Self-organization, resources and strategies in a minority game

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Abstract

We find that the existence of self-organization of the members of a recently proposed minority game, depends on the type of update rules used. The resulting resource distribution is studied in some detail, and a related strategy scheme is considered, as a tool to improve the understanding of the model.

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The emergency of organization inside a population can be the result of local interactions between its members. This type of problems have been under study for a long time, and can be schematically reduced, for instance, to an Ising-like model. A problem that has recently become of interest is the self organization of a population without direct interactions between its members, but with a feedback mechanism related with its collective behavior. The minority game, introduced by Challet and Zhang [1], addresses one of the simplest situations of this kind. In this model every member of a population has to choose from a simple alternative, without knowing what the other members will do. Simple cases are: to buy or sell in a stock market, to select one of two possible routes, etc. At the end of the day, the winners are those ‘agents’ that happen to be in the minority side. Feedback is established by a reward system for winners and losers. In more general terms, these problems are nothing but simple examples of a situation where there is a competition for a limited resource (money, food, free highways, etc.), and individual members of a population adapt their behavior following their (recent) experiences. Arthur was the first to propose this type of approach [2], in what now is known as the El Farol bar problem.

The specific form in which every member of the population makes his choice is generically designated as his ‘strategy’. Different versions of the model are characterized by this strategy selection. In this work we will address the model proposed by Johnson et al. [3]. As in all minority games, there is an odd number of agents $N$, every one choosing between option “0” (e.g. to buy an asset) and option “1” (to sell the asset). After all agents have made their choice, the winners, i.e. those in the minority group, gain a point, while those in the majority group lose a point. A single binary digit, 0 or 1, signals the winner option. Each agent knows beforehand the previous $m$ outcomes of the game, as well as the outcomes of the most recent occurrences (histories) of all $2^m$ possible bit strings of length $m$. Now, Johnson et al. assign to each agent a single number $p (0 \leq p \leq 1)$: given a history, the agent will either choose the same outcome as that stored in the memory, with probability $p$, or will choose the opposite with probability $(1-p)$. Strategies can be modified, following the evolution of the game. Thus, if an agent’s account is below a threshold value $d < 0$, he gets a new strategy, whose value $p’$ is chosen with an equal probability from the interval $(p - r/2, p + r/2)$, where $0 \leq r \leq 2$; in what follows we will use the simpler notation $p \rightarrow p’ = p \pm \Delta p$. Simultaneously (and to some extent, arbitrarily), his account is reset to zero. As we will discuss below, the existence of negative points, combined with the behavior at the threshold, introduce some confusion at the time of considering the resources. In the following, we will refer to this combinations as the $d$-rule.

The work of Johnson et al. has shown that, as a result of the correlations established by those rules, agents self-organize, in such a way that the frequency distribution $P(p)$ becomes strongly peaked around both $p \simeq 0$ and $p \simeq 1$ (see curve $R1$ in Fig. [1]).

Interesting as it is, this work leaves open some questions. First, it would be interesting to check the robustness of self-organization, under changes in the strategy actualization rules. As we will see, it also is of interest in this case to study with some detail the question of the resulting distribution of the resources.

Our notation is as follows. A single realization of the game involves $n_t$ time steps. All results are then averaged over $n_s$ different samples. The total amount of points to be distributed in this process is $T = N * n_t * n_s$. We call $n_+$ ($n_-$) the number of positive
(negative) points, i.e. those assigned to winner (loser) agents. Obviously, \( T = n_+ + n_- \). There is also certain amount of points, \( N_{\text{lost}} \), that are eliminated from the game, namely those assigned to any agent changing his strategy, \( p \to p' \). After all \( n_s \) games are played, there will be \( N_{\text{acc}} \) accumulated points, resulting simply from the sum of all accounts, at the end of every game. Note that, in general, \( T > N_{\text{lost}} + N_{\text{acc}} \), because there are both positive and negative contributions to the accounts. Whenever necessary, we used reflective boundary conditions.

We have made extensive numerical simulations, both with \( p \to p' = p \pm \Delta p \), the original rule \( \equiv R1 \), and also with \( p \to p' = (1 - p) \pm \Delta p \) (rule \( R2 \)), a seemingly minor modification of the actualization rule for the strategies. In the latter case, any loosing agent will change his mind and pick a ‘complementary’ strategy; in other words, if the initial selection was, say, to choose preferentially option “0” then, after losing, the agent will rather prefer option “1”. The resulting distribution functions are shown in Fig.1. Our results for \( R1 \) reproduce (without noise) those of Johnson et al. It is apparent that there are important differences between both cases: while self-organization shows up very clearly for \( R1 \), it is practically absent in \( R2 \), which remains near to its initial (homogeneous) distribution \[4\]. This result shows, apparently, that the presence of self-organization itself depends on the kind of strategy employed.

Consider now the question of the distribution of available resources (i.e. points). We have already mentioned that every agent losing more than \( d \) points gets his account reset to zero, whereby all those points leave the game. This introduces some confusion at the time to interpret our results. The standard interpretation \[4\] only takes into account positive points. If we ignore for a moment the \( d \)-rule, ans simply consider all positive points added, we found that the accumulated earning per time step, \( n_+ / T \), is \( \approx 0.47 \) in both cases, within a small error. Notice that this is very near \( (N - 1) / 2N = 0.495 \), the maximum possible gain with \( N = 101 \). The behavior of both cases, however, is also very different in this regard. In fact, the distribution of positive points earned by an agent, \( C^+(p) \), follows closely the form of the corresponding \( P(p) \) shown in Fig.1, i.e. while earning is concentrated around the extrema for \( R1 \), it is distributed for \( R2 \).

On the other hand, application of the \( d \)-rule modifies sensibly this interpretation. The number of points earned by an agent with strategy \( p \), \( C(p) \), is the algebraic sum of both positive and negative points. Every time \( C(p) < d \), all these points (positive and negative) are discarded. At the end of all games, it is natural to choose the ratio \( G = N_{\text{acc}} / T \) as the magnitude of interest. We find that \( G \) is always positive, but vanishes as \( T \to \infty \). In fact, \( N_{\text{acc}} \) is proportional to both \( N \) and \( n_s \), and our results imply \( G \sim 1 / n_t \). Thus, for instance, for \( n_s = 500 \), and \( n_t = 10^6 \), it already is \( G \sim 0.00002 \). In other words, although there is self-organization, as described by the work of Johnson et al., the net resources distributed between all agents are vanishingly small. It is worth mentioning that this is not the result of adding negative and positive accounts: in the mean they are mostly positive. Simply enough, \( C(p) \) follows closely the behavior of \( P(p) \), but its magnitude vanishes in this limit. A good amount of the points involved in the game turn out to be in \( N_{\text{lost}} \) and, what is more important, \( N_{\text{lost}} \propto n_+ - n_- \). This can be described by telling that in this case there are no winners in the game \[3\]. This situation is still more pronounced in case one uses \( R2 \), the alternative strategy rule.

In fact, it is only because the accounts are adjusted periodically to zero, that they appear
to have mostly positive balance at the end of the games. In their study of this model, D’Hulst and Rodgers \[5\] used the Hamming distance between strategies and concluded that, on average, the number of points earned by an agent, \( C(p, t) \), evolves with time \( t \) following

\[
C(p, t) = -(\frac{1}{2} - \tau(p)) (t - t_0)
\]

until \( C < d \), at which point is set equal to zero. In the above expression \( \tau(p) < 1/2 \) and \( t_0 \) are constants. This is a sawtooth function of \( t \) that is always negative, or vanishes.

This analysis, however, does not take into account properly the role of the \( d \)-rule, as can be seen in the following example. Consider a game where the winner is always the same agent, while all others lose. After \( L \) time steps, the winner agent will have \( L \) points, while the remaining \((N - 1)\) agents will have \(-L\) each. Whenever \(-L < d\), the \( d \)-rule implies that only the winner keeps his points. Therefore, after a while, the net amount of points of all players is necessarily positive.

We now turn our attention to a related type of strategy. Our main concern here is to understand how we can improve the resource distribution, using rules similar to those of Ref. \[3\].

Consider a rule \( p \rightarrow p' \) which is intrinsically asymmetric (rule \( R3 \)). In this case,

\[
p' = p_0 \pm \Delta p
\]

where \( 0 \leq p_0 \leq 1 \) is constant.

Application of Eq.2 will move agents to the neighborhood of \( p_0 \). Eventually, however, there will be a majority of agents in this place, and therefore all others players will win, establishing a stationary state. We want to know the dependence of \( G \) on \( p_0, \Delta p \) and \( r \). Note that it is possible to describe some cases \( p \rightarrow p' \) as a superposition of situations with this type of update scheme. Figure 2(a) illustrates the case \( p_0 = 0.8, r = 0.2 \). The resulting frequency distribution is asymmetric. In this case \( \text{there are winners}, \) namely those agents that manage to have their strategy below 0.5 The left side of Fig.2(a) shows the gain as a function of \( p \). In Fig.2(b), on the other hand, we have the gain \( G \) \( (i.e. \) the integral of that shown in (a)) as a function of \( p_0 \), for a fixed value of \( r \).

Also, and rather unexpectedly, we can see in Fig.2 that \( G \) is almost independent of \( r \), until it approaches \( r \approx 1 \), where there is something analogous to a ‘phase transition’; it probably corresponds to the ‘transition’ between localization around \( p_0 \), and delocalization. It should be emphasized that these results are associated with the use of the \( d \)-rule. Within the standard interpretation, it can be seen that the gain increases with \( r \), at least for \( r \) smaller than \( \approx 1 \).

Finally, it should be pointed out that, although the present version shares the main ideas of the original minority game \[1\], in some respects it does not follow the same behavior. Recent work \[7\] study the model of Callet and Zhang in terms of the variable \( \alpha = P/N \) (in our case, \( P = 2^m \)), and the variance of the time series, \( \sigma^2 = \langle (n_- - n_+)^2 \rangle \) or, more specifically, the reduced variance \( z \equiv \sigma^2/N \). As it is well known, the random agent case is given by \( \sigma^2 = N, \) \( i.e. \) \( z = 1 \). This value is attained for \( \alpha_r \approx 0.2 \). Smaller values of \( \alpha \) produce a worst-than-random answer \( (z > 1) \), while the game output improves if \( \alpha > \alpha_r \) \( (z < 1) \). Moreover, they have identified two ‘phases’, characterized by the behavior of \( z = z(\alpha) \). For \( \alpha < \alpha_c \), the reduced variance decreases with \( \alpha \), but for \( \alpha > \alpha_c \) it becomes an
increasing function of $\alpha$. Numerical simulations give the critical value $\alpha_c \approx 0.34$ A theoretical description of this game has been developed [5], based on an analogy with spin glasses. In any case, it is apparent that, for fixed $N$, the response of the game is strongly dependent with $P$ (i.e. $m$ in our case). We have not completed a systematic study in this respect, but it is rather clear that the present model is, in fact, almost totally independent of the actual value of $m$. On the other hand, we find $z \approx 0.04 - 0.08$ for our three upgrade rules (although $\alpha = 2^{3}/101 \simeq 0.08$). This illustrates an important difference between both formulations.

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[4] It can be argued that there is a slight indication of self-organization near $p = 0$ and 1, but the great majority of the agents remain in between, with an almost uniform distribution. We have made much longer simulations, checking that this behaviour it is not due to a large relaxation time
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FIGURES

FIG. 1. Frequency distribution function for two different strategy actualization rules. \( R1: \)
\[ p \rightarrow p' = p \pm \Delta p; \quad R2: \quad p \rightarrow p' = (1 - p) \pm \Delta p. \]
In both cases \( N = 101, \) \( n_t = 10^5, n_s = 10^4, d = -4, \)
\( m = 3, r = 0.2. \)

FIG. 2. Strategy rule \( p \rightarrow p' = p_0 \pm \Delta p, \) \( N = 101, \) \( n_t = 10^5, n_s = 10^4, r = 0.2, d = -4, m = 3. \)
The line in (b) is only a guide to the eyes.

FIG. 3. Strategy rule \( p \rightarrow p' = p_0 \pm \Delta p. \) \( n_t = 10^5, n_s = 500, N, d, m \) have the same values as in Fig.2. Continuous lines are only a guide to the eyes.
(b) $r = 0.2$

$a$ gain per time step

$p_0 = 0.8$

$P(p)$
$G (%)$

- $p_0 = 1$
- $p_0 = 0.8$
- $p_0 = 0.3$
- $p_0 = 0.5$

$r$

$1E-4$ $1E-3$ $0.01$ $0.1$ $1$