Modulation behaviours and interchangeability of modulators for electrical machines

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Abstract
The authors aimed to investigate the modulation behaviours and the interchangeability of three kinds of modulators in electrical machines, which are short circuited coil, salient pole reluctance and flux barriers, to understand the evolution of the modulator and reveal their individual action mechanism based on the general airgap field modulation theory. The modulation behaviours are systematically analysed and compared, including the modulation principles of the improved short circuited coil, the relationship between salient pole reluctance and pole arc together with slot opening depth etc., based on which the key rule of interchangeable modulators is summarised. Although different modulators possess different magnetic field modulation principles and magnetic conversion capabilities, they share the similar asynchronous behaviours and modulated harmonics distribution. In addition, the detailed topological analysis of traditional brushless doubly fed machine with interchangeable and combined modulators is presented to show the effectiveness of the theoretical investigation. Electromagnetic performances comparison, such as airgap flux density distribution, cross coupling ability, general torque performances and inductance characteristics, of a brushless doubly fed machine with different modulators are provided to reveal the relation between the machine performances and magnetic field conversion capability. Theoretical predictions are verified by 2-D finite element analysis and experimental measurements.

1 INTRODUCTION

With the unprecedented surge and need for electrification of the world, a vast variety of electromagnetic machines have been resurging or emerging to meet various performance requirements, such as the synchronous reluctance machine [1], the permanent magnet brushless machines [2], the vernier machine [3], the switched reluctance machine (SRM) [4], the brushless doubly-fed reluctance/induction machine (BDFR/IM) [5], the stator-PM machines with doubly salient structure [6], the vernier permanent magnet (VPM) machine [7], the magnetic gear (MG) [8] and magnetically geared machine (MGM) [9], the dual-mechanical-port (DMP) machine [10] and so on. Meanwhile, various analytical theories such as the rotating magnetic field theory [11], the cross-field theory [12], the two-reaction theory [13], the unified theory of torque production [14] and the winding function theory [15] are available to analyse their electromagnetic performances. These studies make outstanding contributions to machine development, however, they show weakness in drawing insightful conclusions across the airgap and effectively analysing a large proportion of newly emerged machines with multi-harmonic and multi-port features. To cater these situations, a general airgap field modulation theory (GAFMT) was proposed [16], in which a basic unit machine is normalised into a cascade of three elements, namely the excitation source, modulator and filter (armature winding). The GAFMT offers a brand new and different perspective for machine analysis. The electromagnetic performances can be generalised and theorised, so as to establish a general theory for electrical machines.

Among the three elements in a unit machine, the modulator is a new concept and plays the key role in the GAFMT. There are basically three types of modulators, namely salient pole reluctance (SPR), short circuited coil (SCC) and flux barriers (FBS), determining different characters of modulation principles. The modulation principles are generally classified as synchronous and asynchronous modulation behaviours,
which depend on the relative static or motion between the modulator and the source magnetising magnetomotive force (MMF). Actually, the airgap field modulation behaviours are valid for almost all electrical machines. Taking the flux-switching permanent magnet (FSPM) machine as an example, the SPR stator core consists of modular U-shaped laminated segments between which the circumferentially magnetised PMs are placed. The \( p_f \)-pole-pair source magnetising MMF is first synchronously modulated by the stator poles to produce \((2k - 1)p_f \) principal MMF components, and then is asynchronously modulated by salient rotor poles to produce three kinds of pole pair magnetic components corresponding to the source ones. In addition, the asynchronous modulation behavior in the FSPM machine is made full utilization to make change on the principal pole pair number (PPPN) of the source magnetizing MMF. The similar modulation principles occur in the MGM, the VPM machine, the BDFR/IM and so on.

In [16], the basic framework of the GAFMT has been established and using it as a base, the airgap field modulation principles of the three modulators are respectively investigated in [17–19]. The relationship between the synchronous (asynchronous) modulation behaviour and the torque component is complicated, whose key characteristics are defined and exemplified in [20]. However, certain ideal assumptions are made for simplification to make the theoretical analysis easy to obtain the insightful conclusions clearly. For example, the modulation operator and modulation principle of improved short circuited coils are not considered in [17], and the relationship between salient pole reluctance and pole arc together with slot opening depth is not discussed in [18]. In addition, the three modulators share similar modulation behaviours, that is, asynchronous modulation, but yield different magnetic modulation capabilities. And the interchanged or combined modulators will produce different modulation behaviours, preforming different degrees of influence on the modulation effects. The comprehensive comparisons on their magnetic field conversion capability and detailed description of whether or not different modulators can be interchangeable have not been conducted in [16–20] either.

The authors aim to understand the evolution of the modulators and reveal their individual action mechanism. The nature of modulation behaviours and the interconvertability of three kinds of modulators are investigated in a detailed manner and concluded from the perspective of the airgap field modulation mechanism based on the GAFMT. The modulation principles of the salient pole reluctance, flux barriers and improved short circuited coil, and the relation between salient pole reluctance and pole arc/slot opening depth etc. are introduced in detail, based on which the key laws of interchangeable and combined modulators are summarised. Typical electromagnetic features of a BDFM with different modulators are compared to reveal the relation between the performances and magnetic field cross coupling ability systematically. The analytical results are verified by finite element analysis (FEA) together with experimental measurements.

2 | THE MODULATION BEHAVIOURS OF MODULATORS

2.1 | Short circuited coil

The SCC is a passive device that relies on slip frequency between the source rotating MMF and the modulator to induce coil current [21]. It will produce symmetrical multi-phase alternating current and then establish the corresponding additional MMF. The SCC modulator generally contains the equally spaced squirrel cage as shown in Figure 1 and improved SCC like the nest-loop, series, isometric ring etc. as illustrated in Figures 2 and 3. The equally spaced squirrel cage is equipped in the induction machine (SCIM), while the improved SCC is utilised to achieve better magnetic field conversion capability for the general field modulated machines like the BDFIM. The modulation principle of the squirrel cage is investigated in [17], while the modulation operator and modulation principle of improved SCC are discussed as follows.

2.1.1 | Squirrel cage

The modulation behaviour of equally spaced squirrel cage is illustrated in Figure 1. As introduced in [17], the airgap field harmonics with \( p_f \) will keep the amplitude almost unchanged \((C_p \approx 1.0)\) under ideal assumption, while the harmonics in

![Figure 1](image1.png)

**Figure 1** The modulation behaviour of equally spaced squirrel cage

![Figure 2](image2.png)

**Figure 2** Improved SCC: Common nest–loop coils. (a) with common end ring, (b) isolated nest–loop coils and (c) with common end ring and loop

![Figure 3](image3.png)

**Figure 3** Other improved SCC. (a) series, (b) series and parallel and (c) isometric ring
summation and differential modulation depend on \( p_p, N_{SC} \) and \( \gamma \). The squirrel cage does not change the PPPN and is nothing but special cases of the improved SCC whose asynchronous modulation ability is advanced. Taking SCIM as an example, \( \gamma \) is equal to 1 and \( N_{SC} \) is far greater than \( p_p \) so that the mutual conversion factors can be ignored for simplification. That is the \( p_p \)-pole-pair harmonic plays a major role in the SCIM and it features regular airgap flux density distribution.

2.1.2 | Nest-loop coils

For the nest-loop coils, the common loops have large coil span and play key roles in magnetic field conversion [22]. The common end ring has no positive effect on modulation ability but makes the nest-loop coil structure simple to process, and the topology as shown in Figure 2(a) is the common choice for the BDFIM application [5,23]. It is assumed that the source magnetising MMF is a unit cosine function and all harmonic items are neglected for simplification. The modulated magnetising MMF turns out the summation of source MMF and all the extra components induced by the nest-loop coils. It can be characterised by introducing modulator \( M(N_{SC}, \gamma) \) as:

\[
M(N_{SC}, \gamma)[f(\phi, t)] = f(\phi, t) + \sum_{x=1}^{N_{SC}} \sum_{j=1}^{S} W_{jx} f_{jx}, \quad \phi \in [0, 2\pi]
\]

where \( N_{SC} \) is the number of SCC group, \( \gamma \) is the coil span factor of a specific loop. \( \phi \) is the mechanical angle along the circumference in modulator-fixed reference frame. \( i_{jx} \) and \( W_{jx} \) are the current flowing and winding function of \( x \)-th loop \((x=1, 2, \ldots, S)\) in \( j \)-th nest \((j=1, 2, \ldots, N_{SC})\), respectively. \( S \) is the number of loops within a single nest. Then the modulation principle can be rewritten in Fourier series as:

\[
M(N_{SC}, \gamma)[f(\phi, t)] = C_p \cos \left( p_p \phi - \omega_s t - \phi \right)
\]

\[
+ \sum_{k=1}^{\infty} C_{sum} \cos \left( k \phi + \omega_s t + \left( k + p_p \right) \frac{\pi}{N_{SC}} - \phi \right)
\]

\[
+ \sum_{k=1}^{\infty} C_{dif} \cos \left( k \phi - \omega_s t + \left( k - p_p \right) \frac{\pi}{N_{SC}} - \phi \right)
\]

where \( \omega_s \) is the slip frequency of rotor coil. The subscripts \( \text{sum} \) and \( \text{dif} \) represent summation and differential modulation process, respectively. \( \phi \) is the impedance angle.

\[
\phi = \arctan(\omega_s L/R_s), \quad L_x = 2\pi \mu_0 \sigma f_s / gN_{SC} + L_a
\]

where \( R_s, L_s \) and \( L_a \) are the resistance, inductance and leakage inductance of the \( x \)-th loop in \( j \)-th nest, respectively.

The \( C_p, C_{sum} \) and \( C_{dif} \) are magnetic field conversion factors and represent interactions between \( p_p \)-pole-pair, \((N_{SC} - p_p)\)-pole-pair and \((N_{SC} + p_p)\)-pole-pair harmonics, respectively, where the variable \( l \) is an arbitrary positive integer. The magnetic field conversion factors are the reflection of the modulation effect or ability on the source magnetising MMF distribution, which modifies the relative amplitude of harmonic components or the positions of spectrum lines. There will be three different components with \( p_p, (N_{SC} - p_p) \) and \((N_{SC} + p_p)\) pole pairs in the modulated MMF corresponding to the source \( p_p \)-pole-pair one. The principal and differential modulated harmonics are both forward-rotating, while the summation modulated one is reverse-rotating when observing from the modulator-fixed reference frame \( \phi \). In addition, the \( p_p \)-pole-pair modulated harmonic, that is the principal magnetic field conversion factor \( C_p \) is provided with high amplitude, and the summation and differential modulated harmonics depend on rotor structure \((N_{SC}, \gamma, S, R_s, L_x \text{ and } L_a)\) and stator winding \((p_p)\).

\[
C_{sum/dif, Nest} = 1 - \sum_{x=1}^{S} \frac{2\pi N_{SC}}{\sqrt{R_s^2 + (\omega_s L_x)^2}} \frac{\mu_0 R_s f_s}{g} \left( \frac{\gamma_x}{N_{SC}} \right)^2
\]

\[
\times \left[ \frac{\sin \left( \frac{r_s p_p \pi}{N_{SC}} \right)}{\frac{r_s p_p \pi}{N_{SC}}} \right]^2
\]

\[
C_{sum/dif, Nest} = \sum_{x=1}^{S} \frac{2\pi N_{SC}}{\sqrt{R_s^2 + (\omega_s L_x)^2}} \frac{\mu_0 R_s f_s}{g} \left( \frac{\gamma_x}{N_{SC}} \right)^2
\]

\[
\times \frac{\sin \left( \frac{r_s p_p \pi}{N_{SC}} \right)}{\frac{r_s p_p \pi}{N_{SC}}} \left[ \frac{r_s k \pi}{N_{SC}} \right]^{\text{sum}} - \frac{\sin \left( \frac{r_s p_p \pi}{N_{SC}} \right)}{\frac{r_s p_p \pi}{N_{SC}}} \left[ \frac{r_s k \pi}{N_{SC}} \right]^{\text{dif}}
\]

2.1.3 | Series ring

The modulation function of the series ring can be given as:

\[
M(N_{SC}, \gamma)[f(\phi, t)] = f(\phi, t) + \sum_{j=1}^{N_{SC}} W_j f_j, \quad \phi \in [0, 2\pi]
\]

where \( W_j \) is the winding function of series ring coils.

The modulated MMF of the series ring can be obtained by replacing \( \gamma_x \) in (5) with \( \gamma_x \). The modulation principle can be rewritten in Fourier series similar as (2) but with different magnetic field conversion factors and the distribution factor of \( m \)-order harmonic \( k_{dm} \). The induced current amplitude difference of each loop in the same nest is small, achieving a balanced rotor current distribution, sinusoidal extra MMF along the airgap and strong effect on reduction of unproductive harmonics. In addition, compared with nest-loop, it improves the induced current of the non-outermost loop to
achieve high field conversion efficiency, but will increase the coil equivalent length/reluctance. The modulator and related conversion factors of the isometric ring can be obtained in a similar way;

For the improved SCC with $N_{SC}$ nests in summation modulation process, the typical conversion factors with $N_{SC} \leq 6$ (only typical factors are listed due to the strictly limited space) are calculated based on reference [17] and (4)–(9) and exhibited in Table 1, from which few conclusions can be drawn as:

$$C_{p_{\text{Series}}} = 1 - 2 \pi N_{SC} k_{dn} \frac{a_{S}}{R^{2} + (a_{S}L)^{2}} \frac{\mu_{0} r_{e} d_{ek}}{g} \left( \frac{\gamma_{c}}{N_{SC}} \right)^{2}$$

and

$$C_{\text{sum/diff}_{\text{Series}}} = 2 \pi N_{SC} k_{dn} \frac{a_{S}}{R^{2} + (a_{S}L)^{2}} \frac{\mu_{0} r_{e} d_{ek}}{g} \left( \frac{\gamma_{c}}{N_{SC}} \right)^{2}$$

$$\times \left[ \frac{\sin(\gamma_{c} p_{f} \pi / N_{SC})}{\gamma_{c} p_{f} \pi / N_{SC}} \right]^{2} \sin(\gamma_{c} p_{f} \pi / N_{SC}) \sin(\gamma_{c} k \pi / N_{SC}) \gamma_{c} k \pi / N_{SC} = l N_{SC} \mp p_{f} \sum_{k=1}^{N_{SC}} k_{dn} = \cos(qn \alpha / 2) / q \cos(n \alpha / 2)$$

- The equally spaced squirrel cage earns small mutual magnetic coupling factors and low conversion efficiency;
- The magnetic conversion ability of improved SCC is closely related to the pole pair combination. For the same number of $N_{SC}$, the close ratio pole pair combination produces large mutual conversion factors, but no more than 0.8;
- The effective field harmonic components appear in pairs, and the summation modulation factor is higher and larger than the differential modulation factor, indicating that SCC is not suitable for differential modulation situations;
- The magnetic field conversion factors/abilities are the same when the $N_{SC}$ and $p_{f}$ are both multiplied by the same number, that is, the magnetic field conversion factors are periodic;
- The SCC modulator shows good effect on reduction of unproductive harmonics. For instance, the high-order harmonics of BDFIM with nest-loop coil structure are effectively depressed as given in Table 2;
- The magnetic field conversion factors of the improved SCC are insensitive with pole pair combination but still keep relatively large at far-ratio pole pair cooperation. For instance, the magnetic field conversion factors of series ring at far-ratio cooperation (i.e. $N_{SC} = 5, p_{f} = 1$) will be larger than that of the squirrel cage;
- The series and parallel ring is equivalent to the stack of multiple series rings loops, and it can avoid the adverse effects caused by the forced flow of large current through the inner loop;
- The isometric ring can be treated as the ideal series ring with $\gamma_{c} = 0.5$ and the magnetic field conversion ability is good when the $N_{SC}$ and $k_{dn}$ are carefully designed.

### Table 2

| $N_{SC}$ | $p_{r}$ | $l$ | $C_{p}$ | $C_{\text{sum}}$ | $C_{\text{diff}}$ |
|---------|---------|-----|---------|----------------|------------------|
| 4       | 3       | 1   | ~1.000  | 0.639          | 0.000            |
| 2       |         |     | -0.116  | 0.033          |                  |
| 3       |         |     | 0.047   | -0.039         |                  |
| 4       |         |     | -0.020  | 0.048          |                  |

### Table 1

| $N_{SC}$ | $p_{r}$ | $N_{SC} - p_{r}$ | Squirrel cage | Nest-loop | Series ring | Series and parallel ring | Isometric ring |
|----------|---------|------------------|--------------|-----------|------------|--------------------------|---------------|
|          |         |                  | $C_{\text{sum}}$ | $C_{\text{diff}}$ | $C_{\text{sum}}$ | $C_{\text{diff}}$ | $C_{\text{sum}}$ | $C_{\text{diff}}$ |
| 3        | 1       | 2                | 0.342        | 0.171     | 0.731      | 0.044        | 0.325          | 0.194          | 0.438          | 0.046          | 0.347          | 0.174          |
| 4        | 1       | 3                | 0.270        | 0.162     | 0.639      | 0.000        | 0.317          | 0.216          | 0.405          | 0.110          | 0.336          | 0.202          |
| 5        | 1       | 4                | 0.219        | 0.146     | 0.573      | 0.038        | 0.310          | 0.229          | 0.379          | 0.153          | 0.327          | 0.218          |
| 5        | 2       | 3                | 0.382        | 0.164     | 0.781      | 0.063        | 0.329          | 0.176          | 0.456          | 0.004          | 0.353          | 0.151          |
| 6        | 1       | 5                | 0.182        | 0.130     | 0.525      | 0.069        | 0.306          | 0.237          | 0.360          | 0.184          | 0.321          | 0.229          |
| 6        | 2       | 4                | 0.342        | 0.171     | 0.731      | 0.044        | 0.325          | 0.194          | 0.438          | 0.046          | 0.347          | 0.174          |
purposely designed for other goals such as improving mechanical robust, reducing flux leakage and so on. On the other hand, the salient rotor pole as shown in Figure 4, usually performs asynchronous modulation behaviour on an arbitrary source magnetising MMF, where g is generally a constant representing the airgap length surrounded by stator and rotor pole envelopes. Parameter \( s_{depr} \) represents the rotor slot opening depth, whose relationship between SPR modulator is not discussed in [18]. The improved modulation operator \( M(N_{ST}, \epsilon_{ST})[\cdot] \) and \( M(N_{RT}, \epsilon_{RT})[\cdot] \) can be defined as:

\[
M(N_{ST}, \epsilon_{ST})[f(\phi)] = \begin{cases} f(\phi), \phi \in C^S & \kappa(\phi, t) \neq 0, 2\pi - C^S \\ \kappa(\phi, t) = 0, \phi \in 0, 2\pi - C \end{cases}
\]

\[
M(N_{RT}, \epsilon_{RT})[f(\phi, t)] = \begin{cases} f_{RT}(\theta, t)f(\phi, t), \phi \in C & \kappa(\phi, t) \neq 0, 2\pi - C \\ 0, \phi \in 0, 2\pi - C \end{cases}
\]

where \( C^S \) is the discontinuous interval occupied by rotor (stator) poles in mechanical radians and \( C \) is intersection of the \( C^S \) and \( C^R \). \( \theta \) is mechanical angle along the rotor circumference. \( t_{SR}(\phi) \) is the rotor (stator) slot opening width and \( t_{SR}(\phi) \) is the pole pitch in radians. \( \epsilon_{RT} \) defined in (10) (11)

\[
f_{RT}(\theta, t) = \epsilon_{RT} + \kappa(1 - \epsilon_{RT}) + \sum_{l=1}^{\infty} \left[ \frac{2(1 - \kappa)}{\pi} \sin(l\epsilon_{RT}\pi) \frac{1}{l} \cos[lN_{RT}\phi - \omega t] \right]
\]

where \( \omega \) is the electrical frequency of stator winding current.

\[
\kappa = \frac{t_{dr} - 1.6\beta \epsilon_{RT}}{2t_{dr}(s_{depr}/t_{dr})} = \frac{t_{dr} - 1.6\beta \epsilon_{RT}}{2s_{depr}}
\]

where \( \beta \) is a function of the ratio \( \epsilon_{RT} \) and increases a little with enlarging of the slot opening width. The principal and mutual magnetic field conversion factors can be derived as:

\[
C_p = C_{cp,sp} - \epsilon_{RT} + \kappa(1 - \epsilon_{RT})
\]

\[
C_{sum} = C_{cp,sp} - \epsilon_{RT} + C_{diff} = \left( \frac{3}{2} \right) \left( \frac{1}{\pi} \right)(1 - \kappa) \frac{\sin(l\epsilon_{RT}\pi)}{l}
\]

The finite slot opening depth \( s_{depr} \) (causing MMF drop in iron core) is taken into consideration by proposing the function \( \kappa \) to achieve reasonable theoretical analysis. The radial airgap magnetic field distribution will decrease with the increase of \( s_{depr} \), while the amplitude of \( \kappa \) is large when \( s_{depr} \) is small, that is, the asynchronous modulation effect is weak (the mutual magnetic field conversion factors are low). In other words, the \( s_{depr} \) will directly influence the modulation effect. The function \( \kappa \) can be ignored on being set as zero under ideal conditions (the \( s_{depr} \) is infinite) to gain simple parameter exploration.

For the salient poles with \( N_{RT} \) blocks, the typical magnetic field conversion factors with \( N_{RT} \leq 6 \) are calculated based on (14) (15) and exhibited in Table 3 (\( \epsilon_{RT} = 0.5, s_{depr} = 0.5, g = 0.35 \)), from which following conclusions are drawn:

- The magnetic field conversion factors/capability of the SPR only depends on the topological parameters such as \( \epsilon_{ST} \) and \( s_{depr} \), but has no connection with pole pair cooperation.
- Thus SPR modulator is suitable for application of magnetically geared machines with far-ratio pole pair combination;
- The best asynchronous modulation effect will be achieved at around \( \epsilon_{RT} = 0.5 \) [18], and \( C_{sum} \) and \( C_{diff} \) will increase with the enlargement of \( s_{depr} \), while the \( C_p \) will decrease a little;
- Magnetic field harmonics appear in pairs and the mutual conversion factors are equivalent for the same \( l \), showing weak effect on the reduction of unproductive harmonics;
- The asynchronous modulation behaviour will shift the spectrum lines of MMF harmonic components, while the synchronous modulation behaviour only modifies the relative amplitudes of harmonic components.

\[ \begin{array}{cccc}
\text{N}_{RT} & \rho & \text{N}_{RT} - \rho & \text{SPR (finite } s_{depr}) & \text{SPR (infinite } s_{depr}) \\
3 & 1 & 2 & 0.626 & 0.210 & 0.210 & 0.477 & 0.304 & 0.304 \\
4 & 1 & 3 & 0.626 & 0.210 & 0.210 & 0.477 & 0.304 & 0.304 \\
5 & 1 & 4 & 0.626 & 0.210 & 0.210 & 0.477 & 0.304 & 0.304 \\
5 & 2 & 3 & 0.626 & 0.210 & 0.210 & 0.477 & 0.304 & 0.304 \\
6 & 1 & 5 & 0.626 & 0.210 & 0.210 & 0.477 & 0.304 & 0.304 \\
6 & 2 & 4 & 0.626 & 0.210 & 0.210 & 0.477 & 0.304 & 0.304 \\
\end{array} \]

2.3 Flux barriers

The FBS modulator is adopted in SynRM and BDFRM in general, achieving high saliency and effective rotor topology to
obtain good modulation effect and performances than those of SPR modulator. The flux barriers contain radial (FBSRL) and axial (FBSAL) laminations, where the former one is the simplification of the latter one with good suppression effect on eddy current loss. The normalised resultant modulated MMF for the FBS modulator can be generally expressed as:

\[
M(N_{MB}) \cos(p_f \omega t - \phi) = C_p \cos(p_f \omega t - \phi) + \sum_{k=1}^{\infty} C_{sum} \cos(k \omega t - (k + p_f) \pi/N_{MB}) + \sum_{k=1}^{\infty} C_{dif} \cos(k \omega t + (k - p_f) \pi/N_{MB})
\]

(16)

\[
C_p = C_{pf, pf} = \left(\frac{1}{2}\right) \left[1 - \sin\left(2p_f \pi/N_{MB}\right) / \left(2p_f \pi/N_{MB}\right)\right]
\]

(17)

\[
C_{sum} = C_{pf, lN_{MB} - p_f} = -\sin\left[(lN_{MB} - 2p_f) \pi/N_{MB}\right]/2 \left(lN_{MB} - 2p_f \pi/N_{MB}\right), l = 1, 2, ...
\]

(18)

\[
C_{dif} = C_{pf, lN_{MB} + p_f} = \sin\left[(lN_{MB} + 2p_f) \pi/N_{MB}\right]/2 \left(lN_{MB} + 2p_f \pi/N_{MB}\right), l = 1, 2, ...
\]

(19)

Similarly, for the standard FBS with \(N_{MB}\) magnetic ducts, the typical conversion factors are calculated based on (17) (18) (19) and are exhibited in Table 4. Except those similar to SCC modulator, the following additional conclusions are drawn:

- The FBS modulator shows strong (weak) effect on the reduction of unproductive high-order harmonics under asynchronous (synchronous) modulation. For example, the inherent synchronous operation principle of SynRM (mutual conversion factor 0.5) causes a huge number of unproductive harmonics \(\{N_{MB} - p_f, l = 2, 3, ...\}\) existing in the airgap;
- The modulation principle of FBS modulator is similar to that of the SPR, but there are obvious differences on their characters. The FBS modulator shows better magnetic field modulation effect (higher \(C_p \times C_{sum}\) value contributing to the reluctance torque) and cross coupling capability than those of SPR modulator, while its magnetic conversion ability is closely related to the pole pair combination and is mainly suitable for the summation modulation application.

## 3 | Analysis of the Interchangeability of Modulators

The modulators and modulation behaviours are systematically introduced and analysed in Section 2, based on which the key rule of interchangeable modulators is summarised in this part.

### 3.1 | Modulation characteristics comparison

The three modulators share some common features and stated as follows:

- The modulators can apply to the asynchronous modulation in field modulated machines and the harmonic components with the pole pair \(N_{SC/RM}/RT = p_f\) (mutual conversion factor \(C_{sum}\)) plays a key role in summation modulation. In addition, the asynchronous modulation behaviours suffer from the irregular airgap flux density distribution, relative low-performance magnetic field utilisation or low power factor;
- The modulated MMFs are even extension, whose rotating directions depend on the magnetic field conversion factors.

On the other hand, there are certain differences as follows:

1. **Differences between the modulation principles**
   - The short circuited coil relies on slip frequency between the rotating source MMF and the modulator to induce extra MMF, and it cannot modulate the static source MMF;
   - The salient pole reluctance modulator utilises direct alternative variation of magnetic reluctance to perform modulation on arbitrary static or rotating source magnetising MMF and allows a relatively concise expression for the airgap flux density;
   - The flux barrier modulator is analogous to salient pole reluctance, while the flux lines in flux ducts change direction due to the non-constant airgap magnetic potential and non-magnetic barriers between the flux ducts and iron bridges.

2. **The conversion capability and pole pair combination**

   The close-ratio pole pair combination produces higher mutual magnetic field conversion factors for short circuited coil and flux barriers modulators. Although salient pole reluctance modulator relies only on the topological parameters \(\varepsilon_{S/R/T}\) and \(s_{dep}\), it can be applied to both the close- and far-ratio combinations like the magnetically-geared machines.
3) The differential modulation factor $C_{diff}$

The $C_{sum}$ and $C_{diff}$ are equivalent for the salient pole reluctance for the same $l$. However, for the short circuited coil and flux barriers, the amplitude of differential modulation conversion factor $C_{diff}$ is relatively low, containing few low-order harmonic components under asynchronous modulation. Thus only salient pole reluctance is suitable for differential modulation with high magnetic conversion efficiency.

4) The effective harmonic components

The rotating heterodyne harmonics $(\nu pj \pm lN_{RT})$ can contribute to the effective torque for the salient pole machines, and the rotating direction depends only on the symbol of $(\nu pj \pm lN_{RT})$. Thus salient pole reluctance features theoretically higher utilisation of field harmonics compared to the short circuited coil and flux barriers modulators, where only one component of the heterodyne harmonics $(\nu pj \pm lN_{SC/MB})$ is used. The stator-PM salient pole machines improve average torque by interactions of multiple harmonics while redundant harmonics will cause losses, vibration and acoustic noise [24]. The mechanism of rotor-PM machines like VPM is mono-harmonic.

5) The modulation effect on the fundamental harmonic

The source conversion factor $C_p$ of short circuited coil is around 1.0 and is usually higher than the other two modulators, indicating better fundamental harmonic conversion ability of induction machine with short circuited coil than synchronous machine with reluctance modulators.

3.2 The interchangeability and combination of modulators

Although all the three modulators can be applied to asynchronous modulation behaviours, they cannot be simply interchanged or combined. The key law of interchangeable modulators depends on the modulation behaviour and character of source MMF; as illustrated in Table 5, and the same applies to the combined modulators. In other words, the modulators should be applied to the same modulation behaviour and the acted source MMF has the same character as given in Table 6. To show the effectiveness clearly, several examples of interchangeability or combination of modulators are presented.

When the source MMF is rotating and is asynchronously modulated by modulators, the modulators can be interchanged by SPR, FBS or SCC. Considering BDFRM as an example and as shown in Figure 5, it can be obtained by easily replacing different reluctance modulators. Similarly, the BDFIM can be obtained by replacing the reluctance modulator with SCC as illustrated in Figure 6. The electromagnetic performances of BDFMs with different interchangeable modulators are analogous, but they are usually called the reluctance/induction machine. Moreover, the magnetic field conversion factor $C_{pp}$ of BFBRL and SCC is larger than that of SPR, showing high indirect magnetic coupling between the two independent stator windings. Also, the magnetic field conversion ability of FBSAL is better than that of BFBRL [25].

The similar rule applies for the modulator combination. For example, if the FBSRL, SPR and SCC are all in the asynchronous modulation behaviours and the source MMF is rotating distributed, they can be combined to achieve composite modulator as illustrated in Figure 7, improving the magnetic conversion ability, torque density, inductance characteristics and power factor [26]. The nest-loop SCC significantly improves the rotor cross coupling capacity and increases two extra effective asynchronous torque components [26]. The SPR and FBS modulators can be interchanged directly due to the similar modulation behaviour. However, on the contrary, the SCC cannot be interchanged or combined with SPR or FBS modulator under (synchronous, rotating) condition, since the SCC modulator applies to the asynchronous modulation and cannot modulate static source MMF.

4 COMPARISON OF ELECTROMAGNETIC PERFORMANCES

To more intuitively observe the magnetic field conversion ability of different modulators, a BDFM with different modulators is analysed as an example based on the airgap field modulation mechanism. The main design parameters are listed in Table 7. The topologies and magnetic field distributions at no-load are illustrated in Figure 8.

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**Table 5** Differences between the modulation principles

| Modulator | Modulation behaviour | Acted source MMF | Interchangeability | Combination | Examples |
|-----------|----------------------|------------------|--------------------|-------------|----------|
| SCC       | Synchronous          | Static           | ×                  | ×           | IPM      |
| SRC       | Asynchronous         | Static           | ×                  | ×           |          |
| FBS       | Synchronous          | Rotating         | SPR/FBS            | SPR and FBS | SynRM    |
| FBS       | Asynchronous         | Rotating         | SPR/FBS/SCC        | SPR and FBS and SCC | BDFM    |

**Table 6** The interchangeability/combination of modulators

| Modulation behaviour | Source MMF | Interchangeability | Combination | Examples |
|----------------------|------------|--------------------|-------------|----------|
| Synchronous          | Static     | SPR/FBS            | SPR and FBS | SynRM    |
| Synchronous          | Rotating   | SPR/FBS/SCC        | SPR and FBS and SCC | BDFM    |
4.1 | Magnetic field distribution

The magnetic field distributions of BDFRM with FBSRL and FBSAL under no-load condition are fairly compared. As shown in Figure 9, the radial flux density is an even periodic function (zero average) of ϕ. There are two dominating field harmonics as a result of the asynchronous modulation behaviour performed on the source magnetising MMF: As the two space harmonics are with different pole pairs and rotating speeds, it is observed that the flux density distribution is irregular and may not be divided into \( N_{MB} \) parts averagely at a certain instantaneous period. The MMF drop in the iron core due to saturation can be treated as a discount on the airgap MMF by multiplying the MMF by the constant saturation factor \( k_{sat} \) of less than 1.

As illustrated in Figure 10, the fundamental or second harmonic of FBSAL is lower than FBSRL due to flux leakage and eddy current loss, while its fourth harmonic is higher than FBSRL indicating better magnetic field conversion efficiency than that of FBSRL modulator. It can be seen that, the amplitudes of fourth \((N_{MB} - p_c)\) and 8th \((N_{MB} + p_c)\) harmonics of BDFRM are not equal, that is, the summation and differential modulations achieve different harmonics amplitudes in FBS modulator. In addition, they both show good suppression effect on the unproductive high-order harmonics compared to that of SPR modulator.

4.2 | Cross coupling ability

The cross coupling capability of BDFM essentially characterises the ability of modulation conversion between magnetic fields established by power winding (PW) and control winding (CW). The induced phase voltage in voltage control mode (CW and PW is respectively excited, the excitation voltage is 220 V rms with 50 Hz) is roughly linear and symmetrical to the synchronous speed, as presented in Figure 11. The cross coupling gets nearly lost and the produced torque will decline significantly in the vicinity of the synchronous speed of 500 r/min. In addition, the FBSAL indicates better indirect electromagnetic coupling compared to FBSRL, proving its good magnetic conversion capability. Moreover, the coupling factor \( C \) is proposed to reflect the indirect cross coupling between CW and PW based on the SPR/SCC/FBS rotor since there is no direct electromagnetic coupling between the two windings with different PPPN. Similar to the summation conversion factor, high coupling factor \( C \) indicates the high cross coupling ability from the magnetising (armature) MMF to the armature (magnetising) MMF. As illustrated in Figure 11 (c), the coupling factors between CW and PW for the BDFM with different modulators are pure alternating components and repeat one complete cycle for \( \pi \) electric radian, whose fundamental cycle depends on the equivalent rotor pole pair number \( N_{SC/RT/MB} \).

| Quantity                  | Value          | Quantity                  | \( SW_c \) | \( SW_p \) |
|---------------------------|----------------|---------------------------|------------|------------|
| Stator slot number        | 45             | Pole pair                 | 2          | 4          |
| Outer radius              | 91 mm          | Coil pitch                | 10         | 5          |
| Airgap length             | 0.35 mm        | Turns in series per phase | 255        | 510        |
| Stack length              | 90 mm          | Winding factor            | 0.9406     | 0.9406     |
The main self-inductances of BDFM are rotor position independent constants since the magnetic reluctance variation produces heterodyne harmonics under asynchronous modulation for the field modulated machines, which is different from the character of SynRM [19]. The predicted and FEA data agree well with each other as shown in Figure 12, and the predicted values are calculated based on [19,20]. The unsaturated average value of A-phase self-inductance of CW is around as twice as the mutual inductance of AB- and AC-phase despite opposite sign symbol. The similar rule applies the inductance character of PW. The main self-inductances of PW is greater than that of CW except for that of the SPR modulator.

In addition, further calculations show that relative amplitude of A-phase inductance characters, are in good accordence respectively with the theoretical prediction conversion factors and coupling factor between CW and PW, as illustrated in Table 8. The factor $k_{\text{lea}}$ is proposed to reflect the flux leakage due to the local tip-to-tip leakage of SPR or the iron bridges in FBSRL [27], which can be treated as a discount on the airgap MMF for simplification. The low flux leakage factor $k_{\text{lea}}$ indicates the high conversion efficiency

**FIGURE 8** No-load flux lines and flux density distribution of the BDFM with different modulators (CW is excited and PW is open). (a) SCC, (b) FBSRL, (c) SPR and (d) FBSAL.

**FIGURE 9** Airgap flux density distribution under no-load condition (only control winding is excited): (a) FBSRL and (b) FBSAL.

**FIGURE 10** Spectrum of radial airgap flux density (FBSRL and FBSAL).

| 4.3 | Inductance characteristics |

The main self-inductances of BDFM are rotor position independent constants since the magnetic reluctance variation variation...
from source pole pair $p_f$ to the mutual summation modulation one ($\Delta N_{\text{SC}/RT}/MB - p_f$).

### 4.4 Torque performance comparison

Figure 13 reveals that the electromagnetic torque of BDFM is proportional to the square of input current and the predicted value agrees with the 2-D FEA results. The FBSAL appears to have the biggest torque value with the increased phase current as expected. The FBSAL shows the best overload capability, while the SCC modulator suffers from magnetic saturation above 5 A current due to the large flux leakage factor $k_{\text{leak}}$ (0.5 for the SCC modulator), and the SPR modulator also saturates as a result of weak reduction effect on high-order harmonics (mutual conversion factors $C_{\text{int}}$ and $C_{\text{dif}}$ are equivalent for the same $\ell$).

When CW is excited and PW is connected with resistive load, the average torque is almost kept unchanged, but the
TABLE 8 The inductance characters and conversion factors

| Modulators | Inductance characters | Coupling factor | Conversion factors | Leakage factor |
|------------|----------------------|----------------|-------------------|---------------|
|            | $L_{cw}$ | $L_{pw}$ | $C_{pc}$ | Ratio | $C_{pp}$ | $C_{sum}$ | $k_{leak}$ | Ratio |
| SPR        | 162.682 | 153.341 | 81.118 | 1/0.943/0.499 | 0.477 | 0.477 | 0.304 | 0.2 | 1/1/0.510 |
| FBSRL      | 98.888  | 213.702 | 68.975 | 1/2.161/0.698 | 0.293 | 0.603 | 0.413 | 0.5 | 1/2.058/0.705 |
| FBSAL      | 160.281 | 317.312 | 112.397 | 1/1.987/0.700 | 0.293 | 0.603 | 0.413 | 0.5 | 1/2.058/0.705 |
| SCC        | 218.120 | 232.399 | 78.870 | 1/1.065/0.362 | 1 | 1 | 0.731 | 0.5 | 1/1/0.366 |

FIGURE 13 Torque performance (CW is excited and PW connected with resistive load): (a) with CW current RMS value, (b) Torque versus power angle

FBSAL varies sinusoidally due to the presence of non-magnetic flux barriers without ribs. In addition, the steady-state torque is decomposed by Maxwell stress tensor method, as illustrated in Figure 14, where torque ripple (torque components contributed by ineffective harmonics) can be clearly observed. The torque contributed by the $2p_p$-pole field harmonic is about half that of the $2p_p$-pole fields component when only CW is excited. Also, the torque contributions of the two harmonics are almost equal for the FBSAL modulator as the second ($p_c$) and fourth ($N_{MB} - p_c$) harmonics are nearly equal.

5 | EXPERIMENTAL VERIFICATION

A typical 4/2 BDFRM prototype with FBSRL modulator as shown in Figure 15, with main design parameters listed in Table 7, has been taken as an example to verify the effectiveness of the aforementioned theoretical analysis. Figure 15(a) shows the assembling of the rotor where the flux barriers are not fully laminated and Figure 15(b) illustrates the stator with two standard short-pitch distributed windings. The cross coupling ability is tested in voltage control mode as shown in Figure 16. It can be seen that the cross coupling ability gets nearly lost at around the synchronous speed of 500 r/min, matching well with the FEA simulated values in Figure 11. A simple method is proposed in [28] to measure the inductances of the BDFRM, and it proves to be effective in accurately measuring the self- and mutual-inductances at an arbitrary rotor position and current level.

Figure 17 reveals the measured inductance characteristics. It can be seen that both the self-inductances are nearly constants independent of rotor position, and a little bit different from the performance of a traditional synchronous machine featuring either the FBS or the SPR modulator whose stator winding inductances can be decomposed into a DC constant and an alternating component due to apparent variable reluctance in $dq$-axis direction. Moreover, the coupling factor $C_{pc}$ between PW and CW is a pure alternating component and repeat $N_{MB}$ states in 360° mechanical degree.

Figure 18 illustrates the measured torque performances. The average torque keeps almost the same with the power angle $\alpha$, since the reluctance variation of FBS modulator produces heterodyne harmonics in asynchronous modulation.
of the analytical method, although certain existing error may be due to the end effect [29], iron saturation and measurement error etc.

6 | CONCLUSION

First, the nature of modulation behaviours is revealed followed by the combination or interchangeability of modulators which are comprehensively investigated from the perspective of the airgap field modulation mechanism by the authors. The modulation principles of the flux barriers and improved short circuited coils modulators, the relationship between salient pole reluctance modulator and pole are together with slot opening depth etc. are introduced in detail and fairly compared.

It is of far-reaching significance for understanding and revealing the evolutionary form and independent action mechanism of modulators based on the general rules and concluded as follows. The three modulators share similar asynchronous behaviours and modulated harmonic distributions, while yield different magnetic field modulation principles, conversion capabilities and modulation effect on fundamental harmonic etc. The flux barriers and short circuited coil modulators show good magnetic field modulation effect and good effect on the reduction of unproductive high-order harmonics under asynchronous modulations, while their magnetic conversion ability is closely related to the pole pair combination and are mainly suitable for the summation modulation applications. The salient pole reluctance modulator can be utilised in both the close-ratio and far-ratio pole pair combinations. The key law of interchangeable modulators depends on the modulation behaviour and character of source magnetising MMF. For instance, the salient pole reluctance and flux barriers can be interchanged directly due to the similar modulation behaviour, while the short circuited coil modulator applies to the asynchronous modulation and cannot modulate the static source MMF.

However, to reasonably allocate the ratio between different modulators in a composite structure by making full use of the compound airgap modulation behaviours to enhance the magnetic field coupling capability, thereby improving the magnetic field modulation effect and obtaining the best electromagnetic performance is an important theoretical issue and deserves to be studied further.
**Figure 18** Measured torque performances. (a) average torque, (b) average torque versus power angle (RMS=5 A)

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**REFERENCES**

1. Obie, E.S.: Calculation of inductances and torque of an axially laminated synchronous reluctance motor. IET Electr. Power Appl. 4(9), 783–792 (2009)
2. Wang, Y., et al.: Design of high-torque-density double-stator permanent magnet brushless motors. IET Electr. Power Appl. 5(3), 317–323 (2011)
3. Lee, C.H.: Vernier motor and its design. IEEE Trans. Power Apparatus Syst. 82(66), 343–349 (1963)
4. Rekik, M., et al.: Improvement in the field-weakening performance of switched reluctance machine with continuous mode. IET Electr. Power Appl. 1(5), 785–792 (2007)
5. Han, P., et al.: Brushless doubly-fed machines opportunities and challenges. Chinese J. Electr. Eng. 4(2), 1–18 (2017)
6. Cheng, M., et al.: Overview of stator-permanent magnet brushless machines. IEEE Trans. Ind. Electron. 58(11), 5087–5101 (2011)
7. Li, X., Chau, K.T., Cheng, M.: Comparative analysis and experimental verification of an effective permanent–magnet vernier machine. IEEE Trans. Magn. 51(7) (2015). Article #8203009
8. Atallah, K., Calverley, S., Howe, D.: Design, analysis and realisation of a high-performance magnetic gear. IET Electr. Power Appl. 151(2), 135–143 (2004)
9. Li, X., Cheng, M., Wang, Y.: Analysis, design and experimental verification of a coaxial magnetic gear using stationary permanent–magnet ring. IET Electr. Power Appl. 12(2), 231–238 (2018)
10. Cheng, M., et al.: Emerging multi-port electrical machines and systems: past developments, current challenges and future prospects. IEEE Trans. Ind. Electron. 65(7), 5422–5435 (July 2018)
11. Hansen, K.: The rotating magnetic field theory of AC motors. J. AIEE. 44(2), 170–178 (1925)
12. West, H. R.: The cross-field theory of alternating-current machines. In: Transactions of the American Institute of Electrical Engineers, Vol. XLV, pp. 45, 466–474. IEEE (1926)
13. Park, R.: Two-reaction theory of synchronous machine generalized method of analysis–part I. Trans. AIEE. 48(9), 716–727 (1929)
14. Staton, D., et al.: Unified theory of torque production in ac, dc and reluctance motors. In: Proceedings of 1994 IEEE Industry Applications Society Annual Meeting, Vol. I., pp. 149–156. IEEE (1994)
15. Lipo, T.A.: Winding Distribution in an Ideal Machine, Analysis of Synchronous Machine, 2nd ed., pp. 1–76. CRC Press Boca Raton (2012)
16. Cheng, M., Han, P., Hua, W.: General airgap field modulation theory for electrical machines. IEEE Trans. Ind. Electron. 64(8), 6063–6074 (2017)
17. Wen, H., Cheng, M.: Unified analysis of induction machine and synchronous machine based on the general airgap field modulation theory. IEEE Trans. Ind. Electron. 66(12), 9205–9216 (2019)
18. Cheng, M., et al.: Analysis of airgap field modulation principle of simple salient poles. IEEE Trans. Ind. Electron. 66(4), 2628–2638 (2019)
19. Wen, H., et al.: Analysis of airgap field modulation principle of flux guides. IEEE Trans. Ind. Appl. 56(5), 4758–4768 (2020)
20. Cheng, M., et al.: Analysis of airgap field modulation behavior and torque component in electric machines. Trans. China Electrotechn. Soc. 35(5), 921–930 (2020)
21. Krause, P.C., et al.: Distributed windings in ac machinery. In: El-Hawary, M.E.(Ed.) Analysis of Electric Machinery and Drive Systems, pp. 53–83. Wiley, Hoboken (2013)
22. Broadway, A., Burbidge, L.: Self-cascaded machine a low–speed motor or high-frequency brushless alternator. Proc. Institut. Electr. Eng. 117(7), 1277–1290 (1970)
23. McMahon, R., et al.: Rotor parameter determination for the brushless doubly fed (induction) machine. IET Electr. Power Appl. 9(8), 549–555 (2015)
24. Zhu, X., et al.: Analysis of back–EMF in flux–reversal permanent magnet machines by air-gap field modulation theory. IEEE Trans. Ind. Electron. 66(5), 3344–3355 (2019)
25. Zhang, F., et al.: Overview of research and development status of brushless doubly–fed machine system. Chinese J. Electr. Eng. 2(2), 1–13 (2016)
26. Zhang, F., et al.: Effects of design parameters on performance of brushless electrically excited synchronous reluctance generator. IEEE Trans. Ind. Electron. 65(1), 9179–9189 (2018)
27. Han, P., Zhang, J., Cheng, M.: Analytical analysis and performance characterization of brushless doubly–fed machines with multi–barrier rotors. IEEE Trans. Ind. Appl. 55(6), 5758–5767 (2019)
28. Han, P., et al.: Multi–frequency spiral vector model for the brushless doubly–fed induction machine. In: Proceedings of IEEE international Electr. Mach. Drives Conference, pp. 1–8. IEMDC, Miami, USA (2017)
29. Hua, W., Cheng, M.: Static characteristics of doubly–salient brushless machines having magnets in the stator considering end–effects. Electr. Power Component. Syst. 36(7), 754–770 (2008)

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