We propose experimental schemes to create and probe minimum forms of different topologically ordered states in a plaquette of an optical lattice: Resonating Valence Bond, Laughlin and string-net condensed states. We show how to create anyonic excitations on top of these liquids and detect their fractional statistics. In addition, we propose a way to design a plaquette ring-exchange interaction, the building block Hamiltonian of a lattice topological theory. Our preparation and detection schemes combine different techniques already demonstrated in experiments with atoms in optical superlattices.

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controlling the two optical superlattices independently, different potential bias, $\Delta_x$ and $\Delta_y$, can be introduced along the $x$- and $y$-direction, leading to different site energy offsets $\mu_i$, as well as different vibrational level splittings at the lattice sites.

this state.

The mini-topological liquids we consider here constitute fundamental building blocks of larger topologically ordered states \cite{27,28}. Furthermore, plaquette ring exchange interactions are the basic ingredients of lattice gauge theories \cite{29}, theoretical models describing topological matter. By connecting plaquettes in the appropriate manner a variety of strongly correlated many body states could be achieved \cite{27,28}.

The system. We consider a system of atoms in two internal states $\sigma = \uparrow, \downarrow$, which for the case of e.g. $^{87}$Rb atoms could correspond to the hyperfine states $|F = 1, m_F = +1\rangle$ and $|F = 1, m_F = -1\rangle$. The atoms are loaded into a two dimensional superlattice, which is produced by superimposing a long and a short period lattice \cite{(a)} both in the $x$- and $y$-direction in such a way that an array of disconnected plaquettes is created (see Fig. 1). In the following, we will restrict the discussion to the case of bosonic atoms, though similar results can be obtained in a straightforward manner for the case of fermions.

The dynamics of atoms in a single plaquette is governed by the Hubbard Hamiltonian

$$H = \sum_{i,j,\sigma}t_{ij}(a_{i\sigma}^\dagger a_{j\sigma} + \text{H.c.}) + U\sum_{i,\sigma,\sigma'}n_{i\sigma}n_{i\sigma'} + \sum_{i,\sigma}\mu_i n_{i\sigma},$$

where $a_{i\sigma}$ and $n_{i\sigma}$ are, respectively, the bosonic annihilator and the particle number operator at site $i$ and for spin $\sigma$. By controlling the superlattice structure, the tunneling amplitudes in the $x$- and $y$-direction, $t_x \equiv t_{12} = t_{14}$ and $t_y \equiv t_{23} = t_{14}$, can be tuned independently. Furthermore, the dependence of the offset energies $\mu_i\sigma$ on position and spin state can be designed using additional magnetic offsets or gradient fields. In the following we will make full use of the experimental ability to control these parameters, as already demonstrated in \cite{22,23} for a single double well.

**RVB in a plaquette.** RVB states, in which particles are paired into short-range singlets, are one of the most relevant examples of topological spin liquids \cite{31}. We consider here minimum forms of RVB states consisting of four particles in a plaquette:

$$|\Phi_{\pm}\rangle \propto \left(s_{1,2}^\dagger s_{4,3}^\dagger \pm s_{1,4}^\dagger s_{2,3}^\dagger \right)|0\rangle. \quad (1)$$

Here, $s_{i,j}^\dagger = (a_{i\uparrow}^\dagger a_{j\downarrow}^\dagger - a_{i\downarrow}^\dagger a_{j\uparrow}^\dagger)$ creates a singlet state on sites $i$ and $j$ and $|0\rangle$ is the vacuum state. The states $|\Phi_{\pm}\rangle$ are disordered states with zero local magnetization, $\langle S_i^z \rangle = 0$, for all sites $i$. They are both total singlets, with $S^z \equiv |\Phi_{\pm}\rangle = \sum_i S_i^z |\Phi_{\pm}\rangle = 0$. But they behave differently under rotation of the plaquette by 90°: $|\Phi_{+}\rangle$ is even (has s-wave symmetry), whereas $|\Phi_{-}\rangle$ is odd (has d-wave symmetry). As larger RVB states \cite{32,33}, states $|\Phi_{\pm}\rangle$ exhibit topological order.

In the following we develop a scheme to prepare and detect the state $|\Phi_{+}\rangle$. The state $|\Phi_{-}\rangle$ can be designed in a similar manner. We start with a situation in which we have four particles per plaquette and tunneling is only allowed along the $y$-direction. The system can be then prepared in a valence bond state $|\text{VB}_{y}\rangle = s_{1,4}^\dagger s_{2,3}^\dagger|0\rangle$, with singlets in the vertical bonds (see Fig. 2). By adiabatically turning on tunneling along the $x$-direction we will connect the state $|\text{VB}_{y}\rangle$ to the state $|\Phi_{+}\rangle$. To make this connection possible, we consider a situation in which the tunneling amplitudes $t_x$ and $t_y$ are very small in comparison to the on-site interaction energy $U$. Under these conditions, the particles are site localized and the physics is described by the superexchange interactions between the remaining spins:

$$H_S = J_x(\hat{P}_{1,2} + \hat{P}_{3,4}) + J_y(\hat{P}_{2,3} + \hat{P}_{1,4}) + \ldots \quad (2)$$

Here, $\hat{P}_{i,j} = s_{i,j}^\dagger s_{i,j}$ is the projector onto a singlet state on sites $i$ and $j$. $J_{x(y)} = 4t_{x(y)}^2/U$, and the dots denote higher order terms in $J_{x(y)}$. If $t_x$ is suddenly increased from 0 to $t_y$ the system will resonate with frequency

![FIG. 2: Preparation of a RVB state. Two spin-triplet atom pairs are split into double wells along the $y$-direction in a double well-potential (a). A magnetic field gradient along the $y$-direction is kept for a finite amount of time to turn the spin triplet bonds along the vertical direction into singlet states (b-c). Finally, the tunnel coupling along the $x$-direction is increased adiabatically to connect the bonds, such that a RVB state is formed (d). The symbols for the bonds are explained in Fig. 3](image)
the order of total singlet. The energy gap to this state is always characterized and differentiated from each other using two which can be easily fulfilled in experiments.

For the case of bosons, an initial spin singlet state with an antisymmetric spin wavefunction will lead to half of the population in the first excited state, whereas for a spin triplet state only the lowest vibrational state will be occupied after merging, thus allowing one to distinguish between the two spin states. In order to create the state |\(\Phi_+\)\(\rangle\), \(t_x\) has to be turned on adiabatically. As shown in Fig. 3 the states |\(\Phi_+\rangle\) and |\(\Phi_-\rangle\) are adiabatically connected. The only state to which a transition is not forbidden by symmetry constraints is |\(\Phi_-\rangle\), the other total singlet. The energy gap to this state is always on the order of \(\sim 2J_y\), giving a time scale of tens of ms, which can be easily fulfilled in experiments.

The two RVB states presented in eq. (1), can be characterized and differentiated from each other using two alternative methods.

a) Merging technique. By merging wells along the x- or y-direction we can monitor singlets in that direction via the bandmapping technique outlined in [22]. For example, by merging along the diagonals we could easily discriminate \(|\Phi_+\rangle = |↓↓↓↓\rangle\)

FIG. 3: (a) Eigenstates and eigenergies of the spin Hamiltonian (2) vs. tunnel coupling ratio along the horizontal vs. vertical bond. (b) Legend explaining the different symbols used to characterize spin singlet and spin triplet states of two particles.

2\(J_y/\hbar\) between the two valence bond states |\(\text{VB}_y\rangle\) and |\(\text{VB}_x\rangle = s_{1,2}^{\uparrow\downarrow}s_{4,3}^{\downarrow\downarrow}|0\rangle\). These oscillations could be detected by monitoring singlet pairs in the x- and y-bonds using a bandmapping technique [22,24] after merging the double wells in either the x- or y-direction into a single well. By merging wells along a) Merging technique

FIG. 4: Momentum distribution of the polarized two-particle states resulting from the RVB states |\(\Phi_+\rangle\) (a) and |\(\Phi_-\rangle\) (b) (see text). Such momentum distributions could be observed after releasing the atoms from the optical lattice potentials and a subsequent time-of-flight period.

\[
\frac{1}{\sqrt{3}} \left( t_{1,3}^\uparrow t_{2,4}^\downarrow + t_{1,3}^\downarrow t_{2,4}^\uparrow - t_{1,3}^\downarrow t_{2,4}^\downarrow \right) |0\rangle,
\]

which is made out of triplets in the diagonal bonds, with \(t_{ij}^{s(-)\dagger} \propto a_i^{s(-)\dagger}a_j^{\dagger}\), from |\(\Phi_-\rangle = s_{1,3}^{\uparrow\downarrow}s_{4,3}^{\downarrow\downarrow}|0\rangle\), made out of singlets. Alternatively, we can distinguish these two states by merging along the vertical direction. To do this we first undo the adiabatic path we followed before (see Fig. 3) by decreasing \(t_x\) from t to 0. In this way |\(\Phi_+\rangle\) will be connected to the state \(s_{1,2}^{\uparrow\downarrow}s_{4,3}^{\downarrow\downarrow}|0\rangle\), with singlets in the vertical bonds, whereas |\(\Phi_-\rangle\) will connect to

\[
\frac{1}{\sqrt{3}} \left( t_{1,3}^\uparrow t_{4,3}^\downarrow + t_{1,3}^\downarrow t_{4,3}^\uparrow - t_{1,3}^\downarrow t_{4,3}^\downarrow \right) |0\rangle,
\]

with triplets in the vertical bonds.

b) Conversion into a polarized two-particle state. Any four spin state |\(\Phi\rangle\) with well defined \(S^2 = \sum_i S_i^2 = 0\) can be written as a state of two up particles in a background of spin down particles

\[
|\Phi\rangle = \sum_{x_1,x_2} \psi(x_1,x_2)S_x^{1+}S_x^{2+}|\downarrow\downarrow\downarrow\downarrow\rangle
\]

where \(S_x^{1+}\) is the spin raising operator on site \(x = 1, \ldots, 4\), and |\(\downarrow\downarrow\downarrow\downarrow\rangle = a_1^{\dagger}a_2^{\dagger}a_3^{\dagger}a_4^{\dagger}|0\rangle\). If we remove the background of spin down particles, that is, if we apply the operator \(\sum_{i\neq j} a_i a_j\) to the state (3), we are left with a system of two polarized hard-core bosons with wave function \(\psi(x_1,x_2)\). In practice, spin down particles can be effectively removed by projecting the spin down part of state (3) onto the state \(\sum_{i\neq j} a_i a_j^\dagger|0\rangle\), where |\(0\rangle\rangle\) is the vacuum of down particles. The properties of the resulting two particle state directly reflect those of the spin parent state (3). By observing the momentum distribution of the two particles via a common time of flight experiment, we can read back the spin-spin correlations of the parent spin state. For example, observing a hole at the center of the time of flight picture would be an unambiguous signature of a total spin singlet parent state. To see this, note that a spin state |\(\Phi\rangle\) is a total singlet if and only if \(\langle \Phi | S^+ S^- | \Phi \rangle = 0\), which for the corre-
spin down particles, a Laughlin state of the remaining spin up particles is created (4). This state is an eigenstate of the total angular momentum operator, \( L = \sum_m m a_m^+ a_m \), with eigenvalue \( L = 4 \). Here, the operator \( a^+ = \frac{1}{\sqrt{2}} \sum_{\ell=1}^{4} e^{i \pi \ell / 4N} a^\ell \), creates a particle in a state of angular momentum \( m \). It has also a well defined center of mass angular momentum, \( L_{cm} = 0 \). This vanishing \( L_{cm} \) is a consequence of the s-wave symmetry of the wave function, inherited from the state \(| \Phi_+ \rangle \). The state (1) is indeed a Laughlin quasi-hole state \(| \Phi \rangle \). It contains a quasi-hole at the center of the plaquette, whose characteristic density profile, with a dip at the center, could be observed in a time of flight interference experiment (see Fig. 3).

The equivalence of a long-range RVB state of \( 2N \) spins and a Laughlin state of \( N \) hard-core bosons has been proposed by Laughlin for a triangular two dimensional lattice Hamiltonian \( \mathcal{H} \). Indeed, it is known that this connection is exact for a lattice of spins sitting on a ring and interacting with a long-range interaction, the so called Haldane-Shastry model \( \mathcal{H}_{HS} = \sum_{ij} J_{ij} \hat{P}_{ij} \), with \( J_{ij}^2 = \sin \left( \frac{\pi}{4N} (x_i - x_j) \right) \). A surprising fact in our case is that the Laughlin state (4) appears in the absence of any frustration or long-range interaction (there is no interaction between spins along the diagonals in Hamiltonian \( \mathcal{H} \)).

This conversion technique provides us with a way to fully characterize the states (1). The corresponding time of flight pictures (see Fig. 3) show both a dip at the center, probing their total being singlets, but are quite different in structure, a consequence of the different symmetry of the s-wave and d-wave RVB states. The two particle states obtained from the RVB states (1) turn out to be of special importance by themselves. We show below that the state \(| \Phi_+ \rangle \) can be converted into a Laughlin state, whereas \(| \Phi_- \rangle \) can be transformed into a paired state with d-wave symmetry.

**Laughlin State.** The Laughlin state (1) is one of the best known examples of a topologically ordered state. Here, we describe a way to prepare a Laughlin state of two particles starting with the state \(| \Phi_+ \rangle \) in (1). In contrast to previous schemes our method does not require either rotation \( \mathcal{R} \) of the optical lattice or of the individual wells \( \mathcal{W} \), or the presence of effective magnetic fields \( B \).

The key point to realize that the state \(| \Phi_+ \rangle \) can be written in the form (3) with

\[
\psi(x_1, x_2) = z_1 z_2 (z_1 - z_2)^2, \\
\text{where } z_i = e^{i \bar{x}_i}, \quad x_i = 1, \ldots, 4.
\]  

FIG. 5: Legend of correlated quantum states on a plaquette that are created using the methods described in the text. (a) s-wave RVB state, \(| \Phi_+ \rangle \), (b) d-wave RVB state, \(| \Phi_- \rangle \), (c) Laughlin state, (d) paired state with d-wave symmetry.
The fractional character of spinons becomes more transparent by mapping the spin system into a hard-core boson problem, in the same way that we did above. We consider the triangle obtained by excluding the site η in which one of the spinons is fixed. By removing down particles in this triangle, the spinon state is mapped onto a two-particle state of the form:

\[ \psi_{\eta_2} \propto z_1z_2 \left( \frac{\partial}{\partial z_1} - \bar{\eta}_2 \right) \left( \frac{\partial}{\partial z_2} - \bar{\eta}_2 \right) (z_1 - z_2)^2, \]  

where \( z_i = e^{\frac{2\pi i}{3} x_i} \), and \( x_i = 1, 2, 3 \) enumerates the sites of the triangle in consecutive order. The state (5) describes a fractional \( \frac{1}{3} \) Laughlin quasiparticle located at position \( \eta \). Since the addition of a complete boson is equivalent to a spin flip and creates an excitation with \( S^z = 1 \), the quasiparticle, which constitutes half a boson corresponds to a spinon, with \( S = 1/2 \).

Let us design an experiment to create spinons and probe their fractional statistics by detecting the non-orthogonality of the states \( |\Phi_{\eta_1, \eta_2}\rangle \). A state of this form can be prepared experimentally by starting with the valence bond state \( s^+_{\eta_1\eta_2} s^+_{x_1, x_2}|0\rangle \), created as explained above. If the spin of the particle at site \( \eta_1 \) is flipped this state is transformed into \( \left( a^+_{\eta_1} a^+_{\eta_2} a^+_{x_1} s^+_{x_2}|0\rangle \right) \), which by measuring the spin at \( \eta_1 \) in the \( z \) basis can be finally transformed into \( |\Phi_{\eta_1, \eta_2}\rangle \).

To probe the linear dependence of these three states, we project the down part of each of them onto the state \( \sum \alpha^*_{ij} |0\rangle \), a projection that will yield zero in all cases only for linearly dependent states.

**Paired state with d-wave symmetry.** Cuprate superconductors are known to exhibit pairing with \( d \)-wave symmetry [6]. A single pair with this exotic symmetry is described by the state:

\[ |\chi\rangle = \frac{1}{2} \left( s^+_{1,2} - s^+_{2,3} + s^+_{3,4} - s^+_{4,1} \right) |0\rangle. \]  

Let us design a scheme to create and detect this state. Starting with the four-particle state \( |\Phi_-\rangle \), we first remove particles with spin down to obtain the state \( \frac{1}{2} \left( t^+_{1,2} - t^+_{2,3} + t^+_{3,4} - t^+_{4,1} \right) |0\rangle \), a triplet pair with the desired \( d \)-wave symmetry. It is curious to see that the wave function describing this state, \( \propto z_1z_2(z_1 + z_2)^2(z_1 - z_2)^2 \), corresponds to an excited Laughlin state, with a quasiparticle excitation in the center of the plaquette (the factor \( z_1z_2 \)) and an excitation of the center of mass of two units of angular momentum (the factor \( (z_1 + z_2)^2 \)). This \( L_{cm} = 2 \) directly reflects the \( d \)-wave symmetry of the state. In order to achieve the state (6) the triplet pair has to be transformed into a singlet. This can be done by using the experimental techniques demonstrated in [36]. In order to reveal the \( d \)-wave character of the state (6) we propose a novel technique which exploits the connection between the symmetry of the state and the center of mass angular momentum of the pair. By inverting the process above we transform the state (6) into a spin polarized pair. We then merge the four sites of the plaquette into a single well and convert the pair into a molecule using a photoassociation technique [37]. Since angular momentum of the center of mass is conserved in the merging process, the molecule will carry two units of angular momentum, which will directly reflect the \( d \)-wave symmetry of the state (6).

The paired state with \( d \)-wave symmetry (6) can be transformed into a pair with a non-vanishing \( d \)-wave order parameter through e.g. admixture of a vacuum state. This could be done, by adiabatically increasing the tunnelling to an unoccupied layer of empty plaquettes, below or above the occupied plaquette layer.

**Ring-exchange interactions.** Lattice gauge theories [29] play an essential role in describing topological matter [3]. The minimum lattice gauge Hamiltonian describes a system of four spins in a plaquette and has the form:

\[ H_G = -J [S^x_1 S^x_2 + S^x_3 S^x_4] + J_+ \sum_{<i,j>} S^z_i S^z_j. \]  

It consists of four terms that commute with each other. The first one is a ring-exchange or flux interaction involving the four spins. For \( J_+ \) it favors symmetric states with respect to spin flipping of the whole plaquette. The other ones are charge interactions between neighboring
spins, which for \( J_+ > 0 \) favor states with anti-parallel neighboring spins. As for the case of an infinite lattice the elementary excitations of this Hamiltonian are anyons. Though this is a well known result, for the sake of clarity of our discussion below, let us first briefly explain it for the case of a single plaquette. The ground state of Hamiltonian (7) is a GHZ state of the form

\[
|\square\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\uparrow\downarrow\rangle + |\uparrow\uparrow\downarrow\downarrow\rangle),
\]

a maximally entangled state of four particles. It is indeed the minimum version of a string-net condensate [11], the ground state of \(|\square\rangle\) when extended to an infinite lattice. We can create two types of excitations on top of the state \(|\square\rangle\). They are flux-like or charge-like quasiparticles, (see Fig. 7) depending on which term of the Hamiltonian (7) is excited. For example, a flux-like excitation (fluxon), which we denote by \(|\Box\rangle\), has the form

\[
|\Box\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\uparrow\downarrow\rangle - |\uparrow\uparrow\downarrow\downarrow\rangle).
\]

It can be obtained by applying, for example, the operator \( S^x_1 \) to the state \(|\Box\rangle\). Charge-like excitations, in which two neighboring spins become parallel, are always created in pairs. For example, the state \(|\diamond\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\downarrow\downarrow\rangle + |\uparrow\downarrow\uparrow\downarrow\rangle)\) contains two charge-like quasiparticles, one at the 1-4 bond and the other at the 3-4 bond. This state is obtained by applying the operator \( S^y_1 \) to the state \(|\Box\rangle\). A charge-like quasiparticle can be moved around a flux-like one (see Fig. 8) by applying the ring operator \( S^z_1 S^z_2 S^z_3 S^z_4 |\Box\rangle = S^x_2 S^y_3 S^z_4 S^z_1 |\Box\rangle = -|\Box\rangle\), the wave function picks up a minus sign during the process. Therefore charges and fluxons are relative \( \pm \)-anyons in this model.

In our optical plaquette a Hamiltonian like (7) seems, in principle, is difficult to implement. The reason behind is that four-spin interactions result from fourth order processes (higher order terms denoted by dots in equation 2) in which four tunneling events occur. These processes are usually highly suppressed (~\( t^4/U^3 \)) compared to second order processes (~\( t^2/U \)), leading to dominant next neighbor superexchange interactions [23, 25]. Here, we present a scheme to suppress second order processes in a plaquette, obtaining a dominating four-body interaction. This will allow us to implement Hamiltonian (7) within a certain subspace of the spin Hilbert space.

We consider a situation in which we have applied a magnetic field gradient \( \Delta \) along one of the diagonals (e.g., 1-3) of the plaquette (see Fig. 9). If \( \Delta \gg 4t^2/U \), spin exchange interactions between neighboring sites are suppressed. The only remaining processes are either the ones in which the four spins in the plaquette are flipped (see Fig.) or those in which spins along the diagonal 2-4 are exchanged, giving rise to the Hamiltonian:

\[
H_R = -J_{\Box} (S^x_1 S^z_2 S^z_3 S^z_4 + H.c.) + J_+ \sum_{\langle i,j>} S^z_i S^z_j + J_x (S^z_2 S^z_4 + H.c.) - \Delta \sum_i B_i S^z_i,
\]

where \( J_{\Box} \approx 24t^4/U^3 \), \( J_+ \approx 4t^2/U \), \( J_x \approx 16t^4/U^3 \), and \( B_1 = 0, B_2 = B_4 = 1, B_3 = 2 \).

Within the subspace generated by the states \(|\uparrow\uparrow\rangle\) and \(|\uparrow\uparrow\uparrow\rangle\), Hamiltonian (9) is equivalent to Hamiltonian (7). The string-net condensed state \(|\Box\rangle\) and the flux excitations are also present in this model.
tion □ are therefore eigenstates of our system. Let us show how to prepare these states. We proceed as follows. The plaquette is initially prepared in the state |↑↑↑↑⟩. This can be done by starting with the state |↑↑↑↑⟩ and then spin flipping atoms on the diagonal sites by addressing them with the scheme presented in Fig. 11. In the presence of the magnetic field gradient, the system will evolve under Hamiltonian (7), oscillating between the states |↑↑↑↑⟩ and |↑↑↑↑⟩ with a frequency ω = Jz/ℏ. For typical experimental parameters this frequency is of the order of 20 Hz, and can be resolved experimentally [22, 23]. After an evolution time $T = π/4ω$ the system will be prepared in the state $\frac{1}{\sqrt{2}} (|↑↑↑↑⟩ + i|↑↑↑↑⟩)$, a maximally entangled state that can be easily transformed into either □ or □. This can be done by applying the local phase operator $R_{θ} = e^{iθ(S_{1}^z + S_{2}^z)}$, with $θ = π/4(−π/4)$, which is performed by addressing sites 1 and 3, and letting the system evolve in the presence of a magnetic field $B$ in the z direction for a time $T = θℏ/B$. We can use Hamiltonian (7) together with local addressability of the plaquette sites to artificially create and detect the anyonic quasiparticles of Hamiltonian (7). Our proposal has the same spirit of the one recently proposed in [38], where anyonic states are artificially encoded using four photons. Even though anyonic states are not eigenstates of our system the preparation and detection scheme we present here can be used in cases in which the Hamiltonian (7) may be achieved using other methods [19]. Our scheme follows the idea proposed in [31] for anyon detection in small rotating atomic gases. We prepare the system in the state $\frac{1}{\sqrt{2}} (|□⟩ − i|□⟩)$, a superposition of a non-excited and a flux-excited plaquette. Such superposition state results indeed from time evolution of the state |↑↑⟩ under Hamiltonian (7) after a time $T = π/4ω$. b) Statistical phase accumulation. We then excite a pair of charge-like excitations and move one of them around the plaquette. This operation is performed by the operator $S_{1}^{−}S_{2}^{−}S_{3}^{−}S_{4}^{−}$ (see Fig. 5), which we apply by subsequently addressing and acting on each site of the plaquette. Because of the relative $\frac{1}{2}$-statistical phase of anyons, the state |□⟩ will pick up a minus sign, and the system will end up in the state $\frac{1}{\sqrt{2}} (|□⟩ + i|□⟩)$. c) Detection. We finally let the system evolve under Hamiltonian (7) for a time $T = π/4ω$, obtaining the final state |↑↑⟩. If the excitations happened to be bosons or fermions with trivial statistics, the final state would have been |↑↑⟩. These two states can be easily discriminated by, for example, measuring $S_{1}^{z}$. In conclusion, we have presented a collection of schemes to create and detect instances of topological order in a minimum system: a plaquette filled with two or four particles in an optical lattice potential. Many of these could be directly implemented in current experiments using the presently available manipulation and detection techniques. Furthermore, the plaquette Hamiltonians we have considered could be used as unit operations to, together with an increased optical resolution to resolve individual plaquettes, engineer large scale topological liquids.

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