Light scalars in semileptonic decays of heavy quarkonia

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Abstract

We study the mechanism of production of the light scalar mesons in the $D_s^+ \to \pi^+\pi^-e^+\nu$ decays: $D_s^+ \to s\bar{s}e^+\nu \to [\sigma(600) + f_0(980)]e^+\nu \to \pi^+\pi^-e^+\nu$, and compare it with the mechanism of production of the light pseudoscalar mesons in the $D_s^+ \to (\eta/\eta')e^+\nu$ decays: $D_s^+ \to s\bar{s}e^+\nu \to (\eta/\eta')e^+\nu$. We show that the $s\bar{s} \to \sigma(600)$ transition is negligibly small in comparison with the $s\bar{s} \to f_0(980)$ one. As for the the $f_0(980)$ meson, the intensity of the $s\bar{s} \to f_0(980)$ transition makes near thirty percent from the intensity of the $s\bar{s} \to \eta_s$ ($\eta_s = s\bar{s}$) transition. So, the $D_s^+ \to \pi^+\pi^-e^+\nu$ decay supports the previous conclusions about a dominant role of the four-quark components in the $\sigma(600)$ and $f_0(980)$ mesons.

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At present the nontrivial nature of the well-established light scalar resonances $f_0(980)$ and $a_0(980)$ is denied by very few people. As for the nonet as a whole, even a cursory look at PDG Review \[1\] gives an idea of the four-quark structure of the light scalar meson nonet, $\sigma(600)$, $\kappa(800)$, $f_0(980)$, and $a_0(980)$, inverted in comparison with the classical $P$ wave $q\bar{q}$ tensor meson nonet, $f_2(1270)$, $a_2(1320)$, $K_2^*(1420)$, $\phi_2'(1525)$. Really, while the scalar nonet cannot be treated as the $P$ wave $q\bar{q}$ nonet in the naive quark model, it can be easily understood as the $q^2\bar{q}^2$ nonet, where $\sigma$ has no strange quarks, $\kappa$ has the $s$ quark, $f_0$ and $a_0$ have the $s\bar{s}$ pair. Similar states were found by Jaffe in 1977 in the MIT bag \[2\].

By now it is established also that the mechanisms of the $a_0(980)$, $f_0(980)$, and $\sigma(600)$ meson production in the $\phi$ radiative decays \[3–8\], in the photon-photon collisions \[9, 10\], and in the $\pi\pi$ scattering \[7, 8\] are the four-quark transitions and thus indicate to the four-quark structure of the light scalars \[11\].

In addition, the absence of the $J/\psi \rightarrow \gamma f_0(980)$, $a_0(980)\rho$, $f_0(980)\omega$ decays in contrast to the intensive the $J/\psi \rightarrow \gamma f_2(1270)$, $\gamma f_2'(1525)$, $a_2(1320)\rho$, $f_2(1270)\omega$ decays argues against the $P$ wave $q\bar{q}$ structure of $a_0(980)$ and $f_0(980)$ also \[12\].

It is time to explore the light scalar mesons in the decays of of heavy quarkonia \[13–15\]. The semileptonic decays are of prime interest because they have the clear mechanisms, see, for example, Fig. 1.

As Fig. 1 suggests, the $D_s^+ \rightarrow s\bar{s} e^+\nu$ decay is the perfect probe of the $s\bar{s}$ component in the $\sigma(600)$ and $f_0(980)$ states \[13, 14\].

Below we study the mechanism of production of the light scalar mesons in the $D_s^+ \rightarrow$
π⁺π⁻e⁺ν decays: $D_s^+ \to s\bar{s} e^+\nu \to [\sigma(600) + f_0(980)] e^+\nu \to \pi^+\pi^- e^+\nu$, and compare it with the mechanism of production of the light pseudoscalar mesons in the $D_s^+ \to (\eta/\eta') e^+\nu$ decays: $D_s^+ \to s\bar{s} e^+\nu \to (\eta/\eta') e^+\nu$, in a model of the Nambu-Jona-Lasinio type.

The amplitudes of the $D_s^+ \to P$(pseudoscalar) $e^+\nu$ and $D_s^+ \to S$(scalar) $e^+\nu$ decays have the form

$$M[D_s^+(p) \to P(p_1)W^+(q) \to P(p_1) e^+\nu] = \frac{G_F}{\sqrt{2}} V_{cs} V_{\alpha} L^\alpha,$$

$$M[D_s^+(p) \to S(p_1)W^+(q) \to S(p_1) e^+\nu] = \frac{G_F}{\sqrt{2}} V_{cs} A_{\alpha} L^\alpha,$$

where $G_F$ is the Fermi constant, $V_{cs}$ is the Cabibbo-Kobayashi-Maskava matrix element,

$$V_{\alpha} = f^P_\alpha(q^2)(p + p_1)_\alpha + f^S_\alpha(q^2)(p_1 - p)_\alpha,$$

$$A_{\alpha} = f^S_\alpha(q^2)(p + p_1)_\alpha + f^S_\alpha(q^2)(p_1 - p)_\alpha,$$

$$L_{\alpha} = \bar{\nu} \gamma_{\alpha}(1 + \gamma_5)e, \quad q = (p - p_1).$$

The influence of the $f^P_\alpha(q^2)$ and $f^S_\alpha(q^2)$ form factors are negligible because of the small mass of the positron.

The decay rates in the stable $P$ and $S$ states are

$$\frac{d\Gamma(D_s^+ \to P e^+\nu)}{dq^2} = \frac{G_F^2 |V_{cs}|^2}{24\pi^3} p_1^3(q^2) |f^P_\alpha(q^2)|^2,$$

$$\frac{d\Gamma(D_s^+ \to S e^+\nu)}{dq^2} = \frac{G_F^2 |V_{cs}|^2}{24\pi^3} p_1^3(q^2) |f^S_\alpha(q^2)|^2,$$

$$p_1(q^2) = \sqrt{m^4_D - 2m^2_D(q^2 + m^2_S) + (q^2 - m^2_S)^2}, \quad \text{or} \quad p_1(q^2) = \sqrt{m^4_D - 2m^2_D(q^2 + m^2_S) + (q^2 - m^2_S)^2}.\quad (3)$$

For the $f^P_\alpha(q^2)$ and $f^S_\alpha(q^2)$ form factors we use the vector dominance model

$$f^P_\alpha(q^2) = f^P_\alpha(0) \frac{m^2_D}{m^2_V - q^2} = f^P_\alpha(0) v_V(q^2), \quad f^S_\alpha(q^2) = f^S_\alpha(0) \frac{m^2_A}{m^2_A - q^2} = f^S_\alpha(0) f_A(q^2),\quad (4)$$

where $V = D_s^*(2112)\pm, A = D_{s1}(2460)\pm, [1]$. 

Following Fig. [1] we write $f^P_\alpha(0)$ and $f^S_\alpha(0)$ in the form

$$f^P_\alpha(0) = g_{D^+_cs} F_P g_{s\bar{s}P}, \quad f^S_\alpha(0) = g_{D^+_cs} F_S g_{s\bar{s}S},$$

where $g_{D^+_cs}$ is the $D_s^+ \to c\bar{s}$ coupling constant, $g_{s\bar{s}P}$ and $g_{s\bar{s}S}$ are the $s\bar{s} \to P$ and $s\bar{s} \to S$ coupling constants.

We know the structure of $\eta$ and $\eta'$

$$\eta = \eta_q \cos \phi - \eta_s \sin \phi, \quad \eta' = \eta_q \sin \phi + \eta_s \cos \phi,$$

3
where \( \eta_g = (u \bar{u} + d \bar{d})/\sqrt{2} \) and \( \eta_s = s \bar{s} \). The angle \( \phi = \theta_i + \theta_P \), where \( \theta_i \) is the ideal mixing angle with \( \cos \theta_i = \sqrt{1/3} \) and \( \sin \theta_i = \sqrt{2/3} \), i.e., \( \theta_i = 54.7^\circ \), and \( \theta_P \) is the angle between the flavor-singlet state \( \eta_i \) and the flavor-octet state \( \eta_s \).

So,

\[
g_{s\bar{s}\eta} = -g_{s\bar{s}\eta_s} \sin \phi, \quad g_{s\bar{s}\eta'} = g_{s\bar{s}\eta_s} \cos \phi. \tag{7}
\]

The Particle Data Group \([1]\) gives the \( \theta_P \) band \(-20^\circ \lesssim \theta_P \lesssim -10^\circ \) that gives us the opportunity to extract information about the \( s\bar{s} \to \eta_s \) coupling constant, \( g_{s\bar{s}\eta_s} \), from experiment and to compare with the \( s\bar{s} \to f_0 \) coupling constant, \( g_{s\bar{s}f_0} \), extracted from experiment also. We consider the next set of \( \theta_P \).

\[
\begin{align*}
\theta_P &= -11^\circ: \quad \eta = 0.72\eta_0 - 0.69\eta_s, \quad \eta' = 0.69\eta_0 + 0.72\eta_s \\
\theta_P &= -14^\circ: \quad \eta = 0.76\eta_0 - 0.65\eta_s, \quad \eta' = 0.65\eta_0 + 0.76\eta_s \\
\theta_P &= -18^\circ: \quad \eta = 0.8\eta_0 - 0.6\eta_s, \quad \eta' = 0.6\eta_0 + 0.8\eta_s. \tag{8}
\end{align*}
\]

The amplitude of the the \( D_s^+ \to s\bar{s} e^+\nu \to [\sigma(600) + f_0(980)] e^+\nu \to \pi^+\pi^- e^+\nu \) decay is

\[
M(D_s^+ \to s\bar{s} e^+\nu \to \pi^+\pi^- e^+\nu) = \frac{G_F}{\sqrt{2}} V_{cs} L^\alpha (p + p_1) \alpha g_{D_s^+ c\bar{s}} f_A(q^2) \times e^{i\phi_{\sigma\pi}} \frac{1}{\Delta(m)} \left( F_\sigma g_{s\bar{s}\sigma} D_f(m) g_{\sigma\pi\pi} + F_\sigma g_{s\bar{s}\sigma} \Pi_{\sigma f_0} D_\sigma(m) g_{f_0\pi\pi} \right) + F_{f_0} g_{s\bar{s}f_0} \Pi_{f_0\sigma} (m) g_{\sigma\pi\pi} + F_{f_0} g_{s\bar{s}f_0} D_\sigma (m) g_{f_0\pi\pi}, \tag{9}
\]

where \( m \) is the invariant mass of the \( \pi\pi \) system, \( \Delta(m) = D_{f_0}(m) D_\sigma(m) - \Pi_{f_0\sigma} (m) \Pi_{f_0\sigma} (m) \), \( D_\sigma(m) \) and \( D_{f_0}(m) \) are the inverted propagators of the \( \sigma \) and \( f_0 \) mesons, \( \Pi_{f_0\sigma} (m) = \Pi_{f_0\sigma} (m) \) is the off-diagonal element of the polarization operator, which mixes the \( \sigma \) and \( f_0 \) mesons. All the details can be found in Refs. \([7, 8, 10]\).

The double differential rate of the \( D_s^+ \to s\bar{s} e^+\nu \to [\sigma(600) + f_0(980)] e^+\nu \to \pi^+\pi^- e^+\nu \) decay is

\[
\frac{d^2 \Gamma(D_s^+ \to \pi^+\pi^- e^+\nu)}{dq^2 dm} = \frac{G_F^2 |V_{cs}|^2 g_{D_s^+ c\bar{s}}^2 f_A(q^2)^2 p_1^3(q^2, m)}{24\pi^3} \times \frac{1}{\Delta(m)} \left[ F_\sigma g_{s\bar{s}\sigma} D_f(m) g_{\sigma\pi\pi} + F_\sigma g_{s\bar{s}\sigma} \Pi_{\sigma f_0} (m) g_{f_0\pi\pi} \right] + F_{f_0} g_{s\bar{s}f_0} \Pi_{f_0\sigma} (m) g_{\sigma\pi\pi} + F_{f_0} g_{s\bar{s}f_0} D_\sigma (m) g_{f_0\pi\pi} \tag{10}
\]
where $\rho_{\pi\pi}(m) = \sqrt{1 - 4m_{\pi}^2/m^2}$.

When $\Pi_{s\sigma}(m) = \Pi_{f\sigma}(m) = 0$ and $g_{s\bar{s}\sigma} = 0$

$$
d^2\Gamma(D^+_s \to \pi^+\pi^- e^+\nu) = \frac{G_F^2 |V_{cs}|^2}{24\pi^3} g_{D^+_s cs}^2 |f_A(q^2)|^2 F_1(q^2, m) \frac{2m^2}{\pi} \frac{\Gamma(f_0 \to \pi^+\pi^- m)}{|D_{f_0}(m)|^2}. \quad (11)
$$

When fitting the CLEO \cite{13}, we use the parameters of the resonances obtained in Ref. \cite{8} in the analysis of the $\pi\pi$ scattering and the $\phi \to \gamma(s + f_0) \to \pi^0\pi^0$ decay. So the 44 events in Fig. \cite{2} determine only one parameter $f^\sigma_+ (0)/f^f_0 (0)$. In this case the Adler self consistency condition (the Adler zero at $m^2$ near $(m_{\pi}^2)/2$) determines $f^\sigma_+ (0)/f^f_0 (0) = (F_{r\sigma} g_{s\bar{s}\sigma})/(F_{f_0} g_{s\bar{s}f_0}) = 0.039, 0.014, 0.055, 0.058, 0.032, 0.055$ for six fits from Ref. \cite{8}. So the intensity of the $\sigma(600)$ production is much less than the intensity of the $f_0(980)$ production $((f^\sigma_+ (0)/f^f_0 (0))^2 \leq 0.003)$. That is we find the direct evidence of decoupling of $\sigma(600)$ with the $s\bar{s}$ pair. As far as we know, this is truly a new result, which agrees well with the decoupling of $\sigma(600)$ with the $K\bar{K}$ states, obtained in Ref. \cite{8} $g_{\sigma K + \bar{K}^-}^2/g_{\sigma \pi + \pi^-}^2 = 0.04, 0.001, 0.01, 0.01, 0.003, 0.025$ for six fits. The decoupling of $\sigma(600)$ with the $K\bar{K}$ states means also the decoupling of $\sigma(600)$ with $\sigma_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ because $\sigma_q$ results in $g_{\sigma K + \bar{K}^-}^2/g_{\sigma \pi + \pi^-}^2 = 1/4$. Results of our analysis of the CLEO \cite{13} data are shown in the Table and on Figs. \cite{2} and \cite{3}. The parameters of the $\sigma(600)$ and $f_0(980)$ mesons are taken from Fit 1 of Ref. \cite{8} which describes the spectrum on Fig. \cite{2} better than others $((F_{r\sigma} g_{s\bar{s}\sigma})/(F_{f_0} g_{s\bar{s}f_0}) = 0.039, g_{\sigma K + \bar{K}^-}^2/g_{\sigma \pi + \pi^-}^2 = 0.04)$. So, the CLEO experiment gives new support in favour of the four-quark $(u\bar{d}u\bar{d})$ structure of the $\sigma(600)$ meson.

In the chirally symmetric model of the Nambu-Jona-Lasinio type the coupling constants of the pseudoscalar and scalar partners with quarks are equal to each other, i.e., $g_{s\bar{s}f_0} = g_{s\bar{s}f_0}$, where $f_0s = s\bar{s}$. In approximation when the mass of the strange quark much less the mass of the charmed quark $(m_s / m_c \ll 1)$ $F_{f_0} = F_{f_0} \approx 17$ and we find from the Table (see the last line) that $g_{s\bar{s}f_0}^2 / g_{s\bar{s}f_0}^2 \approx 0.3$. So, the $f_0s = s\bar{s}$ part in the $f_0(980)$ wave function is near thirty percent. Taking into account the suppression of the $f_0(980)$ meson coupling with the $\pi\pi$ system, $g_{f_0\pi^+\pi^-}^2 / g_{f_0 K^+\bar{K}^-}^2 = 0.154$, see Fit 1 in the Table I of Ref. \cite{8}, one can conclude that the $f_0q = (u\bar{u} + d\bar{d})/\sqrt{2}$ part in the $f_0(980)$ wave function is suppressed also. So, the CLEO experiment gives strong support in favour of the four-quark $(s\bar{s}d\bar{d})$ structure of the $f_0(980)$ meson, too.
Table. Results of the analysis of the CLEO \cite{13} data. All quantities are defined in the text.

| \( \frac{F_{\sigma}g_{ss\sigma}}{F_{f_0}g_{ssf_0}} \) | \( \frac{F_{f_0}g^2_{ssf_0}}{F_{\sigma}g^2_{ss\sigma}} \) | \( \frac{F_{\eta}g^2_{ss\eta}}{F_{f_0}g^2_{ssf_0}} \) | \( \frac{F_{\eta'}g^2_{ss\eta'}}{F_{f_0}g^2_{ssf_0}} \) | \( \frac{F_{\eta}g^2_{ss\eta}}{F_{\eta'}g^2_{ss\eta'}} \) | \( \frac{F_{\eta'}g^2_{ss\eta'}}{F_{\eta}g^2_{ss\eta}} \) |
|---|---|---|---|---|---|
| 0.039 | 0.67 | 0.49 | 0.73 | 0.039 | 0.67 | 0.49 | 0.73 |

The \( \eta - \eta' \) mixing

\[ \theta_{\Pi} \]  
\[ -11^\circ \]  
\[ -14^\circ \]  
\[ -18^\circ \]  

\[ \frac{F_{f_0}g^2_{ssf_0}}{F_{\eta}g^2_{ss\eta}} \]  
0.32  
0.29  
0.24  

\[ \frac{F_{f_0}g^2_{ssf_0}}{F_{\eta'}g^2_{ss\eta'}} \]  
0.27  
0.28  
0.31  

FIG. 2: The CLEO data \cite{13} on the invariant \( \pi^+\pi^- \) mass \( (m) \) distribution for \( D_s^+ \to \pi^+\pi^-e^+\nu \) decay with the subtracted backgrounds, which are calculated in Ref. \cite{13}. The dotted line is Fit from Ref. \cite{13}, Fig. 9, corresponding to \( BR(D_s^+ \to f_0(980)e^+\nu) BR(f_0(980) \to \pi^+\pi^-) = (0.20 \pm 0.03 \pm 0.01) \). Our theoretical curve is the solid line.
FIG. 3: The $q^2$ distribution for $BR(D_s^+ \to f_0(980) e^+\nu)$. The axial-vector dominance model, see Eq. (4), describes the CLEO data [13] quite satisfactorily.

Certainly, there is an extreme need in experiment on the $D_s^+ \to \pi^+\pi^- e^+\nu$ decay with high statistics.

Of great interest is the experimental search for the decays $D^0 \to d\bar{u}\, e^+\nu \to a_0^-\,(980)\, e^+\nu \to \pi^-\eta\, e^+\nu$ and $D^+ \to d\bar{d}\, e^+\nu \to a_0^0\,(980)\, e^+\nu \to \pi^0\, \eta\, e^+\nu$ (or the charge conjugate ones), which will give the information about the $a_q^- = d\bar{u}$ (or $a_q^+ = u\bar{d}$) and $a_q^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$ components in the $a_0^-(980)$ and $a_0^0$ wave functions respectively.

No less interesting is also search for the decays $D^+ \to d\bar{d}\, e^+\nu \to [\sigma(600) + f_0(980)]\, e^+\nu \to \pi^+\pi^- e^+\nu$ (or the charge conjugate ones), which will give the information about the $\sigma_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $f_{0q} = (u\bar{u} + d\bar{d})/\sqrt{2}$ components in the $\sigma(600)$ and $f_0(980)$ wave functions respectively.

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[11] In particular, the was shown that the ideal $q\bar{q}$ model prediction $g_{f_0(980)\gamma\gamma}^2 : g_{a_0^0(980)\gamma\gamma}^2 = 25 : 9$ is excluded by experiment in contrast to the simalar prediction for the tensor states $f_2(1270)$ and $a_2(1320)$. We mean here the $f_0(980) = (u\bar{u} + d\bar{d})\sqrt{2}$ and $a_0^0(980) = (u\bar{u} - d\bar{d})\sqrt{2}$ case for equality of the masses: $m_{f_0} = m_{a_0^0}$.
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[17] The study beyond this approximation we hope to carry out subsequently.