Vortices and Quantum tunneling in Current-Biased 0-\(\pi\)-0 Josephson Junctions of \(d\)-wave Superconductors

Takeo KATO and Masatoshi IMADA

Institute for Solid State Physics, University of Tokyo,
7-22-1 Roppongi, Minato-ku, Tokyo 106

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We study a current-biased 0-\(\pi\)-0 Josephson junction made by high-\(T_c\) superconductors, theoretically. When a length of the \(\pi\) junction is large enough, this junction contains a vortex-antivortex pair at both ends of the \(\pi\) junction. Magnetic flux carried by the vortices is calculated using the sine-Gordon equation. The result shows that the magnetic flux of the vortices is suppressed to zero as the distance between the vortices is reduced. By applying an external current, the orientation of the vortices is reversed, and a voltage pulse is generated. The current needed for this transition and generated pulse energy are calculated. Macroscopic quantum tunneling (MQT) in this transition is also studied. The tunneling rate has been evaluated by an effective Hamiltonian with one degree of freedom.

KEYWORDS: high-\(T_c\) superconductor, \(\pi\) junction, sine-Gordon equation, half vortex, macroscopic quantum tunneling

§1. Introduction

Static and dynamical properties of long Josephson junctions (LJJs) has been studied by many authors for practical applications. Particularly, kink propagation in LJJs has been studied both theoretically and experimentally on the basis of the classical (damped) sine-Gordon equation

\[
\phi_{xx} - \phi_{tt} - \sin \phi - \alpha \phi_t + \beta \phi_{xxt} = f.
\]  

(1)

Here, \(\alpha\) and \(\beta\) are damping parameters due to the quasiparticle tunneling loss and the surface loss, and \(f = I/I_0\) is an external current scaled by the critical current \(I_0\), and the spatial variable \(x\) is normalized by the Josephson penetration length \(\lambda_J\). Recently, quantum
effects of a kink have been discussed through interference effects, and macroscopic quantum tunneling (MQT).

In this paper, we consider a long Josephson junction including a so-called π junction, which has negative critical current. In such junctions, the phase difference shows more complicated properties than traditional LJJs. Hence, we expect new phenomena characteristic of the sine-Gordon field with infinite degrees of freedom.

The negative critical current has been discussed first in Josephson junctions, which have an insulator layer with magnetic impurities. More recently, it has been proposed that π junctions can be made by unconventional superconductors with non-s-wave symmetry. Geshkenbein et al. have discussed π junctions formed in heavy electron systems, whereas π junctions in high-Tc superconductors have been proposed to explain the positive paramagnetic Meissner effect. In order to probe the symmetry of the superconducting gap, a number of experiments have been performed, including interference measurements in single crystal YBa$_2$Cu$_3$O$_{7-\delta}$-Pb SQUIDs and direct imaging of magnetic flux in a tricrystal ring geometry. These measurements indicate that π junctions can be realized at a grain boundary of high-Tc superconductors, and that the d-wave symmetry is realized in high-Tc superconductors.

Static behavior of LJJs with both 0 and π junctions (0-π junctions) has been studied theoretically by several authors. They have shown that a half vortex appears spontaneously at a boundary between 0 junction and π junction in sufficiently long junctions. Further, Kuklov et al. have proposed that the half vortex can change its orientation by applying an external current to the junction. They have also conjectured that the half-vortex can be utilized in superconducting memory and logic devise.

In this paper, we consider a 0-π-0 long Josephson junction, which has a positive critical current at $|x| > a$, and has a negative critical current at $|x| < a$. This junction may be realized in systems such as grain boundary Josephson junctions. First, we study static properties of the junction in §2. After that, we consider MQT in this junction to study quantum effects in §3. Summary is given in §4.

§2. Hamiltonian and Static Properties

2.1 Hamiltonian

The Hamiltonian of the 0-π-0 Josephson junction with an external current is given by

$$H = \int_{-\infty}^{\infty} \left( \frac{1}{2} \phi_x^2 + \Theta(x)(1 - \cos \phi) + f\phi \right) dx.$$  (2)
Fig. 1. Two types of solutions of (4) are drawn for a sufficiently large $a$. The minima of the potential for $\phi$ is given as $(2n + 1)\pi$ at $|x| < a$, and as $n\pi$ at $|x| > a$.

Here, $x$ is scaled by $\lambda_3$, and $f = I/I_0$ is an external current, and the Hamiltonian is scaled by an energy scale of the junction, $E_0$. $\Theta(x)$ is a step function defined by

$$\Theta(x) = \begin{cases} -1 & (|x| < a) \\ 1 & (|x| > a) \end{cases}.$$  

Here, to simplify the situation, we assume that the absolute value of the critical current is uniform along the junction. Static behavior of $\phi$ is given by $\delta H/\delta \phi(x) = 0$. From (2), we obtain the sine-Gordon equation

$$\phi_{xx} = \Theta(x) \sin \phi + f.$$  

This equation is also obtained from (1) by taking $\phi_t = 0$. It should be noted that dissipation does not affect static properties.

First, we consider the case with $f = 0$. In stable solutions by minimizing (2), the system prefers to have a uniform phase difference $\phi$ at $2\pi n$ for $x \gg |a|$, while $\phi = (2n + 1)\pi$ is preferred for $|x| \ll a$, where $n$ is an integer. As a result, we obtain two types of solutions, $\phi_1(x)$ and $\phi_2(x)$ as schematically shown in Fig. 1. Here, we assumed without losing generality that both solutions satisfy $\phi(\pm \infty) = 0$.

The physical meaning of these solutions is clear in case of $a \gg 1$. For example, we consider the situation described by $\phi_1(x)$. The induced magnetic field in the junction is proportional to $\phi_x$. Hence, a vortex and an antivortex appear at $x = -a$ and at $x = a$, respectively. The magnetic flux carried by these vortex is $\pm \Phi_0/2$, where $\Phi_0 = \hbar/2e$ is the unit flux. These induced vortices are called half vortices. The other solution $\phi_2(x)$ has the same vortices, except that the orientations of the vortices are reversed from those in $\phi_1(x)$. 


Fig. 2. $\Delta \phi = \phi_1(0) - \phi_1(\infty)$ is shown as a function of $a$, which is a half of the distance between two vortices scaled by $\lambda_1$. The critical current $f_c$ necessary for vortices to change their orientation is also shown. Both varnishes at $a = \pi/4 \approx 0.79$ as $a$ decreases.

2.2 Magnetic flux

The induced magnetic flux is proportional to $\Delta \phi = \phi_1(0) - \phi_1(\infty)$. In the case of $a \gg 1$, we obtain $\Delta \phi = \pi$, where the magnetic flux carried by the vortices is $\pm \Phi_0/2$. The magnetic flux defined by $\Delta \phi$, however, decreases as we put vortex and antivortex closer each other. We have calculated $\phi(x)$ from (4) numerically, and obtained $\Delta \phi$ as a function of $a$. We show the result in Fig. 2. For $a \leq \pi/4 \approx 0.79$, we obtained $\Delta \phi = 0$. This means that the vortices disappear when the distance between vortices is too small. As shown later, the critical value $\pi/4$ can be obtained analytically. For $a \geq \pi/4$, the value of $\Delta \phi$ increases quickly as the value of $a$ becomes large, and for $a \gg 1$, approaches $\pi$.

We expect that this behavior of the flux carried by the vortices may be applied to accurate measurements of $\lambda_1$. When we change the length of $\pi$ junction or the Josephson penetration length $\lambda_1$ by external magnetic field, measurement of magnetic flux carried by vortices will tell us the information about the value of $\lambda_1$. The measurement of $\lambda_1$ using direct imaging by scanning SQUIDs has already been reported by Kirtley et al.\textsuperscript{[4]} Compared with this, we expect that the measurement using the 0-$\pi$-0 Josephson junction is more accurate, because the magnetic flux is sensitive to the ratio $a = d/2\lambda_1$, where $d$ is a length of the $\pi$ junction.

2.3 External current

Next, we study static behavior of the solution in the presence of the external current $f$. We assume that the initial state is described by $\phi_1(x)$. As the external current $f$ adiabatically

\[ $\Delta \phi = \phi_1(0) - \phi_1(\infty)$ \]
increases from zero, the form of $\phi_1(x)$ is modified. The modified solution is denoted as $\phi_1(x, f)$. When the external current takes a critical value $f_c$, the solution $\phi_1(x, f)$ becomes unstable. Then, a transition from $\phi_1(x, f)$ to the other stable solution $\phi_2(x, f)$ occurs. Here, $\phi_2(x, f)$ is a solution modified adiabatically from the initial solution $\phi_2(x)$ by the external current $f$. During the transition, a voltage pulse across the junction is generated. This transition is intuitively understood easily as follows: the vortices exchange their locations each other. After that, the system remains the state described by $\phi_2(x, f)$, as long as $f > 0$. When a negative external current is applied to the junction, a transition from $\phi_2(x, f)$ to $\phi_1(x, f)$ occurs at $f = -f_c$.

The critical current $f_c$ is calculated from (1) numerically. The result is shown also in Fig. 2. For $a > \pi/4$, the critical current $f_c$ increases as $a$ becomes large, and is saturated toward $2/\pi$, which can be obtained analytically as shown later. Note that the critical value $f_c$ is smaller than 1. This means that the exchange of a vortex and an antivortex occurs before the whole junction is driven to a voltage state.

2.4 Limiting cases

Next, we consider two limiting cases: (i) $a = \pi/4 + \lambda$ ($\lambda \ll 1$), and (ii) $a \gg 1$. In these cases, we can perform analytical calculation.

In the case (i), the phase difference $\phi(x)$ satisfies $|\phi(x)| \ll 1$ for all $x$. Then, the Hamiltonian (2) for $f = 0$ up to $\phi^2$ is reduced to

$$H[\phi(x)] = \int_{-\infty}^{\infty} dx \varphi(x) \frac{1}{2} \left(-\frac{d^2}{dx^2} + \Theta(x)\right) \varphi(x),$$

where $\varphi$ represents fluctuations around the trivial solution and is defined by $\phi(x) = 0 + \varphi(x)$. The eigenmodes of the fluctuation are obtained by solving the ‘Schrödinger equation’

$$-\frac{d^2\varphi_n}{dx^2} + \Theta(x)\varphi_n = \varepsilon_n\varphi_n,$$

under the normalization condition

$$\int_{-\infty}^{\infty} dx \varphi_n(x)\varphi_m(x) = \delta_{nm}.$$

Here, $\Theta(x)$ can be regarded as a well-shaped potential with its width $2a$. When $a \leq \pi/4$, the lowest energy $\varepsilon_0$ is positive, and the trivial solution $\phi = 0$ is stable. However, when $a = \pi/4 + \lambda$ ($0 < \lambda \ll 1$), there exists a negative eigenmode $\phi_0$, which means the instability of the trivial solution $\phi(x) = 0$. Hence, when we expand the static solution

$$\phi(x) = \sum_{n=0}^{\infty} C_n\varphi_n(x),$$
the coefficients $C_n$ become 0 for $n \geq 1$, and only $C_0$ is nonzero. For $a = \pi/4 + \lambda (\lambda \ll 1)$, the ground state energy $\varepsilon_0$ is close to zero. By taking $\varepsilon_0 = 0$, the form of the unstable mode $\varphi_0(x)$ is obtained approximately from (3) and (4) as

$$
\varphi_0(x) = \begin{cases} 
\sqrt{\frac{4}{\pi + 4}} \cos x & (|x| < a), \\
\sqrt{\frac{4}{\pi + 4}} \cos a e^{-(|x|-a)} & (|x| > a).
\end{cases}
$$

(9)

To determine $C_0$, we derive an effective Hamiltonian for $C_0$ by substituting $\phi(x) = C_0 \varphi_0(x)$ and (4) to the Hamiltonian (2). By assuming that $C_0$ is small, a simple analysis gives

$$
H = -\frac{\varepsilon_0}{2} C_0^2 + \frac{\pi + 2}{8(\pi + 4)^2} C_0^4 - \sqrt{\frac{32}{\pi + 4}} f C_0
$$

(10)
to the fourth order of $C_0$. Here, $\varepsilon_0 = 8\lambda/(\pi + 4)$. From (10), $\Delta \phi$ and $f_c$ are calculated analytically as

$$
\Delta \phi = \sqrt{\frac{64}{\pi + 4}} \lambda^{3/2}, \quad f_c = \frac{128}{27(\pi + 2)} \lambda^{3/2}.
$$

(11)

In the case (ii), i.e. for $a \gg 1$, we can regard the junction as two independent 0-π junctions. Then, a half vortex and a half antivortex appear at $x = \pm a$. We notice only the half vortex at $x = a$. The behavior of a half vortex in 0-π junctions in the presence of an external current has already been studied by Kuklov et al.[16] They have studied a critical current $f_c$, at which a half vortex changes its orientation with an integer flux being created. In order to study the stability of the static solution $\phi_1(x, f)$, we expand $\phi(x)$ around $\phi_1(x, f)$

$$
\phi(x) = \phi_1(x, f) + \sum_{n=0}^{\infty} C_n' \varphi_n(x)
$$

(12)

with eigenmodes of small fluctuation defined by

$$
-\frac{d^2 \varphi_n}{dx^2} + U'' \varphi_n = E_n \varphi_n, \quad U'' \equiv \frac{\partial^2 U}{\partial \phi^2}(\phi_1(x, f)).
$$

(13)

Here, $U(\phi) = \Theta(x)(1 - \cos \phi)$. At $f = f_c$, the lowest energy $E_0$ becomes zero as in the case (i). It can easily be checked that $\varphi_0 = C \partial_x \phi_1(x, f)$ always satisfies the equation (13) with $E_0 = 0$. Here, the constant $C$ is determined by solving the normalization condition (7). This mode is called the translational mode, because this modulation translates the half vortex along the junction. We should note, however, that both $\varphi_0(x)$ and $\partial_x \varphi_0(x)$ must be continuous at $x = a$. This condition and (4) leads to $\phi_1(a, f_c) = 0$. Further, from the first integral of (4), we obtain

$$
0 - \cos \phi(0) - f \phi(0) = \frac{1}{2} \phi_x(a)^2 - \cos \phi(a) - f \phi(a),
$$

(14)

$$
0 + \cos \phi(\infty) - f \phi(\infty) = \frac{1}{2} \phi_x(a)^2 + \cos \phi(a) - f \phi(a).
$$

(15)
Fig. 3. Sketches of the transition from $\phi_1(x)$ to $\phi_2(x)$ in two limiting cases: (a) $a = \pi/4 + \lambda (\lambda \ll 1)$, and (b) for $a \gg 1$. The gray lines represent intermediate configurations during the transition. The transition occurs symmetrically in the case (a), while the integer flux is generated at either side in the case (b).

From these equations and $\phi_1(a, f_c) = 0$, the critical current for $a \gg 1$ is calculated as $f_c = 2/\pi$. For the critical solution $\phi(x, f_c)$, the normalization constant in $\varphi_0$ is numerically calculated as $C \approx 0.5468$.

As well as in the case (i), only the lowest energy mode $\varphi_0(x)$ becomes unstable for $f > f_c$. The effective Hamiltonian in the case (ii) can be made by substituting $\phi = \phi_1(x, f_c) + C'_0 \varphi_0(x)$ to (2) as

$$H = \pi \delta CC'_0 - \alpha (CC'_0)^3,$$

(16)

to the third order of $C'_0$. Here, $\delta = f_c - f$ denotes the difference between the critical current and the external current, and $\alpha$ is a constant calculated by

$$\alpha = \frac{1}{6} \int_{-\infty}^{\infty} dx \Theta(x) \sin \phi_1(x, f_c) (\partial \phi_1(x, f_c))^3$$

$$= \frac{2}{3} (\phi_\infty \sin \phi_\infty + \cos \phi_\infty - 1) \approx 0.1403,$$

(17)

where $\phi_\infty = \phi(\infty) = -\sin^{-1} f_c$. From the effective Hamiltonian (16), it is shown that there exists a metastable state for $\delta > 0$. Note that (16) is valid only for a small $C'_0$. Hence, (16) can be used only for $\delta \ll 1$ case, where the value of $C'_0$ at a metastable state is small.

2.5 Crossover region

It should be noted that the way to exchange the location of a vortex and an antivortex each other is different in the two limiting cases. In the case (i), the transition from $\phi_1$ to $\phi_2$ occurs by keeping the symmetry $\phi(-x) = \phi(x)$ as shown in Fig. 3(a), because the most unstable mode $\varphi_0(x)$ is symmetric. In the case (ii), the transition occurs symmetrically in the original model (2) as well as in the case (i). However, in the presence of even small spatial inhomogeneities, either of two half vortices begins to move at a lower external current as shown in Fig. 3(b). Then, this half vortex changes its orientation first and creates an integer
vortex in the region $|x| < a$. This integer flux propagates along the $\pi$ junction toward the other half vortex, and combines to the other half vortex to change its orientation.

To study the transition process for an intermediate value of $a$, we focus on the Schrödinger equation for fluctuations $\varphi(x)$,

$$-\frac{d^2\varphi_n}{dx^2} + U''\varphi_n = E_n\varphi_n, \quad U'' \equiv \frac{\partial^2 U}{\partial \phi^2}(\phi(x)).$$

The potential term $U''(\phi(x))$ has two minima at $x = \pm a$ as shown in Fig. 4. The ground state energy $E_0$ and the first excited state energy $E_1$ are also drawn in Fig. 4. As $a$ increases, the wave function of the ground state is modified by keeping the symmetry $\varphi_0(-x) = \varphi_0(x)$ and its nodeless form. Hence, the most unstable mode $\varphi_0(x)$ is connected from the case (i) to the case (ii), smoothly.

When the distance between two wells $2a$ increases, the energy splitting $\Delta = E_1 - E_0$ is suppressed exponentially. For $a \gg 1$, the lowest two eigenstates become almost degenerate, and we can constitute wave functions localized at each well as

$$\varphi_{R(L)} = (\varphi_0(x) \pm \varphi_1(x))/\sqrt{2}.$$  \hspace{1cm} (19)

In the presence of spatial inhomogeneities, the potential energy is modified, and the energy difference between two wells appears. This effect can be studied by the effective Hamiltonian on the two-dimensional Hilbert space spanned by $\varphi_R$ and $\varphi_L$:

$$H_{\text{eff}} = \Delta \sigma_x + \varepsilon \sigma_z.$$  \hspace{1cm} (20)

Here, $\varepsilon$ is the energy difference between wells due to inhomogeneities, and $\sigma_x$ and $\sigma_z$ are Pauli’s matrices. When the energy splitting $\Delta$ is much larger than $\varepsilon$, the wave function of the ground state is symmetric, and given by $\varphi_0(x)$ approximately. Hence, the transition from $\phi_1$ to $\phi_2$ occurs symmetrically as shown in Fig. 3(a). On the other hand, $\Delta$ becomes smaller
Fig. 5. The solid line represents $\Delta E = H[\phi_1(x, f_c)] - H[\phi_2(x, f_c)]$ as a function of $a$. Here, $E_0$ is an energy scale of the junction, and is defined below (21). The curve for $\Delta E/E_0$ approaches $8a$ (gray line) for $a \gg 1$.

than $\varepsilon$ when $a$ is large. Then, the wave function of the ground state is localized in either well, and given by $\varphi_R$ (or $\varphi_L$) approximately. As a result, the transition begins from either side as shown in Fig. 3(b). The qualitative change which is expected to occur in the intermediate region of $a$ is not a transition but a crossover.

2.6 Voltage pulse

In order to estimate a voltage of the pulse, we have calculated numerically the energy difference $\Delta E$ defined as

$$\Delta E/E_0 = H[\phi_1(x, f_c)] - H[\phi_2(x, f_c)], \quad (21)$$

where $E_0 = \Phi_0 I_0 \lambda_J / 2\pi L$ is an energy scale of the junction, and $L$ is the junction length. The result is shown in Fig. 5. The energy difference increases monotonically for $a \geq \pi/4$. When $a$ is large, the curve for $\Delta E/E_0$ approaches $8a$. This feature can be obtained analytically by neglecting the spatial distribution of vortices around the boundary between the 0 junction and the $\pi$ junction. The electric power of the pulse $P$ is estimated as $P = \Delta E/T$, where $T$ is the time scale of the transition from $\phi_1$ to $\phi_2$ or vice versa. Accurate estimate of $T$ is difficult, because we have to solve the sine-Gordon equation dynamically. For $a \gg 1$, however, it is inferred that $T$ is determined by the time for an integer vortex to propagate from $x = a$ to $x = -a$ or vice versa, and is estimated as $T \sim 2a \lambda_J / c$ in the original unit. Here, $c = \lambda_J \omega_c$ is a characteristic velocity of the integer vortex, and $\omega_c \sim \omega_p \times \max(1, \alpha + \beta/3)$.
is a characteristic frequency of the junction. Thus, the pulse voltage $V$ is estimated as

$$V \sim \frac{P}{I} \sim \frac{\Phi_0 \omega_c \lambda J}{L}, \quad (22)$$

where $L$ is the junction length. Here, we used the critical current $f_c = I/I_0 = 2/\pi$. From Fig. 5, it is expected that the pulse voltage is suppressed as $a$ decreases. Hence, the estimate (22) is expected to give an upper limit for $V$.

### 3. Macroscopic Quantum Tunneling

In this section, we consider the transition due to macroscopic quantum tunneling (MQT) from the metastable state $\phi_1$ to the stable state $\phi_2$. In this paper, we neglect dissipation effects, taking $\alpha$ and $\beta$ in (1) as 0. Moreover, we study only limiting cases, which allow us to perform analytical calculation on the basis of the effective Hamiltonian (10) and (16).

We first consider the case $a = \pi/4 + \lambda (\lambda \ll 1)$. We assume that $\delta = f_c - f \ll 1$, and that (10) can be approximated as a cubic potential. For a cubic potential, the tunneling rate $\Gamma$ is calculated as

$$\Gamma = A \exp(-B), \quad B = \frac{7.2V}{h\omega_0}. \quad (23)$$

Here, $V$ is the energy barrier, and $\omega_0$ is the frequency of small oscillations around the metastable state. The prefactor $A$ is the order of $\omega_p$, where $\omega_p$ is the plasma frequency of the junction. From (10), we can estimate the exponent

$$B \approx \frac{121}{\beta^2} \lambda^{-3/8} \delta^{5/4}, \quad (24)$$

where $\beta^2 = \hbar \omega_p/E_0$.

In a similar way, the exponent $B$ is estimated for the case $a \gg 1$ from the effective Hamiltonian (10) as

$$B \approx \frac{99}{\beta^2} \delta^{5/4}. \quad (25)$$

Note that the exponent $B$ is proportional to $\delta^{5/4}$ in both cases as seen in (24) and (25). Therefore, we expect that the $\delta$-dependence in $B$ does not change qualitatively in all the ranges of $a$.

The value of $\beta^2$ can be related to experimental parameters as

$$\beta^2 = \frac{16\pi}{137} \left( \frac{2\lambda_L d}{W^2 \varepsilon_r} \right)^{1/2}, \quad (26)$$

where $\lambda_L$ is the London length, and $\varepsilon_r/d$ is the capacitance per area, and $W$ is the junction width. The typical experimental value for $\beta$ is very small ($\sim 10^{-3}$). Therefore, MQT can be observed only for $\delta \ll 1$, because the tunneling rate $\Gamma$ must be large enough to be observed.
in the laboratory. For example, assuming \( a \gg 1, \beta = 10^{-3} \) and \( A = 10^{10}[1/s] \), we obtain \( \Gamma \sim 2 \times 10^2 [1/s] \) for \( \delta = 10^{-3} \).

The quantum effects were treated here on the basis of the effective Hamiltonians including only one degree of freedom for the field. Other degrees of freedom appears only in the form of plasmons in the semiclassical approximation, while the plasmons do not affect the tunneling rate at sufficiently low temperatures.\(^1\) The semiclassical approximation is justified for \( \beta \ll 1 \). Then, many-body effects characteristic of the sine-Gordon fields do not appear in this junction.

In the above calculation, we have neglected dissipation effects due to quasiparticles. It is, however, expected that damping effect on the junction remains even at sufficiently low temperatures, and strongly affects the tunneling rate, when there exist gapless nodes for quasiparticle excitation in non-s-wave superconductors. Because dissipation on the junction generally suppresses the tunneling rate, our calculation gives an upper limit for \( \Gamma \).\(^2\) Damping effects on MQT in this case with more accurate estimate of the tunneling rate remains for further studies.

§4. Summary

In summary, we studied static properties of magnetic fluxes and their macroscopic quantum tunneling in the 0–\( \pi \)–0 Josephson junction, where two half vortices are formed if the \( \pi \) junction region is long. We calculated the magnetic flux of spontaneously induced vortices, and the critical current needed to make a transition between two degenerate vortex configurations. We also studied quantum tunneling rate for this transition. This MQT may be observed in high-\( T_c \) superconductors under an appropriate condition.

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