DUST SCATTERING IN TURBULENT MEDIA: CORRELATION BETWEEN THE SCATTERED LIGHT AND DUST COLUMN DENSITY

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ABSTRACT

Radiative transfer models in a spherical, turbulent interstellar medium (ISM), in which the photon source is situated at the center, are calculated to investigate the correlation between the scattered light and the dust column density. The medium is modeled using fractional Brownian motion structures that are appropriate for turbulent ISM. The correlation plot between the scattered light and optical depth shows substantial scatter and deviation from simple proportionality. It was also found that the overall density contrast is smoothed out in scattered light. In other words, there is an enhancement of the dust-scattered flux in low-density regions, while the scattered flux is suppressed in high-density regions. The correlation becomes less significant as the scattering becomes closer to being isotropic and the medium becomes more turbulent. Therefore, the scattered light observed in near-infrared wavelengths would show much weaker correlation than the observations in optical and ultraviolet wavelengths. We also find that the correlation plot between scattered lights at two different wavelengths shows a tighter correlation than that of the scattered light versus the optical depth.

Key words: dust, extinction – methods: numerical – radiative transfer – scattering
Online-only material: color figures

1. INTRODUCTION

It is known that dust-scattered light is proportional to the dust column density in optically thin regions. The correlation between the dust-scattered light and tracers of the dust column density, such as far-infrared emission and neutral hydrogen column density, has been extensively examined in near-infrared (NIR), optical, and far-ultraviolet (FUV) wavelengths (Guhathakurta & Tyson 1989; Bowyer 1991; Schiminovich et al. 2001; Witt et al. 2000; Seon et al. 2011; Brandt & Draine 2012; Malinen et al. 2013). However, it was found that the correlation relation shows large scatter (e.g., Ienaka et al. 2013). The correlation slope between the scattered light and the dust tracers was also found to vary from sightline to sightline. Various causes (including spatial variations of the illuminating radiation field, the dust temperature, the dust grain properties, and/or the contribution from emission lines) have been suggested as factors that can explain the large variation in the correlation relationship. Recently, it was found that a substantial fraction of the scatter is attributable to the lognormal probability density function (PDF) of the density in a turbulent interstellar medium (ISM; Seon et al. 2011; Seon 2013).

The ISM is turbulent and the PDF of structure is hierarchical, scale-free, and/or fractal (Elmegreen & Scalo 2004). The PDFs of the densities of the turbulent ISM have been found to be close to lognormal not only in numerical simulations (Vázquez-Semadeni 1994; Klessen 2000; Ostriker et al. 2001; Burkhart & Lazarian 2012), but also in observations (Padoan et al. 1997; Berkhuijsen & Fletcher 2008; Lombardi et al. 2008; Seon 2009; Froebrich & Rowles 2010).

Radiative transfer can significantly differ in homogeneous and clumpy media because photons can easily escape through low-density regions (Witt & Gordon 1996, 2000; Mathis et al. 2002). It is crucial, therefore, to use a realistic density structure of the dusty ISM in radiative transfer models. Most previous radiative transfer models in the dusty ISM have assumed uniform or smoothly varying density distributions (Witt et al. 1992; Bianchi et al. 1996; De Looze et al. 2012), or a two-phase medium to represent a clumpy medium (Witt & Gordon 1996, 2000; Bianchi et al. 2000; Stalevski et al. 2012). Witt et al. (1997) adopted a spectrum of three types of clouds to describe the scattering medium (see also Schiminovich et al. 2001). A more complex algorithm to mimic hierarchically clumped clouds proposed by Elmegreen (1997) has been used for radiative transfer models in the dusty ISM (Mathis et al. 2002). However, this hierarchical model does not satisfy the condition of a lognormal PDF.

The purpose of this Letter is to mimic the ISM density structure such that it matches the current best knowledge as closely as possible, and to investigate the general properties of dust radiative transfers. We use the fractional Brownian motion (fBm) algorithm to simulate the turbulent ISM structure and perform Monte Carlo radiative transfer calculations. This Letter is organized as follows. In Section 2, we review the density structure and radiative transfer code. Section 3 presents the effect of the turbulent density structure on dust scattering. A summary is given in Section 4.

2. MODEL

A lognormal density field in the turbulent ISM can be characterized by the standard deviation ($\sigma_{\ln\rho}$) of the logarithm of the density and the power-law index ($\gamma$) of the density power spectrum (Seon 2012). In hydrodynamic regimes, the variance of the log-density has been found to be related to the sonic Mach number $M_s$ of the medium according to

$$\sigma_{\ln\rho}^2 = \ln(1 + b^2 M_s^2),$$

(1)

where the proportional constant $b$, depending on the type of turbulence forcing mode, is 1/3, 0.4, or 1.0 for solenoidal, natural mixing, and compressive modes, respectively, (Federrath et al. 2008, 2010). Seon (2009) combines the numerical simulation
results of Padoan et al. (2004), Kim & Ryu (2005), and Kritsuk et al. (2006), finding a relationship between $M_s$ and $\gamma$.

$$\gamma = -3.81 M_s^{-0.16},$$  \hspace{1cm} (2)

which may be applicable to solenoidal and natural mixing modes (see also Seon 2012). Therefore, given a Mach number, we can determine the standard deviation and power spectral index.

A lognormal density field $\rho(x)$ is obtained by extending the method in Elmegreen (2002). First, a Gaussian random field $\rho_g(x)$ is generated using the fBm algorithm (Voss 1988; Stutzki et al. 1998). The fBm structures are generated by assigning three-dimensional Fourier coefficients. The Fourier phases are generated such that they are uniformly distributed and the amplitudes are distributed in a Gaussian form with the variance $|k|^{-\gamma}$. The inverse Fourier transform gives a Gaussian random field $\rho_k(x)$. We then multiply the density field with the desired standard deviation $\sigma_{\ln \rho}$ of the logarithmic density and exponentiate the field to obtain $\rho(x) = \exp(\sigma_{\ln \rho}, \rho_k)$. Here, we should note that the power-law spectral index $\gamma$ of the resulting lognormal density field is different from the spectral index $\gamma_k$ of the input Gaussian field. Seon (2012) derived the input spectral index $\gamma_k$ as a polynomial function of $(\gamma, \sigma_{\ln \rho})$. We use this relationship together with Equations (1) and (2) to obtain a random realization of the lognormal density field with the spectral index $\gamma$ and standard deviation $\sigma_{\ln \rho}$ appropriate for a given Mach number.

We assumed the natural mixing mode with $b = 0.4$ in which solenoidal and compressive forces are mixed naturally. A random realization of the lognormal density field with a box size of $N^3 = 256^3$ was generated for each Mach number of $M_s$ = 1.0 and 3.0.

For the dust-scattering simulation, we used the three-dimensional radiative transfer code Monte Carlo radiative transfer (MoCAfe; Seon 2009; Jo et al. 2012; Seon & Witt 2012; Lim et al. 2013; Choi et al. 2013). The direction into which a photon is scattered is randomly determined from a Heneyy–Greenstein phase function, based on the algorithm of Witt (1977). The first scattering of the photons is forced to ensure that every photon contributes to the scattered light (Cashwell & Everett 1959). Photons are assumed to originate at a centrally located, isotropically radiating point source and to escape from the dust cloud at a spherical surface with a diameter equal to the box size. The spherical surface was divided into equal areas using the Hierarchical Equal Area isoLatitude Pixelization scheme with a resolution parameter $N_{\text{side}} = 16$, corresponding to an angular resolution of $\sim 3.7$ (Görski et al. 2005).

The homogeneous optical depth $\tau_{\text{H}}$ for a lognormal density cloud is defined by the optical depth of a cloud with a constant density, but with the same dust mass as the lognormal density cloud within the sphere. For each dust cloud, $\tau_{\text{H}}$ was varied from 0.1 to 1.4 and the asymmetry factor $g$ ranged from 0.0 to 0.99. We obtained the fluxes of scattered light ($F_{\text{scat}}$) measured at every pixel on the sphere and the optical depths ($\tau$) integrated from the center to the spherical surface. We also calculated a series of uniform density models with the optical depth ranging from 0.1 to 1.4. The results for lognormal density clouds were then compared with those of the uniform density clouds. An albedo of $a = 0.5$ is assumed throughout the Letter unless noted otherwise. When other albedos were used, we obtained similar results except that the overall levels of scattered light were scaled up or down according to the albedo. We used $10^8$ photons in each of the models.

A fractal ISM structure is scale-free and contains regions which are much denser than the mean density. Thus, it is essential to use a large number of photons to probe the complex structure through the scattering. A grid with $N^3 = 256^3$ may not be appropriate to resolve the density gradients in small scales. To examine these effects, we used $10^{10}$ photons for a few cases. However, no differences larger than 2% were found. We also generated lognormal density fields with a box size of $N^3 = 512^3$, and binned the data cube into coarser box sizes. The radiative transfer results obtained with box sizes down to $128^3$ were essentially the same as those of $512^3$. However, noticeable differences were found in the models with $N^3 < 32^3$. Therefore, our simulations are well suited for radiative transfer calculations in complex density structures.

3. RESULTS

Figures 1 and 2 show the Mollweide projection maps of $\tau$ and $F_{\text{scat}}$ for the clouds with $\tau_{\text{H}}$ values of 0.1 and 0.5 for each of $M_s$ values of 1 and 3. For each ($M_s$, $\tau_{\text{H}}$) combination, the radiative transfer models with three asymmetry factors ($g = 0.99, 0.5$, and 0.0) are presented to emphasize the differences in the correlations. The three asymmetry factors represent almost-complete forward scattering (0.99), typical forward scattering (0.5) in optical and UV wavelengths, and isotropic scattering (0.0), which may be applicable to NIR wavelengths longer than $\sim 2 \mu$m. The scattered flux is expressed relative to the flux $F_0$ that would be observed in the absence of dust.

As shown in Figures 1 and 2, the correlation weakens gradually as $g$ decreases and $M_s$ increases. The most dramatic change in the correlation is found when $g$ decreases. Therefore, the results are discussed in the order of a decreasing value of $g$. The scattered light for $g = 0.99$ correlates very well with $\tau$, although a slightly weaker correlation is found for large values of $\tau_{\text{H}}$ and $M_s$. When $g = 0.5$, the contrast of scattered light is largely smoothed out. When ($M_s$, $\tau_{\text{H}}$, $g$) = (3, 0.5, 0.5), weak anti-correlations become apparent at locally dense sightlines. When $g = 0$, the correlation between $F_{\text{scat}}$ and $\tau$ is scarcely seen, with even anti-correlations found. When ($\tau_{\text{H}}$, $g$) = (0.5, 0), the positive correlations completely disappear. Instead, weak anti-correlations are clearly shown. An anti-correlation at locally dense sightlines with $\tau > 1$, as shown when ($M_s$, $\tau_{\text{H}}$, $g$) = (3, 0.5, 0), is not unexpected. However, the anti-correlation at sightlines with $\tau < 1$, shown when ($M_s$, $\tau_{\text{H}}$, $g$) = (1, 0.5, 0), cannot be understood generally in that the scattered light is correlated with the optical depth at a low optical depth.

These trends are shown more clearly in Figure 3. For each of the six combinations of ($M_s$, $g$), $\tau_{\text{H}}$ is varied from 0.1 to 1.4 and the resulting correlations are shown in alternating colors. The scattered fluxes, as functions of $\tau$, from uniform density clouds are also shown in solid curves (these are not clearly visible when $g = 0.99$). First, we note large scatter in the correlation plots, which becomes more evident when a medium with a higher value of $M_s$ is considered. Second, the scattered flux in the sightlines with a low $\tau$ ($\lesssim \tau_{\text{H}}$) is enhanced compared to uniform density models, while the flux in the sightlines with a high value of $\tau$ ($\gtrsim \tau_{\text{H}}$) is suppressed. In other words, the correlation slope of each model is shallower than the curves of uniform density models or has negative value. This effect becomes more significant as $M_s$ increases and $g$ decreases. Third, an anti-correlation is found at a lower $\tau_{\text{H}}$ than that expected in uniform density models (Witt et al. 1982). The critical $\tau_{\text{H}}$ at which the anti-correlation occurs is lower for a higher $M_s$ and a lower $g$. 
the models with positive correlation is seen, even in the model with the lowest cases, this gives rise to a stronger (steeper) anti-correlation. Compared to the uniform density models. In the optically thick regions to less dense regions. At the optical thin limit, this effect results in a weaker correlation (with a shallower slope) than uniform density clouds. Anti-correlation is also found at a lower optical depth. To compare this with our models, we examined a cloud with $\tau_H = 0.3$ at $B_3$, assuming that $\alpha = (0.6, 0.6, 0.5), g = (0.6, 0.5, 0.4)$, and $\tau_H = (0.3, 0.2, 0.1)$ in the $B_3, R,$ and $I$ bands, respectively, which are appropriate for the Milky Way dust in

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Mollweide maps of the optical depths and scattered fluxes obtained for the Mach number $M_s = 1$. Maps in the first to fourth rows show the results for the homogenous optical depths of $\tau_H = 0.1$ and 0.5, respectively. For each $\tau_H$, the first column shows maps of the optical depth, and the second, third, and fourth columns present maps of the scattered light obtained when the asymmetry phase factor $(g)$ is 0.99, 0.5, or 0.0, respectively. For each $\tau_H$, the same color contrasts of the scattered fluxes are used for comparison. (A color version of this figure is available in the online journal.)}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Mollweide maps of the optical depths and scattered fluxes obtained for the Mach number $M_s = 3$. (A color version of this figure is available in the online journal.)}
\end{figure}

For models with $(M_s, g) = (3, 0),$ for instance, no obvious positive correlation is seen, even in the model with the lowest $\tau_H (= 0.1)$. When $g = 0$, the anti-correlation appears even in the models with $\tau_H = 0.4$, despite the fact that the values of the local optical depth $\tau$ are typically less than 1.

The above results can be understood as follows. In the case of complete forward scattering $(g = 1)$, the scattered light will continue to propagate along the original incident direction. Therefore, the scattered intensity depends solely on the optical depth in the line of sight. However, as $g$ decreases, the chance for photons to be scattered in other directions increases. Regions with relatively high densities would scatter photons into nearby regions of lower density. Low-density regions would also scatter photons into higher density regions in the vicinity, but this occurs less efficiently than the case of scattering from high-density regions into low-density regions because the scattering probability at a point is proportional to the density at that point. Therefore, the final effect of non-complete forward scattering would be a net flow of radiation from relatively dense regions to less dense regions. At the optical thin limit, this effect results in a weaker correlation (with a shallower slope) than uniform density clouds. Anti-correlation is also found at a lower $\tau_H$ compared to the uniform density models. In the optically thick cases, this gives rise to a stronger (steeper) anti-correlation. Clearly, the effect becomes more significant as $g$ decreases. Moreover, the effect will be more prominent in a medium with high density contrast, i.e., with a higher $M_s$.

We now compare our results with optical observations of cirrus clouds. Guhathakurta & Tyson (1989) found that the correlation plots between the $B_3$ and $R$ bands’ intensities versus the 100 $\mu$m intensity show shallower correlation slopes compared to predictions obtained with a constant density model. However, the correlation slope between the $I$ band and the 100 $\mu$m intensity showed a steeper slope than the theoretical value. This may due to the large contribution of the extended red emission in the $I$ band. The $B$ band correlation plot of the four clouds observed in Witt et al. (2008) shows a slightly steeper slope than the clouds of Guhathakurta & Tyson (1989), but it is still shallower than their constant density model. The $B$ band correlation slope of the cloud observed in Lenaha et al. (2013) is also shallower than the constant density model. Guhathakurta & Tyson (1989) also found that the correlation between the scattered intensities at two different bands shows a tighter correlation than those between the scattered light and the optical depth. To compare this with our models, we examined a cloud with $\tau_H = 0.3$ at $B_3$, assuming that $\alpha = (0.6, 0.6, 0.5), g = (0.6, 0.5, 0.4)$, and $\tau_H = (0.3, 0.2, 0.1)$ in the $B_3, R,$ and $I$ bands, respectively, which are appropriate for the Milky Way dust in
Figure 3. Scattered light vs. optical depth. For each \((M_s, g)\), \(\tau_H\) varied from 0.1 to 1.4, as denoted by the alternating colors in the plot of \((M_s, g) = (1, 0.0)\). Solid curves are the correlation plots of the uniform density clouds.

(A color version of this figure is available in the online journal.)

Figure 4. Correlations between the scattered fluxes at three wavelengths, roughly corresponding to the \(B\), \(J\), \(R\), and \(I\) bands, and the optical depth at the \(B\) band. We assumed \((\tau_H, g, \alpha, F_0) = (0.3, 0.6, 0.6, 1.02), (0.2, 0.5, 0.6, 2.19),\) and \((0.1, 0.4, 0.5, 3.56)\) for the \(B\), \(R\), and \(I\) bands, respectively. Here, \(F_0\) is the input radiation field strength of Mathis et al. (1983).

(A color version of this figure is available in the online journal.)

Draine (2003). We also assumed the radiation field strengths \(F_0 = (1.02, 2.19, 3.56)\) of Mathis et al. (1983) at the three bands, as in Guhathakurta & Tyson (1989). Figure 4 shows that the correlations between the fluxes at the two bands are much stronger than the relationships between the fluxes versus the optical depth. Therefore, the observational results accord well with our results. However, more quantitative comparisons would require more realistic models using external radiation fields.

If observations are made over a wide range of Galactic coordinates, as in the observations of the diffuse Galactic light (DGL), rather than a well-defined single cloud, the obtained correlation would be a result of the complex superposition...
of many clouds with various values of $\tau_H$. To understand the general properties of such observations, we assume a virtual system consisting of clouds with the ten different $\tau_H$ values considered in Figure 3. We also assume that the clouds are exposed to radiation sources with the same luminosity.

In this context, Figure 3 can be regarded as the observed correlation in the virtual system. We can find, in Figure 3, a general correlation between the scattered light $F_{\text{scatt}}$ and the optical depth $\tau$ for a wide range of optical depths. This indicates that the mean value of the scattered flux ($F_{\text{scatt}}$) is roughly proportional to $\tau$. However, large scatter in the correlation is also found. We also note that the spread, measured in terms of the standard deviation $\sigma_F$, of $F_{\text{scatt}}$ increases with $\tau$. In other words, the spread $\sigma_F$ is proportional to $\tau$ and is thus proportional to the mean value of the scattered flux ($F_{\text{scatt}}$). The increase of the flux spread with the mean flux is a property of a lognormal function. We therefore obtained histograms of the scattered fluxes ($F_{\text{scatt}}$) normalized to the fluxes ($F_{\text{H}}$) of uniform density models, which can be regarded as average fluxes. The resulting PDFs are lognormal, as shown in Figure 5. In the figure, the best-fit lognormal functions and the standard deviations ($\sigma_{\log F}$) of the logarithmic fluxes are also shown. The standard deviation increases with an increase in $M_s$ and a decrease in $g$. The same trend is also apparent in Figure 3.

Seon (2013) presented the standard deviations of the dust column density as measured from molecular clouds and the FUV background intensities. The standard deviations $\sigma_{\log F}$ for $M_s = 3$ in Figure 5 are well within the range found from molecular clouds, but are smaller than those estimated from the FUV background. The smaller standard deviations in our models compared to the FUV observations may be due to the lack of consideration of realistic radiation fields in our models, or may imply that the dust clouds responsible for the FUV background are more turbulent than our cloud models. However, we note that our models are too simplified to be quantitatively compared with the observed results of the DGL.

Some basic properties similar to our results were found in previous investigations using a rather simplified medium. Witt & Gordon (1996) dealt with the escape of stellar radiation in a clumpy, two-phase medium. They found that the sharp central peak of the radial intensity distribution observed in the uniform density cloud disappeared in the clumpy medium. Mathis et al. (2002) used hierarchically clumped clouds and found that the optical properties of grains are poorly constrained by observations of reflection nebulae, mainly due to a very large spread in both the scattered fluxes and stellar extinctions. Our results are qualitatively consistent with their results. However, our models show less severe departures from uniform density models. In Mathis et al. (2002), the scattered fluxes at the sightlines with average optical depths ($\tau_{ext} = 2$ in their Figures 1 and 2) were always lower than those of uniform density models. However, in our results, the scattered fluxes at $\tau = \tau_H$ are always centered around the values of uniform density models. The density structures in Mathis et al. (2002) included a large number of empty cells. Their models in which a constant density was added to all cells to avoid zero density, gave rise to higher scattered fluxes than the models with empty cells, mainly due to the contribution of scattered fluxes from the constant density cells. A lognormal density PDF in our models yielded even higher scattered fluxes than the clouds with the constant density cells in Mathis et al. (2002). We also obtained no significant difference, unlike their results, when the central source was embedded within a relatively dense region. This is because we mainly considered optically thin clouds, while Mathis et al. (2002) covered larger optical depths.

4. SUMMARY

Dust scattering in a turbulent, dusty ISM yielded large scatter in the correlation plots between the scattered flux and the optical depth. The slopes of the correlation plots from turbulent media are in general shallower than those expected from uniform
density models. The effect becomes more significant as $M_s$ increases and $g$ decreases. The correlation between the scattered fluxes at two different wavelengths is found to be tighter than the plots of the scattered fluxes versus the optical depth. These results are in good accord with the observed results of the scattered light in the optical and FUV wavelengths. Given that micron-size dust grains are rare in the diffuse ISM, the scattering in NIR wavelengths (especially at $\lambda \gtrsim 2\, \mu m$) would be essentially isotropic, as shown in Draine (2003). Therefore, the scattered light in the NIR would show a much weaker correlation with optical depth than the observations at optical and UV wavelengths. Our future works will deal with various configurations between clouds and radiation field, including the case of a cloud exposed to an external radiation field.

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