Are the pentaquark sum rules reliable?*

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We review and scrutinize the existing mass determinations of the pentaquarks from the exponential Laplace Sum Rules (LSR). We do not find any sum rule window for extracting optimal and reliable results from the LSR, due to the unusual slow convergence of the OPE and to the exceptional important role of the QCD continuum into the spectral function in this channel. Instead, we use in this channel, for the first time, Finite Energy Sum Rules (FESR), which exhibit a nice stability in the QCD continuum threshold \( t_c \), at which one can extract, with a good accuracy, the mass of the lowest resonance. Including the \( D = 7, 9 \) condensate contributions in the OPE, we obtain \( M_\Theta = (1513 \pm 114) \text{ MeV} \), and the corresponding residue \( \Delta^D_\Theta \approx -(0.14 \pm 0.49) \times 10^{-9} \text{ GeV}^2 \), which favours the \( I = 0, J = 1/2 \), and negative parity \( S \)-wave interpretation of the \( \Theta(1540) \). However, our analysis indicates a degeneracy between the unmixed \( D \) and \( S \) resp. states. Contrary to the ordinary mesons, where the lowest dimension current built by two diquarks and one anti-quark describes the \( \Theta \) as a \( I = 0, J^P = 1/2^+ \) S-wave resonance has been proposed by \([3]\) and used by \([8]\). Finally, a study of the \( \Theta \)-pentaquark mass from the exponential Laplace Sum Rule (ESR) is consistent with the FESR results. Some materials of this paper have been presented by R. D. Matheus at the QCD 04 11th International Conference (Montpellier 5-9th July 2004), and by S. Narison at the HEP-MAD 04 2nd High-Energy Physics International Conference (Antananarivo 27th Sept.-2nd Oct. 2004).

1. INTRODUCTION

Recent experimental discovery of the \( \Theta(1540) \) as a narrow \( K \)-nucleon state in \( \gamma \)-nucleus and \( \gamma \)-nucleon processes, \( e^+e^- \) and hadronic machines \([1,2]\) have stimulated renewed theoretical interests in hadron spectroscopy \([3,4]\). In this paper, we shall critically reanalyze the mass determinations of the isoscalar \( I = 0 \), \( \Theta \) pentaquark mass from the exponential Laplace sum rules (LSR) \([5,7]\) within the diquark scenario \([3]\) and propose new analysis using Finite Energy Sum Rule (FESR).

2. THE PENTAQUARK CURRENTS

The basic ingredients in the resonance mass determinations from QCD spectral sum rules \([9,10]\) as well as from lattice QCD calculations are the choice of the interpolating currents for describing the resonance states. Contrary to the ordinary mesons, where the form of the current comes from first principles, there are different choices of the pentaquark currents in the literature. The following analysis also postulates the existence of a strongly bound pentaquark resonance. We shall list below some possible operators describing the isoscalar \( I = 0 \) and \( J = 1/2 \) channel\(^3\), which would correspond to the experimental candidate \( \Theta(1540) \). Defining the pseudoscalar \( (ps) \) and scalar \( (s) \) diquark interpolating fields as:

\[
Q_{ab}^P(x) = \left[ u_a^T(x)C d_b(x) \right],
Q_{ab}^S(x) = \left[ u_a^T(x)C \gamma_5 d_b(x) \right],
\]

where \( a, b, c \) are colour indices and \( \epsilon \) are charge conjugation matrix, the lowest dimension current \( \epsilon \) is \([5]\) (see also \([6]\))\(^5\):

\[
\gamma_{ab}^\Theta = \epsilon^{abcde} \epsilon^{fjg} Q_{af}^s Q_{de}^s C s_j^T T_f,
\]

and the one with one diquark and three quarks is \([7]\):

\[
\gamma_{ab}^\Theta = \frac{1}{\sqrt{2}} \epsilon^{abc} Q_{ab}^s \left\{ u_c \delta_{j3} d_e - (u \leftrightarrow d) \right\}.
\]

This later choice can be interesting if the instanton repulsive force arguments \([15]\) against the existence of a pseudoscalar diquark bound state apply. Alternatively, a description of the \( \Theta(1540) \) as a \( I = 0, J^P = 1/2^+ \) \( P \)-wave resonance has been proposed by \([3]\) and used by \([8]\) in the sum rule analysis:

\[
\gamma_{ab}^\Theta = \left( \epsilon^{abcd} \gamma_c + \epsilon^{abc} \delta_{de} \right) \left( Q_{ab}^s (D^{\mu} Q_{cd}^s) - (D^{\mu} Q_{ab}^s) Q_{de}^s C s_j^T T_f \right).
\]

We have generalized this current by considering its mixing with the following one having the same dimension of an isoscalar \( I = 0 \) as confirmed by the authors in the revised version of their paper. In the following, we shall neglect the isospin breaking discussed in \([14]\), which would come from higher order diagrams in our analysis.

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\(^3\)However, this narrow state has not been confirmed by several experiments with high statistics and very good particle identification, using either \( e^+e^- \) and hadronic initial states.

\(^4\)The isovector \( I = 1 \) current for \( S \)-wave resonance have been proposed by \([11]\). We have also checked (see also \([12]\)) that the tensor diquark current used in \([13]\) is an isovector \( I = 1 \) instead of \( I = 0 \), indicating that the \( \Theta \) is very narrow.

\(^5\)A negative parity state can be obtained by multiplying by \( \gamma_5 \) the diquark operator.
sion and quantum numbers:
\[ s^n_{\text{new}} = e^{abc} e^{def} e^{ijg} Q_{ab}^{\alpha} Q_{de}^{\beta} \gamma_\mu (D^\mu C \bar{s} g) \] .

3. THE QCD SPECTRAL FUNCTIONS

For the QCD spectral sum rules analysis, we shall work here with the two-point correlators:
\[ \Pi^H(q^2) \equiv \frac{1}{\pi} \int d^4x \, e^{ixq} \langle 0 | T \eta^H(x) \eta^H(0) | 0 \rangle , \]
(6)
built from the previous \( \eta \) currents. It possesses the Lorentz decomposition:
\[ \Pi^H(q^2) = \hat{q} A^H(q^2) + B^H(q^2) . \]
(7)

- The QCD expression of the correlators associated to different choices of the currents is known in the literature \[ \text{578} \] to leading order of PT series and including the three first non-perturbative condensate \( (D \leq 5, 6) \) contributions. We have checked the QCD expressions given there and agree with their results. However, at this approximation, we have added some missing contributions in \[ \text{2} \].
- We have not included in the OPE the contribution of the \( D = 2 \) tachyonic gluon mass induced by the resummation of the PT series \[ \text{10} \] bearing in mind that this effect will be negligible, to the accuracy we are working, as illustrated in some examples \[ \text{17} \].
- We have included the \( D = 7, 9 \) contributions into the QCD expression of the spectral function associated to the current in Eq. \[ \text{2} \], which we shall extensively study as a prototype example in this paper. In doing this calculation, we have worked in the chiral limit \( m_s = 0 \), such that for consistencies, we shall use the \( SU(3) \) symmetric value \( \langle \bar{s}s \rangle = \langle \bar{d}d \rangle \) for these contributions. In this particular example, we have checked that the contribution of the four-quark condensate vanishes leading order. In our preliminary results, we also found that its radiative correction though not identically zero gives a negligible contribution. We have also neglected the contributions of the gluon condensate of the type \( g(GGG) \langle \bar{s}s \rangle \) assuming that the theorem in \[ \text{15} \] for the light quark bilinear operators continues to hold for the diquark correlators \[ \text{6} \], which factorize during the evaluation of the QCD expression.

- We have evaluated the new contributions associated to the current \( \eta_{\text{new}} \) in Eq. \[ \text{5} \], where we found that, to leading order in \( \alpha_s \) and in the chiral limit \( m_q \to 0 \), the contribution to the correlator vanishes. This result justifies a posteriori the unique choice of operator for the \( P \)-wave state used in \[ \text{35} \].

4. THE LAPLACE SUM RULES (LSR)

We shall be concerned with the Laplace transform sum rules:
\[ \mathcal{L}^H_{A/B}(\tau) \equiv \int_{t \leq \tau} \frac{dt}{t} \frac{\pi}{\pi} \text{Im}(A^H/B^H(t)) , \]
\[ \mathcal{R}^H_{A/B}(\tau) \equiv -d/d\tau \log \mathcal{L}^H_{A/B}(\tau) . \]
(8)

where \( t \leq \) is the hadronic threshold, and \( H \) denotes the corresponding hadron. The latter sum rule, or its slight modification, is useful, as it is equal to the resonance mass squared, in the usual duality ansatz parametrization of the spectral function:
\[ \frac{1}{\pi} \text{Im}(A^H/B^H(t)) \simeq (\lambda^2_H/\lambda_H^2 M_H) \delta(t - M^2_H) + \]

“QCD continuum” \( \Theta(t - t_c) \),
(9)
where the “QCD continuum comes from the discontinuity of the QCD diagrams, which is expected to give a good smearing of the different radial excitations. \( M_H \) is the residue of the hadron \( H \); \( t_c \) is the QCD continuum threshold, which is, like the sum rule variable \( \tau \), an (a priori) arbitrary parameter. In this paper, we shall look for the \( \tau \)- and \( t_c \)-stability criteria for extracting the optimal results. For illustrating our analysis, we give below the checked and completed LSR of the \( S \)-wave current in Eq. \[ \text{2} \] including the new \( D = 7, 9 \) high-dimensional condensates in \( B \):

\[ \mathcal{L}^H_{A/B} \equiv \frac{\pi}{\pi} \text{Im}(A^H/B^H(t)) , \]
(10)

\[ \mathcal{L}^H_{A/B} = \frac{\tau - 6 E_5}{860160 \pi^8} + \frac{\tau - 6 E_3}{30720 \pi^6} m_s \langle \bar{s}s \rangle + \frac{\tau - 5 E_4}{122880 \pi^7} (\alpha_s G^2) - \frac{\tau - 5 E_2}{36864 \pi^5} m_s g(\bar{s}s G \bar{G}) , \]
(10)

\[ \mathcal{L}^H_{A/B} = \frac{\tau - 5 E_4}{122880 \pi^8} m_s - \frac{\tau - 5 E_4}{15360 \pi^6} \langle \bar{s}s \rangle + \frac{\tau - 4 E_3}{24576 \pi^5} g(\bar{s}s G \bar{G}) - \frac{\tau - 4 E_2}{6144 \pi^4} g(\bar{s}s G \bar{G} G) , \]
(11)

where:
\[ E_n = 1 - \sum_{k=0}^{n} \frac{(t_c)^k}{k!} \]
(12)
\( \rho_n \) being the notation in \[ \text{10} \], while: \( \langle \bar{s}s \rangle , \langle \alpha_s G^2 \rangle \) are respectively the dimension \( D = 3 \) quark and \( D = 4 \) gluon condensates; \( g(\bar{s}s G \bar{G}) \equiv g(\bar{s}s G \bar{G}) m_s (\bar{s}s G \bar{G}) \equiv M^2_0 (\bar{s}s G \bar{G}) \) is the \( D = 5 \) mixed condensate. Throughout this paper we shall use the values of the QCD parameters given in Table 1.

Table 1

| Parameters | References |
|-----------|------------|
| \( m_s (2 \text{ GeV}) = (111 \pm 22) \text{ MeV} \) | \[ \text{10} \, \text{19} \, \text{21} \]| |
| \( \langle \bar{d}d \rangle^{1/3} (2 \text{ GeV}) = (243 \pm 14) \text{ MeV} \) | \[ \text{10} \, \text{19} \, \text{22} \]| |
| \( \langle \bar{s}s \rangle / \langle \bar{d}d \rangle = 0.8 \pm 0.1 \) | \[ \text{10} \, \text{19} \, \text{23} \]| |
| \( \langle \alpha_s G^2 \rangle = (0.07 \pm 0.01) \text{ GeV}^4 \) | \[ \text{10} \, \text{24} \]| |
| \( M^2_0 = (0.8 \pm 0.1) \text{ GeV}^2 \) | \[ \text{10} \, \text{24} \]| |

We study the LSR in Eqs. \[ \text{10} \] and \[ \text{11} \]. We find that all LSR corresponding to different currents present the common features shown in Fig. \[ \text{4} \]:

- The \( B \)-component increases rapidly with \( \tau \). Then, it is useless at that approximation of the OPE.
- \( \text{OPE} \) for \( A \), the mass prediction decreases smoothly when \( \tau \) increases. The OPE converges for \( \tau \leq 0.9 \text{ GeV}^{-2} \) (LHS of the vertical dashed line).
- The QCD continuum contribution dominates over the resonance one in all ranges of $\tau$ where the OPE converges. The vertical line with arrow on each curve shows that the continuum contribution is not appropriate for the determination of the pentaquark masses due to the absence of the resonance into the spectral function. Therefore, it is impossible to find a sum rule window region where both the resonance dominates over the QCD continuum contribution, and where the OPE converges. Intuitively, this feature is expected as the current describing the pentaquark is of higher dimensions, and therefore is more affected by the continuum contribution than the well-known sum rule for ordinary $\bar{q}q$ mesons. The absence of the sum rule window is reflected by the increase of the mass predictions with the $t_c$-values having ad hoc and intuitively.
- During the evaluation of the different QCD diagrams, we do not find (to leading order in $\alpha_s$) any factorization of the $(su)$-$(udd)$ diagram corresponding to a reducible $K$-$N$ continuum diagram, which has nothing to do with the diquark picture. Then, our direct observation does not support the criticisms raised in [26] and refuted in [27] on a possible double counting due to the non-subtraction of the reducible diagram in the existing sum rules analysis of the $\Theta$.
- We conclude from the previous prototype example that the LSR using the simple duality ansatz: resonance+QCD continuum criterion is not appropriate for determining the pentaquark masses due to the absence of the usual sum rule window. Due to the huge continuum contribution ($\approx 85\%$) at relatively large $\tau \approx 1$ GeV$^{-2}$, the LSR cannot strictly indicates the existence of the resonance into the spectral function.
- We have checked (though not explicitly shown in the paper) that the conclusions reached in the paper also apply to the sum rules used in the literature: for the $I = 0$, $S$-wave state; the sum rules used in [26] for the $I = 1$, $S$-wave state; in [8] (current in Eq. (11)) and subsequent uses in [27,24] for the $I = 0$, $P$-wave state; in [13] for the $I = 1$ tensor current and the sum rules used in [8] for studying the $J = 3/2$ states. Indeed, in most LSR, the OPE does not converge at the scale where the results are extracted, while the QCD continuum threshold has been taken arbitrarily or intuitively.
- The above results raise some doubts on the validity of the results obtained so far in the existing literatures. Indeed, if one insists on using the LSR for predicting the $\Theta$ parameters and some other pentaquark states, it is mandatory to introduce a more involved parametrization of the continuum spectral function.

5. Finite Energy Sum Rules (FESR)

In contrast to the LSR, Finite Energy Sum Rules (FESR) [30,31,10] have the advantage to project out a set of constraints for operators of given dimensions (local duality). They also correlate the resonance mass and residue to the QCD continuum threshold $t_c$, so avoiding inconsistencies of the values of these parameters. Also contrary to the LSR, the resonance and QCD continuum contributions are separated from the very beginning. The FESR read:

$$\mathcal{M}_{H}^\Theta(A/B) \equiv \int_{t_c}^{\infty} dt \ t^n \ \text{Im} A^H/B^H|_{\text{EXP}}$$

$$\simeq \int_{t_c}^{\infty} dt \ t^n \ \text{Im} A^H/B^H|_{\text{QCD}}$$  \hspace{1cm} (13)

From the expressions of the spectral function given previously, one can easily derive the FESR constraints. Doing the FESR analysis for the $A(q^2)$ invariant, one can notice that, at the approximation where the OPE is known ($D \leq 6$), one does not have a stability in $t_c$ for different moments $\mathcal{M}_{0,5}^\Theta$ and for different choices of the currents (see Fig. 2). Therefore, we will not consider this invariant in the paper.

The $I = 0$, $S$-wave channel

We illustrate the analysis by the current in Ref. [8] (the other choice [7] in Eq. (3)) has approximately the same dynamics as one can inspect from the QCD expressions). Including the $D = 7$ and $9$ condensate contributions, the two first lowest dimension constraints from the $B(q^2)$ invariant read:

$$\mathcal{M}_{0.0}^{\Theta} = \frac{m_x t_c^6}{88473600\pi^8} - \frac{\langle \bar{s}s \rangle t_c^5}{1843200\pi^6} + \frac{\langle \bar{s}s \rangle G_2}{g_s t_c^4}.$$  \hspace{1cm} (14)

$$\mathcal{M}_{1.5}^{\Theta} = \frac{m_x t_c^7}{103219200\pi^8} - \frac{\langle \bar{s}s \rangle t_c^6}{2211840\pi^6} + \frac{\langle \bar{s}s \rangle G_2}{g_s t_c^5}.$$  \hspace{1cm} (15)

from which one can deduce the mass squared:

$$M_\Theta^2 \approx \frac{\mathcal{M}_{1.5}^{\Theta}}{\mathcal{M}_{0.0}^{\Theta}}.$$

(16)
The behaviour of $M_\Theta$ is shown in Fig. 2 for different truncations of the OPE. One can notice a stability at $t_c \simeq 2.29 \text{ GeV}^2$, where the OPE starts to converge after only the inclusion of the $D = 7 + 9$ condensates, while $D = 7$ alone destroys the stability reached for $D \leq 5$. One can notice the important contribution of the lowest quark and mixed quark-gluon condensates in the OPE, which play a crucial role in this mass determination. To that order, we obtain:

$$M_\Theta \simeq (1513 \pm 20 \pm 10 \pm 40 \pm 30 \pm 95) \text{ GeV},$$

where the errors come respectively from $m_s$, $\langle \bar{q}q \rangle$, $\langle \alpha_s G^2 \rangle$, $M_2^2$, the estimate of the higher dimension condensates and the violation of the vacuum saturation assumption of the $D = 7 + 9$ condensates by a factor $(2 \pm 1)$ like in the $\rho$-meson \[2,30\] and some other channels \[10\]. One can notice that:

- The existence of the $t_c$-stability point makes the superiority of FESR compared to the LSR in this channel. For the LSR, $M_\Theta$ increases with $t_c$. Here, the localisation of the stability point induces here a negligible error.
- The FESR order parameter in the OPE, $t_c \simeq 2.3$ GeV$^2$ is much larger than for the LSR ($\tau^{-1} \leq 1$ GeV$^2$), implying a much better convergence of the OPE for the FESR, and then a much more reliable result than the LSR.
- Working with ratio of higher moments $M_{2,3}^G/M_1^G$, ..., leads to almost the same value of $M_\Theta$. The slight variation is much smaller than the error in Eq. \[17\].
- Truncating the OPE at $D = 5$ like done in the available literature would give a slightly lower value of $M_\Theta$ at the stability point $t_c \approx 3.2$ GeV$^2$ (see Fig. 2), but compatible with the one in Eq. \[17\].
- Contrary to $M_\Theta$, the value and the sign of $\lambda_5^2$ are very sensitive to the truncation of the OPE due to the alternate signs of the condensate contributions in the analysis. The stability in $t_c$ is obtained after the inclusion of the $D = 5$ condensate contributions as shown in Fig. 3. To our approximation $D \leq 9$, the most conservative result is:

$$\lambda_5^2 \approx -(0.14 \sim 0.49) \times 10^{-9} \text{ GeV}^{12},$$

where the range comes from the shift of the $t_c$-stability point from $D = 5$ to $D = 9$ approximation. This result, though inaccurate, suggests that the parity of the $\Theta$ is negative, as indicated by the lattice results given in \[33\]. Improving the accuracy of our result requires more high-dimension condensate terms in the OPE.

### The $I = 1$, S-wave channel

We have also applied FESR in the $I = 1$ S-wave channel with the current \[11\]:

$$\eta^{\Theta}_{11} = \frac{1}{\sqrt{2}} \epsilon^{abc} \left[ Q_{ab}^c Q_{cc}^c + t Q_{ab}^c Q_{cc} - (u \leftrightarrow d) \right] G^T, \tag{19}$$

where $t$ is an arbitrary mixing parameter. To the $D \leq 5$ approximation, the analysis gives almost the value of $M_\Theta$ in Fig. 2 at the same approximation. This result can be interpreted as a consequence of the good realization of the $SU(2)_F$ symmetry for the $u$ and $d$ quarks. Then, we expect that the unmixed $I = 1$ partners of the unmixed $I = 0$ state will be around the 1.5 region if any.

### The $I = 0$, P-wave channel

We do a similar analysis for the $P$-wave current given in Eqs. \[4\] and \[5\], where as we have mentioned in section 3, the contribution from Eq. \[6\] vanishes to leading order in $\alpha_s$. The corresponding FESR up to $D = 5$ condensates are given below \[8\]:

$$M_0^P, S = \frac{m_s t_c^7}{361267200 \pi^8} - \frac{\langle \bar{s}s \rangle t_c^6}{5529600 \pi^6} - \frac{\langle \bar{q}q \rangle t_c^5}{1966080 \pi^4} \tag{20}$$

$$M_1^P, S = \frac{m_s t_c^8}{412876800 \pi^8} - \frac{\langle \bar{s}s \rangle t_c^7}{6451200 \pi^6} - \frac{\langle \bar{q}q \rangle t_c^6}{23592960 \pi^4} \tag{21}$$

To this order, the moments present a similar $t_c$-behaviour as above (see Figs. \[1\] and \[5\]). Both for the mass and residue, the stability point is:

$$t_c \approx 5.5 \text{ GeV}^2.$$  \[22\]

\[7\]At the approximation $D \leq 5$ the LSR does not converge such that analogous results obtained in \[8,26,27\] should be taken with a great care.

\[8\]The inclusion of higher dimension condensates is in progress.
The errors come mainly from the estimate of the unknown $D = 7$, 9 condensates contributions. The mass value obtained for the $P$-wave state of about 2 GeV in Eq. (23) to order $c_\tau$ suggests that there is a destructive interference between two almost degenerate $\bar{N}-\bar{N}$ states below this threshold are naturally narrow. A narrow $S$-wave pentaquark state has been also obtained in [35] and [13] using simple chiral symmetry arguments. In [36] the narrowness of the $\Theta$ is due to a destructive interference between two almost degenerate $\bar{N}$ states, while in [37], it is due to the flavour structure of the $\Theta$, which, after the meson formation, the residual three-quark piece has a little overlap with the octet baryon wave-function.

7. SUMMARY AND CONCLUSIONS

- We have re-analyzed the existing LSR results in the literature. We found that due to the slow convergence of the OPE and to the relative importance of the QCD continuum contribution into the spectral function, the minimal duality ansatz “one resonance + QCD continuum” is not sufficient for finding a sum rule window where the results are optimal. These features penalize all existing sum rule results in the literature, which then become unreliable despite the fact that the mass predictions reproduce quite well the data. However, this apparent good prediction is due to the fact that the intuitive or arbitrary choice of the continuum threshold $t_c$. In fact, in the LSR analysis, the mass prediction increases with $t_c$ though it is a smooth function of $\tau$ as can be seen in Fig. 1.  
- On the contrary, FESR has the advantage to present a good $t_c$-stability and converges faster than the LSR, because the optimal results are obtained at higher scale $t_c \approx (2 \sim 3)$ GeV$^2$ than the one of LSR $\tau^{-1} \leq 1$ GeV$^2$.
- Truncating the OPE at $D = 9$ condensates, at which the OPE starts to converge, we obtain the result in Eq. (17), for the $S$-wave state, which one can compare with the experimental candidate $\Theta(1540)$. 
- By truncating the OPE at $D = 5$, we also find from FESR a good degeneracy between the unmixed $I = 0$ and $I = 1$ $S$-wave states. 
- Similarly, we obtain, from FESR, the mass of the $P$-wave state of about 2 GeV in Eq. (20) to order $D = 5$ of the OPE, but including the estimated effects of $D = 7, 9$ condensates. This mass is $(450 \pm 190)$ MeV higher than the $\Theta(1540)$.
- Finally, an analysis of the $\Theta-K-N$ coupling using vertex sum rules supports results in the literature that the $\Theta$ current defined in previous section. For definiteness, we work with the $S$-wave current given by $\bar{C}\gamma_5 u$. A QCD evaluation of the vertex in the chiral limit $m_s = 0$ shows that the leading and $\alpha_s$ orders perturbative and non-perturbative diagrams give zero contributions. The result then suggests that the $\Theta-K-N$ coupling is of the order $\alpha_s^2$ supporting the experimental observation [1] that the $\Theta(1540)$ is a narrow state. The narrowness of a pentaquark state has been already advocated in the past, from duality arguments, where its decay into $BBB$ baryon states is dynamically favoured, implying that light pentaquark states below this threshold are naturally narrow. 

6. THE $\Theta-K-N$ COUPLING

For studying this coupling, we start from the three-point function:

$$V(p,q) = \int d^4x d^4y e^{i(px+qy)} \langle 0| \eta(0) N(x) K(y) |0 \rangle$$  

where $K(y) \equiv (m_s + m_q)\bar{s}(\gamma_5)u$ is the kaon current, while $N(x) \equiv u(\gamma_5 \gamma_3)du + 2 : u(C\gamma_5 \gamma_3 u :$ is the nucleon interpolating field $\bar{B}$ ($b$ being an arbitrary mixing parameter). $\eta$ is the $\Theta$ current defined in previous section. For definiteness, we work with the $S$-wave current given by $\bar{C}\gamma_5 u$. A QCD evaluation of the vertex in the chiral limit $m_s = 0$ shows that the leading and $\alpha_s$ orders perturbative and non-perturbative diagrams give zero contributions. The result then suggests that the $\Theta-K-N$ coupling is of the order $\alpha_s^2$ supporting the experimental observation [1] that the $\Theta(1540)$ is a narrow state. The narrowness of a pentaquark state has been already advocated in the past, from duality arguments, where its decay into $BBB$ baryon states is dynamically favoured, implying that light pentaquark states below this threshold are naturally narrow. 

- The value of the QCD continuum threshold at which the FESR stabilizes is much higher than the intuitive choice used in the LSR [8] needed to reproduce the experimental mass of the $\Theta$.
- The mass value obtained for the $P$-wave resonance is $(450 \pm 190)$ MeV higher than the $\Theta(1540)$ mass, which suggests that there is a $P$-wave state different from the $\Theta(1540)$ in the region around 2 GeV, which we expect to be discovered experimentally.
- The value and sign of the residue suggest that this $P$-wave state has a negative parity like the $\Theta(1540)$. 

Figure 4. $t_c$-behaviour of $M_P$ including the $D \leq 5$ condensates in the OPE.

The corresponding resonance mass and residue are:

$$M_P \approx 1.99 \pm 0.19 \text{ GeV},$$

$$\lambda_P \approx -(0.7 \sim 7.1) \times 10^{-9} \text{ GeV}^4.$$

Figure 5. $t_c$-behaviour of $\lambda_P^2 \times 10^9 \text{ GeV}^{12}$ including the $D \leq 5$ condensates in the OPE.
\(\Theta(1540)\) is a narrow state.

- Our results seem to favour the case (b) discussed in [23] where the \(\Theta\) resonance is induced in \(KN\) scattering by coupling to a confined channel. A complete program using FESR in different pentaquark channels is in progress and will be published elsewhere.

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