Graceful Exit in String Cosmology

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Abstract

The graceful exit transition from a dilaton-driven inflationary phase to a decelerated Friedmann–Robertson–Walker era requires certain classical and quantum corrections to the string effective action. Classical corrections can stabilize a high curvature string phase while the evolution is still in the weakly coupled regime, and quantum corrections can induce violation of the null energy condition, allowing evolution towards a decelerated phase.

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1 Introduction

String theory predicts gravitation, but the gravitation it predicts is not that of standard general relativity. In addition to the metric fields, string gravity also contains a scalar dilaton, that controls the strength of coupling parameters. An inflationary scenario [1], is based on the fact that cosmological solutions to string dilaton-gravity come in duality-related pairs, an inflationary branch in which the Hubble parameter increases with time and a decelerated branch that can be connected smoothly to a standard Friedmann–Robertson–Walker (FRW) expansion of the Universe with constant dilaton. The scenario (the so called “pre-big-bang”) is that evolution of the Universe starts from a state of very small curvature and coupling and then undergoes a long phase of dilaton-driven kinetic inflation and at some later time joins smoothly standard radiation dominated cosmological evolution, thus giving rise to a singularity free inflationary cosmology.

However, in the lowest order effective action these two branches are separated by a singularity. Additional fields or correction terms need to be added to make this “graceful exit” transition possible. It has been studied intensely in the last few years, but all attempts failed to induce graceful exit. In [2, 3] it was shown that the transition is forbidden for a large class of fields and potentials. In [4] we proposed to use an effective description in terms of sources that represent arbitrary corrections to the lowest order equations and were able to formulate a set of necessary conditions for graceful exit and to relate them to energy conditions appearing in singularity theorems of Einstein’s general relativity [7]. In particular, we showed that a successful exit requires violations of the null energy condition (NEC) and that this violation is associated with the change from a contracting to an expanding universe (bounce) in the “Einstein frame”, defined by a conformal change of variables. Since most classical sources obey NEC this conclusion hints that quantum effects, known to violate NEC in some cases, may be the correct sources to look at.

Because the Universe evolves towards higher curvatures and stronger coupling, there will be some time when the lowest order effective action can no longer reliably describe the dynamics and it must be corrected. Corrections to the lowest order effective action come from two sources. The first are classical corrections, due to the finite size of strings, arising when the fields are varying over the string length scale $\lambda_s = \sqrt{\alpha'}$. These terms are important in the regime of large curvature. The second are quantum loop corrections.
The loop expansion is parameterized by powers of the string coupling parameter $e^\phi = g_{\text{string}}^2$, which in the models that we consider is time dependent. So quantum corrections will become important when the dilaton $\phi$ becomes large, the regime we refer to as strong coupling.

In [5] we were able to find an explicit model that satisfies all the necessary condition and to produce the first example of a complete exit transition. The specific model we present here makes use of both classical and quantum corrections. We allowed ourselves the freedom to choose the coefficients of correction terms which generically appear in string effective actions. Our reasoning for allowing this stems in part from a lack of any real string calculations and in part by our desire to verify, by constructing explicit examples, the general arguments of [4].

2 A Specific Exit Model

String theory effective action in four dimensions takes the following form in the string frame ($S$),

$$S_{\text{eff}} = \int d^4x \left\{ \sqrt{-g} \left[ e^{-\phi} \frac{1}{16\pi\alpha'} (R + \partial_\mu \phi \partial^\mu \phi) \right] + \frac{1}{2} \sqrt{-g} L_c \right\},$$

where $g_{\mu\nu}$ is the 4-d metric and $\phi$ is the dilaton. The Einstein frame ($E$) is defined by the change of variables $g \rightarrow e^{\phi/2} g$, diagonalizing the metric and dilaton kinetic terms and resulting in equations of motion similar to those of standard general relativity.

We are interested in solutions to the equations of motion derived from the action (1) of the FRW type with vanishing spatial curvature $ds^2 = -dt_S^2 + a_S^2(t)dx_i dx^i$ and $\phi = \phi(t)$. The contribution of $L_c$ is contained in the correction energy-momentum tensor $T_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta \sqrt{-g} L_c}{\delta g_{\mu\nu}}$, which will have the form $T^\mu_\nu = \text{diag}(\rho, -p, -p, -p)$. In addition we have another source term arising from the $\phi$ equation, $\Delta_\phi L_c = \frac{1}{2} \frac{1}{\sqrt{-g}} \frac{\delta \sqrt{-g} L_c}{\delta \phi}$.

The 00 equation of motion is quadratic and may be conveniently written,

$$\dot{\phi} = 3H_S \pm \sqrt{3H_S^2 + e^\phi \rho},$$

where $H_S = \dot{a}_S/a_S$, and we have fixed our units such that $16\pi\alpha' = 1$. The choice of sign here corresponds to our designation of (+) and (−) branches.
For the corrections we propose classical, one and two loop terms of a particularly simple form. The first term in (3) is the form of $\alpha'$ corrections examined in [6], the second and third are plausible forms for the one and two loops corrections respectively. The large coefficients account for the expected large number of degrees of freedom contributing to the loop. The signs of these terms are deliberately chosen to force the exit.

$$\frac{1}{2}L_c = e^{-\phi}\left(\frac{R_{GB}^2}{4} - \frac{(\nabla \phi)^4}{4}\right) - 1000(\nabla \phi)^4 + 1000e^{\phi}(\nabla \phi)^4.$$  \hspace{1cm} (3)

We have checked that qualitatively similar evolution is obtained for a range of coefficients, of which (3) is a representative.

We set up initial conditions in weak coupling near the (+) branch vacuum and a numerical integration yields the evolution shown in Fig. 1 in the $\dot{\phi},H_S$ phase space. We have also plotted lines marking important landmarks in the evolution, the (+) and (−) vacuum ($\rho = 0$ in (2)), the line of branch change (+) $\rightarrow$ (−) (square root vanishing in (2)) and the position of the $E|$ bounce ($H_E = 0, \dot{\phi} = 2H_S$). We see that the evolution falls into three distinct phases which we will discuss in turn. \textbf{Phase (i)} The solution begins

Figure 1: The correction induced graceful exit
with a long evolution near the (+) vacuum, this is the inflationary phase. As curvature becomes large we see deviation induced by the \( \alpha' \) corrections in \( \mathcal{S} \) and without influence from other corrections the solution would settle into the fixed point noted in \[6\], marked with a '+' . The solution does cross the line of branch change, but does not execute the \( E \) bounce required by a complete exit, corresponding to the fact it does not violate NEC in the \( E \). Phase (ii) While the Universe sits near the fixed point the dilaton is still increasing linearly, so eventually the loop corrections in \( \mathcal{S} \) will become important. The first to do so is the one loop correction. Since we require further NEC violation to complete the \( E \) bounce, we have chosen the sign of the one loop correction to provide this and in this phase corrections are dominated by this term. As a result a bounce occurs and the evolution proceeds into the \( \rho > 0 \) region. We checked that other forms of loop correction will have the same effect if they are introduced with a coefficient allowing NEC violation. But without further corrections this solution would continue to grow into regions of larger curvature and stronger coupling. We refer to this era as “correction dominated” and we also find there are obstacles to stabilizing the dilaton with standard mechanisms like capture in a potential or radiation production. Phase (iii) To offset the destabilizing NEC violation we have introduced the two loop correction with the opposite sign, allowing it to overturn the NEC violation when it becomes dominant as \( \phi \) continues to grow. Indeed during this phase we see the expansion decelerating, dilaton growth stabilizing, and the corrections vanishing. We have also checked that in this phase the dilaton can be captured into a potential minimum or halted by radiation production. This phase can be smoothing joined to standard cosmologies.

3 Conclusion

Corrections to lowest order string effective action can induce graceful exit. It remains to be seen whether string theory has the predictive power to fix the form and coefficients of these corrections and thus give a definite answer to the question of whether graceful exit indeed proceeds as suggested in our model.
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