NUMERICAL SIMULATION OF A STEPPED CAPILLARY TUBE USED IN RESIDENTIAL HEAT PUMP SYSTEMS

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Keywords: Capillary tubes, Refrigeration, Two-phase flow, Expansion devices, New design
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ABSTRACT
A stepped capillary tube is a new design that consists of two serial-connected capillary tubes with different diameters as an alternative in order to reduce the manufacturing cost of the typical assembly of two capillary tubes and a bypass check valve widely used to achieve different mass flow rates in residential heat pump systems.

One-dimensional numerical modelling of fluid-flow inside stepped capillary tubes was performed, and successfully validated against experimental data. Governing equations (continuity, momentum, energy, and entropy) for describing the fluid flow have been solved by using a fully implicit step-by-step method. A numerical treatment has been codified for considering thermodynamic and flow transitions (subcooled liquid region, metastable liquid region, metastable two-phase region and equilibrium two-phase region). Sudden contraction and enlargement were also considered.

These results demonstrate a robust application of the model developed to predict reliably the mass flow rate through stepped capillary tubes, which enable this tool to be reliably used for the design of this kind of systems.

INTRODUCTION
Residential heat pumps provide heating in winter and cooling in summer. In the cooling mode, the indoor heat exchanger works as an evaporator and the outdoor as a condenser; in the heating mode, the indoor one works as a condenser and the outdoor as an evaporator [1]. The flow directions of the refrigerant under these two modes are opposite, and the mass flow rates required to match with the different heat loads. Due to this, an expansion device to achieve different mass flow rates for both opposite refrigerant flow is required. The most common expansion devices used for residential heat pumps systems due to its simple structure, easy fabrication and low cost is a capillary tube assembly that consists of two serial connected capillary tubes and a bypass check valve. The bypass check valve makes the refrigerant flow through specific capillary tube depending upon the operating mode, resulting in different mass flow rates [1].

The bypass check is almost the half cost of a capillary tube assembly. If the bypass check valve can be saved, the cost of an entire assembly will be significantly reduced. The conventional capillary tube assembly employs two paths to achieve the different mass flow rates, the head load of heating mode is usually smaller than that of cooling mode, so the mass flow rate of heating mode is usually smaller than that of cooling mode. The configuration and the operating principles of both expansion devices are shown in Figure 1 [2].

The capillary tube assembly consists of a main capillary tube, an auxiliary capillary tube and a bypass check valve. As the check valve turns on and off, it directs the refrigerant to two different paths. As shown in Figure 1, in the heating mode, the refrigerant goes through two capillary tubes in series flowing the path of A-B-C-D-E; whereas in the cooling mode, the refrigerant only goes through the auxiliary capillary tube following the path of E-D-A. The flow path in the heating mode
is longer than in the cooling ones resulting in a smaller mass flow rate in the heating mode. The stepped capillary tube proposed by Zhao et al. [2] consists of a main capillary tube and an auxiliary capillary tube in series connection and does not employ a by-pass check valve. The inner diameter of the main capillary tube is larger than the auxiliary capillary tube; in the heating mode the refrigerant goes through both capillary tubes in series flowing the path of A-B-C-D; and a sudden contraction occurs at B-C; whereas in the cooling mode, the refrigerant flow direction is reversed D-C-B-A and a sudden enlargement occurs at C-B [2].

**FIGURE 1. CAPILLARY TUBE ASSEMBLY AND STEPPED CAPILLARY TUBE UNDER OPERATING HEATING AND COOLING OPERATION MODE**

Although the simple designs of a stepped capillary tube, the two-phase flows that occur inside them are somewhat complex, which make difficult to find out a general theoretical model to predict their behaviour with accuracy. Results have indicated that choked and metastable flows can coexist inside capillary tube expansion devices used in heat pump systems [3-4].

The objectives of this study to apply and to validate the numerical simulator for modeling stepped capillary tubes by considering separated flow model and metastable regions. The numerical solution was performed by discretization of the one-dimensional governing equations based on a finite volume formulation. The use of the entropy equation enabled the physical processes produced under critical flow conditions to be successfully detected. The mass flow rate results obtained from this simulation study were compared with experimental measurements for validating the model performance.

**MATHEMATICAL FORMULATION**

A mathematical formulation for two-phase flow inside a control volume (CV) of a tube was originally reported by García-Valladares [5]. The CV is shown schematically in Fig. 2, where ‘i’ and ‘i+1’ represent the inlet and outlet cross section area of the tube, respectively. Taking into account the geometry of tubes (i.e., diameter, length, roughness, inclination angle, etc.), the governing equations were solved by using the following assumptions: (i) one-dimensional flow \([p(z,t), h(z,t), T(z,t),...]\); (ii) adiabatic flow; (iii) neglecting axial heat conduction through the fluid; (iv) constant internal diameter, and (v) uniform surface roughness. The one-dimensional model also requires the selection of the following empirical correlations and boundary conditions to close the numerical problem related with the governing equations and the number of unknown variables:

- The evaluation of the shear stress by means of the following expression: \(\tau_W = \Phi(f \frac{1}{4}) \frac{m^2}{2\rho A^2}\) where \(f\) and \(\Phi\) are the friction factor and a two-phase multiplier, respectively.
- The calculation of the pressure drop by a balance over a given CV using a suitable empirical equations when a singularity process is presented (i.e., a sudden contraction or a sudden enlargement).
- The knowledge of the two-phase flow pattern, which is correctly evaluated by using a suitable correlation for the void fraction \(\varepsilon_\nu\) calculation.
- The definition of proper boundary conditions at the inlet and outlet sections of a stepped capillary tube. At the inlet section, the inlet pressure \(p_{in}\), and either temperature \(T_{in}\) or mass fraction \(x_{in}\) should be given, depending if the inlet fluid is under single- or two-phase flow conditions; whereas at the outlet section, the discharge pressure \(p_{out}\) must be given.

**FIGURE 2. SCHEMATIC DIAGRAM SHOWING A FLOW INSIDE A CV OF THE MODEL**

**Evaluation of empirical coefficients**

The one-dimensional mathematical model required information for representing the thermodynamic and fluid flow regions (e.g., subcooled liquid, metastable liquid, metastable two-phase, and equilibrium two-phase), which were obtained from additional empirical correlations. After a comprehensive review of these correlations, given previously in García-Valladares [5], the following equations were kept and selected for modelling each region:

**Single-phase (subcooled liquid or superheating vapour)**

The friction factor \((f)\) was calculated from the empirical correlations suggested by Churchill [6].
Metastable liquid

This region initiates when the pressure drops down to the saturated pressure condition, and finishes at the beginning of the vaporization process. The pressure of vaporization ($p_v$) at the flashing point was computed by a correlation proposed by Chen et al. [7]:

$$\left(\frac{p_{sat,l} - p_v}{\sigma}\right)_{sat} = 0.679 \left[\frac{\rho_l}{\rho_l - \rho_g}\right] \Re 0.914 \left[\frac{\Delta h_{sc}}{T_c}\right]^{-0.208} \left[\frac{D}{D'}\right]^{-3.18}$$

where $D'$ is a reference diameter given by the following equation: $D' = \sqrt{\frac{KT_{sat,l}}{\sigma}} \times 10^4$.

Constants of equation (1) were determined with R-12 fluid data collected from experimental works carried out with adiabatic capillary tubes under the following operating conditions: $1.44 \times 10^3 < G < 5.09 \times 10^3$ kgm$^{-2}$s$^{-1}$; $0 < \Delta T_m < 17$ K; and $0.66 < D < 1.17 \times 10^{-3}$ m. Bittle and Pate [8] extended the use of this correlation for a wide variety of refrigerants (R12 to R134a, R22, R152a and R410A). Since it constitutes the unique reliable correlation reported in the literature for these purposes, Eq. (1) was implemented into the numerical model.

In the metastable liquid region, the fluid thermodynamic properties were estimated by using the values corresponding to liquid saturated conditions at the fluid pressure. Temperature was determined from the thermodynamic relationship:

$$dh = c_p dT + \frac{\eta h}{\rho_l} dp$$

derived from the energy equation (see García-Valladares et al. [9]). The friction factor parameter was calculated in the same way as for the single-phase region.

Metastable two-phase

In this region, Feburie et al. [10] suggest that the two-phase flow can be analyzed considering three states: superheated or metastable liquid (denoted by the subscript m), saturated liquid (subscript l), and saturated vapor (subscript g). The governing equations of continuity, momentum and energy were used to estimate the mass flow rate, the fluid pressure, and the mean fluid enthalpy, respectively.

A $w$ variable, defined as mass ratio between the sum of saturated liquid and vapour phase and the total phase, i.e., $w=[m_l+m_g]/(m_l+m_g+m_m)$, was used to estimate the superheated liquid mass flow. This variable was computed by means of the correlation proposed by Feburie et al. [10]:

$$\frac{dw}{dz} = 0.02 \left(\frac{P}{P\text{sat,l}}\right)^{1.25} (1-w)^{0.25}$$

The mean fluid enthalpy was calculated from the three fluid phase enthalpies and their mass fractions as follows:

$$h = x_g h_g + x_l h_l + x_m h_m$$

This equation also allows the vapour mass fraction ($x_g$) to be estimated. According to Feburie et al. [10], the temperature of the superheated liquid was assumed constant in this region. When $w$ approaches unity, the superheated liquid vanishes, and the fluid flow process enters to an equilibrium state of two-phase flow.

In the metastable two-phase region, two temperatures should be known: the superheated liquid temperature ($T_m$), and the saturation liquid or gas temperature ($T_{eq}$). For this reason, an average temperature requires to be calculated as follows:

$$T = T_{eq} - \left(\frac{x_g - x_{eq}}{x_{eq}}\right) (T_m - T_{eq})$$

where $x_g$ is the vapour mass fraction, which results from the mean fluid enthalpy equation; and $T_m$ and $T_{eq}$ are the values obtained for the temperature and mass fraction, respectively, if the fluid were in thermodynamic equilibrium.

Due to the lack of specific correlations for the metastable two-phase region, the friction factor was estimated in the same way as for the equilibrium two-phase region.

Two-phase zone under equilibrium conditions

Considering a separated two-phase flow model ($v_{lp}v_{lp}$), the void fraction ($\varepsilon$) was estimated from a semi-empirical equation suggested by Premoli et al. [11]. The friction factor was calculated from the same equation as in the case of the single-phase flow using a correction factor (two-phase friction multiplier) according to Friedel [12].

Singularities

Sudden contraction. The model developed by Schmidt and Friedel [13] was adopted for performing the simulation work. Sudden enlargement. The equation proposed by Chisholm [14] was used.

Global numerical algorithm

The main objective of this numerical work was to determine the mass flow rate at any time instant inside stepped capillary tubes. Due to the high pressure gradients produced at the end of them, a non-uniform concentrated grid at the outlet section (Fig. 3) was designed using the following equation:

$$\Delta z_i = \frac{L}{\tanh(k)} \left[\tanh\left(\frac{k}{n}\right) - \tanh\left(\frac{(i-1)}{n}\right)\right]$$

where $\Delta z_i$ represents the size of the $i$th CV, and $k$ is the concentration factor, which has a value of $k \geq 0$ (for a non-uniform distribution a typical value of $k=2.5$ was adopted).

Discretized equations were then coupled by using a fully implicit step-by-step method in the fluid flow direction. From the known input variables (at the inlet section and the wall boundary conditions), their corresponding values at the outlet section of each CV were iteratively calculated using the discretized equations. This numerical scheme used at the outlet section was considered as the new inlet values for the next CV using a strict convergence condition which was verified in each CV before to advance to the next CV. The numerical procedure is completed until the end of the stepped capillary tube is reached.
For each CV, a set of algebraic equations was obtained by a discretization of the governing equations of continuity, momentum and energy. Transient terms of these equations were also discretized by using the following approximation: \( \frac{\phi}{\partial T} \approx \frac{(\phi - \phi^0)}{\Delta t} \), where \( \phi \) represents a generic dependent variable (\( \phi = h, p, T, \rho, \ldots \)); superscript “o” indicates the value of the previous instant. Average values of different variables were estimated through an arithmetic mean of their values between the inlet and outlet sections, i.e., \( \bar{\phi}_i = (\phi_i + \phi_{i+1})/2 \).

Based on the numerical approaches described above, the governing equations were discretized for estimating the value of the main dependent variables (e.g., mass flow rate, pressure and enthalpy) at the outlet section of each CV. The resulting form of the governing equations was as follows:

For the discretized continuity equation, the outlet mass flow rate was calculated by means of the following equation:

\[
m_{i+1} = m_i - \frac{\Delta z}{\Delta t}(\rho_{tp} - \rho_{tp}^0)\]

(6)

where the two-phase density was determined from: \( \rho_{tp} = \varepsilon_g \rho_g + (1-\varepsilon_g) \rho_l \).

Gas and liquid velocities were calculated in terms of the mass flow rate, as follows:

\[
\begin{align*}
v_g &= \frac{\varepsilon_g m_i}{\rho_g \varepsilon_g A} ; & v_l &= \frac{(1-\varepsilon_g) m_i}{\rho_l (1-\varepsilon_g) A}
\end{align*}
\]

From the discretized momentum equation, the outlet fluid pressure was determined as follows:

\[
p_{i+1} = p_i - \frac{\Delta z}{A} \left( \Phi f \frac{m^2}{4 \rho_{tp} A^2} \Delta D + \rho_{tp}^0 A \sin \theta + \ldots \right)
\]

\[
\quad + \frac{\rho - \rho^0}{\Delta t} \left( \bar{m}_g \bar{v}_g + (1-\varepsilon_g) \bar{v}_l \right) \frac{\Delta z}{\Delta t}
\]

(7)

In the case of a singularity (sudden contraction or sudden enlargement), the equations indicated in the section of singularities are used in order to calculate de pressure drop and after that the corresponding outlet fluid pressure.

From the use of energy and continuity equations, the following discretized equation was finally obtained for estimating the outlet fluid enthalpy under adiabatic flow conditions:

\[
h_{i+1} = \frac{\Delta z}{\Delta t} \left( a m_{i+1} + b m_i + c \Delta \rho \Delta z / \Delta t \right)
\]

(8)

where

\[
a = \left( x_g g + (1-x_g) v_1 \right) \frac{B}{1+1} + g_2 \Delta z - h_i
\]

\[
b = \left( x_g g + (1-x_g) v_1 \right) \frac{B}{1+1} - g_2 \Delta z + h_i
\]

\[
c = 2 \left( \rho_{tp} - \rho_{tp}^0 \right) - \rho_{tp}^0 (h_i - 2 \Delta z) - \left( \rho_{tp}^0 - \rho_{1} \right)^2
\]

At the thermodynamic equilibrium zones (i.e., subcooled liquid, superheated vapour, and equilibrium two-phase regions), temperature, mass fraction, and all the thermophysical properties were estimated by using matrix functions of pressure and enthalpy, which were computed with the REFPROP v.8.0 program [15]:

\[
T = f(p,h) ; \quad x_g = f(p,h) ; \quad \rho = f(p,h) ; \quad \ldots
\]

(9)

The above mentioned conservation equations of mass, momentum and energy, together with the thermophysical properties, were also applicable to transient two-phase flow. Steady- and/or single-phase (liquid or gas) flows were particular cases of this formulation. Moreover, a mathematical formulation in terms of fluid enthalpy provided a more simple scheme for the numerical solution (because only one equation is needed for all the regions), which could be useful for the analysis of a fluid flow composed by a mixture of refrigerants.

The fluid flow inside stepped capillary tube was divided into four regions (if all these appear): subcooled (zone I: when \( \rho \geq \rho_{sat,i} \), \( x_g = 0 \)); metastable liquid (zone II: when \( \rho_{sat,i} > \rho \geq \rho_s, x_g = 0 \)); metastable two-phase (zone III: when \( \rho_s > \rho \geq \rho_{sat,g}, 0 < x_g < x_{sat}, 0 \leq w \leq 1 \)); and a thermodynamic equilibrium two-phase (zone IV: when \( \rho_s > \rho \geq \rho_{sat,g}, x_{sat} < x_g \leq 1 \)).

Using the differentiation conditions among regions, the CV (where the transition occurs) is defined. For evaluating the position of the transition point, the CV was split into two CVs (see García-Valladares et al. [16]). The length of the first CV was calculated from the momentum equation, assigning the pressure condition to define the new thermodynamic region at the outlet section. The length of the second CV was then determined by a simple difference among them. Depending on the empirical correlations used, the friction factors can increase or decrease significantly among regions.

The numerical global algorithm can be summarized as follows: the inlet mass flow rate was iteratively estimated by a Newton-Raphson algorithm for obtaining the critical flow conditions, which were reached when the entropy equation was not valid in the last CV. To check the critical flow conditions, other authors propose the use of the criterion \( dp/dz \rightarrow \infty \). For our purposes, both critical flow criteria were equivalent. A flow diagram for calculating the critical mass flow rate through stepped capillary tube is shown in detail in Fig. 4. In the case of the mass flow rate, a change of variable was adopted to ensure the method convergence [17], using \( m^{-f}_{NR} \) instead of \( m \) (as
independent variable). A factor $f_{NR}$ greater than 3.5 provided a satisfactory fitting in most of applications.

![Flow Diagram](image)

**FIGURE 4. FLOW DIAGRAM FOR CALCULATING THE CRITICAL MASS FLOW RATE ALONG THE STEPPED CAPILLARY TUBE**

After determining the critical flow conditions, critical and discharge fluid pressures were compared. If the critical pressure is greater than or equal to the discharge pressure, the fluid flow will be critical, and therefore, the discharge shock wave needs to be solved; otherwise, the flow will be non-critical. Under such non critical conditions (where the outlet and the discharged pressure are equal), the mass flow rate must be re-evaluated by applying again the Newton-Raphson algorithm using the outlet pressure as a new dependent variable.

**RESULTS AND DISCUSSION**

To characterize the adiabatic flows inside stepped capillary tubes, the following design and thermodynamic parameters were considered (see Fig. 1): (i) Tube geometries for the main and auxiliary capillary tube (length, inner diameter and roughness); (ii) R-22 working refrigerant properties [15]. According to the values measured in copper tubes by Young et al., all the numerical cases were solved by considering a tube roughness of $0.3 \times 10^{-6}$ m [18].

Grid independent solutions were systematically obtained for the following numerical parameters: $n=1000$, $k=2.5$ and $f_{NR}=3.5$. According to the numerical results obtained by the model developed, the cases 1, 2, 4, 5, 7, 8 and 13 (see Table 1) of a stepped capillary tube in the cooling operation mode are working in a non-choke flow.

**TABLE 1. EXPERIMENTAL DATA INCLUDING GEOMETRIES AND WORKING CONDITIONS, AND COMPARISON OF THE NUMERICAL MODEL RESULTS AGAINST EXPERIMENTAL DATA FOR STEPPED CAPILLARY TUBES. FOR ALL THE CASES D1=1.7 mm, D2=1.3 mm, $\Delta T_{evap}=5^\circ C$ AND $T_{cool}=5^\circ C$.**

| Case | Geometry (mm), working conditions ($^\circ C$) | Measured mass flow rate ($kg/h$) | Model mass flow rate ($kg/h$) | Deviation (%) |
|------|---------------------------------------------|---------------------------------|------------------------------|---------------|
| c    | a   | s   | L1 | L2 | $T_{cool}$ | Heat | Cool | Heat | Cool | Heat | Cool |
| 1    | 200 | 600 | 45 | 44.37 | 46.40 | 43.15 | 46.15 | 2.75 | 0.54 |
| 2    | 200 | 600 | 50 | 47.76 | 49.31 | 46.16 | 49.29 | 3.35 | 0.04 |
| 3    | 200 | 600 | 55 | 50.90 | 52.11 | 49.28 | 52.07 | 3.18 | 1.07 |
| 4    | 400 | 600 | 45 | 43.48 | 45.26 | 41.81 | 45.39 | 3.84 | 0.29 |
| 5    | 400 | 600 | 50 | 45.88 | 47.27 | 44.73 | 48.63 | 2.51 | 2.88 |
| 6    | 400 | 600 | 55 | 49.94 | 51.34 | 47.78 | 51.92 | 4.33 | 1.13 |
| 7    | 600 | 600 | 45 | 41.38 | 42.94 | 40.55 | 44.57 | 2.01 | 3.33 |
| 8    | 600 | 600 | 50 | 45.23 | 45.48 | 43.43 | 47.56 | 3.98 | 4.57 |
| 9    | 600 | 600 | 55 | 48.22 | 48.90 | 46.38 | 50.79 | 3.82 | 3.87 |
| 10   | 600 | 200 | 45 | 53.70 | 60.71 | 54.14 | 66.75 | 0.82 | 9.95 |
| 11   | 600 | 200 | 50 | 57.57 | 63.43 | 57.67 | 71.46 | 0.17 | 9.22 |
| 12   | 600 | 200 | 55 | 61.22 | 68.17 | 61.33 | 76.40 | 0.18 | 12.07 |
| 13   | 600 | 400 | 45 | 47.76 | 51.37 | 45.79 | 52.21 | 4.12 | 1.64 |
| 14   | 600 | 400 | 50 | 50.50 | 54.82 | 48.95 | 55.74 | 3.07 | 1.68 |
| 15   | 600 | 400 | 55 | 55.06 | 58.22 | 52.21 | 59.37 | 5.18 | 1.98 |

Average deviation: 2.89 ±2.62

**Results of the comparative analyses**

Fig. 5 shows an excellent degree of correlation between all the numerical and experimental results compared. Average random deviation errors of $\pm 2.9\%$, and $\pm 3.6\%$ were computed for heating and cooling operation mode respectively; the average deviation error of all the experimental data set was $\pm 3.2\%$. 96.7% of the 30 data points evaluated are within an error of $\pm 10\%$, 86.7% are within $\pm 5\%$.

![Graph](image)

**FIGURE 5. NUMERICAL RESULTS VS. EXPERIMENTAL DATA POINTS GIVEN BY ZHAO ET AL. [2].**
Numerical pressure and quality distribution through a stepped capillary tube

In this section the numerical results obtained in the case 9 (Table 1) are used in order to shown the pressure and quality distribution along a stepped capillary tube depending if it is used in the heating or cooling mode.

**CONCLUSION**

A numerical model for analysing stepped capillary tubes expansion devices considering metastable region has been successfully developed. One-dimensional analysis of the governing equations (continuity, momentum, energy and entropy) was carried out. The numerical model implemented was solved on the basis of a finite volume formulation of the governing equations.

Comparison of numerical simulation results were successfully carried out against a mass flow rate experimental data for R-22 for heating and cooling operation mode in a residential heat pump system. An average deviation error of ±3.2% was computed between numerical model and experimental data, which also demonstrates the acceptable capability of the model developed for predicting the fluid flow processes.

**NOMENCLATURE**

- $A$: cross section area [m$^2$]
- $c_p$: specific heat at constant pressure [J kg$^{-1}$ K$^{-1}$]
- CV: control volume
- $D$: diameter [m]
- $f$: friction factor
- $f_{NR}$: change of variable (in the Newton-Raphson algorithm)
- $g$: gravity force [m s$^{-2}$]
- $G$: mass velocity [kg m$^{-2}$ s$^{-1}$]
- $h$: enthalpy [J kg$^{-1}$]
- $k$: mesh concentration factor
- $K$: Boltzmann’s constant 1.380662x10$^{-23}$ [J K$^{-1}$ mol$^{-1}$]
- $L$: length [m]
- $m$: mass flow rate [kg s$^{-1}$]
- $n$: number of CVs
- $p$: pressure [Pa]
- $p_v$: pressure of vaporization [Pa]
- $P$: perimeter [m]
- $s$: entropy [J kg$^{-1}$ K$^{-1}$]
- $t$: time [s]
- $T$: temperature [K]
- $v$: velocity [m s$^{-1}$]
- $w$: mass ratio of total saturated phase to total phase
- $x_g$: mass ratio of total saturated phase to total phase
- $x_v$: mass fraction (vapour quality)
- $z$: axial coordinate

**Greek letters**

- $\delta$: rate of convergence

**FIGURE 6. TYPICAL PRESSURE AND QUALITY DISTRIBUTION ALONG A STEPPED CAPILLARY TUBE OPERATING IN: (a) A HEATING MODE, (b) A COOLING MODE**

Figs 6a-b shown the results for the heating and cooling mode respectively. In these figures, it is possible to see how the sudden contraction (in the heating mode) or sudden enlargement (in the cooling mode) occurs inside the stepped capillary tube at 0.6 m of the inlet section. Choking flow occurs in both cases, it is a limited condition which occurs at the end of some capillary tubes when the velocity of vapour refrigerant phase is increased to sonic velocity due in part to the high gradient of pressure produce at the outlet of capillary tubes. Under this condition, the mass flow rate of refrigerant through the stepped capillary tube corresponds to the critical flow rate, which is the maximum mass flow rate that can be attained by reducing the downstream pressure under given upstream conditions.
Δt temporal discretization step [s]
ΔT_{sc} subcooling degree [K]
Δz spatial discretization step [m]
ε_i void fraction
ϕ generic dependent variable
ϕ two-phase frictional multiplier
θ inclination angle [rad]
µ dynamic viscosity [Pa s]
ρ density [kg m$^{-3}$]
σ surface tension [N m$^{-1}$]
τ shear stress [N m$^{-2}$]

Subscripts

- c critical
d discharge
eq thermodynamic equilibrium
g gas or vapour
l liquid
m superheated or metastable liquid
sat saturation
tp two-phase
w wall
z axial direction

Superscripts

- o previous instant
* previous iteration
- arithmetic average over a CV: $\bar{a} = (a_i + a_{i+1})/2$

$X_{i+1} = X_{i+1} - X_i$

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