Low-lying $ud\bar{s}\bar{s}$ configurations in a non-relativistic constituent quark model

W.L. Wang$^1$, F. Huang$^2$, Z.Y. Zhang$^3$, Y.W. Yu$^3$, and F. Liu$^1$

$^1$Institute of Particle Physics, Huazhong Normal University, Wuhan 430079, China
$^2$CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China
$^3$Institute of High Energy Physics, P.O. Box 918-4, Beijing 100049, China

Abstract

The energies of the low-lying isoscalar and isovector $ud\bar{s}\bar{s}$ configurations with spin-parity $J^P = 0^+, 1^+, \text{ and } 2^+$ are calculated in a non-relativistic constituent quark model by use of the variational method. The contributions of various parts of the quark-quark interacting potentials including the $s$-channel interaction are investigated, and the effect of different forms of confinement potential is examined. The model parameters are determined by the same method as in our previous work, and they still can satisfactorily describe the nucleon-nucleon scattering phase shifts and the hyperon-nucleon cross sections. The parameters of the $s$-channel interaction are fixed by the masses of $K$ and $K^*$ mesons, for which the size parameter is taken to be two possible values. When it is chosen as the same as baryons', the numerical results show that the masses of all the $ud\bar{s}\bar{s}$ configurations are higher than the corresponding meson-meson thresholds. But when the size parameter for the $K$ and $K^*$ mesons is adjusted to be smaller than that for the baryons, the $ud\bar{s}\bar{s}$ configuration with $I = 0$ and $J^P = 1^+$ is found to lie lower than the $K^*K^*$ threshold, furthermore, this state has a very small $KK^*$ component and the interaction matrix elements between this state and $KK^*$ is comparatively small, thus its coupling to the $KK^*$ channel will consequently be weak and it might be regarded as a possible tetraquark candidate.

PACS numbers: 12.39.-x, 21.45.+v

Keywords: Tetraquark state; Constituent quark model

*Electronic address: wlwang@ihep.ac.cn
I. INTRODUCTION

So far, all the observed hadrons can be classified into two types, i.e. “baryons” composed of \(qqq\) and “mesons” composed of \(q\bar{q}\). But in principle, the QCD fundamental theory doesn’t exclude the existence of the states containing more than three quarks, i.e. the so-called multi-quark states. Since Jaffe predicted the \(H\) particle \((uuddss)\) in 1977\(^1\), the research on multi-quark states has always been an attractive topic for nearly three decades in both theoretical and experimental studies. But up to now, there is no convincing evidence of their existence in experiments. In 2003, LEPS Collaboration reported the possible existence of the \(\Theta^+\) pentaquark\(^2\), and subsequently some laboratories also reported the similar results\(^3\). At the same time, several laboratories reported the negative results\(^3\). Although its existence is still questioned, the \(\Theta^+\) particle has motivated a number of theoretical and experimental studies of pentaquarks and further the multi-quark states.

Besides dibaryon and pentaquark, the possible \(ud\bar{s}\bar{s}\) tetraquark is another interesting multi-quark system, and many works have been devoted to the investigation of this state in the past few years\(^4\,5\,6\,7\,8\,9\,10\). In Ref.\(^4\), using the MIT bag model, Jaffe performed a wide study for the spectrum of the \(4q\) states, and he predicted the masses of the isovector \(J^P = 0^+\) and isoscalar \(J^P = 1^+\) \(ud\bar{s}\bar{s}\) states to be 1.55 GeV and 1.65 GeV, respectively. In Ref.\(^5\), Vijande et al. analyzed the \(ud\bar{s}\bar{s}\) systems for both isospin \(I = 0\) and \(I = 1\) channels in a constituent-quark model, and they didn’t find any stable \(ud\bar{s}\bar{s}\) tetraquark state. In Ref.\(^6\), Burns et al. claimed that the \(\Theta^+\) particle suggest there should exist a \(ud\bar{s}\bar{s}\) tetraquark state with \(J^P = 1^-\) and mass around 1.6 GeV. This state has a strong color-magnetic attraction and decays into \(KK\) channel via \(P\)-wave with a width around 10 – 100 MeV. In Ref.\(^7\), Karliner and Lipkin argued that the isoscalar \(J^P = 0^+\) \(ud\bar{s}\bar{s}\) tetraquark state is a cousin of the \(\Theta^+\) pentaquark, and for this state the lowest allowed decay mode is a four-body \(KK\pi\pi\) channel with a very small phase space and a distinctive experimental signature. However, we should note that from the non-clustered quark degree of freedom, the isoscalar \(J^P = 0^+\) \(ud\bar{s}\bar{s}\) state within the spatially symmetric configuration is not allowed due to the Pauli principle. In Ref.\(^8\), Kanada-En’yo et al. studied the \(ud\bar{s}\bar{s}\) system in the framework of the flux-tube quark model, and they pointed that the isoscalar \(J^P = 1^+\) \(ud\bar{s}\bar{s}\) is a stable and low-lying tetraquark state with a mass around 1.4 GeV. It can decay into \(K^*K\) via \(S\)-wave with its width around 20-80 MeV, while the isovector
Recently, Cui et al. calculated the masses of the isoscalar \( J^P = 1^+ \) \( ud \bar{s}s \) systems by using the color-magnetic interaction Hamiltonian with SU(3) flavor symmetry breaking, and they found the \( ud \bar{s}s \) tetraquark lies around 1347 MeV with a narrow width since it can not decay into the \( KK^* \) channel. In Ref. [10], Chen et al. performed a QCD sum rule study for the isovector \( J^P = 0^+ \) \( ud \bar{s}s \) system, and they obtained a tetraquark state with a mass around 1.5 GeV.

To sum up, the existence and properties of the possible \( ud \bar{s}s \) tetraquark state are presently model dependent. Further theoretical and experimental investigations of this state via different approaches seem to be significant and essential.

It is a general consensus that QCD is the underlying theory of the strong interaction. However, as the non-perturbative QCD effect is very important for light quark systems in the low energy region and it is difficult to be seriously solved, people still need QCD-inspired models to be a bridge connecting the QCD fundamental theory and the experimental observations. In the past few years, we have developed a non-relativistic constituent quark model, which has been quite successful in reproducing the energies of the baryon ground states, the binding energy of the deuteron, the nucleon-nucleon (\( NN \)) scattering phase shifts, and the hyperon-nucleon (\( YN \)) cross sections [11]. In this model, the quark-quark interaction contains confinement, one-gluon exchange (OGE) and boson exchanges stemming from scalar and pseudoscalar nonets, with the boson exchange potentials deduced from a linear interacting Lagrangian which is invariable under the chiral transformation [12], and the short range quark-quark interaction is provided by OGE and quark exchange effects.

Actually it is still a controversial problem in the low-energy hadron physics whether gluon or Goldstone boson is the proper effective degree of freedom besides the constituent quark. Glozman and Riska proposed that the Goldstone boson is the only other proper effective degree of freedom [13, 14]. But Isgur gave a critique of the boson exchange model and insisted that the OGE governs the baryon structure [15, 16]. Anyway, it is still an open problem in the low-energy hadron physics whether OGE or vector-meson exchange is the right mechanism for describing the short-range quark-quark interaction, or both of them are important. Thus in Ref. [17] we further extended our original constituent quark model to include the coupling of the quark and vector meson fields. The OGE that plays an important role in the short-range quark-quark interaction in our original model is now nearly replaced by the vector boson exchanges. This model has also been successful in
reproducing the energies of the baryon ground states, the binding energy of the deuteron, and the nucleon-nucleon scattering phase shifts \[17\].

Recently, we have extended our constituent quark model from the study of baryon-baryon scattering processes to the baryon-meson systems and the pentaquark Θ+ state \[12, 18, 19, 20, 21, 22, 23, 24\]. We found that some results are similar to those given by the chiral unitary approach study, such as that both the ΔK system with isospin \(I = 1\) and the ΣK system with \(I = 1/2\) have quite strong attractions \[19, 20, 21\]. In the study of the KN scattering \[12, 18, 19\], we get a considerable improvement not only on the signs but also on the magnitudes of the theoretical phase shifts comparing with other’s previous work. We also studied the structures of the Θ+ particle \[23, 24\], and found that the theoretical mass of Θ+ is much higher than the experimental value, thus we concluded that either the Θ+ particle does not exist or it can not be explained by a five-quark cluster, which is consistent with the high-statistic experimental negative results.

In this paper, using the variational method, we study the structures of the isoscalar and isovector \(ud\bar{s}\) configurations with spin-parity \(J^P = 0^+, 1^+, 2^+\) in our non-relativistic constituent quark model with meson and one gluon exchanges included. The contributions of various parts of the quark-quark interacting potentials including the \(s\)-channel interaction are investigated. The effect of different forms of confinement potential is examined, and the configuration mixing between the states with same quantum numbers is considered. The model parameters are determined by the same method as in our previous work \[20, 21, 22\], and they still can satisfactorily describe the nucleon-nucleon scattering phase shifts and the hyperon-nucleon cross sections. The \(s\)-channel quark-antiquark interaction is complicated, and we take the zero momentum approximation and fix the corresponding parameters by fitting the masses of \(K\) and \(K^*\) mesons. With the size parameter of \(K\) and \(K^*\) taken to be the same as baryons’, the numerical results show that the masses of all the \(ud\bar{s}\) configurations are higher than the corresponding meson-meson thresholds. But when the size parameter for the \(K\) and \(K^*\) mesons is adjusted to be smaller than that for the baryons, the \(ud\bar{s}\) configuration with \(I = 0\) and \(J^P = 1^+\) is found to lie lower than the \(K^*K^*\) threshold, furthermore, this state has a very small \(KK^*\) component and the interaction matrix elements between this state and \(KK^*\) is comparatively small, thus its coupling to the \(KK^*\) channel will consequently be weak and it might be regarded as a possible tetraquark candidate.

The paper is organized as follows. In the next section the framework of the non-relativistic
constituent quark model we used and the wave functions of the $uds\bar{s}$ configurations are briefly introduced. The calculated energies of six $uds\bar{s}$ states are shown in Sec. III, where some discussion is presented as well. Finally, the summary is given in Sec. IV.

II. FORMULATION

A. Model

Our non-relativistic constituent quark model has been widely described in the literature [12, 18, 19, 20], and the details can be found in these references. Here we just give the salient features of our model.

The total Hamiltonian of the $uds\bar{s}$ systems in the model can be written as

$$H = \sum_i T_i - T_G + V_{12} + V_{34} + \sum_{i=1,2, j=3,4} V_{ij},$$

where $T_G$ is the kinetic energy operator for the center-of-mass motion, and $V_{12}, V_{34}$ and $V_{ij}$ represent the quark-quark, antiquark-antiquark and quark-antiquark interactions, respectively,

$$V_{12} = V_{12}^{OGE} + V_{12}^{conf} + V_{12}^{ch},$$

where $V_{12}^{OGE}$ is the OGE interaction,

$$V_{12}^{OGE} = \frac{1}{4} g_1 g_2 (\lambda_1^c \cdot \lambda_2^c) \left\{ \frac{1}{r_{12}} - \frac{\pi}{2} \delta(r_{12}) \left[ \frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{4}{3} \frac{1}{m_1 m_2} (\sigma_1 \cdot \sigma_2) \right] \right\},$$

and the confinement potential $V_{12}^{conf}$, instead of the quadratic form as used in our previous work, is taken as the linear one,

$$V_{12}^{conf} = - (\lambda_1^c \cdot \lambda_2^c) (a_{12}^r r_{12} + a_{12}^d).$$

$V_{12}^{ch}$ represents the effective quark-quark potential induced by the one-boson exchanges. In our original constituent quark model, $V_{12}^{ch}$ includes the scalar boson exchanges and the pseudoscalar boson exchanges,

$$V_{12}^{ch} = \sum_{a=0}^{8} V_{\sigma a}(r_{ij}) + \sum_{a=0}^{8} V_{\pi a}(r_{ij}),$$
and when the model is extended to include the vector boson exchanges, \( V_{12}^{\text{ch}} \) can be written as

\[
V_{12}^{\text{ch}} = 8 \sum_{a=0}^{8} V_{\sigma a} (r_{ij}) + 8 \sum_{a=0}^{8} V_{\pi a} (r_{ij}) + 8 \sum_{a=0}^{8} V_{\rho a} (r_{ij}).
\]

(6)

Here \( \sigma_0, ..., \sigma_8 \) are the scalar nonet fields, \( \pi_0, ..., \pi_8 \) the pseudoscalar nonet fields, and \( \rho_0, ..., \rho_8 \) the vector nonet fields. The expressions of these potentials can be found in the literature [12, 18, 19, 20].

\( V_{34} \) in Eq. (1) represents the antiquark-antiquark interaction,

\[
V_{34} = V_{34}^{\text{OGE}} + V_{34}^{\text{conf}} + V_{34}^{\text{ch}},
\]

(7)

where \( V_{34}^{\text{OGE}} \) and \( V_{34}^{\text{conf}} \) can be obtained by replacing the \( \lambda_1 \cdot \lambda_2 \) in Eqs. (3) and (4) with \( \lambda_0 \cdot \lambda_0^* \), and \( V_{34}^{\text{ch}} \) has the same form as \( V_{12}^{\text{ch}} \).

\( V_{ij} \) in Eq. (1) represents the quark-antiquark interaction,

\[
V_{ij} = V_{ij}^{\text{OGE}} + V_{ij}^{\text{conf}} + V_{ij}^{\text{ch}} + V_{ij}^{s},
\]

(8)

where \( V_{ij}^{\text{OGE}} \) and \( V_{ij}^{\text{conf}} \) can be obtained by replacing the \( \lambda_1 \cdot \lambda_2 \) in Eqs. (3) and (4) with \( -\lambda_i \cdot \lambda_j^* \), and \( V_{ij}^{\text{ch}} \) can be obtained from the G parity transformation:

\[
V_{ij}^{\text{ch}} = \sum_k (-1)^{G_k} V_{ij}^{\text{ch},k},
\]

(9)

with \((-1)^{G_k}\) being the G parity of the \( k \)th meson. \( V_{ij}^{s} \) denotes the s-channel quark-antiquark interaction. For the \( ud\bar{s}s \) system, the s-channel interaction includes \( K \) and \( K^* \) exchanges,

\[
V_{ij}^{s} = V_{s}^{K} + V_{s}^{K^*}.
\]

(10)

Taking the zero momentum approximation, the spatial part of \( V_{ij}^{s} \) can be expressed as a delta-function. To flatten the delta-function, we replace it by the Yukawa function, then \( V_{s}^{K} \) and \( V_{s}^{K^*} \) can be expressed as:

\[
V_{s}^{K} = C^K \left( \frac{1 - \sigma_q \cdot \sigma_q}{2} \right) \left( \frac{2 + 3\lambda_q \cdot \lambda_q^*}{6} \right) \left( \frac{\Lambda^2}{r} \right) e^{-\Lambda r},
\]

(11)

and

\[
V_{s}^{K^*} = C^{K^*} \left( \frac{3 + \sigma_q \cdot \sigma_q}{2} \right) \left( \frac{2 + 3\lambda_q \cdot \lambda_q^*}{6} \right) \left( \frac{\Lambda^2}{r} \right) e^{-\Lambda r},
\]

(12)

where \( C^K \) and \( C^{K^*} \) are treated as parameters and we adjust them to fit the masses of \( K \) and \( K^* \) mesons.
B. Parameters

TABLE I: Model parameters. The meson masses and the cutoff masses: $m_{\sigma'} = 980$ MeV, $m_{\kappa} = 980$ MeV, $m_{\epsilon} = 980$ MeV, $m_{\pi} = 138$ MeV, $m_{K} = 495$ MeV, $m_{\eta} = 549$ MeV, $m_{\rho} = 957$ MeV, $m_{\rho'} = 957$ MeV, $m_{K^*} = 892$ MeV, $m_{\omega} = 782$ MeV, $m_{\phi} = 1020$ MeV, and $\Lambda = 1100$ MeV.

| Parameter | Model I | Model II | Model III |
|-----------|---------|----------|-----------|
| $f_{\text{chv}}/g_{\text{chv}}$ | $0$ | $2/3$ |
| $b_u$ (fm) | 0.5 | 0.45 | 0.45 |
| $m_u$ (MeV) | 313 | 313 | 313 |
| $m_s$ (MeV) | 470 | 470 | 470 |
| $g_u^2$ | 0.766 | 0.056 | 0.132 |
| $g_s^2$ | 0.846 | 0.203 | 0.250 |
| $g_{\text{ch}}$ | 2.621 | 2.621 | 2.621 |
| $g_{\text{chv}}$ | 2.351 | | 1.973 |
| $m_{\sigma}$ (MeV) | 595 | 535 | 547 |
| $a_{uu}^0$ (MeV/fm) | 87.5 | 75.3 | 66.2 |
| $a_{us}^0$ (MeV/fm) | 100.8 | 123.0 | 106.9 |
| $a_{ss}^0$ (MeV/fm) | 152.2 | 226.0 | 196.7 |
| $a_{uu}^0$ (MeV) | $-77.4$ | $-99.3$ | $-86.6$ |
| $a_{us}^0$ (MeV) | $-72.9$ | $-127.9$ | $-109.6$ |
| $a_{ss}^0$ (MeV) | $-83.3$ | $-174.20$ | $-148.7$ |

The harmonic-oscillator width parameter $b_u$ is taken to be 0.50 fm in our original constituent quark model, and when the vector boson exchanges are included, $b_u$ is taken to be 0.45 fm. This means that the bare radius of baryon becomes smaller when more meson clouds are included in the model, which sounds reasonable in the sense of the physical picture. The up (down) quark mass $m_{u(d)}$ and the strange quark mass $m_s$ are taken to be the usual values: $m_{u(d)} = 313$ MeV and $m_s = 470$ MeV. The coupling constant for scalar and pseudoscalar meson field coupling, $g_{\text{ch}}$, is determined according to the relation

$$\frac{g_{\text{ch}}^2}{4\pi} = \left(\frac{3}{5}\right)^2 \frac{g_{NN\pi}^2}{4\pi} \frac{m_{\sigma'}^2}{M_N^2}.$$  \hspace{1cm} (13)
with the empirical value $g_{NN\pi}^2/4\pi = 13.67$. The coupling constant for vector coupling of the vector-meson field is taken to be $g_{\text{chv}} = 2.351$, the same as used in the $NN$ case [17]. The masses of the mesons are taken to be the experimental values, except for $\sigma$ meson. The $m_\sigma$ is adjusted to fit the binding energy of the deuteron. The OGE coupling constants and the strengths of the confinement potential are determined by baryon masses and their stability conditions. All the parameters are tabulated in Table I, where the first set is for our original constituent quark model, the second and third sets are for the models with vector meson exchanges included by taking $f_{\text{chv}}/g_{\text{chv}}$ as 0 and $2/3$, respectively. Here $f_{\text{chv}}$ is the coupling constant for tensor coupling of the vector meson fields.

From Table I one can see that in models II and III, $g_u^2$ and $g_s^2$ are much smaller than the values in model I. This means that when the coupling of quarks and vector meson fields is included in the non-relativistic constituent quark model, the coupling constants of OGE will be greatly reduced. Thus the OGE that plays an important role of the quark-quark short-range interaction in our original constituent quark model is now nearly replaced by the vector-meson exchange. In other words, the mechanisms of the quark-quark short-range interactions in these models are quite different.

We’d like to mention that our previous work concentrated on the hadron-hadron interactions and it can be strictly proved that different forms of confinement potential does not make any visible influence on the theoretical results since the two hadrons are treated as two color-singlet clusters. In this work we adopt a color linear confinement potential to study the $ud\bar{s}\bar{s}$ one-cluster system, and the method of parameters determination is the same as in our previous work. Naturally, the three sets of parameters in Table I still can satisfactorily describe the energies of the baryon ground states, the binding energy of the deuteron, the nucleon-nucleon scattering phase shifts and the hyperon-nucleon cross sections.
C. \textit{ud\bar{s}\bar{s}} configurations

According to the Pauli principle in each quark (antiquark) pair, in the spatial symmetry case only six configurations are permitted for the \textit{ud\bar{s}\bar{s}} system:

\[
I = 1, J^P = 0^+ \implies \begin{cases} 
|1\rangle = \left(\{ud\}^3_1\{\bar{s}\bar{s}\}^3_1\right)_0 \\
|2\rangle = \left(\{ud\}^6_0\{\bar{s}\bar{s}\}^6_0\right)_0
\end{cases}
\]

\[
I = 0, J^P = 1^+ \implies \begin{cases} 
|3\rangle = \left(\{ud\}^3_0\{\bar{s}\bar{s}\}^3_1\right)_1 \\
|4\rangle = \left(\{ud\}^6_1\{\bar{s}\bar{s}\}^6_0\right)_1
\end{cases}
\]

\[
I = 1, J^P = 1^+ \implies |5\rangle = \left(\{ud\}^3_1\{\bar{s}\bar{s}\}^3_1\right)_1
\]

\[
I = 1, J^P = 2^+ \implies |6\rangle = \left(\{ud\}^3_1\{\bar{s}\bar{s}\}^3_1\right)_2
\]

The \textit{s} quark has isospin-zero so the total isospin of the \textit{ud\bar{s}\bar{s}} configuration state is determined by the isospin of the \textit{u, d} quarks. In the above expressions, \{ \} and \[ \] represent the flavor symmetry and antisymmetry, respectively, and the superscript is the representation of the color SU(3) group, the subscript is the spin quantum number. Making a re-coupling calculation, we can express the above six states into two quark-antiquark pairs, including two color octet \textit{q\bar{q}} pairs and two color singlet \textit{q\bar{q}} pairs with KK, KK*, and K*KK* quantum numbers. The corresponding expressions are given as follows:

\[
|1\rangle \equiv \left(\{ud\}^3_1\{\bar{s}\bar{s}\}^3_1\right)_0 = \frac{1}{2} \left((us)^{1}_0(d\bar{s})^{1}_0\right)_0 \sqrt{\frac{1}{12} \left((us)^{1}_1(d\bar{s})^{1}_1\right)_0} - \frac{1}{2} \left((us)^{1}_0(d\bar{s})^{1}_0\right)_0 \sqrt{\frac{1}{6} \left((us)^{1}_1(d\bar{s})^{1}_1\right)_0}, \tag{14}
\]

\[
|2\rangle \equiv \left(\{ud\}^6_0\{\bar{s}\bar{s}\}^6_0\right)_0 = \frac{1}{\sqrt{6}} \left((us)^{1}_0(d\bar{s})^{1}_0\right)_0 \sqrt{\frac{1}{12} \left((us)^{1}_1(d\bar{s})^{1}_1\right)_0} + \frac{1}{\sqrt{2}} \left((us)^{1}_1(d\bar{s})^{1}_1\right)_0 \sqrt{\frac{1}{6} \left((us)^{1}_1(d\bar{s})^{1}_1\right)_0} + \frac{1}{\sqrt{12}} \left((us)^{1}_0(d\bar{s})^{1}_0\right)_0 \sqrt{\frac{1}{2} \left((us)^{1}_1(d\bar{s})^{1}_1\right)_0}, \tag{15}
\]

\[
|3\rangle \equiv \left(\{ud\}^3_0\{\bar{s}\bar{s}\}^3_1\right)_1 = \frac{1}{\sqrt{12}} \left((us)^{1}_0(d\bar{s})^{1}_1\right)_1 \sqrt{\frac{1}{12} \left((us)^{1}_1(d\bar{s})^{1}_1\right)_1} - \frac{1}{\sqrt{6}} \left((us)^{1}_0(d\bar{s})^{1}_1\right)_1 \sqrt{\frac{1}{6} \left((us)^{1}_1(d\bar{s})^{1}_1\right)_1} - \frac{1}{\sqrt{3}} \left((us)^{1}_0(d\bar{s})^{1}_1\right)_1 \sqrt{\frac{1}{3} \left((us)^{1}_1(d\bar{s})^{1}_1\right)_1}, \tag{16}
\]

\[
|4\rangle \equiv \left(\{ud\}^6_1\{\bar{s}\bar{s}\}^6_0\right)_1 = -\frac{1}{\sqrt{6}} \left((us)^{1}_1(d\bar{s})^{1}_1\right)_1 \sqrt{\frac{1}{12} \left((us)^{1}_1(d\bar{s})^{1}_1\right)_1} + \frac{1}{\sqrt{6}} \left((us)^{1}_1(d\bar{s})^{1}_1\right)_1 \sqrt{\frac{1}{6} \left((us)^{1}_1(d\bar{s})^{1}_1\right)_1} - \frac{1}{\sqrt{3}} \left((us)^{1}_1(d\bar{s})^{1}_1\right)_1 \sqrt{\frac{1}{3} \left((us)^{1}_1(d\bar{s})^{1}_1\right)_1}, \tag{17}
\]
\[ |5\rangle \equiv \left( ud_s \bar{s} \bar{s}_1^3 \right)_1 = \sqrt{\frac{1}{6}} ((u\bar{s})_0^0)_{1}^1 + \sqrt{\frac{1}{6}} ((u\bar{s})^1_1)_{0}^1 
- \sqrt{\frac{1}{3}} ((u\bar{s})^8_0)_{1}^1 - \sqrt{\frac{1}{3}} ((u\bar{s})_{0}^8)_{1}^1; \] 
(18)

\[ |6\rangle \equiv \left( ud_s \bar{s} \bar{s}_1^3 \right)_2 = \sqrt{\frac{1}{3}} ((u\bar{s})^1_1)_{1}^1 - \sqrt{\frac{2}{3}} ((u\bar{s})^8_1)_{1}^1; \] 
(19)

where \((u\bar{s})(d\bar{s})\) represents \(\sqrt{1/2 \left[ (u\bar{s})(d\bar{s}) + (d\bar{s})(u\bar{s}) \right]}\) for Eqs. (14), (15), (18) and (19), and denotes \(\sqrt{1/2 \left[ (u\bar{s})(d\bar{s}) - (d\bar{s})(u\bar{s}) \right]}\) for Eqs. (16) and (17).

In the actual calculation, the configuration mixing between the states \(|1\rangle\) and \(|2\rangle\), as well as \(|3\rangle\) and \(|4\rangle\) is considered since these states have same quantum numbers.

### III. RESULTS AND DISCUSSIONS

**TABLE II: Energies (in MeV) of \(ud\bar{s}\) six configurations in our constituent quark models (without configuration mixing).**

| Configurations | Model I | Model II | Model III | Threshold |
|----------------|---------|----------|-----------|-----------|
| \(J^P = 0^+\) |         |          |           |           |
| \(\left( ud_s \bar{s} \bar{s}_1^3 \right)_0\) | 1679    | 1650     | 1650      | KK (990)  |
| \(\left( ud_s \bar{s} \bar{s}_1^6 \right)_0\) | 1828    | 1865     | 1833      | KK (990)  |
| \(J^P = 1^+\) |         |          |           |           |
| \(\left( ud_s \bar{s} \bar{s}_1^3 \right)_1\) | 1698    | 1704     | 1698      | KK* (1387)|
| \(\left( ud_s \bar{s} \bar{s}_1^6 \right)_1\) | 1803    | 1812     | 1798      | KK* (1387)|
| \(\left( ud_s \bar{s} \bar{s}_1^3 \right)_1\) | 1765    | 1750     | 1742      | KK* (1387)|
| \(J^P = 2^+\) |         |          |           |           |
| \(\left( ud_s \bar{s} \bar{s}_1^3 \right)_2\) | 1904    | 1906     | 1885      | \(K^*K^*\) (1784) |

We calculate the energies for six low configurations of \(ud\bar{s}\) system in our non-relativistic constituent quark models. The model parameters we used are shown in Table I, which can reproduce the NN phase shifts reasonably. As mentioned above, the parameters of the s-channel interactions are fixed by fitting the masses of \(K\) and \(K^*\). For reducing the input
parameters, here the size parameter for $K$ and $K^*$ is taken to be the same as that for baryons. The calculated results (without configuration mixing) are given in Table II. From this Table, we can see that the results from model I, II and III are quite similar, although the mechanisms of the quark-quark short-range interactions are different in these models, i.e., one is from OGE and the two others are from vector meson exchanges. This means that the OGE and the vector-meson exchange can give similar contributions in the $ud\bar{s}\bar{s}$ system, the same situation as in the nucleon-nucleon and kaon-nucleon systems [17, 18].

In Ref. [9], using the color-magnetic interaction Hamiltonian with SU(3) flavor symmetry breaking, Cui et al. argued that the strong attractive color-magnetic interaction can reduce the energies of the $ud\bar{s}\bar{s}$ systems, and they found a $I = 0$ and $J^P = 1^+ \, ud\bar{s}\bar{s}$ tetraquark state with a mass around 1347 MeV. In our constituent quark model calculations, in model I, the color-magnetic interactions are attractive in both the isovector $J^P = 0^+ \, ([ud]_0^3\{\bar{s}\bar{s}\})_0^3$ state and the isoscalar $J^P = 1^+ \, ([ud]_1^3\{\bar{s}\bar{s}\})_1^3$ state, while they are repulsive in the other four configurations, and in models II and III, the OGE is largely reduced and the color-magnetic attractions are almost replaced by $\rho$ exchange. Furthermore, in all the models, the $\sigma$ and $\pi$ exchanges provide more attractive interactions in the isovector $J^P = 0^+ \, ([ud]_1^3\{\bar{s}\bar{s}\})_1^3$ state and the isoscalar $J^P = 1^+ \, ([ud]_0^3\{\bar{s}\bar{s}\})_0^3$ state than in the other four configurations. Thus the energies of the $([ud]_0^3\{\bar{s}\bar{s}\})_0^3$ and $([ud]_0^3\{\bar{s}\bar{s}\})_1^3$ states are respectively the lowest one in $J^P = 0^+$ and $J^P = 1^+$ cases in various models. However, due to the high kinetic energies, the attractive interactions are not strong enough to reduce the energies of these two states to be lower than the corresponding meson-meson thresholds (see Table II).

The states $([ud]_1^3\{\bar{s}\bar{s}\})_0^3$ and $([ud]_0^6\{\bar{s}\bar{s}\})_0^6$ in $J^P = 0^+$ case, as well as $([ud]_1^3\{\bar{s}\bar{s}\})_1^3$ and $([ud]_0^6\{\bar{s}\bar{s}\})_0^6$ in $J^P = 1^+$ case have the same quantum numbers. Thus the configuration mixing between them has to be considered. The results are shown in Table III. Comparing it with Table II, one can see that the configuration mixing effect is significant, it can shift the energies over 80 MeV for most configurations. In the isovector $J^P = 0^+$ case, now the energy of the lowest state is reduced about 170–290 MeV in various models, and in the isoscalar $J^P = 1^+$ case, it is about 80–130 MeV. Even though, the energies of these configurations are still higher than their corresponding meson-meson thresholds. In other words, the stable $ud\bar{s}\bar{s}$ tetraquark state cannot yet be obtained.

The confinement potential is phenomenological, and usually it is taken as linear, quadratic or error function form. Here we consider these three various forms of the confinement potential.
TABLE III: Energies (in MeV) of $ud\bar{s}\bar{s}$ states with configuration mixing considered.

|                | Model I | Model II | Model III |
|----------------|---------|----------|-----------|
| $I = 1, J^P = 0^+$ | 1394    | 1483     | 1482      |
|                | 1963    | 1952     | 1924      |
| $I = 0, J^P = 1^+$ | 1567    | 1619     | 1615      |
|                | 1887    | 1878     | 1861      |
| $I = 1, J^P = 1^+$ | 1765    | 1750     | 1742      |
| $I = 1, J^P = 2^+$ | 1904    | 1906     | 1885      |

potential to see the corresponding effects. The results, from our original non-relativistic constituent quark model, are shown in Table IV. In this Table, $r$, $r^2$ and erf represent the confinement potential adopted as linear, quadratic and error function form, respectively. We can see that the energies with the confinement potential taken to be the error function form are always the lowest. But the difference of various confinement potentials is less than about 30 MeV, which means different form of the confinement potential has no significant effect on the energy of the $ud\bar{s}\bar{s}$ state.

TABLE IV: Energies (in MeV) of the $ud\bar{s}\bar{s}$ states in our original constituent quark model with configuration mixing and three different forms (linear, quadratic and error function) of the confinement potential considered.

|                | $r$    | $r^2$    | erf    |
|----------------|--------|----------|--------|
| $I = 1, J^P = 0^+$ | 1394   | 1402     | 1390   |
|                | 1963   | 1996     | 1945   |
| $I = 0, J^P = 1^+$ | 1567   | 1568     | 1567   |
|                | 1887   | 1907     | 1876   |
| $I = 1, J^P = 1^+$ | 1765   | 1770     | 1763   |
| $I = 1, J^P = 2^+$ | 1904   | 1920     | 1895   |

As we know, the s-channel quark-antiquark interaction mechanism is a complicated and unclear problem. In the study of the structure of the $\Theta^+$ particle [23], it has been pointed
TABLE V: Energies (in MeV) of $ud\bar{s}\bar{s}$ mixing states without and with $s$-channel interaction in different models.

|                | without $s$-channel interaction | with $s$-channel interaction |
|----------------|---------------------------------|-----------------------------|
|                | I  | II | III | I  | II | III |
| $I = 1, J^P = 0^+$ | 1699 | 1910 | 1890 | 1394 | 1483 | 1482 |
|                | 2041 | 2001 | 1991 | 1963 | 1952 | 1924 |
| $I = 0, J^P = 1^+$ | 1749 | 1825 | 1828 | 1567 | 1619 | 1615 |
|                | 1984 | 1964 | 1960 | 1887 | 1878 | 1861 |
| $I = 1, J^P = 1^+$ | 1876 | 1922 | 1912 | 1765 | 1750 | 1742 |
| $I = 1, J^P = 2^+$ | 1964 | 1939 | 1935 | 1904 | 1906 | 1885 |

Out that how to treat the $s$-channel interaction reasonably is very important. In the $ud\bar{s}\bar{s}$ system, the effect of the $s$-channel interaction should also be examined. We completely omit this interaction to see its influence, and the results are shown in Table V, compared with those with $s$-channel interaction. One can see that the $s$-channel interactions offer quite strong attractions in all these six $ud\bar{s}\bar{s}$ configurations and thus can reduce the energies of these states about several hundreds MeV in all the models. In this sense, the effect of the $s$-channel interactions is significant and un-negligible in the $ud\bar{s}\bar{s}$ system.

TABLE VI: Energies (in MeV) of $ud\bar{s}\bar{s}$ states with the configuration mixing considered and the size parameter of $K$, $K^*$ taken as 0.4 fm.

|                | Model I | Model II | Model III |
|----------------|---------|----------|-----------|
| $I = 1, J^P = 0^+$ | 1602    | 1573     | 1572      |
|                | 1857    | 1909     | 1882      |
| $I = 0, J^P = 1^+$ | 1577    | 1623     | 1618      |
|                | 1768    | 1833     | 1817      |
| $I = 1, J^P = 1^+$ | 1771    | 1754     | 1745      |
| $I = 1, J^P = 2^+$ | 1821    | 1872     | 1852      |
In the above calculations, the size parameter of $K$ and $K^*$ mesons are taken to be the same as the baryons’ in order to reduce the free parameters. It seems more reasonable to choose the size parameter for the mesons to be smaller than that for the baryons. We then perform a variational method calculation for the energies of the $ud\bar{s}\bar{s}$ states with the size parameter of $K$ and $K^*$ taken to be 0.4 fm, and the results are shown in Table VI. We notice that one of the $I = 0 J^P = 1^+$ $ud\bar{s}\bar{s}$ configurations is a very interesting state in model I. Its energy is 1768 MeV, lower than the threshold of $K^*K^*$, and the corresponding root mean square radius of this state is about 0.57 fm. Especially, the structure of this state is very interesting since it contains very few components of $KK^*$, as can be seen from the expression of its wave function given below:

$$\langle 4' | = -0.17 \left( (u\bar{s})_0^1 (d\bar{s})_1^1 \right)_1^1 + 0.17 \left( (u\bar{s})_1^1 (d\bar{s})_0^1 \right)_1^1 + 0.71 \left( (u\bar{s})_1^1 (d\bar{s})_1^1 \right)_1^1$$

$$- 0.47 \left( (u\bar{s})_0^8 (d\bar{s})_1^1 \right)_1^1 + 0.47 \left( (u\bar{s})_1^8 (d\bar{s})_0^1 \right)_1^1 + 0.0056 \left( (u\bar{s})_1^8 (d\bar{s})_1^1 \right)_1^1.$$  \hspace{2cm} (20)

From this equation it is clear to see that besides 43.8% part of two color octet $q\bar{q}$ pairs, the component of two color singlet $q\bar{q}$ pairs is 50.4% for $K^*K^*$ and only 5.8% for $KK^*$, which means the $K^*K^*$ component is dominate and comparatively the $KK^*$ component is very small in the color-singlet $q\bar{q}$-$q\bar{q}$ part, thus the $I = 0 J^P = 1^+$ $ud\bar{s}\bar{s}$ configuration might have a few possibility decaying into $K$ and $K^*$.

Furthermore, in order to see the effect of the coupling between this $I = 0 J^P = 1^+$ $ud\bar{s}\bar{s}$ state and the $KK^*$ channel, we calculate the interaction matrix elements between the $|4\rangle'$ state and $KK^*$ channel in model I by treating $K$ and $K^*$ as two clusters with a distance $S$, \begin{equation}
\langle V \rangle = \langle 4' | \sum_{i \in K^*} \sum_{j \in K^*} V_{ij} | KK^* \psi(\vec{R} - \vec{S}) \rangle,
\end{equation}

where $\psi(\vec{R} - \vec{S})$ is the relative motion wave function of the two clusters $K$ and $K^*$, and for simplicity we take it as

$$\psi(\vec{R} - \vec{S}) = (\omega \mu / \pi)^{3/4} \exp\left[ -\omega \mu (\vec{R} - \vec{S})^2 / 2 \right],$$  \hspace{2cm} (22)

here $S$ is the generator coordinate which can qualitatively describe the distance between the two clusters. The calculated results are shown in Figure 1, along with the contributions of $\langle (u\bar{s})_0^1 (d\bar{s})_1^1 \rangle_1^1$, $\langle (u\bar{s})_1^1 (d\bar{s})_0^1 \rangle_1^1$ and the two color-octet $q\bar{q}$ pairs components in the $|4\rangle'$ state. In Fig. 1, M denotes the interaction matrix elements, and $S$ can qualitatively describe the distance between the two clusters $K$ and $K^*$.
FIG. 1: The matrix elements of the interaction potential. The solid line represent the matrix elements of interaction potential between the $|4\rangle'$ state and the $KK^*$ channel. The dotted, dash-dotted and dashed lines represent the contributions of $((u\bar{s})_0^1|d\bar{s})_1^1$, $((u\bar{s})_1^1|d\bar{s})_1^1$, and the two color-octet components in $4\rangle'$ state, respectively.

From Fig. 1 one can see that the contribution of color-octet components to the interaction matrix elements is the smallest which is apparent and can be easily understood, while the $((u\bar{s})_0^1|d\bar{s})_1^1$ component also has a small contribution due to the small component in $|4\rangle'$ (smaller than 6%), although the matrix element $\langle((u\bar{s})_0^1|d\bar{s})_1^1|\sum_{j\in K^*} V_{ij}KK^*\psi(\vec{R} - \vec{S})\rangle$ is comparatively big. The solid line in Fig. 1 clearly tells us that the total contribution of $4\rangle'$ to the interaction matrix elements, i.e. $\langle4'|\sum_{j\in K^*} V_{ij}KK^*\psi(\vec{R} - \vec{S})\rangle$, is very small, less than 6 MeV and inclines to zero with the increase in distance between $K$ and $K^*$. Hence, the coupling between this interesting $|4\rangle'$ state and the $KK^*$ channel can be regarded as quite small. This means the $I = 0 \; J^P = 1^+ \; ud\bar{s}\bar{s}$ state has a few possibility decaying into two separate $K$ and $K^*$ via interaction potential. Furthermore, since the energy of this state, 1768 MeV, is lower than the $K^*K^*$ threshold (1784 MeV), it cannot decay into $K^*K^*$ final state. This means this state would possibly have a narrow width, and might be treated as a good candidate for the $ud\bar{s}\bar{s}$ tetraquark state.

As discussed in Ref. [9], it seems worth searching this state in $K^+d \rightarrow p + p + K^- + T^+$ or $J/\psi(\Upsilon) \rightarrow K^- + K^0 + T^+$ channels in the future experiments [Here $T^+$ denotes the
udśś tetraquark state with $I = 0$ and $J^P = 1^+$. The experimental information about the existence of this state will help us test the validity of the application of our constituent quark models to the study of multi-quark states, although these models are quite successful in the investigations of $NN$, $NY$, and $KN$ interactions \[11, 12, 17, 18, 19\].

**IV. SUMMARY**

The structures of $udśś$ states with $J^P = 0^+, 1^+, \text{and } 2^+$ are studied in our non-relativistic constituent quark models with the meson and one gluon exchanges included in the quark-quark interaction potentials. We calculate the energies of six low-lying $udśś$ configurations by use of the variational method. The configuration mixing between the states with same quantum numbers are considered. The effect of different forms of color confinement potentials and the contributions of $s$-channel $q\bar{q}$ interactions are also examined. The results show that the different forms (linear, quadratic, and error function) of the confinement potential just give similar contributions, and the $s$-channel interactions can reduce the energy of the $udśś$ system several hundred MeV. With the model parameters determined by the same method as in our previous work, the calculated energies of all the $udśś$ configurations are higher than the corresponding meson-meson thresholds. But when the size parameter for the mesons is adjusted to be 0.4 fm, a value smaller than that for the baryons, the $udśś$ configuration with $I = 0$ and $J^P = 1^+$ is found to lie lower than the $K^*K^*$ threshold, furthermore, this state has a very small $KK^*$ component and the interaction matrix elements between this state and $KK^*$ is comparatively small, thus its coupling to the $KK^*$ channel will consequently be weak and it might be regarded as a possible tetraquark candidate. A further dynamical calculation would be done in the future work.

**Acknowledgments**

This work was supported in part by the National Natural Science Foundation of China (No. 10475087) and China Postdoctoral Science Foundation (No. 20060400093).

[1] R.L. Jaffe, Phys. Rev. Lett. 38, 195 (1977).
[2] T. Nakano et al. (LEPS Collaboration), Phys. Rev. Lett. 91, 012002 (2003).

[3] For recent reviews see K. Hicks, Int. J. Mod. Phys. A 20, 219 (2005); A.R. Dzierba, C.A. Meyer, and A.P. Szczepaniak, J. Phys. Conf. Ser. 9, 192 (2005).

[4] R.L. Jaffe, Phys. Rev. D 15, 267 (1977); 15, 281 (1977).

[5] J. Vijande, F. Fernandez, A. Valcarce, and B. Silvestre-Brac, Eur. Phys. J. A 19, 383 (2004).

[6] T.J. Burns, F.E. Close, and J.J. Dudek, Phys. Rev. D 71, 014017 (2005).

[7] M. Karliner and H.J. Lipkin, Phys. Lett. B 612, 197 (2005).

[8] Y. Kanada-En’yo, O. Morimatsu, and T. Nishikawa, Phys. Rev. D 71, 094005 (2005).

[9] Y. Cui, X.L. Chen, W.Z. Deng, and S.L. Zhu, Phys. Rev. D 73, 014018 (2006).

[10] H.X. Chen, A. Hosaka, and S.L. Zhu, Phys. Rev. D 74, 054001(2006).

[11] Z.Y. Zhang, Y.W. Yu, P.N. Shen, L.R. Dai, A. Faessler, and U. Straub, Nucl. Phys. A 625, 59 (1997).

[12] F. Huang, Z.Y. Zhang, and Y.W. Yu, Phys. Rev. C 70, 044004 (2004).

[13] L.Ya. Glozman and D.O. Riska, Phys. Rept. 268, 263 (1996).

[14] L.Ya. Glozman, Nucl. Phys. A 663, 103c (2000).

[15] N. Isgur, Phys. Rev. D 61, 118501 (2000).

[16] N. Isgur, Phys. Rev. D 62, 054026 (2000).

[17] L.R. Dai, Z.Y. Zhang, Y.W. Yu, and P. Wang, Nucl. Phys. A 727, 321 (2003).

[18] F. Huang and Z.Y. Zhang, Phys. Rev. C 72, 024003 (2005).

[19] F. Huang and Z.Y. Zhang, Phys. Rev. C 70, 064004 (2004).

[20] F. Huang, D. Zhang, Z.Y. Zhang, and Y.W. Yu, Phys. Rev. C 71, 064001 (2005).

[21] F. Huang and Z.Y. Zhang, Phys. Rev. C 72, 068201 (2005).

[22] F. Huang, Z.Y. Zhang, and Y.W. YU, Phys. Rev. C 73, 025207 (2006).

[23] F. Huang, Z.Y. Zhang, Y.W. Yu, and B.S. Zou, Phys. Lett. B 586, 69 (2004).

[24] D. Zhang, F. Huang, Z.Y. Zhang, and Y.W. Yu, Nuc. Phys. A 756, 215 (2005).