Using the Maple computer algebra system to study mathematical induction

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Abstract. Mathematical induction is one of the main methods of the proof that students study and use in their studies in higher education. This important method, in addition to mathematics, is widely used in computer science and a number of other related disciplines. However, even if the principles of proof using the method of induction are taught, understood and mastered by students well, many students have great difficulty with algebraic transformations necessary for proving, for example, finding the inductive step. The use of the Maple computer algebra system allows students to overcome this obstacle and, more attention and effort to pay to understanding the concept of proofs using the method of mathematical induction. In addition, students can prove more complex algebraic statements, and their activities can be of a pronounced research nature.

1. Introduction

Today, the main distinguishing feature of the modern stage of development of education is its informatization and digitalization. The use of infocommunication technologies (ICT) in education requires a revision of the teaching methods of all academic disciplines, including various disciplines of mathematical orientation, in particular, discrete mathematics. It is also necessary to address the issue of studying the possibilities of various software (its choice) for the effective use of ICT elements in teaching both students and schoolchildren.

The use of computer algebra systems (CAS) in teaching mathematics sets its own kind of logic for mastering a subject, learning to become more professionally oriented, and also makes it possible to introduce a research character into their study. In addition, the use of CAS leads to an increase in motivation for learning and reveals the features of the use of information technology in teaching, as well as the need to use the capabilities of CAS in the study of mathematics is associated with an increase in the requirements imposed by society on the level of mathematical training of graduates. This circumstance is explained by the wide possibilities of the practical application of mathematics.

There are several popular technology tools that are widely used in mathematics education today. Thus, a number of universal packages of symbolic transformations MATHEMATICA, MATLAB or MAPLE are available for mathematics courses and can be used both in educational activities and for scientific research [1].
Some of these packages allow students to achieve a high level of logical and analytical reasoning by visually justifying the hypotheses put forward and further viewing the evidence using a graphical or visual presentation of the results, for example, [2]. It is possible to use CAS to measure and evaluate student performance, including in the study of mathematical concepts. For example, CAC Maple was used to teach integration, double integration [3], number theory [4, 5, 6], construction of complex graphs [7], and mathematical analysis in applied sciences [8]. CAS is also used in teaching other subjects such as physics, biology, and chemistry [9] and computing [10, 11].

At the same time, mathematics is primarily a demonstrative science, and teaching mathematics is a systematic inducement of students to discover their own proofs, and one of the most frequently used proof methods in mathematics, and discrete mathematics in particular, is the method of mathematical induction (MMI). The issues of using this method were considered by the authors in [12,13]. However, the first author uses CAS Maxima as a technological tool for solving the set problem, and in both cases the principle of visibility and the possibility of organizing the research activities of trainees is not fully ensured.

The authors propose to use CAS Maple as a support for the creation of several user-friendly Maplets, which will become an effective didactic tool to help students and teachers in the study of the principle of mathematical induction and will help to visually implement the solution of problems: proving equalities, inequalities and solving problems on divisibility and multiplicity [14].

2. Materials and methods

2.1. Principle of mathematical induction

It is known that if some statement \( A(n) \), depending on the natural parameter \( n \), is true for \( n = 1 \) and for any \( n \) the statement “if \( A(n) \) is true, then \( A(n+1) \) is true”, then statement \( A(n) \) is true for any \( n \). The statement “if \( A(n) \) is true, then \( A(n+1) \) is true” is called an inductive transition.

The proof using the method of mathematical induction is carried out in two stages: Induction base (induction basis, BI): We have \( n = 1 \), i.e. \( A(1) \) is true. Inductive transition (induction step, SHI):

We prove that \( A(k) \Rightarrow A(k+1) \).

Let \( k \) be any natural number and let the statement be true for \( n = k \), i.e. \( 1+3+5+\ldots+(2k-1)=k^2 \). Let us prove that then the statement is also true for the next natural number \( n = k+1 \), that is, that \( 1+3+5+\ldots+(2k-1)+(2k+1)=(k+1)^2 \).

Indeed, \( 1+3+5+\ldots+(2k-1)+(2k+1)=k^2+2k+1=(k+1)^2 \).

Thus, \( A(k) \Rightarrow A(k+1) \). Based on the principle of mathematical induction, we conclude that assumption \( A(n) \) is true for any \( n \in N \).

Most often it is necessary to solve or investigate the following problems using the method of mathematical induction: proof of equalities; proof of inequalities; multiplicity and divisibility problems [15].

2.2. Maplets for proving equalities

To create an interactive application that allows you to demonstrate the principle of mathematical induction in solving equalities, the Maplet package is used - one of the most effective ways to develop an ergonomic graphical user interface (GUI).

The values of the left and right sides of the equality are entered separately. To perform and analyze the induction basis, buttons are provided to find it. Next, the second step is performed to analyze and prove the induction step, for which input windows and buttons for outputting the obtained values are also used. The final stage: obtaining an automated conclusion about the legitimacy or inappropriateness of the hypothesis.

Thus, Maplets “Mathematical induction. Equality” allows students to visually perform actions on the application of the principle of MMI to prove equality, put forward hypotheses and conduct research.
Students (learners) can use the information on the order of using Maplets and the course of solving the problem on using the method of mathematical induction to prove equality.

This application can balance the development of understanding of the material being presented and the development of proof skills. Figure 1 shows an example of a proof of equality: $1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

Figure 1. Proof of Equality $1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

2.3. Investigating polynomial interpolation

This Maplets is based on the CAS Maple function - interp (it is also possible to use the function of the CurveFitting package - PolynomialInterpolation). This function allows you to switch from existing values to their representation as a polynomial. The available windows for entering a sequence of values allows you to find a polynomial describing (interpolating) the entered values. The completion of the search (research) process is the presence of a repeating polynomial value.

Thus, Maplets Series Exploration allows students to imagine the process of moving (if possible) from a dataset to a polynomial describing the nature of the relationship of the input data.

Students (learners) can use the information on how to use Maplets and the course of exploring the values of the series. And this application can also balance the development of understanding of the material being presented and the development of proof skills. Figure 2 shows the interpolation (stepwise obtaining) of the right side of the equality as an example of the study: $1+2+\ldots+n = \frac{n(n+1)}{2}$.

Figure 2. Getting a polynomial step by step $1+2+\ldots+n = \frac{n(n+1)}{2}$. 
2.4. Maplets for solving divisibility and multiplicity problems

The monitored value is entered as a function. The steps to be performed are provided by the algorithm for finding divisibility and multiplicity using the method of mathematical induction. For clarity of presentation of the result of the solution, and since in some cases it is necessary to apply the MMI twice, additional windows and command buttons are provided for obtaining the result.

Thus, Maplets “Mathematical induction. division” allows students to visually perform actions on the application of the MMI principle to prove divisibility and multiplicity of the established value, to put forward hypotheses and conduct their research.

In addition, the Maplets structure provides help in getting to know the order of its use, and the ability to obtain information while solving the problem using the method of mathematical induction in the proof of multiplicity and divisibility.

This application can help you solve divisibility and multiplicity problems.

Figure 3 shows the proof as an example: the expression: \( n^3 + 6n^2 + 29n \) is a multiple of 6.

![Maplets for illustration: expression: \( n^3 + 6n^2 + 29n \) is a multiple of 6.](image)

2.5. Maplets for solving inequality proof problems

The Maplets framework allows you to investigate the hypothesis that the inequality holds. The values are entered separately. The structure corresponds to the algorithm used to prove the inequalities. There is a check of the correctness of the input of the specified functions (representation of the input values in an algebraic form). At the final stage, an automated conclusion is made about the legitimacy or inappropriateness of the hypothesis put forward.

Thus, Maplets “Mathematical induction. inequality” allows students to visually perform actions on the application of the principle of MMI to prove inequalities, put forward various hypotheses and conduct their research.

Students (learners) can also use the information on the order of using Maplets and the progress of solving the problem on using the method of mathematical induction to prove inequality.

And this application can balance the development of understanding of the material being presented and the development of proof skills. Figure 4 shows an example of a proof of the inequality: \( 1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^n} = \sum_{i=0}^{n} \frac{1}{2^i} < 2 \).
3. Conclusion
The MAPLE Computer Algebra System (CAS) can be used as an assistant to perform transformations and proofs using the method of mathematical induction. It was suggested that this system will be useful for schoolchildren, undergraduate and graduate students in the field of mathematics, engineering and physics, as it allows you to perform complex transformations on symbolic data. To perform (demonstrate) proofs using the method of mathematical induction, the following results were obtained: The presented Maplets can be used to conduct experiments with interpolation of a set of given values, to prove and conduct experiments using the method of mathematical induction for identities, inequalities, solving problems of divisibility and multiplicity of numbers.

The use of the MAPLE computer algebra system allows students to spend more time studying and applying the basic properties and principles of the method of mathematical induction. In addition, the capabilities of the Maple computer algebra system allow for visual experiments in mathematics.

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