Solar Cycle Characteristics and Their Relationship with Dynamo Theory

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Abstract.
We try to establish the correlation between different parameters of “butterfly-diagrams” derived from the analysis of solar observational data for the 12–23 solar activity cycles and the values in the models of $\alpha$-$\Omega$-dynamo using RGO – NASA/Marshall data set. We have ascertained that there is a linear relationship between $S$ and $BT/L$ for all the investigated cycles, where $S$ is the mean area of the sunspots (umbrae), $B$ is the mean magnetic field strength, $T$ is duration of a cycle and $L$ is the mean latitude of the sunspots in a cycle.

1. Introduction
It is generally accepted that the 11-year cycle of solar activity is connected with the action of the magnetic dynamo, the mechanism of which is based on the mutual work of the $\alpha$-effect and the differential rotation [1]. It is assumed that Sun magnetic field has two components: the poloidal and the toroidal. The toroidal magnetic field originates from the poloidal one owing to the action of the differential rotation, which is situated inside the convective zone of the Sun. The reverse process of the transformation of the toroidal magnetic field into the poloidal one is realized as a result of the violation of the mirror symmetry of convection in a rotating body. The Coriolis force acting on the arising and diverging (descending and converging) vortices leads to the prevalence of right vortices in the northern hemisphere and left vortices in the southern one. The electromotive force, appearing as a result of the action of the Faraday’s electromagnetic induction, after the averaging by velocity pulsations gain the component which is parallel to the mean magnetic field. It is this component that make close the self-excitation circuit in the Parker dynamo.

The 11-year cycle of solar activity is connected with its spot-producing activity. At the beginning of the solar activity cycle the spots appear at the mean latitudes and during approximately 11 years approach to the equator. Such latitudinal-temporal sunspot distribution looks like butterflies and is called butterfly-diagram or Maunder butterflies.

According to conceptions of dynamo theory, the butterfly appearance is connected with the motion of the toroidal component of the magnetic field (dynamo-wave) from higher latitudes to the equator in each hemisphere. The latitudinal-temporal diagram of the level lines of the magnetic field toroidal component obtained even using simplest dynamo models, qualitatively reproduces observational data of the butterfly-diagram. The form of the butterfly-diagram
Table 1. Cycle durations

|   | Total | Northern “wings” | Southern “wings” |
|---|-------|------------------|------------------|
| 16| 08.1923–08.1933 | 08.1923–08.1933 (0+0=0) | 08.1923–08.1933 (0+0=0) |
| 17| 09.1933–01.1944 | 09.1933–01.1944 (3+0=3) | 10.1933–12.1943 (7+17=24) |
| 18| 02.1944–03.1954 | 07.1944–03.1954 (19+0=19) | 02.1944–03.1954 (1+0=1) |
| 19| 04.1954–09.1964 | 05.1954–09.1964 (1+0=1) | 06.1954–09.1964 (6+0=6) |
| 20| 10.1964–05.1976 | 10.1964–05.1976 (0+0=0) | 04.1965–05.1976 (37+0=37) |
| 21| 06.1976–08.1986 | 06.1976–08.1986 (13+0=13) | 06.1976–08.1986 (0+0=0) |
| 22| 09.1986–04.1996 | 09.1986–04.1996 (5+0=5) | 10.1986–04.1996 (12+0=12) |
| 23| 05.1996–12.2008 | 05.1996–09.2008 (0+24=24) | 05.1996–12.2008 (17+0=17) |

obtained in models is in essential dependence from the controlling model parameters. In [2], [6], [7], [8] it was shown numerically and analytically that meridional circulation directed against the dynamo-wave propagation can essentially slow down its motion. Besides, intensive meridional circulation “blows away” the dynamo-wave to the poles. In [4] it was shown that the turbulent diffusivity coefficient also has ability to influence the duration of solar activity cycle. Also there was shown that the $\alpha$-effect latitudinal profile has influence upon the form of the Maunder butterfly.

Thus, the characteristics of the theoretical latitudinal-temporal diagrams for the magnetic fields of the Sun and other stars significantly depend on the controlling parameters in the dynamo models. On the other hand, the attempt to reproduce the observational latitudinal-temporal distributions for the stars magnetic fields by the choice of the appropriate values and the controlling processes depending on the coordinates and time could give the information about the physics of the investigated process. However, it is should be remembered that the physics of the process will be limited by the bounds of the chosen model.

2. Observations

In this paper we use Greenwich data (RGO – NASA/Marshall) of monthly and daily areas and center coordinates of the sunspot groups during the period 1923–2008, which entirely embraces cycles 16–23 of the solar activity, as well as for the umbrae during the period 1923–1976 (cycles 16–20 respectively). On the basis of this data one forms the butterfly-diagrams for each cycle, separately for the northern and the southern solar hemispheres. For each “wing” of the Maunder butterfly we applied linear approximation.

This linear approximation may be invalidated since at the beginning of a cycle the spots “from the previous cycle” (i.e. the spots with low latitudes) continue to occur for some time. Similarly, at the end of a cycle the spots “from the following cycle” (i.e. the spot with high latitudes) start to arise. Though the relative number of groups with such spots is very small (in relation to the total number of groups in a cycle, which is of the order of twenty thousand), their presence would bring distortion in the linear approximation and would lead to an error in finding the “wing” tilt angles. Therefore, we have deleted the groups that belong to “adjacent” cycles, i.e. ones with low latitudes in the beginning of the cycle and with high latitudes in the end of the cycle. In Table 1 it is shown: total cycle duration $T$ according to NGDC/NOAA data; cycle duration for a northern “wing” separately; the number of groups deleted in the beginning and in the end of a northern “wing”; the same information for a southern “wing”.

As can be seen from the above, the number of the deleted groups is negligible, so their contribution to the mean latitude of the cycle is insignificant. Also it is noteworthy to note
Table 2. The basic parameters of cycles 16–23

|   | $S_{\text{spot}}$, mh | $S_{\text{umb}}$, mh | $B$, Gs | $T$, years | $L_{\text{mean}}$, ° | $L_{\text{north}}$, ° | $L_{\text{south}}$, ° |
|---|----------------------|----------------------|---------|-------------|----------------------|----------------------|----------------------|
| 16 | 707                  | 129                  | 2695    | 10.03       | 14.7                 | 14.95                | 14.42                |
| 17 | 957                  | 178                  | 2400    | 10.41       | 15.3                 | 15.23                | 15.31                |
| 18 | 1185                 | 193                  | 2555    | 10.14       | 15.4                 | 15.48                | 15.28                |
| 19 | 1424                 | 233                  | 2190    | 10.47       | 17.3                 | 17.30                | 17.33                |
| 20 | 846                  | 129                  | 2350    | 11.65       | 14.7                 | 15.30                | 14.19                |
| 21 | 1242                 | —                    | 2230    | 10.23       | 15.0                 | 14.87                | 15.17                |
| 22 | 1174                 | —                    | 2720    | 9.66        | 17.1                 | 17.71                | 16.46                |
| 23 | 796                  | —                    | 2520    | 12.60       | 14.9                 | 14.92                | 14.95                |

that the northern and southern “wings” pass to the following cycle not simultaneously but independently. Besides, the tangents of the tilt angles (found from the linear approximation) for the northern and the southern “wings” are not equal to each other in a general case, which also indicates the independence of the spot latitudes in the opposite hemispheres.

The mean latitude of each “wing” (separately for the north and for the south) was calculated as the sum of products of daily groups latitudes (by the absolute value) by daily groups areas, divided by the total sum of group areas. It is of interest that the mean latitudes of the North and the South are almost equal for each cycle. The mean cycle latitude $L$ was calculated as the arithmetical mean of the North and the South.

Further, we have calculated the mean cycle sunspot (umbra) area $S$ as the mean value of monthly average sunspot (umbra) areas for cycles 16–23 (16–20 in the case of umbrae) in the millions of the solar hemisphere (mh).

At last we used the data of the mean magnetic field strength $B$ for cycles 16–23 [10], which was calculated by A.G. Tlatov on the basis of Mount Wilson Observatory data for the sunspots with areas more than 100 mh (actually, the vast majority of the magnetic flux is concentrated in the spots of such area).

The data for $S$, $B$ and $L$ are summarized in Table 2.

3. Results

The analysis shows that there exists an almost direct dependence ($R = -0.87$) between $S$ and $BT/L$ for all the studied cycles. This dependence can be defined by the following relation: $S = 2539 - 0.89BT/L$, $R = 0.87$, $\sigma = 132$ (Figure 1, (a)).

In case of umbra area the dependence is even more linear ($R = -0.95$), but here we have only five cycles for which both the umbra areas and the magnetic field were calculated (and hence, only five points to plot) (Figure 1, (b)). In the case of umbrae the dependence is following: $S = 500 - 0.20BT/L$, $R = 0.95$, $\sigma = 16$. This is understandable, since it is in the umbrae the greater part of the magnetic field is concentrated.

Let us analyze the obtained result. With other things being equal the more powerful cycle (i.e. the cycle with greater $S$) corresponds either to the smaller value of the magnetic field $B$, or to the smaller cycle duration $T$, or to the greater latitude $L$. It allows to assume that each cycle involves approximately the same amount of magnetic energy, however, the ways of its realization in each cycle are various. So, the increased sunspot activity in the general case leads to the fact that the spots will be generated at higher latitudes, but the magnetic field strength in the spots will be smaller, and the cycle will be transient.
Indeed, all four values are changed in combination, since there is no dependence between any pair of individual values. The only exception is the pair of values $(S, L)$ for which there is a weak dependence with the correlation coefficient $R = 0.74$ (Figure 2). However, this dependence becomes much better, if instead of the mean sunspot area $S$ we take the relative number of groups containing very large spots. In [9] there was calculated the ratio of groups containing spots with an area of more than 800 mh for cycles 19–23 according to the data of Kislovodsk Mountain Solar Station. In Figure 3 there is dependence between this ratio and the mean latitude $L$ ($R = 0.87$). Considering the fact that the spots of large and huge size contains the essential part of the magnetic flux, we can suggest that the meridional circulation, which is responsible for the latitudinal “butterfly wings” shift, is associated with the total energy of the magnetic field.

The mean cycle spot area generally is proportional to the tangent of the “wing” tilt angle, although there is no direct relationship. There is no dependence between the magnetic field strength and the tangent of the tilt angle.

**Figure 1.** The dependence between the mean magnetic field strength, the mean spot (a) and umbra (b) area, the duration and the arithmetical mean latitude for cycles 16–23

**Figure 2.** The dependence between the mean sunspot area and the arithmetical mean sunspot latitude for cycles 16–23

**Figure 3.** The dependence between the ratio of groups with spots of area more than 800 mh and the arithmetical mean sunspot latitude for cycles 19–23
4. Simulations

Here we use a simple dynamo model which is a straightforward generalization of the initial Parker \[5\] migratory dynamo. The governing dynamo equations are derived by averaging the magnetic field over the radius within the thin shell, where the dynamo mechanism operates, and neglecting terms describing the effects of curvature near the pole. In this case the dynamo equations have the form:

\[
\frac{\partial A}{\partial t} = -\frac{\alpha_0 B}{1 + \xi^2 B^2} + \beta \frac{\partial^2 A}{\partial \theta^2} - V \frac{\partial A}{\partial \theta},
\]

(1)

\[
\frac{\partial B}{\partial t} = -D \cos \theta \frac{\partial A}{\partial \theta} + \beta \frac{\partial^2 B}{\partial \theta^2} - \frac{\partial (VB)}{\partial \theta}.
\]

(2)

Here \( B \) represents the toroidal magnetic field, \( A \) is the azimuthal component of the vector potential of the poloidal magnetic field, and \( \theta \) is the latitude measured from the equator. The factor \( \cos \theta \) is the decrease in the differential rotation at higher latitudes [3]. The second equation neglects the small contribution of the \( \alpha \)-effect, i.e. we use so-called \( \alpha \)-\( \Omega \)-approximation. Curvature effects are absent in the diffusion terms. In Eq. 1–2 the parameter \( D \) is the dimensionless dynamo-number (a measure of the intensity of dynamo action), and \( \beta \) is the turbulent diffusivity. We used a simple scheme for the stabilization of the magnetic field growth, namely, the algebraic quenching of the helicity. This scheme assumes that \( \alpha = \alpha_0(\theta)/(1 + \xi^2 B^2) \approx \alpha_0(\theta)/(1 - \xi^2 B^2) \), where \( \alpha_0(\theta) = \sin \theta \) is the helicity in the unmagnetized medium and \( B_0 = \xi^{-1} \) is the magnetic field for which the \( \alpha \)-effect is considerably suppressed.

For reasons of symmetry (\( \alpha(-\theta) = -\alpha(\theta) \)), the equations 1–2 may be considered only for one hemisphere (for example, the northern one) with the conditions of the antisymmetry (the case of dipole symmetry) or the symmetry (the case of quadrupole symmetry) at the equator. In this paper we restrict ourselves to the consideration of dipole symmetry with the simplest kinematic value of the helicity in the unmagnetized medium \( \alpha_0 = \sin \theta \). As boundary condition at the poles we use the following: \( A(-\pi/2, t) = B(-\pi/2, t) = A(\pi/2, t) = B(\pi/2, t) = 0 \), because we find only the solutions with the dipole symmetry.

Following [6], we consider latitudinal profile of the meridional circulation: \( V(\theta) = v \sin 2\theta \). Since in our model the latitude is measured from the equator, the value with a positive sign corresponds to the meridional circulation directed against the dynamo-wave propagation.

Since the parameters of the Mounder butterflies vary from cycle to cycle, we modify them for each cycle but keep them constant during each cycle. Any periodic dependence of their change over time is not revealed. Maybe, this is because the number of cycles studied is small.

According to such simplest dynamo theory the synchronous latitudinal butterfly motion which is due to that the meridional circulation has the same module in both hemispheres. Both the meridional circulation and the turbulent diffusion have an influence upon the butterfly tilt angle. Thus, the fact that the cycles with less duration have a bigger tilt angle is reproduced in the model by increasing the meridional circulation or by the decrease of the turbulent diffusion coefficient.

The increase of the meridional circulation always leads to the butterfly shift to upper latitudes and to the increase of the cycle duration. However, the field amplitude initially decreases and then grows with the further increase of the meridional circulation. When the turbulent diffusion coefficient is decreasing the cycle duration is growing, the field amplitude is increasing, and the butterfly is not shifted by the latitude. Increasing of the dynamo-number module leads to an increase in the magnetic field amplitude but latitude and duration of the cycle does not change. From the dynamo theory it follows that in the observed cycles these parameters should be modified together. Therefore, for example, there is no evident dependence between the mean butterfly latitude and the cycle duration, since according to the dynamo theory the cycle duration would be directly proportional to the mean butterfly latitude in the case that only the meridional circulation should be changed.
The average value of the latitude of a butterfly is located about 15°. The maximum deviation from the mean latitude for the observed cycles is 18%. The maximum deviation of the duration of the 11-year cycle is approximately 11%, the deviation from the mean amplitude of the field is 13%.

To solve Eq. 1–2 we used numerical method of lines in Mathcad 11. One can consider the typical values of the parameters for obtaining 11-year solar cycle: \( D \approx -10000 \), amplitude of the meridional circulation \( \approx 0.5 \) in model units (\( \approx 2 \) meters per second), and \( \beta \approx 1 \). In this case the normalized cycle duration is approximately equal 1 diffusive unit. The observed deviation from the mean latitude can be obtained by changing amplitude of the meridional circulation \( \Delta v \approx 0.1 \) in model units (which gives a deviation of 20% from its mean value) or \( \Delta \beta \approx 0.25 \) (deviation from the mean value of 25%). The observed deviation from the mean duration can be obtained by changing the amplitude of the meridional circulation to \( \Delta v \approx 1.5 \) in model units (deviation from the mean value is 300%) or \( \Delta \beta \approx 0.05 \) (deviation from the mean value is 5%). In order to obtain observed deviation from the mean amplitude of the toroidal magnetic field one needs to change \( D \) (\( \Delta D \approx 2000 \), or 20%), or \( v \) (\( \Delta v \approx 2.5 \), or 500%), or \( \beta \) (\( \Delta \beta \approx 0.01 \), or 1%).

5. Conclusion
The linear dependence \( f(BT/L, S) \) from cycle to cycle possibly is realized due to the fact that the equal quantity of energy is spent on each cycle, but it is redistributed in different ways in each cycle, therefore we can observe the change of the other cycle characteristics. It is the dynamo coefficient that is responsible for the magnetic field generation in the dynamo models. Hence it is quite probable that it does not change from cycle to cycle.

According to the numerical simulations changes of butterfly latitude are most sensitive to changes in the amplitude of the meridional circulation, while changes of cycle duration and amplitude of the magnetic field are most sensitive to changes in the coefficient of turbulent diffusion.

Since the clear periodicity in changes in the characteristics of Mounder butterflies were not found, it can be assumed that the changes of control parameters (dynamo-number, coefficient of turbulent diffusion, and meridional circulation) have some stochastic nature.

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