Breaking discrete symmetries in the effective field theory of inflation

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Abstract. We study the phenomenon of discrete symmetry breaking during the inflationary epoch, using a model-independent approach based on the effective field theory of inflation. We work in a context where both time reparameterization symmetry and spatial diffeomorphism invariance can be broken during inflation. We determine the leading derivative operators in the quadratic action for fluctuations that break parity and time-reversal. Within suitable approximations, we study their consequences for the dynamics of linearized fluctuations. Both in the scalar and tensor sectors, we show that such operators can lead to new direction-dependent phases for the modes involved. They do not affect the power spectra, but can have consequences for higher correlation functions. Moreover, a small quadrupole contribution to the sound speed can be generated.

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Contents

1 Introduction 1

2 System under consideration 3
  2.1 Unitary gauge condition with broken space-time diffeomorphisms 3
  2.2 Background system 4

3 Quadratic operators that break discrete symmetries 7

4 Dynamics of linearized fluctuations 9
  4.1 No propagating vector modes 10
  4.2 Direction-dependent phase in the scalar sector 10
  4.3 Chiral phase in the tensor sector 12

5 Conclusions 14

A Wald’s no-hair theorem in the EFT of inflation 15

B List of allowed operators 16

C Circular polarization conditions 16

1 Introduction

The inflationary paradigm [1–3] provides a convincing explanation for the statistical properties of the temperature fluctuations in the cosmic microwave background (CMB) and of the distribution of galaxies on large scale in our universe. The generic predictions of the simplest models of inflation, based on a single scalar field slowly rolling down on a potential, fit very well current observations [4]. On the other hand, the wealth of available observational data motivates a detailed theoretical analysis of inflationary scenarios. Indeed, more refined theoretical predictions — when tested by observations — allow us to characterize more accurately the mechanism of inflation.

The effective field theory (EFT) of inflation [5, 6] has emerged in recent years as a powerful unifying approach for analyzing broad classes of inflationary scenarios; see [7] for a review. After specifying general properties of the system, such as the number of available degrees of freedom and the symmetries respected (or slightly broken) by the fields that drive inflation, one can write a general, model-independent action governing cosmological fluctuations.\footnote{There is another, top-down EFT which results from integrating out heavy degrees of freedom given a mother theory [8–11].} This action is characterized by operators that can be associated with the amplitude and scale dependence of the correlation functions of the curvature and tensor perturbations. Measuring such cosmological observables can allow us to test broad classes of scenarios associated with the operators that constitute the effective action for inflationary fluctuations. Moreover, a systematic analysis of all possible operators compatible with the given symmetries can also suggest new effects, not predicted or analyzed within the specific
models studied so far, that if supported by observations would motivate new directions for model building.

During inflation, time translation invariance is broken by the homogeneous cosmological background. Motivated by this, most of the works on the EFT of inflation until now has focussed on analyzing setups where the dynamics of perturbations breaks time diffeomorphisms, while maintaining spatial diffeomorphism invariance. On the other hand, we know very little about the nature of the fields driving the inflationary epoch. The simplest case is a single scalar field breaking time reparameterization only. But there is also the possibility of inflationary systems that break also spatial diffeomorphism invariance, like inflationary vector fields [12] (see e.g. [13, 14] for review), or alternatively a set of scalar fields obeying special internal symmetries that acquire space-dependent vacuum expectation values, as in the case of solid/elastic inflation [15, 16] (see also [17, 18]). Motivated by these considerations, we find interesting to exploit the EFT of inflation to explore more general options than those studied so far, also considering the effects of slightly broken spatial diffeomorphism invariance. Such effective description, as we will see, also allows one to describe systems where spatial anisotropies can be generated during inflation, avoiding Wald’s no-hair theorem [19].

The power of EFT is that we do not need to commit on specific models to develop our arguments, that are based on general properties of the action of perturbations, and that allow us to explore generic cosmological consequences of systems that break all diffeomorphisms during inflation. The aim is to identify in a model-independent way novel operators that can lead to new effects associated with inflationary observables, as non-standard correlations among inflationary perturbations.

The analysis of such systems was started in [20] — building on the results of [15, 21] — where it has been shown that, when spatial diffeomorphism invariance is broken, the EFT of inflation allows for operators that modify the usual dispersion relations in the scalar and tensor sectors. They can lead to blue spectrum in the tensor sector, and can induce non-vanishing anisotropic stress in the scalar sector that violates the conservation of the curvature perturbation on large scales. See also [22–24].

In this article we instead focus on the interesting class of operators that break discrete symmetries during inflation. In particular, using the EFT of inflation, we consider the consequences of the leading operators that break discrete symmetries as parity and time-reversal during this epoch. Unless discrete symmetries are imposed by hand on the theory under consideration, such operators will normally be generated by inflationary dynamics or renormalization effects: hence it is interesting to explore their consequences. Parity-violating interactions have been studied in great detail for their consequences in the CMB, starting with [25]. These operators can be associated with the amplification of the amplitude of one of the circular polarization of tensor modes around horizon crossing, leading to distinctive effects associated with $TB$ and $EB$ cross correlations in the CMB [26, 27]. Moreover, parity-violating operators can also affect the scalar sector, leading to statistical anisotropies in the bispectrum, or also explain anomalies in the CMB. The realization of models leading to parity violation during inflation and their observational consequences have been motivated by, for example, pseudoscalars coupled to gauge fields [28] or Chern-Simons modifications of gravity [29, 30]. See e.g. [31–52] for a selection of papers discussing both theoretical and observational aspects of parity violation during inflation.

To the best of our knowledge, there are no studies on the consequences of operators that contain a single derivative of time coordinate: in absence of better name, we say that these operators break time-reversal symmetry during inflation. In this article, using the model-
independent language of the EFT of inflation, we study selected operators that break the aforementioned discrete symmetries, and their effects for the dynamics of linearized perturbations around a homogeneous and isotropic Friedmann-Robertson-Walker (FRW) universe. Both in the scalar and tensor sectors, we show that such operators can lead to new direction-dependent phases for the fluctuations, and moreover a small quadrupole contribution to the effective sound speed. A direction-dependent phase does not affect the power spectrum, but can have consequences for higher correlation functions.

We start in section 2 describing the system that we consider, and on how to describe breaking of spatial diffeomorphisms during inflation using an EFT approach. When spatial diffeomorphisms are broken, background spatial anisotropies can be present during inflation, both in the metric and the energy-momentum tensor (EMT). Then we establish a perturbative scheme based on the hypothesis that such anisotropies are small, and find general conditions to eliminate anisotropies from the background metric while maintaining anisotropies in the EMT. In section 3 we discuss the leading EFT operators, quadratic in perturbations, that break discrete symmetries during inflation, and their general features. In section 4 we discuss their consequences for the dynamics of fluctuations at the level of linearized equations of motion. In section 5 we present our conclusions, followed by three technical appendices.

2 System under consideration

We use an EFT approach to inflation in a setup that does not necessarily preserve spatial diffeomorphism invariance. Such a scenario can be realized if the fields that drive inflation acquire vacuum expectation values (vevs) characterized by spatially non-trivial profiles. This situation can still be compatible with a good degree of spatial homogeneity and isotropy at the background level, if the system has additional internal symmetries, or if the aforementioned vevs are small or combine in such a special way to mostly cancel the background anisotropies. Explicit examples of such situations are solid inflation — where scalars acquire space-dependent vevs or models in which vector fields drive inflation — that can select preferred directions. On the other hand, the EFT approach that we adopt allows us not to commit on a specific model for developing our arguments.

We start discussing a generalized version of the unitary gauge condition during inflation, that is useful for characterizing the dynamics of fluctuations. In such a gauge, all fluctuations are limited to those in the metric. We then continue discussing the background configuration that we adopt, using the language of EFT of inflation. We establish a perturbative scheme that allows us to solve the equations of motion for a general anisotropic background solution, in a setup that potentially breaks all diffeomorphisms and that can violate the hypothesis of Wald’s no-hair theorem. Within our approximations, we then determine conditions that allow us to establish spatial isotropy and homogeneity in the metric. On the other hand the EMT can contain fields that select a preferred spatial direction, and this will be important for characterizing operators that break discrete symmetries.

2.1 Unitary gauge condition with broken space-time diffeomorphisms

We start discussing a generalized version of single field inflation condition, in a context where time and spatial diffeomorphisms can be simultaneously broken. A diffeomorphism transformation acts on the coordinates of space-time as

\[ x^\mu \rightarrow x^\mu + \xi^\mu(t, \mathbf{x}) \]  \hspace{1cm} (2.1)
for an arbitrary function $\xi^\mu = (\xi^0, \xi^i)$. In conformally flat space-time, the function $\xi^i$ can be decomposed into a transverse vector and a longitudinal scalar as $\xi^i = \xi^i_T + \partial^i \xi^L$ with $\partial_i \xi^L_T = 0$.

Inflation corresponds to a period of quasi-de Sitter expansion of the universe. Usually, the source driving inflation breaks the time reparameterization invariance. For example, in models where a single scalar field $\phi(t, x) = \phi_0(t) + \delta \phi(t, x)$ drives inflation, the time diffeomorphism acts non-linearly on the inflaton perturbation:

$$\delta \phi \rightarrow \delta \phi - \dot{\phi}_0 \xi^0.$$  

(2.2)

Since the scalar perturbation transforms non-trivially under diffeomorphisms, it is not gauge invariant. A gauge invariant quantity can be formed by combining it with metric perturbations that transform under time reparameterization. A gauge can then be selected — the so-called unitary gauge — where $\delta \phi = 0$ and all dynamical degrees of freedom are stored in the metric perturbations. In this gauge, the effective Lagrangian for fluctuations contains metric perturbations only, that break explicitly time reparameterization, and leave spatial diffeomorphisms unbroken. See [5] for details. Such a scenario is generally called “single-field” inflation since there is a unique field driving the inflationary expansion, and its background value can be unambiguously used as inflationary clock.

However, we do not really know what was occurring during inflation, and it could very well be that the system of fields driving inflation was richer than a single scalar field, as explained in section 1. To be more general, we consider the case for inflation driven by a set of fields, which we collect within a four-vector $\Psi^\mu$. We can consider a case where the background values $\bar{\Psi}^\mu$ depends not only on time, but they are allowed to depend also on the spatial coordinates. The perturbation $\delta \Psi^\mu$ transforms under the full diffeomorphism transformation (2.1) as

$$\delta \Psi^\mu \rightarrow \delta \Psi^\mu - \bar{\Psi}^{\mu, \nu} \xi^\nu.$$  

(2.3)

We assume that the background value $\bar{\Psi}^\mu$ has a suitable profile so that — by choosing appropriately the gauge function $\xi^\mu$ — a gauge can be selected, where all field perturbations can be consistently set to zero: $\delta \Psi^\mu = 0$. In this gauge the propagating degrees of freedom are limited to the metric fluctuations only. This will constitute the generalized unitary gauge field condition for the setup of fields we consider in this article. A special case of such condition, for broken spatial diffeomorphisms only was discussed in [15]. Our gauge condition generically breaks time as well as spatial diffeomorphism invariance: the resulting action for metric fluctuations consequently breaks all diffeomorphisms. Notice that our generalized single field condition does not imply that inflation is driven by a single background field, and perturbations are not necessarily adiabatic, and anisotropic stress can arise. This has been discussed in the explicit realization in [15, 20].

### 2.2 Background system

If spatial diffeomorphisms are broken, in the absence of specific symmetries we can expect at least a small degree of background anisotropy during inflation. This can be induced by direction-dependent components of the EMT. On the other hand, we can impose specific

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2This generalized single field condition, among other things, implies that interaction among the Goldstone bosons of the broken diffeomorphism invariance can be constrained by non-linearly realized symmetries. The latter will lead to consistency conditions for correlation functions that generalize the ones found in the “standard” case where only time diffeomorphisms are broken. This interesting subject will be developed elsewhere.
conditions, for example associated with symmetries, that reduce or cancel the background anisotropies. In this section, we first start discussing the general case in which a small degree of background anisotropy is admitted in the metric and in the EMT. We adopt a perturbative scheme that allows us to carry on our calculations in a straightforward way, and make manifest how to evade Wald’s no-hair theorem [19] within an EFT approach. Then we discuss an example of conditions that allows us to remove the background anisotropies, yet maintaining direction-dependent profiles for the fields constituting the background EMT. Hence the system still maintains interesting features associated with broken spatial diffeomorphisms, that will become important in the next section when discussing quadratic fluctuations and operators that break discrete symmetries.

During inflation we consider a background metric that is decomposed as homogeneous FRW and anisotropic parts, denoted by $\bar{g}_{\mu \nu}(0)$ and $\bar{g}_{\mu \nu}(a)$ respectively, as

$$\bar{g}_{\mu \nu} = \bar{g}_{\mu \nu}(0) + a^2(\eta) \left( -\frac{1}{\delta_{ij}} \right) + a^2(\eta) \left( \frac{\beta_i}{\beta} \chi_{ij} \right), \quad (2.4)$$

where $\beta_i$ and $\chi_{ij}$ are transverse and traceless. We assume from now on that $\beta_i$ and $\chi_{ij}$ are small: $|\beta_i| \ll 1$ and $|\chi_{ij}| \ll 1$. Hence we consider their contributions only at linearized order in our analysis. In other words, we develop a perturbative scheme in terms of the small quantities parameterizing the background anisotropy in the metric and (as we shall see in a moment) in the EMT. We stress that (2.4) is our background metric. On top of it, we will include inhomogeneous perturbations in the next sections.

We now consider a background EMT that is able to support our small deformation (2.4) of a FRW background metric. For this aim, we start introducing the following anisotropy parameters that will enter in the EMT:

- A vector $\theta_i$, selecting a preferred spatial direction.
- A shear $\sigma_{ij}$, a symmetric, traceless tensor.

To be consistent with the fact that the magnitudes of the anisotropic metric components $\beta_i$ and $\chi_{ij}$ are small, we assume both these anisotropic parameters $\theta_i$ and $\sigma_{ij}$ to be small, and treat them at linearized order in our discussion. We can think of these objects as vevs of some fields, and in realistic cosmological situations at least a mild coordinate dependence is expected. Since we are implementing an EFT approach to describe inflationary fluctuations, we do not need to specify an underlying theory that provides such quantities, and the equations of motion for the fields associated with them. This since by hypothesis the perturbations of EMT can be set zero by an unitary gauge choice, and do not influence the dynamics of the metric perturbations on which we are focusing our attention. We only need to ensure that the EMT constructed using these quantities satisfies the Einstein equations, order by order in a perturbative expansion in the fluctuations.

In the spirit of the EFT of inflation, the matter action that controls the background EMT breaks both time and space reparametrization invariance, and it is then written as

$$S_m = - \int d^4x \sqrt{-g} \left[ \Lambda(\eta) + c_1(\eta)g^{00} + c_2(\eta)\delta_{ij}g^{ij} + d_1(\eta)\theta_i g^{0i} + d_2(\eta)\sigma_{ij} g^{ij} \right]. \quad (2.5)$$

Notice the presence of terms depending on $g^{ij}$ and $g^{0i}$, that are absent in the EFT standard where spatial diffeomorphisms are preserved. Since the degree of anisotropy is assumed to be small, in what follows we only consider contributions at most linear in $\theta_i$ and $\sigma_{ij}$, and in the
metric deformations $\beta_i$ and $\chi_{ij}$. Moreover, we neglect the possible spatial dependence of the coefficients in the previous action. The background EMT associated with the action (2.5) is

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \delta S_m \delta g_{\mu\nu}. \quad (2.6)$$

Combined with the Einstein tensor $G_{\mu\nu}$ — which can be constructed straightforwardly from (2.4) — the Einstein equations impose the following relations to be satisfied at the background level, in a linearized expansion for the anisotropy parameters (from now on we set the Planck mass $M_{Pl} = 1$):

$$3H^2 = c_1 + 3c_2 + a^2\Lambda, \quad (2.7)$$
$$H^2 - H' = c_1 + c_2, \quad (2.8)$$
$$d_1\theta_i = c_2\beta_i, \quad (2.9)$$
$$2d_2\sigma_{ij} = \mathcal{H}\chi_{ij}' + \frac{1}{2}\chi_{ij}'' + 2c_1\chi_{ij}. \quad (2.10)$$

So we learn that in our linearized approximation the background quantity $\beta_i$ in the metric is controlled by $d_1$ and the vector $\theta_i$, while $\chi_{ij}$ is controlled by $d_2$ and the shear $\sigma_{ij}$. A configuration that solves these equations can lead to a solution with a small degree of anisotropy in the background during a quasi-de Sitter inflationary stage. By choosing appropriately the anisotropic parameters $\theta_i$ and $\sigma_{ij}$ such a configuration can avoid Wald’s no-hair theorem [19] and lead to anisotropic inflation, as we discuss in appendix A using the language of the EFT of inflation.

Hence, as a matter of principle, our approach based on the EFT can accommodate a model-independent analysis of inflationary models with anisotropic backgrounds (see e.g. [53, 54] for specific models with these properties).

On the other hand, the general analysis of such system can be very cumbersome, due to several new operators that can contribute. For the rest of this article, we make some additional simplifying assumptions to remove the background anisotropies and facilitate as much as we can our analysis of fluctuations, yet covering some relevant features that are distinctive of our system with broken spatial diffeomorphism invariance. Our requirements are as follows:

1. We impose a residual symmetry [55],

$$x^i \to x^i + \xi^i(t) \quad (2.11)$$

for an arbitrary time-dependent function $\xi^i$. Notice that this symmetry invariance is less restrictive than spatial diffeomorphism, see (2.1). In our context, this residual symmetry is quite powerful. Since the 0\textsubscript{i} component of the metric perturbation transforms non-trivially under this symmetry (see next section), this symmetry eliminates it from our action, if there are no spatial derivatives acting on it. This requires to choose the parameter $d_1 = 0$ in the action (2.5), and consequently (2.9) tells us that the metric anisotropic parameter $\beta^i$ vanishes:

$$\beta^i = 0. \quad (2.12)$$

2. In addition, from now on we set the shear equal to zero,

$$\sigma_{ij} = 0, \quad (2.13)$$
and focus on the effects of the vector $\theta_i$ only. Setting the shear to zero implies a vanishing source in (2.10) for the background anisotropic tensor $\chi_{ij}$. For simplicity, in what follows we choose the solution corresponding to the configuration,

$$\chi_{ij} = 0.$$  

(2.14)

After imposing these two requirements we obtain an isotropic and homogeneous FRW background metric. However, the anisotropic parameter $\theta_i$ contributing to the background EMT can be non-vanishing, and as we shall see next it can play an important role to characterize quadratic operators that break discrete symmetries, in the quadratic action for perturbations.

3 Quadratic operators that break discrete symmetries

In this section we discuss how to build a quadratic Lagrangian for the metric fluctuations in our setup. We mainly concentrate on operators that break discrete symmetries during inflation. We work within the generalized unitary gauge context explained in section 2.1, and consider an homogeneous and isotropic background. The operators that we consider in this section are a selection chosen for the most notable phenomenological consequences. We stress that higher derivative symmetry breaking operators — even preserving spatial diffeomorphisms — can also be included, but ours are the leading ones in a derivative expansion given our symmetry choices. We make use of the background vector $\theta_i$ introduced in the previous section for constructing our quadratic operators, and we work at linearized order on this small quantity.

The linearized perturbations around our isotropic background, $g_{\mu\nu} = \bar{g}_{\mu\nu} + a^2(\eta) h_{\mu\nu}$, can be decomposed into scalar, vector, and tensor sectors:

$$h_{00} = 2A,$$

(3.1)

$$h_{i0} = S_i + \partial_i B,$$

(3.2)

$$h_{ij} = 2\varphi\delta_{ij} + 2\partial_i\partial_jE + \partial_iF_j + \partial_jF_i + \gamma_{ij}.$$  

(3.3)

Under the most general diffeomorphism transformations (2.1), the quantities that appear in the decomposition of $h_{\mu\nu}$ transform as, with $\beta^i = \sigma_{ij} = \chi_{ij} = 0$ [56],

$$A \to A - \partial_\eta \xi^0 - \mathcal{H} \xi^0,$$

(3.4)

$$S_i \to S_i - \partial_\eta \xi^T_i,$$

(3.5)

$$B \to B - \partial_\eta \xi^L + \xi^0,$$

(3.6)

$$\varphi \to \varphi - \mathcal{H} \xi^0,$$

(3.7)

$$E \to E - \xi^L,$$

(3.8)

$$F_i \to F_i - \xi^T_i,$$

(3.9)

$$\gamma_{ij} \to \gamma_{ij}.$$  

(3.10)

As we explained, our setup breaks both space and time diffeomorphism invariance, but we impose invariance under the residual symmetry transformation of (2.11) that ensures that the quadratic action for the metric perturbations does not contain contributions proportional to the metric components $h_{0i}$, if there are no spatial derivatives acting on them.

In appendix B we list the new derivative operators that are allowed by the previous requirements. Here, after discussing the Einstein-Hilbert action and the leading operators that
do not contain derivatives — the mass terms — we concentrate on derivative operators that break discrete symmetries. Our derivative operators can be considered as leading derivative corrections to the mass terms that break spatial diffeomorphisms and discrete symmetries in Lorentz violating theories of massive gravity [55, 57, 58].

**Einstein-Hilbert action and mass terms.** We start with the Einstein-Hilbert action for quadratic fluctuations. Once decomposed into scalar, vector and tensor parts, they read respectively as follows [59]:

\[
S^{(s)} = \int d^4 x \frac{a^2}{2} \left[ -6 (\varphi' - HA)^2 - 2(2A + \varphi)\nabla^2 \varphi + 4(\varphi' - HA) \nabla^2 (B - E') \right],
\]

\[
S^{(v)} = \int d^4 x a^2 \left[ -(S_i - F'_i)\nabla^2 (S_i - F'_i) \right],
\]

\[
S^{(t)} = \int d^4 x a^2 \left[ \gamma_{ij}'^2 - (\nabla \gamma_{ij})^2 \right].
\]

Repeated spatial indices are contracted with \(\delta_{ij}\).

To this action we can include the mass operators that are allowed by our symmetries:

\[
O^{(0)}_1 = -m^2 a^4 h_{ij}^2 = -m^2 a^4 \left[ 12\varphi^2 + 2(\partial_i F_j)^2 + \gamma_{ij}^2 + 8\varphi \nabla^2 E + 4(\nabla^2 E)^2 \right],
\]

\[
O^{(0)}_2 = -m^2 a^4 h_{ii}^2 = -m^2 a^4 \left( 12\varphi + 2\nabla^2 E \right)^2,
\]

\[
O^{(0)}_3 = -m^2 a^4 h_{00}^2 = -m^2 a^4 \left( 4A^2 \right),
\]

\[
O^{(0)}_4 = -m^2 a^4 h_{00} h_{ii} = -m^2 a^4 \left( 12A\varphi + 4A \nabla^2 E \right).
\]

These are the zero-derivative [hence the superscript \((0)\)], leading operators that break diffeomorphism invariance. These operators, and the ones that we meet next, already contain the square root of the metric, and can be included as they stand into the action. For example the operator (3.14) can be included in the action as

\[
\Delta S^{(0)}_1 = \int d^4 x O^{(0)}_1.
\]

These mass terms can lead to a non-vanishing anisotropic EMT, that among other things does not respect the adiabaticity condition and leads to non-conservation of the curvature perturbation on super-horizon scales. See [20] for a discussion on this point.

**Single-derivative operators.** We now consider some novel single-derivative operators, built with or without the anisotropic vector \(\theta_i\), that have the feature to break discrete symmetries in scalar and/or tensor sectors. As discussed in the introduction, there is a rich literature on possible interactions that violate the discrete parity symmetry, and their consequences for the CMB. The novelty of our model-independent approach is the use of EFT for inflation in a context where spatial diffeomorphism invariance can be explicitly broken (see also [6] for a discussion of parity violating operators in an EFT for inflation preserving spatial diffeomorphism invariance). As we discussed, spatial diffeomorphism invariance can be violated in inflationary systems where background fields acquire spatial-dependent background values, as in models with vectors or in solid inflation. If discrete symmetries are not imposed \emph{a priori}, the operators that we consider can be expected to be generated by quantum effects in such inflationary scenarios. For this reason, it is interesting to explore
them and their consequences. Here we introduce a couple of such operators, the ones with the most notable phenomenological consequences that will be studied in the next section.

The lowest dimensional, single derivative operator that breaks parity does not involve anisotropic parameters and reads

\[
O^{(1)}_1 = \mu a^3 \epsilon_{ijk} (\partial_i h_{jm}) h_{km} = \mu a^3 \epsilon_{ijk} \left[ (\partial_i \gamma_{jm}) \gamma_{km} - \partial_i F_j \nabla^2 F_k \right].
\]  

(3.19)

It leads to parity violation in the tensor sector, since it is not invariant under the interchange \(x^i \to -x^i\). \(\mu\) is a mass scale we have included for dimensional reasons.

In addition, there is another interesting single-derivative operator, built with the background vector \(\theta_i\), that contains a single derivative along time:

\[
O^{(1)}_2 = \mu a^3 \epsilon_{ijk} \theta_i \partial_{jm} h'_{km} = \mu a^3 \epsilon_{ijk} \theta_i \left( \gamma_{jm} \gamma_{km} - F_m \partial_j \gamma_{km} - F_m' \partial_k \gamma_{jm} - F_j \nabla^2 F'_k + 2 \partial_j F_k' \nabla^2 E + 2 \partial_k F_j' \nabla^2 E' \right).
\]  

(3.20)

We can say that such an operator breaks time-reversal in the tensor sector, since the contributions within the parenthesis are not invariant under a change of sign in the time direction. Notice that in order to build it we need to use the vector \(\theta_i\) that selects a preferred direction. Recent papers discussed possible phenomenology of scenarios that contain together background anisotropies and parity violation: see e.g. [60–62]. We will see that such an operator can have interesting consequences for the dynamics of the tensor modes.

**Two-derivative operators.** Among the possible two-derivative operators, we focus on two interesting ones that break discrete symmetries:

\[
O^{(2)}_1 = a^2 h'_{ij} \theta_j \partial_k h_{ik} = -a^2 \theta_j \left( 4 \varphi' \partial_j \varphi + 2 \varphi' \nabla^2 F_j + \gamma'_{ij} \nabla^2 F_i - 2 \varphi \nabla^2 F'_j - 2 F'_j \nabla^4 E \right.
\]

\[
+ 4 \varphi' \partial_j \nabla^2 E + 4 \partial_j \varphi \nabla^2 E' + 4 \nabla^2 E' \partial_j \nabla^2 E - F'_i \partial_j \nabla^2 F_i \right),
\]  

(3.21)

\[
O^{(2)}_2 = a^2 h'_{ij} \theta_k \partial_k h_{ij} = -a^2 \theta_k \left( 12 \varphi' \partial_k \varphi + \gamma'_{ij} \partial_k \gamma_{ij} 
\right.
\]

\[
+ 4 \varphi' \partial_k \nabla^2 E + 4 \partial_k \varphi \nabla^2 E' + 4 \nabla^2 E' \partial_k \nabla^2 E - F'_i \partial_k \nabla^2 F_i \right). \]  

(3.22)

Notice that, considering their scalar and tensor parts, such operators are not invariant under an (independent) interchange of spatial and of time coordinates. Hence we can say that these operators break both parity and time-reversal, in the tensor as well as in the scalar sectors. In the next section we will discuss their consequences.

Other single and two derivative operators that can break discrete symmetries are listed in appendix B.

### 4 Dynamics of linearized fluctuations

We now discuss some consequences of the discrete symmetry breaking operators that we presented in the previous section. We concentrate our attention to the dynamics of linearized fluctuations. Within our approximation of small anisotropy parameter \(\theta_i\), we show that
vector degrees of freedom do not propagate. Scalar degrees of freedom acquire a direction-dependent phase. Although this phase factor does not have consequences for the scalar power spectrum, nevertheless it might affect higher order correlators. We also show that small direction dependent contributions to the sound speed can arise. At the quadratic level, the most notable consequences occur in the tensor sector, where we find that some of our new operators lead to a chiral amplification of gravity waves. This is more effective than the one first pointed out in [25] discussing parity-breaking operators, because the modes can be continuously amplified during the whole inflationary epoch.

4.1 No propagating vector modes

At linear order in the anisotropy parameter \( \theta_i \), we can arrange our system such that there are no propagating vector degrees of freedom: the derivative operators of the previous section have been selected, among other things, to ensure this condition. To see this, we include for simplicity a single mass term, proportional to \( m_1^2 \), as given by (3.14), plus a combination of the discrete symmetry breaking operators proportional to \( \theta_i \) that we have introduced in the previous section. The quadratic vector Lagrangian can be expressed as

\[
L^{(v)} = \frac{a^2}{2} \left[ \partial_k \left( S_i - F_i' \right) \partial_k \left( S_i - F_i' \right) \right] - 2m_1^2a^4 (\partial_i F_j)^2 - \theta_i F_j (\cdots),
\]

(4.1)

where the dots contain contributions depending on \( F_k \) or on scalar and tensor fields, that we do not need to specify for our arguments. In the previous expression, the first part comes from the Einstein-Hilbert term, the second from a mass term, while the third part collects the contribution from the new derivative operators discussed in the previous section. In this context, the vector \( S_i \) appears only in the first term of (4.1). It can be readily integrated out, leaving a Lagrangian identical to (4.1) but with the first term missing. The equation of motion for \( F_i \) then reads

\[
\nabla^2 F_i = \frac{\theta_i}{m_1^2} (\cdots),
\]

(4.2)

where again the dots contain contributions of the various fields involved, that we do not need to specify. Substituting (4.2) into (4.1), we find only terms of \( \mathcal{O}(\theta_i^2) \) that are negligible within our approximation. Hence, although typically vector modes propagate in our context, at linearized order in \( \theta_i \), the vector degrees of freedom are not dynamical and will be set to zero.

4.2 Direction-dependent phase in the scalar sector

Let us examine the effects of parity breaking and time-reversal operators in the scalar sector. We consider a quadratic action built in terms of the Einstein-Hilbert contributions, mass terms, and a linear combination of the two-derivative operators \( \mathcal{O}_i^{(2)} \) introduced in (3.21) and (3.22). We set the vector perturbations to zero as seen in the previous subsection. This scalar action contains four scalar degrees of freedom: \( A, B, E \) and \( \phi \). Among them, \( A \) and \( B \) are non-dynamical and can be integrated out, leaving a scalar Lagrangian for \( E \) and \( \phi \). We can proceed as done in [20], further solve the equation of motion for the non-dynamical field \( E \), and plug it into the action. We find at linearized order in \( \theta_i \), a two-derivative operator for \( \phi \) (already present in our expressions for \( \mathcal{O}_i^{(2)} \)) that breaks the discrete parity and time-reversal symmetries:

\[
\mathcal{L}^{(s)} \supset a^2 \phi' \theta_i \partial_i \phi.
\]

(4.3)
Other contributions quadratic or higher in the parameter \( \theta_i \) can be neglected, as done in the previous subsection for vector fluctuations. The scalar field \( \varphi \) in the unitary gauge is the curvature perturbation \( R \), hence its statistics can be directly connected with observable quantities. Here however we limit our attention to understand how the operator (4.3) modifies the mode function for \( \varphi \), viz. \( R \). We consider then the action for the canonically normalized field \( u = z R \) with \( z \propto a \) during quasi-de Sitter expansion:

\[
S^{(\epsilon)} = \int d^4x \frac{1}{2} \left[ u'^2 - (\nabla u)^2 + \frac{z''}{z} u^2 + 2b_1 \theta_i u' \partial_i u \right],
\]

where the last is our new term, weighted by a real coefficient \( b_1 \) that for simplicity we consider as constant. Here we do not explicitly discuss the consequence of the mass terms \( O^{(0)}_i \). Such contributions have been already studied for example in [20] and have been shown to lead to anisotropic stress and non-conservation of the curvature perturbation, generalizing the results first pointed out for solid inflation [15].

The equation of motion for the mode function \( u_k \), that follows from (4.4) once converted to Fourier space, results

\[
u''_k + 2i b_1 \theta_i k_i u'_k + \left( k^2 - \frac{z''}{z} \right) u_k = 0.
\]

At early times the new operator proportional to \( b_1 \) is subdominant, so a standard Bunch-Davies vacuum can be unambiguously defined. It is convenient to express the mode function \( u_k \) as

\[
u_k = e^{-i \theta_i k_i \eta} u^{(0)}_k
\]

so that (4.5) becomes

\[
u^{(0)''}_k + \left( k^2 - \frac{z''}{z} \right) u^{(0)}_k + k^2 \left( b_1 \theta_i \hat{k_i} \right)^2 u^{(0)}_k = 0,
\]

where \( \hat{k}_i \equiv k_i / k \). The last term in the previous expression is quadratic in \( \theta_i \), so it can be neglected for consistency with our approximation (but see the comment at the end of this subsection). Doing so we end with the standard evolution equation in a FRW background, and the solution for \( u^{(0)}_k \) can be expressed in terms of Hankel functions. On top of this, the complete solution for \( u_k \) gains a new direction-dependent contribution to the phase proportional to \( b_1 \) as in (4.6). Such a configuration is only reliable at linearized order in \( \theta_i \), hence on large scales, \( k/(aH) \leq 1 \). For smaller scales, contributions that are non-linear in \( \theta_i \) can become large and change the solution: this fact is important when quantizing the system. Note that the power spectrum remains isotropic because (4.6) is different from the standard solution by a direction-dependent phase, which cancels when computing the power spectrum.

It is also interesting to interpret the role of this phase in coordinate space, making a Fourier transform of (4.6). One finds that

\[
u(\eta, x^i) = u^{(0)}(\eta, x^i + b_1 \eta \theta^i).
\]

Hence its effect amounts to a time-dependent shift of the argument of the scalar mode function in coordinate space. Such shifts cancel when taking correlation functions among scalar fluctuations, due to the translational invariance of these quantities. On the other
hand, they can have non-vanishing physical effects when taking higher order correlations functions between scalar and tensor modes, since the tensor perturbations do not necessarily share the same shifts. It would be interesting to study this topic further.

Let us end this subsection briefly commenting on the last term in (4.7): as we explained above, consistency of our approximations would require to neglect such terms, since at quadratic order in the anisotropy parameter \( \theta \), other contributions of comparable size can arise — for example the coupled terms between scalar, vector and tensor fluctuations — that should be taken into account. Nevertheless, such a particular term would be present, and provide a quadrupole contribution to the scalar sound speed. It would be interesting to study its effects, noticing also that being of positive size it increases the amplitude of the sound speed rendering it larger than one. We leave the analysis of this topic to future work.

4.3 Chiral phase in the tensor sector

We now explore the consequences of our discrete symmetry breaking operators for the tensor sector. We do not consider the mass terms, which were studied e.g. in [20]. We consider here the effects of the single derivative operators \( O^{(1)}_1 \) and \( O^{(1)}_2 \) that break parity and time-reversal. The consequences of the two-derivative operator \( O^{(2)}_i \) are, as can be read from the derivative structure of the tensor perturbations in (3.22), identical to the ones discussed in the previous section on the scalar sector, so we do not analyze them here.

The action for tensor fluctuations is

\[
S^{(t)} = \int d^4 x \frac{a^2}{8} \left[ \gamma_{ij}'' - (\nabla \gamma_{ij})^2 + 2q_1 \mu a \epsilon_{ijk} (\partial_i \gamma_{jm}) \gamma_{km} + 2q_2 \mu a \epsilon_{ijk} \gamma_{jm} \gamma_{km}' \right],
\]

(4.9)

where we have assumed the the coefficients \( q_1 \) and \( q_2 \) are constant dimensionless real parameters: the condition of being real is required by our conventions on the tensor polarizations.

The equation of motion for the tensor degrees of freedom results

\[
\gamma_{ij}'' + 2H \gamma_{ij}' - \nabla^2 \gamma_{ij} - 2q_1 \mu a \epsilon_{kmij} \partial_k \gamma_{mj} + 2q_2 \mu a \epsilon_{kmij} \theta_k \gamma_{mj} + 3q_2 \mu a \mathcal{H} \epsilon_{kmij} \theta_k \gamma_{mj} = 0,
\]

(4.10)

where all indices are contracted with the Kronecker symbol \( \delta_{ij} \). Now, we introduce the circular polarization tensor \( e^{(\lambda)}_{ij}(\hat{k}) \), with \( \lambda = + (-) \) corresponding to the right (left) circular polarization, which satisfies the circular polarization conditions (see appendix C):

\[
e^{(\lambda)}_{ij} k_j = e^{(\lambda)}_{ii} = 0,
\]

\[
\epsilon_{im} e^{(\lambda)}_{ij} k_m = i \lambda k e^{(\lambda)}_{ij},
\]

\[
e^{(\lambda)}_{ij} e^{(\lambda)}_{ij} = 2 \delta_{\lambda \lambda'}.
\]

(4.11)

\( \gamma_{ij} \) can be Fourier expanded in terms of polarization mode functions as

\[
\gamma_{ij}(\eta, x) = \int \frac{d^3 k}{(2\pi)^3} \sum_{\lambda = +, -} \left[ \gamma_{ij}(\eta, k) e^{(\lambda)}_{ij}(\hat{k}) e^{i k \cdot x} + h.c. \right].
\]

(4.12)

Then, we find the equation of motion for the mode function \( \gamma_{ij}(\lambda) \), after contracting with \( e^{(-\lambda)}_{ij} \), as

\[
\gamma_{ij}''(\lambda) + 2H \gamma_{ij}'(\lambda) + k^2 \gamma_{ij}(\lambda) - 2\lambda q_1 \mu a k \gamma_{ij}(\lambda) - 2\lambda q_2 \mu a \hat{\mathcal{H}} \gamma_{ij}(\lambda) = 0,
\]

(4.13)
where we have introduced
\[ \dot{\theta} \equiv \frac{\lambda}{2} e_{ij}^\epsilon t_{im} \theta e_{mj}^{(-\lambda)}. \] (4.14)

Let us discuss the interpretation of $\dot{\theta}$. Using (C.3), we learn that \[ \dot{\theta} = \lambda e_{i}^{(\lambda)} \epsilon_{im} \theta e_{m}^{(-\lambda)}. \] Here $e_{i}^{(\lambda)}$ and $e_{m}^{(-\lambda)}$ are two mutually orthogonal vectors, that are both orthogonal to the direction of the three-momentum $k$. This implies that the cross product $e_{i}^{(\lambda)} e_{m}^{(-\lambda)} \epsilon_{mil} \theta_{l}$ is a vector parallel to $k$: contracting it with the vector $\theta_{l}$ and using (4.11) leads to the identity
\[ \dot{\theta} = i \theta_{i} \hat{k}_{i}. \] (4.15)

Notice at this stage the main difference between the operators proportional to $q_{1}$ and $q_{2}$. The operator proportional to $q_{2}$ is associated with time-derivatives of the mode function $\gamma_{(\lambda)}$ or the scale factor, while the operator $q_{1}$ with space-derivatives. The effect of the contribution of $q_{1}$ corresponds to the known parity-violating operators \[ (\lambda) \] and produces an enhancement/suppression of tensor mode polarization at horizon crossing only. Such effects are well studied in the literature (see as an example the review \[ (13) \]) so we will not study them here. Let us instead concentrate on the consequences of the novel operator $O_{2}^{(1)}$ proportional to $q_{2}$. We rescale the field $\gamma_{(\lambda)}$ in the standard manner as
\[ v_{(\lambda)} \equiv \frac{a}{\sqrt{2}} \gamma_{(\lambda)}. \] (4.16)

The equation of motion for $v_{(\lambda)}$ is then
\[ v_{(\lambda)''} - 2i \lambda q_{2} \mu a \theta_{i} \hat{k}_{i} v_{(\lambda)'} + \left( k^{2} - \frac{a''}{a} - i \lambda q_{2} \mu a \theta_{i} \hat{k}_{i} \right) v_{(\lambda)} = 0. \] (4.17)

Similar to what we did for the scalar sector, it is convenient to rescale
\[ v_{(\lambda)} \equiv e^{i \lambda q_{2} \mu a \theta_{i} \hat{k}_{i} \int a d \eta} v_{(\lambda)_{0}}. \] (4.18)

The equation for $v_{(\lambda)_{0}}$, at linear order in $\theta_{i}$ and so neglecting quadrupolar effects, reduces to the well-known form
\[ v_{(\lambda)_{0}''} + \left( k^{2} - \frac{a''}{a} \right) v_{(\lambda)_{0}} = 0. \] (4.19)

This equation is identical to the standard mode function equation for the tensor perturbations. Furthermore, we notice that, neglecting slow-roll corrections, we can write
\[ \int a d \eta = \frac{N_{e}}{H}, \] (4.20)

with $N_{e}$ being the number of $e$-folds, and $H$ the value of the Hubble parameter during inflation. So the solution for $\gamma_{(\lambda)}$ is given by
\[ \gamma_{(\lambda)} = \exp \left( i \lambda q_{2} \mu a \theta_{i} \hat{k}_{i} \frac{N_{e}}{H} \right) \gamma_{(\lambda)_{0}}. \] (4.21)

Therefore, we again find a phase modulation of the wavefunction — like in the scalar sector — but now the coefficient of this phase depends on the chirality of the specific gravity wave

\[ ^{3} \text{We thank Angelo Ricciardone and Maresuke Shiraishi for pointing out this argument to us.} \]
one is considering, and on the number of e-folds as well. Such a phase does not influence the power spectrum, since it can be read as a “chiral” translation of the modes when expressed in the coordinate space:

$$\gamma(\lambda)(\eta, x^i) = \gamma^{(0)}(\eta, x^i - \lambda q_2 \mu \frac{N_e}{H} \theta^i).$$

(4.22)

A translation in the coordinates does not affect the power spectrum of the tensor modes, since the power spectrum is translationally invariant. On the other hand, since the translation depends on the chirality, it can affect the bispectra among tensor modes with different chirality (as studied for example in [67, 68]), as well as bispectra between tensor and scalar sectors. We hope to return to investigate these topics in the near future.

5 Conclusions

The EFT of inflation has emerged in recent years as a powerful unifying approach for analyzing broad classes of inflationary scenarios. A systematic analysis of effective operators compatible with the symmetries assigned to a system might suggest new effects, not predicted or analyzed within the specific models studied so far, that if supported by observations can motivate new directions for model building.

In this article we have studied the consequences of breaking discrete symmetries during inflation using the model-independent language of EFT. We have developed aspects of EFT of inflation in a context where all diffeomorphisms are broken during the inflationary phase. We have shown that this effective description allows one to describe systems where background spatial anisotropies can be present during inflation. Moreover, we have discussed how to avoid Wald’s no-hair theorem within the model-independent language of EFT of inflation. Then we have focussed on studying the leading operators at the quadratic order in perturbations that break discrete symmetries. We have identified operators that break parity and time-reversal during inflation, and analyzed their consequences for the dynamics of linearized fluctuations. Both in the scalar and tensor sectors, we show that such operators can lead to a new direction-dependent phase for modes involved. Such a directional phase does not affect the power spectrum, but can have consequences for higher correlation functions. Moreover, a small quadrupole contribution to the sound speed can be generated. We stress that using an EFT approach we did not have to commit on specific models to develop our arguments, that are based on general features of the system of fields driving inflation.

Our investigations can be further elaborated in various directions. At theoretical level, it would be interesting to extend our model-independent approach to derive a third order action for perturbations, and study how new discrete symmetry breaking operators and direction-dependent effects can influence non-Gaussian observables. After analyzing novel ramifications of such findings, it will also be important to study actual realizations and concrete models able to generate the new operators that break discrete symmetries in the manner studied here. At observational level, it would be interesting to study distinctive consequences of our discrete symmetry breaking operators for the properties of the CMB, in particular for what respect specific correlations between T, E and B modes, at the level of two- and three-point functions. We leave these interesting questions to future study.

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A Wald’s no-hair theorem in the EFT of inflation

In this appendix we discuss how Wald’s isotropization theorem [19] can be violated using the language of EFT of inflation, justifying the approach developed in section 2.2.

Wald’s theorem states that, under some hypothesis on the EMT that we will review below, the inflationary expansion rapidly reduces the amplitude of background anisotropies to an unobservable level. Here we show that the prerequisites behind the theorem are not necessarily satisfied in our case. We use results and notation of section 2.2. We can write

\[ H' = H^2 (1 - \epsilon), \]  

where \( \epsilon \equiv -a^{-1} H'/H^2 \) with \( H = a H \). Substituting this result into (2.8), we find that \( c_1 + c_2 = \epsilon H^2 \). Using this information, the background EMT can be decomposed as

\[ T_{\mu\nu} = -\Lambda(\eta) \bar{g}_{\mu\nu} + T^{(2)}_{\mu\nu}, \]  

with

\[ T^{(2)}_{00} = \epsilon H^2 + 2c_2, \]
\[ T^{(2)}_{bi} = (\epsilon H^2 - 2c_2) \beta_i, \]  
\[ T^{(2)}_{ij} = (\epsilon H^2 - 2c_2) (\delta_{ij} + 3\chi_{ij}) + H\chi'_{ij} + \frac{1}{2} \chi''_{ij}. \]

Wald’s isotropization theorem states that anisotropies are rapidly suppressed during inflation if the strong and dominant energy conditions are satisfied:

\[ \left( T^{(2)}_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} T^{(2)} \right) t^\mu t^\nu \geq 0 \quad \text{for all time-like vectors } t^\mu, \]  
\[ T^{(2)}_{\mu\nu} \tilde{t}^\mu \tilde{t}^\nu \geq 0 \quad \text{for all future-directed, causal vectors } \tilde{t}^\mu. \]

Time-like vectors \( t^\mu \) satisfy the condition

\[ (t^0)^2 \geq 2\beta_i t^0 t^i + (\delta_{ij} + \chi_{ij}) t^i t^j. \]

In our case, the dominant energy condition reads

\[ (\epsilon H^2 - 2c_2) \left[ (t^0)^2 + 2\beta_i t^0 t^i + (\delta_{ij} + \chi_{ij}) t^i t^j \right] + 4c_2 (t^0)^2 + 2(d_2 \sigma_{ij} - c_2 \chi_{ij}) t^i t^j \geq 0. \]  

In EFT scenarios with no breaking of spatial diffeomorphisms or isotropy, \( c_2 = 0, \sigma_{ij} = 0 \) and \( \chi_{ij} = 0 \). The second line in the above equation would vanish, while the first line would
be positive definite, satisfying in this way the dominant energy condition (A.5). In our more general setup, instead, the second line is non-vanishing, and can render the previous quantity negative. Hence, in general the prerequisites underlying Wald’s theorem can be expected to be violated in our context based on the EFT of inflation. Such situations can be realized in models of inflation with vector fields [63], or solid inflation, as discussed in the recent literature, see e.g. [64–66].

B List of allowed operators

In this appendix we list new derivative operators that satisfy our requirements, besides the ones already presented in the main text and in [20]. To avoid possible ghost pathologies, we do not consider operators that contain time derivatives on $h_{00}$ and $h_{0i}$. Moreover, to satisfy the residual symmetry (2.11) we consider operators containing $h_{0i}$ only when spatial derivatives act on it.

**Single-derivative operators.** The new single-derivative operators are the following:

\[
\begin{align*}
&h_{0i,i}h_{00}, \ h_{0i,j}h_{jj}, \ h_{0i,j}h_{ij}, \ h_{ii}h_{00}, \ h_{ij}h_{jj}, \ h_{ij}h_{ij}, \ \epsilon_{ijk}h_{00,i}h_{jk}, \\
&\theta_{ij}h_{ij,i}h_{00}, \ \theta_{ij}h_{ij,i}h_{00}, \ \theta_{ij}h_{ij,j}h_{kk}, \ \theta_{ij}h_{ij,j}h_{jk}, \\
&\delta_{ij}h_{ij,k}h_{il}, \ \delta_{ij}h_{ij,k}h_{il}, \ \delta_{ij}h_{ij,j}h_{kl}, \ \delta_{ij}h_{ij,j}h_{kl}.
\end{align*}
\]

(B.1)

Note that $\theta_{ij}h_{ij,i}h_{kk}$ and $\theta_{ij}h_{ij,j}h_{jk}$ are allowed but can be made as total derivatives, thus we have omitted these operators.

**Two-derivative operators.** The new two-derivative operators are:

\[
\begin{align*}
&h_{0i,i}h_{ij}^{'}, \ h_{0i,j}h_{ij}^{'}, \ \epsilon_{ijk}h_{ij,j}h_{ik}^{'}, \\
&\theta_{ij}h_{00,i}h_{ij}^{'}, \ \theta_{ij}h_{00,j}h_{ij}^{'}, \ \theta_{ij}h_{ij,j}h_{kk}^{'}, \ \theta_{ij}h_{ij,j}h_{jk}^{'}, \ \theta_{ij}h_{ij,j}h_{kk}^{'}, \ \theta_{ij}h_{ij,j}h_{kk}^{'}, \\
&\delta_{ij}h_{ij,k}h_{il}, \ \delta_{ij}h_{ij,k}h_{il}, \ \delta_{ij}h_{0i,j}h_{kl}^{'}, \ \delta_{ij}h_{0i,j}h_{kl}^{'}.
\end{align*}
\]

(B.2)

C Circular polarization conditions

We explain in more details our conventions for the circular polarization condition for tensor perturbations, (4.11) [13, 69]. The polarization vector $e^{(\lambda)}_{i}(\hat{k})$ perpendicular to $\hat{k}$ can be written as

\[
e^{(\lambda)}_{i}(\hat{k}) = \frac{\hat{\theta}_{i}(\hat{k}) + i\lambda \hat{\phi}_{i}(\hat{k})}{\sqrt{2}},
\]

(C.1)

with $\lambda = \pm$. This vector satisfies

\[
k_{i}e^{(\lambda)}_{i} = 0,
\]

\[
e^{(\lambda)}_{i}(\hat{k}) = e^{(-\lambda)}_{i}(\hat{k}) = e^{(\lambda)}_{i}(-\hat{k}),
\]

(C.2)

By means of such a polarization vector we can construct the polarization tensor as

\[
e^{(\lambda)}_{ij} = \sqrt{2}e^{(\lambda)}_{i}e^{(\lambda)}_{j}.
\]

(C.3)

It is straightforward to prove (4.11) using (C.2).
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