Dark Matter Effects on Stellar Populations in Globular Clusters

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Abstract. According to the classical view of globular clusters, stars inside globular clusters are evolved from the same giant molecular cloud. Then their stars’ chemical compositions must be the same. But recent photometric and spectroscopic studies of globular clusters reveal the presence of more-than-one stellar populations inside globular clusters. This finding challenges our classical view of globular clusters.

In this work, we investigated the possibility of solving multiple stellar populations problem in globular clusters using dark matter assumptions. We showed that the presence of dark matter inside globular clusters changes the physical parameters (e.g. chemical composition, luminosity, temperature, age, etc.) of stars inside them. We supposed that dark matter distributed non-uniformly inside globular clusters. It means stars in high dark matter density environments (like the central region of globular clusters) are more affected by the presence of dark matter. Using this assumption, we showed that stars in different locations of globular clusters (corresponding to different dark matter densities that surrounds stars) follow different evolutionary paths (e.g. on Hertzsprung-Russell diagram). We used this note to infer that the presence of dark matter inside globular clusters can be the reason for the multiple stellar populations.
1 Introduction

Studying rotation curves of spiral galaxies during the 1960s-1980s leads to the conclusion that about 80 percent of the total mass of galaxies is in the form of dark matter (DM) [1–3]. Subsequent series of studies used different methods (e.g. gravitational lensing [4, 5], cosmic microwave background analysis [6], Lyman-alpha forest [7–9], N-body simulations of the universe and galaxies [10], and etc.) to infer that DM must be present in large-scale and small-scale structures (e.g. dwarf galaxies) of the universe.

From DM density profiles of galaxies (e.g. NFW profile), it is conceivable that DM distributed non-uniformly throughout the galaxies. It means in central regions of galaxies, DM density is the highest and by moving toward the edges of the galaxies, the density of DM reduces. Then it is logical to suppose that all astronomical objects inside galaxies, including stars and stellar clusters, are immersed inside DM. Thus, DM will affect the physics of everything inside galaxies.

By passing the time, stars inside galaxies can absorb DM particles from their surrounding. It is predicted that the gathered DM particles inside stars can affect the physics of stars mostly in two ways (see Sec. (2) for more details. In addition, for good review papers in this respect see the papers: [11, 12]):

- DM particles can transfer (whether they annihilate or not) energy between different layers of stars. In this way, the temperature profile, pressure profile, chemical composition, and many other physical parameters of stars must be altered in comparison to the standard stellar evolutionary model.

- If DM particles annihilate inside stars, then they can act as a new source of energy inside stars, beside the energy that come from baryonic matter energy production cycles. If this supposition is true, then DM can alter the luminosity and temperature of stars on the H-R diagram too. In this way, stars with the same mass and with the same initial chemical compositions, but with different DM densities that surround them, will follow different evolutionary paths on the H-R diagram (using this note, we could infer that the non-uniform distribution of DM inside GCs can be considered as a possible solution for the presence of multiple stellar populations problem in GCs (see Sec (4) for more details)).
Globular clusters (GC) are one of the oldest members of our galaxy, the Milky Way [13]. According to the classical view of GCs, it is believed that [14]:

- Stars inside GCs are located from nearly the same distance from the Earth.
- They have a similar chemical composition as they evolved from the same giant molecular cloud.
- They have the same age.

But Nova days spectroscopic and photometric studies of GCs do not accept this classical view for GCs. If this classical view for GCs is correct, then H-R diagrams of typical GCs are anticipated to be a narrow path. But instead, most of the H-R diagrams of GCs are split at least into two separate paths. As an example, Fig. (1) illustrates the color-magnitude diagram of the ω Centauri GC. In this figure, stars with higher values of metallicity are less luminous than the low-metallicity stars. This causes the H-R diagram of the ω Centauri GC to be thicker than what is anticipated according to the classical view of globular clusters. The presence of multiple stellar populations can be detected from spectroscopic analysis of the globular clusters too [15–17]. As an example, the elemental abundances of stars of the ω Centauri GC (or NGC 5139, which is the largest known GC of our Galaxy too), vary from star to star [15, 16].

These findings are challenging our current view of stellar and stellar clusters evolutionary models. Many models have been developed to solve the discrepancies between the observed and predicted theoretical features of GCs. But each model has its own deficiency and no model has been able to solve the multiple stellar populations problem (MSPP) completely. Figure 6 of Ref. [19] summarized a list of some famous models with their successes and shortcomings that are developed to solve the MSPP in GCs.

In current study, we investigated the possibility of solving MSPP using DM assumptions. We used the previously predicted DM effects on stars to provide an alternative possible solution to the MSPP.

In Sec. (2), as mentioned above, we reviewed DM effects on stars. Sec. (3) devoted to discussion on DM content of the GCs and DM density distribution inside GCs. The results of our simulations are presented in Sec. (4). Finally, a discussion about our simulations and results is presented in Sec. (5).

2 Dark Matter effects on stars

By passing the time, stars inside galaxies can absorb and accrete DM particles. According to the definition, capture rate of DM particles by a massive body (like stars, planets, white dwarfs, etc.) is the total number of DM particles that are (after weak interaction with baryonic matters) absorbed and gravitationally bounded by that massive body and in units of time [20]. The capture of DM particles by Hydrogen atoms inside stars can be calculated using Eq. (11) of the paper [21]:

\[
C_{X,H} = \left[ \frac{4\sqrt{6\pi}\rho_X}{m_X v_X^2} \frac{1}{v_0} \exp\left(\frac{-3v_0^2}{2v_X^2}\right) \right] \left[ \sigma_X,ST + \sigma_X,SD \right] \left[ \int_0^R n_H(r)r^2dr \right] \times \left[ \int_0^\infty \exp\left(\frac{-3u^2}{2v_X^2}\right) \sinh\left(\frac{3u v_0}{v_X^2}\right) \left( v_0^2 - \frac{\mu^2_H}{\mu_H} u^2 \right) \theta(v_0^2 - \frac{\mu^2_H}{\mu_H} u^2) du \right]. \tag{2.1} \]

Capture of DM particles by elements heavier than Hydrogen can be calculated using the Eq. (12) of the paper [21]:
Figure 1: (Colour online) Colour-magnitude diagram of \( \omega \) Centauri GC (or NGC 5139, which is the largest known GC of our Galaxy) which depicts the presence of multiple stellar populations. Stars with different [Fe/H] values are color coded. At least three separate generation of stars are detected in this GC [16]. Figure from Ref. [18].

In Eqs. (2.1) and (2.2), \( \rho_\chi \) is DM density in star’s location, \( m_\chi \) is the mass of DM particles, \( m_{n,i} \) is the nuclear mass of the element \( i \), \( m_p \) is the mass of a proton, \( A_i \) is the atomic number of the element \( i \), \( \sigma \) is velocity dispersion, \( v \) is the velocity of the star relative to the DM halo, \( u \) is the velocity of DM particles (velocity distribution of DM particles in the location of stars is usually considered to be a Maxwell–Boltzmanian distribution [12]), \( v_e \) is the escape velocity from the surface of stars, \( \sigma_{\chi,SD} \) is the spin-dependent DM-nucleon scattering cross-section, \( \sigma_{\chi,SI} \) is the spin-independent DM-nucleon scattering cross-section, \( n_H (r) \) is the number density of the Hydrogen atoms at distance \( r \) from the...
centre of the star, \( n_i(r) \) is the number density of the element \( i \) at distance \( r \) from the centre of the star, \( R \) is the radius of the star, \( \theta \) is the step function, \( E_0 = (3\hbar^2)/(2m_{n,i}(0.91m_n^{1/3}+0.3)^2) \) is characteristic coherence energy (see Ref. [20] for more details) and is a constant, \( \mu_i \) and \( \mu_{\pi,i} \) are defined to be: 
\[
\mu_i = m_\chi/m_{n,i}, \quad \mu_{\pi,i} = (\mu_i = 1)/2.
\]
Each of Eqs. (2.1) and (2.2) consisted of four brackets. The first two brackets are constant and can be calculated analytically. The third brackets are functions of the distance from the centres of stars, \( r \). Although it seems impossible to calculate the third brackets analytically, but it is possible to estimate it using the state-of-the-art stellar evolutionary codes. In this study, we embedded Eqs. (2.1) and (2.2) in the last version (version 15140 which is released in December of 2020) of the MESA stellar evolutionary code to calculate the capture rate of DM particles by stars. MESA is a publicly available and open-source code and can simulate the evolution of stars from very low-mass to very high-mass ones (\( 10^{-3} - 10^3 M_\odot \)). For full capabilities of the MESA code see its official papers [22–26].

If DM particles will annihilate inside stars then, they can act as a new source of energy inside stars (beside the energy sources that come from baryonic matter energy production cycles). By multiplying Eqs. (2.1) and (2.2) by the \( m_\chi c^2 \) it is possible to calculate the extra luminosity that is produced by DM particles annihilation:
\[
L_\chi = (\text{Capture rate}) \times m_\chi c^2,
\]
which \( L_\chi \) is the luminosity that is produced through DM particles annihilation, and \( c \) is the speed of light.

### 3 DM in globular clusters

In this section, we want to make an order-of-magnitude estimate of the average DM density inside GCs. Although it is believed that most of the DM content of the GCs has been stripped away by the tidal interactions with their host galaxies, but a typical GC can still keep about 20 percent of its initial DM content [27].

To estimate the average DM density of a typical GC, consider \( \omega \) Centauri GC globular cluster (or NGC 5139) as an example. Its physical parameters are presented in table (1). According to table (1), the total mass of this GC is about \( M = 3.34 \times 10^6 M_\odot \) and its V-band mass-to-light ratio is about \( Y = 2.68 \) \( (M_\odot/L_\odot) \). Being \( Y \) bigger than one \( (Y > 1) \) means that we do not receive any light from about 63 percent (that is \( (2.68-1)/2.68 \times 100 = 63\% \) ) of the total mass of \( \omega \) Centauri GC. Assuming that about 20 percent of the total mass of the \( \omega \) Centauri GC is in the form of DM (and the rest of the dark mass is in the form of white dwarfs, neutron stars, interstellar gas, etc.), then the total DM mass of this GC becomes:
\[
M_{\chi, \omega \text{Centauri}} = \frac{20}{100} \times 3.34 \times 10^6 M_\odot = 0.67 \times 10^6 M_\odot.
\]

Then, the average DM density inside \( \omega \) Centauri GC becomes:
\[
\bar{\rho}_{\chi, \omega \text{Centauri}} = \frac{\frac{1}{2}M_{\chi, \omega \text{Centauri}}}{\frac{4}{3} \pi R^3} = 70 \,(M_\odot/p_c^3).
\]

In Eq. (3.2) \( R \) is the half-light radius of the \( \omega \) Centauri GC. We assumed that half of the total mass of this GC lies within the half-light radius. For this reason, the numerator of the Eq. (3.2) is
Table 1: Physical parameters of some known GCs in the milky way galaxy. Data are from the online catalogue of GCs parameters which is available at: https://people.smp.uq.edu.au/HolgerBaumgardt/globular/parameter.html and their reference papers are: [29–31]

| Name        | Distance * (K pc) | Radius * (pc) | Mass *** (M) | (M/L)V (M/L) | \(\vec{\rho}_\chi\) **** | \(\vec{\rho}_\chi\) ***** |
|-------------|------------------|---------------|--------------|--------------|-----------------|-----------------|
| \(\omega\) Centauri | 5.24             | 10.36         | 3.34 \times 10^6 M\(_\odot\) | 2.68         | 70              | 7 \times 10^3 |
| NGC 6535    | 6.5              | 3.65          | 6.5 \times 10^4 M\(_\odot\) | 3.93         | 6.4              | 4.4 \times 10^3 |
| NGC 6121    | 1.93             | 3.69          | 9.3 \times 10^4 M\(_\odot\) | 2.02         | 44              | 4.4 \times 10^3 |
| NGC 5466    | 16.0             | 14.03         | 5.47 \times 10^4 M\(_\odot\) | 1.52         | 0.47            | 4.7 \times 10^3 |
| NGC 6642    | 8.05             | 1.51          | 6.45 \times 10^4 M\(_\odot\) | 2.79         | 447             | 4.47 \times 10^4 |
| NGC 6316    | 11.6             | 4.77          | 5.09 \times 10^5 M\(_\odot\) | 2.85         | 112             | 1.1 \times 10^3 |
| NGC 3201    | 4.6              | 6.78          | 1.41 \times 10^5 M\(_\odot\) | 2.37         | 11              | 1.1 \times 10^3 |
| NGC 1851    | 11.33            | 2.90          | 2.81 \times 10^5 M\(_\odot\) | 1.91         | 2.7             | 2.7 \times 10^2 |
| Ter 4       | 6.7              | 6.06          | 7.95 \times 10^4 M\(_\odot\) | 15.34        | 8.5             | 8.5 \times 10^2 |
| NGC 5466    | 16.0             | 14.03         | 5.47 \times 10^4 M\(_\odot\) | 1.52         | 0.47            | 47              |

* Distance from the sun
** Half-mass radius of the GCs
*** Total mass of the GCs
**** Estimated average DM density of the GCs
***** Estimated average DM density of the GCs in units of the average DM density in the sun’s location. DM density in the sun’s location was estimated to be about \(\rho_{\chi,\odot} \approx 0.01 (M/pc^3)\) [28]

multiplied to \(\frac{1}{2}\). According to Eq. (3.3), the average DM density inside \(\omega\) Centauri GC is about \(7 \times 10^3\) times more than the DM density at the sun’s location. DM density at the sun’s location was estimated to be about [28]:

\[
\bar{\rho}_{\chi,\odot} = 0.43 (GeV/cm^3) \approx 0.01 (M/pc^3).
\]

Assuming that about 20 percent of the total mass of the milky way’s GCs is in the form of DM, the average DM density for several other GCs is estimated and presented in table (1).

The mass-to-light ratio of the central regions of the \(\omega\) Centauri GC was estimated to be about \((M/L)_V = 6.7(M/pc^3)\) which is higher than the average mass-to-light ratio of the whole GC ([32]). This means DM density in central regions of this GC is higher than the average DM density of the whole GC.

If DM affects the physics of stars inside GCs, then the evolutionary courses of stars must deviate from the standard stellar evolutionary model. The more the DM density that surrounds the stars is, then the more the deviations from the standard stellar evolutionary model must be. In section (4), the results of our simulations showed that stars with the same mass and with the same initial chemical compositions but in different DM density environments (corresponding to different locations inside GCs) follow different evolutionary paths on the H-R diagram. We used these results to propose DM effects as a possible solution for the MSPP in GCs.

The calculations of this section aim to show that the average DM density inside Milky-Way’s GC is usually several orders of magnitude higher than the average DM density that surrounds the sun. So, DM effects on stars inside GCs are significant and must be taken into account.
the rest of this study, we supposed DM distributed non-uniformly inside GCs, i.e. its density is higher in central regions of the GCs. So, stars in different locations of GCs and with the same mass and with the same chemical composition will follow different evolutionary paths on the H-R diagram.

4 Results of simulations

In our simulations, weakly interacting massive particles (WIMP) with masses $m_\chi = 100 \text{ GeV} \text{ c}^{-2}$ considered to be the DM candidate. In Eqs. (2.1) and (2.2), spin-dependent and spin-independent scattering cross-sections considered to be $\sigma_{\chi,SD} = 10^{-38} \text{ cm}^2$ and $\sigma_{\chi,SI} = 10^{-44} \text{ cm}^2$, respectively. These amounts are the maximum magnitudes that are determined through the experimental DM detection experiments [33, 34]. Escape velocity from the surface of the stars are considered to be $v_e = 600 \text{ km sec}^{-1}$ during the evolution of stars. This amount is the approximate escape velocity from the surface of the sun. Although it is possible to calculate the exact value of $v_e$ using MESA code and while each star is evolving, but to reduce the computational time we used this constant value for $v_e$. In addition, the velocity distribution of dark matter particles is assumed to be a Maxwell-Boltzmann distribution with a velocity dispersion $v_\chi = 270 \text{ km sec}^{-1}$ [12]. The velocity of stars relative to the dark matter halo of the GCs is considered to be $v_\star = 220 \text{ K m sec}^{-1}$.

After considering Eq. (2.3) in MESA stellar evolutionary code and running it, the results of the simulations for a one-solar-mass star are presented in Fig. (2). In this figure, blue lines represent the evolutionary paths of a one solar mass star according to the standard stellar evolutionary model (i.e. when DM effects have not been considered into account). So, the blue lines are the same for all sub-plots in Fig. (2).

Red lines in Fig. (2) represent the evolutionary paths of one-solar-mass stars when DM effects are taken into account. Each red line in each sub-plot corresponds to the evolutionary path of a star with different DM density that surrounds the star (i.e. different values for $\rho_\chi$). In Fig. (2) by increasing DM density from Fig. (2-b) to Fig. (2-f), the deviation between red and blue lines increases. So, we can say, the presence of a non-uniform distribution of DM inside GCs causes stars with the same mass and with the same initial chemical composition but with different values of $\rho_\chi$ to follow different evolutionary paths on the observed H-R diagram of GC (e.g. see Fig. (1)).

Fig. (3) represents the results of our simulations for stars with different masses. Like Fig. (2), blue lines represent the evolution of stars according to the standard stellar evolutionary model. Red and green lines represent the evolution of stars when DM assumptions are taken into account. It is conceivable from Fig. (3) that, when DM effects are taken into account, stars with the same mass (in each sub-plot) and with the same initial chemical compositions follow different evolutionary paths on the H-R diagram.

Assuming that DM density distribution inside GCs is not uniform, then stars in different locations of GCs are immersed in different DM density environments. So, we can say, stars with the same mass and with the same initial chemical compositions will follow different evolutionary paths on the H-R diagram. These results might be the solution to the question: why we see different generations of stars inside GCs? We will discuss more about this in Sec. (5).
Figure 2: (Colour online) Evolutionary path of a one-solar-mass star on the H-R diagram and in different DM density environments. In all sub-plots, blue lines represent the evolutionary path of a one-solar-mass star according to the standard model stellar evolutionary model ($\rho_\chi = 0$). So, blue lines are the same in all sub-plots. Red lines represent the evolutionary path of a one-solar-mass star when DM effects are considered into account ($\rho_\chi \neq 0$). From sub-plot 2-b to sub-plot 2-f the density of DM increased. So, from sub-plot 2-b to sub-plot 2-f, the deviation between blue and red lines increased too.

5 Discussions and Conclusions

According to the results of our simulations (our results for stellar simulations in the presence of DM are in agreement with the results of previous works, e.g. see ([12])), the presence of DM inside GCs causes the evolutionary paths of stars to deviate from the standard stellar evolutionary model. The more the DM density around a star is, the more the stars deviate from the standard stellar evolutionary model (see Fig. (2) and Fig. (3)).

It is anticipated that GCs inside galaxies have lost most of their DM content due to the tidal interactions with their host galaxies [35]. But in Sec. (3) we discussed about the possibility that GCs may have kept some portion of their initial DM content. Mass-to-light ratios of most of the Milky way’s GCs are bigger than one (see table (1)). It means we do not receive any light from some mass of the GCs. Assuming that just 20 percent of this dark part of the GCs is in the form of DM, then we estimated the average DM density for some GCs. In almost all cases, $\bar{\rho}_\chi$ is several bigger than the average DM density around the sun (see table (1) for estimated values for $\bar{\rho}_\chi$).

Assuming that DM distributed non-uniformly inside GCs (e.g. like NFW or einasto DM density profile [36]), then stars with the same mass and with the same initial chemical compositions will follow different evolutionary paths on the H-R diagram. As an example, a one-solar-mass star near the central regions of a GC (and with the higher surrounding DM density) will follow different evolutionary path on the H-R diagram in comparison to a one-solar-mass star that evolves near the outer regions of the same GC (see Fig. (2)).
Figure 3: (Colour online) Evolutionary paths of stars with different masses on the H-R diagram. Like Fig. (2), blue lines represent the evolutionary paths of stars according to the standard stellar evolutionary model ($\rho_X = 0$). Red and green lines represent the evolutionary paths of stars when DM effects are taken into account ($\rho_X = \rho_{X,\odot}$ states are represented by the red lines, and $\rho_X = 10^2 \rho_{X,\odot}$ states are represented by the green lines). Sub-plot 3-a represents the evolution of a $1.0 \, M_\odot$ solar mass star and sub-plots 3-b, 3-c, 3-d, 3-e and 3-f represent the evolutions of stars with masses $1.2 \, M_\odot$, $1.4 \, M_\odot$, $1.5 \, M_\odot$, $1.75 \, M_\odot$, and $2.0 \, M_\odot$, respectively. It is conceivable from the figure that, by considering DM effects, the evolutionary paths of stars start to deviate from the standard stellar evolutionary model. Using this note, we suggest that dark matter effects on stars inside globular clusters might be the possible reason for the presence of multiple stellar populations inside globular clusters.

In addition to the location of stars on the H-R diagram, the presence of DM can alter the amount of time that stars spend in each evolutionary phase (e.g. main-sequence phase or red-giant phase) of their evolution. As an example, consider the main-sequence phase. According to definition, it is a phase at which most of the energy sources of stars come from Hydrogen fusion in the core of stars. Depending on the mass of the stars, stars convert Hydrogen to Helium through pp or CNO energy production cycles [37]. Both pp and CNO energy production cycles are a strong function of the temperature (that is, $\varepsilon_{pp} \propto T^4$ and $\varepsilon_{CNO} \propto T^{16}$, [37]). Then, we can say, if DM particles will annihilate inside stars, then they alter the temperature of the core of the stars too. This causes stars to consume Hydrogen atoms at different rates in comparison to the models without DM. So, we infer that the presence of DM affects the elemental abundances of star too. Our overall result from this discussion is that, if the presence of DM can alter the temperature of stars, then it can alter the age, chemical composition, luminosity, and many other physical parameters of stars too.

If the presence of DM alters the rate of pp and CNO energy production cycles inside stars, then we can say that, the chemical compositions of stars can be affected by the presence of DM too. But, because of a lack of our knowledge about the exact physical nature of DM, it is hard to discuss more
about the consequences of DM effects on stars.

Our overall result is that, if the presence of DM alters the luminosity, temperature, chemical composition, age, etc. of stars, then its presence can be considered as a possible solution to the multiple stellar populations problem in GCs.

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7 Data availability

The codes and data underlying this article are available on the GitHub repository website, at https://github.com/eb-hassani/Dark-Matter-effects-on-stars-in-Globular-Clusters-version-2.0

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