Measurement of hybrid content of heavy quarkonia using lattice NRQCD

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Abstract

Using lowest-order lattice NRQCD to create heavy meson propagators and applying the spin-dependent interaction, $c_B \frac{g}{2m_q} \vec{\sigma} \cdot \vec{B}$, at varying intermediate time slices, we compute the off-diagonal matrix element of the Hamiltonian for the quarkonium-hybrid two-state system. Thus far, we have results for one set of quenched lattices with an interpolation in quark mass to match the bottomonium spectrum. After diagonalization of the two-state Hamiltonian, we find the ground state of the $\Upsilon$ to show a $0.0035(1)c_B^2$ (with $c_B^2 \sim 1.5 - 3.1$) probability admixture of hybrid, $|\bar{b}b\gamma\rangle$.

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I. INTRODUCTION

In quantum chromodynamics (QCD), it has been known for some time that mesons \((q\bar{q})\) and baryons \((qqq)\) are not the only composite states for which gauge-invariant propagators may be constructed. Multi-quark states \((qq\bar{q},qqqq\bar{q},\text{etc.})\) and states with gluonic excitations, or hybrids \((q\bar{q}g,qqg,\text{etc.})\), can also form the necessary color singlet. It is possible for these states to have quantum numbers \((J^P C)\) different from those allowed for typical hadrons. Such states are termed “exotics” and while they offer significant promise for hybrid and multi-quark state detection, they will not be the subject of our discussion here. Instead, we focus on “non-exotic” heavy hybrid mesons \((|Q\bar{Q}g\rangle)\), those which have quarkonium-like quantum numbers. The true, heavy meson ground state should thus be a mixture of heavy quarkonium and hybrid:

\[
|\Upsilon\rangle = A_0|Q\bar{Q}\rangle + A_1|Q\bar{Q}g\rangle + \ldots
\]  

Heavy quarks within the bound state move with relatively small velocities and thus we expect the rest mass of the quarks to dominate the energy \((K << m_q)\). Expanding in the quantity \(\frac{1}{m_q}\), one finds a non-relativistic approximation to the heavy-quark Hamiltonian (NRQCD) [1]. For simplicity, we keep only the lowest-order kinetic term of the NRQCD Hamiltonian and the lowest-order spin-dependent term:

\[
H = H_0 + \delta H = m_q \frac{\bar{D}^2}{2m_q} + c_B \frac{g}{2m_q} \vec{\sigma} \cdot \vec{B} + \ldots
\]  

where \(\bar{D}\) is the covariant derivative. Using only the first term in this expansion, the Hamiltonian lacks a spin-flip interaction and there is no mixing between the lowest-lying hybrid and quarkonium states. For the \(J^{PC} = 1^{--}\) system, the total quark spin is 1 in the S-wave state. Our operational definition of the hybrid component of a \(1^{--}\) meson is the component with total quark spin of 0; the gluonic excitation (the \(B_i\) in the meson operator; see below) carries the angular momentum and ensures that the color-octet channel of the quark/anti-quark state contributes to the color-singlet meson propagator. Inclusion of the spin-dependent term allows the mixing of these two states (see Fig. 1). Degeneracies are also lifted with the inclusion of this interaction: e.g., the \(0^{++}/1^{--}\) mass difference is due to the hyperfine spin-spin interaction and is quadratically dependent upon this term. Without the \(\vec{\sigma} \cdot \vec{B}\), these states are degenerate, as are the P-wave states: \(0^{++},1^{++},2^{++}\).

NRQCD has been used previously at this order, and beyond, to study heavy hybrids [2,3]. Recently Drummond et al. [3] reported seeing no quadratic dependence of the \(1^{--}\) hybrid mass (same source and sink operators) upon the normalization of the \(\vec{\sigma} \cdot \vec{B}\) term, although the mass of the \(0^{++}\) hybrid did show effects. The small, if any, dependence of the \(1^{--}\) mass on \(c_B\) can be taken as evidence that the mixing with the S-wave \(Q\bar{Q}\) state is small.

We apply a different strategy, allowing us to measure the effect of the spin-dependent interaction to first order. Rather than apply the \(\vec{\sigma} \cdot \vec{B}\) term at every intermediate time slice in the lattice quark propagator, as has been the usual practice with all terms in the NRQCD expansion [4,5], we restrict it to a particular intermediate time slice, viewing this interaction as a “perturbation” at a specific time in the lattice. Thus, the unperturbed \(Q\bar{Q}\) (quark-antiquark spin one) and unperturbed \(1^{--}\) hybrid (quark-antiquark spin zero) states
propagate without mixing except at the time slice where the perturbation is applied. Then, using the non-exotic hybrid source and the appropriate quarkonium sink (or vice versa), we extract the off-diagonal matrix element of the \( \vec{\sigma} \cdot \vec{B} \) interaction for this two-state system. From this, we estimate the amount of non-exotic hybrid admixture within the true ground-state using a simple two-state mixing model. We expect that such a result may be useful for studies of the creation and decay of heavy quarkonia via color-octet channels (see, e.g., Ref. \[4\]) and models of heavy-quark confinement \[5\].

II. THE METHOD

We construct our lattice meson propagators using the NRQCD approach. This effectively turns what would be a boundary value problem - determining the relativistic quark propagators on a periodic lattice - into an initial value problem since, in the non-relativistic limit, the quarks propagate only forward in time. Using this method, the evolution of the quark propagator in the Euclidean time direction is given by:

\[
G(\vec{x}, t + a) = \left(1 - \frac{aH_0}{2n}\right)^n U^\dagger_t(x) \left(1 - \frac{aH_0}{2n}\right)^n \cdot (1 - \delta_{t',t} a\delta H) G(\vec{x}, t),
\]

where \( H_0 \) and \( \delta H \) are given above and \( n \) is a parameter needed for numerical stability \[6\] \((n > \frac{3}{2m_qa}; \text{we use } n = 2)\). We also use plaquette tadpole improvement of the gauge links. Note that we apply the interaction term, \( \delta H \), at only a single intermediate time step, \( t' \).

We use an incoherent sum of point sources: at the source end, we start with a given quark color and spin at all spatial points, without fixing the gauge; at the sink end, we sum over all the contributions where the quark and anti-quark are at the same spatial point. Since the lattices are not gauge-fixed, we expect the contributions from sources with the quark and anti-quark at different spatial points to average to zero. We combine the quark and anti-quark sources (propagators) with the appropriate spin matrices to construct the meson operators at the source (sink) time slices. The meson operators we use are displayed in Table II. The hybrid operators in Table II involve the color magnetic field, with \( J^{PC} = 1^{+-} \). As can be seen from the quantum numbers, this corresponds to a transverse electric gluon mode in a bag model approach. The color magnetic field is calculated using the “clover formulation” (average of field from 4 plaquettes with corners at these points):

\[
\mathcal{F}_{jk}(x) = \frac{1}{8} [U_j(x)U_k(x + \hat{j})U^\dagger_j(x + \hat{k})U^\dagger_k(x) + U_k(x)U^\dagger_j(x + \hat{k} - \hat{j})U^\dagger_k(x - \hat{j})U_j(x - \hat{j}) + U^\dagger_j(x - \hat{j})U^\dagger_k(x - \hat{k} - \hat{j})U_j(x - \hat{k} - \hat{j})U_k(x - \hat{k}) + U^\dagger_k(x - \hat{k})U_j(x - \hat{k} - \hat{j})U_k(x - \hat{k} + \hat{j})U^\dagger_j(x) - \text{herm. conj.}]
\]

We use this form of the chromo-magnetic field (with the appropriate tadpole factor: \( 1/u_0^4 \)) in the interaction term \( \delta H \). The field used to make the hybrid sources and sinks is constructed in the same fashion, with one exception: we use smeared links (sum of the simple link and
all 3-staples connecting neighboring lattice sites) in place of the simple links in Eq. (4), the object being to improve the overlap with the hybrid ground state.

Since we are working on lattices with a Euclidean metric (i.e., time is imaginary), the propagators should follow decaying exponentials. The form we use to fit the meson correlators, $C(t)$, follows:

$$C(t) = A_0 e^{-m_0 t} + A_1 e^{-m_1 t}. \tag{5}$$

We include the second term to account for excited-state contributions. The propagators were averaged over a set of quenched lattices with Symanzik 1-loop improved gauge action.

To set the physical scale of our lattices, we use the S-P (spin-averaged) mass difference for bottomonium, a quantity which is relatively insensitive to the quark mass. We also create non-zero momentum operators for the $1^{--}$ S-wave meson and determine its kinetic mass from the resulting dispersion relation. An interpolation in $m_q$ is then performed to match this kinetic mass with the experimentally determined value for the mass of the $\Upsilon$. This provides us with an estimate of the bottom quark mass, $m_b$.

For the “mixed” propagators (different source and sink operators), we expect a propagator of the form

$$C_{\text{mix}}(t) = A_0^{1/2} \text{source} A_0^{1/2} \text{sink} \left\langle 1^{--}(H) \left| \frac{g}{2m_q} \vec{\sigma} \cdot \vec{B} \right| 1^{--}(S) \right\rangle e^{-m_0 \text{source} t'} e^{-m_0 \text{sink} (t-t')} + .... \tag{6}$$

Knowing the amplitudes and masses of the source and sink operators from their “unmixed” propagators and fitting this propagator in the region $t > t'$, we can extract the matrix element from the amplitude of $C_{\text{mix}}(t)$ at different values of $t'$. At sufficiently large values of $t'$, we expect less excited-state contamination from the source operator and hope to find a plateau in the value of the matrix element.

For these mixed propagators, we use the tree-level value for the renormalization factor, $c_B = 1$, in the interaction term. The final result for the matrix element, however, should contain the appropriate factor for the given value of the lattice spacing. To address this, we choose a non-perturbative approach. We perform additional spectrum runs, applying the interaction term (with $c_B = 1$ and 2) at all intermediate time slices and for both the quark and anti-quark. By interpolating in the resulting values of the $0^{-+}/1^{--}$ mass difference to that of experiment (or, in the present case of the $b\bar{b}$ system, potential model results), a value for $c_B^2$ may be found.

III. RESULTS

The meson correlators were averaged over 165 quenched, $20^3 \times 64$, $\beta = 8.0$ lattices with Symanzik 1-loop improved gauge action (see Ref. [7] for more details on these lattices). Shown in Fig. 2 are the fit masses for the $1^{--}(S)$ and $0^{++}(P)$ (ground and first-excited state), and the $1^{--}$ hybrid (ground state only). The results of our chosen fits to the correlators appear in Table I. Using the spin-averaged 1S-1P mass difference of 440 MeV for bottomonium, we find an inverse lattice spacing of $a^{-1} = 1604(25)$ MeV [$a = 0.123(2)$ fm].
This differs from a previous determination of the lattice spacing for this set of lattices using the quantity $r_1/a$ from the static quark potential [1]: $a^{-1} = 1449(4) \text{ MeV} [a = 0.1360(3) \text{ fm}]$.

Using two values of the quark mass, $mqa = 2.5$ and $2.8$, we were able to interpolate to a physical quark mass by fixing the kinetic S-wave mass to that of experiment ($M_\Upsilon = 9.46$ GeV). We find a lattice-regularized bottom quark mass of $m_b \approx 4.18$ GeV.

Fits to the “mixed” propagators were also performed and the resulting values for the off-diagonal matrix element of the Hamiltonian appear in Fig. 3, along with the associated jackknife errors. The Hamiltonian was then diagonalized and the admixture of hybrid within the true ground state calculated (see Fig. 4). It may seem odd that the relative errors for $\sin(\theta)$ are much smaller than those for the matrix element. However, the matrix element is quite strongly correlated with the hybrid/S-wave mass difference and since the mixing angle is roughly equivalent to the ratio of these two quantities, the errors for $\sin(\theta)$ tend to be smaller. A plateau is reached in these plots by $t' = 8$. Using the result at $t' = 9$ ($\chi^2/d.o.f. < 1$) and interpolating in the quark mass to $mqa = 2.6$, we find

$$|\Upsilon\rangle = \cos(\theta)|Q\bar{Q}\rangle + \sin(\theta)|Q\bar{Q}_g\rangle = 0.99826(6)|Q\bar{Q}\rangle - 0.059(1)|Q\bar{Q}_g\rangle,$$

(7)
corresponding to $0.0035(1)$ probability admixture of hybrid in the $1^{--}$ bottomonium ground state.

While this result is clearly non-zero, there remain some unresolved issues surrounding the actual numerical value. For one thing, there is the question of the field normalization (i.e., the value for $c_B$). To get a handle on this number, we performed additional spectrum runs with the interaction “turned on” for all times. The results for these propagator fits can be found in Table III. The resulting $0^{--}/1^{--}$ mass differences ($\Delta M_{\Upsilon-\eta_b}$) are found to be $\sim 20$ and $\sim 78$ MeV for $c_B = 1$ and 2, respectively. While there is currently no experimental result for the $\eta_b$ mass, there are some potential model calculations [8,9] which predict this splitting to be in the $\sim 30 - 60$ MeV range. This would imply a value of $c_B^2 \sim 1.5 - 3.1$, corresponding to the mixing angle values $|\sin(\theta)| \sim 0.073 - 0.104$. This range for our value of the mixing angle sits between two values found previously with calculations based upon the MIT bag model [10]: $|\sin(\theta)| = 0.0427$ and 0.1503. While a value of $c_B^2 \sim O(1)$ needed for consistency is encouraging, a more precise numerical result for this parameter remains elusive, mainly due to the fact that the actual $\Upsilon - \eta_b$ mass splitting is not well known.

Comparison of our result for $\sin(\theta)$ with the hybrid mass results of Ref. [3] is not straightforward. Whereas our result can be related to s-channel diagrams where the valence gluon scatters off the quark or anti-quark, there are other spin-dependent contributions to the hybrid mass that cannot be taken into account with our first-order result (see e.g., Ref. [11]), including non-perturbative effects; to include such contributions in the lattice simulation, one needs to apply the $\vec{\sigma} \cdot \vec{B}$ interaction at all time slices. Since we do have hybrid propagators with such a heavy-quark Hamiltonian, we attempted to extract these hybrid masses for a more direct comparison with the results of Ref. [3]. However, as there is a significant contribution to our hybrid propagators from intermediate mixing with the corresponding S-wave states, these fits do not return reliable hybrid masses. An effective mass plot would be better-suited for determining these hybrid masses; however, our lattices lack the time resolution for finding the necessary plateau at short times. Our time resolution is given by
\( a^{-1} \approx 1600 \text{ MeV} \), as compared with \( a^{-1}_t \approx 4500 \text{ MeV} \) in Ref. [3], where they find a plateau in their \( 1^{-+} \) hybrid effective mass plot from \( t \sim 4 - 10 \). This suggests that we would need finer resolution in the \( 1 < t < 4 \) range to observe a similar plateau. It may be for this reason that our former values (without \( \vec{\sigma} \cdot \vec{B} \)) of the hybrid/S-wave mass splittings are quite large: 
\[ \Delta M_{1_H - 1S} = 1.81(12) \text{ GeV}. \]
This is significantly above the 1.644(17) GeV result quoted in Ref. [3] and the recent CP-PACS result of 1.56(18) GeV [12]. However, if we use the lattice spacing as determined by the static quark potential, \( a^{-1} = 1.449(4) \text{ GeV} \) and \( m_a \approx 2.9 \), this splitting (which is relatively insensitive to the quark mass) becomes 1.64(12) GeV. It should also be noted, however, that our heavy quark action is simpler than those used by these groups in that they include the first relativistic correction. The CP-PACS result also includes two flavors of light dynamical quarks.

It would be useful to resolve these issues with a continuum extrapolation as our results thus far are from quenched lattices with only a single value of the coupling. It would also be useful to explore the effects of dynamical quarks.

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### TABLE I. Meson operators.

| $J^P C$ | Operator |
|---------|----------|
| 0$^-$ S-wave ($\eta_b$) | $\bar{Q}Q$ |
| 1$^-$ S-wave ($\Upsilon$) | $\bar{Q}\sigma_i Q$ |
| 0$^+$ P-wave ($\chi_{b0}$) | $\bar{Q}\sigma_i D_i Q$ |
| 1$^+$ P-wave ($\chi_{b1}$) | $\bar{Q}\varepsilon_{ijk}\sigma_j D_k Q$ |
| 2$^+$ P-wave ($\chi_{b2}$) | $\bar{Q}(\sigma_i D_j + \sigma_j D_i - \frac{2}{3}\delta_{ij}\sigma_k D_k) Q$ |
| 0$^-$ hybrid | $\bar{Q}\sigma_i B_i Q$ |
| 1$^-$ hybrid | $\bar{Q}B_i Q$ |

### TABLE II. Fit results and resulting mass differences (with jackknife errors). For each quantity, the first row is for $m_q a = 2.5$, the second is for $m_q a = 2.8$.

| Propagator | Fit range | $m_0 a$ | $\chi^2$/d.o.f. |
|------------|-----------|---------|-----------------|
| $1^-$ (S)  | 9-25      | 0.4920(2) | 13/13 |
| $1^-$ (S)  | 9-25      | 0.4754(2) | 16/13 |
| $0^+$ (P)  | 5-21      | 0.767(4) | 5.4/13 |
| $0^+$ (P)  | 5-21      | 0.749(4) | 5.8/13 |
| $1^-$ (H)  | 1-6       | 1.62(8)  | 0.16/2 |
| $1^-$ (H)  | 1-6       | 1.62(8)  | 0.01/2 |

| Quantity | Mass ($a^{-1}$) | Mass (MeV) |
|----------|-----------------|------------|
| $\Delta M_{1P-1S}$ | 0.275(5) | 440(fixed) |
| $\Delta M_{1H-1S}$ | 1.13(8) | 1800(120) |
| $M_{1^-}^{kinetic}$ | 5.64(6) | 9030(100) |
| $M_{1^-}^{kinetic}$ | 6.36(6) | 10230(100) |
TABLE III. Fit results and resulting mass differences (with jackknife errors) with the interaction term present at all intermediate time slices. For each quantity, the first row is for $c_B = 1$, the second is for $c_B = 2$ ($m_qa = 2.5$).

| Propagator | Fit range | $m_qa$  | $\chi^2$/d.o.f. |
|------------|-----------|---------|-----------------|
| $0^{-+}(S)$ | 8-24      | 0.4736(2) | 20/13           |
|           | 8-24      | 0.4222(2) | 14/13           |
| $1^{--}(S)$ | 8-24      | 0.4859(2) | 12/13           |
|           | 8-24      | 0.4680(3) | 7.4/13          |
| $0^{++}(P)$ | 5-21      | 0.764(5)  | 7.8/13          |
|           | 5-21      | 0.737(5)  | 10/13           |
| $1^{++}(P)$ | 5-21      | 0.762(6)  | 5.8/13          |
|           | 5-21      | 0.737(8)  | 13/13           |
| $2^{++}(P)$ | 5-21      | 0.753(6)  | 2.1/13          |
|           | 5-21      | 0.721(7)  | 6.0/13          |

| Quantity   | Mass ($a^{-1}$) | Mass (MeV) |
|------------|-----------------|------------|
| $\Delta M_{\bar{\chi} - \Upsilon}$ | 0.271(5)       | 440(fixed) |
|           | 0.260(7)       | 440(fixed) |
| $\Delta M_{\Upsilon - \eta_b}$   | 0.01232(8)     | 20.0(4)    |
|           | 0.04583(24)    | 78(2)      |
FIG. 1. Mixing of a $1^{-+}$ hybrid with a $1^{--}$ S-wave via a single application of the $\vec{\sigma} \cdot \vec{B}$ interaction.

FIG. 2. Fit masses (above the zero point, $E_0$) vs. minimum time slice for the $1^{--}$(S) (squares), $0^{++}$(P) (diamonds), and $1^{--}$ hybrid (cross: two-mass fit; fancy plus: single-mass fit) with $m_qa = 2.5$. For the $1^{--}$(S) and $0^{++}$(P), both ground state and first-excited state masses are shown.
FIG. 3. Magnitude of the off-diagonal matrix element of the Hamiltonian (in lattice units) for the $1^-$ S-wave (source) / hybrid (sink) two-state system vs. the time slice, $t'$, at which the interaction ($\delta H$) is applied. The squares are for $m_q a = 2.5$, the diamonds for $m_q a = 2.8$.

FIG. 4. Mixing angle, $\sin(\theta)$, vs. the time slice, $t'$, at which the interaction term ($\delta H$) is applied. The squares are for $m_q a = 2.5$, the diamonds for $m_q a = 2.8$. 