Helical relativistic electron beam and THz radiation

S. Son

18 Caleb Lane, Princeton, NJ 08540

(18) Benjamin Rush Lane, Princeton, NJ 08540

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A THz laser generation utilizing a helical relativistic electron beam propagating through a strong magnetic field is discussed. The initial amplification rate in this scheme is much stronger than that in the conventional free electron laser. A magnetic field of the order of Tesla can yield a radiation in the range of 0.5 to 3 THz, corresponding to the total energy of mJ and the duration of tens of pico-second, or the temporal power of the order of GW.

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A THz electromagnetic (E&M) wave has a range of practical applications [1][4]. In particular, the light wave of the frequency of 1 to 10 THz is under increased attention [3][4]. Around the frequency range of 100 GHz, there exist appropriate technologies such as gyrotron [7–9]. However, these technologies cannot be extended to the range above a few hundred GHz, due to the well-known scaling problem [10]. Other technologies such as the quantum cascade laser [11, 12] and the free electron laser [13] have their own limitations in generating an intense E&M wave. There have been preliminary proposals for generating a THz radiation based on the recent advances in the intense visible laser [14–16], in the context of the inertial confinement fusion [17–19].

In this paper, we propose a scheme to generate a THz radiation, where the energy is extracted from the perpendicular kinetic energy of an relativistic electron beam, in the presence of a strong magnetic field. In this scheme, a relativistic electron beam gets launched to a slightly skewed direction with respect to the magnetic field. The electrons gyrate around the magnetic field and the perpendicular velocity of the electron exhibits a periodic structure (Fig 1). When a certain resonance condition is satisfied, a specific E&M THz wave becomes amplified. In particular, the rate at which the electron energy is extracted is proportional to the electric field strength, so that the terms of $v_x^0$ and $v_y^0$ in the linearized first order

\[ m_e \frac{d\gamma v}{dt} = \frac{v}{c} \times B, \]

where $m_e$ is the electron mass, $v$ is the electron velocity,

\[ \gamma_0^{-2} = 1 - \left( v_0^2 + v_y^2 \right)/c^2 \]

is the relativistic factor, $v_0$ is the parallel (perpendicular) velocity relative to the $z$-axis, and the magnetic field is given as $B = B_0 \hat{z}$. The solution is $v_x^0(t) = v_{0x}$, $v_y^0(t) = v_p \cos(\omega_{ce} t + \phi_0)$, and $v_y^0(t) = -v_p \sin(\omega_{ce} t + \phi_0)$, where $\omega_{ce} = eB_0/\gamma_0 m_e c$. Consider a linearly-polarized E&M wave propagating along the $z$-direction so that $E_x(z, t) = E_1 \cos(kz - ckt)$, $E_y = E_z = 0$, $B_y(z, t) = E_1 \cos(kz - ckt)$, and $B_z = B_0 \hat{z}$. The perturbed motion of the electron is

\[ m_e \left( \frac{d\gamma^0 v^{(1)}}{dt} + \frac{d\gamma v^{(0)}}{dt} \right) = -e \frac{v}{c} \times B_0 \]

\[ = -e \left[ E_1 + \frac{v^{(0)}}{c} \times B_1 \right], \]

where $E_1 = E_1 \cos(kz - ckt) \hat{x}$, and $B_1 = E_1 \cos(kz - ckt) \hat{y}$ is the magnetic field of the E&M wave. Consider the first case when $v_x^0 \gg v_y^0$ and $v_z^0 \gg v_y^0$ so that

\[ v_x^0 \gg v_y^0 \]
Consider the time slice at \( t = 0 \), where the helical structure of the electron velocity is given as in Fig. 1:

\[
\begin{align*}
    v_{0x}(t = 0, z) &= v_p \cos(k_h z)
    \\
    v_{0y}(t = 0, z) &= -v_p \sin(k_h z)
    \\
    v_{0z}(t = 0, z) &= v_{0z},
\end{align*}
\]

where \( k_h = \omega_{ce}/v_{0z} \) is the helix wave vector. This helical structure, formed by the electron gun, has zero phase velocity in the laboratory frame or is a static wave. An electron initially \(( t = 0 )\) located at \( z = z_0 \) evolves as

\[
\begin{align*}
    v_{0x}(t) &= v_p \cos(k_h z_0 + \omega_{ce} t)
    \\
    v_{0y}(t) &= -v_p \sin(k_h z_0 + \omega_{ce} t)
    \\
    v_{0z}(t) &= v_{0z}.
\end{align*}
\]

Since \( E_1(z, t) = E_1 \cos(kz - ckt) \), the ensemble average over the electrons, \( \langle EV \rangle = e \langle E_z(v_{0x} + iv_{0y}) \rangle \), is given as

\[
\langle EV \rangle = \int E_1 v_p \cos(kv_{0z} + ckt + \omega_{ce} t - k_h z)(z) \, dz_0.
\]

The phase angle \( \phi_0(z) \) is given as \( \phi_0(z) = (k_h + k)z \). While the ensemble average \( \langle \rangle \) over the entire beam does cancel out, it does not locally for a fixed value of \( z \). The electrons of the same phase, located in the range \( z_0 - \delta z < z < z_0 + \delta z \) where \( \delta = \pi/2(k + k_H) \), contribute coherently to \( \langle EV \rangle \) so that the local E&M wave is amplified or damped by the these coherently phased electrons. The E&M wave that initially gets amplified by the coherent electrons gets damped by different but coherently phased electrons as it propagates. The frequency at which the E&M wave experiences the change between the amplification and the damping is estimated to be \( \Omega \cong 1/\delta t \), where \( \delta t(c - v_0) = \delta z \). Since \( (c - v_0)k = \omega_{ce} \), \( \delta t \) can be estimate as \( 1/\omega_{ce} \), so that \( \Omega \cong \omega_{ce} \). This local amplification is in contrast with the electrons with random phases for fixed \( z = z_0 \).

The above argument suggests that there exists a local amplification mechanism for the helical plasma, that could be used as a THz generation. Denoting the relativistic factors \( \gamma_m = (1 - v_0^2/c^2)^{-1/2} \) and \( \gamma = (1 - v^2/c^2)^{-1/2} \), the frequency of the amplified wave can be driven from the resonance condition \( k(c - v_0) = \omega_{ce} \) as \( 2\gamma_m^2 \omega_{ce} \). Consider the case \( \gamma_m = 7 \), \( \gamma = 10 \) and \( B_0 = 1 \) T, which correspond to \( 2\gamma_m^2 \omega_{ce} = 300 \) GHz. Consider another when \( \gamma_m = 30 \), \( \gamma = 40 \) and \( B_0 = 1 \) T, \( 2\gamma_m^2 \omega_{ce} = 1.4 \) THz.

Let us estimate the E&M wave growth rate for the amplification. For simplicity, let us use the reference frame
where we move with the electron beam with the same velocity in the z-direction. Assume that $\gamma_m > 1$, $v_{0zm} \ll v_{0xm}$ and $v_{0zm} \ll v_{0ym}$. If $v_{0x}$ ($v_{0y}$) is the perpendicular velocity in the laboratory frame, $v_{0xm} = \gamma_m v_{0x}$ ($v_{0ym} = \gamma_m v_{0y}$) is the perpendicular velocity in the moving frame. If the electron density in the laboratory frame is $n_c$, then it is $n_{em} = n_c/\gamma_m$ in the moving frame. The electron energy loss rate in the moving frame given in Eq. (6) is

$$\frac{de}{dt} \simeq \alpha n_{em} (eE_1 v_{0xm}),$$

where $\alpha$ is a constant of order of 1. By considering the local E&M wave and the local amplification, we obtain from $dE_1/dt = (1/8\pi)(dE^2_1/dt)$ that

$$\frac{dE_1}{dt} \simeq \alpha \pi n_{em} v_{0xm} \cos(\Omega t),$$

where $\Omega \equiv eB_0/m_e c$ is in the moving frame. Eq. (7) shows that the initial growth rate, $(dE_1/dt)/E_1$, is infinite. During the time duration of $1/\Omega$, $E_1$ grows to $E_1(T) = \alpha n_{em} v_{0xm}/\Omega$, and the ratio of the E&M energy intensity to the particle kinetic energy becomes

$$\frac{E_1(T)^2}{n_{em} v_{0xm}^2} \simeq \frac{\alpha}{2} \frac{\omega_{pem}^2}{\Omega^2},$$

where $\omega_{pem}^2 = 4\pi n_{em} e^2/m_e$ and $(\cos(\Omega t)^2) = 1/2$ is used. Eq. (8) suggests that the THz E&M wave gets amplified to the energy intensity comparable to the perpendicular electron kinetic energy density times the ratio $\omega_{pem}^2/\Omega^2$ during the time duration of $\Omega$, which is the maximum energy that could be extracted. In the moving frame, the perpendicular kinetic energy of an electron is $N m_e v_{0xm}^2 = N \gamma_m^2 m_e v_{0x}^2$. The maximum total energy radiating into the THz wave is $E_{1max} = N \gamma_m^2 m_e v_{0x}^2 (\omega_{pem}^2/\Omega^2)$ so that the maximum THz energy in the laboratory frame is $\gamma_m E_{1max} = N \gamma_m^3 m_e v_{0x}^2 (\omega_{pem}^2/\Omega^2)$, where $N$ is the total number of electrons in the beam. In order to extract the appreciable fraction of the electron kinetic energy, the ratio $\omega_{pem}/\Omega$ needs to be maximized. As shown in the non-ideal plasma beam analysis, it is theoretically possible to get $\omega_{pem}/\Omega \simeq 1$.

Let us give a few examples of the practically relevant beam parameters. Consider a 10 pico-second electron beam with $\gamma = 35$, $n_e = 10^{14}$ cm$^{-3}$ and the total number of electrons being $10^{10}$, and assume that the magnetic field is order of 1 T. If the beam gets launched with $v_{p}/v_{0x} = 0.03$, the parallel relativistic factor is $\gamma_m = 25$. The resonant frequency for the THz radiation is roughly 1 THz. In the moving frame, the electron density becomes roughly $4 \times 10^{11}$ cm$^{-3}$, and $\omega_{pem}/\Omega \simeq 0.1$; the beam duration is 250 pico-second. The total energy of the electron is $7 \times 10^{15}$ eV, and at the maximum, a few percents of the total electron kinetic energy can be radiated into the THz E&M wave. As another example, consider a 10 pico-second electron beam with $\gamma = 14$. Assume that the electron density is $10^{14}$ cm$^{-3}$, the total number of electrons is $10^{10}$, and the beam of $v_{p}/v_{0x} = 0.06$ gets launched ($\gamma_m = 11$). Assuming the magnetic field is order of 1 T, the resonant frequency is roughly 0.5 THz. In the moving frame, the electron density is roughly $10^{13}$ cm$^{-3}$, $\omega_{pem}/\Omega \simeq 0.1$, and the beam duration is 100 pico-second. The total energy of the electron is $10^{16}$ eV, and as much as tens of percents of the total electron kinetic energy can be radiated into the THz E&M wave.

To summarize, a scheme of THz generation is discussed, where the spatial helical structure of the relativistic electron beam is used for the amplification, via a physical mechanism similar to that of the FEL. In contrast to the FEL with the magnets, the energy extraction rate from the electrons is not proportional to the intensity, rather it is proportional to the electric field of the E&M wave. This property makes this scheme advantageous, as the THz field can be explosively amplified up to certain amplitude. The overall efficiency is another advantage. A THz radiation with the total energy of a few tens of percents of the total electron beam energy can be as high as gyrotron or magnetron; the only difference is the operating regime, the THz range.

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