Electromagnetic Shocks in Quantum Vacuum

Hedvika Kadlecová,1 Georg Korn,1 and Sergei V. Bulanov1,2,3

1Institute of Physics of the ASCR, ELI–Beamlines project, Na Slovance 2, 18221, Prague, Czech Republic
2National institutes for Quantum and Radiological Science and Technology (QST), Kansai Photon Science Institute, 8–1–7 Umemidai, Kizugawa, Kyoto 619–0215, Japan
3A. M. Prokhorov Institute of General Physics of RAS, Vavilov Str. 38, Moscow 119991, Russia

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The interaction of two counter-propagating electromagnetic waves in vacuum is analyzed within the framework of the Heisenberg-Euler formalism in quantum electrodynamics. The nonlinear electromagnetic wave in quantum vacuum is characterized by the wave steepening, subsequent generation of higher-order-harmonics, electromagnetic shock wave formation with electron-positron pair generation at the shock wave front.

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In contrast to classical electrodynamics where the electromagnetic waves do not interact in vacuum, in quantum electrodynamics (QED) the photon-photon scattering in vacuum occurs via generation of virtual electron-positron pairs resulting in vacuum polarization, Lamb shift, vacuum birefringence, Coulomb field modification, etc. [1]. Photon-photon scattering was observed in collisions of heavy ions accelerated by standard accelerators of charged particles (see review article [2] and the results of the experiments obtained with the ATLAS detector at the Large Hadron Collider [3]). Further study of the process will allow one to test extensions of the Standard Model, in which new particles can participate in the loop diagrams [4].

In the relatively low photon energy limit, for photon energy below the electron rest-mass energy, \(E_\gamma < m_e c^2\), the total photon-photon scattering cross section for non-polarized photons is proportional to sixth power of the photon energy, \(\sigma_{\gamma-\gamma} = (973/10125 \pi) \alpha^2 r_e^4 (\omega / m_e c^2)^6\), it reaches the maximum at \(\hbar \omega \approx 1.5 m_e c^2\), and decreases proportionally to the inverse of square power of the photon energy for \(\hbar \omega > m_e c^2\) (see Ref. [1]). Here \(\alpha = e^2 / \hbar c \approx 1/137\) is the fine structure constant, \(r_e = e^2 / m_e c^2\) is the classical electron radius, \(c\) and \(\hbar\) are the electron charge and mass, \(c\) is speed of light in vacuum, and \(\h\) is the reduced Planck constant. To find the number of photon-photon scattering events in the low frequency limit we estimate the number of photons in the electromagnetic pulse with the amplitude \(E\) in the \(\lambda^3\) volume, where \(\lambda = 2 \pi c / \omega\) is the electromagnetic wave wavelength. It is equal to \(N_\gamma = E^2 \lambda^3 / 4 \pi \hbar \omega\). Using these relationships it is easy to find the number of scattering events per 4-volume \(2 \pi \lambda^3 / \omega\). It reads

\[
N_{\gamma-\gamma} = \sigma_{\gamma-\gamma} N_\gamma^2 \lambda^2 = \frac{973}{10125 \pi} \alpha^2 \left( \frac{E}{E_S} \right)^4,
\]

where \(E_S = m_e^2 c^3 / e \hbar\) is the critical field of quantum electrodynamics. Corresponding electromagnetic radiation intensity equals \(I_S = c E_S^2 / 4 \pi \approx 10^{29} \text{W/cm}^2\). As we see, the number of scatterings does not depend on the electromagnetic wave frequency. It is determined by wave intensity \(I = c E^2 / 4 \pi\) as \(N_{\gamma-\gamma} \propto \alpha^2 (I / I_S)^2\).

The growing availability of high power lasers rises up the interest towards the experimental observation and motivates theoretical studying of such process in the laser-laser scattering [5–12], in the scattering of the XFEL emitted photons [4], and in the interaction of relatively long wavelength high intensity laser light with short wavelength x-ray photons [13]. In the limit of extremely high amplitude of the electromagnetic field with the strength approaching the QED critical field \(E_S\), nonlinear modification of the vacuum refraction index via the polarization of virtual electron-positron pairs supports the electromagnetic wave self-interaction. This process results in decreasing the velocity of counterpropagating electromagnetic waves [14–15]. The co-propagating waves do not change their propagation velocity because the co-propagating photons do not interact, e. g. see Ref. [16]. The finite amplitude wave interaction is demonstrated by the high order harmonics generation [17–20].

Nonlinear properties of the QED vacuum in the long wavelength and low frequency approximation are described by the Heisenberg-Euler Lagrangian [21–24]. They correspond to nonlinear electrodynamics in dispersionless media. In nonlinear media with the refraction index depending on the electromagnetic field the electromagnetic wave can evolve into the configuration with singularities [22]. Singular patterns of the electromagnetic field are discussed also in Ref. [23]. The process of the finite amplitude wave evolution is accompanied by the wave front steepening and formation of shock-like waves, i. e. it is characterized by the processes leading to gradient catastrophes. Appearance of singularities in the Heisenberg-Euler electrodynamics has been noticed in Ref. [24] and indicated in computer simulations presented in Ref. [26]. In this letter, we present and analyze an analytical solution of the Heisenberg–Euler electrodynamics equations describing the finite amplitude electromagnetic...
wave counter-propagating to the crossed electromagnetic field. This configuration corresponds to the collision of the short and long wavelength electromagnetic pulses. It can be considered as a model of interaction of the high intensity laser pulse with the x-ray pulse generated by XFEL.

The Heisenberg–Euler Lagrangian is given by

\[ \mathcal{L} = \mathcal{L}_0 + \mathcal{L}'. \]

Here \( \mathcal{L}_0 = -(1/16\pi)F_{\mu\nu}F^{\mu\nu} \) is the Lagrangian in classical electrodynamics, \( F_{\mu\nu} \) is the electromagnetic field tensor, and \( \mu, \nu \) being the 4-vector of electromagnetic field, and \( \mu = 0,1,2,3 \). In the Heisenberg–Euler theory, the radiation corrections are described by \( \mathcal{L}' \) in the r.h.s. of Eq. (2), which in the weak field approximation is given by \[ \mathcal{L}' = \frac{k}{4} \left\{ (F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4} (F_{\mu\nu}F^{\mu\nu})^2 + \frac{90}{315} (F_{\mu\nu}F^{\mu\nu}) (F_{\mu\nu}F^{\mu\nu})^2 + \left( \frac{13}{16} (F_{\mu\nu}F^{\mu\nu})^2 \right) \right\} \]

where \( k = e^4/360\pi^2 m^4 \). Here and below we use units \( c = \hbar = 1 \). In the Lagrangian (3) the first two terms in the r.h.s. describe four interacting photons and the last to terms correspond to six photon interaction.

For the sake of brevity we consider counter-propagating electromagnetic waves of the same polarization. They are given by the vector potential having one component, \( \mathbf{A} = \mathbf{A}_e \) with \( \mathbf{e}_z \) being a unit vector along the \( z \) axis. We assume that in the light cone coordinates, \( x_+ = (x+t)/\sqrt{2} \) and \( x_- = (x-t)/\sqrt{2} \), the vector potential equals \( \mathbf{A} = W x_+ + a(x_+, x_-) \). The term \( W x_+ \) describes the equal amplitude, \( E_0 = B_0 = -W/\sqrt{2} \), crossed electric and magnetic fields, whose Pointing vector is antiparallel to the \( x \) axis: \( \mathbf{e}_x \) is a unit vector along the \( x \)-axis. In this case, the Lagrangian (2) takes the form

\[ \mathcal{L} = -\frac{1}{4\pi} [(W + w)u - \epsilon_2(W + w)^2 u^2 - \epsilon_3(W + w)^3 u^3] \]

with the functions \( u = \partial_{x_-} a \) and \( w = \partial_{x_+} a \). The electromagnetic field is normalized on the critical QED field, \( E_S = m^2/e \). The dimensionless parameters \( \epsilon_2 \) and \( \epsilon_3 \) in Eq. (4) are equal to \( 2e^2/45\pi = (2/45\pi)\alpha \) and \( 32e^2/315\pi \alpha \). Here \( \alpha = \epsilon^2/\hbar c \approx 1/137 \) is the fine structure constant, i.e. \( \epsilon_2 \approx 10^{-4} \) and \( \epsilon_3 \approx 2 \times 10^{-4} \).

The field equations can be found from the Lagrangian variation: \( \partial_{x_+} (\partial \mathcal{L}/\partial u) + \partial_{x_-} (\partial \mathcal{L}/\partial w) = 0 \). As a result we obtain the system of equations

\[ \partial_{x_+} w - \partial_{x_-} u = 0, \]

\[ [1 - 4\epsilon_2(W + w)u - 9\epsilon_3u^2(W + w)^2]\partial_{x_+} u - [\epsilon_2(W + w)^2 + 3\epsilon_3u(W + w) + \partial_{x_-} u] \]

\[ -[\epsilon_2 u^2 + 3\epsilon_3u(W + w)]\partial_{x_-} w = 0, \]

where the first equation comes from the equality of mixed partial derivatives: \( \partial_{x_+} x a = \partial_{x_-} x a \).

The equations (5) have a solution for which \( u = 0 \) and \( \partial_{x_+} w = 0 \), i.e. \( w \) is an arbitrary function depending on the variable \( x_+ \). This is a finite amplitude electromagnetic wave propagating from the right to the left with the propagation velocity equal to speed of light in vacuum. Its form does not change in time. The electric and magnetic field components are equal to each other \( E = B = -w/\sqrt{2} \), i.e. its a superposition with the crossed electromagnetic field, \( E = B = -1/\sqrt{2}(W + w) \).

Linearizing Eqs. (5, 6), it is easy to find expressions describing the small amplitude wave

\[ u(x_+, x_-) = u_0(x_- + \epsilon_2 W^2 x_+) \]

and

\[ w(x_+, x_-) = \epsilon_2 W^2 u_0(x_- + \epsilon_2 W^2 x_+) + w_0(x_+). \]

In Eqs. (7, 8) the functions \( u_0 \) and \( w_0 \) are determined by the initial conditions. The function \( u(x_+, x_-) \) depends on the light-cone coordinates \( (x_+, x_-) \) in combination

\[ \psi(x_+, x_-) = x_- + \epsilon_2 W^2 x_+. \]

The wave phase \( \psi \) can be rewritten as

\[ \psi(x, t) = |x(1 + \epsilon_2 W^2) - t(1 - \epsilon_2 W^2)|/\sqrt{2}. \]

The constant phase condition shows that the wave propagates from the left to the right with the speed,

\[ v_W = 1 - \epsilon_2 W^2 \approx 1 - 2\epsilon_2 W^2 + 2\epsilon_3^2 W^4, \]

which is less than unity, i.e. the wave phase (group) velocity is below speed of light in vacuum (see also Refs. 6 [13-15] and literature cited therein).

To analyze the nonlinear wave evolution we seek self-similar solution to Eqs. (5, 6) of a simple wave (e.g. see [29]), in which \( w \) can be considered as a function of \( u \) \( w(u) \). With this assumption we obtain from Eqs. (6, 7), the system of equations

\[ \partial_{x_+} u = J\partial_{x_-} u, \]

\[ \partial_{x_-} u = (W + w)^2 (\epsilon_2 + 3\epsilon_3(W + w)u)\partial_{x_-} u \]

\[ -\frac{1}{1 - (W + w)u}[4\epsilon_2 + 9\epsilon_3u(W + w) + 3\epsilon_3u^3J] - \epsilon_2 u^2J^2 \]

where \( J = dw/du \). Equations (12) and (13) are consistent provided the coefficients in front of \( \partial_{x_-} u \) in the right hand sides are equal to each other. This condition yields equation for \( J \). It reads

\[ \epsilon_2 u^2 + 3\epsilon_3(W + w)^3 J^2 - [1 - 4\epsilon_2(W + w)u - 9\epsilon_3u(W + w)^2] J + (W + w)^2[\epsilon_2 + 3\epsilon_3u(W + w)] = 0. \]
Using smallness of the parameters $\epsilon_2$ and $\epsilon_3$ and a relationship $w \approx \epsilon_2 W^2$, which follows from Eq. (8), we obtain expression for the function $J(u)$ in the form of the power series:

$$J(u) = \epsilon_2 W^2 + 4\epsilon_2^2 W^3 u + 3\epsilon_3 W^3 u + \ldots$$  \hspace{1cm} (15)

It gives for the function $w(u)$:

$$w(u) = \epsilon_2 W^2 u + 2\epsilon_2 W^3 u^2 + \frac{3}{2} \epsilon_3 W^3 u^2 + \ldots$$  \hspace{1cm} (16)

As a result we find the electric and magnetic field components in the electromagnetic wave propagating from the left to the right

$$E = (w-u)/\sqrt{2} \approx -\sqrt{2}(1-\epsilon_2 W^2)$$  \hspace{1cm} (17)

and

$$B = (u+w)/\sqrt{2} \approx \sqrt{2}(1+\epsilon_2 W^2)$$  \hspace{1cm} (18)

respectively.

Substitution of this expression to the r.h.s of Eq. (12) results in

$$\partial_x u - [\epsilon_2 W^2 + 4\epsilon_2^2 W^3 + 3\epsilon_3 W^3 u] \partial_x u = 0.$$  \hspace{1cm} (19)

In the variables $x, t$ it can be written for the function

$$\ddot{u} = -2(4\epsilon_2^2 + 3\epsilon_3) W^3 u$$  \hspace{1cm} (20)

as

$$\partial_t \ddot{u} + (v_W + \ddot{u}) \partial_x \ddot{u} = 0.$$  \hspace{1cm} (21)

with the velocity of linear wave, $v_W$, given by Eq. (11).

Solution to this equation can be obtained in a standard way (see Ref. [26]). According to this solution the function $\ddot{u}(x,t)$ transfers along the characteristic $x_0$ without distortion: $\ddot{u} = \ddot{u}_0(x_0)$. The characteristic equation for Eq. (21) is

$$\frac{dx}{dt} = v_W + \ddot{u}$$  \hspace{1cm} (22)

with the solution

$$x = x_0 + (v_W + \ddot{u}_0(x_0))t.$$  \hspace{1cm} (23)

Combining these relationships we obtain the solution to Eq. (21) in the implicate form, where the function $u(x,t)$ should be found from equation

$$\ddot{u} = \ddot{u}_0 \left(x - (v_W + \ddot{u})t\right).$$  \hspace{1cm} (24)

In particular, this expression describes the high order harmonics generation, wave steepening and formation of the electromagnetic shock wave in vacuum.

Various mechanisms for generation of the high order harmonics in QED vacuum are analyzed in Refs. [17, 20].

In particular, the parametric wave interaction process have been considered [17] and the “relativistic oscillating mirror” concept (for details see [27, 28]) have been applied in [20]. Here we formulate perhaps one of the most simple mechanisms. To obtain the scaling of the high order harmonics generation within the framework of this mechanism we choose the initial electromagnetic wave as

$$\ddot{u}_0 = \ddot{a}_1 \cos(k(x-v_W t)),$$  \hspace{1cm} (25)

where $\ddot{a}_1$ and $k = \omega/c$ are the wave amplitude and wavenumber ($\omega$ is the wave frequency), respectively. Using the weakness of nonlinearity ($\ddot{a}_1 \ll 1$) we obtain from Eqs. (24, 25) that

$$\ddot{u}(x,t) = \ddot{a}_1 \cos(k(x-v_W t)) - (\ddot{a}_1^2/2) k \sin(2k(x-v_W t)) \ldots$$  \hspace{1cm} (26)

Taking into account the normalization of the wave amplitude given by Eq. (20) we can find that the ratio of the second harmonic amplitude to the amplitude of the wave with fundamental frequency scales as $(2\epsilon_2^2 W^3 + 3\epsilon_3 W^3)/\ddot{a}_1 k t$. It is proportional to the duration of the electromagnetic wave interaction.

From expression (21) it follows that the electromagnetic field gradient grows with time, i.e. the wave steepening occurs. Differentiating $u(x,t)$ with respect to the coordinate $x$ we obtain

$$\partial_x u = \frac{\partial_x u_0(x_0)}{1 - 2(4\epsilon_2^2 + 3\epsilon_3) W^3 \partial_x u_0(x_0) t},$$  \hspace{1cm} (27)

where the dependence of the Lagrange coordinate $x_0$ on time and on the Euler coordinate $x$ is given by Eq. (23). As seen, the gradient $\partial_x u$ becomes infinite at time

$$t_{br} = \frac{1}{2(4\epsilon_2^2 + 3\epsilon_3) W^3 \partial_x u_0(x_0)}$$  \hspace{1cm} (28)

and at the coordinate $x_0$ where the derivative $\partial_x u_0(x_0)$ has the maximum. This singularity is called “the gradient catastrophe” or/and “the wave breaking”.

The singularity formation during the evolution of a finite amplitude electromagnetic wave in quantum vacuum is illustrated in Figs. 1 and 2. The electromagnetic pulse at $t = 0$ has a form

$$u_0(x_0) = a_0 \exp(-x_0^2/2L^2) \cos(kx_0)$$  \hspace{1cm} (29)

with $L = 4\pi$ and $k = 2$. The parameter $4\epsilon_2^2 W^3 + 3\epsilon_3 W^3$ is assumed to be equal to 0.125 and $a_0 = 1$. As it is clearly seen in Fig. 1 the wave steepens evolves with time. The wave breaking occurs due to the characteristic intersection as it is shown in Fig. 2.

We note that the singularity formed at the electromagnetic wave breaking has a character of the rarefaction like shock wave (the wave steepens and breaks in the backward direction, as shown in Fig. 1) because the wave crest
FIG. 2: Characteristics of Eq. (24) plotted in the plane given by expression (29).

FIG. 1: The function $u(x,t)$ given by Eq. (29) for $u_0(x_0)$ given by expression (29).

propagates with the speed lower that propagation speed of the part of the pulse with lower amplitude.

When wave approaches the wave breaking point the wave steepening is equivalent to the harmonics generation with the higher and high numbers. Due to this fact the above used long-wavelength approximation becomes unapplicable and the Heisenberg-Euler Lagrangian cannot be used for the wave evolution description in the vicinity of the gradient catastrophe. The long-wavelength approximation breaks when the frequencies of the interacting waves, $\omega$, and $\Omega$ become high enough, i.e. when their product becomes of the order of or higher than $\omega, \Omega > m_e^2 c^4/\hbar^2$. At this photon energy level the photon-photon interaction can result in the creation of real electron-positron pairs in the Breit-Wheeler process [29], in the saturation of the wave steepening, and in the electromagnetic shock wave formation. Here $\omega$ and $\Omega$ are the frequencies of high energy photons and of low frequency counter-propagating electromagnetic wave, respectively.

The electromagnetic shock wave separates two regions, I and II, where the function $u(x,t)$ takes the values $u_I$ and $u_{II}$, respectively. The shock wave front (it is an interface between the regions, I and II) moves with the velocity equal to $v_{sw}$, in other words the shock wave front is localized at the position $x_{sw} = v_{sw} t$. Integrating Eq. (21) over infinitely small interval $(-\delta + x_{sw}, x_{sw} + \delta)$ with $\delta \to 0$ we obtain that

$$\{-v_{sw} u + v_W u - (4e_2^2 + 3e_3)W^3 u^2\}_{x=x_{sw}} = 0.$$  \hspace{1cm} (30)

Here $\{f\}_x = f(x + \delta) - f(x - \delta)$ at $\delta \to 0$ denotes the discontinuity of the function $f(x)$ at the point $x$. From Eq. (30) it follows that

$$v_{sw} = v_W - (4e_2^2 + 3e_3)W^3(u_I + u_{II}).$$  \hspace{1cm} (31)

Near threshold $\omega \Omega > m_e^2 c^4/\hbar^2$, the electron-positron creation cross section equals [1, 30]

$$\sigma_{e-p} = \pi v_e^2 \sqrt{\frac{\hbar^2 \omega \Omega}{m_e^2 c^2} - 1}.$$  \hspace{1cm} (32)

The width of the shock wave front can be estimated by of the order of the length $l_{sw} = 1/n_r \sigma_{e-p}$, over which the photon with the energy of the order of $m_e c^2/\hbar$ creates the electron positron pair. The photon density $n_r$ is related to the electromagnetic pulse energy $E_{em}$ as $n_r \approx E_{em}/\hbar \omega \lambda^3 N_{em}$, where $N_{em} = l_{em}/\lambda$ is the electromagnetic pulse length devided by the wavelength. It yields $l_{sw} \approx \hbar \omega \lambda^3 N_{em}/\pi v_e^2 E_{em}$. Since it is assumed that the photon energy is at the threshold of the electron-positron pair creation, the shock wave front width should be of the order of the Compton wavelength, $\lambda_C = \hbar/m_e c$. This condition imposes a constraint from below on the electromagnetic pulse energy. It reads $E_{em} \geq m_e c^2(\lambda/\sigma_r)^2$. For 1\mu m wavelength laser with $N_{em} = 10$ it requires $E_{em} \geq 100$kJ. If $\lambda = 10^{-6} cm^2$, which corresponds to 10 KeV X-ray pulse, we have $E_{em} \geq 10^{-4}J$. The electron-positron pairs created at the electromagnetic shock front being accelerated by the electromagnetic wave emit gamma-ray photons leading to the electron-positron avalanche via the multi-photon Breit-Wheeler mechanism [31] as discussed in Refs. [22, 33] (see also review article [7] and the literature cited therein).

In conclusion, the photon-photon scattering in vacuum is governed by the dimensionless parameter $\alpha (l_{em}/l_S)$ as it is concerning the shock-like configuration formation, high order harmonics generation and the electron-positron and gamma ray flush at the electromagnetic shock wave front. Observation of these phenomena in the high power laser or x-ray interaction with matter implies a high precision measurements as in the experiments [2, 3] or/and achieving the electromagnetic field amplitude approaching the critical QED field $E_S$. One of the ways to reach these regimes is to increase the laser
power. For example observation of one scattered photon per day with 1 Hz laser requires the intensity of the order of $8 \times 10^{27}$ W/cm$^2$, i.e. several hundred kJ laser energy. Another way towards approaching the critical QED field limit is connected to the relativistic flying mirror concept \cite{11} (for relativistic flying mirror theory and experiment see Refs. \cite{28,34,35,36}), where the light intensity can be increased in the nonlinear laser-plasma interaction.

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