Supersymmetric QCD Parity Nonconservation in Top Quark Pairs at the Tevatron

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ABSTRACT

In the supersymmetry (SUSY) models, because of the mass difference between the left- and right-top squarks, the supersymmetric QCD interactions can generate parity violating effects in the production of $t\bar{t}$ pairs. We show that SUSY QCD radiative corrections to the parity violating asymmetry in the production rates of the left- and right-handed top quarks via the $q\bar{q} \rightarrow t\bar{t}$ process can reach about 3% at the Fermilab Tevatron with $\sqrt{s} = 2$ TeV. This could be observable with an integrated luminosity larger than 2 fb$^{-1}$. We also show that in these models the SUSY QCD radiative corrections to the $t\bar{t}$ production rate are small unless gluinos are very light, of the order of 1 GeV.
1 Introduction

In a recent paper [1], we studied the parity violating asymmetry induced from the supersymmetric electroweak (SUSY EW) and Yukawa (SUSY Yukawa) corrections at the one loop level. Two classes of supersymmetry (SUSY) models were considered: the minimal supergravity (mSUGRA) models [2] and the minimal supersymmetric models (MSSM) with scenarios motivated by current data [3, 4]. After sampling a range of values of SUSY parameters in the region that might give large contributions to the parity-violating asymmetry $A$, and which are also consistent with either of the above two classes of models, we found that the asymmetry $A$ due to the one-loop SUSY EW ($\alpha m_t^2/m_W^2$) and SUSY Yukawa corrections for the production process $q\bar{q} \to g \to t\bar{t}$ at the upgraded Tevatron is generally small, less than a few percent. However, the sign can be either positive or negative depending on the values of the SUSY parameters. (The effect from the Standard Model (SM) weak corrections to this asymmetry is typically less than a fraction of percent [5, 6].)

In the supersymmetric Standard Model, some superparticles experience not only the electroweak interaction but also the strong interaction. Although the SM QCD interaction respects the discrete symmetries of charge conjugation (C) and parity (P), the SUSY QCD interactions for superparticles, in their mass eigenstates, need not be C and P invariant. (Needless to say, in the strong interaction eigenstates, the SUSY QCD interaction is C-and P-invariant.) For either the mSUGRA or the MSSM models, the masses of the left-stop (the supersymmetric partner of the left-handed top quark) and the right-stop can be noticeably different due to the large mass of the top quark. This is a general feature of the supersymmetry models in which the electroweak symmetry is broken spontaneously via radiative corrections. Since both the left-stop and the right-stop contribute to the loop corrections for the $t\bar{t}$ pair production process $q\bar{q}, gg \to t\bar{t}$, the different masses of the top-squarks will induce a parity violating asymmetry. It is this effect that we shall study in this paper. Because the $t\bar{t}$ pairs are produced predominantly via the QCD process $q\bar{q} \to t\bar{t}$ at the Tevatron (a p$p$ collider with CM energy $\sqrt{s} = 2$ TeV), we shall concentrate on the SUSY QCD corrections for the $q\bar{q}$ fusion process. We show that the parity violating asymmetry $|A|$ at the Tevatron induced by SUSY EW, SUSY Yukawa and SUSY QCD effects could add up to more than 3% for some of the SUSY models. The SM tree level cross section is 4.34 pb for a 176 GeV top quark. The SM QCD correction increases the rate by about 60% [7], while the SM EW correction is less than a percent [8]. It is expected that there are about 2300 fully reconstructed $b$-tagged $t\bar{t}$ events in the lepton plus jets mode collected by the two experimental groups with a 2 fb$^{-1}$ integrated luminosity [9].
This amounts to a signal at $\sim 90\%$ c.l. (confidence level) with $2 \text{ fb}^{-1}$, or $99\%$ c.l. with $10 \text{ fb}^{-1}$. Thus, a study of $A$ at the Tevatron could yield information about the allowed range of SUSY model parameter space.

2 SUSY QCD Corrections and ParityViolation

I. Squark mixings

In the MSSM the mass eigenstates $\tilde{q}_1$ and $\tilde{q}_2$ of the squarks are related to the (strong) current eigenstates $\tilde{q}_L$ and $\tilde{q}_R$ via the mixing angle $\theta_\tilde{q}$ by

$$\tilde{q}_1 = \tilde{q}_L \cos \theta_\tilde{q} + \tilde{q}_R \sin \theta_\tilde{q}, \quad \tilde{q}_2 = -\tilde{q}_L \sin \theta_\tilde{q} + \tilde{q}_R \cos \theta_\tilde{q}. \quad (1)$$

For the top squarks, the mixing angle $\theta_\tilde{t}$ and the masses $m^2_{\tilde{t}_1, 2}$ can be calculated by diagonalizing the following mass matrix [3],

$$M^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} & m_t m_{\tilde{t}_{LR}} \\ m_t m_{\tilde{t}_{LR}} & M^2_{\tilde{t}_R} \end{pmatrix},$$

$$M^2_{\tilde{t}_L} = m^2_{\tilde{t}_L} + m^2_t + \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \cos(2\beta) m^2_Z,$$

$$M^2_{\tilde{t}_R} = m^2_{\tilde{t}_R} + m^2_t + \frac{2}{3} \sin^2 \theta_W \cos(2\beta) m^2_Z,$$

$$m_{\tilde{t}_{LR}} = -\mu \cot \beta + A_t, \quad (2)$$

where $m^2_{\tilde{t}_L}$ and $m^2_{\tilde{t}_R}$ are the soft SUSY breaking mass terms of the left-stop and the right-stop, $\mu$ is the coefficient of the $H_1 H_2$ mixing term in the superpotential, $A_t$ is the parameter describing the strength of soft SUSY breaking trilinear scalar interaction $\tilde{t}_L \tilde{t}_R H_2$, and $\tan \beta = v_2/v_1$ is the ratio of the vacuum expectation values of the two Higgs doublets. $\theta_W$ is the weak mixing angle, and $m_Z$ is the mass of the $Z$ boson.

II. Renormalized amplitudes and the asymmetry

The effects of parity nonconservation can appear as an asymmetry in the invariant mass $(M_{t\bar{t}})$ distributions as well as in the integrated cross sections $(\sigma)$ for $t_L$ and $t_R$ production. The integrated asymmetry, after integrating over a range of $M_{t\bar{t}}$, is defined by [3]

$$A \equiv \frac{N_R - N_L}{N_R + N_L} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}, \quad (3)$$

where $N_R$ and $N_L$ are the numbers of right-handed and left-handed top quarks. Thus, $\sigma_R = \sigma_{RL} + \sigma_{RR}$ ($\sigma_L = \sigma_{LR} + \sigma_{LL}$) is the cross section for producing a $t\bar{t}$ pair in which the top quark is right- (left-) handed, and the top antiquark is either left- or right-handed.
Some of the one-loop scattering amplitudes of $q\bar{q} \rightarrow t\bar{t}$ were already presented in Refs. \[10, 11\] for calculating the total production rates of $t\bar{t}$ pairs. To calculate the parity violating asymmetry $A$ in the $t\bar{t}$ system, additional renormalized amplitudes are needed. In terms of the tree-level amplitude, $M_0$, and the next-to-leading order SUSY QCD corrections, $\delta M$, the renormalized amplitudes at the one-loop level can be written as $M = M_0 + \delta M$. Denote the momenta of the initial and the final state particles as $q_i(p_i)\bar{q}_m(p_m) \rightarrow t_j(p_j)\bar{t}_l(p_l)$, and the Dirac four-spinor as $u_i \equiv u(p_i)$ ($v_i \equiv v(p_i)$) for particle (anti-particle) $i$. Then, $M_0 = ig_s^2(T^c_{ji}T^c_{lm})J_1 \cdot J_2/\hat{s}$, where $J^\mu_1 = \bar{v}(p_3)\gamma^\mu u(p_4)$ and $J^\mu_2 = \bar{u}(p_2)\gamma^\mu v(p_1)$; $\hat{s}$ is the invariant mass of the $t\bar{t}$ pair; $g_s$ and $T^c_{ij}$ are the gauge coupling and the generator of the group $SU(3)_c$, respectively.

To calculate the parity violating asymmetry induced by the SUSY QCD effects, we follow the method presented in Ref. \[12\], in which the asymmetry was calculated numerically using the helicity amplitude method. To obtain the renormalized scattering amplitudes, we adopt the dimensional regularization scheme to regulate the ultraviolet divergences and the on-mass-shell renormalization scheme \[13\] to define the input parameters. The SUSY QCD corrections to the scattering amplitudes arise from the vertex diagram, the gluon self-energy and the box diagrams, as well as the crossed-box diagrams. The renormalized amplitudes can be written as

$$\delta M = \delta M^{v1} + \delta M^{v2} + \delta M^s + \delta M^{DB} + \delta M^{CB},$$

where $\delta M^{v1}$ and $\delta M^{v2}$ are vertex corrections, $\delta M^s$ is the self-energy correction, and $\delta M^{DB}$ and $\delta M^{CB}$ are the contributions from the box diagrams and crossed-box diagrams, respectively. The results for these separate contributions are

$$\delta M^{v1} = ig_s^2(T^c_{ji}T^c_{lm})\bar{u}(p_2)[F^0_{v1} \cdot J_1 + F^v_{1}\beta_1 + F^v_{3}\beta_1 + F^v_{4}\beta_1 + F^v_{6}\beta_1 \cdot J_1 + (F^A_{v1}\beta_1 + F^A_{3}\beta_1 + F^A_{4}\beta_1 + F^A_{6}\beta_1 \cdot J_1)\gamma_5]v(p_1)/\hat{s},$$

$$\delta M^{v2} = ig_s^2(T^c_{ji}T^c_{lm})\bar{u}(p_3)(F^v_{1}\beta_2 + F^v_{6}\beta_2 \cdot J_2)u(p_4)/\hat{s},$$

$$\delta M^s = F^s_0M_0,$$

$$\delta M^{DB} = ig_s^2\frac{7}{6}(T^c_{ji}T^c_{lm})[F_{1DB}^0(\bar{u}_2u_4\bar{v}_3v_1 - \bar{u}_2\gamma_5u_4\bar{v}_3\gamma_5v_1) + F_{2DB}^0(-\bar{u}_2u_4\bar{v}_3\gamma_\mu v_1 - \bar{u}_2\gamma_\mu u_4\bar{v}_3v_1 - \bar{u}_2\gamma_5u_4\bar{v}_3\gamma_\mu \gamma_5v_1 + \bar{u}_2\gamma_\mu \gamma_5u_4\bar{v}_3\gamma_5v_1)
\ + F_{3DB}^\nu(\bar{u}_2\gamma_\mu u_4\bar{v}_3\gamma_\nu v_1 + \bar{u}_2\gamma_\mu \gamma_5u_4\bar{v}_3\gamma_\nu \gamma_5v_1) + F_{4DB}^0(\bar{u}_2u_4\bar{v}_3v_1 - \bar{u}_2u_4\bar{v}_3\gamma_\nu v_1) + F_{5DB}^0(-\bar{u}_2\gamma_5u_4\bar{v}_3\gamma_\mu v_1 - \bar{u}_2\gamma_\mu \gamma_5u_4\bar{v}_3v_1 - \bar{u}_2u_4\bar{v}_3\gamma_\mu \gamma_5v_1 + \bar{u}_2\gamma_\mu u_4\bar{v}_3\gamma_5v_1)
\ + F_{6DB}^\nu(\bar{u}_2\gamma_\mu u_4\bar{v}_3\gamma_\nu \gamma_\mu v_1 + \bar{u}_2\gamma_\mu \gamma_5u_4\bar{v}_3\gamma_\nu v_1)],$$

\[\text{Ref. 12}\]
\[ \delta M^{CB} = i g_s \frac{1}{3} (T^e_i T^e_m) [F^{CB}_1 (\bar{u}_2 u_3 \bar{v}_4 v_1 - \bar{u}_2 \gamma_5 u_3 \bar{v}_4 \gamma_5 v_1) \\
+ F^{CB}_2 (-\bar{u}_2 u_3 \bar{v}_4 \gamma_\mu v_1 - \bar{u}_2 \gamma_\mu u_3 \bar{v}_4 v_1 - \bar{u}_2 \gamma_5 u_3 \bar{v}_4 \gamma_5 v_1 + \bar{u}_2 \gamma_\mu \gamma_5 u_3 \bar{v}_4 \gamma_5 v_1) \\
+ F^{CB}_3 (\bar{u}_2 \gamma_\mu u_3 \bar{v}_4 \gamma_\nu v_1 + \bar{u}_2 \gamma_\mu \gamma_5 u_3 \bar{v}_4 \gamma_5 v_1) \\
+ F^{CB}_4 (\bar{u}_2 \gamma_5 u_3 \bar{v}_4 v_1 - \bar{u}_2 u_3 \bar{v}_4 \gamma_5 v_1) \\
+ F^{CB}_5 (-\bar{u}_2 \gamma_5 u_3 \bar{v}_4 \gamma_\mu v_1 - \bar{u}_2 \gamma_\mu \gamma_5 u_3 \bar{v}_4 v_1 - \bar{u}_2 u_3 \bar{v}_4 \gamma_\mu \gamma_5 v_1 + \bar{u}_2 \gamma_\mu u_3 \bar{v}_4 \gamma_5 v_1) \\
+ F^{CB}_6 (\bar{u}_2 \gamma_\mu u_3 \bar{v}_4 \gamma_\nu \gamma_5 v_1 + \bar{u}_2 \gamma_\mu \gamma_5 u_3 \bar{v}_4 \gamma_\nu v_1)] \]

where, \( j_1 \equiv J_1^{\mu} \gamma_\mu, \) etc., and the explicit expressions of the form factors \( F^{v}_i, F^{A_\mu}_i, F^{s}_0, F^{DB, CB}_i, F^{DB(\mu, \nu)}_i, \) and \( F^{CB(\mu, \nu)}_i \) are given in the Appendix.

### 3 Numerical Results and Conclusion

In this section, we give our numerical results for a 176 GeV top quark [14]. To avoid numerical instabilities and to take into account the fact that the decay products of the produced top quarks at large scattering angles are better distinguishable from the background, we impose a cut on the transverse momentum \( (p_T) \) and the rapidity \( (y) \) of the top quark and anti-quark:

\[ p_T > 20 \text{ GeV} \quad \text{and} \quad |y| < 2.5. \]  

We note that the parity violating asymmetry \( \mathcal{A} \) would be independent of the parton distribution functions (PDFs) if there were no kinematic cuts imposed in the calculation. Therefore, \( \mathcal{A} \) should not be sensitive to the choice of the PDFs. In this paper we use the MRSA’ PDFs [15] and evaluate both the strong coupling \( \alpha_s \) and the PDFs at the scale \( Q = \sqrt{s} = M_\ell \).

As discussed in the previous section, the SUSY parameters relevant to our study are \( m_{\tilde{t}_i}, m_{\tilde{t}_2}, \theta_\ell \) (or \( m_{\tilde{t}_L}, m_{\tilde{t}_R}, m_{L,R} \)), \( m_{\tilde{b}_R}, m_{\tilde{b}_L} \), and \( m_{\tilde{g}} \). To simplify our discussion, we assume \( m_{\tilde{b}_L} = m_{\tilde{b}_R} = m_{\tilde{t}_L} \), so that there are only four SUSY parameters to be considered, \( m_{\tilde{t}_i}, m_{\tilde{t}_2}, \theta_\ell \) and \( m_{\tilde{g}} \). (The \( SU(2)_L \) gauge symmetry requires that \( m_{\tilde{g}}^2 = m_{\tilde{b}_L}^2 \).

The mSUGRA models predict radiative breaking of the electroweak gauge symmetry induced by the large top quark mass. Consequently, it is possible to have large splitting in the masses of the left-stop and the right-stop, while the masses of all the other (left- or right-) squarks are about the same [17]. For the MSSM models with scenarios motivated by current data [4], a light \( \tilde{t}_1 \) is likely to be the right-stop \( (\tilde{t}_R) \), with a mass at the order of \( m_W \); the other squarks are heavier than \( \tilde{t}_1 \). Since heavy superparticles decouple in loop

\footnote{Using the CTEQ4M PDFs [16] gives a similar result.}
contributions, we expect that a lighter $\tilde{t}_1$ would induce a larger asymmetry. Because the parity-violating effects from the SUSY QCD interactions arise from the mass difference between $\tilde{t}_1$ and $\tilde{t}_2$, it is obvious from Eq. (1) that the largest parity violating effect occurs when $\theta_{\tilde{t}}$ is $\pm \pi/2$ for $m_{\tilde{t}_R} \leq m_{\tilde{t}_L}$. When $\theta_{\tilde{t}} = \pm \pi/4$, the parity asymmetry should be zero. This is evident from the results shown in the Appendix, which indicate that the amplitudes that contribute to $A$ are all proportional to $Z_i = \mp \cos(2\theta_{\tilde{t}})$.

In either the mSUGRA or the MSSM models, the gluinos are usually as heavy as the light squarks, on the order of a few hundred GeV. However, Farrar has argued that light gluinos are still a possibility. If gluinos are light, then a heavy top quark can decay into a stop and a light gluino for $m_{\tilde{g}} < (m_t - m_{\tilde{t}_1})$ such that the branching ratio of $t \to bW^+$ could show a large difference from that ($\sim 100\%$) predicted by the SM. The CDF collaboration has measured the branching ratio of $t \to bW^+$ to be $0.87^{+0.13}_{-0.30}$. At the 1σ level, this implies that a 50 (90) GeV $\tilde{t}_1$ requires the mass of the gluino to be larger than about 120 (80) GeV for $\theta_{\tilde{t}} = \pm \pi/2$. However, at the 2σ level (i.e. 95% c.l.), there is no useful limit on the mass of the gluino.

It is interesting to note that for all the models listed in Table 1, the asymmetry $A$ is negative (i.e. $\sigma_R < \sigma_L$) for $m_{\tilde{g}} < 200\text{GeV}$, and its magnitude can be as large as 3% for models with light $\tilde{t}_1$. The maximal $|A|$ occurs when $m_{\tilde{g}}$ is about equal to $(m_t - m_{\tilde{t}_1})$ because of the mass threshold enhancement. For $m_{\tilde{g}} > 200\text{GeV}$, the asymmetry $A$ becomes positive, with a few percent in magnitude, and monotonically decreases as $m_{\tilde{g}}$ increases. Comparing these results with those induced by the SUSY EW and SUSY Yukawa corrections, it is clear that SUSY QCD interactions can generate a relatively larger parity-violating asymmetry.

The differential asymmetry $A(M_{tt})$ also exhibits an interesting behaviour as a function of the $t\bar{t}$ invariant mass $M_{tt}$. This is illustrated in Table 2 for the first SUSY model in Table 1 ($\left(m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}} \right) = (50, 1033, -1.38)$). As shown, $|A(M_{tt})|$ increases as $M_{tt}$ increases for $m_{\tilde{g}} < 200\text{GeV}$, which is similar to the effects from the SUSY EW and SUSY Yukawa

\footnote{A recent analysis on the possible experimental signature for a light gluino from top quark decay can be found in Ref. \cite{20}.}
Table 1: Parity violating asymmetry $A$ in $p\bar{p} \to t\bar{t} + X$, as a function of $m_{\tilde{g}}$, for four sets of SUSY models labeled by ($m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_t$).

| $m_{\tilde{g}}$ (GeV) | (50, 1033, −1.38) | (90, 1033, −1.38) | (50, 558, −1.25) | (90, 558, −1.25) |
|----------------------|------------------|------------------|------------------|------------------|
| 2                    | −1.10%           | −1.28%           | −0.98%           | −1.13%           |
| 50                   | −1.53%           | −1.77%           | −1.40%           | −1.59%           |
| 100                  | −2.34%           | −2.23%           | −2.21%           | −2.23%           |
| 120                  | −2.86%           | −1.99%           | −2.89%           | −1.99%           |
| 135                  | −3.16%           | −1.79%           | −3.43%           | −1.82%           |
| 150                  | −2.58%           | −1.50%           | −2.80%           | −1.53%           |
| 175                  | −1.18%           | −0.44%           | −1.30%           | −0.56%           |
| 200                  | 0.99%            | 0.97%            | 0.82%            | 0.77%            |
| 225                  | 1.60%            | 1.41%            | 1.40%            | 1.19%            |
| 250                  | 1.53%            | 1.34%            | 1.35%            | 1.17%            |
| 275                  | 1.27%            | 1.16%            | 1.16%            | 1.02%            |
| 300                  | 1.04%            | 0.94%            | 0.95%            | 0.83%            |

Contributions [1]. When $M_{tt}$ is larger than 500 GeV, $|A(M_{tt})|$ can become very large at the expense of the signal event rate. For the model with $m_{\tilde{g}}$ equal to 200 GeV, $A(M_{tt})$ is positive for $M_{tt}$ less than $\sim$ 500 GeV; it reaches its maximal value when $M_{tt}$ is about twice the gluino mass. This is again due to the mass threshold enhancement.

We note that our conclusion for the magnitude of the parity-violating asymmetry is different from that given in Ref. [21]. There the box (and crossed-box) diagram contributions were not included and some of the formulae (Eq. (8) and the form factor $A$ in the Appendix) for calculating the vertex diagram contributions appear to contain misprints. From our calculation, we find that the box diagram contributions are in general small compared to the vertex diagram contributions, and are sizable only for heavy gluinos ($m_{\tilde{g}} > 200$ GeV). Therefore, the fact that we have included these corrections, while Ref. [21] did not, does not constitute the main difference between the two calculations. To resolve this discrepancy, we compare our results with those in the literature for the SUSY QCD radiative correction to the production rate of $q\bar{q} \to t\bar{t}$ at the Tevatron [22]. After correcting the relative sign between the box and crossed-box terms and the apparent misprints in the form factors $F_{12,13}^{DB}$ and $F_{12,13}^{CB}$ in Ref. [22], our results agree with those presented in [22], providing us with confidence in our results. However, our numerical results for the $t\bar{t}$ production rate do not agree with those given in Ref. [21]. For the case considered in [21],

3 Without the cuts in [10], the values of $A$ for the first model in Table 1 are −1.0%, −2.65%, and +0.94% for $m_{\tilde{g}} = 2, 120, 200$ GeV, respectively.

4 These apparent problems in Ref. [22] were also pointed out in Ref. [21].
Table 2:
The differential asymmetry $A(M_{tt})$ and cross section $d\sigma/dM_{tt}$ (in unit of fb/GeV) as a function of $M_{tt}$ for the first SUSY model in Table 1 with various $m_{\tilde{g}}$ values.

| $M_{tt}$ (GeV) | $m_{\tilde{g}} = 2$ GeV | $m_{\tilde{g}} = 120$ GeV | $m_{\tilde{g}} = 200$ GeV |
|---------------|----------------|----------------|----------------|
| 358           | -0.22% 23.0 | -0.73% 16.3 | 0.44% 19.4 |
| 368           | -0.42% 36.6 | -1.33% 26.2 | 0.95% 31.4 |
| 378           | -0.53% 40.2 | -1.63% 29.0 | 1.39% 34.9 |
| 388           | -0.67% 38.7 | -2.00% 28.1 | 2.12% 34.2 |
| 398           | -0.74% 37.2 | -2.17% 27.2 | 3.33% 34.4 |
| 408           | -0.82% 35.0 | -2.38% 25.7 | 3.67% 32.4 |
| 425           | -0.93% 30.9 | -2.64% 22.8 | 2.35% 27.0 |
| 450           | -1.13% 24.1 | -3.07% 18.0 | 1.05% 20.0 |
| 475           | -1.31% 18.3 | -3.40% 13.8 | 0.11% 14.7 |
| 500           | -1.47% 13.7 | -3.68% 10.4 | -0.62% 10.8 |
| 525           | -1.61% 10.3 | -3.81% 7.9 | -1.21% 8.0 |
| 550           | -1.76% 7.7 | -4.11% 5.9 | -1.71% 5.9 |
| 575           | -1.93% 5.7 | -4.34% 4.4 | -2.08% 4.3 |
| 1000          | -3.26% 0.032 | -5.82% 0.026 | -4.66% 0.024 |

Table 3:
The SUSY QCD corrections ($\Delta\sigma$) to the $q\bar{q} \to t\bar{t}$ production rates at the Tevatron with $\sqrt{s} = 2$ TeV as a function of $m_{\tilde{g}}$, for the first SUSY model in Table 1.

| $m_{\tilde{g}}$ (GeV) | 2 | 50 | 100 | 120 | 135 | 150 | 175 | 200 | 225 | 250 | 275 | 300 |
|-----------------|---|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\Delta\sigma$ (pb) | 1.17 | 0.26 | -0.04 | -0.18 | -0.87 | -0.49 | -0.02 | 0.33 | 0.30 | 0.24 | 0.19 | 0.16 |

$m_{\tilde{g}} = 200$ GeV and $m_{\tilde{t}} = m_{\tilde{q}} = 75$ GeV, we obtain a 39%, in contrast to 33%, correction in the total cross section without cuts. Including cuts in (10) only slightly increases the correction to 40%.

For completeness, in Table 3 we show the SUSY QCD corrections $\Delta\sigma$ to the $q\bar{q} \to t\bar{t}$ production rates at the Tevatron with $\sqrt{s} = 2$ TeV, as a function of $m_{\tilde{g}}$, for the first SUSY model in Table 1. The SM tree level cross section, with the cuts in (10), is 4.06 pb. At the upgraded Tevatron, it is expected to measure the cross section of $t\bar{t}$ pair production to $\sim 10\%$ (6%) with a $2 \text{ fb}^{-1}$ ($10 \text{ fb}^{-1}$) integrated luminosity [9]. Therefore, it is a challenging task to detect the difference in rates from SUSY QCD contributions predicted by these models unless gluinos are very light (of the order of 1 GeV). Nevertheless, the parity violating effect induced by the SUSY interactions could be observable with a large integrated luminosity ($\geq 2 \text{ fb}^{-1}$) at the Tevatron.
Up to now, we have only considered the one loop SUSY QCD effects on the parity violating asymmetry \(\mathcal{A}\) in \(tt\) pair production. Amusingly, the parity-violating asymmetry induced by the SUSY QCD interactions can also occur at the Born level. If gluinos are very light, of the order of 1 GeV, this asymmetry can be generated by the tree level process \(\tilde{g}\tilde{g} \to tt\). Unfortunately, its production rate is smaller than the \(gg \to tt\) rate, which is only about one tenth of the \(qq \to tt\) rate at the Tevatron. Hence, it cannot be measured at the Tevatron. However, at the CERN Large Hadron Collider (LHC), the production rate of \(\tilde{g}\tilde{g} \to tt\) is large enough to allow the measurement of the parity-violating asymmetry induced by the SUSY QCD interactions. The asymmetry in the production rates of \(t_L\bar{t}\) and \(t_R\bar{t}\), generated by the \(\tilde{g}\tilde{g}\) fusion process alone, can reach about 10\% for \(M_{t\bar{t}}\) larger than about 500 GeV. We shall present its details and include the effect from the \(gg\) and \(qq\) fusion processes in a future publication [23].

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Appendix

We give here the form factors for the matrix elements appearing in Eqs. (8)-(12). They are written in terms of the conventional one-, two-, three- and four-point scalar loop integrals defined in Ref. [24].

\[
F_{0}^{v1\mu} = \sum_{i=1,2} \frac{3\alpha_s}{8\pi} (2m_\tilde{g}Y_i) C^\mu(-p_2, k, m_{\tilde{g}}, m_{\tilde{g}}) \\
+ \sum_{i=1,2} \frac{\alpha_s}{12\pi} [m_\tilde{g}Y_i((p_2 - p_1)^\mu C_0/2 - C^\mu)(-p_2, k, m_{\tilde{g}}, m_{\tilde{g}}, m_{\tilde{g}})]
\]

\[
F_{1}^{v1} = \sum_{i=1,2} \frac{3\alpha_s}{8\pi} [C^{20} X_i - (m_t^2 + m_{\tilde{g}}^2)C_0 X_i - 2m_t m_{\tilde{g}} C_0 Y_i](-p_2, k, m_{\tilde{g}}, m_{\tilde{g}}) \\
+ \sum_{i=1,2} \frac{\alpha_s}{3\pi} [B_1 X_i - 2m_t m_{\tilde{g}} B_0 Y_i + 2m_{\tilde{g}}^2 B_1 X_i](m_{\tilde{g}}^2, m_{\tilde{g}}, m_{\tilde{g}})
\]

\[
F_{3}^{v1\mu} = \sum_{i=1,2} \frac{3\alpha_s}{8\pi} (m_t C^\mu X_i)(-p_2, k, m_{\tilde{g}}, m_{\tilde{g}})
\]

\[
F_{4}^{v1\mu} = F_{3}^{v1\mu}
\]

\[
F_{6}^{v1\mu} = \sum_{i=1,2} \frac{3\alpha_s}{8\pi} (-2X_i) C^{\mu\nu}(-p_2, k, m_{\tilde{g}}, m_{\tilde{g}})
\]
\[ F^{A_{1\mu\nu}}_1 = \sum_{i=1,2} \frac{3\alpha_s}{8\pi} Z_i (C^{20} + (m_i^2 - m_\bar{g}^2)C_0) (-p_2, k, m_\bar{g}, m_\bar{g}) \]
\[ + \sum_{i=1,2} \frac{\alpha_s}{3\pi} Z_i B_1 (m_\bar{g}^2, m_\bar{g}, m_\bar{g}) \]
\[ F^{A_{1\mu}}_3 = \sum_{i=1,2} \frac{3\alpha_s}{8\pi} Z_i m_\mu C^\mu \]
\[ F^{A_{1\mu}}_4 = -F^{A_{1\mu}}_3 \]
\[ F^{A_{1\mu\nu}}_6 = \sum_{i=1,2} \frac{3\alpha_s}{8\pi} (-2Z_i) C^{\mu\nu} (-p_2, k, m_\bar{g}, m_\bar{g}) \]
\[ + \sum_{i=1,2} \frac{\alpha_s}{2\pi} Z_i \left[ \frac{1}{2} - \frac{1}{2} \frac{C^{\mu\nu} / 2 - C^{\mu\nu}}{\alpha_s} \right] (-p_2, k, m_\bar{g}, m_\bar{g}) \]
\[ F^{\nu} = \frac{3\alpha_s}{4\pi} (C^{20} - m_\bar{g}^2 C_0) (p_2, k, m_\bar{g}, m_\bar{g}) \]
\[ + \frac{2\alpha_s}{3\pi} B_1 (m_\bar{g}^2, m_\bar{g}, m_\bar{g}) \]
\[ F^{\nu\mu} = \frac{3\alpha_s}{4\pi} (-2) C^{\mu\nu} (p_2, k, m_\bar{g}, m_\bar{g}) \]
\[ + \frac{\alpha_s}{6\pi} (-p_2, k, m_\bar{g}, m_\bar{g}) \]
\[ F^s_0 = \frac{3\alpha_s}{2\pi} \left[ (B_{21} + B_1 + \frac{1}{6} - (m_\bar{g}^2 (B_0 + 1) - 2B_{22}) / k^2) (k^2, m_\bar{g}, m_\bar{g}) \right] \]
\[ - (B_{21} + B_1 + \frac{1}{6} - m_\bar{g}^2 B_0 + 2B_{22}) (0, m_\bar{g}, m_\bar{g}) \]
\[ + \sum_{i=1,2} \frac{\alpha_s}{4\pi} \left[ (2B_{22} (k^2, m_\bar{g}, m_\bar{g}) - A_0 (m_\bar{g})) / k^2 - 2B_{22} (0, m_\bar{g}, m_\bar{g}) \right] \]
\[ F^{DB}_1 = \sum_{i=1,2} \frac{\alpha_s}{4\pi} (m_\bar{g}^2 + m_i^2) X_i + 2m_\bar{g} m_i Y_i) D_0 (-p_2, p_4, p_3, m_\bar{g}, m_\bar{g}) \]
\[ F^{DB}_2 = \sum_{i=1,2} \frac{\alpha_s}{4\pi} (m_\mu X_i + m_\bar{g} Y_i) D_0 (-p_2, p_4, p_3, m_\bar{g}, m_\bar{g}) \]
\[ F^{DB\mu}_3 = \sum_{i=1,2} \frac{\alpha_s}{4\pi} X_i D^{\mu\nu} (-p_2, p_4, p_3, m_\bar{g}, m_\bar{g}) \]
\[ F^{DB}_4 = \sum_{i=1,2} \frac{\alpha_s}{4\pi} Z_i (m_i^2 - m_\bar{g}^2) D_0 (-p_2, p_4, p_3, m_\bar{g}, m_\bar{g}) \]
\[ F^{DB\mu}_5 = \sum_{i=1,2} \frac{\alpha_s}{4\pi} Z_i m_\mu D^{\mu} (-p_2, p_4, p_3, m_\bar{g}, m_\bar{g}) \]
\[
F_6^{DB\mu\nu} = \sum_{i=1,2} \frac{\alpha_s}{4\pi} Z_i D^{\mu\nu}(-p_2, p_4, p_3, m_i, m_{\tilde{q}}, m_{\tilde{q}}, m_{\tilde{g}})
\]

\[
F_{1,3,4,6}^{CB} = F_{1,3,4,6}^{DB}(p_2 \to p_1)
\]

\[
F_{2,5}^{CB} = -F_{2,5}^{DB}(p_2 \to p_1)
\]

In the above:

\[
k = p_1 + p_2 = p_3 + p_4, \quad \hat{s} = k^2,
\]

\[
X_i = a_i^2 + b_i^2 = 1, \quad Y_i = a_i^2 - b_i^2 = \mp \sin(2\hat{\theta}_i), \quad Z_i = 2a_ib_i = \mp \cos(2\hat{\theta}_i).
\]

where \( a_1, b_1, a_2, b_2 \) are given by

\[
a_1 = \frac{1}{\sqrt{2}} (\cos \theta_\tilde{t} - \sin \theta_\tilde{t}), \quad b_1 = -\frac{1}{\sqrt{2}} (\cos \theta_\tilde{t} + \sin \theta_\tilde{t}),
\]

\[
a_2 = -\frac{1}{\sqrt{2}} (\cos \theta_\tilde{t} + \sin \theta_\tilde{t}) = b_1, \quad b_2 = -\frac{1}{\sqrt{2}} (\cos \theta_\tilde{t} - \sin \theta_\tilde{t}) = -a_1.
\]

Also,

\[
B'_0 = \frac{\partial B_0(p^2, m_1, m_2)}{\partial p^2}, \quad B'_1 = \frac{\partial B_1(p^2, m_1, m_2)}{\partial p^2}, \quad B'_{22} = \frac{\partial B_{22}(p^2, m_1, m_2)}{\partial p^2},
\]

\[
C^{20} = g_{\mu\nu} C^{\mu\nu} - \frac{1}{2}
\]
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