Implication of the hidden sub-GeV bosons for the \((g - 2)_\mu\), \(^8\text{Be} - ^4\text{He}\) anomaly, proton charge radius, EDM of fermions and dark axion portal

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(Dated: February 19, 2020)

We discuss new physics phenomenology of hidden scalar \((S)\), pseudoscalar \((P)\), vector \((V)\) and axial-vector \((A)\) particles coupled to nucleons and leptons, which could give contributions to proton charge radius, \((g - 2)_\mu\), \(^8\text{Be} - ^4\text{He}\) anomaly and electric dipole moment (EDM) of Standard Model (SM) particles. In particular, we estimate sensitivity of NA64\(\mu\) experiment to observe muon missing energy events involving hidden scalar and vector particles. That analysis is based on \texttt{GEANT4} Monte Carlo simulation of the signal process of muon scattering off target nuclei \(\mu N \rightarrow \mu NS(V)\) followed by invisible boson decay into Dark Matter (DM) particles, \(S(V) \rightarrow \chi\). The existence of light sub-GeV bosons could possibly explain the muon \((g - 2)\) anomaly observed. We also summarize existing bounds on ATOMKI X17\( (J^P = 0^-, 1^\pm)\) boson coupling with neutron, proton and electron. We implement these constraints to estimate the contribution of \(P, V\) and \(A\) particles to proton charge radius via direct 1-loop calculation of Sachs form factors. The analysis reveals the corresponding contribution is negligible. We also calculate bounds on dark axion portal couplings of dimension-five operators, which contribute to the EDMs of leptons and neutron.

I. INTRODUCTION

The measurement of anomalous magnetic moment of muon provides the potential signal of new physics. Indeed, the value of \((g - 2)_\mu\) measured by BNL \(^1\) differs from the prediction of Standard Model (SM) at the level of three standard deviations \(^2\), \(\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (287 \pm 80) \times 10^{-11}\). The existence of light and weakly coupled hidden bosons \(^3\) could be a possible beyond SM explanations of that discrepancy \(^4,5\). In particular, BELLE-II experiment \(^6\) has been already put constraints on hidden vector boson \(Z\) coupled with muons, which can contribute to \((g - 2)_\mu\) anomaly. In addition, \(M^3\) compact muon missing momentum experiment \(^7\) has been proposed recently at Fermilab to examine \((g - 2)_\mu\) puzzle. Moreover, muon fixed target NA64\(\mu\) experiment at CERN SPS \(^8\) plans to collect data after CERN long shutdown in 2021 to test sub-GeV boson contribution into muon \((g - 2)\).

The processes accompanied by the emission and decay of hypothetical hidden boson \(^9\) provide an additional evidence towards the weakly coupled particle interactions beyond SM \(^10\). Namely, ATOMKI Collaboration has been reported recently the \(\sim 6.8\sigma\) and \(\sim 7.2\sigma\) anomalies of \(e^+e^-\) pair excess from electromagnetically transition in \(^8\text{Be}\) and \(^4\text{He}\), respectively. The relevant \(^8\text{Be}\) data have been explained as creation and decay of X17 boson particle with mass \(m_X = 16.70 \pm 0.35 \pm 0.50\) MeV. Furthermore, most favored candidates, that could play the role of the X17 boson \(^11\) have spin-parity \(J^P = 1^+, J^P = 0^-,\) and \(J^P = 1^-\). In particular, in order to explain \(^8\text{Be}\) anomaly, authors of Ref. \(^15\) provided an analysis for excited \(^8\text{Be}\) states and presented anomaly-free extension of SM that contains gauge boson with experimentally favored couplings \(^15\). In addition, in Ref. \(^15\) light pseudoscalar state from Higgs extended sector was suggested to describe relevant \(e^+e^-\) excess in \(^8\text{Be}\) transition with coupling that satisfies existing constraints \(^28,29\). Moreover, authors of Ref. \(^17\) investigated the production of vector boson with primarily axial couplings to quarks that is consistent with experimental data \(^31\), such that new axial field has a mass \(m_X \sim 16.7\) MeV (see e.g., Refs. \(^34\) for recent review) and describes comprehensively nuclear properties of the \(^8\text{Be}(1^+) \rightarrow ^8\text{Be}(0^+\) anomalous transition.

However, in Ref. \(^37\) authors provide dedicated analysis of \(e^+e^-\) pair emission anisotropy in nuclear transitions of \(^8\text{Be}\), which has a possible relevance to that anomaly. Another comprehensive study of \(^8\text{Be}\) anomaly not involving beyond Standard Model explanation was carried out recently in Ref. \(^38\). In particular, 17 MeV excess in the experiment with \(^8\text{Be}\) \(^12\) and \(^8\text{He}\) \(^13\) is explained as background effect due to the quantum phase transition in the \(\alpha\)-like nuclei of \(^8\text{Be}, ^4\text{He}, ^{12}\text{C},\) and \(^{16}\text{O}\).

It is worth mentioning that NA64\(e\) facility at the CERN SPS \(^18\) has excellent opportunity of probing \(^8\text{Be}\) anomaly due to its dedicated searching sensitivity.
for short-lived hidden particles, \( \tau_X \lesssim 10^{-12} \) s. In particular, we expect that NA64e active target facility will be able to probe hidden pseudoscalar \( X^{17} \) boson after CERN LS2 in 2021.

Precise determination of the proton charge radius \( r_p^E \), one of the fundamental quantities of hadron physics, remains unsolved problem for many years. There are three methods of measurement of the proton charge radius from study: (1) cross section of elastic lepton-proton scattering, (2) Lamb shift in atomic hydrogen, and (3) Lamb shift in muonic hydrogen.

The most recent and precise result for the \( r_p^E \) extracted from the elastic electron scattering off proton was obtained by the A1 Collaboration at MAMI \[39\]:

\[
r_p^E = 0.879 \pm 0.005 \pm 0.006 \text{ fm.}
\]

It is in a good agreement with the 2014 CODATA recommended value \( r_p^E = 0.8751 \pm 0.0061 \) fm \[40\]. However, these results are in a sizable disagreement (by 5.6 standard deviations) with most accurate result for the \( r_p^E = 0.84087 \pm 0.00026 \pm 0.00029 \) fm obtained from Lamb shift in \( \mu p \) atom by the CREMA Collaboration at PSI \[41,42\]. In 2019 the proton radius was deduced from measurement of the electronic hydrogen Lamb shift: \( r_p^E = 0.833 \pm 0.010 \) fm \[43\], which led to a conclusion that the electron- and muon-based measurements of the \( r_p^E \) finally agrees with each other. Recently, the PRad Collaboration at JLab \[44\] reported on improved measurement of the proton charge radius from an electron-proton scattering experiment:

\[
r_p^E = 0.831 \pm 0.007(\text{stat}) \pm 0.012(\text{syst}) \text{ fm.}
\]

As stressed in Ref. \[44\], this prediction is smaller than the most recent high-accuracy predictions based on \( ep \) elastic scattering and very close to the results of the precise muonic hydrogen experiments \[41,42\]. Also it was noticed in \[44\] that their prediction is 2.7 standard deviations smaller than the average of all \( ep \) experimental results \[40\]. We note that an independent and a highly-precise measurement proposed by the COMPASS++/AMBER at the M2 beam line of the CERN SPS \[43\] has very strong physical motivation as independent and complimentary experiment to recent observation done by the PRad Collaboration \[44\]. On the other hand, the use of the muon beam in the planned COMPASS++/AMBER experiment \[43\] gives a unique opportunity to test electron-muon universality and to reduce systematic uncertainties and radiative corrections. For discussion of future experiments and overview on proton radius see, e.g., Refs. \[46\]-\[47\].

One should stress that from theoretical point new particles with different spin-parity assignments could contribute to resolving of puzzles in particle phenomenology and to more precise determination of their properties. E.g., one can imagine existence of new particles with different spin-parity assignments, e.g., scalar (\( J^P = 0^+ \)), pseudoscalar (\( J^P = 0^- \)), vector (\( J^P = 1^+ \)), and axial (\( J^P = 1^- \)) particles. Also one can analyze a possible contribution of these states to the \((g-2)_\mu\) anomaly. Note that effects of scalar, pseudoscalar, and vector particles on the Lamb shift in lepton-hydrogen and \((g-2)_\mu\) anomaly have been already discussed and estimated in literature \[48\]-\[53\]. We noticed that one can also estimate the relative contribution of new particles (\(S, P, V, A\)) to the proton charge radius via direct 1-loop calculation of Sachs form factors. From our preliminary analysis it follows that contribution of these particles to the charge radius of proton is negligible.

However, it is instructive to collect existing bounds on \( X^{17} \) boson coupling with SM fermions and calculate contribution of \( X^{17} \) to EDMs of leptons and neutron. The relevant coupling terms originate from dimension-five operators (see e.g., Eq. (3) below). These interactions are motivated by dark-axion portal scenarios, involving couplings of photon, dark photon, and axion-like particle (for details, see e.g., Refs. \[54\]-\[59\]).

Our paper is structured as follows. In Sec. \[II\] we consider effective couplings of sub-GeV bosons with SM fermions. In Sec. \[III\] we estimate sensitivity of NA64\( \mu \) muon active target experiment to probe sub-GeV Vector and Scalar mediator of DM by using comprehensive GEANT4 MC simulation. These bosons can possibly explain \((g-2)_\mu\) anomaly. In Sec. \[IV\] we summarize existing constraints on \(^8\)Be anomaly for hidden \( X^{17}(J^P = 0^-, 1^\pm) \) bosons. In Sec. \[V\] we estimate contribution of \( X^{17}(J^P = 0^-, 1^\pm) \) bosons to proton charge radius directly from Sachs form factors. We conclude, that current information on new particles suggests that their contribution to the charge radius of proton is negligible. In Sec. \[VI\] we also set constraints on dimension-five operator couplings of light bosons which can contribute to EDM of SM fermions. That analysis is motivated by dark axion portal study. Finally, in Sec. \[VII\] we summarize our results.

## II. EFFECTIVE LAGRANGIAN

We consider entirely phenomenological couplings of light bosons to SM particles, which are based on an effective theory approach. Namely, New Physics (NP) Lagrangian involving coupling of nucleons and leptons with scalar \( S \), pseudoscalar \( P \), vector \( V \), and axial \( A \) bosons, which could contribute to the proton radius, muon magnetic moment, and electric dipole moments of electron (muon) and neutron can be written as follows

\[
\mathcal{L}_{\text{NP}} = \sum_H \mathcal{L}_H + \sum_{H_1H_2} \mathcal{L}_{\gamma H_1H_2},
\]

where \( H = S, P, V, A \) and \( H_1H_2 = SV, PV, PA \). Here \( \mathcal{L}_H = HJ_H \), where \( J_H \) is the fermionic currents including effects of \( P \)-parity violation. They are composed of nucleons and fermions as

\[
J_H = \sum_{N=p,n} \bar{N} (g^N_H \Gamma_H + f^N_H \tilde{\Gamma}_H) N
+ \sum_{\ell=e,\mu,\tau} \bar{\ell} (g^\ell_H \Gamma_H + f^\ell_H \tilde{\Gamma}_H) \ell,
\]

where \( \Gamma_S = \tilde{\Gamma}_P = I, \Gamma_P = \tilde{\Gamma}_S = i\gamma^5, \Gamma_V = \tilde{\Gamma}_A = \gamma^\mu, \) and \( \Gamma_A = \tilde{\Gamma}_V = \gamma^\mu\gamma^5 \) are the Dirac spin matrices.
Second term in Lagrangian (1) describes the coupling of new particles with photon (here we list only the terms which contribute to the electric dipole moment):

$$\mathcal{L}_{\gamma SV} = \frac{e}{4M_p} g_{\gamma SV} F_{\mu\nu} V_{\mu\nu} S,$$

$$\mathcal{L}_{\gamma PA} = \frac{e}{4M_p} g_{\gamma PA} A_\mu P,$$

$$\mathcal{L}_{\gamma PV} = \frac{e}{4M_p} f_{\gamma PV} F_{\mu\nu} V_{\mu\nu} P. \quad (3)$$

$g_{\gamma H}^{N(t)}$, $g_{\gamma H_1 H_2}$ and $f_{\gamma H}^{N(t)}$, $f_{\gamma H_1 H_2}$ are the sets of $P$-parity even and $P$-parity odd couplings, respectively. In Appendix we list the expressions for the contributions of new particles to the muon magnetic moment and proton charge radius including both $P$-even and $P$-odd couplings, while in numerical analysis, for the first time we will neglect by the $P$-odd couplings. Later, we derive the constraints of combinations of $P$-even and $P$-odd couplings of new particles using data on electric dipole moments of leptons and neutron. However, we note that constraints on (3) couplings can be motivated by dark axion-portal study [54, 59].

### III. NA64 $\mu$ Experiment for Probing $(g - 2)_\mu$ Anomaly

The NA64$\mu$ is upcoming experimental facility at CERN SPS [11], [60], [61], which aims to examine light hidden sector particles weakly coupled to muons. It will utilize a muon beam at CERN SPS to search for missing energy signatures in the bremsstrahlung process on the active target, $\mu N \rightarrow \mu N E_{\text{miss}}$. That process can be associated with sub-GeV hidden vector boson $V$ invisibly decaying into light dark matter particles, $V \rightarrow \chi\chi$, or neutrinos, $V \rightarrow \nu\nu$. That vector particle is referred to $Z'$-boson, which interacts mainly with $L_{\mu} - L_{\tau}$ currents of SM. In addition, it can serve a sub-GeV vector mediator between SM and DM sector due to the mechanism of relic DM abundance [10], [63], [64]. Furthermore, in Refs. [10], [61], [67], authors considered a scenarios with muon-specific scalar mediator between visible and hidden matter in order to resolve $(g - 2)_\mu$ anomaly and DM puzzle.

In this section we revisit recent [60], [62] expected sensitivity curves of NA64$\mu$ for muon-specific couplings of sub-GeV vector and scalar hidden particles $L \equiv g_{\mu}^S S \mu + g_{\mu}^V V_{\mu} \gamma^\nu \mu$ by using comprehensive GEANT4 Monte-Carlo (MC) simulation. One can expect that relevant scalar coupling originates from UV completed models with vector-like fermions and Higgs extended sector [65], [67].

In Fig. 1 the expected limits of NA64$\mu$ detector are shown for hidden Scalar and Vector boson, we also set benchmark assumption, $g_{\mu}^S = g_{\mu}^V = 0$, such that pseudoscalar and axial vector coupling admixtures don’t contribute to $(g - 2)_\mu$ anomaly. The expected sensitivity of NA64$\mu$ was calculated by using GEANT4 MC simulation of missing energy signal of $E_0 = 100$ GeV muon scattering on target with heavy nuclei $\mu N \rightarrow \mu NS(V)$. The number of produced light bosons can be approximated as follows

$$N_{S(V)} \simeq \text{MOT} \times \frac{\rho N_A}{A} \times L_T \times \sigma_{S(V)}, \quad (4)$$

where MOT is a number of muons accumulated on target, $A$ is a atomic weight of target medium, $N_A$ is Avogadro’s number, $\rho$ designates the target density, $L_T \simeq 40 X_0$ is a typical distance that are passed by muon before producing $S(V)$ with the energy of $E_{S(V)} \gtrsim E_0/2$ in the active lead target of NA64$\mu$ ($X_0 \approx 0.5$ cm), $\sigma_{S(V)}$ is a total production cross-section of light bosons. For $m_{S(V)} \lesssim M_\mu$ that production rate can be approximated in bremsstrahlung-like limit as

$$\sigma_{S(V)} \simeq \langle g^\mu_{S(V)} \rangle^2 / M_\mu^2.$$  

Which implies that relevant sensitivity curves in Fig. III we have a plateau in the light mass region. In Fig. III we require $N_{S(V)} > 2.3$, which corresponds to 90% CL exclusion bound on $g^\mu_{S(V)}$ coupling for the background free case. In particular, a preliminary hadron contamination analysis and study of the detector hermiticity with muon beam [11] show that total background to be at the level $\lesssim 10^{-12}$. It is worth mentioning that muon energy losses in the lead target can be neglected [67], since the muon energy attenuation is small for typical beam energy, $(dE_{\mu}/dz) \simeq 12.7 \times 10^{-3}$ GeV/cm. In the NA64$\mu$ experiment one assumes to utilize two, upstream and downsteam, magnetic spectrometers. These spectrometers, will provide a precise measurements of initial and final muon energies [11]. We suppose that $S(V)$ being produced produced by muons in the target escapes the NA64$\mu$ detector without interaction decaying invisibly into DM particles.

### IV. $^8$Be Anomaly Constraints

It is worth mentioning that nucleon terms in Lagrangian (2) can be referred to hadron-$X$17 boson couplings [14], [15] for the case of parity-violating interaction [34]. In particular, authors of [14], [15] provide a rough estimate of $P$-even hadronic couplings of $X$17 boson as $|f^8_{A}/e| \simeq |g^0_{S}/e| \lesssim 1.2 \times 10^{-5}$ and $|f^8_{V}/e| \simeq |g^0_{V}/e| \lesssim (2 - 10) \times 10^{-3}$ from null result of $^8p$ decay in the ATOMKI experiment [12]. For $P$-odd hadronic couplings of $X$17 vector boson one can expect them to be proportional to quark axial couplings $g^A_{A(p)} \simeq f^A_{V(p)} \sim g^A_{V}$ in a manner of Ref. [17]. Namely, a comprehensive analysis of [17] for both, enhanced isoscalar, $^8\text{Be}^{*}(J^P = 1^+; T = 0) \rightarrow ^8\text{Be}^{*}(J^P = 0^+; T = 0) + X17$, and suppressed isovector, $^8\text{Be}^{*}(J^P = 1^+; T = 1) \rightarrow ^8\text{Be}^{*}(J^P = 0^+; T = 0) + X17$, nuclear transitions implies a conservative bounds $|g^A_{A(p)}| \simeq |f^A_{V(p)}| \lesssim 10^{-5} - 10^{-4}$. The hadronic terms in the Lagrangian (2) involving hidden scalar and pseudoscalar particles can be originated
from extended Higgs sector of SM [70]. In particular, light pseudoscalar can be a valid candidate for $^8$Be anomaly explanation [16]. The relevant Lagrangian reads

$$\mathcal{L} \supset \sum_{q=u,d} \frac{m_q}{v} \bar{P}q_i \gamma_5 q,$$  \hspace{1cm} (5)

where $v = 246$ GeV is the Higgs vacuum expectation value. This implies [16] that the resulting Yukawa-like couplings of $P$ to up and down type quarks are $\xi_u \simeq \xi_d \simeq 0.3$, with $\xi_u$ and $\xi_d$ being a linear combination of nucleon couplings, such that

$$g_P^u \simeq \frac{M_P}{v} (-0.40 \xi_u -1.71 \xi_d),$$  \hspace{1cm} (6)

$$g_P^n \simeq \frac{M_n}{v} (-0.40 \xi_u + 0.85 \xi_d).$$  \hspace{1cm} (7)

Therefore, one has conservative limits, $|g_P^u| \lesssim 2.5 \times 10^{-3}$ and $|g_P^n| \lesssim 5.5 \times 10^{-4}$, which, however depend on nuclear shell model of isospin transition [16]. We note, that Lagrangian [5] doesn’t respect gauge symmetry of SM unbroken gauge group, and therefore can be considered as effective interaction of UV completed model [71].

Now let us consider $^8$Be constraints for light hidden boson from lepton sector, which is described by the second term in the Lagrangian [2]. A numerous well motivated scenarios [72] have been suggested recently for explaining the ATOMKI $e^+e^-$ anomaly, which involve neutral vector boson interacting with leptons. That vector particle decays predominantly via $e^+e^-$ pair, with $\text{Br}(V \rightarrow e^+e^-) \simeq 1$, since its mass doesn’t exceed the masses of any hadronic states. The dominant constraints on vector coupling to electron come from NA48/2 data on $\pi^0 \rightarrow \gamma V(V \rightarrow e^+e^-)$ decay and from NA64 data on $eN \rightarrow eNV(V \rightarrow e^+e^-)$ bremsstrahlung $e^+e^-$ pair emission. In particular, NA48/2 experimental facility provides best upper limit on $X_17(J^P = 1^+)$ mixing with electrons, $\mathcal{L} \supset g_V^e \bar{V}_{\mu} e^+ \gamma^\mu e$, such that the allowed values of coupling are $g_V^e/e \lesssim 1.4 \times 10^{-3}$ at 90% CL. NA64 experiment has been recently set the lower limit on the relevant coupling at 90% CL [18]. Therefore, the existence of $X_17$ vector boson favors the following values of electron mixing $g_V^e/e \gtrsim 6.8 \times 10^{-4}$. The former bound can be resealed for the case of axial-vector coupling admixture, $\mathcal{L} \supset V_{\mu} \bar{e} \gamma^\mu e$, as $\sqrt{(g_V^e)^2 + (g_A^e)^2}/e \gtrsim 6.8 \times 10^{-4}$. It is worth mentioning that one can estimate the projected sensitivity of the NA64 to probe pseudo-scalar particles $X_17(J^P = 0^-)$. The authors of Ref. [10] provide the following limits for reduced Higgs-like coupling of $X_17( J^P = 0^-)$ boson with electrons $\mathcal{L} \supset \xi_P \frac{M_P}{v} \bar{P} \epsilon \gamma_5 e$

$$4.5 \lesssim \xi_P \lesssim 10.0,$$  \hspace{1cm} (8)

which are favored by experimental data from electron and proton beam-dump facilities [28] for $m_P \gtrsim 17$ MeV. The relevant limits $C_{Aff} = \xi_P$ are shown in Fig. (4) of Ref. [28]. These bounds can be transferred to the electron’s coupling in terms of Lagrangian [2] as follows $3.0 \times 10^{-5} \lesssim g_P^e/e \lesssim 6.7 \times 10^{-5}$. It means that $P$ is a relatively long-lived with respect to vector boson $X_17(J^P = 1^+)$, that has the allowed couplings in the

![Graphical representation](image-url)
Neutron \leq \text{Proton} \leq \text{Electron}

Therefore, the NA64 experiment has an excellent prospect for probing of X17, since it will decay mostly into $e^+e^-$ within the fiducial volume of the NA64 ($L_{fid} \sim 7-10$ m) due to large boost factor $E_p/m_p \approx 6 \times 10^3$, with typical decay length of $L_{dec} \approx 14$ m. We note however that our estimate is conservative, therefore one should perform a comprehensive Monte-Carlo simulation for the flux and spectra of hidden pseudo-scalars produced in the target by primary electrons, $eN \to eNP(P \to e^+e^-)$. That investigation will take into account realistic response and efficiency of the NA64 detector. We leave that task for future analysis. In Tab. II we summarize current limits on X17 couplings.

V. COMBINED CONTRIBUTION OF LIGHT BOSONS TO THE PROTON RADIUS

In this section we consider the problem of the proton charge radius. In particular, we discuss direct contribution of these light bosons into the proton radius via the charge Sachs form factor $G_E^p(q^2)$. As we stressed in Sec. II this possibility is quite interesting in the connection to planned precise measurement of the proton charge radius from analysis of the elastic muon-proton scattering. $P$-even electromagnetic vertex function is defined for incoming photon as

$$M_{\text{inv}}^P = \bar{u}(p') \left[ \gamma^\mu F_1(q^2) + \frac{i}{2M_p} \sigma^{\mu\nu} q_\nu F_2(q^2) \right] u(p).$$

Here $F_1$ and $F_2$ are the Dirac and Pauli form factors; $q^2 = -Q^2$. For minimal coupling of photon with proton and charged leptons

$$\mathcal{L}_{\text{em}l} = e A_\mu \left[ \bar{p} \gamma^\mu p - \bar{\ell} \gamma^\mu \ell \right].$$

It is interesting to look at the relative contribution of new hidden particles to both proton charge radius and muon $(g-2)_\mu$ ratio anomaly. We do the direct estimate of the contributions of new particles into the proton charge radius. The proton charge radius is defined as

$$\langle r_p^E \rangle^2 = -6[G_E^p(0)]' = -6[F_1^p(0)]' + \frac{3}{2M_p^2} F_2^p(0),$$

where $F_1^p$ and $F_2^p$ are the Dirac and Pauli electromagnetic form factors of the proton, respectively; $[F(0)]'$ means the derivative at $Q^2 = 0$. Here $F_2^p(0) = \kappa_p$ is the proton anomalous magnetic moment. In particular, using the mass value $m_N = 16.7$ MeV of the hypothetic X17 vector particle observed in the ATOMKI experiment, we get the following leading (logarithmic) contribution to the charge proton radius:

$$\langle \delta r_p^E \rangle^2 \approx 0.014 h_r^{(1)} \text{ fm}^2,$$

where

$$h_r^{(1)} = (g_V^p)^2 - (g_A^p)^2 + (f_V^p)^2 - (f_A^p)^2,$$

is the combination of couplings of vector and axial vectors with proton (see Appendix). Let us estimate that contribution for benchmark couplings shown in Tab. II for X17 boson. In particular, $h_r^{(1)} = 2(g_V^p)^2 - 2(g_A^p)^2 \approx 2.6 \times 10^{-7}$, that yields $\langle \delta r_p^E \rangle \approx 6 \times 10^{-5}$ fm. Therefore we conclude, that current information on new particles suggests that their contribution to the charge radius of proton is negligible.
VI. CONSTRAINTS ON COUPLINGS OF NEW PARTICLES USING DATA ON ELECTRIC DIPOLE MOMENTS OF LEPTONS AND NEUTRON

![Graph showing constraints on coupling combinations](image)

FIG. 3: 90% CL constrains on coupling combinations from EDM of SM fermions. Solid (dashed) green line shows bound on \( g_{SV} f_S \) \( f_S^\mu \) coupling from EDM of muon. Solid (dashed) blue line shows bound on \( g_{SV} g_{SV} f_S \) \( f_S^\mu \) coupling from EDM of electron. Solid (dashed) red line shows bound on \( g_{SV} g_{SV} f_S \) \( f_S^\mu \) coupling from EDM of neutron. Solid orange line represents combined limits on \( g_{SV} f_S \) from electron EDM and CHARM bounds.

In this section we derive the constraints on the combinations of \( P \)-even and \( P \)-odd couplings of new particles using data on electric dipole moments (EDM) of leptons and neutron. The contributions of new particles to EDMs are described by the diagram in Fig. 4 where square vertex is \( P \)-odd and round vertex is \( P \)-even coupling with leptons (neutron). The EDM of spin-\( \frac{1}{2} \) fermion \( \psi \) (neutron or leptons) is defined as \( d^E = D_E(0) \), where \( D_E(q^2) \) is the relativistic electric dipole form factor extracted from full electromagnetic vertex function of corresponding fermion \( \psi \).

\[
M_{inv} = \bar{u}_\psi(p_2) \Gamma^\mu(p_1, p_2) u_\psi(p_1),
\]

\[
\Gamma^\mu(p_1, p_2) = -\alpha^{\mu\nu} g_{\nu\gamma} D_E(q^2) + \ldots
\]

The contributions of individual diagrams in Fig. 4 are given in Appendix.

Using current upper limits/results for the electron, muon, and neutron EDMs:

\[
|d_E^e| < 0.11 \times 10^{-28} \text{ e cm},
\]

\[
d_\mu^e < (-0.1 \pm 0.9) \times 10^{-19} \text{ e cm},
\]

\[
|d_\mu| < 0.30 \times 10^{-25} \text{ e cm},
\]

we get the upper limits for combinations of couplings of new particles, which are displayed in Tab. 4. Let us consider several benchmark limits. Namely, for concreteness we set to zero couplings of dimension-five operators [3], \( g_{SV} = g_{PA} = f_{PV} = 0 \). That yields the following constraints on electron and muon interaction with \( P \) and \( S \) for light bosons masses \( m_V = m_P = m_S \ll M_e \\

\[
|d_e/|c| \approx \frac{|g_{SV} f_S^\mu|}{8\pi^2 M_e} < 5.5 \times 10^{-16} \text{ GeV}^{-1},
\]

or equivalently \( |g_{SV} f_S| < 4.5 \times 10^{-17} \) and \( |g_{SV} f_S^\mu| < 8.5 \times 10^{-6} \). In order to avoid interference between diagrams (1)-(4) in Fig. 4 we now consider a benchmark point \( g_S = g_P = 0 \). That implies the following limits or the product of vector-specific and pseudo-scalar couplings of leptons

\[
|d_\mu/e| = \frac{|g_{SV} f_S^\mu|}{8\pi^2 M_\mu} < 5.0 \times 10^{-7} \text{ GeV}^{-1},
\]

\[
|d_\mu| = \frac{|g_{SV} f_S^\mu|}{16\pi^2 M_p} < 5.0 \times 10^{-7} \text{ GeV}^{-1},
\]

which yield \( |g_{SV} f_S^\mu| < 1.7 \times 10^{-13} \) and \( |g_{SV} f_S^\mu| < 0.2 \times 10^{-3} \). For relatively light hidden bosons \( m_A = m_V = m_P = m_S \ll M_\psi \) one can also obtain corresponding constraints from neutron EDM, \( |g_{SV} f_S^\mu| < 2.3 \times 10^{-10} \) and \( |g_{SV} f_S^\mu| < 4.5 \times 10^{-10} \). Heavy bosons \( m_H \gg M_\psi \) yield the limits on coupling products, which are scaled as \( \sim (m_H/M_\psi)^2 \). These bounds are shown in Fig. 3. One can see from Fig. 3 that the most stringent constraints on couplings come from electron EDM bounds for \( m_H \ll M_e \). Moreover, for the benchmark values of electron coupling with vector, \( g_V/e \simeq 1.4 \times 10^{-3} \), and scalar, \( f_S/e \simeq 3.0 \times 10^{-5} \), one can also estimate the bound on \( g_{SV} \) that is favored by X17-boson existence. In particular, for \( m_H \simeq 16.7 \text{ MeV} \), one has \( g_{SV} \lesssim 7.7 \times 10^{-2} \) from Fig. 3. Corresponding bound...
TABLE II: Upper limits on couplings of new particles from data on EDMs of electron, muon, and neutron

| Coupling combination | Electron | Muon | Neutron |
|----------------------|----------|------|---------|
| $|g\gamma J_P|=|g_s f_s|, m_\gamma \ll M_\phi$ | $< 4.5 \times 10^{-17}$ | $(0.8 - 7.6) \times 10^{-5}$ | $< 2.3 \times 10^{-10}$ |
| $|g_\gamma g_V| = |g_{\gamma A} g_P f_A| = |g_{\gamma SV} g_V f_s|, m_\gamma \ll M_\phi$ | $< 1.7 \times 10^{-13}$ | $(0.2 - 1.4) \times 10^{-3}$ | $< 4.5 \times 10^{-10}$ |
| $|g_\gamma g_V| = |g_{\gamma A} f_A|, M_\gamma \simeq 16.7$ MeV | $\lesssim 7.7 \times 10^{-2}$ | – | $\lesssim 1.5 \times 10^{-3} - 3 \times 10^{-4}$ |

from neutron EDM yields $g_{\gamma VS} \lesssim 1.5 \times 10^{-3} - 3 \times 10^{-4}$ for $g_{\gamma S}^e / e (2 - 10) \times 10^{-3}$ and $f_s^2 / e \simeq 1.8 \times 10^{-3}$ provided in Tab. \[. Here we expect naively that $X_0$ is admixture of vector and pseudo-scalar states which have dark axion portal coupling as in Ref. \[55, 56\]. In particular, one can relate corresponding values of $G_{a\gamma\gamma}$ and $g_{\gamma SV}$ as follows, $G_{a\gamma\gamma} = e g_{\gamma SV} / (2 M_\mu)$. That implies conservative bound on dark axion portal interaction of $X_0$ states $G_{a\gamma\gamma} \lesssim 2.5 \times 10^{-4} - 5 \times 10^{-5}$ GeV$^{-1}$ for $m_\gamma = m_\gamma \simeq 16.7$ MeV. We note that our latter rough estimate is referred to the model, which incorporates consistently both $X_0 (J^P = 0^{-})$ and $X_0 (J^P = 1^{+})$ states for $^8$Be anomaly explanation. The development of that scenario however is beyond the scope of the present paper. Besides, we want to point out that proposed sensitivity for a future measurement of the proton EDM and indirect limit to neutron EDM which the JEDI Collaboration \[57\] plans to obtain at level of $\sim 10^{-29}$ can receive more stringent limit for the couplings by a factor $10^{-3}$. 

It is instructive to obtain constraint on $g_{\gamma} f_s^2$ coupling from combined limit on electron EDM and CHARM experimental bounds for dark axion portal interaction $G_{a\gamma\gamma}$ presented in Ref. \[53, 56\]. The authors of Ref. \[55, 56\] have set severe upper limit on $G_{a\gamma\gamma}$ assuming null result of CHARM experiment to observe $\gamma' \rightarrow a\gamma$ decay within regarding fiducial volume. The latter implies $m_{\gamma'} \gg m_a$, thus contribution of $\gamma'$ and $a$ into $d_e / e$ in that mass range reads as follows

$$d_e / e = \frac{G_{a\gamma\gamma} f_s^2 g_{\gamma}^e}{8 \pi^2} J \left( \frac{m_{\gamma'}}{m_e}, 0 \right). \tag{22}$$

Here we use the notations of Ref. \[55, 56\] denoting indices as $a = S$ and $\gamma' = V$ for axion-like and dark-photon particles respectively, the function $J (m_{\gamma'}/m_e, 0)$ is given by Eq. \[A18\] in Appendix. In particular, for $m_{\gamma'} \gg M_e$ one has

$$|g_{\gamma} f_s^2| < 1.3 \times 10^{-13} \left( \frac{G_{a\gamma\gamma}}{\text{GeV}^{-1}} \right)^{-1} m_{\gamma'}^2 / M_e^2 \log (m_{\gamma'}/M_e^2),$$

which yields $10^{-10} \lesssim |g_{\gamma} f_s^2| \lesssim 10^{-6}$ for the masses in the range $1 \text{ MeV} \lesssim m_{\gamma'} \lesssim 30$ MeV from CHARM experimental constraints in Fig. 3 of Ref. \[55\]. We show corresponding limit in Fig. 3 by solid orange line.

VII. SUMMARY

In this paper we discuss phenomenological aspects of new scalar, pseudoscalar, vector and axial particles coupled to fermions (nucleons and leptons), which could give contributions to proton charge radius and $(g - 2)_\mu$ ratio, $^8$Be anomaly and EDM of fermions. The main conclusions of this paper are:

- We estimate sensitivity of NA64$\mu$ muon active target experiment to probe sub-GeV Vector and Scalar mediator of DM by using comprehensive GEANT4 MC simulation. These bosons can possibly explain $(g - 2)_\mu$ anomaly. In case of NA64$\mu$ null result of observing muon missing energy events associated with hidden vector and scalar particles, $\mu N \rightarrow \mu NS(V)$, one can exclude new sub-GeV bosons as interpretation of $(g - 2)_\mu$ anomaly.

- We summarize existing constraints on $^8$Be anomaly for hidden $X_0 (J^P = 0^{-}, 1^{+})$ bosons. We estimate contribution of these particles to proton charge radius by direct calculation of Sachs form factors. It turns out that the resulting contribution is negligible.

- We also set constraints on couplings of dimension-five operators for light hidden bosons which can contribute to EDM of SM fermions. That novel EDM analysis is motivated by dark axion portal study, which involves axion-photon-dark-photon couplings.

Acknowledgments

We would like to thank S. N. Gninenko, N. V. Krasnikov and M. M. Kirsanov for many fruitful discussions. The work of V. L. was funded by the Carl Zeiss Foundation under Project “Kepler Center für Astro- und Teilchenphysik: Hochsensitive Nachweis- technik zur Erforschung des unsichtbaren Universums (Gz: 0653-2.8/581/2)”, by “Verbundprojekt 05A2017 - CRESST-XENON: Direkte Suche nach Dunkler Materie mit XENON1T/aT und CRESST-III, Teilprojekt 1” (Förderkennzeichen 05A17VTA)”, by CONICYT (Chile) under Grants No. PIA/Basal FB0821 and by FONDECYT (Chile) under Grant No. 1191103. The work of A. S. Zh. was supported by The Tomsk State University competitiveness improvement program.
Appendix A: Contributions of new particles to the muon magnetic moment, proton charge radius and EDM of fermions

Contributions of new particles to the anomalous magnetic moments of proton and charged leptons read

\[
\delta a_S^\psi = \frac{1}{8\pi^2} \int_0^1 dx \frac{(1-x)^2 \left((g_S^\psi)^2 - (f_S^\psi)^2 + x \left[(g_S^\psi)^2 + (f_S^\psi)^2\right]\right)}{(1-x)^2 + x(\mu_S^\psi)^2},
\]

(A1)

\[
\delta a_P^\psi = -\frac{1}{8\pi^2} \int_0^1 dx \frac{(1-x)^2 \left((g_P^\psi)^2 - (f_P^\psi)^2 + x \left[(g_P^\psi)^2 + (f_P^\psi)^2\right]\right)}{(1-x)^2 + x(\mu_P^\psi)^2},
\]

(A2)

\[
\delta a_V^\psi = \frac{1}{8\pi^2} \int_0^1 dx \frac{2x(1-x) \left((g_V^\psi)^2 - 3(f_V^\psi)^2 + x \left[(g_V^\psi)^2 + (f_V^\psi)^2\right]\right)}{(1-x)^2 + x(\mu_V^\psi)^2},
\]

(A3)

\[
\delta a_A^\psi = -\frac{1}{8\pi^2} \int_0^1 dx \frac{2x(1-x) \left(3(g_A^\psi)^2 - (f_A^\psi)^2 + x \left[(g_A^\psi)^2 + (f_A^\psi)^2\right]\right)}{(1-x)^2 + x(\mu_A^\psi)^2},
\]

(A4)

where \( \mu_H^\psi = m_H/M_\psi \), \( \psi = p, \ell^- \). Here we use the standard Feynman propagator for spin-0 particles \( D_{\mu=0}(k^2) = 1/(M^2 - k^2) \) and the one without longitudinal part for spin-1 particles \( D_{\mu=1}(k^2) = -g^{\mu\nu}/(M^2 - k^2) \). Also we drop all occurring ultraviolet divergent terms supposing that they can be removed by appropriate choice of counterterms.

Below we list the corrections from new particles \( S, P, V, A \) to the \( \langle r_p^2 \rangle \):

\[
\langle \delta r_P^E \rangle_S^2 = \frac{1}{8\pi^2 M_p^2} \int_0^1 dx \frac{(1-x)^2 \left(2(g_P^E)^2 - (f_P^E)^2 + x \left[(g_P^E)^2 + (f_P^E)^2\right]\right)}{(1-x)^2 + x(\mu_P^E)^2},
\]

(A5)

\[
\langle \delta r_P^E \rangle_P^2 = -\frac{1}{8\pi^2 M_p^2} \int_0^1 dx \frac{(1-x)^2 \left((g_P^E)^2 - 2(f_P^E)^2 + x \left[(g_P^E)^2 + (f_P^E)^2\right]\right)}{(1-x)^2 + x(\mu_P^E)^2},
\]

(A6)

\[
\langle \delta r_P^E \rangle_V^2 = \frac{1}{8\pi^2 M_p^2} \int_0^1 dx \frac{(1-x) \left((g_V^E)^2 + (f_V^E)^2 + x \left[7(g_V^E)^2 - 6(f_V^E)^2\right]\right)}{(1-x)^2 + x(\mu_V^E)^2},
\]

(A7)

\[
\langle \delta r_P^E \rangle_A^2 = -\frac{1}{8\pi^2 M_p^2} \int_0^1 dx \frac{(1-x) \left(- (g_V^E)^2 - (f_V^E)^2 + x \left[6(g_A^E)^2 - 7(f_A^E)^2\right]\right)}{(1-x)^2 + x(\mu_A^E)^2},
\]

(A8)

Let's consider two limiting cases: (1) \( m_H = m_S = m_P = m_V = m_A \ll M_\psi \), (2) \( m_H = m_S = m_P = m_V =
\[ m_A \gg M_\psi, \text{ where } \psi = p, \mu. \] The total contribution of new particles into \( a^\mu \) and proton charge radius read: Scenario (1):

\[ \delta a^\mu_{\text{tot}} = \frac{1}{16\pi^2} \left[ g_a^{(1)} - 8 h_a^{(1)} \log(\mu_H^2) \right], \]

\[ g_a^{(1)} = 3 \left( (g^S_3)^2 + (f^\mu_p)^2 \right) - \left( (g^p_3)^2 + (f^\mu_p)^2 \right) \]

\[ + 2 \left( (g^p_3)^2 + (f^\mu_p)^2 \right) + 18 \left( (g^p_A)^2 + f^\mu_A \right), \]

\[ h_a^{(1)} = (f^\mu_p)^2 + (g^p_A)^2, \] (A9)

\[ \langle \delta r_p^{E^2} \rangle_{\text{tot}} = \frac{1}{16\pi^2 M_p^2} \left[ g_r^{(1)} + 6 h_r^{(1)} \log(\mu_H^2) \right], \]

\[ g_r^{(1)} = 5 \left( (g^S_3)^2 + (f^\mu_p)^2 \right) - \left( (g^p_3)^2 + (f^\mu_p)^2 \right) \]

\[ - 8 \left( (g^p_3)^2 + (f^\mu_p)^2 \right) + \frac{47}{3} \left( (f^\mu_p)^2 + (g^p_A)^2 \right), \]

\[ h_r^{(1)} = (g^p_3)^2 - (g^p_A)^2 + (f^\mu_p)^2. \] (A10)

Scenario (2):

\[ \delta a^\mu_{\text{tot}} = \frac{1}{16\pi^2 m_H^2} \left[ g_a^{(2)} + h_a^{(2)} \log(\mu_H^2) \right], \]

\[ g_a^{(2)} = - \frac{7}{6} \left( (g^S_3)^2 + (f^\mu_p)^2 \right) + \frac{11}{3} \left( (g^p_3)^2 + (f^\mu_p)^2 \right) \]

\[ + \frac{2}{3} \left( (g^p_3)^2 + (f^\mu_p)^2 \right) \]

\[ h_a^{(2)} = (g^p_3)^2 - (g^p_A)^2 - (f^\mu_p)^2. \] (A11)

\[ \langle \delta r_p^{E^2} \rangle_{\text{tot}} = \frac{1}{8\pi^2 m_H^2} \left[ g_r^{(2)} + h_r^{(2)} \log(\mu_H^2) \right], \]

\[ g_r^{(2)} = - \frac{8}{3} \left( (g^S_3)^2 + (f^\mu_p)^2 \right) \]

\[ + \frac{11}{6} \left( (g^p_3)^2 + (f^\mu_p)^2 \right) + \frac{13}{6} \left( (g^p_3)^2 + (f^\mu_p)^2 \right) \]

\[ - \frac{49}{2} \left( (g^p_A)^2 + (f^\mu_p)^2 \right), \]

\[ h_r^{(2)} = \frac{2}{3} \left( (g^p_3)^2 + (f^\mu_p)^2 \right) - \left( (g^p_A)^2 + (f^\mu_p)^2 \right) \]

\[ + \left( (g^p_3)^2 + (f^\mu_p)^2 \right) \] (A12)

The contributions of individual diagrams in Fig. 4 are given by:

Diagrams 1+2:

\( S(P)-\)boson exchange

\[ d_s^E = \frac{e g_s \gamma S g_v f_S}{16\pi^2 M_p} J(\mu_{\psi}, \mu_{\psi}). \] (A14)

Diagrams 5+6:

\( P\)-boson exchange

\[ d_p^E = \frac{e g_p \gamma P g_f f_p}{16\pi^2 M_p} J(\mu_{\psi}, \mu_{\psi}). \] (A15)

For the second case we get:

\[ I(\mu) = 1 \]

\[ J(\mu, \tau) = \frac{1}{\mu^2 - \tau^2} \]

\[ \int_0^1 dx \frac{x^2(1-x)}{(x^2 + (1-x)\mu^2)}, \] (A17)

\[ \int_0^1 \log \frac{x^2 + \mu^2(1-x)}{x^2 + \tau^2(1-x)} \] (A18)

\[ \int_0^1 \frac{1}{3(\mu^2 - \tau^2)} \log \frac{\mu^2}{\tau^2}. \] (A20)

For the second case we get:

\[ I(\mu) = I(\mu) = \frac{1}{2}. \] (A21)

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