Power Law Plateau Inflation Potential In The RS II Braneworld Evading Swampland Conjecture

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In the recent time, inflationary cosmology is facing an existential crisis due to the proposed Swampland criterion which aims to evade any (meta-)stable de Sitter construction within the String landscape. It is been realised that a single field slow roll inflation is inconsistent with the Swampland criterion unless the inflationary model in realised in some non standard scenario such as Warm inflation or the Braneworld scenario. In \cite{1}, Dimopoulos and Owen introduced a new class of model of inflation dubbed as the power law plateau inflation in the standard cold inflationary scenario. But to realise this model in the standard scenario consistent with observation, they had to introduce a phase of thermal inflation. In this paper we have analysed this model in the braneworld scenario to show that for some choice of the parameters defining the model class, one can have an observationally consistent power law plateau without any phase of thermal inflation. We have also shown that, for the correct choice of model parameters, one can easily satisfy the Swampland criterion. Besides, for a particular choice of the potential one can also satisfy the recently proposed Trans-Planckian Censorship Conjecture.

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I. INTRODUCTION

Cosmological inflation is a paradigm added to the standard hot big bang model, justifying a rapid exponential expansion of the Universe which is required to solve some of the initial condition problems such as ‘horizon’ and ‘flatness’ problem. Apart from the initial motivation to add an inflationary era, it was realised that it is essential to justify the structure formation of the Universe. After the first proposed model of inflation by Guth [3] several different models have been proposed [4–7]. However, with the recent development of observational cosmology, it has became very hard for many of the models to survive the hard reality of observations. It has been argued in some places that inflation is a theoretical paradigm that cannot be refuted by any means which has been advocated by people like Steinhardt [8]. But with the recent theoretical constraints coming from the String theorists, going by the name of Swampland Conjectures (SC) [9–12] or the Trans-Planckian Conjecture (TCC) [13, 14], the whole idea of inflationary cosmology is in an existential crisis. One can obviously argue against the whole notion on which these conjectures are proposed. But String theory being our best hope to have a complete theory including gravity, one might need to be more judicious to even refute or bypass those claims. In this work, we refrain ourself from making any comment on the validity of these conjectures. Rather, with a phenomenologically motivated class of inflationary model, we try to understand the ways to satisfy all the theoretical and observational demands, staying in the premise of the single field models. Single field models of inflation is in loggerheads with both the conjectures at least in the standard cold inflationary scenario. On the other hand, the observational data [15–18] is pointing towards a plateau like potential for the inflationary potential. On that note Dimopoulos and Owen, made an attempt to propose a phenomenological model dubbed as the power law plateau model in the standard cold inflationary scenario. However, it is realised in their paper to have a successful observationally viable model, there is a need of effective number of e-folds ($N_e$) as low as $N_e = 35$. Thus to satisfy that they have introduced a phase of thermal inflation for lower $N_e$ to be allowed. In this paper, we tried to appreciate their model in the Randal-Sundrum brane-world [19] scenario due to the reasons listed below:

• Power law plateau model realised in the RS II brane world [20], is much more robust in the sense that to satisfy the observational bounds on the inflationary observable, there is no need to put by hand the phase of thermal inflation.

• It can give enough number of e-folds to solve the initial motivation of inflation: the flatness problem.

• For almost the whole class of this model in the braneworld, the SC can be well evaded for the correct choice of parameters keeping the observational bounds in mind.

• For particular choice from these class of models, even the TCC can be satisfied.

A. RS II Braneworld

The thermal history of the Universe can differ from the standard scenario once one considers the braneworld scenario [21–23]. The original motivation of such scenario is to solve the hierarchy problem. One of such braneworld model is Randall Sundrum model. The RS II model is considered in our analysis. One of the immediate effects of considering this scenario is the modification of the Friedmann equation. In three brane, the cosmological expansion can be formulated by a more generalized Friedmann equation for an observer which is described by:

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G_N}{3} \rho - \frac{K}{a^2} + \frac{A_4}{3} + \frac{\kappa_5^2}{36} \rho^2 + \frac{\mu}{a^4}. \quad (1.1)$$
where \( a(t) \) is scale factor at time \( t \) and \( \rho \) is matter density in 3 Dimensional (3-D) space, \( G_N \) is the 4 Dimensional (4-D) Gravitational constant and its related to \( \kappa_5 \) which is 5 Dimensional (5-D) Gravitational constant. The last two terms are absent in the standard Friedmann equation. The last term plays key role in the radiation dominated era and dubbed as the ‘dark radiation’. For details reader is advised to study references [24–26]. The fourth term plays key role in the inflationary era and would be the prime focus of our analysis. Again for details reader is advised to study the references [27–32]. In the high energy limit when the Universe is dominated by the scalar field one can rewrite the Friedmann equation as:

\[
H^2 = \frac{V(\phi)}{3M_5^2} \left( 1 + \frac{V(\phi)}{\rho_0} \right).
\]  

(1.2)

Here \( V(\phi) \) is the potential and \( \phi \) is the field and \( \rho_0 \) is the variable which directly depends upon the \( M_5 \) (5 Dimensional Planck Mass) and \( \rho_0 \) is expressed as:

\[
\rho_0 = 12 \frac{M_5^6}{M_P^2}.
\]

(1.3)

The usual slow roll conditions are modified due to the modification in the evolution equation and this will reduced to the standard cosmology when, \( V/\rho_0 << 1 \). In the subsequent section, we will calculate everything in the limit \( V/\rho_0 >> 1 \). As the inflation is only restricted to the brane, the scalar perturbation behaves in similar fashion as the evolution of the standard cosmology after considering the altered Hubble expansion. The presence of extra dimension reshapes the tensor spectrum and gravitational power spectrum [33, 34]. The altered slow roll parameter can be written as:

\[
\epsilon_{RS} = \frac{\ln(H^2/V')}{6H^2}, \quad \eta_{RS} = \frac{V''}{3H^2}.
\]

(1.4)

Here, the prime stands for the derivative with respect to \( \phi \). Thus the observables such as \( P_s \) (Scalar Perturbation), \( n_s \) (Spectral Index), \( r \) (Tensor to Scalar Ratio) gets modified and the slow roll analysis gets modified accordingly.

\[
P_s = \frac{9}{4\pi^2} \frac{H^6}{V'^2}, \quad n_s = 1 - 6\epsilon_{RS} + 2\eta_{RS},
\]

(1.5)

In similar fashion tensor power spectra changes as:

\[
P_T = 8 \left( \frac{H}{2\pi} \right)^2 F(x_0)^2,
\]

(1.6)

Here the extra term \( F(x_0) \) is written :

\[
F(x) = \left( \sqrt{1 + x^2} - x^2 \ln \left[ \frac{1}{x} + \sqrt{1 + \frac{1}{x^2}} \right] \right)^{-1/2},
\]

(1.7)

where, \( x_0 = 2(3H^2/\rho_0)^{1/2} \). This will reduce to standard cosmology when \( x_0 \ll 1 \) and for \( x_0 \gg 1 \) the extra factor can be approximated by \( \sqrt{3x_0/2} \). The tensor to scalar ratio can be written as:

\[
r = P_T/P_s
\]

(1.8)
B. Swampland Conjecture

Swampland Criteria is being proposed by Vafa et al. [2] to evade any (meta-)stable de Sitter constructions within String landscapes. This conjecture make the paradigms of accelerations such as the inflationary epoch or the quintessence dark energy, extremely difficult to survive. It has been shown that the two conjectures if true can rule out all the single field inflationary models at least in the standard cosmology. In recent times another conjecture is proposed namely the Trans-Planckian Conjecture(TCC) [35, 36] which puts these paradigms of single scalar field dominated accelerated expansion theories in more trouble. On that note, the two SC conjectures are[37, 38] :

- SC1: The range traversed by scalar fields in field space has the maximum limit:
  \[
  \left| \frac{\Delta \phi}{M_{Pl}} \right| \leq \Delta \sim O(1) .
  \] (1.9)

- SC2: This criterion limits the gradient of scalar potentials in an EFT as:
  \[
  M_{Pl} \frac{|V'|}{V} \geq c \sim O(1)
  \] (1.10)

SC2 is in direct conflict with the idea of slow roll inflation[39]. As we know the slow roll inflation requires the first slow roll parameter \( \epsilon_s \) defined as: \( \epsilon_s = \frac{M_{Pl}^2 (V'/V)^2}{2} \) has to be less than 1, to have a successful inflationary epoch. It is quite evident that the SC2 and the slow roll requirements are in logger heads. Finally, the TCC demands a very small value of the tensor to scalar ratio \( r \sim 10^{-30} \). Though it was quickly pointed out even with \( r \sim 10^{-8} \) one can satisfy the TCC [40]. Thus keeping all of this things in mind, we explored the class of PLP inflationary model in the RS braneworld.

For the rest of the paper, we have rewritten \( \epsilon_{RS} \) in terms of \( \epsilon_s \) and from conjecture SC2, it implies \( \epsilon_s \sim c^2/2 \sim O(1) \). However, the redefined \( \epsilon_{RS} \) can be less than one to successfully carry out the inflation. The rewritten form is given as:

\[
\epsilon_{RS} = -\frac{\ln(H^2) (2\epsilon_s)^{1/2} V}{6H^2}
\] (1.11)

C. Power-Law Plateau Type Potential

Considering, \( \phi_1 \) and \( \phi_2 \) and \( S \) as the chiral superfields as done in [1] the superpotential can be written as:

\[
W = \frac{S(\phi_1^2 - \phi_2^2)}{2m}
\] (1.12)

where \( m \) is the sub-Planckian scale. Finally, one can get the power law potential (PLP) as:

\[
V = \frac{M^4 \phi^2}{m^2 + \phi^2}
\] (1.13)

where \( M \) is the GUT scale. Thus, one can define a class of model which has the same feature as that in (1.13). The class of potentials can be written as:

\[
V = V_0 \left( \frac{\phi^n}{\phi^n + m^n} \right)^q
\] (1.14)

Here \( m \) is the mass scale, \( \phi \) is the real scalar field. Generally \( n \), and \( q \) are the real parameters. \( V_0 \) is the scale of inflation. Here we assume \( \phi > m \), otherwise it is reduced to to the monomial inflation model \( V \propto \phi^n \) to maintain
the shape of the plateau potential. In the following section we take the assumption \( \frac{m}{\rho} \ll 1 \). We vary \( n, q \) from 1 to 4. This gives us 16 different combination for the different values of \( n, q \) but here we restrict our discussion to only those combination which satisfy both the Swampland conjectures. In the subsequent section we will take \( \epsilon_s = 1 \) which is the second swampland conjecture (SC2). In our analysis the value of \( \phi_i \) is always less than \( M_{Pl} \), thus it is obvious the (SC1) is maintained through out the analysis. In the section II a detailed analysis of PLP inflationary potential in the RS II is carried out. In the section III the reheating phase after the inflation is discussed and finally we have drawn our conclusion in the last section IV.

II. ANALYSIS OF INFLATIONARY OBSERVABLES

For the power law plateau class of inflationary potential, in this section, we have computed \( P_s \), \( n_s \) and \( r \) for different cases as a function of number of e-folds \( (N) \).

A. case \( q = 1 \)

For the case \( n = 1 \) and \( q = 1 \),

\[
n_s = 1 - \frac{2m\rho \left( \frac{3\sqrt{3} \sqrt{2} V_0 N}{3mN\rho + 2^{3/4} m\rho^{3/4} \epsilon_s^{3/4} \sqrt{\frac{\rho}{V_0}} + 2} \right)}{3mN\rho + 2^{3/4} m\rho^{3/4} \epsilon_s^{3/4} \sqrt{\frac{\rho}{V_0}} + 2m^2 N\rho_0}, \quad r = \frac{24 \sqrt{V_0} m^2 \rho}{\left( m\rho \left( 2^{3/4} \sqrt{\rho} \epsilon_s^{3/4} \sqrt{\frac{\rho}{V_0}} + 3N \right) \right)^{4/3}} \tag{2.1}
\]

For the case \( n = 2 \) and \( q = 1 \),

\[
n_s = 1 - \frac{3m^2 \rho_0 \left( \frac{2\sqrt{2} V_0}{\sqrt{\frac{m^{8/3} \rho^{4/3} \epsilon^{2/3}}{V_0} + m^2 N\rho_0}} + 1 \right)}{m^{8/3} \rho^{4/3} \epsilon^{2/3} + m^2 N\rho_0}, \quad r = \frac{12 \sqrt{V_0} m^4 \rho}{\left( m^{8/3} \rho^{4/3} \epsilon^{2/3} V_0 + m^2 N\rho_0 \right)^{3/2}} \tag{2.2}
\]

For \( n = 3 \) and \( q = 1 \),

Following the previous approach we can calculate the \( r \) and \( n_s \):

\[
n_s = 1 - \frac{2m^3 \rho_0 \left( 3 \sqrt{3} \sqrt{2} V_0 \sqrt{\frac{m^{8/3} \rho^{4/3} \epsilon^{5/3}}{V_0} + m^3 N\rho_0}}}{\sqrt{3} \sqrt{m^{8/3} \rho^{4/3} \epsilon^{5/3} + m^3 N\rho_0} + 4}, \quad r = \frac{24 \sqrt{3} V_0^{3/5} m^6 \rho}{\left( m^{8/3} \rho^{4/3} \epsilon^{5/3} V_0 + m^3 N\rho_0 \right)^{8/5}} \tag{2.3}
\]

For \( n = 4 \) and \( q = 1 \),

\[
n_s = 1 - \frac{m^4 \rho_0 \left( 6 \sqrt{3} \sqrt{m^{24/5} \rho_0 \epsilon_s^{3/5} \frac{\sqrt{\rho}}{V_0} + m^4 N\rho_0} \right)}{m^{24/5} \rho_0 \epsilon_s^{3/5} \sqrt{\frac{\rho_0}{V_0} + m^4 N\rho_0} + 5}, \quad r = \frac{12 V_0^{2/3} m^8 \rho}{\left( m^{24/5} \rho_0^{8/5} \epsilon_s^{3/5} \sqrt{\frac{\rho}{V_0} + m^4 N\rho} \right)^{5/3}} \tag{2.4}
\]
FIG. 2.1. Plots of $r$ and $n_s$ as a function $\rho_0$ for fixed values of $V_0 = 10^8\text{GeV}$. In fig (a), $n = 1, q = 1$ and $m = 10^{-6}$, fig (b), $n = 2, q = 1$ and $m = 10^{-4}$, fig (c), $n = 3, q = 1$ and $m = 10^{-3}$ and in fig (d), $n = 4, q = 1$ and $m = 10^{-3}$. The blue line corresponds to $N_e = 55$, the black line corresponds to $N_e = 65$, the green line corresponds to $N_e = 75$. The light pink shaded region corresponds to the 1-$\sigma$ bounds on $n_s$. The violet shaded region corresponds to the 1-$\sigma$ bounds of future CMB observations using same central value for $n_s$. The blue line corresponds to $N_e = 55$, the black line corresponds to $N_e = 65$, the green line corresponds to $N_e = 75$. The light pink shaded region corresponds to the 1-$\sigma$ bounds on $n_s$. The violet shaded region corresponds to the 1-$\sigma$ bounds of future CMB observations using same central value for $n_s$.[41, 42].

B. case $q = 2$

For $n = 1$ and $q = 2$,

$$n_s = 1 - \frac{2m\rho \left( \frac{3 \cdot 2^{5/6} \sqrt{\epsilon} \sqrt[3]{3mN\rho + 2 \sqrt{2} m\rho^{5/2} \epsilon^{3/4} \sqrt{\frac{m}{V_0}}}}{V_0} + 2 \right)}{3m\rho + 2 \sqrt{2} m\rho^{5/2} \epsilon^{3/4} \sqrt{\frac{m}{V_0}}}, \quad r = \frac{24 \cdot 2^{2/3} \sqrt{V_0} m^2 \rho}{\left( m\rho \left( 2 \sqrt{2} \sqrt{\epsilon} \sqrt[3]{3mN\rho + 3N} \right) \right)^{4/3}}$$

(2.5)

For the case $n = 2$ and $q = 2$,

$$n_s = 1 - \frac{3m^2 \rho \left( 2 \sqrt{2} \sqrt{\epsilon} \sqrt[3]{\frac{2 \sqrt{m^8/3} \rho^{4/3} \epsilon^{2/3}}{V_0^2} + 2m^2 N\rho_0}}{V_0} + 1 \right)}{\sqrt{2} \sqrt{m^8/3} \rho^{4/3} \epsilon^{2/3} \sqrt{V_0^2} + 2m^2 N\rho_0}, \quad r = \frac{12 \sqrt{2} \sqrt{V_0} m^4 \rho}{\left( \frac{2 \sqrt{2} \sqrt{m^8/3} \rho^{4/3} \epsilon^{2/3}}{V_0^2} + 2m^2 N\rho \right)^{3/2}}$$

(2.6)

For $n = 3$ and $q = 2$,
FIG. 2.2. Plots of $r$ and $n_s$ as a function of $\rho_0$ for fixed values of $V_0 = 10^6 \text{GeV}$. In fig (a), $n = 1, q = 2$ and $m = 10^{-6}$, fig (b), $n = 2, q = 2$ and $m = 10^{-3}$, fig (c). $n = 3, q = 2$ and $m = 10^{-3}$ and in fig (d), $n = 4, q = 2$ and $m = 10^{-3}$. The blue line corresponds to $N_e = 55$, the black line corresponds to $N_e = 65$, the green line corresponds to $N_e = 75$. The light pink shaded region corresponds to the 1-$\sigma$ bounds on $n_s$. The violet shaded region corresponds to the 1-$\sigma$ bounds of future CMB observations using same central value for $n_s$[41, 42].

\[ n_s = 1 - \frac{2m^3 \rho_0}{\left( \frac{3 \sqrt{2 \pi} \sqrt{\pi} \left( \frac{2^{7/8} \sqrt{2} V m^{15/4} / \rho_0^{5/4} \sqrt{5/8}}{\sqrt{V_0}} + 5m^3 N \rho_0 \right)} + 4 \right) \}, \quad r = \frac{24 \sqrt{2} V_0^{3/5} m^6 \rho}{\left( \frac{2^{7/5} \sqrt{2} V m^{15/8} / \rho_0^{5/8} + 5m^3 N \rho_0} + 5 \right)}^{8/5} } \tag{2.7} \]

For $n = 4$ and $q = 2$,

\[ n_s = 1 - \frac{m^4 \rho}{\sqrt{2} m^{24/5} \rho_0^{5/4} / \sqrt{5/5} \sqrt{V_0} + 3m^4 N \rho}, \quad r = \frac{12 \sqrt{2} V_0^{2/3} m^8 \rho}{\left( \frac{\sqrt{2} m^{24/5} / \rho_0^{5/8} + 3m^4 N \rho} + 5 \right)}^{5/3} } \tag{2.8} \]

C. case $q = 3$

For the case $n = 1$ and $q = 3$,
For the case $n = 2$ and $q = 3$,

\[
 n_s = 1 - \frac{3m^2\rho}{\sqrt[3]{\frac{3m^6/3 \rho^4/3 \varepsilon_3^{2/3}}{\sqrt{V_0}} + 2m^2N\rho}} + 1, \quad r = \frac{12\sqrt[3]{V_0}m^4\rho}{\sqrt[3]{\frac{3m^6/3 \rho^4/3 \varepsilon_3^{2/3}}{\sqrt{V_0}} + 2m^2N\rho}}^{3/2} \tag{2.10}
\]

For $n = 3$ and $q = 3$,

\[
 n_s = 1 - \frac{2m^3\rho}{\sqrt[3]{\frac{3.3^{2/5} m^{15/4} \rho^{4/5} \varepsilon_3^{3/5}}{\sqrt{V_0}} + 5m^3N\rho}} + 4, \quad r = \frac{24 m^6\rho}{\sqrt[3]{\frac{3m^{15/4} \rho^{4/5} \varepsilon_3^{3/5}}{\sqrt{V_0}} + 5m^3N\rho}}^{8/5} \tag{2.11}
\]

For $n = 4$ and $q = 3$,

\[
 n_s = 1 - \frac{m^4\rho}{\sqrt[3]{\frac{6 \sqrt[3]{m^{21/5} \rho^{6/5} \varepsilon_3^{3/5}}}{\sqrt{V_0}} + 3m^4N\rho}} + 5, \quad r = \frac{12 \sqrt[3]{V_0}2^{2/3} m^8\rho}{\sqrt[3]{\frac{6 \sqrt[3]{m^{21/5} \rho^{6/5} \varepsilon_3^{3/5}}}{\sqrt{V_0}} + 3m^4N\rho}}^{5/3} \tag{2.12}
\]

D. case $q = 4$

For the case $n = 1$ and $q = 4$,

\[
 n_s = 1 - \frac{2m\rho}{\sqrt[3]{\frac{3^{\frac{3}{2}} \sqrt[3]{2} \sqrt[3]{2^{3/4} m^{3/2} \rho^{3/2} \varepsilon_3^{3/4}}}{\sqrt{V_0}} + 2mN\rho}} + 2, \quad r = \frac{48 \sqrt[3]{V_0}m^2\rho}{\sqrt[3]{\frac{3^{\frac{3}{2}} \sqrt[3]{2} \sqrt[3]{2^{3/4} m^{3/2} \rho^{3/2} \varepsilon_3^{3/4}}}{\sqrt{V_0}} + 3mN\rho}}^{4/3} \tag{2.13}
\]

For the case $n = 2$ and $q = 4$,

\[
 n_s = 1 - \frac{3m^2\rho}{\sqrt[3]{\frac{3^{\frac{3}{2}} \sqrt[3]{m^{6/3} \rho^{4/3} \varepsilon_3^{2/3}}}{\sqrt{V_0}} + 8m^2N\rho}} + 4, \quad r = \frac{192 \sqrt[3]{V_0}m^4\rho}{\sqrt[3]{\frac{3^{\frac{3}{2}} \sqrt[3]{m^{6/3} \rho^{4/3} \varepsilon_3^{2/3}}}{\sqrt{V_0}} + 8m^2N\rho}}^{3/2} \tag{2.14}
\]
For $n = 3$ and $q = 4$, 

$$n_s = 1 - \frac{2m^3\rho}{\frac{2\sqrt[3]{2^{3/2}\sqrt{m^{15/4}\rho^{3/4}e^{5/8}} + 5m^3N\rho}}{\sqrt[12]{V_0}}} + 4$$

For $n = 4$ and $q = 4$, 

$$n_s = 1 - \frac{m^4\rho}{\frac{2^{2/5}m^{24/5}\rho^{6/5}e^{3/5}}{\sqrt[15]{V_0}} + 3m^4N\rho} + 5$$

$$r = \frac{24\frac{2^{4/5}3^{2/5}V_0^{3/5}m^6\rho}{\frac{2\sqrt[3]{2^{3/2}\sqrt{m^{15/4}\rho^{3/4}e^{5/8}} + 5m^3N\rho}}{\sqrt[12]{V_0}}}^{8/5}}{5m^4N\rho}$$

FIG. 2.3. Plots of $r$ and $n_s$ as a function $\rho_0$ for fixed values of $V_0 = 10^6$GeV. In fig (a), $n = 1$, $q = 3$ and $m = 10^{-6}$, fig (b), $n = 2$, $q = 3$ and $m = 10^{-5}$, fig (c), $n = 3$, $q = 3$ and $m = 10^{-3}$ and in fig (d), $n = 4$, $q = 3$ and $m = 10^{-3}$. The blue line corresponds to $N_e = 55$, the black line corresponds to $N_e = 65$, the green line corresponds to $N_e = 75$. The light pink shaded region corresponds to the 1-$\sigma$ bounds on $n_s$. The violet shaded region corresponds to the 1-$\sigma$ bounds of future CMB observations using same central value for $n_s$ [41, 42].
FIG. 2.4. Plots of $r$ and $n_s$ as a function of $\rho_0$ for fixed values of $V_0 = 10^6\text{GeV}$. In fig (a), $n = 1, q = 4$ and $m = 10^{-6}$, fig (b). $n = 2, q = 4$ and $m = 10^{-3}$, fig (c). $n = 3, q = 4$ and $m = 10^{-3}$ and in fig (d). $n = 4, q = 4$ and $m = 10^{-3}$. The blue line corresponds to $N_e = 55$, the black line corresponds to $N_e = 65$, the green line corresponds to $N_e = 75$. The light pink shaded region corresponds to the $1-\sigma$ bounds on $n_s$. The violet shaded region corresponds to the $1-\sigma$ bounds of future CMB observations using same central value for $n_s$ [41, 42].

| $q$ | $n$ | $m(\text{GeV})$ | $M_5 \times 10^{10}(\text{GeV})$ | $N$ | $r \times 10^{-6}$ | $n_s$ | $A_s \times 10^{-9}$ |
|-----|-----|-----------------|----------------------------------|-----|------------------|-------|------------------|
| 1   | 1   | $10^{12}$       | 3.30                             | 65  | 0.393            | 0.965878 | 2.08             |
|     | 2   | $10^{14}$       | 5.50                             | 55  | 0.049            | 0.964624 | 2.08             |
|     | 3   | $10^{15}$       | 4.16                             | 55  | 0.024            | 0.964117 | 2.08             |
|     | 4   | $10^{16}$       | 3.77                             | 65  | 0.079            | 0.965243 | 2.08             |
| 2   | 1   | $10^{12}$       | 3.22                             | 75  | 0.543            | 0.967437 | 2.07             |
|     | 2   | $10^{15}$       | 3.06                             | 75  | 0.927            | 0.963407 | 2.22             |
|     | 3   | $10^{15}$       | 4.16                             | 65  | 0.024            | 0.968557 | 2.06             |
|     | 4   | $10^{16}$       | 3.66                             | 65  | 0.112            | 0.964426 | 2.09             |
| 3   | 1   | $10^{12}$       | 3.13                             | 75  | 0.752            | 0.966186 | 2.07             |
|     | 2   | $10^{15}$       | 3.01                             | 75  | 1.191            | 0.962066 | 2.09             |
|     | 3   | $10^{15}$       | 2.98                             | 65  | 0.031            | 0.968153 | 2.07             |
|     | 4   | $10^{16}$       | 3.57                             | 65  | 0.143            | 0.964009 | 2.26             |
| 4   | 1   | $10^{12}$       | 3.06                             | 75  | 0.951            | 0.965526 | 2.12             |
|     | 2   | $10^{15}$       | 2.96                             | 75  | 1.441            | 0.965881 | 2.08             |
|     | 3   | $10^{15}$       | 4.03                             | 65  | 0.036            | 0.967838 | 2.04             |
|     | 4   | $10^{15}$       | 3.56                             | 65  | 0.158            | 0.963523 | 2.09             |

TABLE 2.1. Values of different cosmological observables for the different combination of $n, q, m, N$ and $M_5$

### III. REHEATING ANALYSIS

After the end of the inflation an epoch of reheating is required to have the approximate temperature of the universe to start the BBN process [43–48]. First idea of the reheating was coined by Linde[4]. After the end of inflation the
inflaton field oscillates to the minima of the potential and decays into the elementary particles\[49\]. These particles interact with each other through some suitable coupling and reheat the universe and achieves a temperature $T_{re}$ which is reheating temperature. This process is called reheating. This process can be studied either by perturbative reheating or through parametric resonance called preheating\[50, 51\]. In this article, rather than focussing on the actual mechanism behind the reheating for the power law plateau model, we tried to understand epoch indirectly. If one consider the $\omega_{re}$ to be constant during the reheating epoch the relation between the energy density and the scale factor using $\rho \propto a^{-3(1+w)}$ can be established as\[52–55\]

$$\rho_{end} = \left(\frac{a_{end}}{a_{re}}\right)^{-3(1+w_{re})}.$$  \hspace{1cm} (3.1)

Here subscript $end$ is defined as the end of inflation and $re$ is the end of reheating epoch. Replacing $\rho_{end}$ by $(7/6)V_{end}$ following\[57\] one can write

$$N_{re} = \frac{1}{3(1+w_{re})} \ln \left(\frac{\rho_{end}}{\rho_{re}}\right) = \frac{1}{3(1+w_{re})} \ln \left(\frac{7}{6} \frac{V_{end}}{\rho_{re}}\right).$$  \hspace{1cm} (3.2)

We follow the standard relation between density and temperature:

$$\rho_{re} = \frac{\pi^2}{30} g_{re} T_{re}^4.$$  \hspace{1cm} (3.3)

Here $g_{re}$ is the number of relativistic species at the end of reheating. Using (3.1) and (3.2), we can established the relation between $T_{re}$ and $N_{re}$ as:

$$N_{re} = \frac{4}{(1-3w_{re})} \left[61.5475 - \ln \left(\frac{V_{end}^{1/4}}{H_k}\right) - N_{k}\right].$$  \hspace{1cm} (3.4)

Assuming that the entropy is conserved between the reheating and today, we can write

$$T_{re} = T_0\left(\frac{a_0}{a_{eq}}\right) \left(\frac{43}{11g_{re}}\right)^{1/4} = T_0 \left(\frac{a_0}{a_{eq}}\right) e^{N_{RD}} \left(\frac{43}{11g_{re}}\right)^{1/4}.$$  \hspace{1cm} (3.5)

Here $N_{RD}$ is the number of e-folds during radiation era and $e^{-N_{RD}} \equiv a_{re}/a_{eq}$. The ratio $a_0/a_{eq}$ can be written as

$$\frac{a_0}{a_{eq}} = \frac{a_0 H_k}{k} e^{-N_k} e^{-N_{re}} e^{-N_{RD}}.$$  \hspace{1cm} (3.6)

Using the relation $k = a_k H_k$ and using the Eqs. (3.4), (3.5) and (3.6), assuming $w_{re} \neq \frac{1}{3}$ and $g_{re} \approx 226$ (degrees of freedom for a supersymmetric model), we can established the expression for $N_{re}$

$$N_{re} = \frac{4}{(1-3w_{re})} \left[61.5475 - \ln \left(\frac{V_{end}^{1/4}}{H_k}\right) - N_{k}\right].$$  \hspace{1cm} (3.7)

Here we have used Planck’s pivot ($k$) of order 0.05 Mpc$^{-1}$. In a similar way we can calculate $T_{re}$:

$$T_{re} = \left[\left(\frac{43}{11g_{re}}\right)^{1/4} \frac{a_0 T_0}{k} H_k e^{-N_k} \left[\frac{35V_{end}}{\pi^2 g_{re}}\right]^{-1/4} e^{-3(1+w_{re})}\right]^{1/3}.$$  \hspace{1cm} (3.8)

To evaluate $N_{re}$ and $T_{re}$ first we need to calculate the $H_k$, $N_k$ and $V_{end}$ for the given potential. Now $H_k$ can be represented in terms of $r$ and $A_s$ as:

$$H_k = \left[\frac{1}{6} \left(\frac{\rho}{3} \pi^2 A_s r\right)\right]^{1/2}.$$  \hspace{1cm} (3.9)
We restrict our reheating analysis for the class of potentials which satisfies the Trans Planckian Censorship Conjecture [13, 14] like potentials for the choice of \( n = 3, q = 1 \) and \( n = 3, q = 2 \). We have plotted \( N_{re} \) and \( T_{re} \) as a function of \( n_s \), for the mentioned potential it is not possible to write \( r \) as a function of \( n_s \). So here we will use the numerical method calculating \( r \) and \( n_s \). For a range of e-folding and doing the necessary cubic fitting we can write the two equations as:

**For \( n = 3 \) and \( q = 1 \)**

\[
r = -1.62561 \times 10^{-13} N_s^3 + 4.31547 \times 10^{-11} N_s^2 - 4.02308 \times 10^{-9} N_s + 1.43247 \times 10^{-7} \tag{3.10}
\]

\[
n_s = 9.71953 \times 10^{-8} N_s^3 - 0.000269363 N_s^2 + 0.0027286 N_s + 0.879288 \tag{3.11}
\]

**For \( n = 3 \) and \( q = 2 \)**

\[
r = -2.14466 \times 10^{-13} N_s^3 + 5.69341 \times 10^{-11} N_s^2 - 5.30772 \times 10^{-9} N_s + 1.87807 \times 10^{-7} \tag{3.12}
\]

\[
n_s = 9.91911 \times 10^{-8} N_s^3 - 0.000274979 N_s^2 + 0.0027873 N_s + 0.876415 \tag{3.13}
\]

![FIG. 3.1. For the choice of \( n = 3, q = 1 \), Plots of \( N_{re} \) and \( T_{re} \) as a function \( n_s \) for different values of \( w_{re} \). The red line corresponds to \( w_{re} = -1/3 \), the green line corresponds to \( w_{re} = 0 \), the blue line corresponds to \( w_{re} = 2/3 \) and finally the black line corresponds to \( w_{re} = 1 \). The light pink shaded region corresponds to the 1-\( \sigma \) bounds on \( n_s \) from Planck'18. The violet shaded region corresponds to the 1-\( \sigma \) bounds of future CMB observations [41, 42] using same central value for \( n_s \).](image1)

![FIG. 3.2. For the choice of \( n = 3, q = 2 \), Plots of \( N_{re} \) and \( T_{re} \) as a function \( n_s \) for different values of \( w_{re} \). The red line corresponds to \( w_{re} = -1/3 \), the green line corresponds to \( w_{re} = 0 \), the blue line corresponds to \( w_{re} = 2/3 \) and finally the black line corresponds to \( w_{re} = 1 \). The light pink shaded region corresponds to the 1-\( \sigma \) bounds on \( n_s \) from Planck'18. The violet shaded region corresponds to the 1-\( \sigma \) bounds of future CMB observations [41, 42] using same central value for \( n_s \).](image2)

Using the above Eq. (3.10 - 3.13), we can write the Eq. (3.9) as a function of \( n_s \). And from the end of inflation condition \( \epsilon_{RS} = 1 \), one can calculate the \( V_{end} \) and then get \( T_{re} \) and \( N_{re} \) by using Eq. (3.7), (3.8) and (3.9) for different values of equation of state (\( w_{re} \)). The changes of \( T_{re} \) and \( N_{re} \) for different values of \( w_{re} \) is shown in Fig. III. We would like to mention that the merging points for the \( T_{re} \) plot and the \( N_{re} \) plot correspond to the instant reheating scenario thus making \( N_{re} = 0 \).
IV. CONCLUSION

In this article we have studied the power law plateau model in the light of recent CMB observations. We have shown that in the RSII brane world scenario it is easy to evade the Swampland Conjunction and for certain choices of inflationary parameters we can also satisfy the Trans Planckian Censorship Conjecture. For the choice of $n = 1, q = 1$ to $n = 4, q = 4$ all the cosmological observables are well within Planck'18 bounds and Swampland conjecture can be satisfied. We have taken the number of e-foldings between $55 - 75$, which are consistent with the RSII braneworld [56]. For certain choices of $n$ and $q$ it is possible to satisfy the TCC which is shown in the Table (2.1), with tensor to scalar ratio($r$) of the order of $10^{-8}$. We have also studied the reheating epoch. Though the reheating analysis can be done for all the potentials but we have shown for only two cases which also satisfies the TCC. It is worthwhile to mention that to have the swampland condition satisfying model of power law inflation in the braneworld, it is the observational constraints which shows $M_5$ of the order of $10^{-2} M_P$ which is consistent with [57]. Though we have shown that the class of potentials is consistent with observation while satisfying swampland, it will be interesting to get the correct parameter estimation of the class of this model directly from observation. We would like to come back to this in future.

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