A Lattice Universe from M-theory

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ABSTRACT

A recent paper on the large-scale structure of the Universe presented evidence for a rectangular three-dimensional lattice of galaxy superclusters and voids, with lattice spacing \( \sim 120 \) Mpc, and called for some “hitherto unknown process” to explain it. Here we report that a rectangular three-dimensional lattice of intersecting domain walls, with arbitrary spacing, emerges naturally as a classical solution of M-theory.

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1 Introduction

In a recent paper on the large-scale structure of the Universe at the 100-million parsec scale, Einasto et al. report seeing hints of a network of galaxy superclusters and voids that seems to form a three-dimensional lattice with a spacing of about $120 \, h^{-1} \, \text{Mpc}$ (where $h^{-1}$ is the Hubble constant in units of $100 \, \text{km s}^{-1} \, \text{Mpc}^{-1}$). These authors remark that “If this reflects the distribution of all matter (luminous and dark), then there must exist some hitherto unknown process that produces regular structure on large scales.” In this paper we point out that a three-dimensional lattice of orthogonally intersecting domain walls, with arbitrary lattice spacing, naturally appears as a solution of the classical equations of $M$-theory.

Until recently, the best hope for an all-embracing theory that would reconcile gravity and quantum mechanics was based on superstrings: one-dimensional objects whose vibrational modes represent the elementary particles and which live in a universe with ten spacetime dimensions, six of which are curled up to an unobservably small size. Unfortunately, there seemed to be five distinct mathematically consistent string theories and this was clearly an embarrassment of riches if one is looking for a unique Theory of Everything. In the last three years, however, it has become clear that all five string theories may be subsumed by a deeper, more profound, new theory called $M$-theory, which is reviewed in Refs. $\text{[5, 6, 7, 8]}$. $M$-theory is an eleven-dimensional theory in which the extended objects are not one-dimensional superstrings but rather two-dimensional objects called supermembranes and five-dimensional objects called superfivebranes. In the limit of low energies, $M$-theory is approximated by eleven-dimensional supergravity which, ironically enough, was the favourite candidate for superunification before it was knocked off its pedestal by the 1984 superstring revolution.

Like string theory before it, $M$-theory relies crucially on a supersymmetry which unifies bosons and fermions and indeed eleven is the maximum spacetime dimension that supersymmetry allows. The basic objects of $M$-theory are solutions of the supergravity equations of motion which preserve the fraction $\nu = 1/2$ of this supersymmetry. In addition to the supermembrane and superfivebrane there are also other solutions called plane waves and Kaluza-Klein monopoles which also preserve $\nu = 1/2$. Again, seven of the eleven dimensions are assumed to be compactified to a tiny size and, when wrapped around these extra dimensions, these membranes, fivebranes, waves and monopoles appear as zero-dimensional particles, one-dimensional strings or two-dimensional membranes when viewed from the perspective of the physical four-dimensional spacetime. Of particular interest for the present
paper will be the membranes which act as domain walls separating one region of the three-
dimensional uncompactified space from another. Solutions preserving a smaller fraction
\( \nu = 2^{-n} \) of the supersymmetry may then be obtained by permitting \( n = 2, 3 \) of the basic
objects to intersect orthogonally \[10\]. In particular, a solution describing 3 domain walls
intersecting orthogonally in three space dimensions will, if it exists, preserve just \( N = 1 \) of
the maximum \( N = 8 \) supersymmetries allowed in four spacetime dimensions.

Although astrophysicists and cosmologists have considered topological defects \[13\] or
solitons such as monopoles, cosmic strings and domain walls as possible seeds for galaxy
formation, they have traditionally\[12\] eschewed the kind of solitons arising in superstring theory.
This was not without good reason. They were interested in energy scales less than
the order of \( 10^{16} \) GeV typical of the energy at which the strengths of the strong, weak and
electromagnetic forces are deemed to converge in Grand Unified Theories of the elementary
particles. By contrast, the solitons arising in string theory are gravitational in origin, having
as their typical energy scale the Planck energy of \( 10^{19} \) GeV which corresponds to much too
too early an epoch in the history of the universe to be relevant to galaxy formation. However,
recent work by Witten \[14\] indicates that \( M \)-theory differs from traditional superstring
theory precisely in this respect: the phenomenologically most favoured size of the eleventh
compact dimension of \( M \)-theory is such that all four forces converge at a common scale of
\( 10^{16} \) GeV. Thus gravitational effects are much closer to home than previously realised and
topological defects in \( M \)-theory are likely to be of greater cosmological significance than
those of old-fashioned string theory.

\[2\] The lattice universe

We now show that solutions describing three orthogonally intersecting domain walls do
indeed exist and that they may be generalised to describe any number of walls so as to
form a three-dimensional lattice, with arbitrary lattice spacing. (A review of domain walls
in \( N = 1 \) \( D = 4 \) supergravity can be found in \[16\].) We begin by considering a four-
dimensional universe with cartesian coordinates \( x^\mu = (x^0, x^1, x^2, x^3) = (t, x, y, z) \) whose
gravitational field is described by a metric tensor \( g_{\mu \nu}(x) \) and which in addition contains
three massless scalar fields described by the three-dimensional vector \( \vec{\phi}(x) \) and a set of three
3-form potentials \( C^{(a)}_{\mu \nu \rho}, (\alpha = 1, 2, 3) \). The Lagrangian is given by the Einstein Lagrangian

\[1\] See, however, \[11\], where the periodic-like distribution of matter in space \[15\] was noted in the context
of stringy topological defects.
for gravity plus kinetic energy terms for $\vec{\phi}(x)$ and $C_{\mu\nu\rho}^{(\alpha)}$,

$$\mathcal{L} = \frac{1}{2\kappa^2} \sqrt{-g} \left[ R - \frac{1}{2} \partial_{\mu} \vec{\phi} \cdot \partial_{\mu} \vec{\phi} - \frac{1}{48} \sum_{\alpha=1}^{3} e^{-\vec{c}_\alpha \cdot \vec{\phi}} G^{(\alpha)\mu\nu\rho\sigma} G_{\mu\nu\rho\sigma} \right]$$  \hspace{1cm} (1)

where $\kappa^2 = 8\pi G$ and $G$ is Newton’s constant and where $G^{(\alpha)\mu\nu\rho\sigma} = 4\partial_{[\mu} C^{(\alpha)\nu\rho\sigma]}$ are the 4-form field strengths associated with the 3-form potentials. In four spacetime dimensions such 3-forms do not correspond to propagating degrees of freedom but may nevertheless give rise to non-trivial topological effects which are akin to the presence of a cosmological constant [17]. The three vectors $\vec{c}_\alpha$ are constants describing the interaction of the scalars with the field strengths and satisfy the relations

$$\vec{c}_\alpha \cdot \vec{c}_\beta = 1 + 6\delta_{\alpha\beta}$$ \hspace{1cm} (2)

A convenient choice is

$$\vec{c}_1 = (1, 1 + \sqrt{2}, 1 - \sqrt{2}) \comma \vec{c}_2 = (1 - \sqrt{2}, 1, 1 + \sqrt{2}) \ comma \vec{c}_3 = (1 + \sqrt{2}, 1 - \sqrt{2}, 1)$$ \hspace{1cm} (3)

We shall shortly indicate the $M$-theoretic origin of the Lagrangian (1) but first we provide the desired solution of the resulting field equations:

$$\Box \vec{\phi} = -\frac{1}{28} \sum_{\alpha} \vec{c}_\alpha e^{-\vec{c}_\alpha \cdot \vec{\phi}} (G^{(\alpha)})^2 \comma$$
$$\partial_{\mu}(e e^{-\vec{c}_\alpha \cdot \vec{\phi}} G^{(\alpha)\mu\nu\rho\sigma}) = 0 \comma$$
$$R_{\mu\nu} = \frac{1}{2} \partial_{\mu} \vec{\phi} \cdot \partial_{\nu} \vec{\phi} + \frac{1}{4} \sum_{\alpha} e^{-\vec{c}_\alpha \cdot \vec{\phi}} ((G^{(\alpha)})^2_{\mu\nu} - \frac{3}{8} (G^{(\alpha)})^2 g_{\mu\nu})$$ \hspace{1cm} (4)

Let us make the following ansatz for the line element

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = (H_1 H_2 H_3)^{-\frac{1}{2}} (-dt^2 + H_1 dx^2 + H_2 dy^2 + H_3 dz^2)$$ \hspace{1cm} (5)

for the scalars

$$\vec{\phi} = -\frac{1}{2} \sum_{\alpha=1}^{3} \vec{c}_\alpha \log H_\alpha$$ \hspace{1cm} (6)

and for the 4-form field strengths

$$G^{(\alpha)} = H_\beta H_\gamma \partial_\alpha H_{-1} dt \wedge dx \wedge dy \wedge dz$$ \hspace{1cm} (7)

where $\alpha \beta$ and $\gamma$ are all different, and take their values from the set 1, 2 and 3. Here the functions have the coordinate dependences $H_1(x), H_2(y), H_3(z)$. Substituting into the field equations resulting from the Lagrangian above, we find that all are satisfied provided the functions $H_\alpha$ are harmonic:

$$\frac{\partial^2 H_1}{\partial x^2} = \frac{\partial^2 H_2}{\partial y^2} = \frac{\partial^2 H_3}{\partial z^2} = 0$$ \hspace{1cm} (8)
These harmonic conditions can be satisfied by taking $H_\alpha$ to have the form

$$H_1(x) = 1 + \sum_{a=1}^{a=N} M_a |x - x_a|$$

and similarly for $H_2(y)$ and $H_3(z)$. From the form of the metric, we see that if $H_2$ and $H_3$ are temporarily taken to be unity, then the function $H_1$ describes a stack of $N$ parallel domain walls lying in the $(y, z)$ plane at the locations $x = x_a$ whose mass per unit area $M_a$ is also equal to the 3-form charge. Similarly, the functions $H_2$ and $H_3$ by themselves can describe stacks of domain walls in the $(x, z)$ and $(x, y)$ planes respectively. When all three functions are of the general form given above, the solution describes the triple intersection of domain walls lying in the three planes orthogonal to $x$, $y$ and $z$ axes, respectively. Actually, the expressions for the $H_\alpha$ are not harmonic everywhere, since they have delta-function singularities at the locations of the domain walls:

$$\frac{\partial^2 H_1}{\partial x^2} = 2 \sum_{a=1}^{a=N} M_a \delta(x - x_a) , \quad \text{etc.}$$

This may be remedied by including in the field equations a source term for each membrane in a way that preserves the supersymmetry.

3 Higher dimensional origins

The Lagrangian we have been considering is obtained from the bosonic sector of eleven-dimensional supergravity which contains a metric $g_{MN}$ and a 3-from potential $A_{MNP}$ where $M = 0, 1, \ldots , 10$. For simplicity we assume that the seven compactified dimensions have the topology of a seven-dimensional torus. The three scalars $\vec{\phi}$ are a subset of the 7 scalars coming from the diagonal components of the internal metric, while the three 4-form field strengths $G^{(4)}_{\mu\nu\rho\sigma}$ appear after dualising a subset of the 0-form field strengths which arise via a generalised Scherk-Schwarz type of dimensional reduction in which tensor potentials coming both from the eleven-dimensional metric $g_{MN}$ and 3-form potential $A_{MNP}$, are allowed to depend linearly on the compactification coordinates. By retracing these steps it is possible to re-express our four-dimensional solution as an eleven-dimensional solution.

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2In eleven-dimensions, duality relates the elementary singular "electric" membrane, which requires a source term in the $D = 11$ supergravity field equations, to the solitonic non-singular "magnetic" fivebrane, which solves the source-free equations. It thus appears that the process of dualisation can eliminate the need for a source! This confusion is presumably a consequence of the inadequacy of $D = 11$ supergravity to capture the full essence of M-theory.
To see the origin of dilaton vectors satisfying the relations (2), we note that the vectors \( \vec{c}_\alpha \) are just the negatives of the dilaton vectors for the 0-form field strengths that we are dualising in order to obtain the 4-forms \( G^{(\alpha)} \). The 0-form field strengths arise from the Scherk-Schwarz reduction of 1-form fields; in other words, from reductions where the axionic scalar potentials for the 1-form field strengths are allowed linear dependences on the compactification coordinates. Details of such Scherk-Schwarz reductions and subsequent dualisations may be found in Ref. [18, 19, 10]. There are two sources of 0-form field strengths, namely those coming from the Scherk-Schwarz reduction of the antisymmetric tensor in \( D = 11 \), and those coming from the Scherk-Schwarz reduction of the Kaluza-Klein vectors coming from the metric in \( D = 11 \). These 0-forms, denoted by \( F^{(ijkl)} \) and \( F^{(ijk)} \) respectively in [10], have associated dilaton vectors \( \vec{a}_{ijkl} \) and \( \vec{b}_{ijk} \), given by

\[
\vec{a}_{ijkl} = f_i + f_j + f_k + f_\ell - g, \quad \vec{b}_{ijk} = -f_i + f_j + f_k, 
\]

(11)

where

\[
g \cdot g = 7, \quad g \cdot f_i = 3, \quad f_i \cdot f_j = 1 + 2\delta_{ij} \]

(12)

in \( D = 4 \). Note that the indices \( i, j, \ldots \) range over the 7 compactification coordinates \( z_i \) here, starting with \( i = 1 \) for the reduction step from \( D = 11 \) to \( D = 10 \). It is now easy to see that one can indeed find three dilaton vectors that satisfy the conditions (2). There are many different combinations that will work, but they are all equivalent, up to index relabellings, to the two choices

\[
\vec{c}_1 = \vec{a}_{1234}, \quad \vec{c}_2 = \vec{a}_{1567}, \quad \vec{c}_3 = \vec{b}_{125}.
\]

(13)

or

\[
\vec{c}_1 = \vec{a}_{1234}, \quad \vec{c}_2 = \vec{b}_{235}, \quad \vec{c}_3 = \vec{b}_{345}.
\]

(14)

Note that for the first choice (13) we must necessarily choose two 0-forms that originate from \( F_4 = dA_3 \) in \( D = 11 \), and one coming from the metric. In particular we must necessarily use terms which originate, in ten-dimensional type IIA language, from both the NS-NS and R-R sectors of the theory. This means that our intersecting membranes are solutions of the type IIA theory or M-theory, but not of the heterotic or type I strings. However, it is also possible, as in choice (14) to choose the dilaton vectors \( \vec{c}_\alpha \) such that the \( D = 4 \) solution originates only from \( D = 10 \) NS-NS sector. As such it could equally well be regarded as a solution of the \( D = 10 \) heterotic or type I string.

The reason why we chose to work with the dual formulation with 4-form field strengths, rather than the original 0-form field strengths coming from the Scherk-Schwarz reduction, is
that the former description is necessary, in the framework of a four-dimensional theory itself, if one wants to have the possibility of multiple membranes in a stack along each coordinate axis. The reason for this is that a 0-form field strength term of the form $-\frac{1}{2} M^2 e^{\vec{a} \cdot \vec{\phi}}$ in the four-dimensional Lagrangian would lead to an equation of motion that required the associated harmonic function $H$ to have slope $\pm M$, and hence we could only have a solution of the form $H = 1 + M |x - x_0|$. In order to have straight-line segments of different slopes, as is required in (9) in order to describe multiple membranes, it is necessary that the slope can take different values in different regions, and so it must arise as an arbitrary constant of integration (related to the the electric charge of $F_4$) rather than as a fixed, given parameter (i.e $M$) in the Lagrangian.

From the higher-dimensional point of view, the restriction of needing to work in the dualised formulation with 4-form field strengths in $D = 4$ is actually an artificial one. The reason for this is that the 0-form field strengths in the four-dimensional theory themselves arise from the generalised Scherk-Schwarz reductions of standard degree 1 or higher field strengths in higher dimensions. In these reductions, unlike in standard Kaluza-Klein reductions, the associated potentials are allowed linear dependences on certain compactifying coordinates. In our example above, for instance, the 0-form field strength $F_0^{(1234)}$ arises from a generalised Scherk-Schwarz reduction step from $D = 8$ to $D = 7$, where the axion $A_0^{(123)}$ is reduced according to $A_0^{(123)}(x, z_4) \rightarrow A_0^{(123)}(x) + M z_4$. This gives the 0-form field strength term $-\frac{1}{2} M^2 e^{\vec{a}_{1234} \cdot \vec{\phi}}$ in $D = 7$, and lower, dimensions. However the constant $M$ is itself an integration constant from the viewpoint of the theory in $D = 8$, and so provided we trace our four-dimensional solution back to $D = 8$ or a higher dimension, the parameter $M$ of the four-dimensional Lagrangian in the 0-form formulation can be understood as nothing but a free integration constant. Thus again, it can take different values in different regions, and so the possibility of having multiply-charged solutions is regained.

In view of the above, it is therefore useful to re-interpret our intersecting membrane solutions back in higher dimensions. This is easily done by retracing the steps of the Kaluza-Klein and Scherk-Schwarz reductions. Taking our first choice of field strengths corresponding to (13), we find that in $D = 10$ the solution becomes

$$ds_{10}^2 = (H_1 H_2)^{-1/4} H_3^{1/8} \left( -dt^2 + H_1 (dx^2 + dz_3^2 + dz_4^2) + H_2 (dy^2 + dz_6^2 + dz_7^2) + H_3 dz_2^2 + H_1 H_3 dz_3^2 + H_2 H_3 dz_5^2 \right),$$

$$e^{\phi_1} = (H_1 H_2)^{-1/2} H_3^{3/4},$$

$$F_3^{(1)} = -\partial_x H_1 dz_2 \wedge dz_3 \wedge dz_4 - \partial_y H_2 dz_5 \wedge dz_6 \wedge dz_7,$$

$$F_2^{(1)} = -\partial_z H_3 dz_2 \wedge dz_5,$$

$$F_4^{(1)} = -\partial_x H_1 dz_2 \wedge dz_3 \wedge dz_4 - \partial_y H_2 dz_5 \wedge dz_6 \wedge dz_7.$$
where \( \phi_1 \), the first component of the fields \( \vec{\phi} \) in \( D = 4 \), is the dilaton of the type IIA theory. The supersymmetry transformation parameter of this solution satisfies the following relations

\[
(1 - \Gamma_{023567}) \epsilon = 0 ,
(1 - \Gamma_{01324}) \epsilon = 0 ,
(1 - \Gamma_{01234567}) \epsilon = 0 ,
\]

where \( \epsilon \) is given in terms of a constant spinor \( \epsilon_0 \):

\[
\epsilon = (H_1H_2)^{-1/16} H_3^{-1/32} \epsilon_0 ,
\]

where \( \Gamma_M \) are the \( D = 10 \) Dirac matrices, and \( \Gamma_{M_1 \cdots M_n} = \Gamma_{[M_1 \cdots \Gamma_{M_n]}}. \) (Note that the explicit numerical indices 0, 1, 2 and 3 refer to the four-dimensional spacetime, while \( \hat{1}, \hat{2}, \) etc. refer to the reduction steps \( i = 1, 2 \) etc. from \( D = 11 \) to \( D = 10, 9 \) etc.) Therefore, the above solution preserves the fraction \( \nu = 1/8 \) of the supersymmetry. The solution (15) can be viewed as a non-standard intersection of two NS-NS 5-branes and one D6-brane. The harmonic functions here depend on the relative transverse coordinates, which differ from the situation in standard intersections, where the harmonic functions depend only on the overall transverse space. The two 5-brane charges are both carried by the NS-NS 3-form field strength \( F_3^{(1)} \), and the D6-brane charge is carried by the R-R 2-form field strength \( F_2^{(1)} \). The solution with vanishing D6-brane charge was also obtained in [20, 21, 22].

Going back one step further, to \( D = 11 \), the solution becomes

\[
ds_{11}^2 = (H_1H_2)^{-1/3} \left( -dt^2 + H_1(dx^2 + dz_3^2 + dz_4^2) + H_2(dy^2 + dz_6^2 + dz_7^2) 
+ H_3dz_2^2 + H_1H_3dz_5^2 + H_2H_3dz_6^2 + H_1H_2H_3^{-1}(dz_1 + \partial_z H_3dz_5dz_7)^2 \right),
\]

\[
F_4 = -\partial_x H_1 dz_1 \wedge dz_2 \wedge dz_3 \wedge dz_4 - \partial_y H_2 dz_1 \wedge dz_5 \wedge dz_6 \wedge dz_7 .
\]

This solution can be viewed as a non-standard intersection of two M5-branes and one NUT (again with the harmonic functions depending on the relative transverse coordinates rather than those of the overall transverse space).

It is also possible to choose the dilaton vectors \( \vec{c}_\alpha \) such that the \( D = 4 \) solution originates from \( D = 10 \) NS-NS sector as with (14). By retracing the steps of the Kaluza-Klein and Scherk-Schwarz reductions we find that in \( D = 8 \) the solution becomes

\[
ds_8^2 = H_3^{-1/6} \left( -dt^2 + H_1 dx^2 + H_2 dy^2 + H_3 dz_2^2 + H_1H_3 dz_4^2 
+ H_2H_3 dz_5^2 + dz_6^2 + dz_7^2 \right),
\]

\[
F_4 = -\partial_x H_1 dx_1 \wedge dx_2 \wedge dx_3 \wedge dx_4 - \partial_y H_2 dx_1 \wedge dx_5 \wedge dx_6 \wedge dx_7 .
\]
\[ \vec{\phi} = -\frac{1}{2} \vec{a}_{123} \log H_1 - \frac{1}{2} \vec{b}_{23} \log H_2 - \frac{1}{2} \vec{b}_3 \log H_3 , \tag{19} \]

\[ F_1^{(123)} = \partial_x H_1 \, dz_4 , \quad F_1^{(23)} = \partial_y H_2 \, dz_5 , \quad F_1^{(3)} = \partial_z H_3 \, dz_4 \wedge dz_5 , \]

where

\[ \vec{a}_{123} = (1, \sqrt{\frac{2}{3}}, 2\sqrt{\frac{2}{3}}) , \quad \vec{b}_{23} = (0, -\sqrt{\frac{4}{3}}, 2\sqrt{\frac{3}{7}}) , \quad \vec{b}_3 = (0, 0, -\sqrt{\frac{2}{3}}) . \tag{20} \]

\( D = 8 \) is the highest dimension where the metric remains diagonal in the oxidation process.

The solution in \( D = 10 \) takes the form

\[ ds_{10}^2 = H_1^{-1/4} \left( -dt^2 + H_1 \, dx^2 + H_2 \, dy^2 + H_3 \, dz^2 + H_1 H_3 \, dz_4^2 \right. \]
\[ + H_2 H_3 \, dz_5^2 + H_1 H_2 H_3^{-1} (dz_3 + z_4 \partial_z H_3 \, dz_5)^2 + dz_6^2 + dz_7^2 \]
\[ \left. + H_1 H_2^{-1} (dz_2 - z_3 \partial_y H_2 \, dz_5)^2 \right) , \]

\[ e^{\phi_1} = H_1^{-1/2} , \tag{21} \]

\[ F_3^{(1)} = \partial_x H_1 \, dz_4 \wedge (dz_2 - z_3 \partial_y H_2 \, dz_5) \wedge (dz_3 + z_4 \partial_z H_3 \, dz_5) . \]

The \( D = 10 \) supersymmetry transformation parameter satisfies the conditions

\[ (1 + \Gamma_{1234}) \epsilon = 0 , \]
\[ (1 + \Gamma_{3456}) \epsilon = 0 , \tag{22} \]
\[ (1 - \Gamma_{2356}) \epsilon = 0 , \]

where

\[ \epsilon = H_1^{-1/16} \epsilon_0 . \tag{23} \]

Thus, the fraction of the supersymmetry preserved by this solution is also \( \nu = 1/8 \). This can regarded as a solution of the type 1, heterotic or type IIA string. In the latter case, the solution can be further oxidised to \( D = 11 \), where it becomes

\[ ds_{11}^2 = H_1^{-1/3} \left( -dt^2 + H_1 \, dx^2 + H_2 \, dy^2 + H_3 \, dz^2 + H_1 H_3 \, dz_4^2 \right. \]
\[ + H_2 H_3 \, dz_5^2 + H_1 H_2 H_3^{-1} (dz_3 + z_4 \partial_z H_3 \, dz_5)^2 + H_1 H_2^{-1} (dz_2 - z_3 \partial_y H_2 \, dz_5)^2 \]
\[ \left. + H_1 H_2 H_3^{-1} (dz_3 + z_4 \partial_z H_3 \, dz_5)^2 + dz_7^2 \right) , \]

\[ F_4 = \partial_x H_1 \, dz_4 \wedge (dz_2 - z_3 \partial_y H_2 \, dz_5) \wedge (dz_3 + z_4 \partial_z H_3 \, dz_5) \wedge dz_1 . \tag{24} \]

In both \( D = 10 \) and \( D = 11 \) dimensions, the solutions can be viewed as non-standard intersections of a 5-brane with two NUTs. In \( D = 10 \), the 5-brane carries the NS-NS charge.
In this section, we have considered two examples of non-standard intersections in $D = 10$ string theory or $D = 11$ M-theory that can give rise to a four-dimensional lattice universe. The first example is intrinsic to M-theory, while the second example can alternatively be embedded in the heterotic string. More solutions can be obtained by invoking the T-duality of the type IIA and type IIB theories. We shall not enumerate such examples here.

4 Conclusions

The worldwide web solution we have presented is admittedly an idealisation, and by itself would not yet satisfy hard-nosed astrophysicists. First, it represents a static universe with an unbroken supersymmetry, whereas in the real world the universe is expanding and supersymmetry (if it exists at all) is a broken symmetry. In fact these two features are intimately related. The reason we were able to find a stable static three-dimensional lattice, rather than one which is collapsing under its own gravity or one whose tendency to collapse is overwhelmed by expansion, is precisely because of the famous “no-static-force” phenomenon \[12\] of supersymmetric vacua which saturate a Bogomol’nyi-Prasad-Sommerfield bound between the mass and the charge. The mutual gravitational attraction due to gravity $g_{\mu\nu}$ and the massless scalar fields $\phi$ is exactly cancelled by a repulsion due the the 3-forms $C^{(\alpha)}_{\mu\nu\rho}$, which act in many ways like a cosmological constant. (In writing this, of course, we are acutely aware that the introduction of a cosmological constant in order to obtain a static rather than expanding universe was, on his own admission, Einstein’s “greatest blunder”.) A more realistic description must therefore await a satisfactory explanation of how M-theory breaks supersymmetry and, unfortunately, this remains M-theory’s biggest unsolved problem. Of course, our solution is but one of many solutions of M-theory. We make no apology for this. The lattice structure is no more a prediction of M-theory than the Friedman-Robertson-Walker cosmology is a prediction of General Relativity.

The intersecting domain wall configuration is one where the regions of high density are concentrated on the faces of the lattice cubes. One might ask whether there are also solutions where the regions of high density are concentrated on the edges of the cube (intersecting strings associated with 3-form field strengths) or on the vertices of the cube (point singularities associated with 2-form field strengths). The experimental data do not sharply distinguish between these possibilities, since they depend on the overdensity $\delta \rho/\rho$ chosen in making the statistical analysis. We have found no such intersecting string solutions and consider their existence unlikely. There are solutions describing any number of isolated points
which may, in particular, be chosen to lie on a cubic lattice. These are $M$-theoretic generalisations of the well-known Papapetrou-Majumber solutions of general relativity. Once again the mutual attraction due to gravity and scalars is exactly cancelled by a repulsion due to the 1-form potentials.

Another reason why astrophysicists might object is that domain walls whose mass per unit area is too great are ruled out experimentally. In the supersymmetric idealisation presented here, the mass per unit area is also a free parameter depending on the vacuum expectation values of the scalar fields which, for simplicity of presentation, we have arbitrarily set equal to zero. Once again, the actual value of these scalar expectation values must await a resolution of the supersymmetry-breaking problem.

Finally, we are aware that with the success of inflationary models of the universe, topological defects have fallen out of favour as the mechanism for galaxy formation [23], although the issue remains controversial [24]. In any event, it is difficult to see how inflation alone could account for the three-dimensional cubic lattice reported by Einasto et al. [1]. To the best of our knowledge, all attempts to fit this lattice structure data with data on the cosmic microwave background have been based on entirely ad hoc assumptions on the initial spectrum of density perturbations. See, for example, [25]. Moreover, the phenomenology of the kinds of defect appearing in M-theory has yet to be scrutinized. Consequently, in spite of the idealised nature of our solution we hope to have shown that M-theory is indeed a rich source of possible explanations for “hitherto unexplained phenomena” and in particular allows for three-dimensional lattice cosmologies.

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