INFLATIONARY PHASE IN A GENERALIZED BRANS-DICKE THEORY

International Journal of Theoretical Physics, to be published.

DOI 10.1007/s10773-009-9965-5

Marcelo S. Berman\(^{(1)}\) and Luis A. Trevisan\(^{(2)}\)

\(^{(1)}\) Instituto Albert Einstein / Latinamerica - Av. Candido Hartmann, 575 - # 17 80730-440 - Curitiba - PR - Brazil email: msberman@institutoalberteinstein.org

\(^{(2)}\) Universidade Estadual de Ponta Grossa, Demat, CEP 84010-330, Ponta Grossa, PR and Brazil email: luisaugustotrevisan@yahoo.com.br

Abstract

We find a solution for exponential inflation in a Brans-Dicke generalized model, where the coupling “constant” is variable. While in General Relativity the equation of state is \( p = -\rho \), here we find \( p = \alpha\rho \), where \( \alpha < -2/3 \). The negativity of cosmic pressure implies acceleration of the expansion, even with \( \Lambda < 0 \).

PACS 98.80 Hw
Introduction

New evidence for primordial inflation has been recently gathered through cosmic microwave observation (Weinberg, 2008). Barrow (1993) has pointed out the possible relevance of scalar-tensor gravity theories in the study of the inflationary phase during the early Universe. He obtained exact solutions for homogeneous and isotropic cosmologies in vacuum and radiation cases, for a variable coupling “constant” $\omega = \omega(\phi)$, where $\phi$ stands for the scalar field. For accounts on inflation, see, for instance, Linde’s book (Linde, 1990).

In this letter we extend Barrow’s paper by the study of an inflationary exponential phase. This letter can be considered also as a complement to Berman and Som’s paper (Berman and Som, 1989) dealing with the inflationary phase in B.D. original framework, which was followed by a letter by Berman (1989) where he studied the same problem in the context of a B.D. theory endowed with a cosmological constant. For scalar-tensor theories, consult the books by Berman (2007), Faraoni (2004), and Fujii and Maeda (2003). In Berman (2007a), we find a rationale for the existence of a cosmological ”constant”, though we must remember that a negative cosmic pressure may be also responsabilized for accelerated expansion, which includes exponential inflation.

The Field Equations

One way to formulate a scalar-tensor theory of gravitation can be with the following Lagrangian:

$$L_\phi = -\phi R + \phi^{-1} \omega(\phi) \partial_a \phi \partial^a \phi + 16\pi L_m - 2\Lambda(\phi)$$

where $L_m$ is the Lagrangian for matter fields, and $\phi$ is the scalar field. If $\omega = \text{const}$ we obtain the Brans-Dicke (1961) theory. This Lagrangian was adopted by Barrow and Maeda (1990). For a discussion about the Lagrangians of the scalar theories of gravitation, see (Liddle and Wands, 1992). The cosmological term $\Lambda(\phi)$ is taken also to mean time-dependent lambda.
By varying the action associated with (1) with respect to the space-time metric and the scalar field $\phi$, respectively we obtain the generalized Einstein equations and the wave equation for $\phi$ (Barrow, 1993):

$$G_{ab} = -\frac{8\pi}{\phi} T_{ab} - \frac{\omega}{\phi^2} \left[ \phi_a \phi_b - \frac{1}{2} g_{ab} \phi_i \phi^i \right] - \frac{1}{\phi} \left[ \phi_{a;b} - g_{ab} \square \phi \right] - \frac{\Lambda}{\phi} g_{ab} \tag{2}$$

$$[3 + 2\omega] \square \phi = 8\pi T - \left( \frac{d\omega}{d\phi} \right) \phi_i \phi^i + 2\phi \frac{d\Lambda}{d\phi} - 4\Lambda \tag{3}$$

In General Relativity theory, in face of a perfect fluid matter field, from the field equations, it is derived the energy momentum conservation law,

$$T_{;b}^{ab} = 0. \tag{4}$$

In Brans-Dicke theory, Weinberg (1972) has commented that in order to preserve the Principle of Equivalence, the scalar-field does not enter into the conservation equation above, which takes into consideration only the matter-fields. For scalar-tensor theories, as well, this conservation equation is imposed on the same token, but, of course, if we take the field equations, say, for Robertson-Walker’s metric, obtaining an equation for cosmic pressure and other for the energy density, we could combine those equations, along with the scalar-field one, and obtain a generalisation of the kind,

$$G_{;b}^{ab} = 0$$

where the conservation law applies to the right-hand-side of (2).

With Robertson-Walker’s metric,

$$ds^2 = dt^2 - a^2 \left[ (1 - kr^2)^{-1} dr^2 + r^2 d\theta + r^2 \sin^2 \theta d\varphi^2 \right] \tag{5}$$

we find, from (4), (2) and (3), that:

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi\rho}{3\phi} - \frac{\dot{\phi}}{\phi} \frac{\dot{a}}{a} + \frac{\omega}{6} \frac{\dot{\phi}^2}{\phi^2} + \frac{\Lambda}{3\phi} \tag{6}$$

$$\dot{\rho} + \frac{3\dot{a}}{a} (\rho + p) = 0 \tag{7}$$
\[
\ddot{\phi} + \left[ 3 \frac{\dot{a}}{a} + \frac{\dot{\omega}}{2\omega + 3} \right] \dot{\phi} = \frac{1}{3 + 2\omega} \left[ 8\pi(\rho - 3p) - 2\frac{\dot{\Lambda}}{\phi} + 4\Lambda \right],
\]

where overdots stand for time derivatives.

From now on, we consider only spatially flat solutions \((k = 0)\). Let

\[
a = a_0 e^{Ht}
\]

where \(a_0, H\), are constants, and

\[
p = \alpha \rho
\]

\((\alpha = \text{const})\).

In General Relativity, \(\alpha = -1\) for the inflationary phase; here, we must consider also other possibilities. Law (10) stands for a “perfect” gas equation of state. From (7) and (9), we find, using (10),

\[
\rho = \rho_0 e^{-3H(1+\alpha)t}
\]

where \(\rho_0 = \text{const}\).

Remember that \(H = \dot{a}/a\) stands for Hubble’s parameter.

Consider the solution for \(\phi(t)\), and \(\omega(t)\):

\[
\frac{\dot{a}^2}{a^2} = -\frac{\dot{\phi}}{\phi} \frac{\dot{a}}{a},
\]

and,

\[
\frac{8\pi \rho}{\phi} = \frac{\omega \dot{\phi}^2}{6 \phi^2} - \frac{\Lambda}{3\phi}
\]

By summing (12) and (13), we recover expression (6), so that the above two equations, result in a particular solution, though that it has some generality altogether.

We find from (12),

\[
\phi(t) = \phi_0 e^{-Ht}
\]
and,

$$\Lambda = \Lambda_0 e^{-3H(1+\alpha)t}$$

with $\phi_0$, $\Lambda_0$ = constants.

From (13), we get a possible solution with, $\omega \gg 3/2$,

$$\omega \cong \omega_0 e^{-H(2+3\alpha)t}$$

with $\omega_0$ = positive constant, and $\phi_0 > 0$.

The constants must obey the condition, obtained from (12) and (13),

$$8\pi \rho_0 + \frac{1}{3}\Lambda_0 + \frac{1}{6} H^2 \phi_0 \omega_0 = 0$$

The reason for a positive scalar-field, is that gravitation should be kept attractive, i.e., Newton’s gravitational constant is positive. The reason for a positive coupling ”constant”, is that experimental gravitational and astronomical observations require a large positive $\omega$ value.

It is then, highly desirable that $\omega$ grow with time, so we impose,

$$\alpha < -\frac{2}{3}$$

This condition on the equation of state encompasses the case $\alpha = -1$ of G.R.

From the scalar-field equation, of course, we get the fulfilled approximate condition,

$$2H^2 \phi_0 \cong \frac{M}{(3 + 2\omega)} e^{-H(2+3\alpha)t}$$

where,

$$M = \omega_0 H^2 \phi_0^2 (2 + 3\alpha) + 4\Lambda_0 + 6(1 + \alpha)\Lambda_0 - 8\pi \rho_0 (1 - 3\alpha)$$
Conclusion

We have thus, found a new solution for inflation, that deserves attention. On the other hand, it can be shown (Berman and Trevisan, 2009) that condition (16) is necessary for the amplification of gravitational waves during exponential inflation, at least when $\omega$ is constant.

It could be argued that another possible solution would be given by a positive cosmological “constant”, followed by a negative coupling $\omega$. It certainly may be more palatable for string theorists, but it would require some hand waving of the type $\omega_0 < -2$, because otherwise, the observation of slowly moving particles or of time-dilation experiments, which imply (see Weinberg, 1972),

$$G = \left[\frac{2\omega + 4}{2\omega + 3}\right]^{-1},$$

would again carry $G < 0$.

Acknowledgments

MSB thanks two anonymous referees of this journal, who contributed significantly towards the preparation of the final manuscript. MSB is also grateful to Nelson Suga, Marcelo F. Guimarães, Antonio F. da F. Teixeira, and Mauro Tonasse and for the encouragement by Albert, Paula, and Geni. He offers this paper in memoriam of M. M. Som.

References

Barrow, J.D;–Phys. Rev D 46, 5329 (1993)
Linde, A–“Particle Physics and Inflationary Cosmology ”, Harwood Acad. Press, N.Y. (1990).
Berman, M.S; Som, M.M; – Phys.Lett. A 136, 206, (1989)
Berman, M.S – Phys.Lett. A 142, 335,(1989)
Brans, C., Dicke, R.H;– Phys.Rev. 124, 925 (1961)
Barrow, J.D. and Maeda, K.–Nucl.Phys B341, 294 (1990)
Liddle, A.R. and Wands, D–Phys.Rev. D45,2665 (1992)
Berman,M.S, Trevisan, L.A; – Submitted.
Weinberg, S. – *Cosmology*, Oxford University Press, Oxford (2008).

Berman, M.S. – *Introduction to General Relativistic and Scalar-Tensor Cosmologies*, Nova Science Publishers, New York (2007).

Faraoni, V. – *Cosmology in Scalar-Tensor Gravity*, Kluwer, Dordrecht (2004).

Fujii, Y.; Maeda, K. -I. – *The Scalar-Tensor Theory of Gravitation*, CUP, Cambridge (2003).

Berman, M.S. - *Introduction to General Relativity and the Cosmological Constant Problem*, Nova Science Publishers, New York (2007a).

Weinberg, S. - *Gravitation and Cosmology*, Wiley, New York (1972).