Feature Analysis of Fractal Surface Roughness Based on Three-dimensional W-M Function

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Abstract. The three-dimensional W-M fractal function was used to simulate the rough surface, and the different fractal parameters correspond to the different surface morphology. In order to explore the influence rule of fractal dimension D and scale coefficient G on the surface profile, the fractal dimension D value was 2.5, and the scale factor G value was 1E-14. The mean height, standard deviation and spectrum characteristics of the surface contour were analyzed. The results show that under the action of a single fractal parameter, the surface roughness Ra and the surface contour height standard deviation std all decrease with the increase of D, and increase with the increase of G, and the change trend is basically the same. When D is kept constant and only G is a variable, the surface height value of frequency components remain unchanged, the amplitude of each frequency decreases with the decrease of G; G remains unchanged but only D is a variable, with the increase of D, the frequency amplitude changes slowly, and the amplitude decreases.

1. Introduction

The surface topography of machined parts is one of the important indexes to describe the quality of turning process. For a long time, scholars have proposed a lot of 2-D and 3-D characterization methods for the surface profile of machined parts. However, most of the methods are still statistical models based on measurement instruments[1,2]. In fact, the change of contour height of the surface produced by machining process is not a stationary process, but a non-stationary random process[3]. From the micro scale, the machined surface is rough, concave convex and unsmooth, showing a strong complexity. Similarly, at the micro scale, the multi-scale and self similar characteristics can be observed when the surface is magnified many times. Compared with the traditional method, fractal theory can better describe it, and has been widely used in the characterization and simulation of rough surface and its contour[4-7].

For micro scale, rough surface profile is not continuous, but from engineering application, it can be regarded as continuous. At the same time, with the enlargement of the surface, more details can be observed, which shows that the tangent line of each point does not exist. Since the above rough surface has self affine fractal characteristics, a W-M function with self affine fractal characteristics, which is continuous but non differentiable at every point in the scale range, is introduced to characterize the rough surface[8].

For 3-D W-M fractal function, the fractal dimension D and scale coefficient G do not depend on the sampling interval and sampling length, but have great influence on the singularity, amplitude and irregularity of the contour height. In this paper, the rough fractal surface is simulated based on 3-D W-M function, and the relationship between fractal dimension D and scale coefficient G on surface

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roughness and standard deviation of contour height is explored. Through the Fast Fourier Transform of the surface profile height, the spectrum distribution is obtained, and the amplitude frequency characteristics are analyzed to explore the influence of different fractal parameters on it.

2. Simulation of rough surface profile with W-M fractal function

Yan and Komvoulos give W-M function of three-dimensional random fractal surface with isotropy by studying and simulating the fractal characteristics of 3-D rough surface.

\[ z(x,y) = L \left( \frac{G}{L} \right)^{(D-2)} \left( \frac{\ln \gamma}{M} \right)^{\frac{1}{M}} \sum_{n=1}^{M} \sum_{\substack{m \in \mathbb{Z} \setminus \{0\}}}^{\infty} \left( \sqrt[2]{D-3} \gamma \phi_{m,n} - \cos \left[ \frac{2\pi \gamma (x^2 + y^2)^{\frac{1}{2}}}{L} \right] \cos \left( \arctan \left( \frac{y}{x} \right) - \frac{\pi m}{M} \right) + \phi_{m,n} \right) \]  

Where \( z(x,y) \) —— height of rough surface profile;
\( D \) —— fractal dimension, \( 2 < D < 3 \);
\( M \) —— the number of superimposed peaks when reconstructing a surface;
\( \phi_{m,n} \) —— random bits uniformly distributed in the range \( [0, 2\pi] \);
\( G \) —— scale coefficient;
\( L \) —— sampling length;
\( \gamma \) —— a constant greater than 1 is usually taken as 1.5 for random surfaces with normal distribution.

Compared with the traditional statistical parameters, the fractal dimension \( D \) and scale coefficient \( G \) of W-M function are independent of frequency, and the parameter variables independent of time have scale independence [9-11]. In order to facilitate the calculation, the uniform distribution of random phase is set as \( \frac{\pi}{6} \), and the profile of 3-D rough surface is obtained by changing different fractal parameters, as shown in Fig. 1.

![Simulation of three-dimensional rough surface topography under different fractal parameters](image)

(a) \( D=2.3; \ G=1\times10^{-14} \)
(b) \( D=2.5; \ G=1\times10^{-12} \)
(c) \( D=2.8; \ G=1\times10^{-10} \)

Fig. 1 Simulation of three-dimensional rough surface topography under different fractal parameters

It can be clearly seen from Fig. 1 that the amplitude, singularity and irregularity of the contour height of the surface topography are different under different fractal parameters. In order to explore its
influence law, the fractal parameters are selected according to Table 1, and the surface characteristics are analyzed.

| Table 1 Fractal parameters |
|---------------------------|
| \( D \) | \( G \) |
| 2.1  | 1E-14 |
| 2.2  | 1E-13 |
| 2.3  | 1E-12 |
| 2.4  | 1E-11 |
| 2.5  | 1E-10 |
| 2.6  | 1E-9  |
| 2.7  | 1E-8  |
| 2.8  | 1E-7  |
| 2.9  | 1E-6  |

3. Influence of fractal parameters on surface roughness

The surface roughness value of the contour reflects the amplitude of the contour height. The most commonly used method to express the surface roughness of a profile is to obtain the arithmetic mean deviation of the contour height. The profile curve of a section on the surface is selected to represent, and it is expressed by the arithmetic mean of the deviation between each point of the contour in the measurement length and the datum line, that is.

\[
R_s = \frac{1}{L} \int_{0}^{L} \int_{0}^{L} |z(x,y)| dx dy
\]  

(2)

Where \( z(x,y) \) —— contour height of each point;

\( L \) —— measuring length.

In order to study the influence law of a single fractal parameter, the scale coefficient \( G \) is selected as a fixed value 1E-14, and the contour roughness values under different fractal dimensions \( D \) are calculated. The results are shown in Fig. 2.

![Fig. 2 Influence of fractal dimension D on surface roughness](image)

It can be clearly seen from Fig. 2 that under the same scale coefficient \( G \), the surface roughness value of the contour decreases with the increase of fractal dimension \( D \), that is, the larger the fractal dimension \( D \), the smoother the surface. At the same time, when \( D \) changes from 2.1 to 2.2, the change of \( R_s \) is the most significant. Although it shows a decreasing trend, the change range is not obvious.

The fractal dimension \( D \) is selected as a fixed value of 2.5, and the scale coefficient \( G \) is changed. The calculated surface roughness value is shown in Fig. 3.

![Fig. 3 Influence of scale coefficient G on surface roughness](image)

In order to study the influence of scale coefficient \( G \), the fractal dimension \( D \) is selected as a fixed value 2.5, and the scale coefficient \( G \) is changed. The calculated surface roughness value is shown in Fig. 3.
obvious when the scale coefficient does not increase to a certain range. When the value of G starts from 1E-8, the surface roughness increases significantly with the increase of G.

![Fig. 3 Influence of scale coefficient G on surface roughness](image)

4. Influence of fractal parameters on standard deviation of contour height

The surface roughness value reflects the amplitude of the overall contour height. In order to study the height variation of each point on the contour surface, the standard deviation of the height of each point on the surface should be calculated. On the basis of obtaining the arithmetic mean deviation of surface height, the general calculation formula of standard deviation can be expressed as:

$$\text{std} = \sqrt{\frac{1}{E} \sum_{i=1}^{L} \sum_{j=1}^{L} \left( z(x_i, y_j) - R_s \right)^2}$$  \hspace{1cm} (3)

In the same way, the separate action rules of fractal dimension D and scale coefficient G are obtained respectively. Select the fixed value 1E-14 of scale coefficient G to calculate the standard deviation of contour height value under different fractal dimension D, as shown in Fig. 4. The fixed value of fractal dimension D is 2.5, and the standard deviation of contour height under different scale coefficient G is calculated, as shown in Fig. 5.

![Fig. 4 Influence of fractal dimension D on the standard deviation of surface height](image)
It can be clearly observed from Fig. 4 and Fig. 5 that the influence of single fractal parameter, whether fractal dimension $D$ or scale coefficient $G$, on the standard deviation of contour surface height is basically consistent with the above-mentioned surface roughness value. It is shown that the change of fractal parameters has the same effect on the amplitude and singularity of contour surface height.

5. The influence of fractal parameters on the spectrum of contour height

The amplitude frequency characteristics of the contour surface height obtained by the 3-D W-M fractal function are obtained by Fast Fourier Transform. In order to obtain the influence rule of different fractal parameters on spectrum components. Firstly, the fractal coefficient $D$ is selected as the fixed value 2.5, and the value of different scale coefficient $G$ is changed. The Fast Fourier Transform results of the height value of the surface profile are shown in Fig. 6.

According to Fig. 6, the frequency components of surface height values are basically unchanged under different scale coefficients $G$ when the fractal dimension $D$ is taken as 2.5. It can be seen from the amplitude frequency diagram that the amplitude of each frequency decreases with the decrease of scale coefficient $G$. 

![Fig. 6](image1.png)

(a) $D=2.5; G=1E-11$  
(b) $D=2.5; G=1E-12$
The scale coefficient $G$ is selected as the fixed value $1E-14$, and the value of different fractal dimension $D$ is changed. The Fast Fourier Transform result of the surface contour height value is shown in Fig. 7.

It can be seen from Fig. 7 that when the scale coefficient $G$ is set at $1E-14$, the amplitude change of each frequency component tends to be gentle with the increase of fractal dimension $D$. This is different from the rule of changing the scale coefficient $G$ with the fixed fractal dimension $D$. In addition, with the increase of $D$, the amplitude of each frequency component is decreasing, which is different from the previous law.
6. Conclusion
1) The rough surface profile is simulated based on 3-D W-M fractal function. The surface profile changes with different fractal parameters.
2) By selecting the values of fractal dimension $D$ and scale coefficient $G$, the variation rules of surface profile roughness value $Ra$ and surface profile height standard deviation $std$ with fractal parameters are obtained. Under the action of a single fractal parameter, $Ra$ and $std$ decrease with the increase of $D$ and increase with the increase of $G$, and the change trend is basically the same.
3) By selecting the values of fractal dimension $D$ and scale coefficient $G$, when $D$ is the same but $G$ is different, the frequency components of surface height value basically remain unchanged, and the amplitude of each frequency decreases with the decrease of $G$; when $G$ is the same but $D$ is different, with the increase of $D$, the amplitude of each frequency component tends to be flat, and the amplitude shows a decreasing trend.

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