Direct measurement of transcription factor dissociation excludes a simple operator occupancy model for gene regulation

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Transcription factors mediate gene regulation by site-specific binding to chromosomal operators. It is commonly assumed that the level of repression is determined solely by the equilibrium binding of a repressor to its operator. However, this assumption has not been possible to test in living cells. Here we have developed a single-molecule chase assay to measure how long an individual transcription factor molecule remains bound at a specific chromosomal operator site. We find that the lac repressor dimer stays bound on average 5 min at the native lac operator in Escherichia coli and that a stronger operator results in a slower dissociation rate but a similar association rate. Our findings do not support the simple equilibrium model. The discrepancy with this model can, for example, be accounted for by considering that transcription initiation drives the system out of equilibrium. Such effects need to be considered when predicting gene activity from transcription factor binding strengths.

Transcription factors are the major regulators of gene expression. Transcription factor–based regulation of transcription initiation is often described by a simple operator occupancy model, where in the case of repressors it is assumed that transcription is ‘off’ when the repressor is bound and ‘on’ when the promoter is free1,2. In this scenario, the resulting ratio of expression levels with and without repressor, i.e., the repression ratio (RR), becomes

$$RR = \frac{\tau_{on} + \tau_{off}}{\tau_{on}}$$  \hspace{0.5cm} (1)$$

where \(\tau_{off}\) is the average time the repressor is bound and \(\tau_{on}\) is the average time the promoter is free (Supplementary Note). The repression ratio is high when the repressor is bound for a long time (large \(\tau_{off}\)) or when the repressor concentration is high, which leads to fast binding (small \(\tau_{on}\)). This simple equation has a central position in quantitative biology as it relates the state of the cell, i.e., transcription factor concentrations, to change in state, i.e., gene expression.

The equation is therefore used in most synthetic and systems biology studies although the underlying assumptions have not been tested in living cells, where cooperative binding, active transcription, DNA replication and chromosome dynamics could influence gene regulation.

The challenge of testing the operator occupancy model in living cells is to measure the rates of operator association, \(\tau_{off}\), and dissociation, \(\tau_{on}\), directly in live cells rather than inferring them from reporter expression assays3,4. Recently, we developed a direct single-molecule microscopy assay to measure the rate of binding to a single lac operator site in the bacterial chromosome5. Here we present an in vivo version of a biochemical chase assay6, which enables direct measurements of spontaneous dissociation of the lac repressor protein, LacI, from individual chromosomal operator sites (Fig. 1a,b). In our assay, operator-bound fluorescent LacI-YFP dimers that spontaneously dissociate are replaced (chased) by non-fluorescent LacI tetrmers. Non-fluorescent LacI molecules are present in excess (Supplementary Fig. 1a) and prevent rebinding of fluorescent LacI. The spontaneous dissociation process can thus be followed by counting the average number of bound fluorescent molecules per cell over time. To start the experiment with the fluorescent LacI bound, a point mutation has been introduced into the fluorescent LacI such that it cannot bind the inducer isopropyl \(\beta\)-d-1-thiogalactopyranoside (IPTG)7. The presence of IPTG prevents binding of the non-fluorescent LacI until IPTG is removed at the start of the experiment (Supplementary Fig. 1b,c). To ensure that dissociation kinetics were independent of IPTG outflux, we showed that the intracellular concentration of IPTG within 1 min of its removal dropped to a level where non-fluorescent LacI bound effectively (Supplementary Fig. 2 and Supplementary Note), and we subsequently analyzed dissociation kinetics beginning at 1.5 min after the removal of IPTG. An extended analysis of how the finite concentrations of non-fluorescent LacI influenced the results is provided in the Online Methods. The model for replication-induced LacI dissociation is extended in the Supplementary Note. The kinetic assays were performed on E. coli cells residing in a microfluidic growth chamber (Fig. 1c,d), which allowed the cells to
be maintained in a constant state of exponential growth (generation
time of 26 min)\(^8\) as well as allowing rapid medium exchange (in 2 s).
Image acquisition and medium exchange were automated and syn-
crhonized so that the experiment was repeatable with high precision
(Fig. 1e). Cell segmentation and detection of fluorescent spots were
also automated and enabled the mapping of individual molecules onto
an intracellular coordinate system for an arbitrary number of cells
(Fig. 1f). For example, Figure 1g (as well as Supplementary Fig. 3)
shows the probability distribution of the intracellular location of spec-
fically bound LacI-YFP molecules as a function of position in the
cell cycle.

We used the in vivo chase assay to measure the kinetics for two
operators of different strength, the natural lacO\(1\) operator and the
stronger, symmetric artificial lac\(O_{sym}\) operator. The dissociation
curves for the LacI-YFP dimer from the lacO\(1\) and lac\(O_{sym}\) operators
at 37 °C are shown in Figure 2a. The average time LacI stayed bound
to its operator (\(\tau_{off}\)) was 5.3 ± 0.2 (s.e.m.) min for lacO\(1\) and 9.3 ±
0.4 (s.e.m.) min for lac\(O_{sym}\). The average time before the operator
was bound by a repressor (\(\tau_{on}\)) was measured under identical experi-
mental conditions (Fig. 2b) and was 30.9 ± 0.5 (s.e.m.) s for lacO\(1\)
and 27.6 ± 0.6 (s.e.m.) s for lac\(O_{sym}\). Thus, a stronger operator has a slower
dissociation rate but a similar association rate.

We were then ready to ask whether the measured association and
dissociation times could be used to predict the repression ratio using
the simple operator occupancy model, i.e., equation (1), as given by the
model in Figure 3a without any cooperative interaction between LacI
and RNA polymerase (RNAP) (\(\omega = 1\), as defined in Fig. 3a and equa-
tions (3) and (4) in the Online Methods). Combining the association
and dissociation measurements, we calculated that the repression
ratio was expected to be 11.2 ± 0.5 (s.e.m.) for lacO\(1\) and 21.2 ± 0.9

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**Figure 1** The single-molecule chase assay. (a) Outline of the single-molecule chase assay. When fluorescent LacI dimers (yellow) dissociate from the lac operator (red box), they are replaced by non-fluorescent wild-type LacI tetramers (blue) present in excess. (b) Examples of fluorescence images (4-s exposure) taken before and at different time points after the removal of IPTG. Scale bar, 4 μm. The inset image is magnified by 2x relative to the original image. Red circles indicate detected operator-bound LacI-YFP. (c,d) The microfluidic switching chip (d) contains 51 traps as illustrated (c). Each trap harbors ~250 E. coli cells and allows for sustained exponential growth and fast change of medium. (e) Medium switch–induced transcription factor dissociation and association. When medium is switched from high 2-nitrophenyl-β-D-β-fucopyranoside (ONPF; anti-inducer) to high IPTG (inducer), transcription factors dissociate in a few seconds (inset). When medium is switched back, transcription factors associate in ~30 s. The graph shows three switching cycles separated by 6-h recovery periods. (f) Automatically segmented cells using a phase-contrast image. Scale bar, 4 μm. (g) Intracellular positions of bound LacI-YFP molecules (x axis) mapped to the cell replication cycle (y axis). Individual cell replication cycles are synchronized so that the time of 0 min always implies a cell length of 4.25 μm. Horizontal lines mark the average times for cell divisions.

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**Figure 2** Kinetic measurements for individual lac operators. (a) Dissociation curves for lac\(O_{sym}\) and lacO\(1\), \(n = (i, j, k)\) implies \(i\) repetitions 6 h apart for chip 1, \(j\) repetitions for chip 2 and \(k\) repetitions for chip 3. Error bars, ± s.e.m.; \(n = (2, 3, 2)\) (lacO\(1\)) and \(n = (3, 2, 3)\) (lac\(O_{sym}\)). Inset, temperature dependence for dissociation from lacO\(1\). Error bars, ± s.e.m.; \(n = (2, 3, 2)\) (37 °C) and \(n = (2, 2)\) (25 °C). (b) Association curves for lac\(O_{sym}\) and lacO\(1\). Error bars, ± s.e.m.; \(n = (2, 3, 2)\) (lacO\(1\)) and \(n = (3, 2)\) (lac\(O_{sym}\)). Inset, temperature dependence for association with lacO\(1\). Error bars, ± s.e.m.; \(n = (2, 3, 2)\) (37 °C) and \(n = (3)\) (25 and 42 °C).
Figure 3 Models of gene regulation. (a) At equilibrium, the repression ratio only depends on the fraction of time the operator is bound independent of kinetic schemes. Owing to cooperative binding (ω > 1), the ratio can be modulated by other factors. TF, transcription factor; Kd, equilibrium binding (s.e.m.) for lacO (Table 1). The corresponding measurements of the repression ratios for the LacI-Venus dimer based on an enzymatic reporter assay were 10.0 ± 1.3 (s.e.m.) for lacO1 and 29.7 ± 3.4 (s.e.m.) for lacO (Table 1). We conclude that the operator occupancy model accounts for the repression ratio for lacO1 but not for the ratio for lacO, where the observed repression ratio was higher than expected when considering association and dissociation rates alone.

This discrepancy for lacO5 motivated the construction of more complex interaction models. One possibility was an equilibrium model where LacI interacts cooperatively with RNAP or another protein binding near the operator and where the degree of cooperativity depends on the operator sequence. This model is represented (Fig. 3a) using ω = 1.5 and ω = 1 for lacO and lacO2, respectively, and resulted in excellent agreement with the measured repression ratios. Such a difference in cooperativity between lacO1 and lacO2 could be due to the markedly different binding of DNA when LacI is bound to the different operators. Operator sequence–specific interactions between LacI and RNAP have previously been suggested when the operator is positioned upstream of the lacUV5 promoter. Although this equilibrium mechanism is also possible with the operator located downstream of the promoter, a model with operator–specific cooperativity was needed to describe our data. Cellular reaction dynamics are commonly out of equilibrium, and we therefore also considered more simple non-equilibrium schemes. In Figure 3b–d, we outline three such schemes that can increase the repression ratio beyond the ratio predicted by the simple operator occupancy model. We discuss them individually below.

The first non-equilibrium scheme (Fig. 3b) is similar to the scheme with cooperative interaction with RNAP (Fig. 3a) except that active transcription initiation clears the promoter in the absence of LacI. Slow transcription initiation leads to a repression ratio as in the cooperative equilibrium model, whereas fast transcription initiation leads to a reduced repression ratio, as it is possible to synthesize transcripts before the repressor has equilibrated with DNA. Interestingly, we found that the transcription rate for the lac operon with full induction was 5.4 ± 0.5 (s.e.m.) times higher in the strain with the lacO1 sequence than in the strain with the lacO5 sequence next to the promoter (Supplementary Note). This difference in transcription rate, in combination with the measured association and dissociation rates, is sufficient to fully account for the measured repression ratios when ω = 1.5 for both lacO1 and lacO5. The reason for this is that lacO5 is closer to the equilibrium case (with slow transcription) described above, whereas lacO1 is out of equilibrium (with fast transcription) and thus has a lower repression ratio than what is expected from the equilibrium model alone (Fig. 3b). As a consequence, no operator sequence–dependent interaction between LacI and RNAP is needed in this case, as the sequences are transcribed at different rates.

Also in the second non-equilibrium scheme (Fig. 3c), transcription initiation drives the system out of equilibrium but this time without any cooperative binding between RNAP and LacI. In this scheme, RNAP binds to one of the alternative lac promoters next to the operator-bound LacI but does not continue into open complex formation. In contrast, when RNAP binds in the absence of LacI, it proceeds rapidly and irreversibly into transcription, clearing the promoter. Consequently, LacI will most often bind in an RNAP-free promoter region and dissociate from an RNAP-bound operator region. Thus,

Table 1 Comparison of repression ratios from reporter expression assays and direct single-molecule in vivo measurements

| Operator region | Reporter expression assay | t_{off} (s) | t_{on} (min) |
|----------------|--------------------------|------------|-------------|
| lacO1          | 10.0 ± 1.3               | 11.2 ± 0.5 | 30.9 ± 0.5  |
| lacO2          | 29.7 ± 3.4               | 21.2 ± 0.9 | 27.6 ± 0.6  |

Data are shown as mean values ± s.e.m.; n indicates replicates from individual experiments (reporter expression); n = 9 (lacO1) and n = 8 (lacO2). See Figure 2 for details of the single-molecule experiments.

The repression ratio is induced (+ IPTG) divided by repressed (− IPTG) lacZ expression in terms of Miller units (normalized β-galactosidase activity) and is normalized to the lower repression concentrations in the kinetic experiments (Supplementary Fig. 8 and Supplementary Note).

Table 2 Binding kinetics dependence on roadblocks

| Roadblock | t_{off} (s) | t_{on} (min) | Repression ratio |
|-----------|------------|--------------|-----------------|
| Without   | 27.6 ± 0.6 | 9.3 ± 0.4    | 21.2 ± 0.9      |
| With      | 37.1 ± 0.6 | 11.6 ± 1.4   | 19.7 ± 1.1      |

Association and dissociation rates measured for LacI-YFP with or without TetR binding next to one side of the operator lacO5. Data are shown as mean values ± s.e.m.; n = i, j, k implies i repetitions for chip 1, j repetitions for chip 2 and k repetitions for chip 3; n = {4, 2} (t_{off}, lacO5 with roadblock) and n = {4, 3, 3} (t_{on}, lacO5 with roadblock). Data without roadblock are the same as in Table 1.
if a bound RNAP molecule slows down LacI dissociation, this would result in repression beyond that predicted in the equilibrium model, even if the binding strength for LacI is unaltered by the bound RNAP. The average times for LacI association and dissociation are expected to increase by up to a factor of two when a protein is bound next to the lac operator, as sliding along DNA in and out of the operator is blocked from one side. To test this hypothesis, we positioned the tet repressor protein, TetR, next to the lac operator site and measured the times for LacI dissociation and association. We found that the time for association increased by a factor of 1.35 ± 0.04 (s.e.m.) when TetR was bound next to lacO_sym and that the effect on dissociation was similar (Table 2 and Supplementary Fig. 4), as was expected from detailed balance when steady-state binding is not altered. The effect was smaller (f = 1.16 ± 0.03 (s.e.m.)) for lacO_sym, for which the lower binding probability reduced the impact of the diffusion blockade, as the transcription factor will need multiple attempts to bind anyway. If RNAP binds in a closed complex near LacI and blocks sliding in and out of the lac operator, as is likely, then a second operator can lead to faster association rates than we report above, as our association studies were performed under steady-state growth conditions and do not consider local concentration gradients of the repressor.

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ONLINE METHODS

Strain construction. Strains were constructed in a BW2593 background using the λ Red16 or pK03 (ref. 17) protocols. Detailed strain descriptions can be found in the Supplementary Note, Supplementary Figure 7 and Supplementary Table 1.

Growth conditions. Cells were grown in M9 minimal liquid medium supplemented with 0.4% glucose and RPMI amino acids (Sigma). For growth of strains harboring pBAD24 plasmids encoding lacI, lacI-Venus or xylR, the medium was supplemented with carbenicillin (Sigma).

For microfluidics experiments, saturated (overnight) cultures were diluted to 1:200 in 40 ml of medium and grown at 37 °C for 4 h unless otherwise specified. Cells were collected by centrifugation and immediately loaded onto microfluidic chips as previously described18.

Information about growth conditions in other microscopy experiments and expression assays can be found in the Supplementary Note.

Fluorescence microscopy and microfluidics. Microfluidic switching—design and preparation. Microfabrication of the templates and construction of the individual devices were performed in accordance with the protocols described previously18 with the exception that an extra medium port was added to allow for rapid exchange of medium. Inert polystyrene beads of 2 μm in diameter (Sigma–Aldrich) were added to one medium reservoir. Beads allowed for the detection of flow rates and flow directions necessary for determining the induction states of the device during operation.

Relative height differences between medium reservoirs were used to control the pressure gradients and, thereby, flow rates and directions in the device during running and medium exchange. Medium exchange, i.e., anti-correlated elevation/lowering of reservoirs, was automated by using programable linear actuators (Robocylinder, Intelligent Actuators), the control of which was synchronized with image acquisition using a custom-written Java program.

Optical setup. We used a Nikon Eclipse Ti-E microscope (with Nikon’s Apo TIRF 100×/1.49 oil immersion objective) equipped with a dichroic mirror (Chroma t515.5rdc), an excitation filter (Chroma 514/10), an emission filter (Chroma 550/50) and an EMCCD camera (iXon EM+ DU-897 from Andor). The camera was cooled to −80 °C, and the linearized electron-multiplying gain (Chroma t515.5rdc), an excitation filter (Chroma 514/10), an emission filter (Chroma 550/50) and an EMCCD camera (iXon EM+ DU-897 from Andor). The camera was cooled to −80 °C, and the linearized electron-multiplying gain was set to 150. A 2× magnification lens was placed in the emission path before the camera. Fluorescence was excited by a Coherent Innova-304 Ar laser at 514 nm. When measuring association and dissociation rates, the power was 15 W/cm² using 4-s exposures. For single-particle tracking, the power was 650 W/cm² and, for overnight growth experiments, the power was ≤5 W/cm² (see the Supplementary Note for details). A second camera (Scion Corp) was used for external phase-contrast imaging. The microscope was enclosed in an OlkLab cage incubator where the temperature was maintained at 37 ± 0.1 °C, 42 ± 0.1 °C or 25.5 ± 0.3 °C. Image acquisition was controlled by the open-source software µManager19 in combination with custom-written acquisition scripts.

Spot detection. We used a Trouv wavelet three-plane decomposition20 and detected the spots in the second wavelet plane. Significant wavelet coefficients were determined through scale-dependent kσ thresholding where σ is the s.d. of the second wavelet plane, estimated by the MAD estimate21, and k = 3 (association experiments) or k = 4 (dissociation experiments).

LacI-Venus kinetics using automated switching of medium. Experiments were started when cells had grown to fill the whole microfluidic traps. For a fast and well-defined switch of medium, the medium reservoirs were connected to linear actuators and controlled from the computer in parallel with µManager-run imaging acquisition.

For the analysis of operator-bound single LacI molecules in fusion with the fluorescent protein, YFP-derived Venus22 (LacI-Venus), spots were detected as described above. Because the traps of the microfluidic chip were full with densely packed cells, we normalized the number of spots per trap by total cell area.

Association with a single operator. The principle of the experiment was essentially as presented previously22-23 with the exception that the experiment was performed in the microfluidic device to allow for direct comparison with the corresponding dissociation experiment at 37 °C. The experiment was started by switching the medium for the induced cells from one containing IPTG to one containing the competitor ONPF at a 1 mM concentration. The addition of ONPF at high concentration was used to ensure that the association rate was not limited by the time it took for IPTG to leave the cell. Cells were imaged with 4-s exposures with a frame rate of ~0.18 frames/s. Fluorescent spots were counted as described above, and binding curves with data from the same strain were fitted (Igor Pro (v6.12A)) to the single exponential function y = a(1−be−kt), where a and b were independent for each series and k was the same for all series. Experiments were repeated to generate sufficient statistical power to test the hypothesis. For visualization in Figure 2b, the a and b parameters were used to normalize the data points in individual series before calculating the average and s.e.m. for each time point and plotting together with the fitted curve.

In Supplementary Figure 2c, the rate of LacI-Venus association is plotted as a function of the added ONPF concentration, and the plot shows that a 1 mM concentration is saturating. It also shows that LacI binds 1 min faster with the addition of ONPF at a saturating concentration, which suggests that it takes up to 1 min for the intracellular IPTG concentration to drop to a level where LacI can bind the operator. This timing is important for the dissociation assay described below. The relative difference in LacI-Venus concentration between strains is described in the Supplementary Note and Supplementary Figure 8.

Chase assay for the measurement of dissociation rates. In the in vivo chase experiment, LacI-Venus molecules are first bound to individual, single operator sites; then, through competition with non-fluorescent wild-type LacI in excess, they can be seen to dissociate as the number of fluorescent spots decreases. The chase experiment relies on the possibility of inducing binding of non-fluorescent LacI in a well-defined timeframe while LacI-Venus is already bound. To accomplish this, a single point substitution was introduced in the lac repressor gene (encoding LacI p.Asp274Asn), which causes more than a 1,000-fold reduction in IPTG affinity without changing operator binding strength17. The gene (referred to as lacI) was expressed in fusion with Venus, resulting in a chromosomally expressed LacI-Venus that does not dissociate, even in the presence of 1 mM IPTG (Supplementary Fig. 1c). Wild-type LacI was expressed from an arabinose-inducible promoter on the plasmid pBAD24.

The ratio between LacI-Venus and wild-type LacI monomers when the plasmid was uninduced is seen at time 0 in Supplementary Figure 1a. When the plasmid was fully induced for a long time, the competitor copy number became so high that either 1 mM IPTG did not saturate LacI to prevent operator binding or the LacI-Venus–LacI heterodimers, which naturally form (and are dominant when LacI is overexpressed) and bind one IPTG molecule, did not bind the operator. When instead XylR was expressed from pBAD24, LacI-Venus was unaffected by IPTG (Supplementary Fig. 1c).

Before the switch, with IPTG present, LacI-Venus homodimer bound the operator. When IPTG was removed at t = 0, there was a short (1-min) period of increased binding (Supplementary Fig. 1b). This increased binding is probably due to the association of heterodimers (in competition with non-fluorescent wild-type LacI) to available operator sites. Because of this initial association and the time delay required to reduce the intracellular IPTG concentration to a level where non-fluorescent LacI bound (see below and Supplementary Fig. 2), we fit the dissociation process from 1.5 min after switching to medium without IPTG to an exponential decay process that also took into account the fact that the transcription factor was displaced once per generation owing to replication. The implications of the approximation are quantified below and in the Supplementary Note. Experiments were repeated to generate sufficient statistics to test the hypothesis.

Time-dependent excess of non-fluorescent LacI. We induced the expression of non-fluorescent LacI at time 0 (medium containing 1 mM IPTG was switched to medium containing 0.2% arabinose). This switch resulted in a time-dependent increase in the concentration of the non-fluorescent LacI chase molecules (Supplementary Fig. 1a). This time-dependent increase motivated us to calculate how this would influence the measured dissociation kinetics.
The equations that describe the probabilities that an operator is initially bound by a fluorescent molecule (Pf), that it is empty (Pe) or that it is occupied by a non-fluorescent molecule (Pn) were

\[
\begin{align*}
\frac{dP_F(t)}{dt} &= \tau_{on}P_E - \tau_{off}P_F \\
\frac{dP_E(t)}{dt} &= \tau_{off}(P_F + P_N) - \tau_{on}(1 + q(t))P_E \\
\frac{dP_N(t)}{dt} &= q(t)\tau_{on}P_E - \tau_{off}P_N \\
\frac{dP_O(t)}{dt} &= 1, \quad \frac{dP_R(t)}{dt} = 0, \quad \frac{dP_N(t)}{dt} = 0
\end{align*}
\]

(2)

Here q(t) is the fold excess of non-fluorescent transcription factor, which was measured directly by protein blot (Supplementary Fig. 1a) and is closely approximated by q(t) = 4 + t, where t is the time in minutes after the addition of IPTG.

For an infinitely high q, Pf will decay as a pure exponential with rate \( \tau_{off} \) starting from t = 0. For a finite q, the observed dissociation process is slightly slower. When fitting a single exponential function to the solution of \( P_F(t) \), using parameters from Table 1, starting from 1.5 min and ending at 20 min, the dissociation rate is underestimated by up to 11% for lacO1 and by up to 9% for lacO2 when assuming large excess of non-fluorescent LacI.

Models. Cooperative LacI binding. Consider the scheme in Figure 3a,b written in further detail.

\[
\begin{align*}
O & \xrightarrow{k_1 = \tau_{on}^{-1}} \text{O-TF} \\
& \xrightarrow{k_2 = \tau_{off}^{-1}} \text{O-TF} \\
& \xrightarrow{\omega = \omega^{-1}_{on} \omega^{-1}_{off}} \text{O-TF-RNAP} \\
& \xrightarrow{k_1 = \tau_{on}^{-1}} \text{O-RNAP} \\
& \xrightarrow{k_2 = \tau_{off}^{-1}} \text{O-RNAP} \\
\end{align*}
\]

(3)

Here LacI and RNAP bind \( \omega \) times longer when they are binding at the same time. The repression ratio in this non-equilibrium scheme is

\[
RR = 1 + \frac{k_1(k_3 + k_4)k_5 + k_6)}{k_2(k_3 + k_4)}
\]

(4)

If we assume that transcription initiation is slow where \( k_6 \to 0 \) (equilibrium case),

\[
RR = 1 + \frac{ak_1(k_3 + k_4)}{k_2(k_3 + k_4)} \approx 1 + \frac{ak_1}{k_2} \approx 1 + \frac{\omega \tau_{off}}{\tau_{on}}
\]

(5)

If we assume that transcription initiation is fast where \( k_6 \to \infty \) (far from equilibrium case),

\[
RR = 1 + \frac{k_1(k_3 + k_4)}{k_2(k_3 + k_4)} \approx 1 + \frac{ak_1}{k_2} \approx 1 + \frac{ak_1}{k_2} \frac{k_3}{k_1 + k_2 + k_3}
\]

(6)

These are the limiting approximations given in the main text (Fig. 3a,b).

To see what we obtained with specific numbers, we used the measured \( \tau_{on} \) and \( \tau_{off} \) values and assumed that \( a = 1.5, k_3 = 1 \text{ min}^{-1} \) and \( k_4 = 0.1 \text{ min}^{-1} \).

These numbers gave RR = 10.0 and an induced transcription initiation rate of 0.61 min\(^{-1}\) (refs. 24,25) when \( k_6 = 1.7 \text{ min}^{-1} \) for lacO1 and RR = 28.2 and an induced transcription initiation rate of 0.61/5.4 min\(^{-1}\) for lacO2zym when \( k_6 = 0.14 \text{ min}^{-1} \). The value of 5.4 is the measured difference in expression between the induced lac operon controlled by lacO1 and lacO2zym.

Non-equilibrium model with roadblock. Consider the scheme in Figure 3c written in further detail.

\[
\begin{align*}
O & \xrightarrow{k_1 = \tau_{on}^{-1}} \text{O-TF} \\
& \xrightarrow{k_2 = \tau_{off}^{-1}} \text{O-TF} \\
& \xrightarrow{\omega = \omega^{-1}_{on} \omega^{-1}_{off}} \text{O-TF-RNAP} \\
O & \xrightarrow{k_1 = \tau_{on}^{-1}} \text{O-RNAP} \\
& \xrightarrow{k_2 = \tau_{off}^{-1}} \text{O-RNAP}
\end{align*}
\]

(7)

The repression ratio RR is

\[
RR = \frac{k_1(k_3 + k_4)}{k_2(k_3 + k_4 + k_5)} + k_4 + f^{-1}(k_1 + k_2 + k_3) \frac{k_1(k_3 + k_4)}{k_2(k_3 + k_4 + k_5)} + 1
\]

(8)

Assuming that the system is far from equilibrium, such that \( k_6 \gg k_3 + k_4 \) and that the transcription initiation rate is fast enough, such that \( k_1(k_3 + k_4)/(k_1k_6) \ll 1 \), then the repression ratio is

\[
RR = 1 + \frac{k_1(k_3 + k_4)}{k_2(k_3 + f^{-1}(k_1 + k_3 + k_4))}
\]

(9)

Further, assuming that RNAP binding is strong, such that \( k_6 \gg k_7, k_8 \), that the turnover of RNAP is faster than the turnover of the transcription factor, such that \( k_7 \gg k_8 + k_9 \), and that f is not very much smaller than 1, then the repression ratio is

\[
RR = 1 + \frac{k_1}{k_2} \frac{k_3}{k_1 + k_2 + k_3}
\]

(10)