Virtual manipulatives media in mathematical abstraction

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Abstract. By the characteristics of mathematics, which has the object of study is abstract, then the abstraction becomes an important part that can not be separated from mathematics. The development of abstraction theories, either directly or not, will be closely related to the process of learning mathematics in the classroom. Also, the rapid development of virtual manipulatives media in the world of mathematics education can bring a good influence on the learning process that takes place in the classroom. In this regard, this paper aims to examine theories of abstraction in mathematics and its possible implementation in mathematics education which is integrated with virtual manipulatives media to provide a choice to overcome various problems of learning mathematics. In the world of mathematics education, some abstract concepts in mathematics will be more easily learned through empirical abstraction processes such as introducing the concept of positive integers in students. However, when abstract concepts are not allowed to be empirically introduced to the students, the role of virtual manipulatives media can be one of the alternatives to choose from. Also, virtual media manipulatives can be utilized to facilitate the process of reflective abstraction and theoretical abstraction.

1. Introduction
Abstraction is a fundamental process in both mathematics and mathematics education. Abstraction has been known as something that plays an important role in the success of learning mathematics when viewed from a cognitive point of view. However, abstraction is also one of the main reasons for the failure of the mathematics learning process [1]. The phenomenon, not without reason. If viewed from the characteristics of mathematics, which has the object of study is abstract, then it takes a process that can bring students to understand the abstract things in a learning process. Simply put in the context of mathematics education, abstraction can be interpreted as a process of studying ideas, objects, or abstract concepts.

Abstract concepts in mathematics can be learned through the process of empirical abstraction and mathematical abstraction. The process of abstraction takes place through a series of learning activities that involve various aspects of learning [2]. One of the important aspects of the learning process of mathematics is the tools in learning.

According to Namukasa, Stanley & Tuchtie [3], the use of tools in learning more can enable students in learning more. Also, the use of tools in learning in addition to improving student understanding and problem-solving skills can improve students' positive attitudes toward mathematics [4]. A meaningful learning activity, as well as cognitive tools, can actively increase student involvement in the learning process and generate reflection on the concepts and relationships under study.
In the context of mathematics, concrete experience is not defined by its physical form or its characteristics in the real world but by how much its relevance to other mathematical ideas or situations. For example, when a child begins to learn about numbers, he is learning a process of counting. The later "circumstances" are represented in the symbols of numbers 1, 2, 3, and so on. The numbers 1, 2, 3, and so on are not the main objects studied in numbers. The main concept is the invisible relationship between a quantity and the unit used to measure that magnitude [5].

To explain the concept needed tools help. Along with the development of technology, then the form of tools also change. The emergence of tools in the form of virtual manipulatives media one of the main components is the computer, providing convenience for people who are engaged in the world of mathematics or mathematics educators. Virtual manipulatives media can provide an interactive environment so that students can apply or solve mathematical problems to form linkages between mathematical concepts and mathematical operations and can quickly obtain a quick response from their activities. Also, virtual manipulatives media can be used in mathematical abstraction processes, which will be more effective and efficient when using software that is widely available for learning mathematics, such as the concept of drawing graphs, the concept of irrational numbers that are not easy to be taught empirically.

2. Method
This research is a preliminary study, so it does not intend to assess the effectiveness of teaching aids in learning. This preliminary study is a defining stage [6], and this section is part of Determining, Designing, developing, and encumbering or 4D [7]. At the defining stage, it is done to define and define development requirements such as needs analysis of Virtual manipulative media.

3. Result And Discussion

3.1 Abstraction in Mathematics Education
One of the characteristics of mathematics is to have an abstract object of study. So often, math lessons are referred to as theoretical lessons. The object of abstract study may be ideas, ideas, concepts, and relationships [2]. The mathematical object is also called abstract-apart [8]. To be able to understand the various ideas, ideas, concepts, and relationships are required a certain activity or process, the activity or certain is referred to an abstraction. Abstractions can be distinguished into empirical abstractions and theoretical abstractions [8].

In the process of empirical abstraction, the formation of the notion of an abstract object is based on empirical experience. One example is the concept of abstraction delivered by Skemp. According to Skemp, abstractions consist of the same characteristic recognition process followed by the incarnation of similarity into a new mental object. The activity of abstraction begins with a sensitivity to the existence of the same characteristics of the experiences it possesses, then the similarities it obtains serve as the basis for classification. So abstraction is a long-lasting change, as a result of abstraction activity that allows us to recognize new experiences.

Still, in line with Skemp, Piaget also put forward the concept of empirical abstraction and quasi-empirical abstraction. Both abstraction processes are based on the social and physical experience of the child. Since empirical abstraction has a focus on the process of identifying commonly important displays, the concepts resulting from empirical abstraction processes are referred to as abstract-general [8].

Contrary to empirical abstraction, the idea of theoretical abstraction comes from two soviet-based psychologists Vygotsky and Davydov. Theoretical abstraction consists of the formation of concepts to suit some theories. Vygotsky makes a difference in the meaning of concepts in the context of everyday life with concepts in the context of the scientific field. According to Vygotsky, the concept in the context of everyday life is formed through an empirical abstraction process. While the formation of scientific concepts consists of three aspects namely: The establishment of a system of relationships between concepts, A consciousness of one's mental activity, and Penetration into an essence of the object would enrich the reality presented in the concept, not the other way around [8].
A fairly clear example of the difference between empirical abstraction and theoretical abstraction in the learning process, especially geometry can be seen through the following examples:

In learning about the concept of lines, based on the theory of empirical abstraction then the process is, the child recognizes the various forms of representation of the line in advances such as line drawing, the trajectory of the former paper folds and others. In this process, the child recognizes the same characteristics of experiences with the real object. Although the same characteristics are only roughly linear and are produced from various contexts, yet from where the concept will be known.

While theoretical abstraction, the students are introduced in advance about the concept of dots and lines based on the definition. This is done to elicit a generalization process from spatial investigations (e.g., two lines intersecting at a single point). Then see the relationship between the two concepts to recognize the concept of the line.

From the example, it is clear the difference between empirical abstraction and theoretical abstraction. The flow of empirical abstraction processes and theoretical abstractions are different. In individual empirical abstractions form new concepts based on observation and experience whereas in theoretical abstractions new concepts are formed by matching concepts to experiences that have already been formed and stored first in individual thought.

Piaget's theory of reflective abstraction, which focuses on relationships between actions, can be categorized in terms of theoretical abstractions. Similarly, the theory of RBC abstraction model (Recognizing, Building-With, Construction) [9]. In the process of learning mathematics, there are actually three things that occur related to the process of abstraction experienced by students are: They learn an empirical concept, they learn about a mathematical object, and they learn about the relationship between empirical concepts and mathematical objects. Or conversely, they learn about mathematical objects, they learn about empirical concepts, and they learn about the relationship of both.

When the statement is scrutinized, it appears that although there is a difference between the concept of empirical abstraction and theoretical abstraction, both are an important and inseparable part of the learning process of mathematics. In the process of learning mathematics, both processes of abstraction must still occur. Some concepts in mathematics can indeed be easily taught through a process of empirical abstraction, such as the concept of positive numbers, angles, and wake-ups. But some concepts in mathematics such as Irrational numbers, Complex numbers, or when introducing the concept of x0 are empirically difficult to find the equivalent empirically. Based on this, both the empirical abstraction and theoretical abstraction, both fundamental processes in the process of learning mathematics [1].

3.2 Virtual Manipulatives Media

As a result of technological innovations, it also brought new changes in the world of tools as one of the media of mathematics learning that helps students in the real world and mathematics abstraction. This new type of tools learning-based computer technology called the concept of "virtual manipulatives media". Moyer, Bolyard & Spikell [10] defines that virtual manipulatives media is a visual representation of interactive, Web-based dynamic objects and enables us to construct mathematical knowledge. Meanwhile, according to Wikipedia site online, virtual manipulatives media is an interactive, web-based or computer-based tool as a visual representation medium of dynamic objects that allow being used to build math knowledge.

In addition to these two explicit definitions, Gaston & Spicer [11] suggests that there are two types of representations of the World Wide Web that are also called virtual manipulatives media, namely, static and dynamic representations of real props. If it is further examined from the virtual definition of existing media manipulatives, essentially virtual manipulatives media is an interactive program based on computer technology by utilizing visual representations of dynamic objects that can be manipulated as real objects to build mathematical knowledge.

3.3 The Role Of Virtual Media Manipulatives In Process Abstraction

The constructivism approach has dominated the mathematical education paradigm for two decades [12]. According to this understanding, learning should be a process of knowledge construction (including mathematical knowledge) through direct experience by an individual is not a mere transfer of knowledge from others. Thus, empirical abstraction becomes important by the paradigm of
constructivism. Based on this, then many elements of constructivism teaching are expected to trigger the process of abstraction. One of them is, methods involving many activities that activate exploration through hands-on activity by using concrete materials, or concrete media.

Some of the previous articles and literature on abstraction discuss only the interpretation of the definition of abstraction [1, 13, 14, 15, 16]; examine abstractions in the learning process of mathematics, it just does not specifically reveal the linkage of the use of tools in an effort to create potential conditions for the abstraction takes place in the learning process. On the other hand, Bond & Keane [17] who studied the process of spontaneously absorbing thought indirectly utilized virtual manipulatives media in the form of computer games under the name "Green Globs" to study the topic of linear equations. Similarly, Dreyfus et al. examine the process of student abstraction in learning algebra by utilizing a spreadsheet computer program.

When concrete media can provide substantial benefits in the empirical abstraction process, due to its characteristics that provide immediate experience for students, the existence of virtual manipulatives media can be used to assist students in performing theoretical abstractions or turning off the role of concrete media that is sometimes less efficient in its use [18]. This is because of the characteristics of virtual manipulatives media, which is a substitute for real experience, can eliminate the difficulty of using media concrete. Since abstraction is a vertical thinking activity organizing previously constructed mathematical knowledge into a new structure, the reorganization of such knowledge can be achieved through a series of actions concerning objects both mentally and materially.

In the model of RBC abstraction (Recognizing, Building-With, and Constructing) proposed by Dreyfus et al. [9] Recognizing phases or recognition of familiar mathematical structures occurs when a student realizes that the structure is embedded in a given mathematical situation. While the Building-With phase consists of merging artifacts to achieve a goal such as solving a problem or justifying a statement. The Constructing phase, which is the main phase of the abstraction process consists of combining various knowledge artifacts to produce a new structure, which then becomes known as a concept.

Virtual media manipulatives can play a role in creating a mathematical situation in the Recognizing and Building-With phases. For example, the use of DGS (Dynamic Geometric Software) in the process of geometry learning, a variety of flat visual representations that can be presented on the computer screen and can be manipulated by students provides a potential situation in discovering the various properties of the builds, from the empirical abstraction process as well as the Recognizing phase. Then, by performing manipulations that produce a rapid response from the work of the computer, the student can see the linkage between the manipulation done with the response obtained so that the potential to bring up the conjecture. Even at the same time, can test theory with the guidance of a teacher in doing exploration. These activities can be categorized included in the Building-With phase. Even in solving problems, DGS can also be used either as a tool to help solve problems or confirm answers quickly. So hopefully, the Construction phase can be achieved more effectively and efficiently.

To create a potential and effective situation for the process of abstraction takes place in the process of learning mathematics by utilizing virtual manipulatives media needs to be considered several aspects related to the various types of virtual manipulatives media available as well as differences in the characteristics of concepts in mathematics.

Some considerations to consider in choosing the types of virtual manipulatives media that can be used to create a potential situation in the learning process of mathematics, among others; the virtual props can be used for assessment as a reflection of the students' thinking process, the virtual manipulatives of the media provide opportunities for students to vary or reflect on their actions so that the potential to make students predict or provide explanations, the aids provide the possibility of creating gaps in the process thinking, the virtual manipulatives of the media provides a link between images and mathematical symbols, the virtual manipulatives of such media are easy to operate.

The selection of virtual manipulatives media that is not appropriate in the process of learning mathematics can cause the learning process is inefficient or even not effective at all. It should also be noted the suitability between the types of virtual manipulatives media and mathematical concepts that will be taught, do not forget to note also the allocation of available time and availability and condition
of existing media. For online props, it is necessary to note also the connectivity of the network and the ease of accessing the sites available.

4. Conclusions
Abstraction is a fundamental process in the process of learning mathematics related to the formation of concepts. Therefore, in every process of learning, mathematics is always expected to raise the process of abstraction. To create a situation that allows students to be abstract, it is necessary to design an adequate learning strategy. One of the most important components in the learning process of mathematics is the existence of props to bridge the abstractions of mathematical concepts. As technology advances, the concept of visual aids evolves into virtual manipulatives media that have more abstract characteristics than just concrete props. The appearance of these props, potentially to help create a potential situation for the ongoing process of abstraction in mathematics learning. However, the utilization of virtual manipulatives media in triggering the emergence of abstraction needs to pay attention to several aspects to make the process more effective and efficient. This theoretical study sparks further questions about the appropriate types of cyber paraphernalia to teach certain concepts related to the process of abstraction. Further studies are needed to see further how the role of virtual media manipulatives in the learning process of mathematics.

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