Chern-Simons functional and the no-boundary proposal in
Bianchi IX quantum cosmology

Jorma Louko

Department of Physics, University of Wisconsin–Milwaukee,
P.O. Box 413, Milwaukee, Wisconsin 53201, USA
(Revised version, October 1994)

Abstract

The Chern-Simons functional $S_{CS}$ is an exact solution to the Ashtekar-
Hamilton-Jacobi equation of general relativity with a nonzero cosmological
constant. In this paper we consider $S_{CS}$ in Bianchi type IX cosmology with
$S^3$ spatial surfaces. We show that among the classical solutions generated
by $S_{CS}$, there is a two-parameter family of Euclidean spacetimes that have a
regular NUT-type closing. When two of the three scale factors are equal, these
spacetimes reduce to a one-parameter family within the Euclidean Taub-NUT-
de Sitter metrics. For a nonzero cosmological constant, $\exp(iS_{CS})$ therefore
provides a semiclassical estimate to the Bianchi IX no-boundary wave function
in Ashtekar’s variables.

Pacs: 04.60.Kz, 04.60.Ds, 04.60.Gw
I. INTRODUCTION

Kodama [1] has pointed out that the Chern-Simons functional $S_{CS}$ provides an exact solution to the Ashtekar-Hamilton-Jacobi equation of general relativity with a nonzero cosmological constant. One therefore anticipates that when the theory is canonically quantized in the connection representation, the quantum constraint equations should possess a solution of the form $\exp(iS_{CS})$, either exactly or in some approximate semiclassical sense [1–3].

Investigating this anticipation within full general relativity faces the difficult issues of regularization [2,3], and even in minisuperspace models there emerge issues of factor ordering [1,4]. Further, even if a state of the form $\exp(iS_{CS})$ is shown to satisfy the quantum constraints in the sense of (functional) differential equations, one may ask whether this state is normalizable with respect to any inner product that is proposed to define the physical Hilbert space of the theory [4]. Nevertheless, leaving for now such questions aside, one is led to identify a state of the form $\exp(iS_{CS})$ as corresponding, in a semiclassical sense, to the family of classical solutions to the Einstein equations that are generated by regarding $S_{CS}$ as a particular solution to the Ashtekar-Hamilton-Jacobi equation. It is therefore of interest to understand what the properties of this family of classical spacetimes are. For earlier work relating to $S_{CS}$, see Refs. [1–6]. For a supersymmetric generalization, see Ref. [7].

The purpose of the present paper is to investigate $S_{CS}$ in the Bianchi type IX spatially homogeneous cosmological model [8,9] with $S^3$ spatial surfaces. We shall show that among the classical solutions generated by $S_{CS}$, there is a two-parameter family of Euclidean spacetimes that have a regular closing of the NUT-type [10,11]. This implies that, in this model, a wave function of the semiclassical form $\exp(iS_{CS})$ can be regarded as compatible with the no-boundary proposal of Hartle and Hawking [12–14]. We shall also note in passing that when two of the three scale factors are equal, the Euclidean spacetimes corresponding to $S_{CS}$ reduce to a one-parameter family within the Taub-NUT-de Sitter metrics [15–17] and can be given in closed form.

We now proceed to prove these assertions. Except when otherwise stated, the notation follows that of Ref. [1].

II. THE MODEL

The (Lorentzian) spacetime metric is

$$ds^2 = \frac{1}{32\pi^2} \left[ -\sigma_1^2 \sigma_2 \sigma_3 \tilde{n}^2 dt^2 + \sigma_1^{-1} \sigma_2 \sigma_3 (\chi^1)^2 + \sigma_2^{-1} \sigma_3 \sigma_1 (\chi^2)^2 + \sigma_3^{-1} \sigma_1 \sigma_2 (\chi^3)^2 \right], \quad (1)$$

where $\chi^I$ are the left invariant one-forms on $\text{SU}(2) \simeq S^3$ satisfying $d\chi^I = \frac{1}{2} \epsilon_{IJK} \chi^J \wedge \chi^K$. The rescaled lapse $\tilde{n}$ and the components $\sigma_I$ of the inverse densitized triad are functions of $t$ only. The Ashtekar action [3] takes the form [1]

$$S = \int dt \left( -\sigma_I \dot{A}_I - \tilde{n} h \right), \quad (2)$$

where a sum over the repeated index is understood. The Hamiltonian constraint $h(\sigma_I, A_I)$ is given by
\[ h = -\sigma_1 \sigma_2 (A_1 A_2 \mp i A_3) - \sigma_2 \sigma_3 (A_2 A_3 \mp i A_1) - \sigma_3 \sigma_1 (A_3 A_1 \mp i A_2) + 3\lambda \sigma_1 \sigma_2 \sigma_3 . \]   

(3)

Here \( \lambda = \Lambda/(96\pi^2) \), where \( \Lambda \) is the cosmological constant. We shall assume throughout \( \lambda \neq 0 \). The upper and lower signs correspond to the two possible signs of \( i \) in the definition of the Ashtekar connection; we refer to Ref. [1] for the details. Note that \( A_I \) differs by an overall factor of \( i \) from the conventions adopted in Ref. [6].

From Eq. (2), the fundamental Poisson brackets are \( \{ A_I, \sigma_J \} = -\delta_{IJ} \). Thus, if \( A_I \) is regarded as a coordinate, its conjugate momentum is \(-\sigma_I\). A naïve counting indicates that the general solution to the equations of motion contains four constants of integration. It appears unknown whether the global structure of the (Lorentzian) solution space is consistent with the existence of such four constants [18]; we shall, however, not attempt to address these global issues at the level of the present paper.

Dirac quantization [19] with \( A_I \) as the configuration variable leads to the Wheeler-DeWitt-type equation

\[ \hat{h}(i(\partial/\partial A_I), A_I) \psi = 0 \]  

(4)

for the wave function \( \psi(A_I) \). We shall not attempt to discuss the factor ordering in this equation, nor the choice of an inner product [6].

Given a solution to the Wheeler-DeWitt equation (4) with the (approximate) form \( \exp(iS) \), a semiclassical expansion [20] shows that \( S \) (approximately) solves the Hamilton-Jacobi equation

\[ h \left( -(\partial S/\partial A_I), A_I \right) = 0 . \]  

(5)

The wave function is therefore, through semiclassical correspondence, associated with the spacetimes obtained by solving the equations

\[ \sigma_I = -\frac{\partial S}{\partial A_I} , \]  

(6a)

\[ \dot{A}_I = \eta \{ A_I, h \} = -\eta \frac{\partial h}{\partial \sigma_I} . \]  

(6b)

By standard Hamilton-Jacobi theory [21–23], the solutions to (6) for the given \( S \) are a two-parameter family of solutions to the classical equations of motion.

The Chern-Simons functional \( S_{CS} \) takes the form [1]

\[ S_{CS} = -\frac{1}{\lambda} \left[ A_1 A_2 A_3 \mp i \left( \frac{1}{2} \left( A_1^2 + A_2^2 + A_3^2 \right) \right) \right] . \]  

(7)

It is readily seen that \( S_{CS} \) solves the Hamilton-Jacobi equation (3). We are now interested in the corresponding classical solutions.

Inserting \( S_{CS} \) (7) into (3), one obtains after some rearrangement the equations

\[ 1 \text{We have chosen the overall sign in Eq. (7) to differ from that adopted in Ref. [1]. This reflects our using wave functions of the form } \exp(iS) \text{ rather than } \exp(-iS). \]
\[ \begin{align*}
\lambda \sigma_1 &= A_2 A_3 \mp i A_1 , \\
\lambda \sigma_2 &= A_3 A_1 \mp i A_2 , \\
\lambda \sigma_3 &= A_1 A_2 \mp i A_3 ,
\end{align*} \tag{8a} \]

and
\[ \begin{align*}
\dot{A}_1 &= -\lambda \eta \sigma_2 \sigma_3 , \\
\dot{A}_2 &= -\lambda \eta \sigma_3 \sigma_1 , \\
\dot{A}_3 &= -\lambda \eta \sigma_1 \sigma_2 .
\end{align*} \tag{8b} \]

The solutions to Eqs. (8) are (anti-)self-dual metrics, and can therefore not generically be Lorentzian \[\text{[1,5]}\]. We shall seek Euclidean solutions.

We take \( \sigma_I \) to be real. Requiring the spatial metric in Eq. (1) to be positive definite implies that either \( \sigma_I \) are all positive, or one of them is positive and two are negative. As Eqs. (8) are invariant under a simultaneous sign change in any two of the \( \sigma_I \) and the corresponding two \( A_I \), we can without loss of generality take all \( \sigma_I \) positive. We now set \( \eta = \pm i(\sigma_1 \sigma_2 \sigma_3)^{-1/2} \), which makes \( t \) proportional to the Euclidean proper time. We also rewrite \( \sigma_I \) in terms of positive-valued scale factors \( a, b, \) and \( c \), defined by
\[ \begin{align*}
\sigma_1^{-1} \sigma_2 \sigma_3 &= \frac{1}{4} a^2 , \\
\sigma_2^{-1} \sigma_3 \sigma_1 &= \frac{1}{4} b^2 , \\
\sigma_3^{-1} \sigma_1 \sigma_2 &= \frac{1}{4} c^2 .
\end{align*} \tag{9} \]

Note that at the isotropic limit \( a = b = c \), the constant \( t \) surfaces in the metric (1) are round three-spheres with sectional curvature \( 32 \pi^2 / (a^2) \). Writing finally \( A_I = \pm i B_I \), equations (8) take the form
\[ \begin{align*}
\frac{1}{4} \lambda bc &= B_1 - B_2 B_3 , \\
\frac{1}{4} \lambda ca &= B_2 - B_3 B_1 , \\
\frac{1}{4} \lambda ab &= B_3 - B_1 B_2 ,
\end{align*} \tag{10a} \]

and
\[ \begin{align*}
\dot{B}_1 &= -\frac{1}{2} \lambda a , \\
\dot{B}_2 &= -\frac{1}{2} \lambda b , \\
\dot{B}_3 &= -\frac{1}{2} \lambda c .
\end{align*} \tag{10b} \]

At the isotropic limit, \( a = b = c \), Eqs. (10) imply \( B_1 = B_2 = B_3 \), and the solution is easily found \[\text{[1]}\]: the metric is just that of Euclidean space of constant sectional curvature \( \Lambda/3 \), which can be understood as a Euclidean section of (anti-)de Sitter space for \( \Lambda > 0 \) \( (\Lambda < 0) \). We shall now consider solutions that are near the isotropic limit. For this purpose we introduce the Misner-type parameters \[\text{[24]}\]
\[ \begin{align*}
a &= \exp \left( \alpha + \beta^+ + \sqrt{3} \beta^- \right) , \\
b &= \exp \left( \alpha + \beta^+ - \sqrt{3} \beta^- \right) , \\
c &= \exp \left( \alpha - 2 \beta^+ \right) .
\end{align*} \tag{11a} \]
and
\[ B_1 = \frac{1}{2} F \exp \left( g^+ + \sqrt{3}g^- \right), \]
\[ B_2 = \frac{1}{2} F \exp \left( g^+ - \sqrt{3}g^- \right), \]
\[ B_3 = \frac{1}{2} F \exp (-2g^+). \] (11b)

and expand Eqs. (10) to linear order in the anisotropy variables \( \beta^\pm \) and \( g^\pm \). One obtains
\[ \dot{F} = -\lambda e^\alpha, \] (12a)
\[ \lambda e^{2\alpha} = F(2 - F), \] (12b)
\[ \frac{d}{dt} (Fg^\pm) = -\lambda e^\alpha \beta^\pm, \] (12c)
\[ \lambda e^{2\alpha} \beta^\pm = -F(2 + F)g^\pm. \] (12d)

For \( \lambda > 0 \), the solution to Eqs. (12) is
\[ e^\alpha = \frac{1}{\sqrt{\lambda}} \sin(\sqrt{\lambda}t), \] (13a)
\[ F = 1 + \cos(\sqrt{\lambda}t), \] (13b)
\[ \beta^\pm = \beta_0^\pm \left( 2 + \tan^2\left( \frac{\sqrt{3}}{2} \right) \right) \tan^2\left( \frac{\sqrt{3}}{2} \right) \] (13c)
\[ g^\pm = -\beta_0^\pm \tan^4\left( \frac{\sqrt{3}}{2} \right). \] (13d)

where \( \beta_0^\pm \) are constants and the range of \( t \) is \( 0 < t < \pi/\sqrt{\lambda} \). For \( \lambda < 0 \), there are two solutions. One is given by the formulas (13), understood in the sense of analytic continuation in \( \lambda \); the range of \( t \) is \( 0 < t < \infty \). The other solution is
\[ e^\alpha = \frac{-1}{\sqrt{-\lambda}} \sin(\sqrt{-\lambda}t), \] (14a)
\[ F = 1 - \cosh(\sqrt{-\lambda}t), \] (14b)
\[ \beta^\pm = \gamma_0^\pm \left( \coth^2\left( \frac{\sqrt{3}}{2} \right) - 2 \right) \coth^2\left( \frac{\sqrt{3}}{2} \right), \] (14c)
\[ g^\pm = -\gamma_0^\pm \coth^4\left( \frac{\sqrt{3}}{2} \right). \] (14d)

where \( \gamma_0^\pm \) are constants and the range of \( t \) is \( -\infty < t < 0 \).

When \( \beta_0^+ = \beta_0^- = 0 \), the linearized solution (13) reduces for both signs of \( \lambda \) to the exact isotropic solution mentioned above. When \( \beta_0^\pm \) are not both equal to zero, the linearized solution (13) grows out of the domain of validity of the linearized equations at large values of \( \sqrt{\lambda}t \), and the same is true for \( \lambda < 0 \) when \( |\beta_0^\pm| \) are not much smaller than unity. However, for any \( \lambda \neq 0 \) and any values of \( \beta_0^\pm \), the linearized solution (13) is in the domain of validity of the linearized equations for sufficiently small \( t \), and further it becomes asymptotically accurate as \( t \to 0 \). We infer that there exists a two-parameter family of solutions to the exact equations (10), such that these exact solutions are well approximated by (13) at the limit \( t \to 0 \).

When \( \gamma_0^+ = \gamma_0^- = 0 \), the second linearized solution (14) reduces to the exact isotropic solution mentioned above, with the coordinate time \( t \) now running in the opposite direction.
compared with the $\beta_0^\pm = 0$ limit of (13). When $\gamma_0^\pm$ are not both equal to zero, the linearized solution (14) is within the domain of validity of the linearized equations only when $|\gamma_0^\pm| \ll 1$ and $|\sqrt{-\lambda t}| \gg 1$. This linearized solution will not be important for our conclusions.

Recall now that we began by defining the metrics, both Lorentzian and Euclidean, in terms of the 3+1 split expression (1). It can now be verified that inserting the linearized solution (13) into (1) gives a Euclidean metric that can be regularly extended to $t=0$ by adding just one point to the manifold: one can view the new point as the coordinate singularity at the origin of a hyperspherical coordinate system in which $t$ is the radial coordinate. The crucial fact that makes this extension possible is the $O(t^2)$ suppression of the anisotropy (13c) as $t \to 0$. Since $t \to 0$ is the limit where the linearized solution is accurate, we see that the corresponding exact metrics can be similarly extended to $t=0$. In the terminology of Refs. [10,11], the closing of the geometry at $t=0$ is of the NUT-type.

This regular closing of the geometry is precisely the property characterizing the classical solutions that are relevant for the no-boundary proposal of Hartle and Hawking [12–14], in the sense that wave functions satisfying the no-boundary proposal are expected to get their dominant semiclassical contribution from one or more such regular classical solutions [23,25,26]. We therefore conclude that in our model, a wave function of the semiclassical form $\exp(iS_{CS})$ is compatible with a semiclassical estimate to the no-boundary wave function.

## III. DISCUSSION

We have investigated the Chern-Simons functional $S_{CS}$ as a particular solution to the Ashtekar-Hamilton-Jabobi equation in the Bianchi type IX cosmological model with $S^3$ spatial surfaces and a nonzero cosmological constant. We showed that among the classical solutions generated by $S_{CS}$, there is a two-parameter family of Euclidean spacetimes that have a regular NUT-type closing. Hence, in this model, a wave function of the semiclassical form $\exp(iS_{CS})$ in the connection representation of Ashtekar’s variables is compatible with a semiclassical estimate to the no-boundary wave function of Hartle and Hawking. Several comments are now in order.

(1). Recall that we chose the spatial surfaces to be $\text{SU}(2) \simeq S^3$. Another possible choice compatible with the Bianchi IX homogeneity type is $\text{SU}(2)/\mathbb{Z}_2 \simeq \text{SO}(3) \simeq \mathbb{R}P^3$. In this case the Euclidean solutions corresponding to the $t \to 0$ limit of (13) are not regularly extendible. However, they can be extended into spaces that have an orbifold-type singularity [27]. It has been suggested that the no-boundary proposal could be meaningfully broadened to include such geometries [23].

(2). At the limit $\lambda \to 0$, $S_{CS}$ (1) diverges. This is in agreement with the obstacles discussed in Ref. [29] to obtaining a semiclassical estimate to the no-boundary wave function in Ashtekar’s variables for a vanishing cosmological constant. The crux of the problem is that without a cosmological constant, the Hamiltonian constraint (3) reduces to a pure kinetic term.

(3). At the limit $\lambda \to 0$, the linearized solutions (13) and (14) yield small anisotropy approximations to self-dual vacuum solutions, provided that before taking the limit the constants $\beta_0^\pm$ are made proportional to $\lambda^{-1}$ and the constants $\gamma_0^\pm$ proportional to $\lambda^2$. In the terminology of Ref. [11], (13) yields an approximation to metrics with $\lambda_1 = \lambda_2 = \lambda_3 = 1$, 6
and (14) yields an approximation to metrics with \( \lambda_1 = \lambda_2 = \lambda_3 = 0 \).

(4). In our minisuperspace model, the gauge freedom pertaining to the Gauss and diffeomorphism constraints [3] was fixed to begin with, and one can view the expression (7) in essence as the definition of the Chern-Simons functional. In more general contexts one may however raise questions regarding the behavior of the Chern-Simons functional under large gauge transformations. For these issues, see Ref. [6].

(5). In the special case where two of the three scale factors are equal, the classical Euclidean solutions corresponding to \( S_{CS} \) can be found in closed form. To see this, we return to Eqs. (8), taking again \( \sigma_I \) to be positive. We now set \( \sigma_1 = \sigma_2 \) and \( \sigma_i = \pm i \sigma_1^{-2} \), and write again \( A_I = \pm i B_I \). The equations then imply \( B_1 = B_2 \) and reduce to

\[
\begin{align*}
\lambda \sigma_1 &= B_1 (1 - B_3) , \\
\lambda \sigma_3 &= B_3 - B_1^2 , \\
\dot{B}_1 &= -\lambda \sigma_3 \sigma_1^{-1} , \\
\dot{B}_3 &= -\lambda .
\end{align*}
\] (15a)

and

\[
\begin{align*}
\dot{B}_1 &= -\lambda \sigma_3 \sigma_1^{-1} , \\
\dot{B}_3 &= -\lambda .
\end{align*}
\] (15b)

The solution is straightforward to find. The connection is given by

\[
\begin{align*}
B_1^2 &= 1 - 2 \lambda t + k t^2 , \\
B_3 &= 1 - \lambda t ,
\end{align*}
\] (16)

where \( k \) is an integration constant and the additive constant in \( t \) has been chosen in a convenient way. \( \sigma_I \) are then obtained from (15a). Writing finally \( t = 32 \pi^2 \rho \) and \( k = -f \lambda/(32 \pi^2) \), the metric (1) takes the form

\[
\begin{align*}
ds^2 &= \frac{(1 + f \rho) d\rho^2}{\rho [1 - (2 \Lambda/3) \rho - (f \Lambda/3) \rho^2]} + \frac{\rho [1 - (2 \Lambda/3) \rho - (f \Lambda/3) \rho^2]}{(1 + f \rho)} (\chi^3)^2 \\
&\quad + \rho (1 + f \rho) \left( (\chi^1)^2 + (\chi^2)^2 \right),
\end{align*}
\] (17)

which is recognized as a one-parameter family within the two-parameter set of Euclidean Taub-NUT-de Sitter metrics [15–17]. The range of \( \rho \) in which (17) gives a positive definite metric depends on \( \Lambda \) and \( f \) but includes always an open (possibly semi-infinite) interval whose lower end is at \( \rho = 0 \), where the geometry has a regular NUT-type closing [10,11,17]: this corresponds to the linearized solution (13). A metric corresponding to the second linearized solution (14) is recovered from (17) with \( \Lambda < 0 \) and \( f \approx -\Lambda/3 \) at large negative values of \( \rho \). The isotropic solution is obtained with \( f = -\Lambda/3 \), for either sign of \( \Lambda \), and the anisotropic solution found in Ref. [1] is obtained with \( f = \Lambda/3 \).

(6). In the Taub truncation of the Bianchi IX model, in which two of the three scale factors are set equal to begin with, the Hamilton-Jacobi function generating the one-parameter family of solutions (17) in the conventional metric variables can be given in terms of elementary functions [17]. Its form (for a nonvanishing \( \Lambda \)) is considerably more cumbersome than that of the corresponding Chern-Simons functional obtained from (7).

(7). Finally, one must ask whether the connection between the Chern-Simons functional and the no-boundary proposal in Bianchi type IX might reflect more general phenomena. We leave this question a subject to future work.
ACKNOWLEDGMENTS

I would like to thank Abhay Ashtekar, Akio Hosoya, and Hideo Kodama for discussions, and Chris Isham and Gary Gibbons for their hospitality at the London Mathematical Society meeting “Quantum Concepts in Space and Time” (Durham, UK, July 1994), where this work was initiated. This work was supported in part by the NSF grant PHY91-05935.
REFERENCES

* Electronic address: louko@alpha1.csd.uwm.edu.

[1] H. Kodama, Phys. Rev. D 42, 2548 (1990).
[2] B. Brügmann, R. Gambini and J. Pullin, Nucl. Phys. B385, 587 (1992); Gen. Rel. Grav. 25, 1 (1993).
[3] L. N. Chang and C. Soo, Phys. Rev. D 46, 4257 (1992).
[4] G. A. Mena Marugán, “Is the exponential of the Chern-Simons action a normalizable physical state?”, Penn State Report CGPG-94/2-2, gr-qc/9402034.
[5] J. Samuel, Class. Quantum Grav. 5 L123 (1988).
[6] A. Ashtekar, Lectures on Non-Perturbative Canonical Gravity (World Scientific, Singapore, 1991).
[7] T. Sano and J. Shiraishi, Nucl. Phys. B410, 423 (1992).
[8] M. P. Ryan and L. C. Shepley, Homogeneous Relativistic Cosmologies (Princeton University Press, Princeton, 1975).
[9] R. T. Jantzen, Commun. Math. Phys. 64, 211 (1979); in Cosmology of the Early Universe, edited by L. Z. Fang and R. Ruffini (World Scientific, Singapore, 1984).
[10] G. W. Gibbons and C. N. Pope, Commun. Math. Phys. 66, 267 (1979).
[11] G. W. Gibbons and S. W. Hawking, Commun. Math. Phys. 66, 291 (1979).
[12] S. W. Hawking, in Astrophysical Cosmology: Proceedings of the Study Week on Cosmology and Fundamental Physics, edited by H. A. Brück, G. V. Coyne and M. S. Longair (Pontificiae Academiae Scientiarum Scripta Varia, Vatican City, 1982).
[13] J. B. Hartle and S. W. Hawking, Phys. Rev. D 28, 2960 (1983).
[14] S. W. Hawking, Nucl. Phys. B239, 257 (1984).
[15] B. Carter, Commun. Math. Phys. 10, 120 (1968).
[16] D. Kramer, H. Stephani, E. Herlt, and M. MacCallum, Exact Solutions of Einstein’s Field Equations, edited by E. Schmutzer (Cambridge University Press, Cambridge, 1980), Section 11.3.1.
[17] L. G. Jensen, J. Louko, and P. J. Ruback, Nucl. Phys. B351, 662 (1991).
[18] B. Grubisic and V. Moncrief, Phys. Rev. D 49, 2792 (1994).
[19] P. A. M. Dirac, Lectures in Quantum Mechanics (Academic, New York, 1964).
[20] C. Kiefer, Phys. Rev. D 47, 5414 (1993).
[21] L. D. Landau and E. M. Lifshitz, Mechanics, 3rd edition (Pergamon, Oxford, 1976).
[22] H. Goldstein, Classical Mechanics, 2nd edition (Addison-Wesley, Reading, Massachusetts, 1980).
[23] J. J. Halliwell, in Quantum Cosmology and Baby Universes (Jerusalem Winter School for Theoretical Physics, Vol. 7), edited by S. Coleman, J. B. Hartle, T. Piran, and S. Weinberg (World Scientific, Singapore, 1991).
[24] C. W. Misner, in Magic Without Magic, edited by J. R. Klauder (Freeman, San Francisco, 1972).
[25] J. J. Halliwell and J. B. Hartle, Phys. Rev. D 41, 1815 (1990).
[26] J. J. Halliwell and J. Louko, Phys. Rev. D 42, 3997 (1990).
[27] L. Dixon, J. Harvey, C. Vafa, and E. Witten, Nucl. Phys. B274, 285 (1986).
[28] K. Schleich and D. M. Witt, Nucl. Phys. B402, 411 (1993).
[29] J. Louko, Phys. Rev. D 48, 2708 (1993).