Role of correlated two–pion exchange in \(K^+N\) scattering

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Abstract

A dynamical model for S– and P–wave correlated \(2\pi\) (and \(K\bar{K}\)) exchange between a kaon and a nucleon is presented, starting from corresponding \(N\bar{N} \to K\bar{K}\) amplitudes in the pseudophysical region, which have been constructed from nucleon, \(\Delta\)–isobar and hyperon (\(\Lambda, \Sigma\)) exchange Born terms and a realistic meson exchange model of the \(\pi\pi \to K\bar{K}\) and \(K\bar{K} \to K\bar{K}\) amplitude. The contribution in the s–channel is then obtained by performing a dispersion relation over the unitarity cut. In the \(\rho\)–channel, considerable ambiguities exist, depending on how the dispersion integral is performed. Our model, supplemented by short range interaction terms, is able to describe empirical \(K^+\bar{N}\) data below pion production threshold in a satisfactory way.
1. INTRODUCTION

For quite some time, kaons have attracted the attention of nuclear physicists. The reason is that strangeness, a quantum number conserved in strong interactions, attributes a special role to the kaons among the possible projectiles for investigating nuclear structure. Kaons have two properties which make them unique tools. Firstly, they can transfer a new degree of freedom to the nucleus, and secondly, in contrast to pions they come in two forms, kaons ($K$) and antikaons ($\overline{K}$) which differ substantially in their interaction with the nucleus. Kaons with their quark content $u\bar{s}$ ($d\bar{s}$) have strangeness $S=1$; however, nuclear states involving strangeness can only contain hyperons ($\Lambda$, $\Sigma$) which have the same strangeness ($S=-1$) as antikaons (quark content $\bar{u}s$ or $\bar{d}s$). Therefore, the interaction between kaons and nuclei is rather weak, which is demonstrated, e.g., by the large $K^+$ nuclear mean free path of about 5–7 fm for $p_{\text{lab}} \leq 0.8\text{GeV/c}$. Consequently, the $K^+$ meson is a suitable probe for investigating the interior region of nuclei. On the other hand, antikaons have $S=-1$ and their absorption in nuclei can easily produce hypernuclei containing $\Lambda$– or $\Sigma$–hyperons. Since such processes occur with sizeable reaction probability the region of investigation is probably restricted to the nuclear surface.

The successful use of kaons in nuclear structure requires the precise knowledge of the interaction mechanism of kaons with nuclei. Each uncertainty in the theoretical description necessarily leads to uncertainties in the interpretation of empirical results. Since each theoretical model for the kaon–nucleus interaction starts from the free kaon–nucleon interaction and then adds medium modifications in one way or another, a precise knowledge of the free interaction is absolutely essential.

Recently [1] we have presented a meson exchange model for the $K^+N$ interaction, which provides a reasonable description of the empirical $K^+N$ scattering data for laboratory momenta smaller than 0.8 GeV/c. For model B of Ref. [1] the diagrams included are shown in Figs. 1(a) and 1(b); they have been evaluated in time–ordered perturbation theory. By $\sigma_{\text{rep}}$ we denote a very short ranged, phenomenological, repulsive contribution, which has
the analytical form of scalar $\sigma$–exchange with opposite sign and with an exchanged mass of 1.2 GeV. This additional repulsion is required if the $\omega$–coupling constants $g_{NN\omega}$, $g_{KK\omega}$ are restricted to their SU(6) value.

An important ingredient is the contribution arising from $\sigma$– and $\rho$–exchange: $\sigma$–exchange provides the dominant part of the intermediate range attraction and $\rho$–exchange determines, to a large part, the isospin dependence of the interaction. Despite their importance, however, these pieces have been treated so far in a very rough way. In both cases a sharp mass has been used which means their appreciable decay widths have not been taken into account. The $\sigma$ meson is a fictitious particle not observed in nature so both its mass and coupling constant are therefore free parameters. In the case of the $\rho$ meson the coupling strength is obtained as a product of a coupling constant at the $NN\rho$ vertex, which is taken from the Bonn potential \(^2\) and the coupling constant at the $KK\rho$ vertex, which is calculated via SU(3) relations from the (empirically known) $\pi\pi\rho$ coupling constant. Whether this procedure provides a reliable result is doubtful since i) the $\rho$–meson coupling constants used in the Bonn potential are questionable \(^3\) and ii) SU(3) relations are not necessarily valid for unstable particles.

In this simple model, $\sigma$– and $\rho$–exchange essentially stand for the correlated $2\pi$–exchange contribution in the $J^P = 0^+$ ($\sigma$–) and $J^P = 1^-$ ($\rho$–) channel, as illustrated in Fig. 1(c). The purpose of the present paper is to derive this contribution starting from a microscopic model for the $t$–channel reaction $NN \to KK$ with $\pi\pi$ (and $KK$) intermediate states and using a dispersion relation over the unitarity cut. This realistic model of (effective) $\sigma$– and $\rho$–exchange is then used to reconstruct an extended meson exchange model for $KN$ scattering.

Such a microscopic description of correlated $2\pi$–exchange is essential not only for an adequate judgment of the quantitative role of meson exchange in free $K^+N$ scattering, but also for the calculation of kaon–nucleus scattering processes. Effects of medium modifications of meson masses \(^4\) inevitably require an explicit, realistic model for correlated $2\pi$–exchange.

In Sect.2 we outline the basic formalism. Sect.3 contains the essential features of our model for the $NN \to KK$ transition. In Sect.4 we present the results for the correlated
2\pi contribution in $KN$ scattering, in terms of suitably defined effective coupling constants. Furthermore, we compare $KN$ phase shifts and observables derived from the extended model with those obtained before. Sect.5 contains some concluding remarks.

2. FORMALISM

In this section we outline the formalism which we use in order to derive the correlated $2\pi$–exchange contribution for the $KN$ interaction. The procedure is similar to that which was used in $\pi N$ scattering [5].

2.1 $KN \to KN$ and $N \bar{N} \to K \bar{K}$ amplitudes

The scattering amplitude $T$ is related to the $S$ matrix by

$$S_{fi} = \delta_{fi} - i(2\pi)^{-2}\delta^{(4)}(P_f - P_i) \left( \frac{m_N}{E_p} \frac{m_N}{E_{p'}} \right)^{-\frac{1}{2}}(2\omega_q 2\omega_{q'})^{-\frac{1}{2}} T_{fi},$$

where $P_i (P_f)$ is the total four–momentum in the initial (final) state, $E_p \equiv (\vec{p}^2 + m_N^2)^{\frac{1}{2}}$, and $\omega_q \equiv (\vec{q}^2 + m_K^2)^{\frac{1}{2}}$ with $m_N$ ($m_K$) the nucleon (kaon) mass.

In the $s$–channel ($KN \to KN$) $T$ can be written as

$$T_s(p', q', p, q) = \bar{u}(\vec{p}', \lambda')\xi^\dagger(\mu')\zeta^\dagger(\beta)\hat{T}(s, t)u(\vec{p}, \lambda)\xi(\mu)\zeta(\alpha).$$

Here, the Dirac spinor $u(\vec{p}, \lambda)$ with the normalization $\bar{u}u = 1$ describes a nucleon with helicity $\lambda$ and three–momentum $\vec{p}$, while $\xi$ ($\zeta$) is the isospin wave function of a nucleon (kaon). The operator $\hat{T}$ acts in spin and isospin space and depends on the two independent Mandelstam variables $s \equiv (p + q)^2 = (p' + q')^2$ and $t \equiv (p' - p)^2 = (q - q')^2$. The third variable $u$ is related to $s$ and $t$ by $s + t + u = 2m_N^2 + 2m_K^2$. The scattering operator $\hat{T}$ has the following isospin structure:

$$\hat{T}(s, t) = 3\hat{T}^{(+)}\mathbf{1} + 2\hat{T}^{(-)}\vec{\tau}_N \cdot \vec{\tau}_K,$$

where $\vec{\tau}_N$ ($\vec{\tau}_K$) is the isospin operator for the nucleon (kaon) and
\[ \hat{T}^{(\pm)}(s,t) = -[A^{(\pm)}(s,t)I_4 + QB^{(\pm)}(s,t)], \]  

(4)

with \( Q \equiv \gamma^\mu Q_\mu \), \( Q \equiv \frac{1}{2}(q' + q) \), and \( I_4 \) being the four-dimensional unit matrix.

The corresponding t–channel \((NN \to KK)\) amplitude is given by

\[ T_t(q', \bar{q}; \bar{p}, p) = \bar{v}(\bar{p}, \bar{\lambda})\xi^\dagger(\bar{\mu})\zeta^\dagger(\beta) \hat{T}(s,t)u(p, \lambda)\xi(\mu)\zeta(\alpha). \]  

(5)

with \( \bar{p} \equiv -p', \bar{q}' \equiv -q \) and \( \bar{v} (\xi) \) the Dirac spinor (isospin state) of an antinucleon. The Mandelstam hypothesis states now that \( \hat{T} \) (and therefore \( A^{(\pm)} \), \( B^{(\pm)} \) in Eq.(4)) is the same function of \( s \) and \( t \) as in Eq.(2), but in a different kinematical domain, i.e. for \( s = (p - \bar{q})^2 \), \( t = (\bar{p} + p)^2 \), and \( Q = \frac{1}{2}(q' - \bar{q}') \).

### 2.2 Spectral functions

In order to isolate the \( \sigma \) and \( \rho \) contribution we have to perform a partial wave decomposition of the amplitudes in the t–channel:

\[ A^{(\pm)}(s,t) = \sum_J (J + \frac{1}{2}) P_J(x) A^{(\pm)}_J(t) \]  

(6)

(the same for \( B^{(\pm)} \)). The \( P_J(x) \) are the Legendre functions and \( x \equiv \cos\theta_t \). The scattering angle in the t–channel, \( \theta_t \), can be expressed in terms of \( s \) and \( t \):

\[ x = \frac{s + \frac{1}{2} t - m_N^2 - m_K^2}{2\sqrt{\frac{1}{4}t - m_N^2} \sqrt{\frac{1}{4}t - m_K^2}}. \]  

(7)

Conservation of parity and G–parity demands that the sum of spin and isospin must be even in case of a \( 2\pi \) intermediate state. Therefore \( A^{(-)}_J, B^{(+)}_J \) \((A^{(+)}_J, B^{(-)}_J)\) will vanish for even (odd) \( J \) if we only consider \( 2\pi \) intermediate states. This argument does not hold for an intermediate \( KK \) state; there \( A^{(+)}_J, B^{(-)}_J \) \((A^{(-)}_J, B^{(+)}_J)\) are also possible for odd (even) \( J \). However in our model these amplitudes turn out to be negligibly small and can be safely neglected. Then the isospin index \((\pm)\) can be suppressed since \( J \) determines uniquely the isospin state.
Since the $A$ and $B$ amplitudes contain kinematical singularities, one has to define new amplitudes $f^J_{\pm}$, which are free of these singularities. Here the index $\pm$ denotes the helicity of the $N\bar{N}$ state: $\lambda = \pm \frac{1}{2}$; $\bar{\lambda} = \frac{1}{2}$. $(f^{J=0}_{+}(t) = 0)$. In terms of these amplitudes $A$ and $B$ can be written as

$$A^{(\pm)}(s, t) = \frac{8\pi}{p_t^2} \sum J \left[ \frac{1}{2} \right] (p_tq_t)^J \left\{ \frac{m_N}{\sqrt{J(J+1)}} x P_j^f(x) f^J_{\pm}(t) - P_j(x) f^J_{\pm}(t) \right\}$$

$$B^{(\pm)}(s, t) = 8\pi \sum J \left[ \frac{1}{2} \right] (p_tq_t)^{J-1} \frac{\sqrt{J(J+1)}}{P_j^f(x) f^J_{\pm}(t)},$$

with $P_j^f(x) = \frac{d}{dx} P_j(x)$ and $p_t = |\vec{p}| (q_t = |\vec{q}|)$ in the c.m. system of the t–channel process.

The $\sigma$– ($\rho$–) exchange contribution, defined as the correlated $2\pi$–exchange in the scalar (vector) t–channel, is identified as the $J = 0 (J = 1)$ term of Eq. (8).

One now can perform the analytic continuation of the $f$ amplitudes to physical $t$–values in the s–channel ($t \leq 0$), which requires knowledge of the cut structure in the complex $t$ plane. The right hand (unitarity) cut runs from $4 m^2_{\pi}$ to $\infty$, whereas the left hand cut, determined by $\Lambda$ exchange runs from $-\infty$ to $t_{\text{LH}} \equiv 4 m^2_{\Lambda} - \frac{(m^2_{K} + m^2_{\pi} - m^2_{N})^2}{m^2_{\Lambda}} > 4 m^2_{\pi}$. This means that the cuts overlap. In this overlap region the baryon exchange Born term of the reaction $N\bar{N} \rightarrow K\bar{K}$ has an imaginary part. As we are only interested in the correlated $2\pi$ contribution (which we call $f^J_{\pm}$ in the following) we perform the dispersion relation for $f^J_{\pm}$ only over the right hand cut, leaving out the Born term. Then we get for $\sigma$–exchange the following contributions to the invariant amplitudes in the s–channel:

$$A^{(+)}(s, t) = -\frac{4\pi}{p_t^2} f^0_+(t) = -16 \int_{4m^2_{\pi}}^{\infty} \frac{\text{Im} f^0_+(t')dt'}{(t' - t)(t' - 4m^2_{N})}$$

$$B^{(+)}(s, t) = 0.$$  \hspace{1cm} (9)

(For convergence reasons, see Ref. [4], the dispersion relation has to be performed for $\frac{f^0_+(t)}{p_t^2}$, $p_t^2 = \frac{t}{4} - m^2_{N}$.) Similarly we obtain for $\rho$–exchange

$$A^{(-)}(s, t) = 12\pi \frac{q_t}{p_t} x \left( \frac{m_N}{\sqrt{2}} f^1_+(t) - f^1_-(t) \right)$$

$$= 12 \frac{s + \frac{1}{2} t - m^2_{N} - m^2_{\Lambda}}{t - 4m^2_{N}} \sqrt{2m_N} \int_{4m^2_{\pi}}^{\infty} \frac{\text{Im} f^1_+(t')dt'}{t' - t} - 2 \int_{4m^2_{\pi}}^{\infty} \frac{\text{Im} f^1_+(t')dt'}{t' - t},$$

6
\[ B_{\rho}^{\text{(--)}}(s,t) = 6\sqrt{2}\pi \tilde{f}_1^1(t) = 6\sqrt{2} \int_{4m_N^2}^{\infty} \frac{\text{Im} \tilde{f}_1^1(t')}{t' - t} \, dt' . \tag{10} \]

Here \( s \) and \( t \) have to assume physical values of the \( s \)-channel, i.e., \( s \geq (m_N + m_K)^2 \) and \( t \leq 0 \). These amplitudes can be interpreted as meson exchange potentials for which the meson mass \( \sqrt{t'} \) is distributed over the range from \( 2m_\pi \) to \( \infty \). The corresponding coupling constants depend on the mass; they are proportional to the spectral functions \( \text{Im} \tilde{f}_1^1(t') \), \( \text{Im} \tilde{f}_1^{-1}(t') \). Therefore, these spectral functions determine the dynamical behaviour of the exchanges, i.e., they characterize the strength as well as the range of these potentials.

Concerning \( \rho \)-exchange, it was pointed out in Refs. \[5,7\] that there is a considerable uncertainty in the results. The reason is that Eq.\(\text{(10)}\) disperses the helicity amplitudes directly; alternatively, one \(\text{(8)}\) can first construct combinations \( \tilde{\Gamma}_1, \tilde{\Gamma}_2(t) \) corresponding to vector \( (\tilde{\Gamma}_1) \) and tensor \( (\tilde{\Gamma}_2) \) coupling amplitudes, where

\[ \tilde{\Gamma}_1(t) = -\frac{m_N}{t^2 - m_N^2} \left\{ \tilde{f}_1^1(t) - \frac{t}{4\sqrt{2}m_N^2} \tilde{f}_1^{-1}(t) \right\} \]
\[ \tilde{\Gamma}_2(t) = +\frac{m_N}{t^2 - m_N^2} \left\{ \tilde{f}_1^1(t) - \frac{m_N}{\sqrt{2}} \tilde{f}_1^{-1}(t) \right\} , \tag{11} \]

and then perform the dispersion integral, which yields

\[ A_{\rho}^{\text{(--)}}(s,t) = -12\pi \frac{q tp_{\pi \pi}}{m_N} \tilde{\Gamma}_2(t) \]
\[ = -6 \left( s + \frac{1}{2} t - m_N^2 - m_K^2 \right) \int_{4m_N^2}^{\infty} \frac{\text{Im} \tilde{\Gamma}_2(t')}{t' - t} \, dt' \]
\[ B_{\rho}^{\text{(--)}}(s,t) = -12\pi (\tilde{\Gamma}_1(t) + \tilde{\Gamma}_2(t)) \]
\[ = -12 \left( \int_{4m_N^2}^{\infty} \frac{\text{Im} \tilde{\Gamma}_1(t')}{t' - t} \, dt' + \int_{4m_N^2}^{\infty} \frac{\text{Im} \tilde{\Gamma}_2(t')}{t' - t} \, dt' \right) . \tag{12} \]

Both methods would be equivalent if the dispersion integrals could be performed over both the complete left hand and unitarity cut. However, \( \rho \)-exchange is customarily defined via a dispersion integral over the unitarity cut only. Indeed, the additional \( t \)-dependence in \( \tilde{\Gamma}_1 \) apart from the \( t \)-dependence provided by the helicity amplitudes \( \tilde{f}_1^J(t) \) causes the results to be quite different, as will be discussed later.
2.3 Correlated $2\pi$–exchange potentials

Based on Eqs. (2) – (4) the correlated $2\pi$–exchange contributions to the $KN$ scattering amplitude can then be written as

\[ T_{\sigma}^{(\sigma)} = -3 \, \bar{u}(p', \lambda') A_{\sigma}^{(+)}(t) u(p, \lambda) \mathbf{1}, \]

\[ T_{\rho}^{(\rho)} = -2 \{ \bar{u}(p', \lambda')[A_{\rho}^{(-)}(s, t) + Q \mathcal{B}_{\rho}^{(-)}(t)] u(p, \lambda) \} \tau_N \cdot \tau_K, \]

omitting isospin states for convenience. In the c.m. system of the $KN \rightarrow KN$ reaction $s = (E_p + \omega_p)^2$, $t = -(p' - \bar{p})^2$, and $Q = (\omega_p, -\frac{1}{2}(p' + \bar{p}))$. Since we will treat $T_{\sigma,\rho}^{(\sigma,\rho)}$ later as potentials to be iterated in the scattering equation belonging to time–ordered perturbation theory, we apply an off-shell extrapolation of the dispersion relations in the following way:

We first replace the denominator $t' - t$ in analogy to the time–ordered propagator

\[ \frac{1}{t - t'} \to \frac{1}{2\omega_r} \left( \frac{1}{Z - \omega_r - E_p - \omega_p'} + \frac{1}{Z - \omega_r - E_p' - \omega_p} \right) \]

(\omega_r \equiv [t' + (p' - \bar{p})^2]^\frac{1}{2}, Z = E_{pon} + \omega_{pon} is the total c.m. energy), which is motivated by the fact that the dispersion integral sums over exchanges of particles with mass $\sqrt{t'}$. For the additional $t$ dependence in Eqs.(10,12) we keep $t = -(p' - \bar{p})^2$ and $s \equiv (E_p + \omega_p)(E_p' + \omega_p')$. Note that this does not change the on–shell result.

In addition we have to add phenomenological cutoffs in order to generate sufficient convergence in the scattering equation. Again we interpret the correlated $2\pi$ potentials as generated by exchange of particles with mass $\sqrt{t'}$ and define a formfactor

\[ F(t) = \frac{\Lambda_{\sigma,\rho}^2 - t'}{\Lambda_{\sigma,\rho}^2 - t}, \]

which is squared under the dispersion integral. This is analogous to the monopole type form factor we use at each vertex of the ordinary meson and baryon exchange potentials. We use $\Lambda_{\sigma,\rho} = 1850(2400)$ MeV for the model where we perform the dispersion integral for $f_{\pm 1}(t)$ ($\bar{\Gamma}_i(t)$). One should realize that this procedure modifies the original on–shell result.
somewhat. However, it lies in the range of uncertainties which are inherent in the whole procedure, which are discussed further in Sect.4.

With the above extensions the amplitudes in Eq.(13) have a well defined off–shell behaviour with a sufficient fall–off for high momenta. Corresponding potential matrix elements, \( < \vec{p}' \lambda' | V(Z) | \vec{p}\lambda > \), acquire an additional factor \( \kappa = \frac{1}{(2\pi)^3} \sqrt{\frac{m}{E_p}} \sqrt{\frac{1}{2m_\pi^2}} \), i.e. \( < \vec{p}' \lambda' | V_{\sigma,\rho}(Z) | \vec{p}\lambda > = \kappa < \vec{p}' \lambda' | T_s^{(\sigma,\rho)}(Z) | \vec{p}\lambda > \). Unitarization then leads to the scattering amplitude, i.e.

\[
< \vec{p}' \lambda' | T(Z) | \vec{p}\lambda > = < \vec{p}' \lambda' | V(Z) | \vec{p}\lambda > + \sum_{\lambda''} \int d^3p'' < \vec{p}' \lambda' | V(Z) | \vec{p}'' \lambda'' > \frac{1}{Z - E_{p''} - \omega_{p''} + i\epsilon} < \vec{p}'' \lambda'' | T(Z) | \vec{p}\lambda > ,
\]

where \( V \) contains contributions from the diagrams shown in Fig. 1, except that now the \( \sigma \) and \( \rho \) exchange potentials are replaced by the correlated \( 2\pi \) exchange potentials discussed here.

3. MICROSCOPIC MODEL FOR THE N\( \bar{N} \rightarrow K\bar{K} \) PROCESS

In the last section we have outlined a method of obtaining the correlated \( 2\pi \)–exchange contribution to the \( KN \rightarrow KN \) scattering amplitude from the \( N\bar{N} \rightarrow K\bar{K} \) partial wave helicity amplitudes \( f_{J^\pm} \). Since, unlike for the \( N\bar{N} \rightarrow 2\pi \) case (Ref. \[5\]), we cannot rely on quasiempirical information, we have to provide a field–theoretic model for the \( N\bar{N} \rightarrow K\bar{K} \) amplitudes. Anyhow, such a dynamical model has definite advantages when medium modifications of the \( KN \) interaction are considered since it facilitates future investigation of not only possible medium effects due to changes in the kaon and nucleon propagators, but also in the \( N\bar{N} \rightarrow K\bar{K} \) interaction itself.

We will generate the amplitude for the process of Fig. 1(c) in the t–channel by solving the scattering equation in the Blankenbecler–Sugar (BbS) \[9\] reduction scheme:

\[
T_{N\bar{N} \rightarrow K\bar{K}} = V_{N\bar{N} \rightarrow K\bar{K}} + \sum_{aa=\pi\pi,K\bar{K}} T_{aa \rightarrow K\bar{K}} g_{aa} V_{N\bar{N} \rightarrow aa} .
\]
where

$$T_{aa \rightarrow KK} = V_{aa \rightarrow KK} + \sum_{bb=\pi\pi, KK} T_{bb \rightarrow KK} g_{bb} V_{aa \rightarrow bb}.$$  \hspace{1cm} (18)

Here $V_{N\bar{N},aa}$ is the transition interaction from $N\bar{N}$ to $aa = \pi\pi, KK$, $T_{aa,K\bar{K}}$ are transition amplitudes from $\pi\pi$ and $K\bar{K}$ to $K\bar{K}$, and $g_{aa}$ is the free two-particle Green’s function for the $aa$ intermediate state. The ingredients of the dynamical model for the transition interactions $V_{N\bar{N},\pi\pi}$ and $V_{N\bar{N},K\bar{K}}$ are shown in Fig. 2. The potential $V_{N\bar{N},\pi\pi}$ ($V_{N\bar{N},K\bar{K}}$) consists of $N$ and $\Delta$ ($\Lambda$ and $\Sigma$) exchange terms plus $\rho$ meson pole diagrams. $T_{\pi\pi,K\bar{K}}$ and $T_{K\bar{K},K\bar{K}}$ are obtained from the driving terms shown in Fig. 3. Such a model involving the coupled channels $\pi\pi$ and $K\bar{K}$ was constructed by our group [10] based on time-ordered perturbation theory. Here, as in a recent study of $\pi N$ scattering [7], we use a model with essentially the same physical input, which alternatively uses the BbS scheme. The description of the data turns out to be as successful as in Ref. [10]. For more details, the reader is referred to [7].

We stress that all parameters are predetermined: $T_{\pi\pi,K\bar{K}}$ and $T_{K\bar{K},K\bar{K}}$, through the coupled channel calculation, are fixed by $\pi\pi$ while both transition potentials are determined by the quasiempirical $N\bar{N} \rightarrow 2\pi$ information, cf. Ref. [7].

In the c.m. system and in helicity representation Eq. (17) after a partial wave expansion becomes

$$<00|T_{J N\bar{N} \rightarrow K\bar{K}}^J(q,p;t)|\lambda N\lambda_{\bar{N}} > = <00|V_{J N\bar{N} \rightarrow K\bar{K}}(q,p;t)|\lambda N\lambda_{\bar{N}} >$$

$$+ \sum_{aa} \int_0^{\infty} dk k^2 <00|T_{aa \rightarrow K\bar{K}}^J(q,k;t)|00 \rangle <00|V_{N\bar{N} \rightarrow aa}^J(q,k;t)|\lambda N\lambda_{\bar{N}} > (2\pi)^3 \omega_a(k) \left( t - \omega_a^2(k) \right)$$  \hspace{1cm} (19)

with

$$\omega_a(k) = \sqrt{k^2 + m_a^2}.$$  \hspace{1cm} (20)

The $N\bar{N} \rightarrow K\bar{K}$ on-shell amplitudes are related to the helicity amplitudes $f_+^J(t)$ via

$$f_+^J(t) = \frac{p_{on} m_N}{4(2\pi)^2 (p_{on} q_{on})^J} <00|T_{N\bar{N} \rightarrow K\bar{K}}^J(q_{on},p_{on};t)|\frac{11}{2} \frac{2}{2} >$$

$$f_-^J(t) = -\frac{p_{on} m_N}{2(2\pi)^2 \sqrt{t(p_{on} q_{on})}^J} <00|T_{N\bar{N} \rightarrow K\bar{K}}^J(q_{on},p_{on};t)|\frac{1}{2} (-\frac{1}{2}) >$$  \hspace{1cm} (21)
with
\[ q_{on} = \sqrt{\frac{t}{4} - m_K^2} \]
\[ p_{on} = \sqrt{\frac{t}{4} - m_N^2} . \]

(22)

Fig. 4 shows the results for \( \text{Im} \tilde{f}_0 \) and \( \text{Im} \tilde{f}_1 \) in the pseudophysical region \( (t > 4m_{\pi}^2) \), needed as input for Eqs.(9, 10, 12). As expected, the spectral function in the \( \rho \)-channel shows a resonant structure with a maximum at about the \( \rho \)-mass. In the \( \sigma \)-channel the spectral function is much broader than for the \( \rho \); compared to the \( N \bar{N} \rightarrow 2\pi \) case (Ref. [5]) it is weaker and the peak is shifted somewhat to a higher mass (see also the discussion later). Furthermore, the inclusion of intermediate \( K \bar{K} \) states leads to a sizable enhancement of \( \text{Im} \tilde{f}_0 \) whereas its effect is negligible in the \( \rho \)-channel. This is due to the fact that the \( K \bar{K} \) interaction is weak in the vector but strong in the scalar channel. In fact, as discussed in Refs. [10] and [11], the \( K \bar{K} \) interaction generates a \( K \bar{K} \) bound state, the \( f_0(975) \) meson. This state clearly has a strong effect on the shape of \( \text{Im} \tilde{f}_0 \).

4. RESULTS

4.1 Effective coupling constants

Based on the spectral functions of the last section we can now evaluate, in a first step, the (on–shell) invariant amplitudes with the help of Eqs.(9, 10, 12). In practice, the integrals have been evaluated up to \( t_c = 120m_{\pi}^2 \), \( i.e., \) in a region in which the dynamical model can be trusted.

It is instructive to parametrize the result by sharp mass \( \sigma \)- and \( \rho \)-exchange with appropriate \( t \)-dependent coupling constants which can be compared with those used in our former model. Let us start with the \( \sigma \)-channel. From the Lagrangians used in Ref. [11]

\[ \mathcal{L}_{NN\sigma} = g_{NN\sigma} \bar{\psi}_N \psi_N \phi_{\sigma} \]
\[ \mathcal{L}_{KK\sigma} = g_{KK\sigma} m_K \phi_K \phi_{\sigma} \]

(23)
we get for the invariant amplitude arising from $\sigma$–exchange:

$$A^{(+)}\prime(t) = -\frac{2g_\sigma m_K}{m_\sigma^2 - t},$$

(24)

with $g_\sigma \equiv g_{NN\sigma}g_{K\sigma}$. If we now parametrize the result of our correlated $2\pi$–exchange potential (Eq.(9)) in this form (allowing $g_\sigma$ to be $t$–dependent) we will get for the effective coupling constant

$$\frac{g_\sigma(t)}{4\pi} = -\frac{1}{8\pi m_K} A^{(+)}(t) (m_\sigma^2 - t),$$

(25)

The result (with $m_\sigma = 0.6$ GeV) is shown in Fig. 5, together with the result of an updated model of Ref. [1], hereafter referred to as model I, based on the diagrams in Figs. 1(a) and 1(b), with the same value of $m_\sigma$. (Details of this model will be given below.) Obviously our model predictions for the correlated $2\pi$–exchange in the scalar channel is in rough agreement with the $\sigma$–strength used before, which was phenomenologically adjusted to empirical $K^+N$ data. However, our new result has a sizable $t$–dependence, which demonstrates clearly that it cannot be well approximated by sharp mass $\sigma$–exchange with $m_\sigma = 0.6$GeV. Since it grows with $-t$, the contribution is shorter ranged, in complete consistency with the behaviour of the spectral function in Fig. 4(a). Therefore, the effective mass exchanged should be higher. Indeed, if we use $m_\sigma = 0.75$GeV in Eq.(25) the result for $g_\sigma(t)$, shown in the dashed curve of Fig. 5, has almost no $t$–dependence. Note that the analogous effective $\sigma$–mass for the $\pi N$ system [2] is $m_\sigma = 0.55$GeV.

Let us now go to the $\rho$–channel. Starting from the Lagrangians for sharp mass $\rho$–exchange

$$\mathcal{L}_{NN\rho} = \bar{\psi}_N \{g_{NN\rho} \gamma_\mu \tilde{\phi}_\rho^\mu + \frac{1}{4m_N} g_{NN\rho} T_\rho \sigma_{\mu\nu} \times (\partial^\mu \tilde{\phi}_\rho^\nu - \partial^\nu \tilde{\phi}_\rho^\mu)\} \gamma^\mu \psi_N$$

$$\mathcal{L}_{KK\rho} = g_{KK\rho} (\phi_K \gamma^\mu \phi_K)(\tilde{\phi}_\rho)_{\mu}$$

(26)

we get for the invariant amplitudes:

$$A^{(-)}_{\rho}(s, t) = -\frac{g_\rho p_{t x}}{m_N} \frac{2g_T}{m_\rho^2 - t}$$

$$B^{(-)}_{\rho}(t) = \frac{2(g_V + g_T)}{m_\rho^2 - t},$$

(27)
with \( g_V \equiv g_{NN\rho}^V g_{KK\rho} \), \( g_T \equiv g_{NN\rho}^T g_{KK\rho} \). Parametrizing again our correlated \( 2\pi \)-exchange result (Eq.\((10)\) resp. Eq.\((12)\)) in this form we obtain for the effective \( \rho \) coupling constants

\[
\frac{g_V(t)}{4\pi} = + \frac{1}{8\pi} \left( \frac{m_N}{g_{p_t} x} A_{\rho}^{(-)}(s,t) + B_{\rho}^{(-)}(t) \right) (m_\rho^2 - t) \\
\frac{g_T(t)}{4\pi} = - \frac{1}{8\pi} \frac{m_N}{g_{p_t} x} A_{\rho}^{(-)}(s,t) (m_\rho^2 - t) .
\]

(Note that according to Eqs.\((10, 12)\) the \( s \)-dependence drops out.)

The results for \( g_V \) and \( g_T \) based on \( m_\rho = 769 \) MeV are shown in Fig. 6, again together with values used in model I. First we observe that there is a remarkable difference between the two alternatives for doing the dispersion relation in the \( \rho \)-channel, \( i.e.\), Eq.\((10)\) on one hand and Eq.\((12)\) on the other: The second method provides larger tensor \((g_T)\) but smaller vector coupling \((g_V)\). Consequently the ratio \( g_T/g_V \) characterizing the tensor to vector coupling ratio of the (effective) \( \rho \) to the nucleon, \( g_{NN\rho}^T g_{NN\rho}^V \) is much larger \((\sim 5)\) for the second than for the first choice \((\sim 2.5)\). Note also that the results based on the second choice are almost \( t \)-independent; therefore, the result can be well identified with an exchange of an (effective) \( \rho \) meson with the empirical mass. On the other hand, the first method yields a result with a non–negligible \( t \)-dependence, with opposite behaviour for \( g_V \) and \( g_T \). Therefore, given the Lagrangians in Eq.\((26)\), a common mass cannot be assigned to the \( \rho \)-channel result derived from the first choice.

The situation is similar to the \( \pi N \) case (cf. Fig. 9 of Ref. \([7]\)). The only difference is that there all results scale by a factor of (roughly) 3, in distinct disagreement with the SU(3) value of 2 for the \( g_{\pi\pi\rho}/g_{KK\rho} \) ratio. (Note that there is an additional factor 2 due to the factor \( \frac{1}{2} \) in the Lagrangian used in \([7]\).) This discrepancy should be of no surprise since there is no reason to expect that such effective exchanges generated by correlated \( 2\pi \) exchange should fulfill the symmetry relations for exchanges of genuine particles.

In the sharp mass \( \rho \)-model I the \( g_T/g_V \) ratio has been taken from the Bonn \( NN \) model potential \([2]\) to be 6.1, only slightly larger than the correlated result based on Eq.\((12)\). On the other hand, the absolute values are much larger, by about a factor of 2. Note however, that in the following calculation of \( KN \) phase shifts and observables this discrepancy in
the physical $t$ region is much reduced since in this model a $\rho NN$ (and $\rho KK$) formfactor of monopole type with a cutoff mass of only about twice the $\rho$–mass are introduced, which suppress the $\rho$–potentials by about a factor of 2 at $t=0$.

4.2 Model for KN scattering

In this section we confront our correlated $2\pi$–exchange model with the experimental $K^+N$ data. Our starting point is model B of Ref. [1] consisting of the diagrams shown in Figs. 1(a) and 1(b). The baryon exchange diagrams in Fig. 1(b) are now treated in an improved way. First, both time orderings are included instead of only one (cf. Fig. 1(b) of Ref. [1]). Second, pseudovector coupling is used at the NYK vertex, i.e.

$$
\mathcal{L}_{N\Lambda K} = \frac{f_{N\Lambda K}}{m_K} (\overline{\psi}_\Lambda (x) \gamma^5 \gamma^\mu \psi_N (x) + \overline{\psi}_N (x) \gamma^5 \gamma^\mu \psi_\Lambda (x)) \partial^\mu \phi_K (x).
$$

$$
\mathcal{L}_{N\Sigma K} = \frac{f_{N\Sigma K}}{m_K} (\overline{\psi}_\Sigma (x) \gamma^5 \gamma^\mu \psi_N (x) + \overline{\psi}_N (x) \gamma^5 \gamma^\mu \overline{\psi}_\Sigma (x)) \vec{\tau} \partial^\mu \phi_K (x).
$$

Third, in case of $Y^*$–exchange, an extended spin–3/2 propagator is taken. Finally monopole formfactors (Eq.(2.17) of Ref. [1]) are used throughout, except for $NY^* K$ and $N\Delta \rho$ vertices, where a dipole formfactor is used. The new expressions for the potential matrix elements are given in the appendix. In addition, we change $g_{N\Delta \pi}^2$ to its experimental value $\frac{g_{N\Delta \pi}^2}{4\pi} = 0.36$, instead of its quark model value used before. Correspondingly, via SU(6) relations, $\frac{g_{N\Delta \rho}^2}{4\pi}$ now becomes 32.95.

As in Ref. [1], most parameters (coupling constants, cutoff masses) are predetermined: Coupling constants (with the exception of $g_{N\Delta \pi}^2$ and $g_{N\Delta \rho}^2$) and cutoff masses belonging to $NN$ and $N\Delta$ vertices are taken to be precisely the same as those of the (full) Bonn $NN$ potential [2]. Coupling constants at the vertices involving strange baryons ($g_{N\Lambda K}$, $g_{N\Sigma K}$, $g_{NY^* K}$) have been related by the assumption of SU(6) symmetry to the empirical $NN\pi$ coupling, as in our hyperon–nucleon model [12]. The three–meson coupling constants have been determined from the empirical $\pi\pi\rho$ coupling, assuming the same symmetry scheme and ideal mixing. The value of $g_{KK\sigma}$ and some cutoff masses have been slightly readjusted to the
empirical $K^+N$ phase shifts below pion production threshold. This defines our model I. The values of parameters used in this model are given in Table 1; the phase shifts which it yields are shown in the dotted curves of Fig. 7. Obviously, the model based on phenomenological sharp mass $\sigma$– and $\rho$–exchange provides a fair description of the empirical situation, with slight improvements in the $P_{03}$ and $P_{13}$ phase shifts compared to Ref. [1].

We now replace the sharp mass $\sigma$ and $\rho$ contributions by the correlated $2\pi$–exchange potentials based on Fig. 1(c) (model II, $A=$Eq.(10), $B=$Eq.(12)) evaluated off–shell using Eq.(14) and including formfactors (Eq.(15)). In order to avoid double counting we have then to omit the box diagrams in Fig. 1(b) involving two pions since they are already included in Fig. 1(c).

After a slight readjustment of some parameters, cf. Table 2, we obtain phase shift results shown likewise in Fig. 7. As expected from Figs. 5 and 6, some discrepancies occur between the various models. The main point however is that $K^+N$ interactions based on a microscopic evaluation of correlated $2\pi$ exchange are able to provide a reasonable description of empirical phase shifts.

Since the existing phase shift analyses have large error bars and are, in some cases, even contradictory it is instructive to examine the experimental observables directly. Fig. 8 shows our model predictions for the elastic cross sections in the relevant momentum range, while Figs. 9–11 show the differential cross sections and polarizations at some selected momenta. All models are in good agreement with experimental data. Again, there are slight differences between the various model results. The differential cross sections for $K^+p$ suggest an almost complete absence of P–waves, which is best realized in the model involving correlated $2\pi$–exchange evaluated according to Eq.(10).

Finally, Table 3 presents scattering lengths $a_I^{l_S}$ ($l=0,1$) and effective range $r_I^{l_S}$ of our models, which are in overall agreement with the empirical values.
5. SUMMARY

In this paper we have presented a microscopic model for correlated $2\pi$ (and $K\bar{K}$)–exchange between kaon and nucleon, in the scalar–isoscalar ($\sigma$) and vector–isovector ($\rho$) channels. We first constructed a model for the reaction $N\bar{N} \rightarrow K\bar{K}$ with intermediate $2\pi$ and $K\bar{K}$ states, based on a transition in terms of baryon ($N$, $\Delta$, $\Lambda$, $\Sigma$) exchange and a realistic coupled channel $\pi\pi \rightarrow \pi\pi$, $\pi\pi \rightarrow K\bar{K}$ and $K\bar{K} \rightarrow K\bar{K}$ amplitude. The contribution in the $s$–channel is then obtained by performing a dispersion relation over the unitarity cut.

In the $\sigma$–channel, the result can be suitably represented by an exchange of a scalar particle with a mass of 0.75 GeV. The strength turns out to be in rough agreement with the strength of phenomenological $\sigma$–exchange used before [1], which has been adjusted, together with other diagrams, to empirical $K^+N$ data.

In the $\rho$–channel, considerable ambiguities exist, of the same structure as in the $\pi N$ case, depending on how the dispersion integral is performed. In terms of effective coupling constants, the results differ strongly from the values used before in our phenomenological $\rho$–exchange amplitude, which were determined from the Bonn $NN$ potential and SU(3) relations. However, the $\rho$–amplitudes actually used are quite similar since the formfactor applied in the phenomenological $\rho$–exchange of model I brings the contribution close to the results of the dispersion–theoretic results.

This model for correlated $2\pi$–exchange has been suitably extrapolated off–shell, and supplemented by short range terms (generated partly by conventional $\omega$–exchange) and box diagrams involving $\pi\rho$– and $\rho\rho$–exchange developed before. A satisfactory description of the empirical situation is achieved, of the same overall quality as obtained before using phenomenological sharp mass $\sigma$– and $\rho$–exchange.

Such an explicit model for correlated $2\pi$–exchange has not only conceptual advantages compared to a phenomenological treatment in terms of $\sigma$–, $\rho$–exchange, but also offers the possibility to study medium modifications of the $KN$ amplitude in a well–defined way—a topic of high current interest.
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APPENDIX A: POTENTIAL MATRIX ELEMENTS FOR BARYON EXCHANGE

For the baryon exchange diagrams we get the following potential matrix elements:

\[
< \vec{p}' \lambda' | V_Y | \vec{p} \lambda > = - \kappa \frac{f^2_{NYK}}{m_K^2} q_\mu q'_\nu \bar{u}(\vec{p}', \lambda') \gamma^\mu (\bar{p}_r + m_r) \gamma^\nu u(\vec{p}, \lambda) \\
\cdot \frac{1}{2E_r} \left( \frac{1}{Z - E_r - E_p - E_{p'}} + \frac{1}{Z - E_r - \omega_q - \omega_{q'}} \right) F_Y(I) \quad \text{(A1)}
\]

where the isospin factors are \( F_\Lambda = \frac{1}{2}(1 + \vec{\tau}_1 \cdot \vec{\tau}_2) \) and \( F_\Sigma = \frac{1}{2}(3 - \vec{\tau}_1 \cdot \vec{\tau}_2) \).

In case of \( Y^* \)-exchange we get

\[
< \vec{p}' \lambda' | V_{Y^*} | \vec{p} \lambda > = - \kappa \frac{f^2_{NY^*K}}{m_K^2} q_\mu q'_\nu \bar{u}(\vec{p}', \lambda') (\bar{p}_r + m_r) \\
\cdot \left\{ -g^{\mu \nu} + \frac{1}{3} \gamma^\mu \gamma^\nu + \frac{2}{3m_r^2} P_r^\mu P_r^\nu - \frac{1}{3m_r} (p_r^\mu \gamma^\nu - P_r^\nu \gamma^\mu) \right\} u(\vec{p}, \lambda) \\
\cdot \frac{1}{2E_r} \left( \frac{1}{Z - E_r - E_p - E_{p'}} + \frac{1}{Z - E_r - \omega_q - \omega_{q'}} \right) F_{Y^*}(I) \quad \text{(A2)}
\]

with \( F_{Y^*} = \frac{1}{2}(3 - \vec{\tau}_1 \cdot \vec{\tau}_2) \). By \( p(p') \) we denote the four–momentum of the ingoing (outgoing) nucleon, by \( q(q') \) the four–momentum of the ingoing (outgoing) kaon, whereas \( p_r \) stands for the four–momentum of the exchanged hyperon. We choose \( p_r^0 = \epsilon_N - \epsilon_K \) with \( \epsilon_N \equiv \frac{s - m_N^2 - m_K^2}{2\sqrt{s}} \), \( \epsilon_K \equiv \frac{s - m_N^2 + m_K^2}{2\sqrt{s}} \).
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FIGURES

FIG. 1. Contributions to KN scattering. (a),(b): diagrams included in Ref. [1]; (c): correlated $2\pi$ exchange, which was parametrized by (a) in Ref. [1].

FIG. 2. Model for the $N\bar{N} \rightarrow \pi\pi, K\bar{K}$ transition potentials.

FIG. 3. Driving terms building up the coupled channels ($\pi\pi, K\bar{K}$) amplitude.

FIG. 4. The $N\bar{N} \rightarrow K\bar{K}$ helicity amplitudes $\tilde{f}^0_\pm$ (a) and $\tilde{f}^1_\pm$ (b), (c) as a function of $t$ in the pseudophysical region. The solid lines show the model result. The dash–dotted line in (a) shows the result neglecting the $N\bar{N} \rightarrow K\bar{K}$ transition potentials. The vertical solid (dashed) line in (a) indicates the $\delta$ function at $m_\sigma = 600(750)$ MeV representing sharp mass $\sigma$–exchange, the vertical lines in (b) and (c) indicate the $\delta$ function at $m_\rho = 769$ MeV representing sharp mass $\rho$–exchange.

FIG. 5. Effective coupling constant $g_\sigma$ as a function of $-t$. The dash–dotted line shows the $g_\sigma$ used in model I (with $m_\sigma = 600$ MeV), the solid (dashed) line shows the result for correlated $2\pi$–exchange using $m_\sigma = 600(750)$ MeV in the parametrization.

FIG. 6. Effective coupling constants for $\rho$–exchange as a function of $-t$ (with $m_\rho = 769$ MeV). The dotted (double–dotted) line shows $g_V (g_T)$ used in model I, the solid (dash–dotted) line shows $g_V (g_T)$ for correlated $2\pi$–exchange calculated with Eq.(10), the short dashed (long dashed) line $g_V (g_T)$ for correlated $2\pi$–exchange calculated with Eq.(12).

FIG. 7. $KN$ scattering phase shifts for $J = \frac{1}{2}$ and $J = \frac{3}{2}$ as a function of the kaon laboratory momentum. The solid (dash–dotted, dotted) line shows the result of model I A (model II B, model I). Empirical data is taken from [13] (empty circles), [14] (full circles) and [15] (empty squares).

FIG. 8. The same as in Fig. 7 for $KN$ ($I=0,1$) elastic cross sections. Experimental data is taken from [16] (full circles), [17] (empty circles), [14] (empty squares).
FIG. 9. The same as in Fig. 7 for $K^+p$ differential cross sections. Experimental data is taken from [18].

FIG. 10. The same as in Fig. 7 for $K^+n$ differential cross sections. Experimental data is taken from [19].

FIG. 11. The same as in Fig. 7 for $K^+p$ and $K^+n$ polarizations. Experimental data is taken from [20] ($K^+p$) and [21] ($K^+n$).
TABLE I. Vertex parameters used in model I

| Process          | Exch. part. | \( M_r \) or \( m_r \) \(^{a)} \) | \( g_1 g_2/4\pi \) \(^{b)} \) | \( \Lambda_1 \) \(^{c)} \) | \( \Lambda_2 \) \(^{c)} \) |
|------------------|-------------|-------------------------------|-----------------------------|---------------------|---------------------|
| \( KN \rightarrow KN \) | \( \sigma \)  | 600 \( [MeV] \) | 1.300 \( [f_1/g_1] \) | 1.7 \( [GeV] \) | 1.5 \( [GeV] \) |
|                  | \( \sigma_{rep} \)  | 1200 \( [MeV] \) | \(-40 \) \( [f_1/g_1] \) | 1.5 \( [GeV] \) | 1.5 \( [GeV] \) |
|                  | \( \omega \)  | 782.6 \( [MeV] \) | 2.318 \( [0] \) \( [f_1/g_1] \) | 1.5 \( [GeV] \) | 1.5 \( [GeV] \) |
|                  | \( \rho \)  | 769 \( [MeV] \) | 0.773 \( [6.1] \) \( [f_1/g_1] \) | 1.4 \( [GeV] \) | 1.6 \( [GeV] \) |
|                  | \( \Lambda \)  | 1116 \( [MeV] \) | 0.905 \( [GeV] \) | 4.1 \( [GeV] \) | 4.1 \( [GeV] \) |
|                  | \( \Sigma \)  | 1193 \( [MeV] \) | 0.031 \( [GeV] \) | 4.1 \( [GeV] \) | 4.1 \( [GeV] \) |
|                  | \( Y^* \)  | 1385 \( [MeV] \) | 0.037 \( [GeV] \) | 1.8 \( [GeV] \) | 1.8 \( [GeV] \) |
| \( KN \rightarrow K^*N \) | \( \pi \)  | 138.03 \( [MeV] \) | 3.197 \( [f_1/g_1] \) | 1.3 \( [GeV] \) | 0.8 \( [GeV] \) |
|                  | \( \rho \)  | 769 \( [MeV] \) | 0.773 \( [6.1] \) \( [f_1/g_1] \) | 1.4 \( [GeV] \) | 1.0 \( [GeV] \) |
| \( KN \rightarrow K^*\Delta \) | \( \pi \)  | 138.03 \( [MeV] \) | 0.506 \( [f_1/g_1] \) | 1.2 \( [GeV] \) | 0.8 \( [GeV] \) |
|                  | \( \rho \)  | 769 \( [MeV] \) | 4.839 \( [f_1/g_1] \) | 1.3 \( [GeV] \) | 1.0 \( [GeV] \) |
| \( KN \rightarrow K\Delta \) | \( \rho \)  | 769 \( [MeV] \) | 4.839 \( [f_1/g_1] \) | 1.3 \( [GeV] \) | 1.6 \( [GeV] \) |

\(^{a)}\) Mass of exchanged particle.

\(^{b)}\) Product of coupling constants [ratio of tensor to vector coupling].

\(^{c)}\) Cutoff mass.
TABLE II. Vertex parameters used in the models (II A,B) with correlated $2\pi$-exchange$^{d,e}$

| Process          | Exch. part. | $M_r$ or $m_r$ $^a$ | $g_1 g_2 / 4\pi$ $^b$ | $\Lambda_1$ $^d$ | $\Lambda_2$ $^c$ |
|------------------|-------------|---------------------|------------------------|------------------|------------------|
| $KN \rightarrow KN$ | $\sigma_{rep}$ | 1600 (1200)          | -40 (-45)              | 2.1 (1.5)        | 2.1 (1.5)        |
|                  | $\omega$    | 782.6                | 2.318[0]               | 1.5              | 1.5              |
|                  | $\Lambda$   | 1116                 | 0.905                  | 3.5 (4.1)        | 5.0 (4.1)        |
|                  | $\Sigma$    | 1193                 | 0.031                  | 5.0 (4.1)        | 5.0 (4.1)        |
|                  | $Y^*$        | 1385                 | 0.037                  | 2.4 (1.8)        | 2.4 (1.8)        |
| $KN \rightarrow K^* N$ | $\pi$       | 138.03               | 3.197                  | 1.3              | 1.3 (0.8)        |
|                  | $\rho$      | 769                  | 0.773[6.1]             | 1.4              | 1.1 (1.0)        |
| $KN \rightarrow K^* \Delta$ | $\pi$       | 138.03               | 0.506                  | 1.2              | 1.3 (0.8)        |
|                  | $\rho$      | 769                  | 4.839                  | 1.8 (1.6)        | 1.1 (1.0)        |
| $KN \rightarrow K\Delta$ | $\rho$     | 769                  | 4.839                  | 1.8 (1.6)        | 1.5 (1.4)        |

$^a$ Mass of exchanged particle.

$^b$ Product of coupling constants [ratio of tensor to vector coupling].

$^c$ Cutoff mass.

$^d$ We used for correlated $2\pi$-exchange a cutoff $\Lambda_{\sigma,\rho}$ (Eq.(15)) of 1.85 GeV for method A resp. 2.4 GeV for method B.

$^e$ In case the parameters differ for model II A, B, the numbers for B are given in parentheses.
|                | $a_{\frac{1}{2}S}[fm]$ | $a_{\frac{3}{2}S}[fm]$ | $r_{\frac{1}{2}S}[fm]$ |
|----------------|--------------------------|--------------------------|--------------------------|
| experiment$^a$ | 0.03 ± 0.15              | −0.30 ± 0.03             | 0.43 ± 0.22              |
| model I        | 0.057                    | −0.316                   | 0.373                    |
| model II A     | 0.038                    | −0.304                   | 0.261                    |
| model II B     | −0.080                   | −0.333                   | 0.130                    |

$^a$ Empirical data is taken from Refs. [22,23].
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