Neural Ordinary Differential Equations for Intervention Modeling

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Abstract
By interpreting the forward dynamics of the latent representation of neural networks as an ordinary differential equation, Neural Ordinary Differential Equation (Neural ODE) emerged as an effective framework for modeling a system dynamics in the continuous time domain. However, real-world systems often involves external interventions that cause changes in the system dynamics such as a patient being administered with particular drug. Neural ODE and a number of its recent variants, however, are not suitable for modeling such interventions as they do not properly model the observations and the interventions separately. In this paper, we propose a novel neural ODE-based approach (IMODE) that properly model the effect of external interventions by employing two ODE functions to separately handle the observations and the interventions. Using both synthetic and real-world time-series datasets involving interventions, our experimental results consistently demonstrate the superiority of IMODE compared to existing approaches.

1 Introduction
Although we live in continuous time, physical systems (e.g., bouncing ball, patient state) are often observed in a discretized fashion, either regularly or irregularly. For example, while climate sensors can collect information at every hour, patient blood samples are drawn only by a physician’s order. Various approaches have been proposed to handle such time-series data with neural networks, often modifying recurrent neural networks (RNNs) with varying degrees of success (Choi et al. 2016; Du et al. 2016; Lipton, Kale, and Wetzel 2016; Baytas et al. 2017; Che et al. 2018), until Neural Ordinary Differential Equations (Neural ODEs) (Chen et al. 2018) proposed a natural framework to model a system dynamics in a continuous time domain.

Neural ODEs view the forward pass of the vector representation \( h \), often corresponding to the system state, as numerically solving an ordinary differential equation using the time derivative \( \frac{dh}{dt} \) parameterized by a neural network. As this framework provides a natural means to handle both regular as well as irregular time-series data, previous studies have extended Neural ODEs to encode observation sequences. These approaches have demonstrated improved performance in prediction tasks on simulated data, climate records, and medical records (Rubanova, Chen, and Duvenaud 2019; De Brouwer et al. 2019; Jia and Benson 2019; Poli et al. 2019).

Real-world systems, however, are often influenced by external factors (i.e., interventions), such as a patient being administered some medication at a particular time. Depending on the system characteristic, these influences can change the system dynamics instantaneously or in a prolonged manner. While effective in modeling intervention-free dynamics, previous approaches capable of handling discrete or continuous feature evolution such as RNN–decay, GRU–D, and ODE–RNNs (Che et al. 2018; Rubanova, Chen, and Duvenaud 2019) were not designed to handle cases with external interventions. In particular, ODE–RNNs and their derivative architectures (De Brouwer et al. 2019; Jia and Benson 2019) place a strong assumption on the effect of additional observations on the system state; by aggregating input information with either recurrent cells or a multi-layer perceptron (MLP), the system state is directly modified by each observations.

In response, this paper proposes Intervention-Modeling Or-
work for modeling the continuous transformation of the state (Rubanova, Chen, and Duvenaud 2019) uses the RNN to vector field interventions modeling. Flows according to an ODE between observations, and a new h encode a sequence of observations, where the latent state transitions to model time-series data, several approaches extended the Impulsive Systems Motivated by Neural ODEs’ flexibility parameters (Massaroli et al. 2020b). h correspond to the input vector h ∈ Rn, generally corresponding to the input vector x ∈ Rnx or its embedding, the state system h(tk) is obtained by integrating forward the vector field fθ : R × Rnx × Rnx → Rnx, parameterized by θ ∈ Rnx. While we consider θ to be constant over time, the discussion below is directly compatible with time-varying parameters (Massaroli et al. 2020b). Impulsive Systems Motivated by Neural ODEs’ flexibility to model time-series data, several approaches extended the Neural ODE framework to encode a sequence of observations {xk}, whether being regular or irregular. ODE-RNN (Rubanova, Chen, and Duvenaud 2019) uses the RNN to encode a sequence of observations, where the latent state h flows according to an ODE between observations, and a new observation xk directly modifies h according to the RNN update equation. GRU-ODE-Bayes (De Brouwer et al. 2019) specifically uses the GRU (Cho et al. 2014) to encode observations, where a new observation directly modifies h according to the GRU update equation combined with some masking operations. Neural Jump Stochastic Differential Equations (NJDE) (Jia and Benson 2019) combines ODEs with point processes to model stochastic events. In NJDE, given a new observation, an event history representation is directly modified via an MLP, combining some internal state with the observation. Let (T, ≥) be a finite linearly ordered set, called the data time set (typically T = {t0, t1, ..., tN}). We assume an input-output data stream is given as a sequence {xtk}tk∈T. For a compact notation, we denote xtk with xk. The underlying core approach in all three approaches is to update the system state h via a particular non-linear operation when a new observation is given, which can be generalized as an impulsive differential equation (Lakshmikantham, Simeonov et al. 1989; Kulev and Banov 1988) of the type,

\[
\begin{aligned}
\dot{h} &= f_\theta(t, h(t)) & t \neq t_k \\
\dot{h}^+ &= g_\theta(h(t), x_k) & t = t_k
\end{aligned}
\]  

(2)

where \( \dot{h}^+ \) indicates the value of \( h \) after the discrete jump at \( t_k \). Between observations, the system state \( h \) evolves according to continuous dynamics. New observations, on the other hand, trigger a jump of \( h \) to \( h^+ \) as determined by \( g_\theta \). For the rest of the paper, we refer to this broad family of models as Neural Jump Differential Equations (NJDE). Although it is possible to use NJDEs to naively model systems with interventions, by, say, simply concatenating them with observations, this approach is limited as it does not explicitly separate autonomous dynamics of the system from these effects. This, in turn, makes it challenging to properly reconstruct the evolution of a system subject to external forces.

It should be noted that according to Eq. 2, new inputs to NJDEs induce state jumps. In general, states in dynamical systems are not guaranteed to jump in their entirety. For example, in mechanical systems with impacts, only higher-order states jump (e.g., velocities). Moreover, when the underlying dynamics is only partially observable, which is the case in numerous real-world problems (e.g., only a patient’s temperature is measured), interventions cause jumps in a latent representation of the state, ultimately leading to an abrupt change in the dynamics driving the observable. As a result, the state does not jump, but rather continuously changes according to the new, modified underlying dynamics.

## 3 Proposed Approach

With the objective of alleviating the underlying limitations in NJDE-based approaches, we propose IMODE, a novel framework designed to natively accommodate various types of intervention effects, common across application areas. IMODE can properly model both the system’s autonomous dynamics as well as the effect of interventions. We first describe the mathematical framework of IMODE, followed by the discussion on how it can be applied to different types of systems.
where "dress such scenarios, we allow system (rent and the past interventions have a combined effect on the preserving continuity of components to incorporate sporadic observations and interventions, namely \( g^r_{\phi} \) and \( g^\theta_{\phi} \), which induce jumps on \( z_x \) and \( z_a \) while preserving continuity of \( h \). It is often the case that the current and the past interventions have a combined effect on the system (i.e. medications given to patients over time); to address such scenarios, we allow \( g^r_{\phi}, g^\theta_{\phi} \) to leverage information contained in \( h \), in addition to \( z_x \) and \( z_a \).

We train the various components of (3) with a reconstruction loss of the type:
\[
\mathcal{L} := \frac{1}{K} \sum_{k=1}^{K} \left\| x_{t_k} - \hat{x}(t_k) \right\|_2^2 = \frac{1}{K} \sum_{k=1}^{K} \left\| x_{t_k} - \ell_\omega(h(t_k)) \right\|_2^2
\]
(4)
where \( \ell_\omega \) is a trainable decoding function that maps \( h \) back to observation space \( x \). IMODE is therefore trained by solving the following nonlinear program
\[
\min_{\psi, \phi, \omega} \frac{1}{K} \sum_{k=1}^{K} \left\| x_{t_k} - \ell_\omega(h(t_k)) \right\|_2^2
\]
subject to \( h(t) = h(0) + \int_{0}^{t} f^h_{\phi}(h, z_x, z_a) \, dt \)
(5)
\[
\begin{align*}
\dot{z}_x &= f^r_{\phi}(h, z_x, x_{t_k}) & t = t_k \\
\dot{z}_a &= f^\theta_{\phi}(h, z_a, a_{t_k}) & t = t_k \\
\hat{x}(t) &= \ell_\omega(h(t))
\end{align*}
\]

System–Specific Variants of IMODE

The framework of our model allows a flexible implementation depending on the property of the target system. In the following, we give a few concrete examples.

**Switching intervention effect:** For example, a ball in uniform motion can be seen as having constant autonomous dynamics: given no external force, it will continue to move along the same course. However, if it comes in contact with another moving ball, its direction will permanently change. In these scenarios, appropriate architectural choices for model (3) would be, as an example, those provided in Table 4. With no intervention given, the autonomous vector field \( f^\theta_{\phi} \) is solely determined by the current state \( h_{t_k} \). If a collision occurs, the intervention latent state \( z_a \) abruptly changes based on the colliding ball state, thus indirectly affecting \( h \) through...
Table 1: IMODE variant for switching intervention effects.

| continuous dyn. | model | discrete dyn. | model |
|------------------|-------|---------------|-------|
| $f^{h^b}_{\psi}$ | $z_x + z_a$ | $g_0^\psi$ | MLP |
| $f^a_{\theta}$    | 0     | $g_0^a$ | MLP |
| $f^a_{\sigma}$    | 0     | $\ell_{\omega}$ | Id |

Table 2: IMODE variant for decaying intervention effects.

| continuous dyn. | model | discrete dyn. | model |
|------------------|-------|---------------|-------|
| $f^{h^b}_{\psi}$ | MLP   | $g_0^\psi$ | MLP |
| $f^a_{\theta}$    | MLP   | $g_0^a$ | MLP |
| $f^a_{\sigma}$    | $-\alpha z_a$ | $\ell_{\omega}$ | Id |

Table 3: IMODE variant for general purpose settings.

| continuous dyn. | model | discrete dyn. | model |
|------------------|-------|---------------|-------|
| $f^{h^b}_{\psi}$ | MLP   | $g_0^\psi$ | MLP |
| $f^a_{\theta}$    | MLP   | $g_0^a$ | MLP |
| $f^a_{\sigma}$    | MLP   | $\ell_{\omega}$ | MLP |

4 Experiments

We evaluate IMODE with two simulated datasets and one real-world medical records both quantitatively and qualitatively. The two simulated datasets (Moving Ball and Exponential Decay) represent systems with permanent-effect interventions and decaying-effect interventions, respectively. We also use eICU (Pollard et al. 2018), a publicly available electronic health records, to evaluate IMODE’s performance on real-world data. Detailed experimental settings including the hyperparameters are described in Appendix [B]. All datasets and IMODE source code are available at GitHub [F].

Moving Ball & Exponential Decay

We use two simulated datasets, Moving Ball and Exponential Decay to demonstrate IMODE’s capability to model the first two cases described in Section 3: permanent-effect interventions, and changing autonomous dynamics with time-decaying effects of intervention. Observations $x$ and interventions $a$ in Moving Ball consist of 2D positions of the target ball, and 2D positions and 2D velocities of intervening balls, sampled from a contact simulator, where we set all balls to have the same size and mass. Example trajectories can be seen in Figure [E]. Note that the velocity of the target ball is unobserved, forcing the models to infer the velocity based on positions.

In Exponential Decay, observations $x$ consist of 2D positions that follow a deterministic dynamics based on $x$. Interventions $a$ consist of 2D values sampled from $N(0, I)$ with 10% chance at every time unit. The effect of interventions is determined by a non-linear function (i.e. a randomly initialized MLP with ReLU activation) using $x$, $dx/dt$ and $a$ as input. The effect is added to $x$, and the value of an effect is halved at every time unit. Example trajectories can be seen in Figure [E]. Note that the intervention effect is hidden from the model, as only the randomly sampled $a$ is given. We describe the simulation algorithm of Exponential Decay in Appendix [C]. In both Moving Ball and Exponential Decay, the initial position $x_0$ and the initial velocity $dx_0/dt$ are randomly chosen. For both Moving Ball and Exponential Decay, we generated 1,000 samples for training, 100 for validation, and 100 for testing, where all samples have the sequence length of 50.

Baseline Methods We compare IMODE against both RNN-based and ODE-based methods: GRU with time-gap information (GRU-$\Delta t$), GRU with exponentially decaying hidden states (GRU-ODE), ODE-RNN (Rubanova, Chen, and Duvenaud [2019]), and GRU-ODE-Bayes (De Brouwer et al. [2019]). Specifically, GRU-decay can be seen as using prior knowledge of intervention effects, similar to classical intervention models discussed in Section [S]. Note that observations $x$ occur regularly (i.e. no missing value) but the interventions occur irregularly. To feed observations $X$ and interventions $A$ to the baseline models, we align both time-series data by the time and concatenate them. If $a$ does not exist at some timestep $t_k$, we set it to a zero vector. Given in-

[https://github.com/eogns282/IMODE](https://github.com/eogns282/IMODE)

[https://scipython.com/blog/two-dimensional-collisions](https://scipython.com/blog/two-dimensional-collisions)
were measured in the same fashion; given the first
modification details are provided in Appendix D).

Table 1, 2) IMODE

| Methods     | Moving Ball ($\times 10^{-2}$) | Exponential Decay ($\times 10^{-4}$) |
|-------------|-------------------------------|-------------------------------------|
|             | Validation MSE | Test MSE | Validation MSE | Test MSE |
| GRU-$\Delta_t$ | 6.033 (± 0.272) | 5.994 (± 0.102) | 4.381 (± 0.204) | 5.686 (± 3.258) |
| GRU-Decay    | 6.384 (± 0.059) | 5.994 (± 0.102) | 5.135 (± 1.660) | 6.589 (± 5.109) |
| ODE-RNN      | 2.478 (± 0.142) | 2.502 (± 0.328) | 3.342 (± 1.161) | 2.778 (± 1.173) |
| GRU-ODE-Bayes| 2.506 (± 0.436) | 2.597 (± 0.520) | 3.852 (± 2.903) | 3.643 (± 4.996) |
| CRN          | 2.209 (± 0.065) | 2.220 (± 0.299) | 3.716 (± 2.542) | 4.881 (± 3.846) |
| IMODE switch | 1.914 (± 0.173) | 1.794 (± 0.203) | 0.019 (± 0.001) | 0.027 (± 0.005) |
| IMODE decay  | 1.794 (± 0.203) | 1.816 (± 0.203) | 0.142 (± 0.084) | 0.131 (± 0.490) |
| IMODE general| 1.798 (± 0.215) | 1.824 (± 0.230) | 0.039 (± 0.009) | 0.041 (± 0.008) |

Table 4: Validation and test MSE of all models on Moving Ball and Exponential Decay.

put $[X_{0:t_k}; A_{0:t_k}]$, RNN-based models are trained to predict $X_{t_k+1}$.

ODE-based models are trained in the same fashion as IMODE (Eq. 2). We also test three variants of IMODE differing in terms of expressiveness: 1) IMODE switch from Table 1, 2) IMODE decay from Table 2, 3) IMODE general from Table 3. We also use Counterfactual Recurrent Network (CRN) (Bica et al. 2020) as a baseline, the state-of-the-art model that takes interventions into account when modeling observations. CRN, however, models only discrete interventions whereas interventions in Moving Ball and Exponential Decay are continuous. We therefore use a modified CRN (modification details are provided in Appendix D).

Quantitative Evaluation During training, we fed the first 10 true observations (i.e. the 2D positions) to each model, and then made it evolve for the remaining 40 steps, while always using true intervention values $A$ from time 0. Model parameters were updated via the MSE loss between the predicted observations $\hat{X}$ and the true observations $X$. The test MSEs were measured in the same fashion: given the first 10 true positions, and the true intervention information, simulate the remaining 40 steps. We conduct 5-fold cross-validation for all experiments.

As seen in Table 4, all IMODE variants consistently outperform the baseline models for both datasets. Moreover, IMODE general shows robust performance in both Moving Ball and Exponential Decay, demonstrating its capability to learn two significantly different dynamical systems. The performance gap between baselines and IMODE is much larger for Exponential Decay, indicating that IMODE is a suitable framework especially in modeling the system with a global pattern (i.e. autonomous dynamics) and local perturbations (i.e. interventions). It is also noteworthy that the MSEs of all models are shown to be significantly higher for Moving Ball compared to Exponential Decay, probably due to the difficulty of modeling acute changes in the ball dynamics, as seen in Figure 3.

We further tested all models in counterfactual scenarios using both datasets where a single trajectory, after 10 initial steps, divides into two alternative futures (with and without an intervention) and continues for another 10 steps. Example trajectories can be seen in Figure 5. We feed the first 10 steps to the already trained models from Table 4 and then let them simulate the next 10 steps for two alternative futures. As seen from Table 5, IMODE variants again outperform all baselines in these counterfactual scenarios. The fact that IMODE is able to separately learn autonomous dynamics and intervention effects clearly indicates its capability to generalize to alternative cases.

Model Behavior Analysis To confirm that IMODE properly learns the autonomous dynamics and the intervention effect, we visualize test trajectories from Moving Ball (Figure 3) and Exponential Decay (Figure 4), where the first 10 true timesteps are given to the model, and the model simulates the remaining 40 steps while using true interventions. We show the results of IMODE general, ODE-RNN, and RNN-Decay for comparison. In both figures, IMODE clearly outperforms baseline models as it closely follows the true trajectories, while the baselines often diverge. The comparison between Figures 3 and 4 demonstrates the challenging nature of Moving Ball, thus resulting in the higher MSE in Table 4. Whereas Exponential Decay shows smooth and moderate change of dynamics, the changes in Moving Ball are not only discrete but also significant (e.g., ball changing direction in almost 180 degrees).

Figure 3 shows that the autonomous latent state $z_a$ stays rather static, while the intervention latent state $z_i$ jumps when a collision occurs. One can also see from this figure that after the jump, $z_i$ does not decay over time, indicating that IMODE is successfully recording the permanent effects of all the previous interventions. Ideally, $z_a$ should remain constant over time, but minor changes and fluctuations are found, especially in the second trajectory of Figure 3, leaving
Figure 3: Simulated trajectories and $L_2$ norms of latent states of IMODE and baselines for the Moving Ball dataset. The gray dotted lines connect collision points to the corresponding timesteps. The three samples represent medium, light, and heavy interventions, respectively.

Figure 4: Simulated trajectories and $L_2$ norms of latent states of IMODE and baselines for the Exponential Decay dataset. The gray dotted lines connect intervention points to the corresponding timesteps. The three samples represent medium, light, and heavy interventions, respectively.

room for further improvement. Based on the trajectories of $|\text{ODE-RNN}|_2$, ODE-RNN also recognizes the occurrence of collisions, but it fails to treat observations and interventions separately, leading to incorrect simulation. Compared to Moving Ball, Exponential Decay is a completely different system where the state follows its own dynamics while being occasionally perturbed with an exponentially decaying effect. Thanks to its relatively smooth trajectory, even the baseline models tend to stay close to the true trajectory. However, the $L_2$ norm plots of Figure 4 clearly demonstrate the benefit of separately modeling observations and interventions. While $||z_i||_2$ demonstrates a smoothly changing trajectory potentially corresponding to the autonomous system dynamics, $||z_a||_2$ jumps when an intervention occurs and decays over time, indicating that IMODE has properly learned the true intervention effect on the system.

We also provide visual examples for the counterfactual scenarios. The two figures in Figure 5 describe two counterfactual cases in Moving Ball and Exponential Decay respectively. As indicated by the quantitative results in Table 5, IMODE outperforms baseline models in both datasets as it closely follows the two alternative futures while the baselines diverge from the true trajectories in both alternative cases.

eICU Dataset

The eICU Collaborative Research Database (eICU) (Polard et al. 2018) contains publicly available electronic health records (EHR) collected from multiple intensive care units (ICU). In order to correctly evaluate IMODE’s ability to learn the patient’s autonomous dynamics and the effect of interventions (i.e., drugs), we choose a patient with the longest ICU stay whose drugs were given only via IV infusion to remove any confounding factors (e.g., drugs taken orally). We focus on a single patient since every patient has a unique autonomous dynamics and response to drugs determined by hidden factors (e.g., DNA and diet). We leave handling multiple heterogeneous dynamics with a single model as future work. We extract from the EHR three blood pressure features (systolic, diastolic, and mean) measured every 5 minutes as the observation $x$, and the hourly interventions consist of five drug types (norepinephrine, vasopressin, propofol, amiodarone, and phenylephrine) and their dosage. We binned the entire observations into 2.5-hour buckets (30 timesteps containing two interventions) and used 150 buckets for training, 50 for validation and 50 for testing.

Quantitative Evaluation Using the same set of baselines as in the above experiments, we train all models with the reconstruction loss (Eq. 4) in a similar fashion as before; 6 true timesteps are given to the models, and the remaining 24 steps are simulated using true intervention information. We conduct 5-fold cross validation for all models. As can be seen in Table 6, IMODE shows the best test performance, demonstrating its potential applicability to real-world data such as patient vital signs. We also present further analysis of the model behavior in Appendix E.
5 Related Work

Continuous–Depth Learning Continuous–depth learning (Sonoda and Murata 2017, Haber and Ruthotto 2017, Hauser and Ray 2017, Lu et al. 2017, Che et al. 2018, Massaroli et al. 2020a) has recently emerged as a novel paradigm providing a dynamical system perspective on machine learning. This view has inspired design of novel architectures (Chang et al. 2017, Zhu, Chang, and Fu 2018, Demeester 2019, Chang et al. 2019, Cranmer et al. 2020, Massaroli et al. 2020a) as well as guiding the injection of physics–inspired inductive biases (Greydanus, Dzamba, and Yosinski 2019, Köhler, Klein, and Noé 2019). The framework has seen applications to various classes of differential equations (Tzen and Raginsky 2019, Li et al. 2020) and graphs (Poli et al. 2019), along with several analyses regarding computational speedups through regularization (Frilay et al. 2020) or specific numerical methods (Poli et al. 2020).

(Rubanova, Chen, and Duvenaud 2019) demonstrated promising empirical performance across various forecasting datasets by combining RNNs with Neural ODEs. (Yildiz, Heinonen, and Lahdesmaki 2019) refined the architecture through higher–order dynamics and Bayesian networks. (De Brouwer et al. 2019) alternates a filtering and predictions steps to improve performance in settings with highly sporadic observations. (Jia and Benson 2019) models a stochastic event by estimating the occurrence probability with Neural ODEs, where a new event observation updates the event intensity. While the above approaches share similarities with the proposed approach such that a input sequence modifies the internal state, they fail to treat observations and interventions differently, leading to suboptimal performance when modeling external interventions in a given system.

Intervention Modeling Intervention modeling is typically discussed in the context of time-series analysis. Combining the intervention analysis technique with the classical time-series models enables the user to handle time-series data with different types of interventions such as permanent, gradually increasing or decreasing, and complex effects (Glass, Willson, and Gottman 2008). Intervention analysis has been used across diverse domains such as healthcare (Evans 2002, Wagner et al. 2002), economics (Box and Tiao 1975) and policies (Enders and Sandler 1993). More recent studies have been conducted in the context of patient modeling, where models based on a Gaussian Process (GP) and RNNs have been proposed (Schulam and Saria 2017, Soleimani, Subbaswamy, and Saria 2017, Lim 2018, Bica et al. 2020). Considering the restricted model structure that GP assumes, we used the state-of-the-art patient modeling algorithm from Bica et al. as one of the baselines. Causal analysis is also relevant to our work, but it aims to identify the causal relationship between input (usually a mixture of causal factors and confounders) and output (Pearl 2009). On the other hand, intervention modeling focuses on correctly predicting the effect of an external force. As intervention modeling is essentially a time-series problem, Neural ODEs are a natural framework for this task.

6 Conclusions

In this work, we proposed IMODE, a Neural ODE-based framework that can properly model dynamical systems with external interventions. IMODE employs two components where one models the autonomous dynamics of the system, and the other models the intervention effect on the system. Using both simulated and real-world datasets, we quantitatively demonstrated IMODE’s superiority in intervention modeling, as well as in-depth analysis on its behavior. As future work, we plan to apply IMODE in large-scale real-world datasets while extending IMODE to further disentangle the autonomous dynamics and the intervention effects.
Ethical Impact

Although we empirically demonstrated that the proposed framework IMODE is capable of learning separate latent states for both autonomous dynamics and intervention effects, it should be used with caution in real-world applications. As described in Section 5, IMODE is not a causal analysis model, which means that the user must possess domain knowledge as to which variables are observations and which are interventions, and that there are no unobserved confounders that can affect the given dynamical system (as described in the eICU Dataset subsection). For example, we believe IMODE can be used to model patient status in a well-controlled environment such as patients under anesthesia during operation. We are certain more opportunities will follow as we address issues such as scalability and confounding factors in the future.

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A Notation Table
Table 7 describes the notations and their descriptions used throughout this paper.

B Hyperparameters
In the experiments section, we evaluated five baseline methods: RNN-Δ, RNN-Decay, ODE-RNN, GRU-ODE-Bayes and CRN. We trained all baseline models for the same number of epochs in each experiment, and the final models were chosen by the validation loss in each epoch. Additionally, we trained all ODE-based models using the Runge-Kutta fourth-order method and the same delta-times.

Hyperparameters for IMODE In the Moving Ball task, we used the batch size of 32 for 1,000 epochs. When using RNNs or ODE-RNNs for both \( f_\theta \) and \( f_\psi \), the size of the hidden vector was 40. Additionally, when using ODE-RNNs for \( f_\theta \) and \( f_\psi \), their derivative functions were a two-layer MLP with the hidden size of 40 and LeakyReLU as the activation function. For \( f_\psi \), we used the same 40-dimensional two-layer MLP with LeakyReLU activation. We used the RMSprop optimizer with the learning rate of 0.001 and set the delta-time as 0.01. In Exponential Decay, we used the same setting as Moving Ball but used 1,500 epochs.

In the eICU task, we trained for 1,500 epochs with the batch size of 32. We used ODE-RNNs for the functions \( f_\theta \) and \( f_\psi \) with 20-dimensional and 10-dimensional hidden vectors respectively. For \( f_\psi \), we used the 20-dimensional two-layer MLP with LeakyReLU activation. We used the delta-time of 1.0 for the ODE solver.

Baselines For the baselines in our experiments, we follow the general structure and hyperparameters of each model’s available implementation other than small details. Specifically, the models were tuned by the performance in the validation phase in order to obtain the proper batch size and learning rate. We also adjusted the hidden vector dimension of the baseline models to match that of IMODE.

C Algorithm of Exponential Decay
Algorithm 1 describes the pseudo-code for generating simulated samples used in the Exponential Decay task.

D Modification to Counterfactual Recurrent Network
Counterfactual Recurrent Network (CRN) (Bica et al. 2020) has Gradient Reversal Layer (GRL) that suppresses the correct prediction of the treatment type that occurs in the next timestep. This technique cannot be used in Moving Ball and Exponential Decay, because the interventions in those datasets consist of continuous values (i.e., position and velocities of the incoming ball in Moving Ball, and the randomly generated intervention effect in Exponential Decay). Although the GRL component is able to predict No Treatment class as well, since the interventions occur randomly in Moving Ball and Exponential Decay, predicting the binary case of Treatment and No Treatment cannot be done either. Therefore, in the two simulated datasets (Moving Ball and Exponential Decay) that have continuous and unpredictable interventions, we used CRN with \( \lambda = 0 \) (i.e., not using a treatment classifier and GRL).

Algorithm 1 The simulation algorithm of Exponential Decay
1: Input: time unit \( dt \), length of time series \( K \)
2: Initialize observation \( x_0 \), \( dx_0/dt \) are randomly chosen in \([0, \ 1] \), intervention effect \( e_0 = 0 \),
update matrix of dynamics \( (dx/dt) \)
\[
M_v = \begin{bmatrix} 1.5 & 0 \\ 0 & -2.5 \end{bmatrix}
\]
3: for \( k = 1 \) to \( K \) do
4: \( a_k = 0 \)
5: \( x_k = x_{k-1} + dt \ast (dx_{k-1}/dt + e_{k-1}) \)
6: \( dx_k/dt = M_v(dx_{k-1}/dt) \)
7: \( e_k = e_{k-1} \ast 0.5 \)
8: \( \text{intervention} \text{occurs} \sim \text{Bernoulli}(0.1) \)
9: if \( \text{intervention} \text{occurs} \text{then} \)
10: \( a_k \sim N(0, 1) \)
11: \( e_k = e_k + \text{MLP}([x_k, dx_k/dt, a_k]) \)
12: end if
13: end for
14: return \( X_{1:K}, A_{1:K} \)

In the eICU experiment, although the value of intervention is continuous (i.e., the dosage of each treatment), their treatment type and its administration time would be predictable using observational trajectories of patient. Therefore, as the original setting in CRN, we used a treatment classifier to predict treatment types excluding their dosage. Additionally, since the patients can be given multiple treatments simultaneously in the eICU experimental settings, we used the sigmoid activation function in the last layer of the treatment classifier instead of softmax function to predict the multiple treatments at the same time.

E Model Behavior Analysis for eICU
In this section, we visualize the trajectories of eICU samples and their \( L_2 \) norms to study the model behaviors. In Figure 3, the model was given true observations and interventions for the first six steps. Then, each model autoregressively predicted the observations for the remaining 24 timesteps while using true interventions.

As depicted in both Figure 6 and 7, the blood pressure features spikes when there are interventions followed by a gradual decay.

References
Bica, I.; Alaa, A. M.; Jordon, J.; and van der Schaaf, M. 2020. Estimating counterfactual treatment outcomes over time through adversarially balanced representations. In International Conference on Learning Representation.
| Symbol | Description | Domain (and codomain) |
|--------|-------------|----------------------|
| x      | input       | $\mathbb{R}^{n_x}$  |
| a      | intervention| $\mathbb{R}^{n_a}$  |
| h      | continuous latent state | $\mathbb{R}^{n_h}$ |
| z_x    | autonomous latent state | $\mathbb{R}^{n_z,x}$ |
| z_a    | intervention latent state | $\mathbb{R}^{n_z,a}$ |
| $\psi$ | h’s dyn. parameters | $\mathbb{R}^{n_\psi}$ |
| $\theta$ | z_x’s dyn. parameters | $\mathbb{R}^{n_\theta}$ |
| $\phi$ | z_a’s dyn. parameters | $\mathbb{R}^{n_\phi}$ |
| $\omega$ | decoder’s parameters | $\mathbb{R}^{n_\omega}$ |
| $f^h_\psi$ | h’s flow map | $\mathbb{R}^{n_h} \times \mathbb{R}^{n_z,x} \times \mathbb{R}^{n_z,a} \times \mathbb{R}^{n_\psi} \rightarrow \mathbb{R}^{n_h}$ |
| $f^h_\theta$ | z_x’s flow map | $\mathbb{R}^{n_h} \times \mathbb{R}^{n_z,x} \times \mathbb{R}^{n_\theta} \rightarrow \mathbb{R}^{n_z,x}$ |
| $f^h_\phi$ | z_a’s flow map | $\mathbb{R}^{n_h} \times \mathbb{R}^{n_z,a} \times \mathbb{R}^{n_\phi} \rightarrow \mathbb{R}^{n_z,a}$ |
| $g^h_\psi$ | z_x’s jump map | $\mathbb{R}^{n_h} \times \mathbb{R}^{n_z,x} \times \mathbb{R}^{n_x} \times \mathbb{R}^{n_\psi} \rightarrow \mathbb{R}^{n_z,x}$ |
| $g^h_\phi$ | z_a’s jump map | $\mathbb{R}^{n_h} \times \mathbb{R}^{n_z,a} \times \mathbb{R}^{n_a} \times \mathbb{R}^{n_\phi} \rightarrow \mathbb{R}^{n_z,a}$ |
| $\ell_\omega$ | output decoder | $\mathbb{R}^{n_h} \times \mathbb{R}^{n_\omega} \rightarrow \mathbb{R}^{n_x}$ |

Table 7: Notations used throughout the paper.

Figure 6: Three trajectories (mean blood pressure, diastolic blood pressure, and systolic blood pressure) and $L_2$ norms of hidden layers of IMODE and baselines for the eICU dataset. The gray-dotted lines connect intervention points (i.e. when norepineprine was injected) to the corresponding timesteps.

Figure 7: Another set of three trajectories (mean blood pressure, diastolic blood pressure, and systolic blood pressure) and $L_2$ norms of hidden layers of IMODE and baselines for the eICU dataset. The gray-dotted lines connect intervention points (i.e. when norepineprine was injected) to the corresponding timesteps.