Generation of Shock-Wave Disturbances at Plasma-Vapor Bubble Oscillation

N S Kuznetsova, A S Yudin and N V Voitenko
National Research Tomsk Polytechnic University, Institute of High Technology Physics, 30 Lenin avenue, Tomsk, 634050, Russian Federation
E-mail: tevn@hvd.tpu.ru

Abstract. The complex physical and mathematical model describing all steps of plasma-vapor bubble evolution in the system of the water–ground condensed media is presented. Discharge circuit operation, discharge plasma channel expansion, its transformation into the vapor-plasma bubble and its pulsation, pressure wave generation and propagation of the mechanical stress waves in the ground are self-consistently considered in the model. The model allows investigation of the basic laws of stored energy transformation into the discharge plasma channel, next to the plasma-vapor bubble and transformation of this energy to the energy of pressure wave compressing the surrounding ground. Power characteristics of wave disturbances generated by gas-vapor bubble oscillation in liquid depending on the circuit parameters are analyzed for the prediction of the ground boundary displacement. The dynamics of the shock-wave propagation in water–ground condensed media depending on the rate of the plasma channel energy release is investigated. Simulation of the shock-wave phenomena at a plasma-vapor bubble oscillation in condensed media consecutively describes the physical processes underlying technology for producing piles by electro-discharge stuffing. The quantitative model verified by physical experimental tests will allow optimization of pulse generator parameters and electrode system construction of high-voltage equipment.

1. Introduction
Shock-wave disturbances generated by the plasma-vapor bubble expansion and oscillation in different condensed media at electro-blast are widely used in engineering and technology due to ability to manage their properties over a wide range by varying both the parameters of RLC-circuit and the geometrics of electro-discharge gap [1–3]. Short time period (in the range of 10–100 µs) and the high rate of electrical energy release into the discharge channel in liquid cause the high plasma pressure in it (up to $3 \times 10^9$ Pa) and the formation of intensive compression waves in the surrounding liquid. The resulting power in the discharge channel leads to the mechanical work both above the liquid, and in the movement and deformation of the surrounding medium (ground) [7–8]. The correlation of contributions of shock waves at the discharge channel expansion and at the plasma-vapor bubble the expansion and oscillation are not defined at present. The lack of experimental information about the general laws determining the amplitude and profile of the pressure wave generated by plasma channel expansion does not allow estimation of the generated pressure in the condensed media on the basis of measured electrical characteristics of the explosion (current and voltage). Some attempts have been made by the authors of monographs [2, 3, 9], but formulations of their practical tasks did not assume the recording of the shock wave profile.
The experimental studies of the fast processes in the water-ground media are difficult and complicated technically. Investigation of the mechanisms of formation, evolution and dissipation of mechanical perturbation pulses in this case is only possible with the use of mathematical modeling. A comprehensive physical and mathematical model describing the plasma-vapor bubble expansion, generation of the shock-wave disturbances, the dynamics of elastoplastic waves, the formation of fields of mechanical stresses and deformations in solid medium is necessary for studying the basic laws of electrical energy conversion into the mechanical work of plasma-vapor bubble expansion and oscillation.

2. The Physical and Mathematical Model of Electro-Blast Action on the System of Water-Ground Condensed Media

The development of shock-wave disturbances in liquid at electro-blast can be divided into three stages. The first – the plasma channel expansion due to high pressure in it leading to the shock waves generation with a sound and supersonic speeds, pulse movements of liquid volumes with the speeds reaching the hundreds of meters per second [6]. The large resistance to the expansion from the liquid surrounding the channel leads to the sharp pressure rise in the channel, whereby the phenomenon takes on the character of the explosion. The second – the discharge channel degeneration into the plasma-vapor bubble filled with the liquid degradation products, vapors under the different thermodynamic conditions and plasma. The diameter of the spherical (or near-shaped) cavity depends on the released energy and the external hydrostatic pressure in the liquid. The third - the subsequent pulsation of a plasma-vapor bubble with generation of the secondary pressure waves. The plasma-vapor bubble expansion extends to the maximum radius at which all the kinetic energy is transferred to the water. The increased potential energy of the water again transferred to the bubble, causing its compression to a minimum sizes. This cycle is repeated until the bubble energy decreases due to the emission of acoustic shock waves, as well as due to condensation and viscous losses in the medium. Thus, due to the bubble formation and oscillation in liquid there is the hydrodynamic pulse pressure on the wellbore walls, as estimated in [7], it is greater than 50 MPa at a distance of 10 cm from the source of pulsations and decreases to zero at a distance of 0.6–0.8 m.

The current pulse generator operation at its discharge on the gap located in a hole filled with water is analyzed. Formation of plasma channel occurs, its expansion generates a pressure wave in the liquid at electro-blast. The propagating wave is refracted into the ground, is reflected in the liquid, returns to the channel and interacts with it. Due to the energy release in the discharge gap the discharge channel degenerates into a plasma-vapor bubble which under the influence of the changing pressure begins to pulsate. The plasma-vapor bubble oscillation leads to the generation of secondary pressure waves propagating in the liquid and soil. As a result, in the ground-water media a complex, fast-changing wave pattern is formed, which results in the displacement and deformation of the borehole wall (figure 1).

The model is a consistent solution of the transient process equations for the pulse generator's circuit (figure 1), the energy balance equation for the discharge channel, the hydrodynamic equations for description of wave disturbances and state equations for plasma channel and surrounding medium [10].

Electrodynamic oscillating processes in the discharge circuit were characterized by the voltage balance equation:

\[ L \frac{di(t)}{dt} + (R_{ch} + r) i(t) + \frac{1}{C} \int_0^t idt = U_0, \]

where \( L \) is the circuit inductance, \( \mu H \); \( R_{ch} \) is channel resistance, \( \Omega \); \( i(t) \) is the circuit current, \( A \); \( U_0 \) is the charge voltage of capacitor bank, \( V \); \( r \) – the active resistance of the generator, \( \Omega \).

Electrical plasma channel energy conversion into the plasma energy and mechanical work, performed by expanding channel and pulsating plasma-vapor bubble:

\[ \frac{1}{\gamma - 1} \frac{d}{dt} (P_{ch} V_{ch}) + P_{ch} \frac{dv_{ch}}{dt} = i(t)^2 R_{ch}, \]

where \( \gamma \) is the specific heat ratio.
where $\gamma$ is the effective ratio of specific heats; $P_{ch}$ is the channel pressure, Pa; $V_{ch} = \pi r_{ch}^2 l_{ch}$ – channel volume, m$^3$; $r_{ch}$ and $l_{ch}$ are the channel radius and length accordingly, m.

The time dependences of the current and voltage have been calculated according to Kirchhoff’s laws along with the empirical equations for plasma channel resistance. The plasma channel resistance $R_{ch}(t)$ is determined through the integral of current action in the form of Vaičel–Rompe [11]:

$$R_{ch}(t) = A_{mid} \cdot l_{ch} \left( \int_0^t i^2(t) dt \right)^{-\frac{1}{2}},$$

(6)

where $A_{mid}$ is a spark constant (in case of water $A_{mid}=120$ V·s$^{0.5}$·m$^{-1}$ [12]).

Wave propagation is described in the hydrodynamic approximation. The description of the channel expansion and plasma–vapor bubble oscillation in a liquid is based on a system of equations including the differential equations of laws of mass conservation, momentum, and energy in the adiabatic approximation, the equations of media and plasma state, the correlation of the stress tensor, spherical tensor and deformation deviator, equations of wave dynamics [13]. Kirchhoff’s equations and power balance equation of the discharge channel were jointly solved with the state equations of water and soil, the equations of continuity, internal energy, rations for the components of the stress deviator. The system of equations describing the evolution and dissipation of shock-wave disturbances was solved self-consistently and approximated by the difference scheme according to Wilkins method [14]. On the basis of the submitted equations the numerical algorithm and software were developed that allows to carry out the electro-blast computer experiments in the water–ground condensed media.

3. Simulation Results of Shock-Wave Dynamics at Plasma-Vapor Bubble Oscillation

The results of computer investigations of electro-blast in the water–ground media by means of the presented model under different conditions of energy release into the plasma channel are presented below. Figure 2 shows the main electrical parameters and the plasma–vapor bubble dynamics at the different modes of electrical energy release into the channel: I – $U_0=10$ kV, $C=1200$ µF, $r_z=114$ mΩ, $L=11.12$ µH; II – $U_0=20$ kV, $C=168$ µF, $r_z=20$ mΩ, $L=1.06$ µH; the discharge gap length $l_{ch}=50$ mm.

To simulate the well depth of about 7 meters the initial channel pressure was assumed to be 50 MPa. Figure 2 shows that during the electro-blast in the water at the expansion stage the velocity of plasma–vapor bubble expansion reaches the values of about 800 m/s and 1100 m/s respectively for I and II modes of energy release into the plasma channel, and the pressure are in the range of 1.5 GPa to 3.4 GPa, which causes a rapid growth of the bubble parameters. During the active phase of the discharge the pressure is sharply reduced up to 50 MPa, the velocity of plasma–vapor bubble expansion reaches extremum during the first third of the current oscillation period, and then reduced to zero.
Figure 2. Time dependence of the discharge current (a), the plasma–vapor bubble pressure (b), the plasma–vapor bubble radius (c) and its expansion speed (d) in water.

The plasma–vapor bubble radius increases up to 200–300 microseconds, after that the bubble is compressed with the same speed and pressure as in the expansion stage. When the plasma–vapor bubble pressure becomes higher than the pressure in the surrounding liquid, the repeated growth of its radius occurs (figure 2, c). Pulsation period and the maximum bubble radius increases with the rise of power of energy release into the plasma channel. The maximum plasma-vapor bubble radius reaches 20 mm at the power of the channel energy release of about 400 kW (II mode). These values are in a good agreement with the experimental results [15], which confirms the adequacy of the model.

Due to the plasma–vapor formation and oscillation in liquid the pressure of about 57 MPa and 80 MPa effects on the borehole wall respectively for I and II modes of electrical energy release, figure 3, a.

Figure 3. Pressure versus time waveforms, produced on the borehole walls of diameter of 400 mm for different modes of electrical energy release.

Figure 4. Dynamics of channel and wave energy for different modes of electrical energy release.
The secondary pressure waves were caused by the processes of wave reflection from the walls of gas-vapor bubble and borehole. The displacement of the borehole wall was in the first case – in order of 1 mm, in the second – 1.8 mm. Comparison of the results of numerical modeling with the experimental data [16] (the dependence on the bore-hole radius from the discharge conditions) is showed a high degree of agreement. The energy of shock-wave disturbances is thus less than 4 % from the storage electrical energy at I mode of energy release, and about 16 % in the II mode (figure 4).

The plasma–vapor bubble expansion and its subsequent pulsation result to the formation of mechanical compression stresses in the ground. The dynamics of mechanical stresses under the different modes of energy release is shown in figure 5.

![Figure 5](image-url)

**Figure 5.** Distribution diagrams of mechanical stress wave profiles in the ground at plasma-vapor bubble pulsation in liquid: a) – I mode of energy release, b) – II mode of energy release.

The profiles of mechanical stress waves for two modes of the channel energy release are different as for the amplitude parameters and for the propagation velocity. In the first case (figure 5, a) the wave velocity is about 385 m/s, in the second (figure 5, b) – 462 m/s.

4. Conclusion
The obtained results of the gas-vapor bubble dynamics have been testified to the determining role of the influence of the electrical energy release mode into the plasma channel on the generated pressure wave in the water–ground system. It is shown that the presented complex physical and mathematical model of the plasma-vapor bubble dynamics in the condensed media allows to estimate the radius and the expansion rate of gas-vapor bubble, its pressure, the pressure wave profile on the borehole wall, the wave pattern evolution in the ground. The model is applicable in a wide range of parameters and gives a good qualitative and quantitative agreement with the experimental results.

Acknowledgements
This work was supported by the Federal Target Program, project No. 14.575.21.0059 (RFMEFI57514X0059).

References
[1] Bluhm H, Frey W, Giese H, Hoppe P, Schultheis C and Sträbner R 2000 IEEE Trans. On Dielectrics and Electr. Insul. 7 625–636
[2] Naugolnyh K and Roy N 1971 Electric discharges in water (Moscow: Nauka) p 238
[3] Krivitskij E Dynamics of electro explosion in a liquid (Kiev: Naukova Dumka) p 208
[4] Okun I 1971 J. Tech. Phys. 41, 2 292–300
[5] Vinogradov B and Fedin D 2009 Mathematical modeling 1 23-26
[6] Yutkin L 1986 The electrohydraulic effect and its application in industry (St. Petersburg: Engineering) p 253
[7] Bakholdin B and Dzhantimirov H 1998 *Grounds, foundations and geotechnics* 4 47–52
[8] Bunttsen R 1965 *Electric wire explosion* (Moscow: Mir) p 360
[9] Stolovich N 1983 *Electro-explosion energy converters* (Minsk: Science and Technology), p 151
[10] Burkin V, Kuznetsova N and Lopatin V 2009 *J. Tech. Phys.* 5 42–48
[11] Rompe R and Weizel W 1944 *Zs. Physik B* 122 9–12
[12] Sjomkin B, Usov A and Ziniviev N 2000 *Transient processes in electro-discharge installations* (St. Petersburg: Nauka) p 276
[13] Burkin V, Kuznetsova N and Lopatin V 2009 *J. Phys. D: Appl. Phys.* 43 185204
[14] Wilkins M 1964 *Calculation of elastic-plastic flow, methods in computational physics* (New York: Academic) 211–264
[15] Ivanov V, Shvets I and Ivanov A 1982 *Underwater sparks* (Kiev: Naukova Dumka) p 192
[16] Kurets V, Tarakanovskiy E, Filatov G and Yushkov A 2004 *Electronic material processing* 6 70–74.