Effects of Cosmic Strings on Free Streaming

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Abstract

We study the effect of free streaming in a universe with cosmic strings with time-varying tension as well as with constant tension. Although current cosmological observations suggest that fluctuation seeded by cosmic strings cannot be the primary source of cosmic density fluctuation, some contributions from them are still allowed. Since cosmic strings actively produce isocurvature fluctuation, the damping of small scale structure via free streaming by dark matter particles with large velocity dispersion at the epoch of radiation-matter equality is less efficient than that in models with conventional adiabatic fluctuation. We discuss its implications to the constraints on the properties of particles such as massive neutrinos and warm dark matter.

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1 Introduction

Cosmological observations such as cosmic microwave background (CMB), large scale structure and so on become now very precise and can constrain various cosmological parameters with unprecedented accuracy. In addition to the determination of cosmological parameters such as energy densities of baryons, dark matter, and dark energy, Hubble parameter, the scalar spectral index, reionization optical depth and so on, cosmological observations can also constrain unknowns in particle physics. For example, neutrino masses can be severely constrained by cosmology [1, 2]. It is well known that massive neutrinos can erase small scale inhomogeneities via free streaming since massive neutrinos can have large velocity dispersion at the time of radiation-matter equality. Due to this effect, the matter power spectrum exhibits the suppression at small scales, which can be compared to observations of large scale structure to give the constraint on neutrino masses#1.

Another such example is warm dark matter (WDM) particles. WDM particles can also have large velocity dispersion at the epoch of radiation-matter equality like massive neutrinos. WDM scenarios have been extensively studied in the literature in connection with particle physics and astrophysics. As for astrophysical aspects, WDM has been discussed, in particular, as a solution of the problem of small scale structure such as the missing satellite problem and the cusp problem [4, 5]. From the viewpoint of particle physics, there exist well-motivated candidates for WDM such as a light gravitino [6, 7], sterile neutrinos [8] and so on. The properties of WDM such as its mass can be constrained by cosmological observations as the same manner as the case with massive neutrinos since WDM particles also erase the small scale fluctuation via the free streaming effect. It should also be mentioned that superweakly interacting massive particles (superWIMPs) [9] can also erase the small scale structure as WDM particles, thus this kind of model can also be constrained by cosmological observations by studying the damping of matter power spectrum. Therefore, probing the damping of small scale structure can be an important test for particle physics.

Constraints on above mentioned particles such as massive neutrinos and candidates for WDM have been well studied in the framework where cosmic density fluctuation is seeded by conventional adiabatic primordial fluctuation which is motivated by inflation driven by a scalar field. However, cosmic density fluctuation can also be produced by cosmic strings. Since fluctuation seeded by cosmic strings is an incoherent actively generated isocurvature one, this kind of fluctuation cannot produce observed structure of the acoustic peaks in the CMB power spectrum but gives rise to fairly broad acoustic peaks. Thus current observations suggest that cosmic strings cannot be the primary source of density fluctuation today. However, subdominant contribution from fluctuation seeded by cosmic strings is still allowed [10]. Furthermore cosmological scenarios with cosmic strings have been revived for recent years since there have been discussed that cosmic strings can be produced in a wide class of string theory models. In particular, cosmic strings can be

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#1 The mass of neutrino can also affect the CMB power spectrum through the modification of the structure of the acoustic peaks, which can give a severe constraint on them [2, 3].
formed at the end of brane inflation [11][12][13].

In light of these considerations, it is interesting to study the effect of free streaming in a scenario with cosmic strings. Importantly, since cosmic strings produce fluctuation actively, the erasure of small scale inhomogeneities via free streaming can be avoided to some extent, which results in delayed damping of small scale power [14][15][16][17]. Hence one may consider that some contribution from fluctuation seeded by cosmic strings can relax the constrains on WDM or neutrino masses since the constraint on the masses mainly come from the effect of free streaming. This is the issue which we are going to consider in this paper.

In fact, the authors of Ref. [18] have discussed the possibilities of relaxing the constraint on neutrino masses in models with cosmic strings with constant tension. There it was shown by a simple analytic estimate that even with the addition of fluctuation from cosmic strings the constraint on neutrino masses cannot be relaxed. In this paper, first of all, we reconfirm this result with more quantitative analysis by numerically calculating matter power spectrum and CMB anisotropy produced by cosmic strings. We also do similar calculations for WDM and judge whether the constraints on WDM can be relaxed or not.

Furthermore, recently a new class of cosmic strings has been considered, which has time-varying tension. When a scalar field constituting a string couples to another oscillating field or a string is realized in a brane configuration, cosmic strings with time-varying tension naturally appear. Cosmological evolution of such a class of cosmic strings has been investigated in Refs. [19][20] where it was shown that, after some relaxation time, it goes into the scaling regime like strings with constant tension [21][22] and global monopoles [23]. In the scaling regime, the typical length of the cosmic string network grows with the horizon scale. Then, in case that string tension is constant, the ratio of the energy density of infinite strings to that of the background universe is also constant, which generates scale invariant density fluctuations. On the other hand, when the string tension is time-varying, the ratio of the energy density of infinite strings to that of the background universe is not necessarily constant due to the time dependence of the tension. For example, when the tension has the time dependence as $G\mu \propto \tau^n$ with $\tau$ being the conformal time, fluctuations at small scales are enhanced but those at large scales are suppressed for negative values of $n$. Thus the above discussion on the effects of cosmic strings on free streaming may be modified when we consider the case with time-varying tension.

The purpose of this paper is to study to what extent fluctuation from cosmic string with time-varying tension as well as with constant tension can affect to avoid the erasure of small scale inhomogeneities via free streaming by massive neutrinos and WDM. We also discuss the implications of the above mentioned phenomenon to the constraints on the masses of these particles.
2 Free streaming effect in a universe with cosmic strings

Although fluctuation from cosmic strings cannot be the primary source of cosmic density fluctuation today, some contributions from them are still allowed. As mentioned in the introduction, the damping of small scale fluctuation by free streaming can be avoided to some extent when fluctuation is seeded by cosmic strings. Thus, even in models with massive neutrinos or WDM, the erasure of small scale fluctuation can be less efficient by adding some fluctuation produced from cosmic strings, which may have much implications to the constraints on the masses of neutrinos and WDM.

First we discuss this issue for the case with massive neutrinos. In fact, the discussion on the alleviation of the constraint on neutrino masses has already been made in Ref. [18], where it was shown that even with the addition of fluctuation from cosmic strings the constraint on neutrino masses cannot be relaxed. This is explained as follows. In order that fluctuation from cosmic strings can affect the matter power spectrum at small scales where the free streaming effect erases the inhomogeneities, the amplitude of the fluctuation should be as large as that of the conventional adiabatic one. In this case, however, the fluctuation amplitude becomes too large at larger scales where the current CMB measurements are relevant. Notice that the fluctuation from cosmic string becomes an isocurvature one which gives the Sachs-Wolfe (SW) effect on the CMB temperature anisotropies as

$$\left. \frac{\Delta T}{T} \right|_{SW} = 2\Phi,$$

where $\Phi$ is the gravitational potential which appears in a metric perturbation in the conformal Newtonian gauge. On the other hand, the SW effect in the conventional adiabatic case can be written as

$$\left. \frac{\Delta T}{T} \right|_{SW} = \frac{1}{3}\Phi.$$  \hspace{1cm} (2)

Thus for the same magnitude of $\Phi$, the isocurvature fluctuation can give about 6 times larger CMB temperature fluctuation on large scales than that from the adiabatic one. In other words, fixing the amplitude to give the right size to fit the CMB data, matter power spectrum for isocurvature fluctuation should be about 36 times smaller than that for adiabatic fluctuation. Thus, it is impossible to relax the constraint on neutrino masses using the isocurvature fluctuation seeded by cosmic strings without affecting the CMB constraint.

Here we demonstrate this quantitatively by calculating the matter and CMB power spectra in models with massive neutrinos and cosmic strings using the CMBACT code [24]. In Fig. 1, we plot the matter power spectra $P(k)$ for the case with the conventional adiabatic fluctuation $P(k)_{(adiabatic)}$ (green dashed line), cosmic strings with constant tension $P(k)_{(string)}$(purple dotted line) and the total power $P(k)_{(adiabatic)} + P(k)_{(string)}$ (red solid line) assuming neutrino masses as $\sum m_\nu = 5$ eV. For reference, we also plot the data.

\#2Neutrino masses can be constrained from CMB data alone through the modification of the structure
from SDSS [25]. The other cosmological parameters are assumed as $\Omega_m h^2 = 0.139$, $\Omega_b h^2 = 0.021$, $h = 0.66$, $n_s = 1$ and $\tau_{\text{reion}} = 0.091$ where $\Omega_{m,b}$ are the present energy densities normalized by the critical density for matter and baryon, $h$ is the Hubble parameter, $n_s$ is the scalar spectral index and $\tau_{\text{reion}}$ is the reionization optical depth. A flat universe is assumed. The energy density of massive neutrinos are added by reducing that of dark energy. We adopt these values unless otherwise stated in this paper. As seen from the figure, although the matter power spectrum for adiabatic fluctuation is damped on small scales in this case, we can compensate the damping by adding fluctuation from cosmic strings with constant tension $G\mu = 5.2 \times 10^{-6}$. However the CMB power spectrum generated by the cosmic strings with the size of the tension has too much power and contradicts with WMAP observations, as shown in Fig. 2.

In the discussion above, we considered cosmic strings with constant tension. Recently, however, cosmic strings with time-varying tension have been studied in Refs. [19, 20]. In particular, in Ref. [20], the CMB and matter power spectra have been discussed for such models. It was explicitly shown that for the cases where the string tension decreases with time as $G\mu \propto \tau^n$ or $\propto a^n$ with $a$ being the scale factor and $n$ being negative power, the fluctuation on large scales is significantly suppressed and that on small scales is enhanced. Hence we can naively expect that the above argument on the alleviation of the constraint of acoustic peaks and the current bound is $\sum m_\nu < 2.0$ eV at 95% C.L. [2]. However this bound may not hold true in our case with not only adiabatic fluctuation but also isocurvature fluctuation generated by cosmic strings. Thus, for an illustration, we assumed this value for neutrino masses.
on neutrino masses can be modified for such cosmic strings. We studied this issue by calculating the CMB and matter power spectrum using the modified version of CMBACT code where we have introduced the time dependence of the tension as $G\mu \propto \tau^n$ or $G\mu \propto a^n$.

As an example, in Fig. 3 we plot the matter power spectrum in models with massive neutrinos ($\sum m_\nu = 10$ eV) for the cases with with conventional adiabatic fluctuation (green dashed line), cosmic strings with time-varying tension $G\mu \propto \tau^{-0.4}$ (purple dotted line) and the total power spectrum from these fluctuations (red solid line). For comparison, we also plot the case with cosmic strings with constant tension (cyan dash-dotted line). Another case for the time dependence of the string tension with $G\mu \propto a^{-0.2}$ is also shown in Fig. 4. As seen from the figure, fluctuation from cosmic strings with the time dependence $G\mu \propto \tau^{-0.4}$ and $G\mu \propto a^{-0.2}$ can cancel the damping caused by free streaming of massive neutrinos. However, even in these cases, since adiabatic fluctuation with massive neutrinos damps the matter power spectrum on smaller scales than around $k \sim 0.02\text{Mpc}^{-1}$, we have to add the contribution from cosmic strings to compensate the damping around this scale. When we have non-negligible amplitude from cosmic strings around this scale, it generates too much power to fit the CMB data even though the fluctuation on large scales is reduced due to the time dependence of the sting tension. To show this, in Fig. 5 we plot the CMB power spectrum generated by cosmic string with time varying tension $G\mu \propto \tau^{-0.4}$ and $a^{-0.2}$ with the same normalization (string tension) as that in Fig. 3. As seen from the figure, we cannot compensate the erasure of fluctuation on such scales without contributing to the CMB power spectrum significantly, which is obviously inconsistent with current observations, even if we introduce the cosmic strings with time-

![Figure 2: The CMB TT power spectrum in models with massive neutrinos ($\sum m_\nu = 5$ eV) for the case with cosmic strings with constant tension $G\mu = 5.2 \times 10^{-6}$ (solid red line). The data from WMAP3 are also plotted.](image)
varying tension. We note that, depending on the neutrino masses, we can compensate the damping of small scale power via free streaming by choosing the time dependence of the string tension appropriately. However, our conclusion remains unchanged as long as adiabatic fluctuation with massive neutrinos damps the matter power spectrum on small scales which correspond to $\ell \sim \mathcal{O}(100)$.

![Figure 3: Matter power spectrum in models with massive neutrinos ($\sum m_\nu = 10$ eV) for the cases with conventional adiabatic fluctuation (green dashed line), cosmic strings with time-varying tension $G_\mu \propto \tau^{-0.4}$ (purple dotted line) and the total power spectrum from the adiabatic fluctuation plus cosmic strings with time-varying tension (red solid line). For comparison, the case with constant tension $G_\mu = 5.1 \times 10^{-6}$ is also plotted (cyan dash-dotted line).](image)

Next we discuss the effects of cosmic strings on free streaming for the case with WDM. For WDM, since the scales of masses are significantly larger than those of massive neutrinos considered above, the damping scale can be much smaller. Thus in this case, we may have a chance to avoid the erasure of small scale inhomogeneities by free streaming without affecting the CMB power spectrum on large scales by adding isocurvature fluctuation from cosmic strings. In Fig. 6 we plot the matter power spectrum for the cases with conventional adiabatic fluctuation (green dashed line), cosmic strings with constant tension $G_\mu = 7.8 \times 10^{-7}$ (cyan dot-dashed line) and the total power spectrum (red solid line). For reference, we have also plotted the data from SDSS [25] and Lyman alpha [26] which are rescaled to the present time $z = 0$. Here we assumed the pure WDM model. The mass of WDM is taken to be $m_{\text{WDM}} = 103$ eV, which corresponds to a thermally decoupled relic with the relativistic degrees of freedom at the time of decoupling being $g_*(T_D) = 100$. For the case with adiabatic fluctuation, WDM particles erase the structure on smaller scales compared to the case with massive neutrinos as seen from the figure. With the parameter
Figure 4: Matter power spectrum in models with massive neutrinos ($\sum m_\nu = 10$ eV) for the cases with conventional adiabatic fluctuation (green dashed line), cosmic strings with time-varying tension $G\mu \propto a^{-0.2}$ (purple dotted line) and the total power spectrum from the adiabatic fluctuation plus cosmic strings with time-varying tension (red solid line). For comparison, the case with constant tension $G\mu = 5.2 \times 10^{-6}$ is also plotted (cyan dash-dotted line).

Figure 5: The CMB TT power spectrum in models with massive neutrinos ($\sum m_\nu = 10$ eV) for the case with cosmic strings with time-varying tension $G\mu \propto \tau^{-0.4}$ (solid red line) and $G\mu \propto a^{-0.2}$ (blue dashed line). The data from WMAP3 are also plotted.
above, the damping scale is \( k \sim 1\text{Mpc}^{-1} \). On the other hand, for the case with cosmic strings, such a damping becomes moderate. As shown in Fig. 6, the small scale damping for adiabatic fluctuation can be canceled by adding that from cosmic strings with constant tension. Furthermore, the CMB power spectrum generated by the above cosmic strings can satisfy WMAP constraints, as shown in Fig. 7. Thus the constraint on WDM can be relaxed in a scenario with cosmic strings even with constant tension.

![Figure 6: Matter power spectrum in models with WDM with the mass \( m_{\text{WDM}} = 103 \text{eV} \). Here we plot \( P(k) \) from the conventional adiabatic fluctuation (green dashed line), cosmic strings with constant tension \( G\mu = 7.8 \times 10^{-7} \) (cyan dot-dashed line) and the total of them (solid red line). For reference, the data from SDSS [25] (blue points) and Lyman alpha [26] (purple points) which are rescaled to \( z = 0 \) are also plotted.](image)

In addition, we also show matter power spectrum for the case with cosmic strings with time-varying tension \( G\mu \propto \tau^{-1} \), conventional adiabatic fluctuation and the total spectrum of them in Fig. 8. Here we assume the mass of WDM particles as \( m_{\text{WDM}} \sim 72 \text{eV} \). As seen from the figure, we can cancel the damping on small scales in the spectrum for adiabatic case by adding the contribution from cosmic strings with the time-varying tension \( n = -1 \). Importantly, in this case, the contribution from the cosmic string to CMB power spectrum up to \( \ell \sim \mathcal{O}(1000) \) is negligible since large scale fluctuation by the cosmic strings is significantly suppressed due to the time dependence of the string tension (Fig. 9). Hence we can have possibilities of alleviating the constraint on the masses of WDM by adding some fluctuation from cosmic strings with time-varying tension in this case too.
Figure 7: The CMB TT power spectrum in WDM models with the mass $m_{\text{WDM}} = 103$ eV for the case with cosmic strings with constant tension $G\mu = 7.8 \times 10^{-7}$ (solid red line). The data from WMAP3 are also plotted [1].

Figure 8: Matter power spectrum in models with WDM with the mass $m_{\text{WDM}} = 72$ eV. Here we plot $P(k)$ from the conventional adiabatic fluctuation (green dashed line), cosmic strings with time-varying tension $G\mu \propto \tau^{-1}$ (cyan dot-dashed line), and the total matter spectrum from the adiabatic fluctuation plus cosmic string with time-varying tension (solid red line). The case with constant tension $G\mu = 9.8 \times 10^{-7}$ is also plotted for comparison (black dash-dashed line). For reference, the data from SDSS [25] (blue points) and Lyman alpha [26] (purple points) which are rescaled to $z = 0$ are also plotted.
3 Conclusion and Discussion

We studied the erasure of small scale structure via free streaming in models with cosmic strings with time-varying tension as well as with constant tension. Because cosmic strings actively produce incoherent isocurvature fluctuation, free streaming effect is less efficient than that in models with the conventional adiabatic ones because of its nature of the fluctuation. Since the damping of small scale power by free streaming is an important probe to constrain the property of particles with large velocity dispersion at the epoch of radiation-matter equality such as massive neutrinos and WDM, above mentioned effects by cosmic strings can have much implications to the constraints on such particles.

We first studied this issue for massive neutrinos. For the case with constant string tension, it has been already discussed some time ago that, even if we add some contribution from isocurvature fluctuation from cosmic strings, we cannot relax the constraint on neutrino masses. In this paper, we explicitly calculate matter power spectrum and CMB anisotropy generated by strings and reach the same conclusion. Furthermore, we have studied this issue for the case with cosmic strings with time-varying tension. We have shown that even if we introduce cosmic strings with time-varying tension, we cannot alleviate the constraint on the mass without conflicting with the constraint from current CMB observations. This is mainly because adiabatic fluctuation with massive neutrinos damps the matter power spectrum on small scales corresponding to multipoles $\ell \sim \mathcal{O}(100)$ in CMB power spectrum.

We have also discussed the possibilities of relaxing constraints on the masses of WDM
by adding fluctuation produced by cosmic strings with time-varying tension and those with constant tension. We explicitly showed that the damping of the matter power spectrum on small scales from adiabatic primordial fluctuation by free streaming caused by WDM can be canceled by adding fluctuation seeded by cosmic strings for both cases, namely with constant and time-varying tension by choosing appropriate parameters. Importantly we can cancel the damping without conflicting the CMB constraints for both cases, which is not the case for massive neutrinos. Although much more detailed study is needed to state qualitatively to what extent the constraint on WDM particles can be relaxed, in this paper, we pointed out that some contributions from cosmic strings to small scale power can affect the constraints without conflicting with the CMB constraint. Since the time dependence of the string tension is highly model-dependent and can be more complicated than that adopted here, various possibilities can arise and have much more implications to the constraints given from observations probing the damping of small scale fluctuation via free streaming.

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