The Peierls substitution in an engineered lattice potential

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Artificial gauge fields open new possibilities to realize quantum many-body systems with ultracold atoms, by engineering Hamiltonians usually associated with electronic systems. In the presence of a periodic potential, artificial gauge fields may bring ultracold atoms closer to the quantum Hall regime. Here, we describe a one-dimensional lattice derived purely from effective Zeeman-shifts resulting from a combination of Raman coupling and radiofrequency magnetic fields. In this lattice, the tunneling matrix element is generally complex. We control both the amplitude and the phase of this tunneling parameter, experimentally realizing the Peierls substitution for ultracold neutral atoms.

Ultracold atoms subjected to artificial gauge fields can realize phenomena usually in the domain of electronic systems. Prime examples include the quantum Hall effect (for abelian gauge fields), and topological insulators (for non-abelian gauge fields)\textsuperscript{1}. Many of these phenomena are predicted to occur at extremely low temperatures, and adding a lattice potential to an ultracold system can increase the energy scales at which strongly correlated states are expected to emerge\textsuperscript{2}\textsuperscript{3}. Current techniques for generating periodic potentials in ultracold atom systems use optical standing waves created with suitably polarized counterpropagating lasers\textsuperscript{4}. In contrast, we describe a one-dimensional (1D) “Zeeman lattice” for ultracold atoms created with a combination of radiofrequency (rf) and optical-Raman coupling fields, without any optical standing waves. In this lattice, atoms acquire a quantum mechanical phase as they hop from site to site, explicitly realizing the Peierls transformation\textsuperscript{5} in the laboratory frame. Our approach extends existing Raman dressing schemes\textsuperscript{6} by simultaneously generating an artificial gauge field and an effective lattice potential.

Optical lattices generally result from the electric dipole interaction between an atom and the electric field of an optical standing wave, yielding a potential \( V_{\text{dip}}(r) \propto \alpha(\lambda)I(r) \), where \( \alpha(\lambda) \) is the atomic polarizability at laser wavelength \( \lambda \), and \( I(r) \) is the spatial intensity distribution\textsuperscript{4}. In such lattices, the natural units of momentum and energy are given by the single photon recoil momentum \( h \kappa L = 2\pi \hbar /\lambda \) and its corresponding energy \( E_{\text{L}} = \hbar^2 k_{\text{L}}^2 /2m \), where \( m \) is the atomic mass.

Quantum particles with charge \( q \) in a 1D periodic potential (here along \( e_x \)) acquire a phase \( \phi_j = (q / \hbar) \int_{x_{j-1}}^{x_j} A \cdot e_x \, dx \) upon tunneling from site \( j \to j+1 \) in the presence of a vector potential \( A \). For sufficiently strong potentials, this system is described by the tight-binding Hamiltonian

\[
H = -\sum_j [t \exp(i\phi_j) \hat{a}_j^\dagger \hat{a}_{j+1} + \text{h.c.}],
\]

where \( \hat{a}_j^\dagger \) describes the creation of a particle at site \( j \), and \( t \exp(i\phi_j) \) is the complex matrix element for tunneling between neighboring sites. Using the phases \( \phi_j \) to represent the effect of \( A \) is known as the Peierls substitution\textsuperscript{6}, and for uniform phase \( \phi \) the energy is \( E(k_x) = -2t \cos(\pi k_x / k_L - \phi) \), where \( k_x \) is the particle’s crystal momentum.

We realize the Peierls substitution for ultracold atoms by synthesizing a 1D effective Zeeman lattice that allows independent control of both \( t \) and \( \phi \). Previous experiments\textsuperscript{7} (a) controlled the amplitude and sign of \( t \) in driven optical lattices\textsuperscript{7}, or in addition (b) controlled \( \phi \) by means of rotating optical lattices\textsuperscript{8} or Raman-assisted tunneling in an optical superlattice\textsuperscript{9}. Our effective Zeeman lattice technique provides both a periodic potential and an artificial vector potential in the laboratory frame.

The Zeeman lattice arises from a combination of rf and Raman fields that simultaneously couple the spin states \( |m_F\rangle \) of \( ^{87}\text{Rb} \)’s \( F = 1 \) ground level, which are split by \( \hbar \omega_Z \) (Fig. 1ab). In the frame rotating at the rf frequency \( \Delta \omega \) and under the rotating wave approximation, the combined rf-Raman coupling contributes a term

\[
\hat{H}_{\text{rf+R}}(x) = \Omega(x) \cdot \hat{F} + \hat{H}_Q
\]

to the overall Hamiltonian, where \( \hat{F} = (\hat{F}_x, \hat{F}_y, \hat{F}_z) \) is the \( F = 1 \) angular momentum operator; \( \Omega = (\Omega_{\text{rf}} + \Omega_R \cos(2k_L x), -\Omega_R \sin(2k_L x), \sqrt{2} \delta) / \sqrt{2} \), in which \( \Omega_{\text{rf}} \) and \( \Omega_R \) are the rf and Raman coupling strengths, and \( \delta = \Delta \omega - \omega_Z \) is the detuning from Raman resonance; and \( \hat{H}_Q = -e(h^2 I - F^2) / \hbar \) describes the quadratic Zeeman shift. Equation (2) is the Zeeman Hamiltonian for an effective field \( \hat{B}_{\text{eff}}(x) = g \mu_B \hat{B}_z / g_F \mu_B \), where \( \mu_B \) is Bohr’s magneton and \( g_F \) is the Landé \( g \)-factor. This spatially varying effective Zeeman shift produces a 1D lattice potential (Fig. 1c). As atoms tunnel from site to site, \( \hat{B}_{\text{eff}} \) rotates by \( 2\pi \) about \( e_x \) (Fig. 1d) and the atoms acquire a geometrical Berry’s phase\textsuperscript{10} proportional to the enclosed solid angle \( \Theta_B \). The tunneling parameters \( t \) and \( \phi \), obtained from \( E(k_x) \), are non-trivial functions of \( \Omega \).

When \( \Omega_{\text{rf}} \gg \Omega_R, \delta, \epsilon \) the effective Zeeman shift reduces to \( \hbar |\Omega| \approx \hbar (\Omega_{\text{rf}} + \Omega_R \cos(2k_L x)) / \sqrt{2} \), and when
ℏΩ to the nearest neighboring site (points 1 to 5 in H) inversely proportional to ℏB effective Zeeman lattice; as plotted with 87Rb BECs in the lowest energy rf-dressed state \([12]\). Two \(\lambda/33\) nm Raman laser beams, counter-propagating \(e_x\) and \(e_y\), create a 1D lattice potential along \(e_x\), the direction of momentum exchange defined by the Raman beams.

We experimentally characterize the lattice in three ways: (i) we measure the effective mass \(m^* = ℏ^2[d^2E(k_x)/dk_x^2]^{-1}\), which in the tight-binding regime is inversely proportional to \(t\); (ii) we quantify the Peierls phase \(φ\) and test its robustness against fluctuations in \(Ω_\text{rf}\); and (iii) we investigate the diffraction of BECs from our effective Zeeman lattice. In each case, we start with \(^{87}\)Rb BECs in the \([F = 1, m_F = -1]\) state in a crossed optical dipole trap with frequencies \((f_x, f_y, f_z) = (13, 45, 90)\) Hz \([11]\). In the presence of a uniform bias field \(B_0e_y\), we apply an rf magnetic field with frequency \(Δω/2π = g_\mu_B B_0 = 3.25\) MHz and prepare the BEC in the lowest energy rf-dressed state \([12]\). Two \(λ=790.33\) nm Raman laser beams, counter-propagating along \(e_x\) and differing in frequency by \(Δω\), couple the BEC’s internal degrees of freedom with strength \(Ω_R\). The spatially varying eigenvalues of \(H_{\text{eff}}(x)\) (red, blue and green curves) give rise to our \(\lambda/2\) effective Zeeman lattice; as plotted \(β_\text{eff} = 1E_L, \ hΩ_\text{rf} = 10E_L, \ hδ = 2E_L\). Spatial precession of \(B_\text{d}(x)\) (dark arrow) and the solid angle \(θ_\text{B}\) it subtends when an atom tunnels to the nearest neighboring site (points 1 to 5 in c). This geometrical Berry’s phase gives the Peierls phase \(φ\).

\[ Ω_R ≫ Ω_\text{rf}, δ \] we obtain the analogous result \(h|Ω| ≈ h(Ω_R + Ω_\text{ rf} \, \cos(2k_x x))/\sqrt{2}\). In both of these limits, the larger of the two fields defines a natural quantizing axis about which the smaller field spatially modulates \(|Ω|\). For \(Ω_R ≫ Ω_\text{rf}\), this quantizing axis is spatially rotating.

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**FIG. 1. Effective Zeeman lattice.** a-b. A uniform magnetic field \(B_0e_y\) Zeeman-splits the levels in the \(F=1\) ground state manifold of \(^{87}\)Rb by \(\omega_2\), and provides a quadratic Zeeman shift \(c\). In conjunction with an rf magnetic field \(B_\text{d}e_x\), with frequency \(Δω\), a pair of orthogonally-polarized counterpropagating Raman beams with frequencies \(\omega, \omega + Δω\) illuminates the atomic sample. The rf and Raman fields have coupling strengths \(Ω_\text{rf}\) and \(Ω_\text{d}\). For \(Ω_\text{d} ≪ \omega_2\), the smaller field spatially modulates \(|\lambda|/2\). 

\[ \text{Effective Zeeman lattice; as plotted with } 87Rb \text{ BECs in the lowest energy rf-dressed state [12]. Two } \lambda/33 \text{ nm Raman laser beams, counter-propagating along } e_x \text{ and differing in frequency by } \Delta\omega, \text{ couple the BEC’s internal degrees of freedom with strength } \Omega_R \text{, which in the tight-binding regime is inversely proportional to } \Delta\omega/2\pi = g_\mu_B B_0 = 3.25 \text{ MHz and prepare the BEC in the lowest energy rf-dressed state [12]. Two } \lambda=790.33 \text{ nm Raman laser beams, counter-propagating along } e_x \text{ and differing in frequency by } \Delta\omega, \text{ couple the BEC’s internal degrees of freedom with strength } \Omega_R \text{.} \]

\[ \text{Spatially varying eigenvalues of } H_{\text{eff}}(x) \text{ (red, blue and green curves) give rise to our } \lambda/2 \text{ effective Zeeman lattice; as plotted } \beta_{\text{eff}} = 1E_L, \ h\Omega_{\text{rf}} = 10E_L, \ h\delta = 2E_L. \text{ Spatial precession of } B_\text{d}(x) \text{ (dark arrow) and the solid angle } \theta_\text{B} \text{ it subtends when an atom tunnels to the nearest neighboring site (points 1 to 5 in c). This geometrical Berry’s phase gives the Peierls phase } \phi. \]

\[ \Omega_R \gg \Omega_\text{rf}, \delta \text{ we obtain the analogous result } h|\Omega| \approx h(\Omega_R + \Omega_\text{ rf} \cos(2k_x x))/\sqrt{2}. \text{ In both of these limits, the larger of the two fields defines a natural quantizing axis about which the smaller field spatially modulates } |\Omega|. \text{ For } \Omega_R \gg \Omega_\text{rf}, \text{ this quantizing axis is spatially rotating.} \]
The sloshing amplitude $|\Delta k_x|$ is displayed in Fig. 3. For large initial $|\Delta k_x|$ (shaded region) the oscillations are strongly damped, which we attribute to energetic instabilities resulting in depletion of BEC atoms [18]. This is evident from the departure of the oscillation amplitude from the expected value; the experimental range of the dynamical instability, $|\Delta k_x| < 1.1E_L$ (corresponding to $\Delta k_x \gtrsim 0.6k_L$), is shaded in gray and is in agreement with previous observations in conventional optical lattices [18]. Figure 3 displays the tunneling amplitude $t$, obtained from $f^*$. For comparison, a sinusoidal lattice would require a depth $V_0 \approx 8E_L$ to give similar parameters.

Having discussed the behavior of atoms in the lattice’s lowest band, we now explore the full lattice by suddenly turning it on, diabatically projecting a ground state BEC into higher bands. At the beginning of such a pulse ($\tau_{\text{pulse}} \ll \pi\hbar/\sqrt{5E_L}$, where $s = V_0/E_L$), an ordinary periodic potential, would spatially modulate the BEC’s phase [19]: our effective Zeeman lattice induces such a modulation but in a spin-dependent manner. We focus on the $\Omega_R \gg \Omega_{\text{rf}}$ and $\Omega_R \ll \Omega_{\text{rf}}$ tight-binding regimes and investigate the spin and spatial structure of our lattice. Our data extends well beyond the short-time phase modulation regime.

In the absence of either Raman or rf coupling, there is no lattice. As indicated in Fig. 4a, we use two different methods to introduce our lattice on an initial spatially uniform state: (i) starting with an rf-dressed state (with $k_x = 0$), we suddenly ($<1\,\mu\text{s}$) turn on the Raman beams; or (ii) starting with a Raman-dressed state [12] (a superposition of $|m_F = 0, k_x = 0\rangle$ and $|m_F = \pm 1, k_x = \pm 2k_L\rangle$), we suddenly turn on the rf-field.

After holding the lattice on for a time $\tau_{\text{pulse}}$, we suddenly turn off the rf and Raman fields, together with the confining potential. The atoms are projected onto the bare spin-momentum basis and separate in TOF in the presence of a magnetic field gradient (along $e_y$), allowing us to resolve their spin and momentum components.

We observe detectable population in states with momenta up to $|k_x| \lesssim 4k_L$ (Fig. 4). We perform such experiments for $\Omega_R/\Omega_{\text{rf}} \approx 3$ and 5. We minimize the effects of interactions by working with small BECs ($\approx 9 \times 10^4$ atoms). Figures 4c,d show the fraction of atoms in each diffracted order evolving with time. We observe multiple revivals of the initial spin-momentum state and find symmetry in the population dynamics of spin-momentum states with opposite momentum and opposite spin. The curves represent fits to the populations in all spin-momentum components. The parameters from the fits are all within 10% for our calibrated values, demonstrates that the spin-momentum dynamics are well described by the unitary evolution of the initial states under $H_{t+R}$ [14].

Based on this technique for controlling the Peierls phase and inspired by recent proposals for creating flux

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**FIG. 3.** Peierls transformation. a. Peierls phase $\phi$ measured using adiabatic (crosses) and sudden (circles) changes of $\Omega_z$. Vertical lines denote the first Brillouin zone. b. Sloshing amplitude after suddenly changing $\Omega_z$. We observed strong damping of oscillations in the region shaded in gray. c. Tunneling amplitude $t$ measured from oscillation frequency. The rf coupling was modulated as a function of $\Omega_{\text{rf}}$, and subsequently deload all atoms into the $|\phi/\pi\rangle$ state and subsequently deload all atoms into the $|\phi/\pi\rangle$ state. We image this distribution after a 13.1 ms TOF, revealing $k_{\min}$. The Peierls phase, shown as crosses in Fig. 3a, is $\phi/\pi = k_{\min}/k_L$. In the sudden method, we test the robustness of the Peierls phase $\phi$ by first adiabatically loading to $\phi = \pm \pi$ (the condensate sits at the edge of the Brillouin zone) and then suddenly changing both $\Omega_z$ and $\Omega_{\text{rf}}$ to new values (changing both $\phi$ and $t$). This results in momentum space oscillations centered at $k_{\min}$. After a time $\tau$ we release the BEC, and measure as above. We fit the crystal momentum dynamics with $k_x(\tau) = k_{\min} + \Delta k_x \cos(2\pi \tau f^* + \gamma)$, where $\Delta k_x$ is the amplitude, and $\gamma$ is an overall phase-shift whose average value is $0.9(1)\pi$ for these measurements. Figure 3b (circles) shows the measured Peierls tunneling phase as a function of $\Omega_z$.

Measurements from the adiabatic and sudden methods are in good agreement with each other and their expected values (Fig. 3b, dashed curves), highlighting the precise experimental control offered by our rf-Raman induced effective Zeeman lattice. This agreement also demonstrates the robustness of our engineered Hamiltonian to deliberate variations in $\Omega_{\text{rf}}$ of up to $0.25E_L$.
FIG. 4. BEC diffraction from the effective Zeeman lattice. a. Starting with an rf-dressed (Raman-dressed) state, we suddenly turn-on the Raman (rf) field for a variable time $\tau_{\text{pulse}}$. b. Using TOF absorption images of the projected spin-momentum distributions, we count the number of atoms in each diffracted order and determine its fractional population. Panels c, d depict time evolution of these fractions. The curves are fits to the data, calculated from $H_{\text{rf}+\text{R}}$. The fit parameters are: 

- (c.) rf-dressed $\hbar(\Omega_{\text{rf}}, \Omega_{\text{R}}, \Omega_{z}) = (3.57, 11.49, -0.04)E_L$
- (d.) Raman-dressed $\hbar(\Omega_{\text{rf}}, \Omega_{\text{R}}, \Omega_{z}) = (3.06, 15.14, 0.08)E_L$

We realized a 1D lattice potential for ultracold atoms using only rf and Raman transitions, in which the tunneling matrix element is in general complex. This work constitutes a first step towards realizing flux lattices [20], in which the physics of charged particles in strong magnetic fields can be simulated. The tunability of the Peierls phase achieved with our rf-Raman lattice would allow the observation of nonlinear effects of ultracold atoms in 1D periodic potentials, such as atomic density modulations with periodicity larger than the lattice spacing [23].

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Starting with the rf-dressed state described in the main text, we ramped the Raman beams from 0 to \( \Omega_R > 4 E_L / \hbar \) in 70 ms. Then we ramped the rf coupling strength to an adjustable final value \( \Omega_{rf} \) in 2 ms, such that an effective Zeeman lattice was created. We induce sloshing by applying a synthetic electric field \( \mathbf{E} \), achieved by ramping the \( z \)-component of the Zeeman field to \( \hbar \Omega_z \approx 2 E_L \) and then jumping it back to 0 in 2 ms.

## MOMENTUM REPRESENTATION AND DIAGONALIZATION OF \( H_{rt+R}(x) \)

In order to compute the properties of our lattice potential we numerically diagonalized the combined rf-Raman Hamiltonian \( H_{rt+R} \) given the experimental parameters \( \Omega_{rf}, \Omega_R, \Omega_z \). For simplicity we work in the momentum space representation of \( H_{rt+R} \). This representation offers an alternative understanding of the effective Zeeman lattice structure that arises from the combination of rf and Raman coupling fields.

While Raman transitions couple together states with \( m_F \) differing by ±1 and momentum differing by \( \pm 2 k_L \), rf-coupling processes only change \( m_F \) by ±1, leaving the momentum unchanged. The combination of both Raman and rf fields sets appropriate conditions for secondary rf and Raman processes to populate states with higher momentum. The available states under rf-Raman coupling constitute a set of spin-momentum states \( \{ | \Psi_n \rangle \} = \{ | m_F, \hbar(k_x + 2 n k_L) \rangle \} \) where \( n \in \mathbb{Z}; m_F = 0, \pm 1 \). As expected, this basis is that of a lattice.

In the basis \( \{ | \Psi_n \rangle \}, H_{rt+R} \) is a Hermitian block matrix of size 3(2N+1), when \( n \) is restricted to \( n < N \). For our parameters, dimensions larger than 3(2N+1) = 81 provided indistinguishable results. The dimension of this basis is appropriate for our calculations since we observed the population of states with up to \( |n| = 4 \). For clarity, we construct the Hamiltonian by arranging the spin-momentum states in the \( \{ | \Psi_n \rangle \} \) basis with increasing momentum index \( n = -N, -N+1, \ldots, N \). Along the principal diagonal, we have 3 \( \times \) 3 blocks

\[
A_{k_z}(n) = \hbar^2 (k_z + 2 n k_L)^2 + \frac{\Omega_{rf}}{2} F_z - \frac{\Omega_z}{\hbar} F_z + \frac{\epsilon}{\hbar} (h^2 R - F_y^2);
\]

these terms correspond to kinetic energy, rf coupling of spin states with equal momentum, and the real Zeeman interaction, respectively; \( \{ F_x, F_y, F_z \} \) are the matrix representations of the \( F = 1 \) angular momentum operators, and \( I \) is the \( 3 \times 3 \) identity matrix. Above and below the \( A_{k_z}(n) \) blocks, we have \( 3 \times 3 \) blocks \( B = \sqrt{2} \Omega_R (F_z - i F_y) / 4 \) describing the Raman coupling of spin-momentum states differing in momentum by \( \Delta k_z = \pm 2 k_L \).
The diagonalization of \( H_{rf+R}(n) \) as a function of \( k_x \) gives the bandstructure of the combined rf-Raman lattice potential, \( E(k_x)=E(k_x, \Omega_{rf}, \Omega_R, \Omega_z, \phi) \).

**EFFECTIVE ZEEMAN LATTICE PROPERTIES**

We extract the Zeeman lattice properties by fully characterizing the lowest energy band which in the tight binding regime is of the form \( E(k_x)=-2t \cos(\pi k_x/k_L-\phi) \). The tunneling amplitude is given by \( t=\Delta E/4 \), where \( \Delta E \) is the width of the lowest band. The lattice depth was calculated as the depth that a lattice potential would have in order to produce the calculated width \( \Delta E \). The effective mass is defined as \( m^*=\hbar^2[\frac{d^2E(k_x)}{dk^2_x}]^{-1} \), where the derivative is evaluated at the point of interest (e.g. for our \( \Omega_z=0 \) measurements of effective mass displayed in Fig. 2, this was at \( k_x=0 \)). In the tight-binding regime, \( t \) and \( m^*/m \) are inversely proportional to each other \( \pi^2t/E_L=m/m^* \). We obtained the Peierls phase \( \phi \) by computing the shifts the lowest band [with minimum at \( k_{\text{min}}=(\phi/\pi)k_L \)] as a function of the experimental parameters.

**DELOADING**

We deloaded to a single bare spin state by rapidly ramping to \( \Omega_z=0 \), taking \( \hbar \Omega_R \to 0 \) to zero in 500 \( \mu s \) while increasing \( \hbar \Omega_{rf} \to 3E_L \). We then ramped the \( \hbar \Omega_z \to -140E_L \) transferring all atoms into \( |m_F=+1 \rangle \).

**2D LATTICE FOR 1/3 FLUX HOFRSTADTER MODEL**

We numerically solve the bandstructure of the 2D lattice arising from the combination of a period \( a=266 \text{ nm} \) lattice with depth \( V_0 = 22.5E_L \), and a 1D “Zeeman lattice” with \( \Omega_{rf} = 1E_L, \ \Omega_R = 10E_L, \ \epsilon = 0.44E_L \). Here \( E_L \) denotes the Raman recoil, not the short-period lattice recoil.