An Explicit Description of the Violation of CP Symmetry in a $S_N$-symmetric Standard Model

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In this manuscript, the violation of CP symmetry within the standard model (SM) is described explicitly. Starting from the most general pattern of a $M$ matrix and with the help of an interesting condition between the real and imaginary components of a Hermitian matrix, the number of unknown parameters in $M$ is reduced from eighteen down to five. However, such a pattern is still too complicated. Thus, an extra assumption which reveals several $S_N$-symmetric circumstances with complex $M_{CKM}$ is introduced. Though the $M_{CKM}$ elements thus derived do not fit the experimentally detected ones perfectly. However, the very strong CP violation (CPV) in such circumstances provides us a way to account for partly the discrepancy between cosmologically observed Baryon Asymmetry of the Universe (BAU) and that predicted by the SM in particle physics. This is a very interesting but orthodox aspect to describe the violation of CP symmetry and use it to account for the observed BAU.

I. INTRODUCTION

Since the first detection of CP violation (CPV) in 1964 [1], the issue of how CP symmetry was violated attracts our interest very much. However, for over fifty years, an explicit way to describe the violation of CP symmetry is still obscure. If we analyze the origin of CPV thoroughly, considering only the quark sector here, the pattern of quark-mass matrices $M$ is obviously the key to ignite such a violation. Thus, we would like to analyze the CPV problem from a very fundamental basis by starting from a most general pattern of the $M$ matrix and then simplify such a pattern step by step rigorously.

In the standard model (SM) of electroweak interactions "Direct" CP violation is allowed if a complex phase appears in the Cabibbo-Kobayashi-Maskawa (CKM) matrix describing quark mixing, or the PMNS matrix describing lepton mixing. In SM such a complex phase can only be achieved by ranking Yukawa coupling parameters between fermions and Higgs fields suitably. However, how should them be ranked to achieve a CKM matrix with complex elements is still obscure even for now.

Theoretically there is another potential source of a complex phase comes from the vacuum expectation value (VEV) of the Higgs doublet. However, in SM such a phase can always be absorbed into redefinitions of quark fields. Thus, an extension of SM with one extra Higgs doublet was proposed in [2] which is usually referred to as a Two-Higgs-doublet model (2HDM). In this way, people expect phases of those two VEVs may unlikely be rotated away simultaneously if there were a nonzero phase difference between them. However, such an extra Higgs doublet not only failed to solve the CP problem, but also brought in an extra problem, the flavor-changing neutral current (FCNC) problem at tree level.

In fact, most of the derivations in this article were originally proposed for solving the FCNC problem and CP problem in a 2HDM. However, we find they apply to SM as well. As SM alone is enough to derive CPV explicitly, why should one bothers to deal with the extra Higgs doublet and problems it brought in? Thus, this article is to be devoted to the theoretical ignorance of CPV in SM alone.

In SM the Yukawa couplings of $Q$ quarks can be written as

$$-\mathcal{L}_Y = \bar{Q} Y^q u_R + \bar{Q} Y^d d_R + \bar{Q} Y^e e_R + h.c.,$$  \hspace{1cm} (1)

where $Y^q$ are $3 \times 3$ Yukawa-coupling matrices for quark types $q = u, d$ and $e$ is the $2 \times 2$ antisymmetric tensor. $\bar{Q}_i$ are left-handed quark doublets, and $d_R$ and $u_R$ are right-handed down- and up-type quark singlets, respectively, in their weak eigenstates.

When the Higgs doublet $\Phi$ acquires a VEV, $\langle \Phi \rangle = \left( \begin{array}{c} 0 \\ v/\sqrt{2} \end{array} \right)$, Equation (1) yields mass terms for the quarks with $M^q = Y^q v/\sqrt{2}$ the mass matrices for $q = u, d$. The physical states are obtained by diagonalizing $Y^{qd}$ by four unitary matrices $U_{LR}^q$, as $M^q_{\text{diag}} = U_{LR}^q M^q U_{LR}^q = U_{LR}^q (Y^q v/\sqrt{2}) U_{LR}^q$, $q = u, d$. As a result, the charged-current $W^\pm$ interactions couple to the physical $u_L$ and $d_L$ quarks with couplings given by

$$-\mathcal{L}_{W^\pm} = \frac{g}{\sqrt{2}} (\bar{a}_L, \bar{c}_L, \bar{f}_L) Y^q W^\mu \beta \nu V_{\text{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c.,$$  \hspace{1cm} (2)

where

$$V_{\text{CKM}} = U_L \tilde{U}_L = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. $$  \hspace{1cm} (3)

Hereafter, we will neglect the subindex $L$ in $U_L$ if unnecessary.

If one acquires a complex $V_{\text{CKM}}$, following two conditions are necessary:

1. At least one of either $U^u$ or $U^d$ is complex.
2. Even both of them are complex, they must not be the same, i.e., $U^u \neq U^d$. Otherwise, $V_{\text{CKM}}$ in Equation (3) will become a $1_{3\times3}$ identity matrix since $U$ matrices are assumed.
unitary.

These two conditions are necessary but not sufficient for yielding a complex $V_{CKM}$. Yielding complex phases in the CKM matrix needs more brilliant inspirations which are to be presented in what follows.

As $V_{CKM}$ is the product of $U^u$ and $U^d$, obviously these two $U$ matrices determine if $V_{CKM}$ were complex. As $U^u$ and $U^d$ are unitary matrices which diagonalize mass matrices $M^u$ and $M^d$, respectively. It is obvious they are objects derived from $M^u$ and $M^d$. Or, we may say the patterns of $M^u$ and $M^d$ are keys to ignite CP violation, not only in the SM but also in its extensions with extra Higgs doublets. Thus, we will start our investigation here from a most general pattern of the $M$ matrix and then put in constraints to see what will happen to the CPV in various circumstances.

At the beginning of section II, the investigation starts from a most general pattern of $M$ matrix. Then, a Hermitian assumption of $M$ is employed to simplify the pattern. In usual, such an assumption only reduce the number of parameters in a $M$ matrix from eighteen down to nine. However, we amazingly find an interesting condition between the real and imaginary components of $M$ reduces the parameter number further down to five [3]. This is a great advance in the theoretical study of this topic.

Since we have only three quark masses as known factors in a quark type for now. The $M$ matrix at this stage is still too complicated to be solved uniquely, not to mention the CKM matrix. Thus, an assumption of $A = A_1 = A_2 = A_3$ which reveal $S_N$ symmetries among or between quark generations is further employed to achieve several special solutions for the $M$, $U$ and $V_{CKM}$ matrices.

At the end of section II, four special $M$ patterns and their corresponding $U$ matrices are achieved. One of them reveals a $S_3$ symmetry among three quark generations and the other three reveal $S_2$ symmetries between two of the three generations. These $M$ patterns and corresponding $U$ matrices enable us to satisfy the necessary conditions mentioned above and thus a chance to yield CPV.

Even we have already a way to describe the violation of CP symmetry within the SM. However, as to be demonstrated in section III, the CPV derived in such $S_N$-symmetric cases are orders stronger than the SM predicted one in particle physics and orders weaker than the one needed to account for the Baryon Asymmetry in the Universe (BAU) [4] observed cosmologically. At the first glance it looks like a defect of the model. However, if we consider such $S_N$ symmetries should exist only in environments of extremely high temperature. It is not strange at all that CPV derived in this manuscript is different to the one detected in present experiments. Besides, it indicates that the BAU we see today could be remnant left over in some $S_N$-symmetric eras, at least part of them.

Though an explicit description of the violation of CP symmetry within the SM is demonstrated in this manuscript and it hints to several $S_N$-symmetricepochs in the early universe which produced BAU much more than what the SM can give nowadays. However, the model demonstrated here is still not a perfect one. Conclusions of this manuscript and discussions on its future prospects will be given in section IV.

II. THE $M$ MATRICES AND $U$ MATRICES

As mentioned above, $V_{CKM}$ is a product of two $U$ matrices while these $U$ matrices are derived from two $M$ matrices. Obviously the $M$ matrices are key factors to determine if $V_{CKM}$ were complex. Based on this, the orthodox way to derive a complex $V_{CKM}$ is to start from the most general pattern of $M$ matrices and then diagonalize them analytically to achieve the $U$ matrices.

In the standard model with three fermion generations, the mass matrix of any specific fermion type must has the general pattern as

$$M = \begin{pmatrix} A_1 + iB_1 & B_4 + iC_4 & B_2 + iC_2 \\ B_4 + iC_4 & A_2 + iB_2 & B_3 + iC_3 \\ B_2 + iC_2 & B_3 + iC_3 & A_3 + iB_3 \end{pmatrix}, \quad (4)$$

which contains eighteen parameters and $A$, $B$, $C$ and $D$ are all real.

Theoretically, the most orthodox way to derive the CKM matrix is to diagonalize one such matrix for up-type quarks and one for down-type quarks and then derive their corresponding $U$ matrices. However, such a pattern is obviously too complicated to be diagonalized analytically. Not to mention the CKM matrix thus derived will contain thirty six parameters in total, eighteen from the up-type quarks and eighteen from the down-type quarks.

In general, physicists employ various constraints to simplify the matrix pattern down to a manageable. For instance, the Fritzsch ansatz (FA) [5, 6] and its subsequent developments like Cheng-Sher ansatz (CSA) [7], Du-Xing ansatz (DXA) [8], combination of the Fritzsch and the Du-Xing ansatz (DFXA and FXDA), combination of different assignments in the Du-Xing ansatz ($\tilde{X}A$), Non-mixing Top quark Ansatz (NTA) [and references therein][9] and Fukuyama-Nishiuura ansatz (FNA) [and references therein][10] had impose several zeros in the mass matrix as ad hoc constraints to reduce the parameter number in a mass matrix. The goal of all these ansatz is to simplify the pattern of fermion mass matrix. However, constraints so strong are in fact unnecessary. As to be demonstrated in what follows, a very weak assumption that mass matrices are Hermitian is already enough to yield a very simple pattern and it includes almost previous ansatz as special cases in it.
In the standard model, the Lagrangian as a whole is
assumed to be Hermitian. If we assume the Yukawa
couplings or equivalently the fermion mass matrices are also
Hermitian, the pattern of mass matrices will be simplified
evernously. As shown in one of our previous articles [3], such an assumption can reduce those eighteen parameters in
Equation (4) down to only five.

In the first stage, the Hermitian condition acquires that
\( D_j = 0, B_{j=3} = B_j \) and \( C_{j=3} = -C_j, j = 1, 2, 3 \). Thus, Equation (4) now becomes
\[
M = \begin{pmatrix}
A_1 & B_1 + iC_1 & B_2 + iC_2 \\
B_1 - iC_1 & A_2 & B_3 + iC_3 \\
B_2 - iC_2 & B_3 - iC_3 & A_3
\end{pmatrix}
\]
\[
= M_R + iM_F = \begin{pmatrix}
0 & C_1 & C_2 \\
-C_1 & 0 & C_3 \\
-C_2 & -C_3 & 0
\end{pmatrix},
\] (5)

At this stage the number of parameters was reduced from
eighteen down to nine.

In the next stage, since the real component \( M_R \) and the
imaginary component \( iM_F \) of a Hermitian matrix \( M = M_R + iM_F \) must are also Hermitian, respectively. For two Hermitian
matrices which can be diagonalized by the same unitary matrix
\( U \) simultaneously. They must satisfy a very interesting condi-
tion which was originally given in [11] and revised in [3].
\[
M_R(iM_F)^\dagger - (iM_F)M_R^\dagger = 0.
\] (6)

Substituting Equation (5) into Equation (6), we will receive
four equations
\[
B_1C_1 = -B_2C_2 = B_3C_3, 
\]
(7)
\[
(A_1 - A_2) = (B_3C_2 + B_2C_3)/C_1, 
\]
(8)
\[
(A_1 - A_1) = (B_1C_2 - B_2C_3)/C_2, 
\]
(9)
\[
(A_2 - A_3) = -(B_2C_1 + B_1C_2)/C_3, 
\]
(10)
which can be used to further reduce the parameter number
from nine down to five.

However, five parameters are still too many for us to diag-
onalize the \( M \) matrix analytically. In order to further reduce the parameter number, we will employ an extra assumption
\[
A_1 = A_2 = A_3 = A, 
\] (11)
and further summarize the Equations (7)-(10) in
\[
B_1^2 = B_2^2 = B_3^2, \quad C_1^2 = C_2^2 = C_3^2. 
\] (12)

By examining all possible solutions of Equation (12) we
found only four cases satisfy it. In what follows they will be studied respectively as:

**Case 1:** \( B_1 = B_2 = B_3 = B \) and \( C_1 = -C_2 = C_3 = C \)

In this case,
\[
M = M_R + iM_F = \begin{pmatrix}
A & B & B \\
B & A & B \\
B & B & A
\end{pmatrix} + i \begin{pmatrix}
0 & C & -C \\
-C & 0 & C \\
C & -C & 0
\end{pmatrix}, \quad (13)
\]
which has the same pattern as what was achieved in [12, 13]
with a \( S_3 \) permutation symmetry imposed among three quark
generations. That pattern was originally derived to solve the
FCNC problem in a 2HDM. However, we find it also appear
in the SM.

Analytically, the \( M \) matrix is diagonalized and the mass
eigenvalues are given as
\[
M_{\text{diag.}} = \begin{pmatrix}
A - B - \sqrt{3}C & 0 & 0 \\
0 & A - B + \sqrt{3}C & 0 \\
0 & 0 & A + 2B
\end{pmatrix}. \quad (14)
\]

The \( U \) matrix which diagonalize Equation (13) is then given
as
\[
U_1 = \begin{pmatrix}
-1 - i\sqrt{3} & -1 + i\sqrt{3} & 1 \\
2\sqrt{3} & 2\sqrt{3} & \sqrt{3} \\
1 + i\sqrt{3} & 1 - i\sqrt{3} & \sqrt{3}
\end{pmatrix}, \quad (15)
\]
where the sub-index \( k (k=1 \rightarrow 4) \) of \( U_k \) indicates to which
case it corresponds.

**Case 2:** \( B_1 = B_2 = -B_3 = B \) and \( C_1 = -C_2 = -C_3 = C \)

In this case,
\[
M = M_R + iM_F = \begin{pmatrix}
A & B & B \\
B & A & -B \\
B & -B & A
\end{pmatrix} + i \begin{pmatrix}
0 & C & -C \\
-C & 0 & C \\
C & C & 0
\end{pmatrix}, \quad (16)
\]
which possesses a residual \( S_2 \) symmetry between the second
and third generations.

The mass eigenvalues are given as
\[
M_{\text{diag.}} = \begin{pmatrix}
A + B - \sqrt{3}C & 0 & 0 \\
0 & A + B + \sqrt{3}C & 0 \\
0 & 0 & A - 2B
\end{pmatrix}, \quad (17)
\]
and the \( U \) matrix is given as
\[
U_2 = \begin{pmatrix}
1 - i\sqrt{3} & -1 + i\sqrt{3} & 1 \\
2\sqrt{3} & 2\sqrt{3} & \sqrt{3} \\
1 + i\sqrt{3} & 1 - i\sqrt{3} & \sqrt{3}
\end{pmatrix}. \quad (18)
\]
Case 3: $B_1 = -B_2 = B_3 = B$ and $C_1 = C_2 = C_3 = C$

In this case,

$$M = M_R + iM_I = \begin{pmatrix} A & B & -B \\ B & A & B \\ -B & B & A \end{pmatrix} + i \begin{pmatrix} 0 & C & C \\ -C & 0 & C \\ -C & -C & 0 \end{pmatrix},$$

(19)

which possesses a residual $S_2$ symmetry between the first and third generations.

The mass eigenvalues are given as

$$M_{\text{diag.}} = \begin{pmatrix} A + B - \sqrt{3}C & 0 & 0 \\ 0 & A + B + \sqrt{3}C & 0 \\ 0 & 0 & A - 2B \end{pmatrix},$$

(20)

and the corresponding $U$ matrix is given as

$$U_3 = \begin{pmatrix} \frac{-1+i\sqrt{3}}{2\sqrt{3}} & \frac{1+i\sqrt{3}}{2\sqrt{3}} & 1 \\ -\frac{2+i\sqrt{3}}{2\sqrt{3}} & -\frac{2+i\sqrt{3}}{2\sqrt{3}} & 1 \\ \frac{\sqrt{3}}{\sqrt{5}} & \frac{\sqrt{3}}{\sqrt{5}} & \frac{\sqrt{3}}{\sqrt{5}} \end{pmatrix}.$$

(21)

Case 4: $B_1 = -B_2 = -B_3 = -B$ and $C_1 = C_2 = -C_3 = -C$

In this case,

$$M = M_R + iM_I = \begin{pmatrix} A & -B & B \\ -B & A & B \\ B & B & A \end{pmatrix} + i \begin{pmatrix} 0 & -C & -C \\ C & 0 & C \\ C & -C & 0 \end{pmatrix},$$

(22)

which possesses a residual $S_2$ symmetry between the first and second generations.

The mass eigenvalues are given as

$$M_{\text{diag.}} = \begin{pmatrix} A + B - \sqrt{3}C & 0 & 0 \\ 0 & A + B + \sqrt{3}C & 0 \\ 0 & 0 & A - 2B \end{pmatrix},$$

(23)

and the corresponding $U$ matrix is given as

$$U_4 = \begin{pmatrix} \frac{-1+i\sqrt{3}}{2\sqrt{3}} & \frac{1+i\sqrt{3}}{2\sqrt{3}} & 1 \\ \frac{-1-i\sqrt{3}}{2\sqrt{3}} & \frac{-1+i\sqrt{3}}{2\sqrt{3}} & 1 \\ \frac{\sqrt{3}}{\sqrt{5}} & \frac{\sqrt{3}}{\sqrt{5}} & \frac{\sqrt{3}}{\sqrt{5}} \end{pmatrix}.$$

(24)

In all four cases the $U$ matrices are complex which satisfy the first condition mentioned in section I. If we assign different of them to $U^u$ and $U^d$ respectively, the second condition will also be satisfied. Even so, we still can not be sure if $V_{\text{CKM}}$ were complex since the inner product of two complex matrices can still be real, which are to be demonstrated in next section.

In fact, these derivations were originally proposed to solve the FCNC problem in 2HDMs and expect it may ignite the violation of CP symmetry in them [14]. However, we found it also apply to the SM. As the standard model is already enough to give a theoretical origin of CPV, why should we bother ourselves to deal with the FCNC problem in 2HDMs? Thus, we will concentrate on CP problem in the SM in this article. Surely we can still apply them to the extensions of SM with one or even two extra Higgs doublets while the FCNC problem vanishes naturally at tree level. But we will not discuss this issue too much here since that is a different subject.

Besides, it is noteworthy those $S_N$ symmetries revealed above are not ad hoc constraints. They are derived and revealed from that assumption among $A$ parameters in Equation (11). Among them, the $S_3$-symmetric pattern in case 1 is exactly the same as the one derived in our very early articles [12, 13]. That pattern solved the FCNC problem at tree level successfully but not the CP problem. The problem it met was the breach of the condition 2, $U^u \neq U^d$, since at that time we have only one $S_3$-symmetric $U$ matrix for both quark types. However, those $S_2$-symmetric $U$ matrices appear in case 2, 3 and 4 give us a key to ignite the breaking of CP symmetry from the theoretical end.

III. THE CKM MATRIX

As mentioned in section I, if we expect to yield a CP violating phase in CKM matrix, two necessary conditions are to be satisfied. In last section, four complex $U$ matrices were achieved with a Hermitian assumption of $M$ matrices and an assumption among $A$ parameters. If we assign different $U$ matrices to up- and down-type quarks respectively, both conditions are satisfied. In what follows, various assemblies of $U^u$ and $U^d$ are examined and the CKM matrix they yield are presented in TABLE I. Several of them are complex which indicate CPV is yielded.

The full expressions of matrices $1_{3 \times 3}$, $D$, $E$, $F$ and $G$ in TABLE I are presented as what follows

$$1_{3 \times 3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad F = \begin{pmatrix} \frac{3}{2} e^{-i\frac{\pi}{3}} & -\frac{1}{2} e^{-i\frac{\pi}{3}} & -\frac{1}{2} e^{-i\frac{\pi}{3}} \\ -\frac{1}{2} e^{i\frac{\pi}{3}} & \frac{3}{2} e^{i\frac{\pi}{3}} & -\frac{1}{2} e^{i\frac{\pi}{3}} \\ -\frac{1}{2} e^{i\frac{\pi}{3}} & -\frac{1}{2} e^{i\frac{\pi}{3}} & \frac{3}{2} e^{i\frac{\pi}{3}} \end{pmatrix}, \quad G = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}.$$

$$D = \begin{pmatrix} \frac{1}{3} & -\frac{1}{\sqrt{3}} & \frac{\sqrt{3}}{3} \\ -\frac{1}{\sqrt{3}} & \frac{1}{3} & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix},$$

(25)

The matrices $1_{3 \times 3}$, $F$ and $G$ are purely real and obviously CP-conserving. While $D$, $E$ and their complex conjugates are complex, which means they are CP-violating. However, as shown in the $F$ and $G$ assemblies, even if $U^u \neq U^d$ and both of them were complex, $V_{\text{CKM}}$ can still be completely real.
TABLE I. Various assembles of CKM matrix.

| $V_{CKM}$ | $U_{1}^{d}$ | $U_{2}^{d}$ | $U_{3}^{d}$ | $U_{1}^{u}$ |
|-----------|-------------|-------------|-------------|-------------|
| $\bar{U}_{1}^{d}$ | 1,$\times$3 | $D$ | $D^{*}$ | $F$ |
| $\bar{U}_{2}^{d}$ | $D^{*}$ | 1,$\times$3 | $G$ | $E$ |
| $\bar{U}_{3}^{d}$ | $D$ | $G$ | 1,$\times$3 | $E^{*}$ |
| $\bar{U}_{1}^{u}$ | $F$ | $E$ | $E^{*}$ | 1,$\times$3 |

Though we now have a way to describe the violation of CP symmetry from a theoretical end. But the CKM elements derived in Equation (25) do not fit the experimentally detected values very well. Some of them are hundreds times higher than the detected values, say both predicted $|V_{ub}| = 2/3$ in D and E are about 187 times the presently detected value $3.57 \pm 0.15 \times 10^{-3}$ [15]. Besides, the CKM matrices derived in this article contain only numbers rather than any parameters. That leaves us no space to improve the fitting between theoretical predictions and experimental detections. This could be ascribed to the over-simplified matrix patterns caused by the $A = A_{1} = A_{2} = A_{3}$ assumption or equivalently the employed $S_{N}$ symmetries. It hints that if we can throw away the constraints from these symmetries, maybe we can achieve patterns which fit the empirical values better.

As we have already complex CKM matrices given in Table I, it is rational for us to go one step further to estimate the CP strength predicted by such a model. In usual, the strength of CPV within the SM is estimated with the dimensionless CP strength predicted by such a model. In usual, symmetries exist in circumstances with higher temperature. As we do not see any $S_{N}$ symmetries among fermion generations in our present universe. It is natural for us to consider their appearance in early epochs of our universe if the Big-Bang cosmology were correct. Thus, the discrepancy between the $S_{N}$-symmetric CPV and that detected in present experiments is also natural.

In another way, we can imagine that in some very early stages of the universe with extremely high temperature $T$ there were $S_{3}$ symmetry among all fermion types. As $T$ dropped down with the expansion of the universe, some of the fermions degenerated from others and the symmetries were broken down to $S_{2}$. Probably the up-type quarks first and then the down-type quarks follow, but this is not necessary. During a succession of breaking down of $S_{N}$ symmetries, for instance

$$S_{3}^{u} + S_{3}^{d} \rightarrow S_{2}^{u} + S_{2}^{d} \rightarrow S_{2}^{u} + S_{2}^{d}$$

(29)

$$\rightarrow No^{a} + No^{d}$$

at least the $S_{2}^{u} + S_{2}^{d}$ stage is proved to be CP-violating and the strength is strong enough to generate large amount of BAU.

As we do not see any $S_{N}$ symmetries in our present universe. Obviously we are now in the latest stage, $No^{a} + No^{d}$, of Equation (29). That means a completely analytical diagonalization of $M$ matrices without any assumptions like that in Equation (11) is needed. Or, we may expect such a diagonalization will give us a $V_{CKM}$ consistent with the presently detected CKM elements. Unfortunately, this is still unaccomplished for now and will be the next goal of our future investigations.

### IV. DISCUSSIONS AND CONCLUSIONS

Since the discovery of CPV in the decays of neutral kaons, the theoretical origin of CPV was always a puzzle physicists urgent to solve. Through analysis on the constituents of $V_{CKM}$, an explicit way to describe the violation of CP symmetry within the SM is presented in this manuscript. Two necessary but not sufficient conditions for yielding a complex phase in $V_{CKM}$ are stated in section I.

With an interesting condition between the real and imaginary components of a Hermitian $M$ matrix, the number of independent parameters in a $M$ matrix can be reduced from eighteen down to five. In previous investigations this can only be reduced down to nine at most. It is an advance in investigations on the theoretical origin of CPV.

With a further assumption in Equation (11), four $S_{N}$-symmetric $M$ patterns are revealed. They provide us a chance
to satisfy both conditions stated in section I for yielding a complex $V_{CKM}$. Accordingly, several complex $V_{CKM}$ are achieved with CP strength orders stronger than the one now detected experimentally.

At the first glance such a very strong CPV seems to be a defect of the model. However, as we do not see any of the $S_N$ symmetries in our present universe and such symmetries should exist only in circumstances with extremely high temperature. It’s natural for us to consider their appearances in some very early stages of our universe rather than at present. In case of this, the discrepancy between the cosmologically observed BAU and that predicted by the SM of particle physics becomes very natural since the large amount of BAU cannot be accounted for by present SM is in fact legacies of some $S_N$-symmetric eras in early universe.

Though a way to describe the violation of CP symmetry is demonstrated in this manuscript explicitly. However, the $S_N$-symmetric environments it describes are obviously not the one we are living in today. Though the very strong CPV predicted in such $S_N$ circumstances does not coincide with that detected experimentally nowadays. It indicates that there could be $S_N$-symmetric eras in early stages of our universe which may have produced large amount of BAU due to their very strong CPV. As our present universe is obviously in the $(No^+ + No^-)$ stage of Equation (29), Equation (11) which bring in the $S_N$ symmetries is obviously improper since it simplifies the $M$ pattern too much to give a $V_{CKM}$ fitting the presently detected one perfectly. Logically speaking, a $V_{CKM}$ fitting empirical values better could likely be achieved if we can through Equation (11) away and diagonalize the $M$ matrix directly since that will look much more like the world we are living in. Though a complete description of the violation of CP symmetry in our present world is still obscure and the BAU discrepancy is still can not be accounted for completely. At least, we have already moved one step forward on this way.

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