Complementary Constraints on Brane Cosmology

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The acceleration of the expansion of the universe represents one of the major challenges to our current understanding of fundamental physics. In principle, to explain this phenomenon, at least two different routes may be followed: either adjusting the energy content of the Universe – by introducing a negative-pressure dark energy – or modifying gravity at very large scales – by introducing new spatial dimensions, an idea also required by unification theories. In the cosmological context, the role of such extra dimensions as the source of the dark pressure responsible for the acceleration of our Universe is translated into the so-called brane world (BW) cosmologies. Here we study complementary constraints on a particular class of BW scenarios in which the modification of gravity arises due to a gravitational leakage into extra dimensions. To this end, we use the most recent Chandra measurements of the X-ray gas mass fraction in galaxy clusters, the WMAP determinations of the baryon density parameter, measurements of the Hubble parameter from the HST, and the current supernova data. In agreement with other recent results, it is shown that these models provide a good description for these complementary data, although a closed scenario is always favored in the joint analysis. We emphasize that observational tests of BW scenarios constitute a natural verification of the role of possible extra dimensions in both fundamental physics and cosmology.

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I. INTRODUCTION

The idea of a dark pressure-dominated universe is usually associated with unknown physical processes involving either new fields in high energy physics or modifications of gravity at very large scales. If one chooses to follow the former route, then the two favorite candidates for this mysterious component are the energy density stored on the true vacuum state of all existing fields in the Universe, i.e., the cosmological constant (Λ), and the potential energy density associated with a dynamical scalar field (φ), usually called dark energy or quintessence (see, e.g., [1] for some recent reviews on this topic).

The second route in turn is predominantly related to the existence of extra spatial dimensions, an idea that is required in various theories beyond the standard model of particle physics, especially in theories for unifying gravity and the other fundamental forces, such as superstring or M theories. In the eleven-dimensional supergravity model of Hořava and Witten [2], for instance, the ordinary matter fields are assumed to be confined in a submanifold (brane) immersed in a higher dimensional space, usually named bulk. An important development of this idea was subsequently given by Randall and Sundrum [3] who showed that, if our three-dimensional world is embedded in a four-dimensional anti-de Sitter bulk, gravitational excitations are confined close to our submanifold, giving rise to the familiar 1/r^2 law of gravity (see also [4] for further discussions).

From the observational viewpoint, the most important aspect associated with these scenarios resides on the fact that some of their versions can lead to a late time accelerating universe, in agreement with supernova observations [5]. For this reason (and also motivated by a possible explanation for the hierarchy problem from extra-dimension physics), braneworld (BW) cosmologies has become a topic of much interest recently. Sahni and Shatnov [6], for instance, proposed a new class of BW models which admit a wider range of possibilities for the dark pressure than do the usual dark energy scenarios. As shown in Ref. [6], for a subclass of the parameter values the acceleration of the Universe can be a transient phenomena, which could help reconcile the supernova evidence for an accelerating universe with the requirements of string/M-theory [7]. More recently, Maia et al. [8] showed that the dynamics of a dark energy component parameterized by an equation of state p = ωρ (the so-called XCDM parametrization) can be fully described by the effect of the extrinsic curvature of a FRW universe embedded into a five-dimensional, constant curvature de-Sitter bulk.

Another interesting scenario is the one proposed by Dvali et al. [9], which we will refer to it as DGP model. It describes a self-accelerating 5-dimensional BW model with a noncompact, infinite-volume extra dimension whose dynamics of gravity is governed by a competition between a 4-dimensional Ricci scalar term, induced on the brane, and an ordinary 5-dimensional Einstein-Hilbert action (see [10] for details). DGP models have been sucessfully tested in many of their observational predictions, ranging from local gravity to cosmological...
observations [16, 11, 12, 13, 14] (see, however, [15, 16]). From the theoretical viewpoint, the consistency of their self-accelerating solutions is still a matter of debate in the current literature (see, e.g., [17]).

This paper aims at placing new observational constraints on DGP models from the current X-ray observations of rich clusters of galaxies and type Ia supernovae (SNe Ia) data. We emphasize that this particular combination of observational data constitutes an interesting and complementary probe for testing the viability of cosmological scenarios because while X-ray data are very effective to place limits on the clustered matter (dark matter) the new SNe Ia sample tightly constrains the unclustered component (dark pressure). To this end, we use the latest Chandra measurements of the X-ray gas mass fraction in 26 galaxy clusters, as provided by Allen et al. [18] along with the most recent determinations of the baryon density parameter, as given by the WMAP team [19], the latest measurements of the Hubble parameter, $H(z)$, the analysis performed here, therefore, updates the constraints on DGP models because while X-ray data are very effective to place limits on the clustered matter (dark matter) the new SNe Ia sample tightly constrains the unclustered component (dark pressure).

In DGP models, the presence of an infinite-volume extra dimension modifies the Friedmann equation in the following way [10]

$$\left[\frac{\rho}{3M_5^2} + \frac{1}{4r_c^2} + \frac{1}{2r_c}\right]^2 = H^2 + \frac{k}{R(t)^2}, \hspace{1cm} (1)$$

where $\rho$ is the energy density of the cosmic fluid, $k = 0, \pm 1$ is the spatial curvature, $M_5$ is the Planck mass and $r_c = M_5^2/2M_5^3$ ($M_5$ is the 5-dimensional reduced Planck mass) is the crossover scale defining the gravitational interaction among particles located on the brane, i.e., for scales $r < r_c$, the gravitational force experienced by two punctual sources is the usual 4-dimensional $1/r^2$ force whereas for distance scales $r > r_c$ the gravitational force follows the 5-dimensional $1/r^3$ behavior. From the above equation we find that the normalization condition is now given by $\Omega_k + \left[\Omega_m + \Omega_{\gamma} + \Omega_\delta + \Omega_\gamma + \Omega_m \right] = 1$, where $\Omega_m$ and $\Omega_\gamma$ are, respectively, the matter and curvature density parameters (defined in the usual way) and

$$\Omega_c = 1/4r_c^2H_0^2, \hspace{1cm} (2)$$

is the density parameter associated with the crossover radius $r_c$. For a flat universe, the normalization condition above reduces to $\Omega_c = (1 - \Omega_m)^2/4$. As noticed in Ref. [10], the above described cosmology can be exactly reproduced by the standard one plus an additional dark energy component with a time-dependent equation of state parameter $\omega_{eff}(z) = 1/G(z, \Omega_m, \Omega_r) - 1$, where $G(z, \Omega_m, \Omega_r) = \sqrt{4\Omega_{\gamma}/\Omega_m x^3 + 4}(\sqrt{\Omega_{\gamma}/\Omega_m x^3 + \sqrt{\Omega_r/\Omega_m x^3 + 1}})$ and $x' = (1 + z)^{-1}$ (see [28] for a discussion on time-dependent parametrizations for $\omega$).

To perform our statistical analysis in the next section two observational quantities are of fundamental importance, namely, the angular diameter $A(z)$ and luminosity distances $d_L(z)$ – intrinsically related, in a homogeneous and isotropic universe, by $d_L(z)(1 + z)^2 = d_A(z)$. From the above equations, it is straightforward to show that

$$d_A(z) = \frac{H_0^{-2}}{(1 + z)^{3/2}} \times \int_{x}^{1} \frac{dx}{x^{2} F(\Omega_j, x)}, \hspace{1cm} (3)$$

where the function $S_k$ is defined by one of the following forms: $S_k(r) = \sinh(r), r$, and $\sin(r)$, respectively, for open, flat and closed geometries. The dimensionless function $F(\Omega_j, x)$ is given by

$$F(\Omega_j, x) = \left[\Omega_k x^{-2} + (\sqrt{\Omega_{\gamma} + \Omega_m x^3})^{1/2}\right], \hspace{1cm} (4)$$

where $j$ stands for $m$, $\gamma$, and $k$. As one may check, for $\Omega_k = 0$, the limit $1/r_c \rightarrow 0 (\Omega_r \rightarrow 0)$ provides

$$d_A^{SCDM}(z) = \frac{2H_0^{-2}}{(1 + z)^{3/2}} \left[ (1 + z)^{1/2} - 1 \right], \hspace{1cm} (5)$$

which is the standard (SCDM) prediction for the angular diameter distance $d_A(z)$.

III. CONSTRAINTS ON DGP MODELS

A. $f_{gas}$ versus redshift test

The X-ray gas mass fraction test $f_{gas}(z)$ was first introduced in Ref. [24] and further developed in Ref. [25] (see also [21, 26, 27, 28]). This is based on the assumption that rich clusters of galaxies are large enough to provide a fair representation of the baryon and dark matter distributions in the Universe. Following this assumption, the matter content of the Universe can be expressed as the ratio between the baryonic content and the gas mass fraction, i.e., $\Omega_m \propto \Omega_h/f_{gas}$. Moreover, as shown by Sasaki [24] since $f_{gas} \propto d_m^{1/2}$ the model function can be defined as

$$f_{gas}(z) = \frac{b\Omega_b}{(1 + 0.19\sqrt{h})\Omega_m} \left[ 2h \frac{d^{SCDM}_A(z)}{d^{DGP}_A(z)} \right]^{1.5}, \hspace{1cm} (6)$$

where the bias factor $b$ is a parameter motivated by gas dynamical simulations that takes into account the fact that the baryon fraction in clusters is slightly depressed with respect to the Universe as a whole [30], the term
0.19$\sqrt{h}$ stands for the optically luminous galaxy mass in the cluster and the ratio $d_A^{\text{SCDM}}(z_i)/d_A^{\text{DGP}}(z_i)$ accounts for deviations in the geometry of the universe (here modelled by the DGP model) from the default cosmology used in the observations, i.e., the SCDM model (see [18, 25] for more observational details).

In order to constrain the cosmological parameters $\Omega_m$ and $\Omega_\gamma$, we use the latest Chandra measurements of the X-ray gas mass fraction in 26 dynamically relaxed galaxy clusters ($0.07 < z < 0.9$), as provided by Allen et al. [18].

The confidence intervals are determined by using the conventional $\chi^2$ minimization with the Gaussian priors on the baryon density parameter, $\Omega_b h^2 = 0.0224 \pm 0.0009$ [19], on the Hubble parameter, $h = 0.72 \pm 0.08$ [20], and on the bias factor, $b = 0.824 \pm 0.089$ [31], i.e.,

$$
\begin{align*}
\chi^2_{\text{gas}} &= \sum_{i=1}^{26} \left( \frac{f_{\text{gas}}^{\text{mod}}(z_i) - f_{\text{gas},i}}{\sigma_{f_{\text{gas},i}}} \right)^2 + \left( \frac{\Omega_b h^2 - 0.0224}{0.0009} \right)^2 + \\
&\quad + \left( \frac{h - 0.72}{0.08} \right)^2 + \left[ \frac{b - 0.824}{0.089} \right]^2.
\end{align*}
$$

In the above expression, $f_{\text{gas}}^{\text{mod}}(z_i)$ is given by Eq. (6) and $f_{\text{gas},i}$ is the observed values of the X-ray gas mass fraction with errors $\sigma_{f_{\text{gas},i}}$. To plot two-dimensional confidence contours, we have projected our five-dimensional parameter space (defined by the vector $\vec{p} = (\Omega_m, \Omega_\gamma, h, \Omega_b h^2, b)$) into the plane $\Omega_m - \Omega_\gamma$, which is similar to marginalize over the parameters $h$, $\Omega_b h^2$ and $b$ by defining the probability distribution function $\mathcal{L}(\Omega_m, \Omega_\gamma) = \int dh d\Omega_b h^2 d\nu \chi^2 / 2$ (see, e.g., [25]).

Figure 1 shows the behavior of $f_{\text{gas}}^{\text{mod}}$ as a function of the redshift for some selected values of $\Omega_m$ and $\Omega_b h^2 = 0.0224$, $b = 0.824$ and $h = 0.72$. The value of $\Omega_m$ is fixed at 0.3, as suggested by dynamical estimates [31]. The current concordance model, i.e., a flat scenario with 70% of the critical energy density dominated by a cosmological constant, is also shown for comparison.

By comparing our Fig. 2a with Fig. 2 of Ref. [22] (also shown here as Fig. 2b for the sake of comparison), it is clear that X-ray and SNe Ia data provide orthogonal statistics in the plane $\Omega_m - \Omega_\gamma$. This, therefore, suggests that possible degeneracies between these parameters may be broken by combining these two data sets in a joint statistical analysis. To perform such an analysis we make use of the so-called gold SNe Ia sample of Riess et al. [5], which consists of 157 SNe Ia events distributed over the redshift interval $0.01 \leq z \leq 1.7$. This particular sample constitutes the selection of the best observations made so far by the two supernova search teams along with 16 new events observed by the HST. Our approach for SNe Ia data is based on Ref. [22] (see also [32]).

The results of our joint analysis are shown in Figure 2c. Note that the combination of these data sets leads to tight constraints on the $\Omega_m - \Omega_\gamma$ plane. A comparison with Fig. 3 of Ref. [21] (which used the then available 172 SNe Ia taken from Tonry et al. [33] plus 9 X-ray clusters from Allen et al. [26]) shows that the current analysis reduces considerably the area corresponding to the
As expected, the parameter space now is reduced relative to our former analysis, with the best-fit scenario occurring at $\Omega_m = 0.34$ and $\Omega_{\gamma} = 0.26$ ($\chi^2/\nu = 1.11$). These values correspond to an accelerating universe with deceleration parameter $q_0 = -0.8$ and a total expanding age of $t_o \simeq 10.1 h^{-1}$ Gyr. At 95.4% c.l. we found the following intervals: $0.29 \lesssim \Omega_m \lesssim 0.41$ and $0.22 \lesssim \Omega_{\gamma} \lesssim 0.30$. Note that this 2$\sigma$ interval for $\Omega_{\gamma}$ leads to an estimate of the crossover scale $r_c$ in terms of the present Hubble radius $H_o^{-1}$ [see Eq. (2)], i.e.,

$$r_c = 0.98^{+0.08}_{-0.07} H_o^{-1},$$

which is slightly smaller than (but in agreement with) the value obtained in Ref. [21], $r_c = 1.09^{+0.29}_{-0.16} H_o^{-1}$ (99% c.l.). If we restrict our analysis to a flat geometry, i.e., by imposing the normalization condition $\Omega_m = (1-\Omega_m)^2/4$, we obtain $\Omega_m \simeq 0.22$ ($\Omega_{\gamma} \simeq 0.152$), which corresponds to a 9.6$h^{-1}$-Gyr-old universe with $q_0 = -0.45$ and $r_c \simeq 1.28 H_o^{-1}$.

At this point we compare our estimates of the crossover scale $r_c$ with other recent determinations of this quantity from independent methods. We note a reasonable agreement among them. For instance, in Ref. [11] Deffayet et al. used the then available SNe Ia + CMB data to find $r_c \simeq 1.4 H_o^{-1}$ for a flat model with $\Omega_m = 0.3$, in agreement with Avelino and Martins [15] who used a larger sample of 92 SNe Ia events. The current measurements of the angular size of high-z sources require an accelerating universe with a crossover radius of the order of $r_c \simeq 0.94 H_o^{-1}$ [11] whereas the statistics of gravitationally lensed quasars implies $r_c \simeq 1.76 H_o^{-1}$ [12]. The less concordant (but not in disagreement) among the current estimates for $r_c$ comes from age measurements of high-z galaxies, which requires $r_c \lesssim 2.04 H_o^{-1}$ [13]. These results, along with the main estimates of the present paper, are summarized in Table I.

![Figure 2](image-url) FIG. 2: The results of our statistical analyses: Confidence regions (68.3%, 95.4% and 99.7%) in the $\Omega_m - \Omega_{\gamma}$ plane by considering: a) the latest Chandra measurements of the X-ray gas mass fraction in 26 galaxy clusters (0.07 < $z$ < 0.9) plus determinations of the baryon density parameter and measurements of the Hubble parameter; b) the so-called Gold sample of Riess et al. [5] – taken from [22] and shown here for the sake of comparison. Note that X-ray and SNe Ia data provide orthogonal statistics in the plane $\Omega_m - \Omega_{\gamma}$; c) Joint X-ray + SNe Ia + $\Omega_m + \Omega_b h^2 + H_o$ analysis. This combination of data provides $\Omega_m = 0.34^{+0.07}_{-0.05}$ and $\Omega_{\gamma} = 0.26 \pm 0.04$ (at 95.4% c.l.) and clearly favors a closed universe.

| Method                  | Reference | $r_c$ |
|-------------------------|-----------|-------|
| SNe Ia + CMB            | [10]      | 1.4   |
| SNe Ia                  | [15]      | 1.4   |
| Angular size            | [11]      | 0.94  |
| Gravitational lenses    | [12]      | 1.76  |
| High-z galaxies         | [13]      | $\leq 2.04$ |
| SNe Ia + X-ray          | [21]      | 1.09  |
| SNe Ia$^a$ + $\Omega_m$| [22]      | 1.09  |
| X-ray$^c$:              |           |       |
| Arbitrary curvature     | This paper| 0.90  |
| Flat case               | This paper| 1.30  |
| SNe Ia$^a$ + X-ray$^c$: |           |       |
| Arbitrary curvature     | This paper| 0.98  |
| Flat case               | This paper| 1.28  |

$^a$In units of $H_o^{-1}$.

$^b$Most recent SNe Ia data

$^c$Most recent X-ray data

TABLE I: Recent estimates of the crossover radius $r_c$.
IV. FINAL REMARKS

There is increasing evidence for an accelerating universe from various astronomical observations. However, understanding the acceleration mechanism based on fundamental particle physics is still one of the most important challenges in modern cosmology. An unknown dark energy component with negative pressure has usually been invoked as the most feasible mechanism for the acceleration although effects arising from extra dimension physics can mimic dark energy through a modification on the Friedmann equation \[ \ddot{a} + 2 H \dot{a} - \kappa R = 0 \]. In this paper, we have focused our attention on a specific self-accelerating five-dimensional braneworld scenario, the so-called DGP model \[ 5 \].

We analyzed the scenario by using the most recent Chandra measurements of the X-ray mass fraction in galaxy clusters, the WMAP determinations of the baryon density parameter, measurements of the Hubble parameter from the HST, and the current supernova data. As shown, the model provides a good description for the so-called gold SNeIa sample of Ref. \[ 5 \]. However, in order to explain the X-ray data of clusters in the framework of DGP model, a closed universe is necessary (a similar result is also obtained by fitting SNeIa data to the standard \( \Lambda \)CDM model. In this case, CMB data is generally used to match a flat universe.). The combination of the above mentioned data sets leads to very tight constraints on the \( \Omega_m - \Omega_r \) plane. At 95.4\% c.l. we found the following intervals: 0.29 \( \lesssim \Omega_m \lesssim 0.41 \) and 0.22 \( \lesssim \Omega_r \lesssim 0.30 \), which gives a closed universe with the curvature of \( -0.39 \lesssim \Omega_k \lesssim -0.18 \). It is worth mentioning that we might make heavy use of the X-ray gas mass fraction in clusters, which further prefers to a closed universe in DGP model. This kind of analysis depends on the assumption that the \( f_{\text{gas}} \) values should be invariant with redshift, which has been criticised by a minority of workers in the field. For example, a recent comparison of distant clusters observed by XMM-Newton and Chandra satellites with available local cluster samples seems to indicate a possible evolution of the \( M-T \) relation with redshift, which may be indicating that the standard paradigm on cluster gas physics needs to be revised \[ 55 \]. Therefore, to pin down the DGP model from this kind of observations, a more detailed study on X-ray gas mass fraction in galaxy clusters is necessary.

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