Reconstruction of High-order Harmonic Signal in AT Power Supply System

Zhende Wei*, Xiaoqiang Lv
National Grid Jiuquan Power Supply Company Jiuquan, China
aweizhd@gs.sgcc.com.cn
805053041@qq.com, 1833295171@qq.com

Abstract. In the process of power transmission, power quality disturbance is caused by the phenomenon of “excitation inrush current” and “leakage resistance” of the core equipment AT in the power supply system. Harmonic signals contain higher harmonics. In this paper, the power supply system under AT mode is modeled, and a damping trapezoidal algorithm is proposed to suppress the numerical oscillations caused by the coupling between AT windings. A new orthogonal matching pursuit algorithm for multi-spectral interpolation correction is proposed. (OMP-MIC), reconstructing the occurrence of higher harmonic signals, improving reconstruction efficiency and accurately recovering the original signal; finally, verifying the validity of the model and the superiority of the reconstruction algorithm through simulation.

1. Introduction
With the rapid development of high-speed trains in my country, the power quality requirements of the power supply system are getting higher and higher [1]. The power supply modes of high-speed trains mainly include: current-absorbing transformer (BT) power supply mode, autotransformer (AT) power supply mode and direct power supply (TR) mode [2]. Due to the high power of high-speed railways, the traction network current is large. Therefore, we adopt the AT power supply method with the strongest power transmission capability [3,4].

In view of the high-order harmonics in high-speed trains, when the monitoring system is constructed based on Nyquist sampling theorem, it is unable to sample high-order harmonics and consumes a lot of storage space and hardware resources, resulting in data redundancy and waste [5,6]. Compressed sensing (CS) theory proposed [7,8], breaking through the defects of Nyquist sampling theorem, has become a new research hotspot in the field of signal processing. Reference [9] points out that the ratio has a wide range of variation; reference [10] proposes a voltage signal reconstruction method of converter combining intrinsic time scale decomposition (ITD) and improved inner product compressed sensing reconstruction algorithm. The above scholars provide an important theoretical basis for the reconstruction of high-order harmonic signal in electrified railway. However, considering the complexity of harmonic signal in electrified railway, these algorithms have the disadvantages of over estimation of sparsity, decrease of execution efficiency and long reconstruction time in signal reconstruction process.

This paper is based on Matlab/Simulink to build a power supply system model in auto-coupling mode, and considers the circuit and magnetic circuit of the autotransformer in the AT power supply system. A modified damping trapezoidal method is used to eliminate numerical oscillations, and a new type of multi The orthogonal matching pursuit with multi-spectral interpolation correction (OMP-MIC)
algorithm reconstructs the high-order harmonic signals collected in the monitoring system to reduce the signal sparsity K and improve Reconstruction accuracy to achieve accurate restoration of the original signal.

2. Modified Damping Trapezoidal Method

The simulation model built in this article contains a large number of non-linear components, and it will inevitably produce numerical oscillations during operation, that is, abnormal oscillations of non-state variables. In order to maintain the stability and accuracy of numerical calculations and the undistorted images in the simulation process, this paper proposes a new type of damping trapezoidal algorithm. To facilitate discussion, consider the first-order ordinary differential equation as:

\[ \frac{dy}{dt} = f(y, t) \]  

(1)

The essence of solving (1) is to solve the corresponding state quantities \( y_0, y_1, \ldots, y_n \) in order for a set of time series \( t_0, t_1, \ldots, t_n \). Write the numerical integral calculation formula of equation (1) in the time period \([t_n, t_{n+1}]\):

\[ y_{n+1} = y_n + \int_{t_n}^{t_{n+1}} f(y, t) dt \]  

(2)

The design calculation step length is \( h = t_{n+1} - t_n \), and the formula (2) is first differentiated by the backward Euler method:

\[ y_{n+1} = y_n + hf(y_{n+1}, t_{n+1}) \]  

(3)

Then, the implicit trapezoidal integration method is used to differentiate equation (3):

\[ y_{n+1} = y_n + \frac{h}{2} [f(y_{n+1}, t_{n+1}) + f(y_n, t_n)] \]  

(4)

Finally, the implicit trapezoid method and the backward Euler method are mixed according to the ratio of \((1-\alpha)\): \(\alpha\) to form the difference formula of damping trapezoidal method:

\[ y_{n+1} = y_n + \frac{(1+\alpha)h}{2} f(y_{n+1}, t_{n+1}) + \frac{(1-\alpha)h}{2} f(y_n, t_n) \]  

(5)

In the formula, \(\alpha\) is the damping factor. When \(\alpha=0\), the above formula is transformed into implicit trapezoidal method; when \(\alpha=1\), the above formula is converted to backward Euler method; when \(0 < \alpha < 1\), the above formula is converted to damping trapezoidal method.

3. OMP-MIC Algorithm

3.1. Correction of multispectral interpolation

For the harmonic signal with \( N \) sampling points, if the spectrum is \( X(k) \) and the maximum spectral line is \( k_0 \), then any two spectral lines can be used to estimate the \( f' \):

\[ f' = \frac{1}{t} (k_0 + m) \frac{|X(k_0) + m|}{|X(k_0)| + |X(k_0 + m)|} \]  

(6)

It is defined \( (k+1/3, k+2/3) \) as the central area between \( k \) and \( k+1 \). If it meets the following requirements, \( f' \) is considered to be located in the center of quantization frequency point as the frequency estimation value; if not, the signal \( f(n) \) needs to be frequency shifted to the left or right to quantify the frequency unit, \( \delta_k \) as shown in formula (7)

\[ \delta_k = \frac{1}{2} - \frac{|X(k_0 + m)|}{|X(k_0)| + |X(k_0 + m)|} \]  

(7)
When the signal \( f(n) \) deviates, it is assumed that \( k_{m1} \) and \( k_{m2} \) are the spectral line labels of the maximum peak line and the second largest peakline of the harmonic component:

\[
\begin{align*}
    f &= (k_{m1} + \delta + 0.5) \frac{f}{N} \\
    A &= \frac{s(k_{m2}) + s(k_{m1})}{N} \\
    g(\delta) &= 1.356002 + 0.155\delta^2 + 0.0326\delta^3 + 0.0079\delta^4 \\
    \phi &= \text{angle}\{s(k_{m1} + 1)\} + \pi(\delta + 1)
\end{align*}
\]

### 3.2. OMP-MIC algorithm flow

Input: data fusion signal \( y \in R^{M} \), measurement matrix \( \Phi \in R^{M \times N} \), sparsity \( k \), candidate atomic index set \( A \), new selected atomic index \( \sigma \).

Output: original reconstruction signal \( x' \).

Initialization: sparse estimate \( s_{\sigma}^r = 0 \); initial residual value \( r_0 = y \); support matrix \( \Omega_0 = \left[ \right] \); support set \( A_0 = \emptyset \); initial search vector \( g_0 = -\Omega_0 \); initial step size \( \alpha_0 \in [\alpha_m, \alpha_m] \); number of iterations \( t = 1 \).

Cycle:

1. Filter atoms: find the maximum value \( \lambda(k) \) of the inner product between \( r_{k-1} \) and the atom. The index is \( \lambda(k) = \arg \max_{i \in D} \left| \langle r_{k-1}, \Phi_i \rangle \right| \).
2. Update the support set: merge \( \lambda(k) \) with the index set of the previous iteration \( A_{k-1} \) to form a new support set \( A_k = A_{k-1} \cup \{ \lambda(k) \} \).
3. Use the least squares method to wait until the atom sparse estimate \( s_{\lambda}^{\text{base}} = \arg \min \| y - \Phi_{\lambda}^{\text{base}} \|_2 = (\Phi_{\lambda}^{\text{base}})\Phi_{\lambda}^{\text{base}}^{-1} \).
4. Update residual: \( r_t = y - \Phi s_{\lambda}^{\text{base}} \).
5. If \( s_{\lambda}^{\text{base}} \) all non-zero elements are adjacent, skip to step(6); otherwise, perform spectral line interpolation correction at the position of non-adjacent elements, use equation.

6. To estimate the frequency, if it is satisfied \( \frac{1}{3} \Delta f < \left| f - \frac{k}{N} \right| < \frac{1}{2} \Delta f \), it can be used as the frequency estimate, if it is not satisfied, use equations (7) and (8) to correct the frequency, amplitude and phase angle of the spectrum;

\( s_{\lambda}^{\text{base}} \) all non-zero elements in satisfies \( s_{i,j}^{\text{base}} \geq y \cdot \max_{i,j \neq 1} \left( s_{i,j}^{\text{base}} \right) \), then the next cycle is performed, \( t = t + 1 \), otherwise, the cycle ends;

### 4. Simulation Results and Analysis

Further analyze the high-order harmonic signals appearing in the AT power supply system, set the sampling point length \( N = 1024 \), calculate the total signal sparsity \( K = 76 \), select the phase A voltage signal as the high-order harmonic reconstruction signal, and use When the orthogonal matching pursuit (OMP) algorithm reconstructs the original signal, the reconstruction effect is shown in Figure 1.

It can be seen from Figure 1 that when the OMP algorithm is used to reconstruct the original signal with high sparseness, the reconstructed signal is quite different from the original signal, and it is impossible to achieve the fault warning of the high-order harmonics.
Introduce the multi-spectral interpolation (MIC) algorithm to reduce the sparsity $K$ of the original signal, making $K=12$, and then improve the reconstruction effect of the higher harmonic signal, the waveform is shown in Figure 2.

![Fig 1 Original signal reconstruction under OMP algorithm](image1)

![Fig 2 Signal reconstruction under OMP-MIC algorithm](image2)

In order to compare the superiority of the OMP-MIC algorithm over other algorithms, the same sampling length $N$ is used to verify the probability of successful signal reconstruction and the reconstruction time under different algorithms. It can be seen from Figure 3 and Figure. 4 that under the same $N$, the OMP-LIC algorithm achieves a relatively high reconstruction probability, and the reconstruction performance of the SPG and CoSaMP algorithms is worse. When the compression ratio and SNR were 100dB, the reconstruction performance was much better than other recovery algorithms.

![Fig.3 SNR of different recovery algorithms](image3)

![Fig.4 Reconstruction precision of different recovery algorithms](image4)

5. Conclusion

The following conclusions can be obtained from the simulation analysis:

(1) The modified damping trapezoidal algorithm and the combination of the magnetic circuit and the circuit are used to model and analyze the AT power supply system. This model not only solves the numerical oscillation problem well, prevents image distortion, but also collects high-order Harmonic waveform.

(2) When using CS theory to compress and reconstruct high-order harmonic signals, the higher the signal sparsity, the worse the reconstruction effect. The multi-spectral interpolation correction (MIC) algorithm is used to reduce the signal sparsity, thereby improving The reconstruction effect of high-order harmonic signals, and the simulation results show that the OMP-MIC algorithm has a higher probability of successful reconstruction compared with other recovery algorithms.
References

[1] CHEN Xiaojing, Li Kaicheng, Xiao Jian, et.al. A method real-time power quality disturbance classification[J]. Transactions of China Electrotechnical Society, 2017, 32(3): 45-54.

[2] ZHANG Qiming. Analysis of Power System Harmonic Generated by High-Speed Rail’s Traction Load[D] Southwest Jiaotong University, 2015, 17-45.

[3] HE Yangyang, HUANG Kang, WANG Tao, et al. Overview of traction power supply system for rail transportation [J]. Journal of Railway Science and Engineering, 2016, 13 (2):352-361.

[4] CHEN Minwu, CUI Zhaohua, GAO Hongge, et al Study of fault location method of high-speed railway using two-terminal electrical quantities [J]. Journal of Railway Science and Engineering, 2017(10):7-16.

[5] Fan J, Guo S, Zhou X, et al. Faster-Than-Nyquist Signaling: An Overview[J]. IEEE Access, 2017, 5:1925-1940.

[6] Donoho D. Compressed sensing[J]. IEEE Transactions on Information Theory, 2006, 52(4): 1280-1306.

[7] Candes E, Wakin M B. An introduction to compressive sampling[J]. IEEE Signal Processing Magazine, 2008, 25(2): 21-30.

[8] Needell D, Tropp J A. CoSaMP: Iterative signal recovery from incomplete and inaccurate samples. Applied and Computational Harmonic Analysis, 2009, 26(3): 301~321.

[9] Beck A, Teboulle M. A fast iterative shrinkage-thresholding algorithm for linear inverse problems. SIAM Journal on Imaging Sciences, 2009, 2(1): 183~202.

[10] XIN Guoqing, DONG Weiguang, GAo Fengyang, et al. Reconstruction of harmonic signal with inter-harmonics under inductive power supply technology[J/OL]. Transactions of China Electrotechnical Society: 1-10[2020-02-02].https://doi.org/10.19595/j.cnki.1000-6753.tces.191336.