Influence of Mathematical and Astronomical Developments in Medieval Kerala on Vāstuśāstra

Rajasekhar P1 and Babu T Jose2
1Research Scholar, Cochin University of Science and Technology, Kochi, 682 022, Kerala, India
2Emeritus Professor, Cochin University of Science and Technology, Kochi, 682 022, Kerala, India

*Email: raju58vaikom@gmail.com

ABSTRACT

The contribution in the field of mathematics is unparalleled. The concept of zero and the place value system is alone sufficed to place India in a high pedestal. Historians were generally under the impression that Indian supremacy in Mathematics came to an end with Bhaskaracharya (1114–1185) who is also known as Bhaskara II. Recent researches and publications of books like ‘Crest of the peacock’ written by George Gheverghese Joseph, has brought out the marvelous achievements of Southern India, especially Kerala region after the period of Bhaskaracharya which produced many results surpassing the Europeans in its indigenous style till the advent of Western Education system in early 19th Cent. This medieval contribution includes mathematical analysis and first steps in Calculus and many outstanding discoveries in astronomy. These contributions in Mathematics as well as in Astronomy are now grouped and studied under the title “Kerala School”. Accordingly, Sangamagrama Madhava (14th Cent.), doyen of Kerala School, is recognized as the ‘Father of infinitesimal Analysis’. In this paper the attempt is made to analyse the influence of Kerala School in the development of traditional building science and architecture. This branch of knowledge is generally categorized under the term ‘Vāstuśāstra’.

1. Introduction

The South India, witnessed an unprecedented advancement in Mathematics during 1300-1700 and further sustained its glory up to 1850. This part of peninsular India tuck between the mountain ranges and Arabian Sea, in the South-West tip, was politically undisturbed during this period unlike Northern India, which was facing continuous threats and attacks from outside. With fertile land blessed with forty rivers and two monsoon seasons and tropical climate made Kerala a well-known location for pepper and other spices from time immemorial. Thus, the rulers of this part of the country were immensely rich through spice trade and other spices from time immemorial. Thus, the rulers of this part of the country were immensely rich through spice trade and were magnanimous and keen in offering boundless patronage for intellectual pursuits. During this medieval period, Arts, Music, Drama, Literature and Sciences like Mathematics, Astronomy, Vāstuvidyā (Architecture & Building Technology) and Ayurveda (indigenous medicine) flourished with no comparison to other parts of the country.

With regard to Mathematics and Astronomy, remarkable contributions were made by Mādhava (c. 1340 – c.1425), Vācheśīri Parameśvara (c. 1380–1460), Dāmodara (c.1410–c.1500), Kelallūr Nilakaṇṭha (14 June 1444–1544) Jyeṣṭhadeva (c.1500–c.1575)

Citrabhānu (16th century) Śāṅkara Vāriyar (c.1500– c.1560), Acyuta Piśāroṭi (c. 1550 – 7 July 1621), Putumana Somayāji (c.1660-1740). In the field of Mathematics, Infinitesimal Analysis and the concept of Calculus were the important contributions from historical point of view. In the history of mathematics and astronomy, these contributions from Kerala are currently studied under the title ‘Kerala School’. This study has gained worldwide acceptance and acclaim. Mathematical and astronomical inventions served the main purpose of refinement of astronomical theories, its computational techniques and revalidation, revision, reconstruction of various astronomical constants required for the calculations of true longitudes of Sun, Moon, planets and nodes to predict exactly the day-to-day movements of these heavenly objects with respect to a given place and time and predict celestial phenomena like eclipses, conjunction of planets etc. accurately. In the West also, the development of mathematics in the seventeenth century was initially related to Astronomy and it got shifted to other areas of science and technology later. In particular, the Calculus played an important role in deriving many results in Physics and Engineering during
the nineteenth century once Calculus got prime acceptance after Cauchy endeavored wrote Cours d’Analyse (1821). In that work, Cauchy endeavored to make Calculus rigorous and to do away with algebraic approaches. It is to be investigated in a similar manner whether the ‘Kerala School’, where Calculus originated, had made any impact on Engineering and Technology and whether it triggered any research in other applied sciences. By and large, it is generally understood that no remarkable technological advancement was made and the ‘invented mathematical ideas’ remained within the mathematical and astronomical sphere at theoretical level. The class structure which prevailed in the society during the period might have prevented the exchange of knowledge and ideas between different sections of society. By and large, those who work with hand and those who work with head were not connected in the sharing of knowledge. The only notable exception to the above compartmentalization was the field of ‘Architecture and Building Science’. This field broke all the barriers of so called ‘caste’ or ‘class’ structure and even the lower tier worker involved in construction were made literate in Sanskrit, the sacred language specifically to get acquainted with the Science behind it. They were taught Sanskrit to get them trained in the ‘Building Science’. This intentional move might be an outcome to attain perfection in construction and to ensure the desired construction quality and to harvest a flawless end-product. This has also had far reaching social impact.

Incidentally prominent writers of Vāstuśāstra during this period were closely associated with ‘Kerala School’ as evident from the fact that Cennās Nārāyaṇanampūtiripāṭ (b.1428), the author of the most renowned text, ‘Tantrasamuccaya (TS)’ the code of rituals for Kerala temples, has included a chapter on Vāstuvidyā depicting various aspects of temple construction. The author’s knowledge in Mathematics and Astronomy is evident from this chapter. He was a contemporary of Paramēśvara (1360 -1460 AD) who introduced the revolutionary system of ‘Drggaṇitam’ in Indian astronomy. The author of Manuyalaacakandrika (MC), Tirumanaṅgalattu Nilakanṭhan (b. 1510) was a student of the famous mathematician-astronomer Nilakaṇṭha Somayāji (1444-1544 CE) who wrote Tantrasāṅgaṅa (1500CE). He was also a colleague of Jyeṣṭhadeva (b1510 CE) who wrote Yuktiḥbhāga. Another popular treatise named Śilpiṇātra-SR (1635 CE) was written by Śrikuṃmāraṇ. He also authored a text called ‘Bhṛgaśilpiṇaṭra’ in Malayalam verse, which is roughly a concise translation of the former text. Śrikuṃmāraṇ had also association with Trikaṇthiyur, the seat of learning established by Nilakaṇṭha. Thus, it could be inferred that there was close association with the Mathematicians and they themselves had acquired ample knowledge in Mathematics and Astronomy. This should have had definite impact on the epistemological and philosophical approach to the building science as well.

Writing new books on the subject itself is an indication of some innovation that had evolved in the field. Traditionally, new books were written only when new ideas/approaches were to be presented, otherwise scholars ventured to write only interpretations on well-known texts, called ‘vyākhyāna’ to exhibit their command over the subject. Both MC and SR refer to TS as the prime text, which indicates that TS had introduced new concepts that were not seen in earlier texts.

2. Traditional Architecture of Kerala

Traditional Kerala Architecture is unique in many aspects when it is compared to the architecture styles in the rest of Indian sub-continent; so also is the Vāstu literature. The development of Kerala style of temple architecture has taken place in around 200 CE subsequent to the stabilisation of migration of people from Northern India. This happened as a part of establishing sixty four grāmas (settlements) from Southern Karnataka area in the North to Chenganoor in the South extending to about 600 kilometres in Kerala. Three stages can be traced in the evolution of Kerala architecture. The first stage began from 4th Century to 10th Century, the second stage from 10th Century to 13th Century and the last stage subsequent to 13th Century. During these stages, when a new style suited to the geography, climate, need and raw material availability was developed, there was also a need for new reference books. As occurred in other spheres of knowledge i.e. Mathematics, Astronomy, Ayurveda etc., there has been revitalization of Vāstuśāstra also. Most evidently, the use of rock and stone masonry was reduced and style became more timber intensive and airy. Some of the Indian Vāstu literature is as old as Vedic texts. The texts available on the subject were numerous as evident from the mentioning of more than twenty five reference texts in the preamble of MC. However, as mentioned above there were requirements for new reference texts as new ideas and concepts were being evolved. This may be a reason for the inclusion of a chapter in the Codes for Rituals in Kerala Temples called Tantrasamuccaya (TS) written by Cennās Nārāyaṇanampūtiripāṭ (b. 1426 CE). Otherwise the possibility is that Cennās introduced the changes and his ideas were presented in TS. Subsequently Tirumanaṅgalattu Nilakanṭhan (b. 1500 CE) and Śrikuṃmāraṇ (c.1590) wrote MC and SR incorporating developments in the school of thought put forward by Cennās, in TS. In this we can notice parallels in the Astronomical developments in Kerala which commenced with Drggaṇita in the same period. After TS, the second and third stages of Kerala Architectural pursuits were mainly according to TS and MC and SR produced based on same philosophical approach. This could also be considered under the umbrella of ‘Kerala School’
Verses 24 Chapter: 5 states as follows:

mathematical analysis are imperative. Astonishingly, MC
the limits, clear understanding of structural behaviour and
a column in to pedestal, short column etc. and to specify
dimension of column at base. As stated earlier for classifying
of column at top shall be 81/800 of height. (Chapter:
the height of the pillar. This works out to L/D = 80/7=11.2. In
modern structural engineering is as follows depending upon
column in general. The classification of columns according to
and alloys are the main building materials.

The roof elements are mostly timber construction and
The roofing support is essentially wooden framework with
generally made of wood, solid rock or well-built masonry.

Thus the structural system mainly consists of columns, its
base and column capital beams/ wall plates, rafters, ridge
piece and reapers. Of these the column base and columns are
generally made of wood, solid rock or well-built masonry.
The roof elements are mostly timber construction and
rarely solid rock-cut panels. Regarding building materials, Śilpiratna (Chapter: 14:Verse: 1) states that rock, brick, lime,
timber, clay tiles, mud blocks and metals like iron, copper
and alloys are the main building materials.

The vertical load carrying member is called pillar or a
column in general. The classification of columns according to
modern structural engineering is as follows depending upon
their slenderness ratio, i.e. the ratio between the length(L)
and its least lateral dimension(D).

1. Pedestals
2. Short columns
3. Intermediate columns and
4. Long columns.

Previously columns were designed based on fixed ratios
called ‘classical order’ of columns as we can see in the ancient
Roman and Greek design. When we refer to ancient Vāstu
texts also we can only find constant L/D ratios mentioned.
To cite examples Matsyapurāṇa specifies (Chapter 250) that
the dimension of the pillar shall be 7L /80 where L, is the
height of the pillar. This works out to L/D = 80/7=11.2. In
the Bhatātasthā, it is said that minimum lateral dimension
of column at top shall be 81/800 of height. (Chapter:
Verse : 27). This will be approximately 1/10 when lateral
dimension of column at base. As stated earlier for classifying
a column in to pedestal, short column etc. and to specify
the limits, clear understanding of structural behaviour and
mathematical analysis are imperative. Astonishingly, MC
Verse 24 Chapter: 5 states as follows:

Lateral dimension (D) shall be 1/4, 1/5, 1/6, 1/7, 1/8,
1/9, 1/10 or 1/11 of its height(L). While referring to the
clause 25.1.1 of IS 456-2000, Indian Standard: Plain and
reinforced Concrete -Code of Practice, a column or a strut is
a compression member, the effective length of which exceeds
(3) three times the least lateral dimension and hence when
L/D is less than 3 it will be under the category of Pedestals
and not columns. MC has precisely set the lower limit as 1/4. Again according to the same code the clause 25.1.2
states that the compression member may be considered as
short when both the slenderness with respect to the least
is less than 12. Here also MC has set the upper limit as 11
ensuring it is always below twelve to ensure column is ‘short’
to avoid any chance of buckling. When other international
as well as Indian codes are examined we can find that this
limit of 12 or conservatively 11 holds well for all most all
building materials. The British code and American code are
more conservative. The upper limit set by the British Code:
BS 8110-1; 1997 is 10. Referring to clause 4.6.2,masonry
code (IS 1905-1987) it could be seen that the same range
is set.

In this context, Manusyālacandrikā (MC) reveals that the
author has clearly understood the behaviour of the columns
under the application of loads, as they have set lower limit
as well as the upper limit for the short column scientifically.

As stated above, the rule in the MC stating both the lower
limit as 4 and upper limit as 11 is amazing. Is this just
a coincidence? Any chance that Tirumāṅgalatt Nilakāṇṭha

Astonishingly, MC
Verse 24 Chapter: 5 states as follows:

stambhacābdhīṣu shaṭṭhādharavunnavadigrudrābhāgai
ikatāt yāt
stambhābhodistadvasanavadaiarudrāmihinagrata
raḥ

Lateral dimension (D) shall be 1/4, 1/5, 1/6, 1/7, 1/8,
1/9, 1/10 or 1/11 of its height(L). While referring to the
clause 25.1.1 of IS 456-2000, Indian Standard: Plain and
reinforced Concrete -Code of Practice, a column or a strut is
a compression member, the effective length of which exceeds
(3) three times the least lateral dimension and hence when
L/D is less than 3 it will be under the category of Pedestals
and not columns. MC has precisely set the lower limit as 1/4. Again according to the same code the clause 25.1.2
states that the compression member may be considered as
short when both the slenderness with respect to the least
is less than 12. Here also MC has set the upper limit as 11
ensuring it is always below twelve to ensure column is ‘short’
to avoid any chance of buckling. When other international
as well as Indian codes are examined we can find that this
limit of 12 or conservatively 11 holds well for all most all
building materials. The British code and American code are
more conservative. The upper limit set by the British Code:
BS 8110-1; 1997 is 10. Referring to clause 4.6.2,masonry
code (IS 1905-1987) it could be seen that the same range
is set.

In this context, Manusyālacandrikā (MC) reveals that the
author has clearly understood the behaviour of the columns
under the application of loads, as they have set lower limit
as well as the upper limit for the short column scientifically.

As stated above, the rule in the MC stating both the lower
limit as 4 and upper limit as 11 is amazing. Is this just
a coincidence? Any chance that Tirumāṅgalatt Nilakāṇṭha
carrying out a mathematical analysis to find the above limits.
The possibility cannot be ruled out though derivation for the
slenderness limits are not stated in any available medieval
Vāstu texts. There are instances where the scholars of Kerala
School using Calculus for deriving a curve for profiling a
vessel for making a globe rotating synchronous with time
(equivalent to a water clock). While interpreting the Verse 22
in Āryabhatīyam (Part III: Golapāda), Nilakāṇṭha Somayājī
(NS), the teacher of TN, states as follows:

“ kāsthāmaṇyam samātvattaṁ samantataḥ samagurum
laghun golan
pāntataisalalaiṁ bhramayet svadhīya ca kālasamam”
itī...

kāsthāmaṇyam na lohādīmayam..... svayamapabhāya
jīreyat”.

Here the requirement is to profile a vessel with a bottom
opening so as the water level in the vessel falls synchronous
with time. (Note that in a cylindrical vessel water will initially
call rapidly and then it will slow due to reduction of head) For this NS suggests to profile (pariprabhēbhat
kartarcyath) the vessel. He adds that “asya yukthi svayamapi
jnātum laksy, ganiṣṭhitvar durviṣāyatabhātvat”, which
means that the logic (required in equation) in this context
is also derivable (by ourself) as in the case of any other
mathematical problem. Thus it becomes a problem in Hydraulics and applied mathematics. Let us derive the equation for profiling.

Assume a circular vessel with radius $R$ at any height $h$.

Then:

Velocity of draining water $v = \sqrt{2gh}$ where $g$ is acceleration due to gravity and $h$ head (height) of water in the vessel.

Let ‘$a$’ area of small opening at the bottom through which water drains. And $C_d$ be the coefficient of discharge.

Then water drained out in time $dt = \Delta Q = C_d \cdot a \cdot v \cdot dt = C_d \cdot a \cdot \sqrt{2gh} \cdot dt \quad \cdots (1)$

Let drop of height in water level be $\Delta h$ and radius at that height be $R$.

Then $\Delta Q = \pi R^2 \cdot \Delta h$

Equating (1) & (2) $\Delta Q = C_d \cdot a \cdot v \cdot dt = C_d \cdot a \cdot \sqrt{2gh} \cdot dt = \pi R^2 \cdot \Delta h$

Then $\Delta h = \frac{\pi R^2}{C_d \cdot a \cdot \sqrt{2g}} \cdot \frac{\Delta h}{dt}$

But in order to achieve synchronisation, $\frac{\Delta h}{dt}$ shall be constant.

Then $\Delta h = \frac{\pi R^2}{C_d \cdot a \cdot \sqrt{2g}} \cdot \frac{\Delta h}{dt}$

Or

$$h = \frac{C \cdot \pi}{C_d \cdot a \cdot \sqrt{2g}} \cdot CR^4$$

$$R^4 = \frac{2g a^2 \cdot C_d^2}{C^2 \cdot \pi^2} \cdot h = K \cdot h$$

Where $K$ is a constant

$$R = K \cdot \sqrt{h}$$

That the radius ($R$) shall reduce with height ($h$) satisfying the equation $R = K \cdot \sqrt{h}$ where $K$ is constant. Then the water level will drop proportional to time (linearly). This is the profiling (Parinähabhedha) implied by NS. This is a solid proof for the fact that ‘Applied Mathematics and Science’ was also under the reach of Kerala school.

Another instance is available in MC itself. Regarding the determination of true cardinal directions (Geographical East and North), several methods have been discussed in several texts, of which MC states only one method. This method suggests a correction with respect to a popular method to make the E-W and N-S line more precise. The theory behind the minute correction also cannot be derived without proper understanding of the Astronomy behind it. Śilpārṇaṭhā (1635) had stated the following formula:

'Bhabhuja' is $\pm h \tan \phi + \frac{R \sin \lambda \cdot R \omega \cdot \sqrt{h^2 + s^2}}{R \cdot R \cos \phi}$

$\lambda, \phi, \omega, s & R$ are respectively the sun’s instant declination, latitude of the place, maximum declination, height of śāṅku (gnomon), length of shadow, R (Radian). A later renowned Astronomy book belonging to Kerala school, Sadratnamāla (1819 CE) also states same formula for finding directions.

As mentioned earlier, astronomers, mathematicians and Vāstu experts had close associations and most of the scholars had studied all the three subjects. Tirunāṅkalatil Nilakanṭhan’s colleague has written very popular Mathematics text Yuktibhāṣa (1530 CE). In this text while deriving the infinite series expansion for PI , example for demonstrating the properties of similar triangle has cited the relation between the length of the collar pin slot and the rafter in a square pavilion. The literal translation of the portion is as follows:

"Then there is a second antecedent figure (triangle). For that E-W line itself is the Base. From the end of the Base, two segments together, along the side of the square, is Height or opposite side. The second radial line from the centre of the circle becomes Hypotenuse. Thus is the second antecedent figure (triangle). Then consider, the consequent figure corresponding to this. The line from the end of point of the first radial line, perpendicular to the second radial line, is the Base. From the meeting point of this Base to the end of the radial line is the Height. The second segment in the side of the square is the hypotenuse. This is the second consequent figure (triangle). When the second rafter becomes the antecedent hypotenuse, the two spans of the rafters together becomes the antecedent height (opposite side of the right triangle). Hence the length of the second rafter will have more length than the first rafter. Proportionately the length of the slot for the wooden pin also will be more. That (the length of the slot) will be the hypotenuse of consequent figure (triangle) and will be parallel to the eave line, which is the opposite side of the antecedent figure (triangle). How the inclination of the rafter and the inclination of the slot are varying proportionally, in the same way the antecedent figures (triangle) and consequent figures (triangle) considered here are also correlated. Here at the end of the E-W line, the second segment in the side of the square is multiplied by the radius, which is the base of the antecedent figure (triangle) and divided by the second radial line (which is the fourth proportion). The result is the base of the consequent figure (Right triangle). Assuming this base as opposite side (height) and then portion of the second radial line between the meeting point of this height and the centre..."
of the circle as adjacent side (base) and the first radial line as hypotenuse there exists a third triangle here also."

In the above, for understanding the similarity of triangles, an analogy with rafters and slits (holes) provided in the rafters to receive wooden pin (ties between rafters) is given. Here as stated earlier, a square pavilion with pyramidal shaped roof with central ridge piece and rafters originating from centre ridge piece in radial direction to seat on to the wall plate. These rafters will be usually tied by providing another square wooden piece, which is locally called Vala. These wooden pins pass through the rafters, and hence there should be holes (slots) to receive these pins. The outer most line of rafters is called eave - reaper is called valama. The interactions between Vāstu experts and Mathematicians of those days are evident from this quote.

Another noticeable feature in Kerala Vāstu texts is that they have updated the old texts in the light of developments in Mathematics and Astronomy. In the Tantrasamuccaya TS, the value of π is given as \(\frac{355}{113}\), which is a very good approximation. Further value of π given in SR (1635CE) is 3.141592653 (33215/104348) which is correct up to 9 decimal places. Mentioning of a high accuracy value for π is definitely under the influence of Kerala School of Mathematics where Madhava (c. 1340-c.1425) gave a rapidly converging series of π, obtaining the infinite series, using the first 21 terms to compute an approximation of π, he obtains a value correct to 11 decimal places (3.14159265359). After Madhava in 15th cent later books have adopted more accurate values other than 22/7, by finding the convergents according to convenience and need. The continued fraction and convergents were known and widely used in Indian Mathematics as a part of kuttakāra, the solution of indeterminate equations much before its emergence in Europe with the writings of the Dutch mathematician, Christian Huygens (1629-95).

3. Conclusion

Under the influence of Kerala School of Mathematics and Astronomy, Vāstuśāstra in Kerala, which is locally known as ‘Taccuśāstra’ also, underwent many notable revisions and updating. Consequently a new philosophical and epistemological approach evolved in this field of Vāstuśāstra also. This unbroken tradition has made Kerala Vāstu something unique when compared to those existing in other parts of India. The new features which were embedded could be summarized as follows:

1. TS (15th Cent.) codified the newly evolved temple tradition. This also influenced the residential architecture as evident from the preamble of MC where it is explicitly stated that it is written in accordance with TS.
2. As the style evolved, Kerala style was more timber intensive. More emphasis was given to ‘modular’ type of construction which in turn helped for optimization of the use of timber as wood cutting was made easy as there was interrelation between various structural elements such as wall plates, rafters, ridge piece and reapers.
3. Units of measurements were standardised in terms of absolute measurements and the same was popularised.
4. A systematic approach was introduced and meticulously followed for room dimensioning, which ensured systematic and quality construction. Also position of rooms were prescribed according to the local (Kerala) climate and geography deviating from the old texts which might have probably originated in North India.
5. In the design of column beam etc., a mathematical approach was brought in and range for slenderness ratio, span-depth ratio etc. was introduced anticipating certain structural engineering theories introduced three centuries later in the West.
6. Advances in Astronomy were made use in formulating the methods for finding true cardinal direction etc.

Reference

British Standard BS 8110: Part 1: 1997
Building Code Requirements for Structural Concrete (ACI 318-14).
George, G. Joseph, Ed. (2009). Kerala Mathematics- History and its possible transmission to Europe. BR Publishing Corporation, New Delhi.
IS 456 :2000: Code of Practice for Plain and reinforced cement concrete.
IS 883:1994: Design of Structural Timber in Buildings-Code of Practice.
IS 1905-1987: Code of Practice for Structural use of unreinforced masonry.
IS 800: (2007). General Construction in Steel - Code of Practice.
Kanippayoor (2003). Tantra Samuchayam, Dwatheeya Patalam (Translation) 8th Ed, Pachangam Book Stall Kunnamkulum.
Michell, G. (1988). The Hindu temple: An Introduction to its Meaning and Forms. Chicago and London: the University of Chicago Press.
https://doi.org/10.1016/B978-1-4831-4104-6.50004-0
Namboothiri, Cheuvally Narayanan (Tr.). (2008). Vāstuvidyā. DC Books, Kottayam.
Namboothiri, Cheruvally Narayanan (Tr.). (2012). Śilparatna of Śríkumāra. Devi Book Stall.
Pillai, S. U. and Devadas, M. (2005). Reinforced Concrete Design. Tata McGrawHill Publishing Company Limited.
Prabhakar, S. (1979). “The Vastu Vidya of Vishvakarma”, Studies in Indian architecture, Asia Publishing House, Mumbai.
Sastri, T. G. (Ed.). (1917). Manaṣyālayacandrikā of Tirumaṅgalatt Nilakaṇṭha. Trivandrum Sanskrit Series, Travancore Manuscript Publication Department, Trivandrum.
Timoshenko, S. P. and Gere, J. M. (1961). Theory of Elastic Stability, 2 Ed., McGraw-Hill.
Vadakkumkoor (1962). Keraleeya Samskrutha Sahithya Charithram, National Book Stall, Kottayam.