Driven strongly correlated quantum circuits and Hall states: Unified photo-assisted noise and minimal excitations

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We show that the photo-assisted noise generated by arbitrary time-dependent voltages and transmission amplitudes obey a perturbative fluctuation relation (FR) which fully extends the side-band transmission picture in terms of many-body eigenstates. This applies for instance to non-equilibrium (NE) strongly correlated systems or a quantum circuits with a temperature bias, formed by a tunneling or Josephson junction strongly coupled to an electromagnetic environment. It applies as well to a quantum point contact (QPC) in the integer or fractional quantum Hall regime where the FR provides robust methods to determine the fractional charge that have been implemented experimentally [1] or to analyze two-particle collisions in a HOM type geometry [2]. We exploit the FR to revisit the characterisation of minimal excitations generated by lorentzian pulses for a non-linear dc current.

PACS numbers: PACS numbers: 3.67.Lx, 72.70.+m, 73.50.Td, 3.65.Bz, 73.50.-h, 3.67.Hk, 71.10.Pm, 72.10.-d

Time-dependent transport presents a powerful probe of quantum phenomena by introducing multiple parameters or functions under control: frequencies for emitted noise generated by constant forces, or time-dependent forces generating current or noise at low or finite frequencies [2–7]. It has been analyzed in a mesoscopic context through seminal theoretical approaches, such as the Tien-Gordon theory [8–10] or the Landauer-Büttiker scattering approach, associated with the Floquet theory [11–17]. One often refers to the low frequency shot noise induced by an a.c. voltage \( V_{ac}(t) \) at a frequency \( \Omega_0 \) as photo-assisted shot noise (PASN). This is because \( V_{ac}(t) \) can be viewed as a coherent radiation with which electrons exchange an integer number of photons which enhance the PASN. The PASN then obeys a NE fluctuation relation (FR) in terms of its counterpart in the d.c. regime, which is interpreted within the lateral side-band transmission picture. While poissonnian shot noise in the dc regime is common to classical and quantum particles, PASN has the interest to provide a signature of a quantum behavior through rectified current fluctuations.

PASN is also an important tool to explore remarkable collective phenomena and macroscopic manifestation of quantum physics where strong correlations play a crucial role, but for which the side-band transmission picture has been claimed to be inappropriate [18]. Though, Tien-Gordon expressions were unexpectedly recovered within specific models. This is the case in particular for the photo-assisted current in a Tomonaga-Luttinger Liquid (TLL) model (a generic non-Fermi liquid arising in strongly correlated one-dimensional systems) [19, 20], which is covered by our unifying analysis of photo-assisted current [21, 22]. Within the TLL, an explicit computation of the PASN which was achieved without comparison to its Tien-Gordon expression [23, 24].

Here we fully extend the side-band transmission picture to PASN in many-body correlated states [21, 22, 25, 26]. This extension unifies such previous works, based on specific models [7, 23, 24], and goes beyond to a much larger universality domain of strongly correlated circuits. Let’s mention a quantum point contact (QPC) in compressible edge states at arbitrary filling factors in the integer quantum Hall effect (IQHE) or the fractional quantum Hall effect (FQHE) (see Fig.1). It also covers quantum circuits with a conductor strongly coupled to an electromagnetic environment. The conductor can correspond to a QPC (Fig.2), a tunnel, Josephson or NIS junctions (Fig.3), as well as a dual phase-slip Josephson junction (Fig.1). The current operator could be more generally replaced by a generalized force such as a voltage operator or spin current. We exploit the framework of a NE perturbative approach [21, 25] which cannot be coined as Tien-Gordon theory. In particular, it is valid not only for such large family of systems initially thermalized states at arbitrary temperatures, but also for NE states. It covers for instance the SIN junction in a NE diffusive wire studied in Ref. [27] or a quantum circuit with a temperature gradient we’ve studied recently [28]. It also incorporates simultaneous time-dependent tunneling amplitudes and voltages which can be non-periodic, thus generated by fluctuating sources or by pseudo-random lorentzian pulses [23, 30]. In addition, this approach led to new fluctuation NE relations which were not derived in the framework of the Tien-Gordon or scattering approach within the independent electron picture [21, 31].

The FR we show here is especially relevant for two rapidly growing and two fascinating domains where injection and manipulation of controlled quantum electronic or photonic states is a challenge: electronic
quantum optics and quantum electrodynamics of mesoscopic circuits. On the one hand, an ideal test-bed for the former is offered by quantum Hall states. There, Coulomb interactions might play a crucial role, as they are fundamental to understand the FQHE and the emergence of fractional charges [32]. But they can intervene between edge states in the IQHE. This is where electronic quantum optics has been mostly developed, and has been associated with the evolution of a single injected electron on an interacting region. A first step in this direction was initiated by the author [33, 34], implementing a scattering approach for plasmon modes with time-dependent boundary conditions. This predicts charge fractionalisation phenomena [33–37], which plays an important role in decoherence [38, 39], and lay the foundation of non-equilibrium bosonisation [40]. Seminal experimental and theoretical developments [41] emerged with the experimental realisation of the analog of a single photon gun based on a mesoscopic capacitor [42], followed by a pioneering implementation of minimal excitations generated by lorentzian pulses [43–45]. In interferometers [46] such as Hanbury-Brown and Twiss or Hong-Ou-Mandel (HOM) type setups, PASN has been a main tool to explore charge fractionalisation [38, 39], to characterize injected excitations and their statistics [2, 43, 48–50] or for tomography [51, 52]. On the other hand, quantum electrodynamics of mesoscopic circuits [53], based for instance on macroscopic atoms such as Josephson junctions [54], requires understanding of radiation-matter interactions (radiation corresponding to photons in the electromagnetic environment). This gives rise to dynamical Coulomb blockade [55] phenomena, which, in the strong backaction regime, offers a quantum simulation of strongly correlated one dimensional conductors [56–58]. Determining statistics of quantum states for both photons and electrons, and generation of squeezed photon states has been based on free energy noise in the a.c. driven circuit [59, 60].

The NE FRs [25] have already inspired pioneering experimental works testing the NE fluctuation-dissipation relations at free energy noise in a quantum circuit [26]. They have been also exploited for a robust determination of the fractional charge [1, 61], or an analysis the two-particle collisions in a HOM type geometry in the IQHE and the FQHE [2].

The present paper focus on the photo-assisted noise. We report finite frequency noise as well as application to HOM to future papers. We will also show how the NE FR extends to NE initial states, similarly to relations obeyed by the photo-assisted current [22] and the finite frequency noise [31]. We also revisit minimal excitations in non-linear systems and provide an alternative criteria [25] (recovered within the specific model of a TLL [24]).

This is the plan of the paper. We will first recall the family of models and the minimal conditions required by the perturbative approach, as well as some examples. Then we express the PASN in terms of its counterpart in the d.c. regime for relevant energies within the range of validity of the approach. We also consider an initial thermal equilibrium, then present some consequences and applications. We address afterwards three choices of the excess PASN, showing that it can have a negative sign. This is in particular related to the characterisation of minimal excitations we revisit in the last section.

| PASN       | Photo-assisted shot-noise |
|------------|---------------------------|
| NE         | Non-equilibrium            |
| FR         | Fluctuation Relation (non-equilibrium) |
| QPC        | Quantum Point Contact     |
| IQHE       | Integer Quantum Hall Effect |
| FQHE       | Fractional Quantum Hall Effect |
| TLL        | Tomonaga-Luttinger Liquid  |

I. UNIVERSAL PERTURBATIVE THEORY: A SHORT REVIEW OF THE MODEL

We consider the Hamiltonian underlying the NE perturbative approach [21, 22, 25]:

$$\mathcal{H}(t) = \mathcal{H}_0 + \mathcal{H}_A(t),$$

$$\mathcal{H}_A(t) = e^{-i\omega J t} \bar{p}(t) \hat{A} + e^{-i\omega J t} \bar{p}^*(t) \hat{A}^\dagger$$

(1a)

(1b)

where the unperturbed and perturbing terms $\mathcal{H}_0$ and $\hat{A}$ are not specified, nor is the complex function $\bar{p}(t)$, which can be non-periodic, and whose phase $\varphi(t)$ as well as its modulus can depend on time:

$$\bar{p}(t) = |p(t)| e^{-i\varphi(t)}.$$  

(2)
One can fix $\bar{\rho}(t=0) = 1$, so that the stationary regime corresponds to $\bar{\rho}(t) = 1$ for all $t$.

Interestingly, the approach is not restricted to initial thermal states, unless specified [22] [31], but extends to an initial stationary NE density matrix $\bar{\rho}_0$, obeying: $[\bar{\rho}_0, \mathcal{H}_0] = 0$. Then the TD drives encoded into $\bar{\rho}(t)$ can be added on top of constant drives applied to the unperturbed system. The approach also applies to a quantum circuit with temperature gradients, as the one we have studied recently (see Fig.2) [28]. The unspecified perturbing term could be a superposition of operators associated with many positions, channels or elements, so that: $\hat{A} = \sum_i \hat{A}_i$. The sum could be replaced as well by a continuous integral over spatially extended processes. Nonetheless, compared to the dc regime, such an extension is limited by the fact that one must incorporate all time-dependent fields into a single function $p(t)$. The main conditions for the approach are: (i) second order perturbative theory is valid (ii) only time correlators implying $\mathcal{A}$ and its hermitian conjugate do not vanish (see Eq. (36a)). In particular, for superconducting junctions supercurrent has to be made negligible, for instance by a coupling to a dissipative electromagnetic environment or magnetic fields.

A generic situation is that the frequency $\omega_J$ in Eq. (1a) and the phase $\varphi(t)$ in Eq. (2) obey the following Josephson-type relations determined by a charge $e^*$:

$$\omega_J = \frac{e^*}{\hbar} V_{dc}$$

(3a)

$$\partial_t \varphi(t) = \frac{e^*}{\hbar} V(t).$$

(3b)

More generally, the common charge $e^*$ could be replaced by two different effective parameters. We can have $\int dt V(t) \neq 0$, which would add a dc contribution to $V_{dc}$. In a Josephson junction or an SIN junction [62] at energies below the superconducting gap (Fig.3), or in a Josephson junction, one has $e^* = 2e$. In the FQHE, $e^*$ is a fraction of $e$. Nonetheless, the above relations are not systematic. They can be broken especially for NE states, as for the anyon collider [31] [63]. Thus we leave both $\omega_J$ and $\varphi(t)$ as free and unspecified.

We focus on transport associated with a chosen charge operator $\hat{Q}$. We assume that $\hat{Q}$ commutes with $\mathcal{H}_0$ and is translated under the action of $\hat{A}$ by the model-dependent charge $e^*$ [31]: $[\hat{A}, \hat{Q}] = e^* \hat{A}$. Here we have adopted, for simplicity, the same charge $e^*$ as in Eqs. (3a), (3b), though not necessary. Thus the associated current operator reads, in view of Eq. (1a):

$$\hat{I}(t) = \partial_t \hat{Q}(t) = \frac{e^*}{\hbar} \delta \mathcal{H}_A(t) \delta \varphi(t) = -i \frac{e^*}{\hbar} \left( \hat{p}(t) \hat{A} - \hat{p}^*(t) \hat{A}^\dagger \right).$$

(4)

The model does not exclude other charge operators not conserved by $\mathcal{H}_0$, for instance associated with an electromagnetic environment whose Hamiltonian is included in $\mathcal{H}_0$, or coupled to other independent dc voltage sources. Typically, $\mathcal{H}_A(t)$ can refer to a tunneling Hamiltonian between strongly correlated conductors with mutual Coulomb interactions, with $\hat{I}(t)$ the tunneling current. In presence of many tunneling terms with a unique dominant one, the model could apply, for instance to a strongly asymmetric quantum dot. We have given many examples of models in Ref. [22], now illustrated in Figs. [123]. Let us discuss in more details the case of a QPC in the FQHE or IQHE, together with possible limitations of the approach.
A. A driven QPC in the quantum Hall effect

For a QPC in the IQHE or FQHE (see Fig.1), the approach has the advantage to be valid without a specific microscopic description of edge states, not even bosonisation. Nonetheless, in order to absorb inhomogeneous couplings to spatially dependent ac voltages along the edges or the injecting regions in the function \( p(t) \), one might need to require a quadratic Hamiltonian \( H_0 \) with respect to bosonic fields (we don’t need such a requirement for the d.c. regime). In that case, a very useful framework to implement these couplings has been initially developed by the author [33, 34] and has become now largely adopted in electronic quantum optics. It describes the electronic charge propagation in terms of plasmon dynamics which is dictated by Coulomb interactions, and led to predict charge fractionalisation. Based on the equation of motion method for bosonic fields, such a dynamics is solved for given time-dependent boundary conditions depending on the context. On the one hand, a classical a.c. source injects a classical plasmon wave whose time evolution is dictated by a scattering matrix for plasmon modes, which provides the a.c. outgoing electronic currents. On the other hand, for a non-gaussian source such as a QPC, the approach developed in Ref.[34] has been extended to take into account counting statistics for independent injected electrons. This step, referred to as NE bosonisation, has been used to address the anyon collider in the dc regime, to which our approach applies [31]. An open question is whether non-gaussian a.c. quantum sources could as well be included, so that one can still end up with Eq.(1a). This would require an analogous extension of the a. c. boundary conditions in Ref.[34] within NE bosonisation. By allowing for an arbitrary NE stationary density matrix, and for a time-dependent modulus of \( \bar{p}(t) \) which could insert time-dependent boundary conditions, one might be able to characterize injected non-thermal states. If this is the case, the FRs derived here would apply to the anyon collider with a.c. sources, which forms a HOM interferometer as suggested in Ref.[50].

![Figure 2](image-url)

**FIG. 2:** Second example. The NE fluctuation relations apply to a QPC coupled to an electromagnetic environment with inhomogeneous temperature in the opposite conducting and insulating regimes of the quantum phase transition. This is example of such a quantum circuit studied in Ref.[28] to address dynamical Coulomb blockade. The approach applies to current average and noise through the QPC (on the right side of the lower scheme), in case both the potential drop and gate voltage are time-dependent.

II. FLUCTUATION RELATION BETWEEN THE DRIVEN AND DC REGIME

We have previously shown that current average induced by the TD drive \( \bar{p}(t) \), thus with an implicit functional dependence on \( \bar{p}(t), I(\omega_J; t) = \langle \hat{I}_H(t) \rangle \), can be fully expressed in terms of the dc characteristics only \( I_{dc}(\omega_J) \) whether \( \bar{p}(t) \) is periodic [21] or non-periodic [22, 23] (see Eq.10 below). The subscript \( H \) refers to the Heisenberg representation. We will show here that current fluctuations are determined, in a similar fashion, by the NE zero-frequency noise in the dc regime, through a universal fluctuation relation. We address noise at two finite frequencies in the next paper of the series. Here we focus on zero-frequency noise, usually referred...
FIG. 3: Third example. Either a Josephson junction with a small Josephson energy $E_J$ or a NIS junction is strongly connected to an electromagnetic environment. It is possible that one has an additional dc voltage $V'_{dc}$ which enters into the Hamiltonian in Eq. (1a) or in the NE stationary density matrix $\rho_0$.

to by PASN,

$$S_{ph}(\omega_J) = \int \frac{dt}{T_0} \int d\tau \left< \delta \hat{I}_H \left( \frac{t-\tau}{2} \right) \delta \hat{I}_H \left( \frac{t+\tau}{2} \right) \right>, \quad (5)$$

where $\delta \hat{I}_H = \hat{I}_H(t) - \langle \hat{I}_H(t) \rangle$. We have introduced $\Omega_0 = 2\pi/T_0$ where $T_0$ is a long measurement time for non-periodic drives, while it corresponds to the period for periodic drives. We refer to the appendix for more details. We follow two main steps. The first one consists into reducing second order perturbative expressions through only two correlators (see Eq. (36a)) that keep track of unspecified Hamiltonian and initial NE density matrix $\hat{\rho}_0$, and which depend only on time difference because $\hat{\rho}_0$ is stationary. Their Fourier transforms at $\omega_J$, denoted by $I_\rightarrow(\omega_J), I_\leftarrow(\omega_J)$, correspond to dc average currents in two opposite directions induced by a dc frequency $\omega_J$. In particular, we obtain, in the dc regime :

$$I_{dc}(\omega_J) \simeq I_\rightarrow(\omega_J) - I_\leftarrow(\omega_J) \quad (6a)$$
$$S_{dc}(\omega_J)/e^* \simeq I_\rightarrow(\omega_J) + I_\leftarrow(\omega_J). \quad (6b)$$

Here we introduce the NE noise in the dc regime, so that $S_{dc}(\omega_J) = S_{ph}(\omega_J)$ (Eq. (5)) whenever $\bar{\rho}(t) = 1$ in Eq. (1a). Notice that inversion symmetry is not imposed; it would correspond to $I_\rightarrow(\omega_J) = I_\leftarrow(-\omega_J)$, leading to an odd dc current and even shot noise.

The second step consists into reversing these two expressions, so that only the two functions $I_{dc}(\omega_J), S_{dc}(\omega_J)$ determine alternatively and completely time-dependent transport. In particular, following these two steps, we can show that the PASN in Eq. (5) is fully determined by $S_{dc}(\omega_J)$ in Eq. (6b).

$$S_{ph}(\omega_J) = P_{dc}S_{dc}(\omega_J) + \int \frac{d\omega'}{\Omega_0} P(\omega')S_{dc}(\omega' + \omega_J), \quad (7)$$

thus obeys a universal FR between the driven and dc regimes. We have denoted $P_{dc} = |p_{dc}|^2$ and $P(\omega) = |p(\omega)|^2$, where $p_{dc}$ is the weight of an eventual delta function and $p(\omega)$ the regular part of $\bar{\rho}(\omega) = p_{dc}\delta(\omega) + p(\omega)$.

FIG. 4: Fourth example. A dual-phase Josephson junction with a small effective parameter $U_J$. The role of voltage and current are permuted, so that one imposes a time-dependent current and considers the voltage noise across the junction. Average voltage was computed explicitly through Keldysh technique in Ref. [64] and found to obey the relation provided by our perturbative approach [21, 22].
Notice that one recovers the dc regime through the first term on the r. h. s. of Eq. (4) when \( \bar{P}(\omega) = \delta(\omega) \). Such a caution is not required for a periodic \( \bar{P}(t) \) for which \( T_0 = 2\pi\Omega_0 \) is the period.

To our knowledge, such a FR has not been derived so far within the present large context of strongly correlated circuits and NE initial states. It shows that noise under TD drives contains a superposition of the noise evaluated at effective dc voltages \( \omega_J + \omega' \) for all finite frequencies \( \omega' \) of the driving photons, modulated by \( P(\omega') \). \( P(\omega') \) can be viewed as the transfer rate for many-body eigenstates of \( \mathcal{H}_0 \) exchanging the energy \( \hbar\omega' \) with the TD field, as one can check from a spectral decomposition [55]. \( P(\omega') \) becomes an exchange probability when \( |\bar{P}(t)| = 1 \), thus for \( \bar{P}(t) = e^{-i\varphi(t)} \). In this case, it plays a similar role to the \( P(E) \) function of an electromagnetic environment in the classical regime with an average phase \( \varphi(t) \). This picture extends fully the side-band transmission, now given in terms of many-body eigenstates, to a large family of strongly correlated circuits, time-dependent tunneling amplitudes and non-periodic drives. We notice that for \( \omega_J = 0 \), current noise in the driven system is still determined by the NE noise \( S_{dc}(\omega') \) at an effective dc bias \( \omega' \). Moreover, even at \( \omega' = 0 \), \( S_{dc}(0) \) is still a NE noise for initial NE states.

Let us now consider a periodic drive \( \bar{P}(t) \) at a period \( T_0 = 2\pi/\Omega_0 \). Then one lets \( p = \bar{P} \) and \( P_{dc} = 0 \) on the r. h. s. of Eq. (7). The integral reduces to a sum over \( \omega' = l\Omega_0 \) for integer \( l \). Denoting \( \bar{p}(l\Omega_0) = p_l \) and \( \overline{P} = |p_l|^2 \), we get:

\[
S_{ph}(\omega_J) = \sum_l P_l S_{dc}(\omega_J + l\Omega_0). \tag{8}
\]

We notice that when \( |p(t)| \neq 1 \), we have

\[
\sum_l p_l p^*_l = F.T. [ |p(t)|^2 ]_m , \tag{9}
\]

given by the Fourier transform at \( m\Omega_0 \) of \( |p(t)|^2 \). In particular, for \( m = 0 \), we have: \( \sum_l P_l = < |p(t)|^2 > \). A similar equation can be found in Ref. [22] for a non periodic \( p(t) \). When \( |p(t)| = 1 \), we recover the standard orthogonality:

\[
|\bar{p}(t)| = 1 \implies \sum_l p_l p^*_l = \delta_m , \tag{10}
\]

where \( \delta_m \) is the Kronecker sign.

One often deals with excess noise, which has not a single convention as it depends on which reference we choose in both theoretical or experimental contexts. Here we choose the same reference for the driven and dc regime, by substracting \( S_{dc}(0) \) both from the PASN and the dc noise:

\[
\begin{align*}
\Delta S_{dc}(\omega_J) &= S_{dc}(\omega_J) - S_{dc}(0) \tag{11} \\
\Delta S_{ph}(\omega_J) &= S_{ph}(\omega_J) - S_{dc}(0) \tag{12}
\end{align*}
\]

Recall that in a NE setup with couplings to other dc voltages independent from \( \omega_J \), a finite \( S_{dc}(0) \) persists even when all temperatures are set to zero, and is therefore different from the thermal equilibrium noise.

This choice yields, focussing on the periodic case (see Eq. (8)):

\[
\Delta S_{ph}(\omega_J) = \sum_l P_l \Delta S_{dc}(\omega_J + l\Omega_0) + \left( \sum_l P_l - 1 \right) S_{dc}(0) . \tag{13}
\]

### A. Differentials of the noise

The differential noise refers often, in related experimental works, to its derivative with respect to the dc voltage. In view of Eqs. (7), (8), these are given by the differential dc noise. One could also consider derivatives with respect to finite Fourier components of the voltage, given by \( V(\omega) \) for a non-periodic voltage or by \( V_m = V(\omega = m\Omega_0) \) for a periodic drive. Assuming the phase of \( \bar{p}(t) \) obeys Eq. (7b), we have:

\[
\delta \bar{p}(\omega')/\delta V(\omega) = e^{\varphi}(\omega' - \omega)/\omega .
\]

In the periodic case, this reads \( \delta P_l/\delta V_m = e^{\varphi}p_{l-m}/m\Omega_0 \), so that we obtain, from Eq. (7):

\[
m\Omega_0 \frac{\delta S_{ph}(\omega_J)}{\delta V_m} = e^{\varphi} \sum_l p_l p^*_l \left[ S_{dc}(\omega_J + (l + m)\Omega_0) - S_{dc}(\omega_J + l\Omega_0) \right] . \tag{14}
\]
For the non-periodic case, \( \delta S_{ph}(\omega_J)/\delta V(\omega) \) can be written as an integral over \( \omega' \), with a product \( p(\omega')p^*(\omega'+\omega) \).

Let us now take a second derivative with respect to \( V_{-m} \). This brings us back to the photo-assisted noise, yielding a very interesting relation which implies a unique functional of the time-dependent drives:

\[
\frac{(m\Omega_0)^2}{e^2} \frac{\delta^2 S_{ph}(\omega_J)}{\delta V_m \delta V_{-m}} = S_{ph}(\omega_J + m\Omega_0) + S_{ph}(\omega_J - m\Omega_0) - 2S_{ph}(\omega_J). \tag{15}
\]

For non-periodic drive, the same relation holds by differentiating with respect to \( V(\omega)\over V(-\omega) \) and replacing \( m\Omega_0 = \omega \). Though they might be difficult to access in an experimental context, these derivatives allow for interesting applications we will discuss later on. Similar relations hold for the photo-assisted current in Eq.\([16][21]\).

### B. Superpoissonnian noise

Let us first recall the expressions obtained for the photo-assisted current, which we have also interpreted within a many-body side band extension picture \([21][22]\). For non-periodic drives, a similar relation to Eq.\([7]\) holds:

\[
I_{ph}(\omega_J) = P_{dc}I_{dc}(\omega_J) + \int \frac{d\omega'}{\Omega_0} P(\omega')I_{dc}(\omega' + \omega_J). \tag{16}
\]

For periodic drives, it becomes similar to Eq.\([8]\):

\[
I_{ph}(\omega_J) = \sum_l P_l I_{dc}(\omega_J + l\Omega_0). \tag{17}
\]

We notice that in case the dc current is linear and \( \left| \hat{p}(t) \right| = 1 \), \( I_{ph}(\omega_J) \) becomes trivial and determined by the d.c. part of the total voltage we denote by \( V_{dc,tot} \). For instance, in case the relations in Eqs.\([6a][6b]\) hold, \( V_{dc,tot} = V_{dc} + \int dt V(t) \), and we obtain:

\[
I_{dc}(V_{dc}) = G_{dc}V_{dc} \implies I_{ph} = G_{dc}V_{dc,tot}, \tag{18}
\]

where \( G_{dc} \) is the linear conductance.

Now we have shown that the dc noise is super-poissonian \( S_{dc}(\omega_J) \geq e^*|I_{dc}(\omega_J)| \) \([31]\), due to the fact that \( I_{\rightarrow,\leftarrow} \) in Eqs.\([6a][6b]\) are positive. This is obviously the case for an initial thermal equilibrium (see Eq.\([20]\) below), for which d.c. noise is poissonnian at low temperatures. Nonetheless, the deviation form poissonnian noise is not necessarily due to thermal effects. This is because a NE initial distribution might be, for instance, generated rather than additional dc voltages than temperature gradients (see Fig.\([2]\)).

Now by comparing Eq.\([7]\) to Eq.\([16]\) we obtain (also valid for periodic drives) a superpoissonnian PASN \([25]\) :

\[
S_{ph}(\omega_J) \geq e^*|I_{ph}(\omega_J)|. \tag{19}
\]

This inequality allows us to revisit the criteria for minimal excitations (see section \([5]\)).

### C. Initial thermal equilibrium distribution

In this section we specify to an initial thermal density matrix \( \rho_0 \) at temperature \( T \). We have shown, in full generality, that the poissonnian dc shot noise obeys the NE FDR \([25][51][60]\):

\[
S_{dc}(\omega_J) = e^* \coth \left( \frac{\beta \omega_J}{2} \right) I_{dc}(\omega_J). \tag{20}
\]

This FDR persists without requiring inversion symmetry, thus when \( I_{dc}(\omega_J) \neq -I_{dc}(-\omega_J) \). By injecting it into Eq.\([7]\), the PASN becomes totally determined by the NE dc current:

\[
S_{ph}(\omega_J) = e^* P_{dc} \coth \left( \frac{\beta \omega_J}{2} \right) I_{dc}(\omega_J) + e^* \int \frac{d\omega'}{\Omega_0} P(\omega' - \omega_J) \coth \left( \frac{\beta \omega'}{2} \right) I_{dc}(\omega'). \tag{21}
\]
This expression is different from the one given by L. Levitov et al for a QPC in the FQHE [67], which is recovered only if we can ignore the term in $I_{dc}$ and if the dc current is linear: $I_{dc}(\omega) = \hbar g_{dc} \omega / q$. For a periodic drive, we get:

$$S_{ph}(\omega) = e^\omega \sum_l P_l \coth \left( \frac{\beta}{2} (\omega + l\Omega_0) \right) I_{dc}(\omega + l\Omega_0).$$  \hspace{1cm} (22)

One could as well express the excess noise as in Eq.(13) replacing $S_{dc}(0) = 2k_B T G_{dc}$, given now by equilibrium dc noise.

Let us now specify further to the case $\omega_J = n\Omega_0$. When the dc current is linear and that the phase of $\bar{\rho}(t)$ is purely a.c., this corresponds to $n$ injected quasiparticles per cycle (see Eq.(19)). In the low temperature regime, precisely at $\Omega_0 \gg k_BT$, we get:

$$S_{ph}(n\Omega_0) = \sum_l P_l \left| I_{dc}((n + l)\Omega_0) \right| + 2P_{-n}k_B T G_{dc}.$$  \hspace{1cm} (23)

This expression unifies and goes beyond previous works [7, 43] restricted to independent fermions scattered by the QPC, thus with a d.c. linear current, or to a TLL [23]. The second term, interpreted for independent electrons Ref.[13], is viewed as a reduced thermal contribution from the reservoirs. Here the effective voltage vanishes by the emission by a many-body state at energy $n\Omega_0$ of $n$ photons, thus contributes by a thermal contribution weighted by $P_{-n}$. For either sine or Lorentzian pulses, and for small integer $n$, the probabilities $P_{-n}$ have important weights. An advantage of this term could then arise within shot noise spectroscopy. For a given $n$ thus a dc voltage), looking at the unique term in the noise which depends on $T < \hbar \Omega_0 / k_B$, one could have access to $P_{-n}$. Now if we adopt the excess noise in Eq.(13) by subtracting $S_{dc}(0) = 2k_B T G_{dc}$ both in the driven and in dc regime, we can get rid of this thermal term provided $|\bar{\rho}(t)| = 1$.

III. SOME APPLICATIONS

We can transpose to the PASN applications based on the photo-assisted current (see Eqs.(16), [17]) [22, 68], in particular the ones to probe the fractional charge or to detect the third current cumulant of a non-gaussian source. In case the dc current is linear and $|\bar{\rho}(t)| = 1$, the photo-assisted current reduces to Eq.(18), so that the FR for noise in Eq.(7) offers a non-trivial alternative for the applications we discuss here. In case the dc current is non-linear, the recourse to PASN could provide either a complementary or a more convenient tool, depending on the experimental context.

A. Shot-noise spectroscopy and third cumulant detection

In general, the transfer rates $P(\omega)$ in Eq.(7) (or $P_l$ in the periodic case) might be unknown as they can be affected both by interactions as well as by the NE setup or fluctuating sources. Thus one possible advantage of the FR in Eq.(7) would reside in shot-noise spectroscopy, as one could measure the noise both in the dc and driven regime without need to know the underlying model, then extract $P(\omega)$ by varying the dc drive $\omega_J$. The spectroscopy has been discussed in Ref.[69] with explicit expressions obtained for non-interacting electrons and a linear dc current. It might be more facilitated here by the compact form of the FR in Eq.(7) in terms of $S_{dc}$ that could be measured experimentally and have a non-trivial voltage dependence in non-linear systems.

Let’s focus on periodic drives, as an analogous analysis holds for non-periodic ones. Indeed as $P_l = |p_l|^2$ hides the phase of $p_l$, it would be more efficient to consider finite-frequency noise where non-diagonal amplitudes $p_l p^*_l+m$ enter (see Ref.[25]). Interestingly, one can get such non-diagonal terms by taking the derivative of the photo-assisted noise with respect to $V_m$ for finite integer $m$ in Eq.(14). But this would require to measure the derivative on its l.h.s.. It is then expected to be interesting only when the phase of $\bar{\rho}(t)$ is known, but one needs to access its time-dependent amplitude.

Similar relations hold for the photo-assisted current in Eq.(16), which could also be used for spectroscopy provided the dc current is non-linear.
B. Robust determination of the fractional charge

An important family of applications consists in robust methods we have proposed for the determination of the fractional charge in the FQHE, and implemented in recent experimental works [11, 61]. They are based on determining the Josephson frequency, which gives access to the charge $e^*$ in case the relation in Eq. (4a) holds. For that, one could use a possible singularity of the dc noise close to zero dc voltage, which corresponds to a locking: $\omega J = n\Omega J$ in Eq. (7). Such a singularity could be more pronounced by taking the second derivative with respect to $\omega J$, as this would give a series of peaks around $n\omega J$. Nonetheless, in the case of a thermal equilibrium, one needs to consider low enough temperatures to preserve these peaks. Otherwise, as discussed in section IIIC, the contribution of the $n$-th term would reduce to a thermal noise when $|\omega J - n\Omega J| \ll k_B T/h$.

A more direct method, which does not rely on such a singular behavior nor low temperatures, would consist into taking the second differential of the PASN in Eq. (15) with respect to $V_m$ or $V(\omega)$. One has to compare both sides determined by a unique function, the PASN, and to infer the value of $\omega J$ which ensures their equality. This should be easier when one takes the limit of a vanishing a.c. voltage:

$$\left(\frac{m\Omega_0}{e^*}\right)^2 \delta^2 S_{ph}(\omega J) \left|_{V(t) = 0} \right. \frac{\partial^2 S_{ph}(\omega J)}{\partial V_m \partial V_m} = S_{dc}(\omega J + m\Omega_0) + S_{dc}(\omega J - m\Omega_0) - 2S_{dc}(\omega J).$$

We have also proposed a similar method based on photo-assisted current [21] which would be more advantageous. It would be however inefficient in case the dc current is linear (see Eq. (18)), as is often the case in the recent experimental works aiming to determine the fractional charge within the Jain series [11, 61]. This explains the recourse to methods based rather on the noise than the photo-assisted current. It turns out that the dc current measured in these works does not obey a power law behavior as predicted by effective theories. This illustrates precisely the power of the methods we propose, which are independent on the underlying microscopic description of the edge states, as long as it can be cast in the form of Eq. (14).

Finally, we recall that the effective charge $e^*$ could be different from the one which renormalizes the current in Eq. (1), and which would be determined by the d.c. poissonian shot-noise at zero temperature. By the way, the latter method [70] assumes also a thermalized system at zero temperature, with a high voltage which could induce heating. Our FR has the advantage to be more generally valid for non-thermalized NE systems and doesn’t require necessary high d.c. nor a.c. voltages.

C. Two-particle collisions and HOM interferometry

As already noticed in our previous works [25, 71], the approach can be suited to address two-particle collisions in a HOM type geometry with two time-dependent sources operating with a time delay $\tau$. The FR in Eq. (7) or (8) applies to non-periodic or periodic injection encoded into a complex function $\pi(t)$ which replaces $\tilde{p}(t)$ in Eq. (1a), as well as symmetric or asymmetric setups. It has been used in a recent experimental analysis of two-electron collisions [2]. In addition, by allowing for an arbitrary NE stationary density matrix, one can hope to be able to characterize non-gaussian a.c. sources, a step not yet achieved to our knowledge for strongly correlated systems. Though the approach is not expected to be applicable to all possible classical or non-classical sources, it goes beyond previous theoretical works and offers new unified insights reported to a separate paper.

Let’s here discuss briefly a symmetric setup at $\tau = 0$ where the effective drive reduces to unity: $\pi(t) = 1$, and with a vanishing dc drive $\omega J$ in Eq. (1a). Then we have $S_{ph}(0; \tau = 0) = S_{dc}(0)$, thus brings us back to the NE noise in the dc regime. Though natural, this equality has the advantage to give hints for the difference between a NE HOM setup for which $S_{dc}(0)$ is a NE finite noise, and an initially equilibrated system where injection from thermalized sources is encoded in a scalar potential. For the excess noise obtained through subtracting $S_{dc}(0)$ (see Eqs. (11), (12)), this yields a strictly vanishing collisional noise at $\tau = 0$. Such a suppression extends for instance to arbitrary temperatures, which might be different between the sources. It fully generalises the suppression of noise correlations at $\tau = 0$ obtained for a symmetric setup [45], without specifying the microscopic model and by allowing for dissipation processes in Eq. (1a).

IV. SIGN OF THE EXCESS NOISE

The denomination "excess noise" does not guarantee its positive sign. We have analyzed this feature in various contexts while addressing the finite-frequency excess noise in conductors with a non-linear dc current...
Here we will show that various choices of excess noise can lead to its negative sign.

### A. First choice of the excess noise

Let us first consider the choice in Eqs. (11), (12). In the expression of $\Delta S_{ph}(\omega_J)$ in Eq. (13), the second term could be negative, but we assume it to vanish in order to simplify the discussion, i.e. we choose $|\tilde{p}(t)| = 1$. Then the excess noise in the dc regime, which provides all the terms in the sum, could be negative, yielding $\Delta S_{ph}(\omega_J) < 0$ for some values of $\omega_J$.

This is precisely the case in a QPC in the FQHE at a simple fractional filling factor $\nu < 1/2$ (see Fig. 1) in case a description in terms of a chiral TLL and an initial thermal distribution at a temperature $T$ are valid. In the low-temperature regime, $|\omega_J| \gg k_B T/h$, the backscattering noise associated with the QPC reads $S_{dc}(\omega_J) = \gamma (\hbar \omega_J)^{2\nu-1}$, with a prefactor $\gamma$ depending on the backscattering amplitude and $\nu$. As the exponent is negative, $S_{dc}(\omega_J)$ is a decreasing function. Now the perturbative approach is valid for energies above a crossover value $k_B T_B$. Thus to reach equilibrium, at a vanishing $\omega_J$, within the same perturbative regime, one has to keep a finite temperature $T > T_B$ [73, 74] at which one has: $S_{dc}(0) = \gamma (k_B T)^{2\nu-1}$. Therefore, the d.c. excess noise is negative: $\Delta S_{dc}(\omega_J) = \gamma (\hbar \omega_J)^{2\nu-1} - (k_B T)^{2\nu-1} < 0$ for $|\omega_J| \gg k_B T/h$.

Let’s now consider an a.c. voltage with a frequency $\Omega_0 \gg k_B T/h$ and choose $\omega_J = n \Omega_0$ with finite integer $n$. Then each term of the sum on the r. h. s. of (13) is negative, apart from the term at $l = -n$ which vanishes. Thus we obtain (recall that we have adopted here $|\tilde{p}(t)| = 1$, so that $\sum_l P_l = 1$):

$$h |\Omega_0| \gg k_B T \gg k_B T_B \implies \Delta S_{ph}(n |\Omega_0|) < 0. \quad (25)$$

This example is also valid in a quantum circuit formed by a coherent conductor coupled to a resistance $R$, which simulates a TLL with a parameter $K = \nu = (1 + e^2 R h)^{-1}$. A quantum phase transition was shown to hold between the conducting regime above $k_B T_B$ and the insulator one below $k_B T_B$, corresponding to a persistent dynamical Coulomb blockade. The negative excess noise in Eq. (25) is then expected for a resistance $R > h/e^2$.

### B. Second choice of the excess noise

Now one could argue that a positive excess noise would be ensured systematically from an alternative choice of reference: the noise in presence of the same dc voltage and vanishing ac part, $S_{dc}(\omega_J)$. This choice was adopted in Refs. [4][75]. It is more relevant to our situation if we adopt again the example discussed above, thus if $S_{dc}(\omega_J)$ is a decreasing function of $|\omega_J|$, $\Delta S_{ph}(\omega_J) < 0$ while $-\Delta S_{dc}(\omega_J) > 0$. Therefore the sign of $\Delta S_{ph}(\omega_J)$ cannot be established independently on the profile of the a.c. voltage.

Thus we propose a toy model that leads to a critical threshold $\omega_c$ above which the dc current vanishes: $|I_{dc}(\omega) > \omega_c)| = 0$. We also assume one can reach a high frequency of the ac voltage, $\Omega_0 > \omega_c$. If we use Eq. (8) at $\omega_J \ll \Omega_0$, all terms for $l \neq 0$ vanish, and we are left with:

$$S_{ph}(\omega_J) = P_0 S_{dc}(\omega_J) < S_{dc}(\omega_J) \implies \Delta S_{ph}(\omega_J) < 0. \quad (27)$$

This gives a counterexample to a theorem in Ref. [11], as discussed in section V.

### C. Third choice of the excess noise

In view of the super-poisonnian noise in Eq (19), a positive sign is always guaranteed if one defines the excess noise as: $S_{ph}(\omega_J) - e^* |I_{ph}(\omega_J)|$. But such a choice is not so advantageous. If the dc current is nonlinear, one would need to measure the non-trivial photo-assisted current in addition to noise. In addition, subtracting a noise reference is more convenient to get rid of undesirable sources which affect noise in a different manner from the photo-assisted current. For instance, if one takes the zero dc voltage limit, one has $I_{ph}(0) = 0$ if $I_{dc}$
is odd and $P(\omega') = P(-\omega')$, but still a finite $S_{ph}(0)$.

A similar choice was given in Ref. [24]. Restricted to a thermal equilibrium and to the TLL model, that work recovered the super-Poissonian PASN we showed in Ref. [25]. This motivated the authors to define the excess noise as: $S_{ph}(\omega_j) - e^\omega \coth [\beta(\omega_j/2)] I_{ph}(\omega_j)$, whose sign is not anymore well determined. In case the dc current is linear (see Eq. (18)), this amounts to adopt the second choice, Eq. (20). Such a definition was intended to cancel thermal contributions. But it cancels only the contribution of $l = 0$ in Eq. (23), but not the term $P_{-n}S_{dc}(0)$. Our first choice in Eqs. (11), (12) is more convenient for that.

V. REVISITING MINIMAL EXCITATIONS

Now let us address the issue of characterizing minimal excitations. We restrict our discussion to an initial thermal equilibrium and to $|p(t)| = 1$. The second choice we have discussed above, thus defining excess noise by substracting $S_{dc}(\omega_j)$, is at the heart of a theorem established by L. Levitov and collaborators [44, 45]. The latter has the form of a universal inequality, which reads, in terms of our notations:

$$S_{ph}(\omega_j) \geq S_{dc}(\omega_j).$$

(28)

This means that adding an a.c. voltage to a d.c. one increases systematically the PASN through a cloud of created electron-hole pairs. Such a theorem was central in the characterization of minimal excitations (we focus here on "electron" type ones) based on PASN. For that, the authors first imposes a fixed injected charge $<Q> = ne$, controlled by a fixed dc component of the applied time-dependent voltage $V(t)$: $V_{dc} = \int dt V(t)$. They assume that $I(t) = e^2 V(t)/h = \partial_t Q(t)$, so that has $V_{dc} = ne/h$. Secondly, in view of Eq. (28), the voltage which minimizes PASN by injecting well-defined electronic excitations is required to ensure the equality $S_{ph}(\omega_j) = S_{dc}(\omega_j)$, thus the lower bound of the PASN. It is precisely formed by a series of lorentzian pulses centered at $kT_0$ with a width $2W$:

$$V(t) = \frac{V_{dc}}{\pi} \sum_{k=-\infty}^{\infty} \frac{1}{1 + (t - kT_0)^2/W^2}.$$  

(29)

Following our notation, if we adopt the convention that the ac part $V(t) - V_{dc}$ controls the phase of $\tilde{p}(t) = e^{-i \omega(t)}$ in Eq. (1a), its Fourier components $p_l$ of $\tilde{p}(t)$ obey:

$$l + n < 0 \implies p_l = 0.$$  

(30)

Nonetheless, those two steps for such a characterisation require the current to be linear. Therefore they don’t apply to a QPC in the FQHE with a non-linear dc current, contrary to the claim in Ref. [15].

Let us give three reasons for that. First, the injected charge corresponds to the photo-assisted current in Eq. (10), which, for a non-linear dc current, has a non-trivial functional dependence on the a.c. voltage [22].

Second, we have seen that the inequality in (28) is violated in a toy model (see Eqs. (26), (27)). We also expect it to be so in some regimes whenever the d.c. noise decreases with $|\omega_j|$ (see Eq. (25)).

Third, let us adopt the lorentzian pulses, and apply Eq. (30) to the FR in Eq. (8):

$$S_{ph}(\omega_j) = \sum_{l=-n}^{\infty} P_l S_{dc}(\omega_j + l\Omega_0).$$

(31a)

Here we have taken the total dc frequency as being $\omega_j = \omega_{dc} + n\Omega_0$, by adding a free dc frequency $\omega_{dc}$ superimposed on the one fixed by the Lorentzian pulses in Eq. (29). Then we have no reason to recover the equality $S_{ph}(\omega_j) = S_{dc}(\omega_j)$ when the dc current is non-linear, even if we adopt the zero temperature limit of a thermalized system for which one has a poissonnian dc noise. This equality holds for a linear dc current, with the additional thermal term on the r. h. s. of Eq. (23): $2k_B G_{dc}P_{-n}T$.

The inequality in Eq. (19) offers an alternative to Eq. (28). Though restricted to a perturbative regime, it covers a much larger family of non-linear systems and quantum circuits. But an important difference from Eq. (28) is that the photo-assisted current, forming the universal bound, depends as well on the voltage $V(t)$.

Another central fact of our alternative path is that one can still define minimal excitations as those for which the PASN becomes poissonnian, thus equality is reached in Eq. (19). To reach a strict equality, we need zero temperature. We don’t fix the dc voltage which doesn’t fix anymore the transferred charge. It is here easier to adopt the convention that the whole voltage $V(t)$, with both its d.c. and a.c. components, determines the
Here:

\[ S_{ph} = e^\ast (I_{ph,+} - I_{ph,-}) \]  \( (32a) \)

\[ I_{ph} = I_{ph,+} + I_{ph,-} \]  \( (32b) \)

We see that \( \epsilon I_{ph,\epsilon} \geq 0 \) because \( \omega J I_{dc}(\omega J) \geq 0 \) for an initial thermal distribution. We used \( I_{dc}(\omega J) = 0 \). This shows that the poissonnian limit is reached whenever \( I_{ph,\epsilon} = 0 \) for either \( \epsilon = + \) or \( \epsilon = - \). If this has to be ensured whatever the profile of \( I_{dc} \), it requires that \( \bar{P}_t = 0 \) for all \( \epsilon < 0 \). Focussing on \( \epsilon = - \), one can show, using similar arguments to those by L. Levitov et al.\[14\], that the total voltage has the form in Eq.\( (34) \). As a consequence, its dc part is necessarily given by \( V_{dc} = \int dt V(t) = ne/h \) due to analytic properties of \( \bar{p}(t) \) in the complex plane. The 'electron-type' excitations correspond now to the exclusive contribution of \( \bar{p}_l \) with positive \( l \), thus to many-body correlated states which only absorb photons.

If we separate again the d.c. voltage due to \( V(t) \) in Eq.\( (29) \), thus going back to \( p_t = \bar{p}_{l-n} \), we get, from Eq.\( (31a) \):

\[ S_{ph}(n\Omega_0) = e^\ast |I_{ph}(n\Omega_0)| + 2P_{-n}\kappa_B T G_{dc}. \]  \( (34) \)

This almost poissonnian regime indicates that the PASN reduces to the transferred charge given by \( e^\ast |I_{ph}(n\Omega_0)| \), now generated by photon absorption of the many-body ground state. Adding a finite dc frequency on top on \( n\Omega_0 \) can bring us to a superpoissonnian noise. For instance, in case \( n = 1 \) and \( \kappa_B T/h < -\omega_{dc} < \Omega_0 \), we get:

\[ S_{ph}(\Omega_0 + \omega_{dc}) = e^\ast |I_{ph}(\Omega_0 + \omega_{dc})| + 2e^\ast P_{-1}|I_{dc}(\omega_{dc})|. \]

Notice that we still keep the renormalization by a charge \( e^\ast \) in front of the current. This is not the case in the Josephson type frequency where only integer charges enter \( |30| \), contrary to the claim in Ref.\[14 \] in the FQHE.

Our analysis can be extended to both non-periodic and time-dependent \( |\bar{p}(t)| \), but would be more cumbersome to write. In this case, we can show similar analytic properties of \( \bar{p}(\omega) \) that lead also to poissonnian noise. For a non-periodic drive, we have also an additional term proportional to \( P_{dc} \neq 0 \) which contributes to Eqs.\( (16), (7) \). For instance, if \( V(t) \) is formed by a single lorentzian pulse, we get (if we ignore the inverse of the time measurement \( T_0 \))

\[ S_{ph}(n\Omega_0) = 2\kappa_B T G_{dc} + e^\ast |I_{ph}(n\Omega_0)|. \]

The photo-assisted current is explicit in Eqs.\( (41), (42) \) of Ref.\[22 \].

We notice finally that in the case of a NE initial distribution, the inequality in Eq.\( (19) \) remains strict even for Lorentzian pulses.

### A. Discussion and conclusion

We have studied the noise generated by radiation fields operating in a large family of physical systems, such as a QPC in the FQHE or the IQHE with interactions. It covers quantum circuits with a Josephson, NIS or dual phase junctions strongly coupled to an electromagnetic environment. We have related the noise in a universal manner to its counterpart in the dc regime which is already in characterized by a NE distribution before applying the fields. Some previous works based on specific models and initial thermal equilibrium verify our FR. The NE FR we have obtained for the PASN \( [22] \) has been fruitful for an unambiguous determination of the fractional charge within the Jain series \( [1] \), as well as analysis of two-particle collision experiments \( [2] \). In this experimental context, effective theories such a the TLL are not in accordance with the observed d.c. current. The advantage of our approach is that it is robust with respect to the underlying microscopic description and non-universal features, such as edge reconstruction or absence of equilibration. A series of works on the IQHE have addressed the role of interactions between edge states through the plasmon scattering approach \( [33], [34], [36], [70], [77] \). Nonetheless, they are based on the scattering approach for independent electrons through the QPC. The associated dc current is linear as the effect of interactions can be ignored for a local QPC within the effective models. All these hypothesis are not required here, as we can deal with a non-linear dc current, which is the signature of the QPC properties.

The NE FR would be as well relevant to shot-noise spectroscopy as well as to third cumulant detection \( [22], [68] \). We have exploited it to show that photo-assisted noise is super-poissonnian whatever is the NE initial
distribution. We also discussed choices of excess noise which can be negative through some counter-examples. Thus the qualification of "photo-assisted" is not universally relevant: it can be reduced by an a.c. voltage superimposed on a d.c. one. For an initial thermal equilibrium, this feature violates a theorem by L. Levitov and collaborators \[22\], which turns out to be restricted to a linear d.c. current. Such a theorem was the basis to characterize minimal excitations generated by lorentzian pulses. We have provided a different criteria: the latter profile of the a.c. voltage is the one which leads to a poissonian photo-assisted noise.

The NE FRs unify higher dimensional and one-dimensional physics, though the latter is atypical as it is drastically affected even by weak interactions.

Finally, compared to the d.c. regime, additional limitations of the approach arise. These are mainly due to possible couplings to time-dependent forces or boundary conditions which have to be incorporated into a unique complex function, for instance through unitary transformations of the Hamiltonian. Therefore, an interferometer with multiple QPCs driven by different time-dependent voltages is not expected to enter within our domain of validity. The present approach would still offer a test in the limiting cases of identical ac voltages, or of a dominant tunneling through one QPC. In presence of a supplementary tunneling point between edges with a different time-dependent voltage, one needs to check whether translation of the bosonic fields justifies that one ends up with Eq. (1a). This seems more difficult to ensure for multiple mixing points addressed in Ref. \[78\].

Also the present analysis might need to be completed in case \( \hat{I}(t) \) does not correspond to the measured current in the setup under study \[79, 81\]. As the PASN is a zero-frequency noise, we expect the present FR to be generally sufficient for an initial equilibrium distribution, owing to current conservation. In NE setups, it is possible that correlations between chiral edge currents are not necessarily identical with the backscattering PASN at the QPC, an issue we will address in future works.

Finally, an important open question consists into finding the criteria for minimal excitations which would go beyond the second-order perturbation we have carried on.

Acknowledgments: The author thanks I. Taktak, C. Glattli, B. Doucot and P. Degiovanni for discussions. She also thanks E. Sukhorukov for inspiring discussions during a previous collaboration.

VI. APPENDIX

To express the noise in Eq. (5) to second order of perturbation with respect to \( A \), we don’t need an expansion of the S-matrix, as \( S \) is already of second order. Thus we can directly replace \( \delta \hat{H}(t) \) by \( \hat{A}_{H_0}(t) \), or, in Eq. (4), \( \hat{A}_H(t) = e^{iH_0t} \hat{A} e^{-iH_0t} \). Then the effect of the TD drive factorizes:

\[
S(\omega; j, t, \tau) = e^{-i\omega\tau} f \left( \frac{t + \tau}{2} \right) f^* \left( \frac{t - \tau}{2} \right) X_{\rightarrow}(-\tau) + e^{i\omega\tau} f^* \left( \frac{t + \tau}{2} \right) f \left( \frac{t - \tau}{2} \right) X_{\leftarrow}(\tau). \tag{35a}
\]

The two correlators \( X_{\rightarrow}, X_{\leftarrow} \) determine all observables associated with the current in Eq. (4), to second order perturbation, and keep track of unspecified Hamiltonian and initial NE density matrix \( \hat{\rho}_0 \). They depend only on time difference \( \tau \) because \( \hat{\rho}_0 \) is stationary \[22, 31\]:

\[
h^2 X_{\rightarrow}(\tau) = \langle \hat{A}_{H_0}^\dagger(\tau) \hat{A}_{H_0}(0) \rangle \quad h^2 X_{\leftarrow}(\tau) = \langle \hat{A}_{H_0}(0) \hat{A}_{H_0}^\dagger(\tau) \rangle. \tag{36a}
\]

Notice that the Fourier transforms \( X_{\rightarrow, \leftarrow}(\omega) \) are real because both functions verify: \( X^*(\tau) = X(-\tau) \) \[22, 25, 65\]. This implements the first important step underlying the derivation of various NE perturbative relations. Here \( X(\omega) = \int d\omega e^{i\omega\tau} X(\tau) \), as measurement time concerns only integrals over the sum of time arguments but not their difference.

As time-translation invariance is broken, double-Fourier transform with respect to \( t, \tau \) introduces two frequencies, \( \omega, \Omega \):

\[
2S(\omega; j; \omega, \Omega) = \frac{1}{T_0} \int dt \int d\omega e^{i\omega t} e^{i\Omega \tau} S(\omega; j; t, \tau), \tag{37}
\]

where the long measurement time \( T_0 \) for non-periodic drives limits the integration domain for \( t \) over \(-T_0/2, T_0/2\). This is especially useful when \( p(\omega) \) contains a term \( p_{dc} \delta(\omega) \), leading to divergencies in the zero-frequency limit \[22\]. By such a regularisation, additional terms due to \( p_{dc} \) appear in the photo-assisted current and noise. Of course, no such caution is required for periodic drives, for which \( T_0 = 2\pi\Omega_0 \) is the
Fourier transforms of the two correlators in Eq.\(36a\):

\[
S_{ph}(\omega_j) = \int \frac{d\omega'}{\Omega_0} |p(\omega')|^2 \left[ X_\rightarrow(\omega_j + \omega') + X_\leftarrow(\omega_j + \omega') \right]. \tag{38a}
\]

We have defined: \(p(\omega) = \int_{-T_0/2}^{T_0/2} e^{i\omega t} p(t)\). The additional term due to a possible singularity \(p_{dc,\delta}(\omega)\) can be found in a similar fashion as for the photo-assisted current in Ref.\([22]\), and leads to an additional term: \(|p_{dc}|^2 S_{dc}(0)\). We have nonetheless neglected a correction term in \(1/T_0\), which is justified if all time scales are much smaller than \(T_0\).

It is useful to recall how \(X_\rightarrow, X_\leftarrow\) determine as well the expressions of current average and zero-frequency noise in the d.c. regime, i.e. at \(\dot{p}(t) = 1\), for which we choose the subscript \(dc\) \([23, 61, 60]\):

\[
I_{dc}(\omega_j) = \sim e^* \left( X_\rightarrow(\omega_j) - X_\leftarrow(\omega_j) \right) \tag{39a}
\]

\[
S_{dc}(\omega_j)/e^2 \sim X_\rightarrow(\omega_j) + X_\leftarrow(\omega_j). \tag{39b}
\]

Thus, using as well a spectral decomposition, we can view \(X_\rightarrow, X_\leftarrow\) as transfer rates in opposite directions, whose difference yields the d.c current, while their superposition evaluated at two effective d.c. voltages yields the FF noise. For a Josephson junction in series with an electromagnetic environment, they play the role of the \(P(E)\) function for initial thermal states, offering its two counterparts in such a NE circuit.

We stress that, contrary to the majority of previous studies on TD transport, \(X_\rightarrow, X_\leftarrow\) are NE correlators which are not necessarily linked through inversion symmetry. They don’t neither satisfy the detailed balance equations, which would require in addition initial thermal states. Without none of these two restrictions, \(X_\rightarrow, X_\leftarrow\) are, in full generality, two independent functions.

Next, in order to derive the FR relating the noise under the TD drive \(p(t)\) to that in the d.c regime, \(S_{dc}\), we compare the two expressions respectively given by Eq.\(38a\) and Eq.\(39a\).

One can then see that the combination of the NE correlators in the integral of Eq.\(38a\) is nothing but the noise in the d.c regime, evaluated at an effective d.c drive given by \(\omega_j + \omega\). This leads to Eq.(\[7\]).

\[\]

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