Turbulent spots in a Stokes boundary layer

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Abstract. The turbulent spots which form in a boundary layer generated by the harmonic oscillations of an incompressible fluid are investigated by numerical means. In order to allow the formation of turbulent spots, the dimensions of the computational box have been increased with respect to previous numerical investigations (Costamagna et al. (2003)). The boundaries of the spots are identified and the speeds of the head, tail, leftmost and rightmost points, are computed. The computed speeds well compare with those measured in steady boundary layers.

1. Introduction

The boundary layer generated by the harmonic oscillations of a fluid parallel to an infinite fixed plate shows different flow regimes depending on the value of the Reynolds number $R_\delta$ which is defined using the amplitude of the fluid velocity oscillations, the characteristic thickness of the boundary layer and fluid kinematic viscosity.

Previous experimental studies (see for example Hino et al. (1976)) showed that for values of $R_\delta$ smaller than 500 but larger than a value, which is quite sensitive to the particular experimental set up and ranges around 100, ‘small amplitude’ perturbations appear superimposed on the Stokes flow, even though the average velocity profiles exhibit only small deviations from the laminar case. Turbulence has unanimously been observed at Reynolds numbers larger than about 500, the resulting flow being characterized by the sudden, explosive appearance of turbulence bursts towards the end of the accelerating phases of the cycle (Hino et al., 1976). Turbulence is sustained throughout the decelerating phases, while during the early stages of the accelerating phases production of turbulence stops, the disturbances decay and the flow recovers its laminar behaviour. For increasing values of $R_\delta$, turbulence affects larger parts of the cycle, till for $R_\delta$ around 3500 (Jensen et al., 1989) turbulence is present throughout the cycle.

Therefore four flow regimes can be identified: I) the laminar regime; II) the disturbed laminar regime, where “small-amplitude” perturbations appear superimposed on the Stokes flow; III) the intermittently turbulent flow, where bursts of turbulence appear explosively only during the decelerating phases of the cycle; IV) the fully developed turbulent regime, characterized by turbulence presence throughout the whole cycle.

Experimental and numerical investigations in the intermittently turbulent regime have shown that the early stages of turbulence appearance are connected to the formation of coherent vortex structures.

Sarpkaya (1993) identified the dynamics of the vortex structures in an oscillatory boundary layer by means of an experimental investigation. In particular Sarpkaya (1993) used a long cylindrical body immersed in a sinusoidally oscillating fluid which moved parallel to the cylinder.
axis. The ratio between the radius of the cylinder and the thickness $\delta^*$ of the boundary layer was large enough to consider the results to be of significance for the flow over a flat plate. For $R_\delta = 400$, Sarpkaya (1993) observed streamwise vortices which remained perfectly straight, smooth and parallel with no visible interaction. When the Reynolds number was increased ($420 < R_\delta < 460$), other events took place. The streaks interacted, moving towards each other and growing in amplitude, and then they tended to coalesce to form a single streak which became sinuous. Shortly thereafter, the streak began to split into short segments which, in turn, began to lift. Moreover parts of the original pair of streaks acquired larger amplitude and gave rise to ‘pockets’. Finally Sarpkaya (1993) argued the presence of hairpin or horseshoe vortices which, as the velocity of the ambient flow increased, folded back, stretched rapidly and became incoherent structures. When the Reynolds number was further increased to about 460-490, a large number of vortices appeared towards the end of the decelerating phases. Usually these turbulent structures did not survive during the accelerating phases and the flow relaminarized. Further increases of the Reynolds number (490-520) led to more numerous vortex structures which penetrated further into the ambient flow. At higher Reynolds number, turbulence activity spanned over larger times and larger areas, till at values of $R_\delta$ around 800 the identification of coherent structures became difficult. Even at values of $R_\delta$ as large as 1800 there were still some time intervals of the cycle where the flow was in a transitional state or in a partially developed turbulent state.

Costamagna et al. (2003), by means of direct numerical simulations of continuity and Navier-Stokes equations and for values of the Reynolds numbers such that the flow was in the intermittently turbulent regime, observed two-dimensional vortex structures with their axis parallel to the bottom and orthogonal to the direction of fluid oscillations. However, these vortex features were observed only when the Reynolds number is close to the critical value for transition from the disturbed-laminar to the intermittently turbulent regime. For larger values of the Reynolds number, Costamagna et al. (2003) observed turbulence generated by a sequence of events similar to those which take place in steady boundary layers. In particular, at the end of the accelerating phases, low speed streaks appeared close to the bottom. During the early part of the decelerating phases the strength of the low-speed streaks grew. Then, the streaks twisted, oscillated and eventually broke, originating small-scale vortices.

A recent experimental investigation of Carstensen et al. (2010) has revealed the existence of two significant coherent turbulent structures: vortex tubes and turbulent spots. The former (vortex tubes) are essentially two-dimensional vortices, with their axis parallel to the bottom and orthogonal to the direction of fluid oscillations, which extend across the width of the boundary layer. These vortex structures are similar to those observed by Costamagna et al. (2003). Turbulent spots are isolated arrowhead-shaped turbulent areas close to the bed in an otherwise laminar flow. While vortex tubes are observed only at relatively small values of the Reynolds number and induce only small oscillations of the bed shear stress, turbulent spots are observed when the Reynolds number is larger than about 548 and produce single or multiple violent spikes in the bed shear stress.

As the main goal of Costamagna et al. (2003) was to isolate the basic flow unit able to originate turbulence and to study its morphology and dynamics, the computational domain was kept as small as possible and Costamagna et al. (2003) could not observe the ‘turbulent spots’ visualized by Carstensen et al. (2010).

In the present paper, the turbulence spots which are generated in the oscillatory boundary layer are investigated using direct numerical simulations of Navier-Stokes and continuity equations. The numerical approach and the code are those of Costamagna et al. (2003) but a larger computational domain is used. In particular, the size of the computational box is approximately 40 times larger, in order to allow the formation of the turbulent spots which were observed by Carstensen et al. (2010). In the following, the formation, dynamics and geometric
characteristics of the turbulent spots are described.

2. Numerical method

The boundary layer which is generated close to a flat smooth wall by the harmonic oscillations of an incompressible fluid in the direction parallel to the plate is investigated by numerical means. The oscillating pressure gradient which drives the flow close to the wall is prescribed as:

\[ \frac{\partial P^*}{\partial x_1^*} = -\rho^* U_0^* \omega^* \sin(\omega^* t^*); \quad \frac{\partial P^*}{\partial x_2^*} = 0; \quad \frac{\partial P^*}{\partial x_3^*} = 0 \] (1)

where \((x_1^*, x_2^*, x_3^*)\) is a Cartesian coordinate system with the \(x_1^*-\)axis pointing in the direction of fluid oscillations, the \(x_2^*-\)axis is vertical and pointing in the upward direction such that \(x_2^* = 0\) describes the averaged wall location. The characteristic length-scale is \(\delta^*\), which is the order of magnitude of the thickness of the boundary layer and is related to the period of fluid oscillations \((T^* = 2\pi/\omega^*)\) by the relationship:

\[ \delta^* = \sqrt{\frac{\nu^* T^*}{\pi}} \]

where \(\nu^*\) is the kinematic viscosity of the fluid. Finally, \(U_0^*\) denotes the amplitude of the fluid velocity oscillations far from the wall. As in Blondeaux & Vittori (1994) and Costamagna et al. (2003), we consider a smooth wall where an imperfection (waviness) of infinitesimal amplitude is introduced; such waviness of the bottom profile cannot be appreciated from a macroscopic point of view but, as explained in Blondeaux & Vittori (1994), is necessary to trigger turbulence. We introduce the following dimensionless variables:

\[ t = t^* \omega^*; \quad (x_1, x_2, x_3) = \frac{(x_1^*, x_2^*, x_3^*)}{\delta^*}; \quad (u_1, u_2, u_3) = \frac{(u_1^*, u_2^*, u_3^*)}{U_0^*}; \quad p = \frac{p^*}{\rho^*(U_0^*)^2} \] (2)

where \(t^*\) is time, \(p^*\) is pressure and \(u_1^*, u_2^*, u_3^*\) are the fluid velocity components along \(x_1^*\)-, \(x_2^*\)- and \(x_3^*\)-directions, respectively.

The Navier-Stokes and continuity equations in terms of the dimensionless variables read:

\[ \frac{\partial u_i}{\partial t} + \frac{R_\delta}{2} u_i \frac{\partial u_i}{\partial x_j} = -\frac{R_\delta}{2} \frac{\partial p}{\partial x_i} + \delta_{k1} \sin(t) + \frac{1}{2} \frac{\partial^2 u_i}{\partial x_k \partial x_k} \quad i = 1, 2, 3 \] (3)

\[ \frac{\partial u_i}{\partial x_i} = 0 \] (4)

where

\[ R_\delta = \frac{U_0^* \delta^*}{\nu^*}. \]

At the wall, having assumed the amplitude of the wall waviness \((\epsilon^*)\) to be much smaller than the thickness of the laminar boundary layer, the no-slip condition is expanded up to second order with respect to the parameter \(\epsilon = \epsilon^*/\delta^*\), in order to obtain a boundary condition at \(x_2^* = 0\).

The equations are solved numerically by means of a numerical approach which uses standard centered second-order finite difference approximations of the spatial derivatives, while the time-advancement of Navier-Stokes equations is obtained using the fractional-step method. The computational mesh is uniform in the streamwise and spanwise directions while in the cross-stream direction a non-uniform mesh is used, to cluster the grid points in the vicinity of the wall, where velocity gradients are expected to be larger. Details of the numerical approach used in present simulations are available in Orlandi (1989), Vittori & Verzicco (1998) and Costamagna...
Table 1. Numerical parameters of the present simulations.

| test n. | $R_\delta$ | $L_{x1}$ | $L_{x2}$ | $L_{x3}$ | $n_1$ | $n_2$ | $n_3$ |
|---------|------------|----------|----------|----------|-------|-------|-------|
| T1      | 948        | 213.60   | 25.13    | 75.40    | 541   | 65    | 385   |
| T2      | 775        | 226.19   | 25.13    | 94.25    | 572   | 65    | 480   |
| T3      | 1220       | 213.60   | 25.13    | 75.40    | 541   | 65    | 385   |

et al. (2003). A quantitative comparison, described in Costamagna et al. (2003), of the results obtained by means of the numerical approach with the experimental measurements by Jensen et al. (1989), fully supports the numerical procedure.

The aim of the present contribution is that of investigating the formation and characteristics of the turbulent spots which have been detected experimentally by Carstensen et al. (2010).

Figure 9 of Carstensen et al. (2010) shows the formation of a turbulent spot of size comparable to the size of the picture area, which is approximately equal to 38 cm $\times$ 13 cm in the streamwise and spanwise directions, respectively. Considering the value of the physical parameters in the experiment of Carstensen et al. (2010), the displayed area is approximately 218 $\delta^*$ long and 76 $\delta^*$ large. Therefore in the present simulations the size of the computational box is increased with respect to that used by Costamagna et al. (2003) and the number of computational points ($n_1$, $n_2$, $n_3$ in the $x_1$-, $x_2$- and $x_3$- directions, respectively) has been increased consequently. The values of the numerical parameters for each run are given in Table 1.

3. Results

During their experiments in the transitional regime, Carstensen et al. (2010) detected two types of coherent flow structures: vortex tubes and turbulent spots. In particular, for values of $R_\delta$ larger than 548, Carstensen et al. (2010) observed that turbulent spots appear randomly in space around the end of the accelerating phases. Moreover, they observed that turbulent spots emerge from the dynamics of high and low speed streaks which develop twisting and turning motions and then break into smaller structures.

Figures 1 and 2 show the time development of the modulus of the fluctuating part of the spanwise component of fluid velocity for tests T2 and T3. Similarly to the observations of Carstensen et al. (2010), turbulent spots appear randomly in space during the accelerating phases (see figures 1a and 2a), they grow in intensity and size (see figures 1b,c and 2b,c), until they merge with their neighbours (see figures 1d and 2d). During the decelerating phases turbulence pervades the whole boundary layer. During the early stage of the following accelerating phases, turbulence decays and the flow tends to recover a laminar-like behaviour. Comparing the results shown in figures 1 and 2, which are characterized by a different value of $R_\delta$, it can be appreciated that for the higher value of $R_\delta$ turbulent spots appear earlier during the cycle and, as a consequence, they merge with the neighbouring spots already during the accelerating phase. Moreover for the higher value of $R_\delta$, more spots form in the computational box.

To quantify the geometric characteristics of the turbulent spots, a quantitative analysis was performed of a large number of spots, before they merge with their neighbours. First, a preliminary analysis aimed at choosing the criterion to detect the boundaries of the spots was made. The boundaries of a spot were defined and detected as the boundaries of the area where a given quantity exceeds an assigned percentage of the maximum in time and in the $x_2$-direction of the horizontal average of the chosen quantity. We considered: i) the local value of the dimensionless production of turbulent kinetic energy (TKEP), ii) the local value
Figure 1. Time development of $|u'_3|$ for test T2 at $x_2 = 0.32$. a) $t=18.49$ isolines are plotted with $\Delta = 0.003$; b) $t=18.61$ isolines are plotted with $\Delta = 0.005$; c) $t=18.70$ isolines are plotted with $\Delta = 0.005$; d) $t=19.02$ isolines are plotted with $\Delta = 0.007$. The free-stream velocity changes as $-\cos(t)$.

of the dimensionless turbulent kinetic energy (TKE), iii) the modulus of the projection on a horizontal plane of the fluctuating component of the dimensionless shear stress ($|\tau'|$), iv) the absolute value of the fluctuating component of the dimensionless streamwise velocity ($|u'_1|$), v) the absolute value of the fluctuating component of the dimensionless spanwise velocity ($|u'_3|$).

The corresponding threshold values used to detect the boundaries of the spots are $\text{TKE}_{\text{M}}$, $\text{TKE}_{\text{M}}$, $|\tau'_{\text{M}}|$, $|u'_{1\text{M}}|$ and $|u'_{3\text{M}}|$ respectively and their numerical values depend on the test considered. Figure 3 shows, for test T1 at $t=5.64$, the areas in the $x_1-x_3$ plane for $x_2 = 0.16$, where $\text{TKE}$, $\text{TKE}, |u'_1|$ and $|u'_3|$ exceed $\text{TKE}_{\text{M}}, \text{TKE}_{\text{M}}, |u'_{1\text{M}}|$ and $|u'_{3\text{M}}|$ respectively. It can be noticed that the area characterized by large fluctuations of the chosen quantity is of a similar extent if $\text{TKE}$ and $|u'_3|$ are considered, while the regions where $\text{TKE}$ and $|u'_1|$ attain large values are affected by the presence of small fluctuations which are located close to the spot. A similar plot is obtained considering $|\tau'|$.

Table 2 shows the positions of the extreme points of the spot (head, tail, left side and right side) shown in figure 3, obtained by considering the different quantities. The comparison of the values obtained shows that the differences in the location of the head/tail of the spot are small. It should be noted that the threshold values used to detect the boundaries of the spot are twice that used in the plots of figure 3.
**Figure 2.** Time development of $|u'_3|$ for test T3 at $x_2 = 0.32$. a) $t=11.24$, isolines are plotted with $\Delta = 0.003$; b) $t=11.60$, isolines are plotted with $\Delta = 0.003$; c) $t=11.70$, isolines are plotted with $\Delta = 0.003$; d) $t=11.80$, isolines are plotted with $\Delta = 0.007$. The free-stream velocity changes as $-\cos(t)$.

**Table 2.** Positions of the head and tail of the turbulent spot shown in figure 3 at $t = 5.653$ and $x_2=0.16$ and computed using the different criteria defined in the text (Test T1). The threshold values are 200% of the maximum in time and in the $x_2$ direction of the horizontally-averaged quantities.

| criterion | threshold | $x_{1H}$ | $x_{3H}$ | $x_{1T}$ | $x_{3T}$ | $x_{1RS}$ | $x_{3RS}$ | $x_{1LS}$ | $x_{3LS}$ |
|-----------|-----------|----------|----------|----------|----------|-----------|-----------|-----------|-----------|
| TKEP      | 0.0030    | 34.08    | 20.67    | 65.39    | 22.25    | 49.54     | 19.10     | 40.82     | 25.00     |
| TKE       | 0.0206    | 32.10    | 21.06    | 66.18    | 22.25    | 48.35     | 19.88     | 41.21     | 24.61     |
| $|r'|$     | 0.3020    | 36.46    | 22.05    | 62.61    | 20.28    | 55.08     | 19.29     | 42.40     | 23.82     |
| $|u'_3|$   | 0.0922    | 32.50    | 21.06    | 66.18    | 22.25    | 48.74     | 19.88     | 57.85     | 23.62     |
| $|u'_1|$   | 0.2200    | 33.68    | 22.24    | 63.01    | 21.85    | 48.35     | 19.29     | 39.63     | 23.62     |

The positions of the head, tail and lateral points of the spots as a function of time make it possible to evaluate their velocity. Figure 4 shows the evolution of the positions of the head and tail of spots A and B which are first detected during test T1 at $t = 5.3$ and start to merge.
Figure 3. a) regions where TKEP is larger than 0.0015 and isolines ($\Delta_{TKEP}=0.001$); b) regions where TKE is larger than 0.0103 and isolines ($\Delta_{TKE}=0.01$); c) regions where $|u_1'|$ is larger than 0.11 and isolines ($\Delta_{|u_1'|}=0.12$); d) regions where $|u_3'|$ is larger than 0.0461 and isolines ($\Delta_{|u_3'|}=0.045$). Test T1; $t=5.64$ $x_2=0.32$. Flow from right to left.

with their neighbours at $t = 5.8$. The lines in the right panel indicate the least-square fit of the data and approximate the actual positions of the head and tail of the spots with a r.m.s. error which ranges between $1.13\delta^*$ and $3.33\delta^*$. Therefore, it can be concluded that, during the time interval considered, the speed of the head and tail of the spots can be assumed to be practically constant and given by the slope of each line.

The speeds of the head, tail, left side and right side of the spot shown in figure 3 and computed by means of the different methods are given in Table 3. It can be observed that while the values of $u_{1H}$ and $u_{1T}$ obtained considering the different criteria, differ from each other by a few percents of the average value, larger differences are observed for $u_{1LS}$ and $u_{1RS}$. However, it should be noticed that $u_{1LS}$ and $u_{1RS}$ are one to two orders of magnitude smaller than $u_{1H}$ and $u_{1T}$. Moreover it is worthwhile to mention that even though the speeds are given with two significant digits, the second digit should be treated with caution. An estimate of the error comes from the values of $u_{3LS}$ and $u_{3RS}$ which should be equal. On the basis of this preliminary investigation, it was chosen to identify the boundaries of the turbulent spots by using the TKEP criterion. Figure 5a shows the speeds computed for two spots which form during the 3rd half-oscillation cycle of T2. As it can be observed, the vertical distribution of the speeds of the boundaries of the spots shows some oscillations, presumably related to the difficulties in the
Figure 4. a) TKEP at t=5.575, $x_2 = 0.32$, $\Delta_{TKEP} = 0.005R_\delta$; b) position of the head and tail of spot A and spot B, identified by means of the TKEP criterion, plotted versus time. The lines have been obtained by linear regression. The outer flow is directed from right to left. Test T1.

Table 3. Speeds of the head, tail, left side and right side of the turbulent spot shown in figure 3 (spot B in figure 4a) at $x_2=0.16$ computed using the different criteria defined in the text (Test T1).

| criterion | $u_1H$ | $u_1T$ | $u_{1RS} \times 10^2$ | $u_{1LS} \times 10^2$ |
|-----------|--------|--------|----------------------|----------------------|
| TKEP      | 0.70   | 0.58   | 0.60                 | 2.48                 |
| TKE       | 0.72   | 0.55   | 1.74                 | 3.79                 |
| $|\tau'|$   | 0.65   | 0.54   | 0.96                 | 1.93                 |
| $|u_1'|$   | 0.73   | 0.54   | 1.36                 | 3.60                 |
| $|u_3'|$   | 0.72   | 0.66   | 0.25                 | 2.58                 |

Table 4. Averaged speed of the head ($u_{1H}$), tail ($u_{1T}$), right side ($u_{3RS}$) and left side ($u_{3LS}$) of the spots. The caret indicates the speed scaled with the instantaneous outer velocity.

| Test n. | $R_\delta$ | $u_{1H}$ | $u_{1T}$ | $u_{3LS}$ | $u_{3RS}$ | $\hat{u}_{1H}$ | $\hat{u}_{1T}$ | $\hat{u}_{3LS}$ | $\hat{u}_{3RS}$ |
|---------|------------|----------|----------|-----------|-----------|----------------|----------------|----------------|----------------|
| T1      | 948        | 0.68     | 0.41     | 0.02      | 0.03      | 0.87           | 0.53           | 0.03           | 0.04           |
| T2      | 775        | 0.67     | 0.40     | 0.04      | 0.03      | 0.78           | 0.46           | 0.04           | 0.03           |
| T3      | 1220       | 0.51     | 0.33     | 0.02      | 0.03      | 0.94           | 0.61           | 0.04           | 0.05           |

accurate estimate of the position of the borders of the spots. Moreover differences are found if spots forming at different half-cycles are considered (see figure 5b). These differences might be partially due to a possible correlation between the velocity of the boundaries of the spots with the external velocity which is different because the spots appear at slightly different phases of the cycle. Since the variations of the values of the speeds of the boundaries of the spots in the vertical direction are of the same order of magnitude as the random variations discussed above, in the following only the speeds averaged in the $x_2-$ direction are presented.

Table 4 gives the speeds of the spots, averaged in the cross-stream direction and over a large number of spots, for the three runs. Columns from 3 to 6 show $u_{1H}$, $u_{1T}$, $u_{3LS}$ and $u_{3RS}$ while the remaining columns show the corresponding quantities scaled with the instantaneous value of the outer velocity $U'_e ((\hat{u}_{1H}, \hat{u}_{1T}, \hat{u}_{3LS}, \hat{u}_{3RS})=(u_{1H}^*, u_{1T}^*, u_{3LS}^*, u_{3RS}^*)/U'_e)$.
Figure 5. a) Speeds of the head ($u_{1H}$), tail ($u_{1T}$), left side ($u_{3LS}$) and right side ($u_{3RS}$) for two spots in the 3rd oscillating cycle of test T2 as function of the vertical coordinate $x_2$; b) Speeds of the head of different spots at different half cycles together with the average value (thick solid line) as function of the vertical coordinate (test T2).

The speed of the boundaries of the spots is smaller than the external velocity, in accordance with results for steady boundary layers (Schubauer & Klebanoff (1955), Singer (1996)). For the Blasius boundary layer Schubauer & Klebanoff (1955), by means of an experimental investigation of the characteristics of a turbulent spot artificially initiated, observed values of the speeds of the head and tail of the spot equal to 0.88 $U^*_c$ and 0.5 $U^*_c$ respectively. A numerical investigation of a spot, initiated in a Blasius boundary layer by a localised disturbance, showed values of the speeds of the head and tail of the spot equal to 0.97 $U^*_c$ and 0.63 $U^*_c$ respectively (Singer (1996)). Thus, it appears that, even though the present boundary layer is unsteady, the speeds of the head and tail of the spots, normalized with the instantaneous actual velocity are similar to those observed in steady boundary layers. The speeds of the two lateral boundaries of a spot are one order of magnitude smaller than those of the head and tail because they are due to turbulent diffusion only. Similar values of $\hat{u}_{3LS}$ and $\hat{u}_{3RS}$ are obtained for T1, T2 and T3, while significant differences are found for $\hat{u}_{1H}$ and $\hat{u}_{1T}$. In particular $\hat{u}_{1H}$ and $\hat{u}_{1T}$ grow as $R_\delta$ is increased and for run T3 ($R_\delta=1220$) they are larger than those for run T2 ($R_\delta=775$) of approximately 20-30 %. Moreover, it can be appreciated that the expansion rate of the spot in the streamwise direction is significantly larger than that in the spanwise direction.
4. Conclusions

The numerical integration of Navier Stokes and continuity equations in a large computational domain has allowed the formation of the turbulent spots already observed during the experiments of Carstensen et al. (2010). The computed flow fields have allowed the estimate of quantities, such as the local values of turbulent kinetic energy and of turbulent kinetic energy production, which are difficult to obtain on the basis of experimental measurements. Different criteria to detect the boundaries and the speed of the spots, before they merge with their neighbours, have been considered and similar results have been obtained. On the basis of that preliminary investigation, it has been decided to identify the spots as the regions where the local value of turbulent kinetic energy production exceeds an assigned value. It has been observed that the speeds of the head, tail, leftmost side and rightmost side show only small random oscillations in the vertical direction which are of the same order of magnitude as the precision of the estimate of the speed of the spot itself. Therefore only the averaged values in the cross-stream direction are presently discussed. It is observed that the speeds of the head and tail of the spots, when divided by the instantaneous value of the free stream velocity, are in broad agreement with the corresponding measurements of Schubauer & Klebanoff (1955) and Singer (1996) for steady boundary layers. The speeds of the leftmost and rightmost points of the spots are related to turbulent diffusion effects and are one order of magnitude smaller than the speeds of the head and tail. Moreover it has been observed that an increase of the value of the Reynolds number $R_d$ leads to increasing values of the speed of the boundaries of the spots.

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