SUCCESSIVE MERGING OF PLASMOIDS AND FRAGMENTATION IN A FLARE CURRENT SHEET AND THEIR X-RAY AND RADIO SIGNATURES

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ABSTRACT

Based on our recent MHD simulations, a conception of the successive merging of plasmoids and fragmentation in the current sheet in the standard flare model is presented. Then, using a 2.5-dimensional electromagnetic particle-in-cell model with free boundary conditions, these processes are modeled on the kinetic level of plasma description. We recognize the plasmoids that mutually interacted and finally merged into one large plasmoid. Between interacting plasmoids, additional plasmoids and current sheets on smaller spatial scales were formed, congruent with the fragmentation found in MHD simulations. During interactions (merging–coalescences) between the plasmoids, the electrons were very efficiently accelerated and heated. We find that after a series of such merging processes, the electrons in some regions reached the energies necessary for emission in the hard X-ray range. Considering these energetic electrons and assuming a plasma density of $10^9–10^{10}$ cm$^{-3}$ and a source volume equal to the 2007 December 31 flare, we compute the X-ray spectra as produced by the bremsstrahlung emission process. Comparing these spectra with observations, we think that these processes can explain the observed above-the-loop-top hard X-ray sources. Furthermore, we show that the process of fragmentation between two merging plasmoids can generate narrow-band dm-spikes. Formulae for schematic fractal reconnection structures are derived.

Key words: acceleration of particles – plasmas – Sun: flares

Online-only material: color figures

1. INTRODUCTION

It is commonly accepted that plasmoids play a very important role in magnetic field reconnection in solar flares. Their importance was recognized for the first time by Ohyama & Shibata (1998). In the 1992 October 5 flare observed via soft X-rays by the Yohkoh satellite, they analyzed the plasmoid that was ejected upward into the corona during the impulsive phase. Studying the same flare, Kliem et al. (2000) showed that this plasmoid ejected was associated with the drifting pulsating structure (DPS) on radio waves. They proposed a model of this radio emission that was further developed in the papers by Karlický et al. (2002), Karlický (2004), Karlický & Bártá (2007), Bártá et al. (2008a), and Karlický et al. (2010). In this model, in the current sheet, plasmoids are formed from tearing and coalescence processes. As shown by Drake et al. (2005, 2006), Hoshino (2005), Pritchett (2006, 2008), and Karlický (2008), during these processes electrons are very efficiently accelerated. The electrons are then trapped in plasmoids, where they generate Langmuir waves, which through a wave transformation produce the electromagnetic waves recorded on the radio spectrum as DPSs. Due to the limited range of plasma densities in a plasmoid, the DPS is generated in a limited range of frequencies. In the vertical current sheet, the plasmoids move upward or downward or even stay motionless, depending on the form of the surrounding magnetic field (Bárta et al. 2008a, 2008b). Due to a preponderance of divergent magnetic field lines in the upward direction, most plasmoids move upward, and corresponding DPSs drift toward lower frequencies. Nevertheless, in some cases the plasmoids move downward and even interact with the underlying flare arcade, as observed by Kolomanski & Karlický (2007) and Milligan et al. (2010).

Recently, Oka et al. (2010) studied electron acceleration by multi-island coalescence processes in the particle-in-cell (PIC) model with periodic boundary conditions. They found that the most effective acceleration process occurs during the coalescence of plasmoids (“anti-reconnection;” see also Pritchett 2008 and Karlický & Bártá 2007).

Furthermore, Shibata & Tanuma (2001) proposed that the current sheet, stretched by a rising magnetic rope, is fragmented into smaller and smaller plasmoids by the tearing-mode instability in narrower and narrower current sheets (cascading reconnection). Loureiro et al. (2007) and Uzdensky et al. (2010) further developed this research to propose the theory of chain plasmoid instability. Multi-scale magnetic islands have been observed in Earth’s magnetotail by Hoshino et al. (1994). In solar flares, a series of DPSs indicates the same processes (Karlický 2004). An advantage of this concept is that it explains how very narrow current sheets with high current densities (required for anomalous resistivity generation and fast reconnection) are generated. Moreover, many X-points in this model give sufficient volumes for particle acceleration.

In addition to the fragmentation described by Shibata & Tanuma (2001), Bártá et al. (2010a, 2010b) found new fragmentation in the region between two merging plasmoids using MHD simulations. This fragmentation is caused by the tearing-mode instability generated between interacting plasmoids in the current sheet, and this process is repeated on smaller and smaller spatial scales. This fragmentation is driven by the merging of plasmoids.

Considering the above-mentioned processes, in the present paper we focus our attention on two: (1) successive merging of plasmoids into a large plasmoid and (2) the fragmentation process between merging plasmoids. We selected these processes because we think that the successive merging of plasmoids can explain the above-the-loop-top hard X-ray source (as a large stationary plasmoid). On the other hand, the fragmentation can explain the narrow-band dm-spikes. Because both these
phenomena are generated by accelerated electrons, in the following simulations we use the PIC model instead of MHD models (e.g., BártA et al. 2008a, 2008b).

The above-the-loop-top hard X-ray sources constitute one of the most discussed topics in recent years. The most well-known example is that observed in the ~30–50 keV energy range by Masuda et al. (1994). However, such events are very rare (Tomczak 2001; Petrosian et al. 2002; Krucker & Lin 2008). Another very interesting example was recently published by Krucker et al. (2010). They presented a hard X-ray source (with energy up to ~80 keV) that was located 6 Mm above thermal flare loops. They derived the upper limits of the plasma density and source volume as \( n_e \sim 8 \times 10^9 \text{ cm}^{-3} \) and \( V \sim 8 \times 10^{26} \text{ cm}^3 \), respectively. A relatively low plasma density in such hard X-ray sources attracts the attention of scientists. Krucker et al. (2010) concluded that these hard X-ray sources have to be close to the acceleration region, the distribution function of electrons emitting hard X-rays is strongly non-thermal, or the plasma in the source is very hot (up to \( T_e \sim 200 \text{ MK} \)). Several ideas explaining these X-ray sources have been proposed, e.g., the magnetic or turbulent trapping and dense (collisionally thick) coronal sources (see Fletcher 1995; Jakimiec et al. 1998; Veronig & Brown 2004; Park & Fleishman 2010).

The narrow-band dm-spikes (further spikes) are among the most interesting radio bursts because of their exceptionally high-brightness temperatures \( (T_b \approx 10^{21} \text{ K}) \) and short durations \( \lesssim 0.1 \text{ s} \); see the review by Benz (1986). Their observational characteristics have been described in many papers (e.g., Slottje 1981; Karlický 1984; Fu et al. 1985; Stähli & Magun 1986; Benz et al. 1982; Zlobec & Karlický 1998; Mészárosóva et al. 2003). On the other hand, the theoretical models can be divided into two groups: (1) based on the plasma emission and acceleration processes (Kuijpers et al. 1981; Tajima et al. 1990; Wentzel 1991; BártA & Karlický 2001) and (2) based on the electron–cyclotron maser (Holman et al. 1980; Melrose & Dulk 1982; Vlahos & Sharma 1984; Wingée et al. 1988; Aschwanden 1990; Fleishman & Yastrebov 1994). To distinguish between these two types of models, polarization and the harmonic structures of the spikes have also been studied (Güdel 1990; Güdel & Zlobec 1991; Krucker & Benz 1994). Searching for a characteristic bandwidth of individual spikes, Karlický et al. (1996, 2000) found that the Fourier transform of the dynamic spectra of spikes has a power-law form with power-law indices close to \(-5/3\). Based on these results, BártA & Karlický (2001) proposed that the spikes are generated in turbulent reconnection outflows.

This paper is organized as follows. First, we present our model scenario and explain the successive merging and fragmentation processes. Then, using a 2.5-dimensional PIC model, we simulate these processes. The results are then used in the interpretation of the above-the-loop-top hard X-ray sources and narrow-band dm-spikes.

2. MODEL SCENARIO AND SIMULATION MODEL

Figure 1 shows our model scenario, which is based on the “standard” CSHKP (Carmichael, Sturrock, Hirayama, Kopp, and Pneumant) flare model (e.g., Magara et al. 1996 and references therein). In the central part of the current sheet, in agreement with Shibata & Tanuma (2001), we assume fragmentation of the current sheet (stretching–tearing fragmentation). Furthermore, based on the new results of BártA et al. (2010b), we propose that the reconnection plasma outflow (which is downward oriented) accumulates plasmoids in the region just above the flare arcade, where plasmoids can interact efficiently. We think that, in some cases, a large plasmoid can be formed here as a result of successive merging of plasmoids. On the other hand, between the merging plasmoids, new current sheets are formed in which additional (but on smaller and smaller spatial scales) plasmoids are generated (fragmentation between merging plasmoids). We can use the successive merging process to interpret the above-the-loop-top hard X-ray sources, while the second process promises to explain the narrow-band dm-spikes (see Sections 3.1 and 3.2).

For our simulations, we used a 2.5-dimensional (2D3V—2 spatial and 3 velocity components), fully relativistic electromagnetic PIC model (Saito & Sakai 2004; Karlický 2004). The system size is \( L_x \times L_y = 600 \Delta \times 4000 \Delta \), where \( \Delta (=1) \) is a grid size. In the initial state, the Harris current sheet is formed along the line \( x = 0 \Delta \), and its half-width is \( L = 10 \Delta \). In this first study, we consider the neutral current sheet, i.e., the guiding magnetic field \( B_z \) is zero. Thus, the initial magnetic field is

\[
\begin{align*}
& B = (B_x, B_y, B_z), \\
& B_y = B_0 \tanh(x/L), \\
& B_z = 0, B_x = 0.
\end{align*}
\]

The electron–proton plasma with the proton–electron mass ratio \( m_p/m_e = 16 \) is unrealistic but used here to shorten computations. Nevertheless, the electron mass is low enough to separate the dynamics of electrons and protons well. In each numerical cell located far from the current sheet, we initiated \( n_0 = 60 \) electrons and \( n_0 = 60 \) protons. In the current sheet, the initial number density was enhanced to keep the pressure equilibrium. The initial electron temperature was taken to be the same as the entire numerical box, \( T = 10 \text{ MK} \), and the temperature of protons and electrons was the same. The plasma frequency is \( \omega_{pe} \Delta t = 0.05 \) (\( \Delta t \) is the time step that equals 1), the electron Debye length is \( \lambda_D = 0.6 \Delta \), and the electron and proton inertial lengths are \( \delta_e = 10 \Delta \) and \( \delta_i = 40 \Delta \), respectively. To study successive coalescence processes among several plasmoids, we initiated a formation of 10 plasmoids along the current sheet using a cosine perturbation of the electric current density in the sheet, with the vector \( k = 2\pi \cdot 10/4000 = 0.0157 \Delta^{-1} \) and the amplitude corresponding to the current density \( J \) given by the magnetic field in the current sheet \( \mathbf{j} = \nabla \times \mathbf{B} \). We made computations with several initial values of the plasma \( \beta \) parameter. Here, we present the results with \( \beta = 0.07 \), using the free boundary conditions. All computations were performed on the parallel computer.
Figure 2. Global view of magnetic field lines and the corresponding current densities (reddish and blue areas) in the $x$-$y$ computational plane at six different times: at the initial state (a), at $\omega_{pe} t = 1800$ (b), at $\omega_{pe} t = 3500$ (c), at $\omega_{pe} t = 5000$ (d), at $\omega_{pe} t = 6500$ (e), and at $\omega_{pe} t = 8000$ (f). The reddish and blue areas represent current densities with the initial and opposite orientations of the electric current, respectively. The $x$ and $y$ coordinates are expressed in $\Delta$. The proton inertial length is $40\Delta$.

(A color version of this figure is available in the online journal.)

Figure 3. Time evolution of the mean electron energy in the entire numerical box, normalized to its initial mean energy (solid line). The dashed line is the time derivative of the electron energy evolution (in arbitrary units), showing times with maximum electron-energy gains.

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3. RESULTS

A global evolution of the magnetic field lines and the corresponding electric current densities in the system is shown in Figure 2. The reddish and blue areas represent current densities with the initial and opposite orientation of the electric current, respectively. When 10 small plasmoids were formed, at about $\omega_{pe} t = 1800$ (Figure 2(b)), they started to interact and merge into larger plasmoids. Due to the free boundaries used in the system and small asymmetries in the initiation, the plasmoids successively merged into one large plasmoid formed in the bottom part of the system (Figure 2(f)). During this process, the mean energy of electrons increases (Figure 3, full line) to its final value, which is about 6.5 times greater than the initial one. This increase varies with time (see the time derivative of the electron energy evolution in Figure 3, dashed line). As seen here, the maxima of electron-energy gains correspond to times in which the plasmoids mutually merge (compare with the merging of plasmoids presented in Figure 2). On the other hand, weak decreases in mean electron energy after plasmoid mergings are probably due to restructuring of electric currents and electric fields during these phases. Simultaneously with this evolution, the energy spectrum of electrons in the entire computational box extends to higher energies; see Figure 4. Furthermore, Figure 5 presents details of

Figure 4. Energy spectrum of electrons in the entire numerical box at three times: at the initial state (dashed line), at $\omega_{pe} t = 5000$ (dotted line), and at $\omega_{pe} t = 9000$ (solid line).
Figure 5. Detailed view of magnetic field lines in the \( x-y \) computational plane and distribution of numerical electrons (points) with energy greater than 40 keV at four different times: at \( \omega_{pet} = 5200 \) (a), at \( \omega_{pet} = 5600 \) (b), at \( \omega_{pet} = 8000 \) (c), and at \( \omega_{pet} = 9000 \) (d). Note that the upper and lower panels show different parts of the numerical box. The white cross, located in panel (d) at \( x = -100, y = 150 \), shows the region where the distribution functions and the X-ray spectra were computed (see Figures 6(b) and 7(b)). The \( x \) and \( y \) coordinates are expressed in \( \Delta \).

this evolution as well as the distribution of numerical electrons (points) with energies greater than 40 keV at four different times: at \( \omega_{pet} = 5200 \) (a), at \( \omega_{pet} = 5600 \) (b), at \( \omega_{pet} = 8000 \) (c), and at \( \omega_{pet} = 9000 \) (d). As can be seen here, in each merging process, electrons are very efficiently accelerated—the number of numerical electrons increases with time. Then, we computed the normalized (the maximum equals 1) electron (velocity) distribution functions in all three coordinates in the entire computational box (Figure 6(a)). For comparison, in Figure 6(a) the thermal distribution function with a temperature of 60 MK is added. It shows that these distribution functions have clear non-thermal tails. But, as shown by different densities of numerical electrons (points) in Figure 5, this distribution is not the same in the entire numerical box. It depends on the location and size of the region in which the distribution is computed. The most energetic electrons are accumulated between merging plasmoids. To show an example of the distribution functions in the hottest parts of the system, in Figure 5(d) we selected a region (see white cross = the center of the circle of the radius 70\( \Delta \)). In this region, we determined the distribution functions presented in Figure 6(b), which are close to the thermal one; see the fit of this function by the thick full line corresponding to the thermal distribution function with a temperature of 118.7 MK.

In summary, the presented figures show that the successive merging (coalescences) of the plasmoids increases the energy (and the temperature) of accelerated electrons. We found that at some regions and for short times during the coalescence process, the distribution functions deviate from the thermal ones, but very soon these distribution functions are thermalized (i.e., changed to the Maxwellian ones) by fast wave–particle processes (anomalous collisions).

3.1. Successive Merging of Plasmoids and the Above-the-loop-top Hard X-ray Sources

Considering the electron distribution functions shown in Figures 6(a) and (b), we computed their hard X-ray spectra and compared them with that observed during the 2007 December 31 flare (Krucker et al. 2010). In accordance with
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Figure 6. Normalized electron distribution functions (thin solid line represents \( f(v_x) \), dashed line \( f(v_y) \), and dotted line \( f(v_z) \)) computed in the entire computational box at \( \omega_{pe}t = 9000 \) (a) and in the region shown by the white cross in Figure 5(d) at \( \omega_{pe}t = 9000 \) (b). The thick solid line in panel (a) represents the thermal distribution function with a temperature of 60 MK that was added to show non-thermal tails in distributions. The thick solid line in panel (b) represents the distribution function with a temperature of 118.7 MK, which roughly fits the computed distribution functions.

Figure 7. X-ray spectra (a) and (b) (the solid line is the source density \( n_e = 10^9 \text{ cm}^{-3} \) and the dashed line \( n_e = 10^{10} \text{ cm}^{-3} \)) corresponding to the distributions in Figures 6(a) and (b), computed by method (A). The X-ray spectra in panel (b) (the dot-dashed line is the source density \( n_e = 10^9 \text{ cm}^{-3} \) and the triple-dot-dashed line is \( n_e = 10^{10} \text{ cm}^{-3} \)) corresponding to the distribution in Figure 6(b), but computed by method (B) for the temperature 118.7 MK. For comparison, the X-ray spectrum (dotted line) observed during the 2007 December 31 flare (according to Krucker et al. 2010) is added.

For the computation of the hard X-ray spectra, we used two methods: (A) the non-thermal bremsstrahlung method, in which the spectrum was computed as a sum of contributions of the bremsstrahlung emission of all numerical electrons (for details, see relations (10) and (11) in the paper by Karlický & Kosugi 2004); and (B) the thermal bremsstrahlung method for specific plasma temperatures (Tandberg-Hanssen & Emslie 1988). The computed spectra and the observed spectrum are shown in Figures 7(a) and (b). The spectra computed by methods (A) and (B) (Figure 7(b)) are similar. The observed spectrum is between the spectrum computed for the source plasma density \( 10^9 \text{ cm}^{-3} \) and that for \( 10^{10} \text{ cm}^{-3} \), especially in later phases of the model evolution and in the localized region. But the observed spectrum has a different form than that of the computed spectra. Considering our results, we think the observed X-ray power-law spectrum is given by a sum of emissions from many locations with different thermal and non-thermal distribution functions.

3.2. Fragmentation between Merging Plasmoids and Narrow-band dm-spikes

In accordance with the results presented by Bártá et al. (2010b), we studied the structure of the magnetic field in the region between merging large plasmoids. One example of such a structure, formed at time \( \omega_{pe}t = 6300 \), is presented in Figure 8. The reddish areas represent the electric current densities with the initial orientation of the electric current, and the blue ones represent those with the opposite orientation. Red and green lines are positions of \( B_y = 0 \) and \( B_x = 0 \), respectively, and their intersections represent the X- and O-type null points. The figure shows that the initial current sheet is fragmented into several subcurrent sheets and plasmoids. This process is very dynamical: plasmoids appear and merge in very short time intervals; see, e.g., a formation of the plasmoid in the position \( x = -150 \Delta, \ y = 1280 \Delta \), where the \( B_x = 0 \) and \( B_y = 0 \) lines are nearly crossing. In the secondary current sheets, further tearing can exist if they are sufficiently long. Our results thus indicate that the fragmentation cascade seen in a large-scale MHD simulation (Bártá et al. 2010b) continues down to the dissipation scale of the order \( \approx d_i \) (the thickness of the current sheet at the bottom panel of Figure 8 is \( \approx 30 \Delta \)). At this scale, however, no further fragmentation has been observed, so the current structures dissipate “silently” by kinetic plasma processes.

Considering the structures of fragmented current sheets (cascade of plasmoids) obtained via numerical simulations (Figure 8), we constructed the schematic structure of fragmentation. An example of such a structure is shown in Figure 9. The circles indicate plasmoids with positively (+) and...
Figure 8. Fragmentation of the current sheet in the direction perpendicular to the initial current layer, i.e., in the current sheet between two large, interacting plasmoids, at $\omega_{pe} t = 6300$. The reddish areas represent the electric current densities with the initial orientation of the electric current, and the blue areas represent those with the opposite orientation. Red and green lines are positions of $B_y = 0$ and $B_x = 0$, respectively, and their intersections represent the X- and O-type null points.

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negatively (−) oriented electric currents, and the parallel lines indicate boundaries of current sheets. In such a structure, the radius of the plasmoid $R_i$ can be written as

$$R_i = AR_{i+1} + BR_{i+2},$$

where $A$ and $B$ are the number of plasmoids and current sheets in a specific current sheet and $i$ is the index of plasmoids with the same size. The number of plasmoids increases with $i$ as $n_i \sim B^i$. Then, the ratio between subsequent radii of plasmoids in the plasmoid cascade can be expressed by the infinite continued
fraction:

$$\frac{R_i}{R_{i+1}} = \left( A + \frac{B}{A + \frac{B}{A + \ldots}} \right).$$  \hspace{1cm} (3)

Now, assuming that the energy in the plasmoids is proportional to their area, the dependence of the function $E_i = R^2_i \times n_i$ on the k-vector scale ($k_i = 2\pi/R_i$) can be computed. This is the power-law function with the power-law index

$$p = \log(C \cdot B) \log R_i - 2,$$  \hspace{1cm} (4)

where $C$ is the free parameter expressing possible deviations from our assumption about plasmoid energy. For the structure presented in Figure 9 and for $C = 1$, this power-law index is $p = -1.54$.

In the present PIC simulation, due to scale limitations, we can only see the first level of the multi-scale fragmentation process. Therefore, comparing our PIC structure in Figure 8 to the proposed structure (relations (2) and (3)), the values of $A$ and $B$ can be only roughly estimated as $A = 2–3$ and $B = 2–4$. Nevertheless, we believe that in larger, future systems, more levels of fragmentation will be visible and the values of $A$ and $B$ could be more precisely determined.

The power-law dependence was also found in the analysis of the frequency bandwidth of the narrow-band dm-spikes (Karlický et al. 1996, 2000). Therefore, considering this fact, the model scenario shown in Figure 1, and the turbulent-plasma model of spikes by Bárt & Karlický (2001), we propose that the narrow-band dm-spikes are generated in fragmentation processes between merging plasmoids in the reconnection outflow in the region above the flare loop arcade. Although Bárt & Karlický (2001) supposed (silently) MHD/Alfvénic turbulence in the supersonic outflows, our recent simulations indicate that the tearing/coalescence cascade might be more likely a source of turbulence. However, in fully developed MHD turbulence, all plasma wave-modes are present. In the fragmentation region, the plasmoids of all spatial scales can interact and accelerate electrons. These electrons are trapped in plasmoids of different sizes. In each plasmoid, they can generate radio emission in the frequency range corresponding to the range of the plasma density in this plasmoid. Due to the expected power-law dependence of spatial scales on these fragmented plasmoids, the dependence of bandwidths on the resulting radio emission should be a power-law one, as observed in the narrow-band dm-spikes.

To support this idea, in Figure 10 we present the radio spectrum observed during the 2001 March 28 flare by two Ondřejov radiospectrographs (0.8–2.0 and 2.0–4.5 GHz; Jiřička et al. 1993). It shows two positive DPSs that, according to our previous studies, indicate two plasmoids moving downward to the sources of the narrow-band dm-spikes (generated in the region of fragmentation; compare to the model scenario in Figure 1). The mean Fourier spectrum of these spikes is a power law with power-law index $-1.5$.

4. DISCUSSION AND CONCLUSIONS

An important aspect of our model is that we used free boundary conditions, which enabled successive merging of small plasmoids into a large, final plasmoid. In our simulations, we also recognized fragmentation of a current sheet between two merging plasmoids.
We showed that these processes very efficiently accelerate electrons to energies required for emission in the hard X-ray range. Based on this result, we propose that the above-the-loop-top hard X-ray sources are produced by the successive merging of plasmoids in the region with turbulent reconnection outflow, just above the loop arcade. Computexray spectra support this idea. To explain the difference between the slopes of the computed and observed X-ray spectra, we propose that the observed power-law spectrum is the sum of emissions from many locations with different thermal and non-thermal distribution functions.

In our simulations, we used the PIC model. Although the studied processes are self-similar (i.e., they do not depend on scales; see, e.g., the MHD and PIC simulations in the paper by Tajima et al. 1987), the results need to be taken with caution, because the spatial and time scales in the model and real plasmoids differ by several orders of magnitude. On the other hand, it is beyond the ability of any present numerical model to take into account all these scales.

Another aspect of the PIC simulation is that the Coulomb collisions were not considered in our model. Namely, the time interval of our computations is much shorter than the collision time in the above-the-loop-top X-ray sources; e.g., \( \omega_{pe} \tau = 9000 \) for the plasma density \( n_e = 10^9 \text{ cm}^{-3} \) corresponds to \( 5.2 \times 10^{-6} \text{ s} \) and the collision time to \( 0.115 \tau_{Te/0.7}^2 \text{ s} \), where \( \tau_T = \tau_{Te}/10^7 \). On the other hand, the Coulomb collisions are essential for bremsstrahlung X-ray emission. The hard X-ray spectra in the 2007 December 31 flare were detected as the mean ones over intervals of several seconds. Furthermore, the above-the-loop-top hard X-ray source lasted for about 2 minutes. Thus, comparing these times with the collision time, the collision influences not only the observed spectra but also the duration of the X-ray source. Therefore, some re-acceleration of electrons is needed. In the present model, such a re-acceleration is very probable because the plasmoids are generated in a broad range of spatial scales and electrons can travel several times through regions of interacting plasmoids. Although including the Coulomb collisions, which would prolong computations, is beyond the ability of our present PIC model, we think that the observed spectra are formed by competition of the collisions with the re-acceleration of electrons. On the other hand, in the PIC model, the anomalous collisions (wave–particle interactions), which are much more effective than the Coulomb collisions, are present, as confirmed by fast thermalization of non-thermal distribution functions.

We made additional computations with different initial parameters and found that: (1) the energy gain of accelerated electrons increases with a decrease in the plasma beta parameter and (2) an increase in the proton–electron mass ratio \( m_p/m_e \) makes computations longer, but results are similar.

We compared the present simulation to that of the numerical model, whose size was two times smaller \( (L_x \times L_y = 600\Delta \times 2000\Delta) \) and in which only five plasmoids were initiated (contrary to 10 plasmoids in the present simulation). In the numerical model, the final energy of accelerated electrons in the hottest parts of the system was 5.3 times greater than the initial value. In our case, the final energy was 11.8 times greater from the initial temperature, 10 MK, to the final temperature, 118.7 MK, in the hottest parts of the system. Namely, each coalescence process increases the energy of accelerated electrons, and therefore the number of successive coalescence processes determines the final energy.

For calculations of the hard X-ray spectra, presented in Figure 7(b), we used two methods. The obtained results are similar. Small differences arise from varying methods and deviations of the computed distribution functions from the thermal one.

The plasmoids in two dimensions are in reality three-dimensional magnetic ropes. While in two dimensions the trapping of energetic electrons is a natural consequence of the close magnetic field structure of the plasmoid, in three dimensions this structure is only semi-closed. However, we consider the merging processes in the turbulent reconnection outflow; therefore, the magnetic trapping of electrons, similar to that proposed by Jakimiec et al. (1998), is highly probable. Moreover, the coalescence fragmentation process, which generates reverse electric currents (which in three dimensions have to be closed with a finite volume), will contribute to a full trapping of electrons.

In agreement with the conclusions of Krucker et al. (2010), in our model the acceleration region is very close to the hard X-ray source. It enables the re-acceleration of energetic electrons, which lose their energy from collisions. Acceleration regions are found in interacting plasmoids, and also between the plasmoids and the arcade of flaring loops. This model can explain not only the above-the-loop-top hard X-ray sources but also the loop-top sources, because the arcade of loops is, in principle, the “plasmoid” fixed in its half-height in the photosphere.

Considering all aspects of the fragmentation process (power-law spatial scales of plasmoids, effective acceleration of electrons, trapping of electrons in plasmoids, and location in the reconnection plasma outflow), we think that this process can explain the generation of the narrow-band dm-spikes. We supported this idea with the radio spectrum observed during 2001 March 28, which shows DPSs drifting toward the narrow-band dm-spikes. Furthermore, it is known that more than 70% of all groups of dm-spikes are observed during the GOES-rising-flare phases (Jiřička et al. 2001). Although these arguments support the presented idea, further analysis of the narrow-band dm-spikes and their modeling is necessary.

In this first study, we considered only the neutral current sheet, i.e., \( B_z = 0 \). For a more realistic description, we plan to extend our study to cases with a non-zero guiding magnetic field. Similarly to Oka et al. (2010), we found that the processes under study also accelerate protons. However, due to our focus on X-ray and radio emissions, we only studied the acceleration of electrons. The acceleration of protons during these processes will need further study, which we plan to do in the near future.

The presented model is a natural extension of our previous models that explain plasmoid formation, its ejection, and the corresponding DPS. Why do above-the-loop-top hard X-ray sources appear more rarely than DPSs or dm-spikes? We think that the above-the-loop-top hard X-ray source is a large and stationary plasmoid, which needs to be sufficiently dense and in which there is a sufficient amount of energetic electrons. The location of this stationary plasmoid is determined by the surrounding magnetic field and the location where this plasmoid begins to form; see the paper by Bártal et al. (2008b). On the other hand, the plasmoids generating DPSs or dm-spikes need not be so dense and have such an amount of energetic electrons. It is known that the number of energetic electrons needed to generate radio emission is much smaller than that for hard X-ray emission. Namely, the intensity of the radio emission depends on derivatives of the electron distribution function in the momentum space, not on the absolute amount of energetic electrons, as in the case of hard X-ray emission.
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