Developing physical and mathematical models of the water-air mode of gravity drainage network

V Orlov and S Zotkin
Moscow State University of Civil Engineering (National Research University),
129337 Yaroslavskoe shosse 26, Moscow, Russia

E-mail address: zotkinsp@mgsu.ru

Abstract. A physical and mathematical model of the water-air mode of free-flow drainage pipelines has been developed. The solution to the problem of physical modelling of pipeline network functioning is based on the study of the dynamics of changes in the hydraulic and aerodynamic parameters of the system, taking into account the phenomena of heat and mass transfer between liquid and air in the vaulted space of a free-flow pipeline. An algorithm for solving the problem is compiled on the basis of the criteria of hydrodynamic similarity, describing the phenomena of mass transfer as a result of forced air convection by using a mechanical ventilation system above the water surface. Based on the results of applying a special computer-aided calculation program, indicators of the required air exchange rate for effective removal of evaporation (steam) from the vaulted space of a pipeline were determined. As a result of solving the complex problem of studying the water-air mode of free-flow pipeline functioning, the authors compared and analysed the calculated values of the intensity of air exchange produced to remove steam, and to reduce harmful foul-smelling gaseous substances to reach the values of maximum permissible concentrations. The research results are aimed at preventing emissions of gases harmful to human health into the atmospheric air by constructive methods.

1. Introduction
An objective assessment of free-flow drainage system pipelines functioning is impossible without figuring out its physical model, including hydrodynamic and aerodynamic indicators [1]. Physical modeling is the initial stage of solving a complex problem associated with a comprehensive assessment of the water-air mode of pipelines operation [2, 3]. The model is to describe such complex phenomena as intensive release of foul-smelling toxic gases into the vaulted space of free-flow networks, and also make a detailed analysis of heat and mass transfer between liquid and air in the free air space of the pipeline [4]. A specific feature of the sewerage network is that chemical, biochemical and other processes actively take place in pipelines that affect the environmental component during transport and wastewater treatment [5, 6]. It is very difficult to fully cover these phenomena when compiling a model, therefore, when solving problems to study the water-air mode of free-flow pipelines functioning, certain assumptions were made, such as, for example, the condition of equilibrium at the "air-water" phase interface [7, 8].

A correctly built physical model is the basis for a mathematical model and becomes one when describing processes and phenomena using algebraic dependencies [9]. The mathematical model provides a multilateral quantitative analysis by changing the initial data, criteria and restrictions, reflecting the dynamics of changes in real physical processes [10]. Thus, the algorithm for solving
mathematical modeling problems is reduced to a consecutive description of the transformation of the corresponding technological indicators used in the physical model. The general requirement for the developed models, as a rule, boils down to making them available to specialists and unambiguously interpreted by them when solving a specific problem of assessing the sanitary safety of a designed or operated drainage network [11].

2. Materials and methods
The object (material) of the study is free-flow drainage pipelines, and the research method is an automated search for the optimal mode of their operation. The ultimate goal of research is to determine technical measures and indicators that provide the required sanitary and hygienic conditions for humans and the surrounding urban environment by neutralizing (removing) harmful foul-smelling gases and water vapour from the vaulted space of free-flow pipelines [12, 13].

The figure 1 shows a simplified physical model of a drainage network with a limited design area, for example, located near crowded places (shopping centres, entertainment and sports complexes, etc.), where the most significant issues are the localization of gas formation, caused by the presence of foul-smelling substances harmful to human health.

![Figure 1](image)

**Figure 1.** General view of the model illustrating the water-air mode of the drainage network operation: 1- free-flow pipeline, laid with a suitable slope; 2- initial sewer well; 3- autonomous ventilation unit with a diffuser oriented downstream; 4- final sewer well; 5- ventilation riser.

To solve the problem of physical modeling of the water-air mode of a free-flow pipeline network functioning, apart from studying the hydraulic and aerodynamic performance of the system, it is necessary to take into account the phenomena of mass transfer (gas-liquid reactions). It is very difficult to fully cover these phenomena when making up a model, therefore, when solving the problem at this stage of research, certain assumptions and simplifications were made, such as: the system is in equilibrium at the phase boundary (air-water). By this, it can be understood that the gas released from the surface of the water does not react with the transported waste water (there is no diffusion of gases, their adsorption and chemical reactions). But at the same time, the amount of air exchange should be determined to remove foul odors, as well as excess moisture entering the vaulted space of the pipeline as a result of wastewater evaporation. The elimination of these negative phenomena is one of the required tasks in the field of wastewater transport through free-flow
networks, since foul-smelling gases coming through the hatches of inspection wells reduce the people’s life comfort and do damage to their health, and moisture in the vaulted pipelines, together with gas emissions, contributes to corrosion of pipe walls [14].

Input data for the formation of physical and mathematical models is based on such indicators as the diameter of the pipeline, its length, slope and filling. When coming up with models, the results of field measurements are presented, for example, the concentration of harmful gaseous substances in the vaulted space of the pipeline and the coefficient of aerodynamic (air) resistance. The input data also includes normative indicators, for example, the maximum permissible concentration of harmful gaseous substances in the atmosphere of the city. In addition, such input characteristics of atmospheric air as its temperature, kinematic viscosity coefficient and density are subject to processing.

The following target design indicators of the water-air regime must be analyzed and on that basis specialists make a decision on the efficiency of the drainage network: the calculated values of air exchange (I) and depression (II) of the air flow in the pipeline, the values of hydraulic and standard indicators (III). A specially developed computer program facilitated an operational analysis of the three presented design indicators [15].

I. As a basic indicator, assessed by a physical and then a mathematical model, the rate of air exchange in the free air space of a free-flow pipeline is regarded. When solving this problem at this stage of modeling, it is assumed that aerodynamic parameters are determined in conditions of still water in the pipeline with a wide range of fillings, limited by the relative hydraulic stability. In this case, the air exchange value is assigned based on the catalog characteristics of the fan installations used in industry. Ultimately, the value of the assigned air exchange is to be adjusted according to the specific results of calculating the required amount of air to remove foul-smelling gases and excess moisture, taking into account heat and mass shift between liquid and air in the sewer pipe, as well as the real time of neutralization of gaseous substances in the vaulted space.

The amount of air exchange $Q_B$ ($m^3/s$) in a closed space to remove harmful foul-smelling gaseous impurities is determined by the formula (1)

$$Q_B = \frac{M}{C_{pdk} - C_0}$$  

where $M$ is the amount of foul-smelling gaseous substances released from water, mg/s; $C_{pdk}$ stands for the maximum permissible concentration of the gas to be neutralized, (mg/m$^3$); $C_0$ corresponds to the concentration of the gas to be neutralized in the supply air, (mg/m$^3$).

To determine the values of air exchange in the vaulted space to remove excess moisture $Q_{BB}$ ($m^3/s$) and the mass of water evaporating into the air $WW$ (mg/s), it is necessary to simulate the process of heat and mass transfer between air and water in the vaulted part of the pipeline. In this case, it is necessary to start out from the fact that heat and mass transfer can be carried out in two ways, determined by the specific conditions of heat transfer, i.e. natural or forced convection.

Due to the fact that the developed model considers heat transfer as a result of forced convection (based on intensive ventilation), the main hydrodynamic criteria of the model are the Reynolds’s criterion $Re$ (2), the Prandtl’s diffusion criterion $Pr$ (3), the Nusselt’s criterion $Nu$ (4)

$$Re = \frac{VS}{\nu}$$  

$$Pr = \frac{\nu}{D}$$  

$$Nu = 0.008 \cdot Re^{0.9} \cdot Pr^{0.43}$$
where $V_b$ is the air flow rate, (m/s); $S$ is the surface of heat and mass transfer (i.e. the area of the water surface in the pipeline with the corresponding filling), (m$^2$); $v$ is the coefficient of kinematic air viscosity, (m$^2$/s); $D$ is diffusion coefficient, (m$^2$/s).

To determine the diffusion coefficient $D$ in the “air-steam” system at an appropriate temperature $t$ and pressure $H$, it’s possible to use the formula (5)

$$D = D_0 \left( \frac{t + T}{273} \right)^{1.89} \frac{760}{H}$$

where $D_0$ is the value of the diffusion coefficient (at air temperature $t=0^\circ C$ and pressure 760 mm Hg); $T$ is the absolute air temperature in underwater space, K; $H$ is pressure in vaulted space, (mm Hg); $t$ corresponds to pressurized air temperature, $0^\circ C$.

According to Dalton's law, the mass of water evaporating into the air $WW$ (mg/s) can be determined by the formula (6)

$$WW = 0.2778 \cdot \beta \cdot S \cdot \left( p_{nas} - p_{par} \right)$$

where 0.2778 is a conversion factor from (kg/h) to (mg/s); $\beta$ is the coefficient of moisture exchange in the vaulted space, referred to the difference in pressure of steam (mg/m$^2$sPa); $p_{nas}$ is pressure of steam above the water surface at the corresponding air temperature in the vaulted space, (Pa); $p_{par}$ means partial steam pressures in the vaulted space, (Pa).

To determine the coefficient $\beta$, let’s use the similarity equation (7)

$$\beta = Nu \cdot \frac{D}{S^{0.5}}$$

Knowing the values of the $\beta$ coefficient and calculating the $WW$ value, it is possible to determine the amount of air exchange to remove excess moisture $QBB$ (m$^3$/s) in a closed space according to the formula (8)

$$QBB = \frac{WW}{d_y - d_n} \cdot \rho$$

where $\rho$ is the density of the supplied air, (mg/m$^3$); $d_y - d_n$ is the difference between the moisture content of the removed and supplied air, (g/kg) of dry air; $d_y$ is the moisture content of the removed air, (g/kg) of dry air; $d_n$ is the moisture content of the supplied air, (g/kg) of dry air.

The average air flow rate, taking into account its entrainment by the water flow $VCR$ (m/s), can be determined by the formula (9)

$$VCR = (V^2 + V_{w}^2)^{0.5}$$

where $V$ is the flow rate of waste water in the pipeline, (m/s).

Next, the real time $TR$ (s) for the removal of gaseous substances and moisture is calculated in a section of length $l$ between two wells when the rate of the air flows entrained by the water flow changes according to the formula (10)

$$TR = \frac{l}{VCR}$$

II. The second basic indicator for assessing the water-air regime is the value of the depression of the air flow in the vaulted space of the free-flow pipeline when setting up a pressurized ventilation system. To solve the presented problem, the depression can be considered as the pressure of the air medium in the calculated section of the pipeline between the wells (see figure 1). The discharge of pressurized air from the system is carried out at the end of the calculated section both directly onto the...
surface relief through an opening in the well hatch (first case) and through a ventilation riser of the corresponding height and diameter, which is set near the well hatch (second case).

The value of the depression $H_1$ (Pa) in the vaulted space of the pipeline is calculated by the formula (11)

$$H_1 = \frac{KAS \cdot P_{ma} \cdot 1}{\omega_{na}^3} \cdot Q B^2$$

(11)

where $KAS$ is the coefficient of the aerodynamic resistance (N s$^2$/m$^4$), considered equal to 0.3-0.44 for ventilated spaces without obstacles in the path of the air flow; $P_{ma}$ is the perimeter of the vaulted air space (m$^2$), determined by the formula (12)

$$P_{ma} = \pi d - \omega \omega + a$$

(12)

where $d$ is the diameter of the pipeline, (m); $\omega$ is the area of the free flow section in the shape of a segment bounded by a chord and the length of the wetted part of the circle, (m$^2$); $R$ is the hydraulic radius acting as a function of filling, m; $a$ is the chord length of the segment acting as a function of filling, (m); $\omega_{ma}$ is the cross-sectional area of vaulted air space, (m$^2$), determined by the formula (13)

$$\omega_{ma} = \frac{\pi d^2}{4} - \omega$$

(13)

The algorithm of the automated program includes the conversion of the depression value from Pa to MPa or m of water column.

III. The third calculated indicators of the water-air mode of the sewer network functioning include hydraulic and standard parameters: water flow rate $V$ (m/s), air rate in the vaulted space $V_b$ (m/s), water spending rate $Q$ (m$^3$/s), air volume in the vaulted space of the pipe $WB$ (m$^3$), the amount of gaseous substance $M$ (mg/s) emitted per unit time, which is subject to subsequent neutralization to the maximum permissible value when ventilating the vaulted space, etc.

Below are the results of an automated calculation of an exemplary water-air mode of operation of a free-flow pipeline and the interpretation of the data obtained with regard to the required rate of air exchange to remove foul-smelling gases and excess moisture in the calculated section of the pipeline (see figure 1).

3. Results and discussions

The initial data on the object of modeling in a brief version is presented by means of the following digital indicators: the length of the network section is 500 m, the pipeline diameter is 0.4 m, the slope of the route is 0.0025, the assigned air exchange in the pipeline system is 0.00277 m$^3$/s, the concentration of harmful gaseous substances (for example, hydrogen sulfide), respectively, in the vaulted space 4.2 mg/m$^3$ (according to field measurements with instruments) and the maximum permissible concentration in the city environment is 0.008 mg/m$^3$ [16].

Taking into account the unevenness of water consumption during the day, and thus the different mode of heat and mass transfer, the simulation was carried out for three cases of filling the pipeline with water: 0.3 (minimum), 0.5 (calculated) and 0.7 (maximum) [17, 18].

The analysis of the results of the automated calculation was carried out on the basis of the most significant hydraulic and aerodynamic indicators, according to which there is an estimate of the sufficiency of the adopted air exchange for removing harmful gases from the pipeline in the vaulted space and removing excess moisture [19, 20].

The summary data on the results of modeling, taking into account the neutralization of the released hydrogen sulfide reaching $2.22 \cdot 10^{-5}$mg/s to its maximum permissible value in the atmospheric air of 0.008 mg/m$^3$ are shown in table 1.
Table 1. Summary results of automated calculation for various fillings \( h/d \) and assigned air exchange 0.00277 m\(^3\)/s.

| Name of the calculated indicator                              | \( h/d=0.3 \) | \( h/d=0.5 \) | \( h/d=0.7 \) |
|---------------------------------------------------------------|---------------|---------------|---------------|
| Waste water flow rate in the pipeline, m/s                    | 0.678         | 0.786         | 0.849         |
| Air velocity in the vaulted space of the pipeline in case of still water, m/s | 0.0302       | 0.0436        | 0.0781        |
| Air volume in the vaulted space of the pipeline, m\(^3\)       | 0.679         | 0.787         | 0.853         |
| Total mass of gaseous substances entering the vaulted space of the pipeline from the water surface, mg | 45.82         | 31.78         | 17.74         |
| Duration of removal of gaseous substances from the vaulted space of the pipeline with the adopted air exchange in stagnant water conditions, s (h) | 192.45  | 133.48        | 74.52         |
| The real time of moisture removal when changing the air flow rate, in case of its entainment by the water flow, s (h) | 736.44 | 635.1         | 586.13        |
| Total depression of the air environment in case of its surpassing the atmospheric pressure, MPa (m water column) | 0.09801      | 0.09802       | 0.09804       |

When the filling changes from 0.3 to 0.7, an increase in the air rate from 0.679 to 0.853 m/s (25.6%) is observed. At the same time, there is a sharp decrease in the total mass of gaseous substances entering the vaulted space of the pipeline from 192.45 to 74.52 mg due to a decrease in its volume from 45.82 to 17.74 m\(^3\), which for both indicators is 30.72%.

For the same values of the range of fillings and the virtual condition of stagnant water in the pipeline (i.e., without taking into account the flow rate) and air rates in the range of 0.0302-0.853 m/s, as well as the duration of removal of gaseous substances from the vaulted space of the pipeline by ventilation will decrease from 4.6 to 1.8 h, i.e. by 39.1%.

In real conditions, the time for removing moisture from the vaulted space when changing the air flow rate, in case of being entrained by the water flow, will decrease from 0.204 to 0.163 h (21.1%). This indicates that moisture from the vaulted space will be removed from the sewer network faster than harmful gaseous substances, and in the range from 4.6/0.204 = 22.54 to 1.8 / 0.163 = 11.04 times, depending on the filling of the pipeline.

Regarding the air pressure (depression value) for the selection of the ventilation unit, it should be noted that the depression practically does not undergo changes due to low aerodynamic resistances in the vaulted space of the pipeline.

Thus, the air exchange value of 0.00277 m\(^3\)/s adopted in modeling ensures the removal of both hydrogen sulfide and moisture from the vaulted space of the pipeline (including the riser) to its maximum permissible value in the atmospheric air of 0.008 mg/m\(^3\). However, the optimal operation of the ventilation unit will be secured by a relatively equal rate (closeness of values) for moisture and hydrogen sulfide removal. Since the time differences in the given example are significant, the modeling of the water-air mode of the pipeline functioning was carried out with a larger value of the assigned air exchange with automatically tracking the moment of equality of the two indicators. By using an automated program, calculated data were obtained with a new air exchange rate of 0.04 m\(^3\)/s.

To display the results of the automated calculation, figure 2 contains graphs characterizing the change in the range of time values for neutralizing foul odors in accordance with the formula \( TZ=125.5(h/d)+603.18 \) and for removing moisture from the pipeline by using the formula \( TB=132.31(h/d)+124.86 \).

In this case, the real time of moisture removal when changing the air flow rate, in case of being entrained by the water flow, does not change.

If we compare the results of time calculation with the previous air exchange (0.00277 m\(^3\)/s), when the results ranged from 22.54 to 11.04 times, then with the new value of air exchange the
difference ranged from $0.3/0.1722 = 1.74$ up to $0.1/0.0984 = 1.02$. For $h/d=0.8$ the time values will practically coincide.

Thus, the value of air exchange selected by calculation will make it possible to successfully solve the problem of removing foul-smelling gases and moisture within a minimum period of time.

Figure 2. Graphs of the time dependence of the removal of odorous substances (the upper curve) and moisture (the lower curve) at the accepted air exchange.

4. Conclusions

1. Developing physical and mathematical models for the study of the water-air mode of free-flow pipelines functioning, as well as automated calculation, allow to quickly track the dynamics of changes in the parameters of the pipeline system in conditions of intensive release of gases harmful to human health.

2. In a specific example, taking into account the unevenness of water consumption through the day, the possibilities of automated calculation for determining the ranges of hydraulic and aerodynamic parameters of the pipeline system are considered.

3. As the basic parameters for modeling the water-air mode of non-pressurized pipeline functioning, the air exchange assigned by the user, as well as the duration of removing foul-smelling substances (for example, hydrogen sulfide) and moisture from the vaulted space of the pipeline are taken.

4. With the help of an automated complex, it is possible to adjust the amount of air exchange depending on technical circumstances and to minimize the cost indicators of the reconstruction and operation of pipeline networks equipped with ventilation units.

References

[1] Primin O G and Pupyrev E I 2013 Methods for improving the environmental safety of sewer pipelines *Ecology and Industry in Russia* 3 pp 13-17

[2] Fedorov S V, Vasiliev V M and Telyatnikova A M 2018 Development of a fundamental model of the sewer network *Journal of Bulletin of Civil Engineers* 2 (67) pp 168-174

[3] Chupin R V and Nguyen Tuan Anh 2015 Optimal reconstruction of sewerage networks *J. of Water*
Supply and Sanitary Engineering 2 pp 58-68

[4] Parker W J and Ryan H A 2011 Tracer study of headspace ventilation in a collector sewer J. of the Air & Waste Management Association 12 pp 581-592

[5] Gostelow P and Parsons S 2000 Sewage treatment works odour measurement Water Sci. Technol 41 (6) pp 33-40

[6] Kofman V Ya 2012 Hydrogen sulfide and methane in sewer networks Water supply and sanitary engineering 11 pp 72–78

[7] Stuetz R and Frechen F B 2001 Odors in wastewater treatment (London: Published by IWA Publishing SW1H QS) pp 436

[8] Pochwat K, Kida M and Ziembowicz S Odors in sewerage - a description of emissions and of technical abatement measures Environments 6 (89) pp 1-13

[9] Vasiliev V M and Malkov A V 2017 Places of formation of aggressive gases in the sewer network J. of Water Supply and Sanitary Engineering 1 pp 66-74

[10] Malkov AV 2017 Calculation of the required gas exchange rate in the underwater space of the sewer network Bulletin of civil engineers 2 (61) pp 140-144

[11] Vasiliev V M, Morozov G V and Zhukov S V 2018 Problems of operation of sewerage networks and ways to solve them Journal of Water Supply and Sanitary Engineering 7 pp 44-50

[12] Rublevskaya O N 2013 Measures to prevent the spread of unpleasant odors at the facilities of the State Unitary Enterprise "Vodokanal of St. Petersburg" Water supply and sanitary engineering 10 pp 46 – 55 (in rus.)

[13] Wysocka I, Gębicki J and Namieśnik J 2019 Technologies for deodorization of malodorous gases Environ. Sci. Pollut. R. 26 p 9409–9434

[14] Vasiliev V M, Pankova G A and Stolbikhin Yu V 2013 Destruction of sewer tunnels and structures on them caused by microbiological corrosion Water supply and sanitary engineering 9 pp 55 – 61 (in rus.)

[15] Orlov V A, Zotkin S P, Storozhnev A P, Gerasimov V A and Melnik O V 2020 Modeling the water-air mode of free-flow drainage networks functioning Certificate of state registration of a computer program 2020614973

[16] Standard GOST 32673-2014. Rules for the establishment of standards and control of emissions of foul-smelling substances into the atmosphere

[17] Vasilyev V and Stolbikhin Y 2015 Inspecting and monitoring the technical condition of sewage collectors Trans Tech Publications Switzerland, Applied Mechanics and Materials 725-726, pp 1319-24

[18] Joyce J, Hunniford Ch and Plummer A 2013 Implementing vapor phase odor control on large diameter interceptor systems Biosolids and Odor and Corrosion, Conference & Expo pp 1-31

[19] Rodionov A I, Kuznetsov Yu P, Soloviev G S 2005 Protection of the biosphere from industrial emissions. Basics of designing technological processes. (Moscow: Chemistry) p 392

[20] Yushin V V, Popov V M and Kukin P P 2005 Technique and technology for air protection (Moscow: Higher school) p 391