Leptogenesis scenarios for natural SUSY with mixed axion-higgsino dark matter

Kyu Jung Bae, a,b Howard Baer, a Hasan Serce a and Yi-Fan Zhang a

aDepartment of Physics and Astronomy, University of Oklahoma, Norman, OK 73019, U.S.A.
bDepartment of Physics, University of Tokyo, Bunkyo-ku, Tokyo 113-0033, Japan
E-mail: bae@hep-th.phys.s.u-tokyo.ac.jp, baer@nhn.ou.edu, serce@ou.edu

Received October 8, 2015
Revised December 4, 2015
Accepted December 21, 2015
Published January 7, 2016

Abstract. Supersymmetric models with radiatively-driven electroweak naturalness require light higgsinos of mass $\sim 100–300 \text{ GeV}$. Naturalness in the QCD sector is invoked via the Peccei-Quinn (PQ) axion leading to mixed axion-higgsino dark matter. The SUSY DFSZ axion model provides a solution to the SUSY $\mu$ problem and the Little Hierarchy $\mu \ll m_{3/2}$ may emerge as a consequence of a mismatch between PQ and hidden sector mass scales. The traditional gravitino problem is now augmented by the axino and saxion problems, since these latter particles can also contribute to overproduction of WIMPs or dark radiation, or violation of BBN constraints. We compute regions of the $T_R$ vs. $m_{3/2}$ plane allowed by BBN, dark matter and dark radiation constraints for various PQ scale choices $f_a$. These regions are compared to the values needed for thermal leptogenesis, non-thermal leptogenesis, oscillating sneutrino leptogenesis and Affleck-Dine leptogenesis. The latter three are allowed in wide regions of parameter space for PQ scale $f_a \sim 10^{10–10^{12}} \text{ GeV}$ which is also favored by naturalness: $f_a \sim \sqrt{\mu M_P/\lambda_\mu} \sim 10^{10–10^{12}} \text{ GeV}$. These $f_a$ values correspond to axion masses somewhat above the projected ADMX search regions.

Keywords: axions, leptogenesis, supersymmetry and cosmology, baryon asymmetry

ArXiv ePrint: 1510.00724
Contents

1 Introduction 1
  1.1 Electroweak naturalness 1
  1.2 Naturalness in the QCD sector 3
  1.3 Naturalness and the $\mu$-problem 3
  1.4 Dark matter in SUSY with electroweak and QCD naturalness 4
  1.5 Connection to baryogenesis 5

2 Survey of some baryogenesis mechanisms 6
  2.1 Thermal leptogenesis (THL) 6
  2.2 Non-thermal leptogenesis via inflaton decay (NTHL) 7
  2.3 Leptogenesis from oscillating sneutrino decay (OSL) 8
  2.4 Affleck-Dine leptogenesis (ADL) 9

3 Constraints in the $T_R$ vs. $m_{3/2}$ plane for various $f_\alpha$ 11
  3.1 SUSY DFSZ model 13
  3.2 SUSY KSVZ model 16

4 Constraints in the $T_R$ vs. $f_\alpha$ plane for fixed $m_{3/2}$ 16
  4.1 SUSY DFSZ model 16
  4.2 SUSY KSVZ model 19

5 Conclusion 21

1 Introduction

1.1 Electroweak naturalness

The recent discovery of the Higgs boson with mass $m_h \approx 125$ GeV at LHC8 [1, 2] brings with it a puzzle: why is the Higgs so light when its mass is quadratically divergent? Supersymmetry provides an elegant solution to this so-called naturalness problem by providing order-by-order cancellation of quadratic divergences. In fact, in the Minimal Supersymmetric Standard Model or MSSM, the Higgs mass is constrained so that $m_h \lesssim 135$ GeV [3]: the measured mass lies comfortably below this bound. The price to pay for a SUSY solution to the electroweak naturalness problem is that, naively, superpartners also ought to exist at or around the weak scale $m_{\text{weak}}$ as typified by the $W$, $Z$ and $h$ masses: $m_{\text{weak}} \sim 100$ GeV. However, null results from sparticle searches at LHC8 have resulted in mass limits within the multi-TeV regime [4–6]: $m_{\tilde{g}} \gtrsim 1.3$ TeV for $m_{\tilde{g}} \ll m_{\tilde{q}}$ and $m_{\tilde{g}} \gtrsim 2$ TeV for $m_{\tilde{q}} \sim m_{\tilde{q}}$. Furthermore, the somewhat large value of $m_h$ seems to require either well-mixed TeV scale top-squarks or $10$–$100$ TeV top-squarks with small mixing [7, 8]. These rather large sparticle mass values threaten to re-introduce the naturalness question: this time due to log divergences which emerge from the Little Hierarchy $m_{\text{weak}} \ll m_{\text{sparticle}} \sim 2$–$20$ TeV. Some authors go so far as to claim the emergent Little Hierarchy leads to a crisis in physics [9–17]. To proceed at a deeper level, a quantitative discussion of SUSY electroweak naturalness is warranted.
The weak scale as typified by the $Z$-boson mass is directly related to weak scale SUSY 
Lagrangian parameters via the well-known scalar potential minimization condition

$$
\frac{m_Z^2}{2} = \frac{(m_{H_u}^2 + \Sigma^d_u - (m_{H_d}^2 + \Sigma_u^d) \tan^2 \beta)}{(\tan^2 \beta - 1)} - \mu^2
$$

(1.1)

$$
\simeq -m_{H_u}^2 - \mu^2 - \Sigma_u^d(i)
$$

(1.2)

where $m_{H_u}^2$ and $m_{H_d}^2$ are the weak scale soft SUSY breaking Higgs masses, $\mu$ is the supersymmetric higgsino mass term and $\Sigma_u^d$ and $\Sigma_d^u$ contain an assortment of loop corrections (labelled by index $i$) to the effective potential (the complete set of one-loop corrections is given in ref. [18, 19]). The electroweak fine-tuning measure $\Delta_{EW}$ [18, 19] compares the largest contribution on the right-hand-side of eq. (1.2) to the value of $m_Z^2/2$. If they are comparable, then no unnatural fine-tunings are required to generate $m_Z = 91.2$ GeV. The measure $\Delta_{EW}$ has the advantage of being model-independent (in that any model yielding the same weak scale spectra will have the same value of $\Delta_{EW}$). It is also pragmatic: in computer codes that calculate the weak scale SUSY spectra, this is the point where actual fine-tuning occurs - usually in the form of dialing the required value of $\mu$ so as to ensure that $m_Z = 91.2$ GeV. The implications of natural SUSY (SUSY spectra with low $\Delta_{EW} \lesssim 30$ [20]) are as follows [18, 19].

- $\mu \sim 100–300$ GeV (the lighter the better) leading to a set of light higgsinos $\tilde{Z}_1, 2$ and $\tilde{W}_1^{\pm}$. The $\tilde{Z}_1$ is the lightest SUSY particle (LSP) and it is a higgsino-like WIMP which is thermally under-produced as a dark matter candidate.

- The soft term $m_{H_u}^2$ is driven radiatively to small values $\sim -(100–300)^2$ GeV$^2$ at the weak scale (this is known as radiatively-driven naturalness or RNS). This can always occur in models with high scale Higgs soft term non-universality.

- The radiative corrections $\Sigma_u^d$ are $\lessim (100–300)^2$ GeV$^2$. The largest of these usually comes from the top squark contributions $\Sigma_u^d(t_{1,2})$. These contributions can both be small for well-mixed (large $A_t$) top squarks with mass $m_{t_1} \lesssim 4$ TeV and $m_{t_2} \lesssim 10$ TeV. These same conditions lift $m_h$ into the 125 GeV vicinity.

While a value of low $\Delta_{EW}$ seems like a necessary condition for SUSY naturalness, the question is: is it also sufficient? Does it embody high scale fine-tuning as well? The answer given in Ref’s [21, 22] is that, yes, eq. (1.2) provides a complete portrayal of SUSY electroweak fine-tuning.

- An often-invoked alternative known as Higgs mass fine-tuning requires no large cancellations in contributions to the Higgs boson mass: $m_h^2 \sim \mu^2 + m_{H_u}^2 + \delta m_{H_u}^2$ and thus requires $\delta m_{H_u}^2 \lesssim m_h^2$. The value of $\delta m_{H_u}^2$ can be calculated by integrating the renormalization group equation for the soft term $m_{H_u}^2$. A back-of-the-envelope evaluation leads to $\delta m_{H_u}^2 \sim -\frac{3f}{8\pi^2}(m_{Q_3}^2 + m_{U_3}^2 + A_f^2) \ln \left(\frac{\Lambda^2}{m_{SUSY}^2}\right)$ where $\Lambda$ is the high scale usually taken to be $m_{GUT}$ and $m_{SUSY} \sim 1$ TeV. However, this overly simplified expression neglects the fact that $m_{H_u}^2$ itself feeds into the evaluation of $\delta m_{H_u}^2$. In fact, the larger the value of $m_{H_u}^2(\Lambda)$, then the larger the cancelling correction $\delta m_{H_u}^2$. This evaluation violates the fine-tuning rule [22]: to avoid over-estimates of fine-tuning, first combine dependent contributions to any observable quantity. By following the fine-tuning rule, then instead the two terms on the r.h.s. of $m_h^2 = \mu^2 + (m_{H_u}(\Lambda) + \delta m_{H_u}^2)$ should be comparable to $m_h^2$. Since $m_{H_u}^2(\Lambda) + \delta m_{H_u}^2 = m_{H_u}(\text{weak})$, then the Higgs mass fine-tuning conditions lead to the same as those for low $\Delta_{EW}$. 

\[ JCAP01(2016)012 \]
- EENZ/BG fine-tuning \cite{23, 24} \( \Delta_{BG} = \max \left| \frac{\partial \ln m_Z^2}{\partial \ln p_i} \right| \) measures the sensitivity of \( m_Z^2 \) to high scale parameters \( p_i \). The usual application of \( \Delta_{BG} \) is to multi-parameter effective theories where the various \( p_i \) parametrize our ignorance of the nature of the hidden sector which serves as an arena for SUSY breaking. By recognizing that in any SUGRA theory the soft terms are all calculated as multiples of the gravitino mass \( m_{3/2} \), then the \( Z \) mass can be expressed in terms of high scale parameters as \( m_Z^2 \simeq -2 \mu^2 (\Lambda) + a \cdot m_{3/2}^2 \). Since \( \mu \) hardly evolves, then \( a \cdot m_{3/2}^2 \simeq m_{H_u}^2 \) (weak) so that a low value of \( \Delta_{BG} \) leads again to the same requirements as a low value of \( \Delta_{EW} \).

### 1.2 Naturalness in the QCD sector

In QCD, in the limit of two light quarks \( u, d \), one has an approximate global \( U(2)_L \times U(2)_R \) chiral symmetry which can be recast as \( U(2)_V \times U(2)_A \). The vector symmetry leads to well-known \( SU(2) \) of isospin along with baryon number conservation. The axial \( U(2)_A \) symmetry is spontaneously broken and naively leads to four instead of three light pions as pseudo-Goldstone bosons. Weinberg suggested \cite{25} the \( U(1)_A \) symmetry was somehow not respected and indeed this viewpoint was vindicated by 't Hooft’s discovery of the QCD \( \theta \) vacuum. A consequence of this resolution of the \( U(1)_A \) problem is that the QCD Lagrangian should contain a \( C \) and \( CP \)-violating term

\[
L_{QCD} \supset \frac{\bar{\theta}}{32\pi^2} G^A_{\mu\nu} \tilde{G}^{A\mu\nu}
\]

where \( \tilde{G}^{A\mu\nu} \) is the gluon field strength tensor. Measurements of the neutron EDM require \( \bar{\theta} \lesssim 10^{-10} \) which leads to the QCD naturalness problem (also known as the strong \( CP \) problem): why is this term - which should be present - so tiny? Peccei and Quinn \cite{26} introduced an additional global PQ symmetry which is spontaneously broken at scale \( v_{PQ} \sim 10^{10} \) GeV leading to a quasi-visible \cite{27–30} axion \cite{31, 32}. Introduction of the axion field allows the offending \( CP \)-violating term to dynamically settle to zero, thus saving the day for QCD. The required axion field ought to have a mass \( m_a \sim 620 \mu eV \) (\( 10^{10} \) GeV \( f_a/N \)) where \( N \) is the color anomaly of the PQ symmetry (\( N = 1 \) for KSVZ \cite{27, 28} and \( N = 6 \) for DFSZ \cite{29, 30}).

The axion can be produced via axion field coherent oscillations in the early universe and serves as a candidate for cold dark matter \cite{33–38}. In a SUSY context, the axion should be accompanied by the spin-1/2 \( R \)-parity-odd axino \( \tilde{a} \) and the spin-0 \( R \)-even saxion field \( s \). In supergravity models, the soft breaking saxion mass \( m_s \sim m_{3/2} \). The axino mass is more model dependent but is usually also expected to be \( m_{\tilde{a}} \sim m_{3/2} \). The supergravity calculation of axino mass is thoroughly discussed in Ref’s \cite{39–42}. The axion, saxion and axino interactions are all suppressed by the inverse of the axion decay constant \( f_a \) where \( f_a \sim v_{PQ} \).

### 1.3 Naturalness and the \( \mu \)-problem

While the axion plays a crucial role in solving the QCD naturalness problem, it also plays a role in the electroweak naturalness problem. While the soft term \( m_{H_u}^2 \) can be radiatively driven to small negative values by the large top-quark Yukawa coupling, the \( \mu \) parameter in eq. (1.2) also needs to be tamed. Since it is supersymmetric and not SUSY breaking, naively one expects \( \mu \sim m_{GUT} \) or \( M_P \) (the reduced Planck mass). In contrast, naturalness
requires $\mu \sim m_{\text{weak}}$. The SUSY DFSZ axion provides an elegant resolution of this so-called SUSY $\mu$ problem [43, 44] in that the Higgs superfields now carry PQ charge which forbids the appearance of the $\mu$ term in the superpotential. Upon spontaneous breaking of the PQ symmetry, then in the SUSY DFSZ axion model the $\mu$ term is regenerated with a value $\mu \sim \lambda_\mu f_a^2/M_P$. This is in contrast to the SUSY soft terms which gain a mass $m_{\text{soft}} \sim m_{3/2}^2 \sim m_{\text{hidden}}^2/M_P$ where $m_{\text{hidden}}$ is the hidden sector mass scale. In such a case, the emerging Little Hierarchy $\mu \ll m_{3/2}$ is just a consequence of a mismatch between hidden sector and PQ sector intermediate mass scales $f_a \ll m_{\text{hidden}}$. In fact, in models such as the MSY SUSY axion model [45], the PQ symmetry is broken radiatively as a consequence of SUSY breaking leading naturally to $\mu \sim 100$–300 GeV whilst $m_{3/2} \sim 2$–10 TeV [46]. As a by-product of radiative PQ breaking, intermediate scale Majorana masses are also induced $m_N \sim f_a$ leading to see-saw neutrinos [47–50]. In this scenario, then, the PQ breaking scale $f_a$ plays a role in determining the axion, the higgsino and the Higgs masses!

1.4 Dark matter in SUSY with electroweak and QCD naturalness

In a highly natural model where the electroweak sector is stabilized by SUSY, the QCD sector is stabilized by the axion, the $\mu$ problem is resolved by PQ-charged Higgs fields and the Little Hierarchy $\mu \ll m_{3/2}$ emerges from radiative PQ breaking, then one expects dark matter to be composed of an axion-higgsino admixture: i.e., two dark matter particles. The favored axion scale $f_a \sim \sqrt{\mu M_P/\lambda_\mu} \sim 10^{10–12}$ GeV (for $\lambda_\mu \sim 10^{-4}$–1) which is somewhat below the range of $f_a$ currently being explored by the Axion Dark Matter search experiment, ADMX [51]. Regarding the higgsino-like WIMPs, their relic abundance calculation is seriously modified from the usual thermal production picture [52]. To be sure, the WIMPs are still produced thermally, but they can also arise via axino and saxion production in the early universe, followed by cascade decays which terminate in WIMPs. While axinos can be produced thermally, saxions can be produced both thermally and via coherent oscillations. If too many WIMPs are produced via heavy particle decays, they may undergo a re-annihilation process [53, 54]. Furthermore, axions can also be produced thermally or via saxion decays. The latter leads to injection of dark radiation parametrized by the effective number of additional neutrinos present in the cosmic soup: $\Delta N_{\text{eff}}$. Current bounds from the Planck experiment require $N_{\text{eff}} = 3.15 \pm 0.23$ [55]. (To be conservative, here we require merely $\Delta N_{\text{eff}} \lesssim 1$ in our results.) With an assortment of interwoven production and decay processes occurring, an accurate estimate of the ultimate mixed axion-higgsino dark matter content requires simultaneous solution of eight coupled Boltzmann equations which track the abundance of radiation, WIMPs, thermal- and oscillation-produced axions, thermal- and oscillation-produced saxions, axinos and gravitinos [56].

Results vary radically depending on whether one is in a hadronic (KSVZ) [27, 28] SUSY axion model or DFSZ [29, 30] SUSY axion model. In the former KSVZ case, thermal production of axinos and saxions is proportional to the reheat temperature $T_R$ [57–59] while decay modes arise from heavy quark induced loop diagrams due to the superpotential term

$$W_{\text{KSVZ}} \ni m_Q e^{A/f_a} QQ^c$$

where $Q$ stands for intermediate-scale heavy quark superfields with $m_Q \sim f_a$. In SUSY DFSZ, axino and saxion thermal productions are different from those in SUSY KSVZ since the axion superfield has tree level couplings which are proportional to the SUSY $\mu$ parameter [60–62]:

$$W_{\text{DFSZ}} \ni \mu e^{-2A/f_a} H_u H_d.$$
Due to this interaction, thermal production of axions, axinos and saxions is largely independent of $T_R$ unless $T_R \lesssim \mu$ [61]. Decays also dominantly proceed through this tree level coupling so the axino and saxion tend to be shorter-lived than in the KSVZ case.

### 1.5 Connection to baryogenesis

One of the major mysteries of particle physics and cosmology is the origin of the matter-anti-matter asymmetry as embodied by the measurement of the baryon-to-photon ratio $\eta_B$:

$$\eta_B \equiv \frac{n_B}{n_\gamma} \simeq (6.2 \pm 0.5) \times 10^{-10} \quad (95\% \text{ CL}). \quad (1.6)$$

$\eta_B$ is determined both from light element production in Big Bang Nucleosynthesis (BBN) and also from CMB measurements. Alternatively, this is sometimes expressed as the baryon-to-entropy ratio

$$\frac{n_B}{s} \simeq 10^{-10} \quad (1.7)$$

where $s \simeq 7.04n_\gamma$ in the present epoch.

Production of the baryon asymmetry of the Universe or BAU requires mechanisms which satisfy the three Sakharov criteria: 1. baryon number violation, 2. $C$ and $CP$ violation and 3. a departure from thermal equilibrium. Early proposals such as Planck scale or GUT scale baryogenesis seem no longer viable since the BAU would have been inflated away during the inflationary epoch of the Universe. Alternatively, most modern proposals for developing the BAU take place after the end of the inflationary epoch, at or after the era of re-heating which occurs around the reheat temperature $T_R$. In fact, the SM contains all the ingredients for successful electroweak baryogenesis since baryon (and lepton) number violating processes can take place at large rates at high temperature $T > T_{\text{weak}} \sim 100 \text{ GeV}$ via sphaleron processes [64]. Unfortunately, these first order phase transition effects require a Higgs mass $\lesssim 50 \text{ GeV}$, and so has been excluded for many years. By invoking supersymmetry, then new possibilities emerge for electroweak baryogenesis. However, successful SUSY electroweak baryogenesis seems to require a Higgs mass $m_h \lesssim 113 \text{ GeV}$ and a right- top-squark $m_{\tilde{t}_R} \lesssim 115 \text{ GeV}$ [65, 66]. These limits can be relaxed to higher values so long as other sparticle/Higgs masses such as $m_A > 10 \text{ TeV}$. Such heavy Higgs masses are not allowed if we stay true to our guidance from naturalness: after all, eq. (1.2) requires $m_{H_u}^2 / \tan^2 \beta \lesssim m_{Z}^2 / 2$. For heavy Higgs masses, then $m_A \sim m_{H_d}$ and then from naturalness we find $m_A \lesssim 4\text{–}8 \text{ TeV}$ (depending on $\tan \beta$) [67].

In section 2, we survey several leptogenesis mechanisms as the most promising baryogenesis mechanisms: 1. thermal leptogenesis [68–73], 2. non-thermal leptogenesis via inflaton decay [77–82] 3. leptogenesis from oscillating sneutrino decay [83, 87] and 4. leptogenesis via an Affleck-Dine condensate [83, 88–90].

Each of these processes requires some range of reheat temperature $T_R$ and gravitino mass $m_{3/2}$, and indeed some of them run into conflict with the so-called cosmological gravitino problem [91, 92]. In this case, gravitinos can be thermally produced in the early universe at a rate proportional to $T_R^2$ [93]. If $T_R$ is too high then too much dark matter arises from thermal gravitino production followed by cascade decays to the LSP. Also, even if dark matter abundance constraints are respected, if the gravitino is too long-lived, then it may decay after the onset of BBN thus destroying the successful BBN predictions of the light element abundances [94–97].

In the case of natural SUSY with mixed axion-higgsino dark matter, then similar constraints arise from axino and saxion production: WIMPs or axions can be overproduced,
or light element abundances can be destroyed by late decaying axinos and saxions. After a brief review of the several leptogenesis mechanisms in section 2, in section 3 we show constraints on leptogenesis in the $T_R$ vs. $m_{3/2}$ planes assuming a natural SUSY spectrum. In section 4, we show corresponding results in the $T_R$ vs. $f_a$ planes. We vary the PQ scale $f_a$ from values favored by naturalness $f_a \sim 10^{10}$ to much higher values. While thermal leptogenesis mechanism is quite constrained depending on $m_{3/2}$ and $T_R$, the latter three mechanisms appear plausible over a wide range of $T_R$, $m_{3/2}$ and $f_a$ values which are consistent with naturalness. A summary and some conclusions are presented in section 5.

2 Survey of some baryogenesis mechanisms

2.1 Thermal leptogenesis (THL)

Thermal leptogenesis [68–76] is a baryogenesis mechanism which relies on the introduction of three intermediate mass scale right hand singlet neutrinos $N_i$ ($i = 1$–3) so that the (type I) see-saw mechanism [47–50] elegantly generates a very light spectrum of usual neutrino masses. The superpotential is given by

$$W \ni \frac{1}{2} M_i N_i N_i + h_{\alpha i} N_i L_{\alpha} H_u$$

(2.1)

where we assume a basis for the $N_i$ masses which is diagonal and real. The index $\alpha$ denotes the lepton doublet generations and $h_{\alpha i}$ are the neutrino Yukawa couplings. From the see-saw mechanism, one expects a spectrum of three sub-eV mass Majorana neutrinos $m_1$, $m_2$ and $m_3$ and three heavy neutrinos $M_1 < M_2 < M_3$ where in GUT-type theories one typically expects $M_3 \sim 10^{15}$ GeV. If the heavy neutrino masses are hierarchical (as assumed here) like the quark masses, then one might expect $M_1/M_3 \sim m_u/m_t \sim 10^{-5}$ and so perhaps $M_1 \sim 10^{10}$ GeV.

After inflation, then it is assumed the Universe re-heats to a temperature $T_R \gtrsim M_1$ thus creating a thermal population of $N_1$s. The $N_1$ decay asymmetrically as $N_1 \rightarrow LH_u$ vs. $\bar{L} \bar{H}_u$ due to interference between tree and loop-level decay diagrams which include CP violating interactions. The CP asymmetry factor

$$\epsilon_1 \equiv \frac{\Gamma(N_1 \rightarrow LH_u) - \Gamma(N_1 \rightarrow \bar{L}\bar{H}_u)}{\Gamma_{N_1}}$$

(2.2)

is calculated to be [98–100]

$$\epsilon_1 \simeq 3 \frac{M_1}{8\pi \langle H_u \rangle^2} m_{\nu_3} \delta_{\text{eff}}$$

(2.3)

where $\langle H_u \rangle \simeq 174$ GeV $\sin \beta$ and $\delta_{\text{eff}}$ is an effective CP-violating phase which depends on the MNS matrix elements and which is expected to be $\delta_{\text{eff}} \sim 1$. For hierarchical heavy neutrinos, one expects

$$\epsilon_1 \sim 2 \times 10^{-10} \left( \frac{M_1}{10^6 \text{ GeV}} \right) \left( \frac{m_{\nu_3}}{0.05 \text{ eV}} \right) \delta_{\text{eff}}.$$  

(2.4)

The ultimate lepton asymmetry requires evaluation via a coupled Boltzmann equation calculation [101]. Once $N_1$ is in thermal equilibrium, its number-density-to-entropy ratio is simply proportional to $1/g_*$ where the effective degrees of freedom $g_* = 232.5$ for the MSSM. The lepton-number-density to entropy ratio is then given by

$$\frac{n_L}{s} = \kappa \epsilon_1 \frac{n_{N_1}}{s} \simeq \kappa \frac{\epsilon_1}{240}$$

(2.5)
where the co-efficient $\kappa$ accounts for washout effects and the efficiency of thermal $N_1$ production. Numerical evaluations of $\kappa$ imply $\kappa \approx 0.05$–0.3.

The induced lepton asymmetry becomes converted to a baryon asymmetry via $B$- and $L$- violating but $B – L$ conserving sphaleron interactions [64, 107, 108]. The ultimate baryon asymmetry is given by [109]

$$\frac{n_B}{s} \simeq 0.35 \frac{n_L}{s} \simeq 0.3 \times 10^{-10} \left( \frac{\kappa}{0.1} \right) \left( \frac{M_1}{10^9 \text{ GeV}} \right) \left( \frac{m_{\nu_3}}{0.05 \text{ eV}} \right) \delta_{\text{eff}}$$

(2.6)

provided that $T_R$ is large enough that the $N_1$ are efficiently produced by thermal interactions: $T_R \gtrsim M_1$. Naively, this requires $T_R \gtrsim 10^{10}$ GeV although detailed calculations [101] allow for $T_R \gtrsim 1.5 \times 10^9$ GeV. This rather large lower bound on $T_R$ potentially leads to conflict with the gravitino problem and violation of BBN bounds or overproduction of dark matter. In the event that late-decaying relics inject entropy after $N_1$ decay is complete, then $n_L/s$ is modified by an entropy-dilution factor $r$: $n_L/s \rightarrow n_L/(r \times s)$.

It is worth mentioning some variant thermal leptogenesis scenarios that can ameliorate the severe $T_R$ bound. In the simple scenario of thermal leptogenesis, the flavor dependence is normally neglected by assuming the alignment of final state leptons and anti-leptons, i.e., $CP(L) = \bar{L}$. In general, however, one can consider the case in which the final state leptons and anti-leptons are not aligned and thus the flavor effect must be taken into account. Depending on the temperature at which dominant lepton asymmetry is generated, flavor effect can enhance the final asymmetry by up to an order of magnitude [102, 103]. On the other hand, one can also consider the case of nearly degenerate right handed neutrinos rather than a hierarchical spectrum. If the mass difference is as small as its decay width, i.e., $(M_1 - M_2) \sim \Gamma_{N_1}$, the CP asymmetry factor is resonantly enhanced so that a successful leptogenesis scenario is possible with $O(\text{TeV})$ right handed neutrino mass [104–106].

In this work, we examine the viability of various leptogenesis scenarios for natural SUSY with mixed axion-higgsino dark matter. Nonetheless, we do not specify the structure of the neutrino sector. For a clear discussion, we will consider only the simplest scenarios for the thermal leptogenesis. If one considers a specific neutrino sector in which flavor and/or resonant effects are important, then bounds from thermal leptogenesis may be modified.

### 2.2 Non-thermal leptogenesis via inflaton decay (NTHL)

As an alternative to thermal leptogenesis, non-thermal leptogenesis posits a large branching fraction of the inflaton field $\chi$ into $N_1N_1$: $\chi \rightarrow N_1N_1$ which is followed by asymmetric $N_1$ decay to (anti-)leptons as before. In this case, the $N_1$ number-density-to-entropy-density ratio is given by [77–82, 110]

$$\frac{n_{N_1}}{s} \simeq \frac{\rho_{\text{rad}}}{s} \frac{n_{\chi}}{\rho_{\chi}} \frac{n_{N_1}}{n_{\chi}}$$

(2.7)

$$\simeq 3 \frac{T_R}{4 m_{\chi}} \times \frac{1}{2 B_{\nu}} = \frac{3}{2} \frac{B_{\nu} T_R}{m_{\chi}}$$

(2.8)

where $\rho_{\text{rad}}$ is the radiation density once reheating has completed and $\rho_{\chi}$ is the energy density stored in the inflaton field just before inflaton decay. Thus, $\rho_{\text{rad}} \simeq \rho_{\chi}$ and $\rho_{\chi} \simeq m_{\chi} n_{\chi}$. Here also $B_{\nu}$ is the inflaton branching fraction into $N_1N_1$. The lepton-number-to-entropy ratio is then given by $n_L/s \simeq \epsilon_1 n_{N_1}/2$ where $\epsilon_1$ is as in eq. (2.3).
The lepton number asymmetry is converted to a baryon asymmetry via sphaleron interactions as before:
\[
\frac{n_B}{s} \simeq 0.35 \frac{n_L}{s}
\] (2.9)
so that finally
\[
\frac{n_B}{s} \simeq 0.5 \times 10^{-10} B_r \left( \frac{T_R}{10^6 \text{ GeV}} \right) \left( \frac{2M_1}{m_\chi} \right) \left( \frac{m_{\nu_3}}{0.05 \text{ eV}} \right) \delta_{\text{eff}}
\] (2.10)
where $\delta_{\text{eff}}$ is the same phase as given above. The resultant baryon asymmetry can match data provided $m_\chi > 2M_1$ and that the branching fraction is nearly maximal. Under these conditions, a re-heat temperature $T_R \gtrsim 10^6 \text{ GeV}$ is required. For $T_R \lesssim 10^6 \text{ GeV}$, then $\rho_{\text{rad}}$ and consequently $\rho_\chi$ are reduced so that there is insufficient energy stored in the inflaton field to generate the required $n_{N_1}$ number density.

2.3 Leptogenesis from oscillating sneutrino decay (OSL)

In the previous two mechanisms, right-handed neutrinos and sneutrinos are produced by thermal scattering or inflaton decay. On the other hand, for sneutrinos, coherent oscillation can be a dominant production process. The decay of oscillating right-sneutrino produces lepton asymmetry which is given by [87]
\[
n_L = \epsilon_1 M_1 \left| \tilde{N}_{1d} \right|^2,
\] (2.11)
where $\tilde{N}_{1d}$ is the sneutrino amplitude when it decays. The CP asymmetry factor $\epsilon_1$ is the same as thermal leptogenesis which is shown in eq. (2.3).

Once the universe is dominated by sneutrino oscillation, pre-existing relics are mostly diluted away and the universe is reheated again by sneutrino decay at $H = \Gamma_{N_1}$, where $\Gamma_{N_1}$ is the sneutrino decay rate. The decay temperature $T_{N_1}$ is determined by
\[
T_{N_1} = \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{M_P \Gamma_{N_1}},
\] (2.12)
while the entropy density is given by
\[
s = \frac{2\pi^2}{45} g_* T_{N_1}^3,
\] (2.13)
where $g_*$ is the number of degree of freedom at $T = T_{N_1}$. At the time of sneutrino decay, the energy density of sneutrino oscillation is dominantly transferred to radiation energy density, so one can find an addition relation as
\[
\rho_{N_1} = M_1^2 \left| \tilde{N}_{1d} \right|^2 = \frac{\pi^2}{30} g_* T_{N_1}^4.
\] (2.14)
From these relations, one finds the lepton-number-to-entropy ratio:
\[
\frac{n_L}{s} = \epsilon_1 \frac{\rho_{N_1}}{M_1} \frac{1}{s} = \frac{3}{4} \epsilon_1 \frac{T_{N_1}}{M_1}
\]
\[
\simeq 1.5 \times 10^{-10} \left( \frac{T_{N_1}}{10^6 \text{ GeV}} \right) \left( \frac{m_{\nu_3}}{0.05 \text{ eV}} \right) \delta_{\text{eff}}.
\] (2.15)

\footnote{The spin-0 partners of right-handed neutrinos.}
The baryon asymmetry is obtained via sphaleron process, and thus baryon number is given by

\[
\frac{n_B}{s} \simeq 0.35 \frac{n_L}{s}.
\] (2.16)

Thus, enough baryon number can be generated for \( T_{N_1} \gtrsim 10^6 \) GeV.

In this scenario, it is interesting that the effective reheat temperature is \( \mathcal{O}(T_{N_1}) \) for thermal relic particles, since sneutrino domination dilutes pre-existing particles when it decays \[87\].\(^2\) Therefore, in the numerical analyses of sections 3 and 4, we consider \( T_{N_1} \) a reheat temperature for production of gravitinos, axinos and saxions in the case of leptogenesis from oscillating sneutrino decay.

### 2.4 Affleck-Dine leptogenesis (ADL)

The last mechanism for baryogenesis is known as Affleck-Dine (AD) \[88–90\] leptogenesis. AD leptogenesis makes use of the \( LH_u \) flat direction in the scalar potential \[89, 90, 111, 112\]. This direction is lucrative in that it is not plagued by \( Q \)-balls which are problematic for flat directions carrying baryon number \[113\] and also because the rate for baryogenesis can be linked to the mass of the lightest neutrino, leading to a possible consistency check via observations of neutrinoless double beta decay (0\( \nu \beta \beta \)) \[114\].

In the case of the \( LH_u \) flat direction, \( F \)-flatness is only broken by higher dimensional operators which also give rise to neutrino mass via the see-saw mechanism \[47–50\]:

\[
W \ni \frac{1}{2M_i} (L_i H_u)(L_i H_u)
\] (2.17)

where \( M_i \) is the heavy neutrino mass scale.\(^3\) The most efficient direction is that for which \( i = 1 \) corresponding to the lightest neutrino mass: \( m_{\nu_1} \sim \langle H_u \rangle^2 / M_1 \) in a basis where the neutrino mass matrix is diagonal. The Affleck-Dine field \( \phi \) then occurs as

\[
\hat{L}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi \\ 0 \end{pmatrix}, \quad H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}.
\] (2.18)

The scalar potential is given by

\[
V = V_{SB} + V_H + V_{TH} + V_F
\] (2.19)

where

\[
V_{SB} = m_\phi^2 |\phi|^2 + \frac{m_{SU3}}{8M}(a_m \phi^4 + \text{h.c.})
\] (2.20)

\[
V_H = -c_H H^2 |\phi|^2 + \frac{H}{8M}(a_H \phi^4 + \text{h.c.})
\] (2.21)

\[
V_{TH} = \sum_{f_k|\phi|<T} c_k J_k^2 T^2 |\phi|^2 + \frac{9\alpha_s^2(T)}{8} T^4 \ln \left( \frac{|\phi|^2}{T^2} \right)
\] and

\[
V_F = \frac{1}{4M^2} |\phi|^6.
\] (2.22)

\(^2\)It is assumed that inflaton decay after sneutrino oscillation starts. If sneutrino oscillation starts after inflaton decay, effective reheat temperature is given by \( 2T_{N_1}(T_R/T_{RC}) \) where \( T_{RC} \) is the temperature at which sneutrino oscillation starts.

\(^3\)Here \( M_i \) contains neutrino Yukawa coupling, i.e., \( 1/M_i = y_{\nu_i}/M_{N_i} \), so it can be larger than \( M_P \) for small \( y_{\nu_i} \).
The first contribution $V_{SB}$ is the SUSY breaking contribution where $m_{\phi}^2 = (\mu^2 + m_{H_u}^2 + m_{\chi}^2)/2$ \(\text{[115]}\). In natural SUSY, we expect $|\mu| \sim |m_{H_u}| \sim m_Z$ in contrast to $m_L \sim m_{\text{SUSY}} \sim 2-10\,\text{TeV}$ in accord with LHC8 limits.\(^4\) The second contribution arises from SUSY breaking during inflation \([89, 90]\) where $3H^2m_{\text{SUSY}}^2 \simeq |F_\chi|^2$ with $H_I$ being the Hubble constant during inflation and where $F_\chi$ is the inflaton $F$-term which fuels inflation and $\chi$ is the inflaton field. In the expression $V_H$, the coefficient $c_H$ may be $> 0$ for a non-flat Kähler metric (which is to be expected in general). This term provides an instability of the potential at $\phi = 0$ and for $c_H > 0$, then a large VEV of $\phi$ can form with value $\langle \phi \rangle \sim \sqrt{MH_I}$ where $H_I \gg m_\phi$ and where $\arg(\phi) = [(-\arg(a_H) + (2n + 1)\pi)/4$ for $n = 0-3$. The second term in $V_H$ is the Hubble-induced trilinear SUSY breaking term. The term $V_F$ is the up-lifting $F$-term contribution arising from the higher-dimensional operator (2.17). Lastly, the term $V_{TH}$ arises from thermal effects \([116, 117]\). The first term is generated when the light particle species which couple to the AD field are produced in the thermal plasma, while the second term is generated by effective gauge coupling running from heavy effective mass of particles which couple to the AD field. Here, $f_k$ represents the Yukawa/gauge couplings of $\phi$ and $c_k$ is expected $\sim 1$.

The equation of motion for the AD field is given by

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi^*} = 0 \quad (2.24)$$

which is the usual equation for a damped harmonic oscillator. Once the AD condensate forms, then the universe continues expansion and the Hubble-induced terms decrease. The minimum of the potential decreases as does the value of the condensate. When $H$ decreases to a value \([118]\)

$$H_{osc} = \max \left[ m_{\phi}, H_i, \alpha_2 T_R \left( \frac{9MP}{8M} \right)^{1/2} \right] \quad (2.25)$$

(where $H_i = \min \left[ \frac{1}{T}, \frac{M_{P}T_R^2}{M^2}, (c_{i}^{2} f_{4} M_{P}T_R^2)^{1/3} \right]$) then the AD field begins to oscillate, and a non-zero lepton number arises: $n_L = \frac{i}{2}(\phi^* \dot{\phi} - \dot{\phi}^* \phi)$ governed by

$$\dot{n}_L + 3Hn_L = \frac{m_{\text{SUSY}}}{2M} Im(a_m \phi^4) + \frac{H}{2M} Im(a_H \phi^4). \quad (2.26)$$

The first term on the r.h.s. of eq. (2.26) is dominant and using $d/dt(R^3n_L) = R^3\dot{n}_L + 3R^2Hn_L$ we can integrate from early times up to $t = 1/H_{osc}$ to find

$$n_L = \frac{m_{\text{SUSY}}}{2M} Im(a_m \phi^4)t_{osc} \approx \frac{1}{3} (m_{\text{SUSY}} |a_m|) M H_{osc} \delta_{ph} \quad (2.27)$$

where $\delta_{ph} = \sin(4\arg \phi + \arg a_m)$. In the second line of the above equation, we have used the relations, $\phi(t_{osc}) \sim \sqrt{H_{osc} M}$ and $t_{osc} = 2/(3H_{osc})$ for an oscillating field/matter-dominated universe. During the inflaton-oscillation-dominated era, the produced lepton number is diluted by an $(H/H_{osc})^2$ factor. The entropy density at $T_R$ is determined by the relation $3M_{P}^2 H_R^2 = \rho_{\text{rad}} = s \times 3T_R^4/4$ where $H_R$ is the Hubble parameter at $T_R$. The lepton-number-to-entropy ratio is conserved once the era of re-heat is completed:

$$\frac{n_L}{s} = \frac{M T_R}{12 M_{P}^2} \left( \frac{m_{\text{SUSY}} |a_m|}{H_{osc}} \right) \delta_{ph}. \quad (2.28)$$

\(^4\)In gravity mediation, it is natural to have $m_{\text{SUSY}} \sim m_{3/2}$. In our benchmark study in section 3, however, $m_{\text{SUSY}}$ for SUSY particle spectrum is fixed while physical gravitino mass $m_{3/2}$ varies from 1 TeV to 100 TeV.
This quantity has the virtue of being \( T_R \) independent if \( H_{osc} \) is determined by the third (thermal) contribution in eq. (2.25) \[118\]. The lepton asymmetry is then converted to a baryon asymmetry via sphaleron interactions

\[
\frac{n_B}{s} \approx 0.35 \frac{n_L}{s}.
\] (2.29)

Replacing \( M \) by \( (H_u)^2/m_{\chi_1} \), then it is found \[118\] that a baryon-to-entropy ratio \( n_B/s \approx 10^{-10} \) can be developed roughly independent of \( T_R \) for \( T_R \gtrsim 10^9 \) GeV for \( m_{\chi_1} \approx 10^{-9} \) eV and for \( m_{\text{SUSY}}|a_n| \sim 1 \) TeV.

### 3 Constraints in the \( T_R \) vs. \( m_{3/2} \) plane for various \( f_a \)

To compute the mixed axion-WIMP dark matter abundance in SUSY axion models, we adopt the eight-coupled Boltzmann equation computation of Ref’s \[56, 119, 120\]. The Boltzmann equations track the energy densities of the various constituents of the early universe while accounting for 1. Hubble expansion and dilution, 2. particle production and annihilation from scattering reactions, 3. particle production from decay and inverse decay processes, 4. particle disappearance due to decays and 5. particle production via bosonic coherent motion (BCM). The Boltzmann equations allow for species which may be in or out of thermal equilibrium.

A complete explanation of the Boltzmann equations and associated numerical analyses is given in ref. \[56\]. Instead of repeating the same discussion, we briefly show the structure of the relevant Boltzmann equations. The number densities \( (n_i) \) and energy densities \( (\rho_i) \) for thermal species \( (i = a, s, \tilde{a}) \) obey the following equations,

\[
\frac{dn_i}{dt} + 3H n_i = \sum_{j \in \text{MSSM}} (\bar{n}_i n_j - n_i n_j) (\sigma v)_{ij} - \Gamma_i m_i \frac{n_i}{\rho_i} (n_i - \bar{n}_i \sum_{a \rightarrow a + b} B_{ab} \frac{n_a n_b}{\bar{n}_a \bar{n}_b}) + \sum_a \Gamma_a B_i m_a \frac{n_a}{\rho_a} (n_a - \bar{n}_a \sum_{a \rightarrow a + b} B_{ab} \frac{n_a n_b}{\bar{n}_a \bar{n}_b})
\] (3.1)

\[
\frac{d\rho_i}{dt} + 3H (\rho_i + P_i) = \sum_{j \in \text{MSSM}} (\bar{n}_i n_j - n_i n_j) (\sigma v)_{ij} \frac{\rho_i}{n_i} - \Gamma_i m_i (n_i - \bar{n}_i \sum_{a \rightarrow a + b} B_{ab} \frac{n_a n_b}{\bar{n}_a \bar{n}_b}) + \sum_a \Gamma_a B_i \frac{m_a}{2} (n_a - \bar{n}_a \sum_{a \rightarrow a + b} B_{ab} \frac{n_a n_b}{\bar{n}_a \bar{n}_b})
\] (3.2)

where \( B_{ab} \equiv BR(i \rightarrow a + b), B_{ib} \equiv BR(a \rightarrow i + b), B_i \equiv \sum_b B_{ib} \). Here number densities in thermal equilibrium are denoted by \( \bar{n}_i \). The zero temperature decay widths are denoted by \( \Gamma_i \). Among terms on the right-hand-side of eq. (3.1), the first term describes the scattering processes of the species of concern with ordinary MSSM particles. On the other hand, the second term shows particle disappearance (production) via decay (inverse decay) processes while the third term represents particle production (disappearance) via decay (inverse decay) of heavier particles. The same explanation also holds for the \( \rho_i \) equation in eq. (3.2). The BCM components of the axion and saxion are simply determined by their initial energy density and decay widths as follows,

\[
\frac{d n_i^{\text{BCM}}}{dt} + 3H n_i^{\text{BCM}} = -\Gamma_i m_i n_i^{\text{BCM}} n_i^{\text{BCM}} \quad \text{and} \quad \frac{d (\rho_i^{\text{BCM}}/n_i^{\text{BCM}})}{dt} = 0.
\] (3.3)
The initial amplitudes are parametrized as $\theta_i = a_i / f_a$ and $\theta_s = s_i / f_a$. In the following analyses, we consider $\theta_s = 1$ as its natural initial condition while the axion amplitude is adjustable to complete the dark matter density if the Higgsino-like neutralino is underabundant.

In our treatment, one begins at temperature $T = T_R$ and tracks the energy densities of radiation, WIMPs, gravitinos, axinos, saxions (BCM- and thermally-produced) and axions (BCM-, thermally- and saxion decay-produced). Whereas WIMPs quickly reach thermal equilibrium at $T = T_R$, the axinos, saxions, axions and gravitinos do not, even though they are still produced thermally. In SUSY KSVZ, the axino, axion and saxion thermal production rates are all proportional to $T_R$ [57–59] while in SUSY DFSZ model they are largely independent of $T_R$ [61]. The calculation depends sensitively on the particle mass spectrum, on the re-heat temperature $T_R$, on the gravitino mass $m_{3/2}$ and on the PQ model (KSVZ or DFSZ), the PQ parameters $f_a$, the axion mis-alignment angle $\theta_i$, the saxion angle $\theta_s$ and on a parameter $\xi_s$ which accounts for the model-dependent saxion-to-axion coupling [41]. Here, we adopt the choices $\xi_s = 0$ ($s \to aa, \tilde{a}\tilde{a}$ decays turned off) or $\xi_s = 1$ ($s \to aa, \tilde{a}\tilde{a}$ decays turned on).

In order to solve the coupled Boltzmann equations, it is important to know the axino, saxion and gravitino decay rates. The gravitino decay rates are adopted from ref. [121] while the axino and saxion decay rates are given in Ref’s [54, 122] for SUSY KSVZ and in ref. [123] for SUSY DFSZ. The axino decays via loops involving the heavy quark $Q$ field such that $\tilde{a} \to gg\tilde{Z}, \tilde{Z}\gamma$ and $\tilde{Z}Z$ in SUSY KSVZ. In SUSY DFSZ, the axino couples directly to Higgs superfields yielding faster decay rates into gauge/Higgs boson plus gaugino/higgsino states. In SUSY KSVZ, the saxion decays via $s \to gg, \tilde{g}\tilde{g}$ and, when $\xi_s = 1$, also to $aa$ and $\tilde{a}\tilde{a}$ (if kinematically allowed). The decay $s \to aa$ leads to production of dark radiation as parametrized by $\Delta N_{\text{eff}}$. In SUSY DFSZ, the saxion decays directly to gauge- or Higgs-boson pairs or to gaugino/higgsino pairs [123]. If $\xi_s = 1$, then also $s \to aa$ or $\tilde{a}\tilde{a}$. In the case where axinos or saxions decay to SUSY particles (leading to WIMPs), then WIMPs may re-annihilate.

For the SUSY mass spectrum, we generate a natural SUSY model within the context of the 2-extra parameter non-universal Higgs (NUHM2) model with $m_0 = 5\,\text{TeV}$, $m_{1/2} = 0.7\,\text{TeV}$, $A_0 = -8.4\,\text{TeV}$ and $\tan \beta = 10$. We take $\mu = 125\,\text{GeV}$ and $m_A = 1\,\text{TeV}$. The spectrum is generated using IsaSUGRA 7.84 [124]. The value of $m_\tilde{g} = 1.8\,\text{TeV}$ so the model is safely beyond LHC8 constraints. The value of $m_h = 125\,\text{GeV}$ and $\Delta m_{\tilde{Z}} = 20$ so the model is highly natural. Higgsino-like WIMPs with mass $m_{\tilde{Z}_1} = 115.5\,\text{GeV}$ are thermally underproduced so that $\Omega_{\tilde{W}}h^2 = 0.007$ using IsaReD [125]. In all frames, we take $m_a = m_s = m_{3/2}$ as is roughly expected in gravity-mediated SUSY breaking models [41, 42]. Since we take $m_a = m_s$, then $s \to \tilde{a}\tilde{a}$ decays are never a factor in our results.

The results of the dark matter abundance calculation for natural SUSY may be found in Ref’s [56, 119, 120] where the relic abundance of WIMPs and axions are plotted typically versus the PQ scale $f_a$. At low $f_a \sim 10^9$–$10^{11}\,\text{GeV}$, then the axion supermultiplet couplings are sufficiently large that axinos and saxions decay well before neutralino freeze-out so that the thermally-produced neutralino abundance is valid while axions make up the remainder of dark matter via axionic BCM. As $f_a$ increases, then axinos and saxions decay more slowly. If they decay after neutralino freeze-out, then they may add a non-thermal component to the neutralino relic abundance. If a sufficient amount of neutralinos are produced at the axino/saxion decay temperature, then they may re-annihilate yielding again an enhanced abundance. At very large $f_a \sim 10^{13}$–$10^{15}\,\text{GeV}$, then saxion production via BCM can be huge. Saxion decays to SUSY particles may bolster the neutralino abundance to values
far beyond the measured DM abundance in which case the parameter choices are excluded. However, if saxions dominantly decay to SM particles then entropy dilution occurs which can reduce the abundance of any relics present during decay. In either case, saxion decays after the onset of BBN can lead to dissolution of light elements and such cases would be ruled out by BBN limits on late decaying neutral relics \cite{97}. If $\xi_s$ is large, then $s \to aa$ decay can produce large amounts of dark radiation, frequently violating observational limits on $\Delta N_{\text{eff}}$

It is reasonable to ask: is it sufficient to present results based on a single SUSY benchmark point? In our case, it is for the following reasons. We restrict our analysis to natural SUSY where $\mu \sim 100$–$300$ GeV but where the remaining sparticles lie in the multi-TeV regime as required by LHC8 search limits and by the measured value of $m_h$. Now the sparticle mass spectrum enters the dark matter abundance calculation mainly through the decay widths (lifetimes) of the axinos and saxions. For natural SUSY, in the KSVZ case the axino decays dominantly to $\tilde{g}g$ when this mode is open and where $m_{\tilde{g}} \lesssim 2$–$4$ TeV is bounded from above by naturalness \cite{18–20}. Since this decay mode is almost always open, the axino decay width mainly depends on $m_{\tilde{a}}(\equiv m_{3/2})$ and $f_a$ and not on the SUSY spectrum. In SUSY KSVZ, the saxion mainly decays as $s \to gg$ (and $s \to aa$ in the $\xi_s = 1$ case). Thus, the saxion decays are rather independent of natural SUSY spectrum variations. In the DFSZ case, the axino decays to higgsino+Higgs or higgsino+vector boson and since $\mu$ is required small, these decay modes always dominate and again the axino decay pattern depends mainly on $\mu$, $m_{\tilde{a}}$ and $f_a$ and not upon variations in the natural SUSY spectra. In SUSY DFSZ, the saxion decays dominantly into higgsino pairs (or into $aa$ for $\xi_s = 1$) and as these modes are always open, is again quite independent of the natural SUSY spectrum.

In all the ensuing plots, the light-blue region corresponds to the parameter space where all BBN, DM and dark radiation constraints are satisfied. The red region corresponds to BBN excluded region, gray to overproduction of WIMP dark matter and brown to $\Delta N_{\text{eff}} > 1$. Red and brown solid lines show the boundaries of excluded regions due to BBN and dark radiation respectively.

### 3.1 SUSY DFSZ model

Our first results of allowed regions in the $T_R$ vs $m_{3/2}$ plane are shown in figure 1. In frame a), we first take $f_a = 10^{11}$ GeV and $10^{12}$ GeV and show allowed and excluded regions. For lower values of $f_a$, DM density is enhanced by gravitino decay only and BBN constraints are violated by late-decaying gravitinos since axinos and saxions are short-lived. For $f_a < 10^{11}$ GeV, BBN bounds and DM exclusion contours can be read from figure 1 once the region $T_R > f_a$ is omitted. As we increase $f_a$ to $10^{11}$ GeV, then the axino and saxion decay rates are suppressed and they decay later. However, they still typically decay before neutralino freeze-out and thus do not change the picture.

The gray band at the top of frame a) is forbidden due to overproduction of WIMP dark matter due to thermal gravitino production and decay well after WIMP freeze-out. This occurs for $T_R \gtrsim 3 \times 10^{10}$ GeV when $f_a = 10^{11}$. The red-shaded region occurs due to violation of BBN constraints on late-decaying neutral relics. In the case of frame a), this comes again from gravitino production along with decay after the onset of BBN. Here, we use a digitized version of BBN constraints from Jedamzik \cite{97} which appear in the $\Omega_X h^2$ vs. $\tau_X$ plane where $X$ stands for the quasi-stable neutral particle, $\Omega_X h^2$ is its would-be relic abundance had it not decayed and $\tau_X$ is its lifetime. The curves also depend on the $X$-particle hadronic branching fraction $B_h$ and on the mass $m_X$. Ref. [97] presents results for $m_X = 0.1$ and 1 TeV and we extrapolate between and beyond these values for alternative mass cases. Together, the
Although these bounds are not directly applicable to $\xi_{s}$ requires further. In this case, the DM-excluded region expands to the black contours labelled by $m_{3/2}$ fully viable for overproduction of WIMPs for $\xi_{s}$. In frame c), the axion in natural SUSY, then such experiments should also aim for exploration down to $f_{a}$ at high $T_{R}$ efficiency (∆$N_{\text{eff}} > 1$) has extended and imposes an additional excluded region for $m_{3/2} \geq 15$ TeV and $T_{R} \geq 10^{8}$ GeV.

These results have important implications for axion detection. Currently, the ADMX experiment is exploring regions of $f_{a}/N \geq 10^{12}$ GeV. Future plans include an exploration of regions down to $f_{a}/N \geq 10^{11}$ GeV. To make a complete exploration of the expected locus of the axion in natural SUSY, then such experiments should also aim for exploration down to $f_{a}/N \sim 10^{10}$ GeV. For even smaller $f_{a}/N < 10^{10}$ GeV values, then axion BCM-production requires $\theta_{i}$ values very close to $\pi$ and the axion production rates would be considered as fine-tuned [126].

5Thus larger $m_{3/2}$ may be allowed if one can find a UV model to realize the natural SUSY spectrum that we show here.
Figure 1. Plot of allowed regions in $T_R$ vs. $m_{3/2}$ plane in the SUSY DFSZ axion model for $a)$ $f_a = 10^{11}$ and $10^{12}$ GeV, $b)$ $f_a = 10^{13}$ GeV, for $\xi_s = 0$ and 1 and $c)$ $f_a = 10^{14}$ GeV for $\xi_s = 1$. For $f_a = 10^{11}$, $T_R > 10^{11}$ is forbidden to avoid PQ symmetry restoration. We take $m_s = m_{\tilde{a}} \equiv m_{3/2}$ in all plots.
3.2 SUSY KSVZ model

In this subsection, we show baryogenesis-allowed regions in the $T_R$ vs. $m_{3/2}$ plane for the SUSY KSVZ model. We regard the SUSY KSVZ model as less lucrative in that one loses the DFSZ solution to the SUSY $\mu$ problem and the connection with electroweak naturalness. In addition, if the exotic heavy quark field $Q$ is not an element of a complete GUT multiplet, then one loses gauge coupling unification. Nonetheless, it is instructive to view these results for comparison to the SUSY DFSZ case.

In figure 2a), we show results for $f_a = 10^{10}$ GeV. Even for $f_a$ as low as $10^{10}$ GeV, the gray-shaded WIMP-overproduction region occupies the region with $m_{3/2} \lesssim 1.3$ TeV. In this region, since $m_a = m_{3/2}$, then thermal axino production followed by decay after neutralino freeze-out leads to WIMP over production across a wide range of $T_R$ values. This is because the axino decay is suppressed by $Q$-mediated loops as compared to SUSY DFSZ. As $f_a$ is increased to $10^{11}$ GeV (figure 2b)), then the DM-forbidden region expands out to $m_{3/2} \sim 2$ TeV region. For $f_a = 10^{12}$ GeV (figure 2b)), then the DM-forbidden region expands out to $m_{3/2} \sim 4$ TeV. Even for this high value of $f_a$, there is still room for leptogenesis in natural SUSY models for each of the cases of THL, NTHL, OSL and ADL.\footnote{For this case only, we have found that there exists some mild entropy dilution $r$ of $n_L$ due to thermal axino production for $T_R \sim 10^{10}$–$10^{11}$ GeV by up to a factor of 2. Since these $T_R$ values are beyond the lower limit, our plots hardly change. Alternatively, the THL lower bound on $T_R$ may be interpreted as a lower bound on $T_R/r$.}

For the SUSY KSVZ model with $f_a = 10^{13}$ GeV as shown in figure 2c), then the DM-forbidden region has expanded to exclude all viable natural SUSY parameter space except for a tiny slice with $m_{3/2} \sim 15$–$20$ TeV and $T_R < 10^6$ GeV where ADL might still function.

4 Constraints in the $T_R$ vs. $f_a$ plane for fixed $m_{3/2}$

In this section, we examine the DM constraints on baryogenesis in the $T_R$ vs. $f_a$ plane for fixed natural $m_{3/2}$ values to gain further insights on axion decay constant dependence of the constraints for $T_R$ between $10^4$–$10^{12}$ GeV. On these planes, in the yellow region labelled $T_R > f_a$ we expect PQ symmetry to be restored during reheating which leads to generation of separate domains with different $\theta$ values and the appearance of domain walls and associated problems. In this case, axion oscillations including the anharmonic effect must be averaged over separate domains $[36, 38]$. As before, we do not consider this region.

4.1 SUSY DFSZ model

In figure 3, we plot allowed and forbidden regions for baryogenesis in SUSY DFSZ model in the $T_R$ vs. $f_a$ plane for $m_{3/2} = 5$ TeV. In frame a), with $\xi_s = 0$, the gray-shaded region still corresponds to WIMP overproduction and sets an upper limit of $f_a \lesssim 10^{12}$ GeV. The red-shaded region corresponds to violation of BBN constraints from late decaying gravitinos and bounds $T_R$ from above: $T_R \lesssim 2 \times 10^8$ GeV which excludes the possibility of THL. Still, large regions of natural SUSY parameter space are consistent with NTHL, OSL and with ADL. The BBN bound kicks in again at $f_a \sim 6 \times 10^{14}$ due to long-lived saxions. For the case of $\xi_s = 1$ shown in figure 3b), then $s \rightarrow aa$ is turned on. This leads to the brown dark-radiation excluded region at very large $f_a$ values and large $T_R$. In addition, we note for this case that the red-shaded BBN forbidden region has actually expanded compared to frame a). This is because for $\xi_s = 0$, the BCM-produced saxions inject considerable entropy into the cosmic soup at large $f_a$ thus diluting the gravitino abundance. For $\xi_s = 1$, then the saxion decays more quickly leading to less entropy dilution of gravitinos and thus more restrictive
Figure 2. Plot of allowed regions in $T_R$ vs. $m_{3/2}$ plane in the SUSY KSVZ axion model for a) $f_a = 10^{10}$ GeV, b) $10^{11}$ and $10^{12}$ GeV for $\xi_s = 0$ and 1 and c) $f_a = 10^{13}$ GeV for $\xi_s = 0$. For $f_a = 10^{11}$, $T_R > 10^{11}$ is forbidden to avoid PQ symmetry restoration. We take $m_s = m_{\tilde{a}} = m_{3/2}$ in all plots.
BBN bounds. Thus, the BBN constraints are actually more severe for $\xi_s = 1$. In addition, for frame $b)$, we see WIMP overproduction bounds are less severe with $f_a \lesssim 10^{13}$ GeV being required for the allowed regions. These are due to a reduced $s \to$ SUSY branching fractions for the $\xi_s = 1$ case.

In figure 4 we show allowed and excluded regions in the $T_R$ vs. $f_a$ plane for $m_{3/2} = 10$ TeV. In the case of $\xi_s = 0$ shown in frame $a)$, the larger gravitino mass causes the gravitinos to decay more quickly so that BBN constraints are diminished: in this case, the THL scenario with $T_R > 1.5 \times 10^9$ GeV is allowed in contrast to the previous case with $m_{3/2} = 5$ TeV. In addition, broad swaths of parameter space are allowed for the NTHL, OSL and ADL scenarios with $f_a \lesssim 5 \times 10^{12}$ GeV. For larger $f_a$ values, then axino and saxion production followed by late decays leads to too much WIMP dark matter. For the case with $\xi_s = 1$ shown in frame $b)$, we see again the BBN constraints are somewhat enhanced due to diminished entropy dilution of gravitinos at large $f_a$. In addition, a dark-radiation forbidden region has appeared. Most importantly, the DM-allowed region occurs for $f_a \lesssim 10^{14}$ GeV so
that large swaths of parameter space are open for baryogenesis. This is because, since we take $m_{\tilde{a}} = m_s = m_{3/2}$, then the axinos and saxions are also shorter-lived and tend to decay earlier - frequently before WIMP freeze-out - so DM overproduction is more easily avoided.

For even larger values of $m_{3/2}$ up to $m_{3/2} \sim 25$ TeV, we would expect to see a very similar BBN constraint since BBN bounds are not sensitive to any changes in $m_{3/2}$ for $7 \text{ TeV} \lesssim m_{3/2} \lesssim 25$ TeV (see figure 1). As $m_{3/2}$ increases and reaches beyond $m_{3/2} \sim 65$ TeV, then gravitino decays much sooner and does not violate BBN constraints at all. However DM production highly depends on $f_a$ and the DM exclusion picture would look different up to a maximum $f_a$ after which the whole parameter space is excluded by too much DM.

4.2 SUSY KSVZ model

In this subsection, we show corresponding results in the $T_R$ vs. $f_a$ plane for SUSY KSVZ. In figure 5 we show the plane for $m_{3/2} = 5$ TeV and a) $\xi_s = 0$. Here, we see that THL is ruled out due to the severe BBN bounds arising from gravitino production and decay which
restrict $T_R \lesssim 2 \times 10^8$ GeV while the DM restriction rules out $f_a \gtrsim 10^{12}$ GeV. The NTHL, OSL and ADL are still viable baryogenesis mechanisms over a wide range of $T_R$ and $f_a$ values. In frame 5b) for $\xi_s = 1$, the DM forbidden region is similar with a $f_a < 10^{12}$ GeV restriction. However, the BBN restricted region has increased because there is less entropy dilution from saxion decay of the gravitinos abundance. The expanded BBN region lies in the already DM and dark radiation excluded region so provides no additional constraint. Since saxions decay earlier for $\xi_s = 1$ compared to $\xi_s = 0$, then they inject neutralinos at a higher decay temperature $T_D^{\xi_s}$; as a consequence, a small DM-allowed region appears at high $f_a \sim 10^{13}–10^{14}$ GeV and $T_R \sim 10^9$ GeV which is barely consistent with ADL. In figure 6a), we show the same $T_R$ vs. $f_a$ plane with $\xi_s = 0$, but this time for a higher value of $m_{3/2} = m_s = m_\tilde{a} = 10$ TeV. The higher value of $m_{3/2}$ means gravitinos decay more quickly and at higher temperature so that the BBN bound on $T_R$ is given by $T_R \gtrsim 4 \times 10^9$ so that THL is again viable. Also, the DM-allowed region has moved to a higher $f_a$ bound of $f_a \lesssim 2 \times 10^{12}$ GeV. In this frame, all four baryogenesis mechanisms are possible. In figure 6b), we show the same plane for $\xi_s = 1$. Here a prominent dark radiation excluded region
appears at large $f_a \gtrsim 10^{13} - 10^{14}$ GeV, although this region is already excluded by WIMP overproduction and by BBN. The larger saxion width arising from the additional $s \rightarrow aa$ decay mode means the saxion decay at higher temperatures leading to some possible allowed regions appearing at $f_a \sim 10^{14}$ GeV and $T_R \sim 10^5$ GeV which admits ADL. Otherwise, large regions of viable parameter space exists for $f_a \lesssim 2 \times 10^{12}$ GeV and for $T_R \lesssim 4 \times 10^9$ GeV where all four leptogenesis mechanisms are possible.

5 Conclusion

In this paper we have investigated constraints on four compelling baryogenesis-via-leptogenesis scenarios within the framework of supersymmetric models with radiatively-driven naturalness. These models are especially attractive since they contain solutions to the gauge hierarchy problem (via SUSY), the strong CP problem (via the axion), the SUSY $\mu$ problem (for the case of the SUSY DFSZ axion) and the Little Hierarchy problem (where $\mu \sim 100$–200 GeV is generated from multi-TeV values of $m_{3/2}$). The characteristic, unambiguous
signature of such models is the presence of light higgsinos \( \tilde{Z}_{1,2} \) and \( \tilde{W}_1^\pm \) with mass \( \sim \mu \). In these models, the LSP is a higgsino-like WIMP which is thermally underproduced. The remainder of the dark matter abundance is filled by the axion. Indeed, over most of parameter space the axion forms the bulk of dark matter \([127]\).

The RNS spectra can be tested at LHC14 via gluino pair production followed by cascade decays for \( m_{\tilde{g}} \lesssim 2 \text{ TeV} \) (for 300–1000 fb\(^{-1}\) of integrated luminosity) \([128]\). The cascade decay signatures will include an opposite-sign/same-flavor (OSSF) dilepton mass edge bounded by \( m_{\tilde{Z}_2} - m_{\tilde{Z}_1} \sim 10–30 \text{ GeV} \) \([128, 129]\). In addition, a unique same-sign diboson signature arising from wino pair production emerges \([128, 130]\). For naturalness measure \( \Delta_{EW} < 30 \), then \( m_{\tilde{g}} \) may range up to 3–4 TeV so the gluino possibly could be beyond LHC reach. Monojet and monojet-plus-OSSF dilepton signatures are also possible \([131, 132]\). The crucial test of naturalness is light higgsino pair production at a linear \( e^+e^- \) collider such as ILC operating with \( \sqrt{s} > 2m(\text{higgsino}) \) \([133]\). Regarding dark matter detection, while higgsino-like WIMPs make up likely only a fraction of dark matter, naturalness implies a large coupling of WIMPs to the Higgs boson so WIMP direct detection seems guaranteed at ton-scale noble liquid detectors \([126, 134, 135]\). The ADMX axion detection experiment should ultimately be sensitive to most, but not all, of the range of axion mass and coupling considered in this paper \([126]\).

In supersymmetric dark matter models, baryogenesis mechanisms are confronted by the gravitino problem: gravitinos which are thermally produced in the early universe can lead to overproduction of WIMPs or to violations of BBN constraints. In SUSY axion models, there are analogous problems arising from thermal axino production and decay and from thermal- and oscillation-produced saxions. We calculated regions of the \( T_R \) vs. \( m_{3/2} \) plane in the compelling RNS SUSY model with DFSZ axions and \( \xi_s = 0 \) and 1. Our main result is that the region of parameter space preferred by naturalness with \( f_a \sim \sqrt{\mu M_P/\lambda_{\mu}} \sim 10^{10}–10^{12} \text{ GeV} \) supports all four leptogenesis mechanisms. The thermal leptogenesis is perhaps less plausible since its allowed region is nested typically between the constrained region of \( 7 \text{ TeV} < m_{3/2} \) \( (<10 \text{ TeV} \) or \( 20 \text{ TeV} \) if one considers the naturalness in terms of the gravitino mass) and \( 1.5 \times 10^9 \) GeV < \( T_R < 4 \times 10^9 \) GeV. The other NTHL, OSL and ADL mechanisms can freely operate over a broad region of parameter space for \( f_a \lesssim 10^{12} \text{ GeV} \) and \( T_R \gtrsim 10^5 \text{ GeV} \). We also evaluated all constraints in the \( T_R \) vs. \( f_a \) plane for fixed \( m_{3/2} = 5 \) and 10 TeV.

The broad allowed regions of parameter space basically favor the following.

- Multi-TeV values of \( m_{3/2} \) to avoid BBN constraints and to hasten saxion and axino decays. Since \( m_{3/2} \) sets the scale for superpartner masses at LHC, these multi-TeV values of \( m_{3/2} \) are also supported by LHC8 sparticle search constraints and the large value of \( m_h \sim 125 \text{ GeV} \) at little cost to naturalness.

- A value of \( f_a \sim 10^{10}–10^{12} \text{ GeV} \) which suppresses WIMP over production from axino/saxion production. Such values of \( f_a \) lead to axion masses somewhat above the standard search region of ADMX and should motivate future axion search experiments to increase their search region to heavier axion masses.

- Values of \( T_R \sim 10^5–10^9 \text{ GeV} \).

For completeness, we have also evaluated the leptogenesis allowed regions in the SUSY KSVZ model for which an alternative solution to the \( \mu \) problem is needed. The loop-suppressed axino and saxion decay rates typically lead to more stringent constraints in this case although regions of parameter space can still be found where the various leptogenesis mechanisms are still possible.
Acknowledgments

We thank Vernon Barger for discussions and Andre Lessa for earlier collaborations on these topics. KJB also thanks Koichi Hamaguchi for fruitful discussions. This work was supported in part by the US Department of Energy, Office of High Energy Physics. The computing for this project was performed at the OU Supercomputing Center for Education & Research (OSCER) at the University of Oklahoma (OU). KJB is also supported by Grant-in-Aid for Scientific research No. 26104009.

References

[1] ATLAS collaboration, Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys. Lett. B 716 (2012) 1 [arXiv:1207.7214] [nSPIRE].

[2] CMS collaboration, Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, Phys. Lett. B 716 (2012) 30 [arXiv:1207.7235] [nSPIRE].

[3] M. Carena and H.E. Haber, Higgs boson theory and phenomenology, Prog. Part. Nucl. Phys. 50 (2003) 63 [hep-ph/0208209] [nSPIRE].

[4] ATLAS collaboration, Search for squarks and gluinos with the ATLAS detector in final states with jets and missing transverse momentum using $\sqrt{s} = 8$ TeV proton-proton collision data, JHEP 09 (2014) 176 [arXiv:1405.7875] [nSPIRE].

[5] ATLAS collaboration, Search for squarks and gluinos in events with isolated leptons, jets and missing transverse momentum at $\sqrt{s} = 8$ TeV with the ATLAS detector, JHEP 04 (2015) 116 [arXiv:1501.03555] [nSPIRE].

[6] CMS Collaboration, Search for supersymmetry in final states with missing transverse momentum and 0, 1, 2, or $\geq$ 3 b jets in 8 TeV pp collisions, CMS-PAS-SUS-12-016.

[7] H. Baer, V. Barger and A. Mustafayev, Implications of a 125 GeV Higgs scalar for LHC SUSY and neutralino dark matter searches, Phys. Rev. D 85 (2012) 075010 [arXiv:1112.3017] [nSPIRE].

[8] A. Arbey, M. Battaglia, A. Djouadi, F. Mahmoudi and J. Quevillon, Implications of a 125 GeV Higgs for supersymmetric models, Phys. Lett. B 708 (2012) 162 [arXiv:1112.3028] [nSPIRE].

[9] M. Shifman, Frontiers Beyond the Standard Model: Reflections and Impressionistic Portrait of the conference, Mod. Phys. Lett. A 27 (2012) 014006 [arXiv:1211.0004] [nSPIRE].

[10] R. Barbieri, Electroweak theory after the first Large Hadron Collider phase, Phys. Scripta T 158 (2013) 014006 [arXiv:1309.3473] [nSPIRE].

[11] G.F. Giudice, Naturalness after LHC8, PoS(EPS-HEP 2013)163.

[12] G. Altarelli, The Higgs: so simple yet so unnatural, Phys. Scripta T 158 (2013) 014011 [arXiv:1308.0545] [nSPIRE].

[13] N. Craig, The State of Supersymmetry after Run I of the LHC, arXiv:1309.0528 [nSPIRE].

[14] H. Murayama, Future Experimental Programs, Phys. Scripta T 158 (2013) 014025 [arXiv:1401.0966] [nSPIRE].

[15] G.G. Ross, SUSY: Quo Vadis?, Eur. Phys. J. C 74 (2014) 2699 [nSPIRE].

[16] J. Lykken and M. Spiropulu, Supersymmetry and the Crisis in Physics, Sci. Am. 310 (2014) 34.

[17] M. Dine, Naturalness Under Stress, Ann. Rev. Nucl. Part. Sci. 65 (2015) 43 [arXiv:1501.01035] [nSPIRE].
[18] H. Baer, V. Barger, P. Huang, A. Mustafayev and X. Tata, *Radiative natural SUSY with a 125 GeV Higgs boson*, Phys. Rev. Lett. 109 (2012) 161802 [arXiv:1207.3343] [SPIRE].

[19] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev and X. Tata, *Radiative natural supersymmetry: Reconciling electroweak fine-tuning and the Higgs boson mass*, Phys. Rev. D 87 (2013) 115028 [arXiv:1212.2655] [SPIRE].

[20] H. Baer, V. Barger and M. Savoy, *Upper bounds on sparticle masses from naturalness or how to disprove weak scale supersymmetry*, arXiv:1509.02929 [SPIRE].

[21] H. Baer, V. Barger and D. Mickelson, *How conventional measures overestimate electroweak fine-tuning in supersymmetric theory*, Phys. Rev. D 88 (2013) 095013 [arXiv:1309.2984] [SPIRE].

[22] H. Baer, V. Barger, D. Mickelson and M. Padeffke-Kirkland, *SUSY models under siege: LHC constraints and electroweak fine-tuning*, Phys. Rev. D 89 (2014) 115019 [arXiv:1404.2277] [SPIRE].

[23] J.R. Ellis, K. Enqvist, D.V. Nanopoulos and F. Zwirner, *Observables in Low-Energy Superstring Models*, Mod. Phys. Lett. A 1 (1986) 57 [SPIRE].

[24] R. Barbieri and G.F. Giudice, *Upper Bounds on Supersymmetric Particle Masses*, Nucl. Phys. B 306 (1988) 63 [SPIRE].

[25] S. Weinberg, *The U(1) Problem*, Phys. Rev. Lett. 43 (1979) 103 [SPIRE].

[26] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Can Confinement Ensure Natural CP Invariance of Strong Interactions?*, Nucl. Phys. B 166 (1980) 493 [SPIRE].

[27] M. Dine and W. Fischler, *The not-so-harmless axion*, Phys. Lett. B 120 (1983) 137 [SPIRE].

[28] M.S. Turner, *Cosmic and Local Mass Density of Invisible Axions*, Phys. Rev. D 33 (1986) 889 [SPIRE].

[29] T. Goto and M. Yamaguchi, *Is axino dark matter possible in supergravity?*, Phys. Lett. B 276 (1992) 103 [SPIRE].
[40] E.J. Chun, J.E. Kim and H.P. Nilles, Axino mass, *Phys. Lett. B* **287** (1992) 123 [hep-ph/9205229] [inSPIRE].

[41] E.J. Chun and A. Lukas, Axino mass in supergravity models, *Phys. Lett. B* **357** (1995) 43 [hep-ph/9503233] [inSPIRE].

[42] J.E. Kim and M.-S. Seo, Mixing of axino and goldstino and axino mass, *Nucl. Phys. B* **864** (2012) 296 [arXiv:1204.5495] [inSPIRE].

[43] J.E. Kim and H.P. Nilles, The µ-Problem and the Strong CP Problem, *Phys. Lett. B* **138** (1984) 150 [inSPIRE].

[44] E.J. Chun, J.E. Kim and H.P. Nilles, A Natural solution of the mu problem with a composite axion in the hidden sector, *Nucl. Phys. B* **370** (1992) 105 [inSPIRE].

[45] H. Murayama, H. Suzuki and T. Yanagida, Radiative breaking of Peccei-Quinn symmetry at the intermediate mass scale, *Phys. Lett. B* **291** (1992) 418 [inSPIRE].

[46] K.J. Bae, H. Baer and H. Serce, Natural little hierarchy for SUSY from radiative breaking of the Peccei-Quinn symmetry, *Phys. Rev. D* **91** (2015) 015003 [arXiv:1410.7500] [inSPIRE].

[47] P. Minkowski, µ → eγ at a Rate of One Out of 10^9 Muon Decays?, *Phys. Lett. B* **67** (1977) 421 [inSPIRE].

[48] T. Yanagida, Horizontal Symmetry And Masses Of Neutrinos, *Prog. Theor. Phys. 64* (1980) 64.

[49] M. Gell-Mann, P. Ramond and R. Slansky, Complex Spinors and Unified Theories, *Conf. Proc. C 790927* (1979) 315 [arXiv:1306.4669] [inSPIRE].

[50] R.N. Mohapatra and G. Senjanović, Neutrino Mass and Spontaneous Parity Violation, *Phys. Rev. Lett. 44* (1980) 912 [inSPIRE].

[51] A. Kusenko and L.J. Rosenberg, Working Group Report: Non-WIMP Dark Matter, arXiv:1310.8642 [inSPIRE].

[52] H. Baer, K.-Y. Choi, J.E. Kim and L. Roszkowski, Dark matter production in the early Universe: beyond the thermal WIMP paradigm, *Phys. Rept.* **555** (2014) 1 [arXiv:1407.0017] [inSPIRE].

[53] K.-Y. Choi, J.E. Kim, H.M. Lee and O. Seto, Neutralino dark matter from heavy axino decay, *Phys. Rev. D* **77** (2008) 123501 [arXiv:0801.0491] [inSPIRE].

[54] H. Baer, A. Lessa, S. Rajagopalan and W. Sriethawong, Mixed axion/neutralino cold dark matter in supersymmetric models, *JCAP* **06** (2011) 031 [arXiv:1103.5413] [inSPIRE].

[55] PLANCK collaboration, P.A.R. Ade et al., Planck 2015 results. XIII. Cosmological parameters, arXiv:1502.01589 [inSPIRE].

[56] K.J. Bae, H. Baer, A. Lessa and H. Serce, Coupled Boltzmann computation of mixed axion neutralino dark matter in the SUSY DFSZ axion model, *JCAP* **10** (2014) 082 [arXiv:1406.4138] [inSPIRE].

[57] L. Covi, H.-B. Kim, J.E. Kim and L. Roszkowski, Axinos as dark matter, *JHEP* **05** (2001) 033 [hep-ph/0101009] [inSPIRE].

[58] A. Brandenburg and F.D. Steffen, Axino dark matter from thermal production, *JCAP* **08** (2004) 008 [hep-ph/0405158] [inSPIRE].

[59] A. Strumia, Thermal production of axino Dark Matter, *JHEP* **06** (2010) 036 [arXiv:1003.5847] [inSPIRE].

[60] E.J. Chun, Dark matter in the Kim-Nilles mechanism, *Phys. Rev. D* **84** (2011) 043509 [arXiv:1104.2219] [inSPIRE].

[61] K.J. Bae, K. Choi and S.H. Im, Effective Interactions of Axion Supermultiplet and Thermal Production of Axino Dark Matter, *JHEP* **08** (2011) 065 [arXiv:1106.2452] [inSPIRE].
[62] K.J. Bae, E.J. Chun and S.H. Im, Cosmology of the DFSZ axino, JCAP 03 (2012) 013 [arXiv:1111.5962] [SPIRE].

[63] Particle Data Group collaboration, K.A. Olive et al., Review of Particle Physics, Chin. Phys. C 38 (2014) 090001 [SPIRE].

[64] V.A. Kuzmin, V.A. Rubakov and M.E. Shaposhnikov, On the Anomalous Electroweak Baryon Number Nonconservation in the Early Universe, Phys. Lett. B 155 (1985) 36 [SPIRE].

[65] M. Carena, M. Quiros and C.E.M. Wagner, Electroweak baryogenesis and Higgs and stop searches at LEP and the Tevatron, Nucl. Phys. B 524 (1998) 3 [hep-ph/9710401] [SPIRE].

[66] M. Carena, M. Quirós and C.E.M. Wagner, The Baryogenesis Window in the MSSM, Nucl. Phys. B 812 (2009) 243 [arXiv:0809.3760] [SPIRE].

[67] K.J. Bae, H. Baer, V. Barger, D. Mickelson and M. Savoy, Implications of naturalness for the heavy Higgs bosons of supersymmetry, Phys. Rev. D 90 (2014) 075010 [arXiv:1407.3853] [SPIRE].

[68] M. Fukugita and T. Yanagida, Baryogenesis Without Grand Unification, Phys. Lett. B 174 (1986) 45 [SPIRE].

[69] M.A. Luty, Baryogenesis via leptogenesis, Phys. Rev. D 45 (1992) 455 [SPIRE].

[70] B.A. Campbell, S. Davidson and K.A. Olive, Supersymmetric (S)neutrino mass induced baryogenesis, Phys. Lett. B 303 (1993) 63 [hep-ph/9302222] [SPIRE].

[71] W. Buchmüller, P. Di Bari and M. Plümacher, Cosmic microwave background, matter-antimatter asymmetry and neutrino masses, Nucl. Phys. B 643 (2002) 367 [Erratum ibid. B 793 (2008) 362] [hep-ph/0205349] [SPIRE].

[72] W. Buchmüller, P. Di Bari and M. Plümacher, A Bound on neutrino masses from baryogenesis, Phys. Lett. B 547 (2002) 128 [hep-ph/0209301] [SPIRE].

[73] P. Di Bari, News on leptogenesis, AIP Conf. Proc. 655 (2003) 208 [hep-ph/0211175] [SPIRE].

[74] W. Buchmüller, R.D. Peccei and T. Yanagida, Leptogenesis as the origin of matter, Ann. Rev. Nucl. Part. Sci. 55 (2005) 311 [hep-ph/0502169] [SPIRE].

[75] S. Davidson, E. Nardi and Y. Nir, Leptogenesis, Phys. Rept. 466 (2008) 105 [arXiv:0802.2962] [SPIRE].

[76] S. Blanchet and P. Di Bari, The minimal scenario of leptogenesis, New J. Phys. 14 (2012) 125012 [arXiv:1211.0512] [SPIRE].

[77] K. Kumekawa, T. Moroi and T. Yanagida, Flat potential for inflaton with a discrete R invariance in supergravity, Prog. Theor. Phys. 92 (1994) 437 [hep-ph/9405337] [SPIRE].

[78] G. Lazarides, Leptogenesis in supersymmetric hybrid inflation, Springer Tracts Mod. Phys. 163 (2000) 227 [hep-ph/9904428] [SPIRE].

[79] G.F. Giudice, M. Peloso, A. Riotto and I. Tkachev, Production of massive fermions at preheating and leptogenesis, JHEP 08 (1999) 014 [hep-ph/9905242] [SPIRE].

[80] T. Asaka, K. Hamaguchi, M. Kawasaki and T. Yanagida, Leptogenesis in inflaton decay, Phys. Lett. B 464 (1999) 12 [hep-ph/9906366] [SPIRE].

[81] T. Asaka, K. Hamaguchi, M. Kawasaki and T. Yanagida, Leptogenesis in inflationary universe, Phys. Rev. D 61 (2000) 083512 [hep-ph/9907559] [SPIRE].

[82] M. Kawasaki, M. Yamaguchi and T. Yanagida, Natural chaotic inflation in supergravity and leptogenesis, Phys. Rev. D 63 (2001) 103514 [hep-ph/0011104] [SPIRE].

[83] H. Murayama and T. Yanagida, Leptogenesis in supersymmetric standard model with right-handed neutrino, Phys. Lett. B 322 (1994) 349 [hep-ph/9310297] [SPIRE].
M. Dine and A. Kusenko, The Origin of the matter-antimatter asymmetry, *Rev. Mod. Phys.* **76** (2003) 1 [hep-ph/0303065] [inSPIRE].

K. Enqvist and A. Mazumdar, Cosmological consequences of MSSM flat directions, *Phys. Rept.* **380** (2003) 99 [hep-ph/0209244] [inSPIRE].

R. Allahverdi and A. Mazumdar, A mini review on Affleck-Dine baryogenesis, *New J. Phys.* **14** (2012) 125013.

K. Hamaguchi, H. Murayama and T. Yanagida, Leptogenesis from $\tilde{N}$ dominated early universe, *Phys. Rev. D* **65** (2002) 043512 [hep-ph/0109030] [inSPIRE].

I. Affleck and M. Dine, A New Mechanism for Baryogenesis, *Nucl. Phys. B* **249** (1985) 361 [inSPIRE].

M. Dine, L. Randall and S.D. Thomas, Supersymmetry breaking in the early universe, *Phys. Rev. Lett.* **75** (1995) 398 [hep-ph/9503303] [inSPIRE].

M. Dine, L. Randall and S.D. Thomas, Baryogenesis from flat directions of the supersymmetric standard model, *Nucl. Phys. B* **458** (1996) 291 [hep-ph/9507453] [inSPIRE].

K. Jedamzik, Big bang nucleosynthesis constraints on hadronically and electromagnetically decaying relic neutral particles, *Phys. Rev. D* **78** (2008) 065011 [arXiv:0804.3745] [inSPIRE].

K. Jedamzik, Big bang nucleosynthesis constraints on hadronically and electromagnetically decaying relic neutral particles, *Phys. Lett. B* **431** (1998) 354 [hep-ph/9710460] [inSPIRE].

R. Allahverdi and A. Mazumdar, A mini review on Affleck-Dine baryogenesis, *New J. Phys.* **14** (2012) 125013.

K. Hamaguchi, H. Murayama and T. Yanagida, Leptogenesis from $\tilde{N}$ dominated early universe, *Phys. Rev. D* **65** (2002) 043512 [hep-ph/0109030] [inSPIRE].

I. Affleck and M. Dine, A New Mechanism for Baryogenesis, *Nucl. Phys. B* **249** (1985) 361 [inSPIRE].

M. Dine, L. Randall and S.D. Thomas, Supersymmetry breaking in the early universe, *Phys. Rev. Lett.* **75** (1995) 398 [hep-ph/9503303] [inSPIRE].

M. Dine, L. Randall and S.D. Thomas, Baryogenesis from flat directions of the supersymmetric standard model, *Nucl. Phys. B* **458** (1996) 291 [hep-ph/9507453] [inSPIRE].

K. Hamaguchi, H. Murayama and T. Yanagida, Leptogenesis from $\tilde{N}$ dominated early universe, *Phys. Rev. D* **65** (2002) 043512 [hep-ph/0109030] [inSPIRE].

I. Affleck and M. Dine, A New Mechanism for Baryogenesis, *Nucl. Phys. B* **249** (1985) 361 [inSPIRE].

M. Dine, L. Randall and S.D. Thomas, Supersymmetry breaking in the early universe, *Phys. Rev. Lett.* **75** (1995) 398 [hep-ph/9503303] [inSPIRE].

M. Dine, L. Randall and S.D. Thomas, Baryogenesis from flat directions of the supersymmetric standard model, *Nucl. Phys. B* **458** (1996) 291 [hep-ph/9507453] [inSPIRE].

K. Hamaguchi, H. Murayama and T. Yanagida, Leptogenesis from $\tilde{N}$ dominated early universe, *Phys. Rev. D* **65** (2002) 043512 [hep-ph/0109030] [inSPIRE].
[106] A. Pilaftsis and T.E.J. Underwood, *Resonant leptogenesis*, Nucl. Phys. B 692 (2004) 303 [hep-ph/0309342] [nSPIRE].

[107] S.Yu. Khlebnikov and M.E. Shaposhnikov, *The Statistical Theory of Anomalous Fermion Number Nonconservation*, Nucl. Phys. B 308 (1988) 885 [nSPIRE].

[108] J.A. Harvey and M.S. Turner, *Cosmological baryon and lepton number in the presence of electroweak fermion number violation*, Phys. Rev. D 42 (1990) 3344 [nSPIRE].

[109] W. Buchmüller, P. Di Bari and M. Plümacher, *Leptogenesis for pedestrians*, Annals Phys. 315 (2005) 305 [hep-ph/0401240] [nSPIRE].

[110] K. Hamaguchi, *Cosmological baryon asymmetry and neutrinos: Baryogenesis via leptogenesis in supersymmetric theories*, hep-ph/0212305.

[111] H. Murayama and T. Yanagida, *Leptogenesis in supersymmetric standard model with right-handed neutrino*, Phys. Lett. B 322 (1994) 349 [hep-ph/9310297] [nSPIRE].

[112] T. Gherghetta, C.F. Kolda and S.P. Martin, *Flat directions in the scalar potential of the supersymmetric standard model*, Nucl. Phys. B 468 (1996) 37 [hep-ph/9510370] [nSPIRE].

[113] K. Enqvist and J. McDonald, *Q balls and baryogenesis in the MSSM*, hep-ph/9711514.

[114] M. Fujii, K. Hamaguchi and T. Yanagida, *Predictions on the neutrinoless double beta decay from the leptogenesis via the LH_u flat direction*, Phys. Lett. B 538 (2002) 107 [hep-ph/0203189] [nSPIRE].

[115] R. Allahverdi, M. Drees and A. Mazumdar, *Hubble induced radiative corrections and Affleck-Dine baryogenesis*, Phys. Rev. D 65 (2002) 065010 [hep-ph/0110136] [nSPIRE].

[116] R. Allahverdi, B.A. Campbell and J.R. Ellis, *Reheating and supersymmetric flat direction baryogenesis*, Nucl. Phys. B 579 (2000) 355 [hep-ph/0001122] [nSPIRE].

[117] A. Anisimov and M. Dine, *Some issues in flat direction baryogenesis*, Nucl. Phys. B 619 (2001) 729 [hep-ph/0008058] [nSPIRE].

[118] M. Fujii, K. Hamaguchi and T. Yanagida, *Reheating temperature independence of cosmological baryon asymmetry in Affleck-Dine leptogenesis*, Phys. Rev. D 63 (2001) 123513 [hep-ph/0010218] [nSPIRE].

[119] H. Baer, A. Lessa and W. Sreezhawong, *Coupled Boltzmann calculation of mixed axion/neutrino cold dark matter production in the early universe*, JCAP 01 (2012) 036 [arXiv:1110.2491] [nSPIRE].

[120] K.J. Bae, H. Baer and A. Lessa, *Dark Radiation Constraints on Mixed Axion/Neutralino Dark Matter*, JCAP 04 (2013) 041 [arXiv:1301.7428] [nSPIRE].

[121] K. Kohri, T. Moroi and A. Yotsuyanagi, *Big-bang nucleosynthesis with unstable gravitino and upper bound on the reheating temperature*, Phys. Rev. D 73 (2006) 123511 [hep-ph/0507245] [nSPIRE].

[122] H. Baer, S. Kraml, A. Lessa and S. Sekmen, *Thermal leptogenesis and the gravitino problem in the Asaka-Yanagida axion/axino dark matter scenario*, JCAP 04 (2011) 039 [arXiv:1012.3769] [nSPIRE].

[123] K.J. Bae, H. Baer and E.J. Chun, *Mixed axion/neutralino dark matter in the SUSY DFSZ axion model*, JCAP 12 (2013) 028 [arXiv:1309.5365] [nSPIRE].

[124] F.E. Paige, S.D. Protopopescu, H. Baer and X. Tata, *ISAJET 7.69: A Monte Carlo event generator for pp, \( \bar{p}p \) and e⁺e⁻ reactions*, hep-ph/0312045 [nSPIRE].

[125] H. Baer, C. Balázs and A. Belyaev, *Neutralino relic density in minimal supergravity with coannihilations*, JHEP 03 (2002) 042 [hep-ph/0202076] [nSPIRE].
[126] K.J. Bae, H. Baer, V. Barger, M.R. Savoy and H. Serce, *Supersymmetry with radiatively-driven naturalness: implications for WIMP and axion searches*, *Symmetry* 7 (2015) 788 [arXiv:1503.04137] [inSPIRE].

[127] K.J. Bae, H. Baer and E.J. Chun, *Mainly axion cold dark matter from natural supersymmetry*, *Phys. Rev. D* 89 (2014) 031701 [arXiv:1309.0519] [inSPIRE].

[128] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, W. Sreethawong et al., *Radiatively-driven natural supersymmetry at the LHC*, *JHEP* 12 (2013) 013 [Erratum ibid. 1506 (2015) 053] [arXiv:1310.4858] [inSPIRE].

[129] B. Altunkaynak, H. Baer, V. Barger and P. Huang, *Distinguishing LSP archetypes via gluino pair production at LHC13*, *Phys. Rev. D* 92 (2015) 035015 [arXiv:1507.00062] [inSPIRE].

[130] H. Baer, V. Barger, P. Huang et al., *Same sign diboson signature from supersymmetry models with light higgsinos at the LHC*, *Phys. Rev. Lett.* 110 (2013) 151801 [arXiv:1302.5816] [inSPIRE].

[131] H. Baer, A. Mustafayev and X. Tata, *Monojets and mono-photons from light higgsino pair production at LHC14*, *Phys. Rev. D* 89 (2014) 055007 [arXiv:1401.1162] [inSPIRE].

[132] H. Baer, A. Mustafayev and X. Tata, *Monojet plus soft dilepton signal from light higgsino pair production at LHC14*, *Phys. Rev. D* 90 (2014) 115007 [arXiv:1409.7058] [inSPIRE].

[133] H. Baer, V. Barger, D. Mickelson, A. Mustafayev and X. Tata, *Physics at a Higgsino Factory*, *JHEP* 06 (2014) 172 [arXiv:1404.7510] [inSPIRE].

[134] H. Baer, V. Barger and D. Mickelson, *Direct and indirect detection of higgsino-like WIMPs: concluding the story of electroweak naturalness*, *Phys. Lett. B* 726 (2013) 330 [arXiv:1303.3816] [inSPIRE].

[135] H. Baer, V. Barger, D. Mickelson and X. Tata, *Snowmass whitepaper: Exploring natural SUSY via direct and indirect detection of higgsino-like WIMPs*, arXiv:1306.4183 [inSPIRE].