CONSTRAINING A MODEL OF TURBULENT CORONAL HEATING FOR AU MICROSCOPII WITH X-RAY, RADIO, AND MILLIMETER OBSERVATIONS

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ABSTRACT

Many low-mass pre-main-sequence stars exhibit strong magnetic activity and coronal X-ray emission. Even after the primordial accretion disk has been cleared out, the star’s high-energy radiation continues to affect the formation and evolution of dust, planetesimals, and large planets. Young stars with debris disks are thus ideal environments for studying the earliest stages of non-accretion-driven coronae. In this paper we simulate the corona of AU Mic, a nearby active M dwarf with an edge-on debris disk. We apply a self-consistent model of coronal loop heating that was derived from numerical simulations of solar field-line tangling and magnetohydrodynamic turbulence. We also synthesize the modeled star’s X-ray luminosity and thermal radio/millimeter continuum emission. A realistic set of parameter choices for AU Mic produces simulated observations that agree with all existing measurements and upper limits. This coronal model thus represents an alternative explanation for a recently discovered ALMA central emission peak that was suggested to be the result of an inner “asteroid belt” within 3 AU of the star. However, it is also possible that the central 1.3 mm peak is caused by a combination of active coronal emission and a bright inner source of dusty debris. Additional observations of this source’s spatial extent and spectral energy distribution at millimeter and radio wavelengths will better constrain the relative contributions of the proposed mechanisms.

Key words: radio continuum: stars – stars: coronae – stars: individual (AU Microscopii) – submillimeter: stars – turbulence – X-rays: stars

Online-only material: color figures

1. INTRODUCTION

Nearly all low-mass stars are believed to have magnetic fields that influence their surroundings and evolution. Young stars with ages less than 10–20 Myr exhibit high-energy activity in the form of hot coronal loops, flares, accretion shocks, and open-field regions with winds or jet-like outflows (see, e.g., Feigelson & Montmerle 1999; McKee & Ostriker 2007; Güdel & Nazé 2009; Günter 2013). Because the magnetic fields of the star and disk are threaded together with one another, it is often difficult to disentangle the contributions from various proposed sources of activity. On the other hand, the situation may be greatly simplified for older stars that have evolved past the classical T Tauri phase. These stars have lost their dense gas disks, and thus the major remaining contributor to the star’s ultraviolet and X-ray emission is the presence of magnetic coronal loops. In that case, the subsequent evolution of a star’s coronal heating as it starts main-sequence hydrogen burning may no longer involve drastic changes in the source regions, but instead just be the result of a gradual evolution in parameters related to its magnetohydrodynamic (MHD) dynamo (Hartmann & Noyes 1987; Wright et al. 2011; Stelzer et al. 2013).

Pre-main-sequence stars with debris disks are ideal targets for studying the earliest stages of non-accretion-driven magnetic activity. In this paper we focus on AU Microscopii (HD 197481, GJ 803), a nearby M1Ve flare star with a well-resolved debris disk. Having an effective temperature of ∼3500 K, AU Mic probably is not fully convective like the later-type M dwarfs. Thus, its MHD dynamo may be qualitatively similar to those of more massive stars like the Sun. AU Mic is bright in X-rays (Schneider & Schmitt 2010), rich in ultraviolet flaring phenomena (Robinson et al. 2001), and surrounded by an edge-on dust disk with a mass of roughly 1 M⊙ and a spatial extent similar to the Kuiper belt in our solar system (e.g., Kalas et al. 2004; Liu et al. 2004; Augereau & Beust 2006; Wilner et al. 2012).

Recently, MacGregor et al. (2013) observed AU Mic at millimeter wavelengths with the Atacama Large Millimeter/submillimeter Array (ALMA). In addition to the Kuiper-like dust belt, they were able to distinguish a compact central emission component is not yet known. The M dwarf’s stellar photosphere would generate a blackbody flux of only about 60 μJy at λ = 1.3 mm, a factor of five smaller than the observed emission peak. In order for a star-sized blackbody to be responsible for the observed emission, it would need to have a temperature of roughly 17,000 K. This kind of dominant chromospheric emission was predicted by Harper et al. (2013) for cool evolved giants, but those stars have much larger emitting areas and no significant coronal emission. An M dwarf like AU Mic may have a thin chromosphere underneath its hot corona, but its optical depth is not likely to be high enough to generate a photosphere-like blackbody spectrum.

MacGregor et al. (2013) suggested that the central emission peak of AU Mic may be produced by an inner “asteroid belt” of cool dust grains or planetesimals within ∼3 AU of the star. Only about 0.01 M⊙ worth of dust material needs to be present to generate the observed emission peak; this is comparable to the mass of the asteroid belt in our solar system. MacGregor et al. (2013) also computed upper limits on the required temperatures of silicate grains in this proposed belt and found values of 35–75 K. This range is the same order of magnitude as the expected dust temperatures at a distance of ∼3 AU from an M dwarf.

In this paper we propose an alternate explanation for the ALMA central emission component of AU Mic. Since this star has such strong X-ray emission, we explored the possibility that a collection of hot coronal loops on its surface could be...
responsible for similarly strong thermal emission at 1.3 mm. Figure 1 illustrates the suggested MHD circumstellar environment, where magnetic loops with a continuous distribution of sizes are assumed to fill the corona with $\sim 10^6$ K plasma. We aim to create a single model of these loops that reproduces the available observations at X-ray and millimeter wavelengths. White et al. (1994) also explored the use of radio continuum measurements as constraints to models of M dwarf coronal heating. In a way, this paper follows on from the general ideas described by White et al. (1994), but our models are based on self-consistent physical sources of plasma heating inspired by recent advances in understanding the Sun’s well-resolved corona.

In Section 2 of this paper, we describe the model of coronal heating that we apply to AU Mic. Section 3 summarizes how we synthesize observable quantities in the X-ray, radio, and millimeter wavelength bands for later comparison with observations. In Section 4 we present the computed coronal loop parameters for AU Mic as well as synthetic X-ray luminosities and radio/millimeter spectra. Section 5 concludes the paper with a brief summary of the main results and suggestions for future improvements.

2. CORONAL HEATING MODEL

In this section we describe a model of turbulence-driven coronal activity motivated by the present-day Sun. The model adopts a time-steady description of coronal heating and does not attempt to simulate the full range of intermittent and chaotic MHD dynamics that is evident in high-resolution solar images (e.g., Cirtain et al. 2013). Nonetheless, such a phenomenological description of turbulence has been shown to accurately reproduce many properties of coronal heating and wind acceleration for the Sun (Cranmer 2012). Also, Cranmer (2009) showed how this model performs well in predicting the X-ray activity of young stars undergoing active accretion, and Cranmer & Saar (2011) used the model to compute the mass-loss rates of stellar winds for stars with a wide range of ages and masses.

The basic idea is that magnetic field lines at the stellar photosphere are continually jostled by a stochastic source of mechanical energy, and this gives rise to MHD waves that propagate up to larger heights and eventually dissipate. In tandem with wave generation, the random footpoint motions produce an increase in the overall magnetic energy via the twisting, shear, and braiding of field lines (e.g., Parker 1972). The transport and dissipation of magnetic energy from both waves and braiding can be described using the unifying language of turbulent cascade (van Ballegooijen 1986; Gómez et al. 2000; Rappazzo et al. 2008; van Ballegooijen et al. 2011). The presence of radiative cooling in such a system produces a thermal instability that often leads to the coexistence of a cool ($T \lesssim 10^4$ K) chromosphere and a hot ($T \gtrsim 10^6$ K) corona. In the ionized corona, heat conduction also makes for a “thermostat” effect in which the time-steady spatial distribution of $T$ is much smoother than the spatial distribution of the heating rate.

There are several possible sources of stochastic mechanical energy at the photospheric base. Convective granulation surely plays a major role for most low-mass stars (Musielak 2004). For stars undergoing active accretion, an additional source of energy can exist in the form of ripples from the impact of gas clumps falling down from above (Cranmer 2008). It has also been suggested that the presence of planets in a magnetized stellar wind may give rise to MHD fluctuations that propagate back down to the star (e.g., Lanza 2012). For the purposes of this paper, we will treat the total available mechanical energy flux at the photosphere as a free parameter.

We describe the star itself by its fundamental parameters (mass, radius, luminosity, and metallicity) and ignore rotation. The dynamics of the jostled magnetic flux tubes are described
by three additional parameters defined at the photosphere: mass density $\rho_*$, magnetic field strength $B_*$, and the mean energy flux $F_\lambda$ of Alfvén waves that propagate upward as a result of the stochastic motion. To compute the density as a function of density, we used the standard criterion that the Rosseland mean optical depth $\tau_R$ should have a value of $2/3$ in the photosphere, which demands

$$\tau_R \approx \kappa_R \rho_* H_s = 2/3 . \quad (1)$$

The Rosseland mean opacity $\kappa_R$ was interpolated from tables given by Ferguson et al. (2005), and we defined the density scale height in the photosphere as

$$H_s = \frac{k_B T_{\text{eff}}}{\mu m_H} , \quad (2)$$

where $k_B$ is Boltzmann’s constant and $m_H$ is the mass of a hydrogen atom. Cranmer & Saar (2011) found that, for cool-star photospheres, the mean atomic weight $\mu$ is a relatively slowly varying function of $T_{\text{eff}}$; we used their parameterization.

To specify the photospheric magnetic field strength, we assumed that $B_*$ remains linearly proportional to the so-called equipartition field strength (i.e., the value at which magnetic pressure balances gas pressure). We used the proportionality constant found from observations by Cranmer & Saar (2011) to specify

$$B_* = 1.13 \sqrt{\frac{8\pi k_B T_{\text{eff}}}{\mu m_H}} \quad (3)$$

for the footpoints of coronal loops. The Alfvén wave energy flux can be written in terms of the surface velocity amplitude $v_\perp$, of transverse MHD waves, with $F_\lambda = \rho_* v_\perp^2 V_\lambda$, and the Alfvén speed is given by $V_\lambda = B_*/\sqrt{4\pi \rho_*}$. We can make an initial estimate of the expected velocity amplitude using turbulent convection models. Cranmer & Saar (2011) used the models of Musielak & Ulmschneider (2002) for main-sequence and evolved stars. We find below, however, that for AU Mic the Musielak & Ulmschneider (2002) models predict an amplitude that is far too small to explain its observed coronal heating and X-ray activity. (Our understanding of the magnetic dynamos of young, rapidly rotating stars is still incomplete; see Section 5.)

In general, we assume that magnetic flux tubes fill only a fraction $f_\perp$ of the stellar photosphere, so that the mean magnetic flux density of the star would be given by $\langle B \rangle \approx f_\perp B_*$. For the Sun, $f_\perp$ varies between about $10^{-3}$ and $10^{-1}$ over location and activity cycle (Schrijver & Harvey 1989). The local magnetic field strength $B(r)$ inside a flux tube drops rapidly from $B_*$ to $\langle B \rangle$ as a function of increasing height. However, for young stars in the “saturated” part of the age–activity–rotation diagram, Saar (2001) and Cranmer & Saar (2011) found that $f_\perp = 1$ is not a bad approximation to make. We will assume this for AU Mic as well (see additional discussion in Section 4), and this also allows us to assume that $B_*$ remains constant over each coronal loop.

We note that $f_\perp$ describes the filling factor of all magnetic flux tubes, of both polarities, that pass through the stellar surface. By itself, our assumption of $f_\perp = 1$ does not directly imply a prediction for the fraction of the magnetic field lines that are open (presumably to the stellar wind) versus those that are closed. The open/closed fraction depends on both the overall level of gas pressure in the stellar wind and the spatial patterns of imbalance between flux tubes with positive and negative polarities (Close et al. 2003; Cranmer & van Ballegooijen 2010).

Below, we do assume the entire surface is covered by loops of varying lengths, but the coronal heating—and thus the X-ray and radio/millimeter emission—from the longest loops is likely to be not too different from the emission from open-field regions (see, e.g., Schrijver et al. 2004).

To specify the rate of plasma heating, we use a general expression for the rate of energy flux in the cascade from large to small eddies that was derived from analytic and numerical studies of MHD turbulence. Following Cranmer (2009), we define the heating rate (in units of power generated per unit volume) as

$$Q = \rho v_\perp^4 \left( \frac{\lambda_\perp V_\lambda}{v_\perp L} \right)^\alpha , \quad \text{where} \quad \alpha = 2 + 420R \frac{1}{1 + 280R} \quad (4)$$

and $R = v_\perp/L$. The quantity $\lambda_\perp$ is a transverse correlation length for the largest driving eddies, and it is typically found to be of the same order of magnitude as the radius of the flux tube (e.g., Hollweg 1986). By convention, $L$ is the half-length of the coronal loop. Nonzero values of the exponent $\alpha$ describe departures from an ideal Kolmogorov (1941) hydrodynamic cascade; such departures occur because of the specific MHD nature of wave-packet collisions that comprise the eddies in the presence of a strong background field. In closed loops, $\alpha$ can vary between 1.5 and 2. Our expression for the dependence of $\alpha$ on the wave amplitude $v_\perp$ and Alfvén speed $V_\lambda$ was derived from the models of Rappazzo et al. (2008).

Even though Equation (4) utilizes constant values for $v_\perp, \lambda_\perp, L$, and $B_*$ (in the definition of $V_\lambda$), the density $\rho$ is known to decrease rapidly as a function of height in a stellar atmosphere. Thus, because $V_\lambda \propto \rho^{1/2}$, the heating rate itself varies with density as $Q \propto \rho^{(\alpha - 2)/2}$. For $\alpha = 2$ the heating rate is constant (as was assumed by Rosner et al. 1978), and for $\alpha < 2$ the heating is stronger at the high-density footpoints. The values of $\rho$ and $V_\lambda$ at the “coronal base” (i.e., just above the sharp transition region between chromosphere and corona) are not the same as the corresponding values in the photosphere; these quantities are computed self-consistently by the model and are not specified as inputs.

In order to determine the time-steady coronal response to a given heating rate $Q$, we must consider how the injected heat is transported along the field by conduction and lost by radiation. We use the energy balance model of Martens (2010) to solve for the temperature $T$ and electron number density $n_e$ as a function of distance along a given loop of half-length $L$ (see also Rosner et al. 1978; Aschwanden & Schrijver 2002). These quantities are specified by first computing the peak temperature $T_{\text{max}}$ at the top of the loop and the base pressure $P$. Martens (2010) parameterized the heating rate as $Q = hT^\alpha P^b$, where $h$ and $P$ are assumed to be constants as a function of distance. For the heating model of Equation (4), the exponents are given by $a = -b = (\alpha/2) - 1$. Martens (2010) derived two scaling laws that allow one to solve for any two of the four loop quantities ($L, T_{\text{max}}, P,$ and $h$) if the other two are provided. In our case, these scaling laws are given by

$$T_{\text{max}} \propto h^{-2/(2a+3)} L^{2(\alpha+1)/(2a+3)} , \quad P \propto T_{\text{max}}^3 / L , \quad (5)$$

and the unspecified constants of proportionality are described in detail by Martens (2010). These constants depend on the exponent $\alpha$, the heat conductivity coefficient $\kappa = 1.1 \times 10^{-6}$ erg cm$^{-1}$ s$^{-1}$ K$^{-7/2}$, and the assumed properties of radiative cooling. For the latter, we assumed $Q_{\text{cool}} = \chi h^2 T^{-2.5}$.
with a value of $\chi = 3.65 \times 10^{12} \text{ cm}^4 \text{ s}^{-1} \text{ K}^{3/2}$, which we enhanced slightly over the solar value because of the higher metallicity of AU Mic (see Equation (21) of Cranmer & Saar 2011).

In order to model the thermal state of a given loop with half-length $L$, we use Equation (4) to determine $h$ in terms of the other input parameters, and we use the scaling laws given in Equation (5) to solve for $T_{\text{max}}$ and $P$. The density at the coronal base is determined mainly by $P$, but it needs to be specified initially in order to compute $V_A$ and $\alpha$ in Equation (4). Thus, we solve for these quantities iteratively starting with the initial guess that the density is given by $\rho_\ast$. Cycling back through the equations gives rise to a converged and self-consistent value of $\rho$ (and thus also $V_A$ and $\alpha$) typically within 10–15 iterations, and we run it for a total of 50 iterations to ensure accuracy. This converged value of the coronal base density is often three to six orders of magnitude smaller than $\rho_\ast$, which produces typical basal $n_e$ values of $10^{10}$–$10^{11} \text{ cm}^{-3}$.

Once $T_{\text{max}}$ and $P$ are known for a given loop, we solved for the spatial dependence of temperature and density along the loop length $s$. Martens (2010) found that the temperature dependence $T(s)$ can be rewritten as the inverse of an exponential beta function; we used Equation (25) of Martens (2010) to obtain $T(s)$, and then we combined it with the ideal gas equation of state to solve for $n_e(s)$ as a function of $P$ and $T(s)$. The minimum value of $T$ at the coronal base was fixed at $10^6 \text{ K}$.

Lastly, we must take account of the fact that there should be a continuous distribution of loop lengths $L$ across the surface of the star. We specify a normalized probability distribution $N(L)$ that describes the chances of finding a loop at any given value of $L$ at a random location on the star. The shape of $N(L)$ depends on many details about the strength and topology of the magnetic field. The large-scale magnetic geometry of M dwarfs is beginning to be understood observationally (e.g., Gastine et al. 2013), but we know from the Sun that the X-ray and UV emission is dominated by activity on spatial scales much smaller than have been resolved so far on other stars. Thus, we follow Cranmer (2009) and assume a power-law probability distribution, $N(L) \propto L^{-\varepsilon}$, with sharp cutoffs below minimum and maximum loop lengths $L_{\text{min}}$ and $L_{\text{max}}$. The exponent $\varepsilon$ in the loop-length distribution is another key free parameter of this model. The shortest loops are defined geometrically as those for which the central hole in the torus shrinks to zero; i.e., $L_{\text{min}} = \pi r_p/2$, where $r_p$ is the modeled poloidal radius of the loop (see Section 4). The longest loops are assumed to have lengths of order the stellar radius. We note that some young stars may have loops that extend out to even larger heights (Jardine & van Ballegooijen 2005; Aarnio et al. 2012). Nonetheless, it is not known what fraction of the very longest loops remain stable and closed in the presence of a stellar wind. Thus, we set $L_{\text{max}} = R_\ast$ and assume that the coronal emission is relatively insensitive to the details of what happens for the small fraction of the star covered by the longest loops.

### 3. SYNTHESIS OF X-RAY AND RADIO/MILLIMETER EMISSION

To simulate a full distribution of loops covering the surface of AU Mic, we constructed 100 coronal models on a logarithmic grid in $L$ that spans the $\sim 3.5$ orders of magnitude between $L_{\text{min}}$ and $L_{\text{max}}$. We synthesized the observable quantities described below for the 100 models individually—assuming for each that the star is filled with loops of a given length—then we convolved them together using $N(L)$ to produce a result that is weighted properly over the statistical distribution of loop sizes.

We computed the total X-ray luminosity $L_X$ under the optically thin assumption that all radiation escapes from the emitting regions. To maintain continuity with past observations of cool-star X-rays, we chose to use the response function of the ROSAT Position Sensitive Proportional Counter (PSPC) given by Judge et al. (2003). This function has nonzero sensitivity between about 0.1 and 2.4 keV, with a minimum around 0.3 keV that separates the hard and soft bands. Cranmer (2009) presented the temperature-dependent radiative loss rate $\Lambda_X(T)$ that is consistent with this response function, which is then used to estimate the X-ray luminosity,

$$L_X = 4\pi R_\ast^2 \int dz \left[ n_e(z)^2 \right] \Lambda_X(T(z)).$$

We integrated down through the length of each loop, from $z = L$ to $z = 0$, under the simplifying geometric assumption that the loop is oriented vertically. In other words, we assumed that each loop has been “snipped in two” and the two ends are pointing radially upward. We also ignored foreshortening effects that would alter the path lengths through loops close to the stellar limb.

Calculating the emission at radio and millimeter wavelengths was slightly more complicated than the X-ray luminosity because we can no longer assume an optically thin emitting region. Thus, we solved the diffuse term in the formal solution to the equation of radiative transfer by integrating both the optical depth $\tau_v$ and the specific intensity $I_v$ simultaneously, with

$$I_v = \int dz \int \chi_v \frac{2k_B T v^2}{c^2} e^{-\tau_v},$$

and we made the standard assumption that the source function is given by the Rayleigh–Jeans tail of the local Planck function. Because the plasma conditions vary rapidly as a function of position $z$ along the loop, we did not make the other standard assumption that the source function is constant over the emitting area. As above, we integrated down from $z = L$ to $z = 0$, so that the resulting optical depth increases as $z$ decreases.

For wavelengths $\nu$ between 0.01 and 100 cm, we assumed the radiation is dominated by thermal free–free emission (bremsstrahlung) and that the opacity is given by

$$\chi_v = 0.01 n_e^2 T^{-3/2} v^{-2} \ln \left( 4.7 \times 10^{10} \frac{T}{\nu} \right),$$

in cgs units (see Dulk 1985; Güdel 2002). For the Gaunt factor (i.e., the natural logarithm term above), we used a fully ionized approximation that should apply for $T \gtrsim 3 \times 10^6 \text{ K}$. Equation (9) also assumes $\nu \gg \nu_p$, where $\nu_p$ is plasma frequency in Hz; this condition is satisfied easily for the ALMA submillimeter wavelengths of interest and is satisfied marginally for longer radio wavelengths. For a star at distance $D$, we converted the surface-averaged specific intensity $I_v$ to flux $S_v$ by assuming that short loops (with $L \ll R_\ast$) generally dominate the emission and that there is no limb brightening or limb darkening. Thus, we used $S_v = \pi R_\ast^2 I_v / D^2$. 

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RESULTS FOR AU MIC

Table 1 gives the basic stellar parameters that we used either as inputs (upper part) or as after-the-fact constraints (lower part) on the models. The photospheric density $\rho_\lambda$ and magnetic field strength $B_\perp$ were computed using the equations discussed above. The photospheric turbulence length scale $\lambda_\perp$, and the flux tube radius $r_\perp$ were scaled down from canonical solar values of 300 km and 200 km, respectively, by the ratio of photospheric scale heights.

The solar value of $\lambda_\perp$ comes from models of Alfvén wave damping in the fast solar wind (Cranmer & van Ballegooijen 2005) and $r_\perp$ comes from measurements of magnetic bright points (Berger et al. 1995, 2007); both are related closely to the horizontal sizes of magnetic features sitting between the granulation cells. Recent convection models (e.g., Robinson et al. 2004; Magic et al. 2013) show that the diameters of granules remain linearly proportional to the vertical scale height $H_\perp$ over a wide range of stellar parameters. We assume that intergranular features like magnetic flux tube scale with $H_\perp$ similarly as the cells. With the values given above, we found that the conical loops of AU Mic span about 3.5 orders of magnitude in length between $L_{\text{min}} = 190$ km and $L_{\text{max}} = 5.83 \times 10^6$ km.

For AU Mic, we assumed the photospheric magnetic filling factor $f_\perp$ is equal to 1. AU Mic appears to be safely inside the “saturated” regime of stellar activity that is consistent with this assumption. Its rotation period $P_{\text{rot}} \approx 4.8$ days implies a dimensionless Rossby number (i.e., $P_{\text{rot}}/\tau_\epsilon$, where $\tau_\epsilon$ is the convective turnover time) of 0.05–0.1 (Hebb et al. 2007). Figure 7(b) of Cranmer & Saar (2011) shows $f_\epsilon \approx 1$ for a large sample of cool stars in the range of Rossby number. AU Mic also has an X-ray luminosity ratio $L_X/L_{\text{X-ray}}$ of 0.0015, which is in the saturated part of the empirical age–activity–rotation diagram (e.g., Wright et al. 2011). Testa et al. (2004) estimated the filling factor of “solar-like active region” plasma in the corona of AU Mic to be roughly 0.9–1. Although this measurement does not constrain the photospheric value of $f_\star$, it helps to show that the coronal flux tubes eventually fill the entire circumstellar volume.

The one remaining major parameter that is not yet determined for AU Mic is the surface flux of Alfvén waves. Cranmer & Saar (2011) used the models of Musielak & Ulmschneider (2002) to produce a fitting formula that gives this surface flux as a function of a star’s $\tau_\text{eff}$ and log $g$. For the parameters of AU Mic, this formula gives $F_\lambda \approx 2 \times 10^7$ erg cm$^{-2}$ s$^{-1}$, which is equivalent to $v_\perp \approx 0.02$ km s$^{-1}$. On the other hand, both observations of microturbulent broadening (Giampapa et al. 1982) and three-dimensional convection simulations (Wende et al. 2009; Beeck et al. 2011) show vigorous overturning motions with velocities of order 1–2 km s$^{-1}$ for M dwarfs. For the Sun, the granulation velocities and surface wave amplitudes are roughly of the same order of magnitude as one another, but we do not know if this holds true for M dwarfs. Thus, we treat $v_\perp$ as a free parameter, but we also note that values of order 1–2 km s$^{-1}$ may be the most realistic.

We computed models of coronal heating for a grid of loops having lengths $L$ between $10^2$ and $10^6$ km, and footpoint wave amplitudes $v_\perp$ between 0.02 and 200 km s$^{-1}$. Figure 2 shows the resulting dependence of $T_{\text{max}}$ and $P$ on these two parameters. Each of these models had its own iterated value of the $\alpha$ exponent, and the largest values ($\alpha \approx 2$) tended to occur for the lowest velocity amplitudes and the longest loops; the smallest values ($\alpha \approx 1.5$) occurred for the largest amplitudes and shortest loops. The overall dependence of $T_{\text{max}}$ and $P$ on the two varied parameters is close to what is expected from the scaling laws given in Section 2. For example, if we use the average value of $v_\perp$...
\[ \alpha = 1.78 \] for this set of models, the scaling laws give
\[ T_{\text{max}} \propto \nu_{\perp}^{0.37} L^{0.034}, \quad P \propto \nu_{\perp}^{1.12} L^{-0.90}, \] (10)
which agrees well with the plotted results. The curves shown in Figure 2 are not exact power laws because \( \alpha \) itself is not constant.

In order to begin the process of testing our coronal models against real observations of AU Mic, we assembled together the temperature and density distributions for individual loop lengths into surface-averaged models using the power-law length distribution \( N(L) \propto L^{-\epsilon} \) discussed above. Various measurements of solar features have constrained the value of \( \epsilon \) to be of order 2–2.5 (Aschwanden et al. 2000, 2008; Close et al. 2003), and the Cranmer (2009) study of T Tauri star X-ray emission also found that \( \epsilon \approx 2.5 \) produced the most reasonable results. For completeness, we varied \( \epsilon \) between 0 and 2.5, noting that values of the exponent greater than 2.5 produce distributions that are only marginally different from those with \( \epsilon \approx 2.5 \) (i.e., once it is peaked sharply at the shortest length scales, making it even steeper does not change the resulting weighting significantly).

Figure 3(a) shows contours of \( L_X \) computed for the ROSAT PSPC band, and for a large grid of 250 \( \nu_{\perp} \) values by 250 \( \epsilon \) values.\(^2\) For any single value of \( \epsilon \), the X-ray luminosity first increases with increasing \( \nu_{\perp} \), because the temperatures and densities in the loops are also increasing, but then \( L_X \) begins to decrease because the loops become too hot to produce significant emission in the relatively soft PSPC passband. The dotted red contour in Figure 3(a) denotes the parameters that produce agreement with the observed value of \( \log L_X = 29.74 \) (Hünsch et al. 1999).

The contours in Figure 3(b) show constant values of the free–free emission flux \( S_\nu \) computed at \( \lambda = 1.3 \) mm. The thickest contour highlights the ALMA flux measured by MacGregor et al. (2013) for the central emission component of AU Mic. The corresponding observational contour for \( L_X \) is reproduced from Figure 3(a), and it is noteworthy that the two empirical constraints intersect one another (roughly perpendicularly) at a single parameter value. This best-fitting model has a basal velocity amplitude \( \nu_{\perp} = 1.5545 \) km s\(^{-1} \) and a loop distribution power-law exponent \( \epsilon = 1.245 \). We note also that this model has a value of \( \nu_{\perp} \) that falls within the region of likely microturbulence values (1–2 km s\(^{-1} \)) for early M-type dwarfs (Giampapa et al. 1982; Wende et al. 2009; Beeck et al. 2011).

The best-fitting model contains loops with peak temperatures between 3.7 MK (for \( L_{\text{min}} \)) and 7.2 MK (for \( L_{\text{max}} \)). Schneider & Schmitt (2010) produced a fit of the full Chandra spectrum of AU Mic with three plasma components having temperatures of 3.37, 7.78, and 17.3 MK. Our range of modeled \( T_{\text{max}} \) values is reasonably consistent with the first two Chandra components. The hottest observed X-ray component may be related to intermittent nonthermal stellar flaring, which we did not include in our model.

The best-fitting model also exhibits loop-top electron densities between \( 7 \times 10^{11} \) cm\(^{-3} \) (for \( L_{\text{min}} \)) and \( 3 \times 10^{14} \) cm\(^{-3} \) (for \( L_{\text{max}} \)), with the densities increasing as one goes down from the loop tops. Testa et al. (2004) found upper limits on electron densities from Chandra spectroscopy of AU Mic that are close to the above values. For lines of O\( \text{vii} \) and Mg\( \text{xix} \), formed at roughly 2 MK and 7 MK respectively, Testa et al. (2004) found upper limits of \( 5.6 \times 10^{11} \) and \( 5.6 \times 10^{12} \) cm\(^{-3} \). We can also combine our model densities with the \( T_{\text{max}} \) values given above to determine the range of gas pressures in the loops and compare them with observations. We assumed that \( P \) remained constant over the length of each loop, and the values spanned from 760 dyne cm\(^{-2} \) (for \( L_{\text{min}} \)) to 1.8 dyne cm\(^{-2} \) (for \( L_{\text{max}} \)). Del Zanna et al. (2002) found a best fit for the UV-derived differential emission measure of AU Mic to occur for a constant pressure of \( \sim 3 \) dyne cm\(^{-2} \), which falls inside the range of our modeled loops.

We also computed the free–free emission at longer radio wavelengths, for which there are no firm detections of AU Mic in quiescence. Both White et al. (1994) and Leto et al. (2000) gave non-detection upper limits that we can use as further tests of our model. Figure 4(a) carries over the two empirical contours from Figure 3 and also divides the parameter space into two regions: one in which the modeled emission is consistent with the White et al. (1994) upper limit for \( \lambda = 2 \) cm, and one in which the modeled emission exceeds that limit. Our best-fitting solution sits comfortably in the region that is consistent with all of the observed radio upper limits.

\(^2\) We chose not to include the smallest \( \nu_{\perp} \) amplitudes in the grid because there were no combinations of parameters in this region that produced agreement with both the X-ray and millimeter observations. The 250 \( \times \) 250 grid encompasses the ranges \( 0.2 \leq \nu_{\perp} \leq 200 \) km s\(^{-1} \) and \( 0 \leq \epsilon \leq 2.5 \).
maximum value of

$$I_v \approx \frac{2k_B T^2}{c^2}. \quad (11)$$

If the temperature in the emitting region was a constant, this would give a power-law spectrum going as $\lambda^{-2}$. A steepening to this shape is indeed seen around $\lambda \sim 1–5$ cm, but then it flattens out again. This is because the optical depth keeps increasing as $\lambda$ increases, and the level of the $\tau_v = 1$ “radio photosphere” moves up from the coronal base to the tops of the loops. Because $T(s)$ increases from the base to the loop tops, the emitting temperature to be used in Equation (11) is an increasing function of $\lambda$, too. This gives rise to an absolute value of $I_v$ that grows progressively larger than it would have been had the emitting temperature remained constant. Presumably, as $\lambda$ is increased even further, all of the loop tops will become optically thick, the emergent free–free emission will be dominated only by $T_{\text{max}}$, and the spectrum will start declining again as $\lambda^{-2}$.

Lastly, we mention that the resulting values of X-ray and radio/millimeter emissivity do depend on the assumed value of the photospheric magnetic filling factor $f_\ast$. We ran a series of models for the standard parameter choices of $v_L = 1.5545$ km s$^{-1}$ and $\varepsilon = 1.245$, where $f_\ast$ was varied over two orders of magnitude from 0.01 to 1. The X-ray luminosity scales roughly as $L_\text{x} \propto f_\ast^{1.26}$, and the free–free emission flux at $\lambda = 1.3$ mm scales as $S_\nu \propto f_\ast^{1.39}$. These order-unity power-law exponents imply that the uncertainties in the coronal loop parameters determined above may be roughly of the same magnitude as the uncertainty in our knowledge of $f_\ast$ for this star.

Figure 4(b) shows representative radio and millimeter spectra for a set of seven models that fall along the observational $L_\text{x}$ contour shown in Figure 4(a). Each of these models reproduces the observed X-ray luminosity of AU Mic, but only one of them (coincidentally, the one with the lowest value of $v_L$) agrees with the ALMA flux at 1.3 mm. Four of these seven models have radio fluxes that fall below the White et al. (1994) and Leto et al. (2000) upper limits. It is noteworthy that a factor-of-two radio fluxes that fall below the White et al. (1994) upper limits. It is noteworthy that a factor-of-two

Figure 4. (a) Contours showing models that agree with observed $L_\text{x}$ (dotted red curve) and ALMA 1.3 mm flux (thick black curve) for AU Mic, together with the region of parameter space that is consistent with the White et al. (1994) radio upper limit at $\lambda = 2$ cm (green area). Expected values of $v_L$ from M dwarf/microturbulence are shown with thin black dashed lines. (b) Synthesized radio/millimeter spectra for models denoted by circles in panel (a). From bottom to top, the spectra correspond to models that follow the empirical $L_\text{x}$ contour counterclockwise from bottom left to top middle in panel (a). Also shown are upper limits from White et al. (1994, black arrows) and Leto et al. (2000, gold arrows), and the MacGregor et al. (2013) 1.3 mm measurement (black str). (A color version of this figure is available in the online journal.)
dust diffusion. It is at least circumstantial evidence that our loop model, “calibrated” with \( v_L \approx 1.5 \text{ km s}^{-1} \), reflects what is actually going on in the outer atmosphere of AU Mic.

Although our model accurately reproduces the magnitude of the ALMA 1.3 mm central emission peak, we do not yet have a definitive way to distinguish between this model and the idea of an inner asteroid belt proposed by MacGregor et al. (2013). For example, neither model predicts significant mid-infrared excess centered on the star, and indeed none is seen for AU Mic (Liu et al. 2004). It is possible, of course, that AU Mic has both an active stellar corona and a bright inner source of dusty debris. Additional information about the spectral energy distribution of the central peak resolved by ALMA would be extremely helpful to putting limits on the relative contributions of these two suggested explanations for the 1.3 mm emission.

Also, better observational limits on the spatial extent of the central peak would be helpful, since the dust belt is expected to be several orders of magnitude larger in size than the source of coronal emission.

In addition to better observational constraints, there are also several ways that the models can be improved. We limited our approach to investigating the main ingredients that contribute to the ALMA observations: corona emission, loop sizes, and the dust belt. These are. Still, we do not know what the granulation pattern—and the distribution of magnetic flux tubes—really looks like on the surface of a saturated-activity star. It is also possible that rapid rotation strongly affects the distribution of loop lengths (Aarnio et al. 2012). M dwarfs such as AU Mic are also close to the dividing line between having a radiative core and being fully convective. The qualitative properties of rotation, dynamos, and large-scale magnetic fields are believed to change substantially across this dividing line (Mullan & MacDonald 2001; Reiners & Basri 2007; Irwin et al. 2011; Gastine et al. 2013). These “hidden” aspects of stellar interiors may have significant impacts on the high-energy activity of a star like AU Mic and its surrounding dust and debris.

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