Transport coefficients in neutron star environment with the possibility of hadron-quark phase transition

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Abstract

We have attempted to visualize transport coefficients like shear viscosity and electrical conductivity with respect to density of neutron star environment, whose core may expect a hadron-quark phase transition due to acquiring much higher density than nuclear saturation density. By making sandwich between MIT bag model for quark phase and two different effective hadronic models for hadronic phase, we have estimated the transport coefficients of two phases. During sketching of the transport coefficients, we have discussed their detailed density profile of phase-space part and relaxation time part. By calculating shear viscosity to density ratio, we have also explored the nearly perfect fluid domain along the density axis of hadron-quark phase diagram.

1 Introduction

Since the discovery of neutron stars (NSs), they have been one of the most interesting astrophysical objects found in the observable universe. A number of branches of physics play important role to explain the extreme characteristics and properties of the NSs. They are wonderful ‘celestial laboratories’ which provide the scope for studying various topics of current interest. The theoretical computation of the structural properties of NSs is largely dependent on the equation of state (EoS) which
in turn is determined by the composition of NS matter (NSM). The later is still inconclusive from experimental point of view as NSs are characterized by extreme density (about 5-10 times the normal nuclear matter density). Therefore theoretical modeling of NSM and determination of EoS is one of the most active areas of research in the domain of NS physics. However, recent observations of massive pulsars [1, 2] and the data extracted from the detection of gravitational wave (GW170817) from binary NS merger (BNSM) [3] have put some constraints on the EoS of NSs. Theoretical research also suggest the possibility of hadron-quark phase transition in NS cores and the formation of hybrid stars (HSs) [4, 5, 6, 7, 8, 9, 10, 11, 12] and even existence of quark stars (QSs) [4, 13, 14]. For recent review on it, readers can go through the Refs. [15, 16]. This hadron-quark transition density within NS environment is largely depends on the theoretical models and interactions considered. It is grossly linked with very low temperature and high baryon density zone of quantum chromodynamic (QCD) phase diagram, whose first-principles predictions via lattice QCD (LQCD) calculations are missing due to the infamous sign problem [17]. However, at high temperature and low/vanishing baryon density domain of QCD phase diagram is well studied for more than 3 decades [18, 19], see Ref. [20] for its latest status. They conclude a cross-over type phase transition, which has been alternatively realized from estimation of thermodynamic quantities in low and high temperature ranges by hadron resonance gas (HRG) model [21] and finite temperature perturbative QCD (pQCD) calculation [22, 23] respectively. Similar mapping in the low/vanishing temperature and high baryon density domain might be possible [15] by fusing ultra-high density (approximately 10 times larger than the nuclear saturation density) pQCD calculations [24, 25, 26] and low density hadronic model calculations. In this context, present work attempts for comparing the estimations of transport coefficients, obtained from standard MIT Bag model for quark phase [27], and chiral hadronic model [11, 28, 29, 30, 12, 31, 32, 33, 34], relativistic mean field (RMF) model [35, 36, 37, 38] for hadronic phase. Both the types of hadronic models have been explored thoroughly to construct the EoS of dense NSM and successfully determine the structural properties of NSs in the light of recent constraints from various astrophysical observations [11, 28, 29, 30, 12, 37, 38]. The present work is aimed to probe this hadron-quark phase transition via transport coefficients estimations in this high baryon density and low/vanishing temperature domain of QCD phase diagram. The motivation of this kind of investigation comes from the equivalent pattern of LQCD thermodynamics [19, 20] and normalized transport coefficients [39] in the high temperature and low/vanishing baryon density domain of QCD phase diagram. If we analyze the temperature ($T$) profile of thermodynamical quantities like pressure $P$, energy density $\epsilon$, entropy density $s$ and transport coefficients like shear viscosity $\eta$, electrical conductivity $\sigma$ for massless quark matter, then we can find the proportional relations $P = \frac{1}{3}\epsilon = \frac{1}{4}TS \propto T^4$ and $\eta \propto T^4, \sigma \propto T^2$, where $\tau$ is relaxation.
time of massless quark. So their normalized values $P/T^4$, $\epsilon/T^4$, $s/T^3$, $\eta/(\tau T^4)$ and $\sigma/(\tau T^2)$ will appear as horizontal line against $T$-axis and they can be marked as their upper or massless or Stefan-Boltzmann (SB) limits. At very high $T$, these limiting values can be reached. As we go from high to low $T$, the values of thermodynamical quantities and transport coefficients will decrease and their maximum decrement will be noticed around quark-hadron transition temperature [39]. Here we are interested to find similar kind of graphs along baryon density axis.

During literature survey, we get a long list of references [40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59], concentrated on microscopic calculation of transport coefficients for NS system. In terms of specific contribution, sketching the thermodynamical phase space of transport coefficients with respect to density may be considered as an alternative portrait, relevant for qualitative understanding hadron-quark transition in astrophysical environment.

The article is organized as follows. Next in Formalism section (2), we have briefly addressed two different models for hadronic phase and MIT Bag model for quark phase description in three different subsections - (2.1), (2.2) and (2.3), respectively. Then in 4th subsection (2.4), the formalism of relaxation time approximation for transport coefficients are quickly addressed. After getting the final expression of transport coefficients and effective kinematic information of hadronic and quark phases in the formalism part, we have generated their curves in the result section (3) along with detail discussion. At the end - in Sec (4), we have summarized our findings.

2 Framework

In this section, we will address briefly about the two hadronic models and the Bag model for describing hadronic and quark matter respectively. We will adopt two different hadronic model, whose brief details are given in next two subsections (2.1), (2.2) and then in the third subsection (2.3), brief discussion of MIT bag model for quark phase is addressed.

2.1 Hadronic Phase: Hadronic model - 1

Considering only nucleons as the baryonic degrees of freedom, the Lagrangian density for the effective chiral model [32, 33, 34] is given by

$$
\mathcal{L} = \overline{\psi} \left[ \left( i\gamma_\mu \partial^\mu - g_\omega \gamma_\mu \omega^\mu - \frac{1}{2} g_\rho \overrightarrow{p} \cdot \overrightarrow{\rho} \gamma^\mu \right) - g_\sigma \left( \sigma + i\gamma_5 \overrightarrow{\tau} \cdot \overrightarrow{\tau} \right) \right] \psi 
+ \frac{1}{2} \left( \partial^\mu \overrightarrow{p} \cdot \partial^\nu \overrightarrow{p} + \partial^\mu \sigma \partial^\nu \sigma \right) - \frac{\lambda}{4} (x^2 - x_0^2)^2 - \frac{\Lambda B}{6} (x^2 - x_0^2)^3 - \frac{\lambda C}{8} (x^2 - x_0^2)^4
$$
\[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g_{\omega}^2 x^2 \omega_\mu \omega^\mu - \frac{1}{4} R_{\mu\nu} \cdot \overline{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 p_\mu \cdot \rho^\mu, \]

where, \( \psi \) is the nucleon isospin doublet. The nucleons interact with each other via the scalar \( \sigma \) meson, the vector \( \omega \) meson (783 MeV) and the isovector \( \rho \) meson (770 MeV) with corresponding coupling strengths \( g_\sigma, g_\omega \) and \( g_\rho \), respectively. As mean field treatment is considered, the pions do not contribute. The model is based on chiral symmetry with the \( \sigma \) and the pseudoscalar \( \pi \) mesons as chiral partners and \( x^2 = (\pi^2 + \sigma^2) \). The \( \sigma \) field attains a VEV \( \sigma_0 = x_0 \) with the spontaneous breaking of the chiral symmetry at ground state \[60\]. The masses of the nucleons \( (m) \) and the scalar and vector mesons can be expressed in terms of \( x_0 \) as

\[ m = g_\sigma x_0, \quad m_\sigma = \sqrt{2\lambda} x_0, \quad m_\omega = g_\omega x_0, \]

where \( \lambda = (m_\sigma^2 - m_\pi^2)/(2f_\pi^2) \) is derived from chiral dynamics. The \( f_\pi \), being the pion decay constant, relates to the VEV of \( \sigma \) field as \( <\sigma> = \sigma_0 = f_\pi \)

The term \( \frac{1}{2} g_\omega^2 x^2 \omega_\mu \omega^\mu \) implies an explicit dependence of the nucleon effective mass on both the scalar and the vector fields and this is one of the salient features of the present model. The isospin triplet \( \rho \) mesons are incorporated to account for the asymmetric nuclear matter. An explicit mass term for the isovector \( \rho \) mesons is chosen following Refs. \[33, 61, 32, 34\]. The coupling strength of \( \rho \) mesons with the nucleons is obtained by fixing the symmetry energy coefficient \( J = 32 \) MeV at nuclear saturation density \( \rho_0 \). In terms of the baryon density \( \rho \)

and the Fermi momentum \( k_F = (6\pi^2 \rho/\gamma)^{1/3} \) \( (\gamma \) is degeneracy factor of nucleon), the isovector coupling strength is related to \( J \) as

\[ J = \frac{C_\rho k_F^3}{12\pi^2} + \frac{k_F^2}{6\sqrt{(k_F^2 + m^*2)}} \]

where, \( C_\rho = g^2_\rho/m^2_\rho \) and \( m^* \) is the nucleon effective mass. The scalar density is obtained as

\[ \rho_S = <\overline{\psi}\psi> = \frac{\gamma}{2\pi^2} \int_0^{k_F} dk \frac{k^2 m^*}{\sqrt{k^2 + m^*2}} \]

while the baryon density as

\[ \rho = <\psi^\dagger \psi> = \frac{\gamma}{2\pi^2} \int_0^{k_F} dk \frac{k^2}{\sqrt{k^2 + m^*2}}. \]

For symmetric nuclear matter (SNM) i.e, with equal number of neutrons and protons \( (N=Z) \) the spin degeneracy factor \( \gamma = 4 \) while for asymmetric nuclear matter \( \gamma = 2 \). \footnote{Readers should be careful on the same notation, used for \( \rho \) meson and baryon density \( \rho \).}
The nucleon chemical potential is given as

$$\mu_B = \sqrt{k^2 + m^{*2}} + g_\omega \omega_0 + g_\rho I_3 B \rho_{03} ,$$

(6)

where, $I_3 B$ (with $B = n, p$) is the third component of isospin of the individual nucleons and $\omega_0$ and $\rho_{03}$ are the mean field values of the vector and isovector mesons, respectively.

The energy density ($\varepsilon$) and pressure ($P$) is given as

$$\varepsilon = \frac{m^2}{8 C_\sigma} (1 - Y^2)^2 - \frac{m^2 B}{12 C_\omega C_\sigma} (1 - Y^2)^3 + \frac{C m^2}{16 C_\omega C_\sigma} (1 - Y^2)^4 + \frac{1}{2 Y^2} C_\omega \rho^2$$

$$+ \frac{1}{2} m^2 \rho_{03}^2 + \frac{\gamma}{2 \pi^2} \int_0^{k_F} k^2 \sqrt{(k^2 + m^{*2})} \, dk$$

(7)

$$P = -\frac{m^2}{8 C_\sigma} (1 - Y^2)^2 + \frac{m^2 B}{12 C_\omega C_\sigma} (1 - Y^2)^3 - \frac{C m^2}{16 C_\omega C_\sigma} (1 - Y^2)^4 + \frac{1}{2 Y^2} C_\omega \rho^2$$

$$+ \frac{1}{2} m^2 \rho_{03}^2 + \frac{\gamma}{6 \pi^2} \int_0^{k_F} \frac{k^4}{\sqrt{(k^2 + m^{*2})}} \, dk$$

(8)

where, $C_i = g_i^2/m_i^2$ are the scaled couplings ($i = \sigma & \omega; m_i$ being the mass of the mesons) and $Y = m^*/m$. $B$ and $C$ are the coefficients of higher order scalar field terms.

The five model parameters $C_\sigma, C_\omega, C_\rho$ and the coefficients $B & C$ of higher order scalar field terms are determined by reproducing the properties of SNM at saturation density $\rho_0$. The detailed procedure of obtaining these model parameters can be found in [34]. For the present work the parameter set is chosen from [34] and is presented in table 1 below along with the SNM properties yielded by this parameter set.

| model parameters chosen for the present work (adopted from [34]) |
|---------------------------------|--------------|-------------|--------------|---------------|
| $C_\sigma$ (fm$^2$)              | $C_\omega$ (fm$^2$) | $C_\rho$ (fm$^2$) | $B/m^2$ (fm$^2$) | $C/m^4$ (fm$^2$) |
|---------------------------------|--------------|-------------|--------------|---------------|
| 6.772                           | 1.995        | 5.285       | -4.274       | 0.292         |
| $m^*/m$                         | $K$ (MeV)    | $B/A$ (MeV) | $J$ (MeV)    | $L_0$ (MeV) |
|---------------------------------|--------------|-------------|--------------|---------------|
| 0.85                            | 303          | -16.3       | 32           | 87            | 0.153         |

Since the nucleon effective mass $m^*$ for this model depends on both the scalar and vector fields, it is quite high compared to well-known RMF models. Also at
high density, unlike RMF models, the value of \(m^*\) increases after a certain high value of density \[33, 34, 11, 30\]. This is due to the dominance of vector potential at such density. Moreover, at high density the higher order terms of scalar field with coefficients \(B\) and \(C\) and the mass term of the vector field of the present model also become highly non-linear and dominant \[33, 34, 11, 30\] leading to softening of the EoS that passes through the soft band of heavy-ion collision data for the adopted parameter set \[31\]. The nuclear incompressibility \(K\) obtained with the chosen parameter set, though consistent with the results of \[62\] but it is larger than estimated in \[63, 64\]. Although it can be seen from \[31\] that few other parameter sets of the present model yield values of \(K\) which are consistent with \[63, 64\], they cannot be adopted as they yield even softer EoS \[31\] and consequently do not satisfy the maximum mass constraint \[1\] of NSs. The present set satisfies this constraint \[28, 11, 29, 31, 30, 12\]. The other SNM properties like the binding energy per nucleon \(B/A\), the symmetry energy \(J\) and the saturation density \(\rho_0\) match well with the estimates of Refs. \[65, 66\]. The slope parameter \(L_0\) is also quite consistent with the range specified by Refs. \[65, 67, 68\]. The same model parameter set has also been adopted earlier in Refs. \[28, 11, 29, 31, 30, 12, 69\] to successfully investigate different properties of NSs as well as hybrid stars in the light of various constraints specified on their structural properties like maximum gravitational mass, radius and tidal deformability of \(1.4M_\odot\) NS etc.

### 2.2 Hadronic Phase: Hadronic model - 2

In the conventional relativistic mean field (RMF) theory \[70, 71, 72, 73, 74, 75, 76\] \[35, 36, 37, 38\] nucleons are treated as elementary particles and interactions between the nucleons are mediated by the exchange of \(\sigma\), \(\omega\) and \(\rho\) mesons. The \(\sigma\) mesons give rise to the strong attractive force, while the \(\omega\) mesons cause the strong repulsive force between the nucleons. In addition to this, several self and cross interaction terms between the mesons are also considered to yield the saturation properties correctly.

The Lagrangian density for the extended RMF model can be written as,

\[
\mathcal{L} = \mathcal{L}_{\mathcal{N}, \mathcal{M}} + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_{\sigma\omega\rho},
\]

where the Lagrangian \(\mathcal{L}_{\mathcal{N}, \mathcal{M}}\) describing the interactions of the nucleons with mass \(m\) through the mesons is,

\[
\mathcal{L}_{\mathcal{N}, \mathcal{M}} = \sum_{B=n,p} \bar{\psi}_i \left[ i\gamma^\mu \partial_\mu - (m - g_\sigma \sigma) - (g_\omega \gamma^\mu \omega_\mu + \frac{1}{2} g_\rho \gamma^\mu \tau \cdot \rho_\mu) \right] \psi_i.
\]

Here, the sum is taken over the neutrons and protons and \(\tau\) are the isospin matrices. The Lagrangians for the \(\sigma\), \(\omega\), and \(\rho\) mesons including their self interaction
terms can be written as,

\[
\mathcal{L}_\sigma = \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) - \frac{\kappa_3}{6m} g_\sigma m_\sigma^2 \sigma^3 - \frac{\kappa_4}{24m^2} g_\sigma^2 m_\sigma^2 \sigma^4,
\]

\[
\mathcal{L}_\omega = -\frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_{\mu} \omega^{\mu} + \frac{1}{24} \zeta_\omega g_\omega^2 (\omega_{\mu} \omega^{\mu})^2,
\]

\[
\mathcal{L}_\rho = -\frac{1}{4} \rho_{\mu\nu} \rho^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho_{\mu} \rho^{\mu}.
\]  \hspace{1cm} (11)

The \(\omega^{\mu\nu}\), \(\rho^{\mu\nu}\) are field tensors corresponding to the \(\omega\) and \(\rho\) mesons, and can be defined as \(\omega^{\mu\nu} = \partial^\mu \omega^\nu - \partial^\nu \omega^\mu\) and \(\rho^{\mu\nu} = \partial^\mu \rho^\nu - \partial^\nu \rho^\mu\). Here, \(m_\sigma\), \(m_\omega\) and \(m_\rho\) are the masses of \(\sigma\), \(\omega\), and \(\rho\) mesons, respectively. The cross interactions of \(\sigma\), \(\omega\), and \(\rho\) mesons are described by \(\mathcal{L}_{\sigma\omega\rho}\) which can be written as,

\[
\mathcal{L}_{\sigma\omega\rho} = \frac{\eta_1}{2m} g_\sigma m_\sigma^2 \sigma \omega_\mu \omega^{\mu} + \frac{\eta_2}{4m^2} g_\sigma^2 m_\sigma^2 \sigma^2 \omega_\mu \omega^{\mu} + \frac{\eta_3}{2m} g_\sigma m_\rho^2 \sigma \rho_{\mu} \rho^{\mu} + \frac{\eta_4}{4m^2} g_\sigma^2 m_\rho^2 \sigma^2 \rho_{\mu} \rho^{\mu}.
\]  \hspace{1cm} (12)

The field equations derived from the above Lagrangian can be solved self-consistently by adopting mean-field approximation, i.e., the meson-field operators are replaced by their expectation values. The energy density of nuclear matter in terms of the coupling constants and mesonic mean fields is given by

\[
\epsilon = \sum_{i=n,p} \frac{1}{2} \int_0^{k_F} dk \ k^2 \sqrt{k^4 + m_i^2 \sigma_0^2 + \frac{\kappa_3}{6m} g_\sigma m_\sigma^2 \sigma_0^3 + \frac{\kappa_4}{24m^2} g_\sigma^2 m_\sigma^2 \sigma_0^4 + g_\omega \omega_0 \rho_B - \frac{1}{2} m_\omega^2 \omega_0^2 - \frac{1}{24} \zeta_\omega g_\omega^2 \omega_0^4 + \frac{1}{2} g_\rho \rho_0 \rho_3 - \frac{1}{2} m_\rho^2 \rho_0^2 - \frac{\eta_1}{2m} g_\sigma m_\sigma^2 \sigma_0 \omega_0^2 - \frac{\eta_2}{4m^2} g_\sigma^2 m_\sigma^2 \sigma_0^2 \omega_0^2 - \frac{\eta_3}{2m} g_\sigma m_\rho^2 \sigma_0 \rho_0^2 - \frac{\eta_4}{4m^2} g_\sigma^2 m_\rho^2 \sigma_0^2 \rho_0^2}
\]  \hspace{1cm} (13)

where \(m_i = m_i - g_i \sigma\) is the effective mass of the nucleon and the equilibrium densities are defined as \(\rho_B = \rho_p + \rho_n\) and \(\rho_3 = \rho_p - \rho_n\). In equation (13) we have introduced \(\sigma_0 \equiv \langle \sigma \rangle\), \(\omega_0 \equiv \langle \omega \rangle\), and \(\rho_0 \equiv \langle \rho \rangle\). The values of the coupling constants are usually determined in such a way that they yield appropriate values for finite nuclei properties (e.g. binding energy, charge radii) and various quantities associated with the nuclear matter at the saturation density. As the mean field approximation is thermodynamically consistent, at zero temperature the pressure of the system can be obtained by using the following expression,

\[
P = \sum_{i=n,p} \rho_i \mu_i - \epsilon
\]  \hspace{1cm} (14)
where $\mu_B = \sqrt{k^2 + m^2} + g_\omega \omega_0 + g_\rho I_{3B} \rho_0$ is the chemical potential of nucleon with $I_{3B}$ being the isospin 3-component of nucleon.

We use the BSP parametrization [35] for our calculation, which describe the properties of finite nuclei very well. It also satisfy the constraint from $2M_\odot$ NS. This parametrization includes the quartic order cross-coupling between $\omega$ and $\sigma$ mesons to model the high density behaviour of the EoS.

Table 2: BSP parameter sets for the extended RMF model with the nucleon mass $m = 939.2$ MeV.

| $g_\sigma/4\pi$ | $g_\omega/4\pi$ | $g_\rho/4\pi$ | $\kappa_3$ | $\kappa_4$ | $\eta_1$ | $\eta_2$ | $\eta_\rho$ | $\eta_{1\rho}$ | $\eta_{2\rho}$ | $\zeta_0$ | $m_\sigma/m$ | $m_\omega/m$ | $m_\rho/m$ |
|----------------|----------------|-------------|-----------|-----------|---------|---------|----------|----------|----------|--------|-----------|-----------|-----------|
| 0.8764         | 1.1481         | 1.0508      | 1.0681    | 14.9857   | 0.0872  | 0.0     | 0.0      | 53.7642  | 0.0      | 53.7642 | 0.5383   | 0.8333    | 0.8200    |

Table 3: Properties of the nuclear matter at the saturation density.

| $m*/m$ | $B/A$ (MeV) | $\rho_0$ (fm$^{-3}$) | $K$ (MeV) | $J$ (MeV) | $L_0$ (MeV) |
|--------|-------------|----------------------|-----------|-----------|------------|
| 0.60   | -15.9       | 0.149                | 230       | 28.83     | 50         |

2.3 Quark Phase

The MIT bag model [27] with u and d quarks is considered to describe the pure massless quark phase. The simplest form of this model has only one parameter known as the bag pressure $B$ [3] and the perturbative corrections or repulsive effects of the strongly interacting quarks are not considered. According this simple model, energy density and pressure of quark matter are given below

$$\varepsilon_{QM} = B + \sum_f \frac{3}{4\pi^2} \left[ \mu_f k_f \left( \mu_f^2 - \frac{1}{2} m_f^2 \right) - \frac{1}{2} m_f^4 \ln \left( \frac{\mu_f + k_f}{m_f} \right) \right],$$  \hfill (15)

$$P_{QM} = -B + \sum_f \frac{1}{4\pi^2} \left[ \mu_f k_f \left( \mu_f^2 - \frac{5}{2} m_f^2 \right) + \frac{3}{2} m_f^4 \ln \left( \frac{\mu_f + k_f}{m_f} \right) \right],$$  \hfill (16)

where chemical potential

$$\mu_f = (k_f^2 + m_f^2)^{\frac{1}{2}}$$  \hfill (17)

and total density

$$\rho = 2 \times 3 \times \frac{k_L^3}{6\pi^2}.$$  \hfill (18)

Readers should be careful with same notation $B$ used for bag pressure and binding energy.
Here, \( f = u \) and \( d \) are the quark flavors and \( m_f \approx 5 \) MeV is the mass of \( u \) or \( d \) quarks, which is low enough to consider the quark matter massless.

### 2.4 Framework of transport coefficients

Here we will address the standard kinetic theory frameworks of transport coefficients like shear viscosity \( \eta \) and electrical conductivity \( \sigma \), which are basically the part of dissipation component of any many body system or medium or fluid. Let us first derive expression of shear viscosity. Considering NSM as a dissipative fluid, one can express energy-momentum tensor in macroscopic form as

\[
T^{\mu\nu} = T_0^{\mu\nu} + T_D^{\mu\nu},
\]

where ideal \((T_0^{\mu\nu})\) and dissipation \((T_D^{\mu\nu})\) parts in terms of fluid quantities like fluid four velocity \( u^\mu \), energy density \( \epsilon \), pressure \( P \) etc. can be expressed as

\[
T_0^{\mu\nu} = -g^{\mu\nu} + (\epsilon + P)u^\mu u^\nu
\]

and

\[
T_D^{\mu\nu} = \pi^{\mu\nu} + ... = \eta^{\mu\nu\alpha\beta} U_{\alpha\beta} + ...
\]

respectively. In the expression of dissipative part of energy-momentum tensor, \( \eta^{\mu\nu\alpha\beta} \) is shear viscosity tensor. Also, dissipative part can have components of shear, bulk and thermal dissipation but we are dealing here only the shear part. The notation of \((+...)\) indicates the other dissipation components. The picture of relativistic shear viscosity \( \eta \) as a proportional constant between viscous stress tensor \( \pi^{\mu\nu} \) and tangential fluid velocity gradient \( \eta^{\mu\nu\alpha\beta} \) can be compared with Newton-Stoke law, applicable in non-relativistic domain, where shear viscosity is defined as proportional constant between shear stress and velocity gradient.

Considering the baryonic NSM to be composed of nucleons, the microscopic expression of its energy-momentum tensor can be written in kinetic theory framework as

\[
T^{\mu\nu} = \gamma \int \frac{d^3k}{(2\pi)^3} \frac{k^\mu k^\nu}{E} (f_N + f_{\bar{N}})
\]

where, \( f_N \) and \( f_{\bar{N}} \) are assumed as non-equilibrium distribution functions for nucleons \( N \) and anti-nucleons \( \bar{N} \). Now splitting \( f_{N,\bar{N}} \) as the sum of equilibrium distribution \( f_0^{N,\bar{N}} \) and a small deviation \( \delta f_{N,\bar{N}} \) i.e,

\[
f_{N,\bar{N}} = f_0^{N,\bar{N}} + \delta f_{N,\bar{N}}
\]
one can separately identify the microscopic expressions of $T_0^{\mu\nu}$ and $T_D^{\mu\nu}$ parts in terms of particle quantities like particle’s four momenta $k^\mu$, degeneracy factor $\gamma$ etc. as

$$T_0^{\mu\nu} = \gamma \int \frac{d^3 k}{(2\pi)^3} \frac{k^\mu k^\nu}{E} (f_N^0 + f_N^0) \quad (24)$$

and

$$T_D^{\mu\nu} = \pi^{\mu\nu} + \ldots = \gamma \int \frac{d^3 k}{(2\pi)^3} \frac{k^\mu k^\nu}{E} (\delta f_N + \delta f_N) + \ldots \quad (25)$$

Now, with the help of standard relaxation time approximation (RTA) methods of Boltzmann transport equation, one can get \[39\]

$$\delta f_{N,\bar{N}} = \frac{k^\alpha k^\beta}{E} \tau_{N,\bar{N}} \beta f_{N,\bar{N}}^0 \left( 1 - f_{N,\bar{N}}^0 \right) \mathcal{U}_{\alpha\beta} \quad , \quad (26)$$

where $\tau_{N,\bar{N}}$ are relaxation times and $E = \sqrt{k^2 + m^2}$ are energy of nucleon and antinucleon. Now, using Eq. \[26\] in Eq. \[25\] and then compare with Eq. \[21\], we will get the shear viscosity tensor as

$$\eta^{\mu\nu\alpha\beta} = \gamma \int \frac{d^3 k}{(2\pi)^3} \frac{k^\mu k^\nu k^\alpha k^\beta}{E^2} \tau_{N,\bar{N}} \beta \left[ f_{N}^0 \left( 1 - f_{N}^0 \right) + f_{\bar{N}}^0 \left( 1 - f_{\bar{N}}^0 \right) \right] \quad , \quad (27)$$

Using tensor identity \[39 \quad 77 \quad 78\], one can obtain the isotropic expression

$$\eta = \frac{\gamma}{15} \int \frac{d^3 k}{(2\pi)^3} \frac{k^4}{E^2} \tau_{N,\bar{N}} \beta \left[ f_{N}^0 \left( 1 - f_{N}^0 \right) + f_{\bar{N}}^0 \left( 1 - f_{\bar{N}}^0 \right) \right] \quad , \quad (28)$$

Now, if we take our calculation from finite $T$ to $T = 0$ case, then the equilibrium Fermi Dirac distribution takes the form of a step function as

\begin{align*}
  f_{N}^0 &= 1 \text{ if } E < \mu \\
  &= 0 \text{ if } E > \mu \\
  f_{\bar{N}}^0 &= 1 \text{ if } E < -\mu \\
  &= 0 \text{ if } E > -\mu \quad . \quad (29)
\end{align*}

Above conditions implies anti-nucleons don’t contribute anymore in positive energy axis for $T = 0$ case. We have to use the replacement:

$$\frac{\partial f_N^0}{\partial E} = -\beta f_N^0 (1 - f_N^0) \rightarrow \frac{\partial}{\partial E} \theta(\mu - E) = -\delta(E - \mu) \quad . \quad (30)$$
Using those replacements in Eq. (28), we will get

\[
\eta = \frac{\gamma}{15} \int \frac{d^3 \vec{k}}{(2\pi)^3} \frac{k^4}{E^2\tau_N} \delta(E - \mu) \\
= \frac{\gamma}{30\pi^2\tau_N} \left(\frac{\mu^2 - m^*}{\mu}\right)^{5/2}
\]  

(31)

Next, let us come to other transport coefficient - electrical conductivity, which is basically proportional constant between electric current density and field. This macroscopic definition comes from Ohm’s law, which can be expressed in three dimensional notation as

\[
J^i = \sigma^{ij}E_j ,
\]  

(32)

where electrical conductivity tensor \( \sigma^{ij} \) connect electric field \( E_j \) and electric current density \( J^i \). Realizing current due to electric field as a dissipation phenomenon, we can microscopically expressed it as

\[
J^i = q\gamma \int d^3 \vec{k} \frac{k^i}{(2\pi)^3} E \delta f_N ,
\]  

(33)

where \( q \) is electric charge of nucleon, i.e. \( q = 0, e \) for neutron and proton. Here, the RTA methods of Boltzmann transport equation will again help us to give the form of \( \delta f_N \):

\[
\delta f_N = \frac{k_j}{E} \tau_N \beta f^0_N (1 - f^0_N) q E_j ,
\]  

(34)

Using Eq. (34) in Eq. (33) and then compare with Eq. (32), we will get the conductivity tensor as

\[
\sigma^{ij} = q^2 \gamma \int d^3 \vec{k} \frac{k^i k^j}{(2\pi)^3} \frac{E^2}{E^2} \tau_N \beta f^0_N (1 - f^0_N) ,
\]  

(35)

whose isotropic expression at \( T = 0 \) will be

\[
\sigma = \frac{q^2 \gamma}{3} \int d^3 \vec{k} \frac{k^2}{(2\pi)^3} \frac{E^2}{E^2} \tau_N \delta(E - \mu) \\
= \frac{q^2 \gamma}{6\pi^2 \tau_N} \left(\frac{\mu^2 - m^*}{\mu}\right)^{3/2}
\]  

(36)

The final expressions of shear viscosity and electrical conductivity, given in Eqs. (31), (36), are built for NSM. The neutrons will not contribute to electrical conductivity, so \( \gamma = 4 \) for Eq. (31) but \( \gamma = 2 \) for Eq. (36). When we apply these two expressions
for quark phase, then nucleon effective mass $m^*$ will be replaced by quark mass and corresponding degeneracy factor $\gamma$ will also be replaced by $g$ (say). Considering $u$ and $d$ quarks, $g = 3 \times 2 \times 2 = 12$ for Eq. (31) and $g q^2 = 3 \times 2 \left( \frac{4 e^2}{9} + \frac{e^2}{9} \right) = \frac{10 e^2}{3}$ for Eq. (36). The quark phase values of $\eta$ and $\sigma$ will be very close to their massless limits

$$\eta = \frac{g}{30 \pi^2} T Q \mu^4$$
$$\sigma = \frac{g q^2}{6 \pi^2} T Q \mu^2,$$

whose normalized values $\frac{\eta}{\tau_Q \mu^4} = \frac{2}{5 \pi^2} \approx 0.04$ and $\frac{\sigma}{\tau_Q \mu^2} = \frac{5 e^2}{9 \pi^2} = 0.005$ become constants. These massless limits may act as reference line at $T = 0$ and finite $\mu$ case.

The relaxation time can also be calculated microscopically. For degenerate scenario, the medium constituents will occupy all energy levels from $m$ to $\mu$ and ideally they have zero probability to move outside $\mu$. However, in reality, due to very low $T$ instead of exactly $T = 0$, we can have a small deviation from step function type distribution function. So medium constituents, having energy near Fermi energy $\mu$ and velocity near Fermi velocity $v_F = \sqrt{\mu^2 - m^2}$ will participate in momentum transfer scattering process. So we can define relaxation time as

$$\tau_c = 1 / [\sigma_s v_F \rho] = \mu / [\sigma_s \sqrt{\mu^2 - m^2} \rho],$$

where $\rho$ is density of the medium and $\sigma_s = 4 \pi a^2$ is cross section with scattering length $a$. Now depending upon system, we have to use the inputs of medium constituents to calculate their relaxation time.

### 3 Results

After introducing brief formalism of thermodynamics and transport coefficients in earlier section, we will proceed for result generation using their final expressions and will discuss them here one by one. Let us first discuss about thermodynamical quantities like pressure, energy density, enthalpy density etc.

Here, the neutron and proton contribution in the hadronic matter are estimated by considering $\beta$-equilibrium conditions. We obtain the pressure of hadronic phase by using Eqs. (8), (14) for two different hadronic models HM1, HM2, respectively and of quark phase by using Eq. (16). The energy density is calculated using Eq. (7) and Eq. (13) for HM1 and HM2, respectively and Eq. (15) for MQM. We next calculate the enthalpy density individually for hadronic and quark matter and compare them
in Fig. (1). If we take hadronic (red solid line for HM1 and green dash-dotted line for HM2) and quark (blue dotted line) results, then a jump in (normalized) enthalpy density of two phases is noticed. One may connect this fact with first order nature of hadron-quark phase transition in dense sector. Similar kind of jumps will be also expected in different transport coefficients like shear viscosity and electrical conductivity, which are addressed below one by one. We will latter see that enthalpy density will be important quantity to measure fluidity when shear viscosity will be normalized by it and form a dimensionless ratio.

After calculating shear viscosity and electrical conductivity by using Eqs. (31), (36) for hadronic matter and Eq. (37) for quark matter, we have normalized them as \( \eta/(\tau_c \mu^4) \) and \( \sigma/(\tau_c \mu^2) \) to make them dimensionless, where the notation \( \tau_c \) is considered as relaxation time of quark or hadronic matter in general i.e. \( \tau_c = \tau_N \) for hadronic matter and \( \tau_c = \tau_Q \) for quark matter. These \( \eta/(\tau_c \mu^4) \) and \( \sigma/(\tau_c \mu^2) \) are plotted against normalized density in left and right panels of Fig. (3) respectively. Roughly 10-20 times difference between HM1 and HM2 model estimations of \( \eta/(\tau_c \mu^4) \) and \( \sigma/(\tau_c \mu^2) \) are noticed. This is because of the difference in chemical potential (\( \mu \)) and also the
nucleon effective mass ($m^*$) of the two hadronic models. Both chemical potential and specially the effective mass are quite different for HM1 and HM2 as seen from Fig. (2). Re-looking the Eqs. (31), (36), one can visualize the numerical impact of Fig. (2) to Fig. (3). The differences in the order of magnitudes between two hadronic models reflects a numerical band of transport coefficients for hadronic matter along density axis. So we may accept the order of magnitude very roughly and may trust

Figure 2: Chemical potential (left) and effective nucleon mass (right) vs normalized baryon density of hadronic matter with two different hadronic models HM1 and HM2.

Figure 3: Normalized shear viscosity (left) and electrical conductivity (right) with respect to scaled baryon density of hadronic matter and massless quark matter (MQM).
on the qualitative profile along density axis. Similar to enthalpy density case, for both \( \eta/(\tau_c\mu^4) \) and \( \sigma/(\tau_c\mu^2) \) also, we notice a jump between the estimated values of hadronic models and the quark model. For shear viscosity case, this jump is from 0.001 (HM1) or 0.01 (HM2) to 0.06 and for electrical conductivity this jump is from 0.0003 (HM1) or 0.003 (HM2) to 0.5. This fact again can be connected with first order phase transition in terms of transport coefficients.

These estimations of thermodynamical quantity - enthalpy density, and transport coefficients - shear viscosity, electrical conductivity at \( T = 0 \) and \( \mu \neq 0 \) domain can be compared with the estimations thermodynamics and transport coefficients at \( T \neq 0 \) and \( \mu = 0 \) domain, given in Ref. [39] and references therein. The quark-hadron transition at \( T \neq 0 \) and \( \mu = 0 \) domain is understood by estimating thermodynamics of massless quark gluon plasma (QGP) and hot pion gas for high and low temperature zones respectively. However, lattice quantum chromodynamics (LQCD) [18, 19, 20] went into deeper and unfolded the crossover type nature of quark-hadron transition at \( T \neq 0 \) and \( \mu = 0 \) domain. The values of normalized thermodynamical quantities like \( P/T^4 \), \( \epsilon/T^4 \) etc. for massless QGP will act as reference line or upper limits of QCD matter at \( \mu = 0 \) case. LQCD thermodynamics remain quite lower than that limit in low or hadronic temperature domain, which is realized as non-perturbative aspects of QCD. Near transition temperature, their values enhance toward the massless limits but in a smooth crossover way instead of a first order phase transition kind jump. Beyond the transition temperature, LQCD thermodynamical values still remain little suppressed with respect to their massless limits. This small suppression is also realized from the direction of finite temperature perturbative quantum chromodynamics (pQCD) theory, is well studied historically, whose latest status can be found in Ref. [33]. Similar investigation of thermodynamical curves against density or chemical potential at \( T = 0 \) are attempted since a very long time for understanding the properties of NSs, see Ref. [85] for review. Due to problem in LQCD calculations at finite \( \mu \), present knowledge of quark-hadron phase transition along \( \mu \)-axis is not quite converging like the understanding of phase transition along \( T \)-axis. In this context, present work considered effective hadronic model and Bag model to get an order of magnitudes for different thermodynamical quantities of hadron and quark phases along \( \rho \) or \( \mu \)-axis. Along with thermodynamical quantities, normalized transport coefficients \( \eta/(\tau_c\mu^4) \) and \( \sigma/(\tau_c\mu^2) \) against \( \rho \) or \( \mu \)-axis will exhibit thermodynamical phase-space of transportation, which follow similar suppression in hadronic \( \rho \)-domain (at \( T = 0 \)) as noticed for hadronic \( T \)-domain (at \( \mu = 0 \)) in Ref. [39]. In Ref. [39], the normalized coefficients are chosen as \( \eta/(\tau_c T^4) \) and \( \sigma/(\tau_c T^2) \), while in present work they are chosen as \( \eta/(\tau_c\mu^4) \) and \( \sigma/(\tau_c\mu^2) \). In both cases, around \( 10^{1-2} \) suppression is observed in hadronic \( T \) or \( \mu \) domain with respect to their massless limits. Suppressed values of both cases can be realized as non-pQCD effect in phase-space of transportation. This equivalence between normalized phase-space of transport coefficients along
$T$-axis and $\rho$-axis might be considered as an unique finding of present investigation.

Figure 4: Relaxation time with respect to scaled baryon density of hadronic matter and massless quark matter (MQM) for different values of cross-section.

Up to now, the transport coefficients results are normalized by relaxation time. So we basically see the phase space part of transport coefficients only. However, actual estimation of relaxation time will be important to know the absolute values of those transport coefficients. We now calculate the relaxation time for hadronic and massless quark matter using Eq. (38). We will consider effective nucleon mass and two possible order of cross-section $\sigma_s = 4\pi a^2 \approx 5340$ fm$^2$ and 12.56 fm$^2$ for hadronic matter case with scattering lengths $a \approx 20$ fm and 1 fm. Former value of $\sigma_s \approx 5340$ fm$^2$ or $a \approx 20$ fm is isospin averaged cross section/scattering length of NN interactions, taken from Refs. [87, 88]. This value is used for relaxation time calculation of nucleon at finite temperature in Ref. [89]. Here, we have used for finite density system. We also consider another small value of scattering length $a = 1$ fm. In Fig. (4), we have plotted HM1, HM2 and MQM curves for $\tau_c$ by considering two different values of scattering lengths - one is guided from our vacuum scattering interaction strength and another is a guess by assuming that scattering length will decrease with density. As a rough order of magnitude, we get $\tau_c \approx 10 - 0.1$ fm and $\tau_c \approx 0.01 - 0.0002$ fm relaxation times for scattering lengths $a \approx 20$ fm and $a = 1$ fm. One should notice the opposite ranking HM1/HM2 > MQM for $\tau_c$ values with respect to the values of thermodynamic quantities like pressure, enthalpy density or normalized values of thermodynamical phase-space of transport coefficients. This is because of inverse relation of relaxation time with thermodynamic quantity like density $\rho$. 

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Once we obtain the relaxation time for each phase, we calculate the shear viscosity and electrical conductivity by using those density dependent relaxation time values. The normalized estimations of $\eta\mu/(\epsilon + P)$ and $\sigma/\mu$ are presented in left and right panels of Fig. (5) respectively. Former dimensionless quantity is chosen for fluid property $[90]$ measurement of quark and hadronic matter in NS environment. This ratio $\eta\mu/(\epsilon + P) = \eta/\rho$ for NS environment with $T = 0, \mu \neq 0$ can be related with so-called viscosity to entropy density ratio $\eta/s$ for early universe or RHIC/LHC environment with $T \neq 0, \mu = 0$ We know that the data of RHIC and LHC experiments $[91, 92, 93, 94, 95]$ indicate about very small values of $\eta/s$, ever observed in nature $[96]$. Being roughly proportional to the ratio of mean free path to de-Broglie wavelength of medium constituent, the $\eta/s$ of any fluid can never be vanished, because mean free path of any constituent can never be lower than its de-Broglie wavelength. It indicates that quantum fluctuations prevent the existence of perfect fluid in nature and $\eta/s$ of any fluid should have some lower bound, which is also claimed from the string theory calculation $[97]$. Interestingly, the values of $\eta/s$ for RHIC and LHC matter are very close to this quantum lower bound $1/(4\pi)$. So, a natural question come - whether this nearly perfect fluid nature is also expected along $\mu$-axis at $T = 0$ like along $T$-axis at $\mu = 0$ of QCD phase diagram? present
investigation is probably first time aimed to explore the fact but to get more con-
clusive outcome, probably further (alternative) research works are required in future.
The question become more contemporary as recently Ref. [98] has experimentally
expected the value $\eta/s \approx 1/(4\pi)$ for nuclear matter in low energy nuclear physics
experiment. We have plotted $\eta\mu/(\varepsilon + P)$, which is roughly equal to $\eta/\rho$, of hadronic
and quark matter with two values of scattering length in the left panel of Fig. [5]. We
notice that $a \approx 20$ fm results cross the KSS values ($\approx 0.08$), which may not be con-
sidered as physically acceptable order of magnitude. Probably the magnitude of the
cross section/scattering length in vacuum might be majorly modified for finite density
picture, which is missing in present calculation. In this regards, our guess value
scattering $a = 1$ fm provide the acceptable $\eta/\rho$ in hadronic density range, which still
cross the KSS boundary in quark density range. So a safe zone might be scattering
lengths $a < 1$ fm for getting $\eta/\rho > 1/(4\pi)$. For mathematical guidance we can get a
lower limit curve of relaxation time $\tau$ as a function of density or chemical potential for
massless quark matter by imposing the KSS limit. For finite temperature we get [99]
$\tau = 5/(4\pi T)$ by imposing KSS limit $\eta/s = 1/(4\pi)$. Similarly, for finite density, one
can easily find the corresponding expression $\tau = 5/(4\pi\mu)$ by using $\eta$ from Eq. [37], $\rho$
from Eq. [18] and then by imposing KSS limit $\eta/\rho = 1/(4\pi)$. In terms of density $\rho$, the
KSS limit of relaxation time for massless quark matter with $N_f = 2$ flavor will be

$$\tau = \frac{5}{4\pi} \left( \frac{2}{\pi^2} \right)^{1/3} \frac{1}{\rho^{1/3}}.$$  (39)

Another interesting outcome here is that we still see the jump in $\eta/\rho$ is possible
in case of a hadron-quark phase transition as HM2 > HM1 > MQM ranking is not-
iced in terms of $\eta/\rho$. So phase transition from hadronic to quark phase may imply a
reduction in $\eta/\rho$. However, for normalized electrical conductivity $\sigma/\mu$, HM2 > MQM
> HM1 ranking is observed. So the ranking of magnitude for two different phases
might be model dependent. We can’t get any conclusive picture on ranking of magni-
tude. Present work probably reveals that uncertainty. We also don’t go through any
comparison discussion of two hadronic models, rather we intend to show a possible
numerical uncertainty of hadronic model estimations. Hence, future research works
on other existing hadronic models are probably necessary for getting conclusive picture.
Only an order of magnitude difference in transport coefficients for two phases
can be considered as conclusive message in present study.

At the end, we should point out that present transport coefficients estimations
are happening within QCD time scale (fm) as we are considering only the strong
interaction. For realistic calculation of transport coefficients of NS, one should also
include electro-magnetic interaction part, whose time scale far away from QCD time
scale. Most of the earlier Refs. [40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54,
55, 56, 57, 58, 59] have captured this realistic estimations. However, present work is
confined within the QCD time scale for revealing the hadron-quark phase transition pattern in terms of momentum and charge transportation due to QCD interactions.

4 Summary

In summary, we have attempted to visualize the shear viscosity and electrical conductivity of NS environment along the density axis. Inspiration is coming from RHIC or LHC matter transport coefficients calculations, where along temperature axis, we get a cross-over type quark-hadron phase transition. The temperature profile of shear viscosity and electrical conductivity of RHIC or LHC matter mainly contain two parts - one is thermodynamical phase-space part and the relaxation time part. If we exclude the relaxation time part by normalizing it, then we get its beautiful pattern along temperature axis as we noticed for other thermodynamical quantities like pressure, energy density etc. from lattice QCD calculations. From high temperature massless limit values of these transport coefficients for quark matter are gradually suppressed when it goes to low temperature hadronic phase. Similar kind of pattern is observed in the present work when we go from high density quark phase to low density hadronic phase by sandwiching between finite density calculations of hadronic model and massless quark model. These equivalent thermodynamic phase-space profile of transportation along temperature axis and density axis is very interesting finding.

Instead of normalizing relaxation time, if we use its microscopic estimated values, which is generally of the order of fm due to strong interaction, then we can get final profile of the transport coefficients. A long list research can be found for RHIC or LHC matter, where most of them found that shear viscosity to entropy density ratio will decrease first then increase with temperature. In this regards, we find shear viscosity to density ratio decrease in hadronic density domain but suddenly dropped near hadron-quark phase transition and a mild decreasing trend with density is observed in quark phase. Present investigation might be considered as beginning level attempt to visualize fluid properties along density axis. However, many more future research is required for getting better understanding of this domain.

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References

[1] E. Fonseca et al., Refined Mass and Geometric Measurements of the High-Mass PSR J0740+6620, arXiv:2104.00880 (2021).
[2] J. Antoniadis et al., *A Massive Pulsar in a Compact Relativistic Binary*, Science 340, 6131 (2013).

[3] B. P. Abbott et al., *The LIGO Scientific Collaboration and The Virgo Collaboration, GW170817: Observation of gravitational waves from a binary neutron star inspiral*, Phys. Rev. Lett. 119 (Oct, 2017) 161101.

[4] N. K. Glendenning, *Compact Stars: Nuclear Physics, Particle Physics, and General Relativity* (Springer-Verlag, New York, 2000).

[5] F. Ozel et al., *The Massive Pulsar PSR J1614-2230: Linking Quantum Chromodynamics, Gamma-ray Bursts, and Gravitational Wave Astronomy*, Astrophys. J. Lett. 724 (2010) L199.

[6] L. Bonanno and A. Sedrakian, *Composition and stability of hybrid stars with hyperons and quark color-superconductivity*, Astron. Astrophys. 539 (2012) 416.

[7] X. Wu and H. Shen, *Finite-size effects on the hadron-quark phase transition in neutron stars* Phys. Rev. C 96 (2017) 025802.

[8] R. O. Gomes, P. Char, and S. Schramm, *Constraining Strangeness in Dense Matter with GW170817*, Astrophys.J. 877 (2019) 2, 139.

[9] S. Han et al., *Treating quarks within neutron stars*, Phys.Rev.D 100 (2019) 10, 103022.

[10] M. Ferreira, 1, R. C. Pereira, and C. Providencia, *Neutron stars with large quark cores*, Phys.Rev.D 101 (2020) 12, 123030.

[11] D. Sen and T. K. Jha, *Effects of hadron-quark phase transition on properties of Neutron Stars*, J. Phys. G: Nucl. Part. Phys. 46 (2019) 015202.

[12] D. Sen, *Variation of the $\Delta$ baryon mass and hybrid star properties in static and rotating conditions*, Phys.Rev.C 103 (2021) 4, 045804.

[13] P. C. Chu et al., *Quark star matter at finite temperature in a quasiparticle model*, Eur.Phys.J.C 81 (2021) 1, 93.

[14] Q. Wang, T. Zhao, and H. Zong, *On the stability of two-flavor and three-flavor quark matter in quark stars within the framework of NJL model*, Mod.Phys.Lett.A 35 (2020) 39, 2050321.

[15] E. Annala, T. Gorda, A. Kurkela, J. Nattila, and A. Vuorinen, *Quark-matter cores in neutron stars*, Nature Physics 16, 907 (2020).
[16] G. Baym, T. Hatsuda, T. Kojo, P. D. Powell, Y. Song, T. Takatsuka, *From hadrons to quarks in neutron stars: a review*, Reports on Progress in Physics 81 (2018) 056902.

[17] P. de Forcrand, *Simulating QCD at finite density*, PoS LAT 2009 (2009) 010.

[18] F. Karsch, *Lattice QCD at finite temperature: a status report*, Zeitschrift Fur Physik C38, 147 (1988).

[19] S. Borsanyi, G. Endrodi, Z. Fodor et al., *The QCD equation of state with dynamical quarks*, Journal of High Energy Physics, 2010, 77 (2010).

[20] J. N. Guenther, *Overview of the QCD phase diagram*, Eur. Phys. J. A57, 136 (2021).

[21] A. N. Tawfik, *Equilibrium Statistical-Thermal Models in High-Energy Physics*, Int. J. Mod. Phys. A29 (2014), 1430021.

[22] M. Strickland, J. O. Andersen, L. E. Leganger, N. Su, *Hard-thermal-loop QCD Thermodynamics*, Prog. Theor. Phys. Suppl.187, 106 (2011).

[23] J. O. Andersen, M. Strickland, and N. Su, *Three-loop HTL gluon thermodynamics at intermediate coupling*, JHEP 1008, 113 (2010).

[24] A. Kurkela, P. Romatschke and A. Vuorinen, *Cold quark matter*, Phys. Rev. D 81 (2010) 105021.

[25] A. Kurkela and A. Vuorinen, *Cool Quark Matter* Phys. Rev. Lett. 117 (2016) no.4, 042501.

[26] T. Gorda, A. Kurkela, P. Romatschke, M. Säppi and A. Vuorinen, *Next-to-Next-to-Next-to-Leading Order Pressure of Cold Quark Matter: Leading Logarithm*, Phys. Rev. Lett. 121 (2018) no.20, 202701.

[27] A. Chodos et al., *New extended model of hadrons*, Phys. Rev. D 9, 3471 (1974).

[28] D. Sen and T. K. Jha, *Deconfinement of nonstrange hadronic matter with nucleons and $\Delta$ baryons to quark matter in neutron stars*, Int. J. Mod. Phys. D 28 (2019) no.02, 1950040.

[29] D. Sen et al., *Properties of Neutron Stars with hyperon cores in parameterized hydrostatic conditions*, Int.J.Mod.Phys. E27 (2018) 1850097.

[30] D. Sen, *Nuclear matter at finite temperature and static properties of proto-neutron star*, J.Phys. G48 (2021) 025201.
[31] D. Sen, Role of $\Delta s$ in determining the properties of neutron stars in parameterized hydrostatic equilibrium, Int. J. Mod. Phys. D 28, No. 9 (2019) 1950122.

[32] P. K. Sahu and A. Ohnishi, $SU(2)$ Chiral Sigma Model and Properties of Neutron Stars, Prog. Theor. Phys. 104, 1163 (2000).

[33] P. K. Sahu, T. K. Jha, K. C. Panda, and S. K. Patra, Hot Nuclear Matter in Asymmetry Chiral Sigma Model, Nucl. Phys. A733, 169 (2004).

[34] T. K. Jha and H. Mishra, Constraints on nuclear matter parameters of an effective chiral model, Phys. Rev. C 78 (2008) 065802.

[35] B. K. Agrawal, A. Sulaksono and P. -G. Reinhard, Optimization of relativistic mean field model for finite nuclei to neutron star matter, Nucl. Phys. A882, 1-20 (2012).

[36] A. Sulaksono, Naosad Alam and B. K. Agrawal, Core–crust transition properties of neutron stars within systematically varied extended relativistic mean-field model, Int. J. Mod. Phys. E 23, 1450072 (2014).

[37] N. Alam, A. Sulaksono, and B. K. Agrawal, Diversity of neutron star properties at the fixed neutron-skin thickness of $^{208}$Pb, Phys. Rev. C 92, 015804 (2015).

[38] N. Alam, H. Pais, C. Providência, and B. K. Agrawal, Warm unstable asymmetric nuclear matter: Critical properties and the density dependence of the symmetry energy, Phys. Rev. C 95, 055808 (2017).

[39] C. A. Islam, J. Dey, S. Ghosh, Impact of different extended components of mean field models on transport coefficients of quark matter and their causal aspects, Phys. Rev. C103 (2021) 3, 034904.

[40] L. Mestel and F. Hoyle, On the thermal conductivity in dense stars, Proc. Cambridge Philos. Soc. 46, 331 (1950).

[41] A. A. Abrikosov, The conductivity of strongly compressed matter, Sov. Phys. JETP 18, 1399 (1964).

[42] V. Canuto, Electrical conductivity and conductive opacity of a relativistic electron gas, Astrophys. J. 159, 641 (1970).

[43] E. Flowers and N. Itoh, Transport properties of dense matter, Astrophys. J. 206, 218 (1976).
[44] P. S. Shternin and D. G. Yakovlev, *Shear viscosity in neutron star cores*, Phys.Rev.D78:063006,2008.

[45] S. Banik and D. Bandyopadhyay, *Effect of shear viscosity on the nucleation of antikaon condensed matter in neutron stars*, Phys.Rev.D82:123010,2010

[46] S. Banik, R. Nandi, and D. Bandyopadhyay, *Shear viscosity and the nucleation of antikaon condensed matter in protoneutron stars* Phys.Rev.C 84 (2011) 065804.

[47] A. Schmitt and P. Shternin, *Reaction Rates and Transport in Neutron Stars*, Astrophys.Space Sci.Libr. 457 (2018) 455-574.

[48] P. Shternin and M. Baldo, *Transport coefficients of nucleon neutron star cores for various nuclear interactions within the Brueckner-Hartree-Fock approach*, Phys. Rev. D 102, 063010 (2020).

[49] E. McLaughlin et al., *Building a testable shear viscosity across the QCD phase diagram*, arXiv:2103.02090 [nucl-th] (2021).

[50] A. Y. Potekhin et al., *Transport properties of degenerate electrons in neutron star envelopes and white dwarf cores*, Astron.Astrophys.346:345,1999.

[51] P. Shternin, M. Baldo, and P. Haensel, *Transport coefficients of nuclear matter in neutron star cores*, Phys.Rev.C 88 (2013) 6, 065803.

[52] P. Shternin, M. Baldo, and H-J Schulze, *Transport coefficients in neutron star cores in BHF approach. Comparison of different nucleon potentials*, J. Phys.: Conf. Ser. 932, 012042 (2017).

[53] X. L. Shang, P. Wang, W. Zuo, and J. M. Dong, *Role of nucleon-nucleon correlation in transport coefficients and gravitational-wave-driven r-mode instability of neutron stars*, Phys. Lett. B, 811, 135963 (2020).

[54] R. Nandi, and S. Schramm, *Calculation of the transport coefficients of the nuclear pasta phase*, J.Astrophys.Astron. 39 (2018) 40.

[55] P. Shternin and I. Vidana, *Transport Coefficients of Hyperonic Neutron Star Cores*, Universe 7(6), 203 (2021).

[56] G. Baym, C. Pethick, and D. Pikes, *Electrical Conductivity of Neutron Star Matter*, Nature volume 224, pages674–675 (1969).

[57] D. G. Yakovlev and D. A. Shalybkov, *Electrical conductivity of neutron star cores in the presence of a magnetic field*, Astrophysics and Space Science volume 176, pages191–215 (1991).
G. M. Ewart, R. A. Guyer, and G. Greenstein, *Electrical conductivity and magnetic field decay in neutron stars*, The Astrophysical Journal, 202:238-247, 1975.

M. E. Raikh and D. G. Yakovlev, *Thermal and electrical conductivities of crystals in neutron stars and degenerate dwarfs*, Astrophysics and Space Science volume 87, pages193–203 (1982)

V. Koch, *Aspects of Chiral Symmetry*, International Journal of Modern Physics E6, 203-250 (1997).

P. K. Sahu, R. Basu, and B. Datta, *High-Density Matter in the Chiral Sigma Model* Astrophys. J. 416, 267 (1993).

J. R. Stone and P. G. Reinhard, *The Skyrme Interaction in finite nuclei and nuclear matter* Prog. Part. Nucl. Phys. 58, 587 (2007).

E. Khan and J. Margueron, *Determination of the density dependence of the nuclear incompressibility*, Phys. Rev. C 88, 034319 (2013).

U. Garg and G. Colo, *The compression-mode giant resonances and nuclear incompressibility*, Prog. Part. Nucl. Phys. 101 (2018) 55.

M. Dutra et. al., *Relativistic mean-field hadronic models under nuclear matter constraints*, Phys. Rev., C 90, 055203 (2014).

J. R. Stone, N. J. Stone, S. A. Moszkowski, *Incompressibility in finite nuclei and nuclear matter*, Phys. Rev. C 89, 044316 (2014).

F. J. Fattoyev et al., *Neutron Skins and Neutron Stars in the Multimessenger Era*, Phys. Rev. Lett. 120, 172702 (2018).

Z-Y Zhu, E-P Zhou, A. Li *Neutron star equation of state from the quark level in the light of GW170817*, Astrophys.J. 862 (2018) no.2, 98.

D. Sen, N. Alam, and G. Chaudhuri, *Properties of hybrid stars with a density-dependent bag model*, J. Phys. G: Nucl. Part. Phys. 48 (2021) 105201.

J. D. Walecka, *A theory of highly condensed matter*, Annals of Physics, 83, 491 (1974).

J. Boguta and A. R. Bodmer, *Relativistic calculation of nuclear matter and the nuclear surface*, Nucl. Phys. A292, 413 (1977).

B. D. Serot and J. D. Walecka, *Advances in Nuclear Physics*, edited by J. W. Negele and E. Vogt (Plenum, New York, 1986), Vol. 16.
[73] R. J. Furnstahl, C. E. Price, and G. E. Walker, *Systematics of light deformed nuclei in relativistic mean-field models*, Phys. Rev. C 36, 2590 (1987).

[74] H. Muller and B. D. Serot, *Relativistic mean-field theory and the high-density nuclear equation of state*, Nucl. Phys. A 606, 508 (1996).

[75] G. A. Lalazissis, J. König, and P. Ring, *New parametrization for the Lagrangian density of relativistic mean field theory*, Phys. Rev. C 55, 540 (1997).

[76] B. D. Serot and J. D. Walecka, *Recent Progress in Quantum Hadrodynamics*, Int. J. Mod. Phys. E 6, 515 (1997).

[77] P. Chakraborty, J.I. Kapusta, *Quasi-Particle Theory of Shear and Bulk Viscosities of Hadronic Matter* Phys.Rev.C 83 (2011) 014906

[78] S. Gavin, *Transport Coefficients in ultra-relativistic heavy ion collision* Nucl. Phys. A 435 (1985) 826.

[79] R. Nandi and P. Char, *Hybrid stars in the light of GW170817*, Astrophys.J. 857 (2018) 1, 12.

[80] E-P Zhou, X. Zhou, and A. Li, *Constraints on interquark interaction parameters with GW170817 in a binary strange star scenario* Phys.Rev.D 97 (2018) 8, 083015.

[81] M. Orsaria et al., *Quark-hybrid matter in the cores of massive neutron stars*, Phys.Rev.D 87 (2013) 2, 023001.

[82] S. Khanmohamadi, H.R. Moshfegh, and S. A. Tehrani, *Structure and tidal deformability of a hybrid star within the framework of the field correlator method*, Phys.Rev.D 101 (2020) 12, 123001.

[83] J-E Christian, and J. Schaffner-Bielich, *Twin stars and the stiffness of the nuclear equation of state: ruling out strong phase transitions below 1.7n_0 with the new NICER radius measurements*, Phys.Rev.D 101 (2020) 12, 123001.

[84] M. Strickland, J. O. Andersen, L. E. Leganger, N. Su, *Hard-thermal-loop QCD Thermodynamics*, Prog. Theor. Phys. Suppl.187, 106 (2011).

[85] G. Baym, T. Hatsuda, T. Kojo, P. D. Powell, Y. Song, T. Takatsuka, *From hadrons to quarks in neutron stars: a review*, Reports on Progress in Physics 81 (2018) 056902.
[86] T. Hatsuda, M. Tachibana, N. Yamamoto, and G. Baym, New critical point Induced by the axial anomaly in dense QCD, Phys. Rev. Lett. 97, 122001 (2006).

[87] M. M. Nagelset al., Compilation of coupling constants and low-energy parameters Nucl. Phys. B147, 189 (1979).

[88] O. Dumbrajset al., Compilation of coupling constants and low-energy parameters. 1982-edition, Nucl. Phys. B216, 277 (1983).

[89] S. Ghosh, S. Ghosh, S. Bhattacharya, Phenomenological bound on the viscosity of the hadron resonance gas, Phys. Rev. C98 (2018) 045202.

[90] G.S. Denicol, C. Gale, S. Jeon, J. Noronha, Fluid behavior of a baryon-rich hadron resonance gas, Phys. Rev. C 88 (2013) 064901.

[91] PHENIX collaboration, S. S. Adler et al., Elliptic flow of identified hadrons in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, Phys. Rev. Lett. 91 (2003) 182301.

[92] STAR collaboration, J. Adams et al., Azimuthal anisotropy in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV, Phys. Rev. C 72 (2005) 014904.

[93] ALICE collaboration, K. Aamodt et al., Higher harmonic anisotropic flow measurements of charged particles in Pb- Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, Phys. Rev. Lett. 107 (2011) 032301.

[94] P. Romatschke and U. Romatschke, Viscosity Information from Relativistic Nuclear Collisions: How Perfect is the Fluid Observed at RHIC?, Phys. Rev. Lett. 99, 172301 (2007).

[95] U. Heinz and R. Snellings, Collective flow and viscosity in relativistic heavy-ion collisions, Annu. Rev. Nucl. Part. Sci. 63 (2013) 123.

[96] M. Gyulassy and L. McLerran, Nucl. Phys. A 750, 30 (2005).

[97] P. Kovtun, D. T. Son, and O. A. Starinets, Phys. Rev. Lett. 94, 111601 (2005).

[98] D. Mondal et al. Phys. Rev. Lett. 118, 192501 (2017).

[99] J. Dey, S. Satapathy, P. Murmu, S. Ghosh Shear viscosity and electrical conductivity of relativistic fluid in presence of magnetic field: a massless case, Pramana 95 (2021) 3, 125