Patterns of supersymmetry breaking in moduli-mixing racetrack model

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Abstract

We show some structures of moduli stabilization and supersymmetry breaking caused by gaugino condensations with the gauge couplings depending on two moduli which often appear in the four-dimensional effective theories of superstring compactifications.
1 Introduction

Superstring/M-theory is a candidate of unified description for elementally particles and their interactions including gravity. Soft supersymmetry (SUSY) breaking parameters in the four-dimensional (4D) effective theory as well as the 4D Planck scale $M_{Pl}$, gauge couplings and Yukawa couplings are given by the vacuum expectation values (VEVs) of modulus superfields which determine the size and shape of extra dimensions. Then the moduli stabilization and its effect on SUSY breaking is quite relevant to the particle phenomenology and cosmology. Here we show some structures of moduli stabilization and SUSY breaking in a so-called racetrack model with double gaugino condensations where gauge couplings are given by more than one modulus field \[1, 2, 3\] (two moduli in practice).

2 Moduli-mixing racetrack model

The 4D effective supergravity (within type IIB $O3/O7$ framework for concreteness if necessary) is given by the Kähler potential and superpotential

\[ K = -n_T \ln(T + \bar{T}) - n_S \ln(S + \bar{S}), \quad W = A e^{-a f_a} - B e^{-b f_b}, \]

where $f_{a,b} = m_{a,b} S + w_{a,b} T$. The superfields $S$ and $T$ represents the dilaton and the Kähler (size) modulus respectively. The $n_S$ and $n_T$ are some model dependent numbers given typically by $(n_S, n_T) = (1, 3)$, and $m_{a,b}, w_{a,b}$ are respectively the magnetic flux and the winding number of the $D$-brane \[1\] where the gaugino condensation occurs, which generates the nonperturbative superpotential. For example, gaugino condensation on the $D3$, $D7$, magnetized $D7$- and magnetized $D9$-brane ($D3$ is assumed to be located at some singular point where the extended SUSY is reduced) yields the nonperturbative superpotential \[1\] with $f = S, T, |m|S + |w|T$ and $mS - wT$, respectively.

Before arriving at the above effective theory \[1\] for the light moduli $S$ and $T$, we have assumed that the existence of three-form flux, $G_3 = F_3 - 2\pi i S H_3$, in ten-dimensions stabilizes the complex structure (shape) moduli $U$ as $\langle U \rangle \sim 1$ at around the Planck scale in a supersymmetric way, $D_U W_{\text{flux}} = 0$, through the superpotential \[5\]

\[ W_{\text{flux}} = \int_{\text{CY}_3} G_3 \wedge \Omega = f^{RR}(U) + S f^{NS}(U), \]

where $\Omega$ is a holomorphic three-form of the Calabi-Yau (CY) three-fold, and $f^{RR,NS}(U)$ are some functions of $U$ determined by the flux. Note also that due to the flux there is a significantly warped region in the CY space \[6\].

2.1 Single light modulus

If the three-form flux induces a SUSY mass like $W_{\text{flux}} \sim SU$, that is $n_{RR} = n_{NS} = 1$ in

\[ f^{RR,NS}(U) = (U - \langle U \rangle)^{n_{RR,NS}} \tilde{f}^{RR,NS}(U), \quad \tilde{f}^{RR,NS}(\langle U \rangle) \neq 0, \]

(2)
the dilaton is also stabilized \( \langle S \rangle = -j^{RR}(\langle U \rangle)/j^{NS}(\langle U \rangle) \sim 1 \) as well as \( U \), via the global SUSY vacuum conditions \( \partial_U S W_{\text{flux}} = W_{\text{flux}} = 0 \). In this case we replace \( S \) in Eq. (1) by its VEV, \( \langle S \rangle \). Then the effective superpotential becomes

\[
W = A' e^{-aw_a} - B' e^{-bw_bT},
\]

which is in the same form as the racetrack model with single modulus, but the coefficients are exponentially suppressed or enhanced \( A' = Ae^{-aw_a(S)} \), \( B' = Be^{-bw_b(S)} \), where \( a = 8\pi^2/N_a \) and \( b = 8\pi^2/N_b \) for SUSY \((N_a, b)\) gaugino condensation.

The minimum of the scalar potential induced by the above Kähler and superpotential corresponds to a SUSY AdS\(_4\) local minimum with negative vacuum energy and \( a(\text{Re} T) = at_{\text{SUSY}} \sim \ln(M_P/m_{3/2}) \), where \( m_{3/2} \approx 10^{-14}M_P \) is the gravitino mass. To be phenomenologically viable, we uplift the vacuum energy by introducing anti D3-branes at the top of warped region in the CY space \( \mathbb{T} \). Then the SUSY is broken due to the slight shift \( \delta T = (t_{\text{SUSY}} - \langle T \rangle) \ll t_{\text{SUSY}} \) caused by an additional potential energy of \( D3s \).

In this case, the ratio between the VEV of auxiliary component in the chiral compensator \( C \), \( F_C \sim m_{3/2} \), and one in \( T \), \( F_T \), is given by \( (w_a = 0 \text{ for simplicity}) \)

\[
\alpha = \frac{F_C}{\ln(M_P/m_{3/2})} \frac{T + \bar{T}}{F_T} \approx \frac{1}{1 + m_b(S)/w_b t_{\text{SUSY}}}.\]

This corresponds to the ratio between the so-called anomaly mediation and the modulus mediation for the visible sector SUSY breaking. It was shown in Ref. \( \mathbb{T} \) that the particle masses are unified at the scale given by \( \Lambda_m = e^{-2\pi\alpha} \Lambda \) where \( \Lambda \) is a messenger scale of the modulus mediated contribution. This \( \Lambda_m \) is called a mirage messenger scale, and if \( \alpha \sim \mathcal{O}(1) \), the \( \Lambda_m \) is quite lower than the \( \Lambda \). Here we find that \( \alpha \) varies in a wide range with various magnetic flux \( m_b \), compared to \( \alpha \sim 1 \) without the magnetic flux in the original analysis \( \mathbb{T} \).

Another implication is that a runaway structure of the usual racetrack model can be avoided for the negative value of \( w_{a,b} \) in Eq. \( \mathbb{T} \) that may be realized, e.g., by the gaugino condensation on the magnetized D9-brane. In this case, the scalar potential is lifted in the region \( \text{Re} T \gg t_{\text{SUSY}} \). This situation can be a solution to the destabilization/overshooting problem with a finite temperature effect and with an arbitrary initial condition for \( T \).

### 2.2 Two light moduli

When the three-form flux \( G_3 \) does not contain the SUSY mass term \( W_{\text{flux}} \sim SU \), that is \( n_{RR} = n_{NS} = 2 \) in Eq. \( \mathbb{T} \), the dilaton remains as a light modulus as well as \( T \) in Eq. \( \mathbb{T} \). A careful analysis for Eq. \( \mathbb{T} \) shows \( \mathbb{T} \) that a SUSY AdS\(_4\) stationary point is given by

\[
s_{\text{SUSY}} \sim \frac{1}{b - a} \ln \frac{bB}{aA}, \quad t_{\text{SUSY}} \sim \frac{n_T(b - a)}{2ab(\alpha + \beta)},
\]

which is valid within a parameter region satisfying \( |s_{\text{SUSY}}| \gg n_s/2a, n_s/2b \), where the parameters \( a, b, w_a \) and \( w_b \) in Eq. \( \mathbb{T} \) are replaced by \( a/m_a, b/m_b, -m_a\alpha \) and \( m_b\beta \), and the small letters \( s \) and \( t \) stand for \( \text{Re} S \) and \( \text{Re} T \), respectively. In order this SUSY point
to reside in a perturbative region \( s_{\text{SUSY}}, t_{\text{SUSY}} > 1 \) with \( a, b \gg 1 \) and \( A, B \sim 1 \), we need a fine-tuning like \( b - a \sim \mathcal{O}(1), \alpha, \beta \sim \mathcal{O}(a^{-2}), \mathcal{O}(b^{-2}) \). Moreover, this SUSY point is actually a saddle point located in a sharp ‘racetrack’ of the scalar potential which is implied by the hierarchical mass square eigenvalues \( (m_{\perp}^2, m_{\parallel}^2) \) at this point with \( m_{\perp}^2 > 0, m_{\parallel}^2 < 0 \) and \( -m_{\perp}^2/m_{\parallel}^2 \sim \frac{128}{n_T^2} \left( \frac{\ln \frac{bB}{aA}}{\ln \frac{b}{a}} \right)^4 \left( \frac{b}{a} \right)^2 \gg 1 \).

A SUSY breaking AdS\(_4\) local minimum \((s_{SB}, t_{SB})\) exists along the same ‘racetrack’ and given by \( s_{SB} = s_{\text{SUSY}}(1 + \delta_{SB}^s) \) and \( t_{SB} = t_{\text{SUSY}}(1 + \delta_{SB}^t) \) where

\[
\delta_{SB}^s \sim \frac{\sqrt{n_T + 1} - 1}{n_T}, \quad \delta_{SB}^t \sim -\frac{a\alpha + b\beta}{\ln(bB/aA)} \delta_{SB}^t.
\]

By uplifting this local minimum with some lifting potential, we can obtain a Minkowski minimum but with modulus-dominated SUSY breaking \( \alpha \ll 1 \). This is because SUSY is broken before uplifting unlike the previous single modulus case.

Fig. 1 shows a numerical plot of the scalar potential in \((s, t)\)-plane. We can see the sharp ‘racetrack’ structure, and the two stationary points along this.

### 3 Conclusions

We have shown a vacuum structure of the racetrack model with double gaugino condensations where the gauge couplings depend on two typical moduli \( S \) and \( T \) appearing in the superstring compactifications. If the flux induced superpotential \( W_{\text{flux}} \) stabilizes one of them, \( S \), at a high scale, the nonperturbative superpotential in Eq. (3) can have large suppression/enhancement factors \( A' \) and \( B' \) in front of the light modulus contribution. Due to these factors, the mirage messenger scale of mixed modulus-anomaly mediation \([9]\) for the visible sector SUSY breaking can reside in a wide range depending on the magnetic flux of \( D\)-brane \( m_{a,b} \) and also on the three-form flux \( G_3 \) through the vacuum value.
of dilaton \(S\) stabilized by the flux induced superpotential. On the other hand, when both \(S\) and \(T\) remain as light moduli, we have a SUSY breaking local minimum, and then the modulus mediated SUSY breaking is dominant after uplifting it to the Minkowski minimum.

**Acknowledgements**

The author would like to thank Tetsutaro Higaki and Tatsuo Kobayashi for the collaborations [1, 2, 3] which form the basis of this talk, and the organizers of SUSY06 and Summer Institute 2006. The author is supported by the Japan Society for the Promotion of Science for Young Scientists (No.0602496).

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