Movement control of a variable mass underwater vehicle based on multiple-modeling approach

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This paper is one of the few studies dealing with modeling and control of a variable mass underwater vehicle with six degrees of freedom. Since, the mass of fuel changes during the operation, both mass and the center of mass of the vehicle change with time. Therefore, dynamic equations are written around the center of buoyancy and these equations are more complicated than the equations of the fixed mass underwater vehicles. Also, fin angles which are means of path-following (control inputs) do not directly appear in the dynamical equations which cause more complexity in the controller design. In this paper, the multiple-modeling approach is used in the controller design and the control signals are achieved by the weighed combination of multiple controllers. Simulation results reveal the effectiveness of the multiple controllers in set point tracking.

Keywords: variable mass underwater vehicle; guidance; control; multiple-models

1. Introduction

Modeling and control of underwater vehicles are extremely difficult tasks. Many interacting factors which are involved in underwater vehicle dynamics can cause oscillatory or unstable operations (Perstro, 1994). Several control solutions for position or velocity control of underwater vehicles have been proposed in the literature (Açakaya & Gören Sümer, 2014; Goheen & Jeffery, 1990; Guo, Chiu, & Huang, 2003; Healey & Lienard, 1993; Kumar, Kumar, Sen, & Dasgupta, 2009; Pisano & Usai, 2004; Subudhi, Mukherjee, & Ghosh, 2013; Sun & Cheah, 2009; Xu, Pandian, Sakagami, & Petry, 2012); however, all of them have concentrated on autonomous underwater vehicles (AUVs). It is worth noting that, unlike the variable mass underwater vehicles (VMUVs), the masses of AUVs are fixed. Trust forces of the AUVs are supplied by electrical motors, which cause restrictions in the maximum speed and acceleration. Also, the dynamic equations of VMUVs are written around the center of mass (CM). Moreover, electrical motors make the sizes of AUVs limited and lead to slowly maneuvering. In order to resolve these restrictions, fuel motors have been used (in the VMUVs), which have variable mass and lead to variations of plant parameters over time (Binazadeh & Yazdanpanah, 2010).

In this paper, a dynamical model of a VMUV (with six degrees of freedom) is described and the structure of the closed-loop control system is proposed. The control system of the VMUV has three channels: depth, roll and yaw. Each of these channels has guidance (outer) and control (inner) loops. Since, the mass of fuel changes during the operation, both mass and the CM of the vehicle also change with time. Therefore, instead of writing the dynamic equations around the CM, these equations are written around the center of buoyancy (CB) which makes more complexity in dynamic equations. Moreover, the considered VMUV has three main characteristics which make the design of the controller a difficult task. The fin angles which are the control inputs of the VMUV do not appear directly in the dynamical equations of the vehicle. These control inputs affect the hydrodynamic coefficients which appear in hydrodynamic forces and moments. These coefficients have complex relations with respect to the system parameters, its attitude and the control inputs (Binazadeh & Yazdanpanah, 2010). Another important point about this vehicle is the interaction between the yaw and roll channels. When the VMUV tries to change its direction and takes yaw angle, it also rolls, simultaneously. Therefore, these two channels are not independent of each other. Additionally, changes of mass and CM with time leads to complicated time-varying motion equations.

Fortunately, the burning rate of the fuel is approximately known and therefore the time variations of the vehicle dynamics are previously known. Thus, it is quite attractive to use an approach in which local “multiple models” are selected at different regions of the operation and the design procedure is carried out based on these models (Baldi, Ioannou, & Mosca, 2012; Du & Hu, 2013; Rosa & Silvestre, in press).

In this paper after suggestion of the structure of the closed-loop system, the controllers of each channel are designed. In the proposed design procedure, the digital
proportional-integral-derivative (PID) controllers are proposed for the inner loops. Multiple-model approach is also used for the controller design of the outer loops. The multiple controllers are designed for each channel and the control signals are achieved by the weighed combination of these multiple controllers. Furthermore, computer simulations show the superior performance of the multiple controllers in comparison with a single controller.

2. The physical description of the VMUV

In this section, the kinematics and dynamic equations of the VMUV are developed using an inertia-frame and body-frame coordinate systems (see Figure 1). Also, the following notation are required for a description of dynamic equations:

\[ \eta_1 = [x \ y \ z]^T; \quad \eta_2 = [\psi \ \theta \ \psi]^T; \]
\[ v_1 = [u \ v \ w]^T; \quad v_2 = [p \ q \ r]^T; \]
\[ r_1 = [X \ Y \ Z]^T; \quad r_2 = [K \ M \ N]^T; \]

where \( \eta_1 \) and \( \eta_2 \) describe the position and orientation of the vehicle with respect to the inertia frame, \( v_1 \) and \( v_2 \) are the translational and rotational velocities of the vehicle with respect to the body frame, and \( r_1 \) and \( r_2 \) indicate the total forces and moments acting on the vehicle with respect to the body frame, respectively (Goheen & Jeffery, 1990).

2.1. Kinematics equations

The kinematics equations of the underwater vehicle can be expressed as follows (Perstro, 1994):

\[
\dot{x} = u \times \cos \psi \cos \theta - v(\sin \psi \cos \varphi - \cos \psi \sin \theta \sin \varphi) + w(\sin \psi \sin \varphi + \cos \psi \sin \theta \cos \varphi),
\]
\[
\dot{y} = u \times \sin \psi \cos \theta + v(\cos \psi \cos \varphi + \sin \psi \sin \theta \sin \varphi) + w(-\cos \psi \sin \varphi + \sin \psi \sin \theta \cos \varphi),
\]
\[
\dot{z} = -u \times \sin \theta + v \times \cos \theta \sin \varphi + w \times \cos \theta \cos \varphi,
\]
\[
\dot{\psi} = p + \sin \varphi \tan \theta \times q + \cos \varphi \tan \theta \times r,
\]
\[
\dot{\theta} = \cos \varphi \times q - \sin \varphi \times r,
\]
\[
\dot{\psi} = \frac{\sin \varphi}{\cos \theta} q + \frac{\cos \varphi}{\cos \theta} r. \tag{2}
\]

2.2. Dynamic equations

The dynamic equations of the VMUV are written around its CB. These equations include the following translational (surge, sway and heave) and rotational (roll, pitch and yaw) equations. In these equations, \( m \) and \( \dot{m} \) are mass and its first derivation (It is assumed that \( \dot{m} \) is negligible).

**Surge or translation along the x-axis:**

\[
m[\ddot{u} + qw - rv + \ddot{x}_G + \ddot{q}z_G - \ddot{r}y_G + p(qy_G + rz_G) - (r^2 + p^2)x_G] + 2\dot{m}[\ddot{x}_G - \ddot{x} + q(z_G - z_e)]
\]
\[ - r(y_G - y_e) = X + T_x. \tag{3}
\]

**Sway or translation along the y-axis:**

\[
m[\ddot{v} + ru - pw + \ddot{y}_G + \ddot{r}x_G - \ddot{p}z_G + q(rz_G + px_G) - (r^2 + p^2)y_G] + 2\dot{m}[\ddot{y}_G - \ddot{y} + r(x_G - x_e) + p(z_G - z_e)] = Y + T_y. \tag{4}
\]

**Heave or translation along the z-axis:**

\[
m[\ddot{w} + pv - qu + \ddot{z}_G + \ddot{p}y_G - \ddot{q}x_G + r(px_G + qy_G) - (p^2 + q^2)x_G] + 2\dot{m}[-\ddot{x}_G + p(y_G - y_e) - q(x_G - x_e)] = Z + T_z. \tag{5}
\]

where \( T_x, T_y, \) and \( T_z \) are components of the trust force which is generated by the engine (\( T_x \) and \( T_z \) can be assumed negligible) and \( X, Y, \) and \( Z \) are external forces. Also, \( x_G, y_G, \) and \( z_G \) are the position of CM (where just \( x_G \) changes with time) and \( x_B, y_B, \) and \( z_B \) are the position of CB with respect to body frame. The second three equations which show rotational motions are as follow:

**Roll or rotation about the x-axis:**

\[
\dot{p}I_{xx} + qr(I_{zz} - I_{yy}) + pl_{xx} + m[y_G(\dot{w} + pv - qu) - z_G(\dot{v} + ru - pw)] - \dot{m}(y_G \dot{z}_e - z_e \dot{y}_e) + \dot{z}_B[m(rx_G - pz_G) + \dot{m}y_G] + \dot{y}_B[m(\dot{p}y_G - qx_G) + \dot{m}z_G] = K + (y_G T_z - z_e T_y). \tag{6}
\]
Pitch or rotation about the y-axis:

\[ \dot{q}_{ly} + pr(I_{xx} - I_{zz}) + qI_{yy} + m[z_G(\dot{u} + qw - rv)] - x_G(\dot{w} + pu - qw) - \dot{m}(x_G\dot{v} - y_G\dot{w}) + \dot{m}x_G = M + (z_GT_y - x_GT_z). \] (7)

Yaw or rotation about the z-axis:

\[ \dot{r}_{Lz} + pq(I_{yy} - I_{xz}) + rI_{zz} + m[x_G(\dot{v} + ru - pw)] - y_G(\dot{u} + qw - rv) - \dot{m}(x_G\dot{v} - y_G\dot{w}) + \dot{m}x_G = N + (x_GT_y - y_GT_z). \] (8)

where \( I_{xx}, I_{yy}, \) and \( I_{zz} \) are components of the mass moment of inertia of the VMUV and \( K, M \) and \( N \) are external moments. The external forces and moments, which appear in Equations (3)–(8), consist of added mass, hydrostatic and hydrodynamic terms which can be expressed as follows (Perstro, 1994):

\[
\begin{align*}
X &= X_{ad} + X_{HS} + X_{HD}, \\
Y &= Y_{ad} + Y_{HS} + Y_{HD}, \\
Z &= Z_{ad} + Z_{HS} + Z_{HD}, \\
K &= K_{ad} + K_{HS} + K_{HD}, \\
M &= M_{ad} + M_{HS} + M_{HD}, \\
N &= N_{ad} + N_{HS} + N_{HD}.
\end{align*}
\] (9)

Hydrostatic forces and moments: These forces and moments, which are the result of weight and buoyancy of the vehicle, can be expressed as,

\[
\begin{align*}
X_{HS} &= -(W - B) \sin \theta, \\
Y_{HS} &= (W - B) \cos \theta \sin \phi, \\
Z_{HS} &= (W - B) \cos \theta \cos \phi, \\
K_{HS} &= -(y_GW - y_BB) \cos \theta \cos \phi - (z_GW - z_BB) \cos \theta \sin \phi, \\
M_{HS} &= -(z_GW - z_BB) \sin \theta - (x_GW - x_BB) \cos \phi \cos \theta, \\
N_{HS} &= -(x_GW - x_BB) \cos \phi \sin \theta - (y_GW - y_BB) \sin \theta,
\end{align*}
\] (10)

where \( W = mg \) and \( B = \rho \nabla V g \) (\( \rho \) is density of the surrounding fluid and \( \nabla \) is total volume displaced by the vehicle).

Added mass inertia forces and moments: Added mass is a measure of the additional inertia created by the water which accelerates with the vehicle. These forces and moments created by added mass may be expressed as:

\[
\begin{align*}
X_{ad} &= X_u \dot{u} + Z_u \dot{w}q + Z_p \dot{q}^2 - Y_v \dot{rv} - Y_i \dot{r}^2, \\
Y_{ad} &= Y_v \dot{v} + Y_i \dot{r} + X_u \dot{u}r - Z_u \dot{wp} - Z_p \dot{pq}, \\
Z_{ad} &= Z_u \dot{w} + Z_p \dot{q} - X_u \dot{u}q + Y_v \dot{up} + Y_i \dot{rp}, \\
K_{ad} &= K_p \dot{p}, \\
M_{ad} &= M_u \dot{w} + M_p \dot{q} - (Z_p - X_p) \dot{w}w - Y_v \dot{up} + (K_p - N_i) \dot{rp} - Z_p \dot{pq}, \\
N_{ad} &= N_i \dot{v} + N_r \dot{r} - (X_p - Y_i) \dot{w}w + Z_p \dot{wp} - (K_p - M_p) \dot{pq} + Y_i \dot{ur}.
\end{align*}
\] (11)

More details about added mass equations and the definition of its parameters could be found in Perstro (1994).

Hydrodynamic forces and moments: The hydrostatic forces and moments are complex nonlinear functions of the state variables and the angle of control surfaces (fin deflections) of the vehicle and may be expressed as follows:

\[
\begin{align*}
X_{HD} &= dp \cdot S_{ref} \cdot C_x, \\
Y_{HD} &= dp \cdot S_{ref} \cdot C_y, \\
Z_{HD} &= dp \cdot S_{ref} \cdot C_z, \\
K_{HD} &= dp \cdot S_{ref} \cdot L_{ref} \cdot C_l, \\
M_{HD} &= dp \cdot S_{ref} \cdot L_{ref} \cdot C_m, \\
N_{HD} &= dp \cdot S_{ref} \cdot L_{ref} \cdot C_n,
\end{align*}
\] (12)

where \( dp \) is the dynamic pressure, \( S_{ref} \) the reference surface and \( L_{ref} \) the reference length. The vector of hydrodynamic coefficients are defined as \( H = [C_x, C_y, C_z, C_l, C_m, C_n] \). These coefficients have a complex relation with respect to system parameters, its attitude and the control variables (angle of fins). This relation is evaluated using either numerical methods such as computational fluid dynamics or semi-experimental softwares such as DATCOM (Blake, 1998). The former is more precise but very time-consuming. Therefore, the latter was used in the simulations of this paper.

### 2.3. Control inputs

Underwater vehicles have control surfaces that are means of path-following (Figure 2). In the considered VMUV, fin angles are control inputs; however, they do not appear in the dynamic equations (3)–(8), directly. The knowledge of the relationship between these control inputs and state variables is of crucial importance. These control inputs affect the hydrodynamic coefficients which appear in hydrodynamic forces and moments.

If horizontal (vertical) fins are deflected in the same direction, the VMUV will turn in pitch (yaw) direction.
It is desirable to express each of these deviations as a function of one deflection, called elevator ($\delta_e = \delta_2 = -\delta_4$) and rudder ($\delta_r = (\delta_3 - \delta_1)/2$) in the pitch and yaw directions, respectively.

If vertical or horizontal fins are deflected in the opposite direction, the VMUV will roll. This turn is expressed as a function of the so-called aileron deflection ($\delta_a = (\delta_3 + \delta_1)/2$). In the considered VMUV, horizontal fins are mechanically coupled and cannot be deflected in the opposite direction; therefore, roll angle will be produced only by vertical fins. Thus, the input vector is defined as $u(t) = [\delta_r, \delta_e, \delta_a]$. Another important point about this vehicle is the interaction between the yaw and roll channels. When the VMUV tries to change its direction and takes yaw angle, it also rolls, simultaneously. Therefore, these two channels are not independent of each other.

3. Guidance and control

The structure of closed-loop control system for movement control can be divided into the areas of guidance and control (Lin, 1991). The function of the control is fulfilled by the inner loop and the guidance function is fulfilled by the outer loop (Figure 3). In what follows, the guidance and control loops of the considered VMUV are designed for three channels: depth, roll, and yaw. Depth channel is independent of two others; however, as mentioned before, roll and yaw channels effect on each other.

3.1. Depth channel

Pitch control is via the horizontal fins and allows the VMUV to change its depth. The control and guidance loops of the depth channel are shown in Figure 4. In order to control the pitch rate, the inner loop uses the feedback of a rate gyro (RG). The depth of the considered VMUV in the water is also measured by a depth meter (DM) and the depth servo converts the command signal to the fin movement.

3.2. Yaw and roll channels

Yaw and roll channels have influence on each other, thus the related control and guidance loops are implemented as shown in Figure 5. The roll and yaw of the considered vehicle are controlled by two vertical fins. Yaw is controlled by the rudder deflection ($\delta_r$) and roll is controlled by the deflection of aileron angle ($\delta_a$). In Figure 5, $r$ and $p$ represent the yaw and roll rates, respectively, which are measured by RGs. The angle $\psi$ denotes the yaw angle which is measured by a directional gyro (DG) and $\phi$ is the roll angle and is measured by a pendulum (PD).

4. The procedure of controllers design

In this section, controllers design for guidance and control loops are performed. In the design procedure, the digital PID controllers are designed for the inner loops and because of time-varying parameters in the VMUV model; the multiple-modeling approach is used in controller design of the outer loops.

4.1. The inner loop controllers

The controllers of the inner loops (Khi, Kri and Kai in Figures 3 and 4) are digital PID controllers. The dynamics of servo motors is considered as a first-order transfer function and the transfer functions of the feedback trajectories are assumed to be unity. The task is to adjust the coefficients of the digital PID controllers in such a way that the error signals between the output and the input command signals for inner loops are minimized. This problem has been solved by Nonlinear Control Design (NCD) blockset (Nonlinear Control Design Blockset User’s Guide 1993–1997).

As seen in Figure 6, the NCD blockset provides a graphical user interface which automatically converts the time-domain constraints into a constrained optimization
problem and then solves the problem using state-of-the-art optimization routines.

After parameter tuning, the following digital PID controllers are obtained for the inner loop controllers:

\[
\begin{align*}
K_{hi} &= -\frac{1320(z - 0.9091)}{z}, \\
K_{ri} &= -\frac{21(z - 0.9987)(z - 0.0341)}{z(z - 1)}, \\
K_{ai} &= \frac{3280(z - 0.9146)}{z}.
\end{align*}
\]

(13)

4.2. The outer loop controllers

In the design of outer loop controllers, two approaches are used. In the first approach, only one digital controller is designed while in the second approach, multiple controllers are designed and control signals in each instance are archived by the weighed combination of these multiple controllers. Finally, these two approaches are compared.

The first approach (single-controllers): in this case, after designing the inner loop controllers, three linear models (between the inputs \( q_c, r_c, p_c \) and the outputs \( z_c, \psi_c, \varphi_c \) in Figures 4 and 5) are identified based on the black-box approach. Then, linear controllers (\( K_h, K_r \) and \( K_a \)) are designed based on these models. The parameters of these controllers are assumed as the initial guess for the NCD blockset and, with this toolbox, the parameters are tuned such that the VMUV has the desired responses. By this method, the controller of the outer loops in Figures 4 and 5 are designed as follows:

\[
\begin{align*}
K_h &= -\frac{6.8(z - 0.9942)(z - 0.068)}{(z - 0.118)(z - 0.288)}, \\
K_r &= \frac{115(z - 0.9649)}{z}, \\
K_a &= \frac{0.3(z + 0.02)(z - 0.5)}{(z - 0.01)(z - 0.1)}.
\end{align*}
\]

(14)

The second approach (multiple-controllers): the second approach in designing the outer loop controllers is based on the multiple-model technique. Consider the following state-space model:

\[
\dot{x} = f(x, u), \quad y = g(x),
\]

(15)

where \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R}^m \) is the control input vector and \( y \in \mathbb{R}^l \) is the output vector. If the range of system operation (\( \Phi \)) is decomposed into \( N \) operating regimes, \( \Phi_1, \ldots, \Phi_N \subset \Phi \) and the structure of local models and local validity functions are extracted for each of these operating regimes; then, the global approximation is an interpolation
\[ \dot{x} = \sum_{i=1}^{N} f_i(x, u_i) \omega_i(\phi), \]
\[ y = \sum_{i=1}^{N} g_i(x) \omega_i(\phi), \]

where \( \omega_i(\phi) = \rho_i(\phi) / \sum_{i=1}^{N} \rho_i(\phi) \) (for \( i = 1, 2, \ldots, N \)) are the smooth interpolation functions with the property \( \sum_{i=1}^{N} \omega_i(\phi) = 1 \) for all \( \phi \in \Phi \) which provide soft transitions between the operating regimes. Also, \( \rho_i \) s are the model validity functions.

In the considered VMUV, the initial fuel mass of the vehicle is 14/3 kg. According to the rate of fuel burning, time duration of system operating is around 14 s. Therefore, as shown in Figure 7, this time interval is partitioned into three parts with appropriate validity functions (\( \rho_1, \rho_2, \rho_3 \)) and three local linear models are identified in each regime, for each channel. Each of these models is valid for a part of time-interval (a regime) and in some time-intervals, they have overlap.

For each linear model, an appropriate controller is designed and the following structure (Figure 8) is substituted for each of the outer loop controllers (Kh, Kr and Ka). In Figure 8, \( D_1(z) \), \( D_2(z) \) and \( D_3(z) \) are proper local controllers, \( \rho_1, \rho_2 \) and \( \rho_3 \) are validity functions and \( \omega_1, \omega_2 \) and \( \omega_3 \) are interpolation functions.

5. Simulation results

Computer simulations are done to show the performance of the designed controllers. Figures 9–11 show the time response of the outputs of three channels to different set points. In these figures the time responses of “single controller” are related to substituting the outer loops controllers with the proposed controllers in Equation (14) and “multiple controllers” are related to substituting the outer loops controllers with a structure presented in Figure 8. Figure 9 demonstrates the time response of the depth channel to the...
set points 6 and 12 m. The multiple controllers are

\[
D_1(z) = \frac{-7z^2 + 7.4354z - 0.47323}{z^2 - 0.408z + 0.033984},
\]

\[
D_2(z) = \frac{-6.7z^2 + 6.9229z - 0.44967}{z^2 - 0.4218z + 0.030486},
\]

\[
D_3(z) = \frac{-7.2z^2 + 7.6809z - 0.54321}{z^2 - 0.508z + 0.040393}.
\]

Figure 10 shows the time response of the yaw channel to the set points 0.1, 0.2 and 0.3 rad where multiple controllers are

\[
D_1(z) = \frac{112z - 108.56}{z},
\]

\[
D_2(z) = \frac{114.89z - 110.693}{z},
\]

\[
D_3(z) = \frac{115.72z - 111.8493}{z}.
\]

Finally, Figure 11 indicates the time response of the roll channel to the set points 0.02, 0.04 and 0.08 rad where multiple controllers are

\[
D_1(z) = \frac{2.365z^2 - 1.10435z - 0.02230}{z^2 - 0.1128z + 0.0015},
\]

\[
D_2(z) = \frac{2.4z^2 - 1.2095z - 0.02369}{z^2 - 0.1142z + 0.0013},
\]

\[
D_3(z) = \frac{2.56z^2 - 1.3106z - 0.02468}{z^2 - 0.11504z + 0.00145}.
\]

Therefore, simulation results show the better performance of the multiple-modeling approach for all channels.

6. Conclusions

In this paper, the problem of movement control was considered for the VMUV. The considered VMUV had three channels; depth, roll and yaw and each of these channels had guidance and control loops. Since, the mass of fuel changed during the operation, dynamic equations were written around the CB, which increased the model complexity. Because of time variations of model parameters, the multiple modeling approach was used in controller design of guidance loops. Simulation results showed the efficiency of multiple-modeling approaches.

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