Abstract This is a brief review of few relevant topics on tunneling of composite particles and how the coupling to intrinsic and external degrees of freedom affects tunneling probabilities. I discuss the phenomena of resonant tunneling, different barriers seen by subsystems, damping of resonant tunneling by level bunching and continuum effects due to particle dissociation.

1 Particles Moving in Mysterious Ways

1.1 History

The tunnel effect is one of the most subtle phenomenon explained by quantum mechanics, responsible e.g., for the existence of stars and ultimately for the existence of life [1]. The first application of particle incursion into classically forbidden regions was done in nuclear physics. Nuclei, such as $^{210}$Po, emit $\alpha$-particles by tunneling through the Coulomb barrier. This process lacked physical explanation until George Gamow used the tunneling theory to calculate $\alpha$-emission half-lives [2,3]. Tunneling is now a well known physical process that has been incorporated in our everyday lives due to the increasing miniaturization in electronics such as microchips. There have been important Nobel Prizes related to the straightforward use of this effect in industry, such as the tunnel diode [4] or the scanning tunneling microscope [5].

1.2 Resonant Tunneling

Resonant tunneling is a particular kind of tunneling effect, frequently applied to miniaturization such as the resonant diode tunneling device. In its simplest form, resonant tunneling occurs when a quantum level in one side of a barrier has an energy match with a level on the other side of it. If this occurs in a dynamical situation, tunneling is enhanced. In certain situations, the transmission probability is equal to one and the barrier is completely transparent for particle transmission. In the resonant diode tunneling device, two semiconductor layers sandwich another creating a double-humped barrier which enables the existence of quantum levels within. On both sides of the barrier, electrons fill a conducting band on two outside semiconductors. A potential difference between these outer layers allows one conducting band to rise in energy. Resonant tunneling leaking occurs to the levels inside the double-hump barrier [4]. The net effect is the appearance of a current which can be fine tuned. Resonant tunneling devices are compact and allow a fast response because the tunneling between the thin double-humped barrier is a very fast process.
1.3 Composite Particles

Resonant tunneling is not constrained to a particle tunneling through a double-humped barrier. An equivalent process occurs when a composite particle tunnels through a single barrier, if each subsystem composing the particle is allowed to tunnel independently or when the interaction reveals a priori unknown energy states. Then the process is equivalent to the tunneling of one single particle through a double-humped barrier, and resonant tunneling proceeds through the appearance of pseudo, quasi-bound, states (see Fig. 1, left). This effect occurs frequently in atomic, molecular and nuclear systems. A good example is the fusion of loosely-bound nuclei [6]. It also relates to Feshbach resonances when a system uncovers a state in a (closed) channel which is not the same (open) channel where it sits in. This happens because the coupling with at least one internal degree of freedom helps the reaction to proceed via resonant tunneling (see Fig. 1, right).

In the next Sect. 1 give several examples of how intrinsic degrees of freedom manifest in subtle ways, leading to tunneling enhancement or suppression. As the subject is very vast and general, I will concentrate on examples that I had the opportunity to work with during the last decades. They range from particle to molecular physics. Some common characteristics are easy to understand, others not so much. An example of the last case if given for diffusion and dissociation of molecules in optical lattices.

2 Tunneling of Composite Particles

Most theoretical problems involving tunneling are not amenable to analytical solutions. There are several computational methods available for them. Below I will just mention a very well-known method, used to generate some of the calculations displayed in the figures of this review.

2.1 Stalking Microscopic Particles

It is interesting to study the time-dependence of tunneling, although it is prohibitive for most cases of interest due to the long tunneling times, e.g., in nuclear $\alpha$-decay processes. The time evolution of a wave function on a space lattice can obtained by solving the Schrödinger equation by a finite difference method. The wave function $\Psi(t + \Delta t)$ can be calculated from the wave function at time $t$, $\Psi(t)$, by applying the unitary time evolution operator, $U$, i.e., $\Psi(t + \Delta t) = U(\Delta t, t)\Psi(t) = \exp[-iH\Delta t/\hbar]\Psi(t)$, where $H$ is the system Hamiltonian. For a small time step $\Delta t$, one can use the unitary operator approximation, valid to order $(\Delta t)^2$,

$$U(t + \Delta t) = \frac{1 + (\Delta t/2i\hbar)H(t)}{1 - (\Delta t/2i\hbar)H(t)}. \quad (1)$$

Other approximations for $\exp[-iH\Delta t/\hbar]$ such as the Numerov algorithm, etc., can also be used. Any of these numerical procedures requires carrying out matrix multiplications and inversions at each iteration and the

Fig. 1 Left: tunneling of a composite particle through a barrier is often equivalent to the tunneling of a single particle through a double-humped barrier. Right: a Feshbach resonance appears when a preferred tunneling is induced by the coupling of a reaction channel to one or more intrinsic degrees of freedom.