In Brazil, the characterization of solid wood for structural design must be carried out according to the recommendations of the ABNT NBR 7190:1997 standards. Studies about alternatives to improve reliability to test methods are always relevant. Thus, the present study aims to examine the bending modulus of elasticity of samples of three wood species: Canelão (Nectandra membranacea), Corymbia citriodora (Corymbia citriodora) and Angelim (Angelim sp). The results for Canelão and Angelim showed statistical equivalence between the methods at 5% significance level. However, for the Corymbia species, statistical differences of results for bending tests with three and four points were observed. More studies are recommended in order to investigate other wood species, different strength classes and moisture content ranges.

Keywords: Tropical species. Wood characterization. Bending test. Wood stiffness.

No Brasil, a caracterização da madeira maciça para uso estrutural deve ser realizada de acordo com as recomendações da norma ABNT NBR 7190:1997. Estudos sobre alternativas para avaliar a confiabilidade dos métodos de ensaio são relevantes. Assim, este estudo objetiva determinar o módulo de elasticidade na flexão de amostras de três espécies de madeira: Canelão (Nectandra membranacea), Corymbia citriodora (Corymbia citriodora) e Angelim (Angelim sp.). Para tanto, foi planejada uma série de métodos de ensaio com base nos testes de flexão estática a três e a quatro pontos e para as duas faces de cada amostra de madeira. Os resultados foram estatisticamente equivalentes para as espécies Canelão e Angelim a 5.0% de nível de significância. Por outro lado, para a espécie de Corymbia foram identificadas diferenças de resultados nos testes de flexão a três e quatro pontos, e como sugestão foi recomendada a condução de mais estudos para investigar também este efeito em outras espécies de madeira, diferentes classes de resistência e variando a faixa de umidade.

Palavras-chave: Espécies tropicais. Caracterização da madeira. Ensaio de flexão. Rígidez da madeira.

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Introduction

Wood is the only major building material that grows naturally, is renewable and is one of the most popular materials used in the construction sector (SILVA et al., 2012). Wood is an organic material with variations in its chemical, physical and mechanical properties, and such differences may occur within the same species and among different species (CHRISTOFORO et al., 2013; 2015; OBEDE et al., 2012; STOLF et al., 2015). Despite its heterogeneous and anisotropic structure, wood present an excellent resistance/weight ratio sometimes coming up to four times greater than steel (CALIL et al., 2003). In this sense, the knowledge of physical and mechanical properties of wood is essential for timber structures design.

In Brazil, physical-mechanical wood characterization is held according to requirements of ABNT NBR 7190:1997 standard (ABNT, 1997), Appendix B: “Determination of wood properties for structural design”. ABNT (1997)’s requirements help designers to define on how wood can be applied as a material for many different construction purposes (e.g., in the roof structures, window frames, temporary uses in concrete forms, scaffolding, etc.). However, it is important to take into account that the Brazilian standard ABNT NBR 7190:1997 is revised and updated, when necessary, from time to time in order to avoid inconsistencies. Thus, studies focused on different and new possibilities of testing methods are desired to bring out new knowledge that can be considered on future technical review processes of the Brazilian standard and/or international documents.

Several data banks on wooden properties of different species contain mechanical characteristics of which the strength and stiffness are highlighted among the most important ones. Tensile, compression and bending are some examples of typical wood strength properties, while modulus of elasticity represents wood stiffness (Calil et al., 2003), and they are all determined on basis of lab tests and comparison of experimental results with requirements according to standards on the topic (BERTOLINI et al., 2012; OBEDE et al., 2012; SILVA et al., 2012).

Bertolini et al. (2012) determined the influence of the length of wood specimens on the modulus of elasticity obtained in compression parallel to the grain ($E_{c0}$). Based on ABNT (1997) and international standards, comparison of results were performed for 1:3, 1:4 and 1:5 ratios between the side of the cross section (50 mm) and the length of the specimens (150, 200, and 250 mm). Results showed that changing the lengths of the specimens did not affect $E_{c0}$ values at 5% statistical significance level.

Obede et al. (2012) analyzed the influence of wood moisture content in the modulus of elasticity in tensile parallel to the grain of six wood Brazilian species, and proposed mathematical equations to correct test results for 12% of moisture content ($E_{12}$). Results of $E_{12}$ were then compared with the requirements proposed by ABNT (1997), and the main conclusion was there were no significant differences among $E_{12}$.
results and the ABNT (1997) requirements at 5% significance level.

Another study concerning the stiffness property of solid wood was also carried out by Silva et al. (2012). The authors analyzed the influence of wood moisture content on $E_C$ values of six Brazilian wood species of different strength classes and moisture ranges (12, 20, 25 and 30%). Results showed that $E_C$ values for 20-30% moisture range were statistically equivalents. Moreover, a first degree equation was obtained to correlate the studied variables, which leads to statically equivalent estimations of moisture content when compared with the expression suggested by ABNT (1997).

In addition to the above mentioned papers, bending modulus of elasticity is also an important reference to better understand wood stiffness (BRANCHERIAU et al., 2002; CHRISTOFORO et al., 2015). The current Brazilian method according to the ABNT (1997) is based on three-point static bending tests. However, the possible influence of the reference face adopted for the specimen test is neglected by ABNT (1997). Furthermore, the possibility of changing the number of load points for bending tests is also not addressed by ABNT (1997).

On the other hand, there are a number of papers that have evaluated static bending tests with four-points. For instance, Mujika (2006) has verified that flexural moduli obtained by three-point and four-point bending tests were different for the same specimen of epoxy/carbon composite material, with relative differences greater than 5%.

Brancheriau et al. (2002) have performed an analytical study for bending modulus of elasticity of some wood species following the French standards. It was observed that the three-points bending tests underestimated up to 19% the values of bending stiffness relative to the four-point tests. However, this relative difference in results was not continuous following the wood density, and can be also caused by other anatomical differences between the species studied.

Moreover, other relevant studies those have examined the influence of number of load points on bending properties, for various materials, were performed by Theobald et al. (1997), Hayat and Suliman (1998) and Yoshihara (2008).

About the determination of the reference face, i.e., the choice of the upper face of the beam for static bending test, no study on this issue has been published yet. As previously discussed here, the Brazilian standard ABNT (1997) neglects this feature in the current normative document, which has also motivated the present study. Finally, as wood is a heterogeneous and anisotropic material, to assume that the selection of the reference face for bending test is not an important issue may not match the reality.

In this context, this study aims to analyze the influence of different static bending test methods (3 and 4 load points, and choice of the upper face) in the determination of longitudinal modulus of elasticity in specimens of Brazilian wood species.
**Material and methods**

This study was conducted at the Wood and Timber Structures Laboratory (LaMEM), Department of Structural Engineering (SET), São Carlos Engineering School (EESC), São Paulo University (USP). The following wood species were selected: Canelão (*Nectandra membranacea*), Corymbia citriodora (*Corymbia citriodora*) and Angelim (Angelim sp). Static bending tests were carried out assuming six specimens (50 × 50 × 1150 mm) per species, involving a total of 18 specimens.

After that, the modulus of elasticity was calculated for each specimen in two cases: a) according to requirements of ABNT NBR 7190:1997, i.e., three-point model as shown in Figure 1 and by applying Equation 1; and b) four-point model according to Figure 2 and Equation 2, and following the four point static bending model (from the Strength of Materials) adapted to the limits of load and displacement (10% and 50% of the maximum force) provided from the Brazilian standard ABNT NBR 7190:1997. Note that the four point static bending test is the structural model used by the American standard ASTM D 198:1997 to determine the timber modulus of elasticity.

![Figure 1 – Three-point bending tests according to ABNT NBR 7190:1997](image)

**Figure 1 – Three-point bending tests according to ABNT NBR 7190:1997**

\[ E = \frac{\Delta F \cdot L^3}{48 \cdot \Delta \delta \cdot I_z} \quad; \quad \Delta F = F_{50\%} - F_{10\%} \quad; \quad \Delta \delta = \delta_{50\%} - \delta_{10\%} \]  

(1)

Where:
- \( E \): modulus of elasticity, in MPa;
- \( F_{50\%} \): force corresponding to 50.0% of the maximum loading \( F \) applied to specimen, in N;
- \( F_{10\%} \): force corresponding to 10.0% of the maximum loading \( F \) applied to specimen, in N;
- \( \delta_{50\%} \) is the displacement corresponding to 50.0% of maximum loading, in mm;
- \( \delta_{10\%} \) is the displacement corresponding to 10.0% maximum loading, in mm.
- \( I_z \): moment of inertia in relation to the “Z” axis, in mm⁴;
- \( L \): span, in mm.
Influence of testing methods to determine the bending modulus of elasticity of wood

**Figure 2** – Four-point bending tests model adapted from the strength of materials and the Brazilian standard

\[
E = \frac{23 \cdot \Delta F \cdot L^3}{648 \cdot \Delta \delta \cdot I_z} \quad ; \quad \Delta F = F_{50\%} - F_{10\%} \quad ; \quad \Delta \delta = \delta_{50\%} - \delta_{10\%}
\] (2)

Where:
- \(E\): modulus of elasticity, in MPa;
- \(F_{50\%}\): force corresponding to 50.0% of the maximum loading \(F\) applied to specimen, in N;
- \(F_{10\%}\): force corresponding to 10.0% of the maximum loading \(F\) applied to specimen, in N;
- \(\delta_{50\%}\) is the displacement corresponding to 50.0% of maximum loading, in mm;
- \(\delta_{10\%}\) is the displacement corresponding to 10.0% maximum loading, in mm;
- \(I\): moment of inertia of the cross section, in \(\text{mm}^4\);
- \(L\): span, in mm.

The reference moisture content to calculate \(E\) was 12%. According to Equations 1 and 2, all the bending tests were conducted to not exceed the proportionality limit, which explains why \(E\) values were calculated based on forces at 10 and 50% of the force in rupture. Furthermore, all experiments were performed using a Universal Test Machine, 250 kN capacity. Figure 3 shows some wood specimens and a bending test representation.

**Figure 3** – Wood specimens (a) and a four-point bending test (b).
During the static bending tests, the effect of changing the reference face was also analyzed. Thus, all the 18 specimens were tested with force applied in the two faces (Face A and Face B). Finally, statistical analyzes of results were performed using the Minitab 16 software based on Analysis of variance (ANOVA) and Tukey tests as a multiple comparison procedure. The significance level adopted was 5%.

According to the ANOVA results, p-values higher than 5% accept the null hypothesis ($H_0$) and reject it otherwise (MONTGOMERY, 2005). The test was made to check whether the mean value of a response (i.e., bending modulus of elasticity) from a population under investigation was the same or different from the mean value of another population. To validate the ANOVA model, residual analyses were performed to check normality, homogeneity and independence (MONTGOMERY, 2005).

**Results and discussion**

Results of modulus of elasticity (E) are presented in the following two situations: 1) three-point static bending results in Table 1 (forces applied in faces A or B), and mean values of E and statistical results according to Table 2; and 2) four-point bending tests results in Table 3 (results of E per specimen) and Table 4 (means of E and statistical results).

Tables 1 and 3 present the dataset obtained from the experiments with the six specimens per wood species. In Tables 2 and 4, different letters simply mean values statistically different at the 95% level of probability. Brackets contain the coefficients of variation expressed in %.

| Specimens | E (MPa) | E (MPa) |
|-----------|---------|---------|
|            | Face A  | Face B  |
| Canelão - 1| 16982   | 16223   |
| Canelão - 2| 18213   | 17150   |
| Canelão - 3| 16435   | 15069   |
| Canelão - 4| 17670   | 17391   |
| Canelão - 5| 17325   | 16463   |
| Canelão - 6| 17332   | 16446   |
| Corymbia - 1| 18352  | 18038   |
| Corymbia - 2| 19074  | 19372   |
| Corymbia - 3| 18001  | 19331   |
| Corymbia - 4| 19939  | 21117   |
| Corymbia - 5| 17179  | 18069   |
| Corymbia - 6| 18448  | 18191   |
Table 2 - Mean values of E (faces A and B) according to Table 1

| Species   | E (MPa)       |
|-----------|---------------|
|           | Face A | Face B |
| Canelão   | 17325 a (5) | 16463 a (6) |
| Corymbia  | 18499 a (5) | 19020 a (6) |
| Angelim   | 14544 a (10)| 14411 a (13)|

Results in Table 2 show that coefficients of variation did not exceed 13%, therefore, in accordance to the ABNT NBR 7190:1997 recommendation, i.e., maximum 18%. Statistical results show that E values were equivalents at 5% confidence level. In this sense, to use face A or B in the bending tests was not a relevant parameter of studying during the determination of the modulus of elasticity.

According to the Tukey test, equal letters imply treatments with equivalent mean results; thus, letter “a” represents the highest mean results, “b” the second highest mean values, etc. In Table 2, all results show equivalent E results at 5% significance level, which explains why all results show letter “a”.

Table 3 - Values of E in three and four-points in static bending tests

| Specimens   | E (MPa) 3 points | E (MPa) 4 points |
|-------------|------------------|------------------|
| Canelão - 1 | 16982            | 15840            |
| Canelão - 2 | 18213            | 19581            |
| Canelão - 3 | 16435            | 16560            |
| Canelão - 4 | 17670            | 16796            |
| Canelão - 5 | 17325            | 17194            |
| Canelão - 6 | 17332            | 17190            |
| Corymbia - 1 | 18352          | 20595            |
| Corymbia - 2 | 19074           | 20700            |
| Corymbia - 3 | 18001           | 21417            |
| Corymbia - 4 | 19939           | 21994            |
| Corymbia - 5 | 17179           | 18542            |
| Corymbia - 6 | 18448           | 18680            |
From Table 4, it can be observed that coefficients of variation did not exceed the 18% as stated by the ABNT (1997)’s recommendations. From the ANOVA and Tukey results, the number of load points showed no statistically relevant changes in the E values for Angelim and Canelão species. On the other hand, it is noteworthy that for the specimens of Corymbia this conclusion does not apply, because the Tukey test results showed different letters, being results of E for 3 points 9% lower than for 4 points. Therefore, E results can be statistically different when working with 3 or 4 load points in bending tests for Corymbia specimens.

As predicted by other studies in the literature, Mujika (2006) highlighted differences exceeding 5% for results of the bending modulus of elasticity of specimens of composite carbon/epoxy. Higher E results were also observed for 4 point static bending tests. The author has developed mathematical models to correct this difference in E values, reducing such differences for less than 1%. Therefore, even in unnatural materials, the difference in E results can be also relevant, and this information is important especially for design and engineering applications.

In the case of the Corymbia species, a variation of E results lower than 10% as stated in this paper may not be enough to compromise a timber structure design, but higher differences in E results could be a problem in designing wood components with rational use of natural resources. That is why, as a future outlook, it should be important to conduct further and deeper studies concerning the bending modulus of elasticity in three and four-point loads for this wood species.

The reason related to this phenomenon, in the case of the Corymbia species, may be related to the intrinsic heterogeneity of the specimens studied. After all, wood is a natural material that can show significant variations in its chemical, physical and mechanical properties within a set of specimens from a single kind of wood species.
As a suggestion, further tests should be conducted on the influence of load application in 3 and 4 points in static bending to other wood species in order to investigate possible other analogous conclusions as showed here for the *Corymbia citriodora*. Thus, in case of finding more relevant differences in $E$ results, an outlet for the correctness of results may be the establishment of mathematical models.

**Conclusions**

This paper presented a study on the influence of testing methods to determine the bending modulus of elasticity of Brazilian wood species. This paper contributes to increase knowledge on the subject, as well as help decision-making about adopting normative parameters.

Based on the conditions under which this research was carried out, it is possible to infer the following conclusions:

- For static bending tests considering faces A and B, results of $E$ were statistically equivalent for all the three species studied;
- For static bending tests with 3 and 4 points, results of $E$ were also statistically equivalent for Angelim and Canelão wood species. However, a different conclusion was observed for the *Corymbia citriodora*, which showed the statistical difference of $E$ results for 5% of significance level;
- Given the statistical differences for at 3 and 4 point static bending tests for *Corymbia citriodora*, we suggest further testing with this species, and also with other different wood species. If more findings highlight significant differences in results of bending modulus of elasticity, a mitigation measure could be the correction of experimental results using mathematical models. At this point, it is noteworthy that other similar researches in the literature have already been taken this approach, e.g., Brancheriau et al. (2002) and Mujika (2006).
- It is expected that the results presented in this paper serve as informational subsidy in order to produce discussions that promote scientific advancement in timber structure design. In this line, as for bending tests in faces A and B showed no statistically significant differences in results, the test methods currently incorporated into the Brazilian standard ABNT (1997) do not need to be reformulated. However, for the case of static bending tests at 3 or 4 load points, it is advisable to carry out critical analysis of the results presented here in order to also investigate other wood species – including different strength classes and moisture content ranges.
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