Stabilizing Controller Analysis for Delay-dependent Networked Control Systems

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Abstract. This work is devoted to analyse the stabilizing controller and stability in the case of delay-dependent networked control systems (NCSs). By considering the relationship between the network-induced delays and their bound, an improved stability criterion for NCSs is proposed in the derivative of Lyapunov function. A stabilizing state feedback controller is applied to generate the next control information for each subsystem using delayed sensing data in a free-weighting LMI formulation.

Introduction

Feedback control systems that are closed over a real-time communication network have more and more popular and hardware devices for networks have become cheaper and the internet has been more and more common. Although NCSs have several advantages such as low installation cost, easy maintenance and so on. The use of communication networks makes it necessary to deal with network-induced delays. These delays may be unknown and time-varying, and may deteriorate the closed-loop systems[1,2].

As for network-induced delays, they have been deal with by the control theory community in most research work. The performance and stability of control scheme strongly rely on the respect of the specified sampling rates and network-induced delays. Recently, the delay-dependent stabilizing method on the MADB has attracted much attention for the stability of NCSs with network-induced delays[3]. A free-weighting matrix approach[4] is reported to cover the results using Moon el al.’s inequality and the descriptor system approach[5]. As for NCSs, the delays are less than the sampling period for continuous-time systems based on sampling-rate method. Some methods to calculate the MADB for NCSs using Moon el al.’s inequality for both discrete-time plants are proposed in[6].

The stability analysis of controller is dealt with in the presence of the network-induced delays. The object is to prove stability and good performance for NCSs in the presence of timing uncertainties as communication delays. Thus, it can be useful to consider more dynamic solutions.

Stability Analysis

We are interested here in real-time control of systems with communication delays (see Fig.1). There are mainly two sources of delays from the network-induced delays in an NCS: device delays and transmission delays. The computing delays include the time delay at the plant node (P) \( \tau_{sou} \) and the controller node (C) \( \tau_{des} \). The transmission delays include the time delay from sensor node(S) to controller node(C) \( \tau_{sc} \) and from controller node (C) to actuator node (A) \( \tau_{ca} \). The total time delay can be expressed as follows

\[ \tau = \tau_{sou} + \tau_{des} + \tau_{sc} + \tau_{ca} \]  

(1)
Delay-dependent NCS Model

In this section, we will review and improve some results on the modeling of NCSs. We need the following assumptions:

Assumption 1. In an NCS, the total time delay $\tau$ is less than one sampling period $h$.

Assumption 2. In an NCS, the NCS uses the way of single-packet transmission. The data packet loss and noise effect on the NCS are not considered.

Assumption 3. In an NCS, the sensor is assumed to be time-driven, whereas the controller and actuator are event-driven, and controller is time-invariant.

The discrete system equations can be written as:

$$ x(k + 1) = A_\tau x(k - \tau) + Bu(k) $$

where $x(k) \in \mathbb{R}^n$; control input vector $u(k) \in \mathbb{R}^m$; $A$, $A_\tau$ and $B$ are real constant matrices with appropriate dimensions.

The total time delay $\tau$ satisfies:

$$ \tau_1 \leq \tau \leq \tau_2 $$

where $\tau_1$ and $\tau_2$ are minimum and maximum time delay respectively and less than one sampling time.

The memoryless state feedback controller for the NCSs is

$$ u(k) = Kx(k) $$

where $u(k) = 0$, $k \in (-\tau, 0)$.

Lemma 4. (Schur complements) Given constant matrices $M$, $P$ and $Q$, where $P = P^T$ and $Q = Q^T < 0$, then $P - MQ^{-1}M^T < 0$, if only if $\begin{pmatrix} P & M \\ M^T & Q \end{pmatrix} < 0$, or $\begin{pmatrix} Q & M^T \\ M & P \end{pmatrix} < 0$. 

Figure 1. NCSs Control Structure.
Theorem 5. Given \( \tau_i > 0 \) \((i = 1, 2)\), if there exist constant matrices \( P, Q, Z, X, N_1, N_2 \), where \( P = P^T > 0 \), \( Q = Q^T \geq 0 \), \( Z = Z^T \geq 0 \), \( X = \begin{pmatrix} X_{11} & X_{12} \\ * & X_{22} \end{pmatrix} \geq 0 \), \( N_1 \) and \( N_2 \) have appropriate dimensions, hold,

\[
\Phi = \begin{bmatrix}
\Phi_{11} & \Phi_{12} (A-I)^T H \\
* & \Phi_{22}
\end{bmatrix} < 0
\]

\[
\tilde{X} = \begin{bmatrix}
X_{11} & X_{12} & N_1 \\
* & X_{22} & N_2 \\
* & * & Z
\end{bmatrix} \geq 0
\]

then, the system(2) with the time-delay constant(4) is stable when \( u(k) = 0 \).

Proof. Given \( y(k) = x(k+1) - x(k) \), then \( x(k+1) = x(k) + y(k) \).

Choose Lyapunov functional candidates as:

\[
V(k) = V_1(k) + V_2(k) + V_3(k)
\]

\[
V_1(k) = x^T(k)Px(k)
\]

\[
V_2(k) = \sum_{\theta=\tau_2+1}^{\tau_1} \sum_{l=k-\tau_2+1}^{\tau_1} \left( y^T(l)Zy(l) \right)
\]

\[
V_3(k) = \sum_{\theta=\tau_2+1}^{\tau_1} \sum_{l=k-\tau_2+1}^{\tau_1} \left( x^T(l)Qx(l) \right)
\]

where \( P = P^T > 0, Q = Q^T \geq 0 \) and \( Z = Z^T \geq 0 \).

Let \( \Delta V(k) = V(k+1) - V(k) \), then

\[
\Delta V_1(k) = x^T(k+1)Px(k+1) - x^T(k)Px(k) = 2x^T(k)Py(k) + y^T(k)Py(k)
\]

\[
\Delta V_2(k) = \tau_2 y^T(k)Zy(k) - \sum_{l=k-\tau_2+1}^{\tau_1} \left( y^T(l)Zy(l) \right) \leq \tau_2 y^T(k)Zy(k) - \sum_{l=k-\tau_2}^{\tau_1} \left( y^T(l)Zy(l) \right)
\]

\[
\Delta V_3(k) = (\tau_2 - \tau_1 + 1)x^T(k)Qx(k) - \sum_{l=k-\tau_2}^{\tau_1} \left( x^T(l)Qx(l) \right) \leq (\tau_2 - \tau_1 + 1)x^T(k)Qx(k) - x^T(k-\tau)Qx(k-\tau)
\]

The differential function of Lyapunov function \( V(k) \) is written as follows:
\[ \Delta V(k) = 2x^T(k)Py(k) + y^T(k)Py(k) + \tau_2 y^T(k)Zy(k) - \sum_{l=1}^{k-1} \left( y^T(l)Zy(l) \right) + (\tau_2 - \tau_1 + 1)x^T(k)Qx(k) \]

\[ -x^T(k - \tau)Qx(k) \]

Equation (8) and matrices \( N_i (i = 1, 2) \) are used, the written equation is was follows:

\[ 2\left[ x^T(k)N_1 + x^T(k - \tau)N_2 \right] - \left[ x(k) - x(k - \tau) - \sum_{l=k-\tau}^{k-1} y(l) \right] = 0 \quad (9) \]

On the other hand, as for any matrix \( X = \begin{bmatrix} X_{11} & X_{12} \\ X_{12} & X_{22} \end{bmatrix} \geq 0 \), the following equation holds:

\[ \sum_{l=k-\tau}^{k-1} \left( \xi_1^T(l)X\xi_1^T(l) \right) - \sum_{l=k-\tau}^{k-1} \left( \xi_2^T(l)X\xi_2^T(l) \right) = \tau_2 \xi_1^T(l)X\xi_2^T(l) - \sum_{l=k-\tau}^{k-1} \left( \xi_1^T(l)X\xi_2^T(l) \right) \quad (10) \]

where, \( \xi_1(k) = \begin{bmatrix} x^T(k) & x^T(k - \tau) \end{bmatrix} \), add the left sides of equation(9) and equation(10) into \( \Delta V(k) \), hence we have

\[ \Delta V(k) \leq \xi_1^T(k)\Xi \xi_1(k) - \sum_{l=k-\tau}^{k-1} \left( \xi_2^T(k) \Psi \xi_2(k) \right) \]

where

\[ \xi_2(k,l) = \begin{bmatrix} \xi_1^T(k) & y^T(l) \end{bmatrix}, \Xi = \begin{bmatrix} \Phi_{11} + (A - I)^T H(A - I) & \Phi_{12} + (A - I)HA_x \\ \Phi_{12}^T + A_I^T H(A - I) & \Phi_{22} + A_{I}^T H A_x \end{bmatrix} \]

So, an NCS is stable is only if \( \Xi < 0 \) and \( \Psi > 0 \). According to Lemma 4, we know that \( \Xi < 0 \) is equal to \( \Phi < 0 \).

Theorem 6. Given \( \tau_i > 0 (i = 1, 2) \), if there exist constant matrices \( L \), \( W \), \( R \), \( Y \), \( M_1 \), \( M_2 \), \( V \), where \( L = L^T > 0 \), \( W = W^T \geq 0 \), \( R = R^T \geq 0 \), \( Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{12} & Y_{22} \end{bmatrix} \geq 0 \), \( M_1 \), \( M_2 \) and \( V \) have appropriate dimensions, holds:

\[ \Pi = \begin{pmatrix} \pi_{11} & \pi_{12} & \pi_{12} & \pi_{11} \\ * & \pi_{21} & \pi_{23} & \tau_2 \pi_{11} \\ * & * & -L & 0 \\ * & * & * & -\tau_1 R \end{pmatrix} < 0 \quad (11) \]

\[ \tilde{Y} = \begin{pmatrix} Y_{11} & Y_{12} \geq 0 \\ * & Y_{22} & M_1 \\ * & * & M_2 \end{pmatrix} \quad (12) \]

then, the system(2) with the time-delay constraint(4) can be controlled and \( K = VL^{-1} \).

where

\[ \pi_{11} = (\tau_2 - \tau_1 + 1)W + AL + LA^T - 2L + BV + V^TB^T + M_1 + M_1^T + \tau_2 Y_{22}, \pi_{12} = A_I L + M_2^T - M_1 + \tau_2 Y_{12}, \]

\[ (138) \]
\[ \pi_{13} = L(A-I)^T + V^T B^T, \pi_{22} = -W - M_2 - M_2^T + \tau_2 Y_{22}. \]

**Proof.** According to the proof of Theorem 5, the LMI (6) is changed by using Lemma 4.

\[
\Phi = \begin{pmatrix} \Phi_{11} & \Phi_{12} & (A-I)^T P & \tau_2 (A-I)Z \\ * & \Phi_{22} & A_1^T P & \tau_2 A_1^T Z \\ * & * & -L & 0 \\ * & * & * & -\tau_2 Z \end{pmatrix} < 0 \tag{13}
\]

Then, the NCSs (8) under the control of the memoryless state feedback controller and its state equation is

\[ x(k+1) = (A+BK)x(k) + A_x x(k-\tau) \tag{14} \]

As for the system (14), \( A+BK \) is used instead of \( A \) in the LMI (13), then the LMI (13) is

\[
\tilde{\Phi} = \begin{pmatrix} \tilde{\Phi}_{11} & \tilde{\Phi}_{12} & (A+BK-I)^T P & \tau_2 (A+BK-I)Z \\ * & \tilde{\Phi}_{22} & A_1^T P & \tau_2 A_1^T Z \\ * & * & -L & 0 \\ * & * & * & -\tau_2 Z \end{pmatrix} < 0 \tag{15}
\]

where

\[
\tilde{\Phi}_{11} = (-\tau_2 - \tau_i + 1) Q + P(A+BK-I) + (A+BK-I)^T P + N_1 + N_1^T + \tau_2 X_{11}, \quad \tilde{\Phi}_{12} = PA_1 + N_2 - N_2^T + \tau_2 X_{12},
\]

\[
\tilde{\Phi}_{22} = -Q - N_2 - N_2^T + \tau_2 X_{22}.
\]

So, the method of proof for Theorem 5 is used, then

\[
diag\{P^3, P^{-1}, P^{-1}, Z^{-1}\} \bullet \Phi \bullet diag\{P^{-1}, P^{-1}, P^{-1}, Z^{-1}\} < 0 \tag{16}
\]

\[
diag\{P^{-1}, P^{-1}, P^{-1}\} \bullet \tilde{X} \bullet diag\{P^{-1}, P^{-1}, P^{-1}\} \geq 0 \tag{17}
\]

where

\[
Y = diag\{P^{-1}, P^{-1}\} \bullet X \bullet diag\{P^{-1}, P^{-1}\}, \quad L = P^{-1}, \quad W = P^{-1} Q P^{-1}, \quad R = Z^{-1}, \quad M_1 = P^{-1} N_1 P^{-1},
\]

\[
M_2 = P^{-1} N_2 P^{-1}, \quad V = KP^{-1}.
\]

The final conclusion is drawn by calculating the LMIs (16), (17).

**Conclusions**

The stability of NCSs is analysed through an improved stability criteria that proposed in the derivative of Lyapunov function and consider the relationship between the network-induced delay which is time-varying and its bound. A stabilizing state feedback controller is applied in each subsystem of NCSs in a free-weighting LMIs formulation.

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