Improving the Parameters of the Model of the Wheeled Mobile Robot with Constraints by Adaptive Fuzzy Method

Shiqi Wang and Songyi Dian*

College of Electrical Engineering and Information Technology, Sichuan University, Chengdu 610065, China

*scudiansy@scu.edu.cn(Songyi Dian)

Abstract. In the process of motion of the ground, the mobile trolley has branches of motion in the direction of the axis of X, Y and Z. However, considering that the mobile robot only has the ability of adjustment of motion on the axis of X and Y, it is said that the mode of the motion of this kind of mobile robots with constraints is not complete. The method of control used in the past is the constraint method of predictive control. The whole mobile robot can be considered as a system including three inputs and three outputs. The motion of the mobile robot in the direction of the axis of Z can be considered as a kind of disturbance from the inputs with a certain effect on the axis of X and Y, but the control of the output is converted to the three parts of control used in the motion on the axis of X and Y and the control of the input of disturbance. The three systems with one independent input and one independent output are considered as the predictive control on the system including three inputs and three outputs. In this paper, the parameters of the predictive model are optimized in the way similar to an adaptive fuzzy control and based on errors and change of errors.

1. Introduction

In the face of a system of motion of the wheeled mobile robot (WMR) with constraints with multiple inputs and multiple outputs, [1] in order to ensure the robustness and accuracy of tracking, the method of traditional predictive control often needs to find a large number of parameters of the state. This method reduces the anti-interference ability of the prediction model, [2] and because the model is built complex, the structure of the model is complicated and the cost is expensive.

In order to ensure the stability of the system, the adaptive fuzzy method is designed to set the parameters of the model, and the stability of the adaptive adjustment with initial experience is guaranteed by the Lyapunov rule, which can be widely used to meet the condition of the equation of the motion. [3] The stability of control and optimization of parameters are realized in the system.

The method of robust predictive control is introduced including the theoretical basis and the related simulation in references [4] and [5]. The errors and change of errors are used as the reference for the prediction of the initial value of each step of the prediction model. At the same time, the method of robust predictive control with the traditional constraint is mentioned in the reference [6]. The study of the class of problems involving a lot of tuning parameters, is the usual systems of this method. The desired tracking effect is guaranteed when the required environment for control is ideal. Excessive precision leads to the decrease of the anti-interference ability of the whole model. And the stick model is unhelpful to the tuning of parameters in the actual control. With this consideration, there is the present idea of this paper because the unsuitable parameters can cause the system to producing huge
delay and may lead to instability of system. In the tuning of parameters, I refer to the related theory of fuzzy adaptive control [7], [8] hoping parameters of the predictive model be adjusted in real time, the adjustment of the parameters is based on the size of the interference, the errors and the change of errors of the expected values of three systems compared to the values of the output at the previous time, and the adaptive adjustment of the parameters without the initial experience is often unable to get the desired results.

The fuzzy rule is a set where different initial fuzzy rules are selected for the disturbances of different level, and each initial model is always used. The present rule is determined by the errors and the change of the errors of the predictive model. By adjusting the parameters of the fuzzy rules according to tracking, the most suitable fuzzy rules are determined. The specific adaptive law is established by the lyapunov function related to the errors of the single-output and single-output model and the change of errors related to the errors. The adaptive law that needs to be used is the self-adjustment with the initial experience. This adjustment, comparing to the adaptive setting without initial experience, reduces the number of trial and errors in the actual movement, and has the more rapid tracking effect, and the accuracy of the motion is unchanged. Learning from the references [9] and [10] is the extension of this method.

The structure of the paper, in addition to the first part about the introduction, includes these parts as follows. The second part will be the traditional constraint predictive scheme for the specific model of movement of vehicle and the system of complex control. The third part will give an adaptive fuzzy control of simple sine single tracking on the motion of the specific model. The forth part will compare the adaptive tuning of parameters on the predictive control with the robust prediction scheme without initial experience. The simulation gives the result of the compare. Finally, it is the fifth part of the conclusion of the work.

2. Dynamic model of WMR

2.1. Dynamic error model with constraints on the WMR

In the complex system, the model of WMR is as shown in figure 1.

![Figure 1. The model of the WMR without the ability of changing the motion on the axis of Z](image1)

The robust controller needs to firstly obtain the full dynamic error model of the WMR. The robust tracking controller is designed by using the rolling time domain and linear matrix inequalities. When the robot has uncertain parameters in the dynamic model, the position and angle of the direction and speed are gradually tracked, as long as the next one is known in advance. When the desired position is needed, the predictive model parameters for control can be adjusted according to the deviation between the current position and the predictive position. It can realize the rolling optimization of the predictive model, decomposing the motion of the next motion on the axis of Z to the two parts only including the motion happening on the axis X and Y as shown in figure 2.

![Figure 2. The motion on the axis of Z is decomposed into X and Y](image2)
In the formula (1), model of motion is decomposed into three parts on X-Y plane has been shown.
\[
x^* = \frac{\dot{x}}{m} \sin \phi + \beta u \cos \phi
\]
\[
y^* = \frac{\dot{y}}{m} \cos \phi + \beta u \sin \phi
\]
\[
\phi^* = \alpha u_2
\]
\[
x' \sin \phi - y' \cos \phi = 0
\]
(1)
In the formula (2), lagrange multiplier has been given.
\[
\lambda = -m \phi' (x' \cos \phi + y' \sin \phi)
\]
(2)

2.2. Predictive model of the WMR
Since the coefficient of this predictive model is the contribution of each increment of control compared to the output of predictive control, it will require a large number of solutions, and because of the interaction between three inputs and outputs with initial values at the time of K. The process of the whole dynamic error control system as follow:
- The predictive array and the controlling array are set up respectively, then the trajectory of motion is decomposed to be regarded as discrete-state points, and each state point includes three states of angle, horizontal velocity and vertical velocity, and three parts of predictive control are established respectively.
- According to the trajectory of reference, the value of output on the time K is calculated, and the value of control are stored. The current time K is iterated, and the state transformation matrix P and the output transformation matrix Q are solved on the lyapunov rule.
- Calculating the error between desired trajectory of position and current trajectory of position then correcting the parameters of output matrix, and then calculating the predicted trajectory from output value and the given value in the current time K.
- Adding three decomposed parts to the control matrix, and adding bounded interference.
- To limit the three control states.
In the formula (3), (4) and (5), the matrix of rolling optimization has been shown.
\[
\begin{array}{ccc}
\tilde{a}_1 & \cdots & a_n \\
\vdots & \ddots & \vdots \\
\tilde{a}_i & \cdots & a_n \\
\end{array}
\begin{array}{c}
0 \\
\Delta u(k) \\
\Delta u(k+1) \\
\vdots \\
\Delta u(k+m-1) \\
\end{array}
\begin{array}{c}
\tilde{y}_1(k) \\
\tilde{y}_2(k) \\
\vdots \\
\tilde{y}_n(k) \\
\end{array}
\begin{array}{c}
y_v(k+1) \\
y_v(k+2) \\
\vdots \\
y_v(k+m) \\
\end{array}
\]
\[
\Delta u(k) = f(e(k))
\]
\[
y_v(k+1) = y_v(k) + e(k)
\]
(3) (4) (5)

3. Tracking of sine single tracking on the WMR based on adaptive fuzzy control
In the tracking control of the WMR, the fixed parameters of the lyapunov function, the positive diagonal matrix and the error transformation function are determined by the established fuzzy membership function, and the main memberships of five Gauss membership functions are decided on the intervals including (-π/4, -π/12), (-π/6,0), (0, π/6), (π/12, π/4), then initial state and expected tracking sine trajectory are selected.
In the formula (6), the model of the WMR has been given, in(7), the states and the errors in the time domain have been shown, in (8), the input of adaptive control has been given.
\[
x^{(n)} = f(x,x',...,x^{(n-1)}) + g(x,x',...,x^{(n-1)})u
\]
\[
y = x
\]
\[
x(k), x'(k), x^*(k),...x^{(n-1)}(k)
\]
\[
e(k), e'(k), e^*(k),...e^{(n-1)}(k)
\]
\[
x(k), x'(k), x^*(k),...x^{(n-1)}(k)
\]
\[
e(k), e'(k), e^*(k),...e^{(n-1)}(k)
\]
(6) (7)
\[ u^* = \frac{1}{g(x|\theta_x)} \left[ f(x|\theta_x) + y^{(n)} + k^T e \right] \]  

(8)

Fuzzy adaptive control with initial fuzzy experience, as shown in figure 3.

Figure 3. Sine tracking of the WMR

Figure 4. Tracking of the WMR with adjusting parameters

Adjusting the adaptive parameters, the output will be improved, but still basically stable. At this time, even if the initial fuzzy experience is selected a little badly, the control can also achieve stable sine signal tracking of the WMR, as shown in figure 4.

4. Comparison between adaptive scheme and robust predictive scheme

The figure 5 shows the structure of adaptive control with the initial experience. Based on the errors and the change of errors, in the figure 6, comparing the two methods for the motion control of the prediction model, we can see that adaptive method (black) is quicker to following the expected motion than the robust predictive method (red). With the interfere, the used motion is a helical motion composed of two sinusoidal motions on the axis of X and Y.

In the formula(9), the changing rule of errors and change of errors on X in the helical motion has been given, in(10), lyapunov function guaranteeing the stability has been given, in (11), the adaptive rule of the current chosen fuzzy rules has been given.
\[ e^{(n)} = -k^e e + \left[ \hat{f}(x|\theta_f) - f(x) \right] + \left[ \hat{g}(x|\theta_g) - g(x) \right] e_t \]
\[ \Delta = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]
\[ e^i = \Delta e + b \left[ \hat{f}(x|\theta_f) - f(x) \right] + \left[ \hat{g}(x|\theta_g) - g(x) \right] e_t \]
\[ V = \frac{1}{2} e^T P e + \frac{1}{2\gamma_1} (\theta_f - \theta_f^*)^T (\theta_f - \theta_f^*) + \frac{1}{2\gamma_2} (\theta_g - \theta_g^*)^T (\theta_g - \theta_g^*) \]
\[ \Delta^T P + P \Delta = -Q \]
\[ \theta_f^* = -\gamma_1 e^T P b \xi(x) \]
\[ \theta_g^* = -\gamma_2 e^T P b \eta(x) u_t \] (9)

Then, figure 7 shows a direct robust predictive mechanism without initial experience on the helical motion with interfere. figure 8 is an adaptive fuzzy control mechanism with initial experience on the helical motion with interfere, and the whole tracking can be clearly seen. Compared to the several errors of the robust predictive mechanism at the start of the tracking, the signals of the helical motion no longer need to be tried many times at the beginning of the control in the adaptive mechanism, and with the initial experience, the method can achieve a better tracking.

**Figure 7.** Mechanism without initial experience  
**Figure 8.** Adaptive mechanism with initial experience

5. **Conclusion**

The experimental simulation on the constrained WMR shows that the method for control is stable, and the less parameters need to be set, so the cost is lower and the actual tracking effect is ideal and the anti-interference ability is improved. Because of the existence of adaptive law and initial fuzzy experience, the shortage of the control lies in the high requirement in the class of the motion, if the actual motion is beyond the limits of the motion on some direction, there will be a huge deviation on the whole tracking, then because of the limits of adaptive parameter what is set to prevent the adjustment from becoming frequent and violent, it is difficult for the method to return to the needed trajectory. Similar problems are encountered in the simulation, so the initial experience and the possible states of motion in all directions should be taken into account so that the complex trajectory can be perfectly tracked in a certain range. Although the whole adaptive system can be determined to keep stable because of the lyapunov rule, the whole tracking can be influenced deeply by initial parameters, and this is the work that needs to be improved in the future.
Acknowledgments
This work is supported by Fundamental Research Special Funds for Central Universities (20826041A4133).

References
[1] Li, Y., He, L., & Yang, L. M. (2013). Path-following control for multi-axle car-like wheeled mobile robot with nonholonomic constraint. IEEE/ASME International Conference on Advanced Intelligent Mechatronics (Vol.8212, pp.268-273). IEEE.
[2] Saccon, A. (2005). Minimum Time Maneuver for a Nonholonomic Car with Acceleration Constraints: Preliminary Results. Intelligent Control, 2005. Proceedings of the 2005 IEEE International Symposium on, Mediterranean Conference on Control and Automation (pp.1337-1342). IEEE.
[3] Anderson, B. D. O., Agathoklis, P., Jury, E. I., & Mansour, M. (1986). Stability and the matrix lyapunov equation for discrete 2-dimensional systems. Circuits & Systems IEEE Transactions on, 33(3), 261-267.
[4] Li, T. S., Tong, S. C., & Feng, G. (2010). A novel robust adaptive-fuzzy-tracking control for a class of nonlinear multi-input/multi-output systems. IEEE Transactions on Fuzzy Systems, 18(1), 150-160.
[5] Deng, M., Inoue, A., & Bi, S. (2009). Robust non-linear control and tracking design for multi-input multi-output non-linear perturbed plants. Iet Control Theory & Applications, 3(9), 1237-1248.
[6] Wu, T. S., Karkoub, M., Chen, H. S., Yu, W. S., & Her, M. G. (2015). Robust tracking observer-based adaptive fuzzy control design for uncertain nonlinear mimo systems with time delayed states. Information Sciences, 290, 86-105.
[7] Chien, C. J., Hsu, C. T., & Yao, C. Y. (2004). Fuzzy system-based adaptive iterative learning control for nonlinear plants with initial state errors. Fuzzy Systems IEEE Transactions on, 12(5), 724-732.
[8] Li, Y., Deng, J. M., & Wei, M. Y. (2002). Meaning and precision of adaptive fuzzy systems with gaussian-type membership functions. Fuzzy Sets & Systems, 127(1), 85-97.
[9] Boulkroune, A., Tadjine, M., M’Saad, M., & Farza, M. (2010). Fuzzy adaptive controller for mimo nonlinear systems with known and unknown control direction. Fuzzy Sets & Systems, 161(6), 797-820.
[10] Erbatur, K., & Kaynak, O. (2010). Use of adaptive fuzzy systems in parameter tuning of sliding-mode controllers. IEEE/ASME Transactions on Mechatronics, 6(4), 474-482.