Oblique wave scattering by double porous structures

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Abstract. The linear wave scattering by double submerged structures of different structural parameters in oblique sea is investigated by use of eigenfunction expansion method. The important hydrodynamical scattering quantities such as reflection, transmission and dissipation coefficients are obtained and analyzed to examine the effects of various wave and structural parameters. As a special case, the computational results are validated for their accuracy by comparing with known results in the literature in case of single structure. The study reveals that a significant amount of wave energy dissipated in case of double structures as compared to that in the presence of single structure. Moreover, the energy dissipation is found to be independent of relative spacing between the two structures.

1. Introduction

Breakwater wave mechanisms are widely studied in applied ocean research owing to the need for efficiently dissipating incoming wave energy that establishes a tranquil zone near a coast. This protects onshore-offshore marine facilities from wave action. Breakwaters that are porous and submerged in water dissipate energy via friction allowing partial transmission of the wave. Such submerged porous breakwaters are efficient, economical, aesthetic, allow the mobility of water transport over the structure and permeability of the porous structure creates a unique ecosystem suitable for marine life. Before construction of these structures, numerical, analytical and experimental studies are required to optimally determine the wave and structural parameters that efficiently dissipate the incoming energy.

The efficiency for attenuation of the incoming wave energy is characterized by reflection, transmission and dissipation coefficients, which have been studied by several authors. Sollitt and Cross [1] developed a classical mathematical model for describing the effect of a submerged porous structure on wave motion. Since then, many researchers have used and developed the Sollitt and Cross model for dissipating wave energy for different applications. Sulisz [2] expanded that theory to infinite rubble-mound breakwater creating a hybrid model which used boundary element method for the numerical solution. Dalrymple et al. [3] further expanded the model to include oblique wave motion on vertical sided porous breakwater. Lee [4] and Zheng et al. [5] developed boundary element methods for wave interaction with porous and prismatic structures respectively in waters of finite depth. Losada et al. [6] modelled wave interaction with submerged structures using 3-D and 2-D models based on eigenfunction expansion and mild slope equation respectively. Liu and Li [7] developed an analytical solution of wave interaction with a new-type pile-rock breakwater in the context of linear potential theory. Koley et al. [8] studied oblique wave trapping by porous structures near a vertical rigid wall and developed
a multi-domain boundary element method for bottom-standing and surface-piercing porous structures in finite water depth. Koley et al. [9] further examined the interaction of waves with submerged structures having perforated outer-layers on a sloping sea bed. Behera et al. [10] introduced a two-layer fluid in the problem of wave scattering and trapping by bottom-standing and surface-piercing porous structures and developed solutions based on eigenfunction expansion method and BEM (boundary element method). Recently, Koley and Sahoo [11] developed a BEM solution for the interaction of surface gravity waves with a semicircular breakwater near a porous sloping seawall and placed on a porous seabed in water, while Zhao et al. [12] studied oblique wave scattering by a submerged porous breakwater with a partially reflecting side-wall. They discussed the sensitivity of transmission coefficient to relative spacing between the porous breakwater and the side-wall, which affects wave resonance.

Additionally, use of horizontal plates with finite thickness as breakwaters has been studied in various configurations. Yong et al. [13] analysed the hydrodynamics of a submerged two layer horizontal plate breakwater and used matched eigenfunction expansion method to solve for the interaction of water waves with the plates. Cho and Kim [14] studied interaction of oblique monochromatic incident waves with horizontal, inclined and dual porous plates two-dimensional linear potential theory and Darcys law. Using eigenfunction expansion and boundary element methods they found that the performance could be improved by changing the inclination angle of the plate near the free surface. Liu and Li [15] gave an alternative analytical solution for wave interaction with submerged horizontal porous-plate breakwater. Although matched-eigenfunction-expansion method was used to obtain the solution, in this solution no complex water-wave dispersion relations were used making this solution numerically easy to implement.

In this study, oblique wave interaction with double submerged porous structures of different structural parameters is investigated in case of finite water depth. Sollitt and Cross model along with eigenfunction method are used to handle the physical problem. To understand the effects of various wave and structural parameters, reflection, transmission and dissipation coefficients are obtained and analyzed.

2. Mathematical formulation

The consideration of two submerged porous structures in water is represented by schematic in Fig. (2) with the Cartesian co-ordinate system under small amplitude water wave theory. The fluid is assumed to be incompressible, inviscid and having irrotational motion. In water of finite depth $h$, two bottom-standing porous structures of width $b$ are considered, however, with different heights $a_i$, porosities $\epsilon_i$, intertial coefficients $s_i$ and frictional coefficients $f_i$, where $i=1, 2$ represent first and second structures, respectively. The spacing between the structures is $L$ as in Fig. [2]. Total flow field divided into three regions with the water region labeled as A and the two structural regions as B and C. Further, region A is divided into five subregions labeled as 1, 2, 4, 5, 7 and occupying regions are given as follows:

1: $-h \leq z \leq 0; -\infty \leq x \leq 0$,  
2: $-h + a_1 \leq z \leq 0; 0 \leq x \leq b$,  
4: $-h \leq z \leq 0; b \leq x \leq b + L$,  
5: $-h + a_2 \leq z \leq 0; r_1 \leq x \leq r_2$,  
7: $-h \leq z \leq 0; 0 \leq x \leq \infty$,

where $r_1 = b + L$ and $r_2 = 2b + L$. On the other hand, the porous structures that occupy the regions B and C are further labeled as 3, 6 respectively, where

3: $-h \leq z \leq -h + a_1; 0 \leq x \leq b$,  
6: $-h \leq z \leq -h + a_2; r_1 \leq x \leq r_2$,  

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Figure 1. Schematic diagram for wave scattering by two submerged porous structures.

Assuming that the incident wave is propagating at an angle $\theta$ with the $x$-axis and the flow is simple harmonic in time with angular frequency $\omega$, then the velocity potentials in each region are of the form $\Phi_j(x, y, z, t) = \text{Re}\{\phi_j(x, z)e^{-i(k_y y - \omega t)}\}$, where $k_y = k_0 \sin \theta$ and $k_0$ is the progressive wave number in the open water region. Further, the spatial velocity potentials $\phi_j(x, z)$ for $j = 1, 2, 3, 4, 5, 6, 7$ satisfy the Helmholtz equation

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - k_y \right) \phi_j = 0,$$

along with the bottom boundary condition as

$$\frac{\partial \phi_j}{\partial z} = 0 \quad \text{on} \quad z = -h, \quad \text{for} \quad j = 1, 3, 4, 6, 7$$

and linearized free surface boundary condition in the open water region as

$$\frac{\partial \phi_j}{\partial z} - K\phi_j = 0 \quad \text{on} \quad z = 0, \quad \text{for} \quad j = 1, 2, 4, 5, 7,$$

where $K = \omega^2 / g$, $g$ is the gravitational constant. The radiation conditions are given as

$$\phi_1 = \left( A_{10}e^{-ik_0 x} + B_{10}e^{ik_0 x} \right)g_0(k_0, z) \quad \text{as} \quad x \to -\infty,$$

$$\phi_7 = A_{70}e^{-ik_0 x}g_0(k_0, z) \quad \text{as} \quad x \to \infty,$$

where $A_{10}$ is the known incident amplitude, and $B_{10}$ and $A_{70}$ are the unknown constants to be determined, $k_0$ is the real root of the dispersion relation in Region 1 and $g_0(k_0, z)$ is the eigenfunction in Region 1 with $g_0 = \sqrt{k_0^2 - k_y^2}$. The boundary conditions on horizontal interfaces between water and porous structures are given by

$$\frac{\partial \phi_2}{\partial z} = \epsilon_1 \frac{\partial \phi_3}{\partial z}, \quad \phi_2 = (s_1 - if_1)\phi_3, \quad \text{at} \quad z = -h + a_1 \quad \text{for} \quad \text{first structure},$$

$$\frac{\partial \phi_5}{\partial z} = \epsilon_2 \frac{\partial \phi_6}{\partial z}, \quad \phi_5 = (s_2 - if_2)\phi_6, \quad \text{at} \quad z = -h + a_2 \quad \text{for} \quad \text{second structure},$$
The continuity of pressure and velocity at \( x = 0 \) and \( x = b \) are given by

\[
\phi_j = \begin{cases} \phi_2, & j = 1, \\
(s_1 - i f_1) \phi_3, & j = 4, \\
(s_2 - i f_2) \phi_6, & j = 7, \\
\end{cases}
\]

\[
\frac{\partial \phi_j}{\partial x} = \begin{cases} \frac{\partial \phi_2}{\partial x}, & j = 1, \\
\frac{\partial \phi_3}{\partial x}, & j = 4, \\
\frac{\partial \phi_6}{\partial x}, & j = 7, \\
\end{cases}
\]

where \( j = 1 \) for \( x = 0 \) and \( j = 4 \) for \( x = b \). Similarly the continuity of pressure and velocity at \( x = r_1 \) and \( x = r_2 \) is given by

\[
\phi_j = \begin{cases} \phi_5, & j = 1, \\
(s_2 - i f_2) \phi_6, & j = 7, \\
\end{cases}
\]

\[
\frac{\partial \phi_j}{\partial x} = \begin{cases} \frac{\partial \phi_5}{\partial x}, & j = 1, \\
\frac{\partial \phi_6}{\partial x}, & j = 7, \\
\end{cases}
\]

where \( j = 4 \) for \( x = r_1 \) and \( j = 7 \) for \( x = r_2 \). Next, the solutions associated with the boundary value problems are discussed in the following Sections.

3. Analytic method of solution

The spatial velocity potentials \( \phi_j \) for \( j = 1, 2, 3, 4, 5, 6, 7 \) satisfying Eq. (1) along with conditions (2)-(9) are expressed as

\[
\phi_j = \begin{cases} 
A_{10} e^{i q x} g_0(k_0, z) + \sum_{n=1}^{\infty} B_{1n} e^{i q n x} g_n(k_n, z) & \text{for } j = 1, \\
\sum_{n=1}^{\infty} \left\{ A_{4n} e^{i q n x} + B_{4n} e^{i q n x} \right\} g_n(k_n, z) & \text{for } j = 4, \\
\sum_{n=1}^{\infty} A_{7n} e^{i q n x} g_n(k_n, z) & \text{for } j = 7, \\
\sum_{n=0}^{\infty} \left\{ A_{jn} e^{i q n x} + B_{jn} e^{i Q n x} \right\} h_{in}(p_{in}, z) & \text{for } j = 2, 3, 5, 6, \\
\end{cases}
\]

where \( A_{10} \) is the known incident amplitude, and \( A_{jn} \) and \( B_{jn} \) for \( n = 0, 1, 2, 3, \ldots \), being the unknown constants to be determined with \( A_{2n} = A_{3n}, B_{2n} = B_{3n}, A_{5n} = A_{6n}, B_{5n} = B_{6n} \), \( g_n = \sqrt{k_n^2 - k_y^2} \) and \( Q_n = \sqrt{p_{in}^2 - k_y^2} \), and \( i = 1 \) for \( j = 2, 3 \) and \( i = 2 \) for \( j = 5, 6 \). Moreover, the eigenfunctions \( g_n(k_n, z) \) and \( h_{in}(p_{in}, z) \) are given by

\[
g_n(k_n, z) = \frac{ig}{\omega} \cosh k_n(z + h) \cosh k_n h, \quad \text{(11)}
\]

\[
h_{in}(p_{in}, z) = \begin{cases} 
\left( \frac{ig}{\omega} \cosh p_{in}(z + h) - F_{in} \sinh p_{in}(z + h) \right) \cosh p_{in} h - F_{in} \sinh p_{in} h & \text{for regions 2 and 5,} \\
\left( \frac{ig}{\omega} \cosh p_{in}(z + h) - F_{in} \sinh p_{in}(z + h) \right) \cosh p_{in} h - F_{in} \sinh p_{in} h & \text{for regions 3 and 6,}
\end{cases}
\]

where \( F_{in} = \frac{(1 - G_i)}{1 - G_i} \tan h p_{in} a_i \) and \( G_i = \frac{\epsilon_i}{(a_i - i f_i)} \) for \( i = 1, 2 \). Further, in regions 1, 4 and 7, the eigenvalues \( k_n \) satisfy the dispersion relation

\[
\omega^2 = g k_n \tan h k_n h. \quad \text{(13)}
\]
However, in regions 2, 3, 5 and 6, the eigenvalues $p_{in}$ satisfy the dispersion relation

$$ K - p_{in} \tanh p_{in} h = F_i(K \tanh p_{in} h - p_{in}), $$

where $i = 1$ for regions 2 and 3, and $i = 2$ for regions 5 and 6. In Eqs. (11) and (12), the eigenfunctions $g_{in}(k_m, z)$ and $h_{in}(p_{in}, z)$ form a complete set of orthogonal functions in their corresponding domains as in Losada et al. [6]. Using the matching conditions as in Eqs. (8) and (9) along with the orthogonality of the eigenfunctions and truncating the infinite sums after $N$ terms, a system of equation for the determination of the unknowns in Eq. (10) are obtained as

$$
\sum_{n=0}^{N} X_{nm} B_{in} - \sum_{n=0}^{N} \left( Y_{nm} + \alpha_1 Z_{nm} \right) \left( A_{2n} + B_{2n} \right) = -\delta_m X_{0m} A_{10},
$$

$$
\sum_{n=0}^{N} q_n X_{nm} B_{1n} + \sum_{n=0}^{N} Q_n \left( Y_{nm} + \epsilon_1 Z_{nm} \right) \left( A_{2n} - B_{2n} \right) = \delta_m q_0 X_{0m} A_{10},
$$

$$
\sum_{n=0}^{N} X_{nm} \left( A_{4n} e^{-iqn} + B_{4n} e^{iqn} \right) - \sum_{n=0}^{N} \left( Y_{nm} + \alpha_1 Z_{nm} \right) \left( A_{2n} e^{-iQn} + B_{2n} e^{iQn} \right) = 0,
$$

$$
\sum_{n=0}^{N} q_n X_{nm} \left( A_{4n} e^{-iqn} - B_{4n} e^{iqn} \right) - \sum_{n=0}^{N} Q_n \left( Y_{nm} + \epsilon_1 Z_{nm} \right) \left( A_{2n} e^{-iQn} - B_{2n} e^{iQn} \right) = 0,
$$

$$
\sum_{n=0}^{N} X_{nm} \left( A_{4n} e^{-iqr} + B_{4n} e^{iqr} \right) - \sum_{n=0}^{N} \left( Y_{nm} + \alpha_2 Z_{nm} \right) \left( A_{5n} e^{-iQn} + B_{5n} e^{iQn} \right) = 0,
$$

$$
\sum_{n=0}^{N} q_n X_{nm} \left( A_{4n} e^{iqr} - B_{4n} e^{-iqr} \right) - \sum_{n=0}^{N} Q_n \left( Y_{nm} + \epsilon_2 Z_{nm} \right) \left( A_{5n} e^{-iQn} - B_{5n} e^{iQn} \right) = 0,
$$

where $\delta_m = 1$ for $m = 0$ and $\delta_m = 0$ for $m > 0$ with $X_{nm} = \int_{-h}^{0} g_n(k_m, z) g_m(k_m, z) \, dz$

$$ Y_{nm}^i = \int_{-h}^{0} h_{in}(p_{in}, z) g_m(k_m, z) \, dz, \quad Z_{nm}^i = \int_{-h}^{h+a} h_{in}(p_{in}, z) g_m(k_m, z) \, dz \quad \text{for } i = 1, 2 \quad \text{and} \quad \alpha_j = s_j - if_j \quad \text{for } j = 1, 2. $$

The system of equations (15)-(22) are solved to determine the unknown coefficients in the velocity potentials, which will be used to compute various physical quantities of interest for wave scattering by the submerged porous structures.

4. Results and Discussion

The effects of wave and structural parameters on wave scattering by double submerged porous structures are evaluated analytically using MATLAB® 2015a. In this study, inertial coefficients $s_i = 1$, linearized friction coefficients $f_i = 0.5$, porosities $\epsilon_i = 0.437$ for $i = 1, 2$, and incident angle $\theta = 10^\circ$, water depth $h = 5$, spacing between structures $L/h = 10$ and time period $T = 8s$
are kept fixed unless otherwise stated. The results are evaluated for single and double structure scenarios wherever deemed necessary and the parameters given by Losada et al. for wave scattering by a single structure are taken as standard for the comparison of two scenarios. In the present work, the reflection, transmission and dissipation coefficients are symbolized by $K_r$, $K_t$ and $K_d$, respectively and computed using the formulae

$$
K_r = \left| \frac{B_{10}}{A_{10}} \right|, \quad K_t = \left| \frac{A_{70}}{A_{10}} \right| \quad \text{and} \quad K_D = 1 - (K_r^2 + K_t^2). \quad (23)
$$

Figures 2(a) and 2(b) show the comparison between single and double structures for variation of (a) reflection and transmission coefficients, (b) dissipation coefficient against non-dimensional structural width $k_0b$. It is observed that the graph of reflection coefficients follow a oscillatory pattern with an increase in structural width. However, in case of wave transmission and dissipation, oscillatory pattern does not occur. Moreover, with increase in structural width, transmission coefficient decreases and dissipation increases. Further, it is seen that the wave transmission is lower, reflection and dissipation coefficients are higher in case of double structures as compared to that for single structure.

![Figure 2. Comparison of single and double porous structures for (a) $K_r$ and $K_t$ and (b) $K_D$ versus structural width $k_0b$ with $a_1/h = a_2/h = 0.3$, $f_1 = f_2 = 0.5$ and $b/h = 2$.](image)

In Fig. 3(a) reflection and transmission coefficients, and in Fig. 3(b) dissipation coefficient against structural width $k_0b$ are plotted for different values of structural height $a_i/h$. Fig. 3(a) shows that with an increase in structural height, reflection is higher and transmission is lower. As the structure width is increased reflection attains a maximum value, after which $K_r$ falls and stabilizes to a limit. It is also seen that as structural height is increased the optimas of the wave reflection shift towards left, which corresponds to increase in dissipation of wave energy by the structure. Further, 3(a) indicates that as the width of structures increases, wave transmission diminishes to zero. This corresponds to maximum efficiency of energy attenuation as verified by steepness of dissipation coefficient for higher structural width in 3(a) thereby specifying an admissible width of the porous structures for creating a tranquil zone on their leeward side.

Figs. 4(a) and 4(b) depict the variation of reflection and transmission coefficients against structural width $k_0b$ for change in frictional coefficient $f_i$ and the spacing between two structures.
Figure 3. Variations in (a) $K_r$ and $K_t$ (b) $K_D$ versus structural width $k_0b$ for different values of structural height $a_i/h$ with $f_1 = f_2 = 0.5$, $b/h = 2$, $L/h = 10$ and $\theta = 10^\circ$.

$L/h$, respectively. Reflection and transmission follow similar patterns as in 2(a). Irrespective of structural width, an increase in frictional coefficients $f_i$ results in higher reflection and lower transmission. However, in 4(b) transmission $K_r$ remains independent of the change in spacing while there is a right shift of minima in $K_r$ with increase in $L/h$. This has negligible effect on the dissipation of wave energy. Thus, attenuation of wave energy is more dependent on structural parameters of the breakwater than the position of structures relative to each other on the ocean floor.

Figure 4. Variation of $K_r$ and $K_t$ versus structural width $k_0b$ for (a) $f_1 = f_2 = 0.1, 0.5, 0.9$; $a_1/h = a_2/h = 0.3$ and $b/h = 2$ (b) $L/h = 5, 10, 15$; $a_1/h = a_2/h = 0.3$, $f_1 = f_2 = 0.5$ and $b/h = 2$.

Fig. 5(a) plots the dependence of reflection and transmission coefficients on incident oblique
angle $\theta$ in the cases of single and double porous structures. It is seen that the transmission is lower in presence of double structures than that for a single structure. In both the cases, zero wave reflections occur near $\theta = 50^\circ$ which is said to be critical angle. Similar observation were found by Losada et al. [6] in case of single structure. In addition, for double structures, after reaching zero reflection, as $\theta$ increases, the reflection coefficient increases with an oscillatory pattern. In Fig. 5(b) reflection and transmission coefficients are plotted against $\theta$ for different structural heights. The panel shows that an increase in $a_i/h$ results in a higher reflection and lower transmission irrespective of $\theta$. Further the minima of $K_r$ shifts to the right on increasing the value of structural height.

![Figure 5](image)

**Figure 5.** Variation of $K_r$ and $K_t$ versus incident angle $\theta$ for (a) single and double porous structures with $a/h = 0.3$, $f_1 = f_2 = 0.5$, $b/h = 2$ and $L/h = 10$ (b) different structural heights $a_i/h$ with $f_1 = f_2 = 0.5$, $b/h = 2$ and $L/h = 10$.

Figs. 6(a) and 6(b) depict the variation of reflection and transmission coefficients against incident angle $\theta$ for change in frictional coefficient $f_i$ and the spacing between two structures $L/h$, respectively. Reflection and transmission follow similar patterns as in 5(a) and an increase in frictional coefficients $f_i$ results in higher reflection and lower transmission. In addition, the position of zero minimas of $K_r$ remains unchanged. On the contrary, in 6(b) transmission $K_t$ remains independent of the change in spacing $L/h$ for values $\theta \leq 60^\circ$, after which it decreases rapidly. However, with increase in $L/h$, there is an increase in the number of minimas in $K_r$.

Fig. 7 depicts the variation of reflection coefficient against normalized spacing between the two structures $L/\lambda_0$, where $\lambda_0$ is wavelength of the incident wave with $\lambda_0 = 2\pi/k_0$. In Fig. 7(a) only height of the second structure $a_2/h$ is varied while $a_1/h$ is kept fixed. It is seen that the reflection coefficients follow an oscillatory pattern and optima in the wave reflection are observed at a periodic interval. Further, wave reflection increases with an increase in height of the second structure and the optima of $K_r$ shift towards right. In Fig. 7(b) frictional coefficient of the second structure $f_2$ is varied while the structural heights $a_1/h$ and $a_2/h$ are different. In this case also, the reflection coefficient follows an oscillatory pattern and wave reflection increases with an increase in frictional coefficient $f_2$. However, the optima of $K_r$ shift towards left.
Figure 6. Variation of $K_r$ and $K_t$ versus incident angle $\theta$ for (a) different values of frictional coefficient with $a_1/h = a_2/h = 0.3$ $b/h = 2$ and $L/h = 10$ (b) different values of spacing between structures with $a_1/h = a_2/h = 0.3$, $f_1 = f_2 = 0.5$ and $b/h = 2$.

Figure 7. Variation of $K_r$ against normalized spacing $L/\lambda_0$ for (a) different $a_2/h$ with $a_1/h = 0.3$ and $f_1 = f_2 = 0.5$ (b) different $f_2/h$ with $a_1/h = 0.3$, $a_2/h = 0.95$ and $f_1 = 0.5$.

5. Conclusion
In this paper, oblique wave scattering by double submerged porous structures is studied under small-amplitude water wave theory. The results are evaluated analytically using eigenfunction expansion method. Reflection, transmission and dissipation coefficients are evaluated to characterize the behaviour of waves past the structures. This study reveals that in case of double submerged structures, varying structural parameters of the breakwater is more effective in attenuation of wave energy than changing the position of structures relative to each other. A submerged breakwater with larger structural height and frictional coefficient dissipates more wave energy thereby lowering the transmission. Further, having a smaller structure is on the seaward side of an already placed breakwater increases the dissipation significantly thereby being economically viable. The study reveals that a combination of two porous structures acts as a
better wave-scattering alternative to a single structure as double submerged porous structures attenuate a significant amount of extra energy. Even so, the implementation of such a system will require optimization of the corresponding structural factors to suit their marine environment. Overall, by using a suitable combination of structural parameters, a comprehensive coastal protection system consisting of double submerged breakwaters can be developed.

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