Perturbations in tachyon dark energy and their effect on matter clustering

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Abstract. A non-canonical scalar tachyon field is a viable candidate for dark energy and has been found to be in good agreement with observational data. Background data alone cannot completely rule out degeneracy between this model and others. To further constrain the parameters, apart from the distance measurements, we study perturbations in tachyon scalar field and how they affect matter clustering. We consider two tachyon potentials for this study, an inverse square potential and an exponential potential. We study the evolution of the gravitational potential, matter density contrast and dark energy density contrast, and compare them with the evolution in the ΛCDM model. Although perturbations in dark energy at sub-Hubble scales are negligible in comparison with matter perturbations, they cannot be ignored at Hubble and super-Hubble scales ($\lambda_p > 1000$ Mpc). We also study the evolution of growth function and growth rate of matter, and find that the growth rate is significantly suppressed in dark energy dominated era with respect to the growth rate for ΛCDM model. A comparison of these models with Redshift Space Distortion growth rate data is presented by way of calculating $f\sigma_8(z)$. There is a tension of $2.9\sigma$ ($2.26\sigma$) between growth rate data and Planck-2015 (Planck-2018) Cosmic Microwave Background Radiation data for ΛCDM model. We present constraints on free parameters of these models and show that perturbations in tachyon scalar field reduce this tension between different data sets.

Keywords: dark energy theory, cosmological parameters from LSS

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1 Introduction

Cosmological observations, which include observation of Supernova Type Ia [1–4], Baryon Acoustic Oscillations [5–8], Cosmic Microwave Background [9, 10], etc., indicate a late-time acceleration of the Universe. This acceleration can be explained by considering the energy density of the Universe to be dominated by a negative pressure medium [10]. One of the main goals of modern cosmology is to explain, whether the equation of state parameter \( w \) is constant or a dynamical quantity. There is a large number of models which are able to describe the acceleration. The most intuitive is the cosmological constant model (ΛCDM model) [11, 12], with the equation of state parameter \( w = -1 \), in which a constant \( \Lambda \) representing vacuum energy density, is understood to be the reason of the late-time acceleration. Although this model shows good agreement with the observations [1, 8, 10], it suffers from theoretical problems like the fine-tuning problem and the coincidence problem [12–15]. On the other hand observations do not rule out \( w \neq -1 \) and in general the equation of state parameter can be a function of the scale factor.

Dynamical dark energy models are an alternative to ΛCDM model and can have an evolving equation of state parameter. These models include the barotropic fluid models, canonical and non-canonical scalar field models, etc. A varying, fluid dark energy equation of state parameter is considered to be a function of redshift or scale factor. There are two parameters, the present day value of the equation of state parameter, \( w_0 \), and the value of its derivative, \( w'_0 \). Detailed studies of the background evolution and constraints on the parameters for these models have been done in [16–23]. Quintessence scalar field is also a potential candidate for dark energy. Using a slow rolling potential, the late-time accelerated expansion can be achieved. The background cosmology in the presence of the canonical scalar field has been studied in [24–31]. In [32], it was shown that a homogeneous quintessence field with inhomogeneous matter is inconsistent with observation. Therefore the scalar field must be perturbed in the course of evolution of the Universe. The perturbations in the...
quintessence field, its dynamics, and its effect on the evolution of matter clustering have been studied in [32–35].

A potential alternative to the canonical scalar field and the fluid model is a non-canonical scalar field model known as the tachyon model. Tachyon scalar field arises as a decay mode of D-branes in string theory [36–38]. The background cosmology for this model has been studied in [39–41] and it is potentially a good candidate for dark energy. Tachyon scalar field has also been used to explain inflation [42–49]. Since its equation of state becomes dust like in the course of time, it is also considered a viable candidate for dark matter [37, 38, 50–54]. The tachyon model is in good agreement with current observations [55]; data puts tight constraints on cosmological parameters and reduces the fine-tuning problem. It can not however completely distinguish this model from the $\Lambda CDM$ and other models. Perturbation in dark energy can potentially break the degeneracy between models, for instance via the Integrated Sachs-Wolf Effect (ISW effect) as it affects the low $l$ CMB angular power spectrum [56, 57].

In this paper, we analyze the dynamics and nature of tachyon perturbations and their effect on the evolution of matter perturbations. We begin with a homogeneous tachyon scalar field and allow it to get perturbed, as the matter clustering grows with time. In this analysis, we consider two tachyon potentials, an inverse square potential and an exponential potential, and solve linearized Einstein’s equations. The clustering of dark energy is a scale dependent phenomena, it is higher at larger scales, just opposite to the matter clustering which is higher at shorter scales. Dark energy perturbations are insignificant with respect to matter clustering at sub-Hubble scales, and dark energy can be considered homogeneous. At Hubble and super-Hubble scales, dark energy perturbations are significant when compared with the matter perturbation. However, as the present value of the equation of state $w_{\phi 0} \rightarrow -1$, it can be considered homogeneous and this model coincides with the $\Lambda CDM$ model.

We also study the linear growth rate $f(z)$ of matter clustering for these models and compare our theoretical computation with the redshift space distortion (RSD) data. We find that initially, in matter dominated era, growth rate is higher for tachyon model than it is for $\Lambda CDM$ model, but in dark energy dominated era the situation is opposite. This makes tachyon model a better alternate to fit growth rate data. We use the ‘Gold-2017’ RSD data compiled and tabulated in [58] with some additional data from [59]. The growth rate measurements from RSD provide the value of $f \sigma_8(z)$, where $\sigma_8(z)$ is the root mean square fluctuation in the matter power spectrum in a sphere of radius $8 \, h^{-1}$Mpc. In [58], it has been shown that there is a tension of $> 3\sigma$ between ‘Gold-2017’ and Planck-2015 data for $\Lambda CDM$ model. We find that this tension still exists between the RSD data we use and Planck-2018 data for $\Lambda CDM$ model. We show that, for tachyon models, this tension is reduced when equation of state parameter $w_{\phi 0}$ is larger than $-1$ and dark energy is allowed to get perturbed.

In section 2 we present the equations for background tachyon model and introduce two potentials. Perturbations in the tachyon scalar field and the matter part are introduced in section 3. We have discussed our numerical approach in section 4, and the results of our analysis have been shown in section 5. Finally, we summarize our results in section 6.

2 Homogeneous tachyon background

The background evolution of a spatially flat homogeneous and isotropic Universe is described by the metric

$$ds^2 = -dt^2 + a^2(t)[dx^2 + dy^2 + dz^2]. \quad (2.1)$$
Here, \( a(t) \) is the scale factor of expansion. For a system of pressureless matter and tachyon scalar field, the dynamics of background is completely governed by Friedmann equations
\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho, \quad \ddot{a} = -\frac{4\pi G}{3} (\rho + 3P),
\]
where \( \rho = \rho_m + \rho_\phi \) is the total energy density of the Universe. The relativistic component of energy density \( \rho_r \propto a^{-4} \) is negligible and hence we do not include it. The energy density of the matter component is given by \( \rho_m \propto a^{-3} \). The tachyon scalar field is described by a Lagrangian
\[
L_\phi = -V(\phi)\sqrt{1 + g_{\mu\nu}\partial_\mu\phi\partial_\nu\phi},
\]
where \( V(\phi) \) is an arbitrary potential. For the tachyon field, the energy density and pressure are given by
\[
\rho_\phi = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \quad p_\phi = -\frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}.
\]
The equation of state parameter for the tachyon scalar field can then be written as
\[
w_\phi = \frac{p_\phi}{\rho_\phi} = \dot{\phi}^2 - 1,
\]
and the dynamics of the tachyon scalar field are governed by equation
\[
\ddot{\phi} = -(1 - \dot{\phi}^2) \left[ 3H\dot{\phi} + \frac{1}{V(\phi)} \frac{dV}{d\phi} \right].
\]

We work with two different scalar field potentials, one is the inverse square potential
\[
V(\phi) = \frac{n}{4\pi G} \left( 1 - \frac{2}{3n} \right)^{1/2} \phi^{-2},
\]
here \( n \) is a real number defines the amplitude of this potential. The exponential potential given by
\[
V(\phi) = V_a \exp \left( -\phi/\phi_a \right),
\]
where amplitude \( V_a \) and \( \phi_a \) are parameters. The background cosmology have been studied with these tachyon potentials in [39, 40, 55] and these are found to be suitable candidates to generate late time acceleration. The study of cosmological dynamics and the stability analysis have been done in [41, 60, 61] for these potentials.

3 Perturbation in tachyon scalar field

We consider the perturbed FLRW metric to study the perturbations in the matter and the scalar field. If there are no anisotropic components, in the spatial part of energy-momentum tensor, i.e. \( T^i_j = 0 \) if \( i \neq j \), then the perturbations can be described by a line element in longitudinal gauge of the form
\[
ds^2 = -(1 + 2\Phi)dt^2 + a^2(t)(1 - 2\Phi)[dx^2 + dy^2 + dz^2],
\]
where \( \Phi \) is the scalar perturbation. In the Newtonian limit, the metric perturbation \( \Phi \) represents the effective gravitational potential. The dynamical equation for this scalar perturbation \( \Phi \) can be derived by solving perturbed Einstein’s equation \( \delta G^\mu_\nu = 8\pi G \delta T^\mu_\nu \). Here,
the perturbed energy-momentum tensor $\delta T^\mu_\nu$ consists of two parts, one for the matter component $\delta T^\mu_\nu_{\text{(matter)}}$ and other for the scalar field $\delta T^\mu_\nu_{\phi}$. We consider matter as a perfect fluid with energy-momentum tensor

$$T^\mu_\nu_{\text{(matter)}} = (\rho + p)u^\mu u_\nu + pg^\mu_\nu.$$  \hfill (3.2)

Here $\rho$, $p$, and $u^\mu$ are energy density, pressure, and four velocity respectively. The perturbations in the matter field are defined by

$$\rho(t, \vec{x}) = \bar{\rho}(t) + \delta\rho(t, \vec{x}),$$

$$p(t, \vec{x}) = \bar{p}(t) + \delta p(t, \vec{x}),$$

$$u^\mu = \bar{u}^\mu + \delta u^\mu,$$ \hfill (3.3)

where $\bar{u}^\mu = \{1, 0, 0, 0\}$, $\bar{\rho}(t)$ and $\bar{p}(t)$ are the average values of their respective quantities and $\delta u^\mu$ is the peculiar velocity. Substituting these values in equation (3.2), the components of the perturbed energy-momentum tensor of matter are

$$\delta T^0_0 = -\delta\rho,$$

$$\delta T^i_0 = (\bar{\rho} + \bar{p})\delta u^i,$$ \hfill (3.4)

$$\delta T^i_j = \delta p\delta^i_j.$$

The energy-momentum tensor for the tachyon field can be derived from

$$T^\mu_\nu_{\phi} = \frac{V(\phi)\partial^\mu \phi \partial_\nu \phi}{\sqrt{1 + g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi}} + L^\mu_\nu_{\phi},$$ \hfill (3.5)

where for tachyon scalar field the Lagrangian $L_{\phi}$ is given by equation (2.3). We define the perturbation in the scalar field as

$$\phi(t, \vec{x}) = \bar{\phi}(t) + \delta\phi(t, \vec{x}).$$ \hfill (3.6)

Here $\bar{\phi}(t)$ is the average background field. Using equation (3.5) with the metric element of longitudinal gauge from equation (3.1), components of perturbed energy-momentum tensor for tachyon scalar field can be calculated:

$$\delta T^0_0 = -\delta\rho_{\phi} = -\left(\frac{\partial V}{\partial \phi}\right)_{\phi} \delta\phi + \frac{1}{2} \frac{V}\sqrt{1 - \dot{\phi}^2} \left(\frac{2\Phi_{\phi}^2 - 2\dot{\phi}\ddot{\phi}}{1 - \dot{\phi}^2}\right),$$

$$\delta T^i_j = \delta p_{\phi}\delta^i_j = -V(\phi)\sqrt{1 - \dot{\phi}^2} \left(\frac{\Phi_{\phi}^2 - \delta\phi\ddot{\phi}}{1 - \dot{\phi}^2}\right) \delta^i_j - \left(\frac{\partial V}{\partial \phi}\right)_{\phi} \delta\phi \sqrt{1 - \dot{\phi}^2} \delta^i_j,$$ \hfill (3.7)

$$\delta T^i_0 = (\rho_{\phi} + p_{\phi})\delta u^i.$$\hfill (3.8)

We can now solve perturbed Einstein’s equation $\delta G^\mu_\nu = 8\pi G\delta T^\mu_\nu$; where the perturbed energy-momentum tensor are given by equations (3.2) and (3.5). Components of the perturbed Einstein tensor $\delta G^\mu_\nu$ can be calculated using line element (3.1). We retain the terms in the solution of perturbed Einstein’s equations up to first (or linear) order in all perturbed
quantities. We then transform these linearized Einstein equations into the Fourier space or the \(k\)-space, where the perturbed quantities of both the spaces are related by the equation

\[
A(\vec{x}, t) = \int d^3k A(\vec{k}, t)e^{i\vec{k} \cdot \vec{x}}.
\]

Here, \(\vec{k}\) is the wave vector.

In longitudinal gauge, the Fourier transformed Einstein’s equations are given by

\[
\frac{3}{a^2} \ddot{\Phi} + \frac{6}{a} \dot{\Phi} + \frac{k^2}{a^2} \Phi = -4\pi G [\delta \rho_m + \delta \rho_\phi],
\]

\[
\dot{\Phi} + \frac{\dot{a}}{a} \Phi + \left(2 \frac{\ddot{a}}{a} + \frac{a''}{a^2}\right) \Phi = 4\pi G \left[-V(\bar{\phi}) \sqrt{1 - \frac{\dot{\phi}^2}{\phi^2}} \left(\Phi \dot{\phi}^2 - \dot{\phi} \dot{\phi} \Phi\right) \right] \delta \phi \sqrt{1 - \frac{\dot{\phi}^2}{\phi^2}},
\]

\[
\frac{\dot{\Phi}}{a} + \frac{\dot{a}}{a} \Phi = 4\pi G \left(\bar{\rho} a^{-3} v_m + \frac{V(\bar{\phi})}{\sqrt{1 - \frac{\dot{\phi}^2}{\phi^2}}} \delta \phi \right),
\]

where \(v_m\) represents the potential for the matter peculiar velocity, i.e., \(\delta u_i = \nabla_i v_m\). Here although we have used the same symbol for quantities \(\Phi, \delta \phi, \delta \rho_m\) and \(v_m\), as they are in real physical space, they represent the Fourier components of respective quantities in \(k\)-mode of perturbation. The wave number is given by \(k = 2\pi / \lambda_p\), where \(\lambda_p\) is the comoving length of the perturbation. Therefore, the Einstein’s equations given above represent the evolution of the \(k\)-mode of perturbations. Equation (3.10) is the dynamical equation for metric perturbation \(\Phi\). Since matter is pressureless, the dynamics of metric perturbation \(\Phi\) is driven only by perturbation in the scalar field. Here, in these equations, there are two unknown perturbed quantities, \(\Phi\) and \(\delta \phi\). Once these two are determined, then other perturbed quantities like \(\delta \rho_m\) and \(v_m\) can be calculated from equation (3.9) and (3.11). The dynamical equation for the perturbed tachyon scalar field \(\delta \phi\) can be derived by solving the Euler-Lagrangian equation using the Lagrangian function (2.3) for the perturbed scalar field, and in the Fourier space for \(k\)-mode, it is given by

\[
\frac{\delta \phi}{(1 - \frac{\dot{\phi}^2}{\phi^2})} + \left[ 3H + \frac{2\bar{\phi}}{(1 - \frac{\dot{\phi}^2}{\phi^2})^2} \right] \delta \phi + \left[ 3H \frac{\dot{\phi} V'}{V} + \frac{k^2}{a^2} + \frac{\dot{\phi}}{(1 - \frac{\dot{\phi}^2}{\phi^2})} \left(\frac{V'}{V} + \frac{V''}{V}\right) \right] \delta \phi
\]

\[
- \left[ 12H \frac{\dot{\bar{\phi}}}{(1 - \frac{\dot{\phi}^2}{\phi^2})} + \frac{2(2 + \frac{\dot{\phi}^2}{\phi^2})}{(1 - \frac{\dot{\phi}^2}{\phi^2})^2} \right] + \frac{2(\dot{\phi} V')}{V} + \frac{3\dot{\phi}^3 - 4\ddot{\bar{\phi}}}{(1 - \frac{\dot{\phi}^2}{\phi^2})} \Phi = 0,
\]

where the prime represents the derivative with respect to the background scalar field \(\bar{\phi}\). The coupled equations (3.10) and (3.12) form a closed system of equations. Solving these equations together with the background equations, we can find the quantities \(\Phi\) and \(\delta \phi\) and then the respective fractional density contrasts \(\delta = \delta \rho / \bar{\rho}\) of \(k\)-mode for matter and tachyon scalar field can be computed from the following equations

\[
\delta \phi = \frac{V'(\bar{\phi})}{V(\bar{\phi})} \delta \phi - \left(\frac{\Phi \dot{\phi}^2 - \dot{\phi} \dot{\phi} \Phi}{1 - \frac{\dot{\phi}^2}{\phi^2}}\right),
\]

\[
\delta_m = -\frac{1}{4\pi G \rho_m a^{-3}} \left[ \frac{3}{a^2} \Phi + \frac{3}{a} \dot{\Phi} + \frac{k^2}{a^2} \right] - \frac{1}{\rho_m a^{-3}} \left[ \frac{V(\bar{\phi})}{\sqrt{1 - \frac{\dot{\phi}^2}{\phi^2}}} - \frac{V(\bar{\phi})}{\sqrt{1 - \frac{\dot{\phi}^2}{\phi^2}}} \left(\frac{\Phi \dot{\phi}^2 - \dot{\phi} \dot{\phi} \Phi}{1 - \frac{\dot{\phi}^2}{\phi^2}}\right) \right]
\]

\[
= -\frac{1}{4\pi G \rho_m a^{-3}} \left[ \frac{3}{a^2} \Phi + \dot{\Phi} + \frac{k^2}{a^2} \right] - \frac{\delta \phi}{\rho_m a^{-3}} \frac{V(\bar{\phi})}{\sqrt{1 - \frac{\dot{\phi}^2}{\phi^2}}},
\]

(3.13)
To calculate matter density contrast $\delta_m = \delta \rho_m / \rho_m$ we have used equation (3.9). We can see from the above equations that the density contrasts of matter and dark energy are coupled with each other.

The growth of structure, quantified by the linear growth function $D_+^m$, defined as

$$D_+^m = \frac{\delta_m}{\delta_{m0}},$$

(3.14)

The quantity $\delta_{m0}$ is the present value of matter density contrast, and the growth rate, defined as

$$f = \frac{d \ln \delta}{d \ln a}.$$  

(3.15)

4 Numerical approach and methodology

To solve for $a$, $\phi$, $\Phi$ and $\delta \phi$, we need four equations. We choose two background equations, first of the Friedmann equations (2.2) and the dynamical equation of scalar field (2.6). The third equation is the dynamical equation of the perturbed scalar field, equation (3.12) and the fourth one is the dynamical equation for the metric perturbation, the second equation of Einstein’s equations (3.10). We rewrite these equations in the dimensionless form by introducing the following variables

$$x = tH_0, \quad y = \frac{a}{a_{in}}, \quad \psi = \frac{\phi}{\phi_{in}}, \quad \Phi_N = \frac{\Phi}{\Phi_{in}}, \quad \delta \psi = \frac{\delta \phi}{\Phi_{in}}.$$

(4.1)

to above equations to solve them. Derivatives are defined with respect to $x$ as

$$y' = \frac{dy}{dx}, \quad \psi' = \frac{d\psi}{dx}, \quad \Phi'_N = \frac{d\Phi_N}{dx}.$$  

(4.2)

4.1 Dimensionless equations for inverse square potential

In terms of the above dimensionless variables (4.1), the background equations (2.2) and (2.6) with inverse square potential (2.7), take the form

$$y' = \left[ \Omega_{m_{in}}y^3 + \frac{2n(1 - \frac{2}{3n})^{1/2}\psi^2}{\phi_{in}^2H_0^2}\sqrt{1 - \phi_{in}^2H_0^2\psi^2} \right]^{1/2},$$

(4.3)

$$\psi'' = (1 - \phi_{in}^2H_0^2\psi^2) \left[ \frac{2}{\phi_{in}^2H_0^2} - \frac{3y'}{y} \right],$$

(4.4)

where $\Omega_{m_{in}}$ can be linked to the present matter density parameter $\Omega_{m0}$ using the relation

$$\Omega_m = \frac{\Omega_{m0}}{(H/H_0)^2} \left( \frac{a}{a_0} \right)^{-3}.$$  

(4.5)

Here, $a_0$ is the present day value of the scale factor. To solve the above background equations, we need values of the parameters $\Omega_{m_{in}}$, $C_n$, and $\phi_{in}H_0$. Here $C_n = \frac{2n}{3}(1 - \frac{2}{3n})^{1/2}$ is the amplitude of the potential.
Using the variables defined in equation (4.1), with inverse square potential (2.7), the dynamical equation for metric perturbation $\Phi$, equation (3.10), and the dynamical equation of perturbed scalar field $\delta \phi$, equation (3.12) takes the form

$$\Phi'' + 4 \frac{y'}{y} \Phi' + \left\{ 2 \frac{y''}{y} + \left( \frac{y'}{y} \right)^2 \right\} \Phi_N = n \left( 1 - \frac{2}{3n} \right)^{1/2} \left[ \frac{2 \delta \psi}{\phi^2_{\text{in}} H_0^2 \psi^3} \sqrt{1 - \phi^2_{\text{in}} H_0^2 \psi^2} \right] \left[ \frac{1 - \phi^2_{\text{in}} H_0^2 \psi^2}{\psi^2} \right], \quad (4.6)$$

$$\frac{\delta \psi''}{(1 - \phi^2_{\text{in}} H_0^2 \psi^2)} + \left[ \frac{3 y'}{y} + \frac{2 \phi^2_{\text{in}} H_0^2 \psi' \psi''}{(1 - \phi^2_{\text{in}} H_0^2 \psi^2)^2} \right] \delta \psi' + \left[ -\frac{6 y'}{y} \frac{\psi'}{\psi} + \frac{k^2}{a^2_0 H_0^2 y^2} - \frac{2 \psi''}{\psi (1 - \phi^2_{\text{in}} H_0^2 \psi^2)} + \frac{6}{\phi^2_{\text{in}} H_0^2 \psi^2} \right] \delta \psi - \left[ 12 \frac{y'}{y} \frac{\psi'}{\psi} + \frac{2 (2 + \phi^2_{\text{in}} H_0^2 \psi^2)}{(1 - \phi^2_{\text{in}} H_0^2 \psi^2)^2} \psi'' - \frac{4}{\phi^2_{\text{in}} H_0^2 \psi^2} (1 - \phi^2_{\text{in}} H_0^2 \psi^2)^2 \right] \Phi_N + \left[ 3 \phi^2_{\text{in}} H_0^2 \psi^3 - 4 \psi' \right] \left[ \Phi_N / (1 - \phi^2_{\text{in}} H_0^2 \psi^2) \right] \Phi_N' = 0. \quad (4.7)$$

On solving the perturbation equations along with the background using the above initial conditions, we can find the values of $\Phi_N$ and $\delta \psi$ as a functions of redshift or scale factor. Subsequently, the values of density parameters can be calculated using equations

$$\frac{\delta \phi}{\phi_{\text{in}}} = -2 \frac{\delta \psi}{\psi} - \phi^2_{\text{in}} H_0^2 \left( \frac{\psi'^2 \Phi_N - \psi' \delta \psi}{1 - \phi^2_{\text{in}} H_0^2 \psi^2} \right),$$

$$\frac{\delta m}{\phi_{\text{in}}} = -2 \Omega_{m_{\text{in}}} y^{-3} \left[ \frac{y'^2}{y^2} \Phi_N + \frac{y'}{y} \Phi_N + \frac{k^2 / H_0^2}{2 a^2_0 y^2} \Phi_N \right] - \frac{\delta \phi / \phi_{\text{in}}}{\phi_{\text{in}}^2 H_0^2} \left( \frac{2 n}{3} \right)^{1/2} \psi^{-2} \left( 1 - \frac{2}{3n} \right)^{1/2}. \quad (4.8)$$

To derive the above equations we have substituted dimensionless variables defined in equation (4.1) to equation (3.13).

4.2 Dimensionless equations for exponential potential

In terms of the variables defined in equation (4.1), the background equations for exponential potential (2.8) can be written as

$$y' = y \left[ \Omega_{m_{\text{in}}} y^{-3} + \frac{V_{0c} e^{-\phi_{m_{\text{in}}} H_0}}{\sqrt{1 - \phi^2_{\text{in}} H_0^2 \psi^2}} \right]^{1/2}, \quad (4.9)$$

$$\psi'' = \left( 1 - \phi^2_{\text{in}} H_0^2 \psi^2 \right) \left[ \frac{\phi_{m_{\text{in}}}}{\phi_{\text{in}}} H_0^2 - 3 \frac{y'}{y} \right]. \quad (4.10)$$

To solve these background equations, we need value of parameters $\Omega_{m_{\text{in}}}$, $V_{0c}$, $\phi_{m_{\text{in}}} H_0$ and $\phi_{\text{in}} / \phi_{\text{a}}$. On introducing variables defined in equation (4.1), with exponential potential, equa-
tions (3.10) and (3.12) for perturbed quantities $\Phi$ and $\delta \phi$ are

$$
\Phi''_N + \frac{4\ell(y_\ell)}{y_\ell} \Phi'_N + \left\{ \frac{y''}{y} + \left( \frac{y'}{y} \right)^2 \right\} \Phi_N
$$

$$
= \frac{3}{2} V_a e^{-\frac{\phi_{\text{in}}}{\rho_{\text{cr}}}} \sqrt{1 - \phi_{\text{in}}^2 H_0^2} \left[ \phi_{\text{in}} \delta \psi - \frac{\phi_{\text{in}}^2 H_0^2 (\Phi_N y'' - \psi' \delta \psi')}{1 - \phi_{\text{in}}^2 H_0^2 y''} \right],
$$

(4.11)

$$
\frac{\delta \psi''}{(1 - \phi_{\text{in}}^2 H_0^2 y''^2)} + \left[ \frac{3y'}{y} + \frac{2\phi_{\text{in}}^2 H_0^2 y''}{(1 - \phi_{\text{in}}^2 H_0^2 y'')^2} \right] \delta \psi'
$$

$$
+ \left[ -3 \frac{\phi_{\text{in}} y'}{\phi_a y} \psi' + \frac{K^2}{a^2 H_0^2 y''^2} - \frac{\phi_{\text{in}}^2 H_0^2 y''}{(1 - \phi_{\text{in}}^2 H_0^2 y'')^2} \right] \delta \psi
$$

$$
- \left[ 12 \frac{y'}{y} \psi' + \frac{2(2 + \phi_{\text{in}}^2 H_0^2 y'')}{(1 - \phi_{\text{in}}^2 H_0^2 y'')^2} \psi'' - \frac{2 \phi_{\text{in}}^2 H_0^2 y''}{\phi_{\text{in}}^2 H_0^2} + \frac{2 \phi_{\text{in}}^2 H_0^4 y''^4 y''}{(1 - \phi_{\text{in}}^2 H_0^2 y'')^2} \right] \Phi_N
$$

$$
+ \left[ \frac{3 \phi_{\text{in}}^2 H_0^2 y''^3 - 4 \psi'}{(1 - \phi_{\text{in}}^2 H_0^2 y'')^2} \right] \Phi'_N
$$

$$
= 0.
$$

(4.12)

In terms of the dimensionless variables, defined in equation (4.1), the equation for density parameters (3.13) for exponential potential takes the form

$$
\frac{\delta \phi}{\phi_{\text{in}}} = -\frac{\phi_{\text{in}}}{\phi_a} \delta \psi - \phi_{\text{in}}^2 H_0^2 \left( \frac{\psi' \Phi_N - \psi' \delta \psi'}{1 - \phi_{\text{in}}^2 H_0^2 y''} \right),
$$

$$
\frac{\delta m}{\Phi_{\text{in}}} = -\frac{2}{\Omega_{m\text{in}} y^{-3}} \left[ \frac{y'}{y} \Phi_N + \frac{k^2}{2 a_{\text{in}}^2 y^2} \Phi_N \right] - \left[ \frac{\delta \phi}{\phi_{\text{in}}} \frac{\phi_{\text{in}}}{\phi_a} \frac{e^{-\frac{m}{\rho_{\text{cr}}}}}{\phi_{\text{in}} H_0} \right].
$$

(4.13)

5 Results and discussion

We evolve the perturbation equations from redshift $z = 1000$ to the present day. The main assumption we have made is that the dark energy field is initially homogeneous. Equation (3.13) suggests that for this assumption to be valid we need not only to consider $\delta \phi_{\text{in}} = 0$, but also $\dot{\phi}_{\text{in}} = 0$ or equivalently an initial equation of state parameter of dark energy $w_{\phi_{\text{in}}} = -1$. Therefore the analysis, along with constraints on the free parameters we are providing, are subject to this assumption. For background equations, our initial conditions are

$$
y_{\text{in}} = 1, \quad \psi_{\text{in}} = 1,
$$

(5.1)

and $\psi_{\text{in}}'$ can be calculated using relation

$$
\psi' = \frac{\dot{\phi}}{\phi_{\text{in}} H_0} = \sqrt{1 + \frac{w_{\phi_{\text{in}}}}{\phi_{\text{in}} H_0}}.
$$

(5.2)

In [55], it has been shown that with the potentials mentioned in section 2, the constraint on matter density contrast is $\Omega_{m0} = 0.285^{+0.023}_{-0.022}$ at 3σ confidence. On the other hand,
Figure 1. The plots on the left of this figure show evolution of the equation of state of dark energy and the plots on the right show the evolution of the density parameters. The top panels are for the inverse square potential and the panels at the bottom show these quantities for the exponential potential. Red, sky-blue, green and blue colours represent $\phi_0 H_0 = 1.0, 1.5, 2.0$ and $3.0$. In the second column, the solid line is for $\Omega_m$ and the dashed line is for $\Omega_\phi$. Parameters $C_n$ and $V_a/\rho_{cr}$ are tuned for each value of $\phi_0 H_0$ to get $\Omega_{m0} = 0.285$.

Figure 2. In this figure we show the dependence of the present day value of the equation of state parameter, $w_{\phi0}$ (plot on the left), and the deceleration to acceleration transition redshift, $z_{dz}$ (in the right panel) on $\phi_0 H_0$. The red curve is for inverse square potential and blue curve is for the exponential potential. The values of $C_n$ and $V_a/\rho_{cr}$ are the same as in 1.
background data puts only a lower bound $\phi_0 H_0 \gtrsim 0.775$ and all larger values are allowed. Here, $\phi_0$ is the value of the scalar field at present, i.e., $(\dot{\phi})_0$. Constraint on $w_{\phi 0}$ depends on the value of $\phi_0 H_0$, as they are correlated quantities. The tachyon scalar field starts evolution only in the near past, this allow us to assume $\phi_{in} H_0 \approx \phi_0 H_0$ [39]. In this paper, we have done our analysis for the best fit value of $\Omega_m$ and other parameters have been varied. In the case of the exponential potential, differences due to the change in the parameter $\phi_{in}/\phi_a$ can be restored by scaling $\phi_{in} H_0$ appropriately [55]. We have fixed the value of this parameter at $\phi_{in}/\phi_a = 1$.

The evolution of the equation of state of dark energy and the density parameters are shown in figure 1 for both the potentials. Red, sky-blue, green and blue colours represent $\phi_{in} H_0 = 1.0$, 1.5, 2.0 and 3.0. For each value of $\phi_{in} H_0$, we need to tune the amplitude of potential, $C_n = \frac{2n}{3} (1-2/3n)^{1/2}$ for the inverse square potential and $V_a/\rho_{cr}$ for the exponential potential, such that the present value of the matter density parameter matches $\Omega_m = 0.285$. We can see that the equation of state parameter for both the potentials remains at $-1$ in the matter dominated era, and starts evolving as the dark energy begins to dominate. In the right panel of figure 2, we can see that the deceleration to acceleration transition redshift, $z_{da}$, is higher for smaller value of $\phi_{in} H_0$ and gradually decreases as we increase this parameter. Hence for smaller values of $\phi_{in} H_0$, the value of equation of state parameter begin to deviate, or start increasing, from -1 earlier. That is the reason why $w_{\phi 0}$ is larger for these values than it is for the larger value of $\phi_{in} H_0$. For larger $\phi_{in} H_0$, the value of $w_{\phi 0}$ is closer to $-1$. This correlation can be seen in the left panel of figure 2. We can see that for a given value of $\phi_{in} H_0$, $w_{\phi 0}$ relatively closer to $-1$ for the exponential potential than it is for the inverse square potential. The reason for this is that the transition from decelerated to accelerated expansion, for a fixed value of $\phi_{in} H_0$, occurs earlier for the inverse square potential than for the exponential potential. For example, for $\phi_{in} H_0 = 2.0$ the value of the transition redshift $z_{da} = 0.732$ for the inverse square potential and $z_{da} = 0.717$ for the exponential potential. Comparing the panels of figure 2, we can see that there is a linear relation between $w_{\phi 0}$ and $z_{da}$.

The future evolution of $w_{\phi}$ can be seen in figure 1, and it is clear that the $w_{\phi}$ for the inverse square potential becomes constant in future, as for this potential, the equation of state asymptotically approaches $w_{\phi} = 2/3n - 1$ [39–41]. Whereas for the exponential potential, the equation of state increases to $w_{\phi} = 0$ (dust like). For smaller values of $\phi_{in} H_0$, it evolves faster and approaches $w_{\phi} = 0$ relatively earlier than for larger values of $\phi_{in} H_0$. Since in future the dominating component is dark energy, the effective equation of state of the Universe depends only on $w_{\phi}$. For the exponential potential, when $w_{\phi}$ becomes larger than $-1/3$, the Universe once again goes to a decelerating phase. Hence, for the exponential potential, there is no future horizon problem for tachyon model of dark energy [39–41].

The perturbation in the scalar field at initial (at $z = 1000$) is assumed to be negligibly small, compared to $\Phi$ and $\delta_m$. The scalar field can initially be assumed to be homogeneous, and our initial conditions for perturbation are

$$\Phi_{N_{in}} = 1, \quad \delta \phi_{N_{in}} = 0, \quad \delta \phi_{N_{in}}' = 0. \quad (5.3)$$

In [32], it was shown that the gravitational potential does not evolve in the matter dominated era, and starts to decay when dark energy begins to dominate. This fact allows us to assume $\Phi_{N_{in}}'(k) = 0$, for all scales. In figure 3, we show the evolution of the gravitational potential with the scale factor. The gravitational potential is normalized to its initial value; solid lines are for tachyon models and dashed lines are for $\Lambda$CDM model. Different colours represent
Figure 3. Here we show the evolution of gravitational potential with scale factor. The top row is for the inverse square potential and the second row is for the exponential potential. Solid lines correspond to the respective scalar field potential whereas the dashed lines are for \( \Lambda \)CDM model. The plots on the left show for \( \phi_0 H_0 = 1.0 \) and those on the right correspond to \( \phi_0 H_0 = 2.0 \) respectively, for the same values of \( C_n \) and \( V_a / \rho_c \) as in the previous two figures. The red, green, blue, sky-blue, pink, yellow, orange and light-green colours represents the scales of perturbation \( \lambda_p = 50, 100, 500, 1000, 5000, 10000, 20000 \) and 50000 Mpc respectively.

different length scales of the perturbation, \( \lambda_p \), from 50 Mpc to \( 5 \times 10^4 \) Mpc. We solve the set of required equations for each of these fixed scales, introduced using the dimensionless ratio \( \bar{k} = k c / H_0 \), where \( k = 2 \pi / \lambda_p \); with \( H_0 = 70 \) Kms\(^{-1}\)Mpc\(^{-1}\) and \( c = 2.99 \times 10^5 \) Kms\(^{-1}\). The gravitational potential remains a constant during the matter dominated era. As dark energy starts to dominate the energy budget, gravitational potential falls at all length scales. We can see that for \( \Lambda \)CDM model, the gravitational potential falls more rapidly and at the same rate at all scales. For tachyon models, the gravitational potential falls more rapidly at a smaller scales. At super-Hubble scales, its decay slows down in future. In the bottom left panel of figure 3, is can be seen that for the exponential potential, the gravitational potential at super-Hubble scales in future first rises and then become constant. However, as we increase the value of parameter \( \phi_0 H_0 \) (because \( w_{\phi_0} \to -1 \)), this effect of scale dependence decreases, and the difference with respect to the \( \Lambda \)CDM model also decreases. The model with exponential potential is more sensitive to the value of the parameter \( \phi_0 H_0 \), as we can see that increasing this parameter from 1 to 2 decreases the scale dependence effect more significantly.

The evolution of matter density contrast, normalized by the initial value of the gravitational potential is shown in figure 4, for \( \phi_0 H_0 = 1.0 \) and 2.0. Since the gravitational potential remains constant during the matter-dominated era, at sub-Hubble scales the mat-
Figure 4. Evolution of matter density contrast with the scale factor is shown in the above figure. Solid lines correspond to tachyon dark energy model (with inverse square potential in row-1 and with exponential potential in row-2) and the dashed lines are for ΛCDM model. The values of $\phi_{in}H_0 = 1.0$ and 2.0 for column-1 and 2 respectively. The colour scheme for this figure is the same as in 3.

Figure 5. This figure shows the matter density contrast (in the left panel) and the dark energy density contrast (in the right panel) normalized to initial gravitational potential, at present epoch ($z = 0$), as a function of $\phi_{in}H_0$ at the scale of $\lambda_p = 1000$ Mpc. Red and blue colours represent the inverse square and exponential potential respectively.

ter density contrast grows linearly with the scale factor i.e. $\delta_m \propto a$, whereas at Hubble and super-Hubble scale it evolves at a slower rate. In the matter dominated era, there is a very small difference between tachyon model (for both the potentials) and ΛCDM model.
Figure 6. In this figure we show the evolution of linear growth function of matter $D_m^{+} = \frac{\delta_m}{\delta_m^0}$ as a function of the scale factor. Solid lines correspond to tachyon dark energy model with inverse square potential (in row-1) and with exponential potential (in row-2) and the dashed lines are for $\Lambda$CDM model. The colour scheme for scales of perturbation is the same as in the figure 3. Column-1 is for sub-Hubble scales (lines of different scales of particular model have overlapped) and column-2 is for super-Hubble scales.

(dashed lines). In the dark energy dominated era, the evolution of matter density contrast is suppressed. At Hubble and super-Hubble scales, it once again increases (for the inverse square potential) and decreases (for the exponential potential) in future as the gravitational potential seizes to decay. This difference in the behavior of the matter density contrast in future is due to the difference in the evolution of the equation of state parameter and the gravitational potential. Whereas in the $\Lambda$CDM model, the evolution of the matter density contrast remains suppressed in the $\Lambda$ dominated era. The evolution of $\delta_m$ depends on the parameter $\phi_{in} H_0$ (or on $w_{\phi0}$). In the left panel of figure 5, we show the dependence of $\delta_m/\Phi_{in}$ at $z = 0$ at the scale of $\lambda_p = 1000$ Mpc on $\phi_{in} H_0$. For smaller value of $\phi_{in} H_0$ (or larger $w_{\phi0}$), the present day value of $\delta_m(z = 0)$ is small, and as we increase $\phi_{in} H_0$ and $w_{\phi0}$ decreases, the value of $\delta_m(z = 0)$ increases. For larger values of $\phi_{in} H_0$, its value approaches a constant as decrease in $w_{\phi0}$ saturates. For a fixed value of $\phi_{in} H_0$, the value of $\delta_m(z = 0)$ is large for the exponential potential than it for the inverse square potential. For a fixed $\phi_{in} H_0$, the value of $w_{\phi0}$ is smaller for the exponential potential than it is for the inverse square potential. As we increase the value of the parameter $\phi_{in} H_0$ and $w_{\phi0}$ approaches $-1$, the difference between the two potentials decreases.
Figure 7. The evolution of the logarithmic growth rate \( f = \frac{d \ln \delta_m}{d \ln a} \) with redshift is shown here. Solid black, dashed blue and dashed-dot red curves are for ΛCDM model, tachyon model with exponential potential and with inverse square potential respectively. Top panel is for scale of perturbation \( \lambda_p = 50 \) Mpc, whereas bottom left and right panels are for \( \lambda_p = 1000 \) Mpc and 5000 Mpc respectively. For these plots, the value of parameters \( \phi_{in}H_0 = 1.0 \) and \( \Omega_{m0} = 0.285 \).

In figure 6, we show the evolution of linear growth function \( D^+_m = \frac{\delta_m}{\delta_{m0}} \) at sub-Hubble (the plot on the left) and super-Hubble scales (the plot on the right). Here we have taken the value of parameters \( \phi_{in}H_0 = 1.0 \) and \( \Omega_{m0} = 0.285 \). We can see that at sub-Hubble scales, linear growth is scale independent as all lines overlap. At super-Hubble scales, its evolution depends on the scale. In matter dominated era, the linear growth \( D^+_m \) is large for tachyon models than the ΛCDM model at all scales. That is why as dark energy dominates it has to slow down, even more than ΛCDM model to match the present value. This becomes more clear in figure 7, where we show the evolution of growth rate \( f = \frac{d \ln \delta_m}{d \ln a} \) with redshift, at the scale of perturbation \( \lambda_p = 50, 1000 \) and 5000 Mpc, for \( \phi_{in}H_0 = 1.0 \). We can see that the growth rate is higher at shorter scales, and as we increase the scale of perturbation growth rate decreases. We can also see that in matter-dominated era, the growth rate remains a constant for smaller scales (sub-Hubble scales), whereas at Hubble and Supper-Hubble scale it grows linearly and reaches a maximum value. In the dark energy dominated era the growth rate falls at all scales, for all the three models. In the matter-dominated era, the growth rate is larger for tachyon models than the ΛCDM model. As the dark energy starts to dominate, it comes below the ΛCDM model. As we increase the value of \( \phi_{in}H_0 \), the tachyon model approaches the ΛCDM model (because \( w_{\phi_0} \to -1 \)) and this difference decreases.
Figure 8. A comparison of theory with Redshift Space Distortion data. Solid black, dashed blue and dashed-dot red curves are for ΛCDM model, tachyon model with exponential potential and with inverse square potential respectively. From left and right panels represent $\phi_{in}H_0 = 0.8$ and $3.0$ respectively. Other parameters $\Omega_{m0}$ and $\sigma_8(z = 0)$ are fixed to the corresponding best fit values taken from table 2. Data points are taken from table I of [62].

We show the evolution of dark energy perturbations as function of the scale factor in figure 9. The dark energy density contrast is normalized to the initial gravitational potential. The magnitude of the dark energy density contrast is higher at larger scales. This behaviour is opposite to that of the matter density contrast, which is higher in magnitude at smaller scales. As the dark energy dominates and gravitational potential decreases, the growth of the dark energy contrast ceases and becomes constant at super-Hubble scale; this is true for the inverse square potential. For the exponential potential, if the value of parameter $\phi_{in}H_0$ is small, $\delta \phi$ keeps on growing (with smaller rate) in the future. If we increase the value of this parameter, the growth of $\delta \phi$ is suppressed for the exponential potential as well. At Hubble and sub-Hubble scale, the dark energy density contrast reaches its maximum at near present epoch and then decreases in future. For the exponential potential, it first decreases in value and then increases in (far) future.

The evolution of dark energy density contrast can be understood from the equation of $\delta \phi$ in (3.13). At sub-Hubble scales, initially the second of three terms, term $\Phi \dot{\phi}^2$, dominates. Since in matter dominated era the gravitational potential remains a constant, $-\delta \phi/\Phi_{in}$ rises as $\dot{\phi}^2$ or $w_\phi$ increases as a function of the scale factor. In dark energy dominated phase, due to decrease in gravitational potential, $-\delta \phi/\Phi_{in}$ decreases. In future, the fist term (term with scalar field perturbation $\delta \phi$) dominates, and as it rises $-\delta \phi/\Phi_{in}$ rises once again. At super-Hubble scale the $\delta \phi$ rises, but other two terms fall. This results in a net suppression of evolution of $-\delta \phi/\Phi_{in}$. For the exponential potential, with smaller value of $\phi_{in}H_0$, the $\delta \phi$ term dominates in future, and $-\delta \phi/\Phi_{in}$ keeps on rising although with a smaller rate of growth. The density contrast $\delta \phi/\Phi_{in}$ as a function of $\phi_{in}H_0$ is shown in the right panel of figure 5. We can see that for smaller value of this parameter (or larger $w_\phi$), dark energy perturbation is larger. As we increase $\phi_{in}H_0$ and $w_\phi$ approaches $-1$, the factor $\delta \phi/\Phi_{in}$ becomes negligible, and we can consider dark energy as homogeneous. Although, the magnitude of $\delta_m$ is higher than that of $\delta \phi$, we can see in figure 9 that in matter dominated era the slopes of $-\delta \phi/\Phi_{in}$ curves, at all scales, are greater than that of $-\delta_m/\Phi_{in}$ (in figure 4). This implies that in matter dominated era the evolution of the dark energy density contrast is faster than that of the matter density contrast.
Figure 9. Evolution of dark energy density contrasts, normalized by initial value of gravitational potential, with the scale factor. Top row is for the inverse square potential, whereas bottom row is for the exponential potential. We have taken the value of parameter $\phi_{in} H_0 = 1.0$ and 2.0 in Column-1 and 2 respectively. Amplitude of potentials $C_n$ and $V_a/\rho_{cr}$ have tuned to get $\Omega_{m0} = 0.285$ at present. Red, green, blue, sky-blue, pink, yellow, orange and light-green lines represents the scale of perturbation $\lambda_p = 50, 100, 500, 1000, 5000, 10000, 20000$ and 50000 Mpc respectively.

In figure 10, we show the ratio of density contrasts $\delta_\phi/\delta_m$ at present epoch $z = 0$ as a function of $\phi_{in} H_0$ and $w_{\phi0}$. For a fixed scale, if the value of the parameter $\phi_{in} H_0$ is small, say of the order of unity (or the value of $w_{\phi0}$ is away from $-1$), the value of $\delta_\phi/\delta_m$ is larger. As we increase the value of $\phi_{in} H_0$ it decreases monotonically. For example, at $\lambda_p = 1000$ Mpc the value of $(\delta_\phi/\delta_m)_{z=0}$ is $1.172 \times 10^{-4}$ for $\phi_{in} H_0 = 1.0$, and it is $1.069 \times 10^{-5}$ for $\phi_{in} H_0 = 4.0$, for the inverse square potential. Near $w_{\phi0} = -1$ the ratio $\delta_\phi/\delta_m$ decreases sharply. So $\delta_\phi/\delta_m \to 0$ as $w_{\phi0} \to -1$.

In figure 11, we show the variation of $\delta_\phi/\delta_m$ with the scale of perturbation $\lambda_p$. We find that for smaller value of the field, say $\phi_{in} H_0 = 0.8$, at scale of $\lambda_p = 10^5$Mpc, the ratio $(\delta_\phi/\delta_m)_{z=0} = 0.2645$ and 0.1060, for the inverse square and the exponential potential respectively. At these scales, the value of $\delta_m$ is very small, hence the value of $\delta_\phi$ is a considerable fraction of the energy density. This ratio decreases monotonically at smaller scales. For example, at $\lambda_p = 10^5$Mpc the ratio $(\delta_\phi/\delta_m)_{z=0}$ is in the range $10^{-6}$ to $10^{-8}$. While the dark energy density contrast is negligible at smaller scales (sub-Hubble scales), it is significant at Hubble and super-Hubble scales.
Figure 10. In the first row, the plots show the dependence of ratio \( \frac{\delta\phi}{\delta m} \) at \( z=0 \) on \( \phi_{\text{in}}H_0 \) and \( w_{\phi 0} \) for the inverse square potential. The second row is for the exponential potential. Lines from bottom to top represent the scale of perturbation \( \lambda_p = 500, 10^3, 5 \times 10^3, 10^4, 5 \times 10^4 \) and \( 10^5 \) Mpc. Amplitude of potentials \( C_n \) and \( V_a/\rho_{\text{crit}} \) are fixed to get the present day \( \Omega_{m0} = 0.285 \).

Figure 11. In this figure, we have plotted the ratio \( \frac{\delta\phi}{\delta m} \) at \( z=0 \) with respect to the scale for the inverse square potential (in the left panel) and for the exponential potential (in the right panel). For curves from up to down the parameter \( \phi_{\text{in}}H_0 = 0.8, 1.0, 1.5, 2.0, 3.0, 4.0, 5.0, 6.0 \) and 7.0.
Figure 12. Evolution of the equation of state parameter $w_\phi$ for $V(\phi) \propto \phi^{-2}$ (left panel) and for $V(\phi) \propto \exp(-\phi/\phi_a)$ (right panel). Red and blue colours represent initial equation of state parameter of dark energy $w_{\phi_{in}}$ to be $-10^{-5}$ and $-0.5$ respectively. The black curves are for initially homogeneous dark energy with $w_{\phi_{in}} = -1$. We have fixed the value of parameter $\phi_{in} H_0 = 1.0$ and $\Omega_{m0} = 0.285$ for these plots.

5.1 Effect of inhomogeneities in dark energy at early Universe

We also study the effect of deviation of initial equation of state parameter from $-1$ at an early epoch. For this, we vary the value of $w_{\phi_{in}}$ at $z = 1000$ from $-1$ assuming perturbation in scalar field $\delta \phi$ and its derivative $\dot{\delta} \phi$ to be negligibly small. In figure 12, we show the evolution of the equation of state parameter in this scenario for both the potentials. We can see that even if $w_{\phi_{in}}$ deviates from $-1$, the equation of state parameter $w_\phi$ sharply approaches $-1$ with the Hubble expansion of the Universe. We find that only for the cases where $w_{\phi_{in}} \approx 0$ (a fluid like equation of state), $w_\phi$ survives deep into the matter dominated era. There is no effect of the parameter $w_{\phi_{in}}$ on the evolution of $w_\phi$ in later epoch. Equation 3.13 suggests that the deviation of $w_\phi$ from $-1$ (hence $\dot{\phi} \neq 0$) introduces contrast in dark energy through the gravitational potential. Larger the value of $w_{\phi_{in}}$, larger is the dark energy contrast $\delta \phi$ in early epoch. We show results for sub-Hubble scale in figure 13. In the top panels of this figure, we can see that the early perturbations in dark energy go through damped oscillations as the equation of state parameter approaches $-1$. The dark energy contrast $\delta \phi$ decreases in amplitude until it approach the evolution track of $w_{\phi_{in}} = -1$ case. After that, $\delta \phi$ for all values of $w_{\phi_{in}}$ follow the same track. We can see, in row-2 and 3 of the same figure, that at sub-Hubble scales there is no effect of deviation of $w_{\phi_{in}}$ or early dark energy perturbations on matter density contrast $\delta_m$ or linear growth function $D_{m+}^m = \delta_m/\delta_{m0}$ for both the potentials. The reason for this behavior can be understood from the Equation 3.13. In matter dominated era, the ratio of dark energy density to matter density ($\rho_\phi/\rho_m$) is very small. Therefore, at early epoch it does not affect $\delta_m$. At the present epoch $\delta_\phi$ itself very small for all $w_{\phi_{in}}$ in comparison to $\delta_m$ at sub-Hubble scales. Even if we vary $\delta \phi_{in}$, it does not affect the evolution of linear growth function $D_{m+}^m$ at sub-Hubble scales. The effect of perturbation in dark energy (and deviation of $w_{\phi_{in}}$ from $-1$) is considerable only at the Hubble and super-Hubble scales, where the ratio $\delta_\phi/\delta_m$ become significant.
Figure 13. Evolution of $\delta \phi / \Phi_{\text{in}}, \delta m / \Phi_{\text{in}}$ and $\delta_m / \delta_{m0}$ at scale of 50 Mpc. Column-1 if for $V(\phi) \propto \phi^{-2}$ and column-2 is for $V(\phi) \propto \exp(-\phi/\phi_a)$. Colour scheme and the values of parameters $\phi_{\text{in}}H_0$ and $\Omega_{m0}$ are same as in figure 12.
Figure 14. Marginalized Constraints and the likelihood for the parameters $\Omega_{m0}$, $\phi_{in} H_0$ and $\sigma_8(0)$ of the tachyon model with inverse square potential. The 2D-contours filled with blue, green and red colours show 68%, 95% and 99% confidence region respectively. We do not include results for the parameter $w_{\phi_{in}}$ because it is not constrained by the data we use (see sections 5.1 and 5.2 for details).

| $\phi_{in} H_0$ | $\Omega_{m0}$ | $\sigma_8(0)$ | $w_{\phi_{in}}$ |
|-----------------|---------------|---------------|-----------------|
| $[0.001, 10.0]$ | $[0.01, 0.9]$ | $[0.1, 3.0]$  | $[-10^{-10}, -1]$ |

Table 1. Priors ranges for the parameters $\Omega_{m0}$, $\phi_{in} H_0$, $\sigma_8(0)$ and $w_{\phi_{in}}$

5.2 Constraints on the parameters

Observations do not provide a direct measurement of $\delta_m$. Instead, the observational data on the growth of structure measures the product $f\sigma_R(z)$, where,

$$\sigma_R^2(z) = \frac{1}{2\pi^2} \int_0^\infty P(k, z) W_R^2(k) k^2 dk \propto \delta^2(z),$$

is the root mean square fluctuation in linear density field or power spectrum $P(k, z)$ within a sphere of radius $R$ [63]. Taking $R = 8 \, h^{-1}\text{Mpc}$, it can be written as,

$$\sigma_R(z) = \sigma_8(0) \frac{\delta_m(z)}{\delta_m(0)}.$$


Here, $\sigma_8(0)$ is the present value of $\sigma_8(z)$ and it is a parameter. In figure 8, we show the comparison between data and theory. The data points are values of $f\sigma_8(z)$ extracted from redshift space distortion (RSD) measurements. In our analysis we have used 22 data points from redshift 0.02 to 1.944, out of which 18 points are compiled in table III of [58] with their fiducial cosmology and references. This compilation is named as ‘Gold-2017’ data set. We have added four more data points at redshift 0.978, 1.23, 1.526 and 1.944 from [59] for our analysis. All these 22 data points, with the value of $f\sigma_8(z)$, error, fiducial cosmology and corresponding references, are tabulated in table I of [62]. In figure 8, solid black, dashed blue and dashed-dot red curves are for $\Lambda$CDM model, tachyon model with exponential potential and with inverse square potential respectively. Left and right panels are for $\phi_mH_0 = 0.8$ and 3.0 respectively. We set the parameters $\Omega_m0$ and $\sigma_8(0)$ to their corresponding best fit values given in table 2. We can see that the tachyon models (with both the potentials) are in good agreement with the data. There is significant difference between tachyon models and the $\Lambda$CDM model if the parameter $\phi_mH_0$ is small (about order of unity) or large $w_{\phi0}$ (because these two parameters are correlated). As we increase $\phi_mH_0$ and $w_{\phi0}$ approaches $-1$, tachyon models then coincide with the $\Lambda$CDM model.

We now constrain the free parameters of the tachyon field model using Redshift Space Distortion (RSD) data from [58, 62]. For this purpose we find out the maximum likelihood by minimizing $\chi^2$ given by

$$\chi^2 = \sum_{i,j=1}^{N} [X_{th,i} - X_{obs,i}] C_{i,j}^{-1} [X_{th,j} - X_{obs,j}],$$

where $N$ is number of data points and $C_{i,j}$ is the covariance matrix. The quantities $X_{th}$ and $X_{obs}$ are the vectors of theoretical and observed values of the observable $f\sigma_8$ respectively. As suggested in [58], to remove the fiducial cosmology, we scale the theoretical value of $f\sigma_8$ by the ratio

$$r(z) = \frac{H(z)d_A(z)}{H_{\text{fid}}(z)d_A^{\text{fid}}(z)},$$

where $H(z)$ and $d_A(z)$ are the Hubble parameter and the angular diameter distance at redshift $z$ respectively. The observable $X_{th,i} = r(z_i)f\sigma_8(z_i, p)$, where $p$ is the set of parameters. We constrain the parameters $\Omega_m0$, $\phi_mH_0$ and $\sigma_8(0)$. The prior used for these parameters are shown in table 1. Since, the parameter $w_{\phi m}$ does not affect the evolution of $D^+_m = \delta_m/\delta_m0$ at sub-Hubble scale, we do not see any change in the theoretical value of $f\sigma_8$ by varying this parameter. The RSD data set, we have used, does not constrain $w_{\phi m}$. We have checked it by varying $w_{\phi m}$ in the prior range $[-10^{-10}, -1]$ for this parameter. Therefore, we need not include this parameter in our analysis. For the exponential potential, we have fixed $\phi_m/\phi_a = 1.0$, since changes due to variation in this parameter can be compensated by scaling $\phi_mH_0$ appropriately [55].

In figure 14 and 15, we show the marginalized contours of 68%, 95% and 99% confidence region for the tachyon model with inverse square potential and the exponential potential respectively. We also show the one dimensional likelihood for each parameter. We find that the constraints on the parameter $\phi_mH_0 > 0.081$ at 99% confidence level for model with exponential potential have no upper bound on it. This can also be seen in the likelihood function of the parameter $\phi_mH_0$ which becomes constant for larger values. We have checked it for arbitrarily large values of this parameter. For tachyon model with inverse square potential $\phi_mH_0 \gtrsim 0.001$. Since only the square of the parameter $\phi_mH_0$ appears in the equations, we
Figure 15. Marginalized Constraints and the likelihood for the parameters $\Omega_{m0}$, $\phi_{in}H_0$ and $\sigma_8(0)$ of the tachyon model with exponential potential. The 2D-contours filled with blue, green and red colours show 68%, 95% and 99% confidence region respectively. We do not include results for the parameter $w_{\phi_{in}}$ because it is not constrained by the data we use (see sections 5.1 and 5.2 for details). We also show constraints in the $\Omega_{m0}-\sigma_8(0)$ plane and find them to be consistent with the observations. In table 2, we show the best fit values of parameters along with their 68%, 95% and 99% confidence range for tachyon model with both the potentials, as well as for $\Lambda$CDM model. In figure 16, we compare the constraints on $\Omega_{m0}-\sigma_8(0)$ plane for tachyon models with constraint for the $\Lambda$CDM model. Here, the black dot and triangle show the best fit values for Planck-2015 [64] and Planck-2018 [65] respectively. The constraints on $(\Omega_{m0}, \sigma_8)$ for $\Lambda$CDM model are $(0.3156 \pm 0.0091, 0.831 \pm 0.013)$ from Planck-2015 (TT,TE,EE+lowP) at 68% confidence [64] and $(0.3166 \pm 0.0084, 0.8120 \pm 0.0073)$ from Planck-2018 (TT,TE,EE+lowE)
Figure 16. Marginalized constraints on $\Omega_m - \sigma_8(0)$ plane. Blue, green and red colors represent the 68%, 95% and 99% confidence region respectively. Top plot is for ΛCDM model whereas bottom left and right plots are for the tachyon model with inverse square and exponential potentials respectively. The black dot and triangle show the best fit values for Planck-2015 [64] and Planck-2018 [65] respectively.

at 68% confidence [65]. We find that Planck-2015 and Planck-2018 best fit points are at 2.9σ and 2.26σ levels respectively for ΛCDM model. Similar result has also been found between ‘Gold-2017’ growth rate data and Planck-2015 data for ΛCDM model, see [58] for more details. This tension is reduced in the tachyon models. The best fit values of Planck-2015 and Planck-2018 are at 1.93σ and 1.66σ levels respectively for the tachyon model with inverse square potential. For the tachyon model with exponential potential these points are at 2.45σ and 1.86σ levels respectively. Therefore, we can see that inclusion of perturbation in dark energy with $w_{\phi 0} \neq -1$ reduces the tension between RSD data and Planck data.

To compare the models, we calculate Bayesian evidence for all the three models. The Bayesian evidence or model likelihood is defined as [66–68]

$$E \equiv p(d|M) = \int_{\Omega_M} p(d|\theta, M) I(\theta|M) d\theta,$$

(5.8)
where $\theta$ is a vector of parameters of the model $M$. The quantities $p(d|\theta, M)$ and $I(\theta|M)$ are the likelihood function and normalized prior for the parameters respectively. Clearly, the evidence is the average value of the likelihood over entire parameter space. Two models $M_0$ and $M_1$ can be compared using the ratio of posterior probabilities or posterior odds, given by [67, 68]

$$
\frac{p(M_0|d)}{p(M_1|d)} = \frac{p(d|M_0)}{p(d|M_1)} I(M_0) I(M_1). 
$$

Here, the ratio of evidences of the models $B_{01} = p(d|M_0)/p(d|M_1)$ are known as the ‘Bayes factor’. The Bayes factor indicate the change in relative odds between the models after data. If $B_{01} > (\leq)1$ then the model $M_0$ is more (less) favorable than the model $M_1$ by the given data. The Jeffreys’ scale provides an empirically calibrated scale for strength of evidence to compare the two models [69]. A notable property of the evidence is that it does not penalize the parameter which is unconstrained by the data [66], e.g. in our case the initial value of the equation of state $w_\text{in}$. There are other popular and simpler way to compare different models, namely Akaike Information criterion (AIC) and Bayesian Information criterion (BIC) [66–68]. These methods require only the maximum likelihood to compare models [66, 67]. These criterion are derived using various assumptions, e.g. Gaussianity of the posterior distribution. These assumptions are not valid for the tachyon models, as posteriors (particularly for $\phi_\text{in} H_0$) are not Gaussian. Therefore, we do not use AIC or BIC for comparison and rely on evidence calculation and Bayes factor. We find that $B_{01} = 0.996$ and $B_{02} = 1.019$, where ‘0’ stands for $\Lambda$CDM model, ‘1’ for tachyon models with inverse square potential and ‘2’ for tachyon models with exponential potential. For this calculation we take uniform or flat prior for all $\phi_\text{in} H_0$. To constrain the parameters we use 22 RSD data points compiled and tabulated in [58, 62].

Table 2. The table lists the best fit values of $\Omega_m$ and $\sigma_8(0)$ along with their 68%, 95% and 99% confidence ranges for the $\Lambda$CDM model as well as tachyon model with both the potentials. In column-2 we show the lower bound on the parameter $\phi_\text{in} H_0$. To constrain the parameters we use 22 RSD data points compiled and tabulated in [58, 62].

| Model                | $\chi_{\text{min}}$ | $\phi_\text{in} H_0$ | $\Omega_m$ | $\sigma_8(0)$ |
|----------------------|----------------------|------------------------|------------|--------------|
| $\Lambda$CDM         | 12.260               | —                      | 0.235+0.125+0.209+0.306 | 0.836+0.191+0.286+0.401  |
| Tachyon with $V(\phi) \propto \exp(-\phi/\phi_0)$ | 12.252               | > 0.081                | 0.234+0.125+0.209+0.306 | 0.843+0.191+0.286+0.401  |
| Tachyon with $V(\phi) \propto \phi^{-2}$   | 12.255               | $\gtrsim$ 0.001        | 0.231+0.126+0.210+0.307 | 0.853+0.191+0.286+0.401  |

6 Summary and conclusions

In this paper, we have studied perturbations in tachyon scalar field dark energy and their effect on matter clustering. We consider two tachyon scalar field potentials, the inverse square potential and the exponential potential. We begin with a homogeneous dark energy with equation of state $w_\phi = -1$ and evolve our equations with time. The matter and dark energy perturbations are coupled with each other and if the equation of state of dark energy $w_\phi \neq -1$ then dark energy is not distributed homogeneously. Distribution of inhomogeneity
in tachyon dark energy, like in other scalar field models, is a scale dependent phenomenon. The dark energy density contrast \( \delta_\phi \) is higher in magnitude at larger scales then it is at shorter scale, opposite to the matter density contrast \( \delta_m \) which is higher at shorter scales. In matter-dominated era at sub-Hubble scales, \( \delta_m \propto a(t) \) for tachyon models as well as for the \( \Lambda CDM \) model. In dark energy dominated era, its evolution is suppressed. Future evolution of matter density contrast is significantly different in all three models. At super-Hubble scales, \( \delta_m \) rises again for the inverse square potential, and falls for the exponential potential, whereas for the \( \Lambda CDM \) model it remains a constant. In the matter dominated era, dark energy density contrast \( \delta_\phi \) evolves monotonically at same rate at all scales with \( a(t) \). Although the magnitude of \( \delta_\phi \) is much smaller than that of \( \delta_m \) in matter dominated era, its growth rate is higher. We also study the effect of parameters, \( \phi_m H_0 \) and \( w_{\phi 0} \), on the evolution of \( \delta_m \) and \( \delta_\phi \). These two parameters are correlated and as we increase the value of \( \phi_m H_0 \) \( w_{\phi 0} \rightarrow -1 \)(a \( \Lambda CDM \) value).

We have also studied the evolution linear growth function \( D_m^+ = \delta_m/\delta_{m0} \) and the growth rate \( f = d \ln \delta_m/\ln a \). Evolution of \( D_m^+ \), at sub-Hubble scales is scale independent, whereas it depends on scale for larger scales. This is true for for all the three models. At higher redshift (in matter dominated era), the growth rate \( f \) for tachyon models is higher than the \( \Lambda CDM \) model, and as evolution approaches dark energy dominated era, growth rate falls, even below the value for \( \Lambda CDM \) model. To show the agreement between theory and observation, we calculated \( f \sigma_8(z) \) for the three models and compared it with RSD data. We find that the tachyon models are in good agreement with the data. If the value of parameter \( \phi_m H_0 \) is small (or \( w_{\phi 0} \) is large), the tachyon models show significant difference from the \( \Lambda CDM \) model. As \( w_{\phi 0} \rightarrow -1 \), for larger \( \phi_m H_0 \), tachyon models coincide with the \( \Lambda CDM \) model. The tachyon dark energy density contrast, \( \delta_\phi < 10^{-4} \delta_m \) at scales \( \lambda_p < 10^3 \) Mpc with both the potentials. Therefore at these sub-Hubble scales, dark energy inhomogeneities can be neglected. If the dark energy equation of state \( w_{\phi 0} \neq -1 \), then at Hubble and super-Hubble scales, \( \delta_\phi \) become significant. For example at the scale of \( \lambda_p = 10^5 \) Mpc, for \( \phi_m H_0 = 0.8 \) the ratio \( (\delta_\phi/\delta_m)_{z=0} = 0.2645 \) and 0.1060 for the inverse square and the exponential potential respectively. Since at these scales \( \delta_m \) itself very small, \( \delta_\phi \) contributes significantly.

We constrain the free parameters of the \( \Lambda CDM \) model as well as tachyon model with both the potentials using Redshift Space Distortion data. For the tachyon model, we constrain \( \Omega_{m0}, \phi_m H_0 \) and \( \sigma_8(0) \). We find that there is a lower bound on \( \phi_m H_0 \) and all larger values are allowed by the RSD data. This feature has also been seen in analysis with the background data [55]. The smaller value of \( \phi_m H_0 \) implies a larger value of \( w_{\phi 0} \) and a larger \( (\delta_\phi/\delta_m)_{z=0} \). We therefore conclude that growth-rate data allows for perturbations in dark energy. In the \( \Omega_{m0} - \sigma_8(0) \) plane, we find that there is a tension of \( 2.9\sigma \) (2.26\sigma) between the redshift space distortion data and Planck-2015 (Planck-2018) best fit value for \( \Lambda CDM \) model. A similar result has also been reported in [58]. This tension is reduced slightly, when \( w_{\phi 0} \neq -1 \) and perturbations in dark energy are considered, for the tachyon models. This is true for both the potentials. We compare tachyon models with \( \Lambda CDM \) model by calculating the ratio of the Bayesian evidences or the Bayes factor \( B_{01} \). We find that the tachyon models are as good as the \( \Lambda CDM \) model to satisfy the RSD data we use.

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