Signal processing technology and its application research of ground penetrating radar (GPR)

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Abstract. Ground Penetrating Radar (GPR) technology is a kind of rapid and continuous detection method with high precision and intuitive image. It is an electromagnetic wave technology used to determine the distribution of the underground medium spectrum (1 MHZ - 3 GHZ). The applications of GPR is facing the influence of various interference in various complex environments and the main direction of GPR application research is how to remove interference and how to extract the useful information. The signal processing is means of the radar to implement the signal retrieval and information extraction. Based on the analysis of the signal processing technology of GPR used commonly, the paper applies these methods to the processing and interpretation of GPR data in an engineering example and illustrates the purpose and the treatment manner of the various signal processing technology.

1. Introduction

The goal of the signal processing of the GPR is to reduce random interference and to display the reflection wave on GPR image more clearly and to extract the reflection wave velocity, amplitude and waveform etc. parameters. So the signal processing of the GPR can provide the reliable basic data for the reasonable and accurate geological interpretation. Whether the data is processed properly is directly related to the accuracy of the geological interpretation. The commonly used methods of the signal processing of the GPR are pretreatment, filtering, deconvolution, migration homing processing [1-3], and other related signal processing technology. In the process of GPR detection, electromagnetic wave is transmitted underground and is received by the antenna. Inevitably there are various kinds of interference wave. Interference wave and effective wave are different in the spectrum, the transmission speed, the direction and energy etc. So the interference wave is not the real reflection of underground geological body and should be removed or suppressed to highlight the effective wave and to improve the quality and precision of the profile. The digital filtering technology is to solve the above problem and it filters the discrete signal by mathematical computation [4]. The input and output of digital filter are discrete data. Because the record of GPR is composed of a series of discrete time series, it is reasonable to process GPR signal with digital filtering technology. It is an important part of the signal processing of GPR. Its principle is the basis of the study of signal processing method of GPR. Before the digital filter of the GPR signal, the relevant pretreatment is needed to prepare for the subsequent signal processing. After the digital filter, deconvolution can be done to improve resolution ratio. After the routine data processing, sometimes further inversion processing (migration homing) [5][6] is need to provide a more accurate reflection profile image. Of course, in the process of signal processing, some other auxiliary signal processing methods are still need.

2. Classic spectrum analysis
The classic spectrum analysis mainly includes the amplitude spectrum analysis, phase spectrum analysis and power spectrum analysis. Through these spectrum analysis, we can determine the frequency components of the signal and phase and can provide the basis for the filtering processing. With the infinite long signal $x^{(k)}$, $k = 0, \pm 1, \pm 2, \pm 3, \cdots L$, it can be considered that the signal is got through sampling from the signal function $x^{(0)}$. It is a bilateral infinite sequence. The spectrum is defined as:

$$X(f) = \sum_{k=-\infty}^{\infty} x_k e^{-2\pi j f k} \quad (1)$$

In radar exploration data, the spectrum analysis is always aiming at a certain record of a period after discrete sampling. The period of the radar records is limited long sequence of discrete ($mT$), $m = N_1, N_2, \cdots, N_n$, and if the records of $t < N_1$ and $t > N_n$ are zero, the formula for spectrum analysis of the radar records is:

$$X(f) = \sum_{k=-N_1}^{N_1} x_k e^{-2\pi j f t k} \quad (2)$$

Among them $x_k$: the radar records of spectrum analysis; $X(f)$: the spectrum of radar records.

If we calculate the frequency spectrum of radar record directly by the formula (2), there is a large amount of calculation. So the most popular method of the spectrum analysis of the radar records is "fast Fourier transform" (also known as the butterfly algorithm). This algorithm was put forward in 1965 by the Coulomb Toke and has been widely used.

**Figure 1.** The original data from the antenna 100MHz

**Figure 2.** The original data of amplitude spectrum

Figure 1 presents the original data from 100 MHZ antenna. Through the analysis of the amplitude spectrum of original data, it can be seen that the signal frequency components concentrate on a range from 60 MHZ to 200 MHZ, as shown in Figure 2. It can provide the basis for the choice of filter parameters for the next step.
3. Classical one-dimensional filtering algorithm

In one dimension filtering, the input and output signal of filter, the filter impulse response and the frequency response are all univariate functions. The filter in the paper adopts the window function of the FIR system. This method has the following advantages: (1) stable system; (2) easy to realize linear phase; (3) to allow more bandpass filter (or stop band).

3.1 The ideal low-pass digital filter

The frequency response function of Ideal low-pass filter is:

\[ \varphi(w) = \begin{cases} 1 & w \leq w_0 \\ 0 & w > w_0 \end{cases} \]

(3)

So, the unit sampling function of the filter is:

\[ h(n) = \frac{\sin(w_0 n)}{\pi n} \]

(4)

Therefore, \( h(n) \) is sinc function symmetrically based on \( h(0) \). \( h(0) = \frac{w_0}{\pi} \). Such a system is causal, and therefore it is physically impossible. However, if we intercept \( h(n) \), and shift it after the interception, we can get:

\[ h'(n) = h(n - \frac{M}{2}) \quad n = 0,1,2,3,\ldots,M \]

(5)

So \( h'(n) \) is causal and finite long. The length of it is \( M + 1 \).

Let

\[ H(z) = \sum_{n=0}^{M} h'(n)z^{-n} \]

(6)

We can get the transfer function of the designed filter. The frequency response of \( H(z) \) is near \( \varphi(w) \) and it is similar to that of linear.

However, by mapping we can find the following phenomenon. When \( M \) increases, ripples appear in the passband. When \( M \) increases continuously, the ripples will not disappear. The cutoff frequency location is closer to the discontinuous point \( (w_0) \). This phenomenon is called the Gibbs phenomenon. The cause of this phenomenon is the sudden interception of \( h(n) \). The intercept of a infinitely long \( h(n) \) to 0-M means adding a rectangle window with length of \( (M + 1) \) to \( h(n) \). The result of adding the window is the convolution of \( \varphi(w) \) and the rectangular window spectrum. Rectangular window spectrum have bigger side lobe. While the convolution, the side lobe causes the Gibbs phenomenon. So in order to reduce the Gibbs phenomenon, we should select the window function with smaller side lobe.

The above is the design of low pass filter. We can obtain the impulse response function for qualcomm, band pass and band stop filter in the same way.

Qualcomm:

\[ h(n) = \frac{\sin[\pi(n - \frac{M}{2})] - \sin[w_0(n - \frac{M}{2})]}{\pi(n - \frac{M}{2})} \]

(7)

bandpass:

\[ h(n) = \frac{\sin[w_0(n - \frac{M}{2})] - \sin[w_1(n - \frac{M}{2})]}{\pi(n - \frac{M}{2})} \]

(8)

band stop:
Now we choose a window function \( w(n) \) mainly, let
\[
h'(n) = h(n)w(n) \quad (10)
\]

3.2 The window function
The selection of window function \( w(n) \) should meet the following conditions:

1) The nonnegative real even functions. And the function is increasing starting from the center of symmetry.

2) In order to ensure the power spectrum estimation to be asymptotically unbiased, window function should satisfy
\[
w(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} W(e^{j\omega})dw = 1 \quad (11)
\]

3) The window function spectrum should have the minimum bandwidth, the minimum side lobe peak and the largest side lobe attenuation velocity of spectral peak.

The commonly used window functions are: rectangle window, triangle window, Hamming window, Blackman window, Hanning window and Gaussian window, etc.

Figure 3 shows the original profile collected by 900 MHZ antenna. We select the bandpass filter and give the low frequency and high frequency range. Then we filter the original profile for one dimensional only and get Figure 4 after filtering. As shown in the figure, we achieve a very good effect on removing noise.

![Figure 3](image1)

**Figure 3.** the original profile from the antenna 900MHz

![Figure 4](image2)

**Figure 4.** profile after filtering for only one dimensional

4. The Hilbert transform
Hilbert transform method can shift 90° phase accurately for every harmonic wave. For a continuous cycle signal \( x(t) \), the Hilbert transform of its continuous time signal \( x(t) \)

Definition:

\[
\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(t-\tau)}{\tau} d\tau = x(t) \otimes \frac{1}{\pi} \tag{12}
\]

From formula (12) we can get the unit impulse response \( h(t) = \frac{1}{\pi} \times t \). Because the Fourier transform of \( j\hat{h}(t) - \hat{h}(t) \) is a symbol of \( \text{sgn}(w) \), the frequency characteristic of Hilbert transformer is:

\[
H(e^{jw}) = -j \text{sgn}(w) = \begin{cases} -j & x > 0 \\ j & x < 0 \end{cases} \tag{13}
\]

Let \( H(j\omega) = |H(j\omega)| e^{j\phi(\omega)} \), when \( |H(j\omega)| = 1 \)

\[
\phi(\omega) = \begin{cases} -\pi/2 , \omega > 0 \\ \pi/2 , \omega < 0 \end{cases} \tag{14}
\]

The Hilbert transform of signal \( x(t) \) can be seen as output of signal \( x(t) \) through a range of 1 all-pass filter. After the Hilbert transform, the phase shift of negative frequency components is +90° and the phase shift of positive frequency components is -90°.

Figure 5 shows the original section of a cave exploration. In the process of Hilbert transform computation, by calculating the instantaneous amplitude section, we strengthen reflection and absorption contrast of the media on radar wave. Thus to some extent, we can improve the resolution of deep karst cave signal. Figure 6 shows the instantaneous amplitude section of Hilbert transform. It can be seen that contrast of karst cave and cranny greater to the surrounding medium increases obviously. So the identifiable ability of abnormal improves.

Figure 5. the original survey and collection for a cave

Figure 6. the instantaneous amplitude section of Hilbert transform

5. Deconvolution algorithm

Deconvolution is the process of compressing radar wavelet and improving the temporal resolution. To understand deconvolution, first of all we should build a radar trace element. Stratum is composed of
rocks with different lithology and physical properties. Media with different physical properties has different wave impedance characteristics. The difference of the wave impedance between adjacent rock produce electromagnetic wave reflection and the reflected signal is received by the receiving antenna. In this way, the recorded radar signal can be expressed as a convolution model, namely the convolution of stratum impulse response and radar wavelet. Stratum impulse response includes a reflection (reflection coefficient sequence) and all the possible multiple wave. The ideal deconvolution can compress wave let and eliminate the multiple wave, leaving only stratum reflection coefficient on the radar records.

If radar wavelet is unknown, the solution of deconvolution problem is a statistic. Wiener prediction theory provides a method of statistical deconvolution. The model for no noise convolution is:

\[ x(t) = w(t)*e(t) \]  \hspace{1cm} (15)  

If a deconvolution operator is a \( (t) \) this definition, a \( (t) \) and the known radar record convolution produce an estimate of stratum impulse response \( e(t) \), so:

\[ e(t) = a(t) * x(t) \]  \hspace{1cm} (16)  

Put formula (16) into formula (15), can get:

\[ x(t) = w(t)* a(t) * x(t) \]  \hspace{1cm} (17)  

Eliminate \( X(t) \) from both sides, then get the following expression

\[ \delta(t) = w(t)* a(t) \]  \hspace{1cm} (18)  

Among:

\[ \delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{others} \end{cases} \]  \hspace{1cm} (19)  

For the operator \( a(t) \), you can get

\[ a(t) = \delta(t) * w(t) \]  \hspace{1cm} (20)  

\( W(t) \) is the inverse of radar wavelet \( w(t) \). Accordingly, the key of deconvolution arithmetic is to calculate the radar wavelet and its inverse.

When calculating the deconvolution filtering factor, it can be identified by the following equation:

\[ \begin{bmatrix} r_0 & r_1 & L & r_{n-1} \\ r_1 & r_0 & L & r_{n-2} \\ L & L & L & L \\ r_{n-1} & r_{n-2} & L & r_0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ M \\ a_{n-1} \end{bmatrix} = \begin{bmatrix} g_0 \\ g_1 \\ M \\ g_{n-1} \end{bmatrix} \]  \hspace{1cm} (21)  

Among: \( r_i, a_i \) and \( g_i \) are input wavelet autocorrelation, deconvolution filter coefficients and the cross-correlation of expected output and input wavelet.

Predictive deconvolution can use the following equation to calculate filtering factor:

\[ \begin{bmatrix} r_0 & r_1 & L & r_{n-1} \\ r_1 & r_0 & L & r_{n-2} \\ L & L & L & L \\ r_{n-1} & r_{n-2} & L & r_0 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ M \\ a_{n-1} \end{bmatrix} = \begin{bmatrix} r_0 \\ r_1 \\ M \\ r_{n-1} \end{bmatrix} \]  \hspace{1cm} (22)  

For formulas (21) and type (22), we can use Levinlitz recursion method to solve.

Figure 7 shows the original signal profile collected by GPR in a project. After deconvolution processing we can get Figure 8. As can be seen from the results, the resolution of the views of the data is increased significantly after deconvolution, but with some unnecessary inference signal. So, on the basis of deconvolution, filtering processing or wavelet processing is need to achieve a better effect.
6. The wavelet transform

The wavelet analysis has the characteristics of low entropy, multi-resolution, correlation and flexibility. So it is used widely in the field of signal processing. It’s unique for transient non-stationary signal analysis. It plays a very important role for target recognition and detection and signal de-noising. For a long time, Fourier analysis is an important tool for signal processing. The orthogonality of the Fourier series and the simplicity of its function provides a great convenience for Fourier transform and computing. But Fourier transform reflects the overall characteristic of signal or function; in many practical problems what we are concerned about is the signal feature in the local area. In order to inherit the advantages and to overcome the disadvantages of Fourier analysis, people are always looking for new ways; this leads to the emergence of the wavelet analysis.

The definition of CWT (Continuous Wavelet Transform) of function with limited energy \( f(t) \) (namely \( f(t) \in L^2(R) \)) is the integral transform of integral kernel, represented by the following formula:

\[
W_f(a,b) = W_a^b f(a,b) = \int_a^b f(t) \varphi_{a,b}(t) dt = \int_a^b f(t) \frac{1}{\sqrt{a}} \left( \frac{t-b}{a} \right) dt \quad (23)
\]

among: \( a > 0 \) is scale parameter, \( b \) is location parameter, \( \varphi_{a,b}(t) \) is wavelet.

Because \( f(t) \) is a function only available in local range, changing the value of \( a \) can stretch (\( a > 1 \)) or shrink (\( a < 1 \)) the nonzero value range of \( \varphi_{a,b}(t) \).

The contract wavelet is sensitive to the rapid changing signal, so it can analyze the high frequency signal better. On the other hand, the stretch wavelet signal is sensitive to slowly changing signal, so it can analyze the low frequency signal better. If we change the value of \( b \), it will affect the analysis results of sampling point of \( f(t) \).
The result of the continuous wavelet transform includes many wavelet coefficients $c_{j,k}$, which is the function of scale $a$ and location $b$. The definition is as follows:

$$
c_{j,k} < f, j,k >= W f(k/2^j,1/2^j) \ (24)
$$

Each coefficient $c_{j,k}$ can be decomposed into wavelet coefficient of different frequency range and different space area.

Figure 9 shows the detection profile of GPR of a tunnel. From the original image we can see the interface of tunnel lining is vague and the resolution is low. Figure 10 shows the profile after processing by wavelet transform. We can see the reflection signal of tunnel lining interface is improved significantly and signal-to-noise ratio is improved obviously.

![Figure 9. The original section of tunnel lining detection](image1)

![Figure 10. The sections after processing by wavelet transform](image2)

7. Conclusions

With the progress of the digital technology, matched filtering theory and the Fourier transform algorithm is widely used in radar signal processing system. The progress of means of signal processing technology greatly promotes the rapid development of signal processing technology and makes the signal processing system of GPR develop rapidly in the digital, software and modular direction. Its application scope is also more and more widely. The multi-functional development of radar signal processing puts forward new requirements for the development of the theory of signal processing. And the progress of digital radar signal processing technology makes it possible to apply all kinds of signal processing theory to radar signal processing working.
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