Squark-, Slepton- and Neutralino-Chargino coannihilation effects in the low-energy effective MSSM

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We calculate the neutralino relic density within the low-energy effective Minimal Supersymmetric extension of the Standard Model (effMSSM) taking into account slepton-neutralino, squark-neutralino and neutralino/chargino-neutralino coannihilation channels. By including squark (stop and sbottom) coannihilation channels we extend our comparative study to all allowed coannihilations and obtain the general result that all of them give sizable contributions to the reduction of the neutralino relic density. Due to these coannihilation processes some models (mostly with large neutralino masses) fall within into the cosmologically interesting region for relic density, but other models drop out at this region. Nevertheless, the predictions for direct and indirect dark matter detection rates are not strongly affected by these coannihilation channels in the effMSSM.

I. INTRODUCTION

A variety of data ranging from galactic rotation curves to large scale structure formation and the cosmic microwave background radiation imply a significant density $0.1 < \Omega h^2 < 0.3$ [1] of so-called cold dark matter (CDM). Here $\Omega = \rho/\rho_c$ and $\rho_c = 3H^2/8\pi G_N$ is the critical closure density of the universe, $G_N$ is the Newton constant and $h$ is the Hubble constant in units of 100 km/s/Mpc. It is generally believed that most of the CDM is made of weakly-interacting massive particles (WIMPs) [2]. A commonly considered candidate for the WIMP is the lightest neutralino, provided it is the lightest supersymmetric particle (LSP) [3] in the Minimal Supersymmetric extension of the Standard Model (MSSM). Four neutralinos in the MSSM, being mass eigenstates, are mixtures of the bino $\tilde{B}$, wino $\tilde{W}$ and higgsinos $\tilde{H}_d^0$, $\tilde{H}_u^0$, and the LSP can be written as a composition $\chi = \tilde{\chi}_1 = N_{11}\tilde{B} + N_{12}\tilde{W} + N_{13}\tilde{H}_d^0 + N_{14}\tilde{H}_u^0$, where $N_{ij}$ are the entries of the neutralino mixing matrix. In SUSY phenomenology one usually classifies neutralinos as gaugino-like (with $P \approx 1$), higgsino-like (with $P \approx 0$), and mixed, where the gaugino fraction is defined as $P = |N_{11}|^2 + |N_{12}|^2$.

In most approaches the LSP is stable due to R-parity conservation [4]. The neutralino, being massive, neutral and stable, often provides a sizeable contribution to the relic density. The contribution of neutralinos to the relic density is strongly model dependent and varies by several orders of magnitude over the whole allowed parameter space of the MSSM. The neutralino relic density then can impose stringent constraints on the parameters of the MSSM and the SUSY particle spectrum, and may have important consequences both for studies of SUSY at colliders and in astroparticle experiments. In the light of this and taking into account the continuing improvements in determining the abundance of CDM and other components of the Universe, which have now reached an unprecedented precision [5], one needs to perform an accurate enough computation of the WIMP relic abundance, which would allow reliable comparison of theory with observation. Important progress in calculations of the relic density of neutralinos in a variety of supersymmetric models has been already made [6–42].

In the early universe neutralinos existed in thermal equilibrium with the cosmic thermal plasma. As the universe expanded and cooled, the thermal energy is no longer sufficient to produce neutralinos at an appreciable rate, they decouple and their number density scales with co-moving volume. The sparticles significantly heavier than the LSP decouple at an earlier time and decay into LSPs before the LSPs themselves decouple. Nevertheless there may exist some other next-to-lightest sparticles (NLSPs) which are not much heavier than the stable LSP. The number densities of the NLSPs have only slight Boltzmann suppressions with respect to the LSP number density.
when the LSP freezes out of chemical equilibrium with the thermal bath. Therefore they may still be present in the thermal plasma, and NLSP-LSP and NLSP-NLSP interactions keep the LSP in thermal equilibrium resulting in significant reduction of the LSP number density and leading to acceptable values even with a rather heavy sparticle spectrum [26]. These coannihilation processes can be particularly important when the LSP-LSP annihilation rate itself is suppressed [14,29,23]. Any SUSY particle can be involved in the coannihilation process provided its mass is almost degenerate with the mass of the LSP [14,27]. In the low-energy effective MSSM (effMSSM), where one ignores restriction from unification assumptions and investigates the MSSM parameter space at the weak scale [23,21,43,44] there is, in principle, no preference for the next-to-lightest SUSY particle.

The relativistic thermal averaging formalism [15] was extended to include coannihilation processes in [23], and was implemented in the DarkSusy code [24] for coannihilation of charginos and heavier neutralinos. In was found [23] that for higgsino-like LSP such a coannihilation significantly decreases the relic density and rules out these LSPs from the region of cosmological interest.

The importance of the neutralino coannihilation with sfermions was emphasized and investigated for sleptons [26,28], stops [33,34] and sbottoms [31] in the so-called constrained MSSM (cMSSM) [22,16,45] or in supergravity (mSUGRA) models [46]. The most popular mSUGRA model [46] has a minimal set of parameters: $m_0$, $m_{1/2}$, $A_0$, $\tan\beta$ and $\text{sign}(\mu)$. Here $m_0$ is the universal scalar mass, $m_{1/2}$ is the universal gaugino mass and $A_0$ is the universal trilinear mass, all evaluated at $M_{\text{GUT}}$, $\tan\beta$ is the ratio of Higgs field vacuum expectation values and $\mu$ is a Higgs parameter of the superpotential. There are strong correlations of sfermion, Higgs boson and gaugino masses in mSUGRA originating from unification assumptions. In regions of the mSUGRA parameter space where $\chi$ and $\tilde{\chi}$ were nearly degenerate (at low $m_0$), coannihilations could give rise to reasonable values of the relic density even at very large values of $m_{1/2}$, at both low and high $\tan\beta$ [26,31]. In addition, for large values of the parameter $A_0$ or for non-universal scalar masses, the squark masses could become nearly degenerate with the $\chi$, so that squark coannihilation processes can become important as well [33,34]. Therefore due to slepton and squark coannihilation effects, the relic density can reach the cosmologically interesting range of $0.1 < \Omega h^2 < 0.3$.

The influence of coannihilation channels on the LSP proton scalar elastic cross sections was considered in supergravity and D-brain models in [31] and in a mSUGRA-like SUSY model for large $\tan\beta$ (only for stau coanannihilations) in [29].

Having in mind investigation of future prospects for direct and indirect detection of relict LSP we follow the most phenomenological view, not bounded by theoretical restrictions from sfermion/gaugino/Higgs mass unifications, etc. To this end we need maximally general and accurate calculations of the relic density within the low-energy effective MSSM scheme (effMSSM) [21,43]. The high-level tool for these calculations is the DarkSusy code [24]. Unfortunately the code calculates only neutralino with next neutralino(s) and chargino coannihilations (NCC), which is not sufficient, when neutralino-slepton coannihilation (SLC) and neutralino-squark (SQC) coannihilation are claimed to be dominant [26,28,33,34,31].

Contrary to the majority of previous papers (see for example [26,34,30,28,40]), aimed mostly at discovery and demonstration of the importance or dominance of some specific coannihilation channels, our main goal is the investigation of the interplay between different coannihilation channels as well as their consequences for detection of CDM. To this end we performed a comparative study of relevant coannihilation channels, exploration of relevant changes in the relic density and investigation of their consequences for detection of CDM particles were performed in the effMSSM in our previous paper [42]. The present paper extends our investigations [42] to the neutralino-stop and neutralino-sbottom coanannihilations and completes our consideration of the subject. For this purpose we combined our previous code [43] with the DarkSusy code [24] and codes of [26,34], which allows for the first time comparative study of relevant coannihilation channels (NCC, SLC, SQC) in the low-energy effMSSM.

II. THE effMSSM APPROACH

As free parameters in the effMSSM, we use the gaugino mass parameters $M_1, M_2$, the entries to the squark and slepton mixing matrices $m^2_{Q_i}, m^2_{U_i}, m^2_{D_i}, m^2_R, m^2_L$ for the 1st and 2nd generations and $m^2_{Q_3}, m^2_{L}, m^2_B, m^2_R, m^2_L$ for the 3rd generation, respectively; the 3rd generation trilinear soft

$$m^2_{1/2}, A_0, \tan\beta, \text{sign}(\mu)$$

were nearly degenerate (at low $m_0$), coannihilations could give rise to reasonable values of the relic density even at very large values of $m_{1/2}$, at both low and high $\tan\beta$ [26,31]. In addition, for large values of the parameter $A_0$ or for non-universal scalar masses, top or bottom squark masses could become nearly degenerate with the $\chi$, so that squark coannihilation processes can become important as well [33,34]. Therefore due to slepton and squark coannihilation effects, the relic density can reach the cosmologically interesting range of $0.1 < \Omega h^2 < 0.3$.

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couplings $A_t, A_b, A_\tau$; the mass $m_A$ of the pseudoscalar Higgs boson, the Higgs superpotential parameter $\mu$, and $\tan \beta$. To reasonably reduce the parameter space we assumed $m^2_D = m^2_{\tilde{U}} = m^2_{\tilde{Q}}$; $m^2_T = m^2_{\tilde{B}} = m^2_{\tilde{Q}_3}$; $m^2_R = m^2_{\tilde{L}}$; $m^2_{\tilde{t}_3} = m^2_{\tilde{t}_3}$ and have fixed $A_b = A_\tau = 0$ [43]. The third gaugino mass parameter $M_3$ defines the mass of the gluino in the model and is determined by means of the GUT assumption $M_2 = 0.3 M_3$. The remaining parameters define our effMSSM parameter space and were scanned randomly within the following intervals:

$-1 \text{ TeV} < M_1 < 1 \text{ TeV}$, $-2 \text{ TeV} < M_2, \mu, A_t < 2 \text{ TeV}$, $1.5 < \tan \beta < 50$, $50 \text{ GeV} < M_A < 1000 \text{ GeV}$, $10 \text{ GeV}^2 < m^2_{\tilde{Q}}, m^2_{\tilde{L}}, m^2_{\tilde{Q}_3}, m^2_{\tilde{t}_3} < 10^6 \text{ GeV}^2$.

We have included the current experimental upper limits on sparticle masses as given by the Particle Data Group [47]. The limits on the rare $b \to s \gamma$ decay [48] following [49] have also been imposed. The calculations of the neutralino-nucleon cross sections and direct and indirect detection rates follow the description given in [3,43].

The number density is governed by the Boltzmann equation [15,23]

$$\frac{dc}{dt} + 3Hn = -\langle \sigma v \rangle (n^2 - n^2_{eq})$$

with $n$ either being the LSP number density if there are no other coannihilating sparticles, or the sum over the number densities of all coannihilation partners. The index “eq” denotes the corresponding equilibrium value. To solve the Boltzmann equation (1) one needs to evaluate the thermally averaged neutralino annihilation cross section $\langle \sigma v \rangle$. Without coannihilation processes $\langle \sigma v \rangle$ is given as the thermal average of the LSP annihilation cross section $\sigma_{\chi\chi}$ multiplied by relative velocity $v$ of the annihilating LSPs

$$\langle \sigma v \rangle = \langle \sigma_{\chi\chi} v \rangle,$$  

otherwise it is determined as $\langle \sigma v \rangle = \langle \sigma_{\text{eff}} v \rangle$, where the effective thermally averaged neutralino annihilation cross section is obtained by summation over coannihilating particles [15,23]

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n^i_{eq} n^j_{eq}}{n_{eq} n^2_{eq}}.$$  

If $n_0$ denotes the present-day number density of the relics, the relic density is given by

$$\Omega = \frac{m_\chi n_0}{\rho_c}.$$  

For each point in the MSSM parameter space (MSSM model) we have evaluated the relic density of the LSP ignoring any possibility of coannihilation (IGC), taking into account only neutralino-chargino (NCC), slepton (SLC), or squark (SQC) coannihilations separately, and including all of the coannihilation channels (ACC). To this end DarkSusy procedures of $\langle \sigma_{\text{eff}} v \rangle$ evaluation and solution of Boltzmann equation were implemented in our former code [43]. Coannihilations with two-body final states that can occur between neutralinos, charginos, sleptons, stops and sbottoms, as long as their masses are $m_i < 2m_\chi$, were included. The Feynman amplitudes for NCC, SLC and stop coannihilations were taken from DarkSusy [24], [50,26], and [51,34], respectively. The amplitudes for the sbottom coannihilation were obtained on the basis of the stop amplitudes from [51,34]. As in [42], the $\langle \sigma_{\text{eff}} v \rangle$ and $\Omega h^2$ were calculated following the relevant DarkSusy routines [24] to which the codes [50,26], and [51,34] were added in a way that guarantees the correct inclusion of SLC and SQC.

In the case where all squarks, sleptons, neutralinos and charginos are substantially heavier than the LSP ($m_i > 2m_\chi$) and there is no possible coannihilations, the relic density $\Omega h^2 = \Omega h^2_{\text{NCC}} = \Omega h^2_{\text{SLC}} = \Omega h^2_{\text{SQC}}$ is equal to $\Omega h^2$ obtained without any coannihilations. When, for example, at least one of the coannihilation channels (NCC, SQC or SLC) is indeed relevant, the $\Omega h^2_{\text{IGC}}$ (ignoring coannihilation) is calculated with

$$\langle \sigma_{\text{eff}} v \rangle_{\text{IGC}} = \langle \sigma_{\chi\chi} v \rangle \left( \frac{n^i_{eq}}{n_{eq}} \right)^2,$$  

where
where $n^{ca}$ includes all open coannihilation channels. This formula (generalized also for NCC, SLC and SQC) allows a comparative study of all coannihilation channels, always leading to a smaller value for the relic density, $\Omega h^2_{COA}/\Omega h^2_{IGC} < 1$. Here $\Omega h^2_{COA}$ is a common notation for $\Omega h^2_{ACC}$, $\Omega h^2_{NCC}$, $\Omega h^2_{SQC}$ or $\Omega h^2_{SLC}$. We assume $0.1 < \Omega h^2 < 0.3$ for the cosmologically interesting region [1].

III. RESULTS AND DISCUSSIONS

A. Coannihilation effects in the relic density

We performed our calculations in the effMSSM approach given above and the results for the neutralino relic density (scatter plots) are presented in Figs. 1–6. The reduction effect on the relic density (RD) produced by SQC, SLC, NCC and ACC is shown in Fig. 1 as a ratio $\Omega h^2_{COA}/\Omega h^2_{IGC}$. On the basis of our sampling (50000 models tested) the maximum RD suppression factor for the NCC and SLC channels is of the order of $10^{-3}$. Almost the same maximal suppression is found for the squark coannihilation channels. These results depend on the experimental limits imposed on the second-lightest neutralino, chargino and slepton stop and sbottom masses. If there were no limits on their masses, the factor of relative RD reduction due to NCC could reach a maximum value of $10^{-5}$ for models with $m_\chi \approx 40$ GeV [42]. The current experimental limits for $m_{\tilde{\tau}}$, $m_{\tilde{\chi}^\pm}$, $m_{\tilde{t}}$, and $m_{\tilde{b}}$ are 80–90 GeV [47], and therefore the critical LSP mass that enables non-negligible NCC, SLC, and SQC contributions is also of the same order ($m_\chi \geq 80$ GeV).

FIG. 1. Effects of squark-neutralino (SQC), slepton-neutralino (SLC), and neutralino-chargino-neutralino (NCC) coannihilations in effMSSM. Panels a) – d) display ratios $\Omega h^2_{SQC}/\Omega h^2_{IGC}$, $\Omega h^2_{SLC}/\Omega h^2_{IGC}$, $\Omega h^2_{NCC}/\Omega h^2_{IGC}$, and $\Omega h^2_{ACC}/\Omega h^2_{SLC}$ for the case when all coannihilations are included. The maximal reduction factors for all channels (NCC, SQC, and SLC) are of the order of $10^{-3}$. Encircled points mark the cosmologically interesting relic density $0.1 < \Omega h^2_{COA} < 0.3$. In panel a) up-going triangles correspond to stop coannihilations and down-going triangles correspond to sbottom coannihilations. The stop and sbottom contributions are seen to be equal.
From panel a) of the figure one can conclude that stop (up-going triangles) and sbottom (down-going triangles) equally contribute to reduction of the RD due to coannihilations.

The encircled symbols depict some kind of “constructive” reduction, when due to the coannihilations the relic density falls into the cosmologically interesting region $0.1 < \Omega h^2_{\text{COA}} < 0.3$. Other points present the cases when coannihilations too strongly reduce the relic density. One can see that NCC plays the main role in “destructive” reduction of RD, these channels contribute the maximal number of models from the cosmologically interesting region $[23,42]$. Despite this, all coannihilation channels contribute equally much in the “constructive” reduction of RD (there are almost the same number of the circled points in panels a)–c)). Therefore for our random sampling in the effMSSM the NCC, SLC and SQC are indeed relevant ($\Omega h^2_{\text{COA}}/\Omega h^2_{\text{IGC}} < 0.95$) for 2–4% of models if one assumes $0.1 < \Omega h^2_{\text{ACC}} < 0.3$, and the NCC is indeed relevant for about 30% of models if this constraint is relaxed (Fig. 1).

![Fig. 2](image_url)

**FIG. 2.** The same as in Fig. 1, but plotted together. Here $\Omega h^2_{\text{SQC}}/\Omega h^2_{\text{IGC}}$, $\Omega h^2_{\text{SLC}}/\Omega h^2_{\text{IGC}}$, $\Omega h^2_{\text{NCC}}/\Omega h^2_{\text{IGC}}$, and $\Omega h^2_{\text{ACC}}/\Omega h^2_{\text{SLC}}$ are marked with crosses, circles, dots and squares, respectively. Therefore, a square filled with a cross, circle, or dot depicts a model that is affected only by SQC, SLC, or NCC, respectively, while any other coannihilation channel gives a negligible contribution. Such a situation takes place for the majority of models, but there are some (quite few) models, given by empty squares, for which at least two coannihilation channels are relevant. For example, arrows on the right side of the figure demonstrate how reduction of RD proceeds: SLC gives no effect ($\Omega h^2_{\text{SLC}}/\Omega h^2_{\text{IGC}} = 1$), SQC reduces RD with a factor $\Omega h^2_{\text{SQC}}/\Omega h^2_{\text{IGC}} \approx 0.4$, and finally NCC gives the main contribution to RD suppression, $\Omega h^2_{\text{ACC}}/\Omega h^2_{\text{SLC}} \approx \Omega h^2_{\text{NCC}}/\Omega h^2_{\text{IGC}} \approx 0.04$ (the square nearly coincides with the dot).

From Fig. 2 one can see that mainly only one of the coannihilation channels (NCC, SQC, or SLC) dominates in the reduction of RD. The other channels of coannihilation in general play no role or lead only to a much smaller further reduction [42].

Although other coannihilation processes besides NLSP-LSP can in principal be also open (including LSP coannihilation with the next-to-NLSP (NNLSP) and next-to-NNLSP, as well as NLLP-NLSP coannihilations, etc.), Fig. 3 allows the conclusion that the dominant coannihilation channel is defined by the type of the NLSP. If the next neutralino $\tilde{\chi}_2$ or chargino $\tilde{\chi}^\pm$ is the NLSP, then NCC indeed dominates. The SQC dominates when NLSP is the stop or the sbottom. The stau $\tilde{\tau}$ (or another slepton) being the NLSP entails a dominant SLC effect.

Nevertheless, contrary to the NCC+SLC case [42], there are (very few) models where the stau
is the NLSP, but masses of the stau, the stop and the sbottom appear by accident almost the same and all of these sparticles participate in the coannihilation with the LSP and each other. As a result, strongly interacting coannihilation channels with squarks produce a larger reduction as compared to the SLC despite the fact that the stau is indeed the NLSP. The bottom panels of Figs. 3 and 4 display an example of such models (circle with a dot inside), in which the stau is the NLSP but the SLC contribution is smaller than the contribution of SQC ($\Omega h^2_{\text{SLC}}/\Omega h^2_{\text{SQC}} > 1$).

**FIG. 3.** Ratios $\Omega h^2_{\text{NCC}}/\Omega h^2_{\text{SLC}}$, $\Omega h^2_{\text{NCC}}/\Omega h^2_{\text{SQC}}$ and $\Omega h^2_{\text{SLC}}/\Omega h^2_{\text{SQC}}$ versus $m_\chi$. An open circle indicates that the $\tilde{\tau}$ is the NLSP, the star means that the light chargino $\tilde{\chi}^\pm$ is the NLSP, small filled circle marks the model where the second-lightest neutralino $\tilde{\chi}^0_2$ is the NLSP. An open cross indicates that the stop $\tilde{t}$ or the sbottom $\tilde{b}$ is the NLSP. One sees, for example, that if $\tilde{\chi}^0_2$ or $\tilde{\chi}^\pm$ is the NLSP, the NCC necessarily dominates ($\Omega h^2_{\text{NCC}}/\Omega h^2_{\text{SLC}} < 1$, $\Omega h^2_{\text{NCC}}/\Omega h^2_{\text{SQC}} < 1$), while $\tilde{t}$ or $\tilde{b}$ being the NLSPs always leads to dominant SQC ($\Omega h^2_{\text{NCC}}/\Omega h^2_{\text{SQC}} > 1$, $\Omega h^2_{\text{SLC}}/\Omega h^2_{\text{SQC}} > 1$). The same is in general true for SLC. Circles with dots inside depict models where the NLSP is the stau, but SLC does not dominate.
Figure 4 shows that for all coannihilation channels maximal RD reduction factors (less than 0.01) occur for mass differences $m_{NLSP} - m_{LSP} \leq 20$ GeV. Mass difference $m_{NLSP} - m_{LSP} \leq 5$ GeV plays a significant role in RD reduction for SLC and mostly for NCC. In contrast with NCC and SLC, SQC can produce the same RD reduction effect with larger mass difference between squarks and the LSP ($m_{\tilde{q}} - m_{LSP} \approx 150$ GeV) due to the possibility of coannihilation via the strong interaction.

In the case of SQC the small mass difference between the coannihilating stop and sbottom quarks ($m_{\tilde{t}} - m_{\tilde{b}}$ instead of $m_{\tilde{q}} - m_{LSP}$) can produce a dominant effect in RD reduction (triangles at $m_{\tilde{q}} - m_{LSP} \geq 100$ GeV in Fig. 4 illustrate this possibility).

Although we have set the coannihilation opening threshold of $m_i = 2m_\chi$, for NCC and SLC channels, relevant effects occur if the mass difference between the coannihilation partner and the LSP is within 15%. This is in agreement with previous considerations [14,26,30,23,28,34,31]. It was found that for SQC the relevant effects occur if the mass difference between the coannihilating squark and the LSP is within 50% (in general agreement with [34,41]).

In Fig. 5 all calculated relic densities ($\Omega h^2_{IGC}$, $\Omega h^2_{SQC}$, $\Omega h^2_{SLC}$, $\Omega h^2_{NCC}$ and $\Omega h^2_{ACC}$) are depicted in the cosmologically interesting region $0.1 < \Omega h^2_{COA} < 0.3$. A large number of models (mostly with $m_\chi \leq 250$ GeV) are completely unaffected by any kind of coannihilation. When at least one of the coannihilation channels is relevant, the RD decreases and some cosmologically unviable models with $\Omega h^2_{IGC} > 0.3$ enter the cosmologically interesting range $0.1 < \Omega h^2_{COA} < 0.3$, due to NCC (squares with a dot inside), SLC (squares with circles inside), SQC (squares with crosses inside), or due to joint contribution of NCC, SQC, or/and SLC (empty squares).
FIG. 5. Illustration of the shifting of effMSSM models into and out of the cosmologically interesting range $0.1 < \Omega h^2_{\text{COA}} < 0.3$ due to NCC, SQC and SLC. Relic densities $\Omega h^2_{\text{IGC}}$, $\Omega h^2_{\text{SQC}}$, $\Omega h^2_{\text{SLC}}$, $\Omega h^2_{\text{NCC}}$ and $\Omega h^2_{\text{ACC}}$ are marked with stars, crosses, circles, small dots, and squares, respectively. Therefore, a superposition of all those symbols corresponds to a model which is totally untouched by coannihilation.

A star-crossed circle marks a model which is unaffected by SLC and SQC ($\Omega h^2_{\text{SLC}} = \Omega h^2_{\text{SQC}} = \Omega h^2_{\text{IGC}}$), but is shifted down due to NCC. If the corresponding $\Omega h^2_{\text{ACC}}$ (which is equal to $\Omega h^2_{\text{NCC}}$) remains within this range, it is still present in the figure below this star-crossed circle as an square with a black dot inside (see short arrow). By analogy, a square with a circle inside gives a model which is shifted into the region due to SLC only ($\Omega h^2_{\text{ACC}} = \Omega h^2_{\text{SLC}}$), and if the corresponding $\Omega h^2_{\text{IGC}} = \Omega h^2_{\text{NCC}} = \Omega h^2_{\text{SQC}}$ is also in the cosmologically viable range, it is located above the symbol as a crossed star with a dot inside (see long arrow). Quite a large number of models are shifted out of the range $0.1 < \Omega h^2 < 0.3$ due to NCC (star-crossed circles).

There are also models which fall within the less interesting region ($\Omega h^2_{\text{COA}} < 0.1$). The largest amount of models are shifted out due to NCC (star-crossed circles), and a relatively small amount of models is shifted out due to SLC (crossed stars with a dot inside), SQC (circles with a star and a dot inside), both NCC and SLC (crossed stars). There are cosmologically interesting LSPs within the full mass range $20 \text{ GeV} < m_\chi < 720 \text{ GeV}$ (Fig. 5) accessible in our scan whether or not coannihilation channels are included.
FIG. 6. Variation of relic density against gaugino fraction. As in Fig. 5, RD $\Omega h^2_{\text{IGC}}$, $\Omega h^2_{\text{SQC}}$, $\Omega h^2_{\text{SLC}}$, $\Omega h^2_{\text{NCC}}$ and $\Omega h^2_{\text{ACC}}$ are marked with stars, crosses, circles, small dots, and squares, respectively. The NCC reduces the RD especially for models with higgsino-like LSPs and shifts these models out of cosmological interest. The joint effect of NCC, SQC and SLC leaves only LSPs with $P > 0.67$ in the cosmologically interesting region.

Cosmologically interesting LSPs occur with arbitrary compositions when coannihilations are ignored (Fig. 6), the inclusion of NCC rules out all the models with higgsino-like LSPs (star-crossed circles), SLC and SQC further tends to rule out LSPs with mixed composition, so that only LSPs with $P > 0.67$ (there are no squares for $1-P < 2$) remain as dominant CDM candidates. In general our estimations (Fig. 6) are in accordance with previous considerations [26,23], as far as they can be compared with these less general treatments.

B. Coannihilation effects in the detection rates

Now we consider the influence of all possible coannihilation channels (NCC, SQC and SLC) on prospects for indirect and direct detection of CDM neutralinos. The results of our calculations (scatter plots) for cold dark matter observables are presented in Figs. 7–9. We compare the rate predictions for cosmologically interesting LSPs when the RD is evaluated with or without any of coannihilation channels taken into account. We see (Fig. 5) that in most models with $m_\chi \leq 250$ GeV the RD is unaffected by SQC, SLC and NCC, mostly because the difference $m_{\text{NLSP}} - m_\chi$ is too large to yield significant effects, therefore the corresponding detection rates are not influenced (depicted in the figures as a square filled with a star, a cross and a dot simultaneously).

Figure 7 displays the expected indirect detection rates for upgoing muons produced in the Earth by neutrinos from decay products of $\chi\chi$ annihilation which takes place in the core of the Earth or of the Sun.

For $\chi\chi$ annihilation in the Earth upgoing muon detection rates merely lie within the range $10^{-19} \text{ m}^{-2} \cdot \text{yr}^{-1} < \Gamma^\mu < 5 \cdot 10^{-5} \text{ m}^{-2} \cdot \text{yr}^{-1}$ as long as $m_\chi \leq 250$ GeV. When $m_\chi \geq 250$ GeV, some of the models with $0.1 < \Omega h^2_{\text{IGC}} < 0.3$ are removed from the cosmological interesting range ($\Omega h^2_{\text{COA}} < 0.1$; Fig. 5) mainly due to NCC (Fig. 7). Other models with $\Omega h^2_{\text{IGC}} > 0.3$ are shifted into this region (Fig. 5 and Fig. 7) mainly due to SLC and SQC. For $m_\chi \geq 250$ GeV one finds $10^{-19} \text{ m}^{-2} \cdot \text{yr}^{-1} < \Gamma_{\text{ACC}} < 5 \cdot 10^{-7} \text{ m}^{-2} \cdot \text{yr}^{-1}$ when the RD is evaluated with coannihilations taken into account, and $10^{-19} \text{ m}^{-2} \cdot \text{yr}^{-1} < \Gamma_{\text{IGC}} < 4 \cdot 10^{-6} \text{ m}^{-2} \cdot \text{yr}^{-1}$ when coannihilations are neglected. The large values of the detection rates of $\chi\chi$ annihilation in the Earth are decreased (from $10^{-5} \text{ m}^{-2} \cdot \text{yr}^{-1}$ to $10^{-8} \text{ m}^{-2} \cdot \text{yr}^{-1}$) only for heavy LSPs, $m_\chi > 450$ GeV, in accordance with the fact that the corresponding models are removed from the cosmologically interesting range. The SQC does not significantly change this conclusion obtained in [42] for SLC and NCC.
FIG. 7. Indirect detection rate for upgoing muons from $\chi\chi$ annihilation in the Earth (a) and the Sun (b). As in Fig. 5, stars, crosses, circles, small dots, and squares correspond to $0.1 < \Omega h^2_{IGC}, \Omega h^2_{SQC}, \Omega h^2_{SLC}, \Omega h^2_{NCC}, \Omega h^2_{ACC} < 0.3$, respectively. NCC decreases the detection rates for models with $m_\chi \geq 400$ GeV.

FIG. 8. Neutralino-proton scattering cross sections for scalar (spin-independent) interaction (a) and axial (spin-dependent) interaction (b). The notations are as in Fig. 7.

In the case of indirect detection of upgoing muons from $\chi\chi$ annihilation in the Sun one has generally similar behavior for models with $m_\chi \geq 350$ GeV. The only noticeable difference is in the absolute predictions for detection rates for models with $m_\chi > 500$ GeV, where instead
one expects the rates to be $\Gamma_{\mu_{\text{ACC}}} < 3 \cdot 10^{-5} \text{ m}^{-2} \cdot \text{yr}^{-1}$. The highest predicted detection rates of $10^{-1} \text{ m}^{-2} \cdot \text{yr}^{-1}$ are again correlated to a few models which are unaffected by coannihilation [42]. The SQC does not significantly contribute to reduction of the rates.

Figure 8 shows neutralino-proton scattering cross sections for the scalar (spin-independent) and axial (spin-dependent) interactions. As in the previous figures, the models with $m_\chi \leq 250 \text{ GeV}$ are hardly affected by coannihilation, and for the majority of those models both neutralino-proton and neutralino-neutron scattering cross sections reach values $\sigma_{\chi p} \leq 10^{-17} \text{ GeV}^{-2}$ with the maximal cross section of the order of $10^{-15} \text{ GeV}^{-2}$.

Cosmologically interesting models with $m_\chi \geq 250 \text{ GeV}$ were influenced by coannihilations in the way discussed above, and the maximal value of the neutralino-nucleon cross section decreases from $10^{-15} \text{ GeV}^{-2}$ to $10^{-16} \text{ GeV}^{-2}$ for the models with $m_\chi > 500 \text{ GeV}$. All in all independently of neglecting or including of NCC, SQC and SLC, the maximal scalar scattering neutralino-nucleon cross section was large as $10^{-15} - 10^{-14} \text{ GeV}^{-2}$.

The spin-dependent neutralino-nucleon cross sections are typically higher than the spin-independent ones, and we have found the maximal values $10^{-10} \text{ GeV}^{-2}$ for the axial neutralino-proton and $10^{-11} \text{ GeV}^{-2}$ for the axial neutralino-neutron scattering for the models which are unaffected by the coannihilations. The majority of cosmologically interesting models yield axial neutralino-proton scattering cross sections in the range $5 \cdot 10^{-16} \text{ GeV}^{-2} < \sigma_{\chi p} < 2 \cdot 10^{-12} \text{ GeV}^{-2}$ and axial neutralino-neutron scattering cross sections in the range $2 \cdot 10^{-16} \text{ GeV}^{-2} < \sigma_{\chi n} < 8 \cdot 10^{-13} \text{ GeV}^{-2}$ [42]. The SQC contributes to reduction of the cross sections, but not significantly again.

Due to the fact that capture of $\chi$ in the Sun (contrary to the Earth) occurs also via spin-dependent $\chi p$ interaction, there are noticeable correlations between the highest upgoing muon rates from $\chi\chi$ annihilation in the Sun and the highest values of the axial neutralino-proton scattering cross sections (Figs. 7b and 8b)).

Figure 9 shows the expected direct detection event rates calculated for a $^{73}$Ge detector when NCC, SQC, SLC, and ACC are taken into account. For models with $m_\chi \leq 250 \text{ GeV}$ coannihilations of any kind play no role. The estimations of the event rate for models with $m_\chi \geq 400 \text{ GeV}$ are decreased (to about 0.005 event/(kg day)) due to NCC [42].
IV. CONCLUSION

Due to continuing improvements of the accuracy of astrophysical data and the importance of relic density constraints for SUSY models the precision calculation of the neutralino relic density is very desirable. The progress in this direction is very fast. Recently a new sophisticated C code micrOMEGAs on the basis of CompHEP [52] for calculations of the relic density in the MSSM has been presented [27]. It includes all coannihilation channels with neutralinos, charginos, sleptons, squarks and gluinos. The relic density of neutralinos in the mSUGRA was calculated on the basis of annihilation diagrams involving sleptons, charginos, neutralinos and third generation squarks in [37]. This paper is mostly devoted to prospects for SUSY search with various $e^+e^-$ and hadron colliders and pays no attention to the individual contributions of different coannihilation channels. In addition, a full set of exact, analytic expressions for the annihilation of the lightest neutralino pairs [38] as well as slepton-neutralino coannihilations [40] into all two-body tree-level final states in the framework of minimal SUSY is now available. The authors of the DarkSusy have also made new efforts [53] to include all possible channels of coannihilations in their public-available DarkSusy code.

Following this main direction we calculated the neutralino relic density (RD) taking into account slepton-neutralino (SLC), neutralino-chargino/neutralino (NCC), and squark-neutralino (SQC) coannihilation channels within the low-energy effective MSSM. To this end we have implemented in our code [43] the relic density part (with neutralino-chargino coannihilations) of the DarkSusy code [24] supplied with the adopted code of [26] (calculating slepton-neutralino coannihilations) and the generalized (including sbottom-neutralino coannihilations) code of [34] (calculating stop-neutralino coannihilations). With the help of this new code, in contrast with previous considerations, we pay attention to the interplay between different coannihilation channels in the effMSSM.

![FIG. 10. Neutralino-proton scattering cross sections for scalar (spin-independent) interaction. The notations are as in Fig. 7. Expectations for the GENIUS detector [54] and the annual-modulation region of DAMA (shaded region) [55] are also given. The maximal sensitivity of GENIUS and the region of DAMA are located at $40 \leq m_\chi \leq 300$ GeV.](image)

We have shown that in the effMSSM the maximum factors of RD decrease due to NCC as well as due to SQC and SLC can reach $10^{-3}$, as long as the lower experimental limits for $m_{\tilde{\tau}}, m_{\tilde{t}}, m_{\tilde{b}},$ and $m_{\tilde{\chi}^\pm}$, are of the order of 80 GeV. We conclude that SQC, NCC, SLC produce comparable RD reduction effects in the effMSSM. For the majority of models which are affected by coannihilations and which successfully passed all relevant accelerator, cosmological and rare-decay constraints it was observed that either NCC, SQC or SLC alone produces significant reduction of RD while the other coannihilation channels give considerably smaller or zero further reduction. Contrary to NCC and SLC, which produce non-negligible effects only if the NLSP mass is smaller than
1.15mχ, for SQC the relevant NLSP mass could reach 1.50mχ. The type of NLSP determines the dominant coannihilation channel [42]. Due to the fact that the effMSSM more often favors neutralinos and chargedinos, but not sfermions, to be the NLSP (the NLSP-LSP mass differences in general are systematically larger for sfermions than for gauginos), the NCC channel more often dominates in agreement with [23]. Only LSPs with the gaugino fraction P > 0.7 remain CDM candidates of cosmological interest.

Some models for which Ωh^2 > 0.3 when coannihilations is neglected fall within the cosmologically interesting region merely due to SLC and SQC, and some other models shift out of the region below Ωh^2 < 0.1 merely due to NCC. In the effMSSM, contrary to mSUGRA [26], all coannihilations do not impose new cosmological limits on the mass of the LSP [42]. The optimistic predictions for neutralino-nucleon cross sections, indirect and direct detection rates for cosmologically interesting models are almost unaffected by these coannihilations. Only for large mχ ≥ 400 GeV, the respectively high values are reduced, mainly because the NCC excludes the corresponding models from the cosmologically interesting region 0.1 < Ωh^2_{IGC} < 0.3.

Therefore, future increase of the lower mass limits for all possible NLSPs (at Tevatron or LHC) can, in principle, strongly reduce the importance of the effect of any of the coannihilation channels. Furthermore, we would like to note that despite obvious importance of sophisticated RD calculations including complete sets of coannihilation channels, it may happen that coannihilations play no role at least for direct detection of cold dark matter. From Fig. 10 one can see that the field of maximal sensitivity of the best new-generation CDM detectors, like GENIUS [54], as well as the annual-modulation region of DAMA [55] are located at 40 ≤ mχ ≤ 300 GeV, where coannihilation effects are almost invisible.

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