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K. P. Drumev  
*Institute for Nuclear Research and Nuclear Energy Bulgarian Academy of Sciences*

A. I. Georgieva  
*Institute for Nuclear Research and Nuclear Energy Bulgarian Academy of Sciences*

J. P. Draayer  
*Louisiana State University*

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Extended (pseudo-)SU(3) models for nuclear structure

K. P. Drumev\textsuperscript{1}, A. I. Georgieva\textsuperscript{1} and J. P. Draayer\textsuperscript{2}

\textsuperscript{1} Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia, Bulgaria;
\textsuperscript{2} Department of Physics and Astronomy, Louisiana State University, Baton Rouge, LA 70803, USA

Abstract. As a simpler version of an extended (pseudo-)SU(3) model which acts in a model space of more than one oscillator shells, an extended pairing-plus-quadrupole model is introduced. The Hamiltonian consists of quadrupole-quadrupole plus pp-, nn- and pn-isovector and isoscalar pairing terms as well as terms describing the pair-scattering between two shells. Energy spectra and shape parameters for two $N=Z$ nuclear systems having the same nucleon content as the nuclei $^{20}$Ne and $^{60}$Zn which belong to two different areas of the nuclear chart ($ds$ or upper $− fp − gs$ shells) are calculated and the role of the two parts in the interaction, depending on the parameter strengths, is discussed. Both extended models provide for a microscopic description of nuclei while being selective on the choice of the relevant model space.

keywords: models based on group theory, neutron-proton pairing, pairing-plus-quadrupole model, SU(3) model, SO(8) model

1. Introduction

Until recently, SU(3) shell-model calculations - real SU(3)\cite{1} for light nuclei and pseudo-SU(3)\cite{2} for heavy nuclei - have been performed in either only one space (protons and neutrons filling the same shell, e.g. the $ds$ shell) or two spaces (protons and neutrons filling different shells, e.g. for rare-earth and actinide nuclei). Number of interesting and important results for low-energy features like energy spectra, shape description and electromagnetic transition strengths, have been published over the years\cite{3, 4, 5}.

Up to now, SU(3)-based methodologies have not been applied to nuclei with mass numbers $A = 56$ to $A = 100$, which is an intermediate region where conventional wisdom suggests the break down of the assumptions that underpin their use in the other domains. In particular, the $g_{9/2}$ intruder level that penetrates down from the shell above due to the strong spin-orbit splitting appears to be as spectroscopically relevant to the overall dynamics as the normal-parity $f_{5/2}, p_{3/2}, p_{1/2}$ levels. Specifically, in this region the effect of the intruder level cannot be ignored or mimicked through a “renormalization” of the normal-parity dynamics which is how it has been handled to date. Moreover, both protons and neutrons occupy the same oscillator shells which suggests strong proton-neutron correlations. These facts should be addressed with the appropriate choice of a Hamiltonian.

In order to build a complete shell-model theory, opposite-parity levels need to be included in the model space especially if experimentally observed high-spin or opposite-parity states are to
be described. After we shortly introduce the basics of the extended SU(3) shell model which, for the first time, explicitly includes particles from the complete unique-parity sector, we present some results in the case of a simpler Hamiltonian consisting of only the pairing plus quadrupole-quadrupole terms.

2. Extended (pseudo-)SU(3) model

It is a microscopic theory in the sense that both SU(3) generators - the angular momentum and the quadrupole operators - are given in terms of individual nucleon coordinate and momentum variables. Results for the quality of the pseudo-SU(3) symmetry for the nuclei $^{64}$Ge, $^{68}$Se [6] with realistic interactions suggest that one can perform symmetry-adapted calculations thus reducing significantly the size of the model space.

The many-particle basis states

\[ |(a_\pi; a_\nu)\rho(\lambda, \mu)\kappa J, (S_\pi, S_\nu)S; JM\rangle \]  

(1)

are built as SU(3) proton (π) and neutron (ν) strongly-coupled configurations with well-defined particle number and good total angular momentum $J$. Here, the proton and neutron quantum numbers are indicated by $a_\pi = \{a_{sN}, a_{uU}\} \rho_\pi(\lambda_\pi, \mu_\pi)$, and the $a_\nu = N_\sigma N_\tau f_\sigma(f_\sigma)\alpha_\sigma(\lambda_\sigma, \mu_\sigma)$ are the basis-state labels for the four spaces in the model ($\sigma$ stands for $\pi$ or $\nu$, and $\tau$ stands for normal (N) or unique (U) parity levels). In the last expression, $N_{\sigma\tau}$ denotes the number of particles in the corresponding space, $f_\sigma$ - the spatial symmetry label and $\{\lambda_\sigma, \mu_\sigma\}$ - the SU(3) irrep label. Multiplicity indices $\alpha_{\sigma\tau}$ and $\rho_\pi$ count different occurrences of $(\lambda_{\sigma\tau}, \mu_{\sigma\tau})$ in $f_{\sigma\tau}$ and in the product $\{\lambda_{sN}, \mu_{sN}\} \times \{\lambda_{uU}, \mu_{uU}\} \rightarrow \{\lambda_{\pi}, \mu_{\pi}\}$, respectively. First, the particles from the normal and the unique spaces are coupled for both protons and neutrons. Then, the resulting proton and neutron irreps are coupled to a total final set of irreps. The total angular momentum $J$ results from the coupling of the total orbital angular momentum $L$ with the total spin $S$. The $\rho$ and $\kappa$ are, respectively, the multiplicity indices for the different occurrences of $(\lambda, \mu)$ in $(\{\lambda_\pi, \mu_\pi\} \times (\lambda_\nu, \mu_\nu))$ and $L$ in $(\lambda, \mu)$. The basis states (1) used in the model calculations are by construction directly linked to the shell-model Lie algebra $U(4\Omega)$, which contains the SU(3) quantum numbers $(\lambda_{\pi}, \mu_{\pi})$ for the proton and neutron systems.

The most general Hamiltonian that has been used until now for SU(3) model calculations is of the form

\[ H = H_{s.p.} - \frac{\chi}{2} Q.Q - G \sum_{\sigma, \tau} (S^+)^\sigma(\lambda^-)^\sigma + aJ^2 + bK_3^2. \]  

(2)

It includes single-particle terms $H_{s.p.}$ as well as the quadrupole-quadrupole $Q.Q$ and the pairing interaction within a shell, represented by the standard pair-creation $(S^+)^\sigma = \frac{1}{2} \sum_{l,l'j,l''j} (-)^{l'+j-m_j}(a_{l'l''j}^\dagger a_{l''j}^\dagger)^{\sigma\tau}(a_{l'l''j} a_{l''j})^{\sigma\tau}$ and annihilation $(S^-)^\sigma = ((S^+)^\sigma)^\dagger$ operators. The two rotor-like terms $J^2$ and $K_3^2$ (the square of the total angular momentum and its projection on the intrinsic body-fixed axis) are used to “fine tune” the energy spectra, adjusting the moment of inertia of the ground band and the position of the gamma $K = 2^+$ bandhead, respectively. Other interactions that have been used are the Casimir operators of SU(3) of second and third order $C_2$ and $C_3$. Because of the specifics of the systems studied in the past, proton-neutron terms in the interaction have not been needed. The single-particle terms together with the pairing interactions mix the SU(3) basis states, which allows for a realistic description of the energy spectra of the nuclei.

3. Pairing-plus-quadrupole model: results in two shells

Pairing-plus-quadrupole model, introduced by Bohr, Mottelson and Pines [7], and Belyaev [8] is an important tool for the description of the nuclear structure for a couple of reasons but mostly
because it uses a simple Hamiltonian which includes both short-range and long-range terms. Although various calculations in this model can be found [9, 10, 11], it has never been applied for full-space calculations in more than one of the low-lying oscillator shells or for restricted number of particles in higher-lying shells. SU(3) realization of the model up to two (proton and neutron) spaces has been developed [12, 13] and the effects of the quadrupole-quadrupole, identical-particle pairing, and even single-particle interactions have been studied.

When proton-neutron pairing terms are included as well, we consider simultaneously isovector ($S = 0, T = 1$) and isoscalar ($S = 1, T = 0$) pairing which have the SO(8) dynamical symmetry. The problem of the classification of the states for the case of the total pairing is solved in [14] for particles in one shell. It is also based on chain of the shell-model Lie algebra $U(4Ω) ⊂ [U(Ω) ⊂ SO(Ω)] ⊗ [SU_{ST}(4) ⊂ SU_{S}(2) ⊂ SU_{T}(2)]$ and its complementarity to the chains of subalgebras typical for the SO(8) model. For example, the chain $SO(8) ⊂ SO_{ST}(6) ⊂ SO_{S}(3) ⊂ SO_{T}(3)$ has been identified as complementary to the latter and used in the classification of the states for a given number of nucleons. For a certain nucleus (good $T_z$ value), these pairing eigenstates can also be expressed as linear combinations of the SU(3) basis states (1). Solution to the scattering of pairs of particles between two orbitals has also been provided in the special case of seniority $ν = 0$ and 1 [15].

We use a Hamiltonian that is more general by considering additionally identical-particle pair-scattering terms ($τ ≠ τ'$), proton-neutron terms ($πν$) and isoscalar terms which is obviously a more sophisticated form of the well-known pairing-plus-quadrupole model. It has the form
\[ H = \frac{\chi}{2} Q.Q - G \sum_{(JT)} \left\{ \sum_{\sigma,\tau} (S^+)_{\sigma\tau} (S^-)_{\sigma\tau} + \sum_{\sigma,\tau \neq \tau'} (S^+)_{\sigma\tau} (S^-)_{\sigma\tau'} + \sum_{\tau,\tau'} (S^+)_{\pi\nu,\tau} (S^-)_{\pi\nu,\tau'} \right\} \] (3)

where, for simplicity, all pairing terms are taken with the same strength. The sum over \((JT)\) describes the isovector \((J = 0, T = 1)\) and the isoscalar \((J = 1, T = 0)\) both at \(L = 0\) identical-particle pairing (first term) and pair-scattering (second term), and proton-neutron pairing and pair-scattering (third term in the braces).

Next, we present the results from calculations performed in two shells for two \(N = Z\) nuclear systems which we, although not entirely accurate, dub \(^{20}\text{Ne}\) and \(^{60}\text{Zn}\). There are no imposed restrictions on the model space and we do not compare with experimental data because the interaction (3) we use could not pretend to be complete and realistic.

In Figures 1 and 2 (3 and 4), we can see the energy spectrum of the isovector- and the total-pairing interaction for the systems \(^{20}\text{Ne}\) (\(^{60}\text{Zn}\)). By definition, the isovector pairing spectrum does not depend on the spin \(S\) and the isoscalar part does not depend on the isospin \(T\) which makes the difference between the Figures 1 and 2 (correspondingly 3 and 4) almost non observable. The states are clustered in groups for the different values of the total seniority quantum number which can be \(\nu = 0, 2, 4\) for \(J = 0\) but only \(\nu = 2, 4\) for the remaining \(J\) values. While the first excited state in the isovector results is due to developing rotational features in isospace, the one in the total pairing results just reflects the pairing gap in the lower-lying (namely \(ds\)) shell of the model space.

Results from calculations with the Hamiltonian (3) of the ground-state energy for the nuclei are presented in Figure 5. The quadrupole parameter \(\chi\) takes different values while the values \(G = 0.2\) MeV and \(G = 0.1\) MeV for the pairing interaction are fixed as the ones for a typical \(ds\)-shell (fp-shell) nucleus. For low values of the quadrupole parameter \(\chi\) the isovector part has prominent contribution which becomes comparable with the effect from the isoscalar part for values of \(\chi\) bigger than 0.04 MeV.

Finally, we calculate the expectation values of the two shape parameters - the deformation parameter \(\beta\) and the triaxiality parameter \(\gamma\), given by the following formulae [16]:

\[ k\beta = \frac{2}{3}(C_2 + 3)^{1/2} \] (4)

\[ \cos(3\gamma) = \frac{C_3}{2(C_2 + 3)^{3/2}} \] (5)
where \( k = \sqrt{\frac{5}{9\pi}} A < r^2 > \) with \( A \) being the total number of nucleons, \( < r^2 > \) - the mean square radius of the system, and \( C_2 \) and \( C_3 \) are the invariant Casimir operators of second and third order, respectively. The results shown in Figures 6 and 7 for the ground states of the two nuclear systems, reveal a steady rise (fall) in \( \beta \) (\( \gamma \)) for both nuclei with the increase of the parameter \( \chi \). Beyond the point marked with an arrow in Figure 7, the values of \( \chi \) lead to eigenfunctions, composed primarily (at least 50 percent) of the leading irreducible representation.

In conclusion, the study presented here for pairing-plus-quadrupole Hamiltonian in two oscillator shells should serve as a base for more elaborated investigations of these and other interesting nuclear systems. More sophisticated interactions should be developed and tested in order to reveal the role of the particles from the unique-parity sector as well as the effectiveness of various truncations on the model space.

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