Study on Fragmentation Parameters and Similarity Relation of Natural Fragments of Counterattack Projectiles for Active Protection

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Abstract. The purpose of this paper is to explore the fragmentation similarity between small-size cylindrical projectiles and large-size projectiles. On the basis of the modified Payman fragmentation parameter Co, a new fragmentation parameter Cu and the projectile fragmentation similarity relation were derived. Fragmentation tests of large-size projectiles showed that the results of the number and mass distribution of natural fragments were consistent with the theoretical prediction results, which verified the correctness of the fragmentation similarity relation.

1. Introduction

The killing power of munitions mainly depends on the number and mass distribution of fragments, and the fragment distribution is closely related to the projectile material and the detonation mode of explosive[1-5]. Fragmentation test is the most direct and accurate method to investigate the number and mass distribution of fragments, but its downside is high cost and long test cycle[6-9]. In the design of new ammunition, the fragment mass and number distribution under different projectile materials, explosives and geometric sizes can be calculated by fragment prediction technology, which can greatly reduce the cost and duration of a fragmentation test.

The similarity relation is the theoretical basis for simulation tests and the prediction technology in projectile fragmentation studies, attracting increasing interest. To realize this prediction, it is necessary to establish the projectile fragmentation similarity relation, complete the initial projectile development stage through small simulating projectile tests, and then theoretically predict the fragment mass and number distribution generated by an actual projectile according to the similarity relation and simulation test data. Australia and the United States jointly studied the projectile similarity relation[10],
and obtained a weight similarity relation: when the C/M ratio (weight ratio of explosive to metal projectile) is fixed, the modified Payman fragmentation parameter of the projectile basically do not change with the wall thickness. However, their conclusion was obtained under given conditions of the C/M ratio of 0.28 and wall thickness range of 3-6 mm, which has limitations.

This work focuses on exploring the practical fragmentation similarity between small-sized cylindrical projectiles and large-size projectiles. The simulating projectiles used in the test are cylindrical shells producing standard fragments\[1\], which have different C/M ratios\[2\], and whose wall thicknesses ranges from 4mm to 18mm (almost covering the wall thicknesses of actual projectiles of various calibres). It was found that the modified Payman fragmentation parameter is related to the length of the projectile, and a new fragmentation parameter and the similarity relation were obtained.

2. Fragmentation parameter and similarity relation

2.1. Limitations of the modified Payman fragmentation parameter

The modified Payman fragmentation parameter has been widely used to evaluate the fragmentation behavior of cylindrical projectiles with different masses. In general, the logarithm of the cumulative mass percent of fragments larger than the given mass is used to plot the weight of the dimensionless (m/Mr). The slope of the fitting line is defined as the modified Payman fragmentation parameter, and the expression of the fitting line is:

\[ \log P = 2 - C_0 \left( \frac{m}{M_r} \right) \]  

Where m is the lower limit of the selected fragment mass range; Mr is the total weight of recovered fragments; \( \frac{m}{M_r} \) is the lower limit of the dimensionless fragment mass range; P is the cumulative mass percent of fragments, 0 < p ≤ 100; Co is the modified Payman fragmentation parameter.

In order to investigate whether the similarity relation holds in a large range of wall thicknesses and the effect of wall thickness on the fragmentation of projectiles, two series of cylindrical projectiles with different lengths were used in the work. The two series had the same nominal C/M ratio respectively and were in two groups: C/M = 0.18 and C/M = 0.28. The details are as follows.

1A series - small cylindrical projectiles, 8 cm in length, 3-6 mm in wall thickness;
2A series - large cylindrical projectiles, 19.4 cm in length, 8-18 mm in wall thickness;

At the same time, in order to study whether the influence of C/M ratio on fragmentation follows the same rule when the wall thickness is fixed, two cylindrical projectile series with different lengths were used in the test. The C/M ratios of the two series were within 0.18-0.48. The details are as follows.

1B series - small cylindrical projectiles, 8 cm in length, 4 mm in wall thickness;
2B series - large cylindrical projectiles, 19.4 cm in length, 11 mm in wall thickness;

The modified Payman fragmentation parameter are used to process the test data. The test results are shown in Table 1.
Table 1. Parameters of test cylindrical projectiles and the fragmentation parameter

| Group | Wall | Co(C/M=0.18) | Co(C/M=0.28) | Co(C/M=0.38) | Co(C/M=0.48) |
|-------|------|--------------|--------------|--------------|--------------|
| 1A    | 3    | -            | 236          | -            | -            |
|       | 4    | 152          | 208          | 264          | 288          |
|       | 5    | 177          | 239          | -            | -            |
|       | 6    | 173          | 229          | -            | -            |
|       | 8    | 406          | 551          | -            | -            |
| 2A    | 11   | 453          | 520          | 616          | 696          |
|       | 15   | 407          | 474          | -            | -            |
|       | 18   | 423          | -            | -            | -            |
| 1B    | 4    | 152          | 208          | 264          | 288          |
| 2B    | 11   | 453          | 520          | 616          | 696          |

Figure 1 shows the relationship between the modified Payman fragmentation parameter Co and the projectile wall thickness obtained by the linear regression method according to formula (1). It can be seen from the figure that, in the case of C/M = 0.18 and C/M = 0.28, for the small cylindrical and large cylindrical projectiles, the fragmentation parameter Co and wall thicknesses are approximately in a horizontal linear relationship, indicating the same changing trend. However, the fragmentation parameters of small and large cylindrical projectiles are not continuous, and sudden jump occurs. Therefore, using the modified Payman fragmentation parameter Co to describe the relationship between fragmentation and wall thickness can cause an illusion that fragmentation seems increasing rapidly when the wall thickness is larger, and that the similarity relation in the larger wall thickness range does not exist, namely, the law obtained by small simulating projectile test not applicable to large-size projectiles.

![Figure 1](image1.png)

Figure 1. Relationship between the modified Payman fragmentation parameter Co and the wall thickness in the case of fixed C/M ratio

Figure 2 shows the relationship between the modified Payman fragmentation parameter Co and C/M ratio when the wall thickness is fixed at 4 mm and 11 mm respectively. In both cases, there is a linear relationship between them. As the C/M ratio increases, Co increases linearly, and the two lines are approximately parallel lines. However, the fragmentation parameter Co of a large projectile with a
A wall thickness of 11 mm is more than twice that of a small projectile with a wall thickness of 4 mm. Therefore, the relationship between the fragmentation parameter Co and the C/M ratio obtained using small cylindrical projectiles can qualitatively indicate the changing trend of the fragmentation of large projectiles with the C/M ratio, but it can not simply and quantitatively predict the fragmentation parameter of the large-size projectile at a given C/M ratio.

**Figure 2.** Relationship between the modified Payman fragmentation parameter Co and the C/M ratio in the case of fixed wall thickness

A large number of studies found that the fragmentation parameter of large cylindrical projectiles were much higher than that of small cylindrical projectiles, which was not caused by the increase of wall thickness, but by the difference of projectile length. The mass similarity relation obtained in the range of wall thicknesses of 3-6 mm was applicable when the projectile length was the same, which limited the practical application of the similarity relation. It cannot quantitatively predict the fragmentation of projectiles with different lengths and calibres, nor can it correctly explain the theoretical basis of simulation fragmentation tests. Therefore, it is urgent to find a new fragmentation parameter to establish the similarity relation that has extensive application value.

### 2.2. Derivation of per-unit-length modified Payman fragmentation parameter

Different lengths of projectiles correspond to very different modified Payman fragmentation parameter Co. Therefore, in order to derive the new fragmentation parameter, the effect of projectile length on Co is discussed first.

It is assumed that there are n identical cylindrical projectiles (with the same projectile material, state, geometric size, explosive type and C/M ratio), and the cumulative fragment mass distribution of each projectile should be exactly the same under ideal conditions, i.e. the cumulative mass percent of fragments \( P_{1}(>m_{i}) = P_{2}(>m_{i}) = \cdots = P_{n}(>m_{i}) \)

Suppose the cumulative mass of fragments greater than \( m_{i} \) is \( G_{i} \) and the metal weight of each projectile is \( M \), then

\[
P_{1}(>m_{i}) = P_{2}(>m_{i}) = \cdots = P_{n}(>m_{i}) = \frac{G_{i}}{M}
\]
If all fragments of \( n \) projectiles are mixed together and then the fragment mass distribution is calculated, then the total mass of fragments larger than the given mass \( m_i \) is \( nG_i \), the total metal weight of \( n \) projectiles is \( nM \), and the cumulative mass percent of fragments greater than the given mass \( m_i \) is:

\[
P'_{(>m_i)} = \frac{nG_i}{nM} = \frac{G_i}{M} = P_{1(>m_i)} = P_{2(>m_i)} = \cdots = P_{n(>m_i)} \tag{4}
\]

Formula (4) shows that fragments of \( n \) identical cylindrical projectiles are mixed together, and that the distribution of the cumulative mass percent of fragments is the same as that of a single projectile. If \( n \) projectiles are connected as a whole for detonation, i.e. the cylindrical projectile with a length of \( L \) is prolonged to \( nL \), the distribution of the cumulative mass percent of fragments does not change. Therefore, it can be concluded that projectile length change will not affect the distribution of fragment mass percent, but the projectile mass changes with the projectile length, and this inevitably leads to the change of the dimensionless weight \( \frac{m}{M_r} \), so the modified Payman fragmentation parameter \( Co \) also varies. Formula (1) is rewritten to:

\[
Co = - \frac{\log P}{\frac{m}{M_r}} \tag{5}
\]

where \( P \) is the fragment cumulative mass fraction, \( 0 \leq P \leq 1 \).

At a high fragment recovery rate, \( Mr \approx M \). There are two cylindrical projectiles with the lengths of \( L_1 \) and \( L_2 \), and their other parameters are identical. In this case, the distribution of the cumulative mass percent of fragments of the two projectiles is exactly the same, but the metal weight is proportional to the length, namely:

\[
\frac{M_1}{M_2} = \frac{L_1}{L_2} \tag{6}
\]

The modified Payman fragmentation parameter of the first projectile is:

\[
Co_1 = - \frac{\log P}{\frac{m}{M_1}}
\]

The modified Payman fragmentation parameter of the second projectile is:

\[
Co_2 = - \frac{\log P}{\frac{m}{M_2}} = - \frac{\log P}{\frac{m}{M_1}} \cdot \frac{L_2}{L_1} = Co_1 \cdot \frac{L_2}{L_1}
\]

Therefore,

\[
\frac{Co_1}{L_1} = \frac{Co_2}{L_2} \tag{7}
\]

From formula (7), under identical parameters except for length, the modified Payman fragmentation parameter of projectiles is directly proportional to the length of projectile. Let the proportional constant be \( Cu \), then:

\[
Cu = \frac{Co}{L} \tag{8}
\]
From formula (8), \( Cu \) is the modified Payman fragmentation parameter of a cylindrical projectile per unit length, which is independent of length but only related to material, explosive type and C/M ratio. \( Cu \) is a new fragmentation parameter to characterize projectile fragmentation, which can overcome the limitation of Co and is no longer limited by the size of projectile. We define the length of the projectile body in cm, and the unit of \( Cu \) is \( \text{cm}^{-1} \).

2.3. Research on fragmentation similarity relation

This paper uses the new fragmentation parameter \( Cu \) to process the test data, and the conclusions obtained this way are different from those shown in Figure 1 and Figure 2. The specific data are shown in Table 2.

**Table 2. Per-unit-length fragmentation parameter \( Cu \) of test cylindrical projectiles**

| Group | Wall Thickness | \( Cu(C/M=0.18) \) | \( Cu(C/M=0.28) \) | \( Cu(C/M=0.38) \) | \( Cu(C/M=0.48) \) |
|-------|----------------|---------------------|---------------------|---------------------|---------------------|
| 1A    | 3              | -                   | 29.6                | -                   | -                   |
|       | 4              | 19.0                | 25.9                | 33.0                | 36.0                |
|       | 5              | 22.1                | 29.8                | -                   | -                   |
|       | 6              | 21.6                | 28.6                | -                   | -                   |
|       | 8              | 20.9                | 28.5                | -                   | -                   |
| 2A    | 11             | 23.4                | 26.8                | 31.7                | 35.9                |
|       | 15             | 21.0                | 24.5                | -                   | -                   |
|       | 18             | 21.7                | -                   | -                   | -                   |
| 1B    | 4              | 19.0                | 25.9                | 33.0                | 36.0                |
| 2B    | 11             | 23.4                | 26.7                | 31.7                | 35.9                |

Figure 3 shows the relationship between the new fragmentation parameter \( Cu \) and the projectile wall thickness when the C/M ratio is constant. It can be seen from the figure that in the case of C/M=0.18 and C/M=0.28, the new fragmentation parameter \( Cu \) has a linear relationship with the wall thickness. Although the straight line fitted with these two sets of data has a certain slope, considering the large fragmentation test itself has a large dispersion, these two lines can be approximately seen as horizontal straight lines. In the case of C/M=0.38 and C/M=0.48, the \( Cu \) values of cylindrical projectiles with the same C/M ratio are basically the same. Therefore, it can be considered that the per-unit-length modified fragmentation parameter \( Cu \) does not change with the wall thickness at a high C/M ratio.

Combined figure 3, we can get the fragmentation similarity relation: for the projectile material with given composition and heat treatment state, when the explosive type and C/M ratio are fixed, the per-unit-length modified Payman fragmentation parameter \( Cu \) of cylindrical projectiles with different sizes is basically the same.
Figure 3. Relationship between the per-unit-length fragmentation parameter \( Cu \) and the wall thickness in the case of fixed C/M ratio

\( Cu \) is used to represent the fragmentation of projectiles. The relationship between the fragmentation of large and small cylindrical projectiles and the C/M ratio is shown in figure 4. From figure 4, it can be seen that all data points fall on the same line, showing that for a given material-explosive combination, there is a definite linear relationship between the per-unit-length modified Payman fragmentation parameter \( Cu \) and the C/M ratio, which is independent of the projectile size. This relationship can be expressed as follows:

\[
Cu = a + b\left(\frac{C}{M}\right)
\]

(9)

where \( a \) and \( b \) are constants, which can be obtained by small cylindrical projectile tests.

Figure 4. Relationship between the per-unit-length fragmentation parameter \( Cu \) and the C/M ratio
3. Test verification of fragment mass and number distribution of cylindrical projectiles

According to the projectile similarity relation, the fragment mass and number distribution of cylindrical projectiles with different sizes can be quantitatively predicted by using the data obtained from small cylindrical projectile tests.

3.1. Test method

1) The C/M ratio is calculated according to the size of the predicting projectile and the density of the explosive. The fragmentation test is carried out on small cylindrical simulating projectiles with the same material, explosive and C/M ratio, and then Cu is measured.

2) According to the length of the predicting projectile, Co is calculated by formula (8).

3) According to formula (1), the cumulative mass percent of fragments is calculated.

4) Fragment masses in each mass range are calculated; according to formula (5), the cumulative mass percent of fragments $P = 10^{-\frac{\text{Com}_i}{M}}$ can be obtained. Therefore, the fragment mass in the range of $m_i$ to $m_j$ is:

$$\Delta m_{ij} = \left(1 + \alpha\right)M\left(10^{-\frac{\text{Com}_i}{M}} - 10^{-\frac{\text{Com}_j}{M}}\right)$$

(10)

Where $M$ is the metal weight of the predicting projectile (g); $m_i$ and $m_j$ are the lower and upper limits of the selected fragment mass range, $m_i < m_j$; $\alpha$ is the modification coefficient.

The number of fragments in each mass range is calculated according to the following formula:

$$N_{ij} = \frac{\Delta m_{ij}}{a_{m_i} + b_{m_j}}$$

(11)

where $a$ and $b$ are weighting coefficients. In the formula, the denominator is $a_{m_i} + b_{m_j}$ instead of $1/2 \left(m_i + m_j\right)$, which takes into account the fact that the smaller the mass, the bigger the number of fragments, so that the results obtained are closer to the actual situation.

3.2. Validation results

In this work, three kinds of projectile steel 60Si$_2$Mn, 58SiMn and D60 were selected to verify the similarity relation through predicting the fragment mass and number distribution. The parameters of predicting projectiles are shown in Table 3.

| Group    | Material | Outer Diameter (mm) | Inner Diameter (mm) | Length (mm) | Mass (g) | C/M  | Explosive type   |
|----------|----------|---------------------|---------------------|-------------|----------|------|------------------|
| Projectile 1 | 60Si2Mn  | 128.9               | 106.7               | 194         | 6100     | 0.47 | Cast Comp. B     |
| Projectile 2 | 58SiMn   | 110                 | 76                  | 194         | 7416     | 0.18 | TNT              |
| Projectile 3 | 58SiMn   | 119.9               | 89.9                | 194         | 7380     | 0.28 | Cast Comp. B     |
| Projectile 4 | D60      | 108.5               | 86.5                | 194         | 5021     | 0.38 | Cast Comp. B     |

The per-unit-length modified Payman fragmentation parameter $Cu$ of the four groups of projectiles is obtained by small simulating projectile tests. According to the similarity relation, the modified Payman fragmentation parameter $Co$ of each group of projectiles is calculated respectively. The specific data are shown in Table 4.
Table 4. Fragmentation parameters of predicting cylindrical projectiles

| Group     | Material | Cu   | Co   | Explosive type |
|-----------|----------|------|------|----------------|
| Projectile 1 | 60Si2Mn  | 36.2 | 702  | Cast Comp. B   |
| Projectile 2 | 58SiMn   | 7.6  | 147  | TNT            |
| Projectile 3 | 58SiMn   | 24.5 | 475  | Cast Comp. B   |
| Projectile 4 | D60      | 31.7 | 615  | Cast Comp. B   |

Each of the four groups has three projectiles, and their parameters are the same. The mass and number of fragments in each mass range are measured by the 9-level classification method, and the average value is taken. Considering that the per-unit-length modified Payman fragmentation parameter Cu is only related to the projectile material, C/M ratio and explosive type, this paper selected Projectile 1 and Projectile 2 from the verification and comparison results of the above four groups of predicting projectiles, and the prediction results of fragment mass and number as well as the cumulative number of fragments in each mass range are listed in the form of chart. The details are shown in figure 5 and figure 6.

Figure 5. Comparison of the prediction results of fragment mass and number in each mass range and the test results for Projectile 1
c. Cumulative number of fragments (piece)

**Figure 6.** Comparison of the prediction results of fragment mass and number in each mass range and the test results for Projectile 2

It can be seen from figure 5 and figure 6 that the predicted fragment mass and number distribution are in good agreement with the test results. For lethal fragments (≥4g) and effective fragments (≥1g) under the current standards\(^\text{[14]}\), the prediction accuracy for the cumulative number of fragments is higher. The differences between the prediction values and the test values of the total number of fragments greater than or equal to 4g for Projectile 1 and Projectile 2 are 14 and 1 respectively, while those of the total number of fragments greater than or equal to 1g are 42 and 34. The prediction results of Projectile 3 and Projectile 4, which are not charted in the paper, are in good agreement with the test results. This shows that the fragment mass and number distribution in each mass range can be accurately predicted using the projectile similarity relation described in this paper, and the total number of lethal and effective fragments can also be accurately predicted, thus providing important data for the calculation of the lethality of projectiles.

4. Conclusion

(1) The modified Payman fragmentation parameter Co has limitations. The newly derived per-unit-length modified Payman fragmentation parameter Cu has nothing to do with the projectile size, and is only related to the material, explosive and C/M ratio.

(2) When the explosive type and C/M ratio are fixed, the per-unit-length modified Payman fragmentation parameter Cu of cylindrical projectiles with different sizes is the same for the given composition and heat treatment state. This similarity relation provides a reliable theoretical basis for predicting the mass and number of fragments.
(3) For a given material-explosive combination, there is a definite linear relationship between the fragmentation parameter $C_u$ and $C/M$ ratio, which is independent of the projectile size.

5. References

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