Measurement uncertainty of discharge air velocity by graphical integration in closed conduits using Pitot tubes

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Abstract. This paper addresses the measurement uncertainty in the determination of the discharge air velocity, using Pitot static tubes, for the flow-rate measurement in closed conduits related to ventilation installations. This indirect measurement method is supported in local air velocity measurements performed in known positions in the conduit’s cross-section. Graphical integration between extreme measuring points is one of the available techniques used for the calculation of the discharge air velocity. This paper illustrates a Monte Carlo simulation procedure for the measurement uncertainty propagation from the input quantities – local air velocities, measurement positions, conduit dimension and inner wall roughness – to the output quantity – discharge air velocity.

1. Introduction

The air flow-rate is a relevant quantity measured in ventilation installations in buildings and experimental research and testing facilities such as wind tunnels and chambers. Measurements in close conduits are often based in the velocity area standard method using Pitot tubes [1], where several local air velocity measurements are performed in known positions in the conduit’s circular or rectangular cross-section. These measurements support the determination of the discharge velocity, for which standard methods are known: graphical or numerical integration and arithmetic methods. This paper addresses the measurement uncertainty propagation applied to the graphical integration method. A Monte Carlo simulation procedure [2] is described in this paper, being justified by the complexity of the multivariable additive mathematical model using a large number of measuring points (air velocity and position), requiring to account for the local air velocities contributions between extreme measuring points and the peripheral zones in the conduit’s cross-section. The calculation procedure developed was applied to an experimental study concerning the air flow-rate measurement in an aerodynamic wind tunnel used by LNEC for experimental research activities and to traceability of anemometers. Several local air velocities profiles were obtained using a Pitot tube placed on a reference rectangular cross-section of the wind tunnel, being measured together with influence quantities (namely, air temperature and atmospheric pressure) and other relevant input quantities related with the geometry of the setup. The contributions to the combined measurement uncertainties were evaluated and their formulation is described in this paper.

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2. Discharge velocity in a rectangular cross-section by graphical integration

From a mathematical perspective and considering a rectangular cross-section where local air velocity \( v \) is evaluated, the discharge velocity \( U \) is defined by the following equation

\[
U = \int_0^1 \int_0^1 v \, d\left( \frac{h}{H} \right) \, d\left( \frac{l}{L} \right),
\]

(1)

where \( L \) and \( H \) are, respectively, the conduit width and height in the measuring cross-section and \( l \) is the distance between a particular measuring point to the side-wall chosen as origin and \( h \) is the height of a particular point above the bottom.

In the graphical integration method, a representation of the velocity profile is performed in order to determinate the area under the curve which is bounded by the measuring points closest to the conduit wall. The velocity profile in the peripheral zone – defined as the area between the wall and the curve through the measuring points closest to the wall – is also taken into account in the global determination of the discharge velocity and assumed to satisfy the Karman’s conventional power law [3] given by

\[
v_x = v_a \left( \frac{x}{a} \right)^{1/m}
\]

(2)

where \( v_x \) and \( v_a \) are, respectively, the local air velocity at distance \( x \) from the nearest wall and the local air velocity measured in an extreme point at a distance \( a \) from the nearest wall, and \( m \) is a coefficient which depends on the wall roughness and is comprised between four (rough wall) and ten (smooth wall).

The graphical integration method can be summarized in the following steps [1]: (i) plotting the variation curve of the velocity on each horizontal line between the extreme measuring points, as a function of the relative distance \( l/L \); (ii) determining the value of the area bellow the curve between extreme measuring points; (iii) addition of the two terms corresponding to the peripheral zones equal to

\[
\left( \frac{m}{m+1} \right) \left( \frac{a}{L} \right) v_a,
\]

(3)

in order to obtain the mean velocity \( u_i \) on the horizontal measuring line; (iv) plotting the variation curve of \( u_i \) between the extreme horizontal measuring lines as a function of the relative height, \( h/H \), of the corresponding horizontal line; (v) determining the value of the area below the curve between extreme horizontal lines; (vi) addition of the two terms corresponding to the peripheral zones equal to

\[
\left( \frac{m}{m+1} \right) \left( \frac{a'}{H} \right) u_{a'},
\]

(4)

in order to obtain the discharge velocity, being \( u_{a'} \) the mean velocity on the horizontal measuring line closest to the wall (at a distance \( a' \) from the wall).

3. Measurement uncertainty evaluation

3.1. Input experimental data and probabilistic formulation

A total of 20 points were measured along one vertical line of the rectangular cross-section reference of LNEC’s wind tunnel no. 4, using a SI traceable digital micromanometer and a Pitot tube. During the tests, both the air temperature and atmospheric pressure remained approximately constant (15.5 °C and 982.2 mbar, respectively). The conduit height in the measuring cross-section was equal to 250 mm and the distance between extreme measuring points to the conduit wall corresponded to 30 mm. A 10 mm
distance was defined between measuring points. One of the air velocity profiles studied is shown in Figure 1.

![Figure 1.](image)

**Figure 1.** Experimental study of air velocity profile.

The air density and velocity computations were obtained according with the standard models defined in [1] and each of local air velocity estimates presented was based on the average value obtained from five differential pressure sequential measurements, from which a stability value was also estimated and accounted for as an uncertainty component.

Table 1 presents the probabilistic formulation of the input quantities based on the available information and related to: (i) the calculation of the air velocity terms of the peripheral zones through equations (3) or (4); (ii) and the graphical integration process aiming the determination of the the area below one segment of the experimental curve shown in Figure 1.

| Input quantity | Estimate | Probability distribution | Parameterization values | Additional notes |
|----------------|----------|--------------------------|-------------------------|------------------|
| $m$            | 7        | Uniform                  | 3                       | Used only for the peripheral zone calculation |
| $a, h_1$       | 30 mm    | Uniform                  | 2.5 mm                  | Used both for the peripheral zone calculation and the graphical integration |
| $L$            | 250 mm   | Uniform                  | 2.5 mm                  |                  |
| $v_{a, v_1}$   | 7.48 m/s | Gaussian                 | 0.74 m/s                |                  |
| $h_2$          | 40 mm    | Uniform                  | 2.5 mm                  | Used only for the graphical integration |
| $v_2$          | 7.50 m/s | Gaussian                 | 0.62 m/s                |                  |

3.2. Monte Carlo simulations results

The computational simulation algorithm was developed in Octave following the guidelines of the S1-GUM [2] to apply Monte Carlo Method (MCM), using validated computational routines. For each simulation, a total of $10^6$ trials were performed in order to obtain convergent solutions, being examples of probability density functions (pdf) shown in Figures 2 and 3, as well as in Table 2.
Figure 2. Output pdf for the area related to the peripheral zone.

Figure 3. Output pdf for the area below one segment of the curve.

Table 2. Results obtained using MCM

| Quantity                          | Mean   | Mode  | 95% expanded uncertainty | Computational accuracy |
|----------------------------------|--------|-------|--------------------------|------------------------|
| Area of the peripheral zone      | 0.78 m/s | 0.76 m/s | 0.15 m/s               | <0.003 m/s             |
| Area below one segment of the curve | 0.30 m/s | 0.29 m/s | 0.12 m/s               | <0.002 m/s             |

Additional simulations were performed for the remaining segments of the velocity curve shown in Figure 1, revealing similar estimates and uncertainties of the area below the segment. These uncertainties were propagated through the sum of terms (areas of the peripheral zones and areas below all segments of the curve) which contributed for a discharge air velocity expanded measurement uncertainty (95%) of 0.57 m/s, for an estimate equal to 7.34 m/s.

4. Conclusions

This paper showed the application of a Monte Carlo simulation procedure for the determination of the measurement uncertainty of the discharge air velocity, considering only one vertical measurement in the conduit cross-section. Since in experimental practice, additional measurement lines and points are usually considered, the developed computational procedure is particularly suitable for more extensive experimental studies on air flow-rate measurement in closed conduits using Pitot tubes.

Future work will be focus on: (i) the uncertainty contribution of the remaining numerical integration and arithmetic standard methods [1] aiming to accurate determination of the discharge velocity; (ii) covariance studies related to the differential pressure measurements, which are usually performed with the same measurement chain.

5. References

[1] ISO 3966:2008 Measurement of fluid flow in closed conduits – Velocity area method using Pitot static tubes, 2nd edition, International Standard Organization, July 2008.

[2] SI-GUM Evaluation of measurement data. Supplement 1 to the “Guide to the expression of Uncertainty in Measurement”. Propagation of distributions using a Monte Carlo method”, 1st edition, JCGM, 2008.

[3] Sabersky R H, Acosta A J, Hauptmann E G, Gates E M Fluid Flow – A First Course in Fluid Mechanics, 4th edition, Prentice Hall, 1999.