Does a Fermi liquid on a half-filled Landau level have Pomeranchuk instabilities?

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We present a theory of spontaneous Fermi surface deformations for half-filled Landau levels (filling factors of the form \( \nu = 2n + 1/2 \)). We assume the half-filled level to be in a compressible, Fermi liquid state with a circular Fermi surface. The Landau level projection is incorporated via a modified effective electron-electron interaction and the resulting band structure is described within the Hartree-Fock approximation. We regulate the infrared divergences in the theory and probe the intrinsic tendency of the Fermi surface to deform through Pomeranchuk instabilities. We find that the corresponding susceptibility never diverges, though the system is asymptotically unstable in the \( n \to \infty \) limit.

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The discoveries of the integer quantum Hall effect (IQHE) \( \nu \) and the fractional quantum Hall effect (FQHE) \( \nu \) have stimulated many studies on the properties of two-dimensional (2D) strongly correlated electronic systems in a magnetic field. New ideas such as existence of fractionally charged quasiparticles \( \nu \) or the prediction of composite fermions \( \nu \) have broadened our understanding of nature and have profoundly affected physics and other sciences. Theories built on these ideas have also proven very reliable to explain unusual properties of strongly correlated electronic systems in the extreme quantum Hall regime at various filling factors.

Somewhat more poorly-understood are the half-filled states, in particular the anisotropic states in high Landau level-s (LL-s) \( \nu \). The issue of anisotropy was addressed theoretically quite early on \( \nu \). Moessner and Chalker \( \nu \) showed using Hartree-Fock theory that a striped charge density wave (CDW) prevails in the limit of very high LL-s if the interaction is of sufficiently short range relative to the magnetic length. In fact ample theoretical evidence of their existence has accumulated \( \nu \).

While a stripe CDW phase is sufficient to explain anisotropy, other, competing structures may lead to an anisotropic response as well. Indeed it was pointed out in Ref. \( \nu \) that, when the range of the effective interaction is comparable to the magnetic length, uniform states may win over the striped phase and, a few years later, it was shown \( \nu \) that melting of the stripes could lead to a nematic phase. In this state, translational symmetry is completely restored but the system remains anisotropic. Interestingly such smectic and nematic states can be regarded as the ‘missing links’ between the Wigner crystal and Fermi liquid states, based on a proposed general picture of strong correlations (for a brief overview, see \( \nu \)).

In order to explain the emergence of anisotropy in Fermi liquid states at filling factor \( \nu = 2n + 1/2 \) \( n \geq 2 \), there has been a surge of interest in the Pomeranchuk instability (PI) \( \nu \). Through such mechanism, a compressible Fermi liquid state (presumably the half-filled states in high LL-s) may "spontaneously" enter into an anisotropic nematic state characterized by a deformed Fermi surface. In this scenario anisotropy emerges at one-particle level. In fact, the wave function for the nematic state proposed in Ref. \( \nu \) consists of single-particle 2D plane-wave states that form an elliptical Fermi sea \( \nu \). It is also plausible to suggest that anisotropy may emerge at a two-particle level, too. In this second scenario, one would start with a broken rotational symmetry wave function that contains a suitable symmetry-breaking parameter in the two-particle correlation part of the wave function and a Slater determinant of 2D plane waves that form a standard circular Fermi sea \( \nu \).

In this Letter we test the first of the above two scenarios. Specifically, we address the question of formation of a nematic state (or its higher angular momentum generalizations) from the opposite direction in the phase diagram of Ref. \( \nu \), i.e. from the Fermi liquid side. Assuming a circular Fermi liquid ground state as a starting point, our \textit{modus operandi} is to derive an effective electron-electron interaction that takes into account the projection onto the relevant half-filled LL. We then investigate whether this circular Fermi surface is unstable to small, point group symmetry-breaking deformations that are the 2D counterpart of the three-dimensional (3D) PI \( \nu \).

As was discussed more generally in 3D, for several model effective interactions \( \nu \), key requirements to have a PI are: (i) a sharp feature in the interaction potential at some characteristic distance; and (ii) for this characteristic length to be larger than the separation distance between particles in the system. Indeed the range of the effective interaction potential that we derive is longer than the magnetic length. Moreover it has non-monotonic features that develop into a sharp kink at a particular distance that increases as the order of
the LL-s increases. On the other hand, the average distance between particles in a half-filled LL is fixed by the magentic length. Thus both of the above conditions are met asymptotically for sufficiently high LL-s. Testing the ensuing expectation of an intrinsic tendency to a PI for high half-filled LL is the main purpose of this work.

We consider filling factors of the form: \( \nu = 2n + \nu^* \) where \( n = 1, 2, \ldots \) is the index of the uppermost half-filled LL (\( \nu^* = 1/2 \)) assumed to be fully spin-polarized. According to the Halperin-Lee-Read theory \[18\], the \( N^* \) electrons of the half-filled LL form a 2D circular Fermi liquid state (with density \( \rho^* \)) that effectively sees no magnetic field. We have \( \nu^* = 2\pi l_0^2\rho^* \) where the magentic length \( l_0 = \sqrt{\hbar c/eB} \) represents the characteristic length scale in the problem. Since \( \nu^* = 1/2 \) and \( \rho^* = \pi k_F^2/(2\pi)^2 \), the radius of the circular 2D Fermi wave vector is \( k_F = 1/l_0 \). Assuming that kinetic energy is quenched the Hamiltonian, up to a constant, is

\[
\hat{H}_n = \frac{1}{2} \int d^2r \int d^2r' \rho_n(\vec{r}) v(|\vec{r} - \vec{r}'|) \rho_n(\vec{r}') ,
\]

where \( v(|\vec{r} - \vec{r}'|) = e^2/(4\pi \epsilon |\vec{r} - \vec{r}'|) \) is the Coulomb potential. For simplicity we will take the dielectric function \( \epsilon \) to be a constant. Here \( \rho_n(\vec{r}) = \Psi_n^\dagger(\vec{r})\Psi_n(\vec{r}) \) is the density operator. The quantum field operator, \( \Psi_n^\dagger(\vec{r}) \), creates a spinless electron in the \( n^{th} \) LL at position \( \vec{r} \).

It is well known that projection onto the \( n^{th} \) LL contained implicitly in the definition of the density operator \( \rho_n(\vec{r}) \) introduces highly non-trivial physics in the Hamiltonian in Eq. \[11\]. The key idea of our approach is to assume a uniform circular Fermi liquid state and investigate possible instabilities induced by the heavily renormalised interactions.

One can transform the density operator in 2D Fourier space as \( \rho_n(\vec{q}) = \int d^2r e^{i\vec{q}\cdot\vec{r}} \rho_n(\vec{r}) \) and rewrite Eq. \[11\] as

\[
\hat{H}_n = \frac{1}{2} \sum_{\vec{q}} v(\vec{q}) \rho_n(-\vec{q}) \rho_n(\vec{q}) \text{ where } v(\vec{q}) = 2\pi e^2/(4\pi\epsilon|\vec{q}|)
\]

is the 2D Fourier transform of the Coulomb interaction potential. Based on the Hamiltonian theory of Ref. \[19\] we can project the density onto the half-filled LL, which amounts to writing \( \rho_n(\vec{q}) = F_n(q)\overline{\rho}(\vec{q}) \) where \( \overline{\rho}(\vec{q}) \) is the projected density operator. Thus the Hamiltonian can be rewritten in 2D Fourier space as

\[
\hat{H}_n = \frac{1}{2} \sum_{\vec{q}} V_n(q) \overline{\rho}(-\vec{q}) \overline{\rho}(\vec{q}) ,
\]

where \( V_n(q) = F_n(q)^2 v(q) \) represents an effective interaction potential which in real space would be given by \( V_n(r) = \int d^2q/(2\pi)^2 e^{-i\vec{q}\cdot\vec{r}} V_n(q) \). The form factors are \( F_n(q) = L_n(q^2l_0^2/2) \) \( - q^2l_0^2/4 \) where the \( L_n(x) \)'s are Laguerre polynomials.

There are two consequences of the projection onto the half-filled Landau level implicit in the Hamiltonian \[20\]: firstly, the effective interaction potential \( V_n(r) \) is heavily renormalized when compared to the Coulomb interaction. Secondly, the projected density operators themselves have a non-trivial algebra. Here we wish to investigate whether the first of these two features may be sufficient to produce a PI instability in a putative Fermi liquid state. In line with this view, we assume that new fermion creation and annihilation operators \( \psi^\dagger, \psi \) can be introduced in such a way that the projected densities can be written as \( \rho(\vec{q}) = \sum_l \psi_l(\vec{k})^\dagger \psi(\vec{k} + \vec{q}) \) (a form that implies standard operator algebra). Here \( \psi_l(\vec{k}) \) creates a fermion in a plane wave state. One can then apply the mean-field theory of a PI in a 3D continuum in Ref. \[17\] (recently generalised to 2D \[20, 21\]). It starts with a Hartree-Fock ansatz for the ground state, \( |\bar{\varepsilon}\rangle = \prod_k \left( \Theta(\varepsilon_k) + \Theta(-\varepsilon_k) \psi^\dagger(\vec{k}) \right) |0\rangle \), where \( |0\rangle \) is the vacuum. Note that this ansatz is a homogeneous, itinerant state with an arbitrary dispersion relation \( \varepsilon_k \).

For the Hamiltonian in \[20\], this Slater determinant of plane waves affords a rudimentary description of the re-emergence of itinerancy when the kinetic energy has been completely quenched (note that, unlike Refs. \[17, 20, 21\], there is no ‘bare’ contribution to \( \varepsilon_k \) here - it all comes from interactions). The functional form of \( \varepsilon_k \) is our variational parameter. It determines which plane wave states are occupied \( \varepsilon_k \) and is found by minimization of \( \langle \hat{H}_n \rangle \). That yields a self-consistency equation giving the shape of the Fermi surface through \( \varepsilon_k \) = 0. PI-s are point group symmetry-breaking instabilities of the shape of the Fermi surface.

To find an instability equation, it is natural to split \( \varepsilon_k \) in two parts: one that preserves the continuous rotational symmetry of the plane and another one that may break it. We thus write \[21\]

\[
\varepsilon_k = \varepsilon_0(\vec{k}) - \Lambda_\perp(\vec{k}) \cos(l \theta_k),
\]

where \( \varepsilon_0(\vec{k}) \) is the symmetric component of the dispersion relation and \( l = 1, 2, 3, \ldots \) determines the symmetry of the instability. The condition of instability towards a small deformation of the Fermi surface is (ignoring the possibility of a first-order phase transition) \[20, 21\]

\[
V_l \geq \frac{4\pi \hbar v_F^2}{k_F^4},
\]

where \( V_l = 4\pi \int_0^\infty dr V_n(r) J_l(k_F r)^2 \) measures the strength of the electron-electron interaction in the channel with angular momentum \( l = 1, 2, 3, \ldots \). For \( l = 1 \), the PI corresponds to a rigid displacement of the Fermi surface in reciprocal space, without change of either shape or volume. This can never lead to a lowering of the energy in a Galilean-invariant system and therefore this instability cannot take place \[17, 21, 22\]. This also follows from the more explicit form of the instability equation derived below, Eq. \[7\]. On the other hand for \( l = 2, 3, 4 \ldots \) we could have PI-s corresponding to deformations of the Fermi surface possessing \( d \)-wave, \( f \)-wave,
g-wave... symmetry, respectively. The Fermi velocity in Eq. (4) is given by [21]:

\[ v_F^0 = \frac{k^0}{4\pi\hbar} V_1 , \]  

(6)

where we take into account the infinite bare mass \( m \rightarrow \infty \) of our Hamiltonian [see Eq. (2)]. Substituting this into the instability equation (4) yields

\[ V_1 - V_1 \geq 0 . \]  

(7)

The effective interaction can be written as

\[ V_n(r) = \frac{e^2}{4\pi \epsilon} \int_0^\infty dq J_0(q r) \left( J_n \left( \frac{q^2 l_n^2}{2} \right) \right)^2 \exp \left( -\frac{q^2 l_n^2}{2} \right) , \]  

(8)

where \( J_n(x) \) are Bessel functions and \( n = 1, 2, 3, \ldots \). We can write \( V_n(r) = (e^2/4\pi\epsilon l_0) v_n(\zeta) \) where \( \zeta = r/l_0 \) is the natural dimensionless distance. We calculated \( v_n(\zeta) \) exactly for several \( n \) (we do not display such expressions for the sake of brevity). The results are shown in Fig. 1 where we also plot an asymptotic expression for the effective interaction obtained in the limit of high LL-s: [9]

\[ v_n(\zeta) \sim \frac{1}{\zeta} \frac{4}{\zeta^2} \Re \left[ K \left( \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{2n}{\zeta^2}} \right)^2 \right] ; n \gg 1 . \]  

(9)

\( K(x) \) is the complete elliptic integral of the first kind. All interaction potentials tend asymptotically to the Coulomb interaction, \( v_n(\zeta) \sim 1/\zeta \) for large \( \zeta \). This leads to infrared divergences in the theory whose regulation is discussed below. On the other hand, the presence of a length scale in the problem (the magnetic length) is apparent at shorter distances where one notices that as \( n \) increases an increasingly sharp kink develops at the specific distance \( r_n = \zeta_n l_0 \) where

\[ \zeta_n = 2\sqrt{2n} . \]  

(10)

Note in particular that the ultraviolet divergence of the Coulomb interaction at short distance has been suppressed. Interestingly, a sharp feature in the interaction potential at a finite distance \( r_n \) suggests the possibility of a PI provided the dimensionless parameter \( r_n k_F \) is large enough [17]. Since \( r_n k_F = \zeta_n \), Eq. (10) implies an increased tendency towards a PI in high LL-s.

![FIG. 1: Interaction potentials calculated from Eq. (5) alongside the large-\( n \) [Eq. (9)] and large-\( \zeta \) (Coulomb) asymptotes.](image)

![FIG. 2: Dependence of \( I(n,l) \) in Eq. (11) on the cutoff \( \zeta_c \) for \( w = 1 \) (red solid curves), 2 (green long dash) and 4 (blue short dash) for (a) \( n = 1 \); (b) \( n = 2 \); and (c) \( n = 10 \) (note the different scales on the third plot). We employed the asymptotic expression in Eq. (9) to plot (c). The values of angular momentum \( l \) are as indicated.](image)
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dependence of the integral $I(n,l)$ in Eq. (11) evaluated numerically, on $n$. The angular momenta of the instabilities are $l = 2, 3, 4$ and 5 (solid, long-dashed, short-dashed and dotted lines, respectively). Circles represent values obtained using the effective potentials, $v_0(\zeta)$ for $n = 1, 2$ and 3. Diamonds were obtained using the asymptotic formula in Eq. (9). The inset shows the behaviour at very large $l > 10$.

FIG. 3: Dependence of the integral $I(n,l)$ in Eq. (11), evaluated numerically, on $n$. The angular momenta of the instabilities are $l = 2, 3, 4$ and 5 (solid, long-dashed, short-dashed and dotted lines, respectively). Circles represent values obtained using the effective potentials, $v_0(\zeta)$ for $n = 1, 2$ and 3. Diamonds were obtained using the asymptotic formula in Eq. (9). The inset shows the behaviour at very large $l > 10$.

The dependence of the converged values of $I(n,l)$ on $n$ and $l$ is shown in Fig. 3. We note that the PI condition is never met for $n = 1, 2$ and 3. Diamonds were obtained using the asymptotic formula in Eq. (9). The inset shows the behaviour at very large $l > 10$.

The susceptibility to a PI has a non-trivial dependence on the integral is independent of $\zeta_c$ as well as of the width $w$ of the cutoff. Taking the limit $\zeta_c \to \infty$ represents a convenient way to estimate the intrinsic tendency of the system to a PI which is independent of the form and value of the cutoff. Physically, such a cutoff may correspond to, for example, the finite thickness of the device. The crucial point is that the value of the integral $I(n,l)$ in Eq. (11) is an intrinsic feature of the system independent of the form, size and mechanism of the cutoff as long as $\zeta_c$ is much larger than the average separation between particles $\sim k_F^{-1} = l_0$, and $\zeta_c \gg \zeta_n$ where $\zeta_n$ is the distance where the kink of the effective potential in Fig. 3 appears. The dependence of the converged values of $I(n,l)$ on $n$ and $l$ is shown in Fig. 3. We note that the PI condition is never met for $\nu = 5/2, 9/2$ and $13/2$. The susceptibility to a PI has a non-trivial dependence on $n$ with a maximum for $l = 2$ at $n = 2$, while it monotonically increases for $l > 2$ up to $n = 3$. Also note that the system seems most susceptible to a PI in the $l = 2$ channel. From these results we conclude that the system never has an intrinsic PI (i.e. $I(n,2)$ is always $< 0$). On the other hand, for large $n$ we have $I(n,2) \propto 1/n^{0.54}$, which implies that the half-filled level is asymptotically unstable to a PI in the $n \to \infty$ limit.

In summary, we have investigated whether a compressible circular Fermi liquid state in half-filled high LL-s undergoes a phase transition to a non-circular anisotropic nematic phase through a PI. We use the Hamiltonian theory approach [19] to derive a properly LL projected effective electron-electron interaction from which itinerancy emerges at the mean-field level. We look for deformations of the Fermi surface by testing for an intrinsic divergence of the corresponding susceptibility. We find that the susceptibility towards a PI is increasingly large as we move to high LL-s and diverges in the $n \to \infty$ limit. The increased tendency towards a PI is a direct consequence of the length scale $r_n \sim \sqrt{2n}l_0$ present in the effective interaction.

As is well known, there is also a tendency towards stripe formation [4, 7] with whom the PI competes. In real systems, there is a second length scale, $r_c$, responsible for cutting off infrared divergences associated with the long-range nature of the Coulomb interaction. Evidently $r_n$ and $r_c$ would become comparable for large values of $n$. The intrinsic susceptibility to a PI is already quite high in this regime. Thus, even minor extrinsic effects (sample thickness, device configuration, etc.) associated with this second length scale may trigger such an instability. A calculation of the critical value of $n$ at which this transition might take place is beyond the scope of the present analysis. It would require taking device configuration into account as well as comparing the energy of any Fermi liquid state to those obtained for stripe configurations.

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[1] K. von Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. 45, 494 (1980).
[2] D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. 48, 1559 (1982).
[3] R. B. Laughlin, Phys. Rev. Lett. 50, 1395 (1983).
[4] J. K. Jain, Phys. Rev. Lett. 63, 199 (1989).
[5] M. P. Lilly, K. B. Cooper, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 82, 394 (1999).
[6] M. M. Fogler, A. A. Koulakov, and B. I. Shklovskii, Phys. Rev. B 54, 1853 (1996).
[7] R. Moessner and J. T. Chalker, Phys. Rev. B 54, 5006 (1996).
[8] N. Shibata and D. Yoshioka, Phys. Rev. Lett. 86, 5755 (2001).
[9] M. O. Goerbig, P. Lederer and C. Morais Smith, Phys. Rev. B 69, 115327 (2004).
[10] E. Fradkin and S. A. Kivelson, Phys. Rev. B 59, 8065 (1999).
[11] E. Fradkin, S. A. Kivelson, and V. Oganesyan, Science 315, 196 (2007).
[12] I. Ia. Pomeranchuk, JETP 35, 524 (1958).
[13] V. Oganesyan, S. A. Kivelson, and E. Fradkin, Phys. Rev. B 64, 195109 (2001).
[14] Q. M. Doan and E. Manousakis, Phys. Rev. B 75, 195433 (2007).
[15] O. Ciftja and C. Wexler, Phys. Rev. B 65, 205307 (2002).
[16] C. Wexler and O. Ciftja, Int. J. Mod. Phys. B 20, 747 (2006).
[17] J. Quintanilla and A. J. Schofield, Phys. Rev. B 74, 115126 (2006).
[18] B. I. Halperin, P. A. Lee, and N. Read, Phys. Rev. B 47, 7312 (1993).
[19] G. Murthy and R. Shankar, Rev. Mod. Phys. 75, 1101 (2003).
[20] J. Quintanilla, C. Hooley, B. J. Powell, A. J. Schofield, and M. Haque, Physica B 403, 1279 (2008).
[21] J. Quintanilla, M. Haque, and A. J. Schofield, Phys. Rev. B 78, 035131 (2008).
[22] P. Wolfle and A. Rosch, J. Low Temp. Phys. 147, 165 (2007).
[23] The strict $n \to \infty$ limit corresponds to zero magnetic field, where focusing on an isolated, half-filled LL is not justified. This result must therefore be interpreted as an enhanced tendency towards a Pomeranchuk instability as $n$ is increased.