An efficient alternative approach for the solution of an interval integer transportation problem

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Abstract. The purpose of an Original Possible Solution (IFS) of an integer transportation interval problematic constructions an essential part of finding a minimum total transportation interval cost solution. Better initial feasible interval solution will result in fewer iterations achieving the minimum total cost solution for the interval. Various methods are obtainable in the fiction to achieve a better original possible result to the problem of interval transport. A new, effective method with row penalties is proposed in this paper to find an initial feasible interval solution to an interval transport problem. One numerical illustration demonstrates the new process. Thus, our new approach can be regarded as an alternate technique for achieving an original possible interval solution to a problem of integral interval transport.

1. Introduction

An essential component of our modern life is the shipping of goods from where they are produced to markets spend billions worldwide. Nationally, companies of dollars annually in transporting goods. The Interval transport problem (ITP) is a special type of network optimization issue dealing with transporting a homogeneous product from multiple foundations (e.g., factories) to multiple terminuses (e.g., warehouses). The ITP's goal is to find a way to accomplish this transfer of goods at minimum total cost. Gaspard Monge, the French mathematician, formalized the TP. The Russian mathematician and economist Leonid Kantorovich made important contributions in the field during World War II. Tolstoi was among the first to mathematically research the transportation problem. An article entitled "Finding Method for Finding Minimum Total Kilometers in Space Transportation Cargo Planning, in Transportation Planning Collection," Tolstoi published Volume I of the Soviet Union National Transportation Board. Leonid Kantorovich was formulated the transportation problem as linear programming problem and he published a book named “Mathematical methods in the Organization and planning of Production”.

Ganesan et.al have discussed about interval numbers [1, 2]. In this sense, this paper is formulated as interval numbers with these parameters of the transport problem. When various modes of transport are available, then we must transport goods in a cost-effective manner and also in time from sources to destinations by different means. Several methods are available to solve problems related to interval integer transport. Chanas et.al [3] studied the intervals and fuzzy extensions of traditional transport problems. Attanu Sengupta and Pal [4] Proposed interval- problem of multiple penalty factors in transportation. Das et.al [5] addressed multi-objective transport issues with parameters of the rate, source and destination intervals. Oliveira and Antunes explored several models of objective linear programming with coefficients of interval. Fegade et.al [6] used interval and triangular membership
functions to deliver an ideal clarification to the transference problem. Sudhakar and Navaneetha Kumar [7] have proposed a new approach to finding an optimal solution for problems with integral interval transport. Pandian and Natarajan [8] established a innovative approach to discovery an ideal result to the complete transport problems of the integer. Ramesh Kumar and Murugesan [9] have proposed a new optimal approach to the Fuzzy Intervals problem. Juman, Z.A.M.S., and Hoque, M.A. [10] have developed a experimental result procedure to reach the lowest overall cost limits of shipping a homogenous invention through changing difficulties and goods. They also developed a heuristic technique of solution [11] to meet the minimum total cost limits of carrying a identical invention through particular difficulties and goods. Safi and Razmjoo [12] discussed interval parameters for a fixed charge transportation problem. Ramesh et.al [13,14] discussed the linear interval programming and its principle of duality with a generalized arithmetic interval. They also [15] suggested a new two-vehicle solution costing different interval transport problems. Sophia Porchelvi and Anitha [16] presented a comparative analysis of the optimal solution between the transport interval and the problem of transhipment time. Akilbasha et.al have proposed [17] an groundbreaking and precise method for solving problems of complete interval transport of integer. Padma karthiyayini et.al [18] presented an innovative approach for solving the problem of transportation at complete intervals. Gurupada Maity et.al [19] discussed the time variability in sustainable development of multi-interval transport problem. Generally, most current techniques only provide crisp solutions to the problem of interval transportation In this paper we propose an optimal diagonal algorithm under a generalized interval arithmetic to solve the problem of complete interval transportation of integer without converting the crisp equivalent question. An example of the algorithm proposed here.

The structure of this article is given as follows: Recall related basic concepts in segment 2. Segment 3 deals with the some basic definitions and theorems. Segment 4 introduces a Row penalty method under generalized interval arithmetic. Segment 5 gives a numerical example. Conclusions are given in Segment 6

2. Preliminaries

The objective of this segment is to present some explanations, thoughts and findings that are suitable for our additional thought

2.1. Interval numbers

Let \( \tilde{a} = [a_1, a_2] = \{ x \in \mathbb{R} : a_1 \leq x \leq a_2 \} \) be an interval on the real line \( \mathbb{R} \). If \( \tilde{a} = a_1 = a_2 = a \), then \( \tilde{a} = [a, a] = a \) is a real number (or a degenerate interval). We shall use the terms interval and interval number interchangeably. The mid-point and width (or half-width) of an interval number \( \tilde{a} = [a_1, a_2] \) are defined as:

\[
\mu(\tilde{a}) = \frac{a_1 + a_2}{2} \quad \text{and} \quad w(\tilde{a}) = \frac{a_2 - a_1}{2}.
\]

The interval number \( \tilde{a} \) can also be expressed in terms of its midpoint and width as \( \tilde{a} = [a_1, a_2] = (\mu(\tilde{a}), w(\tilde{a})) \).

2.2. Ranking of Interval Numbers

Sengupta and Pal [19] proposed a simple and efficient index for comparing any two intervals on \( \mathbb{R} \) through decision maker’s satisfaction.

**Definition 2.2.1.** Let \( \preceq \) be an extended order relation between the interval numbers \( \tilde{a} = [a_1, a_2], \tilde{b} = [b_1, b_2] \) in \( \mathbb{R} \), then for \( \mu(\tilde{a}) < \mu(\tilde{b}) \), we construct a premise \( \tilde{a} \preceq \tilde{b} \) which implies that \( \tilde{a} \) is inferior to \( \tilde{b} \) (or \( \tilde{b} \) is superior to \( \tilde{a} \)).

An acceptability function \( A_\preceq : \mathbb{R} \times \mathbb{R} \to [0, \infty) \) is defined as:

\[
A_\preceq(\tilde{a}, \tilde{b}) = A(\tilde{a} \preceq \tilde{b}) = \frac{\mu(\tilde{b}) - \mu(\tilde{a})}{w(\tilde{b}) + w(\tilde{a})}, \quad \text{where} \quad w(\tilde{b}) + w(\tilde{a}) \neq 0.
\]

\( A_\preceq \) may be interpreted as the grade
of acceptability of the the first interval number to be inferior to the second interval number. For any two interval numbers \( \tilde{a} \) and \( \tilde{b} \) in \( \mathbb{IR} \) either \( A(\tilde{a} \preceq \tilde{b}) \geq 0 \) (or \( A(\tilde{b} \succeq \tilde{a}) \geq 0 \) (or \( A(\tilde{a} \preceq \tilde{b}) + A(\tilde{b} \preceq \tilde{a}) = 0 \).

If \( A(\tilde{a} \preceq \tilde{b}) = 0 \) and \( A(\tilde{b} \preceq \tilde{a}) = 0 \), then we say that the interval Numbers \( \tilde{a} \) and \( \tilde{b} \) are equivalent (non-inferior to each other) and we denote it by \( \tilde{a} \approx \tilde{b} \). Also if \( A(\tilde{a} \preceq \tilde{b}) \geq 0 \) then \( \tilde{a} \preceq \tilde{b} \) and if \( A(\tilde{b} \preceq \tilde{a}) \geq 0 \),then \( \tilde{b} \preceq \tilde{a} \).

2.3. A New Interval Arithmetic

Ming Ma et al. [20] have proposed a new fuzzy arithmetic based upon both location index and fuzziness index function. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which are the least upper bound and greatest lower bound in the lattice \( L \). That is for \( a, b \in L \) we define \( a \lor b = \max\{a,b\} \) and \( a \land b = \min\{a,b\} \).

For any two intervals \( \tilde{a} = [a_1, a_2], \tilde{b} = [b_1, b_2] \in \mathbb{IR} \) and for \( * \in \{+, -, \times, \div\} \), the arithmetic operations on \( \tilde{a} \) and \( \tilde{b} \) are defined as:

(i). Addition : \( \tilde{a} + \tilde{b} = \langle m(\tilde{a}), w(\tilde{a}) \rangle + \langle m(\tilde{b}), w(\tilde{b}) \rangle = \langle m(\tilde{a}) + m(\tilde{b}), \max\{w(\tilde{a}), w(\tilde{b})\} \rangle \).

In particular ,

(ii). Subtraction : \( \tilde{a} - \tilde{b} = \langle m(\tilde{a}), w(\tilde{a}) \rangle - \langle m(\tilde{b}), w(\tilde{b}) \rangle = \langle m(\tilde{a}) - m(\tilde{b}), \max\{w(\tilde{a}), w(\tilde{b})\} \rangle \).

(iii). Multiplication : \( \tilde{a} \times \tilde{b} = \langle m(\tilde{a}), w(\tilde{a}) \rangle \times \langle m(\tilde{b}), w(\tilde{b}) \rangle = \langle m(\tilde{a}) \times m(\tilde{b}), \max\{w(\tilde{a}), w(\tilde{b})\} \rangle \).

(iv). Division : \( \tilde{a} \div \tilde{b} = \langle m(\tilde{a}), w(\tilde{a}) \rangle \div \langle m(\tilde{b}), w(\tilde{b}) \rangle = \langle m(\tilde{a}) \div m(\tilde{b}), \max\{w(\tilde{a}), w(\tilde{b})\} \rangle \), provided \( m(\tilde{b}) \neq 0 \).

3. Main results

Consider a question of transport at complete intervals through \( m \) bases as well as \( n \) numbers-including endpoints. Let \( \tilde{a}_i \geq \tilde{0} \) exist the obtainability on basis \( i \) and \( \tilde{b}_j \geq \tilde{0} \) exist the requirement at destination \( j \). Let \( \tilde{c}_{ij} \geq \tilde{0} \) stay the component break moving charge from basis \( i \) to terminus \( j \). Let \( \tilde{x}_{ij} \) represent the sum of break elements to be elated from basis \( i \) to terminus \( j \). Currently the difficult is to discovery a possible approach of shipping the obtainable quantity at every basis to fulfill the request at each terminus so that the full break shipping cost is reduced.

3.1. Mathematical formulation of interval transportation problem

The mathematical model of fully interval transportation problem is as follows

Minimize \( Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij} \)

subject to \( \sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{a}_i, \quad i = 1, 2, 3, ..., m \)

\( \sum_{i=1}^{m} \tilde{x}_{ij} = \tilde{b}_j, \quad j = 1, 2, 3, ..., n \)

(3.1)

and \( \tilde{\tilde{a}}_i \geq \tilde{0}, \tilde{\tilde{b}}_i \geq \tilde{0} \) for all \( i \) and \( j \) and \( \tilde{\tilde{a}}_i, \tilde{\tilde{b}}_i, \tilde{\tilde{c}}_{ij}, \tilde{\tilde{x}}_{ij} \) in \( \mathbb{IR} \), where \( \tilde{\tilde{c}}_{ij} \) is the interval unit transportation cost from \( i^{th} \) source to the \( j^{th} \) destination. The goal is to reduce total fuzzy transport costs, in this paper the problem of fuzzy
transport is solved by the interval version of Vogel's and MODI method. This difficult of interval transport is expressly illustrated by the succeeding table of interval transport in table 1 to 7.

Table 1. Interval transportation table.

| Sources | Demand | Destination |
|---------|--------|-------------|
| 1       | \(\tilde{c}_{11}\) | 2           | ... | N           | Supply |
| 2       | \(\tilde{c}_{21}\) | \(\tilde{c}_{12}\) | ... | \(\tilde{c}_{2n}\) | \(\tilde{a}_1\) |
| M       | \(\tilde{c}_{m1}\) | ... | \(\tilde{c}_{m2}\) | ... | \(\tilde{c}_{mn}\) | \(\tilde{a}_n\) |

**Definition 3.1.1.** Fuzzy feasible solution is defined as a set of non-negative portions that satisfy (in the equivalent sense the row and the column limit).

**Definition 3.1.2.** A potential interval explanation to a shipping interval problematic with \(m\) bases and \(n\) endpoints is assumed to be a simple interval explanation if the sum of confident distributions is \((m+n-1)\). If the quantity distribution in a simple interval answer is a lesser amount of than \((m+n-1)\), then the simple feasible solution is entitled debased interval.

**Definition 3.1.3.** An interval possible result is said to be ideal answer if it reduces the general cost of transporting intervals.

### 3.2. Basic Theorems

**Theorem 3.2.1. (Existence of an interval feasible solution)**

The necessary and sufficient condition for the existence of an interval feasible solution to the fully interval transportation problem (3.1) is,

\[
\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j, \quad i = 1, 2, 3, ..., m \quad \text{and} \quad j = 1, 2, 3, ..., n
\]

(Total supply \(\approx\) Total demand).

**Proof: (Necessary condition)**

Let there exist an interval feasible solution to the fully interval transportation problem

\[
\text{Minimize} \quad \tilde{Z} \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{x}_{ij}
\]

subject to

\[
\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i, \quad i = 1, 2, 3, ..., m.
\]

(3.3)

\[
\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j, \quad j = 1, 2, 3, ..., n.
\]

and \(\tilde{x}_{ij} \geq \tilde{0}\) for all \(i\) and \(j\).

From \(\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i, (i = 1, 2, 3, ..., m)\), we have \(\sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{x}_{ij} \approx \sum_{i=1}^{m} \tilde{a}_i\) \n
(3.4)

Also from \(\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j, (j = 1, 2, 3, ..., n)\), we have \(\sum_{j=1}^{n} \sum_{i=1}^{m} \tilde{x}_{ij} \approx \sum_{j=1}^{n} \tilde{b}_j\)

(3.5)

From equations (3.4) and (3.5), we have \(\sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j\)
(Sufficient condition)

Since all \( a_i \) and \( b_j \) are positive, \( x_{ij} \) need be all positive. Hence equation (3.2) profits a feasible solution.

**Theorem 3.2.2.**

The measurement of the source of an \((m \times n)\) entirely interval transportation are \((m+n-1)\). That is a entirely interval transportation problem has only \((m+n-1)\) independent essential restrictions and its simple feasible solution has only \((m+n-1)\) optimistic modules.

**Proof:** Consider a fully interval transportation problem with \( m \) sources and \( n \) destinations,

Minimize \[ \hat{Z} \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \hat{c}_{ij} \hat{x}_{ij} \]

subject to \[ \sum_{j=1}^{n} \hat{x}_{ij} \approx \hat{a}_i, \ i = 1, 2, 3, \ldots, m. \]

\[ \sum_{i=1}^{m} \hat{x}_{ij} \approx \hat{b}_j, \ j = 1, 2, 3, \ldots, n. \]

and \( \hat{x}_{ij} \geq \hat{0} \) for all \( i \) and \( j \).

Let's say that there are \( m \) rows (supply restriction equations) and \( n \) columns (demand constraint equations) for a complete interval transport. Hence there are completely \((m+n)\) equations of constraint. This is due to the condition that \( \sum_{i=1}^{m} \hat{a}_i \approx \sum_{j=1}^{n} \hat{b}_j \) which is the last requirement constraint. So one of \((m+n)\) constraints can always be extracted from the remainder \((m+n-1)\). Therefore there are only \((m+n-1)\) independent constraints, and only \((m+n-1)\) have positive components in its basic feasible solution.

**Theorem 3.2.3.** The values of the simple solution available at intervals are all discrepancies between the partial sum of \( \hat{a}_i \) and the partial \( \hat{b}_j \). That is \( \hat{x}_{ij} = \pm \sum_{k=1}^{m} \hat{r}_k \hat{a}_i \pm \sum_{l=1}^{n} \hat{s}_l \hat{b}_j \), where \( \hat{r}_k, \hat{s}_l \) are either \( \hat{1} = [1,1] \) or \( \hat{0} = [0,0] \).

**Theorem 3.2.4.** The fully interval transportation problem has a triangular basis.

**Proof:** We note that every equation has a basic variable; otherwise, the equation cannot be satisfied for \( \hat{a}_i \neq 0, \ \hat{b}_j \neq 0 \). Suppose every row and column equations has at least two basic variables, since there are \( m \) rows and \( n \) columns, the total number of basic variables in row equations and column equations will be at least \( 2m \) and \( 2n \) correspondingly. Suppose if the entire number of simple variables is \( D \), then clearly \( D \geq 2m, D \geq 2n \).

Case (i). If \( m < n \), then \( m + n + n + n \Rightarrow m + n < 2n \Rightarrow m + n < 2n < D \Rightarrow m + n < D \).

Case (ii). If \( m > n \), then \( m + n + m + n \Rightarrow m + m > m + n \Rightarrow 2m > m + n \Rightarrow m + n < 2m \Rightarrow m + n < 2m \leq D \Rightarrow m + n < D \).

Case (iii). If \( m = n \), then \( m + n = m + n \Rightarrow m + m = m + n \Rightarrow 2m = m + n \Rightarrow m + n = 2m \Rightarrow m + n = 2m < D \Rightarrow m + n < D \).

Thus in all cases \( D \geq m + n \). But the number of basic variables in a fully interval transportation problem is \((m+n-1)\) which is a contradiction.

Hence at least one equation remains, either row or column having just one basic component. Let the \( r^{th} \) equality require only one simple variable. Let \( \hat{x}_{rs} \) be the only simple variable in the \( r^{th} \) row and \( s^{th} \) column, then \( \hat{x}_{rs} \approx \hat{a}_r \). Remove \( r^{th} \) row from the structure of equations and additional \( \hat{x}_{rs} \approx \hat{a}_r \) in \( s^{th} \)
column equation and exchange \( \tilde{b}_s \) by \( \tilde{b}_s' = \tilde{b}_s - \tilde{\alpha}_s \). Afterwards removing the \( r^{th} \) row, the structure has \((m+n-2)\) linearly independent restrictions. Therefore the number of simple variables is \((m+n-2)\).

Repeat the cycle and conclude that there is an equation that only has one simple variable in the reduced set of equations. But if this is in \( s^{th} \) column equation, then it will have two simple variables. That concludes that there is an equation in our original equations system that has at least two basic variables. Start the process of getting to the theorem.

4. Methods of solving Interval Transportation Problem

There are several methods for finding the initial feasible solution to a problem with interval transport. Few of them are North – West Corner Rule (2004), Minimum Cost Method (2004), Juman and Hoque Method (2015), Vogel’s Approximation Method (1958). Using one of the above methods, a minimum total cost solution technique can be implemented to achieve the Minimum Total Cost Solution (MTCS), after finding the Initial Feasible Solution (IFS) of the interval transport problem.

4.1. Maximum overall cost solution of the problem of interval transport

The problem of interval transportation is solved using the transportation algorithm Interval. The transportation interval algorithm consists of two steps

Step 1: Evaluate an initial feasible solution (IFS) for the IFS problem
Step 2: MODI method check for optimality

4.2. Objective of this Research

The main purpose of this research is to develop an alternative approach, in particular for the Juman and Hoque (2015) approach of seeking an original possible solution to a balanced transport problem. Here we have developed a Row Penalty Method (RPM) to obtain an effective IFS (optimum or very close to optimum) to solve the traditional problem of interval transport. The steps in producing the initial feasible solution involving a Row penalty method are defined below:

4.2.1. Algorithm for Row penalty method

Step 1: Present the question of intervals in the transport board.
Step 2: Define the cost of supply, demand and unit transport of all interval parameters in relations of the average and half breadth of the transportation issue. These are in the form of \( \tilde{a} = [\tilde{a}_1, \tilde{a}_2] = (m(\tilde{a}), w(\tilde{a})) \).
Step 3: Test whether the problem is balanced, or not. If it is unbalanced then add dummy request or dummy supplier to resolve the issue with zero cost of transport.
Step 4: Classify the least cost cell for every row of the transportation environment Interval. Allocate the amounts of the particular supply there.
Step 5: Test if the column number for allocated allocation in each of the columns is less than or equal to the respective demand amount (without taking into account any crossed column, if existing). If you do, go to stage 12
Step 6: Since the row covering that portion, measure the transformation among the next lowest and the lowest unit cost for each of the allocations in an unmet column, and classify the smallest of them (recognize the minimum with the highest component price in the tie case).
Step 7: Check if there is a cell (or cells) for each of the unmet column allocations in a row that does not include the next smallest component price parallel to the smallest of the distance among the next smallest and the last component rate in a row. If such a column exists, mark the former uncompleted column to move 10
Step 8: Take out any two unmet columns. Find differences for each of these in the unmet column among the 2nd minimum and the smallest component prices.

Let the smallest of the differences correspond to the lowest unit cost \( c_1C_1 \), and the smallest of the differences correspond to the lowest unit cost \( e_1e_1 \) for the other unmet column. Let \( c_1C_1, c_2C_2 \) and
be the 1st, 2nd and 3rd least unit cost in a row, and, in another row, the 1st, 2nd and 3rd least unit cost. If \( c_2 < c_1 < c_3 \), the unmet column with the lowest unit cost \( c_1 c_1 \) is found. Else defines the other unfulfilled column (with the least unit cost \( e_1 e_1 \)).

**Step 9:** Take any two columns unmet for every. If the two columns do not have such a relation, then pick the column with minimal difference. And move on to phase 10.

**Step 10:** Considering the identified unmet column in step 6, 7, 8 or 9 and parallel to the smallest of the differences between the second least and the least unit cost in a row for each of the allocations in this column, transfer the maximum possible amount of excess demand quantity which can make next column met, from the least unit cost cell to the next least unit cost cell in a row. (If the demand value is smaller than the value of the allocation in the unmet column, transfer the full amount of the allocation). If there remains more excess demand in the selected column, do the same for the next smallest difference of the second least and least unit costs and continue the transferring process until the selected unmet column become met.

**Step 11:** Cross the column that was completely satisfied with the elimination of the quantity of excess demand objective finished, and move to step 5.

**Step 12:** End, and take the current answer as the simple original possible answer.

### 4.2.2. Interval Version of MODI Method

**Step 1:** Begin with the simple feasible solution interval consisting of allocations \((m+n-1)\) to independent positions.

**Step 2:** Determine the values of \((m+n)\) dual variables and all rows and columns such that the following condition is fulfilled for each cell occupied \((i, j)\).

**Step 3:** Total the opportunity cost \( \hat{d}_{ij} \) for each unoccupied cell \((i, j)\) by using the formula:

\[
\hat{d}_{ij} = \hat{c}_{ij} - (\hat{u}_i + \hat{v}_j)
\]

for all \(i, j\).

**Step 4:** Check the sign of the opportunity cost \( \hat{d}_{ij} \) for each unoccupied cell \((i, j)\) and conclude that

(i) If all \( \hat{d}_{ij} \geq 0 \), then the current solution is optimal.

(ii) If at least one \( \hat{d}_{ij} < 0 \), then the current solution is not optimal and an improved solution can be obtained.

In this case, the unoccupied cell with the most negative value of \( \hat{d}_{ij} \) is considered for the new transportation schedule.

**Step 5:** Build a closed path (loop) with the highest negative opportunity cost for the unoccupied cell \((i, j)\). Place a \((+)\) sign in that cell and pass along the rows (or columns) to locate a cell that has been filled. In this cell display a\((-)\) sign and find another occupied cell. Replicate the cycle and instead mark the occupied cells with signs \((+\) and \((-)\). Close the way back to the unoccupied cell you picked.

**Step 6:** Select the smallest number marked with \((-)\) sign among the cells. Allocate this value to the loop's unoccupied cell, and increase and deduct it according to their signs in the occupied cells. Therefore an better solution is obtained by using this approach to measure the overall cost of transportation.

**Step 7:** Check the updated solution additional in place of optimality. The process ends when all \( \hat{d}_{ij} \geq 0 \), for unoccupied cells.

### 5. Numerical example

**Example 5.1.** Consider the subsequent question of transport intervals discussed by G.Padma karthiyayini et.al. [21] They suggested a new approach, namely the upper-wide approach for finding an optimal solution to transportation interval problems where transportation costs, supply and demand are intervals.
Table 2. Interval Transportation table.

|   | D₁   | D₂   | D₃   | D₄   | Supply |
|---|------|------|------|------|--------|
| F₁ | [1,2] | [1,3] | [5,9] | [4,8] | [7,9]  |
| F₂ | [1,2] | [7,10]| [2,6] | [3,5] | [17,21]|
| F₃ | [7,9] | [7,11]| [3,5] | [5,7] | [16,18]|
| Demand | [10,12]| [2,4] | [13,15]| [15,17]| [40,48]|

G.Padma karthiyayini et.al.[12] achieved the ideal interval transportation cost for this problem as [128,252]. Using our method let us explain the same problem. Definite entirely the interval restrictions \( \tilde{a} = [a_1, a_2] \) in expressions of average and breadth as \( \tilde{a} = [a_1, a_2] = \{ m(\tilde{a}), w(\tilde{a}) \} \). Now the problem of transport intervals becomes

Table 3. Interval Transportation table.

|   | D₁   | D₂   | D₃   | D₄   | Supply |
|---|------|------|------|------|--------|
| F₁ | \{1.5,0.5\} | \{2.1\} | \{7.2\} | \{6.2\} | \{8.1\} |
| F₂ | \{1.5,0.5\} | \{8.5,1.5\} | \{4.2\} | \{4.1\} | \{19,2\} |
| F₃ | \{8.1\} | \{9.2\} | \{4.1\} | \{6.1\} | \{17,1\} |
| Demand | \{11,1\} | \{3.1\} | \{14,1\} | \{16,1\} | \{44,4\} |

Using the proposed algorithm, Find the least cost cell for every row of the Transportation Interval environment. Attribute the respective supply amounts there

Table 4. Interval Transportation table.

|   | D₁   | D₂   | D₃   | D₄   | Supply |
|---|------|------|------|------|--------|
| F₁ | \{8.1\} | \{1.5,0.5\} | \{2.1\} | \{7.2\} | \{6.2\} | \{8.1\} |
| F₂ | \{19,2\} | \{1.5,0.5\} | \{8.5,1.5\} | \{4.2\} | \{4.1\} | \{19,2\} |
| F₃ | \{8.1\} | \{9.2\} | \{17,1\} | \{4.1\} | \{6.1\} | \{17,1\} |
| Demand | \{11,1\} | \{3.1\} | \{14,1\} | \{16,1\} | \{44,4\} |

Unmet column | Met column | Unmet column | Met column
The number of the allocations is greater or equal to column requests. Then the cycle continues. Find the rows penalty which corresponds to the allocations in that column. Allocate excess supply inside the row’s total cost cell corresponding to the minimum fine.

**Table 5. Interval Transportation table.**

|       | D_1     | D_2     | D_3     | D_4     | Supply | Penalty |
|-------|---------|---------|---------|---------|--------|---------|
| F_1   | (5,1)   | (3,1)   | (7,2)   | (6,2)   | (8,1)  | (0.5,1) |
|       | (1.5,0.5) | (2,1)   |         |         |        |         |
| F_2   | (19,2)  | (8.5,1.5) | (4,2)   | (4,1)   | (19,2) | (2.5,1) |
|       | (1.5,0.5) |         |         |         |        |         |
| F_3   | (8,1)   | (9,2)   | (17,1)  | (4,1)   | (6,1)  | (17,1)  |
|       |         |         | (4,1)   |         |        | (2,1)   |
| Demand | (11,1) | (3,1)   | (14,1)  | (16,1)  | (44,4) |

The cycle continues to use the row penalty approach until we get the original possible solution (sum of the allocations ≤ demand of the column).

**Table 6. Interval Transportation table.**

|       | D_1     | D_2     | D_3     | D_4     | Supply | Penalty |
|-------|---------|---------|---------|---------|--------|---------|
| F_1   | (5,1)   | (3,1)   | (7,2)   | (6,2)   | (8,1)  | -       |
|       | (1.5,0.5) | (2,1)   |         |         |        |         |
| F_2   | (19,2)  | (8.5,1.5) | (4,2)   | (4,1)   | (19,2) | (2.5,1) |
|       | (1.5,0.5) |         |         |         |        |         |
| F_3   | (8,1)   | (9,2)   | (14,1)  | (3,1)   | (17,1) | (2,1)   |
|       |         |         | (4,1)   | (6,1)   |        |         |
| Demand | (11,1) | (3,1)   | (14,1)  | (16,1)  | (44,4) |

The table below For in-column allocation. check if the number of the column is fewer than or equivalent to the particular amount of demand then take the present result as the original possible simple result.
Table 7. Interval Transportation table.

|          |          |          |          |          |
|----------|----------|----------|----------|----------|
| \(1.5, 0.5\) | \((5, 1)\) | \(2.1\)  | \((3, 1)\) |          |
| \(6.1\)    | \(8.5, 1.5\) | \(4.2\)  | \((2, 2)\) |          |
| \(8.1\)    | \((9, 1)\)  | \(6.1\)  | \((14, 1)\) |          |
| \(v_1 = (2.5, 1)\) | \(v_2 = (2, 1)\) | \(v_3 = (2, 1)\) | \(v_4 = (2, 1)\) |          |

\[ u_1 = (4, 1) \]

\[ u_2 = (4, 1) \]

\[ u_3 = (4, 1) \]

The initial interval transportation cost is

\[ \langle 7.5, 1 \rangle + \langle 6, 1 \rangle + \langle 9, 1 \rangle + \langle 52, 2 \rangle + \langle 56, 1 \rangle + \langle 18, 1 \rangle \]

\[ = \langle 148, 5, 2 \rangle \]

\[ = [146.5, 150.5] \]

Now we have to spread on the interval form of the MODI method to test the optimality

Then all \( \hat{d}_{ij} \geq 0 \). The present solution is an ideal solution on behalf of intervals. Therefore optimum transport intervals cost is

\[ = \langle 7.5, 1 \rangle + \langle 6, 1 \rangle + \langle 9, 1 \rangle + \langle 52, 2 \rangle + \langle 56, 1 \rangle + \langle 18, 1 \rangle \]

\[ = \langle 148, 5, 2 \rangle \]

\[ = [146.5, 150.5] \]

6. Conclusion

Finding an initial feasible solution is the prime prerequisite to achieve an optimal solution to a problem of transportation of the integer interval. Depending on this initial feasible problem solution, the number of iterations can be modified to get the optimum solution. When an original possible solution closes to the minimum total cost, the optimal solution with fewer iterations can be achieved. There are several methods to achieving initial feasible solution. Among them can be considered Vogel's Approximation Method (1958) as a state-of-the-art initial solution provider available in the literature. A new alternative approach is being developed in this research to achieve an effective initial feasible solution to problems of transportation of integer intervals. Only row penalties including assignments are calculated in the implementation of our new system, whereas the row and column consequences are considered in the Vogel Approximation System. A row penalty approach gives in less number of iterations the optimal solution. And in some cases the number of iterations taken by Vogel's Approximation Method (VAM) and new row penalty method to achieve the optimal solution are the same. For the case of a large number of plants and destinations, future work could be carried out to improve this process. This work considers problems related to linear transport. But the problem with transport can also become nonlinear. The existing approach is not valid in such situations. And the new approach introduced can be expanded to include certain variants. We plan to commit ourselves to future work in that direction.

7. References

[1] Ganesan K and Veeramani P 2005 On arithmetic operations of interval numbers International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 13 619-31

[2] Ganesan K 2007 On some properties of interval matrices

[3] Chanas S, Delgado M, Verdegay JL and Vila MA 1993 Interval and fuzzy extensions of classical transportation problems Transportation Planning and Technology 17 203-18
[4] Sengupta A and Pal TK 2003 Interval-valued transportation problem with multiple penalty factors VU Journal of Physical Sciences 9 71-81
[5] Das SK, Goswami A and Alam SS 1999 Multiobjective transportation problem with interval cost, source and destination parameters European Journal of Operational Research 117 100-12
[6] Fegad MR, Jadhav AV and Minley AR 2011 Finding an optimal solution of transportation problem using interval and triangular membership functions Eur. J. Oper. Res. 60 415-21
[7] Sudhakar VJ and Kumar VN 2010 A new approach for finding an optimal solution for integer interval transportation problems Int. J. Open Problems Compt. Math. 3
[8] Natarajan PP 2010 A new method for finding an optimal solution of fully interval integer transportation problems Applied Mathematical Sciences 4 1819-30
[9] Murugesan S and Kumar BR 2013 New optimal solution to fuzzy interval transportation problem International Journal of Engineering Science and Technology 3 188-92
[10] Juman ZA and Hoque MA 2015 An efficient heuristic to obtain a better initial feasible solution to the transportation problem Applied Soft Computing 34 813-26
[11] Juman ZA and Hoque MA 2014 A heuristic solution technique to attain the minimal total cost bounds of transporting a homogeneous product with varying demands and supplies European journal of operational research 239 146-56
[12] Safi MR and Razmjoo A 2013 Solving fixed charge transportation problem with interval parameters Applied Mathematical Modelling 37 18-19
[13] Ramesh G and Ganesan K 2011 Interval linear programming with generalized interval arithmetic Int J Sci Eng Res 2 1-8
[14] Ramesh G and Ganesan K 2012 Duality theory for interval linear programming problems 2
[15] Sudha GR and Ganesan K 2018 Solution of two vehicle cost varying interval transportation problem-a new approach International Journal of Pure and Applied Mathematics 119 363-72
[16] Sophia Porchelvi and M Anitha 2018 Comparative study of optimum solution between interval transportation and interval transhipment problem International Journal of Advanced Science and Engineering 4 764-767
[17] Akilbasha A, Pandian P and Natarajan G 2018 An innovative exact method for solving fully interval integer transportation problems Informatics in Medicine Unlocked 11 95-9
[18] Padma karthiyyayini G, Ananthalakshmi S and Usha Parameswari R 2019 An inventive method for solving fully interval transportation problem SSRG International Journal of Mathematics Trends and Technology 50-57
[19] Maity G, Roy SK and Verdegay JL 2019 Time Variant Multi-Objective Interval-Valued Transportation Problem in Sustainable Development Sustainability 11
[20] Ma M, Friedman M and Kandel A 1999 A new fuzzy arithmetic Fuzzy sets and systems 108 83-90
[21] Oliveira C and Antunes CH 2007 Multiple objective linear programming models with interval coefficients--an illustrated overview European journal of operational Research 181 1434-63