Quantum phenomena explored with neutrons

Yuji Hasegawa and Helmut Rauch

Atominstitut, TU-Wien, Stadionallee2, A-1020 Wien, Austria
E-mail: rauch@ati.ac.at

New Journal of Physics 13 (2011) 115010 (18pp)
Received 28 June 2011
Published 25 November 2011
Online at http://www.njp.org/
doi:10.1088/1367-2630/13/11/115010

Abstract. Neutrons are suitable tools for quantum experiments since they are massive, experience nuclear, electromagnetic and gravitational interaction and are easy to manipulate and to detect. Perfect crystal interferometry opened new possibilities to explore quantum phenomena on a new ground. The $4\pi$-symmetry of spinor wave functions, the spin-superposition law, topological quantum phases and various gravitational effects have been examined using this method. Experiments exploiting contextuality and Kochen–Specker phenomena exhibit intrinsic entanglement in single particle systems. This may have consequences for a deeper understanding of quantum physics and for applications in future quantum communication systems.

Contents

1. Introduction .................................................. 2
2. Neutron interferometry .................................... 3
   2.1. Coherence experiments .................................. 3
   2.2. Post-selection and dephasing experiments .......... 5
   2.3. Topological phases ..................................... 6
   2.4. Confinement-induced phase ............................ 9
3. Tests of quantum contextuality using neutron matter waves 10
   3.1. Neutron interferometry: violation of a Bell-like inequality 10
   3.2. Neutron polarimetry: falsification of a contextual model à la Leggett 13
4. Concluding remarks ......................................... 16
Acknowledgments ............................................. 17
References .................................................... 17

1 Author to whom any correspondence should be addressed.
1. Introduction

Duality in quantum mechanics attaches matter and wave properties to any physical system. Neutrons, as uncharged elementary particles, are proper tools for the investigation of both fundamental properties since they carry well-known particle and wave properties as shown in Table 1. Free neutrons can be produced by various nuclear reactions such as fission, fusion, or spallation and slowed down in a moderator material. They can then be detected with high efficiency by various detectors based also on nuclear reaction, e.g., BF$_3$ or He-3 detectors. Wave and particle features are connected by the de Broglie relation and the Schrödinger equation

$$\lambda = \frac{\hbar}{p},$$

(1.1)

$$\left(-\frac{\hbar^2}{2m} \Delta + V(\vec{r}, t)\right) \psi(\vec{r}, t) = i\hbar \frac{\partial \psi(\vec{r}, t)}{\partial t},$$

(1.2)

where $p$ represents the canonical momentum and $|\psi(\vec{r}, t)|^2$ the probability of finding the particle within a certain region of space and time interval near $\vec{r}$ and $t$, respectively.

In the stationary situation, the total energy is conserved and therefore the time-independent form can be used.

$$-\frac{\hbar^2}{2m} \nabla^2 \psi(r) + \bar{V}(r) \psi(r) = E \psi,$$

(1.3)

$\bar{V}$ denotes the space averaged interaction potential, which can be caused by a nuclear, electromagnetic or gravitational interaction. The related potentials are (e.g., Sears 1989)

$$\bar{V}_n = \int \frac{2\pi}{m} b_c \delta(\vec{r}) d\vec{r} = \frac{2\pi}{m} b_c N,$$

(1.4)

$$\bar{V}_m = -\mu \vec{r} \cdot \vec{B}(r),$$

(1.5)

$$\bar{V}_g = -m \vec{g} \cdot \vec{r} - \vec{\Omega}_g \cdot \vec{L},$$

(1.6)

where $b_c$ and $N$ denote the coherent scattering length and the particle number, respectively; $B$ is the magnetic field strength, $\vec{g}$ the gravitational acceleration, $\vec{\Omega}_g$ the angular velocity of the Earth ($|\vec{\Omega}_g| = 0.727 \times 10^{-4}$ rad s$^{-1}$) and $\vec{L}$ the angular momentum of the neutrons relative to the center of the Earth. Since it makes a difference how a quantum system experiences an excursion in parameter space, a topological effect can be expected as well (Berry 1984, Shapere and Wilczek 1989). In the interaction-free region the solution of the Schrödinger equation is given by a coherent superposition of plane waves resulting in a wave packet

$$\psi(\vec{r}, t) = (2\pi)^{-3/2} \int a(\vec{k}, \omega) e^{i(\vec{k} \cdot \vec{r} - \omega t)} d^3k.$$

(1.7)

The density of states $g(\vec{k}, \omega) = |a(\vec{k}, \omega)|^2$ with $\omega = \omega_k = \hbar k^2/2m$ and the coherence function, with a characteristic width $\Delta_c$ (coherence length), can be defined in analogy to standard optical concepts (Glauber 1963, Mandel and Wolf 1995).

$$\Gamma^{(1)}(\Delta_c, \tau) = \langle \psi^*(\vec{r}, t) \psi(\vec{r} + \Delta_c, t + \tau) \rangle = \int g(\vec{k}, \omega) e^{i(\vec{k} \cdot \vec{r} - \omega t)} d^3k.$$

(1.8)
Table 1. Particle and wave properties of neutrons (Nakamura et al 2010).

| Particle properties | Wave properties |
|---------------------|----------------|
| $m = 1.674928(1) \times 10^{-27}$ kg | $\lambda_c = \frac{\hbar}{mc} = 1.319695(20) \times 10^{-15}$ m |
| $s = \frac{1}{2} \hbar$ | For thermal neutrons: $\lambda = 1.8$ Å, $v = 2200$ m s$^{-1}$ |
| $\mu = 9.6491783(18) \times 10^{-27}$ J T$^{-1}$ | $\lambda_B = \frac{\hbar}{mv} = 1.8 \times 10^{-16}$ m |
| $\tau = 885.7(8)$ s | $\Delta_c = \frac{\lambda}{2\pi} \approx 10^{-8}$ m |
| $R = 0.862(9)$ fm | $\Delta_v = v\Delta t \approx 10^{-2}$ m |
| $\alpha = 11.6(1.5) \times 10^{-4}$ fm$^3$ | $\Delta_d = v\tau = 1.942(5) \times 10^6$ m, $0 \leq \chi \leq 2\pi(4\pi)$ |
| u-d-d: quark structure | $\lambda_c$, Compton wavelength; $\lambda_B$, de Broglie; $\mu_B$, wavelength; $\Delta_c$, coherence length; $\Delta_p$, packet |

In stationary situations ($\omega = \text{const}$) the equations simplify and when the wave packet enters an interaction region the momentum changes (from $k$ to $K$) according to the index of refraction

$$n^2 = \frac{K^2}{k^2} = 1 - \frac{V}{E} = 1 - \frac{\lambda^2 Nb_c}{\pi} = 1 - \left( \frac{\lambda}{\lambda_c} \right)^2.$$  \hfill (1.9)

Ultra-cold neutrons have wavelengths $\lambda \gg \lambda_c$ and they are totally reflected under all angles of incidence and, as a consequence, can be trapped in material and magnetic bottles (Golub et al 1991). Typical values for $\lambda_{ucn}$ are $\lambda_{ucn}(\text{Be}) = 57.1$ nm and $\lambda_{ucn}(\text{Cu}) = 69.4$ nm.

Light optical phenomena, described by the Helmholtz equation, and matter wave optics, described by the Schrödinger equation, are analogous and all known optical phenomena have also been observed for neutrons, i.e. reflection, total reflection, single and double-slit diffraction, grating diffraction, crystal diffraction, diffraction in time and interferometry (Sears 1989, Kaiser and Rauch 1999). These experiments have been extended to even heavier particles such as atoms, molecules and macromolecules (Cronin et al 2009, Gerlich et al 2011).

The basic particle properties of the neutron have been exploited in the past as well. Properties of the nucleon–nucleon force can be investigated by few-body experiments, the weak interaction by measuring the $\beta$-decay products of neutrons and the search for an electric dipole moment is intended to discover new physics beyond the standard model of particle physics (Abele 2008). In this paper, we will focus on recent neutron interferometry experiments but also comment on related ultra-cold neutron and gravity quantization experiments (Nesvizhevsky et al 2002, Jenke et al 2011).

2. Neutron interferometry

2.1. Coherence experiments

Photon and particle beams can be split coherently in ordinary or momentum space, which provides the basis for interferometry experiments where the relative phase becomes a
measurable quantity. A wide, spatially coherent separation of neutron beams is feasible in perfect crystal interferometers (Rauch et al. 1974). In this case nuclear, magnetic and gravitational phase shifts can be created and measured precisely (Rauch and Werner 2000). Coherent beam separation in momentum space is used in spin–echo systems (Mezei 1972, 1980) and has recently been used for interferometric measurements as well (Bouwman et al. 2008).

Perfect crystal interferometers are based on perfect atomic arrangement in silicon crystals. A monolithic design and a stable environment provide the parallelity of the lattice planes throughout the interferometer. Figure 1 shows different interferometers that have been created and used in the course of our experiments. The wave function behind the interferometer in the forward (O) direction is composed of wave functions arising from beam paths I and II where beam path I is transmitted–reflected–reflected and beam path II is reflected–reflected–transmitted. In the case of zero absorption both wave functions have to be equal due to symmetry principles ($\psi_I = \psi_{II}$) and when a phase shift ($\chi = \int k \, ds = (1 - n)kD$) is applied we have $\psi_{II} = e^{i\chi} \psi_I$ and an intensity ($D$ denotes the optical path length)

$$I \propto |\psi_I + \psi_{II}|^2 \propto |\psi_I|^2 (1 + \cos \chi).$$

(2.1)

When the wave packet form of the incident beam (equation (1.7)) and unavoidable small disturbances in the crystal and its geometry are taken into account, one has

$$I_{\text{exp}} \propto A + B |\Gamma(\Delta)| \cos(\chi + \Theta),$$

(2.2)

where $A$, $B$ and $\Theta$ are characteristic parameters, which depend on all imperfections of a specific setup. High-contrast and high-order interference pattern can be observed as shown in figure 2. The periodicity of the interference pattern gives the phase shift $\chi$ and the envelope gives the coherence function $\Gamma(\Delta)$. Equation (2.2) indicates that each interference fringe is slightly different from all the others indicating individuality even in the case of a quasi-periodic phenomenon.

The different interactions mentioned in section 1 cause different phase shifts

$$\chi_n = k(1 - n)D = -Nb\nu\lambda D,$$

(2.3)

$$\chi_m = \pm 2\pi\lambda m\mu B D / h^2,$$

(2.4)
Figure 2. High-order interference pattern (e.g. Lemmel 2011).

\[
\chi_g = -2\pi \lambda m^2 g A_0 \sin \alpha / h^2 + 4\pi m \Omega_g A_0 \cos \Theta_L \cos \alpha / h, \tag{2.5}
\]

\[
\chi_t = -\frac{\Omega_t}{2}, \tag{2.6}
\]

where \(D\) denotes the distance in which the interaction acts on the system, \(A_0\) the area encircled by the coherent beams, \(\alpha\) the angle of rotation of the interferometer around the incident beam axis, \(\Theta_L\) the colatitude angle where the experiment is carried out and \(\Omega_t\) the solid angle of the excursion seen from the center of the Bloch sphere. The Sagnac term in equation (2.5) holds in this form when the incident beam lies in the North–South direction. The topological phase \(\chi_t\) depends on the solid angle \(C\) of the excursion cycle seen from the center of the Bloch sphere.

Measurements of \(\chi_n\) give accurate values for neutron scattering lengths of various elements and isotopes (e.g. Kaiser et al 1979), \(\chi_m\) measurements demonstrated for the first time the \(4\pi\)-symmetry of spinor wave functions (Rauch et al 1975, Werner et al 1975) and the spin superposition law (Summhammer et al 1983) and \(\chi_g\) measurements demonstrated the gravitational law (Colella et al 1975) and the Sagnac effect for elementary particles (Werner et al 1979). The most direct interferometric verification of a topological quantum phase was obtained by rotation of a spin flipper within an interferometer (Allman et al 1997). More details can be found in the book by Rauch and Werner (2000).

### 2.2. Post-selection and dephasing experiments

Such measurements show that much more information can be extracted even behind the interferometer when advanced post-selection procedures are applied (figure 3). Here, we deal with post-selection in momentum space but other parameter spaces can be used as well. At high interference order \((\Delta \gg \Delta_n)\) the interference pattern disappears due to the finite width of the coherence function (equation (2.2)), but at the same time a modulation of the momentum distribution appears (figure 3, Rauch 1993, Jacobson et al 1994). This modulation can be measured with an additional analyzer crystal behind the interferometer. In this case, Schrödinger cat-like non-classical quantum states are produced, which are rather fragile against any fluctuation and dissipation process, and related experiments may help us to understand the transition from a quantum to a classical state (Giulini et al 1996, Haroche and Raimond 2006).

Magnetic noise fields can simulate related decoherence effects even when they are not inherently irreversible and have to be taken rather as dephasing components although the time dependence of the noise field causes multi-photon exchange processes between neutron and field (Summhammer et al 1995). The dephasing can be observed at low order \(\Delta \ll \Delta_n\) by the
reduction of the interference contrast and at high order ($\Delta \gg \Delta_c$) from the smearing of the momentum distribution.

The experimental setup is shown in figure 4 and typical results measured at large phase shifts are shown in figure 5. There is an ongoing discussion whether such noise-induced decoherence phenomena can simulate quantum decoherence processes where an entanglement to the environment is required. Although the noise field causes multi-photon exchange all processes can be retrieved by opposite noise fields or the same field in the other beam. It is still an open question how an inherently irreversible decoherence process can be realized.

It should be mentioned that the topological (geometric) phases, as discussed in the next section, behave completely differently with respect to noisy fields.

2.3. Topological phases

Geometric or topological phases are of interest since they are caused by forceless interactions when the quantum system experiences various excursions in phase space. The field has been
Figure 4. Experimental arrangement to apply Gaussian noise fields to one of the coherent beams and to analyze the momentum distribution by means of an additional analyzer crystal.

Figure 5. Dephasing effect on the momentum modulated spectrum at large phase shifts (left) and dephasing as a function of the strength of the noise field (right) (Sulyok et al 2010).

opened by Pancharatnam (1956) and applied to adiabatic, non-adiabatic, cyclic and non-cyclic excursions in phase space (Berry 1984, Aharonov and Anandan 1987, Shapere and Wilczek 1989). The phase shift in these cases is given by a dynamical ($\alpha$) and a geometric term ($\phi_g$)

$$
\phi = -\frac{1}{\hbar} \int_0^T \langle \psi(t) | H | \psi(t) \rangle \, dt + i \int_0^T \left( \phi(t) \frac{d}{dt} | \phi(t) \right) dt = \alpha + \phi_g
$$

(2.7)
Figure 6. Double-loop interferometer for the measurement of a non-adiabatic and non-cyclic quantum phase (Filipp et al 2005, 2009a). The closure line $C_2-C_1$ follows the geodesic between these points. Measured and calculated values are shown (lower right panel).

with $|\phi(t)\rangle = e^{i\phi} |\psi(t)\rangle$. In the case of an adiabatic excursion the geometric phase becomes half of the solid angle ($\Omega$) of the excursion seen by the center of the Bloch sphere. This has been verified with neutrons with a rather high accuracy ($\phi_g = -0.51(1)\Omega_1$) (Allman et al 1997, Filipp et al 2009a).

Non-cyclic and non-adiabatic phase excursions have attracted attention recently. In this case the end-point of the excursion has to be connected with the start point by a geodesic line (Samuel and Bhandari 1988). Related experiments have been done by means of a double-loop interferometer, a phase shifter with different thicknesses (SP2) and an absorber sheet ($A$ and transmission $T$) where excursions like those shown in figure 6 can be realized (Filipp et al 2005). The dynamical phase can be canceled when the condition $\chi_1 + \chi_2 T = \text{const}$ is fulfilled.

The topological phases may become important for advanced quantum communication, last but not least because they are rather robust against any disturbances as predicted by DeChiara and Palma (2003) and tested experimentally with ultra-cold neutrons by Filipp et al (2009a). This has been done with ultra-cold polarized neutrons trapped within a storage bottle surrounded by Helmholtz coils, which permit a controlled adiabatic rotation of the neutron spin and the application of a noisy field (Filipp et al 2009b). Rotating the spin in the upper and lower hemispheres in opposite directions balances the dynamical phase (spin–echo method) and doubles the dynamical one. First an accurate measurement of the geometric phase has been made without a noisy field ($\chi_t = -0.51(1)\Omega_t$, see equation (2.6)) and then the topological phase has been measured within noisy fields (figure 7). A comparison of the dephasing effect of the dynamical and geometric phases shows completely different behavior (figures 5 and 7). Whereas
the dephasing of the dynamical phase increases with the strength and duration of the noisy field, the geometric phase improves with it.

2.4. Confinement-induced phase

When a quantum system becomes confined within a potential caused by some wall material, the transverse momentum becomes quantized, $k_{n\perp} = n\pi/a$, and this causes a change of the longitudinal momentum as well, $k_{\parallel,n} = \sqrt{2m(E_{\text{in}} - E_{n\perp})}/\hbar^2$, and concomitant phase shifts, $\chi_{n,\text{conf}} = (k_{\parallel,n} - k)D \approx -k_{n\perp}^2 D/2k = -\pi n^2 \lambda D/4a^2$. Related experiments have been proposed (Levy-Leblond 1987, Greenberger 1988) and realized with neutron interferometry (Rauch et al 2002). These experiments have been performed with a stack of silicon slits with a slit width of $a = 22.1 \mu m$, a length of $D = 20 mm$ and a neutron wavelength $\lambda = 0.189(3) nm$. The energy levels excited within these slits have energies $E_{1,\parallel} = 0.4172 \text{ peV}$, $E_{2,\parallel} = 1.669 \text{ peV}$, etc. There are about 360 levels within the potential and their excitation depends on the angle of the incident beam component relative to the surface of the walls. The main contribution to a measurable phase shift comes from the low-lying levels. The situation and typical results are shown in figure 8. As a final result, a phase shift of $\chi_{\text{conf}} = 2.8(4) ^\circ$ has been reported, whereas the theoretical value is $2.5 ^\circ$. The low contrast of the interference pattern with the slits inserted can be explained by the beam attenuation, the variation of the slit width and the mixing of different quantum states. More recent experiments show even larger phase shifts and a stronger discrepancy with calculated values, which will be the subject of further investigations. This is a worthwhile endeavor because the effect is a purely quantum phenomenon and free from van der Waal interactions and Casimir forces (Casimir and Polder 1948).
3. Tests of quantum contextuality using neutron matter waves

3.1. Neutron interferometry: violation of a Bell-like inequality

An EPR–Bell argument (Einstein, Podolsky and Rosen 1935, Bell 1964) is ideal to focus on the conflict between local realistic theories and quantum mechanics. While a number of experimental tests of the violation of Bell’s inequalities (Einstein et al 1935, Bell 1964) have been performed with correlated photon pairs (Tittel et al 1998, Bertlmann and Zeilinger 2002 and references therein), a single-neutron system provides a more interesting subject for such tests (Hasegawa et al 2003, Sponar et al 2010a, b). A class of hidden-variable theories, larger than the local one, is known as the non-contextual hidden-variable theories (NCHVT), where the measured value $v[A]$ of an observable $A$ is assumed predetermined and not affected by a joint (or simultaneous) measurement of an observable $B$ compatible with $A$ (Mermin 1993). Kochen and Specker started studies of non-contextual theories to demonstrate the conflict between these theories and quantum mechanics (Kochen and Specker 1967). An experimentally feasible test consists of joint measurements of commuting observables of single neutrons in an appropriately generated non-factorizable state. The current experiment is an improved version of our previous experiment (Hasegawa et al 2003): a newly developed spin flipper leads to a lower decoherence rate and, consequently, to higher contrast of the interference fringes. As a result, a larger violation than in the previous experiment is observed.

3.1.1. Theory. In the interferometer experiment with polarized neutrons the total wave function represents the entanglement between the spatial part and the spinor part. The
normalized total wave function $|\Psi_{\text{Bell}}\rangle$ can be written as a Bell-like state

$$|\Psi_{\text{Bell}}\rangle = \frac{1}{\sqrt{2}} (|\downarrow\rangle \otimes |\text{I}\rangle + |\uparrow\rangle \otimes |\text{II}\rangle),$$

where $|\uparrow\rangle$ and $|\downarrow\rangle$ denote the up- and down-spin states and $|\text{I}\rangle$ and $|\text{II}\rangle$ denote the two beam paths in the interferometer. A Bell-like inequality for a single-neutron experiment is given in terms of expectation values $E(\alpha; g \chi)$ for the joint measurement of the spin state and the path by (Basu et al 2001)

$$-2 \leq S \leq 2, \quad S := E(\alpha_1, \chi_1) + E(\alpha_1, \chi_2) - E(\alpha_2, \chi_1) + E(\alpha_2, \chi_2). \quad (3.1)$$

Here, $\alpha$ and $\chi$ are associated with the projectors to the spin states,$^1\sqrt{2}$ ($|\uparrow\rangle \pm e^{i\alpha} |\downarrow\rangle$), and the path states,$^1\sqrt{2}$ ($|\text{I}\rangle \pm e^{i\chi} |\text{II}\rangle$).

Quantum theory predicts sinusoidal behavior for the count rate such as

$$N(\alpha, \chi) = \frac{1}{2} [1 + \cos(\alpha + \chi)],$$

and similar behavior is also expected for the resulting expectation value: $E(\alpha; \chi) = \cos(\alpha + \chi)$. According to this dependence of $E$ on $\alpha$ and $\chi$, the Bell-like inequality is not obeyed for various sets of $\alpha$ and $\chi$ values. In particular, the theoretical maximum violation, $S = 2\sqrt{2} \approx 2.82 > 2$, is expected for the set $\alpha_1 = 0, \alpha_2 = \pi/2, \chi_1 = -\pi/4$ and $\chi_2 = \pi/4$.

In a real experiment, one always encounters unavoidable environmental disturbances, imperfect polarization, limited alignments of optical components and so on. All these factors reduce the visibility of the sinusoidal behavior of $N(\alpha s\chi)$, which is characterized by the contrast of the sinusoidal oscillation. The obtained value of $S$ decreases in proportion to these contrasts. That is, the mean contrast should be larger than $\sqrt{2}/2 (\approx 70.7\%)$ in order to allow a demonstration of the violation of the Bell-like inequality.

### 3.1.2. Experiment

Our experimental demonstration of the violation of a Bell-like inequality by means of interferometry with polarized neutrons consists of three steps: (i) generation of a Bell-like neutron state given by $|\Psi_{\text{Bell}}\rangle = \frac{1}{\sqrt{2}} (|\leftarrow\rangle \otimes |\text{I}\rangle + |\rightarrow\rangle \otimes |\text{II}\rangle)$, (ii) manipulation of this state, where the parameters $\alpha$ and $\chi$ are adjusted to select neutrons with certain properties for detection, and (iii) detection of neutrons realized after a polarization analysis in the O-beam to obtain correlation coefficients. In the present experiment a new method is applied to generate the Bell-like state. A schematic view of the experimental setup is shown in figure 9.

In previous experiments by Hasegawa et al (2003, 2006, 2007) a spin-up polarized neutron beam $|\rangle$ enters the interferometer and, after splitting into two beam paths at the first plate of the interferometer, its spin was rotated by $\pi/2$ in one path and by $-\pi/2$ in the other. As a result, the spinor in path I changes to $|\leftarrow\rangle$, while that in path II changes to $|\rightarrow\rangle$, thus yielding a Bell-like state. The new method to generate the Bell-like state consists of two steps: (i) the spin is rotated by $\pi/2$ before entering the interferometer (the spinor $|\uparrow\rangle$ is rotated to $|\rightarrow\rangle$) and (ii) the azimuthal angle of the spin in path I is turned by $\pi$ relative to that in path II. To achieve this, a new spin turner in the interferometer has been developed. In particular, a spin turner
Figure 9. Schematic view of the experimental setup for demonstrating the violation of a Bell-like inequality in single-neutron interferometry. In this experiment, a Mu-metal tube is used to generate a Bell-like state.

for step (ii) is realized by reducing locally the strength of the guide field in beam path I by using appropriate magnetic shielding. A cylindrical tube of Mu metal is used with both ends open, where the neutron beam passes in the axial direction $r$, without touching any material (see figure 9). This soft-magnetic tube weakens the guide field inside and thus reduces the Larmor precession in this region. In our previous experiments, the neutrons in paths I and II passed through a Mu-metal sheet (magnetized perpendicularly to the guide field), which considerably reduced contrast of the interference fringes due to a dephasing effect due to the passage through this sheet. In contrast, no material is placed in the beam with the new method: the new Mu-metal tube indeed caused much less loss of interference contrast.

The experiment was carried out at the high flux reactor of the Institute Laue-Langevin (ILL) (Kroupa et al 2000). The wavelength of the incident beam was tuned to $\lambda_0 = 1.92 \text{Å}$. The beam was polarized vertically by magnetic-prism refractions. A parallel-sided Al plate was used as a phase shifter, to vary $\chi$. A pair of water-cooled Helmholtz coils produced a fairly uniform magnetic guide field $B_0 \hat{z}$ around the interferometer. A super-mirror together with a dc spin rotator, to adjust $\alpha$, enabled us to select neutrons in certain spin directions for detection. The Mu-metal tube was placed in path I of the interferometer. The dimensions of the tube were: length = 13.0 mm, mean diameter = 15.5 mm and wall thickness = 0.10 mm. The measured difference of the azimuthal angle between the spins in paths I and II after the tube was $0.98 \pm 0.05\pi$, with a guide field $B_0 = 2.19 \text{mT}$. The contrasts of the interferometer itself (in empty scans) were nearly 90%.

Since a maximum violation of the Bell-like inequality is expected at the values $\alpha = 0, \pi/2$, the spin-analysis parameter was tuned to $\alpha = 0, \pi/2, \pi$ and $-\pi/2$. The contrasts in the $\chi$ scans for these $\alpha$ values were $C = 0.813(5), 0.717(5), 0.859(5)$ and $0.726(5)$, respectively. A typical set of intensity oscillations is shown in figure 10. An overall mean contrast of $C = 0.778(3)$ was achieved, which significantly exceeds the value in the previous experiment (Hasegawa et al 2003). After fitting the measured count rates to a sinusoidal dependence by the least-squares method, the expectation values were determined. The same measurements were repeated seven
times to reduce statistical errors. We obtained for $S$ in the Bell-like inequality equation (3.1) a value of

$$S = 2.202 \pm 0.007 > 2$$  \hspace{1cm} (3.2)

The error includes statistical and systematic errors, where the main reason for systematic errors was phase instability of the interferometer and inaccuracies in the adjustment of optical elements. This violates the Bell-like inequality by $\sim 29$ standard deviations and so clearly confirms quantum contextuality, while rejecting NCHVT at the same time.

3.2. Neutron polarimetry: falsification of a contextual model à la Leggett

In 2003, Leggett proposed a class of nonlocal (crypto-non-local) hidden-variable theories and proved that his model is incompatible with quantum predictions (Leggett 2003). Experimental tests confirm the incompatibility of such theories (Gröblacher et al. 2007a, b, Paterek et al. 2007, Branciard et al. 2007, 2008). Until now, however, crypto-nonlocal models à la Leggett have been tested only with photon systems. Here we report experiments for a crypto-contextual model with matter-waves of neutrons (Hasegawa et al. 2011). We exploit entanglement between degrees of freedom, i.e. energy-spin, of neutrons and use a neutron polarimeter (Sponar et al. 2010a, 2010b). This apparatus enables neutron optical experiments with high-efficiency manipulations and insensitivity to ambient disturbances.

3.2.1. Theory. For our polarimetric test, we follow the criteria used in the first experimental investigation by Gröblacher et al. (2007a, 2007b). We define the crypto-contextual theory to be tested here as based on the following assumptions:

(i) All values of measurements are predetermined.
(ii) States are a statistical mixture of subensembles.
(iii) The local expectation values after an ideal polarizer show a cosine dependence.

Whereas assumptions (i) and (ii) are common to ordinary non-contextuality assumptions, assumption (iii), namely Malus’ law, demands correlations even between measurements of compatible observables, which is a particular point of crypto-contextual models à la Leggett.
Figure 11. Schematic view of the neutron polarimetric test of the crypto-contextual model à la Leggett (upper panel) together with Bloch sphere (lower panel) descriptions of the observables of spin and energy degree of freedom. Note that the directions of the spin measurements are out of the equatorial plane, whereas the directions of the energy measurements are in the equatorial plane.

Following the path used in previous works (Gröblacher et al 2007a, 2007b, Branciard et al 2007) and assuming full rotational symmetry, an inequality can be used to test crypto-contextual hidden variable (CCHV) models in our experiments. Denoting the measurement settings of observables A and B by $\vec{a}_1, \vec{a}_2$ and $\vec{b}_1, \vec{b}_2$ on the Bloch sphere (see figure 11), respectively, the crypto-contextual Leggett-like inequality is expressed by

$$S_{\text{CCHV}} := |E_1(\vec{a}_1; \varphi) + E_1(\vec{a}_1; 0)| + |E_2(\vec{a}_2; \varphi) + E_2(\vec{a}_2; 0)| \leq 4 - \frac{4}{\pi} |\sin \frac{\varphi}{2}|,$$

where $E_j(\vec{a}_j; \phi)$ represent the expectation value of joint (correlation) measurements at the settings $\vec{a}_j$ and $\vec{b}_j$ with the relative angle $\phi$. For a singlet state, quantum mechanics predicts for the expectation values $E_j(\vec{a}_j; \varphi) = -\vec{a}_j \cdot \vec{b}_j = -\cos \phi$ which gives for the $S$-function $S_{\text{QM}}(\phi) = 2|1 + \cos \phi|$. The maximum violation is expected at $\phi_{\text{max}} \sim 0.1\pi$: a bound of the Leggett-like inequality is given by $S_{\text{CCHV}} = 3.797$, whereas quantum mechanics predicts a value of $S_{\text{QM}} = 3.899$. It is worth noting here that the difference between predictions by the model and quantum mechanics is very small, $\Delta S = 0.102$: this calls for extremely high contrast, or rather high correlation, values, i.e. higher than 97.4%, in the measurements and thus makes the experiments non-trivial and challenging.
3.2.2. Experiment. The experimental setup, together with a Bloch sphere description of the spin and the energy degree of freedom, is depicted in figure 11. The measurement is based on joint measurements of two commuting observables, $A^{\text{spin}}$ for the spin and $B^{\text{energy}}$ for the total energy degree of freedom of neutrons. In the experiment, a maximally entangled Bell-like state

$$|\Psi^{N}_{\text{Bell}}\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle \otimes |E_0\rangle + |\downarrow\rangle \otimes |E_0 - \hbar \omega\rangle)$$

(3.4)

with the spin basis, $|\uparrow\rangle$ and $|\downarrow\rangle$, as well as the energy basis, $|E_0\rangle$ and $|E_0 - \hbar \omega\rangle$, is generated, and followed by successive energy and spin measurements. The experiment was carried out at the research reactor facility TRIGA Mark II of the Vienna University of Technology. The incident beam from the reactor is monochromatized to $\lambda = 1.96 \text{ Å}$ by a highly oriented pyrolytic graphite (PG) monochromator and propagates in the $+y$-direction. Reflected from a bent Co-Ti super-mirror array, the beam is highly polarized (more than 99%). The same technique is employed for analyzing the polarization downstream. A high-efficiency (>99%) BF$_3$ detector is used for measuring the expectation values appearing in equation (3.3) Two identical radio-frequency (RF) spin rotators are employed, each producing a sinusoidally oscillating magnetic field ($\sim 1 \text{ G}$ for a $\pi/2$ rotation) with $\omega = 40 \text{ kHz}$. They are about 20 cm long and made of enameled copper wire wound on PVD pipes (diameter $\sim 4 \text{ cm}$). Both put in a homogeneous and static magnetic guide field ($\sim 13 \text{ G}$) supplied by two rectangular coils in Helmholtz geometry. After tuning the strength of the guide field, scans of the magnetic field amplitude of the RF spin rotators exhibit sinusoidal intensity modulations with more than 99% contrast: high-efficiency manipulation is ensured. The second RF spin rotator (RF2) downstream is mounted on a translation table, which enables precise adjustment of the neutron’s flight-time between the two spin rotators.

The maximally entangled Bell-like state $|\Psi^{N}_{\text{Bell}}\rangle$ is generated by tuning the rotation angle of RF1 to $\pi/2$. The amplitude and the phase of the oscillating magnetic field tuned by RF2 are directly associated with the parameters of the measurement. Additionally, the adjustment of the position of RF2 enables precise tuning of the relative phase between the two energy eigenstates. The spin analyzer filters out the down-component of the spin. As a result, the needed measurement settings on the equator of the Bloch sphere, for the energy, and arbitrary directions for the spin are realized by RF2 together with the analyzer.

For the measurement of the Leggett-like inequality (equation (3.3)) correlation measurements between settings outside the single plane are needed. The parameter $\phi$ is varied, which represents the latitudinal deviation from corresponding points on the equator by varying the phase of RF2 (see figure 11). The polar angle setting is $\pi/2$ for the measurement directions in the (equatorial) plane and $\pi/2 - \phi$ for the direction $\vec{b}_2$. We obtained final sinusoidal oscillations with mean contrasts of 98.5%, which is the highest correlation ever obtained between commuting observables of massive particles.

The maximum discrepancy between the crypto-contextual Leggett-model and quantum mechanics is expected at the angle $\phi_{\text{max}} \sim 0.1 \pi$. We have carried out the measurement by setting $\phi = 0.104\pi$. The $S_{\text{CCHV}}$-value of the crypto-contextual Leggett-like inequality is determined as $S_{\text{CCHV}} = 3.8387(61)$, which is clearly larger than the boundary 3.7921. The violation is more than 7.6 standard deviations. In order to see the tendency of the violations, the parameter $\phi$ is varied around the point of maximum violation: eight different $\phi$-values are chosen between 0 and $0.226\pi$. Figure 12 is the plot of the experimentally determined $S_{\text{CCHV}}$ together with the bound of the contextual model and the quantum mechanical prediction, where a 99% contrast of
Figure 12. $S$-values as a function of the derivation angle $\phi$ to test alternative quantum theory à la Leggett. The red curve is the prediction of quantum mechanics and the broken line gives the boundary provided by the crypto-contextual Leggett model. The $S_{\text{CCHV}}$-value is determined as $3.8387(61)$ at $\phi = 0.104\pi$, which is clearly larger than the boundary $3.7921$ (Hasegawa et al 2011).

the correlation is taken into account. The obtained values fully follow the quantum mechanical predictions. All errors include statistical and systematic errors: the systematic errors of the present experiments are found to be much smaller than those for the interferometric experiments (Hasegawa et al 2011, 2003, Bartosik et al 2009) where such errors mainly result from the phase instability of the interferograms. The present experiment clearly confirms the violation of the crypto-contextual model.

4. Concluding remarks

The wave–particle duality is manifest in neutron interference experiments and many textbook experiments of quantum mechanics have been realized with this technique. To a large extent neutron quantum optics paved the way for optics with even heavier particles. The results are well described by standard quantum mechanics although their epistemological understanding is still under discussion. Expectation values are measured but individual events remain random and the physical phenomenon how the quantum world converts into a classical one remains an open question. Here various dephasing experiments with noisy magnetic fields are described which mimic real decoherence effects, which are seen as the cause for this transition. Employing single-particle interferences, intrinsic entanglements between different degrees of freedom have been observed and various Bell-like inequalities have been tested. These results demonstrate quantum contextuality as an important feature of quantum theory, and the strength of entanglement phenomena in physics. From these results various hidden variable theories can be rejected. Multi-entanglement experiments are related to the Kochen–Specker theorem, which may help to further broaden our understanding of quantum phenomena. It may even become
relevant for modern quantum communication systems, mainly because in the entanglement feature additional information can be stored.

Acknowledgments

We thank all our colleagues for assistance with carrying out the experiments presented here; in particular, we appreciate G Badurek, E Balcar, H Bartosik, A Cabello, K Durstberger-Rennhofer, D Erdösi, S Filipp, D Home, J Klepp, H Lemmel, R Loidl, S Sponar, C Schmitzer and G Sulyok. This work was partially supported by the Austrian FWF (Fonds zur Förderung der Wissenschaftlichen Forschung) through grant numbers P-18943-N2D and P21193-N20.

References

Abele H 2008 Prog. Part. Nucl. Phys. 60 1
Aharonov Y and Anandan J S 1987 Phys. Rev. Lett. 58 1593
Allman B E, Kaiser H, Werner S A, Wagh A G, Rakhecha V C and Summhammer J 1997 Phys. Rev. A 56 4420
Bartosik H, Klepp J, Schmitzer C, Sponar S, Cabello A, Rauch H and Hasegawa Y 2009 Phys. Rev. Lett. 103 040403
Basu S, Bandyopadhyay S, Kar G and Home D 2001 Phys. Lett. A 279 281 arXiv:quantph/9907030
Bell J S 1964 Physics I 195
Berry M V 1984 Proc. R. Soc. A 302 45
Bertlmann R A and Zeilinger A (ed) 2002 Quantum (Un)speakables: From Bell to Quantum Information (Heidelberg: Springer)
Bouwman W G, Plomp J, de Haan V O, Kraan W H, van Well A A, Habicht K, Keller T and Rekveldt M T 2008 Nucl. Instrum. Methods A 586 9
Branciard C, Ling A, Gisin N, Kurtsiefer C, Lamas-Linares A and Scarani V 2007 Phys. Rev. Lett. 99 210407
Branciard C, Brunner N, Gisin N, Kurtsiefer C, Lamas-Linares A, Ling A and Scarani V 2008 Nat. Phys. 4 681
Casimir H B and Polder D 1948 Phys. Rev. 73 360
Colella R, Overhauser A W and Werner S A 1975 Phys. Rev. Lett. 34 1472
Cronin A D, Schmiedmayer J and Pritchard D E 2009 Rev. Mod. Phys. 81 1051
DeChiara G and Palma G M 2003 Phys. Rev. Lett. 91 090404
Einstein A, Podolsky A B and Rosen N 1935 Phys. Rev. 73 777
Filipp S, Hasegawa Y, Loidl R and Rauch H 2005 Phys. Rev. A 72 021602
Filipp S, Klepp J, Hasegawa Y, Plonka-Spehr C, Schmidt U, Geltenbort P and Rauch H 2009a Phys. Rev. Lett. 102 030404
Filipp S, Klepp J, Plonka C, Schmidt U, Geltenbort P, Hasegawa Y and Rauch H 2009b Nucl. Instrum. Methods A 598 571
Gerlich S, Eibenberger S, Tomandl M, Nimrichter S, Hernberger K, Fagan P J, Tuxen J, Mayor M and Arndt M 2011 Nat. Commun. 2 263
Giulini D, Joos E, Kiefer C, Kupsch J, Stamatescu I-O and Zeh H D 1996 Decoherence and the Appearance of a Classical World in Quantum Theory (Berlin: Springer)
Glauber R J 1963a Phys. Rev. 130 2529
Glauber R J 1963b Phys. Rev. 131 2766
Golub R, Richardson D and Lamoreaux S K 1991 Ultra Cold Neutrons (Bristol: Hilger)
Greenberger D M 1988 Physica B 151 374
Gröblacher S, Paterek T, Kaltenbaek R, Brukner Č, Żukowski M, Aspelmeyer M and Zeilinger A 2007a Nature 446 871

New Journal of Physics 13 (2011) 115010 (http://www.njp.org/)
Gröblacher S, Paterek T, Kaltenbaek R, Brukner Č, Żukowski M, Aspelmeyer M and Zeilinger A 2007b Nature 449 252

Haroche S and Raimond J-M 2006 Exploring the Quantum (Oxford: Oxford University Press)

Hasegawa Y, Loidl R, Badurek G, Baron M and Rauch H 2003 Nature 425 45

Hasegawa Y, Loidl R, Badurek G, Baron M and Rauch H 2006 Phys. Rev. Lett. 97 230401

Hasegawa Y, Loidl R, Badurek G, Filipp S, Klee J and Rauch H 2007 Phys. Rev. A 76 052108

Hasegawa Y, Schmitzer C, Bartosik H, Klee J, Sponar S, Durstberger-Rennhofer K and Badurek G 2011 in preparation

Jacobson D L, Werner S A and Rauch H 1994 Phys. Rev. A 49 3196

Jenke T, Geltenbort P, Lemmel H and Abele H 2011 Nat. Phys. 7 468

Kaiser H, Rauch H, Badurek G, Bauspiess W and Bonse U 1979 Z. Phys. A 291 231

Kaiser H and Rauch H 1999 De Broglie wave optics: neutron, atoms and molecules Experimental Physics Bergmann–Schaefer. Vol. 3, Optics ed H Niedrig (Berlin: de Gruyter) p 1043

Kochen S and Specker E P 1967 J. Math. Mech. 17 59

Kroupa G, Bruckner G, Bolik O, Zawisky M, Hainbuchner M, Badurek G, Buchelt R J, Schricker A and Rauch H 2000 Nucl. Instrum. Methods A 440 604

Leggett A G 2003 Found. Phys. 33 1469

Lemmel H 2011 private communication

Levy-Leblond J M 1987 Phys. Lett. A 125 441

Mandel L and Wolf E 1995 Optical Coherence and Quantum Optics (Cambridge: Cambridge University Press)

Mezei F 1972 Z. Phys. 255 146

Mezei F (ed) 1980 Neutron Spin Echo (Berlin: Springer)

Mermin N D 1993 Rev. Mod. Phys. 65 803

Nakamura et al 2010 J. Phys. G: Nucl. Part. Phys. 37 075021

Nesvizhevsky V V et al 2002 Nature 415 297

Pancharatnam S 1956 Proc. Ind. Acad. Sci. A 44 247

Paterek T, Fedrizzi A, Gröblacher S, Jennewein T, Żukowski M, Aspelmeyer M and Zeilinger A 2007 Phys. Rev. Lett. 99 210406

Rauch H 1993 Phys. Lett. A 173 240

Rauch H, Lemmel H, Baron M and Loidl R 2002 Nature 417 630

Rauch H, Treimer W and Bonse U 1974 Phys. Lett. A 47 369

Rauch H and Werner S A 2000 Neutron Interferometry (Oxford: Clarendon)

Rauch H, Zeilinger A, Badurek G, Wilfing A, Bauspiess W and Bonse U 1975 Phys. Lett. A 54 425

Samuel J and Bhandari R 1988 Phys. Rev. Lett. 60 2339

Sears V F 1989 Neutron Optics (Oxford: Oxford University Press)

Shapere A and Wilczek F (ed) 1989 Geometric Phases in Physics (Singapore: World Scientific)

Sponar S, Klee J, Loidl R, Filipp S, Durstberger-Rennhofer K, Bertlmann R A, Badurek G, Hasegawa Y and Rauch H 2010a Phys. Rev. A 81 042113 (arXiv:quant-ph/09074909)

Sponar S, Klee J, Zeiner C, Badurek G and Hasegawa Y 2010b Phys. Lett. A 374 431 (arXiv:quant-ph/09074654)

Summhammer J, Badurek G, Rauch H and Kischko U 1983 Phys. Rev. A 27 2532

Summhammer J, Hamacher K A, Kaise H, Weinfurter H, Jacobson D L and Werner S A 1995 Phys. Rev. Lett. 75 3206

Sulyok G, Hasegawa Y, Klee J, Lemmel H and Rauch H 2010 Phys. Rev. A 81 053609

Tittel W, Brendel J, Zbinden H and Gisin N 1998 Phys. Rev. Lett. 81 3563

Werner S A, Colella R, Overhauser A W and Eagen C F 1975 Phys. Rev. Lett. 35 1053

Werner S A, Staudenmann J-J and Colella R 1979 Phys. Rev. Lett. 42 1103

New Journal of Physics 13 (2011) 115010 (http://www.njp.org/)