CLASSICAL CEPHEID PULSATION MODELS. IX. NEW INPUT PHYSICS

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ABSTRACT

We constructed several sequences of classical Cepheid envelope models at solar chemical composition ($Y = 0.28$, $Z = 0.02$) to investigate the dependence of the pulsation properties predicted by linear and nonlinear hydrodynamic models on input physics. To study the dependence on the equation of state (EOS) we performed several numerical experiments by using the simplified analytical EOS originally developed by Stellingwerf and the recent analytical EOS developed by Irwin. Current findings suggest that the pulsation amplitudes, as well as the topology of the instability strip, marginally depend on the adopted EOS. To compromise between accuracy and numerical complexity we computed new EOS tables using the Irwin analytical EOS. We found that the difference between analytical and tabular thermodynamic quantities and their derivatives are smaller than 2% when adopting suitable steps in temperature and density. To improve the numerical accuracy of physical quantities, we are now adopting bicubic splines to interpolate both opacity and EOS tables. The new approach presents a substantial advantage to avoiding numerical derivatives in both linear and nonlinear models. The EOS first- and second-order derivatives are estimated by means of the analytical EOS or by means of analytical derivatives of the interpolating function. The opacity first-order derivatives are evaluated by means of analytical derivatives of the interpolating function. We also investigated the dependence of observables predicted by theoretical models on the mass-luminosity (ML) relation and on the spatial resolution across the hydrogen and the helium partial ionization regions. We found that nonlinear models are marginally affected by these physical and numerical assumptions. In particular, the difference between new and old models in the location as well as in the temperature width of the instability strip is, on average, less than 200 K. However, the spatial resolution somehow affects the pulsation properties. The new fine models predict a period at the center of the Hertzsprung progression ($P_{\text{HP}} = 9.65 - 9.84$ days) that reasonably agrees with empirical data based on light curves ($P_{\text{HP}} = 10.0 \pm 0.5$ days); and radial velocity curves ($P_{\text{HP}} = 9.95 \pm 0.05$ days); they improve previous predictions by Bono, Castellani, & Marconi.

Subject headings: Cepheids — Galaxy: stellar content — hydrodynamics — stars: evolution — stars: oscillations

On-line material: additional figures, machine-readable tables

1. INTRODUCTION

The equation of state (EOS) and opacity are fundamental physical ingredients for both evolutionary and pulsation models. In particular, hydrodynamic models of variable stars, when compared to static stellar structures, require additional derivatives of thermodynamic quantities (Christy 1969; Stellingwerf 1974, 1982, and references therein). As a consequence, the EOS, opacities, and numerical methods adopted to estimate physical quantities such as pressure, temperature, internal energy, molecular weight, and their derivatives, are crucial to properly compute the physical structure of stellar envelopes (Dorman, Irwin, & Pedersen 1991). Recent helioseismic data uncorked several theoretical investigations aimed at improving the accuracy of input physics currently adopted to construct both solar and stellar models (see, e.g., Christensen-Dalsgaard et al. 1996; DeGiovincenti et al. 1997; Castellani et al. 2002). The EOS is typically provided in tabular form, where thermodynamic quantities are provided for each given chemical composition as a function of temperature and density. The most popular ones are the so-called Mihalas-Däppen-Hummer (MDH) EOS (Mihalas, Däppen, & Hummer 1988; Däppen et al. 1988; Däppen, Anderson, & Mihalas 1987) and the OPAL EOS developed at Livermore (Rogers 1986; Iglesias & Rogers 1995; Rogers, Swenson, & Iglesias 1996) and recently improved in the treatment of input physics (Rogers 2000, 2001; Rogers & Nayfonov 2002).

To overcome the problems introduced by the interpolation across the tables, new EOSs have also been developed in the form of in-line analytical formulae that allow the estimate of thermodynamic quantities as a function of temperature, density, and chemical composition. The most recent EOSs are EFF (Eggleton, Faulkner, & Flannery 1973), CEFF (Christensen-Dalsgaard & Däppen 1992), SIREFF (see, e.g., Guzik & Swenson 1997), and the recent in-line
EOS developed by Irwin\(^1\) (Cassisi, Salaris, & Irwin 2003; Irwin et al. 2003). The reader interested in a detailed discussion concerning pros and cons of the different physical assumptions adopted to derive these EOSs is referred to the reviews by Däppen, Keady, & Rogers (1991), Rogers & Iglesias (1998), and Däppen & Guzik (2000).

Detailed numerical experiments concerning the dependence of pulsation predictions on the EOS were performed by Kanbur (1991, 1992). He found that both linear and nonlinear radiative bump Cepheid models constructed by adopting either the MDH EOS or the canonical Saha EOS present negligible differences. However, more recent EOS calculations, when compared to previous ones, include a more detailed treatment of heavy elements and the most important hydrogen molecules. As a consequence, we decided to investigate the dependence of current pulsation predictions for classical Cepheids on the input physics and, in particular, on the EOS by taking advantage of the analytical EOS developed by Irwin. Moreover, recent theoretical investigations bring forward that the mass-luminosity (ML) relation might play a fundamental role in accounting for actual properties of variable stars (Bono, Castellani, & Marconi 2002; Bono et al. 2002). Therefore, we also plan to test the dependence on this key ingredient as well as on the spatial resolution across the H and the He ionization regions. The theoretical framework adopted for constructing linear radiative and nonlinear convective models has already been described in a series of papers (Bono & Stellingwerf 1994; Bono, Marconi, & Stellingwerf 1999, hereafter BMS99; Bono, Castellani, & Marconi 2000, hereafter BCM00, and references therein). The treatment of the EOS is described in Stellingwerf (1975, 1982), while the treatment of the opacities is discussed in Bono, Incerpi, & Marconi (1996).

In §2 we present the method adopted for handling the opacity tables and the new EOS tables, while in §3 we discuss the comparison between the old EOS by Stellingwerf (1975, 1982) and the new one by A. W. Irwin et al. (2004, in preparation; Cassisi et al. 2003). In §4 we present detailed theoretical investigations aimed at testing the dependence of theoretical observables predicted by hydrodynamic models for Galactic Cepheids on the input physics. To investigate the linear pulsation behavior, we constructed a set of nonadiabatic radiative models. The linear observables, namely, period and blue boundary of the instability strip, are widely discussed in §§3.1 and 3.2. We also constructed nonlinear and_tidependent convective Cepheid models to assess the modal stability and, in turn, to evaluate both the boundaries of the strip as well as the amplitude and morphology of light and velocity curves (§§4.1 and 4.2). Theoretical predictions are also compared to observational data available in the literature. Finally, §4.3 is focused on the Hertzsprung-Progression (HP). A summary of the results is given in §5, together with a brief discussion concerning future plans.

2. INPUT PHYSICS

2.1. The Opacity

The pulsation models are constructed by adopting radiative opacities (OPAL; Rogers & Iglesias 1992; Iglesias & Rogers 1996) for temperatures higher than \(\sim6000\) K and molecular opacities (Alexander & Ferguson 1994, hereafter AF94) at lower temperatures. The method adopted to handle the opacity tables, and to evaluate the opacity derivatives with respect to temperature and density, is a revised version of the method described by Bono et al. (1996). It relies on bicubic interpolating functions with analytical derivatives whose coefficients are computed from the functions and derivatives at grid points (Seaton 1993). In the previous approach, for each given chemical composition, two distinct programs were developed to compute a finer opacity table by interpolating the original OPAL and AF94 tables. Together with the opacity, the opacity derivatives with respect to temperature and density were also calculated and stored. The previous tables were matched together for a temperature equal to 10,000 K. Finally, the opacity and its derivatives were computed by means of a bilinear interpolation on the finer big table.

The revised method only relies on bicubic spline interpolations. The original two sets of opacity tables (OPAL and AF94) are first matched together; then the bicubic spline and its derivatives are computed directly inside the pulsation codes for the entire range of temperature and density covered by the original table. The temperature value we adopted to match the two sets of opacities is \(T = 6300\) K (VandenBerg et al. 2000), since in this temperature range the opacity variations are smooth. Moreover, at lower temperatures, the density range has been extended from \(\log R = 1.0\) up to \(\log R = 7.0\), where \(R = \rho T_\odot^4\), \(T_\odot = 10^{-6}T\), and \(\rho\) is the density. The new method allowed us to decrease by approximately a factor of 10 the CPU time required to construct nonlinear models.

To investigate whether the method adopted to interpolate the opacity tables affects the structure of pulsating models, we performed several numerical experiments across the instability strip. Figure 1 shows the relative difference in opacity between the old and new interpolation methods for

\[\Delta\alpha = \frac{\alpha_{\text{new}} - \alpha_{\text{old}}}{\alpha_{\text{old}}} \]

\[\log T = 7.0, 7.5, 8.0, 8.5, 9.0, 9.5, 10.0, 10.5, 11.0, 11.5, 12.0, 12.5, 13.0\]

\[M = 5, 6, 7, 8, 9, 10, 11, 12, 13\]

Fig. 1.—Relative difference in opacity between the old and new methods adopted to interpolate opacity tables as a function of logarithmic temperature. From top to bottom, data refer to different linear models. See text for more details.

\(^1\)See also ftp://astroftp.phys.uvic.ca/pub/irwin/eos.
The adopted stellar mass ranges from 5 to 13 $M_{\odot}$, while the effective temperature ranges from $T_e = 5600$ to 3800 K. The difference is, on average, smaller than 1% and approaches 2% at log $T \approx 3.8$–4.0 K, i.e., the region where the radiative (OPAL) and the molecular (AF94) opacity tables were matched together (6300 vs. 10,000 K). This indicates that the two methods approach the accuracy of opacity tables that is typically on order of 2% (Seaton 1993). Note that to avoid spurious wiggles in the comparison between the two interpolation methods due to small changes in the zoning of the models (Guzik & Swenson 1997), we computed at first the models with the old method, and then we used the same values of temperature and density of the individual models to interpolate the opacity tables with the new method. The marginal difference between the two interpolation methods is further supported by the fact that two independent sets of linear models constructed by adopting the old and the new interpolation method present a negligible difference in both the pulsation period and the growth rate (see data listed at the top of Table 1).

2.2. The Equation of State

Pulsation models constructed by our group rely on the equation of state developed by Stellingwerf (1975, 1982). It treats equilibrium mixture of $H$, $H^+$, $He$, $He^+$, $He^{++}$, $M$, and $M^+$, where $M$ designates a fictional metal with fixed ionization potential, number abundance, atomic weight, and degeneracy ratio. These parameters were chosen by fitting the electron pressure at cool temperatures given by an exact solution of King IVa ($Y = 0.28$, $Z = 0.02$) chemical composition (Cox, King, & Tabor 1973). This fictional element was included to account for the ionization of Mg, Fe, and Si in cooler Cepheid envelopes (Stellingwerf 1982).

The accuracy in opacity and EOS derivatives is a key feature to construct accurate pulsation models, since the driving mechanisms are connected with envelope regions where the temperature and the density gradients present sudden changes. Therefore, we decided to improve our hydrodynamic codes by adopting the analytical EOS developed by Irwin, since it is based on approximately the same physics adopted in recent EOS, such as MDH and OPAL, and it is quite flexible. We adopted the option suggested by Irwin (EOS) that was constrained by fitting the OPAL and SCVH (Saumon, Chabrier, & van Horn 1995) EOSs. The high numerical complexity typical of in-line routines was partially overcome by selecting some input parameters that allow a sound compromise between speed and accuracy. The flexibility is the main advantage in using an analytical EOS, since it can cope with variations in chemical composition and elemental mixture. Moreover, an analytical EOS provides smooth high-order derivatives of physical quantities, and it can also be easily modified to compute additional thermodynamic quantities. On the other hand, the tabular EOS requires a limited amount of CPU time to interpolate the physical quantities when compared to the in-line EOS. However, the tabular EOS might be limited in the log $T$ versus log $\rho$ coverage as well as in chemical compositions and does not include high-order derivatives of thermodynamic
quantities (Guzik & Swenson 1997; Däppen & Guzik 2000). The latter problem is generally overcome by computing numerical derivatives.

Keeping in mind these problems, we decided to investigate the key features of EOSs available in the literature. The Irwin EOS includes neutral and positive ions of the 20 most abundant atoms, namely, H, He, C, N, O, Ne, Na, Mg, Al, Si, P, S, Cl, A, Ca, Ti, Cr, Mn, Fe, and Ni. On the other hand, the tabular EOSs by MDH and OPAL only include the elements H, He, C, N, O, and Ne, with the last one considered as representative of heavier elements (Rogers, Swenson, & Iglesias 1996; Rogers & Nayfonov 2002). The Irwin EOS allows the use of different solar element mixtures and therefore the possibility to use the same mixture adopted in the calculation of opacity tables. Note that an EOS that accounts for molecular species might play an important role in constructing pulsation models for long-period variables such as semiregular, Miras, and long-period Cepheids. The OPAL project recently provided new EOS tables that include H2, H+2 as well as H−, He+2, and HeH+ (Rogers & Nayfonov 2002), while the Irwin EOS only includes the hydrogen molecules. However, previous authors found that in physical regimes in which the EOS shows rapid changes and the density is lower than log ρ < −8, typical of Cepheid envelopes at temperatures cooler than 60,000 K, first-order properties, such as pressure and internal energy, might be off by as much as a few percent, while second-order properties such as specific heat and adiabatic exponents might be off by as much as 10%.

In this context, it is worth noting that the main difference in dealing with linear and nonlinear pulsation models is that the latter ones require the second derivatives of total pressure (see eqs. [A21]–[A23] in Stellingwerf 1982). These quantities are not generally included in tabular EOSs, and to compute them it is necessary to perform numerical derivatives across the tables. This means that the second derivatives are likely less accurate than the other thermodynamic quantities. To compromise between accuracy and numerical complexity, we decided to use the Irwin EOS to compute several tabular EOSs at fixed chemical composition but with different grid spacings. The resolution in temperature and density of the tabular EOS need to be cautiously treated, since the interpolation across the tables or the estimate of numerical derivatives might be affected by systematic uncertainties if the grid resolution is too coarse (Dorman et al. 1991). We performed several numerical experiments to overcome this problem, and we found that EOS tables with a step ranging from ~0.05 to ~0.1 dex in log T and 0.5 dex in log R works quite well. Figure 2 shows the relative difference in four physical quantities between the analytical and the tabular EOSs by Irwin for a 9 M⊙ model with Te = 4800 K. From top to bottom, the figure shows the relative difference in the adiabatic exponent γ1 (Cox & Giuli 1968), internal energy E, mean molecular weight per particle μ, and specific heat at constant pressure Cp. Data plotted in this figure suggest that the difference is typically smaller than 1%, while for Cp it attains the 2% across the hydrogen ionization region (HIR). Note that to perform the difference, we constructed at first the linear model using the tabular EOS, and then we used the same values of temperature and density to estimate with the analytical EOS the same physical quantities. The current finding is further supported by the fact that the difference in pulsation periods and growth rates between linear Cepheid models constructed by adopting the analytical and tabular EOSs is at most on the order of a few thousandths (0.003; see data listed in the middle of Table 1).

We performed the same test using a wide range of stellar input parameters and the outcome was the same. This means that the current approach allows us to derive thermodynamic quantities and their derivatives with an accuracy that is on average better than 2%. The method adopted for interpolating tabular EOSs and deriving high-order derivatives is the same adopted for the opacity tables (see § 2.1). Note that current method uses bicubic splines that provide the opportunity to derive analytical first- and second-order derivatives instead of estimating them numerically. As a matter of fact, we computed with the analytical Irwin EOS the first derivatives of pressure with respect to temperature and density; then the interpolation with the bicubic splines allowed us to derive analytically the three second-derivatives of pressure. At the same time, we computed with the analytical EOS the specific heat at constant pressure and the interpolation provided the analytical first-derivatives with respect to temperature and density.

2 See also http://www-phys.llnl.gov/Research/OPAL/index.html.
3. LINEAR RESULTS

3.1. Dependence on the EOS

To investigate the dependence of pulsation properties on the EOS, we constructed a survey of linear, nonadiabatic models at solar chemical composition, namely, $Y = 0.28$, $Z = 0.02$. The luminosity of Cepheid envelope models was fixed according to the ML relation recently derived by Bono et al. (2000). This relation relies on several sets of evolutionary models and agrees quite well with similar prescriptions available in the literature. The difference between the current ML relation and the ML relation provided by Alibert et al. (1999) and by Castellani, Chieffi, & Straniero (1992) is, at fixed stellar mass, on average smaller than 0.1 dex. To constrain the effect of the EOS, we selected three Cepheid models that cover a substantial portion of the instability strip. In particular, the model with 5 $M_\odot$ is located close to the fundamental blue edge ($T_e = 5600$ K), the 9 $M_\odot$ model is approximately located in the middle of the strip ($T_e = 4800$ K), while the 13 $M_\odot$ model is close to the red edge ($T_e = 3800$ K).

Current numerical experiments show that the relative difference in $T_e$ and in $C_v$ is generally smaller than the uncertainties of the interpolation scheme (see Fig. 2). However, toward lower temperatures, the difference between the two EOSs increases and becomes on the order of 15% ($T_e$) and 30% ($C_v$) close to the surface of these envelope structures. The relative difference in the mean molecular weight, $\mu$, presents a systematic trend, but it is vanishing (smaller than 1%) throughout the entire envelope. On the other hand, the difference in the internal energy attains vanishing values (smaller than 1%) over a substantial portion of the envelope; it undergoes a sudden increase in the outermost regions ($T \leq 10,000$ K) and approaches a difference on the order of 50%–60% close to the surface. Note that to avoid spurious wiggles in the comparison between the two EOSs, we constructed at first the models with the analytical Stellingwerf EOS, and then we used the same values of temperature and density to interpolate the tabular Irwin EOS.

Even though some physical quantities present a sizable difference between the Stellingwerf and the Irwin EOS, the impact of this difference on linear observables such as pulsation periods and growth rates is negligible. The difference in the linear periods (fundamental, first, and second overtone) is at most a few hundredths (see data listed at the bottom of Table 1). The same outcome applies for the growth rates. The negligible difference between the two sets of models is due to the marginal effect that the outermost layers have on the pulsation properties.

3.2. Blue Boundaries

To investigate the dependence of the blue (hot) edges of the Cepheid instability strip on the EOS, we constructed several sequences of models with stellar masses ranging from 5 to 13 $M_\odot$. As in previous investigations (BMS99), the adopted effective temperature step is 100 K. The current ML relation predicts, for each given stellar mass, a luminosity level that is on average lower than adopted by BMS99 and BCM00. The difference in luminosity is vanishing in the low-mass range but increases when moving toward higher stellar masses. The main intrinsic properties of hydrostatic envelopes and the basic assumption on the numerical approximations adopted for constructing the models have already been discussed in a series of previous papers (see, e.g., Bono & Stellingwerf 1994; BMS99; BCM00). The new models, when compared to the old models, present a finer spatial resolution in the region located between the first helium ionization region ($T \approx 2.1 \times 10^4$ K) and the surface. The number of zones in this region is typically 35, i.e., $\Delta T \approx 450$–520 K, while in the old regions it was $\approx 25$, i.e., $\Delta T \approx 600$–680 K. The inner boundary condition was fixed in such a way that the base of the envelope is located at a distance $r_0 = r/R_{\text{ph}} = 0.03$–0.04 from the star center, where $R_{\text{ph}}$ is the equilibrium photospheric radius, while in the old models it was fixed at $r_0 = r/R_{\text{ph}} \approx 0.1$. The envelope mass ranges from 65% to 40% of the total mass when moving from hotter (5 $M_\odot$) to cooler (13 $M_\odot$) Cepheid models.

To investigate the dependence of the blue edges on the equation of state, we constructed two independent sets of linear models according to the Stellingwerf and the Irwin EOSs. Current numerical experiments show that the location of fundamental (F) and first overtone (FO) linear blue boundaries mildly depend on the adopted EOS. The F edges based on the Irwin EOS are either identical, in the low-mass regime, to the edges based on the Stellingwerf EOS or they present a difference, at most, of 200 K for $M/M_\odot = 9$. The difference for FO edges is even smaller and, at most, $\sim 100$ K. However, it turns out that the spatial resolution and the ML relation affect the linear blue boundaries more than the EOS. In fact, the new F blue edge is on average 200 K hotter than predicted by BMS99, and the difference becomes $\sim 400$ K for stellar masses ranging from 7 to 9 $M_\odot$. Note that the difference in the predicted luminosity for $M/M_\odot = 5$ between Bono et al. (2000) and BMS99 ML relations is vanishing. Therefore, the difference of 300 K between new and old F and FO blue edges is due to the increase in the spatial resolution of the new models.

4. NONLINEAR RESULTS

4.1. The Instability Strip

Nonlinear and time-dependent convective hydrodynamic models supply the unique opportunity to estimate both the blue and red edges of the instability strip, as well as provide robust predictions concerning the pulsation amplitudes and the shape of light and velocity curves along the pulsation cycle. This means that observables predicted by nonlinear models can be soundly compared to actual properties of variable stars. To supply a new homogeneous scenario, we constructed six new sequences of nonlinear models with stellar masses ranging from 5 to 13 $M_\odot$. The nonlinear analysis was performed by perturbing the linear F and FO radial eigenfunctions with a velocity amplitude of 5 km s$^{-1}$. Table 2 summarizes the input parameters, as well as the nonlinear pulsation properties, of the entire set of models. The observables listed in this table have been estimated at full amplitude; i.e., after that the perturbed envelopes approached the nonlinear limit cycle stability (see BMS99 for more details).

To investigate the dependence of nonlinear pulsation models on the EOS we also computed six sequences of pulsation models by adopting the same assumptions and input parameters of the models listed in Table 2, but using the Stellingwerf EOS. The observables predicted by these models are given in Table 3. The top panel of Figure 3 shows the comparison between nonlinear F and FO edges predicted by models that
## TABLE 2

Input Parameters and Nonlinear Observables for First Overtone and Fundamental Cepheid Models at Solar Chemical Composition

| $M^a$ ($M_\odot$) | $\log L^b$ ($L_\odot$) | $T_i^c$ (K) | $\rho^d$ (g cm$^{-3}$) | $\log \tau (R_i^c)^e$ | $\Delta R / R_{ab}^f$ (km s$^{-1}$) | $\Delta M_{bol}^b$ (mag) | $\Delta log g^i$ | $\Delta T_e^j$ (K) |
|-----------------|-----------------|---------|-----------------|----------------|----------------|----------------|--------------|---------------|
| 5.0             | 3.07            | 6100    | 2.0655          | 1.488          | 0.045          | 34.87          | 0.497        | 0.04         | 740          |
| 5.0             | 3.07            | 6000    | 2.1733          | 1.502          | 0.051          | 39.27          | 0.505        | 0.04         | 728          |
| 5.0             | 3.07            | 5900    | 2.2958          | 1.516          | 0.050          | 37.94          | 0.438        | 0.04         | 620          |
| 5.0             | 3.07            | 5800    | 2.4253          | 1.531          | 0.044          | 32.38          | 0.341        | 0.04         | 479          |

**First Overtone**

| 5.0             | 3.07            | 5700    | 3.6840          | 1.548          | 0.115          | 57.88          | 0.742        | 0.10         | 1037         |
| 5.0             | 3.07            | 5600    | 3.8991          | 1.564          | 0.120          | 57.55          | 0.656        | 0.11         | 906          |
| 5.0             | 3.07            | 5500    | 4.1422          | 1.578          | 0.105          | 50.63          | 0.511        | 0.09         | 712          |
| 5.0             | 3.07            | 5400    | 4.3949          | 1.593          | 0.094          | 43.80          | 0.396        | 0.08         | 610          |
| 5.0             | 3.07            | 5300    | 4.6688          | 1.606          | 0.074          | 33.22          | 0.288        | 0.07         | 445          |
| 5.0             | 3.07            | 5200    | 4.9913          | 1.622          | 0.038          | 16.42          | 0.127        | 0.03         | 212          |
| 5.0             | 3.07            | 5100    | 7.0730          | 1.758          | 0.072          | 31.13          | 0.309        | 0.06         | 703          |
| 5.0             | 3.07            | 5000    | 7.4988          | 1.773          | 0.079          | 34.83          | 0.524        | 0.07         | 712          |
| 5.0             | 3.07            | 4900    | 7.9803          | 1.790          | 0.077          | 34.24          | 0.483        | 0.07         | 643          |
| 5.0             | 3.07            | 4800    | 8.4829          | 1.804          | 0.074          | 32.45          | 0.412        | 0.06         | 554          |
| 5.0             | 3.07            | 4700    | 9.0525          | 1.820          | 0.059          | 23.98          | 0.258        | 0.05         | 375          |
| 5.0             | 3.07            | 4600    | 9.3328          | 1.828          | 0.048          | 18.40          | 0.211        | 0.04         | 287          |
| 5.0             | 3.07            | 4500    | 9.6496          | 1.836          | 0.042          | 15.39          | 0.188        | 0.04         | 223          |
| 5.0             | 3.07            | 4400    | 9.9560          | 1.845          | 0.047          | 18.58          | 0.212        | 0.04         | 269          |
| 5.0             | 3.07            | 4300    | 10.3025         | 1.853          | 0.060          | 24.68          | 0.266        | 0.05         | 354          |
| 5.0             | 3.07            | 4200    | 10.6464         | 1.869          | 0.076          | 29.66          | 0.298        | 0.07         | 418          |
| 5.0             | 3.07            | 4100    | 10.9979         | 1.882          | 0.075          | 23.71          | 0.372        | 0.05         | 521          |
| 5.0             | 3.07            | 4000    | 11.3486         | 1.895          | 0.075          | 21.74          | 0.282        | 0.05         | 394          |
| 5.0             | 3.07            | 3900    | 11.6986         | 1.908          | 0.075          | 20.85          | 0.273        | 0.04         | 359          |
| 5.0             | 3.07            | 3800    | 12.0482         | 1.921          | 0.075          | 19.95          | 0.265        | 0.04         | 324          |
| 5.0             | 3.07            | 3700    | 12.3988         | 1.934          | 0.075          | 19.05          | 0.259        | 0.04         | 299          |

**Fundamental**

Notes.— Table 2 is also available in machine-readable form in the electronic edition of the Astrophysical Journal.

$^a$ Stellar mass.

$^b$ Logarithmic luminosity.

$^c$ Effective temperature.

$^d$ Period.

$^e$ Logarithmic mean radius.

$^f$ Fractional radius variation.

$^g$ Radial velocity amplitude.

$^h$ Bolometric amplitude.

$^i$ Amplitude of logarithmic static gravity.

$^j$ Effective temperature variation.
use the Stellingwerf (triangles) or the Irwin (circles) EOSs. Once again, data plotted in this panel disclose that the difference is smaller than 100 K. The same outcome applies for the temperature width, and indeed it decreases by 200 K for stellar masses across the so-called Hertzsprung progression, i.e., $M/25$;

\[55;7\]

while for the other mass values the difference is smaller than 1% across the entire instability strip. These findings, together with the results obtained for the linear blue edges, indicate that the EOS has a marginal effect on the topology of the Cepheid instability strip.

To single out the effect of the ML relation on pulsation properties, we also constructed a new sequence of nonlinear models for $M = 11 M_\odot$ and $\log L/L_\odot = 4.40$ (see Table 4). These models, when compared to the sequence of models for $M = 11 M_\odot$ listed in Table 2, only differ in the luminosity level (4.40 [BCM00] vs. 4.21 [Bono et al. 2000]). The comparison between the observables given in Tables 2 and 4 shows that the dependence of the instability strip on the ML relation is mild. The shift in temperature of the new edges is typically smaller than 200 K, while the pulsation amplitudes attain quite similar values.

To investigate the dependence of the pulsation behavior on the spatial resolution across the HIR, we constructed a new sequence of nonlinear models for $M = 11 M_\odot$ by adopting the same input parameters of models listed in Table 2, but a coarse zoning (see Table 5). Data listed in Tables 2 and 5 disclose that the spatial resolution affects the modal stability, and indeed the F edges shift by $200 K$.

\[3\] In the period range from 6 to 16 days, classical Cepheids present a well-defined bump along both the luminosity and the radial velocity curve. For periods shorter than $9$ days the bump is located along the decreasing branch, while toward longer periods it takes place along the rising branch.
toward hotter effective temperatures. This finding is also supported by the set of models for $M = 5 M_\odot$ listed in Table 3. These models and the set of models for $M = 5 M_\odot$ constructed by BCM00 present the same luminosity, but the comparison displays a shift of 200–300 K in the F edges. Moreover, finer models present an increase in the temperature width of the instability region, namely, 1000 versus 600 K. This difference is even greater (~25%) for the $M = 5 M_\odot$ FO models.

The bottom panel of Figure 3 shows the comparison between the new F and FO edges (circles) and the edges predicted by BCM00 (plus signs). A glance at the data plotted in this panel shows that the topology of the instability strip predicted by old and new models is quite similar.

Figures 4, 5, and 6 display both light and velocity curves as a function of phase over two consecutive pulsation cycles for the entire set of models.4 The comparison between these figures and predictions obtained for the set of models listed in Table 3 further strengthens the evidence that the EOS

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**TABLE 4**

**Input Parameters and Nonlinear Observables for Cepheid Models with $M = 11 M_\odot$ and Solar Chemical Composition, Constructed by Adopting the Irwin EOS, the ML Relation used by BCM00, and a Fine Zoning across Hydrogen and Helium Ionization Regions**

| $M^a$ | $log (L_\odot)^b$ | $T^c_e$ | $P^d$ | $log \tau (R_\odot)^e$ | $\Delta R / R_{ph}\footnote{f}$ | $\Delta U^g$ | $\Delta M_{bol}^h$ | $\Delta T^j$ |
|-------|-----------------|--------|-------|----------------------|----------------|---------|-----------|--------|
| (M$_\odot$) | (2) | (K) | (days) | (5) | (km s$^{-1}$) | (mag) | (mag) | (K) |
| 11.0 | 4.40 | 4800 | 59,322 | 2.364 | 0.446 | 32.48 | 0.632 | 0.47 | 827 |
| 11.0 | 4.40 | 4600 | 68,550 | 2.401 | 0.507 | 41.51 | 0.806 | 0.55 | 916 |
| 11.0 | 4.40 | 4300 | 73,741 | 2.418 | 0.525 | 42.27 | 0.799 | 0.59 | 828 |
| 11.0 | 4.40 | 4100 | 85,242 | 2.449 | 0.546 | 37.21 | 0.663 | 0.64 | 649 |
| 11.0 | 4.40 | 3900 | 91,733 | 2.465 | 0.550 | 32.57 | 0.578 | 0.67 | 579 |
| 11.0 | 4.40 | 4000 | 39,000 | 5.625 | 2.364 | 18.66 | 0.216 | 0.46 | 309 |

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**TABLE 5**

**Input Parameters and Nonlinear Observables for Cepheid Models of $M = 11 M_\odot$ and Solar Chemical Composition, Constructed by Adopting the Irwin EOS, the ML Relation by Bond et al. (2000), and a Coarse Zoning across Hydrogen and Helium Ionization Regions**

| $M^a$ | $log (L_\odot)^b$ | $T^c_e$ | $P^d$ | $log \tau (R_\odot)^e$ | $\Delta R / R_{ph}\footnote{f}$ | $\Delta U^g$ | $\Delta M_{bol}^h$ | $\Delta T^j$ |
|-------|-----------------|--------|-------|----------------------|----------------|---------|-----------|--------|
| (M$_\odot$) | (2) | (K) | (days) | (5) | (km s$^{-1}$) | (mag) | (mag) | (K) |
| 11.0 | 4.21 | 4900 | 36,950 | 2.257 | 0.282 | 42.13 | 0.725 | 0.26 | 984 |
| 11.0 | 4.21 | 4900 | 36,950 | 2.257 | 0.321 | 46.38 | 0.760 | 0.30 | 987 |
| 11.0 | 4.21 | 4600 | 45,776 | 2.309 | 0.376 | 44.98 | 0.648 | 0.37 | 983 |
| 11.0 | 4.21 | 4500 | 49,066 | 2.325 | 0.396 | 42.01 | 0.579 | 0.40 | 763 |
| 11.0 | 4.21 | 4300 | 56,784 | 2.354 | 0.417 | 30.17 | 0.382 | 0.45 | 532 |
| 11.0 | 4.21 | 4200 | 60,955 | 2.368 | 0.408 | 18.66 | 0.216 | 0.46 | 309 |

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* Stellar mass.
* Logarithmic luminosity.
* Effective temperature.
* Period.
* Logarithmic mean radius.
* Fractional radius variation.
* Radial velocity amplitude.
* Bolometric amplitude.
* Amplitude of logarithmic static gravity.
* Effective temperature variation.

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4 Figures 4, 7, 8, and 9, as well as Figures 12 and 13, are only available in the electronic edition of the Journal.
Fig. 4.—Bolometric light curves (left) and radial velocity curves (right) as a function of the pulsation phase for $M = 5 M_\odot$. Dashed and solid lines show the variation over two consecutive cycles of FO and F pulsators, respectively. In the left panel are also plotted the nonlinear periods (days), while in the right panel the adopted effective temperatures (K) are shown. Positive values along the velocity curves denote expansion phases, while negative values denote contraction phases. [See Figs. 7, 8, and 9 in the electronic edition of the Journal for $M = 9$, 11, and 13 $M_\odot$, respectively.]
Fig. 5.—Same as Fig. 4, but for the sequence of models with $M = 6.55 \, M_{\odot}$. [See Figs. 12 and 13 in the electronic edition of the Journal for versions of this model with the fine and coarse zoning across the partial ionization regions.]
Fig. 6.—Same as Fig. 4, but for the sequence of models with $M = 7 \, M_\odot$. 

- $M_{\text{bol}} \, (\text{mag})$
- Velocity (Km/sec)
also marginally affects the morphology of light and velocity curves.

4.2. Pulsational Amplitudes

Nonlinear models also predict the luminosity amplitude, which is a key parameter to study the pulsational properties of classical Cepheids. In particular, the Bailey diagram, i.e., luminosity amplitude versus period, is only marginally affected by reddening and distance uncertainties. Therefore, the comparison between theory and observations in this plane supplies independent constraints on the intrinsic Cepheid parameters to be compared to evolutionary prescriptions. The top panel of Figure 10 shows predicted bolometric amplitudes of Cepheid models constructed by adopting the new (solid line) and the old (dotted line) EOSs as a function of period. The agreement between the two sets of models is very good over the entire mass range. The first overtone models for \( M = 5 \, M_\odot \) and the F models across the HP present a mild difference on the EOS close to the edges of the instability region. Data plotted in the middle panel show the dependence of bolometric amplitudes on the ML relation and spatial resolution when the Stellingwerf EOS is adopted in the computations. As a whole, the bottom panel displays the comparison between new predictions and old results by BCM00. The predicted luminosity amplitudes show a "hook" shape when moving from the blue to the red edge of the instability region, while the models located across the HP present the typical "double-peaked" distribution of bump Cepheids (Bono, Marconi, & Stellingwerf 2000, hereafter BMS00). This trend supports the empirical evidence for Galactic Cepheids originally brought forward by Sandage & Tammann (1971) and Cogan (1980). They found that in the period range \( \log P \approx 0.40-0.86 \) and for \( \log P > 1.1-1.3 \) the largest luminosity amplitudes are attained close to the blue edge, while across the HP (0.85 \( \leq \log P \leq 1.1-1.3 \) ) the maximum is attained close to the red edge (BCM00, BMS00).

4.3. Hertzsprung Progression

The top panel of Figure 11 shows the comparison between predicted \( V \)-band amplitudes and empirical data for Galactic Cepheids collected by Fernie et al. (1995), while the bottom panel displays the comparison between predicted and empirical radial velocities. Theory is in reasonable agreement with observations. The models across the HP deserve a more detailed discussion. According to

![Fig. 10.—Comparison between bolometric amplitudes based on pulsation models constructed by adopting different EOSs (top), different ML relation and spatial resolution (middle), and different EOS, ML relation, and spatial resolution (bottom).](image)

![Fig. 11.—Comparison between predicted and empirical amplitudes for fundamental Galactic Cepheids. The top panel shows the comparison between \( V \)-band amplitudes collected by Fernie et al. (1995) and current models. The bottom panel displays the comparison between different samples of empirical radial velocity amplitudes and theoretical predictions. Note that predicted amplitudes were multiplied by 1.36 (Bersier & Burki 1996).](image)
agreement with Moskalik et al. (2000), who found the velocity amplitude of classical Cepheids located at

\[
\log R = \text{log the velocity amplitude of classical Cepheids located at}
\]

| \(M^a (M_\odot)\) | \(\log L (L_\odot)\) \(b\) | \(T_c^c (K)\) | \(P^d \) \(\text{days}\) | \(\log \mathcal{R} \left( R_\odot \right)\) \(e\) | \(\Delta R/R_{\odot}^f \) \(\text{km s}^{-1}\) | \(\Delta M_{bol}^g \) \(\text{mag}\) | \(\Delta log g^h \) \(\text{K}\) | \(\Delta T^i\) \(\text{K}\) |
|------------------|-------------------|------------|----------------|-----------------|-----------------|----------------|----------------|----------------|
| 6.55.............. | 3.48              | 5600       | 7.4046         | 1.767           | 0.041           | 16.87          | 0.267          | 0.168          | 300            |
| 6.55.............. | 3.48              | 5500       | 7.8451         | 1.783           | 0.057           | 24.06          | 0.353          | 0.255          | 400            |
| 6.55.............. | 3.48              | 5400       | 8.3426         | 1.798           | 0.057           | 23.61          | 0.312          | 0.267          | 350            |
| 6.55.............. | 3.48              | 5350       | 8.6278         | 1.806           | 0.053           | 21.14          | 0.262          | 0.251          | 300            |
| 6.55.............. | 3.48              | 5300       | 8.8797         | 1.814           | 0.047           | 17.98          | 0.218          | 0.228          | 250            |
| 6.55.............. | 3.48              | 5250       | 9.1559         | 1.822           | 0.043           | 15.55          | 0.204          | 0.210          | 200            |
| 6.55.............. | 3.48              | 5200       | 9.4667         | 1.830           | 0.045           | 17.03          | 0.217          | 0.216          | 250            |
| 6.55.............. | 3.48              | 5100       | 10.139         | 1.847           | 0.067           | 26.58          | 0.310          | 0.296          | 350            |
| 6.55.............. | 3.48              | 5000       | 10.789         | 1.863           | 0.086           | 33.01          | 0.363          | 0.348          | 400            |

\(\log (L)\) is the logarithmic luminosity of the Cepheid. \(T_c\) is the central temperature of the model. \(P\) is the period of the Cepheid. \(\log \mathcal{R}\) is the logarithmic static gravity. \(\Delta R/R_{\odot}\) is the relative radial velocity amplitude. \(\Delta M_{bol}\) is the bolometric amplitude. \(\Delta log g\) is the effective temperature variation.

Input Parameters and Nonlinear Observables for Fundamental Cepheid models with \(M = 6.55 \, M_\odot\) and Solar Chemical Composition, Constructed by Adopting the Stellingwerf EOS, as well as the ML Relation and the Zoning Used by BCM00

BMS00, the secondary minimum in the luminosity and in the velocity amplitude of classical Cepheids located at \(P \approx 1\) can be adopted to fix the period of the HP center (\(P_{\text{HP}}\)). They found that for LMC Cepheids \(P_{\text{HP}} = 11.24 \pm 0.46\) days, while current models for Galactic Cepheids suggest that it is located at \(P_{\text{HP}} = 9.65\) days (\(M = 6.55 \, M_\odot\), \(T_e = 5100\) K) and \(P_{\text{HP}} = 9.84\) days (\(M = 7 \, M_\odot\), \(T_e = 5300\) K). Current estimates are in good agreement with empirical data, and indeed Moskalik, Buchler, & Marom (1992) found that the minimum in the Fourier parameters of Galactic Cepheid light curves is roughly equal to \(P_{\text{HP}} = 10.0 \pm 0.5\) days, and in reasonable agreement with Moskalik et al. (2000), who found \(P_{\text{HP}} = 9.95 \pm 0.05\) days using radial velocity curves of 131 Cepheids. This finding confirms the results obtained by BMS00; i.e., an increase in the metal content causes a shift of the HP center toward shorter periods. In fact, for SMC (\(Z \approx 0.004\)) and LMC (\(Z \approx 0.008\)) bump Cepheids, it is located at 11.0 \pm 0.5 and 10.5 \pm 0.5 days, respectively (Beaulieu 1998). Moreover, predicted luminosity and velocity curves plotted in Figures 5 and 6 also disclose that the bump along the light curves crosses the luminosity maximum at shorter periods when compared to the velocity curves. A similar feature was already found by BMS00 for LMC Cepheids.

To constrain the effect of the ML relation and of the spatial resolution on the HP we computed a new sequence of models for \(M = 6.55 \, M_\odot\) by adopting the same input physics and parameters of the models by BCM00 (see Table 6).

Note that for \(M = 7 \, M_\odot\) the models constructed by BCM00 do not show the HP (see their Fig. 11 in the online edition).

Qualitative arguments concerning the shape of the fundamental light curves indicate that the new models for \(M = 5 \, M_\odot\) also show a well-defined bump across the instability region. To assess on a quantitative basis whether theoretical models account for the HP that has been detected among short-period Galactic Cepheids, it is necessary to implement current models with new ones that cover the low-mass range (Bono et al. 2002).

5. SUMMARY AND FINAL REMARKS

We performed several numerical experiments aimed at testing the dependence of pulsation observables predicted by both linear and nonlinear models on input physics. We found that the physical structure of linear models is marginally affected by the interpolation methods, based on bicubic splines, that we are currently using to estimate the opacity and its derivatives. Interestingly enough, we also found that both linear and nonlinear convective models are also marginally affected by the adopted EOS. We constructed several sequences of pulsation models at solar chemical composition (\(Y = 0.28, Z = 0.02\)) using the analytical EOS developed by Stellingwerf and the recent one developed by Irwin. The comparison suggests that the difference in the pulsation amplitudes, as well as in the topology of the instability strip, is negligible.

We selected the Irwin EOS, since it is available in analytical form, and it allows us to change the chemical composition as well as the abundance of individual elements. To compromise between accuracy and numerical complexity, we computed with the analytical EOS several tabular EOSs by changing the grid resolution in temperature and density. The comparison between models constructed with the analytical and tabular EOSs suggests that the difference is marginal once the step ranges from 0.05 to 0.1 dex in log \(T\) and it is equal to 0.5 dex in log \(R.\) Note that the use of bicubic spline interpolations for both opacity and EOS tables provides the unique opportunity to avoid the calculation of numerical derivatives. The EOS first- and second-order derivatives are estimated by means of the analytical EOS or by means of analytical derivatives of the interpolating...
function. The opacity first-order derivatives are evaluated by means of analytical derivatives of the interpolating function.

According to recent theoretical investigations, we performed several tests to single out the dependence of pulsation predictions on the ML relation, as well as on the spatial resolution across the H and the He partial ionization regions. We found that nonlinear models are marginally affected by ML relations available in the literature. Note that current ML relations rely on evolutionary prescriptions (CCS; ABHA; Bono et al. 2000) that neglect convective core overshooting, mass loss, and rotation.

Both the location and the temperature width of the nonlinear instability strip present differences, on average, smaller than 200 K, while the pulsation amplitudes attain quite similar values. Otherwise, the increase in the spatial resolution across the partial ionization regions somehow affects the pulsation properties of Cepheids. In fact, the instability strip based on finer models moves by approximately 200–300 K toward cooler effective temperatures when compared to models based on a coarse zoning. Moreover, fine-zoning models present bolometric amplitudes that are 25% larger than the coarse-zoning models.

As a whole, the differences between current models with new input physics and parameters, and predictions by BCM00 marginally affect the overall trend inside the instability strip, since the slopes of both blue and red edges predicted by the two sets of models are quite similar. This finding is strongly supported by the evidence that the period-radius relation predicted by current models at solar chemical composition is

$$\log R = 1.173 (\pm 0.008) + 0.676 (\pm 0.006) \log P,$$

while the models constructed by BCM00 supply

$$\log R = 1.191 (\pm 0.006) + 0.654 (\pm 0.005) \log P,$$

where the radius R is in solar units and the period P is in days. The difference between these two relations is quite small and ranges from 0.005 to 0.002 dex when moving from log P = 0.5 to 1.0.

Moreover, and even more importantly, we found that spatial resolution also affects $P_{\text{HP}}$, i.e., the pulsation period at the center of the Hertzsprung progression. The new models show that $P_{\text{HP}}$ ranges from 9.65 for $M = 6.55 M_\odot$ to 9.84 days for $M = 7 M_\odot$. These estimates, when compared to models constructed by BCM00, agree reasonably well with empirical estimates based on light curves ($P_{\text{HP}} = 10.0 \pm 0.5$ days; Moskalik et al. 1992), as well as on radial velocity curves ($P_{\text{HP}} = 9.95 \pm 0.05$ days; Moskalik et al. 2000). Preliminary qualitative results indicate that the new models might also account for the HP that has been detected in the short-period range (Kienzle et al. 1999 and references therein). Current models do account for the shift of $P_{\text{HP}}$ toward shorter periods when moving toward more metal-rich stellar systems (BCM00; BMS00). However, more detailed calculations are required to figure out whether current nonlinear pulsation models do account for the double-peaked distribution disclosed by Galactic and Magellanic bump Cepheids in the amplitude versus period plane.

To further constrain the zoning effect, we computed several linear and nonlinear models across the instability strip by increasing the spatial resolution from 35 to $\approx 50$, i.e., $\Delta T \approx 400$ K. We found that the difference between these two sets of models is negligible, and indeed the difference among linear periods and growth rates is smaller than 0.5%–1% across the instability strip. The relative difference among nonlinear observables is even smaller; in particular, the shift in temperature of the instability strip is smaller than 100 K, while the change in luminosity and velocity amplitudes is smaller than $\approx 0.1%$.

The extension of current theoretical framework to Magellanic Cepheids seems quite promising, not only for the intrinsic properties of variable stars, but also to figure out whether the input physics and/or the spatial resolution of pulsation models do affect the topology of the instability strip and, in turn, predictions concerning the PL and the PLC relations. At the same time, we are also interested in performing a more quantitative comparison between predicted and empirical light curves using both the Fourier technique (Ngeow et al. 2003) and the principal component analysis (Kanbur et al. 2002). The main goal of this project is to constrain the accuracy of current pulsation models and, in turn, to figure out whether the decomposition parameters can be safely adopted to estimate intrinsic parameters of classical Cepheids.

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