Beyond Standard Inflationary Cosmology

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The inflationary scenario is not the only paradigm of early universe cosmology which is consistent with current observations. General criteria will be presented which any successful early universe model must satisfy. Various ways, including inflation, will be presented which satisfy these conditions. It will then be argued that if nature is described at a fundamental level by superstring theory, a cosmology without an initial space-time singularity will emerge, and a structure formation scenario which does not include inflation may be realized.

I. INTRODUCTION

A. Goals of Early Universe Cosmology

Explaining the origin and early evolution of the universe has been a goal of cosmology for millenia. However, it is only over the past few decades that cosmology has moved from being a branch of philosophy and theology to being a mainstream area of physics research. This change is due to an explosion of data about the structure of our universe which experimentalists and observers have gathered.

The prime example of data is the Cosmic Microwave Background (CMB). The CMB was discovered serendipitously in the 1960s\textsuperscript{1}. Only in the early 1990s detailed measurements of the black body nature of the spectrum became available\textsuperscript{2}, and the first anisotropies were discovered \textsuperscript{3}. Over the following two decades, rapidly improving measurements of the angular power spectrum \textsuperscript{1} of the CMB anisotropies were made, culminating with the results from the WMAP \textsuperscript{5} and Planck \textsuperscript{6} satellites. To better than 1 part in $10^4$, the temperature of the CMB is isotropic. At a slightly lower level, anisotropies appear. Figure\textsuperscript{1} depicts the map of CMB anisotropies from the WMAP satellite. In this figure, the sky is projected onto a plane in the same way that the surface of the Earth is sometimes projected onto a plane. To quantify the anisotropies, we can expand the map in spherical harmonics and compute the angular power spectrum. The results are shown in Figure\textsuperscript{2} where the horizontal axis is the value of $l$ (or equivalently the angular scale), and the vertical axis gives the amplitude. The figure shows several interesting features. First of all, there are oscillations in the spectrum with a first peak at an angular scale of about $1\degree$. Secondly, at large angular scales the spectrum is quite flat. Finally, on small angular scales the spectrum is suppressed. One of the main goals of early universe cosmology is to provide an explanation for this data.

The CMB is not the only window we have to probe the structure of the universe. Using optical and infrared telescopes we can study the distribution of galaxies and galaxy clusters. X-ray telescopes allow us to map out the distribution of hot gas (gas with a temperature much higher than the surface temperature of the sun) in the Universe. Radio telescopes allow the exploration of cold gas (gas with a temperature lower than the surface temperature of the sun). A new window to probe the universe is emerging: the 21cm window\textsuperscript{2} (radiation at a frequency of 21cm), a window which allows us to map out the distribution of neutral hydrogen, the atom which dominated matter content, up to very large distances and early times. These windows have the added advantage (compared to the CMB) of providing us with three-dimensional maps as opposed to only two-dimensional ones (given a cosmological model).

The second main goal of early universe cosmology is to provide explanations for this data based on physical models which obey the principles of causality. As we will see, Standard Big Bang Cosmology, the paradigm of cosmology until 1980, cannot provide a causal explanation (an explanation from a theory which obeys the principles of Special Relativity, namely that no information can travel faster than the speed of light) of the data, and we have to go back to the very early universe if we want to find explanations.

\textsuperscript{1} The CMB temperature map of the sky can be expanded in terms of spherical harmonics (the analog of Fourier expansion in Euclidean spaces). The spherical harmonics are labelled by two integers $l$ and $m$ with $-l \leq m \leq l$. The coefficient of the $(l, m)$ spherical harmonic gives the amplitude of the CMB anisotropies on an angular scale proportional to $l^{-1}$ and in a direction given by $m$. If we average the square of the coefficients for fixed $l$ over the allowed values of $m$, we obtain what is called the angular power spectrum of CMB anisotropies.

\textsuperscript{2} WMAP stands for Wilkinson Microwave Anisotropy Probe.
An important aspect of cosmology as a branch of physics is that, once we have a model which can address the data, we can make predictions for future observations. As will be discussed below, the current paradigm of early Universe cosmology made predictions concerning data which was not yet available at the time when the model was formulated. As mentioned below, Standard Big Bang cosmology predicted the existence and black body nature of the CMB, and inflationary cosmology predicted an almost spatially flat universe with a spectrum of CMB anisotropies with specific properties. These predictions were confirmed much later once observations became available, thus ruling out various alternatives cosmological models. I believe that any alternative to the current cosmological paradigm must make specific predictions for future observations with which it can be distinguished from the current paradigm. I regard this falsifiability aspect as an important challenge for modern cosmology.

Summarizing, the goals of early universe cosmology are:

- Explain the origin and early evolution of the universe.
- Explain the currently available data on the large-scale structure of the universe based on causal physics.
- Make predictions for future observations.

A few words on the notation. We will mostly consider homogeneous and isotropic space-times in which the line element is given by

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2,$$

where $t$ is physical time (time measured by our physical clocks), $\mathbf{x}$ are comoving spatial coordinates (a coordinate system in which the coordinates of points locally at rest do not change as space expands), and $a(t)$ is the cosmological scale factor. Particles at rest have time-independent comoving coordinates, and the change in $a(t)$ describes the expansion (or contraction) of space. Light travels on curves $\mathbf{x}(t)$ for which $ds^2 = 0$. It is often useful to work with
### B. Problems of Standard Big Bang Cosmology

Cosmology deals with space, time and matter. Standard Big Bang Cosmology (SBB) is based on describing space and time using Einstein’s classical theory of General Relativity, and matter in terms of classical perfect fluids. At the present time (which will be denoted by $t_0$ in the following), matter is dominated by a fluid with vanishing pressure, which in cosmology is called cold matter, while the CMB provides a component which is sub-dominant at the present time.

The equations of motion of General Relativity imply that in the presence of homogeneously distributed matter, the scale factor $a(t)$ cannot be constant. Space is either expanding or contracting. Observations tell us that we live in an expanding universe. Since the energy density in radiation increases faster (namely as $a(t)^{-4}$) as we go back in time than the energy density in cold matter (which increases as $a(t)^{-3}$), there is a time $t_{eq}$ when the energy densities of the two components are equal. Before $t_{eq}$, the universe is dominated by radiation, afterwards by cold matter. It turns out that $t_{eq}$ is the time before which inhomogeneities do not increase in time (up to factors which are logarithmic in}
There is another time which is important in the evolution of the late universe. This is the time $t_{\text{rec}}$ of recombination before which the energy density was larger than the ionization energy of hydrogen. Hence, for $t < t_{\text{rec}}$ the universe was an electromagnetic plasma, and only after which it becomes neutral. The photons (particles of light) which are present at the time $t_{\text{rec}}$ can then travel unimpeded to us. This is in fact the origin of the CMB.

For $t < t_{\text{eq}}$ the scale factor increases as

$$a(t) \sim \left(\frac{t}{t_0}\right)^{1/2},$$

and vanishes at the time $t = 0$. At this time, the curvature and energy densities become infinite. This is the Big Bang singularity of standard cosmology.

The most impressive success of Standard Big Bang Cosmology is that it predicted the existence and black body nature of the CMB. However, the scenario is not able to explain the near isotropy of this radiation. This is the so-called horizon problem and it is illustrated in Figure 3. Here, the horizontal axis indicates comoving spatial coordinates, the vertical axis in conformal time. The lines at 45° represent light rays. The speed of propagation of causation is limited by the speed of light. As indicated in the figure, the region of causal contact between $t = 0$ and the time of recombination is smaller than the region of the last scattering surface (the intersection of our past light cone $l_p$ with the surface at $t = t_{\text{rec}}$) over which the microwave background is observed to be isotropic. The maximal angular scale where isotropy can in principle be explained by causal physics is the angular scale of the first peak in the CMB angular power spectrum.

As Figure 2 shows, there is nontrivial structure in the CMB on angular scales which according to SBB cosmology could never have been in causal contact. Hence, SBB cannot explain the origin of structure on these large scales. This is the formation of structure problem from which the SBB scenario suffers.

In addition, SBB cosmology cannot explain the degree of spatial flatness of the universe which is currently observed. According to SBB cosmology, a spatially flat universe is unstable to the development of curvature in the expanding phase. The near spatial flatness which is currently observed requires a tuning of the relative contribution of the spatial curvature to the total energy density which is of the order of $10^{-50}$ at energy scales corresponding to particle physics grand unification.

Since the SBB scenario is well tested at late cosmological times, any solution of the above-mentioned problems will require new physics during the stages of the very early universe.

C. Preview

In the following Sections we will discuss different scenarios of very early universe cosmology which can provide solutions to the horizon, flatness and structure formation problems. We will compare the current paradigm, the inflationary universe scenario, with a couple of alternatives. The main message will be that there are a number of early universe scenarios, and not just inflation, which are compatible with current observations.

We will then ask the question what kind of picture of the early universe emerges if superstring theory is the correct theory which unifies all forces of nature at high energies. Hints will be discussed which indicate that superstring cosmology will be nonsingular and may not include a phase of inflation.

II. THEORY OF COSMOLOGICAL PERTURBATIONS

A. Basics of Cosmological Perturbation Theory

Observations of CMB anisotropies and large-scale structure concern linear fluctuations about the cosmological background. Hence we must start with a brief summary of how these inhomogeneities are described and how they evolve (see e.g. [8, 9] for details). We here work in the context of Einstein gravity with a matter source. For simplicity, matter is modelled in terms of a scalar field $\varphi$ with a non-trivial background dynamics $\varphi_0(t)$. Since the universe is observed to converge to homogeneity on large scales, the fluctuations can be described in linear theory.

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3 The dominant radiation causes a homogeneous gravitational potential which impedes the growth of density fluctuations in the cold matter.
FIG. 3: Illustration of the horizon problem. The vertical axis is time, the horizontal axis is distance - implicitly using coordinates in which light travels at 45 degree angles. We see light rays (wavy lines) starting at the time $t_{\text{rec}}$ of recombination and reaching us today (time $t_0$). As shown, the distance over which we see the microwave light to be isotropic is much larger than the distance which could have been in causal contact starting at the initial time $t = 0$. 
This means that any fluctuating field can be expanded in plane waves (Fourier modes), and each such mode evolves independently. The Fourier modes can be labelled by the comoving wavenumber \( k \).

Linear fluctuations of geometry and matter can be classified according to how they transform under spatial rotations. Out of the ten degrees of freedom of the metric, four are scalars, four vectors, and two tensors (the two polarization states of gravitational waves). The physics is independent of which coordinates are used, and hence there are four coordinate modes (gauge modes) which can be factored out, leaving only two scalars and two vectors, plus the gravitational waves. For simple matter models such as a scalar field with a homogeneous component which is evolving in time (a \textit{rolling} scalar field) the vector modes are not sourced at linear order in perturbation theory, and hence we will not consider them. We focus on the scalar modes which are sourced by matter. One of the scalar modes vanishes for matter without anisotropic stress, and the remaining mode is determined by the matter fluctuations.

We can choose coordinates in which the scalar metric fluctuations are diagonal, and the metric is

\[
ds^2 = a^2 \left\{ (1 + 2\Phi) \, dt^2 - [(1 - 2\Phi) \, \delta_{ij} + h_{ij}] \, dx^i \, dx^j \right\}, \tag{4}
\]

where \( \Phi \) is a function of space and time and represents the scalar metric fluctuations. The matter field including linear fluctuations is

\[
\varphi(x, t) = \varphi_0(t) + \delta \varphi(x, t). \tag{5}
\]

The equations of motion for the fluctuations can be obtained by expanding the action of matter plus gravity to second order about the background. Since the background satisfies the equations of motion, terms linear in cosmological fluctuations cancel out in the action, leaving the quadratic terms as the leading fluctuation terms. There is only one canonical fluctuation variable \( \zeta(x, t) \) (variable in terms of which the action has a canonical kinetic term). As shown in \cite{11, 12}, this variable is

\[
v = a \left( \delta \varphi + \frac{z}{a} \Phi \right), \tag{6}
\]

where

\[
z = \frac{\alpha \varphi'}{\dot{H}}, \tag{7}
\]

and obeys the equation of motion

\[
v'' + \left( k^2 - \frac{z''}{z} \right) v_k = 0, \tag{8}
\]

where a prime denotes the derivative with respect to conformal time \( \tau \). Here we see the crucial role which the Hubble radius \( H^{-1} \) plays. If the equation of state of matter is constant in time, then the factor \( z''/z \) is of the order \( H^2 \). Hence, on sub-Hubble scales \( k > H \) fluctuations oscillate, while on super-Hubble scales \( k < H \) they are \textit{frozen in} and are \textit{squeezed}, i.e. in an expanding universe the amplitude of the dominant mode of the above equation grows as \( z \sim a \).

The curvature fluctuation \( \zeta \) is in fact given by \( v/z \). What is measured is the power spectrum \( P_\zeta(k) \) of \( \zeta \) which gives the mean square fluctuation of \( \zeta \) on a length scale \( k^{-1} \) and is given in terms of the Fourier modes \( \zeta(k) \) of \( \zeta \) by

\[
P_\zeta(k) = k^3 |\zeta(k)|^2. \tag{9}
\]

If this quantity is independent of \( k \), one has a \textit{scale-invariant} spectrum. More generally, one has \( P(k) \sim k^{n_s-1} \) where \( n_s \) is called the scalar spectral index. The observed spectrum is nearly scale-invariant with a small red tilt \cite{10}.

An initial vacuum spectrum at time \( t_i \) is

\[
v_k(t_i) = \frac{1}{\sqrt{2k}}. \tag{10}
\]

This is a deep blue power spectrum, i.e. there is more power on short wavelengths. If fluctuations originate as vacuum perturbations (which they are assumed to in inflationary cosmology and in a number of alternatives), then the squeezing of the fluctuations on super-Hubble scales must have exactly the right features to convert the initial vacuum spectrum to a scale-invariant one. As will be discussed in the next section, this happens both for inflationary cosmology and for the matter bounce scenario.

Note that the equation of motion for the Fourier mode of the amplitude of gravitational waves is the same as \cite{8} except that the function \( z(\eta) \) is replaced by the scale factor \( a(\eta) \). If the equation of state of matter is constant, then \( z \sim a \) and hence the gravitational waves evolve as the curvature fluctuations.
B. Criteria for a Successful Early Universe Cosmology

The presence of acoustic oscillations in the angular power spectrum of the CMB and in the power spectrum of matter density fluctuations (the so-called baryon oscillation peak) were predicted \cite{13,14} a decade before the development of inflationary cosmology. In these works it was realized that a roughly scale-invariant spectrum of standing wave fluctuations on super-Hubble scales at cosmological times before the time $t_{\text{rec}}$ of recombination would, as the scales enter the Hubble radius and begin to oscillate as described in the previous subsection, would be in good agreement with the data on the distribution of galaxies which was available at the time when the papers were written, and give rise to oscillations in the angular power spectrum of the CMB and in the power spectrum of density fluctuations. The dynamics is illustrated in Figure 4. At that time, however, no models based on causal physics were known for how to produce such primordial fluctuations.

Here we outline the criteria which a successful early universe scenario must satisfy. Let us emphasize again the difference between the horizon and the Hubble radius. The horizon is the forward light cone of a point at the initial time and carries causality information. Beginning at the initial time, it is possible to have causal contact within the horizon. The Hubble radius $H(t)^{-1}$, on the other hand, is a local concept. It separates scales where fluctuations oscillate (sub-Hubble modes) from scales where the inhomogeneities are frozen in and get squeezed (super-Hubble modes).

Since the Hubble radius at $t_{\text{rec}}$ is smaller than the radius of the part of the universe which we probe with the CMB, the condition for an early universe scenario to solve the horizon problem is that the horizon at $t_{\text{rec}}$ be much larger than the Hubble radius at that time (at least two orders of magnitude since the Hubble radius at $t_{\text{rec}}$ corresponds to an angular scale of about $1^\circ$ and we need to explain the absence of order one anisotropies on the angular scales of the full sky). Thus, the first requirement on a successful early universe scenario is that the horizon at late times be much larger than the Hubble radius.

In order to be able to have a causal mechanism of structure formation, scales which are currently observed in cosmology must originate in the early universe with a wavelength smaller than the Hubble radius. This is the second requirement.

Thirdly, the scales which we observe today must propagate for a long time at super-Hubble lengths. This is in order that the fluctuations are squeezed and enter the Hubble radius at late times as standing waves. This is required in order to provide an explanation for the observed acoustic oscillations in the angular power spectrum of the CMB.

Finally, the generation mechanism which acts on sub-Hubble scales must yield a nearly scale-invariant spectrum of cosmological perturbations.

The inflationary universe scenario provides a simple realization of these four criteria. However, there are alternative realizations as will be explained in the following section.

III. PARADIGMS OF EARLY UNIVERSE COSMOLOGY

A. Cosmological Inflation

Cosmological inflation \cite{15,18} assumes that space underwent a period of almost exponential expansion during a finite time interval in the very early universe, starting at some time $t_i$ and ending at a time $t_R$ \footnote{The subscript “R” stands for reheating since at this time the conditions of a hot early universe must be created.}. The first model of inflation was based on a modified gravitational action \cite{19} where higher curvature terms dominate at early times and lead to accelerated expansion. Later models assumed that the accelerated expansion of space is realized in the context of Einstein gravity, but assuming the presence of a slowly rolling \cite{20,22} scalar field whose energy-momentum tensor is dominated by the scalar field potential energy. After the end of inflation at time $t_R$, the evolution of the universe is like in Standard Big Bang cosmology.

Figure 5 represents a space-time sketch of inflationary cosmology. The vertical axis is time, the horizontal axis denotes physical spatial distance. The solid curve labelled $H^{-1}$ represents the Hubble radius, the dashed line is the horizon, and the curve labelled $k$ represents the physical wavelength of a fluctuation mode.

During the phase of accelerated expansion for $[t_i < t < t_R]$, the horizon is growing very fast while the Hubble radius increases only slowly. For almost exponential expansion the horizon increases nearly exponentially while the Hubble radius is almost constant. Only a very short interval of accelerated expansion is needed to solve the horizon problem (the horizon needs to increase by a factor of greater than 100 relative to the Hubble radius).
FIG. 4: Origin of acoustic oscillations in the CMB angular power spectrum: assuming that fluctuations originate as standing waves on scales which are super-Hubble at early times, then different modes perform a different number of oscillations between when they enter the Hubble radius and start to oscillate and the time of recombination. Those which perform $n = 0, 2, 4, \ldots$ quarter oscillations are standing waves with maximal amplitude at $t_{\text{rec}}$ and yield peaks in the anisotropy spectrum, whereas those which perform $n = 1, 3, \ldots$ quarter oscillations only have velocity inhomogeneities at $t_{\text{rec}}$ and yield minima in the angular power spectrum (see [13] from where the figure is adapted).
If the energy scale of inflation corresponds to the scale of particle physics Grand Unification (an energy scale of about $10^{16}$ GeV), then about 60 e-foldings of exponential expansion are required in order that scales which we observe now in cosmology had a wavelength smaller than the Hubble radius at the beginning of the inflationary period. In this case, the second of the general criteria for a successful early universe cosmology is satisfied.

As is evident from Figure 5, the wavelength of fluctuations is larger than the Hubble radius for a long time. Hence, the squeezing of the fluctuations which is required in order to obtain the acoustic oscillations in the CMB angular power spectrum is realized.

Finally, as initially argued in [23] for gravitational waves and in [24] for the curvature perturbations, the power spectrum of fluctuations will be approximately invariant of scale. This is a consequence of the time-translation invariance of the phase of exponential expansion [17, 25]. It is usually assumed that fluctuations originate as quantum vacuum perturbations at early times during the inflationary phase. This assumption can be justified since, in the absence of interactions with the agent driving the accelerated expansion, any initial fluctuations are redshifted away, leaving the matter in a vacuum state. If the agent driving inflation interacts with matter, it is possible to realize inflation while maintaining the dominance of thermal fluctuations. This is the warm inflation scenario [26].

The scale-invariance of the spectrum of curvature fluctuations can also be seen using the general formalism described in the previous section. The curvature fluctuation variable, starting out with a deep blue vacuum spectrum on sub-Hubble scales, gets squeezed on super-Hubble scales. Large wavelength fluctuations exit the Hubble radius earlier and are thus squeezed more. For almost exponential inflation the squeezing is just right to turn the initial vacuum spectrum into a scale-invariant one. To see this, consider first the Hubble crossing condition

$$a^{-1}(t_H(k))k = H.$$  \hfill (11)

Before $t_H(k)$, the amplitude of $v_k$ is constant, afterwards it increases as $z$. Thus, the power spectrum of $\zeta$ at some late time $t$ is

$$P_{\zeta}(k, \eta) = k^3 z^{-2}(\eta) \left( \frac{z(\eta)}{z(\eta_H(k))} \right)^2 |v_k(t_i)|^2 \approx \frac{1}{2} \left( \frac{a(t_H(k))}{z(t_H(k))} \right)^2 H^2$$  \hfill (12)
The $k$-dependence has cancelled out and we thus obtain a scale-invariant spectrum.

Gravitational waves evolve as curvature fluctuations, with the function $z(\eta)$ being replaced by $a(\eta)$. Hence, inflationary cosmology predicts a scale invariant spectrum of gravitational waves whose amplitude is suppressed compared to that of curvature fluctuations by the slow-roll parameter $\epsilon$ which gives the ratio between $z$ and $a$ during the inflationary phase.

Of the early universe scenarios discussed here, inflationary cosmology is the only one which predicted (as opposed to post-dicted) various observational results, e.g. the spatial flatness of the universe, the almost scale-invariance of the spectrum of cosmological perturbations and the near Gaussianity of this spectrum. In fact, inflation predicted a slight red tilt of the spectrum, a tilt which has now been established by the recent CMB observations (see e.g. [33]). Inflation predicts a nearly scale-invariant spectrum of gravitational waves [23] with a slight red-tilt. Note that, whereas the tilt of the spectrum of cosmological perturbations can be made blue by complicating the scalar field model of inflation, the spectrum of gravitational waves has a red tilt unless the matter which is responsible for inflation violates the energy condition $p + \rho \geq 0$, where $p$ and $\rho$ are pressure and energy density, respectively.

A drawback of the inflationary scenario is the trans-Planckian problem for cosmological perturbations [27] [28] (see e.g. [29] for a review with an extended list of references). If the period of inflation lasts only slightly longer than the minimal amount of time it needs to in order to solve the flatness and structure formation problems of SBB cosmology, then the wavelengths of the fluctuation modes which we observe today are smaller than the Planck length at the beginning of inflation. We do not understand the physics on these wavelengths, and hence it is unclear if the initial conditions for the fluctuations which are used are well justified. Some other problems of inflationary cosmology will be discussed in later sections.

B. Matter Bounce

Another paradigm to obtain successful structure formation is the matter bounce scenario [30] [31]. It is assumed that there is new physics [6] which resolves the cosmological singularity. Time runs from $-\infty$ to $\infty$. The universe begins in a contracting phase which is the mirror inverse of the expanding Standard Big Bang cosmology evolution, which is to say that at very early times during contraction, a dark energy component may dominate, followed by a period of matter-dominated contraction, followed by a radiation-dominated phase, and then a nonsingular bounce. The bounce point (minimal value of the scale factor) is $t = 0$.

Figure 6 presents a sketch of the resulting space-time diagram. The vertical axis is conformal time, the horizontal axis represents comoving spatial distance. The Hubble radius is symmetric about $t = 0$. The perpendicular curve indicates the wavelength of a fluctuation. Since there is no origin of time, the horizon is infinite and there is no horizon problem. As obvious from the figure, all scales originate inside the Hubble radius, and hence a causal structure formation scenario is possible. In fact, for scales which exit the Hubble radius during the matter-dominated phase of contraction, an initial vacuum spectrum on sub-Hubble scales evolves into a scale-invariant spectrum on super-Hubble scales. If we take into account the dark energy component in the contracting phase, a slight red tilt of the spectrum results, as in inflationary cosmology [33].

The reason why a scale-invariant spectrum of curvature fluctuations is obtained from initial vacuum perturbations is that the squeezing function in $\eta$ has the same dependence on $\eta$ as in the case of inflation. Hence, the conversion of a vacuum spectrum to a scale-invariant way proceeds as in the case of inflation.

Note that a scale-invariant spectrum of gravitational waves is also generated in the matter bounce scenario. This demonstrates that the often stated claim that a detection of primordial gravitational waves on cosmological scales will confirm inflation is false (see [34] for a detailed discussion of this point). In fact, in the matter bounce scenario the analog of the inflationary slow-roll parameter $\epsilon$ is of order one, and hence there is no suppression of the amplitude of gravitational waves.

The matter bounce scenario faces significant problems. For one, the contracting phase is unstable against anisotropies [35]. In addition, there is no suppression of gravitational waves compared to cosmological perturbations, and hence the amplitude of gravitational waves is predicted to be in excess of the observational bounds. If the cosmological perturbations are boosted in the bounce phase, then non-Gaussianities are induced which are in

\[ \text{At this point, we have not yet observed a stochastic background of gravitational waves, and observing the small tilts predicted by various early universe models will be very challenging.} \]

\[ \text{The new physics could be string theory, loop quantum gravity or some effective field theory which corresponds to Einstein gravity coupled to some matter which violated the usual Hawking-Penrose energy conditions. For a review of bouncing cosmologies see e.g. [29]. We are emphasizing here ways to obtain a spectrum of cosmological fluctuations in agreement with observations, and the specific dynamics of the bounce phase has little to say about this issue.} \]
FIG. 6: Space-time sketch of the matter bounce cosmology. The vertical axis is conformal time, the horizontal axis is comoving distance. The blue solid curve is the Hubble radius, the green vertical line represents the wavelength of a fluctuation mode. The bounce phase (for which new physics is required) lasts from the time when the Hubble radius takes on its minimal value in the contracting phase to the corresponding time during expansion.

excess of observational bounds [36]. Postulating a large graviton mass in the contracting phase can solve both of these problems [37].

C. Pre-Big Bang and Ekpyrotic Scenarios

The Pre-Big-Bang [38] (PBB) and Ekpyrotic scenarios [39] are bouncing cosmologies which avoids the anisotropy and overproduction of gravitational wave problems of the matter bounce scenario. In both of these cosmologies, the contracting phase is dominated by a form of matter whose energy density increases as fast (in the case of the PBB scenario) or faster (in the case of the Ekpyrotic scenario) than the contribution of anisotropies. In the case of the PBB scenario, it is the kinetic energy of the dilaton field (one of the massless degrees of freedom of string theory) which dominates in the contracting phase, in the case of the Ekpyrotic scenario it is the energy density of a new scalar field with negative exponential potential. Both of these scenarios were initially proposed based on ideas in superstring theory but they can also be viewed as effective field theories involving a new scalar field with some special features.

The space-time sketches of the PBB and Ekpyrotic scenarios are similar to that of the matter bounce paradigm (see Figure 6), except that the bounce is not symmetric. In particular, the horizon problem of Standard Big Bang cosmology is solved in the same way as in the matter bounce, and in the same way fluctuations on all scales observed today originate inside the Hubble radius at early times, thus allowing a causal structure formation scenario. However, unlike what happens in the matter bounce scenario, the growth of fluctuations on super-Hubble scales in the contracting phase is too weak to convert an initial vacuum spectrum into a scale-invariant one. The resulting spectrum of curvature fluctuations and gravitational waves is blue (see e.g. [40]), thus not allowing initial vacuum perturbations to explain the observed structures in the universe, and predicting a negligible amplitude of gravitational waves on cosmological scales. As studied in [41] in the case of the PBB scenario, and in [42] in the case of the Ekpyrotic scenario, a scale-invariant spectrum of curvature fluctuations can be obtained by using primordial vacuum fluctuations in a second scalar field.
D. Emergent String Gas Cosmology

Another alternative to cosmological inflation as a theory for the origin of structure in the universe is the emergent scenario as realized in String Gas Cosmology (SGC) [43]. This scenario (see e.g. [44] for reviews) is based, as discussed in the following section, on the idea that there was a long quasi-static phase in the early universe dominated by a thermal gas of fundamental strings. This phase may be past-eternal, or it may be preceded by a previous phase of contraction. At the end of this phase there is a transition to the expanding radiation phase of SBB cosmology.

Figure 7 is a space-time sketch of SGC. The vertical axis is time, with the time \( t_R \) denoting the end of the quasi-static phase, the horizontal axis is physical distance. The curve labelled \( H^{-1} \) is the Hubble radius which is infinite in the Hagedorn phase, falls to a microscopic value at \( t_R \) and then increases linearly in time as in SBB cosmology. Since time goes back to \( -\infty \), there is no horizon problem, as in the bouncing models discussed in previous subsections. As is obvious from Figure 7, all scales originate from inside the Hubble radius at early times, thus allowing for a causal structure formation scenario.

As first realized in [45, 46], thermal fluctuations of a gas of closed strings on a compact space with the topology of a torus lead to a scale-invariant spectrum of curvature fluctuations and gravitational waves. The tilt of the spectrum of curvature fluctuations is predicted to be red (as in the case of inflationary cosmology), but that of the gravitational waves is slightly blue, in contrast to what is obtained in inflation. The amplitude of gravitational waves is suppressed compared to that of curvature fluctuations by the equation of state parameter \( w = p/\rho \) [46].

The emergent string gas cosmology model is an example where the fluctuations are of thermal origin, not of quantum origin as they are in most inflationary models (warm inflation being the exception) and the Ekpyrotic scenario.

E. Comparisons

The main messages from this section is that there are a number of different early universe scenarios which can solve the horizon problem of SBB cosmology and which can generate a spectrum of curvature fluctuations consistent with observations. Inflationary cosmology is not the only game in town. One must, however, admit that inflation is the first scenario proposed and the only one which can claim to have made successful predictions (as opposed to postdictions). Inflation is also the only scenario which is at the present time self-consistent from the point of view of low energy effective field theory (Einstein gravity coupled to low energy quantum field matter). All the other scenarios mentioned above need new physics to obtain essential aspects of the dynamics of the cosmological background, be it a cosmological bounce or a quasi-static phase. Another nice feature which inflation has is that - at least in the
case of large-field inflation - the slow roll trajectory in field space which leads to inflation is a local attractor in initial condition space \[47, 48\]. This feature is shared by the Ekpyrotic scenario, but not some of the other ones.

On the other hand, in all of the alternatives to inflation mentioned above, the physical wavelength of the fluctuations which are currently observed are much larger than the Planck length (as long as the maximal temperature is smaller than the Planck scale) at all times. Hence, the trans-Planckian problem which the theory of cosmological fluctuations in inflationary cosmology suffers from is not present.

Unless model parameters are finely tuned, the energy scale at which inflation takes place is close to the particle physics scale of Grand Unification. This is close to the Planck scale and even closer to the preferred string scale \[49\]. Hence, the extrapolation of low energy physics to the scale of inflation and (to the bounce scale in bouncing cosmologies) needs to be justified. In spite of a large body of work, there have so far not been any convincing realizations of inflation in the context of superstring theory, and there are indications that it might not be possible to embed simple inflation models in string theory \[50\]. Hence, there are good reasons to look beyond standard inflationary cosmology.

Note that the structure formation scenarios described above are not the only ones. The main goal was to present a few very different scenarios and to show how the general criteria for a successful early universe model can be realized.

IV. HINTS FROM SUPERSTRING THEORY

A. Challenges

The goal of superstring theory is to provide a quantum theory which unifies all four forces of nature, including gravity (see e.g. \[49, 51, 52\] for textbook treatments of string theory). If nature is indeed described by superstring theory, then string theory should play a crucial role at the high energy densities which were present in the very early universe. Whichever of the scenarios for structure formation described in the previous section is in fact realized in nature should then be determined by string theory.

It has been shown to be very challenging to obtain an inflationary phase from string theory (see e.g. \[53\] for a detailed discussion). The problem is that in order to obtain a period of slow-roll inflation from simple scalar field potentials, field values in excess of the Planck mass \(m_{pl}\) are required (see e.g. the review in \[48\]). However, for such large field values string effects on the shape of the potential need to be considered, and tend to destroy the required flatness of the potential unless special symmetries of the field theory (e.g. shift symmetry \[54\]) are considered. But even in this case string theory arguments such as the Weak Gravity Conjecture \[55\] tend to invalidate the effective field theory constructions.

B. String Thermodynamics

There are indications that the description of the very early universe in string theory will look very different from what can be obtained by models based on point particle theories (which includes all existing “string-derived” low energy effective field theories). The first evidence for this comes from string thermodynamics. It has been known \[56\] from the earliest days of string theory that there is a maximal temperature \(T_H\) of a gas of closed string in thermal equilibrium, the so-called Hagedorn temperature. The value of this limiting temperature is given by the string scale. Assuming that space is a torus of radius \(R\), then the evolution of the temperature \(T\) as a function of \(R\) is sketched in Figure 8. The length of the plateau region of \(T(R)\) depends on the total entropy of the system: the larger the entropy, the larger the plateau region.

The origin of the \(T(R)\) curve of Figure 8 is easy to understand. On a compact space, there are three types of states of strings: momentum modes which correspond to the centre of mass motion of the string, winding modes (which count the number of times the string wraps the torus), and the string oscillatory modes (whose energies are independent of \(R\)). The number of oscillatory states increases exponentially with the energy of the state. It is this fact which leads to the existence of the limiting temperature. As the energy density in a system is increased, then the energy will go into the excitation of new oscillatory states as opposed to the increase in the temperature.

The energies of the momentum and winding modes are quantized in units of \(1/R\) and \(R\), respectively. For momentum modes we have

\[E_n = n \frac{1}{R},\]  \hspace{1cm} (13)

and for winding modes

\[E_m = mR\]  \hspace{1cm} (14)
(in string units), where \( n \) and \( m \) are integers. In fact, there is one momentum and one winding quantum number for each compact spatial dimension.

The above is the mass spectrum of free string states. It reflects an important symmetry of string theory, the T-duality symmetry which implies that under the transformation

\[
R \rightarrow \frac{1}{R}
\]

(in string units) the spectrum of states is unchanged. This symmetry corresponds to an interchange between momentum and winding quantum numbers. It is a symmetry of string interactions, and it is assumed to be a symmetry of non-perturbative string theory, giving rise to the existence of D-branes [52] (see also [57]). This symmetry allows us to understand the symmetry of the \( T(R) \) curve of Figure 8 about \( \ln R = 0 \): for large values of \( R \) the energy wants to be in the modes which are light for large values of \( R \), namely the momentum modes, while for \( R \ll 1 \) in string units the energy will drift into the winding modes which are the light one in this range of \( R \) values.

Figure 8 immediately leads one to expect that in the context of string theory the cosmological singularity can
be avoided\(^7\), while in the context of Einstein gravity coupled to particle matter a temperature singularity as \(R \to 0\) is unavoidable.

C. T-Duality Symmetry and Doubled Space

What is missing from the previous discussion is the dynamics of the background space-time. This can obviously not be described by the Einstein-Hilbert action since the this action is incompatible with the T-duality symmetry of string theory. Dilaton gravity as described in [38] is a better starting point. In this case the T-duality symmetry of string theory is reflected in the so-called scale factor duality which involves a transformation of both the metric and the dilaton field.

However, there may be a modified background description which is more useful for superstring cosmology. The starting point of this description is the following: In quantum mechanics the position eigenstates \(|x\rangle\) are dual to momentum eigenstates \(|p\rangle\). In a toroidal background, the momenta are discrete, labelled by integers \(n\), and hence

\[
|x\rangle = \sum_n e^{inx} |n\rangle.
\]

(16)

where \(|n\rangle\) are the momentum eigenstates with momentum quantum numbers \(n\). As discussed above, in string theory on a torus the windings are T-dual to momenta, and it is possible [43] to define a T-dual position operator

\[
|\tilde{x}\rangle = \sum_m e^{im\tilde{x}} |m\rangle,
\]

(17)

where \(|m\rangle\) are the eigenstates of winding, labelled by an integer \(m\).

String states with both momenta and windings can be viewed as point particles propagating on a doubled space which is spanned by both the position and the dual position eigenstates. The coordinates of this space are \(X^i\)

\[
X^i = (x^i, \tilde{x}^i).
\]

(18)

If the underlying target space of the strings has \(d\) spatial dimensions, the number of spatial dimensions of the doubled space is \(2d\). This is the same space which is used in the Double Field Theory (see [58] and [59] for a review) approximation to string theory.

Let us consider a torus with radius \(R\). If \(R\) is large in string units, then the light states are the momentum modes and a physical apparatus to measure length will be built from momentum modes. However, if \(R\) is small compared to the string length, then it is the winding modes which are light and hence a physical apparatus will be constructed from the winding modes. Hence, an apparatus measuring the physical length \(l_p(R)\) will be measuring (see Figure 9)

\[
l_p(R) = \begin{cases} R & R \gg 1 \\ \frac{1}{R} & R \ll 1 \end{cases}
\]

(19)

when \(R\) is expressed in units of the string length [43].

The above yields a new interpretation of a dynamical evolution \(R(t)\). The physical interpretation of \(R\) decreasing towards \(R = 0\) is that the dual radius (which is also the physical radius) is increasing to \(R = \infty\) in the dual directions. In double space we can express this dynamics by taking a cosmological metric in double space which is

\[
ds^2 = dt^2 - a(t)^2 dx^2 - a(t)^{-2} d\tilde{x}^2.
\]

(20)

As mentioned above, physical measuring sticks measure length in terms of the coordinates related to the light string modes. Hence, for \(R > 1\) (in string units), length will be measured in terms of \(x\), while for \(R < 1\) it is measured in terms of \(\tilde{x}\). Thus, a physical device will see space as contracting as \(R\) decreases towards \(R = 1\), but for \(R < 1\) it will be seen as increasing. Thus, a physical observer will see no singularity. This argument is elaborated on in a recent paper [60].

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\(^7\) The logic here is that if the temperature remains finite, the curvature should not be able to blow up.
V. DISCUSSION

In spite of the fact that the inflationary scenario has been a model of early universe cosmology with many successes, we may have to look beyond the standard inflationary scenario in order to understand the complete evolution of the early universe. All inflationary practitioners would admit that standard inflation cannot explain the evolution all the way back to the Big Bang as a consequence of the cosmological singularities which it does not avoid [61]. Although it is possible that the correct picture of the very early universe will involve a new phase followed by a period of inflation like we understand inflation today, this needs not be the case: there are a number of alternative early universe scenarios which lead to cosmological structure formation consistent with current observations.

Of all of the structure formation paradigms, inflation may be the only one which is self-consistent at the level of effective point particle field theories. However, if nature is described by string theory, then it may be difficult to embed standard inflation into the model, and an alternative such as string gas cosmology may emerge in a more natural way, as already argued in [69].

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[1] A. A. Penzias and R. W. Wilson, “A Measurement of excess antenna temperature at 4080-Mc/s,” Astrophys. J. 142, 419 (1965). doi:10.1086/148307
[2] H. P. Gush, M. Halpern and E. H. Wishnow, “Rocket measurement of the cosmic-background-radiation mm-wave spectrum,” Phys. Rev. Lett. 65, 537 (1990). doi:10.1103/PhysRevLett.65.537
[3] J. C. Mather et al., “A Preliminary measurement of the Cosmic Microwave Background spectrum by the Cosmic Background Explorer (COBE) satellite,” Astrophys. J. 354, L37 (1990). doi:10.1086/185717
[4] G. F. Smoot et al. [COBE Collaboration], “Structure in the COBE differential microwave radiometer first year maps,” Astrophys. J. 396, L1 (1992). doi:10.1086/186504
[5] G. Hinshaw et al. [WMAP Collaboration], “First year Wilkinson Microwave Anisotropy Probe (WMAP) observations: The Angular power spectrum,” Astrophys. J. Suppl. 148, 135 (2003) doi:10.1086/377225 [astro-ph/0302217].
[6] P. A. R. Ade et al. [Planck Collaboration], “Planck 2013 results. XV. CMB power spectra and likelihood,” Astron. Astrophys. 571, A15 (2014) doi:10.1051/0004-6361/201321573 [arXiv:1303.5075 [astro-ph.CO]].
[7] S. Furlanetto, S. P. Oh and F. Briggs, “Cosmology at Low Frequencies: The 21 cm Transition and the High-Redshift Universe,” Phys. Rept. 433, 181 (2006) doi:10.1016/j.physrep.2006.08.002 [astro-ph/0608032].
