Neutrino CP violation and sign of baryon asymmetry in the minimal seesaw model

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Abstract

We discuss the correlation between the CP violating Dirac phase of the lepton mixing matrix and the cosmological baryon asymmetry based on the leptogenesis in the minimal seesaw model with two right-handed Majorana neutrinos and the trimaximal mixing for neutrino flavors. The sign of the CP violating Dirac phase at low energy is fixed by the observed cosmological baryon asymmetry since there is only one phase parameter in the model. According to the recent T2K and NO\(\nu\)A data of the CP violation, the Dirac neutrino mass matrix of our model is fixed only for the normal hierarchy of neutrino masses.

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1 Introduction

Recent T2K data strongly indicate the CP violation in the neutrino oscillation [1, 2]. The NOνA data also suggest the CP violation [3], which is consistent with the T2K result. We are in the era to develop the flavor structure of Yukawa couplings by focusing on the leptonic CP violation since the CP violating phase of neutrinos will be observed in the near future.

We study the flavor structure in the seesaw model [4]-[6] to reveal the underlying physics of the lepton flavors. For this purpose, it is advantageous to consider the minimum number of parameters in the neutrino mass matrices needed for reproducing the neutrino mixing angles and CP violating phases completely [7]. There are many attempts toward the minimal seesaw model [8]-[21].

In our previous work, we studied the minimal seesaw model taking account of the CP violation of neutrinos [22], where we assumed two right-handed Majorana neutrinos and took the trimaximal mixing pattern of neutrino flavors [23, 24]. The trimaximal mixing is realized by the non-Abelian discrete flavor symmetry [25]-[28]. Since this mixing pattern is the simple framework obtained by the additional rotation of $2-3$ (TM$_1$) or $1-3$ (TM$_2$) families of neutrinos [29, 30] to the tribimaximal mixing (TBM) basis [31, 32], we could find the relations among mixing angles and CP violating phase, so called mixing sum rules [33]-[35]. The relations among mixing angles and CP violating phase have been discussed intensively [36]-[47]. We investigated the flavor structure of the Dirac neutrino mass matrix in our framework of the minimal seesaw model. The desirable candidates of Dirac neutrino mass matrices were obtained in the diagonal basis of the charged lepton mass matrix and the $2 \times 2$ right-handed Majorana neutrino mass matrix for both cases of the normal hierarchy (NH) and the inverted hierarchy (IH) of neutrino masses, respectively. However, we could not determine the sign of the CP violating phase $\delta_{CP}$ [22].

As well known, it is possible to correlate the CP violation at the low energy with the CP violation at the high energy [48, 49] through the leptogenesis [50]. In this work, it is found that the CP violating phase in the minimal seesaw model is directly related to the baryon asymmetry of the universe (BAU). We discuss the correlation between the predicted CP violating phase $\delta_{CP}$ and BAU through the leptogenesis in our minimal seesaw model. Finally, we determine the flavor structure of the Dirac neutrino mass matrix so that it explains both the low energy neutrino experimental data and BAU.

The paper is organized as follows. We summarize our minimal seesaw model [22] in section 2, where the structure of the Dirac neutrino mass matrix is discussed to reproduce TM$_1$ or TM$_2$ in NH or IH. In section 3, we present the framework of the leptogenesis in our minimal seesaw model. In section 4, the correlation between the low energy CP violation and BAU is discussed. The section 5 is devoted to the summary and discussions. Appendix A gives the detailed studies of the Dirac neutrino mass matrices and Appendix B presents the explicit form of the relevant mixing matrix elements in our model.
2 Our minimal seesaw model

Our minimal seesaw model consists of two right-handed Majorana neutrinos $N_1$ and $N_2$, and three left-handed neutrinos in Type I seesaw \cite{7}. The $3 \times 2$ Dirac neutrino mass matrix is constrained by a certain principle in the diagonal basis of both the charged lepton and right-handed Majorana neutrino mass matrices. We have already proposed simple Dirac neutrino mass matrices based on the symmetry of the neutrino mixing matrix, $T M_1$ and $T M_2$. We summarize them in the following subsections.

2.1 $T M_1$: 2-3 family mixing in NH

For the case of $T M_1$ in the NH neutrino masses, the Dirac neutrino mass matrix $M_D$ is given in terms of the Yukawa matrix $Y_{\nu}$ in the basis of the diagonal right-handed Majorana neutrino mass matrix $M_R$ as follows \cite{22}:

\[
M_D = v Y_{\nu} = v \begin{pmatrix} \frac{b+c}{2} & \frac{e+f}{2} \\ b & e \\ c & f \end{pmatrix}, \quad M_R = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} = M_0 \begin{pmatrix} p^{-1} & 0 \\ 0 & 1 \end{pmatrix},
\]

(1)

where $v = 174.1 \text{GeV}$, $M_0$ is the mass scale of the right-handed Majorana neutrino and $p$ is the ratio $M_2/M_1$. We take $e$ and $f$ to be real and $b$ and $c$ to be complex in general by using the freedom of redefinitions of phases in the left-handed lepton fields. By using the type I seesaw, we obtain the left-handed neutrino mass matrix $M_\nu$:

\[
M_\nu = -M_D M_R^{-1} M_D^T,
\]

(2)

which turns to

\[
\hat{M}_\nu = V_{TBM}^T M_\nu V_{TBM} = -\frac{v^2}{M_0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4} ( (b+c)^2 p + (e+f)^2 ) & \frac{3}{2} \sqrt{\frac{3}{2}} ( (c^2-b^2) p - e^2 + f^2 ) \\ 0 & \frac{1}{2} \sqrt{\frac{3}{2}} ( (c^2-b^2) p - e^2 + f^2 ) & \frac{1}{2} ( (b-c)^2 p + (e-f)^2 ) \end{pmatrix},
\]

(3)

where

\[
V_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}.
\]

(4)

This mass matrix $\hat{M}_\nu$ is given in the TBM mixing basis, which is derived in Appendix A. The neutrino mass matrix in Eq. (3) is diagonalized by the rotation of 2 - 3 families as

\[
V_{23} = \frac{1}{A} \begin{pmatrix} A & 0 & 0 \\ 0 & 1 & \nu \\ 0 & -\nu^* & 1 \end{pmatrix}, \quad A = \sqrt{1 + |\nu|^2},
\]

(5)
where $\mathcal{V}$ is given in terms of $b$, $c$, $e$, $f$ and $p$. The PMNS matrix $[51, 52]$ is expressed as

$$U_{\text{PMNS}} = V_{\text{TBM}} V_{23},$$

which gives three mixing angles, one Dirac phase, and one Majorana phase.

In order to remove one complex parameter from the Dirac neutrino mass matrix, we put one zero in the first column of the Dirac neutrino mass matrix (see Appendix A.1). There are three cases with one zero. For case I ($b + c = 0$), the Dirac neutrino mass matrix $M_D$ and the left-handed neutrino mass matrix $\hat{M}_\nu$ are given as

**Case I** $b + c = 0$ :

$$M_D = v Y_\nu = v \begin{pmatrix} 0 & \frac{e+f}{2} \\ b & e \\ -b & f \end{pmatrix},$$

$$\hat{M}_\nu = -\frac{f^2 v^2}{M_0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4}(k+1)^2 & -\frac{1}{2} \sqrt{\frac{3}{2}}(k^2 - 1) \\ 0 & -\frac{1}{2} \sqrt{\frac{3}{2}}(k^2 - 1) & 2B^2 pe^{2i\phi_B} + \frac{1}{2}(k - 1)^2 \end{pmatrix},$$

respectively, where the CP violating phase $\phi_B$ appears in the parameter $b$ such as

$$e = k, \quad \arg[b] = \phi_B, \quad \frac{b}{f} = B e^{i\phi_B},$$

with $k$ and $B$ being real. For case II ($c = 0$), they are

**Case II** $c = 0$ :

$$M_D = v Y_\nu = v \begin{pmatrix} b & \frac{e+f}{2} \\ 0 & e \\ f \end{pmatrix},$$

$$\hat{M}_\nu = -\frac{f^2 v^2}{M_0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4}[\hat{B}^2 p e^{2i\phi_B} + (k+1)^2] & -\frac{1}{2} \sqrt{\frac{3}{2}}[\hat{B}^2 p e^{2i\phi_B} + k^2 - 1] \\ 0 & -\frac{1}{2} \sqrt{\frac{3}{2}}[\hat{B}^2 p e^{2i\phi_B} + k^2 - 1] & \frac{1}{2}[\hat{B}^2 p e^{2i\phi_B} + (k - 1)^2] \end{pmatrix},$$

where

$$e = k, \quad \arg[b] = \phi_B, \quad \frac{b}{f} = \hat{B} e^{i\phi_B},$$

with $k$ and $\hat{B}$ being real. For case III ($b = 0$), they are

**Case III** $b = 0$ :

$$M_D = v Y_\nu = v \begin{pmatrix} \frac{e}{2} & \frac{e+f}{2} \\ 0 & e \\ c & f \end{pmatrix},$$

$$\hat{M}_\nu = -\frac{f^2 v^2}{M_0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4}[B^2 p e^{2i\phi_B} + (k+1)^2] & -\frac{1}{2} \sqrt{\frac{3}{2}}[-B^2 p e^{2i\phi_B} + k^2 - 1] \\ 0 & -\frac{1}{2} \sqrt{\frac{3}{2}}[-B^2 p e^{2i\phi_B} + k^2 - 1] & \frac{1}{2}[B^2 p e^{2i\phi_B} + (k - 1)^2] \end{pmatrix},$$

3
where
\[ \frac{e}{f} = k, \quad \arg[c] = \phi_B, \quad \frac{c}{f} = B e^{i\phi_B}. \] (15)

For three cases, neutrino masses are given follows:

\[ m_1 = 0, \quad m_2^2 m_3^2 = \frac{9v^8}{4M_0^2}(j - k)^4 f^8 B^4 p^2, \]
\[ m_2^2 + m_3^2 = \frac{v^4 f^4}{16M_0^2} [B^4 p^2(5j^2 + 2j + 5)^2 + 2B^2 p(5jk + j + k + 5)^2 \cos 2\phi_B + (5k^2 + 2k + 5)^2] \] (16)

where \( j \equiv b/c = -1 \) and \( j = 0 \) for cases I and III, respectively. For case II, \( Bj \equiv \hat{B} \) with \( j \to -\infty \) and \( B \to 0 \). It is noticed that the PMNS matrix elements are correlated with neutrino masses.

The explicit form of the 2 – 3 flavor mixing \( V \) in Eq. (5) is given in Appendix B for the three cases. We can predict the CP violating measure, the Jarlskog invariant \( J_{CP} \) [53], with \( V \)'s as follows:

\[ J_{CP} \equiv \text{Im} \left[ U_{\alpha 1} U_{\mu 2} U_{e 2} U_{\mu 1}^* \right] = -\frac{1}{3\sqrt{6}A^2} \text{Im}[V^*], \] (17)

where \( U_{\alpha i} \) denotes the PMNS matrix elements of Eq. (6).

It is noted that the Littlest seesaw model by King et al. [54]-[56] corresponds to \( k = -3 \) in case I, that is,

\[ M_D = vY_\nu = v \begin{pmatrix} 0 & f \\ b & 3f \\ -b & -f \end{pmatrix}. \] (18)

### 2.2 TM1: 2 – 3 family mixing in IH

The Dirac neutrino mass matrix \( M_D \) is presented in terms of the Yukawa matrix \( Y_\nu \) for the case of TM1 in the IH neutrino masses with \( m_3 = 0 \). As shown in Appendix A.2, the Dirac neutrino mass matrix and the left-handed Majorana neutrino mass matrices are given as:

\[ M_D = vY_\nu = v \begin{pmatrix} -2b & \frac{e + f}{2} \\ b & e \\ b & f \end{pmatrix}, \] (19)

and

\[ \hat{M}_\nu = -\frac{v^2}{M_0} \begin{pmatrix} 6b^2 p & 0 & 0 \\ 0 & \frac{3}{4}(e + f)^2 & -\frac{1}{2}\sqrt{v^2 (e - f)(e + f)} \\ 0 & -\frac{1}{2}\sqrt{v^2 (e - f)(e + f)} & \frac{1}{2}(e - f)^2 \end{pmatrix}, \] (20)

respectively, where \( m_1 = 6b^2 pv^2/M_0 \) and \( m_3 = 0 \). One can take the elements in the first column of the Dirac neutrino mass matrix real with the freedom of redefinitions of phases, and \( e \) and \( f \) are complex. The relative phase between \( e \) and \( f \) leads to the CP violation in contrast to the case of subsection 2.1.
2.3 TM$_2$: $1 - 3$ family mixing in NH or IH

Finally in section 2, we present the Dirac neutrino mass matrix $M_D$ in terms of the Yukawa matrix $Y_\nu$ in the case of TM$_2$ for both NH and IH neutrino masses. As seen in Appendix A.3, the Dirac neutrino mass matrix and the left-handed Majorana neutrino mass matrix are given as follows:

$$M_D = v Y_\nu = v \begin{pmatrix} b & -e - f \\ b & e \\ b & f \end{pmatrix},$$  

$$\hat{M}_\nu = -\frac{v^2}{M_0} \begin{pmatrix} \frac{3}{2}(e + f)^2 & 0 & \sqrt{3}(e^2 - f^2) \\ 0 & 3b^2p & 0 \\ \sqrt{3}(e^2 - f^2) & 0 & \frac{1}{2}(e - f)^2 \end{pmatrix},$$

respectively, where $m_2 = 3b^2pv^2/M_0$ with $m_1 = 0$ or $m_3 = 0$.

3 Implications on Leptogenesis

Our minimal seesaw model predicts the magnitude of the CP violating phase $\delta_{CP}$. This result encourages us to discuss a cosmological consequence of the type-I seesaw mechanism. The CP violating decays of the heavy Majorana neutrinos can explain the observed BAU by the mechanism of the leptogenesis [50]. Since our seesaw model has only one phase parameter $\phi_B$, we expect that the sign of the lepton asymmetry of the leptogenesis is related with the sign of $\delta_{CP}$ [11, 12, 17, 19, 21, 48].

Let us discuss the numerical implications of our models on the leptogenesis. We assume the lightest right-handed neutrino $N_1$ is much lighter than $N_2$ for simplicity. This condition suffices for the flavor independent analysis. The flavor summed CP asymmetry at the decay of the lighter right-handed neutrino $N_1$ is given as [57, 58, 59]

$$\epsilon_{N_1} = -\frac{1}{8\pi} \sum_j \text{Im} \left[ \frac{(Y_\nu^\dagger Y_\nu)_{j1}}{(Y_\nu^\dagger Y_\nu)_{11}} \right] \left[ f^V \left( \frac{M_j^2}{M_1^2} \right) + f^S \left( \frac{M_j^2}{M_1^2} \right) \right],$$

where $f^V(x)$ and $f^S(x)$ are the contributions from vertex and self-energy corrections, respectively. In the case of the standard model (SM) with right-handed neutrinos, they are given as

$$f^V(x) = \sqrt{x} \left[ (x + 1) \ln \left( 1 + \frac{1}{x} \right) - 1 \right], \quad f^S(x) = \frac{\sqrt{x}}{x - 1}.$$  

For the case of $p = M_2/M_1 \gg 1$, the CP asymmetry is approximately expressed as

$$\epsilon_{N_1} \approx -\frac{3}{16\pi} \sum_j \text{Im} \left[ \frac{(Y_\nu^\dagger Y_\nu)_{j1}}{(Y_\nu^\dagger Y_\nu)_{11}} \right] \frac{1}{p}.$$  

The Yukawa matrix $Y_\nu$ is given as the $3 \times 2$ matrix as discussed in the section 2.
By assuming no pre-existing asymmetry of $N_1$, the final amount of $B - L$ asymmetry $Y_{B-L}$ can be conveniently written as

$$Y_{B-L} = -\epsilon_{N_1} \kappa Y^\text{eq}_{N_1}(T \gg M_1), \quad (26)$$

where

$$Y^\text{eq}_{N_1}(T \gg M_1) = \frac{135 \zeta(3)}{4\pi g^*}, \quad (27)$$

with $g^* = 106.75$ for SM. The parameter $\kappa$ is a suppression factor which accounts for the number density of $N_1$ with respect to the equilibrium value, the out-of-equilibrium condition at the decay, and the thermal corrections. It is approximately given as

$$\frac{1}{\kappa} \approx 3.3 \times 10^{-3} \frac{\bar{m}_1}{v^2} + \left(\frac{\bar{m}_1}{5.5 \times 10^{-4} \text{eV}}\right)^{1.16}, \quad (28)$$

where

$$\bar{m}_1 = \frac{v^2}{M_1} (Y^\dagger_{\nu} Y_{\nu})_{11}. \quad (29)$$

This expression is valid for in the region of $M_1 \ll 10^{14}$GeV and $m_1 \geq 10^{-2} \text{eV}$ [59]. As seen later, our numerical results are presented under these conditions. After the reprocessing by sphaleron transitions, the baryon asymmetry is related to the $B - L$ asymmetry as

$$\eta_B \equiv \frac{n_B}{n_\gamma} = 7.04 \times \frac{28}{79} Y_{B-L}. \quad (30)$$

In order to estimate the lepton asymmetry $\epsilon_{N_1}$, we show the explicit forms given by the Yukawa matrix elements for cases I, II and III as

**case I**: \[ \frac{\text{Im}[(Y^\dagger_{\nu} Y_{\nu})_{21}]^2}{(Y^\dagger_{\nu} Y_{\nu})_{11}} = \frac{1}{2} f^2 (k - 1)^2 \sin 2\phi_B, \quad \bar{m}_1 = 2v^2 \frac{f^2}{M_0} B^2, \quad (31) \]

**case II**: \[ \frac{\text{Im}[(Y^\dagger_{\nu} Y_{\nu})_{21}]^2}{(Y^\dagger_{\nu} Y_{\nu})_{11}} = \frac{1}{20} f^2 (5k + 1)^2 \sin 2\phi_B, \quad \bar{m}_1 = 5 \frac{v^2 f^2}{M_0} \hat{B}^2, \quad (32) \]

**case III**: \[ \frac{\text{Im}[(Y^\dagger_{\nu} Y_{\nu})_{21}]^2}{(Y^\dagger_{\nu} Y_{\nu})_{11}} = \frac{1}{20} f^2 (k + 5)^2 \sin 2\phi_B, \quad \bar{m}_1 = 5 \frac{v^2 f^2}{M_0} B^2, \quad (33) \]

respectively. Therefore, the sign of the CP asymmetry $\epsilon_{N_1}$ in Eq. (23) is fixed by the sign of $(-\sin 2\phi_B)$ in the I, II, III cases with $M_1 \ll M_2$, while the signs of $Y_{B-L}$ and $\eta_B$ in Eqs. (26) and (30) are given by the sign of $\sin 2\phi_B$.

It is easily found that $(Y^\dagger_{\nu} Y_{\nu})_{21}$ vanishes for the Yukawa matrices of Eqs. (19) and (21) in subsections 2.2 and 2.3. Therefore, the CP asymmetry at the decay of the right-handed neutrino $N_1$ is not realized for TM$_1$ of IH and TM$_2$ of both NH and IH. We present numerical results of the leptogenesis for the three cases of TM$_1$ with NH in the next section.
We discuss the correlation between the predicted CP violating phase $\delta_{CP}$ and the observed BAU through the leptogenesis in cases I, II and III for TM$_1$ in NH.

In order to determine the free parameter set ($k$, $\phi_B$, $B\sqrt{p}$, $f^2/M_0$) in the neutrino mass matrices in Eqs. (8), (11) and (14), we use the result of the global analyses in Refs. [60, 61] for five data of the mixing angles and neutrino masses. We can predict the CP violating phase $\delta_{CP}$ by inputting the data within 3$\sigma$ in Table 1 [60]. The CP violating measure, $J_{CP}$ [53] in Eq. (17), is related to the mixing angles and the CP violating phase as

$$J_{CP} = s_{23}s_{12}c_{12}s_{13}c_{13}^2 \sin \delta_{CP},$$

where $c_{ij} (s_{ij})$ denotes $\cos \theta_{ij} \geq 0$ ($\sin \theta_{ij} \geq 0$) and $\delta_{CP}$ is the CP violating Dirac phase in the PDG convention [62]. It is noted that the sign of $J_{CP}$ is given by the sign of $\sin \delta_{CP}$. Although $J_{CP}$ is defined in terms of the mixing matrix elements in Eq. (17), it is directly calculated by using the neutrino mass matrices in Eqs. (8), (11) and (14) for three cases (TM$_1$ in NH). The CP-odd weak basis invariant $J_{CP}$ is given as follows [63 64]:

$$J_{CP} = \text{Tr} \left[ (M_\nu M_\nu^*)^3 \right] = -6i \Delta m_\ell^6 \Delta m_\nu^6 J_{CP},$$

where $M_\ell$ is the charged lepton mass matrix, and $\Delta m_\ell^6$ and $\Delta m_\nu^6$ are

$$\Delta m_\ell^6 = (m_\mu^2 - m_e^2)(m_\tau^2 - m_\mu^2)(m_\nu^2 - m_\mu^2), \quad \Delta m_\nu^6 = (m_2^2 - m_1^2)(m_3^2 - m_2^2)(m_4^2 - m_3^2),$$

respectively. By using this formula and the abbreviation $\Delta m_{ij}^2 \equiv m_j^2 - m_i^2$, we obtain

\begin{align*}
\text{case I :} & \quad J_{CP} = -\frac{3}{8} \frac{f_{12}^2}{M_0^6} (B\sqrt{p})^6 (k - 1)(k + 1)^5 \sin 2\phi_B \frac{v_{12}^2 (\Delta m_{13}^2 - \Delta m_{12}^2) \Delta m_{13}^2 \Delta m_{12}^2}{(\Delta m_{13}^2 - \Delta m_{12}^2) \Delta m_{13}^2 \Delta m_{12}^2}, \\
\text{case II :} & \quad J_{CP} = -\frac{3}{32} \frac{f_{12}^2}{M_0^6} (B\sqrt{p})^6 (5k + 1) \sin 2\phi_B \frac{v_{12}^2 (\Delta m_{13}^2 - \Delta m_{12}^2) \Delta m_{13}^2 \Delta m_{12}^2}{(\Delta m_{13}^2 - \Delta m_{12}^2) \Delta m_{13}^2 \Delta m_{12}^2},
\end{align*}

Table 1: 3$\sigma$ range of the global analysis of the neutrino oscillation experimental data for NH and IH [60].
case III: \[ J_{CP} = \frac{3}{32} \frac{f^{12}}{M_0^6} (B\sqrt{p})^{6} k^{5} (k + 5) \sin 2\phi_B \frac{v^{12}}{(\Delta m_{13}^{2} - \Delta m_{12}^{2}) \Delta m_{13}^{2} \Delta m_{12}^{2}}. \] (39)

It is remarked that the sign of \( J_{CP} \) is classified according to \( k \) because of \( \Delta m_{13}^{2} \gg \Delta m_{12}^{2} > 0 \) as follows:

\[
J_{CP} \sim \begin{cases} 
\sin 2\phi_B \text{ for } -1 \leq k \leq 1 & ; -\sin 2\phi_B \text{ for } k \leq -1, k \geq 1 \text{ in case I} \\
\sin 2\phi_B \text{ for } k \leq -1/5 & ; -\sin 2\phi_B \text{ for } k \geq -1/5 \text{ in case II} \\
\sin 2\phi_B \text{ for } k \leq -5, k \geq 0 & ; -\sin 2\phi_B \text{ for } -5 \leq k \leq 0 \text{ in case III}
\end{cases}
\] (40)

It is noted that the sign of \( \sin 2\phi_B \) must be positive since the observed \( \eta_B \) is positive as discussed below Eq. (33). Therefore, we can investigate the sign of \( \sin 2\phi_B \) in Eqs. (34) and (40). We can predict not only the absolute value of \( \delta \) as seen in Eqs. (25) and (31). Therefore, the numerical result of neutrino masses and the consistent with the Littlest seesaw model with \( \nu \) if we take account of the recent T2K data [1, 2] and the NOνA data [3]. This result is consistent with the Littlest seesaw model with \( k = -3 \) by King [54] where the leptogenesis phase yields an observable neutrino oscillation phase with \( \delta_{CP} \simeq -\pi/2 \).

Let us discuss the case of \( k \leq -1 \) in detail. The parameters of \( f \) and \( p = M_2/M_1 \) depend on the magnitude of \( M_0 \). As mentioned in Section 2, \( f \) is taken to be real positive. We show the allowed region on the plane of \( f \) and \( p \) for both cases of \( M_0 = 10^{14}\text{GeV} \) and \( 10^{15}\text{GeV} \) in Fig. 2(a). The Yukawa coupling \( f \) is predicted in \( f < 1 \), which is preferable in the perturbative calculation of the lepton asymmetry. For the case of \( M_0 = 10^{15}\text{GeV} \), \( p \) and \( f^2 \) are larger ten times compared to those in \( M_0 = 10^{14}\text{GeV} \). This behavior is easily understood as follows. The neutrino masses \( m_3, m_2 \) and \( \tilde{m}_1 \) are proportional to \( f^2/M_0 \) as seen in Eqs. (16) and (31) while the asymmetry parameter \( \epsilon_{N_1} \) depends on \( f^2/p \) for \( p \gg 1 \) as seen in Eqs. (25) and (31). Therefore, the numerical result of neutrino masses and the asymmetry are unchanged if we enlarge \( M_0, f^2 \) and \( p \) ten times simultaneously.

In order to check the availability of the approximate \( \kappa \) in Eq. (28), we show \( \tilde{m}_1 \) versus \( k \) in Fig. 2(b). As seen in this figure, \( \tilde{m}_1 \) is \((4.2 - 7.7) \times 10^{-2}\text{eV}\), which satisfies the condition,
Figure 1: Predictions in case I. The blue and orange dots denote the region of $k < -1$ and $-1 < k < 0$, respectively. The red lines for $\sin^2\theta_{23}$ and $\delta_{CP}$ denote the experimental bounds of $3\sigma$ (global analyses) and $2\sigma$ (T2K) ranges, respectively: (a) $\delta_{CP}$ versus $k$, (b) $\delta_{CP}$ versus $\sin^2\theta_{23}$.

$m_1 \geq 10^{-2}\text{eV}$ in Ref. [59]. The calculated $m_1$'s almost overlap for $M_0 = 10^{14}\text{GeV}$ and $10^{15}\text{GeV}$.

Figure 2: Obtained parameters in case I for $M_0 = 10^{14}\text{GeV}$ (blue dots) and $10^{15}\text{GeV}$ (orange dots) with $k \leq -1$: (a) $p$ versus $f$, (b) $\tilde{m}_1$ versus $k$.

Next, we present our result for case II. In our previous paper [22], it is found that $k$ is restricted in a narrow range, $k = -1.65 \sim -1.10$. The predicted $\delta_{CP}$ is positive around $+50^\circ$ as seen in Fig. 3(a) while the predicted $\sin^2\theta_{23}$ is restricted near the upper bound of the $3\sigma$ range of the experimental data. The case II is disfavored by the recent T2K and NOvA data because the predicted $\delta_{CP}$ to be positive. We show the allowed region on the plane of $f$ and $p$ for both cases of $M_0 = 10^{14}\text{GeV}$ and $10^{15}\text{GeV}$ in Fig. 3(b). The magnitude of $M_0$ is allowed up to $10^{15}\text{GeV}$ within $f \leq 1$. We note that $\tilde{m}_1$ is $(1.8 - 2.4) \times 10^{-2}\text{eV}$, which satisfies the condition, $\tilde{m}_1 \geq 10^{-2}\text{eV}$ in Ref. [59].

Finally, we discuss the case III. The allowed range of $k$ is restricted to be very narrow,
Figure 3: The predictions in case II. (a) The predicted $\delta_{CP}$ versus $\sin^2 \theta_{23}$. The red lines for $\sin^2 \theta_{23}$ and $\delta_{CP}$ denote the experimental bounds of 3$\sigma$ (global analyses) and 2$\sigma$ (T2K) ranges, respectively, (b) $p$ versus $f$ for $M_0 = 10^{14}$GeV (blue dots) and $10^{15}$GeV (orange dots).

$k = -0.86 \sim -0.71$, as shown in our previous paper [22]. We show the predicted $\delta_{CP}$ versus $\sin^2 \theta_{23}$ in Fig. 4(a), where $\delta_{CP}$ is predicted to be negative around $-125^\circ$. However, the case III may be soon excluded in the near future as the mixing angle $\sin^2 \theta_{23}$ is strongly restricted to the lower bound of the 3$\sigma$ range. In order to compare the allowed parameter regions of $f$ and $p$ in case III with that in cases I and II, we show the plot of $f$ and $p$ for both cases of $M_0 = 10^{14}$GeV and $10^{15}$GeV in Fig. 4(b). The magnitude of $M_0$ is allowed up to $10^{15}$GeV in the region of $f \leq 1$. The allowed range of $\tilde{m}_1$ is $(1.9 - 2.1) \times 10^{-2}$eV, which satisfies the condition, $\tilde{m}_1 \geq 10^{-2}$eV in Ref. [59].

Figure 4: The prediction in case III. (a) The predicted $\delta_{CP}$ versus $\sin^2 \theta_{23}$. The red lines for $\sin^2 \theta_{23}$ and $\delta_{CP}$ denote the experimental bounds of 3$\sigma$ (global analyses) and 2$\sigma$ (T2K) ranges, respectively, (b) $p$ versus $f$ for $M_0 = 10^{14}$GeV (blue dots) and $10^{15}$GeV (orange dots).
5 Summary and discussions

We have studied the correlation between the CP violating phase $\delta_{CP}$ and the observed BAU in the minimal seesaw model, where two right-handed Majorana neutrinos are assumed. We have also taken the trimaximal mixing pattern for the neutrino flavor (TM$_1$ or TM$_2$) in the diagonal basis of both the charged lepton and right-handed Majorana neutrino mass matrices. As our model has only one CP violating phase, we have found the clear correlation between the CP violating phase $\delta_{CP}$ and BAU for TM$_1$ in NH of neutrino masses. On the other hand, the lepton asymmetry vanishes for TM$_1$ in IH and TM$_2$ in both NH and IH.

We have discussed the three cases of the Dirac neutrino mass matrix for TM$_1$ in NH. The flavor structure of the Dirac neutrino mass matrix is fixed for case I. The parameter $k$ should be smaller than $-1$ in order to predict a negative $\delta_{CP}$, which is indicated by the recent T2K and NO$\nu$A data. Our result is completely consistent with the Littlest seesaw model [54] which is the specific model of case I with $k = -3$. It is emphasized that our Dirac neutrino mass matrix predicts the negative sign of $\delta_{CP}$ and the observed value of BAU as far as we take $k < -1$ under the condition, $M_1 \ll M_2$.

In our calculation, we have assumed $M_1 \ll M_2$ for the right-handed Majorana neutrino masses. For the case of $M_1 \gg M_2$, one can obtain the numerical result by switching $M_1$ and $M_2$ with the labels 1 and 2 in Eqs. (31), (32) and (33). Note that the left-handed neutrino mass matrix in Eq. (3) is unchanged for $M_0 = M_1$ and $p = M_1/M_2$. Therefore, the relevant lepton asymmetry $\epsilon_{N_2}$ is given by $\text{Im}[(Y_\nu^\dagger Y_\nu)_{12}]^2$ where the Yukawa matrix $Y_\nu$ is expressed as Eqs. (7), (10) or (13). Since $(Y_\nu^\dagger Y_\nu)_{12}$ is the complex conjugate of $(Y_\nu^\dagger Y_\nu)_{21}$, we get the opposite sign of the CP violating phase $\delta_{CP}$ compared with the case of $M_1 \ll M_2$. Namely, the case with $-1 < k < 0$ is preferred in contrast to our result of $k < -1$ in case I. It is also noticed that the case II is favored while the case III is disfavored for $M_1 \gg M_2$. The mass hierarchy between $M_1$ and $M_2$ is an important key parameter to obtain a robust prediction of the correlation between $\delta_{CP}$ and BAU. It may be interesting that the Froggatt-Nielsen mechanism [66] is added to our minimal seesaw model to fix the $M_2/M_1$ ratio.

Finally, we add a comment. Since our Dirac neutrino mass matrices are given at the high energy scale to predict the BAU, we should examine the renormalization group correction for the neutrino mixing matrix. However, it is very small since the lightest neutrino mass $m_1$ vanishes in our model as seen in Ref. [67].

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Appendix

A Minimal seesaw mass matrix

We study the minimal seesaw model in the basis of the diagonal $2 \times 2$ right-handed Majorana neutrino mass matrix $M_R$ given in general as:

$$M_R = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} = M_0 \begin{pmatrix} p^{-1} & 0 \\ 0 & 1 \end{pmatrix},$$

(41)

where $v = 174.1\text{GeV}$, $M_0$ is the mass scale of the right-handed Majorana neutrino and $p$ is the ratio $M_2/M_1$. The relevant Dirac neutrino mass matrix $M_D$ is defined as

$$M_D = v Y_\nu = v \begin{pmatrix} a & d \\ b & e \\ c & f \end{pmatrix},$$

(42)

where $a \sim f$ are complex parameters. By using the type I seesaw mechanism, the left-handed Majorana neutrino mass matrix $M_\nu$ is given by

$$M_\nu = -M_DM_R^{-1}M_D^T = -\frac{v^2}{M_0} \begin{pmatrix} a^2 p + d^2 & abp + de & acp + df \\ abp + de & b^2 p + e^2 & bcp + ef \\ acp + df & bcp + ef & c^2 p + f^2 \end{pmatrix},$$

(43)

By turning the neutrino mass matrix $M_\nu$ to the TBM mixing basis, $M_\nu$ is given as:

$$\tilde{M}_\nu \equiv V^T_{\text{TBM}} M_\nu V_{\text{TBM}} = -\frac{v^2}{M_0} \begin{pmatrix} A_\nu^2 p + D_\nu^2 & \frac{A_\nu B_\nu p + D_\nu E_\nu}{2\sqrt{3}} & \frac{A_\nu C_\nu p + D_\nu F_\nu}{\sqrt{6}} \\ \frac{A_\nu B_\nu p + D_\nu E_\nu}{2\sqrt{3}} & \frac{B_\nu^2 p + E_\nu^2}{3} & \frac{B_\nu C_\nu p + E_\nu F_\nu}{\sqrt{6}} \\ \frac{A_\nu C_\nu p + D_\nu F_\nu}{\sqrt{6}} & \frac{B_\nu C_\nu p + E_\nu F_\nu}{\sqrt{6}} & \frac{C_\nu^2 p + F_\nu^2}{2} \end{pmatrix},$$

(44)

where

$$A_\nu \equiv 2a - b - c, \quad B_\nu \equiv a + b + c, \quad C_\nu \equiv c - b,$$

$$D_\nu \equiv 2d - e - f, \quad E_\nu \equiv d + e + f, \quad F_\nu \equiv f - e.$$

(45)

Based on these formulae, we discuss the additional $2 - 3$ family rotation (TM$_1$) and $1 - 3$ family rotation (TM$_2$) to the TBM mixing for both NH and IH in the following subsections.

A.1 TM$_1$: 2 $-$ 3 family rotation in NH

We consider the case of NH in TM$_1$. Since $(1, 1)$, $(1, 2)$, $(2, 1)$, $(1, 3)$, and $(3, 1)$ entries of the matrix must be zero, conditions for the additional $2 - 3$ rotation to the TBM mixing are

$$A_\nu = 2a - b - c = 0, \quad D_\nu = 2d - e - f = 0,$$

(46)
where \((a, b, c)\) are supposed to be independent of \((d, e, f)\). Under these conditions, the mass matrix Eq. (44) is rewritten as

\[
\hat{M}_\nu = -\frac{v^2}{M_0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4}((b + c)^2p + (e + f)^2) & \frac{1}{2}\sqrt{\frac{3}{2}}((c^2 - b^2)p - e^2 + f^2) \\ 0 & \frac{1}{2}\sqrt{\frac{3}{2}}((c^2 - b^2)p - e^2 + f^2) & \frac{1}{2}(b - c)^2p + (e - f)^2 \end{pmatrix},
\]

(47)

where the lightest neutrino mass \(m_1\) is zero. The Dirac neutrino mass matrix is given as:

\[
M_D = vY_\nu = v \begin{pmatrix} b + c \\ b \\ e + f \end{pmatrix}.
\]

(48)

We discuss the specific three cases from this texture. Putting one zero in the texture Eq. (48), we have three possible patterns as follows:

(I) \(b + c = 0\), (II) \(c = 0\), (III) \(b = 0\).

(49)

Corresponding Dirac neutrino mass matrices are

\[
M_D = \begin{cases} 
  v \begin{pmatrix} 0 & \frac{e + f}{2} \\ b & e \\ -b & f \end{pmatrix} & \text{for (I) } b + c = 0 \\
  v \begin{pmatrix} b & e \\ 0 & f \end{pmatrix} & \text{for (II) } c = 0 \\
  v \begin{pmatrix} c & f \\ b & e \end{pmatrix} & \text{for (III) } b = 0
\end{cases},
\]

(50)

We get another set by switching the first and second columns of the Dirac neutrino mass matrix in Eq. (50). However, the neutrino mass matrix \(\hat{M}_\nu\) is unchanged under the rescale of parameters. Therefore, we show only three cases in Eq. (50). It is noted that the switching the first and second columns of the Dirac neutrino mass matrix leads to the change of the sign for the leptogenesis.

We show the neutrino mass matrix \(\hat{M}_\nu\) for three cases:

Case I : \(\hat{M}_\nu = -\frac{v^2}{M_0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4}(e + f)^2 & -\frac{1}{2}\sqrt{\frac{3}{2}}(e - f)(e + f) \\ 0 & -\frac{1}{2}\sqrt{\frac{3}{2}}(e - f)(e + f) & 2b^2p + \frac{1}{2}(e - f)^2 \end{pmatrix} \),

(51)

Case II : \(\hat{M}_\nu = -\frac{v^2}{M_0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4}[b^2p + (e + f)^2] & -\frac{1}{2}\sqrt{\frac{3}{2}}[b^2p + (e - f)(e + f)] \\ 0 & -\frac{1}{2}\sqrt{\frac{3}{2}}[b^2p + (e - f)(e + f)] & \frac{1}{2}[b^2p + (e - f)^2] \end{pmatrix} \),

(52)
Case III : \( \hat{M}_\nu = -\frac{v^2}{M_0} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{3}{4} [c^2 p + (e + f)^2] & -\frac{1}{2} \sqrt{\frac{3}{2}} [c^2 p + (e - f)(e + f)] \\ 0 & -\frac{1}{2} \sqrt{\frac{3}{2}} [-c^2 p + (e - f)(e + f)] & \frac{1}{2} [c^2 p + (e - f)^2] \end{pmatrix} \). \hfill (53)

### A.2 TM\(_1\): 2 − 3 rotation in IH

We discuss the case of IH in TM\(_1\). In order to give the additional 2 − 3 family rotation to the TBM mixing, the \((1, 2), (1, 3), (2, 1), \) and \((3, 1)\) elements in Eq. \((44)\) should vanish.

These conditions lead to

\[
A_\nu = a + b + c = 0, \quad C_\nu = c - b = 0, \quad D_\nu = 2d - e - f = 0. \quad (54)
\]

The Dirac neutrino mass matrix is

\[
M_D = v Y_\nu = v \begin{pmatrix} -2b & \frac{e + f}{2} \\ b & e \\ b & f \end{pmatrix}. \quad (55)
\]

The neutrino mass matrix \( \hat{M}_\nu \) is given as:

\[
\hat{M}_\nu = -\frac{v^2}{M_0} \begin{pmatrix} 6b^2 p & 0 \\ 0 & \frac{3}{4} (e + f)^2 & -\frac{1}{2} \sqrt{\frac{3}{2}} (e - f)(e + f) \\ 0 & -\frac{1}{2} \sqrt{\frac{3}{2}} (e - f)(e + f) & \frac{1}{2} (e - f)^2 \end{pmatrix}, \quad (56)
\]

where the neutrino mass \( m_3 \) vanishes.

### A.3 TM\(_2\): 1 − 3 rotation in NH or IH

We discuss the case of the additional 1 − 3 family rotation to the TBM mixing. This case is called as TM\(_2\). Then, \((1, 2), (2, 3), (2, 1), \) and \((3, 2)\) elements in Eq. \((44)\) should vanish.

These conditions lead to

\[
A_\nu = 2a - b - c = 0, \quad C_\nu = c - b = 0, \quad E_\nu = d + e + f = 0. \quad (57)
\]

The Dirac neutrino mass matrices is

\[
M_D = v Y_\nu = v \begin{pmatrix} b & -e - f \\ b & e \\ b & f \end{pmatrix}. \quad (58)
\]

and the neutrino mass matrix \( \hat{M}_\nu \) is given as

\[
\hat{M}_\nu = -\frac{v^2}{M_0} \begin{pmatrix} \frac{3}{2} (e + f)^2 & 0 & \frac{\sqrt{3}}{2} (e^2 - f^2) \\ 0 & 3b^2 p & \sqrt{3} (e^2 - f^2) \\ \frac{\sqrt{3}}{2} (e^2 - f^2) & 0 & \frac{1}{2} (e - f)^2 \end{pmatrix}. \quad (59)
\]

The mass eigenvalue \( m_1 \) or \( m_3 \) vanishes for NH or IH, respectively.
B 2 − 3 flavor mixing \( \mathcal{V} \)

The explicit forms of the 2 − 3 flavor mixing \( \mathcal{V} \) in Eq. (5) are given as follows. For case I:

\[
\mathcal{V} = \frac{f^2v^4}{M_0^2} \sqrt{6}[(k^2 - 1)(5k^2 + 2k + 5) + 8B^2 p e^{2i\phi_B} M_0^2 + 3f_{\text{fix}}^4(k + 1)^2(5k^2 + 2k + 5)]
\]

(60)

For case II:

\[
\mathcal{V} = -\frac{f^2v^4}{M_0^2} \frac{\sqrt{6}[(k^2 - 1)(5k^2 + 2k + 5) + 5B^4 p^2 + 2B^2 p(5k + 1)(k \cos 2\phi_B + i \sin 2\phi_B)]}{16m_3^2 - 3f_{\text{fix}}^4 M_0^2}
\]

(61)

For case III:

\[
\mathcal{V} = -\frac{f^2v^4}{M_0^2} \frac{\sqrt{6}[(k^2 - 1)(5k^2 + 2k + 5) - 5B^4 p^2 - 2B^2 p(k + 5)(\cos 2\phi_B + ik \sin 2\phi_B)]}{16m_3^2 - 3f_{\text{fix}}^4 M_0^2}
\]

(62)

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