Generalized Confidence Interval for the Common Coefficient of Variation

J. Behboodian* and A. A. Jafari**

*Department of Mathematics, Shiraz Islamic Azad University, Shiraz, IRAN

email: Behboodian@stat.susc.ac.ir

**Department of Statistics, Shiraz University, Shiraz 71454, IRAN

Abstract

In this article, we consider the problem of constructing the confidence interval and testing hypothesis for the common coefficient of variation (CV) of several normal populations. A new method is suggested using the concepts of generalized p-value and generalized confidence interval. Using this new method and a method proposed by Tian (2005), we obtain a shorter confidence interval for the common CV. This combination method has good properties in terms of length and coverage probability compared to other methods. A simulation study is performed to illustrate properties. Finally, these methods are applied to two real data sets in medicine.

KeyWords: Common Coefficient of Variation; Generalized confidence interval; Generalized variable; Monte Carlo Simulation.

1 Introduction

The coefficient of variation (CV) of a random variable $X$, with mean $\mu \neq 0$ and standard deviation $\sigma$, is defined by the ratio $\frac{\sigma}{\mu}$. This ratio is an important measure of variation and it is useful in medicine, biology, physics, finance, toxicology, business, engineering, and survival analysis, because it is free from the unit of measurement and it can be used for comparing the variability of two different populations.
There are different methods for making inferences about the coefficient of variation. Lehmann (1996) proposed an exact method for a confidence interval of CV. Vangel (1996), and Wong and Wu (2002) obtained approximate confidence intervals. Verrill (2003) reviewed the exact approach that is appropriate for normally distributed data.

Let \( X_{ij}, i = 1, \ldots, k, j = 1, \ldots, n_i \), be independent normal random variables with means \( \mu_i \) and variance \( \sigma_i^2 \). Denote the CV of the \( i \)th population by \( \varphi_i = \frac{\sigma_i}{\mu_i} \). Consider the hypothesis \( H_0 : \varphi_1 = \varphi_2 = \ldots = \varphi_k \). Miller (1991) proposed an asymptotic test statistic for the \( H_0 \). Fung and Tsang (1998) reviewed several parametric and nonparametric tests for the equality of CV in \( k \) populations. Pardo and Pardo (2000) introduced a class of test statistics based on Rényi’s divergence for this problem. Nairy and Rao (2003) proposed three new tests based on the inverse sample CV, i.e. \( \bar{X}/S \), and discussed about the size and power comparison of eight tests.

The assumption of the equality of CV’s is common in biological and agricultural experiments (See Fung and Tsang, 1998). Feltz and Miller (1996) presented one reasonable estimate for the common CV. Ahmed (2002) proposed six asymptotic estimators for the common CV and discussed on the risk behavior of the estimators. The generalized \( p \)-value concept was introduced by Tsui and Weerahandi (1989) and the generalized confidence interval by Weerahandi (1993). By using these concepts, Tian (2005) proposed a generalized \( p \)-value and a generalized confidence interval for the common CV. Verrill and Johnson (2007) obtained confidence bounds on the common CV and a ratio of two CV’s.

In this article, we propose new methods for making inferences about the common CV. In Section 2, we look at the concepts of generalized \( p \)-value and generalized confidence interval. In Section 3, we will first review the method of Tian (2005) and Verrill and Johnson (2007), briefly, and then a new method is given to construct a confidence interval and hypothesis testing for the common CV by using the concept of generalized variable. Then by combining this new method and the proposed method by Tian (2005), we obtain a confidence interval for the common CV that has good properties with respect to other methods. Section 4 is devoted to a simulation study, to compare the lengths and coverage probabilities of the four methods that are given in Section 3. Two real medicine examples are given in Section 5.
2 Generalized \( p \)-value and generalized confidence interval

The concept of generalized \( p \)-value was first introduced by Tsui and Weerahandi (1989) to deal with some nontrivial statistical testing problems. These problems involve nuisance parameters in such a fashion that the derivation of a standard pivot is not possible. See also Weerahandi (1995).

Let \( X \) be a random variable with density function \( f(x \mid \zeta) \), where \( \zeta = (\theta, \eta) \) is a vector of unknown parameters, \( \theta \) is the parameter of interest, and \( \eta \) is possibly a vector of nuisance parameters.

Suppose we have the following hypothesis to test:

\[
H_0 : \theta \leq \theta_0 \quad vs \quad H_1 : \theta > \theta_0,
\]

where \( \theta_0 \) is a specified value.

Let \( x \) be the observed value of random variable \( X \). \( T(X; x, \zeta) \) is said to be a generalized variable if the following three properties hold:

(i) For fixed \( x \) and \( \zeta = (\theta_0, \eta) \), the distribution of \( T(X; x, \zeta) \) is free of the nuisance parameters \( \eta \).

(ii) \( t_{obs} = T(x; x, \zeta) \) does not depend on unknown parameters.

(iii) For fixed \( x \) and \( \eta \), \( P(T(X; x, \zeta) \geq t) \) is either stochastically increasing or decreasing in \( \theta \) for any given \( t \).

If \( T(X; x, \zeta) \) is stochastically increasing in \( \theta \), the generalized \( p \)-value is defined as

\[
p = \sup_{\theta \leq \theta_0} P(T(X; x, \theta, \eta) \geq t^*) = P(T(X; x, \theta_0, \eta) \geq t^*),
\]

where \( t^* = T(x; x, \theta_0, \eta) \).

To derive a confidence interval for \( \theta \), let \( T_c(X; x, \theta, \eta) \) satisfies the following conditions:

(i) The distribution of \( T_c(X; x, \theta, \eta) \) does not depend on any unknown parameters.

(ii) The observed value of \( T_c(X; x, \theta, \eta) \) is free of nuisance parameters.

Then, \( T_c(X; x, \theta, \eta) \) is called a generalized pivotal variable. Further, if \( t_1 \) and \( t_2 \) are such that

\[
P(t_1 \leq T_c(X; x, \theta, \eta) \leq t_2) = 1 - \alpha,
\]

then, \( \Theta = \{ \theta : t_1 \leq T_c(X; x, \theta, \eta) \leq t_2 \} \) gives a \( 100(1 - \alpha) \% \) generalized confidence interval for \( \theta \). For example, if the value of \( T_c(X; x, \theta, \eta) \) at \( X = x \) is \( \theta \), then \( \{T_c(x, \alpha/2), T_c(x, 1 - \alpha/2)\} \) is a \( (1 - \alpha) \) confidence interval for \( \theta \), where \( T_c(x, \gamma) \) stands for the \( \gamma \)th quantile of \( T_c(X; x, \theta, \eta) \).
3 Inferences for $\varphi$

Consider $k$, ($k \geq 2$) independent random samples $(X_{i1}, ..., X_{in_i})$ from $k$ normal populations with means $\mu_i$ and unequal variances $\sigma_i^2$, $i = 1, 2, ..., k$. For the $i$th population, let $\bar{X}_i = 1/n_i \sum_{j=1}^{n_i} X_{ij}$ and $S_i^2 = 1/(n_i - 1) \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_i)^2$ be the sample mean and sample variance, and let $\bar{x}_i$ and $s_i^2$ be the observed value of the sample mean and sample variance, respectively.

Suppose that

$$\varphi_1 = \varphi_2 = ... = \varphi_k = \varphi$$

where $\varphi_i = \frac{\sigma_i}{\mu_i}$ and $\varphi$ is the common CV parameter.

We are interested in developing a confidence interval and hypothesis test for the common CV, based on the sufficient statistics $\bar{X}_i$ and $S_i^2$.

In this section, we first review the method of Tian(2005) and Verrill and Johnson (2007) for this problem. A new method is introduced for hypothesis test and confidence interval, regarding $\varphi$, by using the concept of generalized $p$-value and generalized confidence interval. At the end, by combining this method and the method of Tian (2005), we find a new method which gives a shorter confidence interval.

3.1 Method of Tian

Tian (2005) proposed a generalized pivotal variable of the common CV $\varphi$, by a weighted average of the generalized pivotal variables of CV based on individual samples as

$$T_1 = T_1(\bar{X}, S; \bar{x}, s, \omega) = \frac{\sum_{i=1}^{k} \bar{x}_i \sqrt{n_i - 1}}{\sum_{i=1}^{k} \bar{x}_i} \frac{n_i - 1}{\sqrt{n_i}} - \frac{Z_i}{\sum_{i=1}^{k} \bar{x}_i} s_i - \frac{X_i - \mu_i}{\sigma_i} = \frac{\sum_{i=1}^{k} \bar{x}_i S_i}{n_i - 1} \frac{X_i - \mu_i}{\sigma_i}$$

where $U_i = \frac{(n_i - 1)S_i^2}{\sigma_i^2} \sim \chi^2_{(n_i-1)}$ and $Z_i = \sqrt{n_i} (\bar{X}_i - \mu_i) / \sigma_i \sim N(0,1)$, $i = 1, 2, ..., k$, and $X = (\bar{X}_1, ..., \bar{X}_k)$ and $S = (S_1, ..., S_k)$ with the corresponding observed values $\bar{x}$ and $s$, and $\omega = (\varphi, \sigma_1, ..., \sigma_k)$.

$T_1$ is a generalized pivotal variable for $\varphi$ and can be used to construct a confidence interval and hypothesis test about $\varphi$. 

4
The $(1 - \alpha)$ confidence interval for $\varphi$ is
\[ \{ T_1(\bar{x}, s, \alpha/2), T_1(\bar{x}, s, 1 - \alpha/2) \}, \]
where $T_1(\bar{x}, s, \gamma)$ is the $\gamma$th quantile of $T_1(\bar{X}, S; \bar{x}, s, \omega)$.

Tian (2005) evaluated the coverage properties of this confidence interval by simulation, and showed that the coverage probabilities are close to nominal level.

For testing $H_0 : \varphi \leq \varphi_0$ vs $H_1 : \varphi > \varphi_0$,
the generalized $p$-value based on (5) is
\[ p = P(T_1(\bar{X}, S; \bar{x}, s, \omega) \leq \varphi_0), \] (5)
and for testing the hypothesis $H_0 : \varphi = \varphi_0$ vs $H_1 : \varphi \neq \varphi_0$,
the generalized $p$-value is
\[ p = 2 \min \{ P(T_1(\bar{X}, S; \bar{x}, s, \omega) \leq \varphi_0), P(T_1(\bar{X}, S; \bar{x}, s, \omega) \geq \varphi_0) \}. \] (6)

### 3.2 Method of Verrill and Johnson

Under the hypothesis in (4) the log-likelihood function can be written as
\[ \ln L(\theta) = \sum_{i=1}^{k} \left( -n_i \ln \sigma_i - \sum_{j=1}^{n_i} \left( x_{ij} - \frac{\sigma_i}{\varphi} \right)^2 / (2\sigma_i^2) \right) - \frac{n}{2} \ln 2\pi \]
\[ = \sum_{i=1}^{k} \left( -n_i \ln \sigma_i - \sum_{j=1}^{n_i} \left( (n_i - 1)S_i^2 + n_i(\bar{x}_i - \frac{\sigma_i}{\varphi})^2 \right) / (2\sigma_i^2) \right) - \frac{n}{2} \ln 2\pi, \] (7)
where $\theta^T = \omega = (\varphi, \sigma_1, \ldots, \sigma_k)$.

The Newton estimator of $\theta$ is given by
\[ \theta_{\text{Newt}} = - \left[ \frac{\partial^2 \ln L}{\partial \theta_l \partial \theta_m} \right]^{-1}_{\theta_{n,c}} \left( \begin{array}{c} \partial \ln L / \partial \theta_1 \\ \vdots \\ \partial \ln L / \partial \theta_{k+1} \end{array} \right)_{\theta_{n,c}} + \theta_{n,c}, \]
where $\theta_{n,c}$ is any $\sqrt{n}$-consistent estimator of $\theta$ (Lehmann, 1996).

Verrill and Johnson (2007) obtained an approximate $(1 - \alpha)$ confidence interval for $\varphi$ as

$$
\hat{\varphi} \pm Z_{\alpha/2} \sqrt{\frac{\hat{\varphi}^4 + \hat{\varphi}^2 / 2}{n}}
$$

where $\hat{\varphi}$ is the first element of $\theta_{Newt}$ and $Z_{\alpha/2}$ is appropriate critical value from a standard normal distribution.

### 3.3 A New Method

Under the hypothesis in (4), we have $X_{ij} \sim N(\eta \sigma_i, \sigma_i^2)$, $i = 1, 2, ..., k$, where $\eta = \frac{1}{\varphi}$. We can show that if $\sigma_i^2$'s are known, then the MLE for $\eta$

$$
\hat{\eta} = \frac{\sum_{i=1}^{k} \frac{n_i \bar{X}_i}{\sigma_i}}{n},
$$

where $\hat{\eta} \sim N(\eta, \frac{1}{n})$, and $n = \sum_{i=1}^{k} n_i$.

**Remark.** If we use $S_i^2$ as an estimator for $\sigma_i^2$, then a reasonable estimator for $\varphi$, is

$$
\hat{\varphi} = \frac{n}{\sum_{i=1}^{k} \frac{n_i \bar{X}_i}{S_i}} = \frac{n}{\sum_{i=1}^{k} \frac{n_i}{\varphi_i}},
$$

which is a $\sqrt{n}$-consistent estimator for $\varphi$.

A generalized pivotal variable for estimating $\sigma_i^2$ can be expressed as

$$
R_i = \sigma_i^2 s_i^2 = \frac{(n_i - 1)s_i^2}{U_i}, \quad i = 1, 2, ..., k,
$$

where $U_i = \frac{(n_i - 1)s_i^2}{\sigma_i^2} \sim \chi^2_{(n_i - 1)}$ and $s_i^2$ is an observed value for $S_i^2$.

We define a generalized pivotal variable for the common CV, $\varphi$, based on (10) and (12) as

$$
T_2 = T_2(\bar{X}, S; \bar{x}, s, \omega) = \frac{n}{\sum_{i=1}^{k} n_i \bar{X}_i / s_i - n(\bar{\eta} - \eta)} = \frac{n}{\sum_{i=1}^{k} \frac{n_i \sqrt{U_i} \bar{x}_i}{s_i} - \sqrt{n}Z},
$$

where $\bar{X} = (\bar{X}_1, ..., \bar{X}_k)$ and $S = (S_1, ..., S_k)$ with the corresponding observed values $\bar{x}$ and $s$ and $Z = \sqrt{n}(\bar{\eta} - \eta) \sim N(0, 1)$. 

Since $T_2(\bar{X}, S; \bar{x}, s, \omega)$ satisfies the two conditions (i) the distribution of $T_2(\bar{X}, S; \bar{x}, s, \omega)$ does not depend on any unknown parameters (ii) the observed value of $T_2(\bar{X}, S; \bar{x}, s, \omega)$ is free of the nuisance parameters, we can use (13) for constructing a generalized confidence interval for $\varphi$.

The $(1 - \alpha)$ confidence interval for $\varphi$ is

$$\{T_2(\bar{x}, s, \alpha/2), T_2(\bar{x}, s, 1 - \alpha/2)\},$$

where $T_2(\bar{x}, s, \gamma)$ is the $\gamma$th quantile of $T_2(\bar{X}, S; \bar{x}, s, \omega)$.

For testing

$$H_0 : \varphi \leq \varphi_0 \quad vs \quad H_1 : \varphi > \varphi_0,$$

we use (13) and define

$$T_2'(\bar{X}, S; \bar{x}, s, \omega) = T_2(\bar{X}, S; \bar{x}, s, \omega) - \varphi.$$  \hspace{1cm} (13)

The distribution of $T_2'(\bar{X}, S; \bar{x}, s, \omega)$ is free from nuisance parameters, the observed value of $T_2'(\bar{X}, S; \bar{x}, s, \omega)$, i.e. $t'_\text{obs}$ is zero, and the distribution function of $T_2'(\bar{X}, S; \bar{x}, s, \omega)$ is an increasing function with respect to $\varphi$. Therefore $T_2'(\bar{X}, S; \bar{x}, s, \omega)$ is a generalized variable for $\varphi$ and the generalized $p$-value is

$$p = P(T_2'(\bar{X}, S; \bar{x}, s, \omega) \leq t'_\text{obs}|\varphi = \varphi_0) = P(T_2(\bar{X}, S; \bar{x}, s, \omega) \leq \varphi_0),$$  \hspace{1cm} (14)

and for testing the hypothesis

$$H_0 : \varphi = \varphi_0 \quad vs \quad H_1 : \varphi \neq \varphi_0,$$

the generalized $p$-value based on (14) is

$$p = 2 \min \{P(T(\bar{X}, S; \bar{x}, s, \omega) \geq \varphi_0), P(T(\bar{X}, S; \bar{x}, s, \omega) \leq \varphi_0)\}.$$  \hspace{1cm} (15)

### 3.4 A Combined Method

For the generalized pivotal variable of $\varphi$, we consider a combination of the generalized pivotal variables in (5) and (13) as follows:

$$T_3(\bar{X}, S; \bar{x}, s, \omega) = 0.5T_1(\bar{X}, S; \bar{x}, s, \omega) + 0.5T_2(\bar{X}, S; \bar{x}, s, \omega).$$  \hspace{1cm} (16)

Since (i) the distribution of $T_1$ and $T_2$ does not on any unknown parameters (ii) the observed values of $T_1$ and $T_2$ are equal $\varphi$, therefore $T_3(\bar{X}, S; \bar{x}, s, \omega)$ is a generalized pivotal variable for
common CV $\varphi$, and we can use it to obtain a confidence interval for $\varphi$ and for testing the hypothesis, we define the generalized variable as

$$T_3' = T_3 - \varphi.$$  \hspace{1cm} (17)

### 3.5 A Computing Algorithm

For given $k$ independent sample from normal populations, let $i$th sample contains $n_i$ observations with statistics $x_i$ and $s_i^2$.

The generalized confidence intervals for $\varphi$ and the generalized $p$-value for testing, based on $T_h$’s, $h = 1, 2, 3$ can be computed by the Monte Carlo simulation (See Weerahandi (1995)). The following steps are given for the generalized variable $T_3$ which they are applicable for the generalized variables $T_1$ and $T_2$:

1. generate $U_i \sim \chi^2_{(n_i-1)}$, $i = 1, ..., k$.
2. generate $Z_i \sim N(0, 1)$, $i = 1, ..., k$.
3. generate $Z \sim N(0, 1)$.
4. compute $T_1$ and $T_2$ in (5) and (13).
5. Calculate $T_3 = 0.5T_1 + 0.5T_2$.
6. Repeat steps 1 to 5 for $m$ times and obtain $m$ values of $T_3$.

Let $T_{3(p)}$ denote the 100$p$th percentile of $T_3$’s in step 6. Then $[T_{3(\alpha/2)}, T_{3(1-\alpha/2)}]$ is a Monte Carlo estimate of $1 - \alpha$ confidence interval for $\varphi$.

The generalized $p$-value for testing $\varphi = \varphi_0$ vs $\varphi \neq \varphi_0$ is $2 \min \{ P(T_3 \geq \varphi_0), P(T_3 \leq \varphi_0) \}$ and the probability $P(T_3 \geq \varphi_0)$ can be estimated by the proportions of the $T_3$’s in step 6 that are greater than or equal to $\varphi_0$. Similarly, $P(T_3 \leq \varphi_0)$ can also be estimated.

### 4 Simulation Study

For comparing the coverage probability of the methods introduced in Section 3;

I) Method of Tian (2005)

II) Method of Verrill and Johnson (2007)

III) A method in (13)

IV) Combined method in (17)
a simulation study is performed for $k = 3$ populations. The data of size $n_i$, $i = 1,2,3$, were generated from normal distributions with mean $\mu_i$ and variance $\varphi^2 \mu_i^2$, such that all $k$ populations have common CV $\varphi$. Using 10000 simulations, coverage (C) probability and average of length (L) estimated. Also we used the algorithm in Section 3 by $m = 5000$ for obtaining the generalized confidence intervals. The results are given in Tables 1, 2 and 3.

We observed that

i) The method in (13) and method in (17) produce comparable results to method of Tian (2005) and method of Verrilla and Johnson (2007). Therefore, we must apply the four methods to see which one is the best on the basis of coverage probability and the length of the interval.

ii) The coverage probabilities of Tian (2005) and the one obtained by (17) are close to nominal level.

iii) In some cases the coverage probabilities of the confidence intervals constructed by (13) are generally lower than the nominal level although having a slightly shorter average length for the confidence intervals.

iv) The coverage probabilities of the confidence intervals constructed by the method of Verrill and Johnson (2007) smaller than the nominal level when the sample sizes are small.

5 Two Real Examples

**Example 1.** This is the example used by Tian (2005). Actually Fung and Tsang (1998) showed that the coefficient of variation for MCV in 1995 is not significantly different from that of 1996. We are interested in making inferences about the common coefficient of variation of these data. The sample size, mean, standard deviation and coefficient of variation for MCV are 63, 84.13, 3.390, 0.0406 from 1995 survey; and 72, 85.68, 2.946, 0.0346 from 1996 survey. These results are derived and explained in detail in the above articles.
Table 4. The confidence intervals for the common CV

| method                              | confidence interval       | length |
|-------------------------------------|----------------------------|--------|
| Tian (2005)                         | (0.0347 , 0.0447)         | 0.0100 |
| Verrill and Johnson (2007)          | (0.0324 , 0.0427)         | 0.0103 |
| New Method in (13)                  | (0.0333 , 0.0423)         | 0.0091 |
| Combined Method in (17)             | (0.0333, 0.0425)          | 0.0092 |

The estimate of $\varphi$, by different methods, are: (i) Feltz and Miller (1996), 0.0374 (ii) new method (11), 0.0372 (iii) MLE, 0.0369.

The 95% confidence intervals for the common CV based on the four methods are given in Table 4.

Example 2. The data in Appendix D of Fleming and Harrington (1991) refer to survival times of patients from four hospitals. These data and their descriptive statistics are given in Table 5.

Table 5. Data and descriptive statistics for survival times of patients from four hospitals

| Hospital | Data          | $\bar{x}_i$ | $s_i^2$ | $\hat{\varphi}_i$ |
|----------|---------------|-------------|---------|-------------------|
| Hospital 1 | 176 105 266 227 66 | 168.0       | 6880.5  | 0.4937            |
| Hospital 2 | 24 5 155 54  | 59.5        | 4460.3  | 1.1224            |
| Hospital 3 | 58 64 15  | 45.7        | 714.3   | 0.5853            |
| Hospital 4 | 174 42 305 92 30 82 265 237 208 147 | 154.6       | 8894.7  | 0.6100            |

Nairy and Rao (2003) tested homogeneity of CV’s for the hospitals and they showed that all tests give the same conclusion of accepting the null hypothesis. Therefore we have common coefficient of variation for these data.

The estimate of $\varphi$, by different methods, are: (i) Feltz and Miller (1996), 0.6734 (ii) new method (11), 0.6248 (iii) MLE, 0.6015. The estimate of $\varphi$ based on (11) is close to MLE.

The 95% confidence intervals for the common CV based on four methods are given in Table 6. We observe that the length of the interval based on combined method is shorter than Tian’s method. Also the length of the interval based on the Verrill and Johnson’s method is shorter than other methods but we showed that the coverage probability of this method for small sample size, is less than nominal level.
Table 6. The confidence intervals for the common CV

| method                        | confidence interval      | length   |
|-------------------------------|--------------------------|----------|
| Tian (2005)                  | (-1.7855 , 3.6561)       | 5.4416   |
| Verrill and Johnson (2007)   | (0.4134 , 1.0613)        | 0.6479   |
| New Method in (13)           | (0.4568 , 1.1759)        | 0.7191   |
| Combined Method in (17)      | (-0.5457 , 2.2563)       | 2.8020   |

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| $\varphi = 0.05$ | I          | II         | III         | IV          |
|-----------------|------------|------------|-------------|-------------|
| $\mu_1, \mu_2, \mu_3$ | $n_1, n_2, n_3$ | $C$ | $L$ | $C$ | $L$ | $C$ | $L$ | $C$ | $L$ |
| 5, 5, 5    | 9.50 0.0679 | 0.920 0.0421 | 0.938 0.0441 | 0.952 0.0529 |
| 5, 5, 10   | 0.966 0.0487 | 0.934 0.0336 | 0.931 0.0358 | 0.958 0.0403 |
| 5, 10, 30  | 0.946 0.0255 | 0.926 0.0218 | 0.930 0.0223 | 0.948 0.0233 |
| 10, 10, 10 | 0.953 0.0331 | 0.939 0.0242 | 0.942 0.0279 | 0.951 0.0295 |
| 1, 1, 1    | 10, 20, 20 | 0.955 0.0228 | 0.947 0.0218 | 0.957 0.0207 | 0.952 0.0214 |
| 1, 1, 2    | 10, 20, 30 | 0.952 0.0203 | 0.951 0.0194 | 0.940 0.0188 | 0.953 0.0193 |
| 20, 20, 30 | 0.948 0.0185 | 0.950 0.0181 | 0.939 0.0173 | 0.952 0.0177 |
| 30, 30, 30 | 0.953 0.0158 | 0.953 0.0152 | 0.959 0.0151 | 0.954 0.0153 |
| 1, 1, 1    | 5, 5, 5   | 0.964 0.0687 | 0.933 0.0442 | 0.935 0.0446 | 0.969 0.0535 |
| 1, 1, 2    | 5, 5, 10  | 0.950 0.0504 | 0.931 0.0380 | 0.928 0.0366 | 0.948 0.0415 |
| 1, 5, 10   | 5, 10, 30 | 0.953 0.0254 | 0.940 0.0241 | 0.938 0.0223 | 0.949 0.0232 |
| 1, 5, 10   | 10, 10, 10| 0.955 0.0333 | 0.941 0.0273 | 0.943 0.0281 | 0.951 0.0297 |
| 1, 5, 10   | 10, 20, 20| 0.958 0.0228 | 0.946 0.0201 | 0.948 0.0206 | 0.953 0.0213 |
| 1, 5, 10   | 10, 20, 30| 0.947 0.0203 | 0.950 0.0185 | 0.942 0.0188 | 0.948 0.0192 |
| 1, 5, 10   | 20, 20, 30| 0.946 0.0185 | 0.954 0.0179 | 0.944 0.0173 | 0.946 0.0176 |
| 1, 5, 10   | 30, 30, 30| 0.947 0.0159 | 0.949 0.0150 | 0.947 0.0151 | 0.947 0.0153 |
Table 2: Simulated coverage probability (C) and average length (L) of 95% two sided confidence interval for $\varphi$ (based on 10000 simulation)

| $\varphi = 0.3$ | I    | II   | III  | IV   |
|-----------------|------|------|------|------|
| $\mu_1, \mu_2, \mu_3$ | $n_1, n_2, n_3$ | $C$  | $L$  | $C$  | $L$  | $C$  | $L$  | $C$  | $L$  |
| 5, 5, 5         | 0.967 | 0.6581 | 0.930 | 0.3712 | 0.926 | 0.2956 | 0.956 | 0.4371 |
| 5, 5, 10        | 0.965 | 0.4333 | 0.932 | 0.2523 | 0.931 | 0.2405 | 0.964 | 0.3091 |
| 5, 10, 30       | 0.959 | 0.1842 | 0.930 | 0.1371 | 0.932 | 0.1451 | 0.946 | 0.1556 |
| 10, 10, 10      | 0.955 | 0.2404 | 0.936 | 0.1726 | 0.941 | 0.1831 | 0.954 | 0.1971 |
| 1, 1, 1         | 10, 20, 20 | 0.948 | 0.1581 | 0.948 | 0.1421 | 0.935 | 0.1358 | 0.946 | 0.1383 |
| 10, 20, 30      | 0.953 | 0.1239 | 0.952 | 0.1226 | 0.946 | 0.1127 | 0.949 | 0.1122 |
| 20, 20, 30      | 0.955 | 0.1237 | 0.949 | 0.1247 | 0.943 | 0.1125 | 0.948 | 0.1120 |
| 30, 30, 30      | 0.964 | 0.1057 | 0.953 | 0.0980 | 0.951 | 0.0861 | 0.956 | 0.0972 |
| 5, 5, 5         | 0.968 | 0.6535 | 0.933 | 0.3562 | 0.925 | 0.2942 | 0.966 | 0.4337 |
| 5, 5, 10        | 0.961 | 0.4285 | 0.938 | 0.2141 | 0.927 | 0.2395 | 0.961 | 0.3072 |
| 5, 10, 30       | 0.956 | 0.1876 | 0.928 | 0.1252 | 0.933 | 0.1449 | 0.947 | 0.1558 |
| 10, 10, 10      | 0.960 | 0.2398 | 0.948 | 0.1736 | 0.949 | 0.1831 | 0.958 | 0.1969 |
| 1, 1, 2         | 10, 20, 20 | 0.955 | 0.1582 | 0.953 | 0.1398 | 0.946 | 0.1358 | 0.947 | 0.1386 |
| 10, 20, 30      | 0.956 | 0.1385 | 0.951 | 0.1147 | 0.944 | 0.1225 | 0.947 | 0.1232 |
| 20, 20, 30      | 0.947 | 0.1243 | 0.957 | 0.1421 | 0.940 | 0.1130 | 0.945 | 0.1225 |
| 30, 30, 30      | 0.952 | 0.1059 | 0.953 | 0.1149 | 0.946 | 0.0984 | 0.948 | 0.1037 |
| 5, 5, 5         | 0.961 | 0.6462 | 0.914 | 0.2301 | 0.927 | 0.2932 | 0.960 | 0.4305 |
| 5, 5, 10        | 0.966 | 0.4319 | 0.922 | 0.2415 | 0.928 | 0.2382 | 0.964 | 0.3079 |
| 5, 10, 30       | 0.948 | 0.1910 | 0.928 | 0.1453 | 0.929 | 0.1462 | 0.948 | 0.1576 |
| 10, 10, 10      | 0.956 | 0.2433 | 0.936 | 0.1722 | 0.948 | 0.1844 | 0.946 | 0.1986 |
| 1, 5, 10        | 10, 20, 20 | 0.949 | 0.1569 | 0.943 | 0.1251 | 0.945 | 0.1352 | 0.948 | 0.1376 |
| 10, 20, 30      | 0.952 | 0.1381 | 0.952 | 0.1326 | 0.942 | 0.1220 | 0.952 | 0.1357 |
| 20, 20, 30      | 0.952 | 0.1237 | 0.960 | 0.1362 | 0.950 | 0.1126 | 0.953 | 0.1230 |
| 30, 30, 30      | 0.955 | 0.1064 | 0.953 | 0.1106 | 0.948 | 0.0988 | 0.956 | 0.1022 |
Table 3: Simulated coverage probability ($C$) and average length ($L$) of 95% two sided confidence interval for $\varphi$ (based on 10000 simulation)

| $\varphi = 0.5$ | $\mu_1, \mu_2, \mu_3$ | $n_1, n_2, n_3$ | I | II | III | IV |
|----------------|------------------------|----------------|----|----|----|----|
|                | 5, 5, 5                | 0.968 2.8818  | 0.921 0.7262 | 0.932 0.5887 | 0.966 1.5547 |
|                | 5, 5, 10               | 0.969 1.5770  | 0.930 0.6221 | 0.936 0.4699 | 0.959 0.9118 |
|                | 5, 10, 30              | 0.967 0.5152  | 0.926 0.2451 | 0.935 0.2775 | 0.954 0.3514 |
|                | 10, 10, 10             | 0.956 0.5907  | 0.942 0.3726 | 0.949 0.2331 | 0.953 0.4213 |
| 1, 1, 1        | 10, 20, 20             | 0.951 0.2818  | 0.943 0.2471 | 0.948 0.2328 | 0.945 0.2302 |
|                | 10, 20, 30             | 0.954 0.3322  | 0.946 0.2658 | 0.944 0.2585 | 0.952 0.2768 |
|                | 20, 20, 30             | 0.959 0.2517  | 0.956 0.2217 | 0.951 0.2143 | 0.949 0.2088 |
|                | 30, 30, 30             | 0.957 0.2079  | 0.963 0.1923 | 0.947 0.1849 | 0.948 0.1764 |
|                | 5, 5, 5                | 0.969 2.8874  | 0.925 0.9531 | 0.928 0.5956 | 0.962 1.5651 |
|                | 5, 5, 10               | 0.955 1.5216  | 0.936 0.7216 | 0.925 0.4652 | 0.958 0.8846 |
|                | 5, 10, 30              | 0.967 0.4875  | 0.946 0.2741 | 0.945 0.2744 | 0.958 0.3391 |
|                | 10, 10, 10             | 0.960 0.5884  | 0.952 0.4651 | 0.938 0.3534 | 0.945 0.4170 |
| 1, 1, 2        | 10, 20, 20             | 0.948 0.3416  | 0.953 0.2821 | 0.950 0.2584 | 0.952 0.2579 |
|                | 10, 20, 30             | 0.949 0.2951  | 0.946 0.2212 | 0.941 0.2328 | 0.953 0.2358 |
|                | 20, 20, 30             | 0.948 0.2487  | 0.956 0.2317 | 0.944 0.2127 | 0.946 0.2017 |
|                | 30, 30, 30             | 0.951 0.2083  | 0.952 0.1851 | 0.948 0.1852 | 0.952 0.1772 |
|                | 5, 5, 5                | 0.967 2.8634  | 0.930 0.6528 | 0.936 0.5888 | 0.964 1.5513 |
|                | 5, 5, 10               | 0.966 1.6602  | 0.928 0.6211 | 0.926 0.4633 | 0.964 0.9465 |
|                | 5, 10, 30              | 0.962 0.5244  | 0.940 0.3224 | 0.937 0.2772 | 0.959 0.3559 |
|                | 10, 10, 10             | 0.953 0.5979  | 0.937 0.3852 | 0.945 0.3549 | 0.949 0.4231 |
| 1, 5, 10       | 10, 20, 20             | 0.958 0.3399  | 0.944 0.2634 | 0.942 0.2569 | 0.946 0.2663 |
|                | 10, 20, 30             | 0.953 0.2943  | 0.952 0.2471 | 0.945 0.2337 | 0.951 0.2362 |
|                | 20, 20, 30             | 0.950 0.2515  | 0.957 0.2716 | 0.947 0.2135 | 0.951 0.2084 |
|                | 30, 30, 30             | 0.953 0.2094  | 0.951 0.1928 | 0.952 0.1869 | 0.949 0.1784 |