Effect of Heat Generation (Absorption) on Unsteady MHD Flow Over a Truncated Cone Embedded in a Stable Thermally Stratified Environment

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Abstract
This paper examines the effect of heat generation (absorption) on unsteady free convection flow of an incompressible electrically conducting fluid over a truncated cone embedded in a thermally stratified medium in the presence of a transverse magnetic field. The governing nonlinear partial differential equations have been solved numerically, using an efficient implicit finite difference scheme along with quasilinearization technique. The numerical results for skin-friction coefficients and velocity components in x- and y-directions, Nusselt number and temperature profiles are obtained and analyzed for different values of governing parameters. It is observed that, heat generation increases skin friction and decreases heat transfer coefficients, whereas opposite trend is observed during heat absorption. Also it is found that heat generation has a significant effect on temperature profile.

Keywords: MHD, Free Convection, Skin friction, Heat transfer, Heat generation (absorption), Unsteadiness

Introduction
The study of free convection flows is of much importance because of numerous science and engineering applications such as cooling of nuclear reactors, design of spacecrafts, design of solar energy collectors, power transformers, and steam generators, atmospheric and oceanic circulations. In such type of flow, in contrast to many viscous flows there is no reference velocity and hence Reynolds number is not considered. However, one can introduce Grashof number, which is the ratio of buoyancy force and frictional force, and is a dimensionless number. Some significant studies can be found in Braun.et.al [1], Roy [2], Na and Chiu [3].

The process, in which the fluid density in the ambient fluid is non-uniform and varies with height, is known as stratification of the medium. Usually stratification of the fluid arises due to temperature variation or due to mass transfer process, which gives rise to a density variation in the fluid. Stratification due to temperature variation is termed as thermal stratification. The investigation of
natural convection in a thermally stratified medium is basically an attention-grabbing problem because of the wide range of practical applications. The problem of free convection flow over a cone embedded in a stratified medium has been studied by Tripathi.et.al [4]. Bapuji Pullepu.et.al [5] analysed transient free convective flow over a vertical cone embedded in a thermally stratified medium.

Since the convection currents are suppressed by Lorentz force, which is generated by the external magnetic field, it is used as control mechanism. An electrically conducting fluid flow in an applied transverse magnetic field has various applications for example pumps, MHD generators, accelerators, flow meters and nuclear waste disposal. In view of these applications, investigators like, Srinivas.et.al [6], Prakash.et.al [7], Iranian.et.al [8] have studied the effect of MHD on free convection flows.

Convection flows in the presence of internal heat generation (absorption) has gained significant attention in recent years because of their importance in various applications like geophysical science, fire and safety engineering, nuclear science, chemical engineering etc. Many physical facts occur in free convection, which are improved and annoyed by internal heat generation (absorption), due to which there is a rise in buoyancy force. As a consequence, variation in heat transfer characteristic can be observed. The heat generation (absorption) effects on flow and heat transfer have been investigated by several authors [9-13].

In numerous engineering applications of modern interest, unsteadiness has become a fundamental aspect of boundary layer flow problems. The aim of this paper is to investigate the unsteady non-similar free convective MHD flow over a truncated cone embedded in a stable thermally stratified medium with heat generation (absorption). It may be noted that the effects of magnetic parameter (M), the heat generation (absorption) parameter (Q), unsteady parameter (t*) and stratification parameter (S) etc., over a cone, have been studied individually by several investigators. Hence, in this chapter, prominence is given on the combined effect of all these parameters.

**Governing Equations**

The physical model and coordinate system of the problem is shown in Fig.1. The vertex of the cone is placed at the origin of the coordinate system, x-axis measures the distance along the surface of the body from the apex and y-axis measures distance normally outward. Consider the unsteady free convection boundary layer flow over a truncated cone situated in a thermally stratified medium. The boundary layer is assumed to develop at the leading edge of the truncated cone(x-x_o) which implies that the temperature at the circular base is assumed to be the same as the ambient temperature T_{w}. The surface of the cone is maintained at a constant temperature T_{w} and the temperature of the fluid far from the body surface is given by T_{∞}(x)=T_{∞0}+ax\bar{x}, where a=(dT_{∞}/dx)>0 and \bar{x}=(x^n\cos\gamma) is measured from apex of the cone and is parallel to the direction of the gravity and T_{∞0} is the temperature of the fluid at the vertex of the cone. We also assume that T_{w}>T_{∞0}. The effect of stratification can be expressed in terms of a stratification parameter S, defines as S=[ax^\gamma\cos\gamma]/\Delta T_{w}[1] where x^\gamma is the slant height of the cone and \Delta T_{w} is the temperature difference, T_{w}-T_{∞}. A magnetic field of strength B_0 is applied normal to the surface of the cone and it is assumed that magnetic Reynolds number of the flow is small enough so that the induced magnetic field is negligible. The fluid is assumed to have constant physical properties except for the density variation which is assumed to be important in buoyancy terms.

Under these assumptions, the boundary layer equations with Boussinesq approximation, governing the flow over a cone are:

\[
\frac{\partial}{\partial x}(r u) + \frac{\partial}{\partial y}(r v) = 0
\]  

(1)
\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} + g \beta \cos \gamma (T - T_\infty) - \frac{\sigma B_0^2}{\rho} u \quad (2)
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{Q_e}{\rho c_p} (T - T_\infty) \quad (3)
\]

subject to the boundary conditions

\[
t \leq 0: \quad u = 0, \quad v = 0, \quad T = T_\infty \quad \forall \quad x, \quad y
\]

\[
t > 0: \quad \begin{cases} 
  u = 0, \quad v = 0, \quad T = T_w & \text{at} \quad y = 0 \\
  u = 0, \quad T = T_{\infty_0} & \text{at} \quad x = 0 \\
  u \to 0, \quad T \to T_\infty & \text{as} \quad y \to 0
\end{cases}
\]

\[
(4)
\]

The assumptions considered in developing the Eqn’s (1)-(3) holds in excellent if the boundary layer is assumed thin compared to the local radius of the cone. The local radius to a point in the boundary layer can be replaced by the radius of the truncated cone \( r = x \sin \gamma \), where \( \gamma \) is semi vertical angle of the cone. Eqns. (1) – (4) is valid in the domain \( x_0 \leq x \leq \infty \), where \( x_0 \) is the leading edge of the truncated cone measured from the origin. \( u \) and \( v \) are the components of fluid velocity in the \( x \) and \( y \) directions respectively; \( \theta \) is the kinematic viscosity; \( g \) is the gravitational acceleration; \( \beta \) is the coefficient of thermal expansion; \( \alpha \) is the thermal diffusivity of the fluid; \( \sigma \) is the electric conductivity; \( \rho \) is the density of the fluid. The subscripts \( \infty \) and \( w \) denote the conditions at the free stream and at the wall, respectively.

![Figure 1 Physical Model and Coordinate System](http://www.shanlaxjournals.com)

On introducing the following similarity variables:

\[
\psi = \nu \left( G r_\infty \right) f(\eta, \xi, t^*); \quad \eta = \frac{y}{x} \left( G r_\infty \right) ; \quad r u = \frac{\partial \psi}{\partial y};
\]

\[
 r v = -\frac{\partial \psi}{\partial x}; \quad \xi = \frac{x^*}{x_0} = \frac{x - x_0}{x_0}; \quad G = \frac{T - T_\infty}{T_w - T_{\infty_0}};
\]

\[
 G r_\infty = \frac{g \beta \cos \gamma (T_w - T_{\infty_0}) x^3}{\nu^2}; \quad t^* = \frac{G r_\infty^2 \nu t}{x^2}
\]

\[
(5)
\]
into the Eqn’s (1)-(3), it is observed that continuity equation is automatically satisfied and Eqn’s (2) and (3) reduce, respectively to

\[
F^*\left(\frac{3}{4} + \frac{\xi^2}{1 + \xi^2}\right)f F' - \frac{1}{2}F^2 + G - M - F F_T = \frac{t^*}{2}(F f' - F f_T^2) = \xi\left(F G - F' g_0\right)
\]

(6)

\[
Pr^{-1}G^*\left(\frac{3}{4} + \frac{\xi^2}{1 + \xi^2}\right)f G' - S(1 + S) - (F + G) + Q G - G_T = \frac{t^*}{2}(G f' - F G_T) = \xi\left(F G - G' f_0\right)
\]

(7)

where

\[
u = \frac{\sqrt{G r_T}(x*)}{x*} f'; \quad M = \frac{\sigma B_0^2(x^*)^2}{\rho \nu (G r_T)^{1/2}}; \quad F = f' = \frac{\partial f}{\partial \eta}; \quad Pr = \frac{v}{\alpha};
\]

\[
Q = \frac{Q_0(x^*)^2}{\rho c_p \nu (G r_T)^{1/2}}; \quad \psi = -\frac{v}{\nu (G r_T)^{1/2}} \left[\frac{3}{4} f + \xi f' + \frac{\eta f'}{4} - \frac{t^*}{2} f_T^2\right]
\]

The transformed boundary conditions are:

\[
t^* < 0: \quad F = 0, \quad G = 0 \quad \forall \xi, \eta
\]

(9)

\[
t^* \geq 0: \quad F = 0, \quad G = 1 - S(1 + S)^{-1} \xi (1 + \xi)^{-1} \quad \text{at } \eta = 0
\]

\[
F = 0, \quad G = 0 \quad \text{as } \eta \rightarrow \infty
\]

Here and f are dimensional and dimensionless stream function respectively; \(\eta\) is the pseudo similarity variable; \((\prime)\) denotes the derivative with respect to \(\eta\); M is the magnetic parameter; F and G are dimensionless velocity and temperature of the fluid; Pr is the Prandtl number; \(Gr_{r_T}\) is the local Grashof number; \(t\) and \(t^*\) are dimensional and dimensionless time, respectively; \(x\) is the stream-wise coordinate; \(x^*\) is the distance measured from the leading edge; \(\xi\) is the dimensionless distance; \(T\) is dimensional temperature; \(G\) is dimensionless temperature. The heat generation or absorption parameter \(Q\) appearing in Eqn. (7) is the non-dimensional parameter based on the amount of heat generated or absorbed per unit volume given by \(Q_0(T - T_{*})\), with \(Q_0\) being constant coefficient that may take either positive or negative values. The source term represents the heat generation that is distributed everywhere when \(Q\) is positive (\(Q > 0\)) and the heat absorption when \(Q\) is negative (\(Q < 0\)); \(Q\) is zero, in case no heat generation or absorption.

As we have assumed that the flow is steady for \(t^* = 0\) and becomes unsteady for \(t^* > 0\), the initial conditions for \(F\) and \(G\) at \(t^* = 0\) are given by steady flow equations obtained by putting \(F_T = G_T = 0\)

(10)

in (6) and (7). Consequently, the initial conditions at \(t^* = 0\) can be written as
\[ F'' + \left(3 + \frac{\xi}{4 + \xi} \right) f F' - \frac{1}{2} F^2 + G - M F = \xi \left( F F_{\xi} - F' f_{\xi} \right) \]  \hspace{1cm} (11)

\[ Pr^{-1} G'' + \left(3 + \frac{\xi}{4 + \xi} \right) f G' - S(1 + S)^{-1} (F + G) = \xi \left( F G_{\xi} - G' f_{\xi} \right) \]  \hspace{1cm} (12)

It is worth mentioning here that when \( S = 0 \), Eqns. (11) and (12) reduce to

\[ F'' + \left(3 + \frac{\xi}{4 + \xi} \right) f F' - \frac{1}{2} F^2 + G - M F = \xi \left( F F_{\xi} - F' f_{\xi} \right) \]  \hspace{1cm} (13)

\[ Pr^{-1} G'' + \left(3 + \frac{\xi}{4 + \xi} \right) f G' = \xi \left( F G_{\xi} - G' f_{\xi} \right) \]  \hspace{1cm} (14)

which are identical (without variable viscosity) to those of Srinivas.et.al [6].

The skin friction and heat transfer in the form of Nusselt number can be expressed as

\[ C_f = \frac{2 (x*)^2 \tau_w}{(G r_*) \rho} \quad \text{and} \quad Nu = -\frac{x* q_w}{\Delta T_w} \]  \hspace{1cm} (15)

where

\[ \tau_w = \mu \left( \frac{\partial u}{\partial y} \right) \quad \text{and} \quad q_w = \left( \frac{\partial T}{\partial y} \right) \]

are the shear stress and rate of heat flux at the surface, respectively, where \( \mu \) is dynamic viscosity. Using (5) and (8) in (15), the skin friction and nusselt number can be written as

\[ C_f (G r_*)^{\frac{1}{2}} = 2 (F')_{\eta=0} \quad \text{and} \quad \]  

\[ Nu (G r_*)^{\frac{1}{2}} = -(1 + S)(G')_{\eta=0} \]  \hspace{1cm} (16)

**Results and Discussion**

Using implicit finite difference scheme along with quasi linearization technique[14], we have solve the coupled non-linear partial differential equations (6) and (7) with three independent variables subject to the boundary conditions (9). The non-similar solutions have been obtained for various values of magnetic parameter \( M \), heat generation (absorption) parameter \( Q \) for \( Pr = 0.72 \) (air). For validation of numerical procedure followed in this paper, the results are compared with those of Srinivas.et.al [6], Cebeci and Bradshaw [15], Kays and Crawford [16]. The comparison results are presented in Table I and Fig.2. The numerical results are found to be in excellent agreement.
Table I
Comparison of steady –state results of skin friction and heat transfer parameter for $M=0.0$, $S=0.0$, $Q=0.0$, with those of Cebeci and Bradshaw [15] and Kays and Crawford [16]

| $Pr$ | Present results $F'(0)$ | Cebeci and Bradshaw [15] $F'(0)$ | Present results $G'(0)$ | Kays and Crawford [16] $G'(0)$ |
|------|-------------------------|---------------------------------|-------------------------|---------------------------------|
| 0.1  | 1.2181                  | 1.2104                          | 0.1624                  | 0.1640                          |
| 0.7  | 0.9403                  | -                               | 0.3574                  | -                               |
| 1.0  | 0.9054                  | 0.9081                          | 0.4014                  | 0.4010                          |
| 10.0 | 0.5892                  | 0.5930                          | 0.8218                  | 0.8272                          |

Figure 2 Comparison of steady state results of (a) skin friction and heat transfer coefficients (b) velocity and temperature profiles for $S=Q=0.0$ when $M = 0.5$ and $Pr= 0.1$ with those of Srinivas.et.al [6]

Effect of magnetic parameter ($M$) on skin friction $[C_f (Gr_r)^{1/4}]$ and heat transfer $[Nu(Gr_r)^{1/4}]$ coefficients when $S = 0.5$ (stratification parameter) and $Q = 0.0$ (in the absence of heat generation (absorption) parameter) are illustrated in Fig.3. It is seen that an increase in magnetic parameter ($M$) causes a decrease in both $C_f (Gr_r)^{1/4}$ and $Nu(Gr_r)^{1/4}$. The percentage of decrease in $C_f (Gr_r)^{1/4}$ at $t^* = 1.0$ for varying $M$ from 0.0 to 1.0 is 22.9% and the percentage of decrease in $Nu(Gr_r)^{1/4}$ is 4.2% for the same value of $t^*$ in the range of $0.0 \leq M \leq 1.0$. In Fig.4, corresponding velocity ($F$) and temperature ($G$) profiles are plotted for different values of $M$ with $S = 0.5$ and $t^* = 1.0$. It is observed that there is an increase in temperature profiles and a decrease in velocity profiles as $M$ increases, this is due to the fact that the magnetic field applied normal to the flow of an electrically conducting fluid creates a drag force called Lorentz force which has a tendency to slow down the motion of the fluid. Both momentum and thermal boundary layer thickness increases with an increase in magnetic parameter $M$. Also, it is observed that as $\eta$ increases velocity profiles ($F$) increases [Fig.4(a)] and reaches its maximum value at $\eta = 1.2$ and gradually decreases to the free stream conditions whereas, temperature profiles ($G$) decrease [Fig.4(b)] along the $\eta –$ direction owing to the velocity and temperature profiles of natural convection boundary layer flows.
Figure 3 Effect of magnetic parameter on (a) skin friction and (b) heat transfer coefficients

Figure 4 Velocity and temperature profiles (F & G) for different values of $M$

Figure 5 shows the effect of heat generation ($Q > 0$) on skin friction and heat transfer $[C_f(Gr_x)^{\zeta}, Nu(Gr_x)^{\zeta}]$ coefficients against unsteady parameter $t^*$ when the magnetic parameter $M = 0.5$ and stratification parameter $S = 0.5$ at $\zeta = 0.5$. It is observed that an increase in heat generation parameter ($Q > 0$) leads to increase in $C_f(Gr_x)^{\zeta}$ and decrease in heat transfer rate $Nu(Gr_x)^{\zeta}$, this is because, heat generation increases the fluid temperature and thus temperature gradient at the surface decreases, thereby reducing the heat transfer at the surface [Fig.5(b)] and enhancing the skin friction coefficient [Fig.5(a)]. Also, it is observed that as $t^*$ increases $C_f(Gr_x)^{\zeta}$ increases, while $Nu(Gr_x)^{\zeta}$ decreases monotonically. The percentage of increase in $C_f(Gr_x)^{\zeta}$ for $0.0 \leq Q \leq 0.5$ (heat generation) is 12.03% and on the other hand, the percentage of decrease in $Nu(Gr_x)^{\zeta}$ is 37.8% for the same range $Q$ of at $t^* = 1.0$.

Figure 5 Effect of heat generation parameter ($Q > 0$) on (a) skin friction and (b) heat transfer coefficients
The relevant velocity \((F)\) and temperature \((G)\) profiles are presented in Fig.6. As observed from the figures, the momentum boundary layer thickness decreases [Fig.6(a)] with increase of \(Q\), whereas thermal boundary layer thickness increases [Fig.6(b)] owing to the reduction in the heat transfer rate. Also, it is found that during heat generation \((Q > 0)\) there is slight overshoot in the profiles [Fig.6(b)] i.e. heat transfer gets reversed. This shows that heat generation mechanism creates a layer of hot fluid near the surface and fluid temperature exceeds the surface temperature, this results in the decrease of heat transfer rate from the surface.

![Figure 6 Velocity and temperature profiles \((F & G)\) for different values of \(Q > 0\)](image)

Figure 7 demonstrate the effect of heat absorption parameter \((Q < 0)\) on skin friction \([C_f(Gr_x)^{\frac{1}{4}}]\) and heat transfer \([Nu(Gr_x)^{\frac{1}{4}}]\) coefficients at the stream-wise location \(\xi = 0.5\), when \(S = M = 0.5\). It is found that \(C_f(Gr_x)^{\frac{1}{4}}\) decreases, while \(Nu(Gr_x)^{\frac{1}{4}}\) increases when heat is absorbed \((Q < 0)\). This is due to the fact that heat absorption has the tendency to decrease the temperature of the fluid in the boundary layer owing the increase of particle deposition level, which in turn decreases the skin friction coefficient. The percentage of decrease in \(C_f(Gr_x)^{\frac{1}{4}}\) during heat absorption \((-0.5 \leq Q \leq 0.0)\) is 9.08% [Fig.7(a)], while the percentage of increase in \(Nu(Gr_x)^{\frac{1}{4}}\) is 30.8% [Fig.7(b)] at \(t^* = 1.0\). The consequential velocity \((F)\) and temperature \((G)\) profiles are shown in Fig.8, in which it is observed that, when there is heat absorption \((Q < 0)\), both velocity and temperature profiles decreases inside the boundary layer due to the enhancement of momentum boundary layer thickness and reduction of the thermal boundary layer thickness. The above quantitative study reveals the fact that the effect of heat generation (absorption) and stratification parameter is more pronounced on \(Nu(Gr_x)^{\frac{1}{4}}\) as compared to \(C_f(Gr_x)^{\frac{1}{4}}\). This is expected, because the heat generation (absorption) parameter \(Q\) and the stratification parameter \(S\) are present in energy equation.

![Figure 7 Effect of heat absorption parameter \((Q < 0)\) on (a) skin friction and (b) heat transfer coefficients](image)
Conclusions
The laminar unsteady MHD free convection flow over a truncated cone embedded in a thermally stratified medium in presence of heat generation (absorption) parameter has been studied numerically. The effect of magnetic, and heat generation (absorption) parameters have been considered on the flow and heat transfer characteristics. Our results are compared with available results in literature and are found to be in very good agreement. The subsequent conclusions are drawn from the present analysis:
1. The magnetic parameter reduces the velocity of the fluid.
2. Heat generation increases the thermal boundary layer thickness and in turn, heat transfer rate decreases, whereas heat absorption parameter does the reverse effect.
3. The skin friction coefficient is increased due to the effect of heat generation parameter as a result of which velocity of the fluid is enhanced. Heat absorption has an opposite cause on skin friction coefficient.
4. Unsteadiness has significant effect on both velocity and temperature profiles.

Acknowledgements
The author is thankful to the reviewer for their valuable comments and suggestions.

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