Energy-based evaluation of hardness testing with discrete element method

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Abstract. Present paper focuses on the static hardness testing of porous construction materials. A series of experiments were conducted by the authors, during which the main strength properties of twelve different construction materials (normal and high strength concrete, concrete with supplementary cementing materials (SCM), limestones, ceramics, polymer concrete, etc.) were tested. Compressive strength, Young’s modulus and Brinell hardness of every material was measured and based on an energy-based approach a relationship between the variables was defined. Brinell hardness test is a commonly used method for estimation of strength of solid materials (e.g. metals), however in case of porous materials contradictory results can be found in the literature. Present research is aiming to find a general formula for porous materials with as less restrictions as possible. The Brinell hardness of the materials was determined with DSI (Depth Sensing Indentation) method, which is an updated form of the classical Brinell hardness test. The output of DSI test are the loading-unloading curves belonging to a predefined maximal loading force. Based on these figures the elastic, dissipated and total strain energy can be determined and a formula can be given to estimate strength parameters based on them. The results were validated by using DEM (Discrete Element Method), mainly focusing on concrete and the effect of SCMs. The model was set up based on the experimental data and DSI test was performed with the model as well. The results of the model were compared to the laboratory measurement results.

1. Research objectives

Present research focuses on the static hardness testing of porous construction materials. A series of experiments were conducted by the authors, during which the main strength properties of fifteen different construction materials were tested (Fig. 1). Static hardness testing methods were developed for testing of metals, especially steel. The expressions, which were derived to define hardness, were defined based on the measurements performed on steel samples. The intention of the authors was to investigate the hardness of other construction materials besides steel. First, three metals (aluminum, mild steel, reinforcing bar steel (S235)) were subjected to the tests. Based on literature data, it is well known that hardness testing is applicable for metals. Then the tests were performed on a wide range of concretes (normal strength concrete, high-strength concrete, concrete containing supplementary cementitious materials (SCMs), polymer concrete). Concrete is the material used in the largest quantity by the construction industry, that’s why it was the first choice for testing. It was seen that hardness testing is applicable for materials with silicate frame structure (concretes), thus other calcium silicate hydrate (CSH) based products were applied for testing (cellular concrete block (Ytong), sand-lime brick (Silka)).
Finally, natural stones and ceramics, which can be used as masonry material were subjected for testing. Summarized, the tested materials were the following:

- **Metals**: aluminum, mild steel, reinforcing bar steel (S235).
- **Concretes**: normal strength concrete, high-strength concrete, concrete containing SCMs, polymer concrete.
- **Other CSH based materials**: cellular concrete block (Ytong), sand-lime brick (Silka).
- **Ceramics**: clinker brick, facebrick, clinker tile.
- **Stones**: porous limestone and compact limestone, rhyolite tuff.

![Figure 1. The investigated materials (bottom row: three metals, Ytong block, three ceramics, sand-lime brick, polymer concrete (150×150×150 mm); top row: three concrete and three stone specimens)](image)

The tested properties were the following for every material:

- density,
- particle size distribution,
- compressive strength,
- Young’s modulus,
- and Brinell hardness.

Based on an energy-based approach a relationship between the variables was defined. It was aimed to find a formula, which can be applied for a wide range of construction materials. To get a deeper understanding on the processes inside the material Discrete Element Method (DEM) was applied, which is a powerful tool for testing the macro-behavior of porous materials and with its application the processes inside the material can be visualized, which can lead us to the better understanding of the phenomena.

### 2. Literature review

Hardness testing was the first material testing in the engineering practice, starting with the scratching hardness testing methods in the 17th century [1-3]. The research of the theoretical background of hardness was initialized by the pioneering work of Heinrich Hertz in the 19th century [4]. Hertz’s proposal formed also the basis of the indentation hardness testing methods by Brinell (1900), Rockwell, (1920), Vickers (1924) and Knoop (1934) [5]. All these methods are measuring the size of a residual plastic deformation in the tested specimen caused by the indenting load. Different indenter geometries (pyramid, spherical etc.) can be applied, however from these the spherical indenters can be used for testing both ductile materials (e.g. metals) and brittle materials (e.g. ceramics). The response of materials to the indentation test includes elastic (reversible) and plastic (irreversible) deformations as well as
forming of cone cracks in brittle materials; therefore, the definition of the term ‘hardness’ is not evident [6]. Lately a computer aided version of hardness testing was developed and successfully applied on porous materials as well [7]. The method is called Depth Sensing Indentation (DSI) test and it is mostly applied to evaluate mechanical properties, like compressive strength or Young’s modulus [8-11]. DSI method allows researchers to study the penetration process on an energy basis.

3. Experimental and numerical methods

A wide range of construction materials were applied for material testing. The research is mainly focused on the Depth Sensing Indentation (DSI) testing, which is an updated form of the classical Brinell hardness test (Fig. 2). Brinell hardness test is a commonly used method for estimation of strength of solid materials (e.g. metals), however in case of porous materials contradictory results can be found in the literature [12]. Brinell hardness test (and thus DSI) is a static (very low speed) hardness test method [13]. The main advantage of DSI that it is a non-destructive testing (NDT) method, meaning that during the test does not occur global failure in the material and it can be used for further testing.

![Figure 2. Schematic figure of a Brinell-hardness tester [13]](image)

During the DSI test a polished steel sphere penetrates into the flat surface of the tested material, under a specified maximum load. A theoretical output of the DSI test can be seen in Fig. 3, which shows the relationship of the loading force and indentation depth (or indentation diameter). Based on that the hardness can be defined as follows:

$$ HB = \frac{F}{D \times h} $$  \hspace{1cm} (1)

where $F$ is the loading force in Newton, $D$ is the diameter of the indenter in millimeters and $h$ is the indentation depth in millimeters.

The DSI test has two phases, the loading phase and the unloading phase. Between the two phases, 30 sec holding time was applied. During the loading period, the indenter body penetrates into the surface of the sample at a constant rate until reaching the maximum value of the load. During that phase elastic and plastic deformations as well occurs in the material. During the unloading period, the indenter body moves away from the sample with the same rate. Elastic deformation of the material occurs during unloading and displacements are formed in the opposite direction, than formed during loading. During the unloading phase, the indentation depth is decreasing and in case of a perfectly elastic material it would reach 0. It is theoretically possible to find the elastic properties, including Young’s modulus, of materials from the unloading curve of the indentation characteristics. In the literature only a few
suggestions can be found for the calculation of Young’s modulus based on indentation hardness and most of them applies the Boussinesq problem expressed by Sneddon. Sneddon defined the load vs. indentation depth functions for a linear elastic half-space for various types of indenter bodies [14].

Figure 3. Theoretical relationship of loading force, indentation diameter (or depth) and indentation work

The area under the loading curve in Fig. 3 is equal to the work that is invested until reaching the maximum penetration depth that deforming the material elastically and plastically, agglomerate and set up cracks in it. This work is called the total indentation work:

$$W_t = \int_0^{h_m} F_1 \cdot dh$$

(2)

where $W_t$ is the total work, $h_m$ is the maximum indentation depth, $F_1$ is the value of the loading force in the loading phase. The area under the unloading curve is called the elastic indentation work. This is the amount of work that is recovered during the unloading:

$$W_e = \int_{h_m}^{h_r} -F_2 \cdot dh$$

(3)

where $W_e$ is the elastic work, $h_m$ is the maximum indentation depth, $h_r$ is the residual indentation depth, $F_2$ is the value of the loading force in the unloading phase. The difference between the two works gives the dissipated energy during the indentation:

$$W_d = W_t - W_e$$

(4)

After rearrangement the following expression can be written:

$$\frac{W_d}{W_t} + \frac{W_e}{W_t} = 1$$

(5)

So the depth sensing indentation (DSI) test may be suitable for addition to the introduction of hardness metrics, the energy based examination of the entire load-unload process. The DSI tests were performed on a Zwick Z050 computer-controlled universal tensile testing machine.

Besides the hardness testing compressive strength test was performed on all materials with an Alpha 3-3000 S hydraulic press. Before the tests the specimens were dried to lose their water content which are not in a chemical bonding. For the investigations specimens were prepared, which have the same size and shape (150×150×150 mm cube). After that these specimens were loaded up to failure with the
same loading rate (5 kN/sec) and the maximum force was recorded. The compressive strength results were used to compare them with the hardness test results and to verify the numerical models.

The test methods were modelled with numerical methods. The aim of the numerical modelling was to generate more results without laboratory experiments. As it was mentioned in Section 1, DEM was applied to model the materials. The Discrete Element Method (DEM) is a family of numerical methods for computing the motion and effect of a large number of small particles. Any type of discrete element model is made up of two basic components: the elements and the contacts between them. In the present case a model was set up, which applies rigid elements that represented the aggregates of the material. As it was shown in [15], Brinell hardness testing can be modelled with DEM. Similar models, as presented in [15] were set up and DSI test performed on them. The particle distribution of the models followed the particle size distribution of the original material (e.g. in case of normal concrete: 0-4 mm: 40%; 4-8 mm: 22%; 8-16 mm: 38%). Several different test specimens were created with the same particle size distribution but with different arrangement of the particles. The parameters of the models were set up based on the laboratory compressive strength results. Finally similar curve (Fig. 3) could be plotted from the software as in case of the laboratory tests. A sample model can be seen in Fig. 4.

4. Results and discussion
In Fig. 5 some typical results of the DSI tests can be seen. Based on these figures the materials were arranged into three groups:
- elastic materials (clinker tile, clinker brick),
- elasto-plastic materials (compact limestone, high-strength concrete, polymer concrete, normal-strength concrete, rhyolite tuff, facebrick, concrete with SCM, metals)
- and plastic materials (sand-lime brick, porous limestone, cellular concrete).

![Figure 4. DEM model of a concrete cube](image)

![Figure 5. DSI test results: (a) normal strength concrete – elasto-plastic, (b) clinker tile – elastic and (c) porous limestone - plastic](image)
The arrangement was done based on the shape of the loading-unloading curves and the slope of the loading and unloading paths. If the slope of the unloading path was close to a vertical line, the material was considered as plastic material, while the material was considered to be elastic if the loading and unloading paths were close to each other. Between them the materials were considered to be elasto-plastic. Metals behavior is elasto-plastic, as it is well known based on a wide range of literature data and the method of static hardness testing was developed for such materials. The elastic and dissipated energy both contributes significantly in the total indentation energy in case of metals (e.g. in case of S235 steel $W_t=25124$ Nmm, $W_e=17762$ Nmm, $W_d=7361$ Nmm). Based on the results it can be observed that other materials, mostly concretes show similar behavior, as it can be seen in Fig. 5. Even polymer concrete, which is an unconventional type of concrete, fits into the group of elasto-plastic materials.

The investigations covered a wide range of construction materials. The lowest body density was 516 kg/m$^3$ (cellular concrete block), while the largest was 2502 kg/m$^3$ (compact limestone).

The curves can be described best with the indentation work, which is equal to the area under the curves. Based on the results, it is shown that the theoretical expression (Eq. 5) is true for these measurements. This supports the validity of the measurements.

The relationship of the elastic and dissipated energy with other material properties (Young’s modulus, compressive strength, Brinell-hardness) was investigated. Based on the results, it could be observed that the elastic energy and the compressive strength (presented in Table 1.) has a relationship with high correlation, as it can be seen in Fig. 6. This could be written in a form of a power function.

![Figure 6. Relationship of the compressive strength and the ratio of the total and elastic energy](image)

### Table 1. Compressive strength and Young’s modulus of the investigated materials

| Material                  | Compressive strength [N/mm$^2$] | Young’s modulus [N/mm$^2$] |
|---------------------------|---------------------------------|-----------------------------|
| Aluminum                  | 350                             | 70000                       |
| Mild steel                | 420                             | 200000                      |
| Reinforcing steel bar     | 600                             | 210000                      |
| Normal strength concrete  | 54.2                            | 34578                       |
| High strength concrete    | 106.2                           | 43802                       |
| SCM concrete              | 72.5                            | 36914                       |
| Polymer concrete          | 98.7                            | 40000                       |
| Cellular concrete block   | 2.02                            | 500                         |
| Sand-lime brick           | 19.3                            | 2000                        |
| Clinker brick             | 80.9                            | 35000                       |
| Facebrick                 | 23.3                            | 3000                        |
| Clinker tile              | 233                             | 65000                       |
| Porous limestone          | 12.4                            | 5127                        |
It was also observed that there is a relationship between the elastic energy and the Brinell-hardness of the materials, as it can be seen in Fig. 7. The Brinell hardness (HB) is written in function of the ratio of the total ($W_t$) and the elastic ($W_e$) indentation energy (Eq. 6). This relation was valid for the experimental and numerical results coming from the DEM software as well. The results of the DEM analysis correlated well with the experimental data. The results indicate that there is a strong correlation between the Brinell hardness and the elastic indentation energy of the porous materials. Eq. 6 was applicable for all investigated materials, except the clinker tile, which behaved fully elastically within the applied load range (Fig. 5). Thus it could be stated that an expression was derived between the Brinell hardness and elastic indentation energy of plastic and elasto-plastic materials. Based on our experiment the value of parameter ‘$k$’ in the expression was found to be 3894.2 as it can be seen on Fig. 7.

$$HB = k \cdot \frac{1}{\left(\frac{W_t}{W_e}\right)^2}$$  \hspace{1cm} (6)

**Figure 7.** Relationship of the Brinell-hardness and the ratio of the total and elastic energy

**5. Conclusions**

Wide range of construction materials (concretes, stones, metals, etc.) were subjected to different material tests (compressive strength, Young’s modulus, Brinell hardness) and the results were analyzed with an indentation energy based method. The aim was to find relationship between hardness of the material and other mechanical parameters. Besides the laboratory experiment numerical tests were performed as well, using Discrete Element Method (DEM). Depth Sensing Indentation tests were performed to determine the hardness of the materials. Based on the results the materials were arranged into three groups: elastic, elasto-plastic and plastic (Fig. 5).

The energy-based analysis of the DSI results indicated that there is a linear relationship between the Brinell hardness and the elastic indentation energy of elasto-plastic and plastic construction materials. The measurement results indicate that based on the found expression a materials property can be defined, which is valid for all elasto-plastic and plastic materials.
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