Generation of rectangular optical waves by relativistic clipping

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Abstract
Theoretical results are reported for the reflection and transmission of few-cycle laser pulses on a very thin conducting layer, which may represent the surface current density of the massless relativistic charges of graphene. It is shown that the pulse may undergo violent distortions to the extent that the scattered radiation contains rectangular trains, which are approximate physical realizations of Rademacher functions in the optical or terahertz regime.

1. Introduction
Laser technology has been undergoing very rapid development recently, making it possible to generate extreme (ultrashort, very high intensity) and well-controlled electromagnetic fields in the optical regime. Carrier-envelope phase difference effects (Milosavljević et al. 2006) have become important in both diagnosing and controlling such short, e.g. attosecond, pulses (Krausz and Ivanov 2009). After the real-time observation of electron tunnelling from atoms (Uiberacker et al. 2007), even the tiny delays of photoemission signals stemming from different atomic states have been measured (Schultze et al. 2010), and further studies of the detailed dynamics of photoemission (Nagele et al. 2011) are ongoing. Important experimental results on high-order harmonic generation in bulk crystals (Ghimire et al. 2011) and attosecond time-resolved photoemission from solid targets (Neppl et al. 2012) raise questions concerning the theoretical description of nonlinear electromagnetic responses of condensed matter (see e.g. Zhang and Thumm 2011, Korbman et al. 2012, Földi and Benedict 2012). Recently Wirth et al. (2011) succeeded in synthesizing light transients for producing sub-cycle pulses, with unprecedented control of the amplitudes and phases, which open up a wide range of possibilities for studying very fast processes in atomic and condensed matter systems.

The generation of broad-band radiation and short pulses relies on the highly nonlinear processes induced by intense laser fields. In general, the magnitudes of these nonlinearities (Fedorov 1997) also depend on the target, of course. It is clear that—depending on various parameters—intense fields may cause relatively modest effects, and moderately intense lasers may induce very high-order processes. Here we discuss an example of the latter situation, and show that the collective radiation back-reaction of relativistic convective surface currents driven by a laser field can cause a violent distortion in the scattered radiation. Similar phenomena have already been considered by us to describe the reflection of laser pulses on thin conducting nano-layers (Varró 2004, 2007a, 2007b, 2007c, 2012). The effectively massless charge carriers, electrons and holes in graphene (see Novoselov et al. 2005, Bostwick et al. 2007, Castro Neto et al. 2009), seem to also be good candidates for illustrating the high ‘susceptibility’ we just mentioned. The unique optical properties of graphene (Blake et al. 2007) are remarkable also in the sense that they are directly related to the fine structure constant (Nair et al. 2008), which naturally appears in linear response theory (see e.g. Abergel and Fal’ko 2007, Cserti and Dávid 2010). In the present paper we report on our theoretical results concerning the reflection and transmission of a few-cycle laser pulse on a very thin conducting layer, and illustrate the temporal evolution of the electric field strength of the reflected radiation in the nonlinear regime. By working out a classical description, we show that the pulses may undergo violent distortions, to the extent that the scattered radiation contains...
rectangular trains, being physical realizations of Rademacher functions in the optical or terahertz regime.

In section 2 the matching equations for the electromagnetic fields and the force terms (including the radiation reaction term) in the equation of motion of the massless charges will be considered. In section 3 the nature of the ‘relativistic clipping effect’ will be discussed, and some illustrative numerical examples and figures will be presented. In section 4 a summary including conclusions closes our paper.

2. Matching equations for the electromagnetic fields, and the equation of motion of the massless charges in a thin layer interacting with them

In the present section we derive and solve the matching equations for the electric field and the magnetic induction representing the scattered radiation. First we shall express the reflected and transmitted fields in terms of an unknown surface current, and then we put the total field into the equation of motion of the massless charge elements of which the surface current consists. In this way, the system of equations becomes closed, and the problem is reduced to the solution of the equation of motion for the charges. This approach automatically incorporates a ‘collective radiative back-reaction’ of the complete layer, and delivers a generalization of the Fresnel formulae (Varró 2004, 2007a, 2007b, 2007c, 2012).

The components of a TM (p-polarized) configuration of waves (0, \(E_x, E_y\)) and (\(B_z, 0, 0\)), in the two separated media (in regions 1 and 3), satisfy the following Maxwell equations:

\[
\begin{align*}
\partial_t B_z &= \partial_\phi \cdot E_y, \\
-\partial_t B_y &= \partial_\phi \cdot E_z, \\
\partial_y E_z - \partial_z E_y &= -\partial_\phi B_y.
\end{align*}
\]

The primary wave propagates (in the \(y\)-\(z\) plane, making an angle \(\theta_1\) with the positive \(z\)-axis) towards the interface (\(z = 0\)), which separates the two media of dielectric permittivities \(\varepsilon_{1,3}\). In region 2, which is the thin layer itself, the field configuration generates an induced current; in this region a term \(4\pi j / c\) has to be added to the right-hand side of the first equation of (1). In region 1 we take \(B_z\) as a superposition of the given incoming plane wave pulse \(F\) (of arbitrary temporal variation), and an unknown reflected plane wave \(f_1\). The corresponding electric field components \(E_x\) and \(E_z\) can be derived from (1):

\[
\begin{align*}
B_{1z} &= \frac{F - f_1}{\varepsilon_0} - \frac{f_1}{\varepsilon_0} [I - n_1 (y \sin \theta_1 - z \cos \theta_1) / c], \\
E_{1y} &= \frac{\cos \theta_1 / n_1}{\varepsilon_0} (F + f_1), \\
E_{1z} &= \frac{\sin \theta_1 / n_1}{\varepsilon_0} (F - f_1).
\end{align*}
\]

In region 3 the magnetic induction \(B_{3z}\) is represented by the unknown refracted wave \(g_3\), and, by putting this into (1), the electric field strength can also be obtained:

\[
\begin{align*}
B_{3z} &= \frac{g_3 [I - n_3 (y \sin \theta_3 - z \cos \theta_3) / c],}{}
E_{3y} &= \frac{\cos \theta_3 / n_3}{\varepsilon_0} g_3, \\
E_{3z} &= \frac{\sin \theta_3 / n_3}{\varepsilon_0} g_3.
\end{align*}
\]

The relevant boundary conditions for the tangential components read

\[
\begin{align*}
[E_{1y} - E_{3y}]_{z=0} &= 0, \\
[B_{1z} - B_{3z}]_{z=0} &= (4\pi / c) K_{2y}.
\end{align*}
\]

On the basis of equations (1)–(4), the unknown scattered fields \(f_1\) and \(g_3\) can be expressed in terms of the, by now, unknown induced surface current \(K_{2y}\):

\[
\begin{align*}
f_1(t') &= \left[1 / (c_1 + c_3)\right] (c_1 - c_3) F(t' - c_1 (4\pi / c) K_{2y}(t')), \\
c_1 &= \cos \theta_1 / n_1, \\
c_3 &= \cos \theta_3 / n_3,
\end{align*}
\]

\[
\begin{align*}
g_3(t') &= \left(2 c_1 c_3 / (c_1 + c_3)\right) \\
&\times [F(t') - (2\pi / c) K_{2y}(t')], \\
K_{2y} &= \eta v_y,
\end{align*}
\]

where \(t' = t - y n_1 \sin \theta_1 / c\) denotes the retarded time parameter at the surface. Snell’s law of refraction (\(n_1 \sin \theta_1 = n_3 \sin \theta_3\)) means that this retarded time \(t'\) must be equal to \(t'' = t - y n_3 \sin \theta_3 / c\). Equations (5) and (6) are valid for any (constant) \(n_1,3\), regardless of the nature of \(K_{2y}\). On the other hand, since \(n_1,3\) in general may depend on the frequencies of the Fourier components of the radiation fields, one should separately discuss in each case whether or to what extent the constancy of \(n_1,3\) applies. Anyhow, if the thin layer is in a vacuum (air), as for instance a suspended graphene sheet above a trench, then \(n_1,3 = 1\), and no such problem appears. The unknown surface current \(K_{2y} = \eta v_y\) has been expressed (Varró 2004, 2007a, 2007b, 2007c, 2012) as the product of the surface charge density \(\eta y\) and the velocity \(v_y = \delta_y (t') / \delta t\) associated with the local displacement \(\delta_y(t')\) of the electrons. The original current term \(4\pi j / c\) in the Maxwell equations (from which we obtained \(4\pi K / c\)) could be phenomenologically interrelated to the electric field strength by the constitutive relation \(j = \sigma(\omega) \cdot E\), where \(\sigma(\omega)\) is the conductivity of the layer at a particular frequency of a stationary field (Nair et al. 2008, Cserti 2012). Anyway, it can be shown that the boundary condition (4), and the solutions (5), (6) are consistent with the discontinuity of the displacement field \(D\). By choosing the ansatz \(K_{2y} = \eta v_y\), we can derive \([D_{1z} - D_{3z}]_{z=0} = 4\pi \eta y\) from them. This relation, on the other hand, can also be obtained from the Gauss law, \(\nabla \cdot D = 4\pi \eta y\), where \(\eta y\) is the volume charge density. The form of the surface current we are using corresponds to a convective flow, and we consider \(\eta\) as an average density.

It is interesting to note that in our procedure the unknown surface current in (4) plays the role of a sort of ‘active boundary’, contributing to the matched fields. Of course, the force terms in the equation of motion of the surface charge elements must contain the total field (including the by now unknown scattered ones). In this way, the present formalism automatically accounts for a ‘collective radiation reaction’; in fact, we describe the dispersion of the layer, as a whole.

In order to solve the scattering problem we need the relativistic equation of motion of the massless current elements under the action of the composed fields (2) and (3), which have been expressed in (5) and (6). The equation of motion of an electron in the \(x\)-\(y\)-plane of the thin layer is taken as

\[
\frac{d\pi}{dt} = eE + e / c \times B, \quad v = \frac{\pi}{|\pi|}.
\]
where $e$ is the elementary charge, $c$ the velocity of light in vacuum and $v_F$ is the Fermi velocity of the particle (for graphene $v_F = c/300$ approximately). In (7) we have taken into account the ultrarelativistic dispersion relation $E(p) = v_F|p|$, where $p$ is the momentum of the particle.

In the TM configuration the $\alpha x \times B$ term has zero components in the $x$-$y$-plane. On the other hand, the total electric field has an extra term stemming from the surface current in (5) and (6), which is proportional with the velocity component $v_y$. Since in the present case $\vec{p}_x = \text{const}$, the derived equation of motion contains $v_y$ as the only unknown function,

$$\frac{d\vec{p}_y(t')}{dt'} = \frac{2e_1c_3}{c_1 + c_3} \left[ eF(t') - \frac{2\pi e^2}{c} \eta v_y(t') \right], \quad v_y = \frac{v_F}{|\vec{p}|} \pi_y,$$

where the geometrical factors $c_1$ and $c_3$ have been defined in (5). If one finds the solution of (8), then from (2), (3), (5) and (6) the scattered fields can be obtained. On the right-hand side of (8) the second term in the bracket represents the collective radiation back-reaction, which always represents a damping term, regardless of the sign of the charges. We represent the incoming field as

$$F(t') = -\partial^2 Z_0/c^2 \partial t'^2,$$

$$Z_0 = -(c^2 F_0/\omega_0^2)(t') \cos(\omega_0 t' + \varphi_0),$$

$$f(t) = \exp(-t'/2\tau),$$

where $Z_0$ is the Hertz potential of the incoming laser pulse of central frequency $\omega_0$, field strength $F_0$, carrier-envelope (CE) phase difference $\varphi_0$, and $\tau = \tau_L/2/\sqrt{\log 2}$. In the envelope function $\tau_L$ means the full temporal width at half maximum of the intensity. By introducing the dimensionless variables $\omega_0 t'/2\pi = t'/T \rightarrow t$ and $c \pi / \hbar o_0 \rightarrow q$, equation (8) becomes

$$\frac{d\omega_0 q(t)}{dt} = \frac{2e_1c_3}{c_1 + c_3} \left[ \pi R \mu_0 |F(t)/F_0| ight. - \frac{\alpha \beta \eta \lambda^2 q(t)}{\sqrt{4\pi^2 + q^2(t)}}, \quad \omega_0 t' = t'/T, \quad \alpha = \frac{v_F}{\hbar c}, \quad \beta = \frac{v_F}{c},$$

$$\eta \lambda^2 = \frac{2m_e^2}{\hbar o_0}, \quad \mu_0 = \frac{eF_0}{mc\omega_0} = 10^{-9}S^{1/2}/E_{ph},$$

$$\pi R \mu_0 \approx \mu_0.$$ (10)

In equation (10) $\alpha \approx 1/137$ is the fine structure constant, $\beta \approx 1/300, \lambda_0 = 2\pi e/c/\hbar o_0$ is the central wavelength of the radiation and $m$ is the usual rest mass of the electron. The parameter $\mu_0$ is the ‘dimensionless vector potential’, which has previously been called the ‘dimensionless intensity parameter’. Its numerical value can be calculated from the displayed formula, where $S = |I/\left[W \text{ cm}^{-2}\right]|$ is the intensity, measured in $W \text{ cm}^{-2}$, and $E_{ph} = \hbar o_0/eV$ is the photon energy, measured in electron volts. We see that $\mu_0$ can be much larger than $\mu_0$, at the same intensity, because $R$ is of the order of $10^6$ in the optical region (i.e. $\hbar o_0 = 1 \text{ eV}$). The definition $q \equiv c \pi / \hbar o_0$ corresponds to the scaling of the kinetic momentum by the central photon momentum. We also note that in the geometrical arrangement (p-polarization) under discussion, the $x$-component $q_x$ is a constant of motion.

In section 3 we shall present a couple of numerical examples, showing the temporal evolution of the electric field strength of the scattered radiation, and illustrate the ‘relativistic clipping effect’, caused by the interaction with the charged layer.

3. Numerical examples illustrating the relativistic clipping effect

The first term in the scattered field $f_1(t)$ given by (5) vanishes if $c_3 = c_1$, which condition is equivalent to $\theta_1 + \theta_2 = \pi/2$. This means that if the angle of incidence is the Brewster angle, then only the surface current term $K_2 = \eta v_F$, contributes to the reflected field. By leaving out this term, we get back the Brewster phenomenon (the p-polarized component in the reflected radiation gets to zero), which is described by the usual Fresnel formulae (see e.g. Born and Wolf 1959). We note that this condition is satisfied at any angles of incidence if $n_3 = n_1$, in particular, if the layer is in a vacuum (air). In these cases

$$f_1^{(\text{Brewster})} = -(2\pi \eta \eta \lambda^2)\eta x, \quad f_1^{(\text{max})} = 3016 \text{ (V cm}^{-1}) \times 10^{-12} (\eta \text{ cm}^2).$$

This field has the upper bound $|f_1^{(\text{Brewster})}| \leq 2\pi (\eta \lambda^2/c)\eta x$, which does not depend on the intensity. The numerical value of this upper bound is $f_1^{(\text{max})} = 3016 \text{ (V cm}^{-1})$, if we choose as an illustration the surface density $\eta = 10^{12} \text{ cm}^{-2}$. It is ‘universal’ in the sense that it does not depend either on the frequency or on the wavelength. The field strength 3016 (V cm$^{-1}$) corresponds to the intensity 12 064 W cm$^{-2}$, which is equivalent to the intensity of 5711 K black-body radiation, according to the Stefan–Boltzmann law.

In the first numerical example we take $\lambda_0 = 620 \text{ nm}$, i.e. $\hbar o_0 = 2 \text{ eV}$, and $\eta = 10^{12} \text{ cm}^{-2}$, in which case in the differential equation (10) we have the parameter value $\alpha \beta \eta \lambda^2 = 0.094$. In figure 1 we present the normalized electric field strength of an incoming field of intensity $I = 1 \text{ MW cm}^{-2}$, in which case the driving term in (10) has the numerical value $\mu_0 = \pi R \mu_0 = 0.785$, and $\alpha \beta \eta \lambda^2 / \mu_0 = 0.12$. At this point we note that $\mu_0$ and $\alpha \beta \eta \lambda^2 / \mu_0$ are both proportional to $\lambda^2_0$, thus their relative size does not depend on the wavelength of the incoming radiation. The $x$-component of the scaled momentum, $q_x$, is a constant of motion; we have chosen $q_x = 3 \times q_x(0)$, and then $q_x(t) = q_x(0)/2$. We note that we have carried out the calculations for various initial values but we have not found considerable qualitative changes. In the figures we illustrate the temporal evolution of the electric field strength as a function of the dimensionless time variable $(t - n y \sin \theta/c)/T$ drawn on the abcissa. The
Figure 1. The distortion of a three-cycle p-polarized incoming Gaussian laser pulse (left, (a)) impinging on a graphene layer at Brewster angle (or at any angle when $n_3 = n_1$). We have taken for the CE-phase $\phi_0 = 0$ (cosine pulse), $\lambda_0 = 620$ nm, $I = 1$ MW cm$^{-2}$, and the effective intensity parameter is $\pi R \mu_0 = 0.785$. The damping parameter used in the differential equation (10) equals $\alpha \beta \eta(\lambda_0) = 0.094$. The reflected component is nonzero (right, (b)), its existence cannot come out from the usual Fresnel formulae. The units of the electric field on the ordinate in (b) is 3016 V cm$^{-1}$. After saturation, this ‘universal’, maximum value is reached (for a surface density $\eta = 10^{12}$ cm$^{-2}$, according to the definition in (11)). The distorted shape stems from the ‘clipping effect’ discussed in the main text.

Figure 2. Illustrates the distortion of a five-cycle p-polarized incoming Gaussian laser pulse (left, (a)) impinging on a graphene layer at Brewster angle (or at any angle when $n_3 = n_1$). We have taken $I = 100$ MW cm$^{-2}$, then the effective intensity parameter is $\bar{\mu}_0 = 7.85$. The other parameters are the same as in fig1. The reflected component is shown on the right (b), whose nearly rectangular shape again stems from the ‘clipping effect’. It should be noticed that the reflected signal reaches its maximum already in the rising part of the incoming pulse.

In figure 1(b) the nearly rectangular structure resembles the orthogonal system of functions $\{r_k(x)\}$ in the unit interval introduced by Rademacher (1922):

$$r_0(x) = 1,$$
$$r_k(x) = \text{sign}(\sin \sqrt{k^2} \pi x)(k = 1, 2, \ldots).$$

The spectrum corresponding to the Rademacher functions is proportional to $1/(2n + 1)^2$ or $1/(2n + 2)^2$, depending on parity, where $n$ is the harmonic index. We note that this system (or rather, the derived complete system of the so-called Walsh functions) is used in signal analysis and synthesis in the microwave regime. Here we found possible physical representatives of them in the optical or terahertz regime. This is better illustrated in figures 2 and 3.

In figure 2 we show the temporal behaviour in the case of a larger intensity, $I = 100$ MW cm$^{-2}$; $\bar{\mu}_0 = 7.85$.

Our last example is the scattering of a sub-cycle pulse ($\approx 0.8$-cycle) similar to that produced recently by Wirth et al (2011). The peak field strength shown in their figure 3(b) was $4 \times 10^7$ V cm$^{-1}$, which corresponds to an intensity of $2.122 \times 10^{12}$ W cm$^{-2}$, according to the conversion $F_0/(V \text{ cm}^{-1}) = 27.46 \times \sqrt{I_0/(W \text{ cm}^{-2})}$. Instead of this large intensity, we take $I = 1$ GW cm$^{-2}$ only, and then the effective intensity parameter of the incoming pulse becomes $\bar{\mu}_0 = 24.82$. In figure 3 we show the temporal behaviour of such a pulse and the response of the layer.

As one can see in figures 1–3, according to our description above, the optical response of graphene really causes high nonlinearities. On the other hand, due to this same violent response of the relativistic charges, a very strong radiation damping develops. The interplay of these
two effects, through the ultrarelativistic kinematics, results in an almost universal rectangular temporal evolution of the reflected signal. We note that this structure follows the shifts of the CE-phase, as has recently been discussed by us (Varró 2012). We also note that the tails of the reflected signals can be considerably longer than shown for instance in figure 3(b). Such wake fields (Varró 2007a, 2007b, 2007c) may form slowly decreasing rectified quasi-static fields.

4. Summary

We have presented our recent theoretical results on the reflection and transmission of few-cycle laser pulses on a very thin conducting layer, which has been represented by a classical convective surface current density of the massless relativistic charges. We have shown that the pulses can undergo considerable distortions, even at relatively modest laser intensities. They are deformed to rectangular trains, which may be considered as approximate physical realizations of Rademacher functions in the optical or terahertz regime. In section 2 the matching equations for the electromagnetic fields and the radiation reaction term in the equation of motion have been determined. In section 3 the nature of the ‘relativistic clipping effect’ has been discussed, with inclusion of some illustrative figures. The numerical values of the input parameters have been chosen to correspond to the massless electrons and holes of a graphene monolayer.

We would like to emphasize that it has long been known that the velocity function of ‘ordinary’ massive electrons can also have an essentially rectangular shape at ultrarelativistic ($I \gg 10^{18}$ W cm$^{-2}$) intensities (see, e.g., Varró 2010). Besides, even at relatively moderate intensities, the nonlinearity parameter can also be so large that a similar distortion happens in the induced current in crystals (see the expression for the current density and figure 5(b) in the paper by Ghimire et al. (2011)). However, in each case the radiated field is proportional to the acceleration, not the velocity, as in the present analysis. Roughly speaking, if, for the former case, we took the derivative of the Rademacher functions (with half-cycle constancy regions of the velocity function), then we would receive very short (attosecond) peaks at the (approximate) discontinuity points.

The relativistic clipping effect described above can in principle manifest itself at any frequency—in the terahertz, optical or even higher frequencies—opening a wide range of potential applications. However, for the large frequency responses the model of massless charges may not be relevant. One should also keep in mind that for higher frequencies or and/or larger intensities the linear or and nonlinear photoelectric effect may be an important competing process in real systems, which has been completely left out of the above considerations. Besides, though we think that from the present classical description the essentials of the process can be understood, a quantum calculation is also desirable.

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References

Abergel D S L and Fal’ko V I 2007 Optical and magneto-optical far-infrared properties of bilayer graphene Phys. Rev. B 75 155430
Blake P, Hill E W, Castro Neto A H, Novoselov K S, Jiang D, Yang R, Booth T J and Geim A K 2007 Making graphene visible Appl. Phys. Lett. 91 063124
Born M and Wolf E 1959 Principles of Optics (London: Pergamon)
Bostwick A, Ohia T, Seyller T, Horn K and Rotenberg E 2007 Quasiparticle dynamics in graphene Nature Phys. 3 36–40
Castro Neto A H, Guinea F, Peres N M R, Novoselov K S and Geim A K 2009 The electronic properties of graphene Rev. Mod. Phys. 81 109–62

Cserti J 2012 private communication

Cserti J and Dávid Gy 2010 Relation between Zitterbewegung and the charge conductivity, Berry curvature, and the Chern number of multiband systems Phys. Rev. B 82 201405

Fedorov M V 1997 Atomic and Free Electrons in a Strong Laser Field (Singapore: World Scientific)

Földi P and Benedict M G 2012 Ultrashort pulse induced currents in solids: theoretical approaches Light at Extreme Intensities 2011 (AIP Conf. Proc. vol 1462) (New York: AIP) pp 96–9

Ghimire S, DiChiara A D, Sistrunk E, Agostini P, DiMauro L F and Reis D A 2011 Observation of high-order harmonic generation in a bulk crystal Nature Phys. 7 138–44

Korbman M, Kruchinin S Yu and Yakovlev V S 2013 Quantum beats in the polarization response of a dielectric to intense few-cycle laser pulses New J. Phys. 15 013006

Krausz F and Ivanov M 2009 Attosecond physics Rev. Mod. Phys. 81 163–234

Milošević D B, Paulus G G, Bauer D and Becker W 2006 Above-threshold ionization by few-cycle pulses J. Phys. B: At. Mol. Opt. Phys. 39 R203–62

Nagele S, Pazourek R, Feist J, Doblhoff-Dier K, Lemell C, Tókési K and Burgdörfer J 2011 Time-resolved photoemission by attosecond streaking: extraction of time information J. Phys. B: At. Mol. Opt. Phys. 44 081001

Nair R R, Blake P, Grigorenko A N, Novoselov K S, Booth T J, Stauber T, Peres N M R and Geim A K 2008 Fine structure constant defines visual transparency of graphene Science 320 1308

Nepple S, Ernststorfer R, Cavaliere A L, Menzel D, Barth J V, Krausz F, Kienberger R and Feulner P 2012 Attosecond time-resolved photoemission from core and valence states of magnesium Phys. Rev. Lett. 109 087401

Novoselov K S, Geim A K, Morozov S V, Jiang D, Katsnelson M I, Grigorieva I V, Dubonos S V and Firsov A A 2005 Two-dimensional gas of massless Dirac fermions in graphene Nature 438 197–200

Rademacher H 1922 Einige Sätze über Reihen von allgemeinen Orthogonalfunktionen Math. Ann. 87 112–38

Schulte M et al 2010 Delay in photoemission Science 328 1658–62

Uiberacker M et al 2007 Attosecond real-time observation of electron tunnelling in atoms Nature 446 627–32

Varró S 2004 Scattering of a few-cycle laser pulse on a thin metal layer: the effect of the carrier-envelope phase difference Laser Phys. Lett. 1 42–5

Varró S 2007a Reflection of a few-cycle laser pulse on a metal nano-layer: generation of phase dependent wake-fields Laser Phys. Lett. 4 138–44

Varró S 2007b Scattering of a few-cycle laser pulse by a plasma layer: the role of the carrier-envelope phase difference at relativistic intensities Laser Phys. Lett. 4 218–25

Varró S 2007c Linear and nonlinear absolute phase effects in interactions of ultrashort laser pulses with a metal nano-layer or with a thin plasma layer Laser Part. Beams 25 379–90

Varró S 2010 Intensity effects and absolute phase effects in nonlinear laser–matter interactions Laser Pulse Phenomena and Applications ed F J Duarte (Rijeka: InTech) pp 243–66 chapter 12

Varró S 2012 Graphene-based carrier-envelope phase difference meter Light at Extreme Intensities 2011 (AIP Conf. Proc. vol 1462) (New York: AIP) pp 128–31

Wirth A et al 2011 Synthesized light transients Science 334 195–200

Zhang C-H and Thumm U 2011 Probing dielectric-response effects with attosecond time-resolved streaked photoelectron spectroscopy of metal surfaces Phys. Rev. A 84 063403