Vacuum fluctuations and the spin current in mesoscopic structures with collinear magnetic order

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We show below that the equilibrium spin current does not vanish in the system with the homogeneous magnetization but inhomogeneous anisotropy. This effect has purely quantum origin and is related to the vacuum fluctuations in the magnetic system. The quantum character of this effect makes it substantially different from the earlier studied mechanism of the equilibrium spin current in noncollinear ferromagnets\textsuperscript{9,11}, which can be understood within a model of the classical moment.

The spin current appearing in a shaped mesoscopic structure (e.g., a nanoring) with the collinear magnetic ordering, transfers the angular momentum. The resulting transferred torque acts on the anisotropy axes, tending to arrange them homogeneously. In other words, the spin current produces a mechanical stress in the anisotropy-nonequilibrium system.

The physics of this phenomenon can be better understood if we consider first a simple model with two quantum spins \( S = 1 \) in the magnetic field \( B \) directed along the \( z \) axis, and with the anisotropy axes oriented along \( \mathbf{n}_1 \) and \( \mathbf{n}_2 \) in the \( x - y \) plane for spins \( S_1 \) and \( S_2 \), respectively (Fig. 1). We take \( \mathbf{n}_1 \) along \( x \) and denote the angle between \( \mathbf{n}_1 \) and \( \mathbf{n}_2 \) by \( \phi \). The Hamiltonian is

\[
H = -BS_1z + \frac{\lambda}{2} (\mathbf{n}_1 \cdot S_1)^2 - BS_2z + \frac{\lambda}{2} (\mathbf{n}_2 \cdot S_2)^2 - gS_1 \cdot S_2,
\]

where \( \lambda > 0 \) is the constant of anisotropy, and the last term corresponds to the exchange interaction with the coupling constant \( g \). We assume \( g > 0 \), which corresponds to the ferromagnetic coupling. Obviously, in the ground state both spins are oriented along \( z \).

We can use the basis of eigenfunctions of Hamilto-
nian (1) with \( g = 0 \). For each of the noninteracting spins \( S_1 \) and \( S_2 \), the eigenfunctions can be easily found, and they differ from each other by the transformation \( \exp(i\phi J_z) \), where \( J_z \) is the generator of \( z \)-rotations. In the absence of interaction, \( g = 0 \), the lowest four states of Hamiltonian (1) correspond to energies \( (2\varepsilon_0, \varepsilon_0 + \lambda/2, \lambda) \), where the level \( \varepsilon_0 + \lambda/2 \) is twofold degenerate. We denote these states \([00], [01], [10], [11]\).

Here \( \varepsilon_0 = \lambda/4 - (\lambda^2/16 + B^2)^{1/2} \) is the ground state energy of a single spin.

The essential point is that each of two separated quantum spins in the ground state has the magnitude of the moment along the \( z \) axis smaller than unity, \(|\langle S_{1z}\rangle_0| = |\langle S_{2z}\rangle_0| = 2|\varepsilon_0| B/|\varepsilon_0^2 + B^2| < 1 \). This is the effect of the magnetic anisotropy. As a result, the quantum fluctuations give a different value of the fluctuating moments along the axes \( x \) and \( y \) (principal axes for \( S_1 \)), \( \langle S_{1x}\rangle_0 - \langle S_{1y}\rangle_0 = (\varepsilon_0^2 - B^2)/(|\varepsilon_0^2 + B^2|) \). It leads to the dependence of interaction on a mutual orientation of the \( S_1 \) and \( S_2 \) anisotropy axes.

The interaction \( H_{\text{int}} = -gS_{1z}S_{2z} \) gives nonzero matrix elements

\[
V \equiv \langle 00 | H_{\text{int}} | 00 \rangle = \frac{-4g\varepsilon_0^2 B^2}{6\varepsilon_0^2 + B^2}, \tag{2}
\]

\[
P(\phi) \equiv \langle 00 | H_{\text{int}} | 11 \rangle = \langle 11 | H_{\text{int}} | 00 \rangle^* = g \left( \cos \phi - \frac{2i\varepsilon_0 B}{\varepsilon_0^2 + B^2} \sin \phi \right). \tag{3}
\]

Assuming \( B \gg \lambda \), we can restrict ourselves by considering only the four lowest energy states. Then the ground-state energy \( E_0 \) of the Hamiltonian (1) can be found by the diagonalization of matrix \( 4 \times 4 \), and we obtain

\[
E_0(\phi) = \varepsilon_0 + \lambda/2 - \left[ (\varepsilon_0 - \lambda/2)^2 + |P(\phi)|^2 \right]^{1/2}, \tag{4}
\]

where \( \varepsilon_0 = \varepsilon_0 + V/2 \). Due to the exchange interaction, the energy depends on the mutual orientation of the anisotropy axis. The torque acting on the spins is \( \mathcal{M}(\phi) = -dE_0/d\phi \). In the limit of \( g \to 0 \) we find from (4)

\[
\mathcal{M}(\phi) \approx \frac{g^2 (\varepsilon_0^2 - B^2)^2 \sin 2\phi}{2 (\varepsilon_0^2 + B^2)^2 (\lambda/2 - \varepsilon_0)} \tag{5}
\]

The torque vanishes for parallel or antiparallel configuration of anisotropy axes, and also if they are perpendicular to each other, \( \phi = \pi/2 \).

Let us consider now the model with arbitrary spins \( S \geq 1 \) on a lattice, with a nearest-neighbor exchange interaction. We assume a one-site anisotropy with the anisotropy axis \( \mathbf{n}_i \), smoothly depending on position. The Hamiltonian has the following form

\[
H = -\frac{g}{2} \sum_{<i,j>} \mathbf{S}_i \cdot \mathbf{S}_j + \frac{\lambda}{2} \sum_i (\mathbf{n}_i \cdot \mathbf{S}_i)^2. \tag{6}
\]

We assume that in the ground state the ferromagnet has a uniform magnetization along the axis \( z \), and all vectors \( \mathbf{n}_i \) lie in the \( x-y \) plane. It corresponds to the easy plane ferromagnet with a nonuniform anisotropy and can be realized, for example, in a thin-wall magnetic cylinder (Fig. 2).

Using a local frame with the \( x \) axis along \( \mathbf{n}_i \) in each lattice site, one transforms (1) into

\[
H = -\frac{9}{2} \sum_{<i,j>} S_i^T e^{i(\phi_i - \phi_j)} J_z S_j + \frac{\lambda}{2} \sum_i (S_i^z)^2, \tag{7}
\]

where \( \phi_i \) is the rotation angle, and we use a matrix form of presentation for the spin operators, \( S_i^z = (S_i^x, S_i^y, S_i^z) \).

After the Fourier transformation, \( S_i^j = \sum_q S_i^j e^{iq\mathbf{r}_j} \), and taking the limit of \( q \to 0 \) the Hamiltonian is

\[
H = \frac{a}{2} \sum_q \left[ S_{-q}^y S_q^y + 2i(\mathbf{q} \cdot \mathbf{A}) \left( S_{-q}^x S_q^y - S_{-q}^y S_q^x \right) \right]
\]

\[
+ \mathcal{A}^2 \left( S_{-q}^x S_q^x + S_{-q}^y S_q^y \right) + \frac{\lambda}{2} \sum_i (S_i^z)^2, \tag{7}
\]

where \( \mathbf{A} = \nabla_i \phi_i \) is the gauge potential, \( a = ga_0^2 \), \( a_0 \) is the lattice constant, and we assume that the variation of \( \mathbf{A} \) in space is small at the wavelength of magnons (adiabatic approximation).

We use the Holstein-Primakoff representation of spin operators\(^3\), \( S_i^a = S_i - n_i, S_i^a = \sqrt{2S} (1 - n_i/2S)^{1/2} a_i, S_i^- = \sqrt{2S} a_i^\dagger (1 - n_i/2S)^{1/2}, n_i = a_i^\dagger a_i, \) and restrict ourselves by the harmonic approximation in \( a_i, a_i^\dagger \). Here \( a_i^\dagger, a_i \) are the boson creation and annihilation operators for the magnons. Then we find

\[
H = \frac{S}{2} \sum_q \left\{ \left[ (a_0 \mathbf{q} \cdot \mathbf{A})^2 + \lambda_1 \right] (a_q^\dagger a_q + a_q a_q^\dagger) \right. \\
\left. + \lambda_2 (a_0 q_0 a_q - a_q a_0 q_0^\dagger) \right\} \tag{9}
\]

where \( \lambda_1 = \lambda (1 - 1/2S) \) and \( \lambda_2 = \lambda (1 - 1/2S)^{1/2} \).

In the following, we make a change in Eq. (9) substituting \( \lambda_2 \to \lambda_1 = \lambda (1 - 1/2S) \). This is related to the well-known renormalization of the constant \( \lambda_2 \) due to the magnon interactions\(^1\). It reproduces correctly both \( S \to 1/2 \) and \( S \to \infty \) limits, and leads to the gapless spectrum of magnons in correspondence with the Goldstone theorem\(^1\). The Hamiltonian (9) with \( \lambda_2 \to \lambda_1 \) can be diagonalized using the Bogolyubov-Holstein-Primakoff transformation method\(^1\), and we obtain finally

\[
H = \sum_q \omega_q (b_q^\dagger b_q + 1/2), \tag{10}
\]

where \( b_q^\dagger, b_q \) operators are related to \( a_q^\dagger, a_q \) by the Bogolyubov-Holstein-Primakoff transformation

\[
b_q = \alpha_q a_q + \beta_q a_q^\dagger, \quad b_q^\dagger = \alpha_q^\dagger a_q^\dagger + \beta_q a_q \tag{11}
\]
we obtain polarization along \( z \)

\[
\omega_q = S \left( \hat{J}_q - \hat{J}_{-q} \right) + S \left[ \left( \hat{J}_q + \hat{J}_{-q} \right) \left( \hat{J}_q + \hat{J}_{-q} + 2\lambda_1 \right) \right]^{1/2}.
\]  
(12)

Here we denoted

\[
\hat{J}_q \equiv J(0) - J(q) = a (q + \mathbf{A})^2
\]
and used the standard notation \( J(q) \) for the Fourier transform of exchange interaction.

Using (12),(13) we can find the dependence of momentum \( q \) on \( \mathbf{A} \). It determines the Berry phase acquired by the spin wave, which moves in the gauge potential related to the varying anisotropy. We find that in case of adiabatic motion (see details in Ref.[5]), the Berry phase along a contour \( C \) is

\[
\gamma_B(C) = \oint_C \mathbf{A}(r) \cdot d\mathbf{r},
\]

where \( \mathbf{A} = \mathbf{A} \left[ 1 + S^2 \lambda_1^2 / (\omega_q^0)^2 \right]^{-1/2} \) is the effective gauge potential acting on magnons and \( \omega_q^0 = 2S [aq^2 (aq^2 + \lambda_1)]^{1/2} \). For \( \lambda_1 \rightarrow 0 \), we get \( \mathbf{A} = \mathbf{A} \) but for a large anisotropy, \( \lambda_1 \gg \omega_q^0 / S \), the effective potential \( \mathbf{A} \simeq \mathbf{A} \omega_q^0 / S \lambda_1 \) is small comparing to \( \mathbf{A} \). Thus, the Berry phase of magnons is determined by \( \mathbf{A}(r) \). It generalizes the result of Ref.[5] for arbitrary quantum spins on the lattice.

We consider now the contribution to the total energy of the vacuum fluctuations, \( E_0 = \sum_q (\omega_q / 2) \), corresponding to the second term in (10). The energy \( E_0 \) depends on the gauge potential \( \mathbf{A} \) resulting from the homogeneous anisotropy. To calculate the spin current density \( j^s \) we add to \( \mathbf{A} \) an additional fictitious potential \( \mathbf{A}'(r) \), which we put zero after calculation. Thus, in the previous definition of \( J_q \) we substitute \( \mathbf{A} \rightarrow \mathbf{A} + \mathbf{A}' \). Note that \( \mathbf{A}' \), like the original potential \( \mathbf{A} \), is associated with the spin transformation – rotation around the axis \( z \), which corresponds to the spin current density \( j^s \) with the spin polarization along \( z \).

We use the definition \( j^s(r) = \gamma (\delta E_0 / \delta A'_n(r)) \mathbf{A}' = 0 \) following from the connection between the spin current conservation and the invariance with respect to rotations in the spin space, where \( \gamma \) is the gyromagnetic ratio. Then we obtain

\[
j^s = \frac{2a\gamma S}{\Omega} \sum_q \left\{ q + \mathbf{A} \left[ \hat{J}_q + \hat{J}_{-q} + \lambda_1 \right] \left[ \left( \hat{J}_q + \hat{J}_{-q} \right) \left( \hat{J}_q + \hat{J}_{-q} + 2\lambda_1 \right) \right]^{1/2} \right\}
\]
(14)

where \( \Omega \) is the volume. In the usual bulk or an infinite 2D system, the first term vanishes due to the inversion symmetry \( q \rightarrow -q \), and, taking the limit of \( \mathbf{A} \rightarrow 0 \), we find

\[
j^s = \frac{a\gamma S \mathbf{A}}{\Omega} \sum_q \frac{2aq^2 + \lambda_1}{\left[ aq^2 (aq^2 + \lambda_1)]^{1/2} \right.}
\]
(15)

Here the sum over momentum runs up to \( q_{\text{max}} \simeq \pi / a_0 \). We can estimate it for the 3D case as

\[
1 / \Omega \sum_q \frac{2aq^2 + \lambda_1}{\left[ aq^2 (aq^2 + \lambda_1)]^{1/2} \right.} \simeq 1 / 3 \pi^2 \left[ (q_{\text{max}}^2 + \kappa^2)^{3/2} - \kappa^2 \right]
\]
and 

\[
\omega_q = 2S \left[ (\kappa_{\text{max}}^2 + \kappa^2)^{3/2} - \kappa^2 \right].
\]

Using (15), the spin current flows along the gradient of the anisotropy axis variation. In terms of the continuous anisotropy field, \( \mathbf{n}(r) \), it can be written as

\[
\hat{j}^s \mu = \epsilon^\mu\nu\lambda_1 \nu \left( \partial n^\lambda / \partial r_1 \right),
\]

where \( \mu \) refers to the spin polarization and \( \epsilon^\mu\nu\lambda_1 \) is the unit antisymmetric tensor. The spin current is responsible for the transmission of angular momentum and produces the torque rotating the anisotropy axis at each site to orient all of them in the same direction.

In the case of magnetic rings, the first term in (14) can be nonvanishing, which is related to the Berry phase acquired by a moment moving along the ring[10]. As we assume the magnetic ordering uniform, the Berry phase has a non-geometric origin but is determined by the topology of the ring[10]. In its turn, the nonvanishing Berry phase affects the inversion symmetry with respect to the motion along the ring in opposite directions.

To specify this suggestions, let us consider now the mesoscopic magnetic ring in form of a thin-wall cylinder, with the magnetization along the \( z \) axis of cylinder, like described in Ref.[2]. The hard axis is directed perpendicular to the cylinder surface. Correspondingly, it changes direction along a contour around the ring. We can restrict ourselves by considering only the \( \varphi \)-component of momentum \( q \) corresponding to the around-ring motion (see Fig. 2) because the gauge potential in this case has only one nonvanishing component \( \mathbf{A}_\varphi = 1/R \), where \( R \) is the radius of the ring. Note that \( \mathbf{A}_\varphi \) is a constant, and it fully justifies using the Fourier transformation leading to Eq. (8). The periodic condition imposes the requirement of momentum quantization, which reads as

\[
q_{\varphi n} + \mathbf{A}_\varphi = R = n, \quad n = 0, \pm 1, \ldots.
\]

Then the first term in Eq. (14) gives us the spin current in the ring

\[
I^s = \frac{a\gamma S}{\pi R^2} \sum_n \left[ n - \left( 1 + \frac{S^2 \lambda_1}{(\omega_n^0)^2} \right)^{-1/2} \right]
\]
(17)

where \( \omega_n^0 = 2S \left[ (\varphi n^2 + R^2) (\varphi n^2 + \lambda_1) \right]^{1/2} \). In the limit of \( \beta \equiv \lambda_1 R^2 / a \gg 1 \), which corresponds to the weak
gauge field regime, the main contribution is related to \( n \sim \sqrt{\beta} \gg 1 \), and we can substitute the sum in (17) by the integral. In this approximation, we obtain

\[
I^s \simeq \frac{C\gamma S (\lambda_1 a)^{1/2}}{\pi R},
\]

where

\[
C = \int_{-\infty}^{\infty} dx \left( 1 - \frac{2|x|\sqrt{x^2 + 1}}{\sqrt{1 + 4x^2 (x^2 + 1)}} \right) \simeq 0.754.
\]

As follows from (18), the spin current is zero if the anisotropy constant \( \lambda_1 = 0 \). By evaluating the second contribution of Eq. (14) into the spin current, we find that in this case it is small with respect to (18) in a parameter \( a_0/(R\sqrt{3}) \ll 1 \).

Using (18) we can estimate the magnitude of the mechanical stress in the ring. We take \( \lambda_1 = 3.4 \times 10^{-14} \text{erg} \), corresponding to \( \lambda_1/a_0^3 = 4\pi M^2, M = 10^4 \text{G} \), \( a_0 = 3 \times 10^{-8} \text{cm} \); \( a_0^3 = 4 \times 10^{-14} \text{erg} \), \( 2R = 10 \text{nm} \). Then we get the angular momentum \( I^{\circ} \simeq 3 \times 10^{-16} \text{erg} \). Varying the length of ring \( L \) by \( \delta L \), we get a variation \( |\delta I^{\circ}| = (I^{\circ}/L) \delta L \). Correspondingly the force acting in the cross section of the ring is given by \( f = \delta I^{\circ}/\delta L = I^{\circ}/L \), and the stress \( \sigma = f/S_0 \), where \( S_0 \) is the cross section of the ring. Taking \( S_0 = 5 \times 10^{-14} \text{cm}^2 (5 \times 1 \text{mm}^2) \) we find \( \sigma \simeq 0.05 \text{N/cm}^2 \).

In conclusion, we demonstrated the presence of non-vanishing equilibrium spin currents in magnetic structures with the homogeneous magnetization and varied anisotropy axis. One of the most important consequences of this effect is related to the correct definition of the non-equilibrium spin current and interpretation of spin current measurements.\textsuperscript{16,17,18} The existing attempts to exclude the equilibrium spin current have been concentrated only to a part responsible for the torque acting on spins. In the collinear magnetic system, this part of torque is zero. Nevertheless, as we demonstrated, the spin current can also transmit the angular momentum rotating the anisotropy axes like in the case of magnetic rings. Similar to the magneto-electric effect in magnetically inhomogeneous systems\textsuperscript{2}, the mechanical stress in anisotropy-inhomogeneous systems can be called the \textit{magneto-mechanical} effect.

Our estimation of the stress suggests that the corresponding deformations are too small to be measured experimentally. However, the spin current in the ring can be observed due to the electric polarization, as it was discussed recently.\textsuperscript{5,6,9,11} It should be noted that the combination of magnetoelectric and magnetoelectrical effects in the ferromagnet can be connected with the physics of ferroic materials, which are in scope of great activity now.\textsuperscript{12}

We calculated the spin current at \( T = 0 \), when the excitation of real magnons can be neglected. The effect of temperature is twofold. Due to the excitation of magnons, one can expect an additional contribution to the spin current, related to the Berry phase from the Dirac string.\textsuperscript{10} On the other hand, the magnon decohrence increases with the temperature due to the magnon-phonon interaction, and it suppresses the spin current.

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