Thickness Measurement of Surface Attachment on Plate with Lamb Wave

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Abstract. Aiming at the thickness detection of the plate surface attachment, a nondestructive testing method based on the Lamb wave is presented. This method utilizes Lamb wave propagation characteristics of signals in a bi-layer medium to measure the surface attachment plate thickness. Propagation of Lamb wave in bi-layer elastic is modeled and analyzed. The two-dimensional simulation model of electromagnetic ultrasonic plate - scale is established. The simulation is conducted by software COMSOL for simulation analysis under different boiler scale thickness wave form curve. Through this study, the thickness of the attached material can be judged by analyzing the characteristics of the received signal when the thickness of the surface of the plate is measured.

1. Introduction
In the process of industrial production, there are many circumstances of surface attachments on the plates, such as the sediments in the bottom of the transport tank, the ice on the surface of the aircraft wing and the fouling in the industrial boiler, etc. It is very essential to detect these surface attachment on the plate for the sake of the safe operation of the structures.

In the case of industrial boiler scaling, the calcium and magnesium salts with low solubility will adhere on the inner surface of the furnace and the pipes when the concentration of impurities become saturated after the continuous evaporation or compression of the water. The thermal conductivity of the scales is different, which depends on their chemical composition as well as the existing state [1]. For example, a scale with more porous is very likely to be less thermally conductive than the denser one. Because of the poor thermal conductivities (only one-tenth or even a few percent of that of the steel, and the thermal conductivity of steel is 46.5 ~ 58.2w/m.k), the scales formed by heating would inevitably affect the thermal conductivity of the generating surfaces and lead to the overheating of the metal tube wall. Generally, overheating can cause metal creep, and reduce its strength. Under the situation of high pressure, overheating can also result in the swelling, perforation and rupture of the heat-absorbing surface, directly influence the safety of boiler operation [2-3]. Therefore, it is an effective means to improve the safety and reliability of the structure operation by taking the necessary precautionary measures to carry out the detection and analysis of the boiler fouling and the surface attachments of such plates.

For the measurement of boiler scaling, off-line detection is one of the most widely used method in China, which needs to shut down the system and intercepts pipe samples then dissolves the boiler...
scaling. Subsequently, the information about boiler scaling thickness can be obtained by measuring its weight per unit area. Due to the rapid development of nondestructive testing, a newly common method is the pulse echo method, which can measure the thickness by calculating the time of furnace wall echo and fouling interface echo using high frequency ultrasound [4]. In addition, based on the pulse thermal imaging method, SANJAY et al. carried out the modeling, simulation and experimental testing of the fouling in the boiler tube [5]. The environmental adaptability of these new methods is poor. In recent years, guided wave has been extensively used in the nondestructive testing of layered structures [6, 7], which is based on the propagation characteristics of the guided wave in the layered structures [8]. Massereyd B, et al. used high frequency ultrasonic guided wave to detect the hidden scars of multi-layer structures of spacecraft[9], which proved that the use of high-frequency ultrasonic guided wave detection can avoid the occurrence of interference while detecting tiny cracks.

Plate structure, a kind of waveguide, is widely used in aerospace, land transport and maritime transport, etc. The Lamb wave is very sensitive to the change of material in the thickness measurement of the sheet [10]. For the isotropic materials, to get the contact position, using the Lamb wave auto-positioning method is adopted [11].

In this paper, the dispersion properties of the Lamb wave are utilized to detect the thickness of the surface of the plate. The wave equation of the double elastic structure is established at first, then the variation law of the Lamb wave in the aluminum-scale model is developed. Eventually, Simulation and analysis of aluminum-scale model is completed by simulation software COMSOL.

2. Wave equation of Lamb in a bi-layer elastic medium

For the sake of simplicity, this paper establishes the wave equation of Lamb wave in double elastic medium by establishing aluminum plate and scale bi-layer structure model. As shown in Fig. 1, h1 is the thickness of the aluminum plate and h2 is the thickness of the fouling.

![Figure 1. Aluminum plate and scale bi-layer structure model.](image)

Displacement field within each layer should satisfy the Navier displacement equations of motion, which can be expressed as:

\[
\mu^n \nabla^2 u^n + (\lambda^n + \mu^n) \nabla (\nabla \cdot u^n) = \rho^n \frac{\partial^2 u^n}{\partial t^2} \quad (n=1, 2)
\]  

(1)

Where \(n\) is the type of material (\(n = 1\) for aluminum, \(n = 2\) for scale), \(u\) is displacement, \(\rho\) is density and \(\lambda, \mu\) are the Lame constant of the material.

The generalized displacement field is decomposed by Helmholtz decomposition:

\[
u^n = \nabla \phi^n + \nabla \times \psi^n
\]  

(2)

For Lamb waves, \(u_x^n = 0, \ \psi_y^n = \psi_z^n = 0, \ \psi_x^n = \psi_x^n(y, z)\) and \(\phi^n = \phi^n(y, z)\), denote \(\psi_x^n\) to \(\psi^n\).

Two uncoupled wave equations can be listed as:
In formula (3), \( \nabla^2 \phi^n = \frac{\partial^2 \phi^n}{\partial t^2} - \frac{1}{(c_l^n)^2} \frac{\partial^2 \phi^n}{\partial x^2} = 0 \)

\( \nabla^2 \psi^n = \frac{\partial^2 \psi^n}{\partial t^2} - \frac{1}{(c_s^n)^2} \frac{\partial^2 \psi^n}{\partial x^2} = 0 \)

In formula (3), \( \nabla^2 = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \), Because \( \frac{\partial^2}{\partial y^2} = 0 \), \( c_l^n \) and \( c_s^n \) represents the velocity of the n-layer of longitudinal wave and shear wave, respectively. Therefore, the general solution of (3) can be given by:

\[
\phi^n = C_1^n e^{ik_L^n [z \sin(\theta_L^n) - y \cos(\theta_L^n)]} + C_2^n e^{ik_L^n [z \sin(\theta_L^n + y \cos(\theta_L^n)]} + C_3^n e^{ik_T^n [z \sin(\theta_T^n - y \cos(\theta_T^n)]} + C_4^n e^{ik_T^n [z \sin(\theta_T^n + y \cos(\theta_T^n)]}
\]

\[\psi^n = C_1^n e^{ik_L^n [z \sin(\theta_L^n) - y \cos(\theta_L^n)]} + C_2^n e^{ik_L^n [z \sin(\theta_L^n + y \cos(\theta_L^n)]} + C_3^n e^{ik_T^n [z \sin(\theta_T^n - y \cos(\theta_T^n)]} + C_4^n e^{ik_T^n [z \sin(\theta_T^n + y \cos(\theta_T^n)]}
\]

In which \( k_L^n = \frac{\omega}{c_L^n} \), \( k_T^n = \frac{\omega}{c_T^n} \)

and \( C_1^n \), \( C_2^n \), \( C_3^n \) and \( C_4^n \) are constants.

The general solution on behalf of the potential function into the formula (2), then there are n-layer displacement field:

\[
u_y^n = ik_L^n \cos(\theta_L^n) \{ C_1^n e^{ik_L^n [z \sin(\theta_L^n) + y \cos(\theta_L^n)]} - C_2^n e^{ik_L^n [z \sin(\theta_L^n) - y \cos(\theta_L^n)]} \} + ik_T^n \sin(\theta_T^n) \{ C_3^n e^{ik_T^n [z \sin(\theta_T^n) + y \cos(\theta_T^n)]} + C_4^n e^{ik_T^n [z \sin(\theta_T^n) - y \cos(\theta_T^n)]} \}
\]

\[
u_z^n = ik_L^n \sin(\theta_L^n) \{ C_1^n e^{ik_L^n [z \sin(\theta_L^n) - y \cos(\theta_L^n)]} + C_2^n e^{ik_L^n [z \sin(\theta_L^n) + y \cos(\theta_L^n)]} \} + ik_T^n \cos(\theta_T^n) \{ C_3^n e^{ik_T^n [z \sin(\theta_T^n) + y \cos(\theta_T^n)]} - C_4^n e^{ik_T^n [z \sin(\theta_T^n) - y \cos(\theta_T^n)]} \}
\]

Where the variable field for each layer can be written as:

\[
\varepsilon_{ij}^n = \frac{1}{2} \left( \frac{\partial u_i^n}{\partial x_j} + \frac{\partial u_j^n}{\partial x_i} \right)
\]

We find \( u_x \equiv 0, u_y \) and \( u_z \) is irrelevant with \( x \), then \( \varepsilon_{xx} \equiv 0, \varepsilon_{xy} \equiv 0, \varepsilon_{xz} \equiv 0 \).

In the stress calculation, the strain invariant of the n-th layer can be expressed as
\[ \Delta^n = \nabla \cdot u^n \]  \hspace{1cm} (9)

Substituting \( u^n \) into equation (9), then becomes
\[ \Delta^n = \nabla \cdot (\nabla \phi^n + \nabla \times \psi^n) = \nabla^2 \phi^n \]  \hspace{1cm} (10)

Substituting \( \phi^n \) into Equation (3), we obtain
\[ \Delta^n = -(k_n^n)^2 \phi^n \]
\[ = -(k_2^n)^2 \left\{ C_1^n e^{i \theta_2^n [z \sin(\theta_2^n) + y \cos(\theta_2^n)]} + C_2^n e^{i \theta_2^n [z \sin(\theta_2^n) - y \cos(\theta_2^n)]} \right\} \]  \hspace{1cm} (11)

Apply Hooke's Law:
\[ \sigma_{ij}^n = \lambda^n \Delta^n \delta_{ij} + 2 \mu^n \epsilon_{ij}^n \]  \hspace{1cm} (12)

Where \( \delta_{ij} \) is the Kronecker tensor mark.

The upper surface of the aluminum plate is a free surface with a stress vector of zero, so, we get
\[ \sigma_{yy}^1 = \sigma_{yz}^1 = 0, \hspace{0.5cm} y=0 \]  \hspace{1cm} (13)

The lower surface of the fouling is also a free surface. The stress vector is zero, therefore, It can be expressed as
\[ \sigma_{yy}^2 = \sigma_{yz}^2 = 0, \hspace{0.5cm} y=h_1+h_2 \]  \hspace{1cm} (14)

Each interface requires stress and displacement of the normal and tangential components are continuous, then
\[ u_{y}^1 \big|_{y}, u_{z}^1 \big|_{y}, \sigma_{yy}^1 = \sigma_{yz}^1, \sigma_{yy}^2 = \sigma_{yz}^2, \hspace{0.5cm} y=h_1 \]  \hspace{1cm} (15)

The equations of the eight equations are represented by a matrix:
\[
\begin{bmatrix}
A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} & A_{18} \\
A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} & A_{27} & A_{28} \\
A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} & A_{37} & A_{38} \\
A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} & A_{47} & A_{48} \\
A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} & A_{57} & A_{58} \\
A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} & A_{67} & A_{68} \\
A_{71} & A_{72} & A_{73} & A_{74} & A_{75} & A_{76} & A_{77} & A_{78} \\
A_{81} & A_{82} & A_{83} & A_{84} & A_{85} & A_{86} & A_{87} & A_{88}
\end{bmatrix}
\begin{bmatrix}
C_{11} \\
C_{21} \\
C_{31} \\
C_{41} \\
C_{51} \\
C_{61} \\
C_{71} \\
C_{81}
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]  \hspace{1cm} (16)

The matrix of the coefficient matrix [A] is zero to obtain the dispersion equation of the aluminum-scale double layer structure, that is:
\[
\left| A(\omega, k, \lambda^n, \mu^n, h_0) \right| = 0
\]

(17)

3. Finite element simulation and analysis

In this paper, COMSOL simulation software is used to simulate the aluminum-scale model. The 3D structure can be simulated by applying the appropriate boundary conditions to the 2D model. As shown in Fig. 2, a two-dimensional simulation model of EMAT (Electromagnetic Acoustic Transducer) is established, in which the length of the aluminum and fouling is 450mm, the thickness of aluminum is 2mm and the thickness of the fouling thickness is range from 0.5mm to 3mm. After the constructing of the model, the dielectric parameters must be set. The other parameters of the aluminum plate and scale are shown in Table 1.

| Parameter | \( C_L \) m·s\(^{-1} \) | \( C_T \) m·s\(^{-1} \) | \( \rho \) kg·m\(^{-3} \) | \( \varepsilon \) GN·m\(^{-2} \) | \( V \) |
|-----------|----------------|----------------|---------------|----------------|---|
| Aluminum  | 5092           | 3122           | 2700          | 70             | 0.33 |
| Scaling   | 5600           | 2700           | 2693          | 30             | 0.25 |

Figure 2. 2D simulation model of Aluminum – fouling and EMAT.

As shown in Figure 2, material 1 is an aluminum plate and material 2 is fouling. The alternating current pass through the coil (the odd-numbered coil applies the current \( I \), and the even-numbered coil applies the current-\( I \)) to impose a body load to the aluminum plate. The static bias magnetic field is applied in the y direction, and \( B_y = -B_0 \). The excitation signal is five peak pitch pulse signal.

The frequency of the excitation current in this simulation is 300 kHz. After the simulation was completed, the data needs to be collected for analysis. The optimal acquisition point was selected to describe the corresponding situation after excitation current had input. The optimal collection points were A (150, -0.5), as shown in Figure 3. In this case, Fig. 4 to 7 shows the relationship between displacement Y of the waveform of the point A and the time.
Figure 3. Schematically illustrating the position of the receiver and transmitter, and the propagation path of the wave guide.

Figure 4. In the case of the boiler scaling thickness $h_2 = 1$, the curve on the displacement $Y$ of the waveform of the point A and time.

Figure 5. In the case of the boiler scaling thickness $h_2 = 0.5$. 
Figure 6. In the case of the boiler scaling thickness $h_2 = 2$.

Figure 7. In the case of the boiler scaling thickness $h_2 = 2.5$.

Figure 8. Variation of wave group amplitude with boiler scaling thickness $h_2$. 
Figure 8 shows the changes of the wave amplitude as a function of the fouling thickness. Two signal wave groups are selected for analysis. Signal 1 is a direct wave signal propagating to the right direction after excitation, signal 2 is a left boundary reflected wave signal. The amplitude of the wave group decreases as the fouling thickness increases. In general, with the increase in the scale thickness, Lamb wave energy will be consumed faster, the magnitude of collection points will become increasingly smaller.

4. Summary
In this paper, the thickness of the surface of the plate is measured by the dispersion characteristics of the Lamb wave. Also, the wave equation of the double-layer structure of aluminum plate and scale is obtained by establishing the model of aluminum-scale double-layer elastic medium. Furthermore, the two-dimensional simulation model of electromagnetic ultrasonic plate-scale was established and Simulation analysis was carried out by COMSOL simulation software. The simulation results illustrate that the Lamb wave energy consumes faster as the fouling thickness increases and the amplitude of the signal wave group decreases.

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