Semi-active control of a nonlinear quarter-car model of hyperloop capsule vehicle with Skyhook and Mixed Skyhook-Acceleration Driven Damper controller

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Abstract
A suspension system is one of the integral parts of a hyperloop capsule train, which is used to isolate the car-body from bogie vibration to provide a safer and comfortable service. A semi-active suspension system is one of the best candidates for its advantageous features. The performance of a semi-active suspension system relies greatly on the control strategy applied. In this article, Skyhook (SH) and mixed Skyhook-Acceleration Driven Damper (SH-ADD) controlling algorithms are adopted for a nonlinear quarter-car model of a capsule with semi-active magnetorheological damper. The nonlinear vertical dynamic response and performance of the proposed control algorithms are evaluated under MATLAB Simulink environment and hardware-in-loop-system (HILS) environment. The SH controlled semi-active suspension system performance is found to be better at the first resonance frequency and worse at the second resonance frequency than the passive MR damper, but the mixed SH-ADD controlled semi-active suspension system performs better than the passive at all frequency domains. Taking the root-mean-square (RMS) value of sprung mass vertical displacement as an evaluation criterion, the response is reduced by 58.49% with mixed SH-ADD controller and by 54.49% with the SH controller compared to the passive MR damper suspension.

Keywords
Subspace nonlinear identification, magnetorheological damper, electrodynamic suspension system, Skyhook, mixed Skyhook-Acceleration Driven Damper

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Introduction
Nowadays, many companies across the world are doing researches intensively on a hyperloop capsule to unveil it as the fifth mode of transportation, and there is a high computation between them for the new achievement. A joint team from Tesla and SpaceX released an Alpha study of hypersonic speed ground transportation system concept, which is called hyperloop, in August 2013.1 According to the proposed concept, the pods or capsules of a hyperloop move through an almost-
A hyperloop capsule is a combination of many complex integral subsystems, and one of these subsystems is the suspension system, which is required to isolate a car from undesired guideway disturbance to provide comfort and safety to the passengers. In a conventional passive suspension system, the damping characteristic coefficient is constant, for all types of load and input disturbance, and the damping efficiency is limited, which are a disadvantage. The limitations of passive suspension motivated the investigation of the active suspension system. The hydropneumatics active-suspension system was first introduced in 1954 by Paul Mages at Citroen company then Colin Chapman developed the concept of electronic actuation of hydraulic suspension system in 1980 for the improvement of racing car suspension. Taking Colin Chaman’s idea as a genesis, much researches have been done on an active-suspension system, and the efficiency of a damper is increased tremendously. Although the active suspension system is very efficient in damping, it requires a high amount of energy, and the cost of construction is very high, which made the system prohibitive. As a result of those disadvantages of the active suspension system, a semi-active suspension system emerged as the best compromise between cost (construction and energy cost) and performance (comfort, safety and handling). A semi-active suspension system utilizes a variable damping fluid or another variable dissipation mechanism to vary the damping characteristics of the damper, and it is classified as a magnetorheological (MR) damper or orifice-based damper. In an orifice-based damper, the suspension system consists of a twin-tube viscose damper piston for the variation of the diameter of the orifice. Hence, the damping coefficient varies. In an MR damper, the damping fluid composed of oil and varying percentage of ferrous magnetic particle 20–50 microns in diameter. So, by varying the magnetic field strength the viscosity of the MR fluid will be varied. Nowadays, MR damper, Figure 1, becomes a prominent suspension system for earthmoving vehicles, especially in an automobile industries and high-speed trains because of many advantageous features, such as variable damping characteristics, requiring a small amount of energy to control its damping efficiency and its adaptability for controlling. In the MR damper suspension system, the damping performance is controlled by a small input current or voltage to the MR damper and it acts as a passive damper in the absence of current, so the system is highly reliable in operation and fail-safe. Considering all the above advantages, the secondary suspension system in this paper consists of an MR damper.

Even though the main focus of this paper is on the control of a semi-active secondary suspension system of a capsule, some points on the primary suspension system shall be discussed, to clearly describe the capsule’s suspension system vertical vibration. At high speed, frictional loss and dynamic instability are very high. Due to these reasons, the conventional wheel-axle system is impractical to apply as a suspension system for a hyperloop capsule. To eliminate this problem many researchers have been doing research to invent a new suspension system. One of the potential options, air bearing suspension, was tested but it is found to be not good for onboard passengers’ comfort. Enormous researchers have proposed magnetic levitation and concluded that the system is most appropriate for levitation and propulsion of advanced ground transportation systems such as high-speed train and hyperloop capsule, though the cost of construction is prohibitive.

There are two main competing technologies of a magnetic levitation suspension system. Electromagnetic Suspension (EMS) system is Germany’s technology, which works under the principle of magnetic attraction. In an EMS system, the train has a C-shaped bogie that warps the almost T-shaped track, so that the magnetized attractive force between the lower side of the track and bogie will support the whole mass of the vehicle. In this principle, the maximum acceptable air gap between the two magnetized components is 10–20 mm. Electodynamic Suspension (EDS) system, the Japanese technology, works under the principles of magnetic repulsion. The two main components of this system are a current-carrying coil fixed on the moving vehicle and a conductive band built on the guideway. When the current-carrying coil is moving above the guideway at some velocity the magnetic field produced by the current-carrying coil induces an eddy current to the conductive band and their magnetic fields are in the repulsive state. So that by controlling the amount of current induced to the current-carrying coil, the repulsive force required to float the vehicle over the
The guideway can be controlled. In this system, the vehicle can be levitated from 10 to 100 mm.\textsuperscript{7,8} The detailed working principle of this electromagnetic suspension (EMS) and electrodynamic suspension (EDS) can be referred to on paper by Rakesh et al.\textsuperscript{7} Because of many advantages explained above, Electrodynamically Suspension (EDS) system is used as the primary suspensions system for the capsule train in this work.

Three main types of vehicle models (quarter-car, half-car and full-car model) have been developed for vehicle dynamics characteristics analysis. The quarter-car model mainly focuses on the vertical dynamics of a vehicle for ride comfort improvement and vibration isolation analysis. In a half-car and full-car model, other dynamic characteristics such as the lateral, pitch and yaw motions of a vehicle are studied. In this paper a quarter-car model is used for the performance analysis of the suggested semi-active suspension system controllers.

The next section of this paper is organized as follows. Section 2 is about the analysis of the EDS system nonlinear stiffness coefficient which is applied as a levitation system. Section 3 discusses a nonlinear quarter-car dynamics model of a hyperloop capsule. Section 4 discusses a semi-active suspension system controlling mechanisms. The HILS experimental setup is discussed in section 5, the experimental results are discussed in section 6 and finally the conclusion is given in section 7.

### Equivalent electrodynamic stiffness coefficient

A prominent experimental and numerical analysis for the dependence of electrodynamic suspension lateral and vertical stiffness coefficient on lateral and vertical displacement and forward velocity of the bogie is done by Shunsuke et al.\textsuperscript{9} used a side wall electrodynamic suspension system for the analysis to get the advantage of not requiring the gap controller. In a side wall electrodynamic levitation system, the short circuit current-carrying coil is installed on the guideway sidewall and the superconducting magnet is fixed on the capsule train as shown in Figure 2. The detailed analysis of the EDS stiffness coefficient relationship is beyond the scope of this paper. So those who want to see the detailed analysis and experimental evaluation shall refer to Shunsuke et al. publication. In this work, the lateral displacement of the bogie is not considered. So that, the stiffness coefficient depends on the vertical displacement and forward velocity of the capsule.

The average and oscillation stiffness coefficients as a function of forward velocity and position of the bogie relative to the centre of the current-carrying coil are expressed as follows (equations (1)–(3))

\[
k_{eds, ave}(v_s, z) = (5.43 \times 10^6 - 1.29 \times 10^8 \times z^2) \times e^{-6.18/v_s} \tag{1}
\]

\[
k_{eds, osc}(v_s, z) = \left(6.85 \times 10^5 \times e^{-1.43/v_s} - 1.56 \times 10^7\right) \times \sin(2\pi x/\tau_{lev}) \tag{2}
\]

\[
k_{eds} = k_{eds, ave} + k_{eds, osc} \tag{3}
\]

The maximum acceptable vertical displacement \(z\) of the bogie is \(-0.1<z<0\) m. There are four superconducting coils on each side of a bogie, and the size of the superconductor coil is equal to the size of a levitation coil, as shown in Figure 3. Therefor the pole pitch \(\tau_{lev}\) will be

\[
\tau_{lev} = \frac{1}{8} \times \text{Length of the bogie} \tag{4}
\]
Quarter-car model of a capsule train.

Quarter-car vehicle semi-active suspension system nonlinear dynamic model

This paper focuses on the vertical motion of a vehicle only, so a quarter-car model, as illustrated in Figure 4, is taken for the analysis of the vertical dynamics of the suspension system as well as to design control of semi-active MR damper on the secondary suspension system. \( m_s \) and \( m_u \) represent the car body and bogie/ski (sprung and unsprung) masses of the capsule train respectively. \( k_s \) and \( k_p \) denote the secondary and primary suspension stiffness coefficients, \( c_p \) represents the damping coefficient of the primary damper and \( F_{MRD} \) represents the actuating force produced by semi-active secondary suspension damper, in this case an MR damper. \( z_t \) and \( z_u \) represent the sprung and unsprung mass vertical displacements of a system respectively and \( z_i \) represents the track irregularity, in this case guideway irregularity. The primary suspension stiffness coefficient \( k_p \) (\( \;= \;k_{eds} \)) is nonlinear, and then the dynamic system of quarter-car model as a whole is also nonlinear.

Many kinds of research have been done to find out computationally less complex and more accurate method for the analysis of nonlinear dynamic suspension system. These days, a subspace identification method becomes more popular and advantageous for the generalized parametrization of multiple-input multiple-output (MIMO) systems and many researchers use this method as a millstone idea. Marchesiello and Garibaldi proposed a nonlinear subspace identification (NSI). In this publication, the nonlinear parts are assumed as unmeasured internal feedback force to the linear system with particular attention to reduce the ill-condition of the system. Noel et al. proposed a Grey-box nonlinear State-space modelling method, in which a two-step identification procedure with the integration of nonlinear subspace initialization and optimization is derived for the grey-box. Jie et al. proposed a novel nonlinear separation subspace identification (NSSI) method. In this paper, the nonlinearity is due to the electrodynamic levitation stiffness coefficient. It depends on the vertical and horizontal position and forwarding velocity of the bogie. So that the capsule’s nonlinear dynamics of a quarter-car model is defined as a combination of linear and nonlinear parts, as shown in equation (5).

\[
M\ddot{z}(t) + C_{\text{linear}}\dot{z}(t) + K_{\text{linear}}z(t) + \sum_{i=1}^{n} \mu_i L_i K_{\text{nonlinear}}(v,z,x) * Z(t) = F(t)
\]  

Where \( \mu_i \) represents the weight of nonlinear component that needs to be determined and \( L_i \) indicates the location of the nonlinear component and its value can be \( -1, 0 \) or \( 1 \). \( M \) represents mass matrix of the linear part, \( C_{\text{linear}} \) denotes the linear damping coefficient matrix, \( K_{\text{linear}} \) represents the stiffness matrix of the linear part and \( K_{\text{nonlinear}} (= k_p = k_{eds}) \) is the nonlinear stiffness matrix. \( n \) represents the number of nonlinear components of the system, in our case the nonlinear component is one \( (n = 1) \). So that, equation (5) can be simplified as

\[
M\ddot{z}(t) + C_{\text{linear}}\dot{z}(t) + K_{\text{linear}}z(t) + \mu k_{eds}(v,z,x) * Z(t) = F(t)
\]  

Using equation (6) and Figure 3, the vertical dynamics of the quarter-car model of a capsule is expressed in state space form as in equation (7) below

\[
\begin{align*}
\dot{m}_u \ddot{z}_u &= k_s(z_i - z_u) - c_p(z_u - \dot{z}_i) - \mu k_{eds}(z_u - z_i) + F_{MR} \\
\dot{m}_s \ddot{z}_s &= -k_s(z_i - z_u) - F_{MR}
\end{align*}
\]  

Using the nonlinear separation strategy, the unknown nonlinear EDS stiffness and MR damper force terms are regarded as the internal feedback input to the linear system and it is illustrated as in Figure 5.

\[
\begin{align*}
\dot{m}_u \ddot{z}_u - k_s(z_i - z_u) + c_p(z_u - \dot{z}_i) + \mu k_{eds}(z_u - z_i) - F_{MR} = 0 \\
\dot{m}_s\ddot{z}_s + k_s(z_i - z_u) + F_{MR} = 0
\end{align*}
\]  

Equation (8) can be represented by state space form as below

\[
\begin{align*}
\dot{z}(t) &= A z(t) + B u(t) \\
y(t) &= C z(t) + D u(t)
\end{align*}
\]
Where \( z(t) = \begin{bmatrix} z_u \\ z_y \\ \dot{z}_u \\ \dot{z}_y \end{bmatrix} \) and \( z(t) = \begin{bmatrix} z_u \\ z_y \\ \dot{z}_u \\ \dot{z}_y \end{bmatrix} \):

\[
A = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-k/m_u & k/m_u & -k/m_u & 0 \\
k/m_s & -k/m_s & 0 & 0
\end{bmatrix}
\]
\[
B = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
1/m_u & 0 \\
0 & 1/m_s
\end{bmatrix}
\]
\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
\[
D = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\]
\[
u = \begin{bmatrix}
F_{MR} - \mu k_{eds} (z_u - z_t) + \epsilon p \dot{z}_t \\
-F_{MR}
\end{bmatrix}
\]

The nonlinear weight term \( \mu \) is determined using the method suggested in Jie et al., and the value is found to be 1 in this paper. Using equation (3), the value of the nonlinear stiffness coefficient \( k_{eds} \) can be calculated at any velocity and vertical and forward position of the bogie. Simulation of quarter-car model is done with MATLAB Simulink environment as shown in Figure 6. In the EDS Stiffness subsystem, the nonlinear levitation electrodynamic force will be calculated based on the relative vertical displacement of the bogie to the guideway, as well as forward velocity and position of the bogie. In the system controller subsystem, the amount of required force will be computed based on the relative vertical displacement and velocity of the car-body and the manipulated data is fed to the damper controller. In the Damper controller the required amount of current or voltage is computed and fed to the MR damper. Finally, the nonlinear levitation and damper forces are fed to the linear state space subsystem as an internal feedback input, \( u(t) \).

**Semi-active suspension system**

The damping efficiency of a semi-active suspension system relies greatly on its control system. Therefore, developing an efficient control algorithm is the main focus of a semi-active suspension system. In an MR damper semi-active suspension system, the system consists of two nested controllers (MR damper controller and system controller), as illustrated in Figure 7. The MR damper controller controls the amount of current or voltage input to the MR damper. A system controller computes the desired damping force for a given system condition.

**Damper controller**

The MR Damper controller verifies and controls the amount of input current to the MR damper by taking the relationship between the input current and an MR damper’s output force (\( F_{MRD} \)) and the required force (\( F_{req} \)) by the system into consideration. Therefore, before designing the MR damper controller, the model shall be developed, which describes the nonlinear and force-velocity hysteretic characteristic relationship between the inputs and outputs of the MR damper adequately. In the case of MR damper forward dynamics, the inputs are current, a relative displacement and a relative velocity between the two ends of MR damper, and the output is MR damper force (\( F_{MRD} \)). The nonlinear hysteretic characteristic of the MR damper can be identified by parametric or non-parametric.
modelling techniques. Bouc-Wen model, a parametric one, proposed by Spencer et al.\textsuperscript{14} is quite efficient to express the nonlinear hysteretic properties of the MR damper over a wide range of inputs and we use this model to express our MR damper characteristic in this paper. The model is expressed by equation (10) as shown below.

\[
F_{MRD} = c\dot{z} + kz - f_o + \alpha w, \\
\dot{w} = \delta \dot{w} - \beta |w|^n - \gamma |z| w |w|^{n-1}
\]  

(10)

Where \( c, \alpha, f_o, \delta, \beta, \gamma \), and \( n \) are parameters of Bouc-Wen model, which are going to be identified for a given MR damper through optimization technique of experimental results and \( z \) is the relative displacement between the two ends of an MR damper. A sinusoidal excitation with maximum displacement 250 mm at low frequencies (1 Hz and 2 Hz) fed to the MR damper, and the experimental results \( F_{MRD} \) are recorded at 0.002 s interval for the input current of 0–2 A at a step of 0.2 A. Then the parameters are identified by using a novel genetic algorithm parameter identification technique proposed by Birhan et al.\textsuperscript{18} and the optimized parameters’ value are found to be

\[
c(i) = 6.594 + 5.818 \\
\alpha(i) = 3.658 \times 10^4 + 8661 \\
k = 0.155 \times 10^{-3} \\
f_o = 199.35 \\
\delta = 0.110 \\
\beta = 2.465 \\
\gamma = 45.87 \times 10^3 \\
n = 4.974
\]

Where \( i \) = the input current and the maximum value is 2 A

Using the above stated MR damper model, the MR damper controller provides the required amount of current to track the desired force signal. The MR damper controllers, proposed in many literatures, can be categorized as in the following categories, and each has advantages and disadvantages.

**Heaviside Step Function (HSF):** It is a feedback type an ‘on-off’ control algorithm where the applied current is either 0 or \( i_{\text{max}} \).\textsuperscript{19} It is a simple algorithm, but the current input is always either 0 or the maximum allowable current, which is a disadvantage

\[
i = i_{\text{max}} H \{ (F_{req} - F_{MRD}) / F_{MRD} \}
\]

(11)

Where: \( H(.) \) is a Heaviside function and \( i_{\text{max}} \) is the maximum allowable input current.

**Signum Function Damper Controller (SFDC):**\textsuperscript{20} it is a modified form of the Heaviside function method, in which under certain conditions, the input current can be switched to some discrete value below the maximum allowable amount. But still, the command input current signal is a discrete type, which is a disadvantage for this method.

**Inverse Polynomial Controller:** it is feed forward control law type. Chio and Lee\textsuperscript{21} divided the nonlinear hysteretic characteristics of the MR damper into positive acceleration (lower loop) and negative acceleration (upper loop) and introduced a polynomial function of degree 6 to represent these loops and also the MR damper model. Then, Haiping et al.\textsuperscript{22} control the current input by taking the inverse polynomial of the MR damper proposed by Choi SB et al.\textsuperscript{23} Arias-Montiel et al.\textsuperscript{24} also proposed a polynomial function with degree 2 to model the MR damper. But in both cases, the polynomial function

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**Figure 7.** General layout of semi-active suspension system controller.
model accuracy is not good as compared to the Bouc-Wen model, especially when the nonlinear hysteretic Force-velocity loop is not smooth.

Inverse MR damper Controller: It is also a feed forward control law type and the required current for a given desired force is estimated from the predefined MR damper’s inverse model. Xia\textsuperscript{25} and Boada et al.\textsuperscript{26} proposed a modified neural network technique for the inverse dynamic model of an MR damper to predict the input current for the desired force. This technique requires a high amount of experimental data for training and validation of the network. Additionally, its time delay effect is not promising as a result of its high computational expense.

Weber\textsuperscript{27} proposed a feedforward control law with a control-oriented mapping approach to reduce modeling effort of the inverse MR damper behaviour. The parallel, proportional and integral feedback gain control algorithm still supports the proposed forward control law to minimize the resulting force tracking error. Terzo and Russo\textsuperscript{28} proposed a combined feedback and feed forward control algorithm for his shear mode magnetorheological device. Pang et al.\textsuperscript{29} proposed a novel variable universal fuzzy controller. In his method he used a combined fuzzy neural network and particle swarm optimization for the computation of the required current. Sunil and Rakesh\textsuperscript{30} showed that the semi-active suspension system using MR damper fluid based on Bingham model is better than the passive suspension system for the ride quality improvement.

Continuous State Damper Controller (CSDC): It is a feedback control type and the MR damper current is estimated based on the difference between the desired control force and the actual MR damper force. Using this algorithm, the input current signal can be varied continuously between 0 and $i_{\text{max}}$ as shown in equation (12).\textsuperscript{31}

$$i = \begin{cases} 0, & G_{\text{fd}}(F_{\text{req}} - H_{\beta} F_{\text{MRD}}) \text{sgn}(F_{\text{MRD}}), \\ i_{\text{max}}, & G_{\text{fd}}(F_{\text{req}} - H_{\beta} F_{\text{MRD}}) \text{sgn}(F_{\text{MRD}}) < 0 \\ 0 \leq G_{\text{fd}}(F_{\text{req}} - H_{\beta} F_{\text{MRD}}) \text{sgn}(F_{\text{MRD}}) \leq i_{\text{max}} \\ G_{\text{fd}}(F_{\text{req}} - H_{\beta} F_{\text{MRD}}) \text{sgn}(F_{\text{MRD}}) > i_{\text{max}} \end{cases}$$

Where $G_{\text{fd}}$ and $H_{\beta}$ are the forward and feedback gains respectively, which are an intrinsic value of the controller, identified by trial and error or an optimization technique for tracking the desired force $F_{\text{req}}$. The input signal will be activated when the error signal $(F_{\text{req}} - H_{\beta} F_{\text{MRD}})$ and the MR damper force $(F_{\text{MRD}})$ have the same sign. Because of its low computational expense, which in turn has a minimum time delay effect and commanding continuous input current signal, it is preferable to other techniques.

Because of the above advantages and disadvantages, the continuous state damper controller (CSDC) is used as MR damper controller in this paper, and $G_{\text{fd}}$ and $H_{\beta}$ values are found to be 0.75 and 0.0038, respectively.

### System controller

There are numerous published techniques for controlling semi-active suspension system. Nagarkar et al.\textsuperscript{32} suggested a linear quadratic regulator (LQR) to control the active suspension system of an automobile and he conclude that the active controlled suspension system is better than the passive one. Saad et al.\textsuperscript{33} evaluated the performance of semi-active Bouc-Wen modelled MR damper with a PID controller under different disturbance inputs, and they concluded that the PID controller is efficient in increasing vehicle stability. Papkollu et al.\textsuperscript{34} compared the performance of the active suspension system with PID and $H_{\infty}$ controllers for 2-DOF Passenger-Car model, and they said that PID controller is better than $H_{\infty}$ controller for passenger comfort. Yamin et al.\textsuperscript{35} suggested particle swarm to tune skyhook controller and Sensitive analysis to tune PID controller for semi-active suspension system controlling and form the study they proved that skyhook with particle swarm tuning performs better than PID controller with sensitive analysis tuning. Though skyhook is the most widely used control technique in semi-active suspension system, it is not efficient for high frequency vibration isolation. Savarese M. et al.\textsuperscript{36} proposed a mixed skyhook and acceleration-driven-damper (ADD) to alleviate the drawback of skyhook controller and Liao et al.\textsuperscript{37} emulate this technique for high speed railway vehicle semi-active suspension system control. In a mixed SH-ADD control, Skyhook and ADD controllers are used interchangeably depending on the transition frequency, which is to be identified, to control the semi-active suspension system. Exploring all advantages and disadvantages of the above system control-
\[ F_{\text{sky}} = -C_{\text{sky}} \dot{z}_s \]  

Where, \( C_{\text{sky}} \) is the damping coefficient of the virtual skyhook and \( \dot{z}_s \) is velocity of the sprung mass (car-body).

In real situation, it is impossible to connect the moving car-body to a fixed frame. Therefore, a skyhook damping is to use a semi-active damper which is placed between sprung and unsprung mass as shown in Figure 8. A semi-active suspension system produces a control force of \(-C(I)(\dot{z}_u - \dot{z}_s)\). The semi-active control unit is required to enforce the damper to generate a force only if \( \dot{z}_s \) and \( \dot{z}_u - \dot{z}_s \) have the same sign and if the magnitude of the desired force is within operating range of the damper, that is if

\[ |C_{\text{min}}(\dot{z}_s - \dot{z}_u)| \leq |C_{\text{sky}}\dot{z}_s| \leq |C_{\text{max}}(\dot{z}_s - \dot{z}_u)| \]  

However, when \( \dot{z}_s \) and \( \dot{z}_u - \dot{z}_s \) have opposite signs, the semi-active suspension system has still producing a force which is opposite to the desired force and the only best option it can do is to set the damping coefficient to its minimum value. Therefore, the variable damping coefficient which best matches the demanded control force \( C_{\text{sky}}\dot{z}_s \) is given by

\[ C_{\text{min}} \leq C \leq C_{\text{max}} \]

Where, \( C_{\text{min}}, \text{and} C_{\text{max}} \) are the lower and upper bounds of damping coefficients of the semi-active damper.

Therefore, Karnopp et al. modified the required skyhook force in another way in relation to the relative velocity between car-body and bogie like below

\[ F_{\text{sky}} = \begin{cases} 
C_{\text{max}}(\dot{z}_s - \dot{z}_u), & \dot{z}_u(\dot{z}_s - \dot{z}_u) \geq 0 \\
C_{\text{min}}(\dot{z}_s - \dot{z}_u), & \dot{z}_u(\dot{z}_s - \dot{z}_u) < 0 
\end{cases} \]  

\text{Acceleration-Driven-Damper (ADD):} in this algorithm, the control factor is the acceleration signals of car-body and bogie and the required force is formulated as below

\[ F_{\text{sky}} = \begin{cases} 
C_{\text{max}}(\dot{z}_u - \dot{z}_s), & \dot{z}_s(\dot{z}_u - \dot{z}_s) \geq 0 \\
C_{\text{min}}(\dot{z}_u - \dot{z}_s), & \dot{z}_s(\dot{z}_u - \dot{z}_s) < 0 
\end{cases} \]  

\text{Mixed SH-ADD Controller:} The performance of an SH controller is better than an ADD controller in a low-frequency domain, and ADD is better than SH in a high-frequency domain. Therefore, in a mixed SH-ADD controller the shifting factor is expressed as follows:

\[ F_{\text{sky}} = \begin{cases} 
C_{\text{max}}(\dot{z}_u - \dot{z}_s), & (\dot{z}_s - \rho\dot{z}_u^2 \leq 0) \land (\dot{z}_s(\dot{z}_u - \dot{z}_s) \geq 0) \lor \\
(\dot{z}_s - \rho\dot{z}_u^2 > 0) \land (\dot{z}_s(\dot{z}_u - \dot{z}_s) \geq 0) \\
C_{\text{min}}(\dot{z}_u - \dot{z}_s), & (\dot{z}_s - \rho\dot{z}_u^2 \leq 0) \land (\dot{z}_s(\dot{z}_u - \dot{z}_s) < 0) \lor \\
(\dot{z}_s - \rho\dot{z}_u^2 > 0) \land (\dot{z}_s(\dot{z}_u - \dot{z}_s) < 0) 
\end{cases} \]

Where, \( \rho \) is the frequency response intersection point at which closed-loop systems using skyhook and ADD controller intersect.

\textbf{Experiment arrangement}

The processor unit analyses the quarter-car model of a capsule on a MATLAB Simulink environment and sends MR damper’s ends relative displacement and input current signals to the plant unit (MR damper) through the signal transmitter. According to input signals, the plant unit (MR damper) produces the required amount of force and send it back to the processor unit. The signal flowing in both directions passes through the signal transmitter to convert it from analogue to digital or vice versa, which is appropriate for the receiver. The MR damper current driver delivers the required amount of current to the MR damper based on the information received from the processor unit.
on the signal it receives from the signal transmitter. The load cell records the produced MR damper force and transmits a signal to the transmitter. The general layout of the experimental setup is illustrated in Figure 9.

**Result and discussion**

In this research, a quarter-car model of a hyperloop capsule dynamics is done for the analysis and performance evaluation of the SH and mixed SH-ADD semi-active suspension system. The semi-active suspension system is assumed to be consist of an MR damper with a CSDS MR damper controller and a mixed SH-ADD and SH system controllers. Parameter values of the hyperloop capsule is stated in Table 1.

The guideway irregularity (input disturbance to the system) is assumed to be a white noise signal of normally distributed random number with a maximum amplitude of ± 3 mm as shown in Figure 10.

The model’s simulation is done on MATLAB Simulink alone environment and hardware-in-loop system (HILS) environment using equations (3) and (9) and parameter values (Table 1) to evaluate the performance of the suggested semi-active suspension system controllers. Based on equation (3), the model’s electrodynamic levitation system nonlinear stiffness spring coefficient is found to be as shown in Figure 11 below, and the amount of current fed by the current controller to the MR damper with SH and mixed SH-ADD system controller is shown in Figure 12.

The time domain and frequency domain displacement responses of sprung and unsprung masses under the above two simulation environments are shown in Figures 13 and 14 below. As seen from the time-domain responses of Figures 13 and 14, the displacement response magnitude using the MATLAB Simulink environment is smaller than the HILS environment. The difference comes from the fact that the theoretical MR damper modelling equation may exaggerate the output performance of the MR damper and there is no time lag in the simulation (theoretical) analysis. In the MATLAB Simulink environment, we express the MR damper with the proposed model, which is not 100% identical to the actual MR damper property, but in the HILS environment, we are using the actual MR damper, and there is no MR damper modelling error with this arrangement. But in both simulation environments, the semi-active MR damper performance is better than the passive MR damper. Passive MR damper here in this paper means an MR damper with 0 current input. The performance of the designed semi-active suspension system under the above-stated input disturbance is evaluated using MATLAB Simulink and HILS environments. We assume a capsule’s sprung mass vertical-displacement in MATLAB Simulink and HILS environment as an evaluation criterion. Taking the

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**Table 1. Parameter values of capsule vehicle.**

| S/N | Description                                      | Symbolic representation | Value (unit) |
|-----|--------------------------------------------------|-------------------------|--------------|
| 1   | Sprung mass a capsule                           | \( m_s \)               | 4265.00 kg   |
| 2   | Unsprung mass a capsule                         | \( m_u \)               | 2129.50 kg   |
| 3   | Spring coefficient of secondary suspension system | \( k_s \)               | 372,600 N/m  |
| 4   | Spring coefficient of primary suspension system  | \( k_p \)               | Nonlinear    |
| 5   | Primary damper damping coefficient              | \( c_p \)               | 0.00 Ns/m    |
| 6   | Length of capsule’s bogie                       | \( L_b \)               | 1.80 m       |
| 7   | Pole pitch of a superconducting coil            | \( t_{lev} \)           | 0.45 m       |
| 8   | Forward velocity of a capsule                   | \( v_x \)               | 800 km/h     |

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**Figure 9. General layout of experimental setup (HILS).**

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frequency from 0 to 8 Hz range, the root-mean-square (RMS) value of vertical displacement of the sprung mass is shown in Table 2 for passive and semi-active (under SH and SH-ADD controllers) secondary suspension systems.

From Figure 13(a), we can prove that the SH system controller is better than the passive MR damper at the lower frequency domain and worse than the passive MR damper at the higher frequency region, but the mixed SH-ADD system controller
performances better than the passive MR damper for all frequency values. Even though the performance of the mixed SH-ADD system controller is a little bit lower than the SH controller at the lower frequency region, the overall performance is better than the SH controller, as shown in Table 2. Taking the RMS value in both MATLAB Simulink and HILS environment vertical displacement response, the SH Controlled MR damper performance is improved by 54.49% in the HILS environment and 36.75% in the MATLAB Simulink environment relative to the passive MR damper performance, and the mixed SH-ADD Controlled MR damper performance is improved by 58.22% in the HILS environment and 36.93% in the MATLAB Simulink environment.

The sprung mass’s maximum vertical displacements at the vicinity of the two resonance frequencies, under the HILS environment, are stated in Table 3. The tabulated values show that at the first resonance frequency, both SH-controller and SH-ADD controller has a better vibration reduction effect than the passive Mr damper. At the second resonance frequency, the SH-controller is not good in reducing the vibration compared to the passive damper but the mixed SH-ADD controller is still good at this vicinity also.

Conclusion
This paper has adopted a mixed SH-ADD and skyhook system controllers to control a viscous MR damper
suspension system of a hyperloop capsule train. The performance of a semi-active suspension system highly relies on its control algorithm. The control algorithm shall be as simple as possible with good tracking ability to decrease the response time. The levitation system of the capsule is a nonlinear EDS system. Hence, the nonlinear vertical dynamics of the quarter-car is modelled by NSI method. The dynamic performances of both

![Figure 14. Time and frequency domain of Capsule’s unsprung mass vertical displacement: (a) HILS environment and (b) MATLAB simulink environment.](image)

**Table 2.** RMS value of capsule’s sprung mass vertical displacement under MATLAB Simulink and HILS environment.

| Damper                        | Response in HILS environment | Response in MATLAB simulink environment |
|-------------------------------|------------------------------|----------------------------------------|
|                              | Sprung mass vertical disp. RMS value | Percent of improvement | Sprung mass vertical disp. RMS value | Percent of improvement |
| Passive MR damper ($I = 0.0A$) | 0.3413                       |                            | 0.2120                         |                            |
| MR damper with SH controller  | 0.1550                       | 54.49                      | 0.1341                         | 36.75                      |
| MR damper with SH-ADD controller | 0.1426                       | 58.22                      | 0.1337                         | 36.93                      |
controllers are evaluated under MATLAB Simulink and hardware-in-loop system (HILS) environment. As illustrated in Figure 13(b) and Table 3, however, the skyhook-controlled MR damper performance is much better than the passive damper in the vicinity of first resonance frequency, its performance is worse than passive damper at the second resonance frequency. To improve the performance of the skyhook control system at higher frequency domain the mixed SH-ADD control algorithm is adopted in this paper. The mixed SH-ADD controlled MR damper performance is better than passive damper at all frequency domains. Taking RMS values of HILS response of a sprung mass (Table 2), the skyhook-controlled MR damper improves the vertical displacement of the sprung mass by 54.49%, and the mixed SH-ADD-controlled viscous MR damper improves by 58.22% relative to the passive damper vertical displacement. By providing a small amount of energy ($I \leq 2.4$) as shown in Figure 12, we can dramatically improve the damping effect of the MR damper.

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