DRIVING WHEEL MOVEMENT MATHEMATICAL MODEL

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Abstract. One of the wheeled tractors use effectiveness essential indicators in agricultural production is the indicator of their traction and grip qualities, due primarily to the interaction of the driving wheels with the rolling surface on different soil backgrounds and the slip coefficient. As the result of the wheel propellers interaction with the soil, the latter one is compacted and intensively destroyed. Uneven soil density increases traction resistance and increases the energy consumption for the machine-tractor unit movement (MTU). The possibility of the most effective reduction of hook load fluctuations corresponding to an intensive increase in slipping is provided by the elastic pneumohydraulic element, which has the ability to regulate optimal parameters in a wide range of elastic properties.

1. Introduction

The stable operation and productivity of the MTU improving consists in creating conditions for a smooth change in the forward speed and the formation of traction resistance created by agricultural machines, depending on the agrotechnical requirements and the background of the processed field surface. The wheel propellers interaction with the rolling surface efficiency is mainly determined by the coefficient of adhesion, rolling resistance and skidding of the driving wheels.

The wheeled tractors undercarriage existing designs analysis showed that the possibilities for increasing the efficiency of their work are limited, new technical solutions, that will optimize the operation of the running apparatus, are needed.

At present, a pneumohydraulic elastic element, which has the ability to regulate the optimal parameters in a wide range of elastic properties, is used to increase the undercarriage using efficiency and reduce slipping on universal row-crop tractors.

The use of the controlled transmission with elastic elements in the clutch helps to provide the effective method for the hook load oscillations amplitudes and torsional vibrations in the tractors undercarriage tires reducing in power transmissions [1, 2].

2. Materials and methods

The dynamic loads formation in the wheeled tractor undercarriage, depending on the change in the input signal of the hook force (Pкр), research was carried out on the basis of the driving wheels motion mathematical modeling; the driving wheels uncertainties are limited by the random nature of exogenous factors, reliably described by the theory of random functions [3, 4].

The use of differential calculus methods, operational methods for solving differential equations, methods for transforming structural schemes of feedback mechanisms, the theory of random processes allows, in the presence of analytical or empirical methods for assessing all kinds of influences, to compose adequate mathematical models of the machine-tractor units operation.

Theoretical and experimental researches related to the wheeled tractors undercarriage using efficiency increasing [5, 10] made it possible to formulate recommendations for determining the optimal values: load on the driving wheels, slip coefficient, hook resistance, hook force and movement resistance of the MTU.
3. Results and discussion

When the wheel propeller operates in the driving mode under the influence of the tractor weight, the reaction of the soil and the transmitted torque to the wheel, the tire perceives radial, longitudinal and tangential deformation providing smoothing of the aggregated machines uneven resistance forces and vibrations from the soil surface unevenness. The driving wheel is presented in the form of two mass models with the inertia moments of the wheel disk with a hub and part of the tire $J_1$, and part of the tire with a tread and spade bug $J_2$.

The tractor driving wheel movement is described by the system of equations:

\[
\begin{align*}
J_1 \cdot \dot{\omega}_1 &= M_{ec} \cdot h_{gr} - M_{lsm} \\
J_2 \cdot \dot{\omega}_2 &= M_{lsm} - M_{ld}
\end{align*}
\]

where $M_{ec}$ is the moment of elastic connection in the power transmission, taking into account its elasticity, damping and dry friction; $i_{tr}$ is the transmission gear ratio; $\omega_1$ is the angular speed of the wheel axis; $\omega_2$ is the angular speed of the tire tread.

The tire spinning moment can be calculated by using the formula (1):

\[
M_{lsm} = C_{k2} \cdot (\varphi_1 - \varphi_2) + \alpha_{k2} \cdot (\omega_1 - \omega_2),
\]

where $c_{k2}$ is the torsional stiffness of the drive wheel tire; $\varphi_1$ is the angle of driving wheel axis rotation; $\varphi_2$ is the angle of tire tread rotation; $\alpha_{k2}$ is the driving wheel torsional damping;

The moment $M_{ld}$ from longitudinal deformation, determined by the elasticity and deformation of the tire, can be calculated by using the equation (2):

\[
M_{ld} = (c_x \cdot x + \alpha_x \cdot x) \cdot r_s + R \cdot \alpha_2,
\]

where $c_x, \alpha_x$ are the coefficients of longitudinal stiffness and viscous friction in the tire; $\alpha_2$ is the displacement of the vertical reaction $R$ relative to the wheel axis (rolling arm); $x$ is the longitudinal deformation of the tire; $r_s$ is the dynamic radius of the wheel; $R$ is the vertical reaction to driving wheels.

The pneumatic tire tangential stiffness magnitude, which is usually included in the total stiffness of the power train, we used the longitudinal stiffness of the tire $c_x$. This is due to the fact that when the wheels roll, deformed in the tangential and, therefore, longitudinal direction, the tire elements come out of contact with the supporting surface, and the new tire elements come into contact and perceive this deformation. Then, during the contact time, the driving wheel travels a patch equal to the length of the patch area ($L$) minus the value of the tire longitudinal deformation ($x$). Therefore, the rolling radius taken by the revolutions of the driving wheel on the investigated section of the patch is always greater than the static one. This is due to the fact that the driving wheels, due to the longitudinal compliance of the tire, are displaced in the direction opposite to the movement relative to the center of the tire patch area, with an wheel increasing propulsive force.

Then the driving wheel dynamic rolling radius taken in the calculations will be equal to:

\[
r_{dr} = r_0 - z_0 - z_{um},
\]

where $r_0$ is the radius of the undeformed wheel; $z_0$ is the static deflection of the driving wheel tire; $z_{um}$ is the radial deformation of the driving wheel tires, from the vertical movement of the tractor framework.

And the vertical reaction on the driving wheels is determined by the formula (4):

\[
R = c_z \cdot z_{rd} + \alpha_z \cdot z_{rd} + c_x \cdot z_0,
\]

where $c_z$ is the driving wheels tires radial stiffness; $\alpha_z$ is the driving wheels tires viscous friction radial coefficient; $z_{rd}$ is the radial deformation of the driving wheel tires, from the vertical movements of the tractor framework; $z_0$ is the static deflection of the driving wheel tire.

Knowing the wheel dynamic rolling radius, one can determine the actual speed of the tractor:

\[
V_{dr} = \left[ \omega_2 \cdot r_s \cdot \left( L - \epsilon_1 \cdot R - \epsilon_2 \cdot \frac{Z}{r_0} \cdot \sin \omega_2 \right) - x \right] \cdot \left( L - \delta \right),
\]

where $\omega_2$ is the angular speed of the tire tread; $\epsilon_1, \epsilon_2$ are the coefficients of the tire material deformation from vertical and longitudinal forces; $R$ is a vertical reaction; $L$ is the length of the tire patch area; $\delta$ is the slip coefficient.

The vertical load on the driving wheel axle and the bearing surface reaction create an additional moment on the increasing arm, resistance to movement.

The loading rate on the driving wheels created by the tractor is determined by the formula (6):

\[
Q \approx Q_c + \frac{P_{ap} \cdot h_{ap}}{2L},
\]

where $Q_c$ is the static load on the driving wheel; $P_{ap}$ is current hook force; $h_{ap}$ is the tractor hitch longitudinal rods attachment height.
Taking into account the formula (5):
\[ Q = Q_c + P_{\text{spo}} \left( 1 + V_{\text{tr}} \right) \frac{h_{\text{re}}}{2L}, \]

or
\[ Q = Q_c + P_{\text{spo}} \left[ 1 + V_c (1-\delta) \right] \frac{h_{\text{re}}}{2L}. \]  

(6)

According to the existing method of tractors traction tests, the rate of slipping is determined as the result of tire slippage relative to the patch area.

This slippage is ensured by turning the drive wheel at the angle \( \gamma_{\text{tr}} \):
\[ \gamma_{\text{tr}} = \frac{\delta \cdot S}{(r_o - e)}, \]  

(7)

where \( e \) is the deformation of the tire under the action of a vertical load \( Q \) in the vertical direction, depending on the change in the torque \( M_c \); \( S \) is the distance between the tire spade bugs.

In our calculations, the circular stiffness of the tire \( C_{\text{qim}} \) was determined by the formula (8):
\[ C_{\text{qim}} = \frac{1}{\mu_k}, \]  

(8)

where \( \mu_k \) is the coefficient of the tire circular elasticity.

This means that one can write down that \( M_c = C_{\text{qim}} \gamma_{\text{tr}} \).

Taking into account that the driving wheel torque \( M_c \) is equal to half the tractive moment of the propellers, \( M_c = \frac{1}{2} M_T = (P_{\text{spo}} + P_i)(r_o - e) \), it is possible to determine the value of the resistance to the tractor (\( P_i \)) movement (9) [8] for the slip coefficient determination:
\[ P_i = \frac{1}{2} \left[ \frac{cBk_m^2}{(1-\delta)^2} + \frac{\alpha C_e}{r_o - e} \left( e^2 + \frac{\mu_m M_c r_o}{r_o - e} \right) \right], \]  

(9)

where \( C_e \) is the radial stiffness of the tire single sector; \( c \) is soil hardness according to the Gerstner-Bernstein formula; \( \alpha \) is the coefficient of hysteresis losses; \( B \) is the width of the tire; \( k_m = C_e/Br_o^2 \); \( r_o \) is the reduced tire radius (as equal to \( S/(r_o b + S - b (r_m)) \)), where \( r_o \) is the undeformed wheel tire outer diameter; \( r_m \) is the radius of the tire along the cavity; \( e \) is the deformation of the tire in the vertical direction; \( Q \) is the vertical load on the driving wheel; \( r_o \) is the radius of the wheel disk; \( \mu_k \) is the tire circular elasticity coefficient; \( S \) is the distance between the spade bugs; \( M_c \) is the moment set in the driving wheel from the engine side.

Then one can determine the current slip coefficient corresponding to the conditions of the tractor start-up (10):
\[ \delta = \frac{2C_e \gamma_{\text{mass}}}{(r_o - e)C_\delta} = \frac{2C_e \gamma_a}{i_{\text{tr}} (r_o - e)C_\delta}, \]  

(10)

where \( C_e \) is the tire circular stiffness; \( C_\delta \) is the soil hardness; \( \gamma_{\text{mass}} \) is the twist angle of the driving wheel shaft; \( i_{\text{tr}} \) is the transmission gear ratio; \( r_o \) is the tire outer diameter.

The increase in the speed of the MTU affects the change in the hook load, as well as the change in the tractive moment on the propellers of the tractor and will make it possible to determine the resistance to the tractor movement during acceleration as a variable [6, 7]. Therefore, the force of resistance to the movement of the MTU can be considered not only as the function of time, but also as the function of the unit translational speed:

Then the dynamic hook force \( P_{\text{sp}(i)} \) can be determined by using the following expression (11):
\[ P_{\text{sp}(i)} = \left[ 1 + 0.1 \frac{\Phi_{\text{l}}}{i_{\text{tr}}^2} (r_o - e) \cdot (1-\delta) \left[ (m_e + m) \frac{\Phi_{\text{l}}}{i_{\text{tr}}} (r_o - e) \cdot (1-\delta) \right. \right. \]  

(11)

where \( \Phi_{\text{l}} \) is the angular acceleration of the engine; \( m_e \) is the tractor weight; \( m \) is the mass of the agricultural machine; \( i_{\text{tr}} \) is the transmission ratio.

When in expression (12) \( P_{\text{sp}} \) is taken as a dynamic hook force \( P_{\text{sp}(i)} \), which takes into account the inertia forces of the tractor and agricultural machine translationally moving masses, then on the basis of it, the resistance to the MTU movement can be determined. In this case, the equation (11) takes the form:
$$p_{\text{MTA}} = \frac{1}{1 - 2\mu_{1}Q_{r}} \left[ \frac{eBk_{c}^{2}}{r_{e} - c} + \frac{\alpha C_{1}e^{2}}{1 - 2\mu_{1}Q_{r}} \right].$$

4. Conclusion.

1. The problem of mathematical modeling of the driving wheel motion, the uncertainties of which are limited by the random nature of the tire elastic properties interaction, was realized. The compiled mathematical model with elastic elements makes it possible to emit the process of forming the average hook force in the tractor hitch and its dynamic component, the change in the tractive moment on the propellers when the working bodies of agricultural machines interact with the soil in various operating conditions of the MTU. It also allows to estimate the resistance to the movement of the tractor and the value of the hysteresis losses in the driving wheels equipped with pneumatic tires.

2. Thus, tractors’ slipping, in which the soil layers begin to shift relative to each other at the end of the wheel-soil patch area, can be considered as a limiting threshold for establishing the permissible operating modes of wheeled tractors. Hence, there are two more particular optimization criteria: the cost of the propellers for slipping and the maximum allowable slipping (slipping coefficient), which limits the operation of the elastic element for environmental reasons, soil abrasion. The maximum admissible slip coefficient is $\delta = 15\%$ for the acceleration process.

3. The prospect of a certain reduction in the border of the experimental MTU traction force specific drag coefficient (per unit of speed) was revealed, to 0.11 ... 0.13, in the conditions of performing typical operations, in contrast to the serial one, which is within 0.15 ... 0.245.

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