Gluon-fusion Higgs production at NNLO for a non-standard Higgs sector

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ABSTRACT: We consider an extension of the Standard Model with an arbitrary number of heavy quarks having general couplings to the Higgs boson. We construct an effective Lagrangian integrating out quarks that are heavier than half the mass of the Higgs boson and compute the Wilson coefficient for the effective gluon-Higgs vertex through NNLO. We apply our result to a composite Higgs model with vector-like quarks coupling to the third generation quarks. In the heavy quark-mass approximation, we show that the suppression of the leading-order cross section with respect to the Standard Model does not depend on the number of vector-like multiplets introduced. We analyse the effects of QCD and electroweak corrections through three loops, as well as bottom-quark contributions through two loops.
1. Introduction

The discovery of the sector responsible for electroweak symmetry breaking (EWSB) is the main goal of the LHC. In the minimal description provided by the Standard Model (SM), the electroweak symmetry is broken by an $SU(2)_L$ doublet that acquires a vacuum expectation value. After EWSB, three of the four degrees of freedom associated to the Higgs doublet provide the longitudinal modes of the electroweak gauge bosons, and only one real degree of freedom survives: the Higgs boson. In this minimal description, the couplings of the Higgs boson to the SM particles are fixed by the mass of the particles themselves. Indirect experimental bounds point towards a relatively light Higgs, with a mass of a few hundred GeV. On the other hand, a fundamental scalar receives quadratically divergent contributions from radiative corrections. A light, fundamental Higgs boson then requires very fine-tuned cancellations between the tree-level mass and the higher-order corrections to it.

New physics scenarios try to address this problem introducing some mechanism to protect the mass of the Higgs boson. One of the most studied examples is supersymmetry. In supersymmetry new particles are introduced, in such a way that their contribution to the self-energy of the Higgs boson counterbalances the contribution from the SM particles. Another possibility is that the Higgs boson is not a fundamental scalar, but a composite state of some new, strongly interacting sector [1, 2]. Compositeness would explain the insensitivity of the Higgs boson mass on ultraviolet physics, as loop contributions are cut-off at the compositeness scale. Such a scale is expected to be of the order of a few TeV. Higher values would reintroduce fine-tuning problems, while a too low scale would be hard to reconcile with electroweak and flavour physics constraints. In this framework, the mass of the Higgs boson can be kept naturally light by identifying the Higgs with the pseudo-Goldstone boson of some spontaneously broken global symmetry of the new sector, in analogy to what happens for the pions in QCD. Quarks masses can be generated by mixing the massless fundamental quarks of the SM with heavy composite states of the new sector. This mixing induces a coupling of the SM fermions to the Higgs boson. As in the SM, only heavy quarks couple significantly to the Higgs boson, as they are mainly composite. However, the strength of this coupling is not simply proportional to the mass, but it depends on the details of the model.

The introduction of new particles and the modification of the Higgs couplings can change in a significant way the Higgs-boson phenomenology. At hadron colliders, the main mechanism for the production of the Higgs boson is gluon fusion. This process is mediated by heavy-particle loops and therefore affected both by the introduction of the new quarks, and by the modification of the Higgs couplings. Within the SM, the gluon-fusion channel has been studied thoroughly. The inclusion of next-to-leading order (NLO) [3, 4] and next-to-next-to-leading order (NNLO) [5–7] corrections was necessary in order to match the accuracy expected by the experiments and to achieve a good converge in the perturbative expansion of the cross section in the strong coupling constant $\alpha_s$. At the LHC, these contributions yield an increase of a factor of 2 in the Higgs boson production cross section. In composite Higgs models, the LO production cross section is expected to be fairly
suppressed with respect to the SM value [8, 9]. The actual suppression factor depends on the details of the model. For an $SO(5) \to SO(4)$ symmetry breaking pattern in the strong sector, with composite fermions embedded in the fundamental representation of $SO(5)$ and a global symmetry breaking scale $f = 500$ GeV, the Higgs boson production cross section is expected to be 35% of the SM value. A more detailed study of the Higgs production cross section and branching ratios in the different channels was carried out in [10]. It is interesting to analyse the effect of higher order corrections in this class of models, both to check if their effect is the same as in the SM and to reach the same accuracy as the SM predictions.

In this paper, we construct an effective Lagrangian integrating out the heavy quarks, for which we assume a generic coupling to the Higgs boson. We compute the Wilson coefficient for the gluon-Higgs effective interaction through NNLO. We apply our result to the composite Higgs model described above, where we introduce one or two multiplets of heavy top-partners. We include in our study the full bottom-mass dependence through NLO [4, 11], the two-loop electroweak corrections [12] and the corresponding three-loop mixed QCD and electroweak corrections [13]. These effects are implemented in the program iHixs [14]. We combine them with the NNLO Wilson coefficient that we compute in order to obtain the Higgs production cross section through NNLO in composite Higgs models.

2. The effective Lagrangian

We extend the SM quark sector through new heavy quarks that transform under the fundamental representation of the QCD colour group. The number of heavy quarks, including the top, is $n_h$. The number of light quarks is $n_l$. We take the light quarks to be massless and not to couple to the Higgs boson. We assume instead an arbitrary coupling $Y_i$ of the heavy quarks to the Higgs boson. In the SM, $Y_i = m_i/v$, where $v \approx 246$ GeV is the vacuum expectation value of the Higgs boson and $m_i$ is the mass of the quark. The Lagrangian describing this model is

$$L = L_{QCD}^{n_l} + \sum_{i=1}^{n_h} \bar{\psi}_i (i\not{D} - m_i) \psi_i + L_Y , \quad \text{with} \quad L_Y = -H \sum_{i=1}^{n_h} Y_i \bar{\psi}_i \psi_i . \quad (2.1)$$

Here $D_{\mu}$ is the covariant derivative in the fundamental representation of the colour group and $L_{QCD}^{n_l}$ is the QCD Lagrangian with only the $n_l$ flavours of light quarks.

We focus on the Higgs production from gluon fusion mediated by loops of heavy quarks. When the particles that couple to the Higgs boson are heavier than half the Higgs boson mass, we can integrate them out. In this limit, we can replace the original Lagrangian (2.1) with an effective Lagrangian

$$L^{eff} = L_{QCD}^{n_l} - C_1 H \mathcal{O}_1 . \quad (2.2)$$

The dimension-four local operator $\mathcal{O}_1$ reads [15]

$$\mathcal{O}_1 = \frac{1}{4} G_{\mu\nu} G^{\rho\mu\nu} , \quad (2.3)$$
where $G'^a_{\mu\nu}$ is the field strength tensor in the effective theory, and $C_1$ is the corresponding Wilson coefficient \[16\]. The effective Lagrangian $\mathcal{L}_{QCD}^{\text{eff,nl}}$ appearing in Eq. (2.2) describes the interactions among the light degrees of freedom. It has the same form as $\mathcal{L}_{QCD}^{\text{nl}}$, but with different coupling and field normalizations to account for the missing contributions from heavy quarks loops. The parameters in the effective theory are related to the parameters in the full theory through multiplicative decoupling constants $\zeta_i$. The derivation of the decoupling constants is reviewed in \[17, 18\].

3. Details of the calculation

We compute the Wilson coefficient $C_1$ up to three loops. We start from the bare amplitude $\mathcal{M}^0_{gg \to H}$ for the process $gg \to H$ in the full theory,

$$\mathcal{M}^0_{gg \to H} = \mathcal{M}^{0,a_1a_2}_{\mu_1\mu_2}(p_1,p_2)\epsilon^{\mu_1}_{a_1}\epsilon^{\mu_2}_{a_2}. \tag{3.1}$$

Here $p_1$ and $p_2$ are the momenta of the two gluons and $\epsilon^{\mu}_{a_1}, \epsilon^{\mu}_{a_2}$ are their polarizations. This amplitude is related to the bare Wilson coefficient $C^0_1$ by \[17\]

$$\zeta^0_3 C^0_1 = \frac{\delta^{a_1a_2} \left( g^{\mu_1\mu_2}(p_1 \cdot p_2) - p_1^{\mu_2} p_2^{\mu_1} \right) }{(N^2 - 1)(d - 2)(p_1 \cdot p_2)^2} \mathcal{M}^{0,a_1a_2}_{\mu_1\mu_2}(p_1,p_2) |_{p_1=p_2=0}, \tag{3.2}$$

with $N$ the number of colours and $d = 4 - 2\epsilon$ the dimension of space-time. The superscript “0” denotes bare quantities. The factor $\zeta^0_3$ is the bare decoupling coefficient that relates the bare gluon field $G'^0_{\mu}^{a}$ in the effective theory to the bare gluon field $G^{0,a}_{\mu}$ in the full theory,

$$G^{0,a}_{\mu} = \sqrt{\zeta^0_3} G'^0_{\mu}^{a}. \tag{3.3}$$

We generate the Feynman diagrams $\mathcal{F}$ for the amplitude through three loops using QGRAF \[19\]. Diagrams containing two different heavy mass scales appear for the first time at the three-loop order. We then expand all diagrams in the external momenta $p_1, p_2$ by applying the following differential operator \[20\] to their integrand:

$$\mathcal{D}\mathcal{F} = \sum_{n=0}^{\infty} (p_1 \cdot p_2)^n \left[ \mathcal{D}_n \mathcal{F} \right]_{p_1=p_2=0}, \tag{3.4}$$

with

$$\mathcal{D}_0 = 1, \quad \mathcal{D}_1 = \frac{1}{d} \Box_{12}, \quad \mathcal{D}_2 = -\frac{1}{2(d-1)(d+2)} \left\{ \Box_{11} \Box_{22} - d \Box^2_{12} \right\}, \tag{3.5}$$

and $\Box_{ij} \equiv g^{\mu\nu} \frac{\partial^2}{\partial p_i^\mu \partial p_j^\nu}$. Differential operators of higher orders are not needed for the expansion in the external momenta at leading order.

After Taylor expansion, all the Feynman diagrams are expressed in terms of one-, two- and three-loop vacuum bubbles. We reduce these integrals to a set of five master integrals using the algorithm of Laporta \[21\] and the program AIR \[22\]. The topologies and the
master integrals that we find are the same as in the calculation of the Wilson coefficient for an arbitrary number of heavy quarks with top-like Yukawa interactions of Ref. [18],

\[
I_1 = \int \frac{d^d k_1}{i \pi^{d/2} P_1^1} = - (m_i^2)^{1-\epsilon} \Gamma(-1+\epsilon), \tag{3.6}
\]

\[
I_2 = \int \frac{d^d k_1 d^d k_2 d^d k_3}{i \pi^{d/2} P_1^2 P_2^3 P_3^5 P_4^6} = (m_i^2)^{2-3\epsilon} \frac{\Gamma^2(1-\epsilon)\Gamma(\epsilon)\Gamma^2(-1+2\epsilon)\Gamma(-2+3\epsilon)}{\Gamma(2-\epsilon)\Gamma(-2+4\epsilon)}, \tag{3.7}
\]

\[
I_3 = \int \frac{d^d k_1 d^d k_2 d^d k_3}{i \pi^{d/2} P_1^2 P_2^3 P_3^4 P_4^5}, \tag{3.8}
\]

\[
I_4 = \int \frac{d^d k_1 d^d k_2 d^d k_3}{i \pi^{d/2} P_1^2 P_2^3 P_3^4 P_4^5}, \tag{3.9}
\]

\[
I_5 = \int \frac{d^d k_1 d^d k_2 d^d k_3}{i \pi^{d/2} P_1^2 P_2^3 P_3^4 P_4^5}, \tag{3.10}
\]

with

\[
\begin{align*}
\mathcal{P}_1 &= k_1^2 - m_i^2, \\
\mathcal{P}_2 &= k_2^2 - m_i^2, \\
\mathcal{P}_3 &= k_3^2 - m_i^2, \\
\mathcal{P}_4 &= (k_1 - k_2 + k_3)^2 - m_i^2, \\
\mathcal{P}_5 &= (k_1 - k_2)^2, \\
\mathcal{P}_6 &= (k_2 - k_3)^2,
\end{align*}
\tag{3.11}
\]

The last two master integrals contain two heavy quarks of different mass, \(m_i\) and \(m_j\). They have been computed in [23]. The one-scale master integrals can be found for example in [24].

For completeness, we report the Wilson coefficient in terms of the bare parameters in the full theory,

\[
\zeta^0_3 C_1^0 = \frac{1}{3} \left( \frac{a_s^0 S_t}{\pi} \right) \left[ -\gamma_0^0 + \epsilon \left( \gamma_0^0 + 2 \gamma_1^0 \right) - 2 \epsilon^2 \left( \gamma_2^0 + \gamma_1^0 + \gamma_0^0 \frac{\pi^2}{24} \right) + \mathcal{O}(\epsilon^3) \right]
\]

\[
+ \left( \frac{a_s^0 S_t}{\pi} \right)^2 \left[ -\frac{\gamma_0^0}{4} + \epsilon \left( \gamma_1^0 + \frac{31}{36} \gamma_0^0 \right) + \mathcal{O}(\epsilon^2) \right]
\]

\[
+ \left( \frac{a_s^0 S_t}{\pi} \right)^3 \left\{ -\frac{n_h^2}{32\pi^2} \gamma_0^0 + \frac{1}{\epsilon} \left[ \gamma_0^0 \left( \frac{89}{576} n_h - \frac{13}{24} + \frac{5}{144} n_l + \frac{1}{8} L_1 \right) + \frac{n_h}{16} \gamma_1^0 \right]
\]

\[
+ \gamma_0^0 \left[ \frac{1171}{576} + \frac{103}{64} n_l - n_h \left( \frac{1051}{3456} + \frac{\pi^2}{128} \right) - \frac{89}{96} L_1^0 - \frac{3}{8} L_2^0 \right]
\]

\[
+ \left( \frac{13}{4} + \frac{5}{24} n_l \right) \gamma_0^0 - \frac{3}{16} n_h \gamma_2^0
\]

\[- \sum_{\substack{i<j<n_h \atop i<j<n_h}} \left( y_i^0 - y_j^0 \right) \left( \frac{19}{128} \frac{(m_i^0)^2}{(m_i^0)^2} - \frac{(m_j^0)^2}{(m_j^0)^2} \right)
\]

\[
+ \left( \frac{19}{128} \frac{(m_i^0)^2}{(m_i^0)^2} + \frac{19}{128} \frac{(m_j^0)^2}{(m_j^0)^2} + \frac{43}{96} \right) \log \left( \frac{m_i^0}{m_j^0} \right)
\]
The RHS of Eq. (3.2) is expressed in terms of the bare coupling constant \( \alpha_s \) in the full theory and of the bare masses and Yukawa couplings of the heavy quarks, \( m_i^0 \) and \( Y_i^0 \), \( C_1^0 = C_1(\alpha_s^0, m_i^0, Y_i^0) \). The bare strong coupling in the full theory is related to the bare strong coupling in the effective theory \( \alpha_s^0 \) by the decoupling constant \( \zeta_g^0 \) \cite{17,25},

\[
\alpha_s^0 = (\zeta_g^0)^2 \alpha_s^0 .
\]  

The decoupling constants of the strong coupling and of the gluon field (Eq. (3.3)) in the presence of an arbitrary number of heavy quarks have been derived in \cite{18},

\[
\begin{align*}
+ &\frac{19}{256} \frac{(m_i^0)^6 + (m_j^0)^6}{(m_i^0)^2(m_j^0)^2} \left( \log^2 \left( \frac{m_i^0}{m_j^0} \right) \right) \\
- &\log^2 \left( \frac{m_i^0}{m_j^0} \right) \left( \frac{73}{768} (y_i^0 + y_j^0) + \frac{23}{384} \frac{y_i^0(m_i^0)^2 - y_j^0(m_j^0)^2}{(m_i^0)^2 - (m_j^0)^2} \right) \\
+ &\left( (m_i^0)^2 - (m_j^0)^2 \right) \frac{19(m_i^0)^4 + 24(m_i^0)^2(m_j^0)^2 + 19(m_j^0)^4}{512(m_i^0)^3(m_j^0)^3} \\
\cdot &\left( y_j^0 \log \left( \frac{m_j^0 - m_i^0}{m_j^0 + m_i^0} \right) - y_i^0 \log \left( \frac{m_i^0 - m_j^0}{m_i^0 + m_j^0} \right) \right) \\
- &\frac{19(m_j^0)^6 + 5(m_i^0)^4(m_j^0)^2 - 5(m_i^0)^2(m_j)^0)^4 - 19(m_j^0)^6}{1024(m_i^0)^3(m_j^0)^3} \\
\cdot &\left( 8y_i^0 \text{Li}_3 \left( \frac{m_i^0}{m_j^0} \right) - 8y_j^0 \text{Li}_3 \left( \frac{m_j^0}{m_i^0} \right) - y_i^0 \text{Li}_3 \left( \frac{(m_j^0)^2}{(m_i^0)^2} \right) + y_j^0 \text{Li}_3 \left( \frac{(m_i^0)^2}{(m_j^0)^2} \right) \\
- &2 \log \left( \frac{m_i^0}{m_j^0} \right) y_i^0 \text{Li}_2 \left( \frac{(m_i^0)^2}{(m_i^0)^2} \right) + y_j^0 \text{Li}_2 \left( \frac{(m_j^0)^2}{(m_j^0)^2} \right) - 4y_i^0 \text{Li}_2 \left( \frac{m_j^0}{m_i^0} \right) - 4y_j^0 \text{Li}_2 \left( \frac{m_i^0}{m_j^0} \right) \right) \right] .
\end{align*}
\]  

(3.12)

Here we introduced the notation

\[
S_\epsilon = e^{-\epsilon \gamma_E} (4\pi)^\epsilon ,
\]

\[
L_1^0 = \sum_{i=1}^{n_h} \log(m_i^0) \quad L_2^0 = \sum_{i=1}^{n_h} \log^2(m_i^0) ,
\]

\[
T_0^0 = \sum_{i=1}^{n_h} y_i^0 \quad T_1^0 = \sum_{i=1}^{n_h} y_i^0 \log(m_i^0) \quad T_2^0 = \sum_{i=1}^{n_h} y_i^0 \log^2(m_i^0) ,
\]  

(3.13)

with

\[
y_i^0 = \frac{Y_i^0}{m_i^0} .
\]  

(3.14)

This expression reproduces the result of Ref. \cite{18} when \( Y_i^0 = \frac{m_i^0}{m_i^0} \).

### 3.1 Decoupling and renormalization

The RHS of Eq. (3.2) is expressed in terms of the bare coupling constant \( \alpha_s \) in the full theory and of the bare masses and Yukawa couplings of the heavy quarks, \( m_i^0 \) and \( Y_i^0 \), \( C_1^0 = C_1(\alpha_s^0, m_i^0, Y_i^0) \). The bare strong coupling in the full theory is related to the bare strong coupling in the effective theory \( \alpha_s^0 \) by the decoupling constant \( \zeta_g^0 \) \cite{17,25},

\[
\alpha_s^0 = (\zeta_g^0)^2 \alpha_s^0 .
\]  

(3.15)
\[ \zeta_g^0 = 1 + \left( \frac{\alpha_s^0 S}{\pi} \right) \left\{ -\frac{n_h}{12\epsilon} + \frac{1}{6} L_1^0 - \epsilon \left( \frac{n_h}{144} \frac{\pi^2}{1} + \frac{1}{6} L_2^0 \right) \right\} + \left( \frac{\alpha_s^0 S}{\pi} \right)^2 \left\{ \frac{n_h}{96\epsilon^2} - \frac{n_h}{24\epsilon} \left[ \frac{3}{4} + L_1^0 \right] + \frac{1}{8} L_1^0 + \frac{1}{24} (L_1^0)^2 + \frac{n_h}{24} \left[ \frac{11}{6} + L_2^0 \right] + \frac{n_h^2}{576} \right\} , \]
\[ \zeta_3^0 = 1 + \left( \frac{\alpha_s^0 S}{\pi} \right) \left\{ \frac{n_h}{6\epsilon} - \frac{1}{3} L_1^0 + \epsilon \left( \frac{\pi^2}{72} n_h + \frac{1}{3} L_2^0 \right) \right\} + \left( \frac{\alpha_s^0 S}{\pi} \right)^2 \left\{ \frac{3}{32\epsilon^2} n_h - \frac{n_h + 24 L_1^0}{64\epsilon} + \frac{3}{4} L_2^0 + \frac{1}{16} L_1^0 + n_h \left( \frac{91}{1152} + \frac{\pi^2}{64} \right) \right\} . \tag{3.16} \]

After decoupling, the bare Wilson coefficient is a function of the bare parameters in the effective theory and of the bare masses and Yukawa couplings of the heavy quarks in the full theory, \( C_1^0 = C_1^0(\alpha_s^0, m_i^0, Y_i^0) \). The bare parameters are related to the renormalized ones through multiplicative renormalization constants \( Z_i \) as

\[ \alpha_s^0 = \mu^{2\epsilon} Z_{\alpha_s} \alpha_s^\prime(\mu) , \tag{3.17} \]
\[ m_i^0 = Z_{m_i} m_i(\mu) , \quad Y_i^0 = Z_{Y_i} Y_i(\mu) . \tag{3.18} \]

The primed parameters in Eq. (3.17) are in the effective theory and the unprimed parameters in Eqs. (3.18) are in the full theory. In the \( \overline{MS} \) scheme the strong coupling renormalization constant is

\[ Z_{\alpha_s} = 1 - \frac{\alpha_s^\prime(\mu)}{\pi} \beta_0 + \left( \frac{\alpha_s^\prime(\mu)}{\pi} \right)^2 \left( \frac{\beta_0^2}{\epsilon^2} - \frac{\beta_1^\prime}{2\epsilon} \right) , \tag{3.19} \]

where \( \beta_0^\prime, \beta_1^\prime \) denote the first two coefficients of the \( \beta \) function in the light-flavour theory,

\[ \beta_0^\prime = \frac{1}{4} \left( 11 - \frac{2}{3} n_l \right) , \quad \beta_1^\prime = \frac{1}{16} \left( 102 - \frac{38}{3} n_l \right) . \tag{3.20} \]

The mass renormalization constant reads [26]

\[ Z_{m_i} = 1 - \frac{\alpha_s(\mu)}{\pi} \frac{1}{\epsilon} + \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 \left[ \frac{1}{\epsilon^2} \left( \frac{45}{24} - 2 n_f \right) + \frac{1}{\epsilon} \left( - \frac{101}{48} + \frac{5}{72} n_f \right) \right] , \tag{3.21} \]

where \( n_f \) is the number of active flavours. In the effective theory \( n_f = n_l \) and in the full theory \( n_f = n_l + n_h \). Finally, the Yukawa coupling \( Y_i^0 \) renormalizes as the mass,

\[ Z_{Y_i} = Z_{m_i} . \tag{3.22} \]

We review here the proof of this relation, following Ref. [27].

Let us consider the bare scalar current

\[ j^0(x) = \overline{\psi}_i^0(x) \psi_i^0(x) . \tag{3.23} \]

We relate it to the renormalized scalar current \( j(x) \) through a multiplicative renormalization constant \( Z_{m_i} \),

\[ j^0(x) = Z_{m_i} j(x) . \tag{3.24} \]
Let us recall that the bare and renormalized two-point Green functions for the fermions with an insertion of the scalar current are defined respectively as

\[ G^{0,2}(p_1, p_2) = i^2 \int dx_1 dx_2 e^{ip_1 x_1 - p_2 x_2} \langle T \psi^0_0(x_1) j^0(0) \bar{\psi}^0_0(x_2) \rangle \]

\[ \equiv i^2 \int dx_1 dx_2 e^{ip_1 x_1 - p_2 x_2} \left\{ \int [D \Phi^0] e^{i \int dx \mathcal{L}^0 j^0(0) \psi^0_0(x_1) \bar{\psi}^0_0(x_2)} \right\}, \tag{3.25} \]

\[ G^{2}(p_1, p_2) = i^2 \int dx_1 dx_2 e^{ip_1 x_1 - p_2 x_2} \langle T \psi_1(x_1) j(0) \bar{\psi}_1(x_2) \rangle \]

\[ \equiv i^2 \int dx_1 dx_2 e^{ip_1 x_1 - p_2 x_2} \left\{ \int [D \Phi] e^{i \int dx \mathcal{L} j(0) \psi_1(x_1) \bar{\psi}_1(x_2)} \right\}. \tag{3.26} \]

Here \( \mathcal{L}^0, \mathcal{L} \) are respectively the bare and renormalized QCD Lagrangian and \( \int [D \Phi^0], \int [D \Phi] \) denote the functional integral over the bare and renormalized fields appearing in them. From the second line of both these equalities we see that \( G^{0,2}, G^{2} \) are related to the bare and renormalized quark propagators

\[ S^0(p) \equiv i \int d^4x e^{ipx} \langle T \psi^0_0(x) \bar{\psi}^0_0(0) \rangle = i \int dx e^{ipx} \left\{ \int [D \Phi^0] e^{i \int dy \mathcal{L}^0 \psi^0_0(x) \bar{\psi}^0_0(0)} \right\}, \]

\[ S(p) \equiv i \int d^4x e^{ipx} \langle T \psi_1(x) \bar{\psi}_1(0) \rangle = i \int dx e^{ipx} \left\{ \int [D \Phi] e^{i \int dy \mathcal{L} \psi_1(x) \bar{\psi}_1(0)} \right\}, \tag{3.27} \]

as

\[ G^{0,2}(p, p) = - \frac{\partial}{\partial m_i^2} S^0(p), \quad G^{2}(p, p) = - \frac{\partial}{\partial m_i} S(p). \tag{3.28} \]

Therefore

\[ G^{0,2} = \frac{Z_2}{Z_{m_i}} G^{2}, \tag{3.29} \]

where \( Z_2 \) is the renormalization constant of the quark wavefunction,

\[ \psi^0_i = \sqrt{Z_2} \psi_i. \tag{3.30} \]

On the other hand, comparing Eqs. (3.25) and (3.26) we have

\[ G^{0,2} = Z_2 Z_J G^{2}, \tag{3.31} \]

and therefore

\[ Z_J = Z_{m_i}^{-1}. \tag{3.32} \]

The bare Yukawa Lagrangian \( \mathcal{L}_Y^0 \) has the same form of \( \mathcal{L}_Y \) (Eq. (2.1)), but it contains bare fields and Yukawa couplings. Treating the Higgs field as an external static field, we can use the result for the renormalization of the scalar current \( j^0(x) \) to relate the bare and renormalized Yukawa Lagrangians. We obtain

\[ Z_{Y_i} = Z_{J}^{-1} = Z_{m_i}. \tag{3.33} \]
We finally renormalize the bare Wilson coefficient $C_1^0(\alpha_s, m_i, Y)$ using [15, 28, 29]

$$C_1 = \frac{1}{1 + \alpha_s^2(\mu) \frac{\partial}{\partial \alpha_s(\mu)} \log Z'} \left[ 1 + \frac{\alpha_s^2(\mu)}{\pi} \frac{\beta_0}{\epsilon} + \left( \frac{\alpha_s^2(\mu)}{\pi} \right)^2 \frac{\beta'_1}{\epsilon} \right] C_1^0 . \quad (3.34)$$

Our final result for the renormalized Wilson coefficient reads

$$C_1 = -\frac{1}{3} Y_0 - \frac{11}{12} \alpha_s^2(\mu) \frac{1}{\pi} Y_0 - \frac{1}{3} \left( \frac{\alpha_s^2(\mu)}{\pi} \right)^2 \left\{ -n_l \left( \frac{67}{96} Y_0 + \frac{2}{3} Y_1 \right) + \frac{Y_0}{192} \left( \frac{1877}{77} n_h + \frac{113}{96} n_L + \frac{3}{8} L_2 \right) - \frac{Y_1}{8} \left( \frac{19}{8} \frac{113}{96} n_h + \frac{3}{4} L_1 \right) + \frac{3}{8} n_h X_2 \right\} + \sum_{1 \leq i < n_h} \left[ (y_i - y_j) \left( \frac{57}{68} \frac{m_i^2}{m_j^2} - \frac{m_i^2}{m_j^2} \right) + \frac{57}{128} \frac{m_i^2}{m_j^2} + \frac{57}{128} \frac{m_i^2}{m_j^2} + \frac{43}{32} \log \left( \frac{m_i}{m_j} \right) \right]$$

$$+ \frac{57}{256} \frac{m_i^6 + m_j^6}{m_i^6 + m_j^6} \log^2 \left( \frac{m_i}{m_j} \right)$$

$$- \log^2 \left( \frac{m_i}{m_j} \right) \left( \frac{73}{256} (y_i + y_j) + \frac{23}{128} \frac{y_i m_i^2 - y_j m_j^2}{m_i^2 - m_j^2} \right)$$

$$+ \frac{3}{512} \left( \frac{m_i^2 - m_j^2}{m_i^2 - m_j^2} \right) \left( 19 m_i^4 + 24 m_i^2 m_j^2 + 19 m_j^4 \right) \left( y_j \log \left( \frac{m_j - m_i}{m_j + m_i} \right) - y_i \log \left( \frac{m_i - m_j}{m_i + m_j} \right) \right)$$

$$- \frac{3}{1024} \left( \frac{19 m_i^6 + 5 m_i^4 m_j^2 - 5 m_i^2 m_j^4 - 19 m_j^6}{m_i^6 + m_j^6} \right)$$

$$\cdot \left( 8 y_i L_3 \left( \frac{m_i}{m_j} \right) - 8 y_j L_3 \left( \frac{m_j}{m_i} \right) - y_i L_3 \left( \frac{m_i^2}{m_j^2} \right) + y_j L_3 \left( \frac{m_j^2}{m_i^2} \right) \right)$$

$$- 2 \log \left( \frac{m_i}{m_j} \right) \left( y_i L_2 \left( \frac{m_i^2}{m_j^2} \right) + y_j L_2 \left( \frac{m_j^2}{m_i^2} \right) - 4 y_i L_2 \left( \frac{m_j}{m_i} \right) - 4 y_j L_2 \left( \frac{m_i}{m_j} \right) \right) \right\} . \quad (3.35)$$

Similarly to Eq. (3.13), we defined

$$L_1 = \sum_{i=1}^{n_h} \log \left( \frac{m_i}{\mu} \right) , \quad L_2 = \sum_{i=1}^{n_h} \log^2 \left( \frac{m_i}{\mu} \right) ,$$

$$Y_0 = \sum_{i=1}^{n_h} y_i , \quad Y_1 = \sum_{i=1}^{n_h} y_i \log \left( \frac{m_i}{\mu} \right) , \quad Y_2 = \sum_{i=1}^{n_h} y_i \log^2 \left( \frac{m_i}{\mu} \right) , \quad \text{and} \quad \text{with} \quad y_i = \frac{Y_i}{m_i} . \quad (3.37)$$

For $Y_i = \frac{m_i}{\mu}$ we recover the Wilson coefficient for an arbitrary number of heavy quarks with Standard Model-like Yukawa interactions of [18].
4. Composite Higgs models

In this Section we review briefly the composite Higgs model of Refs. [30, 31].

4.1 The Higgs sector

At low energy, the strong sector responsible for EWSB in composite Higgs scenarios can be described by a non-linear sigma model. Such description allows to decouple the light pseudo-Goldstone boson from other heavy composites of the new sector. We will consider the global symmetry breaking pattern $SO(5) \rightarrow SO(4)$, as this is the minimal pattern that includes custodial symmetry. An additional $U(1)_X$ symmetry is required in order to generate the correct Weinberg angle. The SM electroweak group $SU(2)_L \times U(1)_Y$ is embedded into $SO(4) \times U(1)_X \sim SU(2)_L \times SU(2)_R \times U(1)_X$ so that the hypercharge is given by $Y = T^3_R + X$ [33, 34]. We denote by $f$ the scale at which the global symmetry is broken. This scale is assumed to be larger than the EWSB scale $v \approx 246$ GeV. Too large values of $f$ would introduce a substantial fine-tuning of the model [32]; on the other hand, if the scale of new physics is too low, large contributions to electroweak parameters and flavour physics are expected. We set $f = 500$ GeV, which corresponds to a $\sim 10\%$ fine-tuning [32].

The effective theory becomes strongly coupled at a scale $\sim 4\pi f$. Above this energy, a more fundamental description needs to be introduced. If the coupling constant of the strong sector $g_\rho$ is not maximal, i.e. $g_\rho < 4\pi$, hadronic bound states appear below the strong-coupling scale. The typical mass of these resonances is $m_\rho \simeq g_\rho f$ and acts as a lower, effective cut-off for the effective-theory description. Throughout this work we will assume the presence of such weakly-coupled new states and take as cut-off $\Lambda = m_\rho \simeq 2\pi f$.

The $SO(5) \rightarrow SO(4)$ breaking is realized through a scalar field $\Sigma(x)$ subject to the constraint

$$\Sigma^2(x) = 1.$$  \hspace{1cm} (4.1)

In the non-linear representation

$$\Sigma(x) = \Sigma_0 e^{\Pi(x)/f}, \quad \Pi(x) = -i T^a \hat{h}^a(x) \sqrt{2},$$  \hspace{1cm} (4.2)

where $\Sigma_0 = (0, 0, 0, 0, 1)$ is the vacuum state that preserves $SO(4)$, $T^a$ are the four broken generators and $\hat{h}^a$ the corresponding Goldstone bosons. Eq. (4.2) can be rewritten as

$$\Sigma = \left( \frac{h}{f} \sin \frac{\hat{h}}{f}, \cos \frac{\hat{h}}{f} \right) \equiv \left( \hat{\Sigma}, \Sigma_5 \right),$$  \hspace{1cm} (4.3)

where $\hat{h} = (\phi^0, -\phi^-, \phi^+, \phi^-)$ transforms under the fundamental representation of $SO(4)$ and $h = \sqrt{(h^a)^2}$. The SM Higgs doublets with hypercharge +1/2 and -1/2 are respectively $\Phi = (\phi^+, \phi^0)^T$ and $\bar{\Phi} = i \sigma^2 \Phi^*$. The requirement

$$m_W^2 = \frac{g^2 f^2}{4} \sin^2 \left( \frac{\langle h \rangle}{f} \right) \equiv \frac{g^2 v^2}{4}$$  \hspace{1cm} (4.4)
for the mass of the $W$ boson yields

$$
\frac{v}{f} = \sin\left(\frac{\langle h \rangle}{f}\right) = \sin\left(\frac{\sqrt{2} \langle \Phi \rangle}{f}\right) \equiv s_\alpha . \tag{4.5}
$$

Higgs compositeness and the requirement for canonical normalization of the kinetic term lead to a rescaling of the physical Higgs field by a factor $c_\alpha = \sqrt{1 - v^2/f^2}$. This rescaling yields a reduction by a factor $c_\alpha$ of the couplings of the Higgs with the electroweak gauge bosons. The coupling of the Higgs boson to the fermions is also suppressed, but the suppression depends on how the fermions are embedded into representations of $SO(5)$. We will discuss this aspect in Section 4.2.

The reduction of the couplings between the Higgs and the gauge bosons leads to some sensitivity of the Peskin-Takeuchi $S$ and $T$ parameters [35,36] on the cut-off $\Lambda$. In the SM the Higgs boson regulates the logarithmic divergencies of the gauge bosons self-energies. The reduction of the Higgs boson couplings to the gauge boson spoils this effect. The result is a positive contribution to the $S$ parameter and a negative contribution to the $T$ parameter [32]. The strong dynamics can further affect electroweak precision observables through some higher-order operator. Custodial symmetry is included to protect $T$, while the $S$ parameter receives a further positive contribution. We refer to [32] for a more complete discussion of these effects.

The shifts to the $S$ and $T$ parameters from Higgs compositeness and from UV physics make the model incompatible with EWPT [30–32]. However, other composite states might lie below the cut-off of the effective theory and contribute to these observables. In particular, quark masses arise through mixing of the SM elementary fermions with fermionic bound states of the strong sector. We analyse this scenario and the effects on the electroweak parameters in the next Section.

### 4.2 The fermionic sector

We extend the $SO(5)$ symmetry of the strong sector to the fermion sector and assume that composite top-partners lie below the cut-off of the effective theory description. Many viable choices for the embedding of these states into representations of $SO(5)$ have been considered, both in the context of five-dimensional and effective four-dimensional models [8,32–34]. Following [31], we include fermionic multiplets $\Psi$ that transform under the fundamental representation of $SO(5)$. Their $SO(5)$-invariant mass Lagrangian is

$$
-\mathcal{L}_{SO(5)} = m_\Psi^i \bar{\Psi}^i \Psi^i + \mu_{ij} f(\bar{\Psi}^i \Sigma) (\Sigma^j \Psi^j) , \tag{4.6}
$$

where the indices $i,j$ allow for the possibility of more than one set of fermionic composites and the brackets in the second term indicate the contraction of the $SO(5)$ indices. In Eq. (4.6), $\mu_{ij}$ is a hermitian matrix and $\Sigma$ is the Higgs field defined in (4.3).

For models with one multiplet, only a small region of parameter space is allowed by electroweak precision measurements [30,31]. The introduction of more sets of fermionic composites is inspired by the five-dimensional model presented in [37], which does not seem to suffer from such severe constraints. In [31], it was shown that a four-dimensional
composite Higgs model with two multiplets of fermionic resonances is compatible with EWPT in large regions of parameter space.

A vector (5) of $SO(5)$ decomposes under $SO(4) \sim SU(2)_L \times SU(2)_R$ into a bidoublet $(Q, X)$ plus a singlet $T$,

$$\Psi = (Q, X, T) \Rightarrow (5) = (4) \oplus (1) \simeq (2, 2) \oplus (1, 1).$$ (4.7)

We assign to $\Psi$ an $U(1)_X$ charge of 2/3. In this way the $SU(2)_L$ doublets $Q$ and $X$ get hypercharge 1/6 and 7/6, respectively, and the singlet $T$ acquires hypercharge 2/3. Therefore the SM quarks $q_L$ and $t_R$ have the same quantum numbers as $Q$ and $T$. We can write for them an interaction Lagrangian

$$-\mathcal{L}_{int} = m^i_{tL} q^i_L Q^i + m^i_{tR} T^i_T t_R + h.c..$$ (4.8)

The doublet $X$ introduces a new quark of electromagnetic charge 2/3, which mixes with the top after electroweak symmetry breaking, and an exotic quark of charge 5/3 that does not couple to the Higgs boson.

Using Eqs. (4.6) and (4.8) and expanding $\phi^0$ in (4.3) around its vacuum expectation value, $\phi^0 = \frac{v + i b}{\sqrt{2}}$, we obtain the mass terms and the Yukawa couplings of the quarks. The mass matrix for the quarks of charge 2/3 reads

$$-\mathcal{L}^t_m = \begin{pmatrix} t_L^T_Q \\ Q^L \\ X^L \\ T_L \end{pmatrix}^T \begin{pmatrix} m^T_{tL} & 0 & 0 & 0 \\ 0 & m^2 & 0 & s^2_f \mu \\ 0 & 0 & m^2 & s^2_f \mu \\ m_R & s^2_f \mu & s^2_f \mu & m^2 \end{pmatrix} \begin{pmatrix} t_R \\ Q^R \\ X^R \\ T_R \end{pmatrix} + h.c..$$ (4.9)

The indices $u$ and $d$ denote the upper and lower components of a doublet, respectively. In the case of more fermionic resonances, the mass matrix is to be understood in block form.

Since the mass of the bottom quark is small, we do not expect large effects from bottom compositeness. Instead of generating a bottom mass introducing additional $SO(5)$ multiplets, we adopt a minimal description and introduce an explicit $SO(5)$-breaking term

$$\mathcal{L}^b = \lambda_{b\bar{q}L} \Phi b_R$$ (4.10)

to give a mass to the bottom quark. Therefore the mass matrix for the quarks of charge -1/3 is

$$-\mathcal{L}^b_m = \begin{pmatrix} b_L^T_Q \\ Q^L \\ X^L \\ T_L \end{pmatrix}^T \begin{pmatrix} -s^2_f \mu & 0 & 0 \\ 0 & m^2 \\ 0 & 0 & m^2 \end{pmatrix} \begin{pmatrix} b_R \\ Q^R \\ X^R \\ T_R \end{pmatrix} + h.c..$$ (4.11)

The Yukawa couplings of the top-like quarks are

$$-\mathcal{L}^t_h = h \begin{pmatrix} t_L^T_Q \\ Q^L \\ X^L \\ T_L \end{pmatrix}^T \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & s^2_f \mu & s^2_f \mu & 1-2s^2_f \mu \\ 0 & s^2_f \mu & s^2_f \mu & 1-2s^2_f \mu \\ 0 & 1-2s^2_f \mu & 1-2s^2_f \mu & -2s^2_f \mu \end{pmatrix} \begin{pmatrix} t_R \\ Q^R \\ X^R \\ T_R \end{pmatrix} + h.c..$$ (4.12)
where ⊗ denotes the matrix tensor product.

The most important bounds on the model come from the $S$ and $T$ parameters and the anomalous $Zb_L\bar{b}_L$ coupling. We use the fit to these three quantities that was employed in [31]. The scan over the parameter space is done as in Ref. [47]. We choose $f = 500$ GeV, $m_H = 120$ GeV, and we set the top and bottom masses to $[38, 39]$:

$$m_t = 172 \text{ GeV} \quad \text{and} \quad m_b = 4.16 \text{ GeV}. \quad (4.13)$$

Direct experimental searches impose lower limits on the masses of the new quarks. These analyses however assume that the new quarks decay entirely through one specific channel ($b' \rightarrow tW$ [40, 41], $b' \rightarrow bZ$ [42], $t' \rightarrow bW$ [43], $t' \rightarrow qW$ [44], $X \rightarrow tW$ [45]). Studies carried out in the context of a four-generation Standard Model show that the bounds can be significantly lowered when multiple decay channels are open [46]. Following [47], we impose the limits:

$$m_{5/3} > 365 \text{ GeV}, \quad m_{2/3}, m_{-1/3} > 260 \text{ GeV} \quad (4.14)$$
on the masses of the new quarks of charge $5/3$, $2/3$ and $-1/3$.

5. Higgs production in composite Higgs models

5.1 General LO results

We first compute the contribution from the charge 2/3 quarks to the LO Higgs production in the heavy-mass approximation ($m_q > 2m_H$). This is an interesting analysis, as we can prove that in this approximation the cross section is suppressed with respect to the SM result by a factor that only depends on the scales of the electroweak symmetry breaking $v$ and of the global symmetry breaking $f$. Such result is already known for the case of one multiplet [8]. We show that it holds for any number of multiplets.

Denoting by $\sigma^{CH(SM)}_{app}$ the LO production cross section in the heavy quark approximation in the composite Higgs (Standard) Model, the suppression factor reads:

$$R_g = \frac{\sigma^{CH}_{app}}{\sigma^{SM}_{app}} = \left[ \frac{\cos (2\langle h \rangle / f)}{\cos (\langle h \rangle / f)} \right]^2 = \left( 1 - 2s_\alpha^2 \right)^2 \cdot \left( \frac{1 - s_\alpha^2}{1 - s_\alpha^2} \right). \quad (5.1)$$

For our choice of parameters, $R_g = 35\%$.

Our proof follows the one of Ref. [8]. In the heavy-mass approximation, the SM Higgs production amplitude $M^{SM}_{gg \rightarrow H}$ can be written as (Eqs. (3.1), (3.35))

$$M^{SM}_{gg \rightarrow H} = f(\epsilon, p) \frac{Y_{top}}{m_{top}} = f(\epsilon, p) \frac{1}{v}. \quad (5.2)$$

In this expression, $f(\epsilon, p)$ contains the dependence on the polarization and momentum of the external gluons and $m_{top}, Y_{top}$ are the mass and Yukawa coupling of the top quark, respectively. In the presence of more heavy quarks of mass $m_i$ and Yukawa coupling $Y_i$, this result generalizes to

$$M_{gg \rightarrow H} = f(\epsilon, p) \sum_i \frac{Y_i}{m_i}, \quad (5.3)$$

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so that
\[ R_g^{1/2} = v \sum_i \frac{Y_i}{m_i} = v \text{Tr} \left( M^{-1}Y \right). \] (5.4)

Here \( M \) and \( Y \) denote respectively the matrices of masses and Yukawa couplings. Using
\[ Y = \frac{\partial M}{\partial \langle h \rangle}, \] (5.5)
we can rewrite (5.4) as
\[ R_g^{1/2} = v \text{Tr} \left( M^{-1} \frac{\partial M}{\partial \langle h \rangle} \right) = v \frac{\partial}{\partial \langle h \rangle} \text{Tr} \log M = v \frac{\partial}{\partial \langle h \rangle} \log \det M. \] (5.6)

The dependence of the determinant of the mass matrix on \( \langle h \rangle \) (i.e. on \( s_\alpha, c_\alpha \)) is of the form
\[ \det M = s_\alpha c_\alpha \xi(m_L, m_R, m_\Psi, \mu, f), \] (5.7)
where \( \xi(m_L, m_R, m_\Psi, \mu, f) \) is a function of the parameters indicated. We derive the expression for \( \det M \) and give the explicit form of the function \( \xi \) in Appendix A. Inserting (5.7) into (5.6) and using the definition of \( s_\alpha \) (Eq. (4.5)), we obtain
\[ R_g^{1/2} = v \frac{\partial}{\partial \langle h \rangle} \log \left[ \sin \left( \frac{\langle h \rangle}{f} \right) \cos \left( \frac{\langle h \rangle}{f} \right) \xi \right] = \frac{\cos (2\langle h \rangle/f)}{\cos (\langle h \rangle/f)}. \] (5.8)

As anticipated, this result does not depend on any of the parameter-space details, including the number of fermionic multiplets.

Finite-mass corrections, bottom-quark and electroweak effects modify this result already at LO. In particular, bottom-quark and electroweak contributions are more significant than in the SM. In the SM, the inclusion of the bottom quark lowers the LO Higgs production cross section by about 7%, while electroweak effects give a 5% increase. As we have seen, in the composite Higgs model the contribution from heavy quarks is strongly suppressed. On the other hand, in our description we couple the bottom quark directly to the Higgs boson. As a consequence, its Yukawa coupling is reduced only by about 13% with respect to the SM value\(^1\). One therefore expects a larger reduction of the cross section from bottom quark loops, of the order of 10%. Similarly, the couplings of the gauge bosons to the Higgs are reduced by a factor \( c_\alpha \sim 87\% \) with respect to their SM value. At LO in \( \alpha_s \), the contribution from electroweak corrections should therefore yield an increase of the cross section by about 7%, against the +5% of the SM. These estimates are confirmed by the exact numerical values for the LO production cross section that we report in Table 1.

---
\(^1\)This suppression is related to the mechanism that we adopt to give a mass to the bottom quark. In particular, it can be modified in a scenario where an additional \( SO(5) \) multiplet is introduced and the bottom quark acquires a mass in a similar way as the top quark.
Table 1: Leading-order gluon-fusion cross section in the SM and in the composite Higgs model of Section 4 for the 7 TeV LHC. The notation \((x_{\text{min}} \div x_{\text{max}})\) indicates the range of values that the quantity \(x\) can assume. We report the cross section \(\sigma_{t}^{\text{LO}}\) due to charge 2/3 quarks only (including finite-mass effects), and analyse how it changes with the inclusion of bottom-quark and electroweak corrections \(\sigma_{tb}^{\text{LO}}\) and \(\sigma_{te}^{\text{LO}}\). In the last column we give the total LO cross section \(\sigma^{\text{LO}}\).

|        | \(\sigma_{t}^{\text{LO}}[pb]\) | \(\sigma_{tb}^{\text{LO}}[pb]\) | \(\frac{\sigma_{tb}^{\text{LO}} - \sigma_{t}^{\text{LO}}}{\sigma_{t}^{\text{LO}}}\) | \(\sigma_{te}^{\text{LO}}[pb]\) | \(\frac{\sigma_{te}^{\text{LO}} - \sigma_{t}^{\text{LO}}}{\sigma_{t}^{\text{LO}}}\) | \(\sigma^{\text{LO}}[pb]\) |
|--------|-------------------------------|-------------------------------|---------------------------------|-------------------------------|---------------------------------|-------------------------------|
| SM     | 8.8                           | 8.1                           | -7\%                            | 9.2                           | +5\%                            | 8.6                            |
| CH     | \((2.9 \div 3.2)\)           | \((2.6 \div 2.8)\)           | -10\%                           | \((3.2 \div 3.4)\)           | +7\%                            | \((2.9 \div 3.1)\)           |

Table 2: Gluon-fusion cross section through NNLO in the SM and in the composite Higgs model, with the corresponding scale variation errors. The factor \(R'_{g}\) is defined as in Eq. (5.1), but for the full result through three loops.

| \(\sigma^{\text{SM}}[pb]\) | \(\delta_{\text{scale}}^{(+)}\) \% | \(\delta_{\text{scale}}^{(-)}\) \% | \(\sigma^{\text{CH}}[pb]\) | \(\delta_{\text{scale}}^{(+)}\) \% | \(\delta_{\text{scale}}^{(-)}\) \% | \(R'_{g}\) |
|-----------------------------|-------------------------------|-------------------------------|-----------------------------|-------------------------------|-------------------------------|-----------------------------|
| 17.6                        | +9\%                          | -10\%                         | \((5.9 \div 6.4)\)         | +(6\% \div 12\%)             | -(7\% \div 11\%)             | \((34 \div 37)\) \%         |

5.2 Precise prediction through NNLO

We now compute the Higgs production cross section in the composite Higgs model through NNLO. We include the contribution from the heavy quarks retaining the full mass dependence through two loops. The NNLO corrections are computed in the effective theory approximation according to the Wilson coefficient (3.35). Since all the heavy quarks are integrated out from the low-energy effective theory, the only difference with the SM calculation is in the expression of the Wilson coefficient. The remaining part of the calculation is the same as in the SM [5–7]. Following the approach of [13], the NNLO corrections are normalized to the exact LO cross section according to

\[
\sigma^{\text{NNLO};\text{heavy}} \simeq \sigma_{\text{exact}}^{\text{LO};\text{heavy}} \cdot \left( \frac{\sigma^{\text{NNLO};\text{heavy}}}{\sigma_{\text{exact}}^{\text{LO};\text{heavy}}} \right)_{\text{effective}}.
\]

Since bottom-quark effects are more important than in the SM, we compute them exactly through NLO [4, 11]. We also include the full two-loop SM electroweak corrections of Ref. [12] and the three-loop mixed QCD and electroweak corrections derived within an effective-theory approach in Ref. [13]. Both the electroweak corrections are rescaled by the factor \(c_{a}\) that reduces the coupling of the Higgs to the gauge bosons. All the effects described here are included in the code \texttt{iHixs} [14], which we use for the calculation of our results.

In Table 2 we present the full NNLO cross section in the SM and in the composite Higgs model. The results are similar for the case of one and of two multiplets of composite fermions. We use the MSTW2008 NNLO parton distribution functions [48] and set the renormalization and factorization scales to \(\mu = \mu_{f} = \mu_{r} = m_{H}/2\). The 35\% suppression factor \(R'_{g}\) computed in Section 5.1 is confirmed through NNLO. We estimate the uncertainty
Table 3: NLO and NNLO K-factors in the SM and in the composite Higgs model.

|                  | SM       | CH       |
|------------------|----------|----------|
| $\sigma^{NLO}_{(m)}$ | $+75\%$ | $(77 \div 78)\%$ |
| $\sigma^{NNLO}_{(m)}$ | $+106\%$ | $(108 \div 110)\%$ |

due to higher order corrections by varying the scale $\mu$ in the interval $(m_H/4, m_H)$. This scale variation uncertainty is similar to the SM one.

Finally, in Table 3 we compare the K-factors in the SM and in the composite Higgs model. As in the SM, the K-factors are large and the NNLO result is about twice as big as the LO cross section.

6. Conclusions

We presented the construction of an effective theory for extensions of the Standard Model with an arbitrary number of heavy quarks coupling to the Higgs. We assumed a general form for the Yukawa couplings of these quarks to the Higgs boson. This situation arises for example in the context of composite Higgs model, where the mass of the quarks can be explained through the mixing of the fundamental SM particles with heavy composite fermions. We computed the Wilson coefficient of the effective Higgs-gluon vertex through $O(\alpha_s^3)$. We used our result to compute the Higgs production cross section through NNLO in a composite Higgs model with an $SO(5) \rightarrow SO(4)$ global symmetry breaking pattern and one or two multiplets of composite fermions transforming under the fundamental representation of $SO(5)$. We showed that, in the heavy quark-mass approximation, the LO production cross section is suppressed with respect to the SM value by a factor that depends neither on the details of the parameter space nor on the number of multiplets. As in the SM, the NNLO result is enhanced with respect to the LO cross section by approximately a factor of 2. The scale variation errors also behave in a similar way as in the SM. We included in our result also the full dependence on the bottom quark mass through two loops and the two-loop electroweak and three-loop mixed QCD and electroweak corrections. Both these effects are enhanced with respect to the SM. As in the SM, they give contributions of opposite sign, which cancel.

In this work we applied our result for the Wilson coefficient to a specific beyond-the-Standard Model scenario, but the calculation can be extended to any model with additional new quarks in the fundamental representation of the colour group.

Acknowledgements

We thank Babis Anastasiou and Giuliano Panico for many useful discussions and for their comments on the script, and Achilleas Lazopoulos for providing the preliminary version of iHiXs. We greatly appreciated the hospitality of the ETH theory group during parts of this work. This research is supported by the DOE under Grant DE-AC02-98CH10886.
A. Analytical form of \( \det M \)

We derive here the analytical result for the determinant of the mass matrix of charge 2/3 quarks, \( \det M \), for an arbitrary number of multiplets. Let us recall (Eq. (4.9)) that the mass matrix for the charge 2/3 quarks reads

\[
M = \begin{pmatrix}
0 & m_T^L & 0 & 0 \\
0 & m_\Psi + s_\alpha^2 f_\mu & s_\alpha^2 f_\mu & s_\alpha c_\alpha \sqrt{2} f_\mu \\
0 & s_\alpha^2 f_\mu & m_\Psi + s_\alpha^2 f_\mu & s_\alpha c_\alpha \sqrt{2} f_\mu \\
m_R & s_\alpha c_\alpha f_\mu & s_\alpha c_\alpha f_\mu & m_\Psi + c_\alpha^2 f_\mu \\
\end{pmatrix}.
\]

(A.1)

Successively taking linear combinations of lines/columns of \( M \), we can recast it into the form

\[
M' = \begin{pmatrix}
m_T^L & 0 & 0 & 0 \\
0 & m_R - s_\alpha^2 + 2 s_\alpha^2 m_\Psi & m_\Psi & 0 \\
s_\alpha^2 f_\mu & 0 & m_\Psi & s_\alpha c_\alpha \sqrt{2} f_\mu \\
m_\Psi & 0 & -m_\Psi & 0 \\
\end{pmatrix} \equiv \begin{pmatrix} A & B \\ C & D \end{pmatrix},
\]

(A.2)

where

\[
A = \begin{pmatrix} m_T^L & 0 \\ 0 & m_R \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ -s_\alpha^2 + 2 s_\alpha^2 m_\Psi \end{pmatrix}, \\
C = \begin{pmatrix} s_\alpha^2 f_\mu \\ m_\Psi \end{pmatrix}, \quad \text{and} \quad D = \begin{pmatrix} m_\Psi \\ -m_\Psi \end{pmatrix}.
\]

(A.3)

Because of the properties of determinant,

\[
\det M = \det M' = \det D \det (A - BD^{-1}C) = s_\alpha c_\alpha \left[ \frac{1}{\sqrt{2}} m_T^L W^{-1} m_\Psi \det(f_\mu) \det(W) \right],
\]

(A.4)

where

\[
W = m_\Psi + \frac{1}{T} m_\Psi^{-1} m_\Psi.
\]

(A.5)

The quantity in square brackets in Eq. (A.4) corresponds to the function \( \xi \) introduced in Eq. (5.7).

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