Enhanced $B \to \mu\nu$ Decay at Tree Level as Probe of Extra Yukawa Couplings

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(Dated: February 2, 2022)

With no New Physics seen at the LHC, a second Higgs doublet remains attractive and plausible. The ratio $R_{B}^{\mu/\tau} = B(B \to \mu\nu)/B(B \to \tau\nu)$ is predicted at 0.0045 in both the Standard Model and the two Higgs doublet model type II, but it can differ if extra Yukawa couplings exist in Nature. Considering recent Belle update on $B \to \mu\nu$, the ratio could be up by a factor of two in the general two Higgs doublet model, which can be probed by the Belle II experiment with just a few ab$^{-1}$.

PACS numbers: 12.60.Fr, 13.20.-v, 13.25.Hw, 14.80.Cp

Introduction.— Despite observing the 125 GeV boson $h$ at the Large Hadron Collider (LHC), no New Physics (NP) beyond the Standard Model (SM) has so far emerged. Supersymmetry (SUSY) is not seen, nor extra gauge bosons or any new particle. Except for weaker couplings, any new symmetry must be at rather high scale. In this light, we ought to reexamine the validity of any presumed symmetry. In this Letter we reflect on the $Z_2$ symmetry imposed on two Higgs doublet models (2HDM) to satisfy Natural Flavor Conservation (NFC).

Although extra Higgs bosons remain elusive, the existence of one doublet makes a second doublet plausible. The most popular 2HDM type II, where $u$- and $d$-type quarks receive mass from separate doublets, is automatic in SUSY. The NFC condition removes flavor changing neutral Higgs (FCNH) couplings by demanding each mass matrix comes from only one Yukawa matrix. But this removes all extra Yukawa couplings that naturally should exist in a 2HDM! But with discovery of $m_h < m_t$, it was stressed that FCNH, or extra Yukawa couplings in general, is an experimental issue, as attested by the ATLAS and CMS pursuit of $t \to ch, uh$, and augmented by the hint of $h \to \tau\mu$ in Run 1 data of CMS, although it disappeared with Run 2 data.

In the LHC era, a handful of so-called “flavor anomalies” do exist, but the recent trend is “softening”. Belle announced a new measurement of the $R_D$ and $R_{D^*}$ ratios, or $B(B \to D(\ast)\mu\nu)/B(B \to D(\ast)\mu\nu)$, using semileptonic tag, finding consistency with SM, and the world average tension with SM expectation decreases from 3.8$\sigma$ to 3.1$\sigma$. For the clean $R_K$ ratio, $B(B \to K\mu^+\mu^-)/B(B \to K e^+e^-)$, using Run 2 data taken in 2016 alone, LHCb finds consistency with SM, but is at 1.9$\sigma$ with reanalyzed Run 1 result, although the combined data is still at 2.5$\sigma$ from SM. In any case, these “anomalies” need confirmation with full Run 2 data, as well as Belle II scrutiny. The $R_{B}^{\mu/\tau}$ ratio adds to the list, and could be an early NP probe at Belle II.

The recent Belle remeasurement supersedes the published (6.46 $\pm$ 2.22 $\pm$ 1.60) $\times$ 10$^{-7}$ [12]. The central value dropped slightly, but the improved systematics moves the significance up from 2.4$\sigma$ to 2.8$\sigma$. The $B \to \ell\nu$ decay branching fraction in SM is

$$B(B \to \ell\nu)|_{SM} = |V_{ub}|^2 f_B^2 \frac{G_F^2 m_\ell^2 m_B}{8\pi\Gamma_B} \left( 1 - \frac{m_\mu^2}{m_B^2} \right)^2,$$

where helicity suppression by $m_\ell^2/m_B^2$ makes it quite rare, but also more susceptible to NP effects. Using $f_B = 190$ MeV from FLAG [13], and exclusive value $|V_{ub}|_{excl.} = 3.70 \times 10^{-3}$ from PDG [14], we find $B(B \to \ell\nu)|_{2HDM} \approx 3.92 \times 10^{-7}$. Eq. (1) allows a mild enhancement, echoing an old result from BaBar [15].

The effect in 2HDM II is also well known [16],

$$B(B \to \ell\nu)|_{2HDMII} = r_H B(B \to \ell\nu)|_{SM},$$

where, with $m_{H^\pm}$ the $H^+$ mass and $\tan \beta$ the ratio of vacuum expectation values (v.e.v.) of the two doublets, $r_H \cong 1 - \tan^2 \beta \frac{m_{B^-}^2}{m_{H^+}^2}$. (4)

As $r_H$ is $m_\ell$-independent, as in SM, 2HDM II gives

$$R_{B}^{\mu/\tau} = \frac{B(B \to \mu\nu)}{B(B \to \tau\nu)} = \frac{m_\mu^2 (m_{B^-}^2 - m_{\mu^+}^2)^2}{m_\tau^2(m_{B^+}^2 - m_{\mu^+}^2)^2} \cong 0.0045,$$

as stressed in a recent review [17]. It is also independent of $|V_{ub}|$. Together with $B \to \tau\nu$ being consistent with SM, one usually expects $B(B \to \mu\nu)$ to be SM-like, as reflected in the Belle II Physics Book [18].

Belle II can check whether $R_{B}^{\mu/\tau} \cong 0.0045$ holds. A deviation would not only be beyond SM, but rule out 2HDM II convincingly. We will show that if there exist extra Yukawa couplings, $B(B \to \mu\nu)$ can become enhanced or suppressed, while $B(B \to \tau\nu)$ would be SM-like. Thus, a measurement of $B(B \to \mu\nu)$ plus refining $B(B \to \tau\nu)$ would open up a probe of extra Yukawa couplings that complements the LHC search for $t \to ch(uh)$ and $h \to \tau\mu$, but involving the $H^+$ boson.
Formalism.— Compared with Eqs. 2 and 3, the ℓ index to $\bar{\nu}$ in Eq. 5 was dropped, as the $\bar{\nu}$ flavor is not measured, but is relevant for 2HDM without $Z_2$. Called 2HDM III earlier 19, 2HDM without $Z_2$ followed the Cheng-Sher ansatz 20, that a trickle-down $\sqrt{m_{\ell} m_{\bar{\nu}}}$ mass-mixing pattern may loosen the need for NFC 1 to forbid FCNH. The h boson discovery made a discrete symmetry appear ad hoc 3: existence of $tch$ or $h_{\tau \mu}$ FCNH couplings should be an experimental question.

For each type of charged fermion $F = u, d, \ell$, a second set of Yukawa matrices $\rho_F^\ell$ comes from a second scalar doublet, where some trickle-down flavor pattern helps hide the effects, in particular from FNCH couplings. It was revealed recently 21 that “NFC protection against FCNH can be replaced by approximate alignment, together with a flavor organizing principle reflected in SM itself”, and that the Cheng-Sher $\sqrt{m_{\ell} m_{\bar{\nu}}}$ pattern may be too strong an assumption.

Approximate alignment emerged with LHC Run 1 data: the h boson appears rather close 22 to the SM Higgs, and the two CP-even scalars, $h^0$ and $H^0$, do not mix much; in 2HDM II notation, the mixing angle $\cos(\alpha - \beta)$ is small, which is now affirmed by Run 2 data 24 25. But in 2HDM without $Z_2$ (which we now call g2HDM), $\tan \beta$ is unphysical 26, hence we replace $\cos(\alpha - \beta)$ by $\cos \gamma$ 21. The $tch$ coupling is then $\rho_\ell \cos \gamma$ ($\rho_\ell$ is already constrained to be small 3 27), and $h_{\tau \mu}$ coupling is $\rho_{\mu \tau} \cos \gamma$. Approximate alignment can suppress $t \rightarrow ch$ or $h \rightarrow \tau \mu$ rates, without invoking tiny $\rho_{\ell \tau}$ or $\rho_{\mu \ell}$.

The fundamental $H^+$ Yukawa couplings

$$-\bar{u} (V_R^d R - \rho^u V_L) d H^+ - \bar{\nu} (\rho^\ell R) \ell H^+ + \text{H.c.},$$

are independent of $\cos \gamma$, where $V$ is the CKM matrix, $L, R \equiv (1 \mp \gamma_\tau) / 2$, and $u, d, \ell$ are in matrix notation. They give rise to the branching fraction

$$B(B \rightarrow \ell \bar{\nu}) = B(B \rightarrow \ell \ell \ell) \left| \frac{m_B V_{tb}}{m_{b^*} V_{t\bar{\nu}}} \right|^2,$$

with explicit sum over $\ell$ flavor, and sum over $i$ implied. Expanding $\sum_i \rho_{ib} V_{ui} = \rho_{ib} V_{ui} + \rho_{ib} V_{ui} + \rho_{ib} V_{ui} \approx \rho_{ib} V_{ui}$, since $\rho_{ib}$ and $\rho_{ib}$ are constrained severely at tree level by $B_s$ and $B_d$ meson decays. Expanding $\sum_i \rho_{ib} V_{ub} = \rho_{ib} V_{ub} + \rho_{ib} V_{ub} + \rho_{ib} V_{ub} \approx \rho_{ib} V_{ub}$, as $\rho_{ib}$ is constrained by $D^0$ mixing, and $\rho_{ub}$ is suppressed by mass-mixing hierarchy, with both terms CKM suppressed. Note that our result is not affected by the PMNS matrix in the neutrino sector, so long that it is unitary.

After some rearrangement, the factor becomes

$$B(B \rightarrow \ell \bar{\nu}) = B(B \rightarrow \ell \ell \ell) \left| \frac{m_B V_{tb}}{m_{b^*} V_{t\bar{\nu}}} \right|^2,$$

where $\lambda_\ell = \sqrt{2 m_\ell / v}$ is the lepton Yukawa coupling with $v \approx 246$ GeV, and $m_b = \sqrt{2 m_\ell / v}$ is defined similarly. But since $m_b$ arises from hadronic matrix element, it should be run to $m_{H^+}$ scale. Following PDG 14, we first calculate $M$ running mass $m_{b}(m_b)$ at pole mass, then evolve to $\mu = m_{H^+}$ by $m_{b}(m_b) = \text{C}(a_{\pi}(\mu)/c(a_{\pi}(m_b))) m_{b}(m_b)$, where $c(\pi)$ is taken with four-loop accuracy using $\text{MS}$ three-loop $a_\pi$ at scale $\mu$ in five-flavor scheme.

The first notable thing in Eq. 8 is the $V_{tb}/V_{ub}$ enhancement of the $\rho_{tb}$ effect, which does not arise in 2HDM II. One can ignore $\rho_{tb}/\rho_{ub} \approx O(1)$ in g2HDM, so long that $\rho_{tu}/\rho_{ub}$ does not approach $|V_{ab} / V_{cb}| \sim 0.004$. Note also that, taking $\rho_{tb} = -\lambda_\tau \tan \beta$ and $\rho_{\ell \ell} = -\lambda_\ell \tan \beta \delta_{\ell \ell}$, and setting $\rho_{tu} = 0$, one recovers the $r_H$ factor of 2HDM II.

Results.— To make contact with experiment, we recast in the notation of BaBar’s $R_{D_s}$ paper 28, the form of which the new Belle analysis 11 has followed,

$$B(B \rightarrow \ell \bar{\nu}) = B(B \rightarrow \ell \ell \ell) \left| \frac{m_B V_{tb}}{m_{b^*} V_{t\bar{\nu}}} \right|^2,$$

where $S_L^{\ell \ell}$, at $m_b$ scale, are the ratios of NP Wilson coefficients (of 4-Fermi operators) with SM ones. We note that +$S_L^{\ell \ell}$ contribution, proportional to $\rho_{tb}$ as seen in Eq. 8, is negligible in g2HDM because of the $|V_{tb}/V_{ub}|$ enhancement of $S_L$. Keeping both $S_R$ and $S_L$, Belle had to assume reality to make a 2D plot 11. But, sourced in Yukawa couplings, they are clearly complex.

The correspondence with extra Yukawa couplings is,

$$S_L^{\ell \ell} = m_b m_{\mu} \frac{V_{tb} V_{t\bar{\nu}}}{m_H^2},$$

where $m_\mu$ is at $m_{H^+}$ scale. We note that Yukawa couplings are dimension-4 terms in the Lagrangian. For leptonic $B^+$ decay, QCD correction is easy to match with 4-Fermi operators. But it was the insight on $|\rho_{tb}|$ vs $|\rho_{tu} V_{tb}/V_{ub}|$ that allowed us to drop the $S_R$ term.

Ignoring $\ell = \ell$, i.e. taking $\rho_{\ell \ell}$ as negligible, we get,

$$B(B \rightarrow \mu \bar{\nu}) = B(B \rightarrow \mu \mu \ell) \left| \frac{m_B V_{tb}}{m_{b^*} V_{t\bar{\nu}}} \right|^2,$$

where the $S_L^{\mu \mu}$ effect from diagonal $\rho_{\mu \mu}$ coupling interferes with SM (the “1”), while the $S_L^{\mu \mu}$ effect from off-diagonal $\rho_{\mu \ell}$ adds in quadrature. The $\tau$ equivalent of the former can be found in an early $B \rightarrow \tau \tau \tau$ study 29, but the more detailed follow-up 30 erroneously summed over $\ell$ in amplitude, incorrectly giving neutrino flavor independence, hence missing the second effect. Furthermore, Ref. 30 kept $\tan \beta$ in the formulation, but $\tan \beta$ is not physical in g2HDM 29. In fact, $B \rightarrow \mu \nu$ has not been emphasized in the literature. The one paper that did,
FIG. 1. Assuming the other effect in Eq. (11) is turned off: [left] projection of Belle $B \rightarrow \mu \bar{\nu}$ result \cite{31} in $|S^\mu_L| - \phi_{\mu\mu}$ plane; and [right] $\mathcal{B}(B \rightarrow \mu \bar{\nu})$ vs $|S^\mu_L|$ (blue dashed line). The $\pm 1\sigma$ and $\pm 2\sigma$ allowed regions are in dark and light pink shades, and black dotted (red solid) line denotes Belle central (SM) value. The gray shaded region below SM value in right panel is theoretically inaccessible. The $\overline{\text{MS}}$ mass $\overline{m}_b(m_b)$ is used.

Ref. \cite{31}, not only followed Ref. \cite{30} in keeping tan $\beta$, but assumed the charged lepton Yukawa matrix $\rho$ to be diagonal, thereby leaving out our second term.

The $S^\tau_L$ effect has thus not been discussed before. Without interference it may appear less important, but we will show it is the leading effect. For $B \rightarrow \tau \bar{\nu}$, one simply exchanges $\mu \leftrightarrow \tau$ in Eq. (11).

We parametrize $S^{\ell'\ell} = |S^\ell_L|e^{i\phi_{\ell'\ell}}$, where $\phi_{\ell'\ell}$ is the phase difference between $\rho_{\ell'\ell}\rho^*_{\ell\ell}$ and $V_{\ell'b}$. The phase does not enter the off-diagonal effect, but the two mechanisms must be treated separately. Setting $S^\mu_L = 0$ and taking $m_{\overline{b}}(m_b) = 4.18$ GeV in Eq. (11), Fig. 1 [left] illustrates Eq. (1) in the $|S^\mu_L| - \phi_{\mu\mu}$ plane, where dotted (red solid) line is the Belle central value (SM expectation of $3.92 \times 10^{-7}$), with two different shades illustrating $\pm 1\sigma$ and $\pm 2\sigma$ ranges. The plot is symmetric for $\phi_{\mu\mu} < 0$.

The in-quadrature second term of Eq. (11) is $\phi_{\tau\mu}$ independent. Setting $S^\mu_L = 0$, we plot in Fig. 1 [right] $\mathcal{B}(B \rightarrow \mu \bar{\nu})$ vs $|S^\mu_L|$, which is the blue dashed line. As there is only enhancement, the gray area below SM expectation is inaccessible. Belle data constrain $|S^\mu_L| \lesssim 0.019$, the same as imaginary $S^{\ell'\ell}_L$ in Fig. 1 [left].

Setting $S^{\tau\tau}_L = 0$, Fig. 2 [left] depicts in the $|S^\tau_L| - \phi_{\tau\tau}$ plane the Belle average $\mathcal{B}(B \rightarrow \tau \bar{\nu}) \simeq (9.1 \pm 2.2) \times 10^{-5}$ from PDG, with notation analogous to Fig. 1. Likewise, Fig. 2 [right] plots $\mathcal{B}(B \rightarrow \tau \bar{\nu})$ vs $|S^{\tau\tau}_L|$ with $S^{\tau\tau}_L = 0$. The bands in Fig. 2 appear narrower not so much as an artifact of plotting, but reflects $\mathcal{B}(B \rightarrow \tau \bar{\nu})$ being better measured than $\mathcal{B}(B \rightarrow \mu \bar{\nu})$.

**Interpretation in g2HDM.** — Interpreting in g2HDM that plausibly holds fundamental extra Yukawa couplings, sheds light on the underlying physics. $\mathcal{B}(B \rightarrow \mu \bar{\nu})$ from Belle, Eq. (1), is consistent with SM, which Belle II would measure in due time. \cite{18}. But if one has $+1\sigma$ or even $+2\sigma$ enhancement over the central value, then earlier discovery is possible. From Fig. 1 we find $+1\sigma$ to $+2\sigma$ enhancement correspond to $|S^{\mu\mu}_L| \in (0.006, 0.009)$ for the constructive (negative) case, (0.015, 0.019) for the imaginary case, and (0.038, 0.041) for the destructive (positive) case. The incoherent, second effect of Eq. (11) has the same parameter range as imaginary $S^{\mu\mu}_L$, but for $|S^{\tau\tau}_L| \in (0.015, 0.019)$, i.e. with $\bar{\nu}_\tau$ index.

We take $m_{H^+} = 300$ GeV as benchmark for sake of the largest effect, but also because the usual $m_{H^+}$ bound from 2HDM II does not apply (some discussion can be found in Refs. \cite{27} and \cite{32}), given the many new flavor parameters. From Eq. (10) we find $|S^{\ell'\ell}_L| \simeq 150|\rho_{\ell\ell}^*\rho_{\ell'\ell}|(300 \text{GeV}/m_{H^+})^2$. The constructive case of $S^{\mu\mu}_L \in (-0.009, -0.006)$ needs $|\rho_{\mu\mu}\rho_{\mu\mu}| \simeq (4-6) \times 10^{-5}$, and would grow as $(m_{H^+/300 \text{GeV}})^2$. So, what do we
know, or can infer, about $|\rho_{\mu\mu}|$ and $|\rho_{\mu\tau}|$? Given that $H^+$ effect is normalized to SM, Eq. (8) offers a clue: $\rho_{\mu\mu}$ is “normalized” against $\lambda_\mu \simeq 0.0006$, the charged lepton Yukawa coupling, while $\rho_{\mu\tau}$ is normalized to $m_b \simeq 0.015$, the “effective Yukawa coupling” from $m_b$ evaluated at $m_{H^+}$ scale. The combined $\lambda_\mu m_t \simeq 1 \times 10^{-5}$ suggests $S_{\mu\mu}^L$ falls short of enhancing $B(B \to \mu\bar{\nu})$ even for the most optimistic case, let alone the larger $|S_{\mu\mu}^L|$ needed for imaginary or destructive cases.

It is instructive to take a look at the second mechanism, i.e. via $\tilde{v}$ flavor. $|S_{\mu\tau}^R| \in (0.015, 0.019)$ is the same as imaginary $S_{\mu\mu}^L$, hence $|\rho_{\mu\tau}| \simeq 10^{-4}$ may not appear promising. However, due to the hint for $h \to \tau\mu$ in CMS Run 1 data, up until early 2017, values of $|\rho_{\mu\tau}|$ as large as 0.26 had been entertained. Although the hint disappeared with Run 2 data, it could reflect approximate alignment, i.e. a small cos $\gamma$. As discussed below, if we allow $\rho_{\mu\tau} \sim \lambda_\tau$, then $\lambda_\tau m_t \sim 1.5 \times 10^{-4}$ seems to allow the $\rho_{\mu\tau}$ mechanism to enhance $B \to \mu\bar{\nu}$.

Taking a closer look, we suggest $\rho_{\mu\mu} = \mathcal{O}(\lambda_\mu)$ is reasonable, as $\lambda_\mu$ arises from diagonalizing the mass matrix, but $\rho_{\mu\tau}$ is from an orthogonal combination of the two unknown Yukawa matrices, going through the same diagonalization. To avoid fine tuning, these two Yukawa matrices must each contain the “flavor organization” [21] reflected in mass-mixing hierarchies, hence $\rho_{\mu\mu} = \mathcal{O}(\lambda_\mu)$.

We treat $\rho_{\mu\tau}$ more liberally, as argued above for $|\rho_{\tau\mu}| \sim \lambda_\tau \sim 0.01$, since much larger $\rho_{\mu\tau}$ values have been considered only recently. The most relevant constraint comes from $\tau \to \mu\gamma$, where the two-loop mechanism constrains $|\rho_{\tau\mu}| \lesssim 0.01$ [22] for $\rho_{\mu\mu} \lesssim 1$, but weakens for weaker $\rho_{\mu\mu}$. Finally, having $|\rho_{\tau\mu}| \lesssim |\rho_{\mu\tau}| \sim \lambda_\tau$ is not unreasonable, just as $|\rho_{\mu\ell}|$ could be up to $|\rho_{\mu\tau}| \sim \lambda_\tau$, where $\rho_{\mu\tau}$ and $\rho_{\mu\ell}$ provide two possible CP violating sources for electroweak baryogenesis, which strongly motivates g2HDM. Thus, we suggest $|\rho_{\mu\tau}| \lesssim 0.02$ as reasonable, and its value is in any case an experimental issue.

For the common $\rho_{\mu\tau}$ factor, things are harder to discern. Taking $|\rho_{\mu\mu}| \sim \sqrt{m_t m_b}/v \sim 0.003$ would be a bit small, but it need not be that small, since the direct $t \to uh$ search bound [3] is not so different from $t \to ch$, hence quite forgiving. In lack of a true yardstick, we take $|\rho_{\mu\mu}| \lesssim m_b$ as reasonable.

Thus, even taking $|\rho_{\mu\mu}| \sim 3\lambda_\mu$ and $|\rho_{\mu\tau}| \sim 2\lambda_\mu$, $|\rho_{\mu\mu}| |\rho_{\mu\tau}| \sim 5 \times 10^{-5}$ is only borderline in enhancing $B(B \to \mu\bar{\nu})$ for the most optimistic, constructive case, and in general would not quite suffice. However, even modest $|\rho_{\mu\tau}| \lesssim \lambda_\tau$ and $|\rho_{\mu\mu}| \lesssim m_b$ give $|\rho_{\mu\mu}| |\rho_{\mu\tau}| \lesssim 1.5 \times 10^{-4}$, allowing reasonable outlook for enhancement even if it comes only in quadrature. For higher $m_{H^+}$, e.g. $500$--$600$ GeV, $|\rho_{\mu\mu}|$ and $|\rho_{\mu\tau}|$ in the upper reaches of our suggested range can still give enhancement.

Turning to $B \to \tau\bar{\nu}$, we take $\rho_{\tau\tau} = \mathcal{O}(\lambda_\tau)$ and $\rho_{\mu\tau} \lesssim \lambda_\tau$. For constructive case, we see from Fig. 2[2] that $S_{\tau\tau}^L \in (-0.065, -0.035)$ for $+1\sigma$ to $+2\sigma$ enhancement, which suggests $|\rho_{\tau\tau}| |\rho_{\mu\tau}| \simeq (2.3-4.3) \times 10^{-4}$ for $m_{H^+} \simeq 300$ GeV. But $|\rho_{\tau\tau}| |\rho_{\mu\tau}| \sim \lambda_\tau m_b \sim 1.5 \times 10^{-4}$ falls short. Enhancement from SM is possible in constructive case only for $|\rho_{\tau\tau}| |\rho_{\mu\tau}|$ in the upper reaches of $\sim 6\lambda_\tau m_b$, but gets easily damped by larger $m_{H^+}$. For the second effect, Fig. 2[2] suggests $|S_{\tau\tau}^R| \in (0.15, 0.2)$, much larger than the constructive case. With $\rho_{\tau\tau} |\rho_{\mu\tau}| \lesssim \lambda_\tau m_b$, this mechanism cannot enhance $B \to \tau\bar{\nu}$.

Thus, one expects $B(B \to \tau\bar{\nu})$ in g2HDM to be SM-like, while $B(B \to \mu\bar{\nu})$ could be better enhanced.

Discussion. — $K \to \mu\bar{\nu}$ decay is not constraining, as both coherent and incoherent effects are suppressed by $|V_{ts}V_{ub}|^2 |V_{tb}V_{us}| (m_K^2 m_b / m_B^2 m_s) \sim 0.0003$, while $K \to e\bar{\nu}$ is even SM-like. The same argument goes with pion decays, and the effect in $D^\pm, D_s$ decays is also rather weak. For $B_c$, we do not see how $B_c \to \tau\bar{\nu}$ can be reconstructed. Thus, $B \to \mu\bar{\nu}$ provides the unique probe of extra Yukawa couplings in g2HDM, whereas $B \to \tau\bar{\nu}$ is expected to be SM-like. Taking the $R^{\mu\tau}_B$ ratio eliminates the main uncertainties associated with $|V_{ub}|$. It is interesting that the $S_{\mu\tau}^L$ mechanism could also suppress $B(B \to \mu\bar{\nu})$ (see lower left region of Fig. 1[1]), but would take longer for Belle II to uncover. We note in passing that a deviation in $B(B \to \mu\bar{\nu})$ may also be caused by leptoquarks [33] ($W'$ is overly constrained).

What about $\mu \to e\nu\bar{\nu}$ and $\tau \to e\nu\bar{\nu}$ decays? As these are dominated by $V-A$ theory, the vector currents couple via $g \sim \mathcal{O}(1)$, without helicity suppression. In contrast, since $|\rho_{\mu\tau}| \lesssim |\rho_{\tau\mu}| = \mathcal{O}(\lambda_\tau) \ll g$ are the largest Yukawa couplings that enter, together with $M_W^2 / m_{H^+}^2$ suppression, Nature has quite an effective mechanism in hiding the extra Yukawa coupling effects in the lepton sector. For example, given the extreme lightness and abundance of the electron, $\rho_{\mu\mu}$ and $\rho_{\mu\tau}$ must be very small, we expect $\mu \to e\nu\bar{\nu}$ and $\tau \to e\nu\bar{\nu}$ to be SM-like to high precision. Similar arguments hold for $B \to X_s \ell\nu, \tau\ell\nu$ decays, which are plagued further by hadronic uncertainties. Finally, we have used $\tau \to \mu\gamma$ to constrain $\rho_{\mu\tau}$. As a cross-check, for $\rho_{\mu\tau} \lesssim 2\lambda_\tau \simeq 0.02$, $\rho_{\mu\mu} \lesssim 3\lambda_\mu \simeq 0.0018$, and $m_\nu \gtrsim 300$ GeV (degenerate with $H^+$, ignoring heavier $H^0$), and approximate alignment control of $h^0$ effect, we estimate $B(\tau \to \mu\nu\mu) \lesssim \mathcal{O}(10^{-11})$, which is far below current [14] experimental bound.

Nature does hide well the effect of extra Yukawa couplings in $H^+$ mediated low energy processes. $B \to \mu\bar{\nu}$ is more helicity suppressed than $B \to \tau\bar{\nu}$, with $\rho_{\mu\tau}$ giving $\mu\bar{\nu}$, final state, and $b \to u$ transition giving $V_{tb}/V_{ub}$ enhancement of $\rho_{\mu\tau}$, both of which can happen only in g2HDM. Our imprecise knowledge of $\rho_{\mu\mu}$ and $\rho_{\mu\tau}$ allow for enhancement: $B(B \to \mu\bar{\nu})$ probes the extra Yukawa coupling product $\rho_{\mu\mu}\rho_{\mu\tau}$. But early impressions of enhanced $B \to \tau\bar{\nu}$ [14] trained people to expect NP in $B \to \tau\bar{\nu}$, as reflected in the Belle II Physics Book [18].

Conclusion. — With a second Higgs doublet quite plausible, the existence of extra Yukawa couplings is an experimental issue. The SM and 2HDM II predict the ra-
the ratio $\mathcal{R}_{\mu/\tau}^B = B(\tau \rightarrow \mu \bar{\nu})/B(\tau \rightarrow \nu \bar{\nu})$ to be 0.0045, which offers a unique test. Through $\bar{\nu}_\tau$ flavor, the $\rho_{\mu \tau}$ coupling can enhance $B \rightarrow \mu \bar{\nu}$, while $B \rightarrow \tau \bar{\nu}$ is SM-like. If enhancement of $\mathcal{R}_{\mu/\tau}^B$ is uncovered by Belle II with just a few ab$^{-1}$, then the many extra Yukawa couplings — fundamental flavor parameters associated with a second Higgs doublet — would need to be unraveled.

Acknowledgments. We thank A. Crivellin for communications. This research is supported by grants MOST 106-2112-M-002-015-MY3, 107-2811-M-002-3069, 107-2811-M-002-039, and NTU 108L104019.

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