Giant Magnetoresistance in Boundary Driven Spin Chains

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In solid state physics, giant magnetoresistance is the large change in electrical resistance due to an external magnetic field. Here we show that giant magnetoresistance is possible in a spin chain composed of weakly interacting layers of strongly coupled spins. This is found for all system sizes even down to a minimal system of four spins. The mechanism driving the effect is a mismatch in the energy spectrum resulting in spin excitations being reflected at the boundaries between layers. This mismatch, and thus the current, can be controlled by external magnetic fields resulting in giant magnetoresistance. A simple rule for determining the behavior of the spin transport under the influence of a magnetic field is presented based on the energy levels of the strongly coupled spins.

When giant magnetoresistance was first discovered in 1988 [1, 2], it was observed in alternating layers of ferro- and antiferromagnetic materials where an external magnetic field drastically changed the conductive properties of the material. Later it has also been observed in ferromagnetic layers separated by isolating non-magnetic layers [3–5]. This discovery led to significant improvements in computer engineering, helping to advance, amongst others, memory (RAM) [6], transistors [7] and sensors [8]. The effect of giant magnetoresistance is largely attributed to electron scattering depending on quantum mechanical phenomena like tunneling. Therefore, many alternative information carrier such as thermal [12, 13] and magnetic [14, 15] currents have been proposed. Among the most prominent of these is spin transport in boundary driven spin chains [16–19].

Here a linear chain of nearest neighbor interacting spins are coupled to magnetic reservoirs in both ends thus inducing a net magnetic transport from one reservoir to the other. The current properties are a consequence of the induced steady state which can be engineered into components like diodes [20–22].

Here we show that a class of simple quantum spin systems allow controlled manipulation of the spin current through application of external magnetic fields, i.e. giant magnetoresistance. This is done by considering a generic model composed of weakly interacting layers of strongly coupled spins as an analog to the classical phenomenon. The groups of strongly coupled spins mimic the action of the ferromagnetic layers which are only allowed to interact weakly with one another mimicking the isolating layers. This results in a mismatch of the energy levels of each group causing spin excitations to be reflected at the weakly coupled boundary. A magnetic field can be applied to align these energy levels allowing spin excitations to be exchanged resulting in giant magnetoresistance. Our work demonstrates that this coveted and technologically important effect is present in a surprisingly simple quantum system of interacting spins as compared to the condensed-matter materials typically studied. Moreover, our work extends the realm of study of giant magnetoresistance to quantum spintronics [23] down to mesoscopic or even few-atom system sizes [24]. As we demonstrate below, the effect can be observed with just a few spins and should be realizable using several of the current platforms used to pursue quantum technology beyond classical electronics.

Setup. The general model studied here is a set of \( N \) linear spin-1/2 chains, where the \( i \)'th chain is composed of \( n_i \) spins coupled strongly to each other through the Hamiltonian

\[
H_0 = \sum_{j=1}^{n_i} \sum_{k=1}^{n_{i-1}} U_{ij} \left( \hat{\sigma}_i^x \hat{\sigma}_{i,j+1}^x + \hat{\sigma}_i^y \hat{\sigma}_{i,j+1}^y \right) + \hbar \sum_{j=1}^{n_i} \hat{\sigma}_i^z.
\]

The Pauli matrices for the \( j \)'th spin within the \( i \)'th chain is \( \hat{\sigma}_j^\alpha \) for \( \alpha = x, y, z \), and we are using units where \( \hbar = 1 \). The exchange coupling between spins in the \( i \)'th chain is \( U_i \), and \( \hbar \) sets the spin excitation energy for the spins. We make these strongly coupled chain segments a part of a larger chain by adding two extra spins labeled \( L \) and \( R \). These are described by the Pauli matrices \( \hat{\sigma}_{L,R}^\alpha \) for \( \alpha = x, y, z \). Finally, we couple these two spins and the strongly interacting chains weakly to each other through the Hamiltonian

\[
H_{LR} = \mathcal{J} \left( \hat{\sigma}_{L,1}^x \hat{\sigma}_{R,1}^x + \hat{\sigma}_{L,1}^y \hat{\sigma}_{R,1}^y \right) + \mathcal{J} \left( \hat{\sigma}_{N,0}^x \hat{\sigma}_{R,0}^x + \hat{\sigma}_{N,0}^y \hat{\sigma}_{R,0}^y \right)
\]

\[
\hat{H} = H_0 + H_{LR} + \mathcal{J} \sum_{i=1}^{N-1} \left( \hat{\sigma}_{i,n_i}^x \hat{\sigma}_{i+1,1}^x + \hat{\sigma}_{i,n_i}^y \hat{\sigma}_{i+1,1}^y \right),
\]

where the exchange coupling between chains \( \mathcal{J} \) must be smaller than the inter-chain exchanges \( \mathcal{J} \ll U_i \). An example of such a setup can be seen in Fig. 1.

To study spin transport in the system, we couple it to spin reservoirs through spin \( L \) on the left and spin \( R \) on the right, see Fig. 1. The presence of the reservoirs means that we have an open (non-unitary) quantum system that can be described by a density matrix \( \hat{\rho} \) and the corresponding Lindblad master
The exchange coupling between the spins in the first chain is $J_1$ while the exchange between the spins in the second chain is $U_2$. The exchange between the two chains and outer spins is $J$. The numbering is shown below the spins and the magnetic field is shown with red arrows.

The steady state current for $U_1 \gg J$ and $0 \leq h \leq 2U_1$ can be found to be

$$J(J, U_1, h) \approx \frac{\hbar^2 + 17U_1^2}{\frac{1}{2}(\hbar^2 - U_1^2) + \frac{3}{4}(11\hbar^2 + 43U_1^2)}. $$

Figure 1. Illustration of the model with an example consisting of $N = 2$ chains, the first containing $n_1 = 3$ spins and the second $n_2 = 2$ spins. The setup is coupled to spin reservoirs at each end, one with an abundance of spin excitations (left) and one with an abundance of spin excitation holes (right). The exchange coupling between the spins in the first chain is $U_1$ while the exchange between the spins in the second chain is $U_2$. The exchange between the two chains and outer spins is $J$. The numbering is shown below the spins and the magnetic field is shown with red arrows.

The characteristic of these reservoirs are determined by the parameters $f_{iR}$. We will focus on the case where $f = f_L = -f_R$, and unless otherwise stated $f = 0.5$. One reservoir has an abundance of spin excitations and forces the adjacent spin into a statistical mixture of predominantly up $\langle \hat{\sigma}_z^+ \rangle = f$, while the other has an abundance of excitation holes and forces the adjacent spin into a statistical mixture of predominantly down $\langle \hat{\sigma}_z^- \rangle = -f$. If $f > 0$, on average spin excitations are created on the left, transported through the chain and decays on the right resulting in a current flowing from left to right. However, if $f < 0$, the current will tend to flow from right to left. The reservoirs will induce currents and generally bring the system out of equilibrium. However, after sufficient time it will reach a steady state (ss), $\frac{\partial \rho}{\partial t} = 0$. To quantify the spin transport in the steady state we define the spin current $[20, 21]$

$$\mathcal{J} = \text{tr}(\hat{j}_L \hat{\rho}_{ss}) = \text{tr}(\hat{j}_R \hat{\rho}_{ss})$$

where $\hat{j}_L = 2J (\hat{\sigma}_L^+ \hat{\sigma}_r^- - \hat{\sigma}_L^- \hat{\sigma}_r^+)$ and $\hat{j}_R = 2J (\hat{\sigma}_R^+ \hat{\sigma}_r^- - \hat{\sigma}_R^- \hat{\sigma}_r^+)$. A single chain. First, we study the simplest case with $N = 1$ chain of $n_1 = 2$ spins coupled strongly to each other with coupling strength $U_1$. This gives a total chain of four spins described by $\hat{\sigma}_L, \hat{\sigma}_{1,1}, \hat{\sigma}_{1,2}$ and $\hat{\sigma}_R$ similar to the example in Fig. 1. For this system an analytical solution can be found for $f = 0.5$. The steady state current for $U_1 \gg J$ and $0 \leq h \leq 2U_1$ can be found to be

$$J(J, U_1, h) \approx \frac{\hbar^2 + 17U_1^2}{\frac{1}{2}(\hbar^2 - U_1^2) + \frac{3}{4}(11\hbar^2 + 43U_1^2)}. $$

The exact current $[27]$ is plotted for different values of $U_1$ in Fig. 2 (a). The largest current is obtained for $h = \pm U_1$ where the current is $J(h = \pm U_1) = \frac{4}{3}J$ and thus independent of $U_1$. Furthermore, for no magnetic field $h = 0$ the current is $J(h = 0) \sim \frac{17\hbar^2}{24J}J$ to lowest order in $J/U_1$, and thus heavily suppressed for large $U_1$. We therefore get giant magnetoresistance even for this minimal model. To explain this we first diagonalize $\hat{H}_0$ to obtain the four states $|\uparrow\downarrow\rangle$, $|\Phi_+\rangle$, $|\Phi_-\rangle$ and $|\uparrow\uparrow\rangle$ for spin (1,1) and (1,2) with corresponding energies $E_{\uparrow\downarrow} = 0$, $E_{\Phi_+} = -U_1$, $E_{\Phi_-} = U_1$ and $E_{\uparrow\uparrow} = 0$, where $|\Phi_\pm\rangle = (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)/\sqrt{2}$. Next, we write the total Hamiltonian $\hat{H}$ in the single excitation basis $|\uparrow\downarrow\downarrow\rangle$, $|\Phi_+\downarrow\rangle$, $|\Phi_-\downarrow\rangle$ and $|\uparrow\uparrow\downarrow\rangle$

$$H = 2 \begin{pmatrix}
-\hbar & J \sqrt{2} & J \sqrt{2} & 0 \\
J \sqrt{2} & U_1 & 0 & -J \sqrt{2} \\
J \sqrt{2} & 0 & U_1 & -J \sqrt{2} \\
0 & -J \sqrt{2} & -J \sqrt{2} & -\hbar
\end{pmatrix}. $$

Here $[\hat{L}_L, \hat{R}_L, \hat{R}_R, \hat{L}_R] = [\hat{\rho}_L, \hat{L}_L, \hat{L}_R, \hat{R}_L]$ is the commutator, $\mathcal{L}[\hat{\rho}]$ is the Lindblad superoperator and $\mathcal{D}_{L,R}[\hat{\rho}]$ are dissipative terms describing the action of the baths

$$\mathcal{D}_{L,R}[\hat{\rho}] = \gamma \left[ \frac{1 + f_{L,R}}{2} \left( \hat{\sigma}_{L,R}^+ \hat{\sigma}_{L,R}^- - \frac{1}{2} \left\{ \hat{\sigma}_{L,R}^\dagger \hat{\sigma}_{L,R}^- + \frac{1}{2} \left\{ \hat{\sigma}_{L,R}^\dagger \hat{\sigma}_{L,R}^- \right\} \right) \right) + \frac{1 - f_{L,R}}{2} \left( \hat{\sigma}_{L,R}^+ \hat{\sigma}_{L,R}^- - \frac{1}{2} \left\{ \hat{\sigma}_{L,R}^\dagger \hat{\sigma}_{L,R}^- \right\} \right) \right].$$

$\hat{\sigma}_{L,R}^\dagger = (\hat{\sigma}_{L,R}^\dagger + i\hat{\sigma}_{L,R}^y)/2$, $\gamma$ is the strength of the interaction with the baths, $f_{L,R}$ determines the nature of the interaction and $\{\bullet, \bullet\}$ denotes the anti commutator. The baths are coupled with strength $\gamma = J$, although we note that smaller values of $\gamma$ induce similar effects. The characteristics of these reservoirs are determined by the parameters $f_{L,R}$. We will focus on the case where $f = f_L = -f_R$, and unless otherwise stated $f = 0.5$.
These four states are therefore eigenstates with the diagonal being the corresponding eigenenergies of the Hamiltonian to lowest order in $J/U_1$. For a spin excitation created at one end to propagate to the other end it needs to pass the middle two spins. This is suppressed if the energy of an excitation at either end and an excitation at the middle chain is far from resonance with each other [28]. From the above we see that this is exactly the case for $h = 0$ whereas they become resonant for $h = \pm U_1$. We would therefore expect the maximum spin current here as is also observed in Fig. 2 (a).

Multiple chains. There are two natural extensions of this both of which are explored in Figs. 2 (b) and (c). First, we look at a different number of chains $N$ while keeping $n_1 = 2$. We also set all the strong exchange couplings equal $U_i = U_1$. The individual strongly coupled chains diagonalize just as before, and we therefore still expect the strongest current for $h = \pm U_1$. The current both off ($h = 0$) and at ($h = U_1$) resonance is plotted for a different number of pairs $N$ in Fig. 2 (b). Off resonance the spin current is heavily suppressed at first but then levels out for larger $N$ whereas on resonance the current is almost constant.

Next, we keep $N = 1$ and instead vary $n_1$. Following the same process as before, we first diagonalize $\hat{H}_0$. Let $|n\rangle$ be the single excitation state with spin $(1, n)$ flipped. Keeping to this one excitation basis the Hamiltonian $H_0$ can be written as

$$H_0 = \begin{pmatrix} (2-n_1)h & U_1 & 0 & \cdots & 0 \\ U_1 & (2-n_1)h & U_1 & \cdots & 0 \\ 0 & U_1 & (2-n_1)h & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & (2-n_1)h \end{pmatrix}$$

One can show that the eigen energies become [29]

$$E_k = 4U \cos \left( \frac{\pi k}{n_1 + 1} \right) + (2-n_1)h \quad 1 \leq k \leq n_1.$$
rent is expected for \(-2U < h < 2U\) while a hard drop off should occur for \(|h| > 2U\).

Finally, we look at the more general case with \(N = 2\) chains consisting of \(n_1 = 3\) and \(n_2 = 2\) strongly coupled chains respectively as seen in Fig. 1. At first we keep \(U_1 \neq U_2\). The first chain will then be at resonance with the ends for \(h/U_1 = 0, \pm \sqrt{2}\) while the second chain will be at resonance with the ends for \(h/U_2 = \pm 1\). However, only when both the chains individually are at resonance with each other so that a spin excitation can propagate between them do we expect the largest current. This is the case when both of the conditions are upheld or rather when \(U_2 = \pm \sqrt{2} U_1\) or \(U_2 \sim 0\). To see that this is true, we plot the current as a function of both \(U_2/U_1\) and \(h/U_1\) in Fig. 3 (b) with the expected resonances plotted as dashed lines. Here we see that lines of high current run along the expected lines and that the current is extra large when the resonances meet. To illustrate the role of the single excitation spectrum explored above, we set \(U_1 = U_2 = U\) and plot both the current and the single excitation spectrum as a function of \(h/U\) in Figs. 3 (c) and (d) respectively. Again we plot the expected resonances with dashed lines. The two eigenenergies that are linearly dependent on \(h/U\) corresponds to eigenstates that are close to \(|\uparrow \uparrow \ldots \uparrow\rangle\) and \(|\downarrow \downarrow \ldots \downarrow\rangle\) whereas the others are close to eigenstates that correspond to a spin excitation within the strongly coupled chains. Here it is clearly seen that when the energies of the states describing excitations at the ends cross the energy of the states with excitations within the chains a higher current is observed. Hence, we see that the giant magnetoresistance is attributed to a set of resonance conditions that can be predicted for particular setups. Lastly, we address the question of sensitivity to the nature of the bath parameter \(f\) in Fig. 3(a). Here it is seen that the current depends linearly on \(f\), and therefore the effects studied above will be present for any \(f > 0\).

**Conclusion.** We have shown how a system of weakly interacting layers of strongly coupled spins exhibit the defining quality of giant magnetoresistance, i.e. we can control the spin current in the chain by applying external magnetic fields. This is caused by reflection of spin excitations at the boundaries between the strongly coupled regions when a mismatch in the energy levels is present. We show that the effect is present even in the simplest case of four spins by obtaining an analytical expression for the spin current, and we propose a method for finding large current resonances in a general chain. This provides a simple picture for understanding and predicting giant magnetoresistance in spin chains. The spin model studied here is generic with many implementation possibilities including neutral atoms in optical lattices [30, 31], phosphorous doped silicon surfaces [23, 32] or super-conducting circuits [33].

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