Mathematical modelling skills of prospective mathematics teachers in problem-solving

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Abstract. This paper aims to explain the results of the analysis of mathematical modelling skills of prospective mathematics teacher in problem-solving. The data collection used is a written test. The research subjects were prospective mathematics teachers at Mathematics Education Department of Teaching and Education Faculty of Universitas Sriwijaya. The results of the study state that mathematical modelling skills of prospective mathematics teachers in problem-solving are categorized sufficient.

1. Introduction

Mathematical modeling is the process of representing real problems in everyday life into mathematical form so that finding a solution [1]. According to [2] mathematical modeling is a process of using mathematics to present, analyze and make predictions and insights into the real world. This process can be seen as a mathematical process [3, 4].

The stages of mathematical modeling in order to solve real-world problems consist of (1) understanding the problem; (2) make assumptions; (3) defining variables; (4) search for functional relationships between variables; (5) building models; (6) completing the model; (7) interpret the solution; and (8) verifying model [3, 5]. Whereas according to [6] the stage of mathematical modeling is becoming simpler: (1) understand the problem; (2) prepare the mathematical model of the real model; (3) solve math problems in mathematical models; and (4) interpret the results of mathematics in a real situation. This modeling stage only solves the problems related to the real world. For general problems, the completion process follows the stage of problem solving. According to [7] There are four stages in the problem solving process namely (1) understanding the problem; (2) making a plan; (3) execute the plan; and (4) look back. When viewed from the problem solving and mathematical modeling stages can be said that mathematical modeling is part of the problem solving.

One of skills to solve problems on learning mathematics is mathematical modeling skill. It has an important role in mathematics learning which is (1) to help students in understanding the realistic context (the real world), (2) triggering students’ motivation in the development of more competent mindset, (3) students are able to connecting between real-world problems with mathematics [8]. In line with research of [9] suggests that there are several abilities that can be raised after applying mathematical modeling, namely identifying surrounding events, exploration of real world knowledge with mathematics and discovering ways or strategies in resolving problems. Based on the opinions above then mathematical modeling skill is important to be owned by the students as the first step in solving the problems. Therefore, the ability of teachers and prospective teachers in mathematical modeling is indispensable.

Based on the results of the study [10] stated that mathematics education students still have errors in solving mathematical modeling problems. However, in which stage of mathematical modeling mistakes
by the prospective teacher students do, have not studied in depth. This can mean the importance of examine causes of it. Therefore, analyze mathematical modeling skills of prospective mathematics teachers in problem solving.

2. Method

This research is a descriptive study with the purpose of describing the analysis of prospective mathematics teachers’ modeling skills in problem solving. The research subject is 12 prospective mathematics teachers who are in semester III and V academic year 2017/2018. Data of mathematical modeling skill obtained from written tests consisting of five problems. However, presented in this article is only a result of analysis of the skills of mathematical modeling students in answering one problem that is related to the construction of new airport. The problems used are not validated, because it is valid. What is done just translate the question into Indonesian language. The problem is show below:

“McNairy County is planning to build a new regional airport and wants it to be equally close to each of the Triangular Cities: Adamsville, Selmer, and Leapwood. If Adamsville and Selmer are 50 mi apart, Leapwood and Adamsville are 30 mi apart, and Leapwood and Selmer are 40 mi apart, where should the airport be located?” [11].

After being translated to Bahasa Indonesia, the question is as follows

“Perusahaan penerbangan Angkasa Raya berencana akan membangun sebuah bandara baru dimana jaraknya sama terhadap 3 kota berikut: kota Lingga, kota Praja, dan kota Rasta. Jika jarak antar kota Lingga dan kota Praja 50 km, jarak antar kota Rasta dan kota Lingga 30 km, serta jarak antar kota Praja dan kota Rasta 40 km, dimanakah sebaiknya letak bandara baru tersebut dibangun?”

The student answer sheet is analyzed with regard to mathematical modeling indicators as expressed [6]. The modeling stage of the problem above is (1) understanding the problem: The students are considered to understand the problem if they write: A flight company will build a new airport that is the same distance to all three cities forming a triangle. Known distance Praja and Lingga is 50 km, Lingga and Rasta is 30 km, Praja and Rasta is 40 km. Telling that the three distances between cities form the right triangle as it meets Triple Pythagoras. The question of the problem is to determine the location of the new airport, which is the same from all three cities; (2) prepare a mathematical model of a real model: Mathematical models from this problem can be expressed in a right triangle that showed Figure 1.

![Figure 1. Mathematical model with right triangle.](image)

This is clearly apparent that the distances shown by these cities correspond to triple Pythagoras, $30^2 + 40^2 = 50^2$ and forming right triangle. So the fact that these cities are the corners of the triangle to determine the location of the airport to be built. To determine the location of the new airport, the city of Rasta can be placed at the point of origin at the Cartesian coordinates, with Praja on the ordinate, as well as Lingga on the abscissa or Praja on the abscissa and Lingga on the ordinate. The coordinate system with these cities and the location of the unspecified airport is described as Figure 2.
Figure 2. Model on coordinate plane.

Based on the information on the Figure 2, the distance between the new airport and Praja (BP), the distance between the new airport and Rasta (BR), and the distance between the new airport and Lingga (BL) modeled as follow:

\[ BR = \sqrt{(x-0)^2 + (y-0)^2} \]  
\[ BL = \sqrt{(x-30)^2 + (y-0)^2} \]  
\[ BP = \sqrt{(x-0)^2 + (y-40)^2} \]

(3) solve math problems in mathematical models: Students are considered able to solve math problem if they can declare the following: The distance between Praja and Lingga is the longest, therefore must be the length of the hypotenuse of the triangle, Praja and Rasta should be the corner point of right triangle. Each new airport's distance with three cities must be the same, so

\[ BP = BR \]
\[ \sqrt{(x-0)^2 + (y-40)^2} = \sqrt{(x-0)^2 + (y-0)^2} \]
\[ x^2 + y^2 - 80y + 1600 = x^2 + y^2 \]
\[ -80y + 1600 = 0 \]
\[ -80y = -1600 \]
\[ y = 20 \]

In the same way, BR = BL, or \( x = 15 \). Thus the airport is located at a point (15, 20). Another way to solve this problem, see that the same distance from the R, P, and L is the center of the circumscribed RPL. The center can be determined by intersection of two perpendicular bisector of triangle RPL. The final stage is (4) interpreting math results in the real world: students are considered able to interpret the results of mathematics in the real world when they can declare a point (15, 20), meaning that the new airport is located 15 km to the east and 20 km to the north of Rasta on a Cartesian coordinate system. The new airport distance with all three cities is 25 km.

3. Result and Discussion
The results of the analysis on 12 the answer of research subjects resulted 11 descriptions of student answers, there was 1 student who did not give an answer at all about the problem given. Description of students modelling skills outlined in four steps are understanding the problem, converting the problem into mathematical models in this case the model is right triangle, solve math problem in mathematical model that determines the point location new airport from the right triangle, and interpret the point's location into the real world. The descriptions of each of these stages are outlined below.
3.1. Understanding the problem
Students' understanding of the problem given is not only measured from what is known and asked, but it can be measured also from the analysis of the known element that is the distance between the three cities forming right triangle. From the eleven students who were analysed, there were four students who had stated all the things that were known and asked and had analysed that the known element formed a right triangle. Students who have written what is known and asked completely, but have not analysed that the three cities form a right triangle there are as many as four students. There is one student who does not write down all the known and asked elements, but directly model them in the form of a right triangle as shown in Figure 3 (i). There is one student who only writes the known element, without writing down the question and without giving the analysis that the known thing form the right triangle. While one other student did not write down what was known and asked.

Based on the results of data analysis of students' understanding of the problem can be said that the understanding is good, although the statement that the distance between the three cities forming a right triangle is not well expressed. Understanding the problem is not only see from the writing of what is known and asked, but can be translated in another form. This is shown with the students stating the problem in the right triangle. In general, students have been able to understand the problem by stating the information from the question given to the mathematical form [12]. In addition, this research is also in line with previous research stating that the student's ability is good in understanding the problem even though not all information is disclosed [13].

![Figure 3. Mathematical models from students’ answers](image-url)
3.2. Make Mathematical Model

The expected mathematical model of the problem is the right triangle with a side length of 30 km, 40 km, and 50 km, the side with a length of 50 km is the longest side of the triangle. To determine the distance of a new airport from all three cities using the formula (1), (2), and (3) between two points as stated in the method section above. Based on the results of the analysis of the student's answer sheet, from eleven students who gave the answer, only three students model with the correct right triangle image complete with the sign of right triangle, and there are five students who forming right triangle model but does not give a sign of right triangle. Then two students describe the arbitrary triangle and one student writes in the form of a circle.

Based on the data analysis results can be seen that the students’ ability to model the problems is categorized enough, because it only describes the problem into the form of a right triangle and has not used the sign of right triangle and the student has not been able to write to the form of mathematical models. According to Haines and Croach in [14] making the model from the real world to the mathematical model is one of the difficulties in mathematical modelling. The results of the study [15] stated that students were only sketch problem in the form of images but not using variables in forming mathematical models.

3.3. Solve The Model

Completion of the expected model of the given problem is to be able to determine the point of coordinates where the new airport will be built so that it has the same distance in all three cities. To be able to determine the new airport point, it is assumed that the new airport distance to Praja is equal to the new airport distance to Rasta.

Based on the results of the analysis on the students’ answer sheet, all students do not specify the point of coordinates where the new airport will be built. But the students’ answer have led to the airport distance to each city should be the same. From the eleven students there are five students are correct in determining the point of the airport located in the middle of Praja and Lingga by giving the symbol of the same length line can be seen in the Figure 3 (a). Subsequently two students had correctly determined the point between Praja and Lingga but did not give the symbol the same line length. Then two students draw dots inside the triangle. Two students did not describe the airport point it was in an existing image.

Based on the results of the analysis, all students have not been able to complete the model that determines the coordinate point because it has not written the mathematical model. However, with the help of picture, students are able to determine the new airport point even without the coordinate point. The inability to determine the coordinate point of the problem is because the student has not yet completed in the previous stage, which model the problem to the mathematical form so that the completion stage could not find the answer [16].

3.4. Interpreting to the Real World

Interpreting into the real world in this matter is by determining the direction of the wind and then the distance from the airport to all existing cities. Based on the results of the analysis there are: firstly, two students who have written the airport distance to all cities in the triangle that has been made can be seen in Figure 4.

Secondly, three students did not write their distance when they wrote the same long side symbol. Student didn't write down the same side symbol and not write down the distance.
Based on the analysis results some students have been able to determine the airport distance to each city by drawing the existing triangle. From the answers can be seen students try to guess where the new airport will be built so that it has the same distance in each city. This is one of the strategies in solving the problem of guessing and using images. According [14] to solve the problem in mathematical modelling, students are expected to use various strategies to solve problems. But to determine the interpretation in the coordinates of the point there are no students who answer correctly because it is incomplete in making the mathematical model thus causing the final completion and interpreting the real world so wrong.

4. Conclusion

Based on the analysis of the students’ answer sheet in mathematics modelling can be concluded that the students’ ability to understand the problem is good, because it is able to determine what information is in the matter given. As for the stage of model problems, the ability of students is limited to using images to help with the completion and no one writes by the formula spacing between the two points so that in the stage model students categorized enough. Further at the stage of solving the problem of all students no one wrote the coordinates of the new airport, this is because in the stage making model the problem no one used a two-point distance formula so that it resulted in no one a new airport coordinate point. At the stage of solving the problem is categorized enough because it is able to determine the distance of the airport to three existing cities by using a triangle image. Last on the stage interprets the results, no student who wrote with the coordinates of the point then interpreted with the direction of the wind, but to determine the distance of the airport to three existing cities students are right so the ability of students at this stage is sufficiently categorized.

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