String theory extensions of Einstein-Maxwell fields: the static case

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Abstract

We present a new approach for generating solutions in both the four-dimensional heterotic string theory with one vector field and the five-dimensional bosonic string theory, starting from static Einstein-Maxwell fields. Our approach allows one to construct classes of solutions which are invariant with respect to the total subgroup of three-dimensional charging symmetries of these string theories. The new solution-generating procedure leads to the extremal Israel-Wilson-Perjes subclass of string theory solutions in a special case and provides its natural continuous extension to the realm of non-extremal solutions. We explicitly calculate all string theory solutions related to three-dimensional gravity coupled to an effective dilaton field which arises after an appropriate charging symmetry invariant reduction of the static Einstein-Maxwell system.

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1 Introduction

In string theories the study of the solution spectrum for their effective field theory limits plays an important role [1], [2], [3], [4]. This study includes both a straightforward construction of new solutions [5], [6], [7], [8], [9] and various applications of symmetry based generation procedures [11], [12], [13]. The subject of this paper is related to the former approach; in fact we propose a new possibility of generating string theory solutions in an explicitly symmetry invariant form starting from the well studied system of Einstein-Maxwell fields (see [14] and the references therein). Namely, we indicate a surprising possibility of extending the static solutions of the Einstein-Maxwell theory to the realm of both the four-dimensional heterotic string theory with one vector field [15], [16], [17], [18] and the five-dimensional bosonic string theory [19].

These two theories can be considered together when studying their extremal Israel-Wilson-Perjes type solutions [20]; in this paper we present the continuous generalization of the corresponding results to the field of non-extremal solutions. We show that both the extremal and non-extremal solution subspaces can be represented in the remarkable Einstein-Maxwell form. We explore the close analogy between the heterotic (bosonic) string theory and the Einstein-Maxwell system [21] as some clear leading principle in the study of the two concrete string theories mentioned above. We also perform a consistent charging symmetry invariant reduction of the string theory inspired static Einstein-Maxwell system to the effective three-dimensional Einstein-dilaton theory – a procedure which relates, for example, the Schwarzschild black hole solution to the Nordstrom-Reissner one [14].

The black hole physics seems to be the most promising field of applications of the new approach developed in this article. This statement is based, from one side, on its hidden symmetry invariant property, which leads to the construction of symmetry non-generalizeble asymptotically flat classes of solution, and, from the other side, on its close relation to the well studied field of classical black hole solutions in Einstein-Maxwell theory [22]. The answer to the natural question about the extension of this new approach to the case of string theories with \( d + n > 2 \) will be given in a forthcoming publication [23]. The experience obtained in the field of extremal solutions [20] leads to the separation of all these effective string theories (with arbitrary values of \( d \) and \( n \), excepting the case of \( d = 1, n = 0 \) which does not possess extremal solutions and is equivalent to a double General Relativity system in the stationary case) in two classes: the first class contains the two special theories under consideration (with \( d + n = 2 \)), whereas the second one encloses all the remaining theories (with \( d + n > 2 \)). In fact, the latter systems can also be incorporated into the approach developed below after some simple but important modifications of the presented formalism.
2 3D heterotic string theory: review of new formalism

In this paper we actively explore a new formalism developed in details in [20] for the general case of the $D$-dimensional low-energy heterotic string theory with $n$ Abelian gauge fields (the bosonic string theory corresponds to the special case of $n = 0$). We consider the toroidal compactification of this theory to three spatial dimensions originally performed in [24]–[25] and settled in a convenient form in [26], [21] and [20]. Let us briefly review some elements of this new formalism which are necessary for the further analysis.

Thus, we start with the action for the bosonic sector of the low-energy heterotic string theory [2]:

\[ S_D = \int d^D X |\det G_{MN}|^{\frac{1}{2}} e^{-\Phi} \left( R_D + \Phi_M \Phi^M - \frac{1}{12} H_{MNR} H^{MNR} - \frac{1}{4} F^I_{MN} F^I_{MN} \right), \]

(2.1)

where $H_{MNR} = \partial_M B_{NK} - \frac{1}{2} A^I_M F^I_{NK} + \text{cyclic} \{M, N, K\}$ and $F^I_{MN} = \partial_M A^I_N - \partial_N A^I_M$. Here $X^M$ is the $M$-th ($M = 1, \ldots, D$) coordinate of the physical space-time of signature $(-, +, +, \ldots)$, $G_{MN}$ is the metric, whereas $\Phi$, $B_{MN}$ and $A^I_M$ ($I = 1, \ldots, n$) are the dilaton, Kalb-Ramond and Abelian gauge fields, respectively. To determine the result of the toroidal compactification to three dimensions, let us put $D = d + 3$, $X^M = (Y^m, x^\mu)$ with $Y^M = X^m (m = 1, \ldots, d)$ and $x^\mu = X^{d+\mu} (\mu = 1, 2, 3)$ and introduce the $d \times d$ matrix $G_0 = \text{diag} (-1; 1, \ldots, 1)$, the $(d + 1) \times (d + 1)$ and $(d + 1 + n) \times (d + 1 + n)$ matrices $\Sigma$ and $\Xi$ of the form $\text{diag} (-1, -1; 1, \ldots, 1)$, respectively, and the $(d + 1) \times (d + 1 + n)$ matrix field $Z = Z(x^\lambda)$ together with the three-metric $h_{\mu\nu} = h_{\mu\nu}(x^\lambda)$. In [20] it was shown that the resulting theory after the toroidal compactification of the first $d$ dimensions $Y^m$ can be expressed in terms of the pair $(Z, h_{\mu\nu})$; its effective dynamics is given by the action

\[ S_3 = \int d^3 x h^{\frac{1}{2}} (-R_3 + L_3), \]

(2.2)

where $R_3 = R_3(h_{\mu\nu})$ is the curvature scalar for the three-dimensional line element $ds_3^2 = h_{\mu\nu} dx^\mu dx^\nu$ and

\[ L_3 = \text{Tr} \left[ \nabla Z \left( \Xi - Z^T \Sigma Z \right)^{-1} \nabla Z^T \left( \Sigma - Z \Xi Z^T \right)^{-1} \right]. \]

(2.3)

It is important to note that in the present notations $Y^1$ corresponds to time, thus, we consider stationary equations of motion and $x^\mu$ are the coordinates on the Riemannian three-dimensional space. To translate this ($\sigma$-model) description into the language of the
field components of the heterotic string theory, let us introduce three doublets of \((Z, h_{\mu \nu})\)-
related potentials \((M_\alpha, \tilde{\Omega}_\alpha)\) \((\alpha = 1, 2, 3)\) according to the relations

\[
\begin{align*}
M_1 &= \mathcal{H}^{-1}, \quad \nabla \times \tilde{\Omega}_1 = \vec{J}, \\
M_2 &= \mathcal{H}^{-1} Z, \quad \nabla \times \tilde{\Omega}_2 = \mathcal{H}^{-1} \nabla Z - \vec{J} Z, \\
M_3 &= Z^T \mathcal{H}^{-1} Z, \quad \nabla \times \tilde{\Omega}_3 = \nabla Z^T \mathcal{H}^{-1} Z - Z^T \mathcal{H}^{-1} \nabla Z + Z^T \vec{J} Z,
\end{align*}
\]

(2.4)

where \(\mathcal{H} = \Sigma - Z \Sigma Z^T\) and \(\vec{J} = \mathcal{H}^{-1} \left( Z \Sigma \nabla Z^T - \nabla Z \Sigma Z^T \right) \mathcal{H}^{-1}\). In Eq. (2.4) the scalars \(M_\alpha\) are off-shell quantities, whereas the vectors \(\tilde{\Omega}_\alpha\) are defined on-shell. The scalar and vector potentials forming any doublet have the same matrix dimensionalities; let us represent them in the following block form

\[
\begin{pmatrix}
1 \times 1 & 1 \times d \\
d \times 1 & d \times d
\end{pmatrix}, \quad \begin{pmatrix}
1 \times 1 & 1 \times d & 1 \times n \\
d \times 1 & d \times d & d \times n
\end{pmatrix}, \quad \begin{pmatrix}
1 \times 1 & 1 \times d & 1 \times n \\
d \times 1 & d \times d & d \times n \\
n \times 1 & n \times d & n \times n
\end{pmatrix}
\]

(2.5)

for \(a = 1, 2, 3\) respectively, where, for example, the ‘13’ block components of the potentials \(M_2\) and \(\tilde{\Omega}_2\) are \(1 \times n\) matrices. Afterwards, let us define the following set of scalar and vector quantities:

\[
\begin{align*}
S_0 &= -M_{1,11} + 2M_{2,11} - M_{3,11}, \\
S_1 &= G_0 M_{1,22} G_0 + G_0 M_{2,22} + (M_{2,22})^T G_0 + M_{3,22} + S_0 \left[ -M_{1,12} G_0 - M_{2,12} + (M_{2,21})^T G_0 + M_{3,12} \right], \\
S_2 &= G_0 M_{1,22} - G_0 M_{2,22} G_0 + (M_{2,22})^T - M_{3,22} G_0 - 1 + S_0 \left[ -M_{1,12} G_0 - M_{2,12} + (M_{2,21})^T G_0 + M_{3,12} \right], \\
S_3 &= \sqrt{2} \left[ G_0 M_{2,23} + M_{3,23} + S_0 \left[ -M_{1,12} G_0 - M_{2,12} + (M_{2,21})^T G_0 + M_{3,12} \right] \right] \times \\
&\quad \times \left( M_{2,13} + M_{3,13} \right); \\
\vec{V}_1 &= \left[ -\tilde{\Omega}_{1,12} G_0 + \tilde{\Omega}_{2,12} + (\tilde{\Omega}_{2,21})^T G_0 + \tilde{\Omega}_{3,12} \right]^T, \\
\vec{V}_2 &= \left[ -\tilde{\Omega}_{1,12} G_0 - \tilde{\Omega}_{2,12} G_0 + (\tilde{\Omega}_{2,21})^T - \tilde{\Omega}_{3,12} G_0 \right]^T, \\
\vec{V}_3 &= \sqrt{2} \left( \tilde{\Omega}_{2,13} + \tilde{\Omega}_{3,13} \right)^T.
\end{align*}
\]

(2.6)

In terms of them the heterotic string theory fields read [20]:

\[
ds_D^2 = ds_{d+3}^2 = (dY + V_{1\mu} dx^\mu)^T S_1^{-1} (dY + V_{1\nu} dx^\nu) + S_0 ds_3^2,
\]

4
\[ e^\Phi = |S_0 \det S_1|^{\frac{1}{2}}, \]

\[ B_{mk} = \frac{1}{2} \left( S_1^{-1} S_2 - S_2^T S_1^{-1} \right)_{mk}, \]

\[ B_{m \nu d + \nu} = \left\{ V_{2 \nu} + \frac{1}{2} \left( S_1^{-1} S_2 - S_2^T S_1^{-1} \right) V_{1 \nu} - S_1^{-1} S_3 V_{3 \nu} \right\}_m, \]

\[ B_{d + \mu d + \nu} = \frac{1}{2} \left[ V_{1 \mu}^T \left( S_1^{-1} S_2 - S_2^T S_1^{-1} \right) V_{1 \nu} + V_{1 \mu}^T V_{2 \nu} - V_{1 \nu}^T V_{2 \mu} \right], \]

\[ A^I_m = \left( S_1^{-1} S_3 \right)_m^I, \]

\[ A^I_{d + \mu} = \left( -V_{3 \mu} + S_3^T S_1^{-1} V_{1 \mu} \right)_I. \] (2.7)

At the end of this section we would like to make two remarks. The first one concerns the procedure of constructing the solutions and their final representation; namely, it is convenient to perform the basic construction of the solution in terms of \( Z \) and \( h_{\mu \nu} \) and leave the result in the (2.7) form giving explicit expressions for the quantities \( S_\alpha, \vec{V}_\alpha \) and \( S_0, S_1^{-1}, \det S_1 \). The second one is related to the hidden symmetries of the compactified theory (2.2)-(2.3). It is easy to see that the transformation

\[ Z \rightarrow C_1 Z C_2 \] (2.8)

is a symmetry if \( C_1^T \Sigma C_1 = \Sigma \) and \( C_2^T \Xi C_2 = \Xi \). In [21] and [20] it was shown that this symmetry coincides with the total group of the three-dimensional charging symmetries of the theory. This group does not affect the trivial spatial asymptotics of the fields and forms the base for the three-dimensional generation technique of asymptotically flat solutions of the theory. The \( Z \)-representation, being the matrix potential representation with the lowest possible matrix dimensionality (the \( \sigma \)-model (2.2)-(2.3) is in fact a symmetric space model parameterizing the coset \( O(d + 1, d + 1 + n)/O(d + 1) \times O(d + 1 + n) \), see [23] and [20]) provides a linear realization of the charging symmetry transformations. These facts allow one to construct asymptotically flat classes of solutions in an explicitly charging symmetry invariant form. In the next sections we give concrete illustrations to the remarks formulated above.

### 3 String theories from static Einstein-Maxwell system

As it was pointed out in the Introduction, the close analogy between the heterotic (bosonic) and the Einstein-Maxwell theories will be used as certain underlying principle in the study of the two concrete string theories under consideration. This analogy, or correspondence,
was originally indicated in [21]; its explicit off- and on-shell status was established in [20], whereas some natural applications were given in [27], [28] and [29]. To formulate the correspondence in appropriate terms, let us represent the Einstein-Maxwell theory in a form very similar to the heterotic (bosonic) string theory one (2.2)-(2.3). Namely, let \( E \) and \( F \) be the conventional Ernst potentials of the stationary Einstein-Maxwell theory [30]. Let us express the corresponding three-dimensional Lagrangian

\[
L_3 = L_{EM} = \frac{1}{2} \left| \nabla E - \bar{F} \nabla F \right|^2 - \frac{1}{f} |\nabla F|^2,
\]

(3.1)

where \( f = \frac{1}{2} (E + \bar{E} - |F|^2) \), in terms of the \( 1 \times 2 \) complex potential \( z \) with

\[
z = (z_1, z_2)
\]

(3.2)

and (compare with [31])

\[
z_1 = \frac{1 - E}{1 + E}, \quad z_2 = \frac{\sqrt{2} F}{1 + E}.
\]

(3.3)

The result reads:

\[
L_{EM} = 2 \frac{\nabla z (\sigma_3 - z^+ z)^{-1} \nabla z^+}{1 - z \sigma_3 z^+},
\]

(3.4)

where \( \sigma_3 = \text{diag} (1, -1) \). Now, comparing Eqs. (2.3) and (3.4), it is easy to see that the map

\[
Z \mapsto z, \quad \Xi \mapsto \sigma_3, \quad \Sigma \mapsto 1,
\]

(3.5)

together with the operation interchange \( T \leftrightarrow + \), relates the heterotic (bosonic) and Einstein-Maxwell theories up to the factor ‘2’. However, this factor becomes necessary when considering the explicit on-shell realization of the correspondence (3.5) for the string theories with \( d + n > 2 \). Namely, it is possible to identify the Einstein-Maxwell theory with some special truncation of the string theory with \( d = 1 \) and \( n = 2 \) [27], and after that, to extend this result to the case of an arbitrary theory with \( d + n > 2 \) in a charging symmetry invariant form using the projectional formalism developed in [29]. The above mentioned special truncation is in fact a consistent ansatz with the \( 2 \times 4 \) potential \( Z_s \) parameterized as

\[
Z_s = (Z_{s1}, Z_{s2})
\]

(3.6)

where the block components \( Z_{sa} \) (\( a = 1, 2 \)) are taken in the form

\[
Z_{sa} = \begin{pmatrix}
z_a' & z_a'' \\
-z_a'' & z_a'
\end{pmatrix},
\]

(3.7)
and the complex functions $z_a = z'_a + iz''_a$ correspond to Eq. (3.2). The statement is that in this special case, the heterotic string and the Einstein-Maxwell theories coincide on-shell; in particular the Lagrangians (2.3) and (3.4) are equal if Eqs. (3.6) and (3.7) take place. Finally, the extension of stationary Einstein-Maxwell theory to the case of heterotic string theory with $d + n > 2$ is given by the map

$$Z_* \rightarrow Z = LZ_* R^T,$$

where $L^T \Sigma L = \Sigma_*$, $R^T \Xi R = \Xi_*$ and the matrices $\Sigma_*$, $\Xi_*$ and $\Sigma$, $\Xi$ correspond to the theories with $Z_*$ and $Z$, respectively. It is easy to see that for the two exceptional string theories with $d + n = 2$ this exact on-shell realization (3.4)-(3.8) of the correspondence (3.5) becomes impossible, thus, in this case the relations (3.5) possess a formal character. However, as it is shown below, a surprising on-shell correspondence between the string theories with $d + n = 2$ and the Einstein-Maxwell system nevertheless exists if we appropriately truncate both the string and Einstein-Maxwell theories.

The necessary truncation naturally arises in the framework of the continuous generalization of the general extremal Israel-Wilson-Perjes class of solutions established for the string theories with arbitrary values of the parameters $d$ and $n$ in [20]. This class can be represented in terms of the ansatz

$$Z = \Lambda Q,$$

where $\Lambda$ is a $(d+1) \times 1$ matrix function and $Q$ stands for a $1 \times 3$ constant matrix parameter for the theories with $d + n = 2$. The class of extremal Israel-Wilson-Perjes solutions arises when the (numerical) parameter $\kappa$, defined by the relation

$$\kappa = Q \Xi Q^T,$$

vanishes, the dynamical quantity $\Lambda$ is harmonic and the three-metric $h_{\mu\nu}$ is flat. In the Einstein-Maxwell theory the situation is extremely similar to this one according to the correspondence rule (3.3): the ansatz $z = \lambda q$, where $\lambda$ is a complex function and $q$ is a $1 \times 2$ constant complex parameter, gives the conventional Israel-Wilson-Perjes class of solutions if the parameter $\kappa = q \sigma_3 q^+$ is zero, $\lambda$ is harmonic and the three-metric is again flat. The consideration of this ansatz in the case of an arbitrary value of $\kappa$ naturally leads to a continuous generalization of the extremal class of Israel-Wilson-Perjes solutions to non-extremal classes. In the framework of this continuous extension, both extremal and non-extremal Kerr-Newman-NUT solutions with the electric and magnetic charges belong to the same family of solutions. The main idea of this article is to perform the corresponding continuous generalization of extremal classes of solutions for string theories with $d + n = 2$ to
non-extremal ones by considering the ansatz (3.9) with a non-zero value of the parameter \( \kappa \) (3.10).

This consideration must be performed on-shell. The straightforward substitution of the ansatz (3.9) into the equations of motion derived from Eqs. (2.2) and (2.3), leads to the following system of equations:

\[
\nabla^2 \Lambda + 2\kappa \nabla \Lambda \Lambda^T \left( \Sigma - \kappa \Lambda \Lambda^T \right)^{-1} \nabla \Lambda = 0,
\]

\[
R_{3\mu\nu} = \kappa \text{Tr} \left[ \Lambda_{,\mu} \left( 1 - \kappa \Lambda^T \Sigma \Lambda \right)^{-1} \Lambda^T_{,\nu} \left( \Sigma - \kappa \Lambda \Lambda^T \right)^{-1} \right].
\] (3.11)

It is clear that in the case \( \kappa = 0 \) we come back to the extremal case studied in [20], whereas for \( \kappa \neq 0 \) we have the above announced continuous extension of the formalism to the non-extremal case. Below, in this section, we study the situation in which \( \kappa \neq 0 \); here the equations (3.11) correspond to the effective Lagrangian

\[
L_{\text{eff}} = \kappa \text{Tr} \left[ \nabla \Lambda \left( 1 - \kappa \Lambda^T \Sigma \Lambda \right)^{-1} \nabla \Lambda^T \left( \Sigma - \kappa \Lambda \Lambda^T \right)^{-1} \right].
\] (3.12)

This Lagrangian has a quasi string theory form. To clarify this statement, let us introduce the new dynamical variable

\[
\zeta = (-\kappa)^{1/2} \Lambda^T
\] (3.13)

and consider the special case \( \kappa < 0 \). Let us also unify the two constrained string theories, i.e. the \( d = n = 1 \) and \( d = 2, \ n = 0 \) models restricted by Eq. (3.9), into a single construction, which in fact will correspond to the former system. For this system \( \Sigma = \Xi = \text{diag} (-1, -1, 1) \) and \( \zeta = (\zeta_1, \zeta_2, \zeta_3) \). An evident statement is that the latter system arises in the special case when \( \zeta_3 = 0 \). Thus, it is possible to consider the \( d = n = 1 \) heterotic string theory ansatz as a (consistent) subsystem of the \( d = 2, \ n = 0 \) bosonic string theory one. Finally, both systems are simultaneously described by a single effective theory which, however, is not exactly of the form (2.3) in view of the fact that the string matrix potential \( Z \) must contain at least two rows. Interestingly, both these problems can be solved by the formal introduction of the following \( 2 \times 3 \) potential

\[
Z = \begin{pmatrix} \zeta \\ 0 \end{pmatrix}
\] (3.14)

together with the corresponding matrices \( \Sigma \) and \( \Xi \). In these terms the effective theory takes the form of Eq. (2.3) and can be interpreted in the framework of the \( d = n = 1 \) heterotic
string theory restricted by the relation (3.14) which is a consistent ansatz for the $d = n = 1$ theory.

Now it is possible to apply the Ernst matrix potential approach [21] to obtain an appropriate interpretation of this effective theory in terms of the well known classical systems of gravity. This approach is based on the use of a pair of Ernst matrix potentials $\mathcal{X}$ and $\mathcal{A}$ defined by the relations

$$
\mathcal{X} = 2(\mathcal{Z} + \Sigma)^{-1} - \Sigma, \quad \mathcal{A} = \sqrt{2}(\mathcal{Z} + \Sigma)^{-1} \mathcal{Z},
$$

(3.15)

where $\mathcal{Z} = (\mathcal{Z}_1 \mathcal{Z}_2)$, compare to Eqs. (3.6)-(3.7). In terms of these potentials the Lagrangian (2.3) takes the form

$$
L_3 = \text{Tr} \left[ \frac{1}{4} \left( \nabla \mathcal{X} - \nabla \mathcal{A} \mathcal{A}^T \right) \mathcal{G}^{-1} \left( \nabla \mathcal{X}^T - \mathcal{A} \nabla \mathcal{A}^T \right) \mathcal{G}^{-1} + \frac{1}{2} \mathcal{G}^{-1} \nabla \mathcal{A} \nabla \mathcal{A}^T \right],
$$

(3.16)

where $\mathcal{G} = \frac{1}{2}(\mathcal{X} + \mathcal{X}^T - \mathcal{A} \mathcal{A}^T)$. By substituting (3.14) into (3.15) and then performing the calculations, we arrive to the effective Lagrangian

$$
L_{\text{eff}} = \frac{1}{4F^2} \nabla F^2 + \frac{1}{F} \left( \nabla V^2 - \nabla U^2 \right),
$$

(3.17)

where the scalar potentials $F, V$ and $U$ are

$$
F = \frac{1 - \zeta_1^2 - \zeta_2^2 + \zeta_3^2}{(1 - \zeta_1)^2}, \quad V = \frac{\zeta_2}{1 - \zeta_1}, \quad U = \frac{\zeta_3}{1 - \zeta_1}.
$$

(3.18)

A classical interpretation of the effective theory is now clear: the quantity $|F|$ can be considered as the $-G_{tt}$ component of the static Einstein-Maxwell theory, whereas $\sqrt{2V}$ and $\sqrt{2U}$, as the electric and/or magnetic potentials of that theory. Thus, in the framework of this interpretation one immediately obtains

$$
L_{\text{eff}} = \frac{1}{2} L_{\text{EM}},
$$

(3.19)

where $L_{\text{EM}}$ is the corresponding Einstein-Maxwell Lagrangian in the static case. Note that for $F > 0$ ($F < 0$) the potential $V$ ($U$) must be imaginary and $U$ ($V$) must be real in order to perform such identification. One also must remember that for the $d = n = 1$ theory $U = 0$, whereas for the $d = 2, n = 0$ model this dynamical variable is not restricted. Let us now consider the case $U = 0$ for both string theories. If $F < 0$ we have the conventional interpretation of the effective theory in terms of the static electric (magnetic)
Einstein-Maxwell system; if $\mathcal{F} > 0$ the potential $\mathcal{V}$ becomes essentially imaginary. Actually, the conventional static Einstein-Maxwell theory with either electric or magnetic potential corresponds to an indefinite $\sigma$–model, whereas the case under consideration ($\mathcal{F} > 0$), to a positive definite one.

An interpretation of the effective theory with $\mathcal{U} = 0$ and $\mathcal{F} > 0$ in terms of well known gravity models with strictly real fields takes place when one identifies $\mathcal{F}^{\frac{1}{2}}$ and $\mathcal{V}$ with the $-G_{tt}$ and the rotational metric component, respectively. In turns out that in this case one can establish the following relationship

$$L_{\text{eff}} = 2L_{\text{GR}},$$  \hspace{1cm} (3.20)

where $L_{\text{GR}}$ is the conventional Lagrangian of stationary General Relativity. Note that the two special cases considered above (with negative and positive values of $\mathcal{F}$) are related by the map

$$\mathcal{F} \longleftrightarrow \frac{1}{2}\mathcal{F}, \quad \mathcal{V} \longleftrightarrow i\mathcal{V},$$  \hspace{1cm} (3.21)

which is nothing else than the classical Bonnor transformation [32]. A last remark related to the factors ‘$\frac{1}{2}$’ and ‘2’ is in order: when considering the axisymmetric theory these factors lead to the maps $\gamma \longleftrightarrow \frac{1}{2}\gamma$ and $\gamma \longleftrightarrow 2\gamma$, respectively, where $\gamma = \gamma(\rho, z)$ enters the line element in the Lewis–Papapetrou form

$$ds^2 = e^\gamma (d\rho^2 + dz^2) + \rho^2 d\varphi^2;$$  \hspace{1cm} (3.22)

thus, such maps do not affect our identifications.

At the end of this section let us establish all the hidden symmetries of the effective theory (3.17). In order to do this, let us introduce the $2 \times 2$ matrix

$$M = \begin{pmatrix} f^{-1} & f^{-1}\chi_r \\ f^{-1}\chi_l & f + f^{-1}\chi_l\chi_r \end{pmatrix},$$  \hspace{1cm} (3.23)

where $f = |\mathcal{F}|^{\frac{1}{2}}$, $\chi_r = \mathcal{V} + \mathcal{U}$ and $\chi_l = \text{sign}(\mathcal{F})(\mathcal{V} - \mathcal{U})$. It is easy to see that this matrix parametrizes the group $SL(2, \mathbb{R})$; a less obvious fact is that

$$L_{\text{eff}} = \frac{1}{2}Tr \left(\nabla MM^{-1}\right)^2,$$  \hspace{1cm} (3.24)

i.e., our complete ($\mathcal{U} \neq 0$) effective theory (3.17) coincides with the $SL(2, \mathbb{R})$ principal chiral model coupled to gravity. From this fact it immediately follows that the group of symmetry transformations acts as

$$M \longrightarrow C_i^TMC_r,$$  \hspace{1cm} (3.25)
where the unimodular constant matrices $C_{l,r}$ can be obtained from $M$ by making use of the substitutions $f \rightarrow s_{l,r}$, $\chi_l \rightarrow \beta_l$, and $\chi_r \rightarrow \alpha_{l,r}$. It is worth noticing that the transformations $C_l$ and $C_r$ are independent each other, thus, there is no relation between the $-l$ and $-r$ labeled constant parameters $s_{l,r}$, $\alpha_{l,r}$ and $\beta_{l,r}$. For example, one can fix $C_l = 1$ and then write down the transformations that correspond to the parameters $s_r$, $\alpha_r$ and $\beta_r$; the result reads:

$$f \rightarrow s_r f, \quad \chi_r \rightarrow s_r^2 \chi_r, \quad \chi_l \rightarrow \chi_l; \quad (3.26)$$

$$f \rightarrow f, \quad \chi_r \rightarrow \chi_r + \alpha_r, \quad \chi_l \rightarrow \chi_l; \quad (3.27)$$

$$f \rightarrow \frac{f}{1 + \beta_r \chi_r}, \quad \chi_r \rightarrow \frac{\chi_r}{1 + \beta_r \chi_r}, \quad \chi_l \rightarrow \frac{\chi_l + \beta_r (f^2 + \chi_l \chi_r)}{1 + \beta_r \chi_r}. \quad (3.28)$$

Here the transformation (3.26) plays the role of scaling, whereas (3.27) and (3.28) are shift and Ehlers-type maps, respectively. Of course the matrix $C_r$ generates the transformations which can be obtained from (3.26)–(3.28) using the interchange $r \leftrightarrow l$ in these relations.

Let us now discuss the special case corresponding to the subgroup of charging symmetry transformations. This subgroup preserves the trivial solution of the theory under consideration. Such a solution corresponds to the matrix $M = M_0 = 1$ since for this special value one obtains $Z_0 = 0$ (see Eqs. (3.13)–(3.14),(3.18) and (3.23)). Thus, for the subgroup of charging symmetries one obtains $C_l = C_r^{T -1}$ from Eq. (3.25), or, in terms of the corresponding parameters

$$s_l = \frac{s_r}{s_r^2 + \alpha_r \beta_r}, \quad \alpha_l = -\frac{\beta_r}{s_r^2 + \alpha_r \beta_r}, \quad \beta_l = -\frac{\alpha_r}{s_r^2 + \alpha_r \beta_r}. \quad (3.29)$$

Some remarks on the $d = n = 1$ string theory are in order. It is clear that the effective theory with $\mathcal{U} \neq 0$ corresponds to the string theory with $d = 2$ and $n = 0$. The theory with $d = n = 1$ implies $\mathcal{U} \equiv 0$, which in turn leads to $\chi_l = \chi_r \equiv \chi$, and thus, $M = M^T$ when $\mathcal{F} > 0$. Thus, we deal with the symmetric space model $SL(2, \mathbb{R})/SO(2)$ coupled to gravity which is, in fact, equivalent to the stationary General Relativity theory up to a factor ‘2’ as it was explained above. The transformation rule (3.25) must preserve the symmetric property of $M$, this implies that $C_l = C_r \equiv C \in SL(2, \mathbb{R})$. For these symmetric space the parameters $s_l = s_r \equiv s$, $\alpha_l = \alpha_r \equiv \alpha$ and $\beta_l = \beta_r \equiv \beta$ define the conventional scaling, shift and Ehlers maps. On the other hand, the one–parameter charging symmetry subgroup is given by the transformation

$$z \rightarrow e^{i\epsilon} z, \quad (3.30)$$
where \( z = \frac{1}{1+\xi^2} \), \( E = f + i\chi, \epsilon = 2 \arctan \beta \) (the remaining parameters are related to \( \beta \) as follows \( \alpha = -\beta \) and \( s = \sqrt{1+\beta^2} \)).

We hope to use the principle chiral model (3.23)–(3.24) for the generation of new solutions in a forthcoming article. In the next section we perform a further reduction of the effective theory (3.17) to the \( \sigma \)–model with a single dynamical variable in a charging symmetry invariant form. Our goal is to obtain an effective dilaton gravity system with arbitrary coupling which allows one to consider the general field configurations of Nordstrom–Reissner type in both the \( d = 2, n = 0 \) and the \( d = n = 1 \) string theories.

4 Explicit solutions via 3D dilaton gravity

In order to perform this truncation in the most general form, let us come back to Eq. (3.11) and consider the new consistent ansatz

\[
\Lambda = \Psi \mathcal{P}^T,
\]

where \( \Psi \) is a dynamical function and \( \mathcal{P} \) is a \( 1 \times (d+1) \) constant row. The resulting system of equations of motion reads:

\[
\nabla^2 \phi = 0,
R_{\mu\nu} = \sigma \phi,_{\mu} \phi,_{\nu},
\]

where we have set \( \sigma = \tau \kappa \),

\[
\tau = \mathcal{P} \Sigma \mathcal{P}^T
\]

and

\[
\Psi = \frac{\tanh \left( \sigma \frac{i}{2} \phi \right)}{\sigma \frac{i}{2}}.
\]

The parameter \( \sigma \) plays the role of the dilaton-gravity coupling; we identify the field \( \phi \) with the effective three-dimensional dilaton interacting with the three-metric field \( h_{\mu\nu} \) according to the equations (4.2). Note that, in general, \( \sigma \) has arbitrary sign and for any sign of \( \sigma \) the relation (4.4) is real. Namely, for \( \sigma = 0 \) we understand Eq. (4.4) in the sense of the limit procedure, thus, in this case \( \Psi = \phi \); for \( \sigma < 0 \) Eq. (4.4) can be rewritten as \( \Psi = \tan \left( \frac{\sigma}{(-\sigma)^{\frac{i}{2}}} \phi \right) \). In this section we shall construct solutions for the \( d = n = 1 \) and \( d = 2, n = 0 \)
string theories as extensions of an arbitrary solution \((\phi, h_{\mu\nu})\) of this effective dilaton gravity system. Our extensions will preserve the asymptotical triviality of the seed solutions in the following sense: dilaton gravity solutions with Coulomb behavior at spatial infinity are mapped to string theory solutions possessing the same underlying property. This means that we must perform our extension procedure in such a way that all possible Dirac string peculiarities, which naturally arise in the framework of any more or less general symmetry based solution-constructing technique, will vanish. For example, in General Relativity the Ehlers symmetry transformation \([33]\) generates the parameter NUT from the asymptotically trivial Schwarzshild solution; the resulting family of Taub-NUT solutions \([14]\) possesses a Dirac string peculiarity of rotational type. In the pure General Relativity it is impossible to specify the generation procedure in the above mentioned sense because, in fact, there exists only a one-parametric charging symmetry transformation whose ‘specification’ leads to an identical map. This transformation coincides with the Ehlers symmetry which must be ‘normalized’ to preserve the asymptotical flatness property, see, for example, \([21]\), where this material together with its straightforward generalization to the string theory case is considered in details. Note that in our approach all the classical nonlinear symmetries (like the Ehlers and Harrison transformations) arise in a matrix-valued framework; nevertheless it is possible to identify some string theory symmetries as the Harrison and Ehlers maps in the conventional non-matrix sense, see \([34], [35]\). It is clear that in the string theories under consideration the group of charging symmetries has a much more rich structure \([20]\); moreover, it is possible to relate different string theories labeled by distinct values of the parameters \(d\) and \(n\) in a charging symmetry invariant form \([29]\). All these facts allows one to realize the above mentioned extension program not only in the case of the two special string theories under consideration, but also for the general case. Below we give the corresponding results for the \(d = n = 1\) and \(d = 2, \ n = 0\) theories.

The concrete asymptotically trivial and charged dilaton gravity solution of Nordstrom-Reissner type reads

\[
\phi = \frac{1}{\sqrt{2\sigma^2}} \log \left( \frac{R + m\sigma^{\frac{1}{2}}}{R - m\sigma^{\frac{1}{2}}} \right),
\]

\[
ds^2 = dR^2 + (R^2 - \sigma m^2)(d\theta^2 + \sin^2 \theta d\phi^2),
\]

(4.5)

where \(m\) is an arbitrary real parameter. We note again that the \(\phi\)-relation in Eq. (4.5) is real and well defined for the arbitrary sign of \(\sigma\): if \(\sigma = 0\) then \(\phi = \sqrt{2m/R}\); for \(\sigma < 0\) one also has from Eq. (1.3) that \(\phi = \frac{1}{\sqrt{2(1-\sigma)}} \arctan \left( \frac{2m(-\sigma)^{\frac{1}{2}}R}{R^2 + m^2\sigma} \right)\). Thus, Eqs. (1.1), (1.4) and (1.3) give a correct solution of Eqs. (3.11) which have, as it is easy to see, the standard monopole
behavior at spatial infinity. From Eqs. (4.4) and (4.5) it follows that the parameter $\sqrt{2}m$ plays the role of the formal Coulomb charge of this solution. The solution (4.5) gives a concrete realization of the on–shell field configurations of the general charged dilaton gravity with the trivial asymptotics at spatial infinity. In principle, our further analysis can be related to such general asymptotically flat solutions. However, we shall take into account just the special class of solutions corresponding to the Nordstrom-Reissner one.

Our next step is to calculate the matrix potentials $M_\alpha$ and $\vec{\Omega}_\alpha$ according to Eq. (2.4). In order to get the more symmetric form of the resulting formulas let us redefine the parameter $P$ as $P \rightarrow P_\Sigma$; this map preserves the $\tau$-value, see Eq. (4.3). The result reads:

$$M_1 = \Sigma + \frac{\sinh^2 \left(\sigma \frac{1}{2} \phi\right)}{\tau} p^T p, \quad \vec{\Omega}_1 = 0;$$
$$M_2 = \frac{\sinh \left(\sigma \frac{1}{2} \phi\right) \cosh \left(\sigma \frac{1}{2} \phi\right)}{\sigma \frac{1}{2}} p^T q, \quad \vec{\Omega}_2 = \vec{\omega} p^T q;$$
$$M_3 = \frac{\sinh^2 \left(\sigma \frac{1}{2} \phi\right)}{\kappa} q^T q, \quad \vec{\Omega}_3 = 0,$$

(4.6)

where the vector function $\vec{\omega}$ is defined on–shell by the relation

$$\nabla \times \vec{\omega} = \nabla \phi.$$

(4.7)

The explicit calculation of $\vec{\omega}$ in the case of the solution (4.5) gives the following only non-zero component of this vector function

$$\omega_\phi = \sqrt{2}m \cos \theta.$$

(4.8)

Of course, one obtains the same asymptotical value of $\vec{\omega}$ for the general solution of the dilaton gravity with a leading term of monopole type.

Now let us compute the potentials $S_0$, $S_\alpha$ and $\vec{V}_\alpha$ for the two concrete string theories under consideration. For the $d = n = 1$ theory $P = (P_1, P_2)$ and $Q = (Q_1, Q_2, Q_3)$ are $1 \times 2$ and $1 \times 3$ rows respectively, whereas $\Sigma = -1$ is a $2 \times 2$ matrix and $G_0 = -1$ is a number. In this special case the block segmentations given by Eq. (2.5) coincide with the usual matrix structures and all the calculations are especially simple. So, for the potentials $\vec{V}_\alpha$ one immediately obtains that

$$\vec{V}_1 = (P_1 Q_2 - P_2 Q_1) \vec{\omega}, \quad \vec{V}_2 = (P_1 Q_2 + P_2 Q_1) \vec{\omega}, \quad \vec{V}_3 = \sqrt{2} P_1 Q_3 \vec{\omega}.$$

(4.9)

These vector quantities lead to the appearance of Dirac string peculiarities for the metric (the NUT parameter), gauge (the magnetic charge) and Kalb Ramond fields according to
Eqs. (2.7) and (4.9). In order to remove these peculiarities and to obtain the guaranteed asymptotically flat string theory solutions which correspond to the charged solutions of the effective dilaton gravity system, one must restrict the parameters $\mathcal{P}$ and $\mathcal{Q}$ in such a way that all the quantities $\vec{V}_\alpha$ vanish identically. Let us also keep the arbitrariness of the $\sigma$ value for the restricted class of solutions in order to preserve the possibility of working within dilaton gravity with arbitrary coupling. These conditions lead to a unique choice of the restriction: $P_1 = Q_1 = 0$ and arbitrary values for the remaining parameters $P_2, Q_2$ and $Q_3$. In this special case the calculation of the scalar potentials becomes much simpler and one finally obtains $S_0 = 1$ and

$$S_1 = -\left[\cosh(\frac{\sigma}{2} \phi) + \frac{Q_2}{(1)} \frac{\sinh(\frac{\sigma}{2} \phi)}{(-\kappa)^{\frac{1}{2}}}\right]^2,$$

$$S_2 = -\left[\frac{Q_3}{(-\kappa)^{\frac{1}{2}}} \frac{\sinh(\frac{\sigma}{2} \phi)}{(-\kappa)^{\frac{1}{2}}}\right]^2,$$

$$S_3 = -\sqrt{2}Q_3 \frac{\sinh(\frac{\sigma}{2} \phi)}{(-\kappa)^{\frac{1}{2}}} \left[\cosh(\frac{\sigma}{2} \phi) + \frac{Q_2}{(1)} \frac{\sinh(\frac{\sigma}{2} \phi)}{(-\kappa)^{\frac{1}{2}}}\right],$$

where now $\tau = -P_2^2$ and $\kappa = -Q_2^2 + Q_3^2$. Note that the parameter $P_2$ can be removed from Eqs. (4.3) and (4.10) with the help of the substitutions $\sigma \to -\kappa, \phi \to |P_2| \phi$ and $m \to |P_2| m$. Equations (4.10) define the extension of the arbitrary solution of the dilaton gravity system (4.2) to the case of the $d = n = 1$ theory completely. This extension is automatically free of any string peculiarity of Dirac type ($\vec{V}_\alpha = 0$) (up to construction of this extension). In particular, the solution (4.3) leads to charged asymptotically trivial heterotic string theory fields.

Now let us consider the bosonic string theory case with $d = 2, n = 0$. Here the block structure of the matrix potentials $\mathcal{M}_\alpha$ and $\vec{\Omega}_\alpha$ is not trivial, and it is convenient to parameterize the $1 \times 3$ rows $\mathcal{P}$ and $\mathcal{Q}$ as $\mathcal{P} = (P_1, P)$ and $\mathcal{Q} = (Q_1, Q)$ where $P = (P_2, P_3)$ and $Q = (Q_2, Q_3)$, respectively. The calculation of the $\vec{V}_\alpha$-potentials gives the following result:

$$\vec{V}_1 = (P_1 Q^T - Q_3 \sigma_3 P^T) \vec{\omega}, \quad \vec{V}_2 = (P_1 \sigma_3 Q^T + Q_1 P^T) \vec{\omega},$$

where $\sigma_3 = -G_0$ in this theory. A further removal of the Dirac string peculiarities from the solution leads again to the restriction $P_1 = Q_1 = 0$ and to arbitrary values of $P$ and $Q$ under the same assumptions that in the previous case. Then, one obtains again that $S_0 = 1,$
whereas for the remaining part of the scalar potentials one has the following relations:

\[
S_1 = -\sigma_3 + \sinh^2 \left( \sigma_1^2 \phi \right) \left[ \frac{P^T P}{\tau} + \frac{Q^T Q}{\kappa} \right] - \frac{\sinh \left( \sigma_1^2 \phi \right) \cosh \left( \sigma_1^2 \phi \right)}{\sigma_1^2} \left[ P^T Q + Q P^T \right] \\
S_2 = \sinh^2 \left( \sigma_1^2 \phi \right) \left[ \frac{P^T P}{\tau} + \frac{Q^T Q}{\kappa} \right] \sigma_3 + \frac{\sinh \left( \sigma_1^2 \phi \right) \cosh \left( \sigma_1^2 \phi \right)}{\sigma_1^2} \left[ Q^T P - Q P^T \right] \sigma_3, \tag{4.12}
\]

where now \( \tau = -P \sigma_3 P^T \) and \( \kappa = -Q \sigma_3 Q^T \) (we have also performed the map \( P \rightarrow \sigma_3 P \) which preserves the \( \tau \)-value and leads to some simplifications of the result). The last step is to calculate the quantities \( \det S_1 \) and \( S_1^{-1} \). For \( \det S_1 \), after some algebra, one concludes that

\[
\det S_1 = - \left[ \cosh \left( \sigma_1^2 \phi \right) + \frac{\sinh \left( \sigma_1^2 \phi \right)}{\sigma_1^2} Q \sigma_3 P^T \right]^2, \tag{4.13}
\]

whereas for \( S_1^{-1} \) one simply obtains \( S_1^{-1} = \frac{1}{\det S_1} \sigma_2 S_1^T \sigma_2 \) in view of its \( 2 \times 2 \) matrix nature.

Eqs. (4.12), (4.13) together with the relations \( \vec{V}_\alpha = 0 \) and the above calculated quantity \( S_1^{-1} \) completely define the \( d = 2, n = 0 \) bosonic string theory extension of the effective three-dimensional dilaton gravity. Again, up to construction, this extension is automatically free of any Dirac string peculiarity. This last step completes the program formulated at the end of the previous section; any further analysis will be related to the special choice of the solution of the dilaton gravity system of equations (4.2). In this context we refer to the Nordstrom-Reissner type solution given by Eq. (4.5) as to the typical monopole solution which is widely represented in the classical \( \sigma \)-model gravity theories.

5 Conclusion

The results of this article allows one to transform static electromagnetic solutions of the Einstein-Maxwell theory to the corresponding solutions of the five-dimensional bosonic string theory and also to extend the electric (magnetic) static Einstein-Maxwell fields to the four-dimensional heterotic string theory with one vector field. As an alternative starting theory for the generating procedure it is possible to use the stationary General Relativity. Let us note that in our approach all the classical starting systems arise in some interpretation invariant form. This means that we do not actually start from, for example, General Relativity which represents the string theory ansatz with vanishing matter fields and Kaluza-Klein
metric modes. In fact our Einstein-Maxwell and General Relativity subsystems arise as some formal objects from the symmetry invariant point of view and their physical nature is nothing else than a very special choice of the starting theories in the framework of our formalism. The same situation takes place in the effective three-dimensional dilaton gravity, which we have explored in details as the natural and the simplest physically interesting starting system. In particular, we have presented explicit relations for the extension of the solutions of the dilaton gravity system with arbitrary value of coupling to both the heterotic and bosonic string theories under consideration. More precisely, we have shown that this extension can be performed in a form which is free of any peculiarity of Dirac string type for solutions with a leading term of monopole type at spatial infinity. Note that the removal of these peculiarities means in fact some gauge fixing with respect to the subgroup of charging symmetries. This subgroup forms a total invariance class of extensions which are free of any parameter fixing. Actually, the general dilaton gravity extension scheme is related to arbitrary parameters $P$ and $Q$, which effectively transform as $P \to P \Sigma C_1$ and $Q \to Q C_2$ under the charging symmetry subgroup of transformations (2.8). However, the consistent removal of peculiarities demands the vanishing of the first components of the rows $P$ and $Q$ - a condition which does not hold when applying general charging symmetry transformations. Thus, the charging symmetry invariant class of string theory charged solutions related to the dilaton gravity subsystem definitely contains Dirac string peculiarities, whereas the completely asymptotically trivial string theory field configuration is charging symmetry non-invariant.

As it was briefly mentioned in the Introduction and also partially supported by our Nordstrom-Reissner solution example, the natural and the nearest applications of the new solution generation procedure proposed in this article is black hole physics. We hope to generalize our approach to the case of string theory with arbitrary dimensionality and with arbitrary number of the Abelian gauge fields. The leading principle for this generalization will be related to demanding a continuous and charging symmetry invariant extension of the general Israel-Wilson-Perjes class of solutions [20] to the non-extremal case. In a forthcoming publication [23] we hope to realize this program and to reach, in particular, the critical cases for the heterotic and bosonic string theories, which are the most interesting from the physical point of view.

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