Algorithms for dynamic stabilization of rear-wheel drive two-axis vehicles with a plug-in rear axle

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Abstract. Currently, two-axis front-wheel drive vehicles are widely used systems that provide increased stability and manageability by connecting the rear axle and redistributing torque between the driving axles and between the driving wheels of the rear axle. To a lesser extent, drive schemes for rear-wheel drive two-axis cars with a plug-in front axle are common, and, accordingly, the issues of traffic stabilization of such cars are not fully studied. The purpose of this work is to increase the stability and controllability of rear-wheel drive two-axis machines with a plug-in front axle by redistributing torques between the driving wheels. The analysis of the effectiveness of the 4x4 vehicle traffic stabilization system by redistributing torques between the driving axles and the wheels carried out by simulation methods allowed us to establish that the following stabilization methods are the most effective for cars with a plug-in front axle:
- a 4x4 vehicle with a plug-in front axle that redistributes torques between the driving wheels of the rear axle (the effectiveness of these stabilization methods is approximately the same):
- a 4x4 vehicle with a plug-in front axle and an automatically locked front-wheel differential;
- 4x4 vehicle with a plug-in front axle with individual braking system for individual wheels.

Introduction

The car control stabilization system is designed to maintain the stability and manageability of the car by detecting and eliminating a critical situation in advance. The system allows you to keep the car within the path set by the driver in various driving modes (acceleration, braking, driving in a straight line, in turns, and when rolling freely). Currently, two-axis front-wheel drive vehicles are widely used systems that provide increased stability and manageability by connecting the rear axle and redistributing torque between the driving axles and between the driving wheels of the rear axle [1–10].

To a lesser extent, drive schemes for rear-wheel drive two-axis cars with a plug-in front axle are common, and, accordingly, the issues of traffic stabilization of such cars are not fully studied [11].

The purpose of this work is to increase the stability and controllability of rear-wheel drive two-axis machines with a plug-in front axle by redistributing torques between the driving wheels.

The following schemes of torque control on the driving wheels of the car are considered:
1) rear-wheel drive car without torque control on the driving wheels (base for comparison);
2) a 4x4 car with a plug-in front axle without redistributing torques between the driving wheels of the same axle;
3) a 4x4 vehicle with a plug-in front axle with a reallocation of torques between the driving wheels of the rear;
4) a 4x4 vehicle with a plug-in front axle and an automatically locked front-wheel differential; 
5) 4x4 vehicle with plug-in front axle with individual braking system for individual wheels.

The control algorithm for items 2 and 3 is given in the work [11].

Algorithm for redistributing torques between the driving wheels of the axle due to individual braking of individual wheels

The operation of the system is based on the recognition of the rear axle skidding or demolition of the front axle of the car. To recognize the type of skid, use the results of [12], where the parameter is used as a diagnostic sign of the onset of front or rear axle drift

$$\delta_V = |V_{c1} - V_{c2}|$$, representing the difference in the estimation of linear speeds of the center of mass of the car, first using the linear speed of the center of the front axle (vector $V_{c1}$), and then — the linear speed of the center of the rear axle (vector $V_{c2}$). In the event of a skid the rear axle is braked the outside front wheel. In case of demolition of the front axle, the inner rear wheel is braked.

The formula for determining the braking torque on each wheel is as

$$M_{mi} = h_{ESP}T_{max}, i = 1, \ldots, N,$$

where $T_{max}$ — maximum braking torque developed by the wheel brake mechanism; $N$ — number of car wheels; $h_{ESP} = [0 \ldots 1]$ — the degree of reduction of the effective braking torque on the $i$-th wheel due to the operation of the algorithm to counteract the demolition of the front axle during braking.

Algorithm for determining the value $h_{ESP}, i = 1, \ldots, N$ taking into account the sign rule adopted in the simulation, it should be as follows.

If $\Theta_1 > 0^\circ$ (turn left) & $\delta_V > 0$ (demolition of the front axle), that $h_{ESP1} = h_{ESP3} = h_{ESP4} = 0$;

$$h_{ESP2} = C_1\delta_V + C_2\delta_V.$$ 

If $\Theta_1 > 0^\circ$ (turn left) & $\delta_V < 0$ (the drift of a back axis), that $h_{ESP1} = h_{ESP2} = h_{ESP4} = 0$;

$$h_{ESP3} = C_1\delta_V + C_2\delta_V.$$ 

If $\Theta_1 < 0^\circ$ (turn right) & $\delta_V > 0$ (demolition of the front axle), that $h_{ESP1} = h_{ESP2} = h_{ESP3} = 0$;

$$h_{ESP4} = C_1\delta_V + C_2\delta_V.$$ 

If $\Theta_1 < 0^\circ$ (turn right) & $\delta_V < 0$ (the drift of a back axis), that $h_{ESP4} = h_{ESP2} = h_{ESP3} = 0$;

$$h_{ESP1} = C_1\delta_V + C_2\delta_V.$$ 

In the above formulas $C_1$ and $C_2$ — regulator gain factors that can be adjusted individually for each vehicle.

Creating an additional moment of turning resistance by locking the inter-wheel differential

Front axle skidding

To increase the car's turnability, it is necessary to bring more torque to the wheels of the rear axle. However, it does not make sense to block the differential of the rear axle, since it will create a moment of resistance to turn

$$M_{c2} = \frac{B_k}{2}(R_{c2} + fR_{c2});$$

where $B_k$ — wheel track, $R_{c2}$, $R_{c2}$ — the total longitudinal and vertical reactions on the wheels of the rear axle in the projection of the $X$ and $Z$ axes associated with the center of mass of the car coordinate system respectively, $f$ — rolling resistance coefficient.

The moment of resistance to the turn is harmful in this case, since it is directed against the angular speed of the turn.
Simultaneously with the redistribution of torque between the axles of the car, it is necessary to apply more torque to the external driving wheel of the rear axle in relation to the direction of rotation, which requires a controlled asymmetric differential.

The drift of a back axis
To reduce the turnability property, it is necessary to reduce the torque on the rear driving axle, transferring most of it to the front axle. At the same time, blocking the front axle differential will create a moment of resistance, which is also a moment of counter-rotation, which helps to stabilize the movement of the car.

\[ M_{c1} = \frac{B_r}{2}(R_{c1} + fR_{c1}) \]

where \( R_{c1}, R_{z1} \) — total reactions on the wheels of the front axle in the projection of the \( X \) and \( Z \) axes associated with the center of mass of the car coordinate system respectively.

The algorithm for locking the front axle differential when the rear axle skids is as follows.

If \( \delta_\nu < 0 \) (the drift of a back axis), that the front axle differential is locked. Otherwise, it is unblocked.

Performance criteria for the 4x4 vehicle stabilization system with differential transmission due to individual braking of individual wheels
As criteria for the effectiveness of the stabilization system of a 4x4 vehicle with a differential transmission due to individual braking of individual wheels, you can take the following.

1) Type of vehicle trajectory (no rear axle skid or front axle drift).

2) The case when \( \delta_\nu = 0 \), corresponds to the vehicle's neutral turnability (ideal value). Therefore, the diagnostic parameter \( \delta_\nu \) it can be used to formulate a criterion for the efficiency \( KE \) operation of motion stabilization algorithms:

\[ KE = \frac{1}{n_\nu - 1} \sum_{i=1}^{n_\nu} \delta_{\nu i} \]

where \( n_\nu \) — the number of points in the implementation process \( \delta_{\nu i} \).

Investigation of the effectiveness of the proposed algorithm for the redistribution of torques between the driving axles and wheels by simulation methods
Testing the efficiency of the algorithm for redistributing torques between the driving axles and wheels was carried out on the example of a 4x4 car with a total weight of 2400 kg with a plug-in front axle and a redistribution of torque between the wheels of the rear axle. Was carried out a theoretical study of the motion of the car with the help of mathematical simulation. Features of the mathematical model of motion are considered in [13–20].

We study the movement on the "ice with snow" support base (with the coefficient of interaction of the engine with the support base at full slip of 0.35). Note that the term "support base" refers only to a solid non-deformable support surface. The front wheels of the car are steerable. The steering angle changes from zero to the set value within 3 seconds and then remains unchanged.

The mode of entering a turn and driving in a turn with a fixed radius is simulated. We studied the movement of the car at the entrance to the turn and the movement in the turn, with the speed \( V=25 \) km/h, which the driver tries to maintain constant during the movement.

The trajectory of the rear wheel drive vehicle on ice when making a turn is shown in Fig. 1.

Analysis of the driving paths of a car with a rear drive axle (Fig. 1) shows that when making a maneuver on the support surface "ice with snow" the car loses control (develops a skid of the rear axle).
The path of movement on ice of a 4x4 vehicle with a plug-in front axle without redistributing torques between the driving wheels of one axis when making a turn is shown in Fig. 2.

![Fig. 1. The trajectory of the rear-wheel drive car in a turn on ice](image1)

![Fig. 2. The trajectory of movement on the ice of a 4x4 car with a plug-in front axle without redistributing torques between the driving wheels of the same axis when making a turn](image2)

Analysis of the driving paths of a 4x4 vehicle with a plug-in front axle without redistributing torques between the driving wheels of the same axle (Fig. 2) shows that when performing a maneuver on the "ice with snow" support surface, the car loses control (the rear axle skid develops), although much later than in a rear-wheel drive car.

The trajectory of movement on the ice of a 4x4 vehicle with a plug in front axle with the redistribution of torques between the driving wheels of the rear axle when making a turn is shown in Fig. 3.

When driving a car with a plug-in front axle and redistributing torque between the wheels of the rear axle in the same driving conditions (Fig. 3) the car remains manageable, small deviations from the trajectory can be compensated by steering.

The path of movement on ice of a 4x4 vehicle with a plug in front axle and with an automatically locked front wheel differential when making a turn is shown in Fig. 4.

When driving a vehicle with a plug-in front axle and an automatically locked front-wheel differential under the same driving conditions (Fig. 4) the car remains manageable, small deviations from the trajectory can be compensated by steering.
Fig. 3. The trajectory of a 4x4 vehicle with a plug-in front axle and the redistribution of torque between the wheels of the rear axle on the ice in a turn.

Fig. 4. The trajectory of a 4x4 vehicle with a plug-in front axle and an automatically locked front-wheel differential on ice in a turn.

Fig. 5. The trajectory of a 4x4 vehicle with a plug-in front axle and individual braking system for individual wheels on the ice in a turn.
The path of movement on the ice of a 4x4 vehicle with a plug-in front axle and with a system of individual braking of individual wheels when making a turn is shown in Fig. 5.

When driving a car with a plug-in front axle and individual braking system for individual wheels in the same driving conditions (Fig. 5) the car remains manageable, small deviations from the trajectory can be compensated by steering.

Table 1 shows the values of the KE performance criterion for traffic stabilization algorithms for the vehicles with a plug in front axle discussed above.

**Table 1. Values of the ke performance criterion for traffic stabilization algorithms for vehicles with a plug-in front axle**

| №  | Method of motion stabilization                                                                 | KE, m/s |
|----|-----------------------------------------------------------------------------------------------|---------|
| 1  | Rear-wheel drive car without torque control on the driving wheels (base for comparison)        | 104,3   |
| 2  | A 4x4 car with a plug-in front axle without redistributing torques between the driving wheels of the same axle | 4,18    |
| 3  | A 4x4 vehicle with a plug-in front axle that redistributes torques between the driving wheels of the rear axle | 0,52    |
| 4  | A 4x4 vehicle with a plug-in front axle and an automatically locked front-wheel differential | 0,48    |
| 5  | 4x4 vehicle with a plug-in front axle with individual braking system for individual wheels     | 0,55    |

The analysis of the effectiveness of the 4x4 vehicle traffic stabilization system due to the redistribution of torques between the driving axles and the wheels allowed us to establish that the following stabilization methods are most effective for cars with a plug-in front axle:

1) a 4x4 vehicle with a plug-in front axle with a reallocation of torques between the driving wheels of the rear axle;
2) a 4x4 vehicle with a plug-in front axle and an automatically locked front-wheel differential;
3) 4x4 vehicle with plug-in front axle with individual braking system for individual wheels.

The effectiveness of these stabilization methods is approximately the same.

**Conclusions**

The analysis of the effectiveness of the 4x4 vehicle traffic stabilization system by redistributing torques between the driving axles and the wheels carried out by simulation methods allowed us to establish that the following stabilization methods are the most effective for cars with a plug-in front axle:

- a 4x4 vehicle with a plug-in front axle with a reallocation of torques between the driving wheels of the rear axle;
- a 4x4 vehicle with a plug-in front axle and an automatically locked front-wheel differential;
- 4x4 vehicle with plug-in front axle with individual braking system for individual wheels.

The effectiveness of these stabilization methods is approximately the same.

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