TWO-HADRON INCLUSIVE DIS AND INTERFERENCE 
FRAGMENTATION FUNCTIONS

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We investigate the properties of interference fragmentation functions arising from
the emission of two leading hadrons inside the same jet for inclusive lepton-nucleon
deep-inelastic scattering. Using an extended spectator model we give numerical
estimates for the example of the fragmentation into a proton-pion pair with its
invariant mass on the Roper resonance.

1 Introduction

For the investigation of the nonperturbative nature of quarks and gluons inside
hadrons we mainly rely on the information extracted from distribution (DF) and
fragmentation functions (FF) in hard scattering processes. There are three fundamental
quark DF that completely characterize the quark inside hadrons at leading twist with respect

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1 The presence of FSI allows that in the fragmentation process there are at
least two competing channels interfering through a nonvanishing phase. However,
as shown in the following, this is not enough to generate “T-odd” FF. A
genuine difference in the Lorentz structure of the vertices describing the fragment-
ing processes is needed. This poses a serious difficulty in modelling the
quark fragmentation into one observed hadron because it requires the ability
of modelling the FSI between the hadron itself and the rest of the jet, unless one accepts to give up the concept of factorization. Therefore, here we will consider a hard process, semi-inclusive Deep-Inelastic Scattering (DIS), where the hadronization leads to two observed hadrons inside the same jet. We will determine all possible FF at leading twist, analyzing their symmetry properties. For the hadron pair being a proton and a pion with invariant mass equal to the Roper resonance, we will estimate FF using an extended version of the diquark spectator model. In this case, FSI come from the interference between the direct production of the two hadrons and the decay of the Roper resonance.

2 Why a fragmentation into two hadrons?
Let’s consider the situation of a one-hadron semi-inclusive DIS. Excluding ab initio any factorization breaking mechanism, there are basically two ways to describe the residual interactions of the leading hadron inside the jet: to model microscopically independent interaction vertices that lead to interfering competing channels, or to assume the hadron moving in an external effective potential. In the former case, the difficulty consists in modelling a genuine interaction vertex that cannot be effectively reabsorbed in the soft part describing the hadronization. In the latter, introduction of an external potential in principle breaks the translational and rotational invariance of the problem. Further assumptions can be made about the symmetries of the potential, but at the price of loosing interesting contributions to the amplitude such as those coming from naive “T-odd” FF.

\[ \Delta(k; P_h, S_h) \]

Figure 1: Quark-quark correlation function $\Delta$ for the fragmentation of a quark into a hadron.

In fact, let’s consider the pedagogical example where a quark with momentum $k$ fragments into a leading hadron detected with momentum $P_h$ and mass $M_h$ (see Fig. 1). The hadron is not polarized and does not interact with the rest of the jet, therefore its wave function is described by a free Dirac spinor $u(P_h)$. The jet itself is replaced by a spectator system which, again for sake of simplicity, is assumed to be a structureless on-shell scalar diquark with mass...
$M_D$ and momentum $k - P_h$ in order to preserve momentum conservation at the vertex. All this amounts to describe the remnants of the fragmentation process with a simple propagator line $\delta((k - P_h)^2 - M_D^2)$ for the point-like on-shell scalar diquark $k - P_h$. Then, ignoring the inessential $\delta$ functions, the function $\Delta(k, P_h; S_h = 0)$ describing the hadronization (the soft blob in Fig. 1) becomes

$$\Delta(k, P_h; S_h = 0) \sim -\frac{i}{k - m} u(P_h) \pi(P_h) \frac{i}{k - m} = \frac{k^2 + m}{k^2 - m^2} (P_h + M_h) \frac{k^2 + m}{k^2 - m^2},$$

where in the second line the usual projector for two free fermion spinors has been used. Eq. (1) can be cast in the following linear combination of all the independent Dirac structures of the process allowed by parity invariance,

$$\Delta(k, P_h; S_h = 0) = A_1 M_h + A_2 P_h + A_3 \frac{k}{P} + \frac{A_4}{M_h} \sigma_{\mu\nu} P_h^\mu k^\nu,$$

where the amplitudes $A_i$ are functions of all the scalar combinations of the independent invariants of the process. In particular, $A_4 = 0$. The function $\Delta(k, P_h; S_h = 0)$ must meet the applicable constraints dictated by hermiticity of fields and invariance under time-reversal operations. Hermiticity implies that all $A_i$ amplitudes be real. Since the outgoing hadron is described by a free Dirac spinor, time-reversal invariance also implies $A_4^* = -A_4$. Combining the two constraints gives $A_4 = 0$ in agreement with the previous result deduced just by simple algebra arguments. Therefore, no “$T$-odd” structure is generated. This result holds true if the hadron plane wave is extended in the complex plane to simulate the effect of an uniform external potential. The hadron wave function becomes a plane wave damped uniformly in space through the imaginary part of $P_h$, but this is not enough to generate “$T$-odd” structures in the scattering amplitude.

Let’s now allow for FSI to proceed through a competing channel having a different spinor structure with respect to the free channel. As a simple test case, we assume for the final hadron spinor the replacement $u(P_h) \leftrightarrow u(P_h) + e^{i\phi} \bar{u}(P_h)$, where $\phi$ is the relative phase between the two channels. Inserting this back into Eq. (1) modifies the $\Delta(k, P_h; S_h = 0)$ function according to

$$\Delta(k, P_h; S_h = 0) = [A_1 (k^2 + 1) + B_1 \cos \phi] M_h + A_2 (1 - k^2) P_h + [A_3 + B_3 + B'_3 \cos \phi] \frac{k}{P} + \frac{B_4 \sin \phi}{M_h} \sigma_{\mu\nu} P_h^\mu k^\nu,$$

where the new amplitudes $B_i$ are still scalar combinations of the invariants of the process. In particular, the coefficient of the tensor structure $\sigma_{\mu\nu}$, $B_4 = \frac{B_4 \sin \phi}{M_h} \sigma_{\mu\nu} P_h^\mu k^\nu$, does not have a simple physical interpretation.
2M_h/(k^2 - m^2), is now not vanishing provided that the interference between the two channels, namely the phase \( \phi \), is not vanishing. In this case, a “T-odd” contribution arises and is maximum for \( \phi = \pi/2 \).

These simple arguments show that, in order to model “T-odd” FF in one-hadron semi-inclusive processes without giving up factorization, one needs to relate the modifications of the hadron wave function to a realistic microscopic description of the fragmenting jet. Such a hard task suggests that a more convenient way to model occurrence and properties of “T-odd” FF is to look at residual interactions between two hadrons in the same jet, considering the latter as a spectator and summing over all its possible configurations.

### 3 Quark-quark correlation function

In analogy with semi-inclusive hard processes involving one detected hadron in the final state, the simplest matrix element for the hadronisation into two hadrons is the quark-quark correlation function describing the decay of a quark with momentum \( k \) into two hadrons \( P_1, P_2 \), namely

\[
\Delta_{ij}(k; P_1, P_2) = \sum_X \frac{d^4\zeta}{(2\pi)^4} e^{ik\cdot\zeta} \langle 0| \psi_i(\zeta) a^\dagger_{P_2} a^\dagger_{P_1} |X \rangle \langle X| a_{P_1} a_{P_2} \bar{\psi}_j(0)|0 \rangle ,
\]

where the sum runs over all the possible intermediate states involving the two final hadrons \( P_1, P_2 \). Since the three external momenta \( k, P_1, P_2 \) cannot all be collinear at the same time, we choose for convenience the frame where the total pair momentum \( P_h = P_1 + P_2 \) has no transverse component. The constraint to reproduce on-shell hadrons with fixed mass (\( P^2_1 = M^2_1, P^2_2 = M^2_2 \)) reduces to seven the number of independent degrees of freedom. By defining the light-cone components of a vector \( a \) as \( a^\mu = [a^-, a^+, a_T] \) with \( a^\pm = (a^0 \pm a^3)/\sqrt{2} \), the independent variables can conveniently be reexpressed in terms of the light-cone component of the hadron pair momentum, \( P_h^- \), of the light-cone fraction of the quark momentum carried by the hadron pair, \( z_h = P_h^-/k^- = z_1 + z_2 \), of the fraction of hadron pair momentum carried by each individual hadron, \( \xi = z_1/z_h = 1 - z_2/z_h \), and of the four independent invariants that can be formed by means of the momenta \( k, P_1, P_2 \) at fixed masses \( M_1, M_2 \), i.e.

\[
\tau_h = k^2 \quad \sigma_h = 2k \cdot P_h = \left\{ \frac{M^2_1 + P^2_T}{z_h \xi} \right. + \left. \frac{M^2_2 + P^2_T}{z_h (1 - \xi)} \right\} + z_h (\tau_h + k^2_T)
\]

\[
\sigma_d = 2k \cdot (P_1 - P_2) = \left\{ \frac{M^2_1 + P^2_T}{z_h \xi} - \frac{M^2_2 + P^2_T}{z_h (1 - \xi)} \right\} + z_h (2\xi - 1)(\tau_h + k^2_T) - 4k_T \cdot P_T
\]

\[
M^2_h = P^2_h = 2P^+_h P^-_h = \left\{ \frac{M^2_1 + P^2_T}{\xi} + \frac{M^2_2 + P^2_T}{1 - \xi} \right\} ,
\]

(5)
with $\mathbf{P}_T^2 = \xi (1 - \xi) M_h^2 - (1 - \xi) M_2^2 - \xi M_2^2$.

By generalizing the Collins-Soper light-cone formalism\footnote{Collins, J. C. and Soper, D.
E. (1981).} for fragmentation into multiple hadrons, the cross section for two-hadron semi-inclusive emission can be expressed in terms of specific Dirac projections of $\Delta$ after integrating over the (hard-scale suppressed) light-cone component $k^+$ and, consequently, taking $\zeta$ as light-like. Expressing the integrations in a covariant way\footnote{Collins, J. C. and Soper, D. E. (1981).}, we get

$$\Delta^{[\Gamma]}(z_h, \xi, P_T^+, M_h^2, \sigma_d) = \int d\sigma_h d\tau_h \delta \left( \tau_h + k_T^2 - \frac{\sigma_h}{z_h} + \frac{M_h^2}{z_h^2} \right) \frac{\text{Tr}[\Delta \Gamma]}{8z_h P_T^+}. \quad (6)$$

Using Eq. (5) it is possible to reexpress $\Delta^{[\Gamma]}$ as a function of $z_h, \xi, k_T^2, P_T^2$, $\mathbf{k}_T \cdot \mathbf{P}_T$, where $\mathbf{P}_T$ is (half of) the transverse momentum between the two hadrons in the considered frame. In this manner $\Delta^{[\Gamma]}$ depends on how much of the fragmenting quark momentum is carried by the hadron pair ($z_h$), on the way this momentum is shared inside the pair ($\xi$), and on the “geometry” of the pair, namely on the relative momentum of the two hadrons ($P_T^2$) and on the relative orientation between the pair plane and the quark jet axis ($k_T^2, \mathbf{k}_T \cdot \mathbf{P}_T$, see also Fig. 2).

Figure 2: Kinematics for a fragmenting quark jet containing a pair of leading hadrons.

4 Analysis of interference fragmentation functions

If the polarizations of the two final hadrons are not observed, the quark-quark correlation $\Delta(k; P_1, P_2)$ of Eq. (4) can be generally expanded, according to hermiticity and parity invariance, as a linear combination of the independent Dirac structures of the process

$$\Delta(k; P_1, P_2) = C_1 (M_1 + M_2) + C_2 \slashed{P}_1 + C_3 \slashed{P}_2 + C_4 \slashed{k} + \frac{C_5}{M_1} \sigma^{\mu \nu} P_{1 \mu} k_{1 \nu} + \ldots \quad (7)$$

$$+ \frac{C_6}{M_2} \sigma^{\mu \nu} P_{2 \mu} k_{2 \nu} + \frac{C_7}{M_1 + M_2} \sigma^{\mu \nu} P_{1 \mu} P_{2 \nu} + \frac{C_8}{M_1 M_2} \gamma_5 \epsilon^{\mu \nu \rho \sigma} \gamma_{\mu} P_{1 \nu} P_{2 \rho} k_{\sigma}. \quad (7)$$

5
From the hermiticity of the fields it follows that \( C_i^* = C_i \), \( i = 1, \ldots, 12 \), and, if constraints from time-reversal invariance can be applied, that \( C_i^* = -C_i \), \( i = 5, \ldots, 8 \). It follows that \( C_5 = C_6 = C_7 = C_8 = 0 \), i.e. terms involving \( C_5, \ldots, C_8 \) are naive “T-odd”. Inserting the ansatz (7) in Eq. (6), we get the following Dirac projections

\[
\Delta^\gamma_5(z_h, \xi, k_T^2, P_T^2, k_T \cdot P_T) \equiv D^1 \equiv \int [d\sigma_{h\tau}^h] f_{D^1}(C_2, C_3, C_4)
\]

\[
\Delta^{[\gamma^- \gamma_5]}(z_h, \xi, k_T^2, P_T^2, k_T \cdot P_T) = \frac{\epsilon^{ij}_{\tau\tau}}{M_1 M_2} G^i_T = \int [d\sigma_{h\tau}^h] f_{G^i_T}(C_8)
\]

\[
\Delta^{[\mu^\nu \gamma_5]}(z_h, \xi, k_T^2, P_T^2, k_T \cdot P_T) = \frac{\epsilon^{ij}_{\tau\tau}}{M_1 + M_2} H^\mu_{1^\text{T}} + \frac{\epsilon^{ij}_{\tau\tau}}{M_1 + M_2} H^\nu_{1^\text{T}}
\]

\[
= \int [d\sigma_{h\tau}^h] \left[ f_{H^\mu_{1^\text{T}}}(C_5, C_6, C_7) + f_{H^\nu_{1^\text{T}}}(C_5, C_6) \right], \quad (8)
\]

where \( \epsilon^{\mu\nu}_{\tau\tau} = \epsilon^{-+\mu\nu} \) and the integration is made covariant as in Eq. (6). The functions \( D^1, G^i_T, H^\mu_{1^\text{T}}, H^\nu_{1^\text{T}} \) are the interference FF that arise at leading order for the fragmentation of a current quark into two unpolarized hadrons inside the same jet. The different Dirac structures used in the projections are related

\[
D^1 = \quad G^i_T = \left( \quad \rightarrow \quad \right) - \left( \quad \rightarrow \quad \right)
\]

\[
H^\mu_{1^\text{T}}, H^\nu_{1^\text{T}} = \left( \quad \rightarrow \quad \right) - \left( \quad \rightarrow \quad \right)
\]

Figure 3: Probabilistic interpretation for the leading order FF arising in the decay of a current quark into a pair of unpolarized hadrons.

to different spin states of the fragmenting quark and lead to the nice probabilistic interpretation illustrated in Fig. 3. \( D^1 \) is the probability for an unpolarized quark to produce a pair of unpolarized hadrons; \( G^i_T \) is the difference of probabilities for a longitudinally polarized quark with opposite chiralities to produce a pair of unpolarized hadrons; \( H^\mu_{1^\text{T}} \) and \( H^\nu_{1^\text{T}} \) both are differences of probabilities for a transversely polarized quark with opposite spins to produce a pair of unpolarized hadrons. \( G^1_T \), \( H^\mu_{1^\text{T}} \) and \( H^\nu_{1^\text{T}} \) are (naive) “T-odd” and do not vanish only if there are residual interactions in the final state. In this case, the above constraint from time-reversal invariance cannot be applied. \( G^1_T \) is
chiral even; $H_1^q$ and $H_1^\perp$ are chiral odd and can, therefore, be identified as the chiral partners needed to access the transversity $h_1$. Given their probabilistic interpretation, they can be considered as a sort of “double” Collins effect.

5 Spectator model

So far, the results about the properties of the FF hold true in general for the quark fragmentation into a pair of unpolarized leading hadrons at leading order in $1/Q$. In order to make quantitative predictions, we extend to the present case the formalism of the so-called diquark spectator model, specializing it to the emission of a proton-pion pair. The basic idea of the spectator model is to make a specific ansatz for the spectral decomposition of the quark correlator by replacing the sum over the complete set of intermediate states in Eq. (4) with an effective spectator state with a definite mass $M_D$, momentum $P_D$ and quantum numbers of the diquark. Consequently, the correlator simplifies to

$$
\Delta_{ij}(k; P_p, P_\pi) \sim \frac{\theta(P_D^2)}{(2\pi)^3} \delta \left((k - P_h)^2 - M_D^2\right) \langle 0|\psi_i(0)|\pi,p,D\rangle \langle D,p,\pi|\psi_j(0)|0\rangle ,
$$

(9)

where the additional $\delta$ function allows for a completely analytical calculation of the Dirac projections (6). The quark decay is specialized to the set of diagrams shown in Figs. 4, 5 and their hermitean conjugates, where the interference, necessary to produce the “T-odd” FF, takes place between the channel for direct production from the quark $q$ of the proton-pion pair $(p,\pi)$ and the channel for the decay of the Roper resonance $R$.

![Figure 4: Diagonal diagrams for quark $q$ decay into a proton $p$ and a pion $\pi$ through a direct channel or a Roper resonance $R$.](image)

Assuming that the proton-pion pair has an invariant mass equal to the Roper resonance one, we can neglect the diagrams not containing the Roper $R$, such as the $a$, $c$ and $b$. Moreover, calculations are still in a preliminary stage and the results for $G_1^\perp$ will be shown only. This FF is determined mainly
by the diagram \(\text{b}\), while \(\text{a}\) contributes at most to 10% of the strength. No contribution comes from the diagonal diagram \(\text{c}\), according to the “T-odd” nature of \(G_1^\perp\) related to FSI interferences. In order to calculate the soft hadronic matrix elements of Eq. (9) for diagram \(\text{c}\) we assume the most naive picture of the quark structure, i.e. in the rest frame all quarks are in the \(1/2^+\) orbitals and the diquark can be in a spin singlet state (scalar diquark, indicated by the label \(S\)) or in a spin triplet state (axial vector diquark, indicated by \(A\)).

Taking the spin-1 field propagator for the diquark and the Roper propagator quoted by the PDG\(^8\), the main Feynman rules for diagram \(\text{c}\) are:

- \((Rp\pi)\) vertex: \(\Upsilon_{ij}^{Rp\pi} = f_{Rp\pi} \left[\gamma_5\right]_{ij} \equiv g \left[\gamma_3\right]_{ij}\), where \(g^2/4\pi = 14.3\) is the strong coupling constant of the \(\pi NN\) pseudoscalar interaction. Within the experimental uncertainties, the strong Roper coupling can be assumed equal to the nucleon one, also because the quark content, and therefore the asymptotic form factor, are the same.

- \(qSR/qSp\) vertex: \(\Upsilon_{ij}^{qSR/qSp} = f_{qSR/qSp} \left[\gamma_5\right]_{ij} \equiv N_{qS} \frac{\tau_h - m^2}{\tau_h - \Lambda^2} \left[\gamma_3\right]_{ij}\)

- \(qAR/qAp\) vertex: \(\Upsilon_{ij}^{qAR/qAp, \mu} = f_{qAR/qAp} \left[\gamma_5\gamma^\mu\right]_{ij} \equiv N_{qA} \frac{\tau_h - m^2}{\tau_h - \Lambda^2} \left[\gamma_5\gamma^\mu\right]_{ij}\)

- \((q\pi q)\) vertex: \(\Upsilon_{ij}^{q\pi q} = f_{q\pi q} \left[\gamma_5\right]_{ij} \equiv N_{q\pi} \frac{\tau_h - m^2}{\tau_h - \Lambda^2} \left[\gamma_3\right]_{ij}\)

The introduction of cut-off parameters to exclude large virtualities of the quark \(q\) has been chosen as to kill the pole of the quark propagator\(^9\) while keeping the asymptotic behaviour of FF at large \(z_h\) consistent with the quark counting rule\(^10\). The values themselves of the cut-offs, \(\Lambda = 0.5\) and \(\Lambda_\pi = 0.4\) GeV, as well as the overall normalizations \(N_{qS} = 7.92\) GeV\(^2\), \(N_{qA} = 11.557\) GeV\(^2\) and \(N_{q\pi} = 2.564\) GeV\(^1/2\), are fixed by computing the second moment of \(D_1(z_h)\) and comparing it with available data; they are taken directly from Ref.\(^9\).
6 Numerical Results

We will plot the $\gamma^{-}\gamma_5$ projection $G_1^\perp$ of Eq. (3) for the fragmentation $u \rightarrow p + \pi$. The scalar and axial diquark contributions to the diagram $D_c$ are combined through the ratio 3:1 to keep the SU(4) structure of the proton spin-flavor wave function $\Psi$. The parameters take the values (in GeV) $m = 0.36, M_S = 0.6, M_A = 0.8, M_h \equiv M_R = 1.44, \Gamma_R = 0.35, M_p = 0.938, M_\pi = 0.139$. The special kinematics $k_T \cdot P_T = 0$ is chosen, where the hadron-pair plane is perpendicular to the plane containing the jet axis and the pair leading light-cone direction $P_0^-$ (see Fig. 2). From Eq. (3) it can be shown that $G_1^\perp$ actually becomes function of $z_h, \xi, k_T^2$.

![Diagram](image)

Figure 6: $G_1^\perp(z_h, k_T^2)$ at $\xi = 0.5$ for the fragmentation of a quark $u$ into a proton $p$ and a pion $\pi$. Kinematics is chosen such that the invariant mass of the pair is equal to the Roper resonance and $k_T \cdot P_T = 0$ (see Fig. 2).

In Fig. 3 $G_1^\perp u \rightarrow p + \pi(z_h, k_T^2)$ is shown for $\xi = 0.5$. We have checked that the result is rather insensitive to $\xi$. On the contrary, the maximum sensitivity to the fragmentation mechanism is concentrated around the kinematical range
where the pair takes roughly 70% of the jet longitudinal momentum and has a small transverse momentum with respect to the jet axis. By “cutting” the 3-d surface at constant values of $z_h \geq 0.6$, one can obtain curves that, for increasing $z_h$, get concentrated at lower $k_T^2$ and have an increasingly less important tail. In other words, the more the hadron pair is leading, i.e. it takes most of the jet longitudinal momentum, the more $G_{1}^\perp$ is concentrated around the jet axis with smaller transverse momentum.

Work is in progress to complete the calculation of FF at leading order including also chiral odd $H_{1}^{<}$ and $H_{1}^{\perp}$. Possible asymmetry measurements will also be addressed that allow isolation of each individual FF. In particular, after integration over $k_T$, the combination of $H_{1}^{<}$ and the transversity distribution $h_1$ could be isolated in the cross section for a semi-inclusive DIS on a polarized nucleon target, where an asymmetry can be built by measuring the proton-pion pair at some angle $k_T \cdot P_T$ with respect to the jet axis and then exchanging the mutual position inside the pair, i.e. flipping $P_T \rightarrow -P_T$. This and other possibilities are presently under consideration.

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