Focusing properties of Fresnel zone plates with spiral phase

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Abstract: Focusing properties of Fresnel zone plates with spiral phase with integer and fractional topological charges illuminated by plane wave are studied. Numerical results show that hollow beams can be generated and can also be controlled by the number of the zones and the topological charge, which implies the potential applications of such kind of zone plate in trapping and manipulating particles.

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1. Introduction

Fresnel zone plate (FZP) is an important focusing device especially in the fields of extreme ultraviolet imaging and X-ray imaging [1,2]. The researches have mainly focused on improving zone plate’s spatial resolution and enhancing their transmission efficiency [3–8]. Some interesting work about zone plates, such as photon sieves and fractal zone plates, have
been reported in recent years [9–11]. Spiral zone plates for x-ray microscopy are fabricated to detect phase effects and isotropic edge enhancement [12]. Spiral fractal zone plates for generating vortices are also reported [13].

In this paper we investigate a kind of FZPs with spiral phase in detail as extension of the spiral fractal zone plates. Numerical simulations show that the spiral phase FZPs with integer topological charges are somewhat similar to conventional FZPs in focusing properties and that hollow beams generated by spiral phase FZPs can be controlled by their zone numbers and topological charges. Focusing properties of spiral phase FZPs with fractional charges are also studied. Potential applications of the spiral phase FZPs are discussed.

2. Fresnel zone plate with spiral phase

A FZP with spiral phase, which consists of alternate transparent and opaque zones with radius $r_n = \sqrt{n} r_1$ is shown in Fig. 1(a), where $r_1$ is the radius of first zone. The phase change of each transparent zone in a period is $2\pi p$, where $p$ is the topological charge and the black-white bar indicates the phase in grey scale while the pattern bar denotes the opaque area. Simulations of diffraction patterns of the zone plates illuminated by a plane wave are performed on the basis of the setup shown in Fig. 1(b).

![Fig. 1. The illustration of the (a) structure of Fresnel zone plate with spiral phase and (b) scheme of the setup.](image)

The diffraction intensity in an observation plane can be calculated by Huygens-Fresnel diffraction formula:
\[ I(\rho, \theta, z) = \left| -\frac{ia}{\lambda R} \int \int_{\text{zones}} \exp(iK + i\theta) \rho d\rho d\varphi \right|^2, \] (1)

with
\[ R = \left[ (\rho \cos \theta - r \cos \varphi)^2 + (\rho \sin \theta - r \sin \varphi)^2 + z^2 \right]^{1/2}, \] (2)

where \( A \) is the amplitude of the plane wave, \( \lambda \) is wavelength, \( k = 2\pi / \lambda \) denotes wave number, \((r, \varphi)\) is the cylindrical coordinate in the zone plate plane, \((\rho, \theta)\) is the cylindrical coordinate in the observation plane, and \( z \) is the distance between two planes. In this paper we choose \( \lambda = 633 \text{ nm}, r_1 = 0.67 \text{ mm} \), so the focus of the normal FZP is \( f = 0.7092 \text{ m} \).

3. Numerical simulation results and discussions

The diffraction intensity distribution of a spiral phase FZP with 60 zones with the topological charge \( p = 1 \) along the optical axis \( z \) is presented in Fig. 2(a). Two brightest points at \( z = f/3 = 0.2364 \text{ m} \) and \( z = f = 0.7092 \text{ m} \) indicate the maximum intensity in the propagation direction.
Figures 2(b) and 2(c) are diffraction patterns at $z = f/3$ and $z = f$. The doughnut patterns can be explained as a result of radial Hilbert filtering [14], and it is easy to get $I(r_{0}, \theta, z_{0}) = \text{constant}$ and $I(0, \theta, z_{0}) = 0$ by using Eq. (1) where $r_{0} = \text{constant}$ and $z_{0} = \text{constant}$. Figures 2(d) and 2(e) are corresponding intensity distributions of Figs. 2(b) and 2(c), respectively. It can be found that the distance between two peaks at $z = f/3$ is smaller than that at $z = f$, which can be observed obviously in the diffraction patterns above. Compared with normal FZP, this kind of

Fig. 3. Spiral phase Fresnel zone plates with integer topological charges and their diffraction patterns. (a), (b) Diffraction patterns of spiral phase FZP with topological charge $p = 1$ with 120 and 240 zones; (c), (e) spiral phase FZPs with topological charge $p = 2$ and 3 with 60 zones; (d), (f) the corresponding diffraction patterns of zone plates shown in (c), (e) in focal plane when $z = 0.7092m$. 

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spiral phase FZP has maximum intensity doughnut patterns at \( f/3 \) while a FZP has foci at \( f, f/3 \); the doughnut pattern and hollow core at \( f \) are larger than that at \( f/3 \) while a FZP has larger focus spot at \( f \) than at \( f/3 \). Figures 3(a) and 3(b) are diffraction patterns of a spiral phase FZP with topological charge 1 with 120 zones and 240 zones. The doughnut pattern and hollow core become smaller as the total zone number increases while the focus spot of a FZP becomes smaller as the total zone number increases. All results show that the FZP with spiral phase has the similar focusing properties as the diffraction of the FZP to some extent. It also indicates that the spiral phase FZP can generate hollow beam whose radius and hollow part can be controlled by the total zone number with a changeable aperture.

Figures 3(c) and 3(e) are spiral phase FZPs of 60 zones with topological charges \( p = 2 \) and 3. The following Figs. 3(d) and 3(f) are corresponding diffraction patterns in focal plane \( z = f \), respectively. It shows that the doughnut pattern and hollow part will expand as the integer topological charge increases, which is similar to the vortex generated by spiral phase plate illuminated by laser beam and diffraction pattern generated by spiral zone plate with different topological charges [12,15]. In other words, hollow beams generated by spiral phase FZP are controllable by topological charge and total zone number, which may be useful in trapping and manipulating particles.

Table 1 gives some detailed parameters of the size of doughnut patterns generated by zone plates presented above. Here the radius of doughnut pattern is defined as the length from the nearest minimum of the doughnut point spread function as shown in Figs. 2(d) for example to the center zero point. The width of doughnut pattern is defined as the full width at half maximum of one peak. And the radius of the dark hollow part is defined as the length from the nearer half maximum point to the center zero point. In Table 1 column 2 and 3, one can see that the radius, width of doughnut pattern and the radius of the dark hollow part of zone plate with \( p = 1 \), \( n = 60 \) at \( z = f \) are 3 times of that at \( z = f/3 \). From column 3, 4, and 5, one can conclude that the corresponding parameters of doughnut pattern are proportional to \( 1/\sqrt{n} \) for zone plates with same topological charge at \( z = f \) but with different total zone numbers \( n \). In addition, from column 3, 6, and 7, one can get that the increase of the width of doughnut pattern is the smallest in three listed parameters while the increase of the radius of doughnut pattern is the largest as the topological charge increases.

Table 1. Parameters of doughnut patterns generated by zone plates with different parameters

| Zone plates with spiral phase | \( p = 1 \), \( n = 60 \), \( z = f/3 \) | \( p = 1 \), \( n = 60 \), \( z = f \) | \( p = 1 \), \( n = 120 \), \( z = f \) | \( p = 1 \), \( n = 240 \), \( z = f \) | \( p = 2 \), \( n = 60 \), \( z = f/3 \) | \( p = 3 \), \( n = 60 \), \( z = f/3 \) |
|-----------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| Radius of doughnut pattern/μm | 26.3 | 79.6 | 58.2 | 40.3 | 108.9 | 131.3 |
| Width of doughnut pattern/μm | 11.9 | 35.5 | 25.8 | 17.9 | 40.0 | 43.3 |
| Radius of the dark hollow part/μm | 5.3 | 16.1 | 11.2 | 7.8 | 33.8 | 50.7 |

Furthermore, spiral phase FZPs with fractional topological charges are also studied. Figures 4(a), 4(c), and 4(e) are zone plates with 60 zones with topological charges \( p = 0.25 \), 0.5, and 0.75, respectively. The corresponding diffraction patterns at focal plane of each zone plate are shown in Figs. 4(b), 4(d), and 4(f). Not like in the situation of integer topological charges, the diffraction patterns of zone plates with fractional topological charges will not only change the sizes of the patterns but also the shapes of the patterns. It can be observed that an asymmetric hollow beam appears as the topological charge increases. The area with stronger intensity at the bottom of the hollow beam implies that it can give a force in one direction when the hollow beam is used to trap and manipulate particles. Compared with the diffraction patterns generated by fractional spiral phase filter with a fractional topological charge [16], the similar diffraction patterns generated by spiral phase FZPs with fractional topological charges suggest their potential application in orientation-selective edge enhancement.
Fig. 4. Spiral phase Fresnel zone plates with fractional topological charges and their diffraction patterns. (a), (c), (e) Spiral phase FZPs with topological charge $p = 0.25$, 0.5, and 0.75 with 60 zones; (b), (d), (f) the corresponding diffraction patterns of the zone plates shown in (a), (c), (e) in focal plane when $z = 0.7092m$.

4. Conclusion

In summary, we propose a kind of FZP consisting of alternate transparent and opaque zones with spiral phase. Focusing properties of this zone plates with integer and fractional topological charges are studied. Numerical results show that doughnut hollow beams can be generated by zone plates with integer topological charges and can be controlled by the total zone numbers and topological charges. Asymmetric hollow beams can also be generated by zone plates with fractional topological charges.

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