Determination of fragmentation functions and their application to exotic-hadron search

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We discuss studies on determination of fragmentation functions and an application to exotic-hadron search by using characteristic differences between favored and disfavored functions. The optimum fragmentation functions are determined for pion, kaon, and proton in the leading order (LO) and next-to-leading order (NLO) of the running coupling constant $\alpha_s$ by global analyses of hadron-production data in electron-positron annihilation. Various parametrization results are much different in disfavored-quark and gluon fragmentation functions; however, we show that they are within uncertainties of the determined functions by using the Hessian method for uncertainty estimation. We find that the uncertainties are especially large in the disfavored-quark and gluon fragmentation functions. NLO improvements are explicitly shown in the determination by comparing uncertainties of the LO and NLO functions. Next, we propose to use differences between favored and disfavored fragmentation functions for determining internal quark configurations of exotic hadrons. We make a global analysis for $f_0(980)$ for finding its internal configuration; however, uncertainties are too large to specify the structure at this stage.

§1. Introduction

Semi-inclusive hadron-production processes are important for investigating properties of quark-hadron matters in heavy-ion collisions and for finding the origin of the nucleon spin in lepton-nucleon scattering and polarized proton-proton collisions. Fragmentation functions (FFs) are key quantities in describing such hadron-production processes in high-energy reactions. They indicate hadron-production probabilities from partons.

The FFs have been determined mainly by hadron-production data of $e^+e^-$ reaction. Many measurements were done in the $Z^0$-mass region, whereas lower-energy data are not sufficient. The determination of the FFs is not in an excellent situation in comparison with the one of parton distribution functions (PDFs), which is obvious from the fact that there are huge differences between the FFs of different analysis groups, especially for disfavored-quark and gluon FFs\textsuperscript{1}. It led us to investigate uncertainties of the FFs\textsuperscript{2} as it was done in the PDFs for the nucleon and nuclei\textsuperscript{3,4}. In addition, making global analyses in the leading order (LO) of the running coupling constant $\alpha_s$ and the next-to-leading order (NLO) at the same time, we showed that the role of NLO terms for reducing the uncertainties of the determined FFs. We provided a useful code in calculating the optimum FFs for the pion, kaon, and proton from our global analyses\textsuperscript{2,5}. After our studies, there are works on related
global analyses of the FFs and also on a hadron-model estimate.

Next, we proposed a possible method for exotic-hadron search by using the FFs. In particular, the FFs are usually classified into favored and disfavored functions. The favored means the fragmentation from a quark which exists in a hadron as a constituent in a naive quark model. The disfavored means the fragmentation from a sea quark. Therefore, internal quark configuration should be reflected in both FFs. This fact led us to investigate an interesting suggestion to use the FFs for a possible exotic-hadron search by looking at differences between the favored and disfavored functions.

We explain these works in this article. In Sec. 2, the FFs are defined in $e^+e^-$ annihilation processes. Our global analysis method is explained in Sec. 3, and results are shown in Sec. 4. The idea of using the FFs for exotic-hadron search is introduced in Sec. 5. Our studies are summarized in Sec. 6.

§2. Fragmentation functions in $e^+e^-$ annihilation

The cross section of hadron-$h$ production in the $e^+e^-$ annihilation is described by a $q\bar{q}$-pair production $e^+e^-\to q\bar{q}$ followed by a fragmentation process from $q$ ($\bar{q}$ or gluon emitted from $q$ or $\bar{q}$) to the hadron $h$. The FF is defined by the cross section

$$F^h(z, Q^2) = \frac{1}{\sigma_{\text{tot}}} \frac{d\sigma(e^+e^-\to hX)}{dz},$$

(2.1)

where $\sigma_{\text{tot}}$ is the total hadronic cross section, and $Q^2$ is the virtual photon or $Z^0$ momentum squared in $e^+e^-\to \gamma, Z^0$. It is equal to the center-of-mass energy squared $s$ ($= Q^2$). The variable $z$ is defined by the energy fraction:

$$z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q},$$

(2.2)

where $E_h$ is the hadron energy. Namely, $z$ is the hadron energy scaled to the beam energy ($\sqrt{s}/2$). The fragmentation is described by the summation of all the parton contributions:

$$F^h(z, Q^2) = \sum_i C_i(z, \alpha_s) \otimes D_i^h(z, Q^2).$$

(2.3)

Here, $D_i^h(z, Q^2)$ is a fragmentation function of the hadron $h$ from a parton $i$ ($= g, u, d, s, \cdots$), $C_i(z, \alpha_s)$ is a coefficient function which is calculated in perturbative QCD and the convolution integral $\otimes$ is defined by $f(z) \otimes g(z) = \int_z^1 dy/y f(y)g(z/y)$.

The measurements of the FFs have been done in various $Q^2$, whereas they are parametrized at a fixed $Q^2$ point ($= Q^2_0$) as explained in the next section. The initial functions at $Q^2_0$ are evolved to the experimental $Q^2$ points by the standard DGLAP evolution equations. The equations are essentially the same as the ones for the PDFs by exchanging the splitting functions $P_{qg}$ and $P_{gq}$. There are also some differences between their NLO expressions, for example, in the modified minimal subtraction ($\overline{\text{MS}}$) scheme.
§3. Global analysis method

The FFs are expressed in terms of a number of parameters, which are determined by a $\chi^2$ analysis of the $e^+ + e^- \rightarrow h + X$ data. The initial functions are provided at $Q^2_0$ as

$$D_i^h(z, Q^2_0) = N_i^h z^{\alpha_i^h} (1 - z)^{\beta_i^h},$$  

(3.1)

where $N_i^h$, $\alpha_i^h$, and $\beta_i^h$ are the parameters. An apparent constraint for the parameters is the energy sum rule:

$$\sum_h M_h^i \equiv \sum_h \int_0^1 dz z D_i^h(z, Q^2) = 1,$$

(3.2)

where $M_h^i$ is the second moment of $D_i^h(z, Q^2)$. However, it is almost impossible to confirm this sum since the summation over all the hadrons cannot be taken practically. In our analysis, we tried to be careful that the sum does not significantly exceed 1 even within analyzed hadrons.

There are two types in the FFs: favored and disfavored functions. For the FFs of light quarks ($u, d, s$), they are assumed to be equal if they are favored or disfavored ones. The favored functions are given by

$$D_{\pi^+}^u(z, Q^2_0) = D_{\pi^+}^{\bar{u}}(z, Q^2_0) = N_{\pi^+}^u z^{\alpha_{\pi^+}^u} (1 - z)^{\beta_{\pi^+}^u},$$  

(3.3)

for $\pi^+$. The $\pi^+$ productions from $\bar{u}, d, s,$ and $\bar{s}$ are disfavored processes so that they are assumed to be equal at $Q^2_0$:

$$D_{\pi^+}^u(z, Q^2_0) = D_{\pi^+}^{\bar{d}}(z, Q^2_0) = D_{\pi^+}^s(z, Q^2_0) = N_{\pi^+}^{\bar{u}} z^{\alpha_{\pi^+}^{\bar{u}}} (1 - z)^{\beta_{\pi^+}^{\bar{u}}},$$  

(3.4)

The FFs from a gluon and heavy quarks are defined separately as

$$D_{\pi^+}^g(z, Q^2_0) = N_{\pi^+}^g z^{\alpha_{\pi^+}^g} (1 - z)^{\beta_{\pi^+}^g},$$

$$D_{\pi^+}^c(z, m_c^2) = D_{\pi^+}^{\bar{c}}(z, m_c^2) = N_{\pi^+}^c z^{\alpha_{\pi^+}^c} (1 - z)^{\beta_{\pi^+}^c},$$

$$D_{\pi^+}^b(z, m_b^2) = D_{\pi^+}^{\bar{b}}(z, m_b^2) = N_{\pi^+}^b z^{\alpha_{\pi^+}^b} (1 - z)^{\beta_{\pi^+}^b},$$

(3.5)

where $m_c$ and $m_b$ are charm- and bottom-quark masses. The parameters in Eqs. (3.3), (3.4), and (3.5) are determined so as to fit the data in Table I where experimental collaborations, center-of-mass energies, and the numbers of data are listed for the charged-pion production. It is clear that most data are taken at the $Z^0$ mass.
The FFs for the kaon and proton are parametrized in the similar way by considering favored and disfavored functions. The amounts of data are almost the same as the ones in Table I. The detailed should be found in the original article.

One of our major purposes is to show the uncertainties of the FFs as explained in Sec. 1. The uncertainties have been already estimated in the studies of nucleonic and nuclear PDFs. The same Hessian method is used for the uncertainty estimation. The $\chi^2$ is expanded around the minimum $\chi^2$ point $\hat{\xi}$: $\Delta\chi^2(\xi) = \chi^2(\hat{\xi} + \delta\xi) - \chi^2(\hat{\xi}) = \sum_{i,j} H_{ij} \delta\xi_i \delta\xi_j$, where $H_{ij}$ is called Hessian which is the second derivative matrix, $\xi$ indicates a parameter set, and $\hat{\xi}$ is the set at the minimum $\chi^2$ point. The confidence region is given in the parameter space by supplying a value of $\Delta\chi^2$. Using the Hessian matrix obtained in a $\chi^2$ analysis, we estimated the uncertainty of the FF by $\left[ \delta D^{\pi}_i(z) \right]^2 = \Delta\chi^2 \sum_{j,k} \left[ \partial D^{\pi}_i(z) / \partial \xi_j \right] \hat{\xi} H^{-1}_{jk} \left[ \partial D^{\pi}_i(z) / \partial \xi_k \right] \hat{\xi}$. There are some variations among groups on the appropriate $\Delta\chi^2$ value for showing the uncertainty range in a global analysis. The details are explained in our article about our $\Delta\chi^2$ choice.

§4. Determined fragmentation functions for pion, kaon, and proton

We show obtained FFs of the pion by the $\chi^2$ analyses of the $e^+e^- \rightarrow \pi^\pm X$ data in Fig. 1 where the FF data in the form of Eq. (2.1) and our parametrization result with an uncertainty band are shown at $Q^2 = M_Z^2$. The good agreement with the data indicates that the fit is successful from small- to large-$z$ regions.

Next, each FF is shown for the pion on the left-hand-side of Fig. 2 with uncertainty bands in both LO and NLO (MS). It should be noted that the charm- and bottom-quark FFs are shown at the scale of their mass thresholds $Q^2 = m_c^2$ or $m_b^2$, whereas the others are shown at $Q^2 = 1$ GeV$^2$. The uncertainties are generally larger in the LO, which indicates the FF determination is improved due to NLO terms. We notice that disfavored-quark and gluon FFs have large uncertainties in both LO and NLO.

The NLO functions are then compared with other parametrizations of KKP, Kretzer, AKK, and DSS in Fig. 2. Some disfavored-quark and gluon functions, for example s-quark functions of Kretzer and AKK, are completely different between the analysis groups; however, they are within our uncertainty bands. There are not much differences between the groups in favored- (u), charm-, and bottom-quark functions except for the small-$z$ region. The large discrepancies among the different parametrizations especially in the disfavored-quark and gluon functions were not clearly understood before our work. Therefore, it is important to point out in our studies that they are not due to an inappropriate analysis of some groups and that
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They come from experimental errors and inaccurate flavor decomposition for light quarks. It is simply impossible to determine accurate disfavored-quark and gluon FFs from current experimental data.

For the kaon and proton, our LO and NLO FFs are shown in Fig. 3. We found that both FFs are not better determined than the pion FFs. There is a similar tendency with the pion case that the disfavored-quark and gluon FFs are
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not determined well. The NLO improvement, namely the uncertainty reduction, is clear in the kaon, whereas it is not apparent in the proton. We also found[2] in both kaon and proton that all the different analyses are consistent with each other in spite of their large differences in some functions because they are roughly within our uncertainty bands.

§5. Exotic-hadron search by fragmentation functions

From the analyses of ordinary hadrons, pion, kaon, and proton, we found characteristic differences between the favored and disfavored FFs. In Table I, the second moments are shown for $\pi^+$, $K^+$, and $p$. It is clear that the moments are larger for the favored functions than the ones for the disfavored functions. It indicates that internal quark configuration can be found by looking at flavor dependence of the FFs. This fact suggested us to use the FFs for exotic hadron search[9].

The exotic means a hadron with internal quark configuration other than ordinary $q\bar{q}$ and $qqq$. This topic has been investigated for a long time; however, an undoubted evidence has not been found yet. In the last several years, there have been reports on exotic candidates mainly from Belle and BaBar collaborations in charmed hadrons. In our work, we investigated a possibility that the FFs can be used for an exotic hadron search by using differences between the favored and disfavored FFs.

As one of the exotic mesons, we investigated a possibility of determining quark configuration of $f_0(980)$, which structure has been controversial for many years. According to a simple quark model, it is described by the configuration $(uu + dd)/\sqrt{2}$. However, it is known that theoretical strong decay widths are an order of magnitude larger than the experimental width[13]. Therefore, it is considered to be $s\bar{s}$, tetraquark, or $K\bar{K}$ molecule state. It used to be considered as a glueball candidate, but recent lattice QCD estimates indicate that the lowest scalar-meson mass is about 1700 MeV. There is an indication that the internal configuration could be determined by the radiative decay $\phi \rightarrow f_0\gamma$ and $2\gamma$ decay $f_0 \rightarrow 2\gamma$. However, the FF method could become a better way for judging its internal structure as well as other exotic-meson configurations.

We summarized in Table III how to judge the structure of the $f_0$ meson, especially by the second moments and $z$-dependent functional forms. All the possible configurations are considered in the table. For example, if $f_0$ is an $s\bar{s}$ state, $s\bar{s}$ is formed from $s$ by creating an $s\bar{s}$ pair from a radiated gluon. In the same way, a color neutral $s\bar{s}$ can be formed from a gluon by its splitting into an $s\bar{s}$ pair and a subsequent gluon radiation for color neutrality (see Ref.9 for details). The gluon can be radiated from either $s$ or $\bar{s}$, so that $M_g$ could be larger than $M_s$ as indicated in
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Table III. Possible $f_0(980)$ configurations and their features in FFs at small $Q^2$.

| Type                  | Configuration                  | Second moments | Peak positions |
|-----------------------|---------------------------------|----------------|----------------|
| Nonstrange $qar{q}$ | $(uar{u} + dar{d})/\sqrt{2}$ | $M_u < M_s < M_g$ | $z_u^\text{max} > z_s^\text{max}$ |
| Strange $qar{q}$   | $sar{s}$                      | $M_u < M_s \lesssim M_g$ | $z_u^\text{max} < z_s^\text{max}$ |
| Tetraquark (or $Kar{K}$) | $(uar{s}dar{s} + dar{s}uar{s})/\sqrt{2}$ | $M_u \sim M_s \lesssim M_g$ | $z_u^\text{max} \sim z_s^\text{max}$ |
| Glueball             | $gg$                            | $M_u \sim M_s < M_g$ | $z_u^\text{max} \sim z_s^\text{max}$ |

the table. If $s$ is a favored quark, a significant portion of its energy (namely large $z$) is transferred to $f_0(sar{s})$. It leads to a functional form which is mainly distributed in the large-$z$ region. In the table, this is denoted as $z_u^\text{max} < z_s^\text{max}$. In the same way, the relations in the second moments and the functional forms are listed for other configurations. Our suggestions are intended to give an idea on the criteria, and further details need to be investigated in more sophisticated hadron models.

There are data on the FFs of $f_0$ in the $e^+e^-$ annihilation. We have done a global analysis for determining the FFs in the same way with the analyses in Sec. 3. This is intended to judge the quark configuration of $f_0$ by using the criteria of Table III. The determined second moments are given by $M_u = 0.0012 \pm 0.0107$, $M_s = 0.0027 \pm 0.0183$, and $M_g = 0.0090 \pm 0.0046$. Then, the moment ratio becomes $M_u/M_s = 0.43 \pm 6.73$. From the ratio 0.43, the $f_0$ seems to be mainly an $s\bar{s}$ state. However, it is obvious from its huge error 6.73 that a clear determination is currently not possible.

The obtained FFs from the global analysis are shown in Fig. 4. The up- and strange-quark functions are distributed mainly in the large-$z$ region, which may indicate that $u$ and $s$ could be important constituents of $f_0$. However, the uncertainties of the determined FFs are huge and they are an order of magnitude larger than the FFs themselves. Therefore, it is not possible to draw a conclusion on its structure from the global analysis at this stage. Hopefully, much better data will be reported in the near future possibly from the Belle collaboration[15] for determining structure of exotic hadrons including $f_0(980)$.

Fig. 4. Determined FFs for $f_0(980)$. Note that uncertainty bands are much larger than the FFs.

§6. Summary

Our studies on the fragmentation functions were reported. First, global analyses have been done for pion, kaon, and proton for determining their FFs. In the past, it was not clear why there are large discrepancies among various parametrizations on disfavored-quark and gluon FFs. We clarified that they are consistent with each other by estimating the uncertainties of the FFs by the Hessian method in the sense that all the distributions are within our error bands. We also clarified the role of
NLO terms in reducing the uncertainties by comparing the determined FFs and their uncertainties in the LO and NLO. Our code for calculating the FFs was supplied on our web site.5

Next, we investigated a possibility of using the FFs for finding internal structure of exotic hadrons by using differences between the favored and disfavored FFs. We proposed to use the second moments and $z$-dependent functional forms of the FFs for determining the internal structure. As an example, the $f_0(980)$ meson was studied by considering all the possibilities of $q\bar{q}$, tetraquark, and glueball configurations. A global analysis has been done also for the FFs of $f_0$ by using the current $e^+e^-$ data; however, they were not accurately determined to the level of finding the internal structure, namely a difference between the favored-quark and disfavored-quark (and gluon) FFs. Much more accurate data are needed to discuss the internal structure by the FFs.

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