I. INTRODUCTION

Two-dimensional (2D) hole systems confined to GaAs quantum wells or GaAs/AlGaAs heterostructures often exhibit a strong positive magnetoresistance (MR) at small perpendicular magnetic fields $|B| \lesssim 0.1 \text{ T}$. An example is shown in Fig. 1. The origin of this MR is of renewed interest in light of the puzzling metallic behavior recently observed in a number of 2D systems including GaAs 2D holes [3,4,5]. It has been argued that the positive MR can be explained by a simple two-band model [3,5,6], where the two bands arise from the spin-orbit induced spin-splitting of the valence bands [6,7,8]. Another report [9] relates the MR to weak antilocalization which is expected to occur at low magnetic fields in a system with spin-orbit coupling. There is yet a third possibility: positive MR has also been observed in 2D carrier systems with a one-dimensional periodic [13,14], or nearly periodic [15], potential modulation. This suggests that the unintentional corrugations which are often present at GaAs/AlGaAs surfaces and interfaces may also contribute to the observed MR.

In this paper we report extensive data on 2D holes in GaAs quantum wells and GaAs/AlGaAs heterostructures, grown on (311)A and (100) GaAs substrates, as well as a 2D electron system grown on a (311)A GaAs substrate. The (311)A 2D hole data, which are the main focus of this paper, span a density ($p$) range of $2.5 \times 10^{10} \leq p \leq 4.0 \times 10^{11} \text{ cm}^{-2}$. The results collectively reveal the following trends. At high densities and in the presence of a large electric field perpendicular to the plane of the (311)A 2D hole system, we observe a pronounced positive MR when the current is driven along the [011] direction. A much weaker MR is observed for current along the [233]. As the perpendicular electric field is reduced, the MR diminishes but does not vanish. On the other hand, as the 2D density is lowered the positive MR progressively becomes smaller and vanishes at the lowest measured density. The behavior of the MR for current along [011] cannot be explained by any one of the previously mentioned three mechanisms. The combination of all three, however, is likely to account for the MR. We also measure the temperature dependence of the resistivity at $B = 0$ and discuss its relation to the observed MR.

The paper is organized as follows. Section II describes the experimental setup and the samples used in our study. Section III concentrates on the high-density data, and demonstrates how each of the above-mentioned mechanisms for positive MR is expressed in the data.

**Low-field magnetoresistance in GaAs 2D holes**

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We report low-field magnetotransport data in two-dimensional hole systems in GaAs/AlGaAs heterostructures and quantum wells, in a large density range, $2.5 \times 10^{10} \leq p \leq 4.0 \times 10^{11} \text{ cm}^{-2}$, with primary focus on samples grown on (311)A GaAs substrates. At high densities, $p \gtrsim 1 \times 10^{11} \text{ cm}^{-2}$, we observe a remarkably strong positive magnetoresistance. It appears in samples with an anisotropic in-plane mobility and predominantly along the low-mobility direction, and is strongly dependent on the perpendicular electric field and the resulting spin-orbit interaction induced spin-subband population difference. A careful examination of the data reveals that the magnetoresistance must result from a combination of factors including the presence of two spin-subbands, a corrugated quantum well interface which leads to the mobility anisotropy, and possibly weak anti-localization. None of these factors can alone account for the observed positive magnetoresistance. We also present the evolution of the data with density: the magnitude of the positive magnetoresistance decreases with decreasing density until, at the lowest density studied ($p = 2.5 \times 10^{10} \text{ cm}^{-2}$), it vanishes and is replaced by a weak negative magnetoresistance.

**FIG. 1.** Low-field magnetoresistance data at $T = 30 \text{ mK}$ from sample A, a GaAs (311)A 2D hole system with $p = 3.3 \times 10^{11} \text{ cm}^{-2}$, $E_L = -6 \text{ kV/cm}$, and current along [011]. The dashed line is a fit of the two-band model to the experimental data (see text), with the parameters used for the fit given in the legend.
Section V shows the evolution of the MR as the density is reduced. In Section VI we make a comparison between the MR and the $B = 0$ temperature dependence of the resistivity where metallic behavior is observed. Section VII summarizes our results and conclusions.

II. EXPERIMENTAL DETAILS

We studied the MR in square quantum wells (QWs) as well as in single heterojunction samples, grown by molecular beam epitaxy (MBE) on both (311)A and (100) insulating GaAs substrates. We report here data for samples from six different wafers; a summary of the important sample parameters is given in Table I. The samples are modulation-doped with either Si or Be. Use of Be as a $p$-type (acceptor) dopant for growth on GaAs (100) substrates is standard, although the quality of the 2D hole system may be somewhat compromised because of the fast Be diffusion in GaAs, and a memory effect in the MBE chamber.

Alternatively, one can grow the heterostructure on GaAs (311)A substrates and use Si which, under normal growth conditions, is incorporated as an acceptor on this surface. Such samples are known to show exceptionally high quality as measured, e.g., by their low-temperature mobility $\mu_233$. They comprise the majority of the samples used in our work as well as of others who have studied the metallic behavior in GaAs 2D hole systems $\mu_{011}$, $\mu_{011}$. The GaAs/AlGaAs (311)A interface, however, has quasi-periodic corrugations along the [233] direction $\mu_{011}$, and the corrugations lead to an in-plane mobility anisotropy $\mu_{011}$: mobility is typically larger along the [233] direction by a factor of 2 or 3 compared to the mobility along [011]. This mobility anisotropy is likely related to an anisotropic interface roughness scattering $\mu_{011}$ and as we will show here, plays an important role in the low-field MR.

For comparison, we also studied a modulation-doped (311)A GaAs/AlGaAs heterojunction sample containing 2D electrons. This sample was grown under MBE growth conditions similar to those used for the 2D hole systems: the substrate temperature during the growth of the GaAs/AlGaAs interface and AlGaAs spacer was kept the same as in 2D hole samples, and was reduced only when the dopant (Si) atoms were introduced so that they were incorporated as donors $\mu_{011}$. The interface morphology of the 2D electron system in this sample should therefore closely resemble that of the 2D hole samples.

The samples were patterned with $L$-shaped Hall bars allowing simultaneous measurements of longitudinal ($\rho_{xx}$ and $\rho_{yy}$) and transverse ($\rho_{xy}$) magnetoresistances along two different current directions. On the (311)A samples, the arms of the Hall bar were aligned along $[011]$ and $[233]$, which are respectively the low- and high-mobility ($\mu$) directions. For purposes of discussion in this paper, we will refer to the resistivity of the low-$\mu$ direction as $\rho_{xx}$ and the resistivity of the high-$\mu$ direction as $\rho_{yy}$. On the (100) samples the arms of the Hall bar were aligned along $[011]$ and $[011]$. Measurements were done in dilution and $^3$He refrigerators with base temperatures ($T$) of 30 and 300 mK respectively. We measured the resistivity with a conventional low-frequency lock-in technique using currents of 1-10 nA. Samples A, B, and F had metallic front and back gates so that we could independently control both the 2D hole density and the electric field applied perpendicular to the QW ($E_{\perp}$). $E_{\perp}$ is defined as positive when the electric field is pointing from the substrate towards the front gate.

III. MAGNETORESISTANCE AT HIGH DENSITY

The positive MR we observe in these GaAs 2D hole samples has several remarkable properties. The most striking is that it can be quite large. In Fig. 2, the longitudinal resistivity $\rho_{xx}$ rises by 33% from $B = 0$ to $|B| = 0.1$ T. $\rho_{xx}$ also often shows a cusp, which is sharp to our measurement resolution, at $B = 0$. Furthermore, we have observed the positive MR in samples that display a mobility anisotropy, and then often only, and always much more strongly, in the low-mobility direction (Figs. 2 and 3). We will focus initially on data from 2D hole systems in square QWs grown on (311)A samples, and then present and compare data from the other systems.

Our data reveal that the magnitude of the MR is in-
TABLE I. Summary of sample parameters. Densities and mobilities are for ungated samples.

| Sample | Substrate | Carrier | Dopant | density ($10^{11}$ cm$^{-2}$) | $\mu_{xx}$ ($10^5$ cm$^2$/Vs) | $\mu_{yy}$ ($10^5$ cm$^2$/Vs) | QW structure |
|--------|-----------|---------|--------|-------------------------------|-------------------------------|-------------------------------|--------------|
| A      | (311)A    | holes   | Si     | 2.2                           | 4.0                           | 5.8                           | square       |
| B      | (311)A    | holes   | Si     | 2.2                           | 5.5                           | 8.0                           | square       |
| C      | (100)     | holes   | Be     | 2.0                           | 1.5                           | 3.7                           | square       |
| D      | (311)A    | electrons | Si    | 1.5                           | 2.9                           | 3.2                           | triangular   |
| E      | (311)A    | holes   | Si     | 1.5                           | 0.7                           | 4.2                           | triangular   |
| F      | (311)A    | holes   | Si     | 0.9                           | 3.3                           | 4.9                           | square       |

FIG. 3. Magnetoresistance data at $T = 30$ mK from sample A, a 2D hole system in a square QW grown on a (311)A substrate with $p = 2.3 \times 10^{11}$ cm$^{-2}$, shown for various $E_\perp$. The upper trace in each panel is from the low-$\mu$ ($[01\bar{1}]$) direction, while the lower trace is from the high-$\mu$ ($[\bar{2}33]$) direction.

fluenced by the symmetry of the potential that confines the 2D holes. Figures 2 and 3 show MR data for samples A and B, from two different wafers, as the asymmetry of the QW potential is changed. The mobilities of these samples are typically in excess of 500,000 cm$^2$/Vs, and the high quality is evident from the strong Shubnikov-de Haas oscillations. When the QW is made asymmetric by the application of a perpendicular electric field $E_\perp$, the $B = 0$ spin-splitting grows. The method of tuning $E_\perp$ is described in detail in Ref. [3]. Beating, due to the presence of two spin-subbands with significantly different populations, is evident in the traces at larger $|E_\perp|$. Examples of Fourier transforms of $\rho_{xx}$ vs. $B^{-1}$ are shown in Fig. 3b: the single peak at low $|E_\perp|$ splits into two peaks at larger $|E_\perp|$, giving a quantitative measure of the spin-splitting. The magnitude of the MR feature is correlated with the spin-splitting. Here we show data only for $E_\perp < 0$, which corresponds to an electric field pointing towards the substrate. Data with $E_\perp > 0$ show the same behavior: both the spin-subband densities and the magnitude of the MR are symmetric around $E_\perp = 0$.

As demonstrated in Fig. 4, the magnitude of the MR is affected by changes in the 2D hole density as well. A reduction in the density, with $E_\perp$ held constant, causes a reduction in the magnitude of the MR. There is very little change in its shape (Fig. 4b): and it retains its cusp at $B = 0$ until it is almost indistinguishable from the noise. The behavior of the MR over a wider density and $E_\perp$ range is discussed in Section IV.

The MR is also observed in Be-doped samples grown on GaAs (100) substrates. An example is shown in Fig. 5. Here, positive MR is observed in both $\rho_{xx}$ and $\rho_{yy}$, but it is again much smaller in $\rho_{yy}$. In the (100) samples, the MR feature depends on $E_\perp$ and density in the same way it does in the (311)A samples. The strong similarities between the positive MR observed in (100) and (311)A samples suggest that the feature has a similar origin in both systems.
FIG. 5. Low field magnetoresistance data at $T = 300$ mK from sample C, a 2D hole system grown on a (100) substrate with $p = 2.0 \times 10^{11} \text{cm}^{-2}$. The results of the two-band model fit for the top trace are also shown (dotted curve).

Here we discuss three possible origins for the low-$B$ positive MR. Weak anti-localization [4], a two-band model [5], and a one-dimensional periodic potential modulation [6] have all been invoked to explain positive MR in the past. Our comprehensive set of data shows that some combination of these is necessary to describe the MR in GaAs 2D holes.

A much weaker MR with a similar shape has been seen in 2D holes on (100) substrates and attributed to weak anti-localization [7]. The cusp at $B = 0$ in our data is suggestive of weak anti-localization, but the magnitude of the resistivity change is many orders of magnitude too large to be caused by weak anti-localization. Weak localization and anti-localization typically contribute a correction of order $\sim 0.01 \epsilon^2/h$ to the conductivity of a 2D system. The conductivities of our samples are orders of magnitude larger than $\epsilon^2/h$, so such a correction is far smaller than the total change in the conductance we observe.

In GaAs 2D holes on (100) substrates, Murzin et al. [8] and Yaish et al. [9] have observed a similar MR feature and described it using a simple two-band model. This two-band model initially looks promising in our system as well. We have already seen that the magnitude of the positive MR increases when the spin-subband population difference is increased. Another factor that qualitatively supports the two-band hypothesis is that the MR is completely absent in a 2D electron system in a triangular well grown on a (311)A substrate; 2D electrons in GaAs have a very small spin-orbit coupling so they occupy essentially a single spin-degenerate subband. Figure 6 shows data from a 2D electron system on (311)A, and Fig. 6b shows data from a similar (311)A 2D hole system for comparison.

We have fit the two-band model as used by Yaish et al. [9] to our data. Examples are shown in Fig. 6a, the top panel of Fig. 4, and in Fig. 6b. We multiply the Shubnikov-de Haas frequencies deduced from the Fourier transforms of $\rho_{xx}$ vs. $B^{-1}$ by $h/e^2$ to get the two subband densities (when there is one measured frequency, we use the same density for both subbands), from which we calculate the Hall coefficients $R_{1h} = 1/p_{1h}e$ for the light ($l$) and heavy ($h$) holes [20]. These Hall coefficients are used as inputs for the two-band model. From the fits we extract $S_l$, $S_h$, and $Q$, which can be expressed using elements of the conductivity tensor: $S_l = \sigma_{ll}^{-1} + \sigma_{lh}^{-1}$, $S_h = \sigma_{hh}^{-1} + \sigma_{hl}^{-1}$, and $Q = \lambda \sigma_{ll}^{-1} = \lambda^{-1} \sigma_{hl}^{-1}$. The $\sigma_{ij}$ take into account intraband scattering and the $\sigma_{ij}$ account for interband scattering.

We find that, overall, the model produces a less satisfactory fit to our data than to that of Yaish et al. The values it produces for $S_l$ and $S_h$ are typically within a factor of two of each other, which are reasonable for two spin-subbands. However, the best fit to our data is always for $Q = 0$, which corresponds to no intersubband scattering, and the model always produces a wider MR feature than the data. We have constrained $Q$ such that $Q \geq 0$, since the conductivity due to intersubband scattering cannot be negative. Intersubband scattering can only increase the width of the feature, so the fact that our data shows sharper MR than the two-band model with no intersubband scattering implies that this simple model is not adequately explaining the MR. Furthermore, the two-band model cannot match the cusp seen in the data at $B = 0$.

Positive MR has also been observed repeatedly when a one-dimensional periodic potential is applied to a 2D system (see [11] and references therein.). The qualitative characteristics of our data are similar to those of these
systems. In our case, we have not imposed an intentional periodic potential, but the resistance anisotropy of our system is suggestive of some sort of anisotropic disorder. Transport data in (311)A samples indeed suggest that interface-roughness scattering limits the mobility along the [011] direction, and scanning-tunneling-microscopy (STM) images show one-dimensional corrugations along the [233] direction. These images show two sets of corrugations: one nearly regular with a 32 Å period and a 2 Å amplitude, and one quasi-periodic with a $\sim$ 1000 Å period and up to $\sim$ 15 Å amplitude. For samples grown on (100) substrates, it is initially a surprise that there is a mobility anisotropy at all, since the [011] and [01|] directions are expected to be crystallographically equivalent. However, any slight miscut of the substrate from the ideal (100) direction could break the symmetry, and indeed STM images have shown a one-dimensional quasi-periodic potential, with a period of a few hundred Å, to exist in samples grown on epi-ready GaAs (100) wafers as well. Also, AFM studies of high-quality (100) samples have shown an anisotropic surface roughness that correlates with anisotropy in transport in the 2D layers. We have not independently verified that in our (100) samples the low-mobility direction is indeed perpendicular to the corrugations, but it is a reasonable assumption given the resemblance to the (311)A data.

The similarity between our data and the data of Akabori et al. from a corrugated 2D electron system in InGaAs is remarkable. Most obviously, in both systems the current direction perpendicular to the corrugations has a lower mobility. Also, in both systems, the positive MR is strong for current perpendicular to the corrugations. Furthermore, the amplitude of the Shubnikov-de Haas oscillations is larger for current perpendicular to the corrugations than for current parallel (see our Figs. 2 and 3, and Fig. 3a of Akabori et al.). Finally, the fact that the MR grows when $|E_\perp|$ is made larger is consistent with the hypothesis that the periodic modulation of the interface plays a role. When $E_\perp = 0$, the carrier wavefunction is centered in the QW, so it has little overlap with the barriers at the edges of the QW. When $|E_\perp|$ is increased, the wavefunction is pushed towards one of the barriers, so any interface corrugations present will have a larger effect.

We now provide a quantitative assessment of this hypothesis. Akabori et al. apply the semiclassical model of Beton et al. which relates the period and amplitude of the potential corrugation to the B at which the positive MR saturates. Adopting such a model and assuming corrugation periods of 32 Å and 1000 Å in our (311)A samples, we calculate corrugation amplitudes of 0.002 and 0.07 meV respectively. These values are much smaller than the typical Fermi energy $E_F$ of 2 meV for 2D holes at a density of $3.3 \times 10^{11}$ cm$^{-2}$. This model therefore gives an inconsistent picture, since potential modulations more than an order of magnitude smaller than $E_F$ would not be expected to produce such a large MR. A quantum mechanical model on the other hand, yields potential amplitudes of 0.6 and 0.1 meV respectively. These values are reasonable, and can possibly lead to the observed MR.

The comparisons between the data of Akabori et al. and ours suggest that the interface corrugations in our system are playing a role in the positive MR, but once again, looking at the complete set of data shows that the corrugations alone cannot be the origin of the positive MR in our samples. As mentioned above, data from the 2D electrons on (311)A substrates show no positive MR (Fig. 3a). The growth of the QW in these samples is identical to the growth of the QW in (311)A 2D hole systems, so we expect the interfaces to be the same as well. The only difference in the samples is that, after the QW has already been grown, the substrate temperature is reduced while the Si dopant atoms are being deposited, causing Si to become a donor instead of an acceptor. An important difference between 2D electrons and 2D holes in GaAs is that spin-orbit interaction is strong for the 2D holes, but weak for the 2D electrons. Therefore, it appears that the existence of both interface corrugations and two spin-subbands is necessary for the expression of the positive MR we have observed. We note that in the system of Akabori et al., 2D electrons in InGaAs, there is also a strong spin-orbit interaction.

In short, the data we have shown collectively reveal that a combination of factors is responsible for the low-B positive MR we observe. It appears to be due to the presence of both two occupied subbands and a one-dimensional quasi-periodic potential modulation. Furthermore, neither of these mechanisms is likely to explain the sharp cusp we observed at $B = 0$. It is possible that weak anti-localization may also be playing a role in the very-low-B regime, causing this cusp.

IV. MAGNETORESISTANCE AT LOW DENSITY

In this section we show how the low-B MR of 2D holes evolves as the density is reduced. Figure 7 shows $\rho_{xx}$ traces from sample F for three different $E_\perp$ at each of three densities. Raw data are plotted, without any shifts. The curves at larger $|E_\perp|$ have lower mobility because as $|E_\perp|$ is increased, the wavefunction is pushed to one side of the QW, where in this current direction it is more strongly affected by the interface corrugations. As mentioned above, this could be a factor in the increasing strength of the positive MR as well.

At the highest density $p = 1.2 \times 10^{11}$ cm$^{-2}$, a positive MR is clearly visible only for the largest $|E_\perp|$, $E_\perp = -9.8$ kV/cm. At lower $|E_\perp|$, it is not visible: at low $B$ there is only a weak negative MR. At the intermediate $p$ of $7.0 \times 10^{10}$ cm$^{-2}$, the MR feature is still visible at the highest $|E_\perp|$, but is noticeably smaller. When $p = 2.5 \times 10^{10}$ cm$^{-2}$, there is no positive MR even at the largest $|E_\perp|$ of $-10$ kV/cm. In this range, there is only a negative MR,
which in a similar system has been attributed to weak localization \[24\]. Figure 7 data show that for asymmetric QWs, the positive MR feature can be seen to quite low densities, well below the density at which two frequencies can no longer be resolved in the Shubnikov-de Haas oscillations.

V. TEMPERATURE DEPENDENCE OF THE \( B = 0 \) RESISTIVITY

It is interesting to explore the relation between the MR and the temperature dependence of the \( B = 0 \) resistivity which has been controversial recently \[2,5–11\]. In Fig. 8, we show the metallic \( T \)-dependence of \( \rho_{xx} \) at \( B = 0 \) and the MR feature for comparison. Both the MR and the \( B = 0 \) \( T \)-dependence of \( \rho_{xx} \) are plotted as a fractional change \( \rho_{xx}/\rho_0 \) (where \( \rho_0 \) is \( \rho_{xx} \) at \( B = 0 \) and \( T = 30 \) mK) so that the relative magnitudes can be easily compared. It has been proposed that the \( B = 0 \) \( T \)-dependence of \( \rho_{xx} \) can be explained in the two-band model \[2,5\]. It is only at high \( p \), where the rise in \( \rho \) as \( T \) is increased is smaller than the MR, that our data are consistent with this hypothesis. For \( p = 3.3 \times 10^{11} \text{ cm}^{-2} \), \( \rho_{xx} \) shows a 6\% rise as \( T \) increases from 30 mK to 800 mK, while the positive MR from \( B = 0 \) to 0.1 T is 33\%. As \( p \) is reduced however, the two features show opposite trends, and the data do not support the two-band hypothesis. For smaller \( p \), the change in \( \rho_{xx} \) with increasing \( T \) becomes larger while the MR becomes smaller. For \( p = 1.2 \times 10^{11} \text{ cm}^{-2} \), \( \rho_{xx}/\rho_0 \) at \( T = 0.8 \) K is 3 times larger than the positive MR. At \( p = 7.0 \times 10^{10} \text{ cm}^{-2} \), the MR is only 2\%, and \( \rho_{xx}/\rho_0 \) at \( T = 0.8 \) K has risen to nearly 40\%. At the lowest density, no positive MR is observed, yet there is still strong metallic behavior in the \( T \)-dependence of \( \rho_{xx} \) at \( B = 0 \). There is evidence that the existence of two spin-subbands does play a role in the \( B = 0 \) metallic behavior \[25,26\] throughout the density range in which it is observed, but a comparison of the \( B = 0 \) MR feature with the \( B = 0 \) \( T \)-dependence of \( \rho \) suggests that a simple two-band model with intersubband scattering is not sufficient to explain the relationship, as the metallic behavior is strongest at densities where the MR is very weak or altogether absent.

VI. SUMMARY

In summary, we observe a positive MR in high-density 2D hole systems in GaAs quantum wells. The MR appears to be related to both the spin-splitting in the system \textit{and} to quasi-periodic corrugations perpendicular to the current direction at the interfaces between the quantum wells and the barriers. The data also show a cusp at \( B = 0 \) that is suggestive of weak anti-localization. At a fixed perpendicular electric field, the MR is reduced in size when the 2D hole density is reduced. Finally, the magnitude of the MR is not correlated with the magnitude of the rise in sample resistivity with increasing \( T \) at \( B = 0 \). This implies that the metallic behavior is not driven solely by the factors that lead to the MR.

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FIG. 8. Temperature dependence of $\rho_{xx}$ at $B = 0$ for four densities from samples A and F. The insets show low-$B$ MR traces at $T = 30$ mK. The $y$-axes are all plotted as $\rho_{xx}/\rho_0$, where $\rho_0$ is $\rho_{xx}$ at $B = 0$ and $T = 30$ mK.

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