Effects of load and displacement controlled bending on plastic collapse of pressurized pipes

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Abstract

An accurate determination of the pressure to cause plastic collapse of a pipe is important when considering its structural integrity. This paper explores the way in which initial stresses, induced by either load or displacement controlled bending, influence plastic collapse as a result of internal pressure. Both open ended and closed ended pipes are considered. The effect of initial bending stresses created by far-field displacement is often considered to be similar to load controlled stresses. Using three dimensional finite element analysis it is demonstrated that global collapse pressures for load and displacement controlled conditions may be significantly different. For both pipe end conditions load controlled bending affects the plastic collapse pressure. However, displacement controlled bending has little influence.

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1. Introduction

The determination of the plastic collapse pressure of a pipe is an important step in the in structural integrity assessment of pipework. For example, the R6 structural integrity assessment code (2001) uses the plastic collapse pressure to determine the safety of a component. In addition, Ainsworth (1984) and Kim et al (2002, 2005) suggest that the plastic collapse pressure can be used to evaluate elastic-plastic fracture mechanics parameters using a reference stress approach. The R6 procedure (2001) classifies the stresses contributing to plastic collapse of the structure as primary stresses and those that do not contribute to plastic collapse as secondary stresses. R6 (2001) notes that long-range residual stresses that may have a membrane stresses or through-wall bending stresses are associated with significant elastic follow-up, and therefore should be classified as primary stress, but it is not always the case. Since no details about the relationship between the long-range residual stress and elastic follow-up are given in R6 (2001), it is still not clear whether a long-range residual stress should be considered to be a primary or a secondary stress. Such residual stresses are normally developed from global or imposed boundary constraints that are generated in the fabrication of complex multi-component structures.

Previous work, for example, Kim and Shim et al. (2002, 2003), Miller (1988), Staat et al. (2005, 2013), Shen and Tyson (1997) and Xu et al. (2000) has evaluated the plastic collapse load for closed-end pipes but these solutions have not considered the open-ended condition as well as the influence of residual stress. Miller (1988) defined the global collapse pressure as the load resulting in unbounded plastic strain in the structure and local collapse as the load leading to yielding of the local ligament or local net section. He also gave analytical solutions for the plastic collapse pressure for closed-end pipes containing circumferential cracks. Kim and Shim et al. (2002, 2003) improved the accuracy of the existing analytical solutions of global collapse loads of pipes containing defects under single and combined loading. Staat et al. (2005, 2013) used finite element methods to give solutions for local and global collapse loads of closed-end pipes with flaws. They also treated local net section yielding as local collapse, although Shen and Tyson (1997) realised that the term local collapse is a misnomer since the yielded region is constrained and therefore the component can sustain further increases in pressure.

This paper investigates the influence of long-range bending residual stress on the plastic collapse of uncracked pipes that are either open-ended or closed-ended. Far-field displacement controlled bending represents the long-range bending residual stress. Plastic collapse is defined as the condition when unbounded plastic strain occurs on the pipe as the pressure is increased. Detailed three-dimensional finite element (FE) models were used, together with an elastic, perfectly plastic material model. Results are presented for plastic collapse pressures of pipes subjected to additional fixed load or fixed far-field displacement controlled bending.

2. Finite element analysis

This section describes the finite element analysis undertaken to explore the effects of the pipe end boundary conditions and to determine the plastic collapse pressures.

![Schematic illustration of a pipe containing an external circumferential crack under four-point bending.](image_url)
2.1. Geometry

Fig. 1 illustrates a pressurised pipe under four-point bending. The pipe has an inner radius \( R_i \), outer radius \( R_o \), mean radius \( R_m \), length \( L \), and thickness \( t \). The pipe is assumed to be subjected to internal pressure and either fixed displacement or load controlled bending conditions. For the closed-end condition, an additional axial force is applied to the end of the pipe.

Table 1 shows the geometry parameters for the pipes used in the analysis. The length of the pipe was fixed at 7000 mm. The distance between the two loading points, \( 2l_t \), was 3400 mm and the distance between the two supports, \( 2l_b \), was 6000 mm.

| \( R_o/t \) | \( t \) (mm) | \( R_m \) (mm) | \( R_i \) (mm) | \( R_o \) (mm) | \( L \) (mm) |
|----------|----------|----------|----------|----------|----------|
| 2        | 10       | 20       | 15       | 25       | 7000     |
| 5        | 10       | 50       | 45       | 55       | 7000     |
| 10       | 10       | 100      | 95       | 105      | 7000     |
| 15       | 10       | 150      | 145      | 155      | 7000     |

2.2. Finite element mesh and boundary conditions

Three-dimensional ABAQUS finite element models of pipes were created using C3D20R elements (twenty-node brick elements with reduced integration). Fig. 2 shows the FE mesh of a quarter model of the pipe under four-point bending. A finer mesh was used around the contact region between the supports and the pipe.

For the closed-end condition, an axial tension equivalent to the internal pressure was applied to the end of the pipe to simulate the closed end. A fixed displacement or load was applied to the loading point to introduce the bending stress. After applying bending, the internal pressure was increased from zero until plastic collapse.

2.3. Limit analysis

ASME (1995), Galic et al. (2011) and Muscat et al. (2003) used the twice elastic slope (TES), tangent intersection (TI) and plastic work (PW) methods to determine the global collapse of a component. In this work we use the PW method combined with an elastic perfectly plastic material model with a yield stress, \( \sigma_y = 117 \) MPa and Young’s modulus, \( E = 73100 \) MPa. For this material model the three methods give similar results.

Fig. 3 illustrates the relationship between the internal pressure and plastic work and for an open-ended pipe. Plastic collapse occurs when unbounded plastic work occurs.
3. Plastic collapse pressures

When a pipe is under four-point bending, the onset of yielding is calculated from

\[ M_y = \frac{\pi}{4} \sigma_y (R_o^4 - R_i^4) / R_o \]  

(1)

Where \( M_y \) is the moment at the onset of yielding.

The collapse moment, \( M_o \), is given by Kim et al. (2003) as

\[ M_o = 4\sigma_y R_o^2 t \]  

(2)

If only internal pressure is applied, the lower bound collapse pressure, \( P_o \), for the thin-walled open-ended pipes is

\[ P_o = \sigma_y t / R_i \]  

(3)

For the closed-end pipes under pressure only, Hill (1950) gave the collapse pressure

\[ P_{cc} = \frac{2}{\sqrt{3}} \sigma_y \ln \left( \frac{R_o}{R_i} \right) \]  

(4)

For pressurised pipes with initial bending stress induced by fixed load or displacement controlled loading, a ratio, \( m \), is defined as \( M / M_o \) where \( M \) is the induced bending moment and \( M_o \) is the collapse moment. When the initial bending moment, \( M \), induced by either fixed load or displacement exists, the collapse pressure is defined as \( P_{co} \). In the following figures, \( M_o \), \( P_{co} \) and \( P_{cc} \) were obtained from the FE simulations.

The onset of yielding and the collapse moments for a pipe as a function of the normalised mean radius are shown in Fig. 4a. Both FE and analytical results for pipes under four point bending are presented. The FE results agree well with the analytical solutions. Fig. 4b shows the global collapse pressure for pipes with open ends or closed ends. Since the analytical solution for an open-ended pipe is based on a thin-walled pipe, the results from the FE simulation are higher for thick-walled pipes. For closed-end cases, the analytical solutions agree with the FE results for all pipes. The closed-end pipe induces an additional axial stress which requires higher hoop stress to reach yield. It is observed that the collapse pressures for open-ended pipes are lower than those for closed-end pipes. The following results were derived for thick-walled pipes (\( R_o/t=5 \)) and thin-walled (\( R_o/t=15 \)) pipes.
The collapse pressures pipes as a function of the moment induced by fixed load or fixed displacement controlled bending are shown in Fig. 5. The behaviours of thin- and thick-walled pipes are similar. When the initial moment is induced by the fixed load condition, it can be seen in Fig. 5a and 5b that a lower collapse pressure is found for open-ended pipes compared with closed-end pipes. It is obvious that load controlled bending contributes to the global collapse of a pressurised pipe. The larger the initial moment, the lower the global collapse pressure. However, the results shown in Fig. 5a and 5b show that displacement-controlled bending does not contribute to global collapse. The long-range bending residual stresses (i.e. pre-bending stress induced by fixed displacement conditions) are relaxed by the plasticity accumulated in the pipe when global collapse pressure is approached.
4. Concluding remarks

This paper has presented results from a finite element based study of plastic collapse for pressurised pipes subjected to an initial bending moment induced by fixed load or fixed displacement conditions. The fixed displacement controlled is taken to represent a long-range residual stress.

For the pipes without flaws, the local and global collapse conditions are equivalent. The pressure to cause collapse is affected by the load controlled bending moment but not by the displacement controlled bending moment for both open-ended and closed end conditions. However, the collapse pressures derived from open-ended pipes are lower than those from closed-end pipes if the same initial moments induced either by fixed load or displacement controlled conditions are considered.

This paper demonstrates that long-range residual stress does not contribute to plastic collapse of unflawed pipes. Therefore, classifying long-range residual stress as a primary stress may result in significantly conservative structural integrity assessments.

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