The excitation energy and the nuclear density at the time of breakup are extracted for the $\alpha +^{197}Au$ reaction at beam energies of 1 and 3.6 GeV/nucleon. These quantities are calculated from the average relative velocity of intermediate mass fragments (IMF) at large correlation angles as a function of the multiplicity of IMFs using a statistical model coupled with many-body Coulomb trajectory calculations. The Coulomb component $\vec{v}_c$ and thermal component $\vec{v}_0$ are found to depend oppositely on the excitation energy, IMFs multiplicity, and freeze-out density. These dependencies allow the determination of both the volume and the mean excitation energy at the time of breakup. It is found that the volume remained constant as the beam energy was increased, with a breakup density of about $\rho_0/\gamma$, but that the excitation energy increased 25% to about 5.5 MeV/nucleon.

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In spite of intensive experimental and theoretical efforts, the mechanism of nuclear multifragmentation remains a subject of much interest and debate. The study becomes even more attractive with the availability of more high quality data from powerful $4\pi$ detectors [1].

In this Letter we report on results of the determination of the freeze-out density and excitation energy by studying the average relative velocity $<v_{rel}>$ of IMFs at large correlation angles as a function of the multiplicity of IMFs. We use the Berlin version of a statistical model [2] coupled with the many-body Coulomb trajectory calculations to describe the multifragmentation process. Among the many available multifragmentation models, those based on the simultaneous breakup mechanism have been rather successful in explaining a number of aspects of multifragmentation data [3]. Moreover, statistical phase-space models like ours have a unique ability for calculating many-fragment correlation functions with reliable statistics.

In the simultaneous breakup scenario, one can picture the multifragmentation as follows: A composite nuclear system formed initially in any reaction will expand under thermal and/or compressional pressure. The expansion may take the system into the hydrodynamical instability region where the sound velocity is imaginary[4]. In the instability region the system breaks up into fragments and nucleons. At the same time the further expansion slows down due to the creation of new surfaces of fragments and becomes chaotic.
due to the strong interactions among fragments and nucleons through the exchange of nucleons and momenta. At the end of this chaotic expansion (the instant of freeze-out) the fragments have rearranged themselves such that the entropy of the system has reached its maximum. At the moment of the freeze out, the momentum distribution, spatial distribution, mass distribution and internal excitations of fragments of the system are solely determined microcanonically by the available phase space. After the freeze out, fragments and nucleons move along their Coulomb trajectories. This picture is in agreement with a recent BUU-model simulation of the reaction dynamics leading to multifragmentation[5, 6].

Within the above picture, the most important quantities determining the final state of the multifragmentation events are the excitation energy and the density or size of the system at the freeze out. Ideally, these quantities can be calculated from dynamical models for nuclear reactions. However, the breakup conditions (time scale, entropy and partitioning into fragments) is the most interesting topic of current research because none of the existing dynamical models can handle the breakup properly. The extracted freeze-out density and excitation energy will put strong constraints on the reaction dynamics leading to the formation of hot nuclei and on the nuclear equation of state for low-density nuclear matter. Moreover, to a significant extent, our study itself is a study of the breakup dynamics.
The velocities of the fragments in the final state are sensitive to their spatial distribution at the freeze out due to their long range Coulomb repulsion. The velocities are also sensitive to the excitation energy of the fragmenting system. The charge and mass distributions of fragments, as well as the thermal velocities of the fragments at the freeze out, are mainly determined by the excitation energy of the system. Therefore, the relative velocity distributions are useful tools for studying the breakup process as well as for extracting the freeze-out density and excitation energy. The above discussions show that the two quantities have to be extracted simultaneously. It is interesting to mention that the relative velocity distributions for various fragment classes have been used to distinguish between the simultaneous breakup scenario and the sequential binary decay scenario [7, 8, 9, 10] with contradictory conclusions. By selecting dedicated event classes a deeper insight can be gained. For three-body events from the system Ar + Au at 30 MeV/nucleon it was shown that the relative velocity distribution can be well reproduced assuming the sequential binary decay of a hot system at an excitation energy of about 3 MeV/nucleon[11]. This result can be understood as a deep inelastic collision, followed by the fission of the heavy partner [12]. At higher incident energies, selecting events involving only three heavy fragments, the ALADIN collaboration has very recently also studied the relative velocity distribution from 600 MeV/nucleon Au induced reactions [13], their results suggest a fast
disintegration of the entire highly excited system. A range of the freeze-out radii and excitation energies was extracted simultaneously using a simple fragmentation model coupled with three-body trajectory calculations.

As a first part of this Letter we study the model predictions for the average relative velocity between IMFs with large correlation angles as a function of the total IMF multiplicity, varying the freeze-out density and the excitation energy. In the second part we compare these results with experimental data in order to deduce the freeze-out condition. This method goes far beyond the usual way of comparing the relative velocity distribution for events involving only three fragments or the relative velocity distribution obtained without any constraint on the multiplicity of IMFs. It is evident that the multiplicity and charge of the unobserved fragments have decisive influence on the observed velocity distributions. We want to emphasize that this means we test more than just a three-body dynamics as in ref.\cite{13}. The change of $< v_{\text{rel}} >$ with the IMF multiplicity at a relatively constant excitation energy gives the clue to our method of extracting the source size and excitation energy. This is a clear advantage using the very asymmetric collision system $\alpha + Au$. There is also a qualitative difference between highly energetic $\alpha$ induced fragmentations and heavy-ion induced ones. In the later the variation of the IMF multiplicity is mainly due to the broad variation of the excitation energy. Here it is much more due to the fluctuation of $M_{IMF}$.
at a relatively narrow distribution of excitation energies.

The final velocity of each fragment $\vec{v}_f$ can be decomposed as

$$\vec{v}_f = \vec{v}_0 + \vec{v}_c,$$

where the $\vec{v}_0$ term is the thermal velocity of the fragment at the freeze out. At a fixed excitation energy of the fragmenting system the $\vec{v}_0$ term increases with the multiplicity of IMFs due to the smooth decrease of the average mass of the IMFs. The $\vec{v}_c$ term is the velocity of the fragment gained after moving in the Coulomb field of the rest of the system and thus depends on the radius of the system at the freeze out. The dependence of $\vec{v}_c$ on the multiplicity of IMFs is somewhat more complicated. It is strongly influenced by the existence of a third, larger fragment. This will be discussed in detail later. Therefore, the dependence of $< v_{rel} >$ on the multiplicity of IMFs results from the interplay between the two velocity components of the IMFs just mentioned. They reflect the freeze-out density and the excitation energy. The secondary deexcitation of primary hot fragments will slightly alter the relative velocity distribution. Since the deexcitation occurs late, the influence of the change in the Coulomb field is small. Further, since the emission is assumed to be isotropic, only the width will be changed, but not the mean value.

Necessary inputs to our calculations are the mass and the charge of the fragmenting source. Since this Letter relates to $\alpha +$ Au reactions at 1 and
3.6 GeV/nucleon, cascade model calculations [14] were performed for these reactions, yielding the mass and charge of the residual nucleus, which is the emitting equilibrized source. For the higher incident energy a mean residue mass of $A=150$ and charge of $Z=62$ was deduced, for the lower incident energy the mean residue mass and charge was determined to be $A=170$ and $Z=70$, respectively. The mass and charge of the heavy residue are rather independent of the impact parameter due to the large mass asymmetry. This legitimates the comparison of the experimental results to the statistical calculations using a fixed residue mass, charge and excitation energy. This is an important advantage of the highly asymmetric $\alpha +$ Au system for investigations of this kind.

To see how the dependence of $<v_{rel}>$ on the multiplicity of IMFs is sensitive to the breakup conditions of hot nuclei towards multifragmentation, we study in Fig. 1 (a) the excitation energy dependence of $<v_{rel}>$ at large correlation angles for a given breakup density.

In Fig. 1 (a)-(c), the calculations are done for excitation energies of 4, 5, 8 and 10 MeV/nucleon respectively at a fixed freeze-out radius of $2.1A^{1/3}$ and a residue mass and charge of $A=150$ and $Z=62$. It is seen that for small excitation energies $<v_{rel}>$ decreases with increasing multiplicity of the intermediate mass fragments $M_{IMF}$ and that $<v_{rel}>$ is higher at $E^*=4$ MeV/nucleon than at 5 MeV/nucleon. However, an opposite behaviour is
seen at high excitation energies. Now $\langle v_{rel} \rangle$ increases with increasing $M_{IMF}$ and $\langle v_{rel} \rangle$ is higher at 10 MeV/nucleon than at 8 MeV/nucleon. These features of $\langle v_{rel} \rangle$ can be understood by considering the interplay between the two velocity components of the fragments and their excitation energy dependence. For this purpose we show in Fig. 1 (b) the average maximum charge of the fragments as a function of the multiplicity of IMFs at the corresponding excitation energies. The dashed line at $Z=15$ is the upper boundary of the charge defined for the IMFs. At lower excitation energies (e.g. 4 and 5 MeV/nucleon) the average maximum charge is larger than 15 for a significant range (if not all) of the IMF multiplicities. The fragment with the highest charge in each event accelerates the other fragments and is therefore responsible for the higher values of $\langle v_{rel} \rangle$. The maximum charge of these events decreases strongly with increasing IMF multiplicity. This explains the trend of decreasing $\langle v_{rel} \rangle$ with increasing $M_{IMF}$ observed at lower excitation energies.

At high excitation energies the maximum charge is within the charge range of IMFs for all statistically significant events (seen from Fig. 1 (c)) and the thermal term $\vec{v}_0$ dominates over the Coulomb term $\vec{v}_c$. Since the average mass of IMFs will decrease as the multiplicity of IMFs increases, the $\vec{v}_0$ term and consequently $\langle v_{rel} \rangle$ of IMFs will increase with increasing $M_{IMF}$. The fact that $\langle v_{rel} \rangle$ is higher at higher excitation energies is
due to both the higher temperature reached and the lower average mass of the fragments. To evaluate the statistical significance of each multiplicity of IMFs, we have presented the distribution of the multiplicity of IMFs at the corresponding excitation energies in Fig. 1 (c). The typical rise and fall behaviour of the IMF multiplicities as a function of the excitation energy can be seen between $E^* = 5$ and 10 MeV/nucleon. The average IMF multiplicity at $E^* = 10$ MeV/nucleon is lower than that at 8 MeV/nucleon. This is due to the increasing dissociation into light fragments ($Z < 3$) and nucleons at higher excitation energies.

To see the effects of the freeze-out density or radius on $\langle v_{rel} \rangle$ we show in Fig. 1 (d) $\langle v_{rel} \rangle$ at large correlation angles as a function of $M_{IMF}$ for freeze-out radii of $2.1A^{1/3}, 2.4A^{1/3}$ and $2.6A^{1/3}$ respectively at a fixed excitation energy of 5 MeV/nucleon. The magnitude of $\langle v_{rel} \rangle$ increases rapidly with decreasing freeze-out radius due to the stronger Coulomb repulsion at smaller freeze-out radii. It is interesting to note that at $R_c = 2.6A^{1/3}$ the Coulomb repulsion is so weak that for $M_{IMF} \geq 6$ the thermal term $\vec{v}_0$ starts dominating over the Coulomb term $\vec{v}_c$ and $\langle v_{rel} \rangle$ starts increasing with increasing $M_{IMF}$.

After establishing the various dependencies of $\langle v_{rel} \rangle$ we can compare them with the experimental data. For this purpose the reaction $\alpha + \text{Au}$ is ideally suited, since all fragments originate from one source, the target
residue. Additionally the recoil velocities are small which facilitates the determination of the velocity and angular correlations with high precision. The reaction has been studied at 1 and 3.6 GeV/nucleon incident energy at the synchrophasotron of the JINR in Dubna using the new 4\pi setup “FASA”\cite{15}. In \cite{16} it has been demonstrated that sufficient excitation energy is reached to induce the multifragment breakup of the system into many IMFs.

The multiplicity of the IMFs was measured by a fragment multiplicity detector (FMD) system covering a large part of the 4\pi solid angle. The energy, velocity and mass of single fragments from the event were determined with high precision using time-of-flight telescopes (TOF). The relative angle and velocity of coincident fragments were measured using a position-sensitive parallel-plate avalanche counter (PPAC). For the FMD and the PPAC an IMF is defined as a fragment with a charge number of 3 \leq Z \leq 15. For TOF it is defined correspondingly as a fragment with a mass number of 6 \leq A \leq 30. The large correlation-angle data were taken by three TOF-PPAC combinations covering the correlation angular range of 105^\circ – 155^\circ, 130^\circ – 180^\circ and 150^\circ – 180^\circ respectively. Efficiencies of both the TOFs and PPACs are taken into account in the data analysis and the model calculations.

The comparison to the experimental data is shown in Fig. 2, which gives the average relative velocity of the IMFs at large relative angles, detected
in TOF and PPAC, as a function of the multiplicity of the IMFs measured in the FMD. In this figure the efficiency of the FMD was not corrected but considered in the respective calculations. Figure 2 shows, that a combination of a freeze-out density of \( \rho_0/7 \) and an excitation energy of 4.5 MeV/nucleon for the lower incident energy and the same freeze-out density but a higher excitation energy of 5.5 MeV/nucleon for the higher incident energy can reproduce the experimental data reasonably well. With this set of parameters, our calculations also well reproduce many other aspects of the experiment simultaneously, including the average IMF multiplicity, the width and the shape of the relative velocity distribution as well as the mass distribution of fragments. To estimate fluctuations of \( <v_{rel}> \) originating from fluctuations of the residue charge and excitation energy we have performed calculations for the case of \( E_{beam}/A = 3.6 \) GeV with Z=62, 70 and 79 respectively. By adjusting the excitation energy between 5.5 and 6.6 MeV/nucleon in order to have a similar average IMF multiplicity of about 5.9, the \( <v_{rel}> \) varies by about 4\% which corresponds to a variation of \( R_c \) by about 5\%, yielding an uncertainty of the breakup density of 15\%.

In summary, using a statistical model coupled with many-body Coulomb trajectory calculations we have studied the average relative velocity of intermediate mass fragments as a function of the multiplicity of IMFs at different but constant excitation energies and freeze-out radii of the fragmenting
source. These two quantities can be determined individually because of the complex and different forms of the dependence of $< \nu_{\text{rel}} >$ on the IMF multiplicity at different excitation energies and freeze-out densities. For the $\alpha + ^{197}\text{Au}$ system at beam energies of 1 and 3.6 GeV/nucleon, the freeze-out density was found to be $\rho_c \approx \rho_0/7$. While the mean excitation energy increases from 4.5 MeV/nucleon to 5.5 MeV/nucleon. The freeze-out density found is very similar to the ones deduced from nucleus-nucleus collisions. Conceivable compression effects in the later seem therefore to play a minor role if at all.

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**Figure Captions**

**Fig. 1** (a) Average relative velocity of IMFs at large correlation angles as a function of the multiplicity of IMFs, (b) the average maximum charge of fragments as a function of the multiplicity of IMFs, (c) the multiplicity distribution of IMFs, and (d) average relative velocity as function of the multiplicity of IMFs for different freeze-out radii.

**Fig. 2** Dependence of the average relative velocity of IMFs at large correlation angles on the multiplicity of IMFs measured in the FMD (not corrected for efficiency) for the system α + Au at 1.0 GeV/nucleon (filled squares: experiment, solid line: calculation with $E^*/A = 4.5 MeV$ and $R_c = 2.3 * A^{1/3}$) and 3.6 GeV/nucleon (filled circles: experiment, dashed line: calculation with $E^*/A = 5.5 MeV$ and $R_c = 2.3 * A^{1/3}$) respectively.