GUT effects in the soft supersymmetry breaking terms

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Abstract

In minimal supergravity theories the soft supersymmetry breaking (SSB) parameters are universal (flavor blind) near the Planck scale. Nevertheless, one often assumes universality at the grand-unification scale $M_G \approx 10^{16}$ GeV instead, and corrections to the SSB parameters arising from their evolution between the Planck and GUT scales are neglected. We study these corrections and show that large splittings between the scalar mass parameters can be induced at $M_G$. These effects are model dependent and lead to significant uncertainties in the low-energy predictions of supersymmetric models, in their correlations and in the allowed parameter space.

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The minimal supersymmetric extension of the standard model (MSSM) is a well motivated candidate to describe the physics beyond the standard model \([1]\). The unknown origin of supersymmetry breaking is parametrized by soft supersymmetry breaking (SSB) terms in the lagrangian (that do not reintroduce quadratic divergencies), \(i.e.,\)

\[
-\mathcal{L}_{\text{soft}} = m_i^2 |\Phi_i|^2 + B_{ij} \Phi_i \Phi_j + A_{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} M_\alpha \lambda_\alpha^2 + h.c.,
\]

(1)

where \(\Phi_i (\lambda_\alpha)\) are the scalar (gaugino) fields. Eq. (1) introduces a large number of new arbitrary parameters and is impractical for phenomenological studies. A better situation appears if the supersymmetry is a local symmetry, \(i.e.,\) supergravity. It is then assumed that supersymmetry is broken in a hidden sector which couples only gravitationally to the observable sector. The effective lagrangian for the observable sector below \(M_P \equiv M_{\text{Planck}}/\sqrt{8\pi} \approx 2.4 \times 10^{18} \text{ GeV}\) consists of a global supersymmetric theory with SSB terms as in eq. (1). In the minimal supergravity model, which we assume here, the Kähler potential is flat and one finds that the SSB parameters have universal values at \(M_P\), \(i.e.,\)

\[
m_i^2 \equiv m_0^2, \quad B_{ij} \equiv B_0 \mu_{ij}, \quad A_{ijk} \equiv A_0 Y_{ijk}, \quad M_\alpha \equiv M_{1/2},
\]

(2)

where \(\mu_{ij}\) and \(Y_{ijk}\) are respectively the bilinear and trilinear couplings in the superpotential. The deviations from the universal boundary condition (2) at lower scales are calculated using renormalization group (RG) methods, and given only four soft parameters one can predict the superpartner mass spectrum.

If the MSSM is embedded in a grand-unified theory (GUT) at the scale \(M_G \approx 10^{16} \text{ GeV}\) suggested by coupling constant unification \([2]\), then the evolution of the parameters between \(M_P\) and \(M_G\) depends on the GUT and is strongly model dependent. Nevertheless, it is often assumed that applying (2) at \(M_G\) rather than at \(M_P\) is a good approximation, because \(M_G\) is close to \(M_P\). One then uses the generic MSSM RG equations (RGEs) between \(M_G\) and the weak scale \([3,4]\).

In this letter we will examine the corrections to the SSB parameters arising from their evolution between the Planck and the GUT scales. We will show that these corrections
induce large deviations in the SSB parameters from their universal values. The corrections are typically proportional to $\frac{2}{\pi} m^2 \ln M_P/M_G$ where $\alpha$ and $m^2$ are a generic coupling and soft mass parameter, respectively. Although these corrections are not enhanced by large logarithms, they can be significant due to:

1. The number of particles above $M_G$, $N$, is large as a result of the large symmetry group, and one roughly has $\frac{\alpha}{\pi} \to \frac{N\alpha}{\pi}$. (See also Ref. [5].)

2. Large Yukawa couplings that are typically present in GUTs and that grow with the energy. In addition to the large top Yukawa coupling, one has to introduce extra large couplings to avoid a too large proton decay rate.

Corrections from the gauge sector $[\propto \frac{\alpha_G}{\pi} M_1^2 \ln M_P/M_G]$ can also be important for large $M_1/2$ or if $\alpha_G$ grows with the energy (as in non-minimal GUTs). The above corrections depend on the details of the GUT model and represent uncertainties in the low-energy predictions. Gravitational and other effects could also affect the boundary condition (2) and would only add to the uncertainty.

For definiteness and simplicity we consider the minimal SU(5) model. We will comment on extended models below. The Higgs sector of the model consists of three supermultiplets, $\Sigma(24)$ in the adjoint representation [which is responsible for the breaking of SU(5) down to $SU(3)_c \times SU(2)_L \times U(1)_Y$], and $\mathcal{H}_1(\overline{5})$ and $\mathcal{H}_2(5)$, each containing a SU(2) doublet $H_i$ and a color triplet $H_C$. The matter superfields are in the $\overline{5} + 10$ representations, $\phi(\overline{5})$ and $\psi(10)$.

The superpotential is given by

$$W = \mu_\Sigma \text{tr} \Sigma^2 + \frac{1}{6} \lambda' \text{tr} \Sigma^3 + \mu_{\mathcal{H}} \mathcal{H}_1 \mathcal{H}_2 + \lambda_{12} \mathcal{H}_1 \mathcal{H}_2$$

$$+ \frac{1}{4} h_{tt} \epsilon_{ijklm} \psi^{ij} \psi^{kl} \mathcal{H}_2^m + \sqrt{2} h_{tt} \psi^{ij} \phi_i \mathcal{H}_1 j.$$  

(3)

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1We define $\Sigma = \sqrt{2} T_a w_a$ where $T_a$ are the SU(5) generators with $\text{tr}\{T_a T_b\} = \delta_{ab}/2$, and we only consider Yukawa couplings for the third generation.
In the supersymmetric limit $\Sigma$ develops a vacuum expectation value $\langle \Sigma \rangle = \nu_\Sigma \text{diag}(2,2,2,-3,-3)$ and the gauge bosons $X$ and $Y$ get a mass $M_V = 5g_G\nu_\Sigma$. In order for the Higgs SU(2) doublets to have masses of $\mathcal{O}(m_Z)$ instead of $\mathcal{O}(M_G)$, the fine-tuning $\mu_H - 3\lambda \nu_\Sigma \lesssim \mathcal{O}(m_Z)$ is required and one obtains $M_{H_C} = \frac{\lambda}{g_G} M_V$. Dimension-five operators induced by the color triplet give large contributions $\propto 1/M_{H_C}^2$ to the proton decay rate \cite{ref}. To suppress such operators, the mass of the color triplets has to be large, $M_{H_C} \gtrsim M_V$, implying $\lambda \gtrsim g_G \approx 0.7$. Thus, one-loop corrections proportional to $\lambda$ produce important effects. Below $M_P$ the effective lagrangian also contains the SSB terms

$$-\mathcal{L}_{soft} = m_{H_1}^2 |\mathcal{H}_1|^2 + m_{H_2}^2 |\mathcal{H}_2|^2 + m_{\Sigma}^2 \text{tr}\{\Sigma^\dagger \Sigma\} + m_\phi^2 |\phi|^2 + m_\psi^2 \text{tr}\{\psi^\dagger \psi\}$$

$$+ [B_{\Sigma} \mu_\Sigma \text{tr}\Sigma^2 + \frac{1}{6} A_\lambda \lambda^3 \text{tr}\Sigma^3 + B_H \mu_H \mathcal{H}_1 \mathcal{H}_2 + A_\lambda \lambda \mathcal{H}_1 \Sigma \mathcal{H}_2$$

$$+ \frac{1}{4} A_t h t \epsilon_{ijklm} \psi^{ij} \psi^{kl} \mathcal{H}_2^m + \sqrt{2} A_b h b \psi^{ij} \phi_i \mathcal{H}_{1j} + \frac{1}{2} M_5 \lambda \alpha \lambda + \text{h.c.}] . \quad (4)$$

From $M_P$ to $M_G$ the SSB terms evolve according to the RGEs of the SU(5) model with eq. \cite{ref} as a boundary condition. Thus, we expect a breakdown of universality at $M_G$ for SSB parameters of fields that are in different SU(5) representations. The SU(5) RGEs for the SSB parameters and Yukawa couplings are given by

$$\frac{dm_{10}^2}{dt} = \frac{1}{8\pi^2} [3h_t^2 (m_{H_1}^2 + 2m_{H_2}^2 + A_t^2) + 2h_b^2 (m_{H_1}^2 + m_{10}^2 + m_5^2 + A_b^2) - \frac{72}{5} g_G^2 M_5^2] ,$$

$$\frac{dm_5^2}{dt} = \frac{1}{8\pi^2} [4h_b^2 (m_{H_1}^2 + m_{10}^2 + m_5^2 + A_b^2) - \frac{48}{5} g_G^2 M_5^2] ,$$

$$\frac{dm_{H_1}^2}{dt} = \frac{1}{8\pi^2} [4h_b^2 (m_{H_1}^2 + m_{10}^2 + m_5^2 + A_b^2) + \frac{24}{5} \lambda^2 (m_{H_1}^2 + m_{H_2}^2 + m_\Sigma^2 + A_\lambda^2) - \frac{48}{5} g_G^2 M_5^2] ,$$

$$\frac{dm_{H_2}^2}{dt} = \frac{1}{8\pi^2} [3h_t^2 (m_{H_1}^2 + 2m_{10}^2 + A_t^2) + \frac{24}{5} \lambda^2 (m_{H_1}^2 + m_{H_2}^2 + m_\Sigma^2 + A_\lambda^2) - \frac{48}{5} g_G^2 M_5^2] ,$$

$$\frac{dm_5}{dt} = \frac{1}{8\pi^2} \left[ \frac{21}{20} \lambda^2 (3m_\Sigma^2 + A_\lambda^2) + \lambda^2 (m_{H_1}^2 + m_{H_2}^2 + m_\Sigma^2 + A_\lambda^2) - 20 g_G^2 M_5^2 \right] ,$$

$$\frac{d\lambda}{dt} = \frac{\lambda}{16\pi^2} \left[ \frac{63}{20} \lambda^2 + 3\lambda^2 - 30 g_G^2 \right] , \quad \frac{dA_t}{dt} = \frac{\lambda}{16\pi^2} \left[ \frac{21}{20} \lambda^2 + 3h_t^2 + 4h_b^2 + \frac{53}{5} \lambda^2 - \frac{98}{5} g_G^2 \right] ,$$

$$\frac{dh_t}{dt} = \frac{h_t}{16\pi^2} \left[ 9h_t^2 + 4h_b^2 + \frac{24}{5} \lambda^2 - \frac{96}{5} g_G^2 \right] , \quad \frac{dh_b}{dt} = \frac{h_b}{16\pi^2} \left[ 10h_b^2 + 3h_t^2 + \frac{24}{5} \lambda^2 - \frac{84}{5} g_G^2 \right] . \quad (5)$$

where $t = \ln Q$. The RGE for the gauge coupling is $d\alpha_G/dt = -3\alpha_G^2/2\pi$, and similarly $dM_5/dt = -3\alpha_G M_5/2\pi$. The RGEs for the trilinear SSB parameter $A_t$ can be obtained from the RGEs of the corresponding Yukawa coupling $Y_t$ by
\[
\frac{dY_i}{dt} = \frac{Y_i}{16\pi^2} \left[ a_{ij} Y_j^2 - b g_{ij}^2 \right] \quad \rightarrow \quad \frac{dA_i}{dt} = \frac{1}{8\pi^2} \left[ a_{ij} Y_j^2 A_j - b g_{ij}^2 M_5 \right].
\]  

We can omit the RGEs for $\mu_\Sigma, \mu_H, B_\Sigma$ and $B_H$, which are arbitrary parameters that decouple from the rest of the RGEs.

The evolution of the SSB parameters from $M_P$ to $M_G$ is dictated by a competition between the positive Yukawa terms (i.e., scalar contributions) and the negative gauge terms (i.e., gaugino contributions) in the RGEs. We can distinguish two scenarios: (A) For moderate values of $M_{1/2} \equiv M_5(M_P)$ the contribution from the gauge sector is small. In this case, the RGEs of $m^2_{H_1}$ and $m^2_{H_2}$ have a large contribution proportional to $\lambda^2$ and both masses are diminished as the energy scale decreases. For $h_t \gg h_b$, $m^2_{H_2}$ decreases faster than $m^2_{H_1}$, but also $m^2_{10}$ (for the third family) is diminished in that case. (B) For large values of $M_{1/2}$ the RGEs are dominated by the negative gaugino contribution so that all the SSB parameters increase as the energy scale decreases. The scalar masses are enhanced by an additive factor

\[
\Delta m_i^2 = -\frac{c_i}{3} \left[ 1 - \frac{1}{\left(1 + \frac{3\alpha_G}{2\pi} \ln \frac{M_G}{M_P} \right)^2} \right] M_{1/2}^2,
\]

where $c_i = \frac{72}{10} (\frac{24}{5})$ for $i$ in the 10(5) representation. One has $\Delta m^2_i \approx 0.5(0.3) M_{1/2}^2$.

Examples of scenarios A and B are given in Figs. 1a and 1b, respectively. We see that the violation of the universality of the SSB parameters at $M_G$ can be substantial. In particular, the soft masses of the Higgs fields are typically split from the matter field masses. For $M_{H_C} = 1.4 M_P$ (i.e., $\lambda \approx 1$ at $M_G$) the splitting can be as large as 100%.

In order to analyze the implications of these soft mass splittings in the supersymmetric spectrum and phenomenology, we have to run the SSB parameters from $M_G$ down to $m_Z$ [7]. Below $M_G$ the effective theory corresponds to the MSSM:

\[
W = \mu H_1 H_2 + h_t Q H_2 U + h_b Q H_1 D + h_\tau L H_1 E,
\]

where $Q(L)$ and $U, D(E)$ are respectively the quark (lepton) SU(2) doublet and singlet superfields. The tree-level matching conditions of the SSB parameters between the SU(5) model and the MSSM are $m^2_{H_i}(M_G) = m^2_{H_i}(M_G)$, $m^2_{10}(M_G) = m^2_{Q,U,E}(M_G)$ and
\[ m_5^2(M_G) = m_{D,L}^2(M_G) \]  

One-loop matching conditions will be considered below. In this letter we present only a qualitative analysis of the GUT effects in the low energy quantities. A comprehensive numerical study, together with the details of the numerical procedures, will be given elsewhere.

In scenario A the parameters \( m_{H_i}^2 \) have substantial shifts and the parameters \( m_{H_i}^2(m_Z) \) are modified. The latter enter the minimization conditions of the weak-scale Higgs potential. Therefore, corrections to \( m_{H_i}^2 \) modify the region of the MSSM parameter space that is consistent with electroweak symmetry breaking (EWSB). The \( \mu \) parameter is also affected by the GUT corrections to \( m_{H_i}^2 \) since it is extracted from the minimization conditions of the weak-scale Higgs potential \([1,3,4]\). This leads to corrections to observables that depend on \( \mu \), such as the Higgsino mass, the Higgsino-gaugino mixing and the left-right scalar quark mixing. Also, consistency with constraints from color and charge breaking \([4]\) can be affected.

A priori, one would expect that the Higgs boson masses are also affected. Notice, however, that the latter depend only on the sum \( m_{H_i}^2 + |\mu|^2 \), which is not modified significantly (the shift in \( m_{H_i}^2 \) is approximately compensated by the shift in \( |\mu|^2 \)). The masses of the lightest chargino and neutralinos are typically proportional to \( M_1/2 \), and are only slightly affected.

There are also large deviations for the scalar quark masses of the third generation. As mentioned above, due to the evolution from \( M_P \) down to \( M_G \), \( m_{10}^2 \) can be shifted for large \( h_t \) (e.g., \( \tan \beta = \langle H_2 \rangle / \langle H_1 \rangle \approx 1 \)) from the universal value \( m_0^2 \) (see Fig. 1a). In addition, the evolution of \( m_Q^2 \) and \( m_U^2 \) from \( M_G \) down to \( m_Z \) depends on the value of \( m_{H_2}^2 \) that is sensitive to the GUT physics.

In scenario B, where the gaugino contribution is dominant, all the scalar masses are shifted \([eq. (7)]\) from their universal value at \( M_P \). The scalar quark squared masses, however, receive large corrections in the running from \( M_G \) down to \( m_Z \) \((\approx 6 M_5^2)\) so that the amount \((7)\) represents only a few percent of their values. This is not the case for the scalar leptons, where such corrections are much smaller \(\approx 0.5(0.1) M_5^2 \) for \( \tilde{e}_L (\tilde{e}_R) \) and the increment \((7)\) can even double their masses.

In Figs. 2 and 3 we present examples of the GUT effects in the low-energy predictions. In
Fig. 2 we take a large tan $\beta$ ($\tan \beta = 42$) and show the allowed values (i.e., consistent with EWSB and experimental bounds) of $\mu$ vs. the gluino mass, $M_{\tilde{g}}$, when the evolution from $M_P$ to $M_G$ is considered (triangles) or neglected (filled circles). The correlation between $\mu$ and $M_{\tilde{g}}$, which exists when neglecting the $M_P$ to $M_G$ evolution [3,4], is “smeared” when that evolution is included, and $\mu$ is larger in the latter case. We also find that $m^2_{H_i}(m_Z)$ are often both negative when the GUT effects are considered [$m^2_{H_i}(M_G)$ are diminished together because $h_t \approx h_b$], which is inconsistent with EWSB. The allowed parameter space is then significantly reduced. In Fig. 3 we show the light $t$-scalar mass $m_{\tilde{t}_1}$ vs. $M_{\tilde{g}}$ for $\tan \beta \approx 1$ ($h_t \approx 1$). $\mu$ is now large ($\sim 1$ TeV) and is less sensitive to the GUT effects. Corrections to $m_{\tilde{t}_1}$ are mainly via the diminished $h^2 m^2_{H_2}$ term in the respective RGEs below $M_G$. $\tilde{t}_1$ is therefore heavier and some points which correspond to a tachionic $t$-scalar and are excluded when the $M_P$ to $M_G$ evolution is neglected, can be allowed. Note also that the correlation between $m_{\tilde{t}_1}$ and $M_{\tilde{g}}$ is weakened by the GUT corrections. We find that correlations between predictions are genericly modified due to the model-dependent “smearing” from the $M_P$ to $M_G$ evolution. The correlation between $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ is, however, strengthened because of the heavier $\tilde{t}_1$. Figs. 2 and 3 correspond to scenario B (A) for very large (small to moderate) values of $M_{\tilde{g}}$.

Even if the universal boundary condition (2) for the SSB parameters is taken at $M_G$, there is some arbitrariness in the value of $M_G$ due to mass-splittings between the particles at the GUT scale, i.e., threshold effects. We will distinguish three categories of corrections (details will be given elsewhere). First, we consider logarithmic threshold corrections arising from the mass splitting between different heavy superfields. Threshold corrections to Yukawa and gauge couplings are discussed, for example, in Ref. [8]. The largest contributions to the SSB parameters arise from the SU(2) triplet and singlet components of the $\Sigma$ superfield, $\Sigma_3$ and $\Sigma_1$, and are given by (we identify $M_G = \max \{M_V, M_{HC}\}$)

$$m^2_{H_i}(M_G) = m^2_{H_i}(M_G) + \frac{\lambda^2}{4\pi^2}(m^2_{H_1} + m^2_{H_2} + m^2_{\Sigma} + A_\Sigma^2) \left[ \frac{3}{4} \ln \frac{M_{\Sigma_3}}{M_G} + \frac{3}{20} \ln \frac{M_{\Sigma_1}}{M_G} \right], \quad (9a)$$
\[
\Delta A_{t,b}(M_G) = \frac{\lambda^2}{4\pi^2} A_{\lambda} \left[ \frac{3}{4} \ln \frac{M_{\Sigma_3}}{M_G} + \frac{3}{20} \ln \frac{M_{\Sigma_1}}{M_G} \right].
\] (9b)

Since the masses \(M_{\Sigma_3}\) and \(M_{\Sigma_1} \equiv 0.2 M_{\Sigma_4}\) can be much smaller than \(M_G\), these corrections can be substantial. For \(M_{\Sigma_3} \approx 10^{-2} M_G\) and \(\lambda \approx 1\), we have \(m^2_{H_1}(M_G) \approx 0.6 m^2_0\). The second type of threshold effects are logarithmic corrections due to the boson-fermion mass splitting within a supermultiplet. Such corrections are suppressed by powers of \(m_{\text{soft}}/M_G\) in the Yukawa and gauge coupling boundary conditions, but there is no such suppression for the SSB mass terms, e.g., corrections to \(m^2_{H_1}\) are \(\sim M^2_G \ln \left[ (M^2_G + m^2_{\text{soft}}) / M^2_G \right] \sim m^2_{\text{soft}}\).

Keeping only the terms of \(\mathcal{O}(\lambda^2/4\pi^2)\), we have
\[
m^2_{H_1}(M_G) = m^2_{H_1}(M_G) + \frac{\lambda^2}{4\pi^2} \left[ \frac{18}{20} m^2_{\Sigma} + A_{\lambda} B_{\Sigma} \right] + \frac{6}{8} (m^2_{H_1} + m^2_{H_2} + 2 A_{\lambda} B_{H}) \right],
\] (10)

where the first term comes from corrections of the \(\Sigma_3\) and \(\Sigma_1\) particles, while the second term from corrections of the Higgs color triplets. This represents a \(\mathcal{O}(10\%)\) correction. Lastly, there are scheme-dependent finite one-loop corrections. In the dimensional-reduction scheme they are given by
\[
m^2_{H_1}(M_G) = m^2_{H_1}(M_G) - \frac{\lambda^2}{4\pi^2} \frac{24}{20} (m^2_{H_1} + m^2_{H_2} + m^2_{\Sigma} + A^2_{\lambda}) \right),
\] (11)

Notice that the corrections (10) tend to cancel the corrections (11) for equal SSB parameters. From (9) – (11) one expects an additional \(\mathcal{O}(40\%)\) common correction to \(m^2_{H_1}(M_G)\) that would induce \(\mathcal{O}(5 \text{ -- } 20\%)\) uncertainties in low-energy predictions.

In extended supersymmetric GUTs one expects the corrections to be larger. If large representations are introduced, the positive scalar contribution to the RGEs is larger and therefore the SSB parameters decrease faster with the scale. However, one has to be aware of a possible breakdown of perturbation theory. An interesting scenario occurs in models in which \(H_1\) and \(H_2\) couple with different strength to the other Higgs supermultiplets. For example, in the missing partner SU(5) model \(W = \lambda_1 H_1 \Sigma(75) \Phi(50) + \lambda_2 H_2 \Sigma(75) \Phi(50) + \ldots\), and if \(\lambda_2 > \lambda_1\), the evolution from \(M_P\) to \(M_G\) splits the two Higgs scalar masses. That splitting can now affect the low-energy Higgs boson masses and reduce the degree of fine-tuning that is typically required to achieve EWSB in scenarios with large tan \(\beta\) (in which
the Higgs masses are not split by Yukawa interactions). If we enlarge the symmetry group, the negative term in the RGEs coming from the gaugino contribution is enhanced and can partially cancel the scalar contribution. In models where the rank of the group is larger than the rank of the SM group, e.g., SO(10), we have an additional contribution to the scalar masses that arises from the D-terms \[9\].

To summarize, we have shown that large deviations from universality at $M_G$ can be generated when considering (i) the model-dependent evolution from $M_P$ to $M_G$ and (ii) threshold corrections at $M_G$ (including those from scalar-fermion splittings). We have also shown that the above leads to a modification of the allowed parameter space, smears predicted correlations and affects certain low-energy predictions such as the $\mu$ parameter and the $t$-scalar mass. These corrections have to be considered as uncertainties when analyzing possible future evidence for supersymmetry. On the other hand, such corrections could provide a probe of the high scale.

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FIGURES

FIG. 1. The evolution of the soft mass parameters of the third family, $\phi(\bar{5})$ and $\psi(10)$, the Higgs, $H_i$, and the gaugino between the Planck and grand-unification scales for: (a) scenario A, with $m_0 = A_0 = 400$ GeV and $M_{1/2} = 100$ GeV; and (b) scenario B, with $m_0 = A_0 = 50$ GeV and $M_{1/2} = 450$ GeV. In both cases $m_t = 160$ GeV; $\tan\beta = 1.25$; and the boundary conditions $\lambda = 1$ (i.e., $M_{HC} = 1.4M_V$) and $\lambda' = 0.1$ at $M_G$ are assumed. All masses are in GeV.

FIG. 2. Scatter plot of the $\mu$ parameter vs. the gluino mass within the allowed parameter space for $m_t = 180$ GeV and $\tan\beta = 42$. Triangles (filled circles) correspond to universality [eq. (2)] at the Planck (grand-unification) scale. $\lambda$ and $\lambda'$ are as in Fig. 1. All masses are in GeV.

FIG. 3. Same as in Fig. 2 except the light $t$-scalar mass vs. the gluino mass, and $m_t = 160$ GeV and $\tan\beta = 1.25$. 
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The minimal supersymmetric extension of the standard model (MSSM) is a well motivated candidate to describe the physics beyond the standard model [1]. The unknown origin of supersymmetry breaking is parametrized by soft supersymmetry breaking (SSB) terms in the lagrangian (that do not reintroduce quadratic divergencies), i.e.,

\[ -\mathcal{L}_{soft} = m_i^2 |\Phi_i|^2 + B_{ij} \Phi_i \Phi_j + A_{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} M_\alpha \lambda_\alpha^2 + h.c., \]

where \( \Phi_i (\lambda_\alpha) \) are the scalar (gaugino) fields. Eq. (1) introduces a large number of new arbitrary parameters and is impractical for phenomenological studies. A better situation appears if the supersymmetry is a local symmetry, i.e., supergravity. It is then assumed that supersymmetry is broken in a hidden sector which couples only gravitationally to the observable sector. The effective lagrangian for the observable sector below \( M_P \equiv M_{Planck}/\sqrt{8\pi} \approx 2.4 \times 10^{18} \text{ GeV} \) consists of a global supersymmetric theory with SSB terms as in eq. (1). In the minimal supergravity model, which we assume here, the Kähler potential is flat and one finds that the SSB parameters have universal values at \( M_P \) [1], i.e.,

\[ m_i^2 \equiv m_0^2, \ B_{ij} \equiv B_0 \mu_{ij}, \ A_{ijk} \equiv A_0 Y_{ijk}, \ M_\alpha \equiv M_{1/2}, \]

where \( \mu_{ij} \) and \( Y_{ijk} \) are respectively the bilinear and trilinear couplings in the superpotential. The deviations from the universal boundary condition (2) at lower scales are calculated using renormalization group (RG) methods, and given only four soft parameters one can predict the superpartner mass spectrum.

If the MSSM is embedded in a grand-unified theory (GUT) at the scale \( M_G \approx 10^{16} \text{ GeV} \) suggested by coupling constant unification [2], then the evolution of the parameters between \( M_P \) and \( M_G \) depends on the GUT and is strongly model dependent. Nevertheless, it is often assumed that applying (2) at \( M_G \) rather than at \( M_P \) is a good approximation, because \( M_G \) is close to \( M_P \). One then uses the generic MSSM RG equations (RGEs) between \( M_G \) and the weak scale [3,4].

In this letter we will examine the corrections to the SSB parameters arising from their evolution between the Planck and the GUT scales. We will show that these corrections
induce large deviations in the SSB parameters from their universal values. The corrections are typically proportional to $\frac{\alpha}{\pi} m^2 \ln M_P/M_G$ where $\alpha$ and $m^2$ are a generic coupling and soft mass parameter, respectively. Although these corrections are not enhanced by large logarithms, they can be significant due to:

1. The number of particles above $M_G$, $N$, is large as a result of the large symmetry group, and one roughly has $\frac{\alpha}{\pi} \rightarrow \frac{N\alpha}{\pi}$. (See also Ref. [5].)

2. Large Yukawa couplings that are typically present in GUTs and that grow with the energy. In addition to the large top Yukawa coupling, one has to introduce extra large couplings to avoid a too large proton decay rate.

Corrections from the gauge sector $[\times \frac{\alpha_G}{\pi} M_{1/2}^2 \ln M_P/M_G]$ can also be important for large $M_{1/2}$ or if $\alpha_G$ grows with the energy (as in non-minimal GUTs). The above corrections depend on the details of the GUT model and represent uncertainties in the low-energy predictions. Gravitational and other effects could also affect the boundary condition (2) and would only add to the uncertainty.

For definiteness and simplicity we consider the minimal SU(5) model. We will comment on extended models below. The Higgs sector of the model consists of three supermultiplets, $\Sigma(24)$ in the adjoint representation [which is responsible for the breaking of SU(5) down to SU(3)$_c \times$ SU(2)$_L \times$ U(1)$_Y$], and $\mathcal{H}_1(\bar{5})$ and $\mathcal{H}_2(\bar{5})$, each containing a SU(2) doublet $H_i$ and a color triplet $H_{Ci}$. The matter superfields are in the $\bar{5} + 10$ representations, $\phi(\bar{5})$ and $\psi(10)$. The superpotential is given by

$$W = \mu_\Sigma \text{tr} \Sigma^2 + \frac{1}{6} \lambda' \text{tr} \Sigma^3 + \mu_H \mathcal{H}_1 \mathcal{H}_2 + \lambda \mathcal{H}_1 \Sigma \mathcal{H}_2$$

$$+ \frac{1}{4} h_i \epsilon_{ijklm} \psi^{ij} \psi^{kl} \mathcal{H}_2^m + \sqrt{2} h_i \psi^{ij} \phi_i \mathcal{H}_1. \quad (3)$$

---

1 We define $\Sigma = \sqrt{2} T_a w_a$ where $T_a$ are the SU(5) generators with $\text{tr} \{T_a T_b\} = \delta_{ab}/2$, and we only consider Yukawa couplings for the third generation.
In the supersymmetric limit $\Sigma$ develops a vacuum expectation value $\langle \Sigma \rangle = \nu_\Sigma \text{diag}(2,2,2,-3,-3)$ and the gauge bosons $X$ and $Y$ get a mass $M_Y = 5g_G\nu_\Sigma$. In order for the Higgs SU(2) doublets to have masses of $O(m_Z)$ instead of $O(M_G)$, the fine-tuning $\mu_H - 3\lambda\nu_\Sigma \lesssim O(m_Z)$ is required and one obtains $M_{H_C} = \frac{\lambda}{g_G} M_Y$. Dimension-five operators induced by the color triplet give large contributions $\propto 1/M_{H_C}^2$ to the proton decay rate [6].

To suppress such operators, the mass of the color triplets has to be large, $M_{H_C} \gtrsim M_Y$, implying $\lambda \gtrsim g_G \approx 0.7$. Thus, one-loop corrections proportional to $\lambda$ produce important effects. Below $M_P$ the effective lagrangian also contains the SSB terms

$$
-\mathcal{L}_{so,ft} = m_{\phi}^2 |H_1|^2 + m_{\phi}^2 |H_2|^2 + m_{\phi}^2 |\Sigma|^2 + m_{\phi}^2 |\phi|^2 + m_{\phi}^2 |\psi|^2 + \sum_{ijkl} A_{ijkl} \phi^{ji} \psi^{kl} H_1^m + \sum_{ijkl} B_{ijkl} \phi^{ji} \psi^{kl} H_2^m + \sum_{ijkl} C_{ijkl} \phi^{ji} \psi^{kl} \Sigma^m 
$$

From $M_P$ to $M_G$ the SSB terms evolve according to the RGEs of the SU(5) model with eq. (2) as a boundary condition. Thus, we expect a breakdown of universality at $M_G$ for SSB parameters of fields that are in different SU(5) representations. The SU(5) RGEs for the SSB parameters and Yukawa couplings are given by

$$
\begin{align*}
\frac{dm_{H_1}^2}{dt} &= \frac{1}{8\pi^2} \left[ 3h_1^2 (m_{H_1}^2 + 2m_{H_2}^2 + A_1^2) + 2h_0^2 (m_{H_1}^2 + m_{H_2}^2 + A_0^2) - \frac{72}{5} g_G^2 M_5^2 \right], \\
\frac{dm_{H_2}^2}{dt} &= \frac{1}{8\pi^2} \left[ 3h_2^2 (m_{H_2}^2 + 2m_{H_1}^2 + A_2^2) + 2h_0^2 (m_{H_2}^2 + m_{H_1}^2 + A_0^2) - \frac{48}{5} g_G^2 M_5^2 \right], \\
\frac{dm_{\phi}^2}{dt} &= \frac{1}{8\pi^2} \left[ 3h_\phi^2 (m_{\phi}^2 + 2m_{\theta}^2 + A_\phi^2) + 2h_0^2 (m_{\phi}^2 + m_{\theta}^2 + A_0^2) - \frac{48}{5} g_G^2 M_5^2 \right], \\
\frac{dm_{\psi}^2}{dt} &= \frac{1}{8\pi^2} \left[ 3h_\psi^2 (m_{\psi}^2 + 2m_{\phi}^2 + A_\psi^2) + 2h_0^2 (m_{\psi}^2 + m_{\phi}^2 + A_0^2) - \frac{48}{5} g_G^2 M_5^2 \right], \\
\frac{d\lambda}{dt} &= \frac{\lambda}{16\pi^2} \left[ \frac{63}{20} \lambda^2 + 3\lambda^2 - 30g_G^2 \right], \\
\frac{d\lambda}{dt} &= \frac{\lambda}{16\pi^2} \left[ \frac{21}{20} \lambda^2 + 3h_1^2 + 4h_0^2 + \frac{53}{5} \lambda^2 - \frac{98}{5} g_G^2 \right], \\
\frac{dh_1}{dt} &= \frac{h_1}{16\pi^2} \left[ h_1^2 + 4h_2^2 + \frac{24}{5} \lambda^2 - \frac{96}{5} g_G^2 \right], \\
\frac{dh_2}{dt} &= \frac{h_2}{16\pi^2} \left[ h_2^2 + 4h_1^2 + \frac{24}{5} \lambda^2 - \frac{84}{5} g_G^2 \right], \\
\end{align*}
$$

where $t = \ln Q$. The RGE for the gauge coupling is $d\alpha_G/dt = -3\alpha_G^2/2\pi$, and similarly $dM_5/dt = -3\alpha_G M_5/2\pi$. The RGEs for the trilinear SSB parameter $A_i$ can be obtained from the RGEs of the corresponding Yukawa coupling $Y_i$ by
\[ \frac{dY_i}{dt} = \frac{Y_i}{16\pi^2} \left[ a_{ij}Y_j^2 - b_{ij}^2 \right] - \frac{dA_i}{dt} = \frac{1}{8\pi^2} \left[ a_{ij}Y_j^2 A_j - b_{ij}^2 M_5 \right]. \]  

We can omit the RGEs for \( \mu_\Sigma, \mu_H, B_\Sigma \) and \( B_H \), which are arbitrary parameters that decouple from the rest of the RGEs.

The evolution of the SSB parameters from \( M_P \) to \( M_G \) is dictated by a competition between the positive Yukawa terms (i.e., scalar contributions) and the negative gauge terms (i.e., gaugino contributions) in the RGEs. We can distinguish two scenarios: (A) For moderate values of \( M_{1/2} \equiv M_5(M_P) \) the contribution from the gauge sector is small. In this case, the RGEs of \( m_{H_1}^2 \) and \( m_{H_2}^2 \) have a large contribution proportional to \( \lambda^2 \) and both masses are diminished as the energy scale decreases. For \( h_t \gg h_b, m_{H_2}^2 \) decreases faster than \( m_{H_1}^2 \), but also \( m_{10}^2 \) (for the third family) is diminished in that case. (B) For large values of \( M_{1/2} \) the RGEs are dominated by the negative gaugino contribution so that all the SSB parameters increase as the energy scale decreases. The scalar masses are enhanced by an additive factor

\[ \Delta m_i^2 = -\frac{c_i}{3} \left[ 1 - \frac{1}{\left(1 + \frac{3\alpha_{SU(5)}}{2\pi} \ln \frac{M_5}{M_P} \right)^2} \right] M_{1/2}^2, \]  

where \( c_i = \frac{24}{10} \left( \frac{24}{5} \right) \) for \( i \) in the \( 10(5) \) representation. One has \( \Delta m_i^2 \approx 0.5(0.3)M_{1/2}^2 \).

Examples of scenarios A and B are given in Figs. 1a and 1b, respectively. We see that the violation of the universality of the SSB parameters at \( M_G \) can be substantial. In particular, the soft masses of the Higgs fields are typically split from the matter field masses. For \( M_{H_2} = 1.4 M_Y \) (i.e., \( \lambda \approx 1 \) at \( M_G \)) the splitting can be as large as 100%.

In order to analyze the implications of these soft mass splittings in the supersymmetric spectrum and phenomenology, we have to run the SSB parameters from \( M_G \) down to \( m_Z \) [7]. Below \( M_G \) the effective theory corresponds to the MSSM:

\[ W = \mu H_1 H_2 + h_t Q H_2 U + h_b Q H_1 D + h_\tau L H_1 E, \]  

where \( Q(L) \) and \( U, D(E) \) are respectively the quark (lepton) \( SU(2) \) doublet and singlet superfields. The tree-level matching conditions of the SSB parameters between the \( SU(5) \) model and the MSSM are \( m_{H_1}^2(M_G) = m_{H_2}^2(M_G), m_{10}^2(M_G) = m_{Q,U,E}^2(M_G) \) and
\[ m_5^2(M_G) = m_{D,L}^2(M_G). \] One-loop matching conditions will be considered below. In this letter we present only a qualitative analysis of the GUT effects in the low energy quantities. A comprehensive numerical study, together with the details of the numerical procedures, will be given elsewhere.

In scenario A the parameters \( m_{H_i}^2 \) have substantial shifts and the parameters \( m_{H_i}^2(m_Z) \) are modified. The latter enter the minimization conditions of the weak-scale Higgs potential. Therefore, corrections to \( m_{H_i}^2 \) modify the region of the MSSM parameter space that is consistent with electroweak symmetry breaking (EWSB). The \( \mu \) parameter is also affected by the GUT corrections to \( m_{H_i}^2 \), since it is extracted from the minimization conditions of the weak-scale Higgs potential [1,3,4]. This leads to corrections to observables that depend on \( \mu \), such as the Higgsino mass, the Higgsino-gaugino mixing and the left-right scalar quark mixing. Also, consistency with constraints from color and charge breaking [4] can be affected. A priori, one would expect that the Higgs boson masses are also affected. Notice, however, that the latter depend only on the sum \( m_{H_i}^2 + |\mu|^2 \), which is not modified significantly (the shift in \( m_{H_i}^2 \) is approximately compensated by the shift in \( |\mu|^2 \)). The masses of the lightest chargino and neutralinos are typically proportional to \( M_{1/2} \), and are only slightly affected. There are also large deviations for the scalar quark masses of the third generation. As mentioned above, due to the evolution from \( M_P \) down to \( M_G \), \( m_{10}^2 \) can be shifted for large \( h_t \) (e.g., \( \tan \beta = \langle H_2 \rangle / \langle H_1 \rangle \approx 1 \)) from the universal value \( m_5^2 \) (see Fig. 1a). In addition, the evolution of \( m_Q^2 \) and \( m_U^2 \) from \( M_G \) down to \( m_Z \) depends on the value of \( m_{H_i}^2 \) that is sensitive to the GUT physics.

In scenario B, where the gaugino contribution is dominant, all the scalar masses are shifted [eq. (7)] from their universal value at \( M_P \). The scalar quark squared masses, however, receive large corrections in the running from \( M_G \) down to \( m_Z \) (\( \approx 6 M_Z^2 \)) so that the amount (7) represents only a few percent of their values. This is not the case for the scalar leptons, where such corrections are much smaller [\( \approx 0.5(0.1) M_{1/2} \) for \( \bar{e}_L(\bar{e}_R) \)] and the increment (7) can even double their masses.

In Figs. 2 and 3 we present examples of the GUT effects in the low-energy predictions. In
Fig. 2 we take a large tan $\beta$ ($\tan \beta = 42$) and show the allowed values (i.e., consistent with EWSB and experimental bounds) of $\mu$ vs. the gluino mass, $M_{\tilde{g}}$, when the evolution from $M_P$ to $M_G$ is considered (triangles) or neglected (filled circles). The correlation between $\mu$ and $M_{\tilde{g}}$, which exists when neglecting the $M_P$ to $M_G$ evolution [3,4], is “smeread” when that evolution is included, and $\mu$ is larger in the latter case. We also find that $m_{H_1}^2 (m_Z)$ are often both negative when the GUT effects are considered [$m_{H_1}^2 (M_G)$ are diminished together because $h_t \approx h_b$], which is inconsistent with EWSB. The allowed parameter space is then significantly reduced. In Fig. 3 we show the light $t$-scalar mass $m_{\tilde{t}_1}$ vs. $M_{\tilde{g}}$ for tan $\beta \approx 1$ ($h_t \approx 1$). $\mu$ is now large ($\sim 1$ TeV) and is less sensitive to the GUT effects. Corrections to $m_{\tilde{t}_1}$ are mainly via the diminished $h_t^2 m_{H_2}^2$ term in the respective RGEs below $M_G$. $\tilde{t}_1$ is therefore heavier and some points which correspond to a tachionic $t$-scalar and are excluded when the $M_P$ to $M_G$ evolution is neglected, can be allowed. Note also that the correlation between $m_{\tilde{t}_1}$ and $M_{\tilde{g}}$ is weakened by the GUT corrections. We find that correlations between predictions are genericly modified due to the model-dependent “smeread” from the $M_P$ to $M_G$ evolution. The correlation between $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ is, however, strengthened because of the heavier $\tilde{t}_1$. Figs. 2 and 3 correspond to scenario B (A) for very large (small to moderate) values of $M_{\tilde{g}}$.

Even if the universal boundary condition (2) for the SSB parameters is taken at $M_G$, there is some arbitrariness in the value of $M_G$ due to mass-splittings between the particles at the GUT scale, i.e., threshold effects. We will distinguish three categories of corrections (details will be given elsewhere). First, we consider logarithmic threshold corrections arising from the mass splitting between different heavy superfields. Threshold corrections to Yukawa and gauge couplings are discussed, for example, in Ref. [8]. The largest contributions to the SSB parameters arise from the SU(2) triplet and singlet components of the $\Sigma$ superfield, $\Sigma_3$ and $\Sigma_1$, and are given by (we identify $M_G = \max \{M_V, M_{H_C}\}$)

$$m_{H_1}^2 (M_G) = m_{H_1}^2 (M_G) + \frac{\lambda^2}{4\pi^2} (m_{H_1}^2 + m_{H_2}^2 + m_\Sigma^2 + A_1^2) \left[ \frac{3}{4} \ln \frac{M_\Sigma^4}{M_G} + \frac{3}{20} \ln \frac{M_\Sigma^2}{M_G} \right] , \quad (9a)$$
\[ \Delta A_{\ell b}(M_G) = \frac{\lambda^2}{4\pi^2} A_\lambda \left[ \frac{3}{4} \ln \frac{M_{\Sigma_3}}{M_G} + \frac{3}{20} \ln \frac{M_{\Sigma_1}}{M_G} \right]. \quad (9b) \]

Since the masses \( M_{\Sigma_3} \) and \( M_{\Sigma_1} \approx 0.2M_{\Sigma_3} \) can be much smaller than \( M_G \), these corrections can be substantial. For \( M_{\Sigma_3} \approx 10^{-2}M_G \) and \( \lambda \approx 1 \), we have \( m_{H_i}^2(M_G) \approx 0.6m_\text{soft}^2 \). The second type of threshold effects are logarithmic corrections due to the boson-fermion mass splitting within a supermultiplet. Such corrections are suppressed by powers of \( m_\text{soft}/M_G \) in the Yukawa and gauge coupling boundary conditions, but there is no such suppression for the SSB mass terms, e.g., corrections to \( m_{H_i}^2 \) are \( \sim M_G^2 \ln \left[ (M_G^2 + m_\text{soft}^2)/M_G^2 \right] \sim m_\text{soft}^2 \).

Keeping only the terms of \( O(\lambda^2/4\pi^2) \), we have
\[
m_{H_i}^2(M_G) = m_{H_i}^2(M_G) + \frac{\lambda^2}{4\pi^2} \left[ \frac{18}{20} (m_{H_1}^2 + A_\lambda B_{\Sigma}) + \frac{6}{8} (m_{H_1}^2 + m_{H_2}^2 + 2A_\lambda B_{H}) \right], \quad (10)
\]
where the first term comes from corrections of the \( \Sigma_3 \) and \( \Sigma_1 \) particles, while the second term from corrections of the Higgs color triplets. This represents a \( O(10\%) \) correction. Lastly, there are scheme-dependent finite one-loop corrections. In the dimensional-reduction scheme they are given by
\[
m_{H_i}^2(M_G) = m_{H_i}^2(M_G) - \frac{\lambda^2}{4\pi^2} \frac{24}{20} (m_{H_1}^2 + m_{H_2}^2 + m_\Sigma^2 + A_\lambda^2). \quad (11)
\]
Notice that the corrections (10) tend to cancel the corrections (11) for equal SSB parameters. From (9) - (11) one expects an additional \( O(40\%) \) common correction to \( m_{H_i}^2(M_G) \) that would induce \( O(5 - 20\%) \) uncertainties in low-energy predictions.

In extended supersymmetric GUTs one expects the corrections to be larger. If large representations are introduced, the positive scalar contribution to the RGEs is larger and therefore the SSB parameters decrease faster with the scale. However, one has to be aware of a possible breakdown of perturbation theory. An interesting scenario occurs in models in which \( H_1 \) and \( H_2 \) couple with different strength to the other Higgs supermultiplets. For example, in the missing partner SU(5) model \( W = \lambda_1 H_1 \Sigma(75)\Phi(50) + \lambda_2 H_2 \Sigma(75)\Phi(50) + \ldots \), and if \( \lambda_2 > \lambda_1 \), the evolution from \( M_P \) to \( M_G \) splits the two Higgs scalar masses. That splitting can now affect the low-energy Higgs boson masses and reduce the degree of fine-tuning that is typically required to achieve EWSB in scenarios with large \( \tan \beta \) (in which
the Higgs masses are not split by Yukawa interactions). If we enlarge the symmetry group, the negative term in the RGEs coming from the gaugino contribution is enhanced and can partially cancel the scalar contribution. In models where the rank of the group is larger than the rank of the SM group, e.g., SO(10), we have an additional contribution to the scalar masses that arises from the D-terms [9].

To summarize, we have shown that large deviations from universality at $M_G$ can be generated when considering (i) the model-dependent evolution from $M_P$ to $M_G$ and (ii) threshold corrections at $M_G$ (including those from scalar-fermion splittings). We have also shown that the above leads to a modification of the allowed parameter space, smears predicted correlations and affects certain low-energy predictions such as the $\mu$ parameter and the $t$-scalar mass. These corrections have to be considered as uncertainties when analyzing possible future evidence for supersymmetry. On the other hand, such corrections could provide a probe of the high scale.

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FIG. 1. The evolution of the soft mass parameters of the third family, $\phi(\bar{5})$ and $\psi(10)$, the Higgs, $H_i$, and the gaugino between the Planck and grand-unification scales for: (a) scenario A, with $m_0 = A_0 = 400$ GeV and $M_{1/2} = 100$ GeV; and (b) scenario B, with $m_0 = A_0 = 50$ GeV and $M_{1/2} = 450$ GeV. In both cases $m_t = 160$ GeV; $\tan \beta = 1.25$; and the boundary conditions $\lambda = 1$ (i.e., $M_{H_C} = 1.4 M_V$) and $\lambda' = 0.1$ at $M_G$ are assumed. All masses are in GeV.
FIG. 2. Scatter plot of the $\mu$ parameter v.s. the gluino mass within the allowed parameter space for $m_t = 180$ GeV and $\tan \beta = 42$. Triangles (filled circles) correspond to universality [eq. (2)] at the Planck (grand-unification) scale. $\lambda$ and $\lambda'$ are as in Fig. 1. All masses are in GeV.
FIG. 3. Same as in Fig. 2 except the light $t$-scalar mass $m_{\text{stop}}$ vs. the gluino mass, and $m_t = 160$ GeV and $\tan \beta = 1.25$. 