Constraints on power spectrum of density fluctuations from PBH evaporations

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We calculate neutrino and photon energy spectra in extragalactic space from evaporation of primordial black holes, assuming that the power spectrum of primordial density fluctuations has a strong bump in the region of small scales. The constraints on the parameters of this bump based on neutrino and photon cosmic background data are obtained.

I. INTRODUCTION

It is well known that for a sufficient production of primordial black holes (PBHs) in early Universe the spectrum of density perturbations set down by inflation must be "blue", i.e., it must have more power on small scales. This implies that the spectral index of the scalar perturbations must be larger than 1, in strong contradiction with the latest WMAP results [1]. In particular, inflationary models of hybrid type, in which the inflaton is trapped in a local minimum of the potential and which predict blue spectra seem to be excluded as a possible source of PBHs.

As an alternative one can consider the models in which the power on small scales is boosted by means of a bump in the power spectrum of primordial fluctuations, as suggested, e.g., in [2]. Such a bump is predicted in many scenarios of two-step inflation (see, e.g., [3,4]) and can, in principle, exist even in single-field inflationary models (if the potential has a special form leading to the period of fast-roll at the end of inflation [5].

In the present work we reconsider the problem of constraining the power spectrum of primordial fluctuations calculating the process of the formation of PBHs having small masses \(10^{11} \text{--} 10^{15} \text{g}\). Products of evaporation of these PBHs contribute, in particular, to extragalactic photon and neutrino diffuse backgrounds (which are measured experimentally).

II. PBH MASS SPECTRUM

Using the Press-Schechter formalism [6], the differential mass spectrum of primordial black holes can be written in the form [7,8]:

\[
 n_{BH}(M_{BH}) = \sqrt{2} \pi \rho_i \int \frac{1}{M\sigma_H(M)} \left( \frac{2}{3M} - \frac{1}{\sigma_H(M)} \frac{\partial \sigma_H(M)}{\partial M} \right) \left( \frac{\sigma_H^2(M)}{\sigma_H^2(M)} - 1 \right) \cdot e^{-\frac{(\delta_H^2)^2}{2\sigma_H^2(M)}} \cdot d\delta_H^2(M) / dM. \tag{2.1}
\]

One must note, before the explanation of the notations used in Eq. (2.1), that in the problem studied here it is convenient to specify the spectrum of primordial fluctuations at a fixed time rather than at horizon crossing. It is so because we assume that, at the end of inflation, reheating and subsequent formation of the perturbation spectrum occur almost instantaneously. The connection between density contrasts at a fixed time and at horizon crossing (neglecting, at the moment, a dependence of the gravitational potential on the time) is quite simple [9],

\[
 \left( \frac{\delta \rho}{\rho} \right)_t = \left( \frac{M}{M_{\text{hor}}(t)} \right)^{-2/3} \cdot \left( \frac{\delta \rho}{\rho} \right)_{\text{hor}}. \tag{2.2}
\]

Correspondingly, the mean square deviation (after smoothing the density field on a given mass \(M\)) at horizon crossing, \(\sigma_H(M)\), is given by the formula

\[
 \sigma_H^2(M) = \left( \frac{M}{M_i} \right)^{4/3} \int \left( \frac{k}{a_i H_i} \right)^4 \delta_H^2(k) W^2(kR) T^2(k) \frac{dk}{k} \equiv \left( \frac{M}{M_i} \right)^{4/3} \sigma_R^2(M). \tag{2.3}
\]

Here, \(M_i\) is the horizon mass at the end of inflation, \(a_i\) and \(H_i\) are the scale factor and Hubble parameter at the end of inflation, \(R\) is the comoving smoothing scale, \(R \equiv 1/k_{fl}\), connected with \(M\) by the expression

\[
 \left( \frac{M}{M_i} \right)^{-2/3} = \frac{k_{fl}^3}{(a_i H_i)^2}. \tag{2.4}
\]
FIG. 1. Horizon crossing amplitude $\delta_H(k)$ (dashed line) and smoothed dispersion $\sigma_H(k_{fl})$ (solid line). Spectra are shown for $\Sigma = 3, \delta_0^H = 0.06, k_0 = 2.75\cdot10^{16}\text{Mpc}^{-1}$.

The notations used for the other functions in the integral (2.3) are standard: $\delta_H(k)$ is the horizon crossing amplitude for primordial perturbations of the density contrast, $W(kR)$ and $T(k)$ are window and transfer functions.

The variable $\delta_0^H$ in Eq. (2.1) is a value of the (smoothed) density contrast at horizon crossing, $\rho_i$ is the energy density at the end of inflation. At last, the function $f(M, \delta_R^H)$ connects the values of the smoothing mass $M$, density contrast $\delta_R^H$ and PBH mass $M_{BH}$,

$$M_{BH} = f(M, \delta_R^H; \rho_i).$$

(2.5)

The concrete form of this function depends on the features of the gravitational collapse leading to PBH formation.

It is assumed that the dependence of $\delta_H(k)$ on $k$ at small scales can be parameterized in the form

$$\ln \delta_H(k) = A + (\ln \delta_0^H - A) \exp\left[-\frac{1}{2\Sigma^2}(\ln k - \ln k_0)^2\right].$$

(2.6)

Here, the value of $A$ is known from observations at large scales, $\delta_0^H, k_0$ and $\Sigma$ are free parameters, the values of which should be constrained.

The calculation of $\sigma_H(M)$ was performed using the gaussian window function,

$$W(kR) = \exp\left(-\frac{k^2R^2}{2}\right)$$

(2.7)

and the transfer function, derived in the cosmological perturbation theory,

$$T(k, \tau) = \frac{3}{(\omega_s\tau)^3}(\sin \omega_s \tau - \omega_s \tau \cos \omega_s \tau),$$

(2.8)

$$\omega_s = k_{cs} = \frac{k}{\sqrt{3}},$$

(2.9)

$\tau$ is the conformal time. In the following we will ignore the time dependence of the transfer function, putting

$$T(k, \tau) \approx T(k, \tau = \frac{1}{k_0}) \equiv T(k)$$

(2.10)

(because the most abundant PBH production takes place at the time when the scale $k_0^{-1}$ corresponding to the maximum of $\delta_H(k)$ crosses horizon).

In the model of spherically-symmetric collapse one obtains the expression

$$M_{BH} = (\delta_R^H)^{1/2}M_h,$$

(2.11)
where $M_h$ is the fluctuation (smoothing) mass at the moment of horizon crossing. According to Carr and Hawking [10], $\frac{1}{3} \leq \delta_R^H \leq 1$.

The connection between $M$ and the horizon mass $M_h$ is [7]

$$M_h = M_i^{1/3} M^{2/3}.$$  

(2.12)

From Eqs. (2.11), (2.12) one has the expression for the function $f(M, \delta_R^H; M_i)$ for the Carr-Hawking collapse:

$$M_{BH} = f(M, \delta_R^H; M_i) = (\delta_R^H)^{1/2} M^{2/3} M_i^{1/3}.$$  

(2.13)

In the picture of the critical collapse [11] the corresponding function is

$$f(M, \delta_R^H; M_i) = k_c (\delta_R^H - \delta_c)^{\gamma_c} M^{2/3} M_i^{1/3},$$  

(2.14)

where $\delta_c$, $\gamma_c$ and $k_c$ are model parameters. In this work we will accept the following set of parameters, which is in agreement with recent calculations [12]:

$$\delta_c = 0.45, \quad \gamma_c = 0.36, \quad k_c = 4.$$  

(2.15)

Some results of PBH mass spectrum calculations are shown in Fig. 1 and 2. In Fig. 1, the smoothed dispersion is drawn as a function of the inverse comoving smoothing scale, $k_f = 1/R$. The connection between $k_f$ and horizon mass $M_h$ is determined from Eqs. (2.4) and (2.12) and is

$$M_h = \frac{M_i (a_i H_i)^2}{k_f^7}.$$  

(2.16)

Fig. 2 shows that the resulting PBH mass spectrum strongly depends on the model of the gravitational collapse (at the same values of the fluctuation spectrum parameters and reheating temperature $T_{RH}$).
III. NEUTRINO AND PHOTON SPECTRA FROM PBHS EVAPORATIONS

Evolution of a PBH mass spectrum due to the evaporation leads to the approximate expression for this spectrum at any moment of time:

\[ n_{BH}(m, t) = \frac{m^2}{(3\alpha t + m^3)^{2/3}} n_{BH} \left( (3\alpha t + m^3)^{1/3} \right), \]

where \( \alpha \) accounts for the degrees of freedom of evaporated particles and, strictly speaking, is a function of a running value of the PBH mass \( m \). In all our numerical calculations we use the approximation

\[ \alpha = \text{const} = \alpha(M_{BH}^{\text{max}}), \]

where \( M_{BH}^{\text{max}} \) is the value of \( M_{BH} \) in the initial mass spectrum corresponding to a maximum of this spectrum. Special study shows that errors connected with such an approximation are rather small.

The expression for an extragalactic differential energy spectrum of neutrinos or photons (the total contribution of all black holes) integrated over time is [7]

\[ S(E) = \frac{c}{4\pi} \int dt \frac{a_0}{a} \left( \frac{a_1}{a_0} \right)^3 \int dm \frac{m^2}{(3\alpha t + m^3)^{2/3}} n_{BH} \left( (3\alpha t + m^3)^{1/3} \right) \cdot \varphi(E \cdot (1 + z), m) e^{-\tau(E, z)} \equiv \int F(E, z) d\lg(z + 1). \]

In this formula, \( a_i, a, a_0 \) are cosmic scale factors at \( t_i, t \) and at present time, respectively, and \( \varphi(E, m) \) is a total instantaneous spectrum of the radiation (neutrinos or photons) from the evaporation of an individual black hole. The exponential factor in Eq. (3.3) takes into account an absorption of the radiation during its propagation in space. The processes of neutrino absorption are considered, in a given context, in [7]. In the last line of Eq. (3.3) we changed the variable \( t \) on \( z \) using the flat cosmological model with \( \Omega_\Lambda \neq 0 \) for which

\[ \frac{dt}{dz} = -\frac{1}{H_0(1 + z)} \left( \Omega_m(1 + z)^3 + \Omega_\nu(1 + z)^4 + \Omega_\Lambda \right)^{-1/2}, \]

with \( \Omega_\nu = (2.4 \cdot 10^4 h^2)^{-1}, h = 0.7, \Omega_M = 0.3, \Omega_\Lambda = 1 - \Omega_M - \Omega_\nu. \)

The instantaneous spectra of neutrinos and photons from PBH evaporation were calculated using the photosphere model elaborated in works [13,14]. The decoupling temperature \( T_f \) for photons was taken equal to 120 MeV, and the temperature of the neutrinosphere, \( T_\nu \), is equal to 100 GeV, as in [14]. The neutrino emission from "cold" PBHs, those with \( T_H < 200 \) GeV, was calculated using the standard Hawking formula. The contribution to the black hole neutrino spectra from decays of pions and muons was, at this stage, not taken into account.

The photosphere model of black hole evaporation used here predicts very steep time-integrated spectra of photons and neutrino \( (E^{-4}) \), and in the low energy region the absolute spectrum values (the number of particles) are very...
FIG. 4. Constraints on the horizon-crossing amplitude $\delta_H(k)$ that were obtained in this work. $M_h^0$ is the horizon mass at the moment when fluctuation with comoving wave number $k_0$ enters horizon. Constraints following from both neutrino and gamma ray experiments are shown. Dashed lines correspond to the model of critical collapse with the parameters $\delta_c = 0.45, \gamma_c = 0.36, k_c = 4$, solid lines represent the results obtained using the standard collapse picture.

IV. RESULTS AND DISCUSSIONS

The calculation of the constraints uses two basic observational facts.

1. The differential energy spectrum of the extragalactic photon background at energy $E_{\gamma} = 10$MeV is [16]

$$\sim 10^{-2} GeV^{-1}cm^{-2}s^{-1}sr^{-1}. \quad (4.1)$$

2. According to the data of Super-Kamiokande experiment [17], the electron antineutrino background flux in space is constrained by the inequality

$$\Phi(E_{\bar{\nu}_e} > 19.3 GeV) < 1.2 cm^{-2}s^{-1}. \quad (4.2)$$

The resulting constraints are shown on Fig. 4. The forbidden values of the most important parameter, $\delta_H(k_0)$, lie inside the bordered regions drawn on the figure. The value of the parameter $\Sigma$ characterizing the width of the gaussian distribution in Eq. (2.6) was fixed throughout all calculations ($\Sigma = 3$). The constraints are given as a function of the horizon mass corresponding to the moment of time when the comoving wave number $k_0$ enters horizon. It follows from these results that the constraints are stronger in the case of the standard Carr-Hawking collapse. The second important result is that the constraints based on neutrino emission of PBHs are comparable with those following from photon emission. At the region of small horizon masses (large $k_0$) where the large red shifts are sufficient, the constraints from neutrino emission are stronger.

One should add, at the end, one comment. In this work we calculated the constraints using the rather high value of $T_{RH} (= 10^{10}$GeV). The corresponding horizon mass at the end of inflation, $M_i$, is equal to $\sim 10^{11}$g. The interval
of studied here values of the $k_0$ parameter corresponds to PBH production in super-horizon perturbations. At low $T_{RH}$ the value of $M_i$ will be high and the parameter $k_0$ can be chosen such that black holes will be produced from sub-horizon perturbations (the possibility of PBH production from such perturbations is studied in [18]). In principle, constraints on the power spectrum due to PBH evaporations can be obtained in this case also, using the hypothesis of critical collapse.

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