Dynamical Analysis and Simulation for a T chaotic System with Single Delay

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Abstract. In this paper, a single delay T chaotic system is constructed by adding a delay term according to the dynamic equation of T chaotic system. The bifurcation diagram, complexity, 0-1 test and phase diagram of the system are analyzed by Matlab, thus showing the sensitivity of the new chaotic system to system parameters. Finally, in order to further study the influence of the change of two parameters on the system, the bifurcation space diagram is used to analyze the system dynamics, thus further revealing the realizability and chaotic characteristics of the system.

1. Introduction

In 1963, the first chaotic model was put forward—Lorenz chaotic system [1], after which many classical chaotic systems were put forward, mainly Chen system [2], Lü system [3], Liu system [4], t system [5] and other integer-order systems [6-7] and fractional-order chaotic systems [8-9]. As chaos theory has been widely applied in power electronics, aviation aircraft and other fields [10-13], the research on the dynamic behavior of chaotic systems is still a hot research direction. In 2019, Professor Qi Guoyuan further put forward a criterion of conservative Hamiltonian system at the Committee of Chaos and Non-linear Circuits of China Electronics Association, which provides new ideas and tools for constructing new complex chaotic systems.

According to the algebraic structure classification of chaotic systems proposed by yang qigui and other scholars of south China university of technology, there is a transitional chaotic system, which was proposed by G.Tigan, Yang Qigui and Wang Zhen in 2004, 2008 and 2011 respectively. In order to facilitate unification, this paper takes the system proposed by T et al. as the research object, namely T chaotic system. T chaotic system, Wang Zhen and others have made a series of researches, such as the existence of heteroclinic orbits [14], basic dynamic analysis and circuit realization [15], calculation of homoclinic orbits [16], calculation of periodic orbits [17], search for invariant algebraic surfaces and so on. The author and others have done a series of researches on the modified system, such as T chaotic system with logarithmic [18], complex T chaotic system [19] and T chaotic system with absolute[20]. With the continuous research of chaotic systems and the occurrence of delay phenomena in practical engineering, delayed chaotic systems have also attracted the attention of scientists. However, there are few references on the dynamics behavior of chaotic systems, and most of them are mainly focused on bifurcation analysis of systems.

In this paper, a single delay T chaotic system is proposed. First, the phase diagram, bifurcation diagram, bifurcation space and other characteristics of the system are analyzed by Matlab simulation. Secondly, in order to further verify the influence of the change of parameters on the system, phase
diagrams are used to illustrate. Finally, the bifurcation space diagram is used to illustrate the influence of simultaneous changes of two parameters on the system. The proposed system provides reference value for chaotic encryption and communication security.

2. Chaotic System with delay

In 2011, Ref.[14] proposed a new kind of chaotic dynamics system:

\[
\begin{align*}
\dot{x} &= a(y - x) \\
\dot{y} &= cx - axz \\
\dot{z} &= xy - bz
\end{align*}
\]  

We add a delay term to chaotic system (1) and a new delayed chaotic system given as:

\[
\begin{align*}
\dot{x} &= a(y - x) + x(t - 2) \\
\dot{y} &= cx - axz \\
\dot{z} &= xy - bz
\end{align*}
\]  

Where \( x, y, z \) is variable and \( a, b, c \) is parameter of system. When \( a = 3, b = 1, c = 9 \) , the system (2) is numerically simulated by Matlab, the chaotic attractor of system (2) is shown in figure 1.

![Phase diagrams](image)

(a) Phase diagram of \( x - y \)  
(b) Phase diagram of \( x - z \)  
(c) Phase diagram of \( y - z \)  
(d) Phase diagram of \( x - y - z \)

Figure 1. The chaotic attractor of system
3. **Nonlinear Characteristic Analysis**

Chaotic system is analyzed mainly by bifurcation diagram and Lyapunov exponent (complexity). This paper mainly discusses and studies the changes of the system with three parameters. Parameters are selected $b = 1, c = 9$, changed parameters $a$, when $a \in [1, 10]$, bifurcation diagram of system (2) and global bifurcation diagram of state variable $x(t)$ are shown in Figure 2. As can be seen from Figure 2, when $a \in [0, 1.3) \cup [7.5, 10]$, the system is in a non-chaotic state, and the complexity of this interval system is very small. When $a \in [1.3, 7.5]$, the system is in a chaotic state, and the complexity $C0$ of this interval system is relatively large. The phase diagram under the change of parameter $a$ is shown in figure 3 (a) (b). Figure 3(a) is a periodic state and figure 3(b) is a collect.

**Figure 2. Bifurcation diagram and complexity of system (1) about $x$ with $a$ changes**

![Bifurcation Diagram and Complexity](image1)

**Figure 3. Phase diagrams of the system with parameter $a$ changes**

Fixed parameters $a = 3, c = 9$, changed parameters $b$, when $b \in [0, 3]$, the complexity of the system (2) and the global bifurcation diagram for the state variable $x(t)$ are shown in figure 4. As can be seen that when $b \in [0, 1.5]$, the system is in chaotic state and the complexity of the interval system is relatively large; when $b \in (1.5, 3]$ the system is in non-chaotic state (periodic state) and the corresponding $C0$ of the interval system is very small. The system enters chaos through period-doubling bifurcation routes. It can be concluded from the reduction of parameter $b$ that the system enters chaos by tangent bifurcation routes. In order to verify the path of the system to chaos, the phase diagram under the change of parameter $b$ is given as shown in figure 5. Figure 5(a) is weak chaos and figure 5(b) is period (collect point).
By the same token, when parameters $c$ is changed and $b = 1, a = 3$, the complexity of the system (2) and the global bifurcation diagram for the state variable $x(t)$ are shown in Fig. 6. It is observed from Figure 6(a) that the system leads to chaos through limit-cycles with different periodicities. When $c \in [7, 9.8] \cup (11, 11.8] \cup [13, 15]$, the system is in chaotic state, and the complexity is relatively large as can be seen from figure. 6(b). The phase diagram when parameter $c$ is a specific value is shown in figure 7.
4. Bifurcation space of the system

In this paper, the bifurcation space of the system is described by using the complexity C0 index as the standard quantity. It can be seen from Figure 8 that the parameters of the system are mutually restricted, while the influence of the above research parameters on the system is only a case of parameter change. If so, it can be seen that Figure 2 is consistent with Figure 4. As can be seen from Figure 8, the attraction domains of a and b are relatively small.
6. Conclusion
In this paper, a delay term is added to the T chaotic system, and a single delay T chaotic system is obtained. The basic dynamic behavior of the system is analyzed by bifurcation diagram, LE spectrum, 0-1 test, bifurcation space and other analysis methods. Due to the limited space, Hopf bifurcation and control of the system are the subject of future research. The related work in this paper provides theoretical support for the application of the system in the field of secure communication.

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