A NEW APPROACH TO OBTAINING CLUSTER MASS FROM SUNYAEV–ZEL'DOVICH EFFECT OBSERVATIONS

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Received 2010 December 19; accepted 2011 January 18; published 2011 January 31

ABSTRACT

The accurate determination of cluster total mass is crucial for their use as probes of cosmology. Recently, the Sunyaev–Zel’dovich effect (SZE) has been exploited in surveys to find galaxy clusters, but X-ray or lensing follow-up observations, or empirically determined scaling relations between SZE flux and total mass, have been required to estimate their masses. Here, we demonstrate a new method of mass determination from SZE observations, applicable in the absence of X-ray or lensing data. This method relies on the virial relation and a minimal set of assumptions, following an approach analogous to that used for stellar structure. By exploiting the virial relation, we implicitly incorporate an additional constraint from thermodynamics that is not used in deriving the equation of hydrostatic equilibrium. This allows us to relate cluster total mass directly to the robustly determined quantity, the integrated SZE flux.

Key words: cosmology: observations – dark matter – galaxies: clusters: general – galaxies: clusters: individual (A1835, A1914, CL J1226.9+3332)

1. INTRODUCTION

Clusters of galaxies are thought to be the largest gravitationally bound objects in the universe and therefore good tracers of cosmology. Ongoing cluster surveys, such as those with the Alabama Cosmology Telescope (ACT; Kosowsky 2003; Menanteau et al. 2010), the South Pole Telescope (SPT; Ruhl et al. 2004; Plagge et al. 2010), and Planck (Rosset et al. 2010; Planck Collaboration et al. 2011a, 2011b), have the potential to place tight constraints on cosmological parameters with the clusters they discover. These surveys utilize the Sunyaev–Zel’dovich effect (SZE), which has redshift-independent surface brightness and arises by Compton scattering of cosmic microwave background (CMB) photons off of the hot electrons in clusters of galaxies (Zel’dovich & Sunyaev 1969; Sunyaev & Zel’dovich 1972). However, the interpretation of cluster yields relies on the accurate determination of the scaling between integrated SZE flux and cluster total mass (McCarthy et al. 2003; Motl et al. 2005; Nagai 2006).

Because the SZE intensity varies as the line-of-sight integral of thermal electron pressure, a cluster’s total, integrated SZE flux scales linearly with the volumetric integral of thermal pressure, which is thermal energy (see, e.g., Equation (8)).

To the extent that clusters are virialized and supported hydrostatically by thermal pressure, a cluster’s total SZE flux will closely track its gravitational energy, thereby motivating SZE flux as a proxy for cluster mass. Approaches exploiting this expected tight correlation have traditionally relied on empirical relations between SZE flux and total mass determined from X-ray observations or optical lensing studies.

We present a new approach for mass determination from SZE data alone, applicable in the absence of X-ray or lensing data, that exploits the virial relation and a minimal amount of simplifying assumptions about cluster astrophysics. In Section 2, we describe how SZE observations can constrain thermal energy, and we relate this directly to cluster total mass via the virial theorem. In Section 3, we demonstrate this mass determination on previously published SZE data. In Section 4, we assess how these simplifying assumptions impact this method and offer conclusions.

2. THERMAL ENERGY CONSTRAINTS FROM OBSERVATIONS OF THE SUNYAEV–ZEL’DOVICH EFFECT

The thermal SZE is a small ($\lesssim 10^{-3}$) distortion in CMB intensity caused by inverse Compton scattering of CMB photons by energetic electrons in the hot intracluster medium (ICM; Zel’dovich & Sunyaev 1969; Sunyaev & Zel’dovich 1972). This spectral distortion can be expressed, for dimensionless frequency $x = h\nu/k_B T_{\text{CMB}}$, where $h$ is Planck’s constant, $\nu$ is frequency, $k_B$ is Boltzmann’s constant, and $T_{\text{CMB}}$ is the primary CMB temperature, as the change in intensity $\Delta I_{\text{SZE}}$ relative to the primary CMB intensity normalization $I_0$:

$$\Delta I_{\text{SZE}} I_0 = g(x, T_e) y.$$  (1)

The factor $g(x, T_e)$ in Equation (1) encapsulates the frequency dependence of the SZE intensity. For non-relativistic electrons

$$g(x) = \frac{x^4 e^x}{(e^x - 1)^2} \left( \frac{x^4 e^x + 1}{e^x - 1} - 4 \right).$$  (2)

At low frequencies ($\lesssim 600 \text{ GHz}$), relativistic corrections to Equation (2) are fairly straightforward to apply (see, e.g., Itoh et al. 1998). The Compton $y$ parameter in Equation (1) is defined as

$$y \equiv \frac{k_B \sigma_T}{m_e c} \int n_e T_e d\ell = \frac{\sigma_T}{m_e c^2} \int P_e d\ell,$$  (3)

where $\sigma_T$ is the Thomson scattering cross-section of the electron, $\ell$ is the line of sight, and $m_e c^2$ is an electron’s rest energy, and the primary CMB intensity normalization is $I_0 = 2(k_B T_{\text{CMB}}^4 h c^2)^{-1} = 2.7033 \times 10^8 \text{ Jy Sr}^{-1}$. Note that we have assumed the ideal gas law ($P_e = n_e k_B T_e$) in Equation (3) to relate electron pressure $P_e$ to electron number density $n_e$ and temperature $T_e$. 

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From Equation (3), one can see that resolved observations of the thermal SZE from a cluster can be used to constrain its electron pressure profile \( P_e(r) \). This can be related to the total pressure as \( P_{\text{gas}} = (1 + 1/\mu_e) P_e \), where \( \mu_e = 2/(1 + X) \) is the mean particle weight per electron and \( X \) is the mass fraction of hydrogen. While deep X-ray observations have shown that the distribution of heavy elements in the ICM varies with radius (e.g., Vikhlinin et al. 2005; Peterson et al. 2001), and theoretical studies indicate that helium sedimentation into the cluster core will also impact \( \mu_e \) (e.g., Markovich 2007; Peng & Nagai 2009), we make the simplifying assumption that \( \mu_e = 1.17 \). We assess the impact of this assumption in Section 4.

A common SZE observable used in SZE mass scaling relations is \( Y_{\text{int}} \), the Compton y parameter integrated over some region of the sky, defined as

\[
Y_{\text{int}} \equiv \int y \, d\Omega.
\]  

(4)

Because \( Y_{\text{int}} \) is proportional to the surface brightness of the SZE integrated over a region of the sky, it tracks the integrated cluster SZE flux. For a spherically symmetric electron pressure profile \( P_e(r) \), the spherically integrated version of \( Y_{\text{int}} \) is (e.g., Mroczkowski et al. 2009)

\[
Y_{\text{ sph}}(r) \equiv \sigma_T \int_0^r P_e(r') 4\pi r'^2 \, dr' = 2s(r) \frac{2}{3} \int_0^r P_{\text{gas}}(r') 4\pi r'^2 \, dr' = \frac{2 \sigma_T E_{\text{th}}(r)}{3(1 + 1/\mu_e) m_e c^2}.
\]  

(5)  

(6)  

7)

We have used the fact in Equation (7) that the thermal energy within \( r \) is

\[
E_{\text{th}}(r) = \frac{3}{2} \int_0^r P_{\text{gas}}(r') 4\pi r'^2 \, dr',
\]

(8)

for a monatomic, ideal gas.

Through Equation (7), one can see that the SZE observable \( Y_{\text{sph}} \) relates directly to the thermal energy content of the cluster. To the extent that a cluster is virialized and supported by thermal pressure, this quantity will closely track the gravitational energy \( U_g(r) \) via the virial relation

\[
2E_{\text{th}}(r) = -U_g(r).
\]  

(9)

Using Equation (8), which derives from statistical mechanics, to relate pressure to thermal energy, we note that the virial relation (Equation (9)) is derived (see, e.g., Schwarzschild 1958, chapter 5) from the equation of hydrostatic equilibrium (HSE),

\[
\frac{dP_{\text{gas}}}{dr} = -\frac{\rho_{\text{gas}} G M_{\text{tot}}}{r^2},
\]  

(10)

where \( \rho_{\text{gas}}(r) \) is the gas density as a function of radius \( r \), \( M_{\text{tot}}(r) \) is the total mass within \( r \), and \( G \) is the gravitational constant. The equation of HSE is derived from fluid mechanics, specifically the equations of motion and continuity (see, e.g., Schwarzschild 1958; Sarazin 1988). Mass estimates based on HSE traditionally assume a spherically symmetric mass distribution and the ideal gas law, and therefore adopting the virial relation is no more restrictive than the assumptions typically required for HSE mass determinations. However, Equation (8) provides an additional, key constraint not used in pure HSE mass determinations, which (assuming the ideal gas law) only require two of these three ICM profiles: density, temperature, and pressure. As we discuss in Section 4, the virial mass estimate is proportional to the square root of scalar changes in the pressure profile, while the same changes would result in linear changes in the HSE mass estimate, as can be seen by examining Equation (10).

Adopting the Navarro, Frenk, and White (NFW) profile (Navarro et al. 1997) to describe the total mass distribution (i.e., baryonic + dark matter distribution), the total density is radially distributed as

\[
\rho_{\text{tot}}(r) = \frac{\rho_0}{(r/R_s)(1 + r/R_s)^2},
\]

(11)

and the total mass within \( r \) is

\[
M_{\text{tot}}(r) = \int_0^r \rho_{\text{tot}}(r') 4\pi r'^2 \, dr' = 4\pi \rho_0 R_s^3 \ln(1 + r/R_s) - (1 + r/R_s)^{-1}.
\]  

(12)

Here, \( \rho_0 \) and \( R_s \) are respectively the normalization and scale radius of the NFW profile. The use of an NFW profile is empirically motivated by simulations of dark matter halos, and we note that other mass profiles could be assumed and may in fact provide better alternatives. More recent theoretical studies, for instance, have indicated that the presence of baryons can significantly modify the mass distribution of dark matter (Gnedin et al. 2004; Rudd et al. 2008).

The gas mass \( M_{\text{gas}} \) and total mass \( M_{\text{tot}} \) can be related by defining the gas fraction \( f_{\text{gas}} \equiv M_{\text{gas}}(r)/M_{\text{tot}}(r) \), which could be a function of \( M_{\text{tot}} \), \( r \), \( z \), and cluster merger history. While recently shown to be poor approximation (e.g., Vikhlinin et al. 2006; Pratt et al. 2010), we make the simplifying assumption that \( f_{\text{gas}}(r) \) is a constant. Detailed measurements of \( f_{\text{gas}}(r) \) typically require high significance X-ray data, which are often insufficient or entirely lacking for clusters discovered via the SZE. The assumption of constant \( f_{\text{gas}}(r) \) implies that

\[
\rho_{\text{gas}}(r) = f_{\text{gas}} \rho_{\text{tot}}(r).
\]  

(13)

We assess the impact of this assumption in Section 4.

Using Equations (11), (12), and (13) to solve for the gravitational potential energy, we find

\[
U_g(r) = \frac{4\pi \rho_0 R_s^3}{2(1 + r/R_s)^2} G f_{\text{gas}} \left[ \frac{R_s}{(1 + r/R_s)^2} \right] - \int_0^r \ln(1 + r/R_s) (1 + r/R_s)^{-2} \, dr,
\]

(14)

where we have used the fact that the differential element of gravitational energy for a spherical shell of gas with density \( \rho_{\text{gas}}(r) \) is \( dU_g(r) = -G M_{\text{gas}} dm/r \), where the mass of the gas shell is \( dm = 4\pi \rho_{\text{gas}}(r) r^2 \, dr \).

Combining Equations (7) and (14) through the virial relation (Equation (9)), we have

\[
3(1 + 1/\mu_e) m_e c^2 Y_{\text{sph}}(r) = \frac{16\pi^2 G f_{\text{gas}} \sigma T}{R_s^2} \left[ \frac{R_s}{2(1 + r/R_s)^2} \right] + \int_0^r \ln(1 + r/R_s) (1 + r/R_s)^{-2} \, dr.
\]

(15)

Through this relation one can find the best-fit NFW profile parameters \( \rho_0 \) and \( R_s \) for any observationally constrained \( Y_{\text{sph}}(r) \). We apply this method to interferometric SZE data in Section 3.
Table 1

| Cluster Name      | Model Fit   | \(r_{2500}\) (Mpc) | \(\rho_{1000}/r_{2500}\) (\(10^{-3}\) Mpc\(^{-3}\)) | \(M_{\text{tot}}(r_{2500})\) (\(10^{15}\) M\(_{\odot}\)) | \(\rho_{500}\) (\(10^{-3}\) Mpc\(^{-3}\)) | \(M_{\text{tot}}(r_{500})\) (\(10^{15}\) M\(_{\odot}\)) |
|------------------|-------------|---------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| A1835            | N07 (this work) | 0.71(4.02)         | 8.88(0.78)                                   | 6.55(4.44)                                    | 1.51(0.06)                                   | 18.14(3.17)                                 | 12.57(1.57)                                 |
|                  | A10 (this work)  | 0.70(4.08)         | 8.68(0.69)                                   | 6.33(3.73)                                    | 1.47(0.06)                                   | 16.42(3.38)                                 | 11.57(1.38)                                 |
|                  | N07+SVM (M09)   | 0.68(4.02)         | 8.25(0.14)                                   | 5.64(1.11)                                    | 1.44(0.07)                                   | 17.54(3.00)                                 | 11.00(2.68)                                 |
|                  | Maughan (M09)   | 0.66(4.02)         | 7.88(0.49)                                   | 5.30(0.53)                                    | 1.42(0.07)                                   | 17.41(1.61)                                 | 10.68(1.01)                                 |
| CL J1226+3332.9 | N07 (this work)  | 0.46(4.01)         | 4.15(0.16)                                   | 3.71(1.16)                                    | 1.00(0.02)                                   | 9.94(1.01)                                  | 7.74(0.57)                                  |
|                  | A10 (this work)  | 0.46(4.01)         | 4.27(0.16)                                   | 3.63(1.17)                                    | 0.95(0.02)                                   | 9.46(0.98)                                  | 7.25(0.56)                                  |
|                  | N07+SVM (M09)   | 0.41(4.01)         | 3.56(0.16)                                   | 2.67(0.29)                                    | 0.98(0.07)                                   | 9.74(1.38)                                  | 7.37(0.50)                                  |
|                  | Maughan et al. (2007) | 0.45(4.01)     | 5.06(0.31)                                   | 3.41(0.30)                                    | 0.89(0.07)                                   | 10.59(0.69)                                 | 5.49(0.46)                                  |
| A1914            | N07 (this work)  | 0.64(4.03)         | 4.86(0.95)                                   | 4.32(0.64)                                    | 1.28(0.09)                                   | 7.81(2.36)                                  | 7.08(1.53)                                  |
|                  | A10 (this work)  | 0.62(4.03)         | 4.52(0.83)                                   | 3.95(0.56)                                    | 1.22(0.07)                                   | 6.66(1.76)                                  | 6.18(1.20)                                  |
|                  | N07+SVM (M09)   | 0.67(4.04)         | 6.29(1.03)                                   | 4.97(0.89)                                    | 1.25(0.11)                                   | 11.05(2.44)                                 | 6.62(1.90)                                  |
|                  | Maughan et al. (2008) | 0.62(4.02)     | 5.69(0.37)                                   | 4.31(0.43)                                    | 1.29(0.07)                                   | 10.76(1.03)                                 | 7.49(1.29)                                  |

3. APPLICATION TO OBSERVATIONS WITH THE SUNYAЕV–ZEL’DOVICH ARRAY

We test here the application of Equation (15) to the SZE observations of the three clusters presented in Mroczkowski et al. (2009, hereafter M09), and compare our results with the independent mass determinations presented in M09. These three clusters span a wide range in redshift and dynamical state. A1835, at \(z = 0.25\), is a relaxed, cool-core cluster (e.g., Peterson et al. 2001). A1914, at \(z = 0.17\), shows evidence of being disturbed, with a hot subcluster near the cluster core (Maughan et al. 2008). CL J1226.9+3332 appears somewhat relaxed given its high redshift (\(z = 0.89\); Maughan et al. 2004, 2007), but recent high-resolution SZE observations with MUSTANG have indicated otherwise (Kornout et al. 2010).

M09 derived mass estimates for these clusters using three data fitting methods. The first method relied on SZE observations and X-ray surface brightness data, but ignored the X-ray spectroscopic data. Instead, a density model was fit to the X-ray surface brightness data simultaneously with a pressure profile fit to the SZE data. Temperature information used in fitting the X-ray surface brightness data were derived from these density and pressure profiles, assuming the ideal gas law. The density model used in this method was a core-cut simplification of that used in Vikhlinin et al. (2006, hereafter V06), to which we refer as the “Simplified V06 Model” (SVM). The pressure profile used in this method is an analytic parameterization of the cluster radial pressure profile proposed by Nagai et al. (2007, hereafter N07),

\[
P_e(r) = \frac{P_{e,i}}{(r/r_p)^\gamma [1 + (r/r_p)^\gamma]^{\delta - c}/a},
\]

This profile has the form of a generalized NFW profile and was fit with the slopes fixed at the best-fit values found in N07, which are \((a, b, c) = (0.9, 5.0, 0.4)\).\(^2\) We refer to this X-ray+SZE method, which does not use or require X-ray spectroscopic data, as the “N07+SVM” method.

The second mass estimation method presented in M09 was an independent, X-ray only analysis performed by Ben Maughan following the methods outlined in Maughan et al. (2007, 2008). This state of the art method relies on deep Chandra X-ray observations, fitting both the spectroscopic and surface brightness data with the full density and temperature parameterizations in V06. The X-ray only analyses of each cluster in M09 were published in three separate papers: CL J1226.9+3332 was published in Maughan et al. (2007), A1914 was published in Maughan et al. (2008), and A1835 was published in M09.

The third method fits the SZE and X-ray data jointly, but relied on the assumption of isothermality and was included in M09 only for comparison with earlier works. We ignore this method here, noting that its results were consistent at small radii but increasingly discrepant at large radii.

With the density profile and the temperature or pressure profile from the above methods in hand, the equation of HSE (Equation (10)) was used to solve for each cluster’s total mass. The results of the N07+SVM and X-ray only analyses are reproduced in Table 1.

We compare the results of the M09 mass analyses with those from the SZE-only method presented here. We assume the same \(\Lambda\)CDM cosmology used in M09 (\(\Omega_M = 0.3, \Omega_\Lambda = 0.7\), and \(h = 0.7\)). As in M09, we adopt the N07 profile ((Equation (16) with \((a, b, c) = (0.9, 5.0, 0.4)\)) and that from Arnaud et al. (2010, hereafter A10; Equation (16) with \((a, b, c) = (1.0510, 5.4905, 0.3081)\)).

The SZE data used here were taken with the Sunyaev–Zel’dovich Array (SZA). Briefly, the SZA is an eight element compact array built to image the SZE in clusters through observations at 30 and 90 GHz (see, e.g., Muchovej et al. 2007). At 30 GHz, the SZA is sensitive to radial scales 1–6’ over a 10.6 diameter field of view. At 90 GHz, the SZA measures 20’–120’ radial scales over a 3.5 diameter field of view. These observations are naturally fit in \(u, v\)-space (Fourier space) using a Markov Chain Monte Carlo (MCMC) process, as discussed in M09. The trial model is computed in the plane of the sky by integrating Equation (3), using Equation (16) to describe the pressure. We present here the results assuming both the N07

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\(^2\) The parameters published in N07 are \((a, b, c) = (1.3, 4.3, 0.7)\), but the combination \((a, b, c) = (0.9, 5.0, 0.4)\) was later found to provide a better fit. The planned erratum to N07 is yet to be published, and the corrected parameterization first appeared in M09. However, the corrected parameterization we use here has come to be known as the “Nagai 07 Profile Pressure,” and we adhere to this terminology.
and A10 parameterizations. The sky-plane model is then Fourier transformed for direct comparison with the interferometric data. This has the advantage that the likelihood of the model fit is computed in a basis where error bars are Gaussian. The full details of this method can be found in M09.

The resulting fit N07 and A10 pressure profiles are used to solve for the radial profile of $Y_{\text{tot}}(r)$ (Equation (5)). For each accepted link in the MCMC, we fit the function described by Equation (15) assuming a constant $f_{\text{gas}} = 0.13$ and $\mu_e = 1.17$. The resulting mass profiles, computed using Equation (12), are used to find $r_A$ and $M_{\text{tot}}(r_A)$, which are respectively the radius within which the average density is $\Delta$ times greater than the critical density of the universe at that redshift, and the total mass contained within that radius. As in M09, we report $r_A$, $M_{\text{tot}}(r_A)$, and $Y_{\text{tot}}(r_A)$ for $\Delta = [2500, 500]$, with statistical error bars, in Table 1. We discuss systematics along with our conclusions in Section 4.

4. CONCLUSIONS

We conclude that this method is remarkably consistent—given the simplifying assumptions required to derive total mass using the virial relation and SZE data alone—within the X-ray only and X-ray+SZE mass determination methods in M09 (see Table 1). The assumption of constant $f_{\text{gas}} = 0.13$ has perhaps the largest systematic impact on the derived values of $M_{\text{tot}}(r_A)$ and $r_A$ for overdensity $\Delta$. The radial mass profile $M_{\text{tot}}(r)$ is $\propto f_{\text{gas}}^{-1/2}$, as can be seen by examining the relation between the NFW parameter $r_0$ and $f_{\text{gas}}$ in Equation (14). However, any change in $M_{\text{tot}}(r)$ affects $r_A$ and therefore $M_{\text{tot}}(r_A)$, so the systematic change in the mass at fixed overdensity is larger than a simple rescaling by the inverse square root of the ratio of the correct to the assumed $f_{\text{gas}}$. Fitting the same data with an assumed $f_{\text{gas}} = 0.11$, for example, increases $M_{\text{tot}}(r_A)$ by an average of 12% (rather than the 9% change in the profile $M_{\text{tot}}(r)$).

The assumption that $\mu_e = 1.17$, by contrast, can be expected to have a much smaller impact on the mass determination method presented here. For typical abundance gradients due to metal enrichment, $\mu_e$ varies on the $\sim 1\%$ level, which changes the $(1 + 1/\mu_e)$ factor in Equation (15) on the $\sim 1\%/2\%$ level. Large systematic deviations in metallicity therefore affect the fit $M_{\text{tot}}(r)$ at the $\sim 1\%/4\%$ level. The assumption of a single, constant metallicity is also common in X-ray studies of high-redshift clusters, where the limited number of X-ray counts is insufficient to constrain more than a single spectroscopic bin. On the other hand, helium sedimentation in the absence of magnetic fields, in a cluster undisturbed for 3 Gyr, could increase $\mu_e$ in the core region by $\sim 5\%$ (Peng & Nagai 2009). Using the results of Peng & Nagai (2009), we note that the sedimentation of helium has little effect on the average $\mu_e$ or on $\mu_e(r)$ at large radii.3 The expectation is that mergers and magnetic fields will both suppress helium sedimentation (Peng & Nagai 2009).

Another potential source of bias is due to uncertainties in the calibration of the SZE data. As discussed in Muchovej et al. (2007), the absolute calibration of SZA data is known to better than 10%, and the variation from observation to observation in amplitude of a flux calibrator (in this case Mars) is $\lesssim 5\%$. Calibration errors would result in scalar systematic errors in the fit pressure profile and have a linear impact on $Y_{\text{tot}}$ (Equation (5)). Examining the relation between $Y_{\text{tot}}$ and the NFW parameter $r_0$ in Equation (15), we can see errors in the derived $M_{\text{tot}}(r) \propto Y_{\text{tot}}(r)^{1/2}$, and will impact the mass at fixed overdensity $M_{\text{tot}}(r_A)$ on the $\lesssim 5\%$ level.

As noted in M09, SZA 30 GHz observations are sensitive to radial angular scales $\sim 1\prime-6\prime$, so the largest scale measured is $\approx r_{500}$ for A1835 and is $\sim 0.8r_{500}$ for A1914. The values we report should therefore be treated as extrapolations of the fit, and depend on the assumed N07 or A10 pressure profile.

Other systematics this method could suffer are common to X-ray mass determinations that rely on a cluster’s fit radial temperature and density profiles and assume thermal HSE to derive mass. First, HSE is most readily applied by assuming spherical symmetry. Second, while the gas may be virialized within the potential and supported predominantly by thermal pressure, there is an expectation that $\sim 10%-20\%$ of the total pressure is due to turbulent motions in the ICM (Lau et al. 2009; Battaglia et al. 2010). This is equivalent to including kinetic energy, in addition to thermal, in the virial relation. The interesting question arises as to whether adopting the virial relation, instead of just HSE, could be used to make X-ray mass determinations more robust.

The broader implications of this work are that by solving for the radial thermal energy profile one can estimate cluster mass from any SZE observation that can constrain that cluster’s radial pressure profile. Radial profiles have already been fit to clusters observed by ACT, SPT, and now Planck (Menanteau et al. 2010; Plagge et al. 2010; Planck Collaboration et al. 2011b), and this method may prove particularly useful for providing initial mass estimates for the clusters they discover. A future work, using SZA observations of a more complete sample of clusters, will compare total mass derived in this way with X-ray and lensing mass estimates. We will also consider how to extend this method for the case where the gas fraction varies with cluster radius.

The author wishes to thank Erik Reese for many useful discussions and Marshall Joy for encouraging him to test this method on a handful of clusters. The author also thanks the anonymous referee for comments that helped improve the direction and focus of this Letter. Support for the author was provided by NASA through the Einstein Fellowship Program, grant PF0-110077.

Facilities: SZA

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3 In fact, the effect of helium sedimentation is greater for X-ray surface brightness data than for SZE data. The redistribution of helium nuclei into the core greatly increases the X-ray emissivity, impacting both the charge and number of ions, and the number of electrons, the product of which determines the bremsstrahlung-dominated X-ray emission. The intensity of the SZE is only impacted by a factor proportional to the increase in the number of electrons. Therefore, the mass determinations from X-ray data can be expected to be more biased by helium sedimentation than those based on the SZE. For a good review, see Markovitch (2007).
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ERRATUM: “A NEW APPROACH TO OBTAINING CLUSTER MASS FROM SUNYAEV–ZEL’DOVICH EFFECT OBSERVATIONS” (2011, ApJ, 728, L2)

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Received 2012 January 2; published 2012 February 6

It was recently pointed out that the published version of this Letter neglected the pressure external to the radii of interest (e.g., $r_{2500}$ or $r_{500}$), which reduces the amount of gravitating matter required to contain the intracluster gas within that radius. The virial relation in the case of non-vanishing surface pressure is

$$2E_{\text{kin}}(r) - 3P(r)V = -U(r).$$

The surface pressure term $-3P(r)V$, where $V = 4\pi r^3/3$ is the volume at $r$, provides a small correction that must be taken into account when solving for the mass.

Using this correction, the relation between $Y_{\text{sph}}$, pressure, and the underlying NFW profile is

$$\frac{(1 + 1/\mu_v)}{16\pi^2 G f_{\text{gas}}} \left[ 3 \frac{m_{\text{eff}}^2}{\sigma T} Y_{\text{sph}}(r) - 4\pi r^3 P_v(r) \right] = \left( \frac{\rho_v r_s^2}{2(1 + r/r_s)^2} + \int_0^r \ln(1 + r'/r_s) \frac{dr'}{(1 + r'/r_s)^2} \right).$$

Here the $-4\pi r^3 P_v(r)$ term in the brackets on the left-hand side of the equation is due to the surface pressure correction in Equation (1).

We provide the corrected mass estimates (labeled “this work”) as well as the uncorrected mass estimates in Table 1.

The author acknowledges the diligence of and help from Colin Hill, who brought to his attention the virial relation’s surface pressure term. The author thanks Erik Reese for many useful discussions and Marshall Joy for encouraging him to test this method on a handful of clusters. The author also thanks the anonymous referee for comments that helped improve the direction and focus of this Letter. Support for the author was provided by NASA through the Einstein Fellowship Program, grant PF0-110077.

Facility: SZA

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Table 1: $Y_{\text{sph}}$ and $M_{\text{tot}}$ for Each Model Tested, Computed within $r_{2500}$ and $r_{500}$ Assuming Constant $f_{\text{gas}} = 0.13$

| Cluster Name  | Model Fit | $r_{2500}$ (Mpc) | $Y_{\text{sph}}(r_{2500})$ (10^{-5} Mpc$^{-2}$) | $M_{\text{tot}}(r_{2500})$ (10^{15} M$_{\odot}$) | $r_{500}$ (Mpc) | $Y_{\text{sph}}(r_{500})$ (10^{-5} Mpc$^{-2}$) | $M_{\text{tot}}(r_{500})$ (10^{15} M$_{\odot}$) |
|--------------|-----------|-----------------|---------------------------------|------------------|-----------------|---------------------------------|------------------|
| A1835        | A10       | 0.63^{+0.01}_{-0.01} | 7.64^{+0.52}_{-0.50} | 4.58^{+0.18}_{-0.19} | 0.64^{+0.01}_{-0.00} | 7.64^{+0.48}_{-0.47} | 4.64^{+0.18}_{-0.19} |
| N07 (this work) | A10       | 0.63^{+0.02}_{-0.00} | 7.88^{+0.72}_{-0.70} | 5.30^{+0.53}_{-0.54} | 0.71^{+0.02}_{-0.00} | 8.88^{+0.78}_{-0.76} | 6.55^{+0.44}_{-0.41} |
| N07 (uncorrected) | A10       | 0.70^{+0.02}_{-0.00} | 8.68^{+0.69}_{-0.68} | 6.33^{+0.42}_{-0.40} | 0.70^{+0.02}_{-0.00} | 8.68^{+0.69}_{-0.68} | 6.33^{+0.42}_{-0.40} |
| CL J1226+3332.9 | N07       | 0.39^{+0.01}_{-0.00} | 3.34^{+0.28}_{-0.28} | 2.35^{+0.15}_{-0.16} | 0.94^{+0.02}_{-0.02} | 9.43^{+0.89}_{-0.85} | 6.49^{+0.34}_{-0.34} |
| N07 (this work) | A10       | 0.40^{+0.01}_{-0.00} | 3.54^{+0.28}_{-0.29} | 2.53^{+0.14}_{-0.15} | 0.94^{+0.02}_{-0.02} | 9.17^{+0.88}_{-0.83} | 6.42^{+0.36}_{-0.36} |
| N07 (uncorrected) | A10       | 0.46^{+0.01}_{-0.00} | 4.15^{+0.30}_{-0.29} | 3.71^{+0.16}_{-0.16} | 0.98^{+0.02}_{-0.02} | 9.71^{+1.58}_{-1.29} | 7.37^{+2.50}_{-1.57} |
| A10 (uncorrected) | N07       | 0.46^{+0.01}_{-0.00} | 4.27^{+0.30}_{-0.30} | 3.75^{+0.16}_{-0.17} | 0.98^{+0.02}_{-0.03} | 9.46^{+0.98}_{-0.91} | 7.25^{+0.56}_{-0.55} |
| CL J1226+3332.9 | N07       | 0.60^{+0.02}_{-0.02} | 4.59^{+0.75}_{-0.68} | 3.59^{+0.30}_{-0.30} | 0.60^{+0.02}_{-0.02} | 4.59^{+0.75}_{-0.68} | 3.59^{+0.30}_{-0.30} |
| N07 (this work) | A10       | 0.59^{+0.02}_{-0.02} | 4.37^{+0.69}_{-0.57} | 3.49^{+0.31}_{-0.29} | 0.59^{+0.02}_{-0.02} | 4.37^{+0.69}_{-0.57} | 3.49^{+0.31}_{-0.29} |
| A10 (uncorrected) | N07       | 0.67^{+0.03}_{-0.00} | 6.29^{+0.82}_{-0.82} | 4.97^{+0.43}_{-0.72} | 0.67^{+0.03}_{-0.00} | 6.29^{+0.82}_{-0.82} | 4.97^{+0.43}_{-0.72} |
| A10 (uncorrected) | N07       | 0.63^{+0.02}_{-0.00} | 5.69^{+0.37}_{-0.38} | 4.31^{+0.43}_{-0.33} | 0.63^{+0.02}_{-0.00} | 5.69^{+0.37}_{-0.38} | 4.31^{+0.43}_{-0.33} |
| A10 (uncorrected) | N07       | 0.64^{+0.03}_{-0.00} | 4.86^{+0.95}_{-0.77} | 4.32^{+0.64}_{-0.57} | 0.64^{+0.03}_{-0.00} | 4.86^{+0.95}_{-0.77} | 4.32^{+0.64}_{-0.57} |
| A10 (uncorrected) | N07       | 0.62^{+0.02}_{-0.00} | 4.52^{+0.84}_{-0.64} | 3.95^{+0.56}_{-0.47} | 0.62^{+0.02}_{-0.00} | 4.52^{+0.84}_{-0.64} | 3.95^{+0.56}_{-0.47} |

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Erratum: 2011, ApJ, 728, L2