A full-field optimization approach for iterative definition of yielding for non-quadratic and free shape yield models in plane strain

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Abstract. An advanced strategy for iterative definition of initial yielding based on planar strain distribution is presented. It is shown that full-field DIC measurement of NTR5 samples provides information on initial yielding for plane strain. The interdependency of strain increment and yield locus under assumption of associated flow allows for definition of yield parameters using a non-linear optimization scheme with LS-OPT. Pivotal for research in direction of additional support points for definition of initial yielding was the discovery that definition of yielding based only on tensile and biaxial experiments is not sufficient for aluminum alloy. Special focus was placed on the area of generalized plane strain, which is the most critical stress state. Previous publications illustrated experimental options using cruciform tension and crystal plasticity as support points in generalized plane strain. This publication introduces an additional strategy to determine data for multiaxial stress states without need of additional experiments. The iterative strategy shows promising results for definition of yielding in generalized plane strain. Additionally, it is illustrated that common yield models such as non-quadratic YLD2000-2D and free-shape Vegter are sufficiently capable to describe yielding of aluminum alloy, if their full potential is exploited. The strategy is evaluated on the basis of Nakajima strain distributions and a conclusion is drawn on applicability and predictive capabilities.

1. Introduction

This publication is an extension of the work presented by Hippke et al. (2020) \cite{1} by adding a full field optimization (FFO) based strategy for yielding definition. The underlying idea is to extend the scope of fitting strategies for yield models with the objective to increase model performance. The two experimental strategies previously published significantly improve model performance, but require large amounts of additional testing. FFO can be applied to standard geometries, reducing experimental cost to a minimum and providing an efficient alternative. Performance is evaluated using strain distributions of Nakajima specimens.

Iterative definition of parameters with FFO has become a regularly applied method in constitutive modeling. Güner et al. (2011) \cite{2} pursued an iterative approach but was limited by computational power. Ilg et al. (2018,2019) \cite{3,4} focused on correct representation of the post necking range of tensile experiments using LS-Opt. A more complex application is discussed by Coppeliers et al. (2018) \cite{5}, who applied FFO to a planar cruciform specimen. A great challenge of FFO is the definition of optimization problem. Especially both space and time discretization play a central role in definition of a well posed problem. Strong inhomogeneity of strain fields
may lead to a solution in local minimum, so more flexible optimization algorithms are needed. In order not be dependent on evaluation of derivatives, simulated annealing was chosen as optimization algorithm. The high number of function evaluations is of minor importance, as implicit FEM is used to reduce calculation time of an NTR5 sample to seconds. Belytschko-Tsay elements are used in simulation. The advantages and disadvantages of zero order and first order algorithms are described by Hao et al. (2019) [6].

2. Definition of reference configuration

In order to be able to evaluate performance of optimization based yield definition, a reference configuration is defined. This is achieved through tensile test and bulge experiment for YLD2000-2D, with an exponent of \( m = 8 \), as recommended by Barat et al. [7]. The fitted parameter set is provided in Table 2. To define a reference configuration of the Vegter yield criterion [8], additional plane strain and shear points need to be defined. This is achieved by NTR5 tensile tests and SH shear tests, using a geometry suggested by Roth and Mohr [9]. A challenge is, that only one stress is directly measured. The respective second stresses are derived using assumptions presented as part of the Vegter light model [10]: the stress ratio in plane strain is \( \alpha_{ps} = f_{ps2}/f_{ps1} = 0.5 \) and the stress ratio in shear is \( \alpha_{sh} = -1.0 \). The measured stress in first principle direction is calculated as \( \sigma_1 = F/A \), with the actual cross surface known through measurement of both in-plane plastic strains and volume consistency. The stress is evaluated at 1% of plastic strain and the initial yield stress in plane strain is calculated from tensile data using plastic work equivalence under the assumption of isotropic yielding. The shear stress is defined similarly as \( \tau = F/A \), with the difference that the force is acting parallel to the plane of deformation. The rotation of the specimen is taken into account by measuring the angle between axis of major deformation and the loading axis. The resulting input parameters are identical with the parameters of the Vegter reference configuration in Table 3. Elastic constants are \( E = 68\text{GPa} \) and \( \nu = 0.33 \). The hardening data in rolling direction is given by a Hockett-Sherby hardening law, as detailed in Table 1.

| AA6014-T4 | \( A[Mpa] \) | \( B[Mpa] \) | \( m \) | \( n \) | \( \sigma_y = A - (A - B) \cdot e^{-m \epsilon_{pl}^2} \) |
|-----------|------------|------------|-------|-------|--------------------------------------------------|
| t = 1.014mm | 357.53 | 134.39 | 6.0 | 0.8338 | |

**Table 1.** Elastic constants and hardening parameters for AA6014-T4.

3. NTR5 samples in full-field optimization for plane strain

As optimization for the state of plane strain is desired at a minimal increase of required experimental testing, the NTR5 geometry is selected due to strain paths very close to perfect plane strain. Digital image correlation (DIC) is extracted to acquire 3D strain fields of the geometry. The software ARAMIS by GOM is used, as a direct import of DIC data is possible from ARAMIS into LS-Opt using an xml-database. In order to map DIC data to FE mesh, a set of alignment points is needed, as illustrated in Figure 1 (a) as red triangles. The error measure used in optimization is given in Equation (1). The resulting objective function is to minimize the error between measured \( \bar{\epsilon}_i \) and simulated major and minor strain \( \epsilon_i \). Additionally, the forces are included and as the error values may change order of magnitude due to their respective physical definition, weighing factors \( w_i \) are introduced. The index \( j \) represents time progression and the index \( i \) represents element position. The error values are normalized with the maximum number of measured stages or times \( n \) and the number of spacial evaluation points \( m \). In order to correlate respective strain data between simulation and measurement, a dynamic time warping (DTW) algorithm [11] is used. Great care is taken to define equivalent start and
4. Optimization based definition of yielding

The mathematical formulation of an optimization problem is given in general form in Equation 2. Most commonly, an objective function \( f(x) \) is formulated and then minimized, subject to additional inequality constraints \( g_i(x) \) and equality constraints \( h_j(x) \).

\[
\begin{align*}
\min(f(x)) \\
\text{s.t.} \\
g_i(x) \leq 0; \ i = 1, 2, ..., m \\
h_j(x) = 0; \ j = 1, 2, ..., l
\end{align*}
\]  

(2)

The general optimization problem given in Equation (2) is adjusted to fit the presented problem for YLD2000 as equivalent to the objective function given in Equation (1) with a constraint only for the lower and upper boundary of the exponent, \( 4 \leq m \leq 10 \). The selected starting value is \( m_0 = 6 \).

For the Vegter yield criterion, the variable parameters change to the plane strain stresses in the respective direction. For the example of rolling direction, these are \( f_{ps1,0} \) and \( f_{ps2,0} \). Additional inequality constraints are defined in dependency of tensile and biaxial measurements. This set of constraints is necessary to guarantee stable definition of Bézier elements, as any plane strain point outside these boundaries return a non-convex yield surface. Constraints are given in Equation 3. \( k \in [0, 45, 90] \) indicates the angle to rolling direction, \( \beta_k \) is the transformed R-value in the respective tensile direction with \( \beta_k = -R_k/(1 + R_k) \). The constraints are defined in LS-Opt.
A sequential response surface method (SRSM) is chosen with a quadratic polynomial metamodel and D-optimal sampling for optimization of YLD2000 m-value. For Vegter, the Kriging metamodel with space filling point selection performed more consistently, mostly due to the strong nonlinear interdependency of design variables and response. Adaptive simulated annealing (ASA) is used as optimization algorithm. Major and minor strain responses are compared simultaneously in agreement with Equation (1). The setup is shown in Figure 1 (b) as illustrated in LS-OPT.

4.1. Result for exponent m of YLD2000

The optimization strategy based on NTR5 specimen with the objective to minimize strain error by changing the exponent m resulted in a value of m = 5.89. The resulting yield locus is illustrated in Figure 5. The result is reasonable for an optimization in plane strain and illustrates that the methodology is successful. The optimization took 17 iterations to converge, successfully minimizing the resulting strain error. The evolution of the optimization is shown in Figure 2. As an additional control mechanism, the measured and simulated forces are compared for both YLD2000 and Vegter in Figure 4 (a). The difference between optimum and measured strain distributions is shown in Figure 3, illustrating a near perfect fit for major strain and only small deviations for minor strain.

4.2. Result for plane strain point of Vegter

It is necessary to evaluate the plane strain points of the Vegter yield criterion independently for each relevant angle to rolling direction. This is due to the piecewise definition of the Vegter
Figure 3. Resulting error distances of DTW algorithm: $\Delta \epsilon_{maj}$ (a) and $\Delta \epsilon_{min}$ (b) for YLD2000-2D optimization using NTR5 at the final state.

Table 2. Parameters for YLD2000-2D yield locus fit of AA6014-T4 based on optimization strategies with NTR5 and the reference configuration.

| fit       | m  | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ | $\alpha_5$ | $\alpha_6$ | $\alpha_7$ | $\alpha_8$ |
|-----------|----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| NTR5      | 5.89 | 0.9330     | 1.0670      | 0.8736      | 1.0500      | 1.0240      | 1.0350      | 0.9173      | 2.8800      |
| Reference | 6   | 0.9218     | 1.1005      | 0.9024      | 1.0457      | 1.0189      | 1.0261      | 0.9635      | 1.2346      |

Table 3. Parameters for Vegter yield locus fit of AA6014-T4 based on Reference case and NTR5 full field optimization. Optimization changed only plane strain parameters.

yield criterion with independent reference points for the Bézier elements in rolling direction (RD), diagonal direction (DD) and transverse direction (TD). In consequence, the optimization procedure outlined above is repeated for every direction. The optimization problem remains equivalent, with the objective function given in Equation (1) and constraints given in Equation (3). The resulting parameters are given in Table 3. To illustrate the optimization procedure, the constraints and error distribution are illustrated in Figure 4 (b) for rolling direction. The figure simultaneously displays the calculated optimum. The constraints are taken into account during sampling, successfully avoiding calculation of unreasonable combinations.
**Figure 4.** (a) Resulting force against displacement for optimization result of YLD2000 and Vegter. Force is not part of the optimization and checked afterwards for first validation. Accuracy is sufficient for both optimization results. (b) Optimization of plane strain point with Vegter. Illustration of optimization constraints and accumulated error as well as resulting optimum in rolling direction.

**Figure 5.** Resulting yield locus (a) and derivative in rolling direction (b) for YLD2000 and Vegter based on NTR5 and both reference configurations.
Figure 6. Calculated error of Nakajima sections for reference configurations and optimization based yielding divided into major strain (a) and minor strain (b).

5. Validation based on strain distributions of Nakajima specimen

Based on a database previously published in Hippke et al. (2020) [1], a comparison of strain distributions of Nakajima samples along central sections is made. Please refer to the cited publication for all details of the experiment. Both major and minor strain is compared in Figure 7 for B50, B80 and B100 and in Figure 8 for B120 and B200. The largest differences are apparent for B200 and the minor strain of B120. An average error is calculated to evaluate model performance. The error value is given in Equation (5), where \( n \) represent the number of measured points along a section and \( N \) represents the respective number of the evaluation height. In order not to bias the error calculation toward any geometry, \( N = 3 \) holds for all geometries.

\[
err = \frac{1}{N \cdot n} \sum_{N} \sum_{n} (\epsilon_{\text{sim}} - \epsilon_{\text{exp}})^2
\]  

(5)

The error is calculated separately for major strain and minor strain and is displayed in form of histograms in Figure 6. Additionally, the error values are given in Table 4, including the accumulated error under exclusion of geometry B120. This is chosen to illustrate the dominance of one geometry on the evaluated average.

6. Conclusion

Full field optimization in plane strain has been proven to provide a suitable addition to standard testing when calibrating yield surfaces. The validation on Nakajima specimen illustrates the advantage for a number of loading cases with an increase in predictive capability. The mean error of major and minor strain both show a strong advantage of the optimization based yield formulations. For minor strain, the advantage is slightly reduced by an increased error for B120 specimen. The advantage is independent of yield formulation as it holds for both YLD2000 and Vegter. In consequence, it is encouraged to include additional fitting strategies in standard procedures for definition of yielding. Two other approaches based on cruciform tension and
Figure 7. Resulting measurement and computation of major and minor strain distribution for the Nakajima samples B50, B80 and B100.
Table 4. Accumulated error per geometry and fitting strategy including total error per fitting strategy.

| error [$\times 10^{-4}$] | B50 | B80 | B100 | B120 | B200 | sum | sum* |
|---------------------------|-----|-----|------|------|------|-----|------|
| YLD2000 classic           | 0.31| 0.14| 0.35 | 1.12 | 4.49 | 6.41| 5.29 |
| YLD2000 Opt               | 0.22| 0.14| 0.20 | 0.45 | 0.85 | 1.86| 1.41 |
| Vegter classic            | 0.28| 0.16| 0.35 | 1.12 | 4.49 | 5.50| 4.38 |
| Vegter Opt                | 0.22| 0.20| 0.58 | 0.94 | 0.56 | 2.50| 1.56 |

(a) \hspace{1cm} (b) \hspace{1cm} (c) \hspace{1cm} (d)

Figure 8. Resulting measurement and computation of major and minor strain distribution for the Nakajima samples B120 and B200.
crystal plasticity are discussed in Hippke et al. [1] with very similar results. This emphasises that model performance is more dependent on fitting strategy than on the complexity of the model itself. Additionally, the full field optimization strategy outlined here provides a methodology to increase precision at only very limited experimental cost. A short summary of the conclusion is given below:

- full field optimization based on NTR5 specimen provides sufficient information to significantly increase yield model performance,
- optimization strategy is suitable independent of type of yield model,
- the experimental cost is minimized when using full field optimization on standard geometries,
- precision of model prediction is increased significantly.

Future investigations address the strong influence of B120 geometry and include the shear geometry in full field optimization. Additionally, the known dependency of yield and fracture model provides an exciting field for future research.

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