Electromagnetics of two-dimensional materials with time-varying carrier density

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Abstract. The variation of carrier density in two-dimensional materials, such as graphene, creates opportunities for rapid control of guided wave propagation which can find numerous applications in optoelectronics. A general theoretical approach to calculate the transformation of the surface wave guided by a two-dimensional material is presented. The frequencies of the excited modes and their energies are analyzed. Our results refute the claims of plasmon amplification under rapid density changes that appeared in recent theoretical studies. The difference in the results stems from the different treatment of the microscopic current during the carrier density changes.

1. Introduction
Moving from bulk to low-dimensional structures offers opportunities to explore new fundamental physical properties and to apply them for the creation or significant improvement of various devices. The move to reduce the material dimensionality started back in the 1970s from making semiconductor quantum wells, two-dimensional (2D) structures with quantization in one direction. Later, quantum wires and dots were grown and investigated. Recently, significant interest has been spiked by graphene [1] and a broad range of other 2D materials [2].

Optoelectronics is one of the key areas which can be impacted by new materials, in particular, 2D materials. Graphene sheets with finite carrier density are capable of supporting propagating modes – graphene plasmons [3]. Graphene-based nanowaveguides hold promise as printed interconnects [4]. Strong nonlinearities in graphene can enable all-optical modulation [5]. Applying gate voltage one can also tune graphene conductivity and realize 2D transformation optics using graphene [6]. While thin metal layers can also support plasmons, the conductivity control is an apparent advantage of graphene [6] in addition to lower losses and stronger confinement [7], which can further be improved by making heterostructures [8]. The timescale of conductivity changes can be slow or fast compared to the plasmon period.

The feasibility of controlling graphene properties in time calls for the study of plasmon transformation under time variations of graphene parameters, in particular, for rapid density changes. In general, the subject of wave transformation in time-varying media has a rather long history. Most of the studies considered the transformation of waves in bulk materials [9]. The scattering of surface waves under rapid carrier injection was also investigated for a plasma half-space [10, 11] and a plasma layer [12]. Theoretical studies of graphene plasmon transformation under density changes have also recently appeared [13, 14]. Both of Refs. [13, 14] explored the
transformation within the quasi-static plasmon approximation. Despite the similarity of the addressed problem, the amplitudes of the scattered plasmons obtained in Ref. [13] and Ref. [14] were different. These studies also predicted the appearance of surface plasmon amplification induced by density changes. However, while Ref. [13] obtained the amplification for decreasing carrier density, Ref. [14] obtained it for increasing. Thus, resolving this controversy seems to be timely and compelling.

Here we develop a theoretical approach to describe the surface plasmon transformation after a rapid carrier density change in a 2D material. The approach is based on solving the Maxwell equations supplemented by a material equation for the carrier current [15]. Our approach is not limited by the quasi-static plasmon approximation considered earlier in Refs. [13, 14]. We obtained explicit formulas for the frequencies and amplitudes for the scattered plasmons, as well as for transient bulk radiation. The analytical results are also in a good agreement with finite-difference time-domain (FDTD) simulations [15].

2. Results

The considered problem is illustrated in figure 1. Initially, at \( t < 0 \), a surface wave at frequency \( \omega_1 \) propagates along a 2D material. The plasmon is supported by the surface conductivity \( \sigma \) of the material. In the Drude model and for the fields \( \sim e^{-i\omega_1 t} \) we obtain the surface current

\[
j_x = \sigma E_x, \quad \sigma = i e \Omega_1 / \omega_1,
\]

where collisional damping in \( \sigma \) is neglected for simplicity. The frequency (or Drude) parameter \( \Omega_1 \) depends on the carrier density \( N_1 \) at \( t < 0 \): \( \Omega_1 \sim E_{f1} \sim \sqrt{N_1} \), where \( E_{f1} \) is the Fermi energy. The propagation wavenumber, the group velocity, and the phase velocity of the plasmon depend on the ratio \( \omega_1 / \Omega_1 \). At low frequencies, \( \omega_1 / \Omega_1 \ll 1 \), the plasmon is weakly confined. At high frequencies, \( \omega_1 / \Omega_1 \gg 1 \), it is strongly confined and this regime is often referred to as quasi-static. At \( t = 0 \), the carrier density changes rapidly to \( N_2 \) and the corresponding Drude parameter to \( \Omega_2 \). This change results in the scattering of the plasmon. Here we consider explicitly the case when the carriers are removed: \( N_2 < N_1 \). The jump can conveniently be described by the ratio \( \Omega_2 / \Omega_1 < 1 \). At \( t > 0 \), two new (transmitted and reflected) surface plasmons are excited as well as some transient bulk radiation that propagates away from the sheet.

The frequencies of the excited waves (surface and bulk) can be found using two conditions: (1) the conservation of the spatial dependence for the waves under spatially uniform density change and (2) the dispersion properties of the waves before and after the density change.
The transmitted and reflected at the temporal discontinuity surface plasmons have the same frequency $\omega_2$. Its dependence on the parameter $\Omega_2/\Omega_1$ is shown in figure 2(a). With decreasing $\Omega_2/\Omega_1$ the frequencies $\omega_2/\omega_1$ decrease for all values of $\omega_1/\Omega_1$. In the quasi-static regime the frequency conversion can be described approximately by $\omega_2/\omega_1 \approx \sqrt{\Omega_2/\Omega_1}$.

To find the scattered fields at $t > 0$ one has to solve the Maxwell equations describing the field evolution and the material equation for the carrier current. A common way to solve the temporal scattering problem is to use these equations with some initial conditions at $t = 0^+$. The initial conditions are defined by the initial plasmon and the physical processes during the carrier density change. The change of carrier density, apparently, does not change the fields immediately and, therefore, the fields are continuous at $t = 0$. A particular attention should be paid to how the jump conditions are incorporated into the current equation. In the considered case of density decrease, the removal of carriers should produce a rapid decrease in current related to the change from $\Omega_1$ to $\Omega_2$. The material equation describing the current for $t > 0$ in the Drude model becomes

$$\frac{\partial j_x}{\partial t} = e\Omega_2 E_x, \quad j_x(t = 0^+) = \frac{\Omega_2}{\Omega_1} j_x(t = 0^-). \quad (2)$$

After the initial conditions are known, one can apply the standard Laplace transform technique to solve the initial value problem and to find the amplitudes of all fields.

A convenient way to characterize the temporal scattering is to consider the energy density of the excited modes. These modes include the transmitted plasmon, the reflected plasmon, and the transient bulk radiation. The carrier removal implies that the kinetic energy of the removed carriers is lost, i.e., taken away from the system. Thus, the energy of the initial plasmon should be equal to the energies of the scattered modes (surface and bulk) and the removed kinetic energy. The numerical results presented below satisfy this energy balance.

Figure 2(b) shows how the energies depend on the jump parameter $\Omega_2/\Omega_1$ when the initial plasmon is well confined, $\omega_1/\Omega_1 = 15$. The transmitted energy $W_t$ monotonically decreases with decreasing $\omega_1/\Omega_1$; in contrast, the reflected energy $W_r$ monotonically increases. A significant part $W_l$ of the initial energy is removed from the system. Indeed, for strongly confined plasmons the time-averaged kinetic energy related to the surface current reaches 1/2 of the total energy. The energy $W_b$ of the transient bulk radiation is rather small for large $\omega_1/\Omega_1$.

Figure 2. (a) Frequencies $\omega_2$ of the excited plasmons (normalized to the initial frequency $\omega_1$) for several values of $\omega_1/\Omega_1$. (b) Energies (normalized to the energy of the incident plasmon) of the excited plasmons (transmitted $W_t$ and reflected $W_r$), of the bulk radiation $W_b$, and the kinetic energy of the removed carriers $W_l$ for $\omega_1/\Omega_1 = 15$. All curves in frames (a, b) are plotted as functions of the jump parameter $\Omega_2/\Omega_1$, which changes from 1 to 0 as we increase the number of the removed carriers.
3. Conclusion
To conclude, the temporal scattering of the plasmon guided by a 2D material, such as graphene, under rapid variation of its density was considered. A particular case of carrier removal was investigated. The problem was solved by using the Maxwell equations and the material equation for the 2D current. It was shown that the carrier density decrease leads to the overall decrease of the electromagnetic energy by the amount of the kinetic energy of the removed carriers. The remaining energy is distributed among the excited modes: two surface waves and transient bulk radiation.

The results of our study refute the claims of plasmon amplification under density changes that appeared in recent theoretical studies [13, 14]. The likely reason for such predictions is an incorrect treatment of carrier current during the density change. Indeed, the change of carrier density not only changes the Drude parameter $\Omega$ but also affects the initial conditions for the carrier motion after the discontinuity. In the case of carrier density decrease considered here, the current rapidly drops since some carriers, which contributed to the current, are taken away. In the case of carrier density increase, the created carriers produce zero initial current since they are likely to be injected with isotropically distributed velocities. In contrast, the existing carriers continue moving with the velocities that they had before the injection. This gives rise to the appearance of a two-stream carrier motion. One of the consequences of such dynamics is the excitation the so called free-streaming mode consisting of direct current and corresponding magnetic field [10, 11, 12]. The free-streaming mode also takes a part of the initial energy. Thus, neither carrier removal nor injection can result in plasmon amplification. However, these two cases should be treated using different initial conditions for the microscopic current. The continuity of the electromagnetic fields alone used in Refs. [13, 14] is not sufficient to find the amplitudes of the scattered surface plasmons.

Funding
This work was supported by the Ministry of Science and Higher Education of the Russian Federation, Project No. 3.3854.2017/4.6.

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