Light Gluino Mass and Condensate from Properties of $\eta$ and $\eta'$

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Abstract

We investigate whether known properties of the $\eta'$ meson are consistent with its being the Goldstone boson of the spontaneously broken anomaly-free $R$ symmetry required in the light gluino scenario. We fit the masses and $2\gamma$ decays of the $\eta$ and $\eta'$ mesons, and also their production in radiative $J/\psi$ decays. We find that the $\eta - \eta'$ system is well-described in the light gluino scenario, if $m_{\lambda} \simeq (84 - 144)$ MeV and $\langle \bar{\lambda}\lambda \rangle \simeq -(0.15 - 0.36)$ GeV$^3$. These values are in the range expected when the gluino gets its mass entirely from radiative corrections.

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It has been known for a long time that there exists a window for a long-lived light gluino in SUSY phenomenology [1, 2]. Dimension-3 SUSY breaking operators (including gaugino masses) may naturally be highly suppressed in the MSSM [3], with a number of attractive consequences such as eliminating the CP problem of the MSSM [3, 4] and providing the observed dark matter [5]. Gauginos acquire masses through radiative corrections from electroweak and top-stop loops [3, 7]. Estimates of the constrained soft SUSY breaking parameter space lead to the gluino mass range \( m_\lambda \simeq (0.1 - 1) \text{ GeV} \) [3]. Such a light gluino leads to a slower running of the strong coupling constant and might help to resolve the strong coupling constant discrepancy problem (see [8] and refs. therein).

The Lagrangian of supersymmetric QCD after decoupling heavy squark modes consists of the usual QCD plus light gluinos. Hence, among the usual symmetries of QCD there exists an additional chiral symmetry associated with the chiral \( U(1) \) transformation of the gluino field. A particular linear combination of the currents associated with the quark and gluino chiral rotations is free of the gluon anomaly. We will call this anomaly-free symmetry \( R \) invariance, and the corresponding current the \( R \) current. The \( R \) invariance is spontaneously broken by quark and gluino condensates. It was argued in ref. [10] that supersymmetry must be explicitly broken by a gluino mass of order 1 GeV or more, in order to avoid an unobserved Goldstone boson. However it was suggested in ref. [1] that the required pseudogoldstone boson might be identified with the \( \eta' \); a naive estimate for the required gluino condensate was given in ref. [3]. In this scenario the \( 0^{-+} \) state which gets its mass from the anomaly is expected [2] to have mass approximately 1.5 GeV and is naturally identified as the otherwise-mysterious "extra" singlet pseudoscalar at 1410 MeV [11]. Approximately degenerate with it is a spin-1/2 "glueballino" whose detection is discussed in ref. [12].

Here we investigate in greater detail the possibility that the \( \eta' \) is the required pseudogoldstone boson of spontaneously broken \( R \)-invariance. In addition to giving a more accurate determination of the gluino condensate, we examine whether a non-vanishing gluino condensate is consistent with the \( \gamma\gamma \) decay rates of \( \eta \) and \( \eta' \), and with the relative production rates of \( \eta \) and \( \eta' \) in \( J/\psi \) radiative decays. As in the theory without gluinos, the \( \eta, \eta' \) system must be analyzed together, since mixing
between flavor octet and singlet components is essential for understanding their $\gamma\gamma$ widths. A basic tool in our consideration is the QCD sum rule method [13] (reviewed in refs. [14], [15], [16]).

The paper is organized as follows. We begin by fixing the conventions for the anomaly free axial currents and mixing angles. After this, we derive expressions for the singlet and octet pseudoscalar meson decay constants using the sum rule technique. Next we consider the $2\gamma$ decays of $\eta$ and $\eta'$ mesons and derive sum rules for $J/\psi$ decays into $\eta(\eta')\gamma$. Finally, imposing all available experimental restrictions on these processes, we numerically solve our system of equations, leading to estimates for the light gluino mass and condensate.

Let us start with the definition of the anomaly free axial currents

$$ J_{\mu 5}^{(8)} = \frac{1}{\sqrt{6}}(\bar{u}\gamma_\mu \gamma_5 u + \bar{d}\gamma_\mu \gamma_5 d - 2\bar{s}\gamma_\mu \gamma_5 s) \quad (1) $$

$$ J_{\mu 5}^{(0)} = \frac{1}{\sqrt{3}}(\bar{u}\gamma_\mu \gamma_5 u + \bar{d}\gamma_\mu \gamma_5 d + \bar{s}\gamma_\mu \gamma_5 s - \frac{1}{2}\bar{\lambda}\gamma_\mu \gamma_5 \lambda), \quad (2) $$

where the Majorana spinor $\lambda$ denotes the gluino field. The derivatives of these currents play the role of interpolating operators for pure octet and singlet pseudoscalar meson states. The physical $\eta$ and $\eta'$ mesons are defined through the mixing angle $\theta$ for which we choose the standard parametrization $|\eta\rangle = |\eta^8\rangle \cos \theta - |\eta^0\rangle \sin \theta$, $|\eta'\rangle = |\eta^0\rangle \cos \theta + |\eta^8\rangle \sin \theta$. Following ref. [17], define the decay constants $F_0$ and $F_8$ as

$$ \langle 0 | \partial_\mu J_{\mu 5}^{(0)} | \eta' \rangle = F_0 \cos \theta \ m_\eta^2, \quad \langle 0 | \partial_\mu J_{\mu 5}^{(0)} | \eta \rangle = -F_0 \sin \theta \ m_\eta^2, $$

$$ \langle 0 | \partial_\mu J_{\mu 5}^{(8)} | \eta' \rangle = F_8 \sin \theta \ m_\eta^2, \quad \langle 0 | \partial_\mu J_{\mu 5}^{(8)} | \eta \rangle = F_8 \cos \theta \ m_\eta^2. $$

Consider now the correlator of two axial currents

$$ q^\mu A_{\mu \nu} = iq^\mu \int e^{i q x} \langle 0 | T J_{\mu 5}^{(0)}(x) J_{\nu 5}^{(0)}(0) | 0 \rangle d^4 x = -q_\nu \Pi(q^2 = -Q^2). \quad (3) $$

In accordance with the QCD sum rule approach [13] the Borel transforms of the phenomenological and theoretical parts of the correlator are equated, making the standard decomposition

$$ Im \Pi^{phen}(s) = Im \Pi^{poles}(s) + Im \Pi^{pert}(s) \theta(s - s_0), $$

$$ Im \Pi^{theor}(s) = Im \Pi^{pert}(s) + Im \Pi^{cond}(s). $$
The superscripts “poles”, “pert”, and “cond” label the resonance, perturbative and power (condensate) contributions to the correlator. The result is

\[
\int_0^\infty e^{\frac{s}{M^2}} \Im\Pi^{\text{poles}}(s) ds = \int_0^{s_0} e^{\frac{s}{M^2}} \Im\Pi^{\text{pert}}(s) ds + \int_0^\infty e^{\frac{s}{M^2}} \Im\Pi^{\text{cond}}(s) ds. \tag{4}
\]

The conventional parametrization for the pole and the condensate contributions are the following:

\[
\begin{align*}
\Im\Pi^{\text{poles}}(s) &= \pi \sum_{i=1}^\infty c_i \delta(s - m_i^2), \\
\Im\Pi^{\text{cond}}(s) &= \pi \sum_{j=0}^\infty k_j \delta^{(j)}(s)/j!,
\end{align*}
\]

with \(c_i\)’s being pole residues, \(k_j\)’s being gauge invariant condensates and \(\delta^{(j)}\)’s denoting the \(j\)’th derivatives of the Dirac’s delta function. Substituting these expressions into eq. (4) and expanding in inverse powers of the Borel parameter (for \(M^2 >> s_0\)) we get the sum rules in each order of \(1/M^2\). For practical calculation it is enough to keep only the first two equations of this tower. These equations look like

\[
\begin{align*}
\pi(c_1 + c_2) &= \int_0^{s_0} \Im\Pi^{\text{pert}}(s) ds + \pi k_0, \tag{6} \\
\pi(c_1 m_1^2 + c_2 m_2^2) &= \int_0^{s_0} s \Im\Pi^{\text{pert}}(s) ds - \pi k_1. \tag{7}
\end{align*}
\]

It is not difficult to see that these relations are nothing but the QCD finite energy sum rule (FESR) [18], modified by the condensate contributions [19].

Having set up the general framework, let us turn to the calculations for our application. In the case at hand, the one loop calculation for the singlet correlator in eq. (4) leads to the following results\(^*\):

\[
\Im\Pi^{\text{pert}} = \left(\frac{m_s^2}{2\pi} + \frac{m_\lambda^2}{3\pi}\right) + O(\alpha_s m_s^2, \alpha_s m_\lambda^2);
\]

\[
k_0 = -\frac{2m_s \langle \bar{s}s \rangle}{3} - \frac{m_\lambda \langle \bar{\lambda}\lambda \rangle}{6}; \quad k_1 = -\left(\frac{m_s^2}{6} + m_\lambda^2\right)\frac{\alpha_s}{\pi} G_{\mu\nu}^2.
\]

The constants \(c_1, c_2, m_1\) and \(m_2\) entering the phenomenological part of the sum rules are: \(c_1 = F_0^2 m_\eta^2 \sin^2\theta, \quad m_1 = m_\eta^2, \quad c_2 = F_0^2 m_\eta^2 \cos^2\theta, \quad m_2 = m_\eta^2\). Substituting all these relations into eqs. (6) and (7), determining then \(s_0\) from eq. (4), and plugging

\(^*\)We hereafter neglect the \(u\) and \(d\) quark masses.
back its value into eq. (4), we arrive at the following expression for the singlet axial constant:

$$F_0^2 = \left( \frac{m_s^2}{2\pi^2} + \frac{m_\lambda^2}{3\pi^2} \right) \frac{(1 + 0.1 \tan^2\theta)(1 + \tan^2\theta)}{(1 + 0.33 \tan^2\theta)^2} \times \left( 1 + \sqrt{1 - \frac{2m_s\langle\bar{s}s\rangle/3 + m_\lambda\langle\bar{\lambda}\lambda\rangle/6}{m_{\eta'}^2\left( \frac{m_s^2}{6\pi^2} + \frac{m_\lambda^2}{6\pi^2} \right)} (1 + 0.1 \tan^2\theta) - g} \right) - \left( \frac{2m_s\langle\bar{s}s\rangle}{3m_{\eta'}^2} + \frac{m_\lambda\langle\bar{\lambda}\lambda\rangle}{6m_{\eta'}^2} \right) \frac{(1 + \tan^2\theta)}{(1 + 0.33 \tan^2\theta)^2},$$

(8)

where

$$g = \frac{(m_s^2 + m_\lambda^2)}{m_{\eta'}^2\left( \frac{m_s^2}{6\pi^2} + \frac{m_\lambda^2}{6\pi^2} \right)} \frac{(1 + 0.33 \tan^2\theta)^2}{(1 + 0.1 \tan^2\theta)^2}. $$

Using the results of the above calculation, it is easy to obtain the analogous sum rule for the decay constant $F_8$. One just needs to drop the gluino contribution and choose the proper normalization for the corresponding axial current. The result is:

$$F_8^2 = \frac{m_s^2}{\pi^2} \frac{(1 + 9.4 \tan^2\theta)(1 + \tan^2\theta)}{(1 + 3.1 \tan^2\theta)^2} \times \left( 1 + \sqrt{1 - \frac{8\pi^2\langle\bar{s}s\rangle (1 + 3.1 \tan^2\theta)}{3m_{\eta'}^2m_s^2 (1 + 9.4 \tan^2\theta) - r} \left( \frac{4m_s\langle\bar{s}s\rangle}{3m_{\eta'}^2} \right) (1 + \tan^2\theta)} \right),$$

(9)

where

$$r = \frac{2\pi^2}{3} \frac{\langle\alpha_s G^2_{\mu\nu}\rangle}{m_{\eta'}^4} \frac{(1 + 3.1 \tan^2\theta)^2}{(1 + 9.4 \tan^2\theta)^2}. $$

Generally speaking there are five unknowns in these equations. These are the decay constants $F_0$ and $F_8$, mixing angle $\theta$, gluino mass $m_\lambda$ and the gluino condensate $\langle\bar{\lambda}\lambda\rangle$.

Our strategy hereafter will be to impose the restrictions on these unknowns coming from the $2\gamma$ decays of the $\eta$ and $\eta'$ mesons and from the radiative decays of the $J/\psi$.

Let us turn first to the $2\gamma$ decays of the $\eta$ and $\eta'$ mesons. These decays are governed by the electromagnetic axial anomaly. Since gluinos do not interact with photons in leading order, the gluino part of the singlet current just drops out from the calculation of the corresponding triangle graphs. Therefore the formal expressions for

\footnote{Though $F_8$ is known from chiral perturbation theory \cite{21}, we include it in the list of unknowns to provide a consistency check of our calculations.}
the decay widths are the same as in conventional QCD. However, the interpolating current for the $\eta'$ meson is now free of the gluon anomaly and one need not worry about modification of PCAC as is necessary in QCD [22]. (Because this current is already modified by subtracting the proper amount of gluinos). Taking into account mixings, one obtains

$$\frac{\Gamma(\eta \rightarrow 2\gamma)}{\Gamma(\pi \rightarrow 2\gamma)} = 2\left(\frac{m_\eta^3}{m_\pi^3}\right)\left(\frac{F_\pi \cos \theta}{F_0 \sqrt{6}} - \frac{2F_\pi \sin \theta}{F_0 \sqrt{3}}\right)^2$$

(10)

$$\frac{\Gamma(\eta' \rightarrow 2\gamma)}{\Gamma(\pi \rightarrow 2\gamma)} = 2\left(\frac{m_{\eta'}^3}{m_\pi^3}\right)\left(\frac{2F_\pi \cos \theta}{F_0 \sqrt{3}} + \frac{F_\pi \sin \theta}{F_8 \sqrt{6}}\right)^2.$$  

(11)

The available experimental data for these decay rates are [23]: $\Gamma(\eta' \rightarrow 2\gamma) = (4.26 \pm 0.62)$ keV, $\Gamma(\eta \rightarrow 2\gamma) = (0.51 \pm 0.09)$ keV, $\Gamma(\pi \rightarrow 2\gamma) = (7.74 \pm 0.58)$ eV.

Eqs. (10) and (11) are derived using PCAC and soft meson technique. The latter is a dubious approximation for the $\eta'$ meson‡. To obtain some estimate of the effect of the extrapolation from $m_{\eta'} = 0$ to $m_{\eta'} = 958$ MeV, we apply the interpolation technique of ref. [24]. The result is that the $F_0$’s appearing in eq. (11) are multiplied by the factor $(1 - (m_{\eta'}^2/16\pi^2F_0^2))$. Results of both estimates will be reported below.

Now let us turn to the radiative decays of vector quarkonium. In particular we will concentrate on $J/\psi \rightarrow \eta(\eta')\gamma$. The basic properties of these processes were worked out using the sum rule technique in refs. [25], [26]. To understand the mechanism which dominates the decay, it is relevant to look at the quantum numbers of the particles involved: $J/\psi(1^{--}) \rightarrow PS(0^{-+}) \gamma(1P-)$. In general the spatial parity of the emitted photon may be negative (for “electric” transition) or positive (for “magnetic” transition - see for example [27]). From the quantum numbers of the quarkonium and the pseudoscalar meson, the emitted photon in the present case must have positive spatial parity and consequently should be the “magnetic” type. Keeping this fact in mind, the following three step picture for these decays emerges naturally: the heavy $\bar{c}c$ quark system being in a vector state emits the “magnetic” photon and turns into a pseudoscalar state of two $c$ quarks (a virtual $\eta_c$ meson). Then this virtual $\eta_c$ emits two gluons in a pseudoscalar state which hadronize into the final pseudoscalar meson ($\eta$ or $\eta'$). Using the operator $i\bar{c}\gamma_5c$ for the interpolating field of the $\eta_c$ meson, one

‡ We are grateful to H. Georgi for bringing this point to our attention.
obtains the following ratio:

\[
\frac{\Gamma(J/\psi \rightarrow \eta'\gamma)}{\Gamma(J/\psi \rightarrow \eta\gamma)} = \frac{|\langle 0| i\bar{c}\gamma_5 c|\eta'\rangle|}{|\langle 0| i\bar{c}\gamma_5 c|\eta\rangle|} \left( \frac{m^2_{J/\psi} - m^2_{\eta'}}{m^2_{J/\psi} - m^2_{\eta}} \right)^{\frac{3}{2}}. 
\]  

(12)

It is convenient to introduce the notation \(\langle 0| i\bar{c}\gamma_5 c|\eta'\rangle \equiv \alpha^2_s a_{\eta'}/\pi^2\) and \(\langle 0| i\bar{c}\gamma_5 c|\eta\rangle \equiv \alpha^2_s a_{\eta}/\pi^2\). Eq. (12) and the experimental data on these decays [23] give

\[|a_{\eta'}| \approx (2.48 \pm 0.16)|a_{\eta}|.\]  

(13)

The next step is to relate \(a_{\eta}\) and \(a_{\eta'}\) to properties of the \(\eta\) and \(\eta'\) using sum rules. Therefore consider the two-point correlator

\[P(q^2) = i \int e^{ixq} \langle 0| Ti\bar{c}\gamma_5 c(x)\partial_\mu J_{\mu 5}^0(0)|0\rangle d^4x.\]  

(14)

The calculational procedure is the same one used above for the determination of \(F_0\) and \(F_8\). The phenomenological part is saturated by the physical \(\eta\) and \(\eta'\) meson states. The leading perturbative part is determined by the three-loop diagram shown in fig.1(a). This diagram has been calculated recently in ref. [28], using an expansion in inverse powers of heavy quark mass [29]. In our case this is the leading approximation in \(1/m_c\). The leading contribution to the nonperturbative part arises from the diagrams of fig. 1(b,c,d). We have used the heavy quark mass expansion and dimensional regularization, with the 't Hooft-Veltman prescription [30] for the \(\gamma_5\) matrix in \(D = 4 - 2\epsilon\) dimensional space-time. The calculational technique is based on the integration by parts method of ref. [31].

The results corresponding to the three diagrams of figs. 1b, 1c and 1d are

\[\text{Im } P_b (s + i\varepsilon) = 0; \quad \text{Im } P_c (s + i\varepsilon) = \text{Im } P_d (s + i\varepsilon) = \]

\[\frac{1}{2\sqrt{3}} \left( \frac{\alpha_s}{\pi} \right)^2 \left( \frac{m_s \langle \bar{s}s \rangle}{3m_c} - \frac{9}{24} \frac{m_\lambda \langle \bar{\lambda}\lambda \rangle}{m_c} \right) \epsilon (2G(1,1) - G(2,1)),\]

where \(G(1,1) = 1/\epsilon + ln4\pi - \gamma + 2 + o(\epsilon)\) and \(G(2,1) = -1/\epsilon - ln4\pi + \gamma + o(\epsilon)\) are the \(G\)-functions introduced in ref. [31], and \(\gamma \approx 0.5772\) is the Euler constant. Using these results for the theoretical part, the sum rules are

\[F_0(a_{\eta'}m^2_{\eta'}\cos\theta - a_{\eta}m^2_{\eta}\sin\theta) = \frac{2}{m_c} \frac{(m^2_s - 3m^2_\lambda)}{m_c^{\frac{3}{2}}} \cdot \frac{3}{8\pi^2} \int_0^{s_0} f(s) ds - \]

\[\frac{1}{\sqrt{3}} \left( - \frac{m_s \langle \bar{s}s \rangle}{m_c} + \frac{9}{8} \frac{m_\lambda \langle \bar{\lambda}\lambda \rangle}{m_c} \right) s_0,\]  

(15)
Figure 1: The double plain lines denote the $c$ quark propagators and the single plain lines denote the light quark or gluino propagators.

$$F_0(a_{\eta'}m_{\eta'}^4\cos\theta - a_{\eta}m_{\eta}^4\sin\theta) = \frac{2(m_s^2 - 3m_{\lambda}^2)}{m_c\sqrt{3}} \frac{3}{8\pi^2} \int_0^{s_0} s f(s) ds - \frac{1}{\sqrt{3}} \left( - \frac{m_s\langle \bar{s}s \rangle}{m_c} + \frac{9}{8} \frac{m_{\lambda}\langle \bar{\lambda}\lambda \rangle}{m_c} \right) \frac{s_0^2}{2},$$

(16)

where the function $f(s) = 4s - sln \frac{s}{m_s^2} + \frac{s^2}{m_c^2}(\frac{61}{324} - \frac{7}{108}ln \frac{s}{m_s^2})$ appears as the imaginary part of diagram (1a) \[28\].

In analogy with the singlet case one can write down the same sum rules for the correlator with the nonsinglet current. Taking the proper normalization for this last we get

$$F_8(a_{\eta}m_{\eta}^2\cos\theta + a_{\eta'}m_{\eta'}^2\sin\theta) = \frac{-4m_s^2}{m_c\sqrt{6}} \frac{3}{8\pi^2} \int_0^{s_0^{(8)}} f(s) ds - \frac{1}{\sqrt{6}} \frac{2m_s\langle \bar{s}s \rangle}{m_c} s_0^{(8)},$$

(17)

$$F_8(a_{\eta}m_{\eta}^4\cos\theta + a_{\eta'}m_{\eta'}^4\sin\theta) = \frac{-4m_s^2}{m_c\sqrt{6}} \frac{3}{8\pi^2} \int_0^{s_0^{(8)}} s f(s) ds - \frac{1}{\sqrt{6}} \frac{2m_s\langle \bar{s}s \rangle}{m_c} \frac{s_0^{(8)}2}{2}.$$  

(18)

We now have enough equations to determine the unknown constants. In fact, we have nine equations \((8,11), (13), (15-18)\) and nine unknowns \(\theta, F_0, F_8, m_{\lambda}, \langle \bar{\lambda}\lambda \rangle, a_{\eta}, \)
\(a_{\eta'}, s_0\) and \(s_0^{(8)}\) which depend on \(m_s, \langle \bar{s}s \rangle, \langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \rangle\). For the \(s\) quark mass, recent lattice calculation gives \(m_s(2\text{ GeV}) = (141 \pm 17)\text{ MeV}\) \cite{33}; QCD sum rules give \(m_s(1\text{ GeV}) = (189 \pm 32)\text{ MeV}\) \cite{34} and \(m_s(1\text{ GeV}) = (171 \pm 15)\text{ MeV}\) \cite{33}. In our calculations we take the average of the above three results \(m_s(1\text{ GeV}) = (167 \pm 13)\text{ MeV}\). We take the \(s\) quark condensate from ref. \cite{20} (see also the recent paper \cite{36}) \(\langle \bar{s}s \rangle = (0.7 \pm 0.2)\langle \bar{u}u \rangle = -(0.011 \pm 0.003)\text{ GeV}^3\). There are a number of estimates for the gluon condensate in the literature (see refs. \cite{37},\cite{13}). We take the world average value of these calculations \(\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^2 \rangle = (2.5 \pm 0.9)\times 10^{-2}\text{ GeV}^4\).

Our procedure is this. First we use eq. (10) to find \(F_0\) (or \(F_0(1 - (m_{\eta'}^2/16\pi^2 F_0^2))\)) using the interpolation based on the Brodsky-Lepage analysis) and substitute it into eq. (11). After this we have two equations, (9) and (11), for \(F_8\) and \(\theta\). Solving this system numerically we find \(F_8 \simeq (130 \pm 20(\text{stat.}))\text{ MeV}\) and \(\theta \simeq -(23.5^0 \pm 2.7^0(\text{stat.}))\). The value obtained here for \(F_8\) agrees within error bars with the chiral perturbation theory estimate \(F_8 \simeq 1.25 F_\pi \simeq 116\text{ MeV}\) \cite{21}. Our value for \(\theta\) is also in agreement with previous calculations (see for example ref. \cite{22}). This gives confidence in these values for \(m_s\) and \(\langle \bar{s}s \rangle\), and in the present method combining PCAC and sum rule techniques for calculating the decay constants. We find \(F_0 \simeq (93 \pm 17(\text{stat.}))\text{ MeV}\) with the conventional soft meson extrapolation for the \(\eta'\), and \(F_0 \simeq (136 \pm 13(\text{stat.}))\text{ MeV}\) with the alternative procedure described above.

Next we make use of the data and SR’s for \(J/\psi \to \eta\gamma\) and \(J/\psi \to \eta'\gamma\), namely eqs. (13,17,18) to determine \(a_\eta\), \(a_{\eta'}\) and \(s_0^{(8)}\) numerically using Mathematica. The procedure is trivial but cumbersome and yields: \(|a_{\eta'}| \simeq (202.1 \pm 45.3)\times 10^{-3}\text{GeV}^2\), \(|a_\eta| \simeq (81.6 \pm 18.1)\times 10^{-3}\text{GeV}^2\) and \(s_0^{(8)} \simeq (1.7 \pm 0.2)\text{GeV}^2\). Note that we took from experiment the ratio \(|a_\eta|/|a_{\eta'}|\), but have not obtained their individual values. Thus if we had a reliable independent way of estimating the matrix element \(\langle 0| i\bar{c}\gamma_5 c|\eta_c \rangle \equiv F_c\) appearing in the amplitude for \(J/\psi \to \eta\gamma\) and \(J/\psi \to \eta'\gamma\), we would have a prediction for the overall magnitude of these rates. Unfortunately, \(F_c\) cannot be determined independently with sufficient accuracy to give us a useful test\(^5\).

The results above allow us to turn to the calculation of the basic quantities, the

\(^5\text{M. Shifman, private communication.}\)
gluino mass and condensate. Substituting the quantities obtained earlier into eqs. (8, 15, 16), we have a system of three equations with the last three unknowns:

\[ m_\lambda, \langle \bar{\lambda}\lambda \rangle \text{ and } s_0. \]

The numerical solution of this set provides us with the following results using \( F_0 \) extracted with the conventional soft meson approximation:

\[ s_0 \approx (1.8 \pm 0.2) \text{ GeV}^2, \quad m_\lambda \approx (120 \pm 24) \text{ MeV}, \quad \langle \bar{\lambda}\lambda \rangle \approx -(0.22 \pm 0.07) \text{ GeV}^3. \]

Using \( F_0 \) obtained from the alternative extrapolation in \( m_\eta' \) gives:

\[ s_0 \approx (1.9 \pm 0.2) \text{ GeV}^2, \quad m_\lambda \approx (102 \pm 18) \text{ MeV}, \quad \langle \bar{\lambda}\lambda \rangle \approx -(0.31 \pm 0.05) \text{ GeV}^3. \]

From this range of results we arrive at the final estimate

\[ m_\lambda \approx (84 - 144) \text{ MeV}, \quad \langle \bar{\lambda}\lambda \rangle \approx -(0.15 - 0.36) \text{ GeV}^3. \]

Before concluding, we note that in conventional QCD the \( \eta' \) is identified with the particle which acquires mass due to the axial anomaly. The Witten-Veneziano (WV) formula [41] relates the vacuum topological susceptibility of pure Yang-Mills theory to the decay constant and mass of the \( \eta' \) meson in the large \( N_c \) limit. (For a recent discussion see ref. [42]). In the limit when at least one quark is massless this relation looks like

\[
\left( \frac{g^2}{16\pi^2} \right)^2 \int \langle 0 | T G^a(x) \tilde{G}^a(0) | 0 \rangle d^4 x |_{N_c \to \infty} \] \[= \frac{1}{3} F_0^2 m_\eta'^2 |_{N_c \to \infty}. \quad (21)
\]

In the scenario we have considered here, the \( \eta' \) is a pseudogoldstone boson and a different state acquires its mass from the anomaly. (In supersymmetric YM theory this state is a pure gluino-gluino bound state [43].) It is possible to derive an analogous formula to eq. (21) in the theory with a light gluino. The only difference is that the left hand side should be calculated in supersymmetric YM theory. The right hand side is unchanged, and still depends on the decay constant and mass of the physical \( \eta' \) meson which now contains a light gluino. Unfortunately, the accuracy of the determination of the vacuum topological susceptibility (for a review see ref. [42]) is not sufficient at present to decide which of the two scenarios is realized. A detailed discussion of these and related topics will be presented elsewhere.

To summarize, in this paper we have undertaken an attempt to identify the \( \eta' \) meson with the Goldstone boson of the spontaneously broken, anomaly-free \( R \) symmetry arising in a theory with a light gluino. We have calculated the corresponding
decay constant $F_0$ using the QCD sum rule method. As a consistency check, the analogous calculation for the octet decay constant $F_8$ is performed. The result is in good agreement with what was already known before. The experimental values for the decay rates of $\eta'(\eta) \to 2\gamma$ and $J/\psi \to \eta'(\eta)\gamma$ impose certain restrictions on the values of gluino mass and condensate. Using the whole system of equations coming from sum rules and from quantities determined by experiment, we determined the gluino mass and condensate. The value of the gluino condensate indicates that the scale of the breakdown of $R$-symmetry is about 2.5 times larger than that of chiral symmetry. In view of the factor $9/4$ larger value of the adjoint Casimir compared to the fundamental one, this seems to be of the right magnitude. Note also that the gluino mass value we have found, $m_\lambda \sim 80 - 160$ MeV, is in the range predicted in ref. [3]. If the properties of the $\eta$, $\eta'$ mesons had been incompatible with the light gluino scenario, our procedure would have given nonsensical rather than reasonable values of gluino mass and condensate. Thus our result shows a surprising consistency between the properties of the ground-state pseudoscalar mesons and the light gluino scenario, and favors a gluino mass of order 100 MeV.

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References

[1] G.R. Farrar, Phys. Rev. Lett. 53(1984)1029.

[2] G.R. Farrar, Phys. Rev. D51(1995)3904.

[3] G.R. Farrar, “Phenomenology of Light Gauginos”, RU-95-17, hep-ph/9504294; “Phenomenology of Light Gauginos II. Experimental Signatures”, RU-95-26, hep-ph/9508292.

[4] G.R. Farrar, “SUSY Breaking and Light Gauginos, SUSY95”, RU-95-73. SUSY95, Paris, May 1995.

[5] G.R. Farrar, E.W. Kolb, Phys. Rev. D53(1996)1900.
[6] R. Barbieri, L. Girardello, A. Masiero, Phys. Lett. B127(1983)429.

[7] G.R. Farrar, A. Masiero, “Radiative Gaugino Masses”, hep-ph/9410401.

[8] M.A. Shifman, Int. Journ. Mod. Phys. A11(1996)3195.

[9] I. Affleck, M. Dine, N. Seiberg, Nucl. Phys. B241(1984)493.

[10] M.I. Eides, M.I. Vysotsky, Phys. Lett. 124B(1983)83;
    A.V. Smilga, M.I. Vysotsky, Phys. Lett. 125B(1983)227.

[11] F.E. Close, G.R. Farrar, Zh. Li, ”Determining the Gluonic Content of Isoscalar
    Mesons”, RAL-96-052, RU-96-35, hep-ph/9610280.

[12] G.R. Farrar, Phys. Rev. Lett. 76(1996)4111.

[13] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B147(1979)385,448.

[14] L.J. Reinders, H.R. Rubinstein, S. Yazaki, Phys. Rep. 127(1985)1.

[15] S. Narison, “QCD Spectral Sum Rules”, World Scientific, 1989.

[16] “Vacuum Structure and QCD Sum Rules”, ed. M. Shifman, North-Holland, 1992.

[17] R. Akhoury, J.-M. Frére, Phys. Lett. B220(1989)258.

[18] A.A. Logunov, L.D. Soloviev, A.N. Tavkhelidze, Phys. Lett. 24(1967)181;
    K.G. Chetyrkin, N.V. Krasnikov, A.N. Tavkhelidze, Phys. lett. 76B(1978)83;
    K.G. Chetyrkin, N.V. Krasnikov, Nucl. Phys. B119(1977)174.

[19] A .L. Kataev, N.V. Krasnikov, A.A. Pivovarov, Phys. Lett. 123B(1982)93;
    N.V. Krasnikov, A.A. Pivovarov, N.N. Tavkhelidze, Z. Phys. C19(1983)301.

[20] A.A. Ovchinnikov, A.A. Pivovarov, Phys. Lett. 163B(1985)231.

[21] J.F. Donoghue, B.R. Holstein, Y.-C. R. Lin, Phys. Rev. Lett. 55(1985)2766.

[22] G.M. Shore, G. Veneziano, Nucl. Phys. B381(1992)3.

[23] Particle Data Group, Phys. Rev. D54(1996)1.
[24] S.J. Brodsky, G.P. Lepage, preprint SLAC-PUB-2587, 1980.

[25] M.B. Voloshin, V.I. Zakharov, Phys. Rev. Lett. 45(1980)688.

[26] V.A. Novikov, M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B165(1980)55.

[27] V.B. Berestetski, E.M. Lifshitz, L.P. Pitaevski, “Quantum Electrodynamics”, Moscow “Nauka”, (1989).

[28] K.G. Chetyrkin, A. Kwiatkowski, Nucl. Phys. B461(1996)3.

[29] G.B. Pivovarov, F.V. Tkachov, Preprint INR P-0370 (1984), Moscow;
    K.G. Chetyrkin, V.A. Smirnov, Preprint INR P-518 (1987), Moscow;
    S.G. Gorishny, Nucl. Phys. B319(1989)633;
    F.V. Tkachov, Int. Journ. Mod. Phys. A8(1993)2241.

[30] G. ’t Hooft, M. Veltman, Nucl. Phys. B44(1972)189.

[31] K.G. Chetyrkin, A.L. Kataev, F.V. Tkachov, Nucl. Phys. B174(1980)345;
    K.G. Chetyrkin, F.V. Tkachov, Nucl. Phys. B192(1981)159.

[32] F.J. Gilman, R. Kauffman, Phys. Rev. D36(1987)2761.

[33] C.R. Allton et al., Nucl. Phys. B431(1994)667.

[34] M. Jamin, M. Münz, Z. Phys. C66(1995)633.

[35] K. Chetyrkin, C. Dominguez, D. Pirjol, K. Schilcher, Phys. Rev. D51(1995)5090.

[36] S. Narison, Preprint PM 95/06; hep-ph/9504333.

[37] L.J. Reinders, H. Rubinstein, S. Yazaki, Nucl. Phys. B186(1981)109; J.S. Bell, R.A. Bertlmann, Nucl. Phys. B177(1981)218; K.J. Miller, M.G. Olsson, Phys. Rev. D25(1982)1247,1253; G. Launer, S. Narison, R. Tarrach, Z. Phys. C26(1984)433; R.A. Bertlmann et al., Z. Phys. C39(1988)231; C.A. Dominguez, J. Sola, Z. Phys. C40(1988)63; S. Narison, Preprint PM 95/51; hep-ph/9512348.
[38] S. Narison, Nucl. Phys. proceedings supplement, B40(1995)47.

[39] Th. Ohl, “feynMF: Drawing Feynman Diagrams with LATEX and META-FONT”, (1995).

[40] G. Veneziano, S. Yankielowicz, Phys. Lett. 113B(1982)231.

[41] E. Witten, Nucl. Phys. B156(1979)269;
    G. Veneziano, Nucl. Phys. B159(1979)213.

[42] T. Schäfer, E.V. Shuryak, "Instantons in QCD", hep-ph/9610451.