Tool profile modification of hypoid gear machined by the duplex helical method

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Abstract
To avoid edge contact and stress concentration on the tooth surface of hypoid gear pairs machined by the duplex helical method for the drive axle of heavy-duty vehicles, a four-segment tool profile was designed for the simultaneous modification of the concave and convex surfaces of the pinion. First, a geometric model of the four-segment tool profile was established. Second, a mathematical model of the duplex helical method based on the four-segment tool profile was established, and a method for solving the tooth surface generated by the connecting points of the four-segment tool profile was presented. Finally, loaded tooth contact analysis based on finite element method was used to analyse the meshing performance of the gear pair obtained by the four-segment tool profile modification. The results of the original and modified gear pairs were compared. The tooth surface modification inhibited edge contact, improved the stress distribution, and decreased the maximum contact stress of the tooth surfaces, thereby improving the fatigue and wear life of the gear pair.

Keywords Duplex helical method · Hypoid gear · Edge contact · Tool profile modification · Loaded tooth contact analysis · Finite element analysis

1 Introduction
The hypoid gear pair of the drive axle of heavy-duty vehicles has the operational characteristics of heavy load and high speed. The axial force generated by the heavy load and meshing causes the contact slip and deformation of the tooth surface. Even a well-designed hypoid gear pair may have edge contact, which affects the vibration, noise, and service life of the gear pair. Therefore, a tooth surface of the gear pair with edge contact should be optimised.

Tooth surface modification is an effective method to improve the load meshing performance of gear pairs [1–5], thereby avoiding edge contact and stress concentration during their operation. Tooth surface modification can be conducted using curved or multi-segment tool profiles, instead of straight tool profiles. Particularly, several scholars have conducted research on spiral bevel gears with tapered tooth depth machined by the five-cut method [6–9] and uniform tooth depth machined by the hobbing method [10, 11]. In addition, studies have been conducted for optimisation of the geometric parameters of the tool profile to improve the meshing performance of spiral bevel gear pairs with uniform tooth depth [12–15]. Zhang et al. [16] used a circular arc profile for tooth surface modification to avoid edge contact and severe contact stress for a spiral bevel gear pair with tapered tooth depth machined by the duplex helical method, which simultaneously machines concave and convex surfaces. However, the circular arc tool profile cannot satisfy the requirement for the accurate modification of the entire tooth surface and may cause excessive partial modification. Meanwhile, the use of a multi-segment tool profiles to modify tooth surfaces allows the selection of a corresponding modification quantity for each segment according to the edge contact and stress concentration of the tooth tip and root, thereby providing a more flexible method.

The duplex helical method has the advantages of high processing efficiency and quality consistency; thus, it
has become an alternative that is gradually replacing the five-cut method as the mainstream processing technique for hypoid gear. However, as the duplex helical method involves the simultaneous processing of concave and convex surfaces, it is more difficult to obtain a precise geometry of the tooth surface compared to the five-cut method. Moreover, there are limited studies on the duplex helical method. Therefore, in this study, a new tool profile was designed to modify the tooth surface of hypoid gear machined by the duplex helical method to avoid edge contact and stress concentration caused by heavy. Finally, the finite element method of the loaded tooth contact analysis (LTCA) [17–22] was used to discuss the effects of the modification on the gear pair.

2 Solution for the tooth surface of the hypoid gear machined by the duplex helical method

2.1 Derivation of the pinion tooth surface

For the hypoid gear pair machined by the duplex helical method, the gear is machined by forming which is given in detail in Litvin et al. [6]. The pinion is machined by the duplex helical method. The detailed modeling process of cutting the pinion is described below.

A geometrical schematic of the four-segment tool profile for machining the pinion tooth surface is shown in Fig. 1. The four-segment tool profile consists of the a-, b-, c-, and d-segment tool profiles, and the four-segment tool profile is tangent at the connecting point respectively. Among them, the a-segment tool profile is a parabola used to generate the top of the tooth surface. The b-segment tool profile is a straight line used to generate the working area of the tooth surface. The c-segment tool profile is a parabola used to generate the transition surface of the tooth root. The d-segment tool profile is an arc line of the tool tip used to generate the transition surface of the tooth root arc. Points A and B are the parabolic vertices of the a- and c-segment tool profiles, respectively. The distances \( h_a \) and \( h_c \) are the heights of points A and B in the cutter head, respectively. Point D is the connection point of the c- and d-segment tool profiles. \( DF \) is the common tangent

\[
r_p^a = \left( \frac{\cos \alpha_p - 2a_{pa}(u_p - h_a/\cos \alpha_p)}{\sin \alpha_p + 2a_{pa}(u_p - h_a/\cos \alpha_p) \sin \alpha_p} \cos \theta_p \right) \bigg/ \sqrt{1+4a_{pa}^2(u_p - h_a/\cos \alpha_p)^2} \tag{1}
\]

of the c- and d-segment tool profiles, and \( \alpha_c \) is the pressure angle of the tangent \( DF \). \( DG \) is the vertical line of the tool tip plane. \( x_G \) is the x-axis coordinate of point G in the \( S_p \) coordinate system. \( \alpha_p \) is the blade angle. \( p_{w1} \) is the point width. \( p_f \) is the arc radius of the d-segment tool profile. \( \lambda_p \) is the arc angle of the a-segment tool profile. \( R_{c1} \) and \( R_f \) are the centre radius, and radius of the cutter head at the intersection of the a-segment tool profile and blade plane, respectively. \( R_p \) is the tip radius of the straight tool profile. Figure 2 shows the generating cones for pinion generating tool with four-segment tool profile, and \( \theta_p \) and \( u_p \) are the tooth surface coordinates of the generating surfaces, where Fig. 2a, b correspond to the generating cones of convex and concave, respectively.

As shown in Figs. 1 and 2, vector function \( r_p \) and its unit normal vector \( n_p \) of the generating cone are obtained in the \( S_p \) coordinate system of cutter head after the four-segment cutting edge rotates around the \( z_p \) axis.

The vector function \( r_p \) and its unit normal vector \( n_p \) of the generating cone generated by the a-segment tool profile (parabola) can be expressed as:

\[
r_p^a(u_p, \theta_p) = \begin{bmatrix} (R_p \mp u_p \sin \alpha_p \mp a_{pa}(u_p - h_a/\cos \alpha_p)^2 \cos \alpha_p) \cos \theta_p \\ (R_p \mp u_p \sin \alpha_p \mp a_{pa}(u_p - h_a/\cos \alpha_p)^2 \cos \alpha_p) \sin \theta_p \\ -u_p \cos \alpha_p + a_{pa}(u_p - h_a/\cos \alpha_p)^2 \sin \alpha_p \\ 1 \end{bmatrix}
\]

\[
R_p = R_{c1} \mp (p_{w1}/2) \tag{3}
\]

where \( a_{pa} \) is the parabolic coefficient of the a-segment tool profile, and the upper and lower signs in Eqs. (1) and (2)
Generating cone of concave

Generating cone of convex

(a) Generating cone of convex

(b) Generating cone of concave

correspond to the convex and concave surfaces of the pinion, respectively. Hereafter, ‘?’ expresses the same meaning as that in Eqs. (1) and (2).

The vector function \( r^b p \) and its unit normal vector \( n^b p \) of the generating cone generated by the \( b \)-segment tool profile (straight line) can be expressed as follows:

\[
\begin{align*}
\mathbf{r}^b_p(u_p, \vartheta_p) &= \begin{bmatrix} \left(R_p \mp u_p \sin \alpha_p \right) \cos \theta_p \\ \left(R_p \mp u_p \sin \alpha_p \right) \sin \theta_p \\ -s_p \cos \alpha_p \\
1 \end{bmatrix} \\
n^b_p &= \begin{bmatrix} \cos \alpha_p \cos \theta_p \\ \cos \alpha_p \sin \theta_p \\ \mp \sin \alpha_p \\
1 \end{bmatrix}
\end{align*}
\] (4)

The vector function \( r^c p \) and its unit normal vector \( n^c p \) of the generating cone generated by the \( c \)-segment tool profile (parabola) can be expressed as follows:

\[
\begin{align*}
\mathbf{r}^c_p(u_p, \vartheta_p) &= \begin{bmatrix} \left(R_p \mp u_p \sin \alpha_p \mp a_{pc}(u_p - h_c \cos \alpha_p) \right) \cos \theta_p \\ \left(R_p \mp u_p \sin \alpha_p \mp a_{pc}(u_p - h_c \cos \alpha_p) \right) \sin \theta_p \\ -u_p \cos \alpha_p + a_{pc}(u_p - h_c \cos \alpha_p) \sin \theta_p \\
1 \end{bmatrix} \\
n^c_p &= \begin{bmatrix} \cos \alpha_p - 2a_{pc}(u_p - h_c \cos \alpha_p) \sin \alpha_p \cos \vartheta_p \\ \cos \alpha_p - 2a_{pc}(u_p - h_c \cos \alpha_p) \sin \alpha_p \sin \vartheta_p \\ \mp \sin \alpha_p - 2a_{pc}(u_p - h_c \cos \alpha_p) \cos \alpha_p \\
1 \end{bmatrix}
\end{align*}
\] (6)

where \( a_{pc} \) is the parabolic coefficients of the \( d \)-segment tool profile.

The vector function \( r^d p \) and its unit normal vector \( n^d p \) of the generating cone generated by the \( d \)-segment tool profile (arc line) can be expressed as follows:

\[
\begin{align*}
\mathbf{r}^d_p(\lambda_p, \vartheta_p) &= \begin{bmatrix} \left( R_f \mp p_f \sin \lambda_f \right) \cos \theta_p \\ \left( R_f \mp p_f \sin \lambda_f \right) \sin \theta_p \\ -p_f(1 - \cos \lambda_f) \\
1 \end{bmatrix} \\
n^d_p &= \begin{bmatrix} \sin \lambda_f \cos \theta_p \\ \sin \lambda_f \sin \theta_p \\ \mp \cos \lambda_f \\
1 \end{bmatrix}
\end{align*}
\] (9)

\[
R_f = x_G \pm p_f(1 - \sin \alpha_c)/\cos \alpha_c
\] (10)

The positions of connecting points A and B of the four-segment tool profile in the \( S_p \) coordinate system (Fig. 1) can be determined using \( h_p, h_c \), and \( \alpha_p \). The position of connecting point D (common tangent point) of the \( c \)-segment tool profile (parabola) and \( d \)-segment tool profile (arc line), \( x_D, z_D \), and \( \alpha_c \) can be obtained as follows:

The coordinates of point D in the \( S_p \) coordinate system \((x_D, z_D)\) can be expressed according to Eq. (6)
where \( u_D \) is the position parameter of the \( c \)-segment tool profile (parabola) at point \( D \). Then, the slope of the tangent \( DF \) is:

\[
k_{DF} = \frac{\partial x_D}{\partial u_D} \frac{\partial z_D}{\partial u_D}
\]

(12)

Since the pressure angle of the tangent \( DF \) is \( a_c \),

\[
k_{DF} = \tan a_c
\]

(13)

In addition, as seen in Fig. 1,

\[
\begin{aligned}
x_D &= x_G \\
z_D &= z_G
\end{aligned}
\]

(14)

Finally, Eqs. (11)–(14) can be combined to determine \( u_D, x_G, \) and \( a_c \).

The coordinate systems used for the pinion generation are shown in Fig. 3. \( S_p \) is the coordinate system rigidly fixed to the cutter head. \( S_m \) is the reference coordinate system of the tilt and the swivel angle. \( S_{m1}, S_c, \) and \( S_d \) are the fixed coordinate systems rigidly fixed to the machine tool. \( S_d \) and \( S_1 \) are the moving coordinate systems rigidly fixed to the cradle and pinion, respectively. The installation and adjustment parameters of the cutter head and pinion, \( q_1, \) \( s_1, \) \( \gamma, \) \( e, \) \( x_{hp}, \) \( x_{ph}, \) \( \gamma_{m1}, \) \( x_{g1}, \) and \( h_l \), represent the center roll position, radial distance, tilt angle, swivel angle, work offset, sliding base, machine root angle, machine center to the cross point, and velocity coefficient of helical motion, respectively. When the pinion is machined by the duplex helical method, the cradle rotates around the axis \( z_{m1} \), while moving as a helix along this axis. Simultaneously, the pinion creates a rotary motion around the axis \( x_d \). \( \varphi \) and \( m_{b1} \) are the rotation angle of the pinion and roll ratio of the machine tool, respectively.

According to the coordinate systems applied for the pinion generation shown in Fig. 3, \( r_p \) and \( n_p \) are converted to the \( S_1 \) coordinate system. Consequently, the tooth surface equation \( r_1 \) and its unit normal vector \( n_1 \) of the pinion in the \( S_1 \) coordinate system can be obtained as follows:

\[
\begin{aligned}
r_1(u_p, \theta_p, \varphi) &= M_{l1m1} M_{mlp} r_p(u_p, \theta_p) \\
n_1(u_p, \theta_p, \varphi) &= L_{lm1} L_{mlp} n_p(u_p, \theta_p)
\end{aligned}
\]

(15)

where matrices \( M_{mlp} \) and \( M_{l1m1} \) represent the transformation of the coordinate systems from \( S_1 \) to \( S_{mlp} \) and \( S_{m1} \), respectively. And \( L_{mlp} \) and \( L_{lm1} \) are the third-order submatrix of the \( M_{mlp} \) and \( M_{l1m1} \), respectively. \( M_{mlp}, L_{mlp}, M_{l1m1}, \) and \( L_{lm1} \) can be expressed as:

\[
M_{mlp} = \begin{bmatrix}
\cos q & \sin q & 0 & 0 \\
-\sin q & \cos q & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(16)

\[
M_{l1m1} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \varphi & \sin \varphi & 0 \\
0 & \sin \varphi & \cos \varphi & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(17)

\[
L_{mlp} = M_{mlp}(1 : 3, 1 : 3)
\]

(18)

\[
L_{lm1} = M_{l1m1}(1 : 3, 1 : 3)
\]

(19)

In Eq. (16), \( q = q_1 + \varphi \cdot m_{b1} \). When the duplex helical method is used to machine the pinion, the cradle undergoes an axial movement. Therefore, \( x_b \) in Eq. (17) can be expressed as:

\[
x_b = x_{b0} - h_l \cdot \varphi \cdot m_{b1}
\]

(20)

In addition, to generate a pinion, the meshing equation is satisfied [20]:

\[
f = n_1(u_p, \theta_p, \varphi) \cdot (\partial r_1(u_p, \theta_p, \varphi)(1 : 3)/\partial \varphi)
\]

(21)

By incorporating the meshing in Eq. (21) into Eq. (15), parameter \( \varphi \) can be eliminated and the pinion tooth surface equation can be expressed as:

\[
\begin{aligned}
r_1 &= r_1(u_p, \theta_p, \xi_1) \\
n_1 &= n_1(u_p, \theta_p, \xi_1)
\end{aligned}
\]

(22)

where \( \xi_1 \) represents the machining parameters of the pinion.

2.2 Tooth surface generated by the connecting points of the four-segment tool profile

To obtain the space coordinate points of the tooth surface, the tooth surface should be discretised based on the geometric parameters of the gear pair, that is, the tooth surface is
divided into uniform grid points. Figure 4 shows a discrete diagram of the tooth surface. The relationship between the coordinates \((L, R)\) of the discrete points of the tooth surface on the rotating projection surface and tooth surface equation \(r_1\) can be expressed as Eq. (23):

\[
\begin{align*}
L &= |r_1 \times p| \\
R &= -r_1 \cdot p
\end{align*}
\]  

(23)

where \(p\) is the unit vector in the axial direction of the pinion. Given the coordinate points \((L, R)\), \(r_1\) and \(n_1\) of the corresponding points on the tooth surface can be obtained using Eqs. (22) and (23), respectively.

When a four-segment tool profile is used to modify the tooth surface, the appropriate amount of modification can be selected according to the edge contact and stress concentration at different positions of the tooth surface, which is more flexible than the modification using one tool profile only. However, the tooth surface generated by the four-segment tool profile should be separately solved and connected to obtain a complete tooth surface. Therefore, the tooth surface boundary line generated by the connection points between the tool profiles should be solved.

The connection point \(D\) can be solved using the tooth surface equation corresponding to the \(c\)-segment tool profile. Therefore, the equations for solving the boundary line generated by connecting point \(D\) are as follows:

\[
\begin{align*}
L &= |r_1^c \cdot p| \\
L &= |r_1^d \cdot p|
\end{align*}
\]  

(26)

As \(h_p\), \(h_r\), and \(\alpha_p\) are known parameters, \(u_p\) in Eqs. (23)–(26) were also determined. \(L\) can be determined based on the geometric parameters of the gear pair. Therefore, the conjugate contact points on the boundary line can be obtained by solving Eqs. (24)–(26) while keeping \(u_p\) constant and taking equal points of \(L\). The boundary line can be obtained by connecting the obtained conjugate contact points. A schematic of the boundary line of the four-segment tool profile is shown in Fig. 5. The tooth surface boundary line is a set of conjugate contact points generated by the tool profile connection points on the tooth surface. Figure 1 shows the three connection points of the four-segment tool profile, which are connection point \(A\) of the \(a\)- and \(b\)-segment tool profiles, connection point \(B\) of the \(b\)- and \(c\)-segment tool profiles, and connection point \(D\) of the \(c\)- and the \(d\)-segment tool profiles.

Connection points \(A\) and \(B\) can be solved using the tooth surface equation corresponding to the \(b\)-segment tool profile. Among them, the equations for solving the boundary line generated by connecting point \(A\) are

\[
\begin{align*}
u_p &= h_a \cos \alpha_p \\
L &= |r_1^b \cdot p|
\end{align*}
\]  

(24)

Similarly, the equations for solving the boundary line generated by connecting point \(B\) are

\[
\begin{align*}
u_p &= h_c \cos \alpha_p \\
L &= |r_1^c \cdot p|
\end{align*}
\]  

(25)
surface is discretised according to the boundary line to obtain discrete points \((L, R)\). The tooth surface generated by the cutting edge can be obtained by combining the tooth surface Eqs. (22) and (23), respectively. Finally, the entire tooth surface can be obtained by connecting the tooth surfaces generated by the four-segment tool profile.

### 3 Example analysis

#### 3.1 Geometric parameter design

The hypoid gear pair used in a heavy-duty drive axle was used as the research object. The gear was machined by forming method, and the pinion was modified using the four-segment tool profile. The main geometric and machining parameters of the hypoid gear pair are listed in Tables 1 and 2, respectively. The design parameters of the four-segment tool profile for the machined pinion are listed in Table 3.

The contact pattern and transmission error curve of the hypoid gear pair corresponding to the parameters in Tables 1 and 2 under no load were obtained by the tooth surface contact analysis (TCA) method [23], as shown in Figs. 6 and 7, respectively. After modifying the tooth surface of the pinion according to the four-segment tool profile (Table 3), the deviation diagram between the modified and original tooth surfaces was obtained, as shown in Fig. 8.

#### 3.2 Loaded tooth contact analysis (LTCA)

To study the effect of the four-segment tool profile on the tooth surface modification, LTCA based on finite element analysis (FEA) was used to analyse the loaded meshing performance of the modified and original gear pairs. First, the gear and pinion were obtained according to the principle of tooth surface cutting. Second, the assembly relationship of the gear pair was obtained by TCA, and the gear and pinion were imported into Pro/E software for the assembly and interference rolling inspection. A tooth of the assembled gear and pinion was cut out. Third, the single-tooth gear pair was imported into the HyperMesh software to divide the tooth surface into finite element meshes. Finally, finite element meshes were imported into the Abaqus software, and the material properties, contact conditions, and loading conditions were set for the LTCA. Figure 9 shows the five-tooth

| Geometric parameter     | Gear (mm) | Pinion (mm) |
|-------------------------|-----------|-------------|
| Face width              | 70        | 77.1228     |
| Pinion offset           | /         | 44.450      |
| Module                  | 12.35     | /           |
| Pitch angle (deg)       | 71.762    | 17.825      |
| Face angle of blank (deg)| 72.4885  | 20.717      |
| Root angle (deg)        | 68.8122   | 17.1131     |
| Addendum (mm)           | 3.2982    | 15.6413     |
| Dedendum (mm)           | 18.3265   | 5.9834      |

| Machining parameter     | Gear       | Pinion     |
|-------------------------|------------|------------|
| Blade angle (deg)       | 22.5       | 22.5       |
| Point radius (mm)       | 149.860    | 154.940    |
| Radial distance (mm)    | 175.072    | 178.113    |
| Tilt angle (deg)        | /          | 20.027     |
| Swivel angle (deg)      | /          | −49.588    |
| Work offset (mm)        | /          | 54.277     |
| Machine root angle (deg)| 68.761     | −5.421     |
| Machine center to back (mm)| 1.327  | 3.670      |
| Sliding base (mm)       | /          | 48.065     |
| Ratio of roll           | /          | 4.187      |
| Center roll position (deg)| 47.098  | 59.581     |
| The first order coefficient of helix motion | / | 10.096 |
| Blade edge radius (mm)  | 2.400      | 1.270      |

| Design parameter        | Convex     | Concave    |
|-------------------------|------------|------------|
| Parabolic coefficient \(a_{pa}\) | 0.0003 | 0.0005 |
| Position parameter of the parabola vertex \(h_a\) (mm) | 12.5 | 14 |
| Parabolic coefficient \(a_{pc}\) | 0.003 | 0.0045 |
| Position parameter of the parabola vertex \(h_c\) (mm) | 5 | 5.5 |
| Arc radius of the d-segment tool profile \(p_f\) (mm) | 2.5 | 2.5 |
FEA model of the LTCA. The parameter settings for the LTCA are listed in Table 4. During the simulation, the stress values and cloud diagrams of a tooth surface contact points were obtained. Moreover, the meshing performance of the third pair of teeth of the original and modified gear pairs was analysed. The contact stress cloud diagrams on the tooth surface of the original and modified gear are shown in Figs. 10 and 11, respectively.

As seen in Fig. 10, edge contact occurred on the convex and concave surfaces of the original gear under heavy load with the stress concentrated near the tooth root. Comparing Figs. 10 and 11, the stress concentration at the tip edge contact and root of the convex and concave surfaces of the gear are inhibited. In addition, the contact stress distribution of the tooth surface is more uniform after the tooth surface modification.

For a more detailed comparison of the contact stress distribution of the tooth surfaces of the original and modified gear pairs, the contact pressure of the contact points of the third pair of teeth from the entry point to the exit point were extracted. The contact stress values of the contact points on the tooth surfaces of the original and modified gears are shown in Fig. 12. The contact stress values of the contact points on the tooth surfaces of the original and modified pinions are shown in Fig. 13.

As seen in Fig. 12, the maximum contact stress of the convex and concave surface of the gear decreased from 1747 to 1470 MPa (15.9%) and 1806 to 1447 MPa (19.8%), respectively, after the modification. Similarly, as shown in Fig. 13, the maximum contact stress of the convex and concave surface of the pinion decreased from 1917 to 1403 MPa (26.8%) and 1973 to 1393 MPa (29.4%), respectively, after the modification. Thus, the contact stress of the entry and exit points and contact stress near the tip and root of the tooth surfaces of the gear and pinion decreased after the modification.

The load transmission error curve of the gear pair can be obtained by extracting the actual rotation angles of the gear and pinion during the integrated meshing process of the gear pair. Figure 14 shows the transmission error curves of the original and modified gear pairs obtained by LTCA.

As shown in Fig. 14, the transmission error on the convex and concave surfaces of the gear pair tends to increase after the modification. To avoid the transmission error from increasing too much during the modification, the appropriate amount of modification can be selected according to the actual working conditions.

After modifying the tooth surface of the pinion, the influence of the modification on the bending stress of the tooth root should be analysed. Figure 15 shows the tooth root bending stress cloud diagram of the concave and convex surfaces of the pinion. Figure 16 shows the tooth root bending stress values of the contact points of the original and modified pinions.

![Fig. 6 Results of the TCA: contact pattern of the gear. a Gear convex. b Gear concave](image-url)
As shown in Fig. 16, the tooth root bending stress on the convex and concave surfaces of the pinion increased after the modification. Particularly, the maximum tooth root bending stress of the convex and concave surfaces increased from 327 to 335 MPa and 320 to 332 MPa, respectively. Moreover, the maximum bending stress of the tooth root is observed in the middle part of the tooth surface close to the tooth root.

### 3.3 Tooth surface contact pattern rolling test

To further compare the contact pattern on the tooth surface of the original and modified gear, the eight-axis five-link CNC spiral bevel gear processing machine H650GA (Fig. 17) produced by Changsha Haliang Kaishuai Co., Ltd. was used to perform the gear processing experiments on the original and modified gear pairs. Under a light load, the tooth surface was tested by rolling inspection machine (Fig. 18). Figures 19 and 20 show the contact pattern of the original and modified gear.

Comparing Figs. 19 and 20, the distance from the contact pattern on the convex and concave surface of the gear to the tip increased from 1.3 to 2.8 mm and 1.5 to 2.6 mm, respectively, after the modification. The contact pattern of the tooth surface tends to shrink to the middle of the tooth surface to avoid contact with the edge of the tooth surface, which is consistent with the variation trend of simulation results in Figs. 10 and 11. Furthermore, the effectiveness of the proposed design of the four-segment tool profile was verified.
Table 4 Parameter settings for the LTCA

| Parameter                          | Value        | Parameter                          | Value        |
|------------------------------------|--------------|------------------------------------|--------------|
| Material density (kg/m³)           | 7.85×10³     | Load of the gear (kN•m)            | 1.1          |
| Elastic modulus (MPa)              | 1.88×10⁶     | Speed of the pinion (r/min)        | 1200         |
| Poisson’s ratio                    | 0.3          | Running time (s)                   | 0.24         |

Fig. 10 Contact stress cloud diagram of the Original gear. a Gear convex. b Gear concave

Fig. 11 Contact stress cloud diagram of the modified gear. a Gear convex. b Gear concave
Fig. 12 Contact stress values of the contact points on the tooth surface of the original and modified gears. a Gear convex. b Gear concave

Fig. 13 Contact stress values of the contact points on the tooth surface of the original and modified pinions. a Pinion convex. b Pinion concave

Fig. 14 Transmission error curves of the original and modified gear pairs obtained by LTCA. a Gear convex (Drive). b Gear concave (Coast)
**Fig. 15** Root bending stress cloud diagram of the convex and concave surfaces of the pinion. (a) Pinion convex. (b) Pinion concave

**Fig. 16** Root bending stress values of the contact points of the original and modified pinions. (a) Pinion convex. (b) Pinion concave

**Fig. 17** Eight-axis five-link CNC spiral bevel gear processing machine H650GA

**Fig. 18** Tooth surface contact pattern rolling inspection
**Fig. 19** Contact pattern of the original gear. **a** Gear convex. **b** Gear concave

**Fig. 20** Contact pattern of the modified gear. **a** Gear convex. **b** Gear concave
4 Conclusions

In this study, the tool profile modification of a hypoid gear pair machined by the duplex helical method was studied. A new four-segment tool profile was designed to modify the tooth surface to avoid edge contact under heavy load. Compared with the modification of the tooth surface using a single tool profile, the four-segment tool profile modification is more flexible. The meshing performances of the original and modified gear pairs were analysed by LTCA combined with FEA. The main conclusions are as follows:

1. The geometric model of the four-segment tool profile and mathematical model of the hypoid gear machined by duplex helical method were established. Moreover, a method for solving the boundary line corresponding to the connecting points of the tool profile was presented.

2. The original and modified hypoid gear pairs under heavy load were subjected to LTCA. The results showed the inhibition of the edge contact, improved stress distribution, and reduced maximum contact stress of the tooth surfaces after the modification. In addition, the contact stress of the entrance and exit points of the gear and pinion decreased. Particularly, the maximum contact stresses of the convex and concave surfaces of the gear decreased by 15.9% and 19.8%, respectively, whereas those of the pinion decreased by 26.8% and 29.4%, respectively.

3. After the modification, the transmission error amplitude of gear pair increased, and the tooth root bending stress on the convex and concave surfaces of the pinion increased. The maximum tooth root bending stress was observed in the middle part of the tooth surface close to the tooth root. Therefore, the influence of the modification on the transmission error amplitude and tooth root bending stress should be considered when selecting the modification parameters to improve the meshing performance and enhance the bearing capacity of the gear pair.

Declarations

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