A numerical model on thermodynamic analysis of free piston Stirling engines

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Abstract. In this paper, a new numerical thermodynamic model which bases on the energy conservation law has been used to analyze the free piston Stirling engine. In the model all data was taken from a real free piston Stirling engine which has been built in our laboratory. The energy conservation equations have been applied to expansion space and compression space of the engine. The equation includes internal energy, input power, output power, enthalpy and the heat losses. The heat losses include regenerative heat conduction loss, shuttle heat loss, seal leakage loss and the cavity wall heat conduction loss. The numerical results show that the temperature of expansion space and the temperature of compression space vary with the time. The higher regeneration effectiveness, the higher efficiency and bigger output work. It is also found that under different initial pressures, the heat source temperature, phase angle and engine work frequency pose different effects on the engine’s efficiency and power. As a result, the model is expected to be a useful tool for simulation, design and optimization of Stirling engines.

1. Introduction

The free piston Stirling engine (FPSE) was invented by William Beale in 1964 [1]. In Israel Urieli’s book [2] it introduce that such machines have the advantages of simplicity, low cost and ultra-reliability, and freedom of working gas leakage over conventional Stirling engines. FPSE’s piston and displacer have no side load for the lack of connecting rods, which makes it has potential for a long operating life without the need for lubrication. FPSE can be operate very quietly since all the motions and forces hence the resulting vibrations are in the axial direction to the engine and may be easily isolated [3]. The free piston Stirling engine driving a linear alternator and because of its simple configuration, light weight and maintenance free performance with long operating life, the FPSE is used for producing electricity from solar energy or nuclear [4].

Compared with the free piston Stirling engines, there are more studies on the traditional crank connecting Stirling engines. Cheng [5, 6] has used a theoretical model to predict the thermodynamic behavior of thermal-lag Stirling engine. Dependence of indicated power and thermal efficiency on engine speed has been investigated. Parlak [7] has used a thermodynamic analysis of a gamma type Stirling engine which is performed by using a quasi-steady flow model based on Urieli and Berchowitz’s works. Theoretical ways were used to analyse and optimize the Stirling engines and have found some important results for designing Stirling engines [8-10]. Mohammad H. Ahmadi [11] used the multi-objective optimization method for designing a powered Stirling heat engine which was designed with maximized power, thermal efficiency and minimized pressure loss. In this paper, some
of their research ways will be used to analyse the free piston Stirling engine. Yoshitaka Kato [12] proposed an experimental method about regenerator evaluation and indicated that the efficiency of the regenerator depends on the difference between the temperature of the working fluid on the hot side and that on the cold side and the fluctuation of pressure.

All the time, the isothermal and adiabatic models are used to analyse the Stirling engines. However the isothermal and adiabatic models are both ideal models, they do not take into account the non-isothermal effects, the effectiveness of the regenerator, the main heat losses and the thermal resistance in the heater and cooler. In this study the model is a complete free piston Stirling engine model which includes piston, compression space, cooler, regenerator, heater, expansion space and displacer. Note that the present model takes into account the non-isothermal effects, the effectiveness of the regenerator, the main heat losses and the thermal resistance in the heater and cooler. According to this new model, it not only could analyse the performances of Stirling engines as traditional models but also some new results have been acquired.

2. Numerical thermodynamic model

2.1. Thermodynamic parameters

In the Stirling engine the movements of piston and displacer are sinusoidal. The displacements of the piston and the displacer are \( Y_p(t) \) and \( Y_d(t) \), respectively, given as:

\[
Y_d(t) = A_d \sin(\omega t) \\
Y_p(t) = A_p \sin(\omega t - \theta)
\]

where \( A_p \) and \( A_d \) are the amplitudes of piston and displacer, respectively; \( \omega \) is the angular speed of engine and \( \omega = 2\pi f \); \( \theta \) is the phase angle between piston and displacer.

The volumes of the expansion and the compression space, \( V_e(t) \) and \( V_c(t) \) can be calculated, respectively, in terms of \( Y_p(t) \) , \( Y_d(t) \) and radiuses of piston and displacer:

\[
V_e(t) = \pi r_e^2 \left( L_e - Y_d(t) \right) \\
V_c(t) = \pi r_c^2 \left( L_c - Y_p(t) \right)
\]

where \( r_e, r_c \) are the radiuses of piston and displacer, respectively; \( L_e, L_c \) are the distances from the equilibrium position of piston and displacer to the top surface of the expansion space and the compression space, respectively.

Basing on above equations and the ideal gas equation, the gas mass \( m_{e,h} \) in expansion space (includes the heater) and \( m_{c,k} \) in compression space (includes the cooler) can be calculated as

\[
m_{e,h} = \frac{P(V_e(t) + V_h)}{R T_e} \\
m_{c,k} = \frac{P(V_c(t) + V_k)}{R T_c}
\]

All the time, the mass flow rate is calculated by some empirical formulas. However, in this model the gas mass in expansion space (includes the heater) and the gas mass in compression space (includes the cooler) can be calculated at any time instant. So the mass flow rate can be calculated as

\[
m = \frac{\Delta m}{\Delta t}
\]

2.2. The losses of Stirling engine

This numerical thermodynamic model has considered some main energy losses when the Stirling engine is running. Due to the piston and displacer are made of Titanium alloy, so they are assumed as adiabatic. The loss is given in Fig.1, which include the axial heat conduction loss of regenerator \( \dot{Q}_{\text{loss,ac}} \), the heat conduction loss from cylinder wall to surroundings \( \dot{Q}_{\text{loss,cc}} \), the heat pumping loss inside the piston and cylinder gap \( \dot{Q}_{\text{loss,lcp}} \) and the shuttle loss between cylinder wall and piston \( \dot{Q}_{\text{loss,scp}} \). So the \( \dot{Q}_{\text{loss}} \) can be calculated by

\[
\dot{Q}_{\text{loss}} = \dot{Q}_{\text{loss,ac}} + \dot{Q}_{\text{loss,cc}} + \dot{Q}_{\text{loss,lcp}} + \dot{Q}_{\text{loss,scp}}
\]
All these losses can be calculated by theoretical deduction and empirical formula. The axial heat conduction loss of regenerator transferring from high temperature to low temperature space is

$$Q_{loss.ac} = \frac{k_{rac} A_{rac} (T_e - T_c)}{L_R}$$

(9)

Where $k_{rac}$ is the equivalent thermal conductivity; $A_{rac}$ is the cross sectional area of the regenerator; $L_R$ is the regenerator length. $k_G$ and $k_M$ are the gas conductivity and the metal conductivity, respectively; $\beta$ is the regenerator porosity.

In the expansion space and compression space the high temperature gas transfers heat to cylinder, then the cylinder transfers heat to the surroundings. It is assumed that the internal surface temperature of cylinder equals to the gas temperature what is $T_e$ or $T_c$. The heat conduction loss from cylinder wall to surroundings can be calculated as

$$Q_{loss.cc} = \frac{(T_{e,c} - T_L)}{\ln(r_2/r_1) / 2\pi k_W (L_{p,d} - Y_{p,d}(t)) + R_{cc}}$$

(11)

where $k_W$ is the thermal conductivity of cylinder wall; $R_{cc}$ is heat resistance of convection; $r_1$ and $r_2$ are the inner radius and the outer radius of cylinder wall, respectively; $L_{p,d}$ is the length of piston or displacer.

The heat pumping loss between cylinder wall and piston $Q_{loss.lcp}$ and the shuttle loss between cylinder wall and piston $Q_{loss.scp}$ are very big and it mainly decided by the gap $\delta_G$. However, the gap has the opposite effects to $Q_{loss.lcp}$ and $Q_{loss.scp}$, so an appropriate gap must be found. It is very complicated, so the empirical formulas are used as

$$Q_{loss.lcp} = \frac{2f}{3\pi} \left( \frac{f}{D_{p,d}} \cdot \frac{c_{pg}}{2k_G} \right)^{0.6} \left( \frac{P_{max} - P_{min}}{R \cdot \bar{T}_R} \right)^{1.6} \cdot c_p \cdot L_{p,d} \cdot (T_{e,c} - T_L)$$

(12)

$$Q_{loss.scp} = \frac{k_G \cdot \pi \cdot D_{p,d} \cdot S_{p,d}^2}{4L_{p,d} \delta_G} \cdot (T_{e,c} - T_L)$$

(13)

where $f$ is the frequency of the engine; $\delta_G$ is the gap between piston and cylinder; $D_{p,d}$ is the diameter of the piston or displacer; $S_{p,d}$ is the stroke of the piston or displacer.

2.3. Numerical model in the expansion space

The heat transfer of a Stirling engine is shown in Fig.2. In this model, the heater and the expansion space are regarded as a control volume. Then the energy conservation law is applied to the expansion space and the compression space. In the expansion space and compression space the energy conservation equations are as following

$$\frac{dU_e}{dt} = Q_{in,e} - W_{out,e} - Q_{loss,e} + m_{e,h} \left( h + \frac{v^2}{2} \right)$$

(14)
\[
\frac{dU_c}{dt} = \dot{Q}_{\text{in},c} - \dot{W}_{\text{out},c} - \dot{Q}_{\text{loss},c} + \dot{m}_{c,k} \left( h + \frac{v^2}{2} \right)_c
\]  

(15)

where \( U_e \) and \( U_c \) are the internal energy of expansion space and compression space respectively; \( \dot{m}_{e,h} \) and \( \dot{m}_{c,k} \) are the mass flow rate of expansion space and compression space respectively; \( \dot{Q}_{\text{in},e}, \dot{W}_{\text{out},e} \) and \( \dot{Q}_{\text{loss,e}} \) are the input heat transfer rate, the output power and the heat loss of expansion space, respectively; \( \dot{Q}_{\text{in},c}, \dot{W}_{\text{out},c} \) and \( \dot{Q}_{\text{loss,c}} \) are the input heat transfer rate, the output power and the heat loss of compression space, respectively.

It is too complicated to get the analytical solution, so the numerical way was used to this model. To satisfy the convergence condition, different schemes were used to discretize the equation. To make sure that the accuracy of the results, in the model \( \Delta t = 10^{-6}s \).

2.4. The output power and efficiency

The net output power per cycle is calculated by the following integration:

\[
W_{\text{out}} = \int_t^{t+T} P_e dV_e + \int_t^{t+T} P_c dV_c
\]

(18)

and the total input heat is

\[
Q_{\text{in},e} = \int_t^{t+T} \dot{Q}_{\text{in},e} dt
\]

(19)

where \( T \) is the cycle of the Stirling engine. So, the thermal efficiency of the Stirling engine can be calculated by

\[
\eta_t = \frac{W_{\text{out}}}{Q_{\text{in},e}}
\]

(20)

3. Results and discussion

According above numerical model some useful results have been got. Fig.3 shows the output work and efficiency of the isothermal model, adiabatic model and the new model. It can be seen that the isothermal model and adiabatic model don’t have big differences on the efficiency and the adiabatic model has bigger output work. In the new model, the output work and the efficiency are 124.56W and 18.36% (the main losses have been considered), respectively.
Fig. 4 shows the P-V diagrams of expansion space, compression space and the total space, the expansion space circulation is reverse and the compression space circulation is positive. The net output power per cycle of the expansion space $W_{\text{oute}} > 0$ and the net output power per cycle of the compression space $W_{\text{outc}} < 0$.

Fig. 3. Output work and efficiency of three models

In this model the temperature varies in the two spaces. The heat source and the heat sink temperature are maintained at 900 K and 300 K, respectively. However, from the Fig. 5 the gas temperature in the expansion space is lower than the heat source temperature and varies from 842 K to 872 K, on the contrary, the gas temperature in the compression space is higher than the heat sink temperature and varies from 316 K to 330 K. There is a phase difference about $\pi/2$ between the gas temperature variations of compression space and expansion space.

Fig. 6 shows the effects of regeneration effectiveness on engine’s power and efficiency. It is found that as the regeneration effectiveness increases from 0.6 to 0.95, the output work increases from 57.2 W to 155 W, and the thermal efficiency increases from 5.4% to 30.2%. The higher regeneration effectiveness, the higher efficiency and bigger output work. Usually the regeneration efficiency is about 0.85.

Fig. 7 illustrates the effects of heat source temperature on engine’s efficiency at different initial pressure. When do not considering the heat losses, both the output work and thermal efficiency increase with the increase of heat source temperature. However, when considering the heat losses, the output work increases with the increase of heat source temperature, but the thermal efficiency performs contrarily. It is obvious that the heat losses has a significant effect on the thermal efficiency, but a slight effect on the output work. When the initial pressure is 2 MPa and the heat source temperature varies from 600 K to 1200 K, the output work varies from 75 W to 162 W and the thermal efficiency varies from 19.2% to 17.5%. It can be seen when the initial pressure is 2 MPa or 3 MPa, the thermal efficiency decreases with the increase of heat source temperature. When the initial pressure is 4 MPa, there is an optimal heat source temperature about 900 K, at this temperature the engine gets the maximum efficiency. The efficiency increases with the increase of heat source temperature when the
initial pressure is 5MPa or bigger. So in the real Stirling engine, the output work can be increased by increasing the heat source temperature, but the efficiency cannot in a lower initial charge pressure by this way.

Fig.8 illustrates the effects of frequency on engine’s efficiency at different initial pressure. It can be seen that at different initial pressure the engine has different optimal frequency. When the pressures are 2MPa, 3MPa, 4MPa and 5MPa, the optimal frequencies are about 85Hz, 65Hz, 50Hz and 30Hz, respectively. In order to get the maximum efficiency, the initial pressure must base on the engine’s frequency.

Fig.7. The effects of heat source temperature on engine’s efficiency at different initial pressure.

Fig.8. The effects of frequency on engine’s efficiency at different initial pressure.

Fig.9 illustrates the effects of phase angle on engine’s power and efficiency. It can be seen that the phase angle has a big effect on the output work and thermal efficiency. When the initial pressure is 2MPa and the phase angle from 30° to 120°, the maximum output work and thermal efficiency are about twice of the minimum output work and thermal efficiency, the output work varying from 63W to 124W and the thermal efficiency from 7.5% to 18.5%. When the phase angle is 70°~90° the output work and thermal efficiency get the maximum values. So, a suitable phase angle is very important to design a free piston Stirling engine. However, no matter how big the initial pressure is, the optimal phase angle is about 80°.

Fig.9. The effects of phase angle on engine’s output work and efficiency at different pressure.

Fig.10. The effects of gap on engine’s output work and efficiency.

Fig.10 illustrates effects of the gap between piston or displacer and the inner surface of cylinder wall on the shuttle and leakage losses. When the δGE = 90 μm the shuttle and leakage losses of the displacer is at the least and the minimum losses is 251W. When the δGC = 70 μm the shuttle and leakage losses of the piston is at the least and the minimum losses is 4.5W. When the δGE = δGC =
90 μm it can get the maximum output work and thermal efficiency, though the output work only varies from 123.9W to 124.6W and the efficiency only varies from 16.8% to 18.4%.

4. Conclusions

According this new model, it not only could analyse the performances of Stirling engines as traditional models but also some new results have been acquired. The paper got the result that the expansion space temperature and the compression space temperature changed with the time, however at different initial pressure the temperatures have different changes, the initial pressure is bigger the temperature of expansion space further away the heat source temperature and the temperature of compression space is on the contrary. It is also found that at the different initial pressure, the heat source temperature and engine work frequency pose different effects on the engine efficiency and the output power. It has been found that when the gap $\delta_G = 90 \mu m$, and the phase angle $\theta = 80^\circ$ the maximum output work and thermal efficiency can be obtained. The output work increases with the increase of heat source temperature, but the thermal efficiency have different changes with different initial pressure. Through this model, the output work $W_{out}$ and the thermal efficiency $\eta_t$ of a free piston Stirling engine has been calculated as about 120W and 18%. The temperature, heat power loss and output work are all change with the time. This study offers valuable guides for the design and optimization of free piston Stirling engines with similar construction.

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