Application of Adam-BP neural network in leveling fitting

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Abstract. According to the accuracy of BP (Back Propagation) neural network in leveling fitting, an improved BP neural network model is proposed. In order to overcome the defect of the low convergence rate and local minimal in leveling fitting. The BP neural network is improved by using Relu (Rectified Linear Units) function as network activation function and Adam (Adaptive Moment Estimation) algorithm as network optimization function. The experimental results show that the improved BP neural network can effectively improve the accuracy.

1. Preface

Because GPS has the advantages of high positioning accuracy, all-weather, all-time and short measuring period, it plays an increasingly important role in geodetic surveying. Using GPS data for leveling fitting has become a hot issue on engineering measurement research$^{[1]}$. In the vertical datum, the elevation provided by GPS survey is the GPS geodetic height based on WGS-84 reference ellipsoid, the normal height is used in China. The relationship is as shown in Equation 1.

$$\zeta = H - h$$  \hspace{1cm} (1)

Aiming at the leveling fitting methods, many experts at home and abroad have done a lot of research, put forward a variety of models and corresponding algorithms, and achieved some results. Such as direct method, mathematical fitting method, neural network method, etc. Each model has its own advantages and disadvantages$^{[2]}$. The direct method obtains elevation accuracy with high accuracy, but requires high-precision, high-density gravity data and high-resolution terrain data. Mathematical fitting is a simple and fast conversion method, but it is limited by its own model and has low precision. The BP neural network has no model assumptions in GPS elevation conversion$^{[3]}$, which has the characteristics of adaptive characteristics, avoids the error caused by the model hypothesis, and does not need to use high-precision gravity data to adjust the connection parameters in the neural network. By adjusting the connection parameters and weights of the neural network, any non-linear function can be approached with arbitrary accuracy, so that the conversion result obtains higher precision, which provides a new method for leveling fitting.

Adam-bp neural network is proposed to overcome the drawbacks of the slow convergence speed and the local minimum of the back-propagation algorithm. Through experiments, the GPS leveling fitting accuracy of quadratic polynomial fitting method, BP neural network and Adam-BP neural network are compared and analyzed.
2. Quadratic polynomial fitting method

Quadratic polynomial fitting method is often used in leveling fitting \[^4\]. The expression between elevation anomaly and plane coordinates is as follows:

\[
\zeta = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 x y
\]  

(2)

In the formula, \(a_0, a_1, a_2, a_3, a_4, a_5\) are the parameters to be determined.

Therefore, this area requires at least 6 known points. When there are more than 6 known points, the corresponding error equations can be listed:

\[
V_i = a_0 + a_1 x_i + a_2 y_i + a_3 x_i^2 + a_4 y_i^2 + a_5 x_i y_i - \zeta
\]  

(3)

\[
V = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}, \quad A = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}, \quad \zeta = \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \vdots \\ \zeta_n \end{bmatrix}
\]  

(4)

\[
X = \begin{bmatrix} 1 & x_1 & y_1 & x_1^2 & y_1^2 & x_1 y_1 \\ 1 & x_2 & y_2 & x_2^2 & y_2^2 & x_2 y_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_n & y_n & x_n^2 & y_n^2 & x_n y_n \end{bmatrix}
\]  

(5)

The matrix form is: \(V = XA - \zeta\). The solution of the height anomaly of unknown point is to determine the coefficient of polynomial. Under the condition of \(\sum V^2 = \text{min} \). \(A = (X'X)^{-1}X'\zeta\) can be obtained according to the principle of least squares method, and then the height anomaly of any point can be obtained, thus the normal height can be obtained\[^5\].

3. Basic principles and methods

3.1 BP neural network model

BP neural network is a multi-layer feedforward neural network. Each layer is composed of several neurons. Each neuron in each layer is fully connected. There is no connection between neurons in the same layer. BP neural network is supervised learning, which consists of input layer, hidden layer and output layer\[^6\]. The network structure model is shown in Figure 1.

![BP neural network diagram](image)

Fig 1. BP neural network diagram

3.2 BP neural network model

The process of BP neural network calculation is mainly divided into two stages. The specific steps are shown in Figure 2.
Fig 2. BP neural algorithm flow chart

The first stage is the forward propagation of input signal. After input learning samples, the signal reaches the output layer from the input layer to the hidden layer. The second stage is to update the weights and thresholds between input layer, hidden layer and output layer, and constantly revise the parameters, so as to improve the accuracy of network fitting.

4. Improved BP neural network method

4.1 Use Relu function as activation function

In the neural network, the activation function is to convert the input signal received by each neuron into the output signal. In the neural network, the function of activation function is to convert the input signal received by each neuron into the output signal. By introducing the non-linear activation function, the neuron can realize the non-linear output, approximate any non-linear function, and enhance the learning ability of the network. [7]

In BP neural network, sigmoid function is usually used as the activation function. However, in leveling fitting, the accuracy of leveling fitting is not high because of the problems of large amount of calculation, disappearance of gradient and slow convergence speed. The Relu function is a piecewise linear function [8]. When the input is greater than 0, the value is directly output; when the input is less than or equal to 0, the output is 0. This feature is called single-sided suppression. Therefore, the neural network has sparsity and can better mine data related features and fit training data. Because of the fast convergence speed of the neural network, the Relu function is used as the activation function. The formula is as follows:

\[ h(x) = \begin{cases} 
  x & (x > 0) \\
  0 & (x \leq 0) 
\end{cases} \] \hspace{1cm} (6)

The graph of function is shown in Figure 3.
4.2 Use the ADAM algorithm as an optimization function

In the BP neural network, the process of solving the minimum value of the loss function is called optimization. SGD (Stochastic Gradient Descent) is usually used as the optimization function of BP neural network, but it has the defect of the low convergence rate and local minimal [9]. However, there is a disadvantage that the rate of decline is slow and it is easy to fall into the local minimum point, resulting in low accuracy in the level fitting.

Adam is a learning rate adaptive optimization algorithm [10], which can update the weight of neural network iteratively based on training data. Adam has the following advantages:

1) The gradient first moment (exponential weighting) is added to the momentum, and the momentum is applied to the scaled gradient to make the gradient diagonal invariance suitable for solving the problem of high noise and sparse gradient.

2) The Adam algorithm modifies the first moment (both momentum term) initialized from the origin and the non-central second moment estimate. After the offset correction, each iteration learning rate has a certain range, making the parameters relatively stable. It is robust to the selection of hyperparameters and enables efficient searching of parameter spaces.

The specific algorithm steps of Adam are shown in the following table:

1) Setting step size $\varepsilon$, moment estimation attenuation rate $\rho_1$, $\rho_2$, initial parameters $\theta$

2) Updating biased first moment estimate:

$$ s \leftarrow \rho s + (1 - \rho_1)g $$

(7)

3) Updating biased second moment estimate:

$$ r \leftarrow \rho_2 r + (1 - \rho_2)g \odot g $$

(8)

4) Correcting the deviation of the first moment:

$$ \hat{s} \leftarrow \frac{s}{1 - \rho_1} $$

(9)

5) Correcting the deviation of the second moment:

$$ \hat{r} \leftarrow \frac{r}{1 - \rho_2} $$

(10)

6) Calculation update:

$$ \Delta \theta = -\varepsilon \frac{\hat{s}}{\sqrt{\hat{r} + \delta}} $$

(11)

7) Using update:

$$ \theta \leftarrow \theta + \Delta \theta $$

(12)

5. GPS elevation fitting analysis

5.1 Test area control network

Leveling fitting experiment is carried out with a regional GPS control network[11]. Quadratic polynomial fitting method, BP neural network method and Adam-BP neural network method are used
to fit, and the effects and accuracy of different methods are analyzed. The terrain in this area is flat and the area is about 10.2 km\(^2\). 20 GPS points are selected, and the survey area is a third-class leveling survey. Point distribution is shown in Fig. 4:

![GPS point map](image)

Fig 4. GPS point map

The elevation data is shown in Table 1:

| Number | Gps height h/m | Ellipsoidal height H/m | Height anomaly /m |
|--------|----------------|------------------------|-------------------|
| 1      | 13.717         | 22.668                 | 8.951             |
| 2      | 40.254         | 48.835                 | 8.581             |
| 3      | 10.428         | 19.070                 | 8.642             |
| 5      | 35.870         | 44.424                 | 8.554             |
| 7      | 8.037          | 16.629                 | 8.592             |
| 8      | 29.719         | 38.269                 | 8.550             |
| 9      | 64.141         | 72.589                 | 8.448             |
| 10     | 49.725         | 58.104                 | 8.379             |
| 11     | 27.085         | 35.686                 | 8.601             |
| 12     | 49.393         | 58.189                 | 8.796             |
| 14     | 26.230         | 35.012                 | 8.782             |
| 15     | 30.295         | 38.851                 | 8.556             |
| 16     | 35.862         | 44.336                 | 8.474             |
| 18     | 14.882         | 23.472                 | 8.590             |
| 20     | 23.787         | 32.499                 | 8.712             |
| 4      | 29.622         | 38.123                 | 8.501             |
| 6      | 10.648         | 19.268                 | 8.620             |
| 13     | 16.321         | 24.962                 | 8.641             |
| 17     | 16.662         | 25.203                 | 8.541             |
| 19     | 9.572          | 18.148                 | 8.576             |

5.2 Selection of sample points
Points 4, 6, 7, 13 and 19 were selected as test points, and the remaining 15 points were selected as fitting points. Fitting experiments were carried out on three fitting models: quadratic polynomial, BP neural network and Adam-BP, and the fitting results were compared.\[12\]

5.3 Accuracy comparison of different algorithms
1) Calculate the internal accuracy of GPS level fitting by the following formula:

\[ u = \frac{\pm \sqrt{\nu}}{(n-1)} \]  \hspace{1cm} (13)

In the formula, \( \nu \) is the residual and \( n \) is the number of fitting points.
2) Calculate the external accuracy of GPS level fitting by the following formula:

$$m = \pm \sqrt{v/n}$$

In the formula, $v$ is the residual and $n$ is the number of number of checkpoints.

5.4 Accuracy comparison of different algorithms

The fitting results of the three methods are shown in Table 2.

| Method                  | Quadratic polynomial | BP neural network | Adam-BP neural network |
|-------------------------|----------------------|-------------------|------------------------|
| Internal accuracy /m    | 0.023                | 0.018             | 0.008                  |
| External accuracy /m    | 0.031                | 0.025             | 0.009                  |

6. Conclusion

By comparing and analyzing the effects of quadratic polynomial fitting method, BP neural network method and Adam-BP neural network method in leveling fitting, the following conclusions are drawn:

1) In the case where the number of known points is small and the accuracy is not required, a quadratic polynomial fitting method can be used.

2) Because BP neural network has the advantages of nonlinear mapping ability and small model error, BP neural network accuracy is better than quadratic polynomial fitting method.

3) Adam-BP neural network model, adding momentum and adaptive learning rate based on BP neural network model, so the accuracy in GPS leveling fitting is higher than BP neural network, and the convergence speed is faster, which can avoid falling into local minimum.

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