On the Effectiveness of Using Various Machine Learning Methods for Forecasting Dangerous Convective Phenomena

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Abstract. The paper considers the possibility of thunderstorm forecasting using only dynamical and microphysical parameters of the cloud, simulated by the 1.5D model with further processing by machine learning methods. The problem of feature selection is discussed in two aspects: selection of the optimal values of time and height when and where the output model data are fixed and selection of fixed set of the most representative cloud parameters (features) among all output cloud characteristics. Five machine learning methods are considered: Support Vector Machine (SVM), Logistic Regression, Ridge Regression, boosted k-nearest neighbour algorithm and neural networks. It is shown that forecast accuracy of all five methods reaches values exceeding 90%.

Keywords: Machine learning \cdot Support Vector Machine (SVM) \cdot Logistic Regression \cdot Ridge Regression \cdot Boosted k-nearest neighbour algorithm \cdot Neural networks \cdot Numerical model of convective cloud \cdot Weather forecasting \cdot Thunderstorm forecasting

1 Introduction

In recent decades mathematicians and programmers are working hard to improve existing numerical weather forecasting models. Nowadays machine learning methods are considered to be one of the most promising tool of such improvement.

Machine learning (ML) is a class of artificial intelligence methods which do not try to solve a problem directly, but by training corresponding algorithms in the process of solution of many similar tasks.

Machine learning is used when:

- it is too complicated to compose system of the equations for a problem solution;
- the solution must be adapted to a new dataset;
- the solution needs to be scaled.
Machine learning algorithms are divided into two groups: supervised and unsupervised learning algorithms. Classification and regression belongs to the first group, clustering to the second one.

Clustering (or cluster analysis) is the task of breaking down multiple objects into groups called clusters. Inside each group there should be “similar” objects, and objects of different groups should be as different as possible. The main difference between clustering and classification is that the list of groups is not clearly defined and is determined during the operation of the algorithm.

The classification problem is the task of assigning a sample to one of several pairwise disjoint sets.

Regression or regression analysis is a statistical method for studying the influence of one or more independent variables on a dependent variable.

The use of machine learning methods in meteorology is twofold. On the one hand, “pure” machine learning models are being developed, where certain atmospheric parameters are predicted on the basis of observational data obtained at meteorological stations, weather centers, etc. [1–3]. On the other hand, machine learning methods are used to verify models by establishing relationships between model forecasts and the actual meteorological situation [4, 5].

In our work, we used the so-called “hybrid” approach [6, 7], combining numerical simulation and machine learning methods to identify the dependence of dynamic, microphysical and electrical parameters of convective clouds. This kind of identification is quite important for forecasting thunderstorm with the help of the models which do not have the block describing electrical processes. It should be noted that Semi-empirical methods of Peskov, Yagudin, Reshetov, Lebedeva and others [8] are still used for operational forecasting of such dangerous phenomena as thunderstorms. These methods are based on the calculation of complex coefficients, which are functions of some cloud parameters, determined either using a synoptic map or using the aerological diagram. The use of modern numerical cloud models for the purpose of forecasting is limited, on the one hand, by the lack of computational resources that are required to implement, for example, modern three-dimensional models with a detailed description of the microphysical and electrical characteristics of the cloud. Such models describe with the greatest degree of detail the dynamic, microphysical and electrical processes in the clouds in all the complexity of their interaction and, therefore, should ensure the best quality of forecasts. However, their use is impossible for operational forecasting in small meteorological centers, due to the lack of the necessary computing power there. On the other hand, the use of models of lower dimensionality and lower functionality sets the problem of determining the likelihood of thunderstorm development only by analyzing the calculated values of the dynamic and microphysical characteristics of the cloud, which are the output of the models, as the latter do not have the block describing electrical processes.

Usage of the machine learning methods for establishing relationship between the output of the numerical model and the probability of thunderstorm, hail, heavy rain will provide effective tool for forecasting most dangerous convective phenomena.

Dangerous meteorological events are in the focus of research in the works [9, 10]. The authors suggested using neural networks to simulate the movement of typhoons, which are the developed tropical cyclones, usually formed in the northwestern Pacific
Ocean. Tropical cyclone intensity changes in the western North Pacific was predicted in [9] using the back-propagation neural network. In [10] a generative adversarial network (GAN) was used for prediction the tracks of typhoons using satellite images as inputs. The neural network was trained with the help of time series of satellite images of typhoons which occurred in the Korea Peninsula in the past.

We concentrate on forecasting only dangerous convective phenomena, mainly thunderstorms, using different machine learning algorithms. Discussion is provided of the most effective method for selection of model output data subsequently used as input for machine learning (features selection).

2 Convective Cloud Model

Modelling of a convective cloud has been conducted by using time dependent, one and a half dimensional (1.5-D) hydrodynamic model with a detailed description of microphysical processes. A cloud shape is simulated by two nested cylinders following the approach suggested by Asai and Kasahara [11]. Cloudy region is represented by the inner cylinder while the downdraft flow outside the cloud is represented by the outer cylinder.

Evolution of dynamical cloud characteristics is simulated by numerical solution of the system of partial differential equations. Buoyancy force, gravity, turbulence are taken into account as well as heat generation/consumption ejected during condensation/evaporation of water vapor and freezing/melting of cloud droplets. A vertical component of the velocity, temperature excess in the cloud, relative humidity, mixing ratio of water vapour, mixing ratio of water drops and cloud thickness are the main dynamical cloud characteristics simulated by the model.

Evolution of microphysical cloud characteristics in time and height is simulated by a numerical solution of the set of stochastic equations for mass distribution functions of cloud drops, columnar crystals, plate crystals, dendrites, snowflakes, graupel and frozen drops. The influence of the following processes is taken into account: nucleation, condensation, sublimation, coalescence, freezing, melting and breakup. Spectra of liquid and solid hydrometeors as well as liquid and ice content of a cloud are calculated with the help of distribution functions obtained earlier.

Transition from the continuous partial differential equations to the finite difference equations is conducted using forward-upstream scheme. Averaged value of the vertical velocity is obtained over two mesh points depending upon the sign of the vertical velocity value (positive or negative).

Though dynamical and microphysical processes develop simultaneously, it is not possible to calculate them in a single time step. The only solution is to split them in time using time-splitting method. Dynamical processes are calculated in the first half of the time step, and microphysical processes in the second half of the time step.

Radiosonde sounding data are used as an input or initial conditions for the model. Radiosonde soundings provide vertical distributions of environmental temperature and relative humidity. It is considered that all cloud characteristics with the exception of temperature and mixing ration of water vapour are equal to zero at the top and at the
bottom boundaries of the cylinders. Impulses of temperature and velocity are set at the initial time moment to push the evolution of the simulated cloud.

The model is able to reproduce the whole cycle of cloud evolution if the conditions in the outer atmosphere is suitable for convection development. Besides, calculated values of cloud parameters allow predicting the probability of the development of such dangerous meteorological phenomena as thunderstorms, hails and rain storms.

Detailed description of the model can be found in [12–15].

Input data are collected with the help of integrated information system [16–18]. We need a significant amount of radiosonde soundings for obtaining sufficient training and test data set collections. This task is not a trivial one, as we have to integrate the data about the dangerous phenomenon occurrence and radiosonde data obtained in the place and at the time of the phenomenon observation. It should be mentioned that the problem has not been solved completely, as the collected test data sets appeared to be small enough for such machine learning algorithm as neural networks that resulted in changing of the structure to perceptron complex.

480 radiosonde soundings with and 196 soundings without phenomena have been collected. 220 soundings related to thunderstorms, 174 ones to heavy rains and 86 soundings to light rains. Different machine learning algorithms use different number of data as test and training data sets. For example, 416 records have been formed for neural networks, where 220 samples correspond to the presence of a dangerous convective phenomenon and 196 samples to its absence. The training set contains 333 samples and the test one contains 83 ones.

3 Algorithms for Data Formation and Preprocessing

Solution of machine learning problems require to find an unknown relationship between a known set of objects and a set of answers. In our case the fact of dangerous phenomenon occurrence can be considered as an answer, and the results of numerical modeling, can be considered as an object. Radiosonde sounding data are used as the model input.

The numerical parameters of the simulated clouds are chosen as object features. The numerical model of convective cloud simulates the whole cycle of natural cloud evolution consisting of three stages: stage of development, mature stage and dissipation stage. Moreover, the output results are produced on every time step of simulation and presents the data on every space step, that is on every 200 m. So the problem is what time step and what height should be chosen for taking the data for future use as object features.

We use three approaches for feature selection. The first one, described in detail in [4, 5, 19, 20] is used for the following machine learning algorithms: Support Vector Machine (SVM), Logistic Regression, Ridge Regression and boosted k-nearest neighbour algorithm.

The first approach assumes fixing the numerical parameters at the moment of maximum cloud development and at the height, where the maximum ratio of water droplets is observed. These time moment and height correspond to the mature stage of cloud evolution. Feature selection has been provided by using recursive feature
elimination algorithm with automatic tuning of the number of features selected with cross-validation. As a result, the following 6 simulated cloud parameters have been chosen as the optimal features to be used for subsequent machine learning processing. These parameters are: the vertical component of the velocity, temperature excess in the cloud, relative humidity, mixing ratio of water vapor, mixing ratio of water drops and cloud thickness.

The second approach is used in the works [4, 5] to refine the results obtained with Support Vector Machine (SVM), Logistic Regression and Ridge Regression algorithms.

The second approach suggests:

- to use individual sets of features for each case of chosen time and height;
- to use the parameters obtained at the stages of cloud development and dissipation (time); and at the lower and the higher levels of maximum ratio of water droplets
- to use features, obtained during the whole cloud evolution in a single set of parameters.

Feature selection has been realized using the L1 regularization method (LASSO) to overcome model overfitting.

Five cases with the different values of time and height have been considered.

Case 1 corresponds to the same height and time (mature stage of cloud development, height of maximum ratio of water droplets). L1 regularization method provides the following cloud parameters, to be used as optimal features: vertical component of the velocity, horizontal velocity, temperature excess in the cloud, mixing ratio of water vapor, mixing ratio of water drops, overall density, pressure and cloud thickness.

Case 2 also corresponds by time to the mature stage of cloud development but the height is chosen to be 300 m lower than that chosen for the first case. In this case optimal features differ slightly from the first case. They are: horizontal velocity, temperature excess in the cloud, mixing ratio of water vapor, mixing ratio of water drops, overall density, pressure, maximum horizontal velocity that was achieved during the whole simulated cloud evolution.

Case 3 is similar to the case 2 but the height was chosen to be 300 m higher than that chosen for the Case 1. Obtained optimal features are as follows: vertical component of the velocity, temperature excess in the cloud, mixing ratio of water drops.

Case 4 corresponds to the stage of development, that is 5 min earlier than the time of the Cases 1–3. The height is the same as in the Case 1. Obtained optimal features are the same as in the Case 3 plus overall density and pressure.

Case 5 corresponds to the stage of dissipation, that is 15 min later than the time of the Cases 1–3. The height is the same as in the Case 1. Obtained optimal features are as follows: vertical component of the velocity, temperature excess in the cloud, mixing ratio of water drops, relative humidity, pressure, overall density, mixing ratio of water drops, hail and graupel.

The third approach for feature selection is used for neural network algorithm. The numerical parameters are fixed, similar to the first approach, at the moment of maximum cloud development and at the height, where the maximum ratio of water droplets is observed. The most significant features have been selected using the Recursive Feature Elimination method from the scikit-learn library [21] with Random Forest
algorithm as an estimator. As a result, the following eight features have been chosen: mixing ratio of vapor and aerosol particles, relative humidity, density, temperature excess in the cloud over the temperature in the environment atmosphere, pressure, the vertical component of the velocity, temperature inside the cloud.

4 Forecast Accuracy Using Support Vector Machine (SVM), Logistic Regression and Ridge Regression Algorithms

Forecast accuracy amounts to 97.7%, 98.6% and 98.1% for Support Vector Machine, Logistic Regression and Ridge Regression correspondingly while using the first approach of data formation and preprocessing [4, 5]. Though looking very promising these results need to be checked and clarified.

For this purpose, the second approach is used for data formation and preprocessing with the same machine learning algorithms. It allows to investigate the influence of a cloud evolution stage upon the choice of cloud parameters and to check the accuracy of application of feature elimination method.

The results have been obtained using the Scikit-learn library [21]. They are presented in the Tables 1 and 2.

Table 1. Forecast accuracy of machine learning methods

| Method                  | Case 1 | Case 2 | Case 3 |
|-------------------------|--------|--------|--------|
| Logistic Regression     | 93,7   | 94,6   | 93,2   |
| Support Vector Machine  | 94,1   | 96,1   | 95,0   |
| Ridge Regression        | 94,1   | 95,0   | 94,6   |

The analyses of the presented results shows that Logistic regression produces the lowest forecast accuracy in all Cases in comparison with the two other methods. SVM and Ridge Regression show approximately the same accuracy. Maximum accuracy has been obtained by SVM method in Case 2.

The choice of the time moment of cloud evolution does not influence much upon the forecast accuracy of the three methods. We may use the cloud parameters at any stage of its development. Influence of the height is more noticeable. The best results have been achieved at the height which was 300 m lower than the height of the maximum mixing ratio of cloud droplets.

5 Forecast Accuracy Using Neural Network Algorithm

As it is written above the third approach of data formation and preprocessing is used for neural network algorithm. The data are normalized using the Standard Scaler method from the scikit-learn library, which converts the data to the standard normal distribution.
We consider the only one type of the convective phenomenon, namely thunderstorm. The data set contains 416 records, including 220 samples corresponding to thunderstorm presence and 196 samples to its absence. Training data set contains 333 samples and the test one contained 83. Due to the small amount of data test set is used for validation.

We also create labels for each sample in the data set. Since there are only two cases, the presence and absence of the phenomenon, we can create one label per sample. But we decide to use two labels per sample, one for each case, mainly because we will need to divide the output variables of the neural network at some point. So there are two types of labels: “target 1” and “target 2”. “Target 1” is equal to 1 and “target 2” is equal to zero in a case of a thunderstorm occurrence. “Target 1” is equal to 0 and “target 2” is equal to 1 in a case of a thunderstorm absence.

We investigate 3 types of perceptron structure: classical multi-layer perceptron (Fig. 1) and two types of complexes, consisting of single layer perceptron structures. The use of different perceptron structures is due to a small amount of data used as a training data set. In this case the use of the algorithms based on classical neural networks may be inefficient [22]. To avoid this the method described in [23] is used for increasing the efficiency of our neural network. The method involves separation of the set of input and output variables into several perceptrons (Fig. 2 and 3) with a simpler structure and then their combination into a single perceptron complex.

| Method                  | Case 4 | Case 5 |
|-------------------------|--------|--------|
| Logistic Regression     | 93,7   | 93,7   |
| Support Vector Machine  | 94,1   | 93,7   |
| Ridge Regression        | 93,7   | 94,6   |

Table 2. Forecast accuracy of machine learning methods

Fig. 1. Classical multi-layer perceptron
The network shown in Fig. 1 gives the highest accuracy value on the test data for a classical layer perceptron. This structure has been found experimentally and showed the accuracy of the trained network of 89.1%.

Perceptron complexes presented in Fig. 2 and 3 show the accuracy of 90.0% and 91.6% correspondingly.

Network design and all calculation are provided using Keras deep learning framework [24]. Networks are trained using Backpropagation. The hyperbolic tangent represents the activation function for all layers, Softmax is used as the output function.

The training algorithms for the perceptron complexes (Fig. 2 and 3) are different for the first level perceptrons and the resulting ones. The training and test data sets for the
first level perceptron are constructed on the base of the initial data taking into account
the input and output variables for each current perceptron. All the outputs are stored.
The training and test data sets for the resulting perceptron have been constructed on the
base of the initial data and the calculated output values of the first level perceptrons
taking into account the input and output variables of the perceptron.

We also try to solve the problem of small data set using the approach proposed in
[25]. The author suggests a cross-variation method for the problem solution. All the
data are used as a training data set, except for one sample, which is used to test the
network after training. Then this procedure is repeated so every sample is sequentially
excluded from the initial data set. Thus, each sample can be considered as both training
and test one. After receiving the loss values for all test samples, they are averaged and
an assessment is got of the neural network testing. The disadvantages of this method
are the need to repeat the training many times, which takes a considerable amount of
time, as well as the possible inaccuracy of the estimates of individual sample losses due
to the influence of the stochastic component of the learning process. As a result of
applying this method the estimated accuracy of network is equal to 89.9%.

6 Discussion

In the paper we continue to examine the effectiveness of using various machine
learning methods for forecasting dangerous convective phenomena. Table 3 illustrates
the results obtained both in the previous works [4, 5, 20] and present paper. We have
considered five machine learning methods: Support Vector Machine (SVM), Logistic
Regression, Ridge Regression, boosted k-nearest neighbour algorithm and neural
networks. The table contains the best accuracy which could be possible achieved
independently of the feature selection methods and approaches of data formation.

| Method                              | Forecast accuracy |
|-------------------------------------|-------------------|
| Logistic Regression [4, 5]           | 98.6%             |
| Support Vector Machine [4, 5]       | 97.7%             |
| Ridge Regression [4, 5]             | 98.1%             |
| Boosted k-nearest neighbour algorithm [20] | 99.0%             |
| Neural networks                     | 91.6%             |

The table does not contain the results described in Sect. 4 of the present paper and
shown in Tables 1 and 3 as the accuracy achieved with second approach of data
formation and preprocessing is lower than the accuracy obtained with the help of the
first approach. That means that the stage of cloud evolution is not crucial for the choice
of the most representative cloud parameters (features).
As it can be seen from the Table 3 the best results are achieved using Boosted k-nearest neighbour algorithm [20], the worst results were achieved with the neural networks algorithms.

But all the obtained values of forecast accuracy should be treated as preliminary ones, as the data sets used are relatively small for machine learning methods.

In future the research should be focused on obtaining sufficient number of radiosonde soundings with the corresponding model simulations for formation relevant data sets for training and testing.

For correct comparison, forecast accuracy of the different machine learning methods should be obtained by the same feature selection methods and the same approaches to data formation.

Besides the forecast accuracy all machine learning methods should be compared by their numerical performance. The method which will show the optimal combination of precision and performance should be recommended for the operational forecasting of the dangerous convective phenomena.

7 Conclusions

The possibility of thunderstorm forecasting is considered, based upon numerical modelling with the subsequent processing of the output data by machine learning methods.

The problem of feature selection is discussed in two aspects: selection of the optimal values of time and height when and where the output model data are fixed and selection of the most representative cloud parameters (features). The results obtained with the help of Support Vector Machine (SVM), Logistic Regression and Ridge Regression show low dependence of forecast accuracy upon the height and the time moment of the output data selection.

The possibility of using neural networks for forecasting dangerous convective phenomena is analysed. Neural networks with three different structures are considered. The best achieved accuracy equals to 91.6%.

Comparison of different machine learning methods is provided. It is shown that forecast accuracy of all five methods reaches values exceeding 90%.

The choice of the most effective method should be based upon the investigation of the performance of machine learning method on the training and testing data sets of a larger volume.

References

1. Information on https://www-03.ibm.com/press/us/en/pressrelease/49954.wss
2. Information on https://arnesund.com/2015/05/31/using-amazon-machine-learning-to-predict-the-weather/
3. Information on https://yandex.ru/company/technologies/meteum/
4. Stankova, E.N., Balakshiy, A.V., Petrov, D.A., Shorov, A.V., Korkhov, V.V.: Using technologies of OLAP and machine learning for validation of the numerical models of convective clouds. In: Gervasi, O., et al. (eds.) ICCSA 2016. LNCS, vol. 9788, pp. 463–472. Springer, Cham (2016). https://doi.org/10.1007/978-3-319-42111-7_36
5. Stankova, E.N., Balakshiy, A.V., Petrov, D.A., Korkhov, V.V.: OLAP technology and machine learning as the tools for validation of the numerical models of convective clouds. Int. J. Bus. Intell. Data Min. 14(1/2), 254–266 (2019)
6. Abramovich, K.G., et al.: Guide to forecasting meteorological conditions for aviation, Goskomgidromet, Moscow (1985). (in Russian)
7. Stankova, E.N., Grechko, I.A., Kachalkina, Y.N., Khvatkov, E.V.: Hybrid approach combining model-based method with the technology of machine learning for forecasting of dangerous weather phenomena. In: Gervasi, O., et al. (eds.) ICCSA 2017. LNCS, vol. 10408, pp. 495–504. Springer, Cham (2017). https://doi.org/10.1007/978-3-319-62404-4_37
8. Information on https://www.microsoft.com/en-us/research/wp-content/uploads/2016/11/KD_D_2014_keynote_horvitz.pdf
9. Baik, J.-J., Paek, J.-S.: A neural network model for predicting typhoon intensity. J. Meteorol. Soc. Jpn. 78, 857–869 (2000). https://doi.org/10.2151/jmsj1965.78.6_857
10. Ruettgers, M., Lee, S., Jeon, S., You, D.: Prediction of a typhoon track using a generative adversarial network and satellite images. Sci. Rep. 9 (2019). Article number: 6057. https://doi.org/10.1038/s41598-019-42339-y
11. Asai, T., Kasahara, A.: A theoretical study of the compensating downward motions associated with cumulus clouds. J. Atmos. Sci. 24, 487–497 (1967)
12. Raba, N.O., Stankova, E.N.: Research of influence of compensating descending flow on cloud’s life cycle by means of 1.5-dimensional model with 2 cylinders. In: Proceedings of MGO, vol. 559, pp. 192–209 (2009). (in Russian)
13. Raba, N., Stankova, E.: On the possibilities of multi-core processor use for real-time forecast of dangerous convective phenomena. In: Taniar, D., Gervasi, O., Murgante, B., Pardele, E., Apduhan, B.O. (eds.) ICCSA 2010. LNCS, vol. 6017, pp. 130–138. Springer, Heidelberg (2010). https://doi.org/10.1007/978-3-642-12165-4_11
14. Raba, N.O., Stankova, E.N.: On the problem of numerical modeling of dangerous convective phenomena: possibilities of real-time forecast with the help of multi-core processors. In: Murgante, B., Gervasi, O., Iglesias, A., Taniar, D., Apduhan, B.O. (eds.) ICCSA 2011. LNCS, vol. 6786, pp. 633–642. Springer, Heidelberg (2011). https://doi.org/10.1007/978-3-642-21934-4_51
15. Raba, N.O., Stankova, E.N.: On the effectiveness of using the GPU for numerical solution of stochastic collection equation. In: Murgante, B., et al. (eds.) ICCSA 2013. LNCS, vol. 7975, pp. 248–258. Springer, Heidelberg (2013). https://doi.org/10.1007/978-3-642-39640-3_18
16. Petrov, D.A., Stankova, E.N.: Use of consolidation technology for meteorological data processing. In: Murgante, B., et al. (eds.) ICCSA 2014. LNCS, vol. 8579, pp. 440–451. Springer, Cham (2014). https://doi.org/10.1007/978-3-319-09144-0_30
17. Petrov, D.A., Stankova, E.N.: Integrated information system for verification of the models of convective clouds. In: Gervasi, O., et al. (eds.) ICCSA 2015. LNCS, vol. 9158, pp. 321–330. Springer, Cham (2015). https://doi.org/10.1007/978-3-319-21410-8_25
18. Stankova, E.N., Petrov, D.A.: Complex information system for organization of the input data of models of convective clouds. Vestnik Saint-Petersburg Univ. Ser. 10, Appl. Math. Comput. Sci. Control Processes (3), 83–95 (2015). (in Russian)
19. Stankova, E.N., Ismailova, E.T., Grechko, I.A.: Algorithm for processing the results of cloud convection simulation using the methods of machine learning. In: Gervasi, O., et al. (eds.) ICCSA 2018. LNCS, vol. 10963, pp. 149–159. Springer, Cham (2018). https://doi.org/10.1007/978-3-319-95171-3_13
20. Stankova, E.N., Khvatkov, E.V.: Using boosted k-nearest neighbour algorithm for numerical forecasting of dangerous convective phenomena. In: Misra, S., et al. (eds.) ICCSA 2019. LNCS, vol. 11622, pp. 802–811. Springer, Cham (2019). https://doi.org/10.1007/978-3-030-24305-0_61

21. Information on http://scikit-learn.org/

22. Dudarov, S.P., Diev, A.N., Fedosova, N.A., Koltsova, E.A.: Simulation of properties of composite materials reinforced by carbon nanotubes using perceptron complexes. Comput. Res. Model. 7(2), 253–262 (2017)

23. Dudarov, S.P., Diev, A.N.: Neural network modeling based on perceptron complexes with small training data sets. Math. Methods Eng. Technol. 114–116 (2013)

24. Information on. https://keras.io/

25. Korobkova, S.V.: Problems of the effective approximation of multidimensional functions using neural networks. Bull. South. Fed. Univ. Tech. Sci. 58(3), 121–127 (2006). (in Russian)