Real-Time Observation of the Formation of Dual-Wavelength Mode Locking

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Dual-wavelength mode-locking lasers delivering asynchronous dual-color solitons are excellent sources for dual-comb spectroscopies. Up to now, diverse approaches have been developed to implement dual-wavelength lasers. However, the buildup dynamics of dual-color solitons have not been well revealed. Here, by means of the time-stretch dispersive Fourier transform technique, real-time observation of the buildup process and propagation dynamics of asynchronous dual-color solitons in a mode-locked fiber laser are reported. In addition to the typical phases that have been well discussed on buildup dynamics of conventional solitons, such as raised relaxation oscillation and spectral beating behavior, it is observed that optical or solitons collision plays a key role in facilitating the buildup of dual- or multi-wavelength mode locking. Specifically, when the dual-wavelength solitons carrying the same color photons encounter in the time domain, their collision induces the solitons explosion and brings about the new evolved mode-locking pulse at another waveband. Experimental results agree well with numerical predictions and bring insights to the understanding of nonlinear dynamics in fiber lasers.

1. Introduction

Dual-wavelength mode-locked fiber lasers are of increasing interest due to their convenience in implementing dual-comb metrology applications, such as high-resolution spectroscopy and ranging. By subtly designing the resonant cavity, the fiber laser is enabled to deliver asynchronous pulse trains, which have a slight repetition rate difference of Δf_r, operating at dual wavebands. Since the generated two pulse trains share a segment of or the whole cavity, mutual coherence is significantly enhanced. Without complicated electrical servo systems, fluctuation of Δf_r can be suppressed to mHz level, which is of great interest for dual-comb spectroscopies. To realize the compact, robust, and practical dual-comb source, enormous efforts have been devoted. In theory, Akhmediev et al. predicted the co-existence of dual-frequency solitons in a passively mode-locked fiber laser. After that, Farnum et al. developed the mode-locking theory of dual- or multi-wavelength lasers and numerically demonstrated that the combination of wavelength division multiplexing interactions and the self- and cavity-saturation gain effects play a key role in multi-wavelength oscillation. By implementing various techniques of wavelength-division multiplexing, such as introducing comb filtering effect, building two optical routes utilizing a bidirectional loop cavity, taking advantage of polarization-multiplexing technique and meanwhile, carefully leveraging gain or loss for laser operations at different wavebands, dual-wavelength mode-locked fiber lasers have been extensively developed. In contrast, buildup dynamics for dual-wavelength mode locking is yet to be unveiled.

Time-stretch dispersive Fourier transform (TSDFT) method is an emerging technique for the studies of real-time transient processes in nonlinear optical systems. Here, the spectrum of an optical pulse is mapped through dispersion into a temporal waveform that can be subsequently digitized and processed in real time. The TSDFT technique has been successfully employed to study the buildup and internal dynamics of soliton molecules, behaviors of soliton buildup, and exotic dynamics of soliton explosions. These reports focus on the studies of conventional single-wavelength solitons. More recent experimental observations on dual-wavelength mode-locked fiber lasers have shown that the TSDFT technique is also competent in revealing transient dynamics of dual-color solitons. For example, based on this technique, ‘periodic’ soliton collisions...
as well as ‘periodic soliton explosions’ and dissipative rogue waves were revealed in dual-wavelength mode-locked fiber lasers.\cite{30,31} Similarly, real-time observation of the buildup and collision dynamics of asynchronous vector solitons were systematically investigated.\cite{32,33}

Despite these significant investigations on the dual-wavelength mode-locking fiber lasers, a fundamental question remains elusive: how does noise evolve to dual-wavelength mode-locking solitons? In particular, the formation of a single soliton refers to the universal behavior of many nonlinear systems featuring the scenario of ‘survival of the strongest’.\cite{34} In this respect, the formation of dual-color solitons, which are even asynchronous, may result from different dynamics. In a conventional single-wavelength operation, it was demonstrated that soliton splitting should be responsible for the generation of bound solitons (soliton molecules) and the harmonic mode-locking solitons.\cite{20,24,35} Regarding the dual- or multi-wavelength lasers, several reports have spectrally resolved the collision-induced ultrafast dynamics and observed the collision-induced perturbations.\cite{36,37} Therefore, it is demanded to explore how the solitons collision is involved in the buildup and propagation of dual-wavelength solitons to address these conjectures.

By designing a self-started dual-wavelength mode-locked fiber laser, here we report the first direct observation of dual-wavelength solitons’ transient dynamics from the formation to annihilation by means of the TSDFT technique. Five transition stages are summarized to characterize the buildup dynamics of dual-wavelength mode-locking, starting from raised relaxation oscillation, successively experiencing Q-switching induced beating dynamics, multi-wavelength oscillation, optical and pulse interactions, and dual-wavelength mode-locking. During the evolution process of the asynchronous pulses, solitons collision not only induces the generation of dual- or multi-wavelength solitons but also results in partial or full soliton annihilation. In a word, the origin for the generation of the dual-wavelength pulses also leads to its ruin. At last, stable dual-wavelength mode-locked pulses are obtained, and the transition process of dual-color solitons collision is unveiled and discussed.

2. Results

2.1. Experimental Setup

The experimental setup, containing a dual-wavelength fiber laser oscillator and a TSDFT setting, is shown in Figure 1. We employed an all polarization-maintaining nonlinear amplifying loop mirror-based oscillator to generate the asynchronous dual-wavelength pulse trains. The related mode-locking mechanism relies on the intensity-dependent nonlinear phase shift difference between counter-propagated lasers in the Sagnac loop. It is thus imperative to accumulate sufficient phase difference to initiate self-starting mode-locking. For this purpose, the 1.0 m-long Yb-doped gain fiber is asymmetrically relative to the central 2 × 2 balanced optical coupler. Additionally, a π/2 non-reciprocal phase shifter is used to further reduce the mode-locking threshold. At one port of the central optical coupler, two collimators, an open-air section comprising a quarter-wave plate, and an optical fiber mirror form the Lyot filter to enable the multi-wavelength oscillation. With a pump power above 180 mW injected into the laser cavity through a 1030 nm/980 nm wavelength division multiplexer (WDM), self-started dual-wavelength mode-locking is guaranteed. A 10:90 optical coupler is utilized to output the pulses circulated in the oscillator. Output characteristics of the dual-wavelength mode-locking pulses are presented in Figure S1, Supporting Information (see Supporting Information for output characteristics of the laser and accuracy of TSDFT).

As shown in Figure 1, one port of the output coupler (OC1) is directly used for optical measurement. Signal laser delivered from the other port is firstly enhanced in output power for fiber loss compensation and then sent to the TSDFT setting. Implemented by temporally stretching the pulses through a τ = 18 km long single-mode fiber with a dispersion of $d^{2} \approx -45$ ps (nm km)$^{-1}$,

![Figure 1](Image)

**Figure 1.** Experimental setup containing a self-started dual-wavelength laser oscillator for generating asynchronous dual-wavelength solitons and TSDFT setting for real-time measurement. LD: laser diode; WDM: wavelength division multiplexer; YSF: Yb-doped single-mode fiber; OC1: fiber coupler with 10:90 splitting ratio; OC2: fiber coupler with 50:50 splitting ratio; C1 and C2: collimators; QWP: quarter waveplate; OFM: optical fiber mirror; Amp.: fiber amplifier; SMF: single-mode fiber.
detecting by a high-speed photo-detector with a bandwidth of 14 GHz, and capturing by a real-time oscilloscope with a sampling rate of 80 gigasamples per second and a bandwidth of 33 GHz (Keysight DSAV334A), the TSDFT setting has a resolution of \( \approx 0.08 \text{ nm} \), which is precise enough to outline the spectral profiles of the mode-locked pulse [see Figure S1a, Supporting Information]. The routine to capture the rapid signal during the buildup for dual-wavelength mode locking is as follows. First, by adjusting the rotation angle of the quarter-wave plate, stable self-started dual-wavelength mode locking is guaranteed. Next, the pump is switched on directly to the mode-locking threshold. Upon the appearance of a signal, the oscilloscope triggers and records the real-time signal.

A conceptual representation of the transition for dual-wavelength mode locking and its experimental evidence are depicted in Figure 2a,b, respectively. The pump power is connected to the laser cavity at time \( T = 0 \). When \( 0 < T < 1900 \mu\text{s} \), the population inversion increases in the gain medium. The laser outputs noise emissions. Afterward, stimulated emission becomes dominant, resulting in the generation of Q-switching pulses. The spacing between two adjacent giant pulses is about 30 \( \mu\text{s} \), which is uniform in the Q-switching stage. 100 \( \mu\text{s} \) later, the mode-locking state is observed and it lasts about 1.4 ms. At the mode-locking stage, the temporal interferogram envelope appeared in the middle demonstrates the asynchronous dual- or multi-wavelength pulses collision and interaction. Around 3.4 ms, a drastic pulse explosion occurs and destroys the mode-locking.

2.2. Experimental Results

To gain deep insight into dual-wavelength pulses evolution, we analyzed the shot-by-shot experimental data by segmenting the TSDFT record time series with a 39.2 ns periodicity, which corresponds to the cavity roundtrip time. The whole time interval in Figure 2b was redrawn as a spectrottemporal picture, shown in Figure 3. For conventional solitons, the pulse evolution process can intuitively display the spectral response by converting the time domain to frequency domain coordinates. However, for unusual solitons with complex interactions and internal motions, that could not be applicable.\(^{[24,38]}\) With our laser setup, asynchronous dual- or multi-wavelength solitons, soliton molecules, and harmonic solitons are all obtained when the self-starting mode-locking is initiated. Therefore, the \( y \) axis in this plot still depicts the time within a single roundtrip (i.e., from 0 to 39.2 ns), while the \( x \) axis shows the dynamics across consecutive roundtrips. After the relaxation oscillation and Q-switching (the first 7000 roundtrips), complex soliton patterns...
operating at two- or multi-wavebands coexist in the cavity during the 7000th to the 42000th roundtrips until a pulse explosion interrupts the mode locking.

Since the pulse trains operating at different wavebands have different group velocities, the evolved pulse trajectories criss-cross in Figure 3. We marked the horizontal pulse trajectories distributed at different time positions as 1, 1', ..., corresponding to the harmonic solitons. As for the other trajectories with different slopes, they are marked as 2, 3, ..., respectively, indicating the pulses operating at other wavebands. Inferred from Figure S2, Supporting Information (Supporting Information for output spectrum of Q-switching pulses), which indicates the potential operating wavebands of the laser, all the horizontal trajectories correspond to soliton packets centered at 1034 nm. While the other trajectories with positive or negative slopes correspond to soliton packets operating at right or left adjacent pass bands around 1034 nm of the Lyot filter, respectively.

Throughout Figure 3, formation process of the dual- or multi-wavelength pulses can be comprehended as follows: at the first 7000th roundtrips, the laser experiences relaxation oscillation and Q-switching to form the noise pulses. Then, multiple pulses operating at two wavebands are initiated around the 7000th roundtrip. Owing to fiber dispersion, they collide around the 13500th roundtrip, spurring the generation of newly evolved pulse trajectories with different slopes. From then to the 36000th roundtrip, multi-wavelength mode-locked solitons circulate in the laser cavity, accompanying solitons collision, explosion, and partial annihilation. Around the 42000th roundtrip, a drastic pulse explosion interrupts the mode locking. The laser re-enters Q-switching stage. Conspicuously, solitons collision and explosion induce the generation of asynchronous pulse trains operating at new wavebands, as well, cause the partial or full soliton annihilation.

### 2.3. Dual-Wavelength Solitons Generation

In order to see clearly how the asynchronous dual-wavelength mode-locking pulses evolve from Q-switching pulses, we magnified the A region in Figure 3 as Figure 4a. At the first 90 roundtrips [See Figure 4a], distinct intensity patterns given rise by modulation instability are observed.[39] Then the stimulated amplification enhances the patterns' intensity and brings about a dominant pulse at the timeline of 11.4 ns. Within the next 60 roundtrips, the dominant pulse is significantly broadened due to the self-phase modulation and gradually evolves into a stationary soliton at 1034 nm (the horizontal pulse trajectory). As for the pulse operating at the other waveband, the starting time for the stationary soliton falls behind at the beginning, but the finishing time is ahead of the former one. Similarly, after a rapid spectral broadening within 10 roundtrips, stationary soliton operating at 1037 nm (the up-tilted pulse trajectory) is achieved. It is worth noting that, at the 131th roundtrip, the broadened transient pulse at 1037 nm is extended to impact the evolved pulse at 1034 nm. That should be caused by the noise pulses collision-induced soliton explosion. Several roundtrips later, a temporally dual-wavelength mode locking is achieved.

The energy evolution shown in Figure 4b gives complementary insights into the buildup dynamics of dual-wavelength pulses. The slowly climbing, saturation, burst, decreasing, perturbation, and stabilization of energy evolution correspond to the stimulated amplification, Q-switching, single-pulse formation, solitons collision, and dual-wavelength mode-locking processes of the laser. Figure 4c shows detailed single-shot pulse evolution at several typical roundtrips. To protect the high-speed photo-detector from the giant Q-switching pulses, we attenuated the average laser power after the TSDFT setting. In this condition, several single-shot spectral curves have relatively low signal-to-noise ratios. In the first 75 roundtrips, the single-shot pulse displays a multi-peaks structure, indicating the modulation instability induced intensity patterns. At the 90th roundtrip, the bandwidth of the pulse at 11.4 ns reaches the maximum owing to the self-phase modulation.[40] The pulse at the other waveband is still noise patterns. After that, the horizontal pulses present a relatively moderate buildup process. Output spectra of the pulses gradually broaden with the roundtrips until stable mode-locking is realized. As for the up-tilted pulse, it is dramatically broadened and almost covers the other pulse at the 131th roundtrip. Then, a

![Figure 4](https://www.advancedsciencenews.com)

**Figure 4.** Close-up of the boxed A area in Figure 3. a) Real-time characterization of the entire buildup process of a dual-wavelength mode-locked laser. b) The energy evolution. c) The detailed pulse evolution at several typical roundtrips.
rapid spectral narrowing occurs, and stable mode-locking is attained within five roundtrips. Around the 156th roundtrip, both mode-locking processes for the dual-wavelength pulses finish. The interference patterns observed on the waveform of the horizontal pulses indicate its bound pulse state.

2.4. Multi-Wavelength Solitons Generation

Nevertheless, the chromatic dispersion, which manifests through the wavelength dependence of the refractive index, disturbs the temporarily steady mode-locking state. The train of the pulses comprising the red-shifted components travels faster than that comprising the blue-shifted components in a normal dispersion regime. As a result, when the pulse operating at 1037 nm catches up and passes through the pulse operating at 1034 nm, pulse collision inevitably occurs. In particular, when the two pulse trains carry the same optical components, the collision process features typical nonlinear characteristics, which are generally reified as soliton collision, explosion, annihilation, etc.

Around the 13500th roundtrip, the pulses train operating at 1037 nm collides with that operating at 1034 nm. Figure 5a is the enlarged part of the B regions in Figure 3a, and it elaborates the pulses collision process. At the first 60 roundtrips, the pulse collision solely induces a linear intensity superposition. Several roundtrips later, a drastic pulse distortion, which indicates soliton explosion in the spectral domain, takes place. Then, two newly formed pulse trajectories with different slopes are generated. The pulse collision induces the soliton explosion, which leads to a transient spectral broadening and gives rise to the multi-wavelength oscillation.

Figure 5b presents the pulse energy evolution. At the first 60 roundtrips, the evolution does not show many fluctuations, indicating a linear pulse collision regime. When the pulse collision enters the nonlinear regime, soliton explosion occurs, accompanying rapidly increased pulse energy and spectral broadening. Next, old and new pulses operating at different wavebands begin to rebuild and establish upon the soliton explosion-induced spectral ruins. Four-wavelength mode-locking pulses are evolving into shape within 50 roundtrips. Since multi-wavelength pulses cycles in the resonant cavity, pulses collisions and interactions occur more frequently, as well as the pulse distortions and energy fluctuations [see Figure 5a,b]. After 200 roundtrips, these multi-wavelength pulses walk away, and their interactions become weak. The pulse energy decreases to about half of the maximum value at the 110th roundtrip. Figure 5c shows the single-shot pulse evolution at several typical roundtrips.

2.5. Partial and Full Soliton Annihilation

Subsequently, the four-wavelength mode-locking pulses continuously cycle in the laser cavity, which can be seen in Figure 3. Around the 18000th roundtrip, the pulse with a negative slope collides with the horizontal pulse located at 11.4 ns. After a dramatic nonlinear collision and explosion, the down-tilted trajectory disappears, indicating an evident partial soliton annihilation. Due to the relatively stronger intensity, the horizontal trajectory survives in the soliton collision, corroborating the scenario of ‘survival of the strongest’. Around the 24000th roundtrip, partial soliton annihilation can also be observed. The pulse trajectory with a positive slope survives this time. Around the 27000th roundtrip, a new trajectory marked as pulse 5 with a more positive slope is generated after the collision process between the marked pulse 3 and pulse 1. Around the 42000th roundtrip, a drastic pulse explosion is observed, as enlarged in Figure 6. In this case, triple-wavelength pulses together enter the collision regime and cause full soliton annihilation. The blue curve plotted in Figure 6 presents the energy evolution.

2.6. Self-Started Dual-Wavelength Mode Locking and Dual-Color Solitons Collision

In the above-discussed TSDFT measurement, the drastic pulse explosion destroyed the multi-wavelength mode locking and drove the laser to re-enter Q-switching state. Then, we restarted a measurement, recorded the entire buildup process of solitons from spontaneous noise to dual-wavelength mode-locking state, and redone the TSDFT analysis.

Figure 5. Close-up of the boxed B area in Figure 3. a) Real-time characterization of the entire transformation process for a dual-wavelength operation to evolve into a multi-wavelength operation. b) The energy evolution. c) The detailed pulse evolution at several typical roundtrips.
For the sake of simplification, the noise pulse stage was excluded. As shown in Figure 7, the process starts from Q-switching and ends up with dual-wavelength multi-pulses mode locking. Due to the excessive pump, the initiated mode-locking soliton pattern is complicated, including soliton molecules, harmonic solitons, and multi-wavelength solitons. The buildup process of multi-wavelength mode locking is boxed as A’ region in Figure 7, which is enlarged as Figure S4b, Supporting Information. The buildup dynamics for multi-wavelength mode locking is not as clear as that shown in Figure 4a. However, the involved solitons interactions and collisions are ubiquitous. Pulses operating at different wavebands are labeled as 1, 2, 3, 4, respectively, and their replicas are marked as 1’, 2’, 3’, 4’. When the mode locking is initiated, pulse 1 quickly fades out within 600 roundtrips due to the instability. Around the 9000th roundtrip, pulse 3’ collides with the replicas of pulse 2 at 19 ns. The retarded solitons collision causes soliton explosion, which induces a newly evolved pulse 4 operating at another waveband. More than 15 000 roundtrips later, pulse 3 and its replica both annihilate during the solitons collision. Pulse 4 with its replica and numerous replicas of pulse 2 survived to complete the stable dual-wavelength mode locking. The blue projection on the γ and z plane shows the intensity profile along with the roundtrips. The green curve shows the cross-section of x axis at the last roundtrip. It demonstrates the multipulse nature of the mode-locking pulses.

When the mode-locking was attained, we reduced the pump power to mitigate the multi-pulse mode locking and steer the laser to output dual-solitons trains. Figure 8a displays the resolved spectrotemporal evolution of the dual-solitons collision after the TSDFT process. The well-marked region around the 1000th roundtrip results from the linear temporal superposition of the two pulses, indicating the linear collision regime. The nonlinear solitons collision-induced explosion and recovery lag behind, as shown in the dashed box and further enlarged in Figure 8b. Since the spectral overlap of the dual-solitons exists in between their operating wavebands [See Figure S1a, Supporting Information], more time is needed for the mutual photons to collide with each other. Therefore, the nonlinear solitons collision is retarded. The drastic fluctuation in the energy evolution backs up the dramatic transient process, as shown in Figure 8c. After the soliton explosion, the dual-wavelength pulses recover to their initial states within 20 roundtrips. The narrowband spectral overlap assists the rapid solitons recovery. Figure 8d shows several exemplary single-shot pulses recorded by the high-speed oscilloscope. The successively appeared pulse broadening and crack, narrowing, and rebuilding process can be well distinguished from the consecutive pulse evolution. The patterns appeared at the joint of the two pulses are induced by the coherent interference.

2.7. Numerical Simulations

To confirm and better interpret the buildup dynamics of dual-wavelength mode locking, numerical simulations were investigated using a non-distributed model. Pulse propagation within the fiber sections was modeled with a standard modified nonlinear Schrödinger equation for the slowly varying pulse envelope. The simulated laser cavity and the related parameters can be referred to in Table S1 and Figure S3, Supporting Information (Supporting Information for the simulated laser design). As shown in Figure 9a,b, the evolution process for quantum noise to evolve into dual-wavelength mode-locking pulses is unveiled. The buildup dynamics involve noise pulse generation (the first 230 roundtrips), single-wavelength mode-locking (from the 231st to the 310th roundtrip), dual-wavelength interaction and competition (from the 311st to the 425th roundtrip), and dual-wavelength mode locking (after the 426th roundtrip). Noteworthily, around the 310th roundtrip, the previously formed pulse at 1027 nm collides with the noise pulse at 1033 nm. Then, a rapid mode-locking process at 1033 nm is observed. The formed dual-wavelength pulses exchange and compete for energy until a stable dual-wavelength mode-locking state is reached. The simulation results corroborate our experimental revelations and confirm that the pulses collision plays a key role in facilitating the dual-wavelength mode locking.
Furthermore, the simulated results also predict the pulses collision-induced soliton explosion. Due to chromatic dispersion, the simulated asynchronous dual-solitons periodically collide and walk off. As shown in Figure 9c,d, when the temporal separation of the two pulses is smaller than 5ps, pulses collision takes place, reflecting on the increased pulses intensity and distorted output spectra. When the two pulses overlap in the time domain, the intensity of the collisional wave reaches its maximum of >4. Meanwhile, output spectra of the two pulses are significantly broadened [blue curve, Figure 9d]. The simulated spectral evolution is similar to the TSDFT results of the collision process. The dramatic pulses collision brings about the emissions at neighboring wavebands, which will give rise to the multi-wavelength oscillation [See inset of Figure 9d].

3. Discussions and Conclusions

Asynchronous dual-wavelength mode-locked fiber laser, as a delicate laser design, provides a unique opportunity for implementing dual-comb metrology applications. By virtue of the phase-biased nonlinear amplifying loop mirror, all polarization-maintaining structure, and Lyot filtering effect, we developed the self-started, long-term stable dual-wavelength mode-locking fiber laser\(^{10}\) which is robust enough to resist the mechanical vibrations and thermal changes, and offers an excellent test-bed to research the complex buildup dynamics and interactions of dual- or multi-color solitons.

By capturing the transient formation and interactions via the TSDFT technique, we unveiled the buildup dynamics for dual- or multicolor solitons. As can be seen in Figure 4a, after the processes of relaxation oscillation and Q-switching, a dominant pulse is formed and it gradually evolves into the single-wavelength mode-locking state under the combination work of gain, saturable absorption, nonlinear effects, and dispersion. As for the dual-color solitons generation, the noise pulses or mode-locked pulses at different wavebands collide due to the fiber dispersion. The nonlinear pulse collision induces soliton explosion, which results in transient spectral broadening. Then, the noise pulses take the opportunity to evolve into mode-locking state. As for the multicolor solitons generation, the experimental observation well clarifies that it is spurred by the dual-color solitons collision (See Figure 5).

Regarding the transient pulse collision process, we classified it into two categories: linear and nonlinear pulses collisions. When two pulses solely encounter in the time domain, their collision features linear intensity superposition. However, when two pulse trains that carry the same photons encounter in the time domain, their collision process presents nonlinear characteristics, and we defined it as nonlinear soliton collision. With the experiment results, the nonlinear solitons collision results in solitons explosion. The collision and interaction time depend on the spectral overlap of the dual-wavelength solitons. After that, the dual-solitons recovered to their initial mode-locking states. In some cases, the dual-color solitons collision leads to partial or full soliton annihilation, which might have unfavorable effects on the applications.

By means of the TSDFT technique, we have experimentally tracked the formation and evolution of asynchronously dual- or multiwavelength mode locking in an ultrafast fiber laser for the first time to the best of our knowledge. The experimental results reveal that the buildup dynamics for dual-wavelength
mode locking includes five stages: raised relaxation oscillation, beating dynamics, multiwavelength oscillation, optical collision and interactions, and dual-wavelength mode locking. We demonstrate that solitons collision spurs the noise to evolve into mode-locking state at new wavebands. In contrast, the solitons collision is also an incentive mechanism for partial or full soliton annihilation, which might have devastating impact on related applications. At last, the numerical simulations confirm the experimental revelation. We believe that our studies can both broaden our horizons in complex nonlinear systems and contribute to the single cavity dual-wavelength mode-locked laser designs. The laws we proposed for dual-wavelength mode-locking might also be applied to an anomalous dispersion system, which is a Hamiltonian system, since the pulses collisions are inevitable in the asynchronous dual-wavelength laser systems. These results could also provide some new perspectives on the evolution and dynamics of ultrafast phenomena and bring useful insights into nonlinear science and its applications.

Figure 9. Simulated results of dual-wavelength mode locking: evolutions of a) temporal waveform and b) spectrum at different roundtrips. Blue curve: energy evolution. c) Temporal waveforms and d) spectra of the dual-wavelength pulses at several typical roundtrips during the collision process. Inset: spectral curves in logarithmic scale.

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Conflict of Interest
The authors declare no conflict of interest.

Data Availability Statement
The data used in this study are available from the corresponding authors under reasonable request.

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