Dynamics of barred galaxies: effects of disc height

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ABSTRACT
We study dynamics of bars in models of disc galaxies embedded in realistic dark matter haloes. We find that disc thickness plays an important, if not dominant, role in the evolution and structure of the bars. We also make extensive numerical tests of different N-body codes used to study bar dynamics. Models with thick discs typically used in this type of modelling (height-to-length ratio \(h_z/R_d = 0.2\)) produce slowly rotating, and very long, bars. In contrast, more realistic thin discs with the same parameters as in our Galaxy (\(h_z/R_d \approx 0.1\)) produce bars with normal length \(R_{bar} \approx R_d\), which rotate quickly with the ratio of the corotation radius to the bar radius \(R_c = 1.2\)–1.4 compatible with observations. Bars in these models do not show a tendency to slow down, and may lose as little as 2–3 per cent of their angular momentum due to dynamical friction with the dark matter over cosmological time. We attribute the differences between the models to a combined effect of high phase-space density and smaller Jeans mass in the thin-disc models, which result in the formation of a dense central bulge. Special attention is paid to numerical effects, such as the accuracy of orbital integration, force and mass resolution. Using three N-body codes – GADGET, adaptive refinement tree (ART) and PKDGRAV – we find that numerical effects are very important and, if not carefully treated, may produce incorrect and misleading results. Once the simulations are performed with sufficiently small time-steps and with adequate force and mass resolution, all the codes produce nearly the same results: we do not find any systematic deviations between the results obtained with tree codes (GADGET and PKDGRAV) and with the adaptive mesh refinement (ART) code.

Key words: methods: N-body simulations – galaxies: evolution – galaxies: haloes – galaxies: kinematics and dynamics.

1 INTRODUCTION

Barred galaxies represent a large fraction (≈65 per cent) of all spiral galaxies (e.g. Eskridge et al. 2000; Sheth et al. 2008). Bars are ubiquitous. They are found in all types of spirals: in large lenticular galaxies (Aguerri et al. 2005), in normal spirals such as our Galaxy (Freudenreich 1998) and M31 (Athanassoula & Beaton 2006; Beaton et al. 2007), and in dwarf magellanic-type galaxies (Valenzuela et al. 2007). An isolated stellar disc embedded into a dark matter halo spontaneously forms a stellar bar as a result of the development of global disc instabilities (e.g. Binney & Tremaine 1987, section 6.5). Bars continue to be closely scrutinized because of their connection with the dark matter halo (e.g. O’Neill & Dubinski 2003; Holley-Bockelmann, Weinberg & Katz 2005; Colín, Valenzuela & Klypin 2006; Athanassoula 2007). Because the bars rotate inside massive dark matter haloes, they lose some fraction of the angular momentum to their haloes and tend to slow down with time (Tremaine & Weinberg 1984; Weinberg 1985).

The formation of bars and associated pseudo-bulges (Kormendy & Kennicutt 2004) is often considered as an alternative to the hierarchical clustering model. This appears to be incorrect: recent cosmological simulations indicate that the secular bulge formation is a part (not an alternative) of the hierarchical scenario. The simulations of the formation of galaxies in the framework of the standard hierarchical cosmological model indicate that bars form routinely in the course of assembly of haloes and galaxies inside them (Ceverino, private communication, Mayer, Governato & Kaufmann 2008). The simulations have a fine resolution of ∼100 pc and include realistic treatment of gas and stellar feedback, which is important for the survival of a bar. Bars form relatively late: well after the last major merger (\(\sim 1\)–2 for normal spiral such as our Milky Way), when a collision of gas-rich galaxies brings lots of gas with substantial angular momentum to the central disc galaxy. As the disc accretes the

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cold gas from the halo, it forms a new generation of stars and gets more massive. At some stage, the disc becomes massive enough to become unstable to bar formation. Once the stellar bar forms, it exists for the rest of the age of the Universe.

The cosmological simulations are still in preliminary stages, and it is likely that many results will change as they become more accurate and the treatment of the stellar feedback improves. However, existing cosmological simulations already show us the place and the role of traditional N-body simulations of barred galaxies, which start with an unstable stellar disc. It was not clear whether and how this happens in the real Universe. Now the cosmological hydrodynamic simulations tell us that this is somewhat idealistic, but still a reasonable setup compatible with cosmological models.

Simulations of bars play an important role for understanding the phenomenon of barred galaxies (e.g. Miller 1978; Sellwood 1980; Athanassoula 2003; Valenzuela & Klypin 2003). Numerical models successfully account for many observed features of real barred galaxies (Bureau & Athanassoula 2003; Bureau et al. 2006; Beaton et al. 2007). So, it is very important to assess the accuracy of those simulations. Recent disagreements between results of different research groups (O’Neill & Dubinski 2003; Valenzuela & Klypin 2003; Sellwood & Debattista 2006) prompted us to undertake a careful testing of numerical effects and to compare results obtained with different codes. This type of code testing is routine in cosmological simulations (Frenk et al. 1999; Heitmann et al. 2005, 2008), but it has never been done before for bar dynamics. Testing and comparison of numerical codes are important for validating different numerical models. It was instrumental for the development of precision cosmology. It is our goal to make such tests for N-body models of barred galaxies.

We use four different popular N-body codes: adaptive refinement tree (ART; Kravtsov, Klypin & Khokhlov 1997), GADGET-1 and GADGET-2 (Springel, Yoshida & White 2001; Springel 2005) and PKDGRAV (Wadsley, Stadel & Quinn 2004). ART is an adaptive mesh refinement (AMR) code that reaches high resolution by creating small cubic cells in areas of high density. GADGET and PKDGRAV, on the other hand, are TREE codes that compute forces directly for nearby particles and use a multipole expansion for distant ones. We use the codes to run a series of simulations using the same initial conditions for all codes.

We also address another issue: the effects of disc thickness on the structure and evolution of bars. Only recently have the simulations started to have enough mass and force resolution to resolve the vertical height of stellar discs. We use different codes to show that the disc height plays an important and somewhat unexpected role.

One of the contentious issues in the simulations of bar dynamics is the angular speed and the structure of bars in massive dark matter haloes. The amount and the rate at which bars slow down are still under debate. Debattista & Sellwood (1998, 2000) find in their massive halo models, i.e. those for which the contributions of the disc and the halo to the mass in the central region are comparable, that the bar loses about 40 per cent of its initial angular momentum, $L_z$, in $\sim 10$ Gyr. However, in simulations with much better force resolution and a more realistic cosmological halo setup, Valenzuela & Klypin (2003) and Colín et al. (2006) find a decrease in $L_z$ of only 4–8 per cent in $\sim 6$ Gyr. Debattista & Sellwood (1998, 2000) also find that bars do not significantly slow down for lower density haloes.

Valenzuela & Klypin (2003) presented bar models in which stellar discs were embedded in a cold dark matter Milky Way type halo with realistic halo concentrations $c \sim 15$, where $c$ is the ratio of the virial radius to the characteristic radius of the dark matter halo. These simulations were run with the ART code with high spatial resolution of 20–40 pc. The bars in the models were rotating fast for billions of years. Valenzuela & Klypin (2003) argued that slow bars in previous simulations were an artefact of low resolution. Sellwood & Debattista (2006) used initial conditions of one of the models of Valenzuela & Klypin (2003) and ran a series of simulations using their hybrid, polar-grid code. They found, in most cases, a different evolution than that reported in Valenzuela & Klypin. In particular, contrary to Valenzuela & Klypin’s results, they did not find that the bar pattern speed is almost constant for a long period of time. They attribute the differences to the ART refinement scheme.

While Valenzuela & Klypin (2003) mention numerical effects (lack of force and mass resolution) as the cause for excessive slowing down of bars in earlier simulations, there was another effect, which was not noted by Valenzuela & Klypin (2003): their disc was rather thin, with a scaleheight $h_z$ of only 0.07 of the disc scalelength $R_d$. This should be compared with $h_z \sim 0.2 R_d$ used in most studies of stellar bars (e.g. Athanassoula & Misiriotis 2002; Athanassoula 2003; Martinez-Valpuesta, Shlosman & Heller 2006). Models of Debattista & Sellwood (1998) have $h_z = 0.1 R_d$, but the resolution of their simulations was grossly insufficient to resolve this scale. O’Neill & Dubinski (2003) used $h_z = 0.1 R_d$ for a model, which had very little dark matter in the central disc region: $M_{dm}/M_{disc} \approx 1/4$ inside radius $R = 3 R_d$. Dependence of bar speed on disc thickness was noted by Misiriotis & Athanassoula (2000): thicker discs result in slower bars.

The remainder of this paper is organized as follows. In Section 2, we give a detailed review of the available data on disc scaleheights and present a simple analytical model for the relation of the disc scalelength with the disc scaleheight. We describe our numerical models in Section 3. In Section 4, we give a brief description of the codes and present analysis of numerical effects. Main results are presented in Section 5. We summarize our results in Section 6.

2 DISC HEIGHTS

Disc scaleheight appears to play a very important role in the development of barred galaxies. Thus, it is important to know the range of disc scaleheights in real spiral galaxies. The most accurate measurement of the disc height comes from the Milky Way galaxy. Edge-on galaxies provide another opportunity to measure disc heights, but those measurements are much less accurate because of dust absorption close to the disc plane. There is not much disagreement between different studies regarding the disc thickness of the Milky Way: the exponential scaleheight of the stellar thin disc is $h_z \approx 300$ pc (Gilmore & Reid 1983; Ojha et al. 1999; Jurić et al. 2008). The thick-disc component has a scaleheight of $\sim 1$ kpc, but it has a small fraction of mass. Neutral hydrogen and molecular gas have scales 200 and 50 pc. So, we can estimate the exponential scaleheight of the mass distribution as 250–300 pc. Using the exponential disc scalelength $R_d \approx 3$ kpc (e.g. Dehnen & Binney 1998a; Klypin, Zhao & Somerville 2002), we get the ratio of the scaleheight to the scaleheight $h_z/\langle R_d \rangle \approx 0.1$.

There is some ambiguity in the definitions of the scaleheights related to different forms of light and density profiles. In this paper, we use the isothermal plane approximation for density perpendicular to the plane $\rho_0(z/h_z)$. If the exponential law $\exp(-z/z_d)$ is used, the scaleheight $h_z$ should be matched with the exponential scale $z_d$. Traditionally, this is done by matching densities at large heights. This is not appropriate for our case because we are interested in heights which contain a significant fraction of stellar mass or light. If we match the profiles at the height where the density
declines $e$ times, then $z_d = 1.086h_\ast$ — a small difference. Alternatively, if we match the profiles at the height which has half the mass, then $z_d = 0.79h_\ast$. So, $z_d$ can be slightly larger or smaller than $h_\ast$. Considering other uncertainties in the situation (e.g. the thick-disc component), we neglect the differences between $z_d$ and $h_\ast$.

Observations of edge-on galaxies can be used to estimate the disc heights for other galaxies (e.g. van der Kruit & Searle 1981; Bizyaev & Mitronova 2002; Kregel, van der Kruit & de Grijs 2002; Kregel, van der Kruit & Freeman 2005; Yoachim & Dalcanton 2006). Kregel et al. (2005) gave ratios of exponential heights to exponential lengths for 34 edge-on galaxies measured in $I$ band and found that the median ratio is $h_d/R_d \approx 0.12$. Seth, Dalcanton & de Jong (2005) present fits for brightness profiles in $K$ band (Two-Micron All-Sky Survey images) for two Milky Way type galaxies NGC 891 ($V_{\text{max}} = 214 \text{ km s}^{-1}$) and NGC 4565 ($V_{\text{max}} = 227 \text{ km s}^{-1}$). They find the half-light height to exponential disc scale ratios $z_{1/2}/R_d = 0.072$ and 0.085 for the two galaxies correspondingly. For the five galaxies in the Yoachim & Dalcanton (2006) sample, which had circular velocities in the range $150–200 \text{ km s}^{-1}$, the average ratio was $z_{1/2}/R_d \approx 0.1$.

Estimates of disc heights in edge-on galaxies suffer from substantial absorption close to the plane of the disc (Xilouris et al. 1999; Yoachim & Dalcanton 2006). This makes scaleheights of the thin disc difficult to measure directly and causes the results to be dominated by flux coming from high galactic latitudes, where the thick disc is dominant. In turn, this leads to a substantial overestimation (by a factor of 2 to 3) of the disc heights even in red bands, if one interprets those as estimates of the thin-disc component (Yoachim & Dalcanton 2006).

We can use stellar dynamical arguments to estimate the scaleheights. The idea is to use the ratio of the vertical velocity dispersion $\sigma_z$ to the radial velocity dispersion $\sigma_R$ and the epicycle approximations.

Assuming an exponential stellar disc with the vertical density profile $\text{sech}^2(z/h_z)$, one gets

$$\sigma_v(R) = Q \frac{3.36GZ(S)}{\kappa(R)}, \quad \sigma_R^2(R) = \pi G h_z \Sigma(R),$$

where $Q$ is the Toomre stability parameter, $\Sigma(R)$ is the surface density and $\kappa(R)$ is the epicycle frequency. For galaxies with flat rotation curves and for radii $R > R_d$, we can use $\kappa = \sqrt{2}V_{\text{circ}}/R$. The circular velocity $V_{\text{circ}}$ is defined by the mass distribution given by the sum of three components: disc, bulge and dark matter. It is convenient to parametrize those relative to the total disc mass: $M(R)/M_{\text{disc}} = f_{\text{disc}}(R) + M_{\text{bulge}}/M_{\text{disc}} + M_{\text{dm}}(R)/M_{\text{disc}}$, where $f_{\text{disc}}(R)$ is the fraction of the disc mass inside radius $R$. Combining these relations, we get the following expression for the height-to-length ratio:

$$\frac{h_d}{R_d} = \left(\frac{\sigma_R}{\sigma_z} \frac{3.36Q}{2\pi}\right)^2 \frac{X^3 \exp(-X)}{f_{\text{disc}}(X) + f_{\text{bulge}} + f_{\text{dm}}(X)},$$

where $X = R/R_d$, $f_{\text{bulge}} = M_{\text{bulge}}/M_{\text{disc}}$ and $f_{\text{dm}}(X) = M_{\text{dm}}(X)/M_{\text{disc}}$.

We can now estimate the disc height at different radii. For example, we can get it at $R = 3R_d$ assuming that the mass of dark matter is about equal to the disc mass $f_{\text{dm}} = 1$ (Klypin, Zhao & Somerville 2002; Widrow, Pym & Dubinski 2008) and taking a small bulge $f_{\text{bulge}} = 0.2$. For $Q = 1.5$ (Widrow et al. 2008) and taking $\sigma_z/\sigma_R = 0.5$, we get $h_d = 0.11R_d$, which is consistent with the height of Milky Way disc. Equation (2) gives nearly the same height for $R_d < R < 3R_d$, if we scale the dark matter contribution in such a way that the rotation curve stays constant.

To summarize, the disc scaleheights are relatively small for high surface-brightness galaxies such as our Milky Way with $h_z = 0.1R_d$ being a reasonably accurate estimate. Observations of external edge-on galaxies favour larger heights and do not exclude $h_z = 0.2R_d$. Because of strong absorption in the plane of disc, those observations do not provide accurate enough measurements and should be interpreted only as upper limits on $h_z$ of the thin-disc component.

### 3 Models and Simulations

#### 3.1 Initial Conditions

The setup of initial conditions is described in detail in Valenzuela & Klypin (2003). Here we briefly summarize the most important features. The system of a halo and a disc, with no initial bulge or bar, is generated using the method of Hernquist (1993). In cylindrical coordinates, the density of the stellar disc is approximated by the following expression:

$$\rho_d(R, z) = \frac{M_d}{4\pi h_d^2 R_d^2} e^{-R/R_d} \text{sech}^2 \left(\frac{z}{h_z}\right).$$

#### 3.2 Description of the models

Selection of parameters of our models is motivated by a number of reasons. First, to simplify the comparison with previous results, we chose parameters, which are close to those used in Colín et al. (2006). Indeed, some of our models have exactly the same parameter as models $K_{\text{disc}}$ and $D_{\text{dm}}$ in Colín et al. (2006). In this paper, we...
Table 1. Initial parameters of the models.

| Code     | Name        | $N_{\text{disc}}$ | $N_{\text{total}}$ | $N_{\text{eff}}$ | Force resolution | Time-step | Disc scaleheight $h_z$ |
|----------|-------------|-------------------|--------------------|-----------------|------------------|-----------|-----------------------|
|          |             | ($10^5$)          | ($10^6$)           | ($10^6$)        | (pc)             | ($10^4$ yr) | (pc)                  |
| ART      | K$_{d1}$    | 2.33              | 2.7               | 6.6             | 44               | 1.4       | 200                   |
| ART      | K$_{d2}$    | 2.33              | 2.3               | 6.6             | 86               | 1.9       | 200                   |
| ART      | K$_{d3}$    | 2.00              | 2.2               | 5.9             | 170              | 2.2       | 714                   |
| GADGET-1 | K$_{g1}$    | 1.00              | 1.1               | 2.9             | 280              | 8.6       | 714                   |
| GADGET-2 | K$_{g2}$    | 2.33              | 2.5               | 6.7             | 112              | 2.6       | 200                   |
| GADGET-2 | K$_{g3}$    | 4.67              | 5.0               | 13.8            | 112              | 3.3       | 200                   |
| GADGET-2 | K$_{g4}$    | 2.00              | 2.2               | 5.9             | 140              | 1.4       | 200                   |
| GADGET-2 | K$_{g5}$    | 1.00              | 1.1               | 2.9             | 280              | 29.2      | 714                   |
| GADGET-2 | K$_{g6}$    | 1.00              | 1.1               | 2.9             | 280              | 3.6       | 714                   |
| PKDGRAV  | K$_{p1}$    | 1.00              | 1.1               | 2.9             | 136              | 24.5      | 714                   |
| PKDGRAV  | K$_{p2}$    | 2.33              | 2.5               | 6.7             | 136              | 1.2       | 200                   |
| GADGET-2 | D$_{g1}$    | 2.33              | 2.5               | 6.7             | 112              | 2.6       | 200                   |

Model D: $M_{\text{disc}} = 5 \times 10^{10} M_\odot$, $M_{\text{tot}} = 1.43 \times 10^{12} M_\odot$, $R_d = 3.86$ kpc, $Q = 1.8 \, c = 10$

The disc scaleheight increases in the course of evolution. We find that for thin-disc models it becomes more than double after 5 Gyr of evolution, while for thick disc models the scaleheight increases by a factor of 1.6. As a result, the scaleheight-to-scalelength ratio of evolved thin-disc models is close to the observed $h_z / R_d = 0.1$. The ratio for evolved thick-disc models is, on the other hand, twice the observed one.

Table 1 presents the parameters of the models. The first and the second columns give the name of the code and the name of the model. The capital letter of model name represents the model type (K or D). The first subscript in the model name indicates the code used to make the run: $a$ for ART, $g$ for GADGET and $p$ for PKDGRAV. In Columns (3)–(5), we show the disc, the total (disc + dark matter) and the effective number of particles – the number of particles, which we would need, if we used equal-mass particles. The force resolution (6th column) is twice the smallest cell size for ART code and the spline softening (2.8 times the effective Plummer softening) for TREE codes. Note that the force resolution is the distance at which the force accurately matches the Newtonian force. The force continues to increase even below the resolution resulting in large changes in density. Column 7 shows the smallest time-step of simulations. All codes use variable time-steps. Details are given in the next section.

The number of particles inside a sphere of radius equal to the force resolution is quite substantial. For a typical simulation such as $K_{d1}$ or $K_{d2}$ with the resolution $\sim 100$ pc, there are $\sim 100$ particles inside the resolution radius at the centre of the system at the initial moment. For TREE codes, which keep the resolution constant, the number declines with distance, but it is still large in the plane of the disc. For example, it is $\sim 10$–15 at 8 kpc. The ART code maintains the nearly constant number of particles inside (increasing) radius of the force resolution at the level of 60 particles (see details in Colín et al. 2006).

Particles with different masses are used in our simulations to increase the mass and force resolution in the central disc region. This is done by placing many small-mass particles in the central disc-dominated region and by utilizing large-mass particles in the outer halo-dominated areas. We use four mass species. The first species represents disc and dark matter particles in the central halo region (the central $\sim 40$ kpc region). Both the disc particles and the central dark matter particles have the same mass. More massive particles rarely enter the central $\sim 10$ kpc region. The mass species differ by a factor of 2 between one species and the next.

The mass resolution – the mass of a disc particle or the mass of the smallest dark matter particle – is given by the ratio of the disc mass to the number of disc particles. It is in the range $m_1 = (1–5) \times 10^5 M_\odot$. We present the time-step and the initial scaleheight in the last two columns. The typical number of time-steps for simulations is $(2–4) \times 10^5$. The physical parameters of the models, such as the mass of the disc or concentration of the dark matter halo, are shown in separate rows.

In order to estimate the bar pattern speed $\Omega_p$, we first determine the orientation of the bar by iteratively applying the method of the inertia tensor in the plane of the disc. $\Omega_p$ is obtained subsequently by numerical differentiation: $\Omega_p = \phi / dr$, where $\phi$ is the position angle of the bar. In practice, we use about 10 consecutive snapshots for which the increasing function $\phi$ is available and make a least-squares fit. Then, $\Omega_p$ is given by the slope of the straight line. The bar amplitude $A_2$ is computed similar to Valenzuela & Klypin (2003). For each logarithmically spaced cylindrical bin, we find the amplitude of the second Fourier harmonic. Then, the amplitude is smoothed over the radius and the maximum is taken as the bar amplitude $A_2$. The bar length is defined as outermost radius at which the ratio of axes of isosurface density contours is 1.5. When finding radius of corotation, we use rotational velocity curve of disc particles $V_{rot}(r)$ and find radius, at which it is equal to $\Omega_p r$. 

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4 NUMERICAL EFFECTS

4.1 Codes

Simulations were run with three N-body codes: ART (Kravtsov et al. 1997), GADGET-1 or GADGET-2 (Springel et al. 2001; Springel 2005) and PKDGRAV (Wadsley et al. 2004). ART is an AMR N-body code, which achieves high spatial resolution by refining the base uniform grid in all high-density regions with an automated refinement algorithm. GADGET is a parallel tree code. Here we only use the N-body part of the code. PKDGRAV is another tree code. These two tree codes differ in the characteristics of the gravitational tree algorithm. For instance, GADGET employs the Barnes & Hut tree construction (Barnes & Hut 1986), while PKDGRAV uses a binary KD tree (Bentley 1979). GADGET-2 only uses monopole moments in the multipole expansion, while PKDGRAV advocates a hexadecapole as the optimum choice. (GADGET-1 can be configured to use octopole moments). The codes also differ in the cell-opening criterion. Here, we use an opening angle criterion of $\theta = 0.7$ for PKDGRAV. In the case of GADGET-2 runs, we use a tolerance parameter $\alpha = 0.005$ (see equation 18 in Springel 2005). The main advantage of GADGET-2 as compared with Gadget-1 is a more accurate time-stepping scheme (Springel 2005).

The codes use different variants of leapfrog scheme. An algorithm of integration of trajectories can be written as a sequence of operators, which advance particle positions (called drifts) and change velocities (called kicks). For example, a simple constant-step leapfrog scheme is

$$v(t_{n+1}) = v(t_{n-1/2}) + g(t_n) dt \quad \text{Kick,}$$

$$x(t_{n+1}) = x(t_n) + v(t_{n+1/2}) dt \quad \text{Drift.}$$

Thus, the leapfrog integration is a sequence of $K(dr)D(dr)K(dr)D(dr)$... operators. We call this a KD scheme. Note that the order of operators is not important: the DK scheme is identical to the KD scheme. When the time-step changes with time, the order of operators makes a difference. It is also important to select the moment, at which the time-step should be changed. Following Quinn et al. (1997), we use the operator $S$ to indicate the moment when a new time-step is selected. We also need to specify the time when accelerations (in kicks) and velocities (in drifts) are estimated relative to the moment to which the velocities and coordinates are advanced. We attach the sign ‘+’ (‘-’) to the name of operator to indicate that the operator uses information from the beginning (end) of the time-step. For example, $K_+(dr)$ is $v(t_n) = v(t_{n-1}) + g(t_{n-1}) dt$ and the operator $D_-(dr)$ is $x(t_n) = x(t_{n-1}) + v(t_n) dt$. Operators with subscript ‘0’ are time symmetric: they use information at the middle of the time-step. Using these operators, we can write the algorithms of time-stepping in all our N-body codes:

$$SD_-(dr/2)K_0(dr)D_+(dr/2), \quad \text{GADGET-1}$$

$$SK_+(dr/2)D_0(dr)K_-(dr/2), \quad \text{GADGET-2}$$

$$K_+(dr/2)SK_-(dr/2)D_0(dr), \quad \text{ART}$$

$$D_-(dr/2)SK_0(dr)D_+(dr/2), \quad \text{Quinn et al.}$$

The PKDGRAV code has different integration schemes. The scheme given in equations (12) uses the algorithm described by Quinn et al. (1997). In our simulations with the PKDGRAV code, we use the scheme which is identical to the GADGET-2 code in the sense of both the sequence of stepping and refinement conditions. This is, in fact, the way PKDGRAV is most commonly run (Wadsley et al. 2004). The ART code uses the time-step $dt_1$ from previous moment to start the integration. Then, having information on coordinates, it makes a decision on the value of new time-step. The GADGET-2 and the ART schemes look different, but actually they are identical, which can be seen when one writes the sequence of a few time-steps. The GADGET-1 and the Quinn et al. schemes look very similar, but they are quite different: the position of the $S$ operator makes the Quinn et al. scheme more accurate (Quinn et al. 1997).

Conditions for changing the time-step are different in different codes. In the ART code, the time-step decreases by a factor of 2 when the number of particles exceeds some specified level (typically two to four particles). A cell that exceeds this level is split into eight smaller cells resulting in the drop by $2^3$ times of the number of particles in a cell. This prescription gives scaling of the time-step with the local density $\rho$ as $dt \propto \rho^{-1/3}$. Colín et al. (2006) give more details of the procedure. The time-step in Quinn et al. scheme scales as $dt \propto \rho^{-1/2}$. Zemp et al. (2007) also advocate a scheme with this scaling of the time-step. The GADGET and PKDGRAV codes use a scaling with the gravitational acceleration $dt \propto g^{-1/2}$, which for $\rho \propto r^{-2}$ gives $dt \propto \rho^{-1/4}$. Among all the codes, the Quinn et al. scheme uses the most aggressive prescription for changing the time-step and GADGET has the smallest change in $dt$.

In practice, the time-step $dt$ for the Gadget-2 code is defined by parameter

$$\eta^2 = \frac{|g| dt^2}{2\epsilon},$$

where $|g|$ is the acceleration and $\epsilon$ is the effective Plummer softening. For example, we used $\eta = 0.04$ for $D_{\epsilon 1}$ and $K_{\epsilon 2}$ models. It is $\eta = 0.11$ for $K_{\epsilon 1}$ model. In simulations where we increase the time-step by a factor of 2, we double $\eta$. For the PKDGRAV code, the time-step is defined by a similar expression: $\eta^2 = |g| dt^2/\epsilon_s$, where $\epsilon_s$ is the spline softening. We use $\eta = 0.025$ for model $K_{\epsilon 2}$ and $\eta = 0.050$ for model $K_{\epsilon 1}$.

In all codes, the time-step changes discretely by a factor of 2: $dt = dt_0 2^{-m}$, where $dt_0$ is the maximum time-step and $m$ is an integer defined by local conditions (local density or local acceleration).

4.2 Numerical effects: resolution and time-stepping

In order to obtain accurate results, the trajectories of particles should be integrated with a sufficient precision. There is no reliable method of estimating how small a time-step should be. While there are theoretical arguments what integration schemes should (or should not) be used (Quinn et al. 1997; Preto & Tremaine 1999; Springel 2005), only tests can tell how accurate results are. Simulations of bars have a special reason why the orbits should be accurate. Particles, which make up the bar, move typically on quite elongated trajectories periodically coming close to the centre. When this happens, the acceleration changes substantially, and fast-changing accelerations pose problems for numerical integration. If accuracy of integration is not sufficient, a particle may erroneously change its direction of motion and start moving away from the bar. This artificial scattering on the centre results in a smaller number of particles staying in the bar. Thus, it is important to have not only accurate particle energies, but also accurate phases of trajectories. The latter is more difficult than the former, because different leapfrog schemes used in current N-body codes are known to have problems with accurately tracking orbital phases (e.g. Springel 2005).
The errors go down with decreasing time-step, but the energy errors by themselves show no obvious signs which would indicate that the largest time-step produces a dramatically wrong answer.

It is difficult to predict what actually happens when the time-step is not small enough. In this case, it produced a steep decline in the bar amplitude during the buckling stage. We had the same effect (weakened bar) in art simulations done with too large time-steps. At the same time, it is quite possible to have just an opposite effect: an artificially stronger bar with lower pattern speed. For example, Sellwood & Debattista (2006) ran a model similar to K_{g2} with a grossly insufficient time-step, 1.5 × 10^7 yr; they got a very long and a slow bar. For the same model, Valenzuela & Klypin (2003) used a 10 times smaller step and got a significantly shorter and faster bar.

The accuracy of orbit integration greatly depends on the distribution of density in the central region: the steeper is the density profile, the more difficult is the simulation. In our case, the models K with a thick disc (h_t = 714 pc; e.g. runs K_{g5} and K_{a3}) did not form a dense centre. We find that for these models a relatively large step of dt ∼ 10^5 yr is sufficient. Models K with thin disc (h_t = 200 pc) produce a nearly flat circular velocity curve implying a steep profile, which can be roughly approximated as v ∝ r^{-2} in the central 2 kpc region. These models showed inconsistent results when large time-steps dt > 10^5 yr were used. Only when we changed to much smaller steps (dt ∼ 10^4 yr), results became stable.

The observed effect of time-stepping presented in Fig. 1 is somewhat unexpected. The most difficult and important part of the simulation is the motion of particles in the central region. Taking a typical particle velocity of 200 km s^{-1}, we estimate that it would take 2 × 10^7 yr and 200 time-steps (dt ∼ 10^5 yr) for a particle to cross the central 2 kpc region. One would expect that 200 time-steps is enough to provide reasonably accurate results. Unfortunately, this is not the case as Fig. 1 shows. Indeed, our simple tests described below indicate that in spite of the fact that the energy is reasonably well preserved, the orbital angle is not – trajectories are scattered very substantially when a large time-step is used. This numerical scattering may result in incorrect properties of the bar.

We investigate the situation with the accuracy of orbit integration by studying the motion of particles in an idealized, but realistic case of a spherical logarithmic potential φ(r) = log(r). We implemented four time-integration schemes used in our simulations: a constant leapfrog scheme, and block-schemes with a variable time-step used in art, gadget-2/pkdgav and Quinn et al. codes.

Fig. 2 shows the results of integration of an eccentric orbit with the apocentre/pericentre ratio of about 10:1. The gravitational potential is normalized in such a way that that the binding energy is equal to the time-averaged kinetic energy. In order to find the accuracy of the orbital angle, we run the orbit with a very small time-step: 10^5 steps per period. Then, we find the position angle of a low-accuracy run and compare it with the position angle at the same moment in the small-step run. As compared with a constant-step run, all variable-step integration schemes give smaller errors in the energy conservation, but the errors in the position angle are too large. The constant-step run is definitely better, but even in this case the errors (∼1°) are still large. Decreasing the time-step by a factor of 10 gives much better results as shown by Fig. 3. In this case, even the phase of the orbit is accurately simulated. The difference between the art and the gadget codes is due to the fact that the art code takes more small steps in the central part of the orbit (the total number of...
Figure 2. Errors of orbit integration. We integrate trajectory of a particle moving in a gravitational potential created by isothermal distribution of matter $\phi = \log(r), \rho \propto r^{-2}$. The particle has angular momentum $1/5$ of the circular orbit with the same total energy and moves on an elongated orbit with $r_{\text{max}}/r_{\text{min}} \approx 10$, which is not unusual for orbits in strong bars. The number of time-steps is 200 per radial period, which corresponds to the time-step of $10^{5}$ years when scaled to a realistic galaxy model. The top panels show the error in energy conservation, and the bottom ones are the errors in position angle. The left-hand column is for the leapfrog scheme implemented in GADGET-2 and PKDGRAV codes. The middle (right) column is for ART (Quinn et al.) code. The energy conservation is reasonably small, but errors in the orbital angle are unacceptably large.

Figure 3. Errors of orbit integration. The same as in Fig. 2, but for 2000 time-steps per radial period. This corresponds to a time-step $\sim 10^{4}$ yr in realistic simulations. The errors of integration are very small for both the energy conservation and for the orbital angle.

Figure 4. Dependence of bar properties on the force resolution. Evolution of the pattern speed of the bar (top panel) and the bar amplitude (lower panel) is shown for the high-resolution model $K_{g2}$ with force resolution 112 pc (full curves) and for a low-resolution simulation with 560 pc resolution (dashed curves). The later simulation is the small time-step model presented in Fig. 1 with the full curves. Increased force and mass resolution produce a more concentrated bulge, which weakened the bar. A shorter and weaker bar rotates faster and does not slow down much over many billions of years.

5 RESULTS

Table 2 gives different properties of the simulated models as measured after 5 Gyr of evolution. In Column 2, we present the fraction of the angular momentum lost by the stellar material. The pattern speed $\Omega_p$ and the ratio of corotation radius to the bar radius are presented in Columns 3 and 4. The bar length is given in Column 5. The last three columns give parameters of a double-exponential approximation of the stellar surface density: $\Sigma(r) = \Sigma_{\text{bulge}} \exp(-r/R_{\text{bulge}}) + \Sigma_{\text{disc}} \exp(-r/R_{\text{disc}})$. 
Table 2. Parameters of the models after 5 Gyr of evolution.

| Name  | ΔL/L (Per cent) | Ω₀ (Gyr⁻¹) | R_cor/R_b (kpc) | Bar length R_b (kpc) | Disc scalelength R_d (kpc) | Bulge scalelength R_bulge (kpc) | Bulge/total |
|-------|-----------------|-------------|-----------------|----------------------|---------------------------|-------------------------------|-------------|
|       | (1)             | (2)         | (3)             | (4)                  | (5)                       | (6)                          | (7)          | (8)          |
| Thick-disc models |               |             |                 |                      |                           |                               |             |
| Kₐ³   | 9.9             | 10.0        | 1.7             | 10.8                 | 6.7                       | 1.07                         | 0.38         |
| K₇¹   | 8.3             | 10.3        | 1.7             | 10.4                 | 7.0                       | 0.97                         | 0.32         |
| K₇⁵   | 10.3            | 9.1         | 1.7             | 12.1                 | 6.6                       | 1.10                         | 0.36         |
| K₇⁶   | 8.2             | 10.5        | 1.5             | 11.5                 | 6.5                       | 1.03                         | 0.35         |
| K₇₁   | 8.5             | 10.3        | 1.8             | 10.0                 | 6.4                       | 1.00                         | 0.33         |
| Thin-disc models |               |             |                 |                      |                           |                               |             |
| K₅₁   | 5.1             | 21.0        | 1.25            | 6.7                  | 5.7                       | 0.73                         | 0.26         |
| K₅₂   | 3.3             | 25.4        | 1.16            | 6.5                  | 5.2                       | 0.54                         | 0.20         |
| K₅₂   | 2.7             | 19.8        | 1.15            | 7.5                  | 5.8                       | 0.63                         | 0.23         |
| K₅₃   | 2.6             | 21.7        | 1.20            | 6.7                  | 5.3                       | 0.64                         | 0.21         |
| K₅₄   | 2.1             | 22.8        | 1.22            | 6.0                  | 5.3                       | 0.61                         | 0.21         |
| K₅₂   | 3.1             | 19.3        | 1.23            | 7.4                  | 5.5                       | 0.66                         | 0.22         |
| D₅₁   | 10.1            | 19.5        | 1.4             | 7.6                  | 5.2                       | 0.43                         | 0.35         |

The models are clearly split into two groups: those which started with a thick disc (Kₐ³, K₇¹, K₇⁵ and K₇₁), and the models which started with a thin disc. For example, the ratio of the corotation radius to the bar radius is about 1.7–1.8 for thick-disc models. For thin-disc models, the ratio is visibly smaller: 1.2–1.4. Thin-disc models have shorter bars and less massive bulges. The differences are especially striking when we compare simulations done with the same code and with similar parameters. For example, models K₅₂ and K₅₃ have very similar numerical parameters (time-step, resolution and number of particles). Yet, their parameters at 5 Gyr (and actually at any moment) are drastically different. For example, the pattern speed for model K₅₂ is 2.5 times larger than for model K₅₃.

5.1 Thick-disc models

We start with the analysis of thick-disc K models. It takes ~2–3 Gyr to form a bar; the buckling phase happens at t ∼ 5 Gyr followed by a regime of a constant amplitude and nearly constant pattern speed. Fig. 5 shows the evolution of the bar pattern speed Ω₀ and the amplitude of the bar A₂ for the models. In Fig. 6, we present the total circular velocity profile \( \sqrt{GM(<r)/r} \), the radial and vertical stellar velocity dispersions, and the disc surface density as a function of distance to the centre of the galaxy, R. The comparison is made at 6.5 Gyr for three models: Kₐ³, K₇⁵ and K₇₁.

Fig. 7 shows disc particles seen along the minor axis of the bar for K models with the thick disc K₇⁵ (left-hand panels) and K₅₁ (right-hand panels) at four different epochs. The selected time moments represent different stages of bar evolution. At 4 Gyr, the bar is the strongest, and it has not yet started the buckling stage: the disc is still thin. At 5 Gyr, the system goes through the buckling instability. At this moment, the models look different. The differences die out as the buckling instability proceeds. Once the buckling stage is finished, the models again are very close.

Overall, the models evolve very similarly. Some small differences are observed during the buckling phase. Yet, models tend to converge at the end of the evolution. The degree of agreement between different codes as demonstrated by Fig. 6 is remarkable. Inside the central 5 kpc region, the surface density of the disc deviates from model to model by only per cent. The vertical velocity dispersion over the whole disc deviates not more than 5 km s⁻¹.

This agreement between models demonstrates that the results are code-independent: if simulations are done with sufficiently small time-steps and with similar force and mass resolution, all codes produce nearly the same results. The results also show that there are no problems with any particular code: all codes produce the same ‘answer’.
Dynamics of barred galaxies

5.2 Thin-disc models

Figs 8 and 9 show the evolution of the bar amplitude and the bar pattern speed for thin-disc K models. Again, the simulations behave very similarly, but this time they are very different from the thick-disc models K. The thin-disc models do not show any substantial evolution in the bar pattern speed and in the bar amplitude. Nevertheless, the agreement between the simulations is not as close as in the case of the thick-disc models. It is important to note that there are no systematic differences between results obtained with different codes. The comparison of two runs presented in Fig. 9 is especially striking. For example, the GADGET model $K_{g2}$ shows some small decline in the pattern speed and some oscillations in bar pattern speed and bar amplitude. The same evolution of $\Omega_p$ is demonstrated by the ART model $K_{a1}$, which also has the oscillations.
with a slightly larger amplitude. The bar amplitudes for the two models are also very close over the whole simulated period of time. At the same time, models $K_{\phi}$ (ART) and $K_{\phi}$ (GADGET), which give very similar results, do not show any indication of slowing down of the bar (see Fig. 8).

We believe that this agreement between results produced by different codes indicates that there are real physical differences between the models and that those differences are not of numerical origin. In other words, when started with the same physical initial conditions, the evolution produces different results. In the case of the thin-disc K models, the differences are on the level of $\sim 10$ per cent after 6 Gyr of evolution.

What are the possible reasons for these variations between the models? The $N$-body problem is deterministic: initial conditions uniquely define the outcome of evolution. If one starts with identical initial conditions and uses a perfect code, the results must be unique. Yet, this is too simplistic. An unstable system may have divergent evolutionary tracks. A fluctuation slightly changes the system, but because of instabilities the fluctuation grows and the final answer is different as compared with the evolution of the system without the fluctuation. The barred stellar dynamical models have two stages when a system is unstable: the initial stage of formation of the bar and the buckling instability. In addition, even in the quiet periods, the bar itself is an example of a potentially unstable system. For example, it could start trapping more particles resulting in even stronger bar, which traps even more particles. To a large degree, this is exactly what happened with the bars in thick-disc K models. The bars were expanding until all the discs became the bar.

The source of fluctuations is an interesting issue. In the simulations, the fluctuations are related to the initial random phases and amplitudes to numerical inaccuracies. Yet, we should not forget that our models are only approximations to the reality. In real galaxies, there is no shortage of fluctuations including satellite galaxies and molecular clouds to name a few. Whatever is the source of fluctuations, simulations of the same physical model evolve slightly different and produce slightly different results. This is exactly what the thin-disc models did.

### 5.3 Effects of the phase-space density: thin versus thick discs

The only difference in the initial conditions between the thin- and thick-disc K models is the disc scaleheight. All the other parameters are the same. These seemingly minor variations in initial conditions resulted in a remarkably different evolution and in large differences in the structure of the evolved models. In order to highlight the differences, Fig. 10 shows the angular momentum loss $\Delta L/L$ of the stellar disc and the bar pattern speed for two models run with the GADGET code: one with a thin disc ($K_{\phi}$) and another with a thick disc ($K_{\phi}$). The disc in the thick-disc model loses four times more angular momentum, and its bar rotates 2.5 times slower as compared with the thin-disc model. The dark matter is the same in both models, and it cannot be the reason of the difference.

So, what are the possible reasons for such a large effect of the disc height? The thickness of the stellar disc affects the vertical waves and oscillations (Merritt & Sellwood 1994). This effect is definitely present and will have some impact on the evolution. At the same time, those vertical modes are likely to play a significant role during the buckling stage (Merritt & Sellwood 1994). However, the differences between the models develop too early, and they are too large for the vertical modes to be the culprit. In the absence of a reliable theory of stellar bars, one can only speculate what is going on. One may think about two other effects, which can influence the systems: the Jeans mass and the phase-space density. The random velocities, which define the Jeans mass, tend to prevent collapse of perturbations on small scales. Thus, for the same random velocities, larger Jeans masses imply smaller densities. The phase-space density acts in the same way. Thus, we expect that models with higher phase-space density (or small Jeans mass) will result in denser and more compact central region, which affects the growth of bars in a profound way.

For a fixed surface density, the vertical disc height $h_z$ changes the density in the disc $\rho \propto h_z^{-1}$ and the velocity dispersion $\sigma_z^2 \propto h_z$ (see equation 2). Assuming for simplicity that the Jeans mass scales with the velocities and density as $M_J \propto \sigma_z \sigma_x / \sqrt{\pi}$, we get $M_J \propto h_z$, which is a remarkably strong effect considering that in our models $h_z$ varies by a factor of 3.5. The phase-space density changes even more: $f = \rho / \sigma_z \sigma_x \sigma_y \propto h_z^{-3/2}$. For our thin/thick models, this gives variations by a factor of 7.

The Jeans mass affects the evolution in a similar way as the Toomre stability parameter $Q$: larger $Q$ results in later formation of the bar and in less prominent spiral arms. Comparison of the initial stages of bar formation in Figs 5 and 8 shows the effect: larger $h_z$ (thus, larger $\sigma_z$) results in much delayed formation of the bar.

The phase-space density is a complicated quantity, which is in practice used in the form of a coarse-grained phase-space density. Avila-Reese et al. (2005) present detailed results on the evolution of the phase-space density during the formation and evolution of barred disc models. As the system evolves, the coarse-grained phase-space density $f = \rho / \sigma_x \sigma_y \sigma_z$ decreases. The phase-space density $f$ can be considered as a measure of the degree of compressibility of a gravitational system: for given rms velocities, it defines the real-space density: $\rho = f \sigma_x \sigma_y \sigma_z$. The larger is the phase-space density, the larger is $\rho$. In turn, the density in the central region of a bar is an important factor because according to our results it moderates the growth of the bar and can prevent it from growing excessively. Thus,
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density model
and a larger surface density as compared with the low phase-space
scaling
timesteps of \( d_t \), thin and thick discs, respectively. The comparison is made at
4 Gyr. The thin-disc model (dashed curves) evolves to a more concentrated
structure. The central concentration explains why the bar pattern speed for
this model is higher than in the thick-disc model.
the phase-space density in the central disc region is a fundamental
parameter, which significantly affects the evolution of barred
galaxies.
Indeed, the buildup of mass in the high-phase-space-density mod-
ecls happens in the models. Fig. 11 shows the total (stellar plus dark
matter) circular velocity (top-panel model) and the stellar surface
density (bottom-panel model) profiles after 4 Gyr of evolution. We
see that the model \( K_2 \) (thin disc) has a larger inner circular velocity
and a larger surface density as compared with the low phase-space
density model \( K_5 \). The rms velocities (right panels) are nearly
the same for the two models. Thus, the \( K_2 \) model has a substantially
larger (by a factor of 4) phase-space density. It started with a larger
\( f \) and it ended with a larger \( f \).

6 DISCUSSION
We find that numerical effects – the mass and force resolution, and
the time integration of trajectories – can significantly alter results
of simulations. The time-step of integration must be small enough
to allow accurate integration of expected trajectories. Accuracy of
energy conservation can give a misleading impression that the sim-
ulation is adequate, while it actually makes substantial errors in the
position of orbits. Our experiments with realistic orbits in models
with flat rotation curves indicate that even the best available integra-
tion schemes require \( \sim 2000 \) time-steps per orbit. This requirement
is valid for variable-step schemes with more steps required for a
constant-step schemes. Numerical tests with full-scale dynamical
models confirm the condition. This condition results in very small
time-steps of \( dt \sim 10^4 \) yr. If one uses dimensionless units defined by
scaling \( G = 1, M_{\text{dis}} = 1, R_g = 1 \), then the dimensionless time-step
should be \( dt \approx 5 \times 10^{-4} \). This should be compared with typically
used \( dt \approx 0.01 \) (e.g. Athanassoula & Misiriotis 2002). This time-
step would be insufficient for integration of models with central
mass concentration presented in this paper.\(^2\) A constant time-step
\( dt = 5 \times 10^2 \) yr used by Widrow et al. (2008) is too large for the
models of the Milky Way galaxy studied in their paper.

Once the necessary numerical conditions are fulfilled, the well-
designed and extensively tested codes, which we used in the paper,
produce practically the same results. We do not find any systematic
deviations between the results obtained with \( \text{TREE} \) codes (GADGET
and PKDGRAV) and with the \( \text{ART} \) (ART) code.

Our results contradict conclusions of Sellwood & Debattista
(2006), who studied similar systems and found substantially dif-
f
erent results. They attributed the differences to numerical defects
in the \( \text{ART} \) refinement scheme. Results presented here strongly dis-
agree with this attribution: we do not find any deviations between
variable-resolution \( \text{ART} \) simulations and fixed-resolution \( \text{TREE} \)
models. There are two significant differences between the experiments
of Sellwood & Debattista (2006) and an actual \( \text{ART} \) simulation. First,
in contrast to the experiments, the \( \text{ART} \) algorithm does not system-
atically strengthen forces as a cell is refined. If this would happen,
it would indeed produce a serious error. However, the code is de-
signed in such a way that the force acting on a particle is never
dominated by the nearest neighbour. This is done by enforcing the
condition that either before or after the cell splitting the number of
particles inside a resolution volume (effectively a sphere with ra-
dius of two cells) has many (typically 20–50) particles (Colín et al.
2006). When the code splits the cells, the density in the new cells
does not increase, and, as a result, the force does not increase either.
Secondly, the experiments of Sellwood & Debattista (2006) have
the resolution increasing significantly after 2 Gyr of evolution in
order to stop the decline in bar pattern speed at that time. In our
simulations, any large density increases (a factor of 10) required
for cell splitting have been accomplished well before that time, and
therefore there is no change in resolution occurring that would effect
the bar pattern speed.

Disc height is an important parameter, which is often ignored in
the models of barred galaxies. In the models, which we consider
in this paper, the disc height determines the global properties of
the bars. Fig. 12 shows surface density maps of the models with
different initial disc thickness. Models with thin discs produce short
bars with \( R_{\text{bar}} \approx R_g \), which rotate relatively fast: \( R \approx 1.2–1.4 \) and
which show very little decline of the pattern speed. Models with
thick disc produce long and slow-rotating bar. In order to facilita-
te the comparison with our Galaxy, we rescaled models to have the
evolved disc scalelength 2.65 kpc and to have the circular velocity at
the solar distance 220 km s\(^{-1}\). Any \( N \)-body system has two arbitrary
scaling factors, which can be used to scale the system.

Having scaled the models to fit the disc scalelength, we can
compare other parameters of the models. Because the simulations
with thin discs produce reasonable models, we use one of the models
(\( K_3 \)) and compare it with the Milky Way. Table 3 gives a list of some
parameters. Fig. 13 compares the surface density maps of the Milky
Way (Freudenreich 1998) and the \( K_3 \) model. These comparisons
show that the model fits the Milky Way reasonably well.

\(^2\) Models studied by Athanassoula & Misiriotis (2002) are less concentrated
and do not require a small time-step.
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Figure 12. The isocontours of the surface stellar density for representative models. The thin-disc models \( K_{g3} \) and \( D_{g3} \) develop realistic bars with the ratio of corotation radius to the bar length \( R = 1.2 \) (left-hand panel; \( R_{\text{cor}} = 4.5 \) kpc) and \( R = 1.4 \) (right-hand panel; \( R_{\text{cor}} = 6.1 \) kpc). The thick-disc model \( K_{g5} \) (bottom panel) has a bar that covers the whole disc with the corotation radius 9.3 kpc. Distances in the plot are given in kpc units. All the models are rescaled to have the disc scalelength 3 kpc – the same as for the Milky Way galaxy.

We suggest that the disc height is only an indicator of a more fundamental property – the phase-space density in the central \( (R < R_{d}) \) region. In our models, the phase-space density is uniquely related to the disc height. Initially in our models, the disc height is low and the stability parameter \( Q = 1.3–1.8 \) is constant across the disc. Thus, the phase-space density in the central region is high, and subsequent evolution brings that highly compressible stellar fluid close to the centre, where it forms a nearly flat circular velocity curve. The latter, as we speculate, is responsible for arresting the growth of the bar. This relation between the disc height and the central phase-space density may not be true in general case. For example, O’Neill & Dubinski (2003) and Widrow et al. (2008) consider models with large central \( Q \) and, thus, with a low central phase-space density. Indeed, in their models bars slow down substantially.

The nearly constant pattern speed of bars in the thin-disc models is somewhat puzzling. A bar is a very massive non-axisymmetric object, which rotates inside a non-rotating dark matter halo. As such, one might expect that it should experience dynamical friction and slow down. This is why results of Valenzuela & Klypin (2003), which showed models with little slowing down of bars, were met with scepticism. Simulations presented in this paper confirm the findings of Valenzuela & Klypin (2003) and show that they cannot be related to numerical problems.

Orbital resonances may be responsible for the observed slow dynamical friction. The resonances in barred galaxies have been extensively studied in recent years using \( N \)-body simulations (e.g. Athanassoula 2003; Colín et al. 2006; Ceverino & Klypin 2007; Weinberg & Katz 2007). It is now well established that a large fraction of stellar particles are in resonance with the bar. Some fraction of dark matter particles are also in resonance (Athanassoula 2002, 2003; Colín et al. 2006; Ceverino & Klypin 2007), and these are very important for the dynamical friction between the stellar bar and the dark matter.

There are two types of (exact) resonances in an autonomous Hamiltonian dynamical system: elliptic and hyperbolic (Arnold & Avez 1968). Orbits, which are close to an elliptic resonance, librate around the exact resonance and have the structure of a simple pendulum (Lichtenberg & Lieberman 1983, section 2.4; Murray & Dermott 1999, section 8). Hyperbolic resonances are points on the intersections of separatrixes, which separate domains of elliptical resonances. Orbits close to a hyperbolic resonance are unstable and tend to migrate away from the resonance, while orbits close to an elliptical resonance are stable.

Ceverino & Klypin (2007) study the resonant orbits in simulations of barred galaxies in detail. It appears that the orbits belong to elliptical resonances. These orbits track their resonance: the orbits do not evolve if the resonance does not move, and they follow the resonances if it gradually migrates in the phase space. Thus, the elliptical resonances trap the orbits: if an orbit for whatever reason happens to appear in the domain of the resonance, it will have a tendency to stay with the resonance. This phenomenon is thought to be common in Solar system dynamics (e.g. Malhotra 1993). Resonance trapping explains why the simulations show maxima in the distribution of orbital frequencies at the positions of resonances.

Colín et al. (2006) investigate another aspect of the resonant interaction between the stellar bar and the dark matter. They found that the dark matter particles, which are in resonance with the stellar bar, themselves form a bar, which rotates with the same angular speed and has a very small lag angle \( (\sim 10^\circ) \) as compared with the stellar bar. Colín et al. (2006) argue that the interaction between the dark matter and the stellar bars is the main mechanism for the dynamical friction between the disc and the dark matter. The near alignment of the dark matter and stellar bars means that their interaction is minimized by the resonances.

These results indicate that the resonant interaction between the stellar bar and the dark matter is mostly due to elliptical resonances, and, thus, has a tendency to minimize the transfer of the angular momentum from the disc to the dark matter. Following orbits in such resonances emphasizes the need for conservative time-steps in

Table 3. Comparison of the parameters of the Milky Way and the model \( K_{g3} \).

| Parameter                             | \( K_{g3} \) | Milky Way | Reference                      |
|---------------------------------------|-------------|-----------|--------------------------------|
| Circular velocity (km s\(^{-1}\))     | 220         | 210–230   | Binney & Merrifield (1998, section 10.6) |
| Surface disc density at \( R_{d} \)  (M\(_{\odot}\) pc\(^{-2}\)) | 44.6        | 48 ± 9    | Kuijken & Gilmore (1991)        |
| Vertical rms velocity of stars at \( R_{d} \) (km s\(^{-1}\)) | 14          | 15–20     | Dehnen & Binney (1998b)         |
| Radial rms velocity of stars at \( R_{d} \) (km s\(^{-1}\)) | 38          | 35–40     | Dehnen & Binney (1998b)         |
| Pattern speed \( \Omega_{p} \) (km s\(^{-1}\) kpc\(^{-1}\)) | 50          | 53 ± 3    | Dehnen (1999)                   |
| Bar length (kpc)                      | 3.3         | 3.0–3.5   | Freudenreich (1998)             |
| Total mass inside 60 kpc (10\(^{11}\) M\(_{\odot}\)) | 5.5         | 4 ± 0.7   | Xue et al. (2008)               |
| Total mass inside 100 kpc (10\(^{11}\) M\(_{\odot}\)) | 7.3         | 7 ± 1.5   | Dehnen & Binney (1998a)         |
Dynamics of barred galaxies

Figure 13. The isocontours of the surface stellar density for model $K_g$ (top panel) and the Milky Way galaxy (bottom panel). The $K_g$ model was rescaled to have the evolved disc scalelength 3 kpc, which is close to the scalelength of our Galaxy. The bottom panel shows one of the models from Freudenreich (1998, fig. 14, right-hand panel). The model represents a multiparameter fit to the COBE Diuse Infrared Background Experiment maps of the near-infrared light coming from central regions of our Galaxy. The small circle shows position of the Sun. The $K_g$ model reproduces the length and the flattening of the Milky Way bar.

$N$-body simulations. Also, subtle changes in the underlying global potential could change the relative number of orbits in these resonances leading to disparate results for the slowing of the bar.

7 CONCLUSIONS

(i) Numerical effects can significantly alter results of simulations. The time-step of integration must be small enough to allow accurate integration of expected trajectories. Particular requirements for selecting the time-step depend on a combination of the force resolution and density distribution of the system. For systems with strong density gradients and nearly flat rotation curves, we recommend at least 200 time-steps per shortest orbit or, equivalently, $\eta = 0.020-0.05$ (see equation 13).

(ii) Once the necessary numerical conditions are fulfilled, $N$-body codes produce practically the same results. We do not find any systematic deviation between the results obtained with $TREE$ (GADGET and PKDGRAV) and with the $AMR$ (ART) code.

(iii) Because the dynamical models go through instabilities (bar instability followed by the buckling), models which start with nearly identical initial conditions end up with different global parameters, such as the bar amplitude, length and pattern speed. The differences depend on how strong are the instabilities. Systems deviate substantially during buckling instability and have a tendency to converge once they pass the instability phase.

(iv) The stellar phase-space density in the central $\sim$kpc region plays crucial role in overall evolution of galaxies. Low phase-space density prevents the buildup of central mass resulting in substantial bar growth. In this case, the bar covers most of the mass of the disc and rotates very slowly. Instead, models with high phase-space density create high-density central region, which prevents excessive growth of the bar.

(v) Simulations with realistic initial dark matter distribution and with high phase-space density produce models, which can match remarkably well numerous parameters of the Milky Way galaxy.

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REFERENCES

Aguerri J. A. L., Elias-Rosa N., Corsini E. M., Muñoz-Tuńon C., 2005, A&A, 434, 109
Arnold V. I., Avez, A., 1968, Ergodic Problems of Classical Mechanics. W.A. Benjamin, Inc., New York
Athanassoula E., 2007, MNRAS, 377, 1569
Athanassoula E., Beaton R. L., 2006, MNRAS, 370, 1499
Athanassoula E., Misiriotis A., 2002, MNRAS, 330, 35
Avila-Reese V., Carrilo A., Valenzuela O., Klypin A., 2005, MNRAS, 361, 997
Barnes J., Hut P., 1986, Nat, 324, 446
Beaton R. L. et al., 2007, ApJ, 658, L91
Bentley L., 1979, IEEE Trans. Softw. Eng. SE-5, 4, 333
Binney J., Merrifield M., 1998, Galactic Astronomy/James Binney and Michael Merrifield. Princeton Univ. Press, Princeton, NJ
Binney J., Tremaine S., 1987, Galactic Dynamics. Princeton Univ. Press, Princeton, NJ
Bizyaev D., Mironova S., 2002, A&A, 389, 795
Bureau M., Athanassoula E., 2005, ApJ, 626, 159
Bureau M., Aronica G., Athanassoula E., Dettmar R.-J., Bosma A., Freeman K. C., 2006, MNRAS, 370, 753
Ceverino D., Klypin A., 2007, MNRAS, 379, 1155
Colín P., Valenzuela O., Klypin A., 2006, 644, 687
Debattista V. P., Sellwood J. A., 1998, ApJ, 493, L5
Debattista V. P., Sellwood J. A., 2000, ApJ, 543, 704
Dehnen W., 1999, ApJ, 524, L35
Dehnen W., Binney J., 1998a, MNRAS, 294, 429
Dehnen W., Binney J. J., 1998b, MNRAS, 298, 387

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