Non-flat universe and interacting dark energy model

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Abstract

For non-flat universe of $k \neq 0$, we investigate a model of the interacting holographic dark energy with cold dark matter (CDM). There exists a mixture of two components arisen from decaying of the holographic dark energy into CDM. In this case we use the effective equations of state ($\omega^\text{eff}_\Lambda$, $\omega^\text{eff}_m$) instead of the native equations of state ($\omega_\Lambda$, $\omega_m$). Consequently, we show that interacting holographic energy models in non-flat universe cannot accommodate a transition from the dark energy to the phantom regime.

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1 Introduction

Recent observations from Supernova (SN Ia) [1] and large scale structure [2] imply that our universe is accelerating. Also cosmic microwave background observations [3, 4] provide an evidence for the present acceleration. A combined analysis of cosmological observations shows that the present universe consists of 70% dark energy and 30% dust matter including CDM and baryons.

Although there exist a number of dark energy models, a promising candidate is the cosmological constant. However, one has the two famous cosmological constant problems: the fine-tuning and coincidence problems. In order to solve these problems, we need a dynamical cosmological constant model derived by the holographic principle. The authors in [5] showed that in quantum field theory, the UV cutoff $\Lambda$ could be related to the IR cutoff $L_\Lambda$ due to the limit set by introducing a black hole (the effects of gravity). In other words, if $\rho_\Lambda = \Lambda^4$ is the vacuum energy density caused by the UV cutoff, the total energy of system with the size $L_\Lambda$ should not exceed the mass of the black hole with the same size $L_\Lambda$: $L_\Lambda^3 \rho_\Lambda \leq 2M_p^2 L_\Lambda$. If the largest cutoff $L_\Lambda$ is chosen to be the one saturating this inequality, the holographic energy density is given by $\rho_\Lambda = 3c^2M_p^2/8\pi L_\Lambda^2$ with a constant $c \geq 1$. The lower limit of $c$ is protected by the entropy bound. Here we regard $\rho_\Lambda$ as a dynamical cosmological constant. Taking $L_\Lambda$ as the size of the present universe, the resulting energy is close to the present dark energy [6]. However, this approach with $L_\Lambda = 1/H$ is not complete because it fails to recover the equation of state (EoS) for the dark energy-dominated universe [7]. Further studies in [8, 9, 10, 11] have shown that choosing the future event horizon as the IR cutoff leads to an accelerating universe with $\omega_\Lambda = -1/3 - 2\sqrt{\Omega_\Lambda}/3c$.

On the other hand, the interacting dark energy models provided a new direction to understand the dark energy [12, 13, 14]. The authors in [15] introduced an interacting holographic dark energy model where an interaction exists between holographic energy and CDM. They derived the phantom-phase of $\omega_\Lambda < -1$ using the native EoS $\omega_\Lambda$. However, it turned out that the interacting holographic dark energy model could not describe a phantom regime of $\omega_\Lambda^{\text{eff}} < -1$ when using the effective equation of state $\omega_\Lambda^{\text{eff}}$ [16]. A key of this system is an interaction between two matters. Their contents are changing due to energy transfer from holographic energy to CDM until the two components are comparable. If there exists a source/sink in the right-hand side of the continuity equation, we must be careful to define its EoS. In this case the effective EoS is the only candidate to represent the state of the mixture of two components arisen from decaying of the holographic energy into CDM. This is different from the non-interacting case which is described by
the native EoS. More recently, it was shown that for non-flat universe of \( k \neq 0 \) [17, 18], the interacting holographic dark energy model could not describe a phantom regime of \( \omega^\text{eff}_\Lambda < -1 \) [19].

In this work, we wish to address this issue again because the previous works contain a few of ambiguous points. We solve two coupled differential equations for density parameters \( \Omega_\Lambda \) and \( \Omega_k \) numerically. Furthermore, we introduce a general form of interaction \( Q \) to find the CDM-dominated universe with \( \omega^\text{eff}_m = 0 \) at the far past. We confirm that the phantom-phase is not found from interacting holographic dark energy models.

2 Interacting model in non-flat universe

Let us imagine a universe made of CDM \( \rho_m \) with \( \omega_m = 0 \), but obeying the holographic principle. In addition, we propose that the holographic energy density \( \rho_\Lambda \) exists with \( \omega_\Lambda \geq -1 \). If one assumes a form of the interaction \( Q = \Gamma \rho_\Lambda \), their continuity equations take the forms

\[
\dot{\rho}_\Lambda + 3H(1 + \omega_\Lambda)\rho_\Lambda = -Q, \quad (1)
\]
\[
\dot{\rho}_m + 3H\rho_m = Q. \quad (2)
\]

This shows that the mutual interaction could provide a mechanism to the particle production. Actually, this is a decaying of the holographic energy component into CDM with the decay rate \( \Gamma \). Taking a ratio of two energy densities as \( r_m = \rho_m/\rho_\Lambda \), the above equations lead to

\[
\dot{r}_m = 3Hr_m\left[\omega_\Lambda + \frac{1 + r_m \Gamma}{r_m 3H}\right], \quad (3)
\]

which means that the evolution of the ratio depends on the explicit form of interaction. Even if one starts with \( \omega_m = 0 \) and \( \omega_\Lambda = -1 \), this process is necessarily accompanied by the different equations of state \( \omega^\text{eff}_m \) and \( \omega^\text{eff}_\Lambda \). The decaying process impacts their equations of state and particularly, it induces the negative effective EoS of CDM. Interestingly, an accelerating phase could arise from a large effective non-equilibrium pressure \( \Pi_m \) defined as \( \Pi_m = -\Gamma \rho_\Lambda/3H(= -\Pi_\Lambda) \). Then the two equations (1) and (2) are translated into those of the two dissipatively imperfect fluids

\[
\dot{\rho}_\Lambda + 3H\left[1 + \omega_\Lambda + \frac{\Gamma}{3H}\right]\rho_\Lambda = \dot{\rho}_\Lambda + 3H\left[(1 + \omega_\Lambda)\rho_\Lambda + \Pi_\Lambda\right] = 0, \quad (4)
\]
\[
\dot{\rho}_m + 3H\left[1 - \frac{1}{r_m 3H}\right]\rho_m = \dot{\rho}_m + 3H(\rho_m + \Pi_m) = 0. \quad (5)
\]
The positivity of $\Pi_\Lambda > 0$ shows a decaying of holographic energy density via the cosmic frictional force, while $\Pi_m < 0$ induces a production of the mixture via the cosmic anti-frictional force simultaneously [20, 21]. This is a sort of the vacuum decay process to generate a particle production within the two-fluid model [22]. As a result, a mixture of two components will be created. From Eqs.(1) and (5), turning on the interaction term, we define their effective equations of state as

$$\omega_{\Lambda}^{\text{eff}} = \omega_\Lambda + \frac{\Gamma}{3H}, \quad \omega_m^{\text{eff}} = -\frac{1}{r_m} \frac{\Gamma}{3H}. \tag{6}$$

In this work, we choose the general decay rate of $\Gamma = 3b^2(1 + r_m)^nH$ with the coupling constant $b^2$ and $n \leq 1$ [23]. For $n > 1$, $\omega_\Lambda^{\text{eff}}$ diverges for small $\Omega_\Lambda$, while for $n < 1$, one finds $\omega_m^{\text{eff}} = 0$ for $\Omega_\Lambda = 0$ which is better in agreement with the data. On the other hand, the first Friedmann equation for $k \neq 0$ is given by

$$H^2 = \frac{8\pi}{3M_p^2} \left[ \rho_\Lambda + \rho_m \right] - \frac{k}{a^2}. \tag{7}$$

Differentiating Eq.(7) with respect to the cosmic time $t$ and then using Eqs.(1) and (2), one finds the second Friedmann equation as

$$\dot{H} = -\frac{3}{2} H^2 \left[ 1 + \frac{\omega_\Lambda}{1 + r_m} \right] - \frac{1}{2} \frac{k}{a^2} \left[ 1 + \frac{3\omega_\Lambda}{1 + r_m} \right] \tag{8}$$

which is useful to study the evolution when choosing $L_\Lambda = 1/H$. Let us introduce density parameters

$$\Omega_m = \frac{8\pi\rho_m}{3M_p^2H^2}, \quad \Omega_\Lambda = \frac{8\pi\rho_\Lambda}{3M_p^2H^2}, \quad \Omega_k = \frac{k}{a^2H^2}. \tag{9}$$

which allow to rewrite the first Friedmann equation as

$$\Omega_m + \Omega_\Lambda = 1 + \Omega_k. \tag{10}$$

Then we can express $r_m$ and $r_k = \rho_k/\rho_\Lambda$ in terms of $\Omega_\Lambda$ and $\Omega_k$ as

$$r_m = \frac{1 - \Omega_\Lambda + \Omega_k}{\Omega_\Lambda}, \quad r_k = \frac{\Omega_k}{\Omega_\Lambda}. \tag{11}$$

3 Non-flat universe with the future event horizon

In the case of $\rho_\Lambda$ with Hubble horizon ($L_\Lambda = 1/H$), we always have a fixed ratio $r_m$ of two energy densities. This provides the same negative EoS for both two components [11, 21].
For a null geodesic, we introduce the future event horizon 

\[ L = R_{\text{FH}} = a\chi_{\text{FH}}(t) = a\chi_k^{\text{FH}}(t) \]

with \( \chi_{\text{FH}}(t) = \int_{t}^{\infty} \frac{dt}{a} \). \( (12) \)

Here the comoving horizon size is given by

\[ \chi_k^{\text{FH}}(t) = \int_{0}^{r(t)} \frac{dr}{\sqrt{1 - kr^2}} = \frac{1}{\sqrt{|k|}} \sin^{-1}\left[ \sqrt{|k|} r(t) \right], \]

where leads to \( \chi_k^{\text{FH}=1}(t) = \sin^{-1}r(t) \), \( \chi_k^{\text{FH}=0}(t) = r(t) \), and \( \chi_k^{\text{FH}=-1}(t) = \sinh^{-1}r(t) \). For our purpose, we obtain the comoving radial coordinate \( r(t) \),

\[ r(t) = \frac{1}{\sqrt{|k|}} \sin^{-1}\left[ \sqrt{|k|} \chi_k^{\text{FH}}(t) \right]. \] \( (14) \)

The definition of \( L = ar(t) \) \(^1\) is useful for non-flat universe \( \[17\] \), which leads to

\[ \dot{L} = HL + a\dot{r} = \frac{c}{\sqrt{\Omega_\Lambda}} - \cos ny, \]

where \( \cos ny = \cos y \) for \( k = 1 \), \( y \) for \( k = 0 \), and \( \cosh y \) for \( k = -1 \) with \( y = \sqrt{|k|}R_{\text{FH}}/a \).

Using Eq. \( (14) \) together with \( L = ar(t) \), we rewrite it as \( \cos ny = \sqrt{1 - c^2 \Omega_\Lambda} \) in terms of \( \Omega_k \) and \( \Omega_\Lambda \). \( (15) \)

Using the definition of \( \rho_\Lambda \) and \( (15) \), one finds the equation of state

\[ \dot{\rho}_\Lambda + 3H \left[ 1 - \frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda}}{3c} \cos ny \right] \rho_\Lambda = 0. \] \( (16) \)

From Eqs. \((4), (6)\) and \((16)\), we find the effective equation of state

\[ \omega_{\Lambda}^{\text{eff}}(x) = -\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda(x)}}{3c} \cos ny. \] \( (17) \)

On the other hand, the effective equation of state for CDM is given differently by

\[ \omega_m^{\text{eff}}(x) = -\frac{b^2}{\Omega_\Lambda^{-1}(1 + \Omega_k)} \frac{(1 + \Omega_k)^n}{1 - \Omega_\Lambda + \Omega_k}. \] \( (18) \)

Now we are in a position to derive two coupled equations whose solutions determine the effective equations of state. Eq. \((3)\) leads to one differential equation for \( \Omega_\Lambda \)

\[ \frac{d\Omega_\Lambda}{dx} = -3\Omega_\Lambda(1 - \Omega_\Lambda + \Omega_k)(\omega_\Lambda^{\text{eff}} - \omega_m^{\text{eff}}) + \Omega_k \Omega_\Lambda(1 + 3\omega_\Lambda^{\text{eff}}) \] \( (19) \)

\(^1\)Definitely, \( L = a\chi_{\text{FH}}(t) \) is the proper distance, while \( L = ar(t) \) is the radius of the event horizon measured on the surface of the horizon to define the proper surface area \( [25, 26] \). In this work, we choose \( L = ar(t) \) to define the IR cutoff for non-flat universe.
with $x = \ln a$. The other differential equation for $\Omega_k$ comes from the derivative of $r_k$ in Eq. (11) using Eq. (10) as

$$\frac{d\Omega_k}{dx} = -3\Omega_k(1 - \Omega_A + \Omega_k)(\omega_{\Lambda}^{\text{eff}} - \omega_m^{\text{eff}}) + \Omega_k (1 + \Omega_k)(1 + 3\omega_{\Lambda}^{\text{eff}}).$$

At this point, we compare our equations of (8), (19) and (20) with those in [18]. Using $d\Omega_k/dx = \Omega_k(1+\Omega_k)+3\Omega_k\Omega_\Lambda\omega_\Lambda$, these correspond to (5), (6), and (8) in [18], respectively. Hence our model is the same as in [18]. In the case of $\Omega_k = 0$, equation (16) leads to the well-known form in [23]. From Eqs. (16) and (20), we find a future fixed point of $\Omega_\Lambda$ which satisfies $\omega_{\Lambda}^{\text{eff}} = \omega_m^{\text{eff}}$ and $\Omega_k = 0$. If one drops off the interaction ($\Gamma = b^2 = 0$), the dark energy evolution for a flat universe of $k = 0$ usually proceeds from the past fixed point $\Omega_\Lambda = 0$ to the other future point $\Omega_\Lambda = 1$. If the interaction is turned on, $\Omega_\Lambda$ approaches a fixed asymptotic value less than 1 for large time.

In order to obtain solution, we have to solve the above coupled equations numerically by considering the initial condition at present time$^2$

$$\frac{dH_0}{dx}\bigg|_{x=0} > 0, \quad \Omega_\Lambda^0 = 0.72, \quad \Omega_{k=1}^0 = 0.01/\Omega_{k=0}^0 = 0.0/\Omega_{k=-1}^0 = -0.01.$$
Figure 2: (color online) For $b^2 = 0.2$ and $c = 1$, $k = 0$ evolution of $\Omega_\Lambda$ (black) and the effective equations of state, $\omega^{\text{eff}}_m$ (green) and $\omega^{\text{eff}}_\Lambda$ (blue). The left picture is for an interaction of $n = 1$ and the right picture is for $n = 1/2$.

Figure 3: (color online) For $b^2 = 0.2$ and $c = 1$, $k = -1$ evolution of $\Omega_\Lambda$ (black) and $\Omega_k$ (red) and the effective equations of state, $\omega^{\text{eff}}_m$ (green) and $\omega^{\text{eff}}_\Lambda$ (blue). The left picture is for an interaction of $n = 1$ and the right picture is for $n = 1/2$.

4 Discussions

The noninteracting picture with $L_\Lambda = R_{\text{FH}}$ has the natural tendency such that a ratio $r_m$ of two densities $\rho_m$ and $\rho_\Lambda$ decreases as the universe evolves $[8]$. In this case the energy-momentum conservation is required for each matter separately. Also the natural tendency holds even for the case including an interaction between the holographic dark energy and CDM $[15]$. They used the native EoS $\omega_\Lambda$ to show that $\rho_\Lambda$ can describe the phantom regime. However, we have to use the effective EoS $\omega^{\text{eff}}_\Lambda$ in the presence of the interaction. As are shown in Figs. 1, 2, and 3, two effective equations of state start
Figure 4: (color online) For $b^2 = 0.001$ and $c = 1$, $k = 0$ evolution of $\Omega_\Lambda$ (black) and the effective equations of state, $\omega_m^{\text{eff}}$ (green) and $\omega_\Lambda^{\text{eff}}$ (blue). The left picture is for an interaction of $n = 1$ and the right picture is for $n = 1/2$.

differently. However, two effective equations of state will take the same negative value which is greater than $-1$ in the far future. Also this value could be estimated from the future fixed point.

The vacuum decay picture is still alive even for a dynamical evolution in the interacting holographic dark energy model. This implies that one cannot generate a phantom-like mixture of $\omega_\Lambda^{\text{eff}} < -1$ from an interaction between the holographic dark energy and CDM. In other words, decaying from the holographic dark energy into the CDM never leads to the phantom regime. Figs. 1, 2, and 3 show clearly that the density parameter $\Omega_\Lambda$ approaches 0.78 with $b^2 = 0.2$ and $c = 1$, irrespective of the curvature constant $k$ and the interaction $n$. Furthermore, at $\Omega_\Lambda = 0$, one recognizes the changes from $\omega_m^{\text{eff}} = -0.2$ for $n = 1$ to $\omega_m^{\text{eff}} = 0$ for $n = 1/2$. This implies that a decay rate of $\Gamma = 3b^2\sqrt{1 + r_m}H$ leads to the CDM-dominated universe with $\omega_m^{\text{eff}} = 0$ at the far past. We note that the effect of non-flat universe is trivial because $\Omega_k$ goes to zero for the far past and far future. This means that the non-flat universe of $k \neq 0$ could not induce the phantom phase even one includes an interaction between the holographic dark energy and CDM.

We comment on the fine-tuning and coincidence problems. The holographic energy density $\rho_\Lambda$ could resolve the fine-tuning problem because taking $L_\Lambda = l_p = 1/M_p$ leads to the cosmological vacuum energy $\rho_\Lambda^p \propto M_p^4$. This means that a small system at planck scale provides an upper limit of $\rho_\Lambda \leq \rho_\Lambda^p$, as is naively expected in quantum field theory. On the other hand, as the universe evolves, a larger system will have a smaller energy density. This is a consequence of the holography. Thus the holographic principle may reconcile the quantum field theory at planck scale with the smallness of the present cosmological
vacuum energy density \( \rho_0 \propto M_p^2 H_0^2 = 10^{-123} \rho_p \).

Furthermore, the resulting equilibrium between holographic dark energy and CDM offers a possible resolution to the cosmic coincidence problem. The cosmic coincidence problem states that it is unlikely that the current epoch with sizable amounts of both CDM and dark energy coincides with the rapid transition from CDM-domination to dark energy-domination. Any interacting holographic models using the future event horizon show the decreasing effective equations of state. Considering a decay of the holographic dark energy into CDM, we expect to show the changes for \( k = 0 \) and \( n = 1 \): \( \Omega_\Lambda = 0.0 (\Omega_m = 1.0) \) at the far past; \( \Omega_\Lambda = 0.72 (\Omega_m = 0.28) \) at present; \( \Omega_\Lambda = 0.78 (\Omega_m = 0.22) \) at the far future. If there is no interaction, one finds the natural tendency for dark energy to dominate over CDM as the universe expands: \( \Omega_\Lambda = 0.0 (\Omega_m = 1.0) \) at the far past; \( \Omega_\Lambda = 0.72 (\Omega_m = 0.28) \) at present; \( \Omega_\Lambda = 1.0 (\Omega_m = 0.0) \) at the far future. This means that the interaction makes the slow transition from CDM-domination to dark energy-domination. The natural tendency is compensated by the decay of the holographic dark energy into CDM \([23]\). As is expected from the future fixed point of \( \omega_{\Lambda}^\text{eff} = \omega_{\text{m}}^\text{eff} \), there exist a balance between tendency and decay. Thus we have the effective equation of state \( \omega_\Lambda^\text{eff} = -0.92(n = 1) \) and \( \omega_\Lambda^\text{eff} = -0.93(n = 1/2) \) for an equilibrium mixture.

At this stage, we consider a very weakly coupling of \( b^2 \). According to the interacting quintessence models \([28]\), \( b^2 \) corresponds to the parameter \( c_{\text{OAP}}^2 \) which must be lower than 0.001. In this case, if \( c_{\text{OAP}}^2 > 0.001 \), a baryon-dominated universe would develop before the dark matter-domination. This would hinder tremendously the formation of cosmic structure. In order to see whether this picture is possible to occur within the interacting holographic model, we choose \( b^2 = 0.001 \). For \( k = 0 \) and \( c = 1 \) case, we observe its evolution from Fig. 4. It shows the nearly same form as in Fig.2 except that \( \omega_\Lambda^\text{eff} = -0.98(n = 1) \) and \( \omega_\Lambda^\text{eff} = -0.98(n = 1/2) \) for an equilibrium mixture. This means that the nature of holographic interaction is not changed even for a very small coupling of \( b^2 \). Therefore, we could not find such a condition of \( c_{\text{OAP}}^2 \) in our model. The only limitation on \( b^2 \) comes from the condition of the natural tendency for dark energy: \( d\Omega_\Lambda/dx|_{x=0} > 0 \rightarrow b^2 < b^2_{\text{max}} \), where \( b^2_{\text{max}} \) satisfies \( d\Omega_\Lambda/dx|_{x=0} = 0 \). As an example, we have \( b^2_{\text{max}} = 0.35 \) for \( k = 0, c = 1, n = 1, \) and \( \Omega_\Lambda^0 = 0.72 \).

In addition we have three parameters \( b^2, c, n \) and observational ranges on \( \Omega_\Lambda, \Omega_m, \Omega_k \). Hence it suggests that there is a parameter space which may describe a phantom-regime of \( \omega_\Lambda^\text{eff} < -1 \). However, it is easily proved that this is not the case. Requiring the second-law of thermodynamics of \( \dot{S}_{\text{BH}} = 2\pi R_{\text{FH}} \dot{R}_{\text{FH}} \geq 0 \) leads to the condition of \( c \geq \sqrt{\Omega_\Lambda \cos n} \). On the other hand, the condition for \( \omega_\Lambda^\text{eff} < -1 \) with Eq.(17) implies that \( c < \sqrt{\Omega_\Lambda \cos n} \). Hence two conditions are not compatible. An important parameter to determine \( \omega_\Lambda^\text{eff} \) is c.
Actually the interaction \((b^2 \text{ and } n)\) between holographic dark energy and CDM does not induce a phantom-like matter.

Finally, we mention the recent observations. A lot of data support on the flat universe. Also it would be important to stress on the motivation of considering the non-flat universe with the small \(\Omega_k\) from CMB experiments \[3, 4\] and supernova measurements \[1\]. Our results show that the effect of the non-flat universe becomes trivial because \(\Omega_k\) goes to zero for the far past and the far future, even the non-flat universe contributes small at present. Hence, if the interacting holographic dark energy model is reliable, we anticipate that the curvature term \(\Omega_k\) does not play an important role for determining the future dark energy-dominated universe.

Consequently, it turned out that the interacting holographic energy density could not describe the phantom regime\(^3\).

Acknowledgment

K. Kim and H. Lee were in part supported by KOSEF, Astrophysical Research Center for the Structure and Evolution of the Cosmos at Sejong University. Y. Myung was in part supported by the SRC Program of the KOSEF through the Center for Quantum Spacetime (CQUeST) of Sogang University with grant number R11-2005-021 and by the Korea Research Foundation (KRF-2006-311-C00249) funded by the Korea Government (MOEHRD).

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\(^3\)However, it seems that there were alternatives to provide the phantom phase. The general interaction \[29\], the particle horizon \[30\], and \(c < 1\) case \[27\] were used to derive phantom phase.
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