Magnetization of a homogeneous two dimensional fermion gas with repulsive contact interaction and Rashba spin-orbit potential

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The z axis magnetization of a two dimensional electron gas with contact repulsive interaction and in presence of a Rashba potential is computed by means of quantum field theory at second order. A striking effect of the pure Rashba interaction is that of hindering spin alignment along the z direction. Evidence of transition at critical repulsive interaction coupling constant is still found. The degree of magnetization, however, shows a clear dependence on the spin-orbit interaction strength. Furthermore, the transition to magnetized state appears to be smoothed by the presence of the Rashba interaction.

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I. INTRODUCTION

The magnetization of a homogeneous gas of fermions is a long standing problem, already considered by several authors during the past years. It is well known, for instance, how, in presence of a contact repulsive interaction, both the two and three dimensional spin 1/2 fermion gas undergo a phase transition, magnetizing upon increase of the repulsive potential coupling constant. A first, tentative, experimental proof of itinerant ferromagnetism in cold fermion atoms was recently given by Jo et al. The basic mechanism underlying the phase transition could be well understood in terms of a competition between kinetic energy and repulsion of fermions with opposite spins. For strong repulsive interactions the potential energy reduction induced by the polarization will compensate the corresponding increase of kinetic energy, favoring the appearance of a magnetization.

Though the physical behavior of the above mentioned systems appears to be well understood, the recent interest in spin orbit (SO) interactions both in semiconductor devices and in systems of cold atoms, leads to new questions regarding the appearance of possible unknown spin effects. Among the different SO interactions present in semiconductors, the Rashba interaction certainly plays a fundamental role, especially due to its tunable strength.

The form of the Rashba interaction is:

\[ V_{\text{Rashba}} = \lambda (k_y \sigma_x - k_x \sigma_y). \] (1)

where \( k_{x,y} \) represents the momentum operator and \( \sigma_{x,y} \) are Pauli matrices. This SO interaction generates in asymmetric quantum wells due to the existence of an electric field perpendicular to the plane of confinement. The confinement introduced by the very narrow quantum well, in turn, allows for a two dimensional treatment of the electrons, which are described as effectively moving in the \( x-y \) plane. Moreover, an external gate potential could be applied in order to effectively modify the strength of the interaction \( \lambda \).

Recently, the Rashba interaction has been reproduced in ultracold atoms by means of controlled laser beams externally applied to the system. The realization of SO interactions by means of artificial gauge fields certainly boosted the interest on SO interactions, and a number of theoretical studies appeared in the last few months, regarding SO effects in superfluid or superconducting states. Experimentally, a quasi two dimensional cold atom gas can be realized by means of counterpropagating laser beams along the \( z \) direction with antinodes at half wavelength spacing \( b \). The ultracold atom gas is a particularly favorable system for the study of magnetization properties. The absence of interfering phonons, present in solid state systems, and the elevated degree of control achievable, make it a valuable tool for investigating delicate magnetic properties, such as combined SO and population imbalance effects.

Some of the most relevant questions related to the two dimensional fermion gas in presence of Rashba interaction are those related to its magnetization properties. The interplay of Rashba interaction and magnetization appears indeed not yet well understood. A recent Diffusion Monte Carlo simulation and perturbative analytical approaches for the two dimensional electron gas in presence of both Coulomb repulsion and Rashba interaction revealed negligible two-body effects on the occupation of single particle Rashba states and no appreciable magnetization along the \( z \) axis. In the following we will consider an unpolarized cold atoms assembly with a repulsive two body interaction. Since due to the very low-energy only s-wave scattering is important, the interaction may be modeled by a contact interaction acting only between particles of opposite spins, due to Pauli principle. Therefore, in the action of the uniform 2-D system confined to the \( x-y \) plane it will be described by a term of the form:

\[ V = g \sum_{k_1,k_2,q} \psi_k^\dagger \psi_{k_1+q}^\dagger \psi_{k_2-q}^\dagger \psi_{k_2}^\dagger \psi_{k_1} \] (2)

The coupling constant \( g \) is assumed to be positive, and \( \psi_k^\dagger \) indicate the fermion fields with \( \pm1/2 \) spin component. Experimentally, \( g \) may be modified by exploiting the Feshbach resonance mechanism. Notice that the interaction in Eq. (2) differs from the Coulomb one between electrons in a quantum well, as it lacks the long-range interaction. 


range tail and acts only between particles with opposite spin.

The present paper is organized as follows: In the II section, the path integral computation of the free energy at second order in the coupling constant \( g \) will be illustrated. In particular, results will be derived considering the possible presence of an external Zeeman potential, introducing a chemical potential difference, i.e. an imbalance, between the \( \uparrow \) and \( \downarrow \) populations. Some analytical results will be discussed regarding the case of zero external potential. In Section III numerical results will be shown for the magnetization of the system as a function of both the \( \lambda \) and the \( g \) coupling constants.

II. SECOND ORDER PERTURBATION THEORY

In the following, a path integral derivation of the system free energy will be given, up to second order in the repulsive potential coupling constant \( g \), while exactly including the Rashba effects in the independent particle propagator. We stress that the second order approximation to the free energy will be obtained as an expansion in the two body interaction, starting from the independent particle solutions. This approach is to be preferred in two dimensions with respect to a saddle point approximation. In fact, in absence of Rashba interaction, the stationary points of the action do not correspond to its minima but instead they provide maxima and therefore one cannot proceed to evaluate the partition function by considering only small fluctuations around them, since large fluctuations would be favored instead. A minimum of the energy, however, is correctly recovered by a minimization of the energy calculated by standard perturbation theory, starting from the solution of the non interacting system. This may be understood by expressing the energy at first order in \( g \) in terms of the spin \( \uparrow \) and spin \( \downarrow \) densities:

\[
E/V = \frac{\pi}{m}(n_{\uparrow}^2 + n_{\downarrow}^2) + g \mu n_{\uparrow} - n_{\downarrow}. \tag{3}
\]

In the 2D gas, both the kinetic energy and the repulsive term show quadratic dependence. As a consequence, for a given at \( n = n_{\uparrow} + n_{\downarrow} \) it will be a parabolic function of the net magnetization \( s = n_{\uparrow} - n_{\downarrow} \), with a minimum or a maximum at \( s = 0 \) according to whether \( \frac{\Delta}{m} - g \) is positive or negative, respectively. So the stationary value at \( s = 0 \) is a maximum for large values of \( g \) while the energy is obtained when either \( n_{\uparrow} = n \) or \( n_{\downarrow} = n \), i.e. when the system is completely polarized. The independent particle propagator \( G_0 \) in presence of Rashba SO, could be obtained by inverting the following expression for \( G_0^{-1} \), written in the \( \uparrow, \downarrow \) spin basis:

\[
G_0^{-1}(\omega_n, \mathbf{k}) = \begin{pmatrix} -i\omega_n + \frac{k^2}{2m} - \mu - \Delta \mu & \lambda(k_y + ik_x) \\ \lambda(k_y - ik_x) & -i\omega_n + \frac{k^2}{2m} - \mu + \Delta \mu \end{pmatrix} \tag{4}
\]

where \( \omega_n = \frac{i}{\pi}(2n + 1) \) are fermions Matsubara frequencies, \( \mu = \frac{\mu_+ + \mu_-}{2} \) is the chemical potential of the system, while \( \Delta \mu = \frac{\mu_+ - \mu_-}{2} \). An external potential of the form \(-\Delta \sigma_z \) is included, favoring the occupation of \( \uparrow \) states with respect to \( \downarrow \). The diagonalization of \( G_0^{-1} \) leads to the eigenenergies

\[
\epsilon_{\pm} = \frac{k^2}{2m} \pm \sqrt{\Delta \mu^2 + \lambda^2 k^2} \tag{5}
\]

corresponding to the eigenstates

\[
\phi_{\pm}(\mathbf{k}) = c_{\pm, \mathbf{k}} \begin{pmatrix} \lambda(k_y \mp ik_x) \\ \Delta \mu \mp \sqrt{\Delta \mu^2 + \lambda^2 k^2} \end{pmatrix} \tag{6}
\]

where the normalization constants \( c_{\pm, \mathbf{k}} \) are defined to be real and obey the equation:

\[
c_{\pm, \mathbf{k}}^2 \left( \frac{\lambda^2 k^2}{(\Delta \mu \mp \sqrt{\Delta \mu^2 + \lambda^2 k^2})^2} + 1 \right) = 1. \tag{7}
\]

These states will be hereafter referred to as \( \pm \) for simplicity. Knowing the \( G_0^{-1} \) eigenstates, it is possible to write the transformation matrix

\[
\mathcal{R}_k = \begin{pmatrix} c_{+, \mathbf{k}} & c_{-, \mathbf{k}} \\ -c_{-, \mathbf{k}} & c_{+, \mathbf{k}} \end{pmatrix} \tag{8}
\]

which diagonalizes the independent particle inverse propagator, yielding

\[
G_{0, \text{diag}}^{-1}(\omega, \mathbf{k}) = \mathcal{R}_k G_0^{-1}(\omega, \mathbf{k}) \mathcal{R}_k^\dagger \tag{9}
\]

where \( G_{0, \text{diag}}^{-1} \) is the diagonalized independent particle inverse. We stress that the inclusion of the Rashba interaction in the independent particle propagator allows for a non perturbative treatment of the SO interaction. A perturbative approach is instead employed for the two body repulsive interaction. By using the relation

\[
\bar{\psi}_{\uparrow} \bar{\psi}_{\downarrow} \psi_{\downarrow} \psi_{\uparrow} = \frac{1}{4} \sum_{n=\uparrow, \downarrow} \bar{\psi}_{\alpha} \psi_{\alpha} \tag{10}
\]

it is possible to employ a double Hubbard-Stratonovich transformation after introducing the auxiliary fields \( \rho \) and \( \phi_z \), associated to density and \( z \) magnetization respectively:

\[
e^{-\frac{\lambda}{2} (\bar{\psi}_{\alpha} \delta_{\alpha \beta} \psi_{\beta})^2} = \int d\rho d\phi_z e^{-\frac{\lambda}{2} (\bar{\psi}_{\alpha} \sigma_{\alpha \beta} \psi_{\beta})^2} \tag{11}
\]

The grand canonical partition function can then be expressed as

\[
Z = \int D\phi_z D\rho D\bar{\psi} D\psi e^{-\frac{\lambda}{2} (\bar{\psi}_{\alpha} \sigma_{\alpha \beta} \psi_{\beta}) + \bar{\psi}_{\alpha} [\delta_{\alpha \beta} + g_\phi \phi_z \sigma_{\alpha \beta}] \psi_{\beta}} \tag{12}
\]
Integrating over the fermion fields and expanding the action up to quadratic order in the fields $\phi_z$ and $\rho$ one obtains

$$
S(\phi_z, \rho) = \int d\tau dr \{ g(\phi_z^2 - \rho^2) - Trln G_0^{-1} - g Tr[G_0(\rho \mathbf{1} - \phi_z \sigma^z)] + \frac{g^2}{2} Tr[G_0(\rho \mathbf{1} - \phi_z \sigma^z)] G_0(\rho \mathbf{1} - \phi_z \sigma^z) + O(g^3) \}.
$$

Then, by expressing $G_0$ in terms of $G_{\text{diag}}$ and $\mathcal{R}$ (see [14]), it is possible to rewrite the second line of the above equation as:

$$
\mathbf{L} \cdot \mathbf{\Phi}(k = 0)
$$

where $\mathbf{\Phi} = (\rho, \phi_z)$ and

$$
\mathbf{L}^T = g \beta \sum_k \left( f(\xi_k^\uparrow) + f(\xi_k^\downarrow) \right) f(\xi_k^\downarrow)(1 - 2c_{+k}^2) + f(\xi_k^\uparrow)(1 - 2c_{-k}^2)
$$

Following a similar, though slightly more complex procedure, the third line of [15] is expressed as:

$$
\sum_q \mathbf{Q}_q \mathbf{\Phi}_q
$$

with:

$$
\mathbf{Q}_q = \frac{1}{2} Tr_k \left( \mathbf{M} \mathbf{N} \mathbf{P} \right)
$$

where the trace $Tr_k$ is taken over two fermion momenta $k_1$ and $k_2$ satisfying the relation $k_2 - k_1 = -q$. $\mathcal{M}$ stands for

$$
\mathcal{M}_{k_1, k_2} = \sum_{s_1, s_2 = \pm} m_{s_1, s_2, k_1, k_2} g_{s_1 k_1}^0 g_{s_2 k_2}^0
$$

with similar expressions for $\mathbf{N}$ and $\mathbf{P}$. $G_0^+$ and $G_0^-$ are the + and - components of $G_{\text{diag}}$ and $n$, $m$ and $p$ are defined as follows:

$$
n_{s_1, s_2, k_1, k_2} = \frac{(c_{s_1, k_1} \lambda k_1)^2}{(\Delta \mu + s_1 \sqrt{\Delta \mu^2 + \lambda^2 k_1^2} \Delta \mu + s_2 \sqrt{\Delta \mu^2 + \lambda^2 k_2^2} \Delta \mu + s_1 \sqrt{\Delta \mu^2 + \lambda^2 k_2^2})^2} - (c_{s_1, k_1} c_{s_2, k_2})^2
$$

$$
m_{s_1, s_2, k_1, k_2} = n_{s_1, s_2, k_1, k_2} + d_{1, s_1, s_2, k_1, k_2} + d_{2, s_1, s_2, k_1, k_2}
$$

$$
p_{s_1, s_2, k_1, k_2} = n_{s_1, s_2, k_1, k_2} + d_{1, s_1, s_2, k_1, k_2} - d_{2, s_1, s_2, k_1, k_2}
$$

where $n_{\uparrow, \downarrow}$ are the density components with spin $\uparrow$ or $\downarrow$ and $n$ is the total density, is simply $2\langle \phi^2 \rangle / n$ with:

$$
\langle \phi^2 \rangle = \frac{\int D\phi D\rho e^{-S + J\phi^0} \phi^0}{\int D\phi D\rho e^{-S + J\phi^0} J = 0} = \frac{\partial}{\partial J} \ln \left( \int D\phi D\rho e^{-S + J\phi^0} \phi^0 \right) J = 0
$$

The introduction of the field $J$ corresponds to modifying the vector $\mathbf{L}$ into

$$
\mathbf{L'} = \mathbf{L} + \left( \begin{array}{c} 0 \\ J \end{array} \right)
$$

which then yields

$$
\langle \phi^2 \rangle = \frac{1}{4\beta V} \left[ (0, 1) \mathbf{Q}^{-1} \mathbf{L} + \mathbf{L}^T \mathbf{Q}^{-1} \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \right] = \frac{1}{2\beta^2 V^3} \left[ \left( \frac{g^2}{2} Tr M \right) \left( \sum_k f(\xi_k^+ \lambda) + f(\xi_k^- \lambda) \right) + \left( g\beta V - \frac{g^2}{2} Tr M \right) \cdot \left( \sum_k f(\xi_k^+ \lambda)(1 - 2c_{+k}^2) + f(\xi_k^- \lambda)(1 - 2c_{-k}^2) \right) \right]
$$

The last term on the first line of [13] could also be accounted for by modifying the matrix $Q$ into

$$
\mathbf{Q'} = \text{Tr} \left( \begin{array}{ccc} -g\beta V & 0 & 0 \\ 0 & \frac{g^2}{2} M & 0 \\ 0 & 0 & -g\beta V + \frac{g^2}{2} P \end{array} \right)
$$

The partition function [12] can thus be written as

$$
Z = \int D\phi D\rho e^{-S_0} \int D\phi D\rho e^{-\mathbf{Q}' \mathbf{\Phi}' - \mathbf{L}' \Phi} = e^{-S_0(\text{Det} \mathbf{Q'})^{-1/2}} e^{\mathbf{L}^T \mathbf{Q'}^{-1} \mathbf{L}}
$$

where $S_0$ represents the action of the independent particle system. The logarithm of the determinant of $\mathbf{Q}'$ will then be expanded in $g$ and only terms up to quadratic order will be retained. The above expression contains a multiplicity of terms and certainly appears more complicated than the corresponding formula, obtained in absence of Rashba interaction. The reason of this complication resides in the momentum dependent transformations $\mathcal{R}_k$, which needs to be taken into account due to the spin structure of the Rashba independent particle solutions.

### III. POLARIZATION

The polarization along the $z$ axis defined as

$$
M = \frac{n_{\uparrow} - n_{\downarrow}}{n}
$$

The introduction of the field $J$ corresponds to modifying the vector $\mathbf{L}$ into

$$
\mathbf{L'} = \mathbf{L} + \left( \begin{array}{c} 0 \\ J \end{array} \right)
$$

which then yields

$$
\langle \phi^2 \rangle = \frac{1}{4\beta V} \left[ (0, 1) \mathbf{Q}^{-1} \mathbf{L} + \mathbf{L}^T \mathbf{Q}^{-1} \left( \begin{array}{c} 0 \\ 1 \end{array} \right) \right] = \frac{1}{2\beta^2 V^3} \left[ \left( \frac{g^2}{2} Tr M \right) \left( \sum_k f(\xi_k^+ \lambda) + f(\xi_k^- \lambda) \right) + \left( g\beta V - \frac{g^2}{2} Tr M \right) \cdot \left( \sum_k f(\xi_k^+ \lambda)(1 - 2c_{+k}^2) + f(\xi_k^- \lambda)(1 - 2c_{-k}^2) \right) \right]
$$
From the above formula it follows that if $\Delta \mu = 0$ then $c_{\pm, k} = 1/\sqrt{2}$ and the z polarization, is identically zero.

We stress that this result is not sufficient in order to establish the absence of magnetization for any value of the coupling constant $g$, since it is a direct consequence of the fact that the expectation value of $\langle \sigma_z \rangle$ on both our $+$ and $-$ unperturbed Rashba states (see Eq. (6)) is zero, when $\Delta \mu = 0$. Therefore even when the occupation numbers of these states are different, the total magnetization is zero. To allow a nonzero magnetization, we must start from unperturbed Rashba states where $\langle \sigma_z \rangle$ may be nonzero. This is easily achieved by introducing a variational field favoring the occupation of $\uparrow$ over $\downarrow$ spin states, i.e. substituting $\Delta \mu$ with $\Delta \mu + h$, in all the above formulas. A the end of the calculation one will get $h$ by minimizing $E - h M n$.

As will be discussed in the subsequent section, in general one will find that when $\Delta \mu = 0$, the minimum energy is achieved for $h = 0$ only when the repulsion strength $g$ is below a critical value $g_c$. No magnetization will thus be present when $g < g_c$.

As mentioned above, this property appears to be closely related to the single particle spin properties of the system: in fact, when $h = 0$, the expectation value of $\sigma_z$ over any of the Rashba states (at $\Delta \mu = 0$) is equal to zero due to spin rotation around the particle wave vector axis. Remarkably, in this case the independent particle Rashba states contain no dependence on the parameter $\lambda$, also implying constant $\langle \sigma_z \rangle = 0$ even at $\lambda \to 0$. In order to gain a more complete picture of the whole magnetization properties of the gas, calculations were also performed for $\langle \sigma_x \rangle$ and $\langle \sigma_y \rangle$. In fact, the absence of a $z$ magnetization in presence of Rashba SO coupling does not in principle exclude the appearance of non zero in-plane magnetization. As an example, in the 2D fermi gas in absence of Rashba interaction, due to spin-rotation invariance, magnetization might equally occur along any direction if $\Delta \mu = 0$, while being necessarily oriented along $z$ for $\Delta \mu \neq 0$.

In order to describe the possible occurrence of in-plane magnetization, one may rewrite the contact repulsive interaction (11) as

$$\frac{1}{4} \left( \sum_{\alpha = \uparrow, \downarrow} \bar{\psi}_\alpha \psi_\alpha \right)^2 - \frac{1}{4} \left( \sum_{\alpha, \beta = \uparrow, \downarrow} \bar{\psi}_\alpha \hat{\sigma}_{\alpha \beta} \psi_\beta \right)^2 \quad (27)$$

and consequently introduce two additional auxiliary fields $\phi_x$ and $\phi_y$ as done in (11), resulting in the four dimensional analogues of $L$ (26) and $Q$ (17). Given the complexity of the calculations, these were only performed for the case $\Delta \mu = 0$ at $g < g_c$ and will be discussed only qualitatively. The four dimensional analog of $L$ will again show a single non-vanishing term, corresponding to the $\rho$ field. The new matrix will instead only have non zero off diagonal elements corresponding to the coupling between $\phi_z$ and $\rho$ and between $\phi_x$ and $\phi_y$. As a consequence, both the expectation values of $\phi_x$ and $\phi_y$ are identically zero at $\Delta \mu = 0$. An analogous picture occurs in the Rashba interacting independent particle model, where both $\langle \sigma_x \rangle$ and $\langle \sigma_y \rangle$ average out to zero, due to the 2D rotational symmetry of the Fermi surface.

IV. NUMERICAL RESULTS

The numerical results presented in this section concern magnetization and quasipolarizability properties of the system under study at $T = 0$. In the following we will express lengths in units of $n^{-1/2} (\text{the 2-D particle density})$, and energies in units of $\hbar^2 k^2/m$ where $m$ is the mass of the particles. Due to the elevated computational cost related to the treatment of some of the terms proportional to $g^2$ in the free energy, the results presented in the following were obtained retaining only the terms of the action which are linear in $g$. In absence of SO interaction, the second order terms were shown to shift the transition to lower $g$ and to increase the order of the transition from the first to continuous. Since our system shows a smoothing of the transition already at first order in $g$ due to the presence of SO coupling, we expect that second order terms will only shift $g_c$, the critical coupling strength for the transition, to lower values, without changing the overall features. As already outlined above, at $\Delta \mu = 0$ no $z$ magnetization is present in the system at repulsive couplings below $g_c$. However, a phase transition, analogous to that found in absence of SO, is still present, causing the appearance of non zero magnetization at strong repulsion. Fig. 11 reports the magnetization as a function of the repulsion strength for different values of $\lambda$. While no $\lambda$ dependence is appreciable at small $g$’s, above the transition one observes a decrease of the magnetization at saturation by increasing $\lambda$. Another relevant feature is the smoothing of the magnetization increase upon introduction of SO, the smoothing being more relevant at higher $\lambda$. The Rashba interaction is therefore expected to effectively frustrate $M$ also at $\Delta \mu \neq 0$. This property, in fact, is confirmed by the results reported in Figs. 23. Above $g_c$, a competition will be present between the magnetizing repulsion and the demagnetizing Rashba interaction. Magnetization will thus increase at fixed $\lambda$ by increasing $g$. When $g > g_c$ the variational parameter $h$ introduced above becomes essential for the description of magnetization, since the energy minimum occurs in this case for $h \neq 0$, corresponding to partial spin alignment along the $z$ axis.

Non zero $M$ is obtained, even below criticality, for $\Delta \mu \neq 0$, increasing with the $\Delta \mu$, as from Fig. 8. This result is clearly understandable, given the role of $\Delta \mu$ in energetically favoring the occupation of $s_z = 1/2$ spin states. Notice that the magnetization enhancement due to $\Delta \mu$ decreases as the Rashba coupling constant $\lambda$ increases.

The so-called "quasi-polarization", defined as

$$\xi = \frac{n_+ - n_-}{n_{tot}}$$

(28)
and corresponding to the difference between the Rashba spin + and - density fraction at $\Delta \mu = 0$, shows no difference from that of independent particles, computed by retaining only the kinetic energy terms in the system energy. This also appears in good agreement with QMC results. For non zero $\Delta \mu$ values, however, also the ”quasi-polarization” appears to be affected by the presence of the repulsive interaction, showing dependences on $g$ and $\Delta \mu$ qualitatively similar to those of $M$. No direct relation is observed between $M$ and $\xi$. In fact, non zero $\xi$ is found in presence of zero $M$. At $\Delta \mu \neq 0$, however, non zero $M$ and $\xi$ are found, both increasing with $g$. Again, however, full quasipolarization is consistent with partial magnetization.

V. CONCLUSIONS

The $z$ axis magnetization of the two dimensional fermion gas in presence of contact repulsion and Rashba SO interaction has been computed from quantum field theory. A general expression for the free energy at second order in $g$ is obtained. As a result, we analytically found no magnetization at $\Delta \mu = 0$, at values of the repulsive interaction strength below a critical value $g_c$. Above criticality magnetization is non zero and depends on both the SO strength and $g$. Numerical results were also given at $\Delta \mu \neq 0$, showing that the system develops $z$ spin polarization by increasing $\Delta \mu$. Moreover, also in this case the repulsive interaction appears to enhance the tendency to develop $z$ magnetization. The Rashba interaction acts by frustrating $M$, suggesting a possible application as an effective tool for controlling the system $z$ polarization.

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FIG. 1: (color online) Magnetization plotted in function of the repulsive interaction coupling constant at fixed $\Delta \mu = 0$ for different values of $\lambda$. Units are specified in the text.

FIG. 2: (color online) Magnetization plotted in function of the $\lambda$ SO coupling constant for different values of $\Delta \mu$ at fixed $g = 6.0$. Units are specified in the text.
FIG. 3: (color online) Magnetization plotted as a function of $\Delta\mu$ at different $\lambda$ values and fixed $g = 6.0$. Units are specified in the text.