Asymmetry and dark energy evolution

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Abstract. The well known asymmetry of type Ia supernova data is used to infer general features of an alternative model for dark energy. Using a model that allows DE to vary with redshift as a probe, the results indicate the low redshift transition of the deceleration parameter $q(z)$ is evident only taken data points from one hemisphere of the sky. A related effect is a reconstructed DE density $X(z)$ that shows evolution (it decreases as redshift increases) only towards the hemisphere where $q(z)$ manifest the transition. We discuss on possible theoretical frameworks where these effects can be explained.

1. Introduction

So far the ΛCDM has been a very successful model in confronting the data, not only using type Ia supernova (SNIa), from which it was proposed for the first time as a model [1],[2], but also using a myriad of different observational probes, both geometric and dynamical. In this setup Λ drives the cosmic acceleration. However, although successful, the model forces us to accept uncomfortable facts: we do not know the origin of Λ, and with this we do not know why it has the value we measure, and also, we must accept that we live in a very unique time, just that time where the contribution of Λ and matter have the same order of magnitude.

Among the models that have been proposed as alternatives to the ΛCDM we can mention the dark energy (DE) [3], a fluid that fills all the space and has negative pressure. The source of this DE can be a scalar field as in quintessential models [4]. Another group of alternative models explore a modification of the Einstein - Hilbert action to explain what we observe [5].

Also, there are models that relax the homogeneity assumption of the standard cosmological model, by assuming that we live near the center of a void. Using this assumption it is possible to fit the SNIa data without invoking DE [7]. Although it is a seductive idea, the initial program testing this against the observations seems to indicate that we need a void of a size never observed (gigaparsec in size) [8]. Research in these areas continues [9].

In the context of the homogeneous and isotropic scheme, direct explorations of possible departures from the ΛCDM model have been focused mainly through parameterizing specific physical quantities. We can parametrize the equation of state (EoS) parameter $w = p/\rho$ (where $p$ is the pressure and $\rho$ the energy density) as a function of redshift [10, 11, 12, 13], or we can parametrize the DE density itself [14].

Using the Chevallier-Polarski-Linder (CPL) parametrization [47], [46] $w(z) = w_0 + w_1 z/(1 + z)$, the authors of [10] found that the reconstructed deceleration parameter $q(z)$ shows a low-redshift transition leading to the cosmic slow down of acceleration scenario. Later, this
phenomena was found using new SNIa data [11, 12], also using data from gas mass fraction in galaxy clusters [13], and using different parameterizations [15], showing that this effect – the low redshift transition of the reconstructed $q(z)$ – is telling us something about our local universe, because the evidence for such transition disappears once data from BAO and CMB (high z physics) are added.

A similar tension between low-redshift with high-redshift data manifest in measurements of the Hubble constant $H_0$ [18]. This tension emerges using the ΛCDM model as the “base” model. Although simple extensions of the ΛCDM model were studied, not all of them alleviated the tension. In [19] the authors showed that the assumption of a large local under-density on radial scales of a few Mpc it is enough to alleviate such a tension.

Regarding the low redshift transition, it was noticed that such a transition is a expected output if we are positioned near the center of a void [16]. Actually, it was claimed that such a feature have to be considered a signal of the existence of a void (underdensity) in our neighborhood [17]. Then the fact that both, the tension between low and high redshift data and the inhomogeneous connection for this feature of the reconstructed $q(z)$, make sense only if we assume a local underdensity, means that something in the local distribution of matter is affecting the data, something that we would like to characterize.

In this context, it is interesting to cite the case of a preferred direction of expansion, as for example, the large scale bulk-flow [20, 22, 21, 24, 23], the low multipoles alignment in the CMB angular power spectrum [25], the large scale alignment of quasar polarization [26], and studies using SNIa [27, 32, 33, 28, 34, 35, 30, 31, 29]. So the evidence points towards the necessity to a study further the effects on the cosmological parameters of a inhomogeneous local distribution of matter. In [38] using two different SNIa samples, it was shown that this low-redshift transition of $q(z)$ is evident only towards one hemisphere and is absent in the opposite one. If we interpret the low-redshift feature as a indication of a local void, this last results implies that such a void is only apparent toward one hemisphere, and so not only implying a local inhomogeneous distribution of matter but also having a dipolar anisotropy.

In this work we perform a study of our local universe, using two samples of SNIa, and using parametrizations for the EoS parameter $w(z)$ and also for the DE density $X(z)$. We also discuss the first steps towards to enhance the cosmological model to take into account these local effects.

2. The method

We begin recalling that the co-moving radial coordinate is

$$r(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')}, \tag{1}$$

in a flat universe, where $E(z) = H(z)/H_0$ is the reduced Hubble function. Assuming an EoS parameterized by [47, 46]

$$w(z) = w_0 + \frac{w_1 z}{1 + z}, \tag{2}$$

the reduced Hubble function takes the form

$$E^2(z) = \Omega_m (1 + z)^3 + (1 - \Omega_m) X(z), \tag{3}$$

where $\Omega_m$ comprises both the baryonic and non baryonic DM, and the reconstructed DE density is

$$X(z) = e^{-\frac{3w_1 z}{1 + z}} (1 + z)^{3(1 + w_0 + w_1)}. \tag{4}$$
From this, following previous works \[11, 12\], we can reconstruct the deceleration parameter function

\[
q(z) = (1 + z) \frac{1}{E(z)} \frac{dE(z)}{dz} - 1. \tag{5}
\]

As was shown in \[14\], the low redshift transition of \(q(z)\) at \(2\sigma\) C.L is observed in both the more recent SNIa data and gas mass fraction data from galaxy clusters. The idea here is to study how this picture changes once the position of the SNIa data is taken into account, and restudy both the effects on \(q(z)\) and \(X(z)\).

3. Using the Union 2 sample

The Union 2 dataset \[40\] consist of 557 SNIa in the range \(0.015 < z < 1.4\). Union 2 join the Union dataset \[43\], six SNIa at high redshift found by the Hubble Space Telescope, and low and intermediate redshift data from \[44\] and \[45\], respectively.

In this section we use the result of \[27\] using the hemispherical asymmetry technique, were they found a dipolar asymmetry with a maximum acceleration towards \((l, b) = (309^\circ, 18^\circ)\) and a minimum acceleration towards \((l, b) = (129^\circ, -18^\circ)\), in galactic coordinates.

Given these points, we separate the data into two hemispheres each one with center in the points of maximal (minimal) acceleration. In the hemisphere corresponding to the maximum acceleration we get 116 supernovae and in the other the remaining 441. This huge asymmetry in the number is essentially due to the fact that in the minimum acceleration hemisphere we found the SDSS stripe of SNIa data. The best fit values in each hemisphere are displayed in Table 1.

| \(\chi^2_{\text{min}}/\text{dof}\) | \(\Omega_m\) | \(\omega_0\) | \(\omega_1\) |
|-----------------|----------|----------|----------|
| A 103.65/113   | 0.21 ± 0.64 | -1.3 ± 1.2 | 1.5 ± 3.4 |
| B 427.57/438   | 0.46 ± 0.06 | -0.41 ± 0.53 | -9.2 ± 7.3 |

Table 1. Best fit values of the cosmological parameters for the two hemispheres. Row A for the 116 SNIa in the maximum acceleration hemisphere. Row B the 441 SNIa in the opposite direction. See also Fig.(1).

From inspection of Figure 1 we notice that the transition feature at low redshift is only evident in the direction of the minimum acceleration, meanwhile in the opposite direction the best fit is in agreement with ΛCDM. Following standard procedure (see \[38\] for details) we can estimate the confidence for each set of best fit parameters. In the case of the 116 SNIa in the maximum acceleration hemisphere we get a distance of 0.31\(\sigma\) away from the ΛCDM point. In the other hemisphere – in which we observe the low redshift transition of the reconstructed \(q(z)\) – we get a distance of 1.76\(\sigma\) away from the ΛCDM best fit.

Further, the reconstructed DE density Eq.(4) for the Union 2 sample in hemispheres is display in Fig. 2. As we can see, the reconstructed DE density towards the hemisphere of maximum acceleration does not show any deviation with respect to a cosmological constant, however towards the minimum acceleration zone, this function decrease as a function of the redshift, which is what we expect if we have a void in that direction.

Taking the result of \[16\] as our main hypothesis – that a low redshift transition of \(q(z)\) can be considered a signal for a local under-density – the result we have found indicates the data (Union 2 set) is able to detect such an under-density only in one (hemisphere) direction of the sky. This implies that the assumption of a spherically symmetric model for such under-density is
Figure 1. Here we show the reconstructed deceleration parameter \( q(z) \) using data from two hemispheres. The upper panel showing the one around the maximum acceleration point, and the bottom the one in the opposite direction. See Table 1 for the best fit values for each case.

not a good idea (as is usual working with LTB models). Moreover, this result probably suggest the use of a metric with a dipolar asymmetry.

The low redshift transition apparent in Figure 1 and the decrease in \( X(z) \) in Figure 2, disappear once data from baryon acoustic oscillation (BAO) and cosmic microwave background radiation (CMBR) are taken into account.

It is also worth notice the effect of the number of points used in each hemisphere. The set pointing towards the maximum acceleration has only 116 SNIa. Although the best fit is only 0.3\( \sigma \) away from the \( \Lambda \)CDM values, indicating these data points are in agreement with the concordance model, the sparsity of data points increases the errors considerably. The actual redshift distribution is also worth mentioned in this context. In this hemisphere most of the data, around 65 points, have \( z < 0.2 \), with no data in the range \( 0.2 < z < 0.3 \), with 45 points distributed around redshift \( z \simeq 0.5 \), and almost one point for every redshift bin (\( \Delta z = 0.1 \)) for \( z > 0.8 \). The other hemisphere has a greater number of data points (441) with a redshift distribution that, although decreasing with redshift, it still maintain a large number of data points per redshift bin (\( \Delta z = 0.1 \)) until \( z \simeq 1.4 \).
Figure 2. Here we show the reconstructed DE density $X(z)$ using data from two hemispheres. The upper panel showing the one around the maximum acceleration point, and the bottom the one in the opposite direction. See Table 1 for the best fit values for each case.

4. Analysis using the Loss sample
We have also performed the study of the consequences, in the reconstructed deceleration parameter, of the existence of an axis of maximal asymmetry using the Loss data set [42]. In order to find this axis, we follow the procedure of [27], using a flat ΛCDM as a base model, and start a scan of random axes looking for the maximum variation in the best fit of $\Omega_m$.

Using the Loss sample we found that the direction of maximal acceleration points towards $(l,b) = (309^\circ, 31^\circ)$.

In the hemisphere corresponding to the maximum acceleration, there are 216 supernovae and in the other the remaining 370. Again we observe the asymmetry in the distribution, although less severe than in the case of the Union 2 sample. In fact, using the Union 2 set we have verified the value $(\Delta \Omega_m)_{\text{max}}/\Omega_m = 0.43 \pm 0.06$ from [27]. Using the same procedure with the Loss sample we get $(\Delta \Omega_m)_{\text{max}}/\Omega_m = 0.30 \pm 0.06$.

The best fit values in each hemisphere are displayed in Table 2:

In agreement with the previous analysis using the Union 2 data set, the result using the hemisphere towards the maximum acceleration is consistent with ΛCDM, showing no more than a 0.1σ departure from it (see Fig. 3).

Our results are also in agreement with the previous case, the analysis with data from the hemisphere of minimum acceleration shows a sharp low redshift transition. In this case the best
Table 2. Best fit values of the cosmological parameters using the LOSS sample separating the sky in two hemispheres. Row A for the 216 SNIa in the maximum acceleration hemisphere. Row B the 370 SNIa in the opposite direction. See also Fig.3.

|          | $\chi^2_{\text{min}}$/dof | $\Omega_m$ | $\omega_0$ | $\omega_1$ |
|----------|-----------------------------|------------|-------------|-------------|
| A        | 197.28/213                  | 0.2 ± 1.1  | −0.9 ± 0.9  | 0.2 ± 5.6   |
| B        | 374.76/367                  | 0.39 ± 0.07| −0.47 ± 0.43| −7.6 ± 6.2  |

**Figure 3.** Reconstructed deceleration parameter $q(z)$ using data from two hemispheres using the Loss sample. The panels show the one around the maximum (upper) and minimum (lower) acceleration points respectively. The upper panel show the one around the maximum acceleration point, and the bottom one that around the minimum acceleration point. See Table 2 for the best fit values for each case.

The fit is around 1.5σ away from the ΛCDM value.

It would be interesting to study this effect in the newest data set, as the JLA set [48] comprising more than 740 data points, and also using different parameterizations, as we have performed in [15]. These issues are under study.

There remains to check if this asymmetry in the behavior of the reconstructed deceleration parameter is not due to the anisotropic distribution of data. This issue as been discussed in the past. As we have mentioned before, using the Union 2 dataset, the authors of [27] found
Figure 4. Here we show the reconstructed DE density $X(z)$ using data from two hemispheres. The upper panel showing the one around the maximum acceleration point, and the bottom the one in the opposite direction. See Table 2 for the best fit values for each case.

a preferred axis in the data, but they also checked if such anisotropy can be obtained using simulated isotropic data. Their method suggest that only a 30% of the simulated data can reach such anisotropy, implying that the anisotropy of the Union 2 set is consistent with statistical isotropy. However, the coincidence of the axes (a total of six phenomenological axes identified in [27]) within a small angular region in the sky, makes this dipolar anisotropy a feature that needs further scrutiny. Using the Loss sample, the authors of [31] found a hint for anisotropy – most pronounced in the range $0.015 < z < 0.045$ – but once they take into account large scale velocity perturbations, the results shows no evidence for any anomalous deviation from the isotropic ΛCDM. In [29] the authors studied this issue using the Union 2.1 and JLA set. Although they found that both datasets are statistically anisotropic, the correlation with the anisotropic distribution found turns out to be different: using the Union 2.1 they found a small correlation but using the JLA sample they found a stronger correlation. A similar study [37] using the JLA dataset also showed that by ignoring the velocity covariance may produce a hint of anisotropy from the data. In this context, it is interesting to notice here the result of the work [49] where the authors studied the JLA set, considering all the information from possible systematics (encoded in the covariance) in the analysis, finding a marginal (less than $3\sigma$) evidence for the accepted cosmic acceleration.
5. Theoretical ideas
We have already mentioned a possible theoretical framework where this behavior of \( q(z) \) and \( X(z) \) can be observed. A model with a void towards one hemisphere. In fact, notice that Figures 2 and 4 shows that \( X(z) \) decreases with \( z \) towards one hemisphere and into the other, the DE density is consistent with a constant \( \lambda \), as in the \( \Lambda \text{CDM} \) model. Actually, given the errors, this last solution is also consistent with zero cosmological constant for most of the range of the SNIa data.

Another way to get a decreasing DE density with redshift is considering an explicit interaction term between DE and DM. In the simplest case we have

\[
\dot{\rho}_m + 3H \rho_m = Q, \tag{6}
\]
\[
\dot{\rho}_x + 3H(1 + w_x)\rho_x = -Q, \tag{7}
\]

where \( Q \) is the interaction and let us assume that \( Q = 3H\alpha \rho_x \), as is usual in this kind of models. It is easy to find that \( \rho_x(z) = \rho_{x0}(1 + z)^{3(1 + w_x + \alpha)} \) and

\[
\rho_m(z) = \rho_{m0}(1 + z)^3 + \frac{\alpha}{\alpha + w_x} \rho_{x0} \left[ 1 - (1 + z)^{3(w_x + \alpha)} \right] (1 + z)^3, \tag{8}
\]

so finally the Hubble function can be written as

\[
H^2 = H_0^2 \left[ \Omega_m(1 + z)^3 + \Omega_x \frac{(1 + z)^3}{\alpha + w_x} \left( \alpha + w_x(1 + z)^{3(w_x + \alpha)} \right) \right]. \tag{9}
\]

If someone give us this Hubble function, we could interpreted as the DE density the second term inside the square brackets,

\[
X(z) = \frac{(1 + z)^3}{\alpha + w_x} \left( \alpha + w_x(1 + z)^{3(w_x + \alpha)} \right). \tag{10}
\]

As the redshift increase, this function can decrease (even getting negative values after some redshift point) just by requiring that for example, \( w_x = -0.9 \) and \( \alpha = 0.01 \), which are typical values obtained from observations. In general, given a positive \( \alpha \) (which ensure energy transfer from DE to DM) this behavior is found, sooner or later, for negative values of \( w_x \), with a transition redshift \( z_c = -1 + (\alpha/|w_x|)(1/(\alpha - |w_x|)) \). However, it is not possible to get a deceleration parameter \( q(z) \) – obtained from Eq.(5) using \( H(z) \) from Eq. (9) – that shows the low redshift transition. This is very interesting, because this means that not every \( X(z) \) that decrease with \( z \) implies a low redshift transition in their related \( q(z) \).

6. Discussion
In this paper we have study the effects in the reconstructed deceleration parameter and dark energy density using the well known dipolar asymmetry present in SNIa data. We have used the Union 2 set and the result of [27], a dipolar asymmetry with a maximum acceleration towards \((l,b) = (309^\circ, 18^\circ)\) and a minimum acceleration towards \((l,b) = (129^\circ, -18^\circ)\) is found. The low redshift transition, previously recognized in [10, 11, 12, 13, 14, 15], appears only towards the direction of minimum acceleration, and it is absent towards the opposite direction. This result was obtained separating the data in two hemispheres constructed along the maximal asymmetry axis.

We have also performed a similar analysis using the Loss dataset [42] finding the results displayed in Fig. 3, which are in agreement with those using the Union 2 dataset (see Fig. 1). Assuming the result of [16] as our main hypothesis – i.e., a low redshift transition of \( q(z) \) can be
considered as a signal of a local under-density – our work indicates that the supernova data is able to detect such an under-density only towards one direction in the sky. This result suggests that a better description of the data would be a background inhomogeneous (because the dark energy density varies with $z$) and also with at least a dipolar asymmetry.

As we discussed in previous section, there remains to check if this asymmetry in the behavior of the reconstructed deceleration parameter is not due to the anisotropic distribution of data. However, it seems difficult to explain why all the evidence points towards the CMB dipole axes.

Altogether, the apparent detection of a local under density can be considered as a possible cause of the original tension between low and high redshift observational constraints discussed in [10, 11, 12, 13, 15], which as a solution also agree with the analysis of [19] where they proposed the existence of a local under-density to alleviate the tension in the determination of $H_0$.

Further study on this subject would be interesting. For example, the effects of using different parameterizations, and also the use of different SNIa data sets. Given that the SNIa data used in this work is obtained after the calibration of the whole set (by fitting globally also the $\alpha$, $\beta$ and $M$ parameters, being the last one connected with the value of $H_0$) it would be interesting to study the impact of the hemispherical asymmetry on such a calibration, and then on the cosmological parameters. All these issues are under study.

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