The evolution of the nonsinglet twist-3 parton distribution function

B. Geyer, D. Müller

Fakultät für Physik und Geowissenschaft der Universität Leipzig,
Augustusplatz, D-04109 Leipzig, Germany

D. Robaschik

DESY-IfH,
Platanenallee 6, D-15738 Zeuthen, Germany

Abstract

The twist three contributions to the $Q^2$-evolution of the spin-dependent structure function $g_2(x_{Bj}, Q^2)$ are considered in the non-local operator product approach. Defining appropriate twist three distribution function we derive their evolution equation for the nonsinglet case in leading order approximation. In the limit $x_{Bj} \rightarrow 1$ as well as in the large $N_c$ limit we confirm the result that the evolution of the nonsinglet part of $g_2$ is governed by a Gribov-Lipatov-Altarelli-Parisi equation.

---

1 An earlier version of this paper has been presented at the “3rd Meeting on the Prospects of Nucleon-Nucleon Spin Physics at HERA”, JINR Dubna, 28-29.6.1996.
1 Introduction

Recently, in deep inelastic scattering the first moments of the polarized structure function $g_2(x_{Bj}, Q^2)$ are measured \[1\]. In leading order of the momentum transfer $Q^2$ this structure function is determined by twist two as well as twist three contributions. In comparison with the twist two case the leading order analysis for the twist three part is more subtle due to the appearance of a set of operators mixing under renormalization and constrained by relations between themselves.

Up to now there exist already several papers which determine the local anomalous dimensions \[2, 3, 4\] and the evolution kernels for the distribution functions as well as nonlocal operators \[2, 5, 6\]. In the nonsinglet sector the one-loop result \[6\] for the evolution kernel of light-ray operators was confirmed in \[7\]. It has been checked that the local anomalous dimensions coincide with the results given in \[2, 4\].

The renormalization properties of twist three operators indicate that the evolution equation for the distribution function is more complicated than the Gribov-Lipatov-Altarelli-Parisi (GLAP) equation governing the evolution of twist two parton distribution functions. However, for the nonsinglet case the solution of this equation is known in both limits $N_c \to \infty$ and $x_{Bj} \to 1$ \[10\].

Here we introduce a new twist three distribution function defined in terms of three-particle operators and we give first results about the evolution in the flavor nonsinglet sector. This distribution function depends on an effective momentum fraction and on the position of the gluon field. The evolution is governed by an extended GLAP equation. The moments are derived from twist three operators with definite spin and so that they do not mix with each other. The remaining mixing problem can be treated numerically for the first few moments. In the limit $N_c \to \infty$ this equation reduces to an evolution equation of the GLAP type.

2 Nonlocal operator product expansion

The antisymmetric part (with respect to Lorentz indices) of the hadronic tensor

$$W_{\mu\nu}^A = \epsilon_{\mu\nu\lambda\delta} q^\lambda \left[ \frac{S^\delta}{qP} g_1(x_{Bj}, Q^2) + \frac{qP S^\delta - qS P^\delta}{(qP)^2} g_2(x_{Bj}, Q^2) \right]$$

being relevant for polarized deep inelastic scattering is given by the structure functions $g_1(x_{Bj}, Q^2)$ and $g_2(x_{Bj}, Q^2)$ depending on the Bjorken variable $x_{Bj}$ and the momentum transfer $q$, $q^2 = -Q^2$. The polarization vector of the proton is defined as $S^\delta = \bar{u}(P,S)\gamma^\delta\gamma^5 u(P,S)$. Our conventions are $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and $\epsilon_{0123} = 1$. The hadronic tensor is determined by the absorptive part $W_{\mu\nu}$ of the virtual forward Compton amplitude $T_{\mu\nu}$,

$$T_{\mu\nu} = \frac{i}{\pi} \int d^4x e^{ixq} \langle P,S | T j^\mu(x/2) j^\nu(\bar{x}/2) | P,S \rangle.$$  \hspace{1cm} (2)

In the Bjorken region the leading terms in $Q^2$ of the Compton amplitude correspond to the leading light-cone singularities in the coordinate space so that the light-cone expansion for operator products (OPE) can be applied. This provides the factorization of the structure function in a perturbatively determined coefficient function and a nonperturbative parton distribution function.
The leading order analysis starts with a perturbative investigation of the time ordered product of two electromagnetic currents. (In the following we neglect flavor and color indices):

\[ \{ T j^\mu(x) j^\nu(y) \}^{as} = \epsilon^{\mu\nu\rho\sigma} \partial^\rho D^\sigma (x, y) \bar{\psi}(x) \gamma_\sigma \gamma_5 U(x, y) \psi(y) + (x, \mu \leftrightarrow y\nu) + \cdots, \]

where the path ordered phase factor \( U(x, y) = P \exp \left\{ -ig \int_y^x dw^\mu A_\mu(w) \right\} \) ensures gauge invariance. For simplicity we set \( y = 0 \).

Technically, we apply the light-cone expansion \([11, 12]\) proved by Anikin and Zavialov in renormalized quantum field theory. Heuristically, this nonlocal OPE is obtained by approximating the vector \( x \) by the light-like vector \( \tilde{x} \), defined as \( x = \tilde{x} + a(x, \eta) \eta \), where \( \eta \) denotes a fixed auxiliary vector (e.g. normalized by \( \eta^2 = 1 \)) and \( a \) the corresponding coefficient. In leading order we substitute \( x \to \tilde{x} \) whereas the \( x^2 \)-singularities remain unchanged, i.e., \( x^2 \to \tilde{x}^2 \). In general, the coefficient function will depend on two auxiliary variables \( \kappa_i \), whose range (according to the \( \alpha \)-representation of the contributing Feynman diagrams) is restricted by \( 0 \leq \kappa \leq 1 \). Here we introduce these variables quite trivially through integration over two \( \delta \)-functions. In this intuitive way we get the following light-cone expansion:

\[ \{ T j^\mu(x) j^\nu(0) \}^{as} = \epsilon^{\mu\nu\rho\sigma} \int_0^1 dk_1 \int_0^1 dk_2 \delta(k_1 - 1) \delta(k_2) i \partial_\rho D^\sigma (x) \{ O_\sigma(k_1, k_2) + (k_1 \leftrightarrow k_2) \}, \]

where the path ordered phase factor \( \bar{\psi}(1) \gamma_\sigma \gamma_5 U(1, \tilde{x}, \kappa_2 \tilde{x}) \psi(\kappa_2 \tilde{x}). \)

It turns out (for more details see \([7]\)) that for an application to forward scattering the twist-3 part can be written as

\[ O^\mu_\rho \kappa_1, k_2 = \tilde{x}^\rho O_\rho(k_1, k_2) = \bar{\psi}(1) \tilde{x} \gamma_5 \psi(\kappa_2 \tilde{x}). \]

Using the results of the OPE \([11, 12]\) it is straightforward to obtain the structure function in terms of parton distribution functions. As is well known the longitudinal structure function \( g_1 \) reads

\[ g_1(x_{Bj}, Q^2) = \frac{1}{2} \sum_{q=u,d,..} e^2 q \Delta q_q(x_{Bj}, Q^2). \]

The quark distribution functions (containing quark and antiquark contributions) are defined as matrix elements of leading twist-2 operators,

\[ \Delta q_q(x, Q^2) = \int \frac{d\kappa}{2\pi(\kappa \tilde{x})} \left\langle P, S \left| O_{tw2}(0, \kappa) + (\kappa \to -\kappa) \right| P, S \right\rangle e^{ix\kappa(\tilde{x}P)}, \]

where the renormalization point of the operator is set equal to the momentum transfer \( Q^2 \). The leading order analysis \([8, 9]\) suggests that the structure function \( g_2 \) can be written as

\[ g_2(x, Q^2) = -\bar{g}_2(x, Q^2) + \int_x^1 \frac{dy}{y} \bar{g}_2(y, Q^2) \]
where \( \tilde{g}_2(x, Q^2) = g_1(x, Q^2) + \tilde{g}_2(x, Q^2) \) is decomposed into the twist-2 part given by \( g_1 \) and a remaining twist-3 part \( \tilde{g}_2 \). The twist-3 contribution

\[
\tilde{g}_2(x_{Bj}, Q^2) = \frac{1}{2} \sum_{q=u,d,..} e^2_q \Delta \tilde{q}(x_{Bj}, Q^2),
\]

can be expressed by the distribution function

\[
\Delta \tilde{q}(x, Q^2) = \frac{1}{x} \int \frac{d\kappa}{2\pi i(xP)} S^\rho (P, S) \mathcal{A}_\rho(0, \kappa) - (\kappa \to -\kappa) \big| P, S \bigg\rangle e^{i\kappa(\vec{x}P)}.
\]

where, as will be seen below, unlike \( q_\rho(x, Q^2) \) this function has no simple parton interpretation.

### 3 Evolution kernels for twist-3 light-ray operators

The twist-3 operator \( \tilde{O}_\rho \) is not closed under renormalization. Indeed it will mix with three-particle operators which contain also the gluon field strength. Using the equation of motion, \((i \not\!D - m)\psi = 0\) the operator \( \tilde{O}_\rho \) can be decomposed with respect to gauge invariant twist-3 operators:

\[
\tilde{O}_\rho(\kappa_1, \kappa_2) = \frac{i}{2} (\kappa_2 - \kappa_1) \int_0^1 du \left\{ -2M_\rho(\kappa_1, \kappa_1 \bar{u} + \kappa_2 u) + u^+S_\rho(\kappa_1, \kappa_1 \bar{u} + \kappa_2 u, \kappa_2) \right. \\
+ \bar{u}^-S_\rho(\kappa_1, \kappa_1 \bar{u} + \kappa_2 u, \kappa_2) \bigg\} + \cdots
\]

Here \( \pm S_\rho \) are the nonlocal generalizations of the so-called Shuryak-Vainshtein operators \[13\]:

\[
\pm S_\rho(\kappa, \tau, \kappa_2) = ig\bar{\psi}(\kappa \bar{x}) \not\!F \left[i\not\!\bar{F}_\rho(\tau\bar{x}) \pm \gamma^5 F_\rho(\tau\bar{x}) \right] \bar{\psi}(\kappa_2 \bar{x}),
\]

where \( \not\!F_\alpha_\beta = \frac{1}{2}\epsilon_{\alpha_\beta\mu\nu} F^{\mu\nu} \) is the dual field strength tensor. Furthermore,

\[
M_\rho(\kappa_1, \kappa_2) = m \bar{\psi}(\kappa \bar{x}) \sigma_{\alpha\beta} \bar{x}^\alpha \gamma^5 (\bar{x} D)(\kappa_2 \bar{x}) \psi(\kappa_2 \bar{x}), \quad \sigma_{\alpha\beta} = \frac{i}{2}[\gamma_\alpha, \gamma_\beta]
\]

denotes a mass dependent operator. Besides the equation of motion operators we neglected also trace terms (proportional to \( \bar{x}^\rho \)) and operators which vanish in the forward case (from general principles \[13\] it is known that the equation of motion operators do not contribute to the evolution of physical matrix elements of gauge invariant operators).

For simplicity we will neglect the mass operator. Then the operators \( \pm S_\rho \) which are related to each other by charge conjugation are closed and, furthermore do not mix under renormalization. We calculated the evolution kernels for these operators in light-cone gauge using the Leibbrandt-Mandelstam prescription \[13\]. Since both kernels are related to each other we give only the result for \( -S_\rho \):

\[
\mu^2 \frac{d}{d\mu^2} S_\rho(\kappa_1, \kappa_2) = \frac{\alpha_s(\mu^2)}{4\pi} \int_0^1 dy \int_0^y dz \left\{ (2C_F - C_A) \left[ y \delta(z) S_\rho(\kappa_1 - \kappa_2 y, -\kappa_2 y) \\
- 2z S_\rho(-\kappa_1 y, -\kappa_2 - \kappa_1 z) + K(y, z) S_\rho(\kappa_1 \bar{y} + \kappa_2 y, \kappa_2 \bar{z} + \kappa_1 z) \right] \\
+ C_A \left[ (2\bar{z} + L(y, z)) S_\rho(\kappa_1 y, \kappa_2 - \kappa_1 z) + L(y, z) S_\rho(\kappa_1 - \kappa_2 z, \kappa_2 y) \right] \right\},
\]

\[
K(y, z) = \left[ 1 + \delta(z) \frac{\bar{y}}{y} + \delta(y) \frac{\bar{z}}{z} \right] +, \quad L(y, z) = \left[ \delta(1 - y - z) \frac{y^2}{\bar{y}} + \delta(z) \frac{y}{\bar{y}} \right] +, \quad \delta(\bar{y}) = \frac{7}{4}\delta(\bar{y}) \delta(z).
\]

To condense the notation we used \( \pm S_\rho(\kappa_1, \kappa_2) = \pm S_\rho(\kappa_1, 0, \kappa_2) \) and the standard plus-preservation fulfilling \( \int dy[.]_+ = 0 \) and \( \int dydz[.]_+ = 0 \), respectively; \( C_F = (N_c^2 - 1)/(2N_c) = 4/3 \) and \( C_A = N_c = 3 \) are the usual Casimir operators of \( SU_c(3) \).
4 Definition of twist-3 parton distribution functions

Since the twist-3 operators are in fact three-particle operators it is also necessary to extend the definition of the distribution function \([12]\). A quite natural way is to take the Fourier transform of the operators \(\hat{^3S^\rho(\kappa_1, \kappa_2)}\) sandwiched between polarized proton states. The resulting C-even distribution function

\[
\Delta \hat{Q}^{\text{NS}}(x_1, x_2) = \int \frac{d\kappa_1}{2\pi} \frac{d\kappa_2}{2\pi} \frac{S_\rho}{2(\bar{x}P)^2} \left( P, S \right) ^3S^\rho(\kappa_1, \kappa_2) + S^\rho(\kappa_2, \kappa_1) + \{\kappa_1 \leftrightarrow \kappa_2\} \left| P, S \right) \times e^{i\kappa_1 x_1 (\bar{x}P) + i\kappa_2 x_2 (\bar{x}P)},
\]

depends on the momentum fractions \(x_1, x_2\) and possesses the support property \(|x_i| \leq 1, |x_1 \pm x_2| \leq 1\). Unfortunately, the resulting evolution equation will be very complicated \([2, 5]\). To get a simpler one and to be able to diagonalize it with respect to one variable we take into account that operators with different spin do not mix with each other.

C-even operators with definite spin can be obtained easily from Shuryak-Vainshtein operators by

\[
Y^\rho(\kappa, u) = ^3S^\rho(0, \kappa u, \kappa) + S^\rho(\kappa, \kappa u, 0)
\]

\[
Y_n^\rho(u) = i^{n-2} \frac{d^{n-2}}{d\kappa^{n-2}} Y^\rho(\kappa, u)|_{\kappa=0} = \sum_{k=1}^{n-1} \binom{n-2}{k-1} u^{k-1} \bar{u}^{n-k-1} Y_{n,k}.
\]

The operators \(Y_n^\rho(u)\) possess the definite spin \(n\). In Eq. \([18]\) the variable \(u\) gives (for \(\kappa = 1\)) the position of the gluon field on the light cone. For \(0 \leq u \leq 1\) the gluon field lies between the two quark fields. Note that the gluon field can also be outside of this range so that \(u\) is not restricted to the interval \([0, 1]\).

Consequently we define a new distribution function as Fourier transform with respect to the variable \(\kappa\) only, so that

\[
\Delta \hat{q}^{\text{NS}}(y, u) = \int \frac{d\kappa}{4\pi} \frac{S_\rho}{(\bar{x}P)^2} \left( P, S \right) Y^\rho(\kappa, u) + Y^\rho(-\kappa, u) \left| P, S \right) e^{iyu (\bar{x}P)}
\]

depends on the Fourier conjugate variable \(y\) and the gluon position \(u\). To clarify the meaning of the variable \(y\) we express this distribution function \([20]\) in terms of the distribution function \(\Delta \hat{Q}^{\text{NS}}(x_1, x_2)\),

\[
\Delta \hat{q}^{\text{NS}}(y, u) = \int_{-1}^{1} dx_1 \int_{-1}^{1} dx_2 \theta(1 - |x_1 \pm x_2|) \delta(y + ux_1 - \bar{u}x_2) \Delta \hat{Q}^{\text{NS}}(x_1, x_2)
\]

which gives the support restriction \(|y| \leq \text{Max}(1, |2u - 1|)\), i.e., for \(0 \leq u \leq 1\) the variable \(y\) can be interpreted as an effective momentum fraction.

The evolution equation for the distribution function \([20]\) can be derived from the renormalization group equation \([16]\). Using the transformation \([18]\) and the definition \([20]\) a straightforward calculation provides the following result:

\[
Q^2 \frac{d}{dQ^2} \Delta \hat{q}^{\text{NS}}(y, u, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int \frac{dz}{z} \int dv \tilde{P}(z, u, v) \Delta \hat{q}^{\text{NS}} \left( \frac{y}{z}, v, Q^2 \right).
\]

Here the integration region is determined by both the support of \(\Delta \hat{q}^{\text{NS}}\) and the support of the evolution kernel.
\[ \hat{P}_{qq}(z, u, v) = \frac{2C_F - C_A}{2} \left[ [\Theta_1(z, u, v)K(z, u, v)]_+ + \Theta_2(z, \bar{u}, \bar{v})L(z, \bar{u}, \bar{v}) - \Theta_2(z, u, v)M(z, u, v) \right] + \frac{C_A}{2} \left[ \left[ [\Theta_3(z, u, v)N(z, u, v)]_+ + \left( \frac{u}{v} \rightarrow \bar{u} \right) \right] + \Theta_3(z, u, v)M(z, u, v) - \frac{7}{2} \delta(u-v)\delta(1-z) \right], \]

where \([A(z, u, v)]_+ = A(z, u, v) - \delta(1-z)\delta(u-v) \int_0^1 dz' \int dv' A(z', u, v')\) and the auxiliary functions are defined by:

\[
\begin{align*}
\Theta_1(z, u, v) &= \theta(z)\theta(u-zv)\theta(\bar{u} - z\bar{v}), \\
\Theta_2(z, u, v) &= \theta(-\bar{u}\bar{v})\theta(\{1-vz\}\bar{u})\theta(\{z-u\}u), \\
\Theta_3(z, u, v) &= \theta(\bar{u}z)\theta(\bar{u}\bar{z})\theta(\{vz-u\}u), \\
K(z, u, v) &= z + \left\{ \frac{u^2}{v(v-u)}\delta(u-zv) + \left( \frac{u}{v} \rightarrow \bar{u} \right) \right\}, \\
L(z, u, v) &= -\text{sign}(\bar{u})\frac{\bar{v}u^2}{\bar{u}^2}\delta(u-z), \\
M(z, u, v) &= \frac{2z(1-zv)}{\bar{u}^3}, \\
N(z, u, v) &= \frac{\text{sign}(u)\bar{v}}{u(v-u)} \left\{ \frac{\bar{v}}{u} \delta(1-z) + \frac{u^2}{\bar{v}}\delta(u-zv) \right\}.
\end{align*}
\]

### 5 Solution of the evolution equation

A diagonalization of the evolution equation \((22)\) can be achieved partly by the Mellin transform

\[ \Delta \tilde{q}_n^{NS}(u) = \int_0^1 dx x^{n-2} \Delta \tilde{q}_n^{NS}(x, u). \]

These moments are given by matrix elements of the operators \(Y_n^\rho(u)\) with spin \(n\) and are polynomials in the variable \(u\) of order \((n-2)\) [see Eq. \((19)\)]. Their evolution equation reads

\[ Q^2 \frac{d}{dQ^2} \Delta \tilde{q}_n^{NS}(u, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int dv \hat{P}_n(u, v) \Delta \tilde{q}_n^{NS}(v, Q^2), \]

where the \(n\)-dependent kernel \(\hat{P}_n(u, v) = \int_0^1 dz z^{n-2} \hat{P}_n(z, u, v)\) can be calculated from Eq. \((23)\). For each given \(n\), \(\Delta \tilde{q}_n^{NS}(u)\) can be expanded with respect to the \((n-2)\) eigenfunctions \(e_{n,i}(u)\) of the kernel \(\hat{P}_n(u, v)\). These polynomials can be constructed by diagonalization of

\[ \frac{1}{n-j-1} \int dv \hat{P}_{qq}^n(u, v) v^{n-j-1}u^{j-1} = \sum_{i=1}^{n-1} \binom{n-2}{i-1} \tilde{\gamma}_{ij}^n u^{n-i-1} \bar{u}^{i-1}, \]

where \(\tilde{\gamma}_{ij}^n\) are the known anomalous dimensions of the local operators \(Y_n^\rho\) \([2, 4]\). A general analytical solution of this problem is not known. But, for the first few moments the numerical solution can be obtained easily.

Knowing the \((n-2)\) eigenfunctions \(e_{n,i}(u)\) and eigenvalues \(\lambda_{n,i}\) of the kernel \(\hat{P}_n(u, v)\) it is possible to determine the evolution of the moments

\[ \Delta \tilde{q}_n^{NS}(u, Q^2) = \sum_{i=1}^{n-1} c_i \left( Q_0^2 \right) e_{n,i}(u) \exp \left\{ \int_{Q_0^2}^{Q^2} \frac{dt}{t} \frac{\alpha_s(t)}{2\pi} \lambda_{n,i} \right\}. \]
The nonperturbative coefficients $c_i(Q_0^2)$ at a reference momentum squared $Q_0^2$ have to be determined from the initial value $\Delta \tilde{q}^{NS}(u, Q_0^2)$. Since, assuming the validity of Eq. (10), the twist-3 nonsinglet part of the structure function $g_2$ satisfies

$$x \tilde{g}_2^{NS}(x) = \frac{d}{dx} \Delta \tilde{q}^{NS}(x), \quad \Delta \tilde{q}^{NS}(x, Q^2) = \int_0^1 du u \Delta \tilde{q}^{NS}(x, u, Q^2)$$

(29)

it is obvious that the initial values can not be obtained alone from transversal polarized deep inelastic scattering experiments.

Finally, we show that the above mentioned complications do not appear in the limit $x \to 1$ as well as in limit of large $N_c$. We deal directly with $\Delta \hat{q}^{NS}(x)$. From the definition of the evolution kernel (23) it turns out that $\int_0^1 du u \tilde{P}(z, u, v)$ factorizes in both limits according to $vP(z)$. Thus, from the evolution equation (22) follows for $x \to 1$,

$$Q^2 \frac{d}{dQ^2} \Delta \hat{q}^{NS}(x, Q^2) =$$

$$\frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dz}{z} \left\{ \left[ \frac{2C_F}{1-z} \right]_+ + \left( \frac{3}{2} C_F - C_A \right) \delta(1-z) \right\} \Delta \hat{q}^{NS}\left(\frac{x}{z}, Q^2\right) + O\left(1-x\right),$$

and for large $N_c$,

$$Q^2 \frac{d}{dQ^2} \Delta \hat{q}^{NS}(x, Q^2) =$$

$$\frac{\alpha_s(Q^2)}{4\pi} N_c \int_0^1 \frac{dz}{z} \left\{ \left[ \frac{z^2}{1-z} \right]_+ - z^2 - \frac{5}{2} \delta(1-z) \right\} \Delta \hat{q}^{NS}\left(\frac{x}{z}, Q^2\right) + O\left(\frac{1}{N_c}\right).$$

These equations are of the GLAP type and coincide with the result of [10]. The $1/N_c$ suppressed terms contain contributions from the region $v < 0$ and $v > 1$ and they provide a system of coupled evolution equations.

**Acknowledgment**

We wish to thank J. Blümlein, V. M. Braun, E. A. Kuraev, L. N. Lipatov, and O. V. Teryaev for valuable discussions. D. M. was financially supported by Deutsche Forschungsgemeinschaft (DFG) and the Landau-Heisenberg program.

**References**

[1] D. Adams et al. (SMC), Phys. Lett. B336 (1994) 125; K. Abe et al. (E143), Phys. Rev. Lett. 76 (1996) 587.

[2] A.P. Bukhvostov, E.A. Kuraev, L.N. Lipatov, Yad. Fiz. 38 (1983) 439 [Sov. J. Nucl. Phys. 38 (1983) 263]; Pis'ma Zh. Eksp. Teor. Fiz. 37 (1984) 406 [JETP Lett. 37 (1983) 482]; Yad. Fiz. 39 (1984) 194 [Sov. J. Nucl. Phys. 39 (1984) 121]; Zh. Eksp. Teor. Fiz. 87 (1984) 37 [Sov. Phys. JETP 60 (1984) 22].

[3] Xiangdong Ji, Chihong Chou, Phys. Rev. D 42 (1990) 3637.
[4] J. Kodaira, Y. Yasui, K. Tanaka, T. Uematsu, *QCD corrections to the nucleon’s spin structure function $g_2*, hep-ph/9603377.

[5] P.G. Ratcliffe Nucl. Phys. B264 (1986) 493; A.V.Efremov, O.V.Teryaev, Yadernaya Fizika 39 (1984) 1517.

[6] I.I. Balitsky, V.M. Braun, Nucl. Phys. B311 (1989) 541.

[7] B. Geyer, D. Müller, and D. Robaschik, *Evolution Kernels of Twist-3 Light-Ray Operators in Polarized Deep Inelastic Scattering*, hep-ph 9606320, will be appear in proceedings of the workshop “QCD and QED in Higher Order”, Rheinsberg, April 21-26, 1996”.

[8] S. Wandzura, F. Wilczek, Phys. Lett. B 72 (1977) 195.

[9] R. D. Tangerman, P. J. Mulders, *Polarized twist three distributions $g_2$ and $h_2$ and the role of intrinsic transverse momentum*, hep-ph/9408305, preprint NIKHEF-94-P7.

[10] A. Ali, V.M. Braun, G. Hiller, Phys. Lett. B266 (1991) 117.

[11] S.A. Anikin, O.I. Zavialov, Ann. Phys. 116 (1978) 135.

[12] D. Müller, D. Robaschik, B. Geyer, F.M. Dittes, J. Horesji, Fortschr. Physik 42 (1994) 101, Phys.Lett. 209B 325 (1988); B. Geyer, D. Robaschik, D. Müller, *Light-Ray Operators and their Application in QCD*, SLAC-PUB-6495, 1994 and Proc. Intern. Workshop “Quantum Field Theoretical Aspects of High Energy Physics”, Kyffhäuser Sept. 1993, Ed. B. Geyer and E.M. Ilgenfritz, NTZ Leipzig, 1993, p. 54.

[13] E.V. Shuryak, A.I. Vainshtein, Nucl. Phys. B199 (1982) 951; Nucl. Phys. B201 (1982) 14.

[14] S. Joglekar, B.W. Lee, Annals of Phys. (NY) 97 (1976) 160.

[15] G. Leibbrandt, Phys. Rev. D29 (1984) 1699; S. Mandelstam, Nucl. Phys. B213 (1983) 149.