Generation of cosmic magnetic fields in electroweak plasma

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Abstract

We study the generation of strong magnetic fields in magnetars and in the early universe. For this purpose we calculate the antisymmetric contribution to the photon polarization tensor in a medium consisting of an electron-positron plasma and a gas of neutrinos and antineutrinos, interacting within the Standard Model. Such a contribution exactly takes into account the temperature and the chemical potential of plasma as well as the photon dispersion law in this background matter. It is shown that a nonvanishing Chern-Simons parameter, which appears if there is a nonzero asymmetry between neutrinos and antineutrinos, leads to the instability of a magnetic field resulting to its growth. We apply our result to the description of the magnetic field amplification in the first second of a supernova explosion. It is suggested that this mechanism can explain strong magnetic fields of magnetars. Then we use our approach to study the cosmological magnetic field evolution. We find a lower bound on the neutrino asymmetries consistent with the well-known Big Bang nucleosynthesis bound in a hot universe plasma. Finally we examine the issue of whether a magnetic field can be amplified in a background matter consisting of self-interacting electrons and positrons.

Keywords: magnetic field, Chern-Simons theory, magnetar, early universe

The origin of magnetic fields ($B$ fields) in some astrophysical and cosmological media is still a puzzle for the modern physics and astrophysics. There are multiple models for the generation of strong $B$ fields in magnetars [1]. The observable galactic $B$ field can be a remnant of a strong primordial $B$ field existed in the early universe [2]. Recently the indication on the existence of the inflationary $B$ field was claimed basing on the analysis of BICEP2 data [3]. In the present work we analyze the possibility for the strong $B$ field generation in an electroweak plasma. First, we study the $B$ field generation driven by neutrino asymmetries. Then, we apply our results for the description of strong $B$ fields in magnetars in the early universe. Finally, we analyze the evolution of a $B$ field in a self-interacting electron-positron plasma.

To study the $B$ field evolution we start with the analysis of the electromagnetic properties of an electroweak plasma consisting of electrons $e^-$, positrons $e^+$, neutrinos $\nu$, and antineutrinos $\bar{\nu}$ of all types. These particles are supposed to interact in frames of the Fermi theory.

$$\Pi_{ij}(k) = i\varepsilon_{ij0}k^0\Pi_2 + \ldots,$$

where $\Pi_2 = \Pi_2(k)$ is the new form factor, or the Chern-Simons (CS) parameter, we will be looking for and $k^\mu = (k_0, \mathbf{k})$ is the photon momentum. Here we adopt the notation of [5].

First, we will be interested in the contribution to $\Pi_2$ arising from the interaction of an $e^- e^+$ plasma with a $\nu\bar{\nu}$ gas. In this case the most general analytical expression for $\Pi_2$ can be obtained on the basis of the Feynman diagram shown in Fig. 1. We shall represent $\Pi_2$ as $\Pi_2 = \Pi_2^{\nu\bar{\nu}} + \Pi_2^{(ev)}$, where $\Pi_2^{\nu\bar{\nu}}$ is the contribution of only the neutrino gas and $\Pi_2^{(ev)}$ is the contribution of the $e^- e^+$ plasma with the nonzero temperature $T$ and the chemi-
Figure 1: The Feynman diagram for the one loop contribution to the photon polarization tensor in case of a $e^+e^-$ plasma interacting with a $\nu\bar{\nu}$ gas. The electron propagators are shown as broad straight lines since they account for the densities of background $\nu$ and $\bar{\nu}$.

The expression for $\Pi_2^{(v)}$ can be obtained using the standard quantum field theory technique,[6]

$$\Pi_2^{(v)} = V_5 e^2 \int_0^1 dx \int \frac{d^3 p}{(2\pi)^3} \frac{1}{4e_p} \left( \Pi^0_{\mu\mu} \right),$$

where $e$ is the electron charge, $m$ is the electron mass, $V_5 = (V_R - V_L)/2$, and $V_{R,L}$ are the potentials of the interaction of right and left chiral projections of the $e^+e^-$ field with the $\nu\bar{\nu}$ background. The explicit form of $V_{R,L}$ can be found in [6].

The expression for $\Pi_2^{(v)}$ can be obtained using the technique for the summation over the Matsubara frequencies,[6]

$$\Pi_2^{(v)} = V_5 e^2 \int_0^1 dx \int \frac{d^3 p}{(2\pi)^3} \frac{1}{4e_p} \left( \Pi^0_{\mu\mu} \right),$$

$$\times \left( J_0^0 - (1 - x) \right) \left( \frac{1}{4e_p} \left( p^2 [3 - 5x] - 3x \left( k^2 x(1 - x) + m^2 \right) \right) \right) \left( J_0^+ + J_0^- \right),$$

where

$$J_0^0 = \frac{1}{\exp(\beta E_p + \mu_+)} + 1 + (\mu_+ \rightarrow -\mu_+),$$

$$J_0^+ = \frac{1}{\exp(\beta E_p + \mu_+)} + 1 + (\mu_+ \rightarrow -\mu_+),$$

$$J_0^- = \frac{1}{\exp(\beta E_p + \mu_+)} + 1 + (\mu_+ \rightarrow -\mu_+),$$

$$J_0^+ = \frac{1}{\exp(\beta E_p + \mu_+)} + 1 + (\mu_+ \rightarrow -\mu_+),$$

Here $E_p = \sqrt{p^2 + M^2}$, $\beta = 1/T$, $\mu_+ = \mu + k_0 x$ and $M^2 = m^2 - k^2 x(1 - x)$. To obtain $J_{0,1,2}$ in Eq. (3) we should replace $\mu_+ \rightarrow \mu_+ - k_0 x$ in $J_{0,1,2}$ in Eq. (4). Note that in Eqs. (3) and (4) we assume that $k^2 < 4m^2$, i.e. no creation of $e^+e^-$ pairs occurs.[7]

It is convenient to represent $\Pi_2$ as $\Pi_2 = 2\alpha \Pi_2 V_5 F$, where $F$ is the dimensionless function and $\alpha = e^2/\pi$. The fine structure constant. Using Eqs. (2)-(4), in Fig. 2 we show the behavior of $F$ versus $k_0$ in relativistic plasmas. It should be noted that in the static limit $F(k_0 = 0) \neq 0$. To plot Fig. 2 we take into account the dispersion law of long electromagnetic waves in plasma $k^2 = k^2(T, \mu)$[8] and the fact that an electron acquires the effective mass $m^2_{\text{eff}} = \frac{e^2}{8\pi}(\mu^2 + \pi^2 T^2)$ in a hot and dense matter.[7]. As shown in [6], the nonzero $\Pi_2(0) = 2\alpha \Pi_2(k_0 = 0)$ results in the instability of a B field leading to the exponential growth of a seed field.

We can apply our results for the description of the $B$ field evolution in a dense relativistic electron gas in a supernova explosion. It is known that, just after the core collapse, a supernova is a powerful source of $\nu$, whereas the fluxes of $\nu_{\mu,\tau}$ and $\bar{\nu}_{e,\mu,\tau}$ are negligible.[9]. Thus $V_5 \neq 0$ and we get that $\Pi_2 = 2\alpha \Pi_2 V_5 F(0)$, where $G_F$ is the Fermi constant and $|F(0)| \approx 2$, see Fig. 2(a) since electrons are degenerate. The magnetic diffusion time $t_{\text{diff}} = \sigma \Pi_2 V_5 F(0) \approx 2.3 \times 10^{-2}$ s for $n_e = 3.7 \times 10^{37}$ cm$^{-3}$ and $n_{\nu} = 1.9 \times 10^{37}$ cm$^{-3}$ in the supernova core [6]. Here $\sigma$ is the electron gas conductivity. Thus at $t \sim 10^{-3}$ s $t_{\text{diff}}$ when the flux of $\nu_e$ is maximal, no seed magnetic field dissipates. Therefore the neutrino driven instability can result in the growth of the $B$ field. It should be noted that the scale of the $B$ field turns out to be small $\Lambda \sim 10^{-3}$ cm. However, at later stages of the star evolution $V_5$ diminishes and $\Lambda$ can be comparable with the magnetar radius. Thus our mechanism can be used to explain strong B fields of magnetars.

Figure 2: The function $F$ versus $k_0$ for a $e^+e^-$ plasma interacting with a $\nu\bar{\nu}$ gas. (a) Degenerate relativistic plasma. (b) Hot relativistic plasma.
Now let us apply out results to study the $B$ field evolution in the primordial plasma. At the stages of the early universe evolution before the neutrino decoupling at $T > (2-3) \text{MeV}$, the $e^- e^+$ plasma is hot and relativistic. Assuming the causal scenario, in which $\Lambda < H^{-1}$, where $H$ is the Hubble constant, we get that $|\xi_n - \xi_{\nu_e} - \xi_{\nu_\mu}| > 1.1 \times 10^{-6} \sqrt{\alpha/106.75} \times (T/\text{MeV})^{-1}$, see [6], where $\xi_{\alpha} = \mu_{\alpha}/T$, $g^s$ is the number of relativistic degrees of freedom, and $\mu_{\alpha}$ is the chemical potential of neutrinos of the type $\alpha = \nu_e, \nu_\mu, \nu_\tau$. Here we use that $|F(0)| \approx 0.2$, see Fig. 2(b). Assuming that before the Big Bang nucleosynthesis at $T \sim (2-3) \text{MeV}$ all neutrino flavors equilibrate owing to neutrino oscillations $\xi_{\nu_e} \sim \xi_{\nu_\mu} \sim \xi_{\nu_\tau}$, we get the lower bound on the neutrino asymmetries, which is consistent with the well-known Big Bang nucleosynthesis upper bound on $|\xi_{\alpha}|$, see [9].

Finally, let us examine the issue of whether a $B$ field can be amplified in a $e^- e^+$ plasma self-interacting within the Fermi model, i.e. when a $\nu \bar{\nu}$ gas is not present. In this case the contributions to $\Pi_2$ are schematically depicted in Fig. 3. The analytical expression for $\Pi_2^{(e\mu)}$ can be obtained analogously to the previous case [10].

$$
\Pi_2^{(e\mu)} = \frac{(1 - 4 \sin^2 \frac{\theta_W}{2})}{2 \sqrt{2}} e^2 G_F (n_e - n_\mu) \int_0^1 (1 - x) dx \\
\times \int \frac{d^3 p}{(2\pi)^3} \frac{1}{E^2} \left\{ [J'_e - J'_\mu] - [J'_e - J'_\nu] \frac{1}{E^2} \right\} \\
\times \left\{ p^2 [3 - 2x] - 3 \left[ m^2 (1 + x) + k^2 x^2 \right] \right\},
$$

where $n_{e, \mu}$ are the electron and positron densities, $\theta_W$ is the Weinberg angle, and $J'_0 \equiv J'_{0,2}$ in Eq. (4), with $\mu' = \mu_\tau$. The expressions for $J'_{0,2}$ can be obtained from $J'_{0,2}$ if we make the replacement $\mu' \to \mu'' = \mu + k_0 (1 - x)$ there. As in deriving of Eqs. (3) and (4), here we also assume that $k^2 < 4m^2$.

Let us express $\Pi_2$ in Eq. (5) as $\Pi_2 = \frac{\sin^2 \theta_W}{2} \left\{ (1 - 4 \sin^2 \frac{\theta_W}{2}) G_F (n_e - n_\mu) F \right\}$, where $F$ is the dimensionless function. We shall analyze this function in the static limit $k_0 \to 0$. We mention that, if we neglect $k_0$ in Eq. (5), then $J'_{0,2} = J''_{0,2}$ and $\Pi_2 \to 0$.

The behavior of $F$ for relativistic plasmas is shown in Fig. 4, where one can see that $\Pi_2(0) = 0$. In Fig. 4 we also account for the thermal corrections to the photon dispersion and to the electron mass. It means that a $e^- e^+$ plasma does not reveal the instability of a $B$ field leading to its growth. Therefore, contrary to the claim of [5], one can use this mechanism for neither the explanation of strong $B$ fields of magnetars nor the $B$ field amplification in the early universe.

In conclusion we mention that we have derived the CS term $\Pi_2$ in an electroweak plasma consisting of $e^-$ and $e^+$ as well as $\nu$ and $\bar{\nu}$ of all flavors. These particles are involved in the parity violating interaction. It makes possible the existence of a nonzero CS term. In case of a $e^- e^+$ plasma interacting with a $\nu \bar{\nu}$ background, the CS term is nonvanishing in the static limit when $k_0 = 0$. Therefore, a $B$ field becomes unstable in this system. We have shown that a seed field can be exponentially amplified. This feature of an electroweak plasma in question can be used to explain strong $B$ fields of magnetars and to study the evolution of a primordial $B$ field. We have also demonstrated that there is no $B$ field instability in a self-interacting $e^- e^+$ plasma.

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References

[1] R.C. Duncan and C. Thompson, Asprophys. J. 392, L9 (1992).
[2] A. Neronov and I. Vovk, Science 328, 73 (2010).
[3] C. Bonvin, et al., Phys. Rev. Lett. 112, 191303 (2014).
[4] S. Mohanty, et al., Phys. Rev. D 58, 093007 (1998).
[5] A. Boyarsky, et al., Phys. Rev. Lett. 109, 111602 (2012).
[6] M. Dvornikov and V.B. Semikoz, JCAP 1405, 002 (2014) [arXiv:1311.5267].
[7] E. Braaten, Astrophys. J. 392, 70 (1992).
[8] H.-Th. Janka, et al., Phys. Rept. 442, 38 (2007).
[9] G. Mangano, et al., Phys. Lett. B 708, 1 (2012).
[10] M. Dvornikov, Phys. Rev. D 90, 041702 (2014) arXiv:1405.3059.