Branching of the vortex nucleation period in superconductor Nb microtubes due to an inhomogeneous transport current

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Abstract

An inhomogeneous transport current, which is introduced through multiple electrodes in an open Nb microtube, is shown to lead to a controllable branching of the vortex nucleation period. The detailed mechanism of this branching is analyzed using the time-dependent Ginzburg–Landau equation. The relative change of the vortex nucleation period strongly depends on the geometry of multiple electrodes. The average number of vortices occurring in the tube per nanosecond can be effectively reduced using the inhomogeneous transport current, which is important for noise and energy dissipation reduction in superconductor applications, e.g. for an extension of the operation regime of superconductor-based sensors to lower frequencies.

Keywords: superconducting vortices, vortex dynamics, branching, vortex nucleation period, time-dependent Ginzburg–Landau equation, nanotechnology, computational physics

(Some figures may appear in colour only in the online journal)

Introduction

Control over the vortex dynamics in superconductors provides an efficient tool for studying the superconducting properties of materials and is an important issue for applications. In most cases a particular control technique is realized by the guidance of vortices through predefined trajectories [1–7], for example, by the rectification of the average vortex movement [8–13]. The guidance is usually implemented through an array of artificial pinning sites [1–3, 14, 15], which lead to a redistribution of the local current density. The latter is shown to be crucial for the formation, propagation and stability of vortex patterns in thin superconductor strips and wires [16, 17]. Meanwhile, fabrication of curved superconductor structures provides another possibility to affect the current density distribution as a function of curvature. The most pronounced control over the vortex dynamics by this approach occurs when the curvature radius is of the order of the coherence length $\xi$ [18–20].

Recent technological advances have allowed for the fabrication of rolled-up structures [21–23] that consist of InGaAs/GaAs/Nb layers. Nb shows mechanical homogeneity and its coherence length $\xi$ is comparable with the thickness of the Nb layer in the rolled-up structures. Theoretical investigations have revealed the crucial role of the curved geometry of the rolled-up structures for the vortex dynamics [20, 24]. The combined effect of pinning centers and a curved geometry has been demonstrated to strongly depend on the location of the pinning centers in a superconductor structure [25]. Because of the correlated dynamics of vortices in different parts of a superconductor structure, the pinning centers have an effect over a long distance. This influence leads to the branching of a period [25] of vortex nucleations on different sides of an open superconducting Nb tube relative to the applied magnetic field, which is orthogonal to the axis of the tube. The magnitude of the branching is determined by the positions and strengths of pinning centers and characterizes a certain pattern of the vortex dynamics.
We suggest manipulating the branching in such a superconductor structure, where the current density distribution can be controlled locally near the positions of vortex nucleation. A possible implementation of this idea is to fabricate multiple mutually isolated electrodes [26], through which the transport current is introduced into the structure. A gap between any two neighboring electrodes leads to a transport current discontinuity. In a planar structure in the presence of a homogeneous magnetic field, such discontinuities become points of vortex nucleation or denucleation. However, in a curved structure, such a geometry can be chosen that in the vicinity of the joints between the electrodes the component of the magnetic field normal to the surface of the structure vanishes. In this case, discontinuities of electrodes do not act as points of vortex nucleation.

In planar nanopatterned superconductor films, oscillating magnetoresistance as a function of the magnetic field was attributed to the interaction between thermally excited moving vortices and the oscillating persistent current induced in the loops that constitute a network [27] or to the current-excited moving vortices owing to the interplay of Meissner and transport currents in a mesoscopic superconductor ladder [28]. To the best of our knowledge, there have been no reports on the transport effects due to the moving vortices in curved nanostructures.

In this work, we evaluate the effect of a two-component electrode on the vortex dynamics in open Nb tubes and show a possibility to efficiently control their characteristics, considering branching of the vortex nucleation period as an example.

Methods

Theoretical model

We consider a Nb superconductor open tube [20] of radius $R$ and length $L$ (figure 1(a)) with three paraxial contacts on the edges of the slit. The ‘red’ contact plays the role of an input contact, through which the transport current $I_{in}$ enters. Through the ‘blue’ contact, the current $I_{out}$ leaves the tube. Through the ‘green’ contact, the additional transport current $I_{control}$ enters; it might be positive (in) or negative (out). Its role is to dynamically control the vortex nucleation. At each instant, the sum of all three currents satisfies the condition of continuity:

$$I_{in} + I_{control} + I_{out} = 0. \quad (1)$$

The scheme of control is realized in the following way: the current $I_{control}$ is modified by changing the potential on the control (green) electrode; after that, the potential on the input (red) electrode is adjusted, so that the current $I_{in}$ keeps its constant value. The out (blue) electrode is grounded, so the value $I_{out}$ is determined by the condition of continuity (1).

The system is placed in a magnetic field $\mathbf{B} = -\mathbf{B}_{e}$ (figure 1(a)), which induces Meissner currents circulating at each half-tube [20]. The total current, which is a sum of the Meissner and transport currents, is shown schematically in figure 1(a) by the black lines on the ‘front’ half of the tube. Two of three currents in equation (1) are independent, which allows for different regimes of control over vortex dynamics. In this work, we keep $I_{in}$ constant and change $I_{control}$.

Our model is based on the time-dependent Ginzburg–Landau (GL) equation [29] for the order parameter $\psi$ in the dimensionless form [2]

$$\frac{\partial \psi}{\partial t} = (\nabla - i\mathbf{A})^2 \psi + 2\kappa^2 (1 - |\psi|^2) \psi, \quad (2)$$

$$(\nabla - i\mathbf{A})\psi|_{\text{boundary}} = 0,$$

$$\left(\nabla - i\left(\mathbf{A} + \frac{j}{|\psi|^2}\right)\right)\psi|_{\text{electrodes}} = 0, \quad (3)$$

where the dimensionless variables are determined in [2, 20, 25], $\kappa$ is the GL parameter and $\nabla$ is the gradient operator. In [19] and [30] this approach was used to study...
vortex dynamics in open cylindrical tubes and kinematic vortex–antivortex lines in superconductor stripes, respectively. The temperature-dependent values of magnetic field considered in the current work belong to the 10 mT-range for the temperature $T = 0.95 T_c$, where $T_c$ is the critical temperature. The empirical law, which represents the thermodynamic critical magnetic field as a function of temperature (see equation (1.2) of [29])

$$B_c(T) \approx B_c(0)[1 - (T/T_c)^2],$$

in view of equations (5.18) and (4.62) of [29], is applicable for the first ($B_{c1}$) and second ($B_{c2}$) critical magnetic fields and thereby provides an estimate of the magnetic field range, where vortices occur at different temperatures. The electric field penetration depth [31] is evaluated to be of the order of 10 nm, which is more than two orders of magnitude less than the typical sizes of the structures under consideration. Under these circumstances, the electrostatic potential has a low impact on the period of vortex nucleation.

The normal (with respect to the cylindrical surface) component of the magnetic field, according to figure 1(a), is $B_n = B \sin(x)$. The vector potential is taken in the Landau gauge: $A = Ae_x$, $A = -By$. Transport currents in equation (1) are obtained from the current densities $j$ in the boundary conditions of equation (3) by integrating along the appropriate contacts. The thickness of the superconductor wall, which is necessary to calculate the electron mean free path, constitutes 50 nm [20]. The typical tube radius and length under consideration are more than 10 times larger than the thickness. With these parameters, it is safe to neglect the effect of an induced magnetic field and to use a 2D approximation. Our numerical simulations show that the induced magnetic field is as low as 1% of the applied magnetic field.

Evaluation of thermal effects based on the heat equation with the Joule heating as a source yields a temperature gradient as low as 2 mK $\mu$m$^{-1}$, which is negligibly small for our analysis. The coherence length (taken at the temperature 0.95$T_c$) constitutes $\xi = 56$ nm [20]. The conceptual framework for numerical simulation is represented in detail in [32]. The numerical scheme is based on the explicit finite-difference time-domain method. We trace how the random initial distribution of the order parameter evolves to a quasi-stationary state, which manifests vortex dynamics.

### Results and discussion

**Branching of the vortex nucleation period**

Two full-side electrodes generate symmetrical vortex dynamics—both halves of the tube demonstrate the same characteristic times [20]. In this work, the period of vortex nucleation $\Delta t_2$ is considered to characterize the vortex dynamics, since it has a more pronounced dependence on the current density, than the duration of the vortex motion through the tube $\Delta t_t$. In what follows, we use the denotation $\Delta t_2^{(1)}$ for the half-tube with input and control electrodes (the front side in figure 1(a)) and $\Delta t_2^{(2)}$ for the half-tube containing the output electrode (the back side in figure 1(a)). For a tube of radius $R = 400$ nm with two full-side electrodes carrying the current $I = 1.7$ mA at $B = 10$ mT, the nucleation period $\Delta t_2^{(1)} = \Delta t_2^{(2)} = \Delta t_2^{(sym)} = 1.2$ ns (this value is shown in figure 2 by the black solid line). However, introducing the control electrode of length $L_{control} = 20\xi$ into the tube of length $L = 60\xi$ (so that the input electrode is of length $L_{in} = 40\xi$) violates the inversion symmetry of the modulus of order parameter with respect to the geometric center of the tube. A similar mechanism occurs in vortex ratchets with a pinning potential, which lacks centrosymmetry [33].

In figure 2, the dependence of $\Delta t_2$ on the control current demonstrates a different behavior for each half-tube. We perform simulations for two radii of the tube. In figure 2, the blue line corresponds to $R = 600$ nm and the red one to $R = 400$ nm. An increase of the radius shifts both the $\Delta t_2^{(1)}$ and $\Delta t_2^{(2)}$ curves upwards. As seen from figure 2, a difference between vortex nucleation periods $\Delta t_2^{(1)}$ and $\Delta t_2^{(2)}$ by a factor of as large as 3 is achieved. This difference, as follows from our simulation, is determined by the control current. Because of the inhomogeneity of the current density distribution over the whole tube, a variation of the control current at one side of the tube leads to a change of the vortex nucleation period at the opposite side. A further decrease of the control current (beyond the values given in figure 2) leads to an infinite rise of $\Delta t_2^{(2)}$, which means that vortex nucleation is blocked at one side of the tube and the dynamics occurs completely on at another side of the tube. The available experimental set-ups allow for the time resolution in voltage–time characteristics down to the sub-nanosecond level (for example, 16 ps in NbN...
superconductor single photon detectors [34]). Therefore a direct detection of the vortex-induced voltage spikes in Nb microtubes is feasible.

A variation of the control current from negative to positive values shows a crossover between $D_{t_2}^{(2)}$ and $D_{t_1}^{(1)}$ functions. For the radius $R = 600 \, \text{nm}$ the crossover occurs at $I_{\text{control}} = 0.70 \, \text{mA}$, and for $R = 400 \, \text{nm}$ it occurs at $I_{\text{control}} = 0.62 \, \text{mA}$. The dependence of $D_{t_2}^{(2)}$ on $I_{\text{control}}$ after passing the crossover point is less evident than before: $D_{t_2}^{(2)}$ is even reduced as compared to $D_{t_1}^{(1)}$.

From the practical point of view, it is interesting to evaluate the average number of vortices in a per nanosecond when the dynamics occurs only at one side of the tube. In the tube with radius $R = 400 \, \text{nm}$, the control electrode $L_{\text{control}} = 1120 \, \text{nm}$ and $I_{\text{control}} = -0.5 \, \text{mA}$, which is analyzed here as a typical case, the characteristic times $\Delta t_2^{(2)} = 0.7 \, \text{ns}$ and $\Delta t_1^{(1)} = 0.7 \, \text{ns}$ result in the average number of vortices $n_c \approx 1.43$ per nanosecond. In a tube with full-side electrodes, $n_c \approx 1.67$ per nanosecond, so that the relative difference in the average number of vortices constitutes $\sim 15\%$ for the same input current. Thus, using an inhomogeneous transport current in the tube leads to an effective decrease of the average number of vortices. Such a decrease plays a crucial role in noise and energy dissipation reduction for numerous superconductor applications [7, 8, 33]. In fact, the property $\Delta t_2^{(2)} \to \infty$ implies that using multiple electrodes allows for vortex removal from certain regions of a superconductor sample, which is of practical interest, for example, in order to suppress the $1/f$-noise due to the activated hopping of trapped vortices and thus to extend the operation regime of superconductor-based sensors to lower frequencies [8].

Current density modification by the control electrode

Vortex dynamics occurs under a Magnus force [$j$, $\mathbf{B}$] and depends linearly on the local current density [28]. The geometry of boundaries in mesoscopic superconductors determines the points of nucleation, denucleation and the possible location of vortices [35], and the corresponding current density distribution leads to the non-linear character of vortex...
motion as a function of the control current. The branching of the vortex nucleation period shown in figure 2 is an example of non-linear dynamics. An interpretation of such a behavior of vortices requires to analyze the current density. In figure 3, the key mechanism of the difference between vortex dynamics in the cases with (figure 3(b)) and without (figure 3(a)) a control electrode is illustrated. Near the points of vortex nucleation for both halves of the tube, the current density components are listed in the rectangles. The black and light gray thin arrows on each panel in figure 3 point to the same positions on the tube. As clearly seen, the main reason for the difference in vortex dynamics is a change of the current density component \( j_s \) along the azimuthal direction (a corresponding coordinate \( s = R \phi \) is defined through the azimuthal angle \( \phi \) shown in figure 2). From a comparison of the two bottom panels in figure 3, the decrease of this component at the points of vortex nucleation specified by the black thin arrows on the side of the output electrode \( (j_s^p - j_s^b = 2.49 - 2.39 = 0.1) \) is higher than the current density increase \( (j_s^b - j_s^p = 2.42 - 2.39 = 0.03) \) at the points shown by the light gray thin arrows on the side of the input electrode. At the same time, the \( j_s \) components are similar to each other in both cases.

**The effect of the magnetic field and the length of the control electrode**

The vortex nucleation period is a decreasing function of the magnetic field as shown in figure 4(a) for two cases, without and with the applied control current 0.5 mA. For higher values of the control current, the difference between \( \Delta t_N^{(2)} \) and \( \Delta t_N^{(1)} \) is less pronounced than for lower values of the control current (see, for instance, a crossover point in figure 2 for a fixed magnetic field).

For a lower magnetic field, the absolute value of the difference between the two periods is larger than for a higher magnetic field. For example, \( |\Delta t_N^{(2)} - \Delta t_N^{(1)}| \approx 0.6 \text{ ns} \) for \( B = 8 \text{ mT} \) and \( |\Delta t_N^{(2)} - \Delta t_N^{(1)}| \approx 0.3 \text{ ns} \) for \( B = 20 \text{ mT} \) for the case without a control current. However, the relative difference \( \delta_{21} = |\Delta t_N^{(2)} - \Delta t_N^{(1)}|/\Delta t_N^{(2)} \approx 0.4 \) varies at most by 5% within the considered range of magnetic fields. Since the \( \Delta t_N^{(2)}(B) \) curves saturate with magnetic field growth [20], the relative difference \( \delta_{21} \) remains almost steady for the whole range of magnetic fields where vortex dynamics occur. Qualitatively, the functions \( \Delta t_N^{(2)}(B) \) have the same shape for the control current \( I_{\text{control}} = 0.5 \text{ mA} \) (see the blue lines in figure 4(a)), for which the relative difference \( \delta_{21} \) is about 0.1.

The relative difference \( \delta_{21} \) strongly depends on the length of the control electrode, as shown in figure 4(b). For \( I_{\text{control}} = 0 \text{ mA} \) and the control electrode of length \( L_{\text{control}} = 10 \xi \), the relative difference is \( \delta_{21} \approx 0.14 \), while for the length \( L_{\text{control}} = 30 \xi \), the relative difference is significantly larger: \( \delta_{21} \approx 0.52 \). The main reason for this dramatic change is a decrease of \( \Delta t_N^{(1)} \) more than twice, while \( \Delta t_N^{(2)} \) only slightly depends on \( L_{\text{control}} \). The decrease of \( \Delta t_N^{(1)} \) in figure 4(b) results from the change of the length of the input electrode, correlated with the length of the control electrode. In particular, the input current density \( j_{\text{in}} \) rises in the vicinity of the vortex nucleation point, which leads to an effective reduction of the potential barrier (see figure 6 of [20]). The vortex nucleation period \( \Delta t_N^{(1)} \) decreases, while the corresponding period \( \Delta t_N^{(2)} \) at the opposite side of the tube remains practically unchanged, since the output current density is not affected by changing the length of the input electrode.
Conclusion

In conclusion, the interplay of a curved geometry with an inhomogeneous transport current determines a specific current density distribution, which destroys the inversion symmetry of the order parameter with respect to the geometric center of the tube and leads to a branching of the vortex nucleation period. In particular, using the appropriate electrode arrangement, the vortex dynamics can be blocked on one side of the tube. The weak relative change of the vortex nucleation period (by about 5%) depends on the magnetic field in the range where vortex dynamics occur for the considered control currents. However, this strongly depends on the length of the control electrode (in particular, the relative difference between the characteristic times $\Delta t_{2}^{(0)}$ and $\Delta t_{2}^{(1)}$ can be modified by a factor of 5). Our main conclusion is that the proposed method allows for tuning the frequency of vortex generation at different parts of the tube and provides a reduction of the average number of vortices per nanosecond, which is important for noise and energy dissipation reduction in superconductor applications, for instance, for an extension of the operation regime of superconductor-based sensors to lower frequencies.

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