Stress modelling in natural foundation

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Abstract. Modeling methods are widely used in solving problems of deformable solid mechanics. Modeling of problems in the mechanics of a deformable solid is carried out using similarity criteria. Based on the similarity criteria, the model is made, the loading conditions are determined, and the transition from the values on the model to the values of the full-scale design is performed. The similarity criteria may be obtained either from the similarity theory or with the use of the dimensional analysis. The authors use the system of equations for a mixed problem in the elasticity and plasticity theory as well as the scale method and determine the similarity criteria for modelling the stresses in the foundations of structures. They note the restrictions on the choice of similarity factors for loose soils and the possibilities of the use of centrifugal modelling for the aforesaid scale method. They also note the special features of modelling the coherent soils.

1. Introduction
Modeling issues in the problems of deformable solid mechanics are considered by the authors in the works [1, 2, 3, 4,5]. When solving stress modeling problems, it is necessary to obtain the appropriate similarity criteria that are the basis for the model. Multipliers of similarity in the modeling of problems of mechanics of deformable solids are related with certain relationships. These relations create some limitations when modeling these problems.

2. Problem statement
Let us consider a soil foundation, to the surface of which an uniform load caused by the structure and a side load caused by the depth of foundation are applied.

The problem of determination of stresses in the foundations of structures at any stage of the development of areas of plastic soil deformations has a strict mathematical formulation, and the similarity criteria may be obtained with the help of a simpler apparatus of the similarity theory.

3. Theoretical principles
For the determination of necessary similarity criteria we write out the system of equations for a plane (two-dimensional) mixed problem in the elasticity-and-plasticity theory for the model concerned [6, 7].

The equilibrium equations
\[
\begin{align*}
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} &= 0 \\
\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \gamma &= 0
\end{align*}
\] (1)

The boundary conditions on the surface in the area of the structure and outside it
\[
\sigma_z = p_c, \quad \sigma_z = p_a
\] (2)

The compatibility condition in the elasticity area
\[
\nabla^2(\sigma_x + \sigma_z) = 0
\] (3)

The condition of limiting equilibrium in the plasticity area
\[
\sigma_1 - \sigma_2 = (\sigma_1 + \sigma_2 + 2\sigma_z)\sin\varphi
\] (4)

where \(\nabla^2\) denotes Laplacian:
\[
\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} = 0,
\]

\(\gamma\) - soil density,
\(\sigma_z = c \cdot \cot \varphi\),
\(\varphi\) - angle of the soil internal friction,
\(c\) - specific soil adhesion.

Using the scale method, we introduce similar transformations between the real structure parameters and the model ones:
\[
\begin{align*}
\sigma_x^p &= K_n \cdot \sigma_x^m; \\
\sigma_z^p &= K_n \cdot \sigma_z^m; \\
\tau_{xz}^p &= K_n \cdot \tau_{xz}^m; \\
X^p &= K_x \cdot X^m; \\
Z^p &= K_x \cdot Z^m; \\
\gamma^p &= K_x \cdot \gamma^m; \\
\sin\varphi^p &= K_\varphi \cdot \sin\varphi^m; \\
\sigma_z^p &= K \cdot \sigma_z; \\
p_c^p &= K_p \cdot p_c; \\
p_a^p &= K_p \cdot p_a.
\end{align*}
\] (5)
If we write out the system of equations (1) – (4) for real structure parameters, we can use the relationships (5) and reduce this system to the form:

\[
\begin{align*}
\frac{K_a}{K_f} \left[ \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} \right] &= 0 \\
\frac{K_a}{K_f} \left[ \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_z}{\partial z} \right] + K_f^\gamma &= 0 \\
\frac{K_a}{K_p} \sigma_z &= p_c, \quad \frac{K_a}{K_p} \sigma_x &= p_a \\
\frac{K_a}{K_f} \nabla^2(\sigma_x + \sigma_z) &= 0 \\
K_a(\sigma_1 - \sigma_2) &= \left[ (\sigma_1 + \sigma_2)K_a + 2K_a^2 \sigma_1 \right]K_f^\phi \sin \phi
\end{align*}
\]

(6)

For the congruence of the equations (6) – (9) for real structure parameters with the corresponding model equations, it is necessary and sufficient to fulfil the following conditions:

\[
\frac{K_a}{K_fK_f^\gamma} = 1, \quad K_a = K_p, \quad K_a = K_c, \quad K_a = K_q = 1
\]

(10)

From the second condition in (10) we obtain the formula for the recalculation of stresses from the model parameters to the real structure ones:

\[
\sigma_x^\ast = K_p \cdot \sigma_x^w; \quad \sigma_z^\ast = K_p \cdot \sigma_z^w; \quad \tau_{xz}^\ast = K_p \cdot \tau_{xz}^w
\]

(11)

From (10) we conclude the necessity of the following relationships:

\[
K_c = K_o, \quad K_f = \frac{K_p}{K_f} \quad K_q
\]

(12)

Thus, in the process of modelling the stresses in the foundations of structures, the similarity factors \( K_f \) and \( K_p \) (determining the geometry scale and the force aspect of the model) may be chosen arbitrarily, and the factors \( K_o \), \( K_c \) and \( K_q \) (determining the angle of internal friction, the specific cohesion and the soil density) must be chosen in correspondence with (12).

In case of loose soils the condition (12) takes the form:

\[
K_p = 1, \quad K_q = \frac{K_p}{K_f}
\]

(13)

As it is not possible to measure the own weight of the soil in standard model experiments and in the dies, the value of \( K_q \) should be taken \( K_q = 1 \), and so for the loose soils we obtain the modelling condition \( K_p = K_f \).

Thus, the necessary similarity condition for stress states of loose uniform foundations in real structures and in models is the equality of the similarity factors in geometry scale and the force factor. This similarity condition considerably reduces the value of the practical use of the results of studies of sand soils through dies if the width of the real structure is considerable.
To make the own weight of the soil greater, we can use the method of centrifugal modelling. In this case, the similarity factors in geometry scale and those of the force factor must be chosen in accordance with the modelling condition (13).

It should be noted that the necessity of the first condition in (13) is substantiated by the condition of limiting equilibrium in the plasticity area. That is why, at the stage when there are no areas of limiting stress state or they are negligibly small, it is possible to produce the models from other materials. In such a case, we may use the photo-elasticity method using an optically sensitive material for modelling stresses in the foundations of structures [8]. The increase in the own weight of the material of the model is achieved through the submergence of the model into a dense liquid [9, 10].

Using the submergence method together with the method of centrifugal modelling, we can achieve a stronger effect of the increase in the own weight of the model made from a transparent optically sensitive material [5]. The fixation of stresses in the area of the model (if necessary) is carried out by the “freezing” method [8].

In case of a coherent soil, the modelling conditions (as it was shown earlier) have the form:

\[
K_0 = 1, \quad K_\sigma = K_p = K_f, \quad K_\gamma = \frac{K_\gamma}{K_f}
\]  

The standard experimental study processes do not allow us to change the soil coherence and its own weight in accordance with the modelling conditions, therefore the fulfilment of the conditions (14) with the change of the similarity factor of the geometry scale cannot be guaranteed with any choice of the similarity factor of the force aspect.

But if we write out the system of equilibrium equations in the form

\[
\begin{align*}
\frac{\partial}{\partial x} (\sigma_x + \sigma_z) + \frac{\partial \tau_{xz}}{\partial z} &= 0, \\
\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial}{\partial z} (\sigma_x + \sigma_z) + \gamma &= 0,
\end{align*}
\]

the boundary conditions on the surface in the area of the structure as well as outside it in the form

\[
\sigma_x = p_x + \sigma_z, \quad \sigma_z = p_x + \sigma_z,
\]

the compatibility equation for the elasticity area in the form

\[
\nabla^2 \left[ (\sigma_x + \sigma_z) + (\sigma_z + \sigma_x) \right] = 0
\]

and the equation of limiting state for the areas which are in limiting stress states in the form

\[
(\sigma_x + \sigma_z) - (\sigma_z + \sigma_z) = \left[ (\sigma_x + \sigma_z) + (\sigma_z + \sigma_z) \right] \sin \varphi
\]

we can act in the same way as we did when we derivated the conditions (10) and can obtain the following result: the similarity conditions for coherent soils may be written out as follows:

\[
\frac{K_{\sigma_0}}{K_\sigma K_\gamma} = 1, \quad \frac{K_{\sigma_0 \sigma_z}}{K_\sigma K_p} = 1, \quad K_\psi = 1.
\]

If the soil is coherent, we can replace the action of the adhesion forces with the all-sided uniform coherence pressure \( P_c = c \cdot \tan \varphi \) applied to the free soil sides, that is we can reduce a coherent soil to a loose one.
4. Conclusions

Conditions for modeling stresses in the bases of structures are obtained. The necessary conditions for similarity of the stressed States of loose homogeneous bases and the condition for bringing a connected soil to a loose one are given.

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