Abstract

In our work we extend the ideas of the derivation of the chiral effective theory from the lattice QCD [1] to the case of the random lattice regularization of QCD. Such procedure allows in principle to find contribution of any order into the chiral effective lagrangian. It is shown that an infinite subseries of the chiral perturbation can be summed up into the Born-Infeld term and the logarithmic correction to them.

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1 Introduction and motivation

At low energies strong interactions can be described by effective chiral lagragians. Such effective field approach is very important, both for the physics of the hadron-hadron interaction and for the concept of the low-energy baryon state. Therefore, there is a strong motivation to derive effective chiral theories from QCD.

The first essential contribution to this topic was made in [2]. Using the method of large $N$ expansion, an effective chiral theory was suggested in the form of a series of chiral invariants. The second order theory of such type is the well-known Skyrme model [3] of low-energy baryon states, a phenomenological unified theory for mesons and baryons where the baryon is treated as a topological soliton of nonlinear chiral fields. Another interesting approach was proposed in [4] where the Skyrme model was derived from integration of the chiral anomaly.

All such methods play a very essential role in particle physics and give appropriate description of the behavior of the chiral field at low energy. However, to study the chiral field near the confinement boundary one must analyze the whole series of chiral perturbations. The Chiral Bag Model [5] provides a very good illustration of this problem. In this model the

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boundary between chiral fields and color fields is specified by hand. Although this gives quite good agreement with experimental data, it is not clear what might be the physical mechanism of formation of this chiral bag.

Fortunately, Lattice QCD numerical experiments give us today a lot of interesting information about the behavior of color fields (quarks and gluons) in the strong coupling regime. These data are very essential to understand the physics of the baryon at low energies; in fact the lattice approach has formed empirical and theoretical basis of the baryon string model [6].

Against the background of these facts, it looks astonishing that the Skyrme model should give a good agreement with the experiment data without accounting for the color degrees of freedom at all. Therefore the question of unification of these paradigms: the chiral soliton model and the model of baryon string, becomes very essential. It is natural to begin the search for the way of such unification with analyzing the chiral limit of QCD on the lattice.

2 Why do we need the Random Lattice QCD

Derivation of a chiral effective Lagrangian from lattice QCD has been attempted many times since long ago. The well-known Brezin & Gross trick [7] makes it possible to perform integration of the link matrix in the strong coupling regime and to obtain various first order chiral effective theories [8].

Although at first such approaches led to great success, they have not been very popular, because they do not allow to obtain any corrections to first order results. Lattice regularization breaks the rotational symmetry of the initial theory from the continuous rotation group down to a discrete group of rotations at fixed angles. Hence, lattice regularization approaches give correct results only for those tensors that are invariant with respect to such discrete groups. In particular, using the ordinary Hyper-Cubical (HC) lattice, one can obtain only a first order effective theory, while for corrections this method generates non-rotation invariant (non-Lorentz invariant) terms. Generation of high-order effective field theories requires a more symmetrical lattice.

The problem of breakdown of rotational symmetry on a lattice has been attracting important attention for a long time. It was shown [9] that in 4 dimensions the so-called Body Centered Hyper-Cubical (BCHC) or F4 lattice has the largest discrete symmetry group. (BCHC consists from the all sites of the HC lattice together with centers of its elementary cells.) This property of the BCHC lattice gives a possibility to obtain the next-to-leading (NL) correction to the first order of the chiral perturbation theory [1].

The results of the papers [1] are essential for our analysis, as they confirm the effectiveness of the idea of chiral effective lagrangian derivation from the lattice QCD. Moreover, these results are interesting from phenomenological point of view because, as is well known [3], the NL corrections violate the scale invariance of the prototype (first order) chiral theory that leads to generation of chiral topological solitons (Skyrmions). The Next-Leading order chiral effective theory that was implemented in [1] is in agreement with our phenomenological propositions [10], and in our work we will use methodological ideas from [1] in order to define the behaviour of chiral field near the confinement surface.

As one could see, in order to solve our problem, the Next-Leading order corrections are not enough. This theory has no solutions that look like chiral “bag”. Moreover, as will be
shown later, near the confinement surface (near the source of the chiral field), the influence of high order corrections became larger and larger. But the BCHC lattice method gives the NL corrections only and the further use of this method for the defining of the high order terms leads to generation of non-relativistic (non-rotational) invariant terms. It means that we need a more symmetric lattice than the BCHC lattice.

Unfortunately, a lattice which would be more symmetric than BCHC lattice cannot be constructed in 4 dimension. Moreover any method based on a lattice of a fixed geometry has artifacts coming from priority directions that correspond to basis vectors of the lattice. It is these artifacts that eventually lead to the problems with the rotational (relativistic) invariance rendering the use of the BCHC lattice to be only half measure. For solving our problem a modification of the initial concept of lattice regularization must be performed. We need to find a concept of lattice regularization that has no priority directions. Fortunately this concept is known for a long time and is called the Random Lattice approach [12].

The idea of Random Lattice was proposed originally by Voronoi and Delaunay: today this method is widely used in the modern science. For the quantum field theory the method was modified by Christ, Friedberg and Lee [12]. In these articles it has been shown that in order to restor the Lorentz (rotational) invariance, it is necessary to perform an average over an ensemble of random lattices. As a result one gets the averaging over all possible directions and it is intuitively clear that this procedure leads to the disappearance of the artifacts that cause the violation of the group of the space rotations.

But how to perform such random discretization? This procedure has the tree steps:  
1) Pick $N$ sites $x_i$ at random in the volume $V$.  
2) Associate with each $x_i$ a so-called Voronoi cell $c_i$

$$c_i = \{ x | d(x, x_i) \leq d(x, x_j), \forall j \neq i \}$$

where $d(x, y)$ is a distance between points $x$ and $y$. It means that the Voronoi cell $c_i$ consists of all points $x$ that are closer to the center site $x_i$ than to any other site.

3) Constrict the dual Delaunay lattice by linking the center sites of all Voronoi cells which share a common face.

Now if one considers the the big ensemble of such Voronoi-Delaunay random lattices based on various distributions of sites $x_i$, it possible to prove that the original rotational symmetry is restored [12]. In our work we use this procedure to obtain an effective chiral lagrangian from lattice QCD. This methodological point of view it is a modification of the method proposed in [1] in the case of the Random Lattice approach.

### 3 From Lattice QCD to chiral lagrangians: step by step

Now let me briefly recall a general steps of the algorithm of derivation of the chiral lagrangian from the lattice QCD that was proposed in [1].

**Step 1: Definitions**

The starting point of our analysis is a standard lattice action with Willson fermions

$$Z = \int [DG][D\bar{\psi}][D\psi] \exp\{-S_{pl}(G) - S_q(G, \bar{\psi}, \psi) - S_J\}$$
where:

1) the plaquette gauge field term is defined by

\[ S_{pl} = \frac{2N_c}{g^2} \sum_{pl} \left[ 1 - \frac{1}{N_c} Re \{ G_{x,\mu} G_{x-\mu,\nu} G_{x+\nu,\mu} G_{x,\nu} \} \right], \quad G_{x,\mu} = \exp\left\{ ig \int_{\text{link}} dx' A_\mu(x') \right\}; \]

2) the link fermions term is defined by

\[ S_q = \sum_{x,\mu} \text{tr} \{ \bar{\psi}_b (x + \mu) P^+ \psi^a (x) \}, \quad A_\mu (x)^a_b = \bar{\psi}_b (x) P^+ \psi^a (x + \mu) \]

and \( P^\pm = \frac{1}{2} (r \pm \gamma_\mu); \)

3) the source term is defined by

\[ S_J = \sum_x J_\alpha^\beta (x) M^{\beta}_\alpha (x), \quad M^{\beta}_\alpha = \frac{1}{N_c} \psi^\alpha,\beta (x) \bar{\psi}^{\alpha,\beta} (x). \]

**Step 2: Strong-coupling regime on the lattice and integration over the gauge field**

In order to realize the strong-coupling regime on the lattice let us consider the limit of the large coupling constant \( g \) (\( g \to \infty \)). This limit was widely studied [8] and the main result is that in such limit integration over the gauge field can be performed. (Of course, the direct integration is difficult since there exists the plaquette term \( S_{pl} \), but due to the strong-coupling limit on the first step plaquette contributions are negligible with respect to the contribution from the link integral \( S_q \). The plaquette contributions could be considered in the systematic manner as perturbations in \( 1/g \) [8].)

Let us consider the leading order contribution in this strong-coupling expansion. The integrals over the gauge degrees of freedom can be calculated into the large \( N \) limit by using the standard procedure [8] and the result of these calculations is the following

\[ Z = \int [D\bar{\psi}] [D\psi] \exp\left\{ -N \sum_{x,\mu} \text{tr} \{ F(\lambda(x, \nu)) \} \right\} - S_J \}, \]

where \( \lambda_\nu = -M(x) P^- \lambda (x + \nu) P^+ \) and

\[ F(\lambda) = \text{tr}[(1 - \sqrt{1 - \lambda})] - \text{tr}[(1 - \frac{1}{2} \sqrt{1 - \lambda})]. \]

Interestingly, the function \( F(\lambda) \) has the typical form of the Born-Infeld action with a first logarithmic correction. This is no coincidence. In [11], it was shown by means of very similar technique that the low-energy theory of the IIB superstring has a Born-Infeld form. From the methodological point of view we perform a similar analysis for QCD on the lattice and it is important to note before starting our proof that our result will have a Born-Infeld form too.

**Step 3: Integration over the fermion field and chiral limit**

Our next step is the integration over the fermion degrees of freedom in (1). Using the source technique it was shown [1] that integral (1) can be re-written into the form of an integral over the unitary boson matrix \( M_x \)

\[ Z = \int DM \exp S_{\text{eff}}(M). \]
As a matter of principle, we already performed the transformation from the color lattice
degrees of freedom \((G, \psi)\) to the boson lattice degrees of freedom \((M)\). Now our task is
to realize the continuum limit of expression (2).

This step of our analysis amounts to studying of the stationary points of the lattice action
\(S_{\text{eff}}\). Fortunately this is a very well studied task \cite{13}. This problem is connected with well-
known investigations of the critical behavior of the chiral field on the lattice and with the
problem of the phase transformation on the lattice (for references see issue \cite{14}). In \cite{1}, it
was shown that for our task the stationary point is
\[
\hat{M}_0 = u_0\hat{1}, \quad u_0(m_q = 0, r = 1) = 1/4.
\]

Now one can expressed \(M(x)\) in terms of the pseudoscalar Goldstone bosons
\[
M = u_0 \exp(i\pi_\tau_5/f_\pi) = u_0[U(x)\frac{1 + \gamma_5}{2} + U^+(x)\frac{1 - \gamma_5}{2}]
\]

and the effective action is given in the form of the Taylor expansion around this stationary
point
\[
S_{\text{eff}}(U) = -N \sum_{k=1}^\infty \frac{F^{(k)}(\lambda_0)}{k!} \sum_{x,\nu} \text{tr}[(\lambda_\nu(x) - \lambda_0)^k].
\]  

Let us consider the expansion of the chiral field \(U = \exp(i\pi_\tau_5/f_\pi)\) on the lattice around
the point \(x\) in power of the small step of the lattice \(a\)
\[
U(x + \nu) = U(x) + a(\partial_\nu U(x)) + \frac{a^2}{2}(\partial_\nu^2 U(x)) + \cdots.
\]

And for components of the Taylor expansion (3) one obtain
\[
\begin{align*}
\text{tr}[(\lambda_\nu(x) - \lambda_0)] &= -2\lambda_0 \text{tr}(\alpha) \\
\text{tr}[(\lambda_\nu(x) - \lambda_0)^2] &= 2\lambda_0^2 \text{tr}(\alpha^2) - 4\lambda_0^2 \text{tr}(\alpha) \\
\text{tr}[(\lambda_\nu(x) - \lambda_0)^3] &= -2\lambda_0^3 \text{tr}(\alpha^3) + 6\lambda_0^3 \text{tr}(\alpha^2) \\
\text{tr}[(\lambda_\nu(x) - \lambda_0)^4] &= 2\lambda_0^4 \text{tr}(\alpha^4) - 8\lambda_0^4 \text{tr}(\alpha^3) + 4\lambda_0^4 \text{tr}(\alpha^2) \\
\text{tr}[(\lambda_\nu(x) - \lambda_0)^5] &= -2\lambda_0^5 \text{tr}(\alpha^5) \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots
\end{align*}
\]

where \(\alpha = a^2 \partial_\nu U \partial_\nu U^+ + O(a^4)\).

**Step 4: Problem of rotational symmetry violation: examples of the Hyper-
Cubical and Body Centered Hyper-Cubical lattices**

Expressions (4) are very essential because these are a simplest illustration of all aspects of the
violation of the rotational symmetry on the lattice. For this moment we assume nothing
special about the structure of our lattice. We try to formulate our result as generally as
possible and all information about the lattice contained in the vectors \(\nu\) that correspond to
the basic vectors of the lattice (for example, the vectors \(\nu\) for the Hyper-Cubical lattice are
the Cartesian basic vectors \(\vec{i}, \vec{j}, \vec{k}\) and \(\vec{t}\)). The leading order part can be calculated trivially.
Indeed, using the simple Hyper-Cubical lattice where \(\nu = i, j : i = (1 \ldots 4)\) it is easy to show
that the leading order contribution is the prototype chiral lagrangian
\[
P_{O(p^2)} \sim \text{tr}[\partial_\mu U \partial^\mu U^+].
\]
As I said before the rotational symmetry violation argument does not allow to use the HC lattice calculation for the next-leading order contributions. For obtaining of these contributions a more symmetrical lattice must be used. In [9] it was shown that this lattice is a Body Centered Hyper-Cubical (BCHC) or F4 lattice.

Unfortunately, this method can not be directly used for finding next contributions and the origin of this fact is again the violation of the rotation symmetry but now on the F4 lattice. Moreover, there are no any more symmetrical lattice with fixed positions of sites in 4-dimensional [9]. It means that we need an absolutely different lattice concept that guarantees the restoration of the initial symmetries. Fortunately this concept is known now. This is the Random Lattice concept (RL) [12].

4 Random Lattice in action

The basic idea of the RL is the averaging over the big ensemble of various lattices with random distributions of sites and it is possible to show that such averaging leads to the restoration of the rotational invariance. There are two methods of the realization of such scheme. A first one based on the Christ, Friedberg and Lee (CFL) technique [12].

Commonly CFL technique leads into complicated geometrical analysis. For our task it would be very useful to use the analogy between Random Lattice and Random Surface technique that was revealed recently [15, 17]. The idea is quit simple: for beginning let us consider a lattice with fixed positions (for simplicity it is possible to use the trivial HC lattice, where basis vectors are just \( \mathbf{\bar{v}} = i, j, \ldots \) in a flat space. For a simulation of the Random Lattice let us consider small deformations of the geometry of this space (\( \gamma_{ij} \rightarrow g_{ij} \)) so that one can rewrite the problem of the random lattice averaging in the terms of the random deformations of the geometry of this space [15]. This is a standard quantum gravity problem for which powerful methods of the Matrix Theory could be used.

In our problem we discuss the link integrals that depend on the basis vectors \( \nu \). All such integrals are considered separately for any lattice site \( x_i \). It means that rotation invariance violation artifacts could be avoided by considering only rotation deformations of these basis vectors (translation and re-scaling deformation are left aside in our case).

Let \( R \in SO(4) \) be a rotation operator of 4 dimensional vectors \( \nu \)

\[
\nu'_i = R_{ij} \nu_j .
\]

It is essential to note that our task can be reformulated in the language of the standard Hermitian averaging because \( SO(4) = SU(2) \times SU(2) \). For infinitesimal rotations we obviously have

\[
R_{ij} = \delta_{ij} + H_{ij} ,
\]

where \( H_{ij} \) is a traceless antisymmetric matrix.

Consider the Gaussian Ensemble \( \mathcal{H} \) of such arbitrary rotation. The matrix average of an arbitrary function \( f \) with respect to Gaussian measure is

\[
\langle f \rangle_{\mathcal{H}} = \frac{1}{N_0} \int dH e^{-\text{tr}(H^2)/2} f(H) .
\]
where \( dH \) is the standard Haar measure. The normalization factor \( N_0 \) is fixed by requiring that \( \langle f = 1 \rangle_{\mathcal{S}} = 1 \).

Using the Matrix integration technique one can prove the so-called Matrix Wick’s theorem for traceless antisymmetric matrices \([18, 15, 16, 17]\)

\[
\left\langle H_{ij} H_{kl} \right\rangle_{\mathcal{S}} = \delta_{ij} \delta_{jl} - \delta_{il} \delta_{jk},
\]

\[
\left\langle \prod_{(i,j)} H_{ij} \right\rangle_{\mathcal{S}} = \sum_{\text{pairings}} (-1)^{\kappa} \prod_{\text{pairs}} \left\langle H_{ij} H_{kl} \right\rangle_{\mathcal{S}}, \tag{7}
\]

where the sum extends over all possible pairings and \( \kappa \) is the number of crossings in the pairing. The matrix average of any odd combination of \( H_{ij} \) equals zero due to the parity argument.

It is not hard to prove that the main contribution into the averaging sum over such infinitesimal rotations comes from the original non-deformable lattice (connected with the \( \delta_{ij} \) part in (6)). In order to cancel non-deformable lattice artifacts let us consider the ensemble \( \mathcal{S} \) without this non-deformable contribution: \( \mathcal{S}' = \mathcal{S} - \mathcal{O} \). Using such averaging, for leading order of chiral effective lagrangian one gets

\[
\left\langle \text{tr}(\alpha) \right\rangle_{\mathcal{S}'} = \left\langle \text{tr}(a^2 \partial_\nu U \partial_\nu U^+) \right\rangle_{\mathcal{S}'} = \text{tr}(3a^2 \partial_i U \partial_i U^+) = \text{tr}(3a^2 (L_i L_i)),
\]

where \( L_i = U^+ \partial_i U \), and for the NL correction one obtains

\[
\left\langle \text{tr}(\alpha^2) \right\rangle_{\mathcal{S}'} = \left\langle \text{tr}(a^4 \partial_\nu U \partial_\nu U^+ \partial_\nu U \partial_\nu U^+) \right\rangle_{\mathcal{S}'} = \text{tr}(3a^4 ((L_i L_i)^2 - \frac{1}{2} [L_i, L_j]^2)) = -3a^4 \left( \frac{1}{2} \text{tr}^2(L_i L_i) + \text{tr}^2(L_i L_j) - 4 \text{tr}(L_i L_i L_j L_j) \right) \tag{8}
\]

These results reproduce the HL and BCHL results and it is easy to see that this is just what we expected to receive because this contribution was obtained from many other approaches \([10]\). In such a way, we can apply this procedure to corrections of any orders from (4) and obtain the rotation invariant result due to the pairing. It means that the question about the derivation of the chiral effective lagrangian from Lattice QCD become just a combinatorial task.

It is interesting to point out that our \( SU(2) \)-flavor result for coefficients in the NL order contribution of the Chiral Perturbation Theory (8)

\[
L_1^r = \frac{L_2^r}{2} = -\frac{L_3^r}{4}
\]

is in agreement with experiment data from \( \pi \pi \rightarrow \pi \pi \) scattering \([10]\) and these coefficients turn out to be closed to the model prediction of the \( V \) exchange \([19]\).

In the last part of this section we show an application of our procedure. We will find that an infinite subseries of the chiral perturbation can be summed up into the Born-Infeld form.

If the expression (7) allows us to calculate all terms in expansion (4), let us consider just the first column there. It is easy to show that either of these is proportional to some power
of the leading order contribution (5)

\[ \begin{align*}
\text{tr}[\alpha] & \sim \text{tr}[L_\mu L^\mu] + \cdots \\
\text{tr}[\alpha^2] & \sim \text{tr}[(L_\mu L^\mu)^2] + \cdots \\
\text{tr}[\alpha^3] & \sim \text{tr}[(L_\mu L^\mu)^3] + \cdots \\
\phantom{\text{tr}[\alpha^3]} & \cdots \phantom{\text{tr}[\alpha^3]} \cdots \phantom{\text{tr}[\alpha^3]} \cdots \\
\text{tr}[\alpha^n] & \sim \text{tr}[(L_\mu L^\mu)^n] + \cdots \\
\phantom{\text{tr}[\alpha^n]} & \cdots \phantom{\text{tr}[\alpha^n]} \cdots \phantom{\text{tr}[\alpha^n]} \cdots
\end{align*} \]  

Substituting (9) into (3) and collecting all terms which depend on the power of the prototype lagrangian one obtains the following expression for the effective chiral lagrangian

\[ \mathcal{L}_{\text{eff}} \sim -\text{tr} \left[ 1 - \sqrt{1 - 1/\beta^2} L_\mu L^\mu \right] - \text{tr} \left[ \log(1 - \frac{1}{2}(1 - \sqrt{1 - 1/\beta^2} L_\mu L^\mu)) \right] + \cdots, \]  

where \( \cdots \) are all other terms (in particular the Skyrme term) and \( \beta \) is an effective coupling constant that depends on the value of our stationary point \( u_0 \).

Now let us discuss the result (10). It was obtained that some part of chiral effective action has a Born-Infeld form plus a first logarithmic correction to it. In [20], it was shown that such form of the effective action plays a very essential role in the problem of the chiral bag formation because just these square-root terms generate the step-like distribution solutions that can be interpreted as internal phases in the two-phase model of the low-energy baryon states. Another terms play essential role only on large distances from the confinement surface and can be considered as corrections.

\section{Conclusions}

The aim of this paper is to derive the chiral effective lagrangian from QCD on the lattice at the strong coupling limit. We find that this theory looks like a Born-Infeld theory for the prototype chiral lagrangian. Such form of the effective lagrangian is expected. From the methodological point of view our consideration is very similar to the low-energy theorem in string theory that leads to the Born-Infeld action [11]. Moreover, in [20], it was shown that Chiral Born-Infeld Theory (without logarithmic corrections) has very interesting “bag”-like solutions for chiral fields. It was an additional motivation of our work.

The Chiral Born-Infeld theory is a good candidate for the role of the effective chiral theory and a model for a chiral cloud of baryons. In this model one can find not only spherical “bags”, it is possible also to study the “string”-like, toroidal or “Y-Sign”-like solutions, or some other geometry. The geometry of the confinement surface depends directly on the model of color confinement and it would be very interesting to use, for example, the Lattice QCD simulations for the color degrees of freedom in combination with our model for the external chiral field.

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