Peculiarities of gas bubble growth in magmatic melt under the condition of rapid decompression

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Abstract. The growth mechanism of a single gas bubble in a highly viscous volatile-saturated magmatic melt during its rapid decompression is investigated. A new analytical solution of the problem is shown. It correctly describes the bubble growth dynamics in a wide range of supersaturations achieved as a result of decompression and at all stages of the process including the transitional one. Accounting for this stage is extremely important if we consider highly-viscous magmatic melts. It is shown that after some time bubble growth is determined solely by diffusion, and the process has a self-similar character.

1. Introduction
At this point in time there are a large number of scientific papers devoted to the modeling of volcanic eruptions. They cover both relatively calm extrusive eruptions and destructive explosive ones. This is due to the need for a detailed forecast and for determining the degree of potential danger of a particular volcano.

It is obvious that field observations and experimental settings cannot always answer the many questions arising when describing this phenomenon. Therefore, a consistent and as rigorous as possible mathematical modeling of the eruption process by the methods of mechanics of multiphase media appears relevant. The aim is to understand the mechanisms determining the type and nature of the eruption.

As a rule, such modeling is carried out in a fairly general setting, where attempts are made to take into account full diversity of phenomena accompanying a volcanic eruption. Of course, this is done in a highly simplified form. Other works are focused on individual processes, trying to understand all their nuances [1]. One of them is the process of magma degassing during its rapid decompression. It largely predetermine the structure of a three-phase flow forming inside a conduit (magma degassing is also accompanied by its partial crystallization and amorphization); therefore, it determines the nature and type of eruption.

The study of cavitation processes in liquids and in particular the degassing process has already more than a century of history. This was begun in the works of Rayleigh, Plesset, Skriven, and others. Starting with Sparks’ work [2], the problem of nucleation and the growth of gas bubbles in magmatic melts has become of interest to geophysics. Subsequently, this study resulted in a separate direction due to the specifics of the subject of the study. The fact is that magmas have a number of properties that significantly distinguish them from other liquids. This is an extremely high content of dissolved volatile components, as well as dynamically changing viscosity during the degassing process (by orders of magnitude). All
this imposes certain restrictions on the applicability of various models, especially classical ones, to the description of the process under study.

A series of experiments were conducted in [3, 4], and some theoretical models of bubble growth in igneous melts were presented. Within the framework of the quasi-steady approximation, an approximate analytical solution is obtained for the diffusion stage. Attempts have been made to describe the inertial and transitional stages. The numerical solution of the bubble growth problem in rheolitic and basaltic magmas in a fairly complete formulation was obtained in [5,6]. The features of the nucleation of bubbles in igneous melts are discussed in [7–9]. However, there is still no universal solution that describes the dynamics of bubble growth in a wide range of supersaturations, during the whole time of the process, under highly non-equilibrium conditions and under the forces of non-stationary external effects.

2. Statement of the problem
2.1. Governing equations

Let us consider an arbitrary volume of gas-saturated magma subjected to rapid decompression at the initial moment (the final pressure will be assumed to be constant). Let us suppose that as a result of decompression a supercritical nucleus (gas bubble) was formed in the volume of the melt. The growth mechanism of this bubble we will try to describe in the framework of this article.

The process of bubble growth can be divided into three stages [10, 11]:
— inertial one, when in the process of bubble growth the pressure in it remains almost constant;
— a transitional one. It is characterized by bubble pressure decreasing as it grows. As a result, an equilibrium condition begins to break at the interphase boundary, which causes diffusion processes in the surrounding fluid;
— asymptotic diffusion one, when the gas pressure in the bubble is almost equal to the pressure of the surrounding liquid. So, the bubble growth is solely a result of diffusion.

To describe the dynamics of dissolved volatile concentration field \( C(t, r) \), we write the diffusion equation using the spherical symmetry of the problem:

\[
\frac{\partial C}{\partial t} + \frac{\dot{R} R^2}{r^2} \frac{\partial C}{\partial r} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( D r^2 \frac{\partial C}{\partial r} \right),
\]

where \( t \) is the time; \( r \) is the radial coordinate, with the origin taken at the bubble center; \( R \) is the bubble radius; \( D \) is the diffusion coefficient of volatile in liquid. The convective term appearing in the equation (1) represent a radial motion of the fluid. The fluid velocity is found from the continuity equation (the fluid is assumed to be incompressible).

We assume that at the initial moment of time the melt is saturated up to equilibrium state: \( (C)_{t=0} = C_i \), where \( C_i = C_s(p_i) \). Here \( C_s \) — is the equilibrium concentration of gas dissolved in the melt. According to Henry’s law it is a function of pressure \( p \) (for water dissolved in magma the function is: \( C_s(p) = (K_H p)^{1/2} \), where \( K_H \) is the Henry constant).

The local equilibrium condition is satisfied at the bubble boundary: \( (C)_{r=R} = C_s(p_g) \); far from the bubble, the concentration field is unperturbed: \( (C)_{r=\infty} = C_i \). Gas inside a bubble is considered to be ideal: \( p_g = \rho_g(\Re/\mu_g) T \), where \( \rho_g \) is the density of the gas; \( \Re \) is the universal gas constant; \( \mu_g \) is the molar mass of the gas; \( T \) is the temperature. From now on the use of subscripts “\( i \)” and “\( f \)” represent the initial and final state respectively; subscripts “\( l \)” and “\( g \)” represent liquid and gas phases.

The formulated boundary problem should be supplemented with the equation of material balance:

\[
\frac{dm_g}{dt} = 4\pi R^2 D \rho_l \left( \frac{\partial C}{\partial r} \right)_{r=R},
\]

where \( m_g \) is the volatile mass in the bubble. It also should be supplemented by the dynamics equation:

\[
\rho_l \left( \dot{R} \dot{R} + (3/2) \dot{R}^2 \right) = p_g - p - \frac{2\sigma}{R} - 12 \rho_l \int_R^{\infty} \frac{\eta(r)}{r^4} dr,
\]
Figure 1. Dependencies of characteristic size $R_0$ (solid line) and process time $t_0$ (dashed line) on liquid viscosity $\eta_i$; $\Delta p_i = 100$ MPa.

where $\eta$ is the dynamical viscosity of the liquid; $\sigma$ is the surface tension coefficient. Equation (3) is a modification of Rayleigh equation \[8, 12\] with the consideration of the viscosity gradient resulting from the formation of a diffusion boundary layer around the bubble. Let us note that this effect cannot be ignored when modeling bubble growth in igneous melts because the viscosity changes by orders of magnitude during degassing. The latter can be described by the dependence of the Arrhenius type with a linear dependence of the activation energy on the dissolved volatile concentration \[6\]:

$$
\eta(\bar{C}) = \eta^* \exp \{E_0 \eta(1 - k_\eta \bar{C})/(RT)\},
$$

where $\eta^*$; $E_0 \eta$ and $k_\eta$ are empirically determined coefficients.

2.2. Non-dimensionalization

Let us introduce the dimensionless variables: $\tau = t/t_0$; $\bar{r} = r/R_0$; $\bar{R} = R/R_0$; $\bar{p} = (p - p_f)/\Delta p_i$; $\bar{C} = (C - C_f)/\Delta C_i$; where $\psi = \Delta p_i/p_i$ is relative decompression; $t_0$ is the characteristic time of the process; $R_0$ is the characteristic size; $\rho_g f = \rho_g(p_f)$; $\Delta p_i = p_i - p_f$; $\Delta C_i = C_i - C_f$; $C_f = C_s(p_f)$. Here we choose the characteristic size $R_0$, so that the time of diffusion $t_0^D = R_0^2/D$ and dynamical $t_0^\eta = \eta_i/\Delta p_i$ processes are equal: $t_0^D = t_0^\eta = t_0$. As a result we have: $R_0 = (D\eta_i/\Delta p_i)^{1/2}$, where $\eta_i = \eta(C_i)$. The use of above-introduced characteristic values allows us to exclude the viscosity as a parameter from all dimensionless equations and significantly simplifies the analysis of the problem.

Dependencies of characteristic size $R_0$ and process time $t_0$ on liquid viscosity $\eta_i$ are presented in fig. 1 (all necessary properties of magmatic melt used in the calculations are taken from the work \[8\]). As you can see from the figure, for different magmatic melts (characterized by different viscosity) characteristic values of the size and the process time may differ by a couple orders of magnitude.

3. Analytical solutions of the problem

3.1. Half-analytical solution

To find a solution to the formulated problem (1)–(3) we are using an approach that has proved itself useful when modeling the growth of new phase nuclei in various metastable systems \[13–15\]. To do this, we are using variables ($\tau$, $\chi = \bar{r}/\bar{R}$) instead of ($\tau$, $\bar{R}$). The need for the use of these variables lies in the fact that the interphase boundary becomes stationary, which greatly simplifies the boundary problem. Now, the solution can be obtained analytically \[10\]:

$$
\frac{1 - \bar{C}(\tau, \chi)}{1 - C_s(p_g)} = \frac{I(\chi, \beta)}{I(1, \beta)},
$$

(4)
Figure 2. Dependencies of bubble radius $R$ on time for different values of liquid viscosity: $\eta_i = 10^4$ Pa·s (curve 1); $10^5$ (2); $10^6$ (3); $\Delta p_i = 150$ MPa.

where $I(\chi, \beta) = \int_{\chi}^{\infty} \zeta^{-2} \exp\left\{-\beta \left(\zeta^2/2 + 1/\zeta\right)\right\} d\zeta$; $\beta = \frac{1}{2} \frac{d}{d\tau} \left(\bar{R}^2\right)$ — coefficient (sought-for function of time) characterizing the bubble growth rate. It can be found from the following integro-differential equation:

$$
\frac{d}{d\tau} \left\{\frac{1 - \psi (1 - \bar{p}_g)}{1 - \psi} \bar{R}^3\right\} = 3 \varepsilon \bar{R} \left\{1 - \bar{C}_s(\bar{p}_g)\right\} \cdot \left\{e^{3\beta/2} I(1, \beta)\right\}^{-1},
$$

which should be solved together with the equation of dynamics that have following dimensionless form:

$$
\bar{p}_g - \bar{p} = \frac{6}{R^2} \frac{d}{d\tau} \left(\bar{R}^2\right) \cdot \int_{1}^{\infty} \frac{\bar{\eta}(\chi)}{\chi^4} d\chi,
$$

where $\bar{\eta} = \eta/\eta_i$. Let us note that the dimensionless criterion $\varepsilon = (\rho_l/\rho_g) \Delta C_i$ appears in the equation (5) characterizes the degree of metastability (supersaturation) of the medium. It is essentially an analog of the Jacob number in the problem about the growth of a vapor bubble in a superheated liquid. The equation (6) is written neglecting the inertial terms, which can be done in view of the high viscosity of the magma. The Laplace pressure is also neglected, which is acceptable if we consider bubbles much larger than the critical one. However, if necessary all these terms can be taken into account [16]. It is also noteworthy that the solution obtained can be used under non-stationary external conditions (the pressure $barp$ in the equation (6) is in general a function of time). Thereby, the formulated boundary problem reduces to a solution of system of integro-differential equations, what significantly simplifies the analysis of the problem.

Fig. 2 shows the numerically calculated dependencies of bubble radius on time, constructed for different values of the liquid viscosity (all other conditions being equal). It should be noted that after a certain time, when the dynamics of bubble growth is determined only by diffusion, the effect of viscosity on the overall picture of the process disappears. However, in the initial and transitional stages of the process, viscous effects are decisive. The characteristic time of these stages is, obviously, the greater, the greater the viscosity of the liquid. It is clearly illustrated in Fig. 2. We also note that the semi-analytic solution obtained practically coincides with the numerical one, and it is rather difficult to show their difference in the figure.
3.2. Self-similar solution

As mentioned above and as shown by the numerical analysis of the equations (4)-(6) for the case of ultra-fast decompression (with a fixed final pressure) the gas pressure in the bubble gradually decreases as it grows and asymptotically tends to the pressure of the surrounding fluid: \( \bar{p}_g \to \bar{p} \) (for \( \tau \gg 1 \)). The characteristic relaxation time here is determined mainly by the viscosity of the liquid. The coefficient \( \beta \) tends to a constant value of \( \beta_0 \), depending on supersaturation of the medium \( \varepsilon \). The concentration field of the dissolved volatile in the \( (\tau, \chi) \) variables becomes stationary (which means that the process is self-similar with \( \chi \) being a self-similar variable), and the bubble growth law can be found explicitly [8,17,18]:

\[
\bar{C}(\chi) = 1 - \frac{I(\chi, \beta_0)}{I(1, \beta_0)}; \quad \bar{R}(\tau) = (2\beta_0 \tau)^{1/2}.
\]

Constant \( \beta_0 \) here should be obtained from the following implicit integral equation:

\[
\beta_0 e^{3\beta_0/2} I(1, \beta_0) = \varepsilon.
\]

At this stage, bubble growth occurs exclusively as a result of diffusion. Dependence of coefficient \( \beta_0 \) on \( \varepsilon \) is shown in fig. 3. In the case of high and low supersaturation, explicit approximations can be derived. For \( \varepsilon \ll 1: \beta_0 \approx \varepsilon \). In this case the obtained solution matches the steady-state one (see, for example, [3]), which was expected for low supersaturations. For \( \varepsilon \gg 1: \beta_0 \approx (6/\pi)(\varepsilon + 4/9)^2 \). It is different from the steady-state solution. We can conclude that the use of steady-state solution in the entire range of initial supersaturation and for all times is inappropriate.

Conclusions

The present article describes the growth of a single gas bubble in a highly-viscous volatile-saturated magmatic melt after the rapid decompression. The complex mathematical model of the process is formulated. The half-analytical solution of the problem, valid in a wide range of initial supersaturations and at all stages of the process is obtained. It is shown that for highly viscous liquids, it is imperative to take into account the dynamic stage of the process, when the pressure in the bubble is not yet leveled with the pressure of the surrounding liquid. Therefore, depending on the viscosity of the liquid, the duration of this stage can be in a range from tens to hundreds of seconds. It is found that after a certain amount of time the influence of viscosity disappears and the process becomes self-similar. Afterwards, the growth of the bubble is determined only by diffusion. It is shown that the solution obtained within the framework of the quasi-stationary approximation is valid only for low value of initial supersaturation.
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