Direct Detection of Leptophilic Dark Matter in a Model with Radiative Neutrino Masses

Daniel Schmidt$^{1,*}$, Thomas Schwetz$^{1,†}$ and Takashi Toma$^{1,2,‡}$

$^1$Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany
$^2$Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan

Abstract

We consider an electro-weak scale model for Dark Matter (DM) and radiative neutrino mass generation. Despite the leptophilic nature of DM with no direct couplings to quarks and gluons, scattering with nuclei is induced at the 1-loop level through photon exchange. Effectively, there are charge-charge, dipole-charge and dipole-dipole interactions. We investigate the parameter space consistent with constraints from neutrino masses and mixing, charged lepton-flavour violation, perturbativity, and the thermal production of the correct DM abundance, and calculate the expected event rate in DM direct detection experiments. We show that current data from XENON100 start to constrain certain regions of the allowed parameter space, whereas future data from XENON1T has the potential to significantly probe the model.

$^*$daniel.schmidt@mpi-hd.mpg.de
$^†$schwetz@mpi-hd.mpg.de
$^‡$t-toma@hep.s.kanazawa-u.ac.jp
1 Introduction

The Standard Model (SM) is very successful in describing the fundamental particles of our world. The only solid evidence for its failure so far is the fact that neutrinos have mass, which is a necessity due to the observation of neutrino oscillations [1–4]. Furthermore, the standard cosmological model, the ΛCDM model, provides an excellent description of our Universe, with the exception that within the SM there is no viable candidate for a Dark Matter (DM) particle, which is an important ingredient of the ΛCDM model, supported by observations such as the rotation curves of spiral galaxies [5], WMAP CMB measurements [6] and gravitational lensing [7]. Hence, both neutrinos, as well as DM require an extension of the SM. Often these two phenomena are considered separately, since they might be manifestations of physics from vastly different energy scales. Here we adopt the hypothesis that neutrino mass and DM are related, and both emerge from physics at the TeV scale. In this respect models which generate neutrino masses radiatively [8–14] are intriguing. Loop suppression factors and several powers of Yukawa couplings can bring the scale of neutrino mass generation down to the TeV, and symmetries required to stabilize DM may play a role for neutrinos, for example forbid tree-level mass terms. Recent works in this context can be found in refs. [18–39].

The so-called WIMP hypothesis suggests that DM interacts sufficiently with SM particles in order to generate the relevant abundance due to thermal freeze-out from the primordial plasma. This motivates the direct search for DM in our galaxy by looking for the scattering of DM particles with nuclei in underground detectors. Several direct detection experiments are pursuing such searches, for example the CDMS II [40], XENON100 [41,42], CoGeNT [43], DAMA/LIBRA [44], CRESST-II [45], ZEPLIN-III [46] and KIMS [47, 48] experiments. In typical WIMP models DM interacts directly with quarks, providing DM–nucleus scattering at tree-level [49,50]. Here we are interested in so-called “leptophilic” models, where DM couples directly only to leptons, see e.g. [51]. Even in that case, DM–nucleus interactions can be induced at loop-level due to the exchange of the photon [52]. The resulting effective interactions have been investigated in refs. [52,53]. In the following we will consider a model where the corresponding loop-diagrams induce a magnetic and/or electric dipole moment interaction [54–61].

We consider a model proposed by Ma [10], in which neutrino masses are generated through 1-loop interactions and the particles which propagate in the loop can be DM candidates, being leptophilic by construction. The DM phenomenology of the model and extended versions thereof has been studied in refs. [19–23,33,34] and prospects for collider searches have been studied in refs. [23,27]. We consider the situation that the lightest right handed neutrino is the DM candidate and the second lightest right handed neutrino is almost degenerated with the DM candidate. Under this situation, elastic DM–nucleus scattering is extremely suppressed and inelastic scattering induced by a lepton-
loop coupled to the photon gives the dominant contribution to the event rate in direct
detection experiments. We calculate the event rate in the model and compare it with
XENON100, KIMS and DAMA data. The paper is organized as follows. In Section 2,
we shortly review the model from ref. [10]. We discuss the constraints from neutrino
oscillation data, lepton-flavour violation and the thermal production of the DM relic
abundance. In Section 3, we discuss the inelastic scattering cross section in an effective
theory approach and calculate the event rate. Moreover, monochromatic photons from
the decay of the excited DM state are also discussed. We summarize and conclude in
Section 4. Explicit functions needed in the effective theory approach are listed in the
Appendix A.

2 The Model

2.1 Neutrino masses and mixing

The model proposed by Ma in ref. [10] is a simple extension of the SM, which correlates
neutrino physics and the existence of DM. The added particles to the SM are three right
handed neutrinos $N_i$ ($i = 1, 2, 3$) and one inert Higgs doublet $\eta$. In addition, a discrete
$\mathbb{Z}_2$ symmetry is imposed: odd for the new particles and even for SM particles. The new
invariant Lagrangian is

$$\mathcal{L}_N = \overline{N}_i \gamma^\mu P_R N_i + (D_\mu \eta)^\dagger (D^\mu \eta) - \frac{M_i}{2} N_i^c P_R N_i + h.c.,$$

and the scalar potential $\mathcal{V}(\phi, \eta)$ is

$$\mathcal{V}(\phi, \eta) = m_\phi^2 \phi^\dagger \phi + m_\eta^2 \eta^\dagger \eta + \frac{\lambda_1}{2} (\phi^\dagger \phi)^2 + \frac{\lambda_2}{2} (\eta^\dagger \eta)^2
+ \lambda_3 (\phi^\dagger \phi) (\eta^\dagger \eta) + \lambda_4 (\phi^\dagger \eta) (\eta^\dagger \phi) + \frac{\lambda_5}{2} (\phi^\dagger \eta)^2 + h.c.,$$

where $\phi$ is the SM Higgs doublet. The vacuum expectation value (VEV) of $\eta$ is assumed
to be zero, so that the discrete $\mathbb{Z}_2$ symmetry which guarantees the stability of DM is
an exact symmetry. Thus Dirac neutrino masses are not generated through the Yukawa
couplings in Eq. (1). After electroweak symmetry breaking, the SM Higgs $\phi$ obtains the
VEV $\langle \phi^0 \rangle$ and Majorana neutrino masses are generated radiatively with the effective mass

$$(m_\nu)_{\alpha\beta} \simeq \sum_{i=1}^3 \frac{2\lambda_3 h_{\alpha i} h_{\beta i} \langle \phi^0 \rangle^2}{(4\pi)^2 M_i} I \left( \frac{M_i^2}{M_\eta^2} \right),$$

where $M_i$ are the masses of the right-handed neutrinos $N_i$; $M_\eta^2 \simeq m_\eta^2 + (\lambda_3 + \lambda_4) \langle \phi^0 \rangle^2$,
and the loop function $I(x)$ is defined as

$$I(x) = \frac{x}{1-x} \left( 1 + \frac{x \log x}{1-x} \right).$$
These relations hold for small coupling $\lambda_5$, which is needed in order to obtain the correct neutrino masses, see below. This assumption is justified since an extra $U(1)$ symmetry appears in the limit of $\lambda_5 \to 0$.

As shown in ref. [21], the close to tri-bimaximal mixing of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix is achieved by adopting the following flavour structure for the Yukawa couplings $h_{\alpha i}$ (rows are labeled by $\alpha = e, \mu, \tau$ and columns by $i = 1, 2, 3$):

$$h_{\alpha i} = \begin{pmatrix} 0 & 0 & h'_3 \\
 h_1 & h_2 & h_3 \\
 h_1 & h_2 & -h_3 \end{pmatrix}. \quad (5)$$

This matrix implies $\theta_{23} = \pi/4$, $\theta_{13} = 0$ and $\tan \theta_{12} = \sqrt{2} h'_3/h_3$.\footnote{If recent indications [62–64] for a non-zero value of the mixing angle $\theta_{13}$ should be confirmed [65], corrections to Eq. (5) will be necessary. This will be discussed in the last part of Section 2.} From the current best fit value $\sin^2 \theta_{12} = 0.312^{+0.017}_{-0.015} [63]$ follows $h'_3/h_3 \approx 0.95^{+0.038}_{-0.033}$. At the same time this Yukawa matrix allows to satisfy severe constraints from lepton-flavour violation, see next subsection. We write the Yukawa couplings as $h_i = |h_i| e^{i \varphi_i}$ including the phases $\varphi_i$.

Neutrino masses are given in terms of the model parameters as follows:

$$|(h_1^2 + h_2^2)A_1| \simeq \frac{\sqrt{\Delta m^2_{\text{atm}}}}{2}, \quad |h_3^2A_3| \simeq \frac{\sqrt{\Delta m^2_{\text{sol}}}}{3}, \quad \text{with} \quad A_i \equiv \frac{2\lambda_5 \langle \phi^0 \rangle^2}{(4\pi)^2 M_i} \left( \frac{M_i^2}{M_0^2} \right), \quad (6)$$

where $\Delta m^2_{\text{atm}} = 2.50 \times 10^{-3} \text{eV}^2$ and $\Delta m^2_{\text{sol}} = 7.59 \times 10^{-5} \text{eV}^2$ correspond to the squared-differences of the eigenvalues of the neutrino mass matrix (3), and the mass difference of $N_1$ and $N_2$ is neglected. The third mass eigenvalue is zero due to the flavour structure Eq. (5). From Eq. (6) we can estimate the required sizes for the couplings $h_i$ and $\lambda_5$.

Assuming $I(x) \sim 1$ we obtain

$$\frac{\lambda_5 h_i^2}{10^{-11}} \sim \frac{M_i}{\langle \phi^0 \rangle} \left( \frac{\sqrt{\Delta m^2}}{0.05 \text{eV}} \right). \quad (7)$$

Since $h_i$ cannot be too small because of the DM relic abundance, typically $\lambda_5$ has to be tiny in order to obtain correct neutrino masses. As discuss later, we impose the perturbativity condition $|h_i| < 1.5$ for the Yukawa couplings.

### 2.2 Lepton flavour violation

Further constraints are imposed on the parameters by limits on charged lepton flavour violation. The branching ratios for lepton flavour violating processes $\ell_\alpha \to \ell_\beta \gamma$ are given as

$$\text{Br}(\ell_\alpha \to \ell_\beta \gamma) = \frac{3 \alpha_{\text{em}}}{64\pi G_F M_\eta^4} \left| \sum_{i=1}^{3} h_{\alpha i}^* h_{\beta i} F_2 \left( \frac{M_i^2}{M_\eta^2} \right) \right|^2 \text{Br}(\ell_\alpha \to \ell_\beta \nu_\alpha \bar{\nu}_\beta), \quad (8)$$
Figure 1: Contours of $\text{Br}(\mu \to e\gamma) = 2.4 \times 10^{-12}$ in the $(M_3, M_\eta)$ plane for various choices of $|h_3|$. The region to the left of each contour is excluded by $\mu \to e\gamma$.

where $\alpha_{em} = e^2/(4\pi)$ is the electromagnetic fine structure constant, $G_F$ is the Fermi constant and $M_\eta$ is the mass of $\eta^+$ which we assume to be degenerate with $\eta^0$ for simplicity. The function $F_2(x)$ is given by

$$F_2(x) = \frac{1 - 6x + 3x^2 + 2x^3 - 6x^2 \log x}{6(1 - x)^4}. \quad (9)$$

The flavour structure of Eq. (5) leads to relaxed constraints from lepton flavour violation processes such as $\mu \to e\gamma$ and $\tau \to \mu\gamma$. Because of the two zero’s in (5) it follows from Eq. (8) that only the third right handed neutrino mass $M_3$ and the Yukawa coupling $h_3$ contribute to $\mu \to e\gamma$ process. As a result, $\tau \to \mu\gamma$ gives a more stringent constraint than $\mu \to e\gamma$ for the neutrino Yukawa couplings $h_1$, $h_2$ and the DM mass $M_1$, and we can benefit from the fact that the experimental upper bound $\text{Br}(\tau \to \mu\gamma) < 4.5 \times 10^{-8}$ [66] is much looser than $\text{Br}(\mu \to e\gamma) < 2.4 \times 10^{-12}$ [67]. Contours of $\text{Br}(\mu \to e\gamma) = 2.4 \times 10^{-12}$ are shown for several $|h_3|$ values in Fig. 1. We take $M_3 = 6000 \text{ GeV}$ and $|h_3| = 0.3$ as a benchmark point in the following discussion. As clear from the figure, for this choice all values of $M_\eta$ are allowed, and for $M_\eta \lesssim 1 \text{ TeV}$ we predict $\mu \to e\gamma$ close to the present bound. Eq. (7) implies then $\lambda_5 \sim 10^{-9}$.

Thanks to the restrictions of neutrino oscillation data and lepton-flavour violation there are very few independent parameters left. We can choose the following set of four independent parameters:

$$M_\eta, \quad M_1, \quad \delta \equiv M_2 - M_1, \quad \xi \equiv \text{Im}(h_2^* h_1), \quad (10)$$

with $\delta \ll M_1$. Since we fix $h_3$ and $M_3$ to the benchmark point above in order to satisfy $\mu \to e\gamma$, the relations Eq. (6) determine $\lambda_5$ as well as $|\text{Re}(h_2^* h_1)|$ for a given choice of $M_\eta$.
and $M_1$. However, there is still an undetermined relative phase between $h_1$ and $h_2$, and we define the parameter $\xi$, which will play an important role in the following.

### 2.3 DM relic abundance

We assume that the lightest right handed neutrino $N_1$ is the lightest of the $Z_2$-odd particles, and hence it will be stable and serve as the DM candidate. We assume that it is almost degenerated with the second lightest right handed neutrino $N_2$. The mass degeneracy could be provided by imposing a symmetry for the right handed neutrinos $N_i$. This could be for example the conservation of particle number such that $N_1$ and $N_2$ form a pseudo-Dirac particle [68]. The smallness of the mass splitting is then related to suppressed operators violating the symmetry. The $N_i$ couple to the SM only via the Yukawa interaction with the lepton doublet and therefore our DM is leptophilic.

The relic density and indirect detection of DM in the model have been investigated with the flavour structure of Eq. (5) in refs. [21, 22, 25, 26]. Here we investigate the prospects for direct detection of DM in this setup for the first time.

For the thermal production of DM in this model co-annihilations between $N_1$ and $N_2$ have to be considered, since they are assumed to be highly degenerate, leading to an enhanced effective annihilation cross section [75]. The effective annihilation cross section is written as $\sigma_{\text{eff}} v = a_{\text{eff}} + b_{\text{eff}} v^2 + O(v^4)$. Then the approximate analytic solution of the Boltzmann equation which describes the evolution of the DM density is given by

$$\Omega h^2 \approx \frac{1.07 \times 10^9 x_f [\text{GeV}^{-1}]}{\sqrt{g_\ast} m_{\text{pl}} (a_{\text{eff}} + 3b_{\text{eff}}/x_f)},$$

with $x_f = \frac{M_1}{T_f}$, \hspace{1cm} \text{(11)}

where $g_\ast$ is the number of relativistic degrees of freedom at the time of freeze-out $T_f$ and $m_{\text{pl}} = 1.2 \times 10^{19}$ GeV. WMAP data [6] implies $\Omega h^2 = 0.11260 \pm 0.0036$. Taking into account co-annihilations of $N_1$ and $N_2$ we find for the coefficients $a_{\text{eff}}$ and $b_{\text{eff}}$ in the effective annihilation cross section

$$a_{\text{eff}} = \frac{\xi^2}{2\pi} \frac{M_1^2}{(M_\eta^2 + M_1^2)^2},$$

$$b_{\text{eff}} = \frac{|h_1^2 + h_2^2|^2}{24\pi} \frac{M_1^2 (M_\eta^4 + M_1^4)}{(M_\eta^2 + M_1^2)^4} + \frac{\xi^2}{2\pi} \frac{M_1^2 (M_\eta^4 - 3M_\eta^2 M_1^2 - M_1^4)}{(M_\eta^2 + M_1^2)^4},$$

where the effect of the mass difference between $N_1$ and $N_2$ is assumed to be negligible. The terms proportional to $\xi^2$ come from the co-annihilation process $N_1 N_2 \rightarrow \ell_\alpha \bar{\ell}_\beta$, whereas the $N_1 N_1$ and $N_2 N_2$ annihilations lead to the terms proportional to $h_1^2$ and $h_2^2$, respectively.

\hspace{1cm} \text{\footnote{Another motivation for leptophilic DM may come from cosmic ray observations from the PAMELA [69] and Fermi-LAT [70, 71] experiments, finding an excess of positrons but anti-protons in agreement with expectations. In order to obtain the required count rates, however, the annihilation cross section must be boosted by a mechanism such as Sommerfeld [72] or Breit-Wigner enhancement [73, 74], beyond the model considered here.}}
We observe from Eq. (12) and (13) that the $s$-wave ($a_{	ext{eff}}$-term) is only present due to co-annihilations. If there is no phase difference between $h_1$ and $h_2$, the combination of the neutrino Yukawa couplings $\xi$ vanishes and only $p$-wave annihilation remains. This corresponds to the helicity suppression for a Majorana fermion. Thus co-annihilations and a non-zero phase difference play an important role in obtaining the correct DM relic density.

For the following results we use the micrOMEGAs package [50] to calculate numerically the relic abundance of DM. In addition to $N_1 - N_2$ co-annihilations, also co-annihilations with $\eta$ are important, if $M_\eta$ becomes close to $M_1$. The allowed parameter region in the plane of DM mass and the Yukawa coupling $\xi$, which is consistent with neutrino masses and mixings, lepton flavour violation, and DM relic density is shown in Fig. 2. The allowed region is colored and divided into four regions A, B, C, D, corresponding to different assumptions on the ratio $M_1/M_\eta$. The upper bound on $\xi$ is imposed by requiring perturbativity of the Yukawa couplings. The lower bound on $M_1$ in regions A and B is determined by the limit on $\tau \to \mu \gamma$ together with the relic abundance requirement. There is no allowed parameter region if $M_\eta/M_1 \gtrsim 9.8$ because taking into account perturbativity as well as $\tau \to \mu \gamma$ the annihilation cross section is suppressed by $M_\eta^4$. If $M_\eta/M_1$ comes close to 9.8 we are driven to the left-upper corner of the allowed region in Fig. 2. In the parameter region C and D we have $M_\eta/M_1 < 1.2$ and co-annihilations with $\eta$ become
might play a role if the two terms in Eq. (17) are of comparable size. Not only the phase difference \( \phi \) as \( P \) by the limit on this process. Using Eq. (6) the parameter \( P \) as \( \sin \theta \) with \( \epsilon \). Then we allow a non-zero value \( \sin \theta \). Then we allow a non-zero value \( \sin \theta \) and deviations of \( \theta_{13} \) from \( \pi/4 \) as \( \sin \theta_{23} = 1/\sqrt{2} + \epsilon_4 \), with \( \epsilon_{3,4} \ll 1 \). Diagonalizing the neutrino mass matrix Eq. (3) we obtain at linear order in \( \epsilon_i \)

\[
\epsilon_4 = \frac{1}{\sqrt{2} (h_1^2 + h_2^2)} \frac{\tan \theta_{12} h_3^2 \Lambda_3}{\Lambda_1 - h_3^2 \Lambda_3} \epsilon_3.
\]

(15)

\[
\epsilon_1 h_1 + \epsilon_2 h_2 = \sqrt{2} (h_1^2 + h_2^2) \frac{(h_1^2 + h_2^2) \Lambda_1 - \sec^2 \theta_{12} h_3^2 \Lambda_3}{(h_1^2 + h_2^2) \Lambda_1 - h_3^2 \Lambda_3} \epsilon_3 \equiv P \epsilon_3.
\]

(16)

If we assume that \( \epsilon_1 \), \( \epsilon_2 \) and \( \epsilon_3 \) are real, we obtain from Eq. (8) the following expression for \( \mu \to e\gamma \):

\[
\text{Br}(\mu \to e\gamma) = \frac{3\alpha_{\text{em}}}{64\pi G_F^2 M_\eta^4} \left| P \epsilon_3 F_2 \left( \frac{M_1^2}{M_\eta^2} \right) + \sqrt{2} \tan \theta_{12} \abs{h_3}^2 F_2 \left( \frac{M_3^2}{M_\eta^2} \right) \right|^2.
\]

(17)

Thus a non-zero \( \theta_{13} \) directly gives a contribution to \( \mu \to e\gamma \) and \( \epsilon_3 = \sin \theta_{13} \) is constrained by the limit on this process. Using Eq. (6) the parameter \( P \) is approximately obtained as \( P \approx \sqrt{2} (h_1^2 + h_2^2) \), and we can obtain an upper bound on \( \epsilon_3 \) from \( \mu \to e\gamma \) at each point in Fig. 2. Contours of the upper bound on \( \sin \theta_{13} \) are shown in Fig. 2. The upper bound becomes severe for small DM mass. Recent results of a non-zero \( \theta_{13} \) [65] imply \( \sin \theta_{13} \approx 0.1 \) at 3\( \sigma \). According to Fig. 2 this requires DM masses around the TeV scale with \( \xi \sim \mathcal{O}(0.1 - 1) \).

\[\text{In general, the phase of } P \text{ depends on the phases of the Yukawa couplings } h_1 \text{ and } h_2, \text{ i.e., } \varphi_1 \text{ and } \varphi_2. \text{ For simplicity we set the overall phase of } P \text{ to zero. This phase might play a role if the two terms in Eq. (17) are of comparable size.}\]
In addition to the extension of the Yukawa matrix Eq. (14) we checked also the effect of changing the \( \tau_1 \) and \( \tau_2 \) components into \( h_1 + \epsilon_1 \) and \( h_2 + \epsilon_2 \). The factor \( P \) only changes to \( P \approx 13\sqrt{2}(h_1^2 + h_2^2)/6 \). Moreover, changing the \( \tau_3 \) component of Eq. (14) into \( -(h_3 + \epsilon) \), the deviation \( \epsilon \) is required to be zero up to \( \mathcal{O}(\epsilon) \) from the diagonalization condition of the neutrino mass matrix. Therefore these extensions of the Yukawa matrix do not change the analysis drastically.

3 Direct Detection of Leptophilic DM

3.1 Inelastic Scattering Cross Section

Inelastic scattering occurs through the effective interactions with quarks which come from the 1-loop diagrams shown in Fig. 3. The 3-point vertex effective interactions of \( N_1, N_2 \) and \( \gamma \) which give a dominant contribution to the inelastic scattering are written as

\[
L_{\text{eff}} = i a_{12} \overline{N}_2 \gamma^\mu N_1 \partial_\mu F + i \left( \mu_{12} \right) \overline{N}_2 \sigma^{\mu\nu} N_1 F_{\mu\nu} + i c_{12} \overline{N}_2 \gamma^\mu N_1 A_\mu, \tag{18}
\]

where the factor \( i \) is a conventional factor to obtain real couplings \( a_{12}, c_{12} \) and \( \mu_{12} \), and \( F_{\mu\nu} \) is the electromagnetic field strength. The coefficient \( \mu_{12} \) is known as the transition magnetic moment between \( N_1 \) and \( N_2 \). Elastic scattering does not occur through the effective interactions because the operators \( \overline{N}_1 \gamma^\mu N_1 \) and \( \overline{N}_1 \sigma^{\mu\nu} N_1 \) are identical zero for Majorana fermions. General inelastic scattering of DM has been discussed in refs. [68,78], and inelastic scattering due to the magnetic moment interactions in ref. [57]. Loop induced DM–nucleus scattering for leptophilic DM has been pointed out in ref. [52], and the model considered here is a specific realization of “flavoured” DM discussed in ref. [53], where similar diagrams to the ones from Fig. 3 have been considered. For another recent model for magnetic inelastic DM see ref. [79].

In the model considered here, the coefficients \( a_{12}, c_{12} \) and \( \mu_{12} \) are calculated as

\[
a_{12} = - \sum_\alpha \frac{\text{Im} (h_{\alpha 2}^* h_{\alpha 1})}{2(4\pi)^2 M_\eta^2} I_a \left( \frac{M_1^2}{M_\eta^2}, \frac{m_\alpha^2}{M_\eta^2} \right), \tag{19}
\]
\[
\mu_{12} = - \sum_\alpha \frac{\text{Im} (h_{\alpha 2}^* h_{\alpha 1})}{2(4\pi)^2 M_\eta^2} e 2 M_1 I_m \left( \frac{M_1^2}{M_\eta^2}, \frac{m_\alpha^2}{M_\eta^2} \right), \tag{20}
\]
\[
c_{12} = \sum_\alpha \frac{\text{Im} (h_{\alpha 2}^* h_{\alpha 1})}{2(4\pi)^2 M_\eta^2} q^2 I_c \left( \frac{M_1^2}{M_\eta^2}, \frac{m_\alpha^2}{M_\eta^2} \right), \tag{21}
\]

where \( q^2 \) is the momentum transfer and the explicit forms of the function \( I_a(x,y), I_m(x,y) \) and \( I_c(x,y) \), which come from the loop integrals, are given in Appendix A. Eq. (5) implies that \( \text{Im} (h_{\alpha 2}^* h_{\alpha 1}) = \xi \) and therefore the parameter \( \xi \) responsible for \( N_1 - N_2 \) co-annihilations controls also the effective interactions of DM with nuclei.

From the effective interactions, we can obtain three types of differential scattering cross sections with a nucleus which has atomic number \( Z \), mass number \( A \), mass \( m_A \), spin
$J_A$ and magnetic moment $\mu_A$, see e.g., [53, 57]:

\[
\frac{d\sigma_{CC}}{dE_R} = \frac{Z^2 b_{12}^2 m_A}{2\pi v^2} F^2(E_R),
\]

\[
\frac{d\sigma_{DC}}{dE_R} = \frac{Z^2 \alpha_{em} \mu^2_{12}}{E_R} \left[ 1 - \frac{E_R}{v^2} \left( \frac{1}{2m_A} + \frac{1}{M_1} \right) \right] - \frac{\delta}{v^2} \frac{1}{\mu_{DM}} - \frac{\delta^2}{v^2} \frac{1}{2m_A E_R} \right] F^2(E_R),
\]

\[
\frac{d\sigma_{DD}}{dE_R} = \frac{\mu^2_A \mu^2_{12} m_A}{\pi v^2} \left( \frac{J_A + 1}{3J_A} \right) F^2_D(E_R),
\]

with the coefficient

\[
b_{12} = (a_{12} + c_{12}/q^2)e.
\]

The cross sections Eq. (22), (23) and (24) are called charge-charge (CC), dipole-charge (DC), and dipole-dipole (DD) coupling, respectively. Here $E_R$ is the recoil energy, the parameter $\delta$ is the mass difference between $N_2$ and $N_1$ i.e., $\delta = M_2 - M_1$ and $\mu_{DM}$ is the DM–nucleus reduced mass. Magnetic moments of several nuclei are shown in Tab. 1. $F(E_R)$ is the nuclear form factor for which we use the parametrization

\[
F(E_R) = \frac{3 [\sin(\kappa r) - \kappa r \cos(\kappa r)]}{(\kappa r)^3} e^{-\kappa^2 s^2/2},
\]

with $\kappa = \sqrt{2m_A E_R}$, $r = \sqrt{R^2 - 5s^2}$, $R \approx 1.2A^{1/3}$ fm and $s \approx 1$ fm. $F_D(E_R)$ is the nuclear magnetic form factor and it is not well-known, see e.g., the discussion in ref. [57]. We adopt the following approximation for $F_D(E_R)$. The magnetic moment of a nucleus receives contributions from the spin $\langle S_{n,p} \rangle$ as well as orbital momentum $\langle L_{n,p} \rangle$ of the neutrons and protons:

\[
\mu_A = g_p^s \langle S_p \rangle + g_n^s \langle S_n \rangle + g_p^l \langle L_p \rangle + g_n^l \langle L_n \rangle.
\]

We approximate the magnetic form factor by neglecting the orbital momentum contribu-
Table 1: Magnetic moments for several nuclei in units of $\mu_N$ where $\mu_N = e/2m_p$ is the nuclear magneton [82].

| $^A_Z$A | $^F_{10}$F | $^{23}_{11}$Na | $^{73}_{32}$Ge | $^{127}_{53}$I | $^{131}_{54}$Xe | $^{133}_{55}$Cs | $^{135}_{74}$W |
|--------|-----------|-------------|-------------|-------------|-------------|-------------|-------------|
| $J_A$  | 1/2       | 3/2         | 9/2         | 5/2         | 3/2         | 7/2         | 1/2         |
| $\mu_A/\mu_N$ | 2.629 | 2.218 | -0.879 | 2.813 | 0.692 | 2.582 | 0.118 |

The spin-dependent form factors and $g^s_{p,n}$ factors are taken from refs. [80,81].

In addition to the CC, DC, DD interactions from Eqs. (22), (23), (24) also a charge-dipole coupling exists. However there is an additional suppression factor of $q^2$ compared to the other couplings, thus it can be neglected. The DC coupling is singular at $E_R = 0$. Therefore the predicted event rate of the DC coupling is enhanced at low recoil energies due to the singularity, and we cannot define a total cross section at the zero momentum transfer limit $\sigma_{DC}^0$. This situation is the same as in Coulomb scattering.

### 3.2 Comparison of the Predicted Event Rate with Experiments

We compare the event rate calculated from the effective interactions with XENON100 [42], KIMS [48] and DAMA [44] data. The DD coupling might be important for KIMS or DAMA [57] since in these experiments, the target nuclei are iodine (I) and cesium (Cs) for KIMS, iodine and sodium (Na) for DAMA, which have a large nuclear magnetic moment as can be seen from Tab. 1. The event rate is written as

$$\frac{dR}{dE_R} = \sum_{\text{nuclei}} \frac{\rho_\odot}{M_1 M_{\text{det}}} \int_{v>v_{\text{min}}} \frac{d\sigma}{dE_R} f(v) d^3v, \quad (29)$$

where $\rho_\odot \approx 0.3$ GeVcm$^{-3}$ is the local DM density, $M_{\text{det}}$ is the mass of target material, $v_{\text{min}}$ is the minimum velocity required for DM to scatter off a nucleus with recoil energy $E_R$,

$$v_{\text{min}} = \frac{1}{\sqrt{2m_A E_R}} \left( \frac{m_A E_R}{\mu_{\text{DM}}} + \delta \right), \quad (30)$$

and $f(v)$ is the local DM velocity distribution function in the rest frame of the Earth. It is obtained by a Galilean transformation from a Maxwell-Boltzmann distribution in

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The ratio of spin and orbital contributions to the magnetic moment in Eq. (27) are 0.59 : 0.41 for Sodium, 0.52 : 0.48 for Iodine, 0.96 : 0.04 for Xenon, −0.38 : 1.38 for Cesium. Therefore, neglecting the orbital contribution is an excellent approximation for Xenon. For the other nuclei this introduces an error of about an factor 2 and therefore the limits derived from KIMS and DAMA should be considered only approximate.
Table 2: The energy range and the quenching factor for the experiments XENON100 [42], KIMS [48], and DAMA [44]. For XENON100 we use the same light-yield function $L_{\text{eff}}$ as in ref. [42].

| Experiment | Energy range | Quenching factor |
|------------|--------------|------------------|
| XENON100  | 8.4 − 44.6 keV | −                |
| KIMS       | 3.6 − 5.8 keVee | 0.1 (Cs), 0.1 (I) |
| DAMA       | 2 − 8 keVee    | 0.3 (Na), 0.09 (I) |

The rest frame of the galaxy with the velocity dispersion $v_0 = 220$ km/s and the escape velocity from the galaxy $v_{\text{esc}} = 544$ km/s. The velocity distribution function $f(v)$ is normalized to $\int f(v) d^3v = 1$. The relative velocity of the Earth to the galaxy is $v_e = v_\odot + v_{\text{orb}} \cos \gamma \cos [2\pi(t - t_0)/\text{year}]$ with $v_\odot = v_0 + 12$ km/s, $v_{\text{orb}} = 30$ km/s, $\cos \gamma = 0.51$ and $t_0 = \text{June 2nd}$. We must evaluate the following velocity integrals to predict the event rate:

$$\zeta_1(v_{\text{min}}, v_e) = \int_{v_{\text{min}}}^{\infty} \frac{f(v + v_e)}{v} d^3v,$$

$$\zeta_2(v_{\text{min}}, v_e) = \int_{v_{\text{min}}}^{\infty} v f(v + v_e) d^3v.$$ (31, 32)

The analytic formulas for the DM velocity integrals given in refs. [60, 83] are used. The total predicted event rate in the XENON100, DAMA, and KIMS experiments is obtained by integrating the differential event rate with respect to an appropriate recoil energy range. We use the energy range and the quenching factors shown in Tab. 2. The quenching factor is the ratio of the energy deposited in scintillation light to the total nuclear recoil energy.

In Fig. 4 we illustrate the relative importance of the the CC, DC, DD interactions from Eqs. (22), (23), (24) for the XENON100, KIMS, and DAMA experiments by calculating the total event rate induced from each of the three interaction types separately. We observe from the left panel that typically CC interactions are more important for small masses $M_1$, which follows from the different dependence on the DM mass of $b_{12}$ and $\mu_{12}$. The value $M_1$ where CC becomes subdominant depends on the ratio $M_\eta/M_1$. The right panel of Fig. 4 shows that for XENON100 the DC coupling is more important, whereas for KIMS and DAMA DD dominates, because of the large magnetic moments of iodine and sodium. The features of the DAMA curves around $M_1 \approx 20$ GeV in both panels are a consequence of the presence of the two elements (I and Na) with rather different masses. In general the relative importance of CC, DC, DD depends on the ratio $M_\eta/M_1$ and to a lesser extent on $\delta$. The main conclusion is that depending on the region in the parameter space and depending on the considered experiment, any of the three interaction types can be important and all of them have to be taken into account.

In order to derive constraints on the model we calculate the total event rate for XENON100 and KIMS in the energy range given in Tab. 2 and require that the predicted rate is less than 0.0017, 0.0098 kg$^{-1}$day$^{-1}$ for XENON100 and KIMS, respectively.
Figure 4: Relative contributions of the charge-charge (CC), dipole-charge (DC), and dipole-dipole (DD) interactions to the total predicted event rate in XENON100, KIMS, and DAMA. The left panel shows the contribution from CC relative to the sum of DC and DD, the right panel shows the ratio of the DC and DD contributions. We assume $M_\eta/M_1 = 1.5$ and $\delta = 0$.

The upper bounds are obtained from the observed 3 events with $3\sigma$ of the statistical error in the 48 kg fiducial volume during 100.9 live days exposure in the signal region for XENON100 [42], and from ref. [48] for KIMS. For DAMA we perform a $\chi^2$ fit to the modulation amplitude in bins of observed scintillation energy between 2 and 8 keVee. In Fig. 5 and Fig. 6 we show the bounds from XENON100, KIMS and allowed regions from DAMA for the coefficients $b_{12}$ (see Eq. (22)) and $\mu_{12}$ (see Eqs. (23), (24)), respectively. These bounds are compared to the regions as predicted in the model according to Eqs. (19), (21), (25) for $b_{12}$ and Eq. (20) for $\mu_{12}$. The colored regions correspond to the regions shown in Fig. 2, satisfying constraints from neutrino masses and mixing, charged lepton-flavour violation, the relic DM density, and perturbativity. The ratio $M_\eta/M_1$ is taken in the range $1 \leq M_\eta/M_1 \leq 9.8$, with the same color shading as in Fig. 2. There is no allowed parameter space for $M_\eta/M_1 \gtrsim 9.8$, as discussed earlier.

We observe that the values of $|b_{12}|$ and $|\mu_{12}|$ obtained in this model are too small to account for the signal in DAMA. For very small mass splittings $\delta$ between $N_1$ and $N_2$ some regions of the parameter space are excluded by XENON100 data. The constraints become weaker for larger $\delta$, since increasing inelasticity suppresses the scattering event rate. Relatively large values of $|b_{12}|$ are obtained for close to degenerate $N_1$ and $N_2$ some regions of the parameter space are excluded by XENON100 data. The constraints become weaker for larger $\delta$, since increasing inelasticity suppresses the scattering event rate. Relatively large values of $|b_{12}|$ are obtained for close to degenerate $N_1$ and $N_2$ some regions of the parameter space are excluded by XENON100 data. The constraints become weaker for larger $\delta$, since increasing inelasticity suppresses the scattering event rate.

By comparing Figs. 5 and 6 we observe that the model predicts values of $|\mu_{12}|$ too small to be tested by current direct detection data. The enhancement for the transition magnetic moment $|\mu_{12}|$ for $M_\eta/M_1 \lesssim 1.05$ is less than for $|b_{12}|$ due to a different behavior of the
Figure 5: Bounds from XENON100, KIMS and allowed regions for DAMA in the $(M_1, |b_{12}|)$ plane (charge-charge interaction). The mass difference $\delta$ is taken as 0 keV (the left top panel), 40 keV (the right top panel), 80 keV (the left bottom panel) and 120 keV (the right bottom panel). The shaded regions correspond to the values of $b_{12}$ predicted in the allowed parameter space of the model, as shown in Fig. 2, with the same color shading for different values of the ratio $M_\eta/M_1$.

We conclude that current data from XENON100 start to exclude some parameter space of the model, in case of degenerate configurations $M_1 \simeq M_2 \simeq M_\eta \sim \text{few TeV}$. In Fig. 7 we show the regions excluded from XENON100 overlayed to the globally allowed regions from Fig. 2 as dark blue, by translating the the $|b_{12}|$ constraint into a bound on $\xi$. Furthermore we show in Fig. 7 the estimated sensitivity for XENON1T. Using the sensitivity for the elastic WIMP-nucleon scattering cross section from ref. [84] we estimate that XENON1T will constrain the event rate to be less than $1.59 \times 10^{-5} \text{[kg}^{-1}\text{day}^{-1}]$. (We assume the same nuclear recoil energy range as for XENON100.) Then we compare this number to

loop functions $I_a(x, y)$ and $I_m(x, y)$. 

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Figure 6: Bounds from XENON100, KIMS and allowed regions for DAMA in the \((M_1, |\mu_{12}|)\) plane (dipole-charge and dipole-dipole interaction). The mass difference \(\delta\) is taken as 0 keV (the left top panel), 40 keV (the right top panel), 80 keV (the left bottom panel) and 120 keV (the right bottom panel). The shaded regions correspond to the values of \(\mu_{12}\) predicted in the allowed parameter space of the model, as shown in Fig. 2, with the same color shading for different values of the ratio \(M_\eta/M_1\).

The event rate induced in the model assuming several values for the mass splitting \(\delta\). From Fig. 7 we find that for \(\delta \lesssim 40\) keV future data from the XENON1T experiment [84] will dig deeply into the allowed parameter region of the model. For \(40\) keV \(\lesssim \delta \lesssim 120\) keV the degenerate region \(M_1 \simeq M_2 \simeq M_\eta \sim \text{few TeV}\) will be tested. We note however, that no signal is guaranteed for direct detection. In the \(N_1 - \eta\) co-annihilation region (dark- and light-red regions, where \(M_\eta/M_1 < 1.2\)) no lower bound on the parameter \(|\xi|\) is obtained, leading to arbitrarily small values of \(|b_{12}|\) and \(|\mu_{12}|\), which implies a vanishing signal in direct detection experiments.
3.3 Monochromatic Photon from the Decay of $N_2$

The excited DM state $N_2$ decays to $N_1$ and a photon through the transition magnetic moment. The diagrams of the decay process are shown in Fig. 8 and the decay width is calculated as

$$\Gamma(N_2 \rightarrow N_1 \gamma) = \frac{\mu_{12}^2}{\pi} \delta^3. \quad (33)$$

Notice that the effective interaction $b_{12}$ does not contribute to the decay width since the emitted photon is on-shell. The decay of $N_2$ produces a monochromatic photon of energy $E_\gamma \simeq \delta$. If the decay happens inside a DM detector this monochromatic photon would contribute to the electromagnetic event rate. Although typically such events are rejected in order to search for nuclear recoils one may be able to place constraints on the model.
by requiring that the electromagnetic event rate induced by the decay of $N_2$ has to be less than the observed rate. A similar mechanism has been used in ref. [85] in order to explain the DAMA modulation signal.

Following ref. [85] we estimate the photon induced event rate in the model under consideration for the XENON100 experiment. The excited state $N_2$ is produced by the inelastic scattering with nuclei inside the Earth which is composed of various elements such as Fe, O and Si. The event rate in XENON100 is given by

$$
\frac{dR}{dE_R} = \rho_\odot \sum_{i=\text{nuclei}} \frac{d\sigma_i}{dE_R} v_f(v) \int_{\text{Earth}} d^3r n_i(r) P(r, v),
$$

where $\rho_\odot$ is the mass density of the XENON detector $21.9$ g/cm$^3$, $\sigma_i$ is the total inelastic scattering cross section which includes the charge-charge, dipole-charge and dipole-dipole interactions, and $n_i(r)$ is the number density for the given nucleus $i$ inside the Earth. Note that $E_R$ is the nuclear recoil energy in the $N_1 + A \rightarrow N_2 + A$ scattering process. The contribution of the dipole-dipole interaction is much smaller than the ones from the charge-charge and dipole-charge interactions since the fraction of isotopes with a sizable magnetic moment in the Earth is less than a few %. In Eq. (34), $P(r, v)$ is the probability that an $N_2$ which is produced by the scattering of DM with velocity $v$ at the position $r$ decays inside the XENON100 detector. It is given by

$$
P(r, v) = \frac{1}{4\pi(v - r_{\text{Xe}})^2} e^{-\Gamma |r - r_{\text{Xe}}|/v_f},
$$

where $v_f = \sqrt{v^2 - 2(\delta + E_R)/M_1}$ is the velocity of the produced $N_2$, and $r_{\text{Xe}}$ is the position of XENON detector on the Earth. The total gamma event rate $R_\gamma$ is obtained by integrating Eq. (34) over the recoil energy $E_R$.

In order to obtain a rough estimate of the induced event rate we introduce some approximations. We use the averaged number density of the elements in the Earth $\bar{n} \simeq 9.85 \times 10^{22}$ cm$^{-3}$, the averaged atomic number $\bar{Z} \simeq 29.9$ and the averaged magnetic moment of nuclei $\bar{\mu}_A/\mu_N \simeq 3.46 \times 10^{-2}$, which are calculated by taking into account the structure of the Earth such as the crust, mantle and core [85]. Replacing $n_i(r)$ by its
average, it can be pulled out of the $r$-integral in Eq. (34) and the integration is performed analytically:

$$\int_{\text{Earth}} d^3r P(r, v) = \frac{1}{2} \left[ \frac{v_f}{2\Gamma r_\oplus} \left( e^{-2\Gamma r_\oplus/v_f} - 1 \right) + 1 \right],$$

(36)

where $r_\oplus = 6.4 \times 10^6$ m is the radius of the Earth. The remaining integrations over $v$ and $E_R$ are done numerically.

With this approximation we estimate the total predicted event rate in XENON100 for typical parameters of the model. We find that the maximal rate is approximately $R_{\gamma}^{\text{max}} \approx 2.0 \times 10^{-7}$ kg$^{-1}$day$^{-1}$, when $\delta \approx 40$ keV and $M_1/M_\eta = 1$. This result should be compared with 22 events obtained in the electromagnetic band in the 40 kg fiducial volume during 11.17 live days exposure in the DM search window by XENON100 [41]. Hence, since the predicted event rate is several orders of magnitude smaller we conclude that the monochromatic photon from the $N_2$ decay will not lead to any observable signal in DM direct detection experiments.

## 4 Summary and Conclusions

We have considered a model proposed by Ma [10], providing an economical extension of the Standard Model to accommodate neutrino masses and DM. The Standard Model is extended by three fermion singlets $N_i$ (“right handed neutrinos”) and an inert scalar doublet $\eta$, where the new particles transform odd under a $Z_2$ symmetry, making the lightest of them a stable DM candidate. In our case $N_1$ is the DM particle. We investigate the parameter space of the model consistent with neutrino masses and mixings, bounds on charged lepton-flavour violation, perturbativity, and the correct relic DM abundance due to the thermal freeze-out mechanism. We find that in order to obtain the correct relic DM abundance co-annihilations are always important, either between the two lightest fermion singlets $N_1$ and $N_2$, or between $N_1$ and the inert doublet $\eta$.

In this model DM has no direct couplings to quarks and gluons. Despite this leptophilic nature of DM, scattering off nuclei is possible at 1-loop level by photon exchange. We have calculated the relevant loop processes in an effective field theory approach. One obtains effective charge-charge, dipole-charge, and dipole-dipole interactions between DM and nuclei, leading to a non-vanishing scattering rate in DM direct detection experiments. The scattering is inelastic and in order to obtain a sizable scattering rate $N_1$ and $N_2$ have to be highly degenerate, with mass differences $\delta$ less than few 100 keV. This is consistent with the need for co-annihilations to obtain the correct relic abundance. Although the scattering cross section in this model is too small to account for the DAMA annual modulation signal, we find that for mass differences $\delta \lesssim 120$ keV current data from the XENON100 experiment start to exclude certain regions of the parameter space. The predicted event rate for XENON100 is dominated by the charge-charge interaction. Future data, for example from XENON1T, will significantly dig into the allowed parameter space and provide a stringent test for the model provided $\delta$ is small enough.
Note added.

After this work has been completed the Daya Bay reactor experiment released their
data [65], establishing a non-zero value of $\theta_{13}$ at more than $5\sigma$ with $\sin^2 2\theta_{13} = 0.092 \pm 0.016 \pm 0.005$. For non-zero values of $\theta_{13}$ additional contributions to $\mu \rightarrow e\gamma$ are induced, providing further constraints on the model. Daya Bay data imply $\sin \theta_{13} > 0.1$ at $3\sigma$, which constrains DM masses around the TeV scale, see Fig. 2.

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Appendix A

Explicit Functions for the Effective Interactions

Here we give the explicit functions for the effective interactions. The functions $I_a(x,y)$ and $I_m(x)$ are given as follows,

$$I_a(x,y) = \frac{1}{3} \int_0^1 \frac{3u^2 - 6u + 1}{xu^2 - (1 + x - y)u + 1} du,$$

$$I_m(x,y) = -\int_0^1 \frac{u(1-u)}{xu^2 - (1 + x - y)u + 1} du.$$ (37)

The analytic formulas of these integrations are given as follows.

(i) If $(1 + x - y)^2 - 4x > 0$,

$$I_a(x,y) = \frac{1}{x} \left[ 1 + \frac{3A_+^2 - 6A_+ + 1}{3(A_+ - A_-)} \log \left| \frac{A_+ - 1}{A_+} \right| - \frac{3A_-^2 - 6A_- + 1}{3(A_+ - A_-)} \log \left| \frac{A_- - 1}{A_-} \right| \right],$$

$$I_m(x,y) = \frac{1}{x} \left[ 1 + \frac{A_+(A_+ - 1)}{A_+ - A_-} \log \left| \frac{A_+ - 1}{A_+} \right| - \frac{A_-(A_- - 1)}{A_+ - A_-} \log \left| \frac{A_- - 1}{A_-} \right| \right].$$ (39)

(ii) If $(1 + x - y)^2 - 4x = 0$,

$$I_a(x,y) = \frac{1}{x} \left[ 1 + 2(A_0 - 1) \log \left| \frac{A_0 - 1}{A_0} \right| - \frac{3A_0^2 - 6A_0 + 1}{3A_0(A_0 - 1)} \right],$$

$$I_m(x,y) = \frac{1}{x} \left[ 2 + (2A_0 - 1) \log \left| \frac{A_0 - 1}{A_0} \right| \right].$$ (41)
(iii) If \((1 + x - y)^2 - 4x < 0\),

\[
I_a(x, y) = \frac{1}{x} \left[ 1 + \frac{B_+ + B_- - 2}{2} \log \left( \frac{(B_+ - 1)^2 + (B_- - 1)^2}{B_+^2 + B_-^2} \right) \right. \\
+ \left. \frac{6(B_+ - 1)(B_- - 1) - 4}{3(B_+ - B_-)} \tan^{-1} \left( \frac{B_+ - B_-}{B_+^2 + B_-^2 - B_+ - B_-} \right) \right], \tag{43}
\]

\[
I_m(x, y) = \frac{1}{x} \left[ 1 + \frac{B_+ + B_- - 1}{2} \log \left( \frac{(B_+ - 1)^2 + (B_- - 1)^2}{B_+^2 + B_-^2} \right) \right. \\
+ \left. \frac{2(B_+ - 1)(B_- - 1)}{2(B_+ - B_-)} \tan^{-1} \left( \frac{B_+ - B_-}{B_+^2 + B_-^2 - B_+ - B_-} \right) \right]. \tag{44}
\]

\(A_\pm, A_0\) and \(B_\pm\) are defined as

\[
A_\pm \equiv \frac{1 + x - y \pm \sqrt{(1 + x - y)^2 - 4x}}{2x},
\]

\[
A_0 \equiv \frac{1 + x - y}{2x},
\]

\[
B_\pm \equiv \frac{1 + x - y \pm \sqrt{4x - (1 + x - y)^2}}{2x}.
\]

The function \(I_c(x, y)\) is the same as \(I_m(x, y)\). These functions are continuous and smooth for \(0 \leq x, y \leq 1\). For \(0 \ll y \ll x \ll 1\), these functions approach to

\[
I_a(x, y) \to \frac{1}{2} + \frac{2}{3} \log y, \tag{45}
\]

\[
I_m(x, y) \to -\frac{1}{2}. \tag{46}
\]

Therefore, the obtained parameters \(|b_{12}|\) and \(|\mu_{12}|\) at lowest order agree with the result of ref. [53] where the parameter \(\lambda^2\) in ref. [53] corresponds to \(\text{Im}(h^*_a h_{a1})/2\) in our notation. The difference of the relative sign comes from the definition of the effective operators.

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