An Analytic Model for the Subgalactic Matter Power Spectrum in Fuzzy Dark Matter Halos

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Abstract

Fuzzy dark matter (FDM), a scalar particle coupled to the gravitational field without self-interaction, whose mass range is \( m \sim 10^{-24} \sim 10^{-20} \text{ eV} \), is one of the promising alternative dark matter candidates to cold dark matter. The quantum interference pattern, which is a unique structure of FDM, can be seen in halos in cosmological FDM simulations. In this paper, we first provide an analytic model of the subgalactic matter power spectrum originating from quantum clumps in FDM halos, in which the density distribution of the FDM is expressed by a superposition of quantum clumps whose size corresponds to the de Broglie wavelength of the FDM. These clumps are assumed to be distributed randomly, such that the ensemble average density follows a halo profile such as the Navarro–Frenk–White profile. We then compare the convergence power spectrum projected along the line of sight around the Einstein radius, which is converted from the subgalactic matter power spectrum, to that measured in the strong lens system SDSS J0252 + 0039. While we find that the current observation provides no useful constraint on the FDM mass, we show that future deep, high spatial resolution observations of strong lens systems can tightly constrain FDM with a mass around \( 10^{-22} \text{ eV} \).

1. Introduction

Dark matter is one of the major components in the universe, yet its nature is not fully understood. In the standard Λ-dominated cold dark matter (ΛCDM) cosmology, the energy density of the universe is composed of baryon (~5%), cold dark matter (CDM; ~25%), and dark energy (~70%), according to the cosmic microwave background (CMB) observations (Planck Collaboration et al. 2020) The ΛCDM model can successfully explain the large-scale structure of the universe. However, there are some discrepancies below the scale of ~1 Mpc between the CDM predictions and observations, which are often referred to as small-scale problems (e.g., Bullock & Boylan-Kolchin 2017; Del Popolo & Le Delliou 2017 for a review). These include the core–cusp problem (e.g., McGaugh et al. 2001), the diversity problem (Oman et al. 2015), the missing satellite problem (e.g., Klypin et al. 1999), and the too-big-to-fail problem (e.g., Boylan-Kolchin et al. 2011). It is not yet clear whether these discrepancies on small scales in the ΛCDM universe originate from baryonic feedback, such as supernova explosions, or the unknown nature of dark matter, or even both. While the small-scale crisis might be resolved by including baryonic processes, the possibility of resolving the crisis by changing the nature of dark matter has also been extensively studied.

Fuzzy dark matter (FDM; Hu et al. 2000) is one of the promising dark matter candidates that might resolve both the core–cusp problem and the missing satellite problem. It is a scalar particle coupled only to gravity. The typical mass is around \( 10^{-22} \text{ eV} \). Such a small mass results in its de Broglie wavelength being \( O(1) \text{ kpc} \), which is an important scale for small-scale problems. At the scale below the de Broglie wavelength, its wavelike nature can be seen, while FDM behaves similarly to CDM at the larger scale. FDM simulations reveal the nature of FDM halos (Schive et al. 2014a). The inner region consists of a core, whose size is about the de Broglie wavelength of FDM, while the outer region follows the Navarro–Frenk–White profile. In the FDM halos, granular structures can also be seen, which is a distinctive feature caused by the wavelike nature of FDM. Throughout the paper, we call such granular structures quantum clumps.

There are currently several constraints on the FDM mass range (see Ferreira (2020) for a review). The constraints are obtained from the CMB power spectrum (Hlozek et al. 2015; Hložek et al. 2018) and the Ly forests (Armena et al. 2017; Irišić et al. 2017; Nori et al. 2019; Rogers & Peiris 2021), which provide the most stringent constraints. These are the constraints from observations at the scale of \( k \gtrsim 10^{-2} \text{ Mpc}^{-1} \), suggesting that they do not result directly from the small scale. Future observations of the 21 cm line of neutral hydrogen can reach below that scale (Lidz & Hui 2018; Schneider 2018; Jones et al. 2021), and can tightly constrain FDM. There are many other constraints from various observations, such as the subhalo mass function (Benito et al. 2020; Schutz 2020), dynamical friction (Lancaster et al. 2020), the Milky Way (Church et al. 2019), dwarf spheroidals (Chen et al. 2017; González-Morales et al. 2017), dwarf satellites (Safarzadeh & Spergel 2020), ultra-faint dwarf spheroidals (Hayashi et al. 2021), and rotation curves (Maleki et al. 2020). Some of these constraints exclude the most interesting FDM mass range, around \( 10^{-22} \text{ eV} \). However, since these constraints suffer from
systematic uncertainties, mainly originating from baryon physics, it is important to derive constraints using several different approaches in order to cross-check any possible systematic effects.

Recently, a new framework for studying the subgalactic structure has been proposed. Hezaveh et al. (2016) constructed a framework to estimate the projected matter power spectrum by using strong gravitational lensing. This method of estimating the subgalactic convergence power spectrum from substructures in the lens plane is studied in Diaz Rivero et al. (2018). Bayer et al. (2018) applied these methods to the strong lens system SDSS J0252+0039, which is a galaxy–galaxy strong lens system found in the Sloan Lens ACS Survey (SLACS; Auger et al. 2009). They obtained constraints on the subgalactic matter power spectrum, and discussed the implications for the CDM model. We expect that this method can also be applied to the FDM model. Since an FDM halo is filled with quantum clumps, the FDM model should predict the much higher amplitude of the subgalactic matter power spectrum on the scale of the size of such quantum clumps.

In this paper, we first construct an analytic model for the subgalactic matter power spectrum in FDM halos. We assume that the FDM halos consist of quantum clumps whose sizes are fixed to the de Broglie wavelength of FDM. These clumps are assumed to be distributed randomly, such that the ensemble average of the FDM density reduces to the assumed halo profile. We also include baryon components that are assumed to be independent of the FDM component. We then calculate the subgalactic matter power spectrum under these assumptions. Finally, we compare our model with the constraints from SDSS J0252+0039. Note that Chan et al. (2020) have pointed out that multiple images and flux ratio anomalies in strong lens systems can be observed more often if the halo is composed of FDM. Our work is complementary to their work, in that we focus on the subgalactic power spectrum rather than flux ratio anomalies.

This paper is organized as follows. In Section 2, we present our model for calculating the subgalactic matter power spectrum in FDM halos. We show the dependence of parameters on the subgalactic matter power spectrum. In Section 3, we compare our model with current observational data obtained by a strong gravitational lens system. In addition, we show that future observations can constrain the interesting range of the FDM mass. Finally, we conclude in Section 4.

2. The Subgalactic Matter Power Spectrum in FDM Halos

The properties of FDM halos have been studied by FDM simulations (e.g., Schive et al. 2014a). The simulations reveal a pervasive modulation of the density field on the scale of the de Broglie wavelength, which can be naturally approximated by FDM halos being filled with quantum clumps of sizes that are roughly the de Broglie wavelength of FDM. These clumps produce large matter fluctuations on the scale of the de Broglie wavelength. In order to evaluate these fluctuations, we construct an analytic model of the subgalactic matter power spectrum in FDM halos, including baryon. In our model, FDM halos are assumed to consist of quantum clumps. They are distributed randomly, while the ensemble average density reduces to a specific dark matter profile (e.g., the NFW profile of Navarro et al. (1997)). The density profile of baryon is assumed to be independent of FDM, and to follow a specific baryon profile (e.g., the Hernquist profile of Hernquist (1990)).

In Section 2.1, we give a formulation for the FDM-only case. We then include the baryon profile in Section 2.2. In Section 2.3, we present the results where the dark matter and baryon profiles follow the NFW and Hernquist profiles, respectively, and we discuss the parameter dependence of the power spectrum.

2.1. The FDM-only Case

The distribution of FDM in a halo is determined by quantum clumps. We assume that the mass of each clump $M_c$ at $r'$ is determined from an average local density of the halo $\rho_h(r')$ as

$$M_c(r') = \rho_h(r') V_c,$$

where

$$V_c = \frac{4}{3} \pi \left( \frac{\lambda_c}{2} \right)^3,$$

is the volume of each clump, whose radius is assumed to be given by half of the de Broglie wavelength $\lambda_c = 2 \pi h / m v$, with $m$ being the FDM mass and $v$ being its velocity dispersion. Here, we assume that $v$ can be approximated as a constant value within a halo, which indicates that $V_c$ is constant in our formalism.

Inside each clump, the density profile $\rho_c(r)$ can be described using the normalized mass profile function $u(r - r')$ around the center of the clump at $r'$ as

$$\rho_c(r; r') = M_c(r') u(r - r').$$

The normalization condition is

$$\int_V d^3r \ u(r - r') = \int_{V(r')} d^3r' \ u(r - r') = 1,$$

where $V$ is the total volume of the halo and $V_c(r')$ is a three-dimensional sphere around the point $r'$, which is sufficiently small compared to the size of the halo, but larger than the size of each clump. As discussed later, in this paper we assume a smoothly truncated normalized mass density profile, such that its tail extends much beyond the de Broglie wavelength. This assumption leads to overlaps of FDM mass densities between neighboring clumps.

We assume that these quantum clumps are randomly distributed on the small scale, while the ensemble average of the number density is fixed. Since we assume that the masses of these clumps depend on their location (Equation (1)), the ensemble average of randomly distributed clumps can yield nonuniform density distributions within a halo, such as the NFW density profile, as we discuss later in more detail. The density profile of the FDM halo $\rho(r)$ can be expressed by a superposition of these randomly distributed clumps. Under this assumption, the FDM halo profile is given by

$$\rho_f(r) = \int_V d^3r' \rho_f(r; r') n(r')$$

$$= \int_V d^3r' \rho_h(r') V_c n(r') u(r - r'),$$

where $n(r)$ represents the number density of the center of the quantum clumps. Supposing that each clump is indexed by $j$ and its center by $r_j$, we can rewrite $n(r')$ in terms of the Dirac
The Fourier transform of the density fluctuation is

\[ \tilde{\delta_k} \equiv \int_{S_c} d^2x \, \delta(x) e^{-ik \cdot x} \]
\[ = \frac{1}{\Sigma_h(x)} \int_{S_c \times Z} d^3r \rho_h(r') V_c n(r') \]
\[ \times \int_{S_c \times Z} d^3r' u(r - r') e^{-ik \cdot r'} |_{k_z = 0} \]
\[ = \frac{V_c}{\Sigma_h(x)} \bar{u}_k \int_{S_c \times Z} d^3r \rho_h(r') \]
\[ \times n(r') e^{-ik \cdot r'} |_{k_z = 0}, \]
(12)

where \( k \) represents the two-dimensional wavenumber of the fluctuation and \( K \) is the three-dimensional wavenumber that is related to \( k \), as \( K_x = k_x, \) \( K_y = k_y, \) and \( K_z = 0 \). Note that we ignore the last term in Equation (11), since we are not interested in the fluctuation with \( k = 0 \). We also assume that \( Z \) is sufficiently large compared to the extent of the normalized mass profile function \( u(r) \). The Fourier transform of this function is denoted as \( \bar{u}_k \), and is related to \( \bar{u}_k \) as

\[ \bar{u}_k \equiv \bar{u}_k |_{k_z = 0} \]
(13)

The definition of the subgalactic matter power spectrum is

\[ \langle \tilde{\delta}_k \tilde{\delta}_{k'} \rangle \equiv S_c \delta^{(2)}_{k + k', 0} P(k). \]
(14)

The left-hand side of Equation (14) can be calculated by substituting Equation (12) as

\[ \langle \tilde{\delta}_k \tilde{\delta}_{k'} \rangle \]
\[ = \left( \frac{V_c}{\Sigma_h(x)} \right)^2 \bar{u}_k \bar{u}_{k'} \]
\[ \times \int_{S_c \times Z} d^3r \int_{S_c \times Z} d^3r' (n(r) n(r')) \]
\[ \times \rho_h(r) \rho_h(r') e^{-ik \cdot r} e^{-ik' \cdot r'} |_{k_z = 0, k_z' = 0} \]
\[ = \left( \frac{V_c}{\Sigma_h(x)} \right)^2 \bar{u}_k \bar{u}_{k'} \int_{S_c \times Z} d^3r \rho_h^2(r) \]
\[ \times e^{-i(k + k') \cdot r} |_{k_z = 0, k_z' = 0} \]
\[ = \left( \frac{V_c}{\Sigma_h(x)} \right)^2 \bar{u}_k \bar{u}_{k'} \int_{S_c \times Z} d^3r \rho_h^2(r) \int_{S_c} d^2x \ e^{-i(k + k') \cdot x} \]
\[ \times \left( \frac{V_c}{\bar{u}_k} \right)^2 | \rho_h^2(r) | \int_{S_c} d^2x \ e^{-i(k + k') \cdot x} \]
\[ = S_c \delta^{(2)}_{k + k', 0} \left( \frac{V_c}{\Sigma_h(x)} \right)^2 \int_{S_c} d^2x \ e^{-i(k + k') \cdot x} \]
(15)

In the second equality, we assume there is no correlation between the number densities at different positions, \( \langle n(r) n(r') \rangle = \delta^{(2)}(r - r') \). From Equation (14) and Equation (15), we can finally obtain the subgalactic matter power spectrum in FDM halos,

\[ P_l(k) \equiv \frac{V_c}{r_h(x)} | \bar{u}_k |^2, \]
(16)

where the effective halo size \( r_h(x) \) has a unit of length and is given by

\[ r_h(x) \equiv \left( \frac{\int_{S_c} d^2x \, \rho_h^2(r)}{\int_{S_c} d^2x \, \rho_h^2(r)} \right)^{1/2}. \]
(17)
Here we add a subscript \( f \) on \( P(k) \), since we are only considering the FDM component. Assuming spherically symmetric halos and matter profile functions of each clump, Equation (16) can be further simplified as

\[
P_f(k) = \frac{V_z}{r_h(x)} |\overline{\rho}_h|, \tag{18}
\]

\[
r_h(x) = \frac{\Sigma_h^2(x)}{\int_Z dz \rho_h(r)} = \left( \int_Z dz \rho_h(r) \right)^2 \tag{19}
\]

While the clumps in FDM simulations appear to be nonspherical (e.g., Schive et al. 2014a), we assume them to be spherical for simplicity. The validity of this assumption in our model is discussed in Section 2.3.

Here, we show that the effective halo size \( r_h(x) \) contains the information for the density dispersion along the line of sight. We can rewrite Equation (19) as

\[
r_h(x) = Z \cdot \frac{\left( \int_Z dz \rho_h(r) \right)^2}{\int_Z dz \rho_h^2(r)} = Z \cdot \frac{\overline{\rho}_h^2(x)}{\overline{\rho}_h^2(x) + s^2(x)}, \tag{20}
\]

where \( \overline{\rho}_h(x) \) represents the average halo density along the line of sight and \( s^2(x) \) represents the density dispersion, which are given by

\[
\overline{\rho}_h(x) \equiv \frac{1}{Z} \int_Z dz \rho_h(r), \tag{21}
\]

\[
s^2(x) \equiv \frac{1}{Z} \int_Z dz \rho_h^2(r) - \left( \frac{1}{Z} \int_Z dz \rho_h(r) \right)^2. \tag{22}
\]

Figure 1 shows an example of \( r_h(x) \), assuming an NFW profile as the halo density profile. It is seen that \( r_h(x) \) monotonically increases around the central region, while it monotonically decreases in the outer region. This behavior can be understood with Equation (20). Around the central region, \( s(x) \) determines the increase/decrease of \( r_h(x) \), since the density dispersion along the line of sight is large. In the outer region, the halo size along the line of sight \( Z \) determines the shape of \( r_h(x) \).

\[\text{Figure 1.}\]

2.2. Including Baryon

In Section 2.1, we describe the subgalactic matter power spectrum of FDM-only halos. Since most of the halos contain baryon, however, we also need to consider a baryon profile. We assume that baryon is smoothly distributed with a smooth density profile function \( \rho_b(r) \). The total density \( \rho(r) \) is

\[
\rho(r) = \rho_f(r) + \rho_b(r). \tag{23}
\]

The total projected density \( \Sigma(x) \) is

\[
\Sigma(x) = \Sigma_f(x) + \Sigma_h(x), \tag{24}
\]

where \( \Sigma_b(x) \) is defined as

\[
\Sigma_b(x) = \int_Z dz \rho_b(r). \tag{25}
\]

Since we assume that the baryon component does not contain any randomness, the ensemble averaging of the baryon functions does not change their functional form. We repeat the calculation from Section 2.1 in order to obtain the subgalactic matter power spectrum with baryon,

\[
P(k) = \left( \frac{\Sigma_h(x) + \Sigma_b(x)}{\Sigma_h(x)} \right)^2 P_f(k). \tag{26}
\]

Equation (26) indicates that the power spectrum with baryon is smaller than that without baryon, because the additional contribution of the smooth baryon component smears out the density fluctuations due to FDM.

2.3. Parameter Dependence

We calculate the power spectrum using Equation (26) with specific functions. The halo and baryon profiles are set to the NFW and Hernquist profiles, respectively. Although the NFW profile has two parameters, the total halo mass and concentration parameter, it is known that there is a scaling relation between them (e.g., Ishiyama et al. 2021). Assuming that relation, we need only one parameter, the total halo mass denoted by \( M_h \). The calculations are conducted using the python module COLOSSUS (Diemer 2018). The Hernquist profile has two parameters, the total stellar mass and characteristic radius. The empirical relation between them is known by fitting a sample of 50,000 early-type galaxies (Hyde & Bernardi 2009). We thus use the single parameter \( M_* \) to determine the Hernquist profile. Note that we use the stellar-to-halo mass ratio \( M_*/M_h \) as a parameter instead of the stellar mass \( M_* \). For sufficiently high-mass halos with masses larger than about \( 10^{11} M_\odot \), the stellar-to-halo mass ratio in the FDM model is expected to be the same as the ratio in the CDM model (Cristofari & Ostriker 2019), and is known to be around \( 10^{-3}-10^{-1} \) (Wechsler & Tinker 2018) in this halo mass range.

As mentioned in Section 2.1, the soliton core is ignored in our model. Therefore, our model is not valid around the central region of FDM halos. According to Schive et al. (2014a), the transition radius between the soliton core and NFW profile is around \( 3 r_c \), where \( r_c \) is the core radius that is known to scale with the FDM mass as well as the total halo mass (Schive et al. 2014b). Since we focus on Einstein radii in strong lens systems...
that are typically larger than the transit radius, unless the FDM mass is too small (see also the captions to Figures 2 and 3), we can compare our model with strong lens observations, as we attempt in Section 3. We also note that our model is likely to be invalid around and beyond the virial radius, as we do not include the dark matter distribution outside the virial radius.

The normalized mass profile function $u(r - r')$ is assumed to be a spherical Gaussian function whose radial variance equals half of the de Broglie wavelength. These assumptions of sphericity and the Gaussian radial mass profile are consistent with the findings in Dalal et al. (2021), in which the FDM halo structure derived using the method proposed in Widrow & Kaiser (1993) is found to be well described by a superposition of randomly distributed spherical Gaussian clumps. The Fourier transform of this function in the projected field is

$$\tilde{u}_k = \tilde{u}_k = \exp\left(-\frac{\lambda_c^2 k^2}{8}\right).$$

(27)

In order to calculate the de Broglie wavelength $\lambda_c = 2\pi h/mv$, we set $v$ as

$$v = \sqrt{\frac{3GM_{\text{tot}}}{2R_{\text{vir}}}},$$

(28)

where $G$ is the gravitational constant, $M_{\text{tot}}$ is the total mass that is the sum of the halo mass $M_h$ and the stellar mass $M_\ast$, and $R_{\text{vir}}$ is the virial radius of the halo. Note that $v$ is assumed to be constant within each halo throughout the paper. An additional parameter for calculating $\lambda_c$ is FDM mass $m$. Finally, the position $x$ in the projected field is needed to calculate the power spectrum.

To sum up, four parameters are required to calculate the subgalactic matter power spectrum in FDM halos: the total halo mass $M_h$, the stellar-to-halo mass ratio $M_\ast/M_h$, the FDM mass $m$, and the position $x$. By substituting the normalized mass profile function in Equation (27), Equation (26) becomes

$$P(k) = \left(\frac{\Sigma_h(x)}{\Sigma_h(x) + \Sigma_b(x)}\right)^2 \frac{4\pi \lambda_c^2}{3r_b(x)} \exp\left(-\frac{\lambda_c^2 k^2}{4}\right).$$

(29)

With this model, we show the parameter dependence, especially that of the total halo mass $M_h$ and the FDM mass $m$.

Figure 2 shows the total halo mass dependence. It is found that the power spectrum damps at larger wavenumbers with larger total halo masses. In addition, the amplitude of the plateau region is smaller with larger total halo masses. Since the FDM mass and the de Broglie wavelength scales as $\lambda_c \propto M_h^{-1/3}$, therefore, the damping scale is different for different total halo masses, even if the FDM mass $m$ is fixed. The latter result can be understood as follows. In the plateau region, $P(k) \propto \lambda_c^4/r_b(x) \propto M_h^{-4/3}$, if we approximate $r_b(x) \approx R_{\text{vir}}$.

These results can also be understood qualitatively, as follows. The variance in real space is obtained by

$$\sigma^2 \sim \int d^3k \ P(k) \sim \mathcal{O}\left(\frac{\lambda_c}{r_b}\right) \sim \mathcal{O}\left(\frac{1}{N}\right).$$

(30)

where $N \sim r_b/\lambda_c$ is the number of clumps along the line of sight. From Equation (30), the fluctuation along the line of sight can be approximated as $\mathcal{O}(1/\sqrt{N})$, which is consistent with the naive picture that $\mathcal{O}(1)$ fluctuations of individual clumps are averaged out by $N$ clumps along the line of sight. As the total halo mass becomes larger, the virial radius also becomes larger, and the number of quantum clumps along the line of sight increases. The large number of clumps along the line of sight results in the smaller amplitude of the power spectrum due to averaging.

Figure 3 shows the FDM mass dependence. It is found that the power spectrum damps at larger wavenumbers, and the amplitude in the plateau region is smaller with larger FDM masses. Since the FDM mass and the de Broglie wavelength are related to each other by $\lambda_c \propto m^{-1}$, the power spectrum in the plateau region is proportional to $P(k) \propto m^{-3}$, and the damping scale is $k \propto m$. These results can be understood in the same way as discussed above. As the FDM mass becomes larger, the de Broglie wavelength becomes smaller, leading to a larger number of clumps along the line of sight, and the lower
amplitude of the power spectrum. Since the subgalactic matter power spectrum is sensitive to FDM mass, it can be used to constrain the mass range of FDM.

In addition, Figure 1 in Hezaveh et al. (2016) shows that the subgalactic power spectrum due to CDM subhalos is around \(10^{-6} \, h^{-2} \, \text{kpc}^{-1}\) in the small wavenumber limit, which is much smaller than the subgalactic matter power spectrum in the FDM model in most cases of interest (see also Chan et al. 2020). This suggests that we can obtain interesting constraints on the FDM mass around the typical range from the observations of the subgalactic matter power spectrum, which we discuss in Section 3.

3. Comparison with Observations

Using the formalism that we describe in Section 2, we compare the subgalactic matter power spectrum with real observational data in order to constrain the range of the FDM mass. We first use the current constraints on the subgalactic matter power spectrum (Bayer et al. 2018) that are obtained from the SLACS strong lens system SDSS J0252 + 0039 (Auger et al. 2009). We then discuss the future prospects of constraints obtained from strong lens systems (Hezaveh et al. 2016). In Section 3.1, we define the dimensionless convergence power spectrum. We present a comparison with the current data in Section 3.2, and we discuss the future prospects in Section 3.3.

3.1. Dimensionless Convergence Power Spectrum

We use the dimensionless convergence power spectrum described below when we compare our model with the observations. Consider a lens system with the angular diameter distance from the observer to the lens \(D_\text{ls}\), from the observer to the source \(D_\text{s}\), and from the lens to the source \(D_\text{ds}\). The critical surface mass density \(\Sigma_\text{cr}\) for this lens system is given by

\[
\Sigma_\text{cr} = \frac{1}{4\pi G} \frac{D_\text{ls}}{D_\text{ds}}. \tag{31}
\]

With this critical surface mass density and the projected density field (Equation (24)), we can define a convergence field \(\kappa(x)\) as

\[
\kappa(x) = \frac{\Sigma(x)}{\Sigma_\text{cr}}. \tag{32}
\]

The convergence power spectrum is defined as

\[
\langle \kappa_{k} \kappa_{k'} \rangle \equiv S_\kappa(2k) P_\kappa(k), \tag{33}
\]

where \(\kappa\) is the two-dimensional Fourier transform of the convergence field. Additionally, we define the dimensionless convergence power spectrum that we use for the comparison with the observations as

\[
\Delta_\kappa^2(k) = 2\pi k^2 P_\kappa(k). \tag{34}
\]

From this relation, we can relate the convergence power spectrum and the subgalactic power spectrum in Section 2 as

\[
\Delta_\kappa^2(k) = 2\pi k^2 \left( \frac{\Sigma(x)}{\Sigma_\text{cr}} \right)^2 P(k). \tag{35}
\]

We use this relation to compare the observational results with those of the dimensional convergence power spectrum.

3.2. Current Observations

From Tables 3 and 4 in Auger et al. (2009), we can obtain the information about SDSS J0252 + 0039. The redshifts of the lens and the source are \(z_\text{lens} = 0.280\) and \(z_\text{src} = 0.982\), respectively, from which we obtain \(\Sigma_\text{cr} = 4.0 \times 10^{-9} \, h \, \text{M}_\odot\, \text{kpc}^{-1}\). In order to calculate the dimensionless convergence power spectrum in our model and constrain the FDM mass, we need to determine the NFW profile and the Hernquist profile and its position. We adopt the Salpeter stellar mass \(M_\text{Salp} = 2.0 \times 10^{11} \, h^{-1} \, \text{M}_\odot\) and the scale radius \(a = 2.34 \, h^{-1} \, \text{kpc}\) with the position being set to the observed Einstein radius, \(x = 3.1 \, h^{-1} \, \text{kpc}\). We vary the FDM mass in the range \(m = 10^{-22} \, \text{eV}\) to \(10^{-21} \, \text{eV}\).

3.3. Future Constraints

The black region in Figure 4 shows the excluded region of the subgalactic matter power spectrum at the 68% confidence level expected to be obtained by future ALMA observations of strong lens systems (Hezaveh et al. 2016). We find that the FDM mass with \(m \approx 3.2 \times 10^{-22} \, \text{eV}\) can be excluded by future
ALMA observations of SDSS J0252 + 0039-like strong lens systems if no significant signals of the subgalactic matter power spectrum are measured by such future observations. Note that we can also obtain the lower bound of the FDM mass.

We now discuss the relation between the excluded FDM mass range and the dimensionless convergence power spectrum to be probed by the future observations of SDSS J0252 + 0039-like strong lens systems. Figure 5 shows the relation between the excluded mass range and the future constraints on the dimensionless convergence power spectrum on 3 kpc. It is found that the interesting mass range of the FDM can be constrained by the future observations. Here, we show the constraints obtained from the 3 kpc wavelength, although we can use other wavelengths as well. Given the FDM mass dependence shown in Figure 4, we can constrain the lower masses if we use higher wavelengths or lower wavenumbers, and the higher masses if we use lower wavelengths or higher wavenumbers.

4. Conclusion

In this paper, we first provide an analytic model for the subgalactic matter power spectrum in FDM halos, assuming that the distribution of FDM in halos is described by a superposition of quantum clumps whose size is comparable to the de Broglie wavelength of FDM. We derive the power spectrum projected along the line of sight that can be directly interpreted as the signal only in the context of the standard CDM model. We adopt the mass model parameters of MG0414 + 0534, as shown in Oguri et al. (2014), to repeat the calculation in this paper, and we find that the quantum clumps in the FDM model with a mass of \( m \approx 4 \times 10^{-23} \text{eV} \) well explain the observed signal. However, one caveat is that MG0414 + 0534 has been known as a quasar lens system, exhibiting significant flux ratio anomalies as well as strong perturbations on the lens potential due to a satellite galaxy (e.g., Minezaki et al. 2009), which suggests that the measured subgalactic matter power spectrum might be biased high. The high source redshift of \( z = 2.64 \) also implies that the effects of the line-of-sight structure may be more important than they are for SDSS J0252 + 0039.

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