Geometry of the charge density wave in kagomé metal AV$_3$Sb$_5$

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Kagomé lattice is a fertile platform for topological and intertwined electronic excitations. Recently, experimental evidence of an unconventional charge density wave (CDW) is observed in a Z2 kagomé metal AV$_3$Sb$_5$ (A= K, Cs, Rb). This observation triggers wide interests on the interplay between frustrated crystal structure and Fermi surface instabilities. Here we analyze the lattice effect and its impact on CDW in AV$_3$Sb$_5$. Based on published experimental data, we show that the CDW induced structural distortions is consistent with the theoretically predicted inverse star-of-David pattern, which preserves the $D_{6h}$ symmetry in the kagomé plane but breaks the sixfold rotational symmetry of the crystal due to the phase shift between kagomé layers. The coupling between the lattice and electronic degrees of freedom yields a weak first order structural transition without continuous change of lattice dynamics. Our result emphasizes the fundamental role of lattice geometry in proper understanding of unconventional electronic orders in AV$_3$Sb$_5$.

Kagomé lattice is a corner shared triangle network that contains three sites per unit cell [1]. The electronic interference between the three sublattices gives rise flat band, van Hove singularity (saddle point) and Dirac-fermion in its band structure. It has been predicted that, near the van Hove filling, the combination of high density of state, sublattice interference and non-local Coulomb interaction may yield unconventional Fermi surface instabilities, such as the $p$-wave charge and spin density waves, $d$-wave Pomeranchuk instability and $f$-wave superconductivity [2,7]. Recently, a three-dimensional charge density wave (CDW) that possibly intertwines with superconductivity is observed in a kagomé metal AV$_3$Sb$_5$ (A= K, Cs, Rb) [4, 8-20]. While a three-dimensional 2×2×2 superstructure is experimentally identified [13] [18] [21] [22], the nature of the CDW and its interplay with the lattice structure of freedom are under rigorous investigations.

Figures 1a and b show the crystal structure of AV$_3$Sb$_5$, which has a space group No. 191 (P6/mnm). The V-Sb slab interlaces with the alkali triangle network along the crystal c-axis. Structurally, there are two Sb positions: Sb1 is located at the center of the V-hexagon and Sb2 is sitting above and below the V-triangles. Density functional theory (DFT) calculations found that the ideal kagomé structure is energetically unstable and favors an inverse star-of-David (ISD) structure at zero temperature [12]. While the ISD distortion of the two-dimensional kagomé lattice preserves the $D_{6h}$ symmetry, recent scanning tunneling spectroscopy (STS) studies found that the CDW superlattice peaks break the sixfold rotational symmetry (Fig. [1]), suggesting a chiral CDW or electronic nematicity [10, 15, 18, 28, 30]. Magnetoresistance measurements also found evidence of $C_2$ symmetry that persists into the superconducting phase [15, 30, 31]. Moreover, as we show in Fig. 1, unlike well-known CDW materials [23, 27, 32, 33], CDW in AV$_3$Sb$_5$ fails to induce acoustic phonon anomalies near the CDW wavevector, $Q_{\text{CDW}}$, indicating a strong commensurability effect [13]. Here we explore the CDW by numerically and analytically assessing the structural responses below $T_{\text{CDW}}$ in AV$_3$Sb$_5$. We show that the three-dimensional ISD structure yields a diffraction pattern that is consistent with x-ray scattering (XRD) and STS measurements [9, 10, 15, 28, 29]. Our analysis supports a CDW in AV$_3$Sb$_5$ that preserves the $D_{6h}$ in the kagomé-plane. However, due to the phase shift between kagomé layers, the CDW breaks the sixfold rotational symmetry, $C_6$, and strongly modifies the CDW superlattice peak intensities. Finally, we show that the coupling between CDW and lattice distortion yields a weak first order phase transition [21, 35] that may be responsible for the absence of acoustic phonon anomaly in AV$_3$Sb$_5$ [18].

The diffraction intensity follows, $I(Q) = |F(Q)|^2$, where $F(Q)$ is the scattering amplitude. For a crystalline material, $F(Q)$ can be formulated as:

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Figure 1. Crystal structure and CDW induced lattice distortion of AV₃Sb₅. (a) and (b) side-view and top-view of AV₃Sb₅ structure. DFT calculations find that star-of-David (SD) is a local energy minimum at zero temperature, while ISD is the global minimum. (c) shows the SD and ISD lattice distortion of the V-kagomé lattice. Purple and Black arrows show atomic distortions of V¹⁻⁶ and V⁷⁻¹² (see (b)), respectively. Distortions of V¹⁻⁶ and V⁷⁻¹² are out-of-phase. The ISD pattern corresponds to V¹⁻⁶/V⁷⁻¹² moving toward/outward the center of the V-hexagon. (d) reproduces the STS determination of AV₃Sb₅. (e) and (f) Sb1/Cs distortion that has C₃ᵥ symmetry. In the diffraction calculations, the lattice distortions were set to be 1% deviating from the original positions. The size of the dots proportional to the intensity of the diffraction pattern.

Figure 2. CDW induced lattice distortions in real space and their corresponding diffraction patterns in momentum space. (a) and (b) ISD distortion on V-kagomé sublattice. (c) and (d) V-stretching distortion on V-kagomé sublattice, both of which preserve the D₆h symmetry in the kagomé plane. (e) and (f) Sb1/Cs distortion that has C₃ᵥ symmetry. In the diffraction calculations, the lattice distortions were set to be 1% deviating from the original positions. The size of the dots proportional to the intensity of the diffraction pattern.

\[ F(Q) = \sum_{j} \frac{e^{iQ \cdot R_{j}}}{e^{iQ \cdot r_{j}}} \delta_{Q=G} \sum_{j} \frac{f_{j}(Q)}{e^{iQ \cdot r_{j}}} \]

where \( R_{n} \) and \( G \) are real and reciprocal lattice vectors, respectively. \( r_{j} \) is the \( j^{th} \) atomic position in the unit cell. \( Q \) is the total momentum transfer and \( f_{j}(Q) \) is the atomic form factor, which is derived from a Fourier transformation of local density of state (see Appendix for more details). Below \( T_{\text{CDW}} \), the formation of CDW distorts the high-temperature structure and gives rise to superlattice peaks at \( Q=Q_{\text{CDW}} \). We first consider the DFT predicted ISD distortions of V-kagomé lattice. Figure 2a schematically shows the ISD distortion, where V¹⁻⁶ and V⁷⁻¹² are breathing out-of-phase with respect to the center of V-hexagon. Figure 2b shows the simulated diffraction pattern of ISD shown in Fig. 2a. The scattering region is chosen to match previous XRD measurement at \( L=0 \) plane [9, 18], which captures in-plane atomic distortions. Remarkably, we find that ISD reproduces the key feature of experiment [9, 18], i.e., the CDW peak intensity is significantly larger than the XRD measurement with \( L=0 \) plane [9, 18], which captures in-plane atomic distortions. As we continue to show below, when CDW is three-dimensional (3D), as reported by recent experimental studies [18, 21, 22, 36], the C₆ᵥ symmetry of the CDW superlattice peaks is, however, incompatible with recent STS studies, where only a C₂ symmetry is observed [10, 13, 28, 29].

Since the two-dimensional ISD/SD has the D₆h symmetry, the CDW superlattice peaks are expected to show C₆ᵥ symmetry. Indeed, based on Eq. 1 and the ISD/SD distortion, we find that:

\[ F(0.5, 0, 0) = F(0.5, 0, 0) \]

\[ = F(-0.5, 0.5, 0) \approx -2\pi(\delta + \epsilon) \]

where \( \delta \) and \( \epsilon \) are corresponding to V¹⁻⁶ and V⁷⁻¹² distortions, respectively. Here we assume \( |\delta|, |\epsilon| \ll a_{0} = 5.4949\AA \). This condition is justified by previous XRD measurement, where the CDW superlattice peaks are 3-5 orders smaller than their nearby fundamental Bragg peaks [18]. The C₆ᵥ symmetry of the CDW superlattice peak is, however, incompatible with recent STS studies, where only a C₂ symmetry is observed [10, 13, 21, 22, 36]. As we continue to show below, when CDW is three-dimensional (3D), as reported by recent experimental studies [18, 21, 22, 36], the C₆ᵥ symmetry of the CDW peak intensity naturally breaks down to C₂. To show the rotational symmetry breaking, we use the theoretically refined 2×2×2 ISD structure of AV₃Sb₅ [12]. Due to the π phase shift between adjacent kagomé layers, the crystal symmetry is lowered from D₆h to D₂h [7]. Figure 3a and b show the calculated CDW superlattice peak intensity...
at \( L = 1.5 \) and \( L = 2 \), respectively. In agreement with STS studies \[16\] \[18\] \[21\] \[22\] \[36\], the CDW superlattice peak intensity only shows \( C_2 \) symmetry. In contrast, the fundamental Bragg peak only weakly breaks the \( C_6 \) symmetry due to small lattice distortions. We note that in our simulation, the fundamental Bragg peak intensity is three-orders larger than the CDW peak intensity.

Theoretically, three types of CDW order parameters are predicted for \( AV_3Sb_5 \), the onsite CDW, bond CDW and imaginary bond CDW involving flux or loop currents \[1\] \[5\] \[7\] \[36\] \[37\]. Due to finite electron-phonon coupling, CDW patterns have to respect the point group symmetry of the lattice. For instance, the Sb/Cs1 distortion shown in Fig. 2, is derived from an onsite CDW with \( C_{3h} \) symmetry. Our simulations demonstrate that the DFT calculated \( 2\times2\times2 \) superstructure at zero temperature is consistent with experimental observations and therefore support a CDW with \( D_{6h} \) symmetry in the kagomé plane. Furthermore, as we shown in Fig. 2c-d, the anti-phase breathing of \( V^+ \) and \( V^- \) puts another constraint on the CDW pattern in \( AV_3Sb_5 \) \[37\]. Our results, however, do not explain the observed chiral CDW peak intensity in \( AV_3Sb_5 \) \[10\]. Indeed, the chiral CDW keeps the \( D_{6h} \) symmetry of the kagomé plane \[4\] and hence cannot be distinguished by non-resonant x-ray scattering. Instead, the \( V \) \( L \)-edge resonant x-ray scattering, which selectively enhance electronic excitions from \( V \) 3d-orbital, might be a sensitive probe for this novel electronic order parameter \[38\].

Finally we explore how lattice distortion intertwines with the CDW transition. Previous DFT calculations \[12\] found that the lattice energy is asymmetric with respect to the lattice distortion, \( \eta \), at zero temperature (Fig. 4). Since CDW always couples with lattice distortions through finite electron-phonon coupling, the asymmetric lattice-distortion energy adds a cubic term in the CDW free energy and leads to a weak first order phase transition. To elaborate it further, we consider an Ising-type Landau free energy on a two-dimensional kagomé lattice:

\[
F(T, \psi) - F_0 = A(T)\psi^2 - C\psi^3 + B\psi^4 \quad (3)
\]

where \( \psi \sim \Delta_{CDW} \sim \eta \). Note the linear term in Eq. 3 can be removed by a linear transformation of \( \psi \). \( \Delta_{CDW} \) is the CDW gap in single particle spectral function. \( C \) is a constant that is proportional to the electron-phonon coupling strength. The hysteresis of the first order phase transition is, \( \Delta T = \frac{C^2}{4AB} \). Experimentally, \( \Delta T \sim 1 \) K \[21\] \[35\], suggesting a weak electron phonon coupling in \( AV_3Sb_5 \). Indeed, the calculated electron-phonon coupling constant from ref. \[12\] is in the weak coupling regime and between 0.3~0.46 for \( AV_3Sb_5 \), supporting our conclusion. Due to the weak first order phase transition, the change of the lattice dynamics is discontinuous near \( T_{CDW} \) and possibly intervenes the softening of the CDW phason mode. Together with the strong commensurability effect of the \( 2\times2\times2 \) CDW, the phason gap may remain large above \( T_{CDW} \) and failed to yield acoustic phonon softening near the CDW wavevector \[18\].

In summary, we explored the intricate interplay between lattice geometry and CDW in kagomé metal \( AV_3Sb_5 \). We prove that the ISD distortion reproduces the XRD and STS measurements. We showed that the coupling between lattice distortions and CDW induces a weak first order transition without continuous phonon softening in \( AV_3Sb_5 \).

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Appendix A: Atomic structure factor

The atomic form factor is a Fourier transform of a spatial density distribution of the scattering object. It is defined as:

\[ f(Q) = \int \rho(r) e^{iQ \cdot r} d^3 r \tag{A1} \]

where \( \rho(r) \) is the real space electron density. For non-resonant x-ray scattering, the atomic form factor is well approximated by a sum of Gaussians of the form:

\[ f(Q) = \sum_{i=1}^{4} a_i \exp(-b_i(Q_2^2)) + c \tag{A2} \]

The coefficients in Eq. A2 can be found in [39]. When considering the lattice vibrations, the atomic form factor will be modified to:

\[ f^{DW}(Q) = f(Q)e^{-\frac{1}{2}Q^2 \langle u_0^2 \rangle} = f(Q)e^{-M} \tag{A3} \]

where \( \langle u_0^2 \rangle \) is the time averaged mean of squared atomic displacement. In our calculation, Debye-Waller factor has been neglected.

Appendix B: Structural domain

We consider the 2\( \times \)2 superstructure which involves a \( \pi \)-phase shift between the ISD distorted kagomé layers. Assuming the scattering pattern of the two-dimensional ISD structure is \( I^{2\times2}(Q) \), the scattering intensity of 2\( \times \)2\( \times \)2 can be written as:

\[ I^{2\times2\times2}(Q) = I^{2\times2}(Q) \ast |1 + e^{iQ \cdot Ta}|^2 \tag{B1} \]

\[ = I^{2\times2}(Q) \ast (|1 + e^{iQ \cdot Ta}|^2 + |1 + e^{iQ \cdot Tb}|^2 + |1 + e^{iQ \cdot Tc}|^2) \]

\[ \propto I^{2\times2}(Q) \tag{B2} \]

Therefore, the CDW superlattice peaks determined by multi-domain measurements will be similar to the 2\( \times \)2 CDW.

Appendix C: Star-of-David distortion

Figure 5 compares diffraction patterns of SD and ISD distortions. While the both distortions capture the empirical diffraction selection rules, SD and ISD show subtle differences, for instance, the relative intensity between (H, 13.5, 0) and (H, 12.5 0) are opposite for SD and ISD.

[1] I. Syözi, Statistics of Kagomé Lattice, Progress of Theoretical Physics 6, 306 (1951), https://academic.oup.com/ptp/article-pdf/6/3/306/5239621/6-3-306.pdf
[2] W.-S. Wang, Z.-Z. Li, Y.-Y. Xiang, and Q.-H. Wang, Competing electronic orders on kagomé lattices at van hove filling, Phys. Rev. B 87, 115135 (2013)
[3] M. L. Kiesel, C. Platt, and R. Thomale, Unconventional fermi surface instabilities in the kagome hubbard model, Phys. Rev. Lett. 110, 126405 (2013)
[4] X. Feng, K. Jiang, Z. Wang, and J. Hu, Chiral flux phase in the kagome superconductor av_{3}sb_{5} (2021)
[5] M. M. Denner, R. Thomale, and T. Neupert, Analysis of charge order in the kagome metal av$_3$sb$_5$ (a=r,b,cs) (2021), arXiv:2103.14045 [cond-mat.str-el]

[6] Y.-P. Lin and R. M. Nandkishore, Complex charge density waves at van hove singularity on hexagonal lattices: Haldane-model phase diagram and potential realization in kagome metals av$_3$sb$_5$ (2021), arXiv:2104.02725 [cond-mat.supr-con]

[7] T. Park, M. Ye, and L. Balents, Electronic instabilities of kagome metals: saddle points and landau theory (2021), arXiv:2104.08425 [cond-mat-supr-con]

[8] B. R. Ortiz, L. C. Gomes, J. R. Morey, M. Winiarski, M. Bordelon, J. S. Mangum, I. W. H. Oswald, J. A. Rodriguez-Rivera, J. R. Neilson, S. D. Wilson, E. Ertekin, T. M. McQueen, and E. S. Toberer, New kagome prototype materials: discovery of kv$_3$sb$_5$, rbv$_3$sb$_5$, and cvs$_3$sb$_5$, Phys. Rev. Materials 3, 094407 (2019)

[9] B. R. Ortiz, S. M. L. Teicher, Y. Hu, J. L. Zuo, P. M. Sarte, E. C. Schueler, A. M. M. Abeykoon, M. J. Kroogstad, S. Rosenkranz, R. Osborn, R. Seshadri, L. Balents, J. He, and S. D. Wilson, cvs$_3$sb$_5$: A F2 topological kagome metal with a superconducting ground state, Phys. Rev. Lett. 125, 247002 (2020)

[10] Y.-X. Jiang, J.-X. Yin, M. M. Denner, N. Shumiya, B. R. Ortiz, G. Xu, Z. Guguchia, J. He, M. S. Hossain, X. Liu, J. Ruff, L. Kautzsch, S. S. Zhang, G. Chang, I. Belopolski, Q. Zhang, T. A. Cochran, D. Multer, M. Litskevich, Z.-J. Cheng, X. P. Yang, Z. Wang, R. Thomale, T. Neupert, S. D. Wilson, and M. Z. Hasan, Unconventional chiral charge order in kagome superconductor KV$_3$SB$_5$, Nature Materials 10.1038/s41563-021-01034-y (2021).

[11] Q. Yin, Z. Tu, C. Gong, Y. Fu, S. Yan, and H. Lei, Superconductivity and normal-state properties of kagome metal rbv$_3$sb$_5$ single crystals, Chinese Physics Letters 38, 037403 (2021)

[12] H. Tan, Y. Liu, Z. Wang, and B. Yan, Charge density waves and electronic properties of superconducting kagome metals (2021), arXiv:2103.06325 [cond-mat-supr-con]

[13] H. Zhao, H. Li, B. R. Ortiz, S. M. L. Teicher, T. Park, M. Ye, Z. Wang, L. Balents, S. D. Wilson, and I. Zeljkovic, Cascade of correlated electron states in a kagome superconductor cvs$_3$sb$_5$ (2021), arXiv:2103.03118 [cond-mat-supr-con]

[14] Z. Liang, X. Hou, W. Ma, F. Zhang, P. Wu, Z. Zhang, F. Yu, J. J. Ying, K. Jiang, L. Shan, Z. Wang, and X. H. Chen, Three-dimensional charge density wave and robust zero-bias conductance peak inside the superconducting vortex core of a kagome superconductor cvs$_3$sb$_5$ (2021), arXiv:2103.04760 [cond-mat-supr-con]

[15] H. Chen, H. Yang, B. Hu, Z. Zhao, J. Yuan, Y. Xing, G. Qian, Z. Huang, G. Li, Y. Ye, Q. Yin, C. Gong, Z. Tu, H. Lei, S. Ma, H. Zhang, S. Ni, H. Tan, C. Shen, X. Dong, B. Yan, Z. Wang, and H.-J. Gao, Roton pair density wave and unconventional strong-coupling superconductivity in a topological kagome metal (2021), arXiv:2103.09188 [cond-mat-supr-con]

[16] F. H. Yu, T. Wu, Z. Y. Yang, B. Lei, W. Z. Zhuo, J. J. Ying, and X. H. Chen, Concurrence of anomalous hall effect and charge density wave in a superconducting topological kagome metal (2021), arXiv:2102.10987 [cond-mat.str-el]

[17] K. Y. Chen, N. N. Wang, Q. W. Yin, Z. J. Tu, C. S. Gong, J. P. Sun, H. C. Lei, Y. Uwatoko, and J. G. Cheng, Double superconducting dome and triple enhancement of tc in the kagome superconductor cvs$_3$sb$_5$ under high pressure (2021), arXiv:2102.09328 [cond-mat-supr-con]

[18] H. X. Li, T. T. Zhang, T. Yilmaz, Y. Y. Päi, C. Marinven, A. Said, Q. Yin, C. Gong, Z. Tu, E. Vescovo, C. S. Nelson, R. G. Moore, S. Murakami, H. C. Lei, H. N. Lee, B. Lawrie, and H. Miao, Observation of unconventional charge density wave without acoustic phonon anomaly in kagome superconductors av$_3$sb$_5$ as revealed by optical spectroscopy (2021), arXiv:2104.01015 [cond-mat-supr-con]

[19] Z. X. Wang, Q. Wu, Q. W. Yin, Z. J. Tu, C. S. Gong, T. Lin, Q. M. Liu, L. Y. Shi, S. J. Zhang, D. Wu, H. C. Lei, and N. L. Wang, Unconventional charge density wave and photoinduced lattice symmetry change in kagome metal cvs$_3$sb$_5$ probed by time-resolved spectroscopy (2021), arXiv:2105.11393 [cond-mat-supr-con]

[20] D. W. Song, L. X. Zheng, F. H. Yu, J. Li, L. P. Nie, M. Shan, D. Zhao, S. J. Li, B. L. Kang, Z. M. Wu, Y. B. Zhou, K. L. Sun, K. Liu, X. G. Luo, Z. Y. Yang, J. J. Xing, X. G. Wan, T. Wu, and X. H. Chen, Orbital ordering and fluctuations in a kagome superconductor cvs$_3$sb$_5$ (2021), arXiv:2104.09173 [cond-mat-supr-con]

[21] B. R. Ortiz, S. M. L. Teicher, L. Kautzsch, P. M. Sarte, J. P. C. Ruff, R. Seshadri, and S. D. Wilson, Fermi surface mapping and the nature of charge density wave order in the kagome superconductor cvs$_3$sb$_5$ (2021), arXiv:2104.07230 [cond-mat-supr-con]

[22] C. M. Varma and A. L. Simons, Strong-coupling theory of charge-density-wave transitions, Phys. Rev. Lett. 51, 138 (1983)

[23] H. Miao, D. Ishikawa, R. Heid, M. Le Taccon, G. Fabbris, D. Meyers, G. D. Gu, A. Q. R. Baron, and M. P. M. Dean, Incommensurate phonon anomaly and the nature of charge density waves in cuprates, Phys. Rev. X 8, 011008 (2018)

[24] F. Weber, S. Rosenkranz, J.-P. Castellan, R. Osborn, R. Hott, R. Heid, K.-P. Bohnen, T. Egami, A. H. Said, and D. Reznik, Extended phonon collapse and the origin of the charge-density wave in 2H-nbse$_5$ (2011), Phys. Rev. Lett. 107, 107403 (2011)

[25] F. Weber, S. Rosenkranz, J.-P. Castellan, R. Osborn, G. Karapetrov, R. Hott, R. Heid, K.-P. Bohnen, and A. Alatas, Electron-phonon coupling and the soft phonon mode in tise$_5$, Phys. Rev. Lett. 107, 266401 (2011)

[26] A. Kogar, M. S. Rak, S. Vigg, A. A. Husain, F. Flicker, Y. I. Joe, L. Venema, G. J. MacDougall, T. C. Chiang, E. Fradkin, J. van Wezel, and P. Abbamonte, Signatures of exciton condensation in a transition metal dichalcogenide, Science 358, 1314 (2017)

[27] H. Li, H. Zhao, B. R. Ortiz, T. Park, M. Ye, L. Balents, Z. Wang, S. D. Wilson, and I. Zeljkovic, Rotation symmetry breaking in the normal state of a kagome superconductor kv$_3$sb$_5$ (2021), arXiv:2104.08209 [cond-mat-supr-con]

[28] N. Shumiya, M. S. Hossain, J.-X. Yin, Y.-X. Jiang, B. R. Ortiz, H. Liu, Y. Shi, H. Lei, S. S. Zhang, G. Chang, Q. Zhang, T. A. Cochran, D. Multer, M. Litskevich, Z.-J. Cheng, X. P. Yang, Z. Guguchia,
S. D. Wilson, and M. Z. Hasan, Tunable chiral charge order in kagome superconductor rbv$_3$sb$_5$ (2021), arXiv:2105.00550 [cond-mat.supr-con].

[30] Y. Xiang, Q. Li, Y. Li, W. Xie, H. Yang, Z. Wang, Y. Yao, and H.-H. Wen, Twofold symmetry of c-axis resistivity in topological kagome superconductor cs$_3$sb$_5$ with in-plane rotating magnetic field (2021), arXiv:2104.06909 [cond-mat.supr-con].

[31] S. Ni, S. Ma, Y. Zhang, J. Yuan, H. Yang, Z. Lu, N. Wang, J. Sun, Z. Zhao, D. Li, S. Liu, H. Zhang, H. Chen, K. Jin, J. Cheng, L. Yu, F. Zhou, X. Dong, J. Hu, H.-J. Gao, and Z. Zhao, Anisotropic Superconducting Properties of Kagome Metal {CsV}$_3$Sb$_5$, Chinese Physics Letters 38, 57403 (2021).

[32] T. M. Rice and G. K. Scott, New mechanism for a charge-density-wave instability, Phys. Rev. Lett. 35, 120 (1975).

[33] M. Hoesch, A. Bosak, D. Chernyshov, H. Berger, and M. Krisch, Giant kohn anomaly and the phase transition in charge density wave zrte$_3$, Phys. Rev. Lett. 102, 086402 (2009).

[34] G. Gruner, Density Waves in Solids (CRC Press, Taylor & Francis Group, Boca Raton, 2018) pp. 1–288.

[35] C. Mu, Q. Yin, Z. Tu, C. Gong, H. Lei, Z. Li, and J. Luo, s-wave superconductivity in kagome metal cs$_3$sb$_5$ revealed by $^{121/123}$sb nqr and $^{51}$v nmr measurement (2021), arXiv:2104.06698 [cond-mat.supr-con].

[36] N. Ratcliff, L. Hallett, B. R. Ortiz, S. D. Wilson, and J. W. Harter, Coherent phonon spectroscopy and interlayer modulation of charge density wave order in the kagome metals cs$_3$sb$_5$ (2021), arXiv:2104.10138 [cond-mat.supr-con].

[37] X. Feng, Y. Zhang, K. Jiang, and J. Hu, Low-energy effective theory and symmetry classification of flux phases on kagome lattice (2021), arXiv:2106.04395 [cond-mat.supr-con].

[38] A. J. Achkar, M. Zwiebler, C. McMahon, F. He, R. Sutarto, I. Djianto, Z. Hao, G. D. Gu, A. Revcolevschi, H. Zhang, Y. Kim, J. Geck, and D. G. Hawthorn, Nematicity in stripe ordered cuprates probed via resonant x-ray scattering, Science 351, 1 (2016).

[39] P. J. Brown, A. G. Fox, E. N. Maslen, M. A. O’Keefe, and B. T. M. Willis, Intensity of diffracted intensities, International Tables for Crystallography C, 554 (2006).