A Lightweight Approach to Computing Message Races with an Application to Causal-Consistent Reversible Debugging *

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Abstract. This paper presents a lightweight formalism (a trace) to model message-passing concurrent executions where some common common problems can be identified, like lost or delayed messages, some forms of deadlock, etc. In particular, we consider (potential) message races that can be useful to analyze alternative executions. We consider a particular application for our developments in the context of a causal-consistent reversible debugging framework for Erlang programs.

1 Introduction

Program debugging is generally a difficult task. When we observe a misbehavior during a program execution (the symptom), finding the source of the error is often a challenging task. This is particularly difficult in the context of concurrent and distributed software due to the inherent nondeterminism of executions. As a consequence, reproducing a faulty execution in a debugger is rather difficult. Several debugging techniques have been developed in order to overcome this problem. Typically, programs are instrumented in order to generated a form of log (or trace) of an execution. These logs can the be used to analyze what went wrong in the execution (as in, e.g., post-mortem debugging) or they can be loaded in a debugger in order to reproduce the faulty execution (as in, e.g., record and replay debugging).

In this work, we consider a message-passing concurrent language like Erlang [5]. The language essentially follows the actor model so that, at runtime, an application consists of a number of processes that can only interact through (asynchronous) message sending and receiving. Each process has a (private) mailbox, where messages are stored until they are eventually consumed (or become orphan messages). Executions are typically nondeterministic because of

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1 In practice, some Erlang built-in’s involve shared-memory concurrency, but we will not consider them in this work.
the order in which messages are delivered to processes. Consider, for instance, two processes, A and B, that send messages \(m_A\) and \(m_B\), respectively, to another process C. If A and B are independent (i.e., their actions do not depend on each other), then messages \(m_A\) and \(m_B\) can reach process C in any order, a so-called message race. Here, the order in which this messages are delivered may determine the rest of the execution (and, ultimately, the outcome of the execution). Actually, we might have very unusual interlavings that do not happen during program testing but eventually arise after software deployment, producing unexpected and hard to find errors.

In order to improve software reliability, one can explore all possible interlavings of a program, checking that no errors may arise in any of them. This is the core idea of techniques like stateless model checking [9] and reachability testing [17]. Intuitively speaking, one can proceed as follows:

- First, a random execution of the program is considered.
- Then, message races in this execution are identified, i.e., situations like that of messages \(m_A\) and \(m_B\) described above.
- For each message race, alternative interlavings are considered. These alternatives are used to drive a new execution up to the point where messages are delivered in a different order, then continuing nondeterministically as usual.
- This process is repeated until all possible interlavings of a program have been considered. In practice, dynamic partial order reduction (DPOR) techniques [7] are typically considered to avoid exploring equivalent executions once and again.

In this work, however, we consider a different application of message races in the context of a reversible debugging framework for Erlang [15,16] and the associated tool CauDEr [6]. Reversible debugging allows one to find the source of bugs in a more natural way by exploring a faulty execution from the observed misbehavior back to its cause. Moreover, CauDEr includes a replay mode where particular executions can be reproduced, and explored back and forth, using execution logs. In this context, the computation of message races can be useful to show the user those points in an execution where a different interleaving is possible. In these cases, the user may decide to abandon the execution that was being replayed and explore a different one.

\section{Message-Passing Concurrent Executions and Logs}

Let us start by considering the notion of log as defined by Lanese et al [15,16], which has some similarities with the SYN-sequences of reachability testing [17]. Both formalisms represent a partial order for the executed actions, which contrasts with the interlavings considered, e.g., in stateless model checking [9] and DPOR techniques [7,11]. To be precise, a log maps each process to a sequence of the following actions:

- process spawning, denoted by \texttt{spawn(p)}, where \(p\) is the pid (process identifier, which is unique) of the new process;
Fig. 1: Some possible message-passing diagrams. We have three processes, identified by pids p1, p2 and p3. Solid arrows denote the connection between messages sent and received (similarly to the synchronization pairs of [17]), while dotted arrows represent message delivery. Time, represented by dashed lines, flows from top to bottom.

− message sending, denoted by send(ℓ), where ℓ is a (unique) message tag;
− and message reception, denoted by rec(ℓ), where ℓ is a message tag.

Execution logs represent quite a rough abstraction of an actual execution, but they have enough information to make a message-passing execution essentially deterministic [15,16]. This is why logs are used by the causal-consistent reversible debugger CauDEr [6] as part of an approach to record-and-replay debugging in Erlang.

Unfortunately, logs do not contain enough information for other purposes, like computing message races, blocked processes (a form of deadlock) or distinguishing lost (i.e., not delivered) messages from orphan (i.e., delivered but not consumed) messages. Let us illustrate these issues with an example. Consider the following simple log:

\[
[p1 \mapsto \text{spawn}(p2), \text{spawn}(p3), \text{send}(\ell_1);\ p2 \mapsto \text{rec}(\ell_1); \ p3 \mapsto \text{send}(\ell_2), \text{send}(\ell_3)]
\]

where p1, p2, p3 are pids and \(\ell_1, \ell_2, \ell_3\) are message tags. Here, we can easily see that message \(\ell_1\) has been consumed by process p2 since the log contains the following pair of elements: send(\(\ell_1\)) and rec(\(\ell_1\)). This execution can be represented by the message-passing diagram in Figure 1a, where \(s_i\) denotes a concurrent action of the form send(\(\ell_i\)), \(i = 1, \ldots, 3\), and \(r_1\) denotes rec(\(\ell_1\)).

Unfortunately, the information in the above log is not enough to identify message races. For this purpose, we need to know (at least) the target of each message. For this purpose, we need to know (at least) the target of each message. For instance, we can replace send(\(\ell\)) by send(\(\ell, p\)) in the logs, where \(p\) is the pid of the target process:

\[
[p1 \mapsto \text{spawn}(p2), \text{spawn}(p3), \text{send}(\ell_1, p1);\ p2 \mapsto \text{rec}(\ell_1); \ p3 \mapsto \text{send}(\ell_2, p2), \text{send}(\ell_3, p2)]
\]

Now, we can see that all three messages, \(\ell_1, \ell_2,\) and \(\ell_3\) are addressed to the same process, p2. However, we do not know when these messages were delivered. As a
consequence, the same log might represent both the message-passing diagram in Figure[1]b and that in Figure[1]. Hence, we cannot know where there is a message race for the receive \( r_1 \) between messages \( \ell_1 \) and \( \ell_2 \) (the case in Figure[1]b) or between messages \( \ell_1 \) and \( \ell_3 \) (the case in Figure[1]c). Observe that message \( \ell_2 \) cannot race with \( \ell_1 \) in Figure[1] since \( \ell_2 \) was delivered before \( \ell_1 \). This situation typically denotes that the value of message \( \ell_2 \) does not meet the constraints of receive \( r_1 \) and, thus, was ignored.

Now, we add explicit actions for message delivery; namely, we add deliver(\( \ell \)) to denote the delivery of message \( \ell \), which is represented by \( d_\ell \) in Figure[1]. Then, the log represented in Figure[1]b is as follows:

\[
[ p1 \mapsto spawn(p2), spawn(p3), send(\ell_1, p2); \\
p2 \mapsto deliver(\ell_1), rec(\ell_1), deliver(\ell_2), deliver(\ell_3); \\
p3 \mapsto send(\ell_2, p2), send(\ell_3, p2) ]
\]

while that of Figure[1]c is as follows:

\[
[ p1 \mapsto spawn(p2), spawn(p3), send(\ell_1, p2); \\
p2 \mapsto deliver(\ell_2), deliver(\ell_1), rec(\ell_1), deliver(\ell_3); \\
p3 \mapsto send(\ell_2, p2), send(\ell_3, p2) ]
\]

Finally, we also add explicit events for process termination, exit. For instance, the log represented by the diagram in Figure[1] could now be as follows:

\[
[ p1 \mapsto spawn(p2), spawn(p3), send(\ell_1, p2), exit; \\
p2 \mapsto deliver(\ell_1), rec(\ell_1), deliver(\ell_2), deliver(\ell_3); \\
p3 \mapsto send(\ell_2, p2), send(\ell_3, p2), exit ]
\]

In this way, we can now easily identify the following issues from a log:

- A process \( p \) is blocked (a form of deadlock) if its log does not end with exit, since all processes are assumed to exit eventually in a normal execution.
- A message \( \ell \) is a lost message whenever the log includes send(\( \ell, p \)) but it does not include deliver(\( \ell \)).
- Finally, a message \( \ell \) is an orphan message whenever the log includes deliver(\( \ell \)) but it does not include rec(\( \ell \)).

In the following, we use the term trace for the extended logs (including the modified send as well as the new deliver and exit actions), while we keep the word log for the original notion introduced in [15].

The computation of message races (from a trace) will be shown in the next section. Nevertheless, let us clarify that we follow a lightweight approach to the computation of message races so that they are only potential races. For instance, given the trace [(1)] above (represented in Figure[1]b), we would determine that there is a (potential) message race between \( \ell_1 \) and \( \ell_2 \) for \( r_1 \). However, if message \( \ell_2 \) does not meet the constraints of receive \( r_1 \), the race might be between \( \ell_1 \) and \( \ell_3 \) instead. In the following, we would compute both possibilities and leave the user (or the debugging tool) to determine where a race is indeed an actual race or not. An experimental evaluation to determine the success ratio of this simple strategy is planned. For a more elaborated approach that computes actual message races we refer the interested reader to [21].
3 Message Races and Reversible Debugging

In this section, we formalize an extension of the causal-consistent reversible debugging framework for Erlang [15,16] in order to also compute message races, following the ideas presented so far.

Causal-consistent reversible debugging [8] allows one to inspect the execution of a concurrent program back and forth, similarly to so-called time-travel debugging. Reversible debugging can be useful to debug issues easier by “rewinding” a faulty execution back to the source of the problem (in contrast to traditional debuggers that usually require several runs of the program, possibly including breakpoints). Reversibility is particularly challenging in the context of concurrent and distributed applications since exploring backwards a forward computation in exactly the inverse order is often a poor strategy (e.g., because a huge number of actions can be completely unrelated to the process of interest). Here, causal-consistent reversible debugging can greatly improve the situation. Compared to traditional reversible debuggers, causal-consistent reversible debuggers allow the user to undo the steps of a concurrent execution in any arbitrary order, as long as the steps are causal-consistent, i.e., no action is undone until all the actions that depend on it have already been undone. Therefore, causal-consistent reversibility is essential to avoid exploring a large number of unrelated execution steps.

These notions have been adapted to a message-passing concurrent language like Erlang in [19,14] and materialized in the CauDEr debugger [14,13,10]. The scheme has also been extended to consider replay debugging in [15,16], thus having the advantages of both (causal-consistent) reversible and record-and-replay debugging. A debugging session is typically driven by user requests like “go forward until process $p$ is spawned” or “go backward up to the point where message $\ell$ was sent”, etc. In this context, adding the computation of message races can be very useful for the user to identify possible sources of nondeterminism; furthermore, the generation of so-called race variants will allow one to explore alternative execution paths in a systematic way. For instance, one could introduce new requests like “go forward until a deadlock is detected” or “go forward until an orphan message is produced”, where all race variants are systematically considered. If a problem is eventually found, the user can then use the current requests to explore (back and forth) the buggy execution and try to identify the source of the error. In this way, one could get the best of both worlds, systematic state-space exploration and causal-consistent reversible debugging in a single tool.

3.1 A Tracing Semantics for Erlang

First, we formalize an appropriate semantics for a significant subset of the language Erlang [5] which can be used to produce a trace of an execution as a side-effect. Following [19,14], we consider a layered semantics: an expression semantics
defined on local states (typically including an environment, an expression and a stack, see, e.g., [10]) and a system semantics.

We are not concerned with the details of the expression semantics here. Let us only mention that, given a local state \( ls \), we denote by \( ls \xrightarrow{z} ls' \) one evaluation step, where \( ls' \) is the new local state and \( z \) is a label that denotes the type of evaluation; for global or non-local actions, this label includes the information which is required for the next layer of the semantics (the system semantics) to perform the associated side-effects. The local state typically contains an environment (a substitution), an expression (to be evaluated) and a stack (see [10] for more details). Here, we consider the following labels:

- \( \iota \): a local evaluation (e.g., a function call, an arithmetic operation, etc).
- \( \text{send}(v, p) \): the evaluation requires sending a message \( v \) to process with pid \( p \) as a side-effect.
- \( \text{rec}(\kappa, cs) \): the evaluation requires receiving a message, where \( cs \) are the branches of the receive statement and \( \kappa \) is a sort of future that will be bound (in the system semantics) to the expression in the selected branch.
- \( \text{spawn}(\kappa, ls_0) \): the evaluation requires spawning a new process as a side-effect. The new process will start with the initial local state \( ls_0 \). In this case, variable \( \kappa \) will be bound to the pid of the new process in the system semantics.

As mentioned before, we assume that processes are given a pid (process identifier) that uniquely identifies them in a computation. In the following, we often refer to “process \( p \)” to mean “process with pid \( p \)”. Analogously, every message is wrapped with a tag which is unique too, so that we can distinguish messages even when they have the same value. The domains of pids, \( P \), and tags, \( L \), are disjoint. In this work, we distinguish five (global) actions:

- \( \text{spawn}(p) \), for process spawning;
- \( \text{exit} \), for process exit (termination);
- \( \text{send}(\ell, p) \), for message sending;
- \( \text{deliver}(\ell) \), for message delivery; and
- \( \text{rec}(\ell) \), for message reception.

Here, \( p \in P \) is a pid and \( \ell \in L \) is a message tag. An event is then a pair \( p : a \), where \( a \) is an action, and \( p \) is the pid of the process performing this action. We note that message delivery is attributed to the target of the message.

Let us now consider an instrumented version of the system semantics, called tracing semantics, where transitions are labeled with the associated event. A running system includes a number of processes, which are defined as follows:

\[ \text{receive } cs \text{ end} \] has the form
\[ \text{receive } p_1 \text{[when } g_1 \text{]} \to e_1; \ldots; p_n \text{[when } g_n \text{]} \to e_n \text{ end} \] so that it looks for the oldest message in the process’ mailbox that matches some pattern \( p_i \) and the corresponding guard \( g_i \) holds; then, it continues evaluating \( e_i \). When no message matches any pattern, execution is blocked until a matching message reaches the mailbox.

\[ ^3 \text{We do not specify how these unique identifiers can be computed in a concurrent or distributed setting, but refer the interested reader to, e.g., [12].} \]
Definition 1 (process). A process is denoted by a configuration of the form 
\( \langle p, ls, q \rangle \), where \( p \) is the pid (process identifier) of the process, which is unique in 
a system, \( ls \) is the local state and \( q \) is the process mailbox (a list).

A system is then defined as a pair \( \Gamma; \Pi \), where \( \Gamma \) represents the network (sometimes 
called the global mailbox [14] or the ether [20]) and \( \Pi \) is a pool of processes.

The network, \( \Gamma \), is defined as a collection of queues, one per each pair of (not 
necessarily different) processes. We use the notation \( \Gamma[(p, p') \mapsto qs] \) either as a 
condition on \( \Gamma \) or as a modification of \( \Gamma \), where \( p, p' \) are pids and \( qs \) is a (possibly 
empty) queue; for simplicity, we assume that queues are initially empty for each 
pair of processes. We use list notation for queues, where \([\ ]\) denotes the empty 
queue and \( m:qs \) denotes a queue with first element \( m \) and tail \( qs \); moreover, we 
let \( qs+m \) denote the addition of message \( m \) at the end of queue \( qs \).

The second component, \( \Pi \), is denoted as \( \langle p_1, ls_1, q_1 \rangle \mid \cdots \mid \langle p_n, ls_n, q_n \rangle \), 
where \( \mid \) represents an associative and commutative operator. We often denote 
a system as \( \Gamma; (p, ls, q) \mid \Pi \) to point out that \( \langle p, ls, q \rangle \) is an arbitrary process of 
the pool (thanks to the fact that \( \mid \) is associative and commutative).

The rules of the tracing semantics are shown in Figure 2. A standard semantics 
would be similar to the tracing semantics by removing transition labels and 
unwrapping messages. Let us briefly explain the rules:

- Rule Exit removes a process from the pool when the local state is final, i.e., 
  when the expression to be reduced is a data term. Rule Local just updates the 
  local state of the selected process according to a transition of the expression 
  semantics, while rule Self binds \( \kappa \) to the pid of the current process.
- Rule Spawn updates the local state, binds \( \kappa \) to the pid of the new process and 
  adds a new initial process configuration with local state \( ls_0 \) as a side-effect.
- Rule Send updates the local state and, moreover, adds a new message to 
  the corresponding queue of the network as a side-effect. Evaluating a send 
  statement and adding the message to the network is considered an atomic 
  operation. In contrast to the standard semantics, a message \( v \) is “wrapped” 
  with a message identifier \( \ell \) so that messages can be tracked in a computation.
- Rule Deliver nondeterministically (since \( \Gamma \) might contain several queues with 
  the same target process \( p \)) takes a message from the network and moves it 
  to the corresponding process mailbox.
- Finally, rule Receive consumes a message from the process mailbox using the 
  auxiliary function matchrec that takes the local state \( ls' \), the future \( \kappa \), the 
  branches of the receive expression \( cs \), and the queue \( q \). It then selects the 
  oldest message in \( q \) that matches a branch in \( cs \) (if any), and returns a new 
  local state \( ls'' \) (where \( \kappa \) is bound to the expression in the selected branch), 
  a queue \( q' \) (where the selected message has been removed), and the label of 
  the selected message (which is needed to label the transition step).

Observe that message sending is split into three different actions: send (the 
message is stored in the network), delivery (the message is moved from the 
network to the mailbox of the target process) and receive (the message is consumed 
and removed from the mailbox). This contrasts with the semantics in \[15,16\].
debugger \[14,13,10\] implements both a user-driven strategy (where the message to be delivered, there are several possible strategies. For instance, the CauDEr message delivery is abstracted away (since the simpler notion of log was enough for defining a replay semantics).

Proving that the tracing semantics is a conservative extension of the standard semantics is straightforward, since it is essentially equivalent to the system semantics in \[14\] with the addition of message tags and transition labels.

Note that the tracing semantics has two main sources of nondeterminism: selecting a process to apply a reduction rule, and selecting the message to be delivered from the network (rule Deliver). Regarding the first point, one can for instance implement a round-robin algorithm that performs a maximum number of transitions, then moves to another process, etc. As for the selection of a message to be delivered, there are several possible strategies. For instance, the CauDEr debugger \[14][13][10\] implements both a user-driven strategy (where the user selects one of the available messages) as well as a random selection. Another possibility would be implementing a strategy called instant-delivery, where sent messages are actually stored in the mailbox of the target process (this is actually the common strategy in Erlang runtime environments) \[4\] This strategy can be formalized in our setting by requiring rules Send and Deliver to be applied always consecutively (and atomically) in a derivation.

Given systems \(\alpha_0, \alpha_n\), we call \(\alpha_0 \rightarrow^* \alpha_n\), which is a shorthand for \(\alpha_0 \rightarrow_{p_1:r_1} \ldots \rightarrow_{p_n-1:r_{n-1}} \alpha_n\), \(n \geq 0\), a derivation. One-step derivations are simply called transitions. We use \(\delta, \delta', \delta_1, \ldots\) to denote derivations and \(t, t', t_1, \ldots\) for transitions. A system \(\alpha\) is said initial if it has the form \([: \langle p, ls, [] \rangle]\), where \(p\) is the pid of some initial process and \(ls\) is an initial local state containing the expression

\[
\begin{align*}
\text{(Exit)} & \quad \frac{\text{final}(ls)}{\Gamma; \langle p, ls, q \rangle \mid \Pi \xleftarrow{p: \text{exit}} \Gamma; \Pi} \\
\text{(Local)} & \quad \frac{ls \rightarrow ls'}{\Gamma; \langle p, ls, q \rangle \mid \Pi \xleftarrow{\ell} \Gamma; \langle p, ls', q \rangle \mid \Pi} \\
\text{(Self)} & \quad \frac{ls \xrightarrow{\text{self}(\alpha)} ls'}{\Gamma; \langle p, ls, q \rangle \mid \Pi \xleftarrow{\ell} \Gamma; \langle p, ls', (\kappa \mapsto p) \rangle \mid \Pi} \\
\text{(Spawn)} & \quad \frac{ls \xrightarrow{\text{spawn}(\kappa, ls_0)} ls'}{\Gamma; \langle p, ls, q \rangle \mid \Pi \xleftarrow{p: \text{spawn}(p')} \Gamma; \langle p, ls', (\kappa \mapsto p') \mapsto p \rangle \mid \langle p', ls_0, [] \rangle \mid \Pi} \\
\text{(Send)} & \quad \frac{ls \xrightarrow{\text{send}(v, p') \mapsto \ell} ls'}{\Gamma; \langle p, ls, q \rangle \mid \Pi \xleftarrow{p: \text{send}(v, p') \mapsto \ell} \Gamma; \langle p, ls, q \rangle \mid \Pi} \\
\text{(Receive)} & \quad \frac{ls \xrightarrow{\text{receiv}(\ell, v)} ls' \quad \text{and} \quad \text{match}(ls', \kappa, cs, q) = (ls'', q', \ell)}{\Gamma; \langle p, ls, q \rangle \mid \Pi \xleftarrow{\ell: \text{match}(\ell, v)} \Gamma; \langle p, ls', q'' \rangle \mid \Pi}
\end{align*}
\]

Fig. 2: Tracing semantics


\[4\] Instant-delivery is the default strategy in the Erlang model checker Concurerror \[3\].
to be evaluated. In the following, we assume that all derivations start with an
initial system.

The notion of trace is now formalized as follows:

**Definition 2 (trace).** A trace is a mapping from pids to sequences of actions. Given a trace \( \tau \), we let \( \tau(p) \) denote the sequence of actions associated to process \( p \) in \( \tau \). Also, \( \tau[p \mapsto as] \) denotes that \( \tau \) is an arbitrary trace such that \( \tau(p) = as \); we use this notation either as a condition on \( \tau \) or as a modification of \( \tau \).

We say that an event \( p:a \) occurs in a trace \( \tau[p \mapsto as] \) if action \( a \) occurs in sequence \( as \). Moreover, we say that event \( p_1:a_1 \) precedes event \( p_2:a_2 \) in \( \tau \), in symbols \( p_1:a_1 \prec_\tau p_2:a_2 \), if \( p_1 = p_2 \), \( \tau(p_1) = as \), and \( a_1 \) precedes \( a_2 \) in \( as \); otherwise, the (partial) relation is not defined. Two traces, \( \tau \) and \( \tau' \), are equal, if they are identical up to renaming of pids and tags.

Let \( \delta \) be a derivation of the form \( \alpha_0 \mapsto_{\tau_1} \alpha_1 \ldots \mapsto_{\tau_n} \alpha_{n+1} \), and let \( e'_1, \ldots, e'_m \), \( m \leq n \), be the subset of its non-null labels. Then, we say that the sequence \( e'_1, \ldots, e'_m \) is the trace of the derivation \( \delta \), in symbols \( T(\delta) \).

Note that traces represent a partial order where only the order of actions within a process matters. This contrasts with the notion of interleaving considered, e.g., in stateless model checking [9] and DPOR techniques [7,1], which could be seen as a particular linearisation of a trace.

### 3.2 Adding the Computation of Message Races to CauDEr

In practice, traces can be generated either by implementing an instrumented interpreter (following the tracing semantics) or by instrumenting a program so that its execution produces a trace as a side-effect (see [11]). Currently, the publicly available debugger CauDEr [6] consists of the following two components [15,16]:

- A **tracer**, that instruments a source program so that its execution (in the standard environment) produces a log of the computation as a side-effect.
- A **reversible** debugger, that is able to replay a particular execution given a program and the log of an execution. The user can explore the execution back and forth using requests, e.g., “go forward until the sending of message \( \ell \)”, “go backward up to the point where process \( p \) is spawned”, etc.

In this context, we have first replaced the logging semantics of [15,16] with the tracing semantics shown in Figure 2 so that we produce a trace instead of a (simpler) log. Thanks to this change, message races can now be computed and, moreover, a log can still be extracted from the trace anyway (since it is a simplification of a trace) in order to drive the replay of an execution.

**Definition 3 (log [15,16]).** A log is a mapping from pids to sequences of simple actions of the form \( \text{spawn}(p) \), \( \text{send}(\ell) \) and \( \text{rec}(\ell) \), where \( p \) is a pid and \( \ell \) is a message tag. We use the same notation conventions as for traces.

Given a trace \( \tau \), we let \( \log(\tau) \) be the log obtained from \( \tau \) by removing message delivery and exit actions, as well as replacing every action of the form \( \text{send}(\ell, p) \) by \( \text{send}(\ell) \).
As mentioned before, our traces represent a partial order on the global actions of a message-passing concurrent program. This partial order can be formalized using the well-known happened-before relation \[12\]:

**Definition 4 (happened-before, independence).** Let \( \tau \) be a trace including events \( e_1 = (p_1 : a_1) \) and \( e_2 = (p_2 : a_2) \), \( e_1, e_2 \in \tau \). We say that \( e_1 \) happened before \( e_2 \), in symbols \( e_1 \sim_\tau e_2 \), if one of the following conditions hold:

1. \( p_1 = p_2 \), \( a_1 \neq \) deliver(\( \_ \)), \( a_2 \neq \) deliver(\( \_ \)), and \( e_1 \prec_\tau e_2 \);
2. \( p_1 = p_2 \), \( a_1 = \) deliver(\( \ell \)), \( a_2 = \) deliver(\( \ell' \)), \( \ell \neq \ell' \), and \( e_1 \prec_\tau e_2 \);
3. \( a_1 = \) spawn(\( p_2 \));
4. \( a_1 = \) send(\( \ell, \_ \)) and \( a_2 = \) deliver(\( \ell \));
5. \( a_1 = \) deliver(\( \ell \)) and \( a_2 = \) rec(\( \ell \));
6. \( p_1 = p_2 \) and \( a_2 = \) exit.

Moreover, if \( e_1 \sim_\tau e_2 \) and \( e_2 \sim_\tau e_3 \), then \( e_1 \sim_\tau e_3 \) (transitivity). Two events \( e_1 \) and \( e_2 \) are independent in \( \tau \) if \( e_1 \nprec_\tau e_2 \) and \( e_2 \nprec_\tau e_1 \).\[6\]

Intuitively, our definition for the happened-before relation can be explained as follows: (1) the actions of a given process which are not message deliveries follow a strict order imposed by the program code; (2) the order of message deliveries only matters within the same process; (3) the spawning of a process happens before all the actions of this process; (4) the sending of a message happens before its delivery; (5) message delivery happens before it is consumed by a receive statement; and (6) all actions of a process must precede its exit.

Intuitively speaking, we have a message race whenever there is a receive statement that can nondeterministically consume different messages, depending on the considered derivation (interleaving). A race set collects all such alternative messages (if any). Observe that race sets are defined on traces, i.e., message races do not depend on a particular derivation but on the class of derivations represented by a given trace.

**Definition 5 (race set).** Let \( \tau \) be a well-defined trace with \( e_d = (p: \) deliver(\( \ell \)) \( ) \in \tau \) and \( e_r = (p: \) rec(\( \ell \)) \( ) \in \tau \). Consider a message \( \ell' \neq \ell \), with sending and deliver events \( e'_1 = (p': \) send(\( \ell', p \)) \( ) \in \tau \) and \( e'_d = (p: \) deliver(\( \ell' \)) \( ) \in \tau \), respectively. We say that messages \( \ell \) and \( \ell' \) race for \( e_r \) in \( \tau \) if \( e'_d \not\prec_\tau e_d \) and \( e_d \not\prec_\tau e'_d \).

We let \( \text{race}_\tau(e_r) \) denote a set with a list of message tags \( [\ell_1, \ldots, \ell_n] \) for each process \( p' \) (possibly equal to \( p \)) with at least one racing message, ordered by the corresponding sending actions, i.e., \( p' : \) send(\( \ell_1, p \)) \( \prec_\tau \ldots \prec_\tau p' : \) send(\( \ell_n, p \)).

For convenience, we also let \( [[\text{race}_\tau(e_r)]] \) denote the set of all message tags in \( \text{race}_\tau(e_r) \), i.e., the union of all messages in the computed lists.

Intuitively speaking, the definition above requires the following conditions for messages \( \ell \) and \( \ell' \) to race for the receive statement \( e_r \):

1. The target of both messages is the same (\( p \)).

\[5\] We use \( \_ \) as a placeholder to denote an arbitrary value.

\[6\] Note that the meaning of \( e_1 \not\prec_\tau e_2 \) is “\( e_1 \sim_\tau e_2 \) is not true”.

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2. The delivery of message \( \ell' \) does not precede the delivery of message \( \ell \), because that would point out either that \( \ell' \) has been consumed by another receive statement or that it does not match the constraints of the receive statement \( e_r \) (otherwise, \( e_r \) would have consumed message \( \ell' \) instead of message \( \ell \)). This is the situation for \( \ell_2 \) in the diagram shown in Figure 1c.

3. Finally, we check that the delivery of the consumed message \( \ell (e_d) \) does not happen before the sending of message \( \ell' (e'_s) \). The reason is that \( e_d \sim \tau e'_s \) would prevent the delivery of message \( \ell' (e'_s) \) to happen before the delivery of message \( \ell (e_d) \) in any derivation, thus there would be no way message \( \ell' \) could be consumed by receive \( e_r \).

When there are several racing messages from the same process, they are grouped into a list that follows the order of the corresponding sending events. Note that the lists of messages actually represent potential races since one should still check that they match the constraints of the receive statement (see [21] for a different approach that computes actual message races).

A so-called race variant can easily be computed from the trace and the messages in the race set as follows:

- Let \( \tau \) be a trace including the receive event \( e_r = (p: \text{rec}(\ell)) \). Let \( \ell' \) the considered message in \( \text{race_set}_r(e_r) \).
- Now, we compute a new trace by removing from \( \tau \) all events \( e \) such \( e_r \sim \tau e \) and, then, replacing \( e_r \) by \( \text{rec}(\ell) \). Let \( \tau' \) be the resulting trace.
- Then, the \( \log(\tau) \) can be used to replay a new execution where the receive statement associated to \( e_r \) will consume message \( \ell' \) instead of \( \ell \) (assuming it meets the constraints of the receive).

When exploring race variants, one can try receiving these messages in order until a matching one is found. Here, we are assuming that we do not want to consider traces with delayed messages. Otherwise, one should consider not only the first matching message but all of them. Observe that we do not need to check that message \( \ell \) indeed matches the constraints of the receive \( e_r \) since the considered trace has been obtained from an actual execution.

For this purpose, the replay reversible semantics has also been extended. In the (extended) reversible semantics, we have two transition relations, \( \rightarrow \) and \( \leftarrow \), that represent the forward (replay) semantics and the backward semantics, respectively. The rules, shown in Figures 3 and 4, are similar to those of the tracing semantics shown in Figure 2. The main difference is that, now, there are two new components:

- On the one hand, we add a log \( \omega \) to a system configuration, which will be used in rules Send, Receive, Spawn and Deliver to drive the execution. Here, we use the auxiliary function \( \text{next}_p \) to return either the information from the next action of process \( p \) in the log (and delete this action) or a fresh identifier if the log is already empty (in order to allow the user to continue exploring an execution when the log represents only a prefix). Therefore, the rules can be used either in replay mode or in user-driven mode, similarly to
Fig. 3: Forward reversible (replay) semantics

Fig. 4: Backward reversible semantics
the semantics presented in [10]. We also use the auxiliary function *admissible* to determine the next message that can be delivered according to the current values in the log and the process mailbox.

On the other hand, each process is now augmented with a history, i.e., a list of terms with enough information to undo each forward step. Observe that in rule *Receive* we store a term of the form \( \text{rec}(ls, ℓ, v, i) \). Here, the position \( i \) in the process mailbox is required in order to guarantee reversibility, since the message consumed by a receive statement needs not be the first one in the queue (recall that a receive statement consumes the oldest message that matches the constraints of the receive statement). This contrasts with the approach in [14] where the complete mailbox was stored. An advantage of our approach is that we can have a more general notion of concurrent actions than [14], which was unnecessarily restrictive because of the decision of storing the complete mailbox. To be precise, now, a message delivery and a message reception commute (i.e., are independent) when the considered messages are different. In [14], message reception and message delivery never commute.

The reversible semantics is then given by the relation \( \equiv \), which denotes the union of the forward and backward relations, i.e., \( \equiv = (\rightarrow \cup \leftarrow) \). An essential feature of these transition relations is that they are causal-consistent. Loosely speaking, it means that no action can be undone using \( \leftarrow \) until all the actions that depend on this action have been undone, and that no action can be performed with \( \rightarrow \) (in replay mode, i.e., when the log is not empty) until all the actions that happened before this action have been performed.

While the nondeterministic relation \( \equiv \) models the legal steps in the reversible debugger, a so-called controlled semantics can be defined on top of it. This controlled semantics is used to drive the exploration of a given execution using requests of the form “go forward until the sending of message \( ℓ \)” (in replay mode) or “go backward up to the point where process \( p \) is spawned”, etc.

More details on this approach can be found in [14,15,16]. We claim that the framework of [15,16] can be extended using the rules in Figures 3 and 4, and that all properties proved there remain true (in particular, the causal consistency of the reversible semantics). The complete formalization of this extension is the subject of ongoing work.

4 Related Work

This work stems from the idea of improving causal-consistent reversible replay debugging [15,16] with the computation of message races, since this information might be useful for the user in order to explore alternative execution paths. Causal-consistent replay debugging introduces a logging semantics that produces a log of an execution. This log has many similarities with the SYN-sequences of reachability testing [17]. Both formalisms represent a partial order with the actions performed by a number of processes running concurrently (they basically denote a Mazurkiewicz trace [18]). SYN-sequences are then used to define a
systematic testing algorithm based on the notions of message race and race variant. In contrast to [17], we consider both traces and logs, and our traces are tailored to a language with selective receives by explicitly distinguishing message delivery and reception, which is not done in [17].

Both reachability testing and our approach share many similarities with so-called stateless model checking [9]. The main difference, though, is that stateless model checking works with interleavings. Then, since many interleavings may boil down to the same Mazurkiewicz trace, dynamic partial order reduction (DPOR) techniques are introduced (see, e.g., [7,1]). Intuitively speaking, DPOR techniques aim at producing only one interleaving per Mazurkiewicz trace. This task is more natural in our context since we deal with traces (that represent all derivations which are causal-consistent) and logs (that represents all derivations which are observationally equivalent). Therefore, there is no need to consider DPOR techniques in our approach. Concuerror [3] implements stateless model checking for Erlang, and has been recently extended to also consider observational equivalence [2], thus achieving a similar result as our technique regarding message races despite the fact that the techniques are rather different. A key difference, though, is that Concuerror performs an extra pass on the schedulings, annotating each message delivery with the patterns of the receive statement [2]. In this way, messages that do not match a given receive can be excluded from the computation of message races. This approach would involve adding message values and receive constraints to traces, which contrasts with our more lightweight approach based on computing potential sets of message races (which, nevertheless, suffices in our particular application within reversible debugging).

Another, related approach is the detection of race conditions for Erlang programs presented in [4]. However, the author focuses on data races (that may occur when using some shared-memory built-in operators of the language) rather than message races. Moreover, the detection is based on a static analysis, while we consider a dynamic approach to computing message races.

Finally, regarding our tracing semantics (cf. Section 3), it shares many similarities with the logging semantics introduced in [15,16]. However, the logging semantics abstracts away process mailboxes and message delivery since they are not needed to replay an execution. Our semantics can be seen as a refinement of the logging semantics so that it is now closer to the actual semantics of the language. As argued in Section 2 dealing explicitly with message delivery is essential in our context in order to produce traces that can be used to compute message races.

5 Conclusions and Future Work

In this paper, we have introduced a lightweight formalism to represent concurrent executions in a message-passing language with selective receives. In particular, we distinguish the notion of trace from that of a log, which is essentially a
representation of a Mazurkiewicz trace. Despite the simplicity of traces, they contain enough information to analyze some common error symptoms: blocked processes, lost or delayed messages, and orphan messages. Moreover, they can also be useful to compute (potential) message races, which can then be used within our replay debugger CauDEr \[6\] to explore alternative execution paths. For this purpose, we have shown the details of a particular application of our ideas in the context of causal-consistent reversible (replay) debugging of Erlang programs.

As future work, we plan to work on the complete formalization of the extended framework for causal-consistent reversible replay debugging presented in Section 3.2 following a similar approach as in \[15\] \[16\].

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