The Interface Tension in Quenched QCD at the Critical Temperature*

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We present results for the confinement-deconfinement interface tension $\alpha_{cd}$ of quenched QCD. They were obtained by applying Binder’s histogram method to lattices of size $L_x^2 \times L_y \times L_t$ for $L_t = 2$ and $L = 8, 10, 12$ and $14$ with $L_z = 30$ for $L = 8$ and $L_z = 3L$ otherwise. The use of a multicanonical algorithm and cylindrical geometries have turned out to be crucial for the numerical studies.

1. Introduction

At high temperature a phase transition occurs in QCD. In the quenched approximation (i.e. without any light quarks) this transition is of first order and separates a low temperature confined phase from a high temperature deconfined phase. The dynamics of a system which crosses the transition temperature $T_c$ (as e.g. in the early universe or in heavy ion collisions) depends on the free energy

$$F_{cd} = \alpha_{cd} A$$

(1)

of an interface of area $A$ between regions of confined and deconfined matter. The interface tension $\alpha_{cd} = \sigma_{cd} T_c$ was investigated before in Monte Carlo simulations of lattice systems with $L_t = 2$ using various approaches (see [1]). Lately these results have been questioned based on an application of Binder’s histogram method to cubic spatial volumes $L^3$ with $L = 6, 8, 10, 12$ (see [3]). However, these results might have been plagued by interfacial interactions. Therefore, we present results using the same method but on asymmetric volumes ($L_z > L_x = L_y$) thereby reducing these interactions.

2. The Interfacial Free Energy

We consider SU(3) pure gauge theory with the Wilson action $S$ on a cylindrical lattice of size $L_x \times L_y \times L_z \times L_t$ with $L_x = L_y = L$ and $L_z \geq 3L$ at the critical coupling $\beta_c$ for $L_t = 2$. We use periodic boundary conditions in the direction and $C$–periodic boundary conditions in the spatial directions, i.e.

$$U_\mu(\vec{x} + L_t \vec{e}_i, t) = U_\mu^*(\vec{x}, t), \text{ for } i = x, y, z$$

(2)

$$U_\mu(\vec{x}, t + L_t) = U_\mu(\vec{x}, t)$$

(3)

Because of the $C$–periodic boundary conditions the value of the Polyakov line $\Omega_L(\vec{x}) \equiv tr \left( \prod_{i=1}^{L_t} U_0(\vec{x}, t) \right)$ will satisfy

$$\Omega_L(\vec{x} + L_t \vec{e}_i) = \Omega_L^*(\vec{x}) \text{ for } i = x, y, z$$

(4)

Therefore, no bulk configurations in either of the two deconfined phases that have nonvanishing imaginary part of $\Omega_L \equiv 1/(L^2 L_z) \sum_{\vec{x}} \Omega_L(\vec{x})$ can exist and the probability distribution $P_L(\rho) d\rho$ of $\rho \equiv Re \Omega_L$ takes the form sketched in Fig.\[1.\] The system is most likely in either the one remaining deconfined phase corresponding to $\rho^{(2)}$ or the confined phase at $\rho^{(1)}$. When $\rho$ is increased from $\rho^{(1)}$, bubbles of deconfined phase form. These configurations are suppressed by the interfacial free energy $\sigma_{cd} A$, where $A$ is the surface area of the bubble. It grows until finally its surface is larger than the surface $L^2$ of two planar interfaces which devide the lattice into three parts.

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Figure 1. Schematic probability distribution for the order parameter. The dotted line indicates the multicanonical distribution.

as depicted in the second part of Fig. 2. Since the interface area of the two planar interfaces is independent of $\rho$ the probability $P_L$ is constant in the region where their contributions dominate, i.e. around $\rho^{\text{min}}$. Because of the $C-$periodic boundary conditions these interfaces always separate a region in the confined phase from one in the deconfined phase that has $\text{Im}(\Omega_L) = 0$. Thus the corresponding configurations will be exponentially suppressed by the interfacial free energy of two confined-deconfined interfaces. Taking into account the capillary wave fluctuations of the interfaces as well as their translational degrees of freedom leads to additional power law corrections \cite{8,9} giving

$$P_L^{\text{min}} \propto L_z^2 \cdot L^{d-3} \cdot \exp \left(-2\sigma_{cd} L^{d-1}\right).$$

(5)

for $d-$dimensional spatial volumes. This relation will be used to determine $\sigma_{cd}$ from the distributions obtained on finite lattices.

In order to calculate the probability distribution $P_L(\rho)$, one has to simulate the SU(3) pure gauge theory at the deconfinement phase transition. But because of eq. \ref{eq:5} any standard local updating algorithm will have autocorrelation times $\tau_L$ which increase exponentially with $L^2$ ("supercritical slowing down"). The use of the multicanonical algorithm reduces this effect consider-
with $L = 8, 10, 12,$ and $14$ and $L_z = 30$ for $L = 8$ and $L_z = 3L$ otherwise. Fig. 3 shows the real part of $\Omega_L(z) \equiv 1/L^2 \sum_{x,y} \Omega_L(x, y, z)$ for a typical configuration close to $\rho^{(min)}$ on a $14^2 \times 42 \times 2$ lattice. As expected from section 2, one can identify two interfaces between the confined phase and the deconfined phase. The imaginary part of $\Omega_L$ is always zero. In Fig. 4 the resulting probability distributions are shown. In contrast to the distributions for cubic volumes ($L$ values as before) they all have a region of constant probability in between the two peaks. This supports the scenario developed in section 2.

In order to extract the interface tension we evaluate the quantities

\[ F_L^{(1)} = \frac{1}{2L^2} \ln \frac{P_{L_{\max}}}{P_{L_{\min}}} + \frac{3}{4} \ln L_z - \frac{1}{2} \frac{\ln L}{L^2} \]  

and

\[ F_L^{(2)} = -\frac{1}{2L^2} \ln P_{L_{\min}} + \frac{\ln L_z}{L^2} \]  

where $P_{L_{\max}} = \frac{1}{2}(P_{L_{\max,1}} + P_{L_{\max,2}})$. Note that one expects $P_{L_{\max}} \propto \sqrt{L_z}L^2$. According to eq. 8 both quantities should be linear functions of $1/L^2$. Their intercept with the $y$-axis is $\sigma_{cd}$. We extract $F_L^{(1)}$ and $F_L^{(2)}$ from the probability distributions of Fig. 4. The results are plotted in Fig. 5 together with the corresponding linear fits. For the interface tension we get the value

\[ \frac{\alpha_{cd}}{T_c^3} = 0.10(1). \]  

It agrees within errors with the value obtained by replacing the Polyakov line by the action density. The agreement with \cite{1} is good while \cite{2} quotes a slightly higher value. Still the discrepancy between these results and \cite{3} which used Binder’s histogram method for cubic volumes is reduced considerably and can thus be attributed mainly to interfacial interactions.

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$\ln P_L(\Omega)$

Order Parameter $\Omega$
\[ \text{Re} \left( \Omega_L(z) \right) \]
\[ \ln P_L(\rho) \]
