Coded Random Access: How Coding Theory Helps to Build Random Access Protocols

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Abstract

The rise of machine-to-machine communications has rekindled the interest in random access protocols and their use to support a massive number of uncoordinatedly transmitting devices. The classic ALOHA approach is developed under a collision model, where slots that contain collided packets are considered as waste. However, if the common receiver (e.g., base station) is capable to store the collision slots and use them in a transmission recovery process based on successive interference cancellation, the design space for access protocols is radically expanded. We present the paradigm of coded random access, in which the structure of the access protocol can be mapped to a structure of an erasure-correcting code defined on graph. This opens the possibility to use coding theory and tools for designing efficient random access protocols, offering markedly better performance than ALOHA. Several instances of coded random access protocols are described, as well as a case study on how to upgrade a legacy ALOHA wireless system using the ideas of coded random access.

I. INTRODUCTION

We start with a deceptively simple question: Where and why should we use random access? A concise answer would be: Whenever there is an uncertainty about the set of users that aim to transmit in a given instant. The canonical scenario falling in the above description is the one in which a set of uncoordinated devices aims to transmit over the shared wireless medium to the same receiver at approximately the same time, and the random access mechanisms are needed to break this “symmetry” and enable successful wireless access. As such, random access mechanisms are essential components of any distributed wireless communication systems, typically used in the initial link establishment and in distributed spectrum sharing among networks that interfere, such as two collocated WiFi hotspots. Presently, we are witnessing a revival of research interest in random access mechanisms, driven by the applications in the area of machine-to-machine (M2M) communications in cellular and satellite networks. The need for efficient random access mechanisms in M2M application scenarios is even more pronounced, as they typically assume reception of data from a massive, uncoordinated set of devices.

ALOHA [1] is a rather generic form of random access, typically operating under the assumption that collided packets are irrecoverably lost. Standard variants of the ALOHA protocol aim to maximize the number of collision-free transmissions within a given time interval, which is directly proportional
In recent years there has been a conceptual shift in the theory and practice of slotted ALOHA protocol family, based on the use of successive interference cancellation (SIC) algorithms that enable “unlocking” of the collisions slots. The essence of the proposed modifications is rather simple: active devices transmit replicas the same packet in multiple slots, and, at the same time, SIC is used on the receiving side to remove replicas of already recovered transmissions from collision slots. Recovery and removal of replicas is performed in an iterative, i.e., successive manner, where new iterations are propelled by the transmissions recovered in the previous round, as illustrated in Fig. 1. The exploitation of the collision slots boosts the throughput - in a basic scenario where active devices transmit two replicas in randomly selected slots of a frame, the asymptotic throughput increases to $T_{\text{max}} \approx 0.55$. The true potential of the SIC-enabled slotted ALOHA was revealed in [4], identifying analogies with the erasure-coding framework and establishing the paradigm of coded random access. The objective of this paper is to introduce these new developments, identify the ways in which they can be beneficial for M2M applications and discuss some of the important implementation issues.

II. Basics of Coded Random Access

A. Access Scheme Description

We start by considering coded slotted aloha (CSA) in which the access is organized in contention periods. Each contention period is a frame containing $M$ slots of equal duration, where $M$ is fixed. A set of $N$ users that use the contention periods to communicate with a Base Station (BS), which acts as a common receiver. We are interested in the regime where the user population is large respect to the size of the contention period $N \gg M$, but only a subset $N_a$ of the users is active in a given contention period. A simple model to create the uncertainty in the set of active users can be described as follows. At the beginning of a contention period each user generates a packet to be transmitted with activation probability $p_a$, where $p_a \ll 1$. Since each user becomes active independently from the other users, the number of

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1One may argue that the comparison with classic FSA is unfair, as in FSA a user sends only one packet replica before receiving feedback on the contention outcome. However, it should be noted that in classic FSA a user should also transmit multiple replicas in order to get the data through, the difference is that the retransmission is on-demand, initiated by the feedback.
active users in a contention period can be modeled by a random variable $N_a$, binomially distributed with mean value $\bar{N}_a = p_a N$.

The CSA scheme works as follows. Each active user generates $d$ replicas of the packet, where the repetition rate $d$ is drawn randomly according to a pre-determined probability distribution which is the same for all users. The repetition rate is picked by an active user independently of all other active users and independently of all his previous choices. The $d$ replicas are then transmitted by the user over $d$ slots picked uniformly at random among the $M$ slots of the contention period. Following the example of Fig. 1 the users 2 and 3 picked a repetition rate $d = 2$, while user 1 did not replicate its packet, i.e., its repetition rate is $d = 1$. A packet in a singleton slot is decoded correctly. Each packet is assumed to contain pointers to describe the positions of the other replicas in the contention period sent by the same user. The packet is then re-encoded and re-modulated and the receiver removes its interference contribution from the $d - 1$ slots where the replicas have been sent. The process proceeds iteratively, i.e., recovered packet replicas may allow solving other collisions, as illustrated in Fig. 1.

The rate of the CSA scheme is defined as

$$ R = \frac{1}{\bar{d}} $$

where $\bar{d}$ is the average number of replicas sent per user. It can be interpreted that a lower rate implies higher number of repetitions and the use of more energy per useful bit. The logical load of the channel is defined as the expected number of active users per slot,

$$ G = \frac{N_a}{M} = p_a \frac{N}{M}, $$

i.e., the logical load corresponds to the expected number of new information packets generated during the contention period. The physical load of the channel, i.e., the expected number of all transmitted replicas, is given by $G_{phy} = G \cdot d$. In classic FSA there is only a single replica $d = 1$, implying that the logical and physical loads coincide.

B. Bipartite Graph Representation and Asymptotic Analysis Over a Collision Channel

Fig. 2 shows the graph representation of the CSA scheme for the example on Fig. 1. The access scheme can be represented by a bipartite graph, consisting of a set $N_a$ user nodes (one for each active user), a set of $M$ slot nodes (one for each slot), and a set of edges. An edge connects the $i$th user node (UN) the $j$th slot node (SN) if and only if the user $i$ sends a packet in the $j$th slot. The degree $d$ of a given UN is equal to the number of edges connected to it, each edge corresponding to one of the replicas sent by the user. This graphical representation allows to establish a connection between the SIC procedure and iterative decoding of the codes-on-graphs. The connection is established under the following assumptions:

1) For each slot, the receiver always discriminates between a “silence”, singleton or a collision.
2) When a packet is received in a singleton slot, data are always correctly decoded.
3) Channel estimation and the interference cancellation are ideal.

The first two assumptions are typical of collision channel models. The third assumption simplifies the analysis without substantially affecting the obtained results, as shown in [3]. We also need to outline the main difference with the codes-on-graphs: the degree of a given slot node cannot be controlled and it can even be equal to zero (idle slot). Clearly, if the BS could control the degree of each slot, we would not need random access in the first place, as a single user would be scheduled in each slot.

Under the above three assumptions, the SIC procedure may be described as an instance of the iterative peeling decoder for codes constructed on sparse graphs and transmitted over a binary erasure channel (BEC) [5]. The decoder consists of initializing the status of all UNs to “unknown” and of repeating the following instruction until the status of all UNs has been updated to “known”, in which case decoding

2An efficient way to transport pointers is discussed in Section III.
terminates successfully, or until at some iteration the status of no UN is updated, in which case a decoding failure is declared. The steps are described as follows:

- For all SNs, if the SN has degree 1 then update to “known” the status of the unique UN connected to it.
- Remove all edges connected to the UN and update the degrees of the SNs accordingly.

The way the SIC mimics the peeling decoder is illustrated in Fig. 2b)-d).

The analogy between SIC for CSA and iterative decoding of codes-on-graphs allows to use analysis techniques developed in the field of codes-on-graphs and apply them in the framework of coded random access. In particular, applying asymptotic tools of analysis, such as density evolution or extrinsic information transfer (EXIT) charts, it is possible to show the existence of a thresholding behavior of CSA under SIC. This happens when both the frame size $M$ and the user population size $N$ tend to infinity, but the ratio $\frac{N}{M}$ remains constant. It turns out that there exists a value $G^*$, such that when the logical load is $G \leq G^*$, the SIC procedure almost certainly terminates successfully i.e. each user manages to send the packet to the BS within the contention period. Conversely, if $G > G^*$ then the opposite is true, i.e. there is a fraction of users which will certainly not be decoded. Therefore, $G^*$ acts as a threshold.

It is possible to show that the threshold $G^*$ depends on both the rates that the users select and on the...
probabilities with which these rates are selected. With a suitable selection of the repetition rates and their
associated probability distribution, a threshold as large as \( G^* = 1 \) packet/slot can be achieved. In other
words, the throughput performance becomes equivalent to the perfectly scheduled access! The way the
rate distribution is optimized follows the footsteps of the degree distribution optimization algorithms used
in the design of low-density parity-check (LDPC) codes [6].

As both the threshold \( G^* \) and the rate \( R \) are functions of the repetition rates distribution, one may look
for the maximum achievable threshold \( G^* \) for a given rate \( R \). Note that when repetition coding is used,
the rate is necessarily \( 0 < R \leq 1/2 \), as there are at least two repetitions. Once \( R \) as defined in (1) is
fixed, it can be shown that the threshold \( G^* \) of a CSA scheme is upper bounded by the unique positive
real solution of the equation

\[
G = 1 - e^{-G/R},
\]

as shown in [6]. If the user invests more power per data packet and increases the number of repetitions,
then \( R \) decreases and the right-hand side of (3) increases, which also means that the upper bound increases.

C. Variants of CSA

1) High-Rate CSA from Generic Component Codes: The upper bound resulting from (3) is valid for
every rate \( R \) between 0 and 1. In order to achieve rates \( R > 1/2 \), [6] introduces a generalization of
the CSA protocol that uses generic linear block codes instead of repetition codes. In this more general
setting, a user that is active in a given contention period, splits his packet into \( k \) segments, all of the same
length in bits. The \( k \) segments are then encoded using a linear block code and \( d \) segments are obtained as
output. The linear block code is drawn randomly by the user from a set of component codes, according
to pre-determined probability distribution. The information about the code used to encode the \( k \) segments
may be conveyed in a dedicated header appended to each segment. The component codes may have
different lengths \( d \) but they all have the same dimension \( k \). The rate of this generalized scheme is given
by \( R = k/\bar{d} \), where \( \bar{d} \) is the expected length of the employed component code. With a judicious selection
of \( k \), of the lengths \( d \) of the component codes, and of their probability distribution, any rate \( 0 < R < 1 \)
can be obtained. Note that the choice \( k = 1 \) reduces this generalized framework to the repetition-based
case.

The \( d \) encoded segments, equipped with appropriate pointers in its header, are transmitted by the user
over \( d \) slots picked uniformly at random within the contention period. The contention period is now
organized into \( kM \) slots, each of the same time duration as that of a segment; the time duration of the
contention period is thus the same as in the repetition-based case. The bipartite graph representing the
access scheme is now composed of \( kM \) SNs and \( N_a \) UNs, where now each UN corresponds to \( k \) segments.
On the receiver side SIC is performed similarly to the repetition-based case, the only difference being
the execution of some form of erasure decoding at the generalized UNs at each iteration. In case simple
codes are used, maximum a-posteriori (MAP) erasure decoding may performed. Similar to the case with
repetition, thresholding phenomenon can also be observed for the high-rate CSA schemes.

2) Convolutional CSA: A convolutional variant of the CSA scheme, is based on spatial coupling, a
technique widely used in the field of modern error correcting codes. We present it in a simplified scenario
in which all users exploit the same packet repetition rate \( d \).

In the convolutional CSA, a user becoming active at the beginning of a contention period with \( M \)
slots is allowed to transmit only one replica of its packet in that contention period, as opposed to the
scheme described in Section II-A in which all \( d \) replicas are transmitted by the user in that contention
period. Each of the other \( d - 1 \) packet replicas is transmitted by the user in one of the subsequent \( d - 1 \)
periods. Assuming the average number of active users per contention period is \( \bar{N}_a = p_a N \), on average
there are \( p_a N \) packet replicas in the first contention period (one per active user), \( 2p_a N \) packet replicas
in the second contention period (one per user becoming active at the beginning of the first period and
one per user becoming active at the beginning of the second period), etc. up to the \( d \)-th contention period
in which we expect \( dp_a N \) packet replicas on average. The expected number of replicas in a contention period that comes after the \( d-\)th one “stabilizes” to \( dp_a N \). Thus the expected physical load is \( G_{\text{phy},1} = G \) in the first contention period, see (2), then it is \( G_{\text{phy},2} = 2G \) in the second contention period, etc. and stays \( G_{\text{phy},d} = dG \) from the \( d-\)th period and onwards.

As it can be shown [7], the probability of a collision in a slot that belongs to a given contention period increases with the physical load imposed on that period. Due to the lighter physical load, the first contention period contains a lower number of collisions. The packets received in singleton slots of the first contention period may be used to remove the contribution of interference of their replicas in all \( d-1 \) subsequent contention periods. Therefore, although a slightly higher number of collisions are expected in the second contention period, some of them are resolved by interference cancellation. The resolved collisions are exploited, together with the packets received in the singleton slots from the first and second periods, to resolve further collisions in the third period. This process, when iterated through the chain of contention periods, determines a “chain reaction” which allows to resolve more collisions than those resolved by the scheme in Section II-A for the same repetition rates and probability distribution. Moreover, a thresholding phenomenon is again observed. Specifically, the iterative decoding threshold of the convolutional scheme reaches the theoretical, upper-bound threshold of the block scheme under optimal, maximum a posteriori probability decoding on a priori known graph [3].

D. Frameless CSA

Finally, we introduce frameless ALOHA [8], a variant of the CSA scheme inspired by the rateless codes [9]. Two essential differences to the previously described CSA protocols are:

- When the contention period starts, the active users decide whether or not to transmit a packet on a slot basis, i.e., as the slots of the contention period “appear” on the wireless medium.
- The duration of the contention period is not a-priori determined, but it is adaptive and tuned to the evolution of the contention/packet-recovery process.

In general case, both the user access strategy (i.e., the choice of slot-access probabilities) and the contention termination criterion are subject to optimization. In [10] a simple variant of the scheme was investigated, where the access strategy is “memoryless” and the slot-access probabilities are uniform both over users and slots. The scheme uses a heuristic termination criterion: the receiver monitors both the instantaneous throughput and the fraction of resolved users and, when either of them surpasses a predefined threshold, the contention is terminated through a suitable feedback signal. It was shown that, although asymptotically suboptimal, this approach grants throughputs that are the highest in reported literature for low to moderate number of active users, i.e., when the number of active users is in the range \( 50 - 1000 \).

Fig. 3 illustrates the asymptotic performance of frameless ALOHA, showing the probability of packet recovery, expected throughput and expected recovery delay of recovered packets, as functions of the number of elapsed slots vs the number of active users \( M/N_a \). The slot-access probability in the example is set to \( 3.1/N_a \), the value that maximizes the expected asymptotic throughput [8]. It is seen that the probability of packet recovery increases slowly in the beginning, but rises steeply for some critical value of \( M/N_a \). The same behavior is also observed in iterative BP erasure-decoding of rateless codes. The critical \( M/N_a \) actually defines the (expected) asymptotically optimal length of the contention process with respect to the throughput maximization, as could observed in Fig. 3. Finally, the expected recovery delay for recovered packets shows linear increase until the critical \( M/N_a \). Although this behavior seems favorable, one should take into account that most of the packets are actually not recovered and thus do not contribute to the calculation of the delay. After critical \( M/N_a \), most of the packets become recovered and the expected recovery delay practically saturates.

The principle of adaptive termination favors the “fortunate” instances of packet-recovery process, ending the contention as soon as the terminating conditions are met [10]. Moreover, the adaptive termination

\[3\] We again stress the fact that in CSA the graph is not known a priori due to the randomness of the contention process.
also implies that the packet-recovery process can tune to the actual wireless link conditions and potential imperfect SIC instances, simply disregarding the affected slots and proceeding with the contention process. In other words, frameless CSA is inherently adaptable to the scenarios when the assumptions outlined in Section II-B may not hold. The main drawback is that the moment when the users receive feedback from the BS that terminates the contention is not known a-priori. In scenarios where the uplink and downlink transmissions share the same spectrum, in frameless CSA the BS has to actually contend with the active users when transmitting the feedback, as analyzed in [10]. We conclude by noting that similar arguments apply when comparing advantages/drawbacks of the block coding and rateless coding frameworks.

III. PRACTICAL ISSUES

One of the underpinning assumptions of CSA is that each packet replica is equipped with pointers to the slots containing other replicas transmitted by the same user. However, in practice, it is neither trivial to make the pointers nor the cost of sending many pointers is negligible. A more elegant approach to address this issue is to embed in each replica a user-specific seed of a pseudorandom generator, that is known both to the users and the BS. Once a replica is resolved, the BS can use the knowledge of the generator and the obtained seed to determine all the slots in which the other replicas occur.

Another important practical issue is the estimation of the number of active users in a given contention period. In order to attain the optimal performance, knowledge of number of active users $N_a$ is required both in the framed and frameless variants of CSA. Specifically, in framed CSA the knowledge of $N_a$ should be used to dynamically adapt the duration of the contention period size $M$, in order to guarantee a constant logical load and thus a constant throughput. In frameless CSA, both the optimal slot-access probabilities and the termination criterion depend on $N_a$ [10]. The number of active users in wireless access systems is usually a priori not known and it varies over time, hence its estimate has to be obtained for optimal operation. A variety of estimating algorithms that could be implemented in framed scenarios have been proposed in literature, c.f. [11], [12]. Moreover, an efficient estimation algorithm specifically tailored for frameless version of the scheme was proposed in [13].

IV. CASE STUDY: UPGRADE THE EXISTING SLOTTED ALOHA IMPLEMENTATIONS

Coded random access protocols can be very useful in the context of M2M communication, both in satellite and terrestrial cellular access. Cellular random access is commonly based on the framed slotted
Fig. 4. Example upgrade of framed slotted ALOHA.

ALOHA, providing acceptable performance for human-oriented traffic. However, the M2M traffic has fundamentally different requirements, primarily seen in the massive number of accessing terminals with short reporting deadlines, and the traditional ALOHA may create bottlenecks already in the access network.

We present a short case study, describing how an existing cellular access protocol can be upgraded to reap the advantages of coded random access while preserving the physical-layer behavior of the devices unchanged. The main idea is to group of multiple ALOHA frames and run SIC across the frames. The required modifications on the device side could be reduced to the implementation of the pseudorandom generators that will drive the selection of slots in which the access will be performed. This includes a downlink signaling between the BS and the devices, in order to tune the pseudo-random generators, timers, back-off exponents and other parameters from the actual slotted ALOHA implementation, c.f. [14]. On the other hand, the BS stores the received uplink signals and uses SIC to process them, thereby absorbing the complexity of the upgraded system, which is a highly desirable feature for the simple M2M devices.

Fig. 4 presents an example upgrading of a generic framed slotted ALOHA. In the basic variant, Fig. 4a), active users are allowed to transmit just once per frame and only the transmissions occurring in singleton slots are successfully received, while the successful devices are notified via the next beacon. The unsuccessful users continue transmitting in the subsequent frames, choosing the slots where the repeated transmission take place independently with respect to the choice made in the previous frames. In the example, transmissions of user 1 and user 4 get through in the second frame, and of user 2 and user 3 in the third frame.

In a simple upgrade, Fig. 4b), the active users are also allowed to transmit once per frame, but the choice of the slots in subsequent frames is determined by a predefined function, which depends both on the user ID and the information received from the beacons sent by the BS. Once a user transmission is recovered, the BS retrieves the corresponding user ID, which enables the backtrack and removal of the user transmission from the previous frames and potential resolution of more users. In the example from Fig. 4, the recovery of transmissions of user 1 and user 4 in the second frame allows to recover user 2 and user 3 from the first frame; for the sake of simplicity, we assume that choice of the slots is the same as in Fig. 4a).

Finally, the extension of the concept is presented in Fig. 4c), where users are allowed to repeat the same transmissions in multiple slots of the frame, again according to a predefined function that depends on the user ID and the information received from the BS. In this case, the BS removes the recovered packets both in “forward” and “reverse” direction. We conclude by noting that the application of the concepts described above could be made both in protocols that contend with data and protocols based on access reservation.
V. CONCLUSION

Random access protocols represent an essential element of a wireless communication system that offers access to multiple devices. The classic approach of ALOHA, although essentially inefficient, underpins the majority of the existing wireless access protocols. The change of the collision model and the application of successive interference cancellation has led to coded random access, an innovative approach superior to classic ALOHA. Coded random access is suited for M2M communications as the main burden of the protocol is put on the receiving Base Station. We have shown that the coded random access is tightly related to the erasure codes-on-graphs and we have presented several protocol variants. Considering that the ALOHA approach dominated during the last four decades, we believe that the coded random access opens new grounds for designing communication systems that should embrace a massive number of M2M devices.

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