Higher Derivative Operators as Counterterms
in Orbifold Compactifications.

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Abstract

In the context of 5D N=1 supersymmetric models compactified on $S_1/Z_2$ or $S_1/(Z_2 \times Z'_2)$ orbifolds and with brane-localised superpotential, higher derivative operators are generated radiatively as one-loop counterterms to the mass of the (brane or zero mode of the bulk) scalar field. It is shown that the presence of such operators which are brane-localised is not related to the mechanism of supersymmetry breaking considered (F-term, discrete or continuous Scherk-Schwarz breaking) and initial supersymmetry does not protect against the dynamical generation of such operators. Since in many realistic models the scalar field is commonly regarded as the Higgs field, and the higher derivative operators seem a generic presence in orbifold compactifications, we stress the importance of these operators for solving the hierarchy problem.

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1 Introduction

In recent years the study of radiative corrections from compact dimensions has seen a strong development in the context of field theory approaches to orbifold compactifications. The interest in such studies has theoretical and experimental motivations. First, one cannot exclude the possibility that radiative corrections from compact dimensions may have some experimental signatures in the context of the Large Hadron Collider experiment. Another motivation is that such studies of compactification allow a comparison with the more comprehensive approaches of string theory. Field theory orbifolds can give one-loop results similar to those of the string in the limit $\alpha' \to 0$. This provides a better understanding of compactification and an indication whether string theory can provide an UV completion for effective field theory models. Finally, field theory orbifolds allow us to re-address in a consistent framework well known problems such as the hierarchy problem, supersymmetry or electroweak symmetry breaking, etc, whether or not there is a link with string theory.

An aspect that is often overlooked in the studies of radiative corrections induced by compactification (to the couplings or the masses in the theory) is the role of the higher (dimension) derivative operators. This observation applies to both field theory and string theory orbifolds. The study of such operators is particularly compelling when they are generated as counterterms in the action, to ensure the quantum consistency of the model under study. Specific examples where higher derivative operators are a result of the compactification can be found in [5]-[11]. In general one could assume that the effects of such operators are suppressed at low energies. From a 4D perspective this suppression is ensured by the compactification scale which is the natural 4D cutoff scale. If this scale is low, such operators are less suppressed and affect the radiative corrections. Their effects become manifest as we increase the external momentum $q^2$ of the Green function under study, relative to the compactification scale $1/R$, from $q^2 \ll 1/R^2$ to regions where $q^2 \gg 1/R^2$. Here we assume that $1/R$ and $q^2$ have arbitrary (fixed) values, to allow a general study of the effects of these operators, whether $q^2 R^2 \ll 1$ or $q^2 R^2 \gg 1$.

The importance of the study of higher derivative operators in gauge theories on orbifolds is due to the underlying physics attached to them. In the absence of a UV completion of the effective field theory which provides the framework of our study, such operators can be present with unknown coefficients, thus affecting the predictive power of the models. Further complications can arise, such as the presence of ghosts fields, unitarity violation, non-locality effects, etc, [3, 4] and for these reasons, in general theories with higher derivative operators

\footnote{1for an example see [4].}

\footnote{2For some studies of higher derivative operators see for example [3, 4].}
were not extremely popular. It is thus even more important to address the consequences of the presence of such operators in gauge theories on orbifolds, where such operators are generated radiatively by gauge \cite{5,6,7,11} or Yukawa interactions \cite{8,10,11}. Their investigation has many implications for model building.

One would expect to find out more about the role of higher derivative operators from string calculations. Unfortunately this is not always the case, and sometimes little light is shed on the corrections such operators induce, partly due to the on-shell formulation of the string. More explicitly, in the context of string loop corrections to the gauge couplings \cite{12} higher derivative counterterms can be missed by the string approach, although they can be shown to be present in effective field theories \cite{5,6,7}. This raises questions \cite{9} (also \cite{11}) on the exact matching of the two approaches to compactification: string theory versus effective field theory. Nevertheless, higher derivative operators can be studied even in the absence of any link with string theory or of a compactification of a higher dimensional theory. This can be done in the context of 4D field theories with additional higher dimension operators, where the scale where they become relevant is regarded as the scale of new physics (rather than the compactification scale).

Higher dimensional models of physics beyond the Standard Model (SM) or the Minimal Supersymmetric Standard Model (MSSM) require in general some amount of supersymmetry, for reasons of stability, hierarchy problem etc. However, such models are nevertheless non-renormalisable, and then such operators can be present in the action. It is then interesting to address the extent to which initial supersymmetry protects against the generation of such operators by radiative corrections. In this context of particular interest for the hierarchy problem is the relation between the nature of supersymmetry breaking on 5D orbifolds and the presence of higher derivative operators as loop counterterms to the mass of a scalar field \cite{8,10,11}.

In the present work we address how (brane-localised) higher derivative operators emerge as counterterms to the mass of the scalar field, from radiative corrections induced by (brane-localised) superpotentials in 5D N=1 supersymmetric models. We review models on $S_1/Z_2$, $S_1/(Z_2 \times Z_2')$ investigated in \cite{8,10} and show how higher derivative operators emerge from compactification regardless of the exact details of the mechanism for supersymmetry breaking. Such interaction is generic in the literature, and some models for which our findings may be relevant can be found in refs.\cite{14}-\cite{32}. For this study we consider that after orbifolding the remaining N=1 supersymmetry is broken via F-term breaking, discrete or continuous Scherk-Schwarz breaking or additional orbifolding ($Z_2'$). Our results show that supersymmetry does not protect against the presence of higher derivative counterterms to the mass of the scalar field, even at the one-loop level. The implications for the hierarchy problem are also discussed briefly.
2 Higher derivative operators as counterterms on orbifolds.

The models we consider have 5D N=1 supersymmetry and are compactified on $S_1/Z_2$ orbifolds. To a large extent our considerations also apply to the $S_1/(Z_2 \times Z'_2)$ orbifold. The spectrum of the models will contain representations of this supersymmetry. Vector supermultiplets on $S_1/Z_2$ may be described in a 4D language as made of a vector superfield $V(\lambda_1, A_\mu)$ and adjoint chiral superfield $\Sigma((\sigma+iA_5)/\sqrt{2}, \lambda_2)$ where $\lambda_1, \lambda_2$ are Weyl fermions, $\sigma$ is a real scalar and $A_\mu, A_5$ is the 5D gauge field. A hypermultiplet contains $\Phi(\phi, \psi)$ and $\Phi^c(\phi^c, \psi^c)$ with opposite SM quantum numbers, with $\phi, \phi^c$ as complex scalars and $\psi, \psi^c$ the Weyl fermions. Under the orbifold action $y \to -y$ we impose that the above fields transform as

$$
\Phi(x, -y) = \Phi(x, y), \quad V(x, -y) = V(x, y)
$$

$$
\Phi^c(x, -y) = -\Phi^c(x, y), \quad \Sigma(x, -y) = -\Sigma(x, y).
$$

(1)

Here $\Phi$ is any of the SM fields $Q, U, D, L, E$ of the Standard Model. As a result of (1), the initial 5D N=1 supersymmetry is broken and the fixed points $(y = 0, \pi R)$ of the orbifold have a remaining 4D N=1 supersymmetry. The gauge field is even under the orbifold action so it has a zero mode, which is the massless 4D gauge boson of the model.

In such models one would like to introduce gauge and Yukawa interactions. Here we restrict the discussion to the case of the latter, to show how higher derivative operators are generated at one-loop$^3$. Given the amount of supersymmetry in the bulk and at the fixed points, the only option is to consider a brane-localised superpotential. The interaction is then

$$
\mathcal{L}_4 = \int dy \mathcal{D}(y) \left\{-\int d^2 \theta \left[ \lambda_t Q U H_u + \lambda_b Q D H_d + \cdots \right] + h.c. \right\}.
$$

(2)

The 5D coupling $\lambda_t = f_{5,t}/M^*_u = (2\pi R)^n f_{4,t}$ and $f_{5,t}$ ($f_{4,t}$) is the dimensionless 5D (4D) coupling, $M_*$ is the cutoff of the theory. In the following $Q, U, D$ superfields are assumed to be bulk fields$^4$, so they have mass dimension $[Q] = [U] = [D] = 3/2$. We also introduced the Higgs fields $H_{u,d}$. These can be brane fields when $[H_{u,d}] = 1$ (n = 1) or bulk fields $[H_{u,d}] = 3/2$, (n = 3/2) when they must also have a $H_{u,d}^c$ partner. If $H_{u,d}$ are also bulk fields they satisfy a condition similar to that for $\Phi$ in eq.(1). The above spectrum and interaction define our minimal model$^5$. Such interaction is generic in 5D extensions of the SM or the MSSM and was extensively considered in the past (for such 5D models see [14]-[32]). New effects so far overlooked are presented below.

$^3$Gauge interactions generate higher derivative operators beyond 1-loop in 5D, or in 6D at 1-loop [5, 7, 11].

$^4$Other possibilities for the character bulk/brane of the fields $Q, U$ are considered, see later.

$^5$The presence of both $H_{u,d}$ is to avoid quadratic divergences to the scalar field mass from FI terms [13].
After the orbifold action (1) on the hypermultiplets and vector multiplets, which breaks the 5D N=1 supersymmetry, the remaining 4D N=1 supersymmetry can be broken using

1). F-term supersymmetry breaking.
2). Discrete Scherk-Schwarz twists.
3). Continuous Scherk-Schwarz twists.
4). An additional orbifolding by $Z_2'$, so the orbifold is actually $S_1/(Z_2 \times Z_2')$.

We briefly review these cases, and then address the one-loop correction to the mass of $\phi_{H_u}$ induced by interaction (2); (similar considerations apply for $H_d$).

Case 1). F-term supersymmetry breaking. In this case one considers supersymmetry broken at a distant (hidden) brane located at $y = \pi R$ by

$$L_4 = \int dy \delta(y - \pi R) \left\{ \int d^2 \theta \ M_\ast^2 Z + \text{h.c.} - \int d^4 \theta \left[ \frac{CQ}{M_\ast^2} Q^\dagger Q Z^\dagger Z + \frac{CU}{M_\ast^2} U^\dagger U Z^\dagger Z \right] \right\},$$

where $Z$ is a (gauge singlet) brane field at $y = \pi R$ and $M_\ast$ is the cutoff scale of the model. The bulk fields $Q, U$ feel the supersymmetry breaking via couplings as in the above integral over $d^4 \theta$. When $<Z> \sim F_Z \theta^2$ the bulk fields $\phi_M, M = Q, U$ which have non-zero coupling at $y = \pi R$ brane have the spectrum modified, while their fermionic partners $\psi_{Q, U}$ do not couple to $Z$ (and neither do $\psi^c_{Q, U}, \phi^c_{Q, U}$, due to eq.(1)) and their Kaluza-Klein spectrum is not affected. The Higgs field $H_u$ (also $H_d$) that is considered in this case to be localised at $y = 0$ (to avoid a direct coupling to $Z$) feels supersymmetry breaking at $y = \pi R$ via loops of bulk fields $Q, U$. As a result the Kaluza-Klein modes $\phi_{M,k}$ have their mass shifted by the loop correction and $m_{\psi_{M,k}} \neq m_{\phi_{M,k}}$, for all positive $k$ including the zero modes.

Case 2). Discrete Scherk-Schwarz supersymmetry breaking. In this case 5D fields acquire under a $2\pi R$ shift a phase which is the R-parity charge of the fields

$$Z_{2,R} M(x, y, \theta) = -M(x, y, -\theta), \quad Z_{2,R} M^c(x, y, \theta) = -M^c(x, y, -\theta), \quad M = Q, U$$
$$Z_{2,R} H(x, y, \theta) = H(x, y, -\theta), \quad Z_{2,R} H^c(x, y, \theta) = H^c(x, y, -\theta),$$
$$Z_{2,R} V(x, y, \theta) = V(x, y, -\theta), \quad Z_{2,R} \Sigma(x, y, \theta) = \Sigma(x, y, -\theta)$$

In this case $H_{u,d}$ can be either bulk or brane fields; in the former case the second line in eq.(4) applies and stands for both $H_{a,d}, H_{u,d}^c$. As a result of $\phi_{M,k}$ and $\psi_{M,k}$ ($M = Q, U$) and in particular their zero modes acquire different masses. The field $\phi_{H_u}$ (also $\phi_{H_d}$) - or its zero mode if a bulk field - receives loop corrections via the fields $\phi_{M,k}$ and $\psi_{M,k}$.
**Case 3.** Continuous Scherk-Schwarz supersymmetry breaking. In this case, using the $SU(2)_R$ global symmetry, one can impose continuous twists for the bulk fields

$$\left( \phi_M, \phi_M^\dagger \right) (x, y + 2\pi R) = e^{-2\pi i \omega_M} \left( \phi_M, \phi_M^\dagger \right) (x, y),$$

with $M = Q, U$. A similar transformation exists for $(\lambda_1, \lambda_2)^T$ while $A_N(x, y)$, $N = \mu, 5$ and $(\psi_M, \psi_M^c)^T$ do not acquire any twists under this transformation. As a result, the fields $\phi_{M,k}$, $\phi_{M,k}^c$, will have mass $(k + \omega)/R$ while $\psi_{M,k}$, $(k \geq 0)$ and $\psi_{M,k}^c$ $(k \geq 1)$ have masses $k/R$. For $\omega = 0$ the fields $\psi_{M,k}$ and $\phi_{M,k}$ regain equal masses at all levels.

**Case 4.** An additional orbifolding by $Z_2'$, so the orbifold is actually $S_1/(Z_2 \times Z_2')$. In this case the one-loop analysis is very close to that of Case 2), since the $Z_2'$ action has similarities to $Z_{2,R}$ Scherk-Schwarz supersymmetry breaking.

With these considerations we can present the one-loop results to the mass of the scalar field $\phi_H$ induced by interaction (2). For technical details see [8, 10]. One obtains in all cases described above the following correction to the mass of the scalar field $\phi_H$ (hereafter denoted simply $\phi_H$)\(^6\):

$$- m_{\phi_H}^2(q^2) \bigg|_B = (2 f A_1 t) N_c \sum_{k \geq 0, l \geq 0} \left[ \eta^Q_{k} \eta^U_{l} \right]^2 \int \frac{d^4 p}{(2\pi)^d} \frac{(-1)(p + q)^2 \mu^{4-d}}{((p + q)^2 + m_{\phi_{Q,k}}^2)(p^2 + m_{\phi_{U,l}}^2)} + (Q \leftrightarrow U)$$

$$- m_{\phi_H}^2(q^2) \bigg|_F = (2 f A_1 t) N_c \sum_{k \geq 0, l \geq 0} \left[ \eta^Q_{k} \eta^U_{l} \right]^2 \int \frac{d^4 p}{(2\pi)^d} \frac{2 p.(p + q) \mu^{4-d}}{((p + q)^2 + m_{\psi_{Q,k}}^2)(p^2 + m_{\psi_{U,l}}^2)}$$

with $\mu$ the finite mass scale of the DR scheme. The Kaluza-Klein spectrum used above is

$$m_{\phi_{Q,k}} = \frac{1}{R}(k + c_1), \quad m_{\phi_{Q,k}^c} = \frac{1}{R}(k + c_2), \quad k \geq 0$$

$$m_{\psi_{Q,k}} = \frac{k}{R}, \quad m_{\psi_{U,k}} = \frac{k}{R}, \quad k \geq 0$$

where the coefficients $c_{1,2}$ have values which depend on the type of supersymmetry breaking:

- F - term breaking: $c_1 = 1/2$, $c_2 = 1$,
- Discrete Scherk – Schwarz: $c_1 = 1/2$, $c_2 = 1/2$,
- Continuous Scherk – Schwarz: $c_1 = \omega$, $c_2 = \omega$,
- $S_1/(Z_2 \times Z_2')$: $c_1 = 1/2$, $c_2 = 1/2$, \(^{(8)}\)

\(^{6}\)In the case $\phi_H$ is a bulk field, the result refers to the one-loop correction to the mass of the zero mode.
In \[6\] one has the wavefunction coefficients \(\eta_k^\phi M = 1/\sqrt{2}^k, M = Q, U\). Also \(\eta_k^{FM} = \eta_l^\phi M = 1\) with the exception of the continuous Scherk-Schwarz case when \(\eta_k^{FM} = \eta_l^\phi M = 1/\sqrt{2}\) and when also the two sums in the bosonic contribution are over the whole set \(Z\). The result of the calculation of eq.\[6\] for all cases described is

\[
-m_{\phi H}^2(q^2) = \frac{(2f_{44})^2}{2(4\pi R)^2} N_c \left\{ \int_0^1 dx \left( \frac{2}{\pi} \right) \left[ J_2[0, 0, c] - J_2[c_1, c_2, c] \right] \right\}
\]

with \(c = x(1 - x) q^2 R^2\), \(\kappa_\epsilon = (2\pi R)^j\) and

\[
J_j[c_1, c_2, c] \equiv \sum_{k_1, k_2 \in Z} \int_0^\infty dt \frac{dt}{e^{\pi t}} e^{-\pi t(k_1+1)^2+a_2(k_2+c)^2} = \frac{(-\pi c)^j}{j!} \left[ \frac{2}{\epsilon} \right] + \mathcal{O}(\epsilon^0), \quad j = 1, 2.
\]

\[
a_1 = (1 - x), \quad a_2 = x, \quad c = x(1 - x) q^2 R^2.
\]

Therefore the pole structure of \(J_j, j = 1, 2\) is independent of the coefficients \(c_1, c_2\) which distinguish between the four cases considered. The finite part \(\mathcal{O}(\epsilon^0)\) was also computed in \[8, 10\]. If \(q^2 = 0\) the second line in \[6\] is absent, so \(m_{\phi H}^2(q^2 = 0)\) is given by the first line alone. Further, the divergent part \(c^2/\epsilon \sim q^4 R^2/\epsilon\) in each \(J_2\) cancels in the difference \(J_2[0, 0, c] - J_2[c_1, c_2, c]\), but the divergence \(c/\epsilon\) in both \(J_1[0, 0, c]\) and \(J_1[c_1, c_2, c]\) does not cancel in the second line in \[6\]. One then finds that for all models

\[
m_{\phi H}^2(q^2) = m_{\phi H}^2(0) - \frac{(2f_{44})^2}{28} N_c \left( \frac{q^4 R^2}{2} \right) \left[ \frac{1}{\epsilon} + \ln(2\pi R\mu) \right] + \frac{1}{R^2} \mathcal{O}(q^2 R^2)
\]

Let us address the significance of the above result. First, \(m_{\phi H}^2(0) \sim f_{44}/R^2 + \mathcal{O}(\epsilon)\) where the constant of proportionality depends on the exact details of supersymmetry breaking (coefficients \(c_{1,2}\)). Note that this constant can have a negative sign giving a negative one-loop \(m_{\phi H}^2(0)\). This can induce electroweak symmetry breaking \[13\], if the tree level mass of the scalar field is somehow arranged (by symmetry arguments) to vanish.

However, the one-loop result of eq.\[11\] presents a much broader picture: \(m_{\phi H}^2(q^2)\) is not finite, and the presence of the divergence \(q^4 R^2/\epsilon\) which can only be “seen” at non-zero external momentum in the associated two-point Green function, requires the addition to the Lagrangian of a counterterm with four derivatives. This can only be a brane-localised \((N=1)\) counterterm, since the interaction considered in eq.\[2\] does not allow one to write a bulk \((N=2)\) counterterm that would necessarily involve \(H^c\) (which does not have a Yukawa coupling). Another explanation
why the counterterm is brane-localised can be made using the mixed position-momentum two-point Green functions in 5D [18], which if evaluated at $y = 0$ give precisely the result in eq.(6). Following these considerations the counterterm has the structure (assuming $H_u$ is a bulk field)

$$\int d^4x \, dy \int d^2\theta \, d^2\overline{\theta} \, \delta(y) \lambda_t^2 H_u^\dagger \Box H_u \sim \frac{f_{4,t}^2}{4} \int d^4x \, R^2 \sum_{n,p \geq 0} \phi_{H,n}^\dagger \Box^2 \phi_{H,p}$$

$$\sim f_{4,t}^2 \int d^4x \, R^2 \phi_{H,0}^\dagger \Box^2 \phi_{H,0} + \cdots$$

(12)

with $[\lambda_t] = -3/2$. However, if $H_u$ is a brane field instead ($[H_u] = 1$, $[\lambda_t] = -1$), the counterterm reads

$$\int d^4x \, d^2\theta \, d^2\overline{\theta} \, \lambda_t^2 H_u^\dagger \Box H_u \sim f_{4,t}^2 \int d^4x \, R^2 \phi_{H,0}^\dagger \Box^2 \phi_H + \cdots$$

(13)

We thus find that brane-localised higher derivative operators are generated by interaction (2), as one-loop counterterms to the mass of the (brane or zero mode of the bulk) scalar field. In the absence of a detailed UV completion of the theory, one does not know the overall (finite) coefficient in front of these operators. As a result the predictive power of the models is significantly affected. Moreover, in the case of higher order theories, further complications may arise, such as the generation of ghost fields, unitarity violation, etc. [3]. These observations underline the importance of the study of such operators and the need for a UV completion of such theories.

The generation of higher derivative counterterms is solely due to compactification. To understand this note that in eq.(6) we summed over the whole Kaluza-Klein towers of modes, associated with the compact dimension. This was done to respect the discrete shift symmetry $k_i \rightarrow k_i + 1$ present in eqs.(9), (10), which are just a re-writing of the initial sums in (6). But to illustrate the origin of the one-loop higher derivative counterterms, it is instructive to examine what happens when the two sums in (6) or (10) were truncated each to a finite but otherwise arbitrary Kaluza-Klein level (while respecting the $N = 2$ multiplet structure of the modes). The framework would then be that of a 4D theory with a finite number of Kaluza-Klein states, i.e. a renormalisable one. In that case, the two sums in $J_j$ ($j = 1, 2$) truncated to say $s_1$ and $s_2$, would diverge as $J_j \sim s_1 s_2 / \epsilon$. This is to be compared with the divergence $c^j / \epsilon$, $j = 1, 2$ of “untruncated” $J_j$, eq.(10). Using this, one would obtain from eq.(10) divergences of type $m_{\phi_H}^2(q^2) \sim m_{\phi_H}^2(0) + s_1 s_2 q^2 / \epsilon + \cdots$ but no $q^4 R^2 / \epsilon$ terms. The terms $s_1 s_2 q^2 / \epsilon$ would then account for wavefunction renormalisation only; in addition $m_{\phi_H}^2(0)$ would have the usual 4D quadratic divergence rather than being one-loop finite (as it was when summing over the whole towers). Therefore higher derivative operators, related to the presence of $q^4 R^2 / \epsilon$ only emerge when summing over the whole Kaluza-Klein towers, so they are indeed the result of compactification. Their presence is due to the same reason which enforced a one-loop finite $m_{\phi_H}^2(0)$. 

8
Let us return to the analysis of eq.(11). After adding the counterterm in the action one ends with a similar equation, but the pole $1/e$ is replaced by an unknown, finite coefficient (hereafter denoted $\xi$). In this equation, if $R$ is somehow fixed to a large value (inverse TeV scale) in order to have a small mass for the scalar field (without large fine tuning), the second term in this equation, $\xi q^4 R^2$, becomes more important. Given the unknown value of $\xi$ this affects significantly the predictive power of the models. Conversely, if $R$ is very small ($q^2$ fixed), the role of higher derivative operators is suppressed, but the first term $\sim 1/R^2$ re-introduces the quadratic mass scale (hierarchy) problem at one-loop, familiar from the Standard Model. While this is the general picture, a detailed analysis should also consider the $O(q^2 R^2)$ terms in (10).

Finally, our calculation can be used to re-address previously mentioned studies of the radiative electroweak symmetry breaking induced by towers of Kaluza-Klein modes, which ignored the effect of the higher derivative operators. Our results eqs.(9)-(11) also provide the running of the scalar mass and its UV behaviour under the UV scaling of the momenta, $q^2 \to \rho q^2$, $\rho \gg 1$.

We can now address the role (the initial 5D N=1) supersymmetry plays in this calculation. According to eq.(9), the divergences $q^4 R^2/e$ present in each $J_2$ cancel in the difference in the first line of eq.(9) which contributes to $m^2_{\phi H}(0)$. Thus initial supersymmetry and the summing over the whole Kaluza-Klein towers lead to one-loop finite $m^2_{\phi H}(0)$. Note however that an identical divergence $q^4 R^2/e$ originating from the second line of eq.(9) and due to $J_1$ functions does not cancel, leading to the need for higher derivative counterterms. Thus initial supersymmetry did not protect against the generation at the one-loop level of such counterterms, and this is true regardless of the values of $c_{1,2}$ i.e. of the way supersymmetry was broken (discrete/continuous Scherk-Schwarz mechanism, F-term breaking or $S_1/(Z_2 \times Z'_2)$). Ultimately, such conclusion may not be too surprising if we recall that initial theory, although supersymmetric is nevertheless non-renormalisable. While (initial) supersymmetry ensured a cutoff independent $m^2(0) \sim 1/R^2$, higher derivative operators are present, similarly to any non-supersymmetric effective theory in which the UV cutoff of the theory is replacing the scale $1/R^2$.

These observations are important since one would naively expect that supersymmetry would ensure that after compactification higher derivative operators and their consequences on the mass of the scalar field would be somewhat under control. But the unknown coefficient $\xi$ of these operators also introduces a new parameter i.e. scale in the theory where such operators become important and which is relevant for the loop corrected mass of the scalar field. One has to fix this scale and also $1/R^2$ by some dynamical mechanism, to provide a solution to the hierarchy problem in the context of higher dimensional theories.

The analysis has so far considered a localised superpotential interaction $\lambda_t \overline{Q} \, U \, H_u$ where the fields $Q, U$ were bulk fields, while $H_u$ was either a bulk or a brane field. Other possibilities for the
character brane/bulk of these fields may be allowed. Then dimensional arguments allow us to estimate the order $n$ in perturbation theory when the (localised) higher derivative counterterm $(\lambda t^{n}H_{u}^{\dagger}\Box H_{u})$ is generated. If the interaction has two genuine brane fields and one bulk field, $[\lambda t] = -1/2$. Then, if $H_{u}$ is a brane field, one has $n = 2$ so the local counterterm is generated at the two-loop level. If $H_{u}$ is the (only) bulk field, dimensional arguments give that $n = 3$, thus such counterterm may arise at three-loop only. Similar considerations can be made for the bulk (gauge) interactions using that the gauge coupling also has mass dimension $-1/2$. These observations can be used when building higher dimensional models, to avoid such counterterms at small number of loops.

3 Conclusions.

In 5D N=1 supersymmetric models compactified on $S_{1}/Z_{2}$ or $S_{1}/(Z_{2} \times Z_{2}')$ we discussed the loop corrections that a (brane-localised) superpotential induces to the mass of the (brane or zero mode of the bulk) scalar field. Such interaction is very common in most higher dimensional extensions of the SM or MSSM models and the scalar field is usually regarded as the Higgs field candidate. The analysis investigated the link between the nature of supersymmetry breaking on these orbifolds and the emergence of higher derivative counterterms to the scalar field mass. Gauge interactions can also induce such higher derivative operators at one-loop (for example in 6D [11]), but for the 5D case this arises beyond the one-loop order.

It was found that (brane-localised) higher derivative counterterms to the mass of the Higgs field are generated at the one loop level. As a result the mass of the scalar field depends on the unknown coefficient $\xi$ that such operators come with in the action. The mass of the scalar field behaves like $m_{\phi_{H}}^{2}(q^{2}) = m_{\phi_{H}}^{2}(0) + \xi q^{4}R^{2} + 1/R^{2} O(q^{2}R^{2})$ with $m_{\phi_{H}}^{2}(0) \sim f_{1,1}^{2}/R^{2}$. Note that a somewhat similar structure can emerge even in the SM with additional, higher derivative operators, but with $1/R$ replaced by the UV cutoff of the model. With $q^{2}$ fixed, if $R$ is small one would expect the higher derivative operators have very small effects, and uncertainties induced by the coefficient $\xi$ are suppressed. However, in this case $1/R^{2}$ is very large and one restores the usual mass hierarchy problem of the SM. Alternatively, if $R$ is large (TeV region) one may have a small $m_{\phi_{H}}^{2}(q^{2})$ (at $q^{2} \sim m^{2}$) and thus an electroweak scale mass for the scalar field without large fine tuning. However, in that case $\xi q^{4}R^{2}$ terms become more important and re-introduce uncertainty in the prediction, due to $\xi$ or equivalently, unknown physics above the compactification scale. The value of $\xi$ and $1/R^{2}$ must be fixed by a dynamical mechanism in order to solve the hierarchy problem in higher dimensional supersymmetric models compactified on $S_{1}/Z_{2}$ or $S_{1}/(Z_{2} \times Z_{2}')$. 

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We investigated the relation between supersymmetry breaking and the presence of higher derivative operators. It turned out that such operators are present regardless of the supersymmetry breaking mechanism considered (F-term breaking, discrete or continuous Scherk-Schwarz mechanisms, additional $Z'_2$ orbifolding). Therefore (initial) supersymmetry does not protect against the presence of such operators which were shown to be ultimately due to a divergence identical to that cancelled in $m_{\phi \mu}(0)$ by (initial) supersymmetry.

The presence of the higher derivative operators is directly related to the number of bulk fields involved in the interaction. In our case the counterterms were ultimately generated because of the (two) corresponding Kaluza-Klein sums acting on the loop integral. They emerged from a “mixing” effect between one infinite sum and a winding zero-mode (on the lattice dual to that of Kaluza-Klein modes of the second sum). For this reason such operators can be considered of non-perturbative, non-local origin and are a generic presence in models with extra dimensions.

Gauge theories with higher derivative operators in the action were not in general the most popular in the past, due to the fact that such operators may bring in a host of complications, related to the presence of additional ghost fields, unitarity violation or non-locality effects. Nevertheless, the emergence of such operators from compactification as counterterms to the masses (and/or the couplings) of the models, shows that these operators are very important at the quantum level. To conclude, addressing the role that higher derivative operators play in orbifold models, for the hierarchy problem in particular, is an interesting study given that such operators are a common presence in higher dimensional models.

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