MSW-like enhancements without matter

G. J. Stephenson Jr.,
Department of Physics & Astronomy, University of New Mexico
Albuquerque, New Mexico 87131

and

T. Goldman,
Theoretical Division, Los Alamos National Laboratory
Los Alamos, New Mexico 87545

and

B. H. J. McKellar
School of Physics, University of Melbourne
Parkville, Victoria 3052, Australia

Abstract

We study the effects of a scalar field, coupled only to neutrinos, on oscillations among weak interaction current eigenstates. The effect of a real scalar field appears as effective masses for the neutrino mass eigenstates, the same for $\bar{\nu}$ as for $\nu$. Under some conditions, this can lead to a vanishing of $\delta m^2$, giving rise to MSW-like effects. We discuss some examples and show that it is possible to resolve the apparent discrepancy in spectra required by r-process nucleosynthesis in the mantles of supernovae and by Solar neutrino solutions.
The possibility of neutrino oscillations, the change of the weak interaction content of mass eigenstates during propagation, is influenced by the presence of normal matter through the MSW process \[1, 2\]. This is important in the analysis of the Solar Neutrino problem \[3, 4\] and in the analysis of nucleosynthesis in supernovae \[5\]. These analyses would seem to require different assumptions about the mass spectrum for the neutrinos.

We have recently examined the possibility that, in addition to the Standard Model interactions, neutrinos interact with each other through a weakly coupled, extremely light scalar field \[6\]. In that study we showed that, for a wide range of parameters consistent with known phenomena, it is possible that neutrinos form clouds in the early Universe, that those clouds could influence the evolution of structures on stellar scales, and that such clouds could have observable consequences. In this Letter we wish to present the effects that such clouds or other aggregates of neutrinos could have on oscillation phenomena. In particular, we will discuss the possibility of enhanced oscillations in regions with no normal matter, the interplay with the usual MSW effect and the possibility that this could resolve the apparent contradiction in required spectra mentioned above.

For the purpose of this Letter, we shall confine ourselves to the case of one scalar field, although more complicated scenarios are possible in principle. As will be apparent, this already provides for a great deal of complexity. We are interested in an effective theory for several (taken here as three) neutrino mass eigenstates with vacuum masses \(m_j\), where by vacuum mass we mean the physical mass which an isolated neutrino would have, and a scalar boson \(\phi\) with mass \(m_s\). At this point we shall ignore the weak interaction, although the effects of the Standard Model are reflected in the vacuum masses. The effective Lagrangian is

\[
\mathcal{L} = \sum_j \left[ \bar{\psi}_j (i \partial \! - \! m_j) \psi_j + g_j \bar{\psi}_j \psi_j \phi \right] + \frac{1}{2} \left[ (\partial \phi)^2 - m_s^2 \phi^2 \right]
\]

which gives as the equations of motion

\[
[\partial^2 + m_s^2] \phi = \sum_j g_j \bar{\psi}_j \psi_j \quad (2)
\]

\[
[i \partial \! - \! (m_j - g_j \phi)] \psi_j = 0. \quad (3)
\]

These equations are a special case of the equations of Quantum Hadrodynamics \[8\] developed for the study of relativistic nucleons interacting through the exchange of scalar and vector mesons. Following \[8\], as in \[8\], we invoke the Thomas-Fermi approximation in which it is assumed that the fields locally take on the values of the infinite system appropriate to the neutrino densities at that point.

One may view equation (3) as a definition of an effective mass,

\[
m_j^* = m_j - g_j \phi
\]

\[1\]
and it is useful to scale out the vacuum mass, defining

\[ y_j = \frac{m_j^*}{m_j} \]  \hspace{1cm} (5)

or

\[ y_j = 1 - \frac{g_j}{m_j} \phi \]  \hspace{1cm} (6)

With more than one mass eigenstate, it is possible for some \( m_j^* \) to become negative. The change of sign is no problem for a fermion and can be removed by a redefinition of phase (related to its properties under charge conjugation [8]), we shall retain the possibility of a negative sign as it makes the following discussion easier to follow. When any confusion would arise, we explicitly indicate signs and absolute values, as appropriate.

If we now specialize to spherical distributions of neutrinos and treat the neutrino distributions as zero temperature Fermi gases, equation (2) may be used to generate a radial equation for the \( \phi \) field

\[
\frac{d^2 \phi}{dr^2} + \frac{2}{r} \frac{d \phi}{dr} + \phi = \frac{1}{m^2} \sum_j g_j \bar{\psi}_j \psi_j. \tag{7}
\]

The scalar density \( \bar{\psi}_j \psi_j \) is given by

\[
\bar{\psi}_j \psi_j = \frac{4\pi w}{(2\pi)^3} \int_0^{k_{F_j}} k^2 dk \left| \frac{m_j^*}{\sqrt{k^2 + m_j^{*2}}} \right|
\]

\[
= \frac{4\pi w m_j^3 |y_j|}{(2\pi)^3} \left[ x_{F_j} |e_{F_j}| - y_j^2 \ln \frac{x_{F_j} + |e_{F_j}|}{|y_j|} \right] \tag{8}
\]

where \( x_{F_j} = k_{F_j}/m_j \) is the Fermi momentum of the distribution of \( \nu_j \) in units of its vacuum mass, \( e_{F_j} = \text{sgn}(y_i) \sqrt{x_{F_j}^2 + y_j^2} \) and \( w \) is the number of neutrino states contributing to the scalar density. For Majorana neutrinos \( w = 2 \) and \( w = 4 \) for Dirac neutrinos. As usual, the number density for each neutrino state, \( \rho_j \), is related to the Fermi momentum \( k_{F_j} \) by

\[
\rho_j = \frac{1}{6\pi^2} \frac{k_{F_j}^3}{m_j^3} \tag{9}
\]

\[
= \frac{m_j^3}{6\pi^2 x_{F_j}^3} \tag{10}
\]

It is convenient to use equation (8) to recast equation (7) as an equation for one of the \( y_j \) and to rescale the radial variable as \( z = m_s r \) [9]. We choose to follow the scaled effective mass of the heaviest neutrino, \( y_h \). Defining the ratios

\[
r_j = m_j/m_h \tag{11}
\]
and the parameter

$$K_0 = \frac{g_h^2 w m_h^2}{2\pi^2 m_s^2}$$

we obtain

$$\frac{d^2 y_h}{dz^2} + \frac{2}{z} \frac{dy_h}{dz} = -1 + y_h \left( 1 + \frac{K_0}{2} F \right)$$

(13)

where

$$F = \sum_j \left| \frac{y_j}{y_h} \right| \frac{g_j}{g_h} \left[ |e_{F_j}| x_{F_j} - y_j^2 \ln \left| \frac{|e_{F_j}| + x_{F_j}}{|y_j|} \right| \right]$$

(14)

In these equations, the $|e_{F_j}|$ serve as Lagrange multipliers to fix the total number, $N_j$, of each mass eigenstate and are constant throughout the distribution.  

The details of any distribution about some center depend on many things, including $N_j, m_j$ and $g_j$. For example, as discussed in [3], if the couplings were to be proportional to the vacuum masses, all of the $y_j$ would be the same at every point in the distribution and all of the effective masses would be positive. Since the vacuum masses depend on other interactions as well as the scalar field, this seems an unlikely scenario.

The richness of the system can be demonstrated with another simple model, one in which the various couplings are all equal to the same constant $g$. Beginning for simplicity with two mass eigenstates, let the vacuum mass of the heavier be denoted by $m_h$ and that of the lighter by $m_l$ with a ratio given by

$$r = \frac{m_l}{m_h}.$$  

(15)

In this case, the shift from the vacuum mass to the effective mass is the same for both neutrinos,

$$m_h^* = m_h - g\phi$$  

(16)

$$m_l^* = m_l - g\phi$$  

(17)

$$\Delta y = g\phi/m_h$$  

(18)

$$y_h = 1 - \Delta y$$  

(19)

$$y_l = 1 - \Delta y/r$$  

(20)

$$= 1 - (1 - y_h)/r$$  

(21)

If, at the center of the distribution, the shift is large enough, the effective mass of the lighter neutrino changes sign and can have a magnitude greater than its vacuum mass. In that case, it is energetically favorable for the light neutrino to move to a region with smaller scalar field, so the center of the distribution will have only the heavier neutrino present. As long as $y_l < -|e_{F_l}|$, the density of light neutrinos will be zero. The density of heavy neutrinos goes to zero when $y_h > e_{F_h}$, so, if $1 - (1 - e_{F_h})/r < -|e_{F_l}|$ there will be an annulus with no neutrinos in which $\phi$ decreases and $y_l$ increases. When $y_l = -|e_{F_l}|$, the density
of light neutrinos becomes non-zero, and remains so until \( y_l > +|\epsilon_{F_l}| \). Beyond this radius there are no neutrinos and \( \phi \) falls to zero as \( \exp(-z)/z \).

The importance of this for MSW-like effects is the following. Enhanced transitions between mass eigenstates occur when the propagators are equal for the same momentum, i.e. when

\[
m_h^{*2} - m_l^{*2} = 0
\]

In this example the equality is achieved when

\[
m_l^* = -m_h^*,
\]

which occurs when

\[
y_l = y_{hl} = (1 - r)/2
\]

This condition can be achieved in the region with no neutrinos. As an example of this, we have revisited the case presented as Figure 9 of [3], studying the value of \( [m_h^{*2} - m_l^{*2}] / m_h^2 \). The result, for \( r = 0.1 \), \( \epsilon_{F_h} = 0.43 \) and \( |\epsilon_{F_l}| = 0.9805 \), is shown in Figure 1, in which we display the latter difference as well as the densities of the heavy and the light neutrinos. The density scales are arbitrary and different for ease of viewing. As indicated by equation (24), in this case the vanishing of the mass-squared difference not only occurs without ordinary matter but at a location where the density of neutrinos is zero.

The extension to three mass eigenstates is quite straightforward. Denote the three mass eigenstates by \( m_h \), \( m_m \) and \( m_l \) in an obvious notation. Define

\[
r_j = m_j / m_h
\]

\[
y_l = 1 - (1 - y_h)/r_l
\]

\[
y_m = 1 - (1 - y_h)/r_m.
\]

Now it is possible, in principle, to have three distinct effective level crossings, where \( y_h \) takes the values

\[
y_{hl}^0 = (1 - r_l)/2
\]

\[
y_{hm}^0 = (1 - r_m)/2
\]

\[
y_{ml}^0 = 1 - (r_l + r_m)/2.
\]

Note that this particularly simple pattern arises because of the assumption that all three mass states couple to the scalar field with the same strength. In general, with

\[
\gamma_j = g_j / g_h,
\]

\[
y_j = 1 - \gamma_j / r_j (1 - y_h).
\]
The level crossings now occur for values of \( y_h \) given by

\[
y_{hj}^0 = \frac{(\gamma_j - r_j)}{(1 + \gamma_j)}, \quad \text{and}
\]
\[
y_{ml}^0 = 1 - \frac{(r_m + r_l)}{(\gamma_m + \gamma_l)}. \tag{34}
\]

Since \( y_h > 0 \), there will be no crossing of \( \nu_j \) with \( \nu_h \) if \( \gamma_j < r_j \) and no crossing of \( \nu_l \) with \( \nu_m \) if \( (\gamma_m + \gamma_l) < (r_m + r_l) \). In particular, for the case where the couplings are proportional to the vacuum masses there are no crossings for finite density.

Since this change in the values of the effective masses is due to the presence of a scalar field, it is the same for \( \bar{\nu} \) as for \( \nu \). This is different from the MSW effect which, being an energy shift arising from a vector exchange, has the opposite sign for \( \nu \) and \( \bar{\nu} \). In the presence of both a large density of neutrinos and a large electron density, these two effects will both come into play, allowing the crossings for \( \nu \) and \( \bar{\nu} \) to occur at different radii.

With this remark, one may understand the source of apparent conflict between the MSW effect on Solar neutrinos and the requirements of r-process nucleosynthesis in supernovae [5]. Assume, for this argument, three points. First, the usual prejudice about the spectrum holds. This means that the lightest mass eigenstate has the largest overlap with the electron neutrino. Second, that the density of neutrinos (strictly, the strength of the scalar field) in the Sun is small enough that the usual MSW effect is not severely perturbed. Third, that the density of neutrinos in a supernova is sufficiently large that \( y_l < -y_h \).

With these assumptions, the radius in the supernova at which the neutrinos are degenerate is smaller than the radius at which the anti-neutrinos are degenerate. This means that the light neutrinos are reheated at a smaller radius, hence traverse more matter and are more efficiently recooled, than the light anti-neutrinos. This can provide the required feature that the effective temperature of \( \bar{\nu} \) is higher than that of \( \nu \) at the site of the r-process.

We should emphasize that this effect in supernovae could occur, given a scalar field coupled to neutrinos, even if the coupling were too weak or the range too short to provide stable clouds [6]. The extremely large neutrino densities associated with supernovae would still be able to alter the effective masses and could provide for the degeneracy in \( m^* \) discussed above.

Note that, even if there is no scalar field, the vector interaction with matter could be large enough to produce degeneracy for the anti-neutrinos. In this case, however, the anti-neutrino degeneracy occurs at a smaller radius than the neutrino degeneracy and the desired shift in temperatures is not achieved.

Another phenomenon in which a neutrino density will produce a non-linear MSW-like effect through the standard model has been discussed in the literature [9]. In this case a neutrino interacts with the background neutrinos by \( Z^0 \) exchange. While this effect can be incorporated into the QHD equations,
we have not done so because at the neutrino densities in stars and supernovae, and for the range of parameters we have considered, it is much smaller than the scalar field effect we have discussed above.

In summary, we have shown that the effect of a scalar field coupled to neutrinos, expressed as an effective mass, could lead to a degeneracy in the propagators of different mass eigenstates. In objects with a radially varying neutrino density, be they supernovae, clouds of relic neutrinos or other astronomical structures, these degeneracies would occur at particular radii leading to enhanced transitions between weak interaction eigenstates in the manner of the MSW effect \[1, 2, 3, 4\]. Unlike MSW, this effect is the same for \(\nu\) and \(\bar{\nu}\), so the interplay between the two, depending on both the matter density and the neutrino density in a given region of space, can produce effects which would appear, with MSW alone, to require different mass spectra \[5\].

This work has been supported in part by the United States National Science Foundation, the United States Department of Energy, the Australian Research Council and the Australian DIST.
References

[1] L. Wolfenstein, Phys. Rev. D17, 2369 (1978); D20, 2364 (1979).

[2] S. P. Mikheyev and A. Yu. Smirnov, Nuovo Cimento Soc. Ital. Fis. 9C, 17 (1986).

[3] H. A. Bethe, Phys. Rev. Lett. 56, 1305 (1986).

[4] S. P. Rosen and J. M. Gelb, Phys. Rev. D34, 969 (1986).

[5] G. M. Fuller, J. R. Primack and Y.-Z. Qian, Phys. Rev. D52, 1288 (1995);
Y.-Z. Qian and G. M. Fuller, Phys. Rev. D52, 656 (1995).

[6] G. J. Stephenson Jr., T. Goldman and B. H. J. McKellar, /hep-ph/9603392; Los Alamos preprint LA-UR-96949; University of Melbourne preprint UM-P-96/29.

[7] Brian D. Serot and John Dirk Walecka, Adv. in Nucl. Phys. Vol. 16, 1 (J. W. Negele and Eric Vogt, eds. Plenum Press, NY 1986).

[8] See e. g., W. C. Haxton and G. J. Stephenson Jr., “Double Beta Decay” in Progress in Particle and Nuclear Physics, Vol. 12, 409 (Sir Denys Wilkinson, ed., Pergamon Press, New York 1984).

[9] A. Dolgov, Sov. J. Nucl. Phys. 33, 700 (1981); R. Barbieri and A. Dolgov, Phys. Lett. B237, 440 (1990); Nucl. Phys. B349, 743 (1991); K. Kainulainen, Phys. Lett. B244, 191 (1990); M. J. Thomson and B. H. J. McKellar, Phys. Lett. B259, 113 (1992); K. Enqvist, K. Kainulainen and M. Thomson, Nucl. Phys. B373, 498 (1992); J. Cline, Phys. Rev. Lett. 68, 3137 (1992); X. Shi, D. N. Schramm and B. D. Fields, Phys. Rev. D48, 2568 (1993); B. H. J. McKellar and M. J. Thomson, Phys. Rev. D49, 2710 (1994); V. A. Kostelecký and S. Samuel, Phys. Lett. B318, 127 (1993); Phys. Rev. D49, 1740 (1994); ibid. D52, 621 (1995); ibid. D52, 3184 (1995); R. Foot, M. J. Thomson and R. R. Volkas, University of Melbourne preprint UM-P-95/90, /hep-ph/9509327.
Figure Captions

Figure 1. Densities and $\Delta m^2 = [m^*_h - m^*_l] / m^2_h$ for two neutrino mass eigenstates with constant coupling to the scalar field. $K_0 = 400$ for the heavy neutrino and $m_l / m_h = 0.1$. The long-dashed line is proportional to the density for the heavy neutrino, the dot-dashed line is proportional to the density of the light neutrino and the solid line is proportional to the value of $\Delta m^2$. The scaled Fermi energies are $e_{F_h} = 0.43$ and $|e_F| = 0.9805$, and the numbers of the two neutrino mass eigenstates are approximately equal. Although it is not apparent from the figure the point at which $\Delta m^2 = 0$ lies outside the region in which the heavy neutrinos are found.
Figure 1

$\Delta m^2$ vs. Radius

Two Neutrino Example

Scaled Radial Distance from Sun

Densities and $\Delta m^2$ (arb. units)

- Heavy Neutrino
- Light Neutrino
- Effective $\Delta m^2$