Stochastic Diffusion of Electrons interacting with Whistler-mode Waves in the Solar Wind

Tien Vo¹, Cynthia Cattell¹, and Robert Lysak¹

¹University of Minnesota

November 24, 2022

Abstract

Whistler-mode waves have often been proposed as a plausible mechanism for pitch angle scattering and energization of electron populations in the solar wind. Recent studies reported observations of obliquely propagating and narrowband waves consistent with the whistler mode at 1 AU. Close to 0.3 AU, similar waves have also been observed in PSP data, where evidence of strong scattering of strahl electrons indicates that these waves regulate the electron heat flux. At both radial distances, the wave amplitude can be as high as 10% of the ambient magnetic field. The oblique propagation angle enables resonant interactions without requiring that the electrons counter-stream with the waves. Self-consistent PIC simulations by Roberg-Clark et al (2019) and Micera et al (2020) studied the strahl scattering and subsequent halo formation due to anomalous resonant interactions enabled by oblique whistlers generated from the heat flux fan instability. Observational studies of whistlers near the Sun have also concluded that they are connected to this instability. Cattell and Vo (2021) also demonstrated the same features of the scattering from a particle tracing simulation, one advantage of which is the ability to calculate kinetic quantities such as the diffusion coefficients. Also, the tracing code includes variational calculations to ensure energy conservation in the presence of highly chaotic dynamics. In this study, we investigate in more detail the resonant interactions of electrons with these high amplitude and oblique whistlers. We will show that these waves at 0.3 AU may exceed the stochasticity condition where resonance overlap occurs. Furthermore, the stochastic width around the primary islands might be large enough that diffusion is enabled even before they overlap. In simulations with 1 AU parameters, the particle motion is strongly stochastic where all harmonics significantly overlap, leading to an isotropic pitch angle diffusion which forms the halo population. Our calculations also indicate the presence of higher-order effects, allowing for sub- and super-harmonic resonant interactions.
Stochastic Diffusion of Electrons interacting with Whistler-mode Waves in the Solar Wind (SH15C-2050)

Tien Vo1,2 (Tien.Vo@colorado.edu), Robert Lysak1, and Cynthia Cattell1

1University of Minnesota – Twin Cities
2University of Colorado – Boulder

Abstract

Effects of increasing whistler amplitude and propagation angle on electron diffusion are studied through a variational test particle simulation. We compare the simulation results with theoretical calculations of the resonant width. While high amplitude and oblique whistlers in typical 1 AU solar wind parameters are capable of forming an isotropic population without any additional processes, anomalous interactions with quasi-parallel whistlers can scatter strahl electrons to lower velocities (less than the wave phase velocity) to form a halo population, as long as their amplitude is sufficiently high. We also present a careful treatment of the sensitivity to initial conditions based on calculations of the phase space volume, which is necessary for numerical calculations when the motion is highly stochastic due to resonant interactions with large amplitude waves. These calculations have a wide application in both PIC and test particle simulations.

I. Introduction

Whistler-mode waves are often proposed as a plausible mechanism for the pitch angle scattering of solar wind electrons [1-4]. Recently, self-consistent PIC simulations [5,6] have been used to study the strahl scattering and halo formation in 0.3 AU solar wind conditions (see Fig. 1). In the early stages (a), large amplitude (\(\delta B_0 \sim 0.1\)) oblique whistlers are generated and scatter highly field-aligned electrons. However, the oblique whistler heat flux instability later saturates. Instead, large amplitude quasi-parallel whistlers are generated (c) and continue to isotropize the distribution.



II. Measuring chaos

PIC simulations usually do not need to consider chaotic motion because it is self-consistently averaged in the results. However, when the interest is the kinetic information of the particle, it is necessary to ensure that the conservation laws are properly simulated when the motion is highly stochastic, which occurs when the wave is high amplitude. In our simulations, we use the variational calculations of the Lyapunov characteristic exponents (LCEs) to estimate the expansion of the phase space volume around the particle trajectory in time:

\[
\frac{\Delta V}{V} = \exp(\lambda t) - 1
\]

where \(\vec{h}\) is the total Lyapunov exponent and \(h_i\) is the growth rate in the \(i\)th dimension (see Fig. 2a). The condition for when Liouville theorem (and subsequently, zero order energy conservation) applies is that \(\Delta V\) is small (or ideally, zero).



III. Comparison between interactions with oblique and quasi-parallel whistlers

Resonant electrons oscillate with regular motion inside each resonant “island” bounded by \(\gamma_{\text{res}} \pm \Delta \omega\).

- The islands are destroyed as they gradually overlap when the amplitude increases (see a1-d1), enabling electrons to diffuse stochastically across different harmonics.
- In 1 AU parameters, the islands overlap so much that the electrons are scattered everywhere in phase space.



1. Oblique whistlers (increasing amplitude)

Phase space

Velocity space

2. Quasi-parallel whistlers (increasing propagation angle)

Phase space

Velocity space

Figure 2. (a) A visualization of the expansion of a 2D volume. (b) The time evolution of a 6D LCE spectrum of a typical electron interacting with a whistler wave. (c) A comparison in the volume expansion among simulations with different step sizes.

From Fig. 2c, only simulations with \(\Delta t < 10^{-4}\) have well-behaved volume expansion around the numerical solutions. For consistency, it is then necessary to pick a step size such that all particles in a range of initial conditions have similar and sufficiently small \(\Delta V\). In Fig. 3, we see the effects of increasing amplitude: \(\Delta V\) becomes larger for particles near the resonances. However, they all have the same order of magnitude because the step size is sufficiently small. Otherwise, \(\Delta V\) will be drastically different among different initial conditions.



VI. Conclusions

- The limitation in strahl scattering for whistlers in 0.3 AU solar wind can be explained by the island separations (see Fig. 4a-d).
- Whistlers in 1 AU solar wind can isotropize the VDF on their own (see Fig. 4a-c).
- Large amplitude, quasi-parallel whistlers can isotropize scattered strahl electrons, as long as they have sufficiently high perpendicular velocities after interacting with oblique whistlers (see Fig. 5a-d).
- Considering that the only waves present in Miciura’s simulation are oblique (early stages) and quasi-parallel (later stages) whistlers, we might conclude that large amplitude, quasi-parallel whistlers are essential to the isotropization of the VDF near the Sun, because they help strahl electrons cross different resonant islands of oblique whistlers.

References & Link to Paper

[1] Holden, J. S. et al. 2012, ApJ, 750.
[2] Lopez, G. A. et al. 2013, ApJ, 773, L37.
[3] Lysak, R. A. et al. 2013, ApJ, 780, L21.
[4] Verscharen, D. et al. 2014, ApJ, 796, L16.
[5] Tieleman, D. et al. 2015, ApJ, 800, 136.
[6] Bilodeau, Y. et al. 2017, ApJ, 847, L20.
[7] Miciura, A. et al. 2020, ApJ, 900, L27.
[8] Miciura, A. et al. 2021, ApJ, 901, 42.