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Resilience toward supply disruptions: A stochastic inventory control model with partial backordering under the base stock policy

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ABSTRACT

This paper examines supply-side disruptions using an inventory management framework to gain insights into the economic performance of buyers who use two conventional costing strategies – end-of-cycle and continuous. The proposed model assumes that a single supplier faces full disruptions with a probability, and therefore, fails to procure all the items ordered. Accordingly, the buyer experiences unmet demand, which is assumed to be partially backordered. To make proper replenishment decisions, two base stock $(S,T)$ periodic review optimization models are developed. The objective functions minimize the expected long-run total costs (i.e., ordering, holding, and shortage). After proving their convexity and obtaining the optimal decisions, computational experiments were carried out to investigate the impacts of the change in parameters. The results highlight the importance of selecting a suitable ratio for backorders during supply disruptions, as it leads to lower costs and reduces the inventory obsolescence and overstocking risks. Moreover, the managerial insights derived from this study aid retailers to make better replenishment decisions (in terms of level and frequency) and to be more resilient in times of disruptions. Retailers can benefit from the provided solution algorithm as a computational application.

1. Introduction

As supply chains have become increasingly globalized, concerns in business and academic sectors have raised about their resilience and responsiveness (García-Arca et al., 2020; Liu et al., 2020). Traditionally, supply chain managers have not considered the impacts of disruptions and have not adopted efficient techniques to control or mitigate them (Kamble et al., 2019). As a result, several business failures have been reported. For instance, in 2001, production of the semiconductor in Philips was disrupted by fire, and Ericsson, the buyer, lost about $400 M, leading to its withdrawal from market (Latour, 2001). In 2007, Boeing encountered cash flow loss of nearly $2.5 B due to improper supply of bolts and screws (Boeing CEO blames the industry for 787 bolt Boeing CEO blames industry for 787 bolt shortage, 2007). In 2017, Samsung stopped production lines of Galaxy Note 7 due to defectiveness of batteries delivered by its global supplier (Samsung probe blames batteries for fires, 2017). It is concluded that improper management of disruptions, specifically at the supply side, leads to severe, devastating impacts on businesses.

Recently, with the advent of the COVID-19 pandemic, several firms have bankrupted due to lack of proper business continuity plans (Shih, 2020; Laato et al., 2020). According to a report by the BCI (2020), 73% of firms have experienced supply-side disruptions because of this pandemic. The report states that nearly 20% of managers will plan to hold more inventory levels, and 27% of them will plan to change their supplier base to ensure about consistent delivery of ordered goods. Therefore, new challenges have been arisen to design resilient inventory systems at the time of crisis.

A disruption usually has two kinds of sources (Bakal et al., 2017; Hishamuddin et al., 2014). Interruption of manufacturing machines, transportation problems, supplier inefficiency, and quality problems are internal disruptions (Sarkar et al., 2019). The members in this category may occur frequently but cause lower chances of business devastation. Meanwhile, natural disasters, political instability, labor strikes, industrial accidents, and advent of pandemic diseases go under the category of external disruptions. These disruptions are usually unpredictable and occasional with severe negative impacts. According to a report, fire and transportation failures were the cause of 29% of disruptive events in the first half of 2018 (Supply Chains Disruptions at Highest Rate in 3 Years, 2018). Managers should consider the most probable type of disruption in
their activity field when planning to increase the resilience of the supply chain. For instance, in the case of a supply-side disruption, due to above mentioned events, it is suggested to plan based on differences between suppliers on various cultural, practical, and technical aspects (Choi and Krause, 2006).

Researchers have posed several methods to mitigate the undesired effects of disruptions for buyers. One of the earliest approaches is to contract with several suppliers rather than a single supplier (Yan and Liu, 2009). However, this strategy is not suitable if buyers order a specific or non-standard product. Another similar approach is to consider backup suppliers for supporting supply flow during the time of disruptions (Tomlin, 2009a). Buyers can also allocate a ratio of their warehouse to hold excess inventory (as a buffer) (Atan and Snyder, 2012). Scheduling maintenances is also an effective solution for suppliers to prevent the breakdown of machinery (Pal et al., 2014). However, these approaches may increase the total costs of any firm if not appropriately implemented.

The most straightforward approach is to accept potential risks of disruption and their economic losses and to ignore planning for such probable events (Tomlin, 2006), as Ericsson did in 2001. However, at that time, Nokia, another buyer of the semiconductors, tried to contract with backup suppliers immediately, therefore, successfully prevented its bankruptcy (Lateur, 2001). This well-known instance shows the importance of implementing a proper approach to control the impacts of a disruption (Qi, 2013). Overall, making an appropriate decision when disruption is probable to occur, needs a complete analysis of the business, which has many complexities.

From the inventory management point of view, there are four main questions if there is a probability for supply-side disruptions: (1) how much to order from the supplier to prevent undesired shortages; (2) in which cases the shortages will happen; (3) what ratio of unsatisfied demand should be met (pure backordering/lost sale, or partial backordering); and (4) how these decisions affect the total long-run costs of the inventory system. Accordingly, logistics and operation managers have been concerned about proper management of supply disruptions ever since their probability of occurrence and impact have been increased due to growing interests for globalization and expansion of supply chain networks (Mishra et al., 2020).

Although some attempts have been made toward modeling inventory management strategies, none of them got very far because they lack proper consideration of customer behavior or the inventory system itself for optimizing replenishment plans under such uncertain conditions (Sarkar, 2016; Sarkar et al., 2020). A recent study carried out by Konstantaras et al. (2019) considers pure backordering case, i.e., the situation in which a delay cost imposes on the inventory system if the customer orders could not be materialized on time. However, there are several real cases that a segment of customers will wait to receive the required items, while others will not. In the first case, meeting the demand of the customers is more expensive than the typical situation when the seller has enough inventory at hand, primarily because of the undesired delay costs (Pentico and Drake, 2009). At the same time, studies that account for partial backordering in periodic review inventory models, and apply the effect of supply disruptions, consider the impact of review interval exogenously, using a random variable with a specific probability distribution. This modeling method neglects the optimality of replenishment decisions, and in cases in which the length of review interval is expected to increase, they suggest a higher inventory at hand that again raises the holding costs and the risk of inventory obsolescence. Therefore, such an assumption leads to the inefficiency of the inventory system and the suboptimality of decisions.

Motivated by these concerns, in this paper, an alternate model is examined to reduce ordering, holding, and shortage costs (mix of backorders and lost sales) by considering the optimal frequency and level of the replenishments, particularly in the base stock $(S,T)$ periodic review policy. The proposed model implicitly assumes the randomness of the review interval and captures its uncertainty using the concept of all-or-none strategy, i.e., the case of full disruptions. Accordingly, supply failures lead to an increase in the lead time from zero (in the business-as-usual scenario) to multiplications of the review interval, based on the number of sequential disruptions that are probable to occur. The probability of disruptions is considered according to the experience of decision-makers and based on being somewhat optimistic or pessimistic. Regarding these assumptions, two inventory costing policies are developed (i.e., end-of-cycle and continuous costing) to aid practitioners based on their financial system. Finally, a novel solution algorithm is presented to make the proposed model computationally tractable. The results of this study bring retailers one step closer to solving the problem of supply disruptions and how they can be better adapted for responding to such undesirable conditions.

Based on the above discussions, the main contributions of this paper are as follows: Firstly, developing two efficient stochastic periodic review base stock $(S,T)$ inventory model with partial backorders to deal with disruptions in the supply side. Secondly, investigating the effects of end-of-cycle and continuous costing to aid financial managers who use these conventional costing strategies. Thirdly, describing the policies to set a suitable value for the backordered fraction in different scenarios of supply disruption. Next, designing a practical and tractable solution algorithm for the proposed problem, and finally, deriving managerial insights from the results of the sensitivity analysis.

The rest of this paper is structured as follows: In section 2, the background of this study is reviewed, and the contributions of this study are discussed; in section 3, the definition of the problem and its corresponding assumptions are provided in detail, and two stochastic periodic-review models under two different costing policies are formulated; in section 4, the convexity of proposed objective functions is proved, and a suitable solution method, as well as an algorithm for computational applications, is provided; in section 5, the mathematical model is validated and is analyzed by changing the value of its parameters. Moreover, managerial insights for real-world scenarios are presented. Finally, in section 6, the results of this paper are discussed, and future research directions are suggested.

2. Literature review

Researchers have paid more attention to the uncertainties at the demand side, which typically is not as critical as uncertainties in the supply side (Konstantaras et al., 2019; Schmitt et al., 2015; Noh et al., 2019). As described, a supply disruption is recognized as an event that differs from other types of uncertainties in a supply chain in terms of its negative impacts (Ali et al., 2018). In this section, the studies that have investigated the supply disruptions are reviewed. In the end, the contributions of this paper are described in detail.

2.1. Inventory management under supply disruptions

Initial models that analyzed supply disruptions consider supply unavailability for an unknown period in the future. Parlar and Berkin (1991) introduced an extension to the economic order quantity (EOQ) model and developed an EOQD with two exponential random variables for supply availability (wet period, WP) and unavailability (dry period, DP) cycles. In their model, all shortages result in lost sales. Also, the replenishment occurs when the inventory level drops to zero. Their model acts as the classic economic order quantity model when the length of the DP gets closer to zero. Since then, other pieces of research have provided various extensions of Parlar’s and Berkin’s model to cope with real-world situations in systematic analyses of disruptions. For instance, Parlar and Perry (1996) extended the previous model into a case when two suppliers exist, and when shortages are backordered. However, they solved the model through numerical samples and provided a suboptimal solution. Göürler and Parlar (1997) used the Erlang- $k$ probability distribution function (PDF) and extended the applicability of the previous study. Similar studies have introduced revisions, solution methods, and
extensive of Parlar’s and Berkin’s research (e.g., Berk and Arreola-Risa (1994) and Snyder (2006)).

Another stream of studies followed the idea of Parlar and Berkin (1991) to develop stochastic models to manage the supply disruptions in periodic review inventory systems. Gupta (1996) developed a continuous periodic review \( (r, Q) \) model with Poisson demand and lost sales. He analyzed impacts of the lead-time on reorder point of the buyer. Song and Zipkin (1996) extended a model with variable lead time and order quantity. Parlar (1997) considered the \( (r, Q) \) periodic review model with pure backordering. Arreola-Risa and DeCroix (1998) proposed a stochastic \( (s, S) \) model with partial backordering. They concluded that the relationship between the cost parameters is the critical factor that affects system optimality under disruptions. Ozekici and Parlar (1999) examined an inventory system with an infinite planning horizon and considered that the purchasing costs are linearly related to the lot sizes. They gained an optimal solution for the order up-to-level \( (S) \). Later, Mohebbi (2004) studied continuous-review policy in an inventory control system where demand follows compound Poisson, \( WP \) follows general, and \( DP \) follows hyper-exponential PDFs.

Recently, Garvey and Carnovale (2020) examined the propagation of supply chain disruptions in a news-vendor inventory policy. They concluded that a local risk at a specific member of the network has greater importance than non-local risk. Similar to our study, Saithong and Lekhavat (2020) have analyzed the effects of supply disruptions in a base stock \( (S, T) \) periodic review policy and optimization of the base stock level \( (S) \), but neglected the effect of disruptions on optimal review interval and considered a continuous random variable for supply disruption length.

Focusing on financial features has become an attractive sub-category in inventory management models with disruptions. For instance, Taleizadeh (2017) developed an EOQD with partial backordering, where the supplier requests multiple prepayments from the retailer. Recently, Konstantaras et al. (2019) introduced an EOQD model under base stock periodic review policy with pure backordering and analyzed end-of-cycle and exact continuous costing schemes, which was introduced by Rao (2003) and extended by Lagodimos et al. (2012).

2.2. Strategies to analyze supply disruptions

In the literature, some papers focused on the existence of multiple suppliers in the case of disruptions. Tomlin and Wang (2005) considered a multi-product supply chain model with demand uncertainty and suppliers’ diversity. Tomlin (2006) examined flexible sourcing contracts with multiple unreliable and reliable suppliers. The results of his study show that choosing a reliable supplier is more expensive most of the time. Chopra et al., 2007, Schmitt et al. (2010), and Chen et al. (2012) continued Tomlin’s studies. Dada et al., 2007 studied the existence of reliable and unreliable suppliers using the newsvendor setting. Tomlin (2009a) studied strategies for disruption management and introduced inventory mitigation methods. Tomlin (2009b) examined multi-sourcing strategies by forecasting the supplier’s reliability when data about disruption are updated.

Many researchers examined internal disruptions in which the supplier may not provide the whole or partial quantity desired by the retailer (Grosfeld-Nir and Gerchak, 2004; Yano and Lee, 1995). These models focus on production/procurement (yield) randomness (Xia et al., 2004). Qi et al., 2009 presented an extension to the EOQD model to handle stocks using continuous-review policy where the retailer may encounter both internal and external supply disruptions. Li and Chen (2012) studied internal supply disruptions in a multi-echelon partial backordering inventory system. Wang et al., 2014 explored a condition where disruptions cause breakdowns to production and developed a multi-period model under periodic review policy, random disruptions, and random yields. Paul et al., 2014 suggested an online recovery strategy with partial backordering, in which disruptions in the production sector occur in two-phase for a single product.

Skouri et al. (2014) developed a model where the supplier delivers random, imperfect supply batches for the retailer. They solved a two-dimensional optimization model in which defective items are entirely rejected using the Bernoulli process. Ritha and Francis-Nishandhi (2015) modeled an EOQD under a two-echelon inventory system and concluded that higher supply quality brings cost efficiency for the course. Salehi et al., 2016 developed the first EOQD while accounting for partial backordering with random disruptions. Later, Taleizadeh et al. (2016) proposed a model for equivalent production systems that have three levels in the supply network and utilized a heuristic algorithm to solve the problem. Rezaei (2016) employed sampling inspection plans to find EOQ value when supplied items are imperfect. The study was further investigated by Taleizadeh et al. (2016), where the defective supply batches can be repaired. Recently, Taleizadeh and Dehkordi (2017) extended previous studies and assumed partial backordering condition for the unmet demand. Bakal et al. (2017) investigated the value of information about disruption and observed that disruption risk is more significant when the shortage cost is relatively high. Seygen and Sargut (2019) analyzed a continuous-review inventory system under EOQD conditions and considered that the disruptions might occur randomly for both the supplier and the retailer. Recently, Avci (2019) and Yoon et al. (2020) examined the importance of lateral transshipments and having information sharing between members of echelons to mitigate the impact of disruptions. Also some related research can be found in Taleizadeh et al. (2013, 2016a,b) and Lashgary et al. (2016).

2.3. Gap analysis and our contributions

The position of this paper in the relevant literature is illustrated in Fig. 1. In this study, compared to the study of Pentico and Drake (2009), who developed EOQ with partial backordering, a new base-stock periodic review policy is considered to model the case of supply disruptions (for an excellent review of partial backordering inventory models, see Pentico and Drake (2011)). Moreover, this paper extends Konstantaras et al., 2019 study by considering the feasibility of having partial backorders.

Based on the above discussions, we fill the current gap by presenting an inventory model consisting of cost dimensions such as ordering, holding and shortage (both lost sales and backorders) under the base stock policy. Indeed, no paper has reflected the impact of consumer behavior, costing schemes, and critical parameters in controlling supply disruptions as well as the joint optimization of the base stock \((S, T)\) policy variables in an inventory management framework. To advance this study, a tractable solution algorithm using derivative optimization is presented. Finally, this research examines the application of the model in several scenarios of changes in input parameters, using a set of sensitivity analyses, to illustrate the effectiveness and validity of the proposed approach and to obtain managerial insights.

3. Decision framework

In this section, the properties of the problem under examination are described.

3.1. Problem definition

In the past years, mitigating the impact of the discussed supply-side disruptions has become an increasingly important topic for the managers. Research on the reasons for and consequences of such interruptions has focused on objective measures of cost, environmental and social sustainability (Ahmed and Sarkar, 2019; Mishra et al., 2020; Ullah, and Sarkar, 2020), and measures of resilience (Torabi et al., 2015). Accordingly, there has been little work exploring experiences of the inventory systems and, particularly, to model the consumer behavior
in the cases in which their demand is postponed or completely failed to be met by the retailer. Although there have been some recent attempts at integrating this effect in inventory models with supply-side disruptions, they lack optimization of the system from multiple aspects and in a simultaneous way.

Adoption of extreme cases for order fulfillment, i.e., letting all demand to be lost or delayed to be met has been shown to have negative associations with the total costs of the system (due to goodwill losses and inventory obsolescence risks) and is becoming an area of increasing concern between academia and practitioners. Therefore, addressing this problem will have practical benefits for the retail, logistics, and operation managers and contribute to analyzing the widespread and frequent phenomenon of supply-side disruptions. Moreover, focusing on decision-makers’ experiences can help develop more robust decisions to be more flexible and responsive in scenarios when demand is high, but supply is failed to be delivered on-time (Laato et al., 2020).

This research aims to investigate an inventory system under supply-side disruptions to minimize ordering, holding, and shortage costs for the retailer. The diagram representing the inventory profile in this situation is illustrated in Fig. 2. As can be seen, in a condition where the supply process is reliable, the lots at size $DT$ are sent to the retailer at the time of review.

The supply chain considered for the investigation is two-echelon and consists of a supplier and one retailer (see Fig. 3). The supplier has contracted with the retailer to deliver the items at the requested time (Case 1 in Fig. 3) in the business-as-usual scenario, while the retailer only has inventory and follows the base stock $(S,T)$ periodic review policy. The retailer periodically reviews inventory level and orders products in batches from the supplier. However, due to the improper quality of the supplier’s production process (Sarkar et al., 2017) as an internal disruption or an external disruption such as a disaster that occurs at supply-side (Hishamuddin et al., 2014), he/she may fail to deliver the products for the retailer successfully or on-time. This case gets more practical in situations viability of the supply chains is a concern. The viability policy measures how two firms inter-connectedly face disruptions (such as a pandemic) and how to minimize the effect of disruptions to survive from them (Ivanov and Dolgui, 2020). As can be seen, since the disruption is propagated to the downstream of the supply chain, the retailer faces shortages in an unknown number of periods until the supplier compensates for its failure by procuring previously ordered batches of products with the desired amount and quality (Case 2 in Fig. 3) (Konstantaras et al., 2019). In the first case, the consumer goodwill loss cost is equal to zero; therefore, all demand is met. However, in the second case, a portion of consumers wait to get their desired products, while others leave the firm to shop from other brands. Thus, the problem is to model this case in order to reduce the total long-run costs for the retailer.

### 3.2. Notations

Multiple symbols have been used to display parameters, variables, and functions in the modeling process. As the analyses in inventory management usually do not follow the same rule on indicating model components, proper notations are defined to make it easier for the reader to understand their meaning:
Parameters:

- \( A \): Replenishment cost (per replenishment)
- \( b \): The cost of the backordered demand (per unit period)
- \( \beta \): The portion of maximum expected shortages that are backlogged (percent)
- \( D \): Demand rate (units per unit time)
- \( p \): Disruption or delivery failure probability
- \( g \): The goodwill loss while encountering lost sales (per unit)
- \( h \): The holding cost of an item (per unit period)

Functions:

- \( \mathcal{I}_{\tau} \): Average inventory at hand when an epoch begins with inventory state \( \tau \in \{1, 2, \ldots, m + 1\} \)
- \( \mathcal{B}_{\tau} \): Average backorders when an epoch begins with inventory state \( \tau \in \{m + 1, m + 2, \ldots\} \)
- \( L_{\tau} \): The lost sale quantity when an epoch begins with inventory state \( \tau \in \{m + 1, m + 2, \ldots\} \)
- \( \pi_{\tau} \): Probability of having \( \tau - 1 \) consecutive disruptions before an inventory epoch or interchangeably when an inventory starts with state \( \tau \in \{1, 2, \ldots\} \)
- \( E(.) \): Indicates expected value
- \( ABC \): Long-run expected backordering cost
- \( AFC \): Long-run expected fixed ordering cost
- \( AGC \): Long-run expected goodwill losses cost
- \( AHC \): Long-run expected holding cost
- \( ATC \): Long-run expected total cost

Variables:

- \( S \): The maximum or the order-up-to level [decision variable]
- \( T \): The time interval between two adjacent reviews [decision variable]
- \( m \): The ratio \( S/DT \) [decision variable]
- \( F \): The fraction of the review interval \( \tau = m + 1 \), that demand will be fulfilled
- \( \tau \): State random variable
- \( (*) \): Indicates the optimal value

3.3. Assumptions

Referring to Fig. 3, the following assumptions are considered while developing this model:

- Supply-side disruptions may occur more than one consecutive time, they are independent, and they are detected at the time of the review.
- The probability of supply-side disruptions is calculated by the random variable \( \tau \) used in a geometric PDF (Konstantaras et al., 2019; Skouri et al., 2014) (Equation (1)). This random variable represents the status of the inventory system in terms of the number of delivery failures encountered. Clearly, if no disruption occurs, \( \tau = 1 \), indicating a normal state.

\[
\pi_{\tau} = P(X = \tau) = p^{\tau-1}(1-p) \forall \tau \in \{1, 2, \ldots\} \quad (1)
\]

- At the time of the contract between supplier and retailer, the procurement lead time is promised to be zero. However, during the time of disruptions, the lead time increases from 0 to \( T, 2T, 3T, \ldots \) based on the probability of the random variable \( \tau \) for getting values 1, 2, 3, and so on.
- A constant portion of the unsatisfied demand is to be backlogged, and it generates a combination of lost sales and backorders (Pentico and Drake, 2009; Bijvank and Vis, 2011; Saithong and Lekhavat, 2020). Therefore, shortage costs imposed on the system are categorized as follows:
  a) Backorder cost is accounted per unit of unfulfilled demands and per period of a review interval,
  b) Lost sales cost is calculated when a unit of product cannot be provided for the customer, which leads to the goodwill loss on each unit of a lost sale.
• The demand of customers in the inventory system is considered to be deterministic.
• Residual inventories are kept for sales in further periods.

3.4. Mathematical modeling

In this section, an effective model to derive optimal and closed-form solutions for decision variables (i.e., the length of the review interval \( T \) and the order-up-to level \( S \)) is developed. Periodic-review inventory models, as in this paper, have been predominantly investigated using an end-of-cycle costing scheme (Arrow et al., 1951). This scheme can be useful when the enterprise is least sensitive to the inventory holding costs (Rudi et al., 2009). This applies to the cases where retailers hold an imperishable or reusable product. Another reason is that the responsiveness about the stockouts is more critical than the holding costs that imposes on the system. In this case, managers are concerned about keeping their competitive advantage through responsiveness at an ideal level. However, the end-of-cycle costing neglects the behavior of the system along each period, which is against the assumptions of exact keeping their competitive advantage through responsiveness at an ideal level. The retailer orders the quantity needed to reach the order-up-to level \( S \) at any state of inventory profile, i.e., \( \tau \). The corresponding cost is calculated during the period \( \tau \), and the order-up-to level \( S \) is equal to \( S = 0 \). Considering the unit in \( \$ \), the complete reduction in the level of held inventory. In other words, the complete area under the net stock inventory diagram with positive value is the expected average inventory at hand, according to three segments introduced the expected average inventory at hand by Equation (5):

\[
E_1\left(T_0\right) = \sum_{i=1}^{\infty} \pi I_i = \sum_{i=1}^{m-1} \pi_i \left[ \left( m-\tau \right) DT + I_{\tau} \right] + \pi \sum_{i=m}^{T+1} I_{\tau} \]
\[
= \sum_{i=1}^{m-1} \pi_i \left[ \left( m-\tau \right) DT \right] + \pi \sum_{i=m}^{T+1} I_{\tau} \]

Equation (5) shows that the expected value of average inventory at hand, which is the weighted mean of a large number of independent observations of random variable \( \tau \) multiplied by \( T_0 \), is equal to \( m \tau \). After placing the expected average inventory in \( AHC(1) \), and considering the unit inventory holding cost as well as values of PDF for \( \tau \leq m \) and \( \tau = m \), the long-run expected holding cost is simplified to Equation (6):

\[
AHC(1) = h E_1\left(T_0\right) = h \left[ \left( DT \left( m-\tau \right) + DT \right) + \left( DT \right) \right]
\]

3.5. Fixed ordering cost

The retailer orders the quantity needed to reach the order-up-to level \( S \) at any state of inventory profile, i.e., \( \tau \). The corresponding cost is only related to the length of the review interval. In both costing strategies, the long-run expected fixed ordering cost will be as Equation (3):

\[
AFC = \frac{A}{T}
\]

3.6. Holding cost

A primary difference between the two costing strategies is the method of calculating the holding cost for the periodic review system. For a better description, holding costs are studied in two consecutive sub-sections.

3.6.1. Holding cost in strategy 1

In a continuous costing scheme, \( AHC \) is calculated during the reduction in the level of held inventory. In other words, the complete area under the net stock inventory diagram with positive value is the average inventory at hand. To efficiently model the effect of consecutive supply-side disruptions, suppose that \( m \) is a variable that indicates the value of \( S/DT \). This variable aids to account for the number of epochs that are left before facing a shortage scenario. For instance, if the optimal base stock level \( S \) is equal to \( DT \), then after one epoch, the inventory profile reaches zero. However, if \( S > DT \), then according to \( m \), there may be a leftover inventory of about \( S - mDT \) before experiencing shortages. Referring to Fig. 4, in the case of sequential disruptions, the inventory state random variable \( \tau \) ranges from 1 to \( m+1 \). Based on this backdrop, the formulation for average inventory at hand, according to three segments denoted by Fig. 4, is offered by Equation (4).

\[
\begin{align*}
I_0 & = \frac{1}{T} \left( \left( m-\tau \right) DT^2 + I_{\tau} \right), \quad \tau \leq m - 1 \\
I_{m+1} & = \frac{1}{T} \left( \left( S - mDT \right) T + DT^2 \right), \quad \tau = m \\
I_{2m+1} & = \frac{1}{T} \left( \left( S - mDT \right) \right), \quad \tau = m + 1
\end{align*}
\]

This formulation is created regarding the inventory diagram in Fig. 4, which is consisted of trapezoids, parallelograms, and a triangle. For instance, in segment 1, \( \left[ m-\tau \right] DT^2 \) represents the area of trapezoids above the shaded region, in the observations in which \( \tau \leq m - 1 \). The area of the shaded region is also equal to the case where \( \tau = m \). Let us introduce the expected average inventory at hand by Equation (5):

\[
E_1\left(I_0\right) = \sum_{\tau=1}^{\infty} \pi I_\tau = \sum_{\tau=1}^{m-1} \pi I_\tau + \pi I_m + \pi I_{m+1}
\]

3.6.2. Holding cost in strategy 2

Despite the previous case, the average inventory at hand will only be affected by the leftover stock at the end of an inventory epoch. As illustrated by Fig. 5, the inventory epochs consist of the shaded rectangular shapes. Same as calculation procedure for Equations (4) and (5), the average inventory at hand is derived as follows:
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A. Case under end-of-cycle costing where the retailer faces shortages that consist of backorders and lost sales. To ensure the total long-run backlogging cost is calculated differently for long-run expected cost for the backlogs and lost sales cost. Besides, the long-run expected shortage cost consists of two strategies, they are separated to be described in the following paragraphs.

3.7.1. Backorders cost in strategy 1

According to Fig. 6, during the time interval in state \( m + 1 \), the average of backorders for states after \( (m + 1) \)th cycle, regarding the value of \( S - mDT \) as shown previously by Figs. 4 and 5. Let us show the formulation in the total long-run expected backorders cost Equation will yield to:

\[ ABC(1) = bE_T(\bar{B}_1) = b \left( \rho^n (1 - p) \betaDT - \frac{3DT}{2} \right) + \left( \frac{p^{n+1}}{1 - p} \right) DT \]

3.7. Total shortage cost

In the case of disruption, there is no knowledge about the time when a stockout will occur. However, it is known that in the pessimistic case, shortages will start when the inventory state random variable reaches \( m + 1 \). In other words, when the level of inventory at hand reaches the value \( S - mDT \) as shown previously by Figs. 4 and 5. Let us show the length of \( (m + 1) \)th inventory epoch in which the inventory level is positive by \( FT \). According to Pentico and Drake (2009), \( F \) is a fraction of a cycle where the demand of customers is filled from stock. In this study, the value of \( F \) is computed by Equation (10).

\[ F = \frac{k}{T} = \frac{S}{DT} - m \]

It is noteworthy that the long-run expected shortage cost consists of long-run expected cost for the backlogs and lost sales cost. Besides, the total long-run backlogging cost is calculated differently for two strategies, they are separated to be described in the following subsections.

3.7.1. Backorders cost in strategy 1

According to Fig. 6, during the time interval in state \( m + 1 \), the average of backorders for states after \( (m + 1) \)th cycle, regarding the value of \( S - mDT \) as shown previously by Figs. 4 and 5. Let us show the formulation in the total long-run expected backorders cost Equation will yield to:

\[ ABC(1) = bE_T(\bar{B}_1) = b \left( \rho^n (1 - p) \betaDT - \frac{3DT}{2} \right) + \left( \frac{p^{n+1}}{1 - p} \right) DT \]

3.7. Total shortage cost

In the case of disruption, there is no knowledge about the time when a stockout will occur. However, it is known that in the pessimistic case, shortages will start when the inventory state random variable reaches \( m + 1 \). In other words, when the level of inventory at hand reaches the value \( S - mDT \) as shown previously by Figs. 4 and 5. Let us show the length of \( (m + 1) \)th inventory epoch in which the inventory level is positive by \( FT \). According to Pentico and Drake (2009), \( F \) is a fraction of a cycle where the demand of customers is filled from stock. In this study, the value of \( F \) is computed by Equation (10).

\[ F = \frac{k}{T} = \frac{S}{DT} - m \]

It is noteworthy that the long-run expected shortage cost consists of long-run expected cost for the backlogs and lost sales cost. Besides, the total long-run backlogging cost is calculated differently for two strategies, they are separated to be described in the following subsections.

3.7.1. Backorders cost in strategy 1

According to Fig. 6, during the time interval in state \( m + 1 \), the average of backorders for states after \( (m + 1) \)th cycle, regarding the value of \( S - mDT \) as shown previously by Figs. 4 and 5. Let us show the formulation in the total long-run expected backorders cost Equation will yield to:

\[ ABC(1) = bE_T(\bar{B}_1) = b \left( \rho^n (1 - p) \betaDT - \frac{3DT}{2} \right) + \left( \frac{p^{n+1}}{1 - p} \right) DT \]
3.7.2. Backorders cost in strategy 2

In this case, similar to average inventory at hand, the average backorder is calculated as the total occurred at the end of each epoch (see shaded rectangles in Fig. 7).

The average backorder for each state in strategy 2, and the corresponding expected backorders equal to:

$$B_2^c(\tau) = \begin{cases} \beta(m + 1)(DT - S), & \tau = m + 1 \\ (\tau - m - 1)\beta DT + B_{m+1}, & \tau \geq m + 2 \end{cases}$$

(14)

And,

$$E_2(B_2) = \sum_{\tau=m+1}^{\infty} \pi_{\tau} B_2 + \sum_{\tau=m+2}^{\infty} \pi_{\tau} [(\tau - m - 1)\beta DT + B_{m+1}]$$

(15)

Finally, $ABC(2)$ is calculated as follows:

$$ABC(2) = bE_2(B_2) = b \left[ p^n \beta ((m + 1) DT - S) + \frac{p^{m+1}}{1 - p} \beta DT \right]$$

(16)

3.8. Objective functions

As stated, the total cost function for both end-of-cycle and continuous costing strategies have the same form. Thus, the extended form of the long-run expected total cost objective function is:

$$ATC = AFC + AHC + ABC + AGC = \frac{A}{T} + \sum_{\tau=1}^{\infty} \beta \pi_{\tau} J_{\tau} + \sum_{\tau=1}^{\infty} \beta \pi_{\tau} B_{\tau} + \sum_{\tau=1}^{\infty} g \pi_{\tau} L_{\tau}$$

(20)

The ordering, holding, and shortage costs will be substituted in this formulation, to obtain the dedicated cost functions for both methods.

3.8.1. The objective function in strategy 1

For the continuous costing model, $ATC(1)$ is obtained as follows:

$$s.t. \quad m = \frac{S}{DT}$$

(22)

As can be seen, in this formulation, three variables, i.e., $(S, T, m)$, are used. After factorizing the model parameters and simplifying its form, the optimization model for the continuous costing strategy is obtained in the form below:

$$\text{Min } ATC(1) = \frac{A}{T} + \left\{ \begin{array}{l}
\text{FixedOrderCost} \\
\text{HoldingCost} \\
\text{BackorderCost} \\
\text{LostSalesCost}
\end{array} \right\}$$

(21)

3.7.3. Lost sales cost in both strategies

Referring to Figs. 6 and 7, it can be seen there is no difference in the value of lost sales because this is vertically calculated from the difference between the current position of inventory and its position when $\beta = 1$ at the end of each cycle. As a result:

$$L_i = \begin{cases} (1 - \beta)DT \left( T - \frac{S}{D} - mT \right), & \tau = m + 1 \\ (\tau - m - 1)(1 - \beta)DT + L_{m+1}, & \tau \geq m + 2 \end{cases}$$

(17)

And,

$$E(L_i) = \sum_{\tau=m+1}^{\infty} \pi_{\tau} L_{\tau} + \sum_{\tau=m+2}^{\infty} \pi_{\tau} [(\tau - m - 1)(1 - \beta)DT + L_{m+1}]$$

(18)

Finally, the expected long-run goodwill losses cost is:

$$AGC = gE(L_i) = g \left( \frac{DTp^{m+1}(1 - \beta)}{1 - p} + Dp^n(1 - \beta) \left( T - \frac{S - DTm}{D} \right) \right)$$

(19)

3.8. Objective functions

As stated, the total cost function for both end-of-cycle and continuous costing strategies have the same form. Thus, the extended form of the long-run expected total cost objective function is:
\[ \lambda_3 = \frac{Dh(1 + p)}{2(1 - p)} > 0 \]  
(32)

\[ \lambda_0 = A > 0 \]  
(33)

### 3.8.2. The objective function in strategy 2

The ATC(2) minimizes expected total costs considering the end-of-cycle costing scheme, as follows:

\[
\text{Min } \text{ATC}(2) = \frac{A}{T} + \left\{ \begin{array}{l}
\text{HoldCost} \\frac{DT(m(1-p) + p^n - 1)}{1-p} + (1-p^n)(S-mDT) + \\
\text{BackorderCost} b \left( \frac{p^n \beta (m+1) DT - S}{1-\beta} \right) + S \left( \frac{p^{n+1} (1-\beta) DT + p^n (1-\beta) D}{1-\beta} \right) - S \left( T - m DT \right) / D
\end{array} \right.
\]  
(34)

\[
\text{s.t. } m = S / DT
\]  
(35)

As before, the simplified model will be:

\[
\text{Min } \varphi_2(S, T, m) = -\gamma_1 S p^n + \gamma_2 S + \gamma_3 (\beta) m T + \gamma_4 T p^n - \gamma_5 T + \gamma_6 \frac{1}{T}
\]  
(36)

\[
\text{s.t. } m = S / DT
\]  
(37)

In this case, \( \gamma \) parameters are used, which are defined as follows:

\[
\gamma_1 = h + g(1-\beta) + b\beta > 0
\]  
(38)

\[
\gamma_2 = h > 0
\]  
(39)

\[
\gamma_3 = D(h + b\beta + g(1 - \beta)) > 0
\]  
(40)

\[
\gamma_4 = \frac{D(h + b\beta + (1-\beta)(g(1-p) + p))}{1-p} > 0
\]  
(41)

\[
\gamma_5 = \frac{Dh}{1-p} > 0
\]  
(42)

\[
\gamma_6 = A > 0
\]  
(43)

### 4. Solution method

In this section, the optimal solution vector \( (S, T, m) \) in described models is obtained in closed form. In the end, an algorithm is provided for future computing applications.

#### 4.1. Deriving closed-form equations

Before calculating optimal decision variables, the convexity of the proposed objective functions for any given \( m \) is proved. Then, by utilizing bracket rules, an optimal solution for the decision variable \( m \) is derived. Lastly, by taking the first partial derivatives of the objective functions, other variables are also optimized. However, as will be seen in strategy 2, the classic method of taking partial derivatives is not applicable.

#### 4.1.1. Optimality in strategy 1

Consider a given value for \( m \). It can be proved that taking partial derivatives of \( \varphi_1(S, T, m) \) in Equation (23) concerning \( S \) and \( T \), yields to the following results:

\[
\frac{\partial \varphi_1(S, T)}{\partial T} = - \lambda_4 p^n S + \lambda_3 \beta^n m^2 + \lambda_2 p^n m + \lambda_1 p^n - \lambda_m \frac{1}{T^2}
\]  
(44)

Thus, the optimal value of \( S \) is found. After that, the convexity of \( \varphi_1(S, T, m) \) for any given \( m \) is proved by using the Hessian rule as follows:

\[
\begin{align*}
\frac{\partial^2 \varphi_1(S, T)}{\partial S^2} & = 2 \lambda_4 p^n S - \lambda_3 \beta^n m - \lambda_2 p^n + \lambda_m > 0 \\
\frac{\partial^2 \varphi_1(S, T)}{\partial S \partial T} & = 2 \lambda_4 p^n - \lambda_m \frac{1}{T^2} > 0 \\
\frac{\partial^2 \varphi_1(S, T)}{\partial T^2} & = - \lambda_4 \frac{2 S \beta^n}{T^3} > 0
\end{align*}
\]  
(45)

(46)

(47)

(48)

Afterward, the convexity of \( \varphi_1(S, T, m) \) for any given \( m \) is proved by using the bracket rule and then:

\[
S = \int \frac{(- \lambda_4 p^n + \lambda_3 \beta^n (m + \lambda_2))}{2 \lambda_4 p^n} dt
\]  
(49)

\[
S = \int \frac{(- \lambda_4 p^n + \lambda_3 \beta^n (m + \lambda_2))}{2 \lambda_4 p^n} dt
\]  
(50)

The optimal value of \( S \), as shown in Equation (50), is not in the closed-form and depends on optimal values of \( m \), \( S \), and \( T \). As we need to find the optimal \( m \) which is equal to \( \frac{2 \lambda_4 S}{T^2} \), we use the bracket rule and then:

\[
m \leq \frac{2 \lambda_4 S}{T^2} < m + 1
\]  
(51)

By substituting \( S \) from Equation (50) into Equation (51) and rearranging it, we have:

\[
\frac{\lambda_4 S}{\lambda_3} \leq p^n < \frac{\lambda_4}{\lambda_3 - \lambda_2}
\]  
(52)

By taking logarithm from each side, we will yield to a unique \( m' \) that satisfies the required tolerances. After some calculations, the optimal value of \( m \) will be as follows:
Finally, embedding \( m^* \) and then \( S^*(T; m^*) \) into the objective function in Equation (23) yields:

\[
\varphi_1(T) = \lambda_2 \frac{T}{T^*} + \varphi_1(m^*)
\]

The value of \( \varphi_1(m^*) \) is indicated using the \( \lambda \) parameters, as follows:

\[
\varphi_1(m^*) = \left( \lambda_1 - \lambda_3 \right) \frac{p^*}{\lambda_2} + 2 \lambda_4 m^* - \lambda_5 \frac{1}{p^*} + (2 \lambda_1 \lambda_4 - 4 \lambda_4 \lambda_3)
\]

Since \( p^* \geq \frac{1}{m} \) then \( \varphi_1(m^*) \) is always positive. Finally, we will derive optimal by \( \hat{T} \) differentiating Equation (54) to yield:

\[
\hat{T} = \frac{4 \lambda_1 \lambda_4}{\varphi_1(m^*)}
\]

Moreover, the value of \( F \) for review interval in the state \( r = m + 1 \) is calculated as follows:

\[
F^* = \frac{S^*}{DT^*} - m^*
\]

This value is used as an indicator for the fraction of \( (m + 1) \)th cycle

\[
m^* = \left( \frac{\lambda_2}{\lambda_1} \right) \frac{\log(p)}{\log(\lambda_2)}
\]

Finally, embedding \( m^* \) and then \( S'(T; m^*) \) into the objective function in Equation (23) yields:

\[
\varphi_2(S, T, m) = \frac{\lambda_2}{\lambda_1} \frac{T}{T^*} + \varphi_2(S, T, m)
\]

The value of \( \varphi_2(S, T, m) \) is indicated using the \( \lambda \) parameters, as follows:

\[
\varphi_2(S, T, m) = \left( \lambda_4 - \lambda_3 \right) \frac{p^*}{\lambda_2} + 2 \lambda_4 m^* - \lambda_5 \frac{1}{p^*} + (2 \lambda_1 \lambda_4 - 4 \lambda_4 \lambda_3)
\]

Since \( p^* \geq \frac{1}{m} \) then \( \varphi_2(S, T, m) \) is always positive. Finally, we will derive optimal by \( \hat{T} \) differentiating Equation (54) to yield:

\[
\hat{T} = \frac{4 \lambda_1 \lambda_4}{\varphi_2(S, T, m^*)}
\]

Moreover, the value of \( F \) for review interval in the state \( r = m + 1 \) is calculated as follows:

\[
F^* = \frac{S^*}{DT^*} - m^*
\]

where the demand of customers is filled from stock.

### 4.1.2. Optimality in strategy 2

The formulation of the objective function represented for strategy 2 is linear-wise in \( S \) (Equation (36)). Taking the second-order partial derivative of \( T \) results in:

\[
\frac{d^2 \varphi_2}{dT^2} = \frac{2 \gamma_5}{T^2} > 0
\]

Therefore, \( \varphi_2(S, T, m) \) is convex both on \( S \) and \( T \) for any given \( m \). In this model, differentiating from \( S \) in \( \varphi_2(S, T, m) \) does not aid to yield optimal solutions. Accordingly, we use a method proposed by Konstantaras et al. (2019).

At first, it is determined that for any \( T \), the optimal condition is when \( S/DT = m \in Z^+ \). Without loss of generality, we have already seen that the optimal value of \( T_n \) equals zero (Fig. 5). Hence, considering \( S \in R^+ \) and Figs. 5 and 7, we will have:

\[
T_n + T_{n+1} = (1 - \beta)S + DT(\beta - (1 - \beta)m) = (1 - \beta)(S - mDT) + \beta DT
\]

With the assumption \( x \in [0, 1] \) we have:

\[
T_n = x((1 - \beta)(S - mDT) + \beta DT)
\]

And
Thus, regarding the sign of the element $p^{\theta_T}$ of $DT$, we get:

$$ATC(2) = \frac{A}{\phi_T} + h \frac{DT(m(1-p) + p^T - 1)}{1-p} + \left( (S(1-\beta) - DTm(1-2\beta))p^T \right. \left. + DTp^{\theta_T} - \frac{1}{1-\beta} \right) + g \left( ((m+1)DT-S)p^T(1-\beta) + DTp^{\theta_T}(1-\beta) \right) + \phi_T h(p^T + b)|S(1-\beta) - DTm(1-2\beta)|$$ \hspace{1cm} (62)

As it is seen, the last term in Equation (62) is linear concerning $x$. Thus, regarding the sign of the element $h - p^T(h + b)$, $ATC(2)$ is minimized, even if $x$ gets its supremum values. Hence, we conclude that $S/\phi_T = m \in Z^+$. Replacing the optimal value of $S$ into $\varphi_2(S, T, m)$ Equation (36) converts to:

$$\varphi_2(T, m) = \frac{T_\gamma}{T_1} + \frac{\gamma_T}{T_1} T^\gamma + \frac{\gamma_T}{T_1} \lambda T$$ \hspace{1cm} (63)

where $m \in Z^+$. From now on, we should prove that $\varphi_2(S', T, m)$ is discretely convex concerning the value of $m$. Using the forward difference operator $\Delta$, we have:

$$\Delta \varphi_2(T, m + 1) - \Delta \varphi_2(T, m) = T \varphi_2(T, m) - \varphi_2(T, m) = T (\frac{T_\gamma}{T_1} - \gamma_T \gamma T^\gamma(1-p))$$ \hspace{1cm} (65)

which is equal to zero when $p^T = \frac{T_\gamma}{T_1 \gamma T(1-p)}$. Since we proved the convexity of $\varphi_2(T, m)$ by $m$, we have $\Delta \varphi_2(T, m) \geq 0$. $\forall m \geq m'$. According to this conclusion, the inequalities, $\Delta \varphi_2(T, m') < 0$ and $\Delta \varphi_2(T, m') \geq 0$ are true for $m'$. These findings imply that:

$$p^T \leq \frac{T_\gamma}{T_1 \gamma T(1-p)} < p^T$$ \hspace{1cm} (66)

Finally, we will yield the optimal value for $m$ as follows:

$$m^* = \log \left( \frac{T_\gamma}{T_1 \gamma T(1-p)} \right) \log(p)$$ \hspace{1cm} (67)

By setting $m = m^*$ in $\varphi_2(T, m)$:

$$T^* = \frac{T_\gamma}{\theta_2(m^*)}$$ \hspace{1cm} (68)

where:

$$\theta_2(m^*) = \frac{\gamma_T T^\gamma - \gamma_T T^\gamma + \gamma_T T^\gamma m'}$$ \hspace{1cm} (69)

As indicated before, it is proved that $\theta_2(m^*)$ is typically positive. Therefore, the optimal value of $S$ above will be:

$$S^* = m'DT$$ \hspace{1cm} (70)

And $F'$ will be:

$$F' = \frac{S^* - m^*}{DT} = 0$$ \hspace{1cm} (71)

The significant result is that $F'$ typically equals zero when the end-of-cycle scheme is considered.

4.2. Solution procedure

The following solution procedure and steps should be considered for each strategy to acquire optimal decision variables for the models.

4.2.1. Computing steps in strategy 1 (continuous costing)

1. Compute $\lambda_1, \lambda_2, ... , \lambda_6$ from Equations (25)–(33).
2. Calculate $m^*$ using Equation (53).
3. Calculate $\theta_2(m^*)$ using Equation (55).
4. Compute $S^*$, $T^*$ and $F'$ using Equations (50), (56) and (57).

4.2.2. Computing steps in strategy 2 (end-of-cycle costing)

1. Compute $f_1, f_2, ... , f_6$ from Equations (38)–(43).
2. Calculate $m^*$ using Equation (67).
3. Calculate $\theta_2(m^*)$ using Equation (69).
4. Compute $S^*$, $T^*$ and $F'$ using Equations (70) and (68) and set $F' = 0$.

For a comprehensive illustration of the proposed solution algorithms, the reader is recommended to refer to Fig. 8.

5. Computational experiments

To evaluate the validity and efficiency of the solution algorithm, at first, a numerical example is designed and will be solved for both costing strategies. Second, the effects of change in model parameters will be examined to derive insightful managerial ideas. Third, an extensive set of analyses will be provided that suggest considerations while selecting the value of the partial backordering factor, in each scenario of supply disruptions. Unlike previous studies, these analyses consider all aspects of the problem under study.

5.1. Numerical examples

Consider $D = 250, A = 30, h = 3, g = 2, b = 5, p = 0.15$, and $\beta = 0.7$. For continuous costing strategy, the following steps are taken to reach the optimal value for the decision variables:

Step 1.

$$\lambda_1 = \frac{(1-p)(h + b\beta)}{2D} = \frac{(1-0.15)(3+1(0.7))}{2(250)} = 0.0063,$$

$$\lambda_2 = (1-p)(h + b\beta) = (1-0.15)(3+1(0.7)) = 3.145,$$

$$\lambda_3 = h + b\beta + g(1-\beta) = 3+1(0.7)+2(1-0.7) = 4.3,$$

$$\lambda_4 = h = 3,$$

$$\lambda_5 = \frac{D(1-p)(h + b\beta)}{2} = \frac{250(1-0.15)(3+1(0.7))}{2} = 393.125,$$

$$\lambda_6 = D(g(1-\beta) + h + b\beta) = 250(2(1-0.7) + 3+1(0.7)) = 1075,$$

$$\lambda_7 = \frac{D(2g(1-\beta) + (1+p)(h + b\beta))}{2(1-p)} = \frac{250(2(1-0.7) + (1+0.15)(3+1(0.7)))}{2(1-0.15)} = 802.206.$$
\[ \lambda_k = \frac{Dh(1 + p)}{2(1 - p)} = \frac{250(3)(1 + 0.15)}{2(1 - 0.15)} = 507.353, \]
\[ \lambda_0 = A = 30. \]

Step 2.
\[ \log \left( \frac{m}{\bar{m}} \right) = \log(0.15) = 0 \]

Step 3.
\[ \theta_i(m') = \left( 4\lambda_i \lambda - \lambda_i^2 \right) \frac{\mu^\alpha}{\beta(m)} + 2\lambda_i \lambda m' - \lambda_i^2 \frac{\gamma}{\beta(m)} + \left( 2\lambda_i \lambda - 4\lambda_i \lambda_i \right) \]
\[ = (4(0.0063)(802.206) - (4.3)^2)(0.15)^1 + 2(3.145)(3)(0) \]
\[ - (3)^2 \frac{1}{0.15} + (2(4.3)(3) - 4(0.0063))(507.353) = 5.74 \]

Step 4.
\[ T' = \sqrt{\frac{4\lambda_1 \lambda_0}{\theta_1(m)} \mu^\alpha} = \sqrt{\frac{4(0.0063)(30)}{5.74}} = 0.363, \]
\[ S' = \sqrt{\frac{(-\lambda_1 + \mu^\alpha (\lambda_i m' + \lambda_1))}{2\lambda_i \mu^\alpha}} \cdot 0.363 \cdot \frac{(-3 + 0.15^1)(3.145(0) + 4.3)}{2(0.0063)(0.15^1)} = 37.511, \]
\[ F' = \frac{S'}{DT'} - m = \frac{37.511}{250(0.363)} = 0 = 0.4138. \]

Also, the optimal long-run expected total cost is $\text{ATC}^1 = 160.489$. For End-of-cycle costing the following steps are taken:

Step 1.
\[ r_1 = h + g(1 - \beta) + \beta h = 3 + 2(1 - 0.7) + (0.7) = 4.3, \]
\[ r_2 = h = 3, \]
\[ r_3 = D(h + \beta h + g(1 - \beta)) = 250(3 + 2(1 - 0.7) + (0.7)) = 1075, \]
\[ r_4 = \frac{D(h + \beta h + g(1 - \beta) + g(1 - \beta) + (p) + (1 - p))}{\lambda_1} = 250(3 + 0.7(1) + (1 - 0.15)) = 1251.5, \]
\[ r_5 = D(1 - p) = \frac{250(3)}{1 - 0.15} = 882.353, \]
\[ r_6 = A = 30. \]

Step 2.
\[ m' = \frac{\log \left( \frac{\mu^\alpha}{\theta_1(m)} \right) \cdot \log(0.15)}{\mu^\alpha} = 1. \]

Step 3.
\[ \theta_i(m') = \left( r_f r\mu^\alpha - \gamma \right), \]
\[ \gamma \left( (1075)(1) \right) = 238.1, \]
\[ \gamma = 0. \]

Step 4.
\[ T' = \sqrt{\frac{r_f}{\theta_1(m') \mu^\alpha}} = \sqrt{\frac{4(3)(30)}{238.1}} = 0.7361, \]
\[ S' = \frac{m' DT'}{1} = 1(250)(0.7361) = 184.025, \]
\[ F' = 0. \]

Also, the optimal value of the long-run expected total cost is $\text{ATC}^2 = 81.5115$.

Table 1

| Input | Changes (%) | Optimal decision variables | Percentage of change |
|-------|-------------|---------------------------|----------------------|
| $A$   | 60%         | 0.4592 47.448 0.41331 203.005 | 26.50% 26.49% -0.01% 26.49% |
|       | 30%         | 0.4139 42.7691 0.41333 182.986 | 14.02% 14.02% 0.00% 14.02% |
|       | -30%        | 0.3037 31.3839 0.41335 134.275 | -16.34% -16.36% 0.00% -16.33% |
| $b$   | 60%         | 0.3395 41.6858 0.49114 172.239 | -6.47% 11.13% 18.82% 7.32% |
|       | -30%        | 0.3502 39.7781 0.45435 166.693 | -3.53% 6.04% 9.92% 3.87% |
| $D$   | 60%         | 0.3815 52.0998 0.54626 152.212 | -6.56% -6.19% -13.84% -6.71% |
|       | -30%        | 0.3395 41.1873 0.35615 149.719 | -12.29% 14.02% -0.01% 14.01% |
| $g$   | 60%         | 0.3529 29.3371 0.41331 154.56 | -5.10% -6.19% -13.84% -6.71% |
|       | -30%        | 0.3579 23.7243 0.41338 101.502 | -11.57% 12.91% 27.68% 10.66% |
| $h$   | 60%         | 0.321 42.3529 0.52776 177.591 | -8.56% 6.99% 13.85% 5.74% |
|       | -30%        | 0.3392 39.9063 0.47059 169.702 | -5.10% 38.89% 32.16% 5.31% |
| $p$   | 60%         | 0.4276 86.057 0.80502 192.007 | -17.96% 22.45% 49.26% 19.64% |
|       | -30%        | 0.4276 86.057 0.33253 165.326 | -2.78% -21.79% -19.55% 3.01% |

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In this section, the relative importance of model input parameters is assessed. Since the input factors are typically uncertain, this analysis will help managers to determine which of the settings are more critical while acquiring the output of the model. This analysis is done based on the data and the main results from the mentioned numerical example for both strategies (i.e., $D = 250, A = 500, h = 3$, $g = 4$, $b = 3$, $p = 0.25$, and $\beta = 0.7$).

5.2. Sensitivity analysis

Table 2 reveals the result of the sensitivity analysis for the continuous costing method.

Table 2: Sensitivity analysis on input parameters for the first example under end-of-cycle costing strategy.

| Inputs | Changes (%) | Optimal decision variables | Changes in | ATC* |
|--------|-------------|---------------------------|------------|------|
|        |             | $T^*$                     | $S^*$ | $T^*$ | $S^*$ | ATC* |
| $A$    | 60%         | 0.9311                    | 232.773   | 103.104 | 26.49% | 26.49% | 26.49% |
|        | 30%         | 0.8933                    | 209.8187  | 92.9374  | 14.02% | 14.02% | 14.02% |
|        | –30%        | 0.6159                    | 153.9649  | 68.1974  | –16.33% | –16.33% | –16.33% |
|        | –60%        | 0.4655                    | 116.3865  | 51.5524  | –36.76% | –36.75% | –36.75% |
| $b$    | 60%         | 0.6372                    | 159.2895  | 94.1682  | –13.44% | –13.44% | 15.53% |
|        | 30%         | 0.6813                    | 170.3239  | 88.0675  | –7.44%  | –7.44%  | 8.04%  |
|        | –30%        | 0.8067                    | 201.5677  | 74.3798  | 9.59%   | 9.59%   | –8.75% |
|        | –60%        | 0.9024                    | 225.6063  | 66.4875  | 22.59%  | 22.60%  | –18.43% |
| $D$    | 60%         | 0.5819                    | 232.773   | 103.104  | –20.95% | 26.49%  | 26.49% |
|        | 30%         | 0.6456                    | 209.8187  | 92.9374  | –12.29% | 14.02%  | 14.02% |
|        | –30%        | 0.8798                    | 153.9649  | 68.1974  | 19.52%  | 16.33%  | –16.33% |
|        | –60%        | 1.1369                    | 116.3865  | 51.5524  | 58.12%  | –36.75% | –36.75% |
| $g$    | 60%         | 0.6                          | 165.0034  | 90.9072  | –10.34% | –10.34% | 11.53% |
|        | 30%         | 0.6949                    | 173.7737  | 86.3372  | –5.60%  | –5.59%  | 5.92%  |
|        | –30%        | 0.7855                    | 196.3829  | 76.3814  | 6.71%   | 6.72%   | –6.29% |
|        | –60%        | 0.8465                    | 211.6223  | 70.881   | 15.00%  | 15.00%  | –13.04% |
| $h$    | 60%         | 0.7361                    | 184.0232  | 81.5115  | 0.00%   | 0.00%   | 0.00%  |
|        | 30%         | 0.7361                    | 184.0232  | 81.5115  | 0.00%   | 0.00%   | 0.00%  |
|        | –30%        | 0.7361                    | 184.0232  | 81.5115  | 0.00%   | 0.00%   | 0.00%  |
| $\beta$| 30%         | 0.7948                    | 198.6952  | 75.4925  | 7.97%   | 7.97%   | –7.38% |
|        | –30%        | 0.6887                    | 172.1851  | 87.1155  | –6.43%  | –6.43%  | 6.88%  |
|        | –60%        | 0.6945                    | 162.3723  | 92.3803  | –6.53%  | –11.77% | 13.33% |
| $p$    | 60%         | 0.5563                    | 139.0697  | 107.8596 | –24.43% | –24.43% | 32.32% |
|        | 30%         | 0.6317                    | 157.9203  | 94.9846  | –14.18% | –14.18% | 16.53% |
|        | –30%        | 0.898                     | 224.4927  | 66.8173  | 21.99%  | 21.99%  | –18.03% |
|        | –60%        | 1.211                     | 302.7436  | 49.5469  | 64.52%  | 64.51%  | –39.21% |

The following facts can be concluded from Table 2:

- Increasing $A$ directly causes an increase in $T^*$, $S^*$, and ATC* and contrariwise.
- Increasing $b$ leads to a decrease in both $T^*$ and $S^*$ but affects reversely on ATC* which differs from continuous costing results.
- Increasing $D$ increases the value of $S^*$ and ATC* but decreases $T^*$.
- Increasing $g$ decreases all decision variables except ATC* since it increases, also increasing or decreasing $h$ does not affect all decision variables. This phenomenon is not supported by the exact continuous costing method because the holding costs are substantial (Rudi et al., 2009).
- As the value of $\beta$ increases to reach full backordering case, all decision variables increase but ATC will decrease, which was different in the previous analysis. But the same as before, changes in $p$ lead to opposite results.
- $T^*$ and ATC* are extremely receptive to $p$, $D$ and $A$.
- $S^*$ is extremely receptive to $p$, $D$ and $A$.
- It seems that despite before, changes in $p$ have higher effects on decision variables.

5.2.2. Analysis of simultaneous changes of $(p, \beta)$

In the solved example, it was considered that $p$ is equal to 0.15 (i.e., there is a low chance of disruption). It was also concluded that increasing $\beta$ by 30%, from 0.2 to 0.32, decreases the total costs of the continuous costing system by 14.15% and end-of-cycle costing by 7.38%. However, there may be cases that if the probability of disruption is about to increase, the total costs of the system may increase when the inventory manager selects a higher value for the backordering ratio. This fact indicates that increasing $\beta$ is not always profitable for managers to answer all unsatisfied demand, which was previously considered in the model introduced by Konstantaras et al. (2019). Based on this background, it was decided to do more extensive analyses on the effects of two particular input parameters of the model, i.e., the $(p, \beta)$ pair. Consider $D = 4500$, $A = 200$, $h = 5.2$, $g = 38$, and $b = 4$. According to Fig. 9, the following results are obtained:

For the end-of-cycle costing method, the sensitivity results are shown in Table 2.

* Note that the value of $F^*$ as explained in the modeling section, is equal to 0 for all cases.
Impacts on $m^*$: In case (a-1), it is seen that $m^*$ increases if $p$ goes beyond 0.3. The rate of the rise in $m^*$ is higher when $p$ goes higher. This fact means the buyer who uses the continuous costing strategy will need to have the order-up-to level $S$ higher than $DT$ only when $p$ is high. According to the case (b-1), the effect of the increase in $p$ is higher when end-of-cycle costing is deployed. This happens because this costing strategy neglects the costs of inventory held through each review cycle. Since it mainly considers the leftover stocks. In a general conclusion, $m^*$ is somehow invariant when $p$ is lower or when $\beta$ ranges from 0 to about 20%.

Impacts on $T^*$: In both cases (a-2 and b-2), it is seen that with the increase of disruption probability $p$, the review interval directly decreases. Predicting the effects of change in $\beta$ is more complicated. While in higher $p$ it reduces the value of $T^*$ by an increase, it may act contrariwise to increase $T^*$ in lower $p$ when continuous costing is being used.

Impacts on $S^*$: Different from previous, changes of either $p$ or $\beta$ has no determining effect on $S^*$.

After this, the total long-run costs of the inventory system will be analyzed. For this purpose, 6 cases for each costing strategy are defined to have a simultaneous analysis of all cost parameters of the model. It should be noted that the parameters $A$ and $D$ are omitted here since they are somehow external to the system and mostly determined by the suppliers and the customers. Table 3 denotes the values set for the parameters ($h, b, g$) in these 3! cases.

According to Fig. 10:

- Cases 1 and 2: At first, $h$ is selected as the highest cost parameter. Then it was considered that $b$ is greater or lower than $g$. In these cases, when disruption probability $p$ is lower than 50%, increasing $\beta$ will aid to decrease the total costs of the long-run inventory system. However, when $p$ is greater than or equal to 50%, increasing $\beta$ will lead to undesired imposed total costs on the system. In conclusion, the manager can reliably increase the value of $\beta$ if the disruptions have a greater probability of occurrence (usually greater than 50%).

- Cases 3 and 4: In these cases, since $b$ is the highest cost parameter, probabilities lower than 50% (about 30%) will also lead to an increase in the total costs of the system if $\beta$ increases.

Table 3

| Case number | Parameters in order | 1st order | 2nd order | 3rd order |
|-------------|--------------------|-----------|-----------|-----------|
| 1           | $h \geq b \geq g$  | 5.07      | 3.9       | 3         |
| 2           | $h \geq g \geq b$  | 6.76      | 5.2       | 4         |
| 3           | $b \geq g \geq h$  | 3.38      | 2.6       | 2         |
| 4           | $b \geq h \geq g$  | 5.07      | 3.9       | 3         |
| 5           | $g \geq h \geq b$  | 6.76      | 5.2       | 4         |
| 6           | $g \geq b \geq h$  | 3.38      | 2.6       | 2         |

Fig. 9. Sensitivity analysis by changing $(p, \beta)$ simultaneously for $S^*$, $T^*$ and $m^*$. 
Cases 5 and 6: When $g$ has the highest value between the costs of the inventory system, the value of $\beta$ should be carefully chosen. Sometimes increasing $\beta$ will be profitable for the manager when $p$ is about 10%. However, in higher amounts of $p$, he or she can be confident about the increase in total costs if $\beta$ increases.

As a managerial conclusion for continuous costing strategy, when there are higher probabilities of supply disruptions (greater than about 70%), the manager should lower the firm’s responsiveness to the unsatisfied demands. Therefore, reaching the lost-sales case when $\beta = 0$ may be the most proper strategy for the system.

Let us continue our analysis of the outputs of such tactical decisions in the end-of-cycle case. According to a glance view of Fig. 11, it is seen that in comparison to the continuous costing case, the changes are more uniform. The following results are obtained:

Cases 1 and 2. Against the previous conclusions, it is seen that increasing $\beta$ definitely will increase the costs imposed. However, when $p$ is about 0.1, and $g$ is higher than $b$ (lost sales are worse than the backorders), increasing $\beta$ decreases the total costs.

Cases 3 and 4. In these cases, the previous claim is more accepted. Therefore when $b \geq g \geq h$, and $b \geq h \geq g$, i.e., the unit backordering...
cost is the highest and, the manager will face higher costs by increasing $\beta$.

Cases 5 and 6. In these cases, it is seen that $p$ should be higher than about 50% to select lower values of $\beta$ to ensure cost savings.

5.3. Managerial insights

In this section, crucial insights obtained from computational experiments are described. These findings aid logistics and operation managers to select efficient business continuity and resilience plans during the presence of disruptions. It is noteworthy that the derived insights are applicable where a retailing firm orders and sells imperishable or a reusable item such as furniture. That is because all strategies investigated are based on the base-stock policy, which acts more suitable for these kinds of retailing firms.

5.3.1. Strategies to manage responsiveness

In the time of disruptions, to be more flexible when responding to the customers’ demand, operation managers in the retail side usually
consider holding high levels of inventory in their order fulfillment centers (Sheffi and Rice Jr., 2005). But as our study shows (see Fig. 9, cases a-3 and b-3), keeping high levels of stock at retail outlets does not have a deterministic relation with the changes of the disruption probability, notably when a continuous costing scheme is used. That is, based on the environment that retailers are active, selection of the optimal inventory strategy is case-based. For instance, most of the time, if retailers adopt lower but closer to optimal inventory levels, they can ensure their cost efficiency as well as reduced risk of consumers’ dissatisfaction. This optimal stocking level may differ according to the number of backorders that are imposed on the retail system. For instance, acceptance of a high number of backorders leads to having lower base stock level especially when the probability of supply disruptions is approximately lower than 70%.

At the same time, the consideration of a more realistic probability for the disruption which is backed up with higher experience in the retailing field aids to determine a more proper value for $S$ (inventory level in periodic review inventory systems) during the time of disruptions. As it is seen that in Fig. 9, an increase in the ratio of responded demands ($\beta$) leads to undesired high long-run total costs for the firm in most of the cases. This examination denotes having backordered items usually make the inventory system inefficient about total costs, the fact which is previously neglected in previous studies.

5.3.2. Strategies to manage the inventory system

As shown by cases a-2 and b-2 in Fig. 9, the higher the probability of disruption is, the higher the speed of inventory review and updates will be. The effect of the cost parameters should not also be neglected, as described below:

- If a firm has the highest holding costs and it is more than 50% probability to face supply disruptions, therefore increasing $\beta$ will indeed impose higher total costs, in both costing strategies. Therefore, managers should cope with the lost-sales case (when $\beta = 0$).
- If the backordering unit costs are the highest cost components, managers should choose lost sales case when the probability of disruptions is about 30% or higher. This fact applies to two costing strategies.
- In the last case, if the goodwill losses costs are the highest, more analyses should be done through the model provided in this paper. Sometimes an increase in $\beta$ decreases the total costs in both costing strategies (usually when disruption probability is 30% or lower).

Overall, disruptions, in most cases, will impose higher total costs on the inventory system, which is inevitable for both strategies. Therefore, the experience of the experts while determining parameters, especially the disruption probability, is extremely influential. However, managers could consider the probability parameter $p$ as a controller of “degree of robustness” in decisions derived from this model. As discussed in Bertsimas and Sim (2004), the higher the manager wants to ensure the reliability of the decisions, the higher the price it should be paid in terms of robustness. Then suitable determination of parameter $p$ will lead to lower total costs in the robust case. This fact differs this study from previous literature by considering that consideration of classic backorder case or lost sales case is not an optimal decision for the inventory management system (Gupta, 1996; Skouri et al., 2014; Konstantaras et al., 2019).

5.3.3. An action plan to manage supply disruptions

We suggest logistics and operation managers consider the following concerns carefully; (1) the effects of consideration of suitable disruption probability, which may be alleviated using historical data, or assessing their suppliers’ reliability through more suitable supplier selection and order allocation (SS&OA) models (Torabi et al., 2015), (2) the effects of fulfillment ratio of demands, in which it may be more desirable to let a proportion of demand to be lost. Briefly, according to the findings of this study, the following action plans are suggested:

- Before consideration of probability for disruption at the supply side or usage of data analysis, use proper SS&OA models to select the most reliable supplier between an existing set of them. Then, gather information about the business environment and consult experts to set a suitable value for the probability of supply disruptions, since it is highly impactful on the total long-run costs.
- Do market analysis to understand the amount of consumer loyalty to the brand, to ensure setting a proper value for inventory at hand. That is, to better determine what ratio of customers are willing to wait instead of leaving the brand to buy from the others if a shortage scenario occurs.
- It is not recommended to adopt a full backorder or pure lost sales strategy. As the results of this study suggest, and according to the market analysis, try to set a reasonable value for the backorder ratio to ensure lower long-run costs during the season of activity, according to the proposed model.
- Finally, most of the time, it is recommended to have lower inventory review and order intervals than having higher inventory at hand, especially if the probability of supply disruptions is high.

6. Concluding remarks and future research agenda

When retailers cannot get their desired items at the right time, due to a supply-side disruption, they will usually encounter undesirable shortages in response to demands. This effect intensifies if the impacts of these disruptions are neglected, and they do not proactively plan for such events. These events negatively affect retailers’ economic performance, especially if they last for a long time. In this paper, two novel base stock ($S$, $T$) inventory optimization models with partial back-ordering are developed to investigate the cases in which internal and independent supply-side disruptions occur. The modeling approach was based on the consideration of state variables for each possible disruption in the inventory profile. Two different costing strategies were compared to aid financial managers in setting the parameters of the model.

The results emphasized the fact that decreasing/increasing the backordered fraction of the demand of customers (i.e., reaching the lost sales or the pure backorder strategy) is highly relevant to the probability of supply disruptions. Therefore, the effects of these parameters together (i. e., $p$ and $\beta$) should not be neglected as they impact the trend of increase or decrease in backorders. Additionally, the retailers should not adopt either pure backordering or lost sales strategy during the time of supply-side disruptions. As the results indicate, setting the backorder ratio value between zero and one, decreases the total costs imposed on the inventory system, especially if the goodwill loss costs are higher than other cost parameters. Therefore, the optimal decisions derived from the proposed model dominates the results from previous studies that neglect the effect of partial backorders or optimality of review interval. The results also suggest maintaining a reasonable amount of stock at hand, leads to lower total inventory costs, and decreases the chances of overstocking or inventory obsolesces. This supports the current industrial practice, namely (1) flexibility in ordering from the supplier, (2) flexibility in meeting consumer demand, during the time of disruptions, instead of the concept of redundancy, which appears to be costly and inefficient (Sheffi and Rice, 2005). Overall, the findings challenge the current industrial practice, by relating the optimal inventory level decision to the specific probability of the disruptive events and selection of a suitable response level to consumers, instead of holding more inventory levels or at distributed local warehouses in such conditions.

Our study is among preemptive efforts to create a generalized baseline for handling disruptions in inventory models. However, this area needs further investigation, as there are some limitations associated with the assumptions considered for modeling. For instance, uncertainties and variability in demand or backorder ratio are not found in our model, while it will help the retailing firms act closer to reality in
many practical situations. Another potential weakness is not accounting for conditions where extra ‘constant lead time’ to the ‘undesired lead time observed by disruptions’ exist. Such an assumption leads to set better ordering decisions at the time of the contract between supplier and retailer. Finally, even in the numerical analysis, there are significant limitations due to the sparsely available data that should be alleviated by conducting well-documented investigations based on real case studies to increase the practicality of our research. This would be beneficial to eliminate the mentioned limitations and make the current analysis more robust in future studies.

To shed light on other future research pathways, some suggestions are provided. For instance, to account for the case of partial supply-side disruptions in which the amount of delivery may be a random ratio of the desired ordering amount instead of an all-or-none strategy, which is more realistic in many practical cases. Another is to consider the specific feature of products (e.g., perishability, volatility, or obsolescence) and those which have a salvage value at the end of an inventory cycle. The last one would be consideration of more than one supplier and one retail outlet (i.e., an expanded network) that provides demanded products by the consumers.

It may be interesting to consider the relations between pricing and inventory decisions that do not affect the modeling process in this study and also the previous researches. It is beneficial to develop efficient pricing strategies based on market analysis and the time when demand surges due to a pandemic disease such as COVID-19. Accordingly, the retailers will be able to set optimal prices to maintain their economic efficiency and social responsiveness while accounting for their resilience plans and viability. Last but not least, consideration of the specific impact of different disruptions or other types of events (e.g., those relevant to the transportation, production processes), is another research pathway to optimize logistical activities as well as inventory decisions. Indeed posing optimal strategies will increase the sustainability of the firms while maintaining a suitable level of their resilience.

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