All-optical photon transport switching in a passive-active optomechanical system

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We propose a feasible scheme to realize all-optical photon transport switching in a passive-active optomechanical system, consisting of one ordinary passive cavity, one active cavity and one common movable membrane oscillator of perfect reflection, driven by two strong control fields and two weak probe fields symmetrically. By means of the gain effect of the active cavity, many novel and valuable phenomena arise, such as frequency-independent perfect reflection (FIPR) first proposed in this paper, adjustable photon bidirectional transport, phase-dependent non-reciprocity and so on. The relevant parameters used for controlling the all-optical switching are precisely tunable by adjusting the strengths of control fields and the relative phase of probe fields. Furthermore, tunable fast and slow light can be realized in our system by accurately adjusting relevant parameters which is readily and feasible in experiments. These novel phenomena originate from the effective optomechanical coupling and the gain effect provide a promising platform for photonic devices, quantum network node fabrication and quantum information process (QIP).

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I. INTRODUCTION

Optomechanical systems attracted intense interest and numerous investigations in the past few decades due to the superiority that plenty of quantum phenomena can be exhibited in such macroscopic devices, which provides a promising platform to explore quantum science and the correlation between classical and quantum issues \cite{1, 2}. As an interdiscipline of microphysics and quantum optics, which primarily studying the interactions between the electromagnetic fields and mechanical systems, it has developed rapidly that series of advances born in recent years, e.g., optomechanically induced transparency (OMIT) \cite{3, 4}, multiple quantum entanglement \cite{5, 6}, signal absorption and amplification \cite{7, 8}, photon blocked \cite{9, 10}, optomechanically induced non-reciprocity \cite{11, 12} and so on, which laying a solid foundation for controllable photon transport and quantum information process via macroscopical optomechanical devices.

Recently, quite a lot of creative and satisfactory phenomena arise in the systems active (gain) subparts, especially in so-called parity-time ($PT$)-symmetric systems, exhibiting extraordinary advantages compared with those in conventional passive-passive systems \cite{13–15}. The successful realization of $PT$ symmetry in optical systems, including optomechanical systems, leads to so many novel breakthroughs recently, e.g., mechanical $PT$ symmetry \cite{23, 24}, $PT$-symmetry-breaking induced ultralow threshold chaos \cite{25}, photon laser \cite{26, 27}, optomechanically induced transparency (OMIT) \cite{28, 29} and enhanced photon blocked effect \cite{30}. In a recent study \cite{31}, Liu \textit{et al.} proposed a single-cavity optomechanical system with an active mechanical resonator, by modifying the gain and loss of the mechanical resonator, a tunable optical signal amplification and absorption, ultralow threshold chaos \cite{25}, photon laser \cite{26, 27}, and enhanced photon blocked effect \cite{30}.

II. MODEL AND METHODS

We consider a three-mode optomechanical system composed of a movable membrane oscillator located around the middle position between two fixed mirrors with finite equal transmission. As shown in Fig. 1 the movable membrane oscillator is perfectly reflected so that the system can be regarded in the form of two Fabry-Pérot cav-
Two control and two probe fields are applied to drive the system. We have assumed identical eigenfrequencies \( \omega_0 \) of the system after a frame rotation with respect to the cavity modes. Note here, the left cavity is passive while the right one is active by adding an optical gain medium or an additional external pumping field \( \hat{b}_m \) to it.

We describe the left (right) cavity mode by photon annihilation and creation operators \( \hat{a}_L \) (\( \hat{a}_R \)) with identical eigenfrequency \( \omega_0 \). The left cavity is passive with loss \( \kappa_1 \) due to the photon annihilation (creation) operator \( \hat{b}(\hat{b}^\dagger) \) with eigenfrequency \( \omega_m \). The boson operators mentioned above satisfy the commutation relation \([O_i, O_j^{\dagger}] = \delta_{ij}(O = a_1, a_2, b, \text{ and } i, j = 1, 2)\). Two control and two probe fields are applied to drive the cavity modes on the left and right fixed mirrors symmetrically, with the amplitude \( \varepsilon_{L,R} = \sqrt{2\kappa_1 P_1/(\hbar\omega_c)} \) for the left (right) control field and \( \varepsilon_{L,R} = \sqrt{2\kappa_2 P_2/(\hbar\omega_p)} \) for the left (right) probe field, where \( \kappa_1(\kappa_2) \) is the decay rate of the left passive cavity (right active cavity). In addition, we have assumed identical eigenfrequency \( \omega_c(\omega_p) \) for the control (probe) fields. Then the total Hamiltonian of the system after a frame rotation with respect to the control field frequency can be written as

\[
\mathcal{H} = \Delta_0(a_1^\dagger a_1 + a_2^\dagger a_2) + g_0(a_2^\dagger a_2 - a_1^\dagger a_1)(b^\dagger + b) + \omega_m b^\dagger b + i(a_1^\dagger e^{i\Delta_p t} + a_2^\dagger e^{i\theta} e^{-i\Delta_p t} + \varepsilon_{c,L}a_1^\dagger + \varepsilon_{c,R}a_2^\dagger - \text{h.c.})
\]

where \( \hbar = 1 \) has been set here for convenience. We introduce \( \Delta_p = \omega_p - \omega_c \) the detuning between cavity modes and control fields, \( \Delta_p = \omega_p - \omega_c \). The detuning between probe fields and control fields, \( \theta \) the relative phase between the two probe fields, \( g_0 = \omega_0 \sqrt{\hbar/(2m\omega_m)/L} \) is the optomechanical coupling constant due to the radiation pressure where \( m \) and \( L \) are the effective mass of the mechanical resonator and the free length of the cavities, respectively.

Using the above Hamiltonian, considering relevant dissipation and quantum (thermal) noise, we can further obtain the following Heisenberg-Langevin equations

\[
\begin{align*}
\dot{b} &= -i\omega_m b - ig_0(a_1^\dagger a_2 - a_1^\dagger a_1) - \gamma_m b + \sqrt{2\gamma_m b_m} \\
\dot{a}_1 &= -[\kappa_1 + i\Delta_0 - ig_0(b^\dagger + b)]a_1 + \varepsilon_{c,L}e^{-i\Delta_p t} + \sqrt{2\kappa_0}a_1^\dagger \\
\dot{a}_2 &= -[\kappa_2 + i\Delta_0 + ig_0(b^\dagger + b)]a_2 + \varepsilon_{c,R}e^{-i\Delta_p t} + \sqrt{2\kappa_2}a_2^\dagger
\end{align*}
\]

where \( \gamma_m \) is the damping rate of the mechanical resonator which determines the mechanical quality factor \( Q = \omega_m/\gamma_m \). Moreover, \( b_m \) is the zero-mean-value thermal noise operator of the mechanical resonator and \( a_1^{\dagger}(a_2^{\dagger}) \) is the zero-mean-value quantum noise operator of the left (right) cavity. Setting all the time derivatives in Eq. (2) to be zero, the steady-state values of the system variables can be obtained by neglecting the zero-mean-value noises and the weak probe fields

\[
\begin{align*}
b_s &= -ig_0(|a_{2s}^2|^2 - |a_{1s}^2|^2) \\
a_{1s} &= \frac{\varepsilon_{c,L}}{\kappa_1 + i\Delta_1} \\
a_{2s} &= \frac{\varepsilon_{c,R}}{\kappa_2 + i\Delta_2}
\end{align*}
\]

where \( \Delta_{1,2} = \Delta_0 \mp 2g_0\text{Re}(b_s) \) are the effective detunings between cavity modes and control fields due to the feedback of the motion of the mechanical resonator. Note a relatively weak optomechanical coupling and identical photon number in two cavities jointly lead to a negligible term \( 2g_0\text{Re}(b_s) \) compared with \( \Delta_0 \).

To solve Eq. (2), we can safely express relevant operators as sums of the steady-state mean values and the small quantum fluctuation terms, i.e., \( O = O_s + \delta O \) with \( O = b, a_1, a_2 \). In this way, a set of linearized quantum Langevin equations can be obtained by neglecting all the nonlinear high-order terms of fluctuation operators and the zero-mean-value quantum noise operators after a rotating wave approximation

\[
\begin{align*}
\langle \delta b \rangle &= -ig_0(a_2^{\dagger s}a_2^s - a_1^{\dagger s}a_1^s) - \gamma_m \langle \delta b \rangle \\
\langle \delta a_1 \rangle &= -[\kappa_1 + i\Delta_0 + ig_0a_{1s}]\langle \delta b \rangle + \varepsilon_{c,L}e^{i\theta} e^{-i\Delta_p} \\
\langle \delta a_2 \rangle &= -[\kappa_2 - ig_0a_{2s}]\langle \delta b \rangle + \varepsilon_{c,R}e^{i\theta} e^{-i\Delta_p} \end{align*}
\]

where \( \delta = \Delta_p - \omega_m \approx \omega_p - \omega_0 \) is the detuning between the probe fields and the cavity modes. In addition, here we have moved Eq. (4) into an interaction picture by introducing the transformation

\[
\begin{align*}
\delta b &\rightarrow \delta b e^{-i\omega_m t}, \delta b_m &\rightarrow \delta b_m e^{-i\omega_m t} \\
\delta a_1 &\rightarrow \delta a_1 e^{-i\Delta_p t}, \delta a_1^s &\rightarrow \delta a_1^s e^{-i\Delta_p t} \\
\delta a_2 &\rightarrow \delta a_2 e^{-i\Delta_p t}, \delta a_2^s &\rightarrow \delta a_2^s e^{-i\Delta_p t}
\end{align*}
\]
Moreover, we assume that the three-mode system is driven by two control fields at red-detuned sideband ($\Delta_1 \approx \Delta_2 \approx \omega_m$) and our system is studied in the resolved sideband regime ($\omega_m \gg |\kappa_1|, |\kappa_2|$), the common mechanical resonator possesses a high quality factor $Q = \omega_m/\gamma_m \gg 1$ and the eigenfrequency $\omega_m$ is assumed to be much larger than $g_0|a_{1s}|$ and $g_0|a_{2s}|$ to ensure the validity of the rotating wave approximation we have performed above.

According to Eq. (4), the time-dependent oscillating terms can be removed out if the solutions are assumed to be form: $|\delta t\rangle = \delta O_+ e^{-i\omega t} + \delta O_- e^{i\omega t}$ where $O = b, a_1, a_2$. Note here, the component $\delta O_+$ possesses the same Stokes frequency $\omega_p$ as the probe fields $\epsilon_L$ and $\epsilon_R$ and the component $\delta O_-$ possesses the anti-Stokes frequency $2\omega_c - \omega_p$ via a nonlinear four wave mixing process. Then it is easy to obtain such following results after effortless calculations

$$\delta b_+ = -iG[\kappa L \epsilon e^{i\theta}(k_1 - i\delta) - \epsilon_L(k_2 - i\delta)]$$
$$\delta a_{1+} = \frac{G^2(n_\epsilon \epsilon e^{i\theta} + n^2 \epsilon_L + \epsilon_L(\gamma_m - i\delta))(k_2 - i\delta)}{F_1 + F_2}$$
$$\delta a_{2+} = \frac{G^2(\epsilon_R e^{i\theta} + n_\epsilon \epsilon e^{i\theta})(\gamma_m - i\delta)(k_1 - i\delta)}{F_1 + F_2}$$

(6)

where $F_1 = (\gamma_m - i\delta)(k_1 - i\delta)(k_2 - i\delta)$ and $F_2 = G^2[n^2(k_1 - i\delta)(k_2 - i\delta)]$. We introduce $G = g_0c_{1s}$ the effective optomechanical coupling rate and $n = |a_{2s}/a_{1s}|^2$ the photon number ratio of two cavities. Without loss of generality we set $a_{1s}$ and $a_{2s}$ real in derivative process of Eq. (6).

According to the input-output field theory [37,38], we can straightforward derive the expression of both the left and right output fields $\epsilon_{outL}$ and $\epsilon_{outR}$

$$\epsilon_{outL} + \epsilon_L e^{i\delta t} = 2\kappa_L \delta a_1$$
$$\epsilon_{outR} + \epsilon_R e^{i\theta} e^{i\delta t} = 2\kappa_R \delta a_2$$

(7)

the time-dependent oscillating terms can also be removed out similar to Eq. (6) if the solutions here have the same form $\delta t\epsilon_{outL} = \epsilon_{outL} + \epsilon_{outL} e^{-i\delta t} + \epsilon_{outL} e^{i\delta t}$ and $\delta t\epsilon_{outR} = \epsilon_{outR} e^{-i\delta t} + \epsilon_{outR} e^{i\delta t}$. Note again, the output components $\epsilon_{outL+}$ and $\epsilon_{outR+}$ possess the same frequency $\omega_p$ as the probe fields, the output components $\epsilon_{outL-}$ and $\epsilon_{outR-}$ possess the frequency $2\omega_c - \omega_p$ but not of our interest in this paper. Then according to Eq. (6) and Eq. (7), we have

$$\epsilon_{outL+} = 2\kappa_L \delta a_1 - \epsilon_L$$
$$\epsilon_{outR+} = 2\kappa_R \delta a_2 - \epsilon_R e^{i\theta}$$

(8)

We introduce the transmission rate $T_L = |\epsilon_{outL+}/\epsilon_L|^2$ ($T_R = |\epsilon_{outR+}/\epsilon_L|^2$) and the reflection rate $R_L = |\epsilon_{outL-}/\epsilon_L|^2$ ($R_R = |\epsilon_{outR-}/\epsilon_R|^2$) for the left (right) cavity, respectively, which are used for gauging the intensities of the output fields from both cavities. Then we introduce $\theta_{tr} = \arg[\epsilon_{outL+}/\epsilon_L]$ ($\theta_{tr} = \arg[\epsilon_{outR+}/\epsilon_L]$) and $\theta_{tr} = \arg[\epsilon_{outL+}/\epsilon_R]$ ($\theta_{tr} = \arg[\epsilon_{outR+}/\epsilon_R]$) the phase of the reflection (transmission) field of the left and right cavity, respectively, which are functions of $\omega_p$. In optomechanics, the group delay can be determined by the slope of the output field phase with respect to the probe frequency, i.e., $\tau_m = \partial \theta_{tr}/\partial \omega_p$ ($m = rl, ll, rr, tr$) [31,39]. Note a positive $\tau$ corresponding to a slow light phenomenon while a negative $\tau$ corresponding to a fast light phenomenon on the contrary. One can see the larger the slope, the larger the group delay, thereby the slower the light.

III. RESULTS AND DISCUSSION

Now we will study numerically how the gain effect of the active cavity impacts the photon transport of such an optomechanical system by setting $\kappa_2$ a negative value. To start with, we should consider the assumption we mentioned above that $n = 1$ is reasonable in our system. According to Eq. (5), the photon number ratio can be indicated as $n \approx \epsilon_R^2(\kappa_1^2 + \omega_m^2)/[\epsilon_L^2(\kappa_2^2 + \omega_m^2)]$.
when \( g_0 \ll \omega_m \) is satisfied. Thus we can guarantee the equal photon number in the two cavities by adjusting \( \varepsilon_{cL} \) or (and) \( \varepsilon_{cR} \) even if \( |\kappa_1| \neq |\kappa_2| \), and in turn, this further ensures the validity of \( 2g_0 \Re(b_s) \ll \Delta_0 \) in the preceding part of the text. In this section, we find several novel and fascinating phenomena arise in different parameter regimes which are accessible experimentally, such as (i) \( G \)-dependent photon bidirectional transport; (ii) frequency-independent perfect reflection (FIPR); (iii) phase-dependent non-reciprocity and (iv) fast and slow light.

### A. Photon bidirectional modulator

According to Eq. (6) and Eq. (8), if no probe field is applied to the right active cavity \( (\varepsilon_{cR} = 0) \) and the relevant parameters of our system simultaneously satisfy \( \kappa_1 = -\kappa_2 = \gamma_m = \kappa, n = 1 \), the reflection and transmission rate of the left incident probe field can be written as

\[
R_L = \left| \frac{(\kappa - i\delta)(\kappa^2 + \delta^2) - 2iG^2\delta - 2\kappa G^2}{(\kappa - i\delta)(\kappa^2 + \delta^2) + 2iG^2\delta} \right|^2
\]

\[
T_L = \left| \frac{2\kappa G^2}{(\kappa - i\delta)(\kappa^2 + \delta^2) + 2iG^2\delta} \right|^2
\]

(9)

we have \( R_L = (\kappa^2 - 2G^2)^2/\kappa^4 \) and \( T_L = 4G^4/\kappa^4 \) when the left probe field is resonant with the cavity exactly \( (\delta = 0) \). So one can easily find that the transmission rate is proportional to the effective optomechanical coupling rate \( G \) monotonously while the reflection rate experiences a transition from reduction to growth versus a increasing \( G \). The numerator of \( R_L \) shows that the reflection field vanishes at \( G = \kappa/\sqrt{2} \) and enters the amplification region at the critical point \( G = \kappa \). Note, by increasing \( G \) from 0 to \( \kappa/\sqrt{2} \) the reflection rate decreases from 1 to 0 while the transmission rate increases from 0 to 1. Thus a photon bidirectional modulator can be realized which enables to continuously adjust the intensity ratio between the reflection and transmission field, means one can control the direction of the output field and the intensity of each direction component.

Fig. 4 shows the numerical results of the photon transmission in this case. According to Fig. 2(a), (b) and (c), one can observe that just as we predicted analytically, the transmission rate increases in direct proportion to \( G \) with the resonance condition \( \delta = 0 \) while the reflection rate experiences a transition. Physically, it is because the two cavities exchange energy with the common mechanical oscillator via radiation pressure rather than a direct coupling, there exits a competition between dissipation of the mechanical oscillator and the indirect compensation effect. A relatively weak effective optomechanical coupling rate cannot support a timely and effective compensation from the active cavity to the passive cavity, within this regime, a increasing energy exchange between the left cavity mode and the mechanical mode due to the increasing \( G \) leads to a greater loss of the left passive cavity. However, a big enough \( G \) amount to a fast and strong tunneling between the two cavity modes so that an effective compensation effect arises, the indirect tunneling

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**FIG. 3:** (Color online) Group delay of the reflection field \( \tau_L \) (a) and the transmission field \( \tau_R \) (b) versus normalized probe field detuning \( \delta/\kappa \) for \( G = 0.8\kappa \) (the red dot-dashed line), \( G = \kappa \) (the green solid line) and \( G = 1.2\kappa \) (the blue dot-solid line). All the other parameters are the same as shown in Fig. 2 except for the parameter conditions mentioned in this section.

**FIG. 4:** (Color online) Reflection rate \( R_L \) (red dot-dashed line) and transmission rate \( T_L \) (blue solid line) versus normalized probe field detuning \( \delta/\kappa \) for (a) \( G = \kappa \); (b) \( G = 2\kappa \); (c) \( G = 3\kappa \). Three-dimensional diagrams (d) and (e) corresponds to \( R_L \) and \( T_L \) versus \( \delta \) and \( G \), respectively. All the other parameters are the same as shown in Fig. 2 except for the parameter conditions mentioned in this section.
between the two cavity modes becomes dominant effect, which leads to a enhanced left output field (reflection field). In addition, a coherent perfect-transmission phenomenon proposed in Ref. [32] can be observed in Fig. 2 (d) and (e) near $\delta = \pm 1.3$, so besides adjusting $G$, controllable photon transport can also be realized via adjusting the frequency of probe field. Moreover, both the reflection and transmission field enter the amplification region while $G$ is larger than the critical point $\kappa$ as shown in panel (c). Three-dimensional diagrams in Fig. 2 (d) and (e) exhibit the results in a more intuitive way, from which one can clear that how to adjust the photon transport by changing the strengths of control fields (thereby $G$) and the frequency of probe field (thereby $\delta$). In view of this, a G-dependent photon bidirectional modulator, i.e., an all-optical photon transport switching, can be realized.

In this passive-active system, a tunable fast and slow light can also be realized at $\delta = 0$ by adjusting the control fields. According to the analytical expression above, once the effective optomechanical coupling rate satisfies $G > \kappa/\sqrt{2}$, the reflection rate $R_L$ will be in direct proportion to $G$, so it’s significative to study the fast and slow light phenomenon of the reflection field in this region. In this case, we define $\tau_L = \tau_{tL}$ as the reflection group delay and $\tau_R = \tau_{tR}$ the transmission group delay. In Fig. 3 (a), one can find a slow-to-fast light transition of the reflection field near the critical point $G = \kappa$. The group delay $\tau_L$ is inversely proportional to $G$ means a gradually faster reflected light, furthermore, $\tau_L$ decreases from positive to negative and the transition point is $G = \kappa$ where $\tau_L = 0$ as we predicted. The case in Fig. 3 (b) is slightly different, where the transmission group delay is always negative with $G > \kappa/\sqrt{2}$, illustrating a algate fast light, $\tau_R$ decreases monotonously with a increasing $G$ at $\delta = 0$, so a gradually faster transmission light can also be achieved by increase the strengths of control fields and thereby the optomechanical coupling rate $G$.

**B. Frequency-independent perfect reflection**

Generally, photon transport is sensitive to relevant parameters such as frequency of the probe field and the effective optomechanical coupling. Series of novel phenomena observed in various optomechanical systems, for instance, OMIT in $PT$-symmetric optomechanical systems [28, 29], coherent perfect absorption (CPA) in general optomechanical systems [32, 33], amplification of optical response in $PT$-symmetric-like optomechanical systems [31] and so on, arising only in the specific parameter regions. In this section, however, we will exhibit a parameter-independent novel photon phenomenon named frequency-independent perfect reflection (FIPR) by us.

We now consider the case that two probe fields with identical amplitudes ($\varepsilon_L = \varepsilon_R$) incident upon the left and right cavity respectively. Based on this, when conditions $\kappa_1 = -\kappa_2 = \gamma_m = \kappa$ and $\theta = \pi, n = 1$ are simultaneously satisfied. The transmission and reflection rate can be simplified as

$$R_L = 1$$

$$T_L = \frac{2iG^2\delta - (\kappa - i\delta)^3}{2iG^2\delta + (\kappa - i\delta)^2(\kappa + i\delta)}$$  \hspace{1cm} (10)

where we define $R_L = R_t = T_r$ the reflection rate and
The case in Fig. 3 is much more different compared with that in Fig. 2. Here we define $\tau_L = \tau_{rl} = \tau_{tr}$ the reflection group delay and $\tau_R = \tau_{rl} = \tau_{tr}$ the transmission group delay of the left incident probe field. In panels (a) and (b), similar to Fig. 3, both $\tau_L$ and $\tau_R$ decrease with a increasing $G$ at $\delta = 0$, where $R_L = T_L = 1$, illustrating a tunable fast light with lossless reflection and transmission field can be realized in this system when the probe fields are resonant with cavity modes. Moreover, in panels (c) and (d), both the reflection and transmission group delay are in direct proportion to a increasing $G$ near the optimal value of $\delta$ where the transmission field has the maximal amplification effect according to Fig. 2 ($\delta \sim 1, \delta \sim 2.5$ and $\delta \sim 4$ corresponding to $G = \kappa, G = 2\kappa$ and $G = 3\kappa$, respectively). So we can realize a tunable slow light accompanied by a amplified transmission field and a lossless reflection field. In a word, by adjusting the strengths of control fields and the frequencies of probe fields, one can realize both fast and slow light for both the reflection and transmission fields.

Up to now in this section, we have chosen a fixed relative phase of the probe fields $\theta = \pi$. Then we will show that except for adjusting $G$ and $\delta$, $\theta$ is also an important control parameter for realizing a all-optical switching. If the condition $\varepsilon_L = \varepsilon_R, \kappa_1 = -\kappa_2 = \gamma_m = \kappa, n = 1$ is met, then we have

$$R_L = \frac{1 - 4G^2(1 + e^{i\theta})^2}{\kappa^2},$$

$$T_L = \frac{|e^{i\theta} + 2G^2(1 + e^{i\theta})/\kappa^2|^2}$$

with resonant probe fields ($\delta = 0$). According to Eq. (11), $R_L = 0, T_L = |1 + 4G^2/\kappa^2|^2$ when $\theta = 2m\pi (m = 0, 1, 2, \ldots)$ and $R_L = T_L = 1$ when $\theta = n\pi (n = 1, 3, 5, \ldots)$, as shown in Fig. 2. Moreover, both the reflection and transmission rate can be continuously adjusted via $\theta$ and the amplification effect of the transmission field relies on $G$. Physically, it is because by adjusting the relative phase $\theta$, the interference effect between the two probe fields can be changed, and this also impacts the compensation effect between the two cavity modes. In view of this, a phase-dependent all-optical switching can be realized by adjusting the relative phase of the probe fields and the strengths of the control fields.

IV. CONCLUSIONS

In summary, we have studied theoretically how to control photon transport in a passive-active optomechanical system driven by two strong control fields and two weak probe fields. According to the main text, in virtue of the gain effect of the active cavity, series of novel and valuable phenomena arise, illustrating the realization of a all-optical photon transport switching. By adjusting the strengths of control fields, the frequencies of probe fields and the gain rate of the active cavity, we can realize: (i) a photon bidirectional modulator relying on the effective optomechanical coupling rate $G$; (ii) frequency-independent perfect reflection (FIPR) which subverts tradition so that the complete reflection in whole frequency region becomes possible. (iii) a phase-dependent photon switching by adjusting the relative phase of the probe fields. Furthermore, tunable fast and slow light can be realized in this passive-active optomechanical system by accurately adjusting the input fields and the gain rate of the active cavity in an experimentally reasonable region.

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