Single and double bit quantum gates by manipulating degeneracy

T. Hakioglu\(^{(1)}\), J. Anderson\(^{(2)}\), F. Wellstood\(^{(2)}\)

1. Department of Physics, Bilkent University, Bilkent, 06533 Ankara, Turkey
2. Department of Physics, University of Maryland, College Park, MD 20742
A novel mechanism is proposed for single and double qubit state manipulations in quantum computation with four-fold degenerate energy levels. The principle is based on starting with a four-fold degeneracy, lifting it stepwise adiabatically by a set of control parameters and performing the quantum gate operations on non-degenerate states. A particular realization of the proposed mechanism is suggested by using inductively coupled rf-squid loops in the recently observed macroscopic quantum tunneling regime where the energy eigen levels are directly connected with the measurable flux states. The one qubit and two qubit controlled operations are demonstrated explicitly. The appearance of the flux states also allows precise read-in and read-out operations by the measurement of the flux.

PACS numbers:

Recent advances on the experimental efforts to demonstrate the fundamental single and two bit quantum gates by ion trap[6] and nuclear magnetic resonance (NMR) experiments[7] as well as the observation of the coherent Rabi oscillations in Cooper pair boxes[8] are hinting at new challenges awaiting the realization of the quantum computational devices. It has been shown theoretically that the basic gate operations based on the unitary transformations of the qubits can be performed by single and double qubit manipulations[9]. Therefore it is a necessary first step that a proposed mechanism should demonstrate these fundamental operations as well as a successful read-in and read-out before any decoherence comes into play. A next step which is not detailed in this letter is to construct arrays of independent qubits or coupled qubit pairs to perform parallel gate operations. In our suggested mechanism the state space is four dimensional corresponding to the state space of a pair of qubits. The paired qubit states are inseparable; nevertheless, each qubit can be manipulated individually. The mechanism also allows controlled operations on selected bits rather easily without affecting those bits that need to be unchanged. A more detailed discussion of the proposed mechanism with its specific experimental realizations using rf-squid loops will be published elsewhere.

A. The mechanism

The mechanism is based upon coupling a pair of ideal two level systems with manifestly degenerate energy levels at the energy $E_0$ by two complex coupling parameters as indicated in Fig. 1a. The coupling $C_1$ lifts the four-fold degeneracy to two-fold (Fig. 1b). The second coupling $C_2$ couples the two remaining degenerate levels in each pair and lifts the degeneracy completely (Fig. 1c). The states in the initial four-fold degenerate configuration at $C_1 = C_2 = 0$ are labelled by $|m,n\rangle$, ($m,n = 0, 1$). We consider them to be the eigen states of some underlying non interacting system. In the qubit language below the first (second) bit is $m$ ($n$). The states $|m,n\rangle$ are the components of the four dimensional row vector $([11], [00], [10], [01])$ which is to be considered as the basis for the model Hamiltonian. The Hamiltonian including the degeneracy breaking couplings is given in this four dimensional basis by the Hermitian matrix

$$H_{c_1,c_2} = \begin{pmatrix} E_0 & C_1 & C_2 & 0 \\ C_1 & E_0 & 0 & C_2 \\ C_2 & 0 & E_0 & C_1 \\ 0 & C_2 & C_1 & E_0 \end{pmatrix}.$$ (1)

Without loss of generality we assumed that the couplings are real and positive and that they are applied adiabatically in the order $C_1$ first and $C_2$ second (the order is commutative) with $0 < C_2 \leq C_1$. The diagonalizing matrices for independent couplings are unitary transformations acting on the four dimensional basis. These are $U_1$ for $C_1 \neq 0, C_2 = 0$ and $U_2$ for $C_1 = 0, C_2 \neq 0$ as given by

$$U_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \quad U_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}.$$ (2)

When both couplings are turned on, the Hamiltonian is diagonalized by $U = U_1 U_2$ as $H_4 = \text{diag}(E_0 + C_1 + C_2, E_0 + C_1 - C_2, E_0 - C_1 + C_2, E_0 - C_1 - C_2)$ which correspond to the levels in Fig.1c. For arbitrary couplings $C_1, C_2 \neq 0$ the energy eigenvalues are all nondegenerate and they are distributed symmetrically around the major center $E_0$ and the two minor ones $E_0 \pm C_1$. The single and double qubit quantum gate operations are performed ideally by manipulating the strengths of at least one of the coupling constants and making use of the invariance of the major center under adiabatic changes of both couplings and of the two minor centers under that of $C_2$. In Fig.1c we have indicated the eigenstates with up and down arrows. In the double-SQUID model considered below they correspond to the flux states. These states are connected with the initial basis states $|m,n\rangle$ by

$$\begin{pmatrix} \uparrow \uparrow \\ \downarrow \downarrow \\ \uparrow \downarrow \\ \downarrow \uparrow \end{pmatrix} = U_2 U_1 \begin{pmatrix} 11 \\ 01 \\ 00 \\ 10 \end{pmatrix}.$$ (3)

Note that the ordering of the $|m,n\rangle$ basis is made with respect to the ordering of the energy eigen values of the flux states on the left hand side in (3) which is different than the ordering used for constructing the Hamiltonian matrix (1). For later convenience we introduce the short notation

$$|V\rangle = \begin{pmatrix} \uparrow \uparrow \\ \downarrow \downarrow \\ \uparrow \downarrow \\ \downarrow \uparrow \end{pmatrix}, \quad |v\rangle = \begin{pmatrix} 11 \\ 01 \\ 00 \\ 10 \end{pmatrix}.$$ (4)

B. An rf-SQUID realization of the mechanism

Physically, this suggested four state mechanism can be realized by using inductively coupled rf-squids as depicted in Fig. 2. The SQUIDs must operate in the macroscopic quantum tunneling (MQT) regime[5] which requires temperatures
on the order of mili Kelvin for standard Josephson junctions. The four level system of Fig.1a is constructed by preparing two identical SQUIDs A and B in which each qubit is represented, to a good approximation, by its two lowest energy eigen states (denoted as $|0\rangle$ and $|1\rangle$); in the MQT regime these are nearly degenerate (pseudo-degenerate) in energy (due to the fluctuations in the flux) compared to the next (third) level.

A qubit which is close to being ideal is characterized by an isolated hypothetical two-level system with a high qubit quality ratio $\eta \approx 10^2$ and the potential parameter range above) comprising the best pseudo-degenerate approximation we ever found to the manifest degeneracy in Fig.1b. At this point we read from the vertical axis in Fig.4 that $C_1 = 10^{-3}$ eV. As the double-well potentials are tilted in parallel and simultaneously in both SQUIDs the shift obtained for each eigenenergy is approximately linear in the bias flux $\Phi_{ex}$. This indicates that the coupling can be primarily represented as a first order perturbation in $\Phi_{ex}$. A comparison of the Fig. 1 with the calculations shown in Fig. 4 indicates that the coupled rf-SQUID model is in essence an isolated four-level quantum system. Recently, Schrödinger Cat states have been obtained experimentally by manipulating these two couplings in the MQT regime. For the physically relevant range $0 \leq y_{ex} \leq y_{ex}^\ast$ the centers of the splitting energies in Fig. 4 also respect the manifest conservation principle of the energy centers in Fig. 1. In fact, in the numerical calculations we found a shift smaller than 2% in the energy centers for the whole range $0 \leq y_{ex} \leq y_{ex}^\ast$.

In this paper we will refer to two different representations of the basis states for two coupled SQUIDs. The first, which is the even and the odd states of the mutually noninteracting SQUIDs, is represented by the vector $|\psi\rangle = \{ |11\rangle, |00\rangle, |10\rangle, |01\rangle \}$ as in section A. Here in the state $|mn\rangle$ m represents the parity state of the SQUID A and n represents that of SQUID B. We will designate $|\psi\rangle$ as the two-bit parity basis. The second, which is the set of flux states for the coupled and tilted SQUIDs, i.e. both couplings are nonzero, is represented by $|\psi\rangle = \{ |\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle \}$ which we refer to as the two-bit flux basis.

### C. The quantum gate operations

According to the Hamiltonian (4), the eigen levels in Fig.1b corresponding to $C_1 \neq 0$ and $C_2 = 0$ are the doubly degenerate states $|\psi\rangle = U_1 |\psi\rangle$. Explicitly these are, $(|00\rangle + |11\rangle)/\sqrt{2}$ and $(|10\rangle + |01\rangle)/\sqrt{2}$ for $E_0 + |C_1|$ and, $(|10\rangle - |01\rangle)/\sqrt{2}$ and $(|00\rangle - |11\rangle)/\sqrt{2}$ for $E_0 - |C_1|$. If $C_2$ is also turned on as in Fig.1c, the non-degenerate energy eigenstates become the two-bit flux states defined in Eq. (4) and written explicitly as...
The choice of the basis in which the fundamental quantum gates are to be operated should be determined by the accessibility of the input and output states by the measurement mechanism. This condition actually means that all information in the relative magnitudes and the phases of these states should be measurable. The absolute values of the amplitudes of the flux states are directly measurable whereas their relative phase factors do not couple to the measurement (see below). Since it is crucial to be able to read and write the relative phase between the flux states, we choose the two-bit parity basis $|v\rangle$ instead of the two-bit flux basis $|V\rangle$ to perform the gate operations. The flux basis is used for the I/O operations. Before and after performing the logic gates it is therefore needed to switch between the $|v\rangle$ and the $|V\rangle$ bases. Notice that the matrices $U_1$ and $U_2$ are two-bit Hadamard transformations. In particular, it can be verified that

$$U_1 = 1 \otimes \mathbb{H}, \quad U_2 = \mathbb{H} \otimes 1, \quad \mathbb{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ \end{pmatrix},$$

(6)

where $1$ is the $2 \times 2$ unit matrix. The matrix $\mathbb{H}$ is the one bit Hadamard transformation. The transformation from the natural eigenbasis $|V\rangle$ to the parity basis $|v\rangle$ can therefore be performed by two successive two-bit Hadamard transformations. In this way any phase information of a quantum state encoded in $|v\rangle$ basis contributes to the relative amplitudes of the $|V\rangle$ basis yielding direct accessibility to the measurement. This transformation can be summarized as $|V\rangle \xrightarrow{U_1} |v\rangle \xrightarrow{U_2} |v\rangle$. The effective result of this transformation is that each state in the parity basis is finally assigned a distinct energy eigenvalue as shown below. This transformation is equivalent to effectively assigning each component $|m\rangle$ of the $|v\rangle$ basis an energy eigenvalue. The logic operations are then performed on the $|v\rangle$ basis. We now demonstrate the fundamental single-qubit operations, i.e. the phase flip and the Hadamard transformation.

**Single qubit operations:**

Although the mechanism is based on four states, it is possible to identify two individual flux qubits and perform single bit operations on each one independently. In the parity basis, $|m\rangle$ we assign the first bit as the control (qubit no.1) and the second bit as the target (qubit no.2). By definition, the single qubit operations are performed on the target in the four dimensional state space unconditionally from the state of the control bit. Whereas the controlled operations (two-qubit operations) are performed when the control bit is in a particular, i.e ($0$ or $1$) state. With this convention we now start with the single qubit operations.

### I. The phase flip is a special case of the phase evolution described by

$$Z_{\phi} : \begin{pmatrix} |11\rangle \\ |01\rangle \\ |00\rangle \\ |10\rangle \end{pmatrix} \mapsto e^{-i\phi} \begin{pmatrix} |11\rangle \\ |01\rangle \\ |00\rangle \\ e^{-i\phi} |10\rangle \end{pmatrix}.$$

(7)

The phase flip is obtained in $|\phi\rangle$ at $\phi = \pi/2$ which implies that the state of the second bit is changed as $1 \rightarrow 0$ and $0 \rightarrow 1$ independent from the state of the first bit. It is shown below that this can be obtained as a direct sum of two controlled phase flips.

The phase evolution $Z_{\phi}$ in Eq. (5) is achieved in the two coupled SQUID realization in two steps. We switch the coupling configuration of the system to $C_2 = -C_1$ which makes the states $|01\rangle$ and $|00\rangle$ degenerate at the major center $E_0$. By this switching, the remaining (nondegenerate) states, i.e. $|11\rangle$ and $|10\rangle$ are now at the energies $E_0 + 2C_1$ and $E_0 - 2C_1$ respectively. The four state system is then permitted to freely evolve in this configuration. Degenerate states do not acquire any relative phase with respect to each other. At this first stage the phases acquired by the states are described by

$$\begin{pmatrix} |11\rangle \\ |01\rangle \\ |00\rangle \\ |10\rangle \end{pmatrix} \mapsto e^{iE_0 t} \begin{pmatrix} e^{i\phi} |11\rangle \\ e^{i\phi} |01\rangle \\ e^{-i\phi} |00\rangle \\ e^{-i\phi} |10\rangle \end{pmatrix}.$$

(8)

where $\phi = 2C_1 t$ as the time dependent phase. In the second stage we set $C_2 = -C_1$ which turns the states $|11\rangle$ and $|10\rangle$ to become degenerate at the energy $E_0$ and $|01\rangle$ and $|00\rangle$ nondegenerate at energies $E_0 + 2C_1$ and $E_0 - 2C_1$ respectively. An identical phase evolution is then performed on the states $|01\rangle$ and $|00\rangle$. The net transformation is given by

$$\begin{pmatrix} |11\rangle \\ |01\rangle \\ |00\rangle \\ |10\rangle \end{pmatrix} \mapsto e^{i2E_0 t} \begin{pmatrix} e^{i\phi} |11\rangle \\ e^{i\phi} |01\rangle \\ e^{-i\phi} |00\rangle \\ e^{-i\phi} |10\rangle \end{pmatrix}.$$

(9)

which, after factoring out the overall phase $e^{i2(E_0 t + \phi)}$ is equivalent to Eq. (6). The phase flip is obtained at $\phi = \pi/2$ corresponding to $t_1 = \pi/4C_1$ which can be described as

$$Z_{\phi/2} = \begin{pmatrix} -i e^{-iE_0 t_1} e^{i4H_c_1} & 0 \\ 0 & -i e^{-iE_0 t_1} e^{i4H_c_1} \end{pmatrix} (C_2 \rightarrow -C_1) \times \begin{pmatrix} -i e^{-iE_0 t_1} e^{i4H_c_1} & 0 \\ 0 & -i e^{-iE_0 t_1} e^{i4H_c_1} \end{pmatrix} (C_1 \rightarrow C_1).$$

(10)

where the action of the operators (matrices) is defined rightwards. Notice that, Eq. (10) is manifestly the composition of two controlled phase flips where these two operations commute. Subtracting the overall phase, the first square bracket is equivalent to $|11\rangle \mapsto |11\rangle$, $|10\rangle \mapsto -|10\rangle$. This is the controlled phase flip $1 \rightarrow 1$ and $0 \rightarrow 0$ when the control bit is in the state 1. Similarly, the second bracket denotes the same controlled phase flip when the control bit is in the 0 state. Hence, this method proves an efficient realization of all controlled operations as well (see the double qubit operations below).

### II. The Hadamard ($\mathbb{H}$) transformation is a special case that can be obtained by rotations and phase flips on the Bloch sphere and its realization is very similar to the one discussed already in the phase flip. The idea is to perform two consecutive controlled rotations by coupling the non-degenerate states to an external pulse resonant at the energy difference $4C_1$. Basically the two-bit Hadamard gate can be obtained in the following steps
\[ (C_2 \rightarrow C_1) \]
\[ [R_x(\pi/4) R_z(\pi/4) R_z(\pi/4)] \]
\[ [C_2 \rightarrow -C_1] \]
\[ [R_x(\pi/4) R_z(\pi/4) R_z(\pi/4)] \] \hspace{1cm} (11)

In the first step \([C_2 \rightarrow C_1]\) the inner states, i.e. \([01], [00]\), are made degenerate at the major center \(E_0\). In the second step a Hadamard transformation is performed (up to an overall phase \(e^{-i\pi/2}\)) on the outer states \([11]\) and \([10]\) (for those the first bit is 1) when the inner states (the first bit is zero) are degenerate. In this term, the states \([11]\) and \([10]\) are rotated by \(R_x(\pi/4)\) sandwiched between two phase evolutions at \(\phi = 2 \times \pi/4\) indicated by \(R_z(\pi/4)\). The overall phase accumulated on the four states is \(e^{iE_0 t_2}\), where \(t_2\) is the duration of the pulse given by \(t_2 = \pi/(4\kappa)\) with \(\kappa\) describing the coupling strength of the levels to the external pulse [The coupling mechanism to the external field is similar to the ion trap experiments and will not be repeated here]. The third factor \([C_2 \rightarrow -C_1]\) indicates that \(C_2\) is now switched in the opposite direction swapping the inner and the outer states. The fourth factor in \([11]\) is another Hadamard transformation in the interchanged configuration (when the first bit is 1). The same front factor \(e^{iE_0 t_2}\) as in the step 1 also appears here as an overall phase. After the fourth step the inner and outer states should be swapped back to the initial configuration. Note that in all single and double qubit operations there is an overall phase. After the fourth step the inner and outer states should be swapped back to the initial configuration.

Double qubit operations: The controlled phase flip was considered previously. It basically comprises the first or the second half of the \([Z]/2\) operation in Eq. (8). We will therefore not consider it here again. The other crucial double qubit operation is the Controlled-\(\text{NOT}\) gate.

Controlled \(\text{NOT}\): Ideally, the \(\text{CNOT}^\dagger\) gate can be obtained from \(\text{CZ}\) and \(\text{H}\) by:

\[
\text{CNOT} = \text{H} \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right] \text{H}
\] \hspace{1cm} (12)

It is much easier to obtain \(\text{CNOT}\) in this proposed mechanism, which can be done by a single controlled rotation by \(\pi/2\). This can be written as:

\[
\text{CNOT} = e^{-iE_0 t_2} [C_2 \rightarrow C_1] R_x(\pi/2) [C_2 \rightarrow C_2].
\] \hspace{1cm} (13)

If one ignores the overall phase, Eq. (13) amounts to setting the coupling so that \(C_2 = C_1\), applying the rf-field to induce the rotation \(R_x(\pi/2)\), and finally switching back to the initial configuration of the \(C_1\) and \(C_2\). The full \(\text{CNOT}\) matrix in the \(|v\rangle\) basis is

\[
\text{CNOT} = \begin{pmatrix}
0 & 0 & 0 & i \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
i & 0 & 0 & 0
\end{pmatrix}
\] \hspace{1cm} (14)

which amounts to the flip of the second bit \(|0\rangle \rightarrow i|1\rangle\) and \(|1\rangle \rightarrow i|0\rangle\) when the first bit is in the 1 state.

D. Operational time scales and dephasing

The time scale required for the gate operations can be roughly considered to be independent from the particular gate operation. We consider the operational time as \(t_{op} \simeq (\Delta E)^{-1}\) where \(\Delta E\) describes the energy difference of the states at the configuration where the gate operation is performed. For instance, a typical scale is the phase flip \(t_{op} = t_1 = \pi/4C_1\) can be read off from Fig. 4 at the degeneracy point \(y_{ex} = y_{ex}'\). We find that \(C_1 \sim 5 \times 10^{-3} eV\) which yields \(t_1 \sim 5 ps\). The switching time is possible for less than 1ns between two gate operations. By comparing these two time scales, we estimate that the gate operations are dictated by the switching time.

As the dephasing mechanisms are concerned certain expected sources and their decoherence time scales have been previously analyzed in detail [11]. SQUIDs in the flux regime are known to be insensitive to the charge fluctuations. The decoherence time for the charge fluctuations in the flux dominated regime has been estimated to be \(t_{\phi} \sim 0.1 s\). Among the other sources there are, the quasiparticle tunneling \((t_\phi \sim 1 ms)\) and electromagnetic radiation of the junction \((t_\phi \sim 10^5 s)\). More substantial dephasing effects are known to arise from the near distance dipole-dipole interactions between the SQUIDs \((t_\phi \sim 0.1 ms)\).

The relaxation time in the time dependence of the basis (flux or parity) states is the most crucial element for the I/O operations based on the density matrix measurements. For the parameters considered here it can be calculated by the explicit expression given by Leggett et al. and Weiss Ref. [11] for which we find \(t_{rel} \sim 1 \mu s\).

E. Read-in and read-out

The state of the coupled squids is observed before and after the computation by the flux measurement made by two do-squids each independently facing one SQUID loop as shown in Fig. 2. Before any read-in or read-out made, a back transformation is needed between the operational \(|v\rangle\) basis to the physical flux basis \(|V\rangle\). Suppose that a particular logic operation is denoted by the matrix \(T\) in the basis \(|V\rangle\). The formula

\[
\textbf{V} \rightarrow [U|L| T [U|L|^\dagger]^{-1} \textbf{V}
\] \hspace{1cm} (15)

describes the effect of \(T\) in the basis \(|V\rangle\). Here \(T\) is any desired composition of the single and double qubit operations. Suppose \(T = [Z]\). Using (13) we find that the flux states are transformed by,

\[
[Z]|V\rangle = \begin{pmatrix}
\cos \phi & 0 & 0 & i \sin \phi \\
0 & \cos \phi & i \sin \phi & 0 \\
i \sin \phi & 0 & \cos \phi & 0 \\
i \sin \phi & 0 & 0 & \cos \phi
\end{pmatrix}|V\rangle.
\] \hspace{1cm} (16)

We give a second example from \(T = \text{CNOT}\). Applying (15) to this case we obtain

\[
|V\rangle \rightarrow \frac{1}{2} \begin{pmatrix}
(1 - i) & 0 & 0 & (1 + i) \\
0 & (1 + i) & 0 & -(1 - i) \\
-(1 + i) & 0 & (1 - i) & 0 \\
0 & -(1 - i) & 0 & (1 + i)
\end{pmatrix}|V\rangle.
\] \hspace{1cm} (17)
The unique correspondence between the flux states and the parity basis permits read-in and read-out operations to be performed by measuring the flux as shown in Fig.2. The flux measurements simulate single event quantum mechanics. Suppose an output state to be measured is \(|\psi\rangle = a_1 |\uparrow\uparrow\rangle + a_2 |\downarrow\downarrow\rangle + a_3 |\uparrow\downarrow\rangle + a_4 |\downarrow\uparrow\rangle\). A single flux measurement yields one of the flux states in the output with the probability \(|a_i|^2\) associated with that state. The partial amplitudes of the flux states can therefore be inferred only after repeated measurements. In this context, the final measurement is similar to the quantum computation model using ion traps.

F. Discussion

We demonstrated a new theoretical mechanism for quantum logic gate operations by manipulating degeneracy of a symmetric 4-level system by two variable coupling parameters. A realization of this system using rf-SQUIDs in the flux regime is made in which only one of the couplings is variable. Physical examples in which both couplings vary are very likely to exist but currently unknown to us. In that case an ideal realization of the proposed mechanism is possible with the particularly added feature of Fig.1a namely storing the computed information by suppressing the energetic interference between the states. The realization of the gates for the fundamental single and double qubit operations is also demonstrated. As the scalability of this SQUID realiztion is concerned we have less to say at this moment. Masking and evaporation techniques that are already well established can be used to fabricate arrays of the qubit pairs. The coupling between different pairs can be realized in a similar way as those that are coupled as members of a single pair. We continue our current effort in the direction of simulating two and three such pairs coupled inductively.

ACKNOWLEDGMENTS

TH is thankful to I.O Kulik (Department of Physics, Bilkent University) for discussions and critical reading of the manuscript.

1 DJ Wineland, C Monroe DM Meekhof et al., Proc. R. Soc. Lond. A 454, 411 (1998).
2 IL Chuang, N Gershenfeld, MG Kubinec et al. Proc. R. Soc. Lond. A 454, 447 (1998).
3 Y Nakamura, YA Pashkin and JS Tsai, Nature 398, 786 (1999).
4 D Deutsch, A Barenco and A Ekert, Proc. R. Soc. Lond. A 449, 669 (1995); S Lloyd, Phys. Rev. Lett. 75, 346 (1995); DP DiVincenzo, Phys. Rev. A 51, 1015 (1995); A Barenco, Proc. R. Soc. Lond. A 449, 679 (1995).
5 J. Clarke, AN Cleland, MH Devoret et al., Science 239, 992 (1988); R Rouse, S Han and JE Lukens, Phys. Rev. Lett. 75, 1614 (1995).
6 Jonathan R. Friedman, Vijay Patel, W. Chen, S.K. Tolpygo and J.E. Lukens, Detection of a Schrödinger’s Cat State in an rf-SQUID [cond-mat/0004293].
7 Michael A. Nielsen and Isaac L. Chuang, Quantum Computation and Quantum Information (Cambridge Univ. Press, 2000).
8 Yuriy Makhlin, Gerd Schön and Alexander Shnirman, Rev. Mod. Phys. 73, 357 (2001).
9 J.E. Mooij, T.P. Orlando, L. Levitov et al., Science 285, 1036 (1999); T.P. Orlando, J.E. Mooij, Lin Tian et al., Phys. Rev. B 60, 15398 (1999).
10 L. Tian, L.S. Levitov, C.H. van der Wal et al. in Quantum Mesoscopic Phenomena and Mesoscopic Devices in Microelectronics, Ed. I.O. Kulik and R. Ellialtoğlu, NATO-ASI Series E. (Kluwer Academic Publ. 2000).
11 A.J. Leggett, S. Chakravarty, A.T. Dorsey et al., Rev. Mod. Phys. 59, 1 (1987); U. Weise, Quantum Dissipative Systems (World Sci. 2000).

FIGURES

Fig.1: Lifting the four-fold degeneracy stepwise in the proposed mechanism. In the ordering of the flux states shown in (c) [see Eq.2 for the definition of the flux states] with respect to the energy eigenvalues we considered \(0 < C_2 \leq C_1\). Note that the initial four-fold degeneracy causes the energy centers indicated by the horizontal dashed lines to be independent of the couplings.

Fig.2: Inductive coupling of the two rf-SQUID loops. The dc-SQUIDs have two junctions for balanced read-in and read-out the flux at each rf-SQUID loop independently. Although the details of the mutual field screening is not shown, the circuitry should be designed such that the I/O devices are decoupled from each other completely.

Fig.3: Pseudo-degenerate energy eigenfunctions and the corresponding single well localized flux states. The flux states \(0 \pm 1\) are properly normalized by \(1/\sqrt{2}\). The parameters used are \(\beta_L = 1.1, y_{xv} = \pi, C = 0.4pF, L = 100pH\).

Fig.4: Lifting the degeneracy by tilting the symmetric double-well potential in the coupled SQUID pair. The vertical scale is the numerically calculated eigen energies in eV units.

Fig.5: The transformation from the flux basis \(|V\rangle\) to the parity basis by two Hadamard transformations. After \(U_1\) is applied on the flux basis, the states are \(v_1 = (|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle)/\sqrt{2}, v_2 = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}, v_3 = (|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle)/\sqrt{2}, v_4 = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}\) with the energies \(E_1 = E_0 - C_1 - C_2, E_2 = E_0 - C_1 + C_2, E_3 = E_0 + C_1 - C_2, E_4 = E_0 + C_1 + C_2\).
\( C_1 = C_2 = 0 \)

\( C_1 \neq 0 \) \( \frac{\pi \Phi / \Phi}{C_1 \neq 0} \)

\( C_2 = 0 \) \( C_2 \neq 0 \) \( C_2 = C_1 \neq 0 \)
