Critical behavior of charmonia across the phase transition: A QCD sum rule approach

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(Dated: May 14, 2008)

We investigate medium-induced change of mass and width of $J/\psi$ and $\eta_c$ across the phase transition in hot gluonic matter using QCD sum rules. In the QCD sum rule approach, the medium effect on heavy quarkonia is induced by the change of both scalar and twist-2 gluon condensates, whose temperature dependences are extracted from the lattice calculations of energy density and pressure. Although the stability of the operator product expansion side seems to break down at $T > 1.06 T_c$ for the vector channel and $T > 1.04 T_c$ for the pseudoscalar channel, we find a sudden change of the spectral properties across the critical temperature $T_c$, which originates from an equally rapid change of the scalar gluon condensate characterized by $\varepsilon - 3p$. By parameterizing the ground state of the spectral density by the Breit-Wigner form, we find that for both $J/\psi$ and $\eta_c$, the masses suddenly decrease maximally by a few hundreds of MeV and the widths broaden to $\sim 100$ MeV slightly above $T_c$. Implications for recent and future heavy ion experiments are discussed. We also carry out a similar analysis for charmonia in nuclear matter, which could serve as a testing ground for observing the precursor phenomena of the QCD phase transition. We finally discuss the possibility of observing the mass shift at nuclear matter at the FAIR project at GSI.

PACS numbers: 14.40.Gx,11.55.Hx,12.38.Mh,24.85.+p

I. INTRODUCTION

In-medium change of spectral properties of heavy quarkonia is one of the interesting problems in recent hadron physics. Firstly, the recent relativistic heavy ion collision experiment at the Relativistic Heavy Ion Collider (RHIC) reveals exciting nature of the QCD matter through a number of observations \cite{1,2,3,4,5}. However, there are many open questions in both experimental facts and theoretical understandings of QCD matter. Hence, it is important to establish appropriate experimental observables that reflect consequences of deeper theoretical understandings of the matter. Heavy quarkonia have been regarded as one of the most suitable diagnostic tools in this respect, since the suppression of $J/\psi$ yields would reflect the Debye screening phenomenon caused by the deconfinement phenomenon in the quark-gluon plasma (QGP), as was originally argued by Matsui and Satz \cite{6}. Until now, quarkonium production, especially that of $J/\psi$, in relativistic heavy ion collisions have been extensively studied both experimentally \cite{7,8} and theoretically \cite{9,10}. However, a remarkable progress comes from recent lattice QCD calculations, which indicate that contrary to the earlier expectation the $J/\psi$ will survive as a bound state even in the QGP up to $T \sim 1.6 - 2 T_c$ \cite{11,12,13}, which was anticipated before based on the non-perturbative nature of QGP \cite{14}. Nowadays, the state of matter at this temperature region has been characterized as “strongly coupled” QGP (sQGP). Hence, there will be change of spectral properties even for heavy quark system which has to be considered in interpreting experimental observables.

Secondly, charmonium in a nuclear medium is also an interesting issue. In relativistic heavy ion collisions, we need knowledge of quarkonium-nucleon interaction to discriminate the suppression by QGP from the “cold nuclear matter effect” induced by such an interaction. Furthermore, multi-gluon exchange can lead to an attractive interaction between $c\bar{c}$ and a nucleon, which may result in a bound state of charmonium and a light nuclei, as pointed out by Brodsky \textit{et al.} \cite{9,10}. It should be also noted that the Panda experiment at GSI-FAIR plans reaction of anti-protons with nucleus target, which will yield charmonia in the nuclear matter. It could serve as a testing ground for observing the precursor phenomenon of the QCD phase transition.

In this paper, we investigate change of mass and width of $J/\psi$ and $\eta_c$ induced by strongly interacting hot gluonic matter and by nuclear medium using QCD sum rule. QCD sum rule provides a systematic procedure for studying hadrons from a viewpoint of the asymptotic freedom in QCD \cite{15,17}. Since the QCD sum rule can take non-perturbative effects into account through the condensate terms, it is a suitable theoretical tool of the current study. Indeed, QGP at $T < 3 T_c$ cannot be understood using perturbation theory alone \cite{18}. Furthermore, the sum rule is more promising for heavy quark systems because we do not have to take the quark-antiquark condensate into account unlike light quark systems. In this respect, the sum rule has been applied to charmonium and bottomonium. Shifman \textit{et al.} established the framework in Ref. \cite{16,17} and Reinders \textit{et al.} extended it to deep Euclidean region $Q^2 = -q^2 > 0$ \cite{19}, in the case of vacuum. As for the quarkonia in-medium, One of us together with Furnstahl and Hatsuda have investigated the mass shift of $J/\psi$ in hot hadronic matter \cite{20}, using a QCD sum rule approach, where the temperature effect was intro-

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duced to the perturbative Wilson coefficient through the scattering terms. A consistent formalism at lower density was developed by one of us [21] and independently by Hayashigaki [22] to study the mass shift of $J/\psi$ in nuclear matter.

Along this direction, we investigated the mass shift and width broadening of $J/\psi$ in hot gluonic plasma (GP) [23] just above the phase transition by consistently using the exact temperature dependencies of condensates from lattice calculation. In the present paper, as a subsequent paper to Ref. [23], we present details of the analysis, further application to $\eta_c$ and to spectral changes in nuclear matter.

In the next section, we will give an explanation of the QCD sum rule for heavy quarkonium in medium used in the present work. Section II and IV describe the details of the numerical computations of the sum rule for hot gluonic matter and nuclear medium, respectively. Section V is devoted to discussion and summary.

II. QCD SUM RULE FOR HEAVY QUARKONIUM

In this section, first we review the sum rule for heavy quarkonium in vacuum [19]. Then we introduce the extension to finite temperature and nuclear medium cases, in which medium effect is eventually induced only by the expectation values of gluonic operators without any additional change in the operator product expansion (OPE).

A. Moment sum rule in vacuum

We start with the time-ordered current-current correlation function for $J$ channel

$$\Pi^J(q) = i \int d^4x e^{iqx} \langle T[j^J(x)j^J(0)]\rangle, \quad (1)$$

where we consider $J = P$ (pseudoscalar) and $V$ (vector) current of the heavy quark. Namely, $j^P = i\bar{c}\gamma_5 c$ and $j^V = \bar{c}\gamma_\mu c$, for charm. The expectation value $\langle \cdots \rangle$ is taken for the vacuum. If we go to deep Euclidean region $Q^2 \equiv -q^2 \gg 0$, the product of the current can be expanded via operator production expansion (OPE) [24]. If we denote \(\tilde{\Pi}(q^2)\) such that \(\Pi^{\mu\nu}(q) = (q^\mu q^\nu - q^2 g^{\mu\nu})\Pi(q^2)\) for the vector current, \(\tilde{\Pi}(q^2)\) can be written as

$$\tilde{\Pi}^J(q^2) = \sum_n C_n^J \langle O_n \rangle \quad (2)$$

where \(O_n\) are the operators of mass dimension \(n\) renormalized at scale \(m_f^2\) and \(C_n^J\) are the Wilson coefficient. By virtue of much heavier quark mass than the confinement scale, heavy quark operators, such as \(m_c\bar{c}c\) for dimension 4, are rewritten in terms of gluonic operator with a factor of \(1/m_c\) via heavy quark expansion [24, 25]. Hence, only gluonic operators contribute to the OPE for the heavy quark currents.

On the other hand, the correlation function (2) is related to its imaginary part through the dispersion relation

$$\tilde{\Pi}^J(q^2) = \frac{1}{\pi} \int_{4m_f^2}^\infty \frac{\text{Im} \tilde{\Pi}^J(s)}{s - q^2} ds \quad (3)$$

where we ignore \(+i\varepsilon\) in the denominator of the integrand since \(q^2 = -Q^2 < 0\). Taking \(n\) times derivative of Eqs. (2) and (3) as

$$M_n^J(Q^2) = \frac{1}{n!} \left(\frac{d}{dq^2}\right)^n \tilde{\Pi}^J(q^2) \bigg|_{q^2 = -Q^2}, \quad (4)$$

we obtain the \(n\)-th order moment for the OPE side

$$M_n^J(Q^2)_{\text{OPE}} = A_n^J(\xi)[1 + a_n^J(\xi)\alpha_s + b_n^J(\xi)\phi_b], \quad (5)$$

and that for the phenomenological (dispersion) side

$$M_n^J(Q^2)_{\text{phen.}} = \frac{1}{\pi} \int_{4m_f^2}^\infty \frac{\text{Im} \tilde{\Pi}^J(s)}{s + Q^2} n! ds. \quad (6)$$

Here, we have introduced a dimensionless scale variable \(\xi = Q^2/4m_F^2\). In Eq. (5), \(A_n^J(\xi)\), \(a_n^J(\xi)\), and \(b_n^J(\xi)\) are the Wilson coefficients which correspond to bare loop diagrams, perturbative radiative correction up to order \(\alpha_s\), and scalar gluon condensate, respectively. These coefficients were derived in Ref. [19] and we summarize them in the Appendix.

In evaluation of spectral properties, we take the ratio of the \((n-1)\)-th moment to the \(n\)-th moment and equate the OPE side with the phenomenological side. Then we obtain the sum rule

$$\frac{M_{n-1}^J}{M_n^J}_{\text{OPE}} = \frac{M_{n-1}^J}{M_n^J}_{\text{phen.}}, \quad (7)$$

which relates the hadron properties (r.h.s.) with asymptotically free QCD (l.h.s.)

B. Moment sum rule for the hot gluonic medium

In this paper, we firstly consider the gluonic medium at finite temperature around \(T_c\). Then, the expectation value in Eq. (1) is taken as \(\langle \cdots \rangle = \text{Tr}(e^{-\beta H}0)/\text{Tr}(e^{-\beta H})\). Hereafter, we set both medium and \(\bar{c}c\) at rest. We denote \(q^2 = (\omega, q)\) and take \(q \to 0\) limit. In this case, the transverse and the longitudinal components of the correlation function for the vector channel are simply related with \(\Pi_T = \omega^2 \Pi_{\perp}\) and \(\Pi_L = \Pi_{\perp}/(-3\omega^2)\). We denote the longitudinal component as \(\Pi^V(\omega)\) for the vector channel.

At finite temperature, retarded correlation function is related to the spectral function [24]. In the Euclidean region \(\omega^2 < 0\), the retarded correlation function \(\Pi^R(\omega)\)
becomes $\Pi(\omega^2)$ and the dispersion relation is given by
\cite{20, 27}

$$\Pi^{\dagger}(\omega^2) = \int_0^\infty du^2 \frac{\rho(u)}{u^2 - \omega^2},$$

(8)

where $\rho(u)$ is the spectral function connecting with the imaginary part as

$$\rho(u) = \frac{1}{\pi} \tanh \left( \frac{u}{2T} \right) \text{Im} \Pi^{\dagger}(u^2).$$

(9)

Then Eq. (8) reduces to the vacuum case [Eq. (9)] when $\text{Im} \Pi^\dagger(u^2)$ has nonzero value only at $u \gg T$. Since we are interested in charmonia for which the mass is much larger than temperature considered here, this condition seems to be appropriate one. However, there are formally two additional terms in the finite temperature spectral function \cite{28}. One is the continuum part which also exists in the case of vacuum. Following the prescription in Ref. \cite{19}, we can suppress contribution from this part as described later because this part has finite values beyond some threshold. The other part arising from scattering of the current with quarks in medium is proportional to $\delta(u^2)$ and the contribution grows up with $T$ in the hadronic medium \cite{20}. However, since we are considering the gluonic medium in which there are no (anti-)quarks which annihilate with the current, such a scattering term does not appear. Hence, we can use the same expression of the phenomenological side with the vacuum case [Eq. (9)] for charmonia in the hot gluonic medium.

As for the OPE side, there is an important change from the vacuum to the medium case. Since we have no longer Lorentz invariance, non-scalar operators have non-vanishing value \cite{27}. In the present case, twist-2 gluon operator has leading contribution and the $n$-th order moment of the OPE side [Eq. (5)] should be modified to

$$M_n^J(Q^2)_{\text{OPE}} = A_n^J(\xi)[1 + a_n^J(\xi)\alpha_s + b_n^J(\xi)\phi_b + c_n^J(\xi)\phi_c],$$

(10)

where $c_n$ and $\phi_c$ are the Wilson coefficients and the medium expectation value for the twist-2 operator. Since we are considering the heavy quark systems, only the condensate terms are temperature dependent as long as $T \ll m_c, |Q| \cite{21, 27}$. Hence, the Wilson coefficients are the same as in the vacuum case. In the following, we show that the gluon condensates $\phi_{b,c}$ are written in terms of thermodynamic quantities which can be extracted from lattice QCD data.

If we define these condensate terms as

$$G_0(T) = \left\langle \frac{\alpha_s}{\pi} G_{\mu
u} G^\mu\nu \right\rangle_T,$$

(11)

$$\left( u^\mu u^\nu - \frac{1}{4} g_{\mu\nu} \right) G_2(T) = \left\langle \frac{\alpha_s}{\pi} G_{\mu
u} G^{\mu\nu} \right\rangle_T,$$

(12)

where $u^\mu$ is the 4-velocity of the medium and taken to be $u^\mu = (1, 0, 0, 0)$, explicit forms of $\phi_{b,c}$ are given as

$$\phi_b = \frac{4\pi^2}{9(4m_c^2)} G_0(T),$$

(13)

$$\phi_c = \frac{4\pi^2}{3(4m_c^2)} G_2(T).$$

(14)

Actually it is possible to calculate the condensates \cite{11} and \cite{12} directly using lattice QCD, but we do not adopt such an approach here. The gluon condensates generally consist of the perturbative piece and the non-perturbative piece. At zero temperature, the condensate term appearing in QCD sum rules is the non-perturbative piece only and it is shown that the non-perturbative part extracted from lattice QCD by subtracting the perturbative part is indeed consistent with the value of the condensate determined from QCD sum rules for charmonium \cite{30, 31, 32}. Similar consideration holds also for the finite temperature case \cite{33}, in which we would have to subtract out the perturbative part at $T \neq 0$ if we directly calculated the non-perturbative condensates from lattice QCD. In this paper, since we are putting all the temperature dependencies in the condensates, including the perturbative and non-perturbative contributions, we can just extract total temperature dependencies of the operators from the lattice. This is possible by noting that the scalar gluon condensate and twist-2 gluon condensates are respectively just the trace part and symmetric traceless part of the energy momentum tensor. This energy momentum tensor is well calculated on the lattice from the pressure and energy density of the plasma through the following equation,

$$T^{\alpha\beta} = (\varepsilon + p) \left( u^\alpha u^\beta - \frac{1}{4} g^{\alpha\beta} \right) + \frac{1}{4}(\varepsilon - 3p)g^{\alpha\beta}.$$  

(15)

The scalar condensate can be related to the trace part
through the trace anomaly as

$$T^\mu = \frac{\beta(g)}{2g} G^a_{\mu
u} G^{a\mu\nu}, \quad (16)$$

with $\beta(g)$ being the beta function, $\beta(g) = -\frac{3g^2}{8\pi} (33 - 2N_f)$ for 1-loop, $N_f$ flavors, and $N_c$ colors. Using the above expression with $T^\mu = \varepsilon - 3p$, we obtain

$$G_0(T) = G^{\text{vac}}_0 - \frac{8}{11}(\varepsilon - 3p) \quad (17)$$

where $G^{\text{vac}}_0$ is the value of the scalar gluon condensate in vacuum. As for the twist-2 part, the symmetric traceless part of the energy-momentum tensor is the gluon operator

$$T^{\alpha\beta} = -G^{a\alpha} G^{a\beta}. \quad (18)$$

Hence we can identify the traceless part of the energy momentum tensor to $(\varepsilon + p)$ as given in Eq. (13). From Eq. (12), the twist-2 part becomes

$$G_2(T) = \frac{\alpha_s(T)}{\pi}(\varepsilon + p), \quad (19)$$

so that $G_2(T)$ is proportional to the entropy density of the system $s = (\varepsilon + p)/T$. We extract the temperature dependent quantities $\varepsilon, p$ [32] and $\alpha_s(T)$ [29] from lattice calculations for the pure SU(3) system. In order to construct $G_2$, we need the temperature dependent effective coupling constant. The coupling constant, however, cannot be uniquely determined by lattice QCD [29]. Ref. [29] presented four kinds of the coupling constant extracted from the color singlet heavy quark-antiquark free energy. Two of them are measured in the short distant regime and the others are done in the long distant regime. In the former, one is from the free energy and the other is from the spatial derivative of the free energy (force). Both coupling constants are almost independent of temperature at short distance, $r < 0.1$ fm. While the former goes to negative value at larger distance due to the remnant of the confinement force, the latter shows temperature dependent maximum value, at which the distance is denoted by $r_\text{screen}$. Here, we adopt the latter one, $\alpha_{qq}(r, T)$ at $r = r_\text{screen}$ as one of reasonable coupling constants since it characterizes the relevant length scale for the separation of short distance regime from long distance one. On the other hand, the long distant regime is based on a fit of the free energy to the Debye-screened functional form which has two coupling parameters, Coulomb force strength $\alpha(T)$ and screening $\tilde{\alpha}(T)$. Although both of the coupling constants show reasonable temperature dependencies and agree each other at $T > 6T_c$, we adopt $\tilde{\alpha}(T)$ because the Coulomb force strength is not relevant for characterizing the long distance non-perturbative physics at temperature considered here. Unlike $\alpha_{qq}$, the uncertainty in the result of $\tilde{\alpha}(T)$ is too large. Therefore, we use the 2-loop perturbative running coupling form

$$g_{\text{pert}}^{-2}(T) = \frac{11}{8\pi^2} \ln \left( \frac{2\pi T}{\Lambda_{\text{MS}}} \right) + \frac{51}{8\pi^2} \ln \left[ 2 \ln \left( \frac{2\pi T}{\Lambda_{\text{MS}}} \right) \right], \quad (20)$$

with $T_c/\Lambda_{\text{MS}} \approx 1.14$ and rescale this as $\tilde{\alpha}(T) = 2.095\alpha_{\text{pert}}(T)$ [29]. Here we put $T_c = 264$ MeV [33]. The two coupling constants as a function of temperature are displayed in Fig. 1. As explained later, we will consider only temperature region near $T_c$ in this paper. Hence, $\alpha_{qq}$ is stronger than $\tilde{\alpha}(T)$ throughout analyses in this paper.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{g0_g2.png}
\caption{(Color online) Gluon condensates near $T_c$.}
\end{figure}

The resultant gluon condensates $G_0$ and $G_2$ for two cases of the coupling constant are shown in Fig. 2. For $G_0$, we use $G^{\text{vac}}_0 = (0.35\text{GeV})^4 \simeq 0.015\text{GeV}^4$. We can see that $G_0$ decreases as temperature increases and reaches less than half of the vacuum value at $T/T_c \simeq 1.04$. It becomes negative at higher temperature but remains positive in the temperature region considered here [33].

C. Moment sum rule for the nuclear medium

In this case, the medium consist of nucleons, thus we do not have to worry about the scattering term. As far as we follow the same method to suppress the contribution from the continuum, we can use the same form of the phenomenological side with the vacuum and finite temperature cases.

Thus, since the medium effect is similarly imposed on the gluon condensates, difference in the nuclear matter case from the case of hot gluonic matter is in the explicit form of $\phi_{b,c}$. In order to evaluate the expectation value

\[ \text{footnote: We have renewed the extraction from lattice data by improving the resolution, so that the present values are slightly different from those of Ref. [23].} \]
for the ground state of the nuclear matter, we employ the linear density approximation [30]:

$$\langle O \rangle_{\text{n.m.}} = \langle O \rangle_0 + \frac{\rho_N}{2m_N} \langle N|O|N \rangle,$$  \hspace{1cm} (21)

where $\rho_N$ and $m_N$ are the normal nuclear matter density and the nucleon mass, respectively. The nucleon state $|N\rangle$ is normalized as $\langle N(p')|N(p)\rangle = 2\rho_N(2\pi)^3 \delta^4(p-p')$. Then, the scalar condensate becomes [21]

$$\langle \frac{\alpha_s}{\pi} G^{\alpha\mu \nu} G_{\alpha \mu \nu} \rangle_{\text{n.m.}} = \langle \frac{\alpha_s}{\pi} G^{\alpha \mu \nu} G_{\alpha \mu \nu} \rangle_0 - \frac{8}{9} m_N^0 \rho_N$$  \hspace{1cm} (22)

where $m_N^0 \approx 750$ MeV is the nucleon mass in the chiral limit [37]. The traceless and symmetric twist-2 operator is given as [21],

$$\langle N(p)|\frac{\alpha_s}{\pi} G^{\alpha \sigma} G_{\alpha \sigma} \delta p^\mu \rangle N(p) = - \left( p_\alpha p^\beta - \frac{1}{4} g_\beta p^2 \right) \frac{\alpha_s}{\pi} A_G$$  \hspace{1cm} (23)

where $A_G$ is related to the moment of the gluon distribution function

$$A_G(\mu^2) = 2 \int_0^1 dx x G(x, \mu^2).$$  \hspace{1cm} (24)

Following Ref. [21], we take $A_G(8m_c^2) \approx 0.9$. While $G_2$ at finite temperature is related to the entropy, this correspondence does not hold in the nuclear matter case. Note that Eq. (23) does not contain the quark sector. Using these expressions, the condensate terms which appear in Eq. (6) finally result in [21]

$$\phi_b = \frac{4\pi^2}{9(4m_c^2)^2} \langle \frac{\alpha_s}{\pi} G^{\alpha \mu \nu} G_{\alpha \mu \nu} \rangle_{\text{n.m.}}$$  \hspace{1cm} (25)

$$\phi_c = -\frac{2\pi^2}{3} \frac{\alpha_s A_G}{(4m_c^2)^2 m_N \rho_N}.$$  \hspace{1cm} (26)

The form of $\phi_b$ is the same as the hot gluonic matter case but now the expectation value is taken through Eq. (22). We depict the density dependence of the gluon condensates based on Eqs. (22) and (26) in Fig. 3. The twist-2 case is re-normalized so that it corresponds to the finite temperature case [14]. We can see that the change of the scalar condensate reaches as large as $T = T_c$ case at $\rho \sim 5\rho_0$ but is much smaller at the normal nuclear density. The twist-2 contribution is much smaller than that of the finite temperature case.

D. Phenomenological side

In the phenomenological side, we use a simple prescription which describes the lowest lying resonance in each channel. For charmonium, previous studies [16, 19, 20, 21, 22] focused on the mass and ignored the small but finite width of $J/\psi$ and $\eta_c$. In this case, the imaginary part of the polarization function in Eq. (9) is simply parametrized by

$$\text{Im} \tilde{\Pi}(s) = f_0 (s - m^2) + \text{corrections},$$  \hspace{1cm} (27)

where we ignore the channel subscript $J$. This spectral function immediately leads to the moment

$$M_n(\xi) = \frac{f_0}{\pi (m^2 + Q^2)^{n+1}} [1 + \delta_n(\xi)].$$  \hspace{1cm} (28)

The correction term in Eq. (27) is absorbed in $\delta_n(\xi)$. By taking the ratio as in Eq. (7), we can remove the constant $f_0$ from the equation. To obtain the mass of lowest lying resonance, we need to choose sufficiently large $n$ such that $(1 + \delta_{n=1}(\xi))/(1 + \delta_n(\xi))$ is close to unity. Then the ratio does not depend on the details of the correction term which contains higher resonances and continuum, and the mass is simply given by

$$m^2 \simeq \frac{M_{n-1}(\xi)}{M_n(\xi)} - 4m_c^2 \xi.$$  \hspace{1cm} (29)

Previous analyses rely on this formula.

In this work, we extend the above formulation to include finite width. Here, we employ the simple relativistic Breit-Wigner form

$$\text{Im} \tilde{\Pi}(s) = \frac{f_0 \sqrt{\Gamma}}{(s - m^2)^2 + \Gamma^2} + \text{corrections}.$$  \hspace{1cm} (30)

As in the $\Gamma = 0$ case, we can eliminate the unnecessary constant and the effects of the correction term by taking the ratio of the moment and choosing appropriately large $n$. In the practical analyses of the sum rule, our task is to find values of $(m, \Gamma)$ which satisfy the sum rule [Eq. (7)]. Generally there are infinite numbers of the pairs of $(m, \Gamma)$ because the sum rule provides one equation with respect to the two quantities which we want to know. Hence, without additional constraints, the sum rule can provide only relation between $m$ and $\Gamma$ as in the case of light.
vector mesons [38]. Here, before the practical calculation, we discuss the relation between the mass and the width which comes from the phenomenological side, Eq. (30).

In calculation of the moment ratio of the phenomenological side, we need to compute the dispersion integral in Eq. (1) with the spectral function in Eq. (30). Since the width of the ground state charmonium is much smaller than its mass, we need careful treatment in numerical integration. To achieve good accuracy, we performed Monte-Carlo integration based on the VEGAS algorithm [39]. In our calculation, typical relative numerical uncertainty evaluated from the standard manner in the Monte-Carlo integration is order of $10^{-6}$ for $10^6$ events with $m = 3$ GeV and $\Gamma = 1$ MeV. As expected, this accuracy becomes better as $\Gamma$ increases.

![FIG. 4: (Color online) Moment ratio of the phenomenological side as a function of $\Gamma$. Upper panels are for $\xi = 1$. Left and right panels denote the case of $n = 9$ and $n = 14$, respectively. Lower ones are for $\xi = 3$ with $n = 14$ (left) and $n = 19$ (right).](image)

We begin with fixing $n$ such that the moment ratio of the OPE side takes its minimum value for each temperature. As briefly mentioned before, we need to choose moderately large $n$ so that contribution from excited states and continuum can be sufficiently suppressed. Therefore, this ratio should approach a constant value at adequately large $n$. However, in the OPE side contribution from higher dimensional operators will be important at large $n$. As such $n$ value that the moment ratio takes its minimum value, pole dominance and truncation of the OPE are valid and the ratio is close to the real asymptotic value, as have been extensively studied in the vacuum case [19].

![FIG. 4](image)

We display the moment ratio for the OPE side [Eq. (10)] in Figs. 5 and 6 with the gluon condensates shown in Fig. 3.

![Figure 5](image)

Figure 5 shows the moment ratio for the vector channel. The left and right column show the case in which we use $\alpha_s$ and $m_c$, respectively. Comparing different $\xi$ cases, we can see that the stability of the moment becomes better as $\xi$ increases. But the values of $n$ which give the stability to the moment ratio also becomes larger. As previously reported in [23], the stability is only achieved near $T_c$ and the stronger coupling, which is $\alpha_{qq}$ in this temperature region, gives worse stability. By increasing $\xi$, we can improve the stability a little. While it is achieved only up to $1.04 T_c$ for $\xi = 0$, the moment ratio remains stable up to $1.06 T_c$ for $\xi = 3$.

We can see the similar situation in the pseudoscalar channel depicted in Fig. 6. However, the moment ratio is less stable than the vector case. In the pseudoscalar case, even the best case (using $\tilde{\alpha}$ and $\xi = 3$) can stabilize the moment ratio only up to $1.04 T_c$.

![Figure 6](image)

Note that the lack of stability does not necessarily mean dissociation of the charmonia. The reason of such instability can be clearly seen in the each terms of the OPE [Eq. (10)], of which each term must be much less than unity for convergence. These terms are displayed in Figs. 7-9. We can see that all the coefficients grow up with $n$. An important feature in all the coefficients is that increasing $\xi$ clearly keeps their value smaller. Among these three, only $c_{\phi}(\xi)\phi_c$ always has positive sign and its magnitude increases with temperature. These two
FIG. 5: (Color online) Moment ratio for the OPE side for the vector channel \((J/\psi)\). Each panels show different \(\xi\) and coupling constant case. The symbols stand for different temperature.

FIG. 6: (Color online) Same as Fig. 5 but for the pseudoscalar channel \((\eta_c)\).

features are opposite to \(b_n(\xi)\phi_0\), in which the sign is always negative and the value seems to approach to zero as
temperature increases. In comparing the two channels, one finds that there are no significant differences. Hence, the stability will be determined by a delicate balance between coefficients and its breakdown will be caused by rapid increase of $c_n(\xi)\phi_c$.

Now we proceed to the determination of mass and width. The values of $n$ are listed in Table I. Note that the stability achieved at the highest temperature is ambiguous in some cases; for example, $J/\psi$ of $\xi = 2$ with $\alpha_{qq}$ case is stable at $T/T_c = 1.05$ with $n = 22$. However, as seen in Fig. 5, the moment ratio is almost constant in such large $n$ region and never rises up as lower temperature cases do. Such a vague stability is also seen in other cases. Hence, we note that mass and width values evaluated on the basis of such a stability are less reliable in the analyses below.

Once $n$ and $\xi$ are fixed, we can compute the mass and the width by making use of Eq. (7). For a fixed moment ratio of the OPE side, we firstly compute the mass in the limit of $\Gamma \to 0$ using Eq. (29). By virtue of the monotonic behavior of the moment ratio of the phenomenological side shown in Fig. 4, we can safely calculate the mass in the case of finite width by numerically solving Eq. (7) with Eq. (30).

We plot the relation between the mass shift and the width at various temperatures in Figs. 10 and 11. We can see the almost linear behavior of the width as a function of the mass shift. Note that the vacuum mass differs for different $\xi$. We do not perform fine tuning of the parameters so that the real vacuum mass is reproduced. Although there are some exceptions for the linear
relation, especially small $\xi$ and high temperature cases, these come from the vague stability we mentioned before. Hence, we can conclude that the mass shift and the width have the linear relationship as far as QCD sum rules properly work. The other important aspect is temperature dependence of the mass shift and the width. We cannot know how the mass and the width behave in the real situation, since we cannot simultaneously determine both of the mass and the width within the current framework only. Here, we investigate two extreme cases; $\Gamma \to 0$ limit and $\delta m \to 0$ limit.

The results are shown in Figs. 12 and 13. In these figures, we plot the results of $T > 0.9T_c$. Figure 12 shows the remarkable behavior of the mass shift; The mass does not change up to $T \sim T_c$ but it suddenly begins to decrease across $T_c$. This fact clearly reflects the temperature dependence of the gluon condensates which represent the phase transition. Above $T_c$, the mass decreases with temperature almost linearly. This feature is common for both $J/\psi$ and $\eta_c$. Though small $\xi$ results, especially $\xi = 0$, show more rapid decrease, the curves become almost parallel among large $\xi$ results, as a consequence of the better stability. From the nature of the phenomenological side shown in Fig. 4, this case corresponds to the maximum mass shift. The mass shift

| $J$ | $\alpha_q(T)$ | $\xi$ | $\frac{T}{T_c}$=1 | 1.01 | 1.02 | 1.03 | 1.04 | 1.05 | 1.06 |
|-----|--------------|------|-----------------|------|------|------|------|------|------|
| $J/\psi$ | $\alpha_{qq}$ | 0    | 6   | 6   | 6   | 7   | 9   | N/A | N/A | N/A |
|      |              | 0.5  | 7   | 8   | 9   | 10  | 11  | N/A | N/A | N/A |
|      |              | 1    | 8   | 10  | 10  | 12  | 13  | N/A | N/A | N/A |
|      |              | 1.5  | 10  | 11  | 12  | 13  | 15  | N/A | N/A | N/A |
|      |              | 2    | 11  | 13  | 14  | 15  | 16  | 18  | 23  | N/A |
|      |              | 2.5  | 13  | 15  | 15  | 16  | 18  | 20  | 22  | N/A |
|      |              | 3    | 14  | 16  | 17  | 18  | 19  | 21  | 23  | 29  |
| $\tilde{\alpha}$ |              | 0    | 6   | 7   | 8   | 12  | N/A | N/A | N/A | N/A |
|      |              | 0.5  | 8   | 10  | 11  | 15  | N/A | N/A | N/A | N/A |
|      |              | 1    | 10  | 12  | 14  | 17  | N/A | N/A | N/A | N/A |
|      |              | 1.5  | 12  | 14  | 16  | 19  | N/A | N/A | N/A | N/A |
|      |              | 2    | 14  | 16  | 18  | 21  | N/A | N/A | N/A | N/A |
|      |              | 2.5  | 15  | 18  | 20  | 22  | 29  | N/A | N/A | N/A |
|      |              | 3    | 17  | 20  | 21  | 24  | 29  | N/A | N/A | N/A |
| $\eta_c$ | $\alpha_{qq}$ | 0    | 6   | 7   | 7   | 14  | N/A | N/A | N/A | N/A |
|      |              | 0.5  | 8   | 9   | 10  | 12  | 16  | N/A | N/A | N/A |
|      |              | 1    | 10  | 12  | 13  | 14  | 18  | N/A | N/A | N/A |
|      |              | 1.5  | 12  | 14  | 15  | 16  | 19  | N/A | N/A | N/A |
|      |              | 2    | 14  | 16  | 17  | 18  | 21  | N/A | N/A | N/A |
|      |              | 2.5  | 15  | 17  | 18  | 20  | 22  | 28  | N/A | N/A |
|      |              | 3    | 17  | 19  | 20  | 22  | 24  | 28  | N/A | N/A |

FIG. 9: (Color online) Twist-2 condensate term $c_n(\xi)\phi_c$ with $\alpha_{qq}$. 

TABLE I: List of $n$ values at which the moment ratio takes minimum values.
for $J/\psi$, left: $\alpha_{qq}$, right: $\tilde{\alpha}$

\begin{align*}
\text{FIG. 10: (Color online) Relation between mass shift } & \delta m = m_{\text{vacuum}} - m \text{ and width } \Gamma \text{ for } J/\psi. \text{ As in Figs. 5 and 6, each figure shows different } \xi \text{ case for two cases of the coupling constant, } \alpha_{qq} \text{ and } \tilde{\alpha}. \\
& \text{shows } \sim 50 \text{ MeV reduction from vacuum to } T_c, \text{ and it increases additionally by } \sim 20-50 \text{ MeV as temperature rises by } 0.01T_c. \text{ Consequently, it becomes 100-300 MeV at } T = 1.04T_c. \\
& \text{Similarly, Fig. 13 shows that the width begins to increase with temperature across } T_c \text{ if no mass shift takes place. This also shows almost linear dependence on temperature above } T_c. \text{ Though some exceptions can be seen in the small } \xi \text{ results, which are also indicated in Figs. 10 and 11.}
\end{align*}
and these behaviors come from the vague stability appearing as too large $n$ in Table I. Hence, we can conclude that the width increases linearly with temperature above $T_c$ if the mass remains unchanged. Since we did not do fine tuning of the parameters for each $\xi$, the values of mass shift and width differ for different $\xi$. However,
the qualitative features do not depend on $\xi$ where the stability is reliable. This shows robustness of our analysis. A realistic change at each temperature should be a combined decrease in mass and increase in width, whose values are smaller than their maximal changes obtained here. However, to determine the realistic combination, we need to have an additional constraint between the changes in the width and the mass, or input the thermal width from another calculation.² From Fig. 14 one may think that the dominant contribution to the change of mass and/or width is the scalar gluon condensate which exhibits sudden decrease around $T_c$. However, $G_2$ has also similar behavior since it relates to the entropy density. Though the value of $G_2$ around $T_c$ is smaller than $G_0$ because of prefactor $\alpha_s/\pi$, the relative contribution to the moment becomes larger as $T$ increases. In order to see the contribution clearly, We show the mass shift of $J/\psi$ without $G_2$ term for $\xi = 1$ together with the two different coupling cases in Fig. 14. We can see that almost half of the mass shift is caused by decrease of $G_2$. Clearly larger $G_2$ value in which $\alpha_{qq}$ is adopted as coupling constant leads to larger mass shift. At $T = 1.04T_c$, $\alpha_{qq}$ is about 0.1 larger than $\hat{\alpha}$. This difference makes the mass shift 30 MeV larger in the $\xi = 1$ case. Unfortunately present analysis is limited to the temperature region around $T_c$, the role of twist-2 term will become more important at higher temperature.

IV. NUCLEAR MATTER

In this section, we analyze change of mass and width of the charmonium induced by nuclear medium with the same framework that was implemented in the previous section. Here, we use Eqs. (25) and (26) instead of Eqs. (13) and (14), respectively. With the common parameter set, the condensates are $\phi_b = 1.74 \times 10^{-3}$ for vacuum, $1.64 \times 10^{-3}$ for the nuclear matter, and $\phi_c = -1.28 \times 10^{-5}$.

As previously shown in Ref. [21], the change of mass, which is identical to the change of the moment ratio of the OPE side [Eq. (29)], is not as large as in the hot gluonic matter case. Thus we do not have to worry about the stability of the OPE. Nevertheless, increasing $\xi$ improves the validity of the OPE. We will show the results for $0 \leq \xi \leq 3$ as well as in the hot gluonic matter case to show the robustness and the consistency of the calculation.

### TABLE II: List of $n$ which stabilize the moment ratio for the nuclear matter

| channel | $\xi = 0$ | $\xi = 0.5$ | $\xi = 1$ | $\xi = 1.5$ | $\xi = 2$ | $\xi = 2.5$ | $\xi = 3$ |
|---------|-----------|-------------|-----------|-------------|-----------|-----------|-----------|
| $J/\psi$ | 5         | 7           | 9         | 10          | 12        | 13        | 14        |
| $\eta_c$ | 6         | 8           | 10        | 12          | 14        | 15        | 17        |

![Mass-Width relation in Nuclear Matter](image)

![FIG. 14: (Color online) $\delta m$ of $J/\psi$ without twist-2 term $G_2$. See text for detail.](image)

![FIG. 15: (Color online) Relation between mass shift and width in the nuclear matter.](image)

2 See Ref. [10] for a recent investigation.
the same $n$ value case with the vacuum is almost negligible. i.e., the moment ratio is almost constant around these $n$.

We plot the width $\Gamma$ as a function of the mass shift $\delta m$ in Fig. 15 as well as in the GP case. In both $J/\psi$ and $\eta_c$ cases, smaller $\xi$ than 1.5 show larger mass shift and width broadening but larger $\xi$ results agree each other. From the stability argument, larger $\xi$ results will be more reliable. Then, possible mass shifts are maximally $-7$ MeV for $J/\psi$ and $-4$ MeV $\eta_c$, while maximum widths are $10$ MeV for $J/\psi$ and $6$ MeV for $\eta_c$.

V. DISCUSSION AND SUMMARY

In Sec. III we have shown that mass decreases suddenly across $T_c$ and the shift reaches maximally a few hundred MeV above $T_c$ in the hot gluonic matter. Alternatively, width can also maximally broaden to $\sim 200$ MeV. Although our analysis cannot determine both of mass and width simultaneously, this is a notable result which should be examined in the present and future experiments. In fact, a next to leading order QCD calculation shows that the thermal width of $J/\psi$ slight above $T_c$ is smaller than a few $10$ MeV [41, 42]. Hence a large mass shift will take place. Note that such a large mass shift has been expected from different points view: an AdS/QCD analysis shows a sudden drop of mass at the phase transition [13]. In Ref. [13], although the mass begins to slowly increase at higher temperature, the temperature region investigated in the present paper corresponds to the critical region. Sudden reduction of the asymptotic value of the potential seen in lattice QCD [44] leads to lowering of the bound state energy [15]. Recent lattice QCD calculation based on the maximum entropy method also shows survival of the peak in the spectral function above $T_c$ [44] but the resolution is still insufficient to discuss shift and broadening of the peak. Since our results access only near $T_c$, we are still far from the complete understanding of the behavior of the charmonium in the deconfined medium. In the most plausible picture from the current understanding, charmonia are melting at very high temperature expected in the early stage of the heavy ion collisions at RHIC and LHC. Then the pairs of heavy quark and antiquark form the bound states at a certain temperature which depends on quantum number. The temperature is expected as $\sim 2T_c$ for $J/\psi$ at RHIC [17]. After charmonia are produced, they will dissociate with collisions with partons. If this phase lasts long enough compared to the inverse of the width, the charmonia can decay in the medium. In fact, the lifetime of the partonic medium is about 4-5 fm/c in a hydrodynamic calculation for the central Au+Au collisions at the maximum RHIC energy [45]. This will be much longer at LHC. From Fig. 10, we expect $\sim 200$ MeV $J/\psi$ mass reduction in the case of the small decay width. This shift is larger than experimental mass resolutions ($\sim 35$ MeV for dielectron channel of PHENIX at RHIC [3], 33 MeV for dielectron channel and 75 MeV for dimuon channel of ALICE at LHC [49]).

Alternatively, statistical hadronization near phase boundary has been also examined [50]. In this case, the number of produced charmonium will be enhanced if the notable mass shift occurs. For example, there may be a factor of 2 enhancement for $T = 170$ MeV and $\delta m = -100$ MeV since the enhancement factor is given by $e^{-\delta m/T}$. This enhancement might be observed by comparing particle ratio.

As for the nuclear medium result, we have extended the analysis carried out in Ref. [21] to the one which takes account of finite width. We have also shown the results for different $\xi$ values. Since we have given the relation between the mass shift and width, we can estimate the mass shift in the presence of finite width effect by considering the dissociation cross section of the charmonium by nucleon. Provided the Fermi momentum is $p_F \simeq 250$ MeV and the cross section is $\sigma_{J/\psi - N} \simeq 2$mb, the decay width $\Gamma = \langle \sigma_{J/\psi - N}\nu pN \rangle$ becomes $\sim 1.3$ MeV for charmonium at rest. The cross section may be smaller, because the incident momentum is considered to be small and the process will be near threshold. From this estimate, if we take into account the broadening of the width, the mass shift becomes slightly smaller, by about 0.5 MeV, according to the results shown in Fig. 15 Therefore, this justifies the argument in Ref. [21] that the influence of the decay widths is expected to be small.

The change of spectral properties in the nuclear matter can be experimentally investigated by Panda experiment at GSI-FAIR in which incident anti-proton collide with nuclear target. Here we present some predictions for cross sections of charmonium production through $\bar{p}p$ annihilation and subsequent decay into dileptons or radiative decay in the experiment. We compute the cross sections with the Breit-Wigner formula

$$\sigma_{BW}(s) = \frac{B_{in}B_{out}(2J + 1)4\pi}{(2s_1 + 1)(2s_2 + 1)} \frac{s\Gamma_{tot}^2}{k_{cm}^2(m^2 - s)^2 + s\Gamma_{tot+med}^2},$$

(31)

where $s$, $k_{cm}^2$, and $m$ are the Mandelstam variable, c.m. momentum and mass of charmonium with spin $J$, respectively. $\Gamma_{tot}$ is the total decay width of the charmonium and $\Gamma_{tot+med} = \Gamma_{tot} + \Gamma_{medium}$. $B_{in}$ and $B_{out}$ are the branching fraction of the resonance into the entrance and exit channels. $s_i$ is the spin of the incident particles, which are anti-protons and protons in the present calculation. Since the target protons are in nucleus, we have to take the Fermi motion into account for accurate estimation. We average the Breit-Wigner cross section with respect to target momentum as

$$\sigma_{BW} = \frac{4}{\rho_0} \int_{F_0}^{k_F} k^2 dk d\Omega \sigma_{BW}.\quad(32)$$

In addition to $J/\psi$ and $\eta_c$, we also calculate cross sections for $\chi_c$ which are expected to show larger mass shift $\delta m \simeq -40 \sim -60$ MeV [51]. Parameters in the calcula-
TABLE III: Parameters and results in charmonium production at GSI-FAIR. Cross sections and event per day correspond to
the case of maximum medium width, $\Gamma_{\text{med}} = 20$ MeV.

| Resonance | $m$ [MeV] | $\delta m$ [MeV] | $\Gamma_{\text{tot}}$ | Final State | $\sigma_{\text{BW}}$ at peak | Events per day |
|-----------|----------|------------------|----------------------|-------------|----------------------|---------------|
| $J/\psi$  | 3097     | -7               | 93.4 keV             | $e^+ + e^-$ | 0.435 pb             | 7.5           |
| $\eta_c$  | 2980     | -4               | 25.5 MeV             | $e^+ + e^-$ | 10.7 pb              | 184           |
| $\chi_{c0}$ | 3415     | -60              | 10.4 MeV             | $J/\psi + \gamma$ | 18.0 pb     | 311           |
| $\chi_{c1}$ | 3511     | -60              | 0.89 MeV             | $J/\psi + \gamma$ | 4.5 pb      | 78            |
| $\chi_{c2}$ | 3556     | -60              | 2.05 MeV             | $J/\psi + \gamma$ | 19.8 pb      | 343           |

FIG. 16: (Color online) Cross section of $J/\psi$ production in $\bar{p} - A$ collisions. Upper panel shows smaller medium width
(1 and 5 MeV) cases and lower one shows larger (10 and 20 MeV) cases for with and without mass shift.

Resonances are summarized in Table III $\Gamma_{\text{med}}$ is treated as
a free parameter varied from 1 MeV to 20 MeV.

Results of the cross sections as a function of incident anti-proton energy are shown in Figs. 16-20. We can
clearly see that sharp peaks of the resonances disappear. This is because of the Fermi motion of the target protons in the nucleus. For example, incident energy to create $J/\psi$ (3097) is $E_{\text{lab}} = 4.17$ GeV, but the fluctuation of the target momentum makes it possible to create $J/\psi$ with $3.17 \leq E_{\text{lab}} \leq 5.51$ GeV, in which the minimum and the maximum $E_{\text{lab}}$ correspond to the target momentum along the collision axis $p_{2z} = -k_F$ and $k_F$, respectively. This effect considerably broadens the cross section. Consequently, one needs no fine tuning of incident proton energy to produce charmonium. For $J/\psi$ and $\eta_c$, mass shifts are so small that the peak positions of incident energy do not change. However, mass shift of $\chi_{c1}, \sim -60$ MeV, is sufficiently large to show clear shift of the peak in the cross section. In these calculations, we treat $\Gamma_{\text{med}}$ as a free parameter. It is shown that this parameter affects only on the magnitude of the cross section, which is larger for smaller change from the vacuum width. Hence, though we cannot predict both of mass shift and in-medium width, we can obtain information on both quantities from the experimentally measured cross sections. We summarized the cross sections and expected
event rate at GSI-FAIR, of which luminosity is expected to be $2 \times 10^{32}\text{cm}^{-2}\text{s}^{-1}$, in last two column of Table. We can see that the expected event rates are large enough for the mass shift of $\chi_c$ to be observed.

Finally we address possible improvements of this work. Since we restricted ourselves to the hot medium which consists of gluons only in the first part of this paper, we should take the quarks into account for more realistic estimation.

To consider the quark effects, first we consider the quark operators appearing in the OPE side. We can neglect the light quark contribution to the OPE, because the light quark operators appear in the OPE at order $\alpha_s^2(q^2)$: This is why the light quark condensate can be neglected in the sum rules for heavy quark system. On the other hand, thermal heavy quarks that directly couple to the heavy quark current contribute to the OPE at leading order. This is different from the heavy quark condensates that are perturbatively generated from the gluon condensates, and contribute to the OPE through gluon condensates, whose Wilson coefficients are calculated in the momentum representation. The direct thermal quark contributions are called the scattering terms. However, similar terms also appear in the phenomenological side, which also has free charm quark mode that is not coupled with a light quark in the form of a $D$ meson above $T_c$ as been recently studied in Ref. [52]. Therefore, the scattering term will cancel out between the OPE side and the phenomenological side in the deconfined medium.

Next, we consider the quark operators appearing in the OPE side. We can neglect the light quark contribution to the OPE, because the light quark operators appear in the OPE at order $\alpha_s^2(q^2)$: This is why the light quark condensate can be neglected in the sum rules for heavy quark system. On the other hand, thermal heavy quarks that directly couple to the heavy quark current contribute to the OPE at leading order. This is different from the heavy quark condensates that are perturbatively generated from the gluon condensates, and contribute to the OPE through gluon condensates, whose Wilson coefficients are calculated in the momentum representation. The direct thermal quark contributions are called the scattering terms.
tion to higher temperature. The failure of quasi-particle. One more thing to be done is the extension of the spectral function may play an important role. This of the moment ratio of the OPE side. The twist-2 gluon condensates becomes larger as temperature increases and then leads to the breakdown of the stability in the OPE side including up to dimension 4 (see Fig. 9 and $O(a_s)$). In Fig. 7 we can also see that the expansion is not good at large $n$ that stabilize the moment ratio at higher temperature. These facts suggest the necessity of including higher dimensional operators, which is examined in Ref. [55]. However, we do not know a simple way to extract the temperature dependencies of the higher dimensional operators from the lattice calculation, as was done in the present work for dimension 4 operators. The other way of the extension is to improve the phenomenological side such that it includes temperature dependent continuum contribution. The decrease of the scalar gluon condensates above $T_c$ indicates perturbative contribution becomes more important at higher temperature. If we can construct a more appropriate phenomenological side reflecting the nature of the strongly interacting matter, it will lead to $n$-independent results for physical parameters until the charmonia really dissolve.

We also note that there are some spaces to improve the analyses for nuclear matter. Especially, the present analysis shows the mass shift of $\chi_c$ states are likely to be observed in the forthcoming experiment. However, the current estimate of the mass shift is not a decisive one; we have to take the twist-2 contribution into account for a more accurate estimation.

In summary, we have given a comprehensive analysis on medium-induced change of the spectral properties of $J/\psi$ and $\eta_c$ in the hot gluonic medium and the nuclear medium by making use of QCD sum rules. In the case of the gluonic medium, our analysis shows there must be a notable change of mass or width, or both around $T_c$, caused by the rapid change of the gluon condensates. Although the present formalism is found to be applicable of the resulting change near the critical temperature are remarkably similar between the full and pure gluon QCD; although the slope at $T_c$ is milder for full QCD as a consequence of rapid cross over transition instead of the first order phase transition. Since the change of the condensate sets in at a lower $T/T_c$ in the full QCD case, the mass and width of charmonia might start varying at a lower temperature in the realistic case than in the pure glue theory. As for the twist-2 condensates, results will not be affected so much by taking into account the fermionic part since the effect will be small at this temperature region. Therefore we believe our main argument and the quantitative result will not be alter even in the realistic situation.

It is also important to study change of $\chi_c$ at finite temperature, which may influence the quantitative feature of the sequential melting [54], since non-negligible fraction of $J/\psi$ comes from decay of $\psi'$ and $\chi_c$ in relativistic heavy ion collisions. This can be done by calculating Wilson coefficients for tensor operators for these channels. It should be also noted that the continuum part of the spectral function may play an important role. This will be possible by modeling the medium with a gas of quasi-particle. One more thing to be done is the extension to higher temperature. The failure of $T > 1.06 T_c$ for $J/\psi$ and $T > 1.04 T_c$ for $\eta_c$ originates from the instability

![Figure 20](image1.png)

**FIG. 20:** (Color online) Same as Fig. [16] but for $\chi_{c2}$.

![Figure 21](image2.png)

**FIG. 21:** (Color online) Comparison of the scalar gluon condensate in the pure gauge theory with the one of full QCD. Horizontal errorbars in the full QCD case are drawn by assuming the $2\%$ uncertainty in the conversion from the lattice units to physical temperature [52].
only up to $T \approx 1.06T_c$, the change of mass and width can maximally reach a few hundred MeV. We have discussed its implication for future heavy ion experiment at CERN-LHC. As for the nuclear matter case, we extend the past works to include small but finite width and check the robustness by varying the scale parameter of the theory. We also examined the possibility of detecting such mass shifts in the future experiment at GSI-FAIR. Although $J/\psi$ and $\eta_c$ do not show prominent signals, $\chi_c$ exhibits more promising results. These analyses give the basis of future improvements to study the nature of the strongly interacting matter deeply with charmonia.

Acknowledgments

This work was supported by BK21 Program of the Korean Ministry of Education. S. H. L. was supported by the Korean Research Foundation KRF-2006-C00011 and by the Yonsei University research fund. K. M. would like to thank the members of the high energy physics group of Waseda University for allowing him to use their workstations. We also would like to acknowledge T. Hatsuda for his fruitful comments and discussions.

APPENDIX A: WILSON COEFFICIENTS

Here we list explicit forms of the Wilson coefficients which appear in Eq. (10) and are originally given in Refs. [19] and [21]. In the following, $\rho = \xi/(1 + \xi)$ and $F(a, b, c; x)$ is the hypergeometric function $\,_{2}F_{1}(a, b; c; x)$.

For the pseudoscalar channel,

$$ A^{p}_{n}(\xi) = \frac{3}{8\pi^{2}} \frac{2^{n}(n-1)!}{(2n+1)!} \left[ (4n^{2})^{-n} (1 + \xi)^{-n} F(n, 1/2, n + 3/2; \rho) \right], \quad (A1) $$

$$ a^{p}_{n}(\xi) = \frac{(2n+1)!!}{3 \cdot 2^{n-1} n!} \left[ \frac{\pi}{2(n+1)} \left( \frac{1}{2} - \frac{3}{4\pi} \right) F(n, 1 + n; 1 + 2n; \rho) \right] \frac{1}{F(n, 1/2, n + 3/2; \rho)} - \left( \frac{1}{2} - \frac{3}{4\pi} \right) \frac{\pi}{(1 + \xi)^2} F(n, 1/2, n + 3/2; \rho), \quad (A2) $$

$$ b^{p}_{n}(\xi) = - \frac{n(n+1)(n+2)(n+3)}{2n+3} \left[ (1 + \xi)^{-1} \left( F(n+1, -3/2, n + 5/2; \rho) \right) \right] - \frac{6}{n+3} \frac{F(n+1, -1/2, n + 5/2; \rho)}{F(n, 1/2, n + 3/2; \rho)}, \quad (A3) $$

$$ c^{p}_{n}(\xi) = \frac{4n(n+1)}{(1 + \xi)^{-1}} \frac{F(n+1, -1/2, n + 3/2; \rho)}{F(n, 1/2, n + 3/2; \rho)}. \quad (A4) $$

Similarly, for the vector channel,

$$ A^{V}_{n}(\xi) = \frac{3}{4\pi^{2}} \frac{2^{n}(n+1)(n-1)!}{(2n+3)!} \left[ (4n^{2})^{n} (1 + \xi)^{n} F(n, 1/2, n + 5/2; \rho) \right], \quad (A5) $$

$$ a^{V}_{n}(\xi) = \frac{(2n+1)!!}{3 \cdot 2^{n-1} n!} \left[ \frac{\pi}{2(n+1)} \left( \frac{1}{2} - \frac{3}{4\pi} \right) F(n, 2 + n; 1 + 2n; \rho) \right] \frac{1}{n+1} \frac{\ln(2 + \xi)}{(1 + \xi)^2} \frac{F(n+1, 2 + n, 7/2; \rho)}{F(n, 1/2, n + 5/2; \rho)}, \quad (A6) $$

$$ b^{V}_{n}(\xi) = - \frac{n(n+1)(n+2)(n+3)}{2n+5} \left[ F(n+2, -1/2, n + 7/2; \rho) \right] - \frac{2n}{n+3} \frac{\ln(2 + \xi)}{(1 + \xi)^2} \frac{F(n+1, 1/2, n + 5/2; \rho)}{F(n, 1/2, n + 5/2; \rho)}, \quad (A7) $$

$$ c^{V}_{n}(\xi) = \frac{4n(n+1)}{(1 + \xi)^{-1}} \frac{F(n, 1/2, n + 5/2; \rho)}{F(n, 1/2, n + 5/2; \rho)}. \quad (A8) $$

In Eqs. (A1) and (A5), $m$ is the running quark mass $m = m_{c}(p^{2} = -(\xi + 1)m_{c}^{2})$ which is given by [56].

$$ \frac{m_{c}(\xi)}{m_{c}(\xi = 0)} = 1 - \frac{\alpha_{s}}{\pi} \left[ \frac{2 + \xi}{1 + \xi} \ln(2 + \xi) - 2 \ln 2 \right] \quad (A9) $$
