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Phase transitions for a class of gradient fields. (English) [Zbl 1467.82031]
Probab. Theory Relat. Fields 179, No. 3-4, 969-1022 (2021).

Summary: We consider gradient fields on $\mathbb{Z}^d$ for potentials $V$ that can be expressed as

$$e^{-V(x)} = pe^{-\frac{q x^2}{2}} + (1-p)e^{-\frac{x^2}{2}}.$$

This representation allows us to associate a random conductance type model to the gradient fields with zero tilt. We investigate this random conductance model and prove correlation inequalities, duality properties, and uniqueness of the Gibbs measure in certain regimes. We then show that there is a close relation between Gibbs measures of the random conductance model and gradient Gibbs measures with zero tilt for the potential $V$. Based on these results we can give a new proof for the non-uniqueness of ergodic zero-tilt gradient Gibbs measures in dimension 2. In contrast to the first proof of this result we rely on planar duality and do not use reflection positivity. Moreover, we show uniqueness of ergodic zero-tilt gradient Gibbs measures for almost all values of $p$ and $q$ and, in dimension $d \geq 4$, for $q$ close to one or for $p(1-p)$ sufficiently small.

MSC:
82B26 Phase transitions (general) in equilibrium statistical mechanics
82B05 Classical equilibrium statistical mechanics (general)
82B20 Lattice systems (Ising, dimer, Potts, etc.) and systems on graphs arising in equilibrium statistical mechanics
60K37 Processes in random environments

Keywords:
gradient Gibbs measures; phase transitions; random conductance model

Full Text: DOI arXiv

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