Analyses of characteristics of spatial lightning electromagnetic field at different horizontal distances and observation point heights

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Abstract. The analytic expressions of the spatial lightning electromagnetic field are obtained by solving Maxwell's Equations based on the dipole method. On this basis, the waveforms of the electric and magnetic fields at different heights and distances in the near-zone are presented using the transmission line (TL) model and the channel-base current was described by the impulse function. The result shows that at a same height to the ground, the rising edges of the electromagnetic field waveforms become subdued and the peak values decrease with the increase of the horizontal distance. For the electromagnetic field at the same distance, the rising edge of the waveform becomes steep gradually with the increasing height of the observation point. The peak value of the horizontal electric field increases with increasing height of the observation point and the peak value of the magnetic field basically remains unchanged. The study can provide useful information for protecting the electronic equipment from the destruction of LEMP.

1. Introduction
Lightning is a typical source of electromagnetic hazards in nature. When a lightning happens, the voltage of the lightning return-stroke channel may reach up to millions of volts, and the current in the channel may reach up to hundreds of kiloamperes, with a rise-rate of tens of kiloamperes per microsecond [1-2]. Such powerful transient current can produce a strong electromagnetic radiation around the lightning return stroke channel, which would generate a serious threat to the surrounding buildings and electronic equipments. Especially in the 20th century, with the rapid development of the electronic industry and information technology, the electromagnetic sensitivity of electronic equipment becomes very high and the capacity to resist the lightning electromagnetic interference gradually drops. Therefore, a very weak electromagnetic field is likely to cause hazard to the electronic equipment. The loss caused by lightning electromagnetic pulse (LEMP) gradually appears and increases year by year [3]. Since the spatial lightning electromagnetic field is difficult to be measured directly, we can utilize the lightning return-stroke engineering model [4-8] and conduct a theoretical calculation for the lightning electromagnetic field. M Rubinstein solved the Maxwell's equations of lightning electromagnetic field by using dipole method [9], and some scholars calculated the expressions of LEMP above the ground without considering the mirror current [10-13]. In this paper, the spatial lightning electromagnetic fields are calculated based on the TL model[14-15], and

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the distribution laws of the electromagnetic fields at different heights and distances in near-zone are then discussed.

2. Theoretical derivation of spatial lightning electromagnetic field
For a linear, homogeneous and single media, Maxwell's equations can be written as:

$$\nabla \cdot \varepsilon_0 E = \rho_v, \quad \nabla \cdot \mu_0 H = 0, \quad \nabla \times E = -\frac{\partial \mu_0 H}{\partial t}, \quad \nabla \times H = \mathbf{J} + \frac{\partial \varepsilon_0 E}{\partial t}$$

(1)

Figure 1. Computational model of lightning return stroke electromagnetic fields.

As shown in Figure 1, we simplify the discharge channel as a vertical antenna model with a height of $H$, assuming that the ground was an ideal conductor and the channel current above the return-stroke height is 0. Considering the current infinitesimal in the return-stroke channel as a current dipole, according to the dipole theory, we can obtain the integral expressions of lightning electromagnetic field in the cylindrical coordinate system as following:

$$E_z(r, \phi, z, t) = \frac{1}{4\pi\varepsilon_0} \int_0^H \left[ \frac{3r(z-z')}{R^3} \int_{-\infty}^{\infty} i(t - \frac{R}{c} \frac{d}{dt}) dt + \frac{3r(z-z')}{cR^4} \int_{-\infty}^{\infty} i(t - \frac{R}{c} \frac{d}{dt}) dt + \frac{r(z-z')}{c^2R^3} \int_{-\infty}^{\infty} i(t - \frac{R}{c} \frac{d}{dt}) dt \right] dz'$$

(2)

$$E_r(r, \phi, z, t) = \frac{1}{4\pi\varepsilon_0} \int_0^H \left[ \frac{2(z-z')^2 - r^2}{R^3} \int_{-\infty}^{\infty} i(t - \frac{R}{c} \frac{d}{dt}) dt + \frac{2(z-z')^2 - r^2}{cR^4} \int_{-\infty}^{\infty} i(t - \frac{R}{c} \frac{d}{dt}) dt + \frac{r^2}{c^2R^3} \int_{-\infty}^{\infty} i(t - \frac{R}{c} \frac{d}{dt}) dt \right] dz'$$

(3)

$$H(z, \phi, z, t) = \frac{1}{4\pi} \int_0^H \left[ \frac{r}{R^3} \int_{-\infty}^{\infty} i(t - \frac{R}{c} \frac{d}{dt}) dt - \frac{r}{cR^2} \frac{\partial i(t - \frac{R}{c} \frac{d}{dt}) dt}{\partial t} \right] dz'$$

(4)

Where $R = \left( (z-z')^2 + r^2 \right)^{1/2}$ is the distance between the current dipole and the observation point $P(r, \phi, z)$, $c$ is the velocity of light in free space, $\varepsilon_0$ and $\mu_0$ represent the vacuum permittivity and permeability respectively, $z'$ is the height of lightning current dipole. The first item, second item and third item at the right side of equations (2) and (3) are the electrostatic field, induction field and radiation field, respectively. The first item and the second item at right side of equation (4) are the induction field and radiation field, respectively.

Using the TL model to calculate electromagnetic field:

$$i(z', t) = u(t - z'/v) \cdot i(0, t - z'/v)$$

(5)

In process of solving equations (2)–(4), we will encounter the integral of function $i$ [16]. Considering that $g(x)$ and $q(x)$ are continuous functions and $g(x_0) = 0$, we can obtain:
\[ \int \delta'(g(x))g(x)dx = \int \delta(y)g^{-1}(y) \left. \frac{1}{dg/dx} \right|_{y=g(x)} dy = q(x) \]  

(6)

Applying (6) in (2)~(4), we can get:

\[ E_z(r, z, t) = I^+_0 \int \frac{1}{4\pi \varepsilon_0} \begin{bmatrix} r \left( \frac{1}{R_{ih}^2} - \frac{1}{R_0^2} \right) r^2 \left( \frac{1}{R_{zh}^2} - \frac{1}{R_z^2} \right) + \frac{r}{cR_{zh}} \frac{1}{\sqrt{v-(z-h)/(cR_{zh})}} \end{bmatrix} dr \]

(7)

\[ E_z(r, z, t) = I^-_0 \int \frac{1}{4\pi \varepsilon_0} \begin{bmatrix} r \left( \frac{1}{R_{ih}^2} - \frac{1}{R_0^2} \right) r^2 \left( \frac{1}{R_{zh}^2} - \frac{1}{R_z^2} \right) + \frac{r}{cR_{zh}} \frac{1}{\sqrt{v-(z+h)/(cR_{zh})}} \end{bmatrix} dr \]

(8)

\[ H_x(r, z, t) = I^+_0 \int \frac{1}{4\pi} \begin{bmatrix} \frac{z}{rR_{zh}} \frac{z-h}{z-h} + \frac{1}{rR_{zh}} \frac{1}{\sqrt{v-(z-h)/(cR_{zh})}} \end{bmatrix} dr \]

(9)

Where \( R_0 = (z^2 + r^2)^{1/2} \), \( R_{ih} = \left( (z-h)^2 + r^2 \right)^{1/2} \), and \( R_{zh} = \left( (z+h)^2 + r^2 \right)^{1/2} \). \( I^+_0 \) and \( I^-_0 \) represent the value of the current at the bottom of the channel at a time of \( t-RH/c \) and \( t-Rh/c \), respectively.

The value of \( h \) and \( h_z \) can be obtained by solving the following equation:

\[ t - \left( r^2 + (z-h)^2 \right)^{1/2} / c - h / v = 0, \quad t - \left( r^2 + (z+h_z)^2 \right)^{1/2} / c - h_z / v = 0 \]  

(10)

3. Distribution laws of lightning electromagnetic field at different horizontal distances

The distribution laws of lightning electromagnetic field at different horizontal distances with a same height to the ground are researched by using equations (7)~(9). The lightning current waveform of 8/20 \( \mu \)s is adopted to calculate the spatial lightning electromagnetic field, and the channel-base current is described by the pulse function [17] as following:

\[ i(t) = \frac{I_o}{\xi} \left[ 1 - \exp(-t/\tau_i) \right] \exp\left(-t/\tau_z\right) \]

(11)

Where \( \xi = [n\tau_i / (\tau_i + n\tau_z)]^{1/2} \left[ \tau_i / (\tau_i + n\tau_z) \right]^{1/2} \)

Hence, the lightning current parameters in the pulse function can be set as: \( I_o = 30 \) kA, \( n = 2 \), \( \tau_i = 4.05 \times 10^{-7} \) s, \( \tau_z = 6.8 \times 10^{-5} \) s.

Herein, the channel height \( H \) is 7.5 km and return stroke speed \( v \) is 1.3 \times 10^8 \) m s\(^{-1}\). At the height of \( z = 50 \) m for the observation point, the spatial distribution of the electric and magnetic fields at different horizontal distances are shown in Figures 2~ 4.

As can be seen in Figure 2, there is a positive peak value for the vertical electric field when the height of the observation point is 50 m and the horizontal distance is smaller than 50 m. This peak value will decrease with the increase of the distance and forms a zero-crossing point with the time axis. When the horizontal distance is larger than 50 m, the positive peak value will disappear and the negative peak value will decrease with the increase of the horizontal distance. Moreover, the peak value of the vertical electric field is about 109 kV m\(^{-1}\) at \( r = 5 \) m, and the peak value is only 5.9 kV m\(^{-1}\) when \( r = 500 \) m, with an attenuation of almost 20 times.
Figure 2. Vertical electric field $E_z$ at different horizontal distances when $z=50$ m.

Figure 3. Horizontal electric field $E_r$ at different horizontal distances when $z=50$ m.

Figure 4. Magnetic field $H_\phi$ at different horizontal distances when $z=50$ m.
From Figures 3 and 4, we can find out that the rising edges of the horizontal electric field and magnetic field become gentle with the increase of the horizontal distance. Moreover, the peak value of the field decreases gradually with the increase of the horizontal distance. The peak value at the distance of \(r=500\) m have attenuated about a hundred times than that at the distance of \(r=5\) m.

4. Distribution laws of lightning electromagnetic field at different heights

We keep the parameters of the channel-base current unchanged. Assuming that the horizontal distance from the field point to the source point is \(r=500\) m, the waveforms of lightning electromagnetic field at different heights with a same horizontal distance are shown in Figures 5–7 by using equations (7)–(9), respectively.

Figure 5 shows that when the height of the observation point is less than 80 m, the waveforms of the vertical electromagnetic field at different heights are basically consistent. When the height of the observation point is larger than 80 m, the peak value of the electric field will increase with the increase of height and the rising edge will become steep gradually. When the observation height is larger than 1,000 m, the polarity of the electric field will reverse and the value of electric field after the initial peak will decrease soon and decay to zero in tens of microseconds. As shown in Figure 6, the rising edge and falling edge of the horizontal electric field waveform will become steeper and steeper, and the peak value will increase gradually with the increase of height. As shown in Figure 7, we can conclude that the peak value and falling edge of the magnetic field waveform are nearly unchanged.

![Figure 5. Vertical electric field \(E_z\) at different observation heights when \(r=500\) m.](image-url)
Figure 6. Horizontal electric field $E_r$ at different observation heights when $r=500$ m.

Figure 7. Magnetic field $H_\Phi$ at different observation heights when $r=500$ m.

5. Conclusions

In this paper, the analytic expressions of the spatial lightning electromagnetic field are obtained by using the transmission line (TL) model and the channel-base current is described by the impulse function based on dipole method. On this basis, the spatial lightning electromagnetic field at different observation heights and horizontal distances are calculated. The result shows that:

1) For the electric and magnetic fields at a same height above the ground, the rising edges of the electric and magnetic fields become gentle with the increase of the horizontal distance, and the peak value reduces sharply with the increase of the horizontal distance. These above result shows that the lightning electromagnetic field is much stronger for the observation point which is much closer to the
lightning channel. Thus, the threat of this field to sensitive electronic equipments and systems will also be more serious.

2) For the electric and magnetic fields at a same horizontal distance from the channel, the rising edges of the electric and magnetic fields become steep gradually with increasing height of the observation points. For the vertical electric field at a height of less than dozens of meters, the waveforms are basically consistent. When the height increases to more than dozens of meters, the peak value of the electric field will increases with increasing height of the observation point. When the observation height is larger than 1 km, the polarity of the electric field may reverse. Moreover, the peak value of the horizontal electric field increases with the increase of height, and the peak value and falling edge of the magnetic field waveform are nearly unchanged.

Acknowledgments
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