Modeling the Flexural Performance of Reinforced Concrete Built-up Beams

Ghazwan K. Mohammed1, Kais F. Sarsam1, Iqbal N. Korkess1
1Civil Engineering Department, University of Technology, Baghdad, Iraq.

Abstract. Finite element modeling is used for tracking the flexural response of built-up reinforced concrete beams under the influence of dominant flexural loading. ABAQUS finite element analysis program was used toward this goal, due to its superior capability to represent the mechanical properties of concrete including compressive and tensile strength in strain hardening and softening behaviors; and the features related to steel reinforcement rebar. This study is based on the comparison between the theoretical analysis by the finite element method and the experimental results. The experimental program consists of casting and testing six rectangular cross-sections of simply supported reinforced concrete beams. Two beams are cast as a reference which fully Conventional Concrete (CC) and Reactive Powder Concrete (RPC), the rest four were built-up beams made by the combination of two types of concrete in one element. The study deals with, the load which caused a first crack, ultimate carrying capacity, load and deflection behavior at mid-span of beams, cracks patterns, and damage index of the examined beams. Good agreement was gained with the available experimental results that show the effectiveness of the finite element method for simulating the flexural performance of beams.

1. Introduction
In the last thirty years, the manufacturers of concrete have succeeded in producing a new generation of composites with high mechanical properties and resistance of environmental circumstances. The result of such effort is Reactive Powder Concrete (RPC) which is also referred to as High or Ultra-High-Performance Concrete (HPC, UHPC). RPC has high mechanical properties like a good compressive and tensile strength comparing with CC with more toughness and durability properties [1]. The brittleness is one of the essential imperfections of RPC, which can be avoided by the addition of fibers with considerably increment in ductility, energy absorption capacity, and fracture toughness [1-2]. The high cost of RPC comparing with CC, paved the way to use it in built-up or composite sections with an economic view, reducing the costs and engineering view by getting the benefits of different types of concrete. Many of previous experimental studies have been adduced in the literature related to the flexural performance of conventional and reactive powder concrete built-up reinforced beams [3-5]. The results of these studies indicate that utilizing a layer of RPC in a built-up beam leads to more stiffness, with fewer disfigurements and a reduction in the width of cracks with lower spacing and delay of the localized microcracks formation comparing with original conventionally reinforced concrete beams. This is affected by the layer depth and position and RPC mechanical properties related to the increment of steel fibers volumetric ratio. There is a shortage in the analytical researches about the flexural testing of conventional and reactive powder concrete reinforced built-up beams. A considerable number of analytical studies have been adduced in literature related to beams made with RPC or CC only. Chen et al. [6] present a study concentrate on the prediction of the flexural and shear
behavior of UHPFRC girders. Hamid et al. [7] use ABAQUS software to evaluate the flexural response of CC reinforced beam based on concrete damaged plasticity model. To treat the shortage of numerical studies on this type of beams, a finite element based numerical model was advanced. This study displays the analytical model specifics to track the flexural performance of built-up beams made from two types of concrete.

2. Specimens Configuration and Properties of Materials

Experimental program consists of six beams made from conventional concrete, reactive powder concrete or by a combination of both materials as a built-up beam. A rectangular cross-section of (125 mm in width and 200mm in depth) has been selected for all beams, with a gross length of (1650mm). The layout of the beams is illustrated in Table 1. Where (HL) refers to RPC layer depth and (R) refer to the fraction of this layer with respect to overall beam depth. Three sizes of deformed steel reinforcement bars were used and tested according to the requirements of ASTM A615[8] with nominal diameters, Ø16 for main reinforcement, Ø8 for lateral reinforcement (stirrups), and Ø4 which only employ to support the stirrups. The yield strength of steel bars was 520MPa, 438MPa, and 415MPa respectively. Specifics about dimensions, steel reinforcement, and geometry of the beams are shown in Figure (1). Two concrete mixes have been prepared and used, as shown in Table 2., having components matching the requirements of valid standers [9-11]. Control specimens have been used to compute the mechanical properties of CC and RPC such as cylinders (100mm×200 mm) were tested to get compressive strength according to ASTM C 39/C39M-01[12] and cylinders (150mm×200mm) were tested to get splitting tensile strength and modulus of elasticity according to ASTM C496/C496M-04[13] and ASTM C469-02 [14] respectively. RPC mix contains Micro copper-coated steel fibers straight shape, with diameter 0.2mm, length 13 mm, tensile strength (2800 MPa) and aspect ratio is 65. Mechanical properties of mixes like as compressive strength, splitting tensile strength and modulus of elasticity for both types of concrete are given in Table 3.

### Table 1. Description of tested beams.

| Beams | HL (cm) | R  | Description of test beams              |
|-------|---------|----|----------------------------------------|
| Ref-1 | 0       | 0  | CC beam reference                       |
| Ref-2 | 20      | 1  | RPC beam reference                      |
| B-1   | 5       | 1/4| Built-up beam (RPC in tension)          |
| B-2   | 10      | 1/2| Built-up beam (RPC in tension)          |
| B-3   | 5       | 1/4| Built-up beam (RPC in compression)      |
| B-4   | 10      | 1/2| Built-up beam (RPC in compression)      |

![Figure 1. Beam geometry and reinforcement details.](image)

### Table 2. Components of mixes.

| Type of concrete | CC | RPC |
|------------------|----|-----|
| Cement (Kg/m³)   | 500| 950 |
Table 3. Mechanical properties of CC and RPC.

| Type of concrete | Compressive strength ($f'_c$) MPa | Splitting tensile strength ($f_{ct}$) MPa | Modulus of elasticity ($E_c$) GPa |
|------------------|-----------------------------------|------------------------------------------|-----------------------------------|
| CC               | 40.35                             | 3.501                                    | 29.835                            |
| RPC              | 89.16                             | 10.520                                   | 42.003                            |

3. ABAQUS Finite Element Modeling

This paragraph gives a summary of the details of the beams modeling by the finite element method. The renowned software ABAQUS (version 2019) was used for this target.

3.1. Material Modeling

3.1.1. Compressive and Tensile Behavior of Concrete

ABAQUS concrete damage plasticity model (CDP) is used to track the nonlinear behavior of CC and RPC. The plastic flow theory is a foundation of this model [15]. Yield surface proposed by Lubliner et al. [16] used to establish the yield surface in this model. The imitation of the concrete inelastic behavior in the CDP model is obtained by assuming isotopically damage development joined with plasticity features of concrete in both of compressive and tensile strength. This model supposes that two major mechanisms of failure control; they are the compressive crushing and the tensile cracking [17]. To adopt the parameters of the CDP model, two constitutive models are utilized to represent the uniaxial compressive behavior in strain hardening and softening branches as shown in Figures (2 and 3). Stress-Strain relation for the concrete was constructing according to proposed models by Hognestad [18] and Lu et al. [19] for CC and RPC, respectively, as shown in Appendix A and B.

Figure 2. Hognestad constitutive model of CC [18]  
Figure 3. Lu constitutive model of RPC [19].

Two exponential constitutive models proposed by Belarbi et al. [20] and Danha et al. [21] as shown in Appendix C and D are used for simulating the post-peak softening behavior of concrete in tension after cracking is occurred as shown in Figures (4 and 5), for CC and RPC respectively. All these models are
carried out according to the experimental results of control specimens such as compressive strength, splitting tensile strength, and \( \varepsilon_0 \) which is the maximum strain before compressive crushing. There are other parameters demand to complete the CDP model as ABAQUS user’s manual [17] states, such as modulus of elasticity which was taken from controls specimens results. The dilation angle (\( \psi \)): it was taken equal to 31°, Eccentricity (\( \epsilon \)) it was taken equal to 0.1, \( (fb0/fc0) \) which is defined as fraction between the initial values of equibiaxial and uniaxial compressive yield stress it was taken equal to 1.16, and \( (Kc) \) which is refer to the ratio of the second stress invariant on the tensile meridian it was taken equal to 0.667. The mass density it was taken equal to 2400 kg/m³, and (\( \nu \)), which is referred to Poisson’s ratio it was taken equal to 0.19. Damage parameters of the CDP model were adopted according to Lubliner et al. [16] damage model as shown in Figure (6), which assumes that plastic degradation only happens in the softening domain and the stiffness is commensurate to the material’s cohesion as shown in equations (1) and (2).

\[
\frac{E}{E_0} = \frac{c}{c_{max}} = 1 - d 
\]

\[
d = 1 - \frac{c}{c_{max}}
\]

The previous equations illustrate that the stress and the ultimate strength of concrete in uniaxial direction for both of compression and tension which are denoted by \( (c_{max}) \) abbreviation are commensurate with the yield criteria of the material’s cohesion which is denoted by \( (c) \) letter. Equation (2) can be modified to equation (3). Which \( (f) \) represents the tensile or compressive strength of concrete. The calculated damage parameters, according to equation (3) are shown in Appendix E and F.

\[
d = 1 - \frac{\sigma}{f}
\]
3.1.2. Steel Reinforcement and Plates

Elastic-perfectly plastic model is assumed to represent the steel reinforcement bars (Ø8 & Ø4), backing and loading plates used in the present study, which just defined by yield stress, modulus of elasticity and Poisson’s ratio as shown in Figure (7). Regarding the main reinforcement bar (Ø16), an elastic-plastic model with a strain hardening effect is used. This model required modulus of elasticity, Poisson’s ratio, plastic strain, and its corresponding stress based on the laboratory tensile strength of this rebar. Figure (8). Illustrate the relationship between stress and plastic strain of Ø16 steel bar which used in this study.

![Stress-strain curve for steel bars with Ø (4&8) and plates](image)

**Figure 7.** Stress-strain curve for steel bars with Ø (4&8) and plates

![Stress-plastic strain curve of Ø16 steel bar.](image)

**Figure 8.** Stress-plastic strain curve of Ø16 steel bar.

3.2. Select Elements and Meshing

ABAQUS supply a wide variety of elements. This wealthy element library leads to powerful tools for analyzing many engineering issues; the choosing of element types is essential for accurate modeling.

3.2.1. Solid Element

The concrete, backing, and loading plates were represented by the element type C3D8R that belongs to the continuum (solid) family with 3 degrees of freedom at each node [17]. Analyses can be done by using solid elements such as C3D8R for both of linear or nonlinear problems inclusive of contact areas, big disfigurements, and plasticity. It is a first-order or often named linear element because it has nodes only at corners, and it uses linear interpolation to extract the displacements at any point. The element C3D8R is a reduced integration element, by using only one integration point located at the
centroid of the element instead of four integration points. A reduced integration of this element leads to more susceptible to the hourglass effect. ABAQUS resolved this issue by the use of the artificial “hourglass stiffness,” which introduces a little amount of artificial stiffness in first-order reduced integration elements. In this study according to the layout of specimens as a reference and built-up beams C3D8R used for modeling concrete types by two techniques, for reference specimens this element used as one section for the solid part which represents the concrete material as shown in Figure (9). For built-up specimens, the concrete solid part section is divided into two partitions as desired depth of specified type of concrete to behave as a composite section as shown in Figure (10).

3.2.2. Truss Element
The reinforcing bars have especially mission to transfer normal forces. Thus, truss element T3D2 was used for the steel reinforcement modeling as shown in Figure (11). T3D2 is a slender element that can resist only the axial forces. It has two nodes element with three degrees of freedom at each node in x, y, and z- directions [17]. The same mesh size equal to 12.5 mm was intentionally selected for all parts of the model to guarantee that different materials share the same node and get accurate results. The overall meshed concrete, loading, and backing plates for all specimens are shown in Figure (12).
3.2.3. Loading and Boundary Conditions
Towards full simulation of beams, the modeling of the supports is being as a roller and pin such as experimental work. One range of nodes was selected at the middle of the bottom of the backing plate parallel with beam width, restricted in the U1, U2, U3 direction to simulate the pin support were U1, U2 and U3 are represented by X, Y, and Z directions respectively. In the same way, other range of nodes were selected in another backing plate, restricted in the U1, U2 direction to simulate the roller support as shown in Figure (13). The load control style was used to carry out the analysis by increasing the load on the model during steps until failure occurs. The load was applied to the loading plate in the same way as the boundary, as shown in Figure (14).
4. Results and Discussion

4.1. Loads Results

Loads which caused the first crack and ultimate loads for both of Experimental work and finite element analysis are illustrated in Table 4. The results reflect reasonable conformity between the numerical analysis and experimental data. Regarding loads which caused of the first crack, finite element analysis results indicate to a slight decrease in cracking loads compared with experimental work with a maximum deviation of about 10% for reference beam Ref-1. This decrees are attributed by the constitutive models for tensile strength of concrete which are based on the splitting tensile strength of both types of concrete at 28 days’ age while the beams are experimentally tested at 56 days’ age, this gap between two ages led the concrete to gain a somewhat an increase in mechanical properties. On the other hand, the occurred of many cracks and big deflections in model lead to divergence of the finite element analysis with a very small increment of load and required a large number of increments to complete it. At this moment, the analysis has been stopped automatically by the software with the ultimate load, which is defined as the load applied at last increment. The comparison between the results of ultimate loads indicates to a slight increment of analysis loads with maximum deviation about 6.36% for reference beam Ref-2, this difference may be attributed by the effect of using a high strain hardening constitutive model for the main reinforcement rebar. Generally, the results which have been obtained from experimental work and finite element analysis indicate that using RCP in compression zones is more effective than using it in tension zones, where there is a significant increase in carrying capacity with clear ductile behavior of beams, proportional with an increase of the RPC layer depth.

| Beams | Load of first crack $P_{\text{crack}}$ (kN) | % Dif. | Ultimate load $P_{\text{ultimate}}$ (kN) | % Dif. |
|-------|----------------------------------|-------|----------------------------------|-------|
| FEM.  | EXP.    |       | FEM.    | EXP.    |       |
| Ref-1 | 22.51   | 25    | -10     | 123.65  | 118   | +4.78 |
| Ref-2 | 43.01   | 45    | -4.42   | 194.97  | 183.3 | +6.36 |
| B-1   | 28.63   | 30    | -5.46   | 126.89  | 122.58| +3.51 |
| B-2   | 34.23   | 35    | -2.2    | 141.32  | 137.29| +2.93 |
| B-3   | 23.58   | 25    | -5.68   | 156.82  | 152   | +3.17 |
| B-4   | 24.16   | 25    | -3.36   | 167.7   | 161.8 | +3.64 |

4.2. Load and Mid-span Deflection Curves Results

Comparisons between (load and mid-span deflection curves) which is obtained from finite element analysis and experimental work are shown in Figure (15) through Figure (20). Mid-span deflections were computed at the middle of the length of the beams at the bottom of the tension zone. The curves showed agreeable approval between the finite element and the experimental work over the whole range of behavior. Generally, the finite element (load and mid-span deflection curves) showed stiffer behavior comparing with experimental curves. With a slight decrease in the final deflection, as illustrated in Table 5. That may be justified by using of constitutive models, for concrete have stiffer behavior compared with the experimental results of control specimens in compressive strength strain hardening and tensile strength strain softening, absence of the microcracks (shrinkage and handling cracks). And a fully bond between the concrete and the reinforcement is assumed in the ABAQUS analysis, but this assumption would not be true in experimental work.
Table 5. Experimental and finite element analysis deflections results.

| Beams | $\Delta u_{EX}$ | $\Delta u_{FE}$ | $\Delta u_{FE}/\Delta u_{EX}$ % |
|-------|-----------------|-----------------|-------------------------------|
| Ref-1 | 12              | 11.105          | 92.54                         |
| Ref-2 | 21              | 19.122          | 91.05                         |
| B-1   | 10.45           | 9.156           | 87.61                         |
| B-2   | 9.62            | 9.3             | 96.67                         |
| B-3   | 16.31           | 15.353          | 94.41                         |
| B-4   | 18.92           | 17.388          | 91.90                         |

Figure 15. EXP and FEM load-deflection curves of the Ref-1 beam.
Figure 16. EXP and FEM load-deflection curves of the Ref-2 beam.

Figure 17. EXP and FEM load-deflection curves of the B-1 beam.
Figure 18. EXP and FEM load-deflection curves of the B-2 beam.

Figure 19. EXP and FEM load-deflection curves of the B-3 beam.
4.3. Crack Pattern and Damage Index Results

The finite element analysis by using ABAQUS showed a nonlinear response, resulting from the linear part. This sequence leads to the understanding that concrete start yields with inelastic behavior. While the load is increased, the cracks generate through the beam and begin to move out from the center of the beam towards the supports diagonally. ABAQUS provides the capability to plot crack patterns. Also, tension and compression damages can be a plot by using of DAMAGC and DAMAGET options. Figures (21-26), illustrate the comparison between the experimental and finite element analysis of all beams, which is shown good agreement.
Figure 21. (a) EXP, (b) Crack pattern, (c) Compression damage, (d) Tension damage, of the Ref-1 beam.
Figure 22. (a) EXP, (b) Crack pattern, (c) Compression damage, (d) Tension damage of the Ref-2 beam.

Figure 23. (a) EXP, (b) Crack pattern, (c) Compression damage, (d) Tension damage of the B-1 beam.
Figure 24. (a) EXP, (b) Crack pattern, (c) Compression damage, (d) Tension damage, of the B-2 beam.
Figure 25. (a) EXP, (b) Crack pattern, (c) Compression damage, (d) Tension damage, of the B-3 beam.
5. Conclusion

Even though a set number of beams was tested, yet a regular trend in their response can be observed. And this may not be enough to affirm the repeatability of this research without further laboratory and numerical work. The following conclusions can be drawn based on the results discussed in this research.

- The finite element model shows acceptable agreement in general behavior compared with the experimental data for the same geometrical features of beams such as dimensions, (loading and supporting conditions) and material properties.
- Full interaction among the finite element models components can justify the little difference between the experimental and numerical results.
- Using constitutive models for concrete have stiffer behavior than the experimental results of control specimens in finite element modeling, clarified the somewhat stiffer load-deflection curves behavior. Also using a constitutive model for the main reinforcement rebar a slightly higher in strain hardening than experimental results can justify the increment of ultimate load of the infinite element model.
- Failure mechanism, load-deformation, and load capacity of the beams proved that the finite element analysis could provide accurate foresee with a good capability to simulate cracking and damaging process similar to the experimental work.

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Appendix A

Hognestad Compressive Constitutive Model of CC

\[ \sigma = f_c \left[ 2 \frac{\varepsilon}{\varepsilon_0} - \left( \frac{\varepsilon}{\varepsilon_0} \right)^2 \right] \text{ for } \varepsilon \leq \varepsilon_0 \]
\[ \sigma = f_c \left[ 1 - 0.15 \frac{\varepsilon - \varepsilon_0}{\varepsilon_u - \varepsilon_0} \right] \text{ for } \varepsilon \leq \varepsilon \leq \varepsilon_u \]

Appendix B

Lu compressive constitutive model of RPC

\[ f_c = f'_c \left[ \frac{(E_{it}/E_0)(\varepsilon/\varepsilon_0) - (\varepsilon/\varepsilon_0)^2}{1 + (E_{it}/E_0 - 2)(\varepsilon/\varepsilon_0)} \right] \text{ for } \varepsilon \leq \varepsilon_0 \]
\[ \varepsilon_L = \varepsilon_0 \left[ 0.1 \frac{E_{it}}{E_0} + 0.8 \right] + \sqrt{\left( 0.1 \frac{E_{it}}{E_0} + 0.8 \right)^2 - 0.8} \text{ for } \varepsilon > \varepsilon_0 \text{, } E_{it} = 10,200(f'_c)^{1/3} \]

Appendix C

Belarbi tensile constitutive model of CC

\[ \sigma_r = (E_c \cdot \varepsilon_r) \text{ for } \varepsilon_r \leq \varepsilon_{cr} \]
\[ \sigma_r = (f_{cr} \left( \frac{\varepsilon_{cr}}{\varepsilon_r} \right)^{0.4}) \text{ for } \varepsilon_r > \varepsilon_{cr} \]

Appendix D

Danha tensile constitutive model of RPC

\[ f_t = f_{td} \left[ \frac{a \left( \varepsilon_t / \varepsilon_{td} \right)^b}{c + \left( \varepsilon_t / \varepsilon_{td} \right)^d} \right] \text{ where } a = 169.259, b = -0.404, c = 164.580, d = 3.11 \]

Appendix E

Compressive damage parameters

| Inelastic compressive stress (N/mm²) | Inelastic strain | dc | Inelastic compressive stress (N/mm²) | Inelastic strain | dc |
|-------------------------------------|------------------|----|--------------------------------------|------------------|----|
| CC                                 |                  |    | RPC                                 |                  |    |
| 16.140                             | 0.00007          | 0  | 35.664                               | 0.00012          | 0  |
| 18                                 | 0.00015          | 0  | 40                                   | 0.00025          | 0  |
| 20                                 | 0.00024          | 0  | 45                                   | 0.00040          | 0  |
| 22                                 | 0.00033          | 0  | 50                                   | 0.00055          | 0  |
| 24                                 | 0.00043          | 0  | 55                                   | 0.00071          | 0  |
| 26                                 | 0.00053          | 0  | 60                                   | 0.00088          | 0  |
| 28                                 | 0.00064          | 0  | 65                                   | 0.00107          | 0  |
| 30                                 | 0.00077          | 0  | 70                                   | 0.00128          | 0  |
| 32                                 | 0.00091          | 0  | 75                                   | 0.00152          | 0  |
| 34                                 | 0.00107          | 0  | 80                                   | 0.00184          | 0  |
| 36                                 | 0.00128          | 0  | 89.160                               | 0.00245          | 0  |
| Inelastic Tensile stress (N/mm$^2$) | Inelastic strain | dt | Inelastic Tensile stress (N/mm$^2$) | Inelastic strain | dt |
|------------------------------------|------------------|----|------------------------------------|------------------|----|
| CC                                 |                  |    | RPC                                |                  |    |
| 3.500                              | 0                | 0  | 10.520                             | 0                | 0  |
| 3.250                              | 0.00002          | 0.07143 | 8.690                             | 0.00200          | 0.17394 |
| 3.000                              | 0.00006          | 0.14286 | 7.304                             | 0.00400          | 0.30571 |
| 2.750                              | 0.00010          | 0.21429 | 6.150                             | 0.00600          | 0.41541 |
| 2.500                              | 0.00015          | 0.28571 | 5.110                             | 0.00800          | 0.51424 |
| 2.250                              | 0.00024          | 0.35714 | 4.172                             | 0.01000          | 0.60341 |
| 2.000                              | 0.00036          | 0.42857 | 3.353                             | 0.01200          | 0.68128 |
| 1.750                              | 0.00055          | 0.50000 | 2.665                             | 0.01400          | 0.74665 |
| 1.500                              | 0.00086          | 0.57143 | 2.107                             | 0.01600          | 0.79971 |
| 1.250                              | 0.00142          | 0.64286 | 1.665                             | 0.01800          | 0.84174 |
| 1.000                              | 0.00257          | 0.71429 | 1.320                             | 0.02000          | 0.87457 |
| 0.750                              | 0.00540          | 0.78571 | 1.052                             | 0.02200          | 0.90003 |
| 0.500                              | 0.01509          | 0.85714 | 0.844                             | 0.02400          | 0.91974 |
| 0.250                              | 0.08591          | 0.92857 | 0.683                             | 0.02600          | 0.93506 |
| 0.125                              | 0.48655          | 0.96429 | 0.557                             | 0.02800          | 0.94701 |

### Appendix F

Tensile damage parameters