Global neutrino parameter estimation using Markov Chain Monte Carlo

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(Dated: February 2, 2008)

We present a Markov Chain Monte Carlo global analysis of neutrino parameters using both cosmological and experimental data. Results are presented for the combination of all presently available data from oscillation experiments, cosmology, and neutrinoless double beta decay. In addition we explicitly study the interplay between cosmological, tritium decay and neutrinoless double beta decay data in determining the neutrino mass parameters. We furthermore discuss how the inference of non-neutrino cosmological parameters can benefit from future neutrino mass experiments such as the KATRIN tritium decay experiment or neutrinoless double beta decay experiments.

PACS numbers: 14.60.Pq, 98.80.-k

I. INTRODUCTION

The question of neutrino mass is one of the most profound in modern particle physics. Most plausible models of neutrino mass solve the puzzle of why neutrino masses are so small by introducing a new scale at high energy, and precision studies of neutrino physics therefore hold the potential to investigate physics at scales beyond those reachable in current accelerator experiments. They also make the study of the possible Majorana nature of neutrinos possible (see [1, 2, 3] for a thorough discussion of this). While the neutrino mass differences have now been measured at about 10% precision by oscillation experiments (see e.g. [4, 5]) the absolute mass scale remains unknown and inaccessible to oscillation experiments.

There are, however, several possible paths to measuring the absolute neutrino mass. The kinematical effect of neutrino mass can be probed either via its effect on the beta decay spectrum or via its effect on cosmological structure formation. If neutrinos are Majorana particles a different possibility is to search for neutrinoless double beta decay because the transition probability for this process is proportional to the neutrino mass squared.

In the past year there have been several papers discussing how to unify the data analysis for the various approaches [6, 7]. This is a non-trivial issue, given that completely different physics is involved and that the three probes are actually sensitive to three distinct observables.

Here we present a new Markov Chain Monte Carlo global analysis of neutrino parameters using both cosmological and experimental data. The analysis software is based on the CosmoMC Markov Chain Monte Carlo (MCMC) package for cosmological parameter estimation [7, 8], appropriately modified to incorporate all parameters related to neutrino physics. This approach uses Bayesian inference instead of the frequentist method commonly used in particle physics. The approach is somewhat similar to the MCMC technique developed in [9] to constrain MSSM parameters. However, a key difference is that here we keep the full cosmological parameter estimation which allows for a closer study of the interplay between neutrino data and cosmological parameter estimation.

In Section II we describe the methodology used and in Section III we present the main results for various different assumptions about present and future data, as well as different parameter spaces. Finally we present our conclusions in Section IV.

II. METHODOLOGY

The MCMC Bayesian inference approach has been described in detail for instance in [7, 10]. Based on assumed priors on each parameter it samples the likelihood function using the Markov Chain Monte Carlo method and from that the posterior credible intervals for all parameters can be calculated. Before running the Markov chains it is therefore necessary to specify both the parameters to be used and the priors on all parameters.

For the neutrino physics part we have used the mass of the lightest eigenstate $m_L$ ($m_L = m_1$ for the normal hierarchy, $m_L = m_3$ for the inverted hierarchy), the two mass differences $\Delta m^2_{12}$, $\Delta m^2_{23}$ and the three mixing angles $\vartheta_{12}$, $\vartheta_{23}$, $\vartheta_{13}$. We also assume that neutrinos are Majorana particles so that there are two additional Majorana phases $\phi_2$, $\phi_3$ 1, which together with the mass differences and mixing angles specify the observables related to absolute neutrino mass (assuming only active neutrinos). In total there are then 8 parameters related to the neutrino sector.

There are three separate observables related to the three different types of probes. Cosmology is only sensitive to the neutrino mass, and until the accuracy reaches the 0.05 eV level only to the sum of neutrino masses (see [11, 12] for a thorough discussion of this)

$$\sum m_\nu = m_1 + m_2 + m_3.$$  \hspace{1cm} (1)

1 These parameters are only important when neutrinoless double beta decay data is used.
In terms of the parameters used in CosmoMC this corresponds to \( m_L \), \( \Delta m^2_{21} \) and \( \Delta m^2_{32} \). These are therefore the only parameters which are regarded as “slow” in the sense that a change in one of them requires recalculation of the transfer function for cosmological perturbations.

At the projected level of accuracy of KATRIN the change in the electron energy spectrum can be described using a single effective mass parameter which is essentially the incoherent sum (see e.g. [13])

\[
m_{\beta} = (c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2)^{1/2}.
\]

The parameter actually measured in such experiments is in fact \( m_{\beta}^2 \) which, being a fit-parameter, can be positive or negative when measured.

Conversely, the effective mass measured in neutrinoless double beta decay is the coherent sum [14, 15]

\[
m_{\beta\beta} = |c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\varphi_2} + s_{13}^2 m_3 e^{i\varphi_3}|,
\]

which allows for phase cancelation.

The actual parameter measured in any neutrinoless double beta decay experiment is the half-life \( T_{1/2} \) which is related to \( m_{\beta\beta} \) by the relation [16]

\[
\frac{1}{T_{1/2}} = G^{0\nu} |M^{0\nu}|^2 m_{\beta\beta}^2.
\]

where \( G^{0\nu} \) is a phase-space factor and \( |M^{0\nu}|^2 \) the nuclear matrix element squared. In principle the MCMC code should use the measured \( T_{1/2} \) and the calculated matrix element (both including uncertainties) as parameters instead of \( m_{\beta\beta} \). However, for simplicity we assume a Gaussian error on \( m_{\beta\beta} \) with the estimated error on the matrix element from [16, 17, 18].

In the following we have used cosmological parameters consistent with the “Vanilla” \( \Lambda \) CDM model: \( \Omega_b h^2 \), the physical baryon density, \( \Omega_c h^2 \), the physical CDM density, \( A_s \), the amplitude of primordial fluctuations, \( n_s \), the scalar spectral index, \( \tau \), the optical depth to reionization, and \( H_0 \), the Hubble parameter. Spatial flatness has been assumed so that \( \Omega_\Lambda = 1 - \Omega_c - \Omega_b - \Omega_\gamma \). In total there are then 6 parameters related to the cosmological model.

For the purpose of parameter estimation the publicly available CosmoMC Markov chain Monte Carlo has been modified to perform parameter estimation in this 14-dimensional parameter space. CosmoMC has been set to use the fast/slow parameter scheme. In Table I we give the list of parameters as well as their priors. Note that we also include the two parameters \( \omega \), the dark energy equation of state, and \( \omega_s \), the running of the scalar spectral index, both of which are discussed in Section III D.

For the observables related to neutrino oscillation data we make the simple assumption of Gaussian errors, given by the combination of different experiments. We note that this assumption can easily be changed in the code and replaced with the full likelihood calculation from experimental data.

| parameter | prior | fast/slow |
|-----------|-------|-----------|
| \( \log_{10}(m_L/eV) \) | -2 - 0 | Top Hat slow |
| \( \Delta m^2_{21} \) | \((4 - 12) \times 10^{-5} \) eV$^2$ | Top Hat slow |
| \( \Delta m^2_{32} \) | \((1 - 4) \times 10^{-3} \) eV$^2$ | Top Hat slow |
| \( \sin^2(\theta_{12}) \) | 0-1 | Top Hat fast |
| \( \sin^2(\theta_{13}) \) | 0-1 | Top Hat fast |
| \( \sin^2(\theta_{23}) \) | 0-1 | Top Hat fast |
| \( \phi_2 \) | 0-2\( \tau \) | Top Hat fast |
| \( \phi_3 \) | 0-2\( \tau \) | Top Hat fast |
| \( \Omega_b h^2 \) | 0.005-0.1 | Top Hat slow |
| \( \Omega_c h^2 \) | 0.01-0.99 | Top Hat slow |
| \( \tau \) | 0.01-0.8 | Top Hat slow |
| \( n_s \) | 0.5-1.5 | Top Hat fast |
| \( \log(10^{10} A_\nu) \) | 2.4-4 | Top Hat fast |
| \( h_0 \) | 0.3-1 | Top Hat slow |
| \( w^* \) | 0-2 - 0 | Top Hat slow |
| \( \alpha^*_s \) | -0.2 - 0.2 | Top Hat fast |

TABLE I: Parameters and priors used in the likelihood analysis. Cosmological parameters marked with * are used only in Section III D.

We use the same constraints as in [5], given by

\[
\begin{align*}
\Delta m^2_{21} &= (7.92 \pm 0.71) \times 10^{-5} \text{ eV}^2 \\
\Delta m^2_{32} &= (2.6_{-0.36}^{+0.30}) \times 10^{-3} \text{ eV}^2 \\
\sin^2(\theta_{12}) &= 0.314_{-0.047}^{+0.057} \\
\sin^2(\theta_{23}) &= 0.45_{-0.09}^{+0.16} \\
\sin^2(\theta_{13}) &< 0.03,
\end{align*}
\]

with all errors being 2\( \sigma \). The assumption of Gaussian errors does not significantly alter any of our results.

### III. RESULTS

Based on the approach described in the previous section we have calculated the present bound on neutrino properties using various combinations of data sets from cosmology, tritium decay and neutrinoless double beta decay respectively.

#### A. Cosmological data

Cosmological constraints on \( \sum m_\nu \) have been calculated by many different authors for various assumptions about parameters and using different data sets (see e.g. [19, 20, 21, 22, 23, 24, 25]).

Here we present just one particular example, which is exactly the same as used in [20]. We use the WMAP CMB temperature and polarisation data [28, 27, 24, 23] and the SDSS-LRG and 2dF large scale structure data [30, 31].
the SDSS-LRG baryon acoustic oscillation data \[33\], and the SNI-a data set compiled in \[34\]. Details about the cosmological data can be found in \[26\].

Using only the cosmological data we find a bound of \(\sum m_\nu \leq 0.50\, \text{eV}\) for the minimal \(\Lambda\)CDM model. We note that this is slightly lower than the 0.6 eV found in \[26\] for the same model and data. This is worth noticing because the prior on \(m_L\) is logarithmic in the present study while it was linear in \[26\]. A logarithmic prior on \(m_L\) tends to favour small \(m_L\) values because of the large parameter space volume at negative \(\log_{10} m_L\) and therefore shifts the allowed region slightly down.

This phenomenon is an integral part of Bayesian inference because a prior probability distribution needs to be specified. In frequentist statistics this problem does not occur and the result does not depend on any priors. It should be noted that in the limit of Gaussian statistics the two methods yield exactly the same result.

The phenomenon has been recently been studied in the context of neutrino properties. For example it was shown in \[35\] that Bayesian inference and likelihood maximisation give very different results for cosmological parameters such as the radiation density as long as the likelihood function is non-Gaussian. As more data is added and the likelihood function approaches a Gaussian the two methods converge. The question of Bayesian versus frequentist statistics was studied in \[10\] in the context of KATRIN. For example the difference between a linear and a logarithmic prior on \(m_\beta\) was investigated and found to have some (not crucial) effect. In conclusion, assumptions about priors will have an effect on the posterior distributions as long as the likelihood function is non-Gaussian which is the case for parameters which are not extremely well constrained.

\section*{B. Neutrinoless double beta decay}

The upper bound on the effective neutrino decay provided by the Heidelberg-Moscow (HM) experiment provides an additional and comparable constraint on the absolute neutrino mass scale \[36\].

We use the constraint \(m_{\beta\beta} < 0.27\, \text{eV} \ (90\%)\), based on the nuclear matrix element calculation in \[17\]. Note that this mass range is more restrictive than what was used in \[5\] because the theoretically predicted half-life has been corrected downwards in \[17\] compared to \[10\]. We stress again that the conversion of half-life to effective mass \(m_{\beta\beta}\) depends strongly on the nuclear matrix element and that the bound used here could turn out to be too restrictive.

As can be seen from Table \[11\] adding the HM data does shift the allowed range on \(\sum m_\nu\) and \(m_{\beta\beta}\) down. Since the best fit cosmological model in any case has \(\sum m_\nu = 0\) it has no influence on other parameters such as \(\Omega_\text{b} h^2\) and \(n_s\).

Note that we have not derived any cosmological constraint based on the claimed positive evidence from Heidelberg-Moscow \[37\] \[38\] \[39\]. Using the same assumptions as above on the nuclear matrix element the claimed evidence translates roughly into \(0.25\, \text{eV} < m_{\beta\beta} < 0.5\, \text{eV} \ (90\% \ C.L.)\).

\section*{C. Future constraints - KATRIN and GERDA}

To get a better idea about the future interplay between the three different methods for measuring the absolute mass scale we have performed similar likelihood analyses for the presently available cosmological data together with forecasts for the KATRIN beta decay experiment \[40\] \[41\] and the GERDA neutrinoless double beta decay experiment \[42\] \[43\].

For KATRIN we assume a Gaussian 1σ error on \(m_{\beta\beta}^2\) of \(\sigma(m_{\beta\beta}^2) = 0.025\, \text{eV}^2\), roughly in accordance with what was used in \[6\] \[44\].

For the GERDA neutrinoless double beta decay experiment we assume a Gaussian error on \(m_{\beta\beta}^2\) of 0.01 eV², corresponding roughly to GERDA phase 2 \[12\]. We note that other neutrinoless double beta decay experiments such as MAJORANA and CUORE \[45\] \[46\] will reach roughly the same sensitivity in a broadly comparable time-frame (see e.g. \[14\]).

Using these very rough experimental characteristics of the two experiments we have proceeded to calculate constraints on neutrino parameters using two different assumptions:

(a) In this case we assume no positive detection from either experiment so that the best fit values are \(m_\beta = m_{\beta\beta} = 0\).

(b) Here we assume a positive detection from both experiments, \(m_{\beta\beta}^2 = 0.079\, \text{eV}^2\) and \(m_{\beta\beta}^2 = 0.032\, \text{eV}^2\). The last case would for instance be realised in a model with normal hierarchy, \(m_L = m_1 = x\). For both cases we perform parameter estimation assuming both normal and inverted hierarchy.

In the first case (a) we note that KATRIN alone would significantly tighten the cosmological constraint on \(\sum m_\nu\) and GERDA would improve this even further. \(m_{\beta\beta}\) is, as expected, mainly constrained by adding the GERDA data.

The second case (b) is more interesting from the perspective of combining data sets. The best fit values both correspond to roughly 2σ evidence for non-zero \(m_\beta\) and \(m_{\beta\beta}\) respectively. However, with the combined data \(m_{\beta\beta} = 0\) is excluded at roughly 4.5σ, likewise \(m_L = 0\) is excluded at a similar significance.

This exercise clearly shows the advantage of analysing all neutrino parameters in this global way, instead of simply adding constraints. It should be noted that since \(\sum m_\nu \sim 0\) is the best fit to present cosmological data the case (b) has a best-fit \(\chi^2\) which is higher than case (a) by \(\Delta \chi^2 = 3.1\), i.e. it gives a slightly (not substantially) worse fit to cosmological data.
TABLE II: The mean value and 95% lower and upper credible intervals for various parameters and combinations of data.

In Figs. 1 and 2 we show the likelihood contours for cases (a) and (b) for the assumption of normal hierarchy. In both cases we have used the present uncertainties on the parameters of the mixing matrix (specified in Eq. 5) which means that the non-trivial behaviour for small $m_L, m_{\beta \beta}$ cannot be resolved. Given future improved constraints from reactor or long baseline experiments this region will look significantly different (see e.g. [17, 18] for a thorough discussion of this point).

1. **KATRIN and GERDA only**

To complete this section we have done a parameter study of cases (a) and (b) using only KATRIN and GERDA data, excluding cosmological constraints. The results of this can be seen in Table III. For case (a) where there is no detection from either experiment the combination of cosmological data with KATRIN and GERDA slightly strengthens the bound on parameters, but since they center on the same best fit value there is no marked difference when cosmological data is added.

However, this changes completely when case (b) is studied. Here, KATRIN and GERDA data prefer a higher value for $\beta$ and the combination of all three data sets significantly shift the allowed range for all of the neutrino parameters. We note that other cosmological parameters such as $\Omega h^2$ and $n_s$ are not affected in any way when the minimal $\Lambda$CDM model is assumed. This conclusion does not hold when larger cosmological parameter sets are used, a point discussed in the next subsection.

D. **Extended cosmological models**

In order to illustrate the relation between neutrino experiments and cosmological parameter estimation we have performed the same analysis as before, but now adding two additional cosmological parameters to the fit: $w$, the dark energy equation of state, and $\alpha_s$, the running of the scalar spectral index (giving a total of 16
FIG. 1: 68% and 95% contours for the Cosmo+KATRIN+GERDA data, assuming best fit values of $m_\beta = m_\beta\beta = 0$ (case a). Inverted hierarchy is assumed.

| Parameter | Cosmo+KATRIN+GERDA | KATRIN+GERDA
|-----------|-------------------|----------------|
| Normal hierarchy | $m_{\beta,0} = 0$, $m_{\beta\beta,0} = 0$ | $m_{\beta,0} = 0.28$ eV, $m_{\beta\beta,0} = 0.18$ eV |
| $\log_{10} m_L$ (eV) | $-1.438_{-2.00}^{+0.958}$ | $-1.476_{-2.00}^{+0.968}$ |
| $\sum m_\nu$ (eV) | $0.168_{-0.079}^{+0.342}$ | $0.157_{-0.0789}^{+0.335}$ |
| $m_\beta/\sum m_\nu$ | $0.260_{-0.169}^{+0.322}$ | $0.254_{-0.168}^{+0.322}$ |
| $m_{\beta\beta}$ (eV) | $0.0317_{-0.00}^{+0.0798}$ | $0.0293_{-0.00}^{+0.0757}$ |
| Normal hierarchy | $m_{\beta,0} = 0$, $m_{\beta\beta,0} = 0$ | $m_{\beta,0} = 0.28$ eV, $m_{\beta\beta,0} = 0.18$ eV |
| $\log_{10} m_L$ (eV) | $-0.660_{-0.800}^{+0.552}$ | $-0.571_{-0.670}^{+0.493}$ |
| $\sum m_\nu$ (eV) | $0.674_{-0.648}^{+0.846}$ | $0.817_{-0.648}^{+0.969}$ |
| $m_\beta/\sum m_\nu$ | $0.330_{-0.328}^{+0.332}$ | $0.331_{-0.330}^{+0.332}$ |
| $m_{\beta\beta}$ (eV) | $0.166_{-0.0873}^{+0.244}$ | $0.188_{-0.104}^{+0.275}$ |

TABLE III: The mean value and 95% lower and upper credible intervals for various parameters and combinations of data.

parameters in the MCMC analysis). Particularly $w$ is known to be degenerate with $\sum m_\nu$ and therefore any independent information on $\sum m_\nu$ from experiments is potentially important for dark energy physics. This particular degeneracy has been studied quite extensively in recent literature. The most recent example is [44] where the impact of a positive KATRIN detection of $m_\beta$ on the estimation of $w$ is discussed.

In Fig. 2 we show the degeneracy between $\sum m_\nu$ and $w$ for our case (b), assuming normal hierarchy. The corresponding numbers are shown in Table IV. The results confirm previous findings, i.e. that a strongly negative equation of state for dark energy can be compensated by increasing the neutrino mass [49, 50]. This also means that the allowed region of $w$ for case (b) is shifted to more negative values, in this case only marginally allowing a cosmological constant (the 1D 95% credible interval is $-1.39 < w < -0.99$). The present result compares well with what is obtained in [44], although the assumed best fit values are slightly different. Note also that our treatment of cosmological data is slightly different from [44] because we use the full BAO correlation function instead.
FIG. 2: 68% and 95% contours for the Cosmo+KATRIN+GERDA data, assuming best fit values of $m_\beta = 0.28$ eV, $m_{\beta\beta} = 0.18$ eV (case b). Inverted hierarchy is assumed.

The exercise carried out in this subsection clearly illustrates why cosmological bounds on neutrino properties are model dependent. Note that this degeneracy would be even stronger if only CMB data is considered.

FIG. 3: 68% and 95% contours for the Cosmo(+$w$++$\alpha_s$)+KATRIN+GERDA data in the $\sum m_\nu - w$ plane, assuming best fit values of $m_\beta = 0.28$ eV, $m_{\beta\beta} = 0.18$ eV (case b). Normal hierarchy is assumed.

TABLE IV: The mean value and 95% lower and upper credible intervals for case (b) with the larger cosmological parameter space.

| Parameter        | Cosmo(+$w$++$\alpha_s$)+KATRIN+GERDA |
|------------------|---------------------------------------|
| Normal hierarchy | $m_{\beta,0} = 0.28$ eV, $m_{\beta\beta,0} = 0.18$ eV |
| $\log_{10} m_L$ (eV) | -0.635 $^{0.898}_{-0.325}$ |
| $\sum m_\nu$ (eV)   | 0.730 $^{0.332}_{-0.097}$ |
| $m_\beta/\sum m_\nu$ | 0.328 $^{0.252}_{-0.226}$ |
| $w$ (eV)           | -1.17 $^{1.39}_{-0.99}$ |

of the $A$ functional parametrisation. For comparison, in Fig. 4 we show the $\sum m_\nu - w$ degeneracy for cosmological data only. In this case we find the result $\sum m_\nu < 0.56$ eV and $-1.22 < w < -0.88$, both at 95% C.L., a result completely consistent with what was found in [21] for the same model and data, but using maximisation instead of marginalisation. Note also that in this extended model the best fit $\chi^2$ increases by 2.6 compared to the case where only cosmological data is used (compared to 3.1 in the smaller parameter space discussed above), i.e. in the extended model the inconsistency between cosmology and the assumed positive detection from KATRIN and GERDA is less pronounced.
IV. DISCUSSION

A detailed neutrino parameter estimation study has been carried out using the Markov Chain Monte Carlo technique with the goal of unifying the various techniques for measuring the absolute neutrino mass scale. The MCMC technique is extremely powerful in this regard and allows for a very fast scanning many-dimensional likelihood spaces. In the concrete example here we have used 8 parameters describing the properties of light, active Majorana neutrinos, and 6 further parameters which specify the cosmology.

We find that for present data the combination of cosmological data with the upper limit on $m_{\beta\beta}$ from Heidelberg-Moscow slightly improves the existing cosmological bound on the sum of neutrino masses.

More interestingly we have studied the interplay between various future constraints from cosmology, tritium decay and neutrinoless double beta decay. If all probes come up with a negative result the addition of data sets does not yield any radically new information. However, we have also studied an example in which the upcoming KATRIN and GERDA experiments are both assumed to provide tentative evidence for neutrino mass. In this case the combination of all three types of data allows for a much stronger constraint on neutrino properties than otherwise allowed.

Finally we have also studied how experimental data from tritium decay or neutrinoless double beta decay can help in cosmological parameter estimation, particularly concerning the dark energy equation of state.

It should be noted that in the present analysis only presently available cosmological data has been used. In the same time frame as KATRIN and GERDA new cosmological data will become available and is likely to improve the cosmological neutrino mass bound significantly (see [51, 52, 53, 54, 55, 56, 57, 58, 59] for a non-exhaustive list). In the somewhat longer term cosmological constraints can be potentially be pushed below 0.1 eV sensitivity to $\sum m_\nu$. At the same time neutrinoless double beta decay experiments will have equally improved sensitivity and it will very likely be possible to determine the absolute neutrino mass as well as the nature of the mass hierarchy.

In conclusion, the combination of cosmological data with experimental neutrino data in a global analysis will be extremely useful in the future, when more precise experimental data becomes available.

Acknowledgements

Use of computing resources from the Danish Center for Scientific Computing (DCSC) is acknowledged. Use of the CosmoMC package [7, 8] is acknowledged. Amand Fässler is thanked for discussions on the effective neutrino mass in neutrinoless double beta decay.

[1] R. N. Mohapatra, “Physics of neutrino mass,” eConf C040802. L011 (2004) [New J. Phys. 6, 82 (2004)] arXiv:hep-ph/0411131.
[2] R. N. Mohapatra et al., “Theory of neutrinos,” arXiv:hep-ph/0412099
[3] R. N. Mohapatra and A. Y. Smirnov, “Neutrino mass and new physics,” Ann. Rev. Nucl. Part. Sci. 56, 569 (2006) arXiv:hep-ph/0603118.
[4] M. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle, “Status of global fits to neutrino oscillations,” New J. Phys. 6, 122 (2004) arXiv:hep-ph/0405172.
[5] G. L. Fogli et al., “Observables sensitive to absolute neutrino masses: A reappraisal after WMAP-3y and first MINOS results,” Phys. Rev. D 75, 053001 (2007) arXiv:hep-ph/0608060.
[6] O. Host, O. Lahav, F. B. Abdalla and K. Eitel, “Forecasting neutrino masses from combining KATRIN and the CMB: Frequentist and Bayesian analyses,” arXiv:0709.1317 [hep-ph].
[7] A. Lewis and S. Bridle, “Cosmological parameters from CMB and other data: A Monte-Carlo approach,” Phys. Rev. D 66 (2002) 103511 arXiv:astro-ph/0205436.
[8] A. Lewis, Homepage, http://cosmologist.info
[9] R. R. de Austri, R. Trotta and L. Roszkowski, “A Markov chain Monte Carlo analysis of the CMSSM,” JHEP 0605, 002 (2006) arXiv:hep-ph/0602028.
[10] N. Christensen, R. Meyer, L. Knox and B. Luey, “Bayesian Methods for Cosmological Parameter Estimation from Cosmic Microwave Background Measurements,” Class. Quant. Grav. 18, 2677 (2001) arXiv:astro-ph/0103134.
[11] J. Lesgourgues, S. Pastor and L. Perotto, “Probing neutrino masses with future galaxy redshift surveys,” Phys.
[10] J. Lesgourgues and S. Pastor, “Massive neutrinos and cosmology,” Phys. Rept. 429 (2006) 307 [arXiv:astro-ph/0603494].

[11] S. S. Massod, S. Nasri, J. Schechter, M. A. Tortola, J. W. F. Valle and C. Weinheimer, “Exact relativistic beta decay endpoint spectrum,” arXiv:0706.0897 [hep-ph].

[12] C. Aalseth et al., “Neutrinoless double beta decay and direct searches for neutrino mass,” arXiv:hep-ph/0412300.

[13] S. Hannestad, “Primordial neutrinos,” Ann. Rev. Nucl. Part. Sci. 56 (2006) 137 [arXiv:hep-ph/0602058].

[14] V. A. Rodin, A. Faessler, F. Simkovic and P. Vogel, “Assessment of uncertainties in QRPA on beta beta-decay nuclear matrix elements,” Nucl. Phys. A 766, 107 (2006).

[15] A. Faessler, talk at “The path to neutrino mass” workshop, Aarhus, September 2007 http://astroparticle.phys.au.dk.

[16] M. Cirigli and A. Strumia, “Cosmology of neutrinos and extra light particles after WMAP3,” JCAP 0612 (2006) 013 [arXiv:astro-ph/0610507].

[17] A. Goobar, S. Hannestad, E. Mortsell and H. Tu, “A new bound on the neutrino mass from the SDSS baryon acoustic peak,” JCAP 0606 (2006) 019 [arXiv:astro-ph/0602155].

[18] J. R. Kristiansen, H. K. Eriksen and O. Elgarøy, “Revised WMAP constraints on neutrino masses and other extensions of the minimal Lambda CDM model,” Phys. Rev. D 74, 123005 (2006).

[19] U. Seljak, A. Slosar and P. McDonald, “Cosmological parameters from combining the Lyman-alpha forest with CMB, galaxy clustering and SN constraints,” JCAP 0610, 014 (2006) [arXiv:astro-ph/0604335].

[20] S. Hannestad, “Neutrino masses and the number of neutrino species from WMAP and 2dFGRS,” JCAP 0305, 004 (2003) [arXiv:astro-ph/0303076].

[21] S. Hannestad, “Primordial neutrinos,” Ann. Rev. Nucl. Part. Sci. 56 (2006) 137 [arXiv:hep-ph/0602058].

[22] S. Hannestad, A. Mirizzi, G. G. Raffelt and Y. Y. Wong, “Cosmological constraints on neutrino plus axion dark matter,” JCAP 0708, 015 (2007) [arXiv:astro-ph/07041121] [astro-ph].

[23] S. Hannestad, “Neutrino masses and cosmology,” Astrophys. J. Suppl. 170 (2007) 377 [arXiv:astro-ph/0605449].

[24] G. Hinshaw et al., “Three-year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Temperature analysis,” Astrophys. J. Suppl. 170 (2007) 288 [arXiv:astro-ph/0605451].

[25] L. Page et al., “Three year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Polarization analysis,” Astrophys. J. Suppl. 170 (2007) 335 [arXiv:astro-ph/0603450].

[26] J. Lesgourgues and S. Pastor, “Massive neutrinos and cosmology,” Phys. Rept. 429 (2006) 307 [arXiv:astro-ph/0603494].

[27] S. Hannestad, A. Mirizzi, G. G. Raffelt and Y. Y. Wong, “Cosmological constraints on neutrino plus axion dark matter,” JCAP 0708, 015 (2007) [arXiv:astro-ph/07041121] [astro-ph].

[28] A. Faessler, talk at “The path to neutrino mass” workshop, Aarhus, September 2007 http://astroparticle.phys.au.dk.

[29] M. Tegmark et al., “Cosmological Constraints from the SDSS Luminous Red Galaxies,” Phys. Rev. D 74 (2006) 123507 [arXiv:astro-ph/0608636].

[30] S. Cole et al. [2dFGRS Collaboration], “The 2dF Galaxy Redshift Survey: Power-spectrum analysis of the final dataset and cosmological implications,” Mon. Not. Roy. Astron. Soc. 362 (2005) 505 [arXiv:astro-ph/0501174].

[31] J. Hamann, S. Hannestad, G. G. Raffelt and Y. Y. Wong, “Observational bounds on the cosmic radiation density,” JCAP 0708, 021 (2007) [arXiv:0705.0410 [astro-ph]].

[32] H. V. Klapdor-Kleingrothaus et al., “Latest results from the Heidelberg-Moscow double-beta-decay experiment,” Eur. Phys. J. A 12, 147 (2001) [arXiv:hep-ex/0103062].

[33] S. Hannestad, A. Dietz, H. L. Harney and I. V. Krivosheina, “Evidence for neutrinoless double beta decay,” Mod. Phys. Lett. A 16, 2409 (2001) [arXiv:hep-ex/0201231].

[34] S. Hannestad, A. Dietz and O. Chkvorets, “Search for neutrinoless double beta decay with enriched Ge-76 in Gran Sasso Phys. Lett. B 586, 198 (2004) [arXiv:hep-ex/0404088].

[35] H. Volker Klapdor-Kleingrothaus, “First evidence for neutrinoless double beta decay - and world status of double beta experiments,” arXiv:hep-ph/0512263.

[36] A. Osipowicz et al. [KATRIN Collaboration], “KA-TRIN: A next generation tritium beta decay experiment with sub-eV sensitivity for the electron neutrino mass,” arXiv:hep-ex/0109003.

[37] S. Schonert et al. [GERDA Collaboration], “The GERmanium Detector Array (GERDA) for the search of neutrinoless beta beta decays of Ge-76 at LNGS,” Nucl. Phys. Proc. Suppl. 145, 242 (2005).

[38] J. R. Kristiansen and O. Elgarøy, “Cosmological implications of the KATRIN experiment,” arXiv:0709.4152 [astro-ph].

[39] R. Arditto et al., “CUORE: A cryogenic underground observatory for rare events,” arXiv:hep-ex/0501101.

[40] R. Gaisser et al. [Majorana Collaboration], “White paper on the Majorana zero-neutrino double-beta decay experiment,” arXiv:nucl-ex/0311013.

[41] S. Parziesi et al. [CUORE Collaboration], “CUORE: A cryogenic underground observatory for rare events,” arXiv:hep-ex/0501101.

[42] M. Lindner, A. Merle and W. Rodejohann, “Improved limit on theta(13) and implications for neutrino masses in neutrino-less double beta decay and cosmology,” Phys. Rev. D 73, 053005 (2006) [arXiv:hep-ph/0512143].

[43] S. Hannestad, “Neutrino masses and the dark energy equation of state: Relaxing the cosmological neutrino mass bound,” Phys. Rev. Lett. 95, 221301 (2005).
[50] A. De La Macorra, A. Melchiorri, P. Serra and R. Bean, “The impact of neutrino masses on the determination of dark energy properties,” Astropart. Phys. 27, 406 (2007) arXiv:astro-ph/0608351.

[51] J. Q. Xia, H. Li, G. B. Zhao and X. Zhang, “Probing for the Cosmological Parameters with PLANCK Measurement,” arXiv:0708.1111 [astro-ph].

[52] S. Gratton, A. Lewis and G. Efstathiou, “Prospects for Constraining Neutrino Mass Using Planck and Lyman-Alpha Forest Data,” arXiv:0705.3100 [astro-ph].

[53] S. Hannestad and Y. Y. Y. Wong, “Neutrino mass from future high redshift galaxy surveys: Sensitivity and detection threshold,” JCAP 0707, 004 (2007) arXiv:astro-ph/0703031.

[54] L. Perotto, J. Lesgourgues, S. Hannestad, H. Tu and Y. Y. Y. Wong, “Probing cosmological parameters with the CMB: Forecasts from full Monte Carlo simulations,” JCAP 0610, 013 (2006) arXiv:astro-ph/0606227.

[55] M. Takada, E. Komatsu and T. Futamase, “Cosmology with high-redshift galaxy survey: Neutrino mass and inflation,” Phys. Rev. D 73, 083520 (2006) arXiv:astro-ph/0512374.

[56] J. Lesgourgues, L. Perotto, S. Pastor and M. Piat, “Probing neutrino masses with CMB lensing extraction,” Phys. Rev. D 73, 045021 (2006) arXiv:astro-ph/0511735.

[57] S. Wang, Z. Haiman, W. Hu, J. Khoury and M. May, “Weighing neutrinos with galaxy cluster surveys,” Phys. Rev. Lett. 95, 011302 (2005) arXiv:astro-ph/0505390.

[58] Y. S. Song and L. Knox, “Dark energy tomography,” arXiv:astro-ph/0312175.

[59] S. Hannestad, “Can cosmology detect hierarchical neutrino masses?,” Phys. Rev. D 67, 085017 (2003) arXiv:astro-ph/021106.