Interferometric Processing of Acoustic Information by Using Extended Antennas in Dispersing Media

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Abstract: The theory of interferometric processing of acoustic information by using extended antennas in media with frequency dispersion is presented. The dependence between the two-dimensional spectral density of the two-fold Fourier transform of the interference pattern formed by a moving noise source and the aperture and angular structure of the received field is analyzed. The gain factor, directivity characteristic, and noise immunity of processing are estimated. Depending between the input signal/noise ratio on the antenna element and the maximal range of the noise source is obtained. This maximal range allows stable detection and estimation of direction, radial velocity, range and depth are close to real values. Numerical simulation results are presented and discussed.

Keywords: dispersion, interferogram, hologram, noise source, extended antennas, gain, directional characteristic, noise immunity, limiting offset, numerical simulation

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1. INTRODUCTION

A typical particular case of media with frequency dispersion is an oceanic waveguide with pronounced waveguide dispersion, on which we will focus our attention. One of the poorly developed problems in the processing of hydroacoustic information is to provide noise immunity in the conditions of spatial and temporal variability of the oceanic environment. Difficulties in approaching its solution arise...
from the a priori uncertainty of knowledge about the propagation medium (including random), as well as the weak value of the useful signal against the background of intense localized interference and surrounding noise. It is this situation that is of practical interest. In this regard, an urgent task seems to be the development of processing, which combined high noise immunity with adaptability to changing conditions of wave field propagation, i.e. with a decrease in the requirements for the volume of a priori information about the transfer function of the waveguide.

Theoretical studies, computational and field experiments have shown that such a combination can be provided by information technology for processing an interference pattern (interferogram) [1–5]. It is based on the formation mechanism of the interferogram of a broadband source caused by waveguide dispersion and multimode propagation [6]. Interferometric processing implements the coherent accumulation of spectral intensity along localized bands, which is then subjected to a 2D Fourier transformation. The converted spectral density (which we will conventionally call a hologram) is concentrated in a small area, providing high noise immunity.

At present, most of the main provisions of the theory, practical issues and potential possibilities of interferometric processing using single vector-scalar receivers have been sufficiently worked out. This made it possible to radically solve the problem of localization of low-noise sound sources [7,8]. If the current development of interferometric processing with the use of single vector-scalar receivers undoubtedly confirms its fruitfulness, then the questions remain, what results can be obtained by applying it in relation to extended receiving antennas.

This article is devoted to this issue, in which the results of the theory [1,2] are generalized for the case of using horizontal and vertical receiving linear antennas. The results of numerical simulation are presented.

2. ALGORITHM OF SIGNAL PROCESSING

The waveguide is assumed to be horizontally uniform. Let the number of elements receiving antenna is B, \( b = 1, B \), interelement distance – \( d \). The fields from each element of the antenna are summed, and an interferogram is formed at the output, to which a 2D Fourier transformation is applied.

2.1. HORIZONTAL LINEAR ANTENNA

Let's designate the distance from the element \( Q_b \) to the source \( S \) as \( r_b \) (Fig. 1). Let us choose the first element as a reference \( Q_1 \). The aperture \( L = (B - 1)d \) is assumed to be much less than the distance to the source, \( L \ll r_\rho \), then \( r_b = r_1 - (b - 1)d\sin\theta \), where \( \theta \) – the bearing. Antenna compensation is provided in the direction
of the angle $\theta$. Since we have to compensate for the difference in the distances from the source to the various elements of the antenna, we multiply the field of the $b$-th element by $\exp[i b_n(\omega) (b-1) \sin \theta]$, where $b_n(\omega)$ is the selected horizontal wavenumber at the middle frequency of the source spectrum $\omega_0 = 2\pi f_0$.

The field at the output of the $b$-th element is written as a sum of modes [9]

$$p_m(\omega, r) = \sum_n A_m(\omega, r_c) \exp[i(b_n(\omega) r - (b-1)(b_n(\omega) - b_n(\omega) \eta))] I_m, \tag{1}$$

where

$$\eta = d \sin \theta/2, \quad \eta_* = d \sin \theta_*/2. \tag{2}$$

Here $A_m$ and $b_m$ are the amplitude and horizontal wavenumber of the $m$-mode. Cylindrical field divergence, modal attenuation, source $z_i$ and $z_f$ antenna element depths are formally taken into account by the amplitude dependence of the modes. At the antenna output $p_m(\omega, r)$, the field, neglecting the dependence of the amplitude on the distance $A_m(\omega, r) \approx A_m(\omega, r_0)$, after simple transformations can be represented as

$$p_m(\omega, r) = \sum_n A_m(\omega, r_c) \exp[i(b_n(\omega) r - (b-1)(b_n(\omega) - b_n(\omega) \eta))] I_m, \tag{3}$$

$$I_m = \frac{\sin[B(b_n(\omega) \eta - b_n(\omega) \eta_*)]}{\sin[(b_n(\omega) \eta - b_n(\omega) \eta_*)]]. \tag{4}$$

The antenna interferogram $P_m(\omega, r)$ is $|p_m(\omega, r)|^2$, according to (3), is

$$P_m(\omega, r) = \sum_m \sum_n P_{mn}(\omega, r), \tag{5}$$

where

$$P_{mn}(\omega, r) = A_m(\omega, r) A_n(\omega, r) \exp[i b_{mn}(\omega)(r - (b-1)\eta)] I_{mn}, \tag{6}$$

$$I_{mn} = I_m I_n. \tag{7}$$

Here $b_{mn}(\omega) = b_m(\omega) - b_n(\omega)$. Consider the case of a moving source with a constant radial velocity $w$ (the projection of the velocity in the direction to the antenna). We assume that the distance $r_1$ corresponds to the initial moment of time $t_0 = 0$. Further, in the interferogram (5), we pass from the distance $r_1$ variable to the time variable $t$ and carry out a 2D Fourier transformation. At the output of the integral transformation, the spectral density is determined by the expression

$$F_m(\nu, \tau) = \int_{\nu_{-\Delta\nu/2}}^{\nu_{+\Delta\nu/2}} \int_{\tau_{-\Delta\tau/2}}^{\tau_{+\Delta\tau/2}} P_m(\omega, t) \exp[i(\nu t - \omega \tau)] d\omega d\tau = \sum_{m} \sum_{n} F_{mn}(\nu, \tau), \tag{7}$$

where $\nu = 2\pi f_0$ and $\tau$ are the frequency and time of the hologram, $\Delta\tau$ and $\Delta\omega$ are the observation time and spectrum width. Using the approach for obtaining a hologram of a single receiver [1], we obtain

$$F_m(\nu, \tau) = A_m(\omega, r_c) \delta(\omega, r_c) I_{mn}(\omega, \eta, \eta_*) \Delta \omega \Delta \tau \times \exp\left[i \left(\frac{\nu \tau}{2} - m \eta_\tau\right)\right] \times \exp\left[i \left(m \eta - \frac{\Delta \omega}{2}\right) + \left(\tau - \frac{\Delta \tau}{2}\right) (\nu t - \omega \tau)\right] \times \frac{\sin[(\tau - \frac{\Delta \tau}{2} + \omega t) \eta + \nu t \eta]}{\sin[\nu t \eta]}. \tag{8}$$

Here $\alpha = dh(\omega)/d\omega = h_{\omega}^{(1)} - h_{\omega}^{(0)}$, $l$ is the number of the mode, in the vicinity of which the modes are in phase, $t_*$ the selected time moment in the observation interval $\Delta t$, $0 < t_* < \Delta t$. The introduction of the quantity is useful in interpreting a hologram. In fact $\alpha(m - n) = h_{mn}(\omega), (d\alpha/d\omega)(m - n) = dh_{mn}(\omega)/d\omega$. If we put $B = 1$, then $I_{mn} = 1$, and relation (8) turns into an expression for a single receiver [1].

As in the case of a single receiver, the spectral density of the antenna hologram is localized on a plane $(\tau, \nu)$ in a narrow strip in the form of focal spots, mirror-inverted relative to the origin. This feature is due to the symmetry of function (8) with respect to the permutation of the numbers of interfering
modes: \( F_m(n, \nu, \tau) = F_m(-\nu, -\tau) \). Focal spots are located in the first and third quadrants if the radial velocity \( w < 0 \) (the source is approaching the antenna) and in the second and fourth quadrants when the source is moving away from the antenna \( (w > 0) \). The localization region contains \((M - 1)\) the main maxima with coordinates \((\tau_\mu, \nu_\mu^\star)\), where \( M \) is the number of modes that form the field, \( \mu = 1, M - 1 \) is the number of the focal spot. The peak closest to the origin is due to the interference of neighboring modes and is located at a point \((\tau_\nu, \nu_\nu^\star)\). The coordinates of the adjacent peak caused by the interference of the mode numbers, \((m, m + 2)\), – \((\tau_{mn}, \nu_{mn}^\star)\). Coordinates of the most distant peak from the origin, due to the interference of the first and last modes \((\tau_{M-1}, \nu_{M-1}^\star)\). The \((M - \mu)\) main peaks are summed at the points with coordinates \((\tau_\mu, \nu_\mu^\star)\).

The coordinates of the main maxima \((\tau_\mu, \nu_\mu^\star)\) are located on a straight line \( \tilde{\nu} = \tilde{\epsilon} \tau \) with a slope factor

\[
\tilde{\epsilon} = -\frac{wh_m(\omega_0)}{n_1(dh_m(\omega_0)/d\omega)},
\]

and occupy a band between the values \( \tau = \pm 2\pi\Delta\omega \) and \( \tilde{\nu} = \pm 2\pi / \Delta t \). Outside these bands, the spectral density of the hologram is practically suppressed. The angular coefficients of the interference fringes of the interferogram and the line of the location of the main maxima of the spectral density are related by the relation \( \tilde{\epsilon} = -\Delta\Omega / \Delta t \), where \( \Delta\Omega \) is the frequency shift of the interference maxima of the wave field over time. The frequency shift expresses the condition for maintaining the phase between constructively interfering modes caused by a change in the distance between the source and the antenna [6].

Under the condition \( r_1 >> |(B - 1)| \eta - \nu t | \), as follows from (8), the estimates of the distance and radial velocity of the source

\[
\hat{r}_1 = \kappa_{r\mu} \tau_\mu, \quad \hat{w} = -\kappa_{w\mu} \nu_\mu.
\]

coincide with expressions for a single receiver [1]. Here

\[
\kappa_{r\mu} = \frac{dh_m(n, \omega_0)}{d\omega}, \quad \kappa_{w\mu} = \frac{h_m(n, \omega_0)}{|1|}
\]

– coefficients that determine the frequency and spatial scales of variability of the transfer function of the waveguide [9]. The source parameter estimates obtained as a result of measurements, in contrast to their true values, are indicated by a dot above. The bar above denotes averaging over mode numbers. Adaptive methods of interferometric processing that allow estimating the source distance and radial velocity in the absence of information about the transfer function, i.e. without knowing the coefficients (11), are presented in [3].

Factor

\[
I_{mn} = I_n(\omega_0, B, \eta, \eta_\nu)I_m(\omega_0, B, \eta, \eta_\nu),
\]

determined from formulas (4), (6), characterizes the distribution of the spectral density of the antenna hologram in relation to the hologram of a single receiver. Peculiarities of dependence (12) on bearing \( \theta \) will be considered by the example of the compensation angle \( \theta_0 = 0 \). The main maxima of the functions \( I_{mn} , \max I_{mn} = B \), correspond to the values

\[
\sin \theta = \pm 2k\pi \frac{1}{h_m,\nu(\omega_0) d},
\]

where \( k = 0, 1, ... \) is the order of the spectrum. In the direction of bearing \( \theta = 0 \), \( k = 0 \), a coherent addition of complex
amplitudes of all mode numbers is carried out; \( k = 1, 2, \ldots \) the position of the maxima depends on the mode numbers. The zeros of the function \( I_{m,n} \) correspond to the values of the angles determined by the expression

\[
\sin \theta = \pm \frac{1}{B h_{m,n}(\omega_0) d}.
\]

where \( j \) are integers, except for \( B, 2B, \ldots \). Zeros are located about \( B \) times more often than the main highs. The secondary maxima \( I_{m,n} \) lie approximately in the middle between two adjacent zeros, i.e. fall on values

\[
\sin \theta \approx \pm (2g + 1) \frac{1}{B h_{m,n}(\omega_0) d},
\]

where \( g \) are integers. Secondary maximum levels do not exceed 0.2 of the \( B \) value [10].

Thus, the positions of the main highs do not depend on the number of elements \( B \), between each two main maximums there are \( (B - 1) \) zeros and secondary maxima \( (B - 2) \). As the parameter \( B d \omega_0 \) increases, i.e. the larger the aperture and the higher the frequency range, as follows from (14), the sharpness of the main maxima increases (the width decreases) and the number of bearings at which the spectral density is equal to zero increases. The distance between the major maxima for a particular wavelength is determined by the inter-element distance.

We put the interelement distance, \( d = \lambda(n + 1)/2 \), where \( \lambda \) is the wavelength for which we will take the value \( \lambda = 2\pi/b_0(\omega_0) \). Then, as follows from (13), if \( n = 0 \), then only the main maximum of order zero exists \( (\theta = 0, \pi) \), if \( n = 1 \) – two main maxima of the zero and first orders \( (\theta = 0, \pi/2, \pi, 3\pi/2) \), etc. The number of orders \( k \) is determined by the condition \( 2k \leq n + 1 \). With an increase in the interelement distance and a decrease in the wavelength, the number of main maxima increases. The width of the main maxima, according to (14), does not depend on their spectrum order and is equal to \( \Delta \theta = \lambda/Bd \). By changing the frequency range (at a given inter-element distance), you can adjust the positions of the main maxima. The multimode propagation regime, obviously, leads to the "blurring" of the zeros of the spectral density of the hologram, a shift in the positions of the main maxima (for angles \( \theta \neq 0 \)) determined by relation (13), and an increase in their width. The introduction of the compensation angle \( \theta \) leads to the appearance of a term in the right-hand sides of relations (13)-(15) \( b_0(\omega_0) \sin \theta/b_{m,n}(\omega_0) \).

With respect to a single receiver, the effectiveness of interferometric processing using an antenna is characterized by the gain

\[
\chi = \frac{|G_{an}(B, \theta, \theta)|}{|G_r|},
\]

where

\[
G_{an}(B, \theta, \theta) = \iint |F_{an}(\tau, \nu)| d\tau d\nu,
\]

\[
G_r = \iint |F_{r}(\tau, \nu)| d\tau d\nu
\]

and antenna directivity

\[
D(B, \theta, \theta) = \frac{G_{an}(B, \theta, \theta)}{\max G_{an}}.
\]

Here, the subscript «r» refers to a single receiver, \( U \) – is the spectral density localization area.

From (8), (12) and (16) it follows that \( \chi_{\max} = B^2 \).

Thus, in the case of a horizontal antenna, the processing allows the direction finding of the source both by scanning the directional characteristic (19) and based on processing the antenna hologram for various vector-scalar field components, as it was proposed...
with respect to a single vector-scalar receiver [5].

2.2. Vertical linear antenna

The field at the output of the $b$-th element, according to [9], is represented as

$$ p_b(\omega, r) = \sum_m \Psi_m(z_b) A_m(\omega, r) \exp[ih_m(\omega)r],$$

(20)

where $\Psi_m(z)$ is the eigenfunction of the $m$-th mode; $z_b$ is the depth of the $b$-th element. As above, the cylindrical field divergence, modal attenuation, and source depth $z_b$ are formally taken into account by the mode amplitude. In (20), the slow change in the eigenfunction of frequency is neglected. At the antenna output, the field is

$$ p_m(\omega, r) = \sum_b \sum_m \Psi_m(z_b) A_m(\omega, r) \exp[ih_m(\omega)r].$$

(21)

The interferogram $P_m(\omega, r)$, according to (20), (21), takes the form

$$ P_m(\omega, r) = \sum_b \sum_a \sum_m \sum_n p_m^{(ba)}(\omega, r),$ $

(22)

where

$$ p_m^{(ba)}(\omega, r) = \Psi_m(z_b) \Psi_n(z_a) A_m(\omega, r) A_n(\omega, r) \exp[ih_m(\omega)r].$$

(23)

Let the source move with radial velocity $w$ and assume that the distance $r$ corresponds to the initial moment of time $t_0 = 0$. Passing in interferogram (22) from the distance variable $r$ to the time variable $t$ and applying the $2D$ Fourier transformation to it, we obtain

$$ F_m^{(b)}(\hat{v}, \tau) = \Delta t \int_0^{\Delta t} P_m(\omega, t) \exp[i(\hat{v}t - \omega\tau)] dt d\omega = $$

$$ = \sum_b \sum_a \sum_m \sum_n p_m^{(ba)}(\hat{v}, \tau),$$

(24)

where

$$ q^{(\omega)}(\sigma) \sim 1.5 J^2, \quad \text{where} \quad J = \Delta t/(T + \delta t)$$

If we put $B = 1$, then $z_b = z_a$, and formula (25) turns into the corresponding formula for a single receiver [1].

The qualitative picture of the localization of the two-dimensional spectral density (24) of a vertical antenna, as in the case of a horizontal antenna (7), is similar to a single receiver. When the condition $r >> |w| t_*$ is satisfied, as follows from (25), the distance and radial velocity of the source are determined by expression (10).

The gain $\chi$ is given by expression (16), from which, according to (25), one should expect $\chi \approx B^2$. The equality is fulfilled when the values of the eigenfunctions of the modes of the antenna elements at different depths are equal to each other $\Psi_m(z_b) = \Psi_m(z_a)$, $z_b \neq z_a$. The source direction finding is carried out on the basis of an algorithm as proposed in relation to a single vector-scalar receiver [5].

3. Noise immunity of signal processing

It is proposed to characterize the noise immunity of interferometer processing using a single receiver by the limiting (minimum) input signal-to-noise ratio ($s/n$) $q^{(\omega)}$, when stable detection and estimates of bearing, radial velocity, distance and depth are close to real ones for $s/n$ values $q >> q^{(\omega)}$ [1].
is the number of time intervals (counts), on which the coherent accumulation of spectral maxima of the wave field along the interference fringes is realized [2]. Here $T$ is the duration of the noise realization, $\delta t$ - the interval between samples. Despite the fact that the estimate $q_{lim}^{(r)} = 1.5 f^2$ was established on the basis of a number of physical considerations, and not derived from any more general principles, it was found correctly and repeatedly verified on the results of numerical and field experiments. This made it possible to construct a theory of noise immunity of interferometric processing using a single receiver. Let us generalize the obtained estimate for extended antennas.

Suppose that the noise signal and interference are statistically unrelated random processes, at the input of the antenna elements the interference is not correlated. To satisfy the second condition, it is sufficient to require the fulfillment of the inequality $d > > \lambda/2$. Then the limiting input $s/n$ ratio at the antenna element is estimated as

$$q_{lim}^{(an)} = B q_{lim}^{(r)}/\chi. \tag{26}$$

In the case of a horizontal linear antenna, the highest processing immunity is achieved with a bearing equal to the compensation angle $\theta = \theta_{n}, \min q_{lim}^{(an)} = q_{lim}^{(r)}/B$. For the limiting input $s/n$ ratio of the vertical antenna, we have $q_{lim}^{(an)} \approx q_{lim}^{(r)}/B$.

For the limiting (maximum) removal $r_{lim}$ of the noise source, when the operability of the interferometric processing is preserved, using the same approach as in the case of a single receiver [3], we obtain

$$r_{lim} = \frac{1}{5\sqrt{1.5}} \sqrt{\frac{q\chi}{B}} \Delta t \left[ dh_{im}(\omega_{n})/d\omega\right], \tag{27}$$

With an increase in the input value of $s/n$ at the antenna element, observation time $\Delta t$, antenna elements $B$, gain $\chi$, and center frequency $\omega_{n}$, the maximum range of the noise source increases. The latter is due to the fact that, with increasing frequency, the group velocities of the modes asymptotically tend to a constant value, which does not depend on the mode number [9]. A characteristic feature of relation (27) is the fact that it includes the parameters of the transfer function of the waveguide and therefore the value of the limiting distance $r_{lim}$ is different depending on the choice of the water area. If we put $B = 1$, then formula (27) turns into the corresponding formula for a single receiver [3].

Note that if at each $b$-th receiver we first perform interferometric processing and then sum up the spectral densities of the holograms at the antenna output, then there will be no gain in noise immunity with respect to a single receiver.

4. SIMULATION RESULT

Numerical simulation was performed for a horizontally uniform waveguide with a depth of $H = 70$ m. The distribution of the sound velocity over depth is shown in Fig. 2. Parameters of an absorbing liquid homogeneous bottom: the ratio of the density of bottom and water $\rho = 1.8$, a complex refractive index $n = 0.85(1 + i0.02)$.

The number of antenna elements $B = 21$. The elements of the horizontal antenna are located at the bottom, $z_b = 70$ m, the elements of the vertical antenna are at depths of $z_b = 10 + 2.5(b - 1)$ m, $b = 1, 21$. The inter-element distance is $d = 2.5$ m, which is approximately equal to half the wavelength $\lambda$ at a frequency of $f_0 = 310$ Hz. The noise source, located at a depth of $z_s = 30$ m, was moving away from
the antennas at a speed of \( w = 3 \text{ m/s} \). At the moment in time \( t = 0 \), the distance of the source from the support element of the horizontal antenna and the vertical antenna was \( r = 10 \text{ km} \). The accumulation time \( \Delta t = 60 \text{ s} \), the duration of the random realization \( T = 2 \text{ s} \), the time interval between samples \( \delta t = 0.5 \text{ s} \), so the number of samples \( J = 12 \).

The limiting input ratio \( s/n \) at the antenna elements, taking \( \lim \frac{q_{\text{lim}}}{q_{\text{lim}}/B} \), is equal to \( q_{\text{lim}} = 4.9603 \times 10^{-4} \).

Highlighted horizontal wavenumber \( b_1(\omega_0) \) (Table 1). If the wavenumbers \( b_1(\omega_0) \) of other modes are used as the quality, then the calculation results practically do not change. The results of numerical simulations for horizontal and vertical antennas are shown in Fig. 3-6 and Fig. 7-10, respectively. In order to increase the contrast and information content, average values are cut out on interferograms and holograms.

**Table 1**

| Modes Numbers, \( m \) | 1      | 2      | 3      | 4      | 5      | 6      | 7      |
|------------------------|--------|--------|--------|--------|--------|--------|--------|
| \( b_m \), m\(^{-1} \) | 1.2787 | 1.2737 | 1.2670 | 1.2593 | 1.2504 | 1.2412 | 1.2325 |
| \( d_{hm}/d\omega \), 1/m\(\cdot\)Hz | 0.6798\(\cdot\)10\(^{-3} \) | 0.6807\(\cdot\)10\(^{-3} \) | 0.6820\(\cdot\)10\(^{-3} \) | 0.6838\(\cdot\)10\(^{-3} \) | 0.6853\(\cdot\)10\(^{-3} \) | 0.6847\(\cdot\)10\(^{-3} \) | 0.6861\(\cdot\)10\(^{-3} \) |

Fig. 3 shows interferometric processing when received on an antenna support element. A contrasting interference pattern is observed (Fig. 3a), the spectral density of the hologram (Fig. 3b) is concentrated in six focal spots. The focal spots of the numbers \( \mu = 1,2 \) partially overlap. The directivity characteristic (Fig. 3c), naturally, has circular symmetry.

In Fig. 4 shows the results of interferometric processing at the antenna output in the absence of compensation. Compared to a single receiver (Fig. 3), the contrast of the interferogram decreases (Fig. 4a) and the topology of the spectral density of the hologram changes (Fig. 4b). This is due to the incoherent summation of the fields on the antenna elements. In this case, the positions of the maxima of the focal spots are retained. The directional characteristic (Fig. 4c) has one main maximum corresponding to the zero order of the spectrum. Its width is half the spectral density \( \Delta\theta \approx 9.5^\circ \).

Directional characteristics at different compensation angles are shown in Fig. 5. With an increase in the compensation angle \( \theta^* \), the width of the main maximum increases. The largest width occurs at an angle \( \theta^* = 90^\circ \). These features of the behavior of the directivity characteristic, as evidenced by other modeling results (not reflected in this work), take place with an increase in the wave size \( \delta /\lambda \) while maintaining the number of elements or with a decrease in the number of antenna elements with a constant wave size.

In Fig. 6 shows the dependences of the normalized gain \( \hat{\chi} = \chi /\chi_{\text{max}} \) (16) on the bearing \( \theta \) for different values of
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**Fig. 3.** Normalized interferogram (a), hologram (b) and directional characteristic (c) of the antenna reference element.

**Fig. 4.** Normalized interferogram (a), hologram (b) and directional characteristic (c) of the antenna at the absence of compensation.

**Fig. 5.** Antenna directivity characteristics for compensation angles: a) $\theta_* = 30^\circ$, b) $\theta_* = 60^\circ$, c) $\theta_* = 90^\circ$.

**Fig. 6.** Dependence of the normalized gain $\hat{\chi}$ on the bearing $\theta$ for different values of the compensation angle $\theta_*$: (a) $\theta_* = 0^\circ$, (b) $\theta_* = 30^\circ$, (c) $\theta_* = 60^\circ$, (d) $\theta_* = 90^\circ$. 
compensation angle $\theta$. With an increase in the compensation angle, the width of the maxima increases. These dependencies are essentially another form of representing dependencies in Fig. 5. For this reason, the main maximums in Figs. 5, 6 have the same width. Normalized value of the coefficient $\chi_{\text{max}} = 384$. The maximum gain $B^2$, approximately equal, occurs in the direction of the compensation angle.

In Fig. 7 shows interferograms and holograms of three antenna elements. The configuration of the spectral density distribution regions is different, despite the coherent addition of the fields on the antenna elements. A shift in the position of the maxima of the focal spots is observed. This difference is explained by the different values of the eigenfunctions at the depths of the elements.

This difference is most clearly illustrated in Fig. 8, which shows the dependences of two normalized spectral maxima $F_b^{(1,2)}$ from the antenna element depth. The regions of localization of the spectral density of the first two focal spots in Fig. 7b. Numerical calculations are marked with dots. The distribution of the focal spot maxima has an oscillating form due to

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**Fig. 7.** Normalized interferograms (a, c, e) and holograms (b, d, f) of antenna elements located at depths of $z_b = 10, 37.5, 60$ m, respectively.

**Fig. 8.** Dependences of the normalized spectral maxima of the holograms of the antenna elements on their depth: (a) the first maximum $F_b^{(1)}$; (b) the second maximum, $F_b^{(2)}$.

**Fig. 9.** Normalized interferogram (a) and hologram (b) of the antenna.
The different values of the mode excitation coefficients (eigenfunctions) at the reception horizons. The nature of the change in the oscillations with a change in the position of the focal spot is associated with the interference of different numbers of modes that determine the location of localized regions. The ratio of normalized coefficients $\beta = \frac{F_{\text{max}}^{(1)}}{F_{\text{max}}^{(2)}} = 1.53$.

Fig. 9 shows the behavior of the normalized interferogram and the antenna hologram. Spectral density is predominantly localized in the region of the first focal spot.

In Fig. 10 shows, obtained by numerical simulation (points), the dependence of the normalized gain $\hat{\chi} = \chi(b)/\chi_{\text{max}}$ (16) on the number of elements $b$. The calculated values fit well with the estimated quadratic dependence $\hat{\chi} = b^2/\chi_{\text{max}}$ (dotted line). The value of the coefficient $\chi_{\text{max}} = 362$ is obtained from the condition for normalizing the calculated values.

Table 2 shows the values of the maximum ranges of a noise source using a single receiver and a linear antenna, depending on the input ratio $s/n$, calculated by formula (27). Observation time $\Delta t = 60$ s, value $|d\mu/d\omega| = 6.3 \times 10^{-6}$ s/m (see Table 1), parameter $\chi/B = 17.62$.

5. CONCLUSION

The intensive introduction of interferometric processing in hydroacoustics over the past few years has already made it possible to obtain a number of new results using single receivers, forcing to revise the previously established classical processing methods (matched-field processing), in short, all those areas where wave interference plays a role. These considerations, as well as, of course, to a large extent the importance of physical and applied problems that can be solved using antennas, stimulated the consideration of interferometric processing using extended linear antennas. An expression is obtained for the distribution of the spectral density of the hologram, which determines the gain and directivity characteristic. The noise immunity and the limiting removal of the noise source are estimated. The results obtained significantly expand the field of application of interferometric processing. The material is illustrated by numerical calculations for the low-frequency region of a noise source, which make it possible to clearly understand the efficiency of interferometric processing when working with multi-element antennas.

| Ratio $s/n$, $q$ | Linear Antenna, Elements count $B = 21$ | Single receiver |
|-----------------|------------------------------------------|----------------|
|                 | Limit of noise source distance $r_{\text{lim}}$, km |                 |
| $10^{-3}$       | 49.2                                     | 206.6          |
| $10^{-4}$       | 15.5                                     | 65.0           |
| $10^{-5}$       | 4.9                                      | 20.5           |
| $10^{-6}$       | 1.5                                      | 6.3            |
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