Modeling of inertial-admixture accumulation zones in vortex ring-like flows by fully Lagrangian method

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Abstract. A new combined fully Lagrangian approach to numerical simulation of axially symmetric vortex ring-like flows (in the absence of swirl) is proposed. The method is applicable to simulation of unsteady viscous flows with a dilute admixture of non-colliding particles which do not affect the carrier phase. The novel approach is based on a modification and combination of two Lagrangian methods: the carrier phase parameters are calculated using the vortex method based on diffusion velocity and the dispersed phase parameters (including the concentration) are calculated using the full Lagrangian approach. The application of the method is illustrated by two examples of numerical calculations of transient two-phase vortex ring-like flows.

1. Introduction
Detailed modeling of admixture transport in complex vortex flows of gas-particle mixtures requires high-accuracy numerical methods to capture the specific features of such flows, e.g. discontinuities and singularities in the dispersed-phase concentration field, multiple intersections of particle trajectories, formation of ‘folds’ in the particle continuum, etc. The correct simulation of the flow features faces serious difficulties when conventional Eulerian or Eulerian/Lagrangian methods described in the literature are used. In the present study, we demonstrate the applicability of a new fully Lagrangian approach for meshless modeling of transient axisymmetric vortex flows of a gas-particle mixture with a viscous carrier phase. This combined method, proposed for plane and axisymmetric flows in [1, 2], is based on the modification and combination of two Lagrangian approaches: a vortex method for solving the Navier-Stokes (or Euler) equations for the carrier phase and the full Lagrangian method for calculating the parameters of the dispersed phase [3]. This combined approach reduces the problem of calculating all parameters of both phases to a high-order system of ordinary differential equations (ODE) and allows to avoid the cumbersome procedure of re-meshing the parameters from the Lagrangian to Eulerian grid, typical of standard commonly used Eulerian-Lagrangian approaches. Below we apply the combined fully Lagrangian approach to the numerical simulation of unsteady axisymmetric viscous vortex rings traveling through a cloud of inertial particles. The calculations demonstrate that the method makes it possible to calculate efficiently the particle concentration fields, including the regions of intersecting particle trajectories and local zones of particle accumulation.
2. Problem formulation

Unsteady axisymmetric (without a swirl) two-phase flows are considered using the one-way coupling two-fluid approach [4]. The carrier phase is assumed to be an incompressible viscous fluid (liquid or gas) with viscosity \( \mu \), density \( \rho \), and velocity \( \mathbf{v} \). The inertial dispersed phase consists of non-Brownian identical spherical inclusions (particles or droplets) of constant radii \( \sigma \) and masses \( m \). The volume fraction of the particles is very small, so the interparticle collisions are absent and the dispersed phase is treated as a compressible continuum with zero pressure. The particle continuum is described by the transient fields of velocity \( \mathbf{v}_s \) and particle number density \( n_s \). The interphase momentum exchange is described by the Stokes drag force exerted on a single sphere in a viscous fluid. An additional assumption of small mass concentration of the admixture allows to neglect the feedback effect of particles on the carrier phase and makes it possible to calculate the carrier-flow parameters ignoring the contribution of the particles.

We will apply the meshless fully Lagrangian method with the reference to two examples of modeling the time evolution of two-phase flows induced by the motion of one vortex ring (case I) and two colliding vortex rings (case II) of finite thickness in a viscous fluid at moderate Reynolds numbers (see figure 1). The cloud \( \Omega_p \) of dispersed admixture is initially located in the quiescent fluid. The following scales are used: the length scale is equal to the initial toroidal radius of the vortex ring \( R_0 \) (see figure 1), the time scale is equal to \( \Gamma^{-1} \) (\( \Gamma \) is the total circulation of one ring), the velocity scale is equal to \( \Gamma / R_0 \), then the Reynolds number is \( \text{Re} = \Gamma \rho / \mu \) and the global dimensionless circulation is equal to unity in the case I and zero in the case II. Particles are characterized by the dimensionless inertia parameter, i.e. the inverse Stokes number \( \beta = S t^{-1} = 6 \pi \sigma \mu R_0^2 / m \Gamma \) and the initial dimensional particle number density \( n_0 \), assumed to be uniform.

![Figure 1](image1.png)

Figure 1. Scheme of the flows: in the case I, one vortex ring (left); in the case II, two colliding vortex rings (right).

3. Combined fully Lagrangian method

To simulate transient viscous dispersed flows we apply the combined fully Lagrangian method, originally proposed in [2], which is based on the combination of a meshless vortex method for calculating axisymmetric flows of the viscous carrier phase and the full Lagrangian method for calculating the parameters of the dispersed phase [3].

A transient viscous incompressible carrier-phase flow in an unbounded space is calculated using Navier-Stokes equations in the “vorticity-velocity” variables. The transport equation for the vorticity is approximated by discrete Lagrangian vortex rings, with the vortex diffusion velocity calculated using the ideas of “smoothed-particle hydrodynamics” [5]. The velocity field is then restored from a discrete analog of the Biot-Savart integral. Then, the parameters of the dispersed phase are calculated at each time step, assuming that the parameters of the carrier phase are known. The momentum and mass balance equations of the dispersed admixture are written in Lagrangian coordinates and calculated by the special full Lagrangian approach for the dispersed phase [3], which makes it possible to find the particle concentration along chosen trajectories, including the regions of crossing particle...
trajectories. The combined approach is fully Lagrangian, it reduces the problem of calculating all parameters of both phases to a high-order system of ordinary differential equations (ODE). The ODE systems are solved using the second-order Runge-Kutta method. More details on the derivation and verification of the combined fully Lagrangian method can be found in [2].

4. Results of numerical modelling
Below we present some results of numerical modeling of the two-phase flows for Re = 100 and \( \beta = 1 \). The initial (at a small \( t_0 = 0.01 \)) dimensionless vorticity distribution in the rings was taken as in the Oseen vortices, which the cylindrical coordinates \((r, \varphi, z)\) for the cases I and II respectively reads:

\[
\omega_1(z,r) = \frac{Re}{4t_0} \exp \left[ -\frac{Re}{4t_0} \left( z^2 + (r-1)^2 \right) \right], \quad \omega_2(z,r) = \omega_1(z,r) - \omega_1(z-1,r)
\]

The circular domains of the initial non-zero vorticity were split into \( N \) vortex elements, which were constructed by splitting the equidistant annular elements by a given number of radial elements. The maximum number of the vortex elements in the calculations was as large as several thousand. Figure 2 shows the typical calculations of the velocity pattern formed in the flows with time and restored using the discrete Biot-Savart integral. With increase in time, the section areas of the vortex rings increase due to the vorticity diffusion. In the first case, the vortex ring under study, specified by Oseen’s initial vorticity distribution, travels along the \( z \)-axis with a gradually decreasing velocity. In case II, when the vortices approach sufficiently close the plane of symmetry at \( z = 0.5 \), they start to expand along this plane, with the toroidal radius being increased. In figure 3, for two subsequent instants of time, we present the distribution patterns of the vortex element centers (square dots 2) for the cases I and II.

![Figure 2.](image)

Figure 2. The typical velocity vector patterns in the vortex rings in the plane \((r, z)\) for the cases I (left) and II (right).

Further, we calculated the evolution of a cloud of inertial particles in the vortex flows under study. At the initial instant, the quiescent cloud of the uniform concentration occupied a disk-shaped region \( \Omega_p = \{(r, z): r \in [0, 1.5], z \in [0, 0.4]\} \).

In figure 3, we show the distribution patterns of the chosen Lagrangian particles of the dispersed phase (dots 1, the dot color corresponds to the value of the particle number density at the considered point of space). It is clear that, with time, the particle cloud is elongated and ‘coiled’ onto the vortices. Therewith, the localized high particle concentration zones on the edges of the vortices and zones devoid of particles inside the vortices are formed.

It should be noted that for fairly inertial particles the formation of ‘folds’ in the dispersed continuum is typical. It is associated with the intersections of particle trajectories. This effect is
illustrated in figure 4, where several Lagrangian surfaces are shown, which at the initial instant were flat surfaces \( z = \text{const} \) \((a), (c)\). In figures 4\((b)\) and 4\((d)\), we show the calculated distributions of the particle concentration at the leading and trailing Lagrangian surfaces. Clearly, the local particle concentration is highly non-uniform, it sharply increases with the development of high gradients at the points of the onset of a fold. In the regions of the formation of folds, the multi-valued velocity and concentration fields appear. The Lagrangian method developed makes it possible to model such multi-layer regions and to resolve the dispersed phase parameters (including the number density) with a controlled accuracy.

Figure 3. Locations of the cloud of inertial particles \((1, \text{the color corresponds to the particle number concentration})\) and vortex elements \((2)\) at \( t = 1 \) \((a, c)\) and \( t = 2 \) \((b, d)\) for the cases I \((a, b)\) and II \((c, d)\).

Conclusion
The time evolution of axisymmetric (without a swirl) two-phase (gas-particle) flows are simulated numerically using the new combined fully Lagrangian approach with the reference to two examples of vortex ring-like flows at moderate Reynolds numbers. The approach used in the paper is based on a combination of the full Lagrangian method for the dispersed phase, proposed in [3], and a mesh-free vortex blob method for the carrier phase. The problem of calculation of all parameters in both phases (including the particle number density) is reduced to the solution of a high-order system of ordinary differential equations, describing transient processes in both carrier and dispersed phases. The calculations of transient flows with vortex rings demonstrated the ability of the new method to cope
successfully with the problem of crossing particle trajectories, to reproduce the formation of local particle accumulation zones at the edges of the vortices, and the appearance of ‘folds’ in the particulate medium, which present serious difficulties for standard Eulerian-Lagrangian methods commonly used in the multiphase community.

![Lagrangian surfaces and particle concentration distributions](image)

**Figure 4.** Locations of the Lagrangian surfaces 1-4 and the distribution of the particle concentration on the leading (1) and trailing (4) Lagrangian surfaces for the case I at $t = 1.35$ (a), (b) and for the case II at $t = 1$ (c), (d).

**Acknowledgments**
The work was supported by the Russian Foundation for Basic Research (No. 17-01-00057).

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