Proposal and theoretical formalism for studying baryon radiative decays from $J/\psi \to B^* \bar{B} + \bar{B}^* B \to \gamma B \bar{B}$

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Abstract

With accumulation of high statistics data at BESIII, one may study many new interesting channels. Among them, $J/\psi \to B^* \bar{B} + \bar{B}^* B \to \gamma B \bar{B}$ processes may provide valuable information of the radiative decays of the excited baryons ($N^*, \Lambda^*, \Sigma^*, \Xi^*$), and may shed light on their internal quark-gluon structure. Our estimation for the branching ratios of the nucleon excitations $N^*(1440)$, $N^*(1535)$ and $N^*(1520)$ from the reaction $J/\psi \to N^* \bar{p} + \bar{N}^* p \to p \bar{p} \gamma$, indicates that these processes can be studied at BESIII with $10^{10}$ $J/\psi$ events. Explicit theoretical formulae for the partial wave analysis (PWA) of the $J/\psi \to B^* \bar{B} + \bar{B}^* B$ with $B^* \to B \gamma$ and $\bar{B}^* \to \bar{B} \gamma$ within covariant L-S Scheme are provided.

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1 Introduction

Baryons $B(N, \Lambda, \Sigma, \Xi, \cdots)$ and their excited states $B^*(N^*, \Lambda^*, \Sigma^*, \Xi^*, \cdots)$ are complex systems of confined quarks and gluons. Excited baryons are sensitive to details of quark confinement [1] which is poorly understood within the fundamental theory of strong interactions - Quantum Chromodynamics (QCD). Thus, understanding their structure and determining their properties (masses, decay widths, branching ratios, spins, parities, electromagnetic form factors, magnetic moments, polarizabilities) will provide a better understanding of how confinement works in baryons. Concerning the internal quark-gluon structure of baryons there are various proposed configurations: (a) the classical constituent three quark ($qqq$) states; (b) $qqg$ hybrid states [2]; (c) diquark-quark states [3, 4]; (d) meson-baryon states [5]-[8]; (e) pentaquark with diquark clusters [9]-[13], etc. A series of new experiments on excited nucleon $N^*$ physics with electromagnetic probes have been started at

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modern facilities such as TJNAF [14], ELSA [15], GRAAL [16], SPRING8 [17] and BEPC [18, 19]. In last few years these facilities provided a considerable amount of precise data for various excited nucleon production and decay channels and opened a great opportunity to make quantitative investigations of the baryon structure. To extract properties of $N^*$ resonances partial wave analysis (PWA) is necessary. In this paper, first we show that the radiative decays of baryons can be studied at BESIII with expected $10^{10}$ $J/\psi$ events. Then we provide PWA formulae within covariant L-S Scheme [20] for multi-step chain processes $J/\psi \to B^* \bar{B} + B^* B \to \gamma B \bar{B}$. Because electromagnetic transition rates of excited baryons to their respective ground states offer a stringent test on the quark model dynamics [21, 22], it is therefore highly desirable to study the electromagnetic decay rates from excited baryon states in order to refine the quark model description of the baryons. To date very few electromagnetic transition rates have been measured for the excited baryon resonances [23]. For a detailed discussion of the experimental and theoretical status of the excited baryons and their electromagnetic decays, see the review by Landsberg [21].

2 Estimation of branching ratios for $J/\psi \to B^* \bar{B} + B^* B \to \gamma B \bar{B}$

In hadron spectroscopy, the ground states of the hadron spectrum are now well understood. However, the excited states still prove a significant challenge. The first excited state $N^*(1440)P_{11}$ with positive parity $J^P = 1/2^+$, and the adjacent excited state $N^*(1535)S_{11}$ with negative parity $J^P = 1/2^-$, as well as $N^*(1520)D_{13}$ with $J^P = 3/2^-$ have been identified by using various techniques. Although these four-star resonances are within the energy region of many modern research facilities, their properties including radiative decays are still not well determined. Previous BES experiments already clearly observed these resonances in $J/\psi \to p\bar{p}\eta, \bar{p}n\pi^+ + c.c., pp\pi^0$ [18, 19]. With two orders of magnitude higher statistics at BESIII, the radiative decays of these $N^*$ may also be studied in $J/\psi \to \gamma p\bar{p}$. In fact, this decay channel has already been studied by BESII experiment. A strong narrow peak $X(1860)$ near the threshold in the invariant mass spectrum of proton-antiproton pairs was observed [24]. The branching ratio for $J/\psi \to \gamma pp$ is about $3.8 \times 10^{-4}$ [23], among which the contribution of $J/\psi \to \gamma X(1860) \to \gamma pp$ is about $7.0 \times 10^{-5}$ [24]. The PWA formulae for determining quantum numbers of intermediate resonances decaying to $pp$ are given in Ref.[25]. Due to limited statistics and large background from $J/\psi \to \bar{p}p\pi^0$ channel, no observation of $N^* \to p\gamma$ was reported.

Based on the branching ratios for the reaction $J/\psi \to N^*\bar{p} + \bar{N}^*p$ measured by BESII [19] and branching ratios of $N^* \to p\gamma$ given by PDG [23], we give the
Table 1: The mass (MeV), widths (MeV), and branching ratios ($10^{-6}$) for $J/\psi \rightarrow N^*\bar{p} + \bar{N}^*p \rightarrow p\gamma\bar{p}$ through intermediate $N^*$ states.

| $M_{N^*}$ | $\Gamma$ | $Br(J/\psi \rightarrow N^*\bar{p} + \bar{N}^*p)$ | $Br(N^* \rightarrow p\gamma)$ | $Br(J/\psi \rightarrow p\gamma\bar{p})$ |
|----------|---------|---------------------------------|----------------------------|---------------------------------|
| 938      | 210     | 210 $\sim$ 224[23]             | 19.8 $\sim$ 21.0           |                                 |
| 1440     | 300     | 133 $\sim$ 354[19]             | 350 $\sim$ 480[23]         | 0.046 $\sim$ 0.170             |
| 1535     | 150     | 92 $\sim$ 210[19]              | 1500 $\sim$ 3500[23]       | 0.138 $\sim$ 0.735             |
| 1520     | 115     | 34 $\sim$ 154[19]              | 4600 $\sim$ 5600[23]       | 0.156 $\sim$ 0.862             |

estimation of branching ratios for the reaction $J/\psi \rightarrow N^*\bar{p} + \bar{N}^*p \rightarrow p\bar{p}\gamma$ through the intermediate $N^* = p(938), N^*(1440), N^*(1535)$ and $N^*(1520)$ states as shown in Table 1.

In the estimation of the contribution from the off-shell nucleon pole, we use the following effective lagrangian for the vertex $\gamma pp$ [26]

$$L_{\gamma pp} = -e\bar{\psi}_p (\gamma^\mu A_\mu - \frac{\kappa_p}{2M_p} \sigma^{\mu\nu} \partial_\nu A_\mu) \psi_p,$$

where $\kappa_p = 2.739$ is the magnetic moment of the proton. The following off-shell form factor is assumed

$$F = \frac{\Lambda^4}{\Lambda^4 + (p_{N^*}^2 - m_{N^*}^2)^2},$$

with $\Lambda = 0.8 GeV$. Here we also use the experimental photon energy cut condition $E_\gamma > 25 MeV$. Because of the zero width of proton, the main contribution for $J/\psi \rightarrow p\bar{p} \rightarrow p\bar{p}\gamma$ is from the low energy photon, for example, the branching ratio will be reduced to $6.7 \times 10^{-6}$ for the photon energy cut $E_\gamma > 100 MeV$. The contribution from the off-shell proton pole contribution is well separated from those from $N^*$ contributions on the Dalitz plot.

Due to flavor SU(3) symmetry, the excited hyperons are produced at a similar rate. So the typical branching ratio for the $J/\psi \rightarrow B^*\bar{B} + \bar{B}^*B \rightarrow \gamma B\bar{B}$ processes is about $10^{-7} \sim 10^{-6}$. With expected $10^{10}$ $J/\psi$ events and much improved photon detection at BESIII, these processes can definitely be studied in order to provide unique information on the structure of various excited nucleon and hyperon states, and to give substantial insight into the non-perturbative aspects of the QCD.

### 3 Formalism

Now we present the necessary tools for the construction of covariant L-S scheme for the $B^*\bar{B}M(\bar{B}^*BM)$ and $B^*B(\bar{B}^*\bar{B}\gamma)$ couplings. The partial wave amplitudes
$U^\mu_\nu$ in the covariant L-S scheme can be constructed by using pure orbital angular momentum covariant tensors $\tilde{t}^{(L_{bc})}_{\mu_1 \cdots \mu_{L_{bc}}}$, covariant spin wave functions $\psi (\Psi)$ or $\phi (\Phi)$, metric tensor $g^{\mu\nu}$, totally antisymmetric Levi-Civita tensor $\epsilon_{\mu\nu\lambda\sigma}$ and momentum of parent particle.

For a given hadronic decay process $a \rightarrow bc$, in the L-S scheme on hadronic level, the initial state is described by its 4-momentum $p_\mu$ and its spin state $S_a$; the final state is described by the relative orbital angular momentum state of $bc$ system $L_{bc}$ and their spin states $(S_b, S_c)$. The spin states $(S_a, S_b, S_c)$ can be well represented by the relativistic Rarita-Schwinger spin wave functions for particles of arbitrary spin [27, 28, 29, 30]. The spin-1/2 wavefunction is the standard Dirac spinor $u(p, S)$ or $v(p, S)$ and the spin-1 wave function is the standard spin-1 polarization four-vector $\varepsilon^\mu(p, S)$ for a particle with momentum $p$ and spin projection $S$

$$\sum_{S=0,\pm 1} \varepsilon_\mu(p, S)\varepsilon^*_\nu(p, S) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \equiv -\tilde{g}_{\mu\nu}(p) \quad (2)$$

with $p^\nu \varepsilon_\mu(p, S) = 0$, which states that spin-1 wave function is orthogonal to its own momentum. Here the Minkowsky metric tensor has the form

$$g_{\mu\nu} = diag(1, -1, -1, -1).$$

Spin wave functions for particles of higher spins are constructed from these two basic spin wave functions with C-G coefficients $(J_1, J_{12}; J_2, J_{22}; J, J_z)$ as

$$\varepsilon_{\mu_1 \mu_2 \cdots \mu_n}(p, n, S) = \sum_{S_{n-1}, S_n} (n - 1, S_{n-1}; 1, S_n|n, S)\varepsilon_{\mu_1 \mu_2 \cdots \mu_{n-1}}(p, n - 1, S_{n-1})\varepsilon_\mu(p, S_n) \quad (3)$$

for a particle with integer spin $n \geq 2$, and

$$u_{\mu_1 \mu_2 \cdots \mu_n}(p, n + \frac{1}{2}, S) = \sum_{S_n, S_{n+1}} (n, S_n; \frac{1}{2}, S_{n+1}|n + \frac{1}{2}, S)\varepsilon_{\mu_1 \mu_2 \cdots \mu_n}(p, n, S_n)u(p, S_{n+1}) \quad (4)$$

for a particle with half integer spin $n + \frac{1}{2}$ of $n \geq 1$. For an antiparticle with half integer spin $n + \frac{1}{2}$ of $n \geq 1$, one has

$$v_{\mu_1 \mu_2 \cdots \mu_n}(p, n + \frac{1}{2}, S) = \sum_{S_n, S_{n+1}} (n, S_n; \frac{1}{2}, S_{n+1}|n + \frac{1}{2}, S)\varepsilon_{\mu_1 \mu_2 \cdots \mu_n}(p, n, S_n)v(p, S_{n+1}) \quad (5)$$

For a process $a \rightarrow b + c$, if there exists a relative orbital angular momentum $L_{bc}$ between the particle $b$ and $c$, then the orbital angular momentum $L_{bc}$ state can be represented by covariant tensor wave functions $\tilde{t}^{(L)}_{\mu_1 \cdots \mu_L}$, which are the same as for meson decay [20, 29, 31]

$$\tilde{t}^{(0)} = 1, \quad (6)$$
\[
\tilde{t}_{\mu}^{(1)} = \tilde{g}_{\mu\nu}(p_a)\tilde{r}^\nu \equiv \tilde{r}_\mu ,
\]
\[
\tilde{t}_{\mu\nu}^{(2)} = \tilde{r}_\mu \tilde{r}_\nu - \frac{1}{3}(\tilde{r} \cdot \tilde{r})\tilde{g}_{\mu\nu} ,
\]
\[
\tilde{t}_{\mu\nu\lambda}^{(3)} = \tilde{r}_\mu \tilde{r}_\nu \tilde{r}_\lambda - \frac{1}{5}(\tilde{r} \cdot \tilde{r})((\tilde{g}_{\mu\nu}\tilde{r}_\lambda + \tilde{g}_{\nu\lambda}\tilde{r}_\mu + \tilde{g}_{\lambda\mu}\tilde{r}_\nu) ,
\]
\[
\tilde{t}_{\mu\nu\lambda\sigma}^{(4)} = \tilde{r}_\mu \tilde{r}_\nu \tilde{r}_\lambda \tilde{r}_\sigma - \frac{1}{7}(\tilde{r} \cdot \tilde{r})((\tilde{g}_{\mu\nu}\tilde{r}_\lambda\tilde{r}_\sigma + \tilde{g}_{\nu\lambda}\tilde{r}_\mu\tilde{r}_\sigma + \tilde{g}_{\lambda\mu}\tilde{r}_\nu\tilde{r}_\sigma + \tilde{g}_{\mu}\tilde{r}_\nu\tilde{r}_\lambda)
\]
\[
+ \tilde{g}_{\nu\sigma}\tilde{r}_\lambda\tilde{r}_\mu + \tilde{g}_{\lambda\sigma}\tilde{r}_\mu\tilde{r}_\nu) + \frac{1}{35}(\tilde{r} \cdot \tilde{r})^2((\tilde{g}_{\mu\nu}\tilde{g}_{\lambda\sigma} + \tilde{g}_{\nu\lambda}\tilde{g}_{\mu\sigma} + \tilde{g}_{\lambda\mu}\tilde{g}_{\nu\sigma}) ,
\]
\[
\tilde{t}_{\mu\nu\lambda\sigma\delta}^{(5)} = \tilde{r}_\mu \tilde{r}_\nu \tilde{r}_\lambda \tilde{r}_\sigma\tilde{r}_\delta - \frac{1}{9}(\tilde{r} \cdot \tilde{r})((\tilde{g}_{\mu\nu}\tilde{r}_\lambda\tilde{r}_\sigma\tilde{r}_\delta + \tilde{g}_{\nu\lambda}\tilde{r}_\mu\tilde{r}_\sigma\tilde{r}_\delta + \tilde{g}_{\lambda\mu}\tilde{r}_\nu\tilde{r}_\sigma\tilde{r}_\delta + \tilde{g}_{\mu}\tilde{r}_\nu\tilde{r}_\lambda\tilde{r}_\delta
\]
\[
+ \tilde{g}_{\nu}\tilde{r}_\lambda\tilde{r}_\mu + \tilde{g}_{\lambda}\tilde{r}_\mu\tilde{r}_\nu) + \frac{1}{63}(\tilde{r} \cdot \tilde{r})^2((\tilde{g}_{\mu\nu}\tilde{g}_{\lambda\sigma}\tilde{r}_\delta + \tilde{g}_{\nu\lambda}\tilde{g}_{\mu\sigma}\tilde{r}_\delta + \tilde{g}_{\lambda\mu}\tilde{g}_{\nu\sigma}\tilde{r}_\delta + \tilde{g}_{\mu}\tilde{g}_{\lambda}\tilde{g}_{\sigma}\tilde{r}_\delta
\]
\[
+ \tilde{g}_{\nu}\tilde{g}_{\mu}\tilde{g}_{\lambda}\tilde{g}_{\sigma}\tilde{r}_\delta + \tilde{g}_{\lambda}\tilde{g}_{\mu}\tilde{g}_{\nu}\tilde{r}_\delta + \tilde{g}_{\nu}\tilde{g}_{\mu}\tilde{g}_{\sigma}\tilde{r}_\delta + \tilde{g}_{\mu}\tilde{g}_{\lambda}\tilde{g}_{\sigma}\tilde{r}_\delta + \tilde{g}_{\nu}\tilde{g}_{\lambda}\tilde{g}_{\mu}\tilde{r}_\delta)
\]
\[
(13)
\]
\[
\tilde{t}_{\mu_1\mu_2\ldots\mu_L}^{(L)} = \tilde{r}_{\mu_1} \tilde{r}_{\mu_2} \ldots \tilde{r}_{\mu_L} + \sum_{L=1}^{[L/2]} \sum_{L=1}^{[L/2]} \frac{(-\tilde{r} \cdot \tilde{r})^l}{(2L - 1)(2L - 3)\ldots(2L - 2L + 1)}
\]
\[
\times \frac{1}{2L!}(\tilde{g}_{\mu_1\mu_2\ldots\mu_L}) \tilde{g}_{\mu_1\mu_2\ldots\mu_L} = \mu_1\mu_2\ldots\mu_L \text{ permutation, } (2L)! \text{ term}
\]
\[
\times (\tilde{r}_{\mu_1} \tilde{r}_{\mu_2} \ldots \tilde{r}_{\mu_{L-1}} \tilde{r}_{\mu_{L+1}} \ldots \tilde{r}_{\mu_{L-1}} \tilde{r}_{\mu_{L+1}} \ldots \tilde{r}_{\mu_L}) ,
\]
where \( r = p_b - p_c \) is the relative four momentum of the two decay products in the parent particle rest frame; \( (\tilde{r} \cdot \tilde{r}) = -r^2 \); \([L/2] = n \) when \( L = 2n \) and \( L = 2n + 1 \); and
\[
p_a^{\mu} \tilde{t}_{\mu}^{(1)} = p_a^{\mu} \tilde{t}_{\mu\nu}^{(2)} = p_a^{\mu} \tilde{t}_{\mu\nu\lambda}^{(3)} = 0 ,
\]
\[
\tilde{g}^{\mu\nu}(p_a) = g^{\mu\nu} - \frac{p_a^{\mu} p_a^{\nu}}{p_a^2} , \quad g^{\mu\nu} = \text{diag}(1,-1,-1,-1) .
\]
In the L-S scheme, we need to use the conservation relation of total angular momentum
\[
S_a = S_b + S_c + L_{bc} \quad \text{or} \quad -S_a + S_b + S_c + L_{bc} = 0 .
\]
Besides the parity should be conserved, which means
\[
\eta_a = \eta_b \eta_c (-1)^L ,
\]
where \( \eta_a, \eta_b \) and \( \eta_c \) are the intrinsic parities of particles \( a, b \) and \( c \), respectively. From this relation, one knows whether \( L \) should be even or odd. Then from Eq.(13) one can figure out how many different L-S combinations, which determine the number of independent couplings.

Comparing with the pure meson case [29], here we need to introduce the concept of relativistic total spin of two fermions. For the case of \( a \) as a vector meson, \( b \) as
$B^*$ with spin $n + \frac{1}{2}$ and $c$ as $B$ with spin-$\frac{1}{2}$, the total spin of $bc$ ($S_{bc}$) can be either $n$ or $n + 1$. The two $S_{bc}$ states can be represented as

$$\psi_{\mu_1\ldots\mu_n}^{(n)} = \bar{u}(p_b, S_b)\gamma_5 v(p_c, S_c),$$

$$\psi_{\mu_1\ldots\mu_{n+1}}^{(n+1)} = \bar{u}(p_b, S_b)(\gamma_{\mu_{n+1}} - \frac{r_{\mu_{n+1}}}{m_a + m_b + m_c})v(p_c, S_c) + (\mu_1 \leftrightarrow \mu_{n+1}) + \cdots + (\mu_n \leftrightarrow \mu_{n+1})$$

for $S_{bc}$ of $n$ and $n + 1$, respectively. As a special case of $n = 0$, we have

$$\psi^{(0)} = \bar{u}(p_b, S_b)\gamma_5 v(p_c, S_c),$$

$$\psi^{(1)} = \bar{u}(p_b, S_b)(\gamma_{\mu} - \frac{r_{\mu}}{m_a + m_b + m_c})v(p_c, S_c).$$

Here $r_{\mu}$ term is necessary to cancel out the $\hat{p}$-dependent component in the simple $\bar{u}\gamma_{\mu}v$ expression.

For the case of $a$ as a vector meson, $b$ as excited anti-baryons ($\bar{B}^*$) with spin $n + \frac{1}{2}$ and $c$ as baryons ($B$) with spin-$\frac{1}{2}$, the above equations can be written as

$$\psi_{\mu_1\ldots\mu_n}^{C(n)} = -\bar{u}(p_c, S_c)\gamma_5 v_{\mu_1\ldots\mu_n}(p_b, S_b),$$

$$\psi_{\mu_1\ldots\mu_{n+1}}^{C(n+1)} = \bar{u}(p_c, S_c)(\gamma_{\mu_{n+1}} - \frac{r_{\mu_{n+1}}}{m_a + m_b + m_c})v_{\mu_1\ldots\mu_n}(p_b, S_b) + (\mu_1 \leftrightarrow \mu_{n+1}) + \cdots + (\mu_n \leftrightarrow \mu_{n+1})$$

for $S_{bc}$ of $n$ and $n + 1$, respectively. As a special case of $n = 0$, we have

$$\psi^{C(0)} = -\bar{u}(p_c, S_c)\gamma_5 v(p_b, S_b),$$

$$\psi^{C(1)} = \bar{u}(p_c, S_c)(\gamma_{\mu} - \frac{r_{\mu}}{m_a + m_b + m_c})v(p_b, S_b).$$

For the case of $a$ as excited baryons ($B^*$) with spin $n + \frac{1}{2}$, $b$ as baryons ($B$) and $c$ as a meson, one needs to couple $-S_a$ and $S_b$ first to get $S_{ab} \equiv -S_a + S_b$ states, which are

$$\phi_{\mu_1\ldots\mu_n}^{(n)} = \bar{u}(p_b, S_b)u_{\mu_1\ldots\mu_n}(p_a, S_a),$$

$$\phi_{\mu_1\ldots\mu_{n+1}}^{(n+1)} = \bar{u}(p_b, S_b)\gamma_5 \tilde{\gamma}_{\mu_{n+1}} u_{\mu_1\ldots\mu_n}(p_a, S_a) + (\mu_1 \leftrightarrow \mu_{n+1}) + \cdots + (\mu_n \leftrightarrow \mu_{n+1})$$

for $S_{ab}$ of $n$ and $n + 1$, respectively.

$$\phi^{(0)} = \bar{u}(p_b, S_b)u(p_a, S_a),$$

$$\phi^{(1)} = \bar{u}(p_b, S_b)\gamma_5 \tilde{\gamma}_\mu u(p_a, S_a)$$

with $\tilde{\gamma}_\mu = \tilde{g}_{\mu\nu}(p_a)\gamma^\nu$. 
For the case of $a$ as excited antibaryons ($\bar{B}^*$) with spin $n + \frac{1}{2}$, $b$ as an antibaryon ($\bar{B}$) and $c$ as a meson, as before one needs to couple $-S_a$ and $S_b$ first to get $S_{ab} \equiv -S_a + S_b$ states, which are

$$
\begin{align*}
\phi^{\mu_1\cdots\mu_n}_{(n)} &= \bar{v}_{\mu_1\cdots\mu_n}(p_a, S_a)v(p_b, S_b), \\
\Phi^{\mu_1\cdots\mu_{n+1}}_{(n+1)} &= \bar{v}_{\mu_1\cdots\mu_n}(p_a, S_a)\gamma_5\gamma_{\mu_{n+1}}v(p_b, S_b) + (\mu_1 \leftrightarrow \mu_{n+1}) + \cdots + (\mu_n \leftrightarrow \mu_{n+1})
\end{align*}
$$

(27)

for $S_{ab}$ of $n$ and $n+1$, respectively. As a special case of $n = 0$, we have

$$
\begin{align*}
\phi^{(0)} &= \bar{v}(p_a, S_a)v(p_b, S_b), \\
\Phi^{(1)}_\mu &= \bar{v}(p_a, S_a)\gamma_5\gamma_\mu v(p_b, S_b).
\end{align*}
$$

(29)

### 4 Partial Wave Amplitudes

We consider the following process

$$
J/\psi \rightarrow B^* \bar{B} + \bar{B}^* B \rightarrow \gamma B\bar{B},
$$

(31)

The possible $J^P$ for $B^*$ is $\frac{1}{2}^\pm$, $\frac{3}{2}^\pm$, $\frac{5}{2}^\pm$, $\frac{7}{2}^\pm$. We denote the four momenta of $J/\psi$, $B^*(\bar{B}^*)$ and $\gamma$ by $p_\mu$, $p_{B^*}(p_{\bar{B}^*})$ and $q_\mu$. The orbital spin tensor describing the first and second steps will be denoted by $\tilde{T}_{\mu_1\cdots\mu_L}$ and $\tilde{t}_{\mu_1\cdots\mu_l}$. For the process (31) the general form of the decay amplitude is

$$
M = \varepsilon_\mu(p, S_{J/\psi})e_\nu^*(q, S_\gamma)M^{\mu\nu} = \varepsilon_\mu(p, S_{J/\psi})e_\nu^*(q, S_\gamma)\sum_{i,j} U_{i,j}^{\mu\nu},
$$

(32)

where $\varepsilon_\mu(p, S_{J/\psi})$ is the polarization four vector of the $J/\psi$; $e_\nu(q, S_\gamma)$ is the polarization four vector of the photon; $S_{J/\psi}$ and $S_\gamma$ are the spins of $J/\psi$ and photon; $U_{i,j}^{\mu\nu}$ is the $i$-th $B^*$ and $\bar{B}^*$, $j$-th partial wave amplitude with complex coupling constants to be determined by the experiment. The spin-1 polarization four vector $\varepsilon_\mu(p, S_{J/\psi})$ for $J/\psi$ with four momentum $p_\mu$ satisfies the relation in Eq.(2). For $J/\psi$ production from $e^+e^-$ annihilation, the electrons are highly relativistic, with the result that $J_z = \pm 1$ for the $J/\psi$ spin projection taking the beam direction as the $z$-axis. This limits $S_{J/\psi}$ to 1 and 2. Then one has the following relation

$$
\sum_{S_{J/\psi}=1}^2 \varepsilon_\mu(p, S_{J/\psi})\varepsilon_{\mu'}^*(p, S_{J/\psi}) = \delta_{\mu\mu'}(\delta_{\mu 1} + \delta_{\mu 2}).
$$

(33)

For the photon polarization four vector, there is the usual Lorentz orthogonality conditions. Namely, the polarization four vector $e_\nu(q, S_\gamma)$ of the photon with momenta $q$ satisfies

$$
q^\nu e_\nu(q, S_\gamma) = 0,
$$

(34)
which states that spin-1 wave function is orthogonal to its own momentum. The above relation is the same as for a massive vector meson. However, for the photon, there is an additional gauge invariance condition

\[ \sum_{S} e^*_{\mu}(q, S_\gamma) e_\nu(q, S_\gamma) = -g_{\mu\nu} + \frac{q_{\mu} K_{\nu} + K_{\mu} q_{\nu}}{q \cdot K} - \frac{K \cdot K}{(q \cdot K)^2} q_{\mu} q_{\nu} \equiv -g_{\mu\nu}^{(\perp)} \]  

(35)

with \( K = p - q \) and \( K^\nu e_\nu = 0 \).

Although \( B^*(\frac{1}{2}^+), \frac{3}{2}^\pm, \frac{5}{2}^\pm, \frac{7}{2}^\pm) \) \( B \omega \) couplings have the same structure as the \( B^*(\frac{1}{2}^+), \frac{3}{2}^\pm, \frac{5}{2}^\pm, \frac{7}{2}^\pm) B \gamma \) couplings, the gauge invariance requirement for the \( B^* B \gamma \) couplings reduces the number of independent amplitudes. For example, the partial wave amplitudes for the process \( B^*(\frac{3}{2}^+) \rightarrow B(\frac{1}{2}^+) \gamma \) can be written as

\[ M = (g_1 M_1^\nu + g_2 M_2^\nu + g_3 M_3^\nu) e^*_\nu(q, S_\gamma) , \]  

(36)

where

\[ M_1^\nu = i \phi^{(1)}_{\mu} \epsilon^{\mu\nu\lambda\sigma} \tilde{t}^{(1)}_{\lambda} \hat{\gamma}_{B^* \sigma} , \quad M_2^\nu = \Phi^{(2)}_{\mu\nu} \tilde{t}^{(1)}_{\mu} , \quad M_3^\nu = \Phi^{(2)}_{\mu\lambda} \tilde{t}^{(3)}_{\mu\lambda\nu} . \]  

(37)

Because of the gauge invariance requirement

\[ (g_1 M_1^\nu + g_2 M_2^\nu + g_3 M_3^\nu) q_\nu = 0 , \]  

(38)

we get the following relation

\[ g_2 = -\frac{3}{5} (\tilde{r} \cdot \tilde{r}) \quad g_3 = \frac{3}{5} \frac{(m_{B^*}^2 - m_B^2)^2}{m_{B^*}^2} \quad g_3 , \]  

(39)

which means that there are two independent partial wave amplitudes. Refs.[32, 33] also provided basically equivalent partial wave amplitude formulae for the vertex \( B^* B \gamma \) in the spin-orbital approach. In order to be able to compare our results with conventional helicity amplitudes for the radiative decays of baryon resonances [23], we also give the relation between our coupling constants and helicity amplitudes in the Appendix.

To compute decay width, we need an expression for \(|M|^2\). Note that the square modulus of the decay amplitude, which gives the decay probability of a certain configuration should be independent of any particular frame, that is, a Lorentz scalar. Thus by using the Eqs. (33) and (35), we have

\[ d\Gamma = \frac{(2\pi)^4}{2M_{J/\psi}} |M|^2 d\Phi_3(p, q_\gamma, p_B, p_B) , \]  

(40)

where \( M_{J/\psi} \) is the mass of the \( J/\psi \), and the general form of the matrix element square is

\[ |M|^2 = \frac{1}{2} \sum_{S_{J/\psi}=1}^{2} \sum_{S_B=1}^{2} \sum_{S_{B^*}=1}^{2} \sum_{S_\gamma=1}^{2} |\varepsilon_{\mu}(p, S_{J/\psi}) e^*_\nu(q, S_\gamma) M^{\mu\nu}|^2 \]
\[ = \frac{1}{2} \sum_{\mu=1}^{2} \sum_{s_p} \sum_{s_p} M^{\mu\nu}\left(-g^{(\perp\perp)}_{\nu\nu'})M^{\star\mu\nu'} \right) \]
\[ = \frac{1}{2} \sum_{i,j',\mu=1}^{2} \sum_{s_p} \sum_{s_p} U^{\mu\nu}_{i,j'}\left(-g^{(\perp\perp)}_{\nu\nu'}\right)U^{\star\mu\nu'}_{i',j'}. \tag{41} \]

the standard Lorentz invariant 3-body phase space element \(d\Phi_3\) is given by

\[ d\Phi_3(p; q, p_B, p_B) = \delta^4(p - q - p_B - p_B) \frac{d^3q}{(2\pi)^3} \frac{2m_Bd^3P_B}{2m_Bd^3P_B}. \tag{42} \]

From (31) we see that \(B^*\) and \(\bar{B}^*\) are the intermediate resonances decaying into \(B\gamma\) and \(\bar{B}\gamma\) respectively, therefore we need to introduce into the amplitude the following propagators denoted by \(G_{B^*}\) and \(G_{\bar{B}^*}\) [34, 35]

\[ G_{B^*}\left(\frac{1}{2}\right) = f_{B^*}(\rho_{B^*}, S_{B^*})\bar{u}(\rho_{B^*}, S_{B^*}) = f_{B^*}(\rho_{B^*} + m_{B^*}), \]
\[ G_{\bar{B}^*}\left(\frac{1}{2}\right) = \bar{f}_{B^*}(\rho_{B^*}, S_{B^*})\bar{v}(\rho_{B^*}, S_{B^*}) = \bar{f}_{B^*}(\rho_{B^*} - m_{B^*}), \tag{43} \]
\[ G_{B^*}^{\mu\nu}\left(\frac{3}{2}\right) = f_{B^*}(\rho_{B^*}, S_{B^*})\bar{u}^{\nu}(\rho_{B^*}, S_{B^*}) = f_{B^*}(\rho_{B^*} + m_{B^*})P_{B^*}^{\mu\nu}\left(\frac{3}{2}\right), \]
\[ G_{\bar{B}^*}^{\mu\nu}\left(\frac{3}{2}\right) = \bar{f}_{B^*}(\rho_{B^*}, S_{B^*})\bar{v}^{\nu}(\rho_{B^*}, S_{B^*}) = \bar{f}_{B^*}(\rho_{B^*} - m_{B^*})P_{B^*}^{\mu\nu}\left(\frac{3}{2}\right), \tag{44} \]
\[ G_{B^*}^{\mu\nu\lambda\sigma}\left(\frac{5}{2}\right) = f_{B^*}(\rho_{B^*}, S_{B^*})\bar{u}^{\alpha\beta}(\rho_{B^*}, S_{B^*}) = f_{B^*}(\rho_{B^*} + m_{B^*})P_{B^*}^{\mu\nu\lambda\sigma}\left(\frac{5}{2}\right), \]
\[ G_{\bar{B}^*}^{\mu\nu\lambda\sigma}\left(\frac{5}{2}\right) = \bar{f}_{B^*}(\rho_{B^*}, S_{B^*})\bar{v}^{\alpha\beta}(\rho_{B^*}, S_{B^*}) = \bar{f}_{B^*}(\rho_{B^*} - m_{B^*})P_{B^*}^{\mu\nu\lambda\sigma}\left(\frac{5}{2}\right), \tag{45} \]
\[ G_{B^*}^{\mu\nu\lambda\sigma}\left(\frac{7}{2}\right) = f_{B^*}(\rho_{B^*}, S_{B^*})\bar{u}^{\lambda\alpha}(\rho_{B^*}, S_{B^*}) = f_{B^*}(\rho_{B^*} + m_{B^*})P_{B^*}^{\mu\nu\lambda\sigma}\left(\frac{7}{2}\right), \]
\[ G_{\bar{B}^*}^{\mu\nu\lambda\sigma}\left(\frac{7}{2}\right) = \bar{f}_{B^*}(\rho_{B^*}, S_{B^*})\bar{v}^{\lambda\alpha}(\rho_{B^*}, S_{B^*}) = \bar{f}_{B^*}(\rho_{B^*} - m_{B^*})P_{B^*}^{\mu\nu\lambda\sigma}\left(\frac{7}{2}\right), \tag{46} \]

where

\[ f_{B^*} = \frac{2m_{B^*}}{p_{B^*}^2 - m_{B^*}^2 + im_{B^*}\Gamma_{B^*}}, \]
\[ \bar{f}_{B^*} = \frac{2m_{B^*}}{p_{B^*}^2 - m_{B^*}^2 + im_{B^*}\Gamma_{B^*}}, \tag{47} \]

here \(m_{B^*}, m_{\bar{B}^*}\) and \(\Gamma_{B^*}, \Gamma_{\bar{B}^*}\) are the resonance masses and widths;

\[ P_{B^*}^{\mu\nu}\left(\frac{3}{2}\right) = -g^{\mu\nu} + \frac{1}{3} \gamma^{\mu}\gamma^{\nu} + \frac{2}{3} \frac{p_{B^*}^\mu p_{B^*}^\nu}{m_{B^*}^2} + \frac{1}{3m_{B^*}}(\gamma^{\mu}p_{B^*}^\nu - \gamma^{\nu}p_{B^*}^\mu), \]
\[ P_{\bar{B}^*}^{\mu\nu}\left(\frac{3}{2}\right) = -g^{\mu\nu} + \frac{1}{3} \gamma^{\mu}\gamma^{\nu} + \frac{2}{3} \frac{p_{\bar{B}^*}^\mu p_{\bar{B}^*}^\nu}{m_{\bar{B}^*}^2} - \frac{1}{3m_{\bar{B}^*}}(\gamma^{\mu}p_{\bar{B}^*}^\nu - \gamma^{\nu}p_{\bar{B}^*}^\mu), \]
where

\[ P^{\mu\nu\rho\lambda}_{\sigma}(\frac{7}{2}) = \frac{4}{9}\gamma^\rho\gamma_\lambda P^{(4)\mu\nu\rho\lambda}_{\sigma}, \]

and where

\[ P^{(4)\mu\nu\rho\lambda}_{\sigma} = \frac{1}{24}(\tilde{g}^\tau\tilde{g}^\mu\tilde{g}^\nu\tilde{g}^\alpha \tilde{g}^\sigma + (\rho, \beta, \lambda, \sigma \text{ permutation, 24 terms}) \]

\[ - \frac{1}{84}(\tilde{g}^\tau\tilde{g}^\mu\tilde{g}^\nu\tilde{g}^\alpha \tilde{g}^\sigma + (\tau, \mu, \nu, \alpha \text{ permutation, } \rho, \beta, \lambda, \sigma \text{ permutation, 72 terms}) \]

\[ + \frac{1}{105}(\tilde{g}^\tau\tilde{g}^\mu\tilde{g}^\nu\tilde{g}^\alpha \tilde{g}^\rho\tilde{g}^\lambda \tilde{g}^\sigma \tilde{g}^\mu \tilde{g}^\nu \tilde{g}^\lambda \tilde{g}^\mu + \tilde{g}^\rho\tilde{g}^\lambda \tilde{g}^\beta \tilde{g}^\alpha + \tilde{g}^\rho\tilde{g}^\lambda \tilde{g}^\rho \tilde{g}^\lambda \tilde{g}^\mu \tilde{g}^\nu \tilde{g}^\sigma \tilde{g}^\mu). \]  

(49)

For the different partial wave amplitudes, we use the following notation

\[ (S_{B^*B\bar{B}\bar{B}}, L_{B^*B\bar{B}\bar{B}}, S_{B^*B\bar{B}\bar{B}}), \]

where \( S_{B^*\bar{B}\bar{B}\bar{B}} = S_{B^*} + S_{\bar{B}} \) or \( S_{\bar{B}^*} + S_{B} \); \( L_{B^*\bar{B}\bar{B}\bar{B}} \) is the relative orbital angular momentum between \( B^* \) and \( \bar{B} \) or \( \bar{B}^* \) and \( B \); \( S_{B^*\bar{B}\bar{B}\bar{B}} = -S_{B^*} + S_{\bar{B}} \) or \( -S_{\bar{B}^*} + S_{B} \).

In the following by considering the parity and angular momentum conservations we provide all relevant covariant amplitudes for the process (31). In these amplitudes, 1, 2 and 3 denote the three final state particles \( B, \bar{B} \) and \( \gamma \).

For \( J/\psi(1^-) \rightarrow B^*(\frac{1}{2}^+)\bar{B}(\frac{1}{2}^-) + B^*(\frac{1}{2}^-)B(\frac{1}{2}^+) \rightarrow (1^-)\bar{B}(\frac{1}{2}^-), \) we find two independent covariant amplitudes for a vector meson \( J/\psi(1^-) \) decaying into the \( B^*(\frac{1}{2}^+)\bar{B}(\frac{1}{2}^-) \) and \( B^*(\frac{1}{2}^-)B(\frac{1}{2}^+) \) states, and one independent covariant amplitudes for a excited baryon resonances \( B^*(\frac{1}{2}^+) \) and \( \bar{B}^*(\frac{1}{2}^-) \) decaying into \( \gamma B(\frac{1}{2}^+) \) and \( \gamma \bar{B}(\frac{1}{2}^-). \) All in all we get the following two covariant amplitudes with two independent coupling constants \( g^{i,a} \) and \( g^{i,b} \) which are determined by the experiment

\[ (1, 0, 1) \]

\[ U_{i,1}^{\mu\nu} = g^{i,a} \left( \sum_{S_{B^*}} \bar{\Psi}^{(1)}_{\beta}(\bar{t}_{(13)}^{(1)})\lambda^\epsilon \tilde{\Phi}_{\beta}^{(1)}(\bar{t}_{(3)}^{(1)})_{\lambda}^\epsilon \tilde{p}_{B^*\sigma} f_{B_{\gamma}}^{B^*} \right) - \sum_{S_{B^*}} \bar{\Psi}^{(1)}_{\alpha}(\bar{t}_{(1)}^{(1)})_{\lambda}^\epsilon \tilde{\Phi}_{\alpha}^{(1)}(\bar{t}_{(3)}^{(1)})_{\lambda}^\epsilon \tilde{p}_{B^*\sigma} f_{B_{\gamma}}^{B^*} \right), \]

(50)

\[ (1, 2, 1) \]

\[ U_{i,2}^{\mu\nu} = g^{i,b} \left( \sum_{S_{B^*}} \bar{\Psi}^{(1)}_{\beta}(\bar{t}_{(1)}^{(2)})_{\alpha}^\mu \tilde{\Phi}_{\beta}^{(1)}(\bar{t}_{(2)}^{(1)})_{\alpha}^\mu \tilde{p}_{B^*\sigma} f_{B_{\gamma}}^{B^*} \right) - \sum_{S_{B^*}} \bar{\Psi}^{(1)}_{\alpha}(\bar{t}_{(1)}^{(2)})_{\alpha}^\mu \tilde{\Phi}_{\alpha}^{(1)}(\bar{t}_{(2)}^{(1)})_{\alpha}^\mu \tilde{p}_{B^*\sigma} f_{B_{\gamma}}^{B^*} \right), \]

(51)
For $J/\psi(1^-) \to B^*(\frac{1}{2}^-)B(\frac{1}{2}^-) + B^*(\frac{1}{2}^+)B(\frac{1}{2}^+) \to \gamma B(\frac{1}{2}^+)\bar{B}(\frac{1}{2}^-)$ we get the following two covariant amplitudes with two independent coupling constants $g^{i,a}$ and $g^{i,b}$ which are determined by the experiment

\[ (0, 1, 1) \]
\[ U_{i,1}^{\mu \nu} = g^{i,a} \left( -\frac{2}{3} C_{B^* B} \sum_{S_{B^*}} \Phi^{(1)\nu} \phi^{(0)}(B^* B) \bar{f}_{B\gamma} \right. \]
\[ + \left. -\frac{2}{3} C_{B^* B} \sum_{S_{B^*}} \phi^{C(0)} \phi^{C(1)\nu} \bar{B}_{B^*} \bar{f}_{B\gamma} \right) \]
\[ + \sum_{S_{B^*}} \phi^{(1)} \phi^{(0)} \bar{B}_{B^*} f_{B\gamma} \]
\[ + \sum_{S_{B^*}} \phi^{C(0)} \phi^{C(1)} \bar{B}_{B^*} f_{B\gamma} \]
\[ (52) \]

\[ (1, 1, 1) \]
\[ U_{i,2}^{\mu \nu} = g^{i,b} \left( -\frac{2}{3} C_{B^* B} \sum_{S_{B^*}} i \phi^{(1)\nu} \phi^{(1)\nu} \bar{f}_{B\gamma} \right. \]
\[ + \left. + \frac{2}{3} C_{B^* B} \sum_{S_{B^*}} i \phi^{(1)\nu} \phi^{(1)\nu} \bar{f}_{B\gamma} \right) \]
\[ + \sum_{S_{B^*}} \phi^{(1)} \phi^{(1)} \bar{B}_{B^*} f_{B\gamma} \]
\[ + \sum_{S_{B^*}} \phi^{(1)} \phi^{(1)} \bar{B}_{B^*} f_{B\gamma} \]
\[ (53) \]

One may note that for $J/\psi \to B^* \bar{B} + B^* B \to \gamma B \bar{B}$ with $J^P(B^*(\bar{B}^*)) = \frac{3}{2}^\pm$, \[ \frac{5}{2}^\pm, \frac{7}{2}^\pm \], we get the six covariant amplitudes for i-th $B^*(\bar{B}^*)$ with five independent coupling constants $g_{J/\psi}^{i,a}$, $g_{J/\psi}^{i,b}$, $g_{J/\psi}^{i,c}$, $g_{J/\psi}^{i,d}$ and $g_{J/\psi}^{i,e}$ which are determined by the experiment. Thus for $J/\psi(1^-) \to B^*(\frac{1}{2}^+)B(\frac{1}{2}^-) + B^*(\frac{3}{2}^-)B(\frac{1}{2}^+) \to \gamma B(\frac{1}{2}^+)\bar{B}(\frac{1}{2}^-)$ we get the following six covariant amplitudes

\[ (1, 0, 1) \]
\[ U_{i,1}^{\mu \nu} = g_{J/\psi}^{i,a} \sum_{S_{B^*}} \phi^{(1)\nu} \phi^{(1)\nu} \bar{f}_{B\gamma} \]
\[ - \sum_{S_{B^*}} \phi^{(1)\nu} \phi^{(1)\nu} \bar{f}_{B\gamma} \]
\[ (54) \]

\[ (1, 2, 1) \]
\[ U_{i,2}^{\mu \nu} = g_{J/\psi}^{i,b} \sum_{S_{B^*}} \phi^{(1)\nu} \phi^{(1)\nu} \bar{f}_{B\gamma} \]
\[ - \sum_{S_{B^*}} \phi^{(1)\nu} \phi^{(1)\nu} \bar{f}_{B\gamma} \]
\[ (55) \]

\[ (2, 2, 1) \]
\[
U^{\mu
u}_{i,3} = g^{i,c}_{J/\psi} g^{i,a}_{\gamma} \left( - \sum_{S_{B^*}} \phi^{(1)}_{\alpha} \phi^{(2)}_{\beta} e^{\alpha\mu\delta\tau} T^{(2)}_{(B^*)} \hat{B} \epsilon^{\nu\lambda\sigma} \bar{T}^{(1)}_{(1)\lambda} \hat{B} \epsilon^{\nu\lambda\sigma} \bar{T}^{(1)}_{(1)\lambda} \right), \quad (1, 0, 2)
\]
\[
U^{\mu
u}_{i,4} = g^{i,a}_{J/\psi} g^{i,b}_{\gamma} \left( - \frac{3}{5} C_{B^*B^*} \sum_{S_{B^*}} \phi^{(2)}_{\alpha} \psi^{(1)}_{\beta} \epsilon_{\alpha\mu\delta\tau} \bar{T}^{(1)}_{(1)\lambda} \hat{B} \epsilon^{\nu\lambda\sigma} \bar{T}^{(1)}_{(1)\lambda} \right) \quad f_{B^*} \gamma
\]
\[
U^{\mu
u}_{i,5} = g^{i,b}_{J/\psi} g^{i,b}_{\gamma} \left( - \frac{3}{5} C_{B^*B^*} \sum_{S_{B^*}} \phi^{(2)}_{\alpha} \psi^{(1)}_{\beta} \epsilon_{\alpha\mu\delta\tau} \bar{T}^{(1)}_{(1)\lambda} \hat{B} \epsilon^{\nu\lambda\sigma} \bar{T}^{(1)}_{(1)\lambda} \right) \quad f_{B^*} \gamma
\]
\[
U^{\mu
u}_{i,6} = g^{i,c}_{J/\psi} g^{i,a}_{\gamma} \left( - \frac{3}{5} C_{B^*B^*} \sum_{S_{B^*}} \phi^{(2)}_{\alpha} \psi^{(1)}_{\beta} \epsilon_{\alpha\mu\delta\tau} \bar{T}^{(1)}_{(1)\lambda} \hat{B} \epsilon^{\nu\lambda\sigma} \bar{T}^{(1)}_{(1)\lambda} \right) \quad f_{B^*} \gamma
\]

For \( J/\psi(1^-) \to B^*(3^+ \frac{2}{2}) \bar{B}(\frac{1}{2}^-) + B^*(\frac{3}{2}^+)B(\frac{1}{2}^+) \to \gamma B(\frac{1}{2}^+) \bar{B}(\frac{1}{2}^-) \) we get the following six covariant amplitudes

\[
(1, 1, 1)
\]
\[
U^{\mu
u}_{i,1} = g^{i,a}_{J/\psi} g^{i,b}_{\gamma} \left( - \frac{2}{3} C_{B^*B^*} \sum_{S_{B^*}} \phi^{(1)}_{\alpha} \psi^{(1)}_{\beta} \epsilon_{\alpha\mu\delta\tau} \bar{T}^{(1)}_{(1)\lambda} \hat{B} \epsilon^{\nu\lambda\sigma} \bar{T}^{(1)}_{(1)\lambda} \right) \quad f_{B^*} \gamma
\]

12
\[ \begin{array}{l}
 + \sum_{S_{B^*}} i\phi^{(1)}_{\beta} (1) \phi^{(1)}_{\alpha} \tilde{T}^{(1)}_{(B^*2)\delta} \tilde{T}^{(2)\beta\nu}_{(13)} \epsilon_{\alpha\mu\delta\tau} \hat{\rho}_\tau \hat{f}_{B^*} \\
 - \sum_{S_{B^*}} i\psi^{(1)}_{\alpha} \Phi^{(1)}_{\beta} \tilde{T}^{(1)}_{(B^*1)\delta} \tilde{T}^{(2)\beta\nu}_{(23)} \epsilon_{\alpha\mu\delta\tau} \hat{\rho}_\tau \hat{f}_{B^*} 
\end{array} \]

(60)

\[ \begin{array}{l}
 U^{\mu\nu}_{i,2} = \frac{i}{g_{J/\psi}} g^i_{\gamma} \left( -\frac{2}{3} C_{B^*B\gamma} \sum_{S_{B^*}} \phi^{(1)\nu} (2) \phi^{(2)\mu\alpha} \tilde{T}^{(1)}_{(B^*2)\alpha} \hat{f}_{B^*} \\
 - \frac{2}{3} C_{B^*B\gamma} \sum_{S_{B^*}} \Psi^{(1)\mu\alpha} \phi^{(2)\nu} \tilde{T}^{(1)}_{(B^*1)\alpha} \hat{f}_{B^*} \\
 + \sum_{S_{B^*}} \phi^{(1)\mu\alpha} \tilde{T}^{(1)}_{(B^*2)\alpha} \hat{f}_{B^*} \\
 + \sum_{S_{B^*}} \Psi^{(1)\mu\alpha} \tilde{T}^{(1)}_{(B^*1)\alpha} \hat{f}_{B^*} \right) 
\end{array} \]

(61)

\[ \begin{array}{l}
 U^{\mu\nu}_{i,3} = \frac{i}{g_{J/\psi}} g^i_{\gamma} \left( \sum_{S_{B^*}} \phi^{(1)\nu} (2) \phi^{(2)\mu\alpha} \tilde{T}^{(3)\alpha\delta\mu}_{B} \hat{f}_{B^*} \\
 - \frac{2}{3} C_{B^*B\gamma} \sum_{S_{B^*}} \Psi^{(1)\mu\alpha} \phi^{(2)\nu} \tilde{T}^{(3)\alpha\delta\mu}_{B} \hat{f}_{B^*} \\
 + \sum_{S_{B^*}} \phi^{(1)\mu\alpha} \tilde{T}^{(3)\alpha\delta\mu}_{B} \hat{f}_{B^*} \\
 + \sum_{S_{B^*}} \Psi^{(1)\mu\alpha} \tilde{T}^{(3)\alpha\delta\mu}_{B} \hat{f}_{B^*} \right) 
\end{array} \]

(62)

\[ \begin{array}{l}
 U^{\mu\nu}_{i,4} = \frac{i}{g_{J/\psi}} g^i_{\gamma} \left( -\sum_{S_{B^*}} \phi^{(2)\nu} (1) \phi^{(2)\mu\alpha} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(1)}_{(B^*2)\alpha} \hat{f}_{B^*} \\
 - \sum_{S_{B^*}} \Psi^{(1)\mu\alpha} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(1)}_{(B^*1)\alpha} \hat{f}_{B^*} \\
 + \sum_{S_{B^*}} \phi^{(1)\mu\alpha} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(1)}_{(B^*2)\alpha} \hat{f}_{B^*} \\
 + \sum_{S_{B^*}} \Psi^{(1)\mu\alpha} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(1)}_{(B^*1)\alpha} \hat{f}_{B^*} \right) 
\end{array} \]

(63)

\[ \begin{array}{l}
 U^{\mu\nu}_{i,5} = \frac{i}{g_{J/\psi}} g^i_{\gamma} \left( \sum_{S_{B^*}} \phi^{(2)\nu} (2) \phi^{(2)\mu\alpha} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(1)}_{(B^*2)\alpha} \hat{f}_{B^*} \\
 - \sum_{S_{B^*}} \Psi^{(1)\mu\alpha} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(1)}_{(B^*1)\alpha} \hat{f}_{B^*} \\
 + \sum_{S_{B^*}} \phi^{(1)\mu\alpha} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(1)}_{(B^*2)\alpha} \hat{f}_{B^*} \\
 + \sum_{S_{B^*}} \Psi^{(1)\mu\alpha} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(1)}_{(B^*1)\alpha} \hat{f}_{B^*} \right) 
\end{array} \]

(64)

\[ \begin{array}{l}
 U^{\mu\nu}_{i,6} = \frac{i}{g_{J/\psi}} g^i_{\gamma} \left( \sum_{S_{B^*}} \phi^{(2)\nu} (2) \phi^{(2)\mu\alpha} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(3)\alpha\delta\mu}_{B} \hat{f}_{B^*} \\
 - \sum_{S_{B^*}} \Psi^{(1)\mu\alpha} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(3)\alpha\delta\mu}_{B} \hat{f}_{B^*} \\
 + \sum_{S_{B^*}} \phi^{(1)\mu\alpha} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(3)\alpha\delta\mu}_{B} \hat{f}_{B^*} \\
 + \sum_{S_{B^*}} \Psi^{(1)\mu\alpha} \epsilon^{\alpha\mu\delta\tau} \tilde{T}^{(3)\alpha\delta\mu}_{B} \hat{f}_{B^*} \right) 
\end{array} \]

(65)

For \( J/\psi(1^-) \rightarrow B^*(\frac{3}{2}^+) \bar{B}(\frac{1}{2}^-) + \bar{B}^*(\frac{5}{2}^-) B(\frac{1}{2}^+) \rightarrow \gamma B(\frac{1}{2}^+) \bar{B}(\frac{1}{2}^-) \) we get the following six covariant amplitudes

\( (2, 2, 2) \)
\[ U_{i,1}^{\mu \nu} = g_{J/\psi}^{i,a} g_{\gamma}^{i,a} \left( -\frac{3}{5} C_{B^* B^* B} \sum_{S_{B^*}} i \phi^{(2) \mu \beta} \phi^{(2) \nu \alpha} \epsilon^{\alpha \nu \delta \sigma} T^{(2) \mu} (B^* \gamma) \right) \]
\[ + \frac{3}{5} C_{B^* B^* B} \sum_{S_{B^*}} i \psi^{(C(2) \mu \beta} C^{(2) \nu \alpha} \epsilon^{\alpha \nu \delta \sigma} T^{(2) \mu} (B^* \gamma) \]
\[ + \sum_{S_{B^*}} i \phi^{(2) \mu \beta} \psi^{(2) \nu \alpha} \epsilon^{\alpha \nu \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \]
\[ = \left( -\frac{3}{5} C_{B^* B^* B} \sum_{S_{B^*}} i \phi^{(2) \mu \beta} \Psi^{(3) \mu \alpha \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \right) \]
\[ + \frac{3}{5} C_{B^* B^* B} \sum_{S_{B^*}} i \psi^{(C(2) \mu \beta} C^{(2) \nu \alpha} \epsilon^{\alpha \nu \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \]
\[ + \sum_{S_{B^*}} i \phi^{(2) \mu \beta} \psi^{(2) \nu \alpha} \epsilon^{\alpha \nu \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \]
\[ (3,2,2) \]
\[ U_{i,2}^{\mu \nu} = g_{J/\psi}^{i,b} g_{\gamma}^{i,a} \left( -\frac{3}{5} C_{B^* B^* B} \sum_{S_{B^*}} i \phi^{(2) \mu \beta} \Psi^{(3) \mu \alpha \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \right) \]
\[ + \frac{3}{5} C_{B^* B^* B} \sum_{S_{B^*}} i \psi^{(C(2) \mu \beta} C^{(2) \nu \alpha} \epsilon^{\alpha \nu \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \]
\[ + \sum_{S_{B^*}} i \phi^{(2) \mu \beta} \psi^{(2) \nu \alpha} \epsilon^{\alpha \nu \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \]
\[ = \left( -\frac{3}{5} C_{B^* B^* B} \sum_{S_{B^*}} i \phi^{(2) \mu \beta} \Psi^{(3) \mu \alpha \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \right) \]
\[ + \frac{3}{5} C_{B^* B^* B} \sum_{S_{B^*}} i \psi^{(C(2) \mu \beta} C^{(2) \nu \alpha} \epsilon^{\alpha \nu \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \]
\[ + \sum_{S_{B^*}} i \phi^{(2) \mu \beta} \psi^{(2) \nu \alpha} \epsilon^{\alpha \nu \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \]
\[ (3,4,2) \]
\[ U_{i,3}^{\mu \nu} = g_{J/\psi}^{i,c} g_{\gamma}^{i,a} \left( -\frac{3}{5} C_{B^* B^* B} \sum_{S_{B^*}} i \phi^{(2) \mu \beta} \Psi^{(3) \mu \alpha \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \right) \]
\[ + \frac{3}{5} C_{B^* B^* B} \sum_{S_{B^*}} i \psi^{(C(2) \mu \beta} C^{(2) \nu \alpha} \epsilon^{\alpha \nu \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \]
\[ + \sum_{S_{B^*}} i \phi^{(2) \mu \beta} \psi^{(2) \nu \alpha} \epsilon^{\alpha \nu \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \]
\[ = \left( -\frac{3}{5} C_{B^* B^* B} \sum_{S_{B^*}} i \phi^{(2) \mu \beta} \Psi^{(3) \mu \alpha \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \right) \]
\[ + \frac{3}{5} C_{B^* B^* B} \sum_{S_{B^*}} i \psi^{(C(2) \mu \beta} C^{(2) \nu \alpha} \epsilon^{\alpha \nu \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \]
\[ + \sum_{S_{B^*}} i \phi^{(2) \mu \beta} \psi^{(2) \nu \alpha} \epsilon^{\alpha \nu \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \]
\[ (2,2,3) \]
\[ U_{i,4}^{\mu \nu} = g_{J/\psi}^{i,a} g_{\gamma}^{i,b} \left( -\sum_{S_{B^*}} i \phi^{(3) \beta \gamma \xi} \psi^{(3) \mu \alpha \xi} \epsilon^{\alpha \nu \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \right) \]
\[ + \sum_{S_{B^*}} i \phi^{(3) \mu \alpha \xi} \psi^{(3) \beta \gamma \xi} \epsilon^{\alpha \nu \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \]
\[ (3,2,3) \]
\[ U_{i,5}^{\mu \nu} = g_{J/\psi}^{i,b} g_{\gamma}^{i,b} \left( \sum_{S_{B^*}} i \phi^{(3) \beta \gamma \xi} \psi^{(3) \mu \alpha \xi} \epsilon^{\alpha \nu \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \right) \]
\[ + \sum_{S_{B^*}} i \phi^{(3) \mu \alpha \xi} \psi^{(3) \beta \gamma \xi} \epsilon^{\alpha \nu \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \]
\[ (3,4,3) \]
\[ U_{i,6}^{\mu \nu} = g_{J/\psi}^{i,c} g_{\gamma}^{i,b} \left( \sum_{S_{B^*}} i \phi^{(3) \beta \gamma \xi} \psi^{(3) \mu \alpha \xi} \epsilon^{\alpha \nu \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \right) \]
\[ + \sum_{S_{B^*}} i \phi^{(3) \mu \alpha \xi} \psi^{(3) \beta \gamma \xi} \epsilon^{\alpha \nu \delta \sigma} \tilde{T}^{(2) \mu} (B^* \gamma) \]
\[ \sum_{S_B} i\Phi^{(3)}_{\alpha \beta \gamma} \phi^{(2)}_{\delta \mu \nu} \tilde{T}_{(B^2 \alpha)} \epsilon^{\beta \nu \lambda \sigma} \tilde{t}_{(13)\lambda} \tilde{f}_{B^*}. \] (70)

For \( J/\psi(1^-) \rightarrow B^*(\frac{n}{2}) \tilde{B}(\frac{1}{2}^-) + \tilde{B}^*(\frac{n}{2}) \tilde{B}(\frac{1}{2}^-) \rightarrow \gamma B(\frac{1}{2}^-) \tilde{B}(\frac{1}{2}^-) \) we get the following six covariant amplitudes:

(2, 1, 2)
\[ U_{i,1}^{\mu \nu} = g_{J/\psi} g_{\gamma} \sum_{S_B} i\phi^{(2)}_{\beta \eta} \psi^{(2)}_{\alpha \mu} \tilde{T}_{(B^2 \alpha)} \epsilon^{\beta \nu \lambda \sigma} \tilde{t}_{(13)\lambda} \tilde{f}_{B^*} \] (71)

(2, 3, 2)
\[ U_{i,2}^{\mu \nu} = g_{J/\psi} g_{\gamma} \sum_{S_B} i\phi^{(2)}_{\beta \eta} \psi^{(3)}_{\alpha \beta \gamma} \epsilon^{\beta \nu \lambda \sigma} \tilde{t}_{(13)\lambda} \tilde{f}_{B^*} \] (72)

(3, 3, 2)
\[ U_{i,3}^{\mu \nu} = g_{J/\psi} g_{\gamma} \sum_{S_B} i\phi^{(2)}_{\beta \eta} \psi^{(3)}_{\alpha \beta \gamma} \epsilon^{\beta \nu \lambda \sigma} \tilde{t}_{(13)\lambda} \tilde{f}_{B^*} \] (73)

(2, 1, 3)
\[ U_{i,4}^{\mu \nu} = g_{J/\psi} g_{\gamma} \sum_{S_B} \Phi^{(3)}_{\nu \lambda \sigma} \psi^{(2)}_{\alpha \beta \gamma} \tilde{T}_{(B^2 \alpha)} \epsilon^{\beta \nu \lambda \sigma} \tilde{t}_{(13)\lambda} \tilde{f}_{B^*} \] (74)

(2, 3, 3)
\[ U_{i,5}^{\mu \nu} = g_{J/\psi} g_{\gamma} \sum_{S_B} \Phi^{(3)}_{\nu \lambda \sigma} \psi^{(2)}_{\alpha \beta \gamma} \tilde{T}_{(B^2 \alpha)} \epsilon^{\beta \nu \lambda \sigma} \tilde{t}_{(13)\lambda} \tilde{f}_{B^*} \] (75)

(3, 3, 3)
\[ U_{i,6}^{\mu \nu} = g_{J/\psi} g_{\gamma} \sum_{S_B} \Phi^{(3)}_{\nu \lambda \sigma} \psi^{(3)}_{\alpha \beta \gamma} \epsilon^{\beta \nu \lambda \sigma} \tilde{t}_{(13)\lambda} \tilde{f}_{B^*} \]
\[ -\frac{4}{i}C_{B^*B^\gamma} \sum_{S_{B^*}} i\Phi^C(3)_{\gamma} \Phi^C(3)_{\nu\lambda\sigma} \epsilon^{\alpha\mu\delta\tau} \tilde{\gamma}(3)_{\rho\kappa} \tilde{T}_{(B^*)0\alpha\delta} \tilde{f}_{B^\gamma} \]
\[ + \sum_{S_{B^*}} i\Phi^C(3)_{\gamma} \psi^C(3)_{\nu\lambda\sigma} \epsilon^{\alpha\mu\delta\tau} \tilde{\gamma}(3)_{\rho\kappa} \tilde{f}_{B^\gamma} \]
\[ + \sum_{S_{B^*}} i\psi^C(3)_{\gamma} \Phi^C(3)_{\nu\lambda\sigma} \epsilon^{\alpha\mu\delta\tau} \tilde{\gamma}(3)_{\rho\kappa} \tilde{f}_{B^\gamma} \], \number{76}

For \( J/\psi(1^-) \to B^*(\frac{7}{2}^-)B(\frac{1}{2}^-) + B^*(\frac{7}{2}^+)B(\frac{1}{2}^-) \to \gamma B(\frac{1}{2}^+)B(\frac{1}{2}^-) \) we get the following six covariant amplitudes

\[ (3, 2, 3) \]
\[ U_{\mu \nu}^{i_1} = g_{J/\psi}^{i_a} g_{\gamma}^{i_a} \left( - \sum_{S_{B^*}} i\epsilon^C(3)_{\gamma} \psi^C(3)_{\mu\nu\lambda\sigma} \epsilon^{\alpha\mu\delta\tau} \tilde{\gamma}(3)_{\rho\kappa} \tilde{T}_{(B^*)0\alpha\delta} \tilde{f}_{B^\gamma} \right) \]
\[ (3, 4, 3) \]
\[ U_{\mu \nu}^{i_2} = g_{J/\psi}^{i_b} g_{\gamma}^{i_a} \left( - \sum_{S_{B^*}} i\epsilon^C(3)_{\gamma} \psi^C(3)_{\mu\nu\lambda\sigma} \epsilon^{\alpha\mu\delta\tau} \tilde{\gamma}(3)_{\rho\kappa} \tilde{T}_{(B^*)0\alpha\delta} \tilde{f}_{B^\gamma} \right) \]
\[ (4, 4, 3) \]
\[ U_{\mu \nu}^{i_3} = g_{J/\psi}^{i_c} g_{\gamma}^{i_a} \left( - \sum_{S_{B^*}} i\epsilon^C(3)_{\gamma} \psi^C(3)_{\mu\nu\lambda\sigma} \epsilon^{\alpha\mu\delta\tau} \tilde{\gamma}(3)_{\rho\kappa} \tilde{T}_{(B^*)0\alpha\delta} \tilde{f}_{B^\gamma} \right) \]
\[ (3, 2, 4) \]
\[ U_{\mu \nu}^{i_4} = g_{J/\psi}^{i_a} g_{\gamma}^{i_b} \left( - \frac{5}{9} C_{B^*B^\gamma} \sum_{S_{B^*}} \epsilon^C(3)_{\gamma} \psi^C(3)_{\mu\nu\lambda\sigma} \epsilon^{\alpha\mu\delta\tau} \tilde{\gamma}(3)_{\rho\kappa} \tilde{T}_{(B^*)0\alpha\delta} \tilde{f}_{B^\gamma} \right) \]
\[ (3, 4, 4) \]
\[ U_{\mu \nu}^{i_5} = g_{J/\psi}^{i_b} g_{\gamma}^{i_c} \left( - \frac{5}{9} C_{B^*B^\gamma} \sum_{S_{B^*}} \epsilon^C(3)_{\gamma} \psi^C(3)_{\mu\nu\lambda\sigma} \epsilon^{\alpha\mu\delta\tau} \tilde{\gamma}(3)_{\rho\kappa} \tilde{T}_{(B^*)0\alpha\delta} \tilde{f}_{B^\gamma} \right) \]
For six covariant amplitudes,

\[
\begin{align*}
+ \sum_{S_{B^*}} \Phi_{\beta\lambda\eta\gamma}^{(4)} \psi_{\alpha\delta\mu}^{(3)} \bar{F}_{(B^*2)} \tilde{T}^{(5)}_{\beta\lambda\sigma\nu} \int_{B^*} & \\
+ \sum_{S_{B^*}} \psi_{\alpha\delta\tau}^{(4)} \beta_{\lambda\sigma\gamma} \tilde{T}_{(B^*1)}^{(3)} \int_{B^*} & \end{align*}
\]

(4, 4, 4)

\[
U_{i,6}^{\mu\nu} = g_{J/\psi}^{i_c} g_{\gamma}^{i_b} \left( - \frac{5}{9} C_{B^*B^*} \sum_{S_{B^*}} i \Phi_{\beta\lambda\eta\gamma}^{(4)} \psi_{\alpha\delta\tau}^{(3)} \bar{F}_{(B^*2)} \tilde{T}^{(5)}_{\beta\lambda\sigma\nu} \int_{B^*} \right) ,
\]

For \( J/\psi(1^-) \rightarrow B^*(\frac{7}{2}^-) B(\frac{1}{2}^-) + B^*(\frac{7}{2}^+) B(\frac{1}{2}^+) \rightarrow \gamma B(\frac{1}{2}^+) B(\frac{1}{2}^-) \) we get the following six covariant amplitudes

(3, 3, 3)

\[
U_{i,1}^{\mu\nu} = g_{J/\psi}^{i_a} g_{\gamma}^{i_b} \left( - \frac{4}{7} C_{B^*B^*} \sum_{S_{B^*}} i \phi_{\beta\lambda\eta\gamma}^{(3)} \psi_{\alpha\delta\tau}^{(3)} \bar{F}_{(B^*2)} \tilde{T}^{(5)}_{\beta\lambda\sigma\nu} \int_{B^*} \right) ,
\]

(4, 3, 3)

\[
U_{i,2}^{\mu\nu} = g_{J/\psi}^{i_b} g_{\gamma}^{i_a} \left( - \frac{4}{7} C_{B^*B^*} \sum_{S_{B^*}} i \phi_{\beta\lambda\eta\gamma}^{(3)} \psi_{\alpha\delta\tau}^{(3)} \bar{F}_{(B^*2)} \tilde{T}^{(5)}_{\beta\lambda\sigma\nu} \int_{B^*} \right) ,
\]

(4, 5, 3)

\[
U_{i,3}^{\mu\nu} = g_{J/\psi}^{i_c} g_{\gamma}^{i_b} \left( - \frac{4}{7} C_{B^*B^*} \sum_{S_{B^*}} i \phi_{\beta\lambda\eta\gamma}^{(3)} \psi_{\alpha\delta\tau}^{(3)} \bar{F}_{(B^*2)} \tilde{T}^{(5)}_{\beta\lambda\sigma\nu} \int_{B^*} \right) .
\]
\[ + \sum_{S_{B^*}} \phi^{(3)}_{\beta \delta \lambda} \Psi^{(4)}_{\alpha \delta \rho} \tilde{T}^{(5)\alpha \delta \rho \mu \nu} \bar{T}^{(4)\lambda \sigma \nu} f_{B^*} \]

\[ + \sum_{S_{B^*}} \Psi^{C(\phi)}_{\alpha \delta \rho} \phi^{C(\phi)}_{\beta \lambda \sigma} \tilde{T}^{(5)\alpha \delta \rho \mu \nu} \bar{T}^{(4)\lambda \sigma \nu} f_{B^*} \]  

(3, 3, 4)

\[ U_{i,4}^{\mu \nu} = g_{j/\psi}^{i,a} g_{\gamma}^{i,b} (- \sum_{S_{B^*}} \Phi^{(4)}_{(\beta \gamma \delta \lambda \phi)} \epsilon^{\alpha \delta \rho \tau} \tilde{T}^{(5)\alpha \delta \rho \mu \nu} \bar{T}^{(4)\lambda \sigma \nu} f_{B^*} \]

\[ - \sum_{S_{B^*}} \Psi^{C(\phi)}_{(\beta \gamma \delta \lambda \phi)} \epsilon^{\alpha \delta \rho \tau} \tilde{T}^{(5)\alpha \delta \rho \mu \nu} \bar{T}^{(4)\lambda \sigma \nu} f_{B^*} \]  

(4, 4, 4)

\[ U_{i,5}^{\mu \nu} = g_{j/\psi}^{i,b} g_{\gamma}^{i,b} \left( \sum_{S_{B^*}} i \Phi^{(4)}_{\beta \gamma \delta \lambda \phi} \epsilon^{\alpha \delta \rho \tau} \tilde{T}^{(5)\alpha \delta \rho \mu \nu} \bar{T}^{(4)\lambda \sigma \nu} f_{B^*} \right) \]

\[ - \sum_{S_{B^*}} i \Psi^{C(\phi)}_{\beta \gamma \delta \lambda \phi} \epsilon^{\alpha \delta \rho \tau} \tilde{T}^{(5)\alpha \delta \rho \mu \nu} \bar{T}^{(4)\lambda \sigma \nu} f_{B^*} \]  

(4, 5, 4)

Note that where

\[ \psi^{(n)}_{\mu_1 \cdots \mu_n} = \psi^{(n)}_{\mu_1 \cdots \mu_n} (p_B, S_B; p_{\bar{B}}, S_{\bar{B}}), \]

\[ \phi^{(n)}_{\mu_1 \cdots \mu_n} = \phi^{(n)}_{\mu_1 \cdots \mu_n} (p_B, S_B; p_{\bar{B}}, S_{\bar{B}}), \]

\[ \psi^{C(n)}_{\mu_1 \cdots \mu_n} = \psi^{C(n)}_{\mu_1 \cdots \mu_n} (p_{\bar{B}}, S_{\bar{B}}; p_B, S_B), \]

\[ \phi^{C(n)}_{\mu_1 \cdots \mu_n} = \phi^{C(n)}_{\mu_1 \cdots \mu_n} (p_{\bar{B}}, S_{\bar{B}}; p_B, S_B). \]

and

\[ C_{B^* B_{\gamma}} = \frac{(m_B^2 - m_{B^*}^2)^2}{m_{B^*}^2}, \quad C_{B^* B_{\gamma}} = \frac{(m_{B^*}^2 - m_B^2)^2}{m_B^2}. \]

For the reaction \( J/\psi \rightarrow p\bar{p} \rightarrow \gamma p\bar{p} \) we get following two covariant amplitudes with two independent coupling constants \( g^{p,a} \) and \( g^{p,b} \) which are determined by the experiment.

\[ (1, 0, 1) \]

\[ U_{p,1}^{\mu \nu} = g^{p,a} \left( \sum_{S_{\rho^*}} \Phi^{(1)\mu} \Psi^{(1)\mu} f_{\rho^*} - \sum_{S_{\rho^*}} \Psi^{C(1)\mu} \phi^{C(1)\mu} f_{\rho^*} \right), \]  

(89)

\[ (1, 2, 1) \]

\[ U_{p,2}^{\mu \nu} = g^{p,b} \left( \sum_{S_{\rho^*}} \Phi^{(1)\mu} \tilde{T}^{(2)\mu \rho} f_{\rho^*} - \sum_{S_{\rho^*}} \Psi^{C(1)\mu} \phi^{C(1)\mu} \tilde{T}^{(2)\mu \rho} f_{\rho^*} \right), \]  

(90)
here we use $p^*$ and $\bar{p}^*$ as intermediate states instead of $p$ and $\bar{p}$; one can read $f_{p^*}^{\nu}$ and $\bar{f}_{\bar{p}^*}^{\nu}$ from equation (47), by setting $\Gamma_{p^*}$ and $\Gamma_{\bar{p}^*}$ to zero; moreover

$$
\Phi_{\nu}^{p^*}(p_\nu, S_{p}, p_\nu^*, S_{p^*}) = -e\bar{u}(p_\nu, S_{p})(\gamma^\nu - i\frac{\kappa N}{2M_N}\sigma^{\mu\nu}p_{3\mu})u(p_\nu^*, S_{p^*}),
$$

$$
\Phi_{\nu}^{\bar{p}^*}(p_\nu^*, S_{p^*}, p_\nu, S_{p}) = -e\bar{v}(p_\nu^*, S_{p^*})(\gamma^\nu - i\frac{\kappa N}{2M_N}\sigma^{\mu\nu}p_{3\mu})v(p_\nu, S_{p}),
$$

which are obtained from the effective lagrangian of $NN\gamma$.

5 Conclusion

To provide a consistent and complete picture of baryon resonances, the various possible production and decay channels need to be explored. With estimated branching ratios for contribution of the $N^*(1440)$, $N^*(1535)$ and $N^*(1520)$ to the process $J/\psi \rightarrow \gamma p\bar{p}$, we propose to study radiative decays of excited nucleon and hyperon states through $J/\psi \rightarrow B^*\bar{B} + \bar{B}^*B \rightarrow \gamma BB$ processes at BESIII. We provide explicit partial wave amplitude formulae for these processes with $J^P$ for $B^*$ is $\frac{1}{2}^\pm$, $\frac{3}{2}^\pm$, $\frac{5}{2}^\pm$, $\frac{7}{2}^\pm$. These formulae can be used to perform partial wave analysis of forthcoming high statistics data from BESIII on these channels to extract various useful information on the excited baryons. The BESIII can produce ground state $(N,\Lambda,\Sigma,\Xi)$ and the excited baryon states $(N^*,\Lambda^*,\Sigma^*,\Xi^*)$ via $J/\psi \rightarrow B^*\bar{B} + \bar{B}^*B \rightarrow \gamma BB$, as well as can do further investigations into the dynamics of the excited baryons. We hope that our knowledge about the structure of the excited baryon resonances and about the mechanisms of nucleon and hyperon production will be clarified by the near future studies at BESIII.

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Appendix: The Relation with the Helicity Amplitude

In this appendix, we discuss the relation between amplitudes in the L-S and helicity formalism for $B^* \rightarrow B\gamma$. From Ref.[36], the radiative decay width is related to the
helicity amplitudes $A_{1/2}$ and $A_{3/2}$ as

$$\Gamma_\gamma = \frac{k^2 m_B}{4\pi m_{B^*} 2J + 1} \frac{8}{(|A_{3/2}|^2 + |A_{1/2}|^2)},$$

(91)

where $k$ is the three momentum of photon, and $J$ is the total spin of $B^*$. Let us consider that the photon is moving along the $z$ axis, and the photon is right-handed polarized, in other words, the spin of the photon is along the $z$-axis. $A_{3/2}$ is the spin-3/2 helicity amplitude of the initial $B^*$ in a state with $|J, 3/2>$ and final $B$ in a state with $|1/2, 1/2>$, and $A_{1/2}$ denotes the spin-1/2 helicity amplitude of the $B^*$ with $|J, 1/2>$ and final $B$ with $|1/2, -1/2>$.

In the L-S Scheme the decay amplitude formulae for $B^* \to B\gamma$ are

$$\Gamma_\gamma = \frac{k}{2\pi m_{B^*} 2J + 1} \sum_{s_{B^*}, s_B, s_\gamma} |M|^2$$

$$= \frac{k}{2\pi m_{B^*} 2J + 1} (|M_{(3/2, 1/2, 1)}|^2 + |M_{(1/2, -1/2, 1)}|^2 + |M_{(-1/2, 1/2, -1)}|^2 + |M_{(-3/2, -1/2, -1)}|^2)$$

$$= \frac{k}{\pi m_{B^*} 2J + 1} (|M_{(3/2, 1/2, 1)}|^2 + |M_{(1/2, -1/2, 1)}|^2).$$

(92)

By comparing Eq.(91) with the Eq.(92), we can have the relation between the helicity and L-S amplitudes as follows

$$A_{3/2} = \frac{1}{2k} |M_{(3/2, 1/2, 1)}|^2 = \frac{1}{2k} |(g_1 M_1(3/2, 1/2, 1) + g_2 M_2(3/2, 1/2, 1))|^2,$$

(93)

$$A_{1/2} = \frac{1}{2k} |M_{(1/2, -1/2, 1)}|^2 = \frac{1}{2k} |(g_1 M_1(1/2, -1/2, 1) + g_2 M_2(1/2, -1/2, 1))|^2.$$  

(94)

As an example, now we calculate the $M(s_{B^*}, s_B, s_\gamma)$ for $B^*(3^+ \to B\gamma)$. From Eq.(38), the two dependent amplitudes can be written as

$$M_1 = i\phi^{(1)}(\mu) \epsilon^{\rho\lambda\sigma} \tilde{\epsilon}_\lambda^{(1)} \rho_{B^*} \sigma \epsilon_{\nu}^*, $$

(95)

$$M_2 = \frac{2}{3} (\tilde{r} \cdot \tilde{r}) \Phi^{(2)}(\nu) \tilde{\epsilon}_\lambda^{(1)} \epsilon_{\nu}^* + \Phi^{(2)}(\tilde{r}) \tilde{\epsilon}_\lambda^{(3)} \mu \nu \epsilon_{\nu}^*. $$

(96)

Before starting our calculation of $M(s_{B^*}, s_B, s_\gamma)$, we should define wave functions of particles

$$u(p_B, 1/2) = \sqrt{\frac{E_B + m_B}{2m_B}} (1, 0, \frac{k}{E_B + m_B}, 0),$$

(97)

$$\bar{u}(p_B, -1/2) = \sqrt{\frac{E_B + m_B}{2m_B}} (0, 1, 0, \frac{-k}{E_B + m_B}),$$

(98)

$$u(m_{B^*}, 1/2) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad u(m_{B^*}, -1/2) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix},$$

(99)
where $B^*$ is at rest, in other words, $p_{B^*} = (m_{B^*}, 0, 0, 0)$, $p_B = (E_B, 0, 0, -k)$ and $p_\gamma = (k, 0, 0, k)$. By using Eqs. (3-4), the different states of $\phi_\mu^{(1)}$ and $\Phi_\mu^{(2)}$ can be obtained

$$
\phi_\mu^{(1)}(s_{B^*} = 1/2, s_B = -1/2) = \sqrt{\frac{E_B + m_B}{2m_B}} \sqrt{\frac{1}{3}} \epsilon_\mu m_{B^*}, 1, 1),
$$

$$
\phi_\mu^{(1)}(s_{B^*} = 3/2, s_B = 1/2) = \sqrt{\frac{E_B + m_B}{2m_B}} \epsilon_\mu m_{B^*}, 1, 1),
$$

$$
\Phi_\mu^{(2)}(s_{B^*} = 1/2, s_B = -1/2) = \sqrt{\frac{E_B + m_B}{2m_B}} \frac{1}{3c_\mu(m_{B^*}, 1, 1)\epsilon_\nu(m_{B^*}, 1, 0)
+\epsilon_\mu(m_{B^*}, 1, 0)\epsilon_\nu(m_{B^*}, 1, 1)),
$$

$$
\Phi_\mu^{(2)}(s_{B^*} = 3/2, s_B = 1/2) = -\sqrt{\frac{E_B + m_B}{2m_B}} (\epsilon_\mu(m_{B^*}, 1, 1)\epsilon_\nu(m_{B^*}, 1, 0)
+\epsilon_\mu(m_{B^*}, 1, 0)\epsilon_\nu(m_{B^*}, 1, 1)).
$$

At last, we get the following amplitudes $M(s_{B^*}, s_B, s_\gamma)$

$$
M_1(1/2, -1/2, 1) = \frac{2k}{\sqrt{3}} \sqrt{\frac{E_B + m_B}{2m_B}},
$$

$$
M_1(3/2, 1/2, 1) = \frac{2k}{\sqrt{3}} \sqrt{\frac{E_B + m_B}{2m_B}},
$$

$$
M_2(1/2, -1/2, 1) = \frac{-24k^3}{\sqrt{3}} \sqrt{\frac{E_B + m_B}{2m_B}},
$$

$$
M_2(3/2, 1/2, 1) = \frac{8k^3}{\sqrt{3}} \sqrt{\frac{E_B + m_B}{2m_B}},
$$

where we have used the following relations

\begin{align*}
i
\epsilon_{\mu\nu\alpha\beta} &= \gamma_5 (\gamma_{\mu} \gamma_{\nu} \gamma_{\alpha} \gamma_{\beta} - \gamma_{\mu} \gamma_{\alpha} g_{\nu\beta} + \gamma_{\mu} \gamma_{\beta} g_{\nu\alpha} - \gamma_{\mu} \gamma_{\nu} g_{\alpha\beta})
- \gamma_{\nu} g_{\mu\beta} + \gamma_{\mu} g_{\nu\beta} - \gamma_{\alpha} \gamma_{\beta} g_{\mu\nu} + g_{\nu\mu} g_{\alpha\beta} - g_{\nu\beta} g_{\mu\alpha} + g_{\mu\nu} g_{\alpha\beta} + g_{\mu\beta} g_{\nu\alpha},
\end{align*}

$$
\gamma^5 = \begin{pmatrix} 0 & \sigma_0 \\ \sigma_0 & 0 \end{pmatrix}, \quad \gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}, \quad i = 1, 2, 3.
$$

Now we list the relation between the square of the helicity amplitudes and square of the coupling constants from our amplitudes for $B^*(1^+; 1^2, 2^2; 3^2, 4^2) \rightarrow B\gamma$ as follows

$$
M = g_\gamma^a M_1 + g_\gamma^b M_4,
$$

(112)
\(B^*(\frac{1^+}{2})\):

\[
M_1 = i \phi_\mu^{(1)} \epsilon^{\mu\nu\lambda\sigma} \epsilon_{\nu} \tilde{t}_\lambda \hat{p}_{B^*\sigma},
\]

\(|A_{1/2}|^2 = \frac{E_B + m_B}{2m_B} 4k|g_\gamma^a|^2.\)  

\(B^*(\frac{1^-}{2})\):

\[
M_1 = -\frac{2}{3} (\tilde{r} \cdot \tilde{r}) \phi_\mu^{(1)} \epsilon^{\mu} + \phi_\mu^{(1)} \tilde{t}_\mu^{(2)} \epsilon^{\nu},
\]

\(|A_{1/2}|^2 = \frac{E_B + m_B}{2m_B} \frac{2}{3} k|g_\gamma^a - 2g_\gamma^b|^2.\)

\(B^*(\frac{3^+}{2})\):

\[
M_1 = \frac{3}{5} \phi_\mu^{(2)} \epsilon^{\mu\nu\lambda\sigma} \epsilon_{\nu} \tilde{t}_\lambda \hat{p}_{B^*\sigma},
\]

\(|A_{1/2}|^2 = \frac{E_B + m_B}{2m_B} \frac{16k}{3} |g_\gamma^a|^2.\)

\(B^*(\frac{3^-}{2})\):

\[
M_1 = \frac{2}{3} (\tilde{r} \cdot \tilde{r}) \phi_\mu^{(1)} \epsilon^{\mu} + \phi_\mu^{(2)} \tilde{t}_\mu^{(2)} \epsilon^{\nu},
\]

\(|A_{1/2}|^2 = \frac{E_B + m_B}{2m_B} \frac{2}{3} k|g_\gamma^a - 2g_\gamma^b|^2.\)

\(B^*(\frac{5^+}{2})\):

\[
M_1 = -\frac{3}{5} (\tilde{r} \cdot \tilde{r}) \phi_\mu^{(2)} \epsilon^{\mu\nu\lambda\sigma} \epsilon_{\nu} \tilde{t}_\lambda \hat{p}_{B^*\sigma},
\]

\(|A_{1/2}|^2 = \frac{E_B + m_B}{2m_B} \frac{32k}{5} |g_\gamma^a|^2.\)

\(B^*(\frac{5^-}{2})\):

\[
M_1 = i \phi_\mu^{(2)} \epsilon^{\mu\nu\lambda\sigma} \epsilon_{\nu} \tilde{t}_\lambda \hat{p}_{B^*\sigma},
\]
\[ M_2 = -\frac{4}{7} (\vec{r} \cdot \vec{r}) \Phi^{(3)}_{\mu\nu\lambda} e_{\mu}^* \bar{t}_{\nu\lambda}^{(1)} + \Phi^{(3)}_{\mu\nu\lambda} \bar{t}^{(4)}_{\mu\nu\lambda\sigma} e_{\sigma}^*, \]  
\[ |A_{1/2}|^2 = \frac{E_B + m_B}{2m_B} \frac{8}{5} k^3 |g_{\gamma}^a - 16 g_{\gamma}^b k^2|^2, \]  
\[ |A_{3/2}|^2 = \frac{E_B + m_B}{2m_B} \frac{16}{5} k^3 | - g_{\gamma}^a + 4 g_{\gamma}^b k^2|^2. \]

\[ B^*(\frac{7^+}{2}) : \]
\[ M_1 = i \Phi^{(3)}_{\mu\alpha\beta\gamma} e_{\mu}^* \bar{t}_{\alpha\beta\gamma}^{(1)} \rho_{B^*\sigma}, \]  
\[ M_2 = \Phi^{(4)}_{\mu\nu\lambda\sigma} e_{\mu}^* \bar{t}_{\nu\lambda\sigma}^{(2)} + \Phi^{(4)}_{\mu\nu\lambda\sigma} \bar{t}^{(5)}_{\mu\nu\lambda\sigma\delta} e_{\delta}^*, \]  
\[ |A_{1/2}|^2 = \frac{E_B + m_B}{2m_B} \frac{128}{35} k^5 |g_{\gamma}^a - 20 g_{\gamma}^b k^2|^2, \]  
\[ |A_{3/2}|^2 = \frac{E_B + m_B}{2m_B} \frac{128}{21} k^5 |g_{\gamma}^a + 12 g_{\gamma}^b k^2|^2. \]

\[ B^*(\frac{7^-}{2}) : \]
\[ M_1 = -\frac{4}{7} (\vec{r} \cdot \vec{r}) \Phi^{(3)}_{\mu\nu\lambda} e_{\mu}^* \bar{t}_{(1)\nu\lambda}^{} + \Phi^{(3)}_{\mu\nu\lambda} \bar{t}^{(4)}_{\mu\nu\lambda\sigma} e_{\sigma}^*, \]  
\[ M_2 = i \Phi^{(4)}_{\mu\alpha\beta\gamma} e_{\mu}^* \bar{t}_{\alpha\beta\gamma}^{(1)} \rho_{B^*\sigma}, \]  
\[ |A_{1/2}|^2 = \frac{E_B + m_B}{2m_B} \frac{512}{35} k^7 | - g_{\gamma}^a + 5 g_{\gamma}^b|^2, \]  
\[ |A_{3/2}|^2 = \frac{E_B + m_B}{2m_B} \frac{512}{21} k^7 | - g_{\gamma}^a - 3 g_{\gamma}^b|^2. \]

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