Analysis of the Casimir force measurements in systems with Si gratings

Valery N Marachevsky and Alexandra D Nelson
Department of Theoretical Physics, Saint Petersburg State University, Ulianovskaya 1, Petrodvorets, 198504 St. Petersburg, Russia.
E-mail: alexandranelson94@gmail.com, maraval@mail.ru

Abstract. A comparison of the Casimir force theory and experiments with Si rectangular gratings is performed. We use Rayleigh decompositions to evaluate the Casimir force in Si grating – Au sphere systems for two Si grating samples. Ratios of exact Casimir force to proximity force approximation (PFA) and pairwise additive approximation (PAA) results for the Casimir force are found. The difference in the use of theoretical PFA results and PAA results extracted from experimental data is emphasized.

1. Introduction
The Casimir effect is a quantum fluctuation effect in the presence of boundaries. It was theoretically predicted by H Casimir [1] who derived the formula for the attractive force between two perfectly conducting plates separated by a vacuum slit.

The theory for gratings with 1d spatial periodicity separated by a vacuum slit was developed in [2,3], the Casimir free energy was expressed in terms of grating reflection matrices and Rayleigh coefficients [4]. The theory was found to be in agreement with experiments on the lateral Casimir force between two sinusoidal Au 1d gratings [5,6]. The difference up to 66% in lateral Casimir force results based on Rayleigh decompositions and the proximity force approximation was predicted and observed [5,6]. Details of a theoretical approach to the Casimir effect based on Rayleigh decompositions are given in [7,8]. Proximity force approximation is based on Lifshitz theory [9] for two dielectrics with flat surfaces separated by a vacuum slit. The modal approach to the Casimir forces was developed in [10]. Explicit expressions for the Casimir energy between a sphere and 1d grating in terms of the sphere and grating reflection matrices were derived in [11].

Measurements of the Casimir-Polder potential of Rb atom above a grating (made of parallel Au stripes on the sapphire substrate) in the presence of an additional external repulsive potential created by the laser field were done during diffraction of Bose-Einstein condensates consisting of cold Rb atoms [12], theoretical results based on Rayleigh decompositions and experiment were found to be in a good agreement. General analytic properties of Rayleigh coefficients were established in [13]. Existence of repulsive Casimir-Polder potential for an anisotropic atom in the presence of 20 nm thin 1d Au grating was predicted in [13].

Two systems with gratings from [14] were already studied in [2] where scattering theory was applied to grating-grating systems. However, due to a misprint in notations in [14] the calculations in [2] were applied to grating systems different from the ones used in experiments [14] as one can easily check by reading an Erratum to [14]. Therefore we decided to reanalyze
Figure 1. Si grating and Au half-space separated by a vacuum slit.

Figure 2. Casimir force gradient between Au sphere and Si sample A grating.

theoretical results related to experiments of [14]. In the current paper we present results of theoretical calculations of the Casimir normal force for two systems with Si gratings from [14] at temperature $T = 300\,\text{K}$. Also we perform analysis of experimental data from [14, 15].

2. Comparison of the theory and experiments

In experiments [14] the force gradient between an Au sphere of radius $R$ and Si rectangular grating separated by a vacuum slit was measured. In proximity force approximation (PFA) one can write

$$F'_{\text{sphere-grating}}(L) = 2\pi RF_{\text{flat-grating}}(L),$$

where $F'_{\text{sphere-grating}}(L)$ is a force gradient in sphere – grating geometry, $F_{\text{flat-grating}}(L)$ is a force in half-space – grating geometry, $L$ is a minimum distance between a grating and respective geometry.
Figure 3. Casimir force gradient between Au sphere and Si sample $B$ grating.

Figure 4. Casimir force gradient between Au sphere and Si half-space.

For $L \lesssim 0.5\mu$m and $R = 50\mu$m from [14] the condition $L \ll R$ is satisfied and PFA approximation is accurate. Therefore one can obtain gradient of the force $F'(L)$ in Au sphere – Si grating geometry by making an exact calculation of the force $F(L)$ in Au half-space - Si grating geometry (Figure 1). We follow this route for comparison of the theory and experiments.

The Casimir force between the grating and the half-space is evaluated by making use of scattering theory [2, 7] at temperature $T = 300\,\text{K}$. We use $2N + 1$ terms in every Rayleigh decomposition with $N = 40$ to determine Rayleigh coefficients. A generalized six-oscillator plasma model of Au [16] and one-oscillator model of intrinsic Si [17] are used in calculations. Theoretical results for the Casimir force in Si grating – Au half-space geometry are evaluated with an error 0.1%.
We calculate the Casimir force at an interval of distances $L = [150, 520]$ nm for two sets of parameters for a Si grating – Au half-space system shown on Figure 1: sample A with $b = 510$ nm, $d = 1000$ nm, $h = 1070$ nm (experimental data for the Casimir force derivative $F'(L)$ are plotted on Figure 3b in [14] and on Figure 2 in this paper) and sample B with $b = 191.2$ nm, $d = 400$ nm, $h = 980$ nm (experimental data for the Casimir force derivative $F'(L)$ are plotted on Figure 3c in [14] and on Figure 3 in this paper). Experimental data points are plotted by black dots at distances of experimental measurements on all figures. On Figure 4 we plot theoretical results and experimental data from Figure 3a in [14] for the Casimir force gradient for flat Si surface (half-space) in front of Au sphere. Theoretical results for $F'(L)$ are plotted on Figure 2, Figure 3, Figure 4 by blue solid lines.

We also plot ratios of the Casimir force $F(L)$ to the theoretical force $F_{PFA}(L)$ evaluated in PFA approximation for samples A and B. For geometry of Figure 1 one has

$$F_{PFA}(L) = (b/d)F_{flat}(L) + (1 - b/d)F_{flat}(L + h),$$

(2)

where $F_{flat}(L)$ is a theoretical Lifshitz force [9] for two half-spaces separated by a distance $L$. 

**Figure 5.** Ratio of the Casimir force $F$ to the theoretical force $F_{PFA}$ for Au half-space – Si sample A grating.

**Figure 6.** Ratio of the Casimir force $F$ to the theoretical force $F_{PFA}$ for Au half-space – Si sample B grating.

**Figure 7.** Ratio of the Casimir force $F$ to $F_{PAA}$ for Au half-space – Si sample A grating.

**Figure 8.** Ratio of the Casimir force $F$ to $F_{PAA}$ for Au half-space – Si sample B grating.
Theoretical ratios for samples A and B are plotted by blue solid lines on Figure 5 and Figure 6 respectively. Ratios of experimental Casimir force values to $F_{PFA}$ are shown by black dots at distances of experimental measurements on Figure 5 and Figure 6.

In [14,15] ratios of the measured Casimir force to the Casimir force defined in pairwise additive approximation $F_{PAA}(L) = b/d F_{flat}^e(L)$ were considered (Figure 3d in [14]) where $F_{flat}^e(L)$ is the force in Si half-space – Au half-space system extracted from the experimental force gradient in Si half-space – Au sphere system by the use of (1). To make a connection with ratios $F/F_{PAA}$ considered in [14,15] we extract average values of $F_{flat}^e(L)$ from $F'(L)$ on Figure 4 by the use of (1). Ratios of the theoretical Casimir force to $F_{PAA}$ obtained this way for samples A and B are plotted by blue solid lines on Figure 7 and Figure 8 respectively. Ratios of experimental Casimir force values to $F_{PAA}$ are shown by black dots at distances of experimental measurements on Figure 7 and Figure 8.

Next we perform analysis of experimental data and evaluate random, theoretical and absolute errors of experimental data. Random errors are taken into account by making use of a Student’s distribution [18]. An uncertainty in the distance $\delta L = 0.2$ nm mentioned in [15] contributes to the theoretical error. An uncertainty in theoretical results due to optical data is set 0.5% at all separations as in [19]. Absolute errors of the difference between theory and experiment are plotted by black error bars at distances of experimental measurements with confidence intervals shown at 95%, a single series of measurements for each sample from [14] is considered to find confidence intervals. Mean experimental values are plotted by dashed black lines, they are extracted from the same series of measurements.

It is instructive to compare Figure 5 with Figure 7 and Figure 6 with Figure 8. Theoreticians typically find ratios to the theoretical force $F_{PFA}$ (shown on Figure 5 and Figure 6) while in [14] ratios to the force $F_{PAA}$ obtained from experimental measurements were considered (they are shown by solid squares and hollow circles on Figure 3d in [14] and on Figure 7 and Figure 8 in the current paper). These ratios do not coincide due to the difference between theoretical values (solid blue line) and average experimental values (dashed black line) of the force gradient on Figure 4.

3. Conclusion

We make a comparison of the Casimir effect theory for Si rectangular grating – Au half-space system at temperature $T = 300$ K with experiments from [14]. The proximity force approximation (1) is used to express Casimir force gradient in Au sphere – Si grating system via the force in Au half-space – Si grating system. We use a generalized six-oscillator plasma model of Au and one-oscillator model of intrinsic Si in Casimir force computations. Ratios of the Casimir force values $F$ to the proximity force approximation theoretical values $F_{PFA}$ and pairwise additive approximation values $F_{PAA}$ obtained from experiment are evaluated. The difference in the use of theoretical PFA results and PAA results extracted from experimental data is emphasized. Error analysis of experimental data from [14] is performed, confidence intervals for absolute errors of the difference between theory and experiment are calculated at 95%. It turns out that for both grating samples A and B the theory is consistent with experiment at all separation distances.

Acknowledgments

We thank Ho Bun Chan for providing us experimental data from [14]. This research was supported in part by Saint Petersburg State University grant 11.38.237.2015. Research was carried out using computational resources provided by Resource Center "Computer Center of SPbU" (http://cc.spbu.ru/en).
References

[1] Casimir H B G 1948 Proc. Kon. Ned. Acad. Wetensch. 51 793-5
[2] Lambrecht A and Marachevsky V N 2008 Phys. Rev. Lett. 101 160403
[3] Lambrecht A and Marachevsky V N 2009 Int. J. Mod. Phys. A 24 1789-95
[4] Rayleigh O M 1907 Proc. Roy. Soc. A 79 399-416
[5] Chiu H C, Klimchitskaya G L, Marachevsky V N, Mostepanenko V M and Mohideen U 2009 Phys. Rev. B 80 121402(R)
[6] Chiu H C, Klimchitskaya G L, Marachevsky V N, Mostepanenko V M and Mohideen U 2010 Phys. Rev. B 81 115417
[7] Marachevsky V N 2012 J. Phys. A: Math. Theor. 45 374021
[8] Marachevsky V N 2015 Theor. Math. Phys. 185 1492-501
[9] Lifshitz E M 1956 Soviet Phys. JETP 2 73
[10] Davids P S, Intravaia F, Rosa F S S and Dalvit D A R 2010 Phys. Rev. A 82 062111
[11] Messina R, Maia Neto P A, Guizal B and Antezza M 2015 Phys. Rev. A 92 062504
[12] Bender H, Stehle C, Zimmermann C, Slama S, Fiedler J, Scheel S, Buhmann S Y and Marachevsky V N 2014 Phys. Rev. X 4 011029
[13] Buhmann S Y, Marachevsky V N and Scheel S 2016 Int. J. Mod. Phys. A 31 1641029
[14] Chan H B, Bao Y, Zou J, Cirelli R A, Klemens F, Mansfield W M and Pai C S 2008 Phys. Rev. Lett. 101 030401. Erratum: Chan H B, Bao Y, Zou J, Cirelli R A, Klemens F, Mansfield W M and Pai C S 2011 Phys. Rev. Lett. 107 019901
[15] Chan H B, Bao Y, Zou J, Cirelli R A, Klemens F, Mansfield W M and Pai C S 2010 Int. J. Mod. Phys. A 25 2212-22
[16] Klimchitskaya G L, Mohideen U and Mostepanenko V M 2007 J. Phys. A: Math. Theor. 40 F339-46
[17] Lambrecht A, Pirozhenko I, Duraflour L and Andreucci Ph 2007 EuroPhys. Lett. 77 44006
[18] Rabinovich S G 2000 Measurement Errors and Uncertainties: Theory and Practice (New York: Springer)
[19] Klimchitskaya G L, Chen F, Decca R S, Fischbach E, Krause D E, López D, Mohideen U and Mostepanenko V M 2006 J. Phys. A: Math. Gen. 39 6485-93