Vibration control of cantilever blade based on trailing-edge flap by restricted control input

Ting-Rui Liu¹ and Ai-Ling Gong²

Abstract
Vibration and control of cantilever blade with bending-twist coupling (BTC) based on trailing-edge flap (TEF) by restricted control input are investigated. The blade is a thin-walled structure using circumferentially asymmetric stiffness (CAS) configuration, with TEF embedded and hinged into the host composite structure along the entire blade span. The TEF structure is driven by quasi-steady aerodynamic forces. Vibration control is investigated based on linear matrix inequality (LMI) algorithm using restricted control input (LMI/RCI). Flutter suppression of BTC displacements and the angle of TEF (i.e. the practical control input) are illustrated, with apparently controlled effects demonstrated. The restricted control input signals are used to driven the TEF to explore the scope of the feasibility of the practical TEF angle, which is displayed by a virtual simulation platform. The platform verifies the feasibility of the hardware implementation for the control algorithms.

Keywords
Vibration control, trailing-edge flap, linear matrix inequality, quasi-steady aerodynamic force, virtual simulation

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Introduction
Both wind turbine blades and helicopter blades have always been involved in classical flutter in actuation of linear flow state or quasi-steady fluid, thereby leading to fatigue problems or performance loss. Large-amplitude displacements of equal-amplitude vibrations of BTC motions are important reasons of fatigue damage of blades. How to effectively realize vibration control for BTC motions has always been a topic that needs to be further studied. So far, most of the literature on BTC flutter is based on the study of 2D airfoil structure that subjected to flap-wise bending and twist vibrations. Classical flutter means the combined flap-wise bending and twist motions of an aerofoil in the linear region of its polar curve on the one hand, and refers to a violent unstable dynamic condition in which the blade structure, under the influence of incident aerodynamic loads, undergoes the high-amplitude vibrations due to the coupling of the BTC modes.¹ In most of the previous studies, to understand and investigate the flutter instability mechanism, a typical 2D blade section has been considered to be able to study transient problems of classical flutter. Most of the structural modeling are based on Navier-Stokes (NS) model, interaction model between Computational Fluid Dynamic (CFD) and Computational Structure Dynamic, fluid-structure model such as using Blade Element Momentum theory, or other models directly from third-party aerodynamic codes. Structure modeling and flutter analysis of an individual blade section, subjected to combined flap-wise bending and twist motions, have been investigated by Chaviaropoulos et al.,² with NS model applied to analyze aeroelastic stability. An aeroelastic numerical model was described, which combined a NS-based CFD solver with an elastic model and two coupling schemes, to study the aeroelastic behavior of 2D

¹College of Mechanical & Electronic Engineering, Shandong University of Science & Technology, Qingdao, China
²Business School, Qingdao University of Technology, Qingdao, China

Corresponding authors:
Ting-Rui Liu, College of Mechanical & Electronic Engineering, Shandong University of Science & Technology, No.579, Qianwangang Road, Qingdao, 266590, China.
Email: liutingrui@sdust.edu.cn
Ai-ling Gong, Business School, Qingdao University of Technology, No.2, Changjiang Road, Qingdao, 266520, China.
Email: gongai9999@163.com

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section undergoing classical flutter. The basic characteristics of the aerodynamic and elastic models were also presented together with the coupling schemes, and interaction between CFD and Computational Structure Dynamic was applied to analyze self-sustaining vibrations because the flexibility of blade can interact with the flow around it, with NS Equations integrated in time for analysis of aerodynamic forces. Hayat and Ha performed eigenvalue analyses using commercial quasi-steady aerodynamic codes to evaluate the classical flutter performance of a $5$MW pitch-regulated wind turbine blade with shallow-angled skins configurations.

However, due to the limitations of 2D airfoil research, a few scholars have developed the classical flutter study of 3D blade body. The fluid-structure interaction of a finite aspect ratio, cantilevered, flexible wing was investigated using a cyber-physical system to virtually augment the twist dynamics of the wing, with a sectional view of NACA0018 cyber-physical wing mounted on the wind tunnel experimental subfloor to study the flutter of 3D structure. Due to modeling uncertainty, which is an important, non-negligible issue in analysis of coupled-mode flutter of long-flexible rotating blade, the stochastic flutter problem of 3D blade was investigated by Li and Caracoglia to understand the influence of uncertainty in selected input characteristics, with the classical aerodynamic model derived from Theodorsen theory.

In addition, in recent years, many scholars have studied the structural modeling, vibration analysis and control methods of composite cantilever beams. Among them, a variety of structural enhancement design, as well as a variety of different intelligent control theories, are comprehensively applied to the vibration control of composite materials. Modal analysis using Finite Element studies for a smart hybrid composite beam in cantilever configuration, constituted of LY551 resin by enclosed shape memory alloy wires amidst layers of fibers, was investigated by Noolvi and Nagar, with the overall properties of the composite beam estimated using method of mixtures. A first order shear deformation theory was used to describe mathematical procedure for free vibration analysis of layered composite cantilever beam, with detailed effect of lay-up sequence and length-to-thickness ratio with lay-up angle on natural frequencies of various modes studied by considering four layered composite cantilever beams. A new analytical model for a functionally graded material/shape memory alloy laminated composite cantilever beam subjected to a concentrated tip load was developed, with a high-accuracy numerical solution and a 3D finite element analysis for the composite beam carried out to validate the analytical solution. A novel explicit analytical solution was proposed for obtaining twisting deformation and optimal shape control of smart laminated cantilever composite plates/beams using inclined piezoelectric actuators, with the linear piezoelectricity and plate theories adopted for the analysis using a novel double integral multivariable Fourier transformation method. Theoretical modeling and vibration control for divergent motion of thin-walled pre-twisted composite blade were investigated based on linear quadratic Gaussian controller using Loop Transfer Recovery at plant input, with flutter suppression for divergent instability investigated by using an experimental platform based on hardware-in-the-loop technology to test the effectiveness of the proposed control algorithm.

In recent years, the local TEF structures including mid-span flap (Huang et al.) and gurney flap (Bianchini et al.), have been used by different scholars to reduce the loads of blades and enhance the aerodynamic performance, with Blade Element Momentum theory or CFD theory used to compute the aerodynamic loads. In the present study, a 3D composite cantilever blade exhibiting flap-wise bending and twist displacements uses CAS configuration by skewing angle-plies with respect to the blade axis in the top flange (above the chord) and the bottom flange, so as to realize structural modeling. Simultaneously, a 3D TEF is embedded and hinged into the host composite structure of blade to suppress classical flutter, with a quasi-steady aerodynamic model used to calculate the aerodynamic excitation effect. To maximize load mitigation, a 3D TEF structure distributed along the full length of blade, is hinged to the host structure of the blade. In addition, the aeroelastic equation based on BTC displacements is generally linear (or becomes a linear system after linearization), so from the point of view of intelligent control theory, it is suitable for the strategy of linear theory control. For example, after linearizing the nonlinear equation of classical flutter, the active control of optimized linear quadratic regulator was proposed to suppress the BTC flutter by Chang et al. In the present study, a linear LMI algorithm deduced by Liu is used to drive the TEF structure to suppress the classical flutter.

The novelty in this study encompasses: (a) Vibration and flutter are investigated based on 3D blade body rather than 2D airfoil, and its solution involves the decoupling of partial differential equations using standard Galerkin method (GM). (b) The TEF structure is based on the full-length distribution of the blade, not the local distribution. The TEF is a hollow, lightweight, space aluminum structure so as to ignore the deformation of TEF, as long as the elastic torsional deformation of composite blade is limited to a sufficiently small range, which requires that the performance parameters and structural parameters of composite materials can meet this requirement. (c) Considering the actual range of swing angles of TEF in engineering applications, the TEF structure is driven by LMI using restricted control input (LMI/RCI) based on multiple constraints. (d) An experimental platform for driving TEF verifies the effectiveness of LMI/RCI algorithm. The platform is based on SWOU server.
Dynamic system

Figure 1 shows the CAS-based thin-walled blade with TEF structure. The origin of the axis system (x, y, z) is located at the root of the cantilever blade. The mid-line equation of the thin-walled cross-section is an integration-line equation, suitable for NACA 6315 profile, based on the conformal transformation and series theory. \( \theta \) The BTC displacements are denoted by bending z and twist angle \( \theta \). The blade length is \( L = 1.0 \) m, with wedge length being \( L_0 = 0.1 \) m. The chord length \( c_r \) at different sections of positions \( r \) is different, which can be fitted as a four-order Gaussian function as:

\[
\text{cr}(x) = \sum_{i=1}^{4} a_i \exp\left(-((x - b_i)/c_i)^2\right),
\]

where the coefficients are \( a_i = 0.1187, 0.1173, 0.2407, 0.0978, b_i = 0.2485, 0.3109, 0.6639, 0.4353; c_i = 0.0338, 0.0792, 0.5959, 0.1780 \), respectively, herein, \( i = 1, 2, 3, 4 \). It needs to be explained that the curve to be fitted and synthesized does not mean that the value of chord length of each section is more accurate than that of the original data point, but because in application of subsequent GM analysis, the function expression of chord length is required to be able to perform integral operations along span-wise direction in GM algorithm. As a result, as long as the performance indexes of goodness-of-fit statistics in the fitting process, such as SSE, RMSE, et al., can meet the precision requirements, it can be considered that the fitting method is effective. In addition, in each sectional position \( r \), there exists \( l_r = c_r/6 \), and there is \( l_p = c_p/6 \) at the tip of the blade.

The structural parameters of the blade, the composite properties and the elastic parameters are as follows:

- The practical angle of TEF is denoted by \( \beta \); the thickness of blade section is \( h = 6\rho_h \); ply thickness is \( \rho_h = 0.0191 \) m; the blade density is \( \rho_b = 1201 \) kg/m\(^3\); ply-angle is denoted by \( \theta_p = \pi/3 \); Longitudinal Young’s modulus and Transverse Young’s modulus are denoted by \( E_1 = 25.8 \) GPa, \( E_2 = 8.7 \) GPa, respectively; Shear modulus is denoted by \( G_{12} = 3.5 \) GPa; Poisson’s ratio is \( \nu_{12} = 0.34 \). In addition, the constant wind speed is \( U = 10 \) m/s.

The composite host structure considered is designed by a series of assumptions: (a) The blade is much stiffer in \( y \) direction than in BTC directions. The transverse in-plane \( (x, o, y) \) normal stresses are negligible. (b) Because the aim is to fully analyze the control effects, the structural damping of host structure and non-linearity of composite structure are not considered. (c) The TEF is a hollow, lightweight, space aluminum structure, without considering its internal mechanical behavior.

The purpose in the present study is to investigate the control effects of TEF on vibration, so the structure model of blade is simplified. According to the above assumptions, the structural modeling used in the present study could be obtained with the consideration of the CAS-based composite host structure exhibiting BTC displacements (Ren and Liu\(^{18}\)). Therefore, the equations of motions of blade section (i.e. \( l_1 - l_1 \) section in Figure 1) that integrate the structural model with the quasi-steady aerodynamics model can be described as follows:

\[
GJ\ddot{\theta}'' + K_C z'' - m_1 c_r^2 \theta = M
\]  

(1)

Figure 1. The TEF-based composite blade using CAS configuration.
\[ K_C \theta'''' + EIz'''' + m_c \ddot{z} = F \] (2)

where \( GJ, EI, \) and \( KC \) are the twist stiffness, bending stiffness, and BTC stiffness, which are expressed as 19:

\[
GJ = A_s^2 \int 1/C(s)ds, \quad KC = -A_x B \int (s)z/C(s)ds \quad (s)z/C(s)ds \int 1/C(s)ds
\]

\[
EI = \int \left( A(s) \frac{B_C(s)}{C(s)} \right)^2 z^2 ds + \left( \frac{20}{C(C(s))} \right)^2 \int 1/C(s)ds.
\]

The stiffness coefficients mentioned above can be obtained by calculation of reduced axial stiffness \( A(s) \), coupling stiffness \( B(s) \), shear stiffness \( C(s) \), and sectional area \( A_s \), respectively. The parameters, \( K_m^2 \), and \( m_c \), are denoted by \( K_m^2 = \frac{1}{m_s} \int \rho_c(z^2 + y^2)dydz \), \( m_c = \frac{1}{\rho_c} dhds \), respectively. Herein, \( \gamma \) is the anticlockwise circumferential coordinate along the mid-line of the thin-walled cross-section.

In addition, a new proposed quasi-steady aerodynamic model composed of aerodynamic Lift force \( F_z \) and Moment \( M_g \) has been used to a chord-wise TEF to control the aerelastic system of 2D section. In order to apply the aerodynamic forces to the cantilever structure (i.e. the rotating speed of blade is zero) in the present study, the aerodynamic force \( F_z \) and Moment \( M_g \) used here can be obtained by replacing “inflow velocity \( V_0 \)” and “\( \theta \)” in the aerodynamic expressions of Lift force \( F_z \) and Moment \( M_g \) with “constant wind velocity \( U \)” and “\( \pi/2 - \theta \)” respectively.

\[
F = \frac{1}{2} \rho_a \left\{ \pi b \left[ U(\ddot{z}/U + \dot{\theta}) + \frac{b \dot{\theta}^2}{2} \right] + 2\pi \dot{z} \right.
\]

\[
+ 2\pi U^2 \frac{C_{lb}(\pi/2 - \theta + 1)}{C_{ls}} \frac{\beta - 2\pi U^2 C_{lb}(\pi/2 - \theta)}{C_{ms}} + 2\pi bU\dot{\theta} \left\} \right.
\]

\[
M = \rho_a b \left\{ -\frac{\pi b U(\ddot{z}/U + \dot{\theta})}{4} - \frac{\pi bU\dot{\theta}}{4} - \frac{3\pi b^2 \dot{\theta}}{16} \right.
\]

\[
+ \frac{\rho_a \pi U^2 C_{mag}}{C_{ms}} B \right\} \right.
\]

where \( \rho_a \) is the air density; the lift and moment coefficients per angle of attack are denoted by \( C_{ls} = 6.28 \), \( C_{ms} = (0.5 + c/6)C_{ls} \), and the lift and moment coefficients per angle of TEF are denoted by \( C_{lb} = 3.358, C_{mag} = -0.635 \). Note that the application condition of equations (3)-(4) is the condition \( l_c = c_r/6 \) mentioned above.

Further, the aerodynamic force \( F \) and Moment \( M \) in equations (3)-(4) are quasi-steady theory-based models which were originally derived from Sahjendra and Yoosoon. After modification, the models were applied to TEF structure and used in Liu. In Sahjendra and Yoosoon, the bending/twist coupling motions of 2D elastic wing in actuation of quasi-steady fluid were analyzed, and active flutter control based on TEF was realized. Therefore, \( F \) and \( M \) can be applied to the study of aerelastic behavior under quasi-steady aerodynamic excitation in the present study. It should be specially noted that, if the rotating motion of wind turbine blade is considered, for instant, the constant rotary speed is \( \Omega = (\lambda_0 U)/(L + L_a) \), herein, \( \lambda_0 = 1 \) is the speed ratio of blade tip, then the dynamic stiffening effect and centripetal force need to be considered in formulas (1) and (2), the vertical wind speed should replace the horizontal wind speed in Figure 1, and the aerodynamic forces can be directly expressed in the form, so that the corresponding blade tip displacements can be calculated.

To solve the partial differential equations given by equations (1)-(2), an accurate modal method based on standard GM analysis is implemented by Hodges and Pierce. It is assumed that:

\[
z(x, t) = Z^T(x)q_x, \quad \theta(x, t) = \Theta^T(x)q_\theta;
\]

\[
Z^T(x) = [z_1(x), z_2(x), \ldots, z_i(x), \ldots z_N(x)],
\]

\[
\Theta^T(x) = [\theta_1(x), \theta_2(x), \ldots, \theta_i(x), \ldots, \theta_N(x)]
\]

where the reserved modes, \( q_x \) and \( q_\theta \), are \( N \times 1 \) vectors of generalized coordinates; \( Z^T(x) \) and \( \Theta^T(x) \) are \( 1 \times N \) vectors of suitable modal functions, respectively, and there exists:

\[
z(x) = \frac{(x/L)^4 + \frac{6 + \bar{c}(1-x/L)^2 + \frac{5 - 6x/L + (x/L)^2}{4} + \frac{3}{4}}{4(i+1)(2+i)(3+i)}}{i(i+1)(1+i)(2+i)(3+i)},
\]

\[
\theta_i(x) = \sin(i\pi x/L)
\]

Substitute equations (3)-(6) into equations (1)-(2), and assume \( X = [q_\theta, q_x]^T, \quad q_x = [q_x]^T, \quad q_\theta = [q_\theta]^T \), the integral process of GM is along the span-wise direction, and the integral interval is \([L_0, L + L_a]\). It results in the equations with \( 2N \times 2N \) sub-equation structures as follows:

\[
M_0X + C_0 \dot{X} + K_0X = Q_0\beta
\]

Vibration control based on LMI/RCI

Considering the actual range of swing angles of TEF in engineering applications, as mentioned above, the TEF structure needs to be driven by LMI using strictly defined input signal. However, there are very few LMI algorithms that not only satisfy the control effects but also have restricted control inputs. The LMI/RCI algorithm based on Schur complement theorem is one of these algorithms. To carry on subsequent solving, equation (7) can be transformed into the state-space, with \( 4N + 4N \) sub-equation structures as follows:

\[
\dot{Y} = AX + BB^T
\]
where

\[
A = \begin{bmatrix}
0_{2N \times 2N} & I_{E(2N \times 2N)} \\
-M_0^{-1}K_0 & -M_0^{-1}C_0
\end{bmatrix},
B = \begin{bmatrix}
0_{2N \times 1} \\
M_0^{-1}Q_0
\end{bmatrix}.
\]

**Design and convergence analysis of controller**

The control input signal, that is the angle \( \beta \) of TEF in equation (8), is designed as \( \beta = KY \), herein, 
\( K = [k_1 k_2 \ldots k_{4N}] \). The control goal is to solve the controller \( K \) by designing LMI, so as to realize
\( Y \rightarrow 0 \), \( |\beta| \leq \beta_{\text{max}} \).

The Lyapunov function is designed as follows:

\[
V = Y^TPY
\]

where \( P > 0, P = P^T \). By the proper design of \( P \), the convergence effect of \( Y \) can be effectively adjusted, and the solution of LMI can be facilitated and obtained, then

\[
\dot{V} = Y^TPY + Y^TPY = (AY + B\beta)^TPY
\]

\[+ Y^TP(AY + B\beta) = Y^TQ_1Y + Y^TQ_1Y = Y^TQY \]

where \( Q_1 = P(A + BK), Q = Q_1^T + Q_1 \).

In order to achieve an exponential convergence, that is, \( \dot{V} \leq -\alpha V \), we might as well assume that

\[
\alpha V + \dot{V} = \alpha Y^TPY + Y^TQY = Y^T(\alpha P + Q)Y
\]

let \( \alpha P + Q < 0, \alpha > 0 \), then \( \alpha V + \dot{V} \leq 0 \). According to the inequation solving theorem, the solution of
\( \dot{V} \leq -\alpha V \) is as follows:

\[
V(t) \leq V(0) \exp(-\alpha t) \leq V(0)
\]

if \( t \rightarrow \infty \), then \( V(t) \rightarrow 0 \), \( Y \rightarrow 0 \). It satisfies exponential convergence.

**Design of LMI/RCI**

Since \( V(0) = Y_0^TPY_0 \), if there exists positive definite symmetric matrix \( P \) and condition \( \omega > 0 \), the following inequation holds
\( Y_0^TPY_0 \leq \omega \), then \( V(0) \leq \omega \) is guaranteed, hence \( V(0) \leq \omega \) can be guaranteed.

We might as well let \( K^TK \leq \omega^{-1}\beta_{\text{max}}^2P \), since \( \beta = KY \), then we can obtain:

\[
\beta^2 = (KY)^TKY = Y^TK^TKY \leq Y^T\omega^{-1}\beta_{\text{max}}^2PY
\]

\[= \omega^{-1}\beta_{\text{max}}^2 \leq \beta_{\text{max}}^2 \]

then there exists the inequation \( |\beta| \leq \beta_{\text{max}} \).

Through the above analysis, we might construct two inequalities and solve them according to Schur complement theorem.16 The first one is \( \alpha P + Q < 0 \), then

\[
\alpha P + PA + PBK + A^TP + K^TB^TP < 0
\]

Pre-multiplication \( P^{-1} \) and post-multiplication \( P^{-1} \) are performed on both sides of the sign of inequation above, it can be obtained as follows:

\[
\alpha P^{-1} + AP^{-1} + BK P^{-1} + P^{-1}A^T + K^TB^TP < 0
\]

(15)

The second inequation constructed is \( K^TK - \omega^{-1}\beta_{\text{max}}^2P \leq 0 \). Let \( k_0 = \omega^{-1}\beta_{\text{max}}^2 \), then \( K^TK \leq k_0P \).

According to Schur complement theorem, it can be obtained as follows:

\[
\begin{bmatrix}
k_0P & K^T \\ K & 1
\end{bmatrix} \geq 0
\]

(16)

Similarly, pre-multiplication and post-multiplication with \( \begin{bmatrix} P^{-1} & 0 \\ 0 & 1 \end{bmatrix} \) are performed on both sides of the sign of inequation above, it can be obtained as follows:

\[
\begin{bmatrix}
k_0P^{-1} & P^{-1}K^T \\ KP^{-1} & 1
\end{bmatrix} \geq 0
\]

(17)

Consider equation (15) and equation (17), let \( F = KP^{-1}, W = P^{-1} \), then the first two LMI for the ultimate calculation can be obtained as follows:

\[
\begin{bmatrix}
\alpha W + AW + BF + WA^T + F^TB^T \\ k_0W & F^T
\end{bmatrix} \geq 0
\]

(18)

In order to further strictly limit the input signal, we add two other restriction strategies, as follows:

According to the definition of \( P \), the third LMI can be expressed as:

\[
P > 0, P = P^T
\]

(20)

According to \( Y_0^TPY_0 \leq \omega \) and Schur complement theorem, the fourth LMI can be expressed as:

\[
\begin{bmatrix}
\omega & Y_0^T \\ Y_0 & W
\end{bmatrix} \geq 0
\]

(21)

then the effective controller \( K \) can be obtained by selecting the values of the appropriate \( \beta_{\text{max}} \) and \( \alpha \) to satisfy the four LMI equations (18)–(21) above mentioned.

Therefore, in the process of algorithm implementation, as long as we select the initial maximum value \( u_{\text{max}} = \beta_{\text{max}} \) of input signal \( u \) for iteration, after multiple restrictions, we can get a reasonable and practical input signal \( \beta \), and the range of the practical input signal \( \beta \) must not exceed the value of \( u_{\text{max}} \).

**Numerical analysis and discussion**

The related parameter values of the TEF blade system, including structural parameters and external motion parameters, follow the values given in the section 2. Figure 2 shows the simulation results of uncontrolled twist (\( \theta \))/bending (\( Z \)) displacements of blade tip using TEF under condition of the fixed angle of \( \beta = 0 \), and
displacements without TEF, respectively. It can be seen that the order of magnitude ($10^{-17}$) of the twist displacements is so small that the hardware detection cannot be achieved due to the hardware resolution limits, therefore, the twist vibration does not cause fracture failure at all, that is, the twist displacement does not need to be controlled at all.

The bending displacement using TEF shows almost equal-amplitude vibration of 0.1m. It is noted that the amplitude of 0.1 m is about 1/10 of the blade length $L = 1$ m, which is actually very destructive in engineering when it’s in high-frequency vibration state, therefore, the equal-amplitude vibration control of the bending displacement is absolutely necessary. In the following discussion about the effect of control algorithm, we mainly focus on the control effect of bending displacement using TEF.

It should be stated that the twist displacement with TEF is almost the same as that without TEF, so the influence of TEF on twist displacement can be ignored. The flap-wise displacement in the case of using TEF is obviously smaller than that in the case of no TEF. It can be seen that the influence of TEF on flap-wise displacement is important, which further illustrates the importance of using TEF to control flap-wise displacement in the present study.

**Result of LMI/RCI control**

To highlight the superiority of LMI/RCI algorithm driving the TEF structure, Figure 3 shows the controlled displacements(a), the controlled vibration speeds(b), and the practical control input signals $\beta$ and $\frac{d\beta}{dt}$ (c) using LMI/RCI algorithm. The empirical parameters in the LMI/RCI algorithm are described as: $u = u_{\text{max}} = \beta_{\text{max}} = 1$ rad, where $u_{\text{max}}$ is the initial, restricted input signal set by the theory, $\alpha = 15$, $\omega = 0.1$, and the assumed initial variable vector signal is $Y_0 = 0.01$. Note that the above three empirical parameters need to be carefully adjusted in order to obtain the best control effects. Because of the limited control input signal, that is, the theoretical parameter $u_{\text{max}}$, is the firstly selected parameter, and the other two parameters are adjusted reasonably with the change of $u_{\text{max}}$.

In Figure 3(a), the amplitude of the controlled bending displacement ($z$) is greatly reduced compared with the uncontrolled displacement ($z$) using TEF in Figure 2, and the fluctuation trend shows rapid attenuation, which reflects the effectiveness and superiority of the control algorithm. In Figure 3(b), the variation ranges of the vibration speeds ($dz/dt$) of bending motion are also small, which reflect the stability of vibration control. In addition, compared with the uncontrolled bending ($z$) in Figure 2, the controlled one in Figure 3(a) shows a chattering phenomenon, which is exactly the characteristics of LMI control, however, the amplitudes of the chattering fluctuation are small and do not have a destructive effect on the blade.

The signal fluctuation of the practical control input signal $\beta$ in Figure 3(c), is within the range of $-0.5$ rad to $0.5$ rad, that is $-28.7^\circ$ to $28.7^\circ$, which is not only a physically realizable range, but also far below the preset range, the theoretical $-u_{\text{max}} \sim u_{\text{max}}$ (1rad to 1rad). The effectiveness of physical implementability of TEF angle $\beta$ also reflects the robustness of the control algorithm. In addition, because the initial fluctuation frequency of the input angle signal $\beta$ or the variation of $d\beta/dt$ is relatively large, the driving of the swing process for the TEF can not be realized by conventional mechanical system or hydraulic system, which is due to the friction and hysteresis of the mechanical system, and the difficulty of hydraulic pipeline dynamic configuration, as well as hysteresis of the hydraulic system. In the follow-up work of this study, on the one hand, we intend to use pneumatic transmission to achieve the drive to the TEF angle. Flexible pneumatic pipe is not only flexible and easy to configure, but also pneumatic impact is more conducive to the realization of high
frequency and high velocity for TEF angle signal, on the other hand, we will study the flutter speed of blade based on pneumatic driving, which is a very interesting trend in the concept of flutter suppression phenomenon by using acoustic excitations in aerodynamic flow.

**Discussion of LMI/RCI control**

As mentioned above, suppression for bending flutter is the central content of this study, and the limited range of control input $u_{\text{max}}$ is the focus of discussion due to the physical limitations of the swing angle $\beta$ of physical TEF structure. Simultaneously, the theoretical parameter $u_{\text{max}}$ is the firstly selected parameter, and the other two parameters, $\alpha$ and $\omega$, are adjusted reasonably with the change of $u_{\text{max}}$. Figure 4 shows the controlled bending displacements, the practical control input signals, and the feedback controllers $K$, under the conditions of different cases of $u = u_{\text{max}}$ as follows:

Case 1: $u = u_{\text{max}} = 1.2$ rad, $\alpha = 10$, $\omega = 0.1$;
Case 2: $u = u_{\text{max}} = 1.0$ rad, $\alpha = 10$, $\omega = 0.1$;
Case 3: $u = u_{\text{max}} = 0.8$ rad, $\alpha = 10$, $\omega = 0.1$;
Case 4: $u = u_{\text{max}} = 0.6$ rad, $\alpha = 15$, $\omega = 0.5$;
Case 5: $u = u_{\text{max}} = 0.4$ rad, $\alpha = 15$, $\omega = 0.5$;

From the bending displacements in Figure 4(a), it can be seen that, when the initial, theoretically restricted input is too small, for example, $u = u_{\text{max}} = 0.4$ rad, the LMI/RCI algorithm will lose utility and cannot achieve the purpose of bending vibration suppression. The bending displacement presents an unstable state of rapid divergence. On the contrary, for a large limited input, such as $u = u_{\text{max}} = 1.2$ rad, although it can achieve

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**Figure 3.** The controlled displacements (a), vibration speeds (b), and the practical control input signals $\beta$ and $d\beta$ (c), under conditions of $u_{\text{max}} = 1$ rad, $\alpha = 15$, and $\omega = 0.1$. 

![Figure 3](image_url)
the purpose of flutter suppression, it is still accompanied by a large vibration amplitude. However, the flutter suppression effects are ideal for moderately limited inputs, such as $u = u_{\text{max}} = 1.0$ rad and $u = u_{\text{max}} = 0.8$ rad.

Figure 4(b) shows the practical control input signals $\beta$ in the implementation of the LMI/RCI control process, based on the theoretical, initial input signals $u = u_{\text{max}} = 1.2, 1.0, 0.8, 0.6$, respectively. The practical input angles $\beta$ based on both $u = u_{\text{max}} = 1.0$ rad and $u = 0.8$ rad are within the range of $-0.5$ rad to $0.5$ rad, that is $-28.7$ degree to $28.7$ degree, which is still the aforementioned, physically realizable range. From the amplitudes of the fluctuations of the practical input angles $\beta$, the amplitudes of both $u = u_{\text{max}} = 1.0$ rad and $u = u_{\text{max}} = 0.8$ rad are relatively small, so it can be seen that the reasonable selection of the theoretical input signal $u_{\text{max}}$ is of positive significance. Meanwhile, the fluctuation frequency in the case of $u_{\text{max}} = 1.0$ is minimal, which is relatively more convenient for the physical realization of TEF angle.

The feedback controllers $K = [k_1 k_2 \ldots k_{4N}]$, based on different theoretical, initial inputs, are also illustrated in Figure 4(c), with the $4N = 20$ elements displayed. The first five elements are the corresponding controller signals for twist displacements and the next five elements are corresponding controller signals of bending displacements. The following 10 elements are the corresponding controller signals for the twist speeds and the bending speeds, respectively. It can be seen that the control consumption of twist displacement increases with the increase of modal order, while the control consumption of bending displacement decreases with the increase of modal order. This is a strange phenomenon, which may have something to do with the expressions of the accurate shape functions in equations (4)–(5). In addition to the bending mode of the first order, that is...
the problem of hardware memory limitation and cumulative error of calculation in the hardware, thus resulting in the failure of the algorithm in the hardware implementation. (5) It can be seen that the orders of magnitudes \((10^{-17})\) of the twist displacements in both the uncontrolled case and the controlled case are too small to be measured due to the hardware resolution limits, therefore, the twist displacements do not need to be controlled in practice. (6) In this design, the purpose of the mounting TEF is mainly to suppress the flap-wise deformation of the blade body. Because of the flexibility of the composite structure, if the twist deformation is too large, it will affect the assembly, reliability and safety of TEF. Therefore, in order to make TEF work properly, the uncontrolled twist deformation of the blade body needs to be as small as possible under the action of wind forces. For this purpose, the design of the structural dimensions of the blade body and the selection of the parameters of the composite are crucial. For instance, we consider the length limit of the blade \((L = 1.0\ m)\) and the choice limit of elastic modulus parameters of the composite, as mentioned in section 2.

### Validity of methodology

#### Validity of GM

The LMI/RCI algorithm is based on system equations of equations (7)–(8), and the discretization of partial differential equations given by equations (1)–(2) is realized by GM way, hence the effectiveness of GM needs to be validated. If the aerodynamic terms in equations (1)–(2) are ignored, then the system equations degenerate into free-vibration equations. At this time, the eigenvalues of the system equation (8) can be solved, and the characteristic frequencies can be obtained. In view of the importance of bending motion, this design only examines the eigenvalue problem of bending motion. In addition, Dancila and Armanios\(^\text{19}\) proposed an eigenvalue calculation method for CAS-based cantilever beam, which is an accurate calculation method (CM) for frequencies of bending displacements as follows:

\[
(2\pi)^2 = \frac{-(GJm_kg^2 - EI_mK^2_{mk})}{2m^2K^2_m} \pm \sqrt{(GJm_kg^2 - EI_mK^2_{mk})^2 + 4m^2K^2_m(GJ\cdot EI - K^2_g)g^6} \tag{22}
\]

Herein, parameter \(g\) has two expressions:

\[g = jk_1, \quad k_1 = (2m + 1)\pi/2/L, \quad m = 0, 1, 2;\]

\[g = k_2, \quad k_2 = \text{the solutions of} \cos(k_2L)\cosh(k_2L) = 0.\]

Under conditions of ply-angle \(\theta_m = \pi/3\) and considering free vibration without aerodynamic terms and TEF structure, Table 1 demonstrates the comparisons of the first three-order frequencies \(f(\text{Hz})\) of bending motions between CM and GM, including the related eigenvalues of GM. Consistency of calculation results of the two methods demonstrates the effectiveness of the GM method. To test wider adaptability, Figure 5 illustrates the comparisons of the first three-order frequencies \(f(\text{Hz})\) of bending motions between CM and GM under conditions of different ply-angles from 0\(^\circ\) to 90\(^\circ\) at interval of 15\(^\circ\). Also, the three natural-frequency curves show perfect consistency on changing trends, and the data of the 2nd and 3rd frequencies solved by

| Table 1. Comparisons of the first three-oder frequencies of bending motions. |
|----------------|----------------|----------------|
| \(f\) (Hz)     | CM             | GM             | Related eigenvalues of GM |
| 1st Frequencies | 0.4121         | 0.4281         | \(2.3749 \times 10^{-13} \pm 2.6985i\) |
| 2nd Frequencies | 3.7088         | 3.6802         | \(3.1515 \times 10^{-13} \pm 2.3123 \times 10^i\) |
| 3rd Frequencies | 10.302         | 10.311         | \(1.469 \times 10^{-2} \pm 5.109 \times 10^2i\) |
the two methods almost completely coincide, which further verifies the effectiveness of the GM method.

It should be stated that, as shown in Dancila and Armanios, under CAS condition, the coupling motion is only shown as the coupling state between flap-wise bending and twist, which is exactly suitable for describing the aeroelastic behavior under quasi-steady aerodynamic force, so edge-wise bending is not involved. This is exactly the reason why the frequency structure described in Dancila and Armanios is used to verify the effectiveness of GM method in the present study. However, the edge-wise bending behavior should be taken into account if the stall-induced state is to be studied, then the quasi-stable aerodynamic force is no longer applicable and should be replaced by the aerodynamic force under the stall condition.

Validity of the LMI/RCI algorithm in the hardware implementation

The virtual simulation platform was proposed to verify the feasibility of the hardware implementation of the control algorithm. It was a simulation platform using controller-hardware-in-the-loop technology based on virtual simulation and OPC method and MATLAB/SIMULINK (MS) environment. To avoid the failure of the algorithm in the hardware implementation due to the problem of hardware memory limitation and cumulative error of calculation in the hardware just mentioned, the virtual simulation platform is used in the present study to verify the feasibility of the hardware implementation of the LMI/RCI algorithm.

In virtual simulation platform using virtual simulation and OPC method, the LMI/RCI theory runs completely in the hardware of the Siemens S7-300 CPU; the virtual simulation process of the blade system is realized completely in the MS environment in PC; WinCC software runs in MS and activates a server component named SWOU server, which communicates with S7-300 CPU by MPI Cable on the one hand and the blade system model of the OPC client in MS environment on the other. The OPC toolbox module library in MS has related OPC Read-Write modules. With the help of SWOU Server, the Read-Write operation between S7-300 CPU and MS environment can be realized. The structure diagram of online control and workflow chart of SWOU communication are illustrated in Figure 6.

The specific workflows of SWOU Server are as follows:

1. Build and activate SWOU Server in WinCC and setup program interface DCOM;
2. Create OPC data areas in WinCC to access client objects; the address of data areas should be consistent with those of variable areas in S7-300 CPU;
3. Add objects and groups in OPC client in MS;
4. Mapping data areas in SWOU server to both Simulink workspace and memories of S7-300 CPU;
5. Start the virtual simulation in the MS, observe the running state of CPU, and observe the simulation results in the WinCC interface.

The signals of the controlled bending displacement $z$ and the practical input $\beta$ can be displayed by WinCC. Figure 7 is a WinCC interface, and demonstrates the controlled bending displacement $z$, and the practical input signal $\beta$ under conditions of Case 2 mentioned above.

The WinCC interface shows the processes of dynamic fluctuations of two signals. Compared with the controlled bending displacement $z$ and the practical input signal $\beta$ in Figure 3 under conditions of $\mu_{\text{max}} = 1$ rad, $\alpha = 15$, and $\omega = 0.1$, Figure 7 demonstrates almost the same changing trends of bending displacement $z$ and input signal $\beta$, and shows a frame of picture.
within time range [0, 8s]. Numerical simulation and hardware implementation show perfect consistency.

It should be stated that because of the difference between the actual sampling time of the platform hardware and the theoretical sampling time of the numerical simulation process, the former has the limitation of sampling interval, so the fluctuations of the curves in Figure 7 are relatively smooth (Note that the chattering phenomenon is not visible due to excessive sampling intervals for hardware), and there are no sharp changes of the peak-values of the curve of the practical input \( \beta \) demonstrated in the numerical simulation process.

Conclusions

In this study, vibration and control of BTC blade based on TEF using restricted control input are investigated. Some concluding remarks can be drawn from the results:

1. The CAS-based composite blade is embedded in span-wise TEF structure driven by quasi-steady aerodynamic forces. The blade exhibits the BTC displacements, with the solution of its aeroelastic system investigated using standard GM theory.

2. Vibration control is investigated based on LMI/RCI algorithm. Flutter suppression of BTC displacements and the angle of TEF, that is, the practical control input, are illustrated. Different preset input signals, that is, the theoretical TEF angles, are introduced into LMI/RCI algorithm to realize flutter suppression for bending displacements, with practical control inputs \( \beta \) deduced and demonstrated.

3. The theoretically restricted input signals are used in LMI/RCI algorithm to drive TEF to explore the scope of the feasibility of the practical TEF angles. The theoretical input signal, \( \dot{\alpha}_{\text{max}} \), can neither be too large nor too small, otherwise the control effect is reduced or the control algorithm is completely invalid. (4) The amplitude of the feedback controller \( K \) should not be too large, otherwise the LMI/RCI algorithm will fail in the controller hardware. A virtual simulation platform using SWOU server verifies the feasibility of the hardware implementation of LMI/RCI algorithm.

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ORCID iD

Ting-Rui Liu https://orcid.org/0000-0002-2680-7289

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**Appendix**

**Notation**

| BTC | bending-twist coupling |
| TEF | trailing-edge flap |
| CAS | circumferentially asymmetric stiffness |
| LMI/RCI | linear matrix inequation (LMI) algorithm using restricted control input (RCI) |
| CFD | Computational Fluid Dynamic |
| CM | calculation method |
| GM | Galerkin method |
| MS | MATLAB/SIMULINK |
| OPC | Object Linking and Embedding (OLE) for process control |
| SWOU | Simatic WinCC OPC Unified Architectur (UA) |