Dark energy as a critical phenomenon: a hint from Hubble tension

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Abstract. We propose a dark energy model based on the physics of critical phenomena which is consistent with both the Planck’s CMB and the Riess et al.’s local Hubble measurements. In this model the dark energy density behaves like the order parameter of a generic system which undergoes a phase transition. This means the dark energy is an emergent phenomenon and we named it critically emergent dark energy model, CEDE. In CEDE, dark energy emerges at a transition redshift, $z_c$, corresponding to the critical temperature in critical phenomena. Combining the Planck CMB data and local measurement of the Hubble constant from Riess et al. (2019) we find statistically significant support for this transition with respect to the case of very early transition that represents effectively the cosmological constant. This is understandable since CEDE model naturally prefers larger values of Hubble constant consistent with local measurements. Since CEDE prefers a non-trivial transition when we consider both high redshift Planck CMB data and local Hubble constant measurements, we conclude that $H_0$ tension may be a hint for the substructure of the dark energy as a well-studied properties of critical phenomena. However if we add BAO and SNe datasets then CEDE prefers lower value for $H_0$. This means the $H_0$ tension still exist but it is milder than $\Lambda$CDM’s.

Keywords: dark energy theory, modified gravity

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1 Introduction

Riess et al. [1–3] have devoted efforts to measure and constrain the present Hubble parameter and their most recent result shows $H_0 = 74.03 \pm 1.42$ km/s/Mpc by the analysis of the Hubble Space Telescope observations using 70 long-period Cepheids in the Large Magellanic Cloud. This observation is in 4.4$\sigma$ tension with the prediction of CMB observations by Planck satellite [4]. This tension can be due to systematics, as speculated e.g. by [5], but its chance has become less and less thanks to the independent local measurements of $H_0$ e.g. based on Type II supernovae [6] ($H_0 = 75.8^{+0.2}_{-0.9}$ km/s/Mpc) or based on strong gravitational lensing effects on quasar systems, H0LiCOW, ($H_0 = 73.5^{+1.7}_{-1.8}$ km/s/Mpc) [7–9] or based on a calibration of the Tip of the Red Giant Branch, TRGB, ($H_0 = 69.8 \pm 1.9$ km/s/Mpc) [10] or based on calibration of the SN Ia luminosity using highly-evolved low-mass stars, Miras, ($H_0 = 73.6 \pm 3.9$ km/s/Mpc) [11] or based on calibration of the type Ia supernovae with surface brightness fluctuations, SBF, ($H_0 = 76.5 \pm 4.0$ km/s/Mpc) [12] or by using geometric distance measurements to megamaser-hosting galaxies, MCP, ($H_0 = 74.8 \pm 3.1$ km/s/Mpc) [13], or based on Tully-Fisher relations ($H_0 = 76.0 \pm 2.6$ km/s/Mpc) [14]. In addition it is worth to mention that the $\Lambda$CDM model suffers from some other (mild) tensions e.g. $S_8$ [15], low/high $\ell$ [16] and CMB spatial anomalies [17–19]. Nevertheless, $H_0$ tension seems the most robust one supported by many observations. In near future the situation will be clear and if this tension be real then it is very important to know if there is any theoretical explanation for it.

There are many different theoretical attempts to address the $H_0$ tension [20]. Dynamical dark energy models [21–27] and gravity theories in which gravity changes with redshift [28–32] are examples of the late universe modifications. On the other hand, assuming an early dark energy phase before recombination in order to decrease the sound horizon [33–36] or non-standard recombination scenarios [37–40] are the main topics of the early universe solutions. There have been though some serious doubts on how these early dark energy models can practically resolve the Hubble tension [41].

There are also interacting dark energy models [42–58], in which dark matter and dark energy have an extra non-gravitational interaction in hope for alleviating this tension. However, there remains some doubts and questions about the detection of dark energy dark matter interaction [59]. Furthermore, decaying dark matter can be a remedy for this problem [60–62]. In addition, one can have a higher $H_0$ at the price of extra relativistic degrees
of freedom, parameterized by the $N_{\text{eff}}$ [63, 64]. In [65], one can find a complete review on the $H_0$ tension and its possible solutions.

We have also proposed a model to address the $H_0$ tension based on the idea of critical phenomena in [66–68]. In these works we considered that the dark energy experienced a phase transition in its history. This assumption can be natural due to the discrepancy between early and late cosmology. In [66], this phase transition has been modeled phenomenologically by a tanh function. This behavior for dark energy has been assumed independently in [69–72] as Phenomenological Emergent Dark Energy (PEDE). In this work we would like to generalize our idea in [66] by assigning a more realistic time evolution to dark energy. In the physics of critical phenomena, the Ising model is one of the well-considered models and shares its behavior with many other ones. We will assume that dark energy behaves like an Ising model which results in a very specific time evolution for it.

2 Critically Emergent Dark Energy (CEDE)

Adapted from the literature of critical phenomena, we suppose that the dark energy consists of a sort of self interacting micro-structures which carry a local “order parameter”. Coarse graining of this system of particles (micro-structures) turns it into a continuum. This allows us to attribute a scalar field, $\phi(r, z)$, to be representative of this order parameter in each point of space and time (redshift). We assume the spatial average of $\phi$ is responsible for the background density of the dark energy:

$$\Omega_{DE}(z) = \langle \phi(r, z) \rangle \equiv M(z). \quad (2.1)$$

It is worth to recall that taking spatial average means we are in the mean field approximation. In this approximation, the spatial variance of the field is averaged out while its time dependence is fully considered.\(^1\) For now, we assume that our dark energy is somehow in thermal contact\(^2\) with the radiation and has a temperature proportional to the cosmological redshift, $T_{DE} \propto 1 + z$. At early times or equivalently high temperatures, the local order parameter changes rapidly and there is no long range correlation in the dark energy field. It is completely disordered and its spatial average vanishes. As the universe cools down, order begins to emerge and the field starts to show a long range correlation. After a critical temperature, $T_c$, the average of $\phi$ takes distance from zero and dark energy appears in a continuous phase transition. To formulate this scenario, we use the mathematical description of all kinds of phase transitions, developed by Ginzburg and Landau [73]. It is very important to mention that Ginzburg-Landau description of the phase transition is a kind of effective theory. This means it is true for all the systems which undergo a phase transition even if there is no knowledge about the microscopic details of these systems. So this formalism is very well suited for our purpose where we have not assumed any specific model for DE substructures. Accordingly, we introduce the effective free energy\(^3\)

$$E_F = \int d^3r \left[ \frac{1}{2} (\nabla \phi)^2 + m t \phi^2 + \frac{1}{2} \lambda \phi^4 \right]. \quad (2.2)$$

\(^1\)We will comment on going beyond the mean field approximation at the discussions.

\(^2\)It is important to emphasize that this interaction should be very tiny. Otherwise we expect modification in the behaviour of photons which are very well-constrained. On the other hand, theoretically one can assume that the phase transition in the behaviour of DE is realized by changing of the temperature due to the expansion rate of the universe. In this scenario we could assume $T_{DE} \propto (1 + z)^\beta$. The $\beta$ dependence does not change the general behavior of our model and here we assume $\beta = 1$.

\(^3\)In the literature of critical phenomena it is shown by $L$. But here we used $E_F$ to not make any confusion with the Lagrangian.
and demand that the $E_F$ be always stationary with respect to the variations of $\phi$. In the above expression, $t$ is the reduced temperature, $t \equiv \frac{T - T_c}{T_c}$, and $\gamma$, $m$ and $\lambda$ are some positive constants that are supposed to be determined from observations. As it is evident in (2.2), we have supposed $\mathbb{Z}_2$ symmetry for our effective free energy; i.e. there isn’t any odd power of $\phi$. This symmetry implies that the phase transition is continuous or second order. For the equation of motion of $\phi$ we obtain:

\[- \gamma \nabla^2 \phi(r) + 2 m t \phi(r) + 2 \lambda \phi^3(r) = 0.\tag{2.3}\]

At the first approximation, we consider $\phi$ within the mean field approach: $\phi(r) = \langle \phi(r) \rangle \equiv M$. Hence it should obey the following equation:

\[2 m t M + 2 \lambda M^3 = 0,\tag{2.4}\]

which yields:

\[M = 0 \quad \text{or} \quad M = \pm \sqrt{\frac{m t}{\lambda}} = \pm \sqrt{\frac{m z_c - z}{1 + z_c}}.\tag{2.5}\]

The last equality comes from our assumption for the dark energy temperature, $T_{DE} \propto 1 + z$. The case $M = 0$ corresponds to $T > T_c$ and the two other cases belong to $T < T_c$; after spontaneous symmetry breaking, $M$ takes one of these two possible temporal functionalities.\footnote{Note that what is observable is $m/\lambda$. According to (2.2), the parameter $m$ encodes the interaction strength between DE and other fields (e.g. photons). This means even by setting $m$ to be very small, we expect we can have the phase transition but ignore the effects on the other fields. In this scenario we can set $\lambda$ to get an appropriate value for dark energy density.}

We choose $M$ to take the positive one, because negative values for the dark energy density are not very pleasing philosophically. It is worth to clarify that we assumed quasi-static case for the DE system. This effectively means the interaction has smaller time scale in comparison to the Hubble time.

In addition, spatially flatness of the universe fixes the coefficient $m/\lambda = (1 - \Omega_m - \Omega_r)^2 (1 + z_c)/z_c$ in (2.5) and we obtain

\[\Omega_{DE}(z) = (1 - \Omega_m - \Omega_r) \sqrt{\frac{z_c - z}{z_c}},\tag{2.6}\]

where $\Omega_m$ and $\Omega_r$ are the matter and radiation fractional densities at present time, respectively. So in this step we have the form of the Friedmann equation,

\[H^2 = H_0^2 \left[ \Omega_m (1 + z)^3 + \Omega_r (1 + z)^4 + \Omega_{DE}(z) \right].\tag{2.7}\]

We can deduce the dark energy’s equation of state from the continuity equation, namely,

\[\rho'_{DE}(z) - \frac{3}{1 + z} \rho_{DE}(z) [1 + w_{DE}(z)] = 0,\tag{2.8}\]

where $'$ denotes the derivative with respect to the redshift. By substituting $\rho_{DE}(z)$ from (2.6) into (2.8) we can read $w_{DE}(z)$ as

\[w_{DE}(z) = -1 - \frac{1 + z}{6 (z_c - z)}.\tag{2.9}\]
Figure 1. The equation of state of dark energy versus redshift for a typical transition redshift, $z_c = 2$. The grey lines are horizontal and vertical asymptotes of this function which lie at $w = -1$ and $z = z_c$, respectively. Note that as we discussed in the draft, $\rho_{DE}(z)$ is the physical quantity and well behaved at $z_c$ even if $w_{DE}(z)$ diverges at the critical redshift. The sub-plot inside, is the same function with linear horizontal axis.

This relation holds for any moment after onset of the phase transition and before that time, since the density of dark energy is zero, $w_{DE}$ is not well-defined. Right at the moment of transition, $w_{DE}$ diverges toward $-\infty$ and afterwards, at far future, approaches $-1$ (cf. figure 1). Physically, this means the DE effectively is in a phantom phase and dynamically approaches to the cosmological constant. This is a behavior that we expect from emergent dark energy models [69, 71]. We emphasize that since $w_{DE}$ is not appeared in the physics directly then $w_{DE} = -\infty$ does not give any physically divergences in our model.\footnote{Note that the physics is given by $\rho(a) = \rho_0 \times exp[-3 \int_{a_0}^a (1 + w(a))d\ln a]$ where an infinite $w_{DE}$ at one point cannot make any divergences.} It is worth to add that we could expect this behaviour since at the critical temperature, it is a very natural behaviour in the physics of critical phenomena to see a discontinuity in some of the parameters.

3 Analysis

The parameter space we want to put constraint on, is

$$\mathcal{P} = \{\Omega_b h^2, \Omega_c h^2, 100\Theta_{MC}, \tau, n_s, \ln[10^{10}A_s], a_c\}, \quad (3.1)$$

which shows that our model has six parameters in common with $\Lambda$CDM and an extra free parameter $a_c$ which represents the scale factor of transition.

In order to put constraint on these free parameters, we focus on the following data sets:

- Planck 2018 temperature and polarization angular power spectra, i.e. combination of the Commander, SimALL and plikTT,TE,EE likelihoods. [74]. We refer to this data as CMB.
- The measurement of $H_0$ by Riess et al. [1]. We refer to this data point as \textbf{R19}.

This relation holds for any moment after onset of the phase transition and before that time, since the density of dark energy is zero, $w_{DE}$ is not well-defined. Right at the moment of transition, $w_{DE}$ diverges toward $-\infty$ and afterwards, at far future, approaches $-1$ (cf. figure 1). Physically, this means the DE effectively is in a phantom phase and dynamically approaches to the cosmological constant. This is a behavior that we expect from emergent dark energy models [69, 71]. We emphasize that since $w_{DE}$ is not appeared in the physics directly then $w_{DE} = -\infty$ does not give any physically divergences in our model.\footnote{Note that the physics is given by $\rho(a) = \rho_0 \times exp[-3 \int_{a_0}^a (1 + w(a))d\ln a]$ where an infinite $w_{DE}$ at one point cannot make any divergences.} It is worth to add that we could expect this behaviour since at the critical temperature, it is a very natural behaviour in the physics of critical phenomena to see a discontinuity in some of the parameters.
• BAO volume distance measurements, at $z = 0.32$ (LOWZ) [77], $z = 0.57$ (CMASS) [77], $z = 0.106$ (6dFGS) [75] and $z = 0.15$ (MGS) [76]. Also we used BAO angular diameter distance measurements at $z = 0.44$, $z = 0.60$ and $z = 0.73$ (WiggleZ) [78]. We refer to these datasets just as BAO

• “Pantheon” sample consisted of 1048 type Ia supernovae spanning the redshift range $0.01 < z < 2.3$ [79]. We refer to this dataset as SNe.

We make use of publicly available code CAMB\(^6\) [80], to calculate the predictions of CEDE for the observables described above. For sampling the parameter space, we use CosmoMC [81, 82]. We use GetDist [83] in order to extract the parameter’s posteriors from the MCMC chains and to plot likelihood contours.

4 Results

The first very important step is to see if independent datasets are compatible for a theoretical setup or not. If they are not compatible then we are not allowed to use both of them at the same time (that usually would result to tight constraints for the model parameters). In the framework of $\Lambda$CDM, CMB and R19 are more than $4\sigma$ incompatible which is the Hubble tension. Though CMB is compatible with BAO+SNe datasets in this model. In the following, we will study our CEDE model against different combination of these datasets. To check the compatibility between these datasets we study CEDE just with CMB dataset and then add the other datasets one by one.

4.1 CMB+R19

The main result can be seen in figure 2 where the red likelihood shows the $2-d$ likelihood for $H_0$ and $a_c$ in CEDE. The result is very promising: constraints from CMB data has no tension with R19 data in this model. Obviously, the $1\sigma$ likelihood touches R19’s $1\sigma$; and $2\sigma$ likelihood for $H_0$ in CEDE reaches to upper $2\sigma$ value for R19. In figure 6, we have shown the $2-d$ and $1-d$ likelihoods for our free parameters as well as derived $H_0$ and $\Omega_\Lambda$. It is obvious from the last row that R19 does not show any inconsistencies in our free parameters. Consequently, we can add up CMB and R19 datasets. The likelihoods for CEDE has been shown in figures 2 and 6 in blue. The best fit values for CEDE’s parameters are reported in table 1 in the presence of CMB with and without R19. Note that we have analyzed $\Lambda$CDM with CMB and R19, in order to be able to compare it with CEDE. But it can not be trusted since these two datasets are incompatible in the framework of $\Lambda$CDM.

The main result is that including R19 needs a non-trivial phase transition in DE by excluding lower values for transition scale factor $a_c$. This lower value at $2\sigma$ is around $a_c \sim 0.2$ which is equivalent to $z_c \sim 4$. To explain it let us emphasize that for very small values of $a_c$ the transition occurs at very high redshifts. This means DE starts to grow from zero to a constant $\Omega_\Lambda$ very early. This transition is so fast and occurs far before DE domination. So it has no observational effects. In other words CEDE behaves exactly same as $\Lambda$CDM for very high transition redshifts. The results show that in the framework of CEDE, $\Lambda$CDM can explain CMB as good as CEDE with non-trivial phase transition. But including R19 breaks this degeneracy in favour of a phase transition in the late time behavior of dark energy. To study this fact more clearly we have plotted the dark energy fractional density ($1\sigma$ values) as a function of redshift in figure 3. It is obvious from this plot that having $\Lambda$CDM is valid in CEDE framework when we use only CMB but including R19 excludes it.

\(^6\)https://camb.info/.
Figure 2. The $1 - 2\sigma$ contours for $a_c$ vs $H_0$ is plotted for CEDE model where Planck CMB data is used without and with R19 data, in red and in blue, respectively. The grey bands show the $1 - 2\sigma$ values for $H_0$ as reported by Riess et al. from local measurements. The red contours show that CEDE model has no inconsistency with R19 which means we can combine R19 and CMB data together to get the joint constraints (blue contour). It is worth to mention that CEDE reduces to $\Lambda$CDM when the transition occurs very early (effectively at $a_c$ tends to 0). The red contours show this property very clearly: for very small $a_c$ the $H_0$ value becomes closer to what Planck reports for $\Lambda$CDM.

Figure 3. The fractional density of dark energy is plotted versus the redshift. The dashed black line represents this function for the $\Lambda$CDM model best fit values. The red line region shows the same for $1\sigma$ values of CEDE when it is constrained by only CMB. Obviously, for this case, CEDE includes $\Lambda$CDM as we discussed in the paper. But including R19 excludes the $\Lambda$CDM as it is shown by $1\sigma$ region in blue. While the main plot is logarithmic in redshift, the interior sub-plot is linear up to $z = 4$. 

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Figure 4. The $1 - 2\sigma$ contours for $a_c$ vs $H_0$ is plotted for CEDE model where Planck CMB data is used without and with BAO or SNe datasets, in red, grey and yellow, respectively. It is obvious that the contours for CMB+BAO and CMB+SNe datasets are overlapped which means they are consistent to each other in CEDE model. However, it is also clear that they do not prefer high value for $H_0$.

4.2 CMB+BAO+SNe

Now it is time to check if CMB is consistent with BAO and/or SNe datasets. It is obvious from figure 4 that BAO and SNe are consistent in CEDE with CMB in a way that can be combined together. When we do that then it is clear that CMB+BAO+SNe dataset is not consistent with R19 in the CEDE framework. This means that CEDE cannot make $H_0$ tension relaxed in the presence of BAO+SNe as it is shown in figure 5. However it makes the tension milder, in comparison to $\Lambda$CDM. In figure 6, the $2 - d$ and $1 - d$ likelihoods are shown for all the free parameters in the presence of CMB and BAO and or SNe. The best fit values for CEDE’s parameters for these combinations of datasets can be found in table 1.

5 Discussions

In this work we have shown a specific form of transition in the behaviour of dark energy can be a very promising framework to address the Hubble tension. This specific form of transition is based on the physics of critical phenomena. We named our model “critically emergent dark energy”, CEDE. As it is shown in figure 5, in CEDE there is no tension between CMB from Planck and local $H_0$ measurement by Riess et al. Consequently we can add up these datasets without any worries. In addition we showed in figure 5 that in CEDE framework, the BAO+SNe datasets are compatible with CMB but in this case CEDE does not allow for a high $H_0$ value. This means the $H_0$ tension still exists though it is milder than what we see in $\Lambda$CDM. One way to interpret our results in figure 5 is to say that CMB+R19 and CMB+BAO+SNe datasets’s inconsistency in the CEDE framework is about $\leq 3\sigma$. Due to our results we can say that in CEDE framework BAO+SNe and R19 are not consistent. This means we need to look for other models or there are systematic inconsistencies and or problematic interpretation in the observational datasets. There are a lot of arguments in the
Figure 5. The 1–2σ contours for \(a_c\) vs \(H_0\) is plotted for CEDE model, in red, blue and green where CMB, CMB+R19 and CMB+BAO+SNe datasets are used, respectively. It is obvious that CMB+R19 and CMB+BAO+SNe are not consistent to each other in CEDE framework and \(H_0\) tension is still there. However this tension is about 3σ’s and less than ΛCDM’s one.

| Parameter | Prior | CMB | CMB+R19 | CMB+SNe | CMB+BAO | CMB+BAO+SNe |
|-----------|-------|-----|---------|---------|---------|-------------|
| \(\Omega h^2\) | [0.005, 0.1] | 0.2243±0.0011 | 0.2243±0.0014 | 0.0223±0.0015 | 0.0223±0.0014 | 0.2237±0.0014 |
| \(\Omega \Lambda\) | [0.001, 0.99] | 0.1199±0.0014 | 0.1194±0.0013 | 0.1201±0.0013 | 0.1202±0.0011 | 0.1201±0.0011 |
| 1000\(\Theta_{MC}\) | [0.5, 10] | 1.04093±0.0032 | 1.04098±0.0032 | 1.04092±0.0030 | 1.04090±0.0030 | 1.04091±0.0030 |
| \(\tau\) | [0.01, 0.8] | 0.6539±0.0080 | 0.5539±0.0077 | 0.5450±0.0079 | 0.5460±0.0078 | 0.5450±0.0076 |
| \(n_s\) | [0.8, 1.2] | 0.9764±0.0045 | 0.9764±0.0043 | 0.9650±0.0041 | 0.9650±0.0039 | 0.9651±0.0039 |
| \(\ln[10^{10}A_s]\) | [2, 4] | 3.043±0.017 | 3.042±0.016 | 3.044±0.016 | 3.044±0.016 | 3.044±0.016 |
| \(a_c\) | [0.01, 1] | 0.185±0.014 | 0.325±0.036 | 0.120±0.032 | 0.120±0.032 | 0.120±0.032 |
| \(\Omega_\Lambda\) | [0.0, 0.7] | 0.707±0.014 | 0.735±0.011 | 0.696±0.010 | 0.696±0.0078 | 0.696±0.0078 |
| \(H_0\) [km/s/Mpc] | [0.0, 0.7] | 70.0±1.2 | 73.4±1.4 | 68.59±0.89 | 68.49±0.55 | 68.39±0.55 |

\(\Delta \chi^2\) | — | −0.6 | −17.9 | +0.8 | +0.1 | −1.2 |

Table 1. The best fit values and 68% CL intervals for CEDE parameters when five combinations of data are used. Last row is just the difference in \(\chi^2\) with respect to ΛCDM, when the same data combination is used. Without R19, CEDE is almost as good as ΛCDM with a one more free parameter, \(a_c\). Inclusion of R19 makes our \(\Delta \chi^2\) much more negative and hence, in this case, CEDE is significantly preferred.

The literature\(^7\) about correctness of using R19 as a hint for new physics in late universe. But our unbiased interpretation is that if there is a problem in the data then it can be in the BAO+SNe as well. In the framework of CEDE we cannot make any comment about this point and it should be considered in the future works.

\(^7\)For example, in [88–90], it is argued that the very sharp transition in very late time (\(z_t \ll 0.1\)) cannot be a solution to the Hubble tension. These models can make ambiguities in the SNe’s calibration and the interpretation of R19. We have to say that our CEDE model does not belong to these models and these arguments (at least at the current state) cannot exclude our model.
Figure 6. 68% and 95% parameter constraint contours for CEDE from different combinations of datasets. Note that blue contours (CMB+R19) are narrower than only CMB because of inclusion of the R19; but still over lapping with their contours. Obviously, CEDE is not consistent with R19 in the presence of CMB and BAO and/or SNe.

Theoretically, in the critical phenomena literature, the phase transition in the behaviour of a system means that there is a substructure for that system. So we think the Hubble tension can be a hint for the existence of substructure of dark energy. This implies a large class of interesting features, specially, when one goes beyond mean field approximation. For instance, since these micro-structures interact locally, one can expect the existence of spatial patches of (iso-order parameter) dark energy separated by domain-walls [84, 85]. This form of clustering of dark energy associated with domain-walls has a very rich phenomenology, observationally. CMB lensing, ISW, structure formation and many other cosmological phenomena are affected. This is, mainly, a consequence of time evolution of spatial patches as partly discussed in [66].
It is well-known that the small patches grow till at the end there is a remained big patch [86]. This affects the spatial behavior of dark energy and may be an answer to CMB spatial anomalies [66]. In addition, dark energy domain-walls not only change the trajectory of CMB photons, but also as temperature decrease, they annihilate and produce stochastic gravitational waves [87] which can be detected in the future experiments. Moreover, according to physics of critical phenomena at the critical point, the correlation length becomes infinity and a fractal pattern appears on the dark energy patches. This may cause a very specific fingerprint in the cosmological observables at the redshift of transition. We think due to richness of the physics of critical phenomena, our CEDE model has its very distinguishable observational smoking guns which should be sought in different area of astrophysics and cosmology. Another very interesting path to follow is comparing CEDE with PEDE [69] and GEDE [71] phenomenological models of emergent dark energy as they have effectively very similar properties.

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