Do we need soft cosmology?

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We examine the possibility of “soft cosmology”, namely small deviations from the usual framework due to the effective appearance of soft-matter properties in the Universe sectors. One effect of such a case would be the dark energy to exhibit a different equation-of-state parameter at large scales (which determine the universe expansion) and at intermediate scales (which determine the sub-horizon clustering and the large scale structure formation). Concerning soft dark matter, we show that it can effectively arise due to the dark-energy clustering, even if dark energy is not soft. We propose a novel parametrization introducing the “softness parameters” of the dark sectors. As we see, although the background evolution remains unaffected, due to the extreme sensitivity and significant effects on the global properties even a slightly non-trivial softness parameter can improve the clustering behavior and alleviate e.g. the $f\sigma_8$ tension. Lastly, an extension of the cosmological perturbation theory and a detailed statistical mechanical analysis, in order to incorporate complexity and estimate the scale-dependent behavior from first principles, is necessary and would provide a robust argumentation in favour of soft cosmology.

Introduction

Standard cosmology has been proven very efficient, qualitatively and quantitatively, in describing the Universe evolution and properties at early and late times, as well as at large and small scales. Nevertheless, since cosmology has now become an accurate science, with the appearance of a huge amount of data of progressively increasing precision, slight disagreements, deviations and tensions between theory and observations lead to a large variety of extensions and modifications of the concordance paradigm.

Although in the usual ways of extensions one may add various novel fields, fluids, sectors and their mutual interactions [1], or alter the underlying gravitational theory [2], there is a rather strong assumption that is maintained in all of them, namely that the sectors that constitute the Universe are simple, or equivalently that one can apply the physics, the hydrodynamics and thermodynamics of usual, “hard” matter. Nevertheless, in condensed matter physics it is well known that there is a large variety of “soft” matter forms, which are characterized by complexity, simultaneous co-existence of phases, entropy dominance, extreme sensitivity, viscoelasticity, etc, properties that arise effectively at intermediate scales due to scale-dependent effective interactions that are not present at the fundamental scales [3, 4].

In this Letter we examine the possibility of “soft cosmology”, namely small deviations from the usual framework due to the effective appearance of soft-matter properties in the Universe sectors. We mention that due to the extreme sensitivity and significant effects of softness on the global properties, one does not need to consider a large deviation from standard considerations, since even a very slight departure would be adequate to improve the observed cosmological behavior at the required level. Finally, we stress that new fundamental physics is not directly needed, since the dark energy dynamical evolution and clustering, which is a widely accepted possibility in many scenarios beyond ΛCDM paradigm, is adequate to effectively induce the soft behavior.

Standard Cosmology

Let us briefly review the basics of cosmology [5]. The cosmological principle (the universe is homogeneous and isotropic at large scales) allows to consider the Friedmann-Robertson-Walker (FRW) metric $ds^2 = dt^2 - a^2(t) \delta_{ij} dx^i dx^j$. Concerning the universe content, one considers the usual baryonic matter and radiation (i.e. all Standard Model particles), the dark matter sector, as well as the dark energy sector. The microphysics of dark matter is unknown, and its source may be most probably some particle(s), however it may arise from black holes, from modified gravity, or even from a combination of the above, i.e. a multi-component dark matter [6]. The microphysics of dark energy is unknown too, and it may arise from new fields or matter forms in the framework of general relativity, or it may have an effective nature of gravitational origin due to modifications of gravity.

The next step is to consider that (at least after a particular stage of the universe evolution) cosmological scales are suitably large in order to allow one to neglect the microphysics of the universe ingredients and describe them effectively through fluid dynamics and continuum flow (at earlier stages one should use the Boltzmann equation). Hence, one can ignore the microscopic Lagrangian of the various sectors, and write their energy momentum tensors as $T_{\mu\nu}^{(i)} = (\rho_i + p_i) u_\mu u_\nu + p_i g_{\mu\nu}$, with $\rho_i$ and $p_i$ the energy density and pressure of the fluid corresponding to the $i$-th sector, $u_\mu$ the 4-velocity vector field and $g_{\mu\nu}$ the metric. Note that one can extend the above expression by including viscosity or/and heat flux.

Under the above considerations, any cosmological scenario will be determined by the two Friedmann equations

$$H^2 = \frac{\kappa^2}{3} (\rho_b + \rho_r + \rho_{dm} + \rho_{de}),$$

$$2\dot{H} + 3H^2 = -\kappa^2 (\rho_b + \rho_r + \rho_{dm} + \rho_{de}),$$

where $H = \dot{a}/a$ is the Hubble parameter, $\kappa^2 = 8\pi G/3$ and $G$ is the gravitational constant.
with $\kappa^2 = 8\pi G$, and where $H \equiv \dot{a}/a$ is the Hubble parameter. Additionally, the conservation equation $\nabla^{\mu} T^{(tot)}_{\mu\nu} = \nabla^{\mu} \left( \sum_i T^{(i)}_{\mu\nu} \right) = 0$ in the case of FRW geometry and for non-interacting fluids gives rise to the separate conservation equations $\dot{\rho}_i + 3H(\rho_i + p_i) = 0$, while the extension to interacting cosmology can be realized through phenomenological descriptors $Q_i$ of the interaction with $\sum_i Q_i = 0$ and with $\dot{\rho}_i + 3H(\rho_i + p_i) = Q_i$.

In order for the equations to close we need to impose the equation of state for each sector. The usual consideration is to assume barotropic fluids, in which the pressure is a function of the energy density only, with the simplest case being $p_i = w_i\rho_i$ with $w_i$ the equation-of-state parameter. Lastly, note that the above framework provides $\Lambda$CDM cosmology for $\rho_{de} = -p_{de} = \Lambda/\kappa^2$, with $\Lambda$ the cosmological constant.

The above formulation of cosmological evolution allows one to proceed to a more subtle investigation, and study small perturbations around the FRW background. Focusing without loss of generality to the linear theory of scalar isentropic perturbations in the Newtonian gauge, imposing $ds^2 = -(1 + 2\Psi)dt^2 + a^2(t)(1 - 2\Phi)c^2dx^i dx^i$, then in a general non-interacting scenario which includes the aforementioned sectors the scalar perturbations are determined by the equations \[ \delta_i + (1 + w_i) \left( \frac{\theta_i}{a} - 3\Psi \right) + 3H[c^{(i)}_{\text{eff}}^2 - w_i] \delta_i = 0, \tag{3} \]
\[ \dot{\theta}_i + H \left[ 1 - 3c^{(i)}_{\text{ad}} \right] \theta_i - \frac{k^2 c^{(i)}_{\text{eff}} \delta_i}{(1 + w_i)a} - \frac{k^2 \Psi}{a} = 0, \tag{4} \]
assuming zero anisotropic stress and with $k$ the wavenumber of Fourier modes (in the case of $\Lambda$CDM paradigm the corresponding dark-energy perturbation equations are not considered). In the above equations $\delta_i \equiv \delta \rho_i/\rho_i$ are the density perturbations and $\theta_i$ is the divergence of the fluid velocity. Furthermore, $c^{(i)}_{\text{eff}} \equiv \delta \rho_i/\delta p_i$ and $c^{(i)}_{\text{ad}} \equiv w_i - w_i/3H(1 + w_i)$ are the effective and adiabatic sound speed squares of the $i$-th sector respectively ($c^{(i)}_{\text{eff}}$ determines the amount of clustering). Note that the above equations can be simplified through the consideration of the Poisson equation, which at sub-horizon scales can be written as [7, 8]:
\[ -\frac{k^2}{a^2} \Psi = \frac{3}{2} H^2 \sum_i \left[ \left( 1 + 3c^{(i)}_{\text{ad}} \right) \Omega_i \delta_i \right]. \]
Finally, we note here that in general the above formulation can be applied also in the cases where the dark energy sector is an effective one arising from gravitational modifications.

We close this section by mentioning that one can find a big variety and many versions of the above formalism. However, there is a rather strong assumption that is maintained in all of them, namely that the sectors that constitute the Universe are simple, or equivalently that one can apply the physics of usual matter. In particular, the underlying assumption is that the laws that determine the Universe behavior at large scales can be induced by the laws that determine the interactions between its individual constituents. Focusing on the hydrodynamic description, the use of fluid energy densities and pressures arises from the assumption that we can define fundamental “particles” of the corresponding sector, the collective flow of which gives rise to $\rho_i$ and $p_i$, while all physics below the particle scale has been integrated out.

Soft Cosmology

The above formulation of standard cosmology is definitely correct and can provide a quite successful quantitative description of the Universe evolution. However, the question is: could it miss something on the details? Indeed, in principle there could be two sources of such information loss.

The first is that the integrated-out physics below the “particle” scale could leave different imprints on the physics above the “particle” scale, depending on this coarse-graining scale. For instance the correct integration out of the sub-galaxies physics leaves back a small but non-zero viscosity of the effective baryonic fluid [9], while in all simulations of the galaxies and large-scale structure formation the assumptions on the integrated-out, subgrid physics are important, and the results are quite affected by it [10]. This fact shows that one should use the coarse-grained description with caution, since using the same concept, of e.g. baryonic energy density $\rho_b$ and the effective hydrodynamic description, for scales that differ by orders of magnitude is possible only after a correct integration-out of the neglected physics.

The second source of possible information loss is the assumption that the Universe sectors and interactions are simple and more or less scale-independent. For instance, two regions of the dark-matter fluid will mutual interact in the same way in the intermediate- and late-time universe, or the interaction of two big clusters of dark matter can arise by the superposition of all individual interactions of their sub-clusters, etc. In other words, in the concordance cosmological formulation one assumes that the sectors that constitute the Universe behave as usual, “hard” matter.

“Soft” matter is a research field that has attracted a large amount of interest of the condensed matter community [3, 4], since it has very interesting and peculiar properties far different than those of hard matter. The problem is that there is not a definition of what is soft matter. In particular, the best definition we have is that soft matter is the one that has the properties of soft materials. Examples of soft materials are the polymers (plastic, rubber, polystyrene, lubricants etc), the colloids (paints, milk, ice-cream etc), the surfactants, granular materials, liquid crystals, gels, biological matter (proteins, RNA, DNA, viruses, etc), etc. Although these examples of soft matter are very different from each other, they have some common properties and features that distinguish them from usual, hard, matter. Amongst others these include complexity (new qualitative properties arise at intermediate scales due to interactions that are not present at the fundamental scales), co-existence of phases (they have different phase properties depending on the scale at one
examines them, e.g. at the same time they can be fluid at small scales and solid at large scales), entropy dominance instead of energy dominance, flexibility, extreme sensitivity to reactions, viscoelasticity (they exhibit viscous and elastic properties simultaneously) etc.

In this work we examine the possibility that the dark sectors of the universe may exhibit (intrinsically or effectively) slight soft properties, which could then lead to small corrections to the concordance model. We mention here that the discussion below holds independently of the underlying gravitational theory, i.e. it is valid both in the framework of general relativity as well as in modified gravity, nevertheless in the latter case we have richer possibilities to obtain scale-dependent interactions.

A. Soft Dark Energy – The nature and underlying physics of dark energy is unknown. The basic framework that has been studied in extensive detail is that the dark energy fluid has the same fluid properties at all scales at a given moment/redshift. However, as we saw, in soft matter the complexity that arises at intermediate scales may lead the material to have a different equation of state (EoS) at different scales simultaneously.

As a simple phenomenological model of soft dark energy we may consider the case where dark energy has the usual EoS at large scales, namely at scales entering the Friedmann equations, but having a different value at intermediate scales, namely at scales entering the perturbation equations. In this case the Universe’s expansion history will remain identical to standard cosmology, nevertheless the large-scale structure evolution can deviate from the standard one and can be brought closer to observations. In summary one can obtain richer behavior.

We mention that in the following we focus on sub-horizon scales \( k \gg aH \) and thus to perturbation modes affected only by the intermediate-scale dark-energy EoS. The full analysis, in which different perturbation modes are affected by different EoS according to their scale, will be presented elsewhere.

A first approach on the subject would be to introduce the effective “softness parameter” \( s_{\text{de}} \) of the dark energy sector. This implies that while at cosmological, large scales (ls) dark energy has the usual EoS, namely \( w_{\text{de-ls}} \), at intermediate scales (is) we have

\[ w_{\text{de-is}} = s_{\text{de}} \cdot w_{\text{de-ls}}, \tag{5} \]

and standard cosmology is recovered for \( s_{\text{de}} = 1 \).

For instance let us assume that the large-scale dark energy EoS \( w_{\text{de-ls}} \) is a constant one \( w_{\text{de-ls}} = w_0 \) or e.g. the CPL one \( w_{\text{de-ls}} = w_0 + w_a(1 - a) \). According to (5) at intermediate scales the dark energy EoS \( w_{\text{de-is}} \) is different, either constant \( w_{\text{de-is}} = w_2 \) or time-varying. Hence, the background Universe evolution will remain the same, however since \( c_{\text{eff}}^{(\text{de})} \) will change, through the Poisson equation and (3),(4) we will acquire a different evolution for the matter overdensity \( \delta_m \). Hence, the resulting \( f_{\sigma_8} \) value, with \( f(a) = d \ln \delta_m(a)/d \ln a \) and \( \sigma(a) = \sigma_0 \delta_m(a) / \delta_m(1) \), will be different than the corresponding one of standard cosmology with the above dark energy EoS (note that since the background behavior remains unaffected we do not need to worry about incorporating fiducial cosmology \([11]\)). As we observe, we have a straightforward way to alleviate the \( \sigma_8 \) tension since we can suitably adjust \( w_{\text{de-is}} \) in order to obtain slightly lower \( f_{\sigma_8} \). As a specific example in Fig. 1 we depict the \( f_{\sigma_8} \) as a function of \( z \). The dashed curve is for \( \Lambda \)CDM. The solid curve is for soft dark energy with \( s_{\text{de}} = 1.1 \), i.e. with \( w_{\text{de-is}} = -1.1 \) and \( c_{\text{eff}}^{(\text{de})} = 0.1 \), while dark matter is the standard one (i.e. not soft) with \( w_{\text{dm}} = 0 \). Note that in principle \( s_{\text{de}} \) can be varying too and one could introduce its parametrization, or one could additionally have more complicated situations in which \( w_{\text{de-is}} \) and \( w_{\text{de-is}} \) have different parametrizations. In this first approach on soft dark energy we consider the simplest case of (5).

![FIG. 1: The \( f_{\sigma_8} \) as a function of \( z \). The dashed curve is for \( \Lambda \)CDM. The solid curve is for soft dark energy with \( s_{\text{de}} = 1.1 \), i.e. with \( w_{\text{de-is}} = -1.1 \) and \( c_{\text{eff}}^{(\text{de})} = 0.1 \), while dark matter is standard (i.e. not soft) with \( w_{\text{dm}} = 0 \).](image)

Finally, note that soft materials may exhibit different EoS properties not only at different scales, but at different directions too. In this case, one may think of a dark energy sector that has a different EoS at different directions, namely an anisotropic dark energy. However, such an analysis would require to deviate from FRW and consider explicitly anisotropic geometries such as the Bianchi ones. We will study this possibility in a separate work.

We close this paragraph by referring to one of the properties of soft materials that can be relatively easily quantified, namely viscoelasticity \([13]\). In order to measure it one applies a sinusoidal strain \( \epsilon = \epsilon_0 \sin(\omega t) \) and measures the induced stress, which for small \( \epsilon_0 \) is of the form \( \sigma = \sigma_0 \sin(\omega t + \delta) \). Pure elastic materials have \( \delta = 0 \), pure viscous materials have \( \delta = \pi/2 \), while viscoelastic materials have a non-trivial \( \delta \) value. As a toy model to estimate the viscoelasticity of dark energy we start from the Friedmann equations (1),(2) and we perturb the scale

\[ 1 \text{ The validity of secondary assumptions related to } \sigma_8 \text{ data formalism, such as the irrotational velocity field for the matter fluid \([12]\), should be carefully examined in the case of soft matter, nevertheless for small softness parameters, namely for small deviation from standard matter, one expects them to remain valid too.} \]
factor solution \( a(t) \) as \( a(t) \rightarrow a(t) + \epsilon_0 \sin(\omega t) \), asking to see what will be the extra pressure (i.e. stress) that one should obtain in (2) in order to maintain consistency. One can easily see that for a general dark energy equation of state \( w_{de} \) parametrization the resulting extra pressure will have a non-trivial \( \delta \) (alternatively one could impose both a strain and a non-trivial extra pressure with a given \( \delta \), and reconstruct suitably \( w_{de}(z) \) in order to obtain consistency, i.e. a dark-energy EoS parametrization with a desired viscoelasticity). From this simple and crude argument we deduce that a general dark energy has non-trivial viscoelasticity (we mention that viscoelasticity is used as an extra argument, since by itself is not adequate to characterize a material as soft).

B. Soft Dark Matter – In this subsection we examine the possibility that the dark matter sector exhibits soft properties. In the framework of general relativity the gravitational interaction of dark matter with itself or with baryonic matter cannot produce internal complexity (unless the unknown microphysics of dark matter does impose an intrinsic soft structure). However, even if it is not intrinsic, soft behavior in the dark matter sector can still arise in an effective way due to the presence of non-trivial dark energy. Specifically, if the dark energy is clustering then, even if dark energy is not intrinsically soft, it will induce scale-dependent, qualitatively different intermediate structures in the dark matter clustering, at scales similar to the dark energy clusters. In particular, the interaction between two dark-matter clusters below the dark-energy clustering scale (i.e. two dark-matter clusters with sparse dark energy between them) will be different from the interaction between two dark-matter clusters with a dark-energy cluster between them.

Hence, one will have the effective appearance of screening effects at intermediate scales, and thus of complexity (this is the standard way that complexity appears in the colloids, namely due to the non-trivial, scale-dependent structure of the bulk between them). In summary, one could have a dark matter sector which at large scales, namely at scales entering the Friedmann equations, behaves in the usual dust way, but which at intermediate scales, namely at scales entering the (sub-horizon) perturbation equations, it could slightly deviate from that.

We can introduce the dark matter softness parameter \( s_{dm} \) (standard cosmology is recovered for \( s_{dm} = 1 \)) as:

\[
\frac{w_{dm} - is}{w_{dm} - ls} + 1 = s_{dm} \cdot \left( \frac{w_{dm} - ls}{1} \right),
\]

(mind the difference in the parametrization comparing to soft dark energy, in order to handle the fact that the dark matter EoS at large scales \( w_{dm} - ls \) is 0). Similarly to the example of the previous subsection, the background evolution will remain identical with that of standard cosmology, but the perturbation behavior (at sub-horizon scales) and the large-scale structure can be improved. In order to provide a specific example, in Fig. 2 we depict the \( f\sigma_8 \) as a function of \( z \), in the case of soft dark matter with \( w_{dm} - is = 0.05 \), i.e for dark matter with softness parameter \( s_{dm} = 1.05 \), while \( w_{de} - ls = w_{de} - is = -1 \).

In the above analysis we did not consider the dark energy sector to be soft. Definitely, proceeding to such a possibility would make the induced soft behavior for dark matter easier. Moreover, this would be the case if one considers a mutual interaction between dark matter and dark energy too, since a different than usual dark-energy clustering behavior would be transferred to a different than usual dark-matter clustering behavior, due to the interaction. Finally, deviating from general relativity would provide additional possibilities to induce effective soft properties to the dark matter sector, since dark matter will implicitly interact in a scale-dependent way (one would have the additional screening behavior due to the extra (scalar) graviton degrees of freedom that dress the dark matter in a scale-dependent way, altering its self-interaction [14]).

Let us make a comment here on the clustering features. The clustering behavior of soft matter has been extensively studied, and indeed it has been shown that the resulting spectrum, factorial moments, fractal dimension, etc, depend on the specific intermediate-scale features. For instance the fractal dimension has been experimentally found to cover all the range from 1 to 3 according to different materials (e.g. colloids of gold nanoparticle in aqueous media give \( d_f = 1.75 \pm 0.05 \) for diffusion-limited kinetics and \( d_f = 2.01 \pm 0.10 \) for reaction-limited kinetics) [15, 16]). On the other hand, the large scale structure and the galaxy distribution in the Universe has a fractal dimension \( d_f = 1.63 \pm 0.20 \) [17, 18]. The fact that soft matter clustering exhibits naturally non-trivial dynamics due to its intermediate-scale complexity, could be useful in describing the details of the observed large-scale structure. We mention here that the non-trivial clustering of soft matter changes at scales below the intermediate ones, and hence soft dark matter could alleviate the cuspy halo problem [19], the dwarf galaxy problem [20], and other clustering-related problems that seem to puzzle the standard collisionless dark matter.

We close this subsection by referring to the possibility that the induced effective soft properties on the dark matter sector could lead to a direction-dependent EoS.
Such slightly anisotropic dark matter might have a non-negligible effect on the lensing behavior. The detailed investigation of this possibility is left for a separate work.

C. Soft Inflation – As a last application of soft cosmology we refer to the possibility that the inflation realization could exhibit soft features. Due to the extremely small length and time scales of the inflationary phase one may expect that complexity cannot be formed. Although this is indeed reasonable, due to the extreme sensitivity of the system behavior on even very small soft properties, one could still have the case that slightly non-zero soft features could appear and then affect the inflation observables. For instance, the inflation-driven field/fluid could develop a slight cluster structure during inflation, inside the causal horizon, being either scale-dependent or direction-dependent. The investigation of this possibility requires to deviate from the usual homogeneous and isotropic consideration. Definitely, soft inflation could be thought as less possible than soft dark energy and soft dark matter, however it could still serve as the underlying mechanism of anisotropic inflation [21] and its role on explaining the possible non-trivial CMB anisotropies.

Conclusions
We examined the possibility of “soft cosmology”, namely small deviations from the usual framework due to the effective appearance of soft properties in the Universe sectors. We started by considering the possibility of soft dark energy due to intermediate-scale features that could arise from its unknown microphysics. One effect of such a case would be the dark energy to exhibit a different EoS at large and intermediate scales. As we saw, although the background evolution remains unaffected, even a slight softness at intermediate scales can improve the clustering behavior and alleviate e.g. the $f\sigma_8$ tension.

We proceeded to the examination of soft dark matter, which can effectively arise just due to the dark-energy clustering even if dark energy is not soft. By considering a slightly different equation of state at large and intermediate scales we were able to improve the clustering behavior. Furthermore, in the additional incorporation of soft dark energy, and/or modified gravity, the effective soft properties of dark matter could be richer, due to the extra screening mechanisms. Finally, for completeness we qualitatively presented the case of soft inflation.

We mention that in this work we incorporated softness by phenomenologically introducing a slightly different EoS at different scales. Clearly, in order to incorporate complexity and estimate the scale-dependent behavior of the equation of states from first principles one should revise and extend the cosmological perturbation theory and perform a detailed mesoscopic statistical mechanical analysis. Such a full investigation is necessary and would provide a robust argumentation in favour of soft cosmology (and towards this direction the Boltzmann-equation-based theoretical investigation of soft matter hydrodynamics will be used [22]), nevertheless it lies beyond the scope of this initial investigation and will be performed elsewhere. However, even in the present, phenomenological framework, since the rheology and dynamics of soft matter is different than the usual, hard one, one should confront it in detail with various observational datasets and examine if non-trivial softness parameters are favoured (or even if they exhibit more complex, scale-dependent or direction-dependent features). This detailed observational confrontation will be presented in a separate project.

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