Effect of Fracture Conductivity Attenuation on Two-phase Non-Steady Deliverability

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Abstract. In the actual development of deep low-abundance extra-low-permeability reservoirs, fracture conductivity damage may be harmful to reservoir performance, arising from proppant crush under the effective closure pressure. Inadequate attention has been given to research on how fracture conductivity attenuation is related to proppant behaviour and how conductivity loss affects seepage field and well deliverability. A prediction model of fracture conductivity considering proppant crush is innovatively established. On this basis, a method, called two-way asynchronously coupling analytical solution and numerical simulation at selected time-steps, is formed to investigate how pressure difference affects fracture two-phase non-steady deliverability. Finally, the effect of fracture conductivity attenuation on two-phase unsteady liquid-producing capacity and the recovery of reservoir is analysed. It is found that the larger the pressure difference, the higher the well deliverability, but the more serious the fracture conductivity attenuation, resulting in greater decrease of the well deliverability. Therefore, formulating development technical policy requires choosing the appropriate pressure difference between production and injection.

1. Introduction

Recent years have seen the effective development of deep low-abundance extra-low-permeability reservoirs via long-fracture water flooding with +200m half fracture length. To obtain better deliverability, the pressure difference between injection and production must be maintained at a high level, arising from proppant crush under the effective closure pressure. This method will reduce propped fracture width, porosity and permeability and provide fracture conductivity attenuation. However, the attenuation is largely irreversible. As a result, it is difficult to recover the decreased injection development of long-fractures and achieve sustainable and efficient development.

According to previous research, the long-term fracture conductivity attenuation with proppant is due to the following: proppant embedment, deformation, crush, diagenesis, particle rearrangement, compaction, reservoir particle intrusion, fracturing fluid residuum, fracturing fluid filter cake and incrustation \cite{1-8}. When the effective pressure of the crack is more than a certain critical value, the proppant particles will be broken, resulting in a decrease of the particle size and decrease of the crack width. J.L. Gidley et al. studied the effect of proppant crush and debris migration on the proppant fracture conductivity \cite{9}. Terry Palisch et al. believe that all proppants are not broken in the same way, and different proppant filling layers are affected by debris in different degrees \cite{10}. The influence of
the crush rate of the proppant on the fracture conductivity was analyzed by example calculation. The results show that the fracture conductivity is gradually reduced with the increase of the proppant crush rate; the larger the proppant diameter, the higher the crack closure pressure, and the smaller the normal stiffness of proppant; with an increase of proppant crush rate, the greater the decline of the fracture conductivity. However, there is still a lack of suitable models to quantitatively predict the fracture conductivity attenuation in the development process, which is caused by the crush of the proppant under effective closure pressure. There is also a lack of research on the effect of the fracture conductivity attenuation on the deliverability of production or injection wells. Commercial reservoir numerical simulation software cannot take into account the fracture conductivity attenuation, which leads to large deviations between the development technical policy or development adjustment measures and the actual situation.

In this paper, the attenuation mechanism of fracture conductivity caused by the dynamic change of proppant in the underground is studied by means of laboratory experiments, numerical simulation and theoretical analysis. Applying the principles and knowledge of rock mechanics, seepage mechanics, reservoir engineering, reservoir physics, geometry, applied mathematics and other principles, the relationship between the fracture conductivity attenuation and the proppant granules crush was analyzed to establish a prediction model of diversion conductivity. Finally, the effect of diversion conductivity on the seepage field and the deliverability of wells is studied, which provides theoretical basis and technical support for the effective development of deep low-abundance and extra-low-permeability reservoirs.

2. Prediction model of fracture conductivity considering the proppant crush

Under the effect of effective closing pressure, the proppant particles are broken into sub-particles, and the supporting effect on the crack partially becomes invalid. In the paper, the proppant crush is regarded as an instantaneous process, which is only related to the effective closing pressure. Once the crushing requirements are achieved, the proppant breaks instantaneously and the process is irreversible. (Figure 1)

![Figure 1. Schematic diagram of proppant crush](image)

2.1. Critical breaking pressure

In this paper, only the contact deformation between the proppant particles is considered, regardless of the embedding of two outermost layers of proppant particles. The hydraulic fracture thickness $\theta_{[d]}$ is:

If the particle size is different, the compressive strength is also different. It is generally believed that the compressive strength will be significantly reduced as the particle size increases. In this paper, the critical compressive strength of spherical particles is defined as $\sigma_{cr}$ (unit: MPa):

$$\sigma_{cr} = \sigma_c f_D f_{CN}$$

$$\sigma_c = \sigma_c^{50} \left( \frac{50}{d} \right)^{0.18}$$

$$f_D = D/d + 1$$
3

\[ f_{CN} = (C - 1) e^{\left(\frac{D}{d}\right)^{\left(\frac{C - 3}{4C}\right)}} \]  

(4)

where \( \sigma_e \) is uniaxial compressive strength of a particle; \( \sigma_{c50} \) is uniaxial compressive strength of particles with a diameter of 50 mm; \( f_p \) is particle curvature effect factor; \( d \) is target particle diameter; \( D \) is the average diameter of the surrounding particles in contact with the target particles; \( f_{CN} \) is coordination number effect factor and \( C \) is coordination number, i.e., the number of surrounding particles in contact with the target particles.

2.2. Crushed pattern

When the stress state of the particles exceeds the critical compressive strength, the particles will break down:

\[ \frac{1}{N} \sum_{i=1}^{N} \sigma_i \geq \sigma_{cr} \]  

(5)

Where \( N \) is the number of surrounding particles in contact with the target particle, and \( \sigma_i \) is the normal contact stress in a certain direction on the target particle.

It is assumed that for multi-layer proppant layering, when the crush conditions are fulfilled, only the outermost two-layer proppant particles in contact with the wall of the hydraulic fracture are all crushed. Then, the size and deformation calculation formula of the crushed sub-particles deduced.

2.3. Sub-particle force

For more than three layers of proppant paving, there is a case where the proppant particles are in contact with the sub-particles, and the force is quite complicated. A primitive proppant particle \( B_1 \) was used as the study object. In addition, an adjacent proppant particle \( C_1 \) in contact with the particle \( B_1 \) is broken down into three sub-particles \( C_{1-1}, C_{1-2}, C_{1-3}, B_1 \) and \( C_{1-1}, C_{1-2}, C_{1-3} \) are in contact with each other. According to the principle of mechanical equilibrium, the external load of \( B_1 \) is borne by \( C_{1-1}, C_{1-2}, C_{1-3} \) (Figure 2).

![Figure 2](image)

**Figure 2.** Schematic diagram of forces acting on four spheres in contact with each other.

Since a primitive proppant particle is broken into three isotactic sub-particles, the total proppant of the individual proppant sub-particles is equal to 1/3 of the force of the individual primitive proppant particles:

\[ F_3 = \frac{F}{3} = \frac{(2R)^2}{3} \sigma_{eff} = \frac{4}{3} R^2 \sigma_{eff} \]  

(6)

It can be seen from the above that the particles \( B_1 \) are subjected to the external load \( F_{AB} \) from the sub-particles \( C_{1-1} \) in contact with the wall of the fracture:
Due to the spatial position symmetry, the sub-particles $C_{1-2}$, $C_{1-3}$ suffered the same force as $C_{1-1}$. The radius of the circle of the contact surface between the sub-particles and the fracture wall is:

$$\alpha = \left[ \frac{3\pi F_S (K_1 + K_2)R}{4} \right]^{\frac{1}{3}} = \left[ \pi \sigma_{eff} (K_1 + K_2)R^2 \right]^{\frac{1}{3}}$$

The depth $\alpha_{(se)}$ of the single proppant sub-particles embedded in the wall of the hydraulic fracture is:

$$\alpha_{(se)} = \left[ \frac{9\pi^2 F_S^2 (K_1 + K_2)^2}{16R} \right]^{\frac{1}{3}} = \left[ \frac{\pi^2 (K_1 + K_2)^2 \sigma_{eff}^2 R^4}{R^3} \right]^{\frac{1}{3}}$$

As mentioned earlier, the two-layer sub-particle deformation $\alpha_{(sd)}$ caused by the direct contact of the sub-particles is:

$$\alpha_{(sd)} = \left[ 12\pi^2 \sigma_{eff}^2 (K_1 + K_2)^2 \right]^{\frac{1}{3}} R$$

The geometric relationship shows that the component $\alpha'_{(sd)}$ of the deformation in the direction of the fracture width is:

$$\alpha'_{(sd)} = \frac{\sqrt{6}}{3} \alpha_{(sd)}$$

The contact deformation $\alpha_{(sdd)}$ between the sub-particles and the original particles is:

$$\alpha_{(sdd)} = \left[ \frac{9\pi^2 F_{B_{C_{1-4}}}^2 (K_1 + K_2)^2 (R + R)}{16RR} \right]^{\frac{1}{3}} = \left[ \frac{\pi^2 \beta^2 (K_1 + K_2)^2 (R + R) \sigma_{eff}^2}{R^{\frac{1}{3}}} \right]^{\frac{1}{3}} R$$

The contact deformation $\alpha'_{(sdd)}$ of the sub-particles with the original particles in the direction of the fracture width is:

$$\alpha'_{(sdd)} = \beta \alpha_{(sdd)}$$

As mentioned earlier, the contact deformation $\alpha'_{(s)}$ between the original proppant particles in the direction of the fracture width is:

$$\alpha_{(s)} = \frac{\sqrt{6}}{3} \alpha_{(s)}$$

2.4. Prediction model of fracture conductivity

Based on the study of the above particle crush, the paper established a prediction model of fracture conductivity under the effect of the proppant particle crush.

The initial thickness $\omega_{ini-1}$ of the sub-particle layer is:

$$\omega_{ini-1} = 2R + f(N_p) \frac{2\sqrt{6}}{3} R$$

$$f(N_p) = \begin{cases} 1, & \text{if } N_p \geq 2 \\ 0, & \text{if } N_p = 1 \end{cases}$$

The overlapping thickness $\omega_{1-2}$ between the original particle layer and the sub-particle layer is:

$$\omega_{1-2} = B C_{1-3} - OB = (1 - \beta)(2\sqrt{3} - 2)R$$
The initial thickness $\omega_{i-2}^i$ of the original particle layer is:

$$
\omega_{i-2}^i = g(N_{po}) \left[ (N_{po} - 3) \frac{2\sqrt{6}}{3} R + 2R - 2\omega_{i-2} \right]
$$

(18)

$$
g(N_{po}) = \begin{cases} 
1, & \text{if } N_{po} \geq 3 \\
0, & \text{if } N_{po} \leq 2 
\end{cases}
$$

(19)

After the outermost proppant particles in contact with the fracture wall are broken, the initial width $\omega_{ni}^i$ of the fracture is:

$$
\omega_{ni}^i = \omega_{ni-1}^i + \omega_{ni-2}^i
$$

(20)

If only considering the proppant crush, hydraulic fracture thickness is $\omega_{(c)}$ for:

$$
\omega_{(c)} = \omega_{ni}^i
$$

(21)

In this case, the pore volume $V_{f(c)}$ of hydraulic fractures is:

$$
V_{f(c)} = LH \omega_{(c)} - N_p \frac{4\pi R^3}{3}
$$

(22)

The porosity $\phi_{f(c)}$ of the hydraulic fracture and the corresponding porosity retention rate $\eta_{\phi(c)}$ are:

$$
\phi_{f(c)} = \frac{LH \omega_{(c)} - N_p \frac{4\pi R^3}{3}}{LH \omega_{(c)}}
$$

(23)

$$
\eta_{\phi(c)} = \frac{\phi_{f(c)}}{\phi_0}
$$

(24)

According to the Carman-Kozeny formula, the permeability $K_{f(1)}$ of the sub-particles layer and the permeability of the original particle layer $K_{f(2)}$ are:

$$
K_{f(1)} = \frac{\phi_{f(c)} R^2}{45(1-\phi_{f(c)})^2}
$$

(25)

$$
K_{f(2)} = \frac{\phi_{f(c)} R^2}{45(1-\phi_{f(c)})^2}
$$

(26)

The average permeability $K_{f(c)}$ is:

$$
K_{f(c)} = \frac{K_{f(1)} \omega_{ni-1}^i + K_{f(2)} \omega_{ni-2}^i}{\omega_{(c)}}
$$

(27)

The conductivity of hydraulic fracture $C_{f(c)}$ is:

$$
C_{f(c)} = K_{f(c)} \omega_{(c)}
$$

(28)

The dimensionless conductivity of hydraulic fracture $C_{fd(c)}$ is:

$$
C_{fd(c)} = \frac{K_{f(c)} \omega_{(c)}}{k_m L}
$$

(29)

The retention rate $\eta_{\phi(c)}$ and the loss rate $\eta_{\phi(c)-}$ of the hydraulic fracture conductivity with proppant are:

$$
\eta_{\phi(c)} = \frac{c_{f(c)}}{c_{f0}}
$$

(30)
3. Reservoir model establishment

The actual reservoir development is mainly oil and water flow at the same time and almost no stable conditions. Therefore, it is necessary to predict a two-phase unsteady deliverability with fracture by means of the reservoir numerical simulation method and to analyze the effect of fracture conductivity attenuation on the deliverability.

3.1. Two-way asynchronously coupling analytical solution and numerical simulation at selected time-steps

Considering that these changes occur at each point in time, at each location of the reservoir, the workload of simulating the change of the fracture conductivity at all time nodes is huge. There may be no obvious changes in the fracture conductivity between two adjacent time nodes. Therefore, the two-way asynchronously coupling analytical solution and numerical simulation at selected time-steps is creatively used to address the change of fracture conductivity in reservoir numerical simulation.

It is assumed that there are \( N \) mesh blocks used to characterize half-length of the fracture in the numerical model and \( N \) time steps in the numerical simulation. First, a numerical simulation is carried out without considering the change of hydraulic fracture conductivity. From the numerical simulation software, the pressure values of the \( i \text{th} \) fracture mesh block and its adjacent matrix mesh block at all-time steps are obtained. The effective closure pressure \( \sigma_{eff} \) of the fracture mesh block at each time step is calculated. Then, through the prediction model of the proppant embedment, deformation, crush and diagenesis on the fracture conductivity, the change of the actual conductivity of the \( i \text{th} \) fracture mesh block at different time steps was calculated. If the change in the time step is less noticeable than the conductivity of the previous time step, the time step can be ignored, and the above calculation is repeated for the next time step. If the conductivity of the fractured mesh block is more obvious, the conductivity of the fractured mesh block in the reservoir model needs to be adjusted and then simulated again. Finally, the simulation results will repeat the above steps until all \( N \) fracture mesh blocks are completed at all \( N \) time steps for the calculation and coupling simulations (Figure 3).

\[
\eta_{c(c)} = 1 - \frac{c_f(c)}{c_{f0}}
\]
This method is only applied at specific time points because the conductivity of the target fractured mesh block is adjusted in the reservoir model only when the conductivity is significantly changed. In fact, the conductivity of the fracture mesh block has changed during the selected specific time step. This method takes a hysteresis to the change, which means that the change only affects the reservoir seepage from the next time step, that is, the bidirectional non-synchronous application between the numerical simulation and the analytical model.

If the time step is set appropriately, the method of two-way asynchronously coupling analytic analysis and numerical simulation at selected time steps can meet the accuracy of the engineering application and simulate the change of the fracture conductivity during the development of the reservoir.

3.2. Numerical simulation reservoir model

The mesh of the typical reservoir model is divided into 50×52×1. The mesh size in the X direction is 4.00 m; the mesh size in the Y direction is 0.05 m, 3.95 m, 48 x 4.00 m, 3.95 m, 0.05 m; the mesh size in the Z direction is 5 m. To simulate the impact of heterogeneity on reservoir development, the model is divided into five partitions (Each of the 10 mesh is divided into 1 partition in the X direction). The horizontal permeability of the leftmost two partitions is 5 mD, the horizontal permeability of the remaining three zones is 1 mD and the vertical permeability of all zones is 1 mD. The oil saturation of each layer is 0.65, and the water saturation is 0.35. The initial pressure of the reservoir is 3000 m, the initial pressure of the reservoir is 37.1 MPa and the pressure of bubble point is 9.8 MPa.

To improve the efficiency of the numerical simulation, one quarter of the well pattern of one injection well and one production well is used. The injection and production wells are located at the two corners of the reservoir, and the fractures are located on the two boundaries of the reservoir. The production well and injection well are put into production at the same time keeping the injection pressure difference.

3.3. Changes of long-fracture conductivity caused by pressure change during reservoir development

During the reservoir development, the change in the long-fracture conductivity is determined by a two-way asynchronously coupling analytic analysis and numerical simulation at selected time steps, which modifies the model and simulates the effect of conductivity change on deliverability and development effect on the basis. The numerical simulation scheme is designed for seven levels of pressure difference between production and injection, i.e., 10 MPa, 15 MPa, 20 MPa, 25 MPa, 30 MPa, 35 MPa and 40 MPa. Using the above numerical simulation model, the change of reservoir pressure at fracture mesh blocks and adjacent matrix lattice blocks over time in different schemes can be calculated.

The pressure values of the fractured mesh blocks and adjacent matrix lattice blocks at each time step are substituted into the above-established well prediction model of the long-fracture conductivity attenuation. After the calculation, the attenuation rate of the conductivity of each fracture section in the production well fractures and injection well fractures under pressure difference between production and injection are used in the reservoir numerical model to simulate and predict the well’s two-phase unsteady deliverability and the change regulation of recovery at different pressure differences.

4. Effect of fracture conductivity attenuation.

4.1. Effect of fracture conductivity attenuation on two-phase unsteady liquid-producing capacity

With the fracture conductivity attenuation, the liquid-producing capacity is reduced in the development process(Figure 4); keeping the flowing bottom-hole pressure of the production well and increasing the flowing bottom-hole pressure of the injection well, the greater the pressure difference between production and injection, the larger the oil recovery. However, with the fracture conductivity attenuation, the greater the pressure difference between production and injection, the more serious the fracture conductivity attenuation and the greater the decrease of oil recovery. Compared with the non-attenuation of the fracture conductivity, when the pressure difference is 10 MPa, the attenuation of the
fracture conductivity reduces the deliverability of the production wells by approximately 10%; when the pressure difference increases to 40 MPa, the fracture conductivity attenuation reduces the deliverability of the production wells by approximately 20%.

4.2. Effect of fracture conductivity attenuation on the recovery of reservoir
With the fracture conductivity attenuation, the reservoir recovery is reduced in the development process (Figure 5); keeping the flowing bottom-hole pressure of the production wells and increasing the flowing bottom-hole pressure of the injection wells, the greater the pressure difference between production and injection, the greater the reservoir recovery. However, with the fractures conductivity attenuation, the greater the pressure difference between the injection and production, the greater the decrease of the reservoir recovery. Compared with the non-attenuation of the fracture conductivity, when the pressure difference is 10 MPa, the attenuation of the fracture conductivity reduces the reservoir recovery by approximately 8%; when the pressure difference increases to 40 MPa, the fracture conductivity attenuation reduces the reservoir recovery by approximately 17%.

5. Conclusion
In the paper, research on the effect of the fracture conductivity attenuation on two-phase non-steady deliverability is carried out. The emphasis is on the problem that the existing model cannot consider fracture conductivity and the heterogeneity of the matrix. The fracture is divided into segments, and the forecasting method of the fracture two-phase non-steady deliverability in heterogeneous reservoirs is calculated by the method called two-way asynchronously coupling analytical solution and numerical simulation at selected time-steps.

(1) The fracture conductivity attenuation is not conducive to the development of injection with long-fracturing. During the development process, the formation pressure and the fluid pressure in the seam will change, causing a change in the effective closure pressure. The fracture conductivity
attenuation with time arises from proppant crush under the effective closure pressure. For long-fractures, the effective closure pressure is different in the longitudinal direction of the fracture, and there is a spatial difference in the fracture attenuation conductivity.

(2) According to the attenuation mechanism of the fracture conductivity, the effect of the attenuation mechanism under different pressure difference between production and injection on deliverability of two-phase unsteady single well and the reservoir recovery degree is analysed. The larger the pressure difference, the greater the well deliverability, but the more serious the fracture conductivity attenuation, resulting in a greater decrease of well deliverability.

(3) Formulating development technical policy must consider the fracture conductivity of the two-phase unsteady deliverability and the impact of the development effects, which means choosing the appropriate pressure difference between production and injection.

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