CAN ANY “INVARIANTS” BE REVEALED IN QUASI-PERIODIC PHENOMENA OBSERVED FROM SCORPIUS X-1?

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ABSTRACT

Using a large number of Rossi X-Ray Timing Explorer observations of Scorpius X-1, we present a detailed investigation of the transition layer and relativistic precession models (TLM and RPM, respectively). These models predict the existence of the invariant quantities: an inclination angle $\delta$ of the magnetospheric axis with the normal to the disk for the TLM and a neutron star (NS) mass $M_{\text{NS}}$ for the RPM. Theoretical predictions of both models are tested, and their self-consistency is checked. We establish the following: (1) The inferred $\delta$-value is $55.56 \pm 0.09$. Correlation of the $\delta$-values with the horizontal-branch oscillation (HBO) frequency is rather weak. (2) There is a strong correlation between an inferred $M_{\text{NS}}$ and the HBO frequency in the RPM frameworks. (3) We infer $M_{\text{NS}}$ for different assumptions regarding the relations between the HBO frequency $\nu_{\text{HBO}}$ and the nodal frequency $\nu_{\text{nod}}$. We find that the inferred $M_{\text{NS}} = 2.7 \pm 0.1 M_\odot$ cannot be consistent with any equation of state of NS matter. We conclude that the RPM fails to describe the data while the TLM seems to be compatible.

Subject headings: accretion, accretion disks — relativity — stars: individual (Scorpius X-1) — stars: neutron

1. INTRODUCTION

Kilohertz quasi-periodic oscillations (kHz QPOs) have been discovered by the Rossi X-Ray Timing Explorer (RXTE) in a number of low-mass X-ray binaries (Strohmayer et al. 1996; van der Klis et al. 1996). The presence of two observed peaks with frequencies $\nu_1$ and $\nu_2$ in the upper part of the power spectrum became a natural starting point in modeling the phenomena. Attempts have been made to relate $\nu_1$ and $\nu_2$, and the peak separation $\Delta \nu = \nu_2 - \nu_1$, with the neutron star (NS) spin. In the beat-frequency model (Miller, Lamb, & Psaltis 1998), the kHz peak separation $\Delta \nu$ is considered to be close to the NS spin frequency, and thus $\Delta \nu$ is predicted to be constant. In other words, $\Delta \nu$ is an “invariant” that does not vary when the kHz QPO frequencies change. However, observations of kHz QPOs in a number of binaries (Scorpius X-1, 4U 1728–34, 4U 1608–52, 4U 1702–429, etc.) show that the peak separation decreases systematically when kHz frequencies increase (see a review by van der Klis 2000, hereafter VDK). For Sco X-1, VDK found that the peak separation of kHz QPO frequencies changes from 320 to 220 Hz when the low peak $\nu_1$ changes from 500 to 850 Hz. The lower frequency part of the power spectrum contains two horizontal-branch oscillation (HBO) frequencies $\nu_{\text{HBO}} \sim 45$ Hz and $\nu_{2\text{HBO}} \sim 90$ Hz (presumably the second harmonic of $\nu_{\text{HBO}}$) that slowly increase with the increase of $\nu_1$ and $\nu_2$. (VDK). Any consistent model faces the challenging task of describing the dependences of the peak separation $\Delta \nu$ and HBO frequency $\nu_{\text{HBO}}$ on $\nu_1$ and $\nu_2$.

1.1. QPO Model Description

There are two other QPO models in the literature that infer the relations between $\nu_1$, $\nu_2$, and $\nu_{\text{HBO}}$. The transition layer model (TLM) was introduced by Titarchuk, Lapidus, &Muslimov (1998, hereafter TLM98) to explain the dynamical adjustment of a Keplerian disk to the innermost sub-Keplerian boundary conditions (e.g., at the NS surface). TLM98 argued that a shock should occur where the Keplerian disk adjusts to the sub-Keplerian flow. It was suggested by Osherovich & Titarchuk (1999, hereafter OT99) that the radial oscillations of the fluid element that bounced from the disk shock region (presumably at the adjustment radius) would be seen as two independent oscillations in the radial and vertical directions because of the presence of a Coriolis force in the magnetospheric rotational frame of reference. Simultaneous measurements of the frequencies of the two kHz QPOs and HBO harmonics in a wide frequency range allow one to derive the angle $\delta$. Titarchuk & Osherovich (2001, hereafter TO01) claim that for the sources GX 340+0, Sco X-1, 4U 1702–42, and 4U 0614+09, the inferred angle $\delta$ (see eq. [1] in TO01) stays the same over a significant range of the observed QPO frequencies. The low-frequency branch frequency $\nu_2$, the Kepler frequency $\nu_K$, and the hybrid frequency $\nu_H$, as they are introduced by OT99, are eigenfrequencies of the oscillator. However, these frequencies are revealed in the observations as the resonance frequencies $\nu_{\text{HBO}}$, $\nu_1$, and $\nu_2$ that are broadened as a result of the (radiative) damping in the oscillator (see TLM98, eq. [15]). Furthermore, the resonance frequencies $\nu_{\text{HBO}}$, $\nu_1$, and $\nu_2$ are shifted with respect to the eigenfrequencies $\nu_2$, $\nu_K$, and $\nu_H$. The frequency shift and the random errors of the eigenfrequencies depend on the damping rate of the oscillations, $\lambda$ (see details in § 2). One should keep in mind that the systematic and random errors in the centroid-frequency determination that are due to this resonance shift can be a factor of a few larger than the statistical error in the determination of the centroid frequency. In the present data analysis (as the first approximation), we assume that $\nu_K = \nu_1$, $\nu_0 = \nu_2$, and $\nu_L = \nu_{\text{HBO}}$. We discuss the results of this analysis in § 2.

The relativistic precession model (RP). The RPM is related to high-speed particle motion in strong gravitational fields, leading to oscillations of the particle orbits. Bardeen, Press, & Teukolsky (1972), Okazaki, Kato, & Fukue (1987), Kato (1990), and later Stella, Vietri, & Morsink (1999) studied the precession of the particle orbit under the influence of a strong gravity due to the general relativistic effects. In order to generate these oscillations, one should assume that the particle orbit is not precisely in the equatorial plane of the compact object. The inclination angle between the particle orbit plane and the equatorial plane
can be infinitesimal. For a nonrotating configuration of the central body (in Schwarzschild’s treatment), the angular velocity is calculated using a classical Keplerian formula. In the general case, when the intrinsic angular momentum $\alpha \neq 0$, the azimuthal frequency is expressed in units $G = 1$ and $c = 1$ as follows (Bardeen et al. 1972; Stella et al. 1999):

$$v_n = \frac{(Mr^3)^{1/2}[2\pi(1 + a(Mr^3)^{1/2})]}{r},$$  \hspace{1cm} (1)

Hereafter, we consider a corotating NS and a probe particle, namely, with $v_{n0} > 0$ and $a > 0$. An epicyclic frequency $\nu$ along with $\nu_0$ determine the orbital periastron rotation $v_{per} = v_{n0} - v_n$. A precession frequency of the node line $\nu_{nod}$ is calculated using $\nu$ and a frequency of oscillations in the direction that is perpendicular to the orbital plane, $\nu_{p}$, namely, $\nu_{nod} = |\nu - \nu_0|$. Formulas for $\nu_p$ and $\nu_{nod}$:

$$\nu_p = \nu_0[1 - 4a(Mr^3)^{1/2} + 3a^2r^2]^{1/2},$$

$$\nu_{nod} = \nu_0[1 - 6Mr + 8a(Mr^3)^{1/2} - 3a^2r^2]^{1/2},$$  \hspace{1cm} (2)

have been obtained by Okazaki et al. (1987) and Kato (1990). In the framework of the RPM (e.g., Stella et al. 1999), a frequency of Keplerian rotation $\nu_p$ is related to the higher kHz peak $\nu_h$, and a frequency of the periastron precession $\nu_{per} = \nu_0 - \nu_p$, is related to a lower kHz peak $\nu_l$. The HBO frequency $\nu_{HBO}$ is related to the nodal precession $\nu_{nod}$. Stella et al. (1999) proposed that $\nu_{HBO}$ can be an even harmonic of $\nu_{nod}$. Thus, equations (1) and (2) allow one to find the NS mass $M_{NS}$ and a using the observable frequencies $\nu_{HBO}$, $\nu_p$, and $\nu_{nod}$.

The goal of this Letter is to test the invariant predictions of the TLM and the RPM using extensive RXTE observations of QPO phenomena. In this Letter, we report the results of the detailed data analysis from Sco X-1 collected by RXTE during 4 years of observations from 1996 to 1999. In § 2, we describe the RXTE data that we use to construct the power spectra in the frequency range from $\sim 0.03$ to $\sim 2050$ Hz. In § 2, we also give details of the resonance effect on the eigenfrequency restoration using the observed QPO frequencies and present comparisons of the predictions of the QPO models with the RXTE observations. Summary and conclusions are drawn in § 3.

2. OBSERVATIONS AND DATA ANALYSIS

We used data of from the RXTE proportional counter array instrument (Jahoda et al. 1996) retrieved from the high-energy astrophysics archive of NASA/GSFC. Sco X-1 was observed by RXTE during 11 observations (10056, 10057, 10059, 10061, 20053, 20426, 30035, 30036, 30406, 40020, and 40706) in 1996–1999. Data were collected from either energy channels 0–87 (2–60 keV) or energy channels 0–249 (0.03125 to 2048 Hz) to analyze the Sco X-1 variability. The maximum of the main resonance amplitude for the linear and nonlinear oscillations is not precisely at the eigenfrequency $\nu_0$, but rather it is shifted to the frequency $\nu_0 + \eta$. For a linear oscillator, $\nu_0$ depends on the damping rate $\lambda$ (see LL and TLM98, eq. [15] for the power of the forced oscillations), i.e.,

$$\nu_0 \sim 2050 \pm 200 \text{ Hz},$$

2.1. HBO Frequencies and Their Harmonics

Among all analyzed spectra in which a kHz QPO pair was detected, we selected only those for which an HBO was detected as well and at a significance of more than 4 $\sigma$. The second harmonic $\nu_{2HBO}$ is present for nearly all of these $\nu_{HBO}$. It is worth noting that the significance of the HBO peaks decreases with the frequency that is a well-known effect for quite a few sources noted by VDK. Because of this, we cannot find HBO harmonics for HBO frequencies higher than 48 $\text{Hz}$ ($\nu_{2HBO} \gtrsim 96 \text{ Hz}$) with a reliable level of significance. The ratio $r_{2/1} = \nu_{2HBO}/\nu_{HBO}$, on the average, is slightly below 2, namely, $r_{2/1} = 1.965 \pm 0.004$. The observational appearance of the HBO harmonics is presumably a combination frequency effect along with a resonance in a weakly nonlinear oscillation system (Landau & Lifshitz 1965, hereafter LL).

2.2. Resonance Effect

The resonance in weakly nonlinear systems occurs at the eigenfrequencies of the system $\nu_0$ when the frequency of the driving force $\gamma \equiv p\nu_0/q$ and $p$, $q$ are integers (LL). The main peak resonance power (for $p = 1$ and $q = 1$) is the strongest among all the harmonics because the peak power diminishes very quickly with the increase of $p$ and $q$. The maximum of the main resonance amplitude for the linear and nonlinear oscillations is not precisely at the eigenfrequency $\nu_0$, but rather it is shifted to the frequency $\nu_0 + \eta$. For a linear oscillator, $\nu_0$ depends on the damping rate $\lambda$ (see LL and TLM98, eq. [15] for the power of the forced oscillations), i.e.,

In the Ken’s geometry, $\nu_0 \neq \nu_p$ (for the Schwarzschild’s case with a dimensionless angular momentum $\alpha = 0$, these frequencies coincide, $\nu_0 = \nu_p$).

Two events detected within 6 $\mu$s at two different anodes of the detector are due to charged particles and a high count rate from very bright sources like Sco X-1. In this case, a part of the source flux dominates in double-event data.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Dependence of the inferred $\delta$ angle on the HBO frequency, $\nu_{HBO} = \nu_0$. Upper panel: $\delta$ for a particular observation with a duration of $\sim 3$ ks. Lower panel: Averaged over five HBO frequencies. Approximations by a constant value are shown as solid lines.}
\end{figure}
They found that for GX 5-1 the systematic and random shifts are respectively. But they pointed out the very large and second harmonic frequency $\nu_{2\text{HBO}}$. The average PDS and the major frequencies $\nu_{\text{HBO}}$ (and also $\nu_{\text{HBO}}^2$), $\nu_1$, and $\nu_2$ were used for the analysis of the TLM and the RPM.

### 2.3. Transition Layer Model

For each set of $\nu_{\text{HBO}}$, $\nu_1$, and $\nu_2$, we obtained the $\delta$-value using equation (1) in TO01, where $\nu_{\text{HBO}}$, $\nu_1$, and $\nu_2$ are identified with $\nu_1$, $\nu_2$, and $\nu_3$, respectively. In Figure 1, the dependence of the angle $\delta$ between a normal to the disk and a magnetospheric axis on $\nu_{\text{HBO}}$ is shown. We establish that the strongest dependence of $\delta$ on the QPO peak frequencies involves $\nu_{\text{HBO}}$. Thus, we investigated this dependence only. The model invariants should be kept the same, independent of the QPO frequency branch where it is verified. In Table 1, the approximation of $\delta$ by a constant is present for two cases: $\nu_1 = \nu_{\text{HBO}}$ (see Fig. 1, lower panel) and $\nu_1 = \nu_{\text{HBO}} = \frac{1}{2}(\nu_{\text{HBO}} + \nu_{2\text{HBO}})$ (in the latter case, the second harmonic is taken into account for the $\delta$-determination). For each case, we provide a reduced $\chi^2_{\text{red}}$. The $\sigma_\delta$ rms and the standard deviations of $\sigma_\delta$ are also present in Table 1. We repeated the same procedure for $\delta$ versus $\nu_{\text{HBO}}$.

#### Table 1

| Parameter | $\nu_1 = \nu_{\text{HBO}}$ | $\nu_1 = \nu_{\text{HBO}}$ |
|-----------|-----------------|-----------------|
| $\delta$  | $5.612 \pm 0.007$ | $5.545 \pm 0.006$ |
| $\chi^2_{\text{red}}$ | 5.53 | 2.94 |
| $\delta$  | $5.600 \pm 0.114$ | $5.555 \pm 0.086$ |
| $r(\delta)$ | $0.567$ | $0.359$ |
| Prob. | $2 \times 10^{-4}$ | $3 \times 10^{-2}$ |
| $N$ (bins) | 39 | 39 |
| $r(\delta)$ | $0.738$ | $0.595$ |
| Prob. | $3.7 \times 10^{-2}$ | $1.2 \times 10^{-1}$ |
| $N$ (bins) | 8 | 8 |

Note.—See captions for Figs. 1 and 2: $\nu_{\text{HBO}} = \frac{1}{2}(\nu_{\text{HBO}} + \nu_{2\text{HBO}})$, $\delta$ is the best-fit parameter, $\delta = (1/N) \sum \delta_i$.

#### Fig. 3

Dependence of the inferred NS mass, $M_{\text{NS}}$, on the HBO frequency, $\nu_{\text{HBO}}$. The data used are the same as for Fig. 1. Approximations by a constant value are shown as solid lines.
we use the Spearman nonparametric test for the evaluation of the correlation of $\delta$ versus $\nu_3$ with the HBO frequency. Table 2, it is seen that the RPM is consistent with the statistical fluctuations.

4. Relativistic Precession Model

Now we investigate the RPM invariant, the NS mass $M_{\text{NS}}$, as a function of $\nu_3$ for a given HBO frequency and as a function of $\nu_3$ for a given HBO frequency. The deduced dependences of $M_{\text{NS}}$ on the HBO frequencies are stronger than for the kHz frequencies (similar to the case with $\delta$ for the TLM). In Table 2 and in Figures 3 and 4, we present a mean value of $M_{\text{NS},\text{red}}$, which is the best-fit parameter, $M_{\text{NS}} = (1/N) \sum M_{\text{NS}}$. For each case, we also calculate a probability $\text{Prob.}$ that the derived correlation is a result of the statistical fluctuations.

2.5. Statistical Evaluation of the Suggested Invariants for the TLM and the RPM

In order to check the consistency of the TLM and the RPM, we use the Spearman nonparametric test for the evaluation of the correlation of $\delta$ with HBO frequencies. If the correlation coefficients $r_i = \pm 1$, then there is a perfect linear correlation between two values. In the general case, $|r_i| \leq 1$. For each case, we also calculate a probability $\text{Prob.}$ that the derived correlation is a result of the statistical fluctuations.

From Table 2, it is seen that the RPM is consistent with the statistical mean $\tilde{M}_{\text{NS}}$ and the statistical deviation converge to the real values of the mean and the standard deviation with the increase of a number of points for a given HBO frequency.

7 As it is expected from the probability theory, the statistical mean $\tilde{M}_{\text{NS}}$ and the statistical deviation converge to the real values of the mean and the standard deviation with the increase of a number of points for a given HBO frequency.

8 Particularly, this test is useful for comparison whether or not one of the correlations is stronger than another.

Using the thorough data analysis of RXTE data for Sco X-1, we have presented a detailed investigation of the TLM and the RPM. These models predict the existence of the invariants: $\delta$ for the TLM and $M_{\text{NS}}$ for the RPM. We establish the following: (1) The inferred $\delta$-value of $5^56 \pm 0^09$ is consistent with a constant. Correlation of the $\delta$-values with the HBO frequency is rather weak. (2) There is a strong correlation between an inferred NS mass and an HBO oscillation frequency in the RPM frameworks. (3) The inferred NS mass $M_{\text{NS}} = 2.7 \pm 0.1 M_\odot$ cannot be consistent with any equation of state of NS matter.

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