Three-Body Analysis of Incoherent Photoproduction of $\eta$ Mesons on the Deuteron near Threshold*

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Abstract. The importance of three-body dynamics in the $\eta np$ system in elastic and inelastic $\eta$-deuteron scattering as well as coherent and incoherent $\eta$ photoproduction on the deuteron in the energy region from threshold up to 30 MeV above has been investigated. It is shown that a restriction to first order rescattering with respect to the $NN$- and $\eta N$-final state interactions, i.e., restriction to rescattering in the two-body subsystems, does not give a sufficiently accurate approximation to the $s$-wave reaction amplitude and that higher order terms, as described by the three-body dynamics give very substantial contributions.

1 Introduction

Eta photoproduction on the deuteron is of interest with respect to information on the neutron amplitude, the role of $\eta N$ interaction; With respect to the absolute strength of the neutron amplitude, quasi-free production in the incoherent process is favoured; However, with respect to the relative phase between the elementary amplitudes of proton and neutron, the coherent reaction on the deuteron is better suited because the deuteron acts as an isospin filter and the proton and neutron amplitudes interfere coherently.

2 The Elementary $\eta$ Production Operator

The following pure resonance form with excitation of the $S_{11}(1535)$ is used

$$ t^{(s/v)}_{\gamma\eta} = \frac{k_{\gamma N}}{M_{N^*} + M_N} \frac{e g_{\gamma N^*}^{(s/v)} g_{\eta NN^*}}{W_{\gamma N} - M_{N^*} + \frac{i}{2} \Gamma(W_{\gamma N})} i \sigma \cdot \varepsilon_\lambda. $$

with parameters $M_{N^*} = 1535$ MeV, $\Gamma_{\pi N} = 16$ MeV, $g_{\eta NN^*} = 2.10$, and $g_{\pi NN^*} = 1.19$. The vertex constant $g_{\gamma NN^*}$ is related to the helicity amplitude

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\[ A_{1/2}^N \text{ by } e g_{\gamma NN^*} = \sqrt{2M_N^* (M_N^* + M_N) / (M_N^* - M_N)} A_{1/2}^N. \]

This parametrization yields a good description of the total cross section on the proton using \( A_{1/2}^p = 0.104 \text{ GeV}^{-1/2} \) as is shown in Fig. 1.

**Figure 1.** Total cross section for \( \eta \) production on the proton. Data from Krusche et al., *Phys. Lett.* B 358, 40 (1995).

### 3 Inclusion of Final State Interaction in Incoherent \( \eta \) Photoproduction by Rescattering in 2-Body Subsystems

Near threshold the impulse approximation (IA) yields a very small cross section for \( d(\gamma, \eta)np \) due to the large momentum mismatch and indeed fails drastically yielding a cross section much too low compared to experiment. Therefore, we first have performed an approximate treatment of final state interaction (FSI) by taking into account only complete rescattering in the two-body \( \eta N \) and \( NN \) subsystems of the final state (see Fig. 2).

**Figure 2.** Diagrams for \( d(\gamma, \eta)np \): (a) IA, (b) \( NN \) rescattering, (c) \( \eta N \) rescattering.

In view of the near-threshold region it is sufficient to consider rescattering in \( s \)-waves only. This first order rescattering leads to a considerable improvement as is seen in the left panel of Fig. 3. The spectrum of the outgoing \( \eta \) meson shows at low energies a distinct signature of the final state \( NN \) rescattering exhibiting the prominent \( ^1S_0 \) peak close to threshold of \( NN \)-scattering (see middle and right panels of Fig. 3). Differential cross sections near threshold are shown in Fig. 4 and one notes a considerable improvement over the IA.
Figure 3. Left panel: Total cross section for \(d(\gamma, \eta)np\): dashed: IA, solid: IA + rescattering, dash-dotted: IA + \(NN\) rescattering, data: inclusive \(\gamma d \rightarrow \eta X\) from Krusche et al., Phys. Lett. B 358, 40 (1995). Middel and right panels: \(\eta\)-meson spectra at forward angles: dotted: IA; solid: IA + rescattering; dashed: without \(D\)-wave contribution to \(NN\)-rescattering amplitude. Top abscissa indicates final \(NN\)-excitation energy \(E_{np}\).

Figure 4. Differential cross section for \(d(\gamma, \eta)np\). Dotted: impulse approximation; Dashed: IA plus \(NN\) rescattering; Full: complete result. Exp.: inclusive \(\gamma d \rightarrow \eta X\) Krusche et al., Phys. Lett. B 358, 40 (1995).

However, the left panel indicates that first order rescattering still fails to explain quantitatively the enhancement of the experimental data right above threshold. This is corroborated by very recent more precise near-threshold data by Hejny et al., Eur. Phys. J. A 13, 493 (2002).

4 Three-Body Treatment of Final State Interaction in \(\eta\)-Deuteron Scattering and \(\eta\) Photoproduction

The very strong effect from the hadronic interaction in the final state in first order rescattering suggests that a genuine three-body treatment is required. Defining as channels "\(N^*\)" the channel with one spectator nucleon and "\(d\)" the channel with the \(\eta\) meson as spectator and taking the interactions in separable form, one obtains from the AGS-3-body equations a set of coupled equations for the channel transition amplitudes (see Fig. 5 for a diagrammatic representation), which reads

\[
X_{N^*d} = Z_{N^*d}^{(\eta)} + Z_{N^*d}^{(\eta)} d^{(\eta)} T_d^{(\eta)} X_{d}^{(\eta)} + Z_{N^*d}^{(\pi)} d^{(\pi)} T_d^{(\pi)} X_{d}^{(\pi)} + (Z_{N^*N^*}^{(\eta)} + Z_{N^*N^*}^{(\eta)} ) T_{N^*N^*} X_{N^*d},
\]

\[
X_{d}^{(\eta)} = 2 Z_{dN^*}^{(\eta)} T_{N^*} X_{N^*d}, \quad X_{d}^{(\pi)} = 2 Z_{dN^*}^{(\pi)} T_{N^*} X_{N^*d},
\]
Figure 5. Representation of three-body equations for $\eta$-deuteron scattering.

where the $N^*$ channel comprises two components $N^*(\eta)$ and $N^*(\pi)$ in order to account for the coupling of the $\eta N$-channel to the $\pi N$-channel. Because of the near threshold region only $s$-waves are included. For the $NN$ interaction the tensor force is neglected for reasons of simplicity. In detail, we have taken for the $NN$-channel a driving term $V_d(p, p') = g_d(p) g_d(p')$ with

$$g_d(p) = \frac{g_d^2 \beta_d^2}{p^2 + \beta_d^2}$$

yielding a $t$-matrix of the form

$$t_d(p, p') = g_d(p) \tau_d(E) g_d(p')$$

with

$$\tau_d(E) = -\frac{1}{2 M_N} \left[ 1 + \frac{g_d^2 \beta_d^2}{16\pi \sqrt{EM_N} + i \beta_d} \right]^{-1}$$

Correspondingly, the $t$-matrix for the $\eta N$-channel has the form $t^{(i)}_{\eta N}(p, p') = g^{(i)}_{\eta N}(p) \tau^{(i)}_{\eta N}(E) g^{(i)}_{\eta N}(p')$, where $g^{(i)}_{\eta N}(p) = g_{\eta N} \beta_{\eta N}^2 / (p^2 + \beta_{\eta N}^2)$ and

$$\tau^{(i)}_{\eta N}(W) = \left[ W - M_0 - \Sigma_{\pi}(W) - \Sigma_{\eta}(W) \right]^{-1}.$$

For the values of the actual parameters we refer to [2]. This interaction yields a scattering length $a_{\eta N} = 0.75 + i 0.27$ fm. Results for $\eta d$ scattering are displayed in

Figure 6. Results for $\eta$-deuteron scattering from [2]. Left panel: elastic total cross section, middle panel: spectrum of emitted $\eta$ mesons in inelastic scattering, notation for left and middle panels: dotted: IA; dashed: first order rescattering; solid: three-body calculation. Right panel: various contributions to total $\eta d$ cross section.

Fig. 6. For elastic scattering one notes a rapid increase of the total cross section approaching the threshold (see left panel of Fig. 6), which is explained by the
presence of a virtual pole in the $\eta NN$ system \cite{3}. This feature is not described by first order rescattering. A similar failure is also seen for the inelastic channel (middle panel of Fig. 6). All contributions to the total cross section are shown in the right panel of Fig. 6.

Turning now to $\eta$ photoproduction, an analogous representation as in Fig. 5 holds for the coherent and incoherent photoproduction amplitudes, replacing the incoming $\eta$ by a photon line. Results for total and differential cross sections of the coherent photoproduction are shown in Fig. 7. One readily notes a drastic increase of the total cross section over the mere IA right above threshold due to a strong attraction in the ($S = 1, T = 0$) channel, which the first-order rescattering calculation is not able to reproduce. Also in the differential cross sections in Fig. 7 a large difference between first-order calculation and complete three-body approach is seen. Similar conclusions are reached for the incoherent reaction. The importance of a three-body treatment is demonstrated by the $\eta$ meson spectrum in Fig. 8. Close to threshold, the first order rescattering underestimates significantly the three-body approach, whereas at higher energies it yields an overestimation. Total and differential cross sections are shown in Fig. 9. The inclusive total cross section data exhibit a distinct enhancement near threshold which is reproduced by the 3-body approach (left panel of Fig. 9). This is also the case for the differential cross sections (middle and right panels of Fig. 9). It remains to be seen, whether a more realistic calculation is also able to describe the data.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.png}
\caption{Results for $d(\gamma, \eta)d$ from \cite{2}. Left panel: Total cross section. Middle and right panels: differential cross sections. Dotted: IA; dashed: first order rescattering; solid: complete 3-body model; dash-dot: 3-body model without $\pi$-exchange contribution; Exp.: P. Hoffmann-Rothe et al., \textit{Phys. Rev. Lett.} 78, 4697 (1997).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure8.png}
\caption{$\eta$ meson spectrum for $d(\gamma, \eta)np$ close to threshold from \cite{3}.}
\end{figure}
Figure 9. Results for $d(\gamma, \eta)np$ from [2]: Inclusive data from Hejny et al., *Eur. Phys. J.* A 13, 493 (2002). Left panel: Total cross section: (a) incoherent: dotted: IA; dashed: first order rescattering; dash-dot: complete 3-body model; (b) inclusive: solid. Middle and right panels: Differential cross sections: dotted: IA; dashed: first order rescattering; solid: complete 3-body model.

5 Conclusions and Outlook

The main conclusions are: (i) Near threshold only a three-body approach gives an adequate description of the $\eta NN$ dynamics. The first order rescattering approximation for the final state interaction fails drastically. (ii) For $\eta d$ elastic scattering a very strong enhancement near threshold is found in the three-body calculation which is not born out in first order rescattering. The coupling to the $\pi NN$ channel is relatively unimportant. (iii) With a Yamaguchi-type separable interaction a satisfactory description of experimental data on $\eta$ photoproduction on the deuteron is achieved, in particular the enhancement of the total cross section right above the threshold, and the nearly isotropic angular distribution of the outgoing $\eta$ meson is reproduced in the three-body approach in contrast to the first order rescattering. (iv) The $\eta N$-interaction can be studied in incoherent eta production near threshold. However, a first order rescattering calculation as used, e.g. by A. Sibirtsev et al., *Phys. Rev.* C 65, 044007 (2002), is not reliable for that purpose, because right above threshold a three-body approach is mandatory.

As open problems remain (i) the inclusion of additional two-body effects like meson exchange currents as discussed in the coherent process [4], and (ii) the use of realistic $NN$ interactions with inclusion of the tensor force and the deuteron $D$-wave.

References

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