A new perspective on galactic dynamics

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We derive the radial acceleration of stars in galaxies by using basic features of thermodynamics, statistical mechanics and general relativity. We assume that the “dark” component of the radial acceleration is originated from the reaction of dark energy to the presence of baryonic matter. It can be also explained as the macroscopic manifestation of a huge number of extremely soft bosonic excitations of the dark energy medium with wavelength larger than the size of the cosmological horizon, in thermal equilibrium with de Sitter spacetime. Our formula agrees with the phenomenological relation proposed by McGaugh et al. which, in turns, fits a large amount of observational data and with the MOND theory. We also show that our formula appears as the weak field limit of Einstein’s general relativity sourced by an anisotropic fluid.

The ΛCDM model of standard cosmology accounts for present experimental data about the accelerated expansion of the universe, structure formation, galaxy rotation curves and gravitational lensing effects by postulating the existence of exotic forms of matter and energy (dark matter and dark energy (DE)). Although the predictions of the ΛCDM model explain well the experimental data coming from scale structure and cosmic microwave background, it fails to give a physical explanation of the so-called baryonic Tully-Fisher relation. This formula relates the asymptotic velocity of stars in galaxies to the total baryonic mass through an acceleration parameter of the same order of magnitude of the cosmological constant parameter and an additional term to which we refer to as the MOND regime.

Motivated by these tensions, recently, there have been several attempts to explain the galactic-scale phenomenology commonly attributed to dark matter as due to a ”dark force” (DF) generated by the reaction of DE to the presence of baryonic matter. These approaches typically proceed by making a connection with phenomenological Milgrom’s MOdified Newtonian Dynamics (MOND) in which a critical scale is promoted to a fundamental constant of nature (see however for a critical discussions about this topic). The additional acceleration component of MOND, \( a_{\text{MOND}} = \sqrt{\alpha_B a_B} \), where \( a_B \) is the Newtonian acceleration due to baryonic matter, is identified with that produced by the DF. In particular, from astrophysical observations we know that these DF effects arise when \( a_B \sim a_0 \) which corresponds to a critical scale \( r_0 \sim \sqrt{Gm_B/a_0} \).

The DF scenarios are powerful not only because they avoid to postulate the existence of dark matter and give a physical explanation of the Tully-Fisher and MOND relation, but also because they explain why \( a_0 \) is a fundamental constant of the same order of magnitude of the cosmological acceleration \( H \). On the other hand, they suffer from a serious drawback: when compared with observations, they are able to reproduce the asymptotic MOND formula but their predictions significantly differ from observations in the galactic region.

In order to fully reproduce the observational data about rotational curves of galaxies in the MOND framework, one needs to introduce a phenomenological interpolating function \( F(x) \), \( x = a_B/a_0 \), so that the total radial acceleration can be written as \( a^r = F(x)a_B \). For \( x \gg 1 \), near to the galactic core, the function \( F \) must reproduce standard Newtonian gravity, \( F(\infty) = 1 \), i.e. \( a^r = a_B \), where \( a_B = Gm_B/r^2 \). Instead, for \( x \ll 1 \) we have the MOND regime, \( F(x) \approx \sqrt{x} \) and the radial acceleration is \( a^r = a_{DF} = \sqrt{a_B a_0} = \sqrt{a_0 Gm_B/r^2} \). An interpolating function which satisfies all the conditions above and fits a large amount of observational data coming from galaxies with different shapes (spiral, elliptical, spherical) has been proposed by McGaugh et al., i.e. \( F(x) = \frac{1}{1-e^{x}} \).

Observations of galactic dynamics imply that the total radial acceleration can be split in two components, the Newtonian contribution \( a_B \) due to purely baryonic matter and an additional term to which we refer to as the DF contribution, \( a_{DF} \), i.e. \( a^r = a_B + a_{DF} \), with

\[
\frac{a_B}{a_{DF}} = \frac{1}{a_0} \frac{a_0}{e^{\frac{r_0}{a_0}} - 1}.
\]

This formula shows that the total acceleration only depends on the baryonic matter distribution and, in principle, no dark matter is needed to fit the data. In Eq. (1), \( a_0 \) appears as a fitting parameter and it is of the same order of magnitude of the cosmological acceleration. This allows us to write \( a_0 = \gamma H \), where \( \gamma \) is a dimensionless parameter of order one. The value of \( a_0 \) found by McGaugh et al. corresponds, approximatively, to \( \gamma = 1/2\pi \).

The purpose of this letter is to show that in a DF scenario, Eq. (1) simply follows from basic features of thermodynamics, statistical mechanics and general relativity
(GR) if one assumes that the DE medium responds in a simple and natural way to the presence of baryonic matter. We will also show that the same equation allows for a “metric-covariant uplifting” as it appears as the weak-field limit of Einstein’s general relativity sourced by an anisotropic fluid.

THE MODEL

We consider our universe as made only by DE and baryons. We do not know what DE really is, thus we just consider the simpler case in which DE is described by a cosmological constant (see the discussion below for details). In absence of baryonic matter our universe is described, consistently with GR, by a de Sitter (dS) spacetime with a cosmological horizon $L$. The cosmological acceleration $H$ is related to $L$ by $H = 1/L$. The cosmological horizon, hence the dS spacetime, has an associated Bekenstein-Hawking temperature $T = 1/(2\pi L)$ [31–33]. We consider baryonic matter in the form of a point particle of mass $m_B$ and its gravitational interaction with a test particle at distance $r$ from it in the weak field approximation. In a spherical region of radius $r$, this gravitational interaction will be given by the sum of the usual Newtonian component originated by the baryonic matter $m_B$ and of a DF component originated by the response of the DE to the presence of baryonic matter inside the sphere. DF effects arise when $r$ becomes comparable with the critical length scale $r \sim r_0$. Hence, the total radial acceleration experienced by a test particle is

$$a_r = a_B + a_{DF}.$$  

A possible quantum description of the dS universe is that of a Bose-Einstein condensate of some quantum bosonic gravitational microscopic degrees of freedom [35]. In this context the DF has to be considered as the macroscopic manifestation of bosonic excitations of the DE medium (henceforth called "DF bosonic excitations") with typical energy $\epsilon \sim 1/r$. For instance, this is the case of a corpuscular gravity scenario like that used in Ref. [16] where these bosonic excitations can be considered as spin-2 particles (dark gravitons). In a thermodynamical, quantum mechanical picture, the DF acceleration $a_{DF}$ can be thought as generated by the pressure $P$ of the gas of DF bosonic excitations in the sphere of radius $r$. Thus, we can write the acceleration for unit mass as $a_{DF} \sim P/V \sim PV/\epsilon$, where $V$ is the volume of the sphere. At galactic scales, $r \ll L$, the thermal contribution, $TS$, is negligible so that usual extensive thermodynamics implies $PV \sim N\epsilon$, where $N$ is the number of DF bosonic excitations in the sphere. This leads to

$$a_{DF}(\epsilon) = C\epsilon^2N,$$  

where $C$ is a constant with dimensions of a length, whose value will be determined shortly. The additional DF is attractive and has therefore the same sign of the Newtonian contribution. For simplicity, we only consider the absolute value of forces and accelerations. Notice that the scaling $a \sim \epsilon^2N$ found by Eq. (2) is the extensive counterpart of the sub-extensive behaviour $a \sim \epsilon^2N$ found for the Newtonian term $a_B$ in the radial acceleration [15, 16]. The extensive behaviour $V \sim N, a \sim N$ of our gas of DF bosonic excitations is perfectly consistent with its origin from the constant energy density characterizing the DE.

Assuming that the DF bosonic excitations are in thermal equilibrium with the dS spacetime and that their energy spectrum is non-degenerate, their number $N$ will follow a thermal Bose-Einstein distribution at a temperature $T = 1/(2\pi L)$ and with zero chemical potential: $N(\epsilon) = \frac{1}{\epsilon^2} \approx 1/(2\pi L).$ The MOND regime appears at scales where the number of soft DF bosonic excitations becomes large, $N > > 1$, i.e. when $r_0 \approx r < L$. In this limit, $2\pi L \to 0$ and, at leading order in $2\pi L\epsilon$, we have $N(\epsilon) = (2\pi L\epsilon)^{-1}$. In the same limit the universe becomes DE dominated, being the contribution of baryonic matter negligible. The behaviour of DF bosonic excitations should match that of the dS universe, i.e $\epsilon \sim \epsilon_{DE} = 1/L$ and $a_{DF}$ must become the cosmological acceleration $a_{DF} = H = 1/L$ [15, 16]. This requirement determines the constant $C$ in Eq. (2) to be $C = 2\pi L$. Putting all together we find

$$a_{DF}(\epsilon) = \frac{2\pi L\epsilon^2}{\epsilon^2 - 1}.  \eqno(3)$$

DETERMINATION OF $\epsilon$

DF bosonic excitations are soft excitations of the DE medium. The effect of introducing the baryonic matter will be the generation of a new effective interaction term between the baryonic mass $m_B$ and a test mass at distance $r$, which we have called DF. The energy $\epsilon$ of the bosonic excitations mediating this dark interaction must therefore deviate from the simple Compton form $\epsilon \sim 1/r$. Introducing a dimensionless coupling constant $\alpha$ characterizing this interaction we write $\epsilon \sim \alpha r$, where $\alpha$ can depend only on the quantities entering in the process, i.e. on $m_B, L, G$.

A direct determination of $\alpha$ requires the knowledge of the energy spectrum of the DF bosonic excitations. This in turns requires a detailed comprehension of the microphysics involved in the interaction between baryonic matter and DE. Unfortunately, this is out of reach because

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1. From now on we use natural units $c = \hbar = k_B = 1$, whereas $\ell_p, m_p = 1/\ell_p$ and $G = \ell_p^2$ are the Planck length, the Planck mass and the Newton constant, respectively.

2. Our results can be easily generalized to the case of a spherically symmetric mass distribution $m_B(r)$.
we do not have any clear understanding of the microscopic nature of DE. However, we can circumvent this problem by using, again, the macroscopic effect given in Eq. (2), i.e. the acceleration, produced by DF excitation. Although the thermodynamical origin of this equation would require \( N \gg 1 \), we can extrapolate its validity also for small values of \( N \). In particular for \( N = 1 \), Eq. (2) gives the DF acceleration \( a_{DF} \) produced by a single DF excitation. In view of the fact that \( a_{DF} \) is the reaction of DE to baryonic matter, the simplest and most natural, assumption is that the DF acceleration produced by a single excitation is given exactly by what has generated it, i.e. \( a_{DF} = a_B \). With this assumption Eq. (2) gives

\[
\varepsilon = \sqrt{\frac{Gm_B}{2\pi L}} \frac{1}{r}.
\]

(4)

Using this result for \( \varepsilon \), Eq. (3) matches with the phenomenological acceleration in Eq. (1), where the value of the dimensionless parameter, \( \gamma = 1/2\pi \), is perfectly compatible with the phenomenological value found by fitting a large amount of astrophysical data [39, 30].

Our model also predicts the correct value of the scale \( r_0 \) at which the dark force effects arise. This occurs when \( N(\varepsilon) \approx 1 \). From the Bose-Einstein distribution for \( N(\varepsilon) \) we find that the condition above is satisfied when DF modes have energy \( \varepsilon \approx 1/L \), which using Eq. (4) gives \( r \approx r_0 = \sqrt{Gm_B/a_0} \). In the Newtonian regime, i.e. for \( r \ll r_0 \), we have hard DF excitations with energy \( \varepsilon \gg 1/L \) whose number is exponentially suppressed and the DF effects are switched off. Finally, for \( r_0 \ll r < L \), corresponding to the MOND regime, we have a huge number of extremely soft DF excitations, \( N \gg 1 \), with energy \( \varepsilon \ll 1/L \) and the DF effects are dominant.

**EFFECTIVE FLUID DESCRIPTION**

The relation in Eq. (1) can be also obtained as the weak field limit of a metric theory of gravity, namely GR sourced by an anisotropic fluid. This can be done along the lines of Ref. [15], where an effective fluid description is used and the radial pressure of the fluid describes the radial acceleration produced by the DF, \( a' = a_B + 4\pi Grp_0 \). The pressure profile has to be chosen to match with Eq. (1):

\[
p_{\parallel}(r) = \frac{1}{4\pi} \frac{m_B}{r^3} \frac{1}{\varepsilon^2 \sqrt{Gm_B/a_0} - 1}.
\]

(5)

The full metric solution can be obtained solving Einstein field equations sourced by an anisotropic fluid with a pressure profile given by Eq. (5). The spacetime metric is taken of the form \( ds^2 = -f(r)e^\gamma(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2 \) and it is given by

\[
f = 1 - \frac{2Gm_B}{r}, \quad \gamma = \frac{2G}{r^2} \left( \frac{m_B}{r} \varepsilon^2 \sqrt{Gm_B/a_0} - 1 \right) \cdot
\]

(6)

In the weak field limit of the metric one finds the potential \( \phi = \frac{1}{2}(f e^\gamma) \), from which one can easily derive the form of the DF acceleration and check that it exactly reproduces Eq. (1). In the MOND regime of Eq. (1), i.e. \( a_B/a_0 \to 0 \), one obtains the same expansion of the gravitational potential generated by a point-like particle given in Ref. [15] with the characteristic logarithmic behaviour of MOND and an extremely tiny Machian contribution to the Newtonian potential (see Ref. [15]).

**CONCLUSIONS**

Starting from a DF scenario, using a simple and natural assumption for the reaction of DE to the presence of baryonic matter and basic features of thermodynamics and statistical mechanics, we have derived the phenomenological acceleration profile of stars in galaxies proposed by McGaugh et al. in [28, 39]. Our formula reproduces both the Newtonian and MOND regimes of gravity as observed from the rotational curves data of a large variety of galaxies in the universe. Moreover, it can be also embedded in GR.

The bosonic excitations responsible for the DF effects are soft modes with wavelength of the order of the cosmological horizon, whose number becomes significant only at galactic scales. For this reason, they only affect the gravitational interaction at galactic scales, leaving unaltered the usual Newtonian contribution at smaller scales. Our model also predict a value of the dimensionless acceleration parameter, \( \gamma = 1/(2\pi) \), which is in accordance with the phenomenological results in [9, 30] obtained by fitting a large amount of observational data. The same value has been obtained in alternative derivations linking \( \gamma \) to the temperature of the dS spacetime [36, 37]. Conversely, emergent gravity scenarios, which use area/volume competition effects, predict a slightly different value, \( \gamma = 1/6 \) [14].

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\[ 1 \] A. A. Penzias and R. W. Wilson, “A Measurement of excess antenna temperature at 4080-Mc/s,” *Astrophys. J.* 142 (1965) 419–421.

\[ 2 \] Planck Collaboration, P. A. R. Ade et al., “Planck 2013 results. XVI. Cosmological parameters,” *Astron. Astrophys.* 571 (2014) A16 arXiv:1303.5076 [astro-ph.CO].

\[ 3 \] Supernova Search Team Collaboration, A. G. Riess et al., “Observational evidence from supernovae for an accelerating universe and a cosmological constant,” *Astron. J.* 116 (1998) 1009–1038 arXiv:astro-ph/9805201 [astro-ph].

\[ 4 \] V. C. Rubin, N. Thonnard, and W. K. Ford, Jr., “Rotational properties of 21 SC galaxies with a large range of luminosities and radii, from NGC 4065 /R = 4kpc/ to UGC 2885 /R = 122 kpc/,” *Astrophys. J.* 238 (1980) 471.

\[ 5 \] M. Persic, P. Salucci, and F. Stel, “The Universal rotation curve of spiral galaxies: 1. The Dark matter connection,” *Mon. Not. Roy. Astron. Soc.* 281 (1996) 27, arXiv:astro-ph/9506004 [astro-ph].

\[ 6 \] R. Massey, T. Kitching, and J. Richard, “The dark matter of gravitational lensing,” *Rept. Prog. Phys.* 73 (2010) 086901 arXiv:1001.1739 [astro-ph.CO].

\[ 7 \] R. B. Tully and J. R. Fisher, “A New method of determining distances to galaxies,” *Astron. Astrophys.* 54 (1977) 661–673.

\[ 8 \] S. S. McGaugh, J. M. Schombert, G. D. Bothun, and W. J. G. de Blok, “The Baryonic Tully-Fisher relation,” *Astrophys. J.* 533 (2000) L99–L102, arXiv:astro-ph/0003001 [astro-ph].

\[ 9 \] S. McGaugh, F. Lelli, and J. Schombert, “Radial Acceleration Relation in Rotationally Supported Galaxies,” *Phys. Rev. Lett.* 117 no. 20, (2016) 201101 arXiv:1609.05917 [astro-ph.GA].

\[ 10 \] A. A. Klypin, A. V. Kravtsov, O. Valenzuela, and F. Prada, “Where are the missing Galactic satellites?,” *Astrophys. J.* 522 (1999) 82–92, arXiv:astro-ph/9901240 [astro-ph].

\[ 11 \] B. Moore, S. Ghigna, F. Governato, G. Lake, T. R. Quinn, and P. Tozzi, “Dark matter substructure within galactic halos,” *Astrophys. J.* 524 (1999) L19–L22, arXiv:astro-ph/9907411 [astro-ph].

\[ 12 \] M. Boylan-Kolchin, J. S. Bullock, and M. Kaplinghat, “Too big to fail? The puzzling darkness of massive Milky Way subhaloes,” *Mon. Not. Roy. Astron. Soc.* 415 (2011) L40 arXiv:1103.0007 [astro-ph.CO].

\[ 13 \] M. Boylan-Kolchin, J. S. Bullock, and M. Kaplinghat, “The Milky Way's bright satellites as an apparent failure of LCDM,” *Mon. Not. Roy. Astron. Soc.* 422 (2012) 1203–1218 arXiv:1111.2048 [astro-ph.CO].

\[ 14 \] E. P. Verlinde, “Emergent Gravity and the Dark Universe,” *SciPost Phys.* 2 no. 3, (2017) 016 arXiv:1611.02269 [hep-th].

\[ 15 \] M. Cadoni, R. Casadio, A. Giusti, W. Mueck, and M. Tuveri, “Effective Fluid Description of the Dark Universe,” *Phys. Lett.* B776 (2018) 242–248 arXiv:1707.09945 [gr-qc].

\[ 16 \] M. Cadoni, R. Casadio, A. Giusti, and M. Tuveri, “Emergence of a Dark Force in Coreguscular Gravity,” *Phys. Rev.* D97 no. 4, (2018) 044047 arXiv:1801.10374 [gr-qc].

\[ 17 \] S. Hossenfelder, “Covariant version of Verlinde’s emergent gravity,” *Phys. Rev.* D95 no. 12, (2017) 124018 arXiv:1703.01415 [gr-qc].

\[ 18 \] D.-C. Dai and D. Stojkovic, “Comment on ‘Covariant version of Verlinde’s emergent gravity’,” *Phys. Rev.* D96 no. 10, (2017) 104001 arXiv:1706.07854 [gr-qc].

\[ 19 \] R.-G. Cai, S. Sun, and Y.-L. Zhang, “Emergent Dark Matter in Late Universe on Holographic Screen,” arXiv:1712.09326 [hep-th].

\[ 20 \] M. Milgrom, “A Modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis,” *Astrophys. J.* 270 (1983) 365–370.

\[ 21 \] M. Milgrom, “MOND theory,” *Can. J. Phys.* 93 no. 2, (2015) 107–118 arXiv:1404.7661 [astro-ph.CO].

\[ 22 \] D. C. Rodrigues, V. Marra, A. del Popolo, and Z. Davari, “Absence of a fundamental acceleration scale in galaxies,” *Nat. Astron.* 2 no. 8, (2018) 668–672 arXiv:1806.06803 [astro-ph.GA].

\[ 23 \] Z. Chang and Y. Zhou, “Is there a fundamental acceleration scale in galaxies?,” *arXiv* e-prints (Dec., 2018) arXiv:1812.05002 [astro-ph.GA].

\[ 24 \] M. Milgrom and R. H. Sanders, “Perspective on MOND emergence from Verlinde’s ’emergent gravity’ and its recent test by weak lensing,” arXiv:1612.09582 [astro-ph.GA].

\[ 25 \] F. Lelli, S. S. McGaugh, and J. M. Schombert, “Testing Verlinde’s Emergent Gravity with the Radial Acceleration Relation,” *Mon. Not. Roy. Astron. Soc.* 408 no. 1, (2010) L68–L71, arXiv:1702.04355 [astro-ph.GA].

\[ 26 \] K. Pardo, “Testing Emergent Gravity with Isolated Dwarf Galaxies,” arXiv:1706.00785 [astro-ph.CO].

\[ 27 \] A. Hees, B. Famaey, and G. Bertone, “Emergent gravity in galaxies and in the Solar System,” *Phys. Rev.* D95 no. 6, (2017) 064019 arXiv:1702.04358 [astro-ph.GA].

\[ 28 \] S. McGaugh, “Milky Way Mass Models and MOND,” *Astrophys. J.* 683 (2008) 137–148 arXiv:0804.1314 [astro-ph].

\[ 29 \] R. H. Sanders, “A historical perspective on modified Newtonian dynamics,” *Can. J. Phys.* 93 no. 2, (2015) 126–138 arXiv:1404.0531 [physics.hist-ph].

\[ 30 \] F. Lelli, S. S. McGaugh, J. M. Schombert, and M. S. Pawloski, “One Law to Rule Them All: The Radial Acceleration Relation of Galaxies,” *Astrophys. J.* 836 no. 2, (2017) 152 arXiv:1610.08981 [astro-ph.GA].

\[ 31 \] H. Narmhofer, I. Peter, and W. E. Thirring, “How hot is the de Sitter space?”, *Int. J. Mod. Phys.* B10 (1996) 1507–1520 [603(1996)].

\[ 32 \] S. Deser and O. Levin, “Accelerated detectors and temperature in (anti)-de Sitter spaces,” *Class. Quant. Grav.* 14 (1997) L163–L168 arXiv:gr-qc/9706018 [gr-qc].

\[ 33 \] T. Jacobson, “Comment on ‘Accelerated detectors and temperature in anti-de Sitter spaces’,” *Class. Quant. Grav.* 15 (1998) 251–253 arXiv:gr-qc/9709048 [gr-qc].

\[ 34 \] W. Mueck, “On the number of soft quanta in classical field configurations,” *Can. J. Phys.* 92 no. 9, (2014) 973–975 arXiv:1306.6245 [hep-th].

\[ 35 \] M. Cadoni and M. Tuveri, “In preparation.”

\[ 36 \] L. Smolin, “MOND as a regime of quantum gravity,” *Phys. Rev.* D96 no. 8, (2017) 083523 arXiv:1704.00780 [gr-qc].

\[ 37 \] S. Alexander and L. Smolin, “The Equivalence Principle and the Emergence of Flat Rotation Curves,”
