Reliability Evaluation Method of Torpedo in Service Based on Conditional Reliability

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Abstract. In order to effectively evaluate the reliability index of in-service torpedo, combined with the actual use of torpedo, the reliability evaluation method of in-service torpedo based on conditional reliability is proposed. On the basis of determining the failure distribution under the single state of components, the variation of the reliability distribution of the components after several states are studied. Based on the fault tree structure function, the reliability simulation model of torpedo is established, and the Monte Carlo method is used to establish the simulation solution process, so as to obtain the reliability parameters of torpedo. This method is simple and easy to implement, and has important guiding significance for evaluating the reliability of in-service torpedoes.

1. Introduction
Modern torpedoes are complex underwater precision guided weapons that make up a wide variety of torpedo components, including electronics, machinery, electromechanical, rubber, etc[1]. The torpedo is divided into three typical states: storage, loading and actual operation from the factory to the end of life. During the service, the torpedo will undergo multiple storage, loading and actual navigation processes[2]. For components, the environment is different under different state conditions, the stresses on the components are different, the failure rules of the components are different, and the state experienced by the components also affects the reliability variation law. At present, when reliability analysis of torpedoes or components is carried out, the reliability law of torpedoes under certain conditions is simply considered, and the various states experienced by torpedoes are not considered. It is a simplification of the torpedo life profile and does not conform to the actual process of torpedo use[3,4]. Therefore, it is necessary to study the law of reliability change after torpedoes undergo various states under actual use conditions.

In this paper, based on the idea of the component to the full mine, based on the determination of the failure state of the single state of the component, the reliability variation law of the component after several states is studied. By establishing a full lightning system fault tree model, the system can accurately judge whether the system is faulty according to the logical relationship of the fault tree, and Monte Carlo simulation method is used to simulate and analyze the reliability variation law of the whole mine. This method provides guidance for analyzing the law of reliability change of torpedoes in actual use, and also provides improvement measures for the weak links of torpedo systems.
2. Reliability analysis of components in a single state

The reliability evaluation method of components in a single state needs to be determined according to the type of components. For mechanical components, the stress-strength model method is generally used for evaluation. For electronic components, generally refer to GJB/Z 299C, GJB/Z 108A, MIL-HDBK-217F and other electronic component reliability manuals or use exponential distribution to process data. For components with more experimental data, a conservative binomial distribution reliability assessment method can be used. The specific assessment method is as follows:

(1) Stress-strength model

The test result of the component in a certain state is \( n \) strength test values \( x_1, x_2, \ldots, x_n \); the component stress is a certain amount \( L \).

The mean and variance of the intensity samples are:

\[
\begin{align*}
\bar{x} &= \frac{1}{n} \sum_{i=1}^{n} x_i \\
S_x^2 &= \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2
\end{align*}
\]

Then the reliability point of the component is estimated as

\[
R = \Phi \left( \frac{\bar{x} - L}{S_x} \right)
\]

The reliability lower limit calculation process is as follows:

First calculate the tolerance factor \( K \)

\[
K = \frac{\bar{x} - L}{S_x}
\]

Check the \( K \) coefficient table and solve \( R_L \).

According to the given confidence \( \gamma \), the cumulative test number \( n \) and the tolerance coefficient \( K \) of each test piece of strength, check the GB4885 number table and interpolate to obtain the lower limit of reliability \( R_L \).

(2) Exponential distribution reliability assessment method

Given a confidence \( \gamma \), the lower confidence limit \( R_L \) of the reliability is

\[
R_L = e^{-\frac{(\bar{x}^2 - L)^2}{2T}}
\]

Where \( T \) is the cumulative total test time, \( r \) is the number of failures, and the \( \gamma \) quantile of the \( \chi^2 \) distribution of \( \chi^2 (2r + 2) \).

(3) Binomial distribution reliability assessment method

Given a confidence \( \gamma \), the lower confidence limit \( R_L \) of the reliability is

\[
\sum_{x=0}^{\left(\begin{array}{c} n \\ x \end{array}\right)} \left(\begin{array}{c} n \\ x \end{array}\right) R_L^{x-1} (1 - R_L)^x = 1 - \gamma
\]

Where \( \left(\begin{array}{c} n \\ x \end{array}\right) = \frac{n!}{x!(n-x)!} \).

According to the relationship between the \( \beta \) distribution and the \( F \) distribution quantile, there is

\[
R_L = \left(1 + \frac{f + 1}{s} \frac{F_{2r,2n-r}}{F_{2f+2,2n-2r}}\right)
\]

3. Reliability analysis of components undergoing multiple states

3.1. Conditional reliability
3.1.1. Definition of conditional reliability. When a product unit with a failure time of \( T \) starts to work at \( t = 0 \) and has been working normally until time \( t \), then the reliability of the product unit for normal operation \( x \) time is [5]

\[
R(x|t) = \frac{\Pr(T > x + t | T > t)}{\Pr(T > t)} = \frac{R(x + t)}{R(t)}
\]

(7)

\( R(x|t) \) is called the conditional reliability function of the product unit at time \( t \).

For torpedo equipment, the reliability variation law of a certain type of component in the storage state is \( R_1(t) \), the reliability variation law in the loading state is \( R_2(t) \), and the reliability variation law in the actual navigation state is \( R_3(t) \).

According to the definition of conditional reliability, assuming that the storage time \( t_1 \) is transferred to the loading state after technical preparation, the reliability variation of the component in the loading state is

\[
R'_2(t) = \frac{R_2(t_{20} + t)}{R_2(t_{20})}
\]

(8)

Where \( R_2(t_{20}) = R_1(t_1) \).

It is assumed that the \( t_1 \) time is stored first and the \( t_2 \) time is loaded and then transferred to the actual navigation state, and the reliability change law in the actual navigation state becomes

\[
R'_3(t) = \frac{R_3(t_{30} + t)}{R_3(t_{30})}
\]

(9)

Where \( R_3(t_{30}) = R'_2(t_2) \).

By analogy, the reliability variation of components in any state at any time can be calculated.

3.1.2. The variation of component reliability is an exponential distribution. Let the reliability variation rule of a certain component be an exponential distribution, satisfy \( R_1(t) = e^{-\lambda_1 t} \) in the storage state, \( R_2(t) = e^{-\lambda_2 t} \) in the loading state, and \( R_3(t) = e^{-\lambda_3 t} \) in the actual navigation state.

After transferring to the loading state after the storage time \( t_1 \), the reliability variation law in the loading state is

\[
R'_2(t) = \frac{R_2(t_{20} + t)}{R_2(t_{20})} = e^{-\lambda_1 (t_{20} + t)} e^{\lambda_2 t_{20}} = e^{-\lambda_1 t}
\]

(10)

It is assumed that the \( t_1 \) time is stored first and the \( t_2 \) time is loaded and then transferred to the actual navigation state, and the reliability change law in the actual navigation state becomes

\[
R'_3(t) = \frac{R_3(t_{30} + t)}{R_3(t_{30})} = e^{-\lambda_2 (t_{30} + t)} e^{\lambda_3 t_{30}} = e^{-\lambda_2 t}
\]

(11)

It can be seen that the reliability variation law of the components whose reliability changes obey the exponential distribution under certain conditions is independent of the previous experience.

3.2. Parameter Estimation

For components whose failure distribution function is Weibull distribution and lognormal distribution, it is calculated according to equation (7), and then the distribution is parameterized by least squares method.

3.2.1. Parameter estimation of Weibull distribution. The distribution function of the Weibull distribution is

\[
F(t) = 1 - e^{-t^{m}}, m > 0, t_0 > 0, t \geq 0
\]

Therefore, \( \frac{1}{1-F(t)} = e^{t^{m}}, \) take two logarithms on both sides,
\[
\ln(\ln \frac{1}{1-F(t)}) = m \ln t - \ln t_0
\]  
(12)

Let \( y = \ln(\ln \frac{1}{1-F(t)}) \), \( x = \ln t, b_0 = -\ln t_0 \), then get the regression equation \( y = b_0 + b_1 x \).

According to the test data \( \{\ln t_i, \ln(\ln \frac{1}{1-F(t_i)})\}; i = 1, 2, \ldots, n \} \), the regression coefficients \( b_0, b_1 \) and the correlation coefficient \( r \) are obtained by the least squares method, wherein the value of \( F(t_i) \) can be calculated using an empirical distribution. The resulting parameter is estimated as
\[
\begin{align*}
\hat{m} &= b_1 \\
\hat{t}_0 &= e^{-b_0}
\end{align*}
\]  
(13)

Thus can find \( \hat{\eta} = \hat{t}_0^{\frac{1}{m}} \).

3.2.2 Parameter estimation of lognormal distribution. The distribution function of the lognormal distribution can be written as
\[
F(t) = \int_{-\infty}^{\frac{\ln t - \mu}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx = \Phi(\frac{\ln t - \mu}{\sigma}) = \Phi(Z)
\]
Where \( \Phi(Z) \) is the distribution function of the standard normal distribution,
\[
Z = \frac{\ln t - \mu}{\sigma}, \ln t = \sigma Z + \mu
\]  
(14)

Let \( x = \ln t \), then \( x = \sigma Z + \mu \) is equivalent to \( y = b_0 + b_1 x \). Therefore, for a set of data of \( Z \) and \( x \), the regression coefficient \( b_0, b_1 \) and the correlation coefficient \( r \) can also be calculated by the least squares method, and the lognormal distribution parameter is estimated as
\[
\begin{align*}
\hat{\mu} &= b_1 \\
\hat{\sigma} &= b_0
\end{align*}
\]  
(15)

After obtaining the component failure distribution function after going through multiple states, the fault tree-Monte Carlo method is used to analyze the reliability of the torpedo system.

4. Reliability Analysis of Torpedo System Based on Fault Tree-Monte Carlo
Monte Carlo simulation technology is based on the theory of large numbers and uses simulated sampling to reflect system reliability behavior[6,7]. The reliability parameters of the system are obtained by calculating the statistical characteristics of the sampled samples of the system. It is insensitive to the dimension of the problem, has strong adaptability and ability to solve problems, and Monte Carlo simulation analysis has no limit on the failure distribution of the bottom event[8,9].

4.1. Reliability Simulation Model Based on Fault Tree
The fault tree consists of a bottom event, an intermediate event, a top event, and a logic gate. Each event has two states, which can be represented by 1 and 0[10]. In the case of introducing a time variable, the bottom event in the fault tree can be expressed as
\[
x(t) = \begin{cases} 
1 & \text{event } x \text{ occurrence} \\
0 & \text{event } x \text{ not occurrence}
\end{cases}
\]  
(16)

The logic gates commonly used in fault trees are "and gates" and "or gates". Their logical relationship is

(1) AND gate: When \( x_1(t) \cdot x_2(t) = 1 \), \( x_1(t) = 1 \); when \( x_1(t) \cdot x_2(t) = 0 \), \( x_1(t) = 0 \), the structure function \( \Phi_{\text{AND}}[x(t)] \) of the AND gate is expressed as: \( \Phi_{\text{AND}}[x(t)] = \prod_{i=1}^{n} x_i(t) \)
(2) OR gate: When \( (1 - x_i(t)) \times (1 - x_j(t)) = 1 \), \( x_i(t) = 0 \); when \( (1 - x_i(t)) \times (1 - x_j(t)) = 0 \), \( x_i(t) = 1 \), the structure function \( \Phi_{\text{OR}}[x(t)] \) of the OR gate is expressed as: 
\[
\Phi_{\text{OR}}[x(t)] = 1 - \prod_{i=1}^{n}(1-x_i(t))
\]

The full lightning system has \( n \) basic components, and each basic component failure distribution function \( F_i(t) \) is known. The fault tree represents the logical relationship between the bottom events. Through the structural functions of the above gates, the state event variable \( \Phi_{i}[x(t)] \) of the top event can be obtained from the logical relationship of the bottom event state variables through the gates\([11]\).

\[
\Phi_i[x(t)] = \Phi_i(x_i(t), x_j(t), \ldots, x_n(t))
\]

Where \( x_i(t) \) is the state variable of the \( i \)th bottom event \( x_i \) of the fault tree, and \( n \) is the number of bottom events.

### 4.2. Reliability simulation running process

1. Sample each basic event lifetime using the Monte Carlo method to obtain a sample of each basic event expiration time. The \( i \)th basic event failure time sample value is
\[
t_i = F_i^{-1}(\eta_i)
\]

In the \( j \)th simulation process, the \( i \)th basic event failure time sample is expressed as \( t_i \), then there is
\[
t_i = F_i^{-1}(\eta_i)
\]

Where \( \eta_i \) represents the random number randomly sampled in the \( j \)th simulation of the \( i \)th basic event. It can be obtained from the above formula that the state variable of the \( i \)th basic event at time \( t \) can be expressed as
\[
x_i(t) = \begin{cases} 
1 & t_i \geq t_k_i \\
0 & t_i < t_k_i 
\end{cases}
\]

2. Sweep the fault tree to find the top event failure time \( t_k \). In the \( j \)th simulation run, the sampling produces \( n \) basic event expiration times \( t_{1j}, t_{2j}, \ldots, t_{nj} \). The \( n \) basic event expiration times are sorted from large to small (represented by \( TTF_i \), and the corresponding basic events are also sorted. The basic event corresponding to the smallest failure time in the sorting is set to the invalid state, and the remaining basic events are invalid at this moment. Then check these basic events in turn, and judge whether the system is invalid through the fault tree structure function until the system fails.

3. Distribution statistics of failure numbers are performed by interval statistics method. Let the cumulative failure time be \( T_{\text{max}} \), divide it into \( m \) intervals, each time interval is \( \Delta t = T_{\text{max}} / m \), and count the number of system failures in the \((tr-1, tr]\) interval, denoted by \( \Delta n \). When \( t \leq tr \), the number of failures of the system is

\[
m(t) = \sum_{i=1}^{n} \Delta n = \sum_{j=1}^{N} \varphi_j(t) \quad t \leq tr
\]

### 4.3. Reliability simulation result statistics

Let system lifetime \( \xi \) be a random variable, then the statistical calculation formula of each estimated value of the system is as follows:

1. System cumulative failure probability \( F_s(t) \):
\[
F_s(t) = P(\xi \leq tr) \approx \frac{1}{N} \sum_{j=1}^{N} \varphi_j(t) = \frac{m(tr)}{N}
\]
(2) System reliability $R_s(t_r)$:

$$R_s(t_r) = 1 - F_s(t_r)$$  \hspace{1cm} (23)

(3) Mean time between failures $MTBF$:

$$MTBF = E(\xi) \approx \sum_{n=0}^{\infty} \left[ t_n \frac{1}{N} \sum_{j=1}^{N} \phi_j(t) \right] \quad (tr - 1 < t < tr)$$  \hspace{1cm} (24)

Figure 1. Simulation flow chart

5. Case Analysis

Assume that a torpedo has three sub-systems, each sub-system consists of two key components. Using the reliability analysis method of the components in a single state, the failure distribution functions and parameters of the six components in the storage and loading states can be obtained, as shown in table 1.

Table 1. Failure distribution function and parameters of components in storage and loading states.

| Component | Failure distribution type | Storage status parameter | Loading status parameter |
|-----------|---------------------------|--------------------------|-------------------------|
| X1        | Exponential distribution  | $\lambda = 0.0113$      | $\lambda = 0.0121$     |
| X2        | Exponential distribution  | $\lambda = 0.0132$      | $\lambda = 0.0143$     |
| X3        | Weibull distribution      | $\eta = 41.0451$        | $\eta = 41.0451$       |
| X4        | Weibull distribution      | $\beta = 2.1837$        | $\beta = 2.0120$       |
|           |                           | $\eta = 42.0352$        | $\eta = 42.0456$       |
|           |                           | $\beta = 2.2830$        | $\beta = 2.1857$       |
5.1. Analysis of component reliability changes after undergoing storage

After the torpedo was shipped from the factory, it was stored in the warehouse for 4 months. After the technical preparation, the inspection was completed and transferred to the loading state to study the change of the reliability of each component in the loading state.

Using the relevant contents of Sections 3.1 and 3.2 for calculation and parameter estimation, the parameters of the component failure distribution function under the conditions of post-storage loading are shown in Table 2.

| Component | Failure distribution type | Loading status parameter |
|-----------|---------------------------|--------------------------|
| X1        | Exponential distribution  | $\lambda = 0.0121$      |
| X2        | Exponential distribution  | $\lambda = 0.0143$      |
| X3        | Weibull distribution      | $\eta = 38.7327, \beta = 1.7385$ |
| X4        | Weibull distribution      | $\eta = 39.4728, \beta = 1.8615$ |
| X5        | Lognormal distribution    | $\mu = 3.7621, \sigma = 1.4377$ |
| X6        | Lognormal distribution    | $\mu = 3.6621, \sigma = 1.4830$ |

Since the failure distribution of the component X1 and the component X2 is an exponential distribution, the reliability function in the loading state is unchanged, and the reliability changes of other components are as shown in the figure.

Figure 2. Reliability curve of component X3
In figure 2 - figure 5, curve R1 represents the reliability distribution of the component in the storage state, curve R2 represents the reliability distribution of the component in the loaded state, and curve R22 is curve after fitting R21, which represents the reliability of the component's reloading after 4 months of storage. As can be seen from the figure, the loading reliability of the component after being subjected to the storage state is lower than that of the unstorage state.

5.2. Reliability Analysis of Torpedo System Based on Fault Tree-Monte Carlo Method

In the loaded state, if any component of the torpedo fails, the torpedo system is considered to be malfunctioning. Therefore, the logical relationship between the two components under any subsystem
is OR gate, and the logical relationship between subsystems is OR gate. Establish a fault tree for the torpedo system failure as the top event, as shown in figure 6.

![Fault Tree of Torpedo System](image)

**Figure 6. Fault Tree of Torpedo System**

Monte Carlo simulation can be used to obtain the variation curve of the reliability and failure probability of the torpedo system. As shown in figure 7, the MTBF of torpedo system is calculated to be 14.6 months.

![Torpedo system reliability curve](image)

**Figure 7. Torpedo system reliability curve**

### 6. Conclusion

Combined with the actual use of torpedoes in service, the reliability variation of torpedo components after multiple states is studied. Based on fault tree analysis, the reliability simulation model of torpedo system is established. The Monte Carlo simulation analysis method was used to analyze the reliability of the model, and the reliability parameters of the torpedo system were obtained. This method can objectively analyze the reliability of the system and provide strong support for system reliability assessment and reliability design.

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