Thermodynamics of Quantum Information Flows

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Goal: Find thermodynamics constrains on information flows across an open Q-system

2nd law for open Q-systems

Main result: local 2nd laws with information flows

Previous works on Maxwell’s demon

Derivation of the main result

Application to an autonomous Q-Maxwell demon
2nd law for open systems

\[ \sigma = \Delta S - \sum_{\alpha} \beta_{\alpha} Q_{\alpha} \geq 0 \]

where

- \( \sigma \) – entropy production
- \( S \) – entropy of the system
- \( Q_{\alpha} \) – heat delivered from the reservoir \( \alpha \)
2nd law in differential form

\[ \dot{\sigma} = d_t S - \sum_{\alpha} \beta_\alpha \dot{Q}_\alpha \geq 0 \]

where

- \( \dot{\sigma} \) – entropy production rate
- \( S = -\text{Tr}(\rho \ln \rho) \) – von Neumann entropy of the system
- \( \dot{Q}_\alpha \) – heat flow from the reservoir \( \alpha \)

[Spohn, Lebowitz, Adv. Chem. Phys. 38, 109 (1978)]
Main result: local Clausius inequality

\[ \hat{H}_S = \sum_i \hat{H}_i + \hat{H}_{\text{int}} \]

- Can we define 2nd law for a single subsystem? Yes!
- **Local Clausius inequality**

\[ \dot{\sigma}_i = d_t S_i - \sum_{\alpha_i} \beta_{\alpha_i} \dot{Q}_{\alpha_i} - \dot{I}_i \geq 0 \]

where

- \( \dot{\sigma}_i \) – local entropy production rate
- \( S_i = -\text{Tr}(\rho_i \ln \rho_i) \) – von Neumann entropy of the subsystem \( i \)
- \( \dot{Q}_{\alpha_i} \) – heat flow from reservoir \( \alpha_i \)
- \( \dot{I}_i \) – information flow between the subsystems (defined later)
Maxwell demons

- Maxwell demon – entropy of a stochastic system can be reduced by a feedback control by an intelligent being

- Experimental realizations
  - Molecular ring [Leigh group, Nature 445, 523 (2007)]
  - Single atoms [Raizen group, PRL 100, 093004 (2008)]
  - Colloidal particles [Sano group, Nat. Phys. 6, 988 (2010)]
  - Single-electron boxes [Pekola group, PRL 113, 030601 (2014)]
  - Superconducting circuits [Masuyama group, Nat. Com. 9, 1291 (2018)]

- 2nd laws with mutual information due to nonautonomous feedback
  - [Sagawa, Ueda, PRL 100, 080403 (2008)] (System only)
  - [Sagawa, Ueda, PRL 102, 250602 (2009)] (System + Memory)
Autonomous Maxwell demons

- [Esposito, Schaller, EPL 99, (2012)] (System only)
- [Strasberg et al., PRL 110, 040601 (2013)] (System + Demon)
- Experimental realization – [Koski et al., PRL 115, 260602 (2015)]

No mutual information....

Connection between autonomous and nonautonomous was unclear
Unified framework within stochastic thermodynamics

- Two subsystems: $X$ and $Y$
- Classical rate equation for state probabilities

$$\dot{p}(x, y) = \sum_{x', y'} \left[ W_{x,x'}^{y,y'} p(x', y') - W_{x',x}^{y,y'} p(x, y) \right]$$

$W_{x,x'}^{y,y'}$ – rate of transition $(x', y') \rightarrow (x, y)$

- Bipartite transitions – either in $X$ or $Y$, not simultaneous

$$W_{x,x'}^{y,y'} = \begin{cases} 
  w_{x,x'}^{y,y'} & x \neq x; y = y' \\
  w_{x,x'}^{y,y'} & x = x'; y \neq y' \\
  0 & x \neq x', y \neq y' 
\end{cases}$$

[J. M. Horowitz, M. Esposito, Phys. Rev. X 4, 031015 (2014)]
Local 2nd of thermodynamics

- Mutual information – measure of correlation between subsystems

\[ I = H_X + H_Y - H = \sum_{x,y} p(x, y) \ln \frac{p(x, y)}{p(x)p(y)} \geq 0 \]

where \( H \) is the Shannon entropy.

- Decomposition: \( d_t I = \dot{I}_X + \dot{I}_Y \)

\[ \dot{I}_X = \sum_{x \geq x', y} \left[ w_{x',x}^y p(x', y) - w_{x',x}^y p(x, y) \right] \ln \frac{p(y|x)}{p(y|x')} \]

- Local 2nd law

\[ \dot{\sigma}_i = \dot{H}_i - \beta_i \dot{Q}_i - \dot{I}_i \geq 0 \]

[J. M. Horowitz, M. Esposito, Phys. Rev. X 4, 031015 (2014)]
Limitations

- Classical systems with bipartite structure

- Q-systems without eigenbasis coherences \textbf{and} satisfying $[\hat{H}_{\text{int}}, \hat{H}_i] = 0$
  
  Since rate equations describe transitions between eigenstates of the total Hamiltonian $\hat{H}_S$, the eigenstates of $\hat{H}_S$ must be products of eigenstates of subsystem Hamiltonians $\hat{H}_i$ for the transition matrix to have a bipartite structure

- \textbf{We will now} generalize the concept of autonomous information flow to a generic Markovian open Q-system
Derivation: Key ingredients

- Dynamics described by Lindblad equation

\[ d_t \rho = -i \left[ \hat{H}_{\text{eff}}, \rho \right] + \mathcal{D} \rho \]

- Additivity of dissipation – interaction with each reservoir gives an independent contribution to the dissipation

\[ \mathcal{D} = \sum_{\alpha} \mathcal{D}_{\alpha} \]

- Local equilibration

\[ \mathcal{D}_{\alpha} \rho^{\text{eq}}_{\alpha} = 0 \]

where \( \rho^{\text{eq}}_{\alpha} = Z_{\alpha}^{-1} e^{-\beta_{\alpha} (\hat{H}_S - \mu_{\alpha} \hat{N})} \)
Partial Clausius inequality

- Applying Spohn’s inequality [Spohn, J. Math. Phys. 19, 1227 (1978)]

\[-\text{Tr} \left[ (\mathcal{D}^\alpha \rho) \left( \ln \rho - \ln \rho_{\text{eq}}^\alpha \right) \right] \geq 0\]

one obtains the \textbf{partial Clausius inequality}
[Cuetara, Esposito, Schaller, Entropy 18, 447 (2016)]

\[
\dot{\sigma}_\alpha = \dot{S}^\alpha - \beta_\alpha \dot{Q}_\alpha \geq 0
\]

where

- \(\dot{\sigma}_\alpha\) – partial entropy production rate
- \(\dot{S}^\alpha = -\text{Tr} \left[ (\mathcal{D}^\alpha \rho) \ln \rho \right]\) – rate of change of the von Neumann entropy due to interaction with the reservoir \(\alpha\)
- \(\dot{Q}_\alpha = \text{Tr} \left[ (\mathcal{D}^\alpha \rho) \left( \hat{H}_S - \mu_\alpha \hat{N} \right) \right]\) – heat flow from the reservoir \(\alpha\)

\textbf{Meaning:} interaction with each reservoir gives a non-negative contribution to the entropy production
Local Clausius inequality

\[ \dot{\sigma}_\alpha = \dot{S}^\alpha - \beta_\alpha \dot{Q}_\alpha \geq 0 \]

- Local entropy production rate – sum of \( \dot{\sigma}_{\alpha_i} \) associated with reservoirs \( \alpha_i \) coupled to subsystem \( i \)

\[ \dot{\sigma}_i = \sum_{\alpha_i} \dot{\sigma}_{\alpha_i} = \sum_{\alpha_i} \dot{S}^{\alpha_i} - \sum_{\alpha_i} \beta_{\alpha_i} \dot{Q}_{\alpha_i} = \]

\[ \frac{d}{dt}S_i - d_tS_i + \sum_{\alpha_i} \dot{S}^{\alpha_i} - \sum_{\alpha_i} \beta_{\alpha_i} \dot{Q}_{\alpha_i} \geq 0 \]

- We obtain the Q-analogous the Horowitz-Esposito result

\[ \dot{\sigma}_i = d_tS_i - \sum_{\alpha_i} \beta_{\alpha_i} \dot{Q}_{\alpha_i} - \dot{I}_i \geq 0 \]
Q-Information flow

- Is the information flow, $\dot{I}_i$, related to mutual information? Yes!

$$\sum_i \dot{I}_i = d_t I$$

where $I = \sum_i S_i - S$ is the (multipartite) Q-mutual information between the subsystems.

- Using secular approximation with $[\hat{H}_{\text{int}}, \hat{H}_i] = 0$, we recover the Horowitz-Esposito result.
\[ \hat{H}_S = \sum_{i \in \{1,2\}} \sum_{\sigma \in \{\uparrow, \downarrow\}} \epsilon_i c_i^\dagger c_i^\sigma \\
+ \sum_{i \in \{1,2\}} U_i n_i^\uparrow n_i^\downarrow \\
+ J(\hat{S}_1^x \hat{S}_2^x + \hat{S}_1^y \hat{S}_2^y) \]

- Operation based on coherent spin exchange + spin selective dissipative dynamics (next slide)
- Essentially non-bipartite dynamics: spin exchange simultaneously flip spins in both dots; \([\hat{H}_{\text{int}}, \hat{H}_i] \neq 0\)
- Could not be described by previously existing approaches

K. Ptaszyński, Phys. Rev. E 97, 012116 (2018)
Demon operation

1) $\mu_{L1}$ $\varepsilon_1$ $\mu_{R1}$

2) $\mu_{L1}$ $\varepsilon_2$ $\mu_{R2}$

3) $J$

Massimiliano Esposito

PRL 122, 150603 (2019)
Results

\[ T \dot{\sigma}_1 = -\dot{Q}_1 - T \dot{i}_1 \geq 0 \]
\[ T \dot{\sigma}_2 = -\dot{Q}_2 + T \dot{i}_1 \geq 0 \]

because \( \dot{i}_2 = -\dot{i}_1 \)

\( J \lesssim 100 \) is the “pure” Maxwell demon regime:

- 2 is cooled (\( \dot{Q}_2 > 0 \))...
- ...with a negligible energy flow \( \dot{E}_i \approx 0 \)...
- ...thanks to an information flow \( T \dot{i}_1 > \dot{Q}_2 \)

\[ T = 100, \ V_1 = 60, \ V_2 = -30 \]
Conclusions

- We derived local 2nd laws with information flows for parts of a Markovian Q-systems coupled to several reservoirs.
- This provides a consistent framework for thermodynamics of Q-information flows.
- Applicability of our approach was demonstrated on the example of an autonomous Q-Maxwell demon.

More details: [K. Ptaszyński and M. Esposito, *Thermodynamics of Quantum Information Flows*, PRL 122, 150603 (2019)]
Acknowledgments

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Thank you for your attention!
Stochastic and quantum thermodynamics of driven RLC networks

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Dynamics of RLC networks

Deterministic dynamics:
\[
\frac{dx}{dt} = A(t)H(t) x + B(t)s(t)
\]
\[
x = \begin{bmatrix} q \\ \phi \end{bmatrix} \quad s = \begin{bmatrix} v_E \\ jI \end{bmatrix} \quad H = \begin{bmatrix} C^{-1} & 0 \\ 0 & L^{-1} \end{bmatrix}
\]

Classical stochastic dynamics:
\[
\langle \Delta v(t) \rangle = 0 \\
\langle \Delta v(t) \Delta v(t') \rangle = 2Rk_bT \delta(t - t')
\]
\[
\frac{dx}{dt} = A(t)H(t) x + B(t)s(t) + \sum_r \sqrt{2k_bT} C_r \xi(t)
\]
\[
\langle \xi_i(t) \xi_j(t') \rangle = \delta_{i,j} \delta(t-t') \quad (A)_s = \frac{A + A^T}{2} = -\sum_r C_r C_r^T
\]
The mean values $\langle x \rangle$ and the covariance matrix $\sigma = \langle xx^T \rangle - \langle x \rangle \langle x \rangle^T$ evolve according to:

$$\frac{d\langle x \rangle}{dt} = \mathcal{A}\mathcal{H}(t)\langle x \rangle + \mathcal{B}(t)s(t)$$

$$\frac{d\sigma(t)}{dt} = \mathcal{A}\mathcal{H}(t)\sigma(t) + \sigma(t)\mathcal{H}(t)\mathcal{A}^T + \sum_r 2k_bT_r C_r C_r^T$$

We can identify work and heat currents by analyzing the change of the circuit energy:

$$E = \frac{1}{2} x^T \mathcal{H}(t)x \quad \implies \quad \langle E \rangle = \frac{1}{2} \text{Tr} \left( \mathcal{H}(t) \langle x \rangle \langle x \rangle^T \right) + \frac{1}{2} \text{Tr} \left[ \mathcal{H}\sigma \right]$$

$$\frac{d\langle E \rangle}{dt} = \frac{1}{2} \text{Tr} \left[ \mathcal{H}(t) \frac{d}{dt} \left( \langle x \rangle \langle x \rangle^T + \sigma \right) \right] + \frac{1}{2} \text{Tr} \left[ \frac{d}{dt} \mathcal{H}(t) \left( \langle x \rangle \langle x \rangle^T + \sigma \right) \right]$$

Employing the evolution equation for $\sigma$ and the FD relation, we obtain:

$$\langle \dot{Q} \rangle = \sum_r \left( \langle j_r \rangle \langle v_r \rangle + \text{Tr}\left[ (\mathcal{H}\sigma \mathcal{H} - k_bT_r \mathcal{H})C_r C_r^T \right] \right)$$

Local heat currents?
Local heat currents are actually given by:

\[
\dot{Q}_r = j_r (v_r + \Delta v_r)
\]

If there are no fundamental cut-sets simultaneously involving resistors inside and outside the normal tree, then:

\[
\langle \dot{Q}_r \rangle = \langle j_r \rangle \langle v_r \rangle + \text{Tr}[ (\mathcal{H} \sigma \mathcal{H} - k_b T \mathcal{I} \mathcal{H}) C_r C_r^T ]
\]

If not, \( \langle \dot{Q}_r \rangle \) is divergent.

Some examples:

- This is an artifact of the white noise idealization.
- It indicates that relevant degrees of freedom are not explicitly described.
- This can be solved by taking \( S(\omega) = (R k_b T / \pi) J(\omega) \), with \( J(\omega) \) vanishing for large frequencies or, equivalently, by ‘dressing’ a white noise resistor (analogous to Markovian embedding techniques).

In (a), fluctuations of arbitrarily high frequency in \( R_2 \) can be dissipated into \( R_1 \). In (b) and (c) these fluctuations are filtered out.
Generalization to quantum noise

Classical Johnson-Nyquist noise:
\[ \langle \Delta v(t) \Delta v(t') \rangle = 2 R k_b T \delta(t - t') \implies S(\omega) = \frac{R k_b T}{\pi} \]

Quantum Johnson-Nyquist noise:
\[ S(\omega) = \frac{R}{\pi} \hbar \omega \coth \left( \frac{\hbar \omega}{2 k_b T} \right) = \frac{R}{2\pi} \hbar \omega \left( N(\omega) + 1/2 \right) \]

Semiclassical treatment:
\[
\frac{dx}{dt} = A(t)H(t) x + B(t)s(t) + \sum_r \sqrt{2k_b T_r} C_r \xi(t) \quad S_{\xi_r}(\omega) = \frac{1}{2\pi} \frac{\hbar \omega}{k_b T_r} \left( N_r(\omega) + 1/2 \right)
\]

- We do not promote \( x \) to quantum operators
- We can directly apply this to overdamped circuits

In this way we obtain:
\[
\frac{d}{dt} \sigma(t) = A H(t) \sigma(t) + \sigma(t) H(t) A^T + \sum_r 2k_b T_r \left( \mathcal{I}_r(t) C_r C_r^T + C_r C_r^T \mathcal{I}_r(t)^T \right)
\]

where:
\[
\mathcal{I}_r(t) = \int_0^t d\tau \, G(t, t - \tau) \langle \xi_r(0) \xi_r(\tau) \rangle \quad \frac{d}{dt} G(t, t') - A(t) H(t) G(t, t') = \mathbb{1} \delta(t, t')
\]

This matches the results of a full quantum treatment for circuits that can be directly quantized (in the Markov approximation)
Generalization of Landauer-Büttiker formula for heat

\[ \langle \dot{Q}_r \rangle = \langle j_r \rangle \langle v_r \rangle + \sum_{r'} \int_{-\Lambda}^{+\Lambda} d\omega \, \hbar \omega \, f_{r,r'}(t, \omega) \left( N_{r'}(\omega) + 1/2 \right) \]

Non-diagonal elements:

\[ f_{r,r'}(t, \omega) = \frac{1}{\pi} \text{Tr} \left[ \mathcal{H}(t) \hat{G}(t, \omega) \mathcal{D}_{r'} \hat{G}(t, \omega) ^\dagger \mathcal{H}(t) \mathcal{D}_r \right] \quad (r \neq r') \]

Sum over first index:

\[ \bar{f}_{r'}(t, \omega) = \sum_r f_{r,r'}(t, \omega) = \frac{1}{2\pi} \text{Tr} \left[ \left( \hat{G} ^\dagger \frac{d\mathcal{H}}{dt} \hat{G} - \frac{d}{dt} \left( \hat{G} ^\dagger \mathcal{H} \hat{G} \right) \right) \mathcal{D}_{r'} \right] \]

For static circuits \((\bar{f}_{r'} = 0)\) we recover the usual Landauer-Büttiker formula

\[ \langle \dot{Q}_r \rangle = \langle j_r \rangle \langle v_r \rangle + \sum_{r'} \int_{-\Lambda}^{+\Lambda} d\omega \, \hbar \omega \, f_{r,r'}(\omega) \left( N_{r'}(\omega) - N_r(\omega) \right) \]

General result

We have derived a generalized Landauer-Büttiker formula which is valid for arbitrary circuits, with any number of resistors at arbitrary temperatures, and for arbitrary driving protocols.
A simple circuit-based machine: cooling a resistor

\[ C_1 = C + \Delta C \cos(\omega_d t) \]
\[ C_2 = C + \Delta C \cos(\omega_d t + \theta) \]

Numerical vs analytical results: (High \( T \), \( \tau_0 = \sqrt{LC} \), \( \tau_d = RC \), \( \tau_0 = \tau_d \))

(a) Asymptotic cycle of the heat currents for \( \Delta C/C = 1/2 \) and \( \omega_d/(2\pi) = 10^{-2}/\tau_d \) (dashed lines indicate cycle averages).
(b) Average heat currents versus driving frequency for \( \Delta C/C = 0.5 \).
(c) Average heat currents versus driving strength for \( \omega_d/(2\pi) = 10^{-2}/\tau_d \).

For all cases we took \( \theta = \pi/2 \) and \( T_1 = T_2 = T \).
Low temperature quantum behaviour:

\[
\langle \dot{Q}_c \rangle \sim T^2 \left( \frac{\hbar}{k_b \tau_0} \right)
\]

\[
\langle \dot{Q}_1 \rangle \sim T^2 \left( \frac{\hbar \tau_0}{\tau} \right)
\]

\[
\frac{\tau_d}{\tau_0} = 1, 2, 4
\]
Conclusions

Stochastic and Quantum Thermodynamics of Driven RLC Networks

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Key findings:

- We identified the proper definition of heat under the white noise idealization
- We showed how driven RLC circuits can be used to design thermal machines
- We showed that a semiclassical approach is equivalent to an exact quantum treatment

Ongoing work:

- An analogous (classical) treatment for non-linear devices is under way.