I. INTRODUCTION

The ‘standard’ form of the KPZ equation introduced in 1986 by Kardar, Parisi and Zhang, for modeling nonlinear growth processes, reads

\[
\frac{\partial h(x, t)}{\partial t} = \nu \nabla^2 h(x, t) + \frac{\lambda}{2} (\nabla h(x, t))^2 + \eta(x, t),
\]

where \( h(x, t) \) is a scalar height field (with \( x, t \) as space and time coordinates, respectively), \( \nu \) is a surface tension parameter and \( \lambda \) is a non-linear coupling constant. Here \( \eta(x, t) \) denotes an uncorrelated Gaussian noise with zero mean (white noise in space and time) \([1]\). Therefore the first and second moments of the Gaussian noise are given by

\[
\langle \eta(x, t) \rangle = 0,
\]

\[
\langle \eta(x, t) \eta(x', t') \rangle = 2D \delta^d(x - x') \delta(t - t'),
\]

where \( D \) is a constant amplitude. Besides the noise correlation of \([2]\), various spatially and temporally correlated driving forces have been studied over the years \([2, 3]\).

With respect to spatial correlations a widely studied type of driving forces is power-law correlated noise with Fourier-space correlations \( \sim q^{-2\rho} \) and \( \rho > 0 \) as a free parameter \([4, 5, 6]\). The intriguing observation here is the emergence of a new ‘noise’ fixed point for \( \rho > 1/4 \), additionally to the standard Gaussian and KPZ fixed points.

Recently the KPZ equation with spatially colored and temporally white noise decaying as \( \sim e^{-q^2/(2\xi^2)} \) was studied by a non-perturbative DRG analysis in \([6]\). It was found that for small values of \( \xi \) the KPZ equation behaves in the large-scale limit as if it were stirred by white noise, i.e. with a driving force with vanishing correlation length. Since the non-perturbative RG equations are difficult to solve analytically, the authors of \([6]\) relied on numerical techniques.

In the present paper we study the case of spatially correlated noise, where the correlations are characterized by a ‘sinc’-like profile in Fourier-space. As in \([6]\), these correlations are characterized by a finite correlation length \( \xi \). Unlike \([6]\), we solve the problem analytically by using field-theoretic DRG techniques.

For treating ‘sinc’-type noise, we first generalize the field-theoretic DRG formalism for the KPZ equation in such a way that we can handle homogeneous and isotropic noise distributions, whose correlations in momentum space are given by

\[
\langle \eta(q, \omega) \eta(q', \omega') \rangle = 2D(|q|^2) \delta^d(q + q') \delta(\omega + \omega').
\]

Note that \( D \) does not depend on the frequency, i.e., the noise is spatially colored but temporally white.

For this class of noise correlations, which includes the power-law \( \sim q^{-2\rho} \), Gaussian \( \sim e^{-q^2/2\xi^2} \) and ‘sinc’-type correlations, the field theoretic DRG formalism will be built in the next section. With the theoretical framework laid out in section II, the explicit ‘sinc’-noise excitation will be analyzed in section III. In section IV the results obtained in section III will be discussed.

II. GENERALIZED FIELD THEORETIC RENORMALIZATION GROUP PROCEDURE

A useful tool for building a field theory for stochastic differential equations of type \([1]\) is the effective action \( A[h, h] \), known as the Janssen-De Dominicis response functional \([2, 5]\). Here the action depends on the original height field \( h(x, t) \) and the Martin-Siggia-Rose (MSR) auxiliary field \( h(x, t) \).

To derive the effective action, it is useful to transform \([3]\) into real-space:

\[
\langle \eta(x, t) \eta(x', t') \rangle = 2D(x - x') \delta(t - t'),
\]

where \( D(x) = FT^{-1}\{D(|q|^2)\} \). Here we use the abbreviations \( y = (x, t) \) and \( \int_y \cdots = \int d^d x \int dt \cdots \), the corresponding Gaussian noise proba-
bility distribution can be written as \[9\]
\[\mathcal{W}[\eta] \propto \exp \left[-\frac{1}{2} \int y \int y' \eta(y)\mathcal{M}(y, y')\eta(y')\right], \tag{5}\]
where \(\mathcal{M}(x, t; x', t')\) is the inverse of the ‘covariance operator’
\[\mathcal{M}^{-1}(x, t; x', t') = \langle \eta(x, t)\eta(x', t') \rangle\]
given in \[4\], i.e.
\[\int d^d x' \int dt' \mathcal{M}(x, t; x', t')\mathcal{M}^{-1}(y, \tau; x', t') = \delta^d(x - y)\delta(t - \tau). \tag{6}\]

Eq. (7) can be rewritten in the form \[4, 5, 7, 8, 13\]
\[\langle O[h] \rangle = \int \mathcal{D}[h] \int \mathcal{D}[\bar{h}] O[h] \exp \left[\int d^d x \int dt \bar{h}(x, t) \left(\partial_t h(x, t) - \nu \nabla^2 h(x, t) - \frac{\lambda}{2} (\nabla h(x, t))^2\right)\right] \times \int \mathcal{D}[\eta] \exp \left[-\frac{1}{2} \int d^d x \int dt \left(\int d^d x' \int dt' \eta(x, t)\mathcal{M}(x, t; x', t')\eta(x', t') - \bar{h}(x, t)\eta(x, t)\right)\right]. \tag{7}\]

\[\langle O[h] \rangle \propto \int \mathcal{D}[h] O[h] \mathcal{P}[h]\]
\[= \int \mathcal{D}[h] O[h] \int \mathcal{D}[\bar{h}] e^{-\mathcal{A}[\bar{h}, h]}, \tag{8}\]
with the Janssen-De Dominicis functional \[9, 12, 14, 15\]
\[\mathcal{A}[\bar{h}(x, t), h(x, t)] = \int_y \bar{h}(y) \left(\frac{\partial h(y)}{\partial t} - \nu \nabla^2 h(y) - \frac{\lambda}{2} (\nabla h(y))^2\right) - \int d^d x' \bar{h}(x, t)D(x - x')\bar{h}(x', t). \tag{9}\]

With this functional, one can carry out the usual field-theoretic perturbation expansion in \(\lambda\), see e.g. \[13, 16\].

The KPZ equation is known to be invariant under tilts (Galilei transformation) of the form \[4, 16\]
\[h(x, t) \rightarrow h'(x, t) = h(x + \alpha \lambda t, t) + \alpha \cdot x, \quad \bar{h}(x, t) \rightarrow \bar{h}'(x, t) = \bar{h}(x + \alpha \lambda t, t), \tag{10}\]
where \(\alpha\) is the tilting angle. This symmetry is giving rise to two Ward-Takahashi identities. For this reason the

\[\Gamma_{hh}(k) = i\omega + \nu q^2 + \frac{\lambda^2}{2\nu^2} \int \mathcal{D}(b - p/2)^2/(b - p)^2 \frac{q^2/4 - p^2}{b^2} + q^2/4 + p^2 + \mathcal{O}(\lambda^3), \tag{12}\]
\[\Gamma_{\bar{h}h}(k) = -2D(|q|^2) - \frac{\lambda^2}{2\nu^2} \int \mathcal{D}(b + q/2)^2)D(|q + 2b/2)|^2)R \left[\frac{1}{b^2} + q^2/4 + p^2\right] + \mathcal{O}(\lambda^3), \tag{13}\]
where we used the following abbreviation: $\int_p \cdots = 1/(2\pi)^d \int d^d p \cdots$.

Evaluating (12) and (13) it is essential to avoid mixing ultraviolet and infrared divergences of the integrands. One way to keep those divergences separated is to introduce a so-called normalization point (NP). An indiscriminate, however very useful choice is given by \[16\] \[
\frac{\omega}{2\nu} = \mu^2, \quad q = 0,
\]
where $\mu$ is an arbitrary momentum scale. One advantage of the choice in (14) is that evaluating the integrals in (12) and (13) at $q = 0$ yields the possibility of expanding the general noise amplitude $D(|p\pm q/2|^2)$ about $p$ for $|q| \ll 1$. Hence to order $O(|q|^2)$ the momentum-dependent noise amplitude reads

\[D \left| p\pm \frac{q}{2} \right|^2 = D \left| p \right|^2 \pm \left( p \cdot q \right) D^\prime \left| p \right|^2 + O(|q|^2). \]

and inserting (15) into (12) implies at the NP

\[
\left. \frac{\partial \Gamma_{gh}}{\partial q^2} \right|_{q=0} = \nu - \frac{\lambda^2}{4 \nu^2} \int_p \frac{D(|p|^2)}{p^2 D^\prime(|p|^2)} - \frac{\lambda^2}{2 \nu^2} \int_p \frac{p^2 D^\prime(|p|^2)}{i\mu^2 + p^2}.
\]

The evaluation of (13) at the NP (14) leads with (15) to

\[
\Gamma_{hh} = -2D(|q|^2) - \frac{\lambda^2}{2 \nu^3} \int_p \frac{D(|p|^2)}{p^2 D^\prime(|p|^2)} \frac{p^2}{\mu^4 + p^4}. \]

From (16) and (17) we obtain the renormalization factors

\[
Z_\nu = 1 - \frac{\lambda^2}{4 \nu^3} \int_p \frac{D(|p|^2)}{i\mu^2 + p^2} - \frac{\lambda^2}{2 \nu^2} \int_p \frac{p^2 D^\prime(|p|^2)}{i\mu^2 + p^2},
\]

\[
Z_D = 1 + \frac{1}{D(|q|^2)} \frac{\lambda^2}{4 \nu^3} \int_p \frac{D(|p|^2)}{i\mu^2 + p^2} \frac{p^2}{\mu^4 + p^4}.
\]

These results allow us to compute the Wilson flow functions [4,14,16]

\[
\gamma_D = \mu \frac{\partial}{\partial \mu} \ln Z_D,
\]

\[
\gamma_\nu = \mu \frac{\partial}{\partial \mu} \ln Z_\nu.
\]
where the derivative is taken while keeping $D$ and $\nu$ fixed. Likewise, the $\beta$-function is given by
\[
\beta_g = \mu \frac{\partial}{\partial \mu} g_R, \tag{22}
\]
where
\[
g_R = g Z_0 \mu^{d-2} = g Z_0 \mu^{d-2} \sim \frac{\lambda^2 D}{4 \nu^3} \mu^{d-2}
\]
is a dimensionless effective coupling constant and $D = D(0)$ given in (28).

The dimension of an effective coupling constant in the above form is (see e.g. [16])
\[
[g] = \left[ \frac{\lambda^2 D}{4 \nu^3} \right] \mu^{2-d}. \tag{23}
\]
This explains why $g$ has to be multiplied by $\mu^{d-2}$ to render $g_R$ dimensionless.

With the flow functions (20)–(22) a partial differential renormalization group equation can be formulated. This RG equation may be solved by using the method of characteristics, where a flow parameter $l$ and an $l$-dependent continuous momentum scale $\bar{\mu}(l) = \mu l$ is introduced. Those solutions are then used to formulate a KPZ-specific scaling relation for, say, the two-point correlation function $C(q, \omega)$. This relation reads [16]
\[
C(\mu, D_R, \nu_R, g_R, q, \omega) = q^{-4-2\gamma_\nu+\gamma_D^*} \hat{C} \left( \frac{\omega}{q^2+\gamma_\nu^*} \right), \tag{24}
\]
where the superscript ‘*’ indicates that the Wilson flow functions are evaluated at the stable IR fixed point. A detailed explanation of how the scaling form in (24) is obtained can be found e.g. in [14] [16]. A comparison with the general scaling form for the KPZ two-point correlation function in Fourier-space (see e.g. [11] [16] [21] [22]), i.e.
\[
C(q, \omega) = q^{-d-2\chi-z} \hat{C} \left( \frac{\omega}{q^2} \right), \tag{25}
\]
leads to the following expressions for the dynamical exponent $z$ and the roughness exponent $\chi$ [14] [16]:
\[
z = 2 + \gamma_\nu^*, \tag{26}
\]
\[
\chi = 1 - \frac{d}{2} + \frac{\gamma_\nu^* + \gamma_D^*}{2}. \tag{27}
\]
These general considerations will be used in the next part to obtain the critical exponents $z$ and $\chi$ for the explicit ‘sinc’-noise correlation.

### III. THE KPZ EQUATION WITH ‘SINC’-NOISE CORRELATION

We now apply these results to the case of the ‘sinc’-type noise with the correlations
\[
\langle \eta(q, \omega) \eta(q', \omega') \rangle = 2D |q|^2 \delta^d(q + q') \delta(\omega + \omega') = 2D \frac{\sin(\xi |q|)}{|\xi|} \delta^d(q + q') \delta(\omega + \omega'), \tag{28}
\]
where $D$ is a constant noise amplitude, $q \in \mathbb{R}^d$ and $\xi$ defines the scale of the ‘sinc’-profile.

For simplicity let us consider the case $d = 1$. Here the noise distribution transformed back to real-space is a rectangle with size $2\xi \times D/\xi$ centered at $x = 0$, which tends to $\delta(x)$ (white noise) in the limit $\xi \to 0$ [1].

The first step now is to calculate explicit expressions for the renormalization factors from (18) and (19) for the driving force (28). Thus with
\[
D(|p|^2) = \frac{D \sin(\xi |p|)}{|\xi| |p|}
\]
and
\[
D'(|p|^2) = \frac{dD}{d(|p|^2)}
\]
the renormalization factors read in $d = 1$ to one-loop order
\[
Z_\nu = 1 + \left[ \frac{\lambda^2}{4 \nu^3} \frac{1}{\pi} \int_0^\infty dp \frac{\sin(\xi p)}{\xi p (i\mu^2 + p^2)} - \int_0^\infty dp \frac{\cos(\xi p)}{i\mu^2 + p^2} \right]. \tag{29}
\]
\[
Z_D = 1 + \left[ \frac{\lambda^2}{4 \nu^3} \frac{1}{\pi} \int_0^\infty dp \frac{\sin^2(\xi p)}{\xi^2 (\mu^2 + p^2)} \right]. \tag{30}
\]
The integrals occurring in (29) and (30) can be computed by employing the Residue theorem, which leads to
\[
Z_\nu = 1 + \left[ \frac{\lambda^2}{4 \nu^3} \frac{e^{-\xi \mu}}{\xi \mu^2} \times \left[ \sin \left( \frac{1}{\sqrt{2}} \xi \mu \right) \left( 1 + \frac{\xi \mu}{2 \sqrt{2}} \right) - \frac{\xi \mu}{2 \sqrt{2}} \cos \left( \frac{1}{\sqrt{2}} \xi \mu \right) \right] \right], \tag{31}
\]
\[
Z_D = 1 + \left[ \frac{\lambda^2}{4 \nu^3} \frac{e^{-\sqrt{2} \xi \mu}}{4 \sqrt{2} \xi^2 \mu^3} \times \left[ e^{\sqrt{2} \xi \mu} - \left( \sin(\sqrt{2} \xi \mu) + \cos(\sqrt{2} \xi \mu) \right) \right] \right]. \tag{32}
\]
In Appendix A the derivation of these formulas is explained in greater detail.
A. Small Correlation Length Expansion

Let us now focus on small correlation lengths $\xi \ll 1$ and expand (31) and (32) in $\xi$ up to order $O(\xi^2)$. Introducing the effective coupling constants (2)

$$g = \frac{D \lambda^2}{4 \nu^3}, \quad g_R = \frac{g Z_g}{2\sqrt{2} \mu}; \quad (33)$$

$$g_\xi = \frac{D \xi^2 \lambda^2}{4 \nu^3}, \quad g_\xi R = \frac{g_\xi Z_g \mu}{2\sqrt{2}}. \quad (34)$$

with $Z_g = Z_D Z_v^{-3}$ the one-loop integrals simplify:

$$Z_\nu = 1 + \frac{g}{2\sqrt{2} \mu} - \frac{g_\nu \mu}{12\sqrt{2}}, \quad (35)$$

$$Z_D = 1 + \frac{g}{2\sqrt{2} \mu} - \frac{D \lambda^2}{12 \nu^3} \xi + \frac{g_\nu \mu}{6\sqrt{2}}. \quad (36)$$

With the renormalized dimensionless effective coupling constants from (33), (34) and using (22) one obtains the flow equations

$$\beta_g = g_R \left[ 2g_R + \frac{5}{6} g_\xi R - 1 \right], \quad (37)$$

$$\beta_{g_\xi} = g_\xi R \left[ 2g_R + \frac{5}{6} g_\xi R + 1 \right]. \quad (38)$$

Solving (37) and (38) for their fixed points $(g^*, g_\xi^*)$ yields three different possible solutions, namely

$$(g_\xi^*, g_\xi R) = \begin{cases} (0, 0) & \text{Gaussian,} \\ (0, -\frac{5}{3}) \\ (\frac{2}{5}, 0) & \text{KPZ.} \end{cases} \quad (39)$$

The second one is nonphysical, since $g_\xi R < 0$, and thus there are two valid fixed points for the KPZ equation stirred by 'sinc'-type noise with $\xi \ll 1$. To determine the stability of the two fixed points we carry out a linear stability analysis via the Jacobian of the two flow functions (37) and (38), i.e.

$$\mathcal{J} = \begin{pmatrix} \frac{\partial g^*}{\partial g_R} \beta_g & \frac{\partial g^*}{\partial g_\xi R} \\ \frac{\partial g^*}{\partial g_R} \beta_{g_\xi} & \frac{\partial g^*}{\partial g_\xi R} \beta_{g_\xi} \end{pmatrix}$$

$$= \begin{pmatrix} 4g_R + 5/6 g_\xi R - 1 & 5/6 g_R \\ 2g_R + 5/3 g_\xi R + 1 & 2g_R + 5/3 g_\xi R + 1 \end{pmatrix}. \quad (40)$$

By evaluating (40) at the respective fixed points it turns out that for the Gaussian fixed point $\mathcal{J}$ is indefinite and for the KPZ fixed point $\mathcal{J}$ is positive definite. Since the condition for asymptotic stability in this framework is positive definiteness of $\mathcal{J}$, only the KPZ fixed point is stable in the infrared limit and the Gaussian fixed point is unstable.

To provide a simple graphical representation of the occurring renormalization group flow Fig. 2 was plotted in Wilson’s picture [23]. Parametrizing the scale transformation from the field-theoretic to Wilson’s representation by (see e.g. [16])

$$l_W = -\ln l \quad (41)$$

one obtains the following flow equations

$$\frac{d g^*(l_W)}{d l_W} = -\beta_g \left[ 2g^*(l_W) + \frac{5}{6} g_\xi(l_W) - 1 \right], \quad (42)$$

$$\frac{d g_\xi(l_W)}{d l_W} = -\beta_{g_\xi} \left[ 2g^*(l_W) + \frac{5}{6} g_\xi(l_W) + 1 \right]. \quad (43)$$

The corresponding RG flow is displayed in Fig. 2. The critical exponents $z$ and $\chi$ are obtained via (26) and (27). Here the fixed point values of the Wilson flow functions,

$$\gamma_\nu = -g^* - \frac{g_\xi R}{6} + \mathcal{O}(g_R^2, g_\xi^2 R, g_R g_\xi R), \quad (44)$$

$$\gamma_D = -g^* + \frac{g_\xi R}{3} + \mathcal{O}(g_R^2, g_\xi^2 R, g_R g_\xi R), \quad (45)$$

are given by

$$\gamma_\nu^* = \gamma_\nu(g_R = 1/2, g_\xi R = 0) = -\frac{1}{2}, \quad (46)$$

$$\gamma_D^* = \gamma_D(g_R = 1/2, g_\xi R = 0) = -\frac{1}{2}. \quad (47)$$

Hence the dynamical exponent $z$ and the roughness exponent $\chi$ read

$$z = \frac{3}{2}, \quad \chi = \frac{1}{2}. \quad (48)$$

They are the same as those in the white-noise case and confirm the KPZ exponent identity $z + \chi = 2$ (see e.g. [11, 2, 4, 110, 24]).

B. Arbitrary Correlation Length Calculation

In the following we show that the same result can be derived for arbitrary correlation lengths $\xi$ in $d = 1$ di-
mensions, although the calculations are technically more involved.

Inserting (31), (32) into (21), (20) and expanding to lowest order in the effective coupling constant \( g = D\lambda^2/(4\nu^3) \), the Wilson flow functions \( \gamma \) can be written down as

\[
\gamma_\mu = \mu \frac{\partial \ln Z_\nu}{\partial \mu} = -g_R e^{-\frac{1}{\sqrt{2}} \xi \mu} \left[ \left( 3\xi \mu + 4\sqrt{2} \right) \sin \frac{\xi \mu}{\sqrt{2}} - \xi \mu \left( \sqrt{2}\xi \mu + 3 \right) \cos \frac{\xi \mu}{\sqrt{2}} \right] + O(g_R^2),
\]

\[
\gamma_D = \frac{\partial \ln Z_D}{\partial \mu} = g_R \frac{e^{-\sqrt{2}\xi \mu}}{2\xi^2 \mu^2} \left[ \left( 2\sqrt{2}\xi \mu + 3 \right) \sin \sqrt{2}\xi \mu - 3 \left( e^{\sqrt{2}\xi \mu} - \cos \sqrt{2}\xi \mu \right) \right] + O(g_R^2),
\]

where we introduced the dimensionless form of the renormalized couplings

\[
g_R = \frac{g Z_\rho}{2\sqrt{2}\mu}, \quad Z_\rho = Z_D Z_\nu^{-3}. \tag{51}
\]

The corresponding \( \beta \)-function (22) reads

\[
\beta_g = g_R \left[ \gamma_D(l) - 3\gamma_\nu(l) \right] - 1, \tag{52}
\]

where \( \gamma_i = \gamma_i/g_R \) and \( \gamma_\nu, \gamma_D \) are taken from (49), (50), respectively.

Again the flow of the effective coupling constant is modeled via the flow parameter \( l \) used for the solution of the RG equations by the method of characteristics. This leads to a continuous momentum scale \( \tilde{\mu}(l) \), effective coupling constant \( \tilde{g}(l) \) and thus to an l-dependent flow equation (see e.g. 16)

\[
\beta_g(l) = \frac{d\tilde{g}(l)}{dl}. \tag{53}
\]

Hence a fixed point is characterized by \( \beta_g(l) = 0 \). Applying this fixed point condition to (52) and solving for \( g_R \) leads to two separate infrared fixed point solutions \( g_{R,i}^* \):

\[
g_{R,1}^* = 0,
\]

\[
g_{R,2}^* = \lim_{l \to 0} \frac{1}{\gamma_D(l) - 3\gamma_\nu(l)}. \tag{55}
\]

Here (34) represents the trivial Gaussian fixed point while the second solution in the limit \( l \to 0 \) yields the nontrivial KPZ fixed point,

\[
g_{R,2}^* = \lim_{l \to 0} \frac{1}{\gamma_D(l) - 3\gamma_\nu(l)} = \frac{1}{2}. \tag{56}
\]

Again the fixed points are stable, if \( d\beta_g/dg_R > 0 \). Since (49), (50), (52) imply that

\[
\beta' = \frac{d\beta_g(l)}{d\tilde{g}(l)} = 2\tilde{g}(l) (\gamma_D(l) - 3\gamma_\nu(l)) - 1 \lim_{l \to 0} 4g_R - 1,
\]

we find that:

\[
g_R^* = 0 : \quad \beta_g^* = -1 < 0 \quad \Rightarrow \text{unstable},
\]

\[
g_R^* = \frac{1}{2} : \quad \beta_g^* = 1 > 0 \quad \Rightarrow \text{stable}.
\]

Hence there is one stable infrared fixed point, \( g_R^* = 1/2 \), at which the critical exponents of the KPZ universality class can be calculated.

We obtain the critical exponents in \( d = 1 \) dimensions again as

\[
z = 2 + \gamma^*_R = 2 - \frac{1}{2} = \frac{3}{2}, \tag{58}
\]

\[
\chi = \frac{1}{2} + \frac{-\frac{1}{2} + \frac{1}{2}}{2} = \frac{1}{2}. \tag{59}
\]

\textbf{IV. DISCUSSION}

In the present work we have studied the field-theoretic DRG of the KPZ equation for correlated noise of ‘sinc’-type which is characterized by a finite correlation length \( \xi \).

The fixed points of the KPZ-DRG flow have been calculated in two different manners, namely first for small correlation lengths \( \xi \) and via two effective coupling constants, \( g \) and \( g_\xi \) (see section III A and (33), (34)) and then, using only one effective coupling constant \( g \) (see section III B), for arbitrary values of \( \xi \). Both methods yield the same results, i.e., an unstable Gaussian fixed point (see (39), (54)) and the stable KPZ fixed point (see (39), (56)).

It might be argued, that the second method is somewhat redundant since the ‘small’-\( \xi \) expansion can also be interpreted to be valid for arbitrary values of \( \xi \) in the infrared limit as in this regime \( \mu \to 0 \) and hence
The expansion would then be done for the parameter $\epsilon$. Nevertheless, the method used in section III B is a reassuring confirmation of the results obtained in section III A.

Building on these fixed points, the critical exponents characterizing the KPZ universality class, i.e. the dynamical exponent $z = 3/2$ and the roughness exponent $\chi = 1/2$, were calculated (see [48] and [58], [59]). The values obtained for $z$ and $\chi$ coincide with the 'standard' KPZ exponents in one spatial dimension, where the system is excited by purely white noise (see e.g. [16]).

Hence, for every finite noise correlation length $\xi$ the system behaves to one-loop order as if it was stirred by the 'standard' uncorrelated Gaussian noise from [2].

This result corresponds nicely with the numerical findings of [6], where a different spatial noise correlation was analyzed. The authors there found that for small values of the noise correlation length the KPZ equation acted as

$$\lim_{\xi \to 0} \mu \to \frac{\xi^2}{\mu^2}$$

Combining the findings of [6] with the present ones, we arrive at the conjecture that the large-scale KPZ dynamics is independent of the details of the noise structure, provided that the correlation length $\xi$ is finite.

**Appendix A: Explicit Evaluation of the Renormalization Factors**

To obtain (31), (32) from the expressions in (29), (30), respectively, we use the Residue theorem. To this end the integrals are first rewritten in a more easily accessible form.

1. Evaluation of Eq. (29)

The first integral needed for the calculation of $Z_\nu$ reads

$$\int_0^\infty dp \frac{\sin(\xi p)}{\xi p(i\mu^2 + p^2)}.$$  \hspace{1cm} (A1)

This may be rewritten as

$$\int_0^\infty dp \frac{\sin(\xi p)}{\xi p(i\mu^2 + p^2)} = -\frac{i}{2} \int_{-\infty}^\infty dp \frac{e^{i\xi p}}{\xi p(i\mu^2 + p^2)}.$$  \hspace{1cm} (A2)

The integrand on the r.h.s. in (A2) has three simple poles which are given by $z_{1,2} = \pm \mu e^{i\pi/4}$ and $z_3 = 0$. Those and the chosen integration contour are shown in Fig. 3.

Hence the residue theorem yields

$$\int_z dz e^{i\xi z} \frac{e^{i\xi z}}{\xi (i\mu^2 + z^2)} = \lim_{\epsilon \to 0} \int_{C_1}^R dz e^{i\xi z} \frac{e^{i\xi z}}{\xi (i\mu^2 + z^2)} + \int_{\epsilon}^R dz e^{i\xi z} \frac{e^{i\xi z}}{\xi (i\mu^2 + z^2)} + \int_{-\epsilon}^{-\infty} dz e^{i\xi z} \frac{e^{i\xi z}}{\xi (i\mu^2 + z^2)}$$

$$= 2\pi i \lim_{z \to -\mu e^{i\pi/4}} \frac{\xi (z - \mu e^{i\pi/4})}{(z - \mu e^{i\pi/4})} = -\pi e^{-\frac{\pi\mu^2}{\xi}} \left[ \cos \left( \frac{1}{\sqrt{2}} \xi \mu \right) - \sin \left( \frac{1}{\sqrt{2}} \xi \mu \right) \right].$$

To obtain the integral on the real axis from minus to plus infinity, the contributions of the integrals over the two circular paths have to be computed. Therefore the parametrization

$$z = \epsilon e^{i\varphi} \quad \Rightarrow \quad dz = i \epsilon e^{i\varphi} d\varphi$$

is used, which yields for the integral over $C_1$ with $\epsilon \to 0$:

$$\lim_{\epsilon \to 0} \int_{C_1}^R dz e^{i\xi z} \frac{e^{i\xi z}}{\xi (i\mu^2 + z^2)} = -\lim_{\epsilon \to 0} \int_0^\pi d\varphi \frac{i e^{i\xi \epsilon e^{i\varphi}}}{\xi (i\mu^2 + e^2 e^{2i\varphi})}$$

$$= -\int_0^\pi d\varphi \lim_{\epsilon \to 0} \frac{i e^{i\xi \epsilon e^{i\varphi}}}{\xi (i\mu^2 + e^2 e^{2i\varphi})} = -\frac{\pi}{\xi \mu^2}$$

For the integration over the contour $C_2$ a similar parametrization is used

$$z = R e^{i\varphi} \quad \Rightarrow \quad dz = i R e^{i\varphi} d\varphi.$$  \hspace{1cm} (A3)
The contribution from this integral vanishes for $R \to \infty$:

$$
\lim_{R \to \infty} \left| \int_{C_2} \frac{e^{i\xi z}}{z (\mu^2 + z^2)} \right| = \lim_{R \to \infty} \left| \int_{-\infty}^{\pi} d\varphi \frac{i R e^{i\xi R e^{i\varphi}}}{{\xi (\mu^2 + R^2 e^{2i\varphi})}} \right| \\
\leq \lim_{R \to \infty} \int_{0}^{\pi} d\varphi \frac{e^{i\xi R e^{i\varphi}}}{{\xi (\mu^2 + R^2 e^{2i\varphi})}} \\
= \lim_{R \to \infty} \int_{0}^{\pi} d\varphi \frac{e^{-\xi R \sin \varphi}}{\xi R^2 \left| \frac{\mu^2}{R^2} + e^{2i\varphi} \right|} = 0,
$$

since $\sin \varphi > 0$ for $0 < \varphi < \pi$. Thus in the limits $R \to \infty$ and $\epsilon \to 0$ the residue theorem results in

$$
\int_{-\infty}^{\infty} dz \frac{e^{i\xi z}}{z (\mu^2 + z^2)} = \frac{\pi e^{-\frac{\sqrt{2} \xi \mu}{2 \sqrt{2}}} \left[ e^{\frac{1}{\sqrt{2}} \xi \mu} - \left( \cos \left( \frac{1}{\sqrt{2}} \xi \mu \right) - i \sin \left( \frac{1}{\sqrt{2}} \xi \mu \right) \right) \right]}{\xi \mu^2}. 
$$

The integrand from (A1) is therefore given by

$$
\int_{0}^{\infty} dp \frac{\sin(\xi p)}{\xi (\mu^2 + p^2)} = \frac{\pi e^{-\frac{\sqrt{2} \xi \mu}{2 \sqrt{2}}} \left[ \cos \left( \frac{1}{\sqrt{2}} \xi \mu \right) - \left( \cos \left( \frac{1}{\sqrt{2}} \xi \mu \right) - e^{\frac{1}{\sqrt{2}} \xi \mu} \right) \right]}{\xi \mu^2 \times} \\
\times \left[ \sin \left( \frac{1}{\sqrt{2}} \xi \mu \right) + i \left( \cos \left( \frac{1}{\sqrt{2}} \xi \mu \right) - e^{\frac{1}{\sqrt{2}} \xi \mu} \right) \right]. 
$$

The second integral needed for the evaluation of (29) is given by

$$
\int_{0}^{\infty} dp \frac{\cos(\xi p)}{\mu^2 + p^2}. 
$$

As for the calculation of (A1), the integral will be rewritten according to

$$
\int_{0}^{\infty} dp \frac{\cos(\xi p)}{\mu^2 + p^2} = \frac{1}{2} \int_{-\infty}^{\infty} dp \frac{e^{i\xi p}}{\mu^2 + p^2}. 
$$

The integrand on the r.h.s. of (A6) has two simple poles at $z_{1/2} = \pm \mu e^{i3\pi/4}$. For the integration contour shown in Fig. 4, the residue theorem leads to

$$
\int_{C} dz \frac{e^{i\xi z}}{\mu^2 + z^2} = \int_{-\infty}^{\infty} dz \frac{e^{i\xi z}}{\mu^2 + z^2} + \int_{C_1} dz \frac{e^{i\xi z}}{\mu^2 + z^2} \\
= \frac{2 \pi i}{e^{i3\pi/4}} \left( z - \mu e^{i3\pi/4} \right) \frac{e^{i\xi z}}{\left( z - \mu e^{i3\pi/4} \right) \left( z + \mu e^{i3\pi/4} \right)} \\
= \frac{\pi e^{-\frac{\sqrt{2} \xi \mu}{2 \sqrt{2}}} \left[ \cos \left( \frac{1}{\sqrt{2}} \xi \mu \right) - \sin \left( \frac{1}{\sqrt{2}} \xi \mu \right) - i \left( \cos \left( \frac{1}{\sqrt{2}} \xi \mu \right) + \sin \left( \frac{1}{\sqrt{2}} \xi \mu \right) \right) \right]}{\sqrt{2} \mu}. 
$$

Hence the sought integral reads

$$
\int_{0}^{\infty} dp \frac{\cos(\xi p)}{\mu^2 + p^2} = \frac{\pi e^{-\frac{\sqrt{2} \xi \mu}{2 \sqrt{2}}} \left[ \cos \left( \frac{1}{\sqrt{2}} \xi \mu \right) - \sin \left( \frac{1}{\sqrt{2}} \xi \mu \right) - i \left( \cos \left( \frac{1}{\sqrt{2}} \xi \mu \right) + \sin \left( \frac{1}{\sqrt{2}} \xi \mu \right) \right) \right]}{\sqrt{2} \mu}. 
$$

Taking the real parts [6] of (A4) and (A7) and inserting the results into (29) leads to the expression in (31).

2. Evaluation of Eq. (30)

The integral (30) reads

$$
1 \int_{0}^{\infty} dp \frac{\sin^2(\xi p)}{\mu^2 + p^2} = \frac{1}{2\pi} \left[ \int_{0}^{\infty} dz \frac{1}{\mu^2 + z^2} - \int_{0}^{\infty} dz \frac{\cos(2\xi z)}{\mu^2 + z^2} \right]. 
$$

The integrands of both integrals in (A8) have simple poles at $z_k = \mu e^{i(k\pi/4+\pi/2)}$, with $k = 0, 1, 2, 3$, and the

![Figure 4: This diagram shows the contour $C$ of integration for (A6). $C_1$ denotes a circle about $z = 0$ with radius $R$.](Image)
contour of integration is shown in Fig. 5. The first of the two integrals on the right hand side of (A8) is readily solved with the aid of the residue theorem (again it can be shown that \( \int_{C_1} dz/(\mu^4 + z^4) = 0 \) for \( R \to \infty \):)
\[
\int_0^\infty dz \frac{1}{\mu^4 + z^4} = \frac{1}{2} \int_{-\infty}^\infty dz \frac{1}{\mu^4 + z^4} = \frac{1}{2} \pi i \left[ \lim_{z \to z_0} \frac{z - z_0}{(z - z_0)(z - z_1)(z - z_2)(z - z_3)} + \lim_{z \to z_1} \frac{z - z_1}{(z - z_0)(z - z_1)(z - z_2)(z - z_3)} \right] = \frac{\pi}{2\sqrt{2} \mu^3}.
\]

For the second integral it is again used that
\[
2 \int_0^\infty dz \frac{\cos(2\xi z)}{\mu^4 + z^4} = \int_{-\infty}^\infty \frac{e^{2i\xi z}}{\mu^4 + z^4} (z \in \mathbb{R}).
\]

With the integration contour shown in Fig. 5, we arrive at
\[
\int_{-R}^R dz \frac{e^{2i\xi z}}{\mu^4 + z^4} = 2\pi i \left[ \lim_{z \to z_0} \frac{(z - z_0)e^{2i\xi z}}{(z - z_0)(z - z_1)(z - z_2)(z - z_3)} + \lim_{z \to z_1} \frac{(z - z_1)e^{2i\xi z}}{(z - z_0)(z - z_1)(z - z_2)(z - z_3)} \right]
\]
\[
= \frac{\pi e^{-\sqrt{2} \xi \mu}}{\sqrt{2} \mu^3} \left[ \cos(\sqrt{2} \xi \mu) + \sin(\sqrt{2} \xi \mu) \right].
\]

As \( \int_{C_1} dz e^{2i\xi z}/(\mu^4 + z^4) \) tends to zero for \( R \to \infty \),
\[
\lim_{R \to \infty} \left| \int_0^\pi d\varphi \frac{i Re^{i\varphi} e^{2i\xi R e^{i\varphi}}}{\mu^4 + R^4 e^{4i\varphi}} \right| \leq \lim_{R \to \infty} \left| \int_0^\pi d\varphi \left| \frac{i Re^{i\varphi} e^{2i\xi R e^{i\varphi}}}{\mu^4 + R^4 e^{4i\varphi}} \right| \right| \leq \lim_{R \to \infty} \int_0^\pi d\varphi \frac{e^{-2\xi R \sin \varphi}}{R^3 \frac{\mu^4}{\pi^4} + e^{4i\varphi}} = 0,
\]

it is found that
\[
\int_0^\infty dz \frac{\cos(2\xi z)}{\mu^4 + z^4} = \frac{\pi e^{-\sqrt{2} \xi \mu}}{2\sqrt{2} \mu^3} \left[ \cos(\sqrt{2} \xi \mu) + \sin(\sqrt{2} \xi \mu) \right].
\]

The results from (A9) and (A10), inserted into (30), yield the expression in (32).

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