Generation of the Baryon Asymmetry of the Universe within the Left–Right Symmetric Model

J.-M. Frère\textsuperscript{a}, L. Houart\textsuperscript{b}, J.M. Moreno\textsuperscript{c}, J. Orloff\textsuperscript{†a} and M. Tytgat\textsuperscript{§b}

\textit{a} Theory Division, CERN, CH-1211 Geneva 23, Switzerland
\textit{b} Service de Physique Théorique, CP 225, Université Libre de Bruxelles, Bld. du Triomphe, B-1050 Bruxelles, Belgium
\textit{c} Dept. de Fisica Teorica, Univ. Autonoma de Barcelona, E–08193 Bellaterra, Spain

Abstract

Fermions scattering off first-order phase transition bubbles, in the framework of $SU(2)_L \otimes SU(2)_R \otimes U(1)$ models, may generate the Baryon Asymmetry of the Universe (BAU), either at the $LR$-symmetry-breaking scale, or at the weak scale. In the latter case, the baryon asymmetry of the Universe is related to CP violation in the $K_0–\bar{K}_0$ system.
1. Introduction

Generation of the BAU at a relatively low scale was a natural response to growing fears that sphaleron-like configurations at the weak scale would destroy any pre-established baryon asymmetry. In fact two responses are possible and both find a natural realization in LR models.

In the first case, one assumes that the existing or generated baryon number is protected by some symmetry immune to the weak forces (a typical example is conservation of \((B - L)\) as defined in terms of the usual fermions). This is studied in section 2 below, where a non-vanishing \((B - L)\) is generated at the \(R\) phase transition, i.e. when large masses are induced for bosons associated with the \(SU(2)_R\) group.

Alternatively, the baryon number can be generated at the usual weak scale, while making sure that \(B\)-violating interactions quickly cease to be in equilibrium, so that the newly-born BAU cannot be washed away\(^{[1]}\). Several mechanisms have been proposed to generate the baryon asymmetry at this rather low scale\(^{[2,3,4,5,6]}\). All of them involve extensions of the Standard Model. This may seem strange, since even the minimum standard model already possesses all the required qualitative ingredients (C and CP violation at the standard Lagrangian level, \((B + L)\) violation through sphaleron-like solutions and, depending upon the scalar mass, departure from equilibrium due to a first-order phase transition). The main reason of this failure rests in the smallness of the invariant effective CP-violation parameter \(\delta_{CP} = O(10^{-20})\)^{[2]}, where the heavy suppression results not only from the bare CP-violating phase of the Kobayashi–Maskawa matrix, but also from the products of mixing angles and mass differences needed to reflect the non-degeneracy of the 3-generation mixing structure. As \(\delta_{CP}\) is expected to enter as a factor in the calculation of the baryon asymmetry, this falls short by many orders of magnitude. Present data thus strongly suggest and to some extent legitimize at least some extension of the standard model. Such extensions, usually to the scalar sector, may seem \emph{ad hoc}, and require the introduction of new sources of CP violation,
unrelated to low-energy phenomenology. As we shall see in section 3, LR models offer a natural framework for such extensions, with a scalar structure similar to models already suggested. The gauge bosons associated with $SU(2)_R$ serve as intermediaries, as they amplify the effect of the phases, making them detectable in the usual K-system $\Delta S = 2$ amplitudes\(^7\).

2. Baryon Number Generation at the Right Scale

We consider in this section the possibility of creating the BAU at the $R$-symmetry breaking scale. Various mechanisms have been imagined to produce the asymmetry using $(B - L)$-violating interactions\(^3\), leading to a $B - L \neq 0$ Universe.

This in turn constrains any left-over $(B - L)$-violating interactions at lower temperatures to be out of equilibrium, to prevent them from erasing the BAU\(^8\). Neutrino Majorana masses are a typical example.

In the LR symmetric model, all the ingredients to generate the baryon asymmetry are at hand. The spontaneous breakdown of the $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge symmetry to $SU(2)_L \otimes U(1)_Y$ breaks $C$ and $(B - L)$ (through Majorana masses for the right-handed neutrinos). Some CP violation in the lepton sector is easily included as this phenomenology is rather unconstrained: several $R$-scalar triplets with non-removable phases between their vacuum expectation values, or two triplets and one pseudoscalar singlet easily foot the bill. This extension is sufficient to generate a non-vanishing $(L)$, and has little effect at low energy as the quarks do not couple to the scalar triplets.

The model also needs some $(B + L)$-violating processes. Such processes induced by sphaleron-like configurations at equilibrium will convert a fraction of $L \neq 0$ to $B \neq 0$. Two qualitatively different configurations are possible:

- the usual sphaleron configuration associated with $SU(2)_L$ or rather its instanton-like continuation above the electroweak scale, creates or destroys left-handed fermions;
— on the other hand the topological argument of Manton and Klinkhamer\cite{9} may be extended to the breaking of a $SU(2)_R$ gauge symmetry.

To verify this last statement, let us consider the simple model of $SU(2)_R$ gauge bosons coupled to a triplet complex scalar field ($\Delta_R$). We thus both neglect the couplings (both of the triplet and of the gauge bosons) to the other scalars (bi-doublets) —which anyway do not develop a v.e.v. at the right breaking scale—and the mixing with the $U(1)_{B-L}$ gauge field (just as $\theta_W = 0$ was assumed in ref.[9]). The scalar potential then reads

$$V(\Delta_R) = -\frac{\mu^2}{2} \text{tr} \Delta^\dagger \Delta + \frac{\lambda_1}{4} \left(\text{tr} \Delta^\dagger \Delta\right)^2 + \frac{\lambda_2}{4} \left(\text{tr} \Delta \Delta \right) \left(\text{tr} \Delta^\dagger \Delta^\dagger\right) \quad (2.1)$$

where

$$\Delta = \begin{pmatrix} \delta^+ \\ \delta^0 \\ \delta^+ \end{pmatrix} = \vec{\tau} \cdot \vec{r}. \quad (2.2)$$

$\vec{r} = \vec{r}_1 + i \vec{r}_2$ is a 3-dimensional complex vector and $\tau_a$ are the Pauli matrices. Expressed in term of $\vec{r}$, (2.1) becomes

$$V(\vec{r}_1, \vec{r}_2) = -\mu^2 (\vec{r}_1^2 + \vec{r}_2^2) + \lambda_1 (\vec{r}_1^2 + \vec{r}_2^2)^2 + \lambda_2 \left((\vec{r}_1^2 - \vec{r}_2^2)^2 + 4(\vec{r}_1 \cdot \vec{r}_2)^2\right) \quad (2.3)$$

The potential is minimized by the following solutions:

$$\vec{r}_1^2 = \vec{r}_2^2 = \frac{\mu^2}{4\lambda_1} = \frac{v_R^2}{2} \quad \text{and} \quad \vec{r}_1 \cdot \vec{r}_2 = 0. \quad (2.4)$$

Note that the quartic term in $\lambda_2 > 0$ is necessary to break completely the $SU(2)_R$ symmetry by giving a mass to the otherwise pseudo-Goldstone doubly-charged scalar particles.

The topological argument of Manton\cite{9} now applies. The existence of sphalerons —i.e. classical unstable solutions— is related to the possibility of constructing non-contractible loops in configuration space, beginning and ending in the vacuum.
Taking the configuration of maximal energy on every loop of a given homotopy class and then selecting the infimum of this ensemble, yields (modulo the validity of Morse theory on non-compact spaces) such a sphaleron.

For a given configuration on the loop to have a finite energy, the scalar field must stay in the vacuum at spatial infinity. This defines a mapping of the boundary of the 3-dimensional space, $S^2$ (parametrized by $\theta, \phi$), onto the space of the vacuum configurations of the scalar field, which is $SO(3)$ since the symmetry is completely broken. What is called a closed loop is simply a continuous family of such mappings, parametrized by $\mu \in [0, \pi]$, and shrinking to single points of the $SO(3)$ vacuum-space for $\mu = 0, \pi$. The homotopy classes of such loops are then isomorphic to those of the maps

$$g(\mu, \theta, \phi) : S^3 \rightarrow SO(3) .$$

(2.5)

where $S^3$ is spanned by $\mu, \theta, \phi$.

Since $\pi_3(SO(3)) = \mathbb{Z}$, the necessary topological condition for the existence of sphalerons is fullfilled. On top of this topological argument, much hard work would still be needed to find an explicit solution. Unfortunately indeed, the spherical symmetry of the energy functional, used in the complex doublet case, is lost here (it was linked to the custodial $SU(2)$ symmetry in the limit where $\sin \theta_W \rightarrow 0$ with vanishing Yukawa couplings). This complicates matters considerably, as the full set of coupled non-linear partial derivative equations must now be tackled; we will not pursue this study further in the present paper. Nevertheless, with the topological conditions satisfied, we may consider that at high temperature (well above $v_R$), both left- and right-baryon numbers are violated, while between the $R$ scale and the electroweak breaking scale, only “left” configurations are active.

The last ingredient required to satisfy Sakharov’s\textsuperscript{[1]} conditions is a departure from equilibrium. We consider the generation of $B$ through the Charge Transport Mechanism of Cohen et al.\textsuperscript{[3]}, i.e. the reflection of right-handed neutrinos on walls
of expanding bubbles of $SU(2)_L \otimes U(1)_Y$ “vacuum”*. When bubble walls are thin, this mechanism is very efficient as it exploits a large region in front of the wall for biasing baryon number. In contrast, the mechanism known as spontaneous baryogenesis, is only efficient in the walls, which thus have to be thick[5]. We restrict this study to a strongly first-order phase transition, and hence relatively thin walls. This assumption does not upset any phenomenology as the scalar sector at the $R$ scale is anyway barely constrained.

At the $R$ scale first-order transition, a wall separates two distinct regions: one symmetric under $SU(2)_R \otimes SU(2)_L \otimes U(1)_{B-L}$ (the symmetric region) and the other, symmetric only under $SU(2)_L \otimes U(1)_Y$ (the broken one). Right-handed neutrinos incident from the symmetric region will interact strongly with the wall and may be either reflected or transmitted. If $N_R$ stands for the field associated with right-handed neutrinos, it also describes the left-handed anti-neutrinos (their CP-conjugates). Those are also present in the thermal bath, and similarly interact with the wall. When necessary, we will use the notation $(N_R)^c$ to distinguish them. (In this we follow the tradition, although $N_R^{CP}$ would seem more appropriate).

With $\mathcal{R}_{N_R \rightarrow (N_R)^c}$ defined as the probability of reflection of an $R$-neutrino into anti-$R$-neutrino, we get, under the C, P and T discrete symmetries†

\[
P: \quad \mathcal{R}_{N_R \rightarrow (N_R)^c} \rightarrow \mathcal{R}_{N_L \rightarrow (N_L)^c} \\
C: \quad \mathcal{R}_{N_L \rightarrow (N_L)^c} \rightarrow \mathcal{R}_{(N_R)^c \rightarrow N_R} \\
T: \quad \mathcal{R}_{(N_R)^c \rightarrow N_R} \rightarrow \mathcal{R}_{N_R \rightarrow (N_R)^c}
\] (2.6)

Since only right-handed fields are involved, CPT brings no information, as it merely relates $\mathcal{R}_{N_R \rightarrow (N_R)^c}$ to itself. If CP is not conserved, $\mathcal{R}_{N_R \rightarrow (N_R)^c}$ may differ from $\mathcal{R}_{(N_R)^c \rightarrow N_R}$. As in [3] we introduce CP violation in the reflection on the wall

* Another starting point was used in ref. [10], namely the out-of-equilibrium decay of right-handed Majorana neutrinos.
† We find it easier to picture these symmetries for fermions incident on a spherical bubble rather than on a plane wall.
through a space-dependent unremovable phase in the potential. The Dirac equations for $N_R$ then involves a complex, space-dependent mass $m(z)$:

$$i\gamma^\mu \partial_\mu N_R = m(z)(N_R)^c.$$  

(2.7)

A detailed calculation can be found in [3] in a slightly different version, and we don’t repeat it here. The flux of leptons in the symmetric region is related to the calculated reflection probabilities as

$$f_L = \frac{2}{\gamma} \int_m^{\infty} dk_L \int_0^{\infty} dk_T \left( f^s(k_L, k_T) - f^b(k_L, k_T) \right) \times$$

$$\times \left( \mathcal{R}(k_L)_{N_R \rightarrow (N_R)^c} - \mathcal{R}(k_L)(N_R)^c_{\rightarrow N_R} \right).$$

(2.8)

where $f^s$ and $f^b$ measure respectively the flux from the symmetric and from the broken regions, obtained by boosting the thermal equilibrium distributions from the rest frame of the plasma to the rest frame of the moving wall.

Owing to rapid $(B + L)$-violating interactions in the symmetric region, the generated $L$-number partially converts into a $B$ excess. If thermal equilibrium is satisfied in the symmetric region, this yields [3]

$$(B + L)|_{eq} \approx x (B - L)|_{eq}$$

(2.9)

with $x = O(1)$. Actually, thermal equilibrium is not perfect and one has to solve the Boltzman equation for the system [3]. The maximum predicted baryon-to-entropy ratio in the symmetric region is again

$$\frac{n_B}{s} \sim \frac{f_L}{s} \lesssim 10^{-6}.$$  

(2.10)

This is a peak value, and the exponent is very sensitive to the parameters involved (wall speed and thickness, fermion mass, critical temperature,...), and typical values

$\dagger$ We have extrapolated this number from the weak scale$^{[3]}$ to the $LR$ symmetry-breaking scale. Without substantial entropy production, it is essentially scale-independent.
quoted in ref.[3,4] range down to $10^{-10}$, while the CP angle comes in as a mere factor.

After completion of the phase transition, $(B - L)$ is no longer conserved. The right-handed neutrinos have acquired a Majorana mass through coupling to the v.e.v. of the right scalar triplet that carries a $(B - L)$-charge. Even at scales much below the $R$ transition, some of those interactions feed through to the remaining light sector, for instance by the see-saw induced light Majorana masses of ordinary (left-handed) neutrinos. Moreover, $(B + L)_L$ violating processes are still active, and these conjugated effects endanger the previously created asymmetry.

To simplify the discussion, we will consider an abrupt transition between two qualitatively different equilibrium regimes. The first one was considered above in the “symmetric region”: all interactions are supposed to be at least approximately in equilibrium. This yields, through the charge transport mechanism, a non-zero baryon and lepton number excess for all the flavours of quarks and leptons. The second regime occurs after the $R$ phase transition: there we assume the heavy particles to be out-of-equilibrium, hence eliminating the $R$-gauge bosons, the right-handed Majorana neutrinos and the $R$-scalar-triplet field from the thermal bath. We are then essentially in the regime of the Standard Model above the electroweak breaking scale. The critical difference is the occurrence of $L$-number-violating processes through the virtual exchange of Majorana neutrinos. The effective cross-sections for these processes is

$$
\sigma_{\Delta L_i=2} \approx \frac{m_{\nu_i}^2}{2\pi v_R^4}.
$$

(2.11)

As the density of relativistic species is $n_i = T^3/\pi^2$, the rate $\Gamma_{\Delta L=2} = \langle \sigma n \rangle$ is

$$
\Gamma_{\Delta L_i=2} \approx \frac{1}{\pi^3} \frac{T^3 m_{\nu_i}^2}{v_R^4}.
$$

(2.12)

In a $LR$ model with scalar triplets, the light neutrino mass $m_\nu$ occurs through a
see-saw mechanism\textsuperscript{[11]}. Barring fine tuning, this typically leads to:

\begin{equation}
m_\nu \approx m_{\text{Dirac}}^2/M_R \tag{2.13}
\end{equation}

where \(m_{\text{Dirac}} \approx m_{\text{charged lepton}}\).

Setting \(m_{\nu_e} = m_{\nu_e}^2/M_R\), \(m_{\nu_\mu} = m_{\nu_\mu}^2/M_R\) and \(m_{\nu_\tau} = m_{\nu_\tau}^2/M_R\), we check which of the lepton-number-violating processes fall out of equilibrium at \(T_c \sim M_R \sim 10\) TeV, a phenomenologically relevant scale. Using

\[m_e = 0.5 \text{ MeV}, \ m_\mu = 105 \text{ MeV}, \ m_\tau = 1800 \text{ MeV},\]

we get the comparison with experimental numbers:

\begin{align*}
m_{\nu_e} & \approx 10^{-10} \text{ GeV} \ll 17 \text{ eV} \\
m_{\nu_\mu} & \approx 10^{-6} \text{ GeV} \ll 0.27 \text{ MeV} \tag{2.15} \\
m_{\nu_\tau} & \approx 10^{-4}\text{GeV} \ll 35 \text{ MeV}
\end{align*}

Comparing the rates for \(\Delta L_i = 2, i = e, \mu, \tau\), with the expansion rate of the Universe at \(T \sim T_c \sim M_R\)\textsuperscript{[12]}:

\[\Gamma_{\Delta L_i=2} \leq H = 1.67 g_*/2 T^2/m_{\text{pl}} \tag{2.16}\]

where \(g_* \approx 100\), we find that for \(m_{\nu_i} \leq 10^{-6} \text{ GeV}\) the process is out of thermal equilibrium. This is certainly not the case for \(L_\tau\)-changing processes but might be the case for \(L_e\)- and even for \(L_\mu\)-changing reactions. An approximate conservation in one or two of the leptonic flavours thus appears possible, provided lepton mixing is kept under control (see also ref.[13]). One can rest on low-energy phenomenology to argue that those mixings are much smaller than in the quark sector.

We thus consider a thermal-equilibrium situation with conserved \(L_e\) and \(L_\mu\) and non-conserved \(L_\tau\). Writing all the processes in equilibrium (as are the \((B+L)_L\)-violating processes) generates a set of relations between chemical potentials (see
for example Harvey and Turner in ref. [8]) which leads to

$$(B + L)|_{eq} = -\frac{288N + 30m + 141}{288N + 78m - 147}(B - L)|_{eq}$$  \quad (2.17)$$

where $N$ and $m$ count the number of left scalar triplets and scalar bi-doublets still participating in the thermal bath (we expect $N \rightarrow 0$ if explicit LR symmetry is imposed). This number must be understood as the maximum value the asymmetry may reach, since we have neglected non-equilibrium effects between the two regimes, which can only diminish $B$.

In conclusion, it is possible to generate the baryon number at the $R$ scale through the breaking of $(B-L)$ and both left- and right-$(B+L)$-violating processes. This scale does not need to be very high —$O(10 \text{ TeV})$— as conservation of at least one lepton number suffices to protect the baryon number. The predictive power of this $R$-scale mechanism is unfortunately rather poor, as it requires the introduction of new CP violation in the leptonic sector, without significant low-energy implications. We now turn to the more challenging possibility of creating $B$ at the lower electroweak scale within a LR-symmetric structure.

3. Baryon Number Generation at the Weak Scale

The main excitement about this model is that the CP-violating parameters needed in the scalar sector are transferred by $R$ gauge bosons to the $K$ system and thus become accessible to experiment. As we shall see, this model also meets another challenge faced by baryon number generators in their panic fear of thermodynamical equilibrium situations. It is clear indeed that, unless some protection mechanism exists (e.g. non-vanishing $(B-L)$, as discussed in the previous section) such thermal equilibrium will tend to wipe out any existing or generated baryon number. The way to prevent a freshly generated baryon number from such fate is thus to require that the phase transition occurs briskly, putting the system out of equilibrium. In particular, the order parameter $v(T)$ must move directly to a
large value. The baryon-number-violating processes are then quickly turned off at the onset of the phase transition. In the standard model, this translates, as we recall below, into a small value for the mass of the symmetry-breaking scalar. This low-mass scalar can however be avoided if trilinear couplings between the relevant fields are present in the tree Lagrangian, as is the case in the LR model with spontaneous CP violation.

Consider the Boltzman factor governing the rate of baryon-number violation \( \Gamma_B \):

\[
\Gamma_B = 3\kappa \alpha^4 T^4 \exp(-E_{sph}/T),
\]

(3.1)

where \( \kappa \) incorporates uncertainties in the prefactor (\( \kappa = O(1) \)) and \( E_{sph} \) is the energy of the saddle configuration known as the sphaleron. This energy is related to the \( T \)-dependent order parameter \( v(T) \)

\[
E_{sph} = \alpha_W(T) v(T) B \left( \frac{\lambda(T)}{\alpha_W(T)} \right).
\]

(3.2)

The numerical value of \( B(\lambda/\alpha) \) varies from 1.5 to 2.7 when the ratio of the quartic scalar coupling to the gauge parameter \( \lambda/\alpha \) varies from 0 to \( \infty \). On the other hand, the value of \( v(T_c) \) at the first-order phase transition is \( E/\lambda(T_c) \) where \( E \) is the trilinear coupling coefficient in the effective potential. In the standard model this coupling is absent at tree level, and \( E = O(g^3) \). A big jump in \( v \) at the phase transition thus imposes a small \( \lambda \), whence, as \( m_{higgs} \approx \lambda v(T = 0) \), a small scalar mass.

Imposing \( \Gamma_B \leq H \), where \( H \) is the expansion rate of the Universe at the time of the phase transition leads to the constraint [14],[15]:

\[
mass_{Higgs} \lesssim 35 \text{ GeV},
\]

(3.3)

which lies “uncomfortably close” to the experimental bound at LEP1 [16],

\[
mass_{Higgs} \gtrsim 48 \text{ GeV}.
\]

(3.4)
Considerations of this kind have been applied to extensions of the Standard Model such as the two-Higgs model \[14\] and the minimal supersymmetric model \[17\]. Typically, this still constrains (sometimes very strongly \[17\]) the mass of the lightest scalar. A very interesting model from this point of view is the non-minimal supersymmetric model considered in ref.\[18\]. The constraint on the Higgs mass is considerably relaxed by the appearance of a “tree level” effective cubic coupling. As seen above, the large size of the trilinear coupling removes the requirement of a tenuous quartic coupling. We will argue that the $LR$-symmetric model shares the same nice feature.

However, one thing struck us in most of these constructions, namely the closeness of the proposed models to the scalar structure traditionally used in the $SU(2)\textsubscript{L} \otimes SU(2)\textsubscript{R} \otimes U(1)\textsubscript{LR}$ model. Imbedding the proposed schemes in a fully-fledged $LR$ structure would at the same time offer some justification for the doubling of the scalars, and relate baryon-number violation to more mundane CP parameters\(^*\). What came as a surprise is that the further requirement of spontaneously broken CP also provided the necessary ingredients for a tree-level driven first-order phase transition. Even independently of the extra nicety of this first-order phase transition, the $LR$ model, as remarked by Mohapatra\[^{10}\], offers to our knowledge the only link between our existence as “baryon beings” and the (minute) value of CP violation.

Unfortunately, as we shall see below, this link is not so strong as to predict e.g. the sign of $\epsilon'/\epsilon$ from the dominance of matter over antimatter. This is due to the fact that even the most straightforward model of spontaneous CP violation in $LR$ still allows for (discrete) number of variants, between which the data and the knowledge of the strong matrix elements are currently too unprecise to choose. Nevertheless, it is only a matter of time before this is clarified, and the uniqueness of the link persists.

This situation is in contrast with existing models where the CP violation related

\(^*\) We wish to thank A. Cohen for numerous discussions on this point
to the BAU is disconnected from the $K$-system, and at most affect the electric dipole moment of quarks$^{[19]}$ and leptons$^{[20]}$.

### 3.1. A sketch of the LR model with spontaneous CP violation

The model we consider is CP-conserving$^{[21,22,23]}$ before symmetry breaking of the electroweak scale. Spontaneous CP violation appears through a non-zero physical phase in the v.e.v. of the bi-doublet fields:

$$\langle \phi \rangle = \frac{e^{i\alpha/2}}{\sqrt{2}} \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v & 0 \\ 0 & w \end{pmatrix}$$

This, through Yukawa couplings to the quarks:

$$M^{(u)} = \frac{1}{\sqrt{2}}(v\Gamma + w^*\Delta) = UD^{(u)}U^T$$

$$M^{(d)} = \frac{1}{\sqrt{2}}(w\Gamma + v^*\Delta) = VD^{(d)}V^T,$$

leads to CP violation in the $K_0$–$\bar{K}_0$ system. One may note the particular form of the diagonalization of the mass matrices that results from imposing explicit CP conservation before symmetry breaking. In this case, the $\Gamma$ and $\Delta$ matrices can be chosen to be real and symmetric.

In a particular basis for the quarks fields, the KM matrices for the L and R charged currents read

$$K_L = U^\dagger V, \quad K_R = K_L^*$$

which are thus not independent matrices. In this convention all the phases in $K_L$ are observable: their number is $\frac{n(n+1)}{2}$ for $n$ generations, hence six for the case of interest, all of them related to $\alpha$ (of course, for two generations all the phases
and for three generations all the phases but one, can be rotated away from $K_L$ into $K_R$ but this does not make them unobservable). This is the zero-temperature situation. To look at the behaviour of $\alpha$ at non-zero temperature, more information is needed on the scalar potential, which is unfortunately not very constrained in the LR model. It contains many unknown couplings between the bi-doublet and the two triplets. One can still show\cite{24} that, without fine-tuning, this potential is unable to generate a non-zero value for the phase $\alpha$ after SSB. This is easy to cure through the introduction of a singlet neutral pseudo-scalar \cite{24}, which couples to the bi-doublet in the following way

$$V(\Phi, \eta) = V(\Phi) + V_\eta(\Phi, \eta),$$

$$V_\eta = V(\eta) + iC_1 \eta (\det \Phi - \det \Phi^*) + C_2 \eta^2 tr \left( \Phi^\dagger \Phi \right).$$

Once expressed in polar coordinates, the trilinear term in the potential depends upon $\sin \alpha$ and will compete with terms even under $\alpha \to -\alpha$, leading to $\alpha \neq 0$ for a large range of parameters.

### 3.2. Phase Transition and Baryogenesis

We will not attempt here to pursue the study of the above potential at finite temperature. Nevertheless reliable hints can be obtained from simpler models. In particular we now argue, in analogy with \cite{18}, that a strongly first-order phase transition may take place.

The argument is based on the existence of a trilinear term (required for natural spontaneous CP violation) in the classical potential. The Higgs field, i.e. the field that develops a non-zero v.e.v., is a temperature-dependent combination of $\phi_1^0, \phi_2^0$ and $\eta$, with

$$\langle \phi_1 \rangle = \langle H \rangle \cos \beta_1 \cos \beta_2$$
$$\langle \phi_2 \rangle = \langle H \rangle \cos \beta_1 \sin \beta_2$$
$$\langle \eta \rangle = \langle H \rangle \sin \beta_1,$$
where $\beta_1$ and $\beta_2$ are temperature-dependent mixing angles. These angles will remain sizeable provided the masses and interactions of the involved scalars are not too different. This requires a relatively light $\eta$, associated with the $L$ scale rather than the $R$ scale, as is more usually assumed.

The orthogonal combinations correspond to massive scalars with zero v.e.v. Once substituted in the potential $V_\eta$, (3.10) yields a term trilinear in $H$. We have thus good indications that a strongly first-order transition will arise, without seriously constraining the masses of the light scalars. We refer the reader to ref. [18] for an explicit example in a more tractable model.

A strongly first-order phase transition must proceed through the nucleation of bubbles with thin walls. The spontaneous baryogenesis mechanism of [10] uses $\langle \dot{\alpha} \rangle$ as a source biasing baryon-number-violating processes. In the present case of a thin wall, we turn to the more efficient charge transport mechanism of Cohen, Kaplan and Nelson [4].

They consider the diffusion of a fermion (the top quark, as its Yukawa coupling is the largest) by the expanding bubble of true vacuum. An incident top (massless in the false vacuum) of right chirality ($t_R$) will be reflected as a top of left chirality ($t_L$) or transmitted as a massive top of right helicity*. Under C,P and T, $t_R$ transforms as:

\[
P : \quad \mathcal{R}_{t_R \rightarrow t_L} \rightarrow \mathcal{R}_{t_L \rightarrow t_R},
\]

\[
C : \quad \mathcal{R}_{t_L \rightarrow t_R} \rightarrow \mathcal{R}_{(t_R)^c \rightarrow (t_L)^c},
\]

\[
T : \quad \mathcal{R}_{(t_R)^c \rightarrow (t_L)^c} \rightarrow \mathcal{R}_{(t_L)^c \rightarrow (t_R)^c}.
\]

The CPT theorem imposes $\mathcal{R}_{t_R \rightarrow t_L} = \mathcal{R}_{(t_L)^c \rightarrow (t_R)^c}$. Only if CP is conserved do we get $\mathcal{R}_{t_R \rightarrow t_L} \overset{CP}{=} \mathcal{R}_{(t_R)^c \rightarrow (t_L)^c} \overset{CPT}{=} \mathcal{R}_{t_L \rightarrow t_R}$. If the interaction with the wall violates CP, those two probabilities may differ, as shown in [4]. In the rest frame

* One may also expect to see the top transform into a bottom, which interacts much less with the wall. However it is easy to find a gauge (so-called unitary gauge) in which there is no stationary gauge field configuration. Any top to bottom transition must then be due to virtual effects in the presence of the wall and thus be reduced by some effective $G_W$, a much smaller effect than the tree-level effect of diffusion considered in [4].
of the wall, an observer in the symmetric region sees a flux of particles with a non-zero axial baryon number:

\[
f_A = \frac{2}{\gamma} \int dk_L \int dk_T \left( f^s(k_L, k_T) - f^b(k_L, k_T) \right) \times
\]

\[
\times (\mathcal{R}(k_L)_{t_L \to t_R} - \mathcal{R}(k_L)_{t_R \to t_L})
\]

where \( f^s \) and \( f^b \) refer respectively to the boosted flux from the symmetric and from the symmetry-broken region. The flux \( f_A \) is non zero if CP is violated and if the wall is moving through the thermal background. This flux carries no net baryonic charge, but is easily shown to carry a hypercharge\(^\dagger\).

As hypercharge is conserved in the symmetric region (as well as \( Q, B - L, \ldots \)) but \( B + L \) is not\(^\ddagger\), equilibrium processes will lead to a non-zero baryon density excess in front of the wall. After completion of the phase transition, this excess of \( B \) is transferred to the broken symmetry Universe.

The authors of [4] obtain in this way a baryon to entropy ratio

\[
B \overset{def}{=} \left. \frac{n_B}{s} \right|_{\text{pred.}} \approx 10^{-8} \Delta \alpha
\]

where \( \Delta \alpha \) is the jump of \( \alpha \) from the symmetric to the broken region\(^\S\). Below we will take the approximation \( \Delta \alpha = \alpha(T = 0) \). We argued indeed that the phase transition may be strongly first order, which means that the jump in the order parameters must be close to their \( T = 0 \) value.

\(^\dagger\) Actually \( f_Y = \frac{1}{4} f_A \). It has been critized that Debye screening of the gauged hypercharge would prevent this current to penetrate far in the symmetric region, thus curbing the mechanism. One should however keep in mind that this current, while carrying some hypercharge, is not identical to the latter, but merely refers to the hypercharge in the top sector. Screening of hypercharge can then occur with the help of all quarks equally without seriously hindering the mechanism, as shown in [26]. For brevity we keep referring to an excess \( Y \).

\(^\ddagger\) The rate of baryon-number violation in the symmetric region has been estimated on dimensional ground to be of order \( \Gamma_B \approx \kappa \alpha^4 W T^4 \) where \( \kappa = O(1) \).

\(^\S\) The above result is of course subject to many uncertainties related, on one side to rough estimates concerning baryon-number-violating processes, and on the other side to variations depending on the wall velocity and width themselves.
The above estimate of $B$ agrees with the value obtained from nucleosynthesis
\[4 \times 10^{-11} \leq n_B/s \leq 1.4 \times 10^{-10}.\]

even for values of $\Delta \alpha$ as small as 0.01 to 0.001. Larger values of $\Delta \alpha$ could of
course be accommodated if some later dilution of $B$ occurs (for instance, if the
$B$-violating processes are not fully out of equilibrium in the broken phase).

3.3. Connection to Low-Energy CP violation

It is interesting to compare the $T = 0$ value of $\alpha$ obtained above with the low-
energy CP violation. While the comparison should be made to the full model\[27],
it is informative to first consider the simple case of two quark generations, where
the dependence on the various parameters is considerably easier to exhibit. The $K_L$
matrix is parametrized in the following form:
\[K_L = e^{i\gamma} \begin{pmatrix} e^{-i\delta_2} \cos \theta_c & e^{-i\delta_1} \sin \theta_c \\ -e^{i\delta_1} \sin \theta_c & e^{i\delta_2} \cos \theta_c \end{pmatrix}.\]

The phases appearing in this matrix can be related to $\alpha$ and to the measured
values of quarks masses and mixings. An exact treatment \[27\] largely corrobrates
Chang’s linear development \[22\] which yields
\[\delta_1 \approx \frac{r \sin \alpha}{1 - r^2 \cos^2 \alpha} \left( -\frac{3}{2} \frac{m_c}{m_s} \cos^2 \theta_c - \frac{1}{2} \frac{m_c}{m_d} \sin^2 \theta_c \right)\]
\[\delta_2 \approx \frac{r \sin \alpha}{1 - r^2 \cos^2 \alpha} \left( -\frac{1}{2} \frac{m_c}{m_s} \cos^2 \theta_c + \frac{1}{2} \frac{m_c}{m_d} \sin^2 \theta_c \right)\]

where $r = |\kappa'/\kappa|$. With $m_s \approx 200$ MeV, $m_c \approx 1.4$ GeV, $m_d \approx 8.9$ MeV, and small
$\alpha$, this reduces to by baryogenesis considerations one gets:
\[|\langle \delta_1 - \delta_2 \rangle| \approx 10 |r\alpha|.\]

Those phases can be related in the minimal $LR$ model to the experimentally well-
known value of $\epsilon$:

$$\epsilon_{LR} \approx e^{i\pi/4} 0.36 \sin(\delta_1 - \delta_2) \left(\frac{1.4 \text{ TeV}}{M_R}\right)^2,$$

$$\epsilon_{\text{exp}} = e^{i\pi/4} 2.26 \times 10^{-3}.$$

Some bounds on $r$ arise from the inversion procedure, which re-expresses $\Delta$ and $\Gamma$ (cf. eq. (3.6)) in terms of the measured quark masses and mixings (for details see [27]). One typically gets (if we choose arbitrarily $\kappa'$ to be the smaller of these parameters)

$$r = O\left(\frac{m_b}{m_t}\right).$$

It is to be noted that this limit on $r$, imposed by the existence of the third generation, is a matter of mathematical consistency and must be taken into account even if dominant contributions to the process considered come from the first two generations.

It is then clear that the values of $\alpha$ required to account for $\epsilon$, even in a simple 2-generation scheme, are if anything more than sufficient to account for the observed baryon number of the Universe, thus allowing for some dilution with respect to the mechanism studied.

This is obviously not the complete story, and even the simplest LR model with spontaneous CP violation allows much more freedom in the relation between these parameters. In particular, a discrete set of models is associated with the choice of the relative signs of the masses in ref. [23]; while in the Standard Model we can always redefine $q_R \rightarrow -q_R$ without observable consequences, this is not possible in the LR model. Furthermore, the range of allowed values is considerably larger in the full 3-flavour study —this being in part due to the lack of precise measurements for mixing angles and the unknown mass of the top.

A detailed study of the various observable CP-violating parameters as a function of $\alpha$ ($\epsilon$, $\epsilon'$ and the neutron electric dipole moment) is presented in [27], to
which we refer the reader. From that study it can be seen that the values of $\alpha$ needed for high-temperature CP violation are perfectly compatible with those observables, but that critical tests (e.g. the agreement of the sign of $\xi'$) will require more precise measurements of the Kobayashi–Maskawa mixing angles.

4. Conclusions

The minimal $LR$ model considered in this paper has many attractive features for the phenomenology of CP violation both at low energy and at high temperature. To our knowledge it offers the first link\cite{10} between previously disconnected sectors: the generation of the baryon number of the Universe and the value of $\epsilon$ from the $K^0-\bar{K}^0$ system, even if too many uncertainties (both theoretical and experimental) temporarily prevent us from fully exploiting the real predictive power of the model.

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