Abstract. In order to characterize one of the most favored candidates for dark matter, we calculate the anapole form factor of the lightest neutralino in the Minimal Supersymmetric Standard Model (MSSM) at the one-loop level. As a Majorana fermion, this particle only shows one electromagnetic property, the toroidal dipole moment, which is directly related to the anapole form factor. We obtain the result analytically in terms of two- and three-points Passarino-Veltman scalar functions and evaluate it for a given spectrum of supersymmetric masses and matrix elements. This work is part of a broader project still in progress.

1. Introduction
The most favored candidates for cold dark matter (non relativistic) are the so called WIMPs (Weakly Interactive Massive Particle), with masses between 10 GeV and a few TeV, and with interaction cross sections around the electroweak scale (~ 100 GeV). Currently, the best motivated candidate for WIMP is the lightest supersymmetric particle (LSP) of the supersymmetric models. However, candidates like axions, sneutrinos or gravitinos have not been discarded yet.

Goldberg [1] and Ellis et al. [2] were the first to suggest that the neutral gauginos and higgsinos, with the same quantum numbers, can mix. This superposition is called neutralino, \( \tilde{\chi}_1^0 \); there are four of them and one is the LSP in some models. Any WIMP candidate to dark matter must be an electrically neutral particle without strong interactions. The LSP is stable in supersymmetric models with \( R \) parity, which is a condition of the Minimal Supersymmetric Standard Model (MSSM), where the lightest neutralino is the LSP.

If the Universe is vastly populated by neutralinos, their detection must be possible, directly (via inelastic collisions with nucleons) or indirectly (observing the fermion-antifermion pairs
when neutralinos are annihilated). The expected cross sections for such interactions are determined by the allowed regions in the parameter space of the SUSY model used.

2. Toroidal Dipole Moment

It is well known that the electromagnetic properties of a $j$-spin particle are described by $2(2j + 1)$ form factors. For spin 1/2-particles, several electromagnetic current parametrizations have therefore four independent form factors. Thus, the most general expression for an electromagnetic vertex function is:

$$\Gamma_\mu(q) = f_Q(q^2)\gamma_\mu + f_M(q^2)i\sigma_{\mu\nu}q^\nu - f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu - q_\mu q_\nu)\gamma_5,$$

(1)

where $f_Q(q^2)$, $f_M(q^2)$, $f_E(q^2)$ and $f_A(q^2)$ are called the charge, magnetic dipole, electric dipole and anapole dipole form factors, respectively; $q_\mu = p'_\mu - p_\mu$ is the transferred 4-momentum; and $\sigma_{\mu\nu} = (i/2)\begin{bmatrix} \gamma_\mu, \gamma_\nu \end{bmatrix}$. These form factors are physically observable quantities as $q^2 \to 0$, and their combinations define the well-known magnetic $(\mu)$, electric $(d)$ and anapole $(a)$ dipole moments [3].

The anapole moment was introduced by Zel’’dovich [4] for a T-invariant interaction which does not conserve P-parity and C-parity individually. Subsequently, a more convenient characteristic was pointed out to describe this kind of interaction, the toroidal dipole moment (TDM). The TDM ($\tau$) is a general case of the anapole moment, it coincides with an anapole on the mass-shell of the particle under consideration and has a simple classical analogue. The TDM, as the electric dipole and magnetic dipole moments, is the first term of the third multipole family [3], the toroidal moments. This type of static multipole momenta does not produce any external electromagnetic fields in vacuum but generates a free-field (gauge-invariant) potential which is responsible for topological effects such as the Aharonov-Bohm effect.

The simplest model of TDM (anapole) was given by Zel’dovich as a conventional solenoid folded into a torus and having only a poloidal current (see figure 1). For such a stationary solenoid, having neither an azimuthal (toroidal) component of the current nor electric fields around the torus, there is only a nonzero azimuthal magnetic field inside the torus [5].

Besides not having electrical charge, the neutralino does not have electric and magnetic dipole moments because that would violate CPT conservation (remember it is a Majorana fermion). Thus, the neutralino electromagnetic vertex function is

$$\Gamma_\mu(q) = f_A(q^2)(q^2\gamma_\mu - q_\mu q_\nu)\gamma_5.$$

(2)
In this work, we calculate the anapolar form factor and, therefore, the TDM of the neutralino in the framework of the MSSM at the one-loop level approximation. Using the FeynArts [6] software we obtained 94 Feynman diagrams which contribute to this calculation, 28 vertex corrections, 6 sea-gulls and 60 self-energies. From these amplitudes we isolate the terms that go as $\gamma - H^0, \gamma - h^0, \gamma - A^0$ and $\gamma - G^0$ self-energies do not contribute to the calculation.

3. Results

Using FeynCalc we calculate the amplitude of each of the 94 diagrams for the process $\gamma \rightarrow \chi^0_1\chi^0_1$ at the one-loop level. In figure 2 we show from left to right an example of one of the 28 vertex correction diagrams, one of the 6 sea-gulls and one of the 60 self-energies. From these amplitudes we isolate the terms that go as $\gamma_\mu \gamma_5$. The first result we get is that the $\gamma - H^0, \gamma - h^0, \gamma - A^0$ and $\gamma - G^0$ self-energies do not contribute to the calculation.

For diagrams in figure 2, for example, we get

$$
\Xi_{24} = \sum_i \frac{e^3}{32\pi^2 c_W s_W^2 (q^2 - 4M^2_W)} \left[ \alpha B_0 \left( q^2, M^2_W, M^2_W \right) + \beta B_0 \left( M^2_{\chi_1}, M^2_{\chi_1}, M^2_W \right) + \delta C_0 \left( q^2, M^2_{\chi_0}, M^2_{\chi_0}, M^2_W, M^2_W, M^2_{\chi_1} \right) \right]
$$

$$
\Xi_{34} = \frac{-M^2_W e^3 \epsilon}{16\pi^2 s_W^2 (q^2 - M^2_W)} \left[ 3B_0 \left( 0, M^2_W, M^2_W \right) + 1 \right]
$$

$$
\Xi_{94} = \frac{e^3 \zeta}{\pi^2 s_W^2 (q^2 - M^2_W)} \left[ -\frac{M^2_{\chi_1}}{192} B_0 \left( 0, M^2_{\chi_1}, M^2_W \right) + \frac{q^2 - 4M^2_W}{384} B_0 \left( q^2, M^2_W, M^2_W \right) + \frac{q^2 - 9M^2_W}{576} \right].
$$

respectively, where $\alpha, \beta, \delta, \epsilon$ and $\zeta$ are coefficients that depend on the neutralino mixing matrix elements; $B_0$ and $C_0$ are two- and three-points Passarino-Veltman scalar functions; and

$$
\sum_i \Xi_i = f_A(q^2)q^2 \quad i = 1, 2, ..., 94.
$$

To get $\tau = f_A(0)$ we use l'Hopital's rule. Derivatives of the $C_0$ functions appear in the calculation. That is why we developed an expansion in power series of $q^2$, around $q^2 = 0$, for the generic $C_0$ that appears in every vertex correction, in terms of logarithms of the masses involved. This will be useful for the evaluation of the expressions above. In figure 3, we show the plot of this expansion for $C_0 \left( q^2, M^2_{\chi_0}, M^2_{\chi_0}, M^2_W, M^2_W, M^2_{\chi_1} \right)$.

Using the supersymmetric spectrum given by SuSpect [7] for the mSUGRA model and the parameter values $m_0 = 100\text{GeV}$, $m_{1/2} = 250\text{GeV}$, $\tan\beta = 10$, $\text{sign} \mu = +$ and $A_0 = -100$ we get the following value for the toroidal dipole moment of the lightest neutralino:

$$
\tau = f_A(0) = -1.03899 \times 10^{-5}\text{GeV}^{-2} = -4.05798 \times 10^{-33}\text{cm}^2.
$$
Figure 3. Comparison between exact and approximate solutions of $C_0 \left( q^2, M_{\tilde{\chi}_0^1}^2, M_{\tilde{\chi}_0^1}^2, M_{\tilde{\chi}_0^1}^2, M_{\tilde{\chi}_0^1}^2 \right)$, using the masses we wet using SuSpect for the mSUGRA model and the parameter values given below. The monotonic decreasing line represents the approximate solution valid only when $q^2 \to 0$, while the other represents the exact solution valid for all $q^2$.

4. Connection to Cosmology
In here we assume the neutralino is the main component of dark matter. This work is part of a broader analysis which intends to connect our calculation to cosmology and astrophysics.

For a given cosmological model, the WIMP relic density is determined uniquely by its annihilation cross section into SM particles, if all WIMPs were produced thermally. Under standard cosmology conditions the uncertainty of the current WIMP relic density is essentially given by the uncertainty of the WIMP annihilation cross section [8]. In order to calculate this cross section to percent level accuracy, at least leading radiative corrections will have to be included.

Thus, we can write the 1-loop corrected WIMP annihilation cross section as

$$\sigma = \sigma_0 + \delta\sigma,$$

where $\sigma_0$ is the cross section at the tree level. Part of the one-loop level correction to the neutralino cross section, and, therefore, to its relic density, would be obtained using our results.

We also have the Hamiltonian of interaction due to the anapole moment in the non-relativistic limit:

$$\mathcal{H}_{int} \propto -a (\sigma \cdot \nabla \times B)$$

where $a$ stands for the anapole dipole moment, $\sigma$ is the spin and $B$ is the strength of the magnetic field [5]. Although the value of this correction is very small, it could lead to an observable effect in regions with high density of dark matter.

Finally, there are experimental bounds we can compare our result with. By using a phenomenological approach to WIMPs, Pospelov and ter Veldenius[9] got an upper bound for the anapole form factor of $\left| \frac{e}{m_\text{em}} \right| \sqrt{\frac{S+1}{2S}} \leq 4 \times 10^{-2}$ fm using DAMA experiment data and lower than $3 \times 10^{-2}$ fm using CDMS experiment data. New data from CDMS experiment can provide more restrictive bounds. In particular for mSUGRA (minimal Supergravity) model we can get the TDM of the neutralino in terms of five parameters: $m_0$, the unified mass for scalars, $m_{1/2}$, the unified mass for gauginos, $A_0$, the unified trilinear coupling, $\tan \beta$, the ratio of the vacuum expectation values for the neutral supersymmetric Higgs and the sign of $\mu$, where $\mu$ is the higgsino mass parameter. By scanning over this set of parameters and comparing this
theoretical result with the experimental bounds, we can constrain the allowed regions in the parameter space.

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