Extrasolar Planet Orbits and Eccentricities

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Abstract.
The known extrasolar planets exhibit many interesting and surprising features—extremely short-period orbits, high-eccentricity orbits, mean-motion and secular resonances, etc.—and have dramatically expanded our appreciation of the diversity of possible planetary systems. In this review we summarize the orbital properties of extrasolar planets. One of the most remarkable features of extrasolar planets is their high eccentricities, far larger than seen in the solar system. We review theoretical explanations for large eccentricities and point out the successes and shortcomings of existing theories.

Radial-velocity surveys have discovered ∼ 120 extrasolar planets, and have provided us with accurate estimates of orbital period $P$, semimajor axis $a$, eccentricity $e$, and the combination $M \sin i$ of the planetary mass $M$ and orbital inclination $i$ (assuming the stellar masses are known). Understanding the properties and origin of the distribution of masses and orbital elements of extrasolar planets is important for at least two reasons. First, these data constitute essentially everything we know about extrasolar planets. Second, they contain several surprising features, which are inconsistent with the notions of planet formation that we had before extrasolar planets were discovered. The period distribution is surprising, because no one predicted that giant planets could have orbital periods less than 0.1% of Jupiter’s; the mass distribution is surprising because no one predicted that planets could have masses as large as 10 Jupiters, or that there would be a sharp cutoff at this point; and the eccentricity distribution is surprising because we believed that planets forming from a disk would have nearly circular orbits.

1. MASS AND PERIOD DISTRIBUTIONS

The mass distribution of extrasolar planets is sharply cut off above $10M_J$, where $M_J$ is the Jupiter mass [1, 2]. The absence of companions with masses in the range $10M_J < M < 100M_J$ (the “brown dwarf desert”) provides the strongest reason to believe that the extrasolar planets are formed by a different mechanism than low-mass companion stars, and thus the criterion $M < 10M_J$ is probably the least bad definition of an extrasolar planet. The observed mass distribution for $M < 10M_J$ can be modeled as a power law $dn \propto M^{-\alpha}dM$ with $\alpha = 1.1 \pm 0.1$ [1, 3, 4, 5]; in other words the distribution is approximately flat in log $M$, at least over the range $M_J \lesssim M \lesssim 10M_J$. Weak evidence that this distribution can be extrapolated to smaller masses comes from the solar system: fitting the distribution of the masses of the 9 planets to a power law over the interval from $M_{Pluto}$ to $M_J$ yields $\alpha = 1.0 \pm 0.1$, with a high Kolmogorov-Smirnov (KS) confidence
level (over 99.9%).

The period distribution can also be described by a power law to a first approximation: 
\[ dn \propto P^{-\beta} dP, \]
where \( \beta = 0.73 \pm 0.06 \) [5]. Some analyses [3, 4, 6] find a steeper slope, \( \beta \approx 1 \), probably because they do not correct for selection effects (the reflex motion of the star due to a planet of a given mass declines with increasing period). Stepinski & Black [4] and Mazeh & Zucker [6] stress that the distribution of periods (and eccentricities) is almost the same for extrasolar planets and spectroscopic binaries—only the mass distributions are different—and this striking coincidence demands explanation (see §2.7).

Recall that the standard minimum solar nebula has surface density 
\[ \Sigma(r) \propto r^{-1.5}, \]
[7]. If the formation process assigned this mass to equal-mass planets without migration we would have \( \beta = 0.67 \), consistent with the exponent found above for extrasolar planets.

In Figure 1 we plot the masses and periods of extrasolar planets.\(^{1}\) Although power laws are good first approximations to the mass and period distributions for \( M < 10 M_J \), there have been a number of suggestions of additional structure. (i) Zucker & Mazeh [8] and Udry et al. [9] suggest that there are too few massive planets \( (M \gtrsim 3 M_J) \) on short-period orbits \( (P \lesssim 50 \text{ days}) \), although (ii) Zucker & Mazeh argue that this shortfall is not present if the host star is a member of a wide binary system (denoted by star symbols in Figure 1). In addition, Udry et al. [9] suggest that there is (iii) a deficit of planets in the range 10 d \( \lesssim P \lesssim 100 \text{ d} \); and (iv) a deficit of low-mass planets \( (M \lesssim 0.8 M_J) \) on long-period orbits \( (P \gtrsim 100 \text{ d}) \). We do not find the latter two features persuasive, since their significance has not yet been verified by statistical tests, and of course low-mass planets on long-period orbits produce the smallest reflex velocity and so are most difficult to detect.

Whether or not these features exist, in our view the most impressive feature of the period distribution is that it is described so well by a power law. In particular, the mechanisms that might halt planetary migration often operate most effectively close to the star [10], so it is natural to expect a “pile-up” of planets at orbital periods of a few days; however, there is only weak evidence for an excess of this nature compared to a power-law distribution.

### 2. ECCENTRICITY DISTRIBUTION

One of the remarkable features of extrasolar planets is their large eccentricities: the median eccentricity of 0.28 is larger than the maximum eccentricity of any planet in the solar system. The eccentricity distribution of extrasolar planets differs from that of solar system planets at the 99.9% KS confidence level.

Observational selection effects in radial-velocity surveys could either favor or disfavor detection of high-eccentricity planets: as the eccentricity grows at fixed period the periastron velocity grows, but the time spent near periastron declines. The limited simulations that have been done [12, 13] suggest that these two effects largely cancel, so the eccentricity-dependent selection effects are small. Thus, the observed distribution

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\(^{1}\) The data for figures are taken from [http://www.obspm.fr/planets](http://www.obspm.fr/planets) as of October 2003.
FIGURE 1. Minimum masses versus periods of extrasolar planets. Planets found in wide binary systems (from [11]) are denoted by stars. The absence of planets in the lower right part of the figure is due to the limited sensitivity of the radial-velocity surveys (the dotted line shows an amplitude of 10 m s\(^{-1}\) for a planet on a circular orbit around a solar-mass star).

Planets that form from a quiescent disk are expected to have nearly circular orbits, and thus the high eccentricities of extrasolar planets demand explanation. A wide variety of eccentricity excitation mechanisms has been suggested so far.

2.1. Interactions with the protoplanetary gas disk

Gravitational interactions between a planet and the surrounding protoplanetary disk can excite or damp the planet’s eccentricity. These interactions are concentrated at discrete resonances, the most important of which are given by the relation [14, 15, 16, 17, 18]

\[
\Omega + \frac{\varepsilon}{m}(\Omega - \varpi) = \Omega_p + \frac{\varepsilon_p}{m}(\Omega_p - \varpi_p);
\]

here \(\Omega\) is the mean angular speed, \(\varpi\) is the apsidal precession rate, the integer \(m > 0\) is the azimuthal wavenumber, \(\varepsilon = 0, \pm 1\), and the unsubscripted variables refer to the disk while those with the subscript “p” refer to the planet. Resonances with \(\varepsilon = 0\) are called corotation resonances and those with \(\varepsilon = +1\) and \(-1\) are called outer and inner Lindblad resonances, respectively. Resonances with \(\varepsilon = \varepsilon_p\) are called “coorbital” resonances since the resonant condition (1) is satisfied when the gas orbits at the same angular speed as the planet; coorbital resonances are absent if the planet opens a gap in the disk. Resonances that are not coorbital are called external resonances. Apsidal or secular resonances have \(m = 1, \varepsilon = \varepsilon_p = -1\) so the resonant condition is simply \(\varpi = \varpi_p\);
in this case the collective effects in the disk (mostly pressure) only have to compete with differential precession rather than with differential rotation, so the resonance is much broader.

Interactions at external Lindblad and corotation resonances excite and damp the planet’s eccentricity, respectively [15]. In the absence of other effects, damping exceeds driving by a small margin ($\sim 5\%$). However, corotation resonances are easier to saturate, by trapping the orbital angular speed into libration around the pattern speed; if the corotation resonances are saturated and the Lindblad resonances are not, then the eccentricity can grow.

Interactions at coorbital resonances damp the planet’s eccentricity [17, 18], but these only operate in a gap-free disk. Apsidal resonances can also damp eccentricity [19, 20], but at a rate that depends sensitively on the disk properties [21].

Given these complexities, about all we can conclude is that either eccentricity growth or damping may occur, depending on the properties of the planet and the disk. Numerical simulations have so far provided only limited insight: two-dimensional simulations [22, 23] show eccentricity growth if and only if the planet mass exceeds $10–20M_J$, but the relation of these results to the resonant behavior described above remains unclear.

### 2.2. Close encounters between planets

In the final stages of planet formation dynamical instability can develop, either as the masses of the planets increase due to accretion or as their orbital separation decreases due to differential migration. The instability usually leads one or more planets to be ejected from the system or to collide with one another or with the central star (this last outcome is relatively rare). The surviving planets are left on eccentric orbits [24, 25, 26, 27].

A great attraction of this mechanism is that it makes calculable predictions; unfortunately, the predictions have some difficulty matching the observations: (i) It has been suggested that this mechanism could produce the ‘hot Jupiters’—planets such as 51 Peg B that are found on low-eccentricity, very short-period orbits (a few days) [24]—by close encounters of planets at distances of a few AU, which throw one planet onto a highly eccentric orbit that is later circularized by tidal dissipation. However, the frequency of hot Jupiters is far larger than this mechanism could produce [27]. (ii) The median eccentricity of the surviving planet following an ejection is about 0.6, compared to 0.3 in the observed systems [27]; on the other hand, so far the simulations have focused on equal-mass planets and the eccentricity is likely to be lower when a more massive planet ejects a less massive one. (iii) Collisions between planets lead to a population of collision remnants on nearly circular orbits, which is not observed [21, 27]. To avoid collisions would require that the planets are much more compact than Jupiter (i.e. the escape speed from the planet must be much larger than the orbital speed).

Another potential problem is the predicted mass-eccentricity relation: in this scattering process, low-mass planets are naturally expected to be excited to higher eccentricities. Unfortunately, so far no simulations have looked for mass-eccentricity correlations in a large sample of simulated planetary systems, so we do not know how strong the cor-
FIGURE 2. Eccentricity versus minimum mass for extrasolar planets. Planets found in wide binaries (from [11]) are shown with stars, and two examples with extreme eccentricities (16 Cyg B and HD 80606) are labeled.

relation should be. The eccentricities and masses of the known extrasolar planets show a weak correlation (at about 90% confidence level) in the opposite sense: more massive planets seem to have higher eccentricities (Figure 2).

A related possibility is that the eccentricity is excited by interactions with massive planetesimals [28, 29]. Normally, close encounters with small bodies tend to damp the eccentricity and resonant interactions excite eccentricity. Because this process relies on interactions with solid bodies, rather than gas giant planets, it requires a rather massive planetesimal disk, comparable to the mass of the planet, which in turn requires a gaseous protoplanetary disk that is even more massive, $\sim 0.1 M_\odot$.

### 2.3. Resonant interactions between planets

Many satellites in the solar system, as well as Neptune and Pluto, are locked in orbital resonances. The formation and evolution of these resonances has been thoroughly studied (see for example [30] and references therein).

Typically, resonance capture in the solar system occurs because of convergent outward migration, in which an inner satellite migrates outward faster than an outer satellite, thereby entering a mean-motion resonance. Continued migration after resonance capture excites the eccentricities of the resonant bodies; for example, this process is believed to be responsible for Pluto’s eccentricity of 0.25 [31].

In contrast, the known extrasolar planets have presumably migrated inward. In this case, migration after resonance capture excites eccentricities even more efficiently. For example, a test particle that is captured into the 2:1 resonance with an exterior planet will
be driven to an eccentricity of unity when the planet has migrated inward by a factor of four following capture [32]. Lee & Peale [33] have analyzed the dynamics of the planets in GJ 876, which are locked in a 2:1 resonance; they argue that to avoid exciting the eccentricities above the observed values requires either (i) extremely strong eccentricity damping, or (ii) fine-tuning the resonance capture to occur just before migration stops.

Chiang et al. [34] point out that eccentricities can also be excited by divergent migration, in which the ratio of the semimajor axes of two planets increases; the planets traverse a series of resonances, each of which excites additional eccentricity.

There are at least two concerns with models based on resonant interactions: (i) this mechanism requires at least two planets, whereas only a single planet has been discovered so far in most of the known extrasolar planetary systems; (ii) like close encounters, resonant interactions are expected to excite low-mass planets to higher eccentricities, but the observed mass-eccentricity correlation is small, and if anything in the opposite sense (Figure 2).

2.4. Secular interactions with a distant companion star

A planet’s eccentricity can be excited by secular interactions with a distant companion that does not lie in the planet’s orbital plane—this is sometimes called the Kozai mechanism [35, 36, 37, 38, 39, 40]. This mechanism has the interesting property that the separation and mass of the companion affect the period but not the amplitude of the eccentricity oscillation; thus even a weak tidal force from a distant stellar or planetary companion can excite a large eccentricity. However, the precession rate due to the companion must exceed the precession rate due to all other effects: other planets in coplanar orbits, any residual planetesimal disk, general relativity, etc.

There are several straightforward predictions for this mechanism: (i) There should be no correlation between the planet’s mass and eccentricity, which is marginally consistent with the weak correlation seen in Figure 2. (ii) High-eccentricity planets should be found in binary star systems. This does not appear to be the case (Figure 2), although two of the highest eccentricities, 16 Cyg B and HD 80686, are found in binary systems. Perhaps some of the other high eccentricities are excited by unseen companions—planets or brown dwarfs. (iii) Coplanar multi-planet systems should have low eccentricities, since their mutually induced precession is far larger than the precession due to a distant companion, thereby suppressing the Kozai mechanism. This prediction is not supported by the dozen or so multi-planet systems, which have substantial eccentricities, although in most cases we have no direct evidence that these systems are coplanar (see §3).

2.5. Phenomenological diffusion

Given the limited success of the models we have described so far, it is useful to ask what constraints we can place on the eccentricity excitation mechanism from simple phenomenological models. Here we explore the possibility that the excitation can be modeled as a diffusion process in phase space, with an eccentricity-dependent diffusion
We shall work with the eccentricity vector $\mathbf{e} = (e \cos \varpi, e \sin \varpi)$, since its components are approximately canonical variables for small eccentricities. Let $n(e)de$ be the number of planets with eccentricity vector in the range $de$. Then the diffusion equation in the eccentricity plane can be written as

$$\frac{\partial n(e,t)}{\partial t} = \nabla(D\nabla n) = \frac{1}{e} \frac{\partial}{\partial e} \left[ eD(e) \frac{\partial n(e,t)}{\partial e} \right].$$ 

(2)

We solve this equation for two cases: (i) a single initial burst of formation at $t = 0$ ($n(e,0) \propto \delta(e)/e$); (ii) continuous formation at a constant rate ($n(0,t) = \text{const}$). Since planets are ejected from the system when they reach $e = 1$, there is also a boundary condition $n(1,t) = 0$. We assume that the diffusion coefficient is a power law, $D(e) \propto e^p$.

The best-fit eccentricity distributions that emerge from the diffusion models are plotted in Figure 3. They are shown at late times when the planets in the initial-burst model have diffused away from the center of the eccentricity plane ($e = 0$) and the continuous-formation model has reached its steady state $n(e) \propto e^{1-p} - e$. Both the initial-burst model and the continuous-formation model produce satisfactory fits (with KS confidence level about 60%), so we cannot distinguish between the two types of models by comparing to the current observations; however, the predicted behavior is very different near $e = 0$ so the models should be distinguishable with more data.

In both cases, the best-fit values of $p$ ($p = 1.9$ for the initial-burst model and $p = 1.1$ for the continuous-formation model) suggest that eccentricity diffusion is produced by
a mechanism that is ineffective for circular orbits.

2.6. Propagation of eccentricity disturbances from other planets

Stars in the solar neighborhood approach passing stars to within about 400 AU during a lifetime of $10^{10}$ y. Such encounters cannot excite the eccentricity of a single planet on an orbit like those of the known extrasolar or solar system planets. However, disks of gas and dust around young stars are observed to extend to radii of hundreds of AU, and it is plausible to assume that these disks form planetesimals or planets that survive to the present at radii $\sim 10^2$ AU. If so, then passing stars can efficiently excite the eccentricities of the outer bodies in the system, and the eccentricity disturbance can then propagate inward via secular interactions between the planets or planetesimals, much like a wave. This process is described more fully in the article “Propagation of eccentricity disturbances in planetary systems” [41] elsewhere in these proceedings. The most important disk properties that determine the efficiency of this excitation mechanism are the radial extent of the disk and the steepness of its smeared-out surface-density profile; the total mass of the disk is not a factor. If the surface density is a power law, $\Sigma(r) \propto r^{-q}$, efficient eccentricity excitation at small radii requires a rather flat profile, $q \approx 1$, compared to the minimum solar nebula profile $q \approx 1.5$.

2.7. Formation from a collapsing protostellar cloud

It is conventional to assume that the components of binary star systems form simultaneously from condensations in a collapsing protostellar cloud (and hence have high-eccentricity orbits), whereas planets form later from the protoplanetary disk (and hence initially have low-eccentricity orbits). The two populations of companions seem to be clearly separated by the brown-dwarf desert.

Thus it is remarkable that the eccentricity and period distributions of extrasolar planets and low-mass secondaries of spectroscopic binaries are almost indistinguishable [1, 4]. This observation suggests that perhaps the extrasolar planets, like binary stars, formed in the collapsing protostellar cloud [42, 43, 44].

Simulations by Papaloizou & Terquem [43] suggest that this process preferentially forms either “hot Jupiters” or planets with high eccentricities and large semimajor axes; although Papaloizou & Terquem did not carry out a detailed comparison, it seems likely that, with some tuning, their simulations could reproduce the distributions of periods and eccentricities of the known extrasolar planets. A possible concern is that the lower limit for opacity-limited fragmentation is a few Jupiter masses, and many planets are known to have smaller masses. Terquem & Papaloizou [44] postulate that the low-mass planets formed in a disk and their eccentricities were later excited by encounters with high-mass planets formed by fragmentation.
3. INCLINATION DISTRIBUTION

So far there is little direct evidence that the planets in multi-planet systems have small mutual inclinations, as we might expect if they formed from a disk. Chiang et al. [45] argue that if the apsidal alignment between υ And C and D (Δω = 5° ± 5°) is not a coincidence, then these two planets must have mutual inclination ∼ 20°. Laughlin et al. [46] conclude that stability considerations restrict the mutual inclination between the two planets in 47 UMa to ∼ 40°.

In multi-planet systems with large mutual inclinations the Kozai mechanism can lead to large inclination and eccentricity oscillations, which promote instability by enabling close encounters between planets or with the central star. Thus, even if planetary systems are formed from collapsing clouds, as suggested in §2.7, multi-planet systems may be restricted to a disk-like configuration, containing planets on both prograde and retrograde orbits.

4. CONCLUSIONS

Planetary eccentricities can be excited by a variety of mechanisms, and it is likely that more than one of these plays a role in determining the eccentricity distribution of extrasolar planets. As the planet search programs continue to find planets of lower and lower masses, and longer and longer periods, which resemble more closely the giant planets in the solar system, the issue of why our planets have much lower eccentricities than their extrasolar analogs becomes more acute.

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