A new approach to generate local resonator for the application of acoustic metamaterials

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Abstract. In this paper, we proposed a type of local resonator which works by introducing more than one vibration modes, and thus may provide a new approach to formulate acoustic/elastic metamaterial. Such new concept of resonator is fundamentally different from traditional translational and rotational resonators. The potential application and design of acoustic/elastic metamaterial based on this kind of structures are also illustrated. The acoustic metamaterial filter we developed has unique property that only the waves with specific frequency can pass the material while other waves will be attenuated. We also proposed a kind of plane wave lens which can transfer plane wave with uniformly distributed amplitude profiles into pure plane wave. With these novel resonators, the research and application regarding it are highly anticipated in the near future.

Key words: Acoustic Metamaterials, Resonator, vibration Mode, Wave filter, Plane wave lens

1. Introduction

Acoustic metamaterial is a kind artificial material with periodic local microstructures [1]. Many acoustic metamaterials with local resonators of various topologies have been proposed and studied. Normally, acoustic/elastic metamaterials can be classified as 1-D [2], 2-D [3][4] and 3-D [5] metamaterials according to their dimensions. They can also be divided into groups of single phase and multiphase metamaterials by considering their building components.

To our best knowledge, basically, there are only two types of local resonators, that is translational resonator and rotational resonator, which can introduce negative effective mass density and negative effective elastic modulus, respectively. By the combination of using those two resonators, one can obtain double negative systems [6] [7], which in return attribute to negative refractive index. A variety of designs [8]-[10] based on those resonators have been proposed and implemented to explore and expand its applications.

The objective of this work is to propose a new type of resonator which is based on the second or high vibration modes of substructures. This work is organized into four sections including this instruction: in section 2, two typical concepts of resonators are reviewed and the concept of multi-mode resonator is for the first time proposed and studied analytically; in section 3 a simplified acoustic/elastic wave filter containing beams-masses resonator is formulated. Numerical modelling is carried out to obtain a better understanding of its working mechanism in section 4.
2. Concept of multi-mode resonator

In order to understand the difference between resonator that we suggested and other resonators, lattice models of two normal resonators is illustrated first. As figure 1(a) shows, negative effective mass density can be perfectly explained in a 1D mass-in-mass model, which here we defined as translational resonator, those metamaterials which can be explained based by translational resonator, such as mass-spring system and matrix-core system are actually have similar working mechanics, and thus we classify them as one type of metamaterials. Likewise, as it is shown in figure 1(b) negative effective modulus is mainly achieved by rotational resonator, chiral structure has been proved to be a realistic way to obtain rotational oscillations. Also some researchers have proposed plate-type acoustic/elastic metamaterials which combine those two resonators and successfully archive negative refractive index. For the sake of brevity, detailed explanation of the governing equation and working mechanism of those two types of resonators are not presented here. Basically those two types of resonators have only one vibration mode, high frequency vibration modes are hard to be activated. As it is well known that structures such as beams have more than just one vibration mode, and each vibration mode is corresponding to a constant eigenfrequency. However those eigenfrequencies are usually very high and not practically for metamaterial design which mainly focuses on low frequency wave manipulation. In order to make use of more than one vibration modes at low vibration frequency, as it is shown in figure 1(c), a beam-masses structures which compromising a continues thin slender massless beam and two concentrated masses are proposed. To obtain a simplified dynamic model, the thickness of the beam and the dimension of the masses are ignored. The Young modulus of the beam is denoted by $E$, while the length of the whole beam is $l$, the distance between the concentrated masses and ends of beam is $l/3$, the mass is represented by $m$. The reaction force at the left and right end of the beam are denoted by $F_{r}(t)$ and $F_{l}(t)$ respectively. The first and second vibration modes are demonstrated in the figure by dashed lines, so that the vibration frequency corresponding to the first and second vibration modes are respectively. If the first vibration mode is activated, then the two concentrated masses moves in phase, then the it behave like a simple spring-mass model, thus it functions as an translational resonator. However when the second vibration mode is activated, the motions of the two masses are 180° out of phase, and hence, $F_{r}(t)=-F_{l}(t)$. Then a harmonic moment of force will be generated, this character, we believe, can be used for the purpose of shear wave manipulation.

$$\text{Figure 1. Discrete models of translational resonator (a), rotational resonator (b) and multi-mode resonator (c).}$$

3. A design of elastic wave filter based on multi-mode resonator

To fully understand the dynamic characteristic and demonstrated its potential application, a passive elastic wave band-pass filter based on multi-mode resonator is proposed in figure 2. The basic function of it is to pass signals with frequencies within a specified band, the pass-band, and to reject signals outside this band. This type of filter is fundamentally different from the other acoustic filters which only prohibit signals with frequencies within certain band. The beam-masses resonator presented in previous section is adapted here, and two ends of the slender beam are clamped to the upper and lower
bars respectively which serve as media for longitudinal wave propagation. For convenience, we assume that the disks are taken as rigid and the radiuses of them are neglected. Only the horizontal displacements of those components are considered here.

Figure 2. An acoustic metamaterial bar based on beam-masses resonator.

If a single frequency elastic longitudinal wave propagating through the upper and down bar, then it can be assumed to have:

\[ u_1(x,t) = p_1 e^{i(ax-wt)}, \]
\[ u_2(x,t) = p_2 e^{i(ax-wt)} \]
\[ v_1(t) = q_1 e^{-iw't}, \]
\[ v_2(t) = q_2 e^{-iw't}, \]

where \( p_1, p_2, q_1 \) and \( q_2 \) denote the vibration amplitudes, \( \alpha \) and \( w \) are the wave number and radian frequency respectively. Thus the longitudinal stiffness of the beam segments are approximately estimated as \( k = 81EI/l^3 \), \( E, I \) and \( l \) are elastic modulus, moment of inertia and length of the beam. Since the wavelength \( \lambda \) is assumed to be much larger than the width of unit cell \( d \), and the structure consists of repeated cells, thus only one cell needs to be considered in the analysis. The governing equation can be obtained based on Newton’s law:

\[ EADu''_1 + k(v_1 - u_1) = M\ddot{u}_1 \]
\[ k(u_1 - v_1) + \frac{k}{2} (v_2 - v_1) = m\ddot{v}_1 \]
\[ k(u_2 - v_2) + \frac{k}{2} (v_1 - v_2) = m\ddot{v}_2 \]
\[ EADu''_2 + k(v_2 - u_2) = M\ddot{u}_2 \]

Assume that \( p_1 = p_2 = 1 \), then \( q_1 \) and \( q_2 \) can be represented by \((Mw^2 - E\alpha^2 - k)/k\),

\[
\begin{bmatrix}
Mw^2 - E\alpha^2 - k & k & 0 & 0 \\
k & -1.5*k + m*w^2 & k/2 & 0 \\
0 & k/2 & -1.5*k + m*w^2 & k \\
0 & 0 & k & Mw^2 - E\alpha^2 - k
\end{bmatrix}
\begin{bmatrix}
p_1 \\
q_1 \\
p_2 \\
q_2
\end{bmatrix} = 0
\]

In order to have nontrivial solution to this eigenvalue problem, the determination of the matrix should be zero, and thus the dispersion equation is achieved as:

\[ (Mw^2 - E\alpha^2 - k)^2 + \left(3/2 * k - m*w^2\right)^2 - k^4 = 0 \]

Thus the stiffness of the beam segment is approximately estimated as \( k = 81EI/l^3 \), \( E, I \) and \( l \) are elastic modulus, moment of inertia and length of the beam. To demonstrate an example, a unit cell having the following geometric and material parameters is listed below:

| Width of unit cell: | \( d = 10 \text{ mm} \) |
Length of the beam: \( l = 12 \, \text{mm} \)

Radius of the mass disk: \( r = 2 \, \text{mm} \)

Beam thickness: \( h = 0.2 \, \text{mm} \)

Bar thickness: \( H = 4 \, \text{mm} \)

Young’s Modulus: \( E = 70 \, \text{GPa} \)

Poisson’s ratio: \( \nu = 0.33 \)

Mass density: \( \rho = 2800 \, \text{kg/m}^3 \)

Width of unit cell: \( D = 10 \, \text{mm} \)

Cross-sectional area: \( A = H \ast 1 \)

Mass of bar segment: \( M = \rho HD \ast 1 \)

Mass of Disk: \( m = \rho m r^2 \ast 1 = 2800 \ast 3.14 \ast 8 \times 10^{-9} = 7.03 \times 10^{-5} \)

Moment of inertia of beam section: \( I = \frac{b^3}{12} = \frac{2}{3} \times 10^{-12} \)

\[ \times 10^5 \]

\[ \text{Frequency} \]

\[ \text{Wave number} \]

\[ 0 \]

\[ 2 \]

\[ 4 \]

\[ 6 \]

\[ 8 \]

\[ 0 \]

\[ 50 \]

\[ 100 \]

\[ 150 \]

**Figure 3.** Band diagram.

The band diagram of a single cell is shown in Fig. 3. For an elastic wave having a specific wave number, it can propagate at two different frequencies. The lower branch can be named as in-phase mode because the displacements \( v_1(t) \) is same to \( v_2(t) \), and the upper branch is called the out-phase mode is due to the fact that \( v_1(t) = -v_2(t) \). It should be pointed out that \( p_1 \) can be different than \( p_2 \) here. If we assume that the left end of the upper and bottom bars are subject to the same excitations with same phase and amplitude, then two bars can be seen as an single bar, thus similar to single bar model, when the excitation frequency is above the vibration frequency of in-phase mode the wave will be attenuated, the effect of the resonator is same to translational resonator. This is not the concentration of this work, so the mechanism is not demonstrated here in detail. We mainly focus on
the out of phase mode, which brings in the concept of so called new approach to generate local resonator.

In order to active the second vibration mode of the resonator, we assume that $p_1$ is larger than $p_2$, and $p' = p_1 - p_2$ is defined. For the analysis of a single unit, the lower bar segment is assumed to be fixed on the ground, thus the matrix of the governing equation can be reduced as:

$$\begin{bmatrix}
Mw^2 - EAD\alpha^2 - k & k & 0 \\
k & -1.5 * k + mw^2 & k/2 \\
0 & k/2 & -1.5 * k + mw^2
\end{bmatrix}
\begin{bmatrix}
p' \\
q_1 \\
q_2
\end{bmatrix} = 0 \quad (11)
$$

Dispersion equation is obtained as:

$$(Mw^2 - EAD\alpha^2 - k)[(-1.5 * k + mw^2)^2 - k^2/4] - k^2(-1.5 * k + mw^2) = 0 \quad (12)$$

4. Numerical simulation

In this section, wave propagation and working mechanism of the proposed acoustic wave filter and plane wave lens are numerically studied. The free wave motion properties are reported in the form of band diagrams. In the following, results are presented in normalized frequency.

1. Filtering properties of the acoustic wave filter with finite units.

Previous subsection analyzed the free wave motion characteristics of the acoustic wave filter with infinite subunits, however it is not available in real industry field. In this subsection, a finite length acoustic wave filter is proposed as shown in Fig.4. The parameters of the bar used here are the same as those of the acoustic bar in the previous section, and it contains 8 unit cells in the length direction. In order to get rid of the influence of the reflection of waves at the boundary, Perfect Matched Layer (PML) is applied. A horizontal displacement field which serves as input source is applied at the left edge of the upper bar, and the output is the right edge of the lower bar. The excitation frequency is swept from 0 to 10000Hz, and then it will cover the first two eigenfrequency of the local resonator. The wave filter can be characterized by the frequency response functions of the displacement at the output. Fig.5 shows calculated response spectra at the output. Significant high transmittance appears when the excitation frequency is close to the resonance frequency of the local resonator especially the second eigenfrequency. In order to understand the working mechanism, a snapshot of the deformation of the wave filter when the excitation frequency is and is shown in Fig.5, we can observe that the formation of the high transmittance ratio is due to the local resonance of the resonator.

![Figure 4. Simulation model of a simple Acoustic wave filter consists of beam-masses resonator.](image)

5. Plane wave lens design and its characterization.

Fig. 6 shows a design of acoustic plane wave lens composed of horizontal bars and proposed local resonators, 5 bars are placed parallel to each other, and 8 beam-masses resonators are placed periodically between and welded to each long bar. The 5 edges of the left ends of the bars serve as inputs, and the 5 edges of the right ends serve as outputs. The microstructure parameter used here is same to that in the design of the wave filter. Obviously, the uneven input wave source would not change at the output without the embedded local resonator. In the numerical simulation, PFL is also
adapted here to avoid wave reflection from the boundary. The uneven input and the outputs at two eigenfrequencies are plotted in Fig. 7, an almost pure plane wave are generated at the second resonance frequency. This design can certainly be expanded to large scale, which means a perfect elastic pane wave generator can be obtained with similar designs.

![Longitudinal wave amplitude at output](image1)

**Figure 5.** Frequency responses at the output.

![A simple Acoustic plane wave lens consists of beam-masses resonator.](image2)

**Figure 6.** A simple Acoustic plane wave lens consists of beam-masses resonator.

![Amplitude of input and output of the plane wave generator.](image3)

**Figure 7.** Amplitude of input and output of the plane wave generator.
6. Concluding remarks
In this work, a new approach to generate local resonator is suggested by introducing a beam-masses resonator. The second vibration mode of the local resonator is for the first time used in the design of local resonator. The special Based on this resonator, simple models of wave filter and plane wave generator are proposed and studied analytically and numerically. The reason of high transmittance of the wave filter and the working mechanism of the plane wave generator are discussed. There are more applications such as wave guide and wave hyper-lens for shear wave, we believe, can be obtained based on this type of resonator. This a new approach expanded the way we have to design the local resonator and family of acoustic metamaterials.

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