Mixed Mediation of Supersymmetry Breaking in Models with Anomalous $U(1)$ Gauge Symmetry

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Abstract. There can be various built-in sources of supersymmetry breaking in models with anomalous $U(1)$ gauge symmetry, e.g. the $U(1)$ $D$-term, the $F$-components of the modulus superfield required for the Green-Schwarz anomaly cancellation mechanism and the chiral matter superfields required to cancel the Fayet-Iliopoulos term, and finally the supergravity auxiliary component which can be parameterized by the $F$-component of chiral compensator. The relative strength between these supersymmetry breaking sources depends crucially on the characteristics of $D$-flat direction and also on how the $D$-flat direction is stabilized at a vacuum with nearly vanishing cosmological constant. We examine the possible pattern of the mediation of supersymmetry breaking in models with anomalous $U(1)$ gauge symmetry, and find that various different mixed mediation scenarios can be realized, including the mirage mediation which corresponds to a mixed modulus-anomaly mediation, $D$-term domination giving a split sparticle spectrum, and also a mixed gauge-$D$-term mediation scenario.

1. Introduction

Anomalous $U(1)$ gauge symmetry, which will be referred to $U(1)_A$ in the following, appears often in 4-dimensional (4D) effective theory of string compactification [1]. It accompanies a modulus $T$ which transforms nonlinearly under $U(1)_A$, and thereby breaks $U(1)_A$ through the St"uckelberg mechanism at scales not far below the cutoff scale of 4D effective theory. The holomorphic gauge kinetic function of the model depends on $T$ as $f_a \propto k_a T$ with $k_a$ being a (quantized) real constant, and then the anomaly of the model is cancelled by the Green-Schwarz (GS) mechanism [2]. The nonlinear transformation of $T$ under $U(1)_A$ induces also a modulus-dependent Fayet-Iliopoulos (FI) term [1], which might have an interesting implication to supersymmetry (SUSY) breaking. If SUSY is not broken at the scale of $U(1)_A$ breaking, this FI term should be cancelled by the $D$-term contribution from $U(1)_A$-charged matter fields $X$. Such matter fields also contribute to the $U(1)_A$ breaking, giving a contribution of the order of the FI term to the $U(1)_A$ gauge boson mass-square. In most cases, string models with $U(1)_A$ contain exotic matter fields $\Phi + \Phi^c$ which are vector-like under the standard model gauge group, while chiral under $U(1)_A$, and these exotic matter fields become massive through the Yukawa coupling to $X$.

Therefore, models with anomalous $U(1)$ gauge symmetry involve various built-in sources of SUSY breaking which might affect the soft terms of visible sector fields: the $U(1)_A$ D-term, $F$-components of the GS modulus $T$ and the chiral matters $X$, and the supergravity (SUGRA) auxiliary component. Then there can be modulus mediation through the couplings between $T$ and the visible sector gauge and matter superfields [3, 4], $D$-term contribution to scalar masses.
gauge mediation through the loops of the exotic matter fields $\Phi + \Phi^c$ which get a SUSY breaking mass from $F_X$ and/or $D_A$ [9], and finally anomaly mediation through the SUGRA auxiliary component [10, 11].

Here we discuss the possible mediation pattern of SUSY breaking in models with anomalous $U(1)$ gauge symmetry [12, 13]. As we will see, the relative strength between the built-in mediation schemes crucially depends on the characteristics of the $D$-flat direction which corresponds to a combination of $T$ and $\ln X$. The coefficient of $D$-flat combination is determined by the ratio $R$ between the two contributions to the $U(1)_A$ gauge boson mass-square, the St"uckelberg contribution associated with the non-linear $U(1)_A$ transformation of $T$ and the conventional Higgs contribution from the vacuum values of $X$. Then various mixed mediation scenarios can be realized with different values of $R$, including the mirage mediation which corresponds to a mixed modulus-anomaly mediation [14, 15, 16, 17], $D$-term domination giving a split sparticle spectrum, and also a mixed gauge-$D$-term mediation scenario. The value of $R$ is somewhat sensitive to the detailed form of the K"ahler potential and superpotential of the model, and is determined mostly by how the $D$-flat direction is stabilized at a vacuum with nearly vanishing cosmological constant.

2. Models with anomalous $U(1)$

In models with anomalous $U(1)$ gauge symmetry, the quantum consistency of the model is ensured through the GS mechanism of anomaly cancellation [2]. This mechanism is implemented by a non-linear variation of the GS modulus,

$$T \rightarrow T - \frac{\delta_{GS}}{2} \lambda_A$$

under the $U(1)_A$ gauge transformation $V_A \rightarrow V_A - \frac{1}{2}(\lambda_A + \lambda^*_A)$ where $V_A$ is the $U(1)_A$ vector superfield. In order for the anomaly cancellation to occur, holomorphic gauge kinetic function of the model should contain a $T$-dependent piece, $f_a \ni k_a T$ with $k_a$ being a real constant. Under the normalization of $T$ for which $k_a = \mathcal{O}(1)$, we have

$$\frac{\delta_{GS}}{2} = \mathcal{O}\left(\frac{1}{8\pi^2}\right)$$

as required to cancel the anomalies due to fermion loops. Since the $U(1)_A$ gauge invariance forces the modulus K"ahler potential $K_0$ to be a function of $t_A = T + T^* - \delta_{GS} V_A$, the GS mechanism dynamically induces a FI term:

$$\xi_{FI} = \frac{\delta_{GS}}{2} K'_0,$$

as well as providing the St"uckelberg contribution to the $U(1)_A$ vector boson mass-square:

$$\Delta M^2_V \equiv 2 g_A^2 M_{GS}^2 = \frac{\delta_{GS}^2}{2} g_A^2 K''_0,$$
single such chiral superfield $X$. Under the assumption that the cutoff scale $\Lambda$ of 4D theory is close to $M_{Pl}$, the Kähler potential can be expanded as

\[ K = K_0(t_A) + Z_X(t_A)e^{-2V_X}|X|^2 + \mathcal{O}(|X|^4), \]  

where the $U(1)_A$ charge of $X$ is normalized as $q_X = -1$. Then the $U(1)_A$ D-term is given by

\[ D_A = \xi_{FI} + Z_X|X|^2, \]

where $Z_X = Z_X(1 + \frac{\delta_{GS}}{2}\partial_t \ln Z_X)$. Along the $D$-flat direction, $X$ is triggered to acquire a vacuum expectation value as $Z_X|X|^2 \simeq -\xi_{FI}$, and thus participates in the breaking of $U(1)_A$. Consequently, one linear combination of $T$ and $\ln X$ becomes the longitudinal component of $V_A$, providing a mass

\[ M_V^2 = 2g_A^2 \left( M_{GS}^2 + \left( 1 + \frac{\delta_{GS}}{2}\partial_t \ln Z_X \right) Z_X|X|^2 \right) \]  

to $V_A$. It is thus the ratio between $\xi_{FI}$ and $M_{GS}^2$,

\[ R \equiv -\frac{\xi_{FI}}{M_{GS}^2} = -\frac{2}{\delta_{GS}} \frac{K_0'}{K_0''}, \]

that determines which combination forms the longitudinal component of $V_A$. If the vacuum lies in the region with $R \gg 1$, the longitudinal component mostly comes from $X$. In the opposite limit with $R \ll 1$, it is mostly the GS modulus $T$. (Note that by definition $R \geq 0$.) It is then expected that the pattern of SUSY breaking crucially depends on which field is left at low energy scales after the breaking of $U(1)_A$, and thus on the value of $R$. It should be noted that the FI term is a monotonic function of $t = T + T^*$ because $K_0''$ is positive definite as required for sensible kinetic terms. This implies that the FI term can vanish only at a single value of $t$. However, regardless of the value of $\xi_{FI}$, the GS mechanism always makes the $U(1)_A$ vector superfield massive through the Stäckelberg mechanism.

### 2.1. Supersymmetry breaking

The chiral superfields involved in the $U(1)_A$ breaking should be stabilized eventually at a SUSY breaking minimum, and then their $F$-terms and the $U(1)_A$ D-term are expected to obtain nonzero vacuum expectation values. When combined with the equation of motion $D_A = -\eta^I\partial_I K$, the $U(1)_A$ gauge invariance of the superpotential $W$ leads to the following relation:

\[ m_{3/2} D_A = K_{IJ}\eta^IF^{I*}, \]

where $F^I = -e^{K/2}K^{IJ}(\partial_J W + W\partial_J K)^*$, $m_{3/2} = e^{K/2}W$, and $\eta^I$ denotes the holomorphic Killing vector generating an infinitesimal $U(1)_A$ transformation, specifically $\eta^T = -\delta_{GS}/2$ and $\eta^X = -X$. The above relation tells us that non-vanishing $D$-term requires a $F$-term SUSY breaking. For supersymmetric minimum, however, additional positive contribution to the scalar potential is required to achieve a viable de-Sitter or Minkowski vacuum. In the KKLT moduli stabilization scenario [18], such uplifting is accomplished by introducing an anti-brane stabilized at the tip of throat, yielding an uplifting potential sequestered from the bulk Calabi-Yau space including the visible sector branes. Here we assume that the uplifting sector is sequestered from the GS modulus as well as from the visible sector as in the KKLT compactification, and thus the uplifting potential takes the form [15]

\[ V_{\text{lift}} = e^{2K/3}P, \]
where $\mathcal{P}$ is a positive constant which should be adjusted to make the cosmological constant to be nearly vanishing. Combining (9) with the minimization condition $\partial_t(V_F + V_D + V_{\text{lift}}) = 0$, one can derive the relation [12]

$$
\left( V_F + \frac{2}{3} V_{\text{lift}} + 2|m_3/2|^2 + \frac{1}{2} M_P^2 \right) D_A = -F^I F^J \partial_I \partial_J (\eta^L \partial_L K) + V_{\text{lift}} \eta^I \partial_I g_\lambda^2,
$$

(11)

where $V_F$ and $V_D$ are the $F$-term and $D$-term scalar potentials in the Einstein frame, i.e.

$$
V_F = K_{IJ} F^I F^J - 3e^K |W|^2, \quad V_D = \frac{g_\lambda^2}{2} D_A^2.
$$

(12)

Now using the relations (9) and (11) we find [13]

$$
\frac{F^X}{X} = F^T \partial_T \ln \left( \frac{K_0'}{Z_X} \right) + O \left( \frac{m_{3/2} D_A}{R M_{\text{GS}}^2} \right),
$$

$$
g_\lambda^2 D_A = -\frac{R}{1 + (1 + \frac{2m_{3/2}}{R} \partial_T \ln Z_X) R} |F^T|^2 \partial_T \ln \left( \frac{K_0'}{Z_X} \right) + O \left( \frac{m_{3/2} |F^T|^2}{M_P^2} \right),
$$

(13)

where we have used that all $F$ and $D$-terms are at most of $O(m_{3/2} M_{\text{Pl}})$. We note that the above relations are insensitive to the addition of sequestered uplifting potential, though in some cases the uplifting could be crucial for constructing a SUSY breaking vacuum. One can also see that $|D_A| \leq O(m_{3/2}^2 M_{\text{Pl}}^2/M_P^2)$ holds at the vacuum. For models with $M_V \gg m_{3/2}$, it is found that the $D$-term potential gives a negligible contribution to the vacuum energy density in comparison to the $F$-term potential. Nonetheless, the $D$-term can still be an important source of soft scalar masses.

### 2.2. Mediation of SUSY breaking

Models with $U(1)_A$ can provide various sources of soft terms when SUSY breaking is transmitted to the visible sector. Here are the list of mediation mechanisms that can naturally appear in models with $U(1)_A$.

- **Modulus mediation**

  Modulus mediation [3, 4], which can be considered as a particular version of gravity mediation, is a natural source of soft terms in models with $U(1)_A$. For $f_a \equiv k_a T$, the $F$-term of the GS modulus $T$ generates gaugino masses as

$$
M_a|_{\text{MM}} = \frac{k_a g_a^2(\Lambda)}{2} F^T,
$$

(14)

at $\Lambda$ close to $M_{\text{Pl}}$, where the gauge couplings are given by $g_a^2(\Lambda) = 1/\text{Re}(f_a)$. If $k_a$ is universal as suggested by the unification of gauge couplings in the MSSM, modulus-mediated gaugino masses are also universal at $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV. Similarly the $T$-dependence of matter wave function superfields gives rise to sfermion masses $m_j^2|_{\text{MM}} = O(M_j^2|_{\text{MM}})$. For the case that $T$ corresponds to the dilaton or a Kähler modulus, flavor-conserving soft terms can be naturally obtained since the $T$-dependence is controlled by the matter modular weight. Sparticle masses induced by modulus mediation are of order the gravitino mass or less since $|F^T| \leq O(m_{3/2} M_{\text{Pl}})$. 


• **Anomaly mediation**

Supergravity always mediates SUSY breaking through the conformal anomaly \([10, 11]\). This effect can be easily understood by introducing the chiral compensator superfield \(C\). Since the running gauge and Yukawa couplings have a logarithmic \(C\)-dependence dictated by the super-Weyl invariance, soft terms of order \(F^C/8\pi^2 C\) \(\approx m_3/2/8\pi^2\) are generated by radiative corrections. As the running is mostly due to gauge interactions, anomaly-mediated soft terms naturally preserve flavor. In anomaly mediation, gaugino masses are given by

\[
M_a|_{AM} = \frac{b_a g_a^2(Q) F^C}{8\pi^2 C},
\]

where \(b_a\) are the one-loop beta-function coefficients, and \(Q\) is the renormalization scale. Soft scalar masses are generated at two-loop order by anomaly mediation \([10]\), so

\[
m_f^2|_{AM} = \mathcal{O}(M_a^2|_{AM}).
\]

• **Gauge mediation**

In models with \(U(1)_A\), soft terms can receive a gauge-mediated contribution \([9]\) if there exist messenger fields which obtain a heavy mass by \(X\). For messenger fields \(\Phi + \Phi^c\) which couple to \(X\) through \(y_{\Phi} X \Phi \Phi^c\) in the superpotential, the SUSY breaking by \(F^X\) is transmitted to the MSSM sector by the loops of messenger fields. In order to preserve the gauge coupling unification, the messengers are usually assumed to form a complete GUT multiplet. The resulting gauge-mediated gaugino masses at the messenger scale \(\Lambda_\Phi = y_\Phi |X|\) are given by

\[
M_a|_{GM} = \frac{N_\Phi g_a^2(\Lambda_\Phi) F^X}{8\pi^2 X},
\]

where \(N_\Phi\) is the number of messenger pairs. Gauge mediation induces soft scalar masses also, giving \(m_f^2|_{GM} = \mathcal{O}(M_a^2|_{GM})\). Because the transmission of SUSY breaking occurs through the SM gauge interactions, the induced fermion soft terms are automatically flavor-blind. Note that, if \(|D_A| \geq \mathcal{O}(|F^X/X|^2)\), the usual relation \(m_f^2 \sim M_a^2/N_\Phi\) in gauge mediation can be changed because \(\Phi \Phi^c\) carries a \(U(1)_A\) charge \(q_\Phi + q_{\Phi^c} = 1\), and thus the gauge-mediated soft scalar masses contain the additional contribution \(\delta m_f^2 \sim \sum_i C_i^a g_a^2 N_\Phi \ln(\Lambda/\Lambda_\Phi) \bar{q}_i^A D_A\) with \(C_i^a\) being the quadratic Casimir of \(\bar{f}\) \([19]\).

• **D-term contribution**

In the presence of \(U(1)_A\), soft scalar masses receive the D-term contribution \([5]\)

\[
m_f^2|_{D-term} = -q_i g_a^2 D_A,
\]

where \(q_i\) denotes the \(U(1)_A\) charge of the corresponding matter field. For \(D_A = \mathcal{O}(M_a^2)\) with \(M_a\) in the TeV range, we need to arrange \(q_i\) to be flavor-independent so that the D-term contributions do not cause flavor problem. If \(D_A\) is suppressed by more than the loop factor compared to \(M_a^2\), the flavor structure is not affected by the D-term contribution and thus the matter fields are allowed to carry flavor-dependent \(U(1)_A\) charges. In this case, \(U(1)_A\) may be useful for generating the observed hierarchical structure of fermion masses. Whereas, for \(D_A\) much larger than \(M_a^2\), large flavor violations can be easily avoided as sfermions get heavy mass from \(D_A\).

Let us now discuss the relative strength between these mediation mechanisms by applying the relations \((13)\) between the vacuum expectation values of \(D\) and \(F\)-terms. The relative size of anomaly-mediated contribution depends on the mechanism to achieve the vanishing cosmological
constant. In general, anomaly mediation becomes less important if the SUSY breaking in the $U(1)_A$ sector is enough to construct a viable de-Sitter or Minkowski vacuum. For the case that the Kähler metric of $X$ has no dependence on $T$, we find the $F$ and $D$ terms satisfy

$$F^T \simeq -\frac{\delta_{GS}}{2} R \frac{F^X}{X},$$
$$g_A^2 D_A \simeq \frac{R}{1 + R} \left( 1 + \frac{\delta_{GS}}{2} R \frac{K_0''}{R_0'} \right) \left| \frac{F^X}{X} \right|^2.$$  

This implies

$$M_a|_{MM} \sim R \, M_a|_{GM},$$  

which remains valid also for large $N_{\Phi}$ because anomaly cancellation fixes $k_a \sim N_{\Phi}$. One can thus conclude that, if the vacuum arises at $R \gg 1$, gauge mediation cannot be an important source of soft terms. In contrast, for $R \ll 1$, modulus mediation would become negligible. It is also found that, for $10^{-6} \leq R \leq 10^3$, the $D$-term contribution to soft scalar masses is non-negligible compared to those from modulus and gauge mediations. In particular, for $R$ of order unity, the $D$-term contribution can dominate over other mediations.

3. Stabilization of the GS modulus

Let us now explore the connection between the value of $R$ and the stabilization of the GS modulus $T$. Since the vacuum lies very close to the $D$-flat direction, the $D$-term from the vacuum expectation value of $X$ should cancel the FI term in $D_A$. To understand the implication of the FI term for the modulus stabilization, it is sufficient to consider the simple case that $Z_X$ is independent of $T$, so we take

$$K = K_0(t_A) + |X|^2 e^{-2V_A},$$  

without specifying the form of the modulus Kähler potential $K_0$.

3.1. Non-perturbative Polonyi term

To stabilize the $D$-flat direction, we need a superpotential of \(\{T; X\}\) providing an $F$-term potential for $D$-flat direction. An attractive example would be the non-perturbative Polonyi term induced by some stringy instanton (or hidden gaugino condensation) effects:

$$W = \omega_0 + A e^{-aT} X,$$  

where $\omega_0$ is a constant of $O(m_{3/2} M_{Pl}^2)$, and the gauge invariance requires $a = 2/\delta_{GS} = O(8\pi^2)$. By an appropriate shift of $T$, the coefficient $A$ can always be made to be of order unity without loss of generality. Then the Polonyi term gives a non-trivial $F$-term potential with a (local) minimum at $e^{-aT} X = O(\omega_0)$, while the detailed vacuum structure depends on the form of $K_0$.

For the Kähler potential (20) and the superpotential (21), it is easy to see that the vacuum expectation value of $\arg(e^{-aT} X)$ is simply fixed as

$$\arg(e^{-aT} X) = \begin{cases} \arg \left( \frac{\omega_0}{X} \right) + \pi, & \text{for } R > \frac{2-|X|^2}{2+|X|^2}, \\ \arg \left( \frac{\omega_0}{X} \right), & \text{for } R < \frac{2-|X|^2}{2+|X|^2}. \end{cases}$$  

and this is not affected by the addition of uplifting potential. After replacing $\arg(e^{-aT} X)$ with its expectation value, the scalar potential becomes a function of $t = T + T^*$ and $|X|$. Then the
stationary conditions \( \partial_{n}X(V_F + V_D) = 0 \) lead to
\[
\begin{align*}
\mathcal{G}_A D_A &= -\frac{1}{\mathcal{G}_A K_0'} \left( V_F' + (\partial_t \ln \mathcal{G}_A) V_D \right), \\
\dot{V}_F &= \frac{2 |X|^2}{\mathcal{G}_A K_0''} \left( V_F'^2 - \frac{K_0''}{K_0'} V_F' \right) - 4 \frac{|X|^2}{\mathcal{G}_A K_0''} \dot{V}_F' > 0,
\end{align*}
\]
where the dot and prime denote the derivative with respect to \( \ln |X| \) and \( t \), respectively. The second relation above determines how the \( D \)-flat direction is lifted by the \( F \)-term potential. Neglecting small corrections of \( \mathcal{O}(m_{3/2}^4/M_P^4) \), the condition for the above stationary solution to be a (local) minimum reads
\[
\dot{V}_F + 4 \frac{|X|^2}{\mathcal{G}_A K_0''} \left( V_F'^2 - \frac{K_0''}{K_0'} V_F' \right) - 4 \frac{|X|^2}{\mathcal{G}_A K_0''} \dot{V}_F' > 0,
\]
which corresponds to the requirement that the \( F \)-term potential should provide a positive mass-squared to the \( D \)-flat direction. Note that a supersymmetric field configuration leads to \( \dot{V}_F = V_F' = 0 \) and \( D_A = 0 \).

The field space of \( T \) can be divided into two regions depending on where \( \text{arg}(e^{-atX}) \) is stabilized. Let us first look for a supersymmetric minimum. The equations (23) can have a supersymmetric solution only in the region with \( R > 2 \) where \( \text{arg}(e^{-atX}) \) is fixed at \( \text{arg}(\omega_0/A) + \pi \). This supersymmetric solution is determined by
\[
\begin{align*}
|X|^2 &= -\xi_{FI}, \\
\frac{|A|}{\omega_0} e^{-at/2} &= \frac{1}{1 - \xi_{FI}},
\end{align*}
\]
for which the condition (24) is automatically fulfilled regardless of the detailed form of \( K_0 \). Thus, if exists, SUSY stationary point always corresponds to a local minimum of \( V_F + V_D \). It is also found that there can be at most one supersymmetric solution because \( \xi_{FI} \) is a monotonically increasing function of \( t \).

On the other hand, a non-supersymmetric minimum can appear in the region with \( R < 2 \) where \( \text{arg}(e^{-atX}) \) is fixed at \( \text{arg}(\omega_0/A) \). Since the non-perturbative Polonyi term stabilizes the GS modulus at \( at \gg 1 \), it is plausible to assume that the modulus Kähler potential satisfies
\[
\left| \left( \frac{\delta_{GS}}{2} \right)^{n-2} \frac{\partial^2 K_0}{K_0''} \right| \leq \frac{1}{(at)^{n-2}} \ll 1
\]
for \( n \geq 3 \) at the stationary point. With this assumption, we find that the scalar potential can have a SUSY breaking minimum with \( R < 2 \) at
\[
\begin{align*}
|X|^2 &= -\xi_{FI} + \mathcal{O} \left( \frac{m_{3/2}^2 M_P^2}{M_P^2} \right), \\
\frac{|A|}{\omega_0} e^{-at/2} &= \frac{1}{R} + \mathcal{O} \left( \frac{M_P^2}{M_P^2} \right),
\end{align*}
\]
for \( M_P^2 > M_V > \sqrt{m_{3/2} M_P} \). It is straightforward to show that the above field configuration satisfies well the condition (24) and leads to
\[
(V_F + V_D)_{SB} = e^{K_{\omega_0}^2} \left( -3 + \frac{1}{R} \frac{M_{GS}^2}{M_P^2} + \mathcal{O} \left( \frac{M_{GS}^2}{M_P^2} \right) \right),
\]
where the second term in the brackets is the contribution from $F^X$. Thus the small cosmological constant requires $R = O(M_{\text{Pl}}^2/M_{\text{GS}}^2)$.

In order to have a SUSY breaking minimum in the region with $R > 2$, one needs a quite non-trivial modulus Kähler potential for which the assumption (26) is not valid. This can be inferred from that a non-supersymmetric solution of (23), if exists in the indicated region, is accompanied by a supersymmetric one, which is necessarily a local minimum of the potential. More than three stationary points are difficult to be obtained for the scalar potential generated by single non-perturbative superpotential term.

We now turn our attention to the dependence of the minimum on the property of the modulus Kähler potential. Written as in (25) and (27), the two minimization conditions for $t$ and $|X|$ make it clear how the GS modulus is stabilized by the non-perturbative superpotential. One can see that the function $e^{-at/2}/\sqrt{-\xi_{\text{FI}}}$ monotonically decreases in the region with $R > 1$ as $t$ increases, while it increases monotonically in the other region. In addition, from $R' = -\frac{2}{\delta_{\text{GS}}} \left( 1 + R \frac{\delta_{\text{GS}} K''_{0}}{K''_{0}} \right)$, it is found that $R$ can be approximated as

$$R \simeq -\frac{2}{\delta_{\text{GS}}} (t - t^*_s),$$

for $R \leq O(1)$ and $at_s \geq at \gg 1$, where $t_s$ is the value of $t$ at which the FI term vanishes. Note that, because $a = -2q_X/\delta_{\text{GS}} > 0$, $X$ can maintain the $D$-flatness by cancelling the FI term in the field space $t \leq t_s$. Using these properties, we arrive at that the scalar potential has the following structures depending on the value of $\gamma_s$

$$\gamma_s = \left( \frac{\delta_{\text{GS}}^2 K''_{0}(t^*_s)}{4} \right)^{-1/2} \frac{|A|}{\omega_0} e^{-a t_s^*/2}. \quad (30)$$

(i) For $0.26 > \gamma_s$, there arises only one supersymmetric minimum. To stabilize $X$ at much lower than $M_{\text{Pl}}$, $|A/\omega_0|e^{-at_s^*/2}$ should be much smaller than the unity.

(ii) For $0.52 > \gamma_s > 0.26$, the scalar potential can develop two local minima, one supersymmetric and one SUSY breaking minimum. Both minima have $R$ of order unity and $V_F + V_D < 0$.

(iii) For $\gamma_s > 0.52$, the scalar potential has only a SUSY breaking minimum. To have non-negative vacuum energy at the minimum, $|A/\omega_0| e^{-at_s^*/2} > \sqrt{3}$ is required.

It should be noted that the SUSY preserving minimum arises in the region $t < t_s - \delta_{\text{GS}}$ where $R > 2$ is obtained, whereas the SUSY breaking minimum appears at $t_s - \delta_{\text{GS}} < t < t_s$ where $R < 2$.

### 3.2. Examples

Here we provide two specific examples of the modulus Kähler potential, illustrating how the stabilization of the GS modulus is affected by the FI term. After fixing arg($e^{-aT}X$) at the minimum of the potential, the two stationary conditions in (23) fix $t = T + T^*$ and $|X|$ at a minimum of $V_F + V_D$. The first condition simply corresponds to that the FI term should be cancelled by the $D$-term due to the vacuum expectation value of $X$, i.e. corresponds to the (approximate) $D$-flatness condition. The other condition determines where the minimum is developed along the $D$-flat direction. For

$$K = K_0 (T + T^* - \delta_{\text{GS}} V_A) + e^{-2V_X} X^2,$$

$$W = \omega_0 + A e^{-aT} X,$$

we consider two models

Model I : $K_0 = -n_0 \ln(T + T^* - \delta_{\text{GS}} V_A)$,

Model II : $K_0 = (T + T^* - t_s - \delta_{\text{GS}} V_A)^2$, \quad (32)
with the following superpotential parameters

$$|\omega_0| = e^{-38}, \quad |A| = 1, \quad a = 4\pi^2.$$  \hfill (33)

In Model I, $T$ might correspond to the dilaton or a Kähler modulus whose Kähler potential is well-approximated by the weak coupling form. On the other hand, in Model II, $T$ might correspond to a modulus describing a singular limit of some cycle at which $\xi_F \propto K_0'$ vanishes. Note that $K_0$ is approximated by a quadratic form in the vicinity of the point with $K_0' = 0$, i.e. $t = t_*$ in Model II.

Let us summarize how the GS modulus is fixed by $V_F + V_D$ in these two examples. In Model I with $n_0 = 1$, the potential has one SUSY minimum at $t \approx 2.04$ and $|X| \approx 0.11M_{Pl}$, which yields $R \approx 80.4$. In Model II, $(a)$ for $t_* = 2.42$, one SUSY minimum appears at $t \approx 2.02$ and $|X| \approx 0.14M_{Pl}$ with $R \approx 15.6$, $(b)$ for $t_* = 2.14$, the potential develops two minima: one supersymmetric minimum at $t \approx 2.06$ and $|X| \approx 0.06M_{Pl}$ with $R \approx 2.87$, and one SUSY breaking minimum at $t \approx 2.1$ and $|X| \approx 0.04M_{Pl}$ with $R \approx 1.44$, $(c)$ for $t_* = 1.82$, there is one SUSY breaking minimum at $t \approx 1.82$ and $|X| \approx 1.6 \times 10^{-4}M_{Pl}$ with $R \approx 0.2 \times 10^{-4}$. These results are all in agreement with the general arguments presented in the previous subsection.

4. Mixed mediations

Here are several examples of mixed mediation that can arise naturally in models with anomalous $U(1)$ gauge symmetry [13].

- **Mirage mediation (Mixed modulus-anomaly mediation) for $R \sim O(8\pi^2)$:**
  
  As we have discussed, a simple minded perturbative modulus Kähler potential leads to $R \sim 1/\delta_{GS} \sim 8\pi^2$. In this case, integrating out the massive $U(1)_A$ vector superfield results in an effective theory of the GS modulus $T$ in which the matter moduli weight is shifted by the $U(1)_A$ charge $q_i$, through which the $D$-term contribution to scalar masses can be identified as the modulus-mediation. The non-perturbative superpotential in the effective theory stabilizes $T$ at a SUSY AdS minimum, and thus an uplifting sector is needed as in the KKLT model. After uplifting, the modulus mediation becomes comparable to the loop-induced anomaly mediation [14, 15], leading to the mirage mediation pattern of soft terms [16]. Cancellation between the anomaly mediation and the radiative corrections to modulus mediation results in a significantly compressed low energy SUSY spectrum compared to other popular schemes. A characteristic feature is the mirage unification of gaugino masses at a mirage messenger scale $M_{\text{mirage}}$ which is determined by the relative ratio between anomaly and modulus mediations. In mirage mediation, 1st and 2nd generation sfermion masses also show mirage unification at the same scale $M_{\text{mirage}}$.

  Mirage mediation can be generalized to include contributions from gauge mediation [20, 21], which would be possible in our case if the gauge-charged messengers get masses by a $U(1)_A$ neutral singlet, not by the $U(1)_A$-charged $X$. Sparticle masses are then deflected from the mirage unification trajectory due to the presence of gauge mediation. The mirage unification behavior is maintained for the gauginos, but the mirage messenger scale changes from the pure mirage mediation case. For sfermion masses, mirage pattern is generically lost, and the deviation depends on the size of the gauge mediation contribution.

- **Mixed modulus-gauge-D-term mediation for $O(1/(8\pi^2)^2) \leq R \leq O(8\pi^2)$:**

  The situation can be quite different in models where the modulus Kähler potential yields $R \leq O(8\pi^2)$. Over the range $1/(8\pi^2)^2 \leq R \leq 8\pi^2$, gaugino masses are given by a mixed modulus-gauge mediation with $M_a |_{MM}/M_a |_{GM} \sim F_T/(F_X/8\pi^2 X) \sim R$. On the other hand, sfermion masses are dominated mostly by the $D$-term contribution with a little hierarchy structure $m_{\tilde{j}} \sim \sqrt{D_A} \sim 8\pi^2 M_a/R$ for $R \geq 1$ and $m_{\tilde{j}} \sim \sqrt{D_A} \sim 8\pi^2 \sqrt{R} M_a$ for $R \leq 1$.  

• Mixed gauge-anomaly-D-term mediation for $R \sim O(1/(8\pi^2)^2)$:
  Another interesting mixed mediation can arise when superpotential does not have any $T$-
  dependent nonperturbative term, but contains a Yukawa coupling $\propto X \Phi \Phi^c$ between $X$ and
  the messengers $\Phi + \Phi^c$. In such case, $X$ is stabilized at $\langle X \rangle \sim M_{GS}/8\pi^2$ by the competition
  between the $D$-term potential and the anomaly-mediated tachyonic mass, which results in
  $F^X/8\pi^2 X \sim F^C/8\pi^2 C \sim \sqrt{D_A}$, and thus a mixed gauge-anomaly-$D$-term mediation.

5. Conclusion
There can be various built-in sources of SUSY breaking in models with anomalous $U(1)$ gauge
symmetry: the $U(1)_D$ D-term, the $F$-components of the Green-Schwarz modulus $T$ and the chiral matter $X$ introduced to cancel the Fayet-Illiopoulos term, and also the SUGRA auxiliary component parameterized by the $F$-component of chiral compensator $C$. Generically the visible sector soft masses receive modulus-mediated contribution of the order of $F^T$, $D$-term contribution of the order of $\sqrt{D_A}$, anomaly-mediated contribution of the order of $F^C/8\pi^2 C$, and finally gauge-mediated contribution of the order of $F^X/8\pi^2 X$ if there exist messengers which couple to $X$ to get masses. We have examined the possible pattern of the mediation of SUSY breaking in such situation, and find that various mixed mediation scenarios can be realized for different values of the ratio between the FI term and the St"uckelberg contribution to the $U(1)_A$ gauge boson mass-square.

Acknowledgments
We thank K. S. Jeong and M. Yamaguchi for useful discussions. This work is supported by the KRF Grants funded by the Korean Government (KRF-2008-314-C00064 and KRF-2007-341-C00010) and the KOSEF Grant funded by the Korean Government (No. 2009-0080844).

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