SCOPE: Scalable Composite Optimization for Learning on Spark

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Abstract

Many machine learning models, such as logistic regression (LR) and support vector machines (SVM), can be formulated as composite optimization problems. Recently, many distributed stochastic optimization (DSO) methods have been proposed to solve the large-scale composite optimization problems, which have shown better performance than traditional batch methods. However, most of these DSO methods are not scalable enough. In this paper, we propose a novel DSO method, called scalable composite optimization for learning (SCOPE), and implement it on the fault-tolerant distributed platform Spark. SCOPE is both computation-efficient and communication-efficient. Theoretical analysis shows that SCOPE is convergent with linear convergence rate when the objective function is convex. Furthermore, empirical results on real datasets show that SCOPE can outperform other state-of-the-art distributed learning methods on Spark, including both batch learning methods and DSO methods.

1. Introduction

Many machine learning models can be formulated as composite optimization problems which have the following form with finite sum of some functions:

$$\min_{w \in \mathbb{R}^d} P(w) = \frac{1}{n} \sum_{i} f_i(w),$$

where \( w \) is the parameter to learn (optimize), \( n \) is the number of training instances, and \( f_i(w) \) is the loss function on the training instance \( i \). For example, \( f_i(w) = \log(1 + e^{-y_i x_i^T w}) + \frac{\lambda}{2} \| w \| $$ in logistic regression (LR), and \( f_i(w) = \max\{0, 1 - y_i x_i^T w\} + \frac{\lambda}{2} \| w \| $$ in support vector machine (SVM), where \( \lambda \) is the regularization hyper-parameter and \((x_i, y_i)\) is the training instance \( i \) with \( x_i \in \mathbb{R}^d \) being the feature vector and \( y_i \in \{+1, -1\} \) being the class label. Furthermore, the matrix factorization model (Koren et al., 2009) \( P(U, V) = \sum_{i=1,j=1}^n \frac{1}{2} (R_{ij} - (U^TV)_{ij})^2 \) also has the composite form where the product between \( U \) and \( V \) is used to approximate the original (low-rank) matrix \( R = [R_{ij}]_{i,j=1}^n \). Other more complex cases like deep neural network (DNN) and convolutional neural network (CNN) (Krizhevsky et al., 2012) can also be written as similar forms of composite optimization.

Due to its efficiency and effectiveness, stochastic optimization (SO) has recently attracted much attention to solve the composite optimization problems in machine learning (Zhang, 2004; Xiao, 2009; Bottou, 2010; Duchi et al., 2011; Schmidt et al., 2013; Johnson & Zhang, 2013; Zhang et al., 2013; Shalev-Shwartz & Zhang, 2012; 2014; Lin et al., 2014b; Nitanda, 2014). Existing SO methods can be divided into two categories. The first category is the stochastic gradient descent (SGD) and its variants, such as stochastic average gradient (SAG) (Schmidt et al., 2013) and stochastic variance reduced gradient (SVRG) (Johnson & Zhang, 2013), which try to perform optimization on the primal problem. The second category, such as stochastic dual coordinate ascent (SDCA) (Shalev-Shwartz & Zhang, 2012), tries to perform optimization with the dual formulation. Many advanced SO methods, such as SVRG and SDCA, are more efficient than traditional batch learning methods both in theory and in practice for many large-scale machine learning problems. Although the convergence of most SO methods can only be theoretically guaranteed for convex problems, SO methods have also been widely used in non-convex optimization problems like matrix factorization (Koren et al., 2009) and deep neural networks (Krizhevsky et al., 2012) with promising performance in real applications.
Most traditional SO methods are sequential which means that the optimization procedure is not parallely performed. However, with the increase of data scale, traditional sequential SO methods may not be efficient enough to handle large-scale datasets. Furthermore, in this big data era, many large-scale datasets are distributively stored on a cluster of multiple machines due to the limited storage space on a single machine. Traditional sequential SO methods cannot be directly used for these kinds of distributed datasets. To handle large-scale composite optimization problems, researchers have recently proposed several parallel SO (PSO) methods for multi-core systems and distributed SO (DSO) methods for clusters.

PSO methods perform SO on a single machine with multi-cores (multi-threads) and a shared memory. In PSO, multi-workers read parameters from the shared memory, compute one update vector, and then update the parameters in the shared memory. Besides the computation cost, the inter-thread communication cost will also affect the efficiency of the algorithms. Typically, synchronous strategies with locks will be much slower than asynchronous ones. Hence, recent progress of PSO mainly focuses on designing asynchronous or lock-free optimization strategies (Recht et al., 2011; Liu et al., 2014; Hsieh et al., 2015; J. Reddi et al., 2015; Zhao & Li, 2015; 2016).

DSO methods perform SO on clusters of multiple machines. DSO can be used to handle extremely large problems which are beyond the processing capability of one single machine. In many real applications especially industrial applications, the datasets are typically distributively stored on clusters. Hence, DSO has recently become a hot research topic. Many DSO methods have been proposed, including distributed SGD methods from primal formulation and distributed dual formulation. Representative distributed SGD methods include PSGD (Zinkevich et al., 2010), BAVGM (Zhang et al., 2012) and Splash (Zhang & Jordan, 2015). Representative distributed dual formulations include DisDCA (Yang, 2013), CoCoA (Jaggi et al., 2014) and CoCoA+(Ma et al., 2015). Many of these methods provide nice theoretical proof about convergence and promising empirical evaluations. However, most of these DSO methods are not scalable enough.

In this paper, we propose a novel DSO method, called scalable composite optimization for learning (SCOPE), and implement it on the fault-tolerant distributed platform Spark (Zaharia et al., 2010). The main contributions of SCOPE are outlined as follows:

- SCOPE is based on the master-slave (parameter server) framework. In each epoch where all the training instances are visited once, all the workers (slave machines) separately solve the subproblems only based on the local data native to each machine before communication. As a result, only one communication is needed for each epoch. Hence, SCOPE is communication-efficient.
- Empirical results on real datasets show that SCOPE can outperform other state-of-the-art distributed learning methods on Spark, including both batch learning methods and DSO methods, in terms of scalability.

Please note that some asynchronous methods or systems, such as Parameter Server (Li et al., 2014), Petuum (Xing et al., 2015) and the methods in (Zhang & Kwok, 2014; Zhang et al., 2015), have also been proposed for distributed learning with promising performance. But these methods or systems cannot easily be implemented on Spark with the MapReduce programming model. Hence, these kinds of asynchronous methods are not the focus of this paper. We will leave the design of asynchronous version of SCOPE and the corresponding empirical comparison for future study.

2. SCOPE: Scalable Composite Optimization for Learning

In this section, we present the algorithmic details of SCOPE, and discuss its implementation on Spark.

2.1. Framework of SCOPE

SCOPE is based on a master-slave distributed framework, which is illustrated in Figure 1. More specifically, there is a master machine (called Master) and \( p \) \((p \geq 1)\) slave machines (called Workers) in the cluster. These Workers are called Worker\(_1\), Worker\(_2\), \ldots, and Worker\(_p\), respectively.

![Figure 1. Distributed framework of SCOPE.](image-url)
2.1.1. Data Partition and Parameter Storage

- For Workers: The whole dataset $D$ is distributively stored on all the Workers. More specifically, $D$ is partitioned into $p$ subsets, which are denoted as $\{D_1, D_2, \cdots, D_p\}$ with $D = \bigcup_{k=1}^p D_k$. $D_k$ is stored on Worker $k$. The data stored on different Workers are different from each other, which means that if $i \neq j$, $D_i \cap D_j = \emptyset$.

- For Master: The parameter $w$ is stored on the Master and the Master always keeps the newest version of $w$.

Different Workers can not communicate with each other, which means that if $i \neq j$, Worker $j$ has no access to the data on Worker $i$. This is similar to most existing distributed learning frameworks like MLlib, Splash, Parameter Server, and CoCoA and so on.

2.1.2. Optimization Algorithm

The whole optimization (learning) algorithm is completed cooperatively by the Master and Workers:

- Task of Master: The operations completed by the Master are outlined in Algorithm 1. We can find that the master has two main tasks. The first task is to compute the full gradient after all the local gradient sum $\{z_k\}$ have been received from all Workers, and then send the full gradient to all Workers. The second task is to update the parameter $w$ after all the locally updated parameters $\{\tilde{u}_k\}$ have been received, and then send the updated parameter to all Workers. It is easy to see that the computation load of the Master is lightweight.

- Task of Workers: The operations completed by the Workers are outlined in Algorithm 2. We can find that each Worker has two main tasks. The first task is to compute the sum of the gradients on its local data (called local gradient sum), i.e., $z_k = \sum_{i \in D_k} \nabla f_i(w)$ for Worker $k$, and then send the local gradient sum to the Master. The second task is to train $w$ by using only the local data, after which the Worker will send the locally updated parameters, denoted as $\tilde{u}_k$, to the Master and wait for the newest $w$ from Master.

Here, $w_t$ denotes the global parameter at the $t$th iteration and is stored on the Master. $u_{k,m}$ denotes the local parameter at the $m$th iteration on Worker $k$.

SCOPE is inspired by SVRG (Johnson & Zhang, 2013) which tries to utilize full gradient to speed up the convergence of stochastic optimization. However, the original SVRG in (Johnson & Zhang, 2013) is sequential and to the best of our knowledge there has not appeared any distributed SVRG method in the literature. One natural strategy to design a distributed SVRG method is to adapt the mini-batch SVRG to distributed settings, which is a typical strategy in most distributed SGD frameworks like Parameter Server (Li et al., 2014) and Petuum (Xing et al., 2015). In Appendix A.1, we briefly outline the sequential SVRG (Algorithm 3) and the mini-batch based distributed SVRG (Algorithm 4 and Algorithm 5). We can find that there exist three major differences between SCOPE and SVRG (or distributed SVRG).

The first difference is that in SCOPE each Worker locally performs stochastic optimization by using its native data (the update on $u_{k,m+1}$ for each Worker $k$ in Algorithm 2). On the contrary, SVRG or the distributed SVRG perform stochastic optimization on the Master (the update on $u_{m+1}$ in Algorithm 4) based on the whole dataset, which means that we need to randomly pick up an instance or a mini-batch from the whole dataset $D$ in each iteration.

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**Algorithm 1** Task of Master in SCOPE

Initialization: $p$ Workers, $w_0$.

for $t = 0, 1, 2, \ldots , T$

Send $w_t$ to the Workers;

Wait until it receives $z_1, z_2, \ldots, z_p$ from the $p$ Workers;

Compute the full gradient $z = \frac{1}{p} \sum_{k=1}^p z_k$, and then send $z$ to each Worker;

Wait until it receives $\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_p$ from the $p$ Workers;

Compute $w_{t+1} = \frac{1}{p} \sum_{k=1}^p \tilde{u}_k$;

end for

**Algorithm 2** Task of Workers in SCOPE

Initialization: initialize $\eta$ and $c$.

For the Worker $k$:

for $t = 0, 1, 2, \ldots , T$

Wait until it gets the newest parameter $w_t$ from the Master;

Let $u_{k,0} = w_t$, compute the local gradient sum $z_k = \sum_{i \in D_k} \nabla f_i(w_t)$, and then send $z_k$ to the Master;

Wait until it gets the full gradient $z$ from the Master;

for $m = 0$ to $M - 1$

Randomly pick up an instance with index $i_{k,m}$ from $D_k$:

$u_{k,m+1} = u_{k,m} - \eta(\nabla f_{i_{k,m}}(u_{k,m}) - \nabla f_{i_{k,m}}(w_t)) + z + c(u_{k,m} - w_t)$;

end for

Send $u_{k,M}$ or the average of these $\{u_{k,m}\}$, which is called the locally updated parameter and denoted as $\tilde{u}_k$, to the Master;

end for
of stochastic optimization. The *locally stochastic optimization* in SCOPE can dramatically reduce the communication cost, compared with the distributed SVRG with mini-batch strategy. This will be further discussed in Section 2.3.

The second difference is the update rule of $\mathbf{w}_{t+1}$ in the Master. There are no locally updated parameters in the distributed SVRG with mini-batch strategy, and hence the update rule of $\mathbf{w}_{t+1}$ in the Master for distributed SVRG can not be written in the form of Algorithm 1, i.e., $\mathbf{w}_{t+1} = \frac{1}{p} \sum_{p} \mathbf{u}_{t}$. The third difference is the update rule for $\mathbf{u}_{k,m+1}$ in SCOPE and $\mathbf{u}_{m+1}$ in SVRG or distributed SVRG. Compared to SVRG, SCOPE has an extra term $c(\mathbf{u}_{k,m} - \mathbf{w}_{t})$ with $c > 0$ in Algorithm 2 to guarantee convergence. The strictly theoretical proof will be provided in Section 3. Here, we just give some intuition about the extra term $c(\mathbf{u}_{k,m} - \mathbf{w}_{t})$. Since SCOPE puts no constraints about how to partition training data on different Workers, the data distributions on different Workers may be totally different from each other. That means the the local gradient in each Worker can not necessarily approximate the full gradient. Hence, the term $\nabla f_{k,m} (\mathbf{u}_{k,m}) - \nabla f_{k,m} (\mathbf{w}_{t}) + \mathbf{z}$ is a bias estimation of the full gradient. This is different from SVRG whose stochastic gradient $\nabla f_{k,m} (\mathbf{u}_{m}) - \nabla f_{k,m} (\mathbf{u}_{0}) + \mathbf{z}$ in Algorithm 3 is an unbiased estimation of the full gradient. The bias estimation $\nabla f_{k,m} (\mathbf{u}_{k,m}) - \nabla f_{k,m} (\mathbf{w}_{t}) + \mathbf{z}$ in SCOPE may lead $\mathbf{u}_{k,m+1}$ to be far away from the optimal value $\mathbf{w}^*$. To avoid this, we introduce the technique in the proximal stochastic gradient that adds an extra term $c(\mathbf{u}_{k,m} - \mathbf{w}_{t})$ so that $\mathbf{u}_{k,m+1}$ will not be far away from $\mathbf{w}_{t}$. If $\mathbf{w}_{t}$ is close to $\mathbf{w}^*$, $\mathbf{u}_{k,m+1}$ will also be close to $\mathbf{w}^*$. So the extra term in SCOPE is reasonable for convergence guarantee. At the same time, it would not bring extra computation since the update rule in SCOPE can be rewritten as

$$\mathbf{u}_{k,m+1} = (1 - c\eta)\mathbf{u}_{k,m} - \eta(\nabla f_{k,m}(\mathbf{u}_{k,m}) - \nabla f_{k,m}(\mathbf{w}_{t}) + \mathbf{z}),$$

where $\mathbf{z} = \mathbf{z} - c\mathbf{w}_{t}$ can be pre-computed and fixed as a constant for different $m$.

### 2.2. Mini-Batch for Local Update

During the local update procedure of the Workers in Algorithm 2, the term $\nabla f_{k,m} (\mathbf{u}_{k,m}) - \nabla f_{k,m} (\mathbf{w}_{t}) + \mathbf{z} + c(\mathbf{u}_{k,m} - \mathbf{w}_{t})$ is always a dense vector whatever the training data is sparse or dense because of the inclusion of full gradient. When SCOPE is used for sparse data with very high dimensionality, we can compute a mini-batch of stochastic gradients, instead of computing one gradient using only one training instance, to speedup the local update of each Worker. The update rule can be written as follows:

$$\mathbf{u}_{k,m+1} = \mathbf{u}_{k,m} - \eta(\nabla f_{S_{k,m}} (\mathbf{u}_{k,m}) - \nabla f_{S_{k,m}} (\mathbf{w}_{t}) + \mathbf{z} + c(\mathbf{u}_{k,m} - \mathbf{w}_{t})), $$

where $S_{k,m}$ is a mini-batch of instances randomly picked-up from the local data $D_{k}$, and $\nabla f_{S_{k,m}}(\cdot)$ is the the average sum of gradients computed on the mini-batch $S_{k,m}$.

### 2.3. Communication Cost

Traditional mini-batch based distributed SGD methods, such as the distributed SVRG in Appendix A.1, need to transfer parameter $w$ and stochastic gradients frequently between Workers and Master. For example, the number of communication times is $O(TM)$ for the distributed SVRG in Appendix A.1. Other traditional mini-batch based distributed SGD methods have the same number of communication times. Typically, $M = \Theta(n)$. Hence, traditional methods have $O(Tn)$ number of communication times, which may lead to relatively high communication cost.

Most training (computation) load of SCOPE comes from the inner loop of Algorithm 2, which is done at local Worker without any communication. It is easy to find that the number of communication times in SCOPE is $O(T)$, which is dramatically less than $O(Tn)$ of traditional mini-batch based distributed SGD methods. In Section 3, we will prove that SCOPE has a linear convergence rate in terms of the iteration number $T$. Hence, $T$ is typically not large for many problems. For example, in most of our experiments, we can achieve convergent results with $T \leq 10$. Hence, SCOPE is communication-efficient. SCOPE is a synchronous framework, which means that some waiting time is also needed for synchronization. Because the number of synchronization is also $O(T)$, and $T$ is typically a small number. Hence, the waiting time is also small.

### 2.4. SCOPE on Spark

One interesting thing is that the computing framework of SCOPE is quite suitable for the popular distributed platform Spark. The programming model underlying Spark is MapReduce. In SCOPE, the task of Workers that computing *local gradient sum* $\mathbf{z}_{k}$ and training procedure in the inner loop of Algorithm 2 can be seen as the Map process since both of them only use local data. The task of Master that computing the average for both *full gradient* $\mathbf{z}$ and $\mathbf{w}_{t+1}$ can be seen as Reduce process.

The MapReduce programming model is essentially a synchronous model, which need some synchronization cost.  

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1This mini-batch strategy is typically used to update the global parameter on Master (like that in Algorithm 4), which is different from our mini-batch strategy in Section 2.2 which is used for local update on Workers.
Fortunately, the number of synchronization times is very small as stated in Section 2.3. Hence, both communication cost and waiting time are very small for SCOPE. In this paper, we implement our SCOPE on Spark since Spark has been widely adopted in industry for big data applications, and hence our SCOPE can be easily integrated into the data processing pipeline of those organizations using Spark.

3. Convergence of SCOPE

In this section, we will prove the convergence of SCOPE when the objective functions are convex. We only list some Lemmas and Theorems, the detailed proof of which can be found in the Appendix.

For convenience, we use $w^*$ to denote the optimal solution. $\| \cdot \|$ denotes the $L_2$ norm $| \cdot |_2$. We assume that $n = pq$, which means that each Worker has the same number of training instances and $|D_1| = |D_2| = \cdots = |D_p| = q$. In practice, we can not necessarily guarantee that these $|D_k|$s are the same. However, it is easy to guarantee that $\forall i,j, ([|D_i| − |D_j|]) \leq 1$, which will not affect the performance.

We define $p$ local functions as follows:

$$F_k(w) = \frac{1}{q} \sum_{i \in D_k} f_i(w),$$

where $k = 1, 2, \ldots, p$.

Then we have

$$P(w) = \frac{1}{p} \sum_{k=1}^p F_k(w).$$

3.1. Preliminary

Here we give some assumptions for our convergence proof.

**Assumption 1** (Smooth Gradient). There exists a constant $L > 0$ such that $\forall x, y \in \mathbb{R}^d$ and $i = 1, 2, \ldots, n$, we have

$$\| \nabla f_i(y) − \nabla f_i(x) \| \leq L \| y − x \|.$$

**Assumption 2** (Strongly Convex). For each local function $F_k(\cdot)$, there exists a constant $\mu_k > 0$ such that $\forall x, y \in \mathbb{R}^d$, we have

$$F_k(y) \geq F_k(x) + \nabla F_k(x)^T(y − x) + \frac{\mu_k}{2} \| y − x \|^2.$$

These two assumptions are common ones, which have also been adopted by most existing stochastic optimization algorithms to prove the convergence. Let $\mu = \min\{\mu_1, \mu_2, \ldots, \mu_p\}$. We define the local stochastic gradient in Algorithm 2 as follows:

$$v_{k,m} = \nabla f_{ik,m}(u_{k,m}) − \nabla f_{ik,m}(w_t) + z + c(u_{k,m} − w_t).$$

Then the update rule at local Workers can be rewritten as follows:

$$u_{k,m+1} = u_{k,m} − \eta v_{k,m}. \quad (2)$$

3.2. Proof of Convergence

First, we give the expectation and variance property of $v_{k,m}$ in Lemma 1 and Lemma 2:

**Lemma 1.** The conditional expectation of local stochastic gradient $v_{k,m}$ on $u_{k,m}$ is

$$E[v_{k,m}|u_{k,m}] = \nabla F_k(u_{k,m}) − \nabla F_k(w_t) + z + c(u_{k,m} − w_t).$$

**Lemma 2.** The variance of $v_{k,m}$ has the following property:

$$E[(v_{k,m}|u_{k,m})^2] \leq 3(L^2 + c^2)\|u_{k,m} − w_t\|^2 + 3L^2\|w_t − w^*\|^2.$$

According to Lemma 1, $v_{k,m}$ is a bias estimation of the full gradient. However, $E[v_{k,m}|u_{k,m}]$ contains the full gradient $z$, which reflects the global information.

According to Lemma 2, if both $w_t$ and $u_{k,m}$ are close to $w^*$, both $E[(v_{k,m}|u_{k,m})^2]$ and $E[v_{k,m}|u_{k,m}]$ would decrease to zero. So SCOPE is still a variance reduction method, which is the same as SVRG. Our experiments also confirm this variance reduction property.

**Theorem 1.** Let $\gamma_m = \frac{1}{p} \sum_{k=1}^p E\|u_{k,m} − w^*\|^2$, then we have

$$\gamma_{m+1} \leq \gamma_m − \eta(2\mu + c)\gamma_m + (c\eta + 3L^2\eta^2)\gamma_0.$$

Let $\alpha = 1 − (2\mu + c) < 1$, $\beta = c\eta + 3L^2\eta^2$ and $\alpha + \beta < 1$. We have the following corollaries:

**Corollary 1.** If we take $w_{t+1} = \frac{1}{p} \sum_{k=1}^p u_{k,M}$, then we can get the following convergence result:

$$E\|w_{t+1} − w^*\|^2 \leq (\alpha^M + \frac{\beta}{1 − \alpha})E\|w_t − w^*\|^2.$$

**Corollary 2.** If we take $w_{t+1} = \frac{1}{p} \sum_{k=1}^p \hat{u}_k$ with $\hat{u}_k = \frac{1}{M} \sum_{m=1}^M u_{k,m}$, then we can get the following convergence result:

$$E\|w_{t+1} − w^*\|^2 \leq (\frac{1}{M(1 − \alpha)} + \frac{\beta}{1 − \alpha})E\|w_t − w^*\|^2.$$

According to Corollary 1 and Corollary 2, we can find that SCOPE gets a linear convergence rate for a large $M$. In fact, we can get the following weaker corollary from Theorem 1:
Corollary 3. ∀ M ≥ 1, if we take \( w_{t+1} = \frac{1}{p} \sum_{k=1}^{p} u_{k,M} \), then we can get the following convergence result:

\[
\mathbb{E}[\|w_{t+1} - w^*\|^2] < \mathbb{E}[\|w_t - w^*\|^2].
\]

**Proof.** We prove it by induction. When \( m = 0 \), we can get that \( \gamma_1 < \gamma_0 \) since \( \alpha + \beta < 1 \). Assuming that for \( m = M_0 \), we have \( \gamma_m < \gamma_0 \). Then it is easy to get that \( \gamma_{m+1} < \alpha \gamma_m + \beta \gamma_0 < \alpha \gamma_0 + \beta \gamma_0 < \gamma_0 \). Subsequently, we can get that \( \forall m \geq 1, \gamma_m < \gamma_0 \).

Since \( w_{t+1} = \frac{1}{p} \sum_{k=1}^{p} u_{k,M} \), we can get the final result by the Jensen’s inequality. □

Corollary 3 means that the average of \( u_{k,m} \) is always closer to \( w^* \) than \( w_t \).

4. Experiment

We choose logistic regression (LR) with a \( L_2 \)-norm regularization term to evaluate SCOPE and baselines. Hence, \( P(w) \) is defined as

\[
P(w) = \frac{1}{n} \sum_{i=1}^{n} \left[ \log(1 + e^{-y_i x_i^T w}) + \frac{\lambda}{2} \|w\|^2 \right].
\]

4.1. Dataset

We use four datasets for evaluation. They are MNIST-8M, epsilon, KDD12 and Data-A. The first two datasets can be downloaded from the LibSVM website. MNIST-8M contains 8,100,000 handwritten digits. We set the instances of digits 5 to 9 as positive, and set the instances of digits 0 to 4 as negative. KDD12 is the dataset of Track 1 for KDD Cup 2012, which can be downloaded from the KDD Cup website. Data-A is a dataset from one data mining competition. The information about these datasets is summarized in Table 1. All the data is normalized before training. The regularization hyper-parameter \( \lambda \) is set to \( 10^{-4} \) for the first three datasets which are relatively small, and is set to \( 10^{-6} \) for the largest dataset Data-A. Similar phenomenon can be observed for other \( \lambda \), which is omitted due to space limitation. For all datasets, we set \( \epsilon = 10 \times 10^{-2} \).

| Dataset   | \#instances | \#features | memory | \lambda |
|-----------|-------------|------------|--------|---------|
| MNIST-8M  | 8,100,000   | 784        | 39G    | 1e-4    |
| epsilon   | 400,000     | 2,000      | 11G    | 1e-4    |
| KDD12     | 73,209,277  | 1,427,495  | 21G    | 1e-4    |
| Data-A    | 106,919,093 | 320        | 260G   | 1e-6    |

4.2. Experimental Setting and Baseline

4.2.1. Distributed Platform

We have a Spark cluster of 33 machines (nodes) connected by 10GB Ethernet. Each machine has 12 Intel Xeon E5-2620 cores with 64GB memory. We construct two clusters, a small one and a large one, from the original 33 machines for our experiments. The small cluster contains 9 machines, one master and eight slaves. We use 2 cores for each slave. The large cluster contains 33 machines, 1 master and 32 slaves. We use 4 cores for each slave. In both clusters, each machine has access to 64GB memory on the corresponding machine and one core corresponds to one Worker. Hence, the small cluster has one Master and 16 Workers, and the large cluster has one Master and 128 Workers. The small cluster is for experiments on the three relatively small datasets including MNIST-8M, epsilon and KDD12. The large cluster is for experiments on the largest dataset Data-A. We use Spark1.5.2 for our experiment, and implement our SCOPE in Scala.

4.2.2. Baseline

Because the focus of this paper is to design distributed learning methods for Spark, we compare SCOPE with distributed learning baselines which can be implemented on Spark. More specifically, we adopt the following baselines for comparison:

- **MLlib** (Meng et al., 2015): MLlib is an open source library for distributed machine learning on Spark. It is mainly based on two optimization methods: mini-batch based distributed SGD and distributed lfbgs. We find that the distributed SGD method is much slower than distributed lfbgs on Spark in our experiments. Hence, we only compare our method with distributed lfbgs for MLlib, which is a batch learning method.

- **LibLinear** (Lin et al., 2014a): LibLinear is a distributed Newton method, which is also a batch learning method.

- **Splash** (Zhang & Jordan, 2015): Splash is a distributed SGD method by using the local learning strategy to reduce communication cost (Zhang et al., 2012), which is different from the mini-batch based distributed SGD method.

- **CoCoA** (Jaggi et al., 2014): CoCoA is a distributed dual coordinate ascent method by using the local learning strategy to reduce communication cost, which is

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1. https://www.csie.ntu.edu.tw/~cjlin/libsvmtools/datasets/
2. https://www.kddcup2012.org/
3. http://www.cjlin.github.io/liblinear/
4. http://zhangyuc.github.io/splash
5. http://spark.apache.org/mllib/
6. http://github.com/gingsmith/cocoa
formulated from the dual problem.

- CoCoA+\(^9\) (Ma et al., 2015): CoCoA+ is an improved version of CoCoA. Different from CoCoA which adopts average to combine local updates for global parameters, CoCoA+ adopts adding to combine local updates.

We can find that the above baselines include state-of-the-art distributed learning methods with different characteristics. All the authors of these methods have shared the source code of their methods to the public. We use the source code provided by the authors for our experiment. For all baselines, we try several parameter values to choose the best performance.

### 4.3. Efficiency Comparison with Baselines

We compare SCOPE with other baselines on the four datasets. The result is shown in Figure 2. Each marked point on the curves denotes one update for \( w \) by the Master, which typically corresponds to an iteration in the outer-loop. For SCOPE, good convergence results can be got with number of updates (i.e., the \( T \) in Algorithm 1) less than five. We can find that Splash vibrates on some datasets since it introduces variance in the training process. On the contrary, SCOPE are stable, which means that SCOPE is a variance reduction method like SVRG. It is easy to see that SCOPE have a linear convergence rate, which also conforms to our theoretical analysis. Furthermore, SCOPE are much faster than all the other baselines.

Our SCOPE can also outperform SVRGfoR (Konecný et al., 2015) and DisSVRG in Algorithm 4 and Algorithm 5. Detailed experimental comparisons are moved to the supplementary materials for space saving.

### 4.4. Speedup

We use dataset MNIST-8M for speedup evaluation of SCOPE. Other datasets have similar results, which are omitted here. We use 2 cores for each machine. We evaluate speedup by increasing the number of machines. We stop training when the gap between the objective function value and the optimal value is less than \( 10^{-10} \). The speedup is defined as follows: 

\[
\text{speedup} = \frac{\text{time with } 16 \text{ cores by SCOPE}}{\text{time with } 2 \text{ cores}}
\]

where \( \pi \) is the number of machines and we choose \( \pi = 8, 16, 24, 32 \). We perform the experiments by 5 times and take the average time for the final speedup result.

The speedup result is shown in Figure 3 (a), where we can find that SCOPE has a super-linear speedup. This might be reasonable due to the higher cache hit ratio with more machines (Yu et al., 2014). This speedup result is quite promising on our multi-machine settings since the communication cost is much larger than that of multi-thread setting. The good speedup of SCOPE can be explained by the fact that most training work can be locally completed by each Worker and SCOPE does not need much communication cost. This will be further verified in the following section about synchronization cost.

### 4.5. Synchronization Cost

SCOPE is based on the synchronous MapReduce framework of Spark. One shortcoming of synchronous framework is that there exists synchronization cost besides the computation cost. The synchronization cost includes both communication time and waiting time. Fortunately, the synchronization cost of SCOPE is low because most computation is completed locally and only a small number of synchronization times is needed. Here, we use experiment to verify this.
We use the dataset MNIST-8M for evaluation. The result is shown in Figure 3 (b). The x-axis is the number of cores, the y-axis is the CPU time (in millisecond) per iteration, which is computed as dividing the total time by the number of iterations $T$. Please note that the CPU time includes both computation time and synchronization time (cost). During the training process, if the Workers or Master are computing, we consider the time as computation time. In each synchronization step, we consider the time gap between the completion of the fastest Worker and the slowest Worker as waiting time. If there is communication between Workers and Master, we consider the time as communication time. From Figure 3 (b), we can find that SCOPE does not take too much waiting time and communication time compared to the computation time. We can also find that with the increase of the number of Workers (cores), the synchronization time per iteration does not increase too much, which is promising for distributed learning on clusters with multiple machines.

The speedup in Figure 3 (b) seems to be smaller than that in Figure 3 (a). Both Figure 3 (a) and Figure 3 (b) are correct. Some Workers still perform computation during the waiting time. So there is a repeating part in the waiting time and computation time in Figure 3 (b). Furthermore, the total number of iterations to achieve the same objective value may not be the same for different number of cores.

5. Conclusion

In this paper, we propose a novel distributed stochastic optimization method, called SCOPE, for distributed machine learning on Spark. Theoretical analysis shows that SCOPE is convergent with linear convergence rate for convex cases. Empirical results show that SCOPE can outperform other state-of-the-art distributed methods on Spark, including both batch methods and stochastic methods.

Although this paper focuses on distributed machine learning methods on Spark, our SCOPE can also be implemented on other distributed platforms like MPI clusters, which will be pursued in our future work. Furthermore, it is also interesting to design asynchronous versions of SCOPE. One possible solution is to adapt the idea of the multi-thread asynchronous SVRG in (Zhao & Li, 2016) to SCOPE. In addition, it is also meaningful to perform empirical comparison between the asynchronous SCOPE and existing asynchronous DSO methods like Parameter Server (Li et al., 2014), Petuum (Xing et al., 2015), and the method in (Zhang et al., 2015).

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A. Appendix

A.1. SVRG

The sequential SVRG is outlined in Algorithm 3, which is the same as the original SVRG in (Johnson & Zhang, 2013).

Algorithm 3 Sequential SVRG

Initialization: $w_0, \eta$;

for $t = 0, 1, 2, \ldots$ do

$w_0 = w_t$;

Compute the full gradient $z = \frac{1}{n} \sum_{i=1}^{n} \nabla f_i(u_0)$;

for $m = 0$ to $M$ do

Randomly pick up an $i_m$ from $\{1, \ldots, n\}$;

$u_{m+1} = u_m - \eta(\nabla f_{i_m}(u_m) - \nabla f_{i_m}(u_0) + z)$;

end for

Take $w_{t+1}$ to be $u_M$ or the average of $\{u_m\}$;

end for

The mini-batch based distributed SVRG is outlined in Algorithm 4 and Algorithm 5, with Algorithm 4 for the operations completed by the Master and Algorithm 5 for the operations completed by the Workers.

Algorithm 4 Task of Master in Distributed SVRG

Initialization: $p$ Workers, $w_0, \eta$;

for $t = 0, 1, 2, \ldots, T$ do

$u_0 = w_t$;

Send $w_t$ to the Workers;

Wait until it receives $z_1, z_2, \ldots, z_p$ from the $p$ Workers;

Compute the full gradient $z = \frac{1}{p} \sum_{k=1}^{p} z_k$;

for $m = 0$ to $M$ do

Randomly pick up a mini-batch indices $S_m$ from $\{1, \ldots, n\}$;

According to the partition of the whole dataset, $S_m$ can be partitioned into $p$ subsets, denoted as $\{S_m,k\}_{k=1}^{p}$, where $S_m,k$ denotes the indices of the instances located in Worker $k$ and picked-up in $S_m$. Send $\{S_m,k\}$ to all Workers with $S_m,k$ for Worker $k$;

Send $u_m$ to all Workers;

Wait until it receives $\{\nabla f_{S_m,k}(u_m)\}$ and $\{\nabla f_{S_m,k}(w_t)\}$ from all the Workers;

Compute $\nabla f_{S_m}(u_m) = \sum_{k=1}^{p} \nabla f_{S_m,k}(u_m)$;

Compute $\nabla f_{S_m}(u_0) = \frac{1}{n} \sum_{k=1}^{p} \nabla f_{S_m,k}(u_0) = \sum_{k=1}^{p} \nabla f_{S_m,k}(w_t)$;

Update the parameter: $u_{m+1} = u_m - \eta(\nabla f_{S_m}(u_m) - \nabla f_{S_m}(u_0) + z)$;

end for

Take $w_{t+1}$ to be $u_M$ or the average of $\{u_m\}$;

end for

Algorithm 5 Task of Workers in Distributed SVRG

For the Worker $k$:

for $t = 0, 1, 2, \ldots, T$ do

Wait until it gets the newest parameter $w_t$ from the Master;

Compute the local gradient sum $z_k = \sum_{i \in D_k} \nabla f_i(w_t)$, and then send $z_k$ to the Master;

for $m = 0$ to $M$ do

Wait until it receives the $S_{m,k}$ and $u_m$ from the Master;

Compute $\nabla f_{S_{m,k}}(u_m) = \sum_{i = 1}^{\vert S_{m,k} \vert} \nabla f_i(u_m)$;

Compute $\nabla f_{S_{m,k}}(w_t) = \sum_{i = 1}^{\vert S_{m,k} \vert} \nabla f_i(w_t)$;

Send $\nabla f_{S_{m,k}}(u_m)$ and $\nabla f_{S_{m,k}}(w_t)$ to the Master;

end for

end for