The decays $B \to \psi(2S)\pi(K), \eta_c(2S)\pi(K)$ in the pQCD approach beyond the leading-order

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Abstract

Two body $B$ meson decays involving the radially excited meson $\psi(2S)/\eta_c(2S)$ in the final states are studied by using the perturbative QCD (pQCD) approach. We find that: (a) The branching ratios for the decays involving $K$ meson are predicted as $Br(B^+ \to \psi(2S)K^+) = (5.37_{-2.22}^{+1.85}) \times 10^{-4}, Br(B^0 \to \psi(2S)K^0) = (4.98_{-2.06}^{+1.71}) \times 10^{-4}, Br(B^+ \to \eta_c(2S)K^+) = (3.54_{-3.09}^{+3.18}) \times 10^{-4}$, which are consistent well with the present data when including the next-to-leading-order (NLO) effects. Here the NLO effects are from the vertex corrections and the NLO Wilson coefficients. The large errors in the decay $B^+ \to \eta_c(2S)K^+$ are mainly induced by using the decay constant $f_{\eta_c(2S)} = 0.243_{-0.111}^{+0.079}$ GeV with large uncertainties. (b) While there seems to be some room left for other higher order corrections or the non-perturbative long distance contributions in the decays involving $\pi$ meson, $Br(B^+ \to \psi(2S)\pi^+) = (1.17_{-0.56}^{+0.42}) \times 10^{-5}, Br(B^0 \to \psi(2S)\pi^0) = 0.54_{-0.23}^{+0.20} \times 10^{-5}$, which are smaller than the present data. The results for other decays can be tested at the running LHCb and forthcoming Super-B experiments. (c) There is no obvious evidence of the direct CP violation being seen in the decays $B \to \psi(2S)\pi(K), \eta_c(2S)\pi(K)$ in the present experiments, which is supported by our calculations. If a few percent value is confirmed in the future, it would indicate new physics definitely.

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I. INTRODUCTION

It is well known that $\eta_c(2S)$ and $\Psi(2S)$ are the first radially excited states of the S-wave ground states $\eta_c(1S)$ and $J/\Psi$, respectively. These two excited charmonia states have been observed in the $B$ meson decays in the experiments [1],

$$Br(B^+ \rightarrow \eta_c(2S)K^+) = (3.4 \pm 1.8) \times 10^{-4},$$
$$Br(B^+ \rightarrow \psi(2S)K^+) = (6.27 \pm 0.24) \times 10^{-4},$$
$$Br(B^+ \rightarrow \psi(2S)\pi^+) = (2.44 \pm 0.30) \times 10^{-5},$$
$$Br(B^0 \rightarrow \psi(2S)K^0) = (6.2 \pm 0.5) \times 10^{-4}. \tag{4}$$

Furthermore, the direct CP-violating asymmetries of the two charged decay channels are given by PDG [1], though with large uncertainties:

$$A_{CP}(B^+ \rightarrow \psi(2S)K^+) = (-2.4 \pm 2.3)\%, \tag{5}$$
$$A_{CP}(B^+ \rightarrow \psi(2S)\pi^+) = (3.0 \pm 6.0)\%. \tag{6}$$

In fact, $B$ meson exclusive decays into charmonia have been received a lot of attentions for many years. They are regarded as the golden channels in researching CP violation and exploring new physics. At the same time, they play the important roles in testing the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) triangle. Moreover, these decays are ideal modes to check the different factorization approaches, such as the naive factorization assumption (FA) [2], QCD-improved factorization (QCDF) [3, 4], light-cone sum rules (LCSR) [5], and the perturbative QCD (pQCD) approach [6]. Most of these approaches can not work well when describing these decays, such as $B \rightarrow (J/\Psi, \eta_c, \chi_{c0}, \chi_{c1})K^{(*)}$. Failure of the naive factorization assumption (FA) [2] makes the factorization breakdown, it will strongly interact with the $B\pi$ system, which makes the factorization fail. Fortunately, the transverse size of the charmonium is small in the heavy quark limit, so the authors of Ref. [5] considered that the pQCD method could be used to the decay $B \rightarrow J/\Psi K$, while they found that the leading-twist (twist-2) contributions were too small to explain the data. Then the authors of Ref. [6] calculated the twist-3 contributions, where the divergent integral was parametrized as Eq.(7). But they still could not explain the experimental data. The end-point singularities were also found in the twist-3 amplitudes.

$$\ln \frac{m_B}{\Lambda} (1 + \rho_H e^{i\delta_H}), \quad \ln \frac{m_B}{\Lambda} (1 + \rho_A e^{i\delta_A}), \tag{7}$$

respectively. These non-universal and uncontrollable parameters $\rho_{H,A}$ and $\delta_{H,A}$ will induce large theoretical uncertainties. What’s worse, when the emitted meson is heavy, such as $D$ and charmonia states $J/\Psi, \Psi(2S), \eta_c(2S)$, the QCDF will break down. For example, in the $B^0 \rightarrow D^0\pi^0$ decay, since the $D^0$ meson is not a compact object with small transverse extension, it will strongly interact with the $B\pi$ system, which makes the factorization fail. Fortunately, the transverse size of the charmonium is small in the heavy quark limit, so the authors of Ref. [5] considered that the QCDF method could be used to the decay $B \rightarrow J/\Psi K$, while they found that the leading-twist (twist-2) contributions were too small to explain the data. Then the authors of Ref. [6] calculated the twist-3 contributions, where the divergent integral was parametrized as Eq.(7). But they still could not explain the experimental data. The end-point singularities were also found in the twist-3 amplitudes.
for the decays $B \rightarrow \eta_c(1S)K, \eta_c(2S)K$ \cite{10}, where the QCDF approach was used. The LCSR approach was also insufficient to account for the data of these B meson decays into charmonia \cite{5}.

While under the pQCD approach, the spectator quark and other quarks are connected by one hard gluon (shown in Fig.1). Unlike the QCDF approach, the hard part of the pQCD approach consists of six quarks rather than four. Certainly, there also exist the soft and collinear gluon exchanges between quarks. So the double logarithms $\ln^2 Pb$ will arise from the overlap of the soft and collinear divergences in radiative corrections to the meson wave functions, $P$ being the dominant light-cone component of a meson momentum, $b$ being the conjugate variable of parton transverse momentum $k_T$. One can use the $k_T$ resummation \cite{11} to organize these leading double logarithms for all loop diagrams into a Sudakov factor, which suppresses the long distance contributions in the large $b$ region.

When the end-point region with a momentum fraction $x \rightarrow 0,1$ is important for the hard amplitude, the corresponding large double logarithms $\alpha_s \ln^2 x$ shall appear in the hard amplitude. One can use the threshold resummation \cite{12} to organize this type of double logarithms for all loop diagrams into a jet function, which suppresses the end-point behavior of the hard amplitude. With the Sudakov factor and the jet function, one can evaluate all possible Feynman diagrams for the six-quark amplitude directly, including the nonfactorizable emission diagrams and annihilation type diagrams. But it is difficult to calculate these two kinds of contributions in QCDF approach. The pQCD approach has been used to B meson decays into charmonia such as $B \rightarrow J/\Psi K(\ast)$ in Refs.\cite{6,13}, where the consistent results with the experimental data were obtained.

So we would like to use the pQCD approach to study $B \rightarrow \psi(2S)P, \eta_c(2S)P$ (here $P$ refers to the pseudo-scalar meson $\pi$ or $K$) decays. Except the full leading-order (LO) contributions, the next-to-leading-order (NLO) contributions, which are mainly from the NLO Wilson coefficients and the vertex corrections to the hard kernel, are also included. Certainly, other NLO contributions, such as the quark loops and the magnetic penguin corrections, are available in the literatures \cite{14,15}, but they will not contribute to these considered decays.

We review the LO predictions for the $B \rightarrow \psi(2S)P, \eta_c(2S)P$ decays including those for the main NLO contributions, in Sec. II. We perform the numerical study in Section IV, where the theoretical uncertainties are also considered. Section V is the conclusion.

It is noticed that we will use the abbreviation $\psi$ and $\eta_c$ to denote the mesons $\psi(2S)$ and $\eta_c(2S)$ unless specified in the following.

II. THE LEADING-ORDER PREDICTIONS AND THE MAIN NEXT-TO-LEADING ORDER CORRECTIONS

As previously stated, the pQCD factorization approach has been used to calculate many two-body charmed B meson decays, and has obtained consistent results with experiments. So we use this approach to consider the decays $B \rightarrow \psi(2S)\pi(K), \eta_c(2S)\pi(K)$, and the corresponding effective Hamiltonian can be written as:

$$H_{eff} = \frac{G_F}{\sqrt{2}} \left[ V_{cb}^* V_{cq} (C_1(\mu)O_1^c(\mu) + C_2(\mu)O_2^c(\mu)) - V_{tb}^* V_{tq} \sum_{i=3}^{10} C_i(\mu)O_i(\mu) \right],$$ (8)
where $C_i(\mu)$ are Wilson coefficients at the renormalization scale $\mu$, $q' = d$ ($q' = s$) for $b \to d$ ($b \to s$) transition. $V$ represents for the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, and the four fermion operators $O_i$ are given as:

\begin{align}
O_1^c &= (\bar{q}'_i c_j)_{V-A} (\bar{c}_j b_i)_{V-A}, \\
O_2^c &= (\bar{q}'_i c_i)_{V-A} (\bar{c}_j b_j)_{V-A}, \\
O_3 &= (\bar{q}'_i b_i)_{V-A} (\bar{q}_j q_j)_{V-A}, \\
O_4 &= (\bar{q}'_i b_j)_{V-A} (\bar{q}_j q_i)_{V-A}, \\
O_5 &= (\bar{q}'_i b_j)_{V-A} (\bar{q}_j q_j)_{V+A}, \\
O_6 &= (\bar{q}'_i b_j)_{V-A} (\bar{q}_j q_i)_{V+A}, \\
O_7 &= \frac{3}{2} (\bar{q}'_i b_i)_{V-A} \sum_q e_q(\bar{q}_j q_j)_{V-A}, \\
O_8 &= \frac{3}{2} (\bar{q}'_i b_j)_{V-A} \sum_q e_q(\bar{q}_j q_i)_{V-A}, \\
O_9 &= \frac{3}{2} (\bar{q}'_i b_i)_{V-A} \sum_q e_q(\bar{q}_j q_j)_{V-A}, \\
O_{10} &= \frac{3}{2} (\bar{q}'_i b_j)_{V-A} \sum_q e_q(\bar{q}_j q_i)_{V-A},
\end{align}

with $i, j$ being the color indices. At the leading order, the relevant Feynman diagrams only include the factorizable and non-factorizable emission diagrams, as shown in Fig.1, where we take the decays $B^0 \to \psi(\eta_c)\pi^0$ as examples. If the emission particle is the vector meson $\psi(2S)$, then the amplitude for the factorizable emission diagrams Fig.1(a) and Fig.1(b) can be written as:

\begin{align}
F_{\psi:\psi P}^{V-A} &= 8\pi C_F m_1^4 f_\psi \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1, b_1) \left\{ (1 - r_\psi^2)(1 + (1 - r_\psi^2)x_3) \times \phi_P^A(x_3) - 2x_3 r_p (1 - r_\psi^2)(\phi_P(x_3) + \phi_P^T(x_3)) + r_p ((1 - r_\psi^2)\phi_P^P(x_3) + (1 + r_\psi^2)\phi_P^T(x_3)) \right\} \\
& \quad \alpha_s(t_a) E_c(t_a) h_c(x_1, x_3, b_1, b_3) S_t(x_3) + \alpha_s(t_b) E_c(t_b) h_c(x_3, b_1) S_t(x_1) 2r_p (1 - r_\psi^2 - x_3) \phi_P^P(x_3),
\end{align}

where $r_\psi = m_\psi / m_B$, $r_p = m_B^P / m_B (P = \pi, K)$, the color factor $C_F = 4/3$, and $\phi_P^{A(T)}$ are the twist-2 (twist-3) distribution amplitudes for the meson $\pi$ or $K$. $x_1$ and $x_3$ are the light quark momentum fractions in the $B$ and $P$ mesons, respectively. The evolution factors evolving the Sudakov factors and the jet function $S_t(x)$ are listed as:

\begin{align}
E_c(t) &= \alpha_s(t) \exp[-S_B(t) - S_\pi(t)], \\
S_t(x) &= \frac{2^{1+2c} \Gamma((3/2 + c)}{\sqrt{\pi} \Gamma(1 + c)} [x(1 - x)]^c,
\end{align}
with $c = 0.3$, and $S_B(t), S_\pi(t)$ being the Sudakov factors, which can be found in Ref.\cite{16}. The hard functions $h_e$ is given as:

$$h_e(x_1, x_3, b_1, b_3) = K_0(\sqrt{x_1 x_3 (1 - r_\psi^2)} m_B b_1) \left[ \theta(b_1 - b_3) K_0(\sqrt{x_3 (1 - r_\psi^2)} m_B b_3) + \theta(b_3 - b_1) K_0(\sqrt{x_3 (1 - r_\psi^2)} m_B b_3) \right].$$

The amplitude for the non-factorizable spectator diagrams Fig.1(c) and Fig.1(d) is given as:

$$M_{\psi P}^{V-A} = \frac{32}{\sqrt{6}} \pi C_F m_B^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_2 b_3 \phi_B(x_1, b_1)$$

$$\times \left\{ (r_\psi^2 (2 - r_\psi^2) (x_3 - x_2) - x_3) \phi_P^A(x_3) + 2r_\psi (r_\psi^2 (2x_2 - x_3) + x_3) \phi_P^T(x_3) \right\}$$

$$\times \psi^L(x_2, b_2) + 2 r_\psi \phi_P^P(x_3) \psi(x_2, b_2) \right\} \times \alpha_s(t) E_d(t) h_d(x_1, x_2, x_3, b_1, b_2),$$

where the twist-2(twist-3) distribution amplitudes $\psi^L(x_2, b_2)$ can be found in the next section. $x_2$ is the $c$ quark momentum fraction in $\psi(2S)$ meson. The evolution factor $E_d(t)$ and the hard function $h_d$ are listed as:

$$E_d(t) = \alpha_s(t) \exp[-S_B(t) - S_\pi(t) - S_\psi(t)|_{b_1 = b_2}],$$

$$h_d(x_1, x_2, x_3, b_1, b_2) = \left[ \theta(b_1 - b_2) K_0(\sqrt{x_1 x_3 (1 - r_\psi^2)} m_B b_1) I_0(\sqrt{x_3 (1 - r_\psi^2)} m_B b_2) \right]$$

$$+(b_1 \leftrightarrow b_2) \begin{cases} \frac{K_0(A_d m_B b_2)}{2} H_0^{(1)}(\sqrt{|A_d|^2} m_B b_2) & \text{for } A_d \geq 0 \\ H_0^{(1)}(\sqrt{|A_d|^2} m_B b_2) & \text{for } A_d \leq 0 \end{cases},$$

with the variable $A_d^2 = r_c^2 + (x_1 - x_2)(x_2 r_\psi^2 + x_3 (1 - r_\psi^2))$, and $K_0, I_0$ and $H_0$ being the modified Bessel functions.

If the emission particle is the pseudo-scalar meson $\eta_c(2S)$, then the corresponding amplitude $F_{\eta_c P}^{V-A}$ can be obtained from $F_{\psi P}^{V-A}$ by the replacements of the parameters $f_\psi$ and $r_\psi$ with $f_\eta_c$ and $r_\eta_c$, respectively. While there are many differences between the nonfactorizable spectator amplitudes $M_{\eta_c P}^{V-A}$ and $M_{\psi P}^{V-A}$ because of the different Lorentz structures between the wave functions of $\eta_c(2S)$ and $\psi(2S)$. Here $M_{\eta_c P}^{V-A}$ is listed as following:

$$M_{\eta_c P}^{V-A} = \frac{32}{\sqrt{6}} \pi C_F m_B^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_2 b_3 \phi_B(x_1, b_1)$$

$$\times \left(1 - r_\eta_c^2\right) x_3 \left[ (r_\eta_c^2 - 1) \phi_P^A(x_3) + 2 r_\eta_c \phi_P^T(x_3) \right] \psi(x_2, b_2)$$

$$\times \alpha_s(t) E_d(t) h_d(x_1, x_2, x_3, b_1, b_2),$$

where $r_\eta_c = m_\eta_c / m_B$, and the twist-3 distribution amplitudes of $\eta_c$ meson do not contribute to the amplitude. It is different from the case of $M_{\psi P}^{V-A}$. While the evolution factor $E_d(t)$ and the hard function $h_d$ are similar with those given in Eq.(18) and Eq.(19).
By combining the amplitudes from the different Feynman diagrams, one can get the total decay amplitude for the decays $B \to \psi(\eta_c)\pi$:  
\begin{align*}
M(B \to \psi(\eta_c)\pi) &= F^{V-A}_{\psi(\eta_c)\pi} [V^*_c V_{c\bar{d}} a_2 - V^*_t V_{t\bar{d}} (a_3 + a_5 + a_7 + a_9)] \\
&\quad + M^{V-A}_{\psi(\eta_c)\pi} [V^*_c V_{c\bar{d}} C_2 - V^*_t V_{t\bar{d}} (C_4 - C_6 - C_8 + C_{10})],
\end{align*}
(21)
where $V_{ij}$ is the CKM matrix element and the combinations of Wilson coefficients $a_2 = C_1 + C_2/3, a_5 = C_3 + C_{i+1}/3$ with $i = 3, 5, 7, 9$. The amplitudes $F^{V-A}_{\psi(\eta_c)\pi}$ and $M^{V-A}_{\psi(\eta_c)\pi}$ are given in Eqs. (14) and (17), respectively. For the decays $B \to \psi(\eta_c)K$, the total amplitude can be obtained by replacing $F^{V-A}_{\psi(\eta_c)\pi}, M^{V-A}_{\psi(\eta_c)\pi}, V_{cd}$ and $V_{td}$ with $F^{V-A}_{\psi(\eta_c)K}, M^{V-A}_{\psi(\eta_c)K}, V_{cs}$ and $V_{ts}$, respectively in Eq. (21).

As stated before, the NLO corrections to the hard kernel for the decays $B \to \psi(2S)P$ and $B \to \eta_c(2S)P$ are simpler compared with other B meson decays such as $B \to \pi K, \rho K$. Here only the vertex corrections are need to be considered. Since the vertex corrections can reduce the dependence of the Wilson coefficients on the renormalization scale $\mu$, they play the important roles in the NLO analysis. It is well known that the nonfactorizable contributions are small [17], we concentrate only on the vertex corrections to the factorizable amplitudes, as shown in Fig.2. Furthermore, the infrared divergences from the soft and collinear gluons in these Feynman diagrams can be canceled each other. That is to say, these corrections are free from the end-point singularity in collinear factorization theorem, so we can simply quote the QCDF expressions for the vertex corrections: their effects can be combined into the Wilson coefficients,

\begin{align*}
a_2 &\to a_2 + \frac{\alpha_s C_F}{4\pi N_c} C_2 (-18 + 12 \ln \frac{m_b}{\mu} + f_I), \\
a_i &\to a_i + \frac{\alpha_s C_F}{4\pi N_c} C_{i+1} (-18 + 12 \ln \frac{m_b}{\mu} + f_I), (i = 3, 9), \\
a_i &\to a_i + \frac{\alpha_s C_F}{4\pi N_c} C_{i+1} (6 - 12 \ln \frac{m_b}{\mu} - f_I), (i = 5, 6),
\end{align*}

with the function $f_I$ defined as [18]:

\begin{align*}
f_I &= \frac{2\sqrt{2N_C}}{f_{\psi(\eta_c)}} \int^1_0 dx_2 \psi^{L(\psi)}(x_2) \left[ \frac{3(1 - 2x_2)}{1 - x_2} \ln x_2 - 3\pi i \\
&\quad + 3 \ln (1 - r^2_{\psi(\eta_c)}) + \frac{2r^2_{\psi(\eta_c)} (1 - x_2)}{1 - r^2_{\psi(\eta_c)} x_2} \right],
\end{align*}
(25)

where we have neglected the terms proportional to $r^A_{\psi(\eta_c)}$. Certainly, in the following numerical analysis, the NLO Wilson coefficients will be used in the NLO calculations.

### III. NUMERICAL RESULTS AND DISCUSSIONS

We use the following input parameters for the numerical calculations [1, 19]:

\begin{align*}
f_B &= 210\text{MeV}, f_{\psi(2S)} = 0.296^{+0.008}_{-0.002}\text{GeV}, f_{\eta_c(2S)} = 0.243^{+0.070}_{-0.111}\text{GeV}, \\
M_B &= 5.28\text{GeV}, M_{\psi(2S)} = 3.686\text{GeV}, M_{\eta_c(2S)} = 3.639\text{GeV}, \\
M_{\psi} &= 80.41\text{GeV}, \tau_B^{1\pm} = 1.638 \times 10^{-12}\text{s}, \tau_B^0 = 1.519 \times 10^{-12}\text{s}.
\end{align*}
(26)
Revertizable one contributions. From the numerical results, we find that the contribution (the square of the \( F \) to \( V \)) while the imagine part induced by the vertex corrections is large and about difference from the leading order value is because of using the NLO Wilson coefficient). For the CKM matrix elements, we adopt the Wolfenstein parametrization and the updated values \( A = 0.814, \lambda = 0.22537, \bar{p} = 0.117 \pm 0.021 \) and \( \bar{\eta} = 0.353 \pm 0.013 \). With the total amplitudes, one can write the decay width as:

\[
\Gamma(B \to \psi(\eta_c)P) = \frac{G_F^2}{32\pi m_B} (1 - r_{\psi(\eta_c)}^2)|M(B \to \psi(\eta_c)P)|^2.
\]  

(29)

The wave functions of \( B, \pi \) and \( K \) have been well defined in many works, while those of the two excited charmonia states exist many uncertainties. Here we take the harmonic-oscillator model:  

\[
\psi^{L,v}(x, b) = \frac{f_{2S}}{2\sqrt{2N_c}} N^{L,v} x(1-x) \mathcal{T}(x) e^{-x(1-x) m_c^2/\omega^2} [\omega^2 b^2 + (2x-1)^2]^2,
\]

(30)

\[
\psi^{t}(x, b) = \frac{f_{2S}}{2\sqrt{2N_c}} N^t (2x-1)^2 \mathcal{T}(x) e^{-x(1-x) m_c^2/\omega^2} [\omega^2 b^2 + (2x-1)^2]^2,
\]

(31)

with

\[
\mathcal{T}(x) = 1 - 4b^2 m_c \omega x(1-x) + \frac{m_c(2x-1)^2}{\omega x(1-x)},
\]

(32)

where \( f_{2S} \) refers to the decay constant \( f_{\psi} \) or \( f_{\eta_c} \), the free parameter \( \omega = 0.2 \pm 0.1 \) GeV and the c quark mass \( m_c = 1.275 \pm 0.025 \) GeV. The main errors come from these parameters and \( \omega_b = 0.4 \pm 0.1 \) for \( B \) meson wave function in our calculations.

Using the input parameters and the wave functions as specified in this section, we give the LO and NLO predictions for the considered decays in Table I. For these color-suppressed decays, the amplitudes associated with the Wilson coefficient \( C_1 + C_2/3 \) usually play the dominant roles. It is instructive to check the contributions from these amplitudes at the leading-order and the next-to-leading-order. Here we take the decay \( B^0 \to \psi(2S)K^0 \) as an example, the value of the factorizable color-suppressed amplitude \( F_{\psi K}^{V-A}(a_2) \) is about \(-3.55 \times 10^{-2}\), which will be partly canceled by the real part of nonfactorizable one \( Re(M_{\psi K}^{V-A}(C_2)) \) = \(1.16 \times 10^{-2}\). And the imagine part \( Im(M_{\psi K}^{V-A}) \) is small and only about \(-5.59 \times 10^{-3}\). When the vertex corrections are included, the factorizable color-suppressed amplitude \( F_{\psi K}^{V-A}(a_2) \) becomes a complex number. It’s real part reduces to \(-1.49 \times 10^{-2}\), which will be largely canceled by \( Re(M_{\psi K}^{V-A}(C_2)) \) = \(1.09 \times 10^{-2}\) (the difference from the leading order value is because of using the NLO Wilson coefficient). While the imagine part induced by the vertex corrections is large and about \(-3.42 \times 10^{-2}\). So the total amplitude from the tree operators increases after including the NLO contributions. From the numerical results, we find that the contribution (the square of the

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**FIG. 2:** NLO vertex corrections to the factorizable amplitudes for the decays \( B^0 \to \psi(\eta_c)\pi^0 \).
meson rescattering effects \[21\] in the decays $B \to \psi(\eta_c)K, \psi(\eta_c)\pi$. The first uncertainty comes from the $\omega_b = 0.4 \pm 0.1$ for $B$ meson, the second and the third uncertainties are from the free parameter $\omega = 0.2 \pm 0.1$ and the $c$ quark mass $1.275 \pm 0.025$ GeV for $\psi(2S)/\eta_c(2S)$ meson. The last one comes from the decay constant $f_{\psi(2S)} = 0.296^{+0.003}_{-0.002}$ GeV/$f_{\eta_c(2S)} = 0.243^{+0.079}_{-0.111}$ GeV. The data are listed in the second column [1].

| mode         | data     | LO         | NLO         |
|--------------|----------|------------|-------------|
| $B^+ \to \psi(2S)K^+(\times 10^{-4})$ | $6.27 \pm 0.24$ | $2.39 +1.11+0.18+0.04+0.05$ | $5.37 +1.61+0.90+0.04+0.11$ |
| $B^0 \to \psi(2S)K^0(\times 10^{-4})$ | $6.2 \pm 0.5$ | $2.22 +1.00+0.16+0.03+0.04$ | $4.98 +1.39+0.84+0.04+0.10$ |
| $B^+ \to \psi(2S)\pi^+(\times 10^{-5})$ | $2.44 \pm 0.30$ | $0.47 +0.24+0.04+0.01+0.01$ | $1.17 +0.36+0.22+0.02+0.03$ |
| $B^0 \to \psi(2S)\pi^0(\times 10^{-5})$ | $1.17 \pm 0.17 \pm 0.08$ | $0.22 +0.11+0.03+0.01+0.02$ | $0.54 +0.17+0.10+0.01+0.02$ |
| $B^+ \to \eta_c(2S)K^+(\times 10^{-4})$ | $3.4 \pm 1.8$ | $2.33 +1.02+0.23+0.02+1.76$ | $3.54 +1.23+1.18+0.05+2.68$ |
| $B^0 \to \eta_c(2S)K^0(\times 10^{-4})$ | $2.16 +0.95+0.21+0.02+1.63$ | $0.53 +0.46+0.16+0.02+1.52$ | $0.68 +0.87+0.62+0.42+2.49$ |
| $B^+ \to \eta_c(2S)\pi^+(\times 10^{-6})$ | $5.82 +2.51+0.70+0.06+4.40$ | $9.03 +3.05+0.38+0.11+6.82$ |
| $B^0 \to \eta_c(2S)\pi^0(\times 10^{-6})$ | $2.72 +1.18+0.33+0.03+2.06$ | $4.19 +1.41+1.43+0.06+3.16$ |

амплитуды) из пингвиновых операторов является очень малым в сравнении с тем, что отсутствует в этих распадах. Из наших результатов, мы можем найти, что вклад распадов $B \to \psi(2S)K, \eta_c(2S)K$ является большим, чем у тех распадов $B \to \psi(2S)\pi, \eta_c(2S)\pi$ при одном и том же порядке. Это происходит благодаря тому, что CKM ограничение фактора $\lambda = 0.22$ в пределах. Как было указано в работе [4], что слабая глюонная корреляция подавлена фактором $\Lambda_{QCD}/(m_b\alpha_s)$ вместо $\Lambda_{QCD}/m_b$ в этом распаде, поэтому в пределах и в дрессировке и в высоких порядках можно считать, что приблизительно равны. Наши результаты, показанные в Таблице 1, поддерживают этот аргумент: вклад NLO может дать (52 ± 55)% увеличение для распадов $\eta_c(2S)$ мезон. И увеличение для распадов $\psi(2S)$ мезон является еще большим. Кроме того, слабый мезон рециркуляция эффекты [21] в распадах $B \to \psi(2S)K, \eta_c(2S)K$ может быть не очень важным. И вложение вклад распадов $B^+ \to \psi(2S)\pi^+, B^0 \to \psi(2S)\pi^0$ являются меньше, чем данные даже с вершинами и NLO коэффициенты включены. Также возможны другие более высокие порядки вклады или вклады от Глаубер глюонов в спектральных диаграмах играют важные роли.

В следующем мы обсудим CP-нарушения асимметрии $B \to \psi(\eta_c)\pi, \psi(\eta_c)K$ распады. Для заряженных распадов $B^+ \to \psi(\eta_c)K^+$, нет фаз существования в их распадах амплитуды, так что распады NLO амплитуды этих распадов равны, и это согласуется с экспериментальным значением $A_{CP}(B^+ \to \psi(2S)K^+) = (-2.4 \pm 2.3)\%$ с погрешностью 1σ. Прямые распады CP амплитуды заряженного распада $B^+ \to \psi(2S)\pi^+$, предсказанные pQCD приближение, представлены как:

$$A_{CP}(B^+ \to \psi(2S)\pi^+) = (2.16^{+0.63+0.54+0.19+0.01}_{-0.55-1.03-0.29-0.01})\%,$$ \text{LO} \quad (33)

$$A_{CP}(B^+ \to \psi(2S)\pi^+) = (0.51^{+0.08+0.00+0.01+0.01}_{-0.13-0.08-0.00-0.00})\%,$$ \text{NLO} \quad (34)
As for the neutral decay channels, the effects of $B^0$ and $\psi(2s)$ are considered. The direct and mixing induced CP violating asymmetries are defined as:

$$A_{CP}^{dir} = \frac{|\lambda_{CP}|^2 - 1}{1 + |\lambda_{CP}|^2}, \quad A_{CP}^{mix} = \frac{2Im(\lambda_{CP})}{1 + |\lambda_{CP}|^2},$$

(35)

where the CP-violating parameter $\lambda_{CP}$ is

$$\lambda_{CP} = \eta_f \frac{V^*_{tb}V_{td} \langle f|H_{eff}|B^0 \rangle}{V^*_{tb}V^*_{td} \langle f|H_{eff}|B^0 \rangle} = \eta_f e^{-2i\alpha(\beta)} \frac{\langle f|H_{eff}|B^0 \rangle}{\langle f|H_{eff}|B^0 \rangle},$$

(36)

for $b \to d (b \to s)$ transition and $\eta_f$ is the CP-eigenvalue of the final states. From the left panel in Fig.3 one can find that the dependence of the mixing CP-asymmetry $A_{CP}^{mix}(B^0 \to \psi(2s)\pi^0)$ on the CKM weak phase $\alpha$. If taking the CKM weak phase $\alpha = (85.4^{+3.9}_{-3.8})^\circ \ [1]$, we find that the value of $A_{CP}^{mix}(B^0 \to \psi(2s)\pi^0)$ is $(31.3^{+26.4}_{-22.3})\%$. The mixing induced CP violating asymmetry for the channel $B^0 \to \psi(2s)\pi^0$ is very sensitive to the angle $\alpha$. As to another CKM weak phase $\beta$, there are much more uncertainties: by including earlier $\sin(2\beta)$ measurements $[25]$ and recent results from LHCb $[26]$ and Belle $[27]$, the Heavy Flavor Averaging Group (HFAVG) gives two possible solutions $2\beta = (43.8^{+1.4}_{-1.4})^\circ$ and $2\beta = (136.2^{+1.4}_{-1.4})^\circ [28]$, which is very consistent with our predictions shown in the right panel of Fig.3. If the assumption that $A_{CP}^{dir} = 0$ is relaxed, then $A_{CP}^{mix} = -\eta_f \sqrt{1 - A_{CP}^{dir}} \sin 2\beta$. There are similar results between the decays $B^0 \to \psi(2S)K_S^0(\pi^0)$ and $B^0 \to \eta_c(2S)K_S^0(\pi^0)$ about the mixing induced CP violating asymmetries. As for the direct CP asymmetries of the decays $B^0 \to \eta_c(2S)\pi^0$ and $B^+ \to \eta_c(2S)\pi^+$, they are very small and at $10^{-5}$ order.

FIG. 3: The mixing induced CP violating asymmetry $A_{CP}^{mix}$ and the parameter $A_{CP}^{mix} - \sin(2\beta)$ for the decays $B^0 \to \psi(2s)\pi^0$ (the left) and $B^0 \to \psi(2s)K_S^0$ (the right), respectively.
We study the B meson decays $B \to \psi(2S)K(\pi), \eta_c(2S)K(\pi)$ within the pQCD approach, where the radially excited charmonia states are involved. With the wave functions of these two mesons $\psi(2S)$ and $\eta_c(2S)$ derived from the harmonic-oscillator-model, we find that the branching ratios for the decays $B^+ \to \psi(2S)K^+, \eta_c(2S)K^+$ and $B^0 \to \psi(2S)K^0$ can agree well with the data within errors after including the NLO corrections. While there is still some room left for other high order corrections or the non-perturbative long distance contributions for the decays $B^+ \to \psi(2S)\pi^+$ and $B^0 \to \psi(2S)\pi^0$. The pQCD predictions for the direct CP-violating asymmetries support the present experimental opinion, that is no evidence of direct CP violation being observed in these decays. If a few percent value is confirmed in the future, it would indicate new physics definitely.

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