Secure Communication Coupled semiconductor Laser Based on Rössler Chaotic Circuits

Raied K. Jamal and Dina A. Kafi

Department of Physics, College of Science, University of Baghdad, Baghdad, Iraq

* E-mail: raiedkamel@yahoo.com

Abstract. Synchronization between two Rössler circuits coupled with semiconductor laser for satisfies secure communication was presented. The two circuits are synchronized via one variable dynamic electronic channel, while and the external signal is transmitted by another variable dynamic through an optical fiber. This system exhibit more fixity and flexibility. Experimentally and numerically results are studied by changing the bidirectional coupling factor. The sinusoidal signal was used in secure communications. The results of numerical simulation are excellent agreement with experiment which demonstrate that the system is viable.

Keywords: Nonlinear dynamics; Chaos; Rössler circuits; Secure communication

Introduction

Recently, the synchronization in chaotic systems has received a great interest among scientists from various fields [1-12]. Sending the information secretly is an important case for all institutions, such as banks and the government. For this reason they require communications system which allows them to send information at high speed in a secure matter. The interaction of two nonlinear (in particular, chaotic) systems via coupling typically leads to a variety of significant behaviors, one of the most intriguing of which is synchronization that is the coordination of a particular. The most common forms are complete synchronization in which the two dynamical units evolve following exactly the same trajectory phase synchronization [2,3] when the coordinated property is the phase, lag synchronization [4]. One of the more important achievements of the chaos subject is the application in secure communications. The secure communication using chaos principle is the synchronization of two chaotic systems under suitable conditions if one of the systems is driven by the other. Since Pecora and Carol used as a basic system to demonstrate various properties of dynamical systems, this system is still a source inspiration for researchers [6]. This system has been widely explored with several tools [6]. Since 1976, the Rössler system is well known for its simplicity and its dynamical richness producing chaos [7].

In this work, two Rössler electronic circuits were used to develop a new method for chaos secure communication that coupled with optical fiber and semiconductor laser as source. Firstly, the dynamic behavior of Rössler system was studied when a parameter of control was changed; secondly the synchronization effect in the coupled circuits was investigated. Finally secure communications with chaos is demonstrated experimentally and numerically using the shift keying scheme.

Experimental setup

The setup that used in this approach consists of two Rössler electronic circuits shown in Fig. (1). \( R_c \) is a coupling factor which changed between 1 and 200 kΩ. The operational amplifier A4 (op-amps) is switched on when the voltage \( x \) override 3V. The op-amps 741 types, appropriate value of resistors and capacitors were used. The transmitter circuit and receiver circuit are identical components. Our method consists of three channels, one to synchronize the Rössler circuit and the second one to transmit the information via semiconductor laser source at 850 nm wavelength with single mode optical fiber, as shown in Fig. (2). The function of photodetector (PD) is to convert the optical signal that coming from transmitter circuit via optical fiber to electrical signal at receiver circuit. When the signal is drive out from transmitter circuit and drive in receiver circuit, so every change in any elements of the first circuit affects the second circuit.
The output voltages $x$, $y$ and $z$ encoded signal, and recovered signal are registered with a digital storage oscilloscope. The synchronization of two coupled chaotic Rössler circuits is studied by connecting the output voltage $y$ of the transmitter circuit to the input voltage $y'$ of the receiver circuit via electronic channel. When changing the coupling factor $R_c$ in the transmitter circuit, so the synchronization between two circuit can be find easily. The time series of the output voltages of transmitter circuit $y$ and receiver circuits $y'$, at coupling factor $R_c$ equal to 200 kΩ is shown in Fig. (3), where it are desynchronized. The trajectory in the phase space $(y, y')$ is shown in Fig. (4). Where shown that the two circuits are not synchronized. When changing the coupling factor $R_c$ in transmitter circuit at 1 Ω, the two Rössler circuits become fully synchronized, as shown in Figs (5) and (6). Full synchronization is obtained when the relation between the peaks amplitude of the transmitter and the receiver is a line at 45°. The deviation from this line means an error in the synchronization between the transmitter and receiver circuits.

**Secure Communication Using Rössler Circuit**

Once the two Rössler are synchronized, the $x$ channel of the Rössler is added to the pump current of the semiconductor laser. Since the oscillation frequency of Rössler system (0.4 kHz) is much smaller than the relaxation frequency of the laser (in over of GHz), the output laser intensity will follow the same behavior of the $x$ output Rössler circuit.

When a small external information signal $M(t)$ (see Fig. 7) from function generator device was added to a chaotic output voltage of the transmitter circuit $x_1$ using shift keying scheme, So the total transmitter signal $x_1(t)+M(t)$ is used to synchronize the identical chaotic system via laser. The sinusoidal signal $M(t)$ has 840 Hz frequency and 1 V amplitude. In order to avoid intermittent synchronization behavior, we can take an advantage that the transmitter circuit has three variables $x_1$, $y_1$ and $z_1$ which can be used one of them for synchronization and another for encoded and recovered signal. The sensitivity of the system to a change in dynamics is different for different dynamics. In Rössler system the $y$ dynamic is sensitive to a small perturbation more than $x$ dynamics; therefore the $y$ dynamic is suitable for synchronization, while the $x$ dynamic is used for signal transmission and encoding. The sinusoidal signal $M(t)$ and the chaotic output $x_1(t)$ is shown in Fig. (8).

**Numerical simulation**

The Rössler electronic circuit is describe by the following dimensionless equations [7]:

\[
\begin{align*}
\dot{x} &= -(y + z) \\
\dot{y} &= x + ay \\
\dot{z} &= b + xz - cz
\end{align*}
\]

Where $a$, $b$, and $c \in R$ and they are dimensionless parameters. Modeling approach of chaos was investigated by programming the physical model as shown in the flow chart 1 in Fig. (9). The total simulation time chosen depends strongly on the magnitude of the temporal scales defined by three parameters $a$, $b$ and $c$, depending on initial conditions. To demonstrate the first property of chaos, i.e., aperiodicity, a numerical simulation was done using Berkeley Madonna [with fourth-order Runge-Kutta method] software of ordinary differential equations. Mathematically, the Rössler circuit can be described by the system of equations (1-3).

In Fig. (9), the first block represents the time scale of chaotic signal that ranged from 0 to 200 and it is changeable in any range of time, where the time interval $Dt$ equal to 0.01. The second block represents the Rössler system parameters values that exhibits chaotic dynamics, where $a$ and $b$ equal to 0.2, and $c$ equal to 6.3. The Rössler system equations are written in the third block. Finally the initial condition values of the system are written in the fourth block, where these values are very important to change chaotic system behavior.

To study the synchronization in Rössler model must take two Rössler systems, as shown in the flow chart 2 in Fig. (10). The two Rössler systems called transmitter and receiver unit. Generally there are two main forms of coupling: unidirectional and bidirectional, where the bidirectional coupling was used in our project. In bidirectional coupling the two systems interact and coupled with each other creating a mutual synchronization to get full synchronized.

The term $k'(y_2-y_1)$ is added to the first equation in $y_1$ scale (transmitter unit), where the second Rössler model (receiver unit) different of the first one in initial conditions in $y$ scale, moreover the term $k'(y_2-y_1)$ called synchronization term and the $k$ called the coupling factor. By the same manner,
the term $k'(y_1-y_2)$ is added to first equation in $y_2$ scale. By changing the coupling factor $k$ value gradually the full synchronization will obtain. To exhibit the sensitive dependence of system on initial condition, the two identical systems are taken with same parameter but starting from different initial conditions, where $x_1$ equal to 1 and $x_2$ equal to 1.1. Figure (11) represents the time series in $y_1$ and $y_2$ dynamics where after some period the two dynamics are divergence from each other. To check the identical chaotic signal between these two systems must plot between amplitude of $y_1$ scale versus amplitude of $y_2$ scale, as shown in Fig. (12). It shown diffusion distribution in ($y_1$, $y_2$) plane with great accumulate point (0,0). The synchronization occurs between the transmittance and receiver units when the coupling factor $k$ equal to 0.9, as shown in Fig. (13). It is shown got full synchronization between two systems. The correlation plot at full synchronization between $x_1$ and $x_2$ is shown in Fig. (14), where show linear relation between them, and with the slope of curve was 1.

To model the semiconductor lasers the following equations was used [13-14]:

$$\frac{dE(t)}{dt} = (1 + ia) \left( \frac{g(N(t) - N_0)}{1 + s|E(t)|^2} - \frac{1}{\tau_p} \right) E(t) - \sqrt{2xN(t)}\xi(t)$$  \hspace{1cm} (4)

$$\frac{dN(t)}{dt} = \frac{1}{e - \tau_n} N(t) - \left( \frac{g(N(t) - N_0)}{1 + s|E(t)|^2} \right)|E(t)|^2$$  \hspace{1cm} (5)

Where $E_{tr}(t)$ is the complex amplitude of the electric field and $|E_{tr}(t)|^2$ is the laser intensity. $N_{tr}(t)$ is the average number of carriers. $\xi(t)$ being a Gaussian noise of zero mean and unity intensity is used to model the spontaneous emission process. Typical values used for the parameters of these two equations are shown in reference [13]. In the our case the message is transmitted as pumping intensity is used to model the spontaneous emission process. Typical values used for the parameters of these two equations are shown in reference [13]. In the our case the message is transmitted as pumping intensity is used to model the spontaneous emission process. Typical values used for the parameters of these two equations are shown in reference [13].

To study the secure communications the observably program apply, after adding sinusoidal signal as external message on transmitter unit in $x_1$ scale for Rössler equation. This sinusoidal signal message is $[M(t) = A \sin (2\pi f t)]$, where $A$ and $f$ are dimensionless unit and represent the amplitude and frequency of the sinusoidal signal message respectively. By this way the chaotic signal can be used as a carrier signal and this technique called shift keying scheme. By changing $A$ and $f$ values which can be used to study the significant of system in carrier signal, where there are limitation in $A$ and $f$ values. Figure (15) represents the sinusoidal signal message that has amplitude $A$ equal to 1 and frequency $f$ equal to 0.26. The transmitted chaotic signal (carrier to message), input, and output sinusoidal signals message is shown in Fig. (16), where the input sinusoidal signal message (encoded message) is represented in violet color and output sinusoidal signal message (recovered message) is represented in black color, moreover the transmittance chaotic signal is represented in red color.

Many things are worth mention regarding the implementation of the chaotic communication system which will also be helpful in our result. One point worth point out is the chaotic signal really broad band, since the basic idea was to hide the narrow band message spectrum within the wide band of chaotic signals, therefore the chaotic signal being used as the carrier should have a wide spectrum.

There are two important conditions must considerateness it, the first one is the power of the sent message should be considerably lower than the power of the transmitted chaotic signal, otherwise once again the message signal will be clearly visible in the spectrum, so the secure communication was failed. The second one is frequency of sent message should be with bandwidth of transmittance chaotic signal. Finally, the results of the numerical simulations are in a good agreement with the experimental results. Depending on parameter $R_c$ the master circuits displays homoclinic orbits, Rossler chaos and periodic orbits.

### Conclusion

This work demonstrates the simple method to synchronize two Rössler electronic circuits with change coupling factor. We propose a new communication scheme based on synchronization of chaotic system that using semiconductor laser source as a carrier of signal. To keep the communication more secure, the peak of power spectrum of the information signal had better to be as indistinguishable from these of the neighboring frequencies of the masking variable as possible, that is almost hidden "in the spectrum of the transmitted signal. Both, the experimental and theoretical results show an excellent agreement."
References
[1] T.L. Carroll and L.M. Perora, "Synchronization in chaotic systems" Phys. Rev. Lett., 64 (8), (1990) 821-824.
[2] M.G. Rosenblum, A.S. Pikovsky and J. Kurths, “Phase synchronization of chaotic oscillators”, Phys. Rev. Lett., 76 (11) (1996) 1804-1807.
[3] E.R. Rosa, E. Ott and M.H. Hess, “Transition to phase synchronization of chaos”, Phys. Rev. Lett., 80(8) (1998) 1642-1645.
[4] M.G. Rosenblum, A.S. Pikovsky and J. Kurths, “From phase to lag synchronization in coupled chaotic oscillators”, Phys. Rev. Lett., 78(22) (1997) 4193-4196.
[5] S. Boccaletti, J. Kurths, G. Osipov, D.L. Valladares and S. Zhou, “The synchronization of chaotic systems”, Phys. Rep., 366 (2002) 1-101.
[6] T. Carrol and L. Pecora, “Nonlinear Dynamics in Circuits”, World Scientific Publishing (Singapore, 1995), 89-119.
[7] O.E. Rössler, “An equation for continuous chaos”, Phys. Lett. A, 57(5) (1976) 397-398.
[8] V. Castro, M. Monti, W.B. Pardo, J.A. Walkenstein and E. Rosa, “Characterization of the Rössler system in parameter space”, Int. J. Bifurcation Chaos, 17(3) (2007) 965-973.
[9] T.L. Carroll, “A simple circuit for demonstrating regular and synchronized chaos,” Am. J. Phys., 63(4) (1995) 377-385.
[10] K. Murali and M. Lakshmanan,” Secure communication using a compound signal from generalized synchronicale chaotic systems”. Phys. Lett. A, 241(6) (1998) 303-310.
[11] Raied K. Jamal and D.A. Kafi, “Secure communication by chaotic carrier signal using Lorenz model”, Iraqi J. Phys., 14(30) (2016) 51-53.
[12] Raied K. Jamal and D.A. Kafi, “Lorenz model and chaos masking/adding technique”, Iraqi J. Phys., 14(31) (2016) 51-53.
[13] Ruiz O. F, Soriano C., Pere Colet and Claudio R. Mirasso "Information encoding and decoding using unidirectionally coupled chaotic semiconductor laser subject to filtered optical feedback", IEEE Journal of Quantum Electronic, 45 (2009) 962-968.
[14] Alexander N. Pisarchik and Flavio R. Ruiz-Oliveras, "Optical chaotic communication using generalized and complete synchronization", IEEE Journal of Quantum Electronics, 46 (2009) 279-284.
Fig. (1) Rössler electronic circuit coupled with semiconductor laser (a) Transmitter circuit (b) Receiver circuit.
Fig. (2) Secure communication diagram with chaotic Rössler circuits.
Fig. (3) Time series of voltage $y$ and $y'$, without coupling (desynchronization).

Fig. (4) Phase space trajectory $y$ and $y'$ without coupling (desynchronizing).

Fig. (5) Time series of voltage $y$ and $y'$, with coupling $R_c = 1\,\Omega$ (synchronization).

Fig. (6) Phase space trajectory $y$ and $y'$ with coupling (synchronizing).

Fig. (7) External signal $M(t)$ (sinusoidal signal).

Fig. (8) Chaotic output voltage $x$ with external signal $M(t)$. 
Fig. (9) Flow chart of Rossler model.

\[
t = 0 \text{ to } 200 \\
Dt = 0.01
\]

\[a=0.2, \; b=0.2, \; c=6.3\]

\[
d\dot{x}/dt = -(y+z) \\
d\dot{y}/dt = x + ay \\
d\dot{z}/dt = b + xz - cz
\]

Init(x) = 1 \\
Init(y) = 1 \\
Init(z) = 1

Fig. (10). Flow Chart of synchronization and secure communication by using Rossler model.

\[
t = 0 \text{ to } 500 \\
Dt = 0.01
\]

\[a=0.2, \; b=0.2, \; c=6.3, \; A=1, \; \pi=\pi, \; f=0.26, \; k=0.9\]

\[
M(t) = A\cdot\sin(2\cdot\pi\cdot f \cdot t)
\]

\[
d\dot{x}_1/dt = -(y_1+z_1) + M(t) \\
d\dot{y}_1/dt = x_1 + a \cdot y_1 + k \cdot (y_2-y_1) \\
d\dot{z}_1/dt = b + x_1 z_1 - c z_1
\]

Init(x_1) = 1 \\
Init(y_1) = 1 \\
Init(z_1) = 1

Transmittance Unit

Bidirectional coupling

Receiver Unit

\[
d\dot{x}_2/dt = -(y_2+z_2) \\
d\dot{y}_2/dt = x_2 + a \cdot y_2 + k \cdot (y_1-y_2) \\
d\dot{z}_2/dt = b + x_2 z_2 - c z_2
\]

Init(x_2) = 1.1 \\
Init(y_2) = 1 \\
Init(z_2) = 1
Fig. (11) Time series of two Rössler systems with different initial conditions, where blue line represent $y_1$ and $y_2$ represent red line.

Fig. (12) Correlation plot between $y_1$ and $y_2$ unit, non-synchronization is clear.

Fig. (13) Time series of two Rössler systems with different initial conditions and full synchronization, where blue line represent $y_1$ and $y_2$ represent red line.

Fig. (14) Correlation plot, full synchronization between $y_1$ and $y_2$ unit.

Fig. (15) External signal (Sinusoidal signal).

Fig. (16) Chaotic carrier is modulated with an external signal (red line), signal in (green line) and signal out (violet line) are very identical.