Fitting formulae for photon spectra from WIMP annihilation

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Abstract. Annihilation of different dark matter (DM) candidates into Standard Model (SM) particles could be detected through their contribution to the gamma ray fluxes that are measured on the Earth. The magnitude of such contributions depends on the particular DM candidate, but certain imprints of produced photon spectra may be analyzed in a model-independent fashion. In this work we provide the fitting formulae for the photon spectra generated by WIMP annihilation into quarks, leptons and gauge bosons channels in a wide range of WIMP masses.

1. Introduction

According to present observations of large scale structures, CMB anisotropies and light nuclei abundances, DM cannot be accommodated within the SM of elementary particles. Indeed, DM is not only a required component on cosmological scales, but it is also needed to provide a satisfactory description of rotational speeds of galaxies, orbital velocities of galaxies in clusters, gravitational lensing of background objects by galaxy clusters and the temperature distribution of hot gas in galaxies and clusters of galaxies. The experimental determination of the DM nature will require the interplay of collider experiments and astrophysical observations. Concerning the latter, DM searches use to be classified in direct or indirect ones (see [1] and references in the introduction section in [2]). Concerning direct detection, the elastic scattering of DM particles from nuclei should lead directly to observable nuclear recoil signatures although the weak interactions between DM and the standard matter makes DM direct detection extremely difficult.

On the other hand, DM might be detected indirectly, by observing their annihilation products into standard model particles. Thus, even if WIMPs (Weakly Interacting Massive Particles) are stable, two of them may annihilate into ordinary matter such as quarks, leptons and gauge bosons. Their annihilation in different places (galactic halo, Sun, etc.) produce cosmic rays which could be discriminated from the background through distinctive signatures. A cascade process follows the WIMPs annihilation. In the end, the potentially observable stable particles would be neutrinos, gamma rays, positrons and antimatter that may be observed through different devices. Among them, neutrinos and gamma rays have the advantage of maintaining their original direction due to their null electric charges.
Concerning gamma rays, photon fluxes in specific (mainly SUSY) DM models are usually obtained by software packages such as DarkSUSY and micrOMEGAs based on PYTHIA Monte Carlo event generator [3] after having fixed a WIMP mass for the particular model under consideration.

This communication precisely focuses on providing the fitting functions for the photon production coming from quarks (including $t$ quark), leptons and gauge bosons annihilations channels. This would allow to apply the results to alternative DM candidates for which software packages have not been developed. In addition, our investigation determine the dependence of such spectra on the WIMP mass in a model independent way.

On the other hand, the information about channel contribution and mass dependence can be very useful in order to identify gamma-ray signals for specific WIMP candidates and may also provide relevant information about the photon energy distribution when SM particle pairs annihilate.

Let us remind that the $\gamma$-ray flux from the annihilation of two WIMPs of mass $M$ into two SM particles coming from all possible annihilation channels (labelled by the subindex $i$) is given by:

$$
\frac{d\Phi^{DM}_{\gamma}}{dE_\gamma} = \frac{1}{4\pi M^2} \sum_i \langle \sigma_i v \rangle \frac{dN^i_\gamma}{dE_\gamma} \times \frac{1}{\Delta\Omega} \int_{\Delta\Omega} \int_{\text{l.o.s.}} \rho^2 |r(s)| \, ds,
$$

where $\langle \sigma_i v \rangle$ holds for the thermal averaged annihilation cross-section of two WIMPs into two ($i^{th}$ channel) SM particles and $\rho$ is the DM density. The integral is performed along the line of sight (l.o.s.) to the target and averaged over the detector solid angle $\Delta\Omega$.

2. Procedure

We have used the particle physics PYTHIA software [3] to obtain our results. The WIMPs annihilation is usually split into two separated processes: The first describes the annihilation of WIMPs and its SM output. The second one considers the evolution of the obtained SM unstable products. Due to the typical velocity dispersion of DM, we expect most of the annihilations to happen quasi-statically. This fact allows to state that by considering different center of mass (CM) energies for the obtained SM particles pairs from WIMP annihilation process, we are indeed studying different WIMP masses, i.e. $E_{\text{CM}} \approx 2M$. The procedure to obtain the photon spectra is thus straightforward: For a given pair of SM particles which are produced in the WIMP annihilation, we count the number of photons. The number of simulated collisions in each bin was fixed in order to provide suitable statistics in the number of produced photons. For instance, for the high energy bins many collisions are required to get a significant number of photons, whereas for low-intermediate energy, many photons are usually produced even for a small number of collisions.

3. Results

According to the extensive PYTHIA simulations performed for electroweak gauge bosons, leptons and quarks channels, three different fitting functions were required to fit the photon spectra $\frac{dN^i_\gamma}{dE_\gamma}$: one for light quarks and leptons, one for gauge bosons ($W$ and $Z$) and a final one for $t$ quark. Those expressions depended on both WIMP mass dependent and independent parameters. Concerning the WIMP mass independent parameters, their values nevertheless do depend on the considered annihilation channel whereas for WIMP mass dependent ones, their evolutions with WIMP mass are parametrized by continuous and smooth curves as seen in [2]. Figure 1 shows two of these fits. The resulting expressions for the fitting functions are the following:
3.1. Light quarks and leptons

For \( q\bar{q} \) (except the \( t\bar{t} \) studied separately in section 3.3), \( \tau^+\tau^- \) and \( \mu^+\mu^- \) channels, the most general formula needed to reproduce the \( dN^\gamma_i/dE^\gamma \) simulations may be written as:

\[
dN^\gamma_i/dx = a_1 x^{1.5} \exp \left( -b_1 x^{n_1} - b_2 x^{n_2} - \frac{c_1}{x^{d_1}} + \frac{c_2}{x^{d_2}} \right) + q \ln \left[ p(1 - x^l) \right] \frac{x^2 - 2x + 2}{x} \tag{2}
\]

where the logarithmic term takes into account the final state radiation through a Weizsäcker-Williams expression. Strictly speaking, the \( p \) parameter in the Weizsäcker-Williams term in the previous formula is \((M/m_{\text{particle}})^2\) where \( m_{\text{particle}} \) is the mass of the charged particle that emits radiation. However in our approach, it will be a free parameter to be fitted since the radiation comes from many possible charged particles, which are produced along the decay and hadronization processes. Therefore all the bremsstrahlung effects were encapsulated in a single Weizsäcker-Williams-like term by using \( p \) and \( q \) parameters. \( l \) parameter is equal to one except for the \( \mu^+\mu^- \) channel.

In fact, for the \( \mu^+\mu^- \) channel, the above expression (2) becomes simpler since the exponential contribution is absent. The total gamma rays flux for this channel is thus well fitted by:

\[
dN^\gamma_i/dx = q \ln \left[ p(1 - x^l) \right] \frac{x^2 - 2x + 2}{x} \tag{3}
\]

For both leptons channels the covered WIMP mass range was from 25 to \( 5 \times 10^4 \) GeV. Concerning the mass dependent parameters, they are \( n_1 \) and \( p \) in expression (2) for \( \tau^+\tau^- \) channel (the rest of parameters are mass independent) and \( p \), \( q \) and \( l \) in expression (3) for \( \mu \) lepton. For \( q\bar{q} \) channels, no general rule about mass (in)dependence of parameters can be settled, but specific results channel by channel are presented in [2].

Let us finally mention that the gamma rays from \( e^-e^+ \) pairs, the only contribution is that coming from bremsstrahlung. Therefore, the previous expression (3) is also valid with \( q = \alpha_{\text{QED}}/\pi \), \( p = (M/m_{e^-})^2 \) and \( l \equiv 1 \), parameters choice that obviously corresponds to the well-known Weizsäcker-Williams formula.

3.2. W and Z gauge bosons

For the \( W^+W^- \) and \( ZZ \) channels, the parametrization used to fit the Monte Carlo simulation is:

\[
x^{1.5}dN^\gamma_i/dx = a_1 \exp \left( -b_1 x^{n_1} - \frac{c_1}{x^{d_1}} \right) \left\{ \frac{\ln[p(j - x)]}{\ln p} \right\}^q \tag{4}
\]

This expression differs from expression (2) in the absence of the additive logarithmic contribution that acquires nonetheless a multiplicative behaviour. The exponential contribution is also quite simple with only one positive and one negative power laws. Moreover, \( a_1 \), \( n_1 \) and \( q \) parameters are WIMP mass independent (but they have different values depending on the studied gauge boson channel). The rest of parameters, i.e., \( b_1 \), \( c_1 \), \( d_1 \), \( p \) and \( j \) turned out to be both channel and mass dependent. The covered WIMP mass range was from 100 to \( 10^4 \) GeV for both channels. Nonetheless, at masses higher than 1000 GeV, no significant change in the photon spectra for both channels was observed.

3.3. t quark

Unlike the rest of the \( q\bar{q} \) channels, the parametrization given by (2) is not valid when photon spectra are studied in the \( t\bar{t} \) channel. For this one, the parametrization used to fit the simulations
Figure 1. Photon spectra for $M_{WIMP} = 1$ TeV in the $b\bar{b}$ (left) and $W^+W^-$ (right) channels. Dotted points are PYTHIA simulations and lines correspond to the proposed fitting functions.

is:

$$x^{1.5} \frac{dN_\gamma}{dx} = a_1 \exp \left( -b_1 x^{n_1} - \frac{c_1}{x^{d_1}} - \frac{c_2}{x^{d_2}} \right) \left\{ \frac{\ln[p(1-x^l)]}{\ln p} \right\}^q$$  \hspace{1cm} (5)

In this formula, the exponential contribution is more complicated than the one in expression (4), with one positive and two negative power laws. Again, the additive logarithmic contribution is absent and acquires a multiplicative behaviour. Notice the exponent $l$ in the logarithmic argument, required to provide correct fits in this channel.

The covered WIMP mass range was from 200 to $10^5$ GeV. Nevertheless, at masses higher than 1000 GeV no significant change in the gamma-ray spectra was observed. The mass dependent parameters for this channel are $b_1, n_1, c_2, p, q$ and $l$ whereas $a_1, c_1, d_1$ and $d_2$ are mass independent.

4. Conclusions

We have presented the model-independent fitting functions for the photon spectra coming from WIMPs pair annihilation into Standard Model particle-antiparticle pairs for all the phenomenologically relevant channels. Explicit calculations for all studied channels [2] are available at the website [http://teorica.fis.ucm.es/~PaginaWeb/downloads.html](http://teorica.fis.ucm.es/~PaginaWeb/downloads.html).

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