ABSTRACT
Data augmentation has been proven to be an effective technique for developing machine learning models that are robust to known classes of distributional shifts (e.g., rotations of images), and alignment regularization is a technique often used together with data augmentation to further help the model learn representations invariant to the shifts used to augment the data. In this paper, motivated by a proliferation of options of alignment regularizations, we seek to evaluate the performances of several popular design choices along the dimensions of robustness and invariance, for which we introduce a new test procedure. Our synthetic experiment results speak to the benefits of squared $\ell_2$ norm regularization. Further, we also formally analyze the behavior of alignment regularization to complement our empirical study under assumptions we consider realistic. Finally, we test this simple technique we identify (worst-case data augmentation with squared $\ell_2$ norm alignment regularization) and show that the benefits of this method outstrip those of the specially designed methods. We also release a software package in both TensorFlow and PyTorch for users to use the method with a couple of lines at https://github.com/jyanln/AlignReg.

CCS CONCEPTS
• Computing methodologies → Regularization; Computer vision; Supervised learning.

KEYWORDS
machine learning, data augmentation, robustness, trustworthy

1 INTRODUCTION
Data augmentation, i.e., to increase the dataset size through generating new samples by transforming the existing samples with some predefined functions, is probably one of the most often used techniques to improve a machine learning model’s performance. It has helped machine learning models achieve high prediction accuracy over various benchmarks [e.g., 8, 19, 31, 38, 42, 70].

In addition to improving prediction accuracy, the community has also leveraged data augmentation to help the models learn more robust representations that can generalize to the datasets distributed differently [e.g., 32, 45, 51, 69]. To improve robustness, the community usually designed the transformation functions used to augment the data in correspondence to the transformations we see in the real world [28], such as the changes of image texture or contrast. Thus, models trained with these augmented data are more likely to be invariant to these designed transformations, such as the texture or contrast variations of the input images.

To further help a model learn representations invariant to the transformations, we can regularize the model so that the distance between representations learned by the model from a pair of data (the original one and the transformed counterpart) will be small. This regularization has been used extensively recently to help models learn more robust and invariant representations [e.g., 26, 36, 40, 65]. Motivated by this popularity, this paper mainly studies the behaviors of this regularization, which we refer to as alignment regularization (AR). In particular, we seek to answer the question: how should we use alignment regularization to take advantage of the augmented data to the fullest extent to learn robust and invariant models?

To answer this, we first conduct a range of experiments over image classification benchmarks to evaluate how popular variants of AR contribute to learning robust and invariant models. We test for accuracy, robustness, and invariance, for which we propose a new test procedure. Our empirical study favors the squared $\ell_2$ norm. Our contributions of this paper are as follows.

• With a new invariance test, we show that alignment regularization is important to help the model learn representations invariant to the transformation function, and squared $\ell_2$ norm is considered the favorable choice as assessed by a variety of empirical evaluations (Section 4).
• We formalize a generalization error bound for models trained with AR and augmented data (Section 5).
2 RELATED WORK AND KEY DIFFERENCES

Tracing back to the earliest convolutional neural networks, [60], we notice that even early models for the MNIST dataset have been boosted by data augmentation [1, 39, 50]. Later, the rapidly growing machine learning community has seen a proliferation of data augmentation techniques that have helped models climb the state-of-the-art ladder [49]. Among the augmentation techniques, the most relevant one to this paper is to generate the samples (with constraint) that maximize the training loss along with training [16].

While the above paragraph mainly discusses how to generate the augmented samples, we mainly study how to train the models with augmented samples. For example, instead of directly mixing augmented samples with the original samples, one can consider regularizing the representations (or outputs) of original samples and augmented samples to be close under a distance metric (which we refer to as alignment regularization, AR). Many concrete ideas have been explored in different contexts. For example, $\ell_2$ distance and cosine similarities between internal representations in speech recognition [40], squared $\ell_2$ distance between logits [36], or KL divergence between softmax outputs [65] in adversarially robust vision models, Jensen–Shannon divergence (of three distributions) between embeddings for texture invariant image classification [26]. These are but a few highlights of the concrete and successful implementations for different applications out of a vast collection (e.g., [4, 24, 47, 48, 62, 63, 68, 69]), and we can expect methods permuting these three elements (distance metrics, representation or outputs, and applications) to be discussed in the future. Further, given the popularity of GAN [21] and domain adversarial neural network [17], we can also expect the distance metric generalizes to a specialized discriminator (i.e., a classifier), which can be intuitively understood as a calculated (usually maximized) distance measure, and one example here is the Wasserstein-1 metric [3, 23].

Key Differences: With this rich collection of regularizing choices, which one method should we consider in general? More importantly, do we need the regularization at all? These questions are important for multiple reasons, especially since sometimes AR may worsen the results [35]. In this paper, we first conduct an empirical study to show that AR (especially squared $\ell_2$ norm) can help learn robust and invariant models, we then also derive generalization error bounds to complement our empirical conclusion.

There are also several previous discussions regarding the detailed understandings of data augmentation [10, 15, 20, 30, 46, 64, 66], among which, [64] is probably the most relevant as it also defends the usage of the AR. In addition to what is reported in [64], our work also connects to invariance and shows that another advantage of AR is to learn invariant representations.

3 ACCURACY, ROBUSTNESS, AND INVARIANCE

This section discusses the three major evaluation metrics we will use to test AR. We will first recapitulate the background of accuracy and robustness. Then we will introduce our definition of invariance and our proposed evaluation.

Notations $(X, Y)$ denotes the data, where $X \in \mathbb{R}^{n \times p}$ and $Y \in \{0, 1\}^{n \times k}$ (one-hot vectors for $k$ classes). $(x, y)$ denotes a sample. $f(\theta)$ denotes the model, which takes in the data and outputs the softmax (probabilities of the prediction) and $\theta$ denotes the corresponding parameters. $g(\cdot)$ completes the prediction (i.e., mapping softmax to one-hot prediction). $a(\cdot)$ denotes a function for data augmentation, i.e., a transformation function, $a \in \mathcal{A}$, which is the set of transformation functions of interest. $P$ denotes the distribution of $(x, y)$. For any sample $(x, y)$, we can have $(a(x), y)$, and we use $P_a$ to denote the distribution of these transformed samples. Further, we use $Q_{a(x), \theta}$ to denote the distribution of $f(a(x); \theta)$ for $(x, y) \sim P$. $D(\cdot, \cdot)$ is a distance measure over two distributions. $r(\theta)$ denotes the risk of model $\theta$. $\tilde{r}(\theta)$ denotes the estimation of the term $r(\theta)$.

3.1 Accuracy

The community studying the statistical property of the error bound usually focuses on the expected risk defined as

$$r_p(\tilde{\theta}) = \mathbb{E}_{(x, y) \sim P} [g(f(x; \tilde{\theta})) \neq y],$$

where $[\cdot]$ is a function that returns 1 if the condition holds.

In practice, the error is evaluated by replacing $P$ with a hold-out test dataset, and the accuracy is $1 - r_p(\tilde{\theta})$.

3.2 Robustness

We define robustness as the worst-case expected risk when the test data is allowed to be transformed by functions in $\mathcal{A}$, following [e.g., 22, 52]. Formally, we study the worst-case error as

$$r_p, \mathcal{A}(\tilde{\theta}) = \mathbb{E}_{(x, y) \sim P} \max_{a \in \mathcal{A}} [g(f(a(x); \tilde{\theta})) \neq y],$$

where we use $r_p, \mathcal{A}(\tilde{\theta})$ to denote the robust error as it will depend on $\mathcal{A}$. In practice, the robust error is also evaluated by replacing $P$ with a hold-out dataset.

3.3 Invariance

Further, high robustness performances do not necessarily mean the model is truly invariant to the transformation functions [29], and we continue to introduce a new test to evaluate the model’s behavior in learning representations invariant to the transformations.

**Invariance.** If the model can learn a representation invariant to the transformation functions, it will map the samples of different transformations to the same representation. Intuitively, to measure how invariant a model is to the transformations in $\mathcal{A}$, we can calculate the distances between each pair of the two transformed samples when a sample is transformed with functions in $\mathcal{A}$. Thus, we define the following term to measure invariance:

$$I_p, \mathcal{A}(\tilde{\theta}) = \sup_{a_1, a_2 \in \mathcal{A}} D(Q_{a_1(x), \tilde{\theta}}, Q_{a_2(x), \tilde{\theta}}).$$

We suggest using Wasserstein metric as $D(\cdot, \cdot)$, considering its favorable properties (e.g., see practical examples in Figure 1 of [14] or theoretical discussions in [57]).
In practice, we also need to replace $P$ with a hold-out dataset so that the evaluation can be performed. In addition, we notice that $l_p, g(\tilde{b})$, although intuitive, is not convenient in practice because the evaluated values are not bounded. Thus, we reformulate it into the following invariance test procedure, whose final score will be bounded between 0 and 1 (the higher, the better). Therefore, the score can be conveniently discussed together with accuracy and robust accuracy, which are also bounded between 0 and 1.

**Invariance test.** Given a family of transformation functions used in data augmentation $\mathcal{A} = \{a_1(), a_2(), \ldots, a_t()\}$ of $t$ elements, and a collection of samples (from the hold-out dataset) of the same label $i$, denoted as $X^{(i)}$, the evaluation procedure is as follows. We first generate the transformed copies of $X^{(i)}$ with $\mathcal{A}$, resulting in $X^{(a_1)}, X^{(a_2)}, \ldots, X^{(a_t)}$. We combined these copies into a dataset, denoted as $X^{(i)}$. For every sample $x$ in $X^{(i)}$, we retrieve its $t$ nearest neighbors of other samples in $X^{(i)}$, and calculate the overlap of the retrieved samples with the transformed copies of $x$ by $\mathcal{A}$, i.e., $\{a_1(x), a_2(x), \ldots, a_t(x)\}$. The calculated overlap score will be in $[0, 1]$ in general, but since the identity map is usually in $\mathcal{A}$, this score will usually be in $[1/1, 1]$.

During the retrieval of nearest neighbors, we consider the distance function of the two samples (namely $x$ and $x'$) as

$$d(f(x; \tilde{b}), f(x'; \tilde{b}))$$

, where $\tilde{b}$ is the model we are interested in examining. In the empirical study later, we consider $d(u, v) = ||u - v||_1$, with $u$ and $v$ denoting two vectors. If we use other distance functions, the reported values may differ, but we notice that the rank of the methods compared in terms of this test barely changes.

Finally, we iterate through label $i$ and report the averaged score for all the labels as the final score. A higher score indicates the model $\tilde{b}$ is more invariant to the transformation functions in $\mathcal{A}$.

This invariance test procedure is formally presented in Algorithm 1 below.

### Algorithm 1: Invariance test

**Results:** $I(\tilde{b})$

**Input:** a family of transformation functions $\mathcal{A} = \{a_1(), a_2(), \ldots, a_t()\}$, a hold-out dataset $(X, Y)$, the model of interest $\tilde{b}$, and a distance metric $d()$.

**for every label $i$ do**

identify all the samples from $(X, Y)$ with label $i$, name this set of samples $X^{(i)}$;

**for every $x \in X^{(i)}$ do**

generate $X^{(i)}_x'$ by applying $a()$ to every $x \in X^{(i)}$;

end

$X^{(i)} = X^{(i)}_1 \cup X^{(i)}_2 \cup \ldots \cup X^{(i)}_t$;

**for every $x \in X^{(i)}_x'$ do**

generate set $T = \{a_1(x), a_2(x), \ldots, a_t(x)\}$;

**for every $x' \in X^{(i)}_x'$ do**

calculate the distance between $x$ and $x'$ with $d(f(x; \tilde{b}), f(x'; \tilde{b}))$;

end

retrieve the $t$ nearest neighbors of $x$ out of $X^{(i)}$, and name this set of samples $K_x$;

calculate the score for $x$ with $|T \cap K_x|/|T|$, where $|T|$ denotes the cardinality of the set $T$;

end

calculate the score for label $i$ as the average score across all $x \in X^{(i)}$;

end

calculate the final score $I(\tilde{b})$ as the average score across all the labels;

### 4 EMPIRICAL STUDY

In this section, we conduct experiments to study the relationship between robustness and invariance, as well as how training with AR can help improve the invariance score. In short, our empirical study in this section will lead us to the following three major conclusions:

- High robust accuracy does not necessarily mean a high invariance score and vice versa.
- AR can help improve the invariance score.
- Squared $\ell_2$ norm over logits is considered the empirically most favorable AR option for learning robust and invariant representations.

#### 4.1 Experiment Setup

Our empirical investigation is conducted over two benchmark datasets (MNIST dataset with LeNet architecture and CIFAR10 dataset with ResNet18 architecture) and three sets of transformations.

**Transformation Functions.** We consider three sets of transformation functions:

- **Texture**: we use Fourier transform to perturb the texture of the data by discarding the high-frequency components cut-off by a radius $r$, following [61]. The smaller $r$ is, the fewer high-frequency components the image has. We consider $\mathcal{A} = \{a_0(), a_{12}(\cdot), a_{15}(\cdot), a_{45}(\cdot), a_{60}(\cdot)\}$, where the subscript denotes the radius $r$ except that $a_0(\cdot)$ is the identity map. We consider $\mathcal{A}$ during test time, but only $a_0(\cdot)$ and $a_{60}(\cdot)$ during training.
- **Rotation**: we rotate the images clockwise $r$ degrees with $\mathcal{A} = \{a_0(), a_{12}(\cdot), a_{15}(\cdot), a_{45}(\cdot), a_{60}(\cdot)\}$, where the subscript denotes the degree of rotation and $a_0(\cdot)$ is the identity map. We consider $\mathcal{A}$ during test time, but only $a_{12}(\cdot)$ and $a_{60}(\cdot)$ during training.
- **Contrast**: we create the images depicting the same visual information, but with different scales of the pixels, including the negative color representation. Therefore, we have $\mathcal{A} = \{a_0(x) = x, a_1(x) = x/2, a_2(x) = x/4, a_3(x) = 1-x, a_4(x) = (1-x)/2, a_5(x) = (1-x)/4\}$, where $x$ stands for the image whose pixel values have been set to be between 0 and 1. We consider $\mathcal{A}$ during test time, but only $a_0(\cdot)$ and $a_3(\cdot)$ during training.

**Alignment Regularizations.** We consider the following popular choices of AR (with $u$ and $v$ denoting two vector embeddings):

- **L**: $\ell_1$ norm of the vector differences, i.e., $||u - v||_1$
- **S**: squared $\ell_2$ norm of the vector differences, i.e., $||u - v||_2^2$
- **C**: cosine similarity, i.e., $u^T v / ||u|| \cdot ||v||$
- **K**: KL divergence over a batch of paired embeddings; the second argument are augmented samples
- **W**: Wasserstein-1 metric over a batch of paired embeddings, with implementation following Wasserstein GAN [3, 23]
- **D**: a vanilla GAN discriminator over a batch of paired embeddings, the one-layer discriminator is trained to classify samples vs. augmented samples.

We mainly discuss applying the AR to logits (embeddings prior to the final softmax function). We have also experimented with applying to the final softmax output and the embeddings one layer
Table 1: Test results on MNIST dataset for different ARs over three evaluation metrics and three distribution shifts. B denotes Baseline, i.e., the model does not use any data augmentation; V denotes vanilla augmentation, i.e., the model uses data augmentation but not AR; L denotes \(l_1\) norm; S denotes squared \(l_2\) norm; C denotes cosine similarity; K denotes KL divergence; W denotes Wasserstein-1 metric; D denotes GAN discriminator.

|         | B       | V       | L       | S       | C       | K       | W       | D       |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Accuracy| 99.2±0.0| 99.2±0.0| 99.0±0.1| 99.1±0.0| 99.4±0.0| 68.7±41| 98.7±0.1| 99.1±0.1|
| Robustness| 98.3±0.3| 99.0±0.0| 99.0±0.0| 99.0±0.0| 99.1±0.0| 68.7±41| 98.4±0.1| 98.8±0.1|
| Invariance| 92.4±0.0| 99.2±0.0| 100±0.0| 100±0.0| 99.0±0.0| 76.0±34| 60.7±29| 35.0±67|
| Accuracy| 99.2±0.0| 99.0±0.1| 99.3±0.0| 99.3±0.0| 99.0±0.0| 98.8±0.0| 98.5±0.4| 98.9±0.0|
| Robustness| 28.9±0.6| 93.6±0.3| 95.2±0.1| 95.1±0.1| 93.5±0.1| 94.5±0.2| 93.2±0.8| 93.2±0.7|
| Invariance| 20.6±0.4| 58.3±2.2| 66.8±3.8| 65.4±3.5| 29.1±0.6| 71.9±2.8| 48.7±1.9| 39.3±9.9|
| Accuracy| 99.2±0.0| 98.2±0.3| 99.4±0.0| 99.4±0.0| 99.2±0.0| 98.9±0.0| 98.7±0.0| 99.1±0.0|
| Robustness| 26.0±1.0| 95.4±2.6| 96.8±0.8| 97.4±0.6| 97.9±0.4| 88.4±4.5| 87.2±9.9| 97.7±0.6|
| Invariance| 20.7±1.1| 37.5±0.9| 41.4±0.3| 41.3±0.4| 26.3±1.1| 40.3±0.9| 28.4±1.7| 20.0±0.1|

prior to logits. Both cases lead to substantially worse results, so we skip the discussion.

**Hyperparameters.** We use standard data splits. We first train the baseline models to get reasonably high performances (for MNIST, we train 100 epochs with learning rate set to be \(10^{-4}\); for CIFAR10, we train 200 epochs with learning rate initialized to be \(10^{-3}\) and reduced one magnitude every 50 epochs; batch sizes in both cases are set to be 128). Then we train other augmented models with the same learning rate and batch size etc. The regularization weight is searched with 8 choices evenly split in the logspace from \(10^{-1}\) to 1. For each method, the reported score is from the weight resulting in the highest robust accuracy. We test with three random seeds.

**Evaluation Metrics:** We consider the three major evaluation metrics as we discussed in Section 3:
- **Accuracy:** test accuracy on the original test data.
- **Robustness:** the worst-case accuracy when each sample can be transformed with \(\epsilon \in \mathcal{A}\).
- **Invariance:** the metric to test whether the learned representations are invariant to the transformation functions, as introduced in Section 3.3.

### 4.2 Results and Discussion

Tables 1 and 2 show the empirical results across the three distribution shifts and the three evaluation metrics. First of all, no method can dominate across all these evaluations, probably because of the tradeoff between accuracy and robustness [54, 61, 65]. Similarly, the tradeoff between accuracy and invariance can be expected from the role of the regularization weight: when the weight is small, AR has no effect, and the model is primarily optimized for improving accuracy; when the weight is considerable, the model is pushed toward a trivial solution that maps every sample to the same embedding, ignoring other patterns of the data. This is also the reason that results in Tables 1 and 2 are selected according to the robust accuracy.

Due to the tradeoffs, it may not be strategic if we only focus on the highest number of each row. Instead, we suggest studying the three rows of each test case together and compare the tradeoffs, which is also a reason we reformulate the invariance test in Section 3.3 so that we can have bounded invariance scores directly comparable to accuracy and robust accuracy.

For example, in the texture rows of Table 1, while cosine similarity can outperform squared \(l_2\) norm in accuracy and robustness with 0.3 and 0.1 margins, respectively, it is disadvantageous in invariance with a larger margin (1.0). Similarly, for rotation rows of Table 1, KL-divergence shows the overall highest scores, followed by \(l_1\) norm and squared \(l_2\) norm. For contrast rows, both \(l_1\) norm and squared \(l_2\) norm stand out. Overall, experiments in MNIST suggest the desired choice to be \(l_1\) norm or squared \(l_2\) norm, and we believe squared \(l_2\) norm is marginally better.

On the other hand, the experiments in CIFAR10 in Table 2 mostly favor \(l_1\) norm and squared \(l_2\) norm. Good performances can also be observed from Wasserstein-1 metric for rotation. Also, we notice that squared \(l_2\) norm, in general, outperforms \(l_1\) norm.
Thus, our empirical study recommends squared $\ell_2$ norm for AR to learn robust and invariant models, with $\ell_1$ norm as a runner-up.

5 ANALYTICAL SUPPORT

According to our discussions in Section 3.3, Wasserstein metric is supposed to be a favorable option as AR. However, its empirical performance does not stand out. We believe this disparity is mainly due to the difficulty in calculating the Wasserstein metric in practice.

On the other hand, norm-based consistency regularizations stand out. We are interested in studying the properties of these metrics. In particular, this section aims to complement the empirical study by showing that norm-based AR can lead to bounded robust generalization error under certain assumptions.

Also, our experiments only use two transformation functions from $\mathcal{A}$ during training but are tested with all the functions in $\mathcal{A}$. We also discuss the properties of these two functions and argue that training with only these two functions can be a good strategy when certain assumptions are met.

5.1 Overview of Analytical Results

All together, we need six assumptions (namely, A1-A6). Out of these assumptions, A1 is necessary for using data augmentation, and A4 is necessary for deriving machine learning generalization error bound. A2 and A3 are properties of data transformation functions, and A5 and A6 are technical assumptions. Along with introducing these assumptions, we will also offer empirical evidence showing that these assumptions are likely to hold in practice.

In particular, we will show that

• With A2 holds, $\ell_1$ norm can replace the empirical Wasserstein metric to regularize invariance (Proposition A.2) and then we can derive a bounded robust error if all the functions in $\mathcal{A}$ are available (Theorem 5.1).
• With the above result and A3, we can derive a bounded robust error if only two special functions in $\mathcal{A}$ (which we refer to as vertices) are available (Lemma 5.2).

5.2 Assumptions Setup

5.2.1 Assumptions on Data Augmentation Functions. Our first three assumptions are for the basic properties of the data transformation functions used. These properties are formally introduced as assumptions below.

A1: Dependence-preservation: the transformation function will not alter the dependency regarding the label (i.e., for any $a() \in \mathcal{A}$, $a(x)$ will have the same label as $x$) or the features (i.e., $a_1(x_1)$ and $a_2(x_2)$ are independent if $x_1$ and $x_2$ are independent).

Intuitively, “Dependence-preservation” has two perspectives: Label-wise, the transformation cannot alter the label of the data, which is a central requirement of almost all the data augmentation functions in practice. Feature-wise, the transformation will not introduce new dependencies between the samples.

We consider the label-wise half of this argument as a fundamental property of any data augmentations. It has to be always true for data augmentation to be a useful technique. On the other hand, the feature-wise half of this argument is a fundamental property required to derive the generalization error bounds. Intuitively, we believe this property holds for most data augmentation techniques in practice.

A2: Efficiency: for $\tilde{\theta}$ and any $a() \in \mathcal{A}$, $f(a(x); \tilde{\theta})$ is closer to $x$ than any other samples under a distance metric $d_x(\cdot, \cdot)$, i.e.,

$$d_x(f(a(x); \tilde{\theta}), f(x; \tilde{\theta})) \leq \min_{x' \in \mathcal{X}} d_x(f(a(x); \tilde{\theta}), f(x'; \tilde{\theta})).$$

We define $d_x(\cdot, \cdot)$ to be $\ell_1$ norm.

Intuitively, the efficiency property means the augmentation should only generate new samples of the same label as minor perturbations of the original one. If a transformation violates this property, there should exist other simpler transformations that can generate the same target sample.

A3: Vertices: For a model $\tilde{\theta}$ and a transformation $a()$, we use $P_{a, \tilde{\theta}}$ to denote the distribution of $f(a(x); \tilde{\theta})$ for $(x, y) \sim P$. “Vertices” argues that exists two extreme elements in $\mathcal{A}$, namely $a^+$ and $a^-$, with certain metric $d_x(\cdot, \cdot)$, we have

$$d_x(P_{a^+, \tilde{\theta}}, P_{a^-, \tilde{\theta}}) = \sup_{a_1, a_2 \in \mathcal{A}} d_x(P_{a_1 \circ a_2, \tilde{\theta}}, P_{a_2 \circ a_1, \tilde{\theta}})$$

We define $d_x(\cdot, \cdot)$ to be Wasserstein-1 metric.

Intuitively, “Vertices” suggests that there are extreme cases of the transformations. For example, if one needs the model to be invariant to rotations from $0^\circ$ to $60^\circ$, we consider the vertices to be rotation function (thus identity map) and $60^\circ$ rotation function.

In practice, one usually selects the transformation vertices with intuitions or domain knowledge.

Notice that we do not need to argue that A3 always holds. All we need is that A3 can sometimes hold, and when it holds, we can directly train with the regularized vertex augmentation. Thus, anytime RVA empirically performs well is a favorable argument for A3. To show that RVA can sometimes perform well, we compare the RVA with vanilla (non-regularized) worst-case data augmentation (VWA) method across our synthetic experiment setup. We notice that out of six total scenarios ({texture, rotation, contrast} $\times$ {MNIST, CIFAR10}), RVA outperforms VWA frequently. This suggests that the domain-knowledge of vertices can help in most cases, although not guaranteed in every case.

5.2.2 Assumptions on Background and Generalization Error Bound.

In an abstract manner, when the test data and train data are from the same distribution, several previous analyses on the generalization error can be sketched as (see examples in A4):

$$\mathbb{E}_{\Theta, \delta}(\tilde{\theta}) \leq \hat{\mathbb{E}}_{\Theta}(\tilde{\theta}) + \phi(\Theta, n, \delta)$$

which suggests that the expected risk $\mathbb{E}_{\Theta}(\tilde{\theta})$ can be bounded by the empirical risk $\hat{\mathbb{E}}_{\Theta}(\tilde{\theta})$ and a function of hypothesis space $|\Theta|$ and number of samples $n$. $\delta$ accounts for the probability when the bound holds. $\phi(\cdot)$ is a function of these three terms. Dependent on the details of different analyses, different concrete examples of this generic term will need different assumptions. We use a generic assumption A4 to denote the assumptions required for each example.

Following our main goal to study how alignment regularization and data augmentation help in accuracy, robustness, and invariance, our strategy in theoretical analysis is to derive error bounds for accuracy and robustness, and the error bound directly contains...
terms to regularize the invariance. Further, as robustness naturally bounds accuracy (i.e., $r_P(\bar{\theta}) \leq r_P, A(\bar{\theta})$ following the definitions in (1) and (2) respectively), we only need to study the robust error.

To study the robust error, we need two additional technical assumptions. **A5** connects the distribution of expected robust risk and the distribution of empirical robust risk, and **A6** connects the 0-1 classification error and cross-entropy risk.

**A4**: We list two examples here:
- When **A4** is $\Theta$ is finite, $\mathcal{L}$ is a zero-one loss, samples are i.i.d., $\phi(\theta) = \frac{\sqrt{\log(\Theta)} + \log(1/\delta)}{2n}$,
- When **A4** is “samples are i.i.d., $\phi(\theta) = 2\mathcal{R}(\mathcal{L}) + \sqrt{\log 1/\delta}/2n$, where $\mathcal{R}(\mathcal{L})$ stands for Rademacher complexity and $\mathcal{L} = \{y | \theta \in \Theta\}$, where $l_0$ is the loss function corresponding to $\theta$.

For more information or more concrete examples of the generic term, one can refer to relevant textbooks such as [7].

**A4** stands for the fundamental assumptions used to derive standard generalization bounds. We rely on this assumption as how previous theoretical works rely on them.

**A5**: the distribution for expected robust risk equals the distribution for empirical robust risk, i.e.,

$$\arg\max_{P \in \mathcal{T}(P,A)} r_P(\bar{\theta}) = \arg\max_{P \in \mathcal{T}(P,A)} \tilde{r}_P(\bar{\theta})$$

where $T(P, A)$ is the collection of distributions created by elements in $A$ over samples from $P$.

Eq. (2) can be written equivalently into the expected risk created over a pseudo distribution $P^*$ (see Lemma 1 in [55]), which is the distribution that can sample the data leading to the expected robust risk. Thus, equivalently, we can consider sup$_{P^*} T(P,A)$ $r_P(\bar{\theta})$ as a surrogate of $r_P, A(\bar{\theta})$, where $T(P, A)$ denotes the set of possible resulting distributions. Following the empirical strength of techniques such as adversarial training [44], we introduce an assumption relating the distribution of expected robust risk and the distribution of empirical robust risk (namely, **A5**, in Appendix 5.2). Thus, the bound of our interest (i.e., sup$_{P \in \mathcal{T}(P,A)} r_P(\bar{\theta})$) can be analogously analyzed through sup$_{P \in \mathcal{T}(P,A)} \tilde{r}_P(\bar{\theta})$.

**A5** is likely to hold in practice: Assumption **A5** appears very strong, however, the successes of methods like adversarial training [44] suggest that, in practice, **A5** might be much weaker than it appears.

**A6**: With $(x, y) \in (X, Y)$, the sample maximizing cross-entropy loss and the sample maximizing classification error for model $\theta$ follows:

$$\forall x, \frac{y^T f(x; \bar{\theta})}{\inf_{a \in A} y^T f(a(x); \bar{\theta})} \geq \exp(\|g(f(x; \bar{\theta})) - g(f(x'; \bar{\theta}))\|)$$

where $x'$ stands for the sample maximizing classification error, i.e.,

$$x' = \arg\min_x y^T g(f(x; \bar{\theta}))$$

Also,

$$\forall x, \inf_{a \in A} | y^T f(a(x); \bar{\theta}) | \geq 1$$

Intuitively, although Assumption **A6** appears complicated, it describes the situations of two scenarios:

If $g(f(x; \bar{\theta})) = g(f(x'; \bar{\theta}))$, which means either the sample is misclassified by $\bar{\theta}$ or $A$ is not rich enough for a transformation function to alter the prediction, the RHS of Eq. 5 is 1, thus Eq. 5 always holds (because $A$ has the identity map as one of its elements).

If $g(f(x; \bar{\theta})) \neq g(f(x'; \bar{\theta}))$, which means a transformation alters the prediction. In this case, **A6** intuitively states that the $A$ is reasonably rich and the transformation is reasonably powerful to create a gap of the probability for the correct class between the original sample and the transformed sample. The ratio is described as the ratio of the prediction confidence from the original sample over the prediction confidence from the transformed sample is greater than $e$.

### 5.3 Analytical Support

**Regularized Worst-case Augmentation.** To have a model with a small invariance score, we should probably directly regularize the empirical counterpart of Eq. (3). However, Wasserstein distance is difficult to calculate in practice. Fortunately, Proposition A.2 conveniently allows us to use $\ell_1$ norm to replace Wasserstein metric. With Proposition A.2, now we can offer our main technical result to study the robust error $r_P, A(\bar{\theta})$ (as defined in Eq. (2)).

**Theorem 5.1.** With Assumptions A1, A2, A4, A5, and A6, with probability at least $1 - \delta$, we have

$$r_P, A(\bar{\theta}) \leq \tilde{r}_P(\bar{\theta}) + \sum_i \|f(x_i; \bar{\theta}) - f(x_i'; \bar{\theta})\|_1 + \phi(\theta), n, \delta)$$

and $x' = a(x)$, where $a = \arg\min_{a \in A} y^T f(a(x); \bar{\theta})$. $\phi(\theta), n, \delta)$ is defined in A4.

This technical result immediately inspires the method to guarantee worst-case performance, as well as to explicitly enforce the concept of invariance. The method $a = \arg\min_{a \in A} y^T f(a(x); \bar{\theta})$ is selecting the transformation function maximizing the cross-entropy loss (notice the sign difference between here and the cross-entropy loss), which we refer to as worst-case data augmentation. This method is also closely connected to adversarial training [e.g., 44].

**Regularized Vertex Augmentation.** As $A$ in practice usually has a large number of (and possibly infinite) elements, we may not always be able to identify the worst-case transformation function with reasonable computational efforts. We further leverage the vertex property (boundary cases of transformation functions, discussed as Assumption A3 in the appendix) of the transformation function to bound the worst-case generalization error:

**Lemma 5.2.** With Assumptions A1-A6, assuming there is a $a^+(\cdot) \in A$ where $\tilde{r}_P_{a^+}(\bar{\theta}) = \frac{1}{2}(r_P, a^+(\bar{\theta}) + \tilde{r}_P_{a^-}(\bar{\theta}))$, with probability at least $1 - \delta$, we have:

$$r_P, A(\bar{\theta}) \leq \frac{1}{2}(\tilde{r}_P_{a^+}(\bar{\theta}) + \tilde{r}_P_{a^-}(\bar{\theta})) + \sum_i \|f(a^+(x_i); \bar{\theta}) - f(a^-(x_i); \bar{\theta})\|_1 + \phi(\theta), n, \delta)$$

where $a^+(\cdot)$ and $a^-(\cdot)$ are defined in A3, and $\phi(\theta), n, \delta)$ in A4.

This result corresponds to the method that can be optimized conveniently without searching for the worst-case transformations. However, the method requires good domain knowledge of the vertices (i.e., boundary cases) of the transformation functions.
Table 3: Comparison to advanced rotation-invariant models. We report the accuracy on the test sets rotated. “main” means the resulting images are highly likely to be semantically the same as the original ones. “all” means the average accuracy of all rotations. The underlined scores show that data augmentation and AR can help a vanilla model to compete with advanced methods. The bold scores (highest at each row) show that data augmentation and AR can further improve the advanced methods.

|          | ResNet18 | ResNet50 | ResNet101 |
|----------|----------|----------|-----------|
|          | B RVA RWA | B RVA RWA | B RVA RWA |
| Top-1    | 75.6 100 | 72.2 77.7 | 74.0 78.2 |
| Top-5    | 93.1 100 | 93.8 93.9 | 93.9 94.4 |

Table 4: Comparison to advanced methods on 9 super-class ImageNet classification with different distribution shifts.

|          | Acc. | WAcc. | ImageNet-A | ImageNet-S |
|----------|------|-------|------------|------------|
| Base     | 90.8 | 88.8  | 24.9       | 41.1       |
| SIN      | 88.4 | 86.6  | 24.6       | 40.5       |
| LM       | 67.9 | 65.9  | 18.8       | 36.8       |
| RUBi     | 90.5 | 88.6  | 27.7       | 42.3       |
| RB       | 91.9 | 90.5  | 29.6       | 41.8       |
| RWA      | 92.2 | 91.2  | 28.0       | 42.5       |
| RWA      | 92.8 | 91.6  | 28.8       | 43.2       |

Table 5: The generic methods can also improve standard accuracy.

|          | ResNet | GC | ST | ETN |
|----------|--------|----|----|-----|
|          | Base RVA RWA | Base RVA RWA | Base RVA RWA | Base RVA RWA |
| main     | 45.4 66.5 72.1 | 38.5 72.2 73.8 | 45.9 58.3 62.9 | 56.9 65.1 57.7 |
| all      | 31.2 48.1 52.8 | 26.7 54.4 55.0 | 32.1 40.2 42.7 | 39.5 52.6 46.1 |

Thus, our theoretical discussions have complemented our empirical findings in Section 4 by showing that norm-based regularizations can lead to bounded robust error. There is a disparity that our analytical result is about \( \ell_1 \) norm while our empirical study suggests squared \( \ell_2 \) norm. We conjecture the disparity is mainly caused by the difficulty in passing the gradient of \( \ell_1 \) norm in practice.

6 EXPERIMENTS WITH ADVANCED METHODS

We continue to test the methods we identified in comparison to more advanced methods. Although we argued for the value of invariance, for a fair comparison, we will test the performances evaluated by the metrics the previous methods are designed for. Our method will use the same generic approach and the same transformation functions as in the previous empirical study, although these functions are not necessarily part of the distribution shift we test now. In summary, our method can outperform (or be on par with) these SOTA techniques in the robustness metric they are designed for (Section 6.2). In addition, we run a side test to show that our method can also improve accuracy (Section 6.3).

6.1 Methods

Section 4 and Section 5 lead us to test the following two methods:

- RVA (regularized vertex augmentation): using squared \( \ell_2 \) norm as AR over logits between the original samples and the augmented samples of a fixed vertex transformation function (original samples are considered as from another vertex).
- RWA (regularized worst-case augmentation): using squared \( \ell_2 \) norm as AR over logits between the original samples and the worst-case augmented samples identified at each iteration. Worst-case samples are generated by the function with the maximum loss when we iterate through all the transformation functions.

6.2 Robustness

Rotation. We compare our results with rotation-invariant models, mainly Spatial Transformer (ST) [34], Group Convolution (GC) [12], and Equivariant Transformer Network (ETN) [53]. We also tried to run CGNet [37], but the method does not seem to scale to the CIFAR10 and ResNet level. All these methods are tested with ResNet34 following popular settings in the community. The results are in Table 3. We test the models every 15° rotation from 0° rotation to 345° rotation. Augmentation-related methods use the \( A \) of “rotation” in synthetic experiments, so the testing scenario goes beyond what the augmentation methods have seen during training.

We report two summary results in Table 3. “main” means the average prediction accuracy from images rotated from 300° to 60° (passing 0°), when the resulting images are highly likely to preserve the class label. “all” means the average accuracy of all rotations.

Our results can be interpreted from two perspectives. First, by comparing all the columns in the first panel to the first column of the other three panels, data augmentation and AR can boost a vanilla model to outperform other advanced techniques. On the other hand, by comparing the columns within each panel, data augmentation and AR can further improve the performances of these techniques.

Interestingly, the baseline model with our generic approach (RWA in the first panel) can almost compete with the advanced methods even when these methods also use augmentation and AR (RWA in GC panel). We believe this result strongly indicates the potential of this simple augmentation and regularization method to match the advanced methods.

In summary, RWA can boost the vanilla model to outperform advanced methods. Data augmentation and squared \( \ell_2 \) AR can further improve the performances when plugged onto advanced methods.

Texture & Contrast. We follow [5] and compare the models for a nine super-class ImageNet classification [33] with class-balanced strategies. Also, we follow [5] to report standard accuracy (Acc.), weighted accuracy (WAcc.), a scenario where samples with unusual texture are weighted more, and accuracy over ImageNet-A [27], a
collection of failure cases for most ImageNet trained models. Additionally, we also report the performance over ImageNet-Sketch [58], an independently collected ImageNet test set with only sketch images. As [5] mainly aims to overcome the texture bias, we also use our texture-wise functions in Section 4 for augmentation. However, there are no direct connections between these functions and the distribution shift of the test samples. Also, we believe the distribution shifts here, especially the one introduced by our newly added ImageNet-Sketch, are more than texture, and also correspond to the contrast case of our study.

Following [5], the base network is ResNet, and we compare with the vanilla network (Base), and several methods designed for this task: including StylisedIN (SIN) [18], LearnedMixin (LM) [11], RUBi (RUBi) [9] and ReBias (RB) [5]. Results are in Table 4.

The results favor our generic method in most cases. RVA outperforms other methods in standard accuracy, weighted accuracy, and ImageNet-Sketch, and is shy from ReBias on ImageNet-A. RVA shows the same pattern as that of RVA and further outperforms RVA. Overall, these results validate the empirical strength of data augmentation (even when the augmentation is not designed for the task) and squared $\ell_2$ norm AR for learning robust models.

6.3 Accuracy
Further, these experiments help us notice that the generic technique can also help improve the accuracy, although the technique is motivated by robustness and invariance. Therefore, we follow the widely accepted CIFAR100 test pipeline and test the performances of different architectures of the ResNet family. The results are reported in Table 5, where Base stands for the baseline model with the default accuracy boosting configurations.

For both top-1 and top-5 accuracies and across the three ResNet architectures, our techniques can help improve the accuracy. In addition, we notice that our techniques can help bridge the gap of different architectures within the ResNet family: for example, RVA helps ResNet50 to outperform the vanilla ResNet101.

7 CONCLUSION
In this paper, we seek to answer how to train with augmented data so that augmentation can be taken to the fullest extent. We first defined a new evaluation metric called invariance and conducted a line of empirical studies to show that norm-based alignment regularization can help learn robust and invariant models. Further, we complement our observations with formal derivations of bounded generalization errors. We notice that regularizing squared $\ell_2$ norm between the logits of the originals samples and those of the augmented samples is favorable: the trained model tends to have the most favorable performances in robust accuracy and invariance. In general, the method we recommend is “regularized worst-case augmentation” with squared $\ell_2$ norm as the alignment regularization. One can also consider “regularized vertex augmentation” when extra assumptions on the vertex properties of the transformation functions are met. Lastly, we would like to remind a potential limitation of alignment regularization: it may not always help improve the $i.i.d$ accuracy due to the tradeoff between accuracy and robustness or invariance. In addition, to simplify the procedure of users in leveraging our contribution, we also release a software package in both TensorFlow and PyTorch for users to use our identified methods with a couple lines of code.

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A.3 Proof of Theorem 5.1

**Theorem.** With Assumptions A1, A2, A4, A5, and A6, with probability at least $1 - \delta$, we have

$$r_{\mathcal{P}, \mathcal{A}}(\tilde{\Theta}) \leq \tilde{r}_P(\tilde{\Theta}) + \sum_{i} \|f(x_i; \tilde{\Theta}) - f(x_i'; \tilde{\Theta})\|_1 + \phi(\Theta, n, \delta)$$

and $x' = a(x)$, where $a = \arg \min_{a \in \mathcal{A}} y^T f(a(x); \tilde{\Theta})$.

**Proof.** First of all, in the context of multiclassification, where $g(f(x; \Theta))$ predicts a label with one-hot representation, and $y$ is represented with one-hot representation, we can have the empirical risk written as:

$$\hat{r}_P(\tilde{\Theta}) = 1 - \frac{1}{n} \sum_{(x,y) \sim P} y^T g(f(x; \tilde{\Theta}))$$

Thus,

$$\sup_{P \in T(\mathcal{P}, \mathcal{A})} \hat{r}_P(\tilde{\Theta}) = \sup_{P \in T(\mathcal{P}, \mathcal{A})} \hat{r}_P(\tilde{\Theta}) + \frac{1}{n} \sum_{(x,y) \sim P} y^T g(f(x; \tilde{\Theta}))$$

$$= \frac{1}{n} \sum_{(x,y) \sim P} y^T g(f(x; \tilde{\Theta}))$$

With A6, we can continue with:

$$\sup_{P \in T(\mathcal{P}, \mathcal{A})} \hat{r}_P(\tilde{\Theta}) \leq \hat{r}_P(\tilde{\Theta}) + \frac{1}{n} \sum_{(x,y) \sim P} e(f(x; \tilde{\Theta}))$$

$$- \sum_{(x,y) \sim P} e(f(x; \tilde{\Theta}))$$

If we use $e(\cdot) = -y^T \log(\cdot)$ to replace the cross-entropy loss, we simply have:

$$\sup_{P \in T(\mathcal{P}, \mathcal{A})} \hat{r}_P(\tilde{\Theta}) \leq \hat{r}_P(\tilde{\Theta}) + \frac{1}{n} \sum_{(x,y) \sim P} e(f(x; \tilde{\Theta}))$$

Since $e(\cdot)$ is a Lipschitz function with constant $\leq 1$ (because of A6, Eq.6) and together with the dual representation of Wasserstein metric (See e.g., [56]), we have

$$\sup_{P \in T(\mathcal{P}, \mathcal{A})} \hat{r}_P(\tilde{\Theta}) \leq \hat{r}_P(\tilde{\Theta}) + W_1(\hat{Q}_{x, \tilde{\Theta}}, \hat{Q}_{a(x), \tilde{\Theta}})$$

where $x' = a(x)$, where $a = \arg \min_{a \in \mathcal{A}} y^T f(a(x); \tilde{\Theta}), \tilde{Q}_{x, \tilde{\Theta}}$ denotes the empirical distribution of $f(a(x); \tilde{\Theta})$ for $(x,y) \in (X,Y)$. Note that $r_{\mathcal{P}, \mathcal{A}}(\tilde{\Theta})$, by definition, is a shorthand notation for

$$\sup_{P \in T(\mathcal{P}, \mathcal{A})} r_P(\tilde{\Theta})$$

Further, we can use the help of Proposition B.2 to replace Wasserstein metric with $\ell_1$ distance. Finally, we can conclude the proof with Assumption A5 as how we did in the proof of Lemma B.1. □


### A.4 Proof of Lemma 5.2

**Lemma.** With Assumptions A1-A6, assuming there is a $a'() \in A$ where $\mathbb{P}_{\rho}(\hat{b}) = \frac{1}{2}(\mathbb{P}_{\rho}(a(); \hat{b}) + \mathbb{P}_{\rho}(a(); \bar{b}))$, with probability at least $1 - \delta$, we have:

\[
\mathbb{P}_{\rho}(\hat{b}) \leq \frac{1}{2}(\mathbb{P}_{\rho}(\hat{b}) + \mathbb{P}_{\rho}(\bar{b})) + \sum_{x} ||f(a'(x); \hat{b}) - f(a'(x); \bar{b})||_1 + \phi(\mathbb{B}, n, \delta),
\]

(8)

**Proof.** We can continue with

\[
\sup_{P \in T(\mathcal{P}, \mathcal{A})} \mathbb{P}_{\rho}(\hat{b}) \leq \mathbb{P}_{\rho}(\hat{b}) + W_{1}(\hat{Q}(x, y) - \tilde{Q}_{a}(x, y)),
\]

where $\hat{Q}(x, y)$ denotes the empirical distribution of $f(a(x); \hat{b})$ for $(x, y) \in (X, Y)$, from the proof of Theorem 5.2. With the help of Assumption A3, we have:

\[
d_{\chi}(f(a'(x); \hat{b}), f(a'(x); \bar{b})) \geq d_{\chi}(f(x; \hat{b}), f(x; \bar{b}))
\]

When $d_{\chi}(\cdot, \cdot)$ is chosen as Wasserstein-1 metric, we have:

\[
\sup_{P \in T(\mathcal{P}, \mathcal{A})} \mathbb{P}_{\rho}(\hat{b}) \leq \mathbb{P}_{\rho}(\hat{b}) + W_{1}(Q_{a}(x, y) - \tilde{Q}_{a}(x, y)),
\]

Further, as the LHS is the robust risk generated by the transformation functions within $\mathcal{A}$, and $\mathbb{P}(\hat{b})$ is independent of the term $W_{1}(Q_{a}(x, y) - \tilde{Q}_{a}(x, y))$, WLOG, we can replace $\mathbb{P}(\hat{b})$ with the risk of an arbitrary distribution generated by the transformation function in $\mathcal{A}$. If we choose to use $\mathbb{P}_{\rho}(\hat{b}) = \frac{1}{2}(\mathbb{P}_{\rho}(\hat{b}) + \mathbb{P}_{\rho}(\bar{b}))$, we can conclude the proof with help from Proposition B.2 and Assumption A5 as how we did in the proof of Theorem 5.2. □

**B ADDITIONAL RESULTS FOR COMPARISONS WITH ADVANCED METHODS**

We have also conducted two full ImageNet level experiments. However, due to the limitation of resources, we cannot tune the models substantially. Our current trial suggest that our techniques can improve the vanilla model to compete with SOTA models, limited by our resources, we cannot do wide-range hyperparameters search to outperform them. Also, considering the fact that many of these methods are significantly more complicated than us and also uses data augmentation specially designed for the tasks, we consider our experiments a success indication of the empirical strength of our methods.

**Texture-perturbed ImageNet classification.** We also test the performance on the image classification over multiple perturbations. We train the model over standard ImageNet training set and test the model with ImageNet-C data [25], which is a perturbed version of ImageNet by corrupting the original ImageNet validation set with a collection of noises. Following the standard, the reported performance is mCE, which is the smaller the better. We compare with several methods tested on this dataset, including Patch Uniform (PU) [43], AutoAugment (AA) [13], MaxBlur pool (MBP) [67], Stylized ImageNet (SIN) [25], AugMix (AM) [26], AugMix w. SIN (AMS) [26]. We use the performance reported in [26]. Again, our augmentation only uses the generic texture with perturbation (the $\mathcal{A}$ in our texture synthetic experiments with radius changed to 20, 25, 30, 35, 40). The results are reported in Table 6, which shows that our generic method outperform the current SOTA methods after a continued finetuning process with reducing learning rates.

**Cross-domain ImageNet-Sketch Classification.** We also compare to the methods used for cross-domain evaluation. We follow the set-up advocated by [59] for domain-agnostic cross-domain prediction, which is training the model on one or multiple domains without domain identifiers and test the model on an unseen domain. We use the most challenging setup in this scenario: train the models with standard ImageNet training data, and test the model over ImageNet-Sketch data [58], which is a collection of sketches following the structure ImageNet validation set. We compare with previous methods with reported performance on this dataset, such as InfoDrop [2], HEX [59], PAR [58], RSC [32] and report the performances in Table 7. Notice that, our data augmentation also follows the requirement that the characteristics of the test domain cannot be utilized during training. Thus, we only augment the samples with a generic augmentation set ($\mathcal{A}$ of “contrast” in synthetic experiments). The results again support the usage of data augmentation and alignment regularization.

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**Table 6: Comparison to advanced models over ImageNet-C data. Performance reported (mCE) follows the standard in ImageNet-C data: mCE is the smaller the better.**

|                  | B   | InfoDrop | HEX | PAR | VA | RVA | RSC | VWA | RWA |
|------------------|-----|----------|-----|-----|----|-----|-----|-----|-----|
| Top-1            | 0.1240 | 0.1224 | 0.1292 | 0.1306 | 0.1362 | 0.1405 | 0.1612 | 0.1432 | 0.1486 |
| Top-5            | 0.2408 | 0.256 | 0.2564 | 0.2627 | 0.2715 | 0.2793 | 0.3078 | 0.2846 | 0.2933 |

**Table 7: Comparison to advanced cross-domain image classification models, over ImageNet-Sketch dataset. We report top-1 and top-5 accuracy following standards on ImageNet related experiments.**