Determination of the strong coupling constant from inclusive jet cross section data from multiple experiments

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Abstract Inclusive jet cross section measurements from the ATLAS, CDF, CMS, D0, H1, STAR, and ZEUS experiments are explored for determinations of the strong coupling constant \( \alpha_s(M_Z) \). Various jet cross section data sets are reviewed, their consistency is examined, and the benefit of their simultaneous inclusion in the \( \alpha_s(M_Z) \) determination is demonstrated. Different methods for the statistical analysis of these data are compared and one method is proposed for a coherent treatment of all data sets. While the presented studies are based on next-to-leading order in perturbative quantum chromodynamics (pQCD), they lay the groundwork for determinations of \( \alpha_s(M_Z) \) at next-to-next-to-leading order.

Key words. Jets, QCD Phenomenology, strong coupling constant

1 Introduction

The strong coupling constant, \( \alpha_s \), is one of the least precisely known fundamental parameters in the Standard Model of particle physics. Because of its importance for precision phenomenology at the LHC and elsewhere, large efforts have been undertaken in the past decades to reduce uncertainties in determinations of \( \alpha_s \) [1,2,3,4].

With the advent of modern particle detectors and sophisticated algorithms for their simulation and calibration, jet measurements have become very precise. Many determinations of \( \alpha_s \) in deep-inelastic scattering (DIS) and in hadron-hadron collisions are therefore based on measurements of the inclusive jet cross section, which is directly proportional to \( \alpha_s \) in DIS in the Breit frame and \( \alpha_s^2 \) in hadron-hadron collisions. Using the most precise predictions of perturbative quantum chromodynamics (pQCD) available at the time, all previous \( \alpha_s \) extractions (except for ref. [5]) were performed at next-to-leading order (NLO) in \( \alpha_s \). Their total uncertainty is dominated by the contribution related to the renormalisation scale dependence of the NLO pQCD results. The recent completion of next-to-next-to-leading order (NNLO) predictions for the inclusive jet cross section [6,7] promises a considerable reduction of the renormalisation scale dependence and will allow the inclusion of \( \alpha_s \) results from inclusive jet data in future determinations of the world average value of \( \alpha_s \) [1].

A determination of \( \alpha_s \) at NNLO from jet measurements in hadron-hadron collisions is still not readily achievable, because the new NNLO pQCD calculations are computationally very demanding and cannot yet be repeated quickly for different parton distribution functions (PDFs) or values of \( \alpha_s(M_Z) \). In preparation of such a determination, it is desirable to study a simultaneous analysis of data sets from different processes and experiments. This study includes an investigation of the consistency of the various data sets and an estimation of the reduction of the experimental contributions to the \( \alpha_s \) uncertainty. The groundwork for these two aspects is presented in this article.

We review inclusive jet cross section data over a wide kinematic range, from different experiments for various initial states and centre-of-mass energies, and study their potential for determinations of \( \alpha_s \). The consistency of the diverse data sets is examined and the benefit of their simultaneous inclusion is demonstrated. Different methods for the statistical analysis of the data are compared and one method is proposed for a coherent treatment of all data sets in an extraction of \( \alpha_s(M_Z) \).

The article is structured as follows: The experimental data sets and the theoretical predictions are introduced in sections 2 and 3, respectively. Methods and results from previous \( \alpha_s \) determinations by different experimental collaborations are discussed and employed in section 4. The strategy for a determination of \( \alpha_s \) from multiple data sets and the final result are presented in section 5.
2 Experimental data

The first measurement of the inclusive jet cross section has been performed in 1982 by the UA2 Collaboration at the SppS collider at a centre-of-mass energy of 540 GeV [8]. Further measurements have been conducted at centre-of-mass energies of

- 540 GeV, 546 GeV, and 630 GeV at the pp collider SppS by the UA1 [9,10] and UA2 experiments [11],
- 546 GeV, 630 GeV, 1.8 TeV, and 1.96 TeV at the Tevatron pp collider by the CDF [12,13,14,15,16,17] and D0 experiments [15,19,20],
- 300 GeV and 320 GeV at the ep collider HERA by the H1 [21,22,23,24,25,26,27,28] and ZEUS experiments [29,30,31,32,33,34,35],
- 200 GeV in pp collisions at RHIC by the STAR experiment [36],
- and of 2.76 TeV, 7 TeV, 8 TeV, and 13 TeV in pp collisions at the LHC by the ALICE [37], ATLAS [38,39,40,41,42,43], and CMS experiments [44,45,46,47,48,49,50,51].

While earlier measurements established the inclusive jet cross section as a useful quantity to study QCD, large experimental uncertainties limited their use for QCD phenomenology. When the NLO pQCD corrections were computed [51,52,53,54], studies revealed collinear- or infrared-safety issues in the jet definitions used in the experimental measurements [55]. These issues were subsequently addressed and improved jet definitions were developed [56,57,58,59,60] and applied in recent measurements.

Previously, $\alpha_s(M_2)$ determinations were based on inclusive jet cross section data from individual experiments, as summarised in Table 1. An extraction of $\alpha_s(M_2)$ from multiple inclusive jet cross section data sets has not been performed so far, except in the context of global PDF analyses, in which PDFs and $\alpha_s(M_2)$ are determined simultaneously. These analyses, however, require data for a variety of measured quantities [51,54,55,56,60]. In this article, $\alpha_s(M_2)$ is determined in a fit to multiple inclusive jet cross section measurements from experiments at HERA, RHIC, the Tevatron, and the LHC. The analysis is based on one selected measurement from each, the H1, ZEUS, STAR, CDF, D0, ATLAS, and CMS collaborations, as listed in Table 2.

Whenever experiments provide multiple measurements, we include those measured with a collinear- and infrared-safe jet algorithm ($k_T$ [61] or anti-$k_T$ [62]) and with a larger jet size parameter $R$, which improves the stability of fixed-order pQCD calculations. The STAR experiment published inclusive jet data collected by two different triggers with partially overlapping jet $p_T$ ranges. We choose the data set collected with the trigger covering the higher jet $p_T$ range from 7.6 GeV up to 50 GeV. The measurements from the STAR and D0 experiments are used as the midpoint cone jet algorithms (MP) [63]. The infrared-unsafe of this jet algorithm [54] prohibits NNLO pQCD predictions for these data sets, but it does not affect calculations at NLO. Four new measurements [28,50,42,43] could not be included in this study; they are left for a future extension.

3 Theoretical predictions and tools

Predictions for the inclusive jet cross section in processes with initial-state hadrons are calculated as the convolution of the partonic cross section $\hat{\sigma}$ (computed in pQCD) and the PDFs of the hadron(s). The inclusive jet cross section in hadron-hadron collisions can be written as

$$\sigma_{\text{pQCD,ih}}(\mu_r, \mu_t) = \sum_{i,j} \int dx_1 \int dx_2 f_{i/h_1}(x_1, \mu_t) f_{j/h_2}(x_2, \mu_t) \hat{\sigma}_{ij\rightarrow \text{jet}+X}(\mu_r, \mu_t),$$

where the sum is over all combinations of parton flavors $i$ and $j$ (quarks, anti-quarks, and the gluon). The $f_{i/h_1}$ denote the PDFs for the parton flavours $i$ or $j$ in the initial-state hadrons $h_1$ and $h_2$, and $x_1$ and $x_2$ correspond to the fractional hadron momenta carried by the partons $i$ and $j$, respectively. The partonic cross section $\hat{\sigma}_{ij\rightarrow \text{jet}+X}$ is computed as a perturbative expansion in $\alpha_s$ as

$$\hat{\sigma}_{ij\rightarrow \text{jet}+X}(\mu_r, \mu_t) = \sum_n \alpha_s^n(\mu_r) c_{ij\rightarrow \text{jet}+X}^{(n)}(\mu_t, \mu_t),$$

where the $c_{ij\rightarrow \text{jet}+X}^{(n)}$ are computed from the pQCD matrix elements and the sum is over all orders of $\alpha_s$ taken into account in the perturbative calculation. The renormalisation and factorisation scales are labelled $\mu_r$ and $\mu_t$, respectively. For inclusive jet production in hadron-hadron collisions, the first non-vanishing order (i.e. the leading order, LO) is given by $n = 2$, while $n = 3$ corresponds to the NLO corrections. For inclusive jet production in DIS in the Breit frame the partonic cross sections are convoluted with a single PDF and the LO (NLO) contribution is given by $n = 1$ ($n = 2$). Hence, inclusive jet production in pp, pp, and ep collisions is sensitive to $\alpha_s$ already at LO.

For transverse jet momenta at the TeV scale accessible at the LHC, electroweak (EW) tree-level effects of $\mathcal{O}(\alpha_s \alpha, \alpha^2)$ and loop effects of $\mathcal{O}(\alpha_s^2)$ become sizeable [71]. A recent study of the complete set of QCD and EW NLO corrections has been presented in ref. [72].

Non-perturbative (NP) corrections to the cross section due to multiparton interactions and hadronisation can be estimated by using Monte Carlo (MC) event generators. An overview of MC event generators for the LHC is presented in ref. [73]. The size of this correction depends on the jet size $R$, shrinks with increasing jet $p_T$, and becomes negligible at the TeV scale. The total theory prediction for the inclusive jet cross section is given by

$$\sigma_{\text{theory}} = \sigma_{\text{pQCD}} \cdot c_{\text{EW}} \cdot c_{\text{NP}},$$

where $c_{\text{EW}}$ and $c_{\text{NP}}$ are the correction factors for electroweak and non-perturbative corrections, respectively.

The partonic cross section is computed at NLO accuracy for five massless quark flavours using the NLOJet++ program version 4.1.3 [74,75] within the fastNLO framework at version 2 [76,77] to allow us fast recalculations for varying PDFs, scales $\mu_r$ and $\mu_t$, and assumptions on
Table 1. Summary of previous determinations of $\alpha_s(M_Z)$ from inclusive jet cross sections. The upper part lists the recent $\alpha_s(M_Z)$ extractions from double-differential inclusive jet cross sections by experimental collaborations that are studied in more detail in this work. The middle and lower parts summarise further determinations of $\alpha_s(M_Z)$ by experimental collaborations and by independent authors, respectively. The results in refs. [61] and [67] are reported for approximate NNLO (aNNLO) and NLO used for the pQCD predictions. In ref. [61] only 22 out of the 110 D0 data points were used in the $\alpha_s(M_Z)$ extraction; the decomposition of the uncertainties is only provided for the aNNLO result. In case of ref. [66], we only consider scale, PDF, and NP related uncertainties as theoretical uncertainty for reasons of comparability to the other listed results.

| Publication | data | comment | $\alpha_s(M_Z)$ |
|-------------|------|---------|----------------|
| H1 [27]     | H1   | HERA II, high $Q^2$ | 0.1174 (22)exp (50)theo |
| D0 [61]     | D0   | aNNLO, 22 points | 0.1161 (1)exp (1)theo |
| D0 [61]     | D0   | NLO, 22 points | 0.1202 (2)hot |
| CMS [62]    | CMS  | 7 TeV, 5.0 fb$^{-1}$ | 0.1185 (19)exp (1)theo |
| H1 [21]     | H1   | HERA I, $\sqrt{s} = 300$ GeV | 0.1186 (30)exp (51)theo |
| H1 [24]     | H1   | HERA I, $\sqrt{s} = 320$ GeV | 0.1193 (14)exp (13)theo |
| H1 [26]     | H1   | HERA I, $\sqrt{s} = 320$ GeV, low $Q^2$ | 0.1180 (18)exp (18)theo |
| H1 [5]      | H1   | HERA I+II, NNLO | 0.1152 (20)exp (27)theo |
| ZEUS [30]   | ZEUS | $\sqrt{s} = 300$ GeV, $Q^2 > 500$ GeV$^2$ | 0.1212 (5)exp (5)theo |
| ZEUS [33]   | ZEUS | $d\sigma/dQ^2$, $Q^2 > 500$ GeV$^2$ | 0.1207 (5)exp (5)theo |
| CDF [63]    | CDF  | 1.8 TeV, 87 pb$^{-1}$ | 0.1178 (5)exp (5)theo |
| CMS [60]    | CMS  | 8 TeV, 19.7 fb$^{-1}$ | 0.1164 (5)exp (5)theo |

W.T. Giele et al. [64], CDF [65] 1.8 TeV, 4.2 pb$^{-1}$ | 0.121 (8)exp (5)theo |
B. Malaeucci et al. [66], ATLAS [39] 7 TeV, 37 pb$^{-1}$ | 0.1151 (47)exp (51)theo |
T. Bieköter et al. [67], H1 [27] aNNLO | 0.122 (2)exp (13)theo |
T. Bieköter et al. [67], H1 [27] NLO | 0.115 (2)exp (5)theo |

Table 2. Overview of the inclusive jet data sets used in the $\alpha_s$ determinations. For each data set the process (proc), the centre-of-mass energy $\sqrt{s}$, the integrated luminosity $\mathcal{L}$, the number of data points, and the jet algorithm are listed. In case of ep collider data, the kinematic range may be defined by the four-momentum transfer squared $Q^2$, the inelasticity $y_{\text{DIS}}$, or the angle of the hadronic final state $|\cos \gamma_h|$ of the NC DIS process. In all cases, jets are required to be within a given range of pseudorapidity $\eta$ or rapidity $y$ in the laboratory frame.

| Data | proc | $\sqrt{s}$ [TeV] | $\mathcal{L}$ [fb$^{-1}$] | no. of points | jet algorithm | $p_T$, $E_T$-range [GeV] | other kinematic ranges |
|------|------|-----------------|------------------|--------------|---------------|------------------------|------------------------|
| H1   | ep   | 0.32 | 0.35 | 24 | $k_T$, $R = 1.0$ | $7 < p_T < 50$ | 0.2 < $y_{\text{DIS}}$ < 0.7 |
|      |      |      |      |    |               |            | −1.0 < $\eta$ < 2.5 |
| ZEUS | ep   | 0.32 | 0.082 | 30 | $k_T$, $R = 1.0$ | $E_T > 8$ | $Q^2 > 125$ GeV$^2$ | $|\cos \gamma_h| < 0.65$ |
|      |      |      |      |    |               |            | −2.0 < $\eta$ < 1.5 |
| STAR | pp   | 0.20 | 0.0003 | 9 | MP, $R = 0.4$ | $7.6 < p_T < 48.7$ | $0.2 < |y| < 0.8$ |
| CDF  | pp   | 1.96 | 1.0 | 76 | $k_T$, $R = 0.7$ | $54 < p_T < 527$ | $|y| < 2.1$ |
| D0   | pp   | 1.96 | 0.7 | 110 | MP, $R = 0.7$ | $50 < p_T < 665$ | $|y| < 2.0$ |
| ATLAS| pp   | 7.0  | 4.5  | 140 | anti-$k_T$, $R = 0.6$ | $100 < p_T < 1992$ | $|y| < 3.0$ |
| CMS  | pp   | 7.0  | 5.0  | 133 | anti-$k_T$, $R = 0.7$ | $114 < p_T < 2116$ | $|y| < 3.0$ |
\(\alpha_s(M_Z)\). Jet algorithms are taken either from the FastJet software library [78] or, for jet cross sections in DIS, from NNLOjet++ . The PDFs are evaluated via the LHAPDF interface [79,80] at version 6. The running of \(\alpha_s(\mu_r)\) is performed at 2-loop order using the package CRunDec with five massless quark flavours [81,82]. The minimal subtraction (\(\text{MS}\)) scheme \([83,84,85]\) has been adopted for the renormalisation procedure in these calculations.

For the computation of the inclusive jet cross section in hadron-hadron collisions, the renormalisation and factorisation scales, \(\mu_r\) and \(\mu_F\), are identified with each jet’s \(p_T\), i.e. \(\mu_r = \mu_F = p_T^{\text{jet}}\). In neutral current (NC) DIS, the scales are chosen to be \(\mu_r^2 = \frac{1}{2} (Q^2 + (p_T^{\text{jet}})^2)\) and \(\mu_F^2 = Q^2\) as used by the H1 Collaboration [24]. Alternative scale choices have been discussed with respect to NNLO predictions \([78,87]\), but are beyond the scope of this article.

The EW corrections, \(c_{\text{EW}}\), relevant for the LHC data are provided by the experimental collaborations together with the data, based on ref. [71]. These are considered to have negligible uncertainties. Due to restrictions of the scale choices in this calculation, the leading jet’s transverse momentum, \(p_T^{\text{jet}}\), is used to define the scales \(\mu_r\) and \(\mu_F\). The NP correction factors \(c_{\text{NP}}\), except for the STAR data [88], are also provided by the experimental collaborations, together with an estimate of the corresponding uncertainty [27,32,33,16,20,61,41,46,62].

### 4 Comparison of three extraction methods for \(\alpha_s(M_Z)\)

Commonly, the value of \(\alpha_s(M_Z)\) is determined from inclusive jet cross sections in a comparison of pQCD predictions to the measurements. These \(\alpha_s(M_Z)\) results therefore depend on details of the extraction method such as the treatment of uncertainties in the characterisation of differences between theory and data, or the evaluation and propagation of theoretical uncertainties to the final result.

An overview of previous determinations of \(\alpha_s(M_Z)\) from fits to inclusive jet cross section data is provided in table [1]. We choose the three \(\alpha_s(M_Z)\) determinations performed by the CMS [62], D0 [61], and H1 [27] collaborations listed in the upper part of table [1] for further study.

The three extraction methods differ in the following aspects:

- the definition of the \(\chi^2\) function to quantify the agreement between theory and data,
- the uncertainties considered in the \(\chi^2\) function,
- the strategy to determine the central result for \(\alpha_s(M_Z)\),
- the propagation of the uncertainties to the value of \(\alpha_s(M_Z)\),
- the choice of PDF sets,
- the consideration of the \(\alpha_s(M_Z)\) dependence of the PDFs, and
- the treatment of further theoretical uncertainties.

To study the impact of these differences, we have implemented the three methods in our computational framework and will refer to them as “CMS-type”, “D0-type”, and “H1-type”, respectively. Each method is employed to extract \(\alpha_s(M_Z)\) from each of the individual data sets selected in section [2] cf. also table [2]. The experimental uncertainties and their correlations are treated according to the respective prescriptions by the experiments. The CMS result was obtained with the CT10 PDF set [89], and the D0 and H1 results with MSTW2008 PDFs [90]. The CMS-type and D0-type methods use the entire \(\alpha_s^{\text{PDF}}(M_Z)\) series available for the PDF set, whereas the H1-type method uses a PDF determined with a value of \(\alpha_s^{\text{PDF}}(M_Z) = 0.1180\). The resulting \(\alpha_s(M_Z)\) values are listed in table [3].

In a first step, these results are compared to the ones obtained by the CMS [62], D0 [61], and H1 [27] collaborations as listed in table [1]. All three central results are reproduced, the H1 result exactly, and the CMS and D0 results within +0.0003 and +0.0001. Such small differences can easily be caused already by using different versions of LHAPDF (e.g. changes from version 5 to version 6). The experimental uncertainties of the CMS and H1 analyses are exactly reproduced [4].

In a second step, the \(\alpha_s(M_Z)\) results and their experimental uncertainties are compared to each other and their dependencies on the extraction method and PDFs are studied. The \(\alpha_s(M_Z)\) results determined for each data set are displayed in figure [1] (top row) for the three different extraction methods using CT10 PDFs (left) and MSTW2008 PDFs (right). For the STAR data, \(\alpha_s(M_Z)\) results cannot be determined in case of the CMS-type and D0-type methods with MSTW2008 PDFs, since no local \(\chi^2\) minima are found. In all other cases the \(\alpha_s(M_Z)\) results obtained with MSTW2008 PDFs are rather independent of the extraction method for all data sets. This is different when using CT10 PDFs: While in this case the extraction method has little impact on the \(\alpha_s(M_Z)\) results from HERA data (H1 and ZEUS), it notably affects the results for the LHC data (ATLAS and CMS), and has large effects for the Tevatron data (CDF and D0). In the latter cases, the D0-type method produces significantly lower \(\alpha_s(M_Z)\) results as compared to the other two methods.

The \(\chi^2/n_{\text{dof}}\) values for the \(\alpha_s(M_Z)\) extractions are displayed in figure [1] (bottom row) for the three extraction methods using CT10 PDFs (left) and MSTW2008 PDFs (right). Overall, the fits exhibit reasonable values of \(\chi^2/n_{\text{dof}}\), thus indicating agreement between theory and data. Exceptions are observed for the ZEUS data with rather low values of \(\chi^2/n_{\text{dof}}\) and for the ATLAS data, where the values of \(\chi^2/n_{\text{dof}}\) are large as also observed elsewhere [12,91,92].

The PDF dependence is displayed in figure [2] where the \(\alpha_s(M_Z)\) results for CT10 and MSTW2008 PDFs are compared to each other, both obtained using the H1-type method. While the PDF choice has no significant effect for the results from the H1, ZEUS, and D0 data, smaller variations are seen for the CDF data, and a large dependence for the ATLAS and CMS data. Re-investigating this

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1 For the D0 analysis, the decomposition of uncertainties has been published only for their central result based on approximate NNLO pQCD and hence a comparison of the experimental uncertainty on \(\alpha_s(M_Z)\) for the NLO result is not possible.
Figure 1. Values of $\alpha_s(M_Z)$ with experimental uncertainties obtained using the three extraction methods CMS-type, D0-type, and H1-type. The upper row compares the results for each data set employing the PDF set CT10 (left) or MSTW2008 (right). In addition, the world average value [1] is shown together with a band representing its uncertainty. The bottom row displays the values of $\chi^2/n_{\text{dof}}$ for each fit using the CT10 (left) or MSTW2008 PDF set (right). The colours illustrate the values of $\chi^2/n_{\text{dof}}$. For the STAR data and the MSTW2008 PDF set no local minimum was found in case of the CMS-type and D0-type fits (blank areas).
measurements from different colliders are considered to be taken into account. For the present study, uncertainties of experimental and theoretical uncertainties as a source of potential correlations in this study. For the determination of NP effects and their uncertainties various methods and MC event generators have been employed \[27,32,36,88,16,20,61,41,62\]. While this study. For the determination of NP effects and their uncertainties various methods and MC event generators have been employed \[27,32,36,88,16,20,61,41,62\]. While a consistent derivation of these corrections with corresponding correlations is desirable, this is beyond the scope of this analysis. Hence, the NP correction factors and their uncertainties are considered to be uncorrelated between the different data sets. In contrast, the PDF uncertainties and the uncertainties due to the renormalisation and factorisation scale variations are treated as fully correlated; the relative variations with respect to the nominal scales are performed simultaneously for all data sets.

The method employed for the simultaneous \(\alpha_s(M_Z)\) extraction combines components of the individual methods outlined in the previous section and is referred to as “CMS-type method”. The central \(\alpha_s(M_Z)\) result is found in an iterative \(\chi^2\) minimisation procedure adopted from the H1-type method, where a normal distribution is assumed for the relative uncertainties. The exact \(\chi^2\) formula is given by equation \[4\] of appendix \[5\]. Whereas in the H1-type \(\chi^2\) expression, only experimental uncertainties are taken into account, the common-type method also accounts for the NP and PDF uncertainties in the \(\chi^2\) expression, as in the CMS-type and D0-type methods. This \(\chi^2\) definition treats variances as relative values and thus has advantages, e.g. when numerically inverting the covariance matrix. Moreover, uncertainties of experimental and theoretical origin are put on an equal footing. As in the H1-type method, and in contrast to the CMS-type and D0-type ones, only PDF sets obtained with a fixed value of \(\alpha_s(M_Z) = 0.1180\) are employed in the determination of the central \(\alpha_s(M_Z)\) result, leaving the \(\alpha_s\) dependence of the PDFs to be treated as a separate uncertainty.

Table 3. Values of \(\alpha_s(M_Z)\) with experimental uncertainties obtained using the three extraction methods CMS-type, D0-type, and H1-type together with the CT10 or MSTW2008 PDF set at NLO. The world average value \[1\] is shown together with a band representing its uncertainty.

| Fit method  | CMS-type | D0-type | H1-type |
|-------------|----------|---------|---------|
| PDF set     | MSTW2008 | CT10    | MSTW2008 | CT10    | MSTW2008 | CT10    |
| H1          | 0.1172 (28) | 0.1172 (28) | 0.1161 (27) | 0.1164 (26) | 0.1174 (22) | 0.1180 (22) |
| ZEUS        | 0.1213 (28) | 0.1223 (29) | 0.1210 (12) | 0.1218 (20) | 0.1231 (30) | 0.1236 (30) |
| STAR        | 0.1193 (68) | 0.1205 (11) | 0.1159 (16) | 0.1280 (11) |
| CDF         | 0.1217 (17) | 0.1265 (27) | 0.1202 (10) | 0.1162 (20) | 0.1217 (35) | 0.1265 (37) |
| D0 (22 pts., NLO) | 0.1226 (32) | 0.1237 (36) | 0.1203 (42) | 0.1191 (25) | 0.1219 (50) | 0.1232 (51) |
| ATLAS       | 0.1220 (9) | 0.1258 (15) | 0.1204 (28) | 0.1241 (9) | 0.1206 (15) | 0.1270 (16) |
| CMS         | 0.1162 (14) | 0.1188 (19) | 0.1158 (12) | 0.1162 (19) | 0.1140 (21) | 0.1217 (23) |

Figure 2. Comparison of the \(\alpha_s(M_Z)\) results with their experimental uncertainties obtained using the H1-type extraction methods for CT10 and MSTW2008 PDFs. The world average value \(\overline{\alpha_s}\) is shown together with a band representing its uncertainty.

PDF dependence in the context of a common determination of \(\alpha_s(M_Z)\) as described in the next section, we observe that differences with respect to the updated PDF sets, CT14 [93] and MMHT2014 [60], are reduced.

5 Determination of \(\alpha_s(M_Z)\) from multiple inclusive jet data sets

The analysis of multiple data sets requires their correlations to be taken into account. For the present study, measurements from different colliders are considered to be uncorrelated because of the largely complementary kinematic ranges of the data sets and different detector calibration techniques. Furthermore, investigations with respect to H1 and ZEUS data [58], CDF and D0 data [94], or ATLAS and CMS data [95,96], did not identify a relevant source of experimental correlation. This only leaves theoretical uncertainties as a source of potential correlations in this study. For the determination of NP effects and their uncertainties various methods and MC event generators have been employed [27,32,36,88,16,20,61,41,62]. While a consistent derivation of these corrections with corresponding correlations is desirable, this is beyond the scope of this analysis. Hence, the NP correction factors and their uncertainties are considered to be uncorrelated between the different data sets. In contrast, the PDF uncertainties and the uncertainties due to the renormalisation and factorisation scale variations are treated as fully correlated; the relative variations with respect to the nominal scales are performed simultaneously for all data sets.
In summary, the individual contributions to the total uncertainty of the \( \alpha_s(M_Z) \) result are evaluated as follows: The experimental uncertainty (\( \text{exp} \)) is obtained from the Hesse algorithm \cite{57} when performing the \( \alpha_s(M_Z) \) extraction with only the uncertainties of the measurements included. The NP and PDF uncertainties are derived by repeating the \( \alpha_s(M_Z) \) extraction while successively including the corresponding uncertainty contributions and calculating the quadratic differences. Further sources of systematic effects are considered as follows:

- The “PDFs” uncertainty accounts for the initial assumption of \( \alpha_s^{\text{PDF}}(M_Z) = 0.1180 \) made in the PDF extraction, which is not necessarily consistent with the value of \( \alpha_s(M_Z) \) used in the pQCD calculation. It is calculated as the maximal difference between any of the results obtained with PDF sets determined for \( \alpha_s^{\text{PDF}}(M_Z) = 0.1170, 0.1180, \) and 0.1190, and therefore covers a difference of \( \Delta \alpha_s^{\text{PDF}}(M_Z) = 0.0020 \), which is somewhat more conservative than the recommendation in ref. \cite{59}.

- The “PDFset” uncertainty covers differences due to the considered PDF set. These are caused by assumptions made on the data selection, parameterisation, parameter values, theoretical assumptions, or the analysis method for the PDF determination. It is defined as half of the width of the envelope of the results obtained with the PDF sets CT14 \cite{93}, HERAPDF2.0 \cite{58}, MMHT2014 \cite{60}, NNPDF3.0 \cite{99} and ABMP16 \cite{100,101}.

- The uncertainty due to variations of the renormalisation and factorisation scales customarily is taken as an estimate for the error of a fixed-order calculation caused by the truncation of the perturbative series. It is obtained using six additional \( \alpha_s(M_Z) \) determinations, in which the nominal scales \((\mu_r, \mu_f)\) are varied by the conventional factors of \((1/2, 1/2), (1/2, 2), (1, 1/2), (1, 2), (2, 1), \) and \((2, 2)\). The scale factor combinations of \((1/2, 2)\) and \((2, 1/2)\) are customarily omitted \cite{102,103,104}.

The NP, PDF, PDF\( \alpha_s \), PDFset, and scale uncertainties are added in quadrature to give the theoretical uncertainty (theo). The total uncertainty (tot) further includes the experimental uncertainty.

In the previous section, cf. figure \[4\] it was found that the \( \chi^2/\text{dof} \) values differ significantly from unity for some of the data sets. This necessitates to investigate in further detail the consistency of the data within an individual data set as well as among the different data sets. Moreover, new PDF sets have become available. Therefore, the common-type method is employed to extract \( \alpha_s(M_Z) \) from each individual data set and for each of the PDFs ABMP16, CT14, HERAPDF2.0, MMHT2014, and NNPDF3.0. The resulting \( \chi^2/\text{dof} \) values are displayed in figure \[3\] left. Detailed listings of the \( \alpha_s(M_Z) \) results and their uncertainties are given in appendix \[B\].

For a given data set, \( \chi^2/\text{dof} \) is rather independent of the PDF set used for the predictions and varies between 0.8 and 1.2. These values indicate reasonable agreement of the predictions with the data. Exceptions are rather low values of \( \chi^2/\text{dof} \) around 0.54 found for all PDF sets with the ZEUS data and large \( \chi^2/\text{dof} \) values between 1.9 and 3.5 exhibited by the ATLAS data, also for all PDFs. Exceptionally large \( \chi^2/\text{dof} \) values appear for the Tevatron or LHC data together with theory predictions using the HERAPDF2.0 set, and for the STAR data in conjunction with the ABMP16 PDF set.

To further investigate the consistency among the data sets, a series of \( \alpha_s(M_Z) \) extractions is performed, in which \( \alpha_s(M_Z) \) is determined simultaneously from all data sets but one. This is repeated for each PDF set. The resulting \( \chi^2/\text{dof} \) values are displayed in figure \[3\] right. Apparently, the exclusion of the ATLAS data leads to significantly smaller \( \chi^2/\text{dof} \) values independent of the PDF set used. This hints at a compatibility issue when using all data sets together, which is not present when the ATLAS data set is ignored. Therefore, we choose to exclude the ATLAS data for our main result, which is thus obtained from the CDF, CMS, D0, H1, STAR, and ZEUS inclusive jet data. The choice of the NNPDF3.0 set for the central result yields

\[
\alpha_s(M_Z) = 0.1192(12)_{\text{exp}}(5)_{\text{NP}}(7)_{\text{PDF}}(5)_{\text{PDFs}}(11)_{\text{PDFset}}(50)_{\text{scale}},
\]

with \( \chi^2 = 328 \) for 381 data points. This result is consistent with the world average value of 0.1181 (11) \[1\]. The experimental uncertainty for the extraction from multiple data sets is significantly smaller than each of the experimental uncertainties reported previously for the separate \( \alpha_s(M_Z) \) determinations. Results obtained with different PDF sets constitute the PDFset uncertainty. They are listed in table \[4\] together with the PDFs and PDF\( \alpha_s \) uncertainties as appropriate for the respective PDF set. The corresponding values of \( \chi^2/\text{dof} \) can be read off from row six of figure \[3\] right. Other uncertainties remain unchanged in the leading digit as compared to the ones obtained for the NNPDF3.0 PDFs.

The \( \alpha_s(M_Z) \) values from fits using the various PDF sets given in table \[4\] are found to be consistent within the experimental uncertainty. The NP, PDF, and PDF\( \alpha_s \) uncertainties are smaller than the experimental uncertainty, while the PDFset uncertainty is of a similar size as the experimental one. The scale uncertainty is the largest individual uncertainty and is more than three times larger than any other uncertainty. Results of the \( \alpha_s(M_Z) \) extractions from single data sets, cf. appendix \[B\] from the simultaneous \( \alpha_s(M_Z) \) extraction from all data sets, and the world average \[1\] are compared in figure \[4\] and are seen to be consistent with each other.

The ratio of data to the predictions as a function of jet \( p_T \) for all selected data sets is presented in figure \[5\]. The predictions are computed for \( \alpha_s(M_Z) = 0.1192 \) as obtained in this analysis. Visually, all data sets are well described by the theory predictions.

\footnote{For HERAPDF2.0, the PDF uncertainty does not include the “model” or “parameterisation” uncertainties as those are represented here by the PDFset uncertainty.}


Figure 3. Left: Illustration of the $\chi^2/n_{\text{dof}}$ values for fits to each data set individually. Right: Illustration of the $\chi^2/n_{\text{dof}}$ values for simultaneous fits omitting a single data set at a time. The included or respectively excluded data set is indicated on the $y$ axis and the PDF set on the $x$ axis. The fits are performed for each PDF set in the envelope definition of the PDFset uncertainty.

| PDF set          | $\alpha_s(M_Z)$ | Exp | NP | PDF | PDF$_{\alpha_s}$ | PDFset | Scale | Theo | Total |
|------------------|-----------------|-----|----|-----|------------------|--------|-------|------|-------|
| ABMP16           | 0.1203          | 4   | 3  | 4   | 3                |        |       | -63  | +64   |
| CT14             | 0.1206          | 10  | 2  | 10  | 2                |        |       | -58  | +59   |
| HERAPDF2.0       | 0.1184          | 6   | 2  | 6   | 2                |        |       | -63  | +64   |
| MMHT2014         | 0.1194          | 7   | 3  | 7   | 3                |        |       | -51  | -53   |
| NNPDF3.0         | 0.1192          | 12  | 5  | 7   | 5                | 11     | -48   | -48  | -48   |

Table 4. Values of $\alpha_s(M_Z)$ for the simultaneous fit to the H1, ZEUS, STAR, CDF, D0, and CMS data using the common-type method for various PDF sets. The experimental, NP, PDFset, and scale uncertainties remain mostly unchanged under a change of the PDF set and are quoted only once for NNPDF3.0.

6 Summary and outlook

Inclusive jet cross section data from different experiments at various particle colliders with jet transverse momenta ranging from 7 GeV up to 2 TeV are explored for determinations of $\alpha_s(M_Z)$ using next-to-leading order predictions. Previous $\alpha_s(M_Z)$ determinations reported by the CMS, D0, and H1 collaborations [62,61,27] are taken as a baseline, and these $\alpha_s(M_Z)$ extraction methods, which differ in various aspects, are applied to inclusive jet cross section data measured by the ATLAS, CDF, CMS, D0, H1, STAR, and ZEUS experiments [41,16,62]. Differences among the $\alpha_s(M_Z)$ results due to the extraction technique are found to be negligible in most cases. A new extraction method is proposed, which combines aspects of the baseline approaches above.

In a statistical analysis, data measured by the CDF, CMS, D0, H1, STAR, and ZEUS experiments are found to be well described by pQCD predictions at next-to-leading order, and hence are considered to be mutually consistent. Moreover, the values of $\alpha_s(M_Z)$ determined from each individual data set are found to be consistent among each other. By determining $\alpha_s(M_Z)$ simultaneously from these data, the experimental uncertainty of $\alpha_s(M_Z)$ is reduced to 1.0%, as compared to 1.9% when only the single most precise data set of that selection is considered.

The largest contribution to the uncertainty of $\alpha_s(M_Z)$ originates from the renormalisation scale dependence of the next-to-leading order pQCD calculation. This uncertainty is expected to be reduced once the next-to-next-to-leading order predictions become available for such studies. Furthermore, a reevaluation of the non-perturbative corrections and their uncertainties for all data sets in a consistent manner is recommended for a determination of $\alpha_s(M_Z)$ at high precision. The presented study and the developed analysis framework provide a solid basis for fu-
Figure 4. The $\alpha_s(M_Z)$ values from fits to individual data sets are compared to our simultaneous fit to H1, ZEUS, STAR, CDF, D0, and CMS data, and to the world average value [1]. The inner error bars represent the experimental uncertainty and the outer ones the total uncertainty. For reasons explained in the text, the ATLAS data are excluded from the common fit and only the result of a separate fit is indicated by the open circle.

We thank our colleagues in the CMS, D0, and H1 collaborations for fruitful discussions, and K. Bjoerk and D. Reichelt for early related studies. K. Rabbertz thanks G. Flouris, P. Kokkas, and the colleagues from the PROSA Collaboration. D. Savoie acknowledges the support by the DFG-funded Doctoral School “Karlsruhe School of Elementary and Astroparticle Physics: Science and Technology”. M. Wobisch also wishes to thank the Louisiana Board of Regents Support Fund for the support through the Eva J. Cunningham Endowed Professorship.

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### Inclusive Jet Production

| Experiment | Data / Theory |
|------------|---------------|
| H1 ep \(\sqrt{s} = 319 \text{ GeV}\) | Data / Theory |
| ZEUS ep \(\sqrt{s} = 318 \text{ GeV}\) | Data / Theory |
| STAR pp \(\sqrt{s} = 200 \text{ GeV}\) | Data / Theory |
| CDF \(\sqrt{s} = 1.96 \text{ TeV}\) | Data / Theory |
| D0 pp \(\sqrt{s} = 1.96 \text{ TeV}\) | Data / Theory |
| ATLAS pp \(\sqrt{s} = 7 \text{ TeV}\) | Data / Theory |
| CMS pp \(\sqrt{s} = 7 \text{ TeV}\) | Data / Theory |

**The NLO predictions are computed with the NNPDF3.0 PDF set for the fitted \(\alpha_s(M_Z)\) value of 0.1192, which is determined considering all presented data except for the ATLAS data set. They are complemented with non-perturbative corrections and, where appropriate, with electroweak corrections.**

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**Figure 5.** Ratio of data over theory for the selected inclusive jet cross sections listed in table 2 as a function of jet \(p_T\). The NLO predictions are computed with the NNPDF3.0 PDF set for the fitted \(\alpha_s(M_Z)\) value of 0.1192, which is determined considering all presented data except for the ATLAS data set. They are complemented with non-perturbative corrections and, where appropriate, with electroweak corrections.
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A Definition of the $\chi^2$ expression for the common-type method

In the common-type method the $\chi^2$ expression, which is subject to the minimisation algorithm, is defined as

$$\chi^2 = \sum_{ij} \left( \frac{\log m_i}{t_i} \right) \left[ (V_{\exp} + V_{\PDF} + V_{\NP})^{-1} \right]_{ij} \left( \frac{\log m_j}{t_j} \right),$$

where the sum runs over all data points $i$ and $j$ of the measured cross sections $m_i$, $m_j$ and theory predictions $t_i$, $t_j$. The covariance matrices $V$ represent the relative experimental, PDF, and NP uncertainties. A similar $\chi^2$ definition, taking into account only experimental uncertainties, was employed by the H1 Collaboration. For the calculation of the covariance matrices, all uncertainties are symmetrised, if necessary, by averaging the corresponding “up” and “down” shifts in $\alpha_s(M_Z)$ in quadrature while keeping the sign of bin-to-bin correlations. Uncertainty contributions to the total covariance matrix that are fully correlated across all observable bins in equation (4) can alternatively be expressed in an equivalent form with nuisance parameters.

B Common-type extraction of $\alpha_s(M_Z)$ from single inclusive jet data sets

Detailed results of the common-type method applied to the individual data sets are given in table 5. The result for the H1 data agrees with the value published in ref. [27]. Even though using the full D0 data set with 110 points, the extracted $\alpha_s(M_Z)$ value is consistent with the value achieved by the D0 Collaboration at NLO for a subset of 22 points in ref. [61]. For the CMS measurement, the common-type method leads to a consistent but somewhat lower result than reported in ref. [62] for various PDFs. Our result for the ZEUS data is compatible with the value obtained by the ZEUS Collaboration from a single-differential variant of the measurement in a reduced phase space as published in ref. [43]. With respect to the ATLAS, CDF, and STAR inclusive jet data, this study constitutes the first $\alpha_s(M_Z)$ determination from either data set. Within uncertainties, all $\alpha_s(M_Z)$ values are consistent with each other and with the world average.

The individual uncertainties compare as follows:
- The experimental uncertainty of $\alpha_s(M_Z)$ is roughly comparable between experiments at the same collider. It is largest for the STAR data, and smallest for the ATLAS data.
- The NP uncertainties are found to vary significantly, even between data sets in similar kinematic regions, for instance between CDF and D0. In case of the LHC experiments the NP uncertainties appear to be negligible.
- The PDF uncertainty as estimated with the NNPDF3.0 PDF set is smaller than the experimental uncertainty. For the HERA data, the PDF uncertainty is found to be moderately smaller than for Tevatron or LHC data as observed also with other PDF sets.
- For all data sets, the PDF $\alpha_s$ uncertainty is rather small. This observation justifies to neglect the $\alpha_s$ dependence of the PDFs in the $\alpha_s(M_Z)$ determinations and to assign a separately derived uncertainty instead.
- The PDFset uncertainty constitutes the largest contribution of the PDF related ones.
- The largely dominating scale uncertainty is of similar size in case of HERA and LHC data and somewhat larger for Tevatron data or the STAR experiment.

The results of $\alpha_s(M_Z)$ determinations from single measurements for the alternative PDF sets ABMP16, CT14, HERAPDF2.0, and MMHT2014 PDF sets are provided in columns 2-5 of table 6. The envelope constructed from these four values together with the NNPDF3.0 result constitutes the PDFset uncertainty shown in column seven of table 6. The further columns in table 6 present the PDFset uncertainty constitutes the largest contribution of the PDF related ones. The PDFset uncertainty constitutes the largest contribution of the PDF related ones. The PDFset uncertainty constitutes the largest contribution of the PDF related ones. The PDFset uncertainty constitutes the largest contribution of the PDF related ones. The PDFset uncertainty constitutes the largest contribution of the PDF related ones. The PDFset uncertainty constitutes the largest contribution of the PDF related ones.

The spread among the $\alpha_s(M_Z)$ determinations from a single data set with varying PDF sets is illustrated in figure 6. For each of the individual data sets, the results are mostly consistent. Larger deviations are observed for the Tevatron data when using the ABMP16 and HERAPDF2.0 sets, and for the STAR data in conjunction with the ABMP16 PDF set.

The PDF uncertainty obtained with different PDF sets for the same data set is largest for CT14 and smallest for HERAPDF2.0. These numbers can differ by a factor of up to almost four. Moreover, we observe that in particular for Tevatron and LHC data the ABMP16 and HERAPDF2.0 sets give significantly larger PDF $\alpha_s$ uncertainties than the nominal PDF NNPDF3.0, whereas the CT14 or MMHT2014
| Data set | $\alpha_s(M_Z)$ | Uncertainties (scaled by factor $10^4$) |
|----------|----------------|----------------------------------|
| H1       | 0.1169         | 22 9 8 4 10 +58 +60 +64         |
| ZEUS     | 0.1222         | 30 18 9 3 18 +48 +56 +63         |
| STAR     | 0.1197         | 116 – 50 26 99 +87 +143 +184     |
| CDF      | 0.1238         | 36 13 14 9 46 +83 +97 +104       |
| D0       | 0.1246         | 40 23 21 8 62 +104 +125 +131     |
| ATLAS    | 0.1236         | 16 3 15 8 30 +65 +74 +76         |
| CMS      | 0.1144         | 22 1 14 9 21 +58 +64 +68         |

Table 5. Results of common-type $\alpha_s(M_Z)$ extractions from individual inclusive jet data sets using the NNPDF3.0 PDF set. The values for $\alpha_s(M_Z)$ are provided along with the experimental and theoretical uncertainties. The latter consist of contributions originating from NP effects, the propagation of the PDF uncertainties, the choices of the PDF $\alpha_s(M_Z)$ value and the PDF set, and the scale uncertainty. The quadratic sum of the experimental and theoretical uncertainties is quoted as the total uncertainty. The corresponding $\chi^2/n_{\text{dof}}$ values are displayed in column five of figure 3 left.

**Figure 6.** Comparison of results for $\alpha_s(M_Z)$ obtained with the alternative PDF sets ABMP16, CT14, HERAPDF2.0, and MMHT2014. The values are compared to the value of $\alpha_s(M_Z)$ obtained with the NNPDF3.0 set and to the world average value [1]. The horizontal error bars, attached to the points representing the NNPDF3.0 results, indicate the total uncertainty.

PDFs exhibit in general systematically smaller PDF$\alpha_s$ uncertainties than NNPDF3.0. A possible reason for the observed effects could lie in the different selections of data considered for the PDF determination. For instance, the ABMP16 and HERAPDF2.0 sets do not include any jet data.
Data set | \( \alpha_s(M_Z) \) (scaled by factor 10^4) | PDF uncertainty (scaled by factor 10^4) | PDF\( \alpha_s \) uncertainty (scaled by factor 10^4)
--- | --- | --- | ---
H1 | 0.1155 0.1169 0.1150 0.1168 | 4 11 3 7 | 9 3 8 4
ZEUS | 0.1203 0.1228 0.1192 0.1224 | 5 11 4 7 | 9 1 8 2
STAR | 0.1034 0.1232 0.1129 0.1159 | 22 63 30 37 | 18 5 1 12
CDF | 0.1303 0.1239 0.1329 0.1243 | 13 29 8 19 | 27 1 17 1
D0 | 0.1344 0.1221 0.1289 0.1237 | 19 34 16 23 | 29 2 11 0
ATLAS | 0.1263 0.1211 0.1211 0.1203 | 13 22 13 17 | 11 0 6 3
CMS | 0.1186 0.1165 0.1146 0.1154 | 10 24 14 19 | 14 2 6 2

Table 6. Results of common-type fits to single inclusive jet data sets for varying PDF sets. Listed are the \( \alpha_s(M_Z) \) results and the respective PDF and PDF\( \alpha_s \) uncertainty. For the purpose of a more compact presentation, the employed PDF sets ABMP16 (AB), CT14 (CT), HERAPDF2.0 (H2), and MMHT2014 (MM) are abbreviated here to the two-letter acronyms given in parentheses. Other uncertainty components differ only in the last digit as compared to the results obtained with NNPDF3.0 displayed in table 5. Uncertainties are scaled by a factor of 10^4.