Temperature Compensation Method of Lamb waves on Composite Plate using First Arrival Wave

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Abstract

There are two basic methods of temperature compensation on isotropic plates, Optimal Baseline Selection (OBS) and Baseline Signal Stretch (BSS). The OBS method compensates the temperature using multiple baseline signals, while the BSS method uses one baseline signal and stretch factor. However, in composite materials, the velocity of Lamb waves differs from one direction to another. In this paper, a temperature compensation algorithm is proposed using the First Arrival Wavepacket (FAW) factor considering the propagation direction of Lamb waves in composite plates. The proposed algorithm consists of two parts. First, Fast Fourier Transform (FFT) was used to filter the response signal in the space domain. Second, the signal was compensated by using the FAW factor for each separated signal. And the accuracy of the algorithm was improved by considering the group velocity and phase velocity of the numerical value of Lamb waves. The numerical value is calculated by using the stress-strain relation and the equation of motion for particles in the composite. Finally, the error of the signals was calculated using the Maximum Error of the Subtracted Signal (MESS). Also, the temperature compensation method using the proposed algorithm was verified by comparing the existing OBS/BSS method. Also, the compensated signal was compared with the experimental signal.

1. Introduction

Composite materials have excellent mechanical properties such as high specific strength and stiffness. Based on these advantages, they have been used in the aerospace and automobile industries. However, composite materials are hard to detect invisible damages such as inner crack and delamination due to operational impact. So, if those damages are not detected in advance, structural failure can occur. Therefore, Structure Health Monitoring (SHM) technology for early detection of inner damages has been studied recently. Among the various existing SHM methods, damage detection methods using Lamb waves are widely used because Lamb waves have advantages of the spreading over wide areas with uniform propagation speed. With these advantages, Lamb waves have some complex characteristics. First, the dispersion characteristic is that the group velocity and the phase velocity are changed according to the thickness of the plate and the wave frequency. Second, the multi-mode characteristic is that both anti-symmetric mode and symmetric mode exist at the same time in wave propagation. Finally, the phase shift and amplitude change occurs when the temperature changes. Those characteristics make it difficult to detect damage. To solve this problem, Lu and Michaels used the Optimal Baseline Selection (OBS) method to compensate for the temperature using several
experimental data. Also, Croxford and Wilcox proposed a Baseline Signal Stretch (BSS) method using a single baseline and stretch factor in isotropic plates [1-2]. However, in each OBS/BSS method, the efficiency of temperature compensation decreases due to the influence of the reflected wave due to boundary conditions. To reduce the effect on reflected waves, a temperature compensation method combining the OBS/BSS method is presented and temperature compensation is performed in the time-frequency domain using Fourier transform [3-5]. A combined OBS/BSS method is used to increase temperature compensation efficiency. However, it takes a long time to set the baseline signal and it is impossible to use in anisotropic plates whose speed changes depending on the direction of the lamb waves. Another method of temperature compensation is physical modeling [6-8], which uses the equation of motion of particles in an elastic medium. Through this method, the governing equations of lamb waves were analytically driven on the isotropic plate using the relation of stress-strain. This method was used to improve the accuracy of temperature compensation. Also, Zhang and Roi proposed a numerical value that is obtained by wavelet transform in the time-frequency domain to improve the accuracy of temperature compensation on an isotropic plate [9-10]. However, it is not accurate in anisotropic materials. Therefore, a new temperature compensation method of lamb waves in anisotropic materials is required. In this paper, the numerical lamb waves equations using the Coefficient of Thermal Expansion (CTE) were calculated to compensate for the temperature of the lamb waves in anisotropic materials [11-15]. Also, the group velocity and phase velocity of lamb waves with respect to temperature change were obtained by using the lamb waves equation. And First Arrival Wavepacket (FAW) factor was used to improve the accuracy of the proposed algorithm. Moreover, temperature compensation of the lamb waves was made in the frequency domain through the Fast Fourier Transformation (FFT). Finally, the OBS / BSS method was compared with the developed algorithm through experiments on the composite plate.

2. Theory
In the anisotropic materials, the material properties depend on the fiber direction and stacking sequence. Because of these characteristics, the group velocity and the phase velocity of the lamb waves have different values. With these characteristics, a new definition of lamb waves is required in anisotropic materials, which is different from the definition of lamb waves in isotropic materials. The stress-strain relation with thermo-elastic stress analysis of such a transversely isotropic material can be expressed by the following equation. And the global coordinate of the composite plate is shown in figure 1.

\[
\sigma_{ij} = C_{ijkl} \left( e_{ij} - e^T_{ij} \right) = C_{ijkl} \left( e_{ij} - \alpha_{ij} \right) \text{T}
\]

\[
\sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j} = \rho \ddot{u}_i, \{i = 1, 2, 3\}
\]
where $\alpha$ is the coefficient of thermal expansion, $\varepsilon^T$ and $C$ are the thermal strain and modulus of elasticity. Also, $h^T$ and $\rho^T$ the temperature-dependent half-thickness and density of the medium are can be expressed as:

$$h^T = h(1 + \alpha_{33}\Delta T)$$

$$\rho^T = \frac{\rho}{(1 + \alpha_{11}\Delta T)(1 + \alpha_{22}\Delta T)(1 + \alpha_{33}\Delta T)}$$

where $h$ and $\rho$ are half-thickness and density at the reference temperature. Also, the superscript $T$ represents the effect on temperature. Next, the direction of the lamb waves propagation on the composite plate is taken as an $x$-axis direction. And assume that lamb waves are a plane wave, the particle displacement can be written as:

$$(u_1, u_3) = (u_1^*, u_3^*) e^{i[(k_1 x_1 + k_3 z_1 - \omega t)]} \quad (4)$$

where $u_1^*$ and $u_3^*$ are the amplitude of particle displacement, $k_1$ and $k_3$ are the wavenumber of a transverse and longitudinal wave, and $\omega$ is the angular frequency. Equation (2) and (4) were used to obtain equation (5) and (6). In this case, the $r_\pm^T$ is assumed to be the ratio of 1 and 3 directions of particle displacement.

$$\left[ (C_{11}k_1^2 + C_{55} - \rho^T \omega^2)u_1^* + (C_{13} + C_{55})k_3k_1u_3^* \right] = 0$$

$$\left[ (C_{33}k_3^2 + C_{55} - \rho^T \omega^2)u_3^* + (C_{13} + C_{55})k_1k_3u_1^* \right] = 0$$

$$(u_1, u_3) = (1, r_\pm^T) e^{i[(k_{11} x_1 + k_{33} z_1 - \omega t)]}, \quad r_\pm^T = \frac{u_3^*}{u_1^*} = \frac{\left( \rho^T \omega^2 - C_{11}k_1^2 - C_{55}k_3^2 \right)}{\left( C_{55} + C_{13} \right)k_1k_3} \quad (6)$$

Assume that the boundary condition of lamb waves propagating in the plate is under the stress-free condition at both surfaces. The boundary condition can be written as:

**Figure 1.** Global coordinate of Composite Plate
\[
\sigma_{33} = C_{33} \frac{\partial u_3}{\partial x_3} + C_{13} \frac{\partial u_1}{\partial x_3} = 0 \\
\sigma_{31} = C_{35} \frac{\partial u_5}{\partial x_1} + C_{55} \frac{\partial u_5}{\partial x_1} = 0
\] (7)

Finally, the symmetric and anti-symmetric mode equations for the lamb waves can be driven by solving equation (6) and boundary condition (7). And the lamb waves equation can be written as:

\[
\begin{align*}
\text{Symmetric} &= \tan(k_3 h) = \left( \frac{C_{33} r^T k_{3,} + C_{13} k_1}{C_{33} r^T k_{3,} + C_{13} k_1} \right) \\
\text{Anti symmetric} &= \tan(k_3 h) = \left( \frac{C_{33} r^T k_{3,} + C_{13} k_1}{C_{33} r^T k_{3,} + C_{13} k_1} \right)
\end{align*}
\] (8)

2.1. Lamb Waves Variation with Temperature changes in Composite Plate

The dispersion curve of the lamb waves was obtained using equation (8) as expressed in frequency and wavenumber. And the result of the lamb waves equation is shown in figure 2. Also, the phase velocities of the lamb waves can be expressed by \( v_p = \frac{\partial \omega}{\partial k} \) and the group velocity can be expressed by \( v_g = \frac{\partial \omega}{\partial k} \). In A0 mode, it can be seen that there is almost no change in the group velocity with respect to the frequency change. Therefore, A0 mode is used for damage detection of composite structures with sensitivity to delamination damage [16]. In this paper, the temperature is compensated by using the group velocity change of A0 mode as the FAW factor. In order to find this, the theoretical lamb wave calculated on the composite plate is shown in figure 3 as the variation of lamb waves with temperature changes. This figure shows that both phase velocity and group velocity were slowed down as the temperature increased.

![Figure 2. Dispersion curves of lamb waves propagation at room temperature (24°C): a) model of the group velocity of lamb waves in the composite plate, b) model of phase velocity of lamb waves in the composite plate.](image)
Figure 3. Theoretical dispersion curve when the temperature difference is 0°C ~ 120°C: a) S0 mode group velocity, b) A0 mode group velocity, c) S0 mode phase velocity, d) A0 mode phase velocity

3. Experiment

We performed the experiments to confirm the characteristics of the temperature change of the lamb waves. Also, figure 4 is the simple geography of the composite plate. The composite plate used in the experiment was 1000 mm long, 1000 mm wide, and 5.4888 mm thick. And PZT sensors were used to obtain the data of lamb waves for temperature changes in composite plates. A distance from the actuator to the sensor is 155mm and from sensor to sensor is 76mm. Furthermore, the laminate sequence of composite plates used in the experiment is $\begin{bmatrix} 45/0/-45/0/0/45/0/-45/0 \end{bmatrix}_{2s}$. Continually, the fiber properties according to the direction are shown in Table 1.

| Property               | Value(Gpa) |
|------------------------|------------|
| Elastic modulus (E1)   | 159        |
| Elastic modulus (E2)   | 8.96       |
| Elastic modulus (E3)   | 8.96       |
| Shear modulus (G12)    | 4.69       |
| Poisson’s ratio (ν12,ν13) | 0.316     |

The central frequency of the signal used in the experiment is 250kHz and the sampling frequency is 48MHz. Also, we obtained experimental data of lamb waves which of temperature ranges between 24°C to 36°C using the Pitch-Catch method. As shown in figure 5, when the temperature increased the phase and amplitude increases. Also, we defined the group velocity of the lamb waves obtained from the experiment using the Time of Arrival(TOA). Where TOA was calculated as the maximum amplitude of the First Arrival Wavepacket(FAW).
4. Temperature Compensation Method

In this section, we performed the temperature compensation of the lamb waves using the experimentally and theoretically obtained FAW factor. First, Experiments with composite plates collected Lamb waves data on temperature changes. Then, we defined the group velocity as the speed of the first arrival wave-packet for a temperature range of 24 °C to 36 °C. In addition, as shown in figure 6, The OBS / BSS factor (\(\beta_E\)) was used as the slope of the group velocity with respect to the temperature change. Also, the theoretical FAW factor (\(\beta_T\)) was obtained from the group velocity of A0 mode with respect to the temperature change obtained from the lamb waves equation. Finally, the Fast Fourier Transformation of the baseline data is used to compensate for the temperature change of the lamb waves. Also, the equation can be expressed as:

\[
\omega (\omega_0 T_0, \beta) = \sum_{j=0}^{N} A_j \exp \left( -i \omega \delta T \right) \beta S (\omega \beta)
\]

where \(t_j, A_j\) and \(S\) are the arrival time, amplitude and wave-form of the \(j_{th}\) wavepacket.
The following method was performed to verification of temperature compensation. The Maximum Error of Subtracted Signal (MESS) is used as the verification method for temperature compensation of lamb waves. Furthermore, Roy set the error value below -25dB as standard [6]. According to this criterion, the A0 mode has a temperature compensation range of about 20°C and the S0 mode has a temperature compensation range of 12°C. In addition, figure 7 shows the difference between the FAW factor and OBS / BSS using Maximum error of the subtracted signal the OBS / BSS method using the MESS method. At this time, the max( $E_r$ ) value uses the data after the output signal. According to table 2, similar results for FAW and OBS / BSS at small temperature
differences. However, the results using FAW are excellent at large temperature differences. Also, the verification equation can be expressed as:

\[
E_r = 20 \log \left( \frac{\max | \text{baseline} - \text{current} |}{\max | \text{baseline}|} \right)
\]  

(10)

| Table 2. Result of temperature compensation |
|---------------------------------------------|
|     | P1                   | P2                   | P3                   | P4                   |
|-----|----------------------|----------------------|----------------------|----------------------|
|     | FAW(dB)              | O/B(dB)              | FAW(dB)              | O/B(dB)              |
| 2°C | -60.8851             | -71.3809             | -64.0299             | -58.1281             |
| 4°C | -52.2281             | -62.7054             | -46.9854             | -33.6122             |
| 6°C | -47.4530             | -37.6742             | -42.6850             | -31.3103             |
| 8°C | -45.8386             | -35.2086             | -32.7413             | -28.5117             |
| 10°C| -36.9379             | -28.7979             | -28.3971             | -25.1086             |
| 12°C| -32.5687             | -25.7004             | -26.3257             | -22.0520             |

6. Conclusion

In this paper, we derived the equations of lamb waves for temperature changes and anisotropic materials. Also, the frequency and wavenumber relation of the Lamb waves equation was used to obtain group velocity and phase velocity curves for temperature changes. In addition, the group velocity variation for the temperature change in A0 mode was linearly fitted and used as the FAW factor. Furthermore, data of 24~36 °C were obtained through an experiment on the composite plate using the PZT sensor. In addition, we defined the phase and amplitude changes according to the temperature change of the lamb waves. And Experimental data were used to calculate the stretch factor of the OBS/BSS method. Thus, OBS/BSS method and FAW factor were used to compare the verification of the proposed algorithm. Also, the comparison method used the Maximum Error of the Subtraction Signal of the baseline-signal and the current signal. The comparison shows that both methods perform better temperature compensation at low-temperature differences, but the temperature compensation using the FAW factor is more efficient at higher temperature differences. In addition, OBS / BSS using the experimental method has the disadvantage that it takes a long time to measure the baseline and current signal, the FAW factor takes less time using numerical theory. Therefore, it is expected that more efficient lamb waves temperature compensation will be performed in the composite through the temperature compensation method using the FAW factor.

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