A multi-charged particle model to explain muon $g - 2$, flavor physics, and possible collider signature

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Abstract

We consider a model with vector-like fermions which also provide multi-charged particles. We search for allowed parameter region explaining muon anomalous magnetic moment (muon $g - 2$) and $b \rightarrow s \ell^+ \ell^-$ anomalies, satisfying constraints from the lepton flavor violations and $Z$ boson decays, and discuss collider physics, in a framework of multi-charged particles. While carrying out numerical analysis, we explore the typical size of the muon $g - 2$ and Wilson coefficients to explain $b \rightarrow s \ell^+ \ell^-$ anomalies when all other experimental constraints are satisfied. Furthermore, we discuss a possible extension of our model introducing $U(1)_{\mu - \tau}$ gauge symmetry and investigate possible collider signatures at LHC.
I. INTRODUCTION

Muon anomalous magnetic moment (muon $g - 2$) is analyzed with high precision experimentally and theoretically and it is a promising observable to test/confirm new physics beyond the standard model (SM). In fact, the discrepancy between the standard model (SM) prediction and experiments is $\Delta a_\mu = (26.1 \pm 8) \times 10^{-10}$, \hspace{1cm} (I.1)
where the deviation reaches $3.3\sigma$ with a positive value. In addition to that, the recent theoretical analysis indicates $3.7\sigma$ deviation \cite{2}. Moreover, several upcoming experiments will provide the results with higher precision such as Fermilab E989 \cite{3} and J-PARC E34 \cite{4}. In order to explain the deviation theoretically, several mechanisms have been proposed historically, for example, gauge contributions \cite{5-7}, Yukawa contributions at one-loop level \cite{8}, and Barr-Zee contributions \cite{9} at two-loop level. In particular, if muon $g - 2$ is related to the other phenomenologies such as neutrino mass generations and dark matter, the new Yukawa interactions become important where muon $g - 2$ would be explained at one-loop level through such interactions \cite{8,10-35}. In such case, one has to simultaneously satisfy several constraints of lepton flavor violations (LFVs), such as $\ell_i \rightarrow \ell_j \gamma$, $\ell_i \rightarrow \ell_j k \ell_k$, ($i,j,k,\ell = (e, \mu, \tau)$), and lepton flavor conserving(violating) $Z$ boson decays $Z \rightarrow \ell \ell'$, $Z \rightarrow \nu \bar{\nu}'$ \cite{36}. In particular, $\ell_\mu \rightarrow \ell_e \gamma$ process gives the most stringent constraint where the current upper bound on the branching ratio is $4.2 \times 10^{-13}$ \cite{37}, and its future bound will reach the sensitivity at $6 \times 10^{-14}$ \cite{38}. In addition, $Z$ boson decays will be tested by future experiments such as at CEPC \cite{39}.

Previously, we analyzed models introducing multi-charged fields (scalars and fermions) with general $U(1)_Y$ hypercharges to get positive muon $g - 2$ and explored the parameter region satisfying several experimental constraints \cite{34}. In the previous scenario, we only considered exotic leptons in fermion sector but there would also be extra vector-like quarks possessing exotic hypercharges. Interestingly, existence of such exotic quarks can explain other existing experimental anomalies of semileptonic $B$-meson decay; $B_s \rightarrow \mu \mu$ decay, deviations in the measurements of the angular observable $P_5'$ in the decay of the $B$ meson ($B \rightarrow K^* \mu^+ \mu^-$) \cite{40-44}, the ratio of branching fractions $R_K = BR(B^+ \rightarrow K^+ \mu^+ \mu^-)/BR(B^+ \rightarrow K^+ e^+ e^-)$ \cite{45-47}, and $R_{K^*} = BR(B \rightarrow K^* \mu^+ \mu^-)/BR(B \rightarrow K^* e^+ e^-)$ \cite{48}. Hence, it is worthwhile to
introduce vector-like quarks in the model discussed in ref. [34] and investigate if both muon $g-2$ and $B$-anomalies can be explained simultaneously.

In this paper, we discuss the model introducing multi-charged fields (scalars and fermions), as an extension of the model in ref. [34]. Extra vector-like quarks are introduced and Wilson coefficient is calculated to explain $B$-anomalies. Constraints from meson anti-meson mixing are discussed in addition to LFV and $Z$ decays. Then we explore the parameter region accommodating both muon $g-2$ and $B$-anomalies. Furthermore, we also discuss a possible extension of the model introducing $U(1)_{\mu-\tau}$ gauge symmetry to obtain better fit. Since multi-charged fields can be produced at the Large Hadron Collider (LHC) and the signature of exotic charged particles are also explored. We particularly focused on the LHC signatures of exotic lepton doublet. As we will show in the following sections that a small mass difference between the charged Higgs and the vector-like lepton (VLL) is naturally implied from the muon $(g-2)$, the collider signature of this particular model will contain soft muons. We particularly focused on the signature of two oppositely charged muon and tau pair at LHC.

This paper is organized as follows. In Sec. II, we show setup of the model and formulate the Wilson coefficient for $B$-decay, meson anti-meson mixing, LFV’s, muon $g-2$ and $Z$ boson decays. In Sec. III, we perform numerical analysis searching for the allowed region of parameter space. In Sec. IV, we discuss possible extension of the model introducing $U(1)_{\mu-\tau}$ gauge symmetry and discuss collider physics signature. We conclude in Sec. V.

### II. MODEL SETUP AND CONSTRAINTS

In our set up of the model, we introduce isospin doublet fermions $L'_a \equiv [\psi_a^-, \psi_a^-]^T$, $Q'_a \equiv [g_a^{L-1/3}, g_a^{L-4/3}]^T \equiv [u'_a, d'_a]^T (a = 1)$ and a singly-charged boson $s^+$, as shown in Table I.

|          | $L$ | $e_R$ | $L'$ | $Q'$ | $H$ | $s^+$ |
|----------|-----|-------|------|------|-----|-------|
| $SU(3)$  | 1   | 1     | 1    | 3    | 1   | 1     |
| $SU(2)_L$ | 2   | 1     | 2    | 2    | 2   | 1     |
| $U(1)_Y$ | $-\frac{1}{2}$ | $-1$ | $-\frac{3}{2}$ | $-\frac{5}{6}$ | $\frac{1}{2}$ | $+1$ |

TABLE I: Charge assignments of fields under $SU(2)_L \times U(1)_Y$. The Lagrangian involving the interaction of new particles and SM and the potential is given
by,

\[-\mathcal{L}_Y^n = f_{ia} \bar{L}_{Li} L'_R R_a s^+ + g_{ia} \bar{Q}_{Li} Q'_R R_a s^+ + h_{ij} \bar{L}_{Li} \cdot L_{Li} s^+ + M_{Q^a} \bar{Q}'_a Q'_R R_a + M_{\psi a} \bar{L}'_{La} L'_R + \text{h.c.} \]

\[
= f_{ia}[\bar{\nu}_i P_R \psi_a^- s^+ + \bar{\ell}_i P_R \psi_a^- s^+] + g_{ia}[\bar{u}_i P_R u'_a s^+ + \bar{d}_i P_R d'_a s^+] + h_{ij}[\bar{\nu}_i \bar{P}_L \ell_j s^+ - \bar{\ell}_i \bar{P}_R \nu_j s^+]
\]

\[+ M_{Q^a} \bar{Q}'_a Q'_R + M_{\psi a} \bar{L}'_{La} L'_R + \text{h.c.}, \tag{II.1}\]

\[\mathcal{V} = \mu_H^2 |H|^2 + \mu_3^2 |s^+|^2 + \lambda_H |H|^4 + \lambda_4 |s^+|^4 + \lambda_{Hs} |H|^2 |s^+|^2, \tag{II.2}\]

where $i, a = 1 - 3$ are generation indices, and $(\cdot \equiv i \sigma_2)$. The SM Yukawa term $y_{\ell_i} \bar{L}_{Li} e_{R_i} H$ provides masses for the charged leptons ($m_{\ell_i} \equiv y_{\ell_i} v / \sqrt{2}$) by developing a nonzero vacuum expectation value (VEV) of $H$, which is denoted by $\langle H \rangle \equiv v / \sqrt{2}$. We expect that the interaction term involving $h_{ij}$ plays a role in $s^+$ decay into the SM fields appropriately. However, since this term gives negative contribution to the muon $g - 2$, we assume the scale of $h_{ij}$ is not so large. It implies that we do not discuss LFVs and muon $g - 2$ of this term. The exotic fermion mass eigenvalues are respectively $M_{Q'}, M_\psi$ for $Q', L'$. The mass eigenvalue of $s^+$ is given by

\[m_S = \mu_S^2 + \frac{\lambda_{Hs}}{2} v^2. \tag{II.3}\]

### A. $M - \bar{M}$ mixing

The parameter space of our model get constrained from the neutral meson mixings, where the VLQ’s appear in the loop. The relevant expressions as shown in [49], are

\[\Delta M_Q \approx \frac{m_Q f_Q^2}{3(4\pi)^2} \sum_{a,b=1}^3 \text{Re}[g_{ka} g_{ai}^\dagger g_{jb} g_{be}^\dagger] F_{\text{box}}(M_{Q^a}, M_{Q^i}, m_S), \tag{II.4}\]

\[F_{\text{box}}(m_1, m_2, m_3) = \int_0^1 \frac{z[dx]}{x m_1^2 + y m_2^2 + z m_3^2}, \tag{II.5}\]

where $B_s - \bar{B}_s$ mixing corresponds to $(i, j, k, \ell) = (2, 3, 3, 2)$, $B_d - \bar{B}_d$ mixing corresponds to $(i, j, k, \ell) = (1, 3, 3, 1)$, $K - \bar{K}$ and $D - \bar{D}$ to $(i, j, k, \ell) = (1, 2, 2, 1)$. The neutral meson mixing formulas should be lower than the experimental bounds as given in [49, 50]:

\[\Delta m_K \lesssim 3.48 \times 10^{-15} \text{ [GeV]}, \tag{II.6}\]

\[3.29 \times 10^{-13} \text{ [GeV]} \lesssim \Delta m_{B_d} + \Delta m_{B_d}^{\text{SM}} \lesssim 3.37 \times 10^{-13} \text{ [GeV]}, \tag{II.7}\]

\[1.16 \times 10^{-11} \text{ [GeV]} \lesssim \Delta m_{B_s} + \Delta m_{B_s}^{\text{SM}} \lesssim 1.17 \times 10^{-11} \text{ [GeV]}, \tag{II.8}\]

\[\Delta m_D \lesssim 6.25 \times 10^{-15} \text{ [GeV]}, \tag{II.9}\]
where we have taken 3σ interval and $m_M$ and $f_M$ are the meson mass and the meson decay constant, respectively. The following values of the parameters are used in our analysis: $f_K \approx 0.156$ GeV, $f_{B_d(B_s)} \approx 0.191(0.274)$ GeV [51, 52], $f_D \approx 0.212$ GeV, $m_K \approx 0.498$ GeV, $m_{B_d(B_s)} \approx 5.280(5.367)$ GeV, and $m_D \approx 1.865$ GeV. The SM contributions are given by [53]:

$$2.96 \times 10^{-13} \text{[GeV]} \lesssim \Delta m_{B_d}^{SM} \lesssim 5.13 \times 10^{-13} \text{[GeV]},$$  \hspace{1cm} (II.10)

$$1.06 \times 10^{-11} \text{[GeV]} \lesssim \Delta m_{B_s}^{SM} \lesssim 1.44 \times 10^{-11} \text{[GeV]}.\hspace{1cm} (II.11)$$

Subtracting the SM contributions from the experimental results, and one finds the following bounds:

$$-1.85 \times 10^{-13} \text{[GeV]} \lesssim \Delta m_{B_d} \lesssim 4.05 \times 10^{-14} \text{[GeV]},$$ \hspace{1cm} (II.12)

$$-2.77 \times 10^{-12} \text{[GeV]} \lesssim \Delta m_{B_s} \lesssim 1.07 \times 10^{-12} \text{[GeV]}.\hspace{1cm} (II.13)$$

B. $b \to s\ell_i\bar{\ell}_j$ decay

Effective Lagrangian to estimate the decay $b \to s\ell_i\bar{\ell}_j$ is given by,

$$\mathcal{L} = -\frac{g_2 g_{a3} f_{jib} f_{ab}}{4(4\pi)^2} (\bar{s}\gamma_{\mu} P_L b)(\bar{\ell}_j \gamma^\mu \ell_i - \bar{\ell}_j \gamma^\mu \gamma_5 \ell_i) F_{\text{box}}(M_{Q_a}, M_{\psi_b}, m_s),$$ \hspace{1cm} (II.14)

which corresponds to $\mathcal{O}_9 = -\mathcal{O}_{10}$ [54]. To explain $b \to s\mu\bar{\mu}$ anomalies the required Wilson coefficient $\Delta C^\mu_9$ is given by $-0.68$ as the best fit value [54].

$$\Delta C^\mu_9 \equiv \Delta C^\ell_9 = C^\ell_9 - C^{\ell}_{10},$$ \hspace{1cm} (II.15)

where $C_{SM} \equiv \sqrt{\frac{V_{tb} V_{ts}^*}{2\pi}} G_F \alpha_{em}$. We have to consider a constraint via $B \to \mu\bar{\mu}$ which is

$$0.7 \lesssim \text{Re}[X^e - X^\mu] \lesssim 1.5,$$ \hspace{1cm} (II.16)

where $X^\ell \equiv C^\ell_9 - C^{\ell}_{10}$.

C. Lepton flavor violations and muon anomalous magnetic moment

The Yukawa terms $f_{ij}$ and $g_{ij}$ give rise to $\ell_i \to \ell_j \gamma$ processes at one-loop level. The branching ratio is given by,

$$\text{BR}(\ell_i \to \ell_j \gamma) \approx \frac{48\pi^3 \alpha_{em}}{G_F^2 m_{\ell_i}^2} C_{ij} \left(|a_{L_{ij}}|^2 + |a_{R_{ij}}|^2\right),$$ \hspace{1cm} (II.17)

\footnote{1σ range of $\Delta C^\ell_9$ is $[-0.85, -0.50]$, and 3σ range is $[-1.22, -0.18]$.}
where $G_F \approx 1.166 \times 10^{-5} \text{GeV}^{-2}$ is the Fermi constant, $\alpha_{em}(m_Z) \approx 1/128.9$ is the fine-structure constant, $C_{21} \approx 1$, $C_{31} \approx 0.1784$, and $C_{32} \approx 0.1736$. $a_{L/R}$ can be expressed as,

$$a_{Lij} \approx -m_{\ell_i} \sum_{a=1-3} f_{ja} f_{ai}^\dagger \left[ F(\psi_a^{-}, s^+) + 2F(s^+, \psi_a^{-}) \right],$$

(II.18)

$$a_{Rij} \approx -m_{\ell_i} \sum_{a=1-3} f_{ja} f_{ai}^\dagger \left[ F(\psi_a^{-}, s^+) + 2F(s^+, \psi_a^{-}) \right],$$

(II.19)

$$F(m_1, m_2) \approx \frac{(m_1^2 - m_2^2)\{5m_1^2m_2^2 - 4m_1^4 + 5m_2^4\} - 12m_1^2m_2^2\{-m_2^2 + 2m_1^2\} \ln \left[ \frac{m_1}{m_2} \right]}{12(m_1^2 - m_2^2)^4},$$

(II.20)

where $m_{\psi^{-}} \equiv m_\psi$, and $m_{h^-} \equiv m_h$. The current experimental upper bounds given by [37] are,

$$B(\mu \to e\gamma) \leq 4.2 \times 10^{-13} (6 \times 10^{-14}), \quad B(\tau \to \mu\gamma) \leq 4.4 \times 10^{-8}, \quad B(\tau \to e\gamma) \leq 3.3 \times 10^{-8},$$

(II.21)

where parentheses of $\mu \to e\gamma$ is a future reach of MEG experiment [38].

**The muon anomalous magnetic moment ($\Delta a_\mu$):** The muon anomalous magnetic moment can be estimated through $a_{L,R}$, which is given by

$$\Delta a_\mu \approx -m_\mu(a_L + a_R)_{22}.$$  

(II.22)

The measured value show a $3.3\sigma$ deviation from the SM prediction, given by $\Delta a_\mu = (26.1 \pm 8) \times 10^{-10}$ [1], which is also a positive value.

**D. Flavor-Conserving(Changing) Leptonic Z Boson Decays**

Here, we consider the Z boson decay into two leptons through the Yukawa terms involving $f_{ij}$ at one-loop level [23]. Since some components of $f_{ij}$ are expected to be large in order to obtain a sizeable $\Delta a_\mu$, the experimental bounds on Z boson decays could be of concern at
FIG. 1: Feynman diagrams for $Z \to \ell_i \bar{\ell}_j$ (up) and $Z \to \nu_i \bar{\nu}_j$ (down), where $h \equiv s$, $n = 1$.

one loop level. First of all, the relevant Lagrangian is given by

$$
\mathcal{L} \sim \frac{g_2}{c_w} \left[ \bar{\ell} \gamma^\mu \left( -\frac{1}{2} P_L + s_W^2 \right) \ell + \frac{1}{2} \bar{\nu} \gamma^\mu P_L \nu \right] Z^\mu
+ \frac{g_2}{c_w} \left[ \bar{\psi} \gamma^\mu \psi - \left( -\frac{1}{2} P_L + 2 s_W^2 \right) \bar{\psi}^+ \gamma^\mu \psi^+ \right] Z^\mu,
+ i g_2 s_W^2 \frac{c_W}{c_W} \left( s^+ \partial^\mu s^- - s^- \partial^\mu s^+ \right) Z^\mu,
$$

(II.23)

where $s(c)_W \equiv \sin(\cos) \theta_W \sim 0.23$ stands for the sine (cosine) of the Weinberg angle. The decay rate of the SM at tree level is then given by

$$
\Gamma(Z \to \ell_i \ell_j^\pm)_{SM} \approx \frac{m_Z}{12 \pi} \frac{g_2^2}{c_W^2} \left( s_W^4 - \frac{s_W^2}{2} + \frac{1}{8} \right) \delta_{ij},
$$

(II.24)

$$
\Gamma(Z \to \nu_i \bar{\nu}_j)_{SM} \approx \frac{m_Z}{96 \pi} \frac{g_2^2}{c_W^2} \delta_{ij}.
$$

(II.25)

Combining all the diagrams in Fig. 1, the ultraviolet divergence cancels out and only the finite part remains and is given by,

$$
\Delta \Gamma(Z \to \ell_i \ell_j^\pm) \approx \frac{m_Z}{12 \pi} \frac{g_2^2}{c_W^2} \left[ \frac{|B_{ij}^\ell|^2}{2} - \text{Re}[A_{ij}(B_{ij}^\ell)^*] - \left( -\frac{s_W^2}{2} + \frac{1}{8} \right) \delta_{ij} \right],
$$

(II.26)

$$
\Delta \Gamma(Z \to \nu_i \bar{\nu}_j) \approx \frac{m_Z}{24 \pi} \frac{g_2^2}{c_W^2} \left[ |B_{ij}|^2 - \delta_{ij} \right] / 4,
$$

(II.27)

\[\text{We neglect one-loop contributions in the SM.}\]
where,

\[
A_{ij} \approx s_{W}^{2} \delta_{ij}, \quad B_{ij}^{\nu} \approx \frac{\delta_{ij}}{2} - \frac{f_{ia}f_{aj}^{\dagger}}{(4\pi)^{2}} G^{\nu}(m_{\psi}, m_{S}), \quad B_{ij}^{\nu} \approx \frac{\delta_{ij}}{2} + \frac{f_{ia}f_{aj}^{\dagger}}{(4\pi)^{2}} G^{\nu}(m_{\psi}, m_{S}),
\]

(II.28)

\[
G^{\nu}(m_{\psi}, m_{S}) \approx -s_{W}^{2} \left( -\frac{1}{2} + s_{w}^{2} \right) H_{1}(m_{\psi}, m_{S}) - \left( -\frac{1}{2} + s_{w}^{2} \right)^{2} H_{2}(m_{\psi}, m_{S})
\]

+ \left( -\frac{1}{2} + 2s_{w}^{2} \right) H_{3}(m_{\psi}, m_{S}),
\]

(II.29)

\[
G^{\nu}(m_{\psi}, m_{S}) \approx -s_{W}^{2} \left( -\frac{1}{2} + s_{w}^{2} \right) H_{1}(m_{\psi}, m_{S}) - \frac{1}{2} H_{2}(m_{\psi}, m_{S}) + \left( -\frac{1}{2} + s_{w}^{2} \right) H_{3}(m_{\psi}, m_{S}),
\]

(II.30)

\[
H_{1}(m_{1}, m_{2}) = \frac{m_{1}^{4} - m_{2}^{4} + 4m_{1}^{2}m_{2}^{2}\ln\left[ \frac{m_{2}}{m_{1}} \right]}{2(m_{1}^{2} - m_{2}^{2})^{2}},
\]

(II.31)

\[
H_{2}(m_{1}, m_{2}) = \frac{m_{1}^{4} - 4m_{1}^{2}m_{2}^{2} + 3m_{1}^{2} - 4m_{2}^{2}(m_{1}^{2} - 2m_{2}^{2})\ln[m_{2}] - 4m_{1}^{4}\ln[m_{1}]}{4(m_{1}^{2} - m_{2}^{2})^{2}},
\]

(II.32)

\[
H_{3}(m_{1}, m_{2}) = m_{1}^{2} \left( \frac{m_{1}^{2} - m_{2}^{2} + 2m_{2}^{2}\ln\left[ \frac{m_{2}}{m_{1}} \right]}{(m_{1}^{2} - m_{2}^{2})^{2}} \right).
\]

(II.33)

Notice here that the upper index of B represents \( \psi \equiv \psi^{-} \) for charged-lepton final state, while \( \psi \equiv \psi^{-} \) for the neutrino final state. The current bounds on the lepton-flavor-conserving(changing) Z boson decay branching ratios at 95 % CL are given by [36]:

\[
\Delta BR(Z \rightarrow \text{Invisible}) \approx \sum_{i,j=1-3} \Delta BR(Z \rightarrow \nu_{i}\bar{\nu}_{j}) < \pm 5.5 \times 10^{-4},
\]

(II.34)

\[
\Delta BR(Z \rightarrow e^{\pm}e^{\mp}) < \pm 4.2 \times 10^{-5}, \quad \Delta BR(Z \rightarrow \mu^{\pm}\mu^{\mp}) < \pm 6.6 \times 10^{-5},
\]

\[
\Delta BR(Z \rightarrow \tau^{+}\tau^{-}) < \pm 8.3 \times 10^{-5},
\]

(II.35)

\[
\text{BR}(Z \rightarrow e^{\pm}\mu^{\mp}) < 7.5 \times 10^{-7}, \quad \text{BR}(Z \rightarrow e^{\pm}\tau^{\mp}) < 9.8 \times 10^{-6},
\]

\[
\text{BR}(Z \rightarrow \mu^{\pm}\tau^{\mp}) < 1.2 \times 10^{-5}.
\]

(II.36)

\[
\Delta BR(Z \rightarrow f_{i}\bar{f}_{j}) \quad (i=j) \text{ is defined by}
\]

\[
\Delta BR(Z \rightarrow f_{i}\bar{f}_{j}) \approx \frac{\Gamma(Z \rightarrow f_{i}\bar{f}_{j}) - \Gamma(Z \rightarrow f_{i}\bar{f}_{j})_{SM}}{\Gamma_{Z}^{\text{tot}}},
\]

(II.37)

where the total Z decay width \( \Gamma_{Z}^{\text{tot}} = 2.4952 \pm 0.0023 \text{ GeV} \) [36]. We consider all these constraints in the numerical analysis in the next section.
III. NUMERICAL ANALYSIS

In this section we perform a numerical analysis to satisfy all the phenomena which we have discussed above. In stead of global analysis, we select a specific parametrization, taking $f_{21}, g_{31,21,11} \neq 0$ and zero for the other Yukawa parameters. Because of such choice of parameterization, we do not not need to consider any constraints of LFVs and $B_s \to \mu\mu$.

Once we set $g_{31,21,11} \neq 0, b \to s\mu\bar{\mu}$ and the neutral meson mixing formulas simplify as,

$$
C_9^\mu \sim g_{21}g_{31}^*|f_{21}|^2, \quad \Delta m_K \sim g_{21}g_{11}^*, \quad \Delta m_{B_s} \sim g_{31}g_{21}^*, \quad \Delta m_{B_d} \sim g_{31}g_{11}^*, \quad \Delta m_D \sim g_{11}g_{21}^*. 
$$

(III.1)

Since we would like to increase $C_9^\mu$ as large as possible, while all the meson mixings want to be as tiny as possible, the following hierarchy would be preferable:

$$
g_{11} \ll g_{21} \lesssim g_{31}.
$$

(III.2)

Under this set up, the allowed parameter space to satisfy muon $g - 2$ is shown in Fig. 2. We have also plotted the contours for $\Delta BR(Z \to \mu^+\mu^-)$ and $\Delta BR(Z \to \text{Invisible})$ that give the most stringent bounds. We find the upper bound on $f_{21}$ is 0.6 and 0.8 for $3\sigma$ and $5\sigma$ values of $\Delta C_9^\mu$ respectively. This upper bound is very important to analyze $b \to s\mu\bar{\mu}$ and neutral meson mixings. Then, we estimate $C_9^\mu$ by adopting $f_{21} = 0.8$ in order to maximize the value. However, the maximum $|C_9^\mu|$ we obtain is 0.115 at most that is out of the $3\sigma$ range of experimental result, where we take $M \lesssim 350$ GeV and $M_{Q'} > 1000$ GeV, where $M \equiv M_{\psi_1} = m_S$ and $M_{Q'} \equiv M_{Q'_1}$. The stringent constraint arises from $\Delta m_{B_s}$, because they ($\Delta C_9^\mu$ and $\Delta m_{B_s}$) are proportional to the same combination $g_{31}g_{21}$. We will discuss a possible extension of this model in order to explain sizeable $\Delta C_9^\mu$.

IV. ANALYSIS IN THE $U(1)_{\mu-\tau}$ EXTENDED MODEL

In the previous section, we find that it is difficult to obtain sizeable $\Delta C_9^\mu$ when we impose constraints from $B_s$-$B_s$ mixing and $\Delta BR(Z \to \mu^+\mu^-)$. In this section, we discuss possible extension of the model to accommodate the best fit value of $\Delta C_9^\mu$ without changing original

\footnote{If one extends $g_{ij}$ to be complex, then one can evade the constraint of $\Delta m_{B_s}$ and keep large value of $|\Delta C_9^\mu|$. However, in this case, another experimental bound of CP asymmetry $A_{CP}$ arises and it gives more stringent constraint \[55\].}
structure by introducing local $U(1)_{\mu-\tau}$ symmetry. Under the new $U(1)_{\mu-\tau}$ gauge symmetry, we assign charges to the extra fermion and scalar fields as summarized in Table II, where $x$ and $y$ are free parameter and the SM quarks are not charged under $U(1)_{\mu-\tau}$. Here, we also introduced scalar field $\varphi$ with non-zero VEV to break $U(1)_{\mu-\tau}$ spontaneously. Then we apply the same mechanism in refs. [56, 57] to generate $\Delta C^u_9$ using $Z'$ interaction. The Yukawa interactions relevant to muon $g-2$ and $\Delta C^u_9$ become,

$$-\mathcal{L}_Y \supset f_{2a} \bar{L}_L L'_{R_a} s^+ + g_{ia} \bar{Q}_L Q'_{R_a} s^+ + \text{h.c.} \quad \text{(IV.1)}$$

Note that the first term involves only second generation of the SM lepton due to $U(1)_{\mu-\tau}$ symmetry, and this is the advantage of the extension, since we can suppress LFV without assuming small values of the Yukawa coupling constants.

The additional terms in the scalar potential are,

$$V \supset \mu_\varphi^2 |\varphi|^2 + \lambda_\varphi |\varphi|^4 + \lambda_{H\varphi} |H|^2 |\varphi|^2 + \lambda_{S\varphi} |s^+|^2 |\varphi|^2. \quad \text{(IV.2)}$$

For simplicity, we assume coupling $\lambda_{H\varphi}$ is small so that mixing between $\varphi$ and $H$ is negligible. Under the assumption, the VEV of $\varphi$ is simply given by $v_\varphi \simeq \sqrt{-\mu_\varphi^2/\lambda_\varphi}$. After $\varphi$ developing

FIG. 2: Contours of $\Delta a_\mu$, $\Delta BR(Z \to \mu^+\mu^-)$ and $\Delta BR(Z \to \text{Invisible})$ on $\{M(=m_S=M_\psi), f_{21}\}$ plane. The left-figure is for $3\sigma$ and the right-one is for $5\sigma$ interval of $\Delta BR(Z \to \mu^+\mu^-)$ and $\Delta BR(Z \to \text{Invisible})$. The (light-)yellow region represents $(2\sigma)1\sigma$ region for muon $g-2$. The two additional lines correspond to $\Delta a_\mu = 5 \times 10^{-10}$ and $1 \times 10^{-10}$ for reference. The shaded region with dotted (dot-dashed line) line is excluded by $\Delta BR(Z \to \mu^+\mu^-)$ ($\Delta BR(Z \to \text{Invisible})$).


\[ m_{Z'} = y g' v_\varphi, \]

where \( g' \) is gauge coupling associated with \( U(1)_{\mu-\tau} \).

In our extended setup, we obtain contribution to \( \Delta C_9^\mu \) from diagrams in Fig. 3. Then we obtain the contribution to \( \Delta C_9^{\mu,Z'} \) as in Ref. [56, 57]

\[ \Delta C_9^{\mu,Z'} \sim \frac{x g'^2}{4\pi^2 m_{Z'}^2 C_{\text{SM}}} \sum_{a=1}^3 g_{3a} g_{a2}^* \int [dx] \ln \left( \frac{\Delta[M_{Q_a}, m_S]}{\Delta[m_S, M_{Q_a}]} \right), \]

\[ C_{\text{SM}} \equiv \frac{V_{tb} V_{ts}^* G_F m_{W}}{\sqrt{2} \pi}, \]

\[ \Delta[m_1, m_2] = (x + y - 1)(x m_b^2 + y m_s^2) + x m_1^2 + (y + z)m_2^2, \]

where \([dx]_3 \equiv dx dy dz \delta(1 - x - y - z)\). We can obtain \( \Delta C_9^{\mu,Z'} \sim -1 \) satisfying all the experimental constraints as shown in refs. [56, 57] with \( M_{Q_a} = \mathcal{O}(1) \) TeV, \( m_S = \mathcal{O}(100) \) GeV and \( m_{Z'} = \mathcal{O}(100) \) GeV, where \( Z' \) contribution to muon \( g - 2 \) is small in this region.

Here, similar to the previous analysis, we assume \( g_{31,32,11} \neq 0 \) and the other couplings are
zero, and simplify the above formula supposing \(m_b, m_s \ll m_S, M_{Q'}\) as follows:

\[
\Delta C^\mu, Z'_{9} \simeq \frac{g_{31}g_{12}xg^2}{(4\pi)^2m_{Z'}^2C_{SM}} \left[ \frac{-M_{Q'}^4 + m_s^4 - 4M_{Q'}^2m_s^2\ln \left(\frac{m_{S}}{M_{Q'}}\right)}{2(M_{Q'}^2 - m_s^2)^2} \right],
\]

(IV.5)

where the above quantity is zero when \(m_s = M_{Q'}\). Then, the \(b \to s\mu\bar{\mu}\) anomalies can be explained by \(\Delta C^\mu, Z'_{9} = -1.11\) as the best fit value, \([-1.31, -0.90]\) at 1\(\sigma\), and \([-1.67, -0.46]\) at 3\(\sigma\) interval [54]. Notice that here we do not need to consider the constraint of \(B_s \to \mu\bar{\mu}\) since \(\Delta C_{10}\) is not induced by the \(Z'\) interactions. It is worthwhile considering \(\Delta a_{\mu}^{Z'}\) via \(Z'\), even though it would not definitely be needed because we already have the contribution via \(f_{ij}\) and preferred mass range is lighter than that for the \(B\) anomalies. The formula is given by [58]

\[
\Delta a_{\mu}^{Z'} = \frac{g^2m_{\mu}^2}{4\pi^2}\int_{0}^{1}\frac{x^2(1-x)}{x^2m_{\mu}^2 + (1-x)m_{Z'}^2}.
\]

(IV.6)

Here, we have to consider the most stringent constraint from trident process [6] and its experimental result is given by CCFR [59]. Left plot in Fig. 4 represents allowed parameter space in terms of \(m_{Z'}\) and \(g'\) to explain the \(b \to s\mu\bar{\mu}\) anomalies via \(\Delta C'^{\mu, Z'}_{9}\), where we take the following input parameters \(x = -2, |g_{31}| = [0.1, \sqrt{4\pi}], |g_{21}| = [0.01, 1], |g_{11}| = [0.001, 1], |g'| = [10^{-5}, 1], m_{Z'} = [10^{-3}, 10^3]\) GeV, \(M = [90, 1100]\) GeV, \(m_s = [M - 20, M - 10]\) GeV, \(M_{Q'} = [1000, 2000]\) GeV. Here, the red region satisfies \(\Delta C_9\) value within 3\(\sigma\) interval, while the blue one satisfies it within 1\(\sigma\) interval. We also show the parameter region excluded by the CCFR experiment and the LHC measurement searching for \(pp \to \mu\bar{\mu}Z'(\to \mu\bar{\mu})\) process [60]. We find that parameter region of \(m_{Z'} \lesssim 70\) GeV is excluded by the LHC constraints while heavier \(Z'\) region can accommodate with \(B\) anomalies. Right plot in Fig. 4 shows the parameter region explaining muon \(g - 2\) where the green(red) region satisfies \(\Delta a_{\mu}^{Z'} = (26.1 \pm 24) \times 10^{-10}\) within 1(3)\(\sigma\) interval. The light blue region is excluded by the CCFR experiment. Note that the other input parameters can take any value in the range as discussed above. Although we don’t have overlap region explaining \(B\) anomalies and muon \(g - 2\) by \(Z'\) contribution only we can explain both anomalies where the former one is explained by \(Z'\) and the latter one is explained by one-loop diagram with multi-charged particles as discussed in the previous sections.

Finally, we discuss the implication to collider physics when we introduce \(U(1)_{\mu - \tau}\) sym-
FIG. 4: Left: Allowed points in the parameter space of $m_{Z'}$ and $g'$ to explain the anomaly of $b \to s\mu\bar{\mu}$ where the red points satisfy $\Delta C_9$ within 3σ interval and the blue points satisfy it within 1σ interval. We also show the region excluded by the CCFR and the LHC experiments. Right: The green (red) region satisfies $\Delta a_{\mu}^{Z'} = (26.1 \pm 24) \times 10^{-10}$ at 1(3)σ interval where the light gray region is excluded by the CCFR experiment.

A. Collider physics and constraints

As discussed in the previous sections, in order to explain muon $g - 2$ and the flavor observables together, the mass scale (M) of exotic lepton doublet is required to be less than $\sim 350$ GeV. Here we are interested in the production and decay modes of the doubly charged vector like lepton (VLL) given by,
\[pp \rightarrow \psi^+\psi^-, \psi^+ \rightarrow (\mu^+h^+) \rightarrow \mu^+(\nu l^+),\]
\[\psi^- \rightarrow (\mu^-h^-) \rightarrow \mu^-(\bar{\nu} l^-).\]

Hence the final state is 1 oppositely charged muon pair + 1 oppositely charged lepton \((l)\) + MET. If \(l = e, \mu\), the final state will be a combination of four leptons \((l = e, \mu)\) with two pairs of opposite sign same flavor. As we choose \(f_{21} = 0.8\), \(\psi^{\pm\pm}\) will decay mostly into muon and a charged Higgs. Now the coupling of the charged Higgs with the SM lepton and the neutrino is defined by \(h_{ij}\), and in order to satisfy the LFV and muon \((g-2)\) data we have considered it to be of the order 0.01, allowing a small percentage of the charged Higgs to decay to the SM leptons.

Vector like leptons are constrained from the collider physics experiments. The ATLAS Collaboration performed a search for heavy lepton resonances decaying into a Z boson and a lepton in a multi lepton final state at a center-of-mass energy of 8 TeV [61], constraining singlet VLL model and excluding its mass range of 114–176 GeV. For the doublet VLL model, the L3 Collaboration at LEP placed a lower bound of \(\sim 100\) GeV on additional heavy leptons [62]. Ref. [63] and [64] have shown that the VLL’s in the mass range 120 – 740 GeV are excluded with 95% CL in different multilepton signals. In those analysis, the vectorlike leptons were singly charged and hence it only decays to a SM boson \((H, W, Z)\) and SM leptons whereas in our case, VLL’s decay in to a charged Higgs and muon specifically, followed by the decay of the off-shell or on-shell charged Higgs into neutrino and another SM lepton. Hence the characteristic of our signal is significantly different from Ref. [63] and [64].

In this analysis we choose our selections differently than Ref. [63] and [64]. As a small mass difference between the charged Higgs and the VLL is naturally implied from the muon \((g-2)\), the muon will have a very small \(pT(\sim 10)\) GeV, but the other two leptons will have a much higher \(pT\). This scenario is still allowed for VLL mass \(\lesssim 350\) GeV. There are scenarios [65, 66] when the doubly charged VLLs decays to a \(W^\pm\) and and lepton\((l^\pm)\), giving a final state of 2 oppositely charges lepton pair \((l^\pm)\) + MET. Our model also can produce the same signal but we focused on a more exotic scenario, as proposed by the \(U(1)_{\mu-\tau}\) extended model, where the charged Higgs decays to tau lepton and a neutrino. Hence in this study we select our signal to be 1 oppositely charged muon pair with very small \(pT\) + 1 oppositely charged tau pair with moderate \(pT\) + MET, and we keep the mass difference between the charged Higgs and the VLL \(\sim 10\) GeV. The same final state has also been studied for a more general
model of vector like leptons in Ref. [67]. One of the advantages of VLL with small mass is that the cross section is large which can negate the effect of the suppression due to more than one tau tagging. Moreover, in the VLL signatures studied so far by CMS and ATLAS the assumption was that VLL decays to a W or Z, which is unlikely in our case. As a result, W/Z veto can increase the signal efficiency.

We write the model Lagrangian of Eq. (II.1) in FeynRules (v2.3.13) [68, 69]. We generate the model file for MadGraph5_aMC@NLO (v2.2.1) [70] using FeynRules. Then we calculate the production cross section using the NNPDF23LO1 parton distributions [71] with the factorization and renormalization scales at the central $m_T^2$ scale after $k_T$–clustering of the event. We have computed the signal cross section of $pp \rightarrow Z/\gamma \rightarrow \psi^+\psi^-$, where $p = q, \bar{q}, \gamma$. The cross sections are normalised to the 5 flavor scheme. The inclusion of the photon PDF increases the signal cross section significantly as the coupling is proportional to the charge of the fermion. We plot the the production cross section in Fig. 5 for 13 TeV as well as 27 TeV. After showering events in PYTHIA [72], events are passed through DELPHES 3 [73] for detector simulation. In DELPHES, we choose the isolation cut for leptons to be $\Delta R_{max} = 0.5$, to ensure no hadronic activity inside this isolation cone. While generating the events, we kept the min $p_T$ for muons to be 6 GeV, and also follow other trigger requirements for the soft muons following [74]. The tau tagging efficiency is considered to be 0.6 and the
FIG. 6: The transverse momentum distribution of the leading(1) and the subleading(2) muon (left) and tau pairs (right) in unweighted events of $pp \rightarrow Z/\gamma \rightarrow \Psi^+\Psi^-$ at 13 TeV p-p collision for BP1.

FIG. 7: The transverse missing energy (MET) and sum of all lepton $p_T$ distribution is shown in the left. In the right the distribution is for the ratio of the MET and $m_{\text{eff}}$. Events are unweighted and generated by $pp \rightarrow Z/\gamma \rightarrow \Psi^+\Psi^-$ at 13 TeV p-p collision for BP1.

misidentification efficiency is 0.01.

The $p_T$ distribution of the leading and subleading tau and muon is shown in Fig. 6 for BP1. In Fig. 7 (left) we show the transverse missing energy and $H_T(l) = \sum_i p_T(l)_i$ distribution and (right) the ratio $MET/m_{\text{eff}}$ ($m_{\text{eff}} = E_T + H_T(l) + H_T(j)$), which is effective to reduce the QCD-jet backgrounds. Based on these distributions we select a set of simple cuts on different kinematic variables.
Selection 1:

- Opposite sign same flavor pair of mu and tau ($\mu^+\mu^- + \tau^+\tau^-$),
- $p_T(\mu_1) > 6$ GeV, $p_T(\mu_2) > 6$ GeV, $p_T(\tau_1) > 60$ GeV, $p_T(\tau_2) > 40$ GeV,
- $|\eta(\mu, \tau)| < 2.5$, $\Delta R(l, l) > 0.3$,

Selection 2:

- b-jet veto, $MET > 100$ GeV, $H_T > 150$ GeV,
- $MET/m_{eff} > 0.5$,

Selection 3:

- Z veto with $M_Z \pm 10$ GeV.

We show the signal cross section after the selections in Table. III for three BPs. One can see that for this multilepton channel the cross section is well above 1 fb after the selections. The signal does not suffer much from the Z-veto which is a big advantage for our signal as Z veto is effective to reduce the backgrounds from Z decays. b-jet veto and the requirement of higher ratio of MET and $m_{eff}$ will also be effective to reduce the background for these type of signal. For the discussion on the background of this particular channel one can see Ref. [67]. In general multilepton channel possesses less background compared to the other processes. After Selection 3, the number of events at 150 fb$^{-1}$ is always more than 150 if background is very small, which makes this channel a good candidate look for new physics at 13 TeV LHC run.

V. CONCLUSION

We have analyzed muon $g-2$, LFV’s, Z decays, $\Delta C_9^\mu$ for B-anomalies, and $M-\bar{M}$ mixing in a framework of multi-charged particles which includes exotic scalars, leptons and quarks. Carrying out numerical calculations, we have found that $\Delta C_9^\mu$ would reach at most $\sim -0.1$ when we impose constraints from $Z \rightarrow \nu_i\bar{\nu}_j$ invisible decay, $Z \rightarrow \mu\bar{\mu}$ decay and $B_s-\bar{B}_s$ mixing. This is due to the stringent constraints from $B_s-\bar{B}_s$ mixing and $Z \rightarrow \mu\bar{\mu}$ which restrict relevant Yukawa coupling constants. Then we have discussed possible extension of the model.
introducing $U(1)_{\mu-\tau}$ gauge symmetry in which we can obtain additional contribution to $\Delta C^\mu_y$ associated with $Z'$ interaction without changing basic structure of the model. Finally we study the collider physics focusing on the production of doubly charged leptons using some benchmark points allowed by the numerical analysis. We show that the channel with pairs of oppositely charged muon and tau has some unique features that distinguishes our model signatures from other vector like lepton signatures at LHC. The exotic vector like quarks and the $Z'$ also give interesting collider phenomenology but we keep that for future study.

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