Optimization of calculating operations in designing of mechanical drives

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Abstract. The present article is concerned with optimization of calculation of project design work in the context of determining the optimal structural weight. As an example, the article considers a mechanical double-reduction drive. Using the condition of minimum total center distance, the kinematic diagram of a double-reduction cylindrical reducer was optimized, with consideration of the effect of bending fatigue strength of teeth. In addition, optimal gear ratios were obtained in Excel.

1. Introduction
In drives of lifting devices, special crane-and-hoist type motor and special crane gearboxes, designed to work under conditions of significant overloads during starting, stopping and sudden braking, are used. At equal calculated capacity $N_{ct}$ in the drive, in the operating mode of a lifting device different electric motors and different gearboxes are installed: more powerful electric motors and more powerful gearboxes are needed for heavier duties, less powerful electric motors and less powerful gearboxes are needed for lighter duties; so-called diagonal unification occurs.

Drives of lifting devices are loaded with significant static loads from $Q = 0$ on $Q = Q_{nom}$ and significant dynamic ones, reaching $Q_{din} = Q_{nom}$ at the time the lowering cargo stops during tripping operations.

Loading condition of lifting devices’ drives is determined according to the rules of Federal Mining and Industrial Supervision (Gosgortekhnadzor) of Russia by groups of operating modes: loading classes; application class. Loading classes correspond to modes of operation of mechanisms according to the rules of Gosgortekhnadzor, which are characterized by an indicator of the duty factor [1-7].

| Loading class | Mode of operation | Duty factor |
|---------------|------------------|-------------|
| B1            | Light, L         | PB 15%      |
| B2            | Middle, M        | PB 25%      |
| B3            | Heavy, H         | PB 40%      |
| B4            | Extremely heavy, EH | PB 60%   |

2. Materials and methods
The algorithm for design of a lifting device drive is as follows:

- Calculation of the required static drive power $N_{ct}$. 

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Selection of possible electric motors having one power but different speed \( n_{d.b.}'; n_{d.b.}''; n_{d.b.}''' \)

Calculation of gear ratios of the drive with possible electric motors:

\[
U'_0 = \frac{n_{d.b.}'}{n_{pb}'}; U''_0 = \frac{n_{d.b.}''}{n_{pb}''}; U'''_0 = \frac{n_{d.b.}'''}{n_{pb}'''}
\]  

Selection of potential gearboxes in a drive by: the predetermined mode of operation; the potential speed of the electric motor \( n_{d.b.}'; n_{d.b.}''; n_{d.b.}''' \); the potential gearbox ratios \( u_p', u_p'', u_p''' \) (a gearbox ratio \( u_p \) should be nearest value to the previously calculated \( u_o \)), and according to the calculated static power \( N_{ct} \); designed standard sizes cylindrical double-reduction crane reducers C2 are chosen.

- C2-a-u – possible assembly – output shaft forms – Y.
- a – center distance of the low speed gear; u – a reducer’s ratio);
- Y – climatic category.
- From potential selected gearboxes, you should use the one having the error of ratio \( \Delta u \) less than 10%; if there are several gearboxes satisfying this condition you should select the gearbox, which fits a motor with higher speed.

The ultimately selected gearbox is tested for start with consideration of the effect of static loads, dynamic loads of reciprocating masses and dynamic loads of rotating masses [2,8-11]:

\[
T'_{pusk} = T'_{st} + T'_{din.post} + T'_{din.bp}
\]  

\[
T'_{st} = \frac{Q \cdot D_{db}^2 \cdot n_{d.b}}{2a \cdot U_0 \cdot \eta_0}, \text{ H} \cdot \text{m},
\]

where

\[
Q \quad \text{lifting capacity, H;} \\
D_{db} \quad \text{drum diameter, m;} \\
U_0 \quad \text{overall ratio of the drive;} \\
\eta_0 \quad \text{overall efficiency of the drive;} \\
a \quad \text{parts of line;} \\
T'_{din.post} \quad \text{starting torque of reciprocating masses, reduced to the motor shaft:}
\]

\[
T'_{din.post} = \frac{Q \cdot D_{db}^2 \cdot n_{d.b}}{375t_n \cdot a^2 \cdot u^2 \cdot \eta^2}, \text{ H} \cdot \text{m},
\]

\[
G D_p^2 \quad \text{flywheel moment of the motor rotor, Hm}^2
\]
GD_2 \text{m}^2 - flywheel moment of the coupling, \( H_m \).

Test for start is initiated at \( t=1.5 \) sec; if the condition \( T_{\text{pusk}} \geq T'_{\text{pusk}} \) is not correct, the starting period is to be increased. If this method does not provide the required result, an electric motor of higher power is selected.

The problem of optimal partitioning of the total ratio \( U \) of a cylindrical double-reduction helical gearbox between the gears, which will provide it with minimum size and weight, is under consideration [2, 11-16].

Having denoted the gear ratios of high-speed and low-speed gears by \( u_B \) and \( u_T \), we write the formula for the total gear ratio of the gearbox:

\[
U = u_B u_T .
\]  

(6)

3. Results and discussion

Rational choice of ratio correlation of gears largely determines both weight-size parameters of the gearbox and its operating characteristics.

The kinematic diagram of the gearbox is shown in figure 1. The diagram shows the input and output shafts, as well as gear parts. Pinion 1 and wheel 2 make up a high-speed gear (b) of the reducer, while pinion 3 and wheel 4 make up a low-speed gear (T). The center distances of the steps are denoted by \( a_b \) and \( a_T \).

An proxy criterion for the optimality of the gearbox scheme is the value of total center distance \( a_z \), which should be as low as possible. To minimize \( a_z \), it is necessary to solve a variational problem in which the parameters affecting \( a_z \) are subject to variation [2].

![Kinematic diagram of the gearbox](image)

Figure 1. Kinematic diagram of the gearbox

After its dependence on variable parameters is established, the total center distance will be taken as an objective function and denoted by \( g \). The main variable parameters are the gear ratios \( u_b \) and \( u_T \), which in turn depend on the number of wheel teeth \( z_k \):

\[
u_b = \frac{z_2}{z_1} ; \quad u_T = \frac{z_4}{z_3} .
\]  

(7)

As it is known, reference diameter of helical gears is determined by the number of teeth \( z \), angle \( \beta \) of inclination between teeth and the generatrix of the reference cylinder and the engagement modulus \( m \):

\[
d_k = \frac{m z_k}{\cos \beta} .
\]  

(8)
The total center distances of the reducer’s gears are equal to the sums of reference radii of meshing wheels. Therefore,

\[ a_\Sigma = a_{bb} + a_T = 0.5 \frac{m_b(z_1 + z_2)}{\cos \beta_b} + 0.5 \frac{m_T(z_3 + z_4)}{\cos \beta_T}. \]

Using formulas (7), the objective function will be as follows:

\[ g = a_\Sigma = 0.5 \frac{m_b(z_1(1 + u_b))}{\cos \beta_b} + 0.5 \frac{m_T(z_3(1 + u_T))}{\cos \beta_T}. \] (9)

Now it is necessary to identify the connections that may exist between the parameters used in formula (9), and also to form a system of constraints that the solution of the variational problem must satisfy.

The variational problem of determining the gear ratios \( u_b \) and \( u_T \) of the reducer’s gears that implement the minimum of the total center distance \( a_\Sigma \) is reduced to selection of synthesis parameters that minimize the functional (8) and satisfy the conditions.

Due to a large number of involved variables and constraints, it is difficult to solve the problem; in order to navigate these difficulties, it is possible to scale down the problem which can be circumvented by lowering the dimension of the problem, and for these reasons a number of simplifications must be adopted.

The first of these simplifications is that there is nothing to prevent the number of gear teeth from being assumed to be the same: \( z_1 = z_3 \). In this case, if cutting is carried out without correction, coefficients of tooth shape will also be equal: \( Y_{F1} = Y_{F3} \).

The second simplification can be obtained by setting coefficients of wheels’ width of certain gears:

\[ \psi_{bd/b} = \psi_{bd/T}. \]

The third simplification is obtained by cutting wheels’ teeth of both gears at the same helix angle:

\[ \beta_b = \beta_T. \]

The fourth simplification is related to an assessment of the ratio of permissible bending stresses, the value of which for an individual gear depends mainly on its resource.

According to Lagrange’s method, the problem of finding the minimum of the objective function (9) in the presence of a connection between the varied parameters is equivalent to finding the minimum of a functional of the form

\[ L = g + \lambda \psi, \] (10)

where \( \lambda \) – the Lagrange multiplier, the value of which is determined in the course of solving the variational problem.

Extremum conditions of the function \( L(u_b, u_T) \) is written as

\[ \frac{\partial L}{\partial u_b} = \frac{\partial g}{\partial u_b} + \lambda \frac{\partial \psi}{\partial u_b} = 0; \]

\[ \frac{\partial L}{\partial u_T} = \frac{\partial g}{\partial u_T} + \lambda \frac{\partial \psi}{\partial u_T} = 0. \]

Having performed the mentioned differencing procedures, we obtain:

\[ \frac{m_b z_1}{2 \cos \beta_b} \left[ 1 + \frac{8}{27} (u_b) \left( \frac{19}{27} (1 + u_T) \right) \right] - \lambda u_T = 0; \] (11)
\[
\frac{m_B z_1}{2 \cos \beta_b} (u_b)_{27}^8 - \lambda u_b = 0. \tag{12}
\]

Eliminating the Lagrange multiplier from the system of equations, we obtain the expression

\[
1 + \frac{8}{27} (u_b)_{27}^{19} (1 + u_T) - (u_b)_{27}^{19} u_T = 0,
\]

which, after simple transformations, finally simplifies to

\[
27(u_b)_{27}^{46} + 8u_b - 19U = 0. \tag{13}
\]

The solution to this equation for the given value of the total gear ratio \(U\) is the optimal gear ratio \(u_b\) of the high-speed gear of the reducer. After determining \(u_b\), the gear ratio \(u_T\) of the low-speed gear can be found by the formula

\[
u_T = \frac{U}{u_b}. \tag{14}
\]

To determine the roots of the equation (14), the algorithm of the half-interval method was used and implemented in the Excel program (figure 2). The calculation results - the division of the total gear ratio of the reducer by gears - are presented in table 2.

**Table 2.** Division of the total gear ratio of the reducer by gears.

| \(U\) | 8   | 10  | 12.5 | 16  | 20  | 25  | 31.5 | 40  |
|------|-----|-----|------|-----|-----|-----|------|-----|
| \(u_b\) | 2.535 | 2.911 | 3.341 | 3.888 | 4.456 | 5.105 | 5.875 | 6.790 |
| \(u_T\) | 3.156 | 3.435 | 3.742 | 4.116 | 4.448 | 4.897 | 5.362 | 5.891 |

**Figure 2.** Excel file of calculated data.
The obtained values of the gear ratios are generally consistent with the solution of a similar problem, the formulation of which, however, did not introduce the helical teeth of wheels, and what is more, the effect of the resource on the bending strength of teeth. As can be seen from the table, for gearboxes with a gear ratio \( U < 20 \), the gear ratio of a high-speed gear should be set lower than for a low-speed one, and with \( U \geq 20 \) - vice versa.

In a simplified calculation method, widely presented in academic literature, for cylindrical double-reduction gearboxes with an expanded diagram, a general recommendation is given:

\[
u_T = k \sqrt{U} \ ; \ \nu_b = \frac{\sqrt{U}}{k} ;
\]

where \( k = 0.88 \), independent of \( U \).

In the proposed version, coefficient \( k \) depends on \( U \); \( k = k(U) \), and its values row: \( k(8) = 1.116; k(10) = 1.086; k(12.5) = 1.058; k(16) = 1.029; k(20) = 0.995; k(25) = 0.979; k(31.5) = 0.955; k(40) = 0.931.\)

The dependence of the objective function (9) on the independent parameter, in this case \( \nu_b \), by which the variation was performed, is of particular interest. Similar dependencies for gearboxes with different gear ratios are presented in figure 3. The graph of each curve has a pronounced minimum at value \( \nu_b \) corresponding to the data in table 2. The value of the ordinate of the minimum point is proportional to the total center distance of the gearbox \( a_z \), to find which it is enough to multiply this value by a factor \( 0.5 m_b z_j / \cos \beta_b \).

The graph of each curve has a pronounced minimum at value \( \nu_b \) corresponding to the data in table 2. The value of the ordinate of the minimum point is proportional to the total center distance of the gearbox \( a_z \), to find which it is enough to multiply this value by a factor \( 0.5 m_b z_j / \cos \beta_b \).

\[\frac{2a_z \cos \beta_b}{m_b z_j} U \geq 40 \]
\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 31.5 \]
\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 25 \]
\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 20 \]
\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 16 \]
\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 12.5 \]
\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 10 \]

\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 8 \]

\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 6 \]

\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 4 \]

\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 3.888 \]

\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 3.4 \]

\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 3 \]

\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 2 \]

\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 1.6 \]

\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 1.2 \]

\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 1 \]

\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 0.8 \]

\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 0.6 \]

\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 0.4 \]

\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 0.2 \]

\[\frac{2a_z \cos \beta_b}{m_b z_j} U = 0 \]

**4. Conclusions**

Optimal ratios of gears are obtained.

Development of a calculation file in Excel reduces the time for subsequent calculation operations, when changing the input initial data.

The results obtained in Excel can be the initial data for parametric modeling of mechanical drives.

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