Exact solutions for a universal set of quantum gates on a family of iso-spectral spin chains

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Abstract

We find exact solutions for a universal set of quantum gates on a scalable candidate for quantum computers, namely an array of two level systems. The gates are constructed by a combination of dynamical and geometrical (non-Abelian) phases. Previously these gates have been constructed mostly on non-scalable systems and by numerical searches among the loops in the manifold of control parameters of the Hamiltonian.

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1 Introduction

Let $H(t) = H(R(t))$ be a time dependent Hamiltonian acting on an $n$ dimensional Hilbert space, whose dependence on time is via some control parameters collectively denoted by $R(t)$. Let this hamiltonian have a $k$-dimensional degenerate subspace $V_0(t)$ spanned by the instantaneous eigenvectors $|1(t)\rangle, |2(t)\rangle, \ldots |k(t)\rangle$, with energy $E_0(t)$. When the point $R(t)$ moves around a loop in the space of control parameters, any state $|\psi(0)\rangle \in V_0$ evolves into a state

$$|\psi(t)\rangle \equiv U(t)|\psi(0)\rangle = e^{i\beta(t)\Gamma}|\psi(0)\rangle. \quad (1)$$

Here

$$e^{i\beta(t)} := e^{-i\int_0^t E_0(t')dt'}, \quad (2)$$

is the dynamical phase where $E_0(t)$ is the instantaneous eigenvalue of the subspace $V_0$ and

$$\Gamma := P(e^{\oint_C A(R)·dR}) = T(e^{\oint_C A(t)·dt}), \quad (3)$$

is the non-abelian geometric phase which is the holonomy operator associated with the anti-hermitian connection $A$ given by

$$A_{\mu;ij}(R) = \langle i(R)|\frac{\partial}{\partial R^\mu}|j(R)\rangle, \quad (4)$$

or

$$A_{ij}(t) \equiv A_{\mu;ij}(R)\frac{dR^\mu}{dt} = \langle i(t)|\frac{d}{dt}|j(t)\rangle. \quad (5)$$

Note that the symbols $P$ and $T$ in the first and the second integral of (3) refer respectively to the path ordering and time ordering of the exponential around the loop $C$ in the control manifold. The basic property of a general holonomy operator is that it is independent of the way the loop is traversed in the parameter space. In some special cases it depends only on a few basic geometrical properties of the loop, like its area. In our case which will be discussed in detail later, in which the connection is constant, the holonomy depends only on this constant connection (which is nothing but the tangent vector on the loop at $t = 0$), and the total time needed for traversing the loop. Note that in the above formulas, the parameter $t$ does not necessarily point to time, although we use this word for explicitness. It points to any single parameter which parameterizes the loop $C$.

In the special case when $V_0$ is one dimensional, $\Gamma$ is the abelian geometrical phase and identical to the well known Berry phase.

This general scheme when applied to the field of quantum computation takes the name of holonomic or geometrical quantum computation and the unitary operators thus obtained are called holonomic quantum gates. The problem of exact holonomic implementation of quantum gates is of great interest in the field of quantum computation \([1, 2, 3, 4, 5, 6, 7, 8, 9, 10]\). This is due to the fact that holonomic quantum computation, being geometrical in nature has a degree of stability against a class of errors \([2, 3, 4]\).
In particular it is known that these gates depend only on the loop and not on the speed with which they are traversed. Moreover this stability is related to the robustness of such gates against small perturbations of the traversed loops and against various noises.

In the past few years many theoretical proposals for holonomic implementation of quantum gates have been reported in the literature and some of them have been realized experimentally. At present we can say that there have been only sporadic successes in overcoming one or the other of the many obstacles in the way of a successful implementation of holonomic gates.

Among these problems the requirement of scalability is the most important one. Let us briefly discuss this issue.

It is well known that any unitary gate can be constructed to arbitrary precision from a combination of a universal set of gates. There are many choices for this universal set. One choice is the set \( \{ H, P(\phi), C(\pi) \} \), where \( H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \) is the Hadamard gate, \( P(\phi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} \) is the phase gate and \( C(\phi) = \text{diag}(1, 1, 1, e^{i\phi}) \) is the controlled phase-gate. If we can act by these universal set of gates on any single qubit or any two qubits of our scalable system, then we can say that holonomic quantum gates have been constructed on the scalable system.

We stress that such gates should be implemented on a scalable system, which is a crucial requirement for any viable candidate for quantum computation. A scalable candidate of quantum computer takes the form of an array of two level systems, or qubits. The array of identical systems can of course have higher dimensions but only two of their states will play the role of our computational qubit. Since any operation between remote qubits can be divided into elementary logic gates on adjacent qubits, it is sufficient to enact the universal set of gates only on single qubits and two adjacent qubits.

An essential property of a scalable system is that the two qubit gates be realized on the tensor product of the same space on which the single qubit gates have been realized. However most of the proposals of holonomic computation so far suggested, lack this property. For example it has been shown that by abelian holonomy or Berry phase, one can implement the one-qubit phase gate on a single spin subject to a time varying magnetic field, and the two-qubit conditional phase gate on a pair of coupled spins. On the other hand the Hadamard gate which is needed to be added to the above set if we are to have a universal set of gates, requires non-abelian holonomy, and is much more difficult to realize than the other gates. In fact the proposal for its holonomic realization is based on a completely different system consisting of two degenerate qubits and two ancilla qubits.

In other words the hadamard gate is not implemented on the same qubit on which the phase gate was implemented.

Of particular interest to us are the models based on iso-spectral Hamiltonians
which is reviewed in section (2). Although in this case one can easily calculate the time or path-ordered exponential, the determination of exact solutions (exact loops in the parameter space) which lead to a universal set of quantum gates is difficult. Such an approach has been followed in [9], but with a numerical search among the class of loops for finding the required loop for each member of the universal set. Moreover in the approach of [9], the single qubit gates are constructed on two dimensional subspaces of a three dimensional space, (i.e. $k = 2, n = 3$) and the two qubit gates are constructed on four dimensional subspaces of a five dimensional space, (i.e. $k = 4, n = 5$). This construction has the drawback that a two-qubit gate is not constructed on the tensor product space of two qubits, a requirement which is highly desirable for scalable quantum computation. In a related work Niskanen, Nakahara and Solomaa [8] employ a three state hamiltonian of the form

$$H_{01 \text{ qubit}} = 0 \langle 2 | 2 \rangle = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & \epsilon
\end{pmatrix}$$

(6)

to implement the phase and the Hadamard gate on the degenerate subspace $V_0 = \text{Span} \{\text{|0\rangle, |1\rangle}\}$ and the controlled phase-gate on the degenerate subspace of the Hamiltonian

$$H_{02 \text{ qubit}} = H_{01 \text{ qubit}} \otimes I + I \otimes H_{01 \text{ qubit}}.$$

(7)

This construction can be generalized to $N$-qubit case. The dimension of the full Hilbert space scales as $3^N$. The one and the two qubit hamiltonians of the proposal of [8] are rather abstract and we do not know of any concrete realizations in terms of spins or some other suitable observables.

**Remark:** Perhaps it is not strictly correct to say that the proposal of [13] is not scalable. In fact in this proposal which implement the universal set \{$e^{i \theta \sigma_y}, P(\phi), C(\pi)$\}, one can achieve scalability by going through polynomially more steps and using ancilla bits which do not destroy the scalability. The aim of this paper is to achieve scalability by encoding each qubit in the two lowest states of two adjacent spins in a spin chain as shown in figure (1). This is equivalent to using $sN$ ancilla qubits for the $N$ computational qubits.

What we want to show in this paper is that one can indeed find exact solutions for universal gates on a scalable candidate for qubits. For this aim, we consider a spin chain, and encode each qubit into the Hilbert space of two adjacent spins. We then show that by moving around appropriate loops in families of is-spectral spin chains, (determined by the adjoint action of suitable operators) one can implement a universal set of one and two qubit gates on such a spin chain. Such gates are realized as a combination of dynamical and geometrical non-abelian phases, when the corresponding loops are traversed. For each gate there are a set of parameters in the form of axis of rotations and frequencies which when tuned suitably enact appropriate one and two qubit gates on single and two adjacent qubits.
The structure of this paper is as follows: In section 2 we review briefly the holonomies derived from iso-spectral hamiltonians. In section 3 we describe our model in detail where we show how a universal set of gates can be implemented by a combination of dynamical and geometrical phases on the spin chain. We end the paper with a discussion in section 4.

2 Holonomies of iso-spectral Hamiltonians

In view of the time ordering of the exponential a closed formula for the holonomy operator (4) can not be obtained for most connections and one has to resort to numerical methods for an approximate calculation of this operator. It would be much desirable if we could find connections whose holonomy could be calculated exactly. This obstacle can be partially overcome by restricting ourselves (and hence paying a price of a using a limited source of holonomies) to constant connections for which we have

\[ \Gamma = e^{AT}, \] (8)
where $T$ is the total time required for traversing the loop.

In view of the remark following equation (3), the parameter $T$ is not necessarily the total time needed for traversing the loop, it is only the final value of the parameter $t$ which parameterizes the loop, which in the sequel will be set to unity.

Perhaps the simplest way for obtaining time-independent holonomy operators is to consider iso-spectral family of Hamiltonians. These are the Hamiltonians which are of the form

$$H(t) := e^{Xt}H_0e^{-Xt}, \quad (9)$$

where $X$ is any anti-hermitian operator and $H_0$ is the Hamiltonian at time 0 with the degenerate subspace $V_0$, spanned by the vectors $|1\rangle, |2\rangle, \cdots |k\rangle$, i.e.

$$H_0|i\rangle = E_0|i\rangle, \quad i = 1 \cdots k. \quad (10)$$

From relation (9) we find that at any time $t$, the instantaneous eigenstates will have a simple form

$$|i(t)\rangle = e^{Xt}|i\rangle, \quad H(t)|i(t)\rangle = E_0|i(t)\rangle, \quad i = 1 \cdots k. \quad (11)$$

In this case we will find from (4) that $A$ will be constant, namely

$$\langle i | A | j \rangle = \langle i | X | j \rangle, \quad i = 1 \cdots k. \quad (12)$$

It is important to note that $X$ is an anti-hermitian operator defined on the full Hilbert space and $A$ is an operator defined only on the degenerate subspace and this relation implies only that the projection of $X$ on this subspace is equal to $A$, that is

$$X|V_0 = A. \quad (13)$$

Therefore a large number of operators $X$ can lead to the same connection. Mathematically different loops in the parameter space are specified by their tangent vector at the origin which is nothing but the operator $X$.

By re-scaling the time variable so that $T = 1$, and taking into account the inevitable dynamical phase, we arrive at the final form of the operator $U$ acting on the space of a single qubit,

$$U = e^{-iE_0}e^A. \quad (14)$$

3 Holonomic computation on a spin chain

We take an array of qubits as shown in figure 4 so that the spins within each block, say the spins (1,2) interact according to the following Heisenberg Hamiltonian:

$$H_0 = B(\sigma_{1z} + \sigma_{2z}) + J\vec{s}_1 \cdot \vec{s}_2, \quad (15)$$

and different blocks do not interact with each other or their interaction is so weak that we can consider them effectively non-interacting at time $t = 0$. Later on we will make these blocks interact in order to implement two qubit gates. It can be easily
verified that if we choose the magnetic field so that \( B = 2J \), then the ground state of the Hamiltonian will be doubly degenerate. In fact the spectrum of the Hamiltonian is as follows, where the states \( |\phi_0\rangle \) and \( |\psi_0\rangle \) are the degenerate ground states and \( |\phi_1\rangle \) and \( |\phi_2\rangle \) are the first and the second excited states.

\[
\begin{align*}
|\phi_2\rangle &= |+, +\rangle, & E &= 5J, \\
|\phi_1\rangle &= \frac{1}{\sqrt{2}}(|+, -\rangle + |-, +\rangle), & E &= J, \\
|\phi_0\rangle &= |-, -\rangle, & E &= -3J, \\
|\psi_0\rangle &= \frac{1}{\sqrt{2}}(|+, -\rangle - |-, +\rangle), & E &= -3J.
\end{align*}
\] (16)

We take the code or computational qubits to be the degenerate ground states, namely

\[
|0\rangle \equiv |\phi_0\rangle = |-, -\rangle, \quad |1\rangle \equiv |\psi_0\rangle = \frac{1}{\sqrt{2}}(|+, -\rangle - |-, +\rangle).
\] (17)

In spin notations where \( S \) and \( S_z \) respectively denote the total and the \( z \) component of the two spins, the code qubits are \( |0\rangle = |S = 1, S_z = -1\rangle \) and \( |1\rangle = |S = 0, S_z = 0\rangle \). We suppose that these states are not hard to access and control experimentally. At low temperatures the two spins reside in the degenerate two dimensional subspace which is a desirable situation for the initialization of the computer. If these two states were among the excited states of the Hamiltonian rather than the ground state, we would have been faced with an extra problem of exciting the two spins to these states.

Let the operator \( X \) be of the following form

\[ X = i\mathbf{n} \cdot (\omega_1 \sigma_1 + \omega_2 \sigma_2), \] (18)

which describes the rotation of the spins \( \mathbf{S}_1 \) and \( \mathbf{S}_2 \) around the axis \( \mathbf{n} \) with frequencies \( \omega_1 \) and \( \omega_2 \) respectively. From (4) we find the gauge potential to be:

\[
\begin{align*}
A_{0,0} &= \langle \phi_0 | X | \phi_0 \rangle = -i(\omega_1 + \omega_2)n_z, \\
A_{0,1} &= \langle \phi_0 | X | \psi_0 \rangle = \frac{i}{\sqrt{2}}(\omega_1 - \omega_2)(n_x + in_y), \\
A_{1,0} &= \langle \psi_0 | X | \phi_0 \rangle = \frac{i}{\sqrt{2}}(\omega_1 - \omega_2)(n_x - in_y), \\
A_{1,1} &= \langle \psi_0 | X | \psi_0 \rangle = 0.
\end{align*}
\]

Therefore in this subspace the gauge potential will be given by the following operator

\[ A = i(r_x \sigma_x + r_y \sigma_y + r_z \sigma_z + r_z I), \] (19)

where

\[
\begin{align*}
r_x &= \frac{1}{\sqrt{2}}(\omega_1 - \omega_2)n_x, & r_y &= \frac{1}{\sqrt{2}}(\omega_1 - \omega_2)n_y, & r_z &= \frac{1}{2}(\omega_1 + \omega_2)n_z.
\end{align*}
\] (20)
After the lapse of time $T = 1$ and acquiring the dynamical phase $3J$, the gate

$$u' = e^{i(r_z \sigma_z + r_y \sigma_y + r_z \sigma_z + (r_z + 3J)I)}, \quad (21)$$

will act on the space of single-qubit codes, $|0\rangle$ and $|1\rangle$. In this form the gate $u'$ is not general enough, since the overall phase it applies namely $r_z + 3J$ is not independent of the other parameters. However at the end of any loop we can stop changing the parameters of the Hamiltonian, and only pause for a time interval $\tau$. This lapse of time will add a phase $3J\tau$ to the above phase and we obtain a general unitary gate given by

$$u = e^{i(r_z \sigma_z + r_y \sigma_y + r_z \sigma_z + (r_z + 3J(1+\tau))I)}, \quad (22)$$

In this way by combining dynamical and geometrical phases we can construct any single qubit gate on our code qubits, since the parameters $r_x, r_y, r_z$ and $\tau$ are independent. It is only necessary to choose the parameters $\omega_1, \omega_2, n$ and $\tau$ appropriately. We should emphasize that the specific structure of the degenerate states, namely that it consists of a product state and an entangled state, has been vital in our ability to arrive at a general form of the holonomy $A \equiv X|V_0$ with a simple choice of the operator $X$. Had these two states been product states, we should have used complicated and hence unjustified forms of the operator $X$ to arrive at the same general result.

Now let us explicitly construct the two single qubit gates in the universal set, namely the phase gate $P(\phi)$ and the Hadamard gate $H$.

### 3.1 The phase gate

The phase gate is defined as

$$P(\phi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} = e^{i\phi(1-\sigma_z) + i2m\pi}, \quad (23)$$

where $m$ is an arbitrary integer. Comparison with (22) shows that for this gate we should have

$$r_x = r_y = 0, \quad r_z = -\frac{\phi}{2}, \quad r_z + 3J(1 + \tau) = \frac{\phi}{2} + 2m\pi. \quad (24)$$

From (20) we find that the following choice of the parameters implements this gate:

$$\hat{n} = (0, 0, 1), \quad \omega_1 = \omega_2 = \frac{\phi}{2}, \quad 3J(1 + \tau) = \frac{\phi}{2} + 2m\pi. \quad (25)$$

This is a rotation of both spins around the $z$ axis with equal frequencies followed by a pause for a time interval $\tau$ given as above. The freedom in choosing the integer $m$ guarantees that the time lapse $\tau$ can always be positive as it should be.
3.2 The Hadamard gate

The Hadamard gate is

\[ H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \]  

We now note that \( H \) can be rewritten as follows:

\[ H = \frac{1}{\sqrt{2}} (\sigma_z + \sigma_x) =: k \cdot \vec{\sigma}, \]  

where \( k \) is a unit vector \( k = 1/\sqrt{2}(1, 0, 1) \). Since for any unit vector \( k \), \( e^{i\pi/2 k \cdot \vec{\sigma}} = i k \cdot \vec{\sigma} \), we find that

\[ H = -i k \cdot \vec{\sigma} = -i e^{i\pi/2 k \cdot \vec{\sigma}} = e^{-i\pi/2} e^{i\pi/2 k \cdot \vec{\sigma}} = e^{-i\pi/2 + i\pi/2 (\sigma_z + \sigma_x) + i2m\pi}. \]  

Comparison with (21) shows that the Hadamard gate is produced when we choose the following parameters:

\[ r_x = r_z = \frac{\pi}{2\sqrt{2}}, \quad r_y = 0, \quad r_z + 3J(1 + \tau) = -\pi/2 + 2m\pi. \]  

Comparison with (20) determines the parameters of rotation as follows:

\[ n = (\sqrt{\frac{1}{3}}, 0, -\sqrt{\frac{2}{3}}), \quad \omega_1 = \frac{\pi}{2} \sqrt{3}, \quad \omega_2 = 0, \quad 3J(1 + \tau) = -\frac{\pi}{2} (\sqrt{2} + 1) + 2m\pi. \]  

Thus we have constructed our single-qubit gates on our code space which contains the computational qubits. We now turn to the conditional phase gate to complete our universal set of gates.

3.3 The conditional Phase gate

The controlled phase gate has the following matrix form when the basis vectors of the two qubits are ordered as \( |0, 0\rangle, |0, 1\rangle, |1, 0\rangle \) and \( |1, 1\rangle \):

\[ C(\phi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} = exp \begin{pmatrix} 0 \\ 0 \end{pmatrix} \]

\[ \begin{pmatrix} 0 \\ i(\phi + 2m\pi) \end{pmatrix}. \]  

In view of the demanded scalability, this gate should act on the space \( V_0 \otimes V_0 \), where \( V_0 \) is the space on which the single qubit gates act. The space \( V_0 \otimes V_0 \) is spanned by the states of two qubits, namely

\[ |0, 0\rangle = |\phi \rangle \otimes |\phi \rangle = |-, -, -, -\rangle, \]
\[ |0, 1\rangle = |\phi \rangle \otimes |\psi \rangle = \frac{1}{\sqrt{2}} (|-, +, -, -\rangle - |-, -, -, +\rangle), \]
\[ |1, 0 \rangle = |\psi_0 \rangle \otimes |\phi_0 \rangle = \frac{1}{\sqrt{2}} (|+, -, -, - \rangle - |-, +, -, - \rangle), \]
\[ |1, 1 \rangle = |\psi_0 \rangle \otimes |\psi_0 \rangle = \frac{1}{2} (|+, -, +, - \rangle - |-, +, +, - \rangle - |-, +, -, + \rangle + |-, +, +, - \rangle) \] \eqno(32)

To implement this gate the operator \( X \) should act on the Hilbert space of two adjacent pairs of spins, namely the pairs \((S_1, S_2)\) and \((S_3, S_4)\). Combining the geometric and the dynamical phases we find that the gate which will be implemented on the two qubits is equal to
\[ U_{2\text{qubit}} = e^{A + i(6J(1 + \tau))}, \] \eqno(33)
where \( A = X|V_0 \otimes V_0 \rangle \) and we have used the fact that the energy of the degenerate subspace is now \( 6J \) instead of \( 3J \).

We now take the operator \( X \) to be of the form
\[ X = i\phi (\sigma_2 \sigma_3 + \sigma_2 \sigma_3). \] \eqno(34)

This operator couples the endpoint spins of the two neighboring blocks which hitherto were considered non-interacting.

It is easy to verify that
\[ X|0, 0 \rangle = -i\phi|0, 0 \rangle \] \eqno(35)
\[ X|0, 1 \rangle = -i\phi|0, 1 \rangle \] \eqno(36)
\[ X|1, 0 \rangle = -i\phi|1, 0 \rangle \] \eqno(37)
\[ X|1, 1 \rangle = -i\phi|1, 1 \rangle - 2i\phi|--, +, +, -\rangle. \] \eqno(38)

This will then lead to
\[ A \equiv X|V_0 \otimes V_0 \rangle = \begin{pmatrix} -i\phi & -i\phi \\ -i\phi & -i\phi \end{pmatrix}. \] \eqno(39)

In view of \( \box{83} \) if we now choose \( \tau \) so that \( 6J(1 + \tau) = \phi + 2m\pi \), we will find
\[ A + i6J(1 + \tau) = \begin{pmatrix} 0 & 0 \\ 0 & i(\phi + 2m\pi) \end{pmatrix}. \] \eqno(40)

and hence the conditional phase gate \( C(\phi) \) will be exactly implemented on the two qubits.

This completes our derivation of exact holonomies for a universal set of gates on an array of qubits.
4 Discussion

We have been able to implement a universal set of quantum gates on a scalable system, by combining appropriately the dynamical and non-abelian geometrical phases. Our system consists of array of two-spin blocks each of which is a four dimensional space with a two dimensional degenerate subspace encoding the computational qubits. With these universal set at hand, one can construct any other gate to a sufficient degree of accuracy. The crucial step in this direction has been an appropriate choice of a degenerate subspace of a physical system which should represent the computational qubits. This subspace should be so that the gauge connection projected on it by a simple operator $X$ be general enough to represent an arbitrary general gate. The choice of hamiltonian, so that its degenerate subspace has one entangled state and one product state has been essential in this step. Moreover we stress that the conditional phase gate has been constructed on the tensor product of two such qubits. We should add that the experimental realization of such a proposal is a completely different problem and we do not claim that this proposal is superior to others as far as experimental realization is concerned. We only emphasize the exact and the scalable nature of the proposal and hope that following the basic idea of this paper, namely taking two-spin blocks for representing qubits, other researchers can proceed to more practical and experimentally viable proposals.

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