Exploration of finite ion orbit effects in the ion
cyclotron range of frequencies

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Abstract. The rf-SciDAC collaboration is developing computer simulations to predict the
damping of radio frequency (rf) waves in fusion plasmas. Here we extend self-consistent quasi-
linear calculations of ion cyclotron resonant heating to include the finite drift of ions from
magnetic flux surfaces and rf induced spatial transport. The all-orders spectral wave solver
AORSA is iteratively coupled with a particle based update of the plasma distribution function.

1. Introduction
The next decade will see the construction of ITER[1] and mark a significant step towards nuclear
fusion as a power source. In part, the ITER design relies on radio frequency (rf) power as a
means of auxiliary plasma heating and current drive[2]. The rf-SciDAC group[3] is developing
computer simulations to predict how the ITER rf system will perform while benchmarking
simulation results against present experiments. Here we report progress in developing tools to
utilize leadership class computing facilities to investigate ion cyclotron resonant heating (ICRH)
in tokamak fusion plasmas. Specifically, a self-consistent quasi-linear (QL) approach is presented
to calculate power absorption with no assumption on the size of the Larmor radius, harmonic
number or ion orbit deviation from a magnetic flux surface.

Recent work by Jaeger et al.[4] has demonstrated a QL iterative calculation of ICRH. The All-
ORDers Spectral Algorithm (AORSA) is used to solve for the rf wave fields assuming a Maxwellian
electron distribution and a combination of thermal and non-thermal ion species. The non-
thermal ion species distribution is provided as a flux surface bounce-averaged quantity by the
finite difference Fokker-Planck code CQL3D [5]. CQL3D utilizes the simplifying assumption of zero
ion orbit deviation from a flux surface and takes as input from AORSA a 3D, orbit averaged
velocity space diffusion tensor that describes the rf heating. For the non-Maxwellian species
AORSA evaluates the susceptibility directly from the ion distribution. An iterative coupling of
these codes on the slow heating timescale (ms) has allowed self-consistent calculation of the
power absorption for minority heating scenarios in ITER and the Alcator C-Mod tokamak[6]
and also the high harmonic heating of neutral beam injected ions in NSTX[7] and DIII-D[8]
[4, 3]. However, in many cases a significant fraction of ions have finite orbit widths such that
coupling AORSA to alternative Fokker-Planck code that include these effects is desirable.

Several authors[9, 10, 11, 12] have investigated the importance of including finite ion orbits in
ICRH calculations. The SELFo code[13] combined the LION full-wave solver[14] with the Monte-
Carlo code FIDO[15]. FIDO calculates the orbit averaged ion distribution function in terms of three
invariants of the unperturbed motion using Monte-Carlo collision and rf operators. Murakami

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et al.\cite{16} have also performed self-consistent iterations using the \textsc{task/wm} full-wave code\cite{17} and \textsc{gnet}\cite{18} which solves the linearized drift kinetic equation. Such studies have shown that including finite orbit effects in ICRH simulations gives: (i) a broader collisional power transfer profile since power absorbed at resonance is transferred to the background plasma across the entire orbit width; (ii) additional resonant surface crossings due to orbits deviating from a flux surface; and (iii) rf-induced spatial transport associated with the ion slowing down time on electrons being shorter on the outer part of the orbit or an asymmetric toroidal antenna spectrum.

Previous calculations with finite orbits have utilised a finite Larmor radius assumption in the wave solve. Here we include finite orbit effects in calculations valid to all-orders in the Larmor radius by coupling a Monte-Carlo particle code to \textsc{aorsa}. Section 2 details the construction of a 4D ion distribution from a particle list. In section 3 a simple Monte-Carlo code with rf operator \textsc{(sMC+rf)} is iterated with \textsc{aorsa} for a high power ICRH minority heating scenario on the C-Mod tokamak.

2. Coupling particle and continuum codes
For non-thermal ion species \textsc{aorsa} requires a 4D $f_i(R, z, v_\perp, v_\parallel)$ distribution function with resolution sufficient to capture the 4D phase space gradients. Furthermore, calculation of the plasma dielectric depends on velocity space derivatives according to Eq. 5 of ref.\cite{4}. Both requirements have implications for the number of particles required to resolve important features in 4D. Fortunately, spatial gradients of the distribution have a larger scale size than the rf wave fields such that a lower spatial resolution is required for $f_i$ relative to that required to solve the wave fields. However a weak sensitivity to noise in the velocity space derivatives act to increase the required particle number. For a typical ICRH calculation a suitable distribution function grid resolution is $32 \times 32 \times 64 \times 128$. At this grid resolution, $10^6$ test particles produce a sufficiently accurate representation of the density. However, the velocity space derivatives are somewhat noisy which may lead to spurious values of plasma conductivity and possibly contaminate the wave solution.

Hellsten et al.\cite{13} solved the problem of noise in the velocity space derivatives by using the ‘hat’ function for the particle shape with derivatives reducing to derivatives of the base hat functions. Our approach here is similar. We choose a Gaussian of arbitrary width such that the final distribution is differentiable by numerical means. The Gaussian width is chosen on a per run basis, being a balance between derivative stability and velocity space resolution. In cylindrical coordinates the distribution is a sum over particles according to

$$ f(R', z', v_\perp^k, v_\parallel^l) = \sum_p w_p \delta(R - R') \delta(z - z') f_p $$

where $R$, $z$, $v_\perp$, $v_\parallel$ are the particle locations, $i$, $j$, $k$, $l$ the grid coordinates, $I_0$ is the Bessel function, $w_p$ the particle weight and $\sigma$ the particle width. Since derivatives are only required in the velocity directions the Dirac delta function is sufficient for the spatial dimensions.

For further improvement in resolution, or an equivalent relaxation in the required number of particles, we can take advantage of the separation of particle orbit and rf heating time scales. Each particle orbit can be traced such that it traverses many phase space grid points. The particle weight may be distributed along its orbit. However, with $10^6$ particles this is rarely required and integrating guiding center trajectories in combination with a 4D or even 2D particle shape is very computationally intensive.
Figure 1. H distribution $f_H(v_\perp, v_\parallel)$ at $z = 0$.

Figure 2. H density, perpendicular (black/solid) and parallel (red/dashed) energy per particle and $E^+$. 

3. Initial results using a simple Monte-Carlo code

Prior to coupling with more comprehensive particle codes such as ORBIT-rf [19], here we use a simple Monte-Carlo code, sMC+rf, which integrates the ion guiding center equations of motion in 5D (cylindrical coordinates in space $R, \phi, z$ and velocity $v_\perp, v_\parallel$) under an axisymmetric background magnetic equilibrium produced by the EFIT code[20]. We assume coulomb collisions with a Maxwellian background by including the pitch and energy scattering operators presented by Boozer et al.[21] and the usual diffusive rf scattering operator[9]. Our rf operator includes only the component of the wave electric field rotating with the gyro motion of the particle ($E^+$), assumes a parallel wavenumber $k_\parallel = n_\phi/R$ where $n_\phi$ is the toroidal mode number of the launched wave, and the perpendicular wavenumber is derived from the cold plasma dispersion relation. While most other Monte-Carlo approaches to ICRH make similar assumptions they are in general complicated by coordinate systems that are either constants of the particle motion or field aligned such that larger time steps reduce computational requirements. To avoid the
Figure 3. Flux surface averaged H power absorption calculated according to Eq. 17 of ref.[4]
sMC+rf will extend present simulation capabilities to include the effects of both rf induced spatial transport and finite orbits. Calculations for a minority heating scenario on the C-Mod tokamak were presented with the results showing a significant reduction in the core power absorption after iterating with the particle code relative to the initial Maxwellian. These results must be benchmarked against other code combinations including AORS/AQL and other particle codes such as ORBIT-rf.

An important step for future work is to remove the limitations on the rf heating operator. By utilising the 4D quasi-linear rf diffusion tensor calculated by AORS/AQL the assumptions of a specific \( k_{||} \) and cold plasma \( k_{\perp} \) will be removed. sMC+rf was developed specifically with this in mind. However, the AORS QL operator contains 4 diffusion coefficients at each point in 4D phase space on the rf wave field grid such that it requires tens of GBytes at present resolutions and grow to hundreds of GBytes at anticipated resolutions. Recent modifications to AORS/AQL and sMC+rf utilise the parallel netCDF library[22] to allow efficient reading and writing of this data. However, using this large data set within a particle based update to the distribution function is difficult. The size of the QL operator requires the data to be read and stored across multiple processors such that preventing a communication bottleneck will be a challenge.

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