The effective of localized holes on the long-range antiferromagnetic order in bilayer cuprates is studied, by applying the renormalization group to the appropriate classical non-linear magnetic phase diagram of Ca doped YBa$_2$Cu$_3$O$_6$. The theory accounts quantitatively for the magnetic divided by the temperature $T$.

$$H_{\text{int}} = \frac{D_s}{T} \int dr \sum_{i,p,p'} \beta_{pp'} f_i^{(p)}(r) \cdot \partial_i \mathbf{n}^{(p')}.$$  

Here $\beta_{pp'} = \delta_{pp'} + \Gamma_p \delta_{p,p'-1} (1 - \delta_{pp'}$, $\Gamma_p \sim \gamma_p < 1$, and $f_i^{(p)}(r) = M \sum_j \delta(r - r^{(j)}) \alpha_i(r^{(j)}) \mathbf{m}(\mathbf{r}^{(j)})$. At temperatures lower than the optical spin-wave gap (approximately 70 meV for YBCO) only configurations with $\mathbf{n}^{(1)} \approx \mathbf{n}^{(2)}$ dominate the action $\mathcal{L}$, and the Hamiltonian transforms to

$$H = \frac{1}{2t_v} \int dr |(\nabla \mathbf{n}|^2 + 2 \sum_i r_i^{(p)}(r) \cdot \partial_i \mathbf{n},$$

with $t_v = T/\nu \rho$, and $r^{(p)} \approx \sum_p f^{(p)}/\nu$.

The randomly distributed effective dipoles develop dipole-dipole interactions mediated by the AF spin background. Therefore, at sufficiently low temperatures they either freeze in a random spin glassy way or at least develop exponentially long range spin–glassy correlations. Assuming that the range of these correlations is much larger than that of the AF correlations, which in Hole doped AF is finite at all $T$, justifies treating these moments as quenched. The variables $r_i^{(p)}$ and $\mathbf{a}(r)^{(p)}$ are also quenched. Denoting quenched averages by $\left\langle \ldots \right\rangle$, we write $\left\langle f^{(\nu)}(r) f^{(\nu)}(r') \right\rangle = \lambda_{\nu} \delta_{\mu\nu} \delta_{ij} \delta(r - r')$, with

$$\lambda_{\nu} = M^2 x/6 \nu \equiv A_{\nu} x.$$  

The $\nu$-layer problem is thus mapped onto that of the monolayer one, albeit with rescaled parameters $t_\nu$ and $\lambda_{\nu}$. Both these parameters are marginal in the renormalization group (RG) sense. This allows to employ the RG methods to find the effect of the dipole moments on a two-dimensional (2D) AF. Generalising the results derived in Ref. [9] for LCO, with $\nu = 1$, the 2D correlation length obtained in the one-loop approximation is:

$$\xi_{2D} = C(T, x) \exp(2\pi/3\lambda_{\nu}), \quad t_\nu < \lambda_{\nu},$$

and

$$\xi_{2D} = C(T, x) \exp\left(\frac{2\pi}{t_\nu} \left[1 - \frac{\lambda_{\nu}}{t_\nu} + \frac{\lambda_{\nu}^2}{3 t_\nu^2}\right]\right), \lambda_{\nu} < t_\nu.$$  

Here $C(T, x)$ is a smooth function of its arguments, known for the bilayer ($\nu = 2$) YBCO only at $x = 0$.

The 3D transition temperature $T_N(x)$ of a system consisting of weakly coupled planes may be deduced from the relation

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Here $C(T, x)$ is a smooth function of its arguments, known for the bilayer ($\nu = 2$) YBCO only at $x = 0$.
\[ \alpha \xi_{2D}^2(T_N, \lambda) \sim 1, \]  
with \( \alpha \) generated by an interplane (interbilayer) exchange, or some in–plane spin anisotropy.

Combining Eqs. (6), (8), and (9), we obtain the critical line \( T_N(x) \). At \( T_N(x) < \rho_s A_1 x \), the critical line is expected to be practically vertical, with \( x_c \) given by

\[ x_c^{-1} = 3 A_v L(0, x_c), \]

where \( L(T, x) = (1/4\pi) \ln(\alpha^2 / \alpha C(T, x)) \), \( \alpha \) is the lattice constant. At smaller defect concentrations, i.e. for \( T_N(x) \) higher than \( \rho_s A_1 x \), the critical line is

\[ \frac{T_N(x)}{T_N(0)} = \frac{A_v L(T_N, x)}{1 - [1 - 3 A_v x L(T_N, x)]^{1/3}}. \]

\[ \text{FIG. 1. } \frac{T_N(x)}{T_N(0)} \text{ versus hole concentration } x. \text{ Full lines are theory (Eqs. (8), (9)) for LCO (left line) and YBCO (right line). Symbols are from experiments; LCO: • Ref. [12], + Ref. [13] \text{ V Ref. [14], } \triangle \text{ Ref. [3], YBCO: o Ref. [2], } \triangle \text{ Ref. [3].} \]

The parameter \( A_1 \approx 20 \) for LCO was estimated in Ref. [2] from the temperature dependence of \( \xi \) in slightly doped samples. It follows then from Eq. (6) that \( A_2 \) for YBCO is \( A_2 \approx 10 \). Considering that the logarithmic factor \( L \) does not differ much for different cuprates, we conclude that \( x_c \) in YBCO should be about two times larger than in LCO. Since both \( T_N \) and \( x_c \) depend on \( C(T, x) \) only logarithmically, we neglect in what follows the temperature and concentration dependence of \( C(T, x) \). The prefactor \( C(0, x_c) \) for LCO is approximately 1.26 \( \AA \), while \( \alpha \approx 10^{-4} \). This gives \( L \approx 0.91 \). For YBCO we estimate \( L \) from Eq. (8) for the undoped sample. With \( T_N \approx 410 \) K and \( \rho_s \approx 200 \) K (this follows from the data given in Ref. [3] and the quantum renormalization factors given in Ref. [3]), one gets \( L \approx 0.97 \).