**GPD sum rules: a tool to reveal the quark angular momentum**

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In deeply virtual exclusive electroproduction to leading order accuracy one accesses generalized parton distributions on their cross-over trajectory. Combining Lorentz covariance and analyticity leads to a family of GPD sum rules, guiding us to phenomenological concepts. As an example, we discuss the constraints from the JLAB/Hall A data on the GPD $E$. Its first Mellin moment is the anomalous gravitomagnetic moment, which is the unknown contribution to the quark angular momentum.

1 Introduction

Generalized parton distributions (GPDs) might be viewed as non-diagonal overlap of wave functions. This offers the opportunity to study the partonic content of the nucleon from a new perspective. They are accessible in deeply virtual lepton production of photon and mesons, where the amplitude factorizes at leading twist-two accuracy in a perturbatively calculable hard-scattering part and the GPD. Deeply virtual Compton scattering (DVCS), a subprocess in the lepton production of a photon, is considered as a theoretically clean process.

One of the main reasons to measure these processes is the quest for an understanding of the decomposition of the nucleon spin in quark and gluon angular momenta [2]:

\[ J^Q(\mu^2) + J^G(\mu^2) = \frac{1}{2}, \quad \text{with} \quad J^Q(\mu^2) = \sum_{q=u,d,s} J^q(\mu^2). \]  

(1)

The partonic angular momenta are given by the expectation values of the corresponding gauge invariant parts of the energy momentum tensor and might be further decomposed in spin and orbital angular momenta. They are also expressed by moments of GPDs $H$ and $E$,

\[ J^q(\mu^2) = \frac{1}{2} \left[ A^q(\mu^2) + B^q(\mu^2) \right], \quad \left\{ \begin{array}{c} A^q \\ B^q \end{array} \right\} (\mu^2) = \lim_{\Delta \to 0} \int_{-1}^{1} dx x \left\{ \begin{array}{c} H^q \\ E^q \end{array} \right\} (x, \eta, t = \Delta^2; \mu^2), \]  

(2)

taken in the forward limit, where $\eta \propto \Delta_+^+ \times \Delta_+^-$ is the longitudinal momentum fraction transferred in the $t$-channel. The quantities $A^q$ are nothing but the averaged momentum fractions of unpolarized partons They are already phenomenologically constrained by deeply inelastic scattering measurements. Momentum conservation guarantees that $A$ is normalized to one and the angular momentum sum rule (SR) [1] implies then that the anomalous gravitomagnetic nucleon moment $B$ vanishes:

\[ A \equiv \sum_{q=u,d,s} A^q(\mu^2) + A^G(\mu^2) = 1, \quad B \equiv \sum_{q=u,d,s} B^q(\mu^2) + B^G(\mu^2) = 0. \]  

(3)

*Talk given by D.M. at DIS 2008, 7-11 April 2008, London [1].*
2 Setting up the hunting scheme

Let us first recall the arguments that provide an estimate for the valence quark momentum fractions [2]. The small $x$ behavior of valence PDFs is governed by the intercept $\alpha \approx 1/2$ of $\rho$ and $\omega$ Regge trajectories, while large $x$ counting rules state $a \sim (1-x)^3$ behavior. Taking also into account the large $x$ ratio $u/d \sim 5$, the resulting averaged momentum fractions,

$$\left\{ \frac{u_{\text{val}}}{d_{\text{val}}} \right\}(x, \mu^2) \sim \frac{35}{32} \left( \frac{1-x)^3}{\sqrt{x}} \right) \sim \frac{5 - 3\frac{\eta}{\xi} (1-x)^2}{1}$$

$$\Rightarrow A_{u_{\text{val}}} \sim 0.32, \ A_{d_{\text{val}}} \sim 0.11,$$  

are in good agreement with phenomenological findings at a scale $\mu \sim 2$ GeV.

To estimate the quark angular momenta, we use isospin symmetry to fix the normalization of $E_{u_{\text{val}}}$ in terms of the nucleon magnetic moments. The relevant Regge intercepts are the same as before and counting rules state now $a \sim (1-x)^5$ behavior [3]. The estimates for the anomalous gravitomagnetic moments and angular momenta follow from Eq. (2):

$$B_{u_{\text{val}}} \sim 0.13, \ B_{d_{\text{val}}} \sim -0.15 \Rightarrow J_{u_{\text{val}}} \sim 0.2, \ J_{d_{\text{val}}} \sim 0.$$  

Hence, the valence part of the anomalous gravitomagnetic moment is expected to be small, i.e., $B^Q = -B^C \sim B^{\text{sea}}$, and the ‘unknown’ in the spin SR is the sea (or gluon) contribution.

It was conjectured that $B^C$ is zero [4] and so we would expect that $B^{\text{sea}}$ nearly vanishes. In such a scenario the angular momentum $J^Q \approx A^Q/2 \sim 0.25$ is essentially expressed by the momentum fraction and changes only slightly under evolution. In a covariant two flavor quark model ($B^C = 0$) one has $B^u = -B^d$, usually within a relatively small value of $B^{\text{sea}}$. In the chiral quark soliton model ($\chi$QSM) the role of sea quarks is more pronounced and estimates are compatible with our valence like ones [5]. Lattice measurements are consistent with the estimates [5], too. However, because of systematical errors, including neglecting gluon induced contributions (disconnected diagrams), we consider $B^{\text{sea}}$ as unmeasured. On the other hand the SR estimate [6] states that at least half of the nucleon spin originates from gluons, i.e., $B^C > 0$ and $B^{\text{sea}}$ is negative. We expect that $|B^{\text{sea}}| \lesssim 1/2$, i.e., $0 \lesssim J^Q \lesssim 1/2$.

It is phenomenologically challenging to reveal the anomalous gravitomagnetic moments. So far this has been attempted in a model dependent way, where the GPD $E^T$ is specifically parameterized in terms of $J^g$ within a non-Reggeized ‘vector-meson exchange’ contribution. This flexibility is incompatible with $\chi$QSM results [3]. Alternatively, one might utilize ‘common’ GPD models with $B^{\text{sea}}$ as a free parameter.

3 ‘New’ phenomenological tools: GPD sum rules

The DVCS amplitude is parameterized by Compton form factors (CFFs). To leading order (LO) accuracy their imaginary parts are given by GPDs at the cross-over trajectory $\eta = x$:

$$\Im \mathcal{F}(\xi, t, Q^2) \overset{\text{LO}}{=} \pi F^\mp(x = \xi, \eta = \xi, t, Q^2), \ F^- = \{H, E\}, \ F^+ = \{\bar{H}, \bar{E}\}.$$  

This relation is analogous to the well-known parton interpretation of DIS structure functions, given as linear combination of PDFs. A fixed $t$ dispersion relation allows to evaluate the real part of the CFF from its imaginary part. Hence, to LO accuracy it is expressed by the GPD on the cross-over trajectory and a subtraction constant ($C_E = -C_H, C_{\bar{H}} = C_{\bar{E}} = 0$):

$$\Re \mathcal{F}(\xi, t, Q^2) \overset{\text{LO}}{=} \text{PV} \int_0^1 \frac{dx}{x} \left( \frac{1}{\xi - x} + \frac{1}{\xi + x} \right) F^\mp(x, x, t, Q^2) + C_F(t, Q^2).$$  

\(^{a}\)A valence quark is the difference of quark and anti-quark; the sea is twice the amount of all anti-quarks.
The scale dependence of a CFF is governed by the GPD in the outer region \((\eta \leq x)\) and that radiative corrections extend Eq. (6) to a convolution integral over the outer region.

The CFFs can be evaluated without knowing the GPDs in the central region. Combining operator product expansion and dispersion relation shows that the GPD in this region arises from the Lorentz covariant decomposition of the CFFs in terms of operator matrix elements \[7\]. This can be also derived from an integral equation \[8, 9\], denoted as GPD SR family:

\[
\int_0^1 dx \left( \frac{1}{\xi - x} \mp \frac{1}{\xi + x} \right) [F^\mp(x, \eta = \partial \xi, t, Q^2) - F^\mp(x, \eta = \partial x, t, Q^2)] = C_F(\partial, t, Q^2). \tag{8}
\]

The application of the GPD SR family (8) is manifold \[10\]. They allow us to construct the GPD in the central region from its knowledge in the outer region and the subtraction constant. We note that the subtraction constant is entirely related to the so-called D-term and only contributes to the \(t\)-channel \(J = 0\) angular momentum contribution. In analogy to finite energy SRs, one might write down GPD ones that connect the 'low' and 'high' energy content of the GPD.

Additionally, one might set up a GPD model on its cross-over trajectory by factorizing it into a GPD for \(\eta = 0\) and a skewness function \(S(x, t, t, Q^2)\):

\[
F^\pm(x, x, t, Q^2) = \left[ 1 + S(x, t, Q^2) |F^\pm_{\eta = 0, t, Q^2} \right] F^\pm_{\eta = 0, t, Q^2}. \tag{9}
\]

Taking the limit \(\xi \to 0\) in Eq. (8), one finds a constraint for the skewness function

\[
\int_0^1 dx \frac{1}{x} S(x, t, Q^2) F^\mp(x, \eta = 0, t, Q^2) = \frac{1}{2} C_F(t, Q^2), \tag{10}
\]

where \((0)\) indicates analytic regularization.

The skewness function can be simply evaluated within a given GPD model. To make contact with both phenomenology and lattice measurements, it is more appropriate to consider Mellin moments. The skewness effects might be quantified by deviation factors:

\[
\delta_j(t, \mu^2 |F^\pm) = \int_0^1 dx x^j S(x, t, \mu^2) F^\mp(x, \eta = 0, t, \mu^2) = \sum_{n=2}^{\infty} \frac{f_j^{(n)}(t, \mu^2)}{f_j(t, \mu^2)}. \tag{11}
\]

They are given by a series of local operator matrix elements \(f_{j+n}^{(n)}(t, \mu^2)\) with spin \(j + n + 1\), containing \(n\) total derivatives. The state of the art in lattice measurements is the evaluation of spin-three operator matrix elements, allowing for a first guess of \(\delta_0(\cdots |H) \sim 0.2\).

4 Accessing GPD \(E\) from experimental data

To hunt for the anomalous gravitomagnetic moment one should read formula \[11\] as \[10\]

\[
B(Q^2) \equiv e_1(t = 0, Q^2) = \frac{1}{\pi} \int_0^1 d\xi \xi \lim_{\mu \to 0} 3mE(\xi, t, Q^2) \frac{1}{1 + \delta_1(t = 0, Q^2 |E^-)}. \tag{12}
\]

Certainly, it will be challenging to measure \(3mE\). We emphasize that a measurement of the real and imaginary part allows to utilize the ‘dispersion’ integral \[11\] as a SR. Assuming a Regge-like extrapolation in the small \(\xi\) region, one might then even extract \(3mE\) in the
large $\xi$ region. The deviation factor $\delta_1$ in Eq. (12) reminds us that a ‘measurement’ of the anomalous gravitomagnetic moment requires an understanding of the skewness effect.

The CFF $\mathcal{E}$ might be measured in DVCS on neutron, from the transverse proton spin asymmetry or the beam charge asymmetry at small $\xi$. Presently, one can only obtain a ‘local’ constraint on the GPD $E$. We have modelled valence GPDs at $\eta = 0$, adjusted them to the nucleon form factors and PDFs. We have varied the skewness function, constrained by Eq. (9) and JLAB/Hall A DVCS off proton data [11], and estimated the sea quark contribution to $H$ by a GPD model dependent extrapolation of DVCS measurements in collider kinematics. We have found that $H$ and $\tilde{H}$ contribute only little to the interference term in DVCS off neutron [12] and that the $E$-constraint is mainly given by the experimental error ($\Delta_{\exp} \approx \pm 0.5$):

$$|E_{\text{val}} + 4E_{d\text{val}} + 2E_{\text{sea}}| (\xi, \xi, t) \bigg|_{t = -0.22, \xi = -0.4 \text{GeV}^2} \lesssim \frac{9}{\pi} \frac{4M^2 |\Delta_{\exp}|}{|F_2(t)|} \approx 20. \quad (13)$$

With our ansatz for $E$ and supposing $\delta_0 = 0.2$ we found it likely that the valence contribution in (13) is about $-5 \pm 2$. Being optimistic, we might state that $E_{\text{sea}}$ is constrained by

$$-7 \lesssim E_{\text{sea}} (\xi = 0.22, t = -0.4 \text{GeV}^2) \lesssim 13.$$

This is a rather weak condition, since for this kinematical point the modulus $|E_{\text{sea}}|$ should be in a pessimistic case of a large skewness effect restricted to be $\lesssim 10|B_{\text{sea}}|$. This exceeds the interval of $|B_{\text{sea}}| \lesssim 0.5$, which covers $0 \lesssim J^Q \lesssim 1/2$.

We conclude that dispersion techniques should be employed to reveal GPDs on its cross-over trajectory from present DVCS measurements in fixed target kinematics. This might lead to a better GPD understanding, needed to access the quark angular momentum from dedicated experiments.

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