The influence of the cross-sectional shape of the body on the parameters of flexural-gravity waves during under-ice motion

V L Zemlyak¹, V M Kozin², A S Vasilyev¹ and E G Rogozhnikova³

¹Sholom-Aleichem Primursky State University, 70a Shirokaja Street, Birobidzhan, 679015, Russia
²Institute of Machining and Metallurgy, FEB RAS, 1 Metallurgov Street, Komsomolsk-on-Amur, 681005, Russia
³Amur State University of Humanities and Pedagogy, 17/2 Kirova Street, Komsomolsk-on-Amur, 681000, Russia

E-mail: vellkom@list.ru

Abstract. When solving the problem of a submarine movement near the lower surface of the ice cover, the hull of the submarine, which has a complex geometry, is usually changed to a body of rotation, having a simplified form while maintaining basic characteristics. In turn, the cross-sectional shape of the hull can have a significant effect on the parameters of flexural-gravity waves generated by the movement of the submarine. The paper describes an experimental study of the influence of the cross-sectional shape of the models of submerged bodies having the same elongation, displacement and cross-sectional area, and a different ratio of the hull height to their width on the ice-breaking ability of flexural-gravity waves. It is analysed how the poorly hydrodynamic form of a submerged body affects the parameters of flexural-gravity waves. The nature of ice failure during the movement of submerged bodies of different forms is defined.

1. Introduction

So far, 60 large hydrocarbon deposits have been discovered in the Arctic. According to expert assessments, up to 30% of the world’s undiscovered gas reserves and 13% of oil ones are now concentrated in the region. However, despite the enormous wealth of the Arctic shelf when running projects in this region, it is necessary to solve a whole range of serious problems, primarily environmental, economic and transport-technological ones. Particular attention is paid to the issues of mining and transportation of minerals. Taking into consideration the constant presence of the ice, there is a need to develop special vehicles for transporting minerals and providing supplies. Solving this issue requires new approaches to the methods of extraction and transportation of raw materials, which can be implemented by using submarine vessels. Therefore, engineering companies are studying a possibility of creating submarine vehicles that would be part of an integrated transport system in the Arctic.

The idea of using submarines as means of transport arose almost as soon as they began to be used in the Navy. During the Second World War, Germany and Italy had underwater cargo fleets. The Japanese Navy used 58% of the total number of submarines as cargo carriers [1]. Many countries of the world developed projects of submarine cargo carriers, but none of these projects was ever implemented. The idea of using underwater transports emerged again when the active development of the Arctic shelf began.
More than 60% of the under-ice water zone of the Arctic region does not allow modern submarines to surface using a traditional method of static loading of ice from below. The reason for this is that most submarines are unable to make an emergency ascent in pack ice of more than 2 meters thick, which is crucial in extreme situations.

It is known that when a submerged body moves under the ice surface, flexural-gravity waves (FGW) emerge in the ice cover, at a certain intensity of which partial or complete ice failure can be achieved [2]. However, the process of the generation of flexural-gravity waves during the motion of a submerged body under the ice near its lower surface has been little studied. Usually, when solving such problems at the theoretical level, a submerged body of a simplified form is used. Kheisin [3] considered a plane stationary problem of the vortex motion under a layer of broken ice. Bukatov and Zharkov [4] studied a steady motion of a point source of mass under a floating elastic plate. Sturova [5] offered a solution of a linear stationary problem concerning the steady flow of a sphere with non-viscous liquid under the ice cover, broken ice, a membrane, as well as under the free surface. Korobkin et al. [6] considered a nonlinear problem of unsteady waves generated by a submerged elliptical cylinder moving under the ice cover. The destruction of ice by dynamic loads is considered in the work [7].

However, the hull of modern submarines is of a complex cross-sectional shape; therefore, it is not entirely correct when calculating to use rotation bodies having a cross-section in the form of a circle. The purpose of the research is an experimental study of the influence of the cross-sectional shape of a submerged body on the parameters and ice-breaking ability of flexural-gravity waves. Section 2 provides a description of the equipment used and the procedure for conducting a model experiment. Section 3 presents the results of model experiments. Section 4 is devoted to discussion.

2. Equipment and technique for conducting experiments
To carry out model experiments, the ice tank of the laboratory of ice technology of the Sholom-Aleichem Primorsky State University (Birobidzhan, Russia) [8] with dimensions $14 \times 3 \times 1$ m. The scheme of the ice tank made by means of Autodesk 3ds Max [10] is shown in figure 1.

![Figure 1. General view of the ice tank.](image)

To determine the effect of the cross-sectional shape of the submerged body on the parameters of generated FGW, three models were made in the scale $\lambda=1:120$. All dimensions of the models are in millimetres. The cross-sectional area of the models, their length and width were not changed. The cross-sectional shape and the height of the models were changed (figure 2). Model schemes were made in AutoCAD 2019 [11]. The models submerged bodies were made by means of the method of layer-by-layer printing of a physical object on a Raise3D printer.
Figure 2. Schemes of models a, b, c.

Experimental studies were carried out in winter from 10 to 20 February 2019 and 15 to 31 December 2020. To freeze ice of the set thickness without any impurity, natural cooling of the room was used. The room where the ice tank is located is not heated for the whole period of the experimental studies. There are negligible fluctuations in the room temperature, within the difference of 1°C to 2°C, at indoor temperatures up to -20 °C. Due to the layer of heat-insulating material installed of the tank and compliance with the constant temperature regime, the thickness of the ice field remained constant. After carrying out each model experiment, broken ice pieces are removed and the depth of the model is readjusted.

To conduct the studies the necessary equipment and measuring system to register vibrations of the model ice are applied. When towing models of submerged bodies, the speed of their movement and profiles of FGW are determined. To record the vibrations of the modelled ice we use LAS-Z infra-red non-contact laser movement sensors of Way Con Company (Germany). The sensor detects the ice deflection change depending on the time with the help of the Test viewer program 2.34.

A towing carriage is used to tow models of submerged bodies. It moves with the help of the Servoline 130SPSM14 servo-driver of Zetek Company (Russia) along rail guides.

The strength of the cable provides the tension of the models necessary to exclude submergence changes in the course of motion that is determined experimentally by means of test starts on clear water before carrying out each series of the experiment.

The simulation scale is chosen based on the size of the ice tank, while the length of the tank provides access to the steady motion regime of the model. The width is sufficient to eliminate the influence of the waves reflected from the channel sides on the main principle wave system [9].

Modeling of sea ice cover can be executed by using various ice models and for each of them there are corresponding conditions of similarity. Modeling was carried with partial satisfaction of conditions of similarity [2]:

\[ \lambda_E = \lambda_w = \lambda_h = \lambda_l \]  \hspace{1cm} (1)

Where \( \lambda_E \) is the model scale for Young’s modulus; \( \lambda_w \) is the model scale for deflections; \( \lambda_h \) is the model scale for ice thickness.

In this case, the requirements for similarity conditions with respect to Poisson’s ratio \( \mu \) and density \( \rho \) of the modelled ice are satisfied.

Of the submerged body model must be geometrically similar to the full-scale one and their displacements must be in proportion to the cube of the geometrical scale-ratio:
\[ \frac{L_n}{L_m} = \lambda_n, \quad \frac{D_n}{D_m} = \lambda_l^3 \] (2)

Where \( L_n \) is the length of the full-scale submarine; \( D_m \) is the submerged body model displacement.

The submerged body model motion velocity \( u_m \) is determined by the condition of similarity:

\[ \frac{u_n}{u_m} = \lambda_l^{1/2} \] (3)

Where \( u_n \) is the full-scale submarine velocity.

The parameters of model FGW are converted to full-scale ones in accordance with the geometrical similarity:

\[ \frac{\lambda_n}{\lambda_m} = \frac{w_n}{w_m} = \lambda \] (4)

Where \( \lambda_n \) is the length of the full-scale FGW; \( \lambda_m \) is the length of model FGW; \( w_n \) is the full-scale FGW deflections; \( w_m \) is the model FGW deflections.

When the natural ice cover is used as a model one, the thickness of the modelled ice will be calculated when converted to a full-scale one in accordance with the following relations:

\[ h_n = h_m \lambda_l^{4/3} \left( \frac{[\sigma_u]_n}{[\sigma_u]_m} \right)^{1/3} \] (5)

Where \( h_n \) is the natural ice thickness; \( h_m \) is the modeled ice thickness; \([\sigma_u]_n\) is the natural flexural stresses; \([\sigma_u]_m\) is the modeled flexural stresses.

The determination of the flexural strength of the modelled ice is experimentally performed by testing beams afloat and for that purpose the modelled ice cover is prepared by building up the ice of the required thickness \( h_m = 0.002 \) m, \([\sigma_u]_m = 1.53 \) MPa [12]. The value of the flexural strength of the freshwater ice \([\sigma_u]_n = 0.7 \) MPa is chosen according to the work Petrov [13]. The thickness of the modelled ice cover after conversion to a natural one is \( h_n = 1.55 \) m.

It is known that when modelling sea ice by using freshwater modelled ice, the elasticity modulus of natural ice should be less than the elasticity modulus of modelled ice \( E_n > E_m \) however the ratio \( E/\sigma_u \) for sea ice is practically the same as that for fresh-water ice. To meet this condition in modelling ice is very important [14]. To enhance the visibility of ice fractures, the modelled ice surface is covered with a thin layer of snow, soon after the model field is formed.

The coefficient \( \alpha \) was used as a criterion for ice breaking, which shows the relationship between the angle of inclination of the ice plate and its destruction. It was established in [15] that when the load moves on the ice and when the submerged body moves under the ice, if the maximum value of the ice surface inclination angle exceeds 0.04, then cracks open and ice destruction is observed.

The stages of ice destruction can be estimated using geometric characteristics. Laboratory studies [15] make it possible to determine the minimum angle of the tangent inclination to the deformed surface, sufficient for complete destruction of the ice during bending.

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\[ \alpha = 2\pi w_m/\lambda_m \] (6)

Due to the complexity of the study, that is the duration of the preparation of the modelled ice field and the limited cold period, aiming to identify optional parameters of the experiment such as model displacement velocity, submergence and water depth a series of preliminary test runs have been conducted. The study shows that the most significant results are to be expected at model submerged body speeds equal \( u_m = 1.4 \div 2.2 \) m/s, the model submergence equal to \( h_{svm} = 0.16 \div 0.25 \) m.
3. Results of the model experiments

3.1 Movement of models submarines at a submergence of $h_{svm} = 0.16$ m

Previously, the authors in their work [16] studied the influence of the relative elongation $L_m/B_m$, changes of the cross-sectional area $S_m$ and the displacement $D_m$ of submarine models on the parameters of flexural-gravity waves generated by submarine motion in the ice cover. In this work we analyse the effect of changes in the cross-sectional shape of the models while maintaining their basic characteristics $L_m/B_m$, $S_m$, and $D_m$ on the parameters of FGW. Models a, b, c had a different ratio of hull height to width: $H_m/B_m = 1, 0.915, 0.813$ for models a, b, c, respectively (figure 2).

Figure 3 shows a dependence of ice cover deflections on the movement velocity of models a, b, c when $h_{svm} = 0.16$ m. The graphs show that the greatest deflections were observed when model a was moving, the cross section of which had a circular shape and value $H_m/B_m = 1$. Despite the fact that model b had a greater value $H_m/B_m = 0.915$ than model c, $H_m/B_m = 0.813$, ice deflection values caused by the movement of model b were less by $11\div33\%$ than $w_m$ caused by the movement of model c for different speeds. This is due to the fact that the form of model c is close to rectangular in contrast to the form of model b which is close to oval. Consequently, the form of model c has a significantly worse flow around it. The work [17] shows that if the degree of the turbulence of the vessel boundary layer increases, both the viscosity resistance and especially the form resistance also increase. In turn, the form resistance affects the distribution of water pressure on the surface of the hull, which determines the magnitude of wave impedance, i.e. the height of waves generated by the vessel. Thus, if the surface of the hull has a poorly hydrodynamic form it will cause a transition of the laminar regime of fluid flow around the hull to a turbulent one. This will lead to an increase in both the total and wave resistances [17], i.e. to an increase in the amplitude of waves generated by the vessel.

![Graph of dependence of the deflections $w_m$ of the model ice on the speed of movement of models of submerged bodies at depth $h_{svm} = 0.16$ m; No.1 is model a [16]; No.2 is model b; No.3 is model c.](image)

On the whole, $w_m$ of model ice caused by the motion of models b as compared with $w_m$ of ice caused by the motion of model a decreased by $13\div48\%$ at different speeds. For model c it decreased by $4\div32\%$ compared to model a.

The ice-breaking ability of FGW generated by the motion of models b and c compared to model a also decreased (figure 4). For model b compared with model a, the value of $\alpha$ decreased by $12\div47\%$ at various speeds. For model c compared to model a it decreased by $3\div30\%$.

The form of the model hull also influenced the nature of model ice failure. When moving at subcritical velocities, the area of ice failure was the smallest (figures 5-7). The greatest destructive
effect was observed when models $a$ and $c$ were moving with velocities close to critical values (figure 8). The efficiency of ice breaking and ice crushing caused by the motion of model $b$ was lower.

**Figure 4.** Graph of dependence of $\alpha$ on the speed of movement of models of submerged bodies at depth $h_{svm} = 0.16$ m; No.1 is model $a$ [16]; No.2 is model $b$; No.3 is model $c$.

**Figure 5.** Character of the failure of the model ice from the movement of the model $a$ with a velocity $u_m = 1.43$ m/s ($h_{svm} = 0.16$ m).

**Figure 6.** Character of the failure of the model ice from the movement of the model $b$ with a velocity $u_m = 1.43$ m/s ($h_{svm} = 0.16$ m).

**Figure 7.** Character of the failure of the model ice from the movement of the model $c$ with a velocity $u_m = 1.43$ m/s ($h_{svm} = 0.16$ m).
Figure 8. Character of the failure of the model ice from the movement of the model \( a \) with a velocity \( u_m = 1.65 \) m/s \((h_{svm} = 0.16 \) m\).

Figure 9. Character of the failure of the model ice from the movement of the model \( b \) with a velocity \( u_m = 2.15 \) m/s \((h_{svm} = 0.16 \) m\).

At a supercritical speed, the efficiency of ice failure decreased, the size of ice floes increased, which was connected with a greater curvature of FGW (figure 9).

3.2 Movement of models submarines at a submergence of \( h_{svm} = 0.21 \) m
When the submergence of the models increased, the magnitude of deflections began to decrease significantly. Thus, when the depth of a model submergence is equal to \( h_{svm} = 0.21 \) m, ice deflections decrease by 22–59% compared with submergence depth \( h_{svm} = 0.16 \) m (figure 10). The ice-breaking ability of FGW is reduced by 20–64% (figure 11). The destruction efficiency of ice was significantly reduced. Cracks in the ice cover opened only at speeds close to the critical value (figure 12).

Figure 10. Graph of dependence of the deflections \( w_m \) of the model ice on the speed of movement of models of submerged bodies at depth \( h_{svm} = 0.21 \) m; No.1 is model \( a \) [16]; No.2 is model \( b \); No.3 is model \( c \).
3.3 Movement of models submarines at a submergence of $h_{svm}=0.25$ m

When the submergence is $h_{svm}=0.25$ m, the deflections decrease by 40–65% (figure 13), and the values of the coefficient $\alpha$ is reduced by 41–68% (figure 14). Ice failure takes place only at speeds close to the critical value. The highest destruction efficiency was observed during the motion of model $a$. 

**Figure 11.** Graph of dependence of $\alpha$ on the speed of movement of models of submerged bodies at depth $h_{svm}=0.21$ m; No.1 is model $a$ [16]; No.2 is model $b$; No.3 is model $c$. 

**Figure 12.** Character of the failure of the model ice from the movement of the model $c$ with a velocity $u_m=1.65$ m/s ($h_{svm}=0.21$ m).
Figure 13. Graph of dependence of the deflections $w_m$ of the model ice on the speed of movement of models of submerged bodies at depth $h_{svm}=0.25$ m; No.1 is model a [16]; No.2 is model b; No.3 is model c.

Figure 14. Graph of dependence of $\alpha$ on the speed of movement of models of submerged bodies at depth $h_{svm}=0.25$ m; No.1 is model a [16]; No.2 is model b; No.3 is model c.

4. Conclusion
The study showed
- for models having the same characteristics (relative elongation, displacement, and cross-sectional area) but differing in the ratio of the height of the hull to their width, the greatest ice-breaking effect was observed during the movement of a model having a cross-sectional shape in the form of a circle;
- a decrease in the ratio of the height of the hull to its width leads to a decrease in the ice-breaking ability of FGW;
- a poorly hydrodynamic form of a submerged body close to rectangular increases the resistance of the hull form, which affects the distribution of water pressure on the hull surface and the growth of ice deflections;
- a significant decrease in the ice-breaking ability of FGW is observed when the load submergence increases.

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Appendices
FGW - flexural-gravity waves

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