The Assessment of Intrinsic Credibility

and a New Argument for $p < 0.005$

Leonhard Held
Epidemiology, Biostatistics and Prevention Institute (EBPI)
and Center for Reproducible Science (CRS)
University of Zurich
Hirschengraben 84, 8001 Zurich, Switzerland
Email: leonhard.held@uzh.ch
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Abstract: The concept of intrinsic credibility has been recently introduced to check the credibility of “out of the blue” findings without any prior support. A significant result is deemed intrinsically credible if it is in conflict with a sceptical prior derived from the very same data that would make the effect non-significant. In this paper I propose to use Bayesian prior-predictive tail probabilities to assess intrinsic credibility. For the standard 5% significance level, this leads to a new \( p \)-value threshold that is remarkably close to the recently proposed \( p < 0.005 \) standard. I also introduce the credibility ratio, the ratio of the upper to the lower limit of a standard confidence interval for the corresponding effect size. I show that the credibility ratio has to be smaller than 5.8 such that a significant finding is also intrinsically credible. Finally, a \( p \)-value for intrinsic credibility is proposed that is a simple function of the ordinary \( p \)-value and has a direct frequentist interpretation in terms of the probability of replicating an effect.

Key Words: Confidence Interval; Credibility Ratio; Intrinsic Credibility; Prior-Data Conflict; \( P \)-value; Replication; Significance Test

1 Introduction

The so-called replication crisis of science has been discussed extensively within the scientific community (Ioannidis, 2005; Begley and Ioannidis, 2015). One aspect of the problem is the widespread misunderstanding and misinterpretation of basic statistical concepts, such as the \( p \)-value (Cohen, 1994; Greenland et al., 2016). This has lead to a major rethinking and new proposals for statistical inference, such as to lower the threshold for statistical significance from the traditional 0.05 level to 0.005 (Johnson, 2013; Benjamin et al., 2018). The proposal has created a lot of discussion in the scientific community and the shortcut “\( p < 0.005 \)” has been even shortlisted and highly
commended in the Statistic of the Year competition by the Royal Statistical Society, see http://bit.ly/2yWFPbD.

Two arguments for this step are provided in Benjamin et al. (2018): The first is based on the Bayes factor, the second is based on the false discovery rate. Both arguments are actually not new, Edwards et al. (1963) have already emphasized that the evidence of \( p \)-values around 0.05 against a point null hypothesis, as quantified by the Bayes factor, is much smaller than one would naively expect: “Even the utmost generosity to the alternative hypothesis cannot make the evidence in favor of it as strong as classical significance levels might suggest.” Likewise, Staquet et al. (1979) have already argued that the false positive rate “could be considerably reduced by increasing the sample sizes and by restricting the allowance made for the \( \alpha \) error, which should be set to a 1% level as a minimum requirement.” Benjamin et al. (2018) therefore propose to lower the threshold for statistical significance to 0.005 and to declare results with \( 0.05 > p > 0.005 \) as “suggestive”, emphasizing the need for replication.

In this paper I provide a new argument for this categorization into three levels of evidence. The approach is based on the concept of intrinsic credibility (Matthews, 2018), a specific reverse-Bayes method to assess the credibility of claims of new discoveries. I review and refine the approach and show that, if you dichotomize \( p \)-values into “significant” and “non-significant”, the proposed method naturally leads to a more stringent threshold for intrinsic credibility. For the standard 5% significance level, the new \( p \)-value threshold is 0.0056, remarkably close to the recently proposed \( p < 0.005 \) standard.

To assess intrinsic credibility based on a confidence interval rather than a \( p \)-value, I propose the credibility ratio, the ratio of the upper to the lower limit of a standard confidence interval for the corresponding effect size. I show that the credibility ratio has to be smaller than 5.8 to ensure that a significant finding is also intrinsically credible. In Section 2 I provide a brief summary of the Analysis of Credibility and the specific
concept of intrinsic credibility. The latter is central for the derivation of a threshold for intrinsic credibility, as outlined in Section 3.

Lowering the threshold of statistical significance is only a temporary measure to the replication crisis (Ioannidis, 2018). A more radical step would be to abandon significance thresholds altogether (McShane et al., 2018), leaving \( p \)-values as a purely quantitative measure of the evidence against a point null hypothesis. In this spirit I extend the concept of intrinsic credibility and propose in Section 4 the \( p \)-value for intrinsic credibility, \( p_{IC} \). This new measure can be used to quantify the evidence for intrinsic credibility – without any need for thresholding – and has a direct and useful interpretation in terms of the probability of replicating an effect (Killeen, 2005). Intrinsic credibility is thus directly linked to replication, a topic of central importance in the current debate on research reproducibility (Goodman et al., 2016). I close with some discussion in Section 5.

2 Analysis of Credibility

Reverse-Bayes approaches allow the extraction of the properties of the prior distribution needed to achieve a certain posterior statement for the data at hand. The idea to use Bayes’s theorem in reverse originates in the work by IJ Good (Good, 1950, 1983) and is increasingly used to assess the plausibility of scientific claims and findings (Greenland, 2006, 2011; Held, 2013; Colquhoun, 2017). Matthews (2001a,b) has proposed the Analysis of Credibility, a specific reverse-Bayes method to challenge claims of “significance”, see Matthews (2018) for more recent developments.

Analysis of Credibility is based on a conventional confidence interval of level \( \gamma \), say, for an unknown effect size \( \theta \) with lower limit \( L \) and upper limit \( U \), say. In the following I assume that both \( L \) and \( U \) are symmetric around the effect estimate \( \hat{\theta} \) (assumed to be normally distributed) and that both are either positive or negative, \( i.e. \) the effect is...
significant at significance level $\alpha = 1 - \gamma$. Matthews (2001a,b) proposed assessing the credibility of a statistically significant finding by computing from the data a sceptical prior distribution for the effect size $\theta$, normal with mean zero, that - combined with the information given in the confidence interval for $\theta$ - results in a posterior distribution which is just non-significant at level $\alpha$, i.e. either the $\alpha/2$ or the $1 - \alpha/2$ posterior quantile is zero. It can be shown that the limits $\pm S$ of the corresponding equi-tailed prior credible interval at level $\gamma$ are given by

$$S = \frac{(U - L)^2}{4\sqrt{UL}},$$

(1)

where $S$ is called the scepticism limit and the interval $[-S, S]$ is called the critical prior interval. Note that (1) holds for any level $\gamma$, not just for the traditional 95% level.

It is convenient to express the variance $\tau^2$ of the sceptical prior as a function of the variance $\sigma^2$ (the squared standard error, assumed to be known) of the estimate $\hat{\theta}$, the corresponding test statistic $t = \hat{\theta}/\sigma$, and $z_{\alpha/2}$, the $1 - \alpha/2$ quantile of the standard normal distribution:

$$\tau^2 = \frac{\sigma^2}{t^2/z_{\alpha/2}^2 - 1}.$$  

(2)

where $t^2 > z_{\alpha/2}^2$ is required for significance at level $\alpha$. Equation (2) shows that the prior variance $\tau^2$ can be both smaller or larger than $\sigma^2$, depending on the value of $t^2$. If $t^2$ is substantially larger than $z_{\alpha/2}^2$, then the sceptical prior variance will be relatively small, i.e. a relatively tight prior is needed to make the significant result non-significant. If $t^2$ is close to $z_{\alpha/2}^2$ (i.e. the effect is “borderline significant”), then the sceptical prior variance will be relatively large, i.e. a relatively vague prior is sufficient to make the significant result non-significant.

Two applications of the Analysis of Credibility are shown in Figure 1. Both are based on a confidence interval of width 3, but with different location ($\hat{\theta} = 2.5$ and $11/6 = 1.83$, respectively). Each Figure has to be read from right to left: To obtain a
95% posterior credible interval with lower limit 0 (shown in green), the 95% confidence interval for the unknown effect size θ (shown in red) has to be combined with a sceptical prior with variance (2) (shown in blue).

In this paper I focus on claims of new discoveries without any prior support. To assess the credibility of such “out of the blue” findings, Matthews (2018) suggested the concept of intrinsic credibility, declaring an effect as intrinsically credible if it is in conflict with the sceptical prior (with mean zero and variance (2)) that would make the effect non-significant. This can be thought of as an additional check to ensure that a significant effect is not spurious. Specifically, Matthews (2018) declares a result as intrinsically credible at level γ, if the effect estimate \( \hat{\theta} \) is outside the sceptical prior interval, i.e. \(|\hat{\theta}| > S\). He shows that, for confidence intervals at level \( \gamma = 0.95 \), this is equivalent to the conventional two-sided \( p \)-value being smaller than 0.0127. I refine the definition of intrinsic credibility in the following Section 3 based on the Box (1980) prior-predictive approach, leading to the more stringent \( p \)-value threshold 0.0056 for intrinsic credibility at the 95% level.

### 3 A new threshold for intrinsic credibility

Matthews’ check for intrinsic credibility compares the size of \( \hat{\theta} \) with the scepticism limit (1), so does not take the uncertainty of \( \hat{\theta} \) into account. He compares the estimate \( \hat{\theta} \) with the (sceptical) prior distribution, not with the corresponding prior-predictive distribution. However, use of the latter is the established way to check the compatibility of the data and the prior (Box, 1980; Greenland, 2006). In what follows I will therefore apply the approach by Box (1980) for the assessment of prior-data conflict based on the prior-predictive distribution, with the perhaps slightly unusual feature that the prior has been derived from the data. I argue that there is nothing intrinsically inconsistent in investigating the compatibility of a prior, defined through the data, and
Figure 1: Analysis of intrinsic credibility for two confidence intervals at level $\gamma = 95\%$. In the first example there is conflict between the sceptical prior and the data and the significant result is intrinsically credible at the 95% level ($L = 1$, $U = 4$, credibility ratio = 4, $p_{IC} = 0.021$). In the second example there is less conflict between prior and data and the significant result is not intrinsically credible at the 95% level ($L = 1/3$, $U = 10/3$, credibility ratio = 10, $p_{IC} = 0.09$). The credibility ratio will be described further in Section 3 while the $p$-value $p_{IC}$ for intrinsic credibility will be introduced in Section 4.
the data itself, extending an argument by Cox (2006, Section 5.10) to the reverse-Bayes setting.

The Box (1980) check for prior-data conflict is based on the prior-predictive distribution, which is in our case normal with mean zero and variance $\tau^2 + \sigma^2$ (Spiegelhalter et al., 2004, Section 5.8). The procedure is based on the test statistic $t_{\text{Box}} = \hat{\theta} / \sqrt{\tau^2 + \sigma^2}$ and the (two-sided) tail probability $p_{\text{Box}} = \Pr(\chi^2(1) \geq t^2_{\text{Box}})$ as the corresponding upper tail of a $\chi^2$-distribution with one degree of freedom. Small values of $p_{\text{Box}}$ indicate a conflict between the sceptical prior and the data.

Now suppose we fix the confidence level at the conventional 95% level, i.e. $\gamma = 0.95$. Intrinsic credibility at the 95% level (i.e. $p_{\text{Box}} < 0.05$) can then be shown to be equivalent to the requirement $p < 0.0056$ for the conventional two-sided $p$-value. To derive this result, note that with (2) we have $\tau^2 + \sigma^2 = \sigma^2 / (1 - z_{\alpha/2}^2 / t^2)$ and so $t^2_{\text{Box}} = t^2 - z_{\alpha/2}^2$. The requirement $t^2_{\text{Box}} > z_{\alpha/2}^2$ for intrinsic credibility at level $\gamma = 1 - \alpha$ then translates to

$$t^2 \geq 2 z_{\alpha/2}^2.$$  \hspace{1cm} (3)

This criterion is to be compared with the traditional check for significance, which requires only $t^2 \geq z_{\alpha/2}^2$. It follows directly that the threshold

$$\alpha_{\text{IC}} = 2 \left\{ 1 - \Phi \left( t = \sqrt{2} z_{\alpha/2} \right) \right\},$$  \hspace{1cm} (4)

here $\Phi(.)$ denotes the cumulative standard normal distribution function, can be used to assess intrinsic credibility based on the conventional two-sided $p$-value $p$: If $p$ is smaller than $\alpha_{\text{IC}}$, then the result is intrinsically credible at level $\gamma = 1 - \alpha$. For $\alpha = 0.05$ we have $t = \sqrt{2} \cdot 1.96 = 2.77$ and the threshold (4) turns out to be $\alpha_{\text{IC}} = 0.0056$, as claimed above. For other confidence levels we will obtain other intrinsic credibility thresholds. For example, Clayton and Hills (1993, Section 10.1) prefer to use 90%
confidence intervals “on the grounds that they give a better impression of the range of plausible values”. Then $\gamma = 0.9$ and we obtain the intrinsic credibility threshold $\alpha_{IC} = 0.02$.

Figure 2 compares the new threshold with the one obtained by Matthews (2018, Appendix A.4) (using $t = 1.272 \, z_{\alpha/2}$) for values of $\alpha$ below 10%. The Matthews threshold for intrinsic credibility is larger than the proposed new threshold (4), because it compares the effect estimate $\hat{\theta}$ with the prior distribution (with variance $\tau^2$) and not the prior-predictive distribution (with variance $\tau^2 + \sigma^2$).

![Figure 2: The threshold for intrinsic credibility of significant results as a function of the conventional $\alpha$ level. The blue line corresponds to the proposal by Matthews (2018). The red line is the proposed new threshold.](image)

Intrinsic credibility can also be assessed based on the confidence interval $[L, U]$, rather than the conventional $p$-value $p$. To see this, note that $t_{\text{Box}}^2$ can be written in terms of $L$ and $U$,

$$ t_{\text{Box}}^2 = z_{\alpha/2}^2 \frac{4 \, UL}{(U - L)^2}, $$

(5)
and the requirement $i_{\text{Box}}^2 \geq z^2_{\alpha/2}$ for intrinsic credibility is then equivalent to require that the credibility ratio $U/L$ (or $L/U$ if both $L$ and $U$ are negative) fulfills

$$\frac{U}{L} \leq d = 3 + 2 \sqrt{2} \approx 5.8.$$  \tag{6}$$

To derive the cut-point $d$ in (6), set $U = Ld$. The requirement $i_{\text{Box}}^2 = z^2_{\alpha/2}$ then reduces to

$$1 = \frac{4UL}{(U-L)^2} = \frac{4d}{(d-1)^2},$$

a quadratic equation in $d$ with $d = 3 + 2 \sqrt{2}$ as solution.

Thus, there is a second way to assess intrinsic credibility based on the ratio of the limits of a confidence interval at any level $\gamma$: if the credibility ratio is smaller than 5.8 than the result is credible at level $\gamma$. For example, in Figure 1 the credibility ratio is 4 in the top and 10 in the bottom panel, so the result shown in the top panel is intrinsically credible at level 95%, but the one in the bottom is not.

If the sceptical prior distribution is available, then a third way to assess intrinsic credibility is to compare the prior variance $\tau^2$ to the data variance $\sigma^2$. Comparing (2) with (3) it is easy to see that intrinsic credibility is achieved if and only if the sceptical prior variance $\tau^2$ is not larger than the variance $\sigma^2$ of the effect estimate $\hat{\theta}$. With this in mind we see immediately from Figure 1 that the first result shown in the top panel is intrinsically credible ($\tau^2 < \sigma^2$), whereas the second isn’t ($\tau^2 > \sigma^2$).

4 A $p$-value for intrinsic credibility

A disadvantage of the dichotomous assessment of intrinsic credibility described in the previous section is the dependence on the confidence level $\gamma$ of the underlying confidence interval, or, equivalently, the significance level $\alpha = 1 - \gamma$. However, there is a way to free ourselves from this dependence. In analogy to the well-known duality
of confidence intervals and standard \( p \)-values, I propose to derive the value \( \alpha^* \), say, that just achieves intrinsic credibility, \( i.e. \) where equality holds in (3). This defines the \textit{p-value for intrinsic credibility} \( p_{IC} = \alpha^* \), which provides a quantitative assessment of the evidence for intrinsic credibility. Of course, the \( p \)-value for intrinsic credibility \( p_{IC} \) can also be used to assess intrinsic credibility as described in Section 3: if \( p_{IC} \leq \alpha \), then the result is intrinsically credible at level \( \gamma = 1 - \alpha \).

The \( p \)-value \( p_{IC} \) for intrinsic credibility can be derived by replacing \( \alpha_{IC} \) with \( p \) and \( \alpha \) with \( p_{IC} \) in equation (7) and then solving for \( p_{IC} \):

\[
p_{IC} = 2 \left[ 1 - \Phi \left( t / \sqrt{2} \right) \right]. \tag{7}
\]

Here \( t = \Phi^{-1}(1 - p/2) \) is the standard test statistic for significance where \( p \) is the conventional two-sided \( p \)-value. Note that the test statistic \( t_{IC} = t / \sqrt{2} \) for intrinsic credibility in (7) is a root-2 shrunken version of the test statistic \( t \) for significance.

Figure 3 shows that the \( p \)-value \( p_{IC} \) for intrinsic credibility is considerably larger than the conventional \( p \)-value \( p \), particularly for small values of \( p \). For example, the two confidence intervals shown in Figure 1 have conventional \( p \)-values \( p = 0.0011 \) (top) and \( p = 0.017 \) (bottom), while the corresponding \( p \)-values for intrinsic credibility are \( p_{IC} = 0.021 \) and \( p_{IC} = 0.09 \), respectively. If we are prepared to adapt the “rough and ready” \( p \)-value guide by Bland (2015, Section 9.4) to \( p_{IC} \), then \( p_{IC} = 0.021 \) provides moderate evidence and \( p_{IC} = 0.09 \) only weak evidence for intrinsic credibility.

There is a direct and useful interpretation of \( p_{IC} \) in terms of the probability of replicating an effect (Killeen, 2005), \( i.e. \) the probability that an identically designed but independent replication study will give an estimated effect \( \hat{\theta}_2 \) in the same direction as the estimate \( \hat{\theta}_1 = \hat{\theta} \) from the current (first) study. To see this, note that under an initial uniform prior the posterior for \( \theta \) is \( \theta \mid \hat{\theta}_1 \sim N(\hat{\theta}_1, \sigma^2) \). This posterior now serves as the prior for the mean of the (unobserved) estimate \( \hat{\theta}_2 \mid \theta \sim N(\theta, \sigma^2) \) from
Figure 3: The $p$-value for intrinsic credibility as a function of the $p$-value for significance. The grey dashed line is the identity line.

the second (hypothetical) study, where we assumed the two studies to be identically designed, having equal variances $\sigma^2$. This leads to the prior-predictive distribution $\hat{\theta}_2 \mid \hat{\theta}_1 \sim N(\hat{\theta}_1, 2\sigma^2)$ and the $p$-value for intrinsic credibility (7) can be seen to be twice the probability that the second study will give an estimate $\hat{\theta}_2$ in the opposite direction as the estimate $\hat{\theta}_1$ of the first study:

$$p_{IC} = 2 \left[ 1 - \Phi \left( \frac{t}{\sqrt{2}} \right) \right]$$
$$= 2 \Phi \left( -\frac{t}{\sqrt{2}} \right)$$
$$= 2 \Phi \left( \frac{0 - \hat{\theta}_1}{\sqrt{2}\sigma} \right)$$
$$= 2 \Pr(\hat{\theta}_2 \leq 0 \mid \hat{\theta}_1 > 0).$$

If $\hat{\theta}_1 < 0$, then $p_{IC} = 2 \Pr(\hat{\theta}_2 \geq 0 \mid \hat{\theta}_1 < 0)$.

The probability $\Pr(\hat{\theta}_2 \leq 0 \mid \hat{\theta}_1 > 0)$ is one of the three replication probabilities that
have been considered by Senn (2002) in response to Goodman (1992). The complementar-y probability \( \Pr(\hat{\theta}_2 > 0 | \hat{\theta}_1 > 0) = 1 - p_{IC}/2 \) can be identified as the probability of replicating an effect, \( p_{rep} \), advocated by Killeen (2005) as an alternative to traditional \( p \)-values, see Lecoutre and Poitevineau (2014); Killeen (2015) for further discussion and additional references. Of course, \( p_{rep} \) can only be correct under the assumption that the null hypothesis is false. Nevertheless, Killeen (2005, 2015) argues that \( p_{rep} \) is a useful alternative to traditional \( p \)-values.

In practice, we can thus use \( p_{IC} \) to assess the probability of replicating an effect, assuming that the null hypothesis is false: \( p_{rep} = 1 - p_{IC}/2 \). An intrinsically credible result with \( p_{IC} \leq \gamma \) therefore has \( p_{rep} \geq (1 + \gamma)/2 \). For example, for \( \gamma = 95\% \) we have \( p_{rep} \geq 97.5\% \). For numerical illustration, recall that the \( p \)-values for intrinsic credibility in Figure 1 are \( p_{IC} = 0.021 \) (top) and \( p_{IC} = 0.09 \) (bottom). The corresponding replication probabilities are thus \( p_{rep} = 99.0\% \) and \( p_{rep} = 95.5\% \). In the second example, there is thus a \( p_{rep} = 4.5\% \) chance that an identically designed replication study will give a negative effect estimate.

## 5 Discussion

Based on the Analysis of Credibility, I have shown that, if you dichotomize \( p \)-values into “significant” and “non-significant” at some pre-specified threshold \( \alpha \), the Analysis of Credibility directly leads to a more stringent threshold \( \alpha_{IC} \) for intrinsic credibility. If you prefer to avoid any thresholding of conventional \( p \)-values, a new \( p \)-value for intrinsic credibility, \( p_{IC} \), has been proposed. \( p_{IC} \) is a quantitative measure of the evidence for intrinsic credibility with a direct connection to \( p_{rep} \), the probability of replicating an effect (Killeen, 2005).

The assessment of intrinsic credibility can be thought of as an additional challenge, ensuring that claims of new discoveries are not spurious. Conventionally significant
results with \(0.05 > p > 0.0056\) lack intrinsic credibility, i.e. they are not in conflict with a sceptical prior that would make the effect non-significant. This matches the classification as “suggestive” by Benjamin et al. (2018). Specifically, \(p > 0.0056\) implies \(p_{IC} > 0.05\) and thus \(p_{rep} < 97.5\%\), emphasizing the need for replication. If \(p < 0.0056\), then the result is both significant and intrinsically credible at the 95\% level, so \(p_{IC} \leq 0.05\) and \(p_{rep} \geq 97.5\%\).

The credibility ratio provides a simple and convenient tool to check whether a “significant” confidence interval at any level \(\gamma\) is also intrinsically credible. If the credibility ratio is smaller 5.8, the result can be considered as intrinsically credible at level \(\gamma\). It is noteworthy that the concept of intrinsic credibility does not require to change the original confidence level \(\gamma\). Indeed, the check for credibility is done at the same level as the original confidence level. I have used \(\gamma = 0.95\) by convention, where it follows that the check for intrinsic credibility is equivalent to the requirement \(p < 0.0056\). This implies that in standard statistical reporting there is no need to replace 95\% confidence intervals with 99.5\% confidence intervals, say. However, I suggest to complement or to replace the ordinary \(p\)-value with the proposed \(p\)-value for intrinsic credibility, \(p_{IC}\).

Although derived using a Bayesian approach, the proposed check for intrinsic credibility is based on a standard confidence interval and thus constitutes a Bayes/non-Bayes compromise (Good, 1992). Specifically, it does not require the specification of a prior probability of the null hypothesis of no effect. In fact, this prior probability is always zero. This is in contrast to the calibration of \(p\)-values to lower bounds on the posterior probability of the null, which requires specification of a prior probability. Minimum Bayes factors have also been proposed to calibrate \(p\)-values, see Held and Ott (2018) for a recent review. They have the advantage that they do not require specification of a prior probability of the null hypothesis and provide a direct “forward-Bayes” assessment of the evidence of \(p\)-values. However, the underlying rationale is
still based on a point null hypothesis with positive prior probability, fundamentally
different from the approach proposed here.

The Analysis of Credibility assumes a simple mathematical framework, where like-
lihood, prior and posterior are all normally distributed. This can be justified because
Gaussian approximations are commonly used in the calculation of confidence inter-
vals and statistical hypothesis tests, if the sample size is fairly large (e.g. Spiegelhalter
et al., 2004, Section 2.4). Of course, suitable transformations of the parameter of in-
terest may be needed to achieve normality, for example, confidence intervals for odds
ratios and hazard ratios should be transformed to the log scale. For small studies,
however, the normal assumption for the likelihood may be questionable and the as-
sessment of intrinsic credibility would need appropriate refinement, for example based
on the t-distribution.

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