Towards Deriving Higgs Lagrangian from Gauge Theories

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(March 28, 2022)

Abstract

A new method of deriving the Higgs Lagrangian from vector-like gauge theories is explored. After performing a supersymmetric extension of gauge theories we identify the auxiliary field associated with the “meson” superfield, in the low energy effective theory, as the composite Higgs field. The auxiliary field, at tree level, has a “negative squared mass”. By computing the one-loop effective action in the low energy effective theory, we show that a kinetic term for the auxiliary field emerges when an explicit non-perturbative mechanism for supersymmetry breaking is introduced. We find that, due to the naive choice of the Kähler potential, the Higgs potential remains unbounded from the below. A possible scenario for solving this problem is presented. It is also shown that once chiral symmetry is spontaneously broken via a non-zero vacuum expectation value of the Higgs field, the low energy composite fermion field acquires a mass and decouples, while in the supersymmetric limit it was kept massless by the ’t Hooft anomaly matching conditions.

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I. INTRODUCTION

Spontaneous chiral symmetry breaking induced by the fermion anti-fermion pair condensate is, in general, expected in asymptotically free vector-like gauge theories. The effective Higgs Lagrangian (linear $\sigma$-model) well describes chiral symmetry breaking at low energies, however it is difficult to derive it from the original gauge theory. It is known that an effective Higgs Lagrangian can be derived from the Nambu-Jona-Lasinio model by using the so called auxiliary field method $[1]$. An unsatisfactory feature is the fact that cannot be applied to gauge theories.

There are already many attempts of deriving low energy effective theories associated with a given gauge theory by using supersymmetry $[2–9]$. The hope is to extend some of the “exact results” deduced by Seiberg and Witten in $[10,11]$ for supersymmetric vector-like gauge theories to non-supersymmetric gauge theories.

However, since in Ref. $[2–9]$ the explicit supersymmetric breaking is treated perturbatively, supersymmetry cannot be completely decoupled. Recently in Ref. $[13,14]$ the issue of decoupling supersymmetry has been addressed, and a suitable decoupling procedure has been proposed for supersymmetric QCD like theories with $N_f < N_c$ and $N_c > 2$. In the latter approach the decoupling is able to constrain only the holomorphic part of the QCD like potential which encodes the anomaly structure of the theory. A key point in the analysis of Ref. $[13]$ was to identify the auxiliary field associated with the supersymmetric effective low energy composite operators as the “meson” fields for the ordinary theory, once supersymmetry is broken.

In Reference $[15]$ we have outlined a new method for deriving the Higgs lagrangian. In this paper we provide a more complete discussion of the method and we will also describe in some detail the new techniques for including non-perturbative supersymmetry breaking effects.

Let us consider the $N = 1$ supersymmetric extension for vector-like gauge theories (assuming that the extension preserves asymptotic freedom). We have the following quark chiral superfields for the underlying theory.

$$Q_i^\alpha = \phi_{Q_i}^\alpha + \sqrt{2}\theta \psi_{Q_i}^\alpha + \theta^2 F_{Q_i}^\alpha,$$  
$$\tilde{Q}^\alpha_j = \phi_{Q_j}^\alpha + \sqrt{2}\theta \psi_{Q_j}^\alpha + \theta^2 F_{Q_j}^\alpha,$$  

where $i$ and $j$ are the flavor indices while $\alpha$ is the color index. The low energy effective theory, including only massless fields, can be obtained by correctly reproducing the global symmetries and assuming holomorphy, as shown by Seiberg $[11]$. The relevant massless chiral superfield is the color-singlet “meson” field $M^{ij}$ which couples to the quark bi-linear operator as follows

$$M^{ij} \sim Q_{i}^\alpha \tilde{Q}^{\alpha j} ,$$  

where $M = A + \sqrt{2}\theta \psi + \theta^2 F$. It is instructive to show how the different components in $M^{ij}$ couple to the operators defined in the underlying theory

$$A^{ij} \sim \phi_{Q_i}^\alpha \phi_{Q_j}^\alpha ,$$  
$$\psi^{ij} \sim \psi_{Q_i}^\alpha \phi_{Q_j}^\alpha + \phi_{Q_i}^\alpha \psi_{Q_j}^\alpha ,$$  
$$F^{ij} \sim \phi_{Q_i}^\alpha F_{Q_j}^\alpha + F_{Q_i}^\alpha \phi_{Q_j}^\alpha - \psi_{Q_i}^\alpha \psi_{Q_j}^\alpha ,$$  

$$2$$
where the contraction of the color indices is understood. If we consider massless quarks, the auxiliary field $F_{ij}$ couples only to the quark bi-linear operator, because of the underlying field equation $F_Q = F_{\tilde{Q}} = 0$. Of course, in the supersymmetric limit, the auxiliary field at the effective lagrangian level must be integrated out by using the algebraic equation of motion.

In order to investigate the effects of the squark decoupling in the low energy physics, we introduce a soft supersymmetric breaking term which correctly reproduces the squark mass in the underlying theory. By using the spurion method we add the following term in the Kähler potential

$$L_{\text{soft}} = \int d^4 \theta X \left( Q^i Q + \tilde{Q}^i \tilde{Q} \right),$$

(1.6)
of the fundamental theory. The spurion vector superfield $X$ is a singlet under both gauge and chiral symmetry. There are two possibilities for $X$: $X = \theta^2 m + \bar{\theta}^2 m^\dagger$ or $X = -\theta^2 \bar{\theta}^2 m^2$. If we take the former, the equation of motion for the quark auxiliary field is modified with the respect to the supersymmetric limit, i.e. $F_Q = -mA_Q$ and $F_{\tilde{Q}} = -mA_{\tilde{Q}}$. Therefore, the auxiliary field in the effective theory now couples also to the squark bi-linear operator. However when $m$ is large enough compared with the dynamical generated scale $\Lambda$, we expect the quark bi-linear operator to dominate over the squark one. If we take the latter $X$ choice the equation of motion for the quark auxiliary field remains unchanged and the auxiliary field in the effective theory couples only to the quark bi-linear operator. Here we choose $X = m\theta + m^\dagger \bar{\theta}$, with $m$ real, as spurion.

In this paper we assume the following naive Kähler potential for the effective “meson” superfield

$$K = \frac{1}{(\alpha \Lambda)^2} M_{ij}^\dagger M^{ji},$$

(1.7)

where $\alpha$ is a numerical constant. This will automatically induce a “negative squared mass” for the auxiliary field. At the component level we have

$$L_{\text{eff}} = \int d^4 \theta K = \frac{1}{(\alpha \Lambda)^2} F_{ij}^\dagger F^{ji} + \cdots,$$

(1.8)

which means that the vacuum at $F = 0$ is unstable and a non-zero vacuum expectation value (if $F$ is regarded as a propagating field once supersymmetry is broken) is expected. This fact supports our idea that the auxiliary field can be identified with the composite Higgs field associated with the quark bi-linear operator.

It is known that the effective Kähler potential for $N = 1$ supersymmetric theories cannot be fixed, so that a more general form is expected. However we believe that the Kähler potential in Eq. (1.7) can be considered as the first term in a general low energy expansion [16].

The effect of supersymmetry breaking is incorporated in the effective theory by using the spurion method. The explicit supersymmetry breaking term in the Kähler potential is of the form

$$K_{\text{soft}} = \frac{\beta}{(\alpha \Lambda)^2} X M_{ij}^\dagger M^{ji},$$

(1.9)
at the lowest order in a $m/\Lambda$ expansion and $\beta$ is a numerical constant.

Once defined the Kähler potential in Eqs. (1.7), (1.9) and added the appropriate exact superpotential [10], we then compute the one-loop effective action in the low energy effective theory. It results in new terms to add to the Kähler potential. The loop calculation can also be interpreted as a way of providing more information about the Kähler potential in $N = 1$ theories. If we had an exact Kähler potential (as for example in $N = 2$ supersymmetry), we do not expect loop calculations to generate new contributions.

If we treat supersymmetry breaking effect perturbatively, the one-loop supergraph is infrared divergent. At the component level it is easy to recognize that the origin of such a divergence is the loop associated with the low energy effective scalar field.

However, if we include the supersymmetry breaking effect non-perturbatively, the infrared divergence is automatically regulated since the low energy scalar field becomes massive via $K_{soft}$. The soft breaking mass, actually, works as an infrared cutoff. Although it is possible to include a mass term in the superpropagator for the effective super-field we prefer, for simplicity, to use the mass as an explicit infrared momentum cutoff within the Euclidean momentum cutoff scheme.

The paper is organized as follows. In section II we consider the asymptotically free $N = 1$ supersymmetric gauge theory with number of colors $N_c = 2$ and flavors $N_f = 3$. We use this system as a laboratory to illustrate our new method of deriving the Higgs Lagrangian. In section III we compute the one-loop effective action for the low energy effective theory. By first computing the two point effective action we show that the kinetic term for the auxiliary field is naturally generated thus enforcing our idea of identifying the auxiliary field with the Higgs field.

We then demonstrate, by computing the four-point effective action contribution that, once chiral symmetry is spontaneously broken via a non-zero vacuum expectation value of the Higgs field, the low energy effective fermion field acquires a mass term. In the supersymmetric limit the composite fermion was kept massless by the ’t Hooft anomaly matching conditions [17]. It is also seen that the Higgs potential remains unstable at the one loop level. However by generalizing the Kähler potential we illustrate a possible scenario for generating the complete Higgs potential. In section IV we briefly review the main results and consider possible improvements as well as some extension of the present model.

### II. A SIMPLE MODEL

As starting point we will consider the supersymmetric asymptotically free theory with $SU(2)$ gauge group and three flavors in the fundamental representation. The global quantum symmetry group is $SU(6) \otimes U(1)_R$, where $SU(6)$ is the enlarged flavor group associated with the quark chiral superfield $Q^i_\alpha$ and $U(1)_R$ is the $R$-symmetry. Here $i = 1, 2, \cdots, 6$ labels the flavor index while $\alpha = 1, 2$ the color one. We use the notation of Ref. [18] throughout this paper. According to the ’t Hooft anomaly matching conditions the “meson” chiral superfield

$$M^{ij} \sim \epsilon^{\alpha \beta} Q^i_\alpha Q^j_\beta,$$

(2.1)

which belongs to the $15_A$ representation of $SU(6)$, is a massless field. By saturating at tree level the anomalous as well as non-anomalous Ward-Takahashi identities and imposing holomorphy [10] one can get the effective low energy superpotential
\[ W_{\text{eff}} = -\frac{1}{\Lambda^3} \text{Pf} M, \]  
(2.2)

where \( \Lambda \) is the dynamical scale associated with the underlying supersymmetric gauge theory. To the “exact” super-potential we have the need, in order to completely define the theory, to add the Kähler potential. This is the most undetermined part for \( N = 1 \) supersymmetric gauge theories in contrast with the \( N = 2 \) supersymmetric theories [11] where the full Kähler is claimed to be known. Following the arguments presented in the introduction a reasonable approximation to the full Kähler potential would be to consider the following terms

\[ K_{\text{eff}} = \frac{1}{(\alpha \Lambda)^2} M_{ij}^\dagger M^{ji} + \frac{\beta}{(\alpha \Lambda)^2} XM_{ij}^\dagger M^{ji}, \]  
(2.3)

where \( \alpha \) and \( \beta \) are dimensionless numerical constants. The need for a more general type of Kähler potential will be advocated later in the paper. The second term is a soft supersymmetry breaking term introduced using the spurion method, and provides a mass term for the scalar component of the effective superfield \( M \). We assume the spurion field to be of the form \( X = m(\theta^2 + \bar{\theta}^2) \) with real \( m \). We can get the canonically normalized field by performing the following field rescaling \( M / (\alpha \Lambda) \rightarrow M \). The Lagrangian becomes

\[ \mathcal{L} = \int d^4 \theta \left( M_{ij}^\dagger M^{ji} + \beta XM_{ij}^\dagger M^{ji} \right) - \alpha^3 \int d^2 \theta \text{Pf} M + \text{h.c.}. \]  
(2.4)

By performing the ordinary theta integration we deduce the tree level “potential” for the effective auxiliary field in the limit \( \beta = 0 \)

\[ V(F, A) = -F_{ij}^\dagger F^{ji} + \frac{\alpha^3}{16} \epsilon_{ijklmn} F^{ij} A^{kl} A_{mn} + \text{h.c.}, \]  
(2.5)

where \( F^{ij} \) and \( A^{ij} \) are, respectively, the auxiliary and the scalar components of the chiral superfield \( M^{ij} \). We observe that the auxiliary field has negative squared mass. Since in the supersymmetric limit the auxiliary field does not propagate, i.e. no kinetic term is present in the tree level lagrangian, it is not considered as a physical field and the instability is removed by integrating it out via its equation of motion \( \partial V(F, A) / \partial F = 0 \). In the present approach, where supersymmetry is explicitly broken, we will keep it [13,15] and will show next that via non-perturbative breaking effects a non trivial kinetic term is generated for the auxiliary field, suggesting that it can become a physical field.

III. CALCULATION OF THE ONE-LOOP EFFECTIVE ACTION

In this section we present the actual calculations for the one-loop effective action based on the Lagrangian defined in Eq. (2.4). We employ the supergraph technique [19] with the spurion \( X \) as external field.

A. The Two Point Function

The first term to compute is the one-loop two point function which corresponds to the quadratic term in the effective field associated with the Kähler potential.
The diagram we evaluated is shown in Fig. 1. By using the standard super Feynman rules we obtain

\[
\Gamma_2 = -18 \left( \frac{\alpha^3}{2^3 3!} \right)^2 \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4 i} d^4 \theta_1 d^4 \theta_2 M_{ij}^\dagger(-p, \theta_2) \epsilon^{ijabcd} \epsilon_{klabcd} M_{kl}^\dagger(p, \theta_1)
\]

\[
\frac{1}{k^2(p + k)^2} \delta^4(\theta_2 - \theta_1) \frac{\bar{D}^2(k, \theta_1) D^2(k, \theta_1)}{16} \delta^4(\theta_1 - \theta_2),
\]

where the factor 18 is a symmetric factor and the factor \(\alpha^3/2^3 3!\) is the coupling constant defined in the superpotential. With the help of the following identity

\[
\delta^4(\theta_2 - \theta_1) \frac{\bar{D}^2(k, \theta_1) D^2(k, \theta_1)}{16} \delta^4(\theta_1 - \theta_2) = \delta^4(\theta_2 - \theta_1),
\]

we can integrate over \(\theta_2\) and deduce

\[
\Gamma_2 = 18 \cdot (2 \cdot 4!) \left( \frac{\alpha^3}{2^3 3!} \right)^2 \int \frac{d^4 p}{(2\pi)^4} d^4 \theta M_{ij}^\dagger(-p, \theta) M^{ji}(p, \theta) \int \frac{d^4 k}{(2\pi)^4 i} \frac{1}{k^2(p + k)^2},
\]

where we used the fact that

\[
\epsilon^{ijabcd} \epsilon_{klabcd} = 4! \left( \delta^i_k \delta^j_i - \delta^i_k \delta^j_i \right).
\]

The \(k\) momentum integration can be expressed as

\[
\int \frac{d^4 k}{(2\pi)^4 i} \frac{1}{k^2(p + k)^2} = \int_0^1 dx \int \frac{d^4 k}{(2\pi)^4 i} \frac{1}{[k^2 + x(1 - x)p^2]^2}
\]

\[
= \int_0^1 dx \int_{m}^{\Lambda} \frac{d^4 k_E}{(2\pi)^4} \frac{1}{[k_E^2 + x(1 - x)p_E^2]^2}
\]

\[
\approx \int_0^1 dx \int_{m}^{\Lambda} \frac{d^4 k_E}{(2\pi)^4 k_E^4} \frac{1}{1 - 2x(1 - x)p_E^2 k_E^2}
\]

\[
\approx \int_{m}^{\Lambda} \frac{d^4 k_E}{(2\pi)^4 k_E^4} \frac{1}{3 p_E^2} \int_{m}^{\Lambda} \frac{d^4 k_E}{(2\pi)^4 k_E^6}.
\]

To perform the integral we have analytically continued it over the Euclidean region and assumed the scale \(\Lambda\) as our ultraviolet cutoff. This procedure corresponds to the Euclidean momentum cutoff regularization scheme. We remark that it seems natural to us to consider \(\Lambda\) as a physical ultraviolet cutoff, since the present theory lives naturally below the scale \(\Lambda\).

The appearance of the infrared divergence is due to the scalar loop, hence it is reasonable to identify the supersymmetric breaking mass \(m\) with a physical infrared cutoff. After regularizing the integral we performed a momentum expansion in \(p_E^2\) with \(p_E^2 \ll m < \Lambda\). It resulted in the following expression for the two point function

\[
\Gamma_2 = \frac{3}{2\pi^2} \left( \frac{\alpha}{2} \right)^6 \int d^4 x d^4 \theta \left\{ \ln \left( \frac{\Lambda^2}{m^2} \right) M_{ij}^\dagger(x, \theta) M^{ji}(x, \theta) \right. + \left. \frac{1}{3m^2} M_{ij}^\dagger(x, \theta) \square M^{ji}(x, \theta) \right\}
\]

\[
= \frac{3}{2\pi^2} \left( \frac{\alpha}{2} \right)^6 \int d^4 x d^4 \theta \left\{ \ln \left( \frac{\Lambda^2}{m^2} \right) M_{ij}^\dagger(x, \theta) M^{ji}(x, \theta) + \frac{1}{48m^2} \bar{D}^2 M_{ij}^\dagger(x, \theta) D^2 M^{ji}(x, \theta) \right\},
\]
where
\[ M^{ij}(x, \theta) = \int \frac{d^4 p}{(2\pi)^4} M^{ij}(p, \theta)e^{-ipx}. \] (3.7)

The first term in the brackets of Eq. (3.6) is a correction to the tree level Kähler potential while the second term is the source for the kinetic term of the auxiliary field. By performing the usual theta integration for the second term we get
\[ \mathcal{L}_{\text{kin}} = -\frac{1}{2\pi^2} \left( \frac{\alpha}{2} \right)^6 \frac{1}{m^2} \partial_m F^\dagger_{ij} \partial^m F^{ji}. \] (3.8)

We have shown that the auxiliary field really propagates and can be associated with a physical field. It is worth stressing that this kind of non-perturbative supersymmetry breaking treatment in evaluating the momentum expansion is actually essential for obtaining the previous result. By canonically normalizing the Higgs field
\[ H \equiv \frac{1}{\sqrt{2\pi m}} \left( \frac{\alpha}{2} \right)^3 F, \] (3.9)
we deduce the following Lagrangian
\[ \mathcal{L}_2 = -\partial_m H^\dagger_{ij} \partial^m H^{ji} + \mu^2 H^\dagger_{ij} H^{ji}, \] (3.10)
where
\[ \mu^2 = m^2 \left[ 2\pi^2 \left( \frac{\alpha}{2} \right)^6 + 3 \ln \left( \frac{\Lambda^2}{m^2} \right) \right]. \] (3.11)

Note that the one-loop contribution to the mass term for the Higgs field has the same sign than the tree level term. We observe that the mass instability suggests that the Higgs can have a non-vanishing vacuum expectation value which would break the chiral symmetry group SU(6) to Sp(6). In order to complete the picture we expect the existence of stabilizing terms in the Higgs potential. In the next section we estimate the fourth order term for the Higgs potential by employing the method outlined in this section.

**B. The Four Point Function**

The one-loop diagram for the four point function (quartic term for the effective field in the Kähler potential) is shown in Fig. The explicit evaluation of the Feynman diagram provides
\[ \Gamma_4 = 162 \left( \frac{\alpha^3}{23!} \right)^4 \epsilon_{klabcd} \epsilon^{ijabc'd'} \epsilon_{opa'b'c'd'} \epsilon^{mnad'b'cd} \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{d^4 r}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \]
\[ \int d^4 \theta_1 d^4 \theta_2 d^4 \theta_3 d^4 \theta_4 \frac{M^{kl}(p, \theta_1)M^\dagger_{ij}(- (p + q + r), \theta_2)M^{op}(r, \theta_3)M^\dagger_{mn}(q, \theta_4)}{k^2(k + p)^2(k + p + q)^2(k + p + q + r)^2} \]
\[ \left( -\frac{\bar{D}^2(k, \theta_1)D^2(k, \theta_1)}{16} \right) \delta^4(\theta_1 - \theta_2) \left( \frac{D^2(-(k + p + q), \theta_4)\bar{D}^2(-(k + p + q), \theta_4)}{16} \right) \delta^4(\theta_3 - \theta_4) \delta^4(\theta_2 - \theta_3) \delta^4(\theta_4 - \theta_1), \] (3.12)
where for simplicity we omit the arguments of $D$ and having performed the Fourier transformation, we have

$$\Gamma_4 = 162 \left( \frac{\alpha^3}{2^{3/3!}} \right)^4 \epsilon_{klabcd} \epsilon^{ijabc} \epsilon^{\alpha^p\gamma^q e^d} \epsilon^{mna'b'c'd'} \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{d^4 r}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \delta^4(\bar{\theta} - \theta)$$

$$\int d^4 \theta_1 d^4 \theta_2 \frac{M_{ij}^\dagger(-p + q + r, \theta_3) M^{op}(r, \theta_3)}{k^2(k + p)^2(k + p + q)^2(k + p + q + r)^2} \delta^4(\theta_3 - \theta_1)$$

$$\left\{ \left( \frac{D^2 D^2}{16} M_{mn}(q, \theta_1) M^{kl}(p, \theta_1) \right) \left( \frac{D^2 D^2}{16} \right) \delta^4(\theta_1 - \theta_3) \right\}.$$

where for simplicity we omit the arguments of $D$ and $\bar{D}$. By using the formulae

$$\delta^4(\theta_3 - \theta_1) \frac{\bar{D}_\alpha(k, \theta_1) D_\alpha(k, \theta_1)}{4} \frac{\bar{D}^2(k, \theta_1) D^2(k, \theta_1)}{16} \delta^4(\theta_1 - \theta_3) = \frac{1}{2} \sigma^m_{\alpha \alpha} k_m \delta^4(\theta_3 - \theta_1),$$

$$D^2(k, \theta_1) \frac{\bar{D}^2(k, \theta_1) D^2(k, \theta_1)}{16} = -k^2 D^2(k, \theta_1),$$

together with Eq. (3.12), we deduce

$$\Gamma_4 = 162 \left( \frac{\alpha^3}{2^{3/3!}} \right)^4 \epsilon_{klabcd} \epsilon^{ijabc} \epsilon^{\alpha^p\gamma^q e^d} \epsilon^{mna'b'c'd'} \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{d^4 r}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \delta^4(\bar{\theta} - \theta)$$

$$\int d^4 \theta \frac{M_{ij}^\dagger(-(p + q + r), \theta) M^{op}(r, \theta)}{4} M_{mn}^\dagger(q, \theta_1) M^{kl}(p, \theta_1).$$

Only the first term in the curly brackets gives the quartic term for the auxiliary field with no derivatives. By keeping only the lowest order in momentum expansion we get

$$\Gamma_4' = 162 \left( \frac{\alpha^3}{2^{3/3!}} \right)^4 \epsilon_{klabcd} \epsilon^{ijabc} \epsilon^{\alpha^p\gamma^q e^d} \epsilon^{mna'b'c'd'} \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{d^4 r}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \delta^4(\bar{\theta} - \theta)$$

$$\int d^4 \theta \frac{M_{ij}^\dagger(-(p + q + r), \theta) D^2(p + q, \theta) D^2(p + q, \theta)}{16} M_{mn}^\dagger(q, \theta_1) M^{kl}(p, \theta_1).$$

The momentum integral can be estimated as in the previous section. After contracting over the flavor indices and having performed the Fourier transformation, we have

$$\Gamma_4' = \frac{1}{64\pi^2} \left( \frac{\alpha}{2} \right)^{12} \frac{1}{m^4} \int d^4 x d^4 \theta$$

$$\left[ M_{ij} M_{ij}^\dagger D^2 M_{kl}^\dagger D^2 M_{kl} + M_{ij} M_{kl}^\dagger D^2 M_{ik}^\dagger D^2 M_{kj}^\dagger + 8 M_{ij} M_{ik}^\dagger D^2 M_{kl}^\dagger D^2 M_{li} \right].$$
This leads to the following contribution for the Higgs Lagrangian.

\[ \mathcal{L}_4 = \frac{\pi^2}{8} \left[ (H_{ij}^* H^{ji})^2 + 4H_{ij}^* H^{jk} H_{kl}^* H^{li} \right]. \]  

(3.19)

Unfortunately, the contribution is positive semi-definite and cannot stabilize the Higgs potential. We attribute this negative result to the incompleteness of our naive Kähler potential. We also feel that the problem of deducing the full Higgs potential is as difficult as to completely solve the underlying gauge theory. While the effective superpotential is fixed, the Kähler potential is not for \( N = 1 \) theories, hence, as we shall see, higher order Kähler terms are expected and can generate the correct Higgs potential.

Let us consider, for example, the effects of the following Kähler term

\[ K_{HE} = \gamma \frac{\Lambda^2}{ \int d^4xd^4\theta Tr (M^\dagger MM^\dagger M)}, \]  

(3.20)

written for the canonically normalized effective field \( M \), where \( \gamma \) is a numerical constant. The one-loop diagram for the four point function containing one vertex due to the above term leads to the following contribution for the Higgs potential

\[ \mathcal{L}_{HE} \sim \gamma \frac{m^2}{\Lambda^2} \left( \text{tr} \left( H^\dagger H \right) \right)^2. \]  

(3.21)

We observe that this contribution might have the correct sign (depending on the sign of \( \gamma \)), while has different \( m \)-dependence with respect to the contribution in Eq. (3.19). On general grounds, if we were to consider all the high energy contributions to the Kähler potential, we expect the following form for the Higgs potential

\[ V_H = -m^2 f \left( \frac{m^2}{\Lambda^2} \right) \text{tr} (H^\dagger H) + g \left( \frac{m^2}{\Lambda^2} \right) \left( \text{tr} (H^\dagger H) \right)^2 + h \left( \frac{m^2}{\Lambda^2} \right) \text{tr} (H^\dagger HH^\dagger H), \]  

(3.22)

where \( f, g \) and \( h \) are general functions of \( m^2/\Lambda^2 \). We can encode the obtained results in the previous functions as

\[ f \left( \frac{m^2}{\Lambda^2} \right) = 2\pi^2 \left( \frac{\alpha}{2} \right)^6 + 3 \ln \left( \frac{\Lambda^2}{m^2} \right), \]  

(3.23)

\[ g \left( \frac{m^2}{\Lambda^2} \right) = -\frac{\pi^2}{8} + \mathcal{O} \left( \frac{m^2}{\Lambda^2} \right), \quad h \left( \frac{m^2}{\Lambda^2} \right) = -\frac{\pi^2}{2} + \mathcal{O} \left( \frac{m^2}{\Lambda^2} \right). \]  

(3.24)

The vacuum expectation value for the Higgs field can be written as

\[ v^2 = \frac{m^2 f(m^2/\Lambda^2)}{6g(m^2/\Lambda^2) + h(m^2/\Lambda^2)}, \]  

(3.25)

where we assumed the breaking pattern, \( SU(6) \rightarrow Sp(6) \), namely \( \langle H \rangle = (v/\sqrt{2})J \) with \( J \) being the \( Sp(6) \) invariant matrix normalized as \( J^\dagger J = 1 \). We conjecture that the complete non-perturbative calculation leads to a positive \( v^2 \) which persists in the limit \( m \rightarrow \infty \). The value of \( v \) in this limit should be of the order of the new dynamical scale associated with the gauge theory [13] without squarks (i.e. quarks and a fermion in adjoint representation of the gauge group).
C. The Four Point Function with One \( X \) Insertion

In this section we show that a non zero vacuum expectation value for the Higgs field triggers the appearance of a mass term for the effective fermion fields. In the supersymmetric limit, since chiral symmetry is unbroken, the effective fermion fields are kept massless by the ’t Hooft anomaly matching conditions. However when chiral symmetry is spontaneously broken the anomalies are saturated via Nambu-Goldstone bosons and the fermions need not to be massless.

Let us compute the four point function with one \( X \) insertion whose Feynman diagram is provided in Fig.3. By evaluating the diagram we find

\[
\Gamma^X_4 = 324 \left( \frac{\alpha^3}{23^3!} \right)^4 \epsilon_{k\ellabcd} \epsilon_{\alpha\beta\gamma\delta} \epsilon_{\mu
u\alpha'd'} \epsilon_{\epsilon\kappa\lambda\mu
u} \epsilon_{\sigma\tau} \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{d^4r}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \int d^4\theta_1 d^4\theta_2 d^4\theta_3 d^4\theta_4 d^4\theta_5 \frac{M^{kl}(p, \theta_1) \beta \chi(\theta_2) M^\dag_{ij}(-(p + q + r), \theta_3) M^{op}(r, \theta_4) M^\dag_{mn}(q, \theta_5)}{k^2 k^2(k + p)^2(k + p + q)^2} \delta^4(\theta_1 - \theta_2) \delta^4(\theta_2 - \theta_3) \delta^4(\theta_3 - \theta_4) \delta^4(\theta_4 - \theta_5),
\]

where the factor 324 is a symmetric factor. After integrating over \( \theta_2, \theta_4 \) and \( \theta_5 \) and using the relation

\[
\frac{\bar{D}^2(k, \theta_1) D^2(k, \theta_1)}{16} X(\theta_1) \frac{\bar{D}^2(k, \theta_1) D^2(k, \theta_1)}{16} \delta^4(\theta_1 - \theta_3)
\]

\[
= -k^2 \left\{ \left( \frac{\bar{D}^2}{4} X \right) \frac{D^2}{4} + \frac{1}{2} \left( \bar{D}_a X \right) \bar{D}_a \frac{D^2}{4} + X \bar{D}_a D^2 \frac{16}{4} \right\} \delta^4(\theta_1 - \theta_3),
\]

we obtain

\[
\Gamma^X_4 = -324 \left( \frac{\alpha^3}{23^3!} \right)^4 \epsilon_{k\ellabcd} \epsilon_{\alpha\beta\gamma\delta} \epsilon_{\mu
u\alpha'd'} \epsilon_{\epsilon\kappa\lambda\mu
u} \epsilon_{\sigma\tau} \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{d^4r}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \int d^4\theta_1 d^4\theta_3 \frac{M^\dag_{ij}(-(p + q + r), \theta_3) M^{op}(r, \theta_3) M^\dag_{mn}(q, \theta_1) M^{kl}(p, \theta_1)}{k^2(k + p)^2(k + p + q)^2} \delta^4(\theta_1 - \theta_3) \delta^4(\theta_3 - \theta_1)
\]

\[
= \left\{ \left( \frac{\bar{D}^2(0, \theta_1)}{4} X \right) \frac{D^2(k, \theta_1)}{4} + \frac{1}{2} \left( \bar{D}_a(0, \theta_1) X(\theta_1) \right) \bar{D}_a(k, \theta_1) \frac{D^2(k, \theta_1)}{4} + X \bar{D}_a D^2 \frac{16}{4} \right\} \delta^4(\theta_1 - \theta_3).
\]

Integrating by parts the \( D \) and \( \bar{D} \) operators with momentum \(-k + p + q\), while restricting our attention to the term which provides the fermion mass term, we have
\[
\Gamma_4^X \rightarrow -324 \left( \frac{\alpha^2}{243!} \right)^4 \beta \epsilon_{klabcd} \epsilon_{ijabc} \epsilon_{opald} \epsilon_{mna} \epsilon_{n}^{cd} \int \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} \frac{d^4r}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} (\alpha_3^2 \frac{2}{3} \chi^3) \int \left( M^\dagger_{ij}(-(p+q+r),\theta_3)M^{op}(r,\theta_3) \right) \left( D^2(p+q,\theta_1)M_{mn}(q,\theta_1)M_{kl}(p,\theta_1) \right) \left( 2 \cdot \frac{D^2(0,\theta_1)X(\theta_1)}{4} \right) \delta^4(\theta_3 - \theta_1), \tag{3.29} \right.
\]

Now we can integrate over \( \theta_3 \) and a straightforward calculation provides the following contribution to the effective Lagrangian
\[
\mathcal{L}_{\psi \text{ mass}} = -2 \beta \left( \frac{\alpha}{2} \right)^{12} \frac{1}{m^3} \left\{ F^{ij} F^{kl} \psi_{ij}^{\dagger} \psi_{kl}^{\dagger} + 4 F^{ij} F^{kl} \psi_{ij}^{\dagger} \psi_{jk}^{\dagger} \right\}, \tag{3.30} \right.
\]

where the contraction of the fermion indices is assumed. In terms of the Higgs field defined in Eq. (3.31),
\[
\mathcal{L}_{\psi \text{ mass}} = -4 \beta \left( \frac{\alpha}{2} \right)^{6} \frac{1}{m} \left\{ H^{ij} H^{kl} \psi_{ij}^{\dagger} \psi_{kl}^{\dagger} + 4 H^{ij} H^{kl} \psi_{ij}^{\dagger} \psi_{jk}^{\dagger} \right\}. \tag{3.31} \right.
\]

We learn that the effective fermion field \( \psi \) becomes massive once chiral symmetry is spontaneously broken via a non zero vacuum expectation value of the Higgs field \( H \).

As for the Higgs potential the coefficient in Eq. (3.31) becomes a general function of \( m^2/\Lambda^2 \) once a more general Kähler is employed (see Eq. (3.20)). We then expect the effective fermion field to decouple as \( m \rightarrow \infty \).

In this analysis we used as spurion the field \( X = m(\theta^2 + \overline{\theta}^2) \). In the introduction we have mentioned the possibility of realizing the spurion field as \( X = -\theta^2 \overline{\theta}^2 m^2 \). However in order to generate a term like the one presented in Eq. (3.31) with the new spurion field we need an odd number of operators \( D^2 \) and \( \overline{D}^2 \) acting on the effective fields \( M \) and \( M^\dagger \) and \( X \). Within our approximation for the Kähler potential we cannot generate a fermionic type mass operator (at one-loop) if the latter choice of the the spurion field is made.

\[ \text{IV. CONCLUSIONS} \]

We have explored a new method for deriving the Higgs Lagrangian from vector-like gauge theories. The key idea is to identify the auxiliary field of the effective “meson” superfield associated with the supersymmetrized version of the given gauge theory with the ordinary Higgs field. A similar identification was made in Ref. [13,14] in order to define a suitable decoupling procedure from supersymmetric QCD to ordinary QCD like theories. In this paper we concentrated on providing a more complete discussion of the novel method proposed in Ref. [15] and we also described in some detail the new techniques for including non perturbative supersymmetric breaking effects.

We have then shown, by using as our laboratory the \( SU(2) \) gauge theory with six matter doublets, that once supersymmetry is explicitly broken the auxiliary field, at the one loop level in the effective theory, acquires an ordinary kinetic term while the squared mass term already negative at the tree level remains negative at one loop. This supports the interpretation of the auxiliary field as the Higgs. We also remark that the present model illustrates
how some of the non-holomorphic terms for the non-supersymmetric low energy effective theory generates when supersymmetry decouples.

We have established that a better knowledge of the effective Kähler potential (associated with the full underlying dynamics) is essential to provide a bounded from the below Higgs potential.

We finally successfully generated a mass term for the effective fermion field which was kept massless in the supersymmetric limit by the ’t Hooft anomaly matching conditions. An encouraging feature is that the non zero vacuum expectation value for the Higgs field, associated with spontaneous chiral symmetry breaking, triggers the appearance of the fermion mass. In the resultant effective low energy theory the chiral anomalies are now saturated by the appearance of pseudoscalar Nambu-Goldstone bosons.

We remind the reader that determining the Higgs Lagrangian is as difficult as solving the complete dynamics in the underlying theory. In principle it is possible to extend the present method for deriving the Higgs Lagrangian for arbitrary vector-like gauge theory. However, there are some technical difficulties associated with the fact that the effective superpotential becomes highly involved for general \( N_c \) and \( N_f \). For example if we consider a \( SU(N_c) \) gauge theory with \( N_f = N_c + 1 \) flavors we have to compute a \((N_f - 2)\)-loop superdiagram to generate the kinetic term for the Higgs field. Even more hard to handle are the \( SU(N_c) \) gauge theories with \( N_f \neq N_c + 1 \). Indeed the associated supersymmetric low energy effective theories do not have a simple polynomial superpotential and is not straightforward to define a reliable loop calculation.

It might be very interesting to generalize the present approach to the \( N = 2 \) supersymmetric extension of gauge theories, since the supersymmetric low energy effective action (including the Kähler potential) in this case is completely known. However since the exact low energy effective theory is expressed, for \( N = 2 \) super theories, in terms of magnetic type variables the task would be to correctly identify the fermion condensate operator.

ACKNOWLEDGMENTS

This work has been supported by the US DOE under contract DE-FG-02-92ER-40704. The work by N.K. has also been supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science, and Culture of Japan on Priority Areas (Physics of CP violation) and #09045036 under International Scientific Research Program, Inter-University Cooperative Research.
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FIG. 1. The supergraph for the two point function.

FIG. 2. The supergraph for the four point function.
FIG. 3. The supergraph for the four point function with one $X$ insertion.