Near field in quantum electrodynamics: Green functions, Lorentz condition, "nonlocality in the small", frustrated total reflection

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Investigation of near field of QED requires the refuse from an averaging of the Lorentz condition that smooths out some field peculiarities. Instead of it Schwinger decomposition of the 4-potential with the Bogoliubov method of interaction switching in time and in space regions is considered. At such approach near field is describable by the part of covariant Green function of QED, the fast-damping Schwinger function formed by longitudinal and scalar components of $A_\mu$ none restricted by light cone. This description reveals possibility of superluminal phenomena within the near field zone as a "nonlocality in the small". Some specification of Bogoliubov method allows, as examples, descriptions of near fields of point-like charge and at FTIR phenomena. Precisely such possibilities of nonlocal interactions are revealed in the common QED expressions for the Van-der-Waals and Casimir interactions and in the Förster law.

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I. INTRODUCTION

The usage of 4-component vector $A_\mu$ for the field quantization, when for the Maxwell equations in vacuum only two components are needed, induced serious problems at early stages of QED. These problems were initially obviated by Fermi via averaging the Lorentz condition as $\partial A_\mu/\partial x_\mu |0\rangle = 0$ [1]. This way had led to the indefinite metrics that formally exhausts the problem of exception of superfluous components of $A_\mu$ by an artificial their averaging and leaves the theory completely local, although the used procedure has not a direct physical sense, e.g. [2]. It excludes also near field from consideration, but at that time its effects were out of common interests.

With the discovery of Aharonov-Bohm effect [3] the complexity of real situation could already become more evident: it is impossible to simply cast away two additional components of 4-potential, as the $E$ and $B$ do not describe all features of electromagnetic fields. But moreover, the development of optics into recent decades show greater complexity of near field than was commonly accepted. The most unexpected peculiarities represent the phenomena of interference in the scope of near field (e.g. [4, 5] and references therein) and numerous observations of superluminal signalling (the reviews [6]), which also may be related with near fields. All it requires a returning to general problems of near field and to a revision of common representations for description of these new phenomena, at that possibly of nonlocal nature.

But can exist nonlocal solutions of covariant equations for little distances? If to search solutions of the wave equation as $f(x) = f(vt - r)$, then the non-fading solution exists only for $v \leq c$. This peculiarity strongly forbids superluminal movement as an effective nonlocality on arbitrary distances. However, this equation does not forbid propagation of faster-than-c "perturbations" on short distances, of order of uncertainty principle distances, at condition of their quick damping.

In our article [7] has been established, in the frame of general phenomenological approach, that the transferring of excitation within near field on the distance $\Delta x$ are possible only and only as the instant tunneling, when these distances are inverse relatively energy deficiency to the nearest stable or resonant (quasistable) state $\Delta \hbar \omega$:

$$\Delta x \cdot \Delta \omega \sim \pi c. \quad (1)$$

It defines a "nonlocality in the small" of near field (this "nonlocality" is restrained by the especial form of the energy-time duration uncertainty principle, such additional proof of (1) is given in [8]). Thus the superluminal phenomena can occur, for example, in the domain of anomalous dispersion or at the frustrated total internal reflection (FTIR). And it is one of peculiarities of near field that is not evident at usual approach.

Notice, that the common description of field is related to far fields only: as must be especially underlined, the features of locality were established and many times checked for far fields only. But the locality of near fields never was experimentally checked and possibility of their general or at special conditions nonlocality can not be a priori excluded.
With the purpose of investigating such possibilities we shall try to examine the structure of near fields without averaging of the Lorentz condition. Instead of it the Schwinger method of decomposition on transverse, longitudinal and scalar components will be used [9]. These procedures (Section 3) will lead to revealing the existence of nonlocal terms within the known decomposition of covariant Green functions (Section 2).

Then the most general procedure of interaction on and off switching, the Bogoliubov method [10] for description of QED interactions without the averaged additional condition, will be used (Section 4). It completes the construction of near field in absence averaging procedure and shows its possible nonlocality, manifestations of which quickly disappear with distance.

In the subsequent Sections we try to apply this approach to some real phenomena. It requires special partial specializations of the Bogoliubov method suggested in the Appendix, but contains some arbitrariness and therefore subsequent considerations have rather a hypothetical character.

In the Section 5 a near field of point-like charge will be considered with estimation of its characteristic energies at different distances. In the Section 6 on some similar base the phenomena of FTIR are considered, which reveals the dependence of wave numbers values on distances from the surface and corresponding possibility of interference picture formation leading to effects of near field optics.

Then it will be shown that the Van-der-Waals forces, in the form deduced in [11], and the Förster law of excitation transfer on short distances are describable in the frame of QED via near-field nonlocal interactions. The revealing of such description means that near-field effects are not so exotic as it can seems and that they are responsible for some known phenomena.

The results of executed examinations, which can be considered as some detailisation of the general results of [7, 8], are summed in the Conclusions, where some further perspectives of the offered method are mentioned.

II. DECOMPOSITION OF CANONICAL GREEN FUNCTIONS

Let’s consider the covariant Pauli-Jordan function $D(t, r)$ in the variant of Coulomb gauge suggested by Dzialoshinski and Pitayevski [11]

$$D_{ij}(\omega, r) = (\delta_{ij} + \partial_i \partial_j / \omega^2)D(\omega, r); \quad D_{i0} = D_{0i} = 0.$$  \hspace{1cm} (2)

After differentiation it leads to the representation of Green functions of wave equation ([12], cf. [5],) as

$$D_{ij}(\omega, r) = \{(\delta_{ij} + e_i e_j) - \frac{i}{\omega^2} P_{ij} \cot(\omega r) + \frac{1}{(\omega r)^2} P_{ij}\} D(\omega, r),$$ \hspace{1cm} (3)

here and below $c = \hbar = 1$, $P_{ij} = \delta_{ij} - 3e_i e_j$. (Note that the decomposition of transfer functions is simpler than decomposition of field strengths [13].)

Three terms of (3), which correspond to far, intermediate (or transient) and near fields, are expressed in the $(t, r)$ representation through the Pauli-Jordan function and the generalized singular function $D_N(t, r)(\partial_x \equiv \partial / \partial x)$:

$$D_{ij}(t, r)|_{FF} = (\delta_{ij} + e_i e_j)D(t, r);$$  \hspace{1cm} (4)

$$D_{ij}(t, r)|_{IF} = \frac{1}{4\pi r} P_{ij} \theta(r^2 - t^2) \equiv \frac{1}{r} P_{ij} \partial_t D_N(t, r);$$  \hspace{1cm} (5)

$$D_{ij}(t, r)|_{NF} = \frac{1}{4\pi r^2} P_{ij} \left\{ sgn(t) \theta(t^2 - r^2) + \frac{t}{r} \theta(r^2 - t^2) \right\} \equiv \frac{1}{r^2} P_{ij} D_N(t, r).$$  \hspace{1cm} (6)
The function $D_N(t,r)$, valuable for all subsequent analysis, can be determined immediately by (6) and in more details will be considered below.

This decomposition shows nonlocality of near field, increasing with time, and demonstrates that transitions (5) between near and far fields are concentrated in the space-like domain. It means that near field of any charge can be nonlocal, i.e. it supposes, in particular, the possibility of propagation of quickly relaxing superluminal perturbations within near fields.

Virtual “quanta of near field” and ”dressed”, i.e. free photons are clearly distinguishable in the $(\omega,k)$ representation (here and below $k=|k|$):

\[
D_{ij}(\omega,k)|_{FF} = (\delta_{ij} + e_i e_j) \frac{2}{(2\pi)^3} \varepsilon(\omega) \delta(\omega^2 - k^2);
\]

\[
D_{ij}(\omega,k)|_{LF} = \frac{1}{4\pi i \omega k} P_{ij} \theta(k^2 - \omega^2);
\]

\[
D_{ij}(\omega,k)|_{NF} = \frac{1}{8\pi^2 \omega^2 k} P_{ij} \{ \omega \theta(\omega^2 - k^2) + k \theta(k^2 - \omega^2) \}.
\]

These expressions clearly show that when quanta of far field are the free photons with light speed $c$ in vacuum, the speed of virtual quanta propagation is not restricted. The transitions between them, described by an intermediate field, are evidently tunnel processes with an energy deficiency relative to momentum.

Note, that for functions $D(\omega,k)$, $D^{(1)}(\omega,k)$ and their combinations the gauge (2) actually coincides with the more widespread Coulomb gauge:

\[
D_{ij}(\omega,k) = (\delta_{ij} - k_i k_j/k^2) D(\omega,k); \quad D_{00} = -1/k^2; \quad D_{0i} = 0.
\]

The decomposition similar (3) can be executed, certainly, with singular functions of non-uniform wave equation $D_e(\omega,k)$, etc.

III. SCHWINGER DECOMPOSITION OF 4-POTENTIAL

For uncovering the origin and physical sense of the function $D_N$ some properties of 4-potential construction should be considered without averaging of its 4-divergence over vacuum, i.e. without usage of the Fermi-Lorentz condition.

For this aim the 4-vector $A_\mu$, or its frequencies parts in any system of readout must be covariantly decomposed by the Schwinger method [9] onto the far field $A_\mu^{(f)}$ and two auxiliary fields, longitudinal $\Lambda_{||}$ and scalar (temporal) $\Lambda_0$:

\[
A_\mu(x) = A_\mu^{(f)}(x) + n_\mu (n \partial) \Lambda_0(x) - (\partial_\mu + n_\mu (n \partial)) \Lambda_{||}(x),
\]

\[
n_\mu \text{ is the unit time-like vector, } n_\mu^2 = 1, \ (n \partial) = n_\mu \partial_\mu. \text{ All three fields independently obey the wave equations:}
\]

\[
\Box A_\mu^{(f)}(x) = \Box \Lambda_0(x) = \Box \Lambda_{||}(x) = 0.
\]

Far field $A_\mu^{(f)}$ is transverse and satisfies the classical Lorentz condition:

\[
n_\mu A_\mu^{(f)}(x) = \partial_\mu A_\mu^{(f)}(x) = 0.
\]
Auxiliary fields are defined via $A_\mu$:

$$\partial_\mu A_\mu = (n\partial)(A_0 - A_\parallel)$$

$$n_\mu A_\mu = -(n\partial)A_0(x);$$

(14)

$$\partial_\mu n_\mu A_\mu = -(n\partial)^2 A_\parallel,$$

(15)

they are invariant relative coordinates inversion and changing sign at the time reversing. As the commutator of

$A_\mu^{(f)}$ obeys the Pauli-Jordan function, additional fields form the nonlocal commutators:

$$[A_0(x), A_0(y)] = -[A_\parallel(x), A_\parallel(y)] = iD_N(x-y),$$

(16)

Let’s try to establish the reasons of appearance of this function.

The wave equation in vacuum is the operator record of dispersion identity $p^2 \equiv \omega^2 - |k|^2 = 0$ and functions, that satisfy this identity, are represented by the Fourier integral:

$$G(x) = \int d^4p f(p) \delta(p^2)e^{i(px)}$$

with arbitrary, non-singular at $p^2 = 0$ function $f(p)$.

The partial solutions of corresponding non-uniform equation are represented by the Fourier integral:

$$\overline{D}(x) = \frac{2}{(2\pi)^4} P \int d^4p \frac{1}{p^2} e^{i(px)}.$$ 

(18)

If to demand that (13) and (14) compose one analytical function that corresponds to the Kramers-Kronig dispersion relations and to an opportunity of energy transition from induced oscillations into free waves and back, then $f(p)$ in (13) can consists of step operators $\theta(\pm \omega)$ only. If, however, such fields closely to the source are examining that does not immediately generate far field waves, but can consist from the own or confinement field of source only, this restriction on the form of $f(p)$ in (13) loses its force (therefore the function $D_N$ had been appeared at calculation of the electron self-field [9]). But it simultaneously means that the acceleration of near field (e.g. at charge acceleration) should lead to occurrence of free, far field waves, i.e.

$$\partial_t^2 D_N(x) \rightarrow D(x).$$

(19)

Hence the response functions of near field should be connected to the corresponding Green functions (13) by the determination (possible constant and linear on $t$ terms are omitted):

$$D_N^{(1)}(x) = \frac{1}{2} \int_{-\infty}^{\infty} d^4y n_\mu |x_\mu - y_\mu| D^{(1)}(y),$$

(20)

or, in the evident time-space form, as

$$D_N^{(1)}(t,r) = \frac{1}{2} \int_{-\infty}^{\infty} d\tau |\tau - t| D^{(1)}(\tau, r),$$

(21)

that just corresponds the function, introduced by Schwinger.

The direct calculation, with the Green functions $D(t, r), D^{(1)}(t, r)$ and $D^{(\pm)}(t, r)$ of wave equation in the right-hand side of (16'), leads to such singular functions of near field:
\[ D_N(t, r) = \frac{1}{8\pi r} (|t - r| - |t + r|) \equiv \frac{1}{4\pi} \{ \text{sgn}(t)\theta(t^2 - r^2) + \frac{t}{r} \theta(r^2 - t^2) \}; \]  

(22)

\[ D^{(1)}_N(t, r) = \frac{1}{4\pi^2 r} ((t + r) \ln(t + r) - (t - r) \ln(t - r)) \equiv \frac{1}{4\pi^2} (\ln(t^2 - r^2) - \frac{t}{r} \ln \frac{t - r}{t + r}); \]  

(23)

\[ D^{(\pm)}_N(t, r) = \frac{1}{2} \{ D_N(t, r) \mp i D^{(1)}_N(t, r) \}; \]  

(24)

terms omitted in (16) can be restored for balancing dimensions of arguments of logarithms in (18). Notice that \( D_N(t, r) \) equals zero at \( t = 0 \) just as \( D(t, r) \).

It is remarkable that the temporal change of function (17) occurs completely in the spatial direction:

\[ \partial_t D_N(t, r) = \frac{1}{t} r \nabla D_N(t, r) = -\frac{1}{4\pi r} \theta(r^2 - t^2), \]  

(25)

which emphasizes the nonlocal character of near field in a concordance with the tunnel character of (5').

Some features of these functions can be seen more obviously in the mixed \((\omega, r)\)-representation:

\[ D_N(\omega, r) = -\frac{1}{\omega^2} D(\omega, r) = -\frac{1}{2\pi i} \frac{\sin(\omega r)}{\omega^2 r}; \]  

(26)

\[ D^{(1)}_N(\omega, r) = \text{sgn}(\omega) D_N(\omega, r); \]  

(27)

\[ D^{(\pm)}_N(\omega, r) = \theta(\pm \omega) D_N(\omega, r). \]  

(28)

So, in contrast to \( D^{(1)}(\omega, r) \), these functions are singular at \( \omega = 0 \).

Thus, the near zone of electromagnetic field, as can be asserting, is nonlocal, in part at least, and is formed, in accordance with (12), by two (additional) components of vector \( A_\mu \) or by scalar fields corresponding to their changes. (The opportunity of real replacement of these field components by two scalar fields demands the in-depth analysis and researches.)

But for all that the electric field \( E \), as must be emphasized, remains local:

\[ \langle T(E_i(x)E_k(y)) \rangle = \partial_x \partial_y \langle T(A_i(x)A_k(y)) \rangle \rightarrow D(x - y). \]  

(29)

A little differently the locality of \( E \) and \( H \) fields can be shown by consideration of commutators:

\[ [E_i(x), E_j(y)] = [H_i(x), H_j(y)] = \frac{1}{4\pi i} \{ \partial_i \partial_j - \delta_{ij} \partial^2 \} D_N(x - y); \]  

(30)

\[ [E_i(x), H_j(y)] = \frac{1}{4\pi i} \partial_i \partial_j D_N(x - y), \]  

(31)

double differentiation of the function \( D_N(x) \) repays in \((\omega, r)\) representation the factor \( \omega^{-2} \) responsible for nonlocal effects in (6).

Thus, the nonlocality should be effective, in particular, in such \( A_\mu \)-depending phenomena as the Aharonov-Bohm effect, the Casimir effect, some effects of near field optics, which become apparent at absence of electric and magnetic fields and consequently without the Lorentz force.
IV. ON QUANTUM GENERALIZATION OF LORENTZ CONDITION

The Lorentz-Fermi condition is written in QED as

\[ \partial_\mu A_{\mu}^{(-)} |0\rangle = -iP_\mu(0) A_{\mu}^{(-)} |0\rangle = 0, \]  

(32)

where \( P_\mu(0) \) is the linear 4-momentum.

However the 4-momentum dependence on a degree of inclusion of interaction (the general adiabatic hypothesis) must be taken into account. In the covariant Stueckelberg-Bogoliubov method [10] these features are described by the interaction switching function (FIS) \( g(x) \in [0, 1] \). So, in particular, the 4-momentum depends on a degree of interaction switching as

\[ P_\mu(g) = P_\mu(0) - \int d^4 x \ H(x) \partial_\mu g(x), \]  

(33)

\( H(x) \) is the Hamiltonian of interaction.

Hence the condition (23) must be generalized as:

\[ -iP_\mu(g) A_{\mu}^{(-)} |g\rangle \equiv \partial_\mu A_{\mu}^{(-)} |g\rangle - \int d^4 y \ D^{(-)}(x-y) j_\nu(y;g) \partial_\nu g(x) |g\rangle = 0. \]  

(34)

The additional relation is executable at the Schwinger decomposition of potential without a vacuum averaging. Therefore the performing of inner part of (25) without averaging in near field can be assumed:

\[ \partial_\mu A_{\mu}^{(-)} = \int d^4 y \ D^{(-)}(x-y) j_\nu(y;g) \partial_\nu g(x). \]  

(35)

In the particular system \( n_\mu = \delta_{\mu 0} \) potentials of near field are expressed by (11) as

\[ A_0^{(N)}(x) = A_0 - A_0^{(f)} = -\partial_\mu \Lambda_0(x); \]  

(36)

\[ A^{(N)}(x) \equiv A - A^{(f)} = -\nabla \Lambda_{||}(x). \]  

(37)

It shows that near field is non-vortex (\( B^{(N)} = 0 \)), and its electric component is pure longitudinal:

\[ E^{(N)}(x) = -\partial_t A^{(N)} - \nabla A_0^{(N)} = \partial_\mu \nabla (\Lambda_0 + \Lambda_{||}). \]  

(38)

Scalar potential in the Coulomb gauge is equal zero, i.e. the field \( \Lambda_0(x) \) is stationary and does not take part in the definition (29). Therefore in such gauge

\[ E^{(N)}(x) \equiv -\partial_\mu \nabla \Lambda_{||} = \int d^4 y [\partial_t \nabla D_N^{(-)}(x-y)] j_\mu(y;g) \partial_\mu g(y) \equiv K(x) \otimes_x J(x;g), \]  

(39)
where \( K(x) = \partial_t \nabla D_N^{(-)}(x) \) and \( J(x; g) = j_\mu(y; g) \partial_\mu g(y) \).

After inserting (20) the relation (30) represents the general expression of near field strength and demonstrates the nonlocality of near field, nonzero at any distance even at \( t = 0 \).

As \( D_N^{(-)}(x) = f(t, r) \), the formula (30) shows that near field is determined by the radial component \( E^{(N)}_r(x) \) only (the index \( r \) is below omitted). So in the \((\omega, r)\)-representation the kernel of these forms is simplified:

\[
K(\omega, r) = \frac{\theta(-\omega)}{2\pi i\omega} \frac{\sin(\omega r)}{r},
\]

and

\[
K(\omega, k) = \frac{\theta(-\omega)}{(2\pi)^3 i} \left\{ \frac{4}{(k - i0)^2 - \omega^2} - \frac{1}{\omega k} \ln \frac{k - \omega}{k + \omega} \right\}.
\]

Via the equation \( \nabla E^{(N)}(x) = 4\pi \rho^{(N)}(x) \), with taking into account the wave equation, the expression (30) allows determination of an effective space-time distribution of charges that forms near field, i.e. allows the determination of dynamical form-factor of charge system:

\[
\rho^{(N)}(x) = \frac{1}{4\pi} \int d^4y \; j_\mu(y; g) [\partial_\mu g(y)] \partial_t D^{(-)}(x - y) = \frac{1}{4\pi} \partial_t D^{(-)}(x) \otimes_x J(x; g).
\]

Further analysis of (30) can be possible by substitution of the some expressions of FIS’ suggested in the Appendix.

V. NEAR FIELD OF POINT-LIKE CHARGE

Let’s consider the fixed charge \( Q \) in the origo of coordinates. Its current density

\[
j_\mu(x) = Q \delta_{\mu0} \delta(r)
\]

leads with taking into account the FIS (61) to the generalized current function

\[
J(x) = -\gamma Q \delta(r) \text{sgn}(t) e^{-\gamma |t|}.
\]

In the \((\omega, r)\)- representation

\[
J(\omega, r) = \frac{i\gamma Q \omega}{\pi (\omega^2 + \gamma^2)} \delta(r)
\]

according to (30). It results in the following expressions for radial components of electric field strength in \((\omega, r)\)- and \((\omega, k)\)-representations:

\[
E^{(N)}_r(\omega, r; \gamma) = \frac{\theta(-\omega)\gamma Q}{2\pi^2 (\omega^2 + \gamma^2)} \frac{\sin(\omega r)}{r},
\]

\[
E^{(N)}_r(\omega, k; \gamma) = \frac{\theta(-\omega)\gamma Q}{4\pi^3 (\omega^2 + \gamma^2)} \left\{ \frac{4\omega}{(k - i0)^2 - \omega^2} - \ln \left| \frac{k - \omega}{k + \omega} \right| \right\}.
\]
The inverse FT of (37) is expressed through the integral hyperbolic functions (cf. [14], Exp. (108-9), $\beta = \gamma(t + r)$):

$$E^{(N)}(t, r; \gamma) = \frac{2Q}{(2\pi)^3} \partial_r \left\{ \frac{1}{r} \left[ i\pi e^{-|\beta|} + 2 \{ \text{chi}(|\beta|) \text{sh}(\beta) - \text{shi}(\beta) \text{ch}(\beta) \} \right] - \frac{1}{r} |\beta \rightarrow \overline{\beta}| \right\}.$$  (48)

Let’s estimate the self energy of near field at the moment $t = 0$:

$$W(0) = \frac{1}{8\pi} \int dr \ |E^{(N)}(t = 0, r; \gamma)|^2.$$  (49)

Direct substitution of (38) into (39) leads to an excessively complicated expression. Therefore we shall consider more rough estimations for FT of (37) at $t = 0$ for two frequencies regions taken separately:

$$E^{(N)}_1(t = 0, r; |\gamma| >> \omega) \approx -\frac{Q\gamma}{\pi^2 r^3};$$  (50)

$$E^{(N)}_2(t = 0, r; |\gamma| << \omega) \approx -\frac{Q\gamma}{2\pi^2 r}.$$  (51)

The expression (40) is related to low frequencies, i.e. to large distances, and therefore (39) can be integrated in the limits $(1/\gamma, \infty)$. The expression (40)’ corresponds to high frequencies and after substitution into (39) it can be integrated over $(0, 1/\gamma)$. These integrations give very close expressions and their sum leads to the compound estimation:

$$W(0) \approx Q^2 \gamma / 6\pi^4 + Q^2 \gamma / 8\pi^4 \approx 3 \cdot 10^{-3} Q^2 \gamma$$  (52)

or in the usual units and at $Q \rightarrow e, \alpha = e^2 / hc, [\gamma] = \sec^{-1}$

$$W(0)/mc^2 \approx 3 \cdot 10^{-3} \alpha \lambda_C \gamma / c \approx 3 \cdot 10^{-3} r_0 \gamma / c.$$  (53)

The estimation shows that on the distances of order of the Compton wavelength, $\gamma \rightarrow c/\lambda_C$, the near field energy is of order of some eV’s that corresponds to potentials of ionization. At $c/\gamma$ of the order of Bohr radius, $r_B = \alpha^{-1} \lambda_C$, it gives decimal fractions of eV for near field energy in correspondence with interatomic bonds.

The estimation of charge distribution of near field represents curious. According to (231) and (37) in the $(\omega, r)$-representation

$$\rho^{(N)}(\omega, r) = -\frac{Q\omega^2}{4\pi} g(\omega) D^{(-)}(\omega, r) = \theta(-\omega) \frac{iQ}{(2\pi)^3} \omega^2 \frac{\gamma \omega^2}{\gamma^2 + \sin \omega r}.$$  (54)

or

$$\rho^{(N)}(t, r) = \frac{1}{4\pi} \int d\tau g(\tau) \partial^2_{\tau \tau} D^{(-)}(\tau - t, r).$$  (55)

Its average value over $r$ or $t$ is equal, certainly, to zero, and the maximum is achieved at $\omega r = 0$. On high frequencies, when $\omega >> \gamma$,

$$\rho^{(N)}(\omega, r) \approx Q\gamma D^{(-)}(\omega, r),$$  (56)

i.e. properties of near field come closer to far field features. In the low frequencies field, i.e. at $\omega << \gamma$, an effective charge density in the near field $\rho^{(N)}(\omega, r) \rightarrow 0$. 
VI. FRUSTRATED TOTAL INTERNAL REFLECTION (FTIR)

Let’s consider, via the expression (30), the phenomenon of FTIR of light wave \( \mathbf{E} = \mathbf{E}_0 e^{i\omega t} \) under an angle \( \varphi \) \((\varphi > \varphi_{cr})\) from smooth dielectric surface \((z = 0)\) of medium with polarizability \(\alpha\).

The index of refraction is formally expressed as

\[
n(z) = n_\theta(\varepsilon - z) + 1 \cdot \theta(z) = \frac{1}{2} \left[ (n + 1) - (n - 1) \text{sgn}(z) \right]. \tag{57}
\]

In reality this idealized plane must be substituted by an effective layer of minimal depth, without an instantaneous jump from parameters of one medium to another ones, i.e. by a sufficiently smooth transitive layer between both media. With this aim it is necessary to choose FIS, smooth together with the first derivative, e.g. \( \text{sgn}(z) \rightarrow g(z|\varphi) = \exp(-z^2/\Delta z^2) \) with \( \Delta z^{-1} = 4n(z)\omega \cos \varphi \).

The initial wave induces (or orients) dipoles \( p(t) = \alpha \mathbf{E}(t) \) on the surface “plane”, i.e. induces the "current" \( j_\mu(y; g) \rightarrow (c/n)\delta_{\mu z} \rho(t, r) \).

If to accept that a plane or layer interface of medium is strictly flat, the density of surface charges would be described as

\[
\rho = (p(t)) \cdot \delta'(z) \delta(x) \delta(y) = - (p(t)) z^{-1} \delta'(z) \delta(x) \delta(y), \tag{58}
\]

i.e. as a double electric layer oscillating with a frequency of falling wave.

As this layer must be taken into account at estimations of other parameters of medium also, it is necessary to replace \( \delta(z) \) in the last expression (45) on a \( \delta \)-like function in an agreement with the choice of FIS: e.g. \( \delta(z) \rightarrow \delta(z, \xi) = (\xi/\pi)^{-1} \exp(-z^2/\xi^2) \).

Thus the expression (30) of near field strength at inter-surface layer will be of the order

\[
\mathbf{E}^{(N)}(t, r|\omega) = \alpha |\mathbf{E}_0| \int_{-\infty}^{\infty} d\tau d\xi \ n_\theta^{-1} e^{-i\tau \omega} \delta'(\xi, \Delta z) \left[ \partial_\xi g(\xi|\Delta z) \right] \nabla \partial_\tau D_N^{(-)}(\tau - t; x, y, \xi - z), \tag{59}
\]

where the formal integration over time can be executed. It shows, that frequencies of near field "photons", the evanescent "particles", will coincide with frequencies \( \omega \) of initial field and all their differences should be manifesting only in momenta.

As \( \partial_\xi g(z|\varphi) = -2z\Delta z^{-2} g(z|\varphi) \) and \( \Delta z(\xi) \) has different values for \( \pm \) arguments, the relation (46) can be rewritten as

\[
\mathbf{E}^{(N)}(t, r|\omega) = \frac{2i}{\sqrt{\pi}} \omega e^{-i\omega t} \alpha |\mathbf{E}_0| \left\{ \left[ \frac{1}{\Delta z^2} \int_0^\infty d\xi e^{(-2\xi^2/\Delta z^2)} \nabla D^{(-)}(\omega; x, y, \xi; \xi - z) \right] + \frac{1}{n} \left[ \Delta z \rightarrow \Delta z_2, z \rightarrow -z \right] \right\}, \tag{60}
\]

where it is taken into account that \( \Delta z t = n\Delta z \) does not depend on \( z \).

The distribution of near field, as shows (47), does not depend on depth of its tunneling relatively FTIR "plane", in \( z \)-direction, and occurs instantaneously. For this reason the possible transformation of evanescent waves into extending waves occurs simultaneously in all forbidden depth (cf. [15]).

The expression (47) can be considered as the Fourier convolution over variable \( z \) in the first term and over \( -z \) in the second one. By separation of \( z \)-component of wave vector, \( k = \{k_\perp, q \} \) and with taking into account the Fourier transformation (FT) of "current" factor:

\[
I(q, \Delta z) = \int_0^\infty d\xi e^{(iqs - 2\xi^2/\Delta z^2)} = \Delta z \sqrt{\frac{\pi}{8}} e^{-q^2\Delta z^2/8} \left[ 1 - \text{erf}(-iq\Delta z/\sqrt{8}) \right]. \tag{61}
\]

So we receive the final expression for FT of near field intensity as
\[ E^{(N)}(t, k|\omega) = 2\pi^{-1/2} e^{-i\omega t} |E_0| \Delta z^{-3} k \{ I(-q, \Delta z) + n^{-4} I(q, \Delta z) \} D_N(-\omega; k). \]  

At small values of parameter \( q\Delta z \) the expression in braces in (49) becomes

\[ \{ \ldots \} \rightarrow \sqrt{\frac{\pi}{8}} \Delta z^2 \left[ \exp(-\frac{q^2 \Delta z^2}{8}) + \frac{1}{n^3} \exp(-n^2 q^2 \Delta z^2/8) \right], \]

i.e. it contains only \( q \)-components of wave vector of any magnitude. Therefore “evanescent photons” can possess, at the same frequency, a bigger or smaller momenta (compare [16]).

But for all that, due to the identity of frequencies, they can interfere with photons of inlet radiation. It means that such interference can locate objects, at supervision of interference picture closely enough to the refraction surface, with sizes about \( |k_z - q|^{-1} \), i.e. smaller light wavelengths in vacuum. Just this effect is the physical basis of so-called near field optics [4] (we do not examine the evident complications associated with light polarization).

Let’s note that the use of another FIS, e.g. (67) instead of (65), leads to similar results with replacement \( q^2 \Delta z^2 \) on \( |q\Delta z| \) and insignificant change of numerical factors. The choice between different FIS’ can be made, at the given stage, by a comparison with experiments only.

VII. INTERACTION OF ATOMS IN NEAR FIELD

Energy of non-resonant interaction of two neutral atoms on distances smaller wavelength, but bigger their own sizes [11], is determined by the two-photon exchange (the fourth order of \( S \)-matrix) as

\[ U(r) = \frac{i}{4\pi} \int_{-\infty}^{\infty} d\omega \omega^4 \alpha_1(\omega) \alpha_2(\omega) |D_{ik}(\omega, r)|^2, \]  

where \( \alpha_i(\omega) \) is the polarizability of cooperating atoms, scalar at the \( S \)-states.

The affinity of \( D_{ik}(\omega, r)|_{NF} \) to the matrix element of dipole-dipole interaction is evident. Therefore for such distances and atomic frequencies that \( \omega r \ll 1 \), the calculations with inserting the decomposition (3) into (51) are precisely identical to the procedure [11] and lead to the Van-der-Waals energy of interaction proportional \( R^{-6} \) and to the Casimir energy of interaction of atoms proportional to \( R^{-7} \).

Thus, these interactions are describable by the propagator (6), i.e. they occur in the near field and, at least, are in part transferred superluminally.

However for resonant interaction between identical (motionless) atoms,

\[ A_1^* + A_2 \leftrightarrow A_1 + A_2^*, \]  

matrix element is nonzero still in the second order:

\[ S^{(2)} = -\frac{1}{2} \int dt_1 dt_2 T \{ V(t_1) V(t_2) \}, \]  

where \( V = -E(r_1) d_1 - E(r_2) d_2 \). Therefore instead of (51) we have

\[ U(r) = (i/2\pi) \int_{-\infty}^{\infty} d\omega \omega^2 D_{ik}(\omega, r) Re[\alpha_{ik}(\omega)], \]  

with the tensor of scattering of two-level, for simplicity, systems expressed through matrix elements of dipole momenta:
\[ \alpha_{ik}(\omega) = \frac{(d_i)_{01}(d_k)_{10}}{\omega_0 - \omega - i\Gamma} + \frac{(d_k)_{01}(d_i)_{10}}{\omega_0 + \omega - i\Gamma}. \]  

(68)

By the substitution of \( D_N \) function into (54) it can be shown that the interaction (52) decreases as \( R^{-3} \) (cf. [17]).

The full probability of process (52) in the near field is determined as

\[ W \propto \int_{-\infty}^{\infty} d\omega |D_{ik}(\omega, r)\alpha_{ik}(\omega)|^2 \rightarrow \int_{-\infty}^{\infty} d\omega |d_1|^2 |d_2|^2 |D_{ik}(\omega, r)|_{NF}^2 \tau(\omega)/\Gamma, \]  

(69)

where the expression of duration of scattering process is separating out:

\[ \tau(\omega) = \frac{\Gamma}{2}[(\omega_0 - \omega)^2 + \Gamma^2/4]. \]  

(70)

By carrying out integration in view of \( \delta \)-character of (57), using matrix elements of dipole operators \(|d|^2 = \hbar e^2 f/2m\omega, f \) is the oscillator force, and substituting the expressions of singular functions (6-7), we receive, that the probability of process depends on distance between cooperating atoms as \( R^{-6} \), i.e. it takes the form of the well-known half-empirical Förster law [18] (see, e.g., [19]):

\[ W = \Gamma^{-1}(R_0/R)^6, \]  

(71)

where \( R_0 \) is the so-called Förster radius.

With (6) it follows that the rate of process (52) in the time representation is represented by the square of near field singular function (6):

\[ |D_{il}(t, r)|_{NF}^2 = \frac{1}{(4\pi r^2)^2} \left\{ \frac{\theta(t^2 - r^2)}{r} + \left(\frac{t}{r}\right)^2 \frac{\theta(r^2 - t^2)}{r} \right\}, \]  

(72)

that determines relative probabilities of excitation transfer with subluminal and superluminal speeds.

But the superluminal (instantaneous) interaction is possible, as was established in [7], at the tunneling only. Are there certain processes and certain frequencies ranges in condensed media similar to anomalous dispersion in optics?

In the articles [20] we had shown that at phase transitions of the first kind the liberated latent energy must be converted, at least partially, into the characteristic radiation with frequencies determined by released energy at establishing definite bonds. (This phenomenon is hard for observations, because surrounded substances quickly thermalize the emitted radiation, but its existence is confirmed by some experiments, e.g. [21] and references therein.)

This phenomenon can take place at the sight of discrepancy between energy of relating corresponding bonds and momenta of emitted virtual "photons". Therefore their interaction with neighbors can have some similarity with anomalous dispersion regime, in particular with its instantaneous peculiarity. It allows to assume that the permanent interatomic bonds in condensed state, partially or completely, are instantaneous ones.

VIII. CONCLUSIONS

Let us enumerate the results.

1. The decomposition of canonical Green functions of QED leads to the appearance of propagators of far and near fields and an intermediate one describing transitions between them.

2. Green function of near field corresponds to the Schwinger function, initially introduced for investigation of an electron self-field. The rule of transition from far field functions into near field ones and vice versa is considered.

3. The Schwinger scheme of \( A_\mu \) decomposition allows to discard at examination of near field phenomena a formal vacuum averaging of the classical Lorentz condition. At the same time it shows that the near field function is formed by longitudinal and scalar (temporal) components of the 4-vector \( A_\mu \) or by two additional scalar fields, derivatives of
which represent these components. Thus this scheme demonstrates the physical sense of "surplus" components and shows the inconsistency of their complete formal elimination by introduction of indefinite metrics.

4. The performed analysis has shown, within the frame of QED, that the near zone of $A_{\mu}$ represents the nonlocal, but quickly decreasing field, so that the $E$ and $B$ fields remain local. Hence this analysis requires the introduction of the "nonlocality in the small" only.

It once more underlined the necessity and significance of notions of adiabatical switching on and off interaction for understanding details of QED interactions (cf., e.g. [22]).

5. The general approach to FTIR phenomena is elaborated via introduction, along an analogue with the field theory, the function of interaction switching at wave transition into another medium. Such method can be applied to other phenomena of substance parameters changing. The used approach shows the superluminal features of FTIR. It means that these interactions can be, partly at least, instantaneous. The expression for excitation transfer on small distances (the Förster law) also has the same form.

Let’s recall, in this connection, the continued discussions of the temporal features of tunneling [23] that had induced a number of paradoxes (e.g., [24, 25]). Our consideration shows within the framework of QED, at least, that the tunnel transition must be executable within the scope of near field and, under some conditions, can be instantaneous [7].

All our consideration shows that the instantaneous transferring is not of very exotic, extraordinarily nature; its manifestations can occur in some phenomena, which may be considered as the "nonlocal in the small", and therefore their temporal features should be investigated more carefully and widely. (It seems, for example, that such phenomena as the energy-time entanglements can be also connected with such nonlocality, e.g. [27].)

7. The described features of near zone of point-like charge and, on the other hand, of optical transitive zones as the fields of longitudinal and scalar evanescent "photons" with possibilities of observability of their "superluminal" features, deprives the formal schemes of elimination of "superfluous" fields components, such as introduction the indefinite metrics, their general significance. All this demands anew returning to the principal problems of the QED gauges, opportunities of their transformations and their peculiarities.

8. The revealed "nonlocality in the small" in the context of covariant field theory requires not only more scrupulous further research of its properties, but also the search of similar phenomena as in the QED, so, probably, in the theories of other fields.

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X. APPENDIX: FUNCTIONS OF INTERACTION SWITCHING

In the Bogoliubov theory the temporal FIS’ $g(t, r)$ are not concretized, they must only satisfy the conditions:

- $g(t, r) \in [0, 1]$, which can be slightly generalized as $|g(t, r)| \in [0, 1]$;
- General covariance;
- Limiting conditions: $g(x) \to 0$ at $x \to \pm\infty$;
- $g(-x_{\mu}) = g(x_{\mu})$, that follows CPT invariance of $A_{\mu}$.

The simplest FIS’ satisfying these conditions:

$$g_1(x) = e^{-\gamma |n_{\mu}x_{\mu}|}; \quad g_2(x) = e^{-\gamma^2(n_{\mu}x_{\mu})^2}; \quad g_3(x) = e^{-\gamma^2 |x_{\mu}|^2}. \quad (73)$$

The existence of near field in classical theories allows its independence from $\hbar$. Therefore for classical problems it seems natural to express this parameter by the Thompson radius of electron: $\gamma = c/r_0 \equiv mc^3/e^2$, this form leads to disappearance of interaction and near field at $e \to 0$. For bound electron its expression via the Compton wavelength of electron seems natural: $\gamma' = c/\lambda_C = mc^2/\hbar$.

For the part of our consideration the choice $n_{\mu} = \delta_{\mu0}$ is sufficient:
\[ g_1(x) = e^{-\gamma |t|}; \quad g_2(x) = e^{-\gamma^2 t^2} \] (74)

and \( g_2(x) = g_3(x) \) at \( r = 0 \). Under the FT these FIS’ take the forms:

\[ g_1(\omega) = \gamma / \pi (\omega^2 + \gamma^2); \quad g_2(\omega) = (2\gamma \sqrt{\pi})^{-1} \exp(-\omega^2/4\gamma^2), \] (75)

at \( \gamma \to 0 \) they aspire to \( \delta(\omega) \).

To functions (62) different physical interpretations can be given. So, \( g_2(\omega) \) corresponds to the normal law of probability with similar process of interaction switching, etc. The function \( g_1(\omega) \) corresponds, excluding factor \( \pi \), to the duration of elastic photon scattering on free electron. Such interpretation allows generalization of \( g_1(\omega) \) for interaction with bound electron into two level systems:

\[ g_1(\omega|\omega_0) = \gamma / 2\pi \{[(\omega - \omega_0)^2 + \gamma^2]^{-1} + [(\omega + \omega_0)^2 + \gamma^2]^{-1}\} \] (76)

or, in the \( t \)-representation,

\[ g_1(t|\omega_0) = e^{-\gamma |t|} \cos(\omega_0 t). \] (77)

Let us introduce on the similar basis the space FIS’ determining the switching or alteration of interaction during particle (wave) transitions across determined (flat) space borders. So, at approach of light wave to refracting surface \( z = 0 \) it can be suggested the FIS:

\[ g_1(x) \to g_1(z) = e^{-\kappa |z|}; \quad g_2(x) \to g_2(z) = e^{-\kappa^2 z^2}, \] (78)

where \( \kappa^{-1} \) is the effective width of an intermediate surface layer depending on parameters of both substances and light flux.

At the total internal (or external) reflection of light of frequency \( \omega \) on intersection of media with indices of refraction \( n_1 \) and \( n_2 \) under angle \( \varphi > \varphi_{\text{crit}} \), \( z \)-component of photon momentum \( k_z = kn_1 \cos \varphi \) changes sign and the alteration of momentum is equal

\[ |\Delta k_z| = 2kn_1 \cos \varphi. \] (79)

In accordance with the uncertainty principle this alteration is executed in the layer \( |\Delta z| = 1/2|\Delta k_z| \), therefore the value \( \kappa = 1/|\Delta z| \) and at \( n = n_1/n_2 \)

\[ g_1(t, r|\varphi) = e^{-4n|\omega||z| \cos \varphi}, \quad g_2(t, r|\varphi) = e^{-(4n \omega z \cos \varphi)^2}. \] (80)

Such transient optical layer should exist at all processes of reflection and refraction, since at \( \varphi < \varphi_{\text{crit}} \) and with refraction angle \( \varphi' \) the change of photon momentum is

\[ |\Delta k_z| = k|n_1 \cos \varphi - n_2 \cos \varphi'|, \] (81)

which again leads to (67).

These processes, as it is known, can be of nonlocal character, described usually via surface polaritons (e.g., [28]).
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