Quality Evaluation of the Earth’s Crust Height Movement Kinematic Model in the Northern Part of the Republic of Croatia

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Abstract. The focus of this paper is the quality evaluation of the relative height movement kinematic model of the Earth’s crust that covers the territory of the Republic of Croatia. This model offers necessary data for the calculation of relative vertical displacements and moving speed of Earth’s crust discrete points. Consequentially it can be used for reductions in direct levelling measurements from their surveying epoch to another chosen epoch by the elimination of relative height movement effects. As the quality of the model is not yet unambiguously analysed and determined this paper offers kinematic model quality analysis on the northern part of the Croatian territory. In this area, an indirect quality assessment method is used based on levelling measurements accuracy investigation. The measurements accuracy is determined in levelling networks of the 2nd order that almost completely cover the northern part of Croatia. Original levelling measurements from precise levelling network (2nd order state networks) are corrected for the systematic effect of benchmark’s relative height movements and reduced from their original survey epochs to the mean epoch of so-called Second High Accuracy Levelling Network (1st order state network). Precise levelling network is investigated simultaneously at the level of original and reduced measurements, comparing “a prior” and “a posterior” measurements accuracy criteria. The result suggests more or less successful elimination of the systematic effect of crustal relative height movements from the levelling measurements. Furthermore, the results confirm the hypothesis that the kinematic model can be reliably used for the determination of regional relative height changes and regional trends in crustal movements within the centimetre level of accuracy.

1. Introduction
Recent relative height movement kinematic model of the Earth’s crust that covers the territory of the Republic of Croatia is presented in [1]. This model was created on the basis of relative vertical displacements of the Earth’s crust determined from measurement and processing data of geometric levelling networks of the highest order of accuracy on the Croatian territory [2]. These 1st order networks, which were measured from 1874 to 1973, are Austrian Precise Levelling Network (abb. APN, mean survey epoch 1892.8), Fist High Accuracy Levelling Network (INVT, mean survey epoch 1949.0) and Second High Accuracy Levelling Network (IINVT, mean survey epoch 1971.1).

Height movement kinematic model of the Earth’s crust is realised, for any point of the crust, with three equiangular grid models that contain three specific parameters of kinematic movement. These
are the point’s relative height position $\Delta H_o$ at the initial epoch $t_o$ ($t_o = 1874.0$ – the start of the survey of APN), the point’s speed of vertical movement $v_o$ at the initial epoch $t_o$ and the constant point’s acceleration $a$. These three parameters are attached to the ellipsoidal coordinates of nodes in three equiangular grids, which are defined on the Bessel’s reference ellipsoid with starting meridian in Greenwich. Parameters $\Delta H_o$, $v_o$ and $a$ can be used for the determination of the relative height displacement $\Delta H_t$ between the initial epoch $t_o$ and any other epoch $t_i$ [1] using the expression

$$\Delta H_t = \Delta H_o + v_o(t_i - 1874.0) + \frac{1}{2}a(t_i - 1874.0)^2,$$

or for the determination of the current vertical movement speed $v_i$ in arbitrary epoch $t_i$ with

$$v_i = v_o + a(t_i - 1874.0).$$

From the kinematic point of view, expression (1) describes the uniformly accelerated or decelerated motion of any discrete point of the Earth’s crust, e.g. benchmark, along the axis of the height reference system. Height movement kinematic model of the Earth’s crust can be reliably used inside the time interval 1874-1973 and on the territory of Croatia, Slovenia and Bosnia and Herzegovina. This time and area limits arise from the survey epochs and areas of the underlying levelling networks APN, INVT and IINVT. Details regarding the creation of height movement kinematic model and its usage can be found in [1]. Model is available for download and usage on [3].

2. Height movement kinematic model

Height movement kinematic model of the Earth’s crust can be used in numerous ways. One of the most interesting usages, at least for geodesy, is a possibility to use the expression (1) to calculate relative height displacement of any benchmark between two different epochs $t_A$ and $t_B$ as

$$\Delta H_{AB} = \Delta H_B - \Delta H_A = \frac{1}{2}\Delta t_{AB}(2v_o + a(t_A + t_B - 3748.0)),\quad (3)$$

where $\Delta t_{AB}$ represents the time interval between older epoch $t_A$ and younger epoch $t_B$. If one observes two different benchmarks $R_1$ and $R_2$, which define a levelling line with a measured height difference $\Delta h$, one can furthermore calculate the correction of $\Delta h$ on the basis of the vertical displacements $\Delta H_{AB}$ of these benchmarks. Difference between vertical displacements $\Delta H_{AB}$, calculated as

$$r_{sh} = \Delta H_{R_1} - \Delta H_{R_2},$$

is the correction of measured height difference $\Delta h$ caused by the effect of height movement of the benchmarks $R_1$ and $R_2$, i.e. the effect of the vertical motion of Earth’s crust between two epochs. The reduced height difference can be simply calculated as

$$\Delta h_r = \Delta h + r_{sh}.\quad (5)$$

These basic expressions provide the possibility of using the kinematic model to reduce levelling measurements from their original survey epoch to any other chosen epoch, provided that they are both within the interval 1874-1973.

2.1. Quality of the kinematic model

Probably the most important information about a model is its quality. In the case of the height movement kinematic model of the Earth’s crust, quality of the model mainly depends on the quality of levelling data that were used for its creation [4]. Secondly, it depends on the methods that were used during the creation of the model. A most reliable direct way to assess the quality of the model is to use the model on the completely independent reference set of control data and then to compare the modelled data (results) with referent ones. As in this case there is no such set of reference data available, an indirect method of assessing the quality of the kinematic model was used [5].
This indirect method uses the above-mentioned possibility of reduction of the levelling measurements from their original survey epoch to another one. As it is common with hierarchically configurated state levelling networks, the lower order levelling networks are connected to higher order levelling networks, e.g. the 2nd order network to the 1st order network, the 3rd order network in the 2nd order network etc. When connecting to a higher order network, lower order network considers the data of higher order as absolutely correct (accurate). The problem is, at least in Croatia, that the levelling lines that form the state levelling networks were surveyed in a relatively long time interval. For example, levelling lines that form the levelling network of 2nd order is surveyed 20 to 25 years before the survey of the levelling network of 1st order to which they are being integrated. It is unquestionable that the benchmarks that connect 1st and 2nd order networks have changed their height positions during that period due to the motions of the Earth’s crust. This changes in height positions of benchmarks that form levelling lines are contained in the measured height differences $\Delta h$ and can therefore be qualified as systematic errors caused by the effect of Earth’s crust height movements. It is prudent to believe that the elimination or reduction of this effect from the original measurements would benefit the overall measurement and positioning quality of networks that are being integrated into the networks of a higher order. If it is really the case that this reduction increases the quality of the levelling measurements, this unambiguously proves that the quality of the kinematic model is sufficient for this purpose, at least to some noticeable level.

The described method of indirect quality assessment of the kinematic model was used in [5] on Croatian levelling networks of the highest quality orders, the 1st and the 2nd order networks. Two levelling figures of IINVT (1st order network) were chosen as best representatives of the quite diverse relief and climate structure of Croatia, levelling figures III and V which can be identified in figure 1. Measured height differences of levelling lines of the 2nd order networks were reduced from their respective survey epochs to the mean epoch of IINVT. 2nd order network was then adjusted and accuracy criteria of original and reduced measurements were calculated and compared. But, the results were contradictory to some extent. In figure III the quality increased and in figure V the quality decreased approximately at the same level. Regardless of possible explanations for this incidence that were offered in [5], it is important to extend the evaluation of the quality of height movement kinematic model of the Earth’s crust. The best way to provide concrete evidence of sufficiency or insufficiency of the kinematic model’s quality is to carry out this method of assessment in a much wider area of Croatia. In this work, this methodology is used to test the kinematic model on the whole of the northern part of Croatia, namely in levelling figures I, III, IV, VIII, IX and XII of IINVT network shown in figure 1.

3. Data for the testing
In total, 6 figures of IINVT are included in the assessment of the quality of the kinematic model. First of them is the figure I showed in figure 2. It consists of 10 levelling lines from the 1st order (IINVT) network and 15 levelling lines from the 2nd order network. They form 9 closed levelling loops. The northern part of the figure is not shown due to the scale. Loops I-VIII are completely on the Croatian territory, while loop IX represents the closure of levelling figure I mainly on the territory of the Republic of Slovenia. Blue lines represent the 1st order network while black lines schematically present the 2nd order network. Numbers of benchmarks are accordingly shown in blue and black colour, depending on the network they are a part of. These style and colouring are kept in all of the upcoming figures.
Figure 1. Levelling network of IIINVT

Figure 2. The configuration of levelling networks in figure I of the IIINVT
The second levelling figure that is included in the testing is figure III shown in figure 3. This is one of the two figures that were tested during the quality assessment undertaken in [5], but here it is included as an integral part of northern Croatia. It consists of 11 levelling lines of 1st order and 25 lines of 2nd order network. They form 13 levelling loops which are all completely situated on the Croatian territory.

![Figure 3](image.png)

**Figure 3.** The configuration of levelling networks in figure III of the IINVT [5]

The third levelling figure which is included in the assessment of quality is figure IV shown in figure 4. The figure consists of 11 levelling lines of 1st order and 19 levelling lines of 2nd order network which are forming 13 closed levelling loops. As can be seen in figure 1, a part of the IINVT figure falls on the territory of Bosnia and Herzegovina.

Levelling figure VIII is shown in figure 5. It consists of 12 levelling lines of 1st order and 19 levelling lines of 2nd order network. They form the total of 13 levelling loops which are all completely inside Croatian country borders. Despite the fact that loops III, XII and XIII don’t actually fall inside the levelling figure, they were included in the assessment because they fall inside of the Croatian territory and should therefore not be excluded.

Levelling figures IX and XII are the last of the 6 figures of IINVT that are included in the testing. Because of some specifics in the processing and adjustment of the measurements, these two figures were originally adjusted as a whole [6], and are therefore in this paper also treated as a whole i.e. as
one figure. Figures IX and XII consist of 4 levelling lines of 1st order and 23 levelling lines of 2nd order network that are forming 9 loops in total. Figure 6 shows parts of the IINVT figures IX and XII scaled so that loops on the Croatian territory can be seen and identified. Parts of these levelling figures that fall on the territories of Serbia and Bosnia and Herzegovina, i.e. the loop IX, can be seen in figure 1.

![Figure 4. The configuration of levelling networks in figure IV of the IINVT](image)

4. Reduction of the measured data
Levelling lines of the 2nd order network shown in chapter 3 were all surveyed between 1938 and 1953. Using the height movement kinematic model of Earth’s crust and expressions (3), (4) and (5), originally measured height differences are reduced to the mean survey epoch of 1st order levelling network, the IINVT, which is 1971.1. Two sets of levelling loop misclosures were then calculated, \( W \) on the basis of original and \( W_r \) on the basis of reduced measurements. In calculations, adjusted levelling line height differences of IINVT were used. Results are presented for comparison, together with lengths of the closed loops \( F \), in table 1.
Figure 5. The configuration of levelling networks in figure VIII of the IINVT.

Figure 6. The configuration of levelling networks in figures IX and XII of the IINVT.
Table 1. Levelling loop misclosures of original and reduced measurements.

| Loop | $F$ [km] | $W$ [mm] | $W_r$ [mm] | Loop | $F$ [km] | $W$ [mm] | $W_r$ [mm] | Loop | $F$ [km] | $W$ [mm] | $W_r$ [mm] |
|------|---------|---------|----------|------|---------|---------|---------|------|---------|---------|---------|
| I    | 135.26  | 32.5    | 4.1      | XI   | 161.84  | -13.2   | -10.4   | V    | 137.53  | -40.1   | -34.3   |
| II   | 109.55  | 14.7    | 6.7      | XII  | 99.98   | -28.0   | -3.3    | VI   | 67.01   | -7.6    | -17.8   |
| III  | 120.43  | 25.6    | 9.8      | XIII | 166.77  | -38.3   | -54.9   | VII  | 65.88   | -24.1   | -18.0   |
| IV   | 136.87  | 24.1    | 44.4     |      |         |         |         | VIII | 11.85   | 1.1     | -1.0    |
| V    | 26.82   | -32.6   | -30.6    |      |         |         |         | IX   | 150.05  | 33.1    | 49.8    |
| VI   | 119.38  | -11.1   | -10.1    |      |         |         |         | X    | 13.30   | -0.9    | 3.5     |
| VII  | 11.89   | -3.5    | -4.4     |      |         |         |         | XI   | 157.65  | 7.9     | 2.7     |
| VIII | 254.14  | -65.4   | -18.6    |      |         |         |         | XII  | 51.26   | -15.9   | -15.9   |
| IX   | 681.05  | 15.7    | -1.4     |      |         |         |         | XIII | 96.73   | 23.0    | 16.3    |
|      |         |         |         |      |         |         |         |      |         |         |         |
| Figure I |       |         |         | Figure III |     |         |         | Figure VIII |     |         |         |         |
| I    | 150.25  | 2.2     | 6.8      |      |         |         |         | V    | 127.78  | 0.6     | 15.6    |
| II   | 127.06  | 13.6    | 2.4      |      |         |         |         | VI   | 85.89   | -24.0   | -23.2   |
| III  | 175.50  | -9.4    | -7.3     |      |         |         |         | VII  | 116.60  | 2.6     | 2.9     |
| IV   | 115.73  | -6.2    | -7.5     |      |         |         |         | VIII | 119.24  | 1.7     | 1.6     |
| V    | 108.73  | 45.1    | 29.6     |      |         |         |         | IX   | 111.64  | -14.5   | -14.4   |
| VI   | 200.89  | 39.2    | 38.7     |      |         |         |         |      | 105.06  | 7.0     | 6.8     |
| VII  | 143.06  | 14.1    | 13.9     |      |         |         |         |      | 30.90   | 3.0     | 7.4     |
| VIII | 87.97   | 59.6    | 64.6     |      |         |         |         |      | 48.66   | -8.1    | -2.3    |
| IX   | 5.96    | -13.9   | -13.6    |      |         |         |         |      | 807.33  | 31.7    | 5.6     |
| X    | 66.09   | -64.8   | -59.0    |      |         |         |         |      |         |         |         |

Table 1 shows that in most cases the levelling loop misclosure has decreased in value by reduction of measurements. To be more precise, in 38 out of the 55 loops value of misclosure have decreased by a moderate to high amount. However, in 8 loops reduction led to some noticeable deterioration causing somewhat big increases in loop misclosures which are underlined in table 1. It is also important to state that the sign of misclosures is almost always preserved, i.e. the sign of a misclosure has changed in only 4 cases. All of this leads to believe that the effect of relative height movements of Earth’s crust is to noticeable extent successfully eliminated from original levelling measurements. In figure 7, all of the loops whose misclosures decreased by the elimination of influences caused by height movements of benchmarks are painted in grey. It can be seen that these loops are more or less uniformly distributed among the whole tested area.

Figure 7. Levelling loops in which the loop disclosure decreased
5. Quality evaluation

Accuracy criteria of measurements that form a levelling network of firm geometric configuration, like the ones we have in each of the IINVT network figures, can be done in two ways. One of them is “a prior” criteria, i.e. based on the levelling loops misclosures before the network adjustment, and the other one is “a posterior” criteria, i.e. based on the adjustment of the network. The “a prior” approach considers a levelling network as a set of completely unrelated loops, while the “a posterior” approach observes the network as actually geometrically whole with firmly connected parts. The reason to calculate both of these accuracy criteria is that their comparison can offer insight into the amount of systematic errors contained within measurements.

“A prior” measurement reference probable errors are calculated from loop misclosures $W$ of a network, shown in table 1, as

\[
u_F = \pm \frac{2}{3} \sqrt{\frac{\sum W_i^2}{\sum F_i}},
\]

where loop lengths $F$ are introduced as weights [7]. “A prior” measurement errors of reduced measurements are accordingly calculated from loop disclosures $W_r$. To calculate the “a posterior” probable measurement errors $u_\gamma$ networks of 2nd order were adjusted in each of the IINVT figures included in the testing. Adjustments were made two times, once with the original and once with the reduced measurements. Classic regular adjustment of indirect measurements is applied, using the least squares method and reciprocals of levelling line lengths as weights [8]. Official heights of connecting benchmarks of 1st order are taken over from the State geodetic administration of Croatia.

“A prior” accuracy criteria $u_F$ and “a posterior” criteria $u_\gamma$ of original and reduced measurements, for all of the tested IINVT figures, are shown in table 2. It can be seen that in 4 of the 5 cases both $u_F$ and $u_\gamma$ are lowered by the reduction of measurements indicating measurement accuracy increase. Figure III and figures IX and XII have moderate accuracy increase at the submillimeter level, from 0.1 to 0.3 mm. On the other hand, in figures I and IV the improvements in quality are more significant, at the level of 0.5 to 0.9 mm. But, in figure VIII some deterioration occurs by reduction of measurements which caused the decrease of “a posterior” accuracy by 0.2 mm, while “a prior” accuracy stays the same. The average, calculated as an ordinary arithmetic mean of criteria from all figures, shows that the removal of influences caused by height movements of benchmarks decreases $u_F$ by 0.2 and $u_\gamma$ by 0.4 mm on average. Similar results are made when calculating the “a prior” and “a posterior” criteria of all of the measurements as a whole, not divided by figures. $u_F$ calculated from all of the levelling loops misclosures changes from 1.5 to 1.3 mm by reduction, while $u_\gamma$ decreases from 2.6 to 2.2 mm. These results in total, together with results of misclosures from chapter 4, point to the fact that elimination of a systematic influence of Earth’s crust motion from original levelling measurements can be successfully made with the height movement kinematic model, and with adequate quality, at least on the tested area.

Table 2. Accuracy criteria of original and reduced measurements by figures and their average

| Indicator | $\Delta h$ | $\Delta h$ | Indicator | $\Delta h$ | $\Delta h$ | Indicator | $\Delta h$ | $\Delta h$ |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| Figure I  |           |           | Figure IV |           |           | Figures IX and XII |           |           |
| $u_F$     | 1.5       | 1.0       | $u_F$     | 1.5       | 0.9       | $u_F$     | 0.7       | 0.6       |
| $u_\gamma$| 3.1       | 2.2       | $u_\gamma$| 2.6       | 1.7       | $u_\gamma$| 1.0       | 0.9       |
| Figure III|           |           | Figure VIII|          |           | Average   |           |           |
| $u_F$     | 2.0       | 1.9       | $u_F$     | 1.5       | 1.5       | $u_F$     | 1.4       | 1.2       |
| $u_\gamma$| 3.4       | 3.1       | $u_\gamma$| 2.2       | 2.4       | $u_\gamma$| 2.5       | 2.1       |
6. Results and discussion

The results of this paper resolve the ambiguity of previous assessment of the quality of the Earth’s crust height movement kinematic model. It is shown that the quality of the kinematic model is more or less adequate even for reduction of original levelling measurements from their survey epoch to another arbitrary epoch. However, elimination of a systematic influence of height movements of benchmarks caused by Earth’s crust motion does not always lead to an increase in the quality of measurements. A possible explanation for this incidence can be found in the complex system of mutual correlations between various systematic errors. Anyway, based on the explicit Croatian case, one can conclude that at least a noticeable part of benchmarks height movement effects can be reduced from the 2nd order levelling measurements, which contributes to the general increase of the height positioning quality.

At the same time, it can be also concluded that the Earth’s crust height movement kinematic model can be used to calculate relative height changes of points on Croatian territory between 1874 and 1973 at a reliable centimetre level of accuracy. Next logical step that could further confirm or reject this conclusion is the assessment of the quality of the kinematic model on the area that would be equal to the full model’s coverage. This can be done using the same methodology but for all of the remaining figures of IINVT network on the territory of the Republic of Croatia.

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