Decay mechanisms of superflow of Bose-Einstein condensates in ring traps

Masaya Kunimi and Ippei Danshita

Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

(Dated: October 1, 2018)

We study supercurrent decay of a Bose-Einstein condensate in a ring trap combined with a repulsive barrier potential. In recent experiments, Kumar et al. [Phys. Rev. A 95, 021602(R) (2017)] have measured the dependence of the decay rate on the temperature and the barrier strength. However, the origin of the decay observed in the experiment remains unclear. We calculate the decay rate due to thermally activated phase slips by using the Kramers formula based on the Gross-Pitaevskii mean-field theory. The resulting decay rate is astronomically small compared to that measured in the experiment, thus excluding the possibility of TAPS as the decay mechanism. Alternatively, we argue that the three-body loss can be relevant to the observed decay and predict that one can observe supercurrent decay via TAPS by decreasing the number of atoms.

Introduction.- Systems of ultracold gases confined in ring-shaped traps have served as an ideal platform for studies of superfluidity [1–14]. Their exquisite controllability has led to experimental observations of various fundamental properties of superfluids, including persistent currents [2–4, 12–14], critical velocities [6, 7], and hysteresis loops [10]. Such systems attract growing interest also as an atomic analog of superconducting quantum interference device [15], which constitutes a basic element for atomtronic circuits [16, 17].

A recent experiment performed by the NIST group has raised a puzzling question concerning the superfluidity in a ring-shaped Bose-Einstein condensate (BEC) [14]. In this experiment, they prepared a current-carrying BEC with winding number 1 as an initial state and measured the lifetime of this state. To induce the decay of the persistent current, they exposed the BEC to a repulsive potential barrier, which is generated by a blue detuned laser beam. They found that the observed decay rate is significantly dependent on the barrier strength $U_0$ and the temperature $T$. One might naively expect that this result could be interpreted as decay of the persistent current via thermally activated phase slips (TAPS) [18–20], whose decay rate $\Gamma$ obeys the Arrhenius law, i.e., $\Gamma \propto e^{-E_B/(k_B T)}$. Here $E_B$ denotes the energy barrier separating the state with winding number 1 from that with winding number 0 (see Fig. 1). However, the NIST group presented a rough estimation of $E_B$ to argue that the observed decay rate is inconsistent with the Arrhenius law. Thus, the decay mechanism of the persistent currents remains unclear.

The above mentioned observation of Ref. [14] challenges our common understanding of the superfluidity in the sense that TAPS has been established as a universal decay mechanism applicable to various superfluid systems at finite temperatures, such as superfluid $^4$He [21, 22], superconductors [23], spin superfluids [24], and one-dimensional Bose gases in optical lattices [25, 26]. Resolving this puzzle is important also for engineering and controlling the atomtronic circuits, which require quantitative understanding of the persistent current in the presence of repulsive potential barriers [8].

In this Letter, we first re-examine the possibility of TAPS as a decay mechanism by quantitatively computing the nucleation rate of TAPS without any fitting parameters. We show that the energy barrier estimated in Ref. [14] is much smaller than our quantitative result. Nevertheless, the computed decay rate completely disagrees with the value measured in Ref. [14], thus confirming that TAPS is irrelevant to the observed decay. Alternatively, we find that the three-body loss can induce the decay of superflow in the experimental setup and its timescale is close to the experimental one. This indicates that one may observe decay via TAPS by optimizing some experimental parameters in such a way that TAPS is enhanced. We explicitly identify parameter regions in which TAPS dominates over the three-body loss and predict the decay rate via TAPS as a reference that can be directly compared with future experiments.

Model and methods.- We consider a BEC in a com-
bined potential of a ring trap and a repulsive barrier to mimic the situation in the experiment of Ref. [14]. The interatomic interaction is sufficiently weak so that the dynamics of the superfluid order parameter \( \Psi(r,t) \) is quantitatively described within the mean-field approximation. Specifically, the two-dimensional (2D) Gross-Pitaevskii (GP) equation with a phenomenological three-body loss term [27] is given by

\[
 i\hbar \frac{\partial}{\partial t} \Psi(r,t) = \left[ -\frac{\hbar^2}{2m} \nabla^2 + U(r) + g_{2D}|\Psi(r,t)|^2 \right] \Psi(r,t)
- \frac{i\hbar}{2}L_{3,2D}^{\text{ext}}|\Psi(r,t)|^4 \Psi(r,t),
\]

(1)

where \( m \) is the mass of the atom, \( U(r) = (1/2)m\omega_r^2(r-R)^2 + U_{\text{ext}}(r) \) represents the trap potential and the external potential, \( R \equiv \sqrt{x^2 + y^2} \), \( \omega_r \) is the trap frequency of the radial direction, \( \omega_z \equiv \sqrt{\hbar/(m\sigma_z)} \) is the harmonic oscillator length of the \( z \)-direction, \( \omega_s \equiv \sqrt{\hbar/(m\sigma_s)} \) is the harmonic oscillator length of the \( z \)-direction, \( L_{3,2D}^{\text{ext}} = L_3/(\sqrt{3}\pi a_s^2) \) is the quasi-2D three-body loss rate. We note that the three-body loss term is set to zero except the calculations for Fig. 3. The external potential is created by dithering the Gaussian laser beam. The time-averaged potential \( U_{\text{ext}}(r) \) is given by [28]

\[
 U_{\text{ext}}(r) = \frac{U_0}{2} \left\{ \text{erf} \left[ \frac{\sqrt{2}}{w} \left( x - R + \frac{ld}{2} \right) \right] - \text{erf} \left[ \frac{\sqrt{2}}{w} \left( x - R - \frac{ld}{2} \right) \right] \right\} e^{-2y^2/w^2},
\]

(2)

where \( U_0 \) is the potential strength, \( w \) is the 1/e width of the laser beam, \( ld \) is the length of the dither, and \( \text{erf}(\cdot) \) is the error function.

To obtain excitation spectra, we linearize the GP equation around a stationary solution \( \Psi(r,t) = e^{-i\mu t}\Phi(r) \), where \( \mu \) is the chemical potential. The linearized GP equation, i.e., the Bogoliubov equation is given by

\[
 H_{B} \left[ \begin{array}{c} u_i(r) \\ v_i(r) \end{array} \right] = \epsilon_i \left[ \begin{array}{c} u_i(r) \\ v_i(r) \end{array} \right],
\]

(3)

\[
 H_{B} \equiv \begin{bmatrix} \mathcal{L} & g_{2D}\Phi(r)^2 \\ -g_{2D}\Phi(r)^2 & \mathcal{L}^* \end{bmatrix},
\]

(4)

\[
 \mathcal{L} \equiv -\frac{\hbar^2}{2m} \nabla^2 + U(r) - \mu + 2g_{2D}|\Phi(r)|^2,
\]

(5)

where \( \epsilon_i \) is the excitation energy \( u_i(r) \) and \( v_i(r) \) are eigenfunction of the excited state labeled by \( i \).

In addition to the Bogoliubov equation, we need to diagonalize the following matrix: \( H_{B} \equiv \sigma_3 H_{B} \), where \( \sigma_3 = \text{diag}(+1,-1) \). Let \( \lambda_i \) be an eigenvalue of the matrix \( H_{B} \), which is a real value in contrast to the eigenvalue of the Bogoliubov equation \( \epsilon_i \in \mathbb{C} \).

In the experiment, they prepare the state with the winding number \( W = 1 \) as initial states. Here, we calculate the decay from \( W = 1 \) state (metastable state) to \( W = 0 \) state (ground state). According to the Kramers formula [38, 39], the nucleation rate can be written as

\[
 \Gamma \equiv \Gamma_+ - \Gamma_-,
\]

\[
 \Gamma_\pm \equiv \frac{|\epsilon_{\text{DI}}|/\hbar}{2\pi} \prod_n \sqrt{\frac{\lambda_n^\pm k_B}{|\lambda_n^\pm|}} e^{-\epsilon_{\text{DI}}^\pm/(k_B T)},
\]

(6)

where \( \Gamma_+ (\Gamma_-) \) is a nucleation rate of the decay process (the acceleration process) (see Fig. 11), \( \epsilon_{\text{DI}} \) is the frequency of the unstable mode and \( \lambda_n^\pm \) and \( \lambda_n^\pm \) are the eigenvalues of the matrix \( H_{B} \), \( E_{B}^\pm \) is the energy barrier of the acceleration and the decay process, \( k_B \) is the Boltzmann constant, \( T \) is the temperature of the system. In the product of Eq. (7), we omit the zero-modes (\( \lambda_n = 0 \)). We define the nucleation rate (inverse of the lifetime) is the difference between \( \Gamma_+ \) and \( \Gamma_- \). We note that the Kramers formula is valid for \( E_{B}^\pm \gg k_B T \).

We use the almost same methods to perform the numerical calculations for the GP equation and the Bogoliubov equation as our previous work [26], i.e., the space discretization is performed by the discrete variable representation method [30] and seeking the unstable stationary solution of the GP equation is based on the pseudospectral continuation method [31, 32] and the Newton method. The Numerical meshes are typically used for \( 129 \times 129 \). It is worth emphasizing that although the GP equation has been extensively used for studying superfluidity of ring-shaped BEC [33–37], decay rates via TAPS at 2D, to our knowledge, have never been quantitatively calculated because of the difficulty in finding the unstable solutions.

**Results.** - Here, we show our theoretical results, which are compared to the experimental ones. In the following calculations, we consider \(^{23}\text{Na} \) atom and set the system parameters as the cases I in the Refs. [14, 40], which corresponds to the lowest temperature and the strongest two-dimensionality in the experiment. The specific values are as follows: \( m = 3.82 \times 10^{-26} \) kg, \( (\omega_r, \omega_z)/(2\pi) = (256 \text{ Hz}, 974 \text{ Hz}) \), \( R = 22.4 \mu \text{m} \), \( a_s = 2.75 \text{ nm} \), \( w = 6 \mu \text{m} \), and \( l_d = 21.8 \mu \text{m} \). We use the particle number \( N \approx 3.75 \times 10^5 \), which comes from the ground state particle number at the chemical potential \( \mu = \mu_{\text{exp}} \equiv h \times 2.91 \text{ kHz} \), where \( \mu_{\text{exp}} \) is the chemical potential measured in the experiment.

First, we show the density and phase profiles of the order parameter for the metastable and unstable states in Fig. 2. We see the SV at the low-density region in the unstable solution [Figs. 2 (b) and (d)]. The NIST group assumed that the origin of the energy barrier is the solitonic vortex (SV) [14]. Our results support the validity of their assumption.

Next, we show the energy barrier as a function of the strength of the external potential in Fig. 3. There we
compare our results with those estimated in Ref. [14] and see that the latter is considerably underestimated. This happens because the energy barrier was estimated by the energy of the SV [Eq. (2) in Ref. [14]] for a uniform system. The present system is non-uniform due to the trap potential and the external potential, which significantly modify the quantitative size of the energy barrier.

We then show the lifetime of the superflow due to the TAPS in Fig. 4. We see that the lifetime due to TAPS is astronomically longer than that observed in Ref. [14] (1-10 s). The major origin of such long lifetime is the large energy barrier compared to the temperature. In the present case, the energy barrier is typically $E_B / h \sim 200$ kHz (see Fig. 3) and the temperature is given by $k_B T / h \simeq 0.6$ kHz ($T = 30$ nK). Consequently, we obtain the extremely long lifetime $\Gamma^{-1} \sim e^{E_B / (k_B T)} \sim e^{300}$. From the above results, we conclude that the TAPS is irrelevant to the decay of superflow observed in Ref. [14].

Instead of the TAPS, we investigate various effects, namely three dimensionality, thermal and quantum depletion, light scattering, and three-body loss as an origin of the decay of superflow [41]. Among them, we find that three-body loss is the most relevant.

In order to investigate the effects of the three-body loss, we numerical solve Eq. (1) with $L_{3,2D} \neq 0$. Here, we set $L_3 = 1.1 \times 10^{-30}$ cm$^{-6}$ s$^{-1}$, which is the three-body loss rate of the $^{23}$Na atom [42]. The initial condition is the ground state solution of the non-dissipative GP equation ($L_3 = 0$) for $U_0 = 0$ and $N \simeq 4.40 \times 10^5$, which comes from the ground state particle number at
small particle number experiment. For example, the time that the instability sets in is about longer than that of the experimental decay of superflow.

...the experimental sequences (see Fig. 1 in Ref. [40], and Ref. [44]). We use the fourth order Runge-Kutta method for the time evolution and the pseudo-spectral method for space discretization. The numerical meshes are used for 128 × 128 and the time step ∆t for ∆t = 0.5 μs.

The time dependence of the angular momentum of the z-component per particle is shown in Fig. 6 where the expression of the angular momentum is given by $L_z(t) = \int dr \Psi^*(r, t) l_z \Psi(r, t)$ and $l_z = -i\hbar (x \partial / \partial y - y \partial / \partial x)$. Our results show that the three-body loss induces the decay of superflow. This decay is due to vanishing the energy barrier by decreasing the total particle number via three-body loss. Although we found that the three-body loss can induce the decay of superflow, the decay timescale is longer than that of the experimental decay of superflow. For example, the time that the instability sets in is about 23.3 s for $U_0 = 0.6884 \mu_{\text{exp}}$ and the experimental results show that the lifetime is about 6.7 s for $U_0 = 0.6884 \mu_{\text{exp}}$. In addition to this, our calculations can not explain the discrepancy of the lifetime. An obvious reason for this discrepancy is the lack of finite temperature effects in our GP calculations. These effects should enhance the decay of superflow. Hence, our results should be interpreted as the upper bound of the decay timescale. The hybrid effects of the three-body loss and the finite temperature effects are strong candidate of the origin of the decay of superflow in the NIST experiment.

To confirm whether or not the hybrid effects appear in the experiment, we need to calculate the dynamical simulations including the finite temperature effects and compare them with the experiments. However, it is hard to perform this kind of simulation with currently available theoretical methods. For example, the truncated-Wigner approximation (TWA) and the Zaremba-Nikuni-Griffin (ZNG) formalism are often used for simulating dynamics of BEC at finite temperatures. However, they are not suited for the current problem because the TWA can not accurately capture the long time dynamics and the ZNG formalism can not describe the nucleation process of the vortices due to the thermal fluctuations. Nevertheless, we can at least predict that if the decay is indeed due to the three-body loss, one can experimentally observe the decay via TAPS by suppressing the energy barrier in the following way.

To do this, we seek the parameter region where the lifetime due to the TAPS becomes 1-100 ms by tuning the particle number in the trap. In this time scale, we can neglect the effects of the three-body loss. Figure 6 shows the lifetime of the superflow for $N \approx 2.7 \times 10^4$ [note that the particle number in the experiment is $O(10^5)$], which corresponds to the ground state particle number for $\mu = h \times 0.5$ kHz. Here, other parameters such as trap frequencies are fixed. If these results agree with future experiments, we can conclude that the superflow decay via TAPS occurs without being affected by the three-body loss.

Summary.- We investigated the supercurrent decay of BEC in ring traps to reveal the origin of the decay observed in the experiment of Ref. [14]. First, we reconsider the decay via the TAPS based on the Kramers formula. Our results show that the lifetime via TAPS is astronomically long. This means that the TAPS is not relevant in the experiment of Ref. [14]. Next, we performed the numerical simulations of the GP equation with the three-body loss term and found the decay timescale is close to but slightly longer than the experimental one. These results indicate that the hybrid effect of the thermal fluctuations and the three-body loss can be a dominant cause of the decay observed in the experiment. Finally, we proposed that one can enhance the TAPS by decreasing the energy barrier in order to observe the decay via the TAPS in future experiments.

We lastly emphasize that the effects of the particle loss, to our knowledge, have not been considered in the context of superflow decay before the present work. In this sense, our results open up a new possibility in the study of superfluidity. Specifically, consideration of such effects is expected to be relevant also to advanced superfluid systems, including exciton polariton BEC and ultracold atomic gases with controllable particle losses.

Acknowledgments.- We thank G. K. Campbell for providing us the experimental data and useful comments, and A. Kumar for providing us with the experimental data. M. K. thanks S. Goto for his advice regarding numerical calculations. I. D. thanks L. Mathey for useful discussions. M. K. was supported by Grant-in-Aid for JSPS Research Fellow Grant Number JP16J07240. I. D.

![Figure 6](image.png)

**FIG. 6.** (Color online) Lifetime of the superflow ($\Gamma^{-1}$) for small particle number $N \approx 2.7 \times 10^4$ compared to the experiment. $\mu = h \times 0.5$ kHz. The grey region represents 1 ms $\leq \Gamma^{-1} \leq$ 100 ms.
was supported by KAKENHI grants from JSPS: Grants No. 15H05855 and No. 25220711, and by research grant from CREST, JST.

* E-mail: masaya.kunimi@yukawa.kyoto-u.ac.jp

† Present address: Department of Physics, Kindai University, Higashi-Osaka, Osaka 577-8502, Japan

[1] C. Ryu, M. F. Andersen, P. Cladé, V. Natarajan, K. Helerson, and W. D. Phillips, Phys. Rev. Lett. 99, 260401 (2007).
[2] A. Ramanathan, K. C. Wright, S. R. Muniz, M. Zelan, W. T. Hill, C. J. Lobb, K. Helmerson, W. D. Phillips, and G. K. Campbell, Phys. Rev. Lett. 106, 130401 (2011).
[3] S. Moulder, S. Beattie, R. P. Smith, N. Tammuz, and Z. Hadzibabic, Phys. Rev. A 86, 013629 (2012).
[4] S. Beattie, S. Moulder, R. J. Fletcher, and Z. Hadzibabic, Phys. Rev. Lett. 110, 025301 (2013).
[5] T. W. Neely, A. S. Bradley, E. C. Samson, S. J. Rooney, E. M. Wright, K. J. H. Law, R. Carretero-González, P. G. Kevrekidis, M. J. Davis, and B. P. Anderson, Phys. Rev. Lett. 111, 235301 (2013).
[6] K. C. Wright, R. B. Blakestad, C. J. Lobb, W. D. Phillips, and G. K. Campbell, Phys. Rev. Lett. 110, 025302 (2013).
[7] K. C. Wright, R. B. Blakestad, C. J. Lobb, W. D. Phillips, and G. K. Campbell, Phys. Rev. A 88, 063633 (2013).
[8] C. Ryu, P. W. Blackburn, A. A. Blinova, and M. G. Boshier, Phys. Rev. Lett. 111, 205301 (2013).
[9] C. Ryu, K. C. Henderson, and M. G. Boshier, New J. Phys. 16, 013046 (2014).
[10] S. Eckel, J. G. Lee, F. Jendrzejewski, N. Murray, C. W. Clark, C. J. Lobb, W. D. Phillips, M. Edwards, and G. K. Campbell, Nature (London) 506, 200 (2014).
[11] F. Jendrzejewski, S. Eckel, N. Murray, C. Lanier, M. Edwards, C. J. Lobb, and G. K. Campbell, Phys. Rev. Lett. 113, 045305 (2014).
[12] S. Eckel, F. Jendrzejewski, A. Kumar, C. J. Lobb, and G. K. Campbell, Phys. Rev. X 4, 031052 (2014).
[13] L. Cormon, L. Chomaz, T. Biemaimé, R. Desbuquois, C. Weitenberg, S. Nasimbené, J. Dalibard, and J. Beugnon, Phys. Rev. Lett. 113, 135302 (2014).
[14] A. Kumar, S. Eckel, F. Jendrzejewski, and G. K. Campbell, Phys. Rev. A 95, 021602(R) (2017).
[15] J. Clarke and A. I. Braginski, The SQUID Handbook (Wiley-VCH, Weinheim, 2004).
[16] B. T. Seaman, M. Krämer, D. Z. Anderson, and M. J. Holland, Phys. Rev. A 75, 023615 (2007).
[17] R. A. Pepino, J. Cooper, D. Z. Anderson, and M. J. Holland, Phys. Rev. Lett. 103, 140405 (2009).
[18] J. S. Langer and M. E. Fisher, Phys. Rev. Lett. 19, 560 (1967).
[19] J. S. Langer and V. Ambegaokar, Phys. Rev. 164, 498 (1967).
[20] D. E. McCumber and B. I. Halperin, Phys. Rev. B 1, 1054 (1970).
[21] J. S. Langer and J. D. Reppy, in Progress in Low Temperature Physics, edited by C. J. Gorter (North-Holland, Amsterdam, 1970), Vol. 6, Chap. 1.
[22] J. D. Reppy, J. Low Temp. Phys. 87, 205 (1992).
[23] B. I. Halperin, G. Refael, and E. Demler, Int. J. Mod. Phys. B 24, 4039 (2010).
[24] S. K. Kim, S. Takei, and Y. Tserkovnyak, Phys. Rev. B 93, 020402(R) (2016).
[25] L. Tanzi, S. S. Abbate, F. Cataldini, L. Gori, E. Lucioni, M. Inguscio, G. Modugno, and C. D’Errico, Sci. Rep. 6, 25965 (2016).
[26] M. Kunimi and I. Danshita, Phys. Rev. A 95, 033637 (2017).
[27] Y. Kagan, A. E. Muryshev, and G. V. Shlyapnikov, Phys. Rev. Lett. 81, 933 (1998).
[28] D. S. Petrov, M. Holzmann, and G. V. Shlyapnikov, Phys. Rev. Lett. 84, 2551 (2000).
[29] A. Kumar, private communications.
[30] D. Baye and P.-H. Heenen, J. Phys. A: Math. Gen. 19, 2041 (1986).
[31] H. B. Keller, Lectures on Numerical Methods in Bifurcation Problems (Springer-Verlag, Berlin/Heidelberg/New York/Tokyo, 1987).
[32] M. Kunimi and Y. Kato, Phys. Rev. A 91, 053608 (2015).
[33] J. Brand and W. P. Reinhardt, J. Phys. B: At. Mol. Opt. Phys. 34, L113 (2001).
[34] F. Piazza, L. A. Collins, and A. Smerzi, Phys. Rev. A 80, 021601(R) (2009).
[35] A. C. Mathey, C. W. Clark, and L. Mathey, Phys. Rev. A 90, 023604 (2014).
[36] A. Muñoz Mateo, A. Gallemi, M. Guilleumas, and R. Mayol, Phys. Rev. A 91, 060625 (2015).
[37] K. Snizhko, K. Isaieva, Y. Kuriatnikov, Y. Bidasyuk, S. Vilchinskii, and A. Yakimenko, Phys. Rev. A 94, 063642 (2016).
[38] J. S. Langer, Ann. Phys. 54, 258 (1969).
[39] P. Hänggi, P. Talkner, and M. Borkovec, Rev. Mod. Phys. 62, 251 (1990).
[40] Supplemental materials of Ref. [14].
[41] M. Kunimi and I. Danshita, to be published elsewhere.
[42] D. M. Stamper-Kurn, M. R. Andrews, A. P. Chikkatur, S. Inouye, H.-J. Miesner, J. Stenger, and W. Ketterle, Phys. Rev. Lett. 116, 235302 (2016).
[43] T. Tomita, S. Nakajima, I, Danshita, Y. Takasu, and Y. Takahashi, Sci. Adv. 3, e1701513 (2017).