Multi-objective optimization model of waste tire recycling network

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Abstract—To achieve sustainable development, logistics enterprises need not only to reduce costs, but also to save energy for environmental protection and improve customer service level. The improvement of reverse logistics management level of waste tires is of great significance to improve the efficiency of the automobile industry. In this paper, multi-objective programming is adopted to establish the waste tire recycling network model. The decision variable is whether the network nodes are set or not, the traffic flow between nodes. Constraints include meeting customer demand, balance of flow in and out of logistics nodes, etc. The model is solved by \( \varepsilon \)-constraint. Taking the actual data of the enterprise as an example, the operation results show that the operation cost, carbon emission and customer transportation distance can get an consistence within a certain range. Waste tire logistics enterprises can realize the simultaneous improvement of profit, environmental protection and customer service level.

1 Introduction

With the rapid development of automobile industry, China has become the world's largest automobile production and sales country. With the growth of automobile consumption, the number of used tires is also increasing, so the logistics management of used tires has become a social problem of increasing concern. Waste tire recycling industry is a branch of reverse logistics industry, and its competition is fierce. Waste tire recycling enterprises need to pay attention to not only the short-term benefits including operating costs and carbon emissions, but also the long-term benefits including customer service level.

S. Vercraene et al.\textsuperscript{[1]} established a reverse logistics network model, in which re-manufacturing and return acceptance of recycled products were studied. B. Ayvaz et al.\textsuperscript{[2]} adopted two-stage stochastic programming to establish a network model e-waste recovery, aiming to maximize the profits of the waste recovery companies of electrical and electronic equipment. M. Zohaland and H. Soleimani\textsuperscript{[3]} established a closed-loop supply chain model with multi-objective programming. In this model, both forward logistics and reverse logistics were considered together. M. Rahimi and V. Ghezavati\textsuperscript{[4]} established a reverse logistics network model by two-stage multi-objective stochastic programming. The goal was to maximize profits, maximize social impact and minimize environmental impact. N. Zarbakhshnia et al.\textsuperscript{[5]} established a network model based on the combination of forward and reverse logistics. The objective function of the model was to minimize the operation cost and equipment cost. K. Subulan et al.\textsuperscript{[6]} established a tire closed-loop supply chain model by multi-objective programming, in which environmental problems were considered by life assessment. Y. Zhou and S. Wang\textsuperscript{[7]} established an optimization model of reverse logistics distribution network in which repairing and reproduction of waste products are studied. S. M. Hatefi and F. Jolai\textsuperscript{[8]} established the forward and reverse logistics network model, and the objective function was to minimize the operating cost. The influence of uncertain parameters was considered in the model.

At present, the research on waste tire recycling network and reverse logistics network is mainly from the perspective of logistics enterprises, with the short-term benefit maximization as the objective function. The short-term utility includes maximization of revenue, minimization of operating costs and minimization of carbon emissions (Government subsidies for carbon emissions of enterprises) etc. There are also literature that studies both operating costs and carbon emissions. There are few literature on the combination of long-term strategic objectives and short-term profits. However, the waste tire industry is highly competitive. If the level of customer service is low, customers will choose to cooperate with other reverse logistics enterprises. The excessive loss of customers not only reduces the profits of recycling enterprises, but also reduces their sustainable competitiveness. Therefore, it is necessary to combine the long-term interests and short-term interests of enterprises for research. The purpose of this paper is to make up for the shortcomings of the previous research. On the premise of combining the short-term profits (operating costs and carbon emissions) and long-term...
benefits (service convenience through transportation distance) of the old tire recycling enterprises, the recycling network model is designed and the material dispatching plan is formulated.

2 Mathematical Modeling

2.1 Structure of reverse logistics networks

Fig. 1 below shows the waste tire recycling network. \( I \) represents the customer, that is, the waste tire supplier. \( R \) represents the regional gathering center of the recycling enterprise. \( C \) represents the centralized gathering center of the recycling enterprise. \( P \) represents the re-processing center of the recycling enterprise. \( S \) represents the second-hand market. \( K \) represents recycling center.

Fig. 1 Structure of waste tires recycling network

2.2 Parameters of the model

\( TB_{rc} \): Cost of shipping unit waste tire from \( r \) to \( c \). \( (r \in R, c \in C) \)

\( TB_{cp} \): Cost of shipping unit waste tire from \( c \) to \( p \). \( (c \in C, p \in P) \)

\( TB_{pk} \): Cost of shipping unit waste tire from \( j \) to \( h \). \( (p \in P, k \in K) \)

\( TB_{ps} \): Cost of shipping unit waste tire from \( p \) to \( s \). \( (p \in P, s \in S) \)

\( EB_{rc} \): Amount of carbon discharged for shipping unit waste tire from \( r \) to \( c \). \( (r \in R, c \in C) \)

\( EB_{cp} \): Amount of carbon discharged for shipping unit waste tire from \( c \) to \( p \). \( (c \in C, p \in P) \)

\( EB_{pk} \): Amount of carbon discharged for shipping unit waste tire from \( j \) to \( h \). \( (p \in P, k \in K) \)

\( EB_{ps} \): Amount of carbon discharged for shipping unit waste tire from \( p \) to \( s \). \( (p \in P, s \in S) \)

\( iQ \): Number of waste tires provided by the supplier \( i \).

\( DIS_i \): Distance from the supplier \( i \) to the regional gathering center \( r \).

\( LMAX_a \): Maximum capacity of logistics node \( a \). \( (a=r,c,p) \)

\( b_p \): Proportion of waste tires that can be sold to the second-hand market after being renovated in the re-processing center. \( (p \in P) \)

\( M \): Positive number large enough.

2.3 Decision Variables

\( Z_a \): If node \( a \) is in business, then \( Z_a = 1 \), otherwise \( Z_a = 0 \). \( (a=r,c,p) \)

\( Z_i \): If \( i \) is dispatched to \( q \), then \( Z_i = 1 \), otherwise \( Z_i = 0 \). \( (i \in I, r \in R) \)

\( AX_{rc} \): Amount of waste tires shipped from \( r \) to \( c \). \( (r \in R, c \in C) \)

\( AX_{cp} \): Amount of waste tires shipped from \( c \) to \( p \). \( (c \in C, p \in P) \)

\( AX_{pk} \): Amount of waste tires shipped from \( p \) to \( k \). \( (p \in P, k \in K) \)

\( AX_{ps} \): Amount of waste tires shipped from \( p \) to \( s \). \( (p \in P, s \in S) \)

2.4 Objective Functions

Objective function 1

\[ \min \sum \sum_{r,c} TC_{rc} + \sum \sum_{c,p} EC_{cp} \]  

Objective function 1 minimizes whole shipping cost and amount of carbon emission of the waste tires recycling enterprise.

\[ TC = \sum \sum_{r,c} TB_{rc} \times AX_{rc} + \sum \sum_{c,p} TB_{cp} \times AX_{cp} \]

\[ + \sum \sum_{p,k} TB_{pk} \times AX_{pk} + \sum \sum_{p,s} TB_{ps} \times AX_{ps} \]  

\( TC \) represents the whole shipping cost between nodes of the waste tire recycling network.

\[ EC = \sum \sum_{r,c} EB_{rc} \times AX_{rc} + \sum \sum_{c,p} EB_{cp} \times AX_{cp} \]

\[ + \sum \sum_{p,k} EB_{pk} \times AX_{pk} + \sum \sum_{p,s} EB_{ps} \times AX_{ps} \]  

\( EC \) represents the whole amount of carbon emission between nodes of the waste tire recycling network.

Objective function 2

\[ \min \sum \sum_{i,r} DIS_i \times Z_i \]  

The objective function 2 indicates the minimization of shipping distance among all suppliers.
2.5 Constraints

\begin{equation}
\sum_{r} Z_{r} \geq 1 \quad (5)
\end{equation}

Constraint (5) means that at least one regional gathering center shall be in business to ensure the operation of waste tire recycling system.

\begin{equation}
\sum_{c} Z_{c} \geq 1 \quad (6)
\end{equation}

Constraint (6) means that at least one centralized gathering center shall be in business to ensure the operation of waste tire recycling system.

\begin{equation}
\sum_{p} Z_{p} \geq 1 \quad (7)
\end{equation}

Constraint (7) means that at least one re-processing center shall be in business to ensure the operation of waste tire recycling system.

\begin{equation}
\sum_{i} Z_{ir} = 1 \quad \forall i \in I \quad (8)
\end{equation}

Constraint (8) shows that each waste tire supplier supplies to one and only one regional collection center.

\begin{equation}
\sum_{i} Z_{ir} \leq MZ_{r} \quad \forall r \in R \quad (9)
\end{equation}

Constraint (9) shows that suppliers can only supply to the regional gathering center if it is in business.

\begin{equation}
\sum_{i} Q_{ir} Z_{ir} = \sum_{c} AX_{rc} \quad \forall r \in R \quad (10)
\end{equation}

Constraint (10) shows that the waste tire flowing into a regional collection center must flow out.

\begin{equation}
\sum_{c} AX_{rc} = \sum_{p} AX_{cp} \quad \forall c \in C \quad (11)
\end{equation}

Constraint (11) shows that the waste tire flowing into a centralized gathering center must flow out.

\begin{equation}
b_{p} \left( \sum_{s} AX_{ps} \right) = \sum_{p} AX_{cp} \quad \forall p \in P \quad (12)
\end{equation}

Constraint (12) shows that the part of unqualified waste tires flows into the second-hand market after being renovated in the reprocessing center.

\begin{equation}
\left(1-b_{p}\right) \left( \sum_{s} AX_{ps} \right) = \sum_{p} AX_{cp} \quad \forall p \in P \quad (13)
\end{equation}

Constraint (13) shows that the part of qualified waste tires flows into the recycling center after being renovated in the reprocessing center.

\begin{equation}
\sum_{i} Q_{ir} Z_{ir} \leq LMAX_{r} Z_{r} \quad \forall r \in R \quad (14)
\end{equation}

Constraint (14) shows that the amount of products shipped into a regional gathering center cannot exceed its processing capacity.

\begin{equation}
\sum_{r} AX_{rc} \leq LMAX_{C} Z_{c} \quad \forall c \in C \quad (15)
\end{equation}

Constraint (15) shows that the amount of products shipped into a centralized gathering center cannot exceed its processing capacity.

\begin{equation}
\sum_{c} AX_{cp} \leq LMAX_{P} Z_{p} \quad \forall p \in P \quad (16)
\end{equation}

Constraint (16) shows that the amount of products shipped into a re-processing center cannot exceed its processing capacity.

\begin{equation}
AX_{rc}, AX_{cp}, AX_{pk}, AX_{ps} \geq 0 \quad (17)
\end{equation}

3 Solution of the Model

Pareto efficient solution is required for solving multiobjective programming. ε constraint is a method to get the effective solution of multiobjective programming. The details are below.

\begin{equation}
\min(J_{1}(Z), \cdots, J_{p}(Z)), \text{ Subject to } Z \in G \quad (18)
\end{equation}

By the following formula (19), the multi-objective programming model (18) is changed into the following single objective programming model.

\begin{equation}
\min J_{i}(Z), \text{ Subject to } J_{i}(Z) \leq \varepsilon_{i}(i=2, \cdots, p), \quad Z \in G \quad (19)
\end{equation}

In model (19), \( J_{i}(Z) \leq \varepsilon_{i} \), \( Z_{i} \) is the objective function value of single objective programming (20) below.

\begin{equation}
\min J_{i}(Z)(i=1, \cdots, p), \text{ Subject to } Z \in G \quad (20)
\end{equation}

4 Numerical Example

This model is applied to the logistics management practice of Shanghai Tongfei logistics company. The company is engaged in the recycling, processing and sale of waste tires, and the vertexes of its recycling network are shown in TABLE 1 below.

**TABLE 1. QUANTITY OF VERTEXES IN RECYCLING NETWORK**

| I | R | C | P | K | S |
|---|---|---|---|---|---|
| 12 | 6 | 3 | 2 | 4 | 2 |

The providing quantity of every supplier is 55, \( LMAX_{r}=235 \), \( LMAX_{c}=210 \) and \( LMAX_{p}=215 \). The quantity of products provided by each supplier is 65. Cost of shipping a waste tire from a regional gathering center to a centralized gather center is 65 yuan, from a centralized gathering center to re-processing center is 72yuan. Cost of shipping a renewed tire from a re-processing center to a second-hand market is 83yuan, 91yuan to a recycling center. \( b_{p}=0.75 \).
The following TABLE 2 describes the distance from a supplier to a gathering center.

| TABLE 2. DISTANCE FROM A SUPPLIER TO A REGIONAL GATHERING CENTER (UNIT: KILOMETER) |
|---|---|---|---|---|---|---|
| $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ | $R_6$ |
| $I_1$ | 28 | 29 | 29 | 31 | 30 | 38 |
| $I_2$ | 29 | 31 | 35 | 39 | 41 | 39 |
| $I_3$ | 35 | 37 | 38 | 43 | 39 | 40 |
| $I_4$ | 31 | 34 | 36 | 37 | 38 | 41 |
| $I_5$ | 29 | 29 | 24 | 39 | 34 | 40 |
| $I_6$ | 42 | 28 | 26 | 39 | 53 | 52 |
| $I_7$ | 44 | 38 | 42 | 43 | 33 | 40 |
| $I_8$ | 51 | 78 | 34 | 46 | 59 | 60 |
| $I_9$ | 78 | 47 | 69 | 48 | 39 | 41 |
| $I_{10}$ | 89 | 79 | 69 | 88 | 68 | 69 |
| $I_{11}$ | 82 | 81 | 70 | 89 | 69 | 71 |
| $I_{12}$ | 42 | 43 | 45 | 47 | 51 | 58 |

The calculation results of the multiobjective model are described in TABLE 3 as follows.

| TABLE 3 SOLVING RESULT OF THE MODEL (UNIT OF Z1: YUAN, UNIT OF Z2: Z3: KILOMETER) |
|---|---|---|---|---|---|---|---|
| $\varepsilon_2$ | $\varepsilon_1$ | $\varepsilon_3$ | $Z_1$ | $Z_2$ | $Z_3$ |
| 1.36 | 1.65 | 1.08 | 17589 | 62178 | 15623 |
| $\alpha$ | $\beta$ | $\gamma$ | 2 | 6 | 0 |
| 1.26 | 1.55 | 1.12 | 16859 | 61952 | 15842 |
| $\alpha$ | $\beta$ | $\gamma$ | 6 | 5 | 0 |
| 1.16 | 1.45 | 1.16 | 16102 | 60123 | 15911 |
| $\alpha$ | $\beta$ | $\gamma$ | 3 | 6 | 2 |
| 1.12 | 1.35 | 1.26 | 15851 | 59638 | 15910 |
| $\alpha$ | $\beta$ | $\gamma$ | 2 | 5 | 1 |
| 1.08 | 1.25 | 1.36 | 14832 | 58901 | 15901 |
| $\alpha$ | $\beta$ | $\gamma$ | 5 | 5 | 2 |

From the solution in above TABLE III, it can be revealed that the three objective functions can not achieve consistency within the whole feasible region. For example, in the process of $\varepsilon_2$ decreasing from 1.36$\alpha$ to 1.16$\alpha$ and $\varepsilon_3$ from 1.45$\beta$ to 1.26$\beta$, $Z_2$ goes up, but $Z_1$ and $Z_3$ go down. Therefore, the transportation cost, carbon emission and customer service level of the waste tire recycling enterprises are difficult to reach optimization simultaneously. However, the three objective functions can achieve good consistency in the local range of the feasible region. For example, when $\varepsilon_2$ decreases from 1.16$\alpha$ to 1.08$\alpha$ and $\varepsilon_3$ from 1.45$\beta$ to 1.25$\beta$, the values of the three objective functions go down synchronously. Reduction of transportation cost and carbon emissions and improvement of the customer service level can be achieved at the same time by selecting the logistics vertexes layout and practicing the reasonable transportation schedule. Then the unity of short-term economic benefits and sustainable development of a waste tire recycling enterprise are realized.

5 Conclusions

With the rapid development of China's automobile industry, the number of waste tires is growing rapidly. Waste tire recycling is of great significance to enterprises and society. The development goal of low-carbon economy and the fierce competition of waste tire logistics require recycling enterprises not only to lower cost, but also to enforce the environmental factors and supplier cooperation. In this paper, the model of waste tire recycling network is established by multi-objective programming. On the premise of meeting production requirements, the transportation distance and carbon emission of suppliers are minimized. By this model, waste tire logistics enterprises can reduce the transportation cost, carbon emission and the transportation distance of suppliers, so as to realize the unity of short-term economic benefits, environmental protection and strategic partnership with suppliers.

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