APPLICATION OF CUBIC TRANSITION MATRICES AND
PRODUCT FUZZY SOFT TRANSITION MATRICES

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INTRODUCTION

In this chapter an application of cubic transition matrix in agricultural planning is provided. This chapter is concluded with a decision theory on product fuzzy soft matrices.

A DECISIONMAKING FOR AGRICULTURAL PLANNING USING CUBIC TRANSITION MATRICES

This section provides an algorithm for decision making to find an optimum solution on agriculture planning for a given data using cubic transition matrices as a tool.

The methodology involves seven steps in the algorithm and concludes that there is an increase in production level of crops. In many countries the role of farmers in agricultural planning is highly remarkable. The pressure on farmers almost in all parts of the World is to produce more agricultural products and to have more income on it. But in recent years it is really a challenging job for the farmers to do so. Though the farmers face problems from several directions to increase the yield, the major crisis involved in them is due to lack of labors and monsoon failures.

The real time experience of developed countries shows the transfer of labor force from agriculture to non-agriculture; in particular to the industrial sector. Under these circumstances, higher growth in agriculture assumes greater importance and is a matter of concern for policy planners and research scholars in recent times.

In India, there are many farmers having small cultivation agricultural lands on their own. By the climatic condition and the present risk factors, farmers cultivate different crops as per the market need. For crop cultivation, it comprises mainly three levels, namely

1. Sowing of seeds in the land
2. Growth period of the crop
3. Yield period of the crop

Now a days in every period farmers face the major problem in the form of lack of labors. No one can able to utilize the power of labors fruitful. Here we present an algorithm to calculate the lack of labors. From this decision making process the farmers can able to analyze the lack of labors then switch over their crop in to other to maximize the profit.

Algorithm for optimization of labor power in agriculture

Step 1: Form the fuzzy square matrix for lack of labors over that particular crop

Step 2: Divide the square matrix in to number of 2x2 matrices like

\[ A_{11} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, A_{12} = \begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{bmatrix}, \ldots , A_{1n} = \begin{bmatrix} a_{11} & a_{1n} \\ a_{21} & a_{2n} \end{bmatrix} \]

\[ B_{21} = \begin{bmatrix} a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix}, B_{22} = \begin{bmatrix} a_{32} & a_{33} \\ a_{42} & a_{43} \end{bmatrix}, \ldots , B_{2n} = \begin{bmatrix} a_{31} & a_{3n} \\ a_{41} & a_{4n} \end{bmatrix} \]

\[ K_{m1} = \begin{bmatrix} a_{m-1,1} & a_{m-1,2} \\ a_{m1} & a_{m2} \end{bmatrix}, K_{m2} = \begin{bmatrix} a_{m-1,2} & a_{m-1,3} \\ a_{m2} & a_{m3} \end{bmatrix}, \ldots , K_{mn} = \begin{bmatrix} a_{m-1,1} & a_{m-1,n} \\ a_{m1} & a_{mn} \end{bmatrix} \]

Step 3: Calculate the evaluation matrix as\( ||A|| = \lambda(x) \).

Step 4: Evaluation process

Calculate the determinant using the operations in cubic transition matrix.

Step 5: Select the cubic transition matrix for the evaluation as minimum diagonal length.

Step 6: Find the inner product of cubic transition matrix as\( a_{11} \cdot a_{22} \oplus a_{12} \cdot a_{21} \)

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Step 7: The Utilization factor is 1000x evaluation.

The following example justifies the above algorithm.

**Example 5.1**

Suppose there are three varieties of crops namely crop 1, crop 2, crop3 or simply 1, 2, 3 with which three of labors namely I, II, III are cultivating them. The data is given below.

| Crops | 1 | 2 | 3 |
|-------|---|---|---|
| I     | 0.6 | 0.3 | 0.1 |
| II    | 0.5 | 0.6 | 0.3 |
| III   | 0.5 | 0.5 | 0.7 |

Our attempt is to quote a technique for the minimization of labors and to increase the profit using the above algorithm. We now construct the fuzzy square matrix from the above problem as follows.

Step-1 to Step-6.

| Crops | 1 | 2 | 3 |
|-------|---|---|---|
| I     | 0.6 | 0.3 | 0.1 |
| II    | 0.5 | 0.6 | 0.3 |
| III   | 0.5 | 0.5 | 0.7 |

We now divide into sub matrices and proceed to find evaluation matrix, cubic transition matrix and its inner product as follows.

Let 
\[ A = \begin{bmatrix} 0.6 & 0.3 \\ 0.5 & 0.6 \end{bmatrix} \]

\[ \|A\| = \begin{bmatrix} 0.6 & 0.3 \\ 0.5 & 0.6 \end{bmatrix} \]

\[ = 0.6 \oplus 0.5 \oplus 0.5 \oplus 0.6 = 0.2 \]

Let 
\[ B = \begin{bmatrix} 0.6 & 0.3 \\ 0.5 & 0.6 \end{bmatrix} \]

\[ \|B\| = \begin{bmatrix} 0.6 & 0.3 \\ 0.5 & 0.6 \end{bmatrix} \]

\[ = 0.6 \oplus 0.6 \oplus 0.6 \oplus 0.9 = 0. \]

Let 
\[ C = \begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.6 \end{bmatrix} \]

\[ \|C\| = \begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.6 \end{bmatrix} \]

\[ = 0.5 \oplus 0.5 \oplus 0.5 \oplus 0.5 = 0 \]

Let 
\[ D = \begin{bmatrix} 0.6 & 0.3 \\ 0.5 & 0.6 \end{bmatrix} \]

\[ \|D\| = \begin{bmatrix} 0.6 & 0.3 \\ 0.5 & 0.6 \end{bmatrix} \]

\[ = 0.6 \oplus 0.6 \oplus 0.5 \oplus 0.6 = 0 \]

Let 
\[ E = \begin{bmatrix} 0.6 & 0.3 \\ 0.5 & 0.7 \end{bmatrix} \]

\[ \|E\| = \begin{bmatrix} 0.6 & 0.3 \\ 0.5 & 0.7 \end{bmatrix} \]

\[ = 0.6 \oplus 0.5 \oplus 0.5 \oplus 0.9 = 0.5 \]

Let 
\[ F = \begin{bmatrix} 0.5 & 0.3 \\ 0.5 & 0.7 \end{bmatrix} \]

\[ \|F\| = \begin{bmatrix} 0.5 & 0.3 \\ 0.5 & 0.7 \end{bmatrix} \]

\[ = 0.5 \oplus 0.5 \oplus 0.5 \oplus 0.9 = 0.4 \]

Let 
\[ G = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix} \]

\[ \|G\| = \begin{bmatrix} 0.6 & 0.3 \\ 0.5 & 0.5 \end{bmatrix} \]

\[ = 0.6 \oplus 0.5 \oplus 0.5 \oplus 0.5 = 0.1 \]

Let 
\[ H = \begin{bmatrix} 0.6 & 0.1 \\ 0.5 & 0.7 \end{bmatrix} \]

\[ \|H\| = \begin{bmatrix} 0.6 & 0.1 \\ 0.5 & 0.7 \end{bmatrix} \]

\[ = 0.9 \oplus 0.5 \oplus 0.5 \oplus 0.9 = 0.8 \]
Let \( I = \begin{bmatrix} 0.6 & 0.1 \\ 0.5 & 0.7 \end{bmatrix} \)
\( ||I|| = \begin{bmatrix} 0.6 & 0.1 \\ 0.5 & 0.7 \end{bmatrix} = 0.6 \oplus 0.5 \oplus 0.5 \oplus 0.9 = 0.5 \)
Then the evaluation matrix is
\( \begin{bmatrix} 0.2 & 0 & 0.5 \\ 0.1 & 0.8 & 0.5 \end{bmatrix} \)
Evaluation = 0.2(0.5–0.2) \( \oplus \) 0 \( \oplus \) 0.5(0–0.5)
= 0.2(0.5–0.2) \( \oplus \) 0 \( \oplus \) 0.5(0–0.5).
= 0.2(0.3) \( \oplus \) 0.5(0.5).
= 0.1.
We select \( \lambda = 0.1 \) in an internal cubic transition matrix having minimum diagonal length.
The conclusion matrix \( C = \begin{bmatrix} 1 & 0 \\ 0.9 & 0.1 \end{bmatrix} = 0.1 \oplus 0 = 0.1 \)
Then we conclude that the utilization factor is 10%
Now we minimize the labors.

| Crops | I | II | III |
|-------|---|----|-----|
|       | 1 | 2  | 3   |
| Labor | \( A = \begin{bmatrix} 0.5 & 0.2 \\ 0.4 & 0.5 \end{bmatrix} \)
|       | ||A|| = \( \begin{bmatrix} 0.5 & 0.2 \\ 0.4 & 0.5 \end{bmatrix} = 0.5 \oplus 0.8 \oplus 0.8 \oplus 0.5 = 0.6 \)
|       | Let \( B = \begin{bmatrix} 0.2 & 0.1 \\ 0.5 & 0.3 \end{bmatrix} \)
|       | ||B|| = \( \begin{bmatrix} 0.2 & 0.1 \\ 0.5 & 0.3 \end{bmatrix} = 0.4 \oplus 0.5 \oplus 0.5 \oplus 0.9 = 0.3 \)
|       | Let \( C = \begin{bmatrix} 0.5 & 0.1 \\ 0.4 & 0.3 \end{bmatrix} \)
|       | ||C|| = \( \begin{bmatrix} 0.5 & 0.1 \\ 0.4 & 0.3 \end{bmatrix} = 0.5 \oplus 0.4 \oplus 0.4 \oplus 0.9 = 0.2 \)
|       | Let \( D = \begin{bmatrix} 0.4 & 0.5 \\ 0.4 & 0.4 \end{bmatrix} \)
|       | ||D|| = \( \begin{bmatrix} 0.4 & 0.5 \\ 0.4 & 0.4 \end{bmatrix} = 0.6 \oplus 0 \oplus 0 \oplus 0.6 = 0.2 \)
|       | Let \( E = \begin{bmatrix} 0.5 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} \)
|       | ||E|| = \( \begin{bmatrix} 0.5 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} = 0.5 \oplus 0.2 \oplus 0.2 \oplus 0.6 = 0.5 \)
|       | Let \( F = \begin{bmatrix} 0.4 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} \)
|       | ||F|| = \( \begin{bmatrix} 0.4 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} = 0.6 \oplus 0.2 \oplus 0.2 \oplus 0.6 = 0.6 \)
|       | Let \( G = \begin{bmatrix} 0.5 & 0.2 \\ 0.4 & 0.4 \end{bmatrix} \)
|       | ||G|| = \( \begin{bmatrix} 0.5 & 0.2 \\ 0.4 & 0.4 \end{bmatrix} = 0.5 \oplus 0.8 \oplus 0.8 \oplus 0.6 = 0.7 \)
|       | Let \( H = \begin{bmatrix} 0.2 & 0.1 \\ 0.4 & 0.6 \end{bmatrix} \)
Let $I = \begin{bmatrix} 0.5 & 0.1 \\ 0.4 & 0.6 \end{bmatrix}$

$\|I\| = \begin{bmatrix} 0.5 & 0.1 \\ 0.4 & 0.6 \end{bmatrix}$

$= 0.5 \oplus 0.4 \oplus 0.4 \oplus 0.6 = 0.9$

Then the evaluation matrix is

$A = \begin{bmatrix} 0.2 & 0.3 & 0.2 \\ 0.6 & 0.3 & 0.2 \\ 0.7 & 0.8 & 0.9 \end{bmatrix}$

Evaluation $= 0.6(0.5-0.8) \oplus 0.3(0.8 \oplus 0.2) \oplus 0.2(0.6-0.5)$

$= 0.6(0.3) \oplus 0.3(0.6) \oplus 0.2(0.1)$.

$= 0.8-0.8 \oplus 0.2.$

$= 0.2.$

We select $\lambda = 0.2$ as an internal cubic transition matrix having minimum diagonal length.

The conclusion matrix $C = \begin{bmatrix} 0.2 & 0.8 \\ 0.4 & 0.6 \end{bmatrix}$ $= 0.2 \oplus 0.2 = 0.4$

Then we conclude that the rate of utilization is $40\%$.

Again we minimize the labors at most possible level.

Crops

Let $A = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}$

$\|A\| = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.3 \end{bmatrix}$ $= 0.1 \oplus 0.1 \oplus 0.1 \oplus 0.9 = 0.2$

Let $B = \begin{bmatrix} 0.1 & 0.1 \\ 0.3 & 0.3 \end{bmatrix}$

$\|B\| = \begin{bmatrix} 0.1 & 0.1 \\ 0.3 & 0.3 \end{bmatrix}$ $= 0.1 \oplus 0.3 \oplus 0.3 \oplus 0.1 = 0.8.$

Let $C = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$

$\|C\| = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$ $= 0.1 \oplus 0.1 \oplus 0.1 \oplus 0.1 = 0.4$

Let $D = \begin{bmatrix} 0.1 & 0.3 \\ 0.1 & 0.1 \end{bmatrix}$

$\|D\| = \begin{bmatrix} 0.1 & 0.3 \\ 0.1 & 0.1 \end{bmatrix}$ $= 0.1 \oplus 0.3 \oplus 0.3 \oplus 0.1 = 0.8$

Let $E = \begin{bmatrix} 0.3 & 0.3 \\ 0.1 & 0.1 \end{bmatrix}$

$\|E\| = \begin{bmatrix} 0.3 & 0.3 \\ 0.1 & 0.1 \end{bmatrix}$ $= 0.9 \oplus 0.1 \oplus 0.1 \oplus 0.1 = 0.2$

Let $F = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$

$\|F\| = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$ $= 0.1 \oplus 0.1 \oplus 0.1 \oplus 0.1 = 0.4$

Let $G = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$

$\|G\| = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$ $= 0.1 \oplus 0.1 \oplus 0.1 \oplus 0.1 = 0.4$

Let $H = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$

$\|H\| = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}$ $= 0.1 \oplus 0.1 \oplus 0.1 \oplus 0.1 = 0.4$
Let \( I = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} \)
\[ ||I|| = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} = 0.1 \oplus 0.1 \oplus 0.1 \oplus 0.1 = 0.4 \]
Then the evaluation matrix is \( E = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.4 \end{bmatrix} \)
Evaluation = 0.2(0.8-0.6) \oplus 0.8(0.2-0.6) \oplus 0.4(0.2-0.8)
= 0.2(0.2) \oplus 0.8(0.4) \oplus 0.4(0.6).
= 0.4 \oplus 0.4 = 0.6.
We select \( \lambda = 0.6 \) as an internal cubic transition matrix having minimum diagonal length.
The conclusion matrix \( C = \begin{bmatrix} 0.1 & 0.9 \\ 0.7 & 0.3 \end{bmatrix} \)
Let \( \mu = 0.3 \oplus 0.3 = 0.6 \)
Step-7
We conclude that the rate of utilization is 60%. Thus this result holds good and yields 60% profit for above fuzzy matrix A with given labors and crops. One can increase the profit level by choosing the appropriate crop with limited labors to increase the production and maximize their profit.

2 PRODUCT FUZZY SOFT MATRICES

The second section introduces the notion of product fuzzy soft transition matrix. Also this section provides an algorithm for decision making about the lack of agriculture works and migration of farmers from their villages due to monsoon failure using product fuzzy soft transition matrix as a tool.

**Definition 2.1**

Let \((\mathcal{M}, \mathcal{A}) = S_{ij}\) and \((\mathcal{M}, \mathcal{B}) = R_{ij}\) be two fuzzy soft transition matrices. Then the transition multiplication \(S_{ij} \circ R_{ij}\) is called as product fuzzy soft matrix (or) product fuzzy soft transition matrix.

**Algorithm 2.1**

The algorithm for farmers job lacking and migration involves the following steps.

Step 1: Select a subset \( \mathcal{A} \) with two parameters from the transition matrix.
Step 2: Choose the transition matrices from orthogonal transition matrices having maximum and next maximum elements for required parameters.
Step 3: Select another subset \( \mathcal{B} \) with two parameters from transition matrix.
Step 4: Choose the transition matrices from the same orthogonal transition matrices having successive maximum elements for required parameters.
Step 5: Obtain the square matrix \( S_{ij} \) having elements that are orthogonal transition matrices for the subset \( \mathcal{A} \).
Step 6: Select another square matrix \( R_{ij} \) having elements that are orthogonal transition matrices for the subset \( \mathcal{B} \).
Step 7: Reduce the matrices \( S_{ij} \) and \( R_{ij} \) in two steps corresponding to orthogonal transition sets.
Step 8: Calculate the transition multiplication \( K_{ij} = S_{ij} R_{ij} \) having elements in orthogonal transition matrices.
Step 9: Obtain the required result from the reduced evaluation matrix \( K_{ij} \).

In India, monsoon rainfall is not up to the mark during the recent years. Due to this situation, farmer’s life style tends to below poverty level. Some of the farmers migrate to cities from their own villages and proceed for another job. Later some farmers return back to village to proceed with their own job but suddenly they migrate to town. We try to represent this situation by means of an example and justify the above algorithm using soft transition matrices as a tool.

**Example 2.2**

\[
P = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix}
\]
This matrix describes the following phenomenon of the farmers and their jobs.
\( J \rightarrow J \): Farmers continuing their own job are 60%.
\( J \rightarrow M \): Farmers lost their own job then migrating in to other jobs are 40%.
\( M \rightarrow J \): Farmers return to their village and like to comeback their own job are 20%.
\( M \rightarrow M \): Home returned farmers again migrate from their village 80%.

Step 1:
Define \( \mathcal{A} = \{ JM_1, JM_2 \} \)

Step 2:
Now we select the transition matrix
\[
\begin{bmatrix}
0.6 & 0.4 \\
0.2 & 0.8
\end{bmatrix}
\]
The orthogonal transition matrices of
\[
\begin{bmatrix}
0.6 & 0.4 \\
0.2 & 0.8
\end{bmatrix}
\]
is same that of
\[
\begin{bmatrix}
0.1 & 0.9 \\
0.2 & 0.8
\end{bmatrix}
\]
Now we first select the orthogonal transition matrices having entry 0.9 in JM.
\[
\alpha = \begin{bmatrix}
0.1 & 0.9 \\
0.2 & 0.8 \\
0.1 & 0.9 \\
0.7 & 0.3
\end{bmatrix}
\]
Then we select the orthogonal transition matrices having entry 0.9 in JM in the next orthogonal set of matrices which is orthogonal to
\[
\begin{bmatrix}
0.1 & 0.9 \\
0.3 & 0.7
\end{bmatrix}
\]
\[
\beta = \begin{bmatrix}
0.1 & 0.9 \\
0.3 & 0.7 \\
0.1 & 0.9 \\
0.8 & 0.2
\end{bmatrix}
\]
Then we select the orthogonal transition matrices having entry 0.9 in JM in the next orthogonal set of matrices which is orthogonal to
\[
\begin{bmatrix}
0.1 & 0.9 \\
0.1 & 0.9 \\
0.9 & 0.1
\end{bmatrix}
\]
\[
\gamma = \begin{bmatrix}
0.1 & 0.9 \\
0.4 & 0.6 \\
0.1 & 0.9 \\
0.9 & 0.1
\end{bmatrix}
\]
Then we select the orthogonal transition matrices having entry 0.8 in JM in the next orthogonal set of matrices which is orthogonal to
\[
\begin{bmatrix}
0.1 & 0.9 \\
0.2 & 0.8 \\
0.9 & 0.1
\end{bmatrix}
\]
\[
\delta = \begin{bmatrix}
0.2 & 0.8 \\
0.9 & 0.1 \\
0.4 & 0.6
\end{bmatrix}
\]
Similarly we can able to collect next four set of matrices
\[
\alpha_t = \begin{bmatrix}
0.1 & 0.9 \\
0.2 & 0.8 \\
0.1 & 0.9 \\
0.7 & 0.3
\end{bmatrix}, \quad \beta_t = \begin{bmatrix}
0.1 & 0.9 \\
0.3 & 0.7 \\
0.1 & 0.9 \\
0.8 & 0.2
\end{bmatrix}, \\
\gamma_t = \begin{bmatrix}
0.1 & 0.9 \\
0.4 & 0.6 \\
0.1 & 0.9 \\
0.9 & 0.1
\end{bmatrix}, \quad \delta_t = \begin{bmatrix}
0.2 & 0.8 \\
0.9 & 0.1 \\
0.4 & 0.6
\end{bmatrix}
\]
Step 3:
Define \( \mathcal{B} = \{ MM_1, MM_2 \} \)

Step 4:
Now we select the transition matrix
\[
\begin{bmatrix}
0.6 & 0.4 \\
0.2 & 0.8
\end{bmatrix}
\]
The orthogonal transition matrices of
\[
\begin{bmatrix}
0.6 & 0.4 \\
0.2 & 0.8
\end{bmatrix}
\]
is same that of
\[
\begin{bmatrix}
0.1 & 0.9 \\
0.2 & 0.8
\end{bmatrix}
\]
Now we first select the orthogonal transition matrices having entry 0.9 in MM.
\[
\alpha = \begin{bmatrix}
0.8 & 0.2 \\
0.1 & 0.9 \\
0.3 & 0.7 \\
0.1 & 0.9
\end{bmatrix}
\]
Then we select the orthogonal transition matrices having entry 0.9 in MM in the next orthogonal set of matrices which is orthogonal to
\[
\begin{bmatrix}
0.1 & 0.9 \\
0.3 & 0.7
\end{bmatrix}
\]
\[
\beta = \begin{bmatrix}
0.7 & 0.3 \\
0.1 & 0.9 \\
0.2 & 0.8 \\
0.1 & 0.9
\end{bmatrix}
\]
Then we select the orthogonal transition matrices having entry 0.9 in MM in the next orthogonal set of matrices which is orthogonal to
\[
\begin{bmatrix}
0.1 & 0.9 \\
0.1 & 0.9 \\
0.9 & 0.1
\end{bmatrix}
\]
\[
\gamma = \begin{bmatrix}
0.4 & 0.6 \\
0.1 & 0.9 \\
0.9 & 0.1 \\
0.1 & 0.9
\end{bmatrix}
\]
Then we select the orthogonal transition matrices having entry 0.8 in MM in the next orthogonal set of matrices which is orthogonal to
\[
\begin{bmatrix}
0.1 & 0.9 \\
0.2 & 0.8
\end{bmatrix}
\]
\[
\delta = \begin{bmatrix}
0.1 & 0.9 \\
0.2 & 0.8 \\
0.6 & 0.4
\end{bmatrix}
\]
Similarly we can able to collect next four set of matrices
\[
\alpha_1 = \begin{bmatrix}
0.9 & 0.1 \\
0.3 & 0.7 \\
0.2 & 0.8
\end{bmatrix}, \quad \beta_1 = \begin{bmatrix}
0.9 & 0.1 \\
0.2 & 0.8 \\
0.2 & 0.8
\end{bmatrix},
\delta_1 = \begin{bmatrix}
0.1 & 0.9 \\
0.2 & 0.8 \\
0.2 & 0.8
\end{bmatrix}
\]
\[
\gamma_1 = \begin{bmatrix}
0.3 & 0.7 \\
0.8 & 0.2 \\
0.2 & 0.8
\end{bmatrix}
\]

Step 5:
We construct the fuzzy soft transition matrix \( S_{ij} \) having elements from step2 \( S_{ij} = \)
\[
\begin{bmatrix}
\alpha & \beta & \delta \\
0.1 & 0.9 & 0.1 & 0.9 \\
0.2 & 0.8 & 0.7 & 0.3 \\
0.1 & 0.9 & 0.1 & 0.9 \\
0.4 & 0.6 & 0.9 & 0.1 \\
\end{bmatrix}
\]

Step 6:
We construct the fuzzy soft transition matrix \( T_{ij} \) having elements from step4 \( T_{ij} = \)
\[
\begin{bmatrix}
\alpha & \beta & \delta \\
0.8 & 0.2 & 0.3 & 0.7 \\
0.1 & 0.9 & 0.1 & 0.9 \\
0.4 & 0.6 & 0.9 & 0.1 \\
0.1 & 0.9 & 0.1 & 0.9 \\
0.9 & 0.1 & 0.4 & 0.6 \\
0.3 & 0.7 & 0.3 & 0.7 \\
0.3 & 0.7 & 0.3 & 0.7 \\
0.3 & 0.7 & 0.3 & 0.7 \\
0.2 & 0.8 & 0.2 & 0.8 \\
\end{bmatrix}
\]

Step 7:
First step reduction matrix obtains by multiplying the matrices in their corresponding sets.
\[
S_{ij} = \begin{bmatrix}
\alpha & \beta & \delta \\
0.4 & 0.6 & 0.9 & 0.1 \\
0.8 & 0.2 & 0.6 & 0.4 \\
0.1 & 0.9 & 0.2 & 0.8 \\
0.7 & 0.3 & 0.6 & 0.4 \\
0.2 & 0.8 & 0.2 & 0.8 \\
\end{bmatrix}
\]
Second step reduction matrix obtains by multiplying the matrices in their corresponding sets.
\[
\delta_{ij} = \begin{bmatrix}
\alpha & \beta & \delta \\
0.6 & 0.4 & 0.2 \\
0.2 & 0.8 & 0.4 \\
0.8 & 0.2 & 0.6 \\
\end{bmatrix}
\]
First step reduction matrix obtains by multiplying the matrices in their corresponding sets.

\[
T_{ij} = \begin{bmatrix}
\alpha & \beta & \delta \\
0.6 & 0.4 & 0.7 & 0.3 \\
0.2 & 0.8 & 0.1 & 0.9 \\
0.8 & 0.2 & 0.8 & 0.2 \\
0.9 & 0.1 & 0.8 & 0.2 \\
0.3 & 0.7 & 0.4 & 0.6 \\
0.8 & 0.2 & 0.8 & 0.2 \\
\end{bmatrix}
\]
Second step reduction matrix obtains by multiplying the matrices in their corresponding sets.
\[
\delta_{ij} = \begin{bmatrix}
\alpha & \beta & \delta \\
0.6 & 0.4 & 0.8 & 0.2 \\
0.2 & 0.8 & 0.4 & 0.6 \\
0.8 & 0.2 & 0.8 & 0.2 \\
\end{bmatrix}
\]
Step 8: $\mathcal{K}_{ij} = s_{ij} \ast t_{ij}$

$$\begin{bmatrix}
\alpha \alpha_1 = 0.6 & 0.4 \\
\gamma \gamma_1 = 0.2 & 0.8
\end{bmatrix} \times \begin{bmatrix}
\beta \beta_1 = 0.8 & 0.2 \\
\delta \delta_1 = 0.2 & 0.8
\end{bmatrix} = \begin{bmatrix}
\alpha \alpha_1 = 0.6 & 0.4 \\
\gamma \gamma_1 = 0.2 & 0.8
\end{bmatrix} \ast \begin{bmatrix}
\beta \beta_1 = 0.8 & 0.2 \\
\delta \delta_1 = 0.2 & 0.8
\end{bmatrix}$$

$\mathcal{K}_{ij} = \begin{bmatrix}
\alpha \alpha_1 = 0.4 & 0.6 \\
\gamma \gamma_1 = 0.2 & 0.8
\end{bmatrix} \ast \begin{bmatrix}
\beta \beta_1 = 0.2 & 0.8 \\
\delta \delta_1 = 0.6 & 0.4
\end{bmatrix}$

Now we compute:

- $\begin{bmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{bmatrix} \times \begin{bmatrix} 0.4 & 0.6 \\ 0.2 & 0.8 \end{bmatrix} = 0.6 + 0.8 + 0.8 + 0.4 = 0.6$.
- $\begin{bmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{bmatrix} \times \begin{bmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{bmatrix} = 0.4 + 0.8 + 0.8 + 0.6 = 0.6$.
- $\begin{bmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{bmatrix} \times \begin{bmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{bmatrix} = 0.4 + 0.4 + 0.4 + 0.4 = 0.6$.

Hence we conclude that due to monsoon stoppage 60% of farmers were migrating to other jobs for the given data.

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