Current fluctuations of an interacting quantum dot

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We calculate the counting statistics of electron transfer through an open quantum dot with charging interaction. A dot that is connected to leads by two single-channel quantum point contacts in an in-plane magnetic field is described by a Luttinger liquid with impurity at the Toulouse point. We find that the fluctuations of the current through this conductor exhibit distinctive interaction effects. Fluctuations saturate at high voltages, while the mean current increases linearly with the bias voltage. All cumulants higher than the second one reach at large bias a temperature independent limit.

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The statistical distribution of current fluctuations in nanoscale conductors, the so-called “counting statistics”, has received considerable attention over the past decade - theoretically \cite{7} as well as experimentally \cite{8}. The theory of this distribution is well developed for non-interacting conductors. Effects of electron-electron interactions, however, have only been studied in a few limiting cases so far. Interacting conductors in the tunneling limit of weak transmission have been shown to exhibit Poissonian current fluctuations \cite{4}. Also for weakly interacting conductors at arbitrary transmission the statistics of charge transfer has been found to be qualitatively identical to that of a non-interacting system - it is multinomial \cite{8,9}. In a conductor that has arbitrary transmission combined with strong interactions one expects more profound changes of that statistics. This is indicated by a classical analysis \cite{8} that has already revealed qualitative changes of the distribution of current fluctuations in that regime. In this Letter, we confirm this expectation by a nonperturbative quantum mechanical analysis of non-Gaussian current fluctuations in a strongly interacting nanostructure at arbitrary transmission.

An interacting nanostructure is at low energy scales described by the model of a non-interacting conductor in an electromagnetic environment, that is a quantum conductor with an effective series resistor \cite{8}. Recently, Safi and Saleur \cite{9} have shown, that for a single-channel conductor this model maps onto a Luttinger liquid (LL) with impurity. The statistics of fluctuations for this model has been obtained at zero temperature in the context of tunneling in fractional quantum Hall samples \cite{10}. From a perturbative analysis one expects that the current $I$ in systems described by a LL scales with a power of the applied voltage $V$ \cite{11}. In the limit of a strong impurity, one has $I \propto V^{2g−1}$, where $g$ is the interaction parameter of the LL. The current $I_B$ that is backscattered by a weak barrier obeys $I_B \propto V^{2g−1}$. Evidently then, $g = 1/2$ is a special point in parameter space: The backscattered current becomes independent of the applied voltage. By virtue of the mapping of Ref. \cite{9}, this unusual behavior should be observable in a quantum conductor with series resistance $1/G_0$, where $G_0 = e^2/h$ is the conductance quantum. An open quantum point contact (QPC) is a natural realization of such a series resistor.

Motivated by this, we study transport through the interacting quantum dot (QD) shown in Fig. 1. It is connected to two leads via single-channel QPCs. One of them (QPC 1) is open such as to act as the desired series resistor to a scatterer in QPC 2. An in-plane magnetic field allows only electrons in one particular spin state to pass the QPCs. We address the incoherent case when inelastic relaxation inside the QD is faster than electron escape. The charging energy of the QD is assumed to be much larger than the temperature $T$ and the applied voltage. In our analysis, we employ techniques that have been developed in the context of the Coulomb blockade \cite{12,13,14,15} and charge pumping in almost open QDs \cite{16}. After bosonization, the QD in Fig. 1 is at low energies, indeed, described by a LL at $g = 1/2$ (the Toulouse point). This allows us to study its transport behavior nonperturbatively by renormalization. We find as expected that $I_B$, the current backscattered by QPC 2, saturates with increasing voltage. It is, however, hidden in the total current $I = I_0 - I_B$ under a large background current $I_0 = gG_0V$ of charge carriers that are not backscattered. Analyzing also the fluctuations of the current through the structure, we find that the same physics leads to a saturation with increasing $V$ of non-Gaussian

\textbf{FIG. 1:} QD connected to two leads by single-channel point contacts QPC 1 and QPC 2. QPC 2 introduces backscattering. The current $I$ of charge transferred from lead 1 to lead 2 is measured.
current fluctuations as well. No similar background as $I_o$ is present for these fluctuations, since all electrons that are not scattered produce only Gaussian thermal noise. This, together with recent experimental advances in the detection of non-Gaussian current fluctuations \[13\], brings the observation of this counterintuitive saturation effect within experimental reach. Remarkably, all cumulants higher than the second one are entirely temperature independent in the limit $V \to \infty$. This is in marked contrast with the behavior of non-interacting conductors.

We assume the QD large enough for electrons to relax inelastically before they escape the dot. In this case, the QD can be modeled by one one-dimensional electron mode for each QPC \[12, 17\] subject to a charging interaction inside the QD (at $x > 0$, cf. Fig. \[\ref{fig:fig1}\]). At the energies $\varepsilon$ of interest, that are smaller than the Zeeman energy of electrons in the applied magnetic field, only electrons of one particular spin projection contribute to transport through the QPCs. For typical conductors the electronic spectrum at the relevant energies can be linearized and the Hamiltonian is diagonalized by bosonization. The QD is then described by two bosonic fields $\theta_j$ and $\phi_j$ for every lead $j$ that describe electron density fluctuations \[14\], \[17\]. Moreover, for energies below the charging energy $E_c$ of the QD, $\varepsilon \ll E_c$, the “charge” mode $\theta_1 + \theta_2$ is pinned and the system is described by an effective Hamiltonian $H_{\text{eff}}$ for the “transport” modes $\theta_s = (\theta_1 - \theta_2)/2$ and $\phi_s = (\phi_1 - \phi_2)/2$. $\theta_s(0)$ is the operator of the charge $Q$ transported through the QD, $Q = e\theta_s(0)/\pi$. We have $(\hbar \equiv k_B = 1)$

$$H_{\text{eff}} = \int \! dx \left\{ \frac{v_F}{2\pi} \left[ \frac{1}{2} (\partial_x \phi_s)^2 + 2(\partial_x \theta_s)^2 \right] \right. + \left. \delta(x) \left( 2\lambda \cos 2\theta_s + \frac{eV}{\pi} \theta_s \right) \right\} . \tag{1}$$

$\lambda$ is the backscattering strength of QPC 2.

To quantify the current $I$ through the QD and its fluctuations, we compute a generating function $Z$ that generates moments of the charge $Q = \int_0^\tau dt I(t)$ transferred during time $\tau$,

$$Z(\theta) = \sum_k \frac{\xi_k^{\theta}}{k!} \langle Q_k^\theta \rangle = \exp \left[ \tau \sum_k \frac{\xi_k^{\theta}}{k!} C_k \right] , \tag{2}$$

where $\tau C_k$ is the $k$th cumulant of the distribution. Writing $Z$ as a Keldysh path integral \[18\] and integrating out $\phi_s$ as well as all modes $\theta_j(x)$ at $x \neq 0$, we arrive at

$$Z(\xi) = \int \! D\theta \; e^{S_0 + S_{\text{int}} - \int \! dt 2\lambda (\cos 2\theta^+ - \cos 2\theta^-)} \tag{3}$$

with the free action

$$S_0 = \int \! \frac{\omega}{2\pi} \left[ 4N\theta^+\theta^- - (2N + 1)(|\theta^+|^2 + |\theta^-|^2) \right],$$

for fields $\theta^\pm$ corresponding to the mode $\theta^\pm(0)$ on the forward ($\theta^+$) and the backward ($\theta^-$) part of the Keldysh time contour. $N = [\exp(\omega/T) - 1]^{-1}$ is the Bose-Einstein distribution. The source term

$$S_{\text{int}} = i \int_0^\tau \! dt \left[ \frac{\epsilon^\theta}{2\pi} \partial_t (\theta^+ + \theta^-) - \frac{eV}{\pi} (\theta^+ - \theta^-) \right] \tag{4}$$

couples $V$ and $\xi$ to charge $e\theta/\pi$ and current $-e\partial_t \theta/\pi$, respectively \[19\]. We eliminate $S_{\text{int}}$ by the change of integration variables $\theta \rightarrow \theta^+ - eVt + \epsilon \xi/8$, where $\epsilon = V - 2i\xi T$. The resulting $Z$ can equivalently be obtained by integrating out all modes $\phi(x)$ and $\theta(x)$ at $x \neq 0$ in a path integral corresponding to the Hamiltonian expression

$$Z(\xi) = Z_0(\xi) \langle T_{\pm} \; e^{i \int_0^\tau \! dt H^{-}(t)} e^{-i \int_0^\tau \! dt H^{+}(t)} \rangle \tag{5}$$

with time-dependent Hamiltonians

$$H^\pm(t) = \int \! dx \left\{ \frac{v_F}{2\pi} \left[ \frac{1}{2} (\partial_x \phi)^2 + 2(\partial_x \theta)^2 \right] \right. + \left. 2\lambda \delta(x) \cos \left( 2\theta \pm \frac{1}{2} e\xi - \frac{1}{2} eVt \right) \right\} . \tag{6}$$

$T_{\pm}$ orders operators along the Keldysh contour and

$$Z_0(\xi) = \exp \left[ - \frac{\xi^2}{4\pi} (i V \xi - T_\xi^2) \right] \tag{7}$$

is the generating function in the absence of backscattering. We now follow the standard procedure \[17\] to refermionize $H^\pm$. For this we define new fields

$$\phi_s(x) = \frac{1}{2} [\phi(x) \mp \phi(-x)] + \theta(x) \pm \theta(-x) \tag{8}$$

and a chiral Fermion $b = a^{-1/2}\exp(i\phi_+)$ with a short distance cutoff $a$. $H^\pm$ then have a representation as non-interacting Hamiltonians for $b$. Following Matveev \[14\] we introduce a Majorana fermion $d + d^\dagger$ and define new fermion operators $c$ by the relation $b = (d + d^\dagger)c$. This brings $H^\pm$ into a quadratic form in fermion operators,

$$H^\pm(t) = \int \! \frac{dk}{2\pi} \left\{ \frac{v_F}{2\pi} k c^\dagger_k c_k \right. + \left. \sqrt{\pi} \lambda \left( d + d^\dagger \right) c e^{\pm i\epsilon / 4 - i\epsilon Vt / 2} + h.c. \right\} . \tag{9}$$

Inserting Fermion coherent states, we rewrite Eq. \[15\] in refermionized form as a path integral \[20\]. We again integrate out all modes $c(x \neq 0)$ and are left with an integral over vector fields $c = (c^+(0), c^-(0))$ and $d = (d^+, d^-)$,

$$Z(\xi) = Z_0(\xi) \int \! Dc \; Dd \; \exp \left\{ - i \int \! \frac{d\omega}{2\pi} \left[ d^* G_d^{-1} d + c^* G_c^{-1} c + \sqrt{\pi} \lambda \left( d + d^\dagger \right) e^{i\epsilon / 4 + \pi Vt / 2} + h.c. \right] \right\} . \tag{10}$$
where $\tau^3$ is the third Pauli matrix. $G_d(\omega) = -i\langle T_B d(\omega)d(\omega)\rangle = -\tau^3/\omega$ is the Green function of $d$ and $G_c$ that of $c$ corresponding to the Hamiltonian at $\lambda = 0$. The time dependence of the scattering term has been removed by a gauge transformation $c \to \exp(i e V t/2) c$ that shifts the frequency of $G_c$. Since $G_c$ is an electron Green function $G_c(x, x')$ evaluated at coinciding spatial coordinates $x = x'$, it is linearly related to the semiclassical Keldysh Green function of $c$

$$G_s = \left( \frac{1 - 2f}{2(1 - f)} \right),$$

(11)

$G_c = G_s/4i v_F$. Here, $f(\omega) = [\exp(\omega - eV/2)/T + 1]^{-1}$ is the Fermi distribution function after the gauge transformation.

The action in Eq. (10) is diagonal in frequency and $Z$ consequently factorizes into contributions from different frequencies. The Gaussian integrals result in

$$\ln Z(\xi) = \ln Z_0(\xi) + \tau \int_0^\infty d\omega \ln\left\{1 + \frac{T_B^2}{\omega^2 + T_B^2}\right\}$$

$$\left\{ (e^{i\xi} - 1)f^+(1 - f^-) + (e^{-i\xi} - 1)f^-(1 - f^+) \right\}$$

(12)

with $f^+ = f$, $f^-(\omega) = 1 - f(-\omega)$, and $T_B = a\lambda^2/2v_F$.

We remark that one arrives at an equivalent result by applying the quasiparticle formalism developed by Fendley, Ludwig, and Saleur [21] to a LL at $g = 1/2$.

From Eq. (12) we find for the first three cumulants

$$C_1 = \frac{e^2}{4\pi} V \left[ 1 - \frac{2T_B}{eV} \text{Im} \psi\left( \frac{1}{2} + \frac{2T_B + i eV}{4\pi T} \right) \right],$$

(13)

$$C_2 = 2T_B \frac{dC_1}{dV} - \frac{e}{2T_B} \text{coth}\left( \frac{eV}{2T} \right) \frac{dC_1}{dT_B}$$

$$+ \frac{TT_B}{dV} \frac{d^2C_1}{dT_B^2},$$

(14)

$$C_3 = \frac{1}{2TT_B} \frac{dC_2}{dV} - \frac{e}{2T_B} \text{coth}\left( \frac{eV}{2T} \right) \frac{dC_2}{dT_B}$$

$$+ \frac{2T}{dV} \frac{dC_2}{dT_B} + \frac{e}{2T_B} \text{coth}\left( \frac{eV}{2T} \right) \frac{d^2C_1}{dT_B^2}$$

$$+ \frac{e^2}{4T_B} \sinh^{-2}\left( \frac{eV}{2T} \right) \frac{dC_1}{dT_B},$$

(15)

where $\psi$ is the digamma function. At zero temperature, Eqs. (13) - (15) obey the known relations between the higher order cumulants and $C_1$ derived in Ref. [11].

We first analyze the limit of a large voltage $eV \gg T, T_B$ (while $eV \ll E_c$). In this limit, the backscatterer is weak. Charge transfer is then best understood by singling out two contributions: First, transfer of electrons that are not backscattered. They are responsible for $I_0$. Second, the contribution due to backscattered electrons, generating $I_B$ which converges to a limiting value $I_B^\infty = (e/4)T_B$

In the total current $I = I_1 + I_B$ is, however, hidden under the contribution $I_0 = G_0 V/2$ of unscattered electrons that increases linearly with $V$. Similarly, the large voltage limit $C_2^\infty = G_0(\pi T_T^B/4 + T)$ of the second cumulant, Eq. (14), has these two contributions: the fluctuations due to the backscatterer as well as the Johnson-Nyquist noise $G_0T$ of electrons that are not backscattered. Non-Gaussian current fluctuations, in contrast, allow to specifically probe the backscatterer and its large voltage behavior. This is because unscattered electrons produce purely Gaussian fluctuations. Accordingly we find that all higher order cumulants saturate at large voltages and, remarkably, their limiting values are temperature independent (see Fig. 2 for $C_3$). The generating function in this limit takes the form

$$\ln Z^\infty(\xi) = \tau \left[ \frac{G_0}{2} (-i eV \xi - T^2) + \frac{T_B}{2} (e^{i\xi/2} - 1) \right].$$

The temperature independence of higher order cumulants in the large voltage limit is in stark contrast to the behavior of a non-interacting conductor [11]. It is thus a clear signature of interactions in the QD considered here. It can be understood by noting that a large voltage bias shifts the energies at which electron occupation numbers are thermally smeared far away from the equilibrium Fermi level. Electrons with these high energies are, however, effectively not backscattered and therefore produce purely Gaussian noise. Thermal fluctuations do thus not contribute to higher order cumulants. With present day experimental techniques this anomalous behavior can be observed in $C_3$, the third cumulant of current fluctuations, Eq. (14). It has the temperature independent large voltage limit $C_3^\infty = -(e\pi/8)G_0 T_B$, as shown in Fig. 2. In
the opposite limit of small voltages, we find that at low temperatures $T \ll eV \ll T_B$ (moderately low such that the inelastic processes that justify our model are still operative) the statistics

$$\ln Z(\xi) = \frac{TB}{48\pi} \left( e^{-ie\xi} - 1 \right) \left( \frac{eV}{TB} \right)^3 + O \left( \frac{eV}{TB} \right) .$$

Due to the scaling of the impurity strength with energy, the high and the low voltage limits, Eqs. (10) and (17), correspond to the weak and the strong backscattering limit, respectively. They are Poissonian, in accordance with Refs. [4,10]. At low voltages, as manifest in Eq. (17), charge is transferred in units of the elementary charge. The current scales as $I \propto V^3$, as expected from perturbative calculations [11]. The high voltage statistics, Eq. (16), suggests, that charge is transmitted in packets carrying $\frac{1}{2}$ the elementary charge. This is a direct consequence of the charging interaction of the QD that after every backscattering of an electron induces a positive electric potential on the QD. It attracts electrons from the leads to compensate for the electron that is missing on the QD. In response, an electron flows onto the QD from either lead 1 or from lead 2. This either cancels or completes the transfer of an electron through the QD that was initiated by the backscattering event. Both processes are equally likely in the weak backscattering limit of an almost open contact QPC 2. Therefore only every other backscattering event transfers charge through the QD. Equivalently one can say that every such event transfers only $e/2$, as indicated by Eq. (10).

In the shot noise limit of zero temperature, the third cumulant for non-interacting electrons vanishes at transmission $\Gamma = 1/2$. This is because in this case every electron is transmitted or reflected with the same probability $1/2$ independently of all other electrons. The distribution of transferred charge is consequently symmetric around its mean and its skewness $\mathcal{C}_3$ vanishes. This intuition remains correct for weakly interacting conductors [3]. One expects it to be invalidated, however, by strong interactions that correlate the transfers of different electrons with each other. The inset of Fig. 2 shows that this is, indeed, the case for the QD we consider. We define an effective single-electron transmission $\Gamma_{\text{eff}} = I / (G_0V - I)$ for QPC 2 in series with QPC 1 by Ohm’s law. $\mathcal{C}_3$ vanishes then for $\Gamma_{\text{eff}} \approx 0.26$, in clear contrast with a non-interacting structure.

In conclusion, we have studied current fluctuations in a strongly interacting quantum conductor at arbitrary transmission. While it had been found in Refs. [3,4] that interactions do not qualitatively change the statistics of current fluctuations in perturbative situations, our nonperturbative solution does display features that are qualitatively different from those of non-interacting structures. This makes the measurement of current correlations a promising tool to probe interactions in the QD we considered and most probably in many other strongly interacting conductors. More specifically, we find that the fluctuations of the current saturate at high voltages, while the current itself increases linearly with the applied voltage. Moreover, all cumulants higher than the second one are temperature independent in the high voltage limit. We have discussed in detail how the voltage and temperature dependence of the third cumulant display these and other qualitative interaction effects. Experimental techniques for its measurement are available [3]. Our predictions are thus experimentally testable.

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Eq. (3) can also be obtained from the Hamiltonian of a one-dimensional conductor of spinless electrons with backscattering and a series resistor of resistance $1/G_0$ along the lines of Ref. [9]. Our results apply, therefore, as well to the problem of the environmental Coulomb blockade with series resistance $1/G_0$.

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