Renormalization of the Spin-dependent WIMP scattering off nuclei

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We study the amplitude for the spin-dependent WIMP scattering off nuclei by including the leading long-range two-body currents in the most important isovector contribution. We show that such effects are essentially independent of the target nucleus and, as a result, they can be treated as a mere renormalization of the effective nucleon cross section or, equivalently, of the corresponding effective coupling with values around 25%.

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I. INTRODUCTION

The combined MAXIMA-1 \cite{1}, BOOMERANG \cite{2}, DASI \cite{3} and COBE/DMR Cosmic Microwave Background (CMB) observations \cite{4} imply that the Universe is flat \cite{5} and that most of the matter in the Universe is Dark \cite{6}, i.e. exotic. These results have been confirmed and improved by the recent WMAP data \cite{7}. Combining the data one finds:

$$\Omega_b = 0.0456 \pm 0.0015,$$
$$\Omega_{\text{CDM}} = 0.228 \pm 0.013,$$
$$\Omega_\Lambda = 0.726 \pm 0.015.$$ 

Since any “invisible” non exotic component cannot possibly exceed 40% of the above $\Omega_{\text{CDM}}$ \cite{8}, exotic (non baryonic) matter is required, i.e. there is room cold dark matter candidates or WIMPs (Weakly Interacting Massive Particles).

Even though there exists firm indirect evidence for a halo of dark matter in galaxies from the observed rotational curves, see e.g. the review \cite{9}, it is essential to directly detect such matter. The possibility of such detection, however, depends on the nature of the dark matter constituents and their interactions.

Since the WIMP’s are expected to be extremely non relativistic, with average kinetic energy $\langle T \rangle \approx 50 \text{ keV}(m_{\text{WIMP}}/100 \text{ GeV})$, they are not likely to excite the nucleus, even if they are quite massive $m_{\text{WIMP}} > 100 \text{ GeV}$. Therefore they can be directly detected mainly via the recoiling of a nucleus $(A,Z)$ in elastic scattering. The event rate for such a process can be computed from the following ingredients: i) An effective Lagrangian at the elementary particle (quark) level obtained in the framework of the prevailing particle theory. The most popular scenario is in the context of supersymmetry. Here the dark matter candidate is the LSP (Lightest Supersymmetric Particle) \cite{10–16}. In this case the effective Lagrangian is constructed as described, e.g., in Refs. \cite{10–18}. At least till supersymmetry is discovered in the laboratory (LHC), other approaches are also possible, such as, e.g., technibaryon \cite{19, 20}, mirror matter \cite{21, 22}, Kaluza-Klein models with universal extra dimensions \cite{23, 24} ii) A well defined procedure for transforming the amplitude obtained using the previous effective Lagrangian, from the quark to the nucleon level. To achieve this one needs a quark model for the nucleon, for the coherent mode see, e.g., \cite{18, 25–27}. Similar effects occur in the case of the spin induced matrix elements, iii) knowledge of the relevant nuclear matrix elements \cite{28, 29}, obtained with as reliable as possible many body nuclear wave functions, iv) knowledge of the WIMP density in our vicinity and its velocity distribution.

In the standard nuclear recoil experiments, first proposed more than 30 years ago \cite{30}, one has to face the problem that the reaction of interest does not have a characteristic feature to distinguish it from the background. So for the expected low counting rates the background is a formidable problem. Some special features of the WIMP-nuclear interaction can be exploited to reduce the background problems, such as the modulation effect \cite{31, 32–40} and backward-forward asymmetry expected in directional experiments, i.e. experiments in which the direction of the recoiling nucleus is also observed \cite{41, 42, 53}.

There exists a plethora of direct dark matter experiments with the task of detecting WIMP event rates for a variety of targets such as those employed in XENON10 \cite{54, 55}, XENON100 \cite{55}, XMASS \cite{56}, ZEPLIN \cite{57}, PANDA-X \cite{58}, LUX \cite{59, 60}, CDMS \cite{60}, CoGENT \cite{61}, EDELWEISS \cite{62}, DAMA \cite{63, 64}, KIMS \cite{65} and PICASSO \cite{66, 67}.

For the interpretation of the experimental data, knowledge of the nuclear matrix elements is essential. In connection with nuclear structure aspects, in a series of calculations, e.g. in \cite{58, 68, 69} and references therein, it has been shown that for the coherent contribution, due to the scalar interaction, the inclusion of the nuclear form factor is important, especially in the case of relatively heavy targets. They also showed that the nuclear spin cross sections, which may
be relevant for odd mass targets, are characterized by a single, i.e. essentially isospin independent, structure function and two static spin values, one for the proton and one for the neutron, which depend on the target.

In the present paper we will focus on the evaluation of the spin induced matrix elements. In particular we will explore the currents for spin-dependent WIMP scattering off nuclei, in particular the most isovector channel. We will examine i) the modification of the rates at the one-body level by including a suitable nucleon form factor, which modifies somewhat the universality of the suitably normalized spin isospin structure functions and ii) include the leading long-range two-body currents, which are predicted in chiral EFT \[71\]. We will show that the two body effects lead to a quenching of the isospin one spin matrix elements. Since, however the resulting quenching factor is independent of the target, it can be absorbed into the corresponding nucleon cross section. We have not examined such effects on the isoscalar spin matrix element since this mode, if present at all, is suppressed due to the quark structure of the nucleon.

II. THE DIFFERENTIAL EVENT RATE

The expressions involving the differential are fairly well known (see e.g. \[71\] for a brief review in our notation). For the reader’s convenience we will list the main features here. The event rate can be cast in the form:

\[
\frac{dR}{dE_R} = \frac{dR_0}{dE_R} + \frac{dR_1}{dE_R}\cos\alpha
\]

where the first term represents the time averaged (non modulated) differential event rate, while the second gives the time dependent (modulated) one due to the motion of the Earth (see below). Furthermore for the coherent mode we get:

\[
\frac{dR_0}{dE_R} = \frac{\rho_x}{m_x} \frac{m_t}{A m_p} \left( \frac{\mu_x}{\mu_p} \right)^2 \sqrt{\langle \nu^2 \rangle} > \frac{1}{Q_0(A)} \sigma_{N}^{coh} \frac{dt}{du}_{coh} ,
\]

\[
\frac{dR_1}{dE_R} = \frac{\rho_x}{m_x} \frac{m_t}{A m_p} \left( \frac{\mu_x}{\mu_p} \right)^2 \sqrt{\langle \nu^2 \rangle} > \frac{1}{Q_0(A)} \sigma_{N}^{coh} \frac{dt}{du}_{coh} ,
\]

while for the spin dependent process we get:

\[
\frac{dR_0}{dE_R} = \frac{\rho_x}{m_x} \frac{m_t}{A m_p} \left( \frac{\mu_x}{\mu_p} \right)^2 \sqrt{\langle \nu^2 \rangle} > \frac{1}{Q_0(A)} \sigma_{N}^{spin} \frac{dt}{du}_{spin} ,
\]

\[
\frac{dR_1}{dE_R} = \frac{\rho_x}{m_x} \frac{m_t}{A m_p} \left( \frac{\mu_x}{\mu_p} \right)^2 \sqrt{\langle \nu^2 \rangle} > \frac{1}{Q_0(A)} \sigma_{N}^{spin} \frac{dt}{du}_{spin} .
\]

In the above expressions \(\mu_x,\) \(\mu_p\) is the WIMP-nucleus (nucleon) reduced mass, \(A\) is the nuclear mass number, \(m_x\) is the WIMP mass, \(\rho(\chi)\) is the WIMP density in our vicinity, assumed to be 0.3 GeV cm\(^{-3}\), and \(m_t\) the mass of the target. Furthermore one can show that

\[
\sigma_{A}^{coh} = A^2 \sigma_{N}^{coh}, \quad \left( \frac{dt}{du} \right)_{coh} = \sqrt{\frac{2}{3}} a^2 F^2(u) \Psi_0(a\sqrt{u}), \quad \left( \frac{dh}{du} \right)_{coh} = \sqrt{\frac{2}{3}} a^2 F^2(u) \Psi_1(a\sqrt{u})
\]

with \(\sigma_{N}^{coh}\) the elementary nucleon cross section for the coherent mode. In the case of the spin the expressions for static nuclear cross section \(\sigma_{N}^{spin}\) will be given below and:

\[
\left( \frac{dt}{du} \right)_{spin} = \sqrt{\frac{2}{3}} a^2 F_{11}(u) \Psi_0(a\sqrt{u}), \quad \left( \frac{dh}{du} \right)_{spin} = \sqrt{\frac{2}{3}} a^2 F_{11}(u) \Psi_1(a\sqrt{u}),
\]

The factor \(\sqrt{2/3}\) is nothing but \(\nu_0/\sqrt{\langle \nu^2 \rangle}\) since in Eq. \(2\) \(\sqrt{\langle \nu^2 \rangle}\) appears. In the above expressions \(a = (\sqrt{2} \mu_r b \nu_0)^{-1}\), \(\nu_0\) the velocity of the sun around the center of the galaxy and \(b\) the nuclear harmonic oscillator size parameter characterizing the nuclear wave function. \(u\) is the energy transfer \(E_R\) in dimensionless units given by

\[
u = \frac{E_R}{Q_0(A)} , \quad Q_0(A) = [m_p A b^2]^{-1} = 40A^{-4/3} \text{ MeV}
\]

In the above expressions \(F(u)\) is the nuclear form factor and \(F_{11}(u)\) the isospin 1 spin response function. Note that the parameter \(a\) depends both on the WIMP mass, the target and the velocity distribution. Note also that for a given
energy transfer $E_R$ the quantity $u$ depends on $A$.

The functions $\Psi_0(x)$ and $\Psi_1(x)$ arise from the WIMP velocity distribution, via function $g(u_{min}, v_E(\alpha))$, which depends on the minimum WIMP velocity for a given energy transfer, i.e.

$$v_{min} = \sqrt{\frac{A m_p E_R}{2 \mu_r^2}} \quad (7)$$

For the M-B distribution in the local frame it is defined as follows:

$$g(u_{min}, v_E(\alpha)) = \frac{1}{(\sqrt{\pi} u_0)^2} \int_{u_{min}}^{v_{max}} e^{-(u^2+2uv_E(\alpha)+v_E^2(\alpha))/u_0^2} v \, dv \, d\Omega, \quad v_{max} = v_{esc} \quad (8)$$

$v_E(\alpha)$ is the velocity of the Earth, including the velocity of the Sun around the galaxy, $v_E(\alpha) = \epsilon_0(\hat{v}_s + \delta (\sin \alpha \hat{x} - \cos \alpha \cos \gamma \hat{y} + \cos \alpha \sin \gamma \hat{z}))$ with $\gamma \approx \pi/6$, $\hat{v}_s$ a unit vector in the Sun’s direction of motion, $\hat{x}$ a unit vector radially out of the galaxy in our position and $\hat{y} = \hat{v}_s \times \hat{x}$. $\delta = 0.235$ is the velocity of the earth around the sun divided $v_0$ and $\alpha$ is the phase of the Earth ($\alpha = 0$, around June 3rd). The above upper cut off value in the M-B is usually put in by hand. Such a cut off comes in naturally, however, in the case of velocity distributions obtained from the halo WIMP mass density in the Eddington approach [72], which, in certain models, resemble a M-B distribution [73].

To obtain Eqs (1) and (5) we expand $g(u_{min}, v_E(\alpha))$ in powers of $\delta$, keeping terms up to linear in $\delta \approx 0.135$. We found it convenient to express all velocities in units of the Sun’s velocity $v_0$ to obtain:

$$v_0 \, g(u_{min}, v_E(\alpha)) = \Psi_0(x) + \Psi_1(x) \cos \alpha, \quad x = \frac{u_{min}}{v_0} \quad (9)$$

$\Psi_0(x)$ represents the quantity relevant for the average rate and $\Psi_1(x)$, which is proportional to $\delta$, represents the effects of modulation.

In the case of a M-B distribution these functions take the following form:

$$\Psi_0(x) = \frac{1}{2} \left[ \text{erf}(1-x) + \text{erf}(x+1) + \text{erfc}(1-y_{esc}) + \text{erfc}(y_{esc}+1) - 2 \right] \quad (10)$$

$$\Psi_1(x) = \frac{1}{2} \frac{\delta}{\sqrt{\pi}} \left[ \frac{1}{2} - \frac{\text{erf}(1-x) - \text{erf}(x+1) - \text{erfc}(1-y_{esc}) + \text{erfc}(y_{esc}+1)}{\sqrt{\pi}} \right.\left. + \frac{e^{-(x-1)^2}}{\sqrt{\pi}} + \frac{e^{-(x+1)^2}}{\sqrt{\pi}} - \frac{e^{-(y_{esc}-1)^2}}{\sqrt{\pi}} - \frac{e^{-(y_{esc}+1)^2}}{\sqrt{\pi}} + 1 \right] \quad (11)$$

where erf$(x)$ and erfc$(x)$ are the error function and its complement, respectively, and $y_{esc} = v_{esc}/v_0$, $550 \leq v_{esc} \leq 660$ km/s.

### III. THE SPIN DEPENDENT WIMP-NUCLEUS SCATTERING

The spin dependent WIMP-nucleus elastic scattering cross section can be derived from an interaction which at the quark level can be cast in the form

$$\mathcal{L} = -\Lambda G_F \sqrt{2} J \sum_q A_q \Psi_q \gamma_5 \Psi_q \quad (12)$$

Where the scale parameter $\Lambda$ as well as $J$ depend on the particle model. Thus:

1. One could, of course, make the time dependence of the rates due to the motion of the Earth more explicit by writing $\alpha \approx (6/5)\pi (2(t/T) - 1)$, where $t/T$ is the fraction of the year.
1. Supersymmetry, see, e.g., a review in our notation \[74\].

\[
\Lambda = 2(|C_{31}|^2 - |C_{41}|^2), \quad J = \bar{\chi} \gamma_5 \chi
\]

where \(C_{31}, C_{41}\) are the Higgsino components of the WIMP, which in this case is the Lightest Supersymmetric Particle (LSP). The coefficients \(A_q\) can be read from the space component of the neutral current:

\[
J^Z_\chi = -\bar{q} \gamma_\lambda \left\{ \frac{1}{3} \sin^2 \theta_W - \left[ \frac{1}{2} (1 - \gamma_5) - \sin^2 \theta_W \right] \gamma_3 \right\} q, \quad A_0 = 0, \quad A_1 = \frac{1}{2}, \quad g^0_A(q) = 0, \quad g^1_A(q) = -1.
\]

2. Kaluza Klein theories in models with Universal Extra Dimensions \[24\].

In this case we encounter a number of cases depending on the nature of the WIMP, e.g.

- The WIMP is a gauge boson.
  In this case:

\[
\Lambda \approx \frac{8}{3} \tan^2 \theta_W \left( \frac{m_W}{m_B} \right) \left( \frac{m_p}{m_B} \right)^2 \quad J = (i \epsilon^* \times \epsilon), \quad A_u = \frac{17}{18}, \quad A_d = A_s = \frac{5}{18}, \quad A_0 = \frac{3}{2}, \quad A_1 = \frac{2}{3}
\]

\[
g^1_A(q) = g^1_A(q) = 1
\]

with \(m_p\) the proton mass and \(\epsilon^*\) and \(\epsilon\) the polarization vectors of the initial and final WIMPs (the normalization factors \(1/\sqrt{m_B}\) entering each boson field in the expression of the cross section has already been absorbed into \(\Lambda\)).

- The WIMP is heavy neutrino.
  It is assumed the process proceeds via Z exchange. A viable possibility is if it is a Majorana Fermion. In this case:

\[
\Lambda = 1, \quad \bar{J} = -\bar{\nu} \gamma_5 \nu
\]

The coefficients \(A_q\) are given again by the space component of the neutral current (\(A_0 = 0, \quad A_1 = \frac{1}{2}\), \(g^0_A(q) = 0, \quad g^1_A(q) = -1\)).

In the present work we will not concern ourselves with any specific particle model, but we will, instead, focus on the following points:

1. Going from the quark to the nucleon level, i.e. constructing an effective transition operator at the nucleon level. This involves:

   - The modification of the currents due to the structure of the nucleon.

     The effective couplings \(g^0_A\) (\(g^1_A\)) are obtained by multiplying the corresponding elementary amplitudes obtained at the quark level, as given above, by suitable renormalization factors \(g^0_A\) and \(g^1_A\) given in terms of the quantities \(\Delta q\) given by Ellis \[75,24\], namely \(\Delta u = 0.78 \pm 0.02\), \(\Delta d = -0.48 \pm 0.02\) and \(\Delta s = -0.15 \pm 0.02\), i.e.

\[
g^0_A(N) = g^0_A(q)(\Delta u + \Delta d + \Delta s) = 0.13g^0_A(q), \quad g^1_A(N) = g^1_A(q)(\Delta u - \Delta d) = 1.26g^1_A(q)
\]

The quantities \(g^1_A(q)\) and \(g^1_A(q)\) can be read from the hadronic current. For the neutral current they are unity, so the renormalization of the isovector component of the axial current is the usual one, while the isoscalar one is greatly suppressed, consistent with the EMC effect, i.e. the fact a tiny fraction of the spin of the quarks is coming from the spin of the quarks. It is for this reason that we started our discussion in the isospin basis. In the case of the boson WIMP in Kaluza-Klein theories one finds \[24\]:

\[
g^0_A(N) = \frac{17}{18} \Delta u + \frac{5}{18} (\Delta d + \Delta s) = 0.26, \quad g^1_A(N) = \frac{17}{18} \Delta u - \frac{5}{18} \Delta d = 0.41
\]

Thus in general, barring very unusual circumstances at the quark level, the isovector component is expected to be dominant.
• The nucleon form factor \[23, 70\], which modifies the isovector axial current:

\[
\mathbf{J} = \frac{g_A(q)}{g_A} \Sigma, \quad \Sigma = \frac{\langle \sigma \cdot q \rangle}{q^2 + m_N^2}
\]  

(19)

If we choose as a z-axis the direction of momentum we find:

\[
\mathbf{J}_m = \left\{ \begin{array}{ll}
(1 - \frac{q^2}{q^2 + m_N^2}) \sigma_m, & m = 0 \\
\frac{\sigma_m}{m = \pm 1} &
\end{array} \right.
\]  

(20)

Thus only the longitudinal component of the transition operator is modified by the inclusion of the nucleon form factor. Thus, even at sufficiently high momentum transfers, only 1/3 of the differential rate will be affected.

• possible effects arising from possible exchange currents, which lead to effective 2-body contributions. We will exploit recent work on Chiral Effective Field Theory (EFT) \[70\].

2. Construct the relative Spin Structure functions.

For this purpose and for illustration purposes we will consider the simplest possible nuclear system, which in our opinion is the \( A = 19 \) system \[29\] currently employed as a target by the PICASSO collaboration. The study of other experimentally more interesting targets, see e.g. ref. \[66\], can be found elsewhere \[70\].

The nuclear ME entering the WIMP-nucleus cross section takes the form:

\[
|ME|^2 = a_1^2 S_{11}(u) + a_1 a_0 S_{01}(u) + a_0^2 S_{00}(u), \quad a_0 = A_0 g_A^0(N), \quad a_1 = A_1 g_A^1(N),
\]

(21)

i.e. the parameters \( a_0 \) and \( a_1 \) depend on the parameters \( A_q \) as well as on the quark model for the nucleon. The structure functions

\[
F_{ij}(u) = \frac{S_{ij}(u)}{S_{ij}(0)}
\]

(22)

with the functions \( S_{ij} \) defined in the appendix \[2\] are essentially independent of the isospin. Thus one can simplify the expression involving the nuclear ME entering the WIMP-nucleus cross section and write it as follows:

\[
|ME|^2 = (a_1^2 \Omega_1^2 + a_1 a_0 \Omega_0 \Omega_1 + a_0^2 \Omega_0^2) F_{11}(u),
\]

(23)

where \( \Omega_0 \) and \( \Omega_1 \) are the static nuclear matrix elements involving \( \sigma \) for isospin 0 and 1 respectively defined by

\[
\Omega_p = 2 \sqrt{\frac{j + 1}{j}} \langle jj|S_p|jj\rangle, \quad \Omega_n = 2 \sqrt{\frac{j + 1}{j}} \langle jj|S_n|jj\rangle, \quad \Omega_0 = \Omega_p + \Omega_n \quad \Omega_1 = \Omega_p - \Omega_n
\]

(24)

As it is common practice the nuclear physics parameters \( \Omega_p \) and \( \Omega_n \) (or \( \Omega_0 \) and \( \Omega_1 \)) as well \( F_{11} \) appear in the expression of the rate, while the parameters \( \Lambda \), \( a_1 \) and \( a_0 \) can be absorbed into the nucleon cross sections \( \sigma_p \) and \( \sigma_n \) (or \( \sigma_0(N) \) and \( \sigma_1(N) \)). Leaving aside the factor \( F_{11} \), taken care of explicitly in Eq. \[29\], the static nuclear cross section becomes

\[
\sigma_A^{\text{min}} = \frac{\Omega_1^2 \sigma_1(N) + 2 \text{sign}(a_1 a_0) \Omega_0 \Omega_1 \sqrt{\sigma_0(N) \sigma_1(N)} + \Omega_0^2 \sigma_0(N)}{3}
\]

(25)

with

\[
\sigma_0(N) = \Lambda^2 a_0^2 G_F^2 m_N^2, \quad \sigma_1(N) = \Lambda^2 a_1^2 G_F^2 m_N^2
\]

(26)

and \( m_N \) the nucleon mass.

The structure functions \( F_{ij} \) obtained in this work are shown in Fig. \[4(a)\], while those derived from a previous work

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2 The functions \( S_{ij} \) for the Xe isotopes \[74, 78\] are normalized differently. The functions \( F_{ij} \) extracted from them are independent of this normalization.
are exhibited in Fig. 4(b) and (c). The isospin behavior of the functions $F_{ij}$ in the $A = 129$ is what we expect. We do not understand the stronger isospin dependence in the case of the $A = 131$. Before proceeding further we will interpret the results of the recent work [70], summarized below (see section [V]), in our notation. If we define:

$$r_{ij}(0) = \frac{S_{ij}(0)}{\Omega_i \Omega_j}$$

then

1. for the $^{129}$Xe isotope we take

$$\Omega_p = 0.034, \quad \Omega_n = 1.140, \quad \Omega_0 = 1.174, \quad \Omega_1 = -1.105,$$

and

$$r_{00}(0) = r_{11}(0) = r_{01}(0) = \frac{1}{8\pi} \simeq 0.038 \text{ (1-body)},$$

$$r_{00}(0) = \frac{1}{8\pi}, \quad r_{11}(0) = 0.0240,$$

$$r_{01}(0) = 0.0614 \text{ (one and two-body)}$$

2. for the $^{131}$Xe isotope we have

$$\Omega_p = -0.023, \quad \Omega_n = -0.702, \quad \Omega_0 = -0.776, \quad \Omega_1 = 0.679,$$

and

$$r_{00}(0) = r_{11}(0) = r_{01}(0) = \frac{1}{4\pi} \simeq 0.079 \text{ (1-body)},$$

$$r_{00}(0) = \frac{1}{4\pi}, \quad r_{11}(0) = 0.0487, \quad r_{01}(0) = 0.1155 \text{ (one and two-body)}$$

The above results have been obtained from Eq. (16) as well as the data of Table 1 of Ref. [70].

3. In the case of $^{19}$F we have [29]:

$$\Omega_p = 1.644, \quad \Omega_n = -0.031, \quad \Omega_0 = 1.614, \quad \Omega_1 = 1.675$$

The precise value of the parameters $r_{ij}$ depends on the parameters of the model. For a reasonable choice of these parameters we get:

$$r_{00}(0) = r_{11}(0) = r_{01}(0) = 1 \text{ (1-body)},$$

$$r_{00}(0) = 1, \quad r_{11}(0) = 0.600, \quad r_{01}(0) = 0.775 \text{ (one and two-body)}$$

In the presence of effective two body terms we can write:

$$r_{11}(0)|_{1+2b} = (1 - \delta)^2 r_{11}(0)|_{1b}, \quad r_{01}(0)|_{1+2b} = (1 - \delta) r_{01}(0)|_{1b}$$

From the above analysis we find:

$$\delta \approx 0.229 \text{ (A=129)}, \quad \delta \approx 0.234 \text{ (A=131)}, \quad \delta \approx 0.235 \text{ (A=19)}$$

The corrections due to the presence of two body terms are significant. From the above expressions, since $\delta$ is independent of A, one has two possibilities:
1. Write Eq. (25) in the form

\[
\sigma_A^{\text{spin}} = \frac{\left(\sigma_1(1 - \delta)^2 \Omega_1^2 + \text{sign}(a_0 a_1)\sqrt{\sigma_1} \sqrt{\sigma_0(1 - \delta) \Omega_0 \Omega_1 + \sigma_0 \Omega_1^2}\right)}{3}, 
\]

where the nucleon parameters remain the same but the isovector nuclear spin matrix elements are quenched by a factor of \((1 - \delta)\).

2. View the correction factor like an effective charge.

In this case, due to the nuclear medium, the elementary nucleon amplitudes are modified:

\[
a_0 \rightarrow a_0, \quad a_1 \rightarrow a_1(1 - \delta), \quad \sigma_0 \rightarrow \sigma_0, \quad \sigma_1 \rightarrow \sigma_1(1 - \delta)^2
\]

but the nuclear spin matrix elements remain unchanged. In this case we recover the standard formula

\[
\sigma_A^{\text{spin}} = \frac{\left(\sigma_1 \Omega_1^2 + \text{sign}(a_0 a_1)\sqrt{\sigma_1} \sqrt{\sigma_0 \Omega_0 \Omega_1 + \sigma_0 \Omega_1^2}\right)}{3},
\]

The parameter \(\delta\) depends on the assumed model as shown in table II.

One may express the above results in the proton neutron representation. The above behavior of the form factors \(F_{ij}\) imply that the proton and neutron form factors are almost the same. If one retains the amplitudes unchanged one finds:

\[
\sigma_A^{\text{spin}} = \frac{1}{3} \left[ \sigma_p \left(\Omega_p(1 - \delta) + \Omega_n \frac{\delta}{2}\right)^2 + \sigma_n \left(\Omega_n(1 - \delta) + \Omega_p \frac{\delta}{2}\right)^2 \right. \\
+ \left. 2\text{sign}(a_p a_n) \left(\Omega_p \Omega_n \frac{1 + (1 - \delta)^2}{4} + (\Omega_p^2 + \Omega_n^2) \frac{1 - (1 - \delta)^2}{4}\right) \right]
\]

with

\[
\sigma_p = \frac{\Lambda^2 a_2 G_F^2 m_n^2}{2\pi}, \quad \sigma_n = \frac{\Lambda^2 a_2^2 G_F^2 m_p^2}{2\pi}, \quad a_p = a_0 + a_1, \quad a_n = a_0 - a_1
\]

In other words the two-body terms induce a small neutron (proton) component in those cases in which such a component is negligible in the standard treatment with only one body terms.

On the other hand one can absorb the quenching into the coupling constants, in which case the proton and the neutron cross sections get modified:

\[
a_p = a_0 + a_1(1 - \delta), \quad a_n = a_0 - a_1(1 - \delta), \quad \sigma_p \rightarrow \sigma_p(a_0 + a_1(1 - \delta))^2, \quad \sigma_n \rightarrow \sigma_n(a_0 - a_1(1 - \delta))^2,
\]

while the standard expression for the nuclear cross section is retained, i.e.

\[
\sigma_A^{\text{spin}} = \frac{1}{3} \left[ \sigma_p \Omega_p^2 + \sigma_n \Omega_n^2 + 2\text{sign}(a_p a_n)\sqrt{\sigma_p} \sqrt{\sigma_n} \Omega_p \Omega_n \right]
\]

IV. THE SPIN STRUCTURE FUNCTIONS IN THE CONTEXT OF EFT

In this section we will employ the normalization of the structure functions even though, as shown above, they will be used only in extracting the relevant static spin matrix elements and in particular the quenching factors. Following the recent work of Ref. [70] we have included the currents for spin-dependent WIMP scattering off nuclei at the one-body level as well as the leading long-range two-body currents, which are predicted in chiral EFT. The nuclear matrix elements (ME) entering the WIMP-nucleus cross section can be written as [70]

\[
|ME|^2 = \left| \langle J||T_{L}^{(l)}||J > \right|^2 + \left| \langle J||L_{L}^{(l)}||J > \right|^2, \quad L = \text{odd}
\]

where \(T_{L}^{(l)}\) is the transverse electric and \(L_{L}^{(l)}\) the longitudinal projections of the axial-currents. Since the ground state of \(^{19}\text{F}\) is the \(J = 1/2^+\) then only the \(L = 1\) multipole contributes. As consequence the above operators can be written in the isospin basis in terms of the operators \(T^{(l,L)} = \sqrt{4\pi} \delta_{l1}(q \cdot r_i)Y^{ll}(\hat{r}_i) \times \sigma^{lb}\) as

\[
L_{L=1}^{(l)}(q) = \frac{1}{\sqrt{3}} \frac{1}{2} \sum_{i=1}^{A} \left[ a_0(\sqrt{2} T^{(2,1)} + T^{(0,1)}) + a_1 \tau_3 f_i(q)(\sqrt{2} T^{(2,1)} + T^{(0,1)}) \right]
\]
TABLE I: The parameter $\delta$ entering the suppression factor $f = (1 - \delta)$ for the axial isovector contribution for various values of the model parameters $\rho$, $I(\rho, P = 0)$, $c_3$ and $c_4$.

| $\rho$ | $I$ | $c_3$ | $c_4$ | $\delta$ |
|-------|-----|-------|-------|---------|
| 0.10  | 0.58| -3.2  | 5.40  | 0.256   |
| 0.10  | 0.58| -2.2  | 4.40  | 0.203   |
| 0.10  | 0.58| -3.40 | 3.40  | 0.189   |
| 0.10  | 0.58| -2.40 | 2.40  | 0.136   |
| 0.10  | 0.58| -4.78 | 3.96  | 0.233   |
| 0.11  | 0.59| -3.30 | 3.40  | 0.212   |
| 0.11  | 0.59| -2.30 | 2.40  | 0.152   |
| 0.11  | 0.59| -4.78 | 3.96  | 0.261   |
| 0.11  | 0.59| -3.78 | 2.96  | 0.202   |
| 0.12  | 0.60| -3.20 | 5.40  | 0.318   |
| 0.12  | 0.60| -2.20 | 4.40  | 0.252   |
| 0.12  | 0.60| -3.40 | 3.40  | 0.235   |
| 0.12  | 0.60| -2.40 | 2.40  | 0.169   |
| 0.12  | 0.60| -4.78 | 3.96  | 0.290   |
| 0.12  | 0.60| -3.78 | 2.96  | 0.224   |

and

$$T_{L=1}^{\pi L} (q) = \frac{1}{\sqrt{3} \sqrt{2} \sqrt{4\pi}} \sum_{i=1}^{A} \left[ a_0 (- T^{(2,1)} + \sqrt{2} T^{(0,1)}) + a_1 \tau_3 f_2(q) (- T^{(2,1)} + \sqrt{2} T^{(0,1)}) \right]$$

where

$$f_1(q) = 1 - \delta - \frac{2g_{\pi NN} F_\pi q^2}{2m_N g_4 (m_N^2 + q^2)} - 2c_3 \rho \frac{q^2}{4m_N^2 + q^2}$$

and

$$f_2(q) = 1 - \delta - \frac{q^2}{\Lambda_A^2}$$

The above operators are given in terms of the variable $u = q^2 b^2 / 2$, where $q$ is the momentum transfer and $b$ the harmonic-oscillator length. Using the results of our previous shell model calculation for $^{19}$F [29] the parameter $b$ has been taken the value 1.63 fm. The multi-particle matrix elements of the above operators in the considered sd model space are given in the Appendix.

The resulting $2b$ contribution to the axial-vector WIMP current has been included as a density-dependent renormalization $a_1(1 + \delta)$, with

$$\delta \equiv \frac{\rho}{F_\pi} I(\rho, P = 0) \left( \frac{1}{3} (2c_4 - c_3) + \frac{1}{6m_N} \right) .$$

A typical range of the densities in nuclei $\rho = 0.10...0.12 fm^{-3}$ has been taken into account. This leads to $I(\rho, P = 0) = 0.64...0.66$, using the Fermi-gas mean-value $P^2 = 6k^2_f/5$ where $k_f$ the Fermi momentum, and $I(\rho, P = 0) = 0.58...0.60$ for $P = 0$. The coupling constants $c_3$ and $c_4$ describing the contributions of long-range one-pion-exchange as well as the short-range parts in the two-body currents are taken from Ref. [79]. In Fig. 1 the contour plot is shown for the parameter $\delta$ as function of $\rho$ and $2(c_4 - c_3)$. The curves in the plot correspond to constant values of $\delta$.

A second contribution $\delta_\alpha P$ in the pseudo-scalar component of $2b$ current has been included via a density and momentum dependent modification:

$$\delta_\alpha P(q^2) \equiv 2c_3 \frac{\rho}{F_\pi} \frac{q^2}{4m_N^2 + q^2}$$
FIG. 1: (Color online) Contour plot of the parameter $\delta$ as a function of density $\rho$ and the difference $2c_4 - c_3$ of the parameters $c_3$ and $c_4$. The curves in the plot correspond to constant values of $\delta$ and lighter shading denotes the increase of $\delta$.

where $m_\pi = 138.04$ MeV and $F_\pi = 92.4$ MeV. All the other parameters in Eqs. (40) and (41) of the appendix have been taken the values $\Lambda_A = 1040$ MeV, $g_{\pi \rho \pi} = 13.05$, $g_A = 1.27$. The first $q^2$-dependent term of $f_2(q)$ and $f_1(q)$ arise at the one-body (1b) currents, while the second $q^2$-dependent term in $f_2(q)$ and the momentum independent term $\delta$ are taken from the two body (2b) currents. All the terms of the form factors $f_1(q)$ and $f_2(q)$ are presented in Fig. 2. As it is seen the first $q^2$-dependent term of $f_2(q)$ is negligible. In Fig. 2 the corresponding average value of
the two body terms $\delta$ and $\delta\alpha_1^D$ is presented. At finite momentum $p$ the structure functions $S_{00}$, $S_{11}$ and $S_{01}$ can be

TABLE II: Fits to the structure factors $S_{00}$, $S_{11}$ and $S_{01}$ for spin-dependent WIMP elastic scattering off $^{19}F$ nucleus, including 1b and 2b currents. The fitting functions are given in terms of an exponential $e^{-u}$ multiplied by a forth-order polynomial. The rows give the coefficients of the $u^n$ terms in the polynomial.

| $e^{-u}$ | $S_{00}$ | $S_{11}$ (1b) | $S_{11}$ (1b+2b) | $S_{01}$ (1b) | $S_{01}$ (1b+2b) |
|----------|----------|---------------|------------------|---------------|------------------|
| 1        | 0.10386  | 0.11122       | 0.067219         | 0.10756       | 0.083473         |
| $u$      | -0.13848 | -0.26231      | -0.16378         | -0.20194      | -0.16079         |
| $u^2$    | 0.06994  | 0.36150       | 0.23770          | 0.20083       | 0.16999          |
| $u^3$    | -0.01585 | -0.30632      | -0.20965         | -0.13217      | -0.11902         |
| $u^4$    | 0.00136  | 0.11021       | 0.07718          | 0.04167       | 0.03888          |

FIG. 2: (Color online) Form factors terms inserting in Eqs. (42) and (43), given in terms of a forth-order polynomial multiplied by an exponential $e^{-u}$ (see Appendix). The results of the fitting procedure are given in Table II, while the structure functions $S_{ij}$ for 1b and 1b+2b currents are presented in Fig. 2.

FIG. 3: (Color online) Structure functions $S_{00}, S_{11}$ and $S_{01}$ for $^{19}F$ as a function of $u = q^2b^2/2$, for one body(1b) and one+two body(1b+2b) currents. No two body corrections were considered for $S_{00}$. 
FIG. 4: (Color online) Structure factors $F_{ij}$, $F_{11}$, and $F_{01}$ for $^{19}$F, $^{129}$Xe, and $^{131}$Xe isotopes and as a function of $u = q^2 b^2 / 2$, for 1b+2b currents. No two body corrections were considered for $F_{00}$.

V. DISCUSSION

In the present paper we studied the evaluation of the spin dependent nuclear matrix elements relevant for dark matter searches with a variety of experimentally interesting targets (see recent work [80, 81] on such an analysis). We focused on the isovector part of the axial current, which is the most important for the spin induced cross section.

In particular, the effect on the suitably normalized spin structure functions $F_{ij}$ of the nucleon form factor has been examined. We find that their isospin behavior of the spin structure functions $F_{ij}$ is no longer universal. The isospin one modes fall a bit faster as a function of the energy transfer. This effect, however, is not very significant in the energy transfer of interest to dark matter searches (see Fig. 4). In a recent analysis [82], in which it is shown that it is possible to extract all three nucleon cross sections from the data (the coherent as well as the proton and neutron spin cross sections), the equality of the spin structure functions $F_{ij}$ is important. In the case of the light nucleus $^{19}$F, in which the spin dependent rate maybe more important, we find that even the effect on the integrated structure functions $I_{ij}$, obtained by integrating from zero to the maximum allowed energy transfer, is small:

$$R_{01} = \frac{I_{01}}{I_{00}} = 0.965948, \quad R_{11} = \frac{I_{11}}{I_{00}} = 0.936502$$

The effect will be smaller, if the additional $u$ dependence arising from the velocity distribution, not discussed in this work, is included. This isospin independent function is also a decreasing function of $u$, but its precise form depends on the WIMP mass.

In addition to the standard one body contribution, we examined the leading long-range two-body currents, which are predicted in chiral EFT [70]. We confirm the reduction of the isovector matrix elements, quenching factor, by as large as $15 - 25\%$. The precise value is dependent on model parameters, not precisely known. We have found, however, that this affects only the static spin ME and it is independent of the target nucleus. Thus this effect can be absorbed as a multiplicative factor either in the isovector nuclear matrix elements or better still in the effective
coupling parameters, which depend on the assumed particle model, which anyway are going to be derived from the experimental data, if and when they become available. In summary the standard analysis of the data previously employed is not seriously affected by such effects.

VI. APPENDIX

Since the ground state of $^{19}\text{F}$ is the $J = 1/2^+$ then only the $L = 1$ multipole contributes (Eq. (39)). In our treatment we will separate the couplings from the nuclear operator. As consequence the multi-particle reduced matrix elements of the $\mathcal{L}_{L=1}^5$ and $\mathcal{T}_{L=1}^{el5}$ operators can be written as

$$< J || \mathcal{L}_{L=1}^5 || J > = a_0 < J || \mathcal{L}_{L=1}^{5,0} || J > + a_1 < J || \mathcal{L}_{L=1}^{5,1} || J >$$  (46)

where

$$< J || \mathcal{L}_{L=1}^{5,0} || J > = \frac{1}{\sqrt{3}} \frac{1}{2} \frac{1}{\sqrt{4\pi}} \sum_{\lambda\rho} a_{\lambda\rho}(J)(\sqrt{2} < \lambda||T^{(2,1)}||\rho > + < \lambda||T^{(0,1)}||\rho >)$$  (47)

$$< J || \mathcal{L}_{L=1}^{5,1} || J > = \frac{1}{\sqrt{3}} \frac{1}{2} \frac{1}{\sqrt{4\pi}} f_1(q) \sum_{\lambda\rho} b_{\lambda\rho}(J)(\sqrt{2} < \lambda||T^{(2,1)}||\rho > + < \lambda||T^{(0,1)}||\rho >)$$  (48)

with

$$f_1(q) = 1 - \delta - \frac{2g_{\pi\rho}F_{\pi}q^2}{2m_Ng_A(m_\pi^2 + q^2)} - 2c_3 \frac{\rho^2}{F_{\pi}^2 4m_\pi^2 + q^2}$$  (49)

We also write

$$< J || \mathcal{T}_{L=1}^{el5} || J > = a_0 < J || \mathcal{T}_{L=1}^{el5,0} || J > + a_1 < J || \mathcal{T}_{L=1}^{el5,1} || J >$$  (50)

with

$$< J || \mathcal{T}_{L=1}^{el5,0} || J > = \frac{1}{\sqrt{3}} \frac{1}{2} \frac{1}{\sqrt{4\pi}} \sum_{\lambda\rho} a_{\lambda\rho}(J)(- < \lambda||T^{(2,1)}||\rho > + \sqrt{2} < \lambda||T^{(0,1)}||\rho >)$$  (51)

and

$$< J || \mathcal{T}_{L=1}^{el5,1} || J > = \frac{1}{\sqrt{3}} \frac{1}{2} \frac{1}{\sqrt{4\pi}} f_2(q) \sum_{\lambda\rho} b_{\lambda\rho}(J)(- < \lambda||T^{(2,1)}||\rho > + \sqrt{2} < \lambda||T^{(0,1)}||\rho >)$$  (52)

with

$$f_2(q) = 1 - \delta - 2q^2 \frac{1}{A_N}$$  (53)

The indices $\lambda$ and $\rho$ run over the single particle orbits of the chosen model space. According to Ref. [29] this model space is the sd one. The quantities $a_{\lambda\rho}$ and $b_{\lambda\rho}$ are essentially products of the one body coefficients of fractional parentage (CFP) for the isoscalar and isovector part of the operator respectively. They depend, of course, on the specific interaction and the model space used (see Ref. [29]).

As it is known the reduced matrix elements of the $T^{(l,j)} = \sqrt{4\pi j_l(qr)}[Y^l(r) \times \sigma]^j$ operator is written as

$$T^{(l,j)} = A^{(l,j)}_{\lambda\rho} \int_0^\infty R_{n,\lambda l_\lambda}(r)j_l(qr)R_{n,\rho l_\rho}(r)r^2dr$$  (54)

where

$$A^{(l,j)}_{\lambda\rho} = (-1)^{l_\lambda l_\rho} \hat{l}_\lambda \hat{l}_\rho \hat{j}_l \hat{j}_\rho \int \left( \begin{array}{c} l_\lambda & 1/2 & j_\lambda \\ l_\rho & 1/2 & j_\rho \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{c} l \lambda \ l \rho \\ 1 \ j \end{array} \right) \sqrt{6}$$  (55)
Therefore the matrix elements \([47], [48], [51]\) and \([52]\) are written as

\[
<J|\mathcal{L}_{L=1}^{5,0}|J> = \frac{1}{\sqrt{3}} \frac{1}{2} \sqrt{\frac{2}{3}} \times \left( T_0^{(2,1)}(u) + T_0^{(0,1)}(u) \right)
\]

(56)

\[
<J|\mathcal{L}_{L=1}^{5,1}|J> = \frac{1}{\sqrt{3}} \frac{1}{2} \sqrt{\frac{2}{3}} \times f_1(u) \left( T_1^{(2,1)}(u) + T_1^{(0,1)}(u) \right)
\]

(57)

\[
<J|\mathcal{T}_{L=1}^{5,0}|J> = \frac{1}{\sqrt{3}} \frac{1}{2} \sqrt{\frac{2}{3}} \times \left( - T_0^{(2,1)}(u) + \sqrt{2} T_0^{(0,1)}(u) \right)
\]

(58)

\[
<J|\mathcal{T}_{L=1}^{5,1}|J> = \frac{1}{\sqrt{3}} \frac{1}{2} \sqrt{\frac{2}{3}} \times f_2(u) \left( - T_1^{(2,1)}(u) + \sqrt{2} T_1^{(0,1)}(u) \right)
\]

(59)

with

\[
T_0^{(0,1)}(u) = \left[ \frac{1}{6} \left( \frac{2}{5} A_1 + A_2 \right) u^2 - \frac{2}{3} \left( A_1 + A_2 \right) u + A_1 + A_2 \right] e^{-u/2}
\]

(60)

\[
T_0^{(2,1)}(u) = \left[ \left( - \frac{1}{15} A_1' + \frac{1}{6} \sqrt{\frac{2}{5}} A_2' \right) u^2 + \left( \frac{7}{15} A_1' - \frac{2}{3} \sqrt{\frac{2}{5}} A_2' \right) u \right] e^{-u/2}
\]

(61)

\[
T_1^{(0,1)}(u) = \left[ \frac{1}{6} \left( \frac{2}{5} B_1 + B_2 \right) u^2 - \frac{2}{3} \left( B_1 + B_2 \right) u + B_1 + B_2 \right] e^{-u/2}
\]

(62)

\[
T_1^{(2,1)}(u) = \left[ \left( - \frac{1}{15} B_1' + \frac{1}{6} \sqrt{\frac{2}{5}} B_2' \right) u^2 + \left( \frac{7}{15} B_1' - \frac{2}{3} \sqrt{\frac{2}{5}} B_2' \right) u \right] e^{-u/2}
\]

(63)

The \(T_0^{(l,\lambda)}(u)\) part of the operator represents the isoscalar part while \(T_1^{(l,\lambda)}(u)\) the isovector one. Using the results of our previous shell model calculation for \(^{19}\text{F}\) [29] the coefficients \(A_1, A_2, B_1, B_2, A_1', A_2', B_1'\) and \(B_2'\) have been taken the values

\[
A_1 = 1.19407, \ A_2 = 1.09082, \ B_1 = 1.3695, \ B_2 = 0.999
\]

(64)

and

\[
A_1' = -0.004, \ A_2' = -0.0587, \ B_1' = 0.0774, \ B_2' = 0.06269
\]

(65)

The above operators are given in terms of the variable \(u = q^2 b^2/2\), where \(q\) is the momentum transfer and \(b\) the harmonic-oscillator length.

According to the above matrix elements the structure functions are written

\[
S_{00} = 8\pi <J|\mathcal{L}_{L=1}^{5,0}|J>^2 + <J|\mathcal{T}_{L=1}^{5,0}|J>^2
= \frac{1}{2} \left[ T_0^{(2,1)}(u)^2 + T_0^{(0,1)}(u)^2 \right]
\]

(66)

\[
S_{11} = 8\pi <J|\mathcal{L}_{L=1}^{5,1}|J>^2 + <J|\mathcal{T}_{L=1}^{5,1}|J>^2
= \frac{1}{6} \left[ (2 f_1(u) + f_2(u)) T_1^{(2,1)}(u)^2 + (f_1(u) + 2 f_2(u)) T_1^{(0,1)}(u)^2 \right]
+ 2 \sqrt{2} T_1^{(2,1)}(u) T_1^{(0,1)}(u) \left( f_1(u) - f_2(u) \right)
\]

(67)

and

\[
S_{01} = 8\pi <J|\mathcal{L}_{L=1}^{5,0}|J> < J|\mathcal{L}_{L=1}^{5,1}|J > + < J|\mathcal{T}_{L=1}^{5,0}|J > < J|\mathcal{T}_{L=1}^{5,1}|J >
= \frac{1}{6} \left[ (2 f_1(u) + f_2(u)) T_0^{(2,1)}(u) T_1^{(2,1)}(u) + \sqrt{2} \left( f_1(u) - f_2(u) \right) T_0^{(2,1)}(u) T_1^{(0,1)}(u) \right]
\]

(68)
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