Production of mesoscopic superpositions with ultracold atoms

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We study mesoscopic superpositions of two component Bose-Einstein condensates. Atomic condensates, with long coherence times, are good systems in which to study such quantum phenomenon. We show that the mesoscopic superposition states can be rapidly generated in which the atoms dispersively interact with the photon field in a cavity. We also discuss the production of compass states which are generalized Schrödinger cat states. The physical realization of mesoscopic states is important in studying decoherence and precision measurement.

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I. INTRODUCTION

In 1935, Schrödinger proposed a thought-experiment of a superposition of macroscopically distinct states [1]. Study of macroscopic superpositions is crucial to our understanding the boundary between the quantum and classical world, and the quest for physical realization of macroscopic superposition states is ongoing [2].

In fact, mesoscopic superpositions of photon field have been observed in cavity QED experiment by Brune et al. [3]. They provide a very clear way to create and measure superposition states of photons in a cavity [3]. In this experiment, a two-level atom is prepared as a superposition of the excited and ground states. The microwave photon field is a coherent state |α⟩ of a quantum harmonic oscillator [4], where the mean number of photons is |α|^2. The two-level atom is sent through the cavity and dispersively interacts with the microwave field. In this process, the two opposite phase shifts of photon field depending on two different states of atom are acquired. Consequently, a superposition of photon states is generated.

However, this photon state is a superposition of the two coherent states with only a few quanta [3]. It is still far-reaching to realize robust mesoscopic superpositions because they are extremely fragile. Thus, it is necessary to find a decoherence-free quantum system or system with a low decoherence rate to generate and observe such states. Bose-Einstein condensation of atomic gases [5] is a promising candidate to exhibit this quantum phenomenon. Many fascinating experiments of atomic Bose-Einstein condensates (BEC’s) have been done in which long coherence times have been observed [6, 7]. This may be used to produce a superposition of quantum states so a “big cat” state would be experimentally observed.

In this paper, we will adopt the idea by Brune et al. [3] to generate mesoscopic superpositions of BEC’s with a cavity. Recently, a BEC has been loaded into a magnetic trap using optical tweezers [8] and can also be transported via a magnetic waveguide [9]. It paves a way to transfer a BEC into a cavity. For a two-component condensate, its ground state and low lying excitations behave in the same way of a quantum harmonic oscillator [10]. More than that, this “mesoscopic quantum harmonic oscillator” being made by BEC, is more robust and long-lived than that of the cavity photon field. Indeed, the coherence times of the atomic condensates have been measured in the Ramsey spectroscopy which exceeds 1 s [6]. It is therefore very encouraging that quantum superpositions of this mesoscopic quantum object will be observed.

We consider a two-component condensate sent through a high-Q microwave cavity. The atoms dispersively interact with the photon field in which no absorption or emission of photons occurs in the dynamics. In this paper, we will show that a superposition of the condensates states with two different relative phases [11] are generated during the dispersive interaction between the atoms and the photon field. The detection of these superposition states is also discussed.

We will also investigate the creation of compass states of ultracold atoms with a cavity. Compass states have been proposed by Zurek [12] which are “generalized” Schrodinger cat states with a superposition of more than two distinguishable coherent states. These states can be used to achieve high precision measurement, such as the detection of small displacements or rotations in phase space [13].
II. SYSTEM

\[ H = \Delta J_z - g(a^\dagger J_- + J_+a), \]

where \( a, J_z \) and \( J_\pm \) are the annihilation operator of the microwave cavity field, atomic inversion and atomic transition operators for the condensates respectively. The parameters \( \Delta = \Delta_1 + \Delta_2 - \Omega^2/\Delta_1 \) and \( g = \Omega_\nu \Omega_{\text{rf}}/\Delta_1 \) are the two-photon detuning and the effective two-photon Rabi frequency respectively.

The BEC has been confined in a very tight trap with the transverse size being less than 500 \( \mu \text{m} \) [2]. The wavelength of a microwave field is much longer than the size of the condensates. Therefore, we have considered that all atoms interact with the cavity field in the same form. We also assume that atom-atom interactions are very small compared to the strength of atom-light coupling so that we can neglect the nonlinear terms \( \kappa J^2 \) coming from the self-interactions between the atoms [15].

A. Effective Jaynes-Cummings (JC) Model

In the low-lying excitations regime, we are able to make the approximation based on the Holstein-Primakoff transformation [18]. This allows us to approximate the angular momentum operators into harmonic oscillators:

\[ J_+ \approx \sqrt{N} b^\dagger, \quad J_- \approx \sqrt{N} b \quad \text{and} \quad J_z = b^\dagger b - N/2 \]

as long as the excitations are small, i.e., \( \langle b^\dagger b \rangle /N \ll 1 \) [10]. This means an atomic coherent state with “small” excitations can be approximated to a coherent state of a harmonic oscillator which can be expressed in the form as [4]:

\[ |\alpha \rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{(n!)^{1/2}} |n\rangle, \]

where \( n \) is a quantum number of excitations in the condensates. The degree of excitation of the atomic coherent state can be adjusted by a two-photon Rabi pulse [11].

The condensates can well be approximate as a superposition of the single photon and vacuum states. We initially prepare the photon state as:

\[ |\psi \rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}. \]

This state has been experimentally observed in a microwave cavity [19]. It can be achieved by sending a two-level atom, with an equally superposition of states \( |r \rangle \) and \( |g \rangle \), through a cavity in which the vacuum photon field state \( |0\rangle \) is prepared [19]. After a certain time, the photon field state will become the form in Eq. (3).

If we constrain the photon state in the single excitation and the vacuum state only such that we are able to represent the bosonic oscillators in terms of Pauli operators [20]: \( \sigma^z = a^\dagger a, \sigma^y = i(a^\dagger - a) \) and \( \sigma^x = 1 - 2a^\dagger a \).

After the above transformations, we can readily obtain an effective Hamiltonian as:

\[ H_{JC} = \Delta b^\dagger b + g\sqrt{N}(b^\dagger \sigma_- + \sigma_+ b). \]
It is noteworthy that the Rabi-coupling strength is vastly enhanced by a factor of $\sqrt{N}$. This effective JC Rabi-coupling strength $g\sqrt{N}$ can be increased with 100 times for $N \approx 10^4$. We estimate this effective Rabi frequency can be as high as 100 kHz if $g \approx 1$ kHz. The strong atom-photon coupling strength is crucial to increase the efficiency of the cat states production.

This JC model can be exactly diagonalized which allows us to solve the problem analytically. If the detuning $\Delta$ is large enough with $Ag^2Nn/\Delta^2 \ll 1$, the eigenvalues can be expressed in this approximate form as $[3]$:

(i) $|+\rangle \approx |1, n-1\rangle$ and (ii) $|-\rangle \approx |0, n\rangle$, and the corresponding eigenvalues are (i) $E_+ \approx (n-1)\Delta - g^2Nn/\Delta$ and (ii) $E_- \approx n\Delta + g^2Nn/\Delta$.

III. GENERATION OF SCHRODINGER CAT STATE

Now we propose a scheme to generate the cat state with only a few steps, where the atoms are dispersively coupled to microwave field. We consider the initial state as a product state of the photon and the atoms, i.e.,

$$|\Psi(0)\rangle = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) \otimes |\alpha\rangle. \tag{5}$$

Then, the condensate is transported through the cavity, and the interaction between the atoms and microwave field is switched on for the duration time $t^*$. The state will become $[3]$

$$|\Psi(t^*)\rangle = \frac{1}{\sqrt{2}}(|1, \alpha e^{i\phi}\rangle + |0, \alpha e^{-i\phi}\rangle), \tag{6}$$

for $\alpha = \alpha e^{-i\Delta t^*}$ and $\phi = g^2Nt^*/\Delta$. The smooth transport of atoms through a magnetic waveguide has also been demonstrated which shows negligible atoms losses during the transfer $[3]$.

We can make the unitary transformation of the photon state $U_{\pi/2}$ with a two-level atom as follows $[19]$:

$$|1\rangle \rightarrow (|1\rangle + |0\rangle)/\sqrt{2} \quad \text{and} \quad |0\rangle \rightarrow (|0\rangle - |1\rangle)/\sqrt{2}. \tag{7}$$

This can be achieved by interacting the photon field with a two-level atom appropriately $[19]$. The state is then written as

$$|\Psi\rangle = |1\rangle|\tilde{\Psi}_1\rangle + |0\rangle|\tilde{\Psi}_2\rangle, \tag{8}$$

where

$$|\tilde{\Psi}_{1,2}\rangle = \left(\frac{|\alpha e^{i\phi}\rangle \mp |\alpha e^{-i\phi}\rangle}{\sqrt{2}}\right). \tag{9}$$

A Schrödinger cat state of condensates is readily obtained by either measuring the photon state $|1\rangle$ or $|0\rangle$. The two possible superposition states are

$$|\tilde{\Psi}_{1,2}\rangle = \frac{|\alpha e^{i\phi}\rangle \mp |\alpha e^{-i\phi}\rangle}{N_{1,2}}, \tag{10}$$

where $N_{1,2} = 2 + 2e^{-|\alpha|^2}(1 - \cos 2\phi)\cos(|\alpha|^2\sin 2\phi)$. Physically speaking, this cat state is a superposition of the states with two different relative phases of the two-component condensates. The lifetime $T$ of the cat state is roughly equal to $2T_c/D^2 = D \gg 1 [3]$, where $T_c$ is the characteristic damping time of the condensates and $D = 2|\alpha|$ is “distance” between two coherent states $[3]$. The characteristic damping time of the BEC is about the coherence lifetime of the condensates $T_c$. Obviously, the cat state lifetime is significantly shortened if the magnitude of $|\alpha|^2$ is very large. But the coherence time of the atomic condensates ($T_c \sim 1 \text{ to } 2 \text{ s}$) is relatively long $[7]$, we are able to estimate the cat state’s lifetime $T$ is about 1 to 10 ms if the number of excitations is up to several hundred.

Next we discuss how to detect such superposition states in condensates. We note that the photon and the condensates are entangled in Eq. (5), therefore the photon state can be used to probe the state of atomic condensates. This enables us to measure the superposition states through detecting the photon state. We consider the cat state in Eq. (6) with $\phi = \pi/2$. Let the system undergo a free evolution $(\hat{H}_0 = \Delta \hat{b}^\dagger \hat{b})$ for a small amount of time $\delta t$, where $\Delta' = \Delta_1 + \omega_{cr}$. We switch on the JC interaction again for a duration $t' = \pi\Delta'/2g^2N$ to undo the cat state. After performing $U_{\pi/2}$ on the photon state, up to a global phase, the quantum state will become $[13]$

$$|\Psi_f\rangle = \frac{1}{2}[(e^{2i\Delta'}|\alpha|^2\delta t - 1)|1, \alpha'\rangle + (e^{2i\Delta'}|\alpha|^2\delta t + 1)|0, \alpha'\rangle]. \tag{10}$$

for $\alpha' = \alpha e^{-i\Delta t'}$. The frequency is amplified by a factor $|\alpha|^2$ as the probability of single photon state is $P_1 = [1 - \cos(2|\alpha'|^2\Delta'\delta t)]/2 [13]$. The probability of photon state can be measured by means of a two-level atom $[19]$. Indeed, this method has been proposed to perform Heisenberg limited precision measurement $[13]$ to detect the small rotation of the state in the phase space.

IV. COMPASS STATE PRODUCTION

Having discussed how to generate Schrödinger cat states, we proceed to study another kind of superposition states which is called the compass state $[12, 13]$. It is a generalization to the usual Schrödinger cat state which is a superposition of two coherent states. These compass states play an important role in the study of sub-Planck structure $[12]$.

We follow the similar method in generating cat states to make a compass state. We first create the state in Eq. (6) with $\phi = \pi$, and prepare the photon state in the cavity in the form of Eq. (6). Then, we switch on the effective JC interaction with a duration $t'' = \pi\Delta'/4g^2N$. The frequency shifts of the coherent state are yielded as $|\tilde{\alpha}|^2 e^{\pm \pi/4}$ and $|\tilde{\alpha}^* e^{\pm 3\pi/4}$, for $\alpha = \alpha e^{-i\Delta t''}$. The unitary transformation of the photon state $U_{\pi/2}$ is then performed. After measuring the photon state $|0\rangle$ or $|1\rangle$...
of the cavity, the state finally becomes

$$|\tilde{\Psi}'_{1,2}\rangle \approx \frac{|\tilde{\alpha}^*e^{i\pi/4}\rangle \mp |\tilde{\alpha}^*e^{-i\pi/4}\rangle + |\tilde{\alpha}^*e^{3i\pi/4}\rangle \mp |\tilde{\alpha}^*e^{-3i\pi/4}\rangle}{2}. \quad (11)$$

The states $|\tilde{\Psi}'_1\rangle$ and $|\tilde{\Psi}'_2\rangle$ correspond to the states after the measurement of two photon states $|0\rangle$ and $|1\rangle$ respectively. The state $|\tilde{\Psi}'_{1,2}\rangle$ is a superposition of four different coherent states. The method here can be generalized to produce a superposition of more than four distinguishable coherent states. The successful realization of compass states would allow us to deepen the understanding of the decoherence problems [12].

V. DISCUSSION

In this paper, we have proposed a method to generate and observe a long-lived mesoscopic superpositions of two-component BEC’s with a high-Q microwave cavity. The cat states can be quickly generated in around 100 $\mu$s if $g$ can attain 1 kHz or higher. We require the photon lifetime is much longer than the time scale of the cat states production.

In addition, we point out that our scheme to generate cat states with a cavity is much more efficient than other schemes using Bose-Einstein condensates [21]. In those schemes, using two-component condensates, the generated time is proportional to the self-interaction strength which is very weak [17]. This requires a much longer production time and thus it is very unfavorable to generate cat states.

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