1/m_Q CORRECTIONS
IN HEAVY MESON DECAYS

M. Masip

Departamento de Física Teórica y del Cosmos
Universidad de Granada
18071 Granada, Spain

ABSTRACT

We study the form factor $f^H_+$ ($H = D, B$) involved in $H$–$\pi$ transitions. We use data on $D^0 \to \pi^- l^+ \nu$ to find $f^D_+$ at low recoil. Then we use perturbative QCD methods to calculate at larger recoil the contributions to $f^H_+$ that break heavy quark symmetry (HQS). Using HQS relations we obtain an estimate of $f^B_+$ which includes first order corrections in $1/(2m_Q)$. Comparison with recent data on $B^0 \to \pi^- l^+ \nu$ gives $V_{ub} = 0.0025 \pm 0.0005^{oz} \pm 0.0004^{th}$
1. Introduction.

Heavy quark effective theory (HQET) provides a model independent framework to describe the hadronic interactions of heavy mesons [1]. In the infinite quark mass ($m_Q$) limit, HQET makes manifest extra symmetries of the QCD Lagrangian: a flavor symmetry relating matrix elements of mesons with different heavy quark content, and a spin symmetry that relates amplitudes involving mesons with the same heavy quark content but with different heavy quark spin. Corrections breaking the symmetry are parametrized as an expansion in $1/(2m_Q)$, and will be small if the momenta exchanged between the heavy quark and the light degrees of freedom inside the meson are smaller than $m_Q$.

In particular, the semileptonic decays $H \rightarrow \pi \ell \nu$ (with $H = B, D$) have been studied at first order in $1/(2m_Q)$. For vanishing lepton mass the decay rate depends only on the form factor $f^H_+$,

$$\langle \pi(p) \mid \overline{q} \gamma^\mu Q \mid H(v) \rangle = f^H_+ (m_H v^\mu + p^\mu) + f^H_- (m_H v^\mu - p^\mu),$$

where $v$ is the heavy meson four velocity and $p$ the pion momentum. In these decays the kinematical variable $v \cdot p$ will vary from $m_\pi$ (for a final pion at rest in the rest frame of the initial heavy meson) to $m_H/2 + m_\pi^2/(2m_H)$ (the highest recoil). At first order in $1/(2m_Q)$, $f^H_+$ can be expressed [2]

$$f^H_+ = \frac{\sqrt{m_H}}{v \cdot p + \Delta H} c_Q (A + \frac{1}{2m_Q} B),$$

where $\Delta H = m_{H^*} - m_H$ and $c_Q$ is a renormalization factor. $A$ and $B$ above are functions of $v \cdot p$ that do not depend on the mass of the heavy quark: they will describe $D^{-}\pi$ and $B^{-}\pi$ transitions at the same recoil point $v \cdot p$. In the infinite $m_Q$ limit only $A$ contributes to $f^H_+$ and one has $f^B_+ / f^D_+ = \sqrt{m_B / m_D}$ [3].

At low $v \cdot p$ the momenta exchanged between the heavy quark and the light degrees of freedom during the transition are of order $m_H - m_Q = \overline{\Lambda} \approx \Lambda_{QCD}$. In consequence, the corrections breaking heavy quark symmetry (HQS) will be small, with $B/A = O(\overline{\Lambda})$. At large $v \cdot p$, however, it will be necessary to exchange large momenta to keep the expectator quark inside the outgoing pion. These hard processes will be able to distinguish the mass of the heavy quark and will introduce symmetric and non-symmetric contributions of the same order: $B/A = O(v \cdot p) = O(m_Q)$. Since large $v \cdot p$ is the dominant kinematical region in heavy meson decays, it will not be possible to give there an accurate relation of $f^B_+$ with $f^D_+$ based on HQS only. Our objective in this letter is to determine the value of the contributions that break HQS and grow (relative to the symmetric contributions) with $v \cdot p$. This will allow a relation between the two form factors valid at order $B/A = O(\overline{\Lambda})$ also in the region of large $v \cdot p$. 
The calculation of $f_+^H$ has been attempted using different methods. Soft pion relations based on partial conservation of the axial vector current imply single pole behaviour at low $v \cdot p$ \[2\]. This means

$$f_+^H = \frac{C}{1 - q^2/m_H^2} = \frac{C m_H/2}{\Delta_H + v \cdot p},$$

(3)

where $q^2 = (p_H - p_\pi)^2$ is given in terms of $v \cdot p$ as $q^2 = m_B^2 + m_\pi^2 - 2m_B v \cdot p$ and $C$ is the value of the form factor at $q^2 = 0$. In the soft pion limit \[2\] single pole dominance holds at first order in $1/(2m_Q)$: dominant terms and first order corrections follow separately this dependence on $v \cdot p$ at low recoil. QCD sum rules \[4\] support the single pole behaviour in the whole region of momentum transfer in $B$ and $D$ to $\pi$ decays, and light-cone sum rules \[5\] suggest that order $1/(2m_Q)$ corrections are small, around a 10% in $D$ decays. Analogous results, consistent with single pole dominance, are obtained \[6\] in the combined limits of chiral symmetry and large $N_c$. All these arguments would imply that the functions $A$ and $B$ in Eq. (2) are just constants at $v \cdot p \leq m_B/2$. On the other hand, the calculation of $f_+^H$ using perturbative QCD methods \(\text{see } [7]–[9]\) gives qualitatively different results. In $B$ decays the values obtained seem to be smaller than expected \[7\], and the HQS-breaking contributions are dominant in most of the phase space.

We pretend here to obtain $f_+^B$ from $f_+^D$. This will require the evaluation of non-symmetric corrections (contributions to $B/A$ in Eq. (3)) that grow with $v \cdot p$ and become of order 1 at $v \cdot p = O(m_\pi)$. At large $v \cdot p$ these corrections involve the exchange of large momenta, which suggests that they can be analyzed using perturbative QCD methods. Then our strategy will be the following. First we will use data on $D^0 \to \pi^- l^+ \nu$ to normalize the total form factor at low recoil. Then a perturbative QCD calculation (we will take an error of order $\Lambda$ from soft contributions) will give us $B$ at large $v \cdot p$. From that we will obtain $f_+^B$ for all the values $v \cdot p$ accesible in $B$ decays. This result will be used to evaluate the partial decay rate $\Gamma(B^0 \to \pi^- l^+ \nu)$,

$$\frac{d\Gamma(B^0 \to \pi^- l^+ \nu)}{d(v \cdot p)} = \frac{G_F^2 m_B}{12\pi^3} |V_{ub}|^2 \left[ (v \cdot p)^2 - m_\pi^2 \right] \frac{1}{2} |f_+^B|^2,$$

(4)

and comparison with recent data will give us a value for the CKM mixing $V_{ub}$.

2. Form factor at low recoil.

To find the form factor $f_+^D$ we use $BR(D^0 \to \pi^- l^+ \nu) = (3.8 \pm 1.2) \times 10^{-3}$ \[10\]. The differential decay rate of this mode can be read from Eq. (2) just by replacing $m_B \to m_D$ and $V_{ub} \to V_{us}$. Taking a mean life $\tau_D = (0.415 \pm 0.004) \times 10^{-12}$ s and single pole dominance from $v \cdot p = m_\pi$ to $v \cdot p = m_D/2 + m_\pi^2/(2m_D)$ we obtain

$$f_+^D = \frac{13.8 \sqrt{BR(D^0 \to \pi^- l^+ \nu)}}{v \cdot p + 0.142} = \frac{(0.85 \pm 0.12)}{v \cdot p + 0.142}.$$

(5)
As discussed before, the deviations from single pole behaviour at the recoils in $D$ decays are small, and can be neglected respect to the quoted experimental uncertainty.

3. Perturbative QCD calculation.

In the Brodsky-Lepage formalism \[11\] the heavy–to–light transition amplitude is expressed

$$
\langle \pi(p) | \overline{\Psi} \gamma^\mu Q | H(v) \rangle = \int dx dy \text{Tr}[\overline{\Psi}_\pi(y, p) T_h^\mu(vp, x, y) \Psi_H(x, m_H v)] ,
$$

where $\Psi_i$ ($i = H, \pi$) is an effective quark-antiquark distribution amplitude ($x$ and $y$ are the fraction of longitudinal momentum carried by the quarks), $q$ and $Q = c, b^c$ are light and heavy valence quarks, $T_h^\mu$ is a hard scattering amplitude given at lowest order as the sum of collinear skeleton graphs in Fig. 1, and Tr stands for a trace over spin, flavour and color indices. Contributions from Fock states with extra $q\bar{q}$ pairs will be suppressed by powers of $1/v \cdot p$. The tensor structure of $\Psi_i$ is

$$
\Psi_i(x, p) = \frac{I_C}{\sqrt{3} \sqrt{2}} \frac{1}{\sqrt{2}} \phi_i(x) [m_i + O(\Lambda) + \not{p} \gamma_5] ,
$$

where $I_C$ is the identity in color space and the distribution amplitudes satisfy

$$
\int dx \phi_i(x) = \frac{1}{2\sqrt{6}} f_i
$$

(with this normalization the pion decay constant is $f_\pi = 132$ MeV). The sum of the two diagrams in Fig. 1 gives

$$
\langle \pi(p) | \overline{\Psi} \gamma^\mu Q | H(v) \rangle = -\int d\tilde{x} dy \ g_5^2 \text{Tr}(T^a T^a) \frac{\phi_i(y)}{12} f_\pi \phi_H(\tilde{x}) f_H \text{Tr}[

(\epsilon \overline{\chi} - \not{p}) \gamma_5 \gamma^\mu (\not{p}_Q + m_Q) \gamma_\alpha (m_H + m_H \not{p}) \gamma_5 \gamma^\alpha S \frac{1}{D_g D_q} +

(\epsilon \overline{\chi} - \not{p}) \gamma_5 \gamma_\alpha (\not{p}_q) \gamma^\mu (m_H + m_H \not{p}) \gamma_5 \gamma^\alpha \frac{1}{D_g D_q}]

$$

where now $\phi_i$ are normalized to one, all the $O(\Lambda)$ uncertainties in the mass term of the meson wave functions are contained in the $O(1)$ factor $\epsilon$, $\text{Tr}(T^a T^a) = 4$, and we have written the fraction of momentum carried by the light quark in the heavy meson as $x = \tilde{x} \frac{\Lambda}{m_H}$. $D_g$ and $D_{q,Q}$ are the denominators in the gluon and quark propagators (see below).

At the energies involved in $B$ decays the amplitude above has the tendency to depend strongly on the treatment of infrared regions. A selfconsistent calculation will require taking
into account double-log (Sudakov) corrections \[12\], which exponentiate and suppress contributions where an internal quark or gluon is near the mass shell. Two infrared regions need to be considered.

(i) In Eq. (6) the dependence of $\Psi_i$ and $T^\mu_h$ on the transverse momenta $k_\perp$ of the partons relative to the parent meson have been neglected. However, this is not a good approximation near the end points of $x$ and $y$, where the gluon propagator becomes singular. Using a modified factorization formalism it has been shown \[13\] that as the momentum transfer grows the configurations with a large (in a $1/\Lambda_{QCD}$ scale) quark-antiquark transverse distance $b$ are suppressed by Sudakov corrections and do not contribute to the transition amplitude. Small $b$ corresponds essentially to large $k_\perp$, its Fourier transform variable. When $x, y \to 0$ these values of $k_\perp$ are the typical momenta carried by the exchanged gluon in the scattering amplitude and provide an adequate renormalization and factorization scale. The denominators in the gluon and light quark propagators,

\[
D_g = -2\Lambda (\vec{x} \cdot v \cdot p + \frac{k_\perp^2}{2\Lambda}) \equiv -2\Lambda \Delta_g,
\]
\[
D_q = -2\Lambda (\vec{x} \cdot v \cdot p + \frac{k_\perp^2}{2\Lambda}) \equiv -2\Lambda \Delta_q,
\]

will be regulated at the end points by a generic transverse momentum (acting as an infrared cut off) $k_\perp^2 = (1.2–1.6 \Lambda_{QCD})^2$ \[13\].

(ii) The internal heavy quark in Fig. 1(a) carries a momentum $p_Q = p_1 - y p_2 + k_2$. Then, its propagator will be singular for a certain distribution of parton momenta in the outgoing pion:

\[
D_Q = p_Q^2 - m_Q^2 = -2m_H(y v \cdot p - \Lambda + \frac{\Lambda^2 + k_\perp^2}{2m_H})
\]

It is clear that for $y \approx \Lambda/(v \cdot p)$ the heavy quark will be near the mass shell even if the transverse quark-antiquark distance is forced by Sudakov corrections (and also by the explicit $k_\perp^2$ distribution \[14\]) to be small. However, The heavy–to–light vertex will have double-log corrections, generated by the diagram in Fig. 2(b), that will suppress this infrared region as well. The external light quark in Fig. 1(a) has an offshellness $\Delta_q \approx \Lambda_{QCD}$, then this correction will suppress the configurations where the heavy (internal) quark is also near the mass shell. For $|q_\perp| = v \cdot p$ and $\delta = |q_\perp|/m_Q$ we obtain a Sudakov factor

\[
S \approx \exp \left[ -\frac{\alpha_s(\mu^2)}{6\pi} \left( \ln^2 \frac{2(v \cdot p)^2}{\Lambda_{QCD}^2} - \ln^2 \frac{2v \cdot p(y v \cdot p - \Lambda)}{\Lambda_{QCD}^2} \right) \right],
\]

with $\mu = y v \cdot p - \Lambda$. $S$ provides in the diagram in Fig. 1(a) an effective cut-off of the region in $y$ with $(y v \cdot p - \Lambda) < \Lambda_{QCD}$. 


We can now proceed with the perturbative calculation. In the expressions above we can separate HQS symmetric and nonsymmetric contributions. We write the decay constant and the heavy quark propagator as

\[ f_H \sqrt{m_H} = (f_H \sqrt{m_H})^{(0)} \left(1 + \frac{1}{2m_Q} c_f \right) \]

\[ \frac{1}{D_Q} = \frac{1}{2m_H \Delta_Q} \left(1 - \frac{1}{2m_Q} \frac{\Lambda^2 + k_\perp^2}{\Delta_Q} \right), \]

(13)

with \( \Delta_Q = y \cdot v \cdot p - \Lambda \). Substituting in Eq. (9) we obtain the HQS–breaking contribution to \( f_H^+ \):

\[ B = v \cdot p \int d\tilde{x} \, dy \frac{\pi \alpha_s(\mu^2)}{9} \bar{\phi}_\pi(y) f_\pi \phi_H(\tilde{x}) \left( f_H \sqrt{m_H} \right)^{(0)} \frac{1}{\Lambda \Delta_y} \]

\[ \left\{ \frac{S}{\Delta_Q} [c_f - \frac{\Lambda^2 + k_\perp^2}{\Delta_Q} (4y v \cdot p - 4\Lambda - 4y \epsilon \Lambda + 2\epsilon \Lambda)] + \right. \]

\[ \frac{1}{\Delta_y} [c_f(-\tilde{x} + 2\epsilon) + 4\tilde{x} v \cdot p - 2 \Lambda^2 - 4\tilde{x} \epsilon \Lambda] \right\}, \]

(14)

where \( \mu^2 = 2\Lambda \cdot \tilde{x} \cdot y \cdot v \cdot p + k_\perp^2 \).

To evaluate \( B \) we take \( \Lambda_{QCD} = 0.2 \text{ GeV} \), \( \Lambda = 0.4-0.5 \text{ GeV} \), \( \epsilon = 0-1 \), \( c_f = -1.45 \) and \( (f_H \sqrt{m_H})^{(0)} = 0.34-0.42 \text{ GeV}^{3/2} \) (deduced from \( f_B = 160-200 \text{ MeV} \) and \( (f_B \sqrt{m_B})/(f_D \sqrt{m_D}) = 1.4 \pm 0.1 [13] \)). For the \( \pi \) meson we shall use the asymptotic distribution amplitude \( \bar{\phi}_\pi(y) = 6y(1-y) \) [16]. Due to the inclusion of \( k_\perp^2 \), the result does not depend on the details of the distribution amplitude of longitudinal momentum in the heavy meson. For example, when the fraction \( \tilde{x} \) of momentum carried by the light quark (in \( \frac{\Lambda}{m_H} \) units) varies between 0.5 and 2, at \( v \cdot p = m_B/2 \) we have that \( B \) goes from 0.40 to 0.15. We shall assume a constant distribution between these two values of \( \tilde{x} \) and zero otherwise. At \( v \cdot p = m_B/2 \) the 50% of the result comes from contributions with \( \alpha_s \leq 0.5 \).

In Fig. 3 we plot our perturbative result for \( B \) (dots) for \( v \cdot p \geq m_B/2 \). We observe that at these intermediate and large \( v \cdot p \) \( B \) is constant, which means that contributions to \( f_H^+ \) that break HQS follow a single pole behaviour \( 1/(v \cdot p) \). This is also the behaviour of \( B \) given by the soft contributions at low \( v \cdot p \) [14]. We shall use the perturbative value to normalize \( B \) at \( v \cdot p = m_B/2 \) and will add a constant soft contribution \( B = \Lambda A \) as an error. To estimate \( A \) we will use the experimental form factor deduced from \( D^0 \to \pi^- l^+ \nu \) data. We extrapolate to lower recoils the value \( B = 0.28 \text{ GeV}^{3/2} \) obtained perturbatively and subtract its contribution
to the total form factor in Eq. (6). Then we obtain \( A = 0.52 \text{ GeV}^{1/2} \). This value of \( A \) will be constant as far as single pole dominance is a good approximation \((v \cdot p \approx m_B/2\) according to QCD sum rules \([4, 5]\)).

The errors in the perturbative calculation of \( B \) will come from the variation of \((\Lambda, k_1^2, \alpha)\) (27%), from the error in \((f_H \sqrt{m_H})^{(0)}\) (10%), and from higher twist and higher order in \(\alpha_s\) corrections (25%). There is also a 8% error from order \((1/m_Q)^2\) corrections generated in the diagrams in Fig. 1 (the corrections appearing at higher twist will be smaller, since they will be suppressed by powers of \(\alpha_s\)). Soft contributions introduce an uncertainty \(\Delta B = \overline{\Lambda} A = 0.23 \text{ GeV}^{3/2} \) (82%). Adding the errors in quadrature we obtain our low-recoil estimate of \( B \):

\[
B = (0.28 \pm 0.25^{th}) \text{ GeV}^{3/2}.
\]

At large \( v \cdot p \gg m_B/2 \) the soft contributions would drop and \( B \) would converge to its perturbative value \( B = (0.28 \pm 0.09^{th}) \text{ GeV}^{3/2} \).

Since \( A \) has been deduced from the difference between the experimental form factor and the calculated HQS-breaking correction, its uncertainty will come from the error in \( B \) (a 18%) and from the experimental error in \( f^D_+ \) (a 18%):

\[
A = (0.52 \pm 0.09^{ex} \pm 0.09^{th}) \text{ GeV}^{1/2},
\]

(16)

4. \( f^B_+ \) and determination of \( V_{ub} \).

We can now estimate \( f^B_+ \):

\[
f^B_+ = \frac{\sqrt{m_B}}{vp + \Delta_B} \left[ \frac{\alpha_s(m_c)}{\alpha_s(m_b)} \right]^{2/\beta} \left( A + \frac{1}{2m_b}B \right),
\]

(17)

with \( \beta = 25/3 \). At \( v \cdot p \leq m_B/2 \)

\[
f^B_+ = \frac{(1.39 \pm 0.24^{ex} \pm 0.25^{th})}{vp + 0.046}.
\]

(18)

Finally, using recent data on \( B^0 \to \pi^- l^+ \nu \) from the CLEO Collaboration \([17]\) we can estimate the mixing angle \( V_{ub} \). For \( \text{BR}(B^0 \to \pi^- l^+ \nu) = (1.8 \pm 0.5) \times 10^{-4} \) and \( \tau_{B^0} = (1.56 \pm 0.06) \times 10^{-12} \text{ s} \) we obtain

\[
V_{ub} = 0.0025 \pm 0.0005^{ex} \pm 0.0004^{th},
\]

(19)

where the experimental error comes from uncertainties in the \( D \) and \( B \) to \( \pi l \nu \) branching ratios.
5. Conclusions.

We have analyzed the form factor $f^H_+ (H = D, B)$ describing $H - \pi$ transitions. In the semileptonic decays of a heavy meson the momentum transfer is of order $v \cdot p \approx m_H/2$, so a priori the corrections breaking the HQS are of order 1. We have computed perturbatively these corrections using the Brodsky-Lepage formalism and including in our calculation Sudakov effects, which are necessary to obtain a selfconsistent result at these values of $v \cdot p$.

We normalize the form factor $A$ at low recoil using data on $D^0 \rightarrow \pi^- l^+ \nu$, and we normalize the contributions $B$ breaking HQS using a perturbative calculation. At intermediate and large $v \cdot p$ these contributions follow a single pole dependence, which is also the behaviour followed by soft contributions at lower recoils [3]. We introduce in $B$ an error of order $\Lambda_A$ from soft contributions that does not increase with $v \cdot p$.

Our results for $f^B_+$ and the partial decay rate $\Gamma(B^0 \rightarrow \pi^- l^+ \nu)$ are slightly higher than other estimates. For example,

$$f^B_+(q^2 = 0) = 0.26 \ [18]; \ 0.33 \ [19]; \ 0.27 \ [20]$$

and

$$\frac{\Gamma(B^0 \rightarrow \pi^- l^+ \nu)}{V^{2}_{ub}} \times 10^{13} \ s^{-1} = 0.51 \ [18]; \ 0.74 \ [19]; \ 1.0 \ [20]$$

versus our values 0.52 and 1.8, respectively. We should stress, however, that our values are normalized by the experimental data on $D^0 \rightarrow \pi^- l^+ \nu$. In this sense, it may be significant that typical calculations of the analogous form factor and partial decay rate in $D$ decays fall short respect the observed values:

$$f^D_+(q^2 = 0) = 0.5 \ [18]; \ 0.69 \ [19]; \ 0.67 \ [20]$$

and

$$\Gamma(D^0 \rightarrow \pi^- l^+ \nu) \times 10^{10} \ s^{-1} = 0.39 \ [18]; \ 0.68 \ [19]; \ 0.80 \ [20]$$

versus 0.91 and 0.92 from the experiment (where single pole dominance has been assumed to deduce the value of the form factor).

Since we have actually computed the difference between $f^B_+$ and $f^D_+$ given at first order in the mass of the heavy quark by HQS, the total values of the form factors that we deduce are strongly correlated. It is possible to redo the analysis in terms of the branching ratios and express the mixing $V_{ub}$ as

$$V_{ub} \approx \frac{\sqrt{\text{BR}(B^0 \rightarrow \pi^- l^+ \nu)}}{(97 \pm 16) \sqrt{\text{BR}(D^0 \rightarrow \pi^- l^+ \nu)} - (0.57 \pm 0.50)}.$$
For the central experimental values we obtain $V_{ub} = 0.0025$, with a 18% error from uncertainties in our calculation.

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Figure 1: Heavy-to-$\pi$ hard scattering amplitudes.
Figure 2: Tree-level heavy-to-light vertex and one-loop diagram giving the double-log correction. See [11] for a definition of light-cone variables and Feynman rules.
Figure 3: Form factor $B$. Dots correspond to the perturbative values obtained with our calculation.