Quantisation of $O(N)$ invariant nonlinear sigma model in the Batalin-Tyutin formalism

N. Banerjee  
Saha Institute of Nuclear Physics  
1/AF, Bidhannagar, Calcutta 700064  
India

Subir Ghosh  
Gobardanga Hindu college  
North 24-Parganas, West Bengal, India

and  
R. Banerjee  
S. N. Bose National Center for Basic Sciences  
Sector I, DB 17, Bidhannagar  
Calcutta- 700064, India.

Abstract

We quantise the $O(N)$ nonlinear sigma model using the Batalin Tyutin (BT) approach of converting a second class system into first class. It is a nontrivial application of the BT method since the quantisation of this model by the conventional Dirac procedure suffers from operator ordering ambiguities. The first class constraints, the BRST Hamiltonian and the BRST charge are explicitly computed. The partition function is constructed and evaluated in the unitary gauge and a multiplier (ghost) dependent gauge.
1 Introduction

Over the last few years a method of generalised canonical quantisation of constrained dynamical systems has been developed by Fradkin and collaborators [1, 2] as an alternative to the pioneering formulation of Dirac [3]. This method [1,2], which has been reasonably well established for systems with first class constraints only, has been very recently extended to include systems with second class constraints [4,5]. We shall henceforth refer to this later method as the Batalin-Fradkin (BF) [4] and Batalin-Tyutin (BT) [5] schemes. The basic idea of this method is to convert the second class system into first class by extending the phase space and then to use the familiar machinery valid for first class systems [1,2]. While the BT [5] method remains unexplored (as far as quantisation of specific models is concerned), Some applications of the BF [4] formalism have been reported recently [6–9]. It is noteworthy, however, that these applications are confined to examples like the chiral gauge theories [6,7], the chiral boson theory [8], the massive Maxwell [7] and the massive Yang-Mills [9] theories. In all these models the Dirac brackets among the canonical variables are very simple (i.e. there are no operator ordering problems) and the quantisation can be, and indeed has earlier been [10,11], just carried out by the classical method of Dirac [3]. Such examples are, therefore, pedagogic exercises and do not reveal the complete power or flexibility of either the BF [4] or BT [5] approaches. Furthermore, the quantisation presented in ref. [7] is not a systematic application of the BF or BT method [4,5].

The motivation of this paper is to provide a non-trivial application of the BT procedure [5]. We shall consider the quantisation of the $O(N)$ invariant nonlinear sigma model. This model, which is a second class system, is known to have (quadratic) field dependent Dirac brackets among the canonical variables. Consequently quantisation by Dirac’s [3] procedure is riddled with operator ordering ambiguities. The conventional approach is to work in the configuration space functional integral formalism [11]. In this paper we show that an ambiguity free operator quantisation of the model can be performed by using the BT method [5] of converting the second class system into first class. The involutive (i.e. first class) Hamiltonian contains an infinite number of terms, although the number of additional (unphysical) fields introduced to extend the phase space is finite. A remarkable series of cancellations allows us to express this infinite series as a closed (exponential) form.
Operator ordering problems never arise since we always work in the canonical formalism. The phase space partition function is next constructed and explicitly evaluated for two different choices of gauge. In one case (the unitary gauge) the original (second class) theory is reproduced. In the other case (ghost dependent gauge) a non-trivial structure, which cannot be obtained by conventional [10] phase space approach, is obtained.

2 Quantisation

The $O(N)$ nonlinear sigma model consists of a multiplet of $N$ real scalar fields $n^a, a = 1, \ldots, N$ whose dynamics is governed by the Lagrangian,

$$L = \frac{1}{4}(\partial_\mu n^a)(\partial^\mu n^a)$$  \hspace{1cm} (1)

subjected to a primary constraint,

$$T_1 = n^a n^a - 1 = n^2 - 1 \approx 0.$$  \hspace{1cm} (2)

The canonical Hamiltonian, obtained by a formal Legendre transform from (1), is,

$$H_c = \Pi_a^2 - \frac{1}{4} \partial_i n^a \partial^i n^a$$  \hspace{1cm} (3)

where $\Pi_a$ is canonical momenta,

$$\Pi_a = \frac{\partial L}{\partial \dot{n}^a} = \frac{1}{2} \dot{n}^a.$$  \hspace{1cm} (4)

Secondary constraints, if present, are found by time conserving $T_1$ (2) with the total Hamiltonian [3],

$$H_T = \int dx [H_c + \lambda T_1]$$  \hspace{1cm} (5)

where $\lambda$ is a Lagrange multiplier. Indeed, there is a secondary constraint $T_2$,

$$T_2 = n^a \Pi_a \approx 0.$$  \hspace{1cm} (6)
The constraints $T_1$ and $T_2$ are second class, satisfying the Poisson algebra,
\[ \Delta_{\alpha\beta}(x,y) = \{T_\alpha(x), T_\beta(y)\} = -2\epsilon_{\alpha\beta} n^2 \delta(x-y); \quad \alpha, \beta = 1, 2 \quad (7) \]
and $\epsilon_{\alpha\beta}$ is the antisymmetric tensor normalised as $\epsilon_{12} = -\epsilon_{12} = -1$.

Time conserving $T_2$, consequently, does not yield a new constraint but fixes the multiplier $\lambda$ in (5),
\[ \lambda = \Pi^2 + \frac{1}{4} \partial_i n^a \partial^i n^a \quad (8) \]
so that the total Hamiltonian (5) becomes,
\[ H_T = \int \left[ n^2 \Pi^2 + \frac{1}{4} \partial_i n^a \partial^i n^a (n^2 - 2) \right]. \quad (9) \]

The involutive algebra of the constraints $T_\alpha$ with $H_T$,
\[ \{T_1(x), H_T\} = 4n^2 T_2 \]
\[ \{T_2(x), H_T\} = \frac{1}{4} \partial^i \{(T_1 - 1)\partial_i T_1\} - (\partial_i n^a)(\partial^i n^a)T_1 \quad (10) \]
clearly illustrate the nonlinear features. Indeed the unsystematic approach of ref. [7] becomes untenable due to this involved algebra (10).

The Hamiltonian (9) with the constraints $T_\alpha$ is the starting point of our analysis. The first step is to convert the second class constraints $T_\alpha$ into first class. In doing this we follow the prescription of ref. [5]. The new first class constraints $T'_\alpha$ are given by,
\[ T'_\alpha(n^a, \Pi_a, \phi^\alpha) = \sum_{n=0}^{\infty} T'_\alpha^{(n)}, \quad T'_\alpha^{(n)} \sim (\phi)^n \quad (11) \]
subject to the boundary condition,
\[ T'_\alpha^{(0)} = T'_\alpha(n^a, \Pi_a, 0) = T_\alpha \quad (12) \]
and where $\phi^\alpha$ are the new dynamical variables in the extended phase space $(n^a, \Pi_a) \oplus (\phi^\alpha)$ with the basic poisson algebra [5],
\[ \{\phi^\alpha(x), \phi^\beta(y)\} = \omega^{\alpha\beta}(x, y) \quad (13) \]
with $\omega$ being an antisymmetric invertible matrix,

$$\omega^{\alpha\beta}(x, y) = -\omega^{\beta\alpha}(y, x).$$  \hfill (14)

After (12), the next term in the series (11) is,

$$T^{(1)}_\alpha(x) = \int dy \ X_{\alpha\beta}(x, y) \phi^\beta(y)$$  \hfill (15)

where,

$$\int dz \ dz'[X_{\alpha\mu}(x, z)\omega^{\mu\nu}(z, z')X_{\nu\beta}(z', y)] = -\Delta_{\alpha\beta}(x, y)$$  \hfill (16)

with $\Delta_{\alpha\beta}(x, y)$ defined in (7). The other terms ($n > 1$) in (11) are obtained by a recursion relation [5]. As we shall presently see these are not needed in our analysis.

A possible choice for $\omega^{\alpha\beta}(x, y)$ and $X_{\alpha\beta}(x, y)$ satisfying (14) and (16) is

$$\omega^{\alpha\beta}(x, y) = 2\epsilon^{\alpha\beta} \delta(x - y)$$

$$X_{\alpha\beta}(x, y) = \begin{pmatrix} 1 & 0 \\ 0 & -n^2 \end{pmatrix} \delta(x - y).$$  \hfill (17)

There is a ‘natural arbitrariness’ [4,5] in this choice corresponding to canonical transformations in the extended phase space. The above choice is crucial for simplifying the subsequent algebra. Using (11), (12), (15) and (17), the final expressions for the new constraints are,

$$T'_1 = T_1 + \phi^1$$

$$T'_2 = T_2 - n^2 \phi^2$$  \hfill (18)

which are strongly involutive,

$$\left\{ T'_\alpha(x), T'_\beta(y) \right\} = 0$$  \hfill (19)

indicating that the terms in the series (11) for $n > 1$ are redundant. This completes the conversion of the second class constraints $T_\alpha$ to first class ones $T'_\alpha$. Since the original constraints $T_\alpha$ were in involution with the original Hamiltonian (see (10), the new constraints $T'_\alpha$ obviously violate this property. The next step is, therefore, to compute the new involutive Hamiltonian.
Following ref. [5], the general structure of this Hamiltonian can be expressed as a power series,

\[ H'(n^a, \Pi_a, \phi^a) = \sum_{n=0}^{\infty} H'^{(n)}(n^a, \Pi_a, \phi^a) \sim (\phi)^n \]  

subject to the boundary condition,

\[ H'^{(0)} = H'(n^a, \Pi_a, 0) = H_T \]

where the general expression for \( H'^{(n)} \) is [5]

\[ H'^{(n+1)} = -\frac{1}{n+1} \int dxdydz \left[ \phi^\mu(x) \omega_{\mu\nu}(x, y) X^{\nu\rho}(y, z) G_{\rho}^{(n)}(z) \right] (n \geq 0) \]  

The matrices \( \omega_{\mu\nu}(x, y) \) and \( X^{\nu\rho}(y, z) \) are the inverse matrices of \( \omega^{\mu\nu}(x, y) \) and \( X_{\nu\rho}(y, z) \) respectively, defined in (17). The generating functional \( G_{\rho}^{(n)} \) has the extremely simple form,

\[ G_{\rho}^{(0)} = \{ T_{\rho} , H_T \} \]

\[ G_{\rho}^{(n)} = \{ T_{\rho}^{(n)} , H'^{(n-1)} \}_{(n^a, \Pi_a)} + \{ T_{\rho} , H^{(n)} \}_{(n^a, \Pi_a)} \]  

the genesis of which is contained in the judicious choice (17) so that the series (11) involves only two terms \( T_\alpha \) and \( T_\alpha^{(1)} \). Indeed a glance at the general structure for \( G_{\rho}^{(n)} \) given in equation (2.54) of [5] would convince the reader of the remarkable algebraic simplification achieved in (23). The symbol \( \{ , \}_{(n^a, \Pi_a)} \) appearing there means that the Poisson bracket has to be evaluated with respect to \((n^a, \Pi_a)\). Using the expressions for the original constraints \( T_\alpha \) (2.6) and the Hamiltonian (9) as well as (21) to (23), it is possible to compute all the terms appearing in the power series (20). Contrary to (11), the series (20) turns out to be an infinite series. We find, however, that a chain of systematic cancellations occurs leading to the result,

\[ H' = H_T - 2 \int dx \phi^2 n^2 T_2 + \int dx \phi^2 (n^2)^2 + \sum_{p=1}^{\infty} H^{(p)} \]

where,

\[ H^{(p)} = \int dx_1 dx_2 \ldots dx_p \left[ \frac{(-1)^p}{p!} \left\{ \frac{2^1}{n^2}\right\}(x_1) \{ T_2(x_1) , \frac{1}{2}\left\{ \frac{2^1}{n^2}\right\}(x_2) \{ T_2(x_2) \ldots \right\} \right] \]

\[ + \frac{1}{2}\left\{ \frac{2^1}{n^2}\right\}(x_p) \{ T_2(x_p) , H_0 \} \}_{p-fold} \]  

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and,

\[ H_0 = H_T - \int dx \, n^2(x) \Pi^2(x) \]  \hspace{1cm} (26)

is a function of \( n^a \) fields only. A convenient way to express (25) is, therefore, to use the functional Schrödinger representation \((\Pi_a \rightarrow (-) \frac{\delta}{\delta n^a})\) so that,

\[ H^{(p)} = \frac{1}{p!} \int dx_1 \ldots dx_p \left[ \frac{\phi^1}{2n^2} \left\{ n^a \frac{\tilde{\delta}_L}{\delta n^a} \right\} \right]^p \Pi^2(x) H_0 \]  \hspace{1cm} (27)

where \( \tilde{\delta}_L \) indicates the left derivative. Combining (27) with (24) yields the final Hamiltonian,

\[ H' = \int n^2 \Pi^2 - 2 \int \phi^2 n^2 T_2 + \int \phi^2 n^2 \phi^2 (n^2)^2 + \int \exp \left[ \int \frac{\phi^1}{2n^2} \left\{ n^a \frac{\tilde{\delta}_L}{\delta n^a} \right\} \right] \Pi^2(x) H_0 \]  \hspace{1cm} (28)

which, by construction [5], is strongly involutive with the constraints \( T'_\alpha \):

\[ \{ H', T'_\alpha \} = 0 \]  \hspace{1cm} (29)

This completes the conversion of the second class system (with Hamiltonian \( H_T \) and constraints \( T_\alpha \)) into first class (with Hamiltonian \( H' \) and constraints \( T'_\alpha \)), and is one of the major accomplishments of our paper.

We now make some comments regarding the construction (28): (i) In passing to the quantum theory where Poisson brackets are replaced by commutators, the representation \( \frac{\tilde{\delta}_L}{\delta n^a} \) goes over to \((i\hbar) \frac{\tilde{\delta}_L}{\delta n^a}\), so that (28) is amenable to perturbation theory by just expanding the exponential; (ii) previous attempts [12] to construct an involutive Hamiltonian without extending the phase space led to a nonlocal form whereas (28) is local; (iii) since (28) is strongly involutive (see (29)) it implies [4] that the involutive Hamiltonian \( H' \) is identical to the BRST [13] invariant Hamiltonian. Thus,

\[ H' = H_{BRST} \]  \hspace{1cm} (30)

The BRST invariance is generated by the BRST charge \( Q \) which is given by

\[ Q = \int dx \left[ C^\alpha(x) T'_\alpha(x) + p_\alpha(x) P^\alpha(x) \right] ; \quad \alpha = 1, 2 \]  \hspace{1cm} (31)
where \((C^\alpha, \bar{P}_\beta)\) and \((P^\alpha, \bar{C}_\beta)\) form a canonical ghost (antighost) pair having the opposite Grassman parity as \(T_\alpha\);

\[
\{C^\alpha(x), \bar{P}_\beta(y)\} = \{P^\alpha(x), \bar{C}_\beta(y)\} = \delta^\alpha_\beta \delta(x-y) \tag{32}
\]

while \((p^\alpha, q^\beta)\) is a canonical multiplier pair with identical Grassman parity to \(T_\alpha\):

\[
\{q^\alpha(x), p_\beta(y)\} = \delta^\alpha_\beta \delta(x-y) \tag{33}
\]

The fields \(\bar{P}_\alpha, \bar{C}_\alpha, q_\alpha\) do not occur in (31) but will used later.

The BRST charge generates the following transformations \((\delta_Q \mathcal{O} = \{Q, \mathcal{O}\})\) on the canonical variables in the complete extended space,

\[
\begin{align*}
\delta_Q n^a &= -C^2 n^a \\
\delta_Q \Phi^1 &= 2C n^a \\
\delta_Q \Phi^2 &= 2C^1 n^a + C^2 (\Pi_a - 2n^a \phi^2) \\
\delta_Q \bar{P}_\alpha &= T'_\alpha \\
\delta_Q C_\alpha &= p_\alpha \\
\delta_Q q_\alpha &= -P_\alpha \\
\delta_Q p_\alpha &= 0 \\
\delta_Q \bar{C}_\alpha &= 0 \\
\delta_Q \bar{P}_\alpha &= 0
\end{align*} \tag{34}
\]

under which the BRST invariance of (30) can be explicitly verified. The nilpotency condition \(\delta^2_Q = 0\) is clearly preserved in (34). Finally, the physical space is defined by

\[
Q |\text{phys}\rangle = 0, \quad |\text{phys}\rangle \neq Q |\ldots\rangle \tag{35}
\]

This completes the operator formulation of the model. We next consider the partition function. The first step is to define the gauge fermion operator \(\psi\) given in ref.[4]

\[
\psi = \int dx \left[ P_\alpha q^\alpha + \bar{C}_\alpha \chi^\alpha \right] \tag{36}
\]

where \(P_\alpha, \bar{C}_\alpha, q^\alpha\) have been defined in (32,33) and \(\chi_\alpha\) is the hermitean gauge fixing function with identical Grassman parity as \(T_\alpha\) and satisfy,

\[
\text{det} |\{\chi_\alpha, T'_\beta\}| \neq 0 \tag{37}
\]

The complete unitarising Hamiltonian \(H_U\) is now defined as,

\[
H_U = H_{BRST} + \{\psi, Q\} \tag{38}
\]
which is also BRST invariant since the added term is a BRST total derivative \[14\]. We now rename the variables \(\phi^1\) and \(\phi^2\) as,
\[
\phi^1 \rightarrow 2\phi, \quad \phi^2 \rightarrow \Pi_{\phi}
\]  
so that \((\Pi_{\phi}, \phi)\) can be regarded as a canonically conjugate pair by virtue of (13) and (17). The partition function \(Z\) may now be written as,
\[
Z = \int [\mathcal{D}\mu] e^{iS} \tag{40}
\]
where,
\[
S = \int \left[ \Pi_\alpha \dot{n}^\alpha + \Pi_\phi \dot{\phi} + C^\alpha \dot{P}_\alpha + P^\alpha \dot{C}_\alpha + p_\alpha q^\alpha - H_U \right] \tag{41}
\]
and the measure \([\mathcal{D}\mu]\) includes all the variables occurring in the action.

Let us now explicitly compute \(Z\) in different gauges. First, consider the ‘unitary gauge’ \([4,5]\) where the gauge conditions are just the original set of second class constraints,
\[
\chi_\alpha = T_\alpha \tag{42}
\]
Making the change of variables \(\chi_\alpha \rightarrow \chi_\alpha / \beta\), \(p_\alpha \rightarrow \beta p_\alpha\), \(\bar{C}_\alpha \rightarrow \beta \bar{C}_\alpha\) whose (super) Jacobian is unity \([1, 14]\), and finally taking the limit \(\beta \rightarrow 0\) \([1, 14]\), we obtain,
\[
Z = \int [\mathcal{D}n^\alpha \mathcal{D}\Pi_\alpha \mathcal{D}\phi \mathcal{D}\Pi_\phi] \delta(T_1)\delta(T_2)\delta(\phi)\delta(\Pi_\phi) det | -2n^2| e^{iS} \tag{43}
\]
with,
\[
S = \int \left( \Pi_\alpha \dot{n}^\alpha + \Pi_\phi \dot{\phi} - n^2 \Pi^2 + 2\Pi_\phi n^2 T_2 - \Pi_\phi^2 (n^2)^2 \right) + \exp \left\{ \frac{\phi}{n^2} \frac{\delta}{\delta n^\alpha} \right\} H_0. \tag{44}
\]
Note that due to \(\delta(T_1)\) in (43) the Faddeev-Popov determinant reduces to a constant which can be absorbed in the normalisation. The \(\phi\) and \(\Pi_\phi\) integrals can be trivially performed. Finally \(\delta(T_2)\) is expressed by its corresponding Fourier transform leading to an action,
\[
S = -\Pi^2 + \Pi_\alpha (\dot{n}^\alpha - \xi n^\alpha) - \frac{1}{4} \partial_\mu n^\alpha \partial^\mu n^\alpha \tag{45}
\]
where \(\xi\) is the Fourier variable. The Gaussian integral over \(\Pi_\alpha\) is done yielding,
\[
Z = \int [\mathcal{D}n^\alpha] \delta(n^2 - 1) \exp \left[ i \int \frac{1}{4} (\partial_\mu n^\alpha)(\partial^\mu n^\alpha) \right] \tag{46}
\]
where we have absorbed a trivial Gaussian over $\xi$ into the normalisation. Expression (46) is, therefore, seen to reproduce the original theory (1) subject to the constraint (2).

Finally we show how nontrivial consequences arise from (40) by choosing a multiplier dependent gauge,

$$\chi_1 = \Pi_\phi + p_1, \quad \chi_2 = \phi$$

(47)

As usual, scaling arguments [1,14] once again enforce the constraint $T_2'$ and the gauge condition $\chi_2$ by delta functions. The presence of the multiplier in $\chi_1$ prevents this enforcement for $T_1'$ and $\chi_1$. We find,

$$Z = \int [Dn^aD\Pi_aD\phi D\Pi_\phi Dq^1 Dp_1] \delta(T'_2) \delta_\phi \delta \det - n^2|e^{iS}$$

(48)

where,

$$S = \int (\Pi_\phi \dot{n}^a + \Pi_\phi \dot{\phi} + p_1 \dot{q}^1 - n^2 \Pi^2 + \Pi_\phi^2 (n^2)^2 - \exp \left\{ \frac{d}{dn^a} \frac{\dot{n}^a}{\dot{\phi}} \right\} H_0 + q^1 (n^2 - 1 + 2\phi + p_1 (\Pi_\lambda + p_1))$$

(49)

The $\phi$ integration is trivially done. A Fourier transformed representation for $\delta(T'_2)$ is taken. Doing successively the Gaussians over $p_1$, $\Pi_\alpha$, $\Pi_\phi$ and the Fourier variable $\xi$ yields,

$$Z = \int [Dn^a Dq^1 (\det| - n^2|)^{1/2} e^{iS}$$

(50)

with,

$$S = \int [q^1 (n^2 - 1) + \frac{1}{4n^4} \left\{ n^2 (\dot{n}^a)^2 + (4n^4 - 1) (n^a \dot{n}^a)^2 - (n^a \dot{n}^a)^2 \right\} \dot{q}^1 - \frac{1}{2} \partial_i n^a \partial^i n^a (1 - \frac{n^2}{2})]$$

(51)

Expression (50) apparently differs drastically from (46) but both are expected to yield identical S-matrix elements by the Fradkin-Vilkovisky theorem [1,14]. Indeed the nontrivial structure (50) illustrates the generality of this approach [4,5] since it cannot be obtained by conventional [10, 15] quantisation methods. Note the explicit presence of the Faddeev-Popov determinant in (50) which, in the previous case (46), could be absorbed in the normalisation.
3 Conclusion

To conclude, we have shown how the BT method [5] can be exploited to quantise the $O(N)$ invariant sigma model. In our knowledge this is the first nontrivial (in the sense that the conventional Dirac [3] method is riddled with operator ordering problems) application of the generalised canonical approach [4,5]. Moreover since the present example does not involve the gauge field the conventional Stückelberg [16] or Wess-Zumino [17] approaches of obtaining a first class theory is not straightforward. As is usual in such an explicit analysis, new theoretical insights have been gained. We have seen the necessity of making an intelligent choice in (17) which simplifies the algebra remarkably and allows us to identify a canonically conjugate pair (39) among the new variables. Moreover we find that, contrasted with [4], the recent work [5] is better suited for computational reasons. This is because, by construction, it automatically yields a strongly involutive system which is BRST invariant. There is another important aspect which we wish to stress. The generalised approach [4,5] does not specify the precise Hamiltonian (i.e. canonical (3) or total (9)) to take as the starting Hamiltonian. We have found that the total Hamiltonian (9) is the better choice since it leads to the closed (exponential) form for the involutive Hamiltonian (28). In fact this Hamiltonian is also ideal for perturbative computations. Moreover, in contrast with the earlier work [12], this expression is local. A corresponding analysis originating from the canonical Hamiltonian (3) leads to severe algebraic complications. Since our analysis is quite general it could be employed to quantise other types of nonlinear sigma models ($CP^N$ models, for instance) including their supersymmetric generalisations.

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