Numerical Simulation of Inclination Vibration in Magnetic Induction Micromachines

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Abstract. This paper studies the inclination vibration of an axial-flux magnetic induction micromachine which is supported by hydrostatic thrust bearings. A mechanical model for the rotor and the corresponding fluid-film bearing is combined with an electromagnetic force model to study the linear and nonlinear rotordynamics of the system. Results obtained for the stability show that magnetic induction micromachine would encounter severe instability problem at high speed operations. The model developed here could serve as a useful reference for design optimization and operation scheme.

1. Introduction

Magnetic micromachines can be used to convert rotational mechanical energy into electrical energy and vice versa at the micro-scale. As they are likely to be utilized in paralleled with the micro gas turbine generator [1, 2] to construct high performance power sources that can outperform batteries for use in portable electronics, standalone sensors, robotics, etc., much progress in these microdimensional motors has been recently reported [3-6]. Previous researches into the issue focused on the fabrication, characterization and efficiency enhancement concerning the performance of the electromagnetic circuit of the micromachine, yet a comprehensive assessment of the rotordynamic behaviors of the system was unexplored. In particular, the magnetic micromachine is required to operate at high speed in order to maintain high power density. The cubic square law implies that contact forces such as friction is large at small scales. This means that non-contact bearings are essential to achieve the necessary rotational speeds. Evaluation of the rotordynamics incorporating both the electromechanical interaction effects and the influence of supporting thrust bearings will not only provide insight into the performance of the system but also be useful for design optimization.

The architecture of the magnetic micromachine differs from its macroscale counterpart mainly due to constraints imposed by MEMS fabrication techniques [6]. At the microscale, it is much desirable to implement an extruded planar pattern with an active area in the lithographic plane. Consequently, we investigated in this paper an axial-flux induction micromachine as shown in figure 1. The rotor in the architecture is supported by inherent-restrictor orifice hydrostatic thrust bearings which provide air flow on either side of the rotor to counterbalance against gravity, the attractive magnetic force as well as the unbalanced magnetic torque between the rotor and the stator.

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1. Electromagnetic force and stiffness

Figure 1. Schematic for the three-phase, two-pole, axial-flux induction micromachines. (a) Side view, not to scale. (b) Top view of the stator. (c) Topology of the rotor and stator.

In axial-flux electrical micromachines, the electromagnetic axial force acting upon rotor and stator surface could be very large compared to the inertial of the rotor, but they are balanced when the rotor is paralleled with the stator and there would be no diametrical torque exerted upon them. When the rotor tilts from the equilibrium position, the non-uniform air-gap distorts the air-gap flux density distribution, and gives rise to hazardous tilting torques which might compromise the performance of the machines such as causing vibration and noise emission, speeding up the bearing wearing and even producing a rub between the rotor and stator with concomitant damage of the stator windings and the rotor.

Using the method in [7] and based on the assumption that secondary effects such as slotting, magnetic anisotropy, hysteresis, saturation, etc. are negligible, the electromechanical interactions of the system could be easily evaluated. Figure 2 shows the pull-in force on the rotor. Figure 3 and figure 4 show the direct-coupled and cross-coupled tilting stiffness, respectively.

Figure 2. Variation of the total axial force as a function of rotational speed and nominal clearance when the rotor does not tilt. Inset is a typical distribution of the pull-in force on the rotor.
The induction motor in consideration here is three-phase, two-pole, axial-flux with system parameters as follows: thickness of rotor conductor 5 m, thickness of rotor backiron 500 m, nominal clearance 20 m, thickness of stator conductor 85 m, thickness of stator backiron 1 mm, amplitude of the excitation current 17 A, frequency of the excitation current $4 \times 10^5$ rad/s, conductivity of the rotor $5 \times 10^6$ S/m, permeability of the rotor and stator backiron $10^5$.  

Figure 3. Direct-coupled tilting stiffness due to the electromagnetic interaction.

Figure 4. Cross-coupled tilting stiffness due to the electromagnetic interaction.

2. Rotordynamic characteristics

Suppose that when set into motion the rotor undergoes conical motion only without lateral vibration and the geometric center of the rotor always stays on the bearing centerline. The tilting response of the rotor-thrust-bearing system in terms of yawing and pitching dynamics could be evaluated by the governing equations as follows.

$$I_{x} \ddot{\theta}_x + I_{y} \omega \dot{\theta}_x + k_{sx} \theta_x + k_{sy} \theta_y + c \dot{\theta}_x + T_{x}^{hs} + T_{x}^{hd} + T_{x}^{em} = (I_{x} - I_{y}) \chi \alpha^2 \cos(\omega t)$$

$$I_{y} \ddot{\theta}_y - I_{x} \omega \dot{\theta}_y + k_{sy} \theta_y - k_{sx} \theta_x + c \dot{\theta}_y + T_{y}^{hs} + T_{y}^{hd} + T_{y}^{em} = (I_{y} - I_{x}) \chi \alpha^2 \sin(\omega t)$$

(1)

The notation for the Euler angles $\theta_x$ and $\theta_y$ follows the same convention as that used in [8]. $k_{sx}$ and $k_{sy}$ are the direct-coupled tilting stiffness induced by hydrostatic force and electromagnetic force, $k_{xy}$ is the cross-coupled stiffness due to hydrodynamic effect and electromagnetic effect, $c$ is the thrust
bearing angular damping (we assume that air damping dominates over the electromagnetic dissipation). The hydrostatic coefficients employed here are evaluated by the fluid resistive model in [9] and [10], while the values of hydrodynamic forces, which are induced by the pumping action of the rotor rotation, are determined by solving the Reynolds equation. $T_x^{hs}$ and $T_y^{hs}$ are the nonlinear parts of the hydrostatic forces, which are the function of the supply pressure $P_s$, the nominal clearance $h_0$ and the tilting angles, while $T_x^{hd}$ and $T_y^{hd}$ are nonlinear parts due to hydrodynamic forces that vary with rotor spinning velocity $\Omega$, nominal clearance, tilting angle and tilting rate. $T_x^{em}$ and $T_y^{em}$ are the nonlinear contributions by electromechanical interactions. The nonhomogeneous driving terms on the right hand side of each equation depend on the diametric and polar moments of inertia, denoted by $I_x$ and $I_p$, respectively. Due to imperfections in MEMS fabrication, such as etch non-uniformities and pattern misalignment, the rotor principle axis is not necessarily at the rotor geometric centerline. Such a deviation could be indicated by the dynamic imbalance $\chi$. When the rotational speed approach infinity, the polar principle axis aligns itself with the bearing centerline as a consequence of gyroscopic stiffening which becomes increasingly dominant at high speed.

Now it is possible to investigate the linear dynamics of the system by performing an eigenvalue study of (1). The resulted natural frequency characteristics is shown in figure 5. The reference state of the thrust bearing is as follows. $h_0=3$ m (thrust bearing clearance), $R_t=1$mm (thrust bearing radius), $R_n=0.75$mm (location of thrust bearing nozzles) and $P_s=80$psig.

The above equations have two eigenvalues, one of the forward precession $p_f$ and the other backward precession $p_b$. At low rotational speed, the real parts of both $p_f$ and $p_b$ are negative, indicating that the system is stable. However, as the speed increases, the value of $p_f$ moves away further from the imaginary axis while that of $p_b$ approaches it in the complex plane, making the system less stable. At one rotational speed, the value of $p_b$ crosses the imaginary axis and its real part becomes positive. At this point, the rotor engages in a precession motion with exponentially growing amplitude and eventually crashes onto the bearing pads. Thus this threshold speed is referred to as the stability boundary (SB).

From the above analysis, we could see that the presence of stability boundary at a relatively large slip frequency poses a great challenge for high speed operation of the induction micromachine. The scenario encountered here is similar to that in the classic fluid bearing system where the rotor would be set into violent self-excited vibration above some threshold speed. This phenomenon is denoted as
ω1 is the point at which the rotational frequency coincides with the natural frequency of the backward processional "oil-whip" in the fluid bearing literature, and is produced by the hydrodynamic torque which exerts orthogonally to the rotor tilt. In the thrust-bearing-supported micromachine, the rotor not only experiences such a destabilizing hydrodynamic torque but also suffers from the cross-coupling electromechanical interaction, which would further exacerbate the instability of the system.

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It is interesting to note that beyond the stability boundary, the real part of \( p_b \) becomes positive and gradually increases with the increasing rotational speed. This negative damping ratio sets the rotor into subsynchronous procession at the natural frequency of \( p_b \) with the amplitude growing progressively. When the vibrational amplitude increases to a certain level, the linear model employed above dose not make sense anymore and nonlinearities of the electromechanical interaction and fluid bearing forces begin to take effect. Such nonlinear torques would confine the growing of the processional amplitude so that the motion would eventually ends in a limit cycle. Also, since the rotor is more often than not subject to a dynamic imbalance, which acts as an excitation at the frequency of the rotational speed, the rotor would accordingly undergoes an additional vibration at the frequency of the rotational speed. Figure 6 plots the vibrational characteristic typical in the range of the stable operation, while figure 7 plots the dynamic characteristics of the double frequency procession of the rotor at a mechanical
frequency beyond the stability boundary. This type of dual frequency vibration will take place slightly beyond the stability boundary where the positive real part of $p_b$ is very small. When the speed of the rotor rises furthermore, the increased real part would set the vibrational amplitude into faster growing pace. It is hence difficult for the nonlinear torques to counterbalance this effect, and the rotor would ultimately wind up in crashing.

Figure 7. The axes contrails and time history of rotor response at the mechanical frequency slightly beyond the stability boundary.

3. Conclusions
The inclination dynamics of a rotor-thrust-bearing system with dynamic imbalance has been analyzed using a two-degree-of-freedom rotordynamic model with corresponding electromagnetic and fluid film coefficients. Results obtained from the model show that the system would encounter severe instability problem when the rotational speed exceeds some threshold value. Such instability is brought about by the cross-coupling torque partly arises from hydrodynamic effect and partly from electromechanical interaction. As the stability boundary occurs at a relative large slip frequency, the system would not be able to operation smoothly at a certain designed speed, and the performance of the micromachine would thus be compromised.

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