Satellite probing General Relativity and its extensions and Kolmogorov analysis

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Abstract. - We apply the Kolmogorov statistic to analyse the residual data of two LAGEOS satellites on General Relativistic Lense-Thirring effect, and show that it reveals a tiny difference in the properties of the satellites, possibly related to Yarkovsky-Rubincam effect. The recently launched LAser RElativity Satellite (LARES) can provide constraints to the extensions of General Relativity such as the Chern-Simons (CS) gravity with metric coupled to a scalar field through the Pontryagin density, so an explicit dependence on the frame dragging measurements vs the CS parameter is given.

Introduction. – The satellite testing of General Relativity was instrumental in confirming by now its predictions, particularly, on the Lense-Thirring (LT) effect [1,2]. Even higher-accuracy probing of LT effect is expected by means of the LARES satellite currently on nearly zero-eccentricity geocentric orbit [3,4]. The importance of such high accuracy tests is increased due to indications of the accelerated expansion of the Universe and the puzzle of the dark energy, for which various models have been proposed including extensions of General Relativity. Chern-Simons (CS) gravity which follows from the string theory, is among the discussed ones [5,6], and LARES can improve the constraints on its parameters.

In the present note we apply the Kolmogorov’s statistic [7,9] for the first time to satellite LT data, i.e. to the residual data of the two LAser GEOdynamics Satellites (LAGEOS); also, this illustrates the method, before the LARES data are available. This method which enables studying the correlations vs the degree of randomness in a sequence of numbers, has been already applied for the analysis of the cosmic microwave background (CMB) data of Wilkinson Microwave Anisotropy Probe (WMAP) [10]. That approach has revealed, among other issues, the enhanced degree of randomness of the Cold Spot, a non-Gaussian region in the CMB sky, thus supporting its void nature in the large scale matter distribution; for recent discussion of the Cold Spot by the Planck Collaboration see [12].

Then, we inquire into the explicit quantitative values of the Chern-Simons parameter upon the expected increase of accuracy of measurements by LARES.

Kolmogorov analysis of LAGEOS and LAGEOS 2 residuals. – Before turning to the analysis of the data of the two satellites, we briefly introduce the Kolmogorov statistic [2,9]. The definition of the Kolmogorov parameter for a finite random sequence of real numbers \(x_1, x_2, \ldots, x_n\) sorted in increasing manner \(x_1 \leq x_2 \leq \ldots \leq x_n\) includes the empirical distribution function (CDF)

\[
F_n(x) = \begin{cases} 
0, & X < x_1 \\
k/n, & x_k \leq X < x_{k+1} \\
1, & x_n \leq X
\end{cases}
\]

and the theoretical cumulative distribution function (CDF)

\[
F(X) = n \cdot (\text{probability that } x \leq X).
\]

The parameter \(\lambda_n\) is defined as

\[
\lambda_n = \sqrt{n} \sup_x |F_n(x) - F(x)|
\]

which also is a random variable.

Kolmogorov’s theorem states that the probability

\[
limit_{n \to \infty} P\{\lambda_n \leq \lambda\} = \Phi(\lambda),
\]

P-1
is uniformly converging at \( n \to \infty \) to \( \Phi(\lambda) \):

\[
\Phi(\lambda) = \sum_{k=-\infty}^{+\infty} (-1)^k e^{-2k^2 \lambda^2}, \quad \Phi(0) = 0, \quad \lambda > 0 ,
\]

for any continuous cumulative distribution function. \( \Phi(\lambda) \) is a monotonic function and varies within \( \Phi(0) = 0 \) to \( \Phi(\infty) = 1 \).

We will use this method to analyze the degree of randomness in the residual data, i.e. the difference between the calculated and the observed positions (in angular degrees) of the two LAGEOS satellites. The satellites have been launched on 4 May 1976 (LAGEOS) and 23 October 1992 (LAGEOS 2) and the data have been collected during nearly 11 years (about 4018 days), with temporal step \( t=14 \) days; for details see [1, 2] and refs therein.

We have computed the function \( \Phi \) for the residuals of both LAGEOS satellites’ datasets for Gaussian CDF vs the variation of the standard deviation \( d\sigma \) of the latter (Figure 1). We see that there is a difference in the randomness of the two sets of residuals (about 10 times, cf. [3]) in fitting the Gaussian with respect to those of LAGEOS 2. This indicates that the method is sensitive to a difference in the residual randomness.

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where the dual of the Riemann tensor is

\[ R^a = \epsilon^{abcd} \Gamma^d_{bk} \left( \partial_\theta \Gamma^k_{dl} + \frac{2}{3} \Gamma^k_{dk} \Gamma^d_{ln} \right) \]

satisfying the relation

\[ \Delta_a K^a = \frac{1}{2} R \nabla \]

and (10) turns to

\[ K^a = \epsilon^{abcd} \Gamma^d_{bk} \left( \partial_\theta \Gamma^k_{dl} + \frac{2}{3} \Gamma^k_{dk} \Gamma^d_{ln} \right) = \epsilon^{abcd} \Gamma^d_{bk} \left( \frac{1}{2} R \nabla - \frac{1}{3} \epsilon^{nl} \Gamma^k_{ln} \right) \]

The variation of the action with respect to the metric gives

\[ \delta S = \frac{1}{16 \pi k} \int d^4 x \sqrt{-g} \left[ \left( R^{ab} - \frac{1}{2} g^{ab} R + l C^{ab} - 8 \pi k T^{ab} \right) \delta g^{ab} + \frac{1}{4} R \delta R + g^{ab} \Delta_a \Delta_b \theta - \frac{dV}{db} \right] \delta \theta + \Sigma \]. (13)

The equation of motion for the scalar field \( \theta \) is

\[ \square \theta = \frac{dV}{db} - \frac{1}{4} l R \nabla \theta. \] (14)

Then the modified gravitational field equations are

\[ G^{ab} + l \nabla C^{ab} = 8 \pi T^{ab}, \] (15)

where \( G^{ab} \) is the Einstein tensor and \( C^{ab} \) is the Cotton-York tensor defined as

\[ C^{ab} = v_l \left( \epsilon^{lacd} \nabla_c R^b + \epsilon^{lbc} \nabla_c R^a \right) + v_k \left( \nabla^{l} R^{ab} + R^{ab} \right), \] (16)

and \( v_l \) and \( v_k \) are Chern-Simons velocity and acceleration

\[ v_l = \partial_t \theta = \nabla_l \theta, \] (17)

\[ v_k = \nabla_l v_k = \nabla_l \nabla_k \theta. \] (18)

and \( T^{ab} \) is combined from the matter stress-energy tensor \( T^{ab}_{\text{mat}} \) and scalar field stress-energy tensor \( T^\theta_{\text{th}} \). The latter has the following form

\[ T^\theta_{\text{th}} = (\nabla_a \theta) \left( \nabla_b \theta \right) - \frac{1}{2} g_{ab} (\nabla_c \theta) (\nabla^c \theta) - g_{ab} V(\theta) \] (19)

The Lense-Thirring secular drag rate of the node for a test particle freely orbiting a central rotating mass is [2]

\[ \hat{\Omega}_{GR} = \frac{2 G L}{a^3 (1 - e^2)^3/2}, \] (20)

where \( L \) is the angular momentum of the central mass and \( e \) is the orbital eccentricity of the test particle, i.e. the satellite. The ratio of the drag rates for Chern-Simons theory and for General Relativity is [5]

\[ \frac{\hat{\Omega}_{CS}}{\hat{\Omega}_{GR}} = \frac{15 a^2}{R^2} \frac{2 (m_{CS} R) y_1 (m_{CS} a)}{2 (m_{GR} R) y_1 (m_{GR} a)}, \] (21)
where $j_\ell(x)$ and $y_\ell(x)$ are the first and the second kind spherical Bessel functions, respectively, and $m_{CS} = -3/lk^2 \theta^2$.

LAGEOS satellites being on practically identical orbits with semi-major axes $a = 12271$ km verified the General Relativistic Lense-Thirring effect in the gravitational field of the Earth with an accuracy of about 10% [1]. Since the LARES is expected to produce higher accuracy data, we represent the plot for $n = \dot{\Omega}_{\text{LARES}}$ (in percents) vs the lower limit of the Chern-Simons parameter $m_{CS}$ (in km$^{-1}$) in Figure 2.

Conclusions. – Kolmogorov statistic is used for the analysis of the residuals of the satellites LAGEOS and LAGEOS 2, while measuring the Lense-Thirring effect in Earth’s gravity. A slight difference in the behaviour of $\Phi$ for the two satellites is revealed, which can be due to non-gravitational effect, i.e. Yarkovsky-Rubincam thermal thrust at differently spinning satellites with non-equal stay time in the orbit. This is the first use of this method for satellite measurements on gravitomagnetism, and later it may be applied to higher-accuracy data by LARES. Chern-Simons gravity parameter’s dependence on the increasing accuracy of the measurements expected by LARES is exhibited. The empirical constraints on CS theory are important also due to the attempts to use it to explain the properties of the dark energy, the expansion of the Universe and the processes in galactic nuclei.

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