The rapid-turn inflationary attractor

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We prove the existence of a general class of rapidly turning two-field inflationary attractors. By requiring a large, slowly varying turn rate and the existence of a conserved adiabatic mode, we solve the system completely without specifying any metric or potential, and prove the linear stability of the solution. Several recently studied turning inflation models, including hyperinflation, side-tracked inflation, and a flat field-space model, turn out to be examples of this general class of attractor solutions. Very rapidly turning models are of particular interest since they can be compatible with the swampland conjectures, and we show that the solutions further simplify in this limit.

Introduction – There has been much interest recently in multifield models of inflation with strongly non-geodesic motion [1–20]. Such models can be realised in potentials that are far too steep for slow-roll, and have furthermore been shown to arise as natural attractor solutions in hyperbolic field-spaces [1–10], although they are by no means restricted to such geometries. In this paper we show that two-field models of inflation with a high turn rate can collectively be described by a single attractor solution, and that several notable examples of strongly turning inflation models are in fact examples of this attractor.

When the dimensionless turning rate \( \omega \) satisfies \( \omega \gg O(\epsilon) \), the equations of motion for the two fields drastically simplify, and one may derive a very general solution to the equations of motion, under mild assumptions which will be outlined below. Examples of rapid-turn attractors include hyperinflation, side-tracked inflation, angular inflation, and the flat field-space turning inflation model of [16], a diverse collection of models which demonstrates the universality of the solution.

To find this attractor solution, we look for a general solution to the background and linear perturbation equations of motion with the following properties: (i) The rotation rate is large: \( \omega \gg O(\epsilon) \). (ii) The rotation rate varies slowly: \( \nu \equiv D_N \ln \omega \ll 1 \), which corresponds to an “adiabaticity condition” [17]. (iii) On superhorizon scales, we require the existence of a mode with a conserved curvature perturbation \( \zeta \). Note that we do not assume that the curvature perturbation generated by quantum fluctuations is constant on superhorizon scales; we only assume that a constant \( \zeta \) solution exists.

As we shall see, these conditions, together with \( \epsilon \) and \( \eta \) being small, will allow us to completely solve the background equations of motion without specifying any potential or metric, and the field velocities will be given in terms of the potential and various covariant derivatives of it. Having found the solution, we will then discuss its stability and its behaviour in the limit \( \omega \gtrsim 180 \), when it is compatible with the recently proposed swampland conjectures [21–27]. Finally we will show how particular solutions can easily be recovered by evaluating the various projections of the covariant derivatives of the potential, and we give four examples that illustrate the power of this general approach.

Rapid-turn inflation – The analysis here rests on a vielbein formulation of inflation, using both the standard kinematic basis [28] and the potential gradient basis introduced in [3].

To solve the background evolution we want to find the (geometric scalar) field velocities \( \phi_v \equiv v_a \dot{\phi}^a \) and \( \phi_w \equiv w_a \dot{\phi}^a \) where \( v_a = \sqrt{\phi} |V| \) and \( w_a \) is a (co)vector field orthonormal to \( v^a \). The equations of motion for these velocities are given by [3]

\[
\begin{align*}
\ddot{\phi}_v &= -3H \dot{\phi}_v - V_v + \Omega_v \dot{\phi}_w, \quad \text{(1)} \\
\ddot{\phi}_w &= -3H \dot{\phi}_w - \Omega_v \dot{\phi}_v, \quad \text{(2)}
\end{align*}
\]

where \( V_v = \omega^a V_{va}, V_{ww} = \omega^a w^b V_{ab} \) etc, \( \Omega_v \equiv w_a D_t v^a = (V_v \dot{\phi}_v + V_{ww} \dot{\phi}_w)/V_v \), and \( D_t X^a \equiv \dot{X}^a + \Gamma^a_{bc} X^b \dot{\phi}^c \). We also define \( D_N \equiv H^{-1} D_t \).

The reason for using the gradient basis above is that unlike in the kinematic basis \( (n^a, \delta^a) \), where \( n^a = \dot{\phi}^a/\phi \) and \( \delta^a = -\phi^{-1} \dot{\phi}^a/\phi \), we already know the directions of \( v^a \) and \( w^a \) before we have the full solution, making them much easier to work with. In this paper we are looking at inflation models with a significant turn rate, meaning \( \omega \gg O(\epsilon) \). The turn rate \( \omega \equiv s_n D_N n^a \) will be a crucial quantity to our analysis, and using the gradient basis it may be expressed as

\[
\omega = \dot{\phi}_w V_v / H \dot{\phi}^2, \quad \text{(3)}
\]

which is obtained using the Klein-Gordon equation [3].

However, while the gradient basis is very useful for solving the background equations of motion, when dealing with perturbations the kinematic basis has significant advantages. This is because we can automatically see which perturbations are adiabatic and which correspond to isocurvature modes. For compactness denoting \( d/dN \) by \( \delta \)'s, the equations of motion for the perturbations can be written [14] [28] [31]

\[
\delta \phi'' + [(3 - \epsilon) \delta_j - 2 \omega \epsilon_j] \delta \phi'^j + C(k) \delta_j \delta \phi'^j = 0 \quad \text{(4)}
\]
where \( \epsilon_2 = -\epsilon_1^2 = 1 \) and \( C(k)^4 \) is given by

\[
C(k)^4 = \left( \frac{\mu_n - \omega^2 - \frac{k^2}{aH^2}}{\mu_x + \omega(3 - \epsilon + \nu)} \right),
\]

where we further defined \( \mu_n = \rho^n b M_{ab}/H^2 \), \( \mu_x = n^x b M_{ab}/H^2 \), and \( \mu_s = s^s b M_{ab}/H^2 \) as projections of the dimensionless mass matrix. These are given by

\[
\mu_n = \frac{V_{vv} \dot{\phi}_v^2 + 2V_{vw} \dot{\phi}_w \dot{\phi}_v + V_{ww} \dot{\phi}_w^2}{H^2 \dot{\phi}_v^2} - 2\epsilon(3 - \epsilon + \eta),
\]

\[
\mu_x = \frac{(V_{ww} - V_{vv}) \dot{\phi}_v \dot{\phi}_w + V_{vw} (\dot{\phi}_v^2 - \dot{\phi}_w^2)}{H^2 \dot{\phi}_v^2} - 2\epsilon \omega
\]

\[
\mu_s = \frac{V_{vw} \dot{\phi}_v^2 - 2V_{vv} \dot{\phi}_v \dot{\phi}_w + V_{ww} \dot{\phi}_w^2 + R \dot{\phi}_v^2/2}{H^2 \dot{\phi}_v^2},
\]

where in the middle line we used the definition of the turning rate to tidy up the expression.

**Finding the attractor solution** – We are interested in finding an inflationary solution to the equations of motion with a large and stable turn rate. Moreover, we require that the equations of motion for the perturbations must admit a mode with a conserved curvature perturbation on superhorizon scales. Mathematically, this translates to:

(i) \( \omega \gg \mathcal{O}(\epsilon) \), (ii) \( \nu \ll 1 \), and (iii) \( \delta \phi_n \propto \sqrt{2} \), \( \delta \phi_s = 0 \) being a solution to \( \mathcal{D} \) in the limit \( \frac{k}{\delta H} \ll 1 \). Since it is an inflationary solution, we also need \( \epsilon \) and \( \eta \) to be small.

To start solving the equations of motion, we first look an equation relating \( \epsilon_v \) and \( \epsilon \), which can be derived from \( \eta \equiv \mathcal{D}_N \ln \epsilon \) and the definition of \( \omega \) [18]:

\[
\epsilon_v = \epsilon (1 + \omega^2/9) + \mathcal{O}(\epsilon^2),
\]

where \( \epsilon_v \equiv M^2 V^2/2V^2 \), which can be solved for \( \omega \) when \( \omega > \epsilon, \eta \). Our expression for \( \omega \) in equation [3] then implies

\[
\frac{\dot{\phi}_w}{\phi} = \frac{\omega}{\sqrt{9 + \omega^2}}, \quad \frac{\dot{\phi}_v}{\phi} = \frac{-3}{\sqrt{9 + \omega^2}}.
\]

The proportions of the field velocity that are parallel and orthogonal to the gradient of the potential are consequently uniquely determined by the rotation rate.\[22\]

Moreover, one can rearrange equation [7] to find that the total field velocity is given by

\[
\dot{\phi}^2 = \frac{V_v}{H^2 (\omega^2 + 9)}.
\]

If we can find \( \omega \) in terms of the background quantities \( V, V_v, V_{vw} \) etc, we will thus have a complete solution.

The next step is to compute \( \mathcal{D}_N \omega \) using the Klein-Gordon equation, which gives

\[
\mathcal{D}_N \omega = -\mu_x + \omega (-3 + \epsilon - \eta)
\]

Demanding \( \nu = \mathcal{D}_N \ln \omega \sim \mathcal{O}(\epsilon) \) therefore requires

\[
\mu_x = -3\omega + \mathcal{O}(\epsilon),
\]

and condition (ii) has reduced to a condition on an element of the effective mass matrix.

For there to be a mode corresponding to a conserved curvature perturbation on superhorizon scales \( \frac{k}{\delta H} \ll 1 \), \( \delta \phi_n \propto \sqrt{2} \), \( \delta \phi_s = 0 \) must be a solution to \( \mathcal{D} \). Using equation [10] to simplify equation [4] we see that for this to be the case \( \mu_n \) must satisfy

\[
\mu_n = \omega^2 + (-3 + \epsilon) \eta/2 + \mathcal{O}(\epsilon^2),
\]

and condition (iii) has reduced to another condition on an element of the effective mass matrix.

The requirement that \( \omega \) varies slowly and that a regular adiabatic mode exists has given us two equations, [11] and [12] which fix two of the three independent elements of the effective mass matrix in terms of \( \omega \). For a given model, i.e. a metric and a potential, we may consider the covariant derivatives as fixed (as functions of the fields), but the field momenta are not. The field momenta are however completely determined by the turning rate \( \omega \), which must now ensure that both equations [11] and [12] are satisfied. Using the definitions of \( \mu_x \) and \( \mu_n \), given in equation [4] and the expressions for the velocities in equation [3] the two constraints can be simplified to

\[
\frac{V_{vw}}{H^2} = \frac{3 V_v}{\omega} + \mathcal{O}(\epsilon),
\]

\[
\frac{V_{ww}}{H^2} = \frac{9 V_v}{\omega^2} + \frac{\omega^2 + 9 + \mathcal{O}(\epsilon)}{}.\]

The first of these equations only provides a meaningful constraint on \( \omega \) as long as \( V_{vw}/H^2, V_{vw}/H^2 \gg \mathcal{O}(\epsilon) \), but in many models this is not the case. However, for those models where this is the case, these two equations will constrain where in field-space this inflationary phase may occur. To avoid dealing with case-by-case results, we just solve equation [14] for \( \omega \), to find:

\[
\omega^2 = \frac{V_{vw}}{H^2} \frac{9}{2} \pm \sqrt{\left( \frac{V_{ww}}{2H^2} - \frac{9}{2} \right)^2 - \frac{9V_{vw}}{H^2}}.
\]

Having found an expression for \( \omega \) we are now done - we have now completely determined the background field velocities. We do have two solutions (modulo overall sign), but empirically we find that the + solution is generally dominant, and it is necessarily so if \( V_{vw} \) is either negative or negligible. This is to be expected, because as we shall see shortly, solutions with large values for \( \omega \) are more likely to be stable.

To verify that this is indeed a solution to the background equations of motion, one can use equations [3] [13] and [14] to show that \( \Omega_v = H \omega + \mathcal{O}(\epsilon) \). With this expression, it is easy to verify that all the terms on the right hand sides of equations [1] and [2] cancel up to \( \mathcal{O}(\epsilon) \) corrections. A further technical detail is that what we have really done is to find the leading order parts of the solutions \( \dot{\phi}_v = \dot{\phi}_v + \delta \dot{\phi}_v \) and \( \dot{\phi}_w = \dot{\phi}_w + \delta \dot{\phi}_w \) that we take
to be functions of field-space position only. This gives us an approximate solution, which is valid as long as the necessary corrections are small, i.e., \( \delta \dot{\phi}_i \sim \mathcal{O}(H \dot{\phi}_i \epsilon) \). For this to be the case, the explicit time derivatives of the field velocities must satisfy \( \delta \dot{\phi}_i / dt \sim \mathcal{O}(H \dot{\phi}_i \epsilon) \), which requires the terms \( V_{\nu}, V_{\nu \nu} \), etc to vary slowly along the trajectory. This is a very reasonable condition, given assumption (ii).

**Stability** – To see that this very general solution corresponds to an attractor, we again look at equation (4) in the limit \( k/aH \ll 1 \), but now substitute in our solution. To see whether the solution is stable, we define \( \delta \pi_i = \delta \phi_i \), and look at the eigenvalues of the evolution matrix, i.e. the local Lyapunov exponents. Since we have ignored \( \epsilon \) corrections above, we now need to drop these, and we find that the equations of motion can be written

\[
\begin{pmatrix}
\delta \phi_n' \\
\delta \phi_s' \\
\delta \pi_n' \\
\delta \pi_s'
\end{pmatrix} =
\begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
6 \omega & -3 & 2 \omega & -3 \\
- \mu_s + \omega^2 & -2 \omega & -3
\end{pmatrix}
\begin{pmatrix}
\delta \phi_n \\
\delta \phi_s \\
\delta \pi_n \\
\delta \pi_s
\end{pmatrix}.
\]

(16)

The eigenvalues of the evolution matrix are given by

\[
\lambda = -3, \quad 0, \quad \frac{1}{2} \left(-3 \pm \sqrt{9 - 4 \mu_s - 12 \omega^2}\right).
\]

(17)

As long as the dimensionless entropic mass satisfies \( \mu_s > -3 \omega^2 \), the system has one (near) constant mode (the adiabatic one) and three decaying ones, and is thus stable. Using equations (6) and (13) one can show that the entropic mass is given by

\[
\mu_s = \frac{9 V_{ww}}{H^2 (\omega^2 + 9)} + \frac{V_{vv}}{H^2} + \frac{R \dot{\phi}^2}{2H^2} + \mathcal{O}(\epsilon)
\]

(18)

which must satisfy the above condition.

**Rapid-turn inflation and the swampland** – For inflation to be compatible with the swampland conjectures, the field excursion must be bounded, \( \Delta \phi \leq M_p \equiv (8 \pi G)^{-1/2} \), which implies \( \epsilon \leq 1/2 N^2 \) (where \( N \) is the number of e-folds of inflation), and either the gradient satisfies \( M_p V_v / V \geq c \) or the minimum eigenvalue of the Hessian satisfies \( M_p^2 \min(V_{ab}) / V \leq -c' \), where \( c \) and \( c' \) are \( \mathcal{O}(1) \) coefficients.

If we wish to satisfy the swampland conjectures by having a large gradient, the turn rate will satisfy \( \omega \gtrsim 180 \) [27]. In this limit, the rapid-turn attractor solution drastically simplifies, assuming that \( V_{vv} \) is not parametrically larger than \( V_{ww} \), and we find

\[
\omega^2 \simeq \frac{V_{ww}}{H^2}, \quad \dot{\phi}_v \simeq -\frac{3 H V_v}{V_{ww}}, \quad \dot{\phi}_s^2 \simeq \frac{V_{sv}}{V_{ww}}.
\]

(19)

The two conditions can then be reformulated as

\[
M_p V_v / V \geq c, \quad M_p V_{ww} / V_v \geq 3 c N^2,
\]

(20)

which are straightforward to check.

If instead we want to satisfy the condition on the Hessian, there are fewer simplifications that can be made, since the turn rate is no longer necessarily very large.

For a given model, we do not have full freedom in choosing which of these conditions we want to satisfy, and there are pitfalls that need to be avoided. The spectral index of the power spectrum needs to be matched with observations, which constrains the background solution, potentially ruling out one or both of these options (c.f. [3]). Moreover, in very rapidly turning models \( (\omega \geq \mathcal{O}(100)) \), a tachyonic entropic mass can lead to such large growth of the quantum fluctuations during horizon crossing that perturbative control is lost [10]. This is especially problematic for models realised in hyperbolic field-spaces, where \( R < 0 \). From equations (18) and (19) one can deduce that these models need at least \( V_{ww} \gtrsim V_{vv} \) to avoid this problem.

**Examples of rapid-turn attractors** – In this section we show that several non-standard two-field inflation models are examples of rapid-turn attractors, and use the relations derived above to straightforwardly find the form of the solutions. Hyperinflation and the turning inflation model of [16] present two algebraically simple models where we can easily derive the form of the full solution, and for the latter we also give some numerical examples of the agreement between the attractor model predictions and numerical simulation. Side-tracked inflation and angular inflation, however, are algebraically messy. In the former case we present more numerical examples of the agreement between the attractor predictions and simulations, and in the latter case we show how we can straightforwardly recover the parametric equation for the field trajectory.

The point of this section is not to delve into the details of these models; it is instead to show what a broad class of models can be described by this attractor solution, and to demonstrate how powerful these techniques can be for finding explicit solutions given a metric and potential. In models where \( V_{ww} \) is negligible (e.g. hyperinflation), rapid-turn inflation can occur anywhere in the target space, and equations (18) and (19) immediately give the full solution (i.e. \( \phi_v \) and \( \phi_s \)) which is valid everywhere. In models where \( V_{ww} \) however is not negligible, the expressions for \( \omega \) in equations (13) and (14) must be matched. This provides a constraint relating the two fields, which tells us where in field-space this phase may occur.

**Hyperinflation** – Hyperinflation, which was first introduced by [1], and then further studied in [2, 3], provides a very clean example of a rapid-turn attractor. The usual metric and potential used for this model are

\[
ds^2 = d\phi^2 + L^2 \sinh(\phi/L)^2 d\theta^2, \quad V = V(\phi),
\]

(21)

although more generally it just requires \( V_{ww} = V_v / L \), \( V_{ww} \simeq 0 \) and \( V_{vv} \ll V_{ww} \). Hyperinflation refers to a new phase of inflation that occurs when the potential
becomes sufficiently steep for slow-roll to be geometrically destabilised \[1\] 5. Using the results derived earlier in this paper and the above covariant derivatives of the potential, one immediately finds
\[
\dot{\phi} = -3HL, \quad \dot{\phi}^2 = LV_v, \tag{22}
\]
which is only a consistent solution if \(LV_v > 9H^2L^2\), which is precisely the condition for slow-roll to become unstable \[1\] 5. Its entropic mass, given by \(\mu_s = -\omega^2 + O(\epsilon)\), is very tachyonic in the \(\omega \gg 1\) limit, but the background evolution is nevertheless stable.

A flat field-space model – This example, first introduced in \[16\], is very different from hyperinflation, especially since the field space is not hyperbolic. Here we have the metric and potential
\[
ds^2 = d\rho^2 + \rho^2 d\theta^2, \quad V = V_0 - \alpha \theta + \frac{1}{2} m^2 (\rho - \rho_0)^2. \tag{23}
\]
In this model, the rapid-turn regime appears at \(\rho \gg \rho_0\), where the gradient of the potential is dominated by the \(\rho\)-direction, but where the \(\theta\)-term nevertheless plays an important role. Here we have \(V_\theta \approx m^2 \rho\), \(V_\rho \approx V_{\rho \rho} \approx m^2\), and \(V_{\theta \theta} \approx \alpha / \rho^2\). Equations \[13\] and \[14\] thus give us two expressions for \(\omega\):
\[
\omega \approx 3\rho^2 m^2 / \alpha, \quad \omega \approx \sqrt{3} \mp \rho / \sqrt{V_\theta}. \tag{24}\]
These must match (up to a sign), telling us that this type of inflation can only happen at
\[
\rho^2 = M_P \alpha / \sqrt{3} V_\theta, \quad \Rightarrow \quad \dot{\theta} = \dot{\phi}_\theta / \rho \approx m, \tag{25}\]
in agreement with \[16\]. Moreover, to illustrate the accuracy of the rapid-turn attractor predictions, Figure \[1\] shows the agreement between predictions and simulations for a swampland compatible model with \(\alpha = 5.0 \times 10^{-16}\), \(m = 2.5 \times 10^{-3}\) and \(V_0 = 3.4 \times 10^{-10}\), giving \(\omega \approx 230\).

Side-tracked inflation – Side-tracked inflation is another example of a rapid-turn attractor in hyperbolic geometry, which like hyperinflation can arise after geometric destabilisation of slow-roll \[5\] 7. Looking at the side-tracked inflation model with the so called ‘minimal geometry’, we have
\[
ds^2 = \left(1 + \frac{2x^2}{M^2}\right) d\phi^2 + dx^2, \quad V = U(\phi) + \frac{m_h^2}{2} \chi^2. \tag{26}\]
In this model, \(m_h\) is the mass of a heavy field with \(m_h \gg H\), but despite the size of this mass, slow-roll is destabilised by the negative curvature, and we end up in a so called ‘side-tracked’ phase of inflation, which is another example of a rapid-turn attractor. \(V_\chi\) and \(V_{\chi \chi}\) are non-negligible in this model, so we can use equations \[13\] and \[14\] to find where in field-space side-tracked inflation may happen. Assuming \(M^2 \phi \ll U_\phi\) and that \(U\) dominates the potential energy, we find that this happens at
\[
\frac{2\chi^2}{M^2} = \sqrt{\frac{2}{3} \frac{M_P |U_{\phi}|}{m_h M \sqrt{U}}} - 1, \tag{27}\]
recovering the expression found in \[7\]. Figure \[2\] also illustrates the agreement between the rapid-turn attractor predictions and numerical simulations for a model with a natural inflation potential \(U(\phi)\) with \(M = 0.001 M_P\) and \(m_h / H = 10\).

Angular inflation – An additional attractor model was found recently by \[4\], in the context of \(\alpha\)-attractor models \[33\] \[40\], where the geometry again is hyperbolic. Here we work with a metric and potential of the form
\[
ds^2 = \frac{6\alpha (d\phi^2 + dx^2)}{(1 - \phi^2 - \chi^2)^2}, \quad V = \frac{\alpha}{6} \left( m_\phi^2 \phi^2 + m_\chi^2 \chi^2 \right). \tag{28}\]
Reparametrising this as \(\phi = r \cos \theta, \chi = r \sin \theta\), and defining \(R = m_\phi^2 / m_\chi^2\), they found a new angular attractor solution at \(1 - r \ll 1\) when the parameters \(\alpha\) and \(R\) satisfied \(\alpha \ll 1\) and \(R \gg 1\). Defining \(\delta \equiv 1 - r^2\), one can then solve equations \[13\] and \[14\] (eliminating \(\omega\)) to leading order in \(\delta\), to find its parametrisation by \(\theta\) as written in \[4\] 41:
\[
\delta(\theta) = 1 - r(\theta)^2 = \frac{9\alpha (\cot \theta + R \tan \theta)^2}{2(R - 1)^2}. \tag{29}\]

Conclusions – We have shown that there exists a completely general rapidly turning attractor solution in
two-field inflation. The attractor is not restricted to any particular background geometry or form of the potential, and we have shown how several recently studied non-standard inflationary attractors are in fact examples of this rapid-turn attractor.

Having a large, slowly varying turn rate and demanding the existence of an adiabatic mode with conserved $\zeta$ on superhorizon scales was sufficient to solve the equations of motion in generality. Moreover, we showed that these solutions have two out of three elements in the effective mass matrix constrained up to $O(\epsilon)$ corrections. Only the turn rate and the entropic mass remain unconstrained degrees of freedom. The primordial perturbations generated by these theories can therefore be expected to be determined by these parameters (and their time evolution), potentially simplifying future analyses of swampland-compatible inflation models.

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[41] The current arXiv version (v1) has a typo in equation (2.14), where cot and tan appear with squares. If (2.13) is expanded in $\alpha$, however, these squares do not appear.