The reexamination of thermal expansion of ferromagnetic superconductors and the pressure differential of its superconducting transition temperature-possible application to UGe$_2$

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Abstract. The temperature dependence of thermal expansion of ferromagnetic superconductors below the superconducting transition temperature $T_{sc}$ of a majority spin conduction band is reexamined. In the previous study [to be published in J. M. Phys. B] the volume differential of the kinetic energy of conduction electrons is constant. However, in this study the volume differential of the kinetic energy of conduction electrons is inconstant. The superconducting gap of the majority spin conduction band used in this study has a line node. It is appropriate to UGe$_2$. The pressure differential of its superconducting transition temperature is also investigated. We find that the thermal expansion coefficient has the divergence at the superconducting transition temperature. The thermodynamic Grüneisen’s relation is satisfied.

1. Introduction

Many researchers have paid attention to the ferromagnetic superconductors [1, 2] again since ferromagnetic superconductors UGe$_2$ [3], UCoGe [4] and URhGe [5, 6] was discovered. Recently, Hatayama and Konno [7, 8] investigated the temperature dependence of thermal expansion based on the free energy derived by Linder and Sudbo [9, 10] by Takahashi’s method [11]. However, they assumed that the volume differential of the kinetic energy of conduction electrons is constant. In this study, thermal expansion behaviour is reexamined by our keeping the volume differential of the kinetic energy of conduction electrons inconstant.

Secondly, the pressure differential of the superconducting transition temperature and that of the Curie temperature are studied. Pfleiderer et al. investigated critical behaviour at the transition as a function of hydrostatic pressure [12]. Recently, Shopova and Uzunov [13] looked into the temperature-pressure phase diagram based on the Landau expansion of the free energy. Aso et al. estimated the pressure dependence of Stoner gap based on the Stoner model from the neutron intensities [14]. It seems to us that they do not satisfy the thermodynamic Grüneisen’s relation. The pressure differential of the superconducting transition temperature and that of the Curie temperature based on the free energy derived by Linder and Subo [10] is unanswered. We will obtain the analytical expression of the pressure differential of the superconducting transition temperature of the majority spin conduction band and that of the Curie temperature by the mean field approximation. The thermodynamic Grüneisen’s relation will be satisfied in this study.
This paper is organised as follows. In the next section thermal expansion of the ferromagnetic superconductors will be derived. In section 3, the numerical results will be provided. In section 4, the pressure differential of the superconducting transition temperature and that of the Curie temperature will be derived. Section 5 will be devoted to conclusions.

2. The derivation of thermal expansion of ferromagnetic superconductors

We begin with the following free energy [8, 10]:

\[ F_{\text{coexist}}/N = F_0/N + F_T/N, \]

\[ F_0/N = \frac{IM^2}{2} + \frac{1}{2\pi} \int_0^{2\pi} d\theta \sum_\sigma \frac{\Delta_s^2(\theta)}{2g} - \frac{1}{2\pi} \sum_\sigma \int_0^{E_F} d\varepsilon N(\varepsilon) \frac{E_\sigma(\varepsilon, \theta)}{2}, \]

\[ F_T/N = -\frac{T}{2\pi} \sum_\sigma \int_0^{2\pi} d\theta \int_0^{\infty} d\varepsilon N(\varepsilon) \ln(1 + e^{-E_\sigma(\varepsilon, \theta)/T}) \]

where \( F_0 \) is the ground state energy and \( F_T \) is the thermal part of the free energy. \( E_F \) is the Fermi energy. \( N \) is the number of magnetic atoms, \( N(\varepsilon) \) is the density of state, and \( \Delta_s(\theta) \) is the superconducting gap of the spin \( \sigma \) conduction band. \( \theta \) is an azimuthal angle in the \( k_Fx-k_Fy \) plane and \( k_F \) is the Fermi wave number. \( g \) is the effective attractive pairing coupling constant. \( I \) is the on-site Coulomb coupling constant. \( M \) is the magnetisation. \( \varepsilon \) is the kinetic energy of electrons. \( E_\sigma(\varepsilon, \theta) \) is given by

\[ E_\sigma(\varepsilon, \theta) = \sqrt{(\varepsilon - \sigma IM - E_F)^2 + \Delta_s^2(\theta)}. \]

Thermal expansion of the ferromagnetic superconductors is obtained as follows:

\[ \omega = -K \frac{\partial F_{\text{coexist}}}{\partial V} \]

where \( K \) is the compressibility. The thermal expansion is

\[ \omega/(NE_F) = \omega_0/(NE_F) + \omega_T/(NE_F). \]

\( \omega_0 \) comes from the part of the ground state energy. \( \omega_T \) originates from the thermal part of the free energy. \( \omega_T \) is much smaller than \( \omega_T \) because of \( T_{sc} << T_F \) where \( T_F \) is the Fermi temperature. \( \omega_T \) is negligible. By our noting \( \frac{\partial \varepsilon}{\partial V} = \frac{\partial \ln t}{\partial V} \varepsilon \) where \( t \) is a transfer integral of electrons, \( \omega_0 \) is

\[ \omega_0 = -K \left\{ \frac{1}{2} \frac{E_F}{I} \left( \frac{\partial \ln I}{\partial V} \right) M^2 + \frac{1}{2} E_F \frac{\partial g}{\partial \varepsilon} \left( \frac{1}{g} \right) \sum_\sigma \frac{1}{2\pi} \int_0^{2\pi} d\theta \Delta_s^2(\theta) \right\} 

- N(0)E_F \frac{\partial \ln N(0)}{\partial V} \sum_\sigma \frac{1}{2\pi} \int_0^{2\pi} d\theta \int_0^1 dx \frac{\tilde{E}_\sigma^2(x, \theta)}{2} 

+ \sum_\sigma \frac{\partial \ln t}{\partial V} \int_0^{2\pi} d\theta \int_0^1 dx \frac{x - 1 - \sigma M}{2\sqrt{(x - \sigma M - 1)^2 + \Delta_s^2(\theta)}} 

+ \frac{1}{2} \sum_\sigma A_\sigma \frac{1}{2\pi} \int_0^{2\pi} d\theta \left\{ \sqrt{(\sigma M)^2} - \sqrt{(-\sigma M - 1)^2 + \Delta_s^2(\theta)} \right\} \} \]
with
\[ A_\sigma = -\sigma \frac{\partial \ln I}{\partial V} \tilde{M} - \frac{\partial \ln E_F}{\partial V} \]  

where \( \tilde{M} = IM/E_F \), \( N(0) \) is the density of states at the Fermi energy, \( \tilde{\Delta}_\sigma(\theta) = \Delta_\sigma(\theta)/E_F \), and \( x = \epsilon/E_F \). \( \tilde{E}_\sigma(x, \theta) \) is
\[ \tilde{E}_\sigma(x, \theta) = \sqrt{(x - \sigma \tilde{M} - 1)^2 + \Delta^2_\sigma(\theta)}. \]  

The corresponding thermal expansion coefficient is given by
\[ \alpha = \frac{\partial \omega}{\partial T} \]  

where \( \tilde{M} = IM/E_F \) and \( E_F \) is the Fermi energy.

We assume that the Curie temperature \( T_C \) is much higher than the superconducting transition temperature and that \( T_C \) is much lower than the Fermi temperature \( T_F \) throughout this paper. Correspondingly, the magnetisation is constant. This assumption is valid in UGe\(_2\).

Next, we mention the superconducting gap in order to study the thermal expansion and its coefficient. Harada et al. displayed that the superconducting gap of the up-spin conduction band with the line node is in UGe\(_2\) experimentally [15]. Therefore, we consider the following superconducting gap:
\[ \Delta_\sigma(\theta) = \begin{cases} \Delta_0 \cos \theta (\sigma = 1) \\ 0 (\sigma = 1) \end{cases} \]  

The superconducting order parameter at \( T = 0[K] \) is
\[ \Delta_0(0) = 2.426E_0 \exp(-1/c\sqrt{1 + \tilde{M}(0)}). \]  

\( E_0 \) is the cutoff energy. \( E_0/E_F \) is set to 0.01. The weak-coupling constant \( c = gN(0)/2 \) is set to 0.2. The temperature dependence of the superconducting order parameter \( \Delta_0(T) \) is obtained
\[ \Delta_0(T) = \Delta_0(0) \tanh(1.70 \sqrt{T_{sc}/T - 1}) \]  

with
\[ T_{sc} = 1.134E_0 \exp(-2/c\sqrt{1 + \tilde{M}(T_{sc})}) \]  

where \( T_{sc} \) is the superconducting transition temperature. In the next section the numerical results will be presented.

3. Results

The temperature dependence of the thermal expansion coefficient of the ferromagnetic superconductors is investigated with Eqs.(6), (7), (8), (9) and (10). Fig.1 shows the temperature dependence of the thermal expansion coefficient. The divergence of the thermal expansion coefficient appears at \( T_{sc} \). The thermal expansion coefficient behaves in the similar way to the previous study [8] even though we keep the volume dependence of the kinetic energy of the conduction electrons inconstant. In addition, the thermodynamic Grüneisen’s relation between the temperature dependence of the thermal expansion coefficient and that of the magnetic specific heat is satisfied because we use the same free energy as the free energy when the expression of the specific heat is derived.
Figure 1. The reduced temperature $T/T_F$ dependence of the thermal expansion coefficient of ferromagnetic superconductors when $E_F/I = 0.1$, $\partial \ln I/\partial P = 0.1$, $E_F \partial T_F/\partial V (1/2) = 0.1$, $N(0)E_F \partial M/\partial V = 0.1$, $N(0)E_F = 0.1$, and $N(0)E_F \partial \ln N(0)/\partial V = 0.1$.

4. The pressure differential of the superconducting transition temperature and that of the Curie temperature

The pressure differential of the superconducting transition temperature and that of the Curie temperature are investigated in this section. From Eq. (14), we get the pressure differential of the superconducting transition temperature

$$\frac{\partial T_{sc}}{\partial P} = \frac{\partial T_{sc}}{\partial P} [\ln 1.134E_0] + \frac{2}{c^2 \sqrt{1 + M(T_{sc})}} \left\{ \frac{\partial E_F}{\partial P} - \frac{1}{c(1 + M(T_{sc}))^{3/2}} \frac{\partial M(T_{sc})}{\partial P} \right\},$$

(15)

The pressure differential of the cutoff energy $E_0$ and that of the magnetisation will be estimated if the pressure differential of the superconducting transition temperature is observed.

We proceed to the pressure differential of the Curie temperature. The magnetisation is

$$M = IN(0)M + TN(0) \ln \frac{\cosh \frac{E_F + IM}{2T}}{\cosh \frac{E_F - IM}{2T}}.$$  

(16)

In $T \rightarrow T_C$, this equation is expanded around the small $M$. After that, $M$ is equal to zero at $T = T_C$. We obtain the Curie temperature

$$T_C = \frac{E_F}{\ln(2IN(0) - 1)}.$$  

(17)

$T_C$ differentiate about the pressure $P$. We obtain the pressure differential of $T_C$

$$\frac{\partial T_C}{\partial P} = \frac{\partial E_F}{\partial P} \left[ \ln(2IN(0) - 1) - \frac{2E_F \partial (IN(0))}{\partial P} \right] - \frac{2E_F \partial (IN(0))}{\partial P} \left[ (2IN(0) - 1) \ln(2IN(0) - 1) \right].$$  

(18)

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If the pressure differential of $T_C$ between experiments and the theory is analysed, the pressure differential of $E_F$ and that of $IN(0)$ will be estimated. Eq. (16) is expanded around small $e^{-(E_F-IM)/T}$ and $e^{-(E_F+IM)/T}$ when $T_{sc} < T << T_C << T_F$. We obtain

$$M = 2IN(0)M - TN(0)e^{-E_F/T}(e^{IM/T} - e^{-IM/T}).$$  

(19)

The magnetisation is exponential in the similar way to the result of Aso et al. [14].

5. Conclusions

We have reexamined the temperature dependence of thermal expansion of the ferromagnetic superconductors. The superconducting gap of the up-spin conduction band with the line node are assumed. We find that the divergence of the thermal expansion coefficient exists at the superconducting transition temperature like the previous study. The thermodynamic Gruneisen’s relation between the temperature dependence of the thermal expansion coefficient and that of the magnetic specific heat is satisfied.

We have obtained both the analytical expressions of the pressure differential of the superconducting transition temperature and the Curie temperature. If they are compared with experimental data, the pressure differential of $E_F$ and that of $IN(0)$ will be determined experimentally. In $T_{sc} < T << T_C << T_F$, the magnetisation is exponential. This result is consistent with that of Aso et al. [14].

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