Diversity and Multiplexing Tradeoff in the Uplink of Cellular Systems with Linear MMSE Receiver

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Abstract—In this paper, we extend the diversity and multiplexing tradeoff (DMT) analysis from point-to-point channels to cellular systems to evaluate the impact of inter-cell interference on the system reliability and efficiency. Fundamental tradeoff among diversity order, multiplexing gain and inter-cell interference intensity is characterized to reveal the capability of multiple antennas in cellular systems. And the detrimental effects of the inter-cell interference on the system performance of diversity and multiplexing is presented and analyzed.

Index Terms—DMT, Multi-user, MIMO, MMSE.

I. INTRODUCTION

Reliability and efficiency are two fundamental aspects of wireless communication systems. Due to the limitation on the available channel resources, improvement of either aspect comes at the price of sacrificing the other. To explore the optimal reliability when certain system efficiency is required, numerous channel coding strategies have been studied in single antenna settings. For multiple-input multiple-output (MIMO) fading channels, additional spatial dimension is utilized to combat deep fadings for reliability or increase the available degrees of freedom for spectral efficiency, namely spatial diversity and multiplexing. It has been proved that multiple antennas can provide lower error probability and/or higher data rate than conventional single antenna systems [1]-[4]. And the tradeoff between the error probability and the data rate can be asymptotically characterized by the tradeoff between the diversity order and the multiplexing gain in the high-power regime.

Fundamental diversity and multiplexing tradeoff is first characterized in the seminal work of Zheng and Tse [5], where an optimal maximum-likelihood (ML) scheme is utilized at the receiver for data processing. Later, in [6], Kumar et al. consider a low-complexity MIMO architecture and discuss the DMT performance with suboptimal linear receivers. All these works focus on the DMT performance in point-to-point settings, but in practical cellular scenarios, due to the presence of inter-cell interference, how to characterize the DMT is still an open problem.

In this paper, we will extend the DMT analysis from point-to-point channels to cellular systems. Joint spatial encoding[1] and linear minimum mean square error (MMSE) MIMO transceiver architecture [2] is utilized for practical complexity consideration. To evaluate the impact of the inter-cell interference on the system reliability and efficiency, inter-cell interference factor $\xi$ is introduced to asymptotically represent the inter-cell interference intensity. And the fundamental tradeoff among diversity order $d$, multiplexing gain $r$ and inter-cell interference factor $\xi$ is characterized. Given $M$ transmit antennas and $N$ receive antennas ($N \geq M$), our main result shows that the optimal DMT in cellular systems is $d_{\text{mmse}}^{\star}(r,\xi) = (N - M + 1) (1 - \xi - \frac{r}{M})^+, \text{where } (x)^+ \text{ denotes max}(0, x)$.

The rest of the paper is organized as follows. The system model and definitions are described in section II. Our main result on the optimal tradeoff among diversity order, multiplexing gain and inter-cell interference intensity is derived in section III. In section IV, numerical results and interpretations are presented to provide insights on the impact of inter-cell interference in cellular systems. And section V summarizes our conclusions.

II. SYSTEM MODEL

Consider the uplink of a narrow-band[4] cellular system, with each user having $M$ transmit antennas and each base station having $N$ ($N \geq M$) receive antennas. At each time instance, only one user is scheduled and served in each cell. And simultaneous transmissions in different cells cause the inter-cell interference. It is assumed that there is no cooperation among the cells. For each receiver in the system, only the local-cell signal is concerned and the inter-cell interference is simply treated as noise. We further assume that the interference at a given base station is caused by the active links from its adjacent cells (first-tier cells) only. Suppose each cell has $K - 1$ adjacent cells. Without loss of generality, we focus on one certain cell (so called the reference cell) in the system and assume that the user in the cell communicates with its one certain cell (so called the reference cell) in the system.

1Although the result still holds in the separate encoding cases, it is not the subject of this work.
2For wide-band systems, we can decompose the frequency-selective channel into multiple parallel and independent frequency-flat sub-channels. And, on each sub-channel, the narrow-band assumption is valid.
intended base station under the presence of $K-1$ co-channel interferers.

Let us index the desired user in the reference cell by $K$ and index the interfering users in the adjacent cells by the numbers starting from 1 to $K-1$. At time $t$, given the transmitted signal vector $x_k \in \mathbb{C}^{M \times 1}$ of the $k$-th user, the received signal vector $y \in \mathbb{C}^{N \times 1}$ at the intended base station can be written as

$$ y = \sqrt{\frac{\text{SNR}}{M}} H_k x_k + \sum_{k=1}^{K-1} \sqrt{\frac{\text{INR}(k)}{M}} H_k x_k + n, \quad (1) $$

where $n \in \mathbb{C}^{N \times 1}$ denotes the complex Gaussian noise vector with entries $\sim \mathcal{CN}(0,1)$. SNR denotes the average signal-to-noise ratio at each receive antenna of the base station and INR$(k)$ denotes the average interference-to-noise ratio at each receive antenna of the base station from the $k$-th ($k = 1, 2, ..., K-1$) interfering user. $H_k$ denotes the frequency-flat channel fading matrix between the $k$-th user and the base station. Under rich scattering condition, $H_k$ is assumed to have independent and identically distributed entries with zero mean and unit variance. $T$-block length spatial-temporal codeword $X_k \in \mathbb{C}^{M \times T}$ is chosen uniformly at random from the common codebook and launched at the $k$-th user. An overall power constraint is considered on $C$ as

$$ \frac{1}{|C|} \sum_{j=1}^{|C|} \|X(j)\|_F^2 \leq MT, \quad (2) $$

where $|C|$ denotes the size of the codebook, $X(j)$ denotes the $j$-th codeword and $\| \cdot \|_F$ denotes the Frobenius norm of a matrix. For each transmission block, data stream is first encoded, then interleaved and finally multiplexed into $M$ sub-streams. Thus, vector $x_k$ can be regarded as the spatially encoded sample of $X_k$ at time instant $t$.

In this paper, quasi-static fading environment is considered that each channel matrix is randomly chosen at the beginning but remains constant over the whole block length $T$. And we assume that the channel state information (CSI) between the $k$-th user ($k = 1, 2, ..., K$) and the receiver is perfectly known at the receiver but not available at any of the users. Though, according to [10], generally, the interference channel matrix $H_k$, $k = 1, 2, ..., K-1$, cannot be estimated in cellular systems. In this work, for theoretical analysis, we assume that all the users may periodically send the orthogonalized pilot signals and thus the receivers can obtain the channel matrices accurately. According to [2] and [3], without loss of optimality, we assume the distribution of $x_k$ to be circularly symmetric complex Gaussian with zero mean and covariance matrix $I$, where $I$ is the identity matrix.

We aim to characterize the fundamental tradeoff among the diversity order, multiplexing gain and inter-cell interference intensity. According to [5], we introduce the definition of multiplexing gain $r$ and diversity gain $d$ as follows:

\textit{Definition 1:} A scheme $\mathcal{C}(\text{SNR})$ operating at SNR is said to achieve spatial multiplexing gain $r$ and diversity gain $d$ if the data rate $\mathcal{R}(\text{SNR})$ satisfies

$$ \lim_{\text{SNR} \to \infty} \frac{\mathcal{R}(\text{SNR})}{\log \text{SNR}} = r \quad (3) $$

and the average error probability $P_e(\text{SNR})$ satisfies

$$ \lim_{\text{SNR} \to \infty} \frac{\log P_e(\text{SNR})}{\log \text{SNR}} = -d. \quad (4) $$

Similarly, according to [8], we introduce the inter-cell interference factor $\xi(k)$ to asymptotically measure the inter-cell interference intensity of the $k$-th interferer INR$(k)$ with respect to SNR as

$$ \lim_{\text{SNR} \to \infty} \frac{\log \text{INR}(k)}{\log \text{SNR}} = \xi(k), \quad (5) $$

where $0 < \xi(k) < 1$.

We further introduce the notation of exponential equality $f(\text{SNR}) = \text{SNR}^b$ to denote

$$ \lim_{\text{SNR} \to \infty} \frac{f(\text{SNR})}{\log \text{SNR}} = b. $$

And we say that $f(\text{SNR})$ is exponential equal to $g(\text{SNR})$, i.e. $f(\text{SNR}) \sim g(\text{SNR})$, if

$$ \lim_{\text{SNR} \to \infty} \frac{f(\text{SNR})}{\log \text{SNR}} = \lim_{\text{SNR} \to \infty} g(\text{SNR}). $$

Accordingly, the symbols $\leq$ and $\geq$ are defined.

In this work, we assume that the inter-cell interference INR$(k)$, for $k = 1, 2, ..., K-1$, are exponentially equivalent that

$$ \text{INR}(1) \leq \text{INR}(2) \leq ... \leq \text{INR}(K-1) \leq \text{INR} \leq \text{SNR}^\xi. $$

It can be interpreted as: though the interference caused by the co-channel interferers may differ from each other, they have the same rate of change $\xi$ with respect to the SNR of the desired link. Then, for asymptotic analysis, we rewrite (1) as

$$ y = \sqrt{\frac{\text{SNR}}{M}} H_K x_K + \sqrt{\frac{\text{INR}(k)}{M}} H_k x_k + n. \quad (6) $$

Since for good codes and long block length $T$, the codeword error event can be characterized by the channel outage event at high SNR region, we use the outage probability $P_{out}$ instead of error probability $P_e$ in (4) to capture the diversity behavior. The outage probability is defined as the probability that the mutual information of the channel fails to support the target rate. To be specific, given the mutual information $I$ and the target rate $R$, we have the outage probability $P_{out}$ as

$$ P_{out}(R) = \Pr (I \leq R). \quad (7) $$

In [5], with ML detector, the diversity-multiplexing tradeoff function $d^*(r)$ in a general point-to-point scenario is derived as

$$ d^*(r) = (M - r)(N - r), $$

where $r = 0, 1, ..., \min\{M, N\}$. 
In this work, instead of using ML receiver, we consider the suboptimal linear MMSE receiver and derive the optimal DMT. The MMSE receiver is characterized by a matrix $G$ which minimizes the mean square error between the transmit signal $x$ and its estimation $\hat{x} = Gy$ as

$$G = \arg \min_{G} E \left[ \| \hat{x} - Gy \|^2 \right].$$

Using the orthogonality principle, we can then have $G$ as

$$G = \sqrt{\frac{M}{\text{SNR}}} H^H \left( HH^H + \frac{\text{INR}}{\text{SNR}} RQR^H + \frac{M}{\text{SNR}} I \right)^{-1},$$

where $(\cdot)^H$ denotes the conjugate transpose, $(\cdot)^{-1}$ denotes the matrix inversion, $H = H_K$, $Q = \text{blockdiag} \{ \frac{\text{INR}(i)}{\text{INR}} I \}$ and $R = [H_1, H_2, ..., H_{K-1}]$.

Actually, by applying $G$, the channel matrix of the transmit-receive pair is divided into $M$ parallel pipes, which is alternatively called virtual channels. For each virtual channel, one complex degree of freedom is supported. And, by using Woodbury’s identity \cite{Woodbury}, the SINR of the $i$-th channel $\rho_i$ is shown to be

$$\rho_i = \left( \left[ \left( \frac{\text{INR}}{\text{SNR}} H^H \left( RQR^H + \frac{M}{\text{SNR}} I \right)^{-1} H \right)^{-1} \right]_{i,i} \right)^{-1},$$

where $[A]_{i,j}$ denotes the $i$-th diagonal entry of matrix $A$.

### III. DIVERSITY-MULTIPLEXING TRADEOFF

In this section, we extend the DMT results in \cite{DMT} from point-to-channel systems to cellular systems to evaluate the impact of the interference on system reliability and efficiency. And our main result is given as follows:

**Theorem 1:** Consider the uplink of a cellular system, at a given time $t$, the user in a certain cell communicates with its intended base station under the presence of the co-channel interference from its $K-1$ adjacent cells. And the optimum diversity-multiplexing tradeoff for the transmit-receive pair in the cell is given by:

$$d_{\text{mmse}}^* (r, \xi) = (N - M + 1) \left( 1 - \xi - \frac{r}{M} \right)^+, \quad (10)$$

where joint spatial Gaussian encoding is applied across the transmit antennas and linear MMSE equalizer is utilized at the receiver.

**Proof:** We first give an upper and a lower bound on the outage exponent, then prove the theorem by using the squeeze lemma.

**Lower bound on the outage exponent.** Since joint spatial encoding scheme is utilized, data transmission is in outage only when the aggregate mutual information fails to support the target data rate $\bar{R}$. Given $H$ and $R$, the mutual information of the transmit-receive pair in the cell by using linear MMSE receiver is given by

$$I_{\text{mmse}} (H, R) = - \sum_{i=1}^{M} \log (1 + \rho_i)$$

$$= - \sum_{i=1}^{M} \log \left( \left[ \left( I + \frac{\text{SNR}}{\text{INR}} C \right)^{-1} \right]_{i,i} \right), \quad (11)$$

where

$$C = H^H \left( RQR^H + \frac{M}{\text{INR}} I \right)^{-1} H.$$ \quad (12)

Since the function $- \log(\cdot)$ is convex, applying Jensen’s inequality \cite{Jensen}, we can write as

$$I_{\text{mmse}} (H, R) \geq - M \log \left( \frac{1}{M} \sum_{i=1}^{M} \left[ \left( I + \frac{\text{SNR}}{\text{INR}} C \right)^{-1} \right]_{i,i} \right)$$

$$= - M \log \left( \frac{1}{M} \text{Tr} \left( I + \frac{\text{SNR}}{\text{INR}} C \right)^{-1} \right). \quad (13)$$

Let $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_M$ denote the ordered eigenvalues of $C$, and (13) reduces to

$$I_{\text{mmse}} (H, R) \geq - M \log \left( \frac{1}{1 + \frac{\text{SNR}}{\text{INR}} \lambda_M} \right). \quad (14)$$

Then, the outage probability is given as

$$\Pr_{\text{out}} (\lambda_M, R) = \Pr (I_{\text{mmse}} (H, R) \leq R)$$

$$\leq \Pr \left( - M \log \left( \frac{1}{1 + \frac{\text{SNR}}{\text{INR}} \lambda_M} \right) \leq R \right)$$

$$= \Pr \left( \log \left( \frac{1}{1 + \frac{\text{SNR}}{\text{INR}} \lambda_M} \right) \geq - r \log \frac{\text{SNR}}{M} \right)$$

$$= \Pr \left( \frac{1}{1 + \frac{\text{SNR}}{\text{INR}} \lambda_M} \geq \text{SNR}^{-\frac{r}{M}} \right). \quad (15)$$

And, we can obtain the asymptotic upper bound on the outage probability as

$$\Pr_{\text{out}} (\lambda_M, R) \leq \Pr \left( \frac{\text{INR}}{\text{SNR}} \cdot \frac{1}{\lambda_M} \geq \text{SNR}^{-\frac{r}{M}} \right)$$

$$= \Pr \left( \lambda_M \leq \text{SNR}^{-\frac{r}{M}} \right). \quad (16)$$

It should be noticed that the above asymptotic outage probability upper bound vanishes to zero as $\text{SNR}$ goes to infinity only when $r/M < 1 - \xi$. For $r/M \geq 1 - \xi$, the asymptotic outage probability upper bound approaches 1 and thus the outage exponent lower bound is zero. When $r/M \leq 1 - \xi$, we have

$$\Pr \left( \lambda_M \leq \text{SNR}^{-\frac{r}{M}} \right) = \int_0^{\frac{\text{SNR}}{M} + \xi - 1} f_{\lambda_M} (\lambda_M) \, d\lambda_M.$$
According to [11], the first-order expansion of the marginal probability density function (pdf) of $\lambda_M$ is given by

$$f_{\lambda_M}(\lambda_M) = a_M (N - M + 1) \lambda_M^{N-M} o (\lambda_M^{N-M+1}),$$

(17)

where $a_M$ is the normalizing factor which is independent of $\lambda_M$ and SNR. As such, for $\lambda \ll 1$, (16) can be further written as

$$Pr_{\text{out}}^\text{mmse}(R) \leq \int_0^{SNR R^{-1}} f_{\lambda_M}(\lambda_M) d\lambda_M = a_M SNR (N-M+1) (\lambda + \xi - 1).$$

(18)

Therefore, we obtain a lower bound on the outage exponent as

$$d_{\text{mmse}}^*(r, \xi) \geq (N - M + 1) \left(1 - \xi - \frac{r}{M}\right)^+.$$  

(19)

Upper bound on the outage exponent. Due to the concavity of $\log(.)$ function, by applying Jensen’s inequality on (11), we can obtain

$$I_{\text{mmse}}(H, R) \leq M \log \left( \frac{1}{M} \sum_{i=1}^{M} \left[1 + SNR \frac{C}{INR} I_{i,i}^{-1} \right] \right).$$

(20)

Since $C$ is a hermitian matrix, we consider the decomposition $C = U^H A U$ on $C$, where $U \in \mathbb{C}^{M \times M}$ is unitary and $A \in \mathbb{C}^{M \times M}$ denotes a diagonal matrix with eigenvalues of $C$ on the diagonal. Let $u_i$ denotes the $i$-th column of $U$, then

$$\left( \begin{bmatrix} 1 & SNR \frac{C}{INR} \end{bmatrix} \right)^{-1}_{i,i} = u_i^H \left( \begin{bmatrix} 1 & SNR \frac{A}{INR} \end{bmatrix} \right)^{-1} u_i = \sum_{j=1}^{M} \frac{|u_{j,i}|^2}{1 + SNR \frac{A}{INR} \lambda_j}.$$  

And, according to [6], we have

$$\sum_{i=1}^{M} \left[1 + SNR \frac{C}{INR} \right]^{-1}_{i,i} = \frac{1}{M} \sum_{i=1}^{M} \frac{1}{\sum_{j=1}^{M} \frac{|u_{j,i}|^2}{1 + SNR \frac{A}{INR} \lambda_j}} \leq \frac{1}{M} \sum_{i=1}^{M} \frac{1}{\sum_{i=1}^{M} |u_{M,i}|^2}.$$  

Let $A$ denote the event $\{ \frac{1}{M} \sum_{i=1}^{M} \frac{1}{|u_{M,i}|^2} \leq c \}$, where $c$ is some constant independent of SNR. And it has been proved in [6] that $Pr(A)$ is a non-zero constant with respect to SNR. Hence, a lower bound on the outage probability can be written as

$$Pr_{\text{out}}^\text{mmse}(R) \geq Pr \left( M \log \left( \frac{1}{M} \sum_{i=1}^{M} \frac{1}{|u_{M,i}|^2} \right) \right) \leq R \right) \geq Pr(A) Pr \left( \log \left( \frac{1}{M} \sum_{i=1}^{M} \frac{1}{|u_{M,i}|^2} \right) \right) \leq Pr \left( \frac{1}{M} \sum_{i=1}^{M} \frac{1}{|u_{M,i}|^2} \right).$$  

(21)

It could be immediately verified that the last line of (21) and the last line of (16) are asymptotically equivalent. Therefore, using the same argument as in [18], we obtain the outage exponent upper bound as

$$d_{\text{mmse}}^*(r, \xi) \leq (N - M + 1) \left(1 - \xi - \frac{r}{M}\right)^+.$$  

(22)

Thus, the outage exponent upper bound equals to the lower bound. This completes the proof.

IV. DISCUSSION AND NUMERICAL RESULTS

Theorem 1 in the previous section characterized the fundamental tradeoff among the diversity order, multiplexing gain and inter-cell interference factor. Given the DMT results in point-to-point scenarios as [6]

$$d_{\text{mmse}}^*(r) = (N - M + 1) \left(1 - \frac{r}{M}\right)^+,$$  

(23)

we can rewrite (10) as

$$d_{\text{mmse}}^*(r, \xi) = (N - M + 1) \left(1 - \xi - \frac{r}{M}\right)^+ = d_{\text{mmse}}^*(r) - (N - M + 1) \xi.$$  

(24)

Obviously, the DMT performance in point-to-point scenarios can be characterized as a special case of the DMT performance in cellular systems when $\xi = 0$. For $\xi \neq 0$, according to (24), we can either degrade the diversity order for $(N - M + 1)$ to maintain the multiplexing gain or pull down the degrees of freedom for $M \xi$ but guarantee the diversity order. Actually, this is due to the fact that the introduced interference reduces the minimum distance between the constellation points with a scale factor related to $\xi$. Specifically, if $(N - M + 1) \xi > M \xi$, i.e. $N > 2M - 1$, the diversity order will suffer more loss from the inter-cell interference than the multiplexing gain, and vice versa. In Fig. 1 we plot the function $d_{\text{m}}^*$ $(r, \xi)$ for $M = 2$ and $N = 4$. The slope of the intersection line between the result plane and the $Z$-$X$ plane describes the rate of change of $d$ with respect to $\xi$. And the slope of the intersection line between the result plane and the $Y$-$X$ plane describes the rate of change of $r$ with respect to $\xi$.

Another implication from Theorem 1 gives the fact that the DMT performance has nothing to do with the number of the interferers. To verify this point, Monte Carlo simulation is employed and the corresponding results are shown in Fig. 2 where $M = 2$, $N = 4$, $\xi = 0.5$ and the target rate $R = 5$ bits per channel use. We see that the diversity order keeps unchanged as the number of the interferers increases. And indeed, this observation implies that the DMT analysis cannot fully characterize the tradeoff among the outage probability, data rate and the interference. In Fig. 2 though DMT remains the same for different number of interferers, for a given outage probability, a considerable increment on SNR is required to support the target rate when more interferers are involved. Actually, according to [13], DMT is only a coarse description of the fundamental tradeoff between the outage probability.
and the data rate. It asymptotically captures the rate of change of the outage probability and the data rate versus SNR in the high-power regime, but ignores the constant offset. Thus, though the number of the interferers has no effect on DMT, it affects the constant offset of the outage probability. And the exact impact of the number of the interferers on the system reliability and efficiency would be addressed in our future work.

V. CONCLUSIONS

In this paper, we study the diversity and multiplexing trade-off in the uplink of cellular systems to evaluate the impact of inter-cell interference on the system reliability and efficiency. When suboptimal linear MMSE receiver is utilized, fundamental tradeoff among diversity order $d$, multiplexing gain $r$ and inter-cell interference factor $\xi$ is asymptotically characterized as $d_{mmse}(r, \xi) = (N - M + 1) \left(1 - \frac{\xi}{N}ight)^+$. Given $N > 2M - 1$, it is shown that the diversity order will suffer more loss from the inter-cell interference than the multiplexing gain, and vise versa. We also observe the fact that, due to the weakness of the DMT characterization, the number of the interferers has no effect on DMT. Therefore, a more complete picture of the impact of the interference is needed. In summary, in cellular MIMO systems, we should take both the randomness of the channel and the inter-cell interference into consideration to achieve the optimized data rate and error probability.

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