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Ab initio NCSM/RGM for three-body cluster systems and application to $^4$He+n+n

Abstract We introduce an extension of the ab initio no-core shell model/resonating group method (NCSM/RGM) in order to describe three-body cluster states. We present results for the $^6$He ground state within a $^4$He+n+n cluster basis as well as first results for the phase shifts of different channels of the $^4$He+n+n system which provide information about low-lying resonances of this nucleus.

Keywords ab initio · three-body · resonances · halo nuclei

1 Introduction

The ab initio NCSM/RGM was presented in [1 2] as a promising technique that is able to treat both structure and reactions in light nuclear systems. This approach combines a microscopic cluster technique with the use of realistic interactions and a consistent ab initio description of the nucleon clusters.

The method has been introduced in detail for two-body cluster bases and has been shown to work efficiently in different systems [1 2 3 4]. However, there are many interesting systems that have a three-body cluster structure and therefore can not be successfully studied with a two-body cluster approach.

The extension of the NCSM/RGM approach to properly describe three-body cluster states is essential for the study of nuclear systems that present such configuration. This type of systems appear, e.g., in structure problems of two-nucleon halo nuclei such as $^6$He and $^{11}$Li, resonant systems like $^5$H or transfer reactions with three fragments in their final states like $^3$H$(^3$H,2n)$^4$He or $^3$He$(^3$He,2p)$^4$He.

Recently, we introduced three-body cluster configurations into the method and presented the first results for the $^6$He ground state [5]. Here we present these results as well as first results for the continuum states of $^9$He within a $^4$He+n+n basis.
2 Formalism

The extension of the NCSM/RGM approach to properly describe three-cluster configurations requires to expand the many-body wave function over a basis $|\Psi_{xy}^{JT}\rangle$ of three-body cluster channel states built from the NCSM wave function of each of the three clusters,

$$|\Psi_{xy}^{JT}\rangle = \sum_{\nu} \int dx \int dy \, G^{JT}_\nu(x,y)\hat{A}_\nu|\Phi_{xy}^{JT}\rangle$$

where $\eta_{1,23}$ is the relative vector proportional to the displacement between the center of mass (c.m.) of the first cluster and that of the residual two fragments, and $\eta_{23}$ is the relative coordinate proportional to the distance between the centers of mass of cluster 2 and 3. In eq. (1), $G^{JT}_\nu(x,y)$ are the relative motion wave functions and represent the unknowns of the problem and $\hat{A}_\nu$ is the intercluster antisymmetrizer.

Projecting the microscopic $A$-body Schrödinger equation onto the basis states $|\hat{A}_\nu\rangle$, the many-body problem can be mapped onto the system of coupled-channel integral-differential equations

$$\sum_{\nu} \int dx \int dy \, x^2 y^2 \left[ H_{\nu \nu'}^{JT}(x',y',x,y) - EN_{\nu \nu'}^{JT}(x',y',x,y) \right] G_{\nu \nu'}^{JT}(x,y) = 0,$$

where $E$ is the total energy of the system in the c.m. frame and

$$H_{\nu \nu'}^{JT}(x',y',x,y) = \langle \Phi_{\nu x'y'}^{JT} | \hat{A}_{\nu'} H \hat{A}_{\nu} | \Phi_{\nu x'y'}^{JT} \rangle,$$

$$N_{\nu \nu'}^{JT}(x',y',x,y) = \langle \Phi_{\nu x'y'}^{JT} | \hat{A}_{\nu'} \hat{A}_{\nu} | \Phi_{\nu x'y'}^{JT} \rangle$$

are integration kernels given respectively by the Hamiltonian and overlap (or norm) matrix elements over the antisymmetrized basis states. Finally, $H$ is the intrinsic $A$-body Hamiltonian.

In order to solve the Schrödinger equations (3) we orthogonalize them and transform to the hyperspherical harmonics (HH) basis to obtain a set of non-local integral-differential equations in the hyper-radial coordinate,

$$\sum_{K\nu} \int d\rho_1 d\rho_2 \, u_{K\nu}^{JT}(\rho_1,\rho_2) \frac{u_{K\nu}^{JT}(\rho_1)}{\rho_1^{\alpha/2}} = E \frac{u_{K\nu}^{JT}(\rho_1)}{\rho_1^{\alpha/2}},$$

which is finally solved using the microscopic R-matrix method on a Lagrange mesh. The details of the procedure can be found in [5].

At present, we have completed the development of the formalism for the treatment of three-cluster systems formed by two separate nucleons in relative motion with respect to a nucleus of mass number $A-2$.

3 Application to $^4$He+n+n.

It is well known that $^6$He is the lightest Borromean nucleus [6], formed by an $^4$He core and two halo neutrons. It is, therefore, an ideal first candidate to be studied within this approach. In the present calculations, we describe the $^4$He core only by its g.s. wave function, ignoring its excited states. This is the only limitation in the model space used.

We used similarity-renormalization-group (SRG) [8] evolved potentials obtained from the chiral N$^3$LO NN interaction [10] with $\Lambda = 1.5$ fm$^{-1}$. The set of equations (6) are solved for different channels using both bound and continuum asymptotic conditions. We find only one bound state, which appears in the $J^T = 0^+1$ channel and corresponds to the $^6$He ground state.
Table 1  Ground-state energies of the $^4$He and $^6$He nuclei. Extrapolations were performed with an exponential fit.

| Approach        | $E_{g.s.}(^4\text{He})$ | $E_{g.s.}(^6\text{He})$ |
|-----------------|-------------------------|-------------------------|
| NCSM/RGM (N$_{\text{max}}=12$) | $-28.22$ MeV | $-28.70$ MeV |
| NCSM (N$_{\text{max}}=12$) | $-28.23$ MeV | $-29.75$ MeV |
| NCSM (extrapolated) | $-28.23(1)$ MeV | $-29.84(4)$ MeV |

Fig. 1 The figure in the left shows the probability distribution of the main component of the $^4\text{He}+n+n$ relative motion wave function for the $J^\pi T = 0^+ 1$ ground state, $r_{nn}$ and $r_{\alpha,nn}$ are the distances between the two neutrons and between the $\alpha$ particle and center of mass of the two neutrons, respectively. In the right, the three main components of the radial part of the $^6\text{He}$ g.s. wave functions $u_{K\nu}(\rho)$ for $N_{\text{max}}=6,8,10$, and 12.

**Ground state** The results for the g.s. energy of $^6\text{He}$ within a $^4\text{He}(\text{g.s.})+n+n$ cluster basis and $N_{\text{max}}=12$, $\hbar\Omega = 14$ MeV harmonic oscillator model space are compared to NCSM calculations in table 1. At $N_{\text{max}} \sim 12$ the binding energy calculations are close to convergence in both NCSM/RGM and NCSM approaches. The observed difference of approximately 1 MeV is due to the excitations of the $^4\text{He}$ core, included only in the NCSM at present. Therefore, it gives a measure of the polarization effects of the core. The inclusion of the excitations of the core will be achieved in a future work through the use of the no-core shell model with continuum approach (NCSMC) [11; 12], which couples the present three-cluster wave functions with NCSM eigenstates of the six-body system.

Contrary to the NCSM, in the NCSM/RGM the $^4\text{He}(\text{g.s.})+n+n$ wave functions present the appropriate asymptotic behavior. The main components of the radial part of the $^6\text{He}$ g.s. wave function $u_{K\nu}(\rho)$ can be seen in fig. 1 for different sizes of the model space demonstrating large extension of the system. In the left part of the figure, the probability distribution of the main component of the wave function is shown, featuring two characteristic peaks which correspond to the di-neutron and cigar configurations.

A thorough study of the converge of the results with respect to different parameters of the calculation was presented in [5], showing good convergence and stability.

**Continuum states** The use of three-cluster dynamics is essential for describing $^6\text{He}$ states in the continuum. Therefore, this formalism is ideal for such study. Using continuum asymptotic conditions, we solved the set of equations (6) in order to obtain the low-energy phase shifts for the $J^\pi = 0^+, 1^+, 1^+$ and $2^+$ channels in the continuum.

In our preliminary results, we obtain the experimentally well-known $2^+_1$ resonance as well as a second low-lying $2^+$ resonance recently measured at Ganil [13]. A resonance is also found in the $1^+$ channel while no low-lying resonances are present in the $0^+$ or $1^-$ channels. In fig. 2 some of the preliminary phase shifts for different channels are shown. Results for bigger model spaces and a study of their stability respect to the parameters in the formalism are presently being calculated and will be presented elsewhere.
4 Conclusions

In this work, we present an extension of the NCSM/RGM which includes three-body dynamics in the formalism. This new feature permits us to study a new range of systems that present three-body configurations. In particular, we presented results for both bound and continuum states of $^4\text{He}$ studied within a basis of $^4\text{He}+n+n$. The obtained wave functions feature an appropriate asymptotic behavior, contrary to bound-state \textit{ab initio} methods such as the NCSM.

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Fig. 2 Preliminary diagonal phase shifts for $^4\text{He}$ for different $J^\pi$ channels.