ON THE GEOMETRIC MUMFORD-TATE CONJECTURE FOR SUBVARIETIES OF SHIMURA VARIETIES

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ABSTRACT. We study the image of ℓ-adic representations attached to subvarieties of Shimura varieties $Sh_K(G, X)$ that are not contained in a smaller Shimura subvariety and are without isotrivial components. We show that, for ℓ large enough (depending on the Shimura datum $(G, X)$ and the subvariety), such image contains the $\mathbb{Z}_\ell$-points coming from the simply connected cover of the derived subgroup of $G$. This can be regarded as a geometric version of the integral ℓ-adic Mumford-Tate conjecture.

INTRODUCTION

Following Deligne’s formulation, a Shimura variety is a complex manifold defined by a Shimura datum $(G, X)$, where $G$ is reductive group defined over $\mathbb{Q}$ and $X$ is a $G(\mathbb{R})$-conjugacy class of homomorphisms $h : S \to G(\mathbb{R})$, $S$ denoting the Weil restriction of the complex multiplicative group to the real numbers, satisfying certain axioms. The axioms ensure that this is the base space for a variation of Hodge structures and its connected components are Hermitian symmetric domains. The prototype for all Shimura varieties is the Siegel moduli space of principally polarised (complex) abelian varieties of dimension $g$ with a level structure, in which case $(G, X) = (\text{GSp}_{2g}, \mathcal{H}_g^+)$.

Let $(G, X)$ be a Shimura datum and $K \subset G(\mathbb{A}_f)$ be a neat compact open subgroup, where $\mathbb{A}_f$ denotes the topological ring of finite adeles. From now on we fix, up to ignoring finitely many primes, a faithful linear representation $G \hookrightarrow \text{GL}_{n, \mathbb{Z}}$. The aim of this note is to study the ℓ-adic monodromy of smooth irreducible subvarieties $C \subset Sh_K(G, X)$. To do so we may assume that $C$ is contained in a connected component of a $Sh_K(G, X)$ and that $G$ is of adjoint type (since $Sh_K(G, X)$ is a finite covering of a Shimura variety associated to the quotient of $G$ by its centre). Therefore we write $C \subset S := \Gamma \backslash X^+$ where $X^+$ is a connected component of $X$ and $\Gamma$ is $G(\mathbb{Z}) \cap K$. In this case, as is recalled in section 1.1, the inclusion $C \hookrightarrow S = \Gamma \backslash X^+$ naturally induces a map

$$\pi_1^\text{top}(C(\mathbb{C})) \to \pi_1^\text{top}(S(\mathbb{C})) \to \Gamma;$$

where $\pi_1^\text{top}$ denotes the topological fundamental group (not to be confused with $\pi_1^\text{et}$ that will denote the étale fundamental group, which can be identified with the profinite completion of the topological one). In both cases in the notation of fundamental groups we omit base points and implicitly work ‘up to conjugation’.

Denote by $\Gamma^\text{ad}$ the adelic closure of $\Gamma$ in $G(\mathbb{A}_f)$, and notice that we have the following commutative diagram, where the vertical arrows denote the profinite completion:

$$\begin{array}{ccc}
\pi_1^\text{top}(C(\mathbb{C})) & \longrightarrow & \pi_1^\text{top}(S(\mathbb{C})) \longrightarrow \Gamma \\
\downarrow & & \downarrow \\
\pi_1^\text{et}(C_{\mathbb{C}}) & \longrightarrow & \pi_1^\text{et}(S_{\mathbb{C}}) \longrightarrow \Gamma^\text{ad}
\end{array}$$

Let $\pi_\ell : G(\mathbb{A}_f) \to G(\mathbb{Q}_\ell)$ the projection to the ℓ-th component. We call the image of the map

$$\pi_1^\text{et}(C_{\mathbb{C}}) \to \pi_1^\text{et}(S_{\mathbb{C}}) \to \Gamma^\text{ad} \overset{\pi_\ell}{\longrightarrow} G(\mathbb{Q}_\ell)$$

the ℓ-adic monodromy of $C$ and we denote it by $\Pi_\ell^C$.

We want to show that ℓ-monodromy of subvarieties that are not contained in any smaller Shimura subvariety (usually called Hodge generic) and are without isotrivial components is as large as possible. We now introduce the notion of being without isotrivial components and then state the main theorem.

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Definition 0.1. Let $G$ be of adjoint type, and $C$ an irreducible smooth complex subvariety of a Shimura variety $\text{Sh}_K(G, X)$. We say that $C$ is without isotrivial components if each component of the image of $C$ in $\text{Sh}_K(G, X) \cong \text{Sh}_{K_1}(G_1, X_1) \times \cdots \times \text{Sh}_{K_n}(G_n, X_n)$ has positive dimension, for every decomposition $(G, X) \cong (G_1, X_1) \times \cdots \times (G_n, X_n)$ and $K \cong K_1 \times \cdots \times K_n$ for every compact open subgroup $K \subset G$.

Theorem 0.2. Let $C$ be a Hodge generic, without isotrivial components subvariety of a Shimura variety $\text{Sh}_K(G, X)$. For all $\ell$ big enough (depending only on $(G, X)$ and $C$), we have $G(\mathbb{Z}_\ell)^+ \subset \Pi_C^\ell$. Where $G(\mathbb{Z}_\ell)^+$ is defined as $\text{Im}(\tilde{G}(\mathbb{Z}_\ell) \to G_{\text{der}}(\mathbb{Z}_\ell))$, i.e. the $\mathbb{Z}_\ell$-points coming from the simply connected cover of the derived subgroup of $G$.

In particular, when $G$ is semisimple simply connected, the theorem shows that $\Pi_C^\ell = G(\mathbb{Z}_\ell)$ for all but finitely many $\ell$. For arbitrary $G$ and Hodge generic subvarieties $C \subset \text{Sh}_K(G, X)$ one can not expect that such equality. Indeed this may only happen when the Hodge structure associated to $\tilde{C}$, $h_C : S \to G_\mathbb{R}$ is Hodge maximal, i.e. there is no non-trivial isogeny of connected $\mathbb{Q}$-groups $G' \to G$ such that $h_C$ lifts to a homomorphism $h_C : S \to G' \to G$ (see [Ser94, Definition 11.1] and [CM15, Definition 2.1]). Of course if $G$ is simply connected, there are no such isogenies. Our result says that, for every group $G$, the image will always contain the $\mathbb{Z}_\ell$-points of of the simply connected cover of $G$.

In the case of the moduli space of abelian varieties the main theorem can be read as follows. The notion of being without isotrivial components we proposed in the [Ser94, Definition 11.1] and [CM15, Definition 2.1]). Of course if $G$ is simply connected, there are no such isogenies. Our result says that, for every group $G$, the image will always contain the $\mathbb{Z}_\ell$-points of of the simply connected cover of $G$.

Corollary 0.3. Let $\mathbb{C}/\mathbb{Q}$ be an a smooth irreducible variety and $\eta$ its generic point. Let $A \to C$ be a $g$-dimensional abelian scheme without isotrivial components, then image of the map

$$\pi^+_\ell(C) \to \text{GL}(T_\ell(A_\eta))$$

contains $M(\mathbb{Z}_\ell)^+$ for $\ell$ large enough, where $M$ denotes the Mumford-Tate group of $A$ and $T_\ell(A_\eta)$ the Tate module of $A_\eta$.

To obtain the corollary it is enough to notice that, after fixing a principal polarization and a $n$-level structure, the family $A \to C$ gives rise to a subvariety without isotrivial components of $\text{Sh}_K(\text{GSp}_{2g}, \mathcal{H}_g^\pm)$ and then apply the theorem.

Notations

Given an algebraic group $G, G_{\text{der}} \subset G$ will denote its derived subgroup, $Z(G)$ its center, $\text{ad} : G \to G^{ab} := G/G_{\text{der}}$ its abelianization and $G \to G^{\text{ad}} := G/Z(G)$ its adjoint quotient. If $G$ is semisimple $\lambda : G \to G$ will be the simply connected covering of $G$. Notice that $G(\mathbb{Z}_\ell)^+$ is an open subgroup of $G(\mathbb{Z}_\ell)$, moreover the index of $G(\mathbb{Z}_\ell)^+$ in $G(\mathbb{Z}_\ell)$ is bounded independently of $\ell$. This follows from a standard Galois cohomology argument: an isogeny $\lambda : G \to G$ induces an open map on the $\mathbb{Z}_\ell$-points and the index is uniformly bounded.

For other notations about Shimura varieties we follow Deligne’s works [Del79], [Del71].

1. Monodromy of Subvarieties (After Deligne, André and Moonen)

In this section we compute the geometric monodromy of subvarieties without isotrivial components. We then recall how to produce, starting form a subvariety $C \subset \text{Sh}_K(G, X)$, a Shimura subvariety of $\text{Sh}_K(G, X)$ containing $C$ and such that $C$ becomes Hodge generic and without isotrivial components in such Shimura variety.

Let $S$ be a connected, non-singular complex algebraic variety, $\mathcal{V} = (\mathcal{V}, \mathcal{F}, \Omega)$ be a polarised variation of $\mathbb{Q}$-Hodge structure. Let $\lambda : \tilde{S} \to S$ be the universal covering of $S$ and fix a trivialisation $\lambda^* \mathcal{V} \cong \tilde{S}$. For $s \in S$, let $\text{MT}_s \subset \text{GL}(\mathcal{V}_s)$ be the Mumford-Tate group at $s$, i.e. the smallest $\mathbb{Q}$-algebraic group $M$ such that the map

$$h_s : S \to \text{GL}(\mathcal{V}_s)$$

describing the Hodge-structure on $\mathcal{V}_s$, factors trough $M_{\mathbb{R}}$. Choosing a point $\tilde{s} \in \lambda^{-1}(s) \subset \tilde{S}$ we obtain an injective homomorphism $\text{MT}_s \subset \text{GL}(\mathcal{V})$.

There exists a countable union $\Sigma \subset S$ of proper analytic subspaces of $S$ such that
• for $s \in S - \Sigma$, MT$_s \subset \text{GL}(V)$ does not depend on $s$, nor on the choice of $\tilde{s}$. We call this group the generic Mumford-Tate group of $V$ and we simply write it as $G$;
• for all $s$ and $\tilde{s}$ as above, with $s \in \Sigma$, MT$_s$ is a proper subgroup of $G$, the generic Mumford-Tate group of $V$.

From now on assume that $V$ admits a $\mathbb{Z}$-structure and choose $s \in S$ and $\tilde{s}$ as above. From the local system underlying $V$ we obtain a representation $\rho : \pi_1^{\text{top}}(S(\mathbb{C})) \to \text{GL}(V)$; the connected component of the identity of smallest algebraic group defined over $\mathbb{Q}$ containing the image of $\rho$ is denoted with $M_s$ and called the (connected) monodromy group. Since we fixed a trivialisation for $\lambda^*V$, we have that $M_s \subset \text{GL}(V)$ and it does not depend on the choice of $s$ and $\tilde{s}$. When a point $t \in S$ is such that $\text{MT}_t$ is abelian (hence a torus), we say that $t$ is a special point.

**Theorem 1.1** (Deligne, André, Moonen). Let $s \in S - \Sigma$. We have:

Normality. $M_s$ is a normal subgroup of the derived group $G^{\text{der}}$.

Maximality. Suppose $S$ contains a special point. Then $M_s = G^{\text{der}}$.

**Proof.** The first statement is [Del72, Thm 7.5], the second is [And92, Prop 2]. See also [Moo98, Section 1]. □

We will use Theorem 1.1 to compute the monodromy of subvarieties of Shimura varieties as follows. Consider a Shimura variety defined by a Shimura datum $(G, X)$ where $G$ is a semi-simple algebraic group of adjoint type and assume that $G$ is the generic Mumford-Tate group on $X$. Let $K$ be a neat compact open subgroup of $G(\mathbb{A}_f)$. Let $X^+$ be a connected component of $X$ and $\Sigma = \Gamma \setminus X^+$, with $\Gamma = G(\mathbb{Q}) \cap K$, i.e. the image of $X^+ \times \{1\}$ in $\text{Sh}_K(G, X)^{\text{c}}$. We can choose a faithful representation of $G$ on a $\mathbb{Q}$-vector space $V$. Such representation gives a polarised variation of $\mathbb{Q}$-Hodge structure, denoted by $V$, on the constant sheaf $V_{\Sigma,c}$ on $\Sigma$. Moreover, since $\Gamma$ acts freely on $X^+ \setminus \Sigma$, $V$ descends to a VHS on $\Sigma$. To obtain a $\mathbb{Z}$-structure, we may choose $V_{\mathbb{Z}}$, a $K$ invariant lattice in $V \otimes \mathbb{A}_f$, and define $V_{\mathbb{Z}}$ by $V \cap V_{\mathbb{Z}}$.

**Corollary 1.2.** Let $C$ be a non-singular irreducible Hodge generic subvariety of $S$ containing a special point $t$. Let $c \in C$ be a Hodge-generic point on $C$, since $\pi_1(C, c)$ acts on $V_{\mathbb{Z},c}$ it acts on $V_{\mathbb{Z}}$. Let $\Pi$ be its image in $\text{GL}(V_{\mathbb{Z}})$, it is a finitely generated group. We have $\Pi \subset \Gamma$ and both of them are Zariski dense in $G$.

**Proof.** The action of $\pi_1(C, c)$ on $V$ is obtained by restriction from the VHS $\mathbb{V}$ on $S$, in particular the image of $\Pi$ is contained in the one of $\pi_1(S, s)$, i.e. the image of $\Gamma$ along the faithful representation of $G$ we fixed at the beginning. The connected component of the Zariski closure of $\Pi$ in $\text{GL}(V)$ is $G$, in virtue of the maximality established in Theorem 1.1, since $G$ is the generic Mumford-Tate of $C$ and $C$ contains a point $t$ such that $\Pi$ is commutative. □

### 1.1. Monodromy of subvarieties of Shimura varieties

Following [Moo98, Sections 2.9, 3.6 and 3.7] we discuss the monodromy of an irreducible algebraic subvariety $C$ of a Shimura variety $\text{Sh}_K(G, X)$ replacing $C$ by its smooth locus we may assume that $C$ is also smooth. Since the intersection of two Shimura subvarieties of $\text{Sh}_K(G, X)$ is again a Shimura subvariety, there exists a unique smallest sub-Shimura variety of $\text{Sh}_K(G, X)$ containing $C$, say $S_C$. By definition there exists a $\mathbb{Q}$-group $M_C$ such that $S_C$ is an irreducible component of the image of $X_H \times \eta K$ in $\text{Sh}_K(G, X)$, for some $\eta \in G(\mathbb{A}_f)$. Moreover, fixing a representation of $G$ into $\text{GL}(V)$, we may take for $M$ the generic Mumford-Tate group on $C$. Notice that, by construction, $C$ is Hodge generic in $S_C$. Indeed $C \subset \Sigma$ would contradict the fact that $S$ is the smallest sub-Shimura variety containing $C$.

Let $M^{\text{ad}} = M_1 \times M_2$ be a decomposition of the adjoint group of $M$, consider the corresponding decomposition $X^{\text{ad}}_M = X_{M_1} \times X_{M_2}$ and the induced decomposition of Shimura data

$$(M^{\text{ad}}, X^{\text{ad}}_M) = (M_1, X_{M_1}) \times (M_2, X_{M_2}).$$

Given a compact open $K \subset G(\mathbb{A}_f)$ and $K_i$ compact opens in $M_i(\mathbb{A}_f)$, for $i = 1, 2$, such that $\text{ad}(K) \subset K_1 \times K_2$ we obtain a morphism

$$\text{ad}_{K_1 \times K_2} : \text{Sh}_K(M, X_M) \to \text{Sh}_{K_1 \times K_2}(M^{\text{ad}}, X^{\text{ad}}_M) = \text{Sh}_{K_1}(M_1, X_{M_1}) \times \text{Sh}_{K_2}(M_2, X_{M_2})$$

that we may assume to be finite étale on the irreducible components.

Let $\Gamma \times \mathbb{X}$ be the irreducible component of $\text{Sh}_K(G, X)$ containing $Z$. Fixing a representation $G \subset \text{GL}(V)$ and a $\Gamma$-stable lattice in $V$; we obtain a $\mathbb{Z}$-VHS $\mathbb{V}$, as explained during the proof of Theorem 5.1 in [EY03]. Therefore we may apply Theorem 1.1. Let $n : C^n \to C$ be a normalisation of $C$ and consider $H$ the connected monodromy group associated to the VHS $n^*(\mathbb{V}|_C)$. The monodromy theorem implies that $H$ is a normal subgroup of $M^{\text{der}}$ and, since
Proposition 1.3. Let $C$ be an irreducible component in the preimage of $Z$ in $X_M$. The image of $C$ under the projection $X_M \to X_{H_2}$ is a single point, say $y_2 \in X_{H_2}$. We have that $C$ is contained in the image of $(Y_1 \times \{y_2\}) \times \eta K$ in $\text{Sh}_K(G,X)$ for some connected component $Y_1 \subset X_H$ and a class $\eta K \in G(\mathbb{A}_f)/K$.

To summarise our discussion, from Corollary 1.2 and Proposition 1.3 applied to the smooth locus of $C_z$, we obtain the following.

Theorem 1.4. Let $C \subset \text{Sh}_K(G,X)$ be smooth irreducible. There exists a sub-Shimura datum $(M,X_M) \hookrightarrow (G,X)$ such that

- $C \subset \text{Sh}_K(M,X_M)$;
- $M$ is the Mumford-Tate group of $C$;
- $C$, seen in $\text{Sh}_K(M,X_M)$, is without isotrivial components.

Let $X^+_M$ be the connected component of $X_M$ such that $C \subset \Gamma_M \backslash X^+$, where $\Gamma_M$ is defined as $K_M \cap M(\mathbb{Q})^+$. We have that

- $\pi_1^{\text{top}}(C(\mathbb{C})))$ and $\Gamma$ have the same Zariski closure in the $\mathbb{Z}$-group scheme $GL(V_{\mathbb{Z}})$ (which is $M$ indeed).

Theorem 1.4 and Nori’s theory ([Nor87]) are the main tools we use in the next section to prove the result.

2. Proof of Theorem 0.2

Let $C \subset S = \Gamma \backslash X$ be a subvariety as in the statement of Theorem 0.2 and let $\Pi \subset \Gamma$ be the image of $\pi_1^{\text{top}}(C(\mathbb{C}))) \to \pi_1^{\text{top}}(S(\mathbb{C}))) \to \Gamma$.

Since $C$ is Hodge generic and without isotrivial components, Corollary 1.2 shows that $\Pi$ is Zariski dense in $G_Z$. We recall a theorem of Nori ([Nor87, Theorem 5.1]) about Zariski dense subgroups of semisimple groups.

Theorem 2.1 (Nori). Let $H \subset GL_n/\mathbb{Q}$ be a semisimple group and $\Pi \leq H(\mathbb{Q})$ be a discrete finitely generated Zariski-dense subgroup. Then for all sufficiently large prime numbers $\ell$ (depending only on $H$ and $\Pi$), the reduction modulo-$\ell$ of $\Pi$ contains $H(\mathbb{F}_\ell)^+$. Notice that reducing modulo-$\ell$ makes sense: since $\Pi$ is finitely generated there are only finitely many primes $\ell_1, \ldots, \ell_k$ such that $\Pi$ belongs to $H(\mathbb{Z}[\ell_i^{-1}]) := H \cap GL_n(\mathbb{Z}[\ell_1^{-1}, \ldots, \ell_k^{-1}])$ and the reduction mod $\ell$ is well-defined for $\ell$ large enough.

Since $G$ is of adjoint type we may apply Nori’s theorem to get, for $\ell$ large enough, a chain of inclusions

$$G(\mathbb{F}_\ell)^+ \subset \Pi_\ell \subset G(\mathbb{F}_\ell),$$

where $(-)_\ell$ denotes reduction modulo-$\ell$ and $G(\mathbb{F}_\ell)^+$ denotes the image of $\tilde{G}(\mathbb{F}_\ell) \to G(\mathbb{F}_\ell)$ (here we use that $G$ is of adjoint type).

We are left to lift such chain of inclusions from $\mathbb{F}_\ell$-coefficients to $\mathbb{Z}_\ell$-coefficients. Denote by $\alpha$ the reduction modulo-$\ell$ map:

$$\alpha : G(\mathbb{Z}_\ell) \longrightarrow G(\mathbb{F}_\ell).$$

It is a well known fact that, if $G$ is a connected semisimple group, $\alpha$ is Frattini, i.e. its kernel is the only closed subgroup of $G(\mathbb{Z}_\ell)$ mapping surjectively onto $G(\mathbb{F}_\ell)$. A proof can be found in [LS03, Lemma 16.4.5 (page 403)], see also [Cad15, Section 2.3] and [MVW84, Proposition 7.3].

Since Nori shows that $G(\mathbb{F}_\ell)^+$ can be identified with the subgroup of $G(\mathbb{F}_\ell)$ generated by its $\ell$-Sylow subgroups we have that the index $[G(\mathbb{F}_\ell) : G(\mathbb{F}_\ell)^+]$, for $\ell$-large enough, is prime to $\ell$. From and the fact that $\alpha$ is Frattini, we deduce that the maps

$$\alpha^{-1}(G(\mathbb{F}_\ell)^+) \to G(\mathbb{F}_\ell)^+, \text{ and } \alpha^{-1}(\Pi_\ell) \to \Pi_\ell$$
are Frattini. Therefore the inclusions
\[ G(\mathbb{Z}_\ell)^+ \subset \alpha^{-1}(G(\mathbb{F}_\ell)^+), \quad \text{and} \quad \Pi_c^\ell \subset \alpha^{-1}(\Pi_\ell) \]
are equalities since the $\Pi_c^\ell$ surjects onto $\Pi_\ell$ and $G(\mathbb{Z}_\ell)^+$ onto $G(\mathbb{F}_\ell)^+$. Eventually we have
\[ G(\mathbb{Z}_\ell)^+ \subset \Pi_c^\ell, \]
as desired. This ends the proof of the theorem.

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