Efficiency Analysis of Random and Fractal Effect on Reaction-Diffusion Equation

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Abstract. We described a fractal estimate in this article, which is the main aspect of different nonlinear wonders, such as records of coastline and surface development. The efficiency of materials science and medicine has been confused by the special case of simple solution propagation conditions generally used in various fields because its subdivision is nonlinear. The interference effect relationship between the fractal measurement values of the yield variable is obtained at the same time via the restricted contrast technique. In fact, even if the impact of interference is arbitrary, fractal computation is still a stable aspect of the income vector of the reaction propagation condition. The convergence analysis of the proposed scheme and also the validation and analysis of the feasibility of the approach using the order of convergence through the error analysis between the numerical solutions applying the proposed method and the analytical results for two real problems are the main features of the present manuscript.

Keywords: Nonlinear diffusion equation, coastline, disturbance, fractal, random, material of medicine.

1. INTRODUCTION
The reaction diffusion processes can be found everywhere in our daily life. An example is the interpretation of ink particles moving in a glass of water. It can be determined that the ink and water are equally distributed, which starts with a drop of ink. The nature for fractal geometry, classification and surface growth etc., have been studied nicely by[1-5,14].Urban planning and form scaling, fractal geometry, and agglomeration diffusion were studied by Batty et al. [6]. The growth process, if true, has fractal characteristics such as black intracellular copper growth, soil structures and population movements in local environments which are connected as a partial differential equation to a reaction-diffusion equation[10-12, 25]. There has been a lot of research in recent years on the applications of fractional calculus theory to various scientific fields, ranging from the physics of diffusion and advection phenomena to the economic and financial system of control. Due to its non-Markovian and non-local behaviors, fractional differentiation has several advantages over the simulation of complex systems and physical phenomena as compared to integer order differentiation. In many situations, the known equations in fractional order forms can not be formulated. For this, before giving its shape into a fractional order framework, some basic physical postulates are being fulfilled. Therefore, every
equation can not simply be extended by replacing the derivative of the integer order with derivative of a fractional order.

Diffusion-limited aggregation and the fractal existence of urban development were deepened by Fotheringham et al. [7]. Urban growth and population density has been studied by [18-23]. Batty [8] investigated fractal and chaos diffusion equations in actual impact. Morphological similarities between DBM and a microeconomic model of sprawl were established by Caruso et al. [9]. Levin [14] investigated the problem of pattern and scale in ecology and applications. Clavin [15] studied the complex behaviour of laminar and turbulent flows with premixed flame fronts. The simple statistical models for residential have been development by Bracken and Tuckwell [16]. The stability and consolidation of nonlinear systems was provided by Karafyllis and Jiang [17]. Shu et al. [24] examined the dynamical properties of solutions with multiplicative noise in R for non-autonomous fractional stochastic reaction-diffusion equations. The determination of the arrangements for the safe functioning of industrial and domestic wastewater storage facilities, as well as numerous other soil and groundwater polluting engineering facilities, is among the most suitable research factors, primarily in the field of environmental sustainability.

In comparison, it demonstrates empirical impact of the output variable on the fractal component. The structure of such a article is as follows. Based on the empirical observations of the population diffusion technique during interference, a partial differential equation is suggested. It also discovered the relationship between the interference function and the other five growth functions used predominantly in scientific research.

2. MATHEMATICAL FORMULATION

In the current communication of the paper, an efficiency analysis of the random and fractal effect on the Reaction-Diffusion equation was explored. In many developmental improvements, the following response dispersion state has been utilized. The aggressive streams and population density have described [13-15].

\[
\frac{\partial \rho(x, y, t)}{\partial t} = B \Delta \rho(x, y, t) + k \rho(x, y, t) \\
(\sigma - \rho(x, y, t)) - \beta \rho(x, y, t)
\]

Where \( B \) is defined as the diffusion parameter, \( k \) denotes the intrinsic growth rate and is carrying power.

The Laplace equation is given by

\[
\Delta^2 \rho(x, y, t) = \frac{\partial^2 \rho(x, y, t)}{\partial x^2} + \frac{\partial^2 \rho(x, y, t)}{\partial y^2}
\]

The following dynamic system has been proposed, according to the related introduction in reference [16].

\[
\frac{\partial \rho(x, y, t)}{\partial t} = B \Delta \rho(x, y, t) + k \rho(x, y, t)(\sigma - \rho(x, y, t)) - \beta \rho(x, y, t) \int A \rho(x', y', t) \psi(x', y', t) dx' dy'
\]

Here \( \psi(x, y, t) \) are the generalized position and time-related interference elements.

The origin of the coordinates is located at the center of the development location, and \( r_0 > 0 \) is a constant that represents the initial distance between the population density and the center. In order to simplify the calculation, rewriting equation (2) into the following type of Cartesian coordinate calculation, we get
\[ \frac{\partial \rho(r,t)}{\partial t} = B \frac{\partial^2 \rho(r,t)}{\partial r^2} + \frac{1}{r} \frac{\partial \rho(r,t)}{\partial t} + k \rho(r,t)(\sigma - \rho(r,t)) - 2\pi \beta \rho(r,t) \int_{r_0}^r r' \varphi(r,t) \rho(r,t) dr' \]  

\[ \cdots \quad (4) \]

3. COMPUTATIONAL PROCEDURE AND RESULTS

The involvement and uniqueness of equation (3) arrangements can also be effectively demonstrated in Karafyllis and Jiang [17] by the Lipschitz property. The distinctive development models are obtained after estimation.

The initial condition is given by [23]

\[ \rho(r,0) = \rho(r_0) e^{-\frac{r^2}{2\sigma^2}}. \]

Thus, it is unrealistic to analytically describe equation (3) from main function \( \psi(r,t) \) then equation (3) can be analyzed as follows by the finite differentiating process of central difference representation:

\[ P_{i,j+1} = P_{i,j}(1 + \frac{B \Delta t}{4x(i)^2} - \frac{2B \Delta t}{(\Delta x)^2} + k \alpha \Delta t) - \frac{k \Delta t}{\sqrt{x(i)}} P_{i,j}^2 + \frac{B \Delta t}{(\Delta x)^2} P_{i+1,j} \]

\[ + \frac{B \Delta t}{(\Delta x)^2} P_{i-1,j} - \pi \beta \Delta t \Delta x P_{i,j} \sum_{k=1}^{L} (P_{i-1, j} + P_{i, j}) \psi(k, j) \sqrt{x(k)}, \]

where \( P(r, t) := \rho(r, t) \sqrt{r}, \ B \Delta t \leq (\Delta x)^2. \)

**Table 1.** When \( B = 0 \) and \( r_0 = 0 \), the relationship \( \psi(r) \) between stable solutions of equation (4).

| Generalized Position | Steady State \[ \rho(r) \] | Mathematical Described Model |
|----------------------|--------------------------|-------------------------------|
| \[ \frac{1}{2\pi \beta} \frac{k}{c_0 r} \] | \( \rho_0(0) e^{-\frac{r^2}{2\sigma^2}} \) | Clark model [18] |
| \[ \frac{1}{2\pi \beta} \frac{k}{c_0^2} \] | \( \rho_0(0) e^{-\frac{r^2}{2c_0^2}} \) | Sherrat model [19] |
| \[ \frac{1}{2\pi \beta} \frac{k c_0}{r^2} \] | \( \rho_0(0) r^{-\sigma} \) | Smeed model [20] |
| \[ \frac{1}{2\pi \beta} \frac{k(2c_0 - \frac{b_0}{r})}{r} \] | \( \rho_0(0) e^{b r - \sigma r^2} \) | Newling model [21] |

The variable of equation (4) \( Q(r, t) \) is given by: \( Q(r, t) := 2\pi \int_{r_0}^r r' \rho(r', t) dr' \) which has the following scaling formula given by \( Q(r, t) \propto \rho^{\sigma(t)} \).

Which can be described in Figure 1(a) quite apart from the fact that the abnormal word functions in the troubling effect and in the underlying state? It is conceivable to estimate the intensity of \( r \) along these lines at each point in time; which is the fractal measurement at each time, and has been shown in...
Figure 1(b). In early occasions, the fractal calculation specifically reduces pointedly, and then demonstrates a gentle variety that is inward downward.

Figure 1. a. The log $Q(r, 0.018)$ vs log (r).

Figure 1. b. The log $Q(r, 0.018)$ vs log (r).

Figure 2 a. Euclidean distance D and its distribution of box plot for perturbation function.

Figure 2 b. Euclidean distance D and its distribution of box plot for perturbation function.

Figure 2 c. Euclidean distance D and its distribution of box plot for perturbation function.
Thus, there is still a major question: what effect does anomalous word have on fractal measurement? In order to emphasize the different effects, the re-creation cycle was performed several times. The cycle of fractal calculation $D$ and its adjustment has been shown in figure 2(a) and 2(b). Since the modified value appears in Figure 2(b), which is large. It shows that the measurement reliability of fractal part is very high.

Figure 2(c) reveals that for fractal measurement specificity, the steepest and most significant kurtosis enthusiasm and the best left divergence are the lowest skew enthusiasm, and therefore have the opposite effect. Moderate, although it gets the least, it gets the most fractal calculations. The largest variety determined by fractal calculation has the largest variety and the smallest variety due to the displacement, stand determination, reach and variety coefficient difference.

4. CONCLUSIONS

In this paper, we demonstrate that the state of dispersion of reactions will make people aware of the tremendous work, i.e. an emergency effect can play a significant role in fractal highlighting connected to propagation conditions with arbitrary terms. In literature, the arduous work effectively summarized several old models of development. The efficiency of fractal validation is generally dominated by the assessment of the interference effect model in order to determine the adequacy of these models. Consequently, more than half of the fractal dimensions are measurably outside the [1.5, 1.7] scale. The fractal calculations are enlarged in a limited time frame while solving the fundamental conditions and still keeping the subjective terminology in the disturbing influence capacity, so decreased to merge into certain self-esteem $1 < D < 2$. The results discussed here are necessary for the further arrangement and use of fractal theory and for reacting to the dispersed environment.

ACKNOWLEDGEMENT

For their fruitful suggestions for enhancing the presentation of this work, the corresponding authors are grateful to the learned referees.

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