Goos-Hänchen and Imbert-Fedorov shifts of polarized vortex beams

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Reflection and transmission of plain waves at an interface separating two homogenous isotropic media are described by the well-known Snell’s and Fresnel equations. However, the problem becomes tricky for confined beams with a finite spectral distribution. Interference of partial plain waves propagating at slightly different angles and obeying individual Snell and Fresnel formulas, results in such effects as the longitudinal Goos-Hänchen (GH) [1,2] and transverse Imbert-Fedorov (IF) [3,4] shifts, which displace the output beams within and across the propagation plane, respectively (see Fig. 1).

While the GH shift was explained and calculated soon after its discovery [2], the transverse IF shift was associated by significant controversies over about 50 years. The direct calculation of the IF effect was first performed for the reflected beam [5–7] and later generalized to the transmitted beam [8,9]. It was shown that the IF shift is closely related to the spin angular momentum carried by a polarized beam and conservation of the total angular momentum in the system [10,11] (see also [12–14]). Despite long history of the theoretical studies and experiments [4,15,16], analytical expression for the IF shift of a polarized Gaussian beam was derived in the correct form only recently [13,14], and these results have been confirmed both experimentally [17] and theoretically [18].

In addition to the usual linear shifts, angular GH and IF shifts caused by the beam diffraction have been described [14,18]. Recently, this description of the IF effect has been extended to the case of higher-order vortex beams carrying intrinsic orbital angular momentum [19]. The vortex-induced IF shift is proportional to the vortex charge, but it also significantly depends on the beam polarization. Such IF shift was calculated [19] and measured experimentally [20] for p and s polarizations only, when the spin IF effect vanishes.

The main purpose of this Letter is twofold. First, we derive explicit analytical expressions for both linear and angular IF and GH shifts in the most general case of an arbitrarily polarized vortex beam. We unveil a direct relation between the vortex-dependent IF shift and the angular GH shift [18] and predict novel vortex-induced GH shift related to the angular IF shift. Second, we verify all theoretical results by direct numerical simulations.

We consider the reflection and refraction of an optical beam at an interface separating two media, as shown in Fig. 1. In addition to the coordinate system (x, y, z) attached to the interface, we employ the coordinate systems of individual beams (X^a, Y^a, Z^a), where a = i, r, t denotes incident, reflected and transmitted beams, respectively. The Z^a axis attached to the directions of the a-th beam as determined by the Snell’s law. The incident beam propagates in the (x, z) plane, so that Y^i = y (see Fig. 1). We also define the wave number in the first medium, k, the angle of incidence, θ, the angle of refraction, θ' = sin⁻¹(n sin θ), as well as the relative permittivity ε, permeability μ, and refractive index n = √εμ of the second medium.

We assume that the incident beam is a uniformly polarized Laguerre-Gaussian beam with the waist located at the interface, so that the transverse (X^i, Y^i)-component of its electric field has the form

\[ \mathbf{E}_{\text{trans}}^i \propto \epsilon \left( \mathbf{X}^i + \epsilon \mathbf{Y}^i \right) \mathbf{L}_{n,m}(X^i, Y^i, Z^i). \]  \( \text{(1)} \)
Here \( \mathcal{L}_{n,m} \) is the Laguerre-Gaussian solution of the scalar parabolic wave equation which contains an optical vortex of charge \( m \): \( \mathcal{L}_{n,m} \propto (X^i + i \text{sign}(m) Y^i)^m \) [21], whereas polarization components form the normalized Jones vector in the basis of \( p \) and \( s \) modes: \( (e_\parallel, e_\perp)^T \), \( |e_\parallel|^2 + |e_\perp|^2 = 1 \).

The main characteristics of the reflected and transmitted beams are determined by the Fresnel coefficients, \( R_{\parallel}, R_{\perp}, T_{\parallel}, \) and \( T_{\perp} \). The amplitude reflection and transmission coefficients \( R, T \) and the corresponding energy reflection and transmission coefficients \( Q^r, Q^t \) are given by

\[
R = \sqrt{|R_{\parallel}|^2 + |R_{\perp}|^2}, \quad T = \sqrt{|T_{\parallel}|^2 + |T_{\perp}|^2},
\]

\[
Q^r = |R|^2, \quad Q^t = \frac{n \cos \theta'}{\mu \cos \theta} |T|^2, \quad Q^r + Q^t = 1. \tag{2}
\]

The Jones vectors of the secondary beams are

\[
\begin{pmatrix}
e^r_{\parallel} \\
e^r_{\perp}
\end{pmatrix}
= \frac{1}{R}
\begin{pmatrix}
R_{\parallel} e_{\parallel} \\
R_{\perp} e_{\perp}
\end{pmatrix}, \quad

\begin{pmatrix}
e^t_{\parallel} \\
e^t_{\perp}
\end{pmatrix}
= \frac{1}{T}
\begin{pmatrix}
T_{\parallel} e_{\parallel} \\
T_{\perp} e_{\perp}
\end{pmatrix}. \tag{3}
\]

To derive the lateral shifts of an arbitrarily polarized vortex beam, we use the basic results obtained for polarized Gaussian beams \( m = 0 \). In the regime of the total internal reflection, \( \sin \theta > n \), the Fresnel reflection coefficient becomes complex: \( R_{\parallel,\perp} = \exp(i\varphi_{\parallel,\perp}) \), and the GH, \( \langle X \rangle \), and IF, \( \langle Y \rangle \), shifts are described by the Artmann and Schillings formulas [2, 5]:

\[
\langle X^r \rangle_{\text{tot}} = \frac{1}{k} \left( |e_\parallel|^2 \frac{\partial \varphi_{\parallel}}{\partial \theta} + |e_\perp|^2 \frac{\partial \varphi_{\perp}}{\partial \theta} \right), \tag{4}
\]

\[
\langle Y^r \rangle_{\text{tot}} = -\frac{\cot \theta}{k} \left[ \sigma (1 + \cos \delta) + \chi \sin \delta \right], \tag{5}
\]

where \( \sigma = 2 \text{Im} \langle e_\parallel e_\perp \rangle \) is the helicity of the incident beam (degree of circular polarization), \( \chi = 2 \text{Re} \langle e_\parallel e_\perp \rangle \) is the degree of linear polarization inclined at \( \pi/4 \) with respect to the incident plane, and \( \delta = \varphi_{\perp} - \varphi_{\parallel} \). We note, that the shifts here and below in the paper are given in the coordinate system of the respective beam.

In the regime of partial reflection and transmission, \( \sin \theta < n \), the Fresnel coefficients are real, and the GH shift vanishes, \( \langle X^r \rangle = 0 \), while the IF shifts are given by the equations derived in \([13, 14]\):

\[
\langle Y^r \rangle_0 = -\frac{\cot \theta}{k} \frac{(R_{\parallel} + R_{\perp})^2}{R^2},
\]

\[
\langle Y^t \rangle_0 = -\frac{\cot \theta}{k} \frac{T_{\parallel}^2 + T_{\perp}^2 - 2T_{\parallel}T_{\perp} \cos \theta'/\cos \theta}{T^2}. \tag{6}
\]

In addition, the reflected and transmitted beams undergo angular shifts, which can be considered as the shifts in the \( \text{wave vector space} \). Generalizing results of \([14, 18]\), these shifts can be written as

\[
\langle K^r_x \rangle_0 = -\frac{1}{2D} \frac{d \ln Q^r}{d \theta}, \quad \langle K^t_x \rangle_0 = \frac{\cos \theta}{2D} \frac{d \ln Q^t}{d \theta}, \tag{7}
\]

\[
\langle K^r_y \rangle_0 = \frac{\cot \theta R_{\perp}^2 - R_{\parallel}^2}{2D R^2}, \quad \langle K^t_y \rangle_0 = \frac{\cot \theta T_{\perp}^2 - T_{\parallel}^2}{2D T^2}. \tag{8}
\]

Here we denote \( K^r_x \) and \( K^r_y \) as the \( X^r \) and \( Y^r \) wave vector components in the \( a \)-th beam, \( D = kw_0^2/2 \) is the Rayleigh length, \( w_0 \) is the minimum beam waist, and the derivatives of the energy reflection and transmission coefficients (7) are explicitly calculated for \( p \) and \( s \) polarizations in \([19]\).

Now we consider a polarized incident beam with vortex, \( m \neq 0 \). Such beam carries \textit{intrinsic orbital angular momentum} (AM) \( \mathbf{L}' = m \mathbf{Z}' \) (in units of \( h \)) per one photon \([21]\). From the Snell’s laws it follows that the AMs of the reflected and refracted beams are given by \([19, 22]\):

\[
\mathbf{L}' = -m \mathbf{Z}', \quad L'_r = \frac{1}{2} \left( \frac{\cos \theta}{\cos \theta'} + \frac{\cos \theta'}{\cos \theta} \right) m \mathbf{Z}' . \tag{9}
\]

As was shown by Fedoseyev \([22]\) for a particular cases of \( p \) and \( s \) polarization, the IF shift of vortex beams consists of two contributions. The first one is a \textit{polarization-independent} shift that comes from the difference in the \( z \)-components of intrinsic AM at the reflection and refraction which is compensated by the transverse shift producing an \textit{extrinsic} AM \([10-14]\): \( \langle Y^{r,t} \rangle_{1} = (L'_r - L'_r)/k \sin \theta \). Taking into account that \( L'_x = L' \cos \theta \), \( L'_z = -L' \cos \theta \), and \( L'_r = L' \cos \theta' \), one obtains \([19, 22]\):

\[
\langle Y^{r} \rangle_1 = 0, \quad \langle Y^{t} \rangle_1 = \frac{m}{2k} \tan \theta(1 - n^{-2}). \tag{10}
\]

The second contribution is essentially \textit{polarization-dependent}. Here we demonstrate that it is directly related to the angular GH shift (7). Indeed, as is shown in \([18]\), the angular shift (7) induces an \textit{imaginary} shift in the Gaussian envelope of the beam: \( X_{r,t} \rightarrow X_{r,t} - i D_{r,t} (K^r_{x})_0/k \) (where \( D_{r,t} = D \) and \( D_{r,t} = D \cos^2 \theta'/\cos^2 \theta \) \([14]\)). It can be readily seen that this imaginary shift produces a \textit{real} shift of the vortex in the \textit{orthogonal} direction:

\[
\left[ \gamma^{r,t} X_{r,t} + i \text{sign}(m) \left( Y^{r,t} - \gamma^{r,t} D_{r,t}(K^r_{x})_0 \right) \right]^{[m]} .
\]

Here the coefficients \( \gamma^{r,t} = -1 \) and \( \gamma^{t} = \cos \theta / \cos \theta' \) account for the deformations of the vortex in the secondary beams: charge flip in the reflected beam and an elliptic deformation of the transmitted beam. As a result, the centers of gravity of the reflected and transmitted vortex beams experience the IF shift

\[
\langle Y^{r} \rangle_2 = \frac{mD}{k} (K^r_{x})_0, \quad \langle Y^{t} \rangle_2 = -\frac{m \cos \theta}{k} \frac{D'}{k} (K^t_{x})_0 . \tag{11}
\]

The net IF shift of an arbitrarily polarized vortex beam is the sum of the contributions (6), (10), and (11):

\[
\langle Y^{r,t} \rangle = \langle Y^{r,t} \rangle_0 + \langle Y^{r,t} \rangle_1 + \langle Y^{r,t} \rangle_2. \tag{12}
\]

This is the first main result that describes the total polarization- and vortex-dependent IF shift.

Similarly to the vortex-induced IF effect (11) associated with the angular GH shift (7), there exists a reciprocal effect of the \textit{vortex-induced GH shift} caused by
the angular IF shift (8). In a manner, similar to the discussion above, we obtain

$$\langle X' \rangle = - m \frac{D}{k} \langle K_{xy}^r \rangle_0, \quad \langle X^t \rangle = m \frac{\cos \theta'}{\cos \theta} \frac{D}{k} \langle K_{xy}^t \rangle_0. \quad (13)$$

This is the second main result of this Letter. It predicts a completely new type of the GH shift, which occurs in the regime of partial reflection, in a sharp contrast to the usual GH effect (4). The vortex-induced GH effect vanishes for $p, s$, and circular polarizations of the incident beam and reaches maximal values for linear polarizations inclined at $\pi/4$ angles: $\chi = \pm 1$.

It can be shown that the angular GH and IF shifts, Eqs. (7) and (8), acquire additional factor $(1 + |m|)$ in the case of incident vortex beam:

$$\langle K_{x,xy}^r \rangle = (1 + |m|) \langle K_{x,xy}^t \rangle_0. \quad (14)$$

The above equations (4)–(8) and (10)–(14) describe all the main shifts of polarized vortex beams with an axially-symmetric intensity profile. One may observe that the angular shifts (7), (8), and (14) fulfill conservation laws for the $x$ and $y$ components of the total intrinsic angular momentum in the system [23]:

$$- Q^r \langle K_x^r \rangle \cos \theta + Q^t \langle K_x^t \rangle \cos \theta' = 0, \quad Q^r \langle K_y^r \rangle + Q^t \langle K_y^t \rangle = 0, \quad (15)$$

whereas linear IF shifts (6), (10)–(12) fulfill conservation law for the $z$ component of the total AM [10,11,13,14,22]:

$$Q^r (J_z^r - \langle Y^r \rangle k \sin \theta) + Q^t (J_z^t - \langle Y^t \rangle k \sin \theta) = J_z^0. \quad (16)$$

Here $J_z^0$ is the $z$-components of the total intrinsic AM of the $\sigma$-th beam, which consist of orbital (vortex) and spin (polarization) contributions:

$$J^\sigma = L^\sigma + S^\sigma, \quad S^\sigma = \sigma^r \hat{Z}^\sigma, \quad (17)$$

where $\sigma^i = \sigma$ and $\sigma^{r,t} = 2 \text{Im}(e_{\parallel}^r e_{\perp}^t)$ are the helicities of the corresponding beams.

To verify our results, we perform numerical simulations of the problem based on the plain-wave decomposition of the incident polarized vortex beam and Fresnel-Snell’s formulas applied to each plane-wave component of the beam spectrum. The numerically calculated coordinates of the centers of gravity of the reflected and transmitted beams in the wave-vector and coordinate space are compared with analytical expressions (6)–(8), (10)–(14) in Fig. 2, which shows an excellent agreement.

In conclusion, we have calculated linear and angular GH and IF shifts arising upon the reflection and transmission of a polarized vortex beam at an interface between two media. We have established a close relation between the spin-induced angular shifts and orthogonal vortex-induced linear shifts, and have predicted a new type of the GH shift for a vortex beam, which occurs in the partial-reflection regime when the GH shift for Gaussian beams vanishes. The value and sign of the vortex-induced shifts can be controlled by the vortex charge.

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