Correction to “An Efficient Game Form for Multi-Rate Multicast Service Provisioning”

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This note corrects an error in our paper, “An Efficient Game Form for Multi-Rate Multicast Service Provisioning” (reference [1]), which presents a rate allocation mechanism for multi-rate multicast service provisioning in networks with arbitrary topology and strategic users. The mechanism presented in [1] includes a tax function which is not differentiable with respect to the rate allocations. To obtain a Nash implementable arbitrary topology and strategic users. The mechanism presented includes a tax function which is not differentiable with respect to these allocations. We correct this error as follows.

We consider the problem formulated in [1]. We use the same notation as in [1]. To help the reader, we repeat here the notation used in this note.

\[ G_i := \text{Multicast group } i, \]
\[ \mathcal{N} := \{G_1, G_2, \ldots, G_N\}, \]
\[ (j, G_i) := \text{User } j \text{ in multicast group } i, \]
\[ \mathcal{R}_{(j, G_i)} := \text{Route of user } (j, G_i), \]
\[ \Xi_{(j, G_i)} := \text{The term that appears in the tax function } t_{l}^{j, G_i} \]
\[ G_i^{\text{max}}(l) := \text{Set of users in } G_i \text{ that request the maximum amount of bandwidth at link } l \text{ among all users in } G_i, \]
\[ Q_l := \text{Set of multicast groups that use link } l, \]
\[ x_{G_i}(l) := \text{Bandwidth requested by } G_i \text{ in link } l. \]

**Specification of the game form/mechanism:**

**Message space:** The message space is the same as that of the mechanism presented in [1]. A message of user \((j, G_i), j \in G_i, G_i \in \mathcal{N}\) is of the form

\[ m_{(j, G_i)} = (x_{(j, G_i)}, \pi_{l_{1}}^{j, G_i}, \pi_{l_{2}}^{j, G_i}, \ldots, \pi_{l_{|\mathcal{R}_{(j, G_i)}|}}^{j, G_i}), \]

where \(x_{(j, G_i)}\) denotes the (non-negative) bandwidth user \((j, G_i)\) requests at all the links of his route \(\mathcal{R}_{(j, G_i)}, \) and \(\pi_{l_{j}}^{l_{k}} \geq 0\) denotes the price user \((j, G_i)\) is willing to pay at link \(l_{j}\) of his route \(\mathcal{R}_{(j, G_i)}.\)

**Outcome function:** For any \(m \in \mathcal{M}\), the outcome function is defined as follows:

\[ f(m) = (x_{(i, G_i_1)}, \pi_{(i, G_i_1)}, \pi_{(i, G_i_2)}, \ldots, x_{(k, G_i_N)}, \pi_{(k, G_i_N)})) \]
\[ t_{(j, G_i)} = \sum_{l \in \mathcal{R}_{(j, G_i)}} t_{(j, G_i)}^{l}, \]

where \(t_{(j, G_i)}^{l}\) is tax paid by user \((j, G_i)\) for using link \(l\). The form of \(t_{(j, G_i)}^{l}\) is the same as the tax function defined in [1] excluding the terms denoted by \(\Xi_{(j, G_i)}\). For example, in Part DI where \(G_{i}^{\text{max}}(l) \geq 2\), the tax function in Eq. (14) of [1] is modified as follows:

Let the label of \((j, G_i)\) in \(G_{i}^{\text{max}}(l)\) be \((k, G_{i}^{\text{max}}(l))\). Then:

(i). If \((j, G_i) \in G_{i}^{\text{max}}(l)\),

\[ t_{(j, G_i)}^{l} = \pi_{(k+1, G_{i}^{\text{max}}(l))} x_{(j, G_i)} + \left(\frac{P_{G_{i}^{\text{max}}(l)} - P_{G_{i}^{\text{max}}(l)} - \gamma_{l}}{|G_{i}^{\text{max}}(l)|}\right) \frac{\mathcal{E}_{-G_{i}^{\text{max}}(l)} + x_{(j, G_i)}}{\gamma_{l}} \]

\[ + \frac{\gamma_{l}}{|G_{i}^{\text{max}}(l)|}, \quad (1) \]

where

\[ \gamma_{l} = \max\{0, \sum_{G_i \in Q_l} x_{G_i}(l) - c_l\}, \]

\(c_l\) is the capacity of link \(l, \mathcal{E}_{-G_{i}^{\text{max}}(l)}, P_{G_{i}^{\text{max}}(l)}, P_{G_{i}^{\text{max}}(l)}\) and \(\Gamma_{G_i}\) are defined by equations (18)-(21) in [1], and \(\gamma_{l}, \gamma_{l}\) are positive constants.

(ii). If \((j, G_i) \notin G_{i}^{\text{max}}(l)\) then

\[ t_{(j, G_i)}^{l} = \pi_{(k+1, G_{i}^{\text{max}}(l))} (\mathcal{E}_{-G_{i}^{\text{max}}(l)} + x_{G_i}(l)). \quad (2) \]

This completes the specification of the mechanism.

Based on the above specification, the proof of Lemma 3 in [1] is updated as follows.

**Proof of Lemma 3 in [1]:** Let \(m^\ast\) be a NE of the game induced by the mechanism. Consider \(G_i \in Q_l\), and \((k, G_{i}^{\text{max}}(l)) \in G_{i}^{\text{max}}(l)\). Since user \((k, G_{i}^{\text{max}}(l))\) does not control \(\Gamma_{G_i}\) (as in [1], page 2101), \(\frac{\partial t_{(l, G_{i}^{\text{max}}(l))}}{\partial \pi_{(k, G_{i}^{\text{max}}(l))}} = 0\). By following the same steps as in equations (32-40) of [1], we
obtain:
\[
\frac{\partial h_{(k,G)}^{\text{max}(l))}}{\partial \pi_{((k,G)}^{\text{max}(l))}} \bigg|_{m=m^*} = \frac{-2}{G_{t}^{\text{max}(l))}} P_{G_t^{\text{max}(l))}}^{*} \left( \frac{E_{G_t^{\text{max}(l))}}^{*} + x_{G_t^{\text{max}(l))}}^{*} \right)_{\gamma}
\]
\[
+ \frac{2}{G_{t}^{\text{max}(l))}} \left( P_{G_t^{\text{max}(l))}}^{*} - P_{G_t^{\text{max}(l))}}^{*} - \eta_{t}^{*} \right)
\]
\[
= 0.
\]
Furthermore (as in Eq. (36) in [1]) we note that \(\sum_{G_t \in Q_t} P_{G_t^{\text{max}(l))}}^{*} = \sum_{G_t \in Q_t} P_{G_t^{\text{max}(l))}}^{*} = 0\) over all \(G_t \in Q_t\), and \((k,G_t^{\text{max}(l)) \in G_t^{\text{max}(l))})\), we get
\[
\sum_{G_t \in Q_t} \left( \frac{\partial h_{(k,G)}^{\text{max}(l))}}{\partial \pi_{((k,G)}^{\text{max}(l))}} \bigg|_{m=m^*} \right) = -|Q_t| \eta_{t}^{*} - \sum_{G_t \in Q_t} P_{G_t^{\text{max}(l))}}^{*} \left( \frac{E_{G_t^{\text{max}(l))}}^{*} + x_{G_t^{\text{max}(l))}}^{*} \right)_{\gamma}
\]
\[
= 0.
\]
Suppose \(\sum_{G_t \in Q_t} x_{G_t^{\text{max}(l))}}^{*} - c_t > 0\). Then we must have \(\eta_{t}^{*} > 0\) and \(\sum_{G_t \in Q_t} P_{G_t^{\text{max}(l))}}^{*} \left( \frac{E_{G_t^{\text{max}(l))}}^{*} + x_{G_t^{\text{max}(l))}}^{*} \right)_{\gamma} \geq 0\). But this contradicts (4). Therefore, we must have
\[
\sum_{G_t \in Q_t} x_{G_t^{\text{max}(l))}}^{*} \leq c_t.
\]
Inequality (5) implies,
\[
\eta_{t}^{*} = 0.
\]
Combining (6) along with (4) we obtain
\[
\sum_{G_t \in Q_t} P_{G_t^{\text{max}(l))}}^{*} \left( \frac{E_{G_t^{\text{max}(l))}}^{*} + x_{G_t^{\text{max}(l))}}^{*} \right)_{\gamma} = 0.
\]
Moreover, combining (6) and (7) we obtain
\[
P_{G_t^{\text{max}(l))}}^{*} \left( \frac{E_{G_t^{\text{max}(l))}}^{*} + x_{G_t^{\text{max}(l))}}^{*} \right)_{\gamma} = 0.
\]
for any \(G_t \in Q_t\). Using (8) and (6) in (3) we obtain
\[
P_{G_t^{\text{max}(l))}}^{*} = P_{G_t^{\text{max}(l))}}^{*}.
\]
Since (9) is true for all \(G_t \in Q_t\), it implies
\[
P_{G_t^{\text{max}(l))}}^{*} = P_{G_t^{\text{max}(l))}}^{*} = P_{G_t^{\text{max}(l))}}^{*} = P_{G_t^{\text{max}(l))}}^{*}, \forall G_t \in Q_t
\]
and along with (8) it implies
\[
P_{G_t^{\text{max}(l))}}^{*} \left( \frac{E_{G_t^{\text{max}(l))}}^{*} + x_{G_t^{\text{max}(l))}}^{*} \right) = 0.
\]
Furthermore, because of
\[
\frac{\partial h_{(k,G)}^{\text{max}(l))}}{\partial x_{G_t^{\text{max}(l))}}} = 0
\]
(12)

I. PROPERTIES OF THE MECHANISM

Existence of Nash equilibria (NE): The proof of existence of NE of the game induced by the mechanism is the same as in Theorem 1 of [1].

Feasibility of allocations at NE: Because of the specification of the mechanism and Eq. (6), the allocations corresponding to all NE are in the feasible set.

Budget Balance at any NE: Budget balance at any NE follows by Lemma 4 of [1].

Individual Rationality: Individual rationality follows by Theorem 5 of [1].

Nash implementation: Nash implementation follows by Theorem 6 of [1].

Remark 1. We can achieve budget balance at any feasible allocation as follows. Consider any user \((j,G_t) \notin G_t^{\text{max}(l)}\); then \(t_{l,j,G_t}^{*}\) is given by (2). Consider \(-\pi_{(k+1,G_t^{\text{max}(l))}}^{*}E_{G_t^{\text{max}(l))}}^{*}\) and add it to the tax of user \((s,G_t), s \neq k + 1, s \in G_t^{\text{max}(l)}\). Next, consider \(-\pi_{(k+1,G_t^{\text{max}(l))}}^{*}E_{G_t^{\text{max}(l))}}^{*}\) and add it to the tax of user \((r,G_t^{\text{max}(l))}, G_t \neq G_t, G_t \in Q(l)\). Using the above tax modification and Lemma 4 in [1] we obtain budget balance at any feasible allocation.

REFERENCES

[1] A. Kakhbod and D. Teneketzis, “An efficient game form for multi-rate multicast service provisioning”, IEEE J. on Selected Areas in Communications: Special Issue on the Economics of Communication Networks and Systems, vol. 30, no. 11, pp. 2093-2104, December 2012.

(11)