Optimal translational motion of the elastic telescopic robot arm

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Abstract. Paper aims to implement optimal translational movement (to the final state of absolute quiescence) in the shortest possible time, which is found from the moment ratios in relative motion. An example of an elastic manipulator arm as a system with two degrees of freedom have been described: while frequency of the first tone and total arm length are given, the links lengths and the second tone frequency of the oscillations are determined. The following related tasks are solved: designing an elastic robot arm with given properties; optimal movement analysis of the arm as a superposition of the translational and relative motions.

1. Introduction
The theory of optimal control describes methods for analyzing elastic system oscillations and controlling it[1-6]. The problems of optimal translational and rotational motion of systems with a finite and infinite number of degrees of freedom are still relevant[7-15].

Thus, in an earlier paper [11] it was shown that solving the complete inverse problem of the variational calculus (from the original function to the functional — reversion calculus) while constructing controls for the translational motion of elastic objects leads in some cases to energy savings.

This paper aims to implement optimal translational movement (to the final state of absolute quiescence) in the shortest possible time, which is found from the moment ratios in relative motion. An example of an elastic manipulator arm as a system with two degrees of freedom have been described: while frequency of the first tone and total arm length are given, the links lengths and the second tone frequency of the oscillations are determined.

The following related tasks are solved: designing an elastic robot arm with given properties; optimal movement analysis of the arm as a superposition of the translational and relative motions.

2. Design of the elastic telescopic robot arm with a given frequency of the first tone
At a given frequency of the first tone $w_1$ of the system (with two degrees of freedom), the frequency of the second tone is calculated as $w_2 = nw_1$, where $n = const$, and the parameter $n$ is a result of solving the problem.
Figure 1. Telescopic robot arm schematic view

For the elastic robot arm (figure 1), the frequency determinant is expressed as:

\[
\begin{vmatrix}
  m_1 \delta_{11} \lambda - 1 & m_2 \delta_{12} \lambda \\
  m_1 \delta_{21} \lambda & m_2 \delta_{22} \lambda - 1
\end{vmatrix} = 0
\]  

(1)

The parameters of the determinant (1) \( m_1 \), \( m_2 \) – reduced concentrated weights that are calculated as:

\[
m_1 = \left[ \frac{L_1 \pi (d_1^2 - d_2^2)}{8} + \frac{L_2 \pi d_3^2}{8} \right] \gamma; \quad m_2 = \frac{L_2 \pi d_3^2}{8} \gamma,
\]

where \( \gamma \) is the mass per unit volume (kg/m³); \( d_1 \), \( d_2 \), \( d_3 \) – the diameters of the cross sections (figure 2).

Figure 2. Telescopic robot arm links cross section diameters: a) for link length \( L_1 \); and b) for link length \( L_2 \)

Relative movements found with the use of Mohr integral, calculated by the rule of Vereschagin:

\[
\delta_{11} = \int_i \frac{\bar{M}_i \bar{M}_2 ds}{EJ_i} = \frac{L_1^3}{3EJ_1}, \quad \delta_{12} = \int_i \frac{\bar{M}_i \bar{M}_2 ds}{EJ_1} = \frac{L_2 L_3}{2EJ_1} + \frac{L_1^3}{3EJ_1},
\]

\[
\delta_{22} = \sum_i \int \frac{\bar{M}_i \bar{M}_2 ds}{EJ_i} = \frac{L_2^3 + L_1 L_2^2}{3EJ_1}.
\]  

(2)

Axial moment of inertia of the links cross sections: \( J_1 = \frac{\pi (d_1^4 - d_2^4)}{64} \), \( J_2 = \frac{\pi d_3^4}{64} \), \( \lambda = \frac{1}{w^2} \).

Solving the determinant (1), we obtain:

\[
\lambda^2 + A_1 \lambda + A_2 = 0,
\]  

(3)
where \( A_1 = \frac{m_1\delta_{11} + m_2\delta_{22}}{m_1\delta_{11}m_2\delta_{22} - m_2\delta_{12}m_1\delta_{21}} \), \( A_2 = \frac{1}{m_1\delta_{11}m_2\delta_{22} - m_2\delta_{12}m_1\delta_{21}} \).

Equation (3) can be also represented as
\[
(\lambda - \lambda_1)(\lambda - n^2\lambda_1) = 0,
\]
where \( n = \text{const} \); \( \lambda_2 = n^2\lambda_1 \).

After rearrangement, (4) takes the following form:
\[
\lambda^2 - (n^2 + 1)\lambda + n^2\lambda_1^2 = 0.
\]

A system of nonlinear algebraic equations is obtained from the equality of coefficients in (3) and (5):
\[
A_1 - (n^2 + 1)\lambda_1 = 0, \quad A_2 - n^2\lambda_1^2 = 0.
\]

While operating, parametric fluctuations may appear, that are related to the change of the links length. To prevent this effect, the arm can be adjusted in advance to the desired length. So, additional condition can appear
\[
L_1 + L_2 - L = 0,
\]
where \( L \) – defined overall length of the arm.

Further, using equations (6) and (7), we can formulate various problems to determine the physical or geometric parameters of the arm.

3. The elastic oscillations of the robot arm triggered by the optimal translational vertical motion

Differential equations of forced flexural oscillations of the elastic telescopic arm (relative motion), as systems with two degrees of freedom, composed on the basis of the forces independence principle and Dalamber principle, has the form:
\[
\delta_1m_1\ddot{x}_1 + \delta_1m_2\ddot{x}_2 + x_1 = -F_1(t), \quad \delta_2m_1\ddot{x}_1 + \delta_2m_2\ddot{x}_2 + x_2 = -F_2(t),
\]
where \( F_1(t) = m_1U_1(t), \quad F_2(t) = m_2U_2(t). \)

In equation (8), the distributed lumped masses are reduced to concentrated masses and resistance is not taken into account. Control (acceleration of translational motion) is accepted according to [12]:
\[
\dot{U}'_e(t) = \frac{d^2S_e}{dt^2} = \ddot{S}_e(t) = \frac{L_p^2}{2\pi}\sin pt,
\]
where \( \ddot{S}_e(t) \) – corresponds to the simplest case of translational acceleration of the arm [7, 11, 12].

The translational velocity \( V_e(t) \) and displacement \( S_e(t) \) are calculated by integrating the acceleration (9) with the boundary conditions \( S_e(0) = 0, \quad \dot{S}_e(0) = 0, \quad S_e(T) = L, \quad \dot{S}_e(T) = 0 \), i.e. the motion is carried out from the initial state of rest to the final state of rest.

This random displacement and velocity of the translational motion of the hands:
\[
S_e(T) = \frac{L}{2\pi}(pt - \sin pt), \quad \dot{S}_e(T) = V_e = \frac{L_p^2}{2\pi}(1 - \cos pt),
\]
where $L_*$ is the total translational motion of the arm (vertically) over time $T$; $p = \frac{w_1}{n_1}$, where $w_1$ is the frequency of the first tone of the natural oscillations of the arm; $n_1 = \text{const}$, which in general is obtained from the moment relations [7, 11, 12], i.e. \( x_1(T) = 0, \; \dot{x}_1(T) = 0; \; x_2(T) = 0, \; \dot{x}_2(T) = 0 \), representing a system of transcendental equations.

In [7, 9, 11] the system of equations (8) according to the method of the main coordinates was transformed to two autonomous equations, and then the physical coordinates were expressed in terms of the main ones.

A possible solution to the system of inhomogeneous linear equations (8) can be found as the sum of the general solutions of the homogeneous system and particular solutions of the inhomogeneous. For a homogeneous system, the solutions are written as follows:

\[
\begin{align*}
    x_1 &= A_1 \sin (w_1 t + \alpha_1) + B_1 \sin (w_2 t + \alpha_2), \\
    \dot{x}_1 &= A_1 w_1 \cos (w_1 t + \alpha_1) + B_1 w_2 \cos (w_2 t + \alpha_2) \\
    x_2 &= \rho_1 A_1 \sin (w_1 t + \alpha_1) + \rho B_1 \sin (w_2 t + \alpha_2), \\
    \dot{x}_2 &= \rho_1 A_1 w_1 \cos (w_1 t + \alpha_1) + \rho \rho B_1 w_2 \cos (w_2 t + \alpha_2)
\end{align*}
\]

where \( \rho_1 = -\frac{(\delta_{11} m_2 w_2^2 - 1)}{\delta_{12} m_2 w_2^2} \), \( \rho_2 = \frac{(\delta_{12} m_1 w_1^2 - 1)}{\delta_{12} m_2 w_2^2} \) (12).

Particular solutions of the inhomogeneous system: \( x_1^* = C_1 \sin pt \), \( x_2^* = C_2 \sin pt \), where the constants \( C_1 \) and \( C_2 \) are found from the following system of algebraic equations:

\[
\begin{align*}
    (\delta_{11} m_1 p^2 - 1)C_1 + \delta_{12} m_2 p^2 C_2 + F_1^* &= 0, \\
    \delta_{21} m_1 p^2 C_1 + (\delta_{22} m_2 p^2 - 1)C_2 + F_2^* &= 0
\end{align*}
\]

where \( F_1 = m_1 \frac{L_p p^2}{2\pi} \); \( F_2 = m_2 \frac{L_p p^2}{2\pi} \).

The general solution of the system (8) is now written as follows:

\[
\begin{align*}
    x_1(t) &= A_1 \sin (w_1 t + \alpha_1) + B_1 \sin (w_2 t + \alpha_2) + C_1 \sin (pt), \\
    \dot{x}_1(t) &= A_1 w_1 \cos (w_1 t + \alpha_1) + B_1 w_2 \cos (w_2 t + \alpha_2) + C_1 p \cos (pt), \\
    x_2(t) &= \rho_1 A_1 \sin (w_1 t + \alpha_1) + \rho B_1 \sin (w_2 t + \alpha_2) + C_2 \sin (pt), \\
    \dot{x}_2(t) &= \rho_1 A_1 w_1 \cos (w_1 t + \alpha_1) + \rho \rho B_1 w_2 \cos (w_2 t + \alpha_2) + C_2 p \cos (pt).
\end{align*}
\]

Constants \( A_1, B_1, \alpha_1, \alpha_2 \) are found taking into account initial conditions (movement from a state of relative rest):

\[
\begin{align*}
    x_1(0) = 0, \; \dot{x}_1(0) = 0, \; x_2(0) = 0, \; \dot{x}_2(0) = 0. \quad (15)
\end{align*}
\]

The system of transcendental equations (15) was evaluated numerically using the standard procedure.

3.1. Example

Initial data – geometric and physical parameters of the hand:
\[ L = 3 \text{ m}; \quad L_1 = 2,4542 \text{ m}; \quad L_2 = 0,5458 \text{ m}; \quad m_1 = 3,3877 \text{ kg}; \quad m_2 = 2,7469 \text{ kg}; \quad \omega_1 = 8 \text{ c}^{-1}; \]
\[ \omega_2 = n\omega_1 = 10,62 \text{ c}^{-1}; \quad n = 1,7623; \quad \text{periods of natural oscillations } T_1 = \frac{2\pi}{\omega_1} = 0,7854 \text{ s}; \]
\[ T_2 = \frac{2\pi}{\omega_2} = 0,5916 \text{ s}; \quad \text{required total time of motion } T = T_1 \cdot 3 = 2,3562 \text{ s}; \quad \text{maximum movement of the arm} \quad L_\alpha = 0,2 \text{ m}; \quad p = \frac{\omega_1}{3}. \]

From system (15) found: \( A_1 = -0,03 \), \( B_1 = -0,02 \), \( \alpha_1 = -6,28 \), \( \alpha_1 = 6,32 \).

**Figure 3.** Displacement \( x_1(t) \) and velocity \( \dot{x}_1(t) \) of mass \( m_1 \) in relative motion

**Figure 4.** Displacement \( x_2(t) \) and velocity \( \dot{x}_2(t) \) of mass \( m_2 \) in relative motion

Figures 3 and 4 show graphs \( x_1(t) \), \( \dot{x}_1(t) \), \( x_2(t) \), \( \dot{x}_2(t) \), which indicate the achievement of a state of relative rest at a time \( T = \frac{2\pi}{p} \).

**Figure 5.** Translational motion of the arm

Figure 5 suggests that at the time \( t = T = \frac{2\pi}{p} \) a translational state of rest is achieved. Obviously that the sum of translational and relative state of rest provides absolute state of rest of a manipulator arm while its fast movement for distance \( L_\alpha \) by the time \( T \).

4. **Summary**

The design of a telescopic arm of a manipulator with the consideration of the finite stiffness of the elements, implies that the time of optimal translational motion is consistent with both the period of the first tone of natural oscillations of the hand and the period of the second tone.

If the natural oscillation frequencies of an elastic object differ from each other as whole numbers, this naturally simplifies the choice of a minimum possible time of translational motion, which ensures the reduction of oscillations in all modes at the end of the motion. For the elastic system with N - degrees of freedom, the time of motion is chosen from the common roots of the 2N moment ratios, which constitute a system of transcendental equations.
The task of optimal control of the movement of the telescopic elastic arm when the two operations happen at the same time: the extension of the arm in the radial direction and its translational motion, is still vital. In this case, periods of natural oscillations vary and Coriolis inertia forces appear, which can cause additional hand deformations. Therefore, at present the simplest solution implies that the initial arm length allows reaching the specified end position without combining movements.

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