Network traffic behaviour near phase transition point

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Abstract

We explore packet traffic dynamics in a data network model near phase transition point from free flow to congestion. The model of data network is an abstraction of the Network Layer of the OSI (Open Systems Interconnection) Reference Model of packet switching networks. The Network Layer is responsible for routing packets across the network from their sources to their destinations and for control of congestion in data networks. Using the model we investigate spatio-temporal packets traffic dynamics near the phase transition point for various network connection topologies, and static and adaptive routing algorithms. We present selected simulation results and analyze them.
I. INTRODUCTION

A Packet Switching Network (PSN) is a data communication network consisting of a number of nodes (i.e., routers and hosts) that are interconnected by communication links. Its purpose is to transmit messages from their sources to their destinations. In a PSN each message is partitioned into smaller units of information called packets that are capsules carrying the information payload. Packets are transmitted across a network from source to destination via different routes. Upon arrival of all packets to the destination, the message is rebuilt. PSNs owe their name to the fact that packets are individually switched among routers. Some examples of PSNs are the Internet, wide area networks (WANs), local area networks (LANs), wireless communication systems, ad-hoc networks, and sensors networks. There is vast engineering literature devoted to PSNs, see [1], [2], [3] and the references therein. Wired PSNs are described by the ISO (International Standard Organization) OSI (Open Systems Interconnect) 7 layers Reference Model [2], [3]. We focus on the Network Layer because it plays the most important role in the packet traffic dynamics of the network. It is responsible for routing packets from their sources to their destinations and for control of congestion.

The dynamics of flow of packets in PSN can be very complex. It is not well understood how these dynamics depend on network connection topology coupled with various routing algorithms. Understanding of packet flow dynamics is important for further evolution of PSNs, improvements in their design, operation and defence strategies. Some aspects of these dynamics can be studied using models of data networks. With different goals in mind and at various levels of abstractions, models of PSNs have been proposed and applied (e.g., [1], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13] and the articles therein).

In our work we explore packet traffic dynamics near phase transition point from free flow to congestion. We study how this dynamics is affected by the coupling of network connection topology with routing algorithms. We consider static and adaptive routing. Continuing our work of [10], [14], and [15], we use an abstraction of the Network Layer (see [9], [10]) and a C++ simulator, called Netzwerk-1 (see [10], and [16]) that we developed. Like in real networks our model is concerned primarily with packets and their routing; it is scalable, distributed in space, and time discrete. It avoids the overhead of protocol details present in simulators designed with different goals in mind.
II. PACKET SWITCHING NETWORK MODEL DESCRIPTION

In our PSN model (see, [9], [10]) each message consists only of one packet carrying only the following information: time of creation, destination address, and number of hops taken. Each node can perform the functions of a host and a router. Packets are created randomly and independently at each node. The probability $\lambda$ with which packets are created is called source load. Each node maintains one incoming and one outgoing queue to store packets. The outgoing queues are of unlimited length and operate in a first-in, first-out manner. Each node at each time step routes the packet from the head of its outgoing queue to the next node on its route independently from the other nodes. A discrete time, synchronous and spatially distributed network algorithm implements the creation and routing of packets [9], [10].

We view a PSN connection topology as a weighted directed multigraph $\mathcal{L}$ where each node corresponds to a vertex and a pair of parallel edges oriented in opposite directions represents each communication link. We associate a packet transmission cost to each directed edge, thus parallel edges do not necessarily share the same cost. Here we consider network connection topologies of the type $\mathcal{L} = \mathcal{L}^p(L, l)$, that is, isomorphic to a two-dimensional periodic square lattice with $L$ nodes in the horizontal and vertical directions and $l$ additional links added to this square lattice. Notice, that if a sufficient number $l = l_1 + l_2$ of additional links is added to $\mathcal{L} = \mathcal{L}^p(L, 0)$, with $l_1$ links added in a proper way, then one can obtain a network connection topology of the type $\mathcal{L}^p(L, l_1 + l_2) = \mathcal{L}^p(L, l_2)$, that is isomorphic to a two-dimensional periodic triangular lattice with $L$ nodes in the horizontal and vertical directions and $l_2$ additional links added to this triangular lattice. All links in the network are static for the duration of each simulation run.

In the PSN model, each packet is transmitted via routers from its source to its destination according to the routing decisions made independently at each router and based on a least-cost criterion. We consider routing decisions based on the minimum least-cost criterion, that is, minimum route distance or the minimum route length depending on the cost assigned to each edge of the graph [1], [2]. We consider the following edge cost functions called One ($ONE$), QueueSize ($QS$), or QueueSizePlusOne ($QSP\ O$) [9], [10]. In each PSN model set-up all edge costs are computed using the same type of edge cost function.

Edge cost function $ONE$ assigns a value of one to all edges in the lattice $\mathcal{L}$. This
results in the \textit{minimum hop routing} (minimum route distance) when a least cost routing
criterion is applied, because the number of hops taken by each packet between its source
and its destination node is minimized. This type of routing is called \textit{static routing}, since
the values assigned to each edge do not change during the course of a simulation. The edge
cost function $QS$ assigns to each edge in the lattice $\mathcal{L}$ a value equal to the length of the
outgoing queue at the node from which the edge originates. When this edge cost function is
used a packet traversing a network will travel from its current node to the next node along
an edge belonging to a path with the least total number of packets in transit between its
current location and its destination at this time. The edge cost function $QSP_{O}$ assigns a
summed value of a constant one plus the length of the outgoing queue at the node from
which the edge originates. This cost function combines the features of the previous two
functions. The costs $QS$ and $QSP_{O}$ are derived for each edge from the router’s load. Since
the routing decisions made using $QS$ or $QSP_{O}$ edge cost function rely on the current state
of the network simulation they imply \textit{adaptive or dynamic routing}. In these types of routing
packets have the ability to avoid congested nodes during a network simulation.

Our PSN model uses \textit{full-table routing}, that is, each node maintains a \textit{routing table} of least
path cost estimates from itself to every other node in the network. Because in a PSN model
using edge cost function $ONE$ the costs of edges are static during a simulation, the routing
tables are calculated only at the beginning, they are not updated, and the cost estimates are
the precise least-costs \textsuperscript{10}. When the edge cost function $QS$ or $QSP_{O}$ is used, the routing
tables are updated at each time step using a distributed version of Bellman-Ford least-cost
algorithm \textsuperscript{5}. In both cases the path costs stored in the routing tables are only estimates of
the actual least path costs across the network because only local information is exchanged
and updated at each time step. In our PSN model time is discrete and we observe its state
at the discrete time points $k = 0, 1, 2, \ldots, T$, where $T$ is the final simulation time. At time
$k = 0$, the set-up of PSN model is initialised with empty queues and the routing tables are
computed using the centralised Bellman-Ford least-cost algorithm \textsuperscript{5}.

The discrete time, synchronous and distributed in space PSN model algorithm consists
of the sequence of five operations advancing the simulation time from $k$ to $k + 1$. These
operations are: (1) \textit{Update routing tables}, (2) \textit{Create and route packets}, (3) \textit{Process incoming
queue}, (4) \textit{Evaluate network state}, (5) \textit{Update simulation time}. This algorithm is described
in \textsuperscript{9}, \textsuperscript{10}. 
TABLE I: Critical source load values

|       | $\mathcal{L}^\square_0(16,0)$ | $\mathcal{L}^\square_1(16,1)$ | $\mathcal{L}^\triangle_0(16,0)$ | $\mathcal{L}^\triangle_1(16,1)$ |
|-------|----------------|----------------|----------------|----------------|
| ONE   | 0.115          | 0.020          | 0.140          | 0.030          |
| QS    | 0.120          | 0.125          | 0.155          | 0.160          |
| QSPO  | 0.120          | 0.125          | 0.155          | 0.160          |

III. PACKET TRAFFIC BEHAVIOUR NEAR PHASE TRANSITION POINT

The phase transition from congestion-free to congested state was observed in empirical studies of PSNs [17] and motivated further research (e.g., [13] and the articles therein). Understanding the dynamics of this transition has practical implications leading to more efficient designs of PSNs. In our PSN model, for a particular family of network set-ups, which differ only in the value of the source load $\lambda$, values $\lambda$ for which packet traffic is congestion-free are called sub-critical while values for which traffic is congested are called super-critical. The critical source load $\lambda_c$ is the largest sub-critical source load. The explanation how we estimate its value in simulations is given in [10].

Here, we explore how spatio-temporal dynamics of packet traffic in our PSN model is affected by the network connection topology and edge cost function type for source loads close to the phase transition point of each PSN model set-up. As a case study we consider network connection topologies of the type $\mathcal{L}^\square_0(L, l)$ and $\mathcal{L}^\triangle_0(L, l)$, where $L = 16$ and $l = 0$ or 1. We say that $\mathcal{L}^\square_0(L, 0)$ and $\mathcal{L}^\triangle_0(L, 0)$, that is, the regular network connection topologies are undecorated and we say that they are decorated if an extra link is added to each of them, that is, they are of the type $\mathcal{L}^\square_1(L, 1)$ and $\mathcal{L}^\triangle_1(L, 1)$. We use the following convention if we want to specify additionally what type of an edge cost function our PSN model is using, namely, $\mathcal{L}^\square_0(16, l, ecf)$ and $\mathcal{L}^\triangle_0(16, l, ecf)$, where $l = 0$ or 1, and $ecf = ONE$, or $QS$, or QSPO.

For the considered network connection topologies the estimated critical source load values $\lambda_c$ are provided in Table II. An extensive study about how the values of $\lambda_c$ depend on the network connection topology, the edge cost function type, and the mode of routing table update in the considered PSN model is presented in [10] and reference therein.

Figures of Tables III and IV display spatial distribution of outgoing queue sizes at nodes for various PSN model set-ups. The x- and y- axis coordinates of each figure denote
| $L^p_{\Box}(16, 0, ONE), k = 8000$ | $L^p_{\Box}(16, 0, QS), k = 8000$ | $L^p_{\Box}(16, 0, QSP0), k = 8000$ |
|----------------------------------|----------------------------------|----------------------------------|
| ![Graph](image1.png)             | ![Graph](image2.png)             | ![Graph](image3.png)             |
| $\lambda_c = 0.115$             | $\lambda_c = 0.120$             | $\lambda_c = 0.120$             |
| ![Graph](image4.png)             | ![Graph](image5.png)             | ![Graph](image6.png)             |
| $\lambda_{sup,c} = 0.120$       | $\lambda_{sup,c} = 0.125$       | $\lambda_{sup,c} = 0.125$       |

**TABLE II:** For critical and super-critical source loads spatial distribution of outgoing queue sizes in the PSN model set-ups defined in the table header.

From Table II we observe that the qualitative behaviour of spatial distribution of outgoing queue sizes is very similar for PSN model set-up $L^p_{\Box}(16, 0, ONE)$ and $L^p_{\Box}(16, 0, QSP0)$ when the source loads are $\lambda_c$ and $\lambda_{sup,c}$, respectively. In each case queue sizes are randomly distributed with large fluctuations. Increase in source load results in substantial increase of queue sizes and fluctuations among them (notice, the figures use different scales on z-axis). The qualitative behaviours for $\lambda_c$ and $\lambda_{sup,c}$ are typically observed for other sub-critical source load values and super-critical ones, respectively, in these types of...
\( L_\Delta(16, 0, ONE), k = 8000 \)
\( L_\Delta(16, 0, QS), k = 8000 \)
\( L_\Delta(16, 0, QSP O), k = 8000 \)

| \( \lambda_c = 0.140 \) | \( \lambda_c = 0.155 \) | \( \lambda_c = 0.155 \) |
| \( \lambda_{sup \, c} = 0.145 \) | \( \lambda_{sup \, c} = 0.160 \) | \( \lambda_{sup \, c} = 0.160 \) |

TABLE III: For critical and super-critical source loads spatial distribution of outgoing queue sizes in the PSN model set-ups defined in the table header

PSN model set-ups.

From Tables II and III we observe also a random distribution of queue sizes in each PSN model set-up \( L_\square(16, 0, ecf) \) and \( L_\Delta(16, 0, ecf) \), where \( ecf = QS, QSP O \), and the source loads are \( \lambda_c \). But, the magnitudes of the fluctuations are smaller than those in the case of PSN model set-ups using the edge cost function \( ONE \). For the other sub-critical source load values in the PSN model set-ups \( L_\square(16, 0, ecf) \) and \( L_\Delta(16, 0, ecf) \), where \( ecf = QS, QSP O \), queue size distributions share the same characteristics. However, we observe a big qualitative difference between the distribution of queue sizes of the PSN model set-up \( L_\square(16, 0, QSP O) \) and those of the set-ups \( L_\square(16, 0, QS) \), \( L_\Delta(16, 0, QS) \), and \( L_\Delta(16, 0, QSP O) \), when their respective \( \lambda_{sup \, c} \) source loads are used, see Table II and Table III.

In each of the PSN model set-up \( L_\square(16, 0, QSP O) \) and \( L_\square(16, 0, QS) \) for \( \lambda_{sup \, c} \) (i.e., in
the congested state of the network) we observe spatio-temporal self-organization in the sizes
of the outgoing queues and the emergence of a pattern of peaks and valleys, see Table I and
Table IV. However, the time scale on which the pattern emerges in the PSN model set-up
$\mathcal{L}_\text{□}(16, 0, QS)$ is much longer than the time scale of the PSN model set-up $\mathcal{L}_\text{□}(16, 0, QSPO)$. Also, the difference between sizes of the neighbouring peaks and valleys increases much
closer to time in the PSN model set-up $\mathcal{L}_\text{□}(16, 0, QSPO)$ than the one of $\mathcal{L}_\text{□}(16, 0, QS)$. This could imply that, when the edge cost function $QSPO$ is used, the cost component
$\text{ONE}$ is responsible for the observed qualitative differences in the evolution of the spatio-
temporal packet traffic dynamics between the network models under consideration. A similar
behaviour was observed for values of $L$ other than 16 and super-critical source load values
other than $\lambda_{\text{sup},c}$. In the congested state as the number of packets increases the pattern of
peaks and valleys emerges in spite of the adaptive routing attempts to distribute evenly the
packets among the queues.

Looking at Table III and Table IV we see that adding an extra link to the connection
topology of the PSN model set-up $\mathcal{L}_\text{□}(16, 0, QS)$ speeds up the “peak-valley” pattern emer-
gence in a congested state. This is also true when the extra link has other position and/or
length and $L$ is different than 16, see [10], [15]. The same phenomenon was observed in the
congested states when edge cost function $QSPO$ was used, see [10], [14].

In spite the fact that the considered adaptive routings try to distribute packets evenly
among the network nodes the effect of an extra link on the packet traffic dynamics is much
stronger. It provides a “short-cut in communication” among distant nodes. This speeds
up the emergence of the “peak-valley” pattern in the network models with square lattice
connection topologies and edge cost functions $QS$ and $QSPO$. This is also true when instead
of one extra link a relatively small number of extra links is added, see [10]. If a larger number
of extra links is added the pattern of peaks-valleys does not emerge, see Table III, Table IV
and [14]. We observe rather small differences among the outgoing queue sizes in congested
states of the PSN model set-ups $\mathcal{L}_\text{□}(16, 0, QS)$ and $\mathcal{L}_\text{□}(16, 0, QSPO)$. Also, from Table IV
we notice that when an extra link is added to the periodic triangular network connection
topology, these differences become even smaller, except of the two nodes to which the extra
link is attached. These nodes attract much larger numbers of packets than other nodes
resulting in local congestion. Qualitatively similar behaviour was observed for the PSN
model set-up $\mathcal{L}_\text{△}(16, 1, QSPO)$ in its congested state.
IV. CONCLUSIONS

We investigated the effects of coupling of network connection topology with, respectively, static and adaptive routings on spatio-temporal packet traffic dynamics near phase transition point from free flow to congested state in our PSN model. We observed that for adaptive routings and periodic square lattice network connection topologies patterns of "peaks-valleys" emerge in the distributions of outgoing queue sizes when the network models are in their congested states. The emergence of this type of synchronization is accelerated by addition of an extra link and is destroyed by addition of many links. Synchronization in other types of networks are discussed in [18], [19] and references therein. For our PSN model with adaptive routings and periodic triangular lattice connection topologies we observed that packet traffic is much more evenly distributed. We demonstrated that dynamics of the PSN model depend on the coupling between network connection topology and routing. The presented work is continuation of our research in [10], [14], [15] and contributes to the study of dynamics of complex networks.

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TABLE IV: Spatial distribution of outgoing queue sizes at various times $k$ in the PSN model set-ups defined in the table header.