Positive Streamer in the Surface Dielectric Barrier Discharge in Air: Numerical Modelling and Analytical Estimations

V Soloviev¹, V Krivtsov²

¹Moscow Institute of Physics and Technology, 141700 Dolgoprudniy, Moscow Region, Russia
²Dorodnicyn Computing Center RAS, 119333 Moscow, Russia

E-mail: vic__sol@mail.ru

Abstract. According to performed numerical simulation of the surface dielectric barrier discharge driven by positive polarity nanosecond voltage pulse the discharge in this case evolves as a streamer “flying” above the dielectric surface. The distance between the streamer and dielectric surface does not depend on dielectric barrier parameters and applied voltage value. The developed analytical model for surface streamer evolution confirms these results and explains the physics of this phenomenon. The electric field in front of a stationary streamer head is constant and defined only by ionization rate constant of the gas and its density.

1. Introduction
The study of the surface dielectric barrier discharge (SDBD) in atmospheric air is still of the considerable interest because of its promising applications for flow control aerodynamics [1-3] and for plasma assisted ignition and combustion [4,5]. The SDBD driven by nanosecond voltage pulse is a preferable object of investigation for discharge physics understanding, because this discharge develops as a single microdischarge on the leading front of the voltage pulse, in contrast to a set of microdischarges relevant for a case of AC applied voltage. The advantage of studying the one microdischarge generation is in well known initial conditions and, consequently, in the possibility of more reliable interpretation of simulated results and experimental data.

The effectiveness of the nanosecond SDBD application for fuel ignition is strongly affected by the mode of the discharge: whether it is quasi-uniform or filamentary (constricted) mode of discharge development. In quasi-uniform mode the ignition occurs in a spot near the electrode edge [6], whereas in filamentary mode it is spread to longer distance along the discharge filament [7]. The physics of the SDBD transition from one mode to another is not understood yet. In order to get it, more detailed data regarding the discharge spatial structure and temporary evolution is needed.

According to performed numerical simulations [8-11] a single microdischarge in the SDBD evolves as a sliding transient glow discharge for negative exposed electrode polarity and as a streamer for positive one. Whereas for negative polarity the discharge is adjoined to the dielectric surface, for positive polarity it propagates at some distance above the surface. The gap between the streamer and the dielectric surface is characterized by positive ion charge surplus and extra-high electric field value at the level of 300 kV/cm for atmospheric air [9,12]. The existence of high electric field layers in the SDBD was implicitly confirmed in experiment [12].
This work is addressed to enhance our understanding of one aspect of the SDBD physics – the physics of positive streamer propagation. For this purpose the simplified analytical model for positive streamer maintenance condition is developed and the results of this model are compared with the results of numerical simulations. Both in numerical and analytical study we consider the streamer as a 2D sheet of plasma relevant for quasi-uniform SDBD development.

2. Numerical simulation of positive streamer in air

The 2D numerical simulation for SDBD streamer evolution in atmospheric air has been done using the numerical model developed in [8]. The results in the form of the extra positive charge density contours above the dielectric surface in units of $10^{14}$ cm$^{-3}$ are shown in Fig.1 and demonstrate the streamer “flying” above the dielectric surface. Calculations were done for stepwise voltage amplitude $V = 5$ kV, dielectric thickness $d = 0.1$ mm, and dielectric relative permittivity $\varepsilon = 2.7$.

3. Analytical model

Numerical simulation has shown that all the quantities inside a streamer body, - the $E$-field and the charged particle densities, are almost constant in the streamer cross section and strongly vary only in the $\delta$ - layer near the streamer boundary. Accordingly, for analytical estimation of the quasi-steady streamer maintenance condition we can use 1D approximation for charged particles transport equations and for Poisson equation for electric field. The spatial coordinate $s$ belongs to $E$-field line at the center of the streamer as shown in schematic picture of the streamer head in Fig.2a, where $h$ is a streamer distance from the dielectric surface.

Fig.1 The scheme (a) of electrode layout and (b) the contours of positive charge surplus in $10^{14}$ cm$^{-3}$.

Fig.2 Schematic picture (a) for streamer head and (b) for qualitative solution inside the $\delta$ - layer.
The simplified system of equations reads

\[ \frac{\partial n_i}{\partial t} - \frac{\partial n_i v_e}{\partial s} = v_e n_e, \quad \frac{\partial n_i}{\partial t} = v_e n_e, \quad \frac{dE}{ds} = 4\pi (n_i - n_e). \]  

(1)

Here \( n_i = \) electron density, \( n_e = \) positive ion density, \( \Delta n = n_i - n_e \) is a positive ion surplus, \( v_e = K_e E \) is electron drift velocity, \( K_e = \) electron mobility, \( E = \) gas ionization frequency. We neglect the negative ion formation and the drift of ions, which are not important in the streamer head formation process [13]. The Eqs. (1) are the same as in simplified model of volumetric streamer propagation. After the transition to new coordinate \( \eta = s - v_i t \) moving with streamer head with velocity \( v_i \) these equations have the same solution obtained in [14]. According to this solution the electron-ion multiplication mainly occurs inside the \( \delta = s - v_i t \) layer. The electron-ion density reaching the value \( n_{ih} \) behind the streamer head (Fig.2b) and the \( \delta \) value read [14]

\[ n_{ih} = \frac{1}{4\pi e K_e v_{ih} (1 + v_i / v_{ih})}, \]  

(2)

\[ \delta \approx \frac{2}{\alpha_t (E_h)}. \]  

(3)

The electric field inside the \( \delta \) - layer falls down from \( E_h \) value in front of the streamer head to almost zero in the streamer body. The Eqs. (2) and (3) show that electron-ion density \( n_{ih} \) and \( \delta \) thickness are the functions of electric field \( E_h \) in front of the streamer head. Here \( \alpha_t (E_h) = v_i (E_h) / K_e E_h \) is Townsend coefficient, and \( v_{ih} = K_e E_h \) is electron drift velocity in electric field \( E_h \).

The commonly used expression for \( \alpha_t (E) \) is

\[ \alpha_t (E) = \frac{1}{\lambda_{im}} \exp(-E / E_i), \quad \lambda_{im} = \frac{1}{Ap}, \quad E_i = Bp, \]  

(4)

where \( A \) and \( B \) are the constants defined by gas properties, the book [14] recommends \( A = 15 \text{cm}^{-1} \text{Torr}^{-1} \), \( B = 365 \text{V/cm/Torr} \) for air, \( p \) is a gas pressure. The \( \lambda_{im} \) means the asymptotic for \( E \to \infty \) of ionization length and \( E_i \) is the characteristic \( E \)-field value describing the ionization rate saturation for growing \( E \). The \( \lambda_{im} = 0.9 \cdot 10^{-4} \text{cm} \) and the \( E_i = 277 \text{kV/cm} \) in atmospheric air.

Following the approach realized in [15] for volumetric streamer between pin and plain electrodes we formulate the electron balance equation near the streamer head. The region between the dielectric surface and the 2D-streamer bottom boundary near the streamer head may be considered as a flat region with all parameters variation only in the normal to dielectric surface direction \( s \). In this approximation the number of electrons generated due to gas ionization inside a gap between dielectric surface and streamer per unit streamer length is

\[ N_e (\delta) = \int_0^\delta dN_{ph} \left[ \exp \left( \int_0^\delta \alpha_t (s) ds \right) + 1 \right], \]  

(5)

where

\[ dN_{ph} = \alpha_{ph} ds \left( \frac{1}{2} \frac{\tau_e}{\tau_p} N_e \alpha_{ex} \exp(-\alpha_{ph} (s - \delta)) \right) \]  

(6)

is the number of photoelectrons born on \( ds \) length due to UV radiation from the discharge region. The photo-ionization occurs because \( O_2 \) molecules are ionized by radiation of \( N_2(b^3 \Pi_{u}, b^3 \Sigma_{u}^+, c^1 \Sigma_{u}^+) \) excited molecules in the band 98.0-102.5 nm [16]. Here \( \alpha_{ph} = \) photo-absorption coefficient for this radiation by \( O_2 \) molecules, \( \alpha_{ex} = 0.07 \) is a ratio of excitation rate of aforementioned excited \( N_2 \)
molecules by electron impact to their ionization rate, $\tau^*$ and $\tau_R$ are the total and radiative lifetimes of excited N$_2$ molecules [8,16], $N_i$ is the total number of elections inside a discharge region initialized by $N_{ph}$ photoelectrons and it is defined by equation

$$N_e = N_i(\delta)\exp\left(\int_0^\delta \alpha_i(s)ds\right). \quad (7)$$

The steady discharge development implies that the number of electrons born in multiplication of $N_{ph}$ photoelectrons is greater than $N_e$ number necessary to create these $N_{ph}$ photoelectrons. This condition leads to inequality

$$\chi_{ph}N_e \int_0^\delta \exp\left(-\alpha_{ph}(s-\delta)\right) \left[\exp\left(\int_0^\delta \alpha_i(\xi)d\xi\right) + 1\right] ds \cdot \exp\left(\int_0^\delta \alpha_i(\xi)d\xi\right) \geq N_e,$$  \quad (8)

where $\chi_{ph} = 0.5r^*\alpha_e/\alpha_{ph} \approx 0.1$ cm$^{-1}$ and $\alpha_{ph} \approx 100$ cm$^{-1}$ in atmospheric air. After integration the expression (8) transforms to inequality

$$F(E_h, h, \delta) = \frac{\chi_{ph}}{\alpha_i(E_h)} \exp(\alpha_i(E_h)h) \cdot \exp\left(\int_0^\delta \alpha_i(s)ds\right) \geq 1.$$  \quad (9)

For three unknowns $E_h, h, \delta$ we have only two conditions: the Eq.(3) and the inequality (9).

The third required condition is the $\delta$ - layer formation condition. According to Poisson equation $E_h \approx 4\pi en\delta$. If $F(E_h, h, \delta) \geq 1$, then $n_i$ grows up inside a $\delta$ - layer. This process occurs at a constant $E_h$ and results in $\delta$ decreasing. It continues until the potential difference $\Delta V_e$ on the $\delta$ - layer reaches its minimum value. Assuming linear $E(s)$ dependence inside a $\delta$ - layer and accounting for Eq.(3) and (4) we get

$$\Delta V_e \approx \frac{E_h\delta}{2} = \frac{E_h}{\alpha_i(E_h)} = E_h\lambda_m \exp\left(\frac{E_h}{E_i}\right). \quad (10)$$

From the solution of equation $\partial\Delta V_e / \partial E_h = 0$ for $\Delta V_e$ extremum condition we get

$$E_h \approx E_i.$$  \quad (11)

This value for electric field in front of the streamer head has been confirmed by numerical simulations both for surface [9,12] and volumetric streamers [17,18].

Combining Eqs.(3), (11) and inequality (9) we get the inequality $F(E_h, h, \delta)|_{E_i = E_h - \delta/2 \alpha_i(E_h)} \geq 1$ for $h$ value determination, which gives

$$h \geq -e\lambda_m \left(\ln(e\lambda_m\chi_{ph}) + 1\right). \quad (12)$$

For atmospheric air $h \geq 30\lambda_m \approx 0.03$ mm and coincides with numerical simulations [8,9,12].

4. Conclusions

The proposed simplified analytical model for quasi-steady streamer maintenance condition permitted to find out the distance at which the streamer moves above the dielectric surface in the surface dielectric barrier discharge (SDBD) driven by positive polarity voltage pulse. This distance is managed only by the gas ionization rate constant, namely, it is primarily defined by asymptotic for infinite $E$-field ionization length of the gas. For atmospheric air the streamer remoteness from the dielectric surface is around 0.03 mm.

The electric field in front of the quasi-steady streamer head is managed by another parameter of the gas ionization rate constant (Townsend coefficient); it is equal to characteristic electric field value, at
which the Townsend coefficient starts to saturate in the growing up electric field. It should be emphasized that this result is valid not only for positive surface streamer in SDBD, but for volumetric streamer between pin and plain electrodes as well. The value of this electric field in atmospheric air is near 300 kV/cm.

Both analytical results for streamer remoteness from the dielectric surface and for electric field value in the streamer head are confirmed by numerical simulation.

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