Estimation of localization of point sources from a printed circuit board in the near field

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Abstract.
This article describes algorithms for localizing sources of electromagnetic radiation from a printed circuit board in the near field. The sources of radiation are represented in the form of a set of the simplest oscillators - Hertz dipoles. Comparison of the two methods of localization of radiation sources is carried out using the Tikhonov regularization and the LASSO regression. The model examples show that the application of LASSO regression gives better results.

1. Introduction
The main problem of electromagnetic compatibility is the ability of technical means to function in conditions of unintended disturbances. The most important characteristics of such disturbances are location of their source and radiation power. The most well-known method for detecting radiation sources is to scan the amplitudes of the component fields at each point of space [1, 2]. This method is commonly used for measurements in the far field, but it can also be used in the near field to improve accuracy of measurement and reduce the requirements for the experimental setup [2]. To ensure electromagnetic compatibility of electronic devices, it is important to be able to determine the location of the radiation sources from the printed circuit boards. There are situations where, according to the known value of the field at each measuring point and the distance to the plane of the object under study, it is necessary to determine the regions of the most active radiation. As a rule, to solve such an inverse problem, a model is used in which the sources of radiation from the printed circuit board are replaced by a set of Hertz dipoles [3-5]. Equations for electric and magnetic Hertz dipoles are well known in the near and far fields [6]. Hertz dipoles are located in the grid nodes.

The inverse problem of determining the location of radiation sources by measuring amplitudes of the electromagnetic field is incorrectly posed. As a rule, Tikhonov regularization is used in such cases [7, 8]. However, recently, LASSO regression (least absolute shrinkage and selection operator) is often used in various fields of physics [9-12]. The main advantage of this approach is that the method allows obtaining low-dimensional solutions (with a small number of non-zero coefficients). In this paper, we propose to use LASSO regression to solve the problem of finding the most active regions of radiation from a printed circuit board.

2. Model of equivalent dipoles
Usually, an equivalent model of a printed circuit board is a combination of electric or magnetic dipoles located in the plane of an object. To build a dipole model of a printed circuit board, the results of modelling the tangential components of the electric or magnetic fields in the measurement plane are used [7, 8]. In our model, we use only magnetic dipoles.

Let us consider an elementary dipole with a dipole moment \( \vec{p} \), which is at the point \((x_0, y_0, z_0)\) in the plane of the object. The magnetic field generated by an elementary dipole at the observation point \((x, y, z)\) and located at a distance \(r\), is determined by the expression [6]:

\[
\vec{H}(\vec{r}) = j \omega \left( jk \frac{\vec{p}}{r} \right) \cdot \left( \vec{I}_r \times \vec{I}_r \right) \cdot G(\vec{r}),
\]

Here \( G(\vec{r}) = \frac{e^{-jkr}}{4\pi r} \) is the Green's function in free space, \( \vec{p} = p_x \cdot \vec{I}_x + p_y \cdot \vec{I}_y + p_z \cdot \vec{I}_z \) is the magnetic dipole moment, \( \vec{I}_r = \frac{\vec{r}}{r} \) is the unit radial vector.

If the dipole is located in the XOY plane, the expression has the form:
The expressions for the components of the magnetic field have the form:

\[ \mathbf{H} = j\mu \frac{e^{-\text{j}kr}}{4\pi r^2} \begin{pmatrix} \frac{1}{r} \end{pmatrix} \begin{pmatrix} (z - z_0) \cdot \mathbf{p}_x + \frac{1}{r} \end{pmatrix} \begin{pmatrix} A_x(r) \cdot \mathbf{p}_y \end{pmatrix} \begin{pmatrix} A_y(r) \cdot \mathbf{p}_x \end{pmatrix} \begin{pmatrix} A_z(r) \cdot \mathbf{p}_x \end{pmatrix} \]

Equations (3,4) can also be written briefly as

\[ \begin{bmatrix} 0 & A_x \\ A_y & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p}_x \\ \mathbf{p}_y \end{bmatrix} = \begin{bmatrix} \mathbf{H}_x \\ \mathbf{H}_y \end{bmatrix} \]

The components of the magnetic field and the transition matrix A are assumed to be known, but the dipole moments p are unknown. The system of equations (6) is expressed in matrix form as \( A \cdot p = H \), and its formal solution can be written as \( p = A^{-1} \cdot H \) (where \( A^{-1} \) is the inverse of A). In our case, the matrix A is normally ill-conditioned, and therefore a small change in the right-hand side in (6) will result in a large change in the solution. Thus, the inverse problem of determining the location of radiation sources by measuring amplitudes of the electromagnetic field is incorrectly posed. To solve incorrectly posed problems, regularization procedure is usually used.

### 3. Ridge regression (Tikhonov regularization) and LASSO regression

Regularization is used to find a stable solution of ill-posed problems by taking into account some a priori information. For example, using Tikhonov regularization, a solution with a minimal Euclidean norm is chosen from all possible solutions that ensure a small disparity of equations (6). To find such a solution, it is necessary to minimize the functional \( \Omega(y,\lambda) \):

\[ \Omega(p,\lambda) = \| A \cdot p - H \|_2^2 + \lambda \| p \|_2^2 \]

where \( \lambda \) is the regularization parameter. This optimization problem has an explicit solution [13]:

\[ p_\lambda = (A^H A + \lambda^2 I)^{-1} A^H \cdot H \]

Among other possible methods of regularization, LASSO regression is of particular interest for this task. In this case, a solution with a minimum L1-norm is chosen from possible solutions. Such problem is equivalent to minimizing the functional

\[ \Omega(p,\lambda) = \| A \cdot p - H \|_2^2 + \lambda \| p \|_1 \]

This regularization method makes allows obtaining solutions of low dimensionality (as a rule, most of the coefficients in the solution vector are equal to zero).

There is no explicit solution to the problem of minimizing the functional (9), but there is an effective numerical method [14]. Thus, minimizing the LASSO \( \Omega(p,\lambda) \) functional and choosing the parameter \( \lambda \), we obtain vectors p for two tangential components of the electromagnetic radiation at each point.
4. The application of regularization for the estimation of radiation sources

In real conditions, the sources of electromagnetic radiation on the circuit board are unevenly distributed over the entire plane of the object. Sources are usually located in certain areas. Electromagnetic radiation in this case is often regarded as a superposition of radiation from point uncorrelated sources [7, 8]. A set of thermal noise elements and components of the device can serve as an example of point uncorrelated sources. The number of sources can usually be considered to be much less than the number of grid nodes.

To test the algorithms under consideration, let us define a model with two point sources located close to each other with coordinates $A_1(50; 50)$ and $A_2(50; 60)$ and a radiation power of 6 and 8 conventional units. The grid size is $10,000 \times 10,000$ nodes. The distance from the measurement plane to the object under study is 5 times larger than the distance between the grid nodes. The wavelength is set 300 times higher than the grid spacing. The distribution of the magnetic field generated by the two sources is shown in Fig. 1.

![Figure 1. The module of the magnetic field strength from two closely spaced sources](image1)

The region with the highest radiation intensity is in the centre of Fig. 1. Axes x and y are oriented in the plane of measurement.

To solve the inverse problem and determine the position of the sources, we used Tikhonov regularization and LASSO regression. The estimates of the power distribution of radiation sources obtained in two ways are shown in Fig. 2. Fig. 3 shows the vertical sections of the distributions from Fig. 2. It can be seen that the LASSO regression estimate gives a better source localization than Tikhonov's regularization.

![Figure 2. Estimation of power distribution of sources: A) using Tikhonov regularization; B) using LASSO regression](image2)
Let us estimate quantitatively the quality of reconstruction of the sources location. As can be seen from Fig. 2A and Fig. 3, the point source of radiation after the reconstruction spreads into a bell-shaped distribution. The displacement of the center of gravity of the "bell" and its half-width are good quantitative characteristics of distortions. The position of the center of gravity can be found by formulas

\[ x_0 = \frac{\sum_{i,j} x_i |p_{ij}|^2}{\sum_{i,j}|p_{ij}|^2}, \quad y_0 = \frac{\sum_{i,j} y_i |p_{ij}|^2}{\sum_{i,j}|p_{ij}|^2} \]  

(10)

where the summation is performed in the immediate vicinity of the "bell". Having determined the position of the center of gravity, we calculate the half-width of the "bell" \( \Delta \) in accordance with the following expressions:

\[ \Delta = \sqrt{\Delta_x^2 + \Delta_y^2}, \quad \Delta_x^2 = \frac{\sum_{i,j} (x_i - x_0)^2 |p_{ij}|^2}{\sum_{i,j}|p_{ij}|^2}, \quad \Delta_y^2 = \frac{\sum_{i,j} (y_i - y_0)^2 |p_{ij}|^2}{\sum_{i,j}|p_{ij}|^2} \]  

(11)

Also, the quality characteristic of the localization can be the fraction of the radiation power attributable to the maximum point in the estimated distribution. The larger this fraction, the better the source localization and the greater the conformity of the estimate to the initially assigned distribution.

Table 1 shows the values of numerical parameters characterizing the quality of source localization using Tikhonov regularization and LASSO regression. The calculations were carried out at a signal-to-noise ratio of 500, or 54 dB (normal white noise was added to the model values of the field strength). As can be seen from the table, the use of LASSO regression yields significantly better results.

**Table 1.** The quality of the estimation of the distribution of radiation sources using the Tikhonov regulation and LASSO regression. The two columns indicate the parameters for the first and second sources.

|                        | Displacement of the center of gravity | Range of values, \( \Delta \) | Fraction of the radiation power attributable to the maximum point, \% |
|------------------------|--------------------------------------|------------------------------|----------------------------------|
|                        | 1st | 2nd | 1st | 2nd | 1st | 2nd |
| Tikhonov regularization | (51;50.5) | (51;60.6) | 7.4 | 8.1 | 20.1 | 19.8 |
| LASSO regression       | (51;51) | (51;61) | 0.13 | 0.12 | 85.5 | 87.2 |
5. Conclusion
The article considers two algorithms of estimating the localization of electromagnetic radiation sources from a printed circuit board in the near field. To restore the power distribution of sources from measurements of the field strength, Tikhonov regularization and LASSO regression were used. Comparison on model examples showed that LASSO regression gives a more accurate estimation of source localization than Tikhonov's regularization. Also, estimates of source power distributions using LASSO regression are more resistant to noise.

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