Estimating Most Productive Scale Size in Data Envelopment Analysis with Integer Value Data

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Abstract. The most productive scale size (MPSS) is a measurement that states how resources should be organized and utilized to achieve optimal results. The most productive scale size (MPSS) can be used as a benchmark for the success of an industry or company in producing goods or services. To estimate the most productive scale size (MPSS), each decision making unit (DMU) should pay attention to the level of input-output efficiency, by data envelopment analysis (DEA) method decision making unit (DMU) can identify units used as references that can help to find the cause and solution from inefficiencies can optimize productivity that main advantage in managerial applications. Therefore, data envelopment analysis (DEA) is chosen to estimating most productive scale size (MPSS) that will focus on the input of integer value data with the CCR model and the BCC model. The purpose of this research is to find the best solution for estimating most productive scale size (MPSS) with input of integer value data in data envelopment analysis (DEA) method.

Keywords: Most Productive Scale Size, Data Envelopment Analysis, Integer Value

1. Introduction

The most productive scale size (MPSS) is a measurement that states how resources should be organized and utilized to achieve optimal results. The most productive scale size (MPSS) can be used as a benchmark for the success of an industry or company in producing goods or services. Each decision making unit (DMU) is currently required to increase productivity and high efficiency due to the higher cost required or incurred to run the center of the company. Khodabakhshi (2009) In determining the maximum function of a solution where the value of its objective function is zero, and it is emphasized that the input $\theta^* > 0$ must be a positive integer, so it is clear that the optimal value of the objective function is the non-negative variable which always have $\phi^* \geq \theta^*$. Based on Khodabakhshi’s statement that to maximize a solution from its objective function, it is more emphasized on the value of inputs with a positive integer value to obtain an optimal value, so as to estimate the most productive scale size can be more efficient. To estimate the most productive scale size many alternative strategies and improvement methods can be developed, depending on the aspects of output or inputs used as basic aggregates, there are some companies that only have the input of integer value data, for example companies that have a system of distributing goods or services directly to the final customer and not to process the product, but only have the right to carry out business by using the brand, name, system
and procedures that have been set before in a certain time and area, so that the basic aggregate
to estimate the most productive scale size using only the input of integer value data. Ahn
(2012) Data envelopment analysis (DEA) is a method for evaluating performance that allows
simultaneous analysis of its performance indicators. Performance appraisals are needed to
identify a productivity performance measurement method to be able to determine the efficient
productive scale of any decision making unit (DMU).

Ramanathan (2003) Data envelopment analysis (DEA) with CCR model reflects
(multiplication) technical efficiency and scale efficiency, while the BCC model reflects only the
technical efficiency, so the relative scale efficiency is the ratio on efficiency of CCR model and
BCC model. If the result of the ratio is one then the decision making unit (DMU) operates at
the most productive scale size. If the ratio of results to less than one means there is still a scale
inefficiency in the decision making unit (DMU).

An efficient decision making unit (DMU) with CCR model means technical efficiency and
scale efficiency is optimal. While efficient decision making unit (DMU) with BCC model but
inefficient with CCR model means experiencing inefficiency to scale, that is because decision
making unit (DMU) is only technical efficiency, so that the inefficiency is come from the scale.

Cooper et al. (2011) by data envelopment analysis (DEA), relative efficiency of the decision
making unit (DMU) can be defined as the ratio of the total weighted output divided by the total
weighted input.

The above statement can be concluded that the data envelopment analysis (DEA) method
determines the weights for each input and output of each decision making unit (DMU), where
they have non-negative and universal, meaning each decision making unit (DMU) in the existing
sample should be able to use the same weighted output / total weighted input and the ratio
should be no more than one (total weighted output / total weighted input = 1), because if
the value dual is more or less than one then the decision making unit (DMU) is deemed to be
inefficient in relative or inefficiency, otherwise if its dual value is equal to one, or its efficiency
value is 100%, and then the decision making unit (DMU) is considered efficient.

Based on its performance assessment, the data envelopment analysis (DEA) method can
identify units that are used as references that can help to find the cause and solution from
inefficiency that optimize productivity, which is a major advantage in managerial applications.
Therefore, one possible method to estimate the most productive scale size of an efficient on a
decision making unit (DMU) is the data envelopment analysis (DEA) method, since the data
envelopment analysis (DEA) method can evaluate the performance of the decision making unit
(DMU) and analyzed the evaluation results of the relative efficiency of each comparable decision
making unit (DMU). In addition to generating the efficiency value of each decision making unit
(DMU), data envelopment analysis (DEA) is also able to show units that are in reference for
inefficiency units. So in this research, the data envelopment analysis (DEA) method was chosen
to estimate the most productive scale size of the that will focus on the input of integer values
data which is expected to find the best solution for estimating most productive scale size of
efficiently.

2. Most Productive Scale Size (MPSS)
Khezrimotlagh et al. (2012) DEA is a nonparametric method in proposed research operations to
measure the efficiency and productivity of each DMU with multiple inputs and outputs that can
only be determined by integer values such as number of employees, books, magazine numbers
and total passengers.

Moghaddas and Vaez-Ghasemi (2014) In order to find a point of most productive scale size
(MPSS) the analysis will be limited to find the intersection between the CCR border and the
combination of the result point after scaling the inefficiency DMU or experience of increasing
return to scale or decreasing return to scale using the model equation of proposed model is as
follows:

\[
\left(\frac{\theta X_0 - S}{\sum_{j=1}^{n} \lambda_j^*}, \frac{Y_0 - S}{\sum_{j=1}^{n} \lambda_j^*}\right) = (\tilde{X}_0, \tilde{Y}_0)
\]  

(1)

Cooper et al. (2011) Figure 1 below shows that the efficiency limits of BCC represented by solid lines and CCR efficiency limits represented by a dotted line from a nonzero technical evaluation point and returns to DMU performance scale simultaneously. It is illustrated with \(A = (1, 1)\) as the initial coordinate point in the figure below using the formula (1) to obtain MPSS.

![Figure 1. Most Productive Scale Size (MPSS)](image)

Unit B weighs from formula (1) to adjust to unit A, it will be the coordinate points of units B and C. Takes into unit calculations B and C as corresponding to the MPSS point for unit A, and completes with the proposed model where the value of \(s^- = 0\) and \(s^+ = 0\) which produces the point with coordinates \(3/2, 2\) and \(3, 4\) where it is the most productive scale size of efficient measure, while unit E after evaluation the efficiency level turns out that unit E has the potential to be the most productive scale size by scaling. Unit E’ is the result of the first scaling that produces the coordinate point \(7/2, 9/2\), after examining it turns out that unit E’ has not reached the most productive scale size, so that unit E’ needs to do the second scaling that produces unit E” Coordinates \(27/8, 9/2\).

3. Data Envelopment Analysis (DEA)

Khoveyni and Eslami (2013) Fare and Grosskopf stated there is an alternative model to estimate the most productive scale size (MPSS) based on the optimal solution of CCR and BCC is constant return to scale (CRS) also called CCR model and variable return to scale (VRS) called the BCC model.
Cooper et al. (2011) returns to scale (RTS) status in data envelopment analysis (DEA) are as follows:

- If $\theta^*_\text{CCR} = \theta^*_\text{BCC}$ or $\sum \lambda^*_j = 1$ This case is called *Constant Return to Scale* (CRS).
- If $\theta^*_\text{CCR} < \theta^*_\text{BCC}$ or $\sum \lambda^*_j < 1$ This case is called *Increasing Return to Scale* (IRS).
- If $\theta^*_\text{CCR} > \theta^*_\text{BCC}$ or $\sum \lambda^*_j > 1$ This case is called *Decreasing Return to Scale* (DRS).

### 3.1. Constant Return to Scale (CRS)

Jahanshahloo and Khodabakhshi (2003) This model shows a change in the amount of output that is proportional to the change in the sum of all inputs used. For example if the capital or input is added by x times then the output also increases by x times.

Cooper et al. (2006) the linear form of the CCR model are as follows:

$$\text{Max} \sum_r u_r y_{ro}$$

s.t.

$$\sum_r u_r y_{ro} - \sum_i v_i x_{io} \leq 0$$
$$\sum_i v_i x_{io} - 1$$
$$u_r, v_i \geq 0$$
$$x_{ij} \in \mathbb{Z}^+ \ \forall i, j$$

### 3.2. Variable Return to Scale (VRS)

Jahanshahlo and Khodabakhshi (2003) In this model there are two types of variable return to scale, that are:

- **Decreasing Return to Scale**
  Decreasing return to scale occurs when the number of output changes is not proportional (smaller) than the input change.

- **Increasing Return to Scale**
  Increasing return to scale occurs when changes of all inputs will result in larger output changes than the proportion of input changes.

Wang and Lan (2013) Banker, Charmes, and Cooper express the linear form of the BCC model as follows:

$$\text{Min} \theta - \epsilon (\sum_{i=1}^m S_i^- + \sum_{r=1}^m S_r^+)$$

s.t.

$$\sum_{j=1}^n \lambda_j x_{ij} + S_i^- = \theta x_{io} \quad i = 1, \ldots, m$$
$$\sum_{j=1}^n \lambda_j y_{rj} - S_i^+ = y_{ro} \quad r = 1, \ldots, s$$
$$\lambda_j \geq 0 \quad j = 1, \ldots, n$$
$$x_{ij} \in \mathbb{Z}^+ \ \forall i, j$$

The efficiency values of BCC is obtained by running the above model for each DMU. The performance efficiency measures of BCC is called pure technical efficiency, which is related to the values obtained from the model allowing the resultant variables, so that the existing scale can be eliminated. In general, the CCR efficiency rating for each DMU will not exceed the efficiency of BCC, which is clearly intuitive as the BCC model analyzes each DMU locally rather than globally.

3.3. Return to Scale (RTS)
Cooper et al. (2011) Linear form of RTS to determine \( \sum_{j=1}^{n} \lambda_j^* \) in equation (3) used to scaling process in DMU of increasing return to scale or decreasing return to scale improvement are as follows:

\[
\begin{align*}
\text{Max} & \quad \sum_{j=1}^{n} \lambda_j - \epsilon \left( \sum_{i=1}^{m} S_i^- + \sum_{r=1}^{m} S_r^+ \right) \\
\text{s.t.} & \quad \sum_{j=1}^{n} \lambda_j x_{ij} + S_i^- = \theta x_{io}, \quad i = 1, \ldots, m \\
& \quad \sum_{j=1}^{n} \lambda_j y_{rj} - S_i^+ = y_{ro}, \quad r = 1, \ldots, s \\
& \quad \lambda_j \geq 0 \quad j = 1, \ldots, n \\
& \quad x_{ij} \in Z^+ \quad \forall i, j
\end{align*}
\]

4. Application
In this research, the data to be sampled is taken from 10 DMU data that have been research by Moghaddas and Vaez-Ghasemi. Each company has two input value data and two output value data. For input \((I1)\) that is data of processed material, \((I2)\) is employee data, while output \((O1)\) is data from customer satisfaction, and \((O2)\) is data from sales. This research focus on the only input of integer value data, so that the data used only inputs derived from employee data. The following will present the efficiency evaluation results of each DMU using DEA with the CCR model and BCC model to estimate the most productive scale size. The 10 DMU data sets presented in Table 1 of the data will be determined which DMUs is efficient and require scaling to be productive. The completion will be done using DEA method with CCR and BCC model.
Table 1. Evaluation result of efficiency input-output

| DMU | X  | Y₁  | Y₂  | CCR  | BCC  | RTS    |
|-----|----|-----|-----|------|------|--------|
| 1   | 24 | 4.1 | 3.6 | 0.95 | 1.00 | Increasing |
| 2   | 32 | 4.0 | 3.2 | 0.69 | 1.00 | Increasing |
| 3   | 30 | 4.4 | 4.2 | 0.81 | 0.67 |        |
| 4   | 35 | 4.4 | 4.4 | 0.70 | 0.57 |        |
| 5   | 25 | 3.6 | 4.0 | 0.84 | 0.80 |        |
| 6   | 24 | 3.9 | 4.2 | 0.92 | 0.83 |        |
| 7   | 33 | 4.5 | 4.5 | 0.76 | 0.61 |        |
| 8   | 32 | 3.6 | 3.7 | 0.63 | 1.00 | Increasing |
| 9   | 20 | 3.6 | 3.8 | 1.00 | 1.00 | Constant |
| 10  | 36 | 4.5 | 4.6 | 0.69 | 0.56 |        |

Table 2. Evaluation result of DMU after scaling

| DMU | X  | Y₁  | Y₂  | CCR  |
|-----|----|-----|-----|------|
| 1   | 21.66 | 3.91 | 3.42 | 1.00  |
| 2   | 15.24 | 2.76 | 2.21 | 1.00  |
| 8   | 12.70 | 2.27 | 2.33 | 1.00  |

Table 3. DMU with input of integer value data

| DMU | X  | Y₁  | Y₂  | CCR  |
|-----|----|-----|-----|------|
| 1   | 22 | 3.97 | 3.47 | 1.00  |
| 2   | 15 | 2.72 | 2.18 | 1.00  |
| 8   | 13 | 2.32 | 2.37 | 1.00  |

From Table 1 it can be seen that DMU 9 has operated on the best scale size efficiency, where $\theta_{CCCR}^* = \theta_{BCC}^*$ or $\sum \lambda_j^* = 1$. Where as DMU 1, 2, and 8 are inefficiency to scale, where $\theta_{CCCR}^* < \theta_{BCC}^*$ or $\sum \lambda_j^* < 1$, so scaling is required to make DMU 1, 2, and 8 feasible to operate on most productive scale size. For the scaling process on the DMU scale which increases above it is required $\lambda_j^*$ by using equation (4), so for DMU 1 it is known $\lambda_j^* = 1.05$, for DMU 2 it is known $\lambda_j^* = 1.45$, and for DMU 8 is known $\lambda_j^* = 1.59$ after that substitute $\lambda_j^*$ in formula (1). After the scaling of the new DMU data is re-evaluated using DEA with CCR model which results can be seen in table 2 which turns out DMU 1, 2, and 8 the objective function is one, which means that the DMU is efficiency and feasible to operate on the best scale size. Because this research focuses on integer value data, the input data which is still in the form of real numbers is changed to form integer and whose results can be seen in table 3.
5. Conclusion

Based on the results of the efficiency evaluation using DEA with CCR model and BCC model to estimate the most productive scale size of the problem samples taken from the 10 DMU data focus on integer value data of Moghaddas research, it appears that DMU 9 is a DMU operating at the best scale size, where $\theta^{*}_{CCR} = \theta^{*}_{BCC}$ or $\sum \lambda_{j}^{*} = 1$ and slack $= 0$. Where as DMU 1, 2, and 8 are inefficient to scale, where $\theta^{*}_{CCR} < \theta^{*}_{BCC}$ or $\sum \lambda_{j}^{*} < 1$, so scaling is required to make DMU 1, 2, and 8 feasible to operate on most productive scale size. This research aim to find the best solution to estimate the most productive scale size with the input of integer value data in DEA, with the limitation on the input of integer value data can be more easily to find the most productive scale size and simultaneously affirm the validity of DEA method is as a fairly efficient means to estimate the most productive scale size.

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