The Doublet Extension of Tensor Gauge Potentials and a Reassessment of the Non-Abelian Topological Mass Mechanism

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Abstract

A well-defined local gauge structure for non-Abelian two-form gauge fields was introduced some years ago. This was achieved by introducing doublet group representations and doublet-assembled connections. We provide a summarized version of this formalism, in order to recall its features and applications. We also build up doublet-extended gauge-invariant actions for bosonic and fermionic matter, and discuss the appearance of novel topological quantities in these doublet-type gauge models. A partner action for higher spin fields appears in the doublet version of the fermionic matter sector. As an application of the formalism, a Chern-Simons and an Yang-Mills action in four dimensions may both be rigorously defined. We carry out this task and show that, in this doublet framework, their combination constitutes a consistent (power-counting renormalizable and unitary) non-Abelian generalization of the Cremmer-Scherk-Kalb-Ramond theory with topological mass.

1 Introduction

The (Abelian) Kalb-Ramond field [1, 2] (KR), \( B_{\mu\nu} \), is a two-form field which appears in the low-energy limit of String Theory [3], in Quantum Gravity and Supergravity [4] and in several other frameworks in Particle Physics [5]. In particular, attempts to assign mass to gauge field models in four dimensions take into account this sort of field accompanied by a one-form gauge field [6, 7, 8], in order to recreate the topological mass mechanism of three dimensions. In this sense, it becomes widely accepted that the electroweak breaking and the mass generation for the matter should be originated from a more fundamental description. The topological mass mechanism sets out to address this question: the mass for the gauge boson is not generated by means of a breaking; we rather have extra physical degrees of freedom inherited from a more fundamental level. We attempt here to deepen the investigation of properties of the 2-form gauge potential, by trying to better understand its non-Abelianisation, the possibility to realise its non-minimal coupling to matter and to work out its behaviour in the ultraviolet regime and quantum consistency.

The Abelian symmetry of the KR field is similar to that of a 1-form gauge field [8]:

\[
B_{\mu\nu} \rightarrow B_{\mu\nu} + \frac{1}{2}(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu),
\]

(1)

where \( \beta_\mu \) is a 1-form parameter. The old question is how to associate the parameter \( \beta_\mu \) to the manifold of a general Lie group [9, 10]? From the physical point of view, it is essential to ask whether a *genuine* gauge theory may be formulated for this field, i.e, if the two-form gauge potential may be stated as a
connection on some group manifold. This is important because, as it is well-known, this structure would
be crucial for the identities which determine the renormalisability or the eventual finiteness of physical
models. In particular, in ref. [11], it was argued that massive (non-Abelian) gauge models [6, 7] based on
a gauge KR field, $B_{\mu\nu}$, and a usual one-form, $A_{\mu}$, are ill-defined in four spacetime dimensions. The first
task of this article is to clarify the group structure underneath non-Abelian KR fields and to show that
non-Abelian theories based on them may be formulated in a similar way to the Yang-Mills-Chern-Simons
theories in 2 + 1 dimensions, which are known to be finite [12]. We however accommodate the KR in an
extended scenario, which we shall refer to as the Doublet Formalism (DF).

Another crucial question that arises is how to define a minimal coupling of this field with matter
fields, with the interaction with gauge fields appearing by replacing partial derivatives of the matter
fields by covariant ones in the free Lagrangian. This is directly related to the charge conservation laws
via Noether’s theorem. To do this, local gauge transformations for matter fields need to be defined. Up
to now, this is unknown for transformations which involve a 1-form parameter. Some alternatives for
these questions were proposed [13], where its expected applications in gravitation with torsion and p-form
cosmology are emphasized [13, 14, 15, 16, 17].

The problem was discussed first considering possible representations through singlet tensor/spinor
spaces, such that the KR field could be built in the connection [18, 19, 20]. However, in ref. [19] (also
in [21]) it has been shown that many difficulties involving Lorentz invariance of physical models appear
in the non-Abelian case, and furthermore, the extension to non-flat spacetimes seems to be hard. A
mathematical framework where these difficulties are solved was finally proposed in Ref. [22] some years
ago. According to this, one may have a well defined two form gauge field and tackle all these problems,
the basic proposal to achieve this is to relax the conventional representations of groups, imposed in the
preceding approaches. This allows us to construct well defined gauge models for KR fields, which may be
minimally coupled with matter fields in a natural way. Once more, the simplest solution of the problem
arises from considering doublets of tensors of different ranks as a representation for a Lie group. This
kind of idea has been successfully used before, to solve other algebraical issues related to dualities and
the Hodge map in field theory [23]. In fact, by considering a doublet field representation, we are able to
include a 1-form parameter in an \textit{exponential-like} symmetry/transformation law:

\[ \delta \left( \begin{array}{c} \phi \\ \phi_{\mu} \end{array} \right) = \left( \begin{array}{c} i\alpha \phi \\ i\alpha \phi_{\mu} + i\beta_{\mu} \phi \end{array} \right) = i \left( \begin{array}{cc} \alpha & 0 \\ \beta & \alpha \end{array} \right) \left( \begin{array}{c} \phi \\ \phi_{\mu} \end{array} \right) \] (2)

where the variation of the fields is proportional to themselves and to the group parameters [22]. These
simple expressions solve the problem of writing this kind of transformation law in a simple and satisfactory
way, and are the key to define the group operations involving an 1-form parameter. Notice that, without
a doublet representation (and a scalar parameter $\alpha$), individual fields ($\phi$ and $\phi_{\mu}$) can never be combined
with an 1-form $\beta_{\mu}$ to give a tensor of the same type, and to define their variations. Clearly, (2) is
the most general rule where both $\beta$ and a minimal number of matter fields appear linearly and in a
Lorentz-invariant way, such as was argued in refs. [19, 21]. This idea may sound technically trivial but
it is meaningful, it has never been used before as the cornerstone for a gauge principle generating the
two-form field. In this approach everything can be expressed in a manifestly covariant form, and the
generalization to curved spacetime becomes rather immediate. In this paper, we describe this formalism
in detail, simplifying and clarifying the language of our original proposal [22] in order to make it useful;
and finally apply it to construct a well-defined non-Abelian generalization of the topologically massive
CSKR theory.

This work is organized as follows: in Section 2, we explicitly find out the Lie group corresponding to
these transformations and build up the covariant derivative with the generic tensor field being part of the
connection is defined; we also summarize some of the useful ingredients of the DF. In Section 3, gauge
theories such as Yang-Mills and Chern-Simons in four dimensions are discussed, and the simplest gauge-
invariant actions for matter minimally coupled to the gauge doublet is presented. Finally, in Section 4, we
study the consistency of the Yang-Mills-Chern-Simons model in four dimensions. Concluding Remarks
are cast in Section 5.
2 A Summary of the Doublet Formalism.

Let \((M, g_{\mu\nu})\) be a general \(D\)-dimensional space time and \(G\) be a Lie group whose associated algebra is \(G\); \(\tau^a\) are the matrices representing the generators of the group with \(a = 1, \ldots, \dim G\); \(\tau^{abc}\) are the structure constants ([\(\tau^a, \tau^b\)] = \(\tau^{abc}\)).

As mentioned, consider the general transformations

\[
\begin{align*}
\delta \phi &= i \alpha \phi \\
\delta \phi_{\mu} &= i \alpha \phi_{\mu} + i \beta_{\mu} \phi
\end{align*}
\]

(3)

where the doublet of parameters \((\alpha, \beta)\) consists of two Lie algebra valued 0- and 1-forms respectively.

Let us denote the doublet of fields by \(\Phi \equiv (\phi, \phi_{\mu})\). Thus, this transformation may be formally expressed as

\[
\delta_{\alpha,\beta} \Phi = (I \alpha + \sigma \beta) \Phi
\]

(4)

where \(I, \sigma\) are \(2 \times 2\) matrices, satisfying the algebra:

\[
I \sigma = \sigma, \quad I^2 = I, \quad \sigma^2 = 0
\]

(5)

From expression (2), the simplest representation of this algebra is

\[
I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.
\]

(6)

(7)

Notice that there is another interesting representation by taking \(\sigma\) to be a Grassman parameter.

The product of two elements of the algebra is well defined and is naturally given by the usual matrix product

\[
\delta_{\alpha',\beta'} \delta_{\alpha,\beta} \Phi = I(\alpha' \alpha) + \sigma(\beta' \alpha + \alpha' \beta) \Phi,
\]

(8)

This is the most general associative product (such that the upper component of the matrix (2) vanishes) we can construct in this way [22].

Thus, the algebra elements may alternatively be expressed in terms of doublets, ordered pairs of 0- and 1-forms. The product of doublets reads as:

\[
(\alpha, \beta_{\mu})(\phi, \phi_{\mu}) = (\alpha \phi, \alpha \phi_{\mu} + \beta_{\mu} \phi)
\]

(9)

thus the symmetry transformation is

\[
(\delta \phi, \delta \phi_{\mu}) = i (\alpha, \beta_{\mu})(\phi, \phi_{\mu}) = (i \alpha \phi, i \alpha \phi_{\mu} + i \beta_{\mu} \phi),
\]

and the identity represents as \((1,0)\).

Using the expression (3) for the infinitesimal transformation, it may be easily shown that a group element may be expressed as usually

\[
g = \exp i \Gamma
\]

(10)

where the algebra parameter in this case writes

\[
\Gamma = (I \alpha^a + \sigma \beta^a_{\mu}) \tau^a.
\]

(11)

Notice that by virtue of the property \(\sigma^2 = 0\), expression 10 reduces to be linear in \(\beta\), however the exponential form is mnemonically more natural, and easier for computations and algebraic manipulations.

3We assume them in a matricial representation of the algebra.

4e. g. in the case of a separated group structure \(G = G(\alpha) \times G(\beta)\), such that \([\alpha, \beta] = 0\), it expresses \(g(\alpha, \beta_{\mu}) = (e^{i\alpha}, i \beta_{\mu} e^{i\alpha})\).
2.1 General doublets of \((1, r + 1)\)-tensors as gauge connections.

Let us consider the most general representations of doublets introduced in Ref. [22], which consist in pairs of tensors \((\phi^a_p, \phi^a_{p'}) \in \Pi^q_p \times \Pi^q_{p'}\) (\(\Pi^q_p\) denotes the standard set of tensors of type \((q, p)\).) So, the symmetry transformation can be built over doublets of arbitrary order using the same idea, in any of the spaces \((\phi^a_p, \phi^a_{p'}) \in \Pi^q_p \times \Pi^q_{p'}\), \(\forall p, q, q', r\) which takes values in a representation of the Lie group. For simplicity, let us take arbitrary pairs of tensors with coincident covariant type \(\Phi = (\phi^a, \phi^a)\) and \((\alpha, \beta, r) \in \Pi_0 \times \Pi_r\ (r \equiv p' - p)\), and the \(r\)-generalized connection reads as \(\mathcal{A} = (A_1, B_{r+1}) \in \Pi_1 \times \Pi_{r+1}\) and so on. In this case, in view of (2), the symmetry can be written as below:

\[
\delta \left( \frac{\phi_p}{\phi_{p+r}} \right) = i \left( \begin{array}{cc} \alpha & 0 \\ \beta_r & \alpha \end{array} \right) \left( \frac{\phi_p}{\phi_{p+r}} \right). \tag{12}
\]

For simplicity consider now \(q = 0\). We introduce the partial derivative of a \((0, p)\) tensor \(T_p\) as a \((0, p + 1)\) tensor given by

\[
T_p = T_{\mu_1...\mu_p} dx^{\mu_1} \otimes ... \otimes dx^{\mu_p},
\]

so that

\[
\partial T_p := \partial_\mu T_{\mu_1...\mu_p} dx^{\mu_1} \otimes ... \otimes dx^{\mu_p}.
\]

So, we can define the partial derivative of a doublet as the doublet consisting of the partial derivatives

\[
\partial(\phi_p, \phi_{p+r}) \equiv ((\partial \phi_p)_{p+1}, (\partial \phi_{p+r})_{p+r+1}). \tag{13}
\]

It is easy to verify that this definition is consistent with the Leibnitz rule for the product of doublets.

The tensor product of two doublets of arbitrary orders and types, is the simple generalization of the rule (9): \((A, B)(A', B') = (A \otimes A', A \otimes B + B \otimes A')\). Next, one can define the [nonexterior] covariant derivative of a \((p, p')\)-doublet, \(\Phi = (\phi_p, \phi_{p'})\), as a \((p + 1, p' + 1)\)-doublet:

\[
D\Phi = \partial \Phi - i A \Phi = (\partial \phi_p - i A_1 \phi_p, \partial \phi_{p+r} - i A_1 \phi_{p+r} - i B_{r+1} \phi_p), \tag{14}
\]

where the connection must be a doublet valued on any Lie algebra:

\[
\mathcal{A} \equiv (A_1, B_{r+1}) = (A^a_1, B^a_{r+1}) \tau^a \tag{15}
\]

of order \(r = p - p'\). These are what we wish call Doublet-Connections, and constitutes one of the most meaningful achievement of the present approach since any-rank tensor canonically appears in a gauge connection on an arbitrary Lie group, in a way that avoid all the old conflicts with Lorentz invariance [19] [21].

Imposing that \(gD\Phi = D'\Phi'\), and using \(D' = \partial - i A'\) and \(\Phi' = g\Phi\), we obtain the transformation law for the connection:

\[
A' = g(\alpha, \beta)A g(-\alpha, -\beta) - i(\partial g(\alpha, \beta))g(-\alpha, -\beta), \tag{16}
\]

whose infinitesimal expression is

\[
\delta A = \partial (\alpha, \beta) - i [A, (\alpha, \beta)] = D(\alpha, \beta) \tag{17}
\]

(The canonical curvature tensor \(\mathcal{F} \in \Pi_2 \times \Pi_{2+r}\) transforms as \(\mathcal{F} = g(\alpha, \beta)\mathcal{F} g(-\alpha, -\beta)\) ; which reads, in terms of the doublet components:

\[
\delta A = \partial \alpha - i [A, \alpha], \tag{18}
\]

\[
\delta b = \partial \beta - i [B, \alpha] - i [A, \beta]. \tag{19}
\]

So for instance, in terms of tensor components, the curvature tensor \(\mathcal{F} = (F_2, h_3) \in \Pi_2 \times \Pi_3\) results as

\[
F_{\mu\nu} = 2 \partial[\mu A_\nu] + i [A_\mu, A_\nu] \tag{20}
\]

\(^5\)In other similar formalisms, only forms and exterior calculus are considered.
where brackets [ , ] stand for anti-symmetrization over the indices $\mu, \nu$, and the symbol | before the $\rho$ index means that $\rho$ is not to be anti-symmetrized.

Let us remark that DF [22] is more general than often definitions of the $B$-field and its associate curvature $G$: recall it describes connections $(A_1, B_p)$, where $B$ may be general contravariant tensors (or spinors [31]) rather only $p$-forms. We have noticed that DF, when particularized to exterior calculus for pairs of $p, p + 1$-rank forms, is similar to the generalized exterior calculus [29]; and the totally symmetric case was studied recently [32]. This allows us to describing all the tensor (and spinor) types in a connection.

The problem of defining self/(anti-self)-duality relations for arbitrary forms, space time dimension and signature, may be solved by introducing doublets of this type (Ref...) and generalizing the Hodge operation accordingly, which allowed to avoid the conflict with the sign of the contraction of two Levi-Civita tensors.

Let a $d$-dimensional space-time with signature $s$, and a generic forms doublet $\Phi \equiv (\phi_p, \phi_q)$ in the space $\Lambda_{p,q} \equiv \Lambda_p \times \Lambda_q$. Thus, one may define a Hodge-type operation for these objects$^6$ by means of

$$^*\Phi \equiv (S_q^* \phi_q, S_p^* \phi_p),$$

$^6$Which clearly includes the case $p = d/2$ described above.

$$G_{\mu\nu\rho} = 2\partial_{[\mu} B_{\nu\rho]} + 2i(B_{[\mu|\rho} A_{\nu]} - A_{[\mu} B_{\nu\rho]}) , \tag{21}$$

where $\mu, \nu, \rho$ are not totally anti-symmetric and contain more components, then, the so-called Kalb-Ramond gauge field must be identified with its totally anti-symmetric part: $B_{\mu\nu} \equiv B_{[\mu|\nu]}$, and the totally antisymmetric part of the curvature doublet is the usual $(2,3)$-form pair: $(F, H) \equiv (F_{[\mu|\nu]}, H_{[\mu|\nu]})$, where

$$H_{\mu\nu\rho} = 2\partial_{[\mu} B_{\nu\rho]} + 2i(B_{[\mu|\rho} A_{\nu]} - A_{[\mu} B_{\nu\rho]}) = d \wedge B + A \wedge B - B \wedge A. \tag{22}$$

So the Kalb Ramond field may be found as a component of the exterior connection in the exterior covariant derivative, defined as the totally anti-symmetric part of the covariant derivative defined above (where tensor products $\otimes$ shall be substituted by exterior ones $\wedge$). For arbitrary rank:

$$D \wedge \Phi \equiv \partial \wedge \Phi - iA \wedge \Phi = (d \phi_p - iA_1 \wedge \phi_p, d \phi_{p+r} - iA_1 \wedge \phi_{p+r} - iB_{r+1} \wedge \phi_p) , \tag{23}$$

and the doublet field strength is the curvature of this derivative:

$$F \equiv (F_2, H_{r+2}) = (d \wedge A_1 + A_1 \wedge A_1, d \wedge B_{r+1} + A_1 \wedge B_{r+1} - B_{r+1} \wedge A_1). \tag{24}$$

This satisfy the Bianchi identities:

$$D \wedge F = 0. \tag{25}$$

Using (23) this splits as:

$$(d - iA) \wedge F = 0 \quad , \quad (d - iA) \wedge H - iB \wedge F = 0 . \tag{26}$$

These relations are a consequence of the Jacobi identities, which are closely the associative property of the algebra. We should recall however, that DF admits natural generalizations to non-associative algebras through the introduction of a metric in the base manifold (more details may be found at the original article [22]).

### 2.2 Hodge map and integration in the DF.

The formalism above requires some extra structure in order to define topological actions and gauge theories in general. Concepts as the Hodge operation and integration on manifolds shall also be extended according to this formalism in a consistent way.

The problem of defining self/(anti-self)-duality relations for arbitrary forms, space time dimension and signature, may be solved by introducing doublets of this type (Ref...) and generalizing the Hodge operation accordingly, which allowed to avoid the conflict with the signal of the contraction of two Levi-Civita tensors.

Let a $d$-dimensional space-time with signature $s$, and a generic forms doublet $\Phi \equiv (\phi_p, \phi_q)$ in the space $\Lambda_{p,q} \equiv \Lambda_p \times \Lambda_q$. Thus, one may define a Hodge-type operation for these objects$^6$ by means of

$$^*\Phi \equiv (S_q^* \phi_q, S_p^* \phi_p),$$

$^6$Which clearly includes the case $p = d/2$ described above.
where $S_p$ has unit norm and it is properly defined in order to get consistency with:

$$^*(*(\Phi)) = \Phi. \quad (28)$$

But the double dual operation, for a generic $p$-form $A$ depends on the signature $(s)$ and dimension of the spacetime in the form: $^*(\Phi) = (\Phi)^{s+d-p}$. Therefore, (27) and (28) imply:

$$(-1)^{s+p(d-p)} S_{d-p} S_p = 1. \quad (29)$$

* applied to doublets is defined such that its components are interchanged with a supplementary factor (27) satisfying (29), which is fulfilled if

$$S_{d-p} S_p = (-1)^{s+p(d-p)} \quad (30)$$

is chosen. Below, we finally fix $S_p$ through physicalness requirements on the actions.

Notice that there is a well defined notion of self (anti-self)-duality for doublets, since

$$^*\Phi = \pm \Phi \quad (31)$$

is consistent with the requirement (28). This operation may be expressed schematically in terms of the algebra elements $(I, \sigma)$ as:

$$^*I \sim \sigma \quad ^*\sigma \sim I \quad (32)$$

up to signal factors $S_{p,q}$ that depend on the tensor coefficients.

### 2.3 Integration

Apart from this one may define integration of doublets though the simplest prescription, which is, given a doublet of forms $(A_{d-1}, B_d)$ being fields defined on a $d$-dimensional manifold $M$, to be the integral of the component whose type coincides with $d$, namely

The present formalism admits more general definitions of integration that might be useful in certain context related to topological objects [in preparation](we are going to discuss a bit this subject in the following section). In fact we naturally can define a *pair* of integrals of $(A_p, B_d)$ for a doublet of manifolds $(\Sigma_p, M)$, provided that $\Sigma_p$ is a $p$-dimensional submanifold of $M$,

$$I \equiv \int_{(\Sigma_p, M)} (A_p, B_d) = \left( \int_{\Sigma_p} A_p, \int_M B_d \right) \quad (33)$$

In order to define actions, one may combine the components of $I \equiv (I_p, I_d)$ to define one number as the integral of $(A_p, B_d)$ over $(\Sigma_p, M)$ in several ways. The natural meaningful prescription is the sum

$$I \equiv I_p + I_d = \int_{(\Sigma_p, M)} (A_p, B_d) \equiv \int_{\Sigma_p} A_p + \int_M B_d \quad (34)$$

In contexts where there is not a natural submanifold $\Sigma_p$, the only geometric (relativistic invariant) definition, reduces just to the second term:

$$I \equiv \int_M (A_p, B_d) = I_d = \int_M B_d. \quad (35)$$

In the particular case $p = d - 1$, there is a natural (relativistic) $\Sigma_p = \partial M$, then one can define the integral over $M$ by means of:

$$\int_M (A_{d-1}, B_d) = \int_{\partial M} A_{d-1} + \int_M B_d \quad (36)$$

This integration allows us to define actions where the boundary terms are often neglected, in such a context or in a boundary-less manifold, the relevant part of the integral reduces to the second term. However in the context of topological configurations and invariants, both term shall be important.
2.4 Topological Numbers

Let us consider a fiber bundle with a \((d \geq 4)\)-dimensional base manifold \(M_d\), whose principal bundle is given by the action of an arbitrary Lie Group \(G\). Consider in addition, a four-dimensional submanifold \(M_4\) embedded into \(M_d\). So in DF, one may define a gauge connection \(A = (A, B)\), where \(B\) is a totally anti-symmetric \((d - 2)\)-form.

According to the definition (33), the gauge invariant integral

\[
I \equiv \int \text{tr} F \wedge F = \left( \int_{M_4} \text{tr} (F \wedge F) , \int_{M_d} \text{tr} (F \wedge H + H \wedge F) \right) = (I_4, I_d)
\]

is a meaningful doublet of topological quantities. The first component is the instanton number in four dimensions, but the second one is a new one.

Notice that this quantity is defined on a \(d\)-dimensional manifold but clearly, although independent, it is closely related to the number of instantons in the \(M_4\)-projected Yang Mills theory \(^7\). As for instantons, it is not difficult to show that \(I_d\) is quantized (at least for an Abelian group), and it shall carry relevant topological information. In a forthcoming work, we shall study topological configurations where this number can be physically interpreted [in preparation][MBC, Helayel, unpublished].

The charge doublet (37) is conserved. In fact, the related topological current density may be expressed as:

\[
J \equiv {^*}L = (S_3{^*}L_3, S_d{^*}L_d)
\]

where

\[
\mathcal{L} \equiv (L_3, L_d) = \left( \text{Ad}A - \frac{2i}{3} A \wedge A \wedge A , A \wedge dB + B \wedge dA - i2 A \wedge A \wedge B \right),
\]

which may be written in terms of the doublet connection in the suggestive form:

\[
\mathcal{L} = \text{tr} \left( \text{Ad}A - \frac{2i}{3} A \wedge A \wedge A \right).
\]

The exterior derivative of this doublet is precisely the integrand of (37)

\[
\mathcal{I} = d\mathcal{L}
\]

which, by virtue of the Bianchi identities, shows that the total divergence of (38) vanishes.

3 Gauge invariant actions.

3.1 The Chern-Simons theory and non-Abelian BF models.

The Chern-Simons action in \(d\) dimensions may be rigorously defined in DF, and notably, the relation with the paradigmatic three-dimensional case is manifest \([22]\).

Consider the connection \(A = (A, B)\) on an arbitrary Lie group, we can define the integral doublet (33):

\[
\mathcal{I}_{CS}[A] \equiv \text{tr} \int \left( \text{Ad}A - \frac{2i}{3} A \wedge A \wedge A \right).
\]

The integrand in this expression is in fact the doublet of \((3, d)\)-form given explicitly by Eq. (39). Notice that the first component corresponds to the canonical Chern-Simons theory for a one form gauge field.

\(^7\)We are tempted to suggest a relation of duality between them.
A in three dimensions, while as is manifest, the doublet as a whole is what we shall identify as its generalization to arbitrary $d$ dimensions in DF.

Finally, we take the integration (35) in order to define the $d$-dimensional Chern-Simons action:

$$S_{CS}[A] \equiv -k \int \text{tr} \left( [A \wedge dB + B \wedge dA] - i2[A \wedge A \wedge B] \right).$$  

(43)

where $k$ denotes the inverse of the coupling constant. This is a well defined gauge invariant topological theory, and may be recognized as the generalization of a so-called BF theory for an arbitrary non-Abelian Lie group. The second term in the R.H.S. is not obvious and it is necessary to get the gauge invariance (17). It is indeed straightforward to check out that $S_{CS}$ is gauge invariant (up to a total derivative) as expected.

The equations of motion read:

$$\mathcal{F} = dA - iA \wedge A = (F, H) = 0.$$  

(44)

$B \wedge F$ theories are similar to Chern-Simons in three dimensions and they are often associated by analogy [23], however, the actual connection between both never has been clearly established so far [22]. In the present framework this is indeed defined as a genuine Chern-Simons theory for a doublet connection.

In the particularly meaningful $d = 4$ case, one can define the invariant integral as (36), and the Chern Simons adopts the more general form:

$$S_{CS}[A] \equiv \int_{\partial M_4} \text{tr} \left( AdA - \frac{2i}{3} A \wedge A \wedge A \right) + \int_{M_4} \text{tr} \left( [A \wedge dB + B \wedge dA] - i2[A \wedge A \wedge B] \right),$$  

(45)

which includes a boundary term that precisely is a Chern Simons theory for the gauge field $A_\mu$.

Therefore, BF theory (rigorously generalized here to non-Abelian groups) and Chern Simons theory, are manifestly part of a same structure in DF.

### 3.2 Pure Yang-Mills theories and dynamics for the $B$-field

Next, we may canonically define Yang Mills theories for doublet connections in $d$ dimensions $A = (A_1, B_{r+1})$, $r \leq d-2$. The field strength tensor is $\mathcal{F} = (F_2, H_{r+2})$, and by virtue of 27, the curvature dual is:

$$^*\mathcal{F} = (S_{r+2}^*H, S_2^*F),$$  

(46)

which is a doublet $(d - r - 2, d - 2)$-form. Therefore,

$$\mathcal{F} \wedge ^*\mathcal{F} = (S_{r+2}F \wedge ^*H, S_2F \wedge ^*F + S_{r+2}^*H \wedge H)$$  

(47)

is a $(d - r, d)$-form. Therefore, the pure Yang-Mills action can be defined as the integral (35) of this expression, which is gauge invariant for construction:

$$S_{YM} \equiv \int \text{tr} \mathcal{F} \wedge ^*\mathcal{F} = \int \text{tr} (S_2 F \wedge ^*F + S_{r+2}^*H \wedge H).$$  

(48)

This may be written also as:

$$S_{YM} \equiv \int d^{d-2}x \text{tr} \left( S_{d-r} S_2 F \cdot F + S_d H \cdot H \right),$$  

(49)

For a boundaryless $M_4$, or asymptotic decay of $A$, this action agrees with (43) for dimension $d$. 

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8For a boundaryless $M_4$, or asymptotic decay of $A$, this action agrees with (43) for dimension $d$. 

---
where the dot represents the ordered contraction over all the spacetime indices: \( P \cdot Q \equiv P^{\mu_1...\mu_n} Q^{\mu_1...\mu_n} \).

Using (30), we have:

\[
S_{YM} = \int dx^d \, \text{tr} \left( (-1)^{s+2(d-2)} \, F \cdot F + (-1)^{s+(r+2)(d-r-2)} \, H \cdot H \right). \tag{50}
\]

The equations of motion (source-free) read:

\[
D \wedge * F = 0, \tag{51}
\]

in components,

\[
(d - iA_1) \wedge (sH)_{d-r-2} , \, S_2 (\ast F)_{d-2} = 0, \tag{52}
\]

where * is the standard contraction by the Levi-Civita tensor. Using (23) this splits in two equations:

\[
(d - iA_1) \wedge \ast = 0 , \, S_2 (d - iA_1) \wedge \ast F - i S_{r+2} B_{r+1} \wedge \ast H = 0. \tag{53}
\]

Contracting these equations by the totally anti-symmetric Levi-Civita tensor, they adopt the more familiar divergence type form:

\[
(\partial - iA_1) \cdot H = 0 , \, \partial - iA_1 \cdot F - i (d - 1)^{r(d-r)} B_{r+1} = 0. \tag{54}
\]

### 3.3 Matter Fields

**Bosonic matter:**

The Hodge operation and integration are the need ingredients to define general global (and local) gauge invariant actions in a exterior calculus language. The standard free action for a complex scalar (a 0-form) field \( \phi \) in the adjoint representation of a group \( G \) expresses as

\[
S_{scalar}(\phi) \equiv \frac{1}{2} \, \text{tr} \int d^4 x \, d\phi \wedge * d\phi + \frac{m^2}{2} \int \text{tr} \, \bar{\phi} \wedge * \phi \tag{55}
\]

in this language. Interacting terms may always be written as:

\[
S_{int} \equiv \sum_n \frac{\lambda_n}{n!} \, \text{tr} \int (\bar{\phi} \wedge * \phi)^n. \tag{56}
\]

The rule to form gauge invariant action for bosonic matter fields in a four-dimensional Lorentzian space time, consists in doing the substitution:

\[
\phi \to \Phi = I \phi + \sigma \phi_\mu = (I \phi^a + \sigma \phi^a_\mu) \tau^a. \tag{57}
\]

Plugging this into the action (55), using the algebra (5), (27), and the integration defined above, we obtain:

\[
S(\phi, \phi_\mu) = \frac{(-1)}{2} \, \text{tr} \int d^4 x \left( \partial_\mu \bar{\phi} \partial^\mu \phi - \partial_\mu \bar{\phi}_\nu \partial^\mu \phi_\nu \right) + \frac{m^2}{2} \left( \bar{\phi} \phi + \bar{\phi}_\mu \phi^\mu \right). \tag{58}
\]

For \( p \)-form matter fields the rule is the same, the action (55) is well defined for any type of the singlet \( \phi \in \Lambda_p \); then, one shall extend this to a doublet as (57) \( \phi \to \Phi = (\phi, \phi_r) \in \Lambda_p \otimes \Lambda_{p+r} \), with \( p, r \) arbitrary.

\[
\phi \to \Phi \equiv I \phi + \sigma \phi_r = (I \phi^a_{\mu_1...\mu_p} + \sigma \phi^a_{\mu_1...\mu_{p+r}}) \tau^a. \tag{59}
\]
Then the free action in $d$ dimensions becomes
\[
S(\Phi) = \frac{1}{2} \tr \int d^d x \left( S_{d-p-1} S_{p+1} \partial_{\mu} \overline{\phi}_{[\mu_1 \ldots \mu_p]} \phi^{[\mu_1 \ldots \mu_p]} + \phi^{[\mu_1 \ldots \mu_{p+1}]} \partial_{\mu} \overline{\phi}_{[\mu_1 \ldots \mu_{p+1}]} \right) + \frac{m^2}{2} \left( (S_{d-p} S_p) \overline{\phi}_{[\mu_1 \ldots \mu_p]} \phi^{[\mu_1 \ldots \mu_p]} + (S_{d-p-r} S_{p+r}) \overline{\phi}_{[\mu_1 \ldots \mu_{p+r}]} \phi^{[\mu_1 \ldots \mu_{p+r}]} \right),
\]  
(60)
where the signal in front of each term is determined from the formula (30).

Locally invariant actions are obtained from this one by minimal substitution of the covariant derivative $d \rightarrow D$ that includes standard gauge one form $A_1$ and the new tensor field $B_{1+r}$.

**Fermionic matter:**

By performing the formal replacement $p = 1/2$ in the rule (59) above, we obtain the doublet of fields we have to consider to construct gauge invariant actions that include Dirac fields in the matter sector. So then,
\[
\psi_{(1/2)} \rightarrow \Psi \equiv \int \psi_{(1/2)} + \sigma_{\xi_{(1/2)+r}} = (I \psi^a + \sigma^{a}_{\xi_{[\mu_1 \ldots \mu_r]}}) \gamma^a.
\]  
(61)
Particularizing now for the simplest $d = 4$ case, $\psi \equiv \psi_{1/2}$ is a spinor field, while its partner $\xi_{(1/2)+1}$ carries spin $3/2$ which behaves as a (massive) Rarita-Schwinger-type field, as it shall become clear in what follows below.

The global invariance under doublet gauge transformations is manifest whenever we write the Dirac action in the language of forms:
\[
S_{Dirac} \equiv \int \tr \overline{\psi} \gamma \wedge^\star(d\psi) + im \int \tr \overline{\psi} \wedge^\star \psi,
\]  
(62)
where the gamma matrices are thought as components of a one-form (whose components are *matrices*) $\gamma \equiv \gamma_{\mu}$, in order to simplify the notation; the spinor indices are all contracted in the standard way, and the trace contracts the group internal indices, and finally, the Hodge operation $\star$ acts only on the tensor indices $\mu, \nu, \ldots$.

In the mass term the product $\wedge$ does not play any role while we do not substitute $\psi$ by the doublet $\Psi$. Then plugging (61) into this action we obtain the globally invariant fermionic action:
\[
S_f(\Psi) \equiv \int \tr \overline{\Psi} \gamma \wedge^\star(d\Psi) + im \int \tr \overline{\Psi} \wedge^\star \Psi,
\]  
(63)
which, by using the DF algebra and integration, reads
\[
S_f(\Psi) = S_{Dirac}(\psi) + \int \tr \overline{\zeta} \gamma \wedge^\star(d\zeta) + im \int \tr \overline{\zeta} \wedge^\star \zeta,
\]  
(64)
where $\zeta$ has to be thought as a one form, while the spinor index is implicit (and contracted with $\gamma$’s as standard). This sector is new and may be written in components as
\[
S(\zeta) = \int \tr \overline{\tilde{\zeta}_{\mu \nu}} (\partial^\mu \zeta^\nu) \, dx^4 + im \int \tr \overline{\zeta} \zeta^\mu,
\]  
(65)
This action gives dynamics to the spin-$3/2$ field. Let us now understand how close it is related to the Rarita-Schwinger field and how the spin-$1/2$ mode propagates. The equation of motion is
\[
\gamma^\mu \partial_{\mu} \zeta_{\nu} = im \zeta_\nu,
\]  
(66)
which looks like as a “dual” to a conventional Rarita-Schwinger field. Notice that, by choosing the Rarita-Schwinger gauge: $\gamma^\mu \zeta_{\mu} = 0$, this equation of motion reduces to the Dirac equation for the component fields:
\[
\partial \zeta_\nu = im \zeta_\nu,
\]  
(67)
which describes the propagating modes. In fact, Eq. (66) defines a way of generalizing the Dirac equation \((r = 0)\), to any \((1/2 + r)\)-form \(\zeta\):
\[
\gamma^\mu (\partial_\mu \zeta_{\mu_1 \ldots \mu_r}) = i m \zeta_{[\mu_1 \ldots \mu_r]} .
\] (68)
By imposing the Rarita-Schwinger-like gauge as
\[
\gamma^\mu \zeta_{\mu_1 \ldots \mu_r} = 0 \quad \forall i = 1, \ldots, r,
\] (69)
eq (68) reduces to:
\[
\gamma^\mu \partial_\mu \zeta_{\mu_1 \ldots \mu_r} = i m \zeta_{\mu_1 \ldots \mu_r} .
\] (70)

4 The Model: An Yang-Mills-Chern-Simons Action

A remarkable no-go theorem on the topological mass mechanism in \(d = 4\) dimensions has been presented in ref. [11]. The argument is based on the non-existence of power-counting renormalizable and unitary gauge theories, constructed as consistent deformations from the Cremmer-Scherk-Kalb-Ramond Abelian theory to a non-Abelian one. However, it is interesting to analyze this theory in view of the gauge group structure clarified here. Since an important ingredient of this negative result is the impossibility of closing the algebra, we hope that the well defined group structure underneath DF might be crucial in solving that problem.

As shown above, the Cremmer-Scherk-Kalb-Ramond model may be rigorously defined in DF (and generalized to non-Abelian gauge symmetry) as a Yang-Mills-Chern-Simons theory (Eqs. (43),(48)):
\[
S(A) := m S_{CS} + S_{YM} ,
\] (71)
where \(m\) is a relative constant which has mass unit. The structure of this theory is indeed completely similar to YMCS in \(2 + 1\) dimensions, which are known to be finite [12].

Because of that, we propose the action (71) in \(d = 4\) as a candidate to a well defined model with topological mass mechanism. This action results a non-Abelian BF one, which reads as
\[
S(A, B) \equiv m \int \mathrm{tr} \left( [A \wedge dB + B \wedge dA] - i 2 [A \wedge A \wedge B] \right) + \int \mathrm{tr} \left( F \wedge * F - * H \wedge H \right) ,
\] (72)
where \(A = (A_\mu , B_{\mu \nu})\). Notice that a pure Chern-Simons boundary term could be added in a gauge invariant way, according to expression (45), but we are not going to consider it here.

4.1 Propagators

To compute the propagators for the gauge-field excitations, we need to concentrate on the bosonic Lagrangian in terms of the physical fields \(A_\mu\) and \(B_{\mu \nu}\). For the sake of reading off these propagators, we refer to the bilinear sector of the Lagrangian, whose kinetic piece can be cast like below:
\[
L_0 = \mathrm{tr} \left( \frac{1}{2} F_{\mu \nu} F^{\mu \nu} - \bar{\psi}_\mu \psi^\mu + 2 m A_\mu \bar{\psi}^\mu \right) ,
\] (73)

In order to invert the wave operator, we have to fix the gauge so as to make the matrix non-singular. This is accomplished by adding the gauge-fixing terms,
\[
L_{A_\mu} = \frac{1}{2 \alpha} \mathrm{tr} (\partial_\mu A^\mu)^2 ;
\] (74)
\[
L_{B_{\mu \nu}} = \frac{1}{2 \beta} \mathrm{tr} (\partial_\mu B^{\mu \nu})^2 .
\] (75)
To read off the gauge-field propagators, we shall use an extension of the spin-projection operator formalism presented in [?, ?]. The propagators we are looking for, were already computed in Ref. [?] which may be expressed as explicitly, in terms of matrix elements,

$$<AA> = \frac{2i}{(4m^2 + \Box)} \Theta_{\mu\nu} + \frac{2i\alpha}{\Box} \Omega_{\mu\nu} \sim k^{-2},$$  \hfill (76)

$$<AB> = -\frac{2m}{(4m^2 + \Box)} \epsilon^{\mu\nu\rho\sigma} \partial_\rho \Theta_{\sigma k} \sim k^{-3},$$  \hfill (77)

$$<BB> = \frac{2i}{(4m^2 + \Box)} (P_1^1)_{\alpha\beta\gamma k} + \frac{i}{(16m^{-1} - (8\beta)^{-1}\Box)} (P_1^1)_{\alpha\beta\gamma k} \sim k^{-2},$$  \hfill (78)

which were expressed in terms of the algebra of projectors:

$$(P_1^1)_{\mu\nu,\rho\sigma} = \frac{1}{2} (\Theta_{\mu\rho} \Theta_{\nu\sigma} - \Theta_{\mu\sigma} \Theta_{\nu\rho}),$$  \hfill (79)

$$(P_1^1)_{\mu\nu,\rho\sigma} = \frac{1}{2} (\Theta_{\mu\rho} \Omega_{\nu\sigma} - \Theta_{\mu\sigma} \Omega_{\nu\rho} - \Theta_{\nu\rho} \Omega_{\mu\sigma} + \Theta_{\nu\sigma} \Omega_{\mu\rho}),$$  \hfill (80)

where $\Theta_{\mu\nu}$ and $\Omega_{\mu\nu}$ are, respectively, the transverse and longitudinal projection operators, given by:

$$\Theta_{\mu\nu} = \eta_{\mu\nu} - \Omega_{\mu\nu},$$  \hfill (81)

and

$$\Omega_{\mu\nu} = \partial_\mu \partial_\nu.$$

The other operator coming from the Kalb-Ramond sector, $S_{\mu\nu\kappa}$, is defined in terms of Levi-Civita tensor as

$$S_{\mu\nu\kappa} = \epsilon_{\lambda\mu\nu\kappa} \partial^\lambda.$$

Let us write down the interaction terms that appear in the CS lagrangian:

$$-m \int \text{tr} ( [A \wedge dB + B \wedge dA] - i2[A \wedge A \wedge B] ) .$$  \hfill (84)

The first term is nothing but the third one in Eq. (73), then it is encoded in the propagators. The second term is the single CS vertex. We also have the vertex terms from the YM sector (48):

$$\int \text{tr} ((-1)^s A \wedge A \wedge s (A \wedge A) + 2(-1)^s s (dA) \wedge (A \wedge A) + 2 s (dB) \wedge (A \wedge B + B \wedge A) + 4 s (A \wedge B) \wedge B \wedge A) .$$  \hfill (85)

From now on, all these terms, which will provide the different vertex diagrams, will be unambiguously referred by the respective subscripts $AAB, AAAA \equiv 4A, AAA \equiv 3A, ABB, AABB$.

### 4.2 Power-counting rule for the primitive divergences

we have

$$\delta = 2I_{AA} + 2I_{BB} + I_{AB} - 4(V - 1) + V_{3A} + V_{BBB},$$  \hfill (86)

where $V = V_{AAB} + V_{3A} + V_{4A} + V_{BBB} + V_{AABB}$.

$$2I_{AA} + I_{AB} + E_A = 2V_{AAB} + 3V_{3A} + 4V_{4A} + V_{BBB} + 2V_{AABB}$$  \hfill (87)
\[ 2I_{BB} + I_{AB} + E_B = V_{AAB} + 2V_{BBA} + 2V_{AABB} \]  

(88)

then we have
\[ \delta = 4 - (I_{AB} + E_A + E_B + V_{AAB}) \]  

(89)

which shows renormalizability for very generic diagrams, except for very special ones given by:
\[ (I_{AB} + E_A + E_B + V_{AAB}) < 4 \]  

(90)

Sufficient number of external lines and/or external vertex, already guarantee that this last inequality is violated, and the diagram is then renormalizable.

### 4.3 BRST invariance

To close our discussion on the consistency of the model, we give a quick glance at the BRST transformations so as to fix the action for the ghost sector: our proposal for the ghosts and BRST transformations has the same structure as those for a YM theory, readily adapted here to the language of doublets. In fact, consider the following algebra-valued doublets of fields, in addition to a gauge-field doublet \( A = (A^a, B^a_{\mu}) \tau^a \):
\[
\Delta = (\eta, \eta_{\mu}) = (\eta^a, \eta^a_{\mu}) \tau^a ; \quad \bar{\Delta} = (\bar{\eta}, \bar{\eta}_{\mu}) = (\bar{\eta}^a, \bar{\eta}^a_{\mu}) \tau^a ; \quad \Theta = (b, b_{\mu}) = (b^a, b^a_{\mu}) \tau^a
\]

(91)

and the group invariant constant field:
\[
\Lambda = (\lambda, \lambda_{\mu}) ; \quad \partial_{\mu} \Lambda = 0,
\]

(92)

where \( \Delta, \bar{\Delta}, \Lambda \) are anti-commuting fields (i.e, its components are Grassman numbers). Then, the BRST transformations may be defined for doublets by:
\[
\delta_{BRST} A = \Delta \Lambda ,
\]

(93)
\[
\delta_{BRST} \Delta = i \Delta \wedge \Delta ,
\]

(94)
\[
\delta_{BRST} \bar{\Delta} = \Theta ,
\]

(95)
\[
\delta_{BRST} \Theta = 0 .
\]

(96)

where the R.H.S of (94) reads: \( i \Delta \wedge \Delta = i(\eta \wedge \eta, \eta \wedge \eta_{\mu} + \eta_{\mu} \wedge \eta) = i \tau^{abc} (\eta^b \eta^c, \eta^b \eta^c_{\mu} + \eta^c \eta^c_{\mu}) \). By virtue of these transformations, the Lie algebra properties, and the doublet algebra (5), results:
\[
\delta^2_{BRST} = 0.
\]

So, the action for ghost sector action to be proposed reads:
\[
S_{\text{ghost}}[A, \Delta, \bar{\Delta}, \Theta] = \int dx^4 \text{tr} \left[ \left( b(\partial_{\mu} A^\mu) + \frac{\alpha}{2} b^2 \right) + \left( b_{\mu}(\partial_{\mu} B^{\mu\nu}) + \frac{\beta}{2} b_{\nu} b^{\nu} \right) - \bar{\Delta} \partial^\mu D_{\mu} \Delta \right].
\]

(97)

One can readily show that the BRST variation of this action vanishes using (93), ... (96), and the identity: \( \delta_{BRST} D_{\mu} \Delta = 0 \), which is a consequence of the Jacobi identity in DF. The Jacobi identity is a consequence of that the elements (10) form a Lie group, which may be easily verified using the DF product rules.

Therefore, the total action \( S_T \equiv m S_{CS} + S_{YM} + S_{\text{ghost}} \) satisfy the Slavnov-Taylor identities,
\[
\delta_{BRST} S_T = 0
\]

(98)

which then ensures that our power-counting model is also unitary.
4.4 Discussion.

By adopting an extended approach, referred to as tensor gauge field doublet formalism, we have found a possibility to build up the non-Abelian generalization of the CSKR model which opens up a viable path for a relevant topological mass mechanism in four dimensions. This construction favors a result that contrasts with a previous No-Go theorem [11].

In ref. [11], it is crucially argued that there is not a consistent deformation of the CSKR model; this essentially means that any consistent (non-Abelian) action, say $S$, deformed from the Abelian one $S_{CSKR}$, must satisfy (98) in order to be unitary. Indeed, our model constitutes an example of that.

We think that the objection raised in [11] on the inconsistency of these type of models may be by-passed in the formalism presented here, since one assumption for the no-go result is that the non-Abelian extension of the Kalb-Ramond field, $B$ is uncharged. In mathematical language, this may be expressed by means of the transformation law:

$$\delta B = D\beta \equiv d\beta + [A, \beta].$$

(99)

In our approach, however, which is based upon the doublet structure of the algebra, this brings up a novel term related precisely to the charge of $B$, namely

$$\delta B = d\beta + [A, \beta] + [B, \alpha].$$

(100)

The $[B, \alpha]$-term is genuinely due to our doublet formalism, and it plays an important role for the consistency of the theory.

5 Concluding Comments

In this work, we have presented a sort of extended algebraic structure and discussed some of its main tools in details, in order to build up general quantum field theories with $p$-tensors/forms as genuine gauge fields of conventional gauging methods associated to matter lagrangians. We have attempted at the construction of a consistent non-Abelian topologically massive gauge model in four dimensions and we argue it is a consistent quantum field-theoretic model. As a next step, we have to include the matter sector with the aim of reproducing/fiting the main features of the Standard Model [27]. Another issue whose result will be reported in a forthcoming work concerns the Goldston theorem, which, in the context of the DF, presents interesting aspects and involves a spin-1 Goldstone boson along with the conventional scalar one whenever there occurs spontaneous symmetry breaking. The role of the would-be spin-1 Goldstone bosons of the DF may be another problem to be understood in connection with the gravitational Higgs mechanism.

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