Branching ratios of $B_c \to AP$ decays in the perturbative QCD approach

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Abstract

In this paper we calculate the branching ratios (BRs) of the 32 charmless hadronic $B_c \to AP$ decays ($A = a_1(1260), b_1(1235), K_1(1270), K_1(1400), f_1(1285), f_1(1420), h_1(1170), h_1(1380)$) by employing the perturbative QCD(pQCD) factorization approach. These considered decay channels can only occur via annihilation type diagrams in the standard model. From the numerical calculations and phenomenological analysis, we found the following results: (a) the pQCD predictions for the BRs of the considered $B_c$ decays are in the range of $10^{-6}$ to $10^{-8}$, while the CP-violating asymmetries are absent because only one type tree operator is involved here; (b) the BRs of $\Delta S = 0$ processes are generally much larger than those of $\Delta S = 1$ ones due to the large CKM factor of $|V_{ud}/V_{us}|^2 \sim 19$; (c) since the behavior for $^1P_1$ meson is much different from that of $^3P_1$ meson, the BRs of $B_c \to A(^1P_1)P$ decays are generally larger than that of $B_c \to A(^3P_1)P$ decays; (d) the pQCD predictions for the BRs of $B_c \to (K_1(1270), K_1(1400))\eta^{(0)}$ and $(K_1(1270), K_1(1400))K$ decays are rather sensitive to the value of the mixing angle $\theta_K$.

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I. INTRODUCTION

Unlike the ordinary light $B_q (q = u, d, s)$ mesons, the $B_c$ meson is the only heavy meson consisting of two heavy quarks $b$ and $c$ and plays a special role in the precision test of the standard model (SM) \[1\]. Moreover, a large number of $B_c$ meson events will be collected with the running of Large Hadron Collider (LHC) experiments and this will provide great opportunities for both theorists and experimentalists to study the perturbative and nonperturbative QCD dynamics, final state interactions, etc.

In two recent works \[2, 3\], the pure annihilation $B_c \rightarrow PP, PV/VP, VV$ decays (here $P$ and $V$ stand for the light pseudoscalar and vector mesons) have been studied by employing the SU(3) flavor symmetry and the pQCD factorization approach \[1, 6\], respectively.

In the present work, we will study the two body charmless hadronic $B_c \rightarrow AP$ decays (here $A$ denotes the light axial-vector mesons), which can only occur via annihilation type diagrams in the SM. First of all, the size of annihilation contributions is an important issue in the $B$ meson physics, and has been studied extensively, for example, in Refs. \[4, 5, 7–10\]. Secondly, the internal structure of the axial-vector mesons has been one of the hot topics in recent years \[11–13\]. Although many efforts on both theoretical and experimental sides have been made \[14–20\] to explore it through the studies for the relevant decay rates, the CP-violating asymmetries, polarization fractions and the form factors, etc., we currently still know little about the nature of the axial-vector mesons.

In the quark model, there are two different types of light axial vector mesons: $3P_1$ and $1P_1$, which carry the quantum numbers $J^{PC} = 1^{++}$ and $1^{-+}$, respectively. The $1^{++}$ nonet consists of $a_1 (1260)$, $f_1 (1285)$, $f_1 (1420)$ and $K_{1A}$, while the $1^{-+}$ nonet has $b_1 (1235)$, $h_1 (1170)$, $h_1 (1380)$ and $K_{1B}$ \[1\]. In the SU(3) limit, these mesons can not mix with each other. Because the $s$ quark is heavier than $u, d$ quarks, the meson $K_1 (1270)$ and $K_1 (1400)$ are not purely $1^3P_1$ or $1^1P_1$ state, but a mixture of $K_{1A} (3P_1$ state) and $K_{1B} (1^1P_1$ state). Analogous to $\eta - \eta'$ system, the flavor-singlet and flavor-octet axial-vector mesons can also mix with each other. It is worth mentioning that the mixing angles can be determined by the relevant data, but unfortunately, there is no enough data now for these mesons which leaves the mixing angles basically free parameters.

In this paper, we will calculate the branching ratios of the 32 non-leptonic charmless $B_c \rightarrow AP$ decays by employing the low energy effective Hamiltonian \[21\] and the pQCD factorization approach based on the framework of $k_T$ factorization theorem. By keeping the transverse momentum $k_T$ of the quarks, the pQCD approach is free of endpoint singularity and the Sudakov formalism makes it more self-consistent. In the pQCD approach one can do the quantitative calculations of the annihilation type diagrams directly, which can be seen, for instance, in Refs. \[3–5, 7, 9\].

The pure annihilation $B_c \rightarrow PP, PV/VP, VV$ decays and $B_c \rightarrow AP$ decays considered in Refs. \[2, 3\] and in this paper generally have very small branching ratios: at the order of $10^{-6}$ to $10^{-9}$. According to the discussions as given in Ref. \[2\], the charmless hadronic $B_c$ decays with decay rates at the level of $10^{-6}$ could be measured at LHC experiments with the accuracy required for the phenomenological analysis, while it may be difficult to

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1 For the sake of simplicity, we will adopt the forms $a_1$ and $b_1$ to denote the non-strange axial-vector mesons $a_1 (1260)$ and $b_1 (1235)$, respectively, in the following section. We will also use $K_{1}$ to denote $K_{1} (1270)$ and $K_{1} (1400)$ for convenience unless otherwise stated.
measure those $B_c$ decays if their branching ratios are much less than $10^{-6}$.

The paper is organized as follows. In Sec. II, we present the formalism of the considered $B_c$ meson decays. Then we perform the analytic calculations for considered decay channels by using the pQCD approach in Sec. III. The numerical results and phenomenological analysis are given in Sec. IV. Finally, Sec. V contains a short summary and some discussions.

II. FORMALISM

In the pQCD approach, the decay amplitude of the two body decay $B_c \to M_1 M_2$ ($M_1, M_2$ stand for the two final state mesons) can be written conceptually as the convolution,

$$A(B_c \to M_1 M_2) \sim \int d^4k_1 d^4k_2 d^4k_3 \, \text{Tr} \left[ C(t) \Phi_{B_c}(k_1) \Phi_{M_1}(k_2) \Phi_{M_2}(k_3) H(k_1, k_2, k_3, t) \right],$$

where $k_i$'s are momenta of light quarks included in each mesons, and Tr denotes the trace over Dirac and color indices. $C(t)$ is the Wilson coefficient which results from the radiative corrections at short distance. In the above convolution, $C(t)$ includes the harder dynamics at larger scale than $m_{B_c}$ scale and describes the evolution of local 4-Fermi operators from $m_W$ (the W boson mass) down to $t \sim \mathcal{O}(\sqrt{\Lambda m_{B_c}})$ scale, where $\Lambda \equiv m_{B_c} - m_b$. The function $H(k_1, k_2, k_3, t)$ describes the four quark operator and the spectator quark connected by a hard gluon whose $q^2$ is in the order of $\sqrt{\Lambda m_{B_c}}$ hard dynamics. Therefore, this hard part $H$ can be perturbatively calculated. The function $\Phi_M$ is the wave function which describes hadronization of the quark and antiquark to the meson $M$. In the present work, since the $B_c$ meson is composed of two heavy quarks $b$ and $c$, we will take the nonrelativistic approximation form $\delta(x - m_c/m_{B_c})$ for the distribution amplitude $\phi_{B_c}(x)$. For light meson $A$ and $P$, we adopt the light-cone distribution amplitudes directly, which will be displayed in Appendix A. While the function $H$ depends on the processes considered, the wave function $\Phi_M$ is independent of the specific processes. Using the wave functions determined from other well measured processes, one can make quantitative predictions here.

Since the $b$ quark is rather heavy, we work in the frame with the $B_c$ meson at rest, i.e., with the $B_c$ meson momentum $P_1 = (m_{B_c}/\sqrt{2})(1, 1, 0, 0)$ in the light-cone coordinates. For the charmless hadronic $B_c \to AP$ decays, we assume that the $A (P)$ meson moves in the plus(minus) $z$ direction carrying the momentum $P_2$ ($P_3$), and with the polarization vector $\epsilon_2^L$ for the $A$ meson. Then the two final state meson momenta can be written as

$$P_2 = \frac{m_{B_c}}{\sqrt{2}}(1, r_A^2, 0, 0), \quad P_3 = \frac{m_{B_c}}{\sqrt{2}}(0, 1 - r_A^2, 0, 0),$$

respectively, where $r_A = m_A/m_{B_c}$ and the mass of light pseudoscalar mesons ($K, \pi$ and $\eta^{(')}$) has been neglected. For the axial-vector meson $A$, its longitudinal polarization vector, $\epsilon_2^L$, can be defined as

$$\epsilon_2^L = \frac{m_{B_c}}{\sqrt{2}m_A}(1, -r_A^2, 0, 0);$$

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Putting the (light-) quark momenta in $B_c$, $A$ and $P$ mesons as $k_1$, $k_2$, and $k_3$, respectively, we can choose

$$k_1 = (x_1 P_1^+, 0, k_{1T}), \quad k_2 = (x_2 P_2^+, 0, k_{2T}), \quad k_3 = (0, x_3 P_3^-, k_{3T}).$$

Then, for $B_c \rightarrow AP$ decays, the integration over $k_1^-$, $k_2^-$, and $k_3^+$ will lead to the decay amplitudes in the pQCD approach,

$$A(B_c \rightarrow AP) \sim \int dx_1 dx_2 dx_3 b_1 b_2 b_3 \cdot \text{Tr} \left[ C(t) \Phi_B(x_1, b_1) \Phi_A(x_2, b_2) \Phi_P(x_3, b_3) H(x_i, b_i, t) S_t(x_i) e^{-S(t)} \right]$$

where $b_i$ is the conjugate space coordinate of $k_{iT}$, and $t$ is the largest energy scale in function $H(x_i, b_i, t)$. The large logarithms $\ln(m_W/t)$ are included in the Wilson coefficients $C(t)$. The large double logarithms $\ln^2 x_i$ are summed by the threshold resummation $23$, and they lead to $S_t(x_i)$ which smears the end-point singularities on $x_i$. The last term, $e^{-S(t)}$, is the Sudakov form factor which suppresses the soft dynamics effectively $24$. Thus it makes the perturbative calculation of the hard part $H$ applicable at intermediate scale, i.e., $m_{B_c}$ scale. We will calculate analytically the function $H(x_i, b_i, t)$ for the considered decays at leading order (LO) in $\alpha_s$ expansion and give the convoluted amplitudes in next section.

For these considered decays, the related weak effective Hamiltonian $H_{eff} [21]$ is given by

$$H_{eff} = \frac{G_F}{\sqrt{2}} \left[ V_{cb}^* V_{ud} (C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)) \right],$$

with the current-current operators $O_{1,2}$,

$$O_1 = \bar{u}_\beta \gamma^\mu (1 - \gamma_5) D_\alpha c_\beta \gamma^\mu (1 - \gamma_5) b_\alpha,$$

$$O_2 = \bar{u}_\beta \gamma^\mu (1 - \gamma_5) D_\beta \bar{c}_\alpha \gamma^\mu (1 - \gamma_5) b_\alpha,$$

where $V_{cb}, V_{ud}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, ”D” denotes the light down quark $d$ or $s$, and $C_i(\mu)$ are Wilson coefficients at the renormalization scale $\mu$. For the Wilson coefficients $C_{1,2}(\mu)$, we will also use the leading order (LO) expressions, although the next-to-leading order calculations already exist in the literature $21$. This is the consistent way to cancel the explicit $\mu$ dependence in the theoretical formulae. For the renormalization group evolution of the Wilson coefficients from higher scale to lower scale, we use the formulae as given in Ref. $[5]$ directly.

### III. ANALYTIC CALCULATIONS IN THE PQCD APPROACH

In this section, we will calculate the decay amplitudes for 32 charmless hadronic $B_c \rightarrow AP/PA$ decays. Analogous to $B_c \rightarrow PV/VP$ decays in Ref. $[3]$, there are four kinds of annihilation Feynman diagrams contributing to these considered decays, as illustrated in Fig. $[1]$. By analytical evaluation of the two factorizable annihilation $(fa)$ diagrams
Fig. 1(a) and 1(b), we find the corresponding decay amplitude

\[ F_{fa}^{AP} = -8\pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \]
\[ \times \left\{ h_{fa}(1-x_3, x_2, b_2) E_{fa}(t_a) \left[ x_2 \phi_A(x_2) \phi_P^A(x_3) + 2r_A r_0^P \phi_P^P(x_3) \right] \right. \]
\[ \left. \times \left( (x_2 + 1) \phi_A^s(x_2) + (x_2 - 1) \phi_A^t(x_2) \right) + h_{fa}(x_2, 1-x_3, b_2, b_3) E_{fa}(t_b) \right\} , \]

where \( \phi_A, \phi_A^s, \phi_A^t \) and \( \phi_P^{A,P,T} \) denote the distribution amplitudes of the axial-vector and pseudoscalar mesons, \( r_0^P = m_0^P/m_{B_c} \) with \( m_0^P \) standing for the chiral scale of pseudoscalar meson(\( P \)), and \( C_F = 4/3 \) is a color factor. In Eq. (8), the terms proportional to \( (r_A r_0^P)^2 \) have been neglected because they are small: less than 7% numerically. The function \( h_{fa}, \) the scales \( t_i \) and \( E_{fa}(t) \) can be found in Appendix B of Ref. [3].

For the two nonfactorizable annihilation (\( na \)) diagrams Fig.1(c) and 1(d), all three meson wave functions are involved. The integration of \( b_3 \) can be performed using \( \delta \) function \( \delta(b_3 - b_2) \), leaving only integration of \( b_1 \) and \( b_2 \). The corresponding decay amplitude is

\[ M_{na}^{AP} = -\frac{16\sqrt{6}}{3} \pi C_F m_{B_c}^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \]
\[ \times \left\{ h_{na}(x_2, x_3, b_1, b_2) E_{na}(t_c) \left[ (r_c - x_3 + 1) \phi_A(x_2) \phi_P^A(x_3) + r_A r_0^P (\phi_A^s(x_2)) \right] \right. \]
\[ \times \left( (3r_c + x_2 - x_3 + 1) \phi_P^P(x_3) - (r_c - x_2 - x_3 + 1) \phi_P^T(x_3) \right) + \phi_A^t(x_2) \right) \]
\[ \times \left( (r_c - x_2 - x_3 + 1) \phi_P^P(x_3) + (r_c - x_2 + x_3 - 1) \phi_P^T(x_3) \right) \}
\[ \times \left. \left[ (r_b + r_c + x_2 - 1) \phi_A(x_2) \phi_P^A(x_3) + r_A r_0^P (\phi_A^s(x_2)) \right( (4r_b + r_c + x_2 - x_3 - 1) \phi_P^P(x_3) - (r_c + x_2 + x_3 - 1) \phi_P^T(x_3) \right) \right] \}
\[ \times \phi_A^t(x_3) - (r_c + x_2 + x_3 - 1) \phi_P^A(x_3) \}
\[ \times \right\} \left. \right| h_{na}(x_2, x_3, b_1, b_2) \right\} , \]

where \( r_b = m_B/m_{B_c}, r_c = m_c/m_{B_c} \) and \( r_b + r_c \approx 1 \) for \( B_c \) meson.

By exchanging the position of the final state mesons \( A \) and \( P \), we can obtain the phenomenological topology for \( B_c \to PA \) decays easily. The corresponding decay amplitudes for this type of decay channels can be obtained directly by the following replacements in Eqs. (8) and (9),

\[ \phi_A \leftrightarrow \phi_P^A, \quad \phi_A^s \leftrightarrow \phi_P^P, \quad \phi_A^t \leftrightarrow \phi_P^T, \quad r_A \leftrightarrow r_0^P. \]

Before we put the things together to write down the decay amplitudes for the studied decay modes, we give a brief discussion about the \( K_{1A} - K_{1B}, f_1 - f_8 \) and \( h_1 - h_8 \) mixing.
The physical states $K_1(1270)$ and $K_1(1400)$ are the mixtures of the $K_{1A}$ and $K_{1B}$. $K_{1A}$ and $K_{1B}$ are not mass eigenstates, and can be mixed together due to the strange and nonstrange light quark mass difference. The mixing of $K_{1A}$ and $K_{1B}$ can be written as

\[
|K_1(1270)\rangle = |K_{1A}\rangle \sin \theta_K + |K_{1B}\rangle \cos \theta_K, \tag{11}
\]

\[
|K_1(1400)\rangle = |K_{1A}\rangle \cos \theta_K - |K_{1B}\rangle \sin \theta_K. \tag{12}
\]

If the SU(3) flavor symmetry between $(u, d, s)$ quark was an exact symmetry, $K_{1A}$ and $K_{1B}$ would not be mixed with each other. As mentioned in the introduction, the mixing angle $\theta_K$ still not be well determined because of the poor experimental data. In this paper, for simplicity, we will adopt two reference values as that used in Ref. [13]: $\theta_K = \pm 45^\circ$.

Analogous to the $\eta$-$\eta'$ mixing in the pseudoscalar sector, $f_1(1285)$ and $f_1(1420)$ (the $1^3P_1$ states) will mix in the form of

\[
\begin{pmatrix}
  f_1(1285) \\
f_1(1420)
\end{pmatrix} = \begin{pmatrix}
  \cos \theta_3 & \sin \theta_3 \\
  -\sin \theta_3 & \cos \theta_3
\end{pmatrix} \begin{pmatrix}
f_1 \\
f_8
\end{pmatrix} \tag{13}
\]

Likewise, the $h_1(1170)$ and $h_1(1380)$ ($1^1P_1$ states) system can be mixed in terms of the pure singlet $|h_1\rangle$ and octet $|h_8\rangle$,

\[
\begin{pmatrix}
h_1(1170) \\
h_1(1380)
\end{pmatrix} = \begin{pmatrix}
  \cos \theta_1 & \sin \theta_1 \\
  -\sin \theta_1 & \cos \theta_1
\end{pmatrix} \begin{pmatrix}
h_1 \\
h_8
\end{pmatrix} \tag{14}
\]

where the component of $|f_1\rangle, |h_1\rangle$ and $|f_8\rangle, |h_8\rangle$ can be written as

\[
|f_1\rangle, |h_1\rangle = \frac{1}{\sqrt{3}}(|\bar{q}q\rangle + |\bar{s}s\rangle),
\]

\[
|f_8\rangle, |h_8\rangle = \frac{1}{\sqrt{6}}(|\bar{q}q\rangle - 2|\bar{s}s\rangle), \tag{15}
\]

where $q = (u, d)$. The values of the mixing angles for $1^3P_1$ and $1^1P_1$ states are chosen as [13]:

\[
\theta_3 = 38^\circ \text{ or } 50^\circ; \quad \theta_1 = 10^\circ \text{ or } 45^\circ. \tag{16}
\]

By putting all things together, we can write down the general expression of the total decay amplitude for the considered decays:

\[
\mathcal{A}(B_c \to AP) = V_{cb}^* V_{ud} \left\{ f_{B_c} F_{fa}^{AP/(PA)} a_1 + M_{fa}^{AP/(PA)} C_1 \right\}, \tag{17}
\]

where $a_1 = C_1/3 + C_2$. Now it is straightforward to present the explicit expressions of the decay amplitudes for all 32 considered $B_c \to AP$ decays.
(1) For $\Delta S = 0$ processes,

\[
\mathcal{A}(B_c \rightarrow \pi^+ a_1^0) = V_{cb}^* V_{ud} \left\{ f_{Bc} F_{fa}^{\pi\eta} a_1 + M_{na}^{\pi\eta} C_1 \right\} \]

\[
- \left[ f_{Bc} F_{fa}^{\pi\eta} a_1 + M_{na}^{\pi\eta} C_1 \right] \right\} / \sqrt{2} ,
\]

\[
\mathcal{A}(B_c \rightarrow a_1^+ \pi^0) = -\mathcal{A}(B_c \rightarrow \pi^+ a_1^0) = V_{cb}^* V_{ud} \left\{ f_{Bc} F_{fa}^{\pi\eta} a_1 + M_{na}^{\pi\eta} C_1 \right\}

\[
- \left[ f_{Bc} F_{fa}^{\pi\eta} a_1 + M_{na}^{\pi\eta} C_1 \right] \right\} / \sqrt{2} ,
\]

\[
\mathcal{A}(B_c \rightarrow a_1^+ \eta) = V_{cb}^* V_{ud} \cos \phi \left\{ f_{Bc} F_{fa}^{\eta\eta} a_1 + M_{na}^{\eta\eta} C_1 \right\}

\[
+ \left[ f_{Bc} F_{fa}^{\eta\eta} a_1 + M_{na}^{\eta\eta} C_1 \right] \right\} / \sqrt{2} ,
\]

\[
\mathcal{A}(B_c \rightarrow a_1^+ \eta') = V_{cb}^* V_{ud} \sin \phi \left\{ f_{Bc} F_{fa}^{\eta\eta} a_1 + M_{na}^{\eta\eta} C_1 \right\}

\[
+ \left[ f_{Bc} F_{fa}^{\eta\eta} a_1 + M_{na}^{\eta\eta} C_1 \right] \right\} / \sqrt{2} ,
\]

\[
\mathcal{A}(B_c \rightarrow \pi^+ b_1^0) = V_{cb}^* V_{ud} \left\{ f_{Bc} F_{fa}^{\pi\eta} a_1 + M_{na}^{\pi\eta} C_1 \right\}

\[
- \left[ f_{Bc} F_{fa}^{\pi\eta} a_1 + M_{na}^{\pi\eta} C_1 \right] \right\} / \sqrt{2} ,
\]

\[
\mathcal{A}(B_c \rightarrow b_1^+ \pi^0) = -\mathcal{A}(B_c \rightarrow \pi^+ b_1^0) = V_{cb}^* V_{ud} \left\{ f_{Bc} F_{fa}^{\pi\eta} a_1 + M_{na}^{\pi\eta} C_1 \right\}

\[
- \left[ f_{Bc} F_{fa}^{\pi\eta} a_1 + M_{na}^{\pi\eta} C_1 \right] \right\} / \sqrt{2} ,
\]

\[
\mathcal{A}(B_c \rightarrow b_1^+ \eta) = V_{cb}^* V_{ud} \cos \phi \left\{ f_{Bc} F_{fa}^{\eta\eta} a_1 + M_{na}^{\eta\eta} C_1 \right\}

\[
+ \left[ f_{Bc} F_{fa}^{\eta\eta} a_1 + M_{na}^{\eta\eta} C_1 \right] \right\} / \sqrt{2} ,
\]

\[
\mathcal{A}(B_c \rightarrow b_1^+ \eta') = V_{cb}^* V_{ud} \sin \phi \left\{ f_{Bc} F_{fa}^{\eta\eta} a_1 + M_{na}^{\eta\eta} C_1 \right\}

\[
+ \left[ f_{Bc} F_{fa}^{\eta\eta} a_1 + M_{na}^{\eta\eta} C_1 \right] \right\} / \sqrt{2} ,
\]

\[
\mathcal{A}(B_c \rightarrow \pi^+ f_1(1285)) = V_{cb}^* V_{ud} \left\{ \frac{\cos \theta_3}{\sqrt{3}} \left[ f_{Bc} (F_{fa}^{\pi\eta} + F_{fa}^{\eta\eta}) a_1 

\right.

\[
+ (M_{na}^{\pi\eta} + M_{na}^{\eta\eta}) C_1 \right\} + \frac{\sin \theta_3}{\sqrt{6}} \left[ f_{Bc} (F_{fa}^{\eta\eta} + (M_{na}^{\eta\eta} + M_{na}^{\pi\eta}) C_1 \right] \right\} ,
\]

\[
\mathcal{A}(B_c \rightarrow \pi^+ f_1(1420)) = V_{cb}^* V_{ud} \left\{ \frac{-\sin \theta_3}{\sqrt{3}} \left[ f_{Bc} (F_{fa}^{\pi\eta} + F_{fa}^{\eta\eta}) a_1 

\right.

\[
+ (M_{na}^{\pi\eta} + M_{na}^{\eta\eta}) C_1 \right\} + \frac{\cos \theta_3}{\sqrt{6}} \left[ f_{Bc} (F_{fa}^{\eta\eta} + (M_{na}^{\eta\eta} + M_{na}^{\pi\eta}) C_1 \right] \right\} ,
\]
\[ A(B_c \to \pi^+ h_1(1170)) = V_{cb}^* V_{ud} \left\{ \frac{\cos \theta_1}{\sqrt{3}} \left[ f_{B_c}(F_{fa}^{\pi h_1^a} + F_{fa}^{h_1^a \pi}) a_1 + (M_{na}^{\pi h_1^a} + M_{na}^{h_1^a \pi}) C_1 \right] \right. \]
\[ + \frac{\sin \theta_1}{\sqrt{6}} \left[ f_{B_c}(F_{fa}^{\pi h_1^a} + F_{fa}^{h_1^a \pi}) a_1 + (M_{na}^{\pi h_1^a} + M_{na}^{h_1^a \pi}) C_1 \right] \left\} \right. , \tag{28} \]
\[ A(B_c \to \pi^+ h_1(1380)) = V_{cb}^* V_{ud} \left\{ -\frac{\sin \theta_1}{\sqrt{3}} \left[ f_{B_c}(F_{fa}^{\pi h_1^a} + F_{fa}^{h_1^a \pi}) a_1 + (M_{na}^{\pi h_1^a} + M_{na}^{h_1^a \pi}) C_1 \right] \right. \]
\[ + \frac{\cos \theta_1}{\sqrt{6}} \left[ f_{B_c}(F_{fa}^{\pi h_1^a} + F_{fa}^{h_1^a \pi}) a_1 + (M_{na}^{\pi h_1^a} + M_{na}^{h_1^a \pi}) C_1 \right] \left\} \right. , \tag{29} \]
\[ A(B_c \to K^0 K_1(1270)^+) = V_{cb}^* V_{ud} \left\{ \sin \theta_K \left[ f_{B_c} F_{fa}^{K^0 K_1^A} a_1 + M_{na}^{K^0 K_1^A} C_1 \right] \right. \]
\[ + \cos \theta_K \left[ f_{B_c} F_{fa}^{K^0 K_1^B} a_1 + M_{na}^{K^0 K_1^B} C_1 \right] \left\} \right. , \tag{30} \]
\[ A(B_c \to K^0 K_1(1400)^+) = V_{cb}^* V_{ud} \left\{ \cos \theta_K \left[ f_{B_c} F_{fa}^{K^0 K_1^A} a_1 + M_{na}^{K^0 K_1^A} C_1 \right] \right. \]
\[ - \sin \theta_K \left[ f_{B_c} F_{fa}^{K^0 K_1^B} a_1 + M_{na}^{K^0 K_1^B} C_1 \right] \left\} \right. , \tag{31} \]
\[ A(B_c \to K_1(1270)^0 K^+) = V_{cb}^* V_{ud} \left\{ \sin \theta_K \left[ f_{B_c} F_{fa}^{K_1^A K^0} a_1 + M_{na}^{K_1^A K^0} C_1 \right] \right. \]
\[ + \cos \theta_K \left[ f_{B_c} F_{fa}^{K_1^B K^0} a_1 + M_{na}^{K_1^B K^0} C_1 \right] \left\} \right. , \tag{32} \]
\[ A(B_c \to K_1(1400)^0 K^+) = V_{cb}^* V_{ud} \left\{ \cos \theta_K \left[ f_{B_c} F_{fa}^{K_1^A K^0} a_1 + M_{na}^{K_1^A K^0} C_1 \right] \right. \]
\[ - \sin \theta_K \left[ f_{B_c} F_{fa}^{K_1^B K^0} a_1 + M_{na}^{K_1^B K^0} C_1 \right] \left\} \right. . \tag{33} \]

(2) For $\Delta S = 1$ processes,
\[ A(B_c \to K^0 a_1^+) = \sqrt{2} A(B_c \to K^+ a_1^0) = V_{cb}^* V_{us} \left\{ f_{B_c} F_{fa}^{K^0 a_1} a_1 + M_{na}^{K^0 a_1} C_1 \right\} , \tag{34} \]
\[ A(B_c \to K^0 b_1^+) = \sqrt{2} A(B_c \to K^+ b_1^0) = V_{cb}^* V_{us} \left\{ f_{B_c} F_{fa}^{K^0 b_1} a_1 + M_{na}^{K^0 b_1} C_1 \right\} , \tag{35} \]
\[ A(B_c \to K_1(1270)^0 a_1^+) = \sqrt{2} A(B_c \to K_1(1270)^+ a_1^0) = V_{cb}^* V_{us} \left\{ \sin \theta_K \left[ f_{B_c} F_{fa}^{K_1^A a_1} a_1 + M_{na}^{K_1^A a_1} C_1 \right] \right. \]
\[ + \cos \theta_K \left[ f_{B_c} F_{fa}^{K_1^B a_1} a_1 + M_{na}^{K_1^B a_1} C_1 \right] \left\} \right. , \tag{36} \]
\[ A(B_c \to K_1(1400)^0 a_1^+) = \sqrt{2} A(B_c \to K_1(1400)^+ a_1^0) = V_{cb}^* V_{us} \left\{ \cos \theta_K \left[ f_{B_c} F_{fa}^{K_1^A a_1} a_1 + M_{na}^{K_1^A a_1} C_1 \right] \right. \]
\[ - \sin \theta_K \left[ f_{B_c} F_{fa}^{K_1^B a_1} a_1 + M_{na}^{K_1^B a_1} C_1 \right] \left\} \right. , \tag{37} \]
\[
\mathcal{A}(B_c \to K^+ f_1(1285)) = V_{cb}^* V_{us} \left\{ \frac{\cos \theta_3}{\sqrt{3}} \left[ f_{B_c}(F_{fa}^{Kf_1^u} + F_{fa}^{f_1^uK})a_1 \\
+ (M_{na}^{Kf_1^u} + M_{na}^{f_1^uK})C_1 \right] + \frac{\sin \theta_3}{\sqrt{6}} \left[ f_{B_c}(F_{fa}^{Kf_1^u} \\
- 2F_{fa}^{f_1^uK})a_1 + (M_{na}^{Kf_1^u} - 2M_{na}^{f_1^uK})C_1 \right] \right\}, \tag{38}
\]

\[
\mathcal{A}(B_c \to K^+ f_1(1420)) = V_{cb}^* V_{us} \left\{ -\frac{\sin \theta_3}{\sqrt{3}} \left[ f_{B_c}(F_{fa}^{Kf_1^u} + F_{fa}^{f_1^uK})a_1 \\
+ (M_{na}^{Kf_1^u} + M_{na}^{f_1^uK})C_1 \right] + \frac{\cos \theta_3}{\sqrt{6}} \left[ f_{B_c}(F_{fa}^{Kf_1^u} \\
- 2F_{fa}^{f_1^uK})a_1 + (M_{na}^{Kf_1^u} - 2M_{na}^{f_1^uK})C_1 \right] \right\}, \tag{39}
\]

\[
\mathcal{A}(B_c \to K^+ h_1(1170)) = V_{cb}^* V_{us} \left\{ \frac{\cos \theta_3}{\sqrt{3}} \left[ f_{B_c}(F_{fa}^{Kh_1^u} + F_{fa}^{h_1^uK})a_1 \\
+ (M_{na}^{Kh_1^u} + M_{na}^{h_1^uK})C_1 \right] + \frac{\sin \theta_3}{\sqrt{6}} \left[ f_{B_c}(F_{fa}^{Kh_1^u} \\
- 2F_{fa}^{h_1^uK})a_1 + (M_{na}^{Kh_1^u} - 2M_{na}^{h_1^uK})C_1 \right] \right\}, \tag{40}
\]

\[
\mathcal{A}(B_c \to K^+ h_1(1380)) = V_{cb}^* V_{us} \left\{ -\frac{\sin \theta_3}{\sqrt{3}} \left[ f_{B_c}(F_{fa}^{Kh_1^u} + F_{fa}^{h_1^uK})a_1 \\
+ (M_{na}^{Kh_1^u} + M_{na}^{h_1^uK})C_1 \right] + \frac{\cos \theta_3}{\sqrt{6}} \left[ f_{B_c}(F_{fa}^{Kh_1^u} \\
- 2F_{fa}^{h_1^uK})a_1 + (M_{na}^{Kh_1^u} - 2M_{na}^{h_1^uK})C_1 \right] \right\}, \tag{41}
\]

\[
\mathcal{A}(B_c \to K_1(1270)^+\eta) = V_{cb}^* V_{us} \left\{ \sin \theta_K \left[ f_{B_c}(\cos \phi F_{fa}^{K_1A\eta} - \sin \phi F_{fa}^{\eta K_1A})a_1 \\
+ (\cos \phi M_{na}^{K_1A\eta} - \sin \phi M_{na}^{\eta K_1A})C_1 \right] + \cos \theta_K \left[ f_{B_c}(\cos \phi F_{fa}^{K_1A\eta} - \sin \phi F_{fa}^{\eta K_1A})a_1 \\
+ (\cos \phi M_{na}^{K_1A\eta} - \sin \phi M_{na}^{\eta K_1A})C_1 \right] \right\}, \tag{42}
\]

\[
\mathcal{A}(B_c \to K_1(1400)^+\eta) = V_{cb}^* V_{us} \left\{ \cos \theta_K \left[ f_{B_c}(\cos \phi F_{fa}^{K_1A\eta} - \sin \phi F_{fa}^{\eta K_1A})a_1 \\
+ (\cos \phi M_{na}^{K_1A\eta} - \sin \phi M_{na}^{\eta K_1A})C_1 \right] - \sin \theta_K \left[ f_{B_c}(\cos \phi F_{fa}^{K_1A\eta} - \sin \phi F_{fa}^{\eta K_1A})a_1 \\
+ (\cos \phi M_{na}^{K_1A\eta} - \sin \phi M_{na}^{\eta K_1A})C_1 \right] \right\}. \tag{43}
\]
\[ \mathcal{A}(B_c \rightarrow K_1(1270)^+\eta') = V_{cb}^* V_{us} \left\{ \sin \theta_K \left[ f_{B_c}(\sin \phi F_{fa}^{K_1A\eta}) + \cos \phi F_{fa}^{\eta K_1A}\right] a_1 \\
+ (\sin \phi M_{na}^{K_1A\eta} + \cos \phi M_{na}^{\eta K_1A}) C_1 \right\} + \cos \theta_K \left[ f_{B_c}(\sin \phi F_{fa}^{K_1\eta}) + \cos \phi F_{fa}^{\eta K_1B}\right] a_1 \\
+ (\sin \phi M_{na}^{K_1\eta} + \cos \phi M_{na}^{\eta K_1B}) C_1 \right\} \right) \\
\mathcal{A}(B_c \rightarrow K_1(1400)^+\eta') = V_{cb}^* V_{us} \left\{ \cos \theta_K \left[ f_{B_c}(\sin \phi F_{fa}^{K_1A\eta}) + \cos \phi F_{fa}^{\eta K_1A}\right] a_1 \\
+ (\sin \phi M_{na}^{K_1A\eta} + \cos \phi M_{na}^{\eta K_1A}) C_1 \right\} - \sin \theta_K \left[ f_{B_c}(\sin \phi F_{fa}^{K_1\eta}) + \cos \phi F_{fa}^{\eta K_1B}\right] a_1 \\
+ (\sin \phi M_{na}^{K_1\eta} + \cos \phi M_{na}^{\eta K_1B}) C_1 \right\} \right) \right) \ . \tag{44} \]

\[ \mathcal{A}(B_c \rightarrow K_1(1400)^+\eta') = V_{cb}^* V_{us} \left\{ \cos \theta_K \left[ f_{B_c}(\sin \phi F_{fa}^{K_1A\eta}) + \cos \phi F_{fa}^{\eta K_1A}\right] a_1 \\
+ (\sin \phi M_{na}^{K_1A\eta} + \cos \phi M_{na}^{\eta K_1A}) C_1 \right\} - \sin \theta_K \left[ f_{B_c}(\sin \phi F_{fa}^{K_1\eta}) + \cos \phi F_{fa}^{\eta K_1B}\right] a_1 \\
+ (\sin \phi M_{na}^{K_1\eta} + \cos \phi M_{na}^{\eta K_1B}) C_1 \right\} \right) \ . \tag{45} \]

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will calculate the branching ratios for those considered 32 charmless hadronic $B_c \rightarrow AP$ decay modes. The input parameters and the wave functions to be used are given in Appendix A. In numerical calculations, central values of input parameters will be used implicitly unless otherwise stated.

For $B_c \rightarrow AP$ decays, the decay rate can be written as

\[ \Gamma = \frac{G_F^2 m_{B_c}^3}{32\pi} (1 - r_A^2) |\mathcal{A}(B_c \rightarrow AP)|^2 \tag{46} \]

where the corresponding decay amplitudes $\mathcal{A}$ have been given explicitly in Eqs. (18-45). With the complete decay amplitudes as given in last section, by employing Eq. (46) and the input parameters and wave functions as given in Appendix A we calculate and present the pQCD predictions for the CP-averaged branching ratios of the considered decays with errors, as shown in Tables I-IV. The dominant errors come from the uncertainties of charm quark mass $m_c = 1.5 \pm 0.15$ GeV, the combined Gegenbauer moments $a_i$ of the relevant meson distribution amplitudes, and the chiral enhancement factors $m_0^\pi = 1.4 \pm 0.3$ GeV and $m^K_0 = 1.6 \pm 0.1$ GeV, respectively.

Based on the numerical results as given in Tables I-IV, we have the following remarks:

- The pQCD predictions for the CP-averaged branching ratios of considered $B_c$ decays vary in the range of $10^{-6}$ to $10^{-8}$. There is no CP violation for all these decays within the standard model, since there is only one kind of tree operator involved in the decay amplitude of all considered $B_c$ decays, which can be seen from Eq. (47).

- Among the considered $B_c \rightarrow AP$ decays, the pQCD predictions for the branching ratios of those $\Delta S = 0$ processes are generally much larger than those of $\Delta S = 1$ channels (one of the two final state mesons is a strange meson), the main reason is the enhancement of the large CKM factor $|V_{ud}/V_{us}|^2 \sim 19$ for those $\Delta S = 0$ decays as generally expected. For $B_c \rightarrow (a_1^+, b_1^+)(\pi^0, K^0)$ decays, however, the difference is not so large, because the enhancement due to the CKM factor is partially canceled by the differences between the magnitude of individual decay amplitude $|F_{fa}^{a_1^+\pi^0}|$ and $|F_{fa}^{b_1^+K^0}|$. 

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• For $B_c \rightarrow (a_1, b_1)\pi$ decays, the same component of $\bar{u}u - \bar{d}d$ is involved in both axial-vector $(a_1^0, b_1^0)$ and the pseudoscalar $\pi^0$ meson at the quark level. We therefore find the same branching ratios for $B_c \rightarrow \pi^+ a_1^0$ and $B_c \rightarrow a_1^+ \pi^0$, and for $B_c \rightarrow \pi^+ b_1^0$ and $B_c \rightarrow b_1^+ \pi^0$, respectively.

• From the numerical results as shown in Table II one can see that:
\[
\begin{align*}
Br(B_c \rightarrow b_1\pi) &\sim 14 \times Br(B_c \rightarrow a_1\pi), \\
Br(B_c \rightarrow b_1K) &\sim 16 \times Br(B_c \rightarrow a_1K).
\end{align*}
\]
This pattern agrees well with that as given in Ref. [14, 16].

• Unlike $B_c \rightarrow (a_1, b_1)(\pi, K)$ decays, we find that
\[
Br(B_c \rightarrow a_1(\eta, \eta')) \sim Br(B_c \rightarrow b_1(\eta, \eta')).
\]
The main reason is that the suppressed factorizable annihilation amplitudes cancel each other for $B_c \rightarrow a_1\eta^{(0)}$ decays, while the enhanced nonfactorizable ones cancel each other for $B_c \rightarrow b_1\eta^{(0)}$ decays.

• For $B_c \rightarrow \overline{K}^0(K_1(1270)^+, K_1(1400)^+)$ and $B_c \rightarrow (\overline{K}^0(K_1(1270)^0, K_1(1400)^0)K^+$ decays, their BRs strongly depend on the value of the mixing angle $\theta_K$ of the $K_{1A}$-$K_{1B}$ system. From Table II, one can see that
\[
\frac{Br(B_c \rightarrow \overline{K}^0 K_1(1400)^+)}{Br(B_c \rightarrow \overline{K}^0 K_1(1270)^+)} \approx \frac{Br(B_c \rightarrow K^+ \overline{K}_1(1400)^0)}{Br(B_c \rightarrow K^+ \overline{K}_1(1270)^0)} \approx 2,
\]
for $\theta_K = 45^\circ$, while
\[
\frac{Br(B_c \rightarrow \overline{K}^0 K_1(1400)^+)}{Br(B_c \rightarrow \overline{K}^0 K_1(1270)^+)} \approx \frac{Br(B_c \rightarrow K^+ \overline{K}_1(1400)^0)}{Br(B_c \rightarrow K^+ \overline{K}_1(1270)^0)} \approx \frac{1}{2},
\]
for $\theta_K = -45^\circ$. This means that one can determine the sign and size of $\theta_K$ after enough $B_c$ events become available at the LHC experiment.

**TABLE I:** The pQCD predictions of branching ratios (BRs) for $B_c \rightarrow (a_1, b_1)P$ decays. The source of the dominant errors is explained in the text.

| $\Delta S = 0$ | BRs | $\Delta S = 0$ | BRs |
|---------------|-----|---------------|-----|
| $\text{Decay modes}$ | BRs(10^{-7}) | $\text{Decay modes}$ | BRs(10^{-6}) |
| $B_c \rightarrow \pi^+ a_1^0$ | $3.0^{+0.1}_{-0.3}(m_c)^{-1.7}(a_1)^{+1.5}(m_0)$ | $B_c \rightarrow \pi^+ b_1^0$ | $4.3^{+1.9}_{-1.4}(m_c)^{-1.5}(a_1)^{-0.9}(m_0)$ |
| $B_c \rightarrow a_1^+ \pi^0$ | $2.9^{+0.1}_{-0.3}(m_c)^{-2.2}(a_1)^{-1.4}(m_0)$ | $B_c \rightarrow b_1^+ \pi^0$ | $4.3^{+2.0}_{-1.9}(m_c)^{-2.0}(a_1)^{-0.1}(m_0)$ |
| $B_c \rightarrow a_1^+ \eta$ | $6.8^{+2.4}_{-1.2}(m_c)^{-2.1}(a_1)^{-0.9}(m_0)$ | $B_c \rightarrow b_1^+ \eta$ | $0.6^{+0.3}_{-0.1}(m_c)^{-0.2}(a_1)^{-0.0}(m_0)$ |
| $B_c \rightarrow a_1^+ \eta'$ | $4.6^{+1.6}_{-0.8}(m_c)^{-1.1}(a_1)^{-0.0}(m_0)$ | $B_c \rightarrow b_1^+ \eta'$ | $0.4^{+0.2}_{-0.1}(m_c)^{-0.1}(a_1)^{-0.0}(m_0)$ |

| $\Delta S = 1$ | BRs | $\Delta S = 1$ | BRs |
|---------------|-----|---------------|-----|
| $\text{Decay modes}$ | BRs(10^{-8}) | $\text{Decay modes}$ | BRs(10^{-7}) |
| $B_c \rightarrow a_1^+ K^0$ | $3.4^{+1.2}_{-1.0}(m_c)^{-2.3}(a_1)^{-0.6}(m_0)$ | $B_c \rightarrow b_1^+ K^0$ | $5.4^{+0.9}_{-0.3}(m_c)^{-3.2}(a_1)^{-0.2}(m_0)$ |
| $B_c \rightarrow K^+ a_1^0$ | $1.7^{+0.6}_{-0.3}(m_c)^{-1.1}(a_1)^{-0.1}(m_0)$ | $B_c \rightarrow K^+ b_1^0$ | $2.7^{+0.5}_{-0.1}(m_c)^{-1.1}(a_1)^{-0.1}(m_0)$ |
TABLE II: Same as Table I but for $B_c \to (K_1(1270), K_1(1400)) (\pi, K, \eta, \eta')$ decays.

| $\Delta S = 0$ | Decay modes | $\theta_K = 45^\circ$ | BRs(10$^{-7}$) | $\theta_K = -45^\circ$ | BRs(10$^{-7}$) |
|---------------|-------------|---------------------|---------------|---------------------|---------------|
| $B_c \to \overline{K}^0 K_1(1270)^+$ | 8.2$^{+1.1}_{-0.7}(m_c) + 16.3(a_i) + 0.9(m_0)$ | 17.4$^{+3.2}_{-4.1}(m_c) + 25.2(a_i) + 0.0(m_0)$ |
| $B_c \to \overline{K}^0 K_1(1400)^+$ | 17.3$^{+3.1}_{-2.3}(m_c) + 24.6(a_i) + 0.0(m_0)$ | 8.1$^{+1.1}_{-0.9}(m_c) + 16.9(a_i) + 0.0(m_0)$ |
| $B_c \to K_1(1270)^0 K^+$ | 15.8$^{+3.5}_{-3.3}(m_c) + 19.8(a_i) + 1.0(m_0)$ | 32.0$^{+14.4}_{-7.3}(m_c) + 20.2(a_i) + 0.9(m_0)$ |
| $B_c \to K_1(1400)^0 K^+$ | 31.7$^{+14.3}_{-7.2}(m_c) + 20.0(a_i) + 0.0(m_0)$ | 15.7$^{+7.0}_{-3.4}(m_c) + 15.2(a_i) + 1.6(m_0)$ |

| $\Delta S = 1$ | Decay modes | $\theta_K = 45^\circ$ | BRs(10$^{-8}$) | $\theta_K = -45^\circ$ | BRs(10$^{-8}$) |
|---------------|-------------|---------------------|---------------|---------------------|---------------|
| $B_c \to K_1(1270)^0 \pi^+$ | 6.8$^{+5.1}_{-3.3}(m_c) + 6.5(a_i) + 0.8(m_0)$ | 5.9$^{+1.5}_{-0.7}(m_c) + 3.5(a_i) + 0.6(m_0)$ |
| $B_c \to K_1(1400)^0 \pi^+$ | 5.8$^{+1.5}_{-0.6}(m_c) + 3.8(a_i) + 0.0(m_0)$ | 6.8$^{+5.0}_{-3.3}(m_c) + 4.5(a_i) + 0.7(m_0)$ |
| $B_c \to K_1(1270)^+ \pi^0$ | 3.4$^{+2.6}_{-1.6}(m_c) + 3.3(a_i) + 0.3(m_0)$ | 3.0$^{+0.7}_{-0.4}(m_c) + 1.1(a_i) + 0.3(m_0)$ |
| $B_c \to K_1(1400)^+ \pi^0$ | 2.9$^{+0.7}_{-0.5}(m_c) + 0.9(a_i) + 0.0(m_0)$ | 3.4$^{+2.5}_{-1.7}(m_c) + 2.2(a_i) + 0.0(m_0)$ |

TABLE III: Same as Table I but for $B_c \to (f_1(1285), f_1(1420))(\pi, K)$ decays.

| $\Delta S = 0$ | Decay modes | $\theta_3 = 38^\circ$ | BRs(10$^{-8}$) | $\theta_3 = 50^\circ$ | BRs(10$^{-8}$) |
|---------------|-------------|---------------------|---------------|---------------------|---------------|
| $B_c \to \pi^+ f_1(1285)$ | 52.4$^{+9.3}_{-7.8}(m_c) + 23.2(a_i) + 21.7(m_0)$ | 44.8$^{+7.0}_{-6.7}(m_c) + 19.1(a_i) + 19.8(m_0)$ |
| $B_c \to \pi^+ f_1(1420)$ | 8.5$^{+0.5}_{-0.1}(m_c) + 6.0(a_i) + 0.7(m_0)$ | 16.0$^{+2.8}_{-2.0}(m_c) + 11.5(a_i) + 0.2(m_0)$ |

| $\Delta S = 1$ | Decay modes | $\theta_3 = 38^\circ$ | BRs(10$^{-8}$) | $\theta_3 = 50^\circ$ | BRs(10$^{-8}$) |
|---------------|-------------|---------------------|---------------|---------------------|---------------|
| $B_c \to K^+ f_1(1285)$ | 1.6$^{+0.7}_{-0.5}(m_c) + 4.4(a_i) + 0.5(m_0)$ | 1.5$^{+1.9}_{-0.5}(m_c) + 3.9(a_i) + 0.5(m_0)$ |
| $B_c \to K^+ f_1(1420)$ | 7.4$^{+0.3}_{-0.0}(m_c) + 3.2(a_i) + 0.4(m_0)$ | 7.5$^{+0.3}_{-0.0}(m_c) + 3.2(a_i) + 0.4(m_0)$ |

- For the $\Delta S = 1$ $B_c \to K_1\pi$ decays, their decay rates have a very weak dependence on the value of mixing angle $\theta_K$:

\[
Br(B_c \to K_1(1270)^0 \pi^+) \approx Br(B_c \to K_1(1400)^0 \pi^+) \approx 6 \times 10^{-8},
\]

\[
Br(B_c \to K_1(1270)^+ \pi^0) \approx Br(B_c \to K_1(1400)^+ \pi^0) \approx 3 \times 10^{-8},
\]

for both $\theta_K = 45^\circ$ and $-45^\circ$. This point will also be tested at LHC.
The theoretical predictions for the branching ratios of $B_c\to(h_1(1170), h_1(1380))((\pi, K)$ decays.

| $\Delta S$ | BRs($10^{-8}$) | BRs($10^{-8}$) |
|------------|----------------|----------------|
| $\theta_1 = 10^\circ$ | $\theta_1 = 45^\circ$ |
| $B_c \to \pi^+ h_1(1170)$ | $6.2^{+28.6}_{-28.0} (m_\pi)^{+32.1}_{-16.9} (a_1)^{+28.8}_{-8.0} (m_0)$ | $49.4^{+28.6}_{-10.9} (m_\pi)^{+33.7}_{-6.9} (a_1)^{+20.7}_{-5.5} (m_0)$ |
| $B_c \to \pi^+ h_1(1380)$ | $1.2^{+2.4}_{-1.0} (m_\pi)^{+3.7}_{-0.5} (a_1)^{+0.5}_{-0.0} (m_0)$ | $12.4^{+3.9}_{-3.7} (m_\pi)^{+5.5}_{-2.2} (a_1)^{+5.5}_{-2.2} (m_0)$ |

- For $B_c \to K_1\eta^{(*)}$ decays, the pQCD predictions have a strong $\theta_K$ dependence:
  \[
  \frac{Br(B_c \to K_1(1400)^+\eta)}{Br(B_c \to K_1(1270)^+\eta)} \approx 1.6, \\
  \frac{Br(B_c \to K_1(1400)^+\eta^*)}{Br(B_c \to K_1(1270)^+\eta^*)} \approx 4.3, \\
  \frac{Br(B_c \to K_1(1270)^+\eta)}{Br(B_c \to K_1(1400)^+\eta)} \approx 0.6, \\
  \frac{Br(B_c \to K_1(1270)^+\eta^*)}{Br(B_c \to K_1(1400)^+\eta^*)} \approx 0.6. 
  \]
  \[
  \frac{Br(B_c \to K_1(1400)^+\eta)}{Br(B_c \to K_1(1270)^+\eta)} \approx 1, \\
  \frac{Br(B_c \to K_1(1400)^+\eta^*)}{Br(B_c \to K_1(1270)^+\eta^*)} \approx 1, \\
  \frac{Br(B_c \to K_1(1270)^+\eta)}{Br(B_c \to K_1(1400)^+\eta)} \approx 4.3, \\
  \frac{Br(B_c \to K_1(1270)^+\eta^*)}{Br(B_c \to K_1(1400)^+\eta^*)} \approx 4.3. 
  \]
  for $\theta_K = 45^\circ$, while
  for $\theta_K = -45^\circ$. It is easy to see that these $B_c \to K_1\eta^{(*)}$ decays are sensitive to the mixing angle $\theta_K$. Analogous to the $B_c \to K^*(\eta, \eta')$ decays [3], the above four decays are dominated by the factorizable annihilation diagrams.

- The theoretical predictions for the branching ratios of $B_c \to K_1(1270)P$ and $B_c \to K_1(1400)P$ for $\theta_K = 45^\circ$, as listed in the column two of Table II, are roughly exchanged with respect to those of the third column for the choice of $\theta_K = -45^\circ$. Such simple relation comes from the fact that the two states $K_1(1270)$ and $K_1(1400)$ can go one into another as a mixture of $K_{1A}$ and $K_{1B}$ states when one sets the mixing angle $\theta_K = 45^\circ$ or $-45^\circ$ respectively, as can be seen from Eqs. (11,12),
  \[
  |K_1(1270))|_{\theta_K=45^\circ} = |K_1(1400))|_{\theta_K=-45^\circ}, \\
  |K_1(1400))|_{\theta_K=45^\circ} = -|K_1(1270))|_{\theta_K=-45^\circ}. 
  \]
  This relation further leads to the following relations between the decay amplitudes of $B_c \to K_1P$:
  \[
  a(B_c \to K_1(1270)P)|_{\theta_K=45^\circ} = a(B_c \to K_1(1400)P)|_{\theta_K=-45^\circ}, \\
  a(B_c \to K_1(1400)P)|_{\theta_K=45^\circ} = -a(B_c \to K_1(1270)P)|_{\theta_K=-45^\circ}. 
  \]
and finally we obtain the special pattern of branching ratios as listed in Table II. The small differences in corresponding decay rates are due to the difference in the masses of $K_1(1270)$ and $K_1(1400)$ mesons. The numerical relations as shown in Eqs. (49-50) and (53-54) are also induced by the same mechanism.

- For the four $B_c \rightarrow f_1(K, \pi)$ decays, one can see from Table III that

$$\frac{Br(B_c \rightarrow \pi^+ f_1(1285))}{Br(B_c \rightarrow \pi^+ f_1(1420))} \approx \begin{cases} 6.2 & \text{for } \theta_3 = 38^\circ \\ 2.8 & \text{for } \theta_3 = 50^\circ \end{cases}$$  (57)

and

$$\frac{Br(B_c \rightarrow K^+ f_1(1285))}{Br(B_c \rightarrow K^+ f_1(1420))} \approx 0.2$$  (58)

for $\theta_3 = 38^\circ$ and $50^\circ$. The relations in Eqs. (57,58) can be understood as follows: (a) since $f_1(1285)$ and $f_1(1420)$ are the mixed states of $f_1$ and $f_8$ (see Eq. (13) ) and both $\sin \theta_3$ and $\cos \theta_3$ are positive for $\theta_3 = 38^\circ$ and $50^\circ$, the contribution from the common component ($\bar{q}q$) of $f_1$ and $f_8$ will interfere constructively (destructively) for $B_c \rightarrow \pi^+ f_1(1285)$ ($B_c \rightarrow \pi^+ f_1(1420)$) decay, this results in the large difference for the decay rate of the two decays; (b) for the two $\Delta S = 1$ decays, however, the new component ($\bar{s}s$) will provide additional contributions to the considered decays. Furthermore, the contributions from ($\bar{s}s$) and $\bar{q}q$ interfere constructively for $B_c \rightarrow K^+ f_1(1420)$, but destructively for $B_c \rightarrow K^+ f_1(1285)$ decay.

- The pQCD predictions for $B_c \rightarrow (h_1(1170), h_1(1380))(K, \pi)$ decays, as given in Table IV, can be explained in a similar way as for $B_c \rightarrow (f_1(1285), f_1(1400))(K, \pi)$ decays.

- Since the LHC experiment can measure the $B_c$ decays with a branching ratio at $10^{-6}$ level [2], our pQCD predictions for the branching ratios of $B_c \rightarrow K(K_1(1270), K_1(1400))$ and $b_1 \pi$ decays could be tested in the forthcoming LHC experiments.

It is worth stressing that the theoretical predictions in the pQCD approach still have large theoretical errors induced by the still large uncertainties of many input parameters, e.g. Gegenbauer moments $a_i$. For most considered pure annihilation $B_c$ decays, it is hard to observe them even in LHC due to their tiny decay rate. Their observation at LHC, however, would mean a large non-perturbative contribution or a signal for new physics beyond the SM.

We here calculated the branching ratios of the pure annihilation $B_c \rightarrow AP$ decays by employing the pQCD approach. We do not consider the possible long-distance (LD) contributions, such as the re-scattering effects, although they may be large and affect the theoretical predictions. It is beyond the scope of this work.

V. SUMMARY

In short, we studied the charmless hadronic $B_c \rightarrow AP$ decays by employing the pQCD factorization approach based on the $k_T$ factorization theorem. These considered decay
channels can occur only via the annihilation diagram in the SM and they will provide an important platform for testing the magnitude of the annihilation contribution and understanding the content of the axial-vector mesons.

The pQCD predictions for CP-averaged branching ratios are displayed in Tables (I-IV). From our numerical evaluations and phenomenological analysis, we found the following results:

- The pQCD predictions for the branching ratios vary in the range of $10^{-6}$ to $10^{-8}$. The $B_c \rightarrow K^0(K_1(1270)^+, K_1(1400)^+)$ and other decays with a decay rate at $10^{-6}$ or larger could be measured at the LHC experiment.

- For $B_c \rightarrow AP$ decays, the branching ratios of $\Delta S = 0$ processes are generally much larger than those of $\Delta S = 1$ ones. Such differences are mainly induced by the CKM factors involved: $V_{ud} \sim 1$ for the former decays while $V_{us} \sim 0.22$ for the latter ones.

- Since the behavior for $^1P_1$ meson is much different from that for $^3P_1$ meson, the branching ratios of pure annihilation $B_c \rightarrow A(^1P_1)P$ are basically larger than that of $B_c \rightarrow A(^3P_1)P$, which can be tested in the LHC and Super-B experiments.

- The pQCD predictions about the branching ratios of $B_c \rightarrow K_1\eta(\prime)$ and $K_1K$ decays are rather sensitive to the value of the mixing angle $\theta_K$. One can determine $\theta_K$ through the measurement of these decays if enough $B_c$ events become available at the LHC experiment.

- The pQCD predictions still have large theoretical uncertainties, mainly induced by the uncertainties of the Gegenbauer moments $a_i$ in the meson distribution amplitudes.

- Because only tree operators are involved, the CP-violating asymmetries for these considered $B_c$ decays are absent naturally.

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Appendix A: Input parameters and distribution amplitudes

The masses (GeV), decay constants (GeV), QCD scale (GeV) and $B_c$ meson lifetime (ps) are
\[
\begin{align*}
\Lambda_{\text{MS}}^{(f=4)} &= 0.250, & m_W &= 80.41, & m_{B_c} &= 6.286, & f_{B_c} &= 0.489, \\
m_{a_1} &= 1.23, & f_{a_1} &= 0.238, & m_{b_1} &= 1.21, & f_{b_1} &= 0.180, \\
m_{K_{1A}} &= 1.32, & f_{K_{1A}} &= 0.250, & m_{K_{1B}} &= 1.34, & f_{K_{1B}} &= 0.190, \\
f_{f_1} &= 0.245, & m_{f_1} &= 1.28, & f_{f_2} &= 0.239, & m_{f_2} &= 1.28, \\
f_{b_1} &= 0.180, & m_{h_1} &= 1.23, & f_{h_2} &= 0.190, & m_{h_2} &= 1.37, \\
m_{0}^\sigma &= 1.4, & m_{0}^K &= 1.6, & m_{0}^n &= 1.08, & m_{0}^n &= 1.92, \\
m_{b} &= 4.8, & f_{\pi} &= 0.131, & f_K &= 0.16, & \tau_{B^+_c} &= 0.46. \quad (A1)
\end{align*}
\]

For the CKM matrix elements, here we adopt the Wolfenstein parametrization for the CKM matrix, and take $A = 0.814$ and $\lambda = 0.2257$, $\bar{\rho} = 0.135$ and $\bar{\eta} = 0.349$ [19].

For the distribution amplitudes of pseudoscalar mesons, we adopt the same forms as that used in the literature (See Ref. [3] and references therein).

The twist-2 distribution amplitudes for the longitudinally polarized axial-vector $3P_1$ and $1P_1$ mesons can be parameterized as [13,18]:
\[
\phi_A(x) = \frac{f}{2\sqrt{2N_c}} \left\{ 6x(1-x) \left[ a_0^\parallel t + a_1^\parallel t + a_2^\parallel t \left( 5t^2 - 1 \right) \right] \right\}, \quad (A2)
\]

As for twist-3 LCDAs, we use the following form:
\[
\begin{align*}
\phi_A^\parallel(x) &= \frac{3f}{2\sqrt{2N_c}} \left\{ a_0^\parallel t^2 + \frac{1}{2} a_1^\parallel t^2 \right\}, \\
\phi_A^\perp(x) &= \frac{3f}{2\sqrt{2N_c}} \frac{d}{dx} \left\{ x(1-x) \left( a_0^\perp t + a_1^\perp t \right) \right\}. \quad (A4)
\end{align*}
\]

where $f$ is the decay constant and $t = 2x - 1$. It should be noted that for the distribution amplitudes of strange axial-vector mesons $K_{1A}$ and $K_{1B}$, $x$ stands for the momentum fraction carrying by s quark.

Here, the definition of these distribution amplitudes $\phi_A(x)$ satisfy the following relation:
\[
\int_0^1 \phi_{3P_1}(x) = \frac{f}{2\sqrt{2N_c}} \int_0^1 \phi_{1P_1}(x) = a_0^{||,3P_1} \frac{f}{2\sqrt{2N_c}}. \quad (A5)
\]

where we have used $a_0^{||,3P_1} = 1$.

The Gegenbauer moments have been studied extensively in the literatures (see Ref. [13] and references therein), here we adopt the following values:
\[
\begin{align*}
da_2^{a_1} &= -0.02 \pm 0.02, & a_1^{a_1} &= -1.04 \pm 0.34, & a_1^{b_1} &= -1.95 \pm 0.35; \\
da_2^{f_1} &= -0.04 \pm 0.03, & a_1^{f_1} &= -1.06 \pm 0.36, & a_1^{h_1} &= -2.00 \pm 0.35; \\
da_2^{f_2} &= -0.07 \pm 0.04, & a_1^{f_2} &= -1.11 \pm 0.31, & a_1^{h_2} &= -1.95 \pm 0.35; \\
a_1^{K_{1A}} &= 0.00 \pm 0.26, & a_2^{K_{1A}} &= -0.05 \pm 0.03, & a_0^{K_{1A}} &= 0.08 \pm 0.09; \\
a_1^{K_{1B}} &= -1.08 \pm 0.48, & a_0^{K_{1B}} &= 0.14 \pm 0.15, & a_1^{K_{1B}} &= -1.95 \pm 0.45; \\
a_2^{K_{1B}} &= 0.02 \pm 0.10, & a_1^{K_{1B}} &= 0.17 \pm 0.22. \quad (A6)
\end{align*}
\]
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