Lyotropic isotropic to columnar phase transition in RNA solutions
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ABSTRACT
RNA oligomers exhibit lyotropic chiral nematic and columnar liquid crystal phases. The lyotropic chiral nematic and columnar phases with special emphasis on the isotropic to columnar phase transition in RNA solutions are described based on Landau theory. A free energy is constructed based on the order parameters to describe the chiral nematic and columnar liquid crystal phases and isotropic to columnar phase transition. The lyotropic isotropic to columnar phase transition is found to be always first order. The birefringence has been calculated in the isotropic phase of isotropic to columnar phase transition. Theoretical observation of the existence of the lyotropic isotropic to columnar phase transition in RNA solutions is confirmed with experimental results.

1. Introduction
RNA is generally found in organisms as a single-stranded chain of nucleotides. RNA is a linear polymer of nucleotides linked by a ribose-phosphate backbone. Polymerization of nucleotides occurs in a condensation reaction in which phosphodiester bonds are formed. RNA helices intrinsically resist bend or twist deformations. Generally, RNA is fairly rigid and possesses high flexibility. Some RNA are also moderately flexible. RNA is an important precursor to DNA. The self-assembly of RNA NTP (rNTPs) is a template of the RNA world and the origins of life. RNA contains a Ribose sugar, which has two hydroxyl groups make the RNA less stable in solution because of their propensity for hydrolysis [1]. RNA has a higher tilt of its bases as well as a shorter rise for the base pairs [1]. The three most commonly studied of RNA are messenger RNA (mRNA), transfer RNA (tRNA), and ribosomal RNA (rRNA), which are present in all organism.

It had been showed experimentally that lyotropic liquid crystal (LLCs) phases can be found both in DNA and RNA oligomers in electrolyte solution [2–7]. These LLCs phases are found at critical concentrations in the short DNA (sDNA) as well as long DNA (IDNA) fragments. It had been found that the appearance of liquid crystal ordering was a general form of spontaneous ordering in DNA solutions. The liquid crystal ordering is accompanied by the segregation of DNA liquid crystal domains when duplex-forming oligomers are mixed with unduplexed DNA strands. The formation of the liquid crystal ordering depends on the oligomers length, concentration and temperature. Some DNA/RNA form chiral nematic phase depending on the chemical structure of the monomeric units. B-DNA form chiral nematic phase. B-DNA helices are right-handed and form a left-handed cholesteric phase. But the relation between molecular and macroscopic chirality is unknown. In a series of self-complementary
sDNA duplex-forming palindromic oligomers, Nakata et al. [7] experimentally studied the lyotropic isotropic-nematic-columnar (I-N-C$_U$) phase sequence and phase transitions among them. The isotropic to cholesteric phase transition for a right handed double helix ultrashort DNA for a large number of sequences ranging from 8 to 20 bases was studied by Zanchetta et al. [8]. Rossi et al. [9] also observed the isotropic to cholesteric phase transition in mixtures of natural D-DNA and L-DNA oligomers. RNA oligomers also produce liquid crystals phases similar to DNA oligomers. It had been found that at high concentrations, short sequences of RNA (six to 12 nucleotides long) could also spontaneously develop into ordered liquid crystal phases. The formation of these liquid crystal phases become much easier when added magnesium ions or polyethylene glycol. Lyotropic columnar liquid crystal phase of RNA mononucleoside tripentaphosphates in aqueous solution had been found [10]. Using the polarized light microscopy and x-ray diffraction method, Haden extensively studied the various liquid crystal phases of RNA mononucleosides [10]. Three distinct phases Isotropic (I), Columnar (C$_U$), and Columnar G-quartets (G) were observed [10]. The G-quartets phase is formed by guanosine monophosphate in solution. When four rGTP molecules bond to one another by eight H-bonds form G-quartets. The G-quartets stacks on one another to form tetraplex columns that exhibit columnar liquid crystal phase. The columnar liquid crystal phase is formed by both G-quartets and G-C base-pairs, i.e. rCTP and rGTP [10]. So the lyotropic I-C$_U$ phase transition is confirmed in this study. The presence of columnar liquid crystal phase is confirmed from the presence of focal conics and/or fan like textures [10]. The selection of these phases was based on the analysis of polarized light microscopy data of rCTP and rGTP samples. The orientation of the optical axis of the liquid crystal columns was determined by the sign of birefringence [10]. Finally it was confirmed that the constituents of RNA (rNTPs) form base-pairs that then form columnar liquid crystal. The columnar liquid crystal was also formed in mixtures of the complementary RNA base-pairs rCTP and rGTP. Todisco et al. [11] studied highly concentrated ordered liquid crystal phases in solutions of short RNA oligomers using nuclear magnetic resonance (NMR) spectroscopy method. They found that solutions of RNA oligomers produce chiral nematic (N$^*$) and C$_U$ liquid crystal ordering in a wide range of chain lengths, concentrations and temperatures. Two coexistence phases N$^*$+C$_U$ and I+C$_U$ were observed. Naturally I-N$^*$-C$_U$ triple point can be observed in temperature – concentration phase diagram in RNA solutions. Zanchetta [12] showed that concentrated solutions of self-complementary RNA oligomers exhibit lyotropic N$^*$ and C$_U$ phases. He measured the cholesteric pitch in nRNA for a particular oligomers length and found that cholesteric pitch is shorter in nRNA than in nDNA [12]. Naturally the I-N$^*$-C$_U$ triple point is formed in RNA solutions. The solutions of very short double-stranded B-DNA and A-RNA, down to six base pairs in length, can also self-organize into N$^*$ and C$_U$ liquid crystal phases [13].

Several Monte Carlo simulations and analytical works [14–22] were undertaken to describe the lyotropic liquid crystals phase transitions in DNA solutions. The formation of the liquid crystal order (nematic and columnar phases) in self-assembling systems was discussed by several authors using Monte Carlo simulations [14,15,18,19]. Hentschke et al. [16] studied the I-N-C$_U$ phase sequence of DNA solutions using molecular dynamics and Monte Carlo simulations study. They pointed out the disappearance of the nematic phase in I-N-C$_U$ triple point as the temperature is increased. Kuriabova et al. [17] also studied the I-N-C$_U$ phase sequence of DNA solutions based on a coarse-grained model. In this study they observed the I-N-C$_U$ triple point both by Monte Carlo simulations and analytic theory. Michele et al. [20] studied I-phase transition using Monte Carlo simulations in an equilibrium bulk solution of blunt ended DNA duplexes undergoing reversible self-assembly into chains. The theoretical predictions of their results confirm the experimental results. In our previous works [21,22], we studied the isotropic to cholesteric phase transition and I-N-C$_U$ phase sequence of DNA solutions using Landau mean-field theory.

There is practically no theoretical studies on the lyotropic liquid crystals phase transitions in RNA solutions available in the literature. So in the present work, we study the lyotropic N$^*$ and C$_U$ phases with special emphasize on the I-C$_U$ phase transition in RNA solutions. We have constructed a Landau free energy density based on the order parameters to study the lyotropic I-C$_U$ phase transition in RNA solutions. Our approach is similar to our previous work [22].

2. Theory

We first define the order parameters to construct the Landau free energy density to describe the lyotropic I-C$_U$ phase transition in RNA solutions. The RNA molecules carry charge. So RNA molecules have both dipole and quadrupole moments [23,24]. Ahmad and Sarai [23] analyzed the of electric moments of RNA-binding proteins. Porschke and Antosiewicz [24] studied the structure of six
different tRNA molecules in solution by electro optical measurements and by bead model simulations. They found that the optical and the electrical anisotropy of tRNA is relatively small, compared with DNA double helices. The dipole moment of RNA is defined as $p = \sum_i q_i r_i$. Where $q_i$ and $r_i$ are the charge and position vector of atom $i$. Now analogous to standard liquid crystal tensor order parameter, we define the RNA order parameter as $Q_{li} = \frac{1}{2} \frac{2\rho_0^2}{\rho} - \frac{1}{2} \delta_{li}$. This is analogous to the tensor order parameter in nematic liquid crystal phase [25]. Now if we assume that both the liquid crystal tensor order and RNA tensor are aligned along the same axis, then RNA has a scalar order parameter $S_{RNA}$ similar to nematic order parameter $S$. $S_{RNA}$ represents the internal configurational nematic order of a single RNA chain. $\phi$ is the another order parameter describes the local concentration difference of the two component systems, i.e. water-RNA system [21]. The columnar phase is a twodimensional lattice order which is condensed from the nematic phase by locating the centers randomly along the discrete column axes [22]. The translational invariance is broken in two directions in the columnar phase. The two main classes of the columnar phases are hexagonal and rectangular columnar phases. The symmetries of these two phases are $D_{6h}$ (hexagonal) and $D_{2h}$ (rectangular). The uniaxial nematic phase has symmetry $D_{\infty h}$. Naturally, both the orientational and translational order parameters exist in the $C_U$ phase. The orientational order parameter in the $C_U$ phase is a tensor order parameter and can be described by $Q_{ij}^\phi(r) = \frac{S(r)}{2} (3n_in_j - \delta_{ij})$ [26]. Here $S(r)$ is only nonzero when $r$ is very close to the column axis. Since the $C_U$ phase is condensed from the nematic phase, the orientational order parameter can be written as $S = S_N + S(r)$, where $S_N$ is the order parameter in the chiral nematic phase. The chiral nematic order parameter proposed by de Gennes [25] is a symmetric, traceless tensor described by $Q_{ij} = \frac{S(r)}{2} (3n_in_j - \delta_{ij})$. Here $n = (n_x, n_y, n_z)$ and is a sinusoidal function. In the geometry which is considered here the chiral nematic director is assumed to be $n_i = e_x \cos k_0 z + e_y \sin k_0 z$. Here the direction of the helix axis is in the $z$ direction. $k_0$ is the wave vector. Thus there is a possibility of the lyotropic chiral nematic to columnar phase transition in RNA solutions. The translational order parameter in the $C_U$ phase is described by [27] $\delta \rho = \frac{1}{2} \sum_q \chi_q e^{iqr} + c.c.$.

Here $q$ lies in the plane of the lattice [27].

Considering above described order parameters, the free energy density near the I-$C_U$ phase transition in RNA solutions can be written as

$$F = F_0 + \frac{1}{2} a S^2 - \frac{1}{3} b S^3 + \frac{1}{2} L_2 S^2 k_0 - \frac{1}{2} L_3 S k_0 + \frac{1}{4} c S^4$$

$$+ \frac{1}{2} m |\delta\rho|^2 + \frac{1}{2} n |\delta\rho|^4 + \frac{1}{2} u S^2_{RNA} - \frac{1}{3} v S^3_{RNA}$$

$$+ \frac{1}{4} w S^4_{RNA} + \frac{1}{2} \lambda_3 |\delta\rho|^2 S_{RNA}$$

$$\delta S S_{RNA} + \eta_1 S \phi + \eta_2 S_{RNA} \phi + \eta_3 |\delta\rho|^2 \phi$$

$$+ k_\beta T \phi (ln\phi - 1) - \mu \phi$$

(1)

where $F_0$ is the free energy density of the isotropic phase. $\mu$ is the chemical potential difference between two components. $k_\beta$ is the Boltzmann constant. $L_2$ and $L_3$ are the orientational elastic constants. The term proportional to $L_3$ is responsible for the formation of a helical ground state in the chiral nematic phase. We assume $L_3 > 0$, $\lambda_2, \lambda_3, \eta_1, \eta_2, \eta_3$, and $\delta$ are coupling constants. Here we assume the anisotropy of the RNA molecules remains small in the liquid crystal phases where $S$ is large. This assumption is described by the local coupling terms $\delta S S_{RNA}$ and $\frac{1}{2} \lambda_3 |\delta\rho|^2 S_{RNA}$. Physically, $\delta$ and $\lambda_2$ represent the change in the microscopic interactions which stabilize the liquid crystalline phases. We have neglected the coupling term $|\delta\rho|^2 S$ in the free energy density [1] since this term can’t exists in the isotropic phase. For the thermodynamic stability, we assume $b > 0$, $c > 0$, $n > 0$, $\nu > 0$, $w > 0$, $\lambda_1 < 0$, $\lambda_2 > 0$, $\eta_1 < 0$, $\eta_2 < 0$, $\eta_3 < 0$ and $\delta > 0$. The term $k_\beta T \phi (ln\phi - 1)$ represents the entropy of isotropic mixing. The term $-\mu \phi$ represents the internal energy contribution. The term $\frac{1}{2} u S^2_{RNA}$ is the entropic cost of imposing RNA order on liquid crystal. We define $u = k_\beta T \phi$.

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First we minimize Equation (1) with respect to $\phi$ for a fixed chemical potential. After the minimization of Equation (1) with respect to $\phi$ for a fixed chemical potential, we get

$$\phi = \phi_0 e^{-\frac{\eta_1 S + \eta_2 S_{RNA} + \eta_3 |\delta\rho|^2 + \delta S S_{RNA}}{\frac{1}{2} u S^2_{RNA}}}$$

(2)

Substitution of $\phi$ from Equation (2) into Equation (1), we get
\[ F = F_0 + \frac{1}{2} a_1 S^2 - \frac{1}{3} b_1 S^3 + \frac{1}{2} L S^2 k_0^2 - \frac{1}{2} L_3 S^2 k_0 + \frac{1}{4} c S^4 \]
\[ + \frac{1}{4} m_1 |\delta \rho|^2 + \frac{1}{4} n_1 |\delta \rho|^4 + \frac{1}{2} u S_{RNA}^3 - \frac{1}{3} v S_{RNA}^3 \]
\[ + \frac{1}{4} w S_{RNA}^3 + \frac{1}{2} \lambda_2 |\delta \rho|^2 S_{RNA}^2 + \frac{1}{2} \lambda_3 |\delta \rho|^2 S^2 \]
\[ + \delta S_{RNA} - k_B T \phi_0 e^{-\frac{(n_1 + n_2 + n_3) \phi_0}{k_B T}} \]
\[ (3) \]

Now if \( \frac{n_1}{k_B T}, \frac{n_2}{k_B T}, \frac{n_3}{k_B T} \) and \( \frac{1}{2} S_{RNA}^2 \) are small in the region of interest, the exponential in Equation (3) can be expanded. Then by the expansion of \( e^{-\frac{c S^4}{k_B T}} \) up to quartic terms free energy density [3] can be written as

\[ F = F_0 + \frac{1}{2} a_1 S^2 - \frac{1}{3} b_1 S^3 + \frac{1}{2} L S^2 k_0^2 - \frac{1}{2} L_3 S^2 k_0 - \frac{1}{4} c S^4 \]
\[ + \frac{1}{4} m_1 |\delta \rho|^2 + \frac{1}{4} n_1 |\delta \rho|^4 + \frac{1}{2} u S_{RNA}^3 + \frac{1}{3} v S_{RNA}^3 + \frac{1}{4} w S_{RNA}^3 \]
\[ + \frac{1}{2} \lambda_2 |\delta \rho|^2 S_{RNA}^2 + \frac{1}{2} \lambda_3 |\delta \rho|^2 S^2 \]
\[ + \delta^* S_{RNA} - n_1 \phi_0 \frac{S^2}{k_B T} + \eta_2 \phi_0 S_{RNA} - k_B T \phi_0 \]
\[ (4) \]

where

\[ a_1 = a - \frac{2 \phi_0 \eta_1^2}{k_B T}, \]
\[ u_1 = k_B T \phi_0 - \frac{2 \phi_0 \eta_1^2}{k_B T}, \]
\[ v_1 = v + 3 \eta_2 \phi_0, \]
\[ w_1 = w - k_B T \phi_0, \]
\[ m_1 = m + 2 \eta_2 \phi_0, \]
\[ n_1 = n - \frac{4 \eta_2 \phi_0}{k_B T}, \]
\[ \delta^* = \delta - \frac{2 \eta_2 \phi_0}{k_B T}, \]
\[ \lambda_1 = \frac{4 \eta_2 \phi_0}{k_B T}, \]
\[ \lambda_2^* = \lambda_2 - \frac{4 \eta_2 \phi_0}{k_B T}. \]

The value of \( S_{RNA} \) in the RNA phase can be calculated by the minimization of the free energy density [4] with respect to \( S_{RNA} \) for \( S = 0 \) and \( |\delta \rho| = 0 \) which can be expressed as

\[ u_1 S_{RNA} - v_1 S_{RNA}^3 + w_1 S_{RNA}^3 + \eta_2 \phi_0 = 0 \]
\[ (5) \]

To find the variation of \( S_{RNA} \) with temperature in the RNA phase, we have plotted Equation (5) for a fixed concentration. Figure 1 shows the variation of \( S_{RNA} \) with temperature for a fixed concentration \( \phi_0 = 0.1 \) and fixed oligomer length for a parameter values \( u_0 = 0.1 \) J/m³, \( v = 0.6 J/m³, w = 6.4 J/m³ \) and \( \eta_2 = -0.8 \times 10^{-11} J/m³ \). As can be seen from Figure 1, \( S_{RNA} \) decreases with temperature.

We now calculate the effect of \( S_{RNA} \) on the \( N^\ast \) and \( C_U \) phases. In the following we neglect the higher order terms like \( S_{RNA}^3 \) and \( S_{RNA}^4 \) in Equation (4) for the simplification of the calculation. Then the minimization of Equation (4) with respect to \( S_{RNA} \), we get

\[ S_{RNA} = \frac{\left( \delta^* S + \frac{1}{2} \lambda_2 |\delta \rho|^2 + \eta_2 \phi_0 \right)}{\left( u_1 - 2 \eta_1 \phi_0 k_B T S - 2 \eta_3 \phi_0 |\delta \rho|^2 \right)} \]
\[ (6) \]

The values of \( S \) and \( |\delta \rho|^2 \) can be calculated from Equations (11) and (12) respectively.

Now the substitution of Equation (6) into Equation (4) and elimination of \( k_0 \) gives

\[ F = F_0 + \frac{1}{2} a_1 S^2 - \frac{1}{3} b_1 S^3 + \frac{1}{2} L S^2 k_0^2 - \frac{1}{2} L_3 S^2 k_0 + \frac{1}{4} c S^4 \]
\[ + \frac{1}{4} m_1 |\delta \rho|^2 + \frac{1}{4} n_1 |\delta \rho|^4 + \frac{1}{2} u S_{RNA}^3 + \frac{1}{3} v S_{RNA}^3 + \frac{1}{4} w S_{RNA}^3 \]
\[ + \delta^* S_{RNA} - \frac{n_1 \phi_0}{u_1} \frac{S^2}{k_B T} + \eta_2 \phi_0 S_{RNA} - k_B T \phi_0 \]
\[ \frac{(n_1 + n_2 + n_3) \phi_0}{k_B T} \]
\[ + \lambda_1 \frac{4 \eta_2 \phi_0}{k_B T}, \]
\[ \lambda_2^* = \lambda_2 - \frac{4 \eta_2 \phi_0}{k_B T}. \]

Figure 1. Variation of \( S_{RNA} \) with temperature for a fixed RNA concentration.
where

\[ a_1^* = a_1 - \frac{\delta_1^2}{u_1} - 2\delta_1^2 - 4\eta_1 \eta_2 \delta_1^2 \phi_0^2 - \frac{L_1^2}{4L}, \]

\[ b_1^* = b + \frac{3\eta_1 \delta_1^2 \phi_0^2}{u_1^2} - 6\eta_1^2 \eta_2 \delta_1^2 \phi_0 \frac{(k_B T)^2}{u_1^3}, \]

\[ c_1^* = c - \frac{16\eta_1^2 \eta_2 \delta_1^2 \phi_0 \frac{(k_B T)^2}{u_1^3}}, \]

\[ m_1^* = m_1 - \frac{\lambda_1^* \eta_2 \phi_0}{u_1} + \frac{\eta_2 \lambda_2^* \phi_0}{u_1}, \]

\[ n_1^* = n_1 - \frac{\lambda_1^* \lambda_2^* \phi_0}{u_1} + \frac{\lambda_2^* \lambda_3^* \phi_0}{u_1}. \]

The conditions of the first order \( I-N^*, I-C_U \)

and \( N^*-C_U \) transitions in terms of the orientational order parameters can be expressed as

\[ F_{N^*}^C(S_N) = F_0, F_{N^*}^C(S_N) = 0, F_{N^*}^C(S_N) \geq 0 \quad (8) \]

\[ F_{C_U}(S) = F_N^C(S), F_{C_U}(S) = 0, F_{C_U}(S) \geq 0 \quad (9) \]

\[ F_{C_U}(S) = F_0, F_{C_U}(S) = 0, F_{C_U}(S) \geq 0 \quad (10) \]

Where \( S_N \) and \( S \) are the orientational order parameter of the \( N^* \) and \( C_U \) phases respectively. \( F_{N^*} \)

and \( F_{C_U} \) are the free energy densities of \( N^* \) and \( C_U \) phases. A schematic phase diagram of the \( I, N^* \)

and \( C_U \) phases and the \( I-N^*-C_U \) triple point in RNA solutions is shown in Figure 2. This analysis agrees with results of Todisco et al. [11].

Now we discuss the lyotropic \( I-C_U \) phase transition in details.

### 2.1. \( I-C_U \) phase transition

Minimization of Equation (7) with respect to \( |\delta \rho| \), the value of the translational order parameter in the \( C_U \) phase can be written as

\[ |\delta \rho|^2 = \frac{1}{n_1} (m_1^* + \lambda_1^* S + \lambda_3^* S^2) \quad (11) \]

Equation (11) shows that a nonzero real value of \( \delta \rho \)

exists only when \( m_1^* + \lambda_1^* S + \lambda_3^* S^2 < 0 \). Substitution of \( |\delta \rho|^2 \) from Equation (11) into Equation (7), we get

\[ F = F_0^* + \frac{1}{2} a^* S^2 - \frac{1}{3} b^* S^3 + \frac{1}{4} c^* S^4 \]

Figure 2. A schematic phase diagram in the vicinity of the \( I-N^*-C_U \) triple point in RNA solutions.
where the renormalized coefficients are
\[ a^* = a_0 - \frac{\lambda^2_1}{4n_1^3}, \]
\[ b^* = b_0 - \frac{\lambda^2_1}{2n_1^2}, \]
\[ c^* = c_0 + \frac{\lambda^2_2}{n_1^2}, \]
\[ F_{0}^{*} = F_{0} - \frac{m_1^2}{4n_1^3}. \]

So the lyotropic I-C_{U} phase transition is described by the free energy density [12]. The lyotropic I-C_{U} phase transition is always a first order transition because the presence of the cubic invariant b* in the free energy density [12]. The stronger or weaker of the first order character of the lyotropic I-C_{U} phase transition solely depends on the magnitude of the value of b*. The variation of the orientational and translational order parameters with temperature for a fixed concentration \( \phi_0 = 0.2 \) and fixed oligomer length can be plotted using Equations (11) and (12). This is shown in Figure 3. This is done for a particular set of phenomenological parameters values, \( a_0 = 0.1 \text{ J/m}^3, b = 0.7 \text{ J/m}^3, c = 3.2 \text{ J/m}^3, m_0 = 0.1 \text{ J/m}^3, n = 2.2 \text{ J/m}^3, \eta_2 = 0.5 \times 10^{-5} \text{ J/m}^3, \eta_3 = -0.7 \times 10^{-5} \text{ J/m}^3, \delta = 0.9 \text{ J/m}^3, \lambda_2 = 0.9 \text{ J/m}^3, \lambda_3 = -0.3 \text{ J/m}^3. \) Figure 3 clearly shows that both the orientational and transitional order parameters jump simultaneously at the phase transition point. So we conclude that the free energy density [1] and renormalized free energy density [12] describe the first-order character of the lyotropic I-C_{U} phase transition. Naturally coexistence phase of the isotropic and C_{U} phases can be observed which will be bounded by the loss of stability line of the C_{U} phase. The C_{U} phase is stable only when \( a^* < 0 \) i.e.

\[ T < T_{I-C_\text{U}}(\phi_0) \]

where

\[ T_{I-C_\text{U}}(\phi_0) = \frac{1}{a^*} \left[ \frac{a^*_0 + L^3}{4L} + \frac{m_0 T^*_1 V_1}{A_2} - K_1 \right. \]
\[ + \frac{m_0 T^*_2 V_2}{K_2} \]
\[ + \frac{\phi_0}{\phi_0} \left. + \frac{m_0 T^*_3 V_3}{K_3 \phi_0 - K_4 \phi_0^2} \right] \]

(14)

where the coefficients \( A_2, K_1, K_2, K_3, K_4, V_1, V_2 \) and \( V_3 \) are defined in Appendix I.

The conditions defined in Equation (10) describes the first order lyotropic I-C_{U} phase transition line.

### 2.2. Birefringence

To calculate the birefringence in the isotropic phase of the lyotropic I-C_{U} phase transition, we have to add the electric field terms \( -E^2 S \) and \( -E^2 S_{\text{RNA}} \) into the free energy density [4]. After adding those terms into the free energy density [4] and elimination of \( S_{\text{RNA}} \) and \( \Delta \rho \), we get the free energy density as

\[ F = \frac{1}{2} a^* S^2 + \left( \eta_1 \phi_0 - \frac{\eta_2^* \phi_0 - 2 \eta_2^* \phi_0}{u_1} - \frac{m_1^2 \lambda^*_1}{2n_1^3} \right) S \]
\[ - \left( \frac{\delta^*}{u_1} + \frac{\lambda^*_1 A^*_2}{2u_1 n_1^3} - 1 \right) E^2 S \]
\[ - \frac{\lambda^*_2 \lambda^*_3}{2u_1 n_1^3} E^2 S^2 + O(E^3, E^4) \]

(15)

In Equation (15), we have neglected the terms like \( S^3, S^4, \ E^2 |\Delta \rho|^2, \ E^2 \) and \( E^4 \) since up to the quadratic terms are sufficient enough to calculate the birefringence in the isotropic phase.

Minimization of Equation (15) with respect to \( S \), the electric field dependence of \( S \) can be written as

\[ S(E) = \left( \frac{\eta_1 \phi_0 - \frac{\eta_2^* \phi_0 - 2 \eta_2^* \phi_0}{u_1} - \frac{m_1^2 \lambda^*_1}{2n_1^3}}{a^* - \frac{m_1^2 \lambda^*_1}{u_1 n_1^3} E^2} \right) \]

(16)

Equation (16) can be simplified as

\[ S(E) = \frac{E^2}{a^* \left( B_0 - \frac{B \lambda^*_1 \lambda^*_3}{u_1 n_1^3} \right)} \]

where

\[ B_0 = \frac{B \lambda^*_1 \lambda^*_3}{u_1 n_1^3} \]

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3. Conclusion

We have presented a Landau theory analysis to describe the lyotropic N° and C\textsubscript{U} phases with a special emphasize on the I-C\textsubscript{U} phase transition in RNA solutions. The effect of RNA concentrations on the lyotropic I-C\textsubscript{U} phase transition has been discussed. The conditions of the lyotropic I-C\textsubscript{U} phase transition have been derived. The lyotropic I-C\textsubscript{U} phase transition is always a first order transition. The RNA concentration dependence of birefringence in the isotropic phase of the transition has been calculated. Theoretical observation of the existence of the lyotropic I-C\textsubscript{U} phase transition in RNA solutions is confirmed with experimental results.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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