Integrated Facility Location and Production Scheduling in Multi-Generation Energy Systems

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Abstract

In this paper, we investigate the energy system design problems with the multi-generation technologies, i.e., simultaneous generation of multiple types of energy. Our results illustrate the economic value of multi-generation technologies to reduce spatio-temporal demand uncertainty by risk pooling both within and across different facilities.

Keywords: Multi-generation systems, sustainable operations, facility location, flexible production systems, second-order conic programming.

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1 Introduction

Energy systems are multi-dimensional: Electricity, heat, oil, natural gas and even biofuels, are distributed to meet demand with supply. An interesting feature of the energy systems is that their dimensionalities are intertwined: Multiple types of energy can be generated simultaneously; Consequently, the optimal design for the value chain of one type of energy, i.e., from generation to distribution, should not be independent from another. This, however, poses great challenges to policy makers and system planners.

In this paper, we investigate the energy systems design problems when multi-generation technologies are available. Co-generation of heat and electricity is a best-known example of such technologies, which increases overall energy efficiency compared with separate production of heat and power [14]. By making more efficient use of fuel inputs and renewable sources, multi-generation technologies also allow low carbon emission [17]. A prominent trend in multi-generation technologies is that, different energy solutions and devices are increasingly installed at the users’ premises to supply their local multi-energy needs [4].

Our research contributes to the state-of-the-art from a system perspective. In this paper, we address the following two research questions:

- How to design an energy production network which integrates macro-level strategic decisions (facility location, multi-generation technology investment, etc.) and micro-level operational decisions (production planning, energy transportation etc.)?

- What is the economic value of the multi-generation technologies under demand uncertainties for energy?

To be specific, the first research question explores methodologies for the system design problems. The second research question is motivated by the fact that both inter-temporal and spatial demand uncertainties pose fundamental challenges to the design and optimization of energy systems. For example, adequate cooling demand in the summertime and thermal demand in the wintertime are needed to make their joint generation economically feasible [17]. From this perspective, we not only make methodological contributions to tackle this challenge, but also offer insights on the value of the multi-generation technologies to policy-makers.

The rest of this paper is organized as follows. Section 2 reviews relevant literature. Section 3
introduces our model setup. In Section 4 we carry out the analysis. In Section 5 we describe our computational method, and provide numerical examples. Section 6 concludes this paper with a discussion of the future research directions.

2 Related Work

Multi-generation technologies in power and energy systems become increasingly attractive to the research community. A holistic multi-energy system assessment (gas, heat and power) is conducted in [5]. The flexibility design problem to cope with the uncertainty in wind power generation is addressed in [9]. In addition, investment in system flexibility helps the integration of wind power to the combined heat and power network [3]. The existing literature often relies on variations of the unit commitment model [8, 15, 16, 18, 19], or dynamic control [13], to solve a dynamic scheduling problem. However, this approach restricts its application to the micro-level operational decisions due to its computational complexity. In contrast to the aforementioned traditional approach, we model the production scheduling component using an asymptotic proxy, which implicitly implements a dynamic allocation of production capacity, while abstracting away from the scheduling details. This formulation is capable of being integrated with both macro- and micro-level decisions in energy systems.

Facility location problems have been studied extensively in the operations research community. The integrated approach of network design is proposed in [22], which is shown to be superior than solving location and capacity planning problem separately. Our model is also related to the literature on the transportation cost minimization [26], and service scheduling problem [20]. The integration of the process flexibility into the network design has been studied by [11, 12].

Our model is also related to network flexibility design literature, such as [7] and the references therein. Nevertheless, we avoid any a priori flexibility assumption by taking an engineering approach (stochastic programming with chance constraints). Note that this methodology has been also applied to the unit commitment problems [23, 25]. The classic solution to such stochastic program is by scenario sampling [2], or robust optimization [10]. In this paper, we adopt an approximation that is analogous to the “square-root staffing rule” from the queueing theory [24], and derive a tractable approximation as mixed-integer second-order conic program (MISOCP), which is less conservative and better interpreted than the existing approaches.
3 Model

**Energy network.** We consider an energy network consisting of energy production facilities (*plants*, denoted by a set *I*), to supply and satisfy local demands (*customers*, denoted by a set *J*) of multiple types of energy. A plant may use different energy production units (*equipments*, denoted by a set *L*) to meet local demands from customers. Customers generate demands for different *types of energy* (denoted by a set *K*).

**Customers.** The customers’ energy demands follow Poisson distribution. We assume that orders from different demand locations are independent. We parameterize the demand uncertainty by a *scenario* *ω*, which is drawn from the sample space Ω (collections of all scenarios). Thus, we can write the demand from customer *j* for energy *k* under a given scenario *ω* as a Poisson random variable *λ*$_{jk}$(*ω*). The expectation of a customer *j*’s demand for energy *k*, which is the mean for the corresponding Poisson random variable, is denoted by *Λ*$_{jk}$. Potential revenue from satisfying unit demand for energy *k* is *V*$_k$, regardless of customer.

**Plants.** A plant is to be placed at a chosen location, and the setup cost for the plant at location *i* is *f*$_i$. We use a binary decision variable *Z*$_i$ to decide whether a plant should be in potential location *i*. We can generate the distance between customer *j* and facility location *i* as *d*$_{ij}$. The transportation cost *φ*$_k$(*d*$_{ij}$) can be any bounded function of distance *d*$_{ij}$, and also depending on the energy type *k*. The cost of unit equipment *l* to be invested at location *i* is denoted by *g*$_{il}$, where the set of all equipments is *L*. *X*$_{il}$ is a decision variable representing the number of equipment *l* available at location *i*. Equipments can be placed at location *i* only when a plant has been set up at this location, i.e.,

\[ X_{il} \leq MZ_i, \forall i \in I, l \in L, \]

where *M* is an arbitrarily large positive number, e.g., the maximum number of equipments exhausting all investment.

**Multi-generation technology.** Since an equipment represents a multi-generation unit, we use *π*(*k*) to denote the set of equipments which can be used to produce energy *k*, while *π$^{-1}$(l)* denotes the types of energy which can be produced by equipment *l*. To model a *flexible* energy production system, we assume a many-to-many mapping between the types of energy and multi-generation units. Equipments differ in their flexibility. The production capacity at each plant for energy *k* is determined by both the dedicated production units and the multi-generation units which can be used to produce energy *k*.
Production scheduling. We define $Y_{ijk}(\omega)$ as a binary variable if energy $k$ from customer $j$ should be produced from location $i$. Without loss of generality, the production rates of different equipments are deterministic and are normalized to one. A feasible production schedule should assign at least one plant to produce energy $k$ for customer $j$, i.e.,

$$\sum_{i \in I} Y_{ijk}(\omega) = 1, \forall j \in J, k \in K, \omega \in \Omega.$$ 

At the planning stage, the demands are not realized yet. The planner needs to consider the uncertainty so that the equipments have sufficient capacity for demands with high guarantee. Let $\xi \subseteq 2^K$ be any subset of energy (wherein the notation $2^K$ represents the power set of $K$). The aggregate production capacity for this set of energy is $\sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il}$, while the aggregate demand for this set of energy is $\sum_{k \in \xi} \sum_{j} Y_{ijk}(\omega) \lambda_{jk}(\omega)$. For small $\alpha$, the service guarantee constraints (resource constraints) require that:

$$p \left\{ \sum_{k \in \xi} \sum_{j \in J} Y_{ijk}(\omega) \lambda_{jk}(\omega) \leq \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il} \right\} = 1 - \alpha, \forall i \in I, \xi \subseteq 2^K,$$

which intuitively means that the aggregate capacity exceeds demand with high probability. For given equipment investment decisions $\{X_{il}\}'s$, these service guarantee constraints pin down feasible production schedule decisions $Y_{ijk}(\omega)$.

Note that this formulation does not require the knowledge of which equipment is producing energy for a particular customer. From a queueing theory perspective, it is similar to a multi-server queueing model, which implies that, a state-dependent dynamic scheduling policy will be required to implement the optimization solution in practice [1]. We will provide further discussion on this issue in section 4.2.

Hierarchy of decision making. Macro-level strategic decisions include facility location and equipment investment. At the beginning, a planner would decide how many plants are necessary, and where they should be located. In addition, investment for different equipments are made. Micro-level operational decisions include production and transportation planning. After customers place orders, demands for energy are realized. The planner decides which plant should produce energy to satisfy the demands. Once production plans are scheduled, the corresponding transportation costs are determined.

Objective. The planner maximizes the aggregate revenue subtracted by the aggregate cost:

$$E_P \sum_{i \in I, j \in J, k \in K} (V_k - \phi_k(d_{ij}))Y_{ijk}(\omega) \lambda_{jk}(\omega) - \sum_{i \in I, l \in L} g_{il}X_{il} - \sum_{i \in I} f_i Z_i.$$ 5
Since the revenue is constant for each type of energy, it is equivalent to adopt a cost-minimization framework, including transportation costs, equipment investment costs, and plant setup costs, respectively. The expectation is taken over probability $P$ associated with the sample space $\Omega$.

We summarize the nomenclature in this paper as follows.

$\Omega$ The set of all possible realization of uncertainty.

$P$ Probability measure for the uncertainty.

$I$ The set of all potential locations for energy production.

$J$ The set of all customers (demand locations).

$K$ The set of all types of energy.

$L$ The set of all equipments.

$V_k$ Revenue from unit demand for energy $k$.

$d_{ij}$ Distance between customer $j$ and plant $i$.

$\phi(\cdot)$ Transportation cost function.

$g_{il}$ Cost of equipment $l$ at location $i$.

$f_i$ Setup cost for a plant at potential location $i$.

$\lambda_{jk}(\cdot)$ Demand at $j$ for energy $k$ (for given state or scenario).

$\Lambda_{jk}$ Expected demand at $j$ for energy $k$.

$Z_i$ Binary decision variables for a plant at location $i$.

$Y_{ijk}(\cdot)$ Whether energy $k$ at $j$ is to be produced from $i$ (for given state or scenario).

$X_{il}$ Number (integer) of equipment $l$ available at location $i$.

$\pi(k)$ The set of equipments to produce energy $k$.

$\pi^{-1}(l)$ The types of energy produced by equipment $l$.

4 Analysis

In general, the problem is intractable due to the uncountable set of scenarios $\Omega$. We now add more structure to the probability space. Suppose that the demands are determined indirectly by some finite underlying states. In addition, we define $Y_{ijk}(s)$ as a binary decision variable whether the demand of energy $k$ from customer $j$ should be satisfied from the production at location $i$ when the state is $s$. The set of all states is $S$ and each state takes place independently with probability $p_s$. Intuitively, a “scenario” is a random draw that can be viewed as “raw data”, while a state represents market condition. The production decisions would be pooling for each state. The motivation to use “state” in our formulation instead of “scenario” is to estimate average demand intensity under
Lemma 1. Denote \( \Lambda_{jk}(s) = E[\lambda_{jk}(\omega|s)] \), and assume that \( \Lambda_{jk}(s) \to \infty \). The resource constraint can be replaced by

\[
\Phi^{-1}(1 - \alpha) \sqrt{\sum_{k \in \xi} \sum_{j \in J} Y_{ijk}(s) \Lambda_{jk}(s)^2 + \sum_{k \in \xi} \sum_{j \in J} Y_{ijk}(s) \Lambda_{jk}(s) \leq \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il}, \forall i \in I, s \in S, \xi \subseteq 2^K,}
\]

wherein \( \Phi(\cdot) \) is the standard normal distribution function.

Under such a resource investment policy, the system performance is analogous to the quality- and efficiency-driven regime (QED) in queueing theory \[24\]. Note that we need to assign at least one plant to produce energy \( k \) for customer \( j \) for every state instead of every scenario, i.e., we require that \( \sum_{i \in I} Y_{ijk}(s) = 1, \forall j \in J, k \in K, s \in S \). We summarize our model as follows:

\[
\text{Minimize } \sum_{i \in I, j \in J, k \in K, s \in S} \phi_k(d_{ij}) Y_{ijk}(s) \Lambda_{jk}(s) p_s + \sum_{i \in I, l \in L} g_{il} X_{il} + \sum_{i \in I} f_i Z_i,
\]

subject to \((RE-\xi)\), and

\[
\sum_{i \in I} Y_{ijk}(s) = 1, \forall j \in J, k \in K, s \in S
\]

\[
X_{il} \leq M Z_i, \forall i \in I, l \in L
\]

\[
Y_{ijk}(s), Z_i \in \{0, 1\}, X_{il} \in \mathbb{Z}.
\]

4.1 Flexibility Structures due to Multi-Generation Technologies

In general, the capacity constraints RE-\( \xi \) are needed for all possible combinations of energy types. We need to eliminate the redundant constraints to reduce the size of the optimization programs.

Proposition 1. Consider a partition \( \xi_t \subset \xi \), for some \( t = 1, 2, \ldots, t < \infty \), wherein \( \{\xi_t\} \)’s are collectively exhaustive and mutually exclusive. If \( \cap_{t=1,2,\ldots} (\cup_{k \in \xi_t} \pi(k)) = \emptyset \), then RE-\( \xi \) is redundant.

\[\square\]

Example 1 (Dense chaining). Suppose we have \( |L| \) types of equipments and \( |K| = |L| \) types of energy. We consider a particular dense flexibility structure. Each equipment can be used to produce \( |L| - 1 \) types of energy, i.e., \( \pi^{-1}(l) = \{1, 2, \ldots, l - 1, l + 1, \ldots, |L|\} \), \( \forall l = 1, 2, \ldots, |L| - 1 \). For \( \forall i \in I, s \in S \), the number of binding resource constraints can’t be greater than \( |L| + 1 \).
Example 2 (Star-flexibility). Suppose we have $|L|$ types of equipment and $|K| = |L| - 1$ types of energy. Equipment $l = 1, 2, ..., |L| - 1$ can only be used to produce one type of energy each, i.e., $\pi^{-1}(\pi(k)) = k$, $\forall k = 1, 2, ..., |L| - 1$. Equipment $l = |L|$ could be used to produce all types of energy, i.e., $\pi^{-1}(|L|) = K$. For $\forall i \in I, s \in S$, the number of binding resource constraints can’t be greater than $2^{L-1} - 1$.

4.2 Capacity Allocation: Responsive vs. Anticipative

We propose an alternative model in which the planner explicitly allocates the production capacity for every equipments. Define $\Delta_{ilk}(s)$ as the proportion of time each equipment $l$ is scheduled for energy $k$ at location $i$. The model (P-1) can be modified as:

\[
\begin{align*}
\text{Minimize} & \quad \sum_{i \in I, j \in J, k \in K, s \in S} \phi_k(d_{ij}) Y_{ijk}(s) \Lambda_{jk}(s) p_s + \sum_{i \in I, l \in L} g_{il} X_{il} + \sum_{i \in I} f_i Z_i, \\
\sum_{i \in I} Y_{ijk}(s) &= 1, \forall j \in J, k \in K, s \in S \quad (P-2) \\
X_{il} &\leq MZ_i, \forall i \in I, l \in L \\
\Phi^{-1}(1-\alpha) \sqrt{\sum_{j \in J} Y_{ijk}(s) \Lambda_{jk}(s) + \sum_{j \in J} Y_{ijk}(s) \Lambda_{jk}(s)} &\leq \sum_{l \in \pi(k)} X_{il} \Delta_{ilk}(s), \forall i \in I, k \in K, s \in S \\
\sum_{k} \Delta_{ilk}(s) &\leq 1, \forall i \in I, l \in L, s \in S \\
Y_{ijk}(s), Z_i &\in \{0, 1\}, X_{il} \in \mathbb{Z}, \Delta_{ilk}(s) \in [0, 1].
\end{align*}
\]

In what follows, we compare this model (P-2) with (P-1).

**Proposition 2.** The aggregate cost under anticipative allocation (P-2) provides an upper bound for that under responsive allocation (P-1).
realization of demands). Thus, the planner does not make full use of the risk pooling instrument, and (P-2) yields a higher aggregate cost.

5 Computational Techniques and Numerical Examples

(P-1) is a mixed integer program, which is difficult to solve. However, we can show that it can be solved easily under reformulation.

**Proposition 3.** *(P-1)* is equivalent to the following tractable MISOCP.

\[
\begin{align*}
\text{Minimize} & \quad \sum_{i \in I, j \in J, k \in K, s \in S} \phi_k (d_{ij}) Y_{ijk} (s) \Lambda_j k (s) p_s + \sum_{i \in I, l \in L} g_{il} X_{il} + \sum_{i \in I} f_i Z_i, \\
\sum_{i \in I} Y_{ijk} (s) & = 1, \forall j \in J, k \in K, s \in S \\
X_{il} & \leq M Z_i, \forall i \in I, l \in L \\
\sum_{k \in \xi} \sum_{j \in J} Y_{ijk}^2 (s) \Lambda_j k (s) & \leq t_i^2 (s), \forall i \in I, s \in S
\end{align*}
\]

\[
\begin{align*}
\left\| t_i (s) + \frac{\Phi^{-1(1-\alpha)}}{2} \right\|_2 \\
\leq \frac{1}{2} \left( 1 + \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il} + \left[ \Phi^{-1(1-\alpha)} \right]^2 \right), \forall i \in I, s \in S, \xi \subseteq 2^K
\end{align*}
\]

\[Y_{ijk} (s), Z_i \in \{0, 1\}, t_i (s) \in \mathbb{R}^+, X_{il} \in \mathbb{Z}.\]

Using this computational technique, we conduct numerical experiments to illustrate the economic value of the multi-generation technologies in lieu of the spatial and inter-temporal demand uncertainty for energy: the average demand is drawn from a two-type distribution wherein we interpret the demand difference between two states as “inter-temporal” while the difference among locations as “inter-spatial”. We observe the following impacts of the multi-generation technology from the numerical experiments:

- It reduces uncertainty by pooling capacity both within and across different facilities.
• It can compensate the loss in de-centralization due to the increase in the transportation cost.
• It can amplify the risk-pooling effects due to centralization triggered by increasing setup costs.

The readers are referred to the Appendix for details of these major observations summarized in the main paper.

6 Conclusion

In this paper, we investigate the system design problems in energy systems when multi-generation technologies are available. Our modeling framework can be adapted to incorporate extensions. For example, in many cases, we would like to improve over the existing infrastructure instead of designing the entire system. This case can be handled by replacing the corresponding decision variables with known parameters. In addition, some operational rules may have strategic impacts, such as some upper and lower limits for production capacity and equipment start-up costs. These can also be taken care of with additional operational constraints in the optimization problem.

In terms of future research, it will be fruitful to apply our methodology by focusing on a particular sector, so that more structural results can be generated to develop a problem-specific optimization algorithm. It will be also desirable to implement our methodology in case studies for a specific energy market. Furthermore, we assume in this paper that the demand is uncertainty (state-dependent) while the energy production is stationary. For the integration of certain sustainable energy, such as wind power, the energy supply can also be uncertain (state-dependent). Another interesting direction is to incorporate energy storage as an alternative instrument for energy supply chain flexibility design.

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A Appendix. Proofs.

In this appendix, we first provide the detailed proofs of the main results.

Proof of Lemma 1. With the scenario aggregation that \( \Lambda_{jk}(s) = E[\lambda_{jk}(\omega|s)] \), we can re-write the resource constraint as

\[
p \left\{ \sum_{k \in \xi} \sum_{j \in J} Y_{ijk}(s) \lambda_{jk}(\omega|s) \leq \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il} \right\} = 1 - \alpha, \forall s \in S. \tag{1}
\]

Next we show that this constraint can be replaced by

\[
\Phi^{-1}(1 - \alpha) \sqrt{\sum_{k \in \xi} \sum_{j \in J} Y_{ijk}(s) \Lambda_{jk}(s)} + \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il}, \forall i, s \in S, \xi \subseteq 2^K, \tag{RE-\xi}
\]

as \( \Lambda_{jk}(s) \to \infty \), wherein \( \Phi \) is the distribution function for standard normal random variable. To see this, we replace the service guarantee by Gaussian approximation:

\[
p \left\{ \sum_{k \in \xi} \sum_{j \in J} Y_{ijk}(s) \lambda_{jk}(\omega|s) \leq \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il} \right\} = p \left\{ \frac{\sum_{k \in \xi} \sum_{j \in J} Y_{ijk}(s)[\lambda_{jk}(\omega|s) - \Lambda_{jk}(s)]}{\sqrt{\sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il}^2 - \sum_{k \in \xi} \sum_{l \in \pi(k)} \sum_{j \in J} Y_{ijk}(s) \Lambda_{jk}(s)}} \right\}, \tag{2}
\]

for \( \forall i, s \in S, \xi \subseteq 2^K \). Let a feasible investment level be to choose the minimum \( \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il} \) that is larger than \( \sum_{k \in \xi} \sum_{j} Y_{ijk}(s) \Lambda_{jk}(s) + \Phi^{-1}(1 - \alpha) \sqrt{\sum_{k \in \xi} \sum_{j} Y_{ijk}(s) \Lambda_{jk}(s)} \), the right-hand side is \( 1 - \alpha \) due to the Central Limit Theorem. \( \square \)

Proof of Proposition 1. For \( \forall \xi, \sum_{k \in \xi} \sum_{j} Y_{ijk}(s) \Lambda_{jk}(s) + \Phi^{-1}(1 - \alpha) \sqrt{\sum_{k \in \xi} \sum_{j} Y_{ijk}(s) \Lambda_{jk}(s)} \)

\[
= \left( \sum_{k \in \xi_1} + \sum_{k \in \xi_2} + \ldots \right) \sum_{j} Y_{ijk}(s) \Lambda_{jk}(s) + \Phi^{-1}(1 - \alpha) \sqrt{\left( \sum_{k \in \xi_1} + \sum_{k \in \xi_2} + \ldots \right) \sum_{j} Y_{ijk}(s) \Lambda_{jk}(s)}
\]

\[
\leq \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il}, \forall i, s \in S. \tag{3}
\]

Therefore, RE-\( \xi \) is redundant. Note that the converse is not true, i.e., if \( \cap_{k \in \xi} \pi(k) \neq \emptyset \), then it is still possible that RE-\( \xi \) is redundant. \( \square \)
Proof of Proposition 2. To prove that this is a upper bound, it suffices to check the feasibility of (P-2)’s solution to (P-1)’s constraints:

\[
\sum_{k \in \xi} \sum_{j} Y_{ijk} (s) \Lambda_{jk} (s) + \Phi^{-1} (1-\alpha) \sqrt{\sum_{k \in \xi} \sum_{j} Y_{ijk} (s) \Lambda_{jk} (s)} 
\]

\[\leq \sum_{k \in \xi} \sum_{j} Y_{ijk} (s) \Lambda_{jk} (s) + \Phi^{-1} (1-\alpha) \sum_{k \in \xi} \sum_{j} Y_{ijk} (s) \Lambda_{jk} (s) \]

\[\leq \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il} \Delta_{ilk} (s) \]

\[\leq \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il}, \forall i \in I, \xi \subseteq 2^K, s \in S. \quad (4) \]

The first inequality is due to the property of square-root function (or Jensen’s inequality). The second inequality holds via the feasibility of (P-2). The third inequality is true since \( \Delta_{ilk} (s) \leq 1 \). Since any solution of (P-2) is feasible for (P-1), (P-2) is more restrictive and thus returns a higher cost. \( \Box \)

Proof of Proposition 3. For convenience, we write (P-1) as follows:

\[
\text{Minimize } \sum_{i \in I, j \in J, k \in K, s \in S} \phi_k (d_{ij}) Y_{ijk} (s) \Lambda_{jk} (s) p_s + \sum_{i \in I, l \in L} g_{il} X_{il} + \sum_{i \in I} f_i Z_i, \]

subject to

\[
\Phi^{-1} (1-\alpha) \sqrt{\sum_{k \in \xi} \sum_{j \in J} Y_{ijk} (s) \Lambda_{jk} (s) + \sum_{k \in \xi} \sum_{j \in J} Y_{ijk} (s) \Lambda_{jk} (s)} \leq \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il}, \forall i \in I, s \in S, \xi \subseteq 2^K.
\]

\[
\sum_{i \in I} Y_{ijk} (s) = 1, \forall j \in J, k \in K, s \in S \quad \text{(P-1)}
\]

\[X_{il} \leq M Z_i, \forall i \in I, l \in L \]

\[Y_{ijk} (s), Z_i \in \{0, 1\}, X_{il} \in Z. \]

Since \( Y_{ijk} (s) \)'s are binary, they will be equivalent to \( Y_{ijk}^2 (s) \), by which we can re-write the resource constraints as

\[
\Phi^{-1} (1-\alpha) \sqrt{\sum_{k \in \xi} \sum_{j \in J} Y_{ijk}^2 (s) \Lambda_{jk} (s) + \sum_{k \in \xi} \sum_{j \in J} Y_{ijk}^2 (s) \Lambda_{jk} (s)} \leq \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il}, \forall i \in I, s \in S, \xi \subseteq 2^K. \quad (5)
\]
We can introduce variable $t_i(s)$ such that

$$\sum_{k \in \xi} \sum_{j \in J} Y_{ijk}(s) \Lambda_{jk}(s) = t_i^2(s),$$

we can re-write the resource constraints as

$$\Phi^{-1}(1-\alpha) t_i(s) + t_i^2(s) \leq \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il}, \forall i \in I, s \in S, \xi \subseteq 2^K.$$  \hspace{1cm} (6)

Adding $[\Phi^{-1}(1-\alpha)/2]^2$ on both sides:

$$\left[t_i(s) + \frac{\Phi^{-1}(1-\alpha)}{2}\right]^2 \leq \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il} + \left[\frac{\Phi^{-1}(1-\alpha)}{2}\right]^2, \forall i \in I, s \in S, \xi \subseteq 2^K.$$

Now the problem becomes:

Minimize

$$\sum_{i \in I, j \in J, k \in K, s \in S} \phi_k(d_{ij}) Y_{ijk}(s) \Lambda_{jk}(s) p_s + \sum_{i \in I, l \in L} g_{il} X_{il} + \sum_{i \in I} f_i Z_i,$$

subject to

$$\sum_{i \in I} Y_{ijk}(s) = 1, \forall j \in J, k \in K, s \in S \quad \text{(P-1')}$$

$$X_{il} \leq M Z_i, \forall i \in I, l \in L$$

$$\sum_{k \in \xi} \sum_{j \in J} Y_{ijk}(s) \Lambda_{jk}(s) = t_i^2(s), \forall i \in I, s \in S$$

$$\left[t_i(s) + \frac{\Phi^{-1}(1-\alpha)}{2}\right]^2 \leq \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il} + \left[\frac{\Phi^{-1}(1-\alpha)}{2}\right]^2, \forall i \in I, s \in S, \xi \subseteq 2^K.$$  \hspace{1cm} (7)

$$Y_{ijk}(s), Z_i \in \{0, 1\}, t_i(s) \in \mathbb{R}^+, X_{il} \in \mathbb{Z}.$$

We shall now prove that (P-1') and (P-3) are equivalent by contradiction. We write down (P-3) for your convenience.

Minimize

$$\sum_{i \in I, j \in J, k \in K, s \in S} \phi_k(d_{ij}) Y_{ijk}(s) \Lambda_{jk}(s) p_s + \sum_{i \in I, l \in L} g_{il} X_{il} + \sum_{i \in I} f_i Z_i,$$

subject to

$$\sum_{i \in I} Y_{ijk}(s) = 1, \forall j \in J, k \in K, s \in S \quad \text{(P-3)}$$
\[ X_{il} \leq M Z_i, \forall i \in I, l \in L \]
\[ \sum_{j \in J} Y_{ijk}^2(s) \Lambda_{jk}(s) \leq t_i^2(s), \forall i \in I, s \in S \]
\[ \left[ t_i(s) + \frac{\Phi^{-1}(1 - \alpha)}{2} \right]^2 \leq \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il} + \left[ \frac{\Phi^{-1}(1 - \alpha)}{2} \right]^2, \forall i \in I, s \in S, \xi \subseteq 2^K, \]
\[ Y_{ijk}(s), Z_i \in \{0, 1\}, t_i(s) \in \mathbb{R}^+, X_{il} \in \mathbb{Z}. \]

Firstly, any solution of \((P-1')\) satisfies \((P-3)\), since \((P-3)\) is a relaxation. Suppose that a solution of \((P-3)\) does not satisfy \((P-1')\), then \[ \sum_{k \in \xi} \sum_{j \in J} Y_{ijk}^2(s) \Lambda_{jk}(s) = t_i^2(s), \forall i \in I, s \in S, \] is not strictly binding: \[ \exists i \in I, s \in S \] such that \[ \sum_{k \in \xi} \sum_{j \in J} Y_{ijk}^2(s) \Lambda_{jk}(s) < t_i^2(s). \] Then, suppose that \[ t_i^2(s) = \sum_{k \in \xi} \sum_{j \in J} Y_{ijk}^2(s) \Lambda_{jk}(s) + \varepsilon \] for some \(\varepsilon > 0\). However, this can not be optimal for \((P-3)\), since we can further decrease \(t_i(s)\).

As
\[ \left[ t_i(s) + \frac{\Phi^{-1}(1 - \alpha)}{2} \right]^2 \leq \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il} + \left[ \frac{\Phi^{-1}(1 - \alpha)}{2} \right]^2, \tag{9} \]
we can reduce some of the \(X_{il}\) and thus the objective function. \textit{Reductio ad absurdum}, solution of \((P-3)\) satisfy \((P-1')\), and thus they are equivalent.

Finally, we need to show that
\[ \Phi^{-1}(1 - \alpha) t_i(s) + t_i^2(s) \leq \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il}, \forall i \in I, s \in S, \xi \subseteq 2^K, \tag{10} \]
is a second-order cone. We have already demonstrated that it is equivalent to
\[ \left[ t_i(s) + \frac{\Phi^{-1}(1 - \alpha)}{2} \right]^2 \leq \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il} + \left[ \frac{\Phi^{-1}(1 - \alpha)}{2} \right]^2, \forall i \in I, s \in S, \xi \subseteq 2^K, \tag{11} \]
which is quadratic, and can be re-written as:
\[ \left\| \frac{1}{2} \left( 1 - \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il} - \frac{\Phi^{-1}(1 - \alpha)}{2} \right)^2 \right\| \leq \frac{1}{2} \left( 1 + \sum_{k \in \xi} \sum_{l \in \pi(k)} X_{il} + \left[ \frac{\Phi^{-1}(1 - \alpha)}{2} \right]^2 \right), \tag{12} \]
\[ \square \]
B Numerical Results.

We assume that there are three different types of energy, i.e., \( K = \{A, S, H\} \). There are four different equipments, i.e., \( L = \{A, S, H, F\} \), where \( F \) indicates an equipment with full flexibility. The plants are located potentially in \( J \), a set of 18 cities (LA, SD, SJ, SF, LB, OAK, SAC, FRE, SA, ANA, RIV, STKN, HB, GNDL, BKD, FMT, MOD, SB). Demands are also from the same potential locations, i.e., \( I = J \). There are two possible states of the world, i.e., \( S = \{H, L\} \).

Transportation costs are generated proportional to the distance \( d_{ij} \) from \( [6] \), i.e., \( \phi_k (d_{ij}) = \phi d_{ij} \) and the scaling coefficient \( \phi = 2 \). We assume that the investment coefficient \( \Phi^{-1} (1 - \alpha) = 0.2 \).

We use heterogeneous service rates as a generalization of the model, wherein (P1) can be easily adapted by the following modification to RE-\( \xi \) constraints:

\[
\Phi^{-1} (1 - \alpha) \sqrt{\sum_{k \in \xi} \sum_{j \in J} Y_{ijk} (s) \Lambda_{jk} (s) + \sum_{k \in \xi} \sum_{j \in J} Y_{ijk} (s) \Lambda_{jk} (s) \leq \sum_{k \in \xi} \sum_{l \in \pi(k)} \mu_{il} X_{il}, \forall i \in I, s \in S, \xi \subseteq 2^K,
\]

wherein the service rates can depend on location \( i \), the equipment \( l \), and potentially the energy type \( k \), without changing the structure of this optimization problem. We assume that \( \mu_A = 500, \mu_S = 460, \mu_H = 420 \). The multi-generation technologies can be used to produce all three types of energy, and the corresponding service rate is \( \mu_F = 400 \). Two states are realized with equal probability, i.e., \( \Pr(H) = \Pr(L) = 0.5 \). The setup costs for potential locations are proportional to the real estate prices in that city.

We consider a revenue maximizing framework, wherein valuations for energy consumption are heterogeneous, i.e., \( V_A = 800, V_S = 725, V_H = 600 \). Equipment costs are homogeneous across all locations, i.e., \( g_{iA} = 90000, g_{iS} = 70000, g_{iH} = 50000, g_{iF} = 100000, \forall i \in I \). Finally, we generate aggregate demand in proportion to the population in each city, and split across different types of energy. We choose to split in a way to generate both spatial and inter-temporal (state-dependent) heterogeneity (see appendix for the demand distribution).

Table 1 and Table 2 summarize the optimal equipment quantities for the chosen locations under both high and low spatial demand heterogeneity. We compare the results with and without multi-generation technologies. Under high spatial heterogeneity, it would be optimal to invest in the multi-generation technologies more than that under low spatial heterogeneity. In addition, with the investment in multi-generation technologies, the number of equipments needed is smaller than that in the non-flexible case. Such improvement is more significant when spatial heterogeneity is high. We summarize the insight as follows.
Table 1: Optimal equipments quantities, with multi-generation technologies.

| Location Chosen | High spatial heterogeneity | Low spatial heterogeneity |
|-----------------|----------------------------|---------------------------|
| # Equipment H   | 5  1  2  2                | 0  5  3  2                |
| # Equipment A   | 4  0  2  1                | 0  4  2  1                |
| # Equipment S   | 0  0  0  0                | 0  2  1  1                |
| # Multi-generation units | 3  1  2  1   | 1  1  1  0               |

Table 2: Optimal equipments quantities, without multi-generation technologies.

| Location Chosen | High spatial heterogeneity | Low spatial heterogeneity |
|-----------------|----------------------------|---------------------------|
| # Equipment H   | 1  6  4  3                | 5  1  3  2                |
| # Equipment A   | 1  6  4  2                | 5  1  2  1                |
| # Equipment S   | 1  2  2  1                | 2  1  1  1                |
Observation 1. The multi-generation technologies reduce uncertainty by pooling capacity both within and across different facilities.

Table 3: Sensitivity analysis for transportation cost coefficient, with multi-generation technologies

| $\phi$ | 1   | 1.5 | 2   | 2.5 | 3   |
|--------|-----|-----|-----|-----|-----|
| Total # open sites | 3   | 5   | 7   | 7   | 7   |
| Total # multi-generation units | 2   | 2   | 5   | 5   | 5   |
| Total # equipments H | 9   | 9   | 9   | 9   | 9   |
| Total # equipments A | 15  | 15  | 14  | 14  | 14  |
| Total # equipment S | 4   | 4   | 3   | 3   | 3   |
| Total equipment cost ($) | 2280000 | 2280000 | 2420000 | 2420000 | 2420000 |
| Total transportation cost ($) | 437045 | 452159 | 335854 | 378496 | 454195 |
| Total net revenue ($) | 4648697 | 4483583 | 4359888 | 4277246 | 4201547 |

Table 4: Sensitivity analysis for transportation cost coefficient, without multi-generation technologies

| $\phi$ | 1   | 1.5 | 2   | 2.5 | 3   |
|--------|-----|-----|-----|-----|-----|
| Total # open sites | 3   | 4   | 6   | 6   | 6   |
| Total # equipments H | 9   | 10  | 11  | 11  | 11  |
| Total # equipments A | 17  | 17  | 17  | 17  | 17  |
| Total # equipment S | 5   | 5   | 6   | 6   | 6   |
| Aggregate equipment cost ($) | 2330000 | 2380000 | 2500000 | 2500000 | 2500000 |
| Aggregate transportation cost ($) | 469570 | 582149 | 498881 | 623601 | 748321 |
| Aggregate net revenue ($) | 4516172 | 4303593 | 4166861 | 4042141 | 3917421 |

Table 3 and Table 4 compare the optimal facility location and equipment investment decisions with different transportation costs, with and without the multi-generation technologies. With the increase of the transportation cost coefficient $\phi$, it is optimal to set up more plants so that the demands can be satisfied by the nearest location. With the increasing number of the open sites,
more multi-generation units are needed because the production capacity is more de-centralized, and the system suffers more from demand uncertainty both within and across different locations. Surprisingly, the aggregate transportation cost decreases when $\phi$ increases from 1.5 to 2.

**Observation 2.** *Investment in the multi-generation technologies can compensate the loss in de-centralization due to the increase of the transportation cost coefficient. Consequently, the overall transportation costs could decrease.*

The increase of the setup cost has the opposite effect to the transportation cost coefficient, summarized in the following observation and the remaining two tables.

Table 5: Sensitivity analysis for set up cost, with multi-generation technologies.

| Set up cost      | Very low | Low | Medium | High | Very high |
|------------------|----------|-----|--------|------|-----------|
| Total # open sites | 7        | 4   | 3      | 3    | 2         |
| Total # multi-generation units | 5        | 5   | 2      | 2    | 3         |
| Total # equipments H | 9        | 11  | 9      | 9    | 10        |
| Total # equipments A | 14       | 13  | 15     | 15   | 14        |
| Total # equipment S | 3        | 2   | 4      | 4    | 3         |
| Aggregate equipment cost ($) | 2420000 | 2360000 | 2280000 | 2280000 | 2270000 |
| Aggregate transportation cost ($) | 335854 | 605403 | 776704 | 925164 | 1377640 |
| Aggregate net revenue ($) | 4359888 | 3931859 | 3677198 | 3482578 | 3095602 |

**Observation 3.** *The multi-generation technologies amplify the risk-pooling effects due to centralization triggered by the increase of the setup costs.*

C Demand Data.

In this appendix, we provide the demand data we use in the numerical experiment. The demand is drawn from a two-type distribution wherein we interpret the demand difference between states “H” and “L” as “inter-temporal” while the difference among locations as “inter-spatial”. We also include the location-dependent setup costs used in our calculations.
Table 6: Sensitivity analysis for set up cost, without multi-generation technologies.

| Set up cost        | Very low | Low  | Medium | High | Very high |
|--------------------|----------|------|--------|------|-----------|
| Total # open sites | 6        | 4    | 4      | 3    | 2         |
| Total # equipments H | 11      | 10   | 10     | 10   | 10        |
| Total # equipments A | 17      | 17   | 17     | 17   | 17        |
| Total # equipment S | 6       | 6    | 6      | 6    | 6         |
| Aggregate equipment cost ($) | 2500000 | 2450000 | 2450000 | 2450000 | 2450000 |
| Aggregate transportation cost ($) | 498881  | 683499  | 683499  | 945895 | 1366350 |
| Aggregate net revenue ($) | 4166861 | 3771763 | 3501403 | 3291847 | 2926894 |

Table 7: Setup costs for each potential locations.

| City     | LA    | SD    | SJ    | SF    | LB    | OAK   |
|----------|-------|-------|-------|-------|-------|-------|
| Setup Cost ($) | 214880 | 178320 | 238000 | 328400 | 151600 | 159280 |
| City     | SAC   | FRE   | SA    | ANA   | RIV   | STKN  |
| Setup Cost ($) | 106800 | 86000  | 152720 | 165040 | 113200 | 78000  |
| City     | HB    | GNDL  | BKD   | FMT   | MOD   | SB    |
| Setup Cost ($) | 233200 | 202800 | 93040  | 230000 | 85040  | 79280  |
Table 8: Demands under high spatio-temporal heterogeneity.

| States | Demands (A) | Demands (S) | Demands (H) | Demands (A) | Demands (S) | Demands (H) |
|--------|-------------|-------------|-------------|-------------|-------------|-------------|
| LA     | 189.19      | 26.2764     | 47.2975     | 42.0422     | 157.658     | 63.0634     |
| SD     | 134.929     | 18.7401     | 33.7322     | 18.7401     | 93.7005     | 74.9604     |
| SF     | 43.3426     | 126.416     | 10.8356     | 108.356     | 18.0594     | 54.1782     |
| SJ     | 263.103     | 36.5421     | 65.7758     | 36.5421     | 182.71      | 146.168     |
| LB     | 148.185     | 20.5812     | 108.356     | 20.5812     | 102.906     | 82.3248     |
| OAK    | 24.7964     | 144.646     | 37.1947     | 144.646     | 20.6637     | 41.3274     |
| SAC    | 438.266     | 2556.55     | 657.399     | 2191.33     | 730.443     | 730.443     |
| FRE    | 343.386     | 47.6925     | 85.8465     | 76.308      | 286.155     | 114.462     |
| ANA    | 41.7298     | 121.712     | 10.4324     | 104.324     | 17.3874     | 52.1622     |
| RIV    | 104.055     | 303.494     | 26.0138     | 260.138     | 43.3563     | 130.069     |
| SA     | 163         | 22.6389     | 40.75       | 36.2222     | 135.833     | 54.3334     |
| HB     | 25.78       | 150.383     | 38.6699     | 150.383     | 21.4833     | 42.9666     |
| STKN   | 104         | 303.332     | 25.9999     | 259.999     | 43.3332     | 130         |
| GNDL   | 117.681     | 16.3446     | 29.4203     | 16.3446     | 81.723      | 65.3784     |
| BKD    | 146.195     | 852.802     | 219.292     | 852.802     | 121.829     | 243.658     |
| FMT    | 220.02      | 641.726     | 55.0051     | 550.051     | 91.6752     | 275.026     |
| MOD    | 180.157     | 525.458     | 45.0392     | 450.392     | 75.0654     | 225.196     |
| SB     | 49.5317     | 144.467     | 12.3829     | 123.829     | 20.6382     | 61.9146     |

22
Table 9: Demands under low spatio-temporal heterogeneity.

| States | Both H and L | Demands (A) | Demands (S) | Demands (H) |
|--------|--------------|-------------|-------------|-------------|
| LA     |              | 115.616     | 91.9674     | 55.1804     |
| SD     |              | 76.8344     | 56.2203     | 54.3463     |
| SF     |              | 75.8495     | 72.2376     | 32.5069     |
| SJ     |              | 149.823     | 109.626     | 105.972     |
| LB     |              | 84.3829     | 61.7436     | 59.6855     |
| OAK    |              | 84.7212     | 82.6548     | 39.261      |
| SAC    |              | 1314.8      | 1643.5      | 693.921     |
| FRE    |              | 209.847     | 166.924     | 100.154     |
| ANA    |              | 73.0271     | 69.5496     | 31.2973     |
| RIV    |              | 182.096     | 173.425     | 78.0413     |
| SA     |              | 99.6112     | 79.2361     | 47.5417     |
| HB     |              | 88.0815     | 85.9332     | 40.8138     |
| STKN   |              | 181.999     | 173.333     | 77.9998     |
| GNDL   |              | 67.0129     | 49.0338     | 47.3993     |
| BKD    |              | 499.498     | 487.315     | 231.475     |
| FMT    |              | 385.036     | 366.701     | 165.015     |
| MOD    |              | 315.275     | 300.262     | 135.118     |
| SB     |              | 86.6804     | 82.5528     | 37.1488     |