Ionization-recombination instability in a photo-ionized nebula

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Abstract. In this work, we consider the thermal (radiation) instability in a weakly ionized plasma with continuous ionization and recombination. The situation can be visualized in the case of envelopes of planetary nebulae, which are envelopes of ionized plasmas surrounding a red giant stars. Various observations report continuous photo-ionization of these plasmas by the high energetic streams of photons emanating from the parent star. Recently, it has been shown that thermal instability can be a probable candidate in such plasmas for existence of small scale structures (viz. striations) whose kinematic age is much smaller than the parent nebula. We therefore report a systematic study of these plasmas with photo-ionization and determine the instability domain. We have shown that in many cases the system bifurcates to an overstable (growing wave) state from a condensation instability (monotonic) and vice versa.

1. Introduction
Resolved H-II regions such as planetary nebulae have complex dynamical structures, the origin and evolution of which remains controversial. These are small condensations, the existence of which is not due to gravitation as their gravitational mass is far below that is required for self-gravitational condensations. Thermal instability is long thought to be a relevant astrophysical process which is responsible for existence of smaller, non-gravitational condensations such as those found in solar prominences, interstellar clouds, and planetary nebulae (PNe). Earlier works on thermal instability include those of Parker[1], Weymann[3], and Field[4], among which, Field’s seminal work on thermal instability is considered to be a comprehensive treatise on the subject till date. It was Parker[1], who first gave the physical reasoning leading to thermal instability in reference to the solar prominences. He argued that when the thermal balance of the medium is between the temperature-independent heating process and the temperature-dependent radiative losses, if by some mechanism in a certain area of the medium, the losses exceed the energy gain, the cooler-than-average area looses further energy and its temperature drops below the average ambient temperature of the medium. This process is known as thermal condensation (instability) which helps in understanding the existence of a cold and dense localized region in pressure equilibrium with its hot surroundings[4]. Some of the recent works have addressed the process of thermal instability in a gravitating medium undergoing expansion with [2] and without dusts [5]. Several other authors have considered occurrence of thermal instability in different contexts and parameter regimes. An analysis of the thermal instability of an optically thin plasma in the nonlinear regime is considered by Steele et. al. [6] Several recent works involving numerical simulations, address the issues of dense structure formation in a hot protogalactic environment.
[7, 8] and the interplay between thermal and magnetorotational instability (MRI) in interstellar media [9, 10].

The classical H-II regions such as the envelope surrounding one or several hot stars have intricate and inhomogeneous structures ranging from small-scale filaments to large globules. Highly resolved nebulae such as Eagle nebula (M16), Orion nebula (M42), and Helix nebula are such examples. They also suffer severe phoionization resulting from highly energetic photons emanating from the hot stars. These ionization may result in a ionization shock in many cases and some of the intricate structures are believed to be the result magnetohydrodynamic (MHD) activities of the ionization front (IF) [12, 11]. In this paper, we have coupled the ionizing radiative transfer and recombination to the magnetohydrodynamics (IRMHD) in a simplified model with any assumption to ionization and thermal equilibrium. This model captures the essential physics of thermal instability along with the IRMHD. Recently, Krumholz, Stone, and Gardiner [11] have considered the IRMHD, numerically, in case of giant molecular clouds (GMC), where they have considered the magnetic energy of the GMCs to be at par with the gravitational potential and kinetic energies [13, 14, 15].

In Section II, we describe our model equations with background cooling and expansion. In Section III, we apply the linear perturbation theory to examine the thermal condensation modes. In Section IV, we consider the case of static dust charge with the help of a WKB formalism and derive the linear dispersion relation. We numerically examine the dust-charge fluctuation dynamics in Section V. Finally, we summarise our observations in Section VI.

2. Physical model

We consider a weakly ionized plasma with a considerable presence of neutrals such as found in some planetary nebulae (PNe). As photo-ionization is ubiquitous most of the PNe, we have included the photo-ionization and recombination physics into the MHD equations. Below, we write down the equations for a collisional plasma in presence of neutrals. The equations are namely, the equations of continuity, momentum, and energy conservation for both electron and ions (the plasma) and the neutrals along with Maxwell’s equations in single-fluid MHD formalism,

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = \alpha_{\text{ion}} - \alpha_{\text{rec}},
\]

\[
\frac{\partial \rho_n}{\partial t} + \nabla \cdot (\rho_n \mathbf{v}_n) = -\alpha_{\text{ion}} + \alpha_{\text{rec}},
\]

\[
\rho \frac{d\mathbf{v}}{dt} = -\nabla p - \frac{1}{4\pi} (\mathbf{j} \times \mathbf{B}) - \rho \mu (\mathbf{v} - \mathbf{v}_n),
\]

\[
\rho_n \frac{d\mathbf{v}_n}{dt} = -\nabla p_n + \rho \mu (\mathbf{v} - \mathbf{v}_n),
\]

\[
\frac{1}{(\gamma - 1)} \frac{dp}{dt} + \frac{\gamma p}{(\gamma - 1)} (\nabla \cdot \mathbf{v}) = \nabla \cdot (\chi \nabla T) - \rho \mathcal{L} (\rho, T) - \mu \left( p - \frac{\rho}{\rho_n} p_n \right),
\]

\[
\frac{1}{(\gamma_n - 1)} \frac{dp_n}{dt} + \frac{\gamma_n p_n}{(\gamma_n - 1)} (\nabla \cdot \mathbf{v}_n) = \mu \left( p - \frac{\rho_n}{\rho} p_n \right),
\]

The subscript ‘n’ refers to the neutrals and the \( \alpha \)s are the ionization and recombination terms,

\[
\alpha_{\text{ion}} = \xi \rho_n \phi, \quad \alpha_{\text{rec}} = \rho \tau_L^{-1},
\]

where \( \phi \) is the photon flux, \( \xi \) is the photo-ionization cross-section, and \( \tau_L \) is the ion loss-time due to recombination. The quantity \( \mu \) denotes the collision frequency of the ions with the
neutrals and the generalized heat-loss function $L(\rho, T)$ represents the total energy loss (energy loss minus energy gain) per unit mass per unit time, exclusive of thermal conduction\[4\]. Here, we have assumed that the thermal radiation from the neutrals is very negligible and the thermal conductivity is mainly due to the electrons (plasma). The other symbols have their usual meanings.

2.1. Photo-ionization and Recombination
We consider the photo-ionization and recombination of a pure hydrogen nebula. A classic example of such a nebula is the Rosette nebula with an outer radius of $\sim 8$ pc, which is powered by some 31 stars with spectral classifications ranging from O-4 to B-3.

2.1.1. Photo-ionization
In order to estimate the photo-ionization rate, we adopt the on-the-spot approximation \[16, 11\] where any recombination of the hydrogen atom to the ground state is assumed to release an ionizing photon with a short mean free path. We also assume that recombinations to the excited states of the hydrogen atom release photons with long enough mean free path so that they can escape the plasma. The basic photo-ionization reaction is,

$$H + h\nu \rightarrow H^+ + e,$$

where a hydrogen atom absorbs an UV photon with energy $h\nu > 13.6$ eV causing the electron to be free (photo-ionization) with kinetic energy of $(h\nu - 13.6$ eV). Though the photo-ionization cross-section $\xi(\nu)$ can calculated exactly, the process is complicated. To a good degree of approximation we can write \[17, 11\],

$$\xi(\nu) \approx 6.3 \times 10^{-18} \left(\frac{\nu_{\text{ion}}}{\nu}\right)^3 \text{cm}^2,$$

where $\nu_{\text{ion}} = 3.29 \times 10^{15}$ Hz. The photo-ionization rate, $\tau_{\text{I}}^{-1}$ can be written as,

$$\tau_{\text{I}}^{-1} = \int_{\nu_{\text{ion}}}^{\infty} \frac{4\pi J_\nu}{h\nu} \xi(\nu) d\nu,$$

where $J_\nu$ is the mean specific intensity of the ionizing photons, which can be defined as,

$$4\pi J_\nu = \left(\frac{R_*}{R}\right)^2 \pi F_\nu = \frac{L_\nu}{4\pi R^2},$$

at a mean distance $R$ from the central star of radius $R_*$ of the PNe with $\pi F_\nu$ as the surface flux and $L_\nu$ as the luminosity of the star. As an example, the photo-ionization rate $\tau_{\text{I}}^{-1}$ of a PNe with $n_H \sim 10$ cm$^{-3}$, at a distance of $R = 5$ pc from the central star can be calculated to be $\approx 10^{-8}$ s$^{-1}$ for a O-6 star of effective surface temperature of $T_* = 40,000$ K, for which the photon flux is given by,

$$4\pi R^2 \phi = \int_{\nu_{\text{ion}}}^{\infty} \frac{L_\nu}{h\nu} d\nu \sim 5 \times 10^{48} \text{ photon s}^{-1}.$$

2.1.2. Recombination
Radiative recombination is the inverse process of photo-ionization,

$$H^+ + e \rightarrow H + h\nu.$$

With the on-the-spot approximation, the recombination rate $\tau_{\text{L}}^{-1}$ is thus proportional to $n_{H^+}n_e$ [16],

$$\tau_{\text{L}}^{-1} = \mu_H \alpha_H n_e n_{H^+},$$
where $\mu_H \approx 2.34 \times 10^{-24}$ g is the mean gas mass per hydrogen atom, assuming a standard cosmic abundance, $n_{e, H^+}$ are the number densities of electron and hydrogen nuclei, and $\alpha_H$ is the recombination coefficient, which can be evaluated using a fitting formula [16, 18],

$$\alpha_H \sim 4 \times 10^{-13} \left( \frac{10^4 \text{K}}{T} \right)^{0.73} \text{cm}^3\text{s}^{-1}.$$

The recombination (loss) time, is then given by,

$$\tau_L = \frac{1}{n_e \alpha_H} \approx \frac{3 \times 10^{12}}{n_e} \text{s} \approx \frac{10^5}{n_e} \text{yr},$$

so that for, say, $n_{H^+} \sim 10 \text{ cm}^{-3}$ and $n_e \sim 10^4 \text{ cm}^{-3}$, the recombination rate, $\tau_L^{-1} \sim 3.2 \times 10^{-9} \text{ s}^{-1}$.

### 2.2. Heating and Cooling

In a photo-ionized nebula, the heating and cooling are mainly due to ionization and recombination. At high collision energies, heating due to ionization of hydrogen can be determined from the Lotz formula,

$$q(T) = 5.85 \times 10^{-11} T^{1/2} e^{-157809/T} \text{cm}^3\text{s}^{-1},$$

where $q(T)$ is the heating rate coefficient. Heating rate due to photo-ionization of hydrogen can be written as,

$$G_{\text{ph}} = n \int_{\nu_{\text{ion}}}^{\infty} 4\pi J_\nu \left( \frac{\nu - \nu_{\text{ion}}}{\nu} \right) \xi(\nu) d\nu.$$  

On the other hand, cooling is normally proceeds collisional excitations followed by photon emissions, where the photons escape. The cooling rate for hydrogen recombination can be written as,

$$L_{\text{rec}} = n_e n_{H^+} kT \tau_L^{-1},$$

where the ‘bar’ denotes the average over the kinetic energy. A thermal balance occur when $G_{\text{ph}} = L_{\text{rec}}$. Additional cooling can occur due to free-free transition,

$$L_{\text{FF}} = 1.42 \times 10^{-27} T^{1/2} g_{\text{FF}} n_e n_{H^+} \text{erg cm}^{-3}\text{s}^{-1},$$

where $g_{\text{FF}} \approx 1$ is the Gaunt factor. From the cooling curves for hydrogen nebula, a thermal balance at $n_e \sim 10^4 \text{ cm}^{-3}$ occur at about 14,000 K.

### 3. Dispersion Calculations

We use a small perturbation of the form $\sim \exp(\sigma t + i \mathbf{k} \cdot \mathbf{r})$, so that the linearized equations can be written as,

$$\sigma p_1 + i \rho_0 (k \cdot v_1) = \xi \rho_{n1} \phi - \rho_1 \tau_L^{-1},$$

$$\sigma \rho_{n1} + i \rho_{n0} (k \cdot v_{n1}) = -\xi \rho_{n1} \phi + \rho_1 \tau_L^{-1},$$

$$\rho_0 \sigma v_1 = -ik p_1 + \frac{i}{4\pi} [B_1 (k \cdot B_0) - k (B_1 \cdot B_0)] - \rho_0 \mu (v_1 - v_{n1}),$$

$$\rho_{n0} \sigma v_{n1} = -ik p_{n1} + \rho_0 \mu (v_1 - v_{n1}),$$

$$\sigma B_1 = ik \times (v_1 \times B_0),$$

$$\frac{\sigma}{(\gamma - 1)} p_1 + \frac{i \gamma \rho_0}{(\gamma - 1)} (k \cdot v_1) = -k^2 \chi T_1 - \rho_0 L_{\chi} - \mu \left( p_1 - \rho_1 \rho_{n0} - \rho_0 \rho_{n0} p_1 + \rho_0 \rho_{n0} p_1 \right),$$

$$\frac{\sigma}{(\gamma n - 1)} p_{n1} + \frac{i \gamma n \rho_{n0}}{(\gamma n - 1)} (k \cdot v_{n1}) = \mu \left( p_1 - \rho_1 \rho_{n0} - \rho_0 \rho_{n0} p_1 + \frac{\rho_0 \rho_{n0}}{\rho_{n0}^2} \rho_{n1} \right).$$
In the above equations, the perturbed and equilibrium quantities are denoted with a subscript ‘1’ and ‘0’, respectively. These equations will be closed with the help of the linearized pressure equations,
\[ \frac{p_{1,n1}}{p_{0,n0}} = \frac{\rho_{1,n1}}{\rho_{0,n0}} + \frac{T_{1,n1}}{T_{0,n0}}, \]  
for both plasma and the neutral particles. In what follows, we shall assume \( \gamma_n \sim \gamma = 5/3 \).

To derive the linear dispersion relation, we first take a scalar product of \( \mathbf{k} \) with Eqs. (24-26) and eliminate the term \( (\mathbf{B}_1 \cdot \mathbf{B}_0) \). We then solve Eqs. (22) and (23) for \( (\mathbf{k} \cdot \mathbf{v}_{1,n1}) \). After eliminating the perturbed plasma temperature \( T_1 \) with the help of Eq. (29), we finally derive the dispersion relation,
\[
\begin{vmatrix}
D_{11} & c_s^2(\gamma - 1) + \mu \delta_p \delta_p^2 & \omega_L + \sigma + \mu(\gamma - 1) & -\mu(\gamma - 1)\delta_p \\
D_{21} & c_s^2[\tau_L^{-1} + \mu\gamma(\gamma - 1)] & -c_s^2[\mu \delta_p \gamma^{-1}(\gamma - 1) + \sigma + \tau_L^{-1}] & -\mu(\gamma - 1) & \sigma + \mu \delta_p(\gamma - 1) \\
\sigma(\mu + \tau_L^{-1}) - \mu \sigma \tau_L^{-1}(1 + \delta_p) & \sigma(\mu + \delta_p + \eta \gamma^{-1}) & 0 & k^2 \\
\end{vmatrix} = 0,
\]  
where
\[
D_{11} = c_s^2(\gamma - 1)(\omega_p - \omega_L) - \sigma - \tau_L^{-1} - \mu \delta_p \delta_p,
\]  
\[
D_{21} = \sigma(\mu + \tau_L^{-1} + \mu) + \omega_p^2(1 + \tau_L^{-1}/\sigma) + \mu \sigma \tau_L^{-1}(1 + \delta_p),
\]  
\[
D_{22} = -\sigma(\mu \delta_p + \tau_L^{-1}) + \tau_L^{-1}(\mu + \mu \delta_p + \omega_p^2/\sigma),
\]  
\[
D_{33} = k^2 + k^2(\hat{e}_k \cdot \hat{e}_B)^2 \frac{\omega_p^2}{\eta \sigma^2}(1 + \mu \delta_p/\sigma),
\]  
\[
D_{44} = k^2(\hat{e}_k \cdot \hat{e}_B)^2 \frac{\mu \omega_p^2}{\eta \sigma^2} \delta_p.
\]  
In the above expressions, \( \delta_p = \rho_p/\rho_{0n} \) is the ratio of the plasma density to the neutral density and \( \delta_p = (\gamma - 1)p_{0n}/\gamma p_0 \) is the ratio of the gas pressures. The photo-ionization rate \( \tau_L^{-1} = \xi \phi \) and \( \eta = 1 + \mu(1 + \delta_p)/\sigma \). Sound velocities corresponding to plasma and the neutrals are given by \( c_s,n = \gamma p_{0n}/\rho_{0n} \) and frequencies corresponding to density and temperature perturbations are expressed as \( \omega_{p,T} = k \nu_A \). The Alfvén frequency \( \omega_A = kv_A \), where \( \nu_A = B_0^2/4\pi \rho_0 \) is the Alfvén velocity. The quantity \( \omega_L = \omega_T + k^2 c_s/k_K \). The unit vectors along the respective vectors are denoted by \( \hat{e} \).

We have chosen to normalize the dispersion relation Eq. (30) with respect to the isothermal perturbation. So, the wave number \( k \), different frequencies \( \omega_s \), and velocities are normalized by \( k_p, k_p c_s, \) and \( c_s \), and the respective dimensionless quantities are denoted with a ‘*’. The dimensionless dispersion relation can now be written as,
\[
\begin{vmatrix}
D_{11} & \frac{\hat{\tau}_L^{-1} + \hat{\mu} \delta_p \delta_p^2}{3} & 3 \delta_p(\hat{\omega}_L + \hat{\sigma} + \frac{2}{3}\hat{\mu}) & -\hat{\mu} \delta_p^2 \delta_p \\
\delta_p(\hat{\tau}_L^{-1} + \frac{2}{3}\hat{\mu}) & -\frac{2}{5}\hat{\mu} \delta_p - \hat{\sigma} - \hat{\tau}_L^{-1} & -\frac{2}{5}\delta_p \hat{\mu} & \delta_p \delta_p \left(\frac{3}{2} \hat{\sigma} + \hat{\mu} \delta_p\right) \\
D_{31} & D_{32} & 0 & 3 \delta_p \delta_p k^2 \\
D_{41} & D_{42} & D_{43} & D_{44} \\
\end{vmatrix} = 0,
\]  
(36)
with

\[ D_{11} = \delta_p \left( \frac{3}{5} - \frac{3}{5}\hat{\omega}_L - \hat{\sigma} - \hat{\tau}^{-1} - \hat{\mu} \delta_p \delta_p \right), \]

\[ D_{31} = -\delta_p \left[ \hat{\sigma} (\hat{\mu} + \hat{\tau}^{-1}) + \hat{\mu} \hat{\tau}^{-1} (1 - \delta_p) \right], \]

\[ D_{32} = \hat{\sigma}^2 + \delta_p \hat{\sigma} \hat{\mu} + \hat{\sigma} \hat{\tau}^{-1} + \hat{\mu} \hat{\tau}^{-1} (1 + \delta_p), \]

\[ D_{41} = \delta_p (\hat{\sigma}^2 + \hat{\sigma} \hat{\tau}^{-1} + \hat{\sigma} \hat{\mu} + \hat{\omega}_A (1 + \hat{\tau}^{-1} \hat{\sigma}) + \hat{\mu} \hat{\tau}^{-1} (1 + \delta_p)), \]

\[ D_{42} = -\hat{\sigma} \hat{\mu} \delta_p - \hat{\tau}^{-1} (\hat{\mu} \hat{\delta}_p \hat{\omega} + \hat{\sigma}), \]

\[ D_{43} = \frac{3}{5} \delta_p \left( k^2 + \hat{k}^2 \cos^2 \theta \hat{\omega}_A^2 \frac{1 + \delta_p \hat{\mu} / \hat{\sigma}}{\hat{\sigma}^2 [1 + \hat{\mu} (1 + \delta_p) / \hat{\sigma}]} \right), \]

\[ D_{44} = \frac{3}{2} \delta_p^2 \delta_p \hat{k}^2 \cos^2 \theta \hat{\omega}_A^2 \frac{\hat{\mu}}{\hat{\sigma}^2 [1 + \hat{\mu} (1 + \delta_p) / \hat{\sigma}]}, \]

### 3.1. Collisionless limit

#### 3.1.1. Without magnetic field \((\mu = 0, B = 0)\)

We consider the simplest case of collisionless plasma \((\mu = 0)\) in the absence of any ambient magnetic field \((B = 0)\). In this limit, the dispersion relation Eq.(36) becomes a six-order polynomial which decouples into simple acoustic mode,

\[ \sigma = \pm i k c_s, \]

and a fourth-order polynomial,

\[ \hat{\sigma}^4 + [(\hat{\tau}^{-1} L + \hat{\tau}^{-1} I) + (\alpha + \beta k^2)] \hat{\sigma}^3 + [\hat{k}^2 + (\hat{\tau}^{-1} L + \hat{\tau}^{-1} I) (\alpha + \beta k^2)] \hat{\sigma}^2 \\
+ \hat{k}^2 \left[ \frac{3}{5} (\alpha + \beta k^2) + \hat{\tau}^{-1} - \hat{\tau}^{-1} + \frac{3}{5} \hat{\sigma} + \frac{3}{5} \hat{k}^2 \right] \hat{\sigma} + \frac{3}{5} \hat{k}^2 (\alpha + \beta k^2 - 1) = 0. \]

The cut-off wave number for a growing condensation mode is given by,

\[ k < k_c = \sqrt{\frac{1}{\beta} (1 - \alpha)}, \]

which is same as the Field’s criterion for \(\tau^{-1} L = \tau^{-1} I = 0\). It is also easily understood as the basic criterion for thermal instability due to radiation remains unaffected by the photo-ionization and recombination. However, as more and more ions recombine, it increases the ambient cooling rate of the plasma and for finite recombination rate, we expect the growth rate of the condensation mode to decrease as the temperature difference between the cooling region and the surrounding becomes less. This can be confirmed by looking at Fig.1, where we show the growth rates of the condensation mode with wave number for different recombination rates. We also plot the maximum growth rate of the mode with different ratio of the recombination to the phot-ionization rate which conforms to our explanation.

#### 3.1.2. With magnetic field \((\mu = 0, B \neq 0)\)

In presence of ambient magnetic the linear dispersion relation reduces to a sixth degree polynomial, the sign of the last term of which determines the cut-off for the condensation mode,

\[ \frac{3}{5} \hat{\tau}^{-1} L \hat{k}^4 \hat{\omega}_A \cos^2 \theta (\alpha - 1 + \hat{k}^2 \beta). \]
Figure 1. (a) The growth rates of the condensation mode are shown with respect to the dimensionless wave number. Values of $\alpha$ and $\beta$ are respectively 0.45 and $3.75 \times 10^{-3}$. As can be seen, the cut-off wave number remains same for different levels of ionization and recombination. (b) The maximum growth rate is shown against the ration of recombination and ionization rates.

Figure 2. (a) Growth rates of condensation mode (solid) and acoustic modes (dashed) for parallel ($\theta = 0$) and perpendicular ($\theta = \pi/2$) propagation.

As we can see that for parallel propagation, Field’s criterion remain unchanged. However, in presence of ionization, a new acoustic mode comes into existence (see the first panel of Fig.2). As can be seen from the figure, the condensation mode is unaffected by ionization-recombination.

For perpendicular propagation, the dispersion relation reduces to a fifth order polynomial. In this case, the condensation mode does not have cut-off and as shown in the second panel of Fig.2, there is a bifurcation of the perturbed modes.

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References
[1] E. N. Parker, Astrophys. J. 117, 431 (1953).
[2] M. P. Bora and M. Buzar Baruah, Phy. Plasmas (2008).
[3] R. Weymann, Astrophys. J. 132, 452 (1960).
[4] G. B. Field, Astrophys. J. 142, 531 (1965).
[5] A. J. Gomez-Pelaez and F. Moreno-Insertis, Astrophys. J. 569, 766 (2002).
[6] C. D. C. Steele, M. H. Ibez, and E. Sira, Phys. Plasmas 7, 3781 (2000).
[7] C. H. Baek, H. Kang, J. Kim, and D. Ryu, Astrophys. J. 630, 689 (2005).
[8] C. H. Baek, D. Ryu, H. Kang, and J. Kim, Astrophys. J. 643, L83 (2006).
[9] R. A. Piontek and E. C. Ostriker, Astrophys. J. 601, 905 (2004).
[10] R. A. Piontek and E. C. Ostriker, Astrophys. J. 629, 849 (2005).
[11] Mark R. Krumholz1, James M. Stone, and Thomas A. Gardiner, ApJ (2007).
[12] R. J. R. Williams, RevMexAA (Serie de Conferencias), 15, 184-189 (2003).
[13] Crutcher, R. M. 1999, ApJ, 520, 706.
[14] Crutcher, R. M. 2005, in IAU Symposium 227: Massive Star Birth: A Crossroads of Astrophysics, ed. R. Cesaroni, M. Felli, E. Churchwell, & M. Walmsley, 98–107.
[15] Heiles, C. & Crutcher, M. 2005, Magnetic Fields in Diffuse HI and Molecular Clouds (Berlin: Springer), in press, astro-ph/0501550.
[16] Osterbrock, D. E. 1989, Astrophysics of gaseous nebulae and active galactic nuclei (University Science Books).
[17] Bertoldi and Draine 1996Bertoldi, F. and Draine, B. T. 1996, ApJ, 458, 222.
[18] Rijkhorst, E.-J., Plewa, T., Dubey, A., and Mellema, G. 2005, astro-ph/0505213.