Strong field electrodynamics of a thin foil

Sergei V. Bulanov, Timur Zh. Esirkepov, and Masaki Kando
Kansai Photon Science Institute, JAEA, Kizugawa, Kyoto 619-0215, Japan

Stepan S. Bulanov
University of California, Berkeley, California 94720, USA

Sergey G. Rykovanov
Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

Francesco Pegoraro
Physics Department, University of Pisa, Pisa 56127, Italy

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Exact solutions describing the nonlinear electrodynamics of a thin double layer foil are presented. These solutions correspond to a broad range of problems of interest for the interaction of high intensity laser pulses with overdense plasmas such as frequency upshifting, high order harmonic generation and high energy ion acceleration.

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I. INTRODUCTION

High power laser irradiation of various targets, such as solid, cluster or gas targets, has been used for a number of years in order to study a broad range of mechanisms of high energy ion and electron acceleration [1, 2], high and low energy photon generation [3, 4], and to explore problems of interest for modeling processes relevant to fundamental physics [4, 7] and astrophysics [8].

When a high-intensity laser pulse interacts with a very thin foil target, which can be modelled as a thin slab of overdense plasma, features appear that are not encountered either in underdense or in overdense plasmas as noted in the current literature, see e.g., Refs. [3, 4]. These features provide novel regimes for ion acceleration [10–14], relativistic high order harmonics generation [15–17], light frequency upshifting [20–26], and laser pulse shaping [9, 27–31]. They become important when the foil thickness is shorter than, or of the order of, both the laser wavelength and the plasma collisionless skin depth.

The thin foil model developed in Refs. [9, 27, 31] has the advantage of being an exactly solvable nonlinear boundary problem in electrodynamics describing the effects of a strong radiation friction force (see [2, 34]).

In this paper we present a set of exactly solvable equations describing the nonlinear electrodynamics of a thin double layer foil when the effects of the charge separation electric field and of the radiation back reaction are taken into account. Within the framework of the thin foil approximation we shall address the generation of high order harmonics, when the thin foil models a relativistic oscillating mirror [13], the frequency upshifting during the head-on collision of an electromagnetic wave with a relativistic foil, corresponding to the case of a relativistic flying mirror [20], and the ion acceleration when the radiation pressure of the electromagnetic wave pushes the electron layer pulling forwards the ions according to the radiation pressure acceleration regime [10].

II. EQUATIONS OF 1D ELECTRODYNAMICS

Let us consider a one-dimensional model of the interaction of a laser pulse with thin foil targets. Each foil comprises two layers: an ion layer with positive electric charge and that the layer thickness and density are the same for all layers.

The only parameter describing the electrodynamics of the thin layer, $\omega_0 c e/n_\alpha$ and $\omega_0$ respectively, the particle velocity and momentum are normalized on $c/\omega_0$ respectively, the density unit is $n_{\alpha\sigma}$, and the electromagnetic (EM) field is normalized on $\omega_0 \alpha \sigma/\epsilon$. The particle velocity and momentum are normalized on $c$ and $m_\alpha c$ where $\alpha$ denotes the species in the $\alpha\sigma$ layer. Here $n_{\alpha\sigma} = m_e \omega_0^2/4\pi e^2$ is the critical density for an EM wave with frequency $\omega_0$. In these expressions $c$ is the speed of light in vacuum, $e$ and $m_e$ are the electron charge and mass, respectively.

Then the only parameter describing the electrodynamical properties of the $\alpha\sigma$ layer will be the normalized areal charge density $\varepsilon_{\alpha\sigma}$, which expressed in terms of the dimensional layer density and thickness is given by

$\varepsilon_{\alpha\sigma} = \frac{e \alpha \sigma}{\epsilon c}$.

* Also at Prokhorov Institute of General Physics, Russian Academy of Sciences, Moscow 119991, Russia and Moscow Institute of Physics and Technology, Dolgoprudny, Moscow region 141700, Russia
The electromagnetic field obeys the Maxwell equations,
\[ \partial_x \partial_x A^\nu = \frac{4\pi}{c} j^\nu, \]
with the four-vector of the electric current density equal to
\[ j^\nu = \sum_\alpha j_\alpha^\nu, \]

\[ \alpha \text{th } \text{layer is determined by the equation} \]
\[ \delta(x) \text{ is the Dirac delta function and } Z_\alpha = \pm 1. \]
The \( \alpha \)th layer velocity is \( v_x = v_{1,\alpha} e_1 + v_{2,\alpha} e_2 + v_{3,\alpha} e_3 \) and \( e_1, e_2, e_3 \) are unit vectors in the \( x, y \) and \( z \) directions, \( x_\alpha(t) \) is the \( \alpha \)th layer coordinate.

Using the results of Refs. \[ 9, 32, 33, 35 \] we can write the solution to the wave equation which yields
\[ E_\alpha(x, t) = Z_\alpha \epsilon_\alpha \left[ s(x, \tilde{t}_\alpha) e_1 + \frac{v_{2,\alpha}(\tilde{t}_\alpha) e_2 + v_{3,\alpha}(\tilde{t}_\alpha) e_3}{1 - s_\alpha(x, t_\alpha)v_{1,\alpha}(t_\alpha)} \right], \]

\[ B_\alpha(x, t) = -Z_\alpha e_\alpha s_\alpha(x, \tilde{t}_\alpha) \frac{v_{3,\alpha}(\tilde{t}_\alpha) e_2 - v_{2,\alpha}(\tilde{t}_\alpha) e_3}{1 - s_\alpha(x, t_\alpha)v_{1,\alpha}(t_\alpha)} \]

for the electric and magnetic field formed by a single \( \alpha \)th layer, where \( s_\alpha(x, t_\alpha) = \text{sgn}(x - x_\alpha(t_\alpha)) \) with the signum function \( \text{sgn}(x) = +1 \) for \( x > 0 \) and \( \text{sgn}(x) = -1 \) if \( x < 0 \). For given longitudinal \( v_{1,\alpha} \) and transverse \( v_{2,\alpha} e_2 + v_{3,\alpha} e_3 \) components of the particle velocity, these expressions describe the EM wave emitted by the thin layer, which acts as a 1D electric charge. Here and below the retarded time is determined by the equation
\[ \tilde{t}_\alpha = t - |x - x_\alpha(\tilde{t}_\alpha)|. \]

These relationships can also be easily derived with the Liénard-Wiechert potentials \[ 36 \] for the 1D four-vector of the electric current density.

Taking into account that the transverse components of the fields \( E_{\alpha,l} \) and \( B_{\alpha,l} \) at the \( \alpha \)th layer, \( x = x_\alpha(t) \) and \( \tilde{t}_\alpha = t \), are equal to the average of their values at both sides,
\[ E_{\alpha,l} = Z_\alpha \epsilon_\alpha \frac{v_{2,\alpha} e_2 + v_{3,\alpha} e_3}{1 - v_{1,\alpha}^2}, \]
\[ B_{\alpha,l} = -Z_\alpha e_\alpha v_{1,\alpha} \frac{v_{3,\alpha} e_2 - v_{2,\alpha} e_3}{1 - v_{1,\alpha}^2}, \]
we can write the expression for the EM acting on the \( \alpha \)th layer as the sum of the external and self-action fields: \( E + E_{\alpha,l} \) and \( B + B_{\alpha,l} \), where
\[ E = -\partial_t A_\alpha(x, t) + \sum_{\alpha' \neq \alpha} E_{\alpha'}(x_\alpha, \tilde{t}_{\alpha,\alpha'}(t)) \]
and
\[ B = e_x \times \partial_x A_\alpha(x, t) + \sum_{\alpha' \neq \alpha} B_{\alpha'}(x_\alpha, \tilde{t}_{\alpha,\alpha'}(t)). \]

Here \( \tilde{t}_{\alpha,\alpha'}(t) \) should be found from equation
\[ \tilde{t}_{\alpha,\alpha'}(t) = t - |x_\alpha(t) - x_{\alpha'}(\tilde{t}_{\alpha,\alpha'})|. \]

The vector potential \( A_{0,\perp} \), normalized on \( m_\epsilon c^2 / e \), corresponds to the external EM field. In particular it describes the EM pulse incident on the target. In these expressions \( \partial_x \) and \( \partial_t \) denote partial derivatives with respect to the coordinate \( x \) and the time \( t \).

Using the above obtained relationships we can write the equations of the \( \alpha \)th layer motion in components as
\[ \dot{p}_{1,\alpha} = Z_\alpha \mu_\alpha \left( E_1 + \frac{p_{2,\alpha} B_3 - p_{3,\alpha} B_2}{\gamma_\alpha} \right) \]
\[ -\epsilon_\alpha \frac{p_{1,\alpha}^2 + p_{2,\alpha}^2 + p_{3,\alpha}^2}{\gamma_\alpha (\gamma_\alpha^2 - p_{1,\alpha}^2)}, \]

\[ \dot{p}_{2,\alpha} = Z_\alpha \mu_\alpha \left( E_2 - \frac{p_{1,\alpha} B_3}{\gamma_\alpha} \right) - \epsilon_\alpha \frac{p_{2,\alpha}}{\gamma_\alpha}, \]

\[ \dot{p}_{3,\alpha} = Z_\alpha \mu_\alpha \left( E_3 + \frac{p_{1,\alpha} B_2}{\gamma_\alpha} \right) - \epsilon_\alpha \frac{p_{3,\alpha}}{\gamma_\alpha}. \]

Here, \( \mu_\alpha = m_\epsilon/m_\alpha \), a dot, ‘,’ denotes time derivative, \( p_{1,\alpha} \) and \( p_{2,\alpha} e_2 + p_{3,\alpha} e_3 \) are the longitudinal and perpendicular momenta of the of the particles in the \( \alpha \)th layer. The layer coordinate \( x_\alpha(t) \) depends on time according to equation \( \dot{x}_\alpha = p_{1,\alpha}/\gamma_\alpha \) where \( \gamma_\alpha = \sqrt{1 + p_{1,\alpha}^2 + p_{2,\alpha}^2 + p_{3,\alpha}^2} \) is the relativistic Lorentz factor.

The longitudinal and perpendicular components of the electric field are equal to \( E_1 = e_1 (\epsilon_1 \cdot E) \) and \( E - E_1 \), respectively. The last terms on the r.h.s. of Eqs. \[ 13, 14 \] and \[ 15 \] are the longitudinal and perpendicular components of the 1D electrodynamics radiation friction force, respectively.

Multiplying Eqs. \[ 13 - 15 \] by \( v_\alpha \) and adding them, we obtain the equation
\[ \frac{dE_\alpha}{dt} = Z_\alpha \epsilon_\alpha v_\alpha - \epsilon_\alpha \frac{p_{2,\alpha}^2 + p_{3,\alpha}^2}{1 + p_{2,\alpha}^2 + p_{3,\alpha}^2}, \]

where \( E_\alpha \) is a kinetic energy of the \( \alpha \)th layer. As we see the rate of radiative energy losses depends only on
the momentum component along the layer. The rate of energy loss vanishes at $p_{2,\alpha}^2 + p_{3,\alpha}^2 = 0$ and it is limited by the value of $\epsilon_\alpha$, because the layer electric field cannot exceed $2\pi c n l$ (in dimensional units). We shall return to this issue below.

In the above formulated 1D electrodynamics the EM wave is normally incident on the target. However, as is well known, by choosing proper initial conditions for the transverse component of the layer momentum, $p_{2,\alpha}$ and $p_{3,\alpha}$ in Eqs. (13), (14) and (15), we obtain a solution for an obliquely incident wave in the boosted frame of reference (see Refs. 3, 13, 57, 38), provided initially all the sheets are at rest and stationary and the (two) pulses are in vacuum (outside the foils).

This 1D electrodynamics system of equations for the EM field and layer motion can also be considered as an extension of Dawson’s electrostatic 1D plasma model 39 to the electromagnetic case with self-action (radiation reaction) taken into account. We notice here that in the case of a rotating electric field Eqs. (14) and (15) are reduced to the equations analysed in Ref. 34.

For analytical considerations and numerical integration of Eqs. (13) – (15) it is convenient to take the vector potential $A_{0,\perp}$ to propagate in the positive $x$ direction, i.e., to depend on $t - x$ and to introduce the function

$$h_\alpha = \gamma_\alpha - p_{1,\alpha}$$

and the variable

$$\tau_\alpha = t - x_{1,\alpha},$$

since, in the limit of week radiation friction $\epsilon_\alpha \to 0$ and vanishing longitudinal electric field $E_1$, the function $h_\alpha$ is an integral of motion. Using these variables we can present Eqs. (13) – (15) in the implicit form

$$\frac{dh_\alpha}{d\tau_\alpha} = -Z_\alpha \mu_\alpha E_1 - \epsilon_\alpha \frac{p_{2,\alpha}^2 + p_{3,\alpha}^2}{2h_\alpha},$$

$$\frac{dx_{1,\alpha}}{d\tau_\alpha} = \frac{p_{2,\alpha}}{h_\alpha},$$

$$\frac{dx_{2,\alpha}}{d\tau_\alpha} = \frac{p_{2,\alpha}}{h_\alpha},$$

$$\frac{dx_{3,\alpha}}{d\tau_\alpha} = \frac{p_{3,\alpha}}{h_\alpha},$$

$$\frac{d\tau_\alpha}{dt} = \frac{2h_\alpha^2}{1 + p_{2,\alpha}^2 + p_{3,\alpha}^2 + h_\alpha^2},$$

with

$$p_{1,\alpha} = \frac{1 + p_{2,\alpha}^2 + p_{3,\alpha}^2 - h_\alpha^2}{2h_\alpha}.$$

$$p_{2,\alpha} = a_{2,\alpha} - \epsilon_\alpha \left(x_{2,\alpha} - x_{2,\alpha}|_{\tau_\alpha = -x_{1,\alpha}}\right),$$

$$p_{3,\alpha} = a_{3,\alpha} - \epsilon_\alpha \left(x_{3,\alpha} - x_{3,\alpha}|_{\tau_\alpha = -x_{1,\alpha}}\right),$$

and

$$\gamma_\alpha = \frac{1 + p_{2,\alpha}^2 + p_{3,\alpha}^2 + h_\alpha^2}{2h_\alpha}.$$

III. SINGLE ELECTRON LAYER ACCELERATION BY THE LASER LIGHT PRESSURE

A. Limit of week radiation friction

In order to elucidate the basic properties of the 1D electrodynamics formulated above we consider the motion of a single electron layer in the plane EM wave $a_{0,\perp}(t - x)$. In this case the longitudinal component of the electric field, $E_1$, in the r.h.s. of Eq. (13) vanishes, and the electric and magnetic fields are equal to $E = -\partial_x a_{0,\perp}(t - x)$ and $B = e_1 \times \partial_x a_{0,\perp}(t - x)$, respectively, with given $a_{0,\perp}$. The electric and magnetic fields are taken at $x = x_\alpha$.

In the case without radiation losses, when $\epsilon_\alpha = 0$, Eqs. (28 – 27) yield the well known results 43.

$$h_\alpha = \text{constant},$$

$$p_{2,\alpha} = a_{0,2}(\tau_\alpha),$$

$$p_{3,\alpha} = a_{0,3}(\tau_\alpha).$$

(28)

If the layer before interacting with the EM pulse is at rest $h_\alpha = 1$. Then for $p_{1,\alpha}$, $\gamma_\alpha$, $x_{1,\alpha}$ and $\tau_\alpha$ we have

$$p_{1,\alpha} = \frac{1}{2}(a_{0,2}^2(\tau_\alpha) + a_{0,3}^2(\tau_\alpha)),$$

$$\gamma_\alpha = 1 + p_{1,\alpha} = 1 + \frac{1}{2}(a_{0,2}^2(\tau_\alpha) + a_{0,3}^2(\tau_\alpha)),$$

$$x_{1,\alpha}(\tau_\alpha) = \frac{1}{2} \int_{-\infty}^{\tau_\alpha} dr' (a_{0,2}^2(r') + a_{0,3}^2(r')),$$

(30)

and

$$t = \tau_\alpha + x_{1,\alpha}(\tau_\alpha).$$

(32)

As a result of the interaction of the electron layer with a finite duration electromagnetic pulse, its kinetic energy, $E_{kin,\alpha} = \gamma_\alpha - 1$, increases from zero to a maximum value equal to $a_{m,\alpha}^2/2$ and then decreases to almost zero (an exponentially small value for a pulse longer than its wavelength) after the electromagnetic pulse has overtaken the layer. Here $a_{m,\alpha}$ is the maximum amplitude of the pulse. This fact is referred to as the Lawson – Woodward theorem 44, 45. The layer displacement from the initial position is equal to

$$\xi_{1,\alpha} = \frac{1}{2} \int_{-\infty}^{+\infty} dr' (a_{0,2}^2(r') + a_{0,3}^2(r')).$$

(33)
In the limit of small but finite radiation losses we can find the radiation scattered by the layer. Considering \( \epsilon_\alpha \) as the parameter of a perturbation expansion, we calculate the reflected and transmitted waves by using Eqs. [4], [6] and [7], in which the layer velocity components and \( t_\alpha \) are obtained from Eqs. [28]–[32] for a pulse linearly polarized along the 2-direction. This yields for the electric field of the wave scattered in forward direction

\[
E_{2,\alpha}(x,t) = -\epsilon_\alpha a_2 \sin(\tau_\alpha) \bigg|_{\tau_\alpha = t - x},
\]

i.e. the transmitted wave is \((1 - \epsilon_\alpha) a_2 \sin(\tau_\alpha)\). The backward scattered wave, which is the wave reflected from the receding layer, is given by

\[
E_{2,\alpha}(x,t) = -\frac{\epsilon_\alpha a_2 \sin(\tau_\alpha)}{1 + a_2^2 \sin^2(\tau_\alpha)} \bigg|_{\tau_\alpha = t + x}.
\]

Here for the sake of brevity we consider the interaction of the layer with a sinusoidal electromagnetic wave given by \( \mathbf{a}_1 = a_2 \sin(t - x) \mathbf{e}_2 \) for \( t > 0 \) and zero before. Fig. 1 shows the waves emitted in the forward and backward directions, respectively.

![FIG. 1. (Color online) Emitted in the forward direction (a) and backward reflected (b) waves for \( a_2 = 25 \). The electric field amplitude is divided by \( \epsilon_\alpha \). (c) Local structure of the electric field \( E_2 \) and of the longitudinal velocity of the electron layer \( v_1 \).](image)

Due to the double Doppler effect the wavelength of the wave reflected back by the receding layer (Fig. 1 b) is larger than the incident wavelength. In addition, the reflected wave is not sinusoidal. The minimal electric field where \( \partial E_{2,\alpha}(x,t)/\partial x = 0 \) is equal to \( \epsilon_\alpha/a_2 \) in the limit \( a_2 \gg 1 \). In this limit every half-period the wave profile becomes singular at the point where \( \partial x/\partial \tau_\alpha(x,t) = 0 \). In the vicinity of the singular point \( t + x \) and \( E_2 \) depend on \( \tau_\alpha \) as \( t + x \approx \tau_\alpha - a_2^2 \tau_\alpha^3/3 \) and \( E_2 \approx a_2 \tau_\alpha - a_2^2 \tau_\alpha^3 \), which gives

\[
E_2 \approx a_2(t + x) - \frac{2}{3} a_2^2(t + x)^3.
\]

The electric field reaches the maximum \( E_{2,m} = \sqrt{\frac{2}{3}} a_2 \) at \((t + x)_m = 1/\sqrt{2a_2} \) with the maximum width equal to \( \delta(t + x)_m = 1/\sqrt{2a_2} \). As it is seen in Fig. 1 c), we show the local structure of the electric field in the wave and the corresponding time dependence of the longitudinal velocity of the layer emitting the wave, spikes of the electric field are formed in the reflected wave when the layer stops, i.e. at \( v_1 = 0 \).

**B. Frequency spectrum of the reflected EM radiation**

The EM wave reflection from the electron layer accelerated by the wave is a simple model of a relativistic oscillating mirror. In the case of a linearly polarized pulse with \( a_{0,2}(\tau) = a_2 \sin \tau \) and \( a_{0,3}(\tau) = 0 \), the reflected periodic EM wave takes the form with spikes shown in Fig. 1 b). It can be represented by the Fourier series

\[
E(\tau) = \sum_{n=1}^{\infty} b_n \sin(n \tau)
\]

with Fourier coefficients \( b_n(a) \) that vanish for even harmonic numbers \( n \) and that can be expressed in terms of hypergeometric functions.

\[
b_n = \epsilon_\alpha \frac{\pi a_2}{1 + a_2^2} \times \\
\left[ _3F_2 \left( \left\{ \frac{1}{2}, 1, 1 \right\}, \left\{ \frac{3}{2}, \frac{1}{2} \right\}; \frac{1}{2} + \frac{n}{2} \right) ; \frac{a_2^2}{1 + a_2^2} \right] \\
- _3F_2 \left( \left\{ \frac{1}{2}, 1, 1 \right\}, \left\{ \frac{1}{2} - \frac{n}{2}, \frac{3}{2} + \frac{n}{2} \right\}; \frac{a_2^2}{1 + a_2^2} \right). \tag{37}
\]

Here \( \text{pFq}(\{a_p\}, \{b_q\}, z) \) is the regularized hypergeometric function equal to \( \text{pFq}(\{a\}, \{b\}, z)/(\Gamma(b_1)...\Gamma(b_q)) \). In Fig. 2 we plot the dependence of \( b_n \) on the amplitude \( a \) for \( n = 1, 2, 3, 5, 7, 9 \).

In the frame of reference where the layer is on average at rest the electric field spikes of the back reflected wave shown in Fig. 1 b) are formed at the moment when the mirror reaches its maximum velocity in the backward direction. The velocity of this frame is is equal to

\[
v_f = \frac{1}{2\pi} \int_{-\pi}^{\pi} a_{0,2}^2(\tau) + a_{0,3}^2(\tau) \frac{a_2^2}{2 + a_{0,2}^2(\tau) + a_{0,3}^2(\tau)} d\tau. \tag{38}
\]
FIG. 2. (Color online) Dependence of $b_n$ on the wave amplitude $a$ for $n = 1, 2, 3, 5, 7, 9$. In the case of the linearly polarized wave with $a_{0,2}(\tau) = a_0 \sin \tau$, the layer moves on average with the velocity $v_f = 1 - 1/(1 + a_0^2/2)^{1/2}$. The spike width and amplitude in this frame of reference changes according to the Lorentz transformation rules.

C. Finite radiation friction force effect

In general case, if $\epsilon_\alpha \neq 0$, the radiation losses lead to a finite acceleration of the layer. Now we assume that the laser radiation has the form of a Gaussian electromagnetic pulse with vector potential

$$a_{0,\perp}(x,t) = \exp \left[ -\frac{(t-x)^2}{t_{EM}^2} \right] \times (a_2 \cos (t-x)e_2 + a_3 \sin (t-x)e_3).$$  \hspace{1cm} (39)

Numerical integration of Eq. 19 using relationships 24–26 yields the dependence of the longitudinal momentum $p_1$ on the variable $\tau_\alpha$ for different values of the parameters of the electromagnetic pulse and of the charged layer.

In Fig. 3 we plot the longitudinal momentum $p_1$ versus $\tau_\alpha$ for different values of the parameter $\epsilon_\alpha$: 1. $\epsilon_\alpha = 0.03$; 2. $\epsilon_\alpha = 0.04$; 3. $\epsilon_\alpha = 0.045$; 4. $\epsilon_\alpha = 0.05$; 5. $\epsilon_\alpha = 2.5$.

Lawson – Woodward theorem the effects of the finite radiation friction force modify the electric charge dynamics due to its acceleration by the radiation pressure [36]. This is seen in the curves 3, 4, and 5 in Fig. 3 as a “re-acceleration” of $p_1(\tau_\alpha)$.

FIG. 3. (Color online) Dependence of the longitudinal momentum $p_1$ on the variable $\tau_\alpha$ for different values of the parameter $\epsilon_\alpha$: 1. $\epsilon_\alpha = 0.03$; 2. $\epsilon_\alpha = 0.04$; 3. $\epsilon_\alpha = 0.045$; 4. $\epsilon_\alpha = 0.05$; 5. $\epsilon_\alpha = 2.5$.

FIG. 4. (Color online) Dependence of the longitudinal momentum $p_1$ on the parameter $\epsilon_\alpha$ for different values of the electromagnetic pulse amplitude: 1. $a_0 = 1.25$; 2. $a_0 = 2.5$; 3. $a_0 = 5$; 4. $a_0 = 7.5$.

As we see, in the limit of very low $\epsilon_\alpha$ (curves 1 and 2) the layer momentum dependence on $\tau_\alpha$ follows approximately according to Eqs. 24. For larger values of $\epsilon_\alpha$ (curves 3 and 4) as a result of the layer interaction with a finite width electromagnetic pulse the momentum does not vanish at $\tau_\alpha \rightarrow \infty$, i. e. the Lawson – Woodward theorem is not valid. When the parameter further increases (curve 5) the maximum value of the longitudinal momentum becomes lower. This fact is illustrated in Fig. 4, where the layer momentum dependence on $\epsilon_\alpha$ is shown for different laser pulse amplitudes.

When the interaction of the charged layer with the electromagnetic wave occurs in the regime beyond the

IV. RELATIVISTIC OSCILLATING MIRROR

The Relativistic Oscillating Mirror (ROM) concept has been proposed in Ref. 15 as a mechanism of high order harmonic generation when an overdense plasma is irradiated by a relativistically intense laser radiation. The
elucidating the basic features of the ROM concept. In
emagnetic wave provides a good theoretical model
layers of electrons and ions irradiated by a high intensity
found in the review articles [3]. A thin foil made of two
ROM theory and its experimental demonstration can be
ation. A detailed discussion of the main features of the
relativistic gamma factor associated with the mirror mo-
momentum
FIG. 5. (Color online) Dependence of the longitudinal mo-
moment and coordinate dependence on time presented in Fig.
frequency spectrum extends into the high frequency range
when the wave reflects from the relativistic mirror its fre-
quency is upshifted to a range determined
and the wave breaks up into short wave packets. The re-
flected wave frequency is upshifted to a range determined
by a factor approximately equal to 4\gamma_M^2, where \gamma_M is the
relativistic gamma factor associated with the mirror mo-
tion. A detailed discussion of the main features of the
ROM theory and its experimental demonstration can be
found in the review articles [2]. A thin foil made of two
layers of electrons and ions irradiated by a high intensity
electromagnetic wave provides a good theoretical model
elucidating the basic features of the ROM concept. In
generation of high frequency radiation in thus interaction
regime was experimentally demonstrated in Refs. [18].
Within the framework of the ROM concept, attention
is paid to the fact that under the laser field action the
light is reflected oscillates periodically back and forth forming in other
words an oscillating mirror. Due to the Doppler effect
when the wave reflects from the relativistic mirror its fre-
quency spectrum extends into the high frequency range
and the wave breaks up into short wave packets. The re-
flected wave frequency is upshifted to a range determined
by a factor approximately equal to 4\gamma_M^2, where \gamma_M is the
relativistic gamma factor associated with the mirror mo-
tion. A detailed discussion of the main features of the
ROM theory and its experimental demonstration can be
found in the review articles [2]. A thin foil made of two
layers of electrons and ions irradiated by a high intensity
electromagnetic wave provides a good theoretical model
elucidating the basic features of the ROM concept. In

\[ \mathbf{E}(x) = -\epsilon_s \text{sgn}(x) \mathbf{e}_1. \] (40)

We consider an electromagnetic pulse whose form is
given by Eq. (39), normally incident on the foil. The
the amplitude and the duration of this linearly polarized
short pulse are \(a_0 = 25\) and \(t_{\text{EM}} = 5\pi\), respectively. The
ion layer is assumed to be at the rest at \(x = 0\).
In the numerical integration, in the expression for the
restoring electric field \(E(x)\), we replace the discontinu-
ous function \(\text{sgn}(x)\) by \(\text{Tanh}(x/l)\) with the plasma layer
thickness equal to \(l = 0.01\). Before the laser pulse hits
the target \(t \to -\infty\) the electrons are located at \(x = 0\)
with \(p_0 = 0\).

A. Opaque mirror

We take the dimensionless parameter \(\epsilon_e\), that charac-
terizes both the radiation losses and the electric charge
separation electric field, equal to \(\epsilon_e = 50\). This choice
corresponds to the limit when \(a_0 \ll \epsilon_e\), and thus in this
case the foil is almost opaque for the incident EM radi-
ation. The electric charge separation field is relatively
strong which results in the electron layer oscillations re-
mainning in close proximity of the ion layer. Figs. [4] and [5]
illustrate the main features of the linearly polarized EM
pulse interaction with the opaque foil target. As we see
in Fig. [7] the electron layer oscillates at the front of the
ion layer due to the combined effect of the reflected elec-
tromagnetic pulse and of the restoring force due to the
ion layer: the net displacement at the end of the pulse in-
teraction is much smaller than the oscillation amplitude.
The average longitudinal momentum of the electron layer
is also almost zero. The reflected and transmitted waves
plotted in Fig. [8 resemble the incident EM pulse [39].
The EM wave is almost completely reflected with the
maximum amplitude of the reflected wave equal to 24.4.
The transmitted wave calculated as the superposition of the
incident wave and of the wave emitted forwards by
the electron layer, which almost cancel each other, has
its maximum amplitude equal to 0.6.

B. Transparent mirror

The case of a transparent foil target with \(a_0 \gg \epsilon_e\) is
shown in Figs. [9] and [10] for \(a_0 = 25\) and \(\epsilon_e = 5\).
In this regime of the EM wave interaction with the
double layer target the radiation pressure pushes the el-
teon layer forwards. The interaction is nonadiabatic with
respect to the longitudinal “sawtooth” oscillation excita-
tion which are seen in the longitudinal electron momen-
tum and coordinate dependence on time presented in Fig.
FIG. 7. (Color online) Time dependence of the longitudinal electron momentum, $p_1(t)$, (red curve) and of the layer coordinate, $x_1(t)$, (blue curve) for $a_0 = 25$, $t_{EM} = 5\pi$ and $\epsilon_e = 50$.

FIG. 8. (Color online) Reflected, $E_2(t + x)$, (blue curve) and transmitted, $E_2(t - x)$, (red curve) waves for $a_0 = 25$, $t_{EM} = 5\pi$ and $\epsilon_e = 50$.

FIG. 9. (Color online) Time dependence of the longitudinal electron momentum, $p_1(t)$, (red curve, 1) and of the layer coordinate, $x_1(t)$, (blue curve, 2) for $a_0 = 25$, $t_{EM} = 5\pi$ and $\epsilon_e = 5$.

FIG. 10. (Color online) Reflected, $E_2(t + x)$, (blue curve) and transmitted, $E_2(t - x)$, (red curve) waves for $a_0 = 25$, $t_{EM} = 5\pi$ and $\epsilon_e = 5$.

Similar oscillations have been noticed in Ref. [17]. In contrast to the opaque case, the net layer displacement at the end of the interaction of the pulse with the layer is not small and provides the initial condition for the “saw-tooth” oscillations that are a periodic sequence of hyperbolic motions of the electric charge in the homogeneous electric field [36] due to the ion layer. Within an oscillation half cycle the electron layer momentum depends on time as

$$p_1(t) = p_m - \epsilon_e t$$

in the time interval $0 < t < t_m$, where $t_m$ is the half-cycle duration equal to $2p_m/\epsilon_e$. The time dependence of the layer coordinate is given by

$$x_1(t) = \frac{1}{\epsilon} \left[ \sqrt{1 + p_m^2} - \sqrt{1 + (p_m - \epsilon_e t)^2} \right]$$

The maximum of the electron layer momentum $p_m$ and the maximum of the layer displacement $x_m$ are related to each other as

$$p_{1,m} = \sqrt{(1 + \epsilon_e x_{1,m})^2 - 1}.$$  

In order to find $x_{1,m}$ in the limit $a_0 \gg \epsilon_e$ we can use expression [43], which for the Gaussian linearly polarized EM pulse [39] yields

$$x_{1,m} = a_0^2 \left[ 1 - \exp \left( -t_{EM}^2 / 2 \right) \right] \sqrt{\pi t_{EM}}.$$  

The condition of nonadiabatic interaction is $t_m > t_{EM}$. The electron kinetic energy found from Eqs. [43] and [45] is given by

$$\frac{\epsilon_e}{m_e c^2} = \gamma_e - 1 = \epsilon_e x_{1,m} \approx \frac{\pi}{32} a_0^2 \epsilon_e t_{EM},$$

which for $a_0^2 \epsilon_e t_{EM} \gg 1$ is well above the quiver energy of an electron moving in the EM wave. The excitation
of the sawtooth oscillations can be regarded as the efficient collisionless heating of the electrons. This in fact can be an underlying mechanism of the electron energization during high intensity laser radiation interaction with a thin foil target observed in the computer simulations presented in Ref. 1 (see Fig. 1 (b) therein).

The transmitted and reflected waves shown in Fig. 10 have approximately of the same amplitude level because the receding relativistic mirror becomes less transparent while it is accelerated in the forward direction [5,13,14]. This results in a relative enhancement of the reflected wave amplitude. The spectra of the reflected and transmitted radiation contain high order harmonics. The reflected wave has the form of ultrashort spikes. The distance between them corresponds to the stretched wavelength of the incident light due to the double Doppler effect, because part of the wave interaction with the oscillating electron layer occurs under the conditions of reflection from a receding mirror. We note that the strongest spike at the rear of the reflected pulse is formed due to interaction with the sawtooth oscillations.

V. RELATIVISTIC FLYING MIRROR

A method to generate high frequency radiation based on the concept of the Relativistic Flying Mirror (RFM) considers a thin plasma shell travelling close to the speed of light as a relativistic mirror. The reflected light undergoes frequency upshift, compression and intensification due to a relativistic double Doppler effect. Various schemes were described [15,20,23,24,26,1] and experimentally demonstrated [21,29] as a proof of the feasibility of this concept.

A. The shape of a pulse reflected from a relativistic flying mirror

Using a double layer thin foil target as a RFM model, we consider the configuration of two counter propagating pulses. The first EM pulse driver pushes the electron layer forwards with relativistic velocity. The second pulse is relatively weak and propagates in the opposite direction. As a result of its head-on collision with the RFM a portion of the photons from this pulse is back reflected. This process is accompanied by the frequency upshifting of the reflected photons and by the modulation of the reflected pulse. When the ponderomotive force of the driver EM pulse is substantially larger than the force from the electric field due to the electric charge separation, i.e. when \( \epsilon_c \ll a_0 \), the motion of the relativistic electron layer can be described by Eqs. (28–32). In the case when the electron layer is accelerated by a linearly polarized EM pulse, as analysed in Ref. [3], the phase of the reflected part of the weaker EM wave is given by

\[
\psi_r(u) = \omega_r \left( u + \frac{a_0^2}{2} u - \frac{a_0^2}{4\omega} \sin 2\omega u \right)
\]

with \( u = t - x \) and \( \omega_r \) the frequency of the EM source pulse. The reflected pulse frequency given by a derivative of the phase \( \psi_r \) with respect to time is

\[
\omega_r(u) = \omega_r \left( 1 + a_0^2 \sin^2 \omega u \right)
\]

The frequency upshifting factor \( g = \omega_r/\omega_s \) depends on the longitudinal velocity of the mirror, \( v_1 = p_1/\gamma \) as

\[
g = \frac{\gamma + p_1}{\gamma - p_1}.
\]

If the source pulse frequency is equal to the driver pulse frequency, \( \omega_s = \omega = \omega_0 \), the reflected pulse frequency, \( \omega_r = \omega_0(1 + a_0^2 \sin^2 \omega_0 u) \), changes from \( \omega_0 \) to \( \omega_0(1 + a_0^2) \). The wave amplitude is modulated accordingly. The reflected radiation consists of a sequence of short high frequency pulses.

Fig. 11 shows the results of the numerical integration of Eqs. (13–15). The Gaussian EM pulse driver is linearly polarized with \( a_2 = 15 \), \( a_3 = 0 \) and \( t_{EM,d} = 5 \tau \). The source EM pulse is linearly polarized in the perpendicular plane, with \( a_2 = 0 \), \( a_3 = 1 \) and \( t_{EM,s} = 150 \tau \).

In Fig. 11a) we plot the time dependence of the longitudinal momentum, \( p_1(t) \), of the electron layer (red curve), of the layer coordinate, \( x_1(t) \), (blue curve) and of the factor \( g(t) \) (black curve) for the driver EM pulse with \( a_2 = 15 \), \( t_{EM,d} = 5 \tau \) and \( \epsilon_c = 0.1 \). The electron layer, while oscillating back and forth, moves on average forwards with a relativistic velocity. The frequency upshifting factor \( g \) oscillates synchronously with the layer momentum \( p_1 \). According to expressions (29–32) and (48) the factor \( g \) and the layer longitudinal momentum are related in the limit \( a_0 \gg \epsilon_c \) as \( g = 1 + 2p_1 \), i.e. the factor \( g \) scales with the layer energy as \( g = \sim 2\gamma \). For the chosen EM pulse driver amplitude equal to 15 the maximum value of the factor \( g \) is 226.

In Fig. 12 we present the frequency spectrum of the driver and source pulses. Fig. 12a) shows the dependence of the absolute value of the Fourier transform of the \( E_2 \) component of the electric field, corresponding to the incident and transmitted electromagnetic of the driver pulse. In Fig. 12b) we plot the dependence of the absolute value of the Fourier transform of the \( E_3 \) component of the electric field, which corresponds to the incident and reflected electromagnetic of the source pulse. The spectrum of the reflected radiation is enriched by the high order harmonics. It has a form of the plateau, which extends to the value of the order of \( \omega_{max} = 226 \).

The reflected EM pulse as seen in Fig. 11b) is approximately shorter by a factor \( g = 226 \) than the source pulse incident on the foil. Fig. 11c) shows that the reflected wave breaks up into a train of high frequency pulses, which are frequency modulated (see Fig. 11d)), i.e. in general the frequency upshifting and shortening of the reflected pulse is accompanied by the generation of high order harmonics.

The amplitude of the reflected EM pulse is proportional to the amplitude of the incident radiation, \( a_s \),
FIG. 11. (Color online) a) Time dependence of the longitudinal electron layer momentum, $p_1(t)$, (red curve), of the layer coordinate, $x_1(t)$, (blue curve) and of the factor $g(t)$ (black curve) for the driver EM pulse with $a_2 = 15$, $t_{EM,d} = 5\pi$ and $\epsilon_e = 0.1$. Counterpropagating source pulse: b) Reflected, $E_3(t-x)$, (blue curve) and transmitted, $E_3(t+x)$, (red curve) waves for $a_3 = 1$, $t_{EM,s} = 150\pi$ and $\epsilon_e = 0.1$. c) Reflected pulse (blue curve) and frequency upshifting factor $g(t)$ (black curve). d) Close up of the reflected pulse (blue curve) and frequency upshifting factor $g(t)$ (black curve).

FIG. 12. (Color online) The frequency spectrum of the driver and source pulses. a) The dependence of the absolute value of the Fourier transform of the $E_2$ component of the electric field, corresponding to the incident and transmitted electromagnetic of the driver pulse. b) The dependence of the absolute value of the Fourier transform of the $E_3$ component of the electric field, which corresponds to the incident and reflected electromagnetic of the source pulse.

FIG. 13. (Color online) Counterpropagating driver and source pulses of equal amplitude and duration. a) Time dependence of the longitudinal electron layer momentum, $p_1(t)$, (red curve), the layer coordinate, $x_1(t)$, (blue curve), the factors $g_+(t)$ (black curve) and $g_-(t)$ (green curve), for the driver EM pulse with $a_2 = 15$, $a_3 = 15$, $t_{EM,d} = t_{EM,s} = 3\pi$ and $\epsilon_e = 0.1$. b) Reflected pulse $E_3(t-x)$ (blue curve) and frequency upshifting factor $g_+(t)$ (black curve).
times the factor \( g \) and times the reflection coefficient \( \rho \). The reflection coefficient can be found as in Refs. [3, 12, 14, 21]. In the frame of reference co-moving with the electron layer where the longitudinal momentum component vanishes, \( p_{1} = 0 \), the equation for the electric field, \( \mathbf{E} = \mathbf{E}_{0} + \mathbf{E}_{\perp} \), according to Eqs. (4) and (10) can be written in the form

\[
\mathbf{E}' = \mathbf{E}_{0}' + \frac{2\pi \epsilon nl}{c} \mathbf{v}_{\perp}',
\]

where a prime denotes the electric field and the electron velocity in the co-moving frame of reference and \( \mathbf{v}_{\perp}' = v_{\perp}^{0}e_{2} + v_{\perp}^{0}e_{3} \). Here we use dimensional variables, i.e. \( 2\pi \epsilon nl \) instead of \( \epsilon_{r} \), in order to clearly show that the areal charge density, \( \epsilon nl \), is Lorentz invariant while the electric field and the electron velocity are not invariant. In the head-on collision configuration of the EM pulse interaction with the electron layer when \( v_{1} < 0 \) the electric field in the boosted frame is larger than that in the laboratory frame of reference by a factor of \( \sqrt{(1 + |v_{1}|)/(1 - |v_{1}|)} \approx 2\gamma \).

Since in any frame of reference the electron velocity cannot exceed the speed of light in vacuum, there are two limiting cases depending on the value of \( E_{0}'/2\pi \epsilon nl \approx 4\gamma^{2}a_{0}/\epsilon_{r} \). In the case of a weak EM wave, when this ratio is much smaller than unity, from Eq. (49) it follows that the amplitudes of the incident, \( E' \), and reflected, \( 2\pi \epsilon nl \mathbf{v}_{\perp}'/c \), waves are almost equal to each other, i.e. the reflection coefficient is of the order of unity. In the opposite limit, when \( E_{0}'/2\pi \epsilon nl \gg 1 \), the amplitude of the reflected EM wave in the boosted frame of reference is of the order of \( 2\pi \epsilon nl \). This yields a constraint on the upper limit of the EM radiation intensity measured in the laboratory frame of reference, when the wave is reflected by a thin electron layer of areal density \( nl \) moving with relativistic gamma-factor \( \gamma \), as in the above considered case or in the flying mirror configuration discussed in Ref. [22]: \( I_{0} \leq 16\pi \epsilon \epsilon_{r} \lambda^{2}/\gamma^{2} \). For example, for an \( 10^{-2}\mu \text{m} \), \( n = 10^{23}\text{cm}^{-3} \) electron layer moving with gamma-factor equal to \( 10^{2} \) this yields \( I_{0} \approx 5 \times 10^{22}\text{W/cm}^{2} \).

In Fig. 13 we illustrate the regime when two EM pulses with equal amplitudes and perpendicular polarizations interact with a thin foil target. The amplitudes of the driver and source pulses are equal to \( a_{2} = 15 \), \( a_{3} = 15 \), \( t_{EM,s} = t_{EM,d} = 3\pi \) and \( \epsilon_{r} = 0.1 \). In Fig. 13 a), we plot the time dependence of the electron layer coordinate \( x(t) \), momentum component \( p_{1}(t) \) and of the frequency upshifting factors \( g_{1} \) and \( g_{-} \), for the waves reflected to the right and to the left hand side directions, respectively. As seen, during the interaction of the two EM pulses colliding head-on the electron layer undergoes irregular jiggles. The frequency upshifting factors are not as large as in the previous case presented in Fig. 11.

The reflected EM wave shown in Fig. 13 b) has a non-sinusoidal form and is much less regular than in the case of a source pulse with a finite but not too large amplitude described by Figs. 13 c) and d).

B. Head on interaction of an EM pulse with an electron layer in the regime of sawtooth oscillations

As noticed above [15], if the electron layer is driven by an electromagnetic wave with amplitude \( a_{0} \) the frequency upshifting factor for a counterpropagating pulse cannot exceed the value \( g \leq (1 + a_{0}^{2})^{-1} \). However, the \( g \) factor can be substantially enhanced by imposing a delay between the driver and the source pulses in such a way that the counterpropagating source pulse gets reflected by the electron layer in the phase when the layer undergoes “sawtooth” oscillations. According to Eq. (46) the mirror Lorentz factor scales as \( a_{0}^{2} \), \( \epsilon_{r} \), \( t_{EM} \), i.e. the the frequency upshifting factor may be of the order of \( \approx 4a_{0}^{4}t_{EM}^{2} \gamma^{2} \). In other words the longitudinal velocity of the electron layer during the phase of “sawtooth” oscillations, i.e. after the end of the driver EM pulse, is substantially larger than the velocity of the oscillations driven by the ponderomotive force as clearly seen in Fig. 9. By choosing the delay time between the driver and the counter propagating source pulse in a such way that the source pulse collides with the electron layer at the “sawtooth” oscillation phase, we can provide conditions for a much higher frequency upshifting and intensification of the back-reflected radiation in the regime as shown in Fig. 14. The source pulse is polarized in the plane perpendicular to the driver pulse polarization plane. Its amplitude is equal to 0.001 and its width is \( t_{EM,s} = 40\pi \). The delay time between the driver and source pulses is 230\pi. In Fig. 14 a) the time dependence of the longitudinal electron layer momentum, \( p_{1}(t) \), (red curve), the layer coordinate, \( x_{1}(t) \), (blue curve) and of the factor \( g(t) \) divided by 100 (black curve) for a driver EM pulse with \( a_{2} = 25 \), \( t_{EM,d} = 5\pi \) and \( \epsilon_{r} = 1 \). b) Reflected pulse \( E_{3}(t - x) \) (blue curve). c) Incident source pulse \( E_{3}(t + x) \) (green curve).
by 100 factor \( g(t) \) (black curve) for the driver EM pulse with \( \alpha_2 = 25, t_{EM,d} = 5\pi \) and \( \epsilon_e = 1 \) are shown. The frequency upshifting factor reaches its maximum \( \approx 1.2 \times 10^5 \) at \( t = 725 \). The reflected source pulse shown in Fig. 14 b) has amplitude \( E_R = 0.4 \) approximately \( 4 \times 10^3 \) times larger than that of the incident wave (see Fig. 14 c)). Its width and wavelength are shortened by a factor \( g \).

The form of the reflected EM pulse resembles that of the frequency upshifting factor \( g(t) \) (see Figs. 14 a) and b)).

VI. ION ACCELERATION

In Ref. [10] the radiation pressure exerted by an ultraintense electromagnetic pulse on a quasineutral plasma foil has been proposed as a very efficient acceleration mechanism capable of providing ultrarelativistic ion beams. In this radiation pressure dominant acceleration (RPDA) regime, the ions move forward under the push of the pulse pressure with almost the same velocity as the electrons. A fundamental feature of this acceleration process is its high efficiency, as the ion energy per nucleon turns out to be proportional in the ultrarelativistic limit to the electromagnetic pulse energy.

Recently the RPDA regime of laser ion acceleration has attracted great attention e.g., see review articles [1]. In Ref. [17] the stability of the accelerated foil has been analyzed. A foil accelerated to relativistic energies by a laser pulse can also act as a relativistic flying mirror for frequency upshift and intensification of a reflected counterpropagating light beam [18]. An indication of the effect of the radiation pressure on bulk target ions is obtained in experimental studies of thin solid targets irradiated by ultraintense laser pulses [19].

Below we consider a double layer (ion and electron) thin foil target irradiated by the EM radiation. For the sake of simplicity we assume that the electron layer motion is described by Eqs. 13 - 15. In the ion layer equations of motion we neglect its interaction with the EM wave retaining only the electrostatic force due to the electric field produced by the electron layer. The electrostatic approximation for the ion layer motion can be used provided the parameter \( \epsilon E/m_i \omega_c \) is small, i.e., in the case of a one micron wavelength laser, for a light intensity below \( \approx 10^{24} \text{W/cm}^2 \). At this limit the classical electrodynamics paradigm must be changed and quantum effects must be included [20].

The results of the numerical integration of the equations of motion of the electron and ion layers irradiated by a strong EM wave are shown in Fig. 15. Figure 15 a) presents typical regimes of ion acceleration for a linearly polarized electromagnetic pulse with amplitude \( a = 10 \) and duration \( t_{EM} = 5\pi \) interacting with a foil with \( \epsilon_e = 1.5 \). At the initial stage \( -25 < t < 25 \) the time dependence of the ion and electron coordinates corresponds to a strong charge separation. As seen in Fig. 15 a), the electron layer pushed by the radiation pressure of the EM wave pulls the ion layer. Then both layers move forwards with the same average velocity and with the electron layer moving back forth around the ion layer performing sawtooth oscillations. This phenomenon can explain the efficient electron heating during the RPDA ion acceleration observed in the PIC simulations presented in Ref. [11]. In addition these oscillations cause oscillations of the ion energy \( \gamma_i - 1 \) around its average value.

The parametric dependence on the EM pulse amplitude and on the target surface density of the energy of the accelerated ions is illustrated in Figs. 15 b) and c). The dependence on time of the ion energy for different values of the EM pulse amplitude and varying parameter \( \epsilon_e \) is not monotonic as seen in Figs. 15 b) and c) and can be explained by the sawtooth oscillations of the electron and ion layers. A finite value of the parameter \( \epsilon_e \) provides an efficient coupling between the EM pulse and the electron-ion foil target. In the case without radiation friction shown in Fig. 15 d) the ion acceleration is less efficient.

VII. CONCLUSIONS, DISCUSSIONS, MAIN RESULTS

The theory of the interaction of relativistically strong electromagnetic fields with foil targets used in the present paper is based on the thin layer model of the one-dimensional electrodynamics of charged particles. It describes the 1D motion of electric charges in the self-consistent electromagnetic field incorporating the charge self-action or, in other words, the effect of the radiation friction force.

Within this framework, the generation of high order harmonics in the relativistic regime occurs through the electromagnetic wave reflection, or collective backward scattering also called nonlinear collective Thomson scattering, at the electron layer driven by the electromagnetic wave. The back reflected radiation takes the form of a train of ultrashort single-cycle electromagnetic pulses that are formed at the moment of maximal negative velocity of the layer.

The radiation scattered by the thin foil target in the backward and that scattered in the forward direction have different frequency spectra.

In the nonadiabatic regime of interaction with the ion-electron layer target a short electromagnetic pulse excites relatively low frequency sawtooth oscillations with amplitude substantially larger than the amplitude of the oscillations of the electron layer driven by the electromagnetic pulse. These sawtooth oscillations provide a mechanism of collective electron heating. They also generate extremely short spikes in the reflected EM wave.

In the configuration when two electromagnetic beams irradiate the thin foil target the electron layer driven by strong enough electromagnetic wave plays the role of a relativistic flying mirror for the second pulse. The electromagnetic radiation reflected from the relativistic mirror counterpropagating is intensified and its frequency is
FIG. 15. (Color online) Ion acceleration by the radiation pressure. a) Time dependence of the the electron (red curve) and ion (blue curve) layer coordinates and of the ion energy $m_i c^2 (\gamma_i - 1)$ (black) for an EM pulse with $a_2 = 10$, $t_{EM} = 5\pi$ and $\epsilon_e = 1.5$. b) Normalized ion energy $\gamma_i - 1$ v.s. time for $a_2 = 400$ and the parameter $\epsilon_e$ varying from 45 to 250 from bottom to top with the step equal to 5. c) Normalized ion energy $\gamma_i - 1$ v.s. time for $\epsilon_e = 45$ and the EM pulse amplitude $a_2$ varying from bottom to top from 100 to 450 with the step equal to 10. d) Time dependence of the the electron (red curve) and ion (blue curve) layer coordinates and of the ion energy $m_i c^2 (\gamma_i - 1)$ (black) for the case without radiation friction.

substantially increased. The reflected radiation takes the form of a sequence of the high frequency short bunches of electromagnetic radiation.

Since the longitudinal velocity of the electron layer during the phase of the “sawtooth” oscillations is substantially larger than the velocity of the oscillations driven within the electromagnetic pulse driver, by choosing the delay time between the driver and the counter propagating source pulse in a such way that the source pulse collides with the electron layer at the “sawtooth” oscillation phase, we can provide conditions for high frequency upshifting and intensification of the back-reflected radiation.

Under the radiation pressure of the electromagnetic wave the electron layer becomes separated from the ion layer that moves in the electric field due the charge separation. As a result, while the electron layer undergoes back and forth sawtooth oscillations around the ion layer, on average both layers move together. The ion acceleration rate grows higher with higher amplitude of the incident electromagnetic wave. It also depends on the radiation friction which is responsible for the coupling of the electromagnetic field with the electron layer because it provides the wave back scattering and thus the momentum transfer from the electromagnetic field to the charge particles. If the radiation friction force effects are not taken into account the ion acceleration rate is substantially lower for the same electromagnetic pulse amplitude.

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