Static and dynamic characteristics of angular velocity and acceleration transducers based on optical tunneling effect

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Abstract. Theoretical and experimental analysis of quasi-linear conversion function of angular velocity and acceleration microoptoelectromechnical (MOEM) transducers based on optical tunneling effect (OTE) are conducted. Equivalent oscillating circuit is developed and dynamic characteristics of angular velocity and acceleration MOEM–transducers are investigated.

1. Introduction
Angular velocity and acceleration transducer, constructed with structure «optical components of total internal reflection (TIF) – medium – sensing element (SE)» are suggested to apply for improving effectiveness of navigation and mobile objects control system with satisfied modern requirements [1-2]. Thus optical power, reflected from the structure «TIF – medium – SE», is received by photodetector and supports information about measuring result [3].

While developing such MOEM–transducers it is needed the complete description of actual conversion function of the measuring value (angular velocity and acceleration) in the change of optical output power in static and dynamic modes. The validity of the derived characteristics of MOEM–transducers should be based on the comparison results of theoretical calculations and experimental studies.

2. Correction of static characteristic of angular velocity and acceleration MOEM–transducers based on OTE according to experimental studies results
The operating principle of angular velocity and acceleration MOEM–transducers is based on dependence of optical source reflective coefficient on the value of operating gap between SE and TIF base. Therefore, the reflected part of optical power, reached to photodetector, is an informative value [4].

Experimental studies by using semi-natural setup has been conducted to verify the conversion function of MOEM–transducers based on OTE and determine the correction type of theoretical calculations (figure 1).

Semi–natural setup is comprised of FOT–930 Multifunction Loss Tester 1, fiber optic cables 2, TIF – prism made up of quartz 3, piezoceramic SE in form of round plate 4, load transferring ball 5, aluminum beam 7 with hanging cable 6. FOT-903 is used as optical source in generation mode with wavelengths $\lambda_1=1.3\mu\text{m}$ and $\lambda_2=0.85\mu\text{m}$ while supplying constant optical power source $P_{OS}=2.045\mu\text{W}$. Beam 7, fixed at one end, can change load $F_{H}$ by changing mass $m_H$ 8 and its hanging position. This varying load $F_H$, acting on SE center, leads to change of operating gap $d$ between SE and TIF base.
Figure 1. Semi–natural setup for investigation of MOEM–transducers based on OTE: a – structure, b – external view

From optical source 1, signal enters to the structure «TIF–medium–SE» via optical fiber cables 2. While reducing operating gap $d$ from the influence of load $F_H$ most of the optical energy penetrates to SE and is absorbed, which leads to decrease the optical input signal of photodetector $P_{PD}$.

While neglecting the mass of beam 7 (light compared to the suspended mass 8) load $F_H$ acting on the SE of MOEM–transducer is calculated by formula:

$$F_H = m_H \cdot g \cdot \left( \frac{L_1 + L_2}{L_1} \right).$$ (1)

SE of those MOEM–transducers based on OTE is clamped on the perimeter of circular end, and the largest deflection is observed at its center. Under the influence of the central load $F_H$ magnitude of SE deflection $y_F(F_H)$ is function of parameters $y_F(F_H) = f(R_{SE}, h_{SE}, \nu, E_{SE})$ [5], where $R_{SE}$, $h_{SE}$ – radius and thickness of SE; $\nu$ – Poisson’s ratio; $E_{SE}$ – Young’s modulus of SE material.

Magnitude of operating gap $d(F_H)$ between SE and TIF base is defined by initial gap $d_0$ and deflection of SE $y_F(F_H)$:

$$d_F(F_H) = d_0 - y_F(F_H).$$ (2)

The output optical power with the conversion function of such MOEM–transducer based on OTE in account of optical loss is determined by the following formula [6]:

$$P_{PD}(F_H) = P_{OS} \cdot k_{opt} \cdot R[d_F(F_H), \theta, \lambda],$$ (3)

where $k_{opt}$ – optical loss; $\lambda$ – wavelength of optical source; $\theta$ – angle of incidence, $R[d_F(F_H), \theta, \lambda]$ – reflectivity of structure «TIF – medium – SE».

Experimental studies of conversion function is conducted with following parameters: $R_{SE} = 5\text{mm}$, $h_{SE} = 1\text{mm}$, $\nu = 0.17$, $E_{SE} = 72.5\text{GPa}$ (figure 2).
Figure 2. Dependence of conversion function of MOEM–transducer based on OTE on load $F_H$: 1, 3 – theoretical calculation results at $\lambda_1=1.3\,\mu\text{m}$, $\lambda_2=0.85\,\mu\text{m}$; 2, 4 – experimental results at $\lambda_1=1.3\,\mu\text{m}$, $\lambda_2=0.85\,\mu\text{m}$ accordingly.

From comparing of theoretical calculation and experimental results it should follow that the analytical formula to calculate the conversion function can support to be quasilinear and can be considered as acceptable for dependent description of output signal on the external influence to MOEM–transducer based on OTE, while ensuring not more than 15% of error.

To improve the accuracy of MOEM–transducer model is necessary to select the correction coefficient of theoretical calculations based on the experimental results, e.g., least squares method. The correction coefficient of conversion function must be satisfied in the condition with $n$ experiments:

$$
\sum_{i=1}^{n} \left[ P_{PD\,EXP}(F_{H,i}) - K_{CORR}(F_{H,i}) \cdot P_{PD\,THEO}(F_{H,i}) \right]^2 \rightarrow \text{min} , \quad (4)
$$

where $P_{PD\,EXP}(F_{H,i})$, $P_{PD\,THEO}(F_{H,i})$ – magnitude of optical power by means of experiments and theoretical calculation, corresponding value of load $F_{H,i}$; $K_{CORR}(F_{H,i})$ – correction coefficient of conversion function, $K_{CORR}(F_{H,i})=f(F_{H,i}, f_1, f_2, \ldots, f_{k+1})$, $f_1, f_2, \ldots, f_{k+1}$ – variables of polynomial coefficient in $k$ times.

Theoretical calculation of conversion function uses correction coefficient, described polynomial with $3^{rd}$ order: $K_{CORR}(F_{H}) = (1.0137+0.00424\cdot F_{H} - 0.00017\cdot F_{H}^2 + 0.00001\cdot F_{H}^3)\cdot 10^{-5}$ to provide the error not more than 2%.

3. Dynamic behaviors of angular velocity and acceleration MOEM–transducers based on OTE

Block diagram is composed to study the dynamic mode of angular velocity and acceleration MOEM–transducers based on OTE (figure 3.), which includes: a movable part - SE, which converts the measuring value $\Psi$ (angular velocity $\Omega_x$ or acceleration $a_z$) according to varying of operating gap $d_{\Psi}$; modulator (M), which converts optical power source $P_{OS}$ to output optical power of photodetector $P_{PD}$; converter “current-voltage” (I-U), which converts current $I_{PD}$ into voltage, supplied to low pass filter (LPF) and generate output voltage; $U$ - amplifier in feedback circuit (OC); PA - piezoactuator that converts the voltage $U_{OC}$ with movement of $d_U$.

Figure 3. Block diagram of angular velocity and acceleration MOEM–transducers based on OTE with feedback circuit
Using feedback circuit can reduce the transfer ratio of conversion, which allows to increase the maximum measuring value to desired measuring range by adjusting operating gap of transducers [7]. In contravention of this part sensitivity and linearity of the MOEM–transducers conversion function can be decreased.

According to the block diagram of MOEM–transducers a transfer function \( W_z(p) \) in form of operators is:

\[
W_z(p) = W_{se}(p) \cdot \frac{W_{m}(p) \cdot W_{eb}(p)}{1 + W_{m}(p) \cdot W_{eb}(p) \cdot W_{y}(p) \cdot W_{pa}(p)},
\]

where \( W_{se}(p) \) – transfer function of SE; \( W_{m}(p) = P_{OS} k_{m}(p) \) – transfer function of M; \( W_{eb}(p) = W_{pd}(p) \cdot W_{i}(p) \cdot W_{t}(p) \) – transfer function of electronic block (EB); \( W_{pd}(p) = k_{pd}(T_{pd} s + 1) \), \( W_{i}(p) = k_{i}/(T_{i} s + 1) \), \( W_{t}(p) = k_{t}/(T_{t} s + 1) \), \( W_{pa}(p) = k_{pa}/(T_{pa} s + 1) \) – transfer function of PD, I-U, LPF, Y, PA respectively; \( k_{m}(p) \) – transfer ratio of M [1/m]; \( T_{pd}, k_{pd} \) – time constant, transfer ratio of PD[A/W]; \( T_{i}, k_{i} \) – time constant, transfer ratio of I-U [Ohms]; \( k_{s} = 1/[1+(\omega_{0}/\omega_{s})^2] \), \( \omega_{s} \) – transfer ratio, frequency pole of LPF; \( T_{y}, k_{y} = R_{y}/R_{1} \) – time constant, gain of Y; \( T_{pa}, k_{pa} \) – time constant, transfer ratio of PA[m/V].

SE of MOEM–transducers based on OTE can be considered as a mechanical oscillating system which can be described with second order differential equation [8]:

\[
J \cdot \frac{d^2 y}{dt^2} + K_{GD} \cdot \frac{dy}{dt} + G_{siff} \cdot y = Q(t),
\]

where \( J, G_{siff} \) – inertial magnitude and stiffness of SE, \( K_{GD} \) – coefficient of gas dynamic damping due to viscous friction of the medium of structure \( \omega_{s} \) – oscillating velocity.

Note that an arbitrary external factor \( Q(t) \), acting on the SE of MOEM–transducers, can be expanded in Fourier series [9]:

\[
Q(t) = \frac{1}{2} a_{0} + \sum_{k=1}^{\infty} A_{k} \cos(k\omega t + \theta_{k})
\]

where \( A_{k}, \omega_{k}, \theta_{k} \) – amplitude, circular frequency, initial phase \( k \) times harmonics oscillation, \( a_{0} \) – coefficient of Fourier series.

Let us assume that the investigated system is satisfied by filter condition, which do not effect with higher harmonics significantly the basic, i.e. can be neglected. Thus, the equation (7) can be written as:

\[
Q(t) = \frac{1}{2} a_{0} + A_{1} \cos(\omega t + \theta_{1})
\]

By following equation (8), the external factor \( Q(t) \) according to the principle of superposition can be separately considered as a constant and harmonious action. Then, the external factor \( Q(t) \), acting on the acceleration MOEM–transducer can be assumed constant, and for angular velocity MOEM–transducer - harmonic.

Angular velocity and acceleration MOEM–transducers are acoustomechanical oscillating system with distributed parameters, consisting of SE and the air gap between SE and prism base. To analyze the dynamic characteristics of MOEM–transducers, which are defined by properties of acoustomechanical oscillatory system, conveniently and efficiently by using well known methods of electrical engineering and circuit theory. The criteria of equivalence are governed by the laws of energy conservation. In addition, each factor of the differential equation (6) is replaced by combination of equivalent elements of electrical oscillating RLC–circuit, in which the value of inertial \( J \) and, gas dynamic damping coefficient \( K_{GD} \) and stiffness of SE \( G_{siff} \) are corresponded to inductive \( L_{Es} \), capacitive \( C_{E} \) and active electrical resistance \( R_{E} \).
Equivalence, acting on external factor of SE corresponds to EMF $U_{d}(t)$, and oscillating velocity of system $y$ - current $I_{E}(t)$ (figure 4).

![Figure 4](image)

Figure 4. SE of angular velocity ($a$) and acceleration ($b$) MOEM–transducers based on OTE and equivalent schematic ($c$) of oscillating RLC-circuit

Under the influence of acceleration $a$, acceleration MOEM–transducer based on OTE (Figure 4a) is deflected along the $OY$ axis [2], which is corresponding to oscillating RLC-circuit (Figure 4c) with equivalent parameters: $U_{d}(t)=U_{d}(m_{a}+m_{add})a_{t}$; $L_{E}=b_{a}h_{a}/12$; $R_{E}=K_{G}G=-2\mu h_{a}^{3}h_{a}^{3}/[h_{a}^{3}((h_{a}^{2}+b_{a}^{2})]$; $1/C_{E}=G_{m}(128 E_{a}b_{a}h_{a}^{4})/I_{a}^{2}$; where $\mu = f(p_{a}, T_{e}, \rho_{c}, v_{c}, k_{b})$ – coefficient of dynamic viscosity, depend on density $\rho_{c}$, pressure $p_{a}$, average flow rate $v_{c}$, ambient temperature $T_{e}$ and Boltzmann constant $k_{b}$ (at temperature $T_{e}=20^\circ$C, here $\mu = 1.83\cdot 10^{-6}$kgm$^{-1}$s$^{-1}$ can be selected for air); $l_{a}$, $h_{a}$, $b_{a}$, $m_{a}$, $E_{a}$ – length, thickness, width, mass of SE and Young's modulus of four-beam fused silicon SE, $m_{add}$ – additional mass accordingly; and, electrical current $I_{E}(t)=dd_{a}(t)/dt$ – rate of central part deflection of SE. By the Kirchhoff’s voltage law:

$$L_{E}a \frac{dI_{E}(t)}{dt} + R_{E}a \cdot I_{E}(t) + \frac{1}{C_{E}a} \int I_{E}(t) \cdot dt = U_{a}.$$  

(9)

Differentiating equation (9) with respect to time and multiplying by $1/L_{E}$, obtain:

$$\frac{d^{2}I_{E}(t)}{dt^{2}} + 2\alpha \frac{dI_{E}(t)}{dt} + a_{0}^{2}I_{E}(t) = 0,$$  

(10)

where $\alpha = R_{E}/(2L_{E}a)$ – attenuation coefficient of oscillation in RLC-circuit; $a_{0} = 1/(L_{E}aC_{E}a)$ – natural oscillating frequency in RLC-circuit.

At the initial condition: current $I_{E}(t)|_{t=0}=U_{a}/R_{E}$ and the rate of change of current is different from zero $L_{E}a \frac{dI_{E}(t)}{dt}|_{t=0} = -U_{a}$, the total current $I_{E}(t)$ in an oscillatory RLC-circuit varies according to formula [10]:

$$I_{E}(t) = \frac{U_{a}}{R_{E}} - \frac{U_{a}}{L_{E}a} \cdot \frac{e^{-\alpha t}}{\sqrt{\alpha^{2} - a_{0}^{2}}} \cdot \frac{\sinh \left(\sqrt{\alpha^{2} - a_{0}^{2}} \cdot t\right)}{\sqrt{\alpha^{2} - a_{0}^{2}}},$$  

(11)

Consequently, the transfer function $W_{SE}(p)$ of SE of acceleration transducer based on OTE may be defined by formula:

$$W_{SE}(p) = L \left( \frac{\partial I_{E}(t)}{\partial a} \right) = L \left( \frac{\partial}{\partial a} \left( \frac{U_{a}}{R_{E}} - \frac{U_{a}}{L_{E}a} \cdot \frac{e^{-\alpha t}}{\sqrt{\alpha^{2} - a_{0}^{2}}} \cdot \frac{\sinh \left(\sqrt{\alpha^{2} - a_{0}^{2}} \cdot t\right)}{\sqrt{\alpha^{2} - a_{0}^{2}}} \right) \right),$$  

(12)

where $L$ – Laplace transform.

Under the influence of measuring angular velocity $\Omega$ SE of angular velocity MOEM–transducers based on OTE (Figure 4b) occurs small deviations $d_{a}(t)$ due to Coriolis effect [11]. This schematic can be seen as an oscillatory RLC-circuit (Figure 4c) with sinusoidal action at equivalent parameters: $Q(t)=U_{0}\sin(\omega t)=[I_{G}F_{0}(\Omega_{e})/(2\pi)] \sin(\omega t)$, where $F_{0}(\Omega_{e})=\pi d_{a}U_{ext} \Omega_{e}^{2}[\rho(\Omega_{e}h_{a}^{3}/2(4\pi m_{add})]$, amplitude of Coriolis force; $R_{E}=\mu I_{G}^{3}(h_{a}^{3})/18 b_{a}^{2} h_{a}^{2} (I_{a}^{3}/2 + 4 h_{a}^{3})$; $1/C_{E}=2 E_{a}h_{a}^{3} b_{a}^{3}/2 l_{a}^{3}$; $L_{E}=\mathcal{J}(p_{a} h_{a}^{3} b_{a}^{3} h_{a}^{3}/24) + (m_{add} l_{a}^{3})/4$; where $l_{a}$, $h_{a}$, $b_{a}$, $m_{a}$, $E_{a}$, $d_{a}$ – length, thickness, width, mass of SE; Young’s modulus of piezomodule with feedback effect of piezoceramic SE, $U_{ext}$ – excitation voltage with frequency $\omega$ accordingly; and electrical current $I_{d}(t)=d\phi_{a}(t)/dt$ – rate of change of angular
displacement of SE along $OY$ axis (for small value $\varphi_0(t) = \sin[\varphi_0(t)]$ is accepted, that $\varphi_0(t) = 2 \cdot d_0(t)/l_0$). Then second order differential equation for such oscillatory RLC-circuit is:

$$\frac{d^2 I_{E\Omega}(t)}{dt^2} + 2\alpha \frac{dI_{E\Omega}(t)}{dt} + \omega_0^2 I_{E\Omega}(t) = \frac{\alpha U_0}{L_{E\Omega}} \sin(\alpha t).$$  \hspace{1cm} (13)

Similarly, the general solution of equation (13) can be written as

$$I_{E\Omega}(t) = \frac{U_0}{L_{E\Omega} \cdot \sqrt{\left(\omega_0^2 - \omega^2\right) + 4\alpha^2 \omega^2}} \cdot \frac{\alpha U_0}{L_{E\Omega} \cdot \sqrt{\alpha^2 - \omega_0^2}} \cdot e^{-\alpha t} \cdot \sinh\left(\sqrt{\alpha^2 - \omega_0^2} \cdot t\right),$$  \hspace{1cm} (14)

where $\Delta \varphi = \arctan\left(\frac{(\omega^2 - \omega_0^2)/(2\alpha \omega)}{\omega_0}\right)$ – phase shift.

Transfer function $W_{SE\Omega}(p)$ of SE of angular velocity MOEM–transducers based on OTE is:

$$W_{SE\Omega}(p) = L \left\{ \frac{\varphi_0(t)}{2 \cdot I_{\Omega}} \frac{\partial}{\partial \Omega} \right\} = \frac{L}{2 \cdot I_{\Omega}} \left\{ \frac{\varphi(t)}{\partial \Omega} \right\} = \frac{L}{2 \cdot I_{\Omega}} \left\{ \frac{U_0 \cdot \alpha \sin(\alpha t - \Delta \Phi)}{\sqrt{\left(\omega_0^2 - \omega^2\right) + 4\alpha^2 \omega^2}} - \frac{U_0 \cdot e^{-\alpha t}}{\sqrt{\alpha^2 - \omega_0^2}} \cdot \sinh\left(\sqrt{\alpha^2 - \omega_0^2} \cdot t\right) \right\}. $$  \hspace{1cm} (15)

Transient process modelling of angular velocity and acceleration MOEM–transducers based on OTE is shown in figure 5 with the following parameters of functional elements: PD $T_{PD}=1 \mu s$, $k_{PD}=0.4$ А/$W$; $T_{I-U}=10 \mu s$, $k_{I-U}=4$ kOhm; $T_{Y}=10 \mu s$, $k_{Y}=100$; $k_{f}=1$; $T_f=10 \mu s$, $T_{PA}=3 \mu s$, $R_{OS}=0.5$ mW, $\lambda=1.3$ μm, $\theta=47^\circ$.

**Figure 5.** Transient processes of angular velocity (a) and acceleration (b) MOEM–transducers based on OTE 1- composed damping, 2- composed setting, 3 - resultant output voltage.

In dynamic mode the considered transducers are stable and possess only small oscillatory transients (line 1). When the permissible dynamic error $\Delta \delta$ is not exceed 5%, output characteristics (line 3) of angular velocity and acceleration MOEM–transducers are included in the steady state (lines 2) with a short duration transient $\tau_{pp} \sim 1$ ms.

4. Conclusion

In this study angular velocity and acceleration MOEM–transducers based on OTE, providing quasi-linearity conversion function is investigated. An experimental study of this transducer conversion function is described. Correction coefficient of conversion functions based on comparing of theoretical calculations and experimental studies is defined and it supports to improve the accuracy of the calculation of the described model.

The block diagram is determined and equivalent oscillatory RLC-circuit is composed for the study of dynamic characteristics angular velocity and acceleration MOEM–transducers based on OTE. It is shown that the transition process of described transducers have oscillated character with permissible magnitude of transition process duration $\tau_{pp}$ is around 1 ms, which allows to apply those transducers in mobile objects control systems.
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References
[1] Eric U, William B and Spillman Jr 2011 Fiber Optic Sensors: An Introduction for Engineers and Scientists (New Jersey: John Wiley & Sons, Inc) 512 p
[2] Busurin V I, Naing H L and Tuan P A 2015 Physics proccedia 73 pp 198-204
[3] Born M and Volf E 1983 Osnovi optiki (Moscow: Nauka) 721 p
[4] Busurin V I, Zhelgov M A, Korobkov V V and Tuan P A 2015 Mikro-optoelectronnicheeski dvuchoevoi datchik uglowi i lineino uckorenir (Patent No 2566384)
[5] Raspakov V Ya 2007 Mikromechanicheskie pribori (Moscow: Mashinostroenie) 400 p
[6] Busurin V I, Gorshkov B G and Korobkov V V 2012 Volokonno – opticheskie informasionno-izmeritelnite sistemi (Moscow: Moscow Aviation Institute Press) p 168
[7] Min H B 2000 Handbook of sensors and actuators: micro mehnical transducers pressure sensors, accelerometers and gyroscopes (Elsevier Press) 392 p
[8] Fraden J 2004 Handbook of modern sensors. Physics designs and application (NewYork: Springer-Verlag) 579 p
[9] Kim D P 2010 Teoriya avtomaticheskovo upravlenir:Linianie sistemi (Moscow: Fizmalit) 312 p
[10] Dorf R C and Bishop R H 2008 Modern Control Systems (Upper Saddle River: Prentice Hall) 1018 p
[11] Busurin V I and Tuan P A 2016 Avtometriya 52 (2) pp 124-30