Low temperature specific heat and thermal conductivity in superconducting UTe$_2$

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The measurements (Phys.Rev.B 100, 220504(R) (2019)) do not detect noticeable thermal conductivity in superconducting UTe$_2$ in the T=0 limit. At the same time the same crystals exhibit a large residual density of states comparable with its normal state value. The improvement of samples quality leads to the augmentation of critical temperature of transition to the superconducting state accompanied by the decreasing of residual specific heat ratio $C(T)/T$ at $T \rightarrow 0$. There is presented an analytic derivation of this inverse correlation property in concrete cases of triplet superconducting states with node-less and point-nodes order parameters allowed by symmetry in UTe$_2$. The obtained explicit formulas for the residual thermal conductivity are in reasonable correspondence with observations.

I. INTRODUCTION

The recently discovered uranium superconductor UTe$_2$ [1,2] has rather unusual qualities such as reentrant superconductivity at extremely high magnetic fields [3,4]. In the early experiments the bulk superconductivity appeared at $T_c \approx 1.6$ K and characterised by the sharp specific heat jump larger than the weak coupling BCS value $\Delta C/\gamma T_c = 1.43$ and by remarkably large residual ratio $C/T = \gamma_{sc}$ at $T \rightarrow 0$ which is equivalent to approximately one half of its normal state $\gamma_n$ value [1,2]. The little variation of residual density of states in UTe$_2$ between the samples with slightly different $T_c$ led to the assumption that the large residual electronic density of states is likely an intrinsic, disorder-insensitive property of UTe$_2$ as if its superconducting state was nonunitary such that only half of the electrons participate in spin-triplet pairing, while the remainder continue to form a normal Fermi liquid. At the same time, the thermal conductivity measurements [2] do not show noticeable thermal conductivity in the T=0 limit for the same crystals that exhibit a large residual density of states. Then, however, the higher $T_c \approx 1.77$ K was registered and there was established the inverse correlation between $T_c$ and $\gamma_{sc}$ [3,4]. At last in the highest quality crystals with RRR=57 there was found the superconducting transition temperature $T_c = 2K$ and the lowest residual heat capacity with $\gamma_{sc}$ noticeably smaller than $\gamma_n$ [3,4].

The inverse correlation between $T_c$ and $\gamma_{sc}$ has also been observed in other unconventional superconductors UPt$_3$ [10] and CeCoIn$_5$ [11]. The residual density of states originates from the electron scattering on impurities and crystal imperfections forming so called gapless superconducting state. Discovered first by Abrikosov and Gor’kov [12] in conventional superconductors with paramagnetic impurities this phenomenon in unconventional superconductors with ordinary impurities has been investigated in the papers [13,14] (see also the textbook [15]). The temperature dependence of the thermal conductivity in unconventional superconductors including T=0 limit has been studied in articles [13,16,17] expounded in the textbook [15].

The superconducting properties of UTe$_2$ are strongly dependent on the synthesis route. The relatively small RRR ratio points on the significant presence of some crystal imperfections probably originating from the non-stoichiometric composition of this material. These type of imperfections not spoiling the crystal periodicity on the large scales serve as the electron scattering centres. Here, I consider the property of inverse correlation between the critical temperature and residual density of states as well the zero temperature limit of the thermal conductivity based on the theory of resonant scattering in concrete application to the possible superconducting states in UTe$_2$. The problem of compatibility of finite residual density of states and small residual thermal conductivity is discussed.

The paper is organised as follows. In the next section there is obtained the explicit expressions for the residual density of states in triplet superconducting states with node-less and point-nodes order parameters. It is shown that in both cases the residual density of states in practically pure crystal may represent a significant part of its normal state value. Then in the next section there is established the T=0 limit of thermal conductivity. The residual thermal conductivity in clean enough specimens in both cases of superconducting states with node-less and point-nodes order parameter turns out to be comparable with values observed experimentally [3].

II. RESIDUAL DENSITY OF STATES

UTe$_2$ has an orthorhombic structure. In view of abnormally large upper critical field in UTe$_2$ one must discuss the superconducting states with triplet pairing. The superconducting states with triplet pairing in a metal with orthorhombic structure are related to one of four different representations of the point group $D_{2h}$: $A_u$, $B_{1u}$, $B_{2u}$, $B_{3u}$. The corresponding order parameters
of superconducting states are
\[
\mathbf{d}_0^i(\mathbf{k}) = \Delta^i_0(\varphi_{0x}\hat{x} + \varphi_{0y}\hat{y} + \varphi_{0z}\hat{z}),
\]
\[
\mathbf{d}_{B1}^i(\mathbf{k}) = \Delta_1(\varphi_{1y}\hat{x} + \varphi_{1x}\hat{y} + \varphi_{1w}\hat{z}),
\]
\[
\mathbf{d}_{B2}^i(\mathbf{k}) = \Delta_2(\varphi_{2x}\hat{x} + \varphi_{2w}\hat{y} + \varphi_{2z}\hat{z}),
\]
\[
\mathbf{d}_{B3}^i(\mathbf{k}) = \Delta_3(\varphi_{3w}\hat{x} + \varphi_{3z}\hat{y} + \varphi_{3y}\hat{z}).
\]

Here, \( \Delta_i, \ i = 0, 1, 2, 3 \) are the order parameter amplitudes. The functions \( \varphi_{ix}(\mathbf{k}), \varphi_{iy}(\mathbf{k}), \varphi_{iz}(\mathbf{k}), \varphi_{iw}(\mathbf{k}) \) transform as \( k_x, k_y, k_z, k_x k_y k_z \) correspondingly. The order parameter of \( A \) state has finite value at any point on the Fermi surface, whereas the order parameter of \( B_i \) state for \( i = 1, 2, 3 \) has the point nodes at \( \mathbf{k} \) along \( z, y, x \) direction correspondingly. To do the calculations analytically we will consider the superconducting states with more simple order parameters possessing the same properties:

\[
\mathbf{d}_A(\mathbf{k}) = \Delta_A(\hat{k}_x\hat{x} + \hat{k}_y\hat{y} + \hat{k}_z\hat{z}), \quad (1)
\]

for unit representation \( A \) and

\[
\mathbf{d}_B(\mathbf{k}) = \Delta_B(\hat{k}_y\hat{x} + \hat{k}_x\hat{y}). \quad (2)
\]

for non-unit representations \( B_i \).

The critical temperature in a dirty unconventional superconductor is determined by the Abrikosov-Gorkov equation

\[
\ln \frac{T_{c0}}{T_c} = \psi \left( \frac{\Gamma}{2\pi T_c} + \frac{1}{2} \right) - \psi \left( \frac{1}{2} \right), \quad (3)
\]

where \( \Gamma \) is the scattering rate of electrons on impurities. The superconductivity disappears at

\[
\Gamma_c = \frac{\pi}{2\gamma} T_{co} = \frac{\Delta_{00}}{2}, \quad (4)
\]

where \( T_{co} \) is the critical temperature in the absence of impurities, \( \Delta_{00} \) is the order parameter amplitude in the absence of impurities at \( T=0 \), \( \gamma = e^C \approx 1.78, C \approx 0.577 \) is the Euler constant.

Found in the Born approximation \( \ref{12} \) in the limit \( T \to 0 \) the finite density of states at Fermi level appears in very dirty limit starting from the scattering rate

\[
\Gamma_g = 2\Gamma_c \exp \left( -\frac{\pi}{4} \right) \approx 0.91\Gamma_c. \quad (5)
\]

We will be interested in the opposite situation of almost clean superconductor, where the gapless superconducting state is realised in case of resonant scattering on impurities with the scattering rate

\[
\Gamma = \frac{n_i}{\pi N_0} \ll \Gamma_c. \quad (6)
\]

Here, \( N_0 \) is the electron density of states at the Fermi level on one spin projection.

For small \( \Gamma \) the critical temperature is

\[
T_c \approx T_{c0} - \frac{\pi}{4} \Gamma, \quad (7)
\]

that is

\[
\Gamma = \frac{8\gamma}{\pi^2 T_c \tau}, \quad (8)
\]

where

\[
\tau = \frac{T_{c0} - T_c}{T_{c0}} \ll 1.
\]

Let us write now the residual density of states for the superconducting states corresponding to unit representation \( \ref{11} \) and to anyone of non-unit representation \( \ref{2} \).

### A. State A

The nonzero density of states at \( E = 0 \) is derived following procedure described in \( \ref{18} \)

\[
N_0(E = 0) = N_0 \left[ \left( \frac{\Gamma}{\Delta_0} \right)^2 + \frac{\Gamma}{\Delta_0} \sqrt{\left( \frac{\Gamma}{\Delta_0} \right)^2 + 4} \right]^{-1/2} \left[ 2 + \left( \frac{\Gamma}{\Delta_0} \right)^2 + \frac{\Gamma}{\Delta_0} \sqrt{\left( \frac{\Gamma}{\Delta_0} \right)^2 + 4} \right]. \quad (9)
\]

The order parameter amplitude at \( T \to 0 \) and small \( \Gamma \) is \( \ref{18} \)

\[
\Delta_0 \approx \Delta_{00} \left( 1 - \frac{\pi \Gamma}{4 T_{c0}} \right). \quad (10)
\]

Hence, at small \( \Gamma \) the residual density of states is

\[
N_0(E = 0) \approx \sqrt{\frac{\Gamma}{\Delta_{00}}} = \frac{2\sqrt{\gamma}}{\pi} N_0 \sqrt{\tau}. \quad (11)
\]

### B. State B

The calculation of the residual density of states for a state \( B_i \) coincides with corresponding treatment for the superconducting state with structure of \( A \)-phase of superfluid \( ^3 \)He presented in the textbook \( \ref{12} \). It is

\[
N_0(E = 0) = N_0 \sqrt{\frac{\pi \Gamma}{2\Delta_{00}}}. \quad (12)
\]

In case of weak disorder the difference between \( \Delta_{00} \) and \( \Delta_0 \) is small. Hence,

\[
N_0(E = 0) \approx N_0 \sqrt{\frac{\pi \Gamma}{2\Delta_{00}}} = \sqrt{\frac{2\gamma}{\pi}} N_0 \sqrt{\tau}. \quad (13)
\]

We see that the zero temperature density of states can be finite even at small enough \( \tau \). Moreover, in the samples with higher critical temperature the residual density of states \( N_0(E = 0) \) is smaller that is in qualitative correspondence with experimental observations \( \ref{8} \).
III. LOW TEMPERATURE THERMAL CONDUCTIVITY

In neglect of vertex corrections the thermal conductivity in a superconducting state in \( i = x, y, z \)-direction is

\[
\kappa_i = \frac{N_0 v_F^2}{4 T^2} \int_0^\infty \frac{dE}{\cosh^2(E/2T)} \left| |k_i|^2 \left[ |t|^2 + t^2 - 2|d(k)|^2 \right] \right|^2.
\]

The integral over angles in Eq. (14) at \( \Gamma \) we find

\[
\kappa \approx N_0 v_F^2 T \frac{\sqrt{\pi} \Delta_0}{2} I, \tag{15}
\]

The solution of this equation at \( \Gamma \ll \Delta_0 \) and \( E \to 0 \) is

\[
t \approx \frac{E}{2} + i \sqrt{\Delta_0 \Gamma}. \tag{16}
\]

The integral over angles in Eq. (14) at \( E \ll \sqrt{\Delta_0 \Gamma} \) is

\[
\frac{1}{4 \pi} \int_0^\infty \frac{dE}{\cosh^2(E/2T)} \left| |k_i|^2 \left[ |t|^2 + t^2 - 2|d(k)|^2 \right] \right|^2 \approx 2 \int \frac{dE}{4 \pi} \left| |k_i|^2 \left( \Delta_0^2 + \Delta_0 \Gamma \right)^{3/2} \right|^2 \approx \frac{2 \Gamma}{3 \Delta_0^2}. \tag{17}
\]

Then the thermal conductivity near the zero temperature is

\[
\kappa = \frac{N_0 v_F^2 \Gamma}{6 T^2 \Delta_0^2} \int_0^\infty \frac{dE}{\cosh^2(E/2T)} = \frac{N_0 v_F^2 \Gamma T}{6 \Delta_0^2} I, \tag{18}
\]

where

\[
I = \int_0^\infty \frac{z^2 dz}{\cosh^2 \frac{z}{2}} = \frac{2 \pi^2}{3}.
\]

Comparing the obtained result with the thermal conductivity in normal state

\[
\kappa_n = \frac{N_0 v_F^2 \Gamma}{6 \Gamma} I, \tag{19}
\]

we find

\[
\frac{\kappa_A(T \to 0)}{\kappa_n(T = T_c)} = \left( \frac{\Gamma}{\Delta_0} \right)^2 T \frac{T_c}{T}. \tag{20}
\]

B. State B

The equation for the function \( t(E) \) in the state B is

\[
t = E + 2 \Gamma \Delta_0 \left( \frac{\cos \frac{t + \Delta_0}{t - \Delta_0}}{\sqrt{t}} \right)^{-1}. \tag{21}
\]

The solution of this equation at \( \Gamma \ll \Delta \) and \( E \to 0 \) is

\[
t \approx \frac{E}{2} + i \sqrt{\frac{2}{\pi} \Delta_0 \Gamma}. \tag{22}
\]

The integral over angles in Eq. (14) at \( E \ll \sqrt{\Delta_0 \Gamma} \) is

\[
\frac{1}{4 \pi} \int_0^\infty \frac{dE}{\cosh^2(E/2T)} \left| |k_i|^2 \left[ |t|^2 + t^2 - 2|d(k)|^2 \right] \right|^2 \approx 2 \int \frac{dE}{4 \pi} \left| |k_i|^2 \left( \Delta_0^2 (k_x^2 + k_y^2) + 2 \Delta_0 \Gamma \right)^{3/2} \right|^2 \approx \frac{2 \sqrt{\pi} \sqrt{T}}{\Delta_0}, \tag{23}
\]

Thus, the residual thermal conductivity in clean enough specimens in both cases of superconducting states with node-less and point-nodes order parameter turns out to be comparable with with values observed experimentally [3].
IV. CONCLUSION

The problems of residual density of states and residual thermal conductivity in triplet superconducting states with node-less and point-nodes order parameters allowed by symmetry in UTe$_2$ have been investigated. There was shown that in all cases the resonant scattering on impurities leads to the formation of the gapless superconducting states with large enough density of states at zero energy. The increasing of scattering rate on impurities suppresses the critical temperature and leads to the augmentation of the residual density of states.

The calculations were performed in clean enough limit when critical temperature of transition to the superconducting states is slightly deviates from the corresponding critical temperature in perfect crystal, that corresponds to the scattering rate much smaller than the zero temperature amplitude of the order parameter $\Gamma \ll \Delta_0$. The comparison of the derived explicit formulas for the residual thermal conductivity with corresponding experimental values is in the reasonable correspondence with this demand. The residual thermal conductivity in clean enough specimens in both cases of superconducting states with node-less and point-nodes order parameter turns out to be comparable with with values observed experimentally. Still, the superconducting state with point node order parameter is preferable in view of non-exponential decreasing of thermal conductivity and the penetration depth at finite temperature reported in the paper [5].

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