Charged Higgs and Neutral Higgs pair production
of weak gauge bosons fusion process in $e^+e^-$ collision

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Abstract

Pair production of the neutral and charged Higgs boson is a unique process which is a signature of two Higgs doublet model. In this paper, we study the pair production and their decays of the Higgs in the neutrinophilic Higgs two doublet model. The pair production occurs through $W$ and $Z$ gauge bosons fusion process. In the neutrinophilic model, the vacuum expectation value (VEV) of the second Higgs doublet is small and is proportional to the neutrino mass. The smallness of VEV is associated with the approximate global U(1) symmetry which is slightly broken. Therefore, there is a suppression factor for the U(1) charge breaking process. The second Higgs doublet has U(1) charge and its single production from the gauge boson fusion violates the U(1) charge conservation and is suppressed strongly to occur. In contrast to the single production, the pair production of the Higgses conserves U(1) charge and the approximate symmetry does not forbid it. To search for the pair productions in collider experiment, we study the production cross section of a pair of the charged Higgs and neutral Higgs bosons in $e^+e^-$ collision with center of energy from 600 (GeV) to 2000 (GeV). The total cross section varies from $10^{-4}$ (fb) to $10^{-3}$ (fb) for degenerate (200 GeV) charged and neutral Higgses mass case. The background process to the signal is gauge bosons pair $W^+ + Z$ production and their decays. We show the signal over background ratio is about $2\% \sim 3\%$ by combining the cross section ratio with ratios of branching fractions.
I. INTRODUCTION

While LHC already have started constraining many new physics models, there are a few aspects in the beyond standard models for which future $e^+e^-$ collider [1, 2] can make unique search scenarios because of its clean environment. In this paper, we study the signature of the neutrinophilic two Higgs doublet model [3] in $e^+e^-$ collision by focusing on the pair production and their decays of the charged Higgs and neutral Higgs bosons.

In the neutrinophilic model, a second Higgs doublet is introduced and the neutrino masses are generated from the tiny VEV (vacuum expectation value) of the second Higgs doublet. The new U(1) global symmetry is introduced. The second Higgs doublet and right-handed neutrinos have the U(1) charge $+1$ and the other fields do not have that charge. The U(1) global symmetry is approximate and is broken explicitly by the soft breaking bilinear term with respect to the second Higgs doublet and to the standard model like Higgs doublet. The tiny VEV of the second Higgs generated is proportional to the coefficient of the mass dimension two in the bilinear term.

In the model, any U(1) charge violating process is suppressed by the tiny VEV. It also implies that the probability amplitude is suppressed and is proportional to neutrino mass. An example of suppressed process is a single second Higgs production with gauge bosons fusion. In contrast to the single second Higgs production, the pair production of the second Higgs is the U(1) charge conserving process. Therefore, they are not suppressed. The processes in this category are $Z^*(\gamma^*) \rightarrow H^+H^-$, $W^+W^- \rightarrow H^+H^-$ and $W^+Z \rightarrow H^+X$ ($X = A, h$), where $H^+$, $A$, and $h$ denote the charged Higgs, CP odd Higgs and CP even Higgs in the second Higgs doublet, respectively.

In LHC set up, the charged Higgs pair production $p+p \rightarrow Z^*(\gamma^*) \rightarrow H^+H^-$ is studied in [4]. In [5], vector boson fusion into the light CP even Higgs pairs is studied at the LHC. In [6], di-Higgs production in various scenario is discussed. In [7], the standard model Higgs boson pair production is studied. Also see reference [8] for the ratio of the cross section of the single Higgs boson and the pair production cross section in the context of the standard model.

In our work, in $e^+e^-$ collision, the pair production of the charged Higgs($H^+$) and neutral Higgs ($X$) in the second Higgs doublet is studied. We derive the pair production cross section; $e^+e^- \rightarrow \nu e^- + e^- + H^+X$ ($X = A, h$).
The paper is organized as follows. In section 1, we set up the Lagrangian which is used in the calculation of charged Higgs and neutral Higgs production. In section 2, we derive the expression of the cross sections for the pair production from $e^+ + e^-$ collision. In section 3, the cross sections including the various differential cross sections are numerically computed and they are compared to the standard model background cross section. In section 4, the decays of the charged Higgs and neutral Higgs are discussed and the dependence on the charged lepton flavor in the final state is studied. Section 5 is devoted to the summary.

II. TWO HIGGS DOUBLET MODEL WITH SOFTLY BROKEN GLOBAL SYMMETRY

In this section, we present the Lagrangian to set up the notation and also to display the interaction terms which are relevant to the calculation in later sections. The Higgs potential is given by

$$V_{\text{tree}} = \sum_{i=1,2} \left( m_i^2 \Phi^\dagger_i \Phi_i + \frac{\lambda_i}{2} (\Phi^\dagger_i \Phi_i)^2 \right) - (m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}) + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi^\dagger_2 \Phi_2) + \lambda_4 |\Phi_1^\dagger \Phi_2|^2.$$  

(1)

Two Higgs doublets in the unitary gauge are parameterized as

$$\Phi_1 = \begin{pmatrix} 0 \\ v \cos \beta \sqrt{2} \end{pmatrix} + \begin{pmatrix} -\sin \beta H^+ \\ \sin \gamma_h \cos \gamma H - i \sin \beta A \sqrt{2} \end{pmatrix},$$

$$\Phi_2 = \begin{pmatrix} 0 \\ v \sin \beta \sqrt{2} \end{pmatrix} + \begin{pmatrix} \cos \beta H^+ \\ \cos \gamma_h \sin \gamma H + i \cos \beta A \sqrt{2} \end{pmatrix}.$$  

(2)

The new U(1) charge for $\Phi_1$ ($\Phi_2$) is 0(+1). The term proportional to $m_{12}$ is U(1) breaking term. H and h denote CP even Higgses. A denotes a CP odd Higgs. In our notation, H is close to the standard model like Higgs, a different notation from $[3]$. In most of the present paper, we follow the notation of $[9]$. $\tan \beta$ is the ratio of two VEVs and is given approximately as

$$\tan \beta = \frac{m_{12}^2}{m_A^2}.$$  

(3)

$v^2$ is the squared sum of two VEVs. $\gamma$ is a mixing angle of CP even Higgses given by $[9]$,

$$\tan 2\gamma = \frac{-4m_{12}^2 + 2\sin 2\beta(\lambda_3 + \lambda_4)v^2}{(3(-\lambda_1 \cos \beta^2 + \lambda_2 \sin^2 \beta) + \cos 2\beta(\lambda_3 + \lambda_4))v^2 - 2(m_{11}^2 - m_{22}^2)}.$$  

(4)
Then one can write the covariant derivative terms for the two doublets, which includes the electroweak interactions of the Higgs with gauge bosons,

$$\sum_{i=1,2} D_{\mu} \Phi^+_i D^{\mu} \Phi_i \cong gM_W (W_\mu^+ W_{\mu}^- + \frac{1}{2c^2_W} Z^\mu Z_{\mu}) (\sin(\beta + \gamma)h + \cos(\beta + \gamma)H)$$

$$+ \frac{g^2}{2} s_W (A_{\mu} - t_W Z_{\mu}) [(H^+ W_{\mu}^- + H^- W_{\mu}^+)(h \cos(\beta + \gamma) - H \sin(\beta + \gamma))]$$

$$- i(H^+ W_{\mu}^- - H^- W_{\mu}^+ A)$$

$$+ i \frac{g \cos(\beta + \gamma)}{2 \cos \theta_W} Z_{\mu} (\partial^\mu H^- H^+ - \partial^\mu H^+ H^-)$$

$$+ \frac{g \cos(\beta + \gamma)}{2 \cos \theta_W} (\partial_{\mu} hA - \partial_{\mu} Ah) Z^\mu$$

$$+ \left\{ i \frac{g}{2} \cos(\beta + \gamma) W_{\mu}^+ (h \partial_{\mu} H^- - \partial_{\mu} h H^-) + \frac{g}{2} W_{\mu}^+ (H^- \partial_{\mu} A - A \partial_{\mu} H^-) + h.c. \right\}. \quad (5)$$

One notes that a single CP even Higgs boson (h or H), could be produced by the gauge boson fusion process $W^+ + W^- (Z + Z) \to h$ or H. There is no single CP odd Higgs A production from gauge boson fusion. Absence of the term like $AW_{\mu}^+ W_{\mu}^-$ is due to CP symmetry. We also note that CP even Higgs h is mostly the real part of the down component of the second Higgs $\Phi_2$. Its coupling to gauge boson pair operators $W_{\mu}^+ W_{\mu}^-$ and $Z_{\mu} Z_{\mu}$ is suppressed as $\sin(\beta + \gamma)$. Since $\sin \beta$ and $\sin \gamma$ are suppressed to be zero in the vanishing limit of U(1) breaking term $m_{12}$, the gauge boson fusion to $h$ is forbidden in the limit. As for the decays of charged Higgs and neutral Higgs, the Yukawa coupling to right handed neutrino is important. Assigning the U(1) charge +1 to right handed neutrino $\bar{\nu}_i$, it is written in terms of mass eigen states as,

$$\mathcal{L}_Y = -y_{\nu ij} \bar{\nu}_i \Phi_2 \nu_R^j \theta$$

$$\cong -y_{\nu ij} \bar{\nu}_i \frac{(m_{\nu j})}{v} \cos \gamma h - \sin \gamma H$$

$$+ \nu_R^j \frac{(m_{\nu i})}{v} \gamma_5 \nu_i \cot \beta A$$

$$+ \sqrt{2} \cot \beta \nu_R^j (\frac{m_{\nu j}}{v}) \nu_i \sin \beta A$$

$$+ h.c., \quad (6)$$

where $m_{\nu}$ denote neutrino masses and V denotes Maki Nakagawa Sakata (MNS) matrix.

III. CROSS SECTION FOR $e^+ + e^- \to \bar{\nu} + e^- + W^{*+} + Z^* \to \bar{\nu} + e^- + H^+ + A$

In this section, we present the formulae for the cross section of $e^+ + e^- \to \bar{\nu} + e^- + W^{*+} + Z^* \to \bar{\nu} + e^- + H^+ + A$. (See Fig.1.) We define,

$$\sigma_{H^+ X} \equiv \sigma(e^+ + e^- \to \bar{\nu} e^- + H^+ + X); X = A, h. \quad (7)$$
FIG. 1: Feynman diagram of charged Higgs $H^+$ and CP odd Higgs $A$ production in $e^+e^-$ collision. The production occurs through $W^+$ and $Z$ fusion which is shown in the circle.

We write the cross section for $H^+A$ production,

$$\sigma_{H^+A} = \frac{1}{2s_{e^+e^-}} \int \frac{d^3q_H^+}{(2\pi)^3} \int \frac{d^3q_e^-}{(2\pi)^3} \int \frac{d^3q_\nu}{(2\pi)^3} \frac{d^3q_{H^+}}{2E_{H^+}} \frac{d^3q_e^-}{(2\pi)^3} \frac{d^3q_\nu}{2E_\nu} \times \frac{1}{4} \sum_{\text{spin}} |M|^2 (2\pi)^4 \delta^4(p_{e^+} + p_e - q_{H^+} - q_A - q_e - q_\nu). \quad (8)$$

$s_{e^+e^-}$ is the center of mass (cm) energy of $e^+$ and $e^-$ collision. $p_{e^+}$ and $p_e$ denote the momentum of positron and electron of the initial state. $q_{e^+}, q_A$ and $q_\nu$ are momentum of the final states; i.e., electron, charged Higgs, neutral Higgs and anti-neutrino respectively.

The transition amplitude $M$ is given by,

$$M = -T_{A\mu\nu} \frac{1}{(p_Z^2 - M_Z^2)(p_W^2 - M_W^2)} \frac{g^2}{2\sqrt{2}c_W} u(q_e^-)\gamma^\nu(L + 2s_W^2)u(p_e)v_{e^+}(p_{e^+})\gamma^\mu L v_\nu(q_\nu). \quad (9)$$

where $p_Z = p_e - q_e$ and $p_W = q_{H^+} + q_A - p_Z$. $L$ denotes the chiral projection $L = \frac{1 - \gamma_5}{2}$. $s_W(c_W)$ denotes sine (cosine) of the Weinberg angle. $T_{A\mu\nu}$ denotes the off shell amplitude for $W^{+\ast}_\mu + Z^{\ast}_\nu \to A + H^+$ production. It corresponds to the circle in Fig.1 and the Feynman diagrams which contribute to $T_{A\mu\nu}^{\ast}$ are shown in Fig.2 ∼ Fig.5. The second rank tensor $T_{A\mu\nu}$ is given as, (On-shell case is shown in $[10]$.)

$$T_{\mu\nu} = iT_{A\mu\nu} = \frac{g^2}{2 \cos \theta_W} (a_A g_{\mu\nu} + d_A q_{A\nu} q_{H^+\mu} + b_A q_{H^+\nu} q_{A\mu}). \quad (10)$$
where we introduce the real amplitude $T^*_{\mu \nu} = T_{\mu \nu}$. $a_A$, $b_A$ and $d_A$ in Eq. (10) are given as,

$$a_A = \frac{s^2_W + \frac{p^2_Z - q^2_W}{M^2_Z} \frac{M^2_A - M^2_{H^+} - M^2_W}{s_{H^+A} - M^2_W}}{c^2_W \frac{t_A - u_A + p^2_Z - p^2_W}{s_{H^+A} - M^2_W}} + c^2_W,$$

$$b_A = -\frac{2 \cos 2\theta_W}{u_A - M^2_{H^+} - \frac{2(\cos 2\theta_W + 1)}{s_{H^+A} - M^2_W}},$$

$$d_A = \frac{2 \cos^2(\beta + \gamma)}{t_A - M^2_h} + \frac{2(\cos 2\theta_W + 1)}{s_{H^+A} - M^2_W},$$

(11)

with $t_A = (q_{H^+} - p_W)^2$, $u_A = (p_W - q_A)^2$ and $s_{H^+A} = (q_{H^+} + q_A)^2$. The spin averaged amplitude squared is given as,

$$\frac{1}{4} \sum_{\text{spin}} |M|^2 = \frac{g^4}{32c^2_W} \frac{1}{(p^2_Z - M^2_Z)(p^2_W - M^2_W)^2} T_{\mu \nu} T^*_{\rho \sigma} L^e_{\nu \sigma} L^e_{\mu \rho},$$

(12)
where $L^\nu_\epsilon$ is a leptonic tensor of the neutral current and $L^\mu_\epsilon$ is the one of the charged current. They are written in terms of the symmetric part $S$ and the anti-symmetric part $A$.

\[
L^\nu_\epsilon = S^\nu_\epsilon + iA^\nu_\epsilon;
\]
\[
S^\nu_\epsilon = (2 + 8s^2_W + 16s^4_W)(p^\nu_e q_\epsilon - g^\nu_\epsilon \cdot q_\epsilon + p^\nu_\epsilon q'_\epsilon),
\]
\[
A^\nu_\epsilon = (2 + 8s^2_W)e^{\nu_\epsilon_3^2}p_{\epsilon_3 q_\epsilon},
\]
\[
L^\mu_\epsilon + \bar{\epsilon} = S^\mu_\epsilon + iA^\mu_\epsilon
\]
\[
S^\mu_\epsilon = 2(q^\mu_\epsilon P^\rho_\epsilon - g^\mu_\epsilon q_\rho \cdot p_{\rho +} + q^\rho_\epsilon p^\rho_\epsilon),
\]
\[
A^\mu_\epsilon = 2e^{\epsilon_3^2}q_\rho p_{\rho +}.
\]

We define the transpose matrix as $T^t_{\mu\nu} = T_{\nu\mu}$. In terms of them, one can write the differential cross section as,

\[
d\sigma_{H^+ A} = \frac{g^4}{64e^2 W s_{e+}} \left( \frac{1}{-4096\pi^2} \left( ((p_+ - q_\epsilon)^2 - M^2_Z)\right)^2 \right)\]
\[
(T^t_{\mu\nu} S^\nu_\epsilon T^t_{\sigma\rho} S^\mu_\epsilon + T^t_{\mu\nu} A^\nu_\epsilon T^t_{\sigma\rho} A^\mu_\epsilon) d^{12} Ph,
\]

where $d^n Ph$ denotes $n$ dimensional phase space integral. For $n = 12$, it is defined as,

\[
d^{12} Ph = \frac{d^3 q_A d^3 q_{H+} d^3 q_\epsilon d^3 q_\nu}{E_A E_{H+} E_\epsilon E_\nu} \delta^4(p_{e+} + p_\epsilon - q_\nu - q_{H+} - q_A).
\]

In center of mass frame of $e^+e^-$ collision, the amplitude is independent of the rotation around the beam axis. One can also set the direction of the $e^+$ beam to $z$ direction and the momentum of electron in the final states in $yz$ plane. Therefore after one integrates the azimuthal angle and the anti-neutrino momentum, one obtains $d^8 Ph$ as,

\[
d^8 Ph = 2\pi d \cos \theta_\epsilon d \cos \theta_{eH} d \phi_{eH} d \cos \theta_{eHA} d \phi_{eHA} \]
\[
\frac{q^2_\epsilon d q_\epsilon q^2_{H+} d q_{H+} q^2_\nu d q_\nu}{E_\epsilon E_{H+} E_{H+} E_\nu} \delta(\sqrt{s} - E_{H+} - E_A - E_\epsilon - E_\nu).
\]

The momentum of electron $q_\epsilon$ in final states is specified by a polar angle ($\theta_\epsilon$) in the orthogonal frame in which positron momentum is chosen as z axis.

\[
P_{e+} = \frac{\sqrt{s_{e+} e}}{2} e_3, \quad P_\epsilon = -\frac{\sqrt{s_{e+} e}}{2} e_3,
\]
\[
q_\epsilon = |q_\epsilon| (\sin \theta_\epsilon e_2 + \cos \theta_\epsilon e_3),
\]
\[
e_1 = e_2 \times e_3.
\]
One can define a new orthogonal coordinate spanned by the basis vectors $\mathbf{e}'_i (i = 1 \sim 3)$. 

$$
e'_3 = \frac{\mathbf{q}_e}{|\mathbf{q}_e|} = \sin \theta_e \mathbf{e}_2 + \cos \theta_e \mathbf{e}_3,$$

$$
e'_2 = -\sin \theta_e \mathbf{e}_3 + \cos \theta_e \mathbf{e}_2,$$

$$
e'_1 = \mathbf{e}_1. \quad (19)$$

$\theta_e$ and $\phi_{eH}$ denote the momentum direction of the charged Higgs relative to that of the electron in the final state.

$$\mathbf{q}_H^+ = |\mathbf{q}_H^+|(\sin \theta_e \cos \phi_{eH} \mathbf{e}'_1 + \sin \theta_e \sin \phi_{eH} \mathbf{e}'_2 + \cos \theta_e \mathbf{e}'_3). \quad (20)$$

Finally $(\theta_{eHA}, \phi_{eHA})$ denote the direction of momentum for the neutral Higgs $A$. $\theta_{eHA}$ is a polar angle measured from the direction $\mathbf{q}_e + \mathbf{q}_H^+.$

$$\mathbf{q}_A = |q_A|(\sin \theta_{eHA} \cos \phi_{eHA} \mathbf{e}''_1 + \sin \theta_{eHA} \sin \phi_{eHA} \mathbf{e}''_2 + \cos \theta_{eHA} \mathbf{e}''_3), \quad (21)$$

$$
e''_3 = \frac{\mathbf{q}_e + \mathbf{q}_H^+}{|\mathbf{q}_e + \mathbf{q}_H^+|}, \quad e''_1 = \frac{\mathbf{q}_e \times \mathbf{q}_H^+}{|\mathbf{q}_e \times \mathbf{q}_H^+|}, \quad e''_2 = e''_3 \times e''_1. \quad (22)$$

In terms of the angles defined, the phase space integration is written,

$$d^8 Ph = 2\pi d \cos \theta_e d \cos \theta_e d \phi_{eH} d \cos \theta_{eHA} d \phi_{eHA} \frac{q_e^2 dq_e q_H^2 dq_H^+ q_A^2 dq_A}{E_e E_H^+ E_A E_\bar{\nu}} \delta(\sqrt{s} - E_H^+ - E_A - E_q - E_\bar{\nu})$$

$$E_\bar{\nu} = \sqrt{|\mathbf{q}_e + \mathbf{q}_H^+|^2 + q_A^2 + 2 \cos \theta_{eHA} q_A |\mathbf{q}_e + \mathbf{q}_H^+|}, \quad (23)$$

where we denote $q_A = |\mathbf{q}_A|, q_H^+ = |\mathbf{q}_H^+|$ and $q_e = |\mathbf{q}_e|$. The integration over the variable $\cos \theta_{eHA}$ is carried out and we obtain,

$$d^7 Ph = 2\pi d \cos \theta_e d \cos \theta_e d \phi_{eH} d \phi_{eHA} \frac{q_A dq_A q_H^2 dq_H^+ dq_e}{E_A E_H^+ E_\bar{\nu}} \frac{1}{|\mathbf{q}_e + \mathbf{q}_H^+|} \times \theta(E_\bar{\nu}^0 - ||\mathbf{q}_H^+ + \mathbf{q}_e|| - q_A)|| \theta(||\mathbf{q}_e + \mathbf{q}_H^+|| + q_A - E_\bar{\nu}^0), \quad (24)$$

where,

$$E_\bar{\nu}^0 = \sqrt{s_e e^-} - E_e - E_A - E_H^+. \quad (25)$$
The step functions in Eq. (24) imply the phase space boundaries. Using Eq. (24), the differential cross section is,

\[
d^7\sigma_{H^+A} = \frac{dq_e dq_{H^+} dq_A}{32e_W^2 \sqrt{s} 4096\pi^3} \left| \frac{1}{((p_e - q_e)^2 - M_Z^2)((p_\nu^+ - q_\nu)^2 - M_W^2)} \right|^2 \]

\[
(T_{\mu\nu}^\sigma \epsilon_{e\rho}^\sigma T_{\nu\rho}^{\bar{e}+p} + T_{\mu\nu}^{A\gamma} \epsilon_{e\rho}^\sigma T_{\nu\rho}^{A_{e+\bar{p}}}) \frac{q_A}{E_{H^+}} q_{e+H^+} \left| q_e + q_{H^+} \right| \theta(E_\nu^0 - \left| q_{H^+} + q_e \right| - q_A - E_\nu^0) \right. \tag{26}
\]

\[
\Delta\sigma_{H^+A}(\theta_e) = \int_{\cos \theta_e - \frac{\Delta \cos \theta_e}{2}}^{\cos \theta_e + \frac{\Delta \cos \theta_e}{2}} \frac{d\sigma_{H^+A}}{d \cos \theta_e} d \cos \theta_e, \quad \Delta \cos \theta_e = 0.2, \tag{31}
\]

We carry out the rest of integration numerically.

**IV. NUMERICAL RESULTS**

In this section, we present the numerical results for the cross sections. We have carried out the phase space integrations by using the montecarlo program, bases [11]. We have studied the three sets of the charged Higgs and neutral Higgs masses.

\[
(m_{H^+}, m_A) = (300, 200), (200, 300), (200, 200) \text{(GeV)}. \tag{28}
\]

As shown in [9], for those input values of charged Higgs and neutral Higgs masses, the radiative corrections to the VEVs, \( \beta \) and \( \nu \) are within 10%.

We have shown the total cross sections \( \sigma_{H^+A} \) with respect to the center of mass energy \( \sqrt{s_{e^+e^-}} \) of \( e^+e^- \) collision in Fig.IV. Then we have plotted the following one dimensional differential cross sections; Fig.7 \sim Fig.11.

\[
\Delta\sigma_{1H^+A}(q_e) = \int_{q_e - \frac{\Delta q_e}{2}}^{q_e + \frac{\Delta q_e}{2}} \frac{d\sigma_{H^+A}}{dq_e} dq_e, \quad \Delta q_e = 50(\text{GeV}), \tag{29}
\]

\[
\Delta\sigma_{2H^+A}(q_{H^+}) = \int_{q_{H^+} - \frac{\Delta q_{H^+}}{2}}^{q_{H^+} + \frac{\Delta q_{H^+}}{2}} \frac{d\sigma_{H^+A}}{dq_{H^+}} dq_{H^+}, \quad \Delta q_{H^+} = 50(\text{GeV}), \tag{30}
\]

\[
\Delta\sigma_{3H^+A}(\cos \theta_e) = \int_{\cos \theta_e - \frac{\Delta \cos \theta_e}{2}}^{\cos \theta_e + \frac{\Delta \cos \theta_e}{2}} \frac{d\sigma_{H^+A}}{d \cos \theta_e} d \cos \theta_e, \quad \Delta \cos \theta_e = 0.2, \tag{31}
\]

\[
\Delta\sigma_{4H^+A}(\cos \theta_{eH}) = \int_{\cos \theta_{eH} - \frac{\Delta \cos \theta_{eH}}{2}}^{\cos \theta_{eH} + \frac{\Delta \cos \theta_{eH}}{2}} \frac{d\sigma_{H^+A}}{d \cos \theta_{eH}} d \cos \theta_{eH}, \quad \Delta \cos \theta_{eH} = 0.2, \tag{32}
\]

\[
\Delta\sigma_{5H^+A}(\phi_{eH}) = \int_{\phi_{eH} - \frac{\Delta \phi_{eH}}{2}}^{\phi_{eH} + \frac{\Delta \phi_{eH}}{2}} \frac{d\sigma_{H^+A}}{d \phi_{eH}} d \phi_{eH}, \quad \Delta \phi_{eH} = \frac{\pi}{5}. \tag{33}
\]
FIG. 6: The gauge boson pair production cross section \( \sigma_{WZ} \) for \( e^+e^- \to W^+Z+\nu\bar{\nu}e^+e^- \) (solid line) and the Higgs pair production cross sections \( \sigma_{H+A} \) for \( e^+e^- \to H^+A+\nu\bar{\nu}e^+e^- \). The horizontal axis denotes center of mass energy; \( \sqrt{s_{ee}} \) (GeV) of \( e^+e^- \) collision. The long dashed line with the cross symbol \( \times \) corresponds to the case \((m_{H^+}, m_A) = (200, 200)\) (GeV). The dotted line with the boxes \( \Box \) corresponds to \((m_{H^+}, m_A) = (300, 200)\) (GeV) and the short dashed line with asterisks \( \ast \) corresponds to \((m_{H^+}, m_A) = (200, 300)\) (GeV).

For comparison, we have also computed gauge boson production cross section. We have used the formulae in [12] for \( W^+Z \to W^+Z \) scattering amplitude.

\[
\sigma_{WZ} \equiv \sigma_{SM}(e^+e^- \to \nu\bar{\nu}e^+e^- + W^+ + Z).
\]  

We have plotted \( \sigma_{WZ} \) in Fig. [V] as well as the differential ones; \( \Delta\sigma_{iWZ}(i = 1 \sim 5) \) for the weak gauge boson pair \((W^+ \text{ and } Z)\) production in the standard model. See Fig. [V] ~ Fig. [III]. It can be a background process to the Higgs pair production. Explicitly, we write the differential cross section \( \Delta\sigma_{iWZ} \) \((i = 1 \sim 5)\), which are defined analogous to the ones defined
for the case of Higgs production in Eq.(29) \sim Eq.(33).

\[ \Delta \sigma_{1W}(q_e) = \int_{q_e - \Delta q_e}^{q_e + \Delta q_e} \frac{d\sigma_{WZ}}{dq_e} dq_e, \quad \Delta q_e = 50(\text{GeV}), \quad (35) \]

\[ \Delta \sigma_{2W}(q_W) = \int_{q_W - \Delta q_W}^{q_W + \Delta q_W} \frac{d\sigma_{WZ}}{dq_W} dq_W, \quad \Delta q_W = 50(\text{GeV}), \quad (36) \]

\[ \Delta \sigma_{3WZ}(\cos \theta_e) = \int_{\cos \theta_e - \Delta \cos \theta_e}^{\cos \theta_e + \Delta \cos \theta_e} \frac{d\sigma_{WZ}}{d \cos \theta_e} d \cos \theta_e, \quad \Delta \cos \theta_e = 0.2, \quad (37) \]

\[ \Delta \sigma_{4WZ}(\cos \theta_{eW}) = \int_{\cos \theta_{eW} - \Delta \cos \theta_{eW}}^{\cos \theta_{eW} + \Delta \cos \theta_{eW}} \frac{d\sigma_{WZ}}{d \cos \theta_{eW}} d \cos \theta_{eW}, \quad \Delta \cos \theta_{eW} = 0.2, \quad (38) \]

\[ \Delta \sigma_{5WZ}(\phi_{eW}) = \int_{\phi_{eW} - \Delta \phi_{eW}}^{\phi_{eW} + \Delta \phi_{eW}} \frac{d\sigma_{WZ}}{d \phi_{eW}} d \phi_{eW}, \quad \Delta \phi_{eW} = \frac{\pi}{5}, \quad (39) \]

We summarize what one can read from figures of the cross sections. (Fig IV \sim Fig III)

- The total cross section for Higgs pair production \( \sigma_{H^+ A} \) increases as the center of mass energy of \( e^+ e^- \) collision grows until it reaches to 2000 (GeV). Even in the case for the lightest masses of Higgs pair which we have chosen, the cross section is at most 0.001 fb. Compared with gauge boson pair production \( \sigma_{WZ} \), the ratio \( \frac{\sigma_{H^+ A}}{\sigma_{WZ}} \) is order of \( \sim 10^{-3} \).

- The differential branching fraction with respect to the electron momentum in final states and with respect to the charged Higgs spectrum, they are limited by phase space and for lighter Higgs pair masses, the momentum of electron is larger.

- The distribution of the direction of the electron in the final states is strongly peaked at \( \cos \theta_e = -1 \). This implies the electron scattered into the forward direction with respect to the incoming electron. This happens because virtuality of \( Z^* \) boson is minimized in this case.

- About the azimuthal \( \phi_{eH} \) angle distributions, we find that the charged Higgs momentum more likely lies within the range \( 0 \leq \phi_{eH} \leq \pi \) than in \( \pi \leq \phi_{eH} < 2\pi \).
FIG. 7: The differential cross sections $\Delta \sigma_{1H^+A}$ and $\Delta \sigma_{1WZ}$ as functions of the momentum $q_e$(GeV) for the final state electron. We have chosen the width of each bin as $\Delta q_e = 50$(GeV). The solid line marked with the plus sign $+$ corresponds to $e^+ + e^- \rightarrow W^+ + Z + \bar{\nu}_e + e^-$. The other lines denote the three cases for $e^+ + e^- \rightarrow H^+ + A + \bar{\nu}_e + e^-$. The long dashed line marked with cross symbol $\times$ corresponds to the case $(m_{H^+}, m_A) = (200, 200)$(GeV). The dotted line marked with the boxes; $\Box$ corresponds to $(m_{H^+}, m_A) = (300, 200)$(GeV) and the short dashed line marked by asterisks $*$ corresponds to $(m_{H^+}, m_A) = (200, 300)$(GeV). The center of mass energy is 1000 (GeV).

V. THE SIGNATURE OF THE CHARGED HIGGS AND THE NEUTRAL HIGGS PAIR PRODUCTION

As we have seen from the studies of the previous section, the cross section and the differential cross sections of the Higgs pair production are much smaller than gauge boson pair production. Considering the smallness, one may wonder if such Higgs pair production and their decays have the distinct signals. Here we consider the charged lepton flavor dependence of the charged Higgs decays into anti-lepton and neutrino. Note that the dominant neutral Higgses decay channel is neutrino and anti-neutrino pair when the neutral Higgs
FIG. 8: The differential cross section $\Delta \sigma_{2H^+A}$ with respect to the charged Higgs momentum $q_{H^+}$. The horizontal axis denotes $q_{H^+}$ (GeV). The long dashed line marked with cross symbol $\times$ corresponds to the case $(m_{H^+}, m_A) = (200, 200)$ (GeV). The dotted line marked with the boxes; $\Box$ corresponds to $(m_{H^+}, m_A) = (300, 200)$ (GeV) and the short dashed line marked by asterisks $\ast$ corresponds to $(m_{H^+}, m_A) = (200, 300)$ (GeV). The center of mass energy is 1000 (GeV) and the width of each bin ($\Delta q_{H^+}$) is 50 (GeV). For comparison, we also show the solid line with the plus sign $+$ for $W, Z$ pair production cross section, $\Delta \sigma_{2WZ}$ as a function of the momentum of $W$ boson in final state $q_W$ (GeV). For the cross section, the horizontal axis denotes the $W$ boson momentum.

and charged Higgs are degenerate as $|m_A - m_{H^+}| < m_W$. We study the degenerate case. In this case, the neutral Higgs decay products are invisible and the visible decay product is a charged anti-lepton $l^+$ from the charged Higgs decay. Therefore, the whole process starting from $e^+e^-$ collision to Higgs decays looks like,

$$
e^+ + e^- \rightarrow \bar{\nu}_e + e^- + H^+ + A \rightarrow \bar{\nu}_e + e^- + l^+\nu_l + \nu_k\bar{\nu}_k.$$

(40)
FIG. 9: The differential cross sections $\Delta \sigma_{3H+A}$ for $e^+ + e^- \to H^+ + A + \overline{\nu}_e + e^-$ with respect to $\cos \theta_e$ where $\theta_e$ denotes the angle between the final electron momentum and the initial positron momentum. The long dashed line marked with cross symbol $\times$ corresponds to the case $(m_{H^+}, m_A) = (200, 200)$ (GeV). The dotted line marked with the boxes; $\square$ corresponds to $(m_{H^+}, m_A) = (300, 200)$ (GeV) and the short dashed line marked by asterisks $\ast$ corresponds to $(m_{H^+}, m_A) = (200, 300)$ (GeV). The center of mass energy is 1000 (GeV) and the width of each bin $(\Delta \cos \theta_e)$ is 0.2. For comparison, we show the cross section $\Delta \sigma_{3WZ}$ of the process $e^+ + e^- \to W^+ + Z + \overline{\nu}_e + e^-$ with solid line. We use the formulae for the $W + Z \to W + Z$ scattering in [12]. The center of mass energy of $e^+ e^-$ collision is 1000 (GeV).

One finds the same final state as in Eq. (40) in gauge bosons pair production process of $e^+ e^-$ collision as follows; By replacing the charged Higgs boson with $W^+$ boson and the neutral Higgs boson $A$ with $Z$ boson in Eq. (40), the decay channels $Z \to \nu_k \overline{\nu}_k$ and $W^+ \to l^+ \nu_l$ lead to the same final state as that of Eq. (40).

$$
e^+ + e^- \to \overline{\nu}_e + e^- + W^+ + Z$$
$$\to \overline{\nu}_e + e^- + l^+ \nu_l + \nu_k \overline{\nu}_k.$$  \hspace{1cm} (41)
FIG. 10: Differential cross sections for $\Delta \sigma_{4H+A}$ and $\Delta \sigma_{4WZ}$. The horizontal axis corresponds to $\cos \theta_{eH}$ and $\cos \theta_{eW}$. $\theta_{eH}(\theta_{eW})$ is an angle between the momentum of the final electron and the one of the charged Higgs boson ($W$ boson). The solid line marked with the plus sign + corresponds to $WZ$ production. The other three lines are Higgs pair production. Among them, the long dashed line marked with the cross symbol $\times$ corresponds to the case $(m_{H^+}, m_A) = (200, 200)$(GeV). The dotted line marked with the boxes; $\square$ corresponds to $(m_{H^+}, m_A) = (300, 200)$(GeV) and the short dashed line marked by asterisks $\ast$ corresponds to $(m_{H^+}, m_A) = (200, 300)$(GeV). The center of mass energy is 1000 (GeV) and the bin widths; $\Delta \cos \theta_{eH}$ and $\Delta \cos \theta_{eW}$ are 0.2.

Since Eq.(41) has a common final state with Eq.(40), they look indistinguishable. However as pointed in [3], the branching fraction of the charged Higgs decay into anti-lepton is flavor non-universal and depends on the lepton family. They are written in terms of the neutrino mixings and masses which precise data except lightest neutrino mass and CP violating phase is now available. Since the $W$ boson decay into anti-lepton is flavor blind, we study the lepton flavor dependence of charged Higgs decay by taking the ratio with the weak gauge boson
FIG. 11: Differential cross sections $\Delta \sigma_{5H^+A}$ and $\Delta \sigma_{5WZ}$. The horizontal line denotes the azimuthal angles $\phi_{eH}$ and $\phi_{eW}$ (radian). The solid line marked with the plus sign $+$ corresponds to $WZ$ production. The other three lines are Higgs pair production. Among them, the long dashed line marked with cross symbol $\times$ corresponds to the case $(m_{H^+}, m_A) = (200, 200)$ (GeV). The dotted line marked with the boxes; $\Box$ corresponds to $(m_{H^+}, m_A) = (300, 200)$ (GeV) and the short dashed line marked by asterisks $\ast$ corresponds to $(m_{H^+}, m_A) = (200, 300)$ (GeV). The center of mass energy is 1000 (GeV) and the bin widths; $\Delta \phi_{eH}$ and $\Delta \phi_{eW}$ are $\frac{\pi}{5}$.

Pair production and decay branching fractions. The ratio we define is

$$r_l = \frac{\sum_{X=h,A} \sigma_{H^+X} Br(X \to \nu \bar{\nu}) Br(H^+ \to l^+ \nu_l)}{\sigma_{WZ} Br(Z \to \nu \bar{\nu}) Br(W^+ \to l^+ \nu_l)}.$$  \hspace{1cm} (42)

where we used the shorthand notation, $Br(X \to \nu \bar{\nu}) = \sum_k Br(X \to \nu_k \bar{\nu}_k)$, for $X = h, A, Z$. Using the notations, one can write $r_l$ as,

$$r_l = \frac{2\sigma_{H^+A} Br(A \to \nu \bar{\nu}) Br(H^+ \to l^+ \nu_l)}{\sigma_{WZ} Br(Z \to \nu \bar{\nu}) Br(W^+ \to l^+ \nu_l)},$$ \hspace{1cm} (43)

where we use the fact that the production cross section for CP even and CP odd Higgs with $U(1)$ charge is almost identical to each other; i.e., $\sigma_{H^+A} \simeq \sigma_{H^+h}$. (See appendix A). We also
FIG. 12: The ratio of the cross sections of Higgs pair production and gauge boson pair production; 
\[ \frac{\sigma_{H^+A} + \sigma_{H^+h}}{\sigma_{WW}} \] as functions of center of mass energy of \( e^+e^- \) collision; \( \sqrt{s_{e^+e^-}} \)(GeV). The solid line corresponds to the case for \( (m_H^+, m_A) = (300, 200) \)(GeV). The dashed line corresponds to the degenerate case, \( m_A = m_{H^+} = 200 \)(GeV). The dotted line corresponds to the case \( (m_H^+, m_A) = (200, 300) \)(GeV).

use the branching fractions satisfy

\[ Br(A \rightarrow \nu \bar{\nu}) = Br(h \rightarrow \nu \bar{\nu}) = 100\%. \]  

(44)

We have shown the ratio of cross sections in Fig. 12. When Higgs masses are degenerate \( m_A = m_{H^+} = 200 \) (GeV), the ratio of the cross section is about \( 1.4 \times 10^{-3} \) for \( \sqrt{s_{e^+e^-}} = 1000 \)(GeV). In what follows, we use this value as benchmark point for the ratio of the cross sections in Eq. (43). The other branching fractions which appear in Eq. (43) are quoted from Particle Data Group (PDG) [13],

\[ Br(W^+ \rightarrow \tau^+\nu) = 11.25 \pm 0.20\%, \]
\[ Br(W^+ \rightarrow \mu^+\nu) = 10.57 \pm 0.15\%, \]
\[ Br(W^+ \rightarrow e^+\nu) = 10.75 \pm 0.13\%, \]
\[ Br(Z \rightarrow \nu\bar{\nu}) = 20.00 \pm 0.06\%. \]  

(45)
Using the numerical values, one can write \( r_l(l = e, \mu, \tau) \) as,

\[
\begin{align*}
    r_e & = 0.465 \times \text{Br}(H^+ \to e^+\nu) \frac{2\sigma_{H^+A}}{\sigma_{WZ}}, \\
    r_\mu & = 0.473 \times \text{Br}(H^+ \to \mu^+\nu) \frac{2\sigma_{H^+A}}{\sigma_{WZ}}, \\
    r_\tau & = 0.444 \times \text{Br}(H^+ \to \tau^+\nu) \frac{2\sigma_{H^+A}}{\sigma_{WZ}},
\end{align*}
\]

where \( \text{Br}(H^+ \to l\nu) \) in % unit should be substituted. The charged Higgs can decay into charged leptons and neutrino. In contrast to the leptonic decay of W boson, the branching fractions for each flavor of charged lepton are obtained from Eq.(46)

\[
\text{Br}(H^+ \to l^+\nu_l) = \frac{\sum_{i=1}^{3} m_i^2|V_{li}|^2}{\sum_{i=1}^{3} m_i^2} \times 100%.
\]

We update the branching fraction to each lepton flavor mode using the recent results on \(|V_{e3}|\). For normal hierarchy case, the branching fractions are written as,

\[
\text{Br}(H^+ \to l^+\nu_l) = \frac{m_1^2 + \Delta m_{2\text{sol}}^2 |V_{l2}|^2 + (\Delta m_{2\text{sol}}^2 + \Delta m_{2\text{atm}}^2)|V_{l3}|^2}{3m_1^2 + 2\Delta m_{2\text{sol}}^2 + \Delta m_{2\text{atm}}^2} \times 100%.
\]

In the formulae of Eq.(48), \( m_1 \) denotes the lightest neutrino mass. For inverted hierarchical case, they are written as,

\[
\text{Br}(H^+ \to l^+\nu_l) = \frac{m_3^2 + \Delta m_{2\text{atm}}^2 (|V_{l1}|^2 + |V_{l2}|^2) - \Delta m_{2\text{sol}}^2 |V_{l1}|^2}{3m_3^2 + 2\Delta m_{2\text{atm}}^2 - \Delta m_{2\text{sol}}^2} \times 100%,
\]

where \( m_3 \) denotes the lightest neutrino mass. We have used the following values for the mixing angles and mass squared differences quoted from Table 13.7 in the review section of Neutrino Mass, Mixing, and Oscillation of [13], \( \sin^2 \theta_{12} = 0.306 \), \( \sin^2 \theta_{23} = 0.42 \), \( \sin^2 \theta_{13} = 0.021 \), \( m_{2\text{atm}}^2 = 2.35 \times 10^{-3} (\text{eV}^2) \) and \( m_{2\text{sol}}^2 = 7.58 \times 10^{-5} (\text{eV}^2) \). In Fig.13 we have shown \( r_l \) (\( l = e, \mu, \tau \)) for normal hierarchical case as functions of the lightest neutrino mass \( m_1 \). In Fig.14 we have shown \( r_l \) for inverted hierarchical case as functions for the lightest neutrino mass \( m_3 \). As we have seen from Fig.13 and Fig.14 we can expect \( 2\% \sim 3\% \) lepton flavor dependence from charged Higgs decay. We summarize the flavor dependence.

- For normal hierarchical case, for \( 0 \leq m_1 < 0.05\text{(eV)} \) \( r_\tau > r_\mu >> r_e \). For larger \( m_1 \) until 0.2 eV, \( r_\mu \sim r_e \sim r_\tau = 0.02 \).

- For inverted hierarchical case, \( r_e > r_\mu > r_\tau \) for \( 0 < m_3 < 0.2 \text{ eV} \).
VI. CONCLUSIONS AND DISCUSSIONS

In this paper, we study the pair production of charged Higgs and neutral Higgs in the neutrinophilic two Higgs doublet model. The pair production process is not suppressed by the U(1) charge conservation. In other words, the approximate global symmetry allows the pair production to occur.
We study the total cross section for the pair production in $e^+e^-$ collision. The pair production occurs through W boson and Z boson fusion. We study the pair production and the decays for degenerate mass of charged Higgs and neutral Higgs as well as non-degenerate case. The cross section increases from $10^{-4}$ fb to $10^{-3}$ fb as the cm energy of $e^+e^-$ varies from 1 (TeV) to 2 (TeV). The cross section is compared with that of $W, Z$ pair production. We show the Higgs pair production is about $10^{-3}$ times smaller than the pair production cross section of the gauge bosons. Therefore if Z decays invisibly into neutrino pairs and W boson decays into anti-lepton and neutrino, the gauge boson pair production and their decays becomes a background to the signal. When the charged Higgs ($H^+$) and neutral Higgs ($X = A, h$) are degenerate as $|m_{H^+} - m_X| < M_W$, which is favored from the electroweak precision data, the charged Higgs dominantly decays into anti-lepton and neutrino and neutral Higgs decays dominantly into neutrino and anti-neutrino pair. Compared with them, W and Z decay branching ratio in the same final state is smaller than that of Higgs decays and is flavor blind. Therefore, by studying the charged anti-lepton flavor in the final state, we may distinguish the Higgs pair production and their decays from that of gauge bosons. We expect $2\% \sim 3\%$ flavor dependence which is null for the gauge bosons decays. Depending on the normal or inverted hierarchy of the mass spectrum of neutrinos, the order of $r_e, r_\mu$ and $r_\tau$ changes. We show the differential cross sections with respect to electron, charged Higgs momentum. The differential cross sections with respect to the angles of electron and charged Higgs in the final states are also shown. They are also important to identify the signals.

Appendix A: Amplitude for $W^+ + Z^* \rightarrow H^+ + h$

In this appendix, we have shown the off-shell charged Higgs and CP even neutral Higgs (h) boson production amplitude for gauge boson fusions $W^+ + Z^* \rightarrow H^+ + h$.

$$T_{\mu\nu} = \frac{g^2 \cos(\beta + \gamma)}{2 \cos \theta_W} (a_h g_{\mu\nu} + d_h q_{h\nu} q_{H^+\mu} + b_h q_{H^+\nu} q_{h\mu}), \quad (A1)$$

where we compute the four Feynman diagrams corresponding to , the contact interaction (Fig. 2), the s channel $W^+$ exchange (Fig. 3) u channel charged Higgs exchange (Fig. 4), and t channel CP odd Higgs (A) exchanged diagram (Fig. 5). $a_h, b_h$ and $d_h$ in Eq. (A1) are given
as,

$$a_h = -s_W^2 - \frac{p_W^2 - p_Z^2}{M_Z^2} \frac{M_h^2 - M_{H^+}^2 - M_W^2}{s_{H+h} - M_W^2} - c_W^2 \frac{t_h - u_h + p_Z^2 - p_W^2}{s_{H+h} - M_W^2},$$

$$b_h = \frac{2 \cos 2\theta_W}{u_h - M_{H^+}^2} + \frac{2(\cos 2\theta_W + 1)}{s_{H+h} - M_W^2},$$

$$d_h = -\frac{2}{t_h - M_A^2} - \frac{2(\cos 2\theta_W + 1)}{s_{H+h} - M_W^2} ,$$

(A2)

with $t_h = (q_{H^+} - p_W)^2$, $u_h = (p_W - q_h)^2$ and $s_{H+h} = (q_{H^+} + q_h)^2$. By taking the vanishing limit of the U(1) breaking term; i.e., $m_{12} \to 0$, $\beta$ and $\gamma$ vanishes. Note also in this limit, one can show $m_h = m_A$ and $-iT_{A\mu\nu} = T_{h\mu\nu}$ with the appropriate replacement $q_A \to q_h$. (See Eq.(10).) Therefore in this limit the production amplitudes for $H^+A$ and $H^+h$ are identical to each other, $\sigma_{H^+A} = \sigma_{H^+h}$.

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