N=2 Supersymmetry and String-Loop Corrected Magnetic Black Holes

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Abstract

We study string-loop corrections to magnetic black hole. Four-dimensional theory is obtained by compactification of the heterotic string theory on the manifold $K3 \times T^2$ or on a suitable orbifold yielding $N=1$ supersymmetry in 6D. The resulting 4D theory has $N=2$ local supersymmetry. Prepotential of this theory receives only one-string-loop correction. The tree-level gauge couplings are proportional to the inverse effective string coupling and decrease at small distances from the center of magnetic black hole, so that loop corrections to the gauge couplings are important in this region. We solve the system of spinor Killing equations (conditions for the supersymmetry variations of the fermions to vanish) and Maxwell equations. At the string-tree level, we reproduce the magnetic black hole solution which can be also obtained by solving the system of the Einstein-Maxwell equations and the equations of motion for the moduli. String-loop corrections to the tree-level solution are calculated in the first order in string coupling. The resulting corrections to the metric and dilaton are large at small distances from the center of the black hole. Possible smearing of the singularity at the origin by quantum corrections is discussed.
1 Introduction

There are two ways to obtain classical solutions in supersymmetric theories: one can either solve the equations of motion derived from the effective action which, for bosonic fields, are of the second order in derivatives, or "spinor Killing equations" resulting from the requirement that supersymmetry variations of the fermionic fields vanish. The latter are of the first order in derivatives. The first method, in general, provides a larger set of solutions which can include non-supersymmetric ones. The second way leads to supersymmetric solutions with partially broken supersymmetry.

In this paper, we discuss the string-loop corrections to magnetic black-hole solutions, namely, the contributions from the higher-genera topologies of the string world sheet, by following the second approach.

The 4D string effective action is obtained by dimensional reduction of 6D, $N = 1$ supersymmetric string effective action on the two-torus. For this class of compactifications, 4D theory is $N=2$ supergravity interacting with matter. As a concrete example of this construction, we have in view heterotic string theory compactified on the manifold $K3 \times T^2$ or its suitable orbifold limit, although we do not rely on any specific properties of the model.

Due to $N=2$ supersymmetry, prepotential of the theory receives only one-string-loop corrections (from string world sheets of torus topology) [1, 2]. There are explicit calculations of the loop-corrected prepotential [1, 2, 3, 4], but for the present study only its general structure is important.

First, solving the string-tree-level "spinor Killing equations" for gravitino and gaugini, we obtain the known spherically-symmetric magnetic black hole solutions [5, 6]. The tree-level gauge couplings are proportional to the inverse effective string coupling and decrease at small distances from the origin, so that the loop corrections to the gauge couplings are important in this region. As a technical simplification, we consider tree-level solutions with a special relation between the magnetic charges, in which case the moduli related to the metric components of the internal two-torus are constants. Next, using the loop-corrected prepotential, in the first order in string coupling, we find the loop-corrected gauge couplings, solve the Maxwell equations for the gauge fields and the loop-corrected "spinor Killing equations" for the moduli.

We obtain a family of solutions for the loop corrections to the tree-level metric and dilaton of magnetic black hole which depend on one parameter.

For a special choice of parameter, in the loop-corrected metric extrapolated to the region of small distances from the origin, the singularity at the origin is smeared by string-loop correction.

The set of solutions of the "spinor Killing equations" is contained in the two-parameter set of solutions of the Einstein-Maxwell equations and the equations of motion for the moduli derived from the loop-corrected effective action.

2 Heterotic versus $N = 2$ pictures

4D effective string theories obtained by two-torus compactification of 6D, $N = 1$ string effective actions share a number of universal properties. The resulting theory is $N = 2$ supersymmetric dilatonic supergravity interacting with matter. The bosonic part of the universal sector of this theory written in a holomorphic section admitting the prepotential in the standard form of $N = 2$ special geometry [1, 2, 3, 4, 5, 6, 7, 8, 9] is

$$I_4 = \int d^4 x \sqrt{-g} \left[\frac{1}{2} R + (\bar{\mathcal{N}}_{I,J} \mathcal{F}^{-I} \mathcal{F}^{-J} - \mathcal{N}_{I,J} \mathcal{F}^{+I} \mathcal{F}^{+J}) + k_{ij} \partial_\mu z^i \partial^\mu \bar{z}^j + \ldots \right].$$ 

(1)
Here $N_{IJ}$ are the gauge coupling constants,

$$\mathcal{F}_{\mu\nu}^\pm = \frac{1}{2}(\mathcal{F}_{\mu\nu} \pm \frac{i}{2} \varepsilon_{\mu\nu\rho\lambda} \mathcal{F}^\rho\lambda) = \frac{1}{2}(\mathcal{F}_{\mu\nu} \pm i\sqrt{-g}^* \mathcal{F}_{\mu\nu}).$$

Here $^*\mathcal{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\lambda} \mathcal{F}^\rho\lambda$, where $\varepsilon_{\mu\nu\rho\lambda}$ is the flat antisymmetric tensor.

The moduli $z^i$ are identified below, $k_{i\bar{j}}$ is the Kähler metric

$$k_{i\bar{j}} = \frac{\partial^2 K}{\partial z^i \partial \bar{z}^j}.$$

If a holomorphic section of $N = 2$ theory admits a prepotential, all the couplings are defined via this function. In the case of the $N = 2$ symmetric compactification, the prepotential receives only one-loop corrections and is of the form

$$F = -\frac{X^1 X^2 X^3}{X^0} - iX^0 h^{(1)}(-i\frac{X^2}{X^0}, -i\frac{X^3}{X^0}) + \ldots$$

where

$$\begin{align*}
\frac{X^1}{X^0} &= z^1 = iy_1 = i(e^{-\phi} + ia_1), \\
\frac{X^2}{X^0} &= z^2 = iy_2 = i(e^{\gamma+\sigma} + ia_2), \\
\frac{X^3}{X^0} &= z^3 = iy_3 = i(e^{\gamma-\sigma} + ia_3),
\end{align*}$$

and dots stand for contributions from other moduli. Here and below $I, J = 0, \ldots, 3$ and $i, j = 1, 2, 3$.

The moduli $z^i$ and the vector fields are identified by comparing the action with that resulting from compactification of the universal sector of the $6D$ theory

$$I_6 = \int d^6 x \sqrt{-G^{(6)}} e^{-\Phi} \left[ R^{(6)} + \left( \partial \Phi \right)^2 - \frac{H^2}{12} \right] + \ldots$$

on the two-torus. Here

$$G^{(6)} = \begin{pmatrix} G_{\mu\nu} + A^m_{\mu} A^n_{\nu} G_{mn} & A^m_{\mu} G_{mn} \\ A^n_{\nu} G_{mn} & G_{mn} \end{pmatrix},$$

where $\mu, \nu = 0, \ldots, 3$ and $m, n = 1, 2$. Here $A^n_{\mu} = G^{mn} G_{mn}$ The second pair of vector fields are the components $B_{mn}$ of the antisymmetric field $B$.

Dimensional reduction of the action on the two-torus yields the $4D$ action

$$I_4 = \int d^4 x \sqrt{-G} e^{-\phi} \left[ R + \left( \partial \phi \right)^2 - \frac{(H)^2}{12} - \frac{1}{4} \mathcal{F}(LML) \mathcal{F} + \frac{1}{8} Tr(\partial ML \partial ML) \right],$$

where

$$M = \begin{pmatrix} G^{-1} & -B G^{-1} \\ -BG^{-1} & G \end{pmatrix}, \quad L = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}.$$

The metric of the two-torus is parametrized as

$$G_{mn} = e^{2\sigma} \begin{pmatrix} e^{2\gamma-2\sigma} + a_3^2 & -a_3 & 0 \\ -a_3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$
\[
\phi = \Phi - \frac{1}{2} \ln \det(G_{mn}).
\]

The dilaton \( \phi \) can be split into the sum of the constant part and a term vanishing at spatial infinity \( \phi = \phi_0 + \phi_1 \). In string perturbation theory, higher order contributions enter with the factor \( e^{\frac{1}{2} \chi \phi} \), where \( \chi \) is the Euler characteristic of the string world sheet. The exponent \( e^{\phi_0} \equiv \epsilon \) can be considered as a string-loop expansion parameter. In the following, we include the factor \( \epsilon \) in string-loop corrections, and use the notation \( \phi \) for the non-constant part of the dilaton.

The moduli (\( n \)) are equal to conventional moduli \( S, T, U \):

\[
(y_1, y_2, y_3) = (S = e^{-\phi} + ia_1, T = \sqrt{G} + iB_{12}, U = \frac{(\sqrt{G} + iG_{12})}{G_{22}}).
\]

Here \( a_1 \) is the axion \( \partial_{\mu}a_1 = -H^{\mu
u\lambda}e^{-2\phi}\sqrt{-g}e_{\mu\nu\lambda}, \) the antisymmetric tensor is \( B_{mn} = a_2 \epsilon_{mn} \).

The tree-level magnetic black hole solutions we discuss in this paper have \( a_i = 0 \).

The gauge part of the action (\( \ref{3} \)) with \( G_{12} = 0 \) and \( B_{12} = 0 \) is

\[
-\frac{1}{4}G_{11}(F^{(1)}_1)^2 - \frac{1}{4}G_{22}(F^{(1)}_2)^2 - \frac{1}{4}G_{11}(F^{(2)}_1)^2 - \frac{1}{4}G_{22}(F^{(2)}_2)^2.
\]

(9)

It is convenient to relabel the vector fields in correspondence with the moduli with which they form the superfields

\[
A^1_\mu = \sqrt{8} \hat{A}^0_\mu, \quad B_{1\mu} = \sqrt{8} \hat{A}^1_\mu, \quad A^2_\mu = \sqrt{8} \hat{A}^2_\mu, \quad B_{2\mu} = \sqrt{8} \hat{A}^3_\mu.
\]

The factor \( \sqrt{8} \) appears because of different normalization of the vector fields in (\( \ref{1} \)) and (\( \ref{3} \)).

Let us turn to the \( N = 2 \) supersymmetric action (\( \ref{1} \)). In sections which admit the prepotential, the coupling constants in the action (\( \ref{1} \)) are calculated using the formula

\[
N_{IJ} = \tilde{F}_{IJ} + 2i \frac{(\text{Im} F_{IK} X^K)(\text{Im} F_{JL} X^L)}{(X^K \text{Im} F_{JL} X^L)},
\]

(11)

where \( F_I = \partial_{X_I} F, F_{IJ} = \partial_{X_I X_J} F, \) etc. Using the loop-corrected prepotential and the formula (\( \ref{1} \)), we calculate the loop-corrected gauge couplings \( N_{IJ} \)

\[
N_{00} = iy^3 \left( -1 + \frac{n}{4y^3} \right), \quad N_{01} = -\frac{n + 2v}{4y_1} - ia_1 \frac{y_2 y_3}{y_1},
\]

\[
N_{02} = -\frac{n + 2v - 2y_3 h y + 4y_2 h_2}{4y_2} - ia_2 \frac{y_1 y_3}{y_2},
\]

\[
N_{03} = -\frac{n + 2v - 2y_3 h y + 4y_3 h_3}{4y_3} - ia_3 \frac{y_1 y_2}{y_3},
\]

\[
N_{11} = -i \frac{y^3}{y_1^2} \left( 1 + \frac{n}{4y^3} \right), \quad N_{12} = iy_3 \frac{2y_2 h y - n}{4y^3} + a_3, \quad N_{13} = iy_2 \frac{2y_3 h y - n}{4y^3} + a_2,
\]

\[
N_{22} = -i \frac{y^3}{y_2^2} \left( 1 - \frac{y_2 h_2 y_3}{y^3} + \frac{n}{4y^3} \right), \quad N_{33} = -i \frac{y^3}{y_3^2} \left( 1 - \frac{y_3 h_2 y_3}{y^3} + \frac{n}{4y^3} \right)
\]

(12)

\[
N_{23} = iy_1 \frac{2y_2 h y - 4y_2 h_2 y_3 - n}{4y^3} + a_1.
\]
Here we used the notations: \( y^3 = y_1 y_2 y_3, \ h y = h_a y_a = h_2 y_2 + h_3 y_3, \ h_a = \partial_{y_a} h, \ h_{ab} = \partial_{y_a} \partial_{y_b} h \)
and
\[
v = h - y_a h_a, \quad n = h - h_a y_a + y_a h_{a} y_b, \quad y_2 h y = y_2 h_{a} y_b.
\] (13)

In the following, we consider purely real tree-level moduli \( y_i \) which, in particular, is the case for black-hole solutions. The imaginary parts of the moduli \( y_i \) can appear at the first order in the string coupling constant. Since, the expressions of the first order in string coupling are calculated by substituting the tree-level moduli, in (12) and below, if it does not cause confusion, we use notations \( y_i \) for the real parts of the moduli \( y_i \).

In sections which do not admit a prepotential (including that which naturally appears in compactification of the heterotic string action), the gauge couplings are calculated by making a symplectic transformation of the couplings calculated in a section with the prepotential. In particular, in the section associated with compactification of the 6D heterotic string on the two-torus, the string-tree level symplectic transformation which connects the couplings is \[ \text{[2][12]} \]

\[
O = \begin{pmatrix} A & B \\ C & D \end{pmatrix},
\] (14)

where
\[
A^T C - C^T A = 0, \quad B^T D - D^T B = 0, \quad A^T D - C^T B = 1.
\] (15)

The gauge couplings are transformed as
\[
\hat{\mathcal{N}} = (C + D N)(A + B N)^{-1}.
\] (16)

Here and below the expressions with hats refer to the section associated with the heterotic string compactification. At the one-loop level, we look for the symplectic transformation in the form
\[
A = \text{diag}(1,0,1,1) + \epsilon(a_{ij}), \quad B = \text{diag}(0,1,0,0) + \epsilon(b_{ij}),
\]
\[
C = \text{diag}(0,-1,0,0) + \epsilon(c_{ij}), \quad D = \text{diag}(1,0,1,1) + \epsilon(d_{ij}),
\] (17)

where \( a \equiv \epsilon(a_{ij}), b \equiv \epsilon(b_{ij}), c \equiv \epsilon(c_{ij}) \) and \( d \equiv \epsilon(d_{ij}) \) are constant matrices and below the factor \( \epsilon \) is not written explicitly.

Since we perform calculations in the first order in string coupling constant, we can consider corrections to the matrices of symplectic transformation and corrections to the gauge couplings due to the one-loop term in the prepotential independently. From the relations (16) we have
\[
\hat{\mathcal{N}}_{(0)IJ} + \hat{\mathcal{N}}_{(1)IJ} = (c + d N_{(0)})(A + B N)^{-1}_{(0)} - \hat{\mathcal{N}}_{(0)}(a + b N_{(0)})(A + B N)^{-1}_{(0)},
\] (18)

where the subscripts (0) and (1) refer to the tree-level and first-order terms correspondingly.

In the heterotic string compactification, the loop corrections to the gauge couplings appear with the factor \( e^{\epsilon \phi} \). The couplings \( N_{00}, N_{22} \) and \( N_{33} \) are proportional to \( e^{-\phi} \), whereas \( N_{11} \) contains the factor \( e^{\phi} \). Thus, admissible structures of the one-loop corrections to the tree-level gauge couplings \( \hat{\mathcal{N}}_{IJ} \) in the section connected with the heterotic string compactification can be of the form \( \frac{N_{00}}{N_{00}} + c N_{11} \) and \( c N_{11} \), where \( c \) is one of the entries \( a, b, c \) and \( d \). Examining the relations (15), we find that admissible non-zero matrix elements are \( c_{ij} \) with \( c_{1i} = c_{i1} = 0 \) and \( d_{11} \).
Calculating the couplings in the holomorphic section associated with the heterotic string compactification, we have

\[
\hat{N}_{IJ} = \begin{pmatrix}
N_0 + c_0 - \frac{N_0 c_1}{N_{11}} & N_2 + c_2 - \frac{N_0 N_1 c_1}{N_{11}} & N_3 + c_3 - \frac{N_0 N_1 c_1}{N_{11}} \\
\frac{N_2 c_0 - N_1 c_1}{N_{11}} & \frac{N_2 c_0 - N_1 c_1}{N_{11}} & \frac{N_2 c_0 - N_1 c_1}{N_{11}} \\
N_3 c_0 - \frac{N_2 N_1 c_1}{N_{11}} & \frac{N_3 c_0 - N_2 N_1 c_1}{N_{11}} & \frac{N_3 c_0 - N_2 N_1 c_1}{N_{11}}
\end{pmatrix}.
\]  

(19)

The terms of the form \(\frac{N_0 N_1 c_1}{N_{11}}\) are of the next order in string coupling.

The field equations and Bianchi identities for the gauge field strengths are

\[
\partial_\mu \left( \sqrt{-g} \text{Im} \, G_I^{-\mu \nu} \right) = 0 \\
\partial_\mu \left( \sqrt{-g} \text{Im} \, F^{\mu \nu} \right) = 0,
\]

where \(G_I^{-\mu \nu} = \hat{N}_{IJ} F^{-\mu \nu} \). Eqs. (20) and are invariant under the symplectic transformations (14) of general form.

From the symplectic transformation of the field strengths

\[
\left( \begin{array}{c} \hat{F}^- \\ \hat{G}^- \end{array} \right) = O \left( \begin{array}{c} F^- \\ G^- \end{array} \right),
\]

(21)

with the matrices (17) we obtain the relations between the field strengths

\[
\hat{F}^- = F^-,
\hat{G}^- = G^-.
\]

(22)

Substituting in the relation \(\hat{G}^- = \hat{N}_{IJ} \hat{F}^- \) the couplings (19) and relations between the field strengths (22) (any \(I = 0, 1, 2, 3\) can be used ), we obtain the relation

\[
F^{-1} = -\frac{\hat{N}_{10}}{N_{11}} \hat{F}^0 + \frac{1}{N_{11}} \hat{F}^{-1} - \frac{\hat{N}_{12}}{N_{11}} \hat{F}^{-2} - \frac{\hat{N}_{13}}{N_{11}} \hat{F}^{-3}
\]

(23)

which does not contain arbitrary constants \(d_{11}\) and \(c_{ij}\).

The Kähler potential is invariant under symplectic transformations and its part which depends on the moduli \(y_i\) is given by

\[
K = -\ln[(y_1 + \bar{y}_1 + V)(y_2 + \bar{y}_2)(y_3 + \bar{y}_3)],
\]

(24)

where the Green-Schwarz function \(V\) is 3, 2

\[
V(y_2, \bar{y}_2, y_3, \bar{y}_3) = \frac{Re h^{(1)} - Re y_2 Re \partial_{y_3} h^{(1)} - Re y_3 Re \partial_{y_2} h^{(1)}}{Re y_2 Re y_3}.
\]

(25)
3 Supersymmetry transformations

To write the supersymmetry transformations, one introduces the expressions (for example, [9, 10, 11, 12, 13])

\[ S_{\mu\nu} = X^I \text{Im} N_{1J} F^{-J}_{\mu\nu}, \]  
\[ T^-_{\mu\nu} = 2ie^{K/2} X^I \text{Im} N_{1J} F^{-J}_{\mu\nu} \]

and

\[ G^{-i}_{\mu\nu} = -k^{ij} f^I_I \text{Im} N_{1J} F^{-J}_{\mu\nu}. \]  
\[ f^I_I = (\partial_i + \frac{1}{2} \partial_i K)e^{K/2} X^I. \]

Supersymmetry transformations of the chiral gravitino \( \psi_{\alpha\mu} \) and gaugino \( \lambda^\alpha \) are (for example, [9, 10, 11, 13])

\[ \delta \psi_{\alpha\mu} = D_{\mu} \epsilon_{\alpha} + T^-_{\mu\nu} \gamma^\nu \epsilon_{\alpha\beta} \epsilon_{\beta}, \]  
\[ \delta \lambda^\alpha = i\gamma^\mu \partial_{\mu} \epsilon_{\alpha} + G^{-i}_{\mu\nu} \gamma^\mu \epsilon_{\alpha\beta} \epsilon_{\beta}, \]

where

\[ D_{\mu} \epsilon_{\alpha} = (\partial_{\mu} - \frac{1}{4} w^{\hat{a}\hat{b}} \gamma_{\hat{a}} \gamma_{\hat{b}} + \frac{i}{2} Q_{\mu}) \epsilon_{\alpha}. \]

Here \( w^{\hat{a}\hat{b}} \) is the spin and \( Q_{\mu} \) is the Kähler connection. Here \( \hat{a}, \hat{b}, ... \) are the tangent space indices, \( a, b, ... \) are the curved space indices.

Requiring that supersymmetry variations of spinors vanish, we obtain a system of supersymmetric Killing equations for the moduli. We look for a solution of this system with supersymmetry parameter satisfying the relation \( \epsilon^\alpha = \gamma_0^\epsilon_{\alpha\beta} \epsilon_{\beta}. \) The \( \mu = 0 \) component of equation \( \delta \psi_{\alpha0} = 0 \) takes the form

\[ (\frac{1}{2} w^{\hat{a}\hat{b}} \gamma_0 \gamma_5 - T^-_{0n} \gamma^{\hat{a}n}) \epsilon_{\alpha\beta} \epsilon_{\beta} = 0. \]  
\[ (30) \]

Here \( e^{\mu\hat{b}} \) is the inverse vielbein.

In this paper we are interested in static spherically-symmetric solutions of the field equations. The metric is

\[ ds^2 = -e^{2U} dt^2 + e^{-2U} dx^i dx_i. \]  
\[ (31) \]

The only non-vanishing components of the spin connection \( w^{\hat{a}\hat{b}}_0 \) are \( w^{\hat{a}\hat{b}}_0 = \frac{1}{2} \partial_{\hat{b}} e^{2U}. \) The vielbein \( e^{\hat{b}}_\mu \) is \( e^{\hat{b}}_\mu = \delta^{\hat{b}}_\mu e^{U}. \) To have a nontrivial solution for the supersymmetry parameter, we must require that

\[ \frac{1}{2} w^{\hat{a}}_0 - e^U T^-_{0n} = 0. \]  
\[ (32) \]

Using the relations

\[ G^-_{mn} = i\epsilon_{mnpq} G^{-p0} \]

and

\[ G^-_{\mu\nu} \gamma^\mu \gamma^\nu \epsilon_{\alpha} = 4G^-_{0n} \gamma^0 \gamma^n \epsilon_{\alpha} \]
valid for any self-dual tensor and chiral spinor, the condition the gaugini supersymmetry transformation to vanish is written as

$$(i\gamma^n \partial_n z^i \gamma^0 + 4G_{0n}^{-i} \gamma^0 \gamma^n) e^{\alpha\beta} \epsilon_\beta = 0. \quad (33)$$

There is a nontrivial solution provided

$$i\partial_n z^i + 4e^{-U} G_{0n}^{-i} = 0. \quad (34)$$

The factor $e^{-U}$ is due to the relation $\gamma_0 = -\gamma^0 = -e_0^0 \gamma^0 = -e^0 U$. Convoluting the equation (34) with the functions $f^I$ and using the relation of special $N=2$ geometry

$$k^i f_I^i \bar{f}_J^j = - \frac{1}{2} (ImN)^{IJ} - e^K X^1 X^J,$$

it is obtained in the form (cf. [20, 18])

$$i f_I^i \partial_n z^i + 4e^{-U} \left( \frac{1}{2} F_{0n}^{-I} + e^K X^I (X^J Im N_{JL} F_{0n}^{-L}) \right) = 0. \quad (35)$$

Combining the Eqs.(32) and (35) it is possible to present the latter as (cf. [22, 23])

$$e^{-K/2} \left[ e^U \partial_i (e^K X^I) \partial_n z^i - (e^K X^I) \partial_n e^U \right] = 2i F_{0n}^{-I} \quad (36)$$

4 Solution of spinor Killing equations for magnetic black hole

Next, we solve the combined system of the equations for the gauge field strengths and the moduli. First, we look for a string-tree-level solution with the metric in the form (31), two magnetic fields $\hat{F}_{00}^\mu$ and $\hat{F}_{10}^\mu$ and purely real moduli $y_i$ (3). This means, that we consider configurations with diagonal metrics $G_{mn}$, vanishing tensor $B_{mn}$ and vanishing axion $a_1$. Such configurations appear as solutions of the "chiral null models" [5, 6]. In the next section we shall solve the equations including the string-loop corrections.

Solving the system of Maxwell equations and Bianchi identities in the section associated with the heterotic string compactification, we have

$$\hat{F}_{0n}^{-0} = P_0^0 \frac{i}{2} e^{2U} x^n, \quad \hat{F}_{0n}^{-1} = P_0^1 \frac{i}{2} e^{2U} x^n. \quad (37)$$

$$K = - \ln 8y_1 y_2 y_3. \quad (38)$$

The tree-level expression for the symplectic-invariant combination $T_{0n}^{-}$ (26) which enters the gravitini supersymmetry transformations is

$$T_{0n}^{-} = 2ie^{K/2} (ImN_{00} F_{0n}^{-0} + iy_1 ImN_{11} F_{0n}^{-1})$$

$$= 2ie^{K/2} (-y_1 y_2 y_3 \hat{F}_{0n}^{-0} - y_1 \hat{F}_{0n}^{-1}) = \left( \frac{y_1 y_2 y_3}{8} \right)^{1/2} \left( P_0^0 + P_1^0 \frac{y_1 y_2 y_3}{8} \right) e^{2U} x^n. \quad (39)$$

The gravitini Killing equation (32) takes the form

$$\frac{1}{4} \partial_n e^{2U} - \left( \frac{y_1 y_2 y_3}{8} \right)^{1/2} e^{3U} (P_0^0 + P_1^0 \frac{y_1 y_2 y_3}{8}) x^n = 0. \quad (40)$$
As in (10), the factor \( y \) and the moduli \( \text{moduli} \) The charges \( P \) equations (20) which we rewrite as considerable technical simplifications in solution of the loop-corrected spinor Killing equations. The charges \( P \) correction to the prepotential and its derivatives are also independent of coordinates, resulting in are calculated by substituting the tree-level moduli. For the constant moduli We look for a solution in the first order in string coupling constant. The loop corrections Our next aim is to solve the Maxwell equations and the loop-corrected spinor Killing equations.

5 Solution of the loop-corrected spinor Killing equations

The tree-level gaugini equations (35) written in the section with the prepotential are

\[
I = 0 : \quad \frac{ie^{K/2}}{2} \partial_\mu \ln y_1 y_2 y_3 - 4e^{-U} \left( \frac{1}{2} \tilde{F}_{\mu}^{00} + e^K S_{\mu 0} \right) = 0,
\]

\[
I = 1 : \quad \frac{y_1 e^{K/2}}{2} \partial_\mu \ln \frac{y_2 y_3}{y_1} + 4e^{-U} \left( \frac{\tilde{F}_{\mu}^{01}}{2N_{11}} - iy_1 e^K S_{\mu 0} \right) = 0,
\]

\[
I = 2 : \quad \frac{y_2 e^{K/2}}{2} \partial_\mu \ln \frac{y_1 y_3}{y_2} + 4e^{-U} \left( -iy_2 e^K S_{\mu 0} \right) = 0,
\]

\[
I = 3 : \quad \frac{y_3 e^{K/2}}{2} \partial_\mu \ln \frac{y_1 y_2}{y_3} + 4e^{-U} \left( -iy_3 e^K S_{\mu 0} \right) = 0.
\]

Here using (23) and (22) we expressed the field strengths in the section with the prepotential through the strengths in the heterotic section.

The system of equations (10) and (11) is solved by a general magnetic black hole with two arbitrary magnetic charges. The background consist of a metric, dilaton and moduli \([5, 6]\). In the following, we shall consider a particular extremal solution

\[
e^{-2U} = 1 + \frac{P}{r}, \quad y_1 = e^{-\phi} = \left( 1 + \frac{P}{r} \right)^{-1},
\]

(42)

The charges \( P^0 \) and \( P^1 \) are expressed via a charge \( P \)

\[
P^0 = \frac{y_2 y_3}{y_2 y_3}, \quad P = \sqrt{8y_2 y_3} P^0
\]

(43)

and the moduli \( y_2, y_3 \) are real constants. The metric components of the torus \( T^2 \) are

\[
G_{11} = y_2 y_3 = e^{2\gamma}, \quad G_{22} = y_2 y_3 = e^{2\sigma}.
\]

As in (11), the factor \( \sqrt{8} \) appears because of different normalizations of the gauge terms in the actions (1) and (3).

5 Solution of the loop-corrected spinor Killing equations

Our next aim is to solve the Maxwell equations and the loop-corrected spinor Killing equations. We look for a solution in the first order in string coupling constant. The loop corrections are calculated by substituting the tree-level moduli. For the constant moduli \( y_2, y_3 \), the loop correction to the prepotential and its derivatives are also independent of coordinates, resulting in considerable technical simplifications in solution of the loop-corrected spinor Killing equations.

In the holomorphic section associated with the heterotic string compactification, the Maxwell equations (21) which we rewrite as

\[
\partial_\mu (\sqrt{-g} Im \tilde{N}_{1j} \tilde{F}^j + Re \tilde{N}_{1j}^* \tilde{F}^j)^\mu = 0,
\]

(44)

with the required accuracy have the form

\[
J = 0 : \quad \partial_\mu (\sqrt{-g} Im \tilde{N}_{00} \tilde{F}^0 + Re \tilde{N}_{00}^* \tilde{F}^0 + Re \tilde{N}_{01}^* \tilde{F}^{10 r} = 0,
\]

(45)

\[
J = 1 : \quad \partial_\mu (\sqrt{-g} Im \tilde{N}_{11} \tilde{F}^1 + Re \tilde{N}_{10}^* \tilde{F}^0 + Re \tilde{N}_{11}^* \tilde{F}^{11 r} = 0,
\]

(46)

\[
J = 2 : \quad \partial_\mu (\sqrt{-g} Im \tilde{N}_{22} \tilde{F}^2 + Re \tilde{N}_{20}^* \tilde{F}^0 + Re \tilde{N}_{21}^* \tilde{F}^{11 r} = 0,
\]

(47)

\[
J = 2 : \quad \partial_\mu (\sqrt{-g} Im \tilde{N}_{33} \tilde{F}^3 + Re \tilde{N}_{30}^* \tilde{F}^0 + Re \tilde{N}_{31}^* \tilde{F}^{11 r} = 0.
\]

(48)
Here \( \mathcal{F}^{0r} = \mathcal{F}_{\partial \varphi} \) and \( \mathcal{F}^{0r} = -\mathcal{F}^{or} \). Only the diagonal gauge couplings \( \hat{N}_{11} \) contain terms of zero-order in string coupling. The tree-level gauge field strengths acquire corrections of the first order in string coupling, and also the gauge fields, absent at the tree level, are generated.

First, we solve the equations (47) and (48):

\[
\hat{F}^{20r} = \frac{C_2 - (\text{Re} N_{20} + c_{20}) P^0 - \text{Re} \frac{N_{21}}{N_{11}} P^1}{\sqrt{-g} \text{Im} N_{22}},
\]

\[
\hat{F}^{30r} = \frac{C_3 - (\text{Re} N_{30} + c_{30}) P^0 - \text{Re} \frac{N_{31}}{N_{11}} P^1}{\sqrt{-g} \text{Im} N_{33}}.
\]

Eqs. (43) and (44) yield

\[
\hat{F}^{00r} = \frac{C_0 - c_{00} P^0 - a_1 P^1}{\sqrt{-g} \text{Im} N_{00}},
\]

\[
\hat{F}^{10r} = \frac{C_1 - d_{11} P^1 - a_1 P^0}{\sqrt{-g} \text{Im} N_{11}}
\]

where \( \sqrt{-g} = e^{-2u} r^2 \) and \( C_I \) are arbitrary constants of the first order in string coupling. All the electric fields \( \hat{F}^I \) are of the first order in string coupling.

One can also introduce magnetic fields \( \mathcal{F}^2 \) and \( \mathcal{F}^3 \) with the charges of the first order in string coupling. However, since these fields enter the Maxwell equations multiplied by the coupling constants of the first order in string coupling, at the required level of accuracy, these terms are omitted from the equations. By the same reason the terms with the constants \( c_{22} \) and \( c_{33} \) do not appear in solutions also.

Let us calculate the symplectic invariant combination \( S_{0n} \) in the first order in string coupling constant. We have

\[
S_{0n} = \left\{ P^0 [\text{Im} N_{00} + y_i \text{Re} N_{i0} + i (a_1 y_2 y_3 + a_2 y_1 y_3 + a_3 y_1 y_2)] - P^1 y_1 \right. \\
- \left. [(y_2 C_2 + y_3 C_3) + i (C_0 - c_{00} P^0 + (C_1 - d_{11} P^1) y_2 y_3)] \right\} \frac{i e^{2u} x^n}{r^3}.
\]

Only the the couplings \( N_{00} \) and \( N_{0i}, i = 1, 2, 3 \) enter the final expression yielding

\[ I \text{m} N_{00} + y_i \text{Re} N_{i0} = -(y_1 y_2 y_3 + 2v + h_i y_i). \]

All the terms containing second derivatives of the prepotential have canceled.

Since gaugino spinor Killing equations are linear in derivatives of the moduli, and axions \( a_i \) are of the first order in string coupling, the equations for real parts of the moduli decouple from the equations for axions. In the first order in string coupling, the Kähler potential (24) is also independent of the axions \( a_i \). The constants \( C_0 \) and \( C_1 \) enter only the imaginary parts of spinor Killing equations. In the following we shall discuss only the real parts of spinor Killing equations, i.e. equations for the real parts of the moduli. The equations for the axions will be considered elsewhere.

Using the Kähler potential (24), we calculate the combinations \( B^I_n = f^I_i \partial_n z^i \) which enter the spinor Killing equations (35) for the moduli \( z_i \). We have

\[
B^0_n = -\frac{1}{2} e^{K/2} \left( 1 - \frac{V}{2 y_1} \right) \partial_n \ln y^3
\]

\[
B^i_n = i y_i \left( B^0_n + e^{K/2} \partial_n \ln y_i \right), \quad i = 1, 2, 3.
\]
Here we substituted
\[ V = e^{-2\gamma v}. \]

All the expressions are calculated in the first order in string coupling. In particular, all the factors multiplying the Green-Schwarz function \( V \) are taken in the leading order in string coupling.

Let us introduce the notations for the loop-corrected metric and moduli. We shall split the functions \( \phi, \gamma \) and \( \sigma \) which appear in the moduli (3) into the tree-level parts \( \phi_0, \gamma_0 \) and \( \sigma_0 \) and those of the first order in string coupling: \( \phi_1, \gamma_1 \) and \( \sigma_1 \). We have \( \phi = \phi_0 + \phi_1 \), etc. The function \( 2U \) in the metric will be written as \( 2U_0 + u_1 \). At the tree level (see (42),
\[ e^{-2U_0} = e^{\phi_0} = f_0 = 1 + \frac{P}{r} \]

From (43), we have
\[ P^0 = \frac{Pe^{-\gamma_0}}{\sqrt{8}}, \quad P^1 = \frac{Pe^{\gamma_0}}{\sqrt{8}} \] (53)

For the Kähler potential we obtain
\[ e^K = \frac{f_0 e^{-2\gamma_0}}{8} \left[ 1 + \left( \phi_1 - 2\gamma_1 - \frac{Vf_0}{2} \right) \right]. \] (54)

Let us turn to the function \( S_{\varphi, \varphi} \). The functions \( a_i \) are of the first order in the string coupling constant. The terms containing the factors \( a_i \) are imaginary. Because the spinor Killing equations for the moduli (34) are linear in derivatives of the moduli, the equations for the imaginary parts of the moduli decouple from those for the real parts. In this section, solving the equations for the real parts of the moduli, the imaginary terms in the equations will be omitted.

Because the tree-level moduli are constants, the terms \( 2v + h_a y_a \) and \( y_a C_a \), which are of the first order in string coupling, are also constants.

The function \( T_{\varphi, \varphi} \) which enters the gravitini equation (32), we write in the form
\[ T_{\varphi, \varphi} = \frac{f_0 e^{-2\gamma_0}}{8} \left[ 1 + \left( \phi_1 - 2\gamma_1 - \frac{Vf_0}{2} \right) \right]. \] (55)

The factors \( f_0 \) appear because in the expressions of the first order in string coupling the modulus \( y_1 \) can be substituted by its tree-level value \( f_0^{-1} \).

Using the expression for the Kähler potential (54) and expanding in (55) all the terms to the first order in string coupling, we finally obtain
\[ T_{\varphi, \varphi} = f_0^{-1/2} \left[ 1 + \left( \frac{\phi_1}{2} + u_1 + \left( \frac{3V}{4} + C \right) f_0 \right) \right] \frac{x^n}{r^3}. \] (56)

where the constant \( C \) is
\[ C = \frac{1}{2} \left( h_a y_a + \frac{C_a y_a}{P^0} \right) e^{-2\gamma_0}. \] (57)

Substituting the expression (56), we obtain the gravitino spinor Killing equation in the form
\[ \frac{1}{4} \partial_n [f_0^{-1} (1 + u_1)] - \frac{P}{4} f_0^{-2} \left[ 1 + \left( \frac{3u_1}{2} - \frac{\phi_1}{2} + \left( \frac{3V}{4} + C \right) \right) \right] \frac{x^n}{r^3} = 0. \] (58)
In this equation the combination of the tree-level terms vanishes; the remaining part of the first order in string coupling is

\[ \frac{u_1'}{q'} + \frac{u_1 - \phi_1}{2} + \left( \frac{3V}{4} + C \right) f_0 = 0. \]  

(59)

Here \( q' = f_0' / f_0 \).

Let us turn to the gaugino spinor Killing equations (35). Substituting the expression

\[ e^K S_{0n} = -\frac{1}{4} \left( 1 - \gamma_1 + \left( \frac{V}{2} + C \right) f_0 \right) \left( \frac{P e^{-q_0} i}{\sqrt{8}} \frac{e^{2U x^n}}{r^3} \right), \]

for the combination \( \frac{1}{2} F_{0n}^{-0} + e^K S_{0n} \) we have

\[ \frac{1}{2} F_{0n}^{-0} + e^K S_{0n} = \frac{1}{4} \left( 1 + \gamma_1 - \left( \frac{V}{2} + C \right) f_0 \right) \left( \frac{P e^{-q_0} i}{\sqrt{8}} \frac{e^{2U x^n}}{r^3} \right). \]

(61)

For the combination \( \frac{1}{2} F_{0n}^{-1} - i y_1 e^K S_{0n} \) we obtain

\[ \frac{1}{2} F_{0n}^{-1} - i y_1 e^K S_{0n} = -\frac{i y_1}{4} \left( 1 - 3\gamma_1 + \left( \frac{V}{2} + C \right) f_0 \right) \left( \frac{P e^{-q_0} i}{\sqrt{8}} \frac{e^{2U x^n}}{r^3} \right). \]

(62)

With the accuracy of the terms of the first order in string coupling, the loop-corrected expressions for \( B^0_n \) and \( B^1_n \) are

\[ B^0_n = \frac{q' f_0^{1/2} e^{-q_0}}{2\sqrt{8}} \left[ 1 + \frac{\phi_1' - 2\gamma_1'}{q'} + \frac{\phi_1 - u_1}{2} - \frac{3V f_0}{4} \right] \frac{x^n}{r}, \]

\[ B^1_n = -i \frac{q' f_0^{1/2} e^{-q_0}}{2\sqrt{8}} \left[ 1 + \frac{\phi_1' + 2\gamma_1'}{q'} - \frac{\phi_1 - u_1}{2} + \frac{V f_0}{4} \right] \frac{x^n}{r}. \]

(63)

Using the expressions (61)-(63), we verify that in the gaugino spinor Killing Eqs. (35) with \( I = 0 \) and \( I = 1 \) the leading-order terms cancel, and the remaining parts of the first order in string coupling are

\[ I = 0 : \quad \frac{\phi_1' - 2\gamma_1'}{q'} + \frac{\phi_1 - u_1}{2} - 2\gamma_1 - \left( \frac{V}{4} - C \right) f_0 = 0, \]

\[ I = 1 : \quad \frac{\phi_1' + 2\gamma_1'}{q'} + \frac{\phi_1 - u_1}{2} + 2\gamma_1 - \left( \frac{V}{4} - C \right) f_0 = 0. \]

(64)

Eqs. (64) split into the following system

\[ \frac{\phi_1'}{q'} + \frac{\phi_1 - u_1}{2} - \left( \frac{V}{4} - C \right) f_0 = 0 \]

\[ \gamma_1' + q' \gamma_1 = 0 \]

(65)

Let us consider the gaugino spinor Killing equations equations (35) with \( I = 2 \) and \( I = 3 \). Substituting in the expressions for the loop-corrected couplings (12) and the field strengths (19), we have

\[ F_{0n}^{-2} = \frac{P^0_{y_1 y_3}}{y_1 y_3} \left( \frac{v}{2} + h_2 y_2 + \frac{C_2 y_2}{P^0} \right) \frac{1}{2} e^{2U x^n} = y_2 P^0 f_0 \left( \frac{V}{2} + L_2 \right) \frac{1}{2} e^{2U x^n} \]

(66)
and similar expression for $\mathcal{F}_{0n}^{-3}$ obtained by substitution $2 \to 3$. The field strengths $\mathcal{F}^{-2,3}$, absent at the string tree level, are of the first order in the string coupling. Here we introduced

$$L_2 = \left( h_2y_2 + \frac{C_2y_2}{P_0} \right) e^{-2\gamma_0}, \quad L_3 = \left( h_3y_3 + \frac{C_3y_3}{P_0} \right) e^{-2\gamma_0}. \quad (67)$$

Subtracting Eq.(12) with $I = 2$ from that with $I = 0$ (the same for $I = 3$), and using the expressions (52) for the combinations $B_n^i$, we have

$$ie^{K/2} \frac{\partial_n y^2}{y_2} + 4e^{-U} \left( \hat{F}_{0n}^{-2} - \frac{1}{2} \hat{F}_{0n}^{-0} - 2e^K S_{0n} \right) = 0. \quad (68)$$

Substituting the expressions for the field strengths $\hat{F}_{0n}^{-0}$, $\hat{F}_{0n}^{-2}$ and Eq.(60) for $e^K S_{0n}$, and keeping the terms of the first order in the string coupling, we obtain

$$\gamma_1' + \sigma_1' + (C - L_2 - \gamma_1 f_0^{-1}) \frac{P}{P'} = 0,$$
$$\gamma_1' - \sigma_1' + (C - L_3 - \gamma_1 f_0^{-1}) \frac{P}{P'} = 0. \quad (69)$$

The sum of the Eqs. (69) is

$$\gamma_1' + \gamma_1 q' + (2C - L_2 - L_3) f_0' = 0. \quad (70)$$

Substituting the expressions (57) and (67) for $C$ and $L_a$, we find that

$$2C - L_2 - L_3 = 0, \quad (71)$$

so that Eq.(70) coincides with the second Eq.(65).

Let us solve the system of equations for the functions $u_1$ and $\phi_1$. Adding and subtracting the gravitini Eq.(53) and the first Eq.(65), we obtain the solution

$$u_1 + \phi_1 = c_1 - \left( \frac{V}{2} + 2C \right) f_0,$$
$$u_1 - \phi_1 = c_2 - \frac{V f_0}{f_0} = \frac{V f_0}{2}, \quad (72)$$

where $c_{1,2}$ are arbitrary constants. Requiring that at large distances from the center of the black hole the metric and dilaton are asymptotic to the Lorentzian metric and constant dilaton equal to unity, we have

$$c_1 = \frac{V}{2} + 2C, \quad c_2 = \frac{V}{2}, \quad (73)$$

and we obtain

$$u_1 = - \left( \frac{V}{2} + C \right) \frac{P}{r} - \frac{V}{4} \frac{P}{r + P}, \quad \phi_1 = -C \frac{P}{r} + \frac{V}{4} \frac{P}{r + P}. \quad (74)$$

At the tree level, magnetic black hole solution is the extremal BPS saturated configuration [5, 6]. Provided supersymmetry is unbroken in perturbation theory, the loop-corrected solution must have the same properties. ADM mass can be obtained from the $r \to \infty$ asymptotics of the metric. Using (74) we have

$$M_{ADM} = 2P \left( 1 - \frac{3V}{4} - C \right).$$
BPS mass is determined from the asymptotics of the function \( T^- \) which can be written as

\[
T^-_{\mu \nu} = (F^I F^I_{\mu \nu} - X^I G_{1 \mu \nu}).
\]

Asymptotics of the fields \( F^I \) and \( G^I \) are proportional to electric and magnetic charges, correspondingly, and the asymptotics of \( T^- \) yields the expression for the BPS mass \( M_{BPS} = |Z_\infty| = e^{k/2} |n_1 X^I - m^I F^I_\infty| \). Taking the asymptotics of the of the function \( T^- \) (56), we find the BPS mass which is equal to the ADM mass.

A particular possibility is to take \( C = -\frac{3V}{4} \), in which case the central charge retains the tree-level form. Correction to the metric becomes

\[
u_1 = \frac{V}{4} \left( \frac{P}{r} - \frac{P}{r + P} \right). \tag{75}
\]

With required accuracy, the loop-corrected metric can be considered as the leading and the first-order terms in the expansion of the expression

\[
g_{ii} = -g^{00} = 1 + \frac{P}{r + \epsilon V^4} \tag{76}
\]

in powers of the string-loop counting parameter \( \epsilon \). In the metric (76) the singularity at the point \( r = 0 \) is smeared by the loop correction.

### 6 Discussion

In our previous study [17], solving the system of the loop-corrected Einstein and Maxwell equations, we obtained a two-parameter set of solutions for the loop corrections to the metric and dilaton

\[
u_1 = A_1 \frac{P}{r} - A_2 \frac{P}{r + P}, \quad \phi_1 = \left( A_1 + \frac{V}{2} \right) \frac{P}{r} + A_2 \frac{P}{r + P}. \tag{77}
\]

The one-parameter family of solutions (74) is contained in the set (77). A nontrivial check of consistency of both calculations is that in both cases the coefficients at the terms \( \frac{P}{r} \) in the expressions for the metric and dilaton differ by \( \frac{V}{2} \).

Near the locations of the enhanced symmetry points in the moduli space, the second derivatives of the prepotential have logarithmic singularities [1], [2]. In particular, for \( y_2 \sim y_3 \),

\[ h^{(1)}(y_2, y_3) = (y_2 - y_3)^2 \log(y_2 - y_3)^2. \]

Although the loop-corrected gauge couplings (12) contain second derivatives of the prepotential, the final expressions for the metric and moduli depend on the Green-Schwarz function \( V \) which contains only the first derivatives of the prepotential and thus is regular at the points of enhanced symmetry. Note also, that the Green-Schwarz function is positive [3] (this can be verified by explicit calculations) as can be seen from the form of the Kähler potential for the moduli which is a regular function at finite values of the moduli.

Our solution for the loop corrections is valid for all \( r \) for which is valid the perturbation expansion in string coupling. In particular, since the dilaton increases at small distances, we can use both the tree-level and a loop-corrected solution for \( \frac{r}{P} > \epsilon V \). However, if we extrapolate the above expression for the loop-corrected metric to the region of small \( r \), it can be seen that
singularity at the origin is smeared. The crucial point is that the Green-Schwarz function \( V \) is positive [3].

Our treatment of spinor Killing equations is similar in spirit to [18, 19]. However, in these papers were discussed only tree-level spinor Killing equations. Another distinction is that usually an emphasis was made on the form of solution at the stabilization point [21], whereas we were interested in full coordinate dependence of solution.

Our approach is different from that in papers [20] based on the assumption that there is a "small" modulus which can be used as an expansion parameter for the loop-corrected action. In string-loop perturbative expansion, a natural expansion parameter is associated with the dilaton, and the loop correction to the tree-level prepotential is independent of the modulus \( y_1 \equiv S \).

Finally, in perturbative approach, we neglected the terms of the form \( O(e^{2\pi S}) \), and the duality properties of the full theory [24] cannot be checked in this approximation.

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