same activity occurs repeatedly. In Table 2 this would be only \( \langle e_5, e_6 \rangle \). We can then define a replacement function \( r((e_1, \ldots, e_k)) \) where the new event \( e' \) gets \( \pi_x(e') = \pi_x(e_k) \) for all attributes \( x \) defined for \( e_k \), i.e., we replace the sequence by the last event. We could also define \( \pi_{\text{item}}(e') = \{ \pi_{\text{item}}(e_i) \mid 1 \leq i \leq k \} \) to collect the items that were involved in these events into a set.

Exercise 23. What is the difference between aggregation where any subsequence is replaced by the last event and projection with \( \psi_5(e) \)?

All three event log pre-processing operations selection, projection, and aggregation always result in single event log. This allows to apply them in arbitrary combinations. For example, first project onto all events where “Bob” is involved and then aggregate on \( \pi_{\text{action}}(e) \).

As in any data analysis, identifying which event log pre-processing operations to apply for the analysis question at hand is an iterative process. Process mining software supports this iterative process by letting the analyst interactively build a stack of filtering operations that can be modified and re-arranged alongside a visualization of the outcome of the filtering operation. Figure 3 shows the “Filter Event Log” plugin of ProM.

Other pre-processing operations on event data are
Fig. 3. Interactive event log filtering in ProM, accessible via the “Filter Event Log” plugin. The shown event data has been filtered using two filters executed one after the other.

- Clustering the cases in the event log $L$ into multiple sub-logs $L_1, \ldots, L_k$ so that cases in a sub-log $L_i$ have similar trace variants and cases in different sub-logs $L_i \neq L_j$ have maximally different trace variants. Technically, clustering is repeated selection. However, the selection criteria are not based on selection predicates; see [8] for a survey on available clustering techniques.
- Event data abstraction is a form of aggregation where arbitrary patterns in the event data are aggregated into higher level events; see [3] for an overview.

6 Event logs have limitations

Event logs as defined in this document face severe limitations.

- The timestamp information in event data is often not reliable. For example, if events are only recorded on day-level granularity and three events $e_1, e_2, e_3$ occurred on the same day, then their order $e_1, e_2, e_3$ in the event table may not be the order in which they occurred. When creating an event log, we have to pick on ordering of these events to build a trace, but it may be the wrong one. A possible solution is to define a trace $\pi_{\text{trace}}(e)$ not as a sequence $\langle e_1, \ldots, e_n \rangle$ of events, but as a strict partial order $(E, <)$ where $e_i < e_j$ are ordered only if $\pi_{\text{time}}(e_i) < \pi_{\text{time}}(e_j)$. Events with the same time-stamp remain unordered.
- Structured event logs only order event data according to a single case identifier. However, we have seen that even basic event data contains multiple entity identifiers that are in 1:n and n:m relationships to each other. For example in Table 2 customer A7001 is involved in 3 orders and deliver 623 is involved in 2 orders. The data structure of the structured event log cannot capture these relations. Graph-based data structures such as [4][5] allow tracing the behavior of multiple objects together.
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