Experimental investigation of Popper’s proposed ghost-diffraction experiment

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In an effort to challenge the Copenhagen interpretation of quantum mechanics, Karl Popper proposed an experiment involving spatially separated entangled particles. In this experiment, one of the particles passes through a very narrow slit, and thereby its position becomes well-defined. This particle therefore diffracts into a large divergence angle; this effect can be understood as a consequence of the Heisenberg uncertainty principle. Popper further argued that its entangled partner would become comparably localized in position, and that, according to his understanding of the Copenhagen interpretation of quantum mechanics, the “mere knowledge” of the position of this particle would cause it also to diffract into a large divergence angle. Popper recognized that such behaviour could violate the principle of causality in that the slit could be removed and the partner particle would be expected to respond instantaneously. Popper thus concluded that it was most likely the case that in an actual experiment the partner photon would not undergo increased diffractive spreading and thus that the Copenhagen interpretation is incorrect. Here, we report and analyze the results of an implementation of Popper’s proposal. We find that the partner beam does not undergo increased diffractive spreading. Our work resolves many of the open questions involving Popper’s proposal, and it provides further insight into the nature of entanglement and its relation to the uncertainty principle of correlated particles.

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I. INTRODUCTION

Among different theories of physics, quantum mechanics plays an essential role in describing atomic and subatomic phenomena. Despite the fact that its formalism passes every rigorous experimental test, quantum mechanics has lead to many controversies since its inception. In a seminal paper, Einstein, Podolsky and Rosen (EPR) conceived a gedankenexperiment using entangled particles to argue against the completeness of quantum mechanics [1]. They made use of a pair of particles that are assumed perfectly correlated in both position and momentum - the EPR state. Their concern was that their thought experiment appeared to allow for the simultaneous reality of conjugate quantities, in apparent conflict with the Heisenberg uncertainty principle (HUP). The modern answer to the EPR argument is that quantum mechanics is a truly nonlocal theory [2], although it is nonetheless consistent with causality in that information cannot be transmitted faster than the speed of light, i.e. it obeys a no-signaling principle [3].

In addition to the EPR work, Karl Popper outlined an experiment that challenged the predictions of quantum mechanics based on its Copenhagen interpretation [4,5]. He designed an experiment that he believed had two possible outcomes, as shown in Fig. 1. The first outcome would conflict with relativistic causality and the concept that information cannot travel faster than the speed of light, that is, with the no-signaling principle. The second outcome would be in conflict with Popper’s understanding of the predictions of the Copenhagen interpretation of quantum mechanics, and specifically with the concept that not all information about a single quantum system can be acquired simultaneously, that is, with the Heisenberg uncertainty relation between conjugate quantities. Popper favoured causality over the Copenhagen interpretation and tended to favour the second outcome.

In this article, we present experimental evidence that the second outcome is in fact what is observed, thus confirming that there is no violation of causality. Nonetheless, our observation of the second scenario does not challenge the standard interpretation of quantum mechanics. We present theoretical arguments that show that this result is in fact consistent both with relativistic causality and with the formalism of quantum mechanics.

Since the time of Popper’s proposal, many scientists have looked into the questions he raised [6–13]. Notably, in 1987, Collett and Loudon proposed a more realistic model of Popper’s experiment where the source, instead of being a point source, is extended in the transverse direction of space [9]. They concluded that the transverse uncertainty of the source causes the diffraction spread on side B to decrease with the size of the slit on side A, hence reconciling quantum mechanics with causality. Ghirardi et al. have a radically different view of Popper’s experiment [13]. They claim that the local actions performed at the plane of the slit on side A are completely uncorrelated with the diffraction spread on side B. In 1999, Kim and Shih [10] reported experimental results showing that the “ghost” slit does not induce increased diffractive spreading. However, suspicion with regards to the explanation of their results was raised by Short [11], who formulated the results of the Popper experiment in terms of conditional uncertainties. Further theoretical work has questioned whether Popper’s thought experiment truly addresses either the Copenhagen interpretation or the HUP [12,13].

II. THEORY

The thought experiments of EPR and of Popper are closely related to the HUP, one of the fundamental concepts of quan-
The HUP states that there is a fundamental limit to the accuracy with which one can gain simultaneous knowledge of conjugate quantities, in our case the position and momentum of a single particle as described by $\Delta x \Delta p \geq \hbar/2$. Here $\Delta x$ and $\Delta p$ stand for uncertainty in position and momentum, respectively, and $\hbar$ is the reduced Planck constant.

According to the arguments of EPR, either quantum mechanics is an incomplete theory or else it allows for simultaneous reality of the conjugate quantities position $x$ and momentum $p$ of a quantum system. However, Popper considered a somewhat different situation: he considered the implications of the HUP on the position and momentum of the second particle when in both cases the measurement on the first particle is made on the position degree of freedom. In this situation, one must consider both position-position and position-momentum correlations. Popper thought that this type of inferred measurement could lead to a violation of the uncertainty principle. Indeed, in the same manner in which the standard uncertainty principle has a lower bound, the uncertainty principle relevant to Popper’s thought experiment is

$$\Delta(x_B|x_A) \Delta(p_B|x_A) \geq \hbar/2.$$  \hspace{1cm} (1)

This equation should be interpreted as the uncertainty product for particle $B$ given a measurement of the position of particle $A$. Below, we show that this inequality can be saturated for the case of a pure two-photon entangled state that exhibits strong EPR correlations.

In our implementation of Popper’s experiment, the entangled particles are generated through spontaneous parametric downconversion (SPDC), the theory of which is well-known \cite{14, 16}. Here, we consider only the horizontal distribution along $x$, which is perpendicular to the real slit, and sum our two-dimensional data over the $y$ degree of freedom.

At the output of the crystal, the two-photon mode function takes the form \cite{14, 15}

$$\Psi(x_B, x_A) = N \exp\left(-\frac{(x_A + x_B)^2}{4\sigma_p^2}\right) \exp\left(-\frac{(x_A - x_B)^2}{4\sigma_q^2}\right).$$ \hspace{1cm} (2)

where $N = (\pi \sigma_p \sigma_q)^{-1}$ is a normalization constant, $\sigma_p$ is the $1/e^2$ width of the intensity of the collimated Gaussian-distributed pump, and $\sigma_q$ is the width of the position correlations, which is a function of the length of the crystal (see Materials and Methods for more details). Upon detection of a photon in arm $A$ at position $x_A = 0$, the conditional state of photon $B$ takes the form of a Gaussian distribution:

$$|\Psi(x_B|x_A = 0)|^2 = N' \exp\left(-\frac{x_B^2}{2\sigma_q^2}\right),$$ \hspace{1cm} (3)

where $N'$ is a normalization constant. The standard deviation (SD) of the associated probability distribution is equal to $\Delta(x_B|x_A = 0) = \sigma_q$. We used the fact that the pump width $\sigma_p (\approx 450 \mu m)$ is much greater than that of the mode width of the SPDC position correlations $\sigma_q (\approx 10 \mu m)$, i.e. $\sigma_p \gg \sigma_q$. The conditioned mode function of photon $B$ can be treated as a pure state (see Eq. (3) and undergoes diffraction like a pure state. Using the Fourier transform of the position wave function of photon $B$, we find the conditional state of photon $B$ in momentum space as

$$|\Psi(p_B|x_A = 0)|^2 = N'' \exp\left(-2p_B^2\sigma_q^2/\hbar^2\right).$$ \hspace{1cm} (4)

where $p_B$ is the momentum of photon $B$, and $N''$ is a normalization constant, respectively. The SD of the probability distribution $|\Psi(p_B|x_A = 0)|^2$ is equal to $\Delta(p_B|x_A = 0) = \hbar/(2\sigma_q)$. This distribution is identical to that of the unconditioned Gaussian distribution, and thus, for the case of the entangled state \cite{2}, the quantum formalism predicts that scenario $B$ from Fig. 1 will occur.
the conditioned standard deviations associated with Eqs. (3) and (4), we obtain
\[ \Delta(x_B|x_A = 0) \Delta(p_B|x_A = 0) = \hbar/2, \] (5)
a result that is consistent with the uncertainty principle in the form of Eq. (1). This result is to be expected since Eq. (3) and Eq. (4) are conjugate quantities related by Fourier transforms. It follows that for the initial state (2), the presence of a slit in臂B does not lead to increased diffusive spreading of photon臂B as the slit width is decreased. Therefore, quantum mechanics leads to the prediction of the second scenario (B) shown in Fig. 1.

![Schematic of our implementations of Popper’s experiment](image)

**FIG. 2. Schematic of our implementations of Popper’s experiment.** Entangled photon pairs are generated through spontaneous parametric downconversion (SPDC) in a β-barium borate (BBO) crystal using type-I phase matching. The real slit on side \( A \) is placed in an image plane of the crystal, and a large-area (bucket) detector (BD) is placed after the slit; this detector registers those photons that pass through the slit and thereby determines their position with high accuracy. Photon \( B \) goes through an image-preserving delay line, and then is captured by an intensified CCD (ICCD) camera that is triggered by the photons that pass through the real slit on side \( A \) (see Materials and Methods for more details).

**III. EXPERIMENTAL RESULTS**

Figure 2 shows the schematic representation of our implemented Popper’s thought experiment. Generated photon pairs are split out by means of a 50/50 non-polarizing beam splitter. They are sent into a 10-μm-wide slit that located at the image plane of the downconversion crystal in arm A and an image-preserving delay line in arm B, respectively. Photons that pass through the slit in arm A are registered by a bucket detector. This detector registers those photons that pass through the slit and thereby determines their position with high accuracy. We make measurements on a photon in arm B that are conditioned on the detection of a photon in arm A.

A triggered intensified CCD camera placed in arm B in the image plane of the real slit and thus also in the image plane of the crystal is used to measure in coincidence the width of the ghost slit (see Materials and Methods for more details). In Fig. 3, the frame at \( z = 0 \) mm shows an image of the ghost slit. The width of this image, expressed in terms of its SD, is \( \Delta(x_B|x_A \text{ within slit}) = (19 \pm 1) \mu m \). The predicted width of the image is given by the convolution of the transmission function of the real slit (10 μm wide) with the point-spread function of the imaging system, which is given by the mode function \(|\Psi(x_B, x_A)|^2\) taken to be a function of \( x_B \). This calculated width is about 12 μm. The measured 19 μm width of the ghost slit is greater than the calculated width of 12 μm because of non-ideal position correlations of the two photons, which we believe arise from misalignment or aberrations in our imaging system. Nonetheless, to within a factor of two, we confirm Popper’s expectation that the position of photon \( B \) is measured with “approximately the same precision” as that of photon \( A \). Of course, a complication raises from the fact that width of an image is often specified in terms of its SD. For a uniform transmission distribution, e.g. for a slit of width \( d \), this SD is equal \( d/\sqrt{12} \). However, in our analysis we take the physical width of the slit to be the measure of the position of photon \( A \).

Using a lens with a focal length of 75 mm, we next place the ICCD camera in the far-field (Fourier transform plane) of the ghost slit and record the distributions of the singles and coincidence counts, as shown in Fig. 3(B). The standard deviations of these distributions are respectively \( \Delta(p_B)/\hbar = (0.048 \pm 0.001) \mu m^{-1} \) and \( \Delta(x_B|x_A \text{ within slit})/\hbar = (0.046 \pm 0.006) \mu m^{-1} \). The widths of the two distributions are very similar. We conclude from these results that the detection of photon \( A \) after the real slit does not affect the far-field distribution of photon \( B \). To express this thought more quantitatively, we note that for these data we find an uncertainty product of
\[ \Delta(p_B|x_A \text{ within slit}) \Delta(x_B|x_A \text{ within slit}) = (0.87 \pm 0.16) \hbar. \] (6)

This product is greater than \( \hbar/2 \) and is thus consistent with the uncertainty principle. The measured uncertainty product does not saturate the uncertainty principle of Eq. (5). As mentioned before, we attribute this increased uncertainty to aberrations and misalignment along the optical axis.

In Fig. 4(A), we compare the measured width of the conditional diffraction from the ghost slit (black points) to the theoretical width of a propagating Gaussian beam (black curve): \( a^2(z) = a^2(0) \sqrt{1 + (z/z_R)^2} \), where \( z_R \) is the Rayleigh range associated with the conditioned mode-function of photon \( B \). The theory fits very well for all points other than the point at \( z = 0 \), and we attribute this difference also to a slight misalignment along the optical axis. We also show the width of the singles counts on side \( B \) (red points) as a function of propagation distance. The width of the singles and the coincidence counts asymptotically approach one another upon propagation, and as confirmed by Fig. 4(B) match perfectly at the far-field of the ghost slit.

**IV. DISCUSSION**

The results from our experiment confirm that the second scenario outlined in Fig. 1(B) prevails. This outcome is precisely what Popper expected on the basis of the argument that it conforms to the principle of causality. Popper was correct
FIG. 3. **Experimental observation of ghost diffraction from a slit.** (A) Coincidence images from the ICCD showing ghost diffraction from a slit upon propagation. Images were recorded between the near-field of the BBO crystal \( z = 0 \) mm and the far-field of the crystal \( z \rightarrow \infty \) (not shown). Here we show five images at 10 mm increments from the ghost slit. (B) Comparison between the ghost diffraction (conditioned \( p_B|x_A \) within slit) and singles (unconditioned \( p_B \)) in the far-field \( (z \rightarrow \infty) \). The black points show data for the conditional case, measured in coincidence; the red points show data for the unconditional case. Note that the conditional and unconditional distribution of photon \( B \) are essentially identical showing that placing a slit in arm \( A \) does not influence the momentum distribution of photon \( B \).

FIG. 4. **Measured transverse width of particle \( B \)'s field as a function of distance from a ghost slit.** The black points show data for the conditional widths; the red points show data for the unconditional widths. The black line is a fit to the conditioned data using Gaussian beam propagation. The error bars correspond to a 95% confidence region of the width of a Gaussian distribution fitted to the recorded transverse distributions.

that the events in the near-field of the real slit on side \( A \) cannot influence the outcome of events in the far field of side \( B \) \cite{19}. We find that the presence or absence of the real slit in arm \( A \) leads to no difference between the conditional and unconditional distributions on side \( B \) in the far field. We can express this thought somewhat differently as follows. The rate of singles counts measured at any position on side \( B \) is the same whether or not a position measurement is made on side \( A \). In contrast, for coincidence measurements the distribution on side \( B \) is narrower than the singles distribution in the near-field of the ghost slit but has the same width in the far-field. Thus, the presence or absence of the slit on side \( A \) does not affect the photon distribution in singles or in coincidence in the far field for side \( B \). We have also shown theoretically that the quantum formalism is consistent with the second scenario, which is causal. This is in contrast to what Popper thought based on his understanding of the Copenhagen interpretation of quantum mechanics, which would preclude to scenario 2. One possible flaw in Popper’s reasoning of the physical situation is that he assumed that the source of particle pairs was both perfectly correlated in transverse momentum and had a very small transverse extent \cite{9}. Ghirardi et al. argue that the laws of quantum mechanics prohibit the simultaneity of these two properties \cite{13}. Thus, if the position of the source is known very well, then according to \cite{13} no correlations in momentum are possible.

In the context of Popper’s experiment, the relevant uncertainty relation involves uncertainties in the position and the momentum of photon \( B \) conditioned on the position of photon of \( A \) \cite{11}. Quantum theory predicts that this product is equal to \( h/2 \) and saturates the HUP. In our experiment, we obtained a value of 0.87\( h \) for this product because of imperfections in our laboratory setup.

V. **CONCLUSIONS**

In conclusion, we have implemented Popper’s proposed experiment with entangled photon pairs generated through SPDC. Our measurements of the diffraction from the ghost image of a narrow slit closely matches the predictions of the corresponding model, derived from the theory of SPDC. Our results are consistent both with the quantum formalism, i.e. the HUP and with causality (the no-signaling principle) and confirm that there is no spread in the momentum distribution of one entangled photon due to presence of the slit for the other photon. We find that the conditional HUP, \( \Delta(p_B|x_A = 0) \Delta(x_B) \geq h/2 \), is validated theoretically and experimentally.
where $\phi_k$ determines the mode of the singles is \( \exp[-(x A + z B)^2/(4\sigma_a^2)] \), while the term that mostly determines the mode of the singles is \( \exp[-(x A + x B)^2/(4\sigma_e^2)] \), where $\sigma_e$ will be defined later (it is a function of $\sigma_e$). Indeed, if we assume perfect position correlations, $\delta(x A - x B)$, the intensity profile of the singles is exactly given by the term \( \exp[-(x A + x B)^2/(4\sigma_e^2)] \).

### Appendix B: Experimental setup

In the experiment a frequency-tripled quasi CW mode-locked Nd:YAG laser (not shown) with a repetition rate of 100 MHz and average output power of $P = 150$ mW at $\lambda = 355$ nm is used to pump a 3 mm thick $\beta$-barium borate (BBO) crystal cut for type-I degenerate phase matching. The generated photon pairs (photon $A$ and photon $B$) via SPDC are split out by means of a 50/50 non-polarizing beam splitter (BS) and sent into an actual slit (path $A$) and a delay line (path $B$), respectively. Photon $A$ is imaged on a 10 $\mu$m slit (SL) via a 4f-system with a unit magnification, and then coupled into a 200 $\mu$m core diameter multimode optical fibre. The coupled photons are detected by a silicon avalanche photodiode (APD) and used to trigger an intensified charge coupled device (ICCD) camera. On the other side, photon $B$ is sent to a delay line, which is made of a polarizing beam splitter (PBS), three 4f-system and a high-pass filter (F). The two photons of a given pair are separated by a 50/50 non-polarizing beam-splitter (BS). The mode of photon $A$ is imaged with unit magnification to a 10-µm-slit and then collected by a multimode optical fiber with a 200 $\mu$m core diameter that is connected to a single-photon detector that triggers the ICCD camera. Photon $B$, initially vertically polarized, is reflected by the polarization beam-splitter (PBS) towards an image-preserving delay line. Photon $B$ hits the final mirror of the delay line and makes its way back with a horizontal polarization because of its two passes in the quarter-wave plate. Photon $B$ thus traverses the PBS and reaches the ICCD camera, which records photon $B$ in the far-field of the BBO crystal. The BBO crystal is imaged with a magnification of one to the plane indicated by $z = 0$ mm.

![Schematic of our implementation of Popper’s experiment.](image)

**FIG. A1.** Schematic of our implementation of Popper’s experiment. Entangled photon pairs are generated through SPDC at a $\beta$-barium borate (BBO) type-I crystal. The pump beam is filtered out by a high-pass filter (F). The two photons of a given pair are separated by a 50/50 non-polarizing beam-splitter (BS). The mode of photon $A$ is imaged with unit magnification to a 10-µm-slit and then collected by a multimode optical fiber with a 200 $\mu$m core diameter that is connected to a single-photon detector that triggers the ICCD camera. Photon $B$, initially vertically polarized, is reflected by the polarization beam-splitter (PBS) towards an image-preserving delay line. Photon $B$ hits the final mirror of the delay line and makes its way back with a horizontal polarization because of its two passes in the quarter-wave plate. Photon $B$ thus traverses the PBS and reaches the ICCD camera, which records photon $B$ in the far-field of the BBO crystal. The BBO crystal is imaged with a magnification of one to the plane indicated by $z = 0$ mm.

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