Hopping Conductivity Enhanced by Microwave Radiation

Z. Ovadyahu
The Racah Institute of Physics
The Hebrew University, Jerusalem 91904, Israel
E-mail: zvi@vms.huji.ac.il

Abstract. Hopping conductivity is enhanced when exposed to microwave (MW) fields. Data taken on several Anderson-localized systems and granular-aluminium are presented to illustrate the generality of the phenomenon. It is suggested that the effect is due to a field-enhanced hopping, which is the ac version of a non-ohmic effect familiar from studies in the dc transport regime.

1. Introduction
Non-ohmic effects are commonly encountered in hopping conductivity. Deviations from linear transport conditions in these systems are actually difficult to completely avoid. Linear response conditions are expected when the applied field \( F \) obeys:

\[
F \leq \frac{k_B T}{e L}
\]

Here, \( k_B \) is Boltzmann-constant; \( e \) is the electron charge, and \( T \) the temperature. \( L \) is the spatial scale over which the electron gains energy from the field \( F \) before it is dissipated into the bath, usually by phonon emission, thus \( L=L(T) \). As the temperature gets lower \( L \) becomes larger, and maintaining Ohmic conditions by is possible only by reducing the potential drop across the sample. The need to ensure good signal-to-noise makes this harder at low temperatures.

Theoretical models to describe the modification to the conductance by a non-ohmic dc field predict an excess conductance \( \Delta G \) that, at small fields, is exponential with \( e F L / k_B T \).

The effect of electromagnetic waves on hopping conductivity has been given much less attention. Of particular interest is the case of microwave (MW) radiation where the wavelength \( \lambda \) is larger than a typical sample-size [4]. When exposed to MW fields, hopping conductance is enhanced which, perhaps naturally, was associated with heating [5]. Later studies, exploiting some unique properties of electron-glasses, suggested however that this is an adiabatic, non-equilibrium effect [6]. A characteristic feature of the results, which makes it difficult to reconcile the effect with simple heating, is the sub-linear dependence of \( \Delta G \) on the microwaves power \( P \).

In this paper we report on MW-excitation results obtained on films of different hopping systems; beryllium, granular-aluminum, In\(_2\)O\(_{3-x}\) (crystalline indium-oxide), and In\(_x\)O (amorphous indium-oxide). In all cases the \( \Delta G \) at \( T\approx4K \) turn out to be sub-linear with the MW power in agreement with the results of previous studies. It is further shown that the functional dependence of \( \Delta G(P) \) is consistent with the current-voltage characteristics measured independently (at low frequency) on the
same sample. The MW-enhanced conductance is then conjectured to be a non-ohmic effect, of a similar nature as the field assisted-hopping phenomenon that is responsible to deviations from linear transport. The temperature dependence of the MW-induced $\Delta G$ is shown to be consistent with this conjecture.

2. Experimental
Several materials were used to prepare samples for measurements in this study. These were thin films of $\text{In}_2\text{O}_3$, $\text{In}_x\text{O}$, and granular-aluminum. Lateral size of the samples was typically 1mm, and thickness varied between 50 to 210 Å. Samples from these materials were similarly prepared by e-gun evaporation of $\text{In}_2\text{O}_3$ or aluminum pellets unto room temperature glass substrates in partial oxygen pressures of 4-6 $\times 10^{-5}$ mbar and rates of 0.3-1 Å/second. The $\text{In}_2\text{O}_3$ samples were obtained from the as-deposited amorphous films by crystallization at 525 K.

Conductivity of the samples was measured using a two terminal ac technique employing a 1211-ITHACO current preamplifier and a PAR-124A lock-in amplifier. The ac voltage bias was small enough to ensure near-ohmic conditions (except for the I-V measurements). Unless otherwise mentioned, measurements reported here were performed at $T \approx 4.1$ K. Complementary details of sample preparation, characterization, and measurements techniques are given elsewhere [7].

The high-power synthesizer HP8360B, was used with power up to 25dBm (~320 mW) for the MW excitation. The frequency range used in this work was 2-4 GHz. The output of the synthesizer was fed to the sample chamber via a coaxial cable ending with a short antenna.

3. Results and Discussion
The single most characteristic feature of the MW-enhancement phenomenon is the functional form of the excess conductance $\Delta G$ versus the radiation power $P$. In more than 120 samples studied in our laboratory $\Delta G$ increased with $P$ less than linearly. Typical results of $\Delta G(P)$ scans, illustrating the sub-linear form of the response, are shown in figure 1:

![Figure 1: The fractional change of the conductance for three different electron-glasses as function of the MW power. Data taken at $T=4.1$ K](image)

While we cannot rule out linearity at sufficiently small $P$, it is noted that $\Delta G(P)$ is sub-linear even for $\Delta G/G \leq 10^{-2}$. This was the first indication that the origin of the effect is not likely to be ‘heating’ [6]. As more data became available, a correlation was found between the sensitivity of $G$ to the voltage used in the conductance measurement, and the magnitude of $\Delta G$ produced by a given power of MW; in particular, samples that showed ohmic behavior up to high voltages, exhibited very small MW-induced $\Delta G$. It was then natural to look for a correlation in the functional form of the two effects, which turned out to be the key feature in the search for the underlying mechanism.
Figure 2 illustrates that $\Delta G(P^{1/2})$ is indeed similar to the $G(V)$ derived from the I-V characteristics of the sample. This similarity was observed in other samples. We are led therefore to consider the following heuristic picture: The MW radiation induces field across the sample (picked-up by the wires connected to the sample for the two-terminal conductance measurement). Once the potential difference associated with this field is greater than the voltage employed for measuring $G$, the conductance of the sample is modified in a similar vein as in the process of measuring $G(V)$ directly (by dc or low frequency). In other words, the conductance monitored by a low-bias, low-frequency technique reflects the reduction of the hopping barriers induced by the high-bias associated with at the MW frequency. In this picture the reason for the sub-linearity of $\Delta G(P)$ is just the less-than-quadratic dependence of the excess conductance on an applied field. This mechanism is plausible when the wave-length $\lambda$ is larger than the sample size which clearly is the case here ($\lambda\approx10$-30 cm for the MW frequencies used while sample size is $\approx1$ mm).

Figure 2: The fractional change of the conductance for (a) a typical granular-aluminium sample as function of the MW power, and (b) the same sample as function of the applied voltage (measured at $f=73$ Hz), and compared with $\Delta G(P^{1/2})/G$ based on the MW data by scaling the x-axis of $\Delta G(P^{1/2})$ by a constant factor of $\approx65$. The black dashed line depicts $exp(F)$ dependence, which is shown to fits only a narrow range near the onset of non-ohmicity (see text).

Figure 3: Temperature dependence of the MW enhanced conductance made on a typical amorphous indium-oxide film under a constant power of 25dBm. Note the shift of the $\Delta G(f)$ peak positions with temperature. This made it necessary to record a range of frequencies. The value for $\Delta G$ used in the plot of figure 4 was obtained by averaging over the range monitored here.
Another set of data that are in line with the conjecture that the MW-enhanced conductance is a non-ohmic effect, is the temperature dependence of the excess conductance (at constant MW power). In these experiments, a range of frequencies was scanned at each temperature. This was done to cater for drifts in the cavity characteristics with the concomitant shift in conductance-response peaks (see figure 3 below). Figure 4 shows $\Delta G(T)$ due to the MW radiation normalized by $G(T)$ at zero field. This was obtained from the data of figure 3 in conjunction with the resistance versus temperature of this sample (inset to figure 4).

The conductance for fields that just take the system away from the ohmic regime is given [1,2,3] by $G=G_0 \exp[eFL(T)/k_BT]$. For small $\Delta G/G$ this yields $\Delta G/G \approx eFL(T)/k_BT$. It has been suggested that $L(T)$ is the hopping length [3] or the percolation radius [2]. The fit to the data in figure 4 gives $\Delta G/G \propto T^{-1.81}$. A power-law dependence is actually in agreement with either of these theoretical expectation: considering the hopping-law exhibited by the sample in figure 4, and if $L(T)$ is the hopping length, one expects $\Delta G/G \propto T^{-1.5}$ while $\Delta G/G \propto T^{-2}$ if $L(T)$ is the percolation radius. It would be interesting to see a similar set of data on other systems that exhibits hopping conductivity as it might help to distinguish between the different theoretical approaches.

![Figure 4: The fractional change of the conductance versus temperature for the sample in figure 4 (see text for details). The inset shows the temperature dependence of the sample resistance and some of its hopping parameters extracted on basis of $R_f=R(0) \exp(T_0/T)^{1/2}$.](image)

While the temperature dependence is apparently not at variance with our picture, the magnitude of the effect appears to be problematic: By conjecture $\Delta G/G$ is a functional of $eFL(T)/k_BT$, and to obtain $\Delta G/G$ of order of 0.1-1 (which is the common range at the maximum power at 4 K), this functional has to be of order 1. For a typical field of 1-10 V/cm, it would appear that an unreasonably large $L$ is required.

This problem is not specific to the MW effect; it is a problem of non-ohmicity in general. This, in fact, is a common occurrence in hopping conductivity: Values of $L$ that are larger by 1-3 orders of magnitude than reasonably expected are reported in the literature [8]. These include results for samples doped by nuclear-transmutation [9] (presumed to be free of technical inhomogeneities). This is also the case in all our samples. For example, deviations from Ohm’s law for the sample in figure 2 are already evident at $V\leq 50$ mV, which in terms of eq.1 yield $L\approx 10\ \mu m$. This is clearly much too large a scale for a hopping sample at 4 K. It should be remarked that the origin of the discrepancy may not be that of $L$ but rather a result of inhomogeneous field distribution. There seems to be some evidence for the latter in studies of non-ohmic behavior as function of sample-size [10].
For demonstrating the similarity between $G(V)$ and the MW-induced $\Delta G$ we have used the measured $G(V)$ rather than the theoretical [1,2,3] expression. These models, predicting $\ln(G/G_0) \propto F$ (which incidentally, depict $\Delta G$ as sub-linear with $P$), might show agreement with our experiments (and indeed with other experiments) only over a very limited range. In [9] for example, a fit with the theory may be obtained only for $\Delta G/G \leq 0.08$, which is unconvincing for an exponential dependence. This is also the range over which this expression may fit our data while it progressively overestimates them for $\Delta G/G \geq 0.1$ (see figure 2b above). The model of Apsley and Hughes [11] which predicts $\ln(G/G_0) \propto F^2$ may be valid near the very onset for non-ohmicity (where, for symmetry reasons, the quadratic dependence is expected), but it is even further from the experimental dependence over the relevant range. It is noted that the lack of agreement with theory is not related to the use of ac technique in the experiments (by Grannan et al [9] and in our experiment) while the theories refer to a dc measurement; It turns out that the functional form of $G(V)$ measured by dc is essentially similar to that measured by a low frequency ac technique [12].

There are two additional issues that should be clarified. The first concerns the mapping of $\Delta G(P^{1/2})$ onto $G(V)$ by rescaling the voltage axis. The procedure, exemplified above in figure 2, was tried on many samples with similar success. However this procedure never gave a perfect registry of the two curves. However, it is not clear that the two measurements should fit better than they do; The $G(V)$ measured at different frequencies will, in general, be different. The reason is that the Miller-Abrahams resistors that make up the hopping system are actually impedances with value that depend on frequency. As a result, the local potential drops across the sample will in general be different at different frequencies.

The second issue is more subtle. Applying a low-frequency, non-ohmic field across any of the samples described here does more than just increase their conductance. When the field is allowed to act on the sample for a certain time the conductance is not stationary; it slowly increases with time, and after the field is set back to achieve linear-response, the conductance does not immediately resume its original equilibrium value. Rather, it is initially higher by a certain $\Delta G$, which slowly relaxes to zero [13]. By contrast, when the MW is applied with amplitude such that the resulting $\Delta G$ is a large as achieved by the respective low-frequency field, the conductance does not show these glassy features [6]. The lack of these non-equilibrium features when the sample is “stressed” by MW is illustrated in figure 5 for three different materials:

Figure 5: Response of three samples to MW exposure of different power (marked in dBm over time segments when MW is turned on). Note that, after the initial jump, the conductance does not keep on increasing while the MW is on, and it promptly resumes its ‘equilibrium’ value when the MW is
turned off even at the highest power. Data taken at T = 4.1 K, at f = 2.530, 2.241, and 2.401 GHz for (a), (b), and (c) respectively.

In contrast with the similarity in terms of enhancing the hopping conductance, there is evidently a fundamental difference between the MW and the low-frequency field mechanisms. This difference is presumably associated with the ability of the electronic system to absorb energy from the field; the slow relaxation of $\Delta G$ that follows exposure to the field is the telling sign of stored excess energy. The latter is accumulated during the time the field is on and this is indicated by the slow build-up of excess conductance. Note that, by itself, a conductance increase does not necessarily imply energy absorption as the process may be adiabatic. Specific experiments to elucidate this issue by using fields with different frequencies will be presented elsewhere.

Acknowledgments
Illuminating discussions with A. Frydman and A. Goldman are gratefully acknowledged. I am indebted to M. Pollak and B. Shapiro for useful comments and critical remarks. This work was supported by a grant administered by the US-Israel Science Foundation and by the Israeli Academy of Sciences.

References
[1] Hill R M, 1971, Phil. Mag. 24 1307
[2] Shklovskii B I, 1976 Fiz. Tech. Polouprovodn. 10 1440 [Sov. Phys. Semicond. 10 855]
[3] Pollak M and Riess I, 1976 J. Phys. C 9 2339
[4] Zvyagin I P, 1978 Phys. Stat. Solidi 88 149
[5] Ben-Chorin M, Ovadyahu Z, and Pollak M, 1993 Phys. Rev. B 48 15025
[6] Ovadyahu Z, Phys. Rev. Lett. 2009 102 206601
[7] Vaknin A, Ovadyahu Z, and Pollak M, 2002 Phys. Rev. B 65, 134208
[8] Rosenbaum T F, Andres K, and Thomas G A, 1980 Solid State Commun. 35 663
Faran O, and Ovadyahu Z, 1988 Solid State Comm., 67, 823
[9] Grannan S M, Lang A E, Haller E E, and Beeman J W, 1992 Phys. Rev. B 45 4516
[10] Kowal D and Ovadyhau Z, 2008 Physica C, 468, 322
[11] Apsley N and Hughes H P, 1974 Philos. Mag. 30, 963
[12] Ovadyahu Z (unpublished)
[13] Orlyanchik V, Vaknin A, and Ovadyahu Z, 2002 Phys. Stat. Sol., b230, 67
Orlyanchik V, and Ovadyahu Z, 2004 Phys. Rev. Lett., 92, 066801