Bearing Fault Diagnosis Based on Empirical Wavelet Transform and Singular Value Decomposition

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Abstract. Aiming at the problem of noise in the signal in bearing fault diagnosis, a diagnosis method based on empirical wavelet transform and singular value decomposition was proposed. Firstly, the vibration signal of the fault bearing outer ring was decomposed by empirical wavelet transform, and then the signal was reconstructed by singular value decomposition. The combination of the two not only decomposed the frequencies contained in the respective components, but also eliminated noise components and had high reliability in analyzing signals. The spectrum of the signal was analyzed to identify the fault signal in the Hilbert spectrum and compared to the theoretically calculated frequency. The experimental results show that the method can accurately determine bearing faults.

Introduction

The bearing is the core component of the motor drive equipment, which functions to reduce friction and transmit load. Therefore, the diagnosis of bearing faults in motor diagnosis is particularly important [1]. In practical applications, factors such as interference from environmental noise and complex vibration transmission paths of the detection machine make early failure signals more difficult. Therefore, effectively analyzing the fault signal is the key to bearing fault diagnosis. In order to overcome the problem of noise in the signal, many test methods have been proposed, and the wavelet transform method is widely used. Gilles [2] proposed an empirical wavelet transform (Abbreviation EWT) to divide the spectrum of the signal, and constructs an orthogonal bandpass filter bank in different partitions through the corresponding bandpass filter to extract the amplitude modulation and frequency modulation components. Some scholars compared the empirical wavelet transform with empirical mode decomposition (Abbreviation EMD) and ensemble empirical mode decomposition (Abbreviation EEMD) to verify the validity of wavelet transform [3-5]. Wavelet transform has good time domain and frequency domain localization characteristics, which can realize the separation of signals in different frequency bands and at different times [4-7]. Modal aliasing of the test signal still exists due to the presence of the same frequency signal in the tester signal and the environment.

In this paper, empirical wavelet transform [8,9] and singular value decomposition [10,11] are combined to analyze the spectrum of the signal and applied to the fault diagnosis of bearings.

Empirical Wavelet Transform

The empirical wavelet transform is to use the spectral information of the original signal to obtain the form of the required bandpass filter, so as to adaptively segment the Fourier spectrum of the original signal and construct a bandpass filter adapted to process the original signal. First, the original signal Fourier spectrum is divided, and the frequency of the original signal Fourier spectrum is assumed to be \( \omega(\omega \in (0,\pi)) \). Considering the number of maxima to be determined, the frequency domain is divided into \( N \) In a frequency band interval in which bandwidths are not equal, \( \omega_n \) is a boundary of each frequency band section. Assuming that these bands are \( \Lambda_n (\Lambda_n = [\omega_{n-1}, \omega_n]) \), then \( \bigcup_{n=1}^N \Lambda_n = [0,\pi] \). Assuming that the boundary bandwidth of each band section \( \tau_n \), it is said that \( \omega_n \) is the center angular frequency, and the area with the band width of \( 2\tau_n \) is the transition section. As shown in Fig. 1.
The empirical wavelet is the filter defined on each band $\Lambda_n$. The empirical wavelet transform uses the Meyer-type orthogonal wavelet basis construction theory to construct the wavelet basis function. The empirical scale function $\hat{\phi}_n(\omega)$ and the empirical wavelet function $\hat{\psi}_n$ are as shown in Eq.1 and Eq.2.

$\tau_n = \gamma \omega_n, 0 < \gamma < \min \frac{\omega_{n+1} - \omega_n}{\omega_{n+1} + \omega_n}, \beta(x) = x^4(35 - 84x + 70x^2 - 20x^2)$.

$$\hat{\phi}_n(\omega) = \left\{ \begin{array}{ll}
1, & |\omega| \leq (1 - \gamma)\omega_n \\
\cos \left[ \frac{\pi}{2} \beta \left( \frac{1}{2\gamma\omega_n} \right) (|\omega| - (1 - \gamma)\omega_n) \right], & (1 - \gamma)\omega_n \leq |\omega| \leq (1 + \gamma)\omega_n \\
0, & \text{otherwise}
\end{array} \right. (1)$$

$$\hat{\psi}_n(\omega) = \left\{ \begin{array}{ll}
1, & (1 + \gamma)\omega_n \leq |\omega| \leq (1 - \gamma)\omega_{n+1} \\
\cos \left[ \frac{\pi}{2} \beta \left( \frac{1}{2\gamma\omega_n} \right) (|\omega| - (1 - \gamma)\omega_{n+1}) \right], & (1 - \gamma)\omega_n \leq |\omega| \leq (1 + \gamma)\omega_{n+1} \\
\sin \left[ \frac{\pi}{2} \beta \left( \frac{1}{2\gamma\omega_n} \right) (|\omega| - (1 - \gamma)\omega_{n+1}) \right], & (1 - \gamma)\omega_n \leq |\omega| \leq (1 + \gamma)\omega_n \\
0, & \text{otherwise}
\end{array} \right. (2)$$

Assuming that the Fourier transform is $F(\cdot)$ and the inverse transform is $F^{-1}(\cdot)$, a similar method to the traditional wavelet transform is used to define the empirical wavelet transform. The approximation coefficient $W_0(n, t)$ and the detail coefficient $W_x(n, t)$ of the empirical wavelet transform are the inner product of the scale function of the signal $x(t)$ and the wavelet function, respectively. As shown in Eq.3 and Eq.4. Reconstructed signal in Eq.5.

$$W^x_x(n, t) = \langle x(t), \psi_n(t) \rangle = F^{-1} \left[ x(\omega) \hat{\psi}_n(\omega) \right] (3)$$

$$W^x_x(0, t) = \langle x(t), \phi_1(t) \rangle = F^{-1} \left[ x(\omega) \hat{\phi}_1(\omega) \right] (4)$$

Reconstructed signal:

$$x_c(t) = W^x_x(0, t) \ast \phi_1(t) + \sum_{n=1}^{N} W^x_x(n, t) \ast \psi_n(t) (5)$$

**Singular Value Decomposition**

Singular value decomposition is a kind of matrix orthogonal variation, which essentially reduces the rank of the matrix. The one-dimensional time series signal is sampled, assuming that $x(i)$ is a one-dimensional digital signal sequence ($i = 1, 2, \cdots, N$). The Hankel matrix is constructed with $x(i)$. The Hankel matrix has the following Eq. 6. $s \in R^{m \cdot n}; 1 \leq n < N, m = N - n + 1$.

$$s = \begin{bmatrix}
x(1) & x(2) & \cdots & x(n) \\
x(2) & x(3) & \cdots & x(n+1) \\
\vdots & \vdots & \ddots & \vdots \\
x(N-n+1) & x(N-n+2) & \cdots & x(N)
\end{bmatrix} (7)$$
Perform singular value decomposition on the Hankel matrix, so that

$$S = UAV^T$$  \hspace{1cm} (8)

As shown in Eq.8: U, V are orthogonal matrices; A is the diagonal matrix of m*n , which is $A = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$. The decomposition of the signal is realized by singular value decomposition, and the Eq.8 is changed to the Eq.9.

$$S = \lambda_1 u_1 v_1^T + \lambda_2 u_2 v_2^T + \cdots + \lambda_k u_k v_k^T$$  \hspace{1cm} (9)

Constructs a set of signals that are noise signals caused by useful signals, frequency overlap, and measurement machines.

$$s = s_1 + s_2$$  \hspace{1cm} (10)

In Eq.10, $s_1$ is a useful signal, and $s_2$ is a noise signal. When the singular value decomposition of the signal $s$ is constructed to construct the Hankel matrix $S$, it can be decomposed into Hankel matrices $S_1$ and $S_2$ respectively constructed by $s_1$ and $s_2$. That is Eq.11.

$$S = S_1 + S_2$$  \hspace{1cm} (11)

There is only one difference in data between rows and rows of the matrix $S_1$, and there is a high correlation between rows and rows, and the rank $r(S_1) \ll k$ of the matrix $S_1$. For the matrix $S_2$, although the row and the row are also different by one value, although the noise of the frequency overlap has a certain correlation, the measurement noises are independent of each other, and the noise between the measurement noise and the frequency is relatively independent. Thereby the matrix $S_2$ is a full rank matrix. Since there is a certain frequency band difference between the frequency of the useful signal and the noise frequency, the useful signal and noise can be discriminated by the singular value distribution law of the different frequency signals.

**Experimental Verification**

The actual signal is used to verify the effectiveness of the method. Using a university bearing test data, the test object is the SKF6205-2RS rolling bearing. The bearing speed is 1796r/min, the sampling period is 1s, and the sampling frequency is 12kHz. The bearing data parameters are shown in the following Table 1.

| name                                    | Numerical value |
|-----------------------------------------|-----------------|
| Rolling bearing diameter [mm]           | 7.940           |
| Section diameter [mm]                   | 39.039          |
| Number of rolling elements              | 9               |
| pressure angle                          | 0               |

Calculation formula for fault frequency of bearing outer ring $f_0$. As shown in Eq. 12.

$$f_0 = \frac{1}{2} Z \left( 1 - \frac{d}{D} \cos \alpha \right) f$$  \hspace{1cm} (12)

Combined with the type (12) and Table 1, the bearing outer ring fault frequency type 107.305Hz. The outer ring fault data time domain diagram is shown in Fig. 2. When the outer ring of the bearing is faulty, the fluctuation and amplitude of the bearing vibration are very large. Because the time domain contains noise, it is difficult to analyze the fault characteristics. Perform a Fourier transform on the signal to obtain the Fourier spectrum, as shown in Fig. 3.
In Fig. 3, it is difficult to find the bearing outer ring fault characteristic $f_0$ and its frequency multiplication in the Fourier spectrum, and it is not found that the sideband with the swing frequency is centered at $f_0$. Therefore, the bearing state cannot be judged from the Fourier spectrum.

The empirical wavelet transform is performed on the signal, and then the obtained useful signal is subjected to singular value decomposition and noise reduction processing. As shown in Fig. 4.

Finally, the Fourier transform is performed on the processed signal to obtain the Fourier spectrum. As shown in Fig. 5. In Fig. 5, the low-frequency component of the vibration signal is modulated by the high-frequency component, and the Fourier spectrum has only the high-frequency component, and the low-frequency component cannot be directly found therefrom. The Hilbert transform is performed on the processed signal to obtain an envelope spectrum. As shown in Fig. 6.

In Fig. 6, in the envelope spectrum of empirical wavelet transform and singular value decomposition processing, the bearing outer ring fault frequency, the bearing rotation frequency and
its multiplication frequency can be found, which is the same as the theoretical value. It can be determined that the fault is the bearing outer ring fault.

Conclusion
This paper proposes a method based on the combination of empirical wavelet transform and singular value decomposition for fault diagnosis. For the frequency aliasing problem in empirical wavelet decomposition, the singular value decomposition is introduced, and the spectrum map is analyzed to extract the fault characteristics. The combination of the two can not only decompose the frequencies contained in the respective components but also remove the noise components, and has high reliability in analyzing signals.

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