COSMIC MICROWAVE BACKGROUND TEMPERATURE AT GALAXY CLUSTERS

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ABSTRACT

We have deduced the cosmic microwave background temperature in the Coma Cluster (A1656, $z = 0.0231$) and in A2163 ($z = 0.203$) from spectral measurements of the Sunyaev-Zeldovich (SZ) effect over four passbands at radio and microwave frequencies. The resulting temperatures at these redshifts are $T_{\text{Coma}} = 2.789^{+0.060}_{-0.065}$ K and $T_{\text{A2163}} = 3.377^{+0.101}_{-0.102}$ K, respectively. These values confirm the expected relation $T(z) = T_0(1 + z)$, where $T_0 = 2.725 \pm 0.002$ K is the value measured by the COBE Far Infrared Absolute Spectrometer experiment. Alternative scaling relations that are conjectured in nonstandard cosmologies can be constrained by the data; for example, if $T(z) = T_0(1 + z)^{-\alpha}$ or $T(z) = T_0[1 + (1 + d)z]$, then $\alpha = -0.16^{+0.34}_{-0.12}$ and $d = 0.17 \pm 0.36$ (at 95% confidence). We briefly discuss future prospects for more precise SZ measurements of $T(z)$ at higher redshifts.

Subject headings: cosmic microwave background — cosmology: observations — galaxies: clusters: individual (A2163, Coma)

1. INTRODUCTION

The (present) cosmic microwave background (CMB) temperature was precisely measured by the Far Infrared Absolute Spectrometer (FIRAS) on board the COBE satellite. $T_0 = 2.725 \pm 0.002$ K, in the frequency range 2–20 cm$^{-1}$ (Mather et al. 1999). These measurements essentially rule out all cosmological models in which the CMB spectrum is non-Planckian at $z = 0$. Models with a purely blackbody spectrum but with a different dependence than in the standard model are, however, unconstrained by the FIRAS database. Also unconstrained are models with spectral distortions that are now negligible but were appreciable in the past. A specific example is the relation $T(z) = T_0(1 + z)^{-\alpha}$, where $\alpha$ is a parameter of the theory (see, e.g., Lima, Silva, & Viegas 2000). More generally, models in which ratios of some of the fundamental constants vary over cosmological time are also of considerable interest.

So far, $T(z)$ has been determined mainly from measurements of microwave transitions in interstellar clouds that contain atoms and molecules that are excited by the CMB (as reviewed by LoSecco, Mathews, & Wang 2001). The temperature has been determined in the Galaxy as well as in clouds at redshifts up to $z \sim 3$ (Levshakov et al. 2002). These measurements are affected by substantial systematic uncertainties stemming from the unknown physical conditions in the absorbing clouds (Combes & Wiklind 1999).

The possibility of determining $T(z)$ from measurements of the Sunyaev-Zeldovich (SZ) effect had been suggested long ago (Fabbri, Melchiorri, & Natale 1978; Rephaeli 1980). For general reviews of the effect and its cosmological significance, see Rephaeli 1995a and Birkinshaw 1999.) The proposed method is based on the steep frequency dependence of the change in the CMB spectral intensity, $\Delta I$, due to the effect, and the weak dependence of ratios $\Delta I(\nu)/\Delta I(\nu')$ of intensity changes measured at two frequencies ($\nu$, $\nu'$) on properties of the cluster (Rephaeli 1980). Because of this, and the fact that—in the standard cosmological model—the effect is essentially independent of $z$, SZ measurements have the potential of yielding much more precise values of $T(z)$ than can be obtained from ratios of atomic and molecular lines. With the improved capability of reasonably precise spectral measurements of the SZ effect, the method can now be used to measure $T(z)$ in nearby and moderately distant clusters. Here we report first results from spectral analysis of SZ measurements in the Coma and A2163 clusters of galaxies.

2. $T(z)$ FROM SZ

The CMB intensity change due to Compton scattering in a cluster can be written in the form

$$\Delta I = \frac{2k^3}{\hbar^2 e^2} \int \frac{x^4 e^x}{(e^x - 1)^2} dx \left[ \theta f_1(x) - \beta R(x, \theta, \beta) \right],$$

where $x = \hbar \nu/kT$ is the nondimensional frequency, $\theta = kT e/mc^2$ is the electron temperature in units of the electron rest energy, and $\beta$ is the line-of-sight component of the cluster (peculiar) velocity in the CMB frame in units of $c$. The integral is over the Compton optical depth, $\tau$. Both the thermal (Sunyaev & Zeldovich 1972) and kinematic (Sunyaev & Zeldovich 1980) components of the effect are included in equation (1), separately in the first two (additive) terms and jointly in the function $R(x, \theta, \beta)$. In the nonrelativistic limit (which is valid only at low electron temperatures and frequencies), the spectral dependence of $\Delta I$ is fully contained in the product of the $x$-dependent prefactor times the function $f_1(x) = x(e^x - 1)/(e^x - 1) - 4$. The more exact treatment of Compton scattering in clusters necessitates a relativistic calculation (Rephaeli 1995b) owing to the high electron velocities. The function $R(x, \theta, \beta)$ includes the additional spectral, temperature, and (cluster) velocity dependence that is obtained in a relativistic treatment. This function
can be approximated by an analytic expression that includes terms to orders $\theta^0$ and $\beta^2\theta$:

$$R(x, \theta, \beta) = \theta^2[f_1(x) + \theta f_2(x) + \theta^2 f_3(x) + \theta^3 f_4(x)]$$

$$- \theta^2[g_1(x) + \theta g_2(x)] + \beta^2[1 + \theta g_1(x)]. \quad (2)$$

The spectral functions $f_i$ and $g_i$ were determined by Itoh, Kohyama, & Nozawa (1998), Itoh et al. (2002), and Shimon & Rephaeli (2002). For our purposes here, this analytic approximation is sufficiently exact even close to the crossover frequency. The nonrelativistic limit, $R = 0$, applies if the sum of all these terms can be ignored at the desired level of accuracy.

The $\nu$ dependence of $\Delta I$ is fully determined by the functions $\nu = \nu(z)$ and $T = T(z)$. The temperature-redshift relation may assume various forms in nonstandard cosmologies; here we consider two examples. In the first, $T(z) = T(0)(1 + z)^{\Delta_{\nu0}}$, where $a$ is taken to be a free parameter with the standard scaling $\nu = \nu_0(1 + z)$ unchanged. With these relations, the non-dimensional frequency obviously depends on $z$, $x = x_0(1 + z)^a$, if $a \neq 0$; here $x_0 = h\nu_0/kT(0)$. Another functional form that seems also to be of some theoretical interest is $T(z) = T(0)[1 + (1 + d)z]$ (LoSecco et al. 2001), for which $x = x_0(1 + z)/[1 + (1 + d)z]$. Obviously, in the standard model $a = d = 0$.

For a slow-moving ($\beta < 10^{-5}$) cluster, the expression for $\Delta I$ in the nonrelativistic limit depends linearly on the Comptonization parameter, $y = \int_0^\infty d\nu \Delta I$, which includes all dependence on the cluster properties. A ratio of values of $\Delta I$ at two frequencies is then essentially independent of these cluster properties. In the more general case, the first term in the square brackets in equation (1) still dominates over the other two, except near the crossover frequency (whose value generally depends on $T_0$, except in the nonrelativistic limit where $x_0 = 3.83$; Rephaeli 1995b; Nozawa et al. 1998; Shimon & Rephaeli 2002), where the sum of the temperature-dependent terms vanishes. For values of $x$ outside some range (roughly, $3.5 < x < 4.5$), the dependence of $\Delta I$—and particularly, a ratio of values of $\Delta I$—on $\beta$ is very weak since the observed temperature range in clusters corresponds to $0.006 < \theta < 0.03$, whereas typically $\beta < 0.002$.

3. DATA ANALYSIS

We have analyzed results of SZ measurements in the Coma Cluster (A1656) and A2163. Measurements of Coma, $z = 0.0231 \pm 0.0017$ (Struble & Rood 1999) were made with the Millimetre and Infrared Testagrigia Observatory (MITO; De Petris et al. 2002) telescope in 20 hr of integration. We also use the result of measurements at 32 GHz made with the Owens Valley Radio Observatory (OVRO) 5.5 m telescope (Herbig et al. 1995; Mason, Myers, & Readhead 2001). A2163, $z = 0.203 \pm 0.002$ (Arnaud et al. 1992), was observed with the Sunyaev-Zeldovich Infrared Experiment (SUZIE) array (Holzapfel et al. 1997a) and with both the OVRO and Berkeley-Illinois-Maryland Association (BIMA) interferometric arrays (LaRoque et al. 2002).

When observing a cluster, the SZ part of the measured signal is

$$\Delta S_i = G_i \Delta I_{i0} \int_0^{\infty} [\Delta f_0(\theta, \beta) - G_i \Delta I(0)\epsilon_i(\nu) d\nu, \quad (3)$$

where $G_i$ is the responsivity of the $i$th photometric channel, $\Delta I_{i0}$ is the corresponding throughput, and $\epsilon_i(\nu)$ is the spectral efficiency. The full measured signal includes also contributions from the atmosphere, CMB anisotropies, and—at very high frequencies—also emission from dust. Multifrequency observations allow us to remove contributions from both the primary CMB anisotropy and the kinematic SZ effect, as has been attempted in the analysis of MITO measurements of the Coma Cluster (De Petris et al. 2002).

The ratio of signals in two different photometric channels $i$ and $j$ is

$$\frac{\Delta S_i}{\Delta S_j} = \frac{G_i \Delta I_{i0}}{G_j \Delta I_{j0}} \int_0^{\infty} \frac{[x^2e^{x}(e^{-x} - 1)]^2}{[x^4e^{x}(e^{-x} - 1)]^2} \left( \int_0^\infty d\nu [\theta f_1(x) - \beta + R(x, \theta, \beta)] \epsilon_i(\nu) d\nu \right) dv. \quad (4)$$

The main dependence on the cluster properties in $\nu$ cancels out when $\beta$ is negligible. Multifrequency observations that include measurements at the crossover frequency (e.g., MITO) afford effective separation of the thermal and kinematic components, exploiting their very different spectral shapes. This was demonstrated in the analysis of MITO measurements of the Coma Cluster (De Petris et al. 2002). Since the above ratio depends weakly on the cluster velocity, the residual uncertainty due to velocities of even $\pm 500$ km s$^{-1}$ can be ignored in comparison with other errors. The ratio is also weakly dependent on the gas temperature at a level that we found to correspond to $\sim 1\%$ uncertainty in the estimation of the CMB temperature (for a typical observational error in $T_0$). Moreover, the uncertainty associated with the absolute calibration, $G_i$, is largely removed once we fit data from several photometric channels, as long as they are calibrated with a source with a known spectrum (e.g., a planet) even if its absolute calibration is uncertain. Only relative uncertainties among the various spectral channels are important; these include differences in angular response and in atmospheric transmittance. A standard blackbody source with a precisely calibrated temperature is therefore not required. We expect these considerations to imply that the precision of CMB temperature measurements via the SZ effect will not be appreciably affected by most of the known systematic errors. The level of precision in the measurement of $T(z)$ is limited largely by other observational uncertainties, as discussed below.

The responsivity $G_i$ of each channel is usually determined from detailed observations of very well measured sources such as planets (mostly Jupiter or Saturn) and the Moon. While the temperature uncertainty of these sources can be as large as 10%, their spectra are relatively well known. For a source at a temperature of $T_0$ and with a throughput $\Omega_s$, the signal in the Rayleigh-Jeans part of the spectrum is

$$\Delta S_i = G_i \Delta I_{i0} T_0 \frac{2k}{c} \int_0^{\infty} v^2 \epsilon_i(v) \eta(v) dv. \quad (5)$$

Since we are interested in the ratio of the signals in two channels, $T_i$ drops out from the final expression, and the uncertainty in the ratio of values of the brightness temperature in the two channels depends only on the emissivity of the planet, $\eta(v)$, convolved with the spectral efficiencies of the channels, $\epsilon_i(v)$. Note that the throughputs of individual channels, $\Delta I_{i0}$, are usually made almost equal by an appropriate choice of the optical layout.
The main uncertainty in the MITO measurements is due to imprecise knowledge of the bandwidth and transmittance of the filters. Therefore, we have precisely measured the total efficiency of our photometer by means of a lamellar grating interferometer. The response of the photometer is measured when it is illuminated with a laboratory blackbody at different temperatures ranging from liquid N$_2$ to room temperature. The effect of the uncertainty in our bandwidths has two main consequences. The first is the error in the ratio of responsivities of channel $i$ and channel $j$, $G_i/G_j$. We estimate this error by computing the ratio of expected signals evaluated by convolving Moon and Jupiter spectra with the spectra of our filters and taking into account spectroscopic uncertainties; we obtain a final error estimate that is less than 1%. The second consequence of bandwidth uncertainty enters in the integrations over the SZ spectral functions in equation (4). However, this latter error is smaller than the former and is practically negligible.

Also quite precisely measured were the angular responses of the telescope and the photometer in the four channels. We have studied differences in the response of the four channels to both a point source, such as Jupiter, and an extended source, such as the Moon, as they cross the field of view. The optical layout of our photometer is such that all four detectors observe the same sky region. The differences among channels are less than 1% for extended sources. Finally, we have also studied the change in relative efficiency of our observations under different atmospheric conditions. The simulations show a change of $\sim$1 mK in the estimated CMB temperature if the water vapor content changes from 0.5 to 1.5 mm, as determined by convolving theoretical atmospheric spectra with our filters. The uncertainties in determining the water vapor content, related to our procedure of measuring the atmospheric transmission through a sky-dip technique, are at a level of 30%. However, when the effect of this error is propagated through our procedure and analysis we estimate that it amounts to a negligible error in the final result for the CMB temperature.

In conclusion, systematic errors contribute less than 3% to the observed MITO signals and are quite negligible with respect to the 10% contribution associated with the residual noise of the four detectors. For the OVRO measurement, we use the total error as specified by Mason et al. (2001); the statistical weight of this low-frequency measurement is such that it contributes only $\sim$12% to the final results.

Our assessment of the errors in the SuZIE and OVRO and BIMA measurements of A2163 (LaRoque et al. 2002) is far less certain than those in the MITO measurements, since we do not know the exact spectral responses of their filters and the atmospheric conditions during each of these observations. We have used Gaussian profiles for the SuZIE filters, with peak frequencies and FWHM values as given by Holzapfel et al. (1997a). For the OVRO and BIMA data point, we also take a Gaussian profile with a peak frequency at 30 and 1 GHz bandwidths. We have employed data provided by SuZIE, for which a correction due to thermal dust emission has been taken into account. The Coma and A2163 data are collected in Tables 1 and 2. Finally, the value we used for the electron temperature is 8.25 $\pm$ 0.10 keV, as measured with XMM in the central region of Coma (Arnaud et al. 2001); the value for A2163 is 12.4 $\pm$ 0.3 keV (Holzapfel et al. 1997b).

In both sets of cluster observations, we have minimized the difference between theoretical and experimental ratios (properly weighted by statistical errors) with $T$ as a free parameter. The results are $T_{\text{Coma}}(0) = 2.726_{-0.064}^{+0.080}$ K which, when multiplied by $(1+z)$ with $z = 0.0231 \pm 0.0017$, yields

$$T_{\text{Coma}} = 2.789_{-0.065}^{+0.090} \text{ K}. \quad \text{(6)}$$

The main uncertainty is in the SZ observations, with only a small error ($7 \times 10^{-4}$ K) due to the uncertainty (0.1 keV) in the electron temperature. For A2163, we obtain $T_{A2163}(0) = 2.807_{-0.085}^{+0.084}$ K, and multiplying by $(1+z)$ with $z = 0.203 \pm 0.002$ yields

$$T_{A2163} = 3.377_{-0.012}^{+0.010} \text{ K}. \quad \text{(7)}$$

The larger error in $T_c$ ($\sim$2 keV) translates to an uncertainty of $1.5 \times 10^{-3}$ K in the deduced CMB temperature at the redshift of A2163. The mean of the above two values for $T(0)$ is $(T) = 2.766_{-0.038}^{+0.038}$ K, in good agreement with the COBE value.

Using the COBE value of the temperature and the two values deduced here for $T(z)$ at the redshifts of Coma and A2163, we fit the three data points by the relation $T(z) = T(0)(1+z)^{\alpha}$: the best fit is shown by the dashed line in Figure 1. It corresponds to $\alpha = -0.16_{-0.2}^{+0.32}$ at a 95% confidence level (CL). With the alternative scaling $T(z) = T(0)[1+(1+d)z]$, we obtain a best-fit value of $d = 0.17 \pm 0.36$ at the 95% CL.

These first SZ results for $T(z)$ are clearly consistent with the standard relation, although the most probable values of both $a$ and $d$ indicate a slightly stronger $z$ dependence. Our values for these two parameters are quite similar to those deduced by

### Table 1

| Experiment | Frequency (GHz) | Bandwidth (GHz) | $\Delta f$ (MJ sr$^{-1}$) | $\Delta T$ ($\mu$K) |
|------------|----------------|----------------|--------------------------|---------------------|
| OVRO ...... | 32.0           | 13.0           | $-0.0159 \pm 0.0028$     | $-520 \pm 93$       |
| MITO1 ...... | 143            | 30             | $-0.068 \pm 0.014$       | $-179.3 \pm 37.8$   |
| MITO2 ...... | 214            | 30             | $-0.016 \pm 0.039$       | $-33.4 \pm 81.2$    |
| MITO3 ...... | 272            | 32             | $0.075 \pm 0.013$        | $169.8 \pm 35.1$    |

### Table 2

| Experiment | Frequency (GHz) | Bandwidth (GHz) | $\Delta f$ (MJ sr$^{-1}$) | $\Delta T$ ($\mu$K) |
|------------|----------------|----------------|--------------------------|---------------------|
| OVRO+BIMA  ...... | 30             | 1              | $-0.048 \pm 0.006$       | $-1777 \pm 222$     |
| SuZIE1 ............ | 141.59         | 12.67          | $-0.380 \pm 0.037$       | $-1011.3 \pm 98.0$  |
| SuZIE2 ............ | 216.71         | 14.96          | $-0.103 \pm 0.077$       | $-213.0 \pm 159.3$  |
| SuZIE3 ............ | 268.54         | 25.66          | $0.295 \pm 0.105$        | $662.2 \pm 235.7$   |
The method employed here to measure \( T(z) \) can potentially yield very precise values that will tightly constrain alternative models for the functional scaling of the CMB temperature with redshift and thereby provide a strong test of nonstandard cosmological models. In order to improve the results reported here, multifrequency measurements of the SZ effect in a significantly larger number (\( \sim 20 \)) of clusters are needed. The clusters should include nearby ones in order to better understand and control systematic errors. Exactly such observations are planned for the next 2–3 yr with the upgraded MITO project (Lamagna et al. 2002) and the stratospheric BOOST experiment (P. Lubin 2002, private communication) and OLIMPO experiment (Masi et al. 2002). These experiments will employ sensitive bolometric arrays at four frequency bands with high spatial resolution.

As detector noise is reduced, the uncertainty in the electron temperature may contribute the dominant relative error. For example, an uncertainty of \( \sim 2 \) keV—as in the case of A2163—becomes dominant when detector noise is reduced to a level 10 times lower than the values used here. Therefore, it will be necessary to select clusters for which the gas temperature is precisely (to a level of \( \pm 0.1 \) keV) known. With the large number of clusters that have already been—or will be—sensitively observed by the XMM and Chandra satellites, it should be possible to optimally select the SZ cluster sample.

With the currently achievable level of precision in intracluster gas temperature measurements, the present technique makes it possible to determine \( a \) and \( d \) with an uncertainty that is not less than 0.03 on both parameters, even with ideal (i.e., extremely high sensitivity) SZ measurements. The projected sensitivities of the future Planck and Herschel satellites will enable reduction of the overall error in the values of these two parameters by more than an order of magnitude. In addition to limits on alternative CMB temperature evolution models, such a level of precision will open new possibilities of testing the time variability of physical constants. For example, it will be feasible to measure a possible variation of the fine-structure constant, thereby providing an alternative to the current principal method, which is based on spectroscopic measurements of fine-structure lines in quasar absorption spectra.

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