Abnormal Structure of Fermion Mixings
in a Seesaw Quark Mass Matrix Model

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Abstract

It is pointed out that in a seesaw quark mass matrix model which yields a singular enhancement of the top-quark mass, the right-handed fermion-mixing matrix $U_{uR}$ for the up-quark sector has a peculiar structure in contrast to the left-handed one $U_{uL}$. As an example of the explicit structures of $U_{uL}$ and $U_{uR}$, a case in which the heavy fermion mass matrix $M_F$ is given by a form $[(\text{unit matrix})+(\text{rank-one matrix})]$ is investigated. As a consequence, one finds observable signatures at projected high energy accelerators like the production of a fourth heavy quark family.

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I. INTRODUCTION

Why is the top-quark mass \( m_t \) so singularly enhanced compared with the bottom-quark mass \( m_b \) (while keeping \( m_u \sim m_d \))? Why does only the top-quark have a mass of the order of the electroweak scale \( m_W \)? Recently, it has been pointed out [1,2] that a seesaw quark mass matrix model [3] can give a natural answer to these questions.

In the conventional seesaw mass matrix model [3], we assume vector-like heavy fermions \( F_i \) in addition to the conventional three-family quarks and leptons \( f_i \) (\( f = u, d, \nu, e; \ i = 1, 2, 3 \)). These fermions \( f \) and \( F \) belong to \( f_L = (2,1) \), \( f_R = (1,2) \), \( F_L = (1,1) \) and \( F_R = (1,1) \) of SU(2)\(_L\) × SU(2)\(_R\). The fermions \( F \) acquire masses \( M_F \) at a large energy scale \( \mu \sim \lambda m_0 \). The symmetries SU(2)\(_L\) and SU(2)\(_R\) are broken at the energy scales \( \mu \sim m_0 \) and \( \mu \sim \kappa m_0 \), respectively. Then, the 6 \times 6 mass matrix \( M \) for the fermions \( (f, F) \) is given by

\[
M = \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} = m_0 \begin{pmatrix} 0 & Z_L \\ \kappa Z_R & \lambda Y \end{pmatrix},
\]

(1.1)

where matrices \( Z_L \), \( Z_R \) and \( Y \) are dimensionless matrices with the order of one. For \(|\lambda| \gg |\kappa| \gg 1\) and \( \det M_F \neq 0 \), the 3 \times 3 mass matrix for fermions \( f \), \( M_f \), is approximately given by the well-known “seesaw” expression [4]

\[
M_f \simeq -m_L M_F^{-1} m_R ,
\]

(1.2)

so that the fermion masses \( m_i^f \) (\( i = 1, 2, 3 \)) are suppressed by a factor \( \kappa/\lambda \) to the electroweak scale \( m_0 \). This was one of the motivations for considering a seesaw mechanism for quarks [3] before the discovery of the top quark [5]. However, the observation of the large top-quark mass has demanded that top-quark mass should be of the order of \( m_0 \) without the factor \( \kappa/\lambda \).

Recently, it has been found [1,2] that the seesaw mass matrix with \( \det M_F = 0 \) can yield fermion masses \( m_i^f \) and \( m_i^F \equiv m_{i+3}^f \) (\( i = 1, 2, 3 \)) with the following order:

\[
\begin{align*}
m_1, m_2 & \sim (\kappa/\lambda)m_0 , \\
m_3 & \sim m_0 \sim O(m_L) , \\
m_4 & \sim \kappa m_0 \sim O(m_R) , \\
m_5, m_6 & \sim \lambda m_0 \sim O(M_F) .
\end{align*}
\]

(1.3)

Note that the third fermion mass does not have the factor \( \kappa/\lambda \). Therefore, if the heavy fermion mass matrix \( M_F \) takes \( \det M_F = 0 \) in the up-quark sector, we can
understand why only the top-quark has a mass of the order of $m_L$ without the suppression factor $\kappa/\lambda$. This was first explicitly derived by Fusaoka and the author [1] on the basis of a special seesaw mass matrix model, “democratic seesaw mass matrix model”, where $M_F$ is given by the form $[(\text{unit matrix})+(\text{a rank-one matrix})]$, and then generalized by Morozumi et al. [2].

In the present paper, we will point out that in such a model the right-handed fermion-mixing matrix $U_R$ has a peculiar structure in contrast to the left-handed one $U_L$, i.e., as if the third and fourth rows of $U_R$ are exchanged each other in contrast to $U_L$. In Sec. II, we will discuss general properties of the fermion mass spectrum and the mixing matrices $U_L$ and $U_R$ in the would-be seesaw mass matrix (1.1) with $\text{det} M_F = 0$. In Sec. III, in order to see the more explicit relations between the Cabibbo-Kobayashi-Maskawa (CKM) [6] matrices $V_L$ and $V_R$, we investigate a case with constraints $Z_R^T = Z_L$ and $Y^T = Y$ which are not so restrictive and which most models can satisfy. In Sec. IV, as an explicit example of $U_L$ and $U_R$, we evaluate the mixing matrices for the case of the “democratic seesaw mass matrix model” which can yield realistic quark masses and CKM matrix $V_L$. The final section V will be devoted to the summary.

II. GENERAL STRUCTURES OF THE FERMION MIXING MATRICES

The mixing matrices $U_L$ and $U_R$ are obtained by diagonalizing the following Hermitian matrices $H_L$ and $H_R$, respectively:

$$H_L \equiv MM^\dagger = \begin{pmatrix} m_L m_L^\dagger & m_L M_F^\dagger \\ M_F m_L^\dagger & M_F M_F^\dagger + m_R m_R^\dagger \end{pmatrix} = m_0^2 \begin{pmatrix} Z_L Z_L^\dagger & \lambda Z_L Y^\dagger \\ \lambda Y Z_L^\dagger & \lambda^2 Y Y^\dagger + \kappa^2 Z_R Z_R^\dagger \end{pmatrix},$$  \hfill (2.1)

$$H_R \equiv M^\dagger M = \begin{pmatrix} m_R^\dagger m_R & m_R^\dagger M_F \\ M_F^\dagger m_R & M_F^\dagger M_F + m_L^\dagger m_L \end{pmatrix} = m_0^2 \begin{pmatrix} \kappa^2 Z_R Z_R^\dagger & \kappa \lambda Z_R^\dagger Y \\ \kappa \lambda Y^\dagger Z_R & \lambda^2 Y^\dagger Y + Z_L^\dagger Z_L \end{pmatrix}.$$  \hfill (2.2)

For $\text{det} M_F \neq 0$, we can obtain the well-known seesaw expression (1.2) since the
$3 \times 3$ Hermitian matrices $H^f_L$ and $H^f_R$ for fermions $f_i$ are approximately given by

\[ H^f_L \simeq m_L m_L^\dagger - m_L M^f_F (M^f_F M^f_F + m_R m_R^\dagger)^{-1} M^f_F m_L^\dagger \simeq -m_L M^f_F m_R (m_L M^f_F m_R)^\dagger, \]

\[ H^f_R \simeq m_R^\dagger m_R - m_R^\dagger M^f_F (M^f_F M^f_F + m_L^\dagger m_L^\dagger)^{-1} M^f_F m_R \simeq -(m_L M^f_F m_R)^\dagger m_L M^f_F m_R. \]

(2.3)

The $6 \times 6$ mixing matrices $U_L$ and $U_R$ are approximately given by

\[ U_L \simeq \begin{pmatrix} A_L & 0 \\ 0 & B_L \end{pmatrix} \begin{pmatrix} 1 & -m_L M^f_F \dagger \\ M^f_F m_L^\dagger & 1 \end{pmatrix} = \begin{pmatrix} A_L & -A_L m_L M^f_F \dagger \\ B_L M^f_F m_L^\dagger & B_L \end{pmatrix}, \]

(2.5)

\[ U_R \simeq \begin{pmatrix} A_R & 0 \\ 0 & B_R \end{pmatrix} \begin{pmatrix} 1 & -m_R^\dagger M^f_F \dagger \dagger \\ M^f_F m_R & 1 \end{pmatrix} = \begin{pmatrix} A_R & -A_R m_R^\dagger M^f_F \dagger \dagger \\ B_R M^f_F m_R & B_R \end{pmatrix}, \]

(2.6)

where the $3 \times 3$ unitary matrices $A$ and $B$ are defined by

\[ -A_L m_L M^f_F \dagger m_R A_R^\dagger = D_f, \quad B_L M^f_F B_R^\dagger = D_F, \]

(2.7)

$D_f = \text{diag}(m^f_1, m^f_2, m^f_3)$ and $D_F = \text{diag}(m^f_4, m^f_5, m^f_6) \equiv \text{diag}(m^f_1, m^f_2, m^f_3)$. The mixing matrix $U_R$ has a structure similar to $U_L$ except for the point that the off-diagonal elements $(U_L)_{ik}$ and $(U_L)_{ki}$ $(i = 1, 2, 3; k = 4, 5, 6)$ have a suppression factor $1/\lambda$, while $(U_R)_{ik}$ and $(U_R)_{ki}$ have a suppression factor $\kappa/\lambda$.

On the other hand, for the case of $\det M_F = 0$, the seesaw expression (1.2) is not valid any longer. For the case of $\det M_F = 0$, without losing generality, we can choose a heavy fermion basis where the mass matrix $M_F$ is given by a diagonal form

\[ M_F = \lambda m_0 \begin{pmatrix} 0 & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}, \]

(2.8)

where $*$ denote elements with the order of one. Then the Hermitian matrices (2.1)
and (2.2) take the following textures:

\[
H_L = m_0^2 \begin{pmatrix}
* & * & * & 0 & \sim \lambda & \sim \lambda \\
* & * & * & 0 & \sim \lambda & \sim \lambda \\
* & * & * & 0 & \sim \lambda & \sim \lambda \\
0 & 0 & 0 & \sim \kappa^2 & \sim \kappa^2 & \sim \kappa^2 \\
\sim \lambda & \sim \lambda & \sim \lambda & \sim \kappa^2 & \sim \kappa^2 & \sim \kappa^2 \\
\sim \lambda & \sim \lambda & \sim \lambda & \sim \kappa^2 & \sim \kappa^2 & \sim \kappa^2 \\
\end{pmatrix}, \quad (2.9)
\]

\[
H_R = m_0^2 \begin{pmatrix}
\sim \kappa^2 & \sim \kappa^2 & \sim \kappa^2 & 0 & \sim \kappa \lambda & \sim \kappa \lambda \\
\sim \kappa^2 & \sim \kappa^2 & \sim \kappa^2 & 0 & \sim \kappa \lambda & \sim \kappa \lambda \\
\sim \kappa^2 & \sim \kappa^2 & \sim \kappa^2 & 0 & \sim \kappa \lambda & \sim \kappa \lambda \\
0 & 0 & 0 & \sim \kappa \lambda & \sim \kappa \lambda & \sim \kappa \lambda \\
\sim \kappa \lambda & \sim \kappa \lambda & \sim \kappa \lambda & \sim \lambda^2 & * & * \\
\sim \kappa \lambda & \sim \kappa \lambda & \sim \kappa \lambda & \sim \lambda^2 & * & * \\
\end{pmatrix}. \quad (2.10)
\]

Note that \((H_L)_{33} \ll (H_L)_{44}\), while \((H_R)_{33} \gg (H_R)_{44}\). This causes the exchange between the third and fourth rows in \(U_R\) in contrast to \(U_L\). As a result, the mixing matrix \(U_R\) has matrix elements of the order

\[
U_R = \begin{pmatrix}
* & * & * & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} \\
* & * & * & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} \\
\sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & * & * & * \\
* & * & * & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} \\
\sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & * & * & * \\
\sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & \sim \frac{\kappa}{\lambda} & * & * & * \\
\end{pmatrix}, \quad (2.11)
\]

in contrast to

\[
U_L = \begin{pmatrix}
* & * & * & \sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} \\
* & * & * & \sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} \\
* & * & * & \sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} \\
\sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} & * & * & * \\
\sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} & * & * & * \\
\sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} & \sim \frac{1}{\lambda} & * & * & * \\
\end{pmatrix}. \quad (2.12)
\]

The structures (2.11) and (2.12) mean that the dominant components of the fermions
in the up-quark sector are given by

\[ u_L \simeq (2, 1), \quad u_R \simeq (1, 2), \]
\[ c_L \simeq (2, 1), \quad c_R \simeq (1, 2), \]
\[ t_L \simeq (2, 1), \quad t_R \simeq (1, 1), \]
\[ t'_L \simeq (1, 1), \quad t'_R \simeq (1, 2), \]
\[ u_{5L} \simeq (1, 1), \quad u_{5R} \simeq (1, 1), \]
\[ u_{6L} \simeq (1, 1), \quad u_{6R} \simeq (1, 1), \]
\[ \text{(2.13)} \]

of SU(2)_L \times U(2)_R, where we have denoted the fermion \( u_4 \) as \( t' \). We should notice that \( t \) and \( t' \) have exceptional structures differently from other fermions \( f \) and \( F \).

We consider that in the down-quark sector the seesaw expression (1.2) is well satisfied, so that the mixing matrices \( U_L \) and \( U_R \) are given by normal structure as (2.12). Then, the CKM matrix \( V_L \) is given by

\[
V_L = \begin{pmatrix}
* & * & * & \sim \frac{1}{\chi} & \sim \frac{1}{\chi} & \sim \frac{1}{\chi} \\
* & * & * & \sim \frac{1}{\chi} & \sim \frac{1}{\chi} & \sim \frac{1}{\chi} \\
* & * & * & \sim \frac{1}{\chi} & \sim \frac{1}{\chi} & \sim \frac{1}{\chi} \\
\sim \frac{1}{\chi} & \sim \frac{1}{\chi} & \sim \frac{1}{\chi} & \sim \left(\frac{\chi}{\lambda}\right)^2 & \sim \left(\frac{\chi}{\lambda}\right)^2 & \sim \left(\frac{\chi}{\lambda}\right)^2 \\
\sim \frac{1}{\chi} & \sim \frac{1}{\chi} & \sim \frac{1}{\chi} & \sim \left(\frac{\chi}{\lambda}\right)^2 & \sim \left(\frac{\chi}{\lambda}\right)^2 & \sim \left(\frac{\chi}{\lambda}\right)^2 \\
\sim \frac{1}{\chi} & \sim \frac{1}{\chi} & \sim \frac{1}{\chi} & \sim \left(\frac{\chi}{\lambda}\right)^2 & \sim \left(\frac{\chi}{\lambda}\right)^2 & \sim \left(\frac{\chi}{\lambda}\right)^2 \\
\end{pmatrix}, \quad \text{(2.14)}
\]

while the CKM matrix \( V_R \) for the right-handed weak currents is given by

\[
V_R = \begin{pmatrix}
* & * & * & \sim \frac{\kappa}{\chi} & \sim \frac{\kappa}{\chi} & \sim \frac{\kappa}{\chi} \\
* & * & * & \sim \frac{\kappa}{\chi} & \sim \frac{\kappa}{\chi} & \sim \frac{\kappa}{\chi} \\
* & * & * & \sim \frac{\kappa}{\chi} & \sim \frac{\kappa}{\chi} & \sim \frac{\kappa}{\chi} \\
\sim \frac{\kappa}{\chi} & \sim \frac{\kappa}{\chi} & \sim \frac{\kappa}{\chi} & \sim \left(\frac{\chi}{\lambda}\right)^2 & \sim \left(\frac{\chi}{\lambda}\right)^2 & \sim \left(\frac{\chi}{\lambda}\right)^2 \\
\sim \frac{\kappa}{\chi} & \sim \frac{\kappa}{\chi} & \sim \frac{\kappa}{\chi} & \sim \left(\frac{\chi}{\lambda}\right)^2 & \sim \left(\frac{\chi}{\lambda}\right)^2 & \sim \left(\frac{\chi}{\lambda}\right)^2 \\
\sim \frac{\kappa}{\chi} & \sim \frac{\kappa}{\chi} & \sim \frac{\kappa}{\chi} & \sim \left(\frac{\chi}{\lambda}\right)^2 & \sim \left(\frac{\chi}{\lambda}\right)^2 & \sim \left(\frac{\chi}{\lambda}\right)^2 \\
\end{pmatrix}, \quad \text{(2.15)}
\]

where the factors \((1/\lambda)^2\) and \((\kappa/\lambda)^2\) come from the reason that the heavy fermions \( F_i \) are \((1, 1)\) of SU(2)_L \times SU(2)_R in the present model.

**III. CKM MATRICES \( V_L \) AND \( V_R \)**

In order to see these relations (2.14) and (2.15) explicitly, we consider a
model with additional constraint

\[ Z_R^T = Z_L, \quad Y^T = Y. \]  \hfill (3.1)

The constraint (3.1) are not so restrictive, and most seesaw mass matrix model will satisfy this constraint.

For the down-quark sector in which the seesaw expression (1.2) is valid, from (2.1) – (2.6), we obtain the relations

\[
\begin{align*}
U_R^{Rd} &\simeq (U_L^{dd})^*, & U_R^{RD} &\simeq \kappa(U_L^{dD})^*, \\
U_R^{DR} &\simeq \kappa(U_L^{Dd})^*, & U_R^{DD} &\simeq (U_L^{DD})^*,
\end{align*}
\]  \hfill (3.2)

where the $3 \times 3$ matrices $U_{ab}$ ($a, b = f, F$) are defined by

\[
U^f = \begin{pmatrix}
U_{ff} & U_{fF} \\
U_{Ff} & U_{FF}
\end{pmatrix}.
\]  \hfill (3.3)

For the up-quark sector with $\det M_F = 0$, the seesaw expression (1.2) [therefore, (2.5) and (2.6)] is not valid any longer. However, when we define $\bar{U}_R = P_{34} U_R$, where

\[
P_{34} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix},
\]  \hfill (3.4)

we can see that the mixing matrix $\bar{U}_R$ has a structure similar to $U_L$, because $\bar{U}_R H_R \bar{U}_R^\dagger = (DP_{34})^\dagger DP_{34} = \text{diag}(m_1^2, m_2^2, m_3^2, m_4^2, m_5^2, m_6^2)$ has the structure similar to $U_L H_L U_L^\dagger = D^2 = \text{diag}(m_1^2, m_2^2, m_3^2, m_4^2, m_5^2, m_6^2)$ apart from the exchange of the coefficients, $1 \leftrightarrow \kappa$ [see (2.9) and (2.10)]. Therefore, we obtain the relations

\[
\begin{align*}
\bar{U}_R^{Ru} &\simeq (U_L^{u})^*, & \bar{U}_R^{uU} &\simeq \kappa(U_L^{uU})^*, \\
\bar{U}_L^{R} &\simeq \kappa(U_L^{L})^*, & \bar{U}_L^{U} &\simeq (U_L^{u})^*,
\end{align*}
\]  \hfill (3.5)

similarly to (3.2).
Since the CKM mixing matrix $V_L$ for the left-handed weak currents is given by

$$V_L = \begin{pmatrix} U_{uu}^L & U_{ud}^L \\ U_{Uu}^L & U_{UU}^L \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} U_{dd}^{L\dagger} & U_{Db}^{L\dagger} \\ U_{Ud}^{L\dagger} & U_{UU}^{L\dagger} \end{pmatrix} = \begin{pmatrix} U_{uu}^L U_{dd}^{L\dagger} & U_{uu}^L U_{Db}^{L\dagger} \\ U_{Uu}^L U_{Ud}^{L\dagger} & U_{UU}^L U_{UU}^{L\dagger} \end{pmatrix}, \quad (3.6)$$

the CKM mixing matrix $V_R$ for the right-handed weak currents is given by

$$V_R = P_{34} \begin{pmatrix} \bar{U}_{uu}^R & \bar{U}_{ud}^R \\ \bar{U}_{Uu}^R & \bar{U}_{UU}^R \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} U_{dd}^{R\dagger} & U_{Db}^{R\dagger} \\ U_{Ud}^{R\dagger} & U_{UU}^{R\dagger} \end{pmatrix} = P_{34} \begin{pmatrix} \bar{U}_{uu}^R U_{dd}^{R\dagger} & \bar{U}_{uu}^R U_{Db}^{R\dagger} \\ \bar{U}_{Uu}^R U_{Ud}^{R\dagger} & \bar{U}_{UU}^R U_{UU}^{R\dagger} \end{pmatrix} = P_{34} \begin{pmatrix} (V_{ud}^L)^* & \kappa (V_{ud}^L)^* \\ \kappa (V_{Ud}^L)^* & \kappa^2 (V_{Ud}^L)^* \end{pmatrix}. \quad (3.7)$$

Therefore, we find

$$V_{ij}^R = (V_{ij}^L)^* \quad i = u, c; \quad j = d, s, b, \quad (3.8)$$

As seen from (3.8), the right-handed weak-interaction structure of $t'$ is the same as the left-handed weak interaction structure of $t$.

In general, in the left-right symmetric model [7], the $W_R$-exchange diagrams can sizably contribute to the $K^0$-$\overline{K^0}$ mixing [8]. However, in the present model, although the right-handed weak currents for $u$ and $c$ can contribute to the $K^0$-$\overline{K^0}$ mixing as pointed out in Ref. [7], those for $t$ and $t'$ are negligibly small, because $t_L$ ($t_R'$) is doublet of SU$(2)_{L(R)}$, while $t_R$ ($t_L'$) is almost singlet of SU$(2)_{R(L)}$. Also the contributions for $u_5$ and $u_6$ are negligibly small because of the suppression factors $(1/\lambda)^2$ and $(\kappa/\lambda)^2$. For example, the $K^0$-$\overline{K^0}$ mixing amplitude via $(t, t', W_L, W_R)$ is suppressed by a factor $(1/\lambda)^2$ compared with that via $(t, t, W_L, W_L)$. The next leading term to the diagram $(t, t, W_L, W_L)$ is a diagram $(t', t', W_R, W_R)$ which is suppressed by a factor $(1/\kappa)^2$ compared with the diagram $(t, t, W_L, W_L)$. For $\kappa \geq 10$, the contributions are negligibly small.

Since the fourth up-quark $t'$ has a comparatively light mass (of the order of $m_{W_R}$), we can expect the observation of $t'$-production via the reaction $d + u \to t' + d$ with the $W_R$ exchange, for example, at LHC. Since we consider $m(W_R) \simeq \kappa m(W_L)$, we obtain

$$\sigma(p + p \to t' + X) \simeq \frac{1}{\kappa^4} \sigma(p + p \to t + X). \quad (3.9)$$
The decay width of $t'$, $\Gamma_{t'}$, is given by

$$\frac{\Gamma_{t'}}{\Gamma_t} \simeq \frac{m_t^5}{m_t^4} \frac{m_W^4}{m_W^3} \simeq \kappa,$$  \quad (3.10)

from (3.8).

**IV. NUMERICAL EXAMPLE FOR A SPECIFIC MODEL**

Thus, as far as the mass matrix (1.1) satisfies the form (3.1), the CKM matrix $V_R$ can be related to $V_L$ irrelevantly to the explicit structures of $Z$ and $Y$. However, in order to see the effects of the flavor-changing neutral currents (FCNC), we need the explicit forms of $U^{u}_{L(R)}$ and $U^{d}_{L(R)}$ separately, because the left- (right-) handed FCNC among the conventional fermions $f_i$ ($i = 1, 2, 3$) are proportional [9] to the matrices (see Appendix)

$$C_{L(R)}^f = U^L_{fF}(U^L_{fF})^\dagger.$$  \quad (4.1)

So, it is interesting to see the explicit structures of $U_L$ and $U_R$ for a realistic model which can give reasonable quark masses and the CKM mixing matrix parameters.

As an example, we choose the democratic seesaw mass matrix model [1], where $M_F$ is given by the form [(unit matrix) +(a rank-one matrix)]:

$$M_F = \lambda m_0 Y_f = \lambda m_0(1 + 3 b_f X),$$  \quad (4.2)

where $1$ and $X$ are the $3 \times 3$ unit matrix and a rank-one matrix with the condition $X^2 = X$, respectively, and $b_f$ is an $f$-dependent complex parameter. The name “democratic” [10] comes from the following assumption: the matrices $Z_L$ and $Z_R$ are given by a diagonal form in the heavy fermion basis on which the matrix $X$ is democratic, i.e.,

$$X = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$  \quad (4.3)

For simplicity, we assume that the matrices $Z_L$ and $Z_R$ have a common structure except for their phases:

$$Z_L = P(\delta_L)Z, \quad Z_R = P(\delta_R)Z,$$  \quad (4.4)

$$P(\delta) = \text{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}),$$  \quad (4.5)
where the matrix $Z$ is a real-parameter matrix

$$Z = \begin{pmatrix}
z_1 & 0 & 0 \\
0 & z_2 & 0 \\
0 & 0 & z_3
\end{pmatrix},$$

(4.6)

with $z_1^2 + z_2^2 + z_3^2 = 1$, and it is universal for all fermion sectors (up- and down-, quark and lepton sectors). In order to obtain input values for the parameters $z_i$, we assume that the parameter $b_f$ takes the value $b_e = 0$ in the charged lepton sector, so that the parameters $z_i$ are given by

$$\frac{z_1}{\sqrt{m_e}} = \frac{z_2}{\sqrt{m_\mu}} = \frac{z_3}{\sqrt{m_\tau + m_\mu + m_\tau}} = \frac{1}{\sqrt{m_e + m_\mu + m_\tau}},$$

(4.7)

from $M_e = m_0(\kappa/\lambda)P(\delta_L^e - \delta_R^e) \cdot Z \cdot 1 \cdot Z$. The ansatz of the democratic $M_F$ was motivated by the successful relation [11]

$$\frac{m_u}{m_c} \approx \frac{3 m_e}{4 m_\mu},$$

(4.8)

(independently of $\kappa/\lambda$ under $\lambda \gg \kappa$) for $b_u = -1/3$ and $b_e = 0$. In Ref. [1], the value of $\kappa/\lambda$ has been fixed as $\kappa/\lambda = 0.02$ by the relations for $b_u \simeq -1/3$

$$m_c \simeq 2 \frac{m_\mu \kappa}{m_\tau \lambda} m_0, \quad m_t \simeq \frac{1}{\sqrt{3}} m_0,$$

(4.9)

(note that the expression of $m_t$ does not contain the suppression factor $\kappa/\lambda$). The value of $\beta_d \equiv \text{arg}(-b_d)$ has been chosen as $\beta_d = \pi/10$ from the relations for $b_d \simeq -1$

$$m_s \simeq 2 \left| \sin \frac{\beta_d}{2} \right| \frac{m_\mu \kappa}{m_\tau \lambda} m_0, \quad m_b \simeq \frac{1}{2} \frac{\kappa}{\lambda} m_0.$$

(4.10)

Then, we can find in Ref. [1] that these parameter values can successfully provide all of the quark mass ratios and CKM matrix parameters. It is worth while noting that the model can yield $m_t \gg m_b$ with keeping $m_u \sim m_d$ by adjusting only one complex parameter $b_f$ and without choosing hierarchically different values between $b_u$ and $b_d$. 
In order to evaluate the CKM matrices $V_L$ and $V_R$, it is convenient to define the matrix $\overline{M}$ which are given by

$$\overline{M} = P^\dagger(\delta_L)MP^\dagger(\delta_R) = m_0 \begin{pmatrix} 0 & Z \\ \kappa Z & \lambda Y \end{pmatrix}, \quad (4.11)$$

where the $6 \times 6$ phase matrix $P(\delta)$ is defined by

$$P(\delta) = \text{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}, 1, 1, 1), \quad (4.12)$$

(we have used the same notation with the $3 \times 3$ phase matrix (4.5)). The unitary matrices $U_L$ and $U_R$ are related to

$$U_L = \overline{U}_L P^\dagger(\delta_L), \quad U_R = \overline{U}_R P(\delta_R), \quad (4.13)$$

where

$$U_L M U_R^\dagger = \overline{U}_L \overline{M} \overline{U}_R^\dagger = D. \quad (4.14)$$

Then, the CKM matrices $V_L$ and $V_R$ are given by

$$V_L = U_L^u P_0 U_L^{d\dagger} = \overline{U}_L^u P(\delta_L - \delta_L^d) P_0 \overline{U}_L^{d\dagger},$$

$$V_R = U_R^u P_0 U_R^{d\dagger} = \overline{U}_R^u P(\delta_R - \delta_R^d) P_0 \overline{U}_R^{d\dagger}, \quad (4.15)$$

where

$$P_0 = \text{diag}(1, 1, 1, 0, 0, 0). \quad (4.16)$$

Since the constraint (3.1) means $\delta_{Ri}^f = \delta_{Li}^f \equiv \delta_i^f$, so that $V_L$ and $V_R$ are given by

$$V_L = \overline{U}_L^u P_0(\delta) \overline{U}_L^{d\dagger}, \quad V_R = \overline{U}_R^u P_0^f(\delta) \overline{U}_R^{d\dagger}, \quad (4.17)$$

where

$$P_0(\delta) = \text{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3}, 0, 0, 0), \quad (4.18)$$

with $\delta_i = -(\delta_i^u - \delta_i^d)$.

The observed CKM matrix parameters are roughly described by $(\delta_1, \delta_2, \delta_3) = (0, 0, \pi)$ [1] and more precisely by $(\delta_1, \delta_2, \delta_3) = (0, 0, \pi - \pi/30)$ [12]. However, in Refs. [1] and [12], only the $3 \times 3$ part of $V_L$ has been investigated. Here, we show the numerical results of the $6 \times 6$ mixing matrices $U_L$ and $U_R$ for the case of $\kappa/\lambda = 0.02, \beta_d = \pi/10, \text{and } \delta_3 = \pi - \pi/30$ (we take $\kappa = 10$ temporarily according to Ref.[1],...
but the results are almost insensitive to the value of $\kappa$:

$$
U_L^u = 
\begin{pmatrix}
+0.9994 & -0.0349 & -0.0084 & -0.0247\frac{1}{\lambda} & +6 \times 10^{-5}\frac{1}{\lambda} & +4 \times 10^{-6}\frac{1}{\lambda} \\
+0.0319 & +0.9709 & -0.2373 & -0.2051\frac{1}{\lambda} & -0.4346\frac{1}{\lambda} & +0.0259\frac{1}{\lambda} \\
+0.0165 & +0.2369 & +0.9714 & +0.8989\frac{1}{\lambda} & +0.8431\frac{1}{\lambda} & -0.0444\frac{1}{\lambda} \\
+0.0093\frac{1}{\lambda} & +0.1114\frac{1}{\lambda} & -1.0364\frac{1}{\lambda} & +0.5774 & +0.5774 & +0.5772 \\
-0.0118\frac{1}{\lambda} & +0.1649\frac{1}{\lambda} & +0.0209\frac{1}{\lambda} & -0.7176 & +0.6961 & +0.0215 \\
-0.0064\frac{1}{\lambda} & -0.1011\frac{1}{\lambda} & +0.7927\frac{1}{\lambda} & -0.3894 & -0.4267 & +0.8163
\end{pmatrix},
$$

(4.19)

$$
U_R^u = 
\begin{pmatrix}
+0.9994 & -0.0349 & -0.0084 & -0.0247\frac{\kappa}{\lambda} & +6 \times 10^{-5}\frac{\kappa}{\lambda} & +4 \times 10^{-6}\frac{\kappa}{\lambda} \\
+0.0319 & +0.9709 & -0.2373 & -0.2051\frac{\kappa}{\lambda} & -0.4346\frac{\kappa}{\lambda} & +0.0259\frac{\kappa}{\lambda} \\
+0.0256\frac{\kappa}{\lambda} & +0.3459\frac{\kappa}{\lambda} & -0.0747\frac{\kappa}{\lambda} & +0.5773 & +0.5773 & +0.5774 \\
+0.0165 & +0.2369 & +0.9713 & +0.3274\frac{\kappa}{\lambda} & +0.2716\frac{\kappa}{\lambda} & -0.6159\frac{\kappa}{\lambda} \\
-0.0118\frac{\kappa}{\lambda} & +0.1649\frac{\kappa}{\lambda} & +0.0209\frac{\kappa}{\lambda} & -0.7176 & +0.6961 & +0.0215 \\
-0.0064\frac{\kappa}{\lambda} & -0.1011\frac{\kappa}{\lambda} & +0.7927\frac{\k}{\lambda} & -0.3894 & -0.4267 & +0.8161
\end{pmatrix},
$$

(4.20)

$$
|U_L^d| = 
\begin{pmatrix}
0.9772 & 0.2061 & 0.0506 & 0.0490\frac{1}{\lambda} & 0.0007\frac{1}{\lambda} & 4 \times 10^{-5}\frac{1}{\lambda} \\
0.2118 & 0.9540 & 0.2124 & 0.2063\frac{1}{\lambda} & 0.0646\frac{1}{\lambda} & 0.0035\frac{1}{\lambda} \\
0.0137 & 0.2179 & 0.9759 & 0.4335\frac{1}{\lambda} & 0.4809\frac{1}{\lambda} & 0.5251\frac{1}{\lambda} \\
0.0118\frac{1}{\lambda} & 0.1649\frac{1}{\lambda} & 0.0209\frac{1}{\lambda} & 0.7176 & 0.6961 & 0.0215 \\
0.0064\frac{1}{\lambda} & 0.1010\frac{1}{\lambda} & 0.7927\frac{1}{\lambda} & 0.3895 & 0.4268 & 0.8162 \\
0.0046\frac{1}{\lambda} & 0.0660\frac{1}{\lambda} & 0.2706\frac{1}{\lambda} & 0.5773 & 0.5773 & 0.5774
\end{pmatrix},
$$

(4.21)

where for $U_L^d$, for simplicity, we have shown only the magnitudes. Since the mixing matrix elements of $U_R^d$ are given by the relation (3.5) with good approximation, here we have dropped the numerical result of $U_R^d$.

From (4.17), the $6 \times 6$ CKM matrix $V_L$ is given by

$$
|V_L| = 
\begin{pmatrix}
0.9756 & 0.2196 & 0.0028 & 0.0174\frac{1}{\lambda} & 0.0038\frac{1}{\lambda} & 0.0046\frac{1}{\lambda} \\
0.2193 & 0.9749 & 0.0388 & 0.1615\frac{1}{\lambda} & 0.0910\frac{1}{\lambda} & 0.1283\frac{1}{\lambda} \\
0.0105 & 0.0374 & 0.9992 & 0.0188\frac{1}{\lambda} & 0.7940\frac{1}{\lambda} & 0.2473\frac{1}{\lambda} \\
0.0780\frac{1}{\lambda} & 0.3253\frac{1}{\lambda} & 0.9873\frac{1}{\lambda} & 0.0399\left(\frac{1}{\lambda}\right)^2 & 0.8104\left(\frac{1}{\lambda}\right)^2 & 0.2879\left(\frac{1}{\lambda}\right)^2 \\
0.0332\frac{1}{\lambda} & 0.1534\frac{1}{\lambda} & 0.0560\frac{1}{\lambda} & 0.0269\left(\frac{1}{\lambda}\right)^2 & 0.0331\left(\frac{1}{\lambda}\right)^2 & 0.0052\left(\frac{1}{\lambda}\right)^2 \\
0.0626\frac{1}{\lambda} & 0.2638\frac{1}{\lambda} & 0.7517\frac{1}{\lambda} & 0.0331\left(\frac{1}{\lambda}\right)^2 & 0.6182\left(\frac{1}{\lambda}\right)^2 & 0.2212\left(\frac{1}{\lambda}\right)^2
\end{pmatrix},
$$

(4.22)
We have again dropped the results of $V_R$ since the numerical results satisfies (3.7) very well.

The numerical results of $C_L$ and $C_R$, to which the contributions of FCNC are proportional, are given as follows:

\[
C^u_L = \begin{pmatrix}
2.43314 \times 10^{-9} & -2.013 \times 10^{-8} & -8.845 \times 10^{-8} \\
-2.013 \times 10^{-8} & 9.263 \times 10^{-7} & 2.208 \times 10^{-6} \\
-8.848 \times 10^{-8} & 2.208 \times 10^{-6} & 6.084 \times 10^{-6}
\end{pmatrix},
\]

(4.23)

\[
C^u_R = \begin{pmatrix}
2.43314 \times 10^{-7} & 2.013 \times 10^{-6} & 0.0002840 \\
2.013 \times 10^{-6} & 9.264 \times 10^{-5} & 0.007087 \\
0.0002840 & 0.007087 & 1.000
\end{pmatrix},
\]

(4.24)

\[
|C^d_L| = \frac{1}{\kappa^2} |C^d_R| = \begin{pmatrix}
9.615 \times 10^{-9} & 4.026 \times 10^{-8} & 8.525 \times 10^{-8} \\
4.026 \times 10^{-8} & 1.870 \times 10^{-7} & 3.514 \times 10^{-7} \\
8.525 \times 10^{-8} & 3.514 \times 10^{-7} & 2.780 \times 10^{-6}
\end{pmatrix}.
\]

(4.25)

Thus, the matrix elements of $C_L$ and $C_R$ are suppressed by factors $(1/\lambda)^2$ and $(\kappa/\lambda)^2$, respectively, except for $(C^u_R)_{3i} = (C^d_R)_{i3} (i = 1, 2, 3)$. We see that the FCNC in the present model are harmless to the $K^0\overline{K}^0$ and $D^0\overline{D}^0$ mixings. However, the elements related to the top-quark have sizable values of $C^u_R$:

\[
(C^u_R)_{tc} = 0.000709, \quad (C^u_R)_{tu} = 0.000284.
\]

(4.26)

The observability of the single top-quark productions via FCNC with (4.26) will be discussed elsewhere [9].

**V. SUMMARY**

In conclusion, we have pointed out that in a seesaw quark mass matrix model which yields a singular enhancement of the top-quark mass, the $6 \times 6$ mixing matrix $U^u_R$ for the right-handed up-quark sector has a peculiar structure, i.e., as if the third and fourth rows of $U^u_R$ are exchanged in contrast to the left-handed mixing matrix $U^u_L$. This means that top quark $t$ and the fourth up-quark $t'$ have approximately components $t_L = (2, 1)$, $t_R = (1, 1)$, $t'_L = (1, 1)$ and $t'_R = (1, 2)$ of SU(2)$_L \times$SU(2)$_R$, although other fermions have $f_L = (2, 1)$, $f_R = (1, 2)$, $F_L = (1, 1)$ and $F_R = (1, 1)$.

For a model with the constraint (3.1) which is a likely case, the CKM mixing matrices $V_L$ and $V_R$ satisfy the relation (3.8). Observation of $t'$ with mass $m_{t'} \simeq \kappa m_t$ (of the order of $m_{W_R}$) is expected at a future collider with a few TeV energy.
As an explicit example of $U_L$ and $U_R$, we have investigated a model where $M_F$ is given by a form (4.2). The numerical results, of course, satisfy the general relations (3.5) and (3.7). The matrix elements of $C = U_{fF}(U_{fF})^\dagger$ to which FCNC are proportional have been evaluated. The contributions of FCNC are harmless to the $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ mixings. On the other hand, the elements related to the top-quark have sizable values $(C_{R}^{u})_{tc} = 0.00709$ and $(C_{R}^{u})_{tu} = 0.000284$.

The present model which can successfully give quark mass ratios and CKM matrix parameters can also provide fruitful new physics. It seems that the model is worth investigating not only phenomenologically but also theoretically.

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APPENDIX: STRUCTURE OF FCNC

When the mass matrix $M$ given in (1.1) is transformed as

$$\bar{\psi} L M \psi R + h.c. = \bar{\psi} L D \psi R + h.c. ,$$

(A1)

where $\psi = (f, F)^T$, and $\psi' = U \psi$ is the mass-eigenstates, the vertex $\bar{\psi} A \Gamma^{AB} \psi B$ ($A, B = L, R$) is also transformed into $\bar{\psi} A \Gamma'^{AB} \psi B'$, where

$$\Gamma'^{AB} = U A \Gamma^{AB} U_B^\dagger .$$

(A2)

For simplicity, hereafter, we drop the indices $A, B$. Correspondingly to (3.3), we denote the $6 \times 6$ matrix $\Gamma$ in terms of $3 \times 3$ matrices $\Gamma_{ab}$ ($a, b = f, F$) as

$$\Gamma = \left( \begin{array}{cc} \Gamma_{ff} & \Gamma_{fF} \\ \Gamma_{Ff} & \Gamma_{FF} \end{array} \right) .$$

(A3)

Our interest is in the physical vertex $\Gamma_{ff}'$, which is given by

$$\Gamma_{ff}' = \sum_{a} \sum_{b} U_{fa} \Gamma_{ab} U_{fb}^\dagger ,$$

(A4)

where $U_{ab}^\dagger = (U_{ab})^\dagger = (U^\dagger)_{ba}$, because $(\Gamma_{ff}')_{ij}$ with $i \neq j$ mean transitions between $f_i$ and $f_j$, i.e., appearance of the FCNC.
In our SU(2)\(_L\)×SU(2)\(_R\)×U(1)\(_Y\) gauge model, the neutral currents \(J^\mu_L = g^Z_L \bar{\psi} \Gamma^\mu_L \psi\), which couple with the left-handed weak boson \(Z_L\), are given by

\[
\Gamma^\mu_L = \left( \begin{array} {cc} c_L & 0 \\ 0 & c_L^F \end{array} \right) \cdot \frac{1}{2} \gamma^\mu (1 - \gamma_5) + \left( \begin{array} {cc} d_L & 0 \\ 0 & d_L^F \end{array} \right) \cdot \frac{1}{2} \gamma^\mu (1 + \gamma_5) ,
\]  

where

\[
c_L = \pm \frac{1}{2} - \sin^2 \theta_L Q_f , \\
c_L^F = - \sin^2 \theta_L Q_F , \\
d_L = \pm \frac{1}{2} h_L - \sin^2 \theta_L Q_f , \\
d_L^F = - \sin^2 \theta_L Q_F , \]

\[
\sin^2 \theta_L = 1 - m^2_{W_L}/m^2_{Z_L} ,
\]

\[
h_L = - \frac{\sin^2 \theta_L}{1 - \varepsilon/\cos^2 \theta_L \cos^2 \theta_L} \epsilon ,
\]

\[
\varepsilon = m^2_{W_L}/m^2_{W_R} ,
\]

the factor \(\pm \frac{1}{2}\) takes +\(\frac{1}{2}\) and −\(\frac{1}{2}\) for up- and down-fermions, respectively, and \(Q_f (Q_F)\) is charge of the fermion \(f \text{ (F)}\). Using the unitary condition for \(U_{ab}, U_{ff}U_{ff}^\dagger + U_{ff}U_{ff}^\dagger = 1\), we can express the physical vertex \(\Gamma^\prime_{Lff}\) as

\[
\Gamma^\prime_{Lff} = \left( c_L^L U_{ff}^L U_{ff}^L + c_L^F U_{ff}^F U_{ff}^F \right) \cdot \frac{1}{2} \gamma^\mu (1 - \gamma_5) + \left( d_L^L U_{ff}^L U_{ff}^L + d_L^F U_{ff}^F U_{ff}^F \right) \cdot \frac{1}{2} \gamma^\mu (1 + \gamma_5) \\
\quad + \left[ c_L^L - (c_L^L - c_L^F) U_{ff}^L U_{ff}^L \right] \cdot \frac{1}{2} \gamma^\mu (1 - \gamma_5) \\
\quad + \left[ d_L^L - (d_L^L - d_L^F) U_{ff}^L U_{ff}^L \right] \cdot \frac{1}{2} \gamma^\mu (1 + \gamma_5) .
\]

Similarly, for the neutral current \(J^\mu_R = g^Z_R \bar{\psi} \Gamma^\mu_R \psi\), which couples with the right-handed weak boson \(Z_R\), we obtain

\[
\Gamma^\mu_R = \left[ c_R - (c_R - c_R^R) U_{ff}^R U_{ff}^R \right] \cdot \frac{1}{2} \gamma^\mu (1 + \gamma_5) \\
\quad + \left[ d_R^L - (d_R^L - d_R^R) U_{ff}^L U_{ff}^L \right] \cdot \frac{1}{2} \gamma^\mu (1 - \gamma_5) ,
\]

where

\[
c_R = \pm \frac{1}{2} - \sin^2 \theta_R Q_f , \\
c_R^F = - \sin^2 \theta_R Q_F ,
\]
\[ d_f^R = \pm \frac{1}{2} h_R - \sin^2 \theta_R Q_f, \quad (A14) \]
\[ d_f^F = -\sin^2 \theta_R Q_f, \quad (A14) \]
\[ \sin^2 \theta_R = 1 - \frac{m_{W_R}^2}{m_{Z_R}^2}, \quad (A15) \]
\[ h_R = -\frac{\sin^2 \theta_R}{1 - \varepsilon \cos^2 \theta_R}, \quad (A16) \]
\[ g_R^Z = g_L^Z \frac{\sin \theta_L}{\sin \theta_R \cos \theta_R} \sqrt{\frac{1 - \varepsilon \cos^2 \theta_R}{1 - \varepsilon \cos^2 \theta_L}} \]
\[ = \frac{e}{\cos \theta_L \sin \theta_R \cos \theta_R} \sqrt{\frac{1 - \varepsilon \cos^2 \theta_R}{1 - \varepsilon \cos^2 \theta_R / \cos^2 \theta_L}}. \quad (A17) \]

Thus, the FCNC are induced by the second terms \( U_{fF} U_{fF}^\dagger \) with magnitude \((c^f - c^F)[(d^f - d^F)]\). Therefore, we have denoted these matrices as \( C_L^f \equiv U_{fL}^L U_{fF}^L \) and \( C_R^f \equiv U_{fR}^R U_{fF}^R \) in (4.1).

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