Modelling of SDG’s Achievement in East Java Using Bi-responses Nonparametric Regression with Mixed Estimator Spline Truncated and Kernel

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Abstract. The Government of Indonesia has grouped 17 objectives of SDG’s into 4 pillars, among others: social pillars, economic pillars, the pillars of environmental development and the pillars of legal development and Governance. Some strategic indicators are used to know the achievement of SDG’s especially achievement in social and economic pillars such as the Human Development Index (HDI) and Gini Ratio. There are several variables affecting then. A method used to model the effects of its variables is regression analysis. The results of the initial exploration through the scatter plot shows a pattern that changes and disconnects at a certain point for a variable IPM with a variable that is allegedly influencing it. With a scatter plot, it can be found that between the Gini ratio variables with some of its predictive variables, does not have a specific pattern. Bi-responses nonparametric regression is an analysis that appropriates with that case. Bi-responses nonparametric regression analysis is a type of nonparametric regression analysis used when the pattern between two response variables and some of the predictor variables is unknown or if there is no prior information regarding the pattern of the relationship. Most of researchers use bi-responses nonparametric regression approached by spline truncated, kernel and etc. Most of the research, use kernel and spline truncated because of their advantages such as: have a good capability to develop model from unknown pattern data and have a good visual representation. The first aim of this paper is obtaining the estimator of mixed estimator spline truncated and kernel in bi-responses nonparametric regression with weighted least square. The second aim is gaining a best model to explain the relationship indicators of SDG’s achievement with some variables affecting them. To obtain the best model, it is necessary to select the optimal of knots point and bandwidth parameters using a smallest Generalized Cross Validation (GCV).

Keywords: Bi-responses Nonparametric Regression, Kernel, Spline Truncated, SDG’s, HDI, Gini Ratio

1. Introduction
The Government of Indonesia has grouped into 17 objectives of SDG’s into 4 pillars, among others: social pillars, economic pillars, the pillars of environmental development and the pillars of legal
development and governance. Some strategic indicators are also able to be used to see the achievement of SDG’s especially achievement in social and economic pillars such as the Human Development Index (HDI) and Gini Ratio. Some indicators that are usually used as a benchmark for the development of a region is to look at the value of Human Development Index and Gini Ratio of the region. The Human Development Index (HDI) is a summary measure of average achievement in key dimensions of human development: a long and healthy life, being knowledgeable and have a decent standard of living. The HDI is the geometric mean of normalized indices for each of the three dimensions. Gini ratio is a statistical measure of distribution developed by the Italian statistician Corrado Gini in 1912. It is often used as a gauge of economic inequality, measuring income distribution or, less commonly, wealth distribution among a population. The coefficient ranges from 0 (or 0%) to 1 (or 100%), with 0 representing perfect equality and 1 representing perfect inequality. Values over 1 are theoretically possible due to negative income or wealth. Based on Indonesia Statistic data, HDI in East Java in 2017 reached 0.27 and Gini Ratio reached 0.418 in 2017. Its value are influenced by many factors, including the level of education, open unemployment rate and per capita GDP.

There are many ways to model the relationship between HDI and Gini Ratio with several variables that affecting them. Bi-responses nonparametric regression is one of nonparametric regression type that is suitable for that case. Nonparametric regression is a regression that is used to know causality relationships between response variables with predictor variables of unknown form functions or not available prior information about the relationship form of both variables type. Bi-responses nonparametric regression is a nonparametric regression involving two interconnected response variables without any causality in them.

In recent years, nonparametric regression analysis with spline truncated and kernel estimators are reasonably desirable by most researchers. Spline truncated estimator is capable of handling smooth data/function as well as data that has changed behavior in certain sub-sub intervals. Kernel estimator has a good ability to model data that has no specific pattern, more flexible, simply mathematical form and can achieve a relatively fast level of convergent [1], [2]. Because of them, many researchers have combined both of them in nonparametric regression analysis. Mixture estimator spline truncated and kernel are used when one of predictor variables have arbitrary patterns and other predictor variables do not follow a specific pattern.

Several methods are implemented to obtain parameter estimator in nonparametric regression with mixture estimator spline truncated and kernel. Some of researchers use maximum likelihood method to estimate the parameter of nonparametric regression with mixture estimators kernel and spline truncated [3], [4]. Penalized least square can be used to estimate its parameter [5]. Others method, such as least square method, is capable to estimate them [6]. Some of researchers use spline truncated or kernel function for estimating the parameter of bi-responses nonparametric regression [7], [8], [9], [10], [11], [12].

In this study, we build the bi-responses nonparametric regression model by developing the multi-responses nonparametric model proposed by [13], [14] to the two responses model with mixed estimator spline truncated and kernel. Weighted least square is a method that chosen in this paper to estimate the parameter. Same as other least square, in weighted least square, it needs to optimize with variance-covariance error-matrix as weighted matrix \( W \). For obtaining the best model, it is necessary to select the optimal of knots point and bandwidth parameters using a Generalized Cross Validation (GCV). The optimal of knots point and bandwidth are obtained from the smallest GCV value.

2. Sustainable Development Goals
The Sustainable Development Goals (SDG’s), also known as the Global Goals, were adopted by all United Nations Member States in 2015 as a universal call to action to end poverty, protect the planet and ensure that all people enjoy peace and prosperity by 2030. The SDG’s build on the success of the Millennium Development Goals (MDGs) and aim to go further to end all forms of poverty. The new Goals are unique in that they call for action by all countries, poor, rich and middle-income to promote prosperity while protecting the planet. They recognize that ending poverty must go hand-in-hand with
strategies that build economic growth and addresses a range of social needs including education, health, social protection, and job opportunities, while tackling climate change and environmental protection. The 17 sustainable development goals (SDGs) to transform our world can be seen in table below

| Goal | Description                                                                 | Goal | Description                                                                 |
|------|------------------------------------------------------------------------------|------|------------------------------------------------------------------------------|
| 1    | End poverty in all its forms everywhere                                       | 10   | Reduce inequality within and among countries                                 |
| 2    | End hunger, achieve food security and improved nutrition and promote sustainable agriculture | 11   | Make cities and human settlements inclusive, safe, resilient and sustainable |
| 3    | Ensure healthy lives and promote well-being for all at all ages               | 12   | Ensure sustainable consumption and production patterns                       |
| 4    | Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all | 13   | Take urgent action to combat climate change and its impacts                 |
| 5    | Achieve gender equality and empower all women and girls                       | 14   | Conserve and sustainably use the oceans, seas and marine resources for sustainable development |
| 6    | Ensure availability and sustainable management of water and sanitation for all | 15   | Protect, restore and promote sustainable use of terrestrial ecosystems, sustainably manage forests, combat desertification, and halt and reverse land degradation and halt biodiversity loss |
| 7    | Ensure access to affordable, reliable, sustainable and modern energy for all  | 16   | Promote peaceful and inclusive societies for sustainable development, provide access to justice for all and build effective, accountable and inclusive institutions at all levels |
| 8    | Promote sustained, inclusive and sustainable economic growth, full and productive employment and decent work for all | 17   | Strengthen the means of implementation and revitalize the global partnership for sustainable development |
| 9    | Build resilient infrastructure, promote inclusive and sustainable industrialization and foster innovation | 10   | Reduce inequality within and among countries                                 |

3. Method

3.1. Bi-responses Nonparametric Regression
In estimation of regression curves, there are three regression approaches, namely parametric regression, nonparametric regression and semiparametric regression [1], [2]. A nonparametric regression can ignore the data assumptions that should be related to a particular distribution. However, errors occurring in nonparametric regression should remain normal, identical, and independent distributions. A nonparametric regression approach has been widely used among others are spline truncated, kernel, local polynomial and etc. [7], [11]. Nonparametric regression is used to know causality relationships between response variables with predictor variables of unknown form functions or not available prior information about the relationship form of both variables type. Bi-responses nonparametric regression
is a nonparametric regression involving two interconnected response variables without any causality in them.

Consider a paired data set \( y_{1i}, y_{2i}, x_{1i}, \ldots, x_{pi}, t_{1i}, \ldots, t_{qi} \) and the relations between predictor variables \( x_{1i}, \ldots, x_{pi} \) and \( t_{1i}, \ldots, t_{qi} \), response variables, \( y_{1i} \) and \( y_{2i} \), are assumed to follow additive nonparametric regression model:

\[
y_{ji} = \mu(x_{1i}, \ldots, x_{pi}, t_{1i}, \ldots, t_{qi}) + \epsilon_{ji} = \sum_{k=1}^{p} \mu(x_{ki}) + \sum_{l=1}^{q} \mu(t_{li}) + \epsilon_{ji}
\]  

(1)

where \( i = 1, 2, \ldots, n \) and \( j = 1, 2 \). The shape of regression curve \( \mu(x_{1i}, \ldots, x_{pi}, t_{1i}, \ldots, t_{qi}) \) is assumed to be unknown and smooth, meaning continuous and differentiable. Random \( \epsilon_{ji} \) has normal distribution.

3.2 Generalized Cross Validation (GCV)

Bi-responses nonparametric regression with mixed estimator spline truncated and kernel really depends on knot point. Furthermore, bandwidth also affect the result of that estimators. In order to gain the best model, it is needed to choose the most optimum knot points and bandwidth. Generalized Cross Validation (GCV) is one of the methods to obtain that purpose. The optimum knot points and bandwidth is gained from the smallest GCV. The formula of GCV is

\[
GCV(K, \alpha) = \frac{n^{-1} \| y - \tilde{\mu}_{n, \alpha}(x, t) \|^2}{\left( n^{-1} \text{trace} \left[ I - A(K, \alpha) - D(\alpha) \right] \right)^{1/2}}
\]  

(2)

4. Result

4.1. Bi-responses Nonparametric Regression with Mixed Estimator Spline Linear Truncated and Kernel

Equation 1 contains from two different components. First component \( \sum_{k=1}^{p} \mu(x_{ki}) \) is approached by linear spline truncated and second component \( \sum_{l=1}^{q} \mu(t_{li}) \) is approached by kernel Nadaraya-Watson. Both functions are assumed smooth, in the sense of continuous and differentiable. Bi-responses nonparametric regression model with spline truncated estimator can be described by:

\[
y_{ji} = \sum_{k=1}^{p} \mu(x_{ki}) + \epsilon_{ji} = \beta_{0j} + \sum_{k=1}^{p} \left( \beta_{jk} x_{ki} + \sum_{r=1}^{M} \gamma_{jkr} \left( x_{ki} - A_{jkr} \right) \right) + \epsilon_{ji}
\]  

(3)

where \( M \) is the number of knot and \( A \) is the point of knot.

Function of \( \sum_{l=1}^{q} \mu(t_{li}) \) is approached by kernel Nadaraya-Watson and can be describe by following equation:
Equation (4) is given by:

\[ \sum_{i=1}^{q} \mu(t_i) = \sum_{i=1}^{q} \left( n^{-1} \sum_{t=1}^{n} \left( \frac{K_{\alpha_0}(t_i - t_t)}{n^{-1} \sum_{t=1}^{n} K_{\alpha_0}(t_i - t_t)} \right) y_j \right) \]

\[ K_{\alpha}(t - t_i) = \frac{1}{\alpha} K \left( \frac{t - t_i}{\alpha} \right) \]

where \( \alpha \) is a bandwidth and \( K \) is a Kernel function. Type of Kernel function are Kernel Gaussian, Kernel Uniform, Kernel Epanechnikov, Kernel Triweights or others Kernel [6].

4.2 Estimator of bi-responses nonparametric regression model with spline truncated and kernel

Based on equation (1),(2) and (3), the model of bi-response nonparametric regression model with spline truncated and kernel, can be described as following below:

\[ y_{ji} = \beta_{0j} + \sum_{k=1}^{p} \left( \beta_{jk} x_{ki} + \sum_{r=1}^{M} \gamma_{jkr} \left( x_{ki} - A_{jkr} \right)_+ \right) + \sum_{i=1}^{q} \mu(t_i) + \varepsilon_{ji} \]

\[ = \beta_{0j} + \sum_{k=1}^{p} \left( \beta_{jk} x_{ki} + \sum_{r=1}^{M} \gamma_{jkr} \left( x_{ki} - A_{jkr} \right)_+ \right) + C_{ji} \]

where \( C_{ji} = \sum_{i=1}^{q} \mu(t_i) + \varepsilon_{ji} \)

From the equation (5), it can be shown as a matrix form to simplify the equation. Then, matrix form for equation (5) is

\[ \begin{bmatrix} y_{11} \\ \vdots \\ y_{1n} \\ y_{21} \\ \vdots \\ y_{2n} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} \begin{bmatrix} \bar{D} \\ \bar{E} \end{bmatrix} \]

\[ \tilde{y} = Z \tilde{\beta} \]

\( Z \) is the matrix that contains \( A, B \) and \( 0 \) matrix. They are all of predictor variables include the intersep and its dimensions are \( 2n \times (2(m+2)p) \). \( \tilde{y} \) is a vector that consist all of response variables and its dimensions are \( 2n \times 1 \). \( \tilde{\beta} \) is a vector that has two component and its dimensions are \( (2(m+2)p) \times 1 \). \( \bar{D} \) is a vector that consist of all parameters from \( y_1 \). \( \bar{E} \) is a vector that consist of all parameters from \( y_2 \).

Equation (6) has a long form equation if it describes every \( q \) predictor variables that approached by kernel. Therefore, it is able to change in matrix form to simplify the equation. If \( \sum_{i=1}^{q} \mu(t_i) \) that is change into \( \tilde{g} \) and its dimension are \( 2n \times 1 \), so matrix form of equation (3) is

\[ \tilde{g} = D_1(\lambda_1)y + \ldots + D_q(\lambda_q)y \]

\[ = D(\lambda)y \]
where \( D(\lambda) = D_1(\lambda_1) + \ldots + D_q(\lambda_q) \).

Consider that \( \varepsilon_j \) is vector of error \( \tilde{\varepsilon} \) and \( \sum_{k=1}^p \mu(x_{ki}) \) is \( \tilde{f} \), vector of spline truncated, the matrix form of equation (5) is:

\[
\tilde{y} = \tilde{\mu} + \tilde{\varepsilon} = \tilde{f} + \tilde{g} + \tilde{\varepsilon}
\]  

(8)

In the modelling, the most important thing is how to obtain the parameter of the model. In the bi-responses nonparametric regression, there is a relationship non causality between \( y_1 \) and \( y_2 \). Furthermore, it has to use weighted least square to obtain the parameter. Same as other least square, in weighted least square, it needs to optimize \( \varepsilon \) with variance-covariance error matrix as weighted matrix \( W \).

The first step to do weighted least square is changing the equation 10 into:

\[
\tilde{\varepsilon} = \tilde{Y}^* \cdot Z\tilde{\beta} \quad ; \quad \tilde{Y}^* = \left( I - D(\lambda) \right) \tilde{y}
\]

(9)

Shape estimator obtained from completing optimization \( \min_{\tilde{\beta}} \left\{ (\tilde{Y}^* - Z\tilde{\beta})^T W (\tilde{Y}^* - Z\tilde{\beta}) \right\} \) then with a calculation obtained by the equation:

\[
Q = (\tilde{Y}^* - Z\tilde{\beta})^T W (\tilde{Y}^* - Z\tilde{\beta})
\]

\[
= \tilde{\gamma}^T W \tilde{\gamma}^* - 2\tilde{\beta}^T Z^T W \tilde{\gamma}^* + \tilde{\beta}^T Z^T W Z \tilde{\beta}
\]

By using the obtained partial derivatives and the result is equated to zero obtained equation:

\[
\frac{\partial (Q)}{\partial \tilde{\beta}} = 0
\]

\[-2Z^T W \tilde{\gamma}^* + 2Z^T W Z \tilde{\beta} = 0
\]

\[
\tilde{\beta} = (Z^T W Z)^{-1} Z^T W \tilde{\gamma}^*
\]

According to equation (11), the estimator of parameter \( \tilde{\beta} \) is

\[
\tilde{\beta} = (Z^T W Z)^{-1} Z^T W \tilde{\gamma}^*
\]

(10)

where \( A = (Z^T W Z)^{-1} Z^T W (I - D(\lambda)) \)

From the equation (10), to obtain the estimator of \( \tilde{\mu} \), a calculation obtained by the equation:

\[
\tilde{\mu} = \tilde{f} + \tilde{g}
\]

\[
= Z\tilde{\beta} + D(\lambda) \tilde{y}
\]

\[
= ZA\tilde{y} + D(\lambda) \tilde{y}
\]

\[
= C\tilde{y}
\]

(11)

where \( C = ZA + D(\lambda) \)
5. Data Application

Bi-responses nonparametric regression with mixed estimator spline truncated and kernel will be applied to the Human Development Index ($Y_1$) and Gini Ratio ($Y_2$) data with the observation unit consisting of 38 districts/cities in the East Java province. The predictor variables used are morbidity rate ($X_1$), pure participation rate of senior high school/ APM SMA ($X_2$) and per capita gross domestic product ($X_3$). Data exploration for showing the relationship between predictors and response variables can be shown in figure 1.a – 2.c.

![Scatterplot of IPM vs morbidity](image1.png)

**Figure 1.a** Scatter plot $Y_1$ and $X_1$

![Scatterplot of IPM vs APM SMA](image2.png)

**Figure 1.b** Scatter plot $Y_1$ and $X_2$

![Scatterplot of IPM vs PDRB per capita](image3.png)

**Figure 1.c** Scatter plot $Y_1$ and $X_3$

![Scatterplot of Gini Ratio vs morbidity](image4.png)

**Figure 2.a** Scatter plot $Y_2$ and $X_1$

![Scatterplot of Gini Ratio vs APM SMA](image5.png)

**Figure 2.b** Scatter plot $Y_2$ and $X_2$

![Scatterplot of Gini Ratio vs PDRB per capita](image6.png)

**Figure 2.c** Scatter plot $Y_2$ and $X_3$
In figure 1.a – 2.c, show that there is an unknown relationship pattern between each response variables and all of predictor variables. So it is modeled as nonparametric. In figure 1, the pattern of the relationship between $Y_1$ and $X_2$ tends to change behavior at several points. In figure 2, the relationship between $Y_2$ and $X_2$ have same pattern like $Y_2$ and $X_2$. So that in theory, it can be approached with the spline truncated function. The pattern of the relationship between $Y_1$ and $X_1$, $Y_1$ and $X_3$, $Y_2$ and $X_1$, $Y_2$ and $X_3$ seen in Figure 1.a – 2.c tends not to follow a certain pattern, so it is modeled as a nonparametric Kernel. So that the appropriate model to explain the relationship between predictor variables and the two response variables in East Java, Human Development Index and Gini Ratio data are bi-response nonparametric regression with mixture estimators spline truncated and kernel.

Based on the bi-responses nonparametric regression with mixture estimators spline truncated and kernel model then variations on its function are carried out to see which functions are more suitable. In this paper, it uses only two knot points and 100 iterations. The top 10 results are shown in the following table:

| No | $A_{111}$ | $A_{112}$ | $A_{211}$ | $A_{212}$ | $a_{11}$ | $a_{12}$ | $a_{21}$ | $a_{22}$ | GCV     |
|----|-----------|-----------|-----------|-----------|---------|---------|---------|---------|---------|
| 1  | 0.791     | 0.527     | 0.764     | 0.659     | 0.021   | 43.522  | 0.017   | 35.609  | 5.754   |
| 2  | 0.580     | 0.764     | 0.553     | 0.817     | 0.021   | 43.522  | 0.017   | 35.609  | 5.762   |
| 3  | 0.369     | 0.553     | 0.764     | 0.606     | 0.021   | 43.522  | 0.017   | 35.609  | 5.783   |
| 4  | 0.606     | 0.791     | 0.580     | 0.843     | 0.021   | 43.522  | 0.017   | 35.609  | 5.784   |
| 5  | 0.395     | 0.580     | 0.791     | 0.632     | 0.021   | 43.522  | 0.017   | 35.609  | 5.887   |
| 6  | 0.791     | 0.527     | 0.764     | 0.659     | 0.041   | 87.043  | 0.034   | 71.217  | 5.888   |
| 7  | 0.527     | 0.738     | 0.580     | 0.527     | 0.021   | 43.522  | 0.017   | 35.609  | 5.891   |
| 8  | 0.580     | 0.764     | 0.553     | 0.817     | 0.041   | 87.043  | 0.034   | 71.217  | 5.895   |
| 9  | 0.817     | 0.553     | 0.791     | 0.685     | 0.021   | 43.522  | 0.017   | 35.609  | 5.907   |
| 10 | 0.369     | 0.553     | 0.764     | 0.606     | 0.041   | 87.043  | 0.034   | 71.217  | 5.912   |

In that table above, with 100 iteration, obtained the smallest GCV is 5.754. Then, we are able obtain the best model for IPM and Gini Ratio. The model for $y_1$ is:

$$\hat{y}_1 = 4.259 + 20.851 x_{2i} - 32.775 (x_{2i} - 0.527)_+ - 13.160 (x_{2i} - 0.791)_+$$

$$+ \sum_{i=1}^{38} \frac{1}{0.021} K_{\frac{t_i - t_{i1}}{0.021}} y_{1i} + \sum_{i=1}^{38} \frac{1}{43.522} K_{\frac{t_i - t_{i3}}{43.522}} y_{1i}$$

Next, the model for $y_2$ is:

$$\hat{y}_2 = 0.000000016 + 0.058 x_{2i} + 0.000028 (x_{2i} - 0.659)_+ + 0.002 (x_{2i} - 0.764)_+$$

$$+ \sum_{i=1}^{38} \frac{1}{0.017} K_{\frac{t_i - t_{i1}}{0.017}} y_{2i} + \sum_{i=1}^{38} \frac{1}{35.609} K_{\frac{t_i - t_{i3}}{35.609}} y_{2i}$$
That model above has coefficient of determination ($R^2$) = 99.544% and MSE (mean square error) = 3.578. Therefore, the model can explain the relationship between two response variables and three predictor variables worth 99.544%.

6. Conclusion

Based on this paper, it can be concluded that:

1. The equation of bi-responses nonparametric regression

\[ y_{ji} = \mu(x_{i1}, \ldots, x_{ip}, t_{i1}, \ldots, t_{iq}) + \varepsilon_{ji} \]

\[ = \sum_{k=1}^{p} \mu(x_{i1}) + \sum_{l=1}^{q} \mu(t_{il}) + \varepsilon_{ji} \]

2. The estimation of spline truncated and kernel is obtained by optimized:

\[ \min_{\beta} \left\{ (\tilde{y} - Z\beta)^T W (\tilde{y} - Z\beta) \right\} \]

That optimization above give the result of mixed spline truncated and kernel in bi-response nonparametric regression:

\[ \hat{\mu} = f + g = ZA\tilde{y} + D(\lambda) \tilde{y} = C\tilde{y} \quad ; \quad C = ZA + D(\lambda). \]

3. The model for $y_1$ is:

\[ \tilde{y}_1 = 4.259 + 20.851x_{2i} - 32.775(x_{2i} - 0.527)_+ - 13.160(x_{2i} - 0.791)_+ \]

\[ + \frac{1}{0.021} \sum_{i=1}^{38} K \left( \frac{t_{i1} - t_{1i}}{0.021} \right) y_{1i} + \frac{1}{43.522} \sum_{i=1}^{38} K \left( \frac{t_{i2} - t_{2i}}{43.522} \right) y_{1i} \]

Next, the model for $y_2$ is:

\[ \tilde{y}_2 = 0.00000016 + 0.058x_{2i} + 0.000028(x_{2i} - 0.659)_+ + 0.002(x_{2i} - 0.764)_+ \]

\[ + \frac{1}{0.017} \sum_{i=1}^{38} K \left( \frac{t_{i1} - t_{1i}}{0.017} \right) y_{2i} + \frac{1}{35.609} \sum_{i=1}^{38} K \left( \frac{t_{i2} - t_{2i}}{35.609} \right) y_{2i} \]

That model above has coefficient of determination ($R^2$) = 99.544% and MSE (mean square error) = 3.578. Therefore, the model can explain the relationship between two response variables and three predictor variables worth 99.544%.
Acknowledgments
This paper is a part of author’s Master thesis under the supervision of the second and third author and has been fully supported and funded by Badan Pusat Statistik (BPS) Republik Indonesia.

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