Unsteady Blood Flow with Nanoparticles Through Stenosed Arteries in the Presence of Periodic Body Acceleration

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Abstract. The effects of nanoparticles such as Fe\textsubscript{3}O\textsubscript{4}, TiO\textsubscript{2}, and Cu on blood flow inside a stenosed artery are studied. In this study, blood was modelled as non-Newtonian Bingham plastic fluid subjected to periodic body acceleration and slip velocity. The flow governing equations were solved analytically by using the perturbation method. By using the numerical approaches, the physiological parameters were analyzed, and the blood flow velocity distributions were generated graphically and discussed. From the flow results, the flow speed increases as slip velocity increases and decreases as the values of yield stress increases.

1. Introduction
Artery stenosis (or atherosclerosis) is an arterial disease caused by accumulation of fats and fibrous tissue in the lumen of arterial wall [1]. The presence of stenosis is detrimental to a blood circulatory system. The study of blood flow in stenosed arteries has been reported by many researchers. Chakravarty and Mandal [2] studied the two-dimensional blood flow through tapered arteries with stenosis. Liu et al. [3] reported that stenosis restricted the flow field at the end of stenotic and tapered arteries. Sankar and Lee [4] investigated analytically the pulsatile flow model of Herschel Bulkley fluid in stenosed arteries with the help of regular perturbation method. Regular perturbation method was used by Akbar et al. [5] in order to study the characteristics of Reiner-Rivlin model in tapered arteries with stenosis. Later, Akbar [6] analyzed the tangent hyperbolic blood flow through tapered artery with mild stenosis. Recently, the slip effects on the wall of tapered artery with mild stenosis have been studied by Ijaz and Nadeem [7] by treating the working fluid as Newtonian.

Flow cases involving different stenosed arteries have been studied by several researchers. Mandal et al. [8] investigated the effects of periodic body acceleration on blood flow. They considered the arterial wall as an elastic cylindrical tube containing a stenosis on the lumen. Mishra et al. [9] modelled the blood flow through a composite stenosis in an artery with permeable wall. Yadav and Kumar [10] studied the effects of length of stenosis on the resistance of blood flow (Bingham plastic model) through a generalized artery with multiple stenosis. They found that the flow resistance decreased as the shape parameter increased and the size of stenosis decreased. The study of blood flow subjected to magnetic
field in arterial stenosis (with slip effect) has been investigated by Hazarika and Sharma [11]. They observed that as the Hartman number increased, both fluid velocity and wall shear stress would decrease. Also, they claimed that increased Reynolds number would lead to the increases of fluid velocity and wall shear stress. Siddiqui et al. [12] have modelled the blood as non-Newtonian Bingham plastic fluid model. The model was used to study the flow through stenosed artery in the presences of slip velocity and body acceleration.

The impacts of nanoparticles such as copper, titanium and aluminum on mild stenosis were studied by Ahmed and Nadeem [13]. Rahbari et al. [14], investigated the magnetic nonNewtonian blood flow (consisting of nanoparticles) in the permeable arteries. Their result showed that the decrease of Brownian motion parameter would increase the thermophoresis rate as well as the blood velocity. The impact of nanoparticles on the stenosed blood flow subjected to magnetic field has been studied by Shahzadi and Nadeem [15]. By varying the flow parameters such as velocity, streamlines, pressure rise and pressure gradient have been examined. Hayat et al. [16] reported the impact of external magnetic field on the entropy generation of nanoparticles in peristaltic flow (in rotating frame of reference). The associated heat transfer process due to radiation has been studied as well. The dispersion of titanium magneto-nanoparticles in peristaltic blood flow through a uniform tube has been reported by Bhatti et al. [17]. The Sisko fluid model has been considered in representing the blood.

As highlighted by Siddiqui et al. [12], modelling of nanoparticles in blood flow is gaining popularity nowadays. In the current work, we have studied the blood flow through an arterial stenosis (with nanoparticles) in the presences of periodic body acceleration and oscillating pressure gradient acting in the axial direction. The effects of nanoparticles on the blood velocity have been analyzed numerically using Mathcad software. The proposed mathematical model could serve as a promising diagnostic tool.

2. Formulation of the problem

A stenosis is generated in the arterial wall of radius $\overline{R}(\overline{z})$ as shown in Figures 1. The size of stenosis is expressed in axial direction $'z'$ and height in which its growth model (see Eq. (1)) follows that reported by Siddiqui and Shah [12]. We assumed that the blood flowed in the axial direction and nanoparticles were uniformly distributed throughout the blood.

\[
\frac{\overline{R}(\overline{z})}{\overline{R}_0} = \begin{cases} 
1 - \overline{\psi} \left[ \overline{L}_0^{m-1} \left( \overline{z} - \overline{d} \right) - \left( \overline{z} - \overline{d} \right)^m \right] & , \quad \overline{d} \leq \overline{z} \leq \overline{d} + \overline{L}_0 \\
1 & , \quad \text{otherwise}
\end{cases}
\]

(1)
where $\tilde{R}(\bar{z})$ is the radius of the artery in the stenosed region, $R_0$ is the radius of the normal artery, $L_0$ is the length of the stenosis, $d$ is the region of the stenosis and $\bar{z} = \frac{z}{L_0}$, $\xi = \frac{\xi_s}{R_0}$ such that $\xi_s/R_0 < 1$.

For an unsteady blood flow in artery shown in Fig. 1, it is assumed that: (1) there is a force due to the relative motion between fluid and nanoparticles; (2) the motion of nanoparticles is governed by Newton’s second law; and (5) the driving force added to the fluid flow is a pulsating pressure gradient, i.e.,

$$\rho_{nf} \frac{\partial \bar{u}}{\partial \bar{t}} = -\frac{\partial \bar{p}}{\partial \bar{z}} - \frac{1}{\tau} \frac{\partial}{\partial \bar{r}} (\bar{r} \bar{F}(\bar{t})),$$

where $\rho_{nf}$ is the effective density of nanofluid, $\bar{u}$ is the axial velocity along the $z$-direction, $\bar{t}$ is the time and $\tau$ is the shear stress.

According to Siddiqui et al. [12] and Akbarzadeh [17], the flow of blood is generated by the pressure gradient $\frac{\partial \bar{p}}{\partial \bar{z}}$ due to the heart pumping action and the body acceleration $\bar{F}(\bar{t})$, is assumed to be given by a harmonic formula as follows:

$$-\frac{\partial \bar{p}}{\partial \bar{z}} (\bar{z}, \bar{t}) = A_0 + A_1 \cos (\bar{\omega_p} \bar{t}),$$

where $A_0$ is the steady-state part of the pressure gradient, $A_1$ is the amplitude of the pressure fluctuation giving rise to the systolic and diastolic pressures, $\omega_p = 2\pi f_p$ is the heart pressure frequency, where $f_p$ is the pulse rate frequency. While

$$\bar{F}(\bar{t}) = A_g \cos (\bar{\omega_g} \bar{t} + \phi),$$

where $A_g$ is the amplitude of the acceleration, $\omega_g = 2\pi f_g$ is the frequency, where $f_g$ is the pulse rate frequency and $\phi$ is the lead angle of the body acceleration with respect to the pressure gradient. Must be noted that, the effect of gravity in radial direction is neglected. For Bingham plastic fluid, the stress tensor $\tau$ is defined as:

$$\tau = \tau_y - \mu_{nf} \frac{\partial \bar{u}}{\partial \bar{r}} \quad \text{for} \quad \bar{r} \geq \tau_y,$$

$$\frac{\partial \bar{u}}{\partial \bar{r}} = 0 \quad \text{for} \quad \bar{r} < \tau_y,$$

where $\mu_{nf}$ is the effective dynamic viscosity of the nanofluid and $\tau_y$ is the yield stress. The boundary conditions are:

$$\bar{u} = \bar{u}_s \quad \text{at} \quad \bar{r} = \bar{R}(\bar{z}),$$

$$\bar{r} \text{ is finite} \quad \text{at} \quad \bar{r} = 0,$$

where $\bar{u}_s$ is the slip velocity at the stenotic walls.
The dimensionless form of the governing equations can be obtained by introducing the following dimensionless variables:

\[ R(z) = \frac{R(z)}{R_0}, \quad u = \frac{\bar{u}}{A_0 R_0/4 \mu_f}, \quad \theta = \frac{\bar{\theta}}{A_0 R_0/2}, \quad r = \frac{\bar{r}}{R_0}, \quad t = \frac{\bar{t}}{\bar{r}_p}, \]

\[ A = \frac{A_1}{A_0}, \quad B = \frac{A_2}{A_0}, \quad \omega = \frac{\bar{\omega}_p}{\bar{r}_p}, \quad \xi_s = \frac{\bar{\xi}_s}{\bar{r}_0}, \quad \tau = \frac{\bar{\tau}}{A_0 R_0/2}, \]

Using the above dimensionless variables, Eqs. (2)-(7) can be written in non-dimensional form:

\[ \frac{\alpha^2 \rho_{nf} \partial u}{\rho_f \partial t} = 4 \left[ (1 + A \cos t) + B \cos (\omega t + \varphi) \right] - \frac{2}{r} \frac{\partial}{\partial r} \left( r \tau \right), \]

\[ \tau = \theta - \frac{1}{2 \mu_f} \frac{\partial u}{\partial r}, \text{ for } \tau \geq \tau_r, \]

\[ \frac{\partial u}{\partial r} = 0, \quad \text{ for } \tau < \theta. \]

where \( \alpha^2 = \frac{\bar{\omega}_p^2 \rho_{nf}}{\mu_f} \), \( \alpha \) is the Womersley frequency parameter and \( \mu_f \) is the dynamic viscosity of blood.

The boundary conditions are:

\[ u = u_s, \text{ at } r = R(z), \]

\[ \tau \text{ is finite at } r = 0. \]

The effective nanofluid particles \( \rho_{nf} \) is:

\[ \rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s, \]

where \( \phi \) is the solid volume fraction and \( \rho_f \) is the density of blood, \( \rho_s \) is the density of dispersed copper nanoparticles.

In view of Eqs. (14), Eqs. (9)-(11) yields

\[ \alpha^2 \frac{\partial u}{\partial t} = 4 D_2 \left[ (1 + A \cos t) + B \cos (\omega t + \varphi) \right] - \frac{2}{r} \frac{\partial}{\partial r} \left( r \tau \right), \]

\[ \tau = \theta - \frac{1}{2 (1 - \phi)^{2.5}} \frac{\partial u}{\partial r}, \quad \text{ for } \tau \geq \tau_r, \]

\[ \frac{\partial u}{\partial r} = 0, \quad \text{ for } \tau < \theta, \]

where \( D_1 = (1 - \phi) + \phi \frac{\rho_s}{\rho_f} \) and \( D_2 = \frac{1}{D_1}. \)

The thermophysical properties of bio-fluid (blood) and nanoparticles [13] at 25°C are presented in Table 1.

| Physical properties | Based fluid (Blood) | Fe3O4 | TiO2 | Cu |
|---------------------|---------------------|-------|------|----|
| \( \rho (\text{kg/m}^3) \) | 1063                | 5200  | 4250 | 8933 |
3. Solution technique

Let us consider the following perturbed solutions for velocity $u$ and shear stress $\tau$ in the following form:

$$u(z,r,t) = u_0(r,t) + \alpha^2 u_1(r,t) + ...$$  \hspace{1cm} (18)

$$\tau(z,r,t) = \tau_0(r,t) + \alpha^2 \tau_1(r,t) + ...$$  \hspace{1cm} (19)

Substituting $u$ and $\tau$ into Eqs. (15)-(17) and collecting the coefficients of equal power of $\alpha^2$ yields:

$$\frac{\partial}{\partial r} (r\tau_0) = 2r \left[ (1 + A \cos t) + B \cos (\omega t + \varphi) \right]$$  \hspace{1cm} (20)

$$\frac{\partial}{\partial r} (r\tau_1) = -\frac{r}{2D_2} \frac{\partial u_0}{\partial t}.$$  \hspace{1cm} (21)

Integrating Eq. (20) and using the boundary condition (13), we obtain

$$\tau_0 = g(t)r,$$  \hspace{1cm} (22)

where $g(t) = [(1 + Acos t) + B \cos(\omega t + \varphi)]$.

Substituting Eqs. (18) and (19) into Eq. (16) and collecting the coefficients of equal power of $\alpha^2$, we obtain

$$\frac{\partial u_0}{\partial r} = 2 (1 - \phi)^{2.5} \left( \theta - \tau_0 \right)$$  \hspace{1cm} (23)

$$\frac{\partial u_1}{\partial r} = -2 (1 - \phi)^{2.5} \tau_1.$$  \hspace{1cm} (24)

Integrating Eq. (23), and using Eq. (22) and boundary conditions (12), we obtain

$$u_0 = u_s + (1 - \phi)^{2.5} \left\{ (1 - R) [2\theta - g(t)(r + R)] \right\}.$$  \hspace{1cm} (25)

Similarly, the solutions of $u_1$ and $\tau_1$ can be obtained by using Eqs. (24) and (21)

$$u_1 = -\frac{1}{16D_2} \left\{ (1 - \phi)^{6.25} g'(t) \left[ r^4 - 4r^2R^2 + 3R^4 \right] \right\}$$  \hspace{1cm} (26)

$$\tau_1 = \frac{1}{8D_2} (1 - \phi)^{2.5} g'(t) r \left( r^2 - 2R^2 \right).$$  \hspace{1cm} (27)

Hence

$$u(z, r, t) = u_s + (1 - \phi)^{2.5} \left\{ (1 - R) [2\theta - g(t)(r + R)] \right\}$$  \hspace{1cm} (28)

$$-\alpha^2 \frac{1}{16D_2} \left\{ (1 - \phi)^{6.25} g'(t) \left[ r^4 - 4r^2R^2 + 3R^4 \right] \right\} + ...$$

$$\tau(z, r, t) = g(t) r + \alpha^2 \frac{1}{8D_2} (1 - \phi)^{2.5} g'(t) r \left( r^2 - 2R^2 \right) + ...$$  \hspace{1cm} (29)
4. Results and discussions

The obtained results have been compared with those generated from the physiological parameters reported in [9]. As shown in Figure 2, Equation (30) reported by Siddiqui et al. [9] has been used to describe the motion of blood with nanoparticles. In this work, we have developed a mathematical model that represents the non-Newtonian blood flow through a stenosed artery.

![Figure 2. Comparison of blood velocity profiles in the stenosed artery.](image)

The influences of nanoparticles (Fe$_3$O$_4$, TiO$_2$, Cu) as drug carriers in the blood flow stream have been incorporated in the model as well. By assuming that the Womersley number is small, the approximate solutions of axial velocity can be determined. Figure 3 shows the distribution of axial velocity for different values of slip velocity while fixing the other parameters. As the slip velocity increase, the blood velocity increases as well. The axial velocity distribution $u(z,r,t)$ is reported in Figure 4 for different values of yield stress. For $\theta=0$, the velocity profiles of all fluids have been plotted. As theta increases, the axial velocity decreases due to the existence of nanoparticles.
Figure 3. Variation of velocity distribution $u(z,r,t)$ with $z$ for different values of slip velocity.
5. Conclusions
In this research, we have developed a mathematical model to describe the flow and velocity distribution of blood in a stenosed artery. The effects of nanoparticles (Fe$_3$O$_4$, TiO$_2$, Cu), oscillating pressure gradient, periodic body acceleration, yield stress and slip velocity have been incorporated in the model as well. The flow governing equations have been solved numerically by using MATHCAD software. The findings of the current work are:

- The blood flow velocity in stenosed artery can be controlled via nanoparticles.
- Blood velocity increases with respect to the slip velocity.
- The stenosed blood flow velocity decreases as yield stress increases.

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